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Performance Analysis on Spatial MFSK Modulation with Energy Detection

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\section*{Abstract}
Noncoherent multiple-input multiple-output (MIMO) detection in fast fading environments has received attention in recent years since less influence by factors such as phase fluctuations and the low requirements for channel estimation and synchronization. Spatial MFSK modulation with energy detection is different from conventional noncoherent MIMO in that it can obtain higher spatial multiplexing, but with the introduction of the nonlinear square-law operation, the analysis of its detection performance needs to be solved. This paper analyzes the theoretical symbol error rate (SER) performance of the Spatial MFSK modulation with energy detection. The noise of the MIMO system by energy detection conform to the generalized gamma distribution. Based on this distribution, the optimal decision rule of the system and the theoretical SER formula are derived. Numerical results show that the theoretical SER formula fits well with the simulation results of the system under the condition of high signal-to-noise ratio (SNR).

\textbf{Keywords:} Noncoherent MIMO, Spatial MFSK modulation, Energy detection, Generalized gamma distribution, SER

environments has gradually attracted attention. The moving speed of high-speed railways can reach 500km/h. In this scenario, some high-reliability and low-latency applications, such as driverless, high-definition video, and video conferencing, have been difficult to carry out properly. However, in the IMT-2020 standard, there is no specific measure to solve the above problems in ITU-R.M.2410.

The IMT-2020 standard divides 5G into three main application scenarios: enhanced mobile broadband (eMBB), massive machine type communications (mMTC), and ultra reliable low latency communications (uRLLC). The uRLLC needs to meet the requirements of high reliability, low latency, and high mobility. It is able to provide more stable service and lower transmission delay for high-speed mobile users, with end-to-end delay in the order of 1ms. Nonetheless, the channel estimation causes partial transmission delay, and pilot symbols of coherent communication allocate a portion of the time and frequency slots. The channel estimation complexity is proportional to the number of antennas and users, which ultimately increases the system delay. In addition, channel estimation needs to use orthogonal training sequences, otherwise it will cause pilot pollution problems. In high-speed mobile environments, the channel coherence time reduces and the channel fading coefficient changes faster, so that channel estimation is difficult for
coherent systems. On the contrary, noncoherent systems have the advantages of low complexity, low power consumption, insensitivity to channel changes and high mobility. Nevertheless, there is a performance loss of the MIMO based on energy detection at low SNR compared with coherent detection, so the theoretical performance analysis of its capacity and SER becomes a problem to be solved objectively.

Noncoherent MIMO is insensitive to time-varying channels, and the receiving equipment is simple, which has become one of the main research fields of MIMO in recent years. The main research directions of noncoherent MIMO focus on noncoherent Grassmannian MIMO \cite{1-5}, differential detection MIMO \cite{6-11}, and energy detection MIMO \cite{12-22}. Among them, Grassmannian MIMO demands no or only partial CSI for demodulation, which applies to high SNR and high-speed mobile scenarios \cite{1}. However, the capacity of Grassmannian MIMO is limited by channel coherence time and complex encoding \cite{2}. In \cite{3}, a systematic unitary space-time code was proposed by continuously rotating the data signal in a high-dimensional space, while its design and decoding complexity are relatively high. In \cite{4}, an optimal design for the unitary space-time code was presented by iteratively searching for unitary space-time code contained in the Grassmann manifold set, while the traversal is difficult and the decoding complexity increases exponentially as the bit transfer rate accelerates. For 2x2 MIMO system, a structured unitary space-time code was proposed to obtain the full diversity gain, lower encoding and decoding complexity. However, the encoding fails to take energy efficiency into account, resulting in low encoding gain \cite{2}. Noncoherent MIMO with differential detection could also reduce the dependence on the channel information. Differential space-time modulation (DSTM) is a noncoherent MIMO scheme that does not require accurate CSI \cite{6}. This technique is an extended form of differential phase shift keying (DPSK) modulation in a multi-antenna system, which uses the phase difference between two adjacent transmission code blocks to carry the transmitted information and completes the transmission of information without the CSI \cite{7}. The research on differential space-time code mainly bases on orthogonalized differential space-time block code (DSTBC), which has low computational complexity because of orthogonality. Differential decoding at the receiver is considered as detecting a single symbol and does not require estimating CSI \cite{8}. In \cite{9}, a multi-symbol differential detection technique was presented, the corresponding decision matrix was designed and its BER performance was analyzed. Unfortunately, differential detection requires a quasi-static fading channel with consistent channel states between adjacent symbols, which is not necessarily true in high-speed mobile environments. The Doppler shift brought about by high-speed mobility will cause the phase of the far-field MIMO channel to change rapidly. Noncoherent MIMO with energy detection is proposed to reduce the sensitivity of Doppler. It is insensitive to phase rotation and has high robustness, low complexity, and low synchronization requirements in non-ideal CSI demodulation. Existing research on noncoherent MIMO focused on energy detection mainly focuses on how to obtain the maximum diversity gain \cite{10}. On the other hand, the MIMO with energy detection of spatial multiplexing has gradually begun to attract attention since its performance improvement \cite{11}, which can improve the communication spectrum efficiency under low complexity conditions.

