Spatial velocity distribution around an endless chain of spherical dust particles

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Abstract. This paper presents a numerical model in which selfconsistent distributions of plasma space charge, ion velocity and potential are calculated in a proximity of the infinite chain consisting of spherical dust particles. In the observed model this chain is under an influence of an external electric field. As a result of calculation spatial distributions of ions velocity and plasma potential around the isolated dust particles are obtained. The data obtained reveal that for the electric field of low intercity local maximum in the potential and velocity distribution linearly increases with the rise of the external field strength. However, with further amplification of external electric field the potential maximum starts to decline in contrast to the velocity local maximum.

1. Introduction
Dust particles are solid micronized objects injected into the gas discharge plasma. Such particles are charged to large negative values $Z_d = 10^3 - 10^5$ e. Studies related to dust grains have shown that dust particles form ordered structures called dust crystals [1–8], the simplest of which are the dust chains. However, these studies, continuously ignore spatial velocity distribution of ions around the dust particle.

A number of numerical models were developed to investigate the plasma potential distribution around isolated dust particles. These methods could be generally broken down into the following three categories: methods of molecular dynamics (MD) [9], [10], Particle-In-Cell (PIC) [11–15] and linear response (LR) [16–18]. The next logical step is to investigate plasma distribution around dust particle chains since their structure is the simplest. However, the LR methods prove to be inapplicable to a number of dust grains greater than one and PIC and MD methods appear to be quite cumbersome.

To date there are successful numerical models which accurately describe the selfconsistent potential distribution around the dust particle crystal [19]. However, in order to simplify calculations, substantial simplifications are implemented [19].

Hence on the basis of the model presented in papers [20–22] a new numerical model was created. In this model the selfconsistent distributions of the plasma space charge, ion velocity distribution and potential are determined around an infinite chain of spherical dust particles.
2. Model
Numerical model presented in this paragraph is modification of the model used in [20]–[22]. General distinctions of the new modification will be revised below and compared to that of [20]–[22].

![Schematic representation of the simulated system of an infinite dust particle chain.](image)

The volume where ions’ trajectories are calculated is a regular quadrilateral prism (see figure 1). Its larger size $L = 40 \lambda_i$, while its smaller length is equal to $D$, where $D$ is the distance between dust particles, $\lambda_i = (kT_i/4\pi e^2 n_\infty)^{1/2}$ is the ion Debye length, $n_\infty$ is the ion density of the unperturbed plasma and $T_i$ is the ion temperature. In the center of the computational volume is a sphere $r_0$.

The method of calculation of ion trajectories remains unchanged and coincides with one used in [20–22].

New geometry of the computational domain is extremely inconvenient for dividing its space into cells in coordinates ($\rho$, $z$) with volume of each individual cylinder $V_{\text{cyl},k,l} = 2\pi \rho_k \Delta \rho_k \Delta z_l$. The dependences on coordinate $\phi$ can be neglected since the task is cylindrically symmetric. The time $T_{k,l}$ that the observed ion spent in the segment $k, l$ is summed up and normalized to the volume of the segment $V_{k,l}$:

$$n_i(k,l) = \frac{T_{k,l}}{V_{k,l}}. \quad (1)$$

The value of $n_i(k,l)$ is proportional to the ion density $n_i(r, \theta)$ in a segment $k, l$. The proportionality coefficient is determined by the ratio:

$$A = \frac{n_i}{\langle n_i(k,l) \rangle_{\text{border}}}. \quad (2)$$

where $\langle n_i(k,l) \rangle_{\text{border}}$ is the average accumulated time in the boundary segments of the system.

In a similar way the statistics on the average velocity spatial distribution is accumulated:

$$u_{\alpha}(k,l) = \sum_{t_n} \frac{u_{\alpha,n} dt}{T_{k,l}}. \quad (3)$$

where $dt$ is the time step chosen for calculating the Newton equations and $u_{\alpha}$ is the current value of the $\alpha$-component of the observed ion velocity.
The initial potential distribution is a superposition of the Debye-Hückel potentials of all considered dust particles and the external electric field. In a dimensionless form it can be written as:

\[ U_0(r, \theta) = \frac{\tilde{Q}}{r} e^{-r} - \sum_k \frac{\tilde{Q}}{r_k,2} e^{-n_{k,2}} - \sum_k \frac{\tilde{Q}}{r_k,1} e^{-n_{k,1}} - \tilde{E}z, \]

\[ r_{k,1} = (D^2 + r^2 + 2kDz), \quad r_{k,2} = (D^2 + r^2 - 2kDz), \]

where \( r_{k,1}, r_{k,2} \) are the distances from neighbouring dust particles to the ion under observation, \( \tilde{E} = e\lambda E/kT_i \) is the dimensionless strength of the external electric field, \( \tilde{Q} = e^2Z_d\lambda/kT_i \) is the dimensionless dust particle charge.

Plasma space charge and potential distributions are calculated in the area bounded by a central or main prism with only one dust particle. The principle of this method is that larger sides of the main prisms are connected with larger sides of other prisms, where the plasma space charge and potential distributions are identical meaning that the periodic conditions for ion density and system potential are simulated. Hence the boundary conditions on larger sides of the calculation area are as follows:

\[ U(\rho, D) = U(\rho, -D), \]

\[ n(\rho, D) = n(\rho, -D), \]

where \( n(\rho, z) = (n(\rho, z) - n_d(\rho, z))/n_e \) is the dimensionless space charge distribution, \( n(\rho, z) \) is the ion density and \( n_d(\rho, z) \) is the electron density.

