Drilling Fluid Leakage Model Based on the Dual-Medium Theory

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Abstract. In view of the lost circulation in carbonate formation, based on the dual-medium model, a two-dimensional transient model for fracture-porous formation is established. All the factors that influence the leakoff rate in the fracture-porous formation are analyzed. The result shows that the leakoff rate increases as the normal stiffness of the crack decreases; the leakoff rate increases as the initial fracture aperture increases; The larger the pressure difference is, the larger the drilling fluid leakage is; The larger the porosity is, the more drilling fluid flows from fracture to matrix, the slower the leakoff descent is; The larger the permeability of matrix is, the larger of leakoff rate is at the initial stage of leakage.

1. Introduction
Lost circulation is one of the most common downhole problems during the drilling operation, especially in the fractured reservoir. According to statistics, in the development process of Fuling shale gas field, lost circulation occurs in 67.2% wells. Drilling fluid leakage increases the cost of drilling and even causes downhole accidents [1-2]. By analyzing the influence of different factors on the leakage rate of drilling fluid, we can judge and evaluate the development of fractures in the formation, and provide reference for subsequent sealing operations.

Several examples and mud loss modele has been made in the literature. Lietard O et al. considered incompressible drilling fluid having Bingham rheology. They developed a model of radial mud flow into a smooth fracture with constant aperture and infinite length [3]. Shahri M P et al. considered influence of the angel of the facture on the drilling fluid loss rate [4]. Based on the model of Lietard O, Majidi.R et al. developed a model which regards drilling fluid as Herschel-Buckley fluid [5-7]. Li daqi et al. developed a model considering the roughness of the fracture [8].

All the models above don’t consider the influence of matrix. If the formation with higher permeability, it will make larger error. Based on the dual-medium theory, a new mud leakage model is presented in the paper.

2. Model Formation
A square fracture, which height is equal to its width, is considered. The well is in the middle of the fracture. The fracture is horizontal so the gravity effect does not need to be taken into account. At the beginning of mud leakage, the fracture aperture is same in all points of fracture and the surface of fracture is smooth, which means the influence of roughness of fracture surface does not need to be considered.

According to the law of conservation of mass and Reynolds equation, the mass balance equation for an incompressible fluid is given by [9]:
\[-\nabla (\rho w v_f) = 2 \rho q_t + \frac{\partial \rho w}{\partial t}\]  

(1)

Where \( w \) is the fracture aperture, m; \( v_f \) is the fluid flow velocity in the fracture, m/s; \( q_t \) is the flow rate between fracture and rock matrix, m³/s; \( \rho \) is the drilling fluid density, kg/m³.

At the present, most of drilling fluid is water-based mud, which can be regarded as a kind of incompressible fluid. For most of drilling fluid, power-law rheology is acceptable, which can be described as follow:

\[ \tau = Ky^n \]  

(2)

where \( \tau \) is the shear stress, Pa; \( K \) is the consistency index, Pa·sⁿ; \( \gamma \) is the shear rate, s⁻¹; \( n \) is the power law exponent.

For the power-law fluid, the drilling fluid velocity in the fracture can be expressed as follows:

\[ v_{fx} = \frac{n}{1+n} \left( \frac{1}{2n+1K} \right)^{\frac{1}{n}} w^{\frac{n+1}{n}} \left( \frac{\partial p_f}{\partial x} \right)^{\frac{1}{n}} \]  

(3)

\[ v_{fy} = \frac{n}{1+n} \left( \frac{1}{2n+1K} \right)^{\frac{1}{n}} w^{\frac{n+1}{n}} \left( \frac{\partial p_f}{\partial x} \right)^{\frac{1}{n}} \]  

(4)

Where \( n \) is the power exponent; \( K \) is the consistency index, Pa·sⁿ; \( v_{fx} \) is the drilling fluid velocity in the fracture in the \( x \)-direction, m/s; \( v_{fy} \) is the drilling fluid velocity in the fracture in the \( y \)-direction, m/s; \( p_f \) is the fracture pressure, MPa.

Compared with the fracture, the permeability of rock matrix is small, so the flow of drilling fluid in the rock matrix can be ignored. The only function of rock matrix is storing drilling fluid. The control equation can be expressed as follow:

\[ q_m = \phi \frac{\partial p_m}{\partial t} \]  

(5)

Where \( \phi \) is the porosity of rock matrix; \( p_m \) is the formation fluid pressure, MPa; \( q_m \) is the flow rate from fracture to the rock matrix, m³/s.