Noncoherent MIMO based on energy detection has better robustness to synchronization and Doppler shift in high-speed mobility, which can avoid frequent channel estimation problems under time-varying channels. The modulation methods of noncoherent MIMO based on energy detection mainly include multi-ary amplitude shift keying (ASK), frequency shift keying (FSK), and pulse position modulation (PPM) \cite{12-13}, where amplitude modulation is mostly adopted. In \cite{14-17}, closed-form BER expressions were given for a single-input multiple-output (SIMO) communication system modulated by ASK in additive white Gaussian noise (AWGN) channels, lognormal channels, and Rayleigh channels, respectively. In \cite{18}, the optimal system performance was obtained by optimal constellation design in the MIMO Rayleigh channels when the amplitude of the multilevel ASK is approximate as a geometric sequence at high SNR. In \cite{19}, a noncoherent system based on energy detection was presented, which uses non-negative pulse amplitude modulation (PAM) to decode the sending signal by averaging the reception signal energy of all antennas. The SER of a SIMO system with energy detection was deduced, on this basis, a minimum distance constellation was proposed to design the transmit constellation points of the SIMO system. This scheme can significantly improve the performance of the system compared to conventional ASK modulation or equidistant power level modulation in \cite{22}. In \cite{23}, two MIMO systems with energy detection were proposed. One is the instantaneous channel energy based on the probability density function, which uses noise hardening and the decision threshold to demonstrate the instantaneous channel energy of a MIMO based on energy detection at high SNR. Whereas, the other is the average channel energy based on the chi-square cumulative distribution function, which has better performance when the spatial degrees of freedom are large enough. In a massive MIMO system based on energy detection, Gaussian approximation is performed on the gamma distribution subject to the received signal. The distribution curve by Gaussian approximation has a good match with the curve of the original received signal. Furthermore, the system capacity boundary of
Gaussian approximation [24] and the theoretical BER formula in the closed form [25] are deduced.

The paper is organized as follows: Section 2 summarize the method and contributions of this paper. Section 3 describes the channel environments in which the system is in a high-speed mobility scenario and the theoretical basis for the system to have better Doppler robustness. Section 4 describes the system model and derives the equivalent system model through energy detection. Furthermore, the signal and noise by equivalence are analyzed separately. Section 5 analyzes the distribution of noise by energy detection. Under this condition, the theoretical SER formula of the system is derived. Numerical results are presented in Section 6. Section 7 gives some concluding remarks.

2 Method

The MIMO based on energy detection by ASK modulation has limited spatial performance since the limited range of amplitude modulation. In this paper, we adopt the spatial MFSK modulation with better performance and analyze it theoretically. We equivalent the system model through nonlinear processing. The receiving end performs noncoherent detection through real channel equivalent channel parameters. The distribution of noise by energy detection and the theoretical expression of the average symbol error probability are derived. The optimal decision rule of the system is deduced, however, for the high complexity of ML detection with optimal judgment, ML is simplified through multi-antennas joint minimum Euclidean distance detection. At the same time, the distribution problem of signal-dependent noise that accompanies minimum Euclidean distance detection is analyzed.

3 Channel environment

For a communication system with $N_t$ transmitting and $N_r$ receiving antennas, the channel conforms to the typical Rice channel characteristics in the context of high-speed mobile line-of-sight (LOS) environments as communication transmission applications [26]. $H(t)$ can be divided into LOS and diffuse portions. Assuming the LOS component is in a static phase with uniform phase distribution. There is

$$H(t) = \sqrt{1 - a}H^*(t) + aH^t(t),$$

(1)

when $a = 1$, the receive signal is the multipath scattering part of the transmitted signal, and the channel model is the Rayleigh channel. When $a = 0$ or $a = 1$, the receive signal is regarded as the direct part of the transmitted signal, and the channel characteristics tend to be stable [27]. Considering a far-field environment where the antenna array size is much smaller than the propagation distance, it can be deemed that all concurrent transmission channels of the system experience the approximate same Doppler shift [28], that is

$$e^{j\sigma_{d,n}(t)} = e^{j2\pi f_d(t)} = e^{j\omega(t)},$$

(2)

where $f_{d,n}$ represents the Doppler shift from the $n$th transmitting antenna to the $l$th receiving antenna.

At $a = 0$ or $a = 1$, $H(t) \approx H^*(t)$, that is

$$H(t) \approx H^*(t) = e^{j\omega(t)} \left[ h_{l,q}e^{j\theta_{l,q}} \begin{array}{c} h_{1} \end{array} \begin{array}{c} M \end{array} \begin{array}{c} h_{N_r} \end{array} \begin{array}{c} e^{j\theta_{N_r}} \end{array} \right] = \left[ \begin{array}{c} h_{l} \end{array} \begin{array}{c} M \end{array} \begin{array}{c} h_{N_r} \end{array} \right],$$

(3)

where $h_{l}$ is the $l$th row of $H(t)$ for $l = 1, N_r$. The fading coefficient $h_{n}e^{j\theta_{n}}$ is independent and identically distributed ($iid$), there is

$$h_{n}e^{j\theta_{n}} : CN (0, 1).$$

(4)