In this model the distribution of self-consistent potential is calculated by the formula

\[ U(r, \theta) = \frac{\tilde{Q}}{r} - \sum_k \frac{\tilde{Q}}{r_k,2} e^{-n_{k,2}} + \sum_k \frac{\tilde{Q}}{r_k,1} e^{-n_{k,1}} - \tilde{E}z, \]

To calculate the self-consistent potential distribution of the solution region, the following calculation algorithm is implemented:

1) Ion trajectories are calculated for the initial potential \( U_0(\rho, z) \) (1) and the data of time ions spent in segments \((l, k)\) is accumulated
2) A new potential \( U(\rho, z) \) is calculated (7).
3) New dust particle charge \( \tilde{Q} \) is derived from the condition of equality of ion and electron fluxes to the surface of dust grain.
4) The calculation procedure for ion trajectories is repeated. The iterative process is repeated until \( U(\rho, z), n(\rho, z), \) and \( \tilde{Q} \) stop changing.

3. Results

The data presented in the current paragraph are obtained for the following set of parameters: ion temperature \( T_i = 273 \) K, temperature ratio \( T_e/T_i = 100 \), ion mean free path for the resonant charge exchange collisions \( l_i = 5 \lambda_i \), dust particles radii \( r_0 = 1 \mu m \), and for the interval of external electric field values \( E = 0-40 \). In the current model, interparticle distance is taken as constant, however, in a more general case, \( D \) is determined by interparticle forces and momentum, transmitted to the dust particle from scattering of ions and electrons.

Figures 2 and 3 show the spatial distributions of potential and ion radial velocity measured on the \( z \)-axis on which the chain of dust particles lies, \( \rho = 0 \). The distributions for the three adjacent dust particles are also shown. For convincie, the potential \( U(\rho, z) \) is represented with a minus sign and the ion radial velocity spatial distribution is presented in the form \( u_1(\rho=0,z)-u_d, \) where \( u_d \) is a drift velocity induced by
the external electric field. Presented distributions satisfy the periodicity conditions, established at the boundaries of solution region.

The local maximum arising in the potential distribution behind the dust grain at distance \( \sim 4 - 6 \lambda_i \) is the head part of the potential wake forming behind the dust particle. Appearance of the wake behind dust particles is one of the most prominent phenomena of dust plasma physics [23], [24]. With an increase of the external electric field the local potential maximum forms between the two dust particles, which grow with the field intensity. Similarly, when the external electrostatic field is amplified in the spatial distribution of the ion radial velocity, a local maximum appears in front of the dust particle, which also increases with increasing field strength. Its appearance is explained by trapped ions orbiting around the dust particle, and its value is always less than the value of drift velocity. Thus, no contradiction arises here.

Figures 4 and 5 show the angular dependence of the potential and ion radial velocity spatial distributions. The interval of angles considered is \( \theta = 0 - 180^\circ \), which represents a complete set of data, given the cylindrical symmetry of this problem with respect to the z-axis. The data presented in Figures 4 and 5 are measured for the external electrostatic field strength \( \tilde{E} = 20 \). The potential distribution turns out to be practically symmetric, while the spatial velocity distribution is distorted by the ion drift velocity induced by the influence of an external electrostatic field on the ions. Thus, the velocity spatial distribution around a chain of dust particles does not significantly affect the potential around each specific dust particle.

The influence of the external electrostatic field on the potential and velocity fields around a dust particle chain can be characterized by the dependence of the local maxima of these distributions on the strength of electric external field. These results are presented in Figures 6 and 7, which depict the dependences of the local maxima of the spatial distributions of the potential and ion radial velocity on \( \tilde{E} \), respectively.
Figure 4. Potential distribution $U(\rho,z)$ presented for different angles and for the value of external electric field values $\vec{E} = 20$.

Figure 5. Ion radial velocity spatial distribution $u_r(\rho,z)$ presented for different angles and for the value of external electric field values $\vec{E} = 20$.

Figure 6. Dependence of local maximum of potential spatial distribution $U(\rho,z)$ on the external electric field $\vec{E}$.

Figure 7. Dependence of local maximum of ion radial velocity spatial distribution $u_r(\rho,z)$ on the external electric field $\vec{E}$.

The data presented in Figure 6 show that in the system consisting of the chain of spherical dust particles the anisotropy grows linearly when the external electrostatic field appears. However, when $\vec{E} > 1$ there is a sharp decline in the maximum value, the speed of that decline increases with the increasing electric field strength. In contrast, the local maximum of spatial distribution of the ion radial velocity, previously presented in Figure 3, continues to grow, despite the field overcoming the value $\vec{E} > 1$. Thus, it becomes clear that the ion dynamics in such a system depends weakly on the ion spatial distribution, and it is completely determined by the Coulomb potential of dust particles and the external electric field vector.

**Conclusion**

This paper presents a numerical model that allows determining selfconsistent distributions of the potential and ion radial velocity near the system consisting of an infinite chain of spherical dust particles.
The direct results of this model calculation, which is the spatial distribution of the potential and ion radial velocity, are demonstrated. It is shown that, as a result of ion focusing, a local maximum in the potential forms behind the dust particle, and a local maximum in the velocity distribution forms in front of the dust particle. These local maxima are parts of the wake, an oscillating structure that is usually observed when an isolated dust particle enters the plasma stream.

The dependence of the wake local maxima of the electrostatic field is demonstrated. It is found that the anisotropy of the potential increases linearly with increasing field strength, and then quickly declines, while the local maximum of spatial velocity distribution does not interrupt its growth. It may be concluded that the spatial distribution of the ion cloud potential induced by a dust particle is weakly related to the ion dynamics in a system consisting of a chain of spherical dust particles.

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