The fracture aperture has a great influence on the loss rate of drilling fluid. Fracture aperture is not invariable during the leakage. Fracture aperture will response to the change of pressure in the fracture. It will cause error if the width of fracture is assumed to be constant. When the drilling fluid flows into the fracture, the pressure in the fracture will increase, and then the fracture aperture will increase as well. At the present, there are two kinds of fracture deformation models, one is exponential model, and the other one is linear model. In this paper, linear deformation model is adopted. The formula is as follow [10]:

\[ w = w_o + \frac{p_f - p_o}{K_n} \]  

(6)

Where \( w_o \) is the fracture aperture when \( t=0 \), m; \( p_f \) is the pressure in the fracture, MPa; \( p_o \) is the formation fluid pressure when \( t=0 \), MPa; \( K_n \) is the normal fracture stiffness, Pa/m.

Substituting (3)(4)(6) into (1)(5), the following equations are obtained:

\[ \nabla \cdot \left[ w \left( \frac{n}{2n+1} \right)^{\frac{1}{n}} w^{\frac{n+1}{n}} \left( \frac{\partial p_f}{\partial x} \right)^{\frac{1}{n}} \right] - q_f = \frac{1}{K_n} \frac{\partial p_f}{\partial t} \]  

\[ q_m = \phi \frac{\partial p_m}{\partial t} \]  

(7)

Where \( q_f \) and \( q_m \) are the flow rate between fracture and rock matrix, their value are equal to each other, but signs are opposite.

Flow between fracture and matrix is slow, so it can be regarded as steady seepage. Based on the dual-medium theory, the formula of power-law fluid flow between fracture and matrix can be expressed as follows [11]:

\[ -\nabla (\rho w v_f) = 2 \rho q_t + \frac{\partial \rho w}{\partial t} \]  

(1)
\[
\begin{align*}
q_f &= a \left[ \frac{k_m (p_f - p_m)}{\mu_{eff} dz/2} \right]^{\frac{1}{n}} \\
q_m &= -a \left[ \frac{k_m (p_f - p_m)}{\mu_{eff} dz/2} \right]^{\frac{1}{n}} \\
\mu_{eff} &= K \frac{\sqrt{\frac{1}{12}}}{n} \left( 9 + \frac{3}{n} \right)^n (150k_m \phi)^{\frac{1-n}{2}}
\end{align*}
\] \tag{8}

Where \(a\) is the shape factor; \(dz\) is the height of cap rock, m; \(n\) is the power exponent; \(K\) is the consistency index, Pa·s\(^n\); \(k_m\) is the rock matrix permeability, um.

3. Initial and Boundary Conditions

Compared with the permeability of fracture, the permeability of rock matrix is very small. To reduce count quantity, it’s assumed that drilling fluid can only flow from wellbore to the fracture but not flow from wellbore to the rock matrix [12-14].

\[
\nabla \cdot \left[ \frac{W}{2n+1} \left( \frac{1}{2^n+1} K \right) \frac{1}{n} \left( \nabla p_f \right)^{\frac{1}{n}} \right] - q_f + \frac{Q_f}{dx \cdot dy} = \frac{1}{k_n} \frac{\partial p_f}{\partial t}
\] \tag{10}

\[
Q_f = 2\pi w \left( \frac{n}{2n+1} \right) \left( \frac{w}{\pi} \right)^{\frac{1-n}{n}} \left( \frac{1-n}{K (n^2-n-1-n)} \right)^{\frac{1}{n}} (p_w - p_f)^{\frac{1}{n}}
\] \tag{11}

Where \(Q_f\) is the drilling fluid loss rate from wellbore to the fracture, m\(^3\)/s; \(r_w\) is the supply radius, m; \(r_e\) is the wellbore radius, m; \(p_w\) is the wellbore pressure, MPa.

At the time \(t=0\), it’s assumed that the pressure all over the fracture is same, and the fracture pressure is equal to the formation fluid pressure, which can be expressed as follows:

\[
p_m = p_f
\] \tag{12}

Reservoir boundaries are considered impermeable and thus act like no-flow boundary:

\[
\frac{\partial p}{\partial x} = 0 \quad x = 0, x = L_x
\]

\[
\frac{\partial p}{\partial y} = 0 \quad y = 0, y = L_y
\] \tag{13}

4. Mesh Generation and Stimulation

In this paper, rectangle central grid is adopted. At the beginning of the leakage, the leakage rate is large and the pressure near the wellbore changes fast. In order to guarantee the calculation speed and accuracy, the variable density grid is used in the paper. The grid near the wellbore should be smaller and the grid far from the wellbore should be bigger. In the initial stage of leakage, the time step should be as small as possible to ensure the accuracy of calculation.