The Doppler shift is supposed to be approximately constant within one symbol period of the transmit signal, it is shown that

$$e^{j\omega(t)} = e^{j\omega}.$$
Assume that the frequency modulation signal \( e^{i2\pi f_1 t + \pi} \) performs square-law energy detection at frequency \( f_1 \) and \( f_2 \), respectively. At the same time, the \( f_1 - f_2 = \frac{1}{T} \) (\( T \) is the duration of a symbol), the frequency are orthogonal to each other, we have

\[
\varepsilon_1 = \left| \frac{1}{T} \int_0^T e^{i2\pi f_1 t + \pi} e^{i2\pi f_2 t} dt \right|^2 = \left| e^{i\pi} \right|^2 = 1
\]

\[
\varepsilon_2 = \left| \frac{1}{T} \int_0^T e^{i2\pi f_1 t + \pi} e^{i2\pi f_2 t} dt \right|^2 = \left| e^{i\pi \sin[\pi(f_1 - f_2)T]} e^{i\pi(f_1 - f_2)^2T} \right|^2 = 0.
\]

The result of energy detection of the \( e^{i2\pi f_1 t + \pi} \) at a frequency matching its frequency point is one. On the contrary, the result is zero. Through the square-law processing, the random phase interference from the carrier is eliminated, in the meantime, the signal orthogonal characteristic minimizes the inter-signal interference. So that it has the characteristic of anti-Doppler frequency offset and phase offset in high-speed mobile environments. Of course, the Doppler shift will also cause the signal frequency offset, which can be ignored under the condition that the Doppler frequency deviation is far less than the symbol rate.

### 4 System model

Consider an energy detection system based on spatial MFSK modulation with \( N_t \) transmit and \( N_r \) receive antennas. The transmitted data stream is modulated by MFSK after V-BLAST encoding, upconverted and sent in parallel through different transmitting antennas. The transmitted signal is received in a noncoherent manner at the receiving end. Within a symbol interval, the \( N_r \times 1 \) received signal vector is represented by

\[
Y = HS + n,
\]

where \( H \) is the \( N_r \times N_t \) channel matrix, \( s \) is the \( N_r \times 1 \) transmitted information-bearing symbol vector through MFSK modulation, and \( n \) is the \( N_r \times 1 \) AWGN vector, which is independent of the channel matrix \( H \). The AWGNs \( n_1, n_2, \cdots, n_{N_r} \) are iid, each having a \( \mathcal{CN}(0, \sigma_n^2) \) distribution, there is

\[
Y = [Y_1, Y_2, \cdots, Y_{N_r}]^T
\]

\[
H = [h_1, h_2, \cdots, h_{N_t}]^T
\]

\[
n = [n_1, n_2, \cdots, n_{N_r}]^T
\]

The \( w \)th symbol of the \( n \)th transmitting antenna through MFSK modulation is denoted as

\[
S_{nw}(t) = e^{i2\pi f_{bw} t},
\]

where \( f_{bw} \) denotes the frequency of the \( w \)th symbol on the \( n \)th transmitting antenna, and \( b_{nw} = \{0,1,\cdots, M-1\} \) indicates the transmitted symbol mapped by V-BLAST encoding. Moreover, the modulated information-bearing symbol matrix is
\[
\mathbf{s} = \begin{bmatrix}
    s_{11}(t) & L & s_{L1}(t) \\
    M & s_{M1}(t) & M \\
    s_{N1}(t) & L & s_{N,N}(t)
\end{bmatrix} = [\mathbf{s}_1, \mathbf{s}_2, \mathbf{L}, \mathbf{s}_N],
\]

(11)

where \( \mathbf{s}_n \) represents the \( n \)th column of the sending symbol matrix.

The \( w \)th symbol received by the \( l \)th receiving antenna in a symbol interval is recorded as

\[
y_{lw} = h_l \mathbf{s}_w + n_{lw} = \sum_{n=1}^{N} h_n e^{in\phi_l} s_{nw}(t) + n_{lw}(t).
\]

(12)

The \( y_{lw} \) performs energy detection after frequency correlation at \( f_i, i = [L, L, \ldots] \) and \( f_i - f_m = \frac{1}{T} \), we get

\[
r_{i,f_i} = \left| \frac{1}{T} \int_{0}^{T} y_{lw}(t) e^{j2\pi f_i t} dt \right|^2
\]

\[
= \left| h_l \frac{1}{T} \int_{0}^{T} s_w e^{j2\pi f_i t} dt + \frac{1}{T} \int_{0}^{T} n_{lw} e^{j2\pi f_i t} dt \right|^2
\]

\[
= \left| h_l X_{w,f_i} + n_{lw} \right|^2
\]

\[
= \sum_{n=1}^{N} h_n e^{in\phi_l} e^{j\omega X_{nw,f_i}} + 2Re(\bar{n}_{lw} \sum_{n=1}^{N} h_n e^{in\phi_l} e^{j\omega X_{nw,f_i}}) + \left| n_{lw} \right|^2
\]

(13)

where \( X_{w,f_i} = [X_{w,f_i}, X_{w,f_i}, L, X_{Nw,f_i}]^T \), according to \((6)\) and \((7)\), \( X_{nw,f_i} \in (0, 1) \), furthermore, we can define as

\[
u_i = 2Re(\bar{n}_{lw} \sum_{n=1}^{N} h_n e^{in\phi_l} e^{j\omega X_{nw,f_i}}) + \left| n_{lw} \right|^2.
\]

