The influence of corotation on high-energy synchrotron emission in Crab-like pulsars

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ABSTRACT
For Crab-like pulsars, we consider the synchrotron mechanism under the influence of relativistic effects of rotation in order to study the production of very-high-energy (VHE) pulsed radiation. The process of quasi-linear diffusion (QLD) is applied to prevent the damping of synchrotron emission due to extremely strong magnetic fields. By examining the kinetic equation governing QLD, apart from the synchrotron radiative force we take into account the so-called reaction force, which is responsible for corotation and influences plasma processes in the zone near to the light cylinder (LC) surface. We have found that the relativistic effects of rotation significantly change the efficiency of QLD. In particular, examining magnetospheric parameters typical for Crab-like pulsars, it has been shown that, unlike the situation in which relativistic effects of rotation are not important at the LC surface, relativistic electrons may produce photons via the synchrotron mechanism even in the TeV domain. It is shown that VHE radiation is strongly correlated with relatively low-frequency emission.

Key words: acceleration of particles – radiation mechanisms: non-thermal – pulsars: general.

1 INTRODUCTION
According to the standard pulsar model, in particular the so-called polar-cap model (Sturrock 1971), particles are uprooted from the star’s surface and accelerate along the magnetic field lines (Ruderman & Sutherland 1975) inside a gap zone. In general, it is believed that these particles produce non-thermal radiation, which is interpreted in terms of the synchrotron mechanism (Pacini 1971; Shklovsky 1970) and inverse Compton scattering (Blandford, Netzer & Woltjer 1990) respectively. Generally speaking, due to strong synchrotron losses, relativistic electrons quickly lose their perpendicular momentum and transit to their ground Landau states. Therefore, particles may be approximately described as moving one-dimensionally along the field lines, which in turn means that the synchrotron mechanism does not contribute to high-energy radiation processes. On the other hand, under certain conditions due to quasi-linear diffusion (QLD) the pitch angles might be recreated, leading to efficient synchrotron emission. This method has already been applied to pulsars (Machabeli & Usov 1979; Malov & Machabeli 2001; Chkheidze & Machabeli 2007; Chkheidze & Lomiashvili 2008; Chkheidze 2009; Machabeli & Osmanov 2009, 2010) and active galactic nuclei (Osmanov 2010; Osmanov & Machabeli 2010).

In the context of QLD, the recent detection of VHE radiation from the Crab pulsar (Aliu et al. 2008) could be very important. The MAGIC Cherenkov telescope, operating during 2007 October–2008 February, has discovered pulsed emission above 25 GeV that reveals several characteristic features. The coincidence of signals from different energy bands ranging from radio to VHE (>25 GeV) domains (Aliu et al. 2008) deserves special interest. The authors conclude that, according to the data, polar-cap models must be excluded from the possible scenarios for the radiation, which must happen far out in the magnetosphere. Analysis of the MAGIC data also implies that the location of the aforementioned VHE and low-energy emission must be the same. By taking into account the synchrotron reaction force, we constructed the kinetic equation governing QLD and the results of the MAGIC data have been interpreted (Machabeli & Osmanov 2009). We argued that the observed VHE radiation is produced by the synchrotron mechanism, having properties that are in a good agreement with observations. The following work (Machabeli & Osmanov 2010) was related to the problem of curvature radiation and inverse Compton scattering in the context of the same observations. We have shown that these mechanisms do not contribute to the VHE domain detected by MAGIC.

In general, the magnetic field in a pulsar magnetosphere is huge and ranges from $10^6$ G (far out in the magnetosphere) to $10^{12}$ G close to the neutron star’s surface. Therefore, the magnetospheric plasma is in the frozen-in condition and is forced to follow the magnetic field lines. This means that effects of corotation in plasma acceleration and emission processes could be very important. In general, it is believed that the high-energy radiation comes from an area located relatively close to the light cylinder (LC) surface, where corotation is extremely significant. For this purpose it is reasonable...
to consider QLD by taking the relativistic effects of rotation into account and seeing how corotation influences the above-mentioned processes.