The model developed in the previous section and comprising the nonlinear partial differential equation (7), describes radial mud flow in a fracture and the associated ballooning phenomenon. The equation was integrated numerically by using an explicit finite difference scheme [15, 16]. Newton Raphson method is used to solve the problem [17].

5. Case Study

The effect of various factors on the drilling fluid leakage rate are analysed in this section. The inputs for the calculation are given in Table 1.
Table 1. Basic Date for the Calculation

| Parameter Name                  | Value          | Parameter Name              | Value          |
|--------------------------------|----------------|-----------------------------|----------------|
| Fracture Length in x Direction | 100m           | Permeability                | $1 \times 10^{-3}$um$^2$ |
| Fracture Length in y Direction | 100m           | Drilling Fluid Density      | $1.2 \times 10^3$kg/m |
| Formation Height               | 0.5m           | Consistency Index           | 0.8Pa·s$^n$    |
| Normal Fracture Stiffness      | $10^6$Mpa/m    | Power Law Exponent          | 0.5            |
| Initial fracture aperture      | 0.5mm          | Formation pressure          | 50Mpa          |
| Formation porous               | 0.1            | Borehole Pressure           | 45Mpa          |

The effect of normal fracture stiffness on mud leakage is illustrated in figure 1. According to equation (6), normal fracture stiffness has a direct influence on the width of fracture. Higher normal fracture stiffness means that the rock is harder and is difficult to be compressed. In the case of the same value of pressure difference between fracture and formation, the bigger normal fracture stiffness is, the narrower the fracture is.

The effect of initial fracture aperture on mud leakage is illustrated in figure 2. Initial fracture aperture has a great influence on the initial drilling fluid leakage rate. The larger the fracture aperture is, the faster drilling fluid loss at the beginning. But after a few seconds, once the fracture is filled with drilling fluid, the drilling fluid leakage rate will be almost same. That is because once the fracture is filled with drilling fluid, the initial fracture aperture will have limited influence on the mud leakage rate. On the other hand, pressure difference between fracture and formation, permeability of formation and other influence factors will play a leading role.

The effect of pressure difference between borehole and formation on mud leakage rate is illustrated in figure 3. As figure 3 shows, larger pressure difference between borehole and formation will increase the peak in the mud leakage curve. From figure 3, we can see that the larger the pressure difference between borehole and formation is, the higher the mud leakage rate is at the beginning. Compared with figure 2, influence of pressure difference on the leakage rate will last longer than fracture aperture. This is because pore pressure increases more slowly; it will take longer time to get a balance between fracture pressure and pore pressure.

The effect of the rock matrix porosity on mud loss rate is illustrated in figure 4. As figure 4 shows, the matrix porosity does not influence the peak in the mud loss curve, which means the value of the rock matrix porosity has no effect on the seepage rate between fracture and rock matrix. On the other hand, the larger the matrix permeability is, the slower the drilling fluid flow rate decrease. That is because that the fluid pressure in the matrix will increase slower if the matrix porosity is higher.

The effect of the rock matrix permeability on mud leakage rate is illustrated in figure 5. As figure 5 shows, at the beginning, larger permeability promotes the fluid flow in the rock matrix, and the pore fluid pressure increases slower, which makes the pressure difference between fracture and rock matrix keep large. After a while, rock with large permeability will first be filled with drilling fluid and pore fluid pressure will increase, then the mud leakage rate will decrease.
6. Conclusions

Based on the dual-medium theory, a model of mud leakage into a single isolated deformable horizontal fracture was developed for a power-law drilling fluid. Numerical simulation with the model has shown that:

1. Increasing normal stiffness of fracture decreases the mud leakage rate.
2. Increasing initial fracture aperture increases the height of the peak of the mud loss curve, but after the beginning, the impact of initial fracture aperture is small.
3. Increasing pressure difference increases the height of the peak of the mud loss curve.
4. The larger the porosity of matrix is, the larger the mud leakage at the beginning is.
5. Permeability of matrix has little influence on the leakage rate at the beginning, but larger permeability will make the leakage last longer in a larger value.
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