(14)

The equivalent signal part by energy detection can be obtained as

\[
\sum_{n=1}^{N} h_n e^{in\phi_l} e^{j\omega X_{nw,f_i}} = \sum_{n=1}^{N} \sum_{j=1}^{N} h_n h_j \cos(\theta_{nj} - \theta_j) X_{nj} X_{ji}^*,
\]

(15)

In \((15)\), the \( l \)th row vector of the equivalent channel matrix can be expressed as

\[
\mathbf{\tilde{h}}_l = [h_{l1}, h_{l2}, 2h_{l1} h_{l2} \cos(\theta_l - \theta_2), L, 2h_{l1} h_{l2} h_{N_l} \cos(\theta_l - \theta_{N_l}), h_{N_l}].
\]

(16)

and the equivalent transmitting data vector is

\[
\mathbf{\tilde{X}}_{w,f_i} = [X_{w,f_i}^2, L, X_{w,f_i}^2, X_{w,f_i}^2, L, X_{Nw,f_i}^2, X_{Nw,f_i}^2]^T.
\]

(17)

where \( X_{nw,f_i} \in (0, 1) \). According to \((13)-(17)\), the transceiver system by energy detection at \( f_i \) can be equivalent to a real model, which is given by

\[
r_{i,f_i} = \mathbf{\tilde{h}}_l \mathbf{\tilde{X}}_{w,f_i} + u_i.
\]
furthermore, there is
\[ \mathbf{r}_{j} = \mathbf{f}_{w, j} + \mathbf{u}, \] (19)
where
\[
\begin{align*}
\mathbf{r}_{j} & = [r_{1,j}, r_{2,j}, \ldots, r_{N_r,j}]^T, \\
\mathbf{f}_{w} & = [f_{w,1}, f_{w,2}, \ldots, f_{w,N_t}]^T, \\
\mathbf{u} & = [u_1, u_2, \ldots, u_{N_c}]^T.
\end{align*}
\]

The equivalent noise component by energy detection is given by (14), there is
\[
\begin{align*}
u_{lw} = & \text{Re}(v_{lw}^*) e^{j\vartheta_{j}} \mathbf{X}_{m,w} + |v_{lw}|^2, \\
\end{align*}
\]
where $v_{lw}$ is the result of $n_{lw}$ through correlation at $f_j$, $i = [1, L, M]$. Since frequency is orthogonal to each other, $v_{lw}$ is iid at each frequency, we can get
\[
\begin{align*}
u_{lw} & \sim \mathcal{CN}(0, \frac{\sigma^2}{M}). \tag{20}
\end{align*}
\]
Furthermore, we have
\[
\begin{align*}
E(u_i) & = E[2\text{Re}(v_{lw}^* \sum_{n=1}^{N_r} h_{n} e^{j\vartheta_{j}} e^{j\vartheta_{j}} \mathbf{X}_{m,w}) + |v_{lw}|^2] = \frac{\sigma^2}{M}, \tag{21}
\end{align*}
\]
where $E(\cdot)$ denotes the expectation operator. Since $r_{l,j} = |h_{l} \mathbf{X}_{w,f} + v_{lw}|^2$ obeys the generalized gamma distribution, we can further deduce as
\[
\begin{align*}
E(r_{l,j}) & = \frac{\text{Re}(\mathbf{X}_{w,f})}{\sigma_n^2} + \frac{\sigma_n^2}{M} \tag{22} \\
E(r_{l,j}^2) & = \left(\frac{\text{Re}(\mathbf{X}_{w,f})}{\sigma_n^2} + \frac{\sigma_n^2}{M}\right)^2 + \frac{4\sigma_n^2}{M} \left(\frac{\text{Re}(\mathbf{X}_{w,f})}{\sigma_n^2} + \frac{\sigma_n^2}{M}\right) + \frac{2\sigma_n^2}{M}, \quad l = 1, 2, L, N_r. \tag{23}
\end{align*}
\]
Therefore, the diagonal and off-diagonal elements of the covariance matrix of the equivalent noise vector $\mathbf{u}$ are derived as
\[
\begin{align*}
\text{var}(u_i) & = E(u_i^2) - E^2(u_i) \\
& = E[r_{l,j}^2] - E^2(r_{l,j}) \\
& = E(r_{l,j}^2) - 2\text{Re}(\mathbf{X}_{w,f}) E(r_{l,j}) + (\text{Re}(\mathbf{X}_{w,f}))^2 - E^2(r_{l,j}) \\
& = \frac{2\sigma_n^2}{M} \text{Re}(\mathbf{X}_{w,f}) + \frac{\sigma_n^2}{M^2} \tag{24}
\end{align*}
\]
and
\[
\begin{align*}
\text{cov}(u_i, u_k) & = E(u_i u_k) - E(u_i)E(u_k) \\
& = E[(r_{l,j} - \text{Re}(\mathbf{X}_{w,f}))(r_{k,j} - \text{Re}(\mathbf{X}_{w,f}))] - E(u_i)E(u_k) \\
& = E(r_{l,j} - \text{Re}(\mathbf{X}_{w,f})) E(r_{k,j} - \text{Re}(\mathbf{X}_{w,f})) + (\text{Re}(\mathbf{X}_{w,f}))^2 - E(u_i)E(u_k) \\
& = 0, \quad l \neq k, \quad l, k = 1, 2, L, N_r, \tag{25}
\end{align*}
\]
where $\text{var}(\cdot)$ denotes variance operator and $\text{cov}(\cdot)$ denotes covariance operator.