In the present paper we study the role of corotation in QLD and we show that in the very vicinity of the LC surface, in contrast to QLD, the effects of corotation attempt to decrease the pitch angles and dampen the subsequent synchrotron emission.

The paper is organized as follows. In Section 2 we consider QLD by taking the effects of corotation into account, in Section 3 we present our results and in Section 4 we summarize them.

2 MAIN CONSIDERATIONS

In this section we present the model, generalizing our previous approach by taking corotation into account and seeing how the efficiency of QLD depends on it.

Generally speaking, pulsar magnetospheres are composed of low- and high-energy particles, respectively. Therefore, in the framework of the paper, for simplicity we assume that the magnetosphere consists of two components: (a) the plasma component with Lorentz factor $\gamma_b$ and (b) the beam component with Lorentz factor $\gamma_p$.

According to the mechanism of QLD, the following transverse mode is generated:

$$\omega \approx k c (1 - \delta), \quad \delta = \frac{\omega_P^2}{4 \omega_B^2 \gamma_b^2},$$

where $k$ denotes the modulus of the wavevector, $c$ is the speed of light, $\omega_P = \sqrt{4 \pi n e^2 / m}$ is the plasma frequency, $e$ and $m$ are electron charge and the rest mass, respectively, $n_p$ is the plasma density, $\omega_B = eB / mc$ is the cyclotron frequency and $B$ is the magnetic induction. It can be shown that the aforementioned waves are excited if the cyclotron resonance condition is satisfied (Kazbegi, Machabeli & Melikidze 1992):

$$\omega - k_x v - k_y u \pm \frac{\omega_B}{\gamma_b} = 0.$$  \hspace{1cm} (2)

We denote the wavevector’s longitudinal (parallel to the background magnetic field) and transverse (perpendicular to the background magnetic field) components by $k_x$ and $k_y$ respectively, $u \equiv e\gamma_b \nu_p / m\omega_p$ is the drift velocity of electrons, $\nu$ is the component of velocity along the magnetic field lines and $\rho$ is the field-line curvature radius. In this paper we consider the beam particles to be resonance particles. One can show from equations (1) and (2) that the excited cyclotron wave is characterized by the following frequency (Malov & Machabeli 2001):

$$\nu \approx \frac{\omega_B}{2 \pi \gamma_b \rho}.$$  \hspace{1cm} (3)

Generally speaking, particles moving in a magnetic field experience two forces, one of which, $G$, has the components (Landau & Lifshitz 1971)

$$G_x = -\frac{mc^2}{\rho} \gamma_p \psi, \quad G_y = \frac{mc^2}{\rho} \gamma_p \psi^2,$$

and is responsible for the conservation of the adiabatic invariant, $I = 3cp^2 / 2eB$, and the second of which is the synchrotron radiative force (Landau & Lifshitz 1971)

$$F_x = -\alpha \psi(1 + \gamma_b^2 \psi^2), \quad F_y = -\alpha \psi^2;$$  \hspace{1cm} (5)

where $\alpha = 2c^2 \omega_B^2 / (3c^2)$. However, one can show that for Crab-like pulsars the radiation reaction force, $|F|$, exceeds $|G|$ by many orders of magnitude (Machabeli & Osmanov 2009, 2010).

On the other hand, apart from the synchrotron radiative force, relativistic electrons also experience a reaction force responsible for the corotation. This force can be estimated by a simple mechanical analogy introduced by Machabeli & Rogava (1994) and reconsidered by Rogava, Dalakishvili & Osmanov (2003). In the framework of this approach, instead of magnetic field lines and particles sliding along them, one considers a corotating pipe with a bead inside it. If we assume that the magnetic field lines are straight, then the reaction force acting on a single particle from the beam component can be given by (Rogava et al. 2003)

$$R = \frac{dp}{dt} + \Omega p, \quad \Omega = \frac{\omega}{\gamma_b},$$  \hspace{1cm} (6)

where $\Omega$ is the angular velocity of rotation, $r$ is the radial coordinate, and

$$p_x \equiv \gamma_p m\nu, \quad p_y \equiv \gamma_p m\nu, \quad \Omega = \frac{\omega}{\gamma_b}.$$  \hspace{1cm} (7)

and

$$p_x \equiv \gamma_p m\nu, \quad p_y \equiv \gamma_p m\nu, \quad \Omega = \frac{\omega}{\gamma_b}.$$  \hspace{1cm} (8)

are the longitudinal and transversal components of momentum respectively.