In summary, the energy detection system based on spatial MFSK modulation is equivalent to a real system model, which is written as
\[
\mathbf{r} = \mathbf{f}_{w,f} + \mathbf{u}, \tag{26}
\]
where $\tilde{\mathbf{X}}_w = [\tilde{\mathbf{X}}_{w,1}, \mathbf{L}, \tilde{\mathbf{X}}_{w,f_w}]$, $\tilde{\mathbf{X}}_w$ is the equivalent data component of the $w$th transmit symbol by energy detection at $M$ frequency points, respectively. $\mathbf{H}$ is a real equivalent channel matrix with the dimension $N_r \times M(N_s + 1)$, which can be estimated by traditional linear channel estimation methods. When linear detection algorithms such as zero forcing (ZF) and minimum mean square error (MMSE) are adopted, the number of receive antennas is constrained by $N_r \geq \frac{M(N_s + 1)}{2}$. So that $\mathbf{H}$ is invertible and the weighted channel matrix $(\mathbf{H} \mathbf{H}^H)^{-1}$ can be obtained.

5 Analysis of detection performance

The energy detection system based on spatial MFSK modulation is equivalent to a real system model. Next, we perform a theoretical analysis of the detection performance of the system. The transmitted data stream is mapped to a symbol vector $\mathbf{c}_m$ by V-BLAST encoding, and its signal set is represented as

$$C = \{c_1, c_2, \mathbf{L}, c_{3N_s}\}.$$  \hspace{1cm} (27)

where

$$\mathbf{c}_m = \begin{bmatrix} b_1 \\ M \\ b_{N_s} \end{bmatrix} \quad b_1, L, b_{N_s} \in \{0, 1, \ldots, M-1\}.$$  \hspace{1cm} (28)

where the $\mathbf{c}_m$ is modulated by MFSK into a frequency modulation signal vector $\mathbf{s}_m$, as in (10). According to (6) and (7), the $\mathbf{s}_m$ is correlated at $f_i$ to obtain a vector with elements of 0 or 1, that is

$$\mathbf{X}_m = \begin{bmatrix} a_1 \\ M \\ a_{N_s} \end{bmatrix} \quad a_1, L, a_{N_s} \in \{0, 1\}.$$  \hspace{1cm} (29)

If the transmission symbol vector is $\mathbf{c}_m$, it can obtain from (13) that

$$r_{i,f_i} = |\mathbf{h}_i \mathbf{X}_{m,f_i} + v_{i,m}|^2 = \left| \sum_{n=1}^{N_s} h_n e^{j\phi_n} e^{j\theta X_{n,m,f_i}} + v_{i,m} \right|^2,$$

where $|\mathbf{h}_i \mathbf{X}_{m,f_i} + v_{i,m}|$ obeys the Rice distribution. Through the square processing, $r_{i,f_i}$ conforms to the generalized gamma distribution, which can be written as

$$r_{i,f_i} : \text{Gamma}(1,2\sigma_n^2/M, \sum_{n=1}^{N_s} h_n e^{j\phi_n} e^{j\theta X_{n,m,f_i}})^2).$$  \hspace{1cm} (30)

The probability density function of $r_{i,f_i}$, as well as the likelihood function, can be indicated as

$$f(r_{i,f_i} | \mathbf{h}_i, \mathbf{X}_{m,f_i}) = \frac{1}{2\sigma_n^2/M} I_0 \left( \frac{\sum_{n=1}^{N_s} h_n e^{j\phi_n} e^{j\theta X_{n,m,f_i}}}{\sigma_n^2/M} \right) \sqrt{r_{i,f_i}} \exp \left( -\frac{r_{i,f_i} + \sum_{n=1}^{N_s} h_n e^{j\phi_n} e^{j\theta X_{n,m,f_i}}}{2\sigma_n^2/M} \right),$$  \hspace{1cm} (31)
where $I_0(\cdot)$ denotes the modified Bessel function of the zeroth order and the first kind. Owing to the frequencies of MFSK are orthogonal to each other, the likelihood functions at each frequency are independent of each other. Thus, the joint likelihood function is the product of the likelihood functions at each frequency. The $c_{m}$ that maximizes the sum of the logarithmic joint likelihood function is the signal that is correctly decided, that is, 

$$
\hat{c} = \arg \max_{c_{m} \in C} \sum_{i=1}^{N_c} \sum_{k=1}^{M} \ln f(r_{ik}, | h_i, X_{m,k,f_i} |)
$$

$$
= \arg \max_{c_{m} \in C} \sum_{i=1}^{N_c} \sum_{k=1}^{M} \ln \left( \frac{1}{2\sigma_i^2/M} \right) + \ln \left[ I_0 \left( \sqrt{\frac{r_{ik} + \sum_{i=1}^{N_c} \sum_{k=1}^{M} h_i e^{j\theta_k} e^{j\phi X_{m,k,f_i}}}{\sigma_i^2/M} } \right) \right] - \frac{r_{ik} + \sum_{i=1}^{N_c} \sum_{k=1}^{M} h_i e^{j\theta_k} e^{j\phi X_{m,k,f_i}}}{2\sigma_i^2/M}.
$$

(32)

Since $\ln(\frac{1}{2\sigma_i^2/M})$ and $-\frac{r_{ik} + \sum_{i=1}^{N_c} \sum_{k=1}^{M} h_i e^{j\theta_k} e^{j\phi X_{m,k,f_i}}}{2\sigma_i^2/M}$ do not rely on the search parameter $c_{m}$, we remove them from the decision variable. The decision criterion (32) can be written as

$$
\hat{c} = \arg \max_{c_{m} \in C} \sum_{i=1}^{N_c} \sum_{k=1}^{M} \ln \left[ I_0 \left( \sqrt{\frac{r_{ik} + \sum_{i=1}^{N_c} \sum_{k=1}^{M} h_i e^{j\theta_k} e^{j\phi X_{m,k,f_i}}}{\sigma_i^2/M} } \right) \right] - \frac{r_{ik} + \sum_{i=1}^{N_c} \sum_{k=1}^{M} h_i e^{j\theta_k} e^{j\phi X_{m,k,f_i}}}{2\sigma_i^2/M}.
$$

(33)

Therefore, the (33) is the optimal decision rule for the energy detection system based on spatial MFSK modulation. Since the Bessel function is nonlinear, the implementation complexity of the receiver is high. Approximate processing of the Bessel function at high SNR can simplify the optimal decision rule. $I_0(\cdot)$ is expanded as

$$
I_0(\cdot) \approx \frac{e^x}{\sqrt{2\pi x}} \left[ 1 + \frac{1}{8x} + \frac{1 \times 9}{2! (8x)^2} + \frac{1 \times 9 \times 25}{3! (8x)^3} + \cdots \right],
$$

(34)

when the absolute value of the argument inside $I_0(\cdot)$ is large, $I_0(\cdot)$ is approximated as

$$
I_0(x) \approx \frac{e^x}{\sqrt{2\pi x}},
$$

$$
\ln[I_0(x)] \approx \ln(e^x) = x - \frac{1}{2} \ln(2\pi x)
$$

(35)

Applying (35) to (33), the optimal decision rule (33) can be rewritten as

$$
\hat{c} = \arg \max_{c_{m} \in C} \sum_{i=1}^{N_c} \sum_{k=1}^{M} \left( 2 \sum_{i=1}^{N_c} \sum_{k=1}^{M} h_i e^{j\theta_k} e^{j\phi X_{m,k,f_i}} \right) \left| \sqrt{r_{ik} + \sum_{i=1}^{N_c} \sum_{k=1}^{M} h_i e^{j\theta_k} e^{j\phi X_{m,k,f_i}}} \right|^2.
$$

(36)

The probability of correct decision when the $c_{m} \in C$ is transmitted, denote by $P_{c_{m}}$, is obtained from (36) as

$$
P_{c_{m}} = P \left( \sum_{i=1}^{N_c} \sum_{k=1}^{M} 2|h_i X_{m,f_i} + v_m| - |h_i X_{m,f_i}|^2 > \sum_{i=1}^{N_c} \sum_{k=1}^{M} 2|h_k X_{k,f_i} + v_m| - |h_k X_{k,f_i}|^2 \right)
$$

(37)

$$
P_{c_{m}} = P \left( \max_{k \in \{k|m \neq k\}} \sum_{i=1}^{N_c} \sum_{k=1}^{M} 2|h_i X_{k,f_i} + v_m| - |h_i X_{k,f_i}|^2 < \sum_{i=1}^{N_c} \sum_{k=1}^{M} 2|h_k X_{m,f_i} + v_m| - |h_k X_{m,f_i}|^2 \right)
$$

(38)

Under the high SNR, $|h_i X_{m,f_i} + v_m|$ in (38) can be processed as follows

$$
|h_i X_{m,f_i} + v_m| = \left( |h_i X_{m,f_i}|^2 + 2 \text{Re}(v_m(h_i X_{m,f_i}^*)^*) + |v_m|^2 \right)^{1/2}
$$
the Taylor series expansion of \( \{1 + \frac{2\text{Re}(v_{lm}(h_m x_{m,l}))}{|h_m x_{m,l}|^2}\}^{\frac{1}{2}} \) is given by

\[
\left\{1 + \frac{2\text{Re}(v_{lm}(h_m x_{m,l}))}{|h_m x_{m,l}|^2}\right\}^{\frac{1}{2}} = 1 + \left\{\frac{\text{Re}(v_{lm}(h_m x_{m,l}))}{|h_m x_{m,l}|^2}\right\} - \frac{1}{4} \left\{\frac{\text{Re}(v_{lm}(h_m x_{m,l}))}{|h_m x_{m,l}|^2}\right\}^2 + R_n(),
\]

where the \( R_n() \) is Lagrangian remainder of Taylor series expansion.