Considering a single-particle approach, one can show that the Lorentz factor of a particle moving along a corotating straight magnetic field yields the equation (Rieger & Mannheim 2000)

$$\gamma_b = \frac{1}{\sqrt{m \left(1 - \frac{r^2}{c^2} \right)}},$$  \hspace{1cm} (9)

where

$$m = \frac{1 - r_0^2 / r_*, \nu_0^2 / c^2}{\left(1 - r_0^2 / r_*^2 \right)}, \quad r_* \equiv \frac{c}{\Omega}.$$  \hspace{1cm} (10)

$r_0$ and $\nu_0$ are the initial position and initial radial velocity of the particle respectively and $r_*$ is the light-cylinder radius. From equation (9) we see that the closer to the LC, the greater the Lorentz factor. Therefore the dynamical effect of the corotation becomes extremely efficient near to this area (see equation 6), where the reaction force can be approximated as (see equation A6 in the Appendix)

$$R \approx 2\pi n_\perp \epsilon c^3 \gamma_b^{3/2}.$$  \hspace{1cm} (10)

As we have mentioned, in the Crab pulsar magnetosphere one may neglect the effects of $G$ in comparison with $F$. Therefore, the process of QLD is mainly influenced by the synchrotron radiative force $F$ and the reaction force $R$. The synchrotron radiation reaction force acting on a particle attempts to decelerate it; therefore relativistic electrons lose their perpendicular momentum, leading to an inevitable decrease of the pitch angles. In spite of the fact that, in contrast to $F$, the reaction force plays an accelerating role in electron dynamics, in the context of pitch angles its role is the same. In particular, as we have already discussed, the above-mentioned reaction force is a direct consequence of corotation. This means that $R$ is constructed in such a way that particles always must stay on the magnetic field lines; therefore, the reaction force also attempts to decrease the pitch angles. On the other hand, QLD that arises through the influence of waves back on particles attempts to widen the range of the pitch angles. The dynamical process saturates when

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1 In the local frame of reference, particles experience the centrifugal force, whereas considering dynamics in the laboratory frame one has to examine the reaction force acting on the electrons from the magnetic field lines.
the effects of the above-mentioned forces are balanced by diffusion. For $\gamma_0 \psi \gg 1$, it is easy to show that in the quasi-stationary case ($\partial/\partial t = 0$) the corresponding kinetic equation governing QLD can be written as follows (Malov & Machabili 2001):

$$\frac{1}{p_1} \frac{\partial}{\partial \psi} \left[ \frac{\partial}{\partial \psi} \left( \frac{\rho}{p_1} \frac{\partial}{\partial \psi} \chi \right) \right] = \frac{\partial}{\partial \psi} \left[ \chi D_{\perp 1} \frac{\partial}{\partial \psi} \chi \right],$$

(11)

where $\chi = \chi(\psi)$ is the distribution function of particles with respect to the pitch angles,

$$D_{\perp 1} = -\frac{\pi e^2 n_b c}{4\psi},$$

(12)

is the diffusion coefficient, $n_b = R/P_c e$ is the density of the beam component and $P$ is the pulsar rotation period. The solution of equation (12) can be written as follows:

$$\chi(\psi) = \chi_0 e^{-A_1 \psi(1-A_2 \psi^3)},$$

(13)