At high SNR, the high-order term of (40) is small and can be ignored. The first two terms of (40) are brought into (39) to obtain as

\[
|h_m x_{m,l}| + \text{Re}(v_{lm}),
\]

and

\[
\text{Re}(v_{lm}) : N(0, \frac{\sigma_m^2}{2M}).
\]

Substituting (41) in (38), we get

\[
p_c \approx p\{ \max_{k \in \mathbb{L}, l \neq (m)} \sum_{i=1}^{N_l} 2\text{Re}(v_{lm}) - \text{Re}(v_{lm}) - |h_m x_{m,l} - h_x x_{m,l}|^2 < 0 \}.
\]

Define

\[
g = \sum_{i=1}^{N_l} \sum_{l=1}^{N} 2\text{Re}(v_{lm}) - \text{Re}(v_{lm}) - |h_m x_{m,l} - h_x x_{m,l}|^2,
\]

furthermore, there is

\[
g : N \left( \sum_{i=1}^{N_l} \sum_{l=1}^{N} \left( \text{Re}(v_{lm}) - \text{Re}(v_{lm}) - |h_m x_{m,l} - h_x x_{m,l}|^2 \right) \right).
\]

Applying the Gaussian distribution normalized result of (44) to (43), the probability of correct decision can be represented as

\[
p_c = \phi\left( \sqrt{\frac{\min_{k \in \mathbb{L}, l \neq (m)} \sum_{i=1}^{N_l} \sum_{l=1}^{N} \text{Re}(v_{lm}) - \text{Re}(v_{lm}) - |h_m x_{m,l} - h_x x_{m,l}|^2}{2\sigma_n^2/M}} \right),
\]

where \( \phi() \) is the distribution function of the standard Gaussian distribution. The probability of error decision when \( c_m \in C \) is transmitted, denoted by \( p_{e_m} \), is given by

\[
p_{e_m} = 1 - p_c.
\]
where \( Q(\cdot) \) denotes the Gaussian Q-function. The average SER under equiprobable signaling is obtained as

\[
P_e = \frac{1}{M^{N_t}} \sum_{m=1}^{M^{N_t}} p_{e_m} = \frac{1}{M^{N_t}} Q\left( \min_{k \in \{1, N_t\}} \sum_{l=1}^{N_t} \sum_{m=1}^{M} \frac{\|h_k, X_{l,m} - h_l, X_{l,m}\|^2}{2\sigma_n^2/M} \right)
\]

(47)

6 Results and discussion

In this section, the theoretical average SER compares with the numerical results. The simulation demonstrates the accuracy of the received signal distribution by energy detection. Furthermore, we analyze the signal-dependent noise problem of minimum Euclidean distance detection. The (33) is the optimal decision rule. It is optimal to use the obeyed distribution to make a decision, but the complexity is high. Thus, it can be done by using the minimum of the joint Euclidean distance of multiple antennas at M frequency points, which can be defined as

\[
\hat{c} = \arg \min_{m \in \{1, \ldots, M\}} \sum_{l=1}^{N_t} \sum_{m=1}^{M} \| h_{l, f_l} - h_{l, X_{l,m}} \|^2.
\]

(48)

When using the minimum Euclidean distance to make a decision, it is necessary to consider the distribution of signal-dependent noise.

Figure 1 and Figure 2 are the distributions of the received signal when the transmitted signal is \( \mathbf{s}_1, \mathbf{s}_2 \) and \( \mathbf{s}_3 \). According to the optimal decision rule, when the received signal falls in the left region of the decision boundary \( b_1 \), the transmitted signal is judged to be \( \mathbf{s}_1 \). When the received signal falls in the region between the decision boundary \( b_1 \) and \( b_2 \), the judgment transmitted signal is \( \mathbf{s}_2 \). When the received signal falls in the area to the right of the judgment boundary \( b_2 \), it is decided to send the signal as \( \mathbf{s}_3 \). At low SNR, the variance of each distribution is relatively large, so that there are obvious overlap regions between adjacent distributions. At the same time, large noise will cause deviations in the center position of each distribution, causing the distance between the centers of each distribution to change based on the original distance. The above changes affect the accuracy of the minimum Euclidean distance detection. When the received signal falls at position A, according to the optimal decision rule, the transmitted signal should be determined as \( \mathbf{s}_3 \). While according to the minimum Euclidean distance, the influence of large noise makes \( d_1 < d_2 \). Therefore, it will be misjudged as \( \mathbf{s}_2 \). As shown in Figure 2, the reduction of variance at high SNR makes distributions more concentrated, and the overlap regions between neighboring distributions become smaller, which can improve the overall performance of the minimum Euclidean distance detection. Of course, the isometric constellation design of the transmitting signal can reduce the overlap regions between neighboring distributions and improve the accuracy of the minimum Euclidean distance detection.

In the energy detection system based on spatial MFSK modulation with 3 transmitting and receiving antennas, all transmitting signals are correlated at \( f_1 \) to obtain 8 signal vectors with elements of 0 and 1. Figure 3 and Figure 4 respectively show the comparison between the numerical and the theoretical formula (31) of the traversal signal \( f(l, f_{l, f}) \) of the transmitting signal constellation at \( f_1 \) under low and high SNR. Numerical results show that the simulation and theoretical formula match well. It can be illustrated from the figure that as the SNR increases, the overlap regions between adjacent distributions decrease, which can improve the overall performance of the minimum Euclidean distance detection.