As is clear from this expression, electrons are differently distributed for different pitch angles. Let us note that the reaction force and the synchrotron radiative force have opposite directions, since $F_\perp$ is responsible for deceleration of electrons whereas $R$ accelerates them. It is clear that close to the LC the value of $|R| \propto \gamma_0^{-1/2}$ is very high and for very small pitch angles exceeds $|F_\perp|$; therefore, higher values of pitch angles lead to higher values of the distribution function. By increasing $\psi$, however, the corresponding reaction force does not change (see equation 10), whereas the synchrotron radiative force is very sensitive to this change, $F_\perp \sim \psi^3$. Therefore at a certain pitch angle, $\psi_0$, these forces will balance each other and the distribution function will reach its maximum value. In particular, one can straightforwardly show that the expression for the distribution function (see equation 13) peaks at

$$\psi_0 = \frac{1}{\sqrt{4A_2}} = \left( \frac{2mc\Omega}{\alpha \gamma_0 \gamma_\perp} \right)^{1/3},$$

(14)

which indeed corresponds to $R = F_\perp$. Here, we have taken into account the following relation: $\tilde{m} \approx 1/\gamma_0$ ($r_0/r_c \ll 1$, see equation 9).

It is reasonable to estimate the steepness of the distribution function. For simplicity we consider the case $\gamma_0 = 1$. Then, examining the Crab pulsar’s ($P \approx 0.0332$ s) magnetospheric parameters close to the LC surface: $B \approx 1.7 \times 10^7$ G, $\gamma_0 = 10^3$, one can see that at the peak, $\psi_0 \approx 0.02$ rad, the distribution function exceeds its value at $\psi = 0$ by many orders of magnitude. This means that most of the particles will be characterized by the peak value of the pitch angle, and therefore its average value can be estimated as $\psi_0$. We have taken into account that $B = B_{\alpha} R_{\alpha}/r_{\perp}^2$, where $B_{\alpha} \approx 10^6$ cm is the pulsar’s radius, $B_{\alpha} \approx 1.8 \times 10^{12} \sqrt{P \times P_{15}}$ G is the magnetic induction on the star’s surface and $P_{15} \approx 10^{15} P$ ($P_{15} = 421$ for the Crab pulsar, see Manchester & Taylor 1980).

Since we consider centrifugally accelerated particles, our approach is based on the assumption that electrons corotate with the spinning pulsar, which is valid only inside a certain zone (corotation zone with radius $r_c$), because in this region the magnetic field is strong enough to channel the flow. This means that, our approach is valid if the following condition is satisfied: $(r_c - r_\perp)/r_\perp \ll 1$. It is clear that corotation takes place if the magnetic energy density exceeds the beam energy density; therefore, $r_c$ can be estimated by the condition $B(r_c)^2/(8\pi) \approx mc^2 n_b \gamma_\perp(r)$. If we consider a range of $\gamma_0 = (1; 10^4)$, then by taking equation (9) into account one can see that for $\gamma_\perp \approx 10^7$ one has $(r_c - r_\perp)/r_\perp \sim (10^{-11}; 10^{-7}) \ll 1$, which means that corotation is violated in a region very close to the LC surface and for almost the whole course of motion the spinning magnetic field channels the electrons.

When relativistic particles move in a magnetic field they emit electromagnetic waves, which in our case correspond to the following photon energies (Rybicki & Lightman 1979):

$$\epsilon_{\gamma\nu} \approx 1.7 \times 10^{-20} B \gamma^2 \sin \psi_0, \approx 10^{-16} \times B^{1/3} \times \gamma_0^{11/6} \times \gamma_b^{-2/3}. \quad (15)$$

3 DISCUSSION

In this section we apply our method to Crab-like pulsars and see what changes in the process of QLD when corotation is taken into account.