Figure 5 shows the comparison between the numerical results using the minimum Euclidean distance detection (48) and the theoretical formula of average symbol error probability (47) in the Spatial 4FSK modulation with energy detection. Numerical results show a good match between the simulation and the theoretical formulation at high SNR. Under the condition that the number of MFSK modulation is constant, the SER performance gradually improves as the number of antennas increases. The reason is that
as the number of receiving antennas increases, the diversity gain of the receiving end will increase. The decision criterion is to adopt multi-antenna joint detection, as the number of antennas increases, the reliability of detection will be enhanced. Therefore, the SER of the system will be improved.

Figure 6 shows the performance comparison of the MIMO system using 4FSK and 4ASK modulation with energy detection. The spatial performance of the system is limited since the limited distance space of amplitude modulation. Therefore, the performance of MIMO-4FSK based on energy detection is better than MIMO-4ASK.

Under the high-speed railway channel, Figure 7 shows the Doppler robustness analysis of the spatial 4FSK modulation with energy detection and spatial QPSK modulation with coherent detection with the number of transmitting and receiving antennas are 4. As the Doppler shift increases, the performance of the MIMO-QPSK deteriorates sharply. In a high-speed mobile environment, the channel changes rapidly, and the channel coherence time becomes shorter, which makes real-time channel estimation difficult. The error performance of coherent detection will rapidly decline because of the lack of real-time tracking of phase changes. In comparison, the MIMO-4FSK based on energy detection has better Doppler robustness. The square-law energy detection can eliminate the phase interference, so that those have little effect on the energy detection system. But a large Doppler shift will cause the signal frequency offset, resulting in frequency mismatch, the error performance of the system will also decrease.

7 Conclusion
This paper analyzes the robustness of Spatial MFSK modulation with energy detection to Doppler in high-speed mobile scenarios. Energy detection can avoid the phase synchronization problem, reduce the complexity of the training sequence, and improve the robustness of the system to Doppler. Through the nonlinear processing of energy detection, the system is equivalent to a real model, and the equivalent signal and noise are analyzed. The theoretical formula of the distribution of the received signal by energy detection is simulated numerically, which demonstrates that the numerical results and the theoretical formula match well. The theoretical formula of average symbol error probability is derived, which fit well with the numerical results of minimum Euclidean distance detection at high SNR.

Abbreviations
MIMO: Multiple-input multiple-output; SER: Symbol error rate; SNR: Signal-to-noise ratio; eMBB: enhanced mobile broadband; mMTC: massive machine type communications; uRLLC: ultra reliable low latency communications; CSI: Channel state information; DSTM: Differential space-time modulation; DSTBC: Differential space-time block code; ASK: Amplitude shift keying; FSK: Frequency shift keying; PPM: Pulse position modulation; SIMO: Single-input multiple-output; AWGN: Additive white Gaussian noise; PAM: Pulse amplitude modulation; ML: Maximum likelihood; LOS: line-of-sight; ZF: Zero forcing; MMSE: Minimum mean square error.

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Authors’ contributions
All authors made contributions in the discussions, analyses, and implementation of the proposed solution. LSJ drafted the manuscript. All authors read and approved the final manuscript.

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Availability of data and materials
Not applicable

Declarations

Competing interests
The authors declare that they have no competing interests.

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Figures

Figure 1 Signal-dependent noise distribution at low SNR. The figure compares the distributions of the received signal when the transmitted signal is $s_1$, $s_2$ and $s_3$ at low SNR.

Figure 2 Signal-dependent noise distribution at high SNR. The figure compares the distributions of the received signal when the transmitted signal is $s_1$, $s_2$ and $s_3$ at high SNR.

Figure 3 The $f(r_{i,j} | h, X_{i,j})$ of the joint constellation at $f$, under low SNR. The figure compares $f(r_{i,j} | h, X_{i,j})$ by the theoretical analysis and the simulation at low SNR.

Figure 4 The $f(r_{i,j} | h, X_{i,j})$ of the joint constellation at $f$, under high SNR. The figure compares $f(r_{i,j} | h, X_{i,j})$ by the theoretical analysis and the simulation at high SNR.

Figure 5 Simulated and theoretical results for SER of spatial 4FSK modulation with energy detection. The figure compares the SER performances by the theoretical analysis and the simulation.

Figure 6 Performance comparison between MIMO-4FSK and MIMO-4ASK based on energy detection. The figure compares the performances of two different modulation modes based on energy detection.

Figure 7 Performance comparison between noncoherent MIMO-4FSK and coherent MIMO-QPSK in high-speed railway channels. The figure compares the Doppler robustness of the spatial 4FSK modulation and spatial QPSK modulation under high-speed railway channels.
Figures

Figure 1

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Figure 2
Figure 3

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Figure 4

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Performance comparison between MIMO-4FSK and MIMO-4ASK based on energy detection. The figure compares the performances of two different modulation modes based on energy detection.
Figure 7

Performance comparison between noncoherent MIMO-4FSK and coherent MIMO-QPSK in high-speed railway channels. The figure compares the Doppler robustness of the spatial 4FSK modulation and spatial QPSK modulation under high-speed railway channels.