In Fig. 1 we show the dependence of photon energies on the Lorentz factors of the beam electrons for three different initial values of beam Lorentz factor. The set of parameters is $P \approx 0.0332$ s, $B \approx 1.7 \times 10^7$ G, $\gamma_0 = (1, 10^2, 10^4)$. As is clear from the plots, by increasing $\gamma_0$ the corresponding emission energy increases as well. This is a natural result, because more energetic electrons emit more energetic photons (see equation 15). A second feature concerns the behaviour of $\epsilon_{\gamma\nu}$ versus $\gamma_0$. It is evident that the higher the initial beam Lorentz factor the lower the emission energy. Indeed, as one can see from equation (14) the pitch angle behaves as $\gamma_0^{-2/3}$, therefore for higher initial beam Lorentz factors one obtains lower values of the pitch angles and hence the corresponding synchrotron energy becomes lower.

The relativistic effects of rotation could also be interesting in the context of the recently detected VHE pulsed emission (>25 GeV) from the Crab pulsar. According to the interpretation presented in (Machabili & Osmanov 2009, 2010), the aforementioned radiation is formed on length-scales of the order of $10^8$ cm via QLD. As has shown, in order to explain the data the value of the beam Lorentz factor must be of the order of $3 \times 10^6$. For centrifugally accelerated electrons the effects of corotation become significant in the zone near to the LC surface. In particular, from equation (10) it is clear that the reaction force is proportional to $\gamma_b^{2/3}$, which in turn asymptotically increases on reaching the LC surface (see equation 9). The corresponding length-scale of a layer on the LC zone

\[ L = \frac{\gamma_0^{1/3} R_{\alpha}}{\gamma_\perp^{2/3}} \]

Figure 1. The synchrotron emission energy versus the Lorentz factor of beam electrons. The set of parameters is $P \approx 0.0332$ s, $B \approx 1.7 \times 10^7$ G, $\gamma_0 = (1, 10^2, 10^4)$.
in which the effects are intensified by corotation can be estimated as follows:

\[ \lambda \approx \frac{\gamma_b}{d\gamma_b/dr} \approx \frac{r_c}{2\gamma_b}, \]

where we have taken into account equation (9). As we see from this expression, the layer is quite thin, \( \lambda \ll r_c \). Unlike the papers by Machabeli & Osmanov (2009, 2010) in which the effects of corotation are not significant and VHE radiation (>25 GeV) can be achieved by relativistic particles with \( \gamma_b \sim 3 \times 10^4 \), a part of the magnetosphere located very close to the LC surface may guarantee the aforementioned emission energy for lower Lorentz factors. In particular, from Fig. 1 it is clear that the synchrotron process supported by QLD produces 25-GeV photons for \( \gamma_b \sim 5.3 \times 10^5 (\gamma_{b0} = 1) \), \( \gamma_b \sim 8 \times 10^4 (\gamma_{b0} = 10^2) \) and \( \gamma_b \sim 1.2 \times 10^4 (\gamma_{b0} = 10^3) \), which are much lower than \( 3 \times 10^5 \). QLD is so efficient that under certain conditions it can provide TeV photons as well. As one can see from the figure, the relativistic electrons with Lorentz factors \( 5 \times 10^5 (\gamma_{b0} = 1) \) may provide TeV emission (1.5 TeV).

According to the present method, the process of QLD depends on the excitation of unstable cyclotron waves, which correlate with the high-energy radiation. In Fig. 2 we show the behaviour of synchrotron emission energy versus the cyclotron frequency. The set of parameters is the same as in Fig. 1. As is seen from the plots, VHE radiation is strongly connected with relatively low-energy radiation, starting from microwave \( (5 \times 10^{10} \text{ Hz}) \) to optical \( (10^{14} \text{ Hz}) \) domains respectively.

The investigation shows that very close to the LC area, where the effects of corotation are extremely important, QLD becomes so efficient that under favourable conditions it may provide VHE radiation for relatively lower energy electrons than the regime considered by Machabeli & Osmanov (2009, 2010). The aim of the present paper was to demonstrate for Crab-like pulsars that the relativistic effects of rotation are of fundamental importance for studying VHE emission via QLD. Studying rotationally driven QLD for 1-s pulsars is also an interesting problem, but it is outside the scope of the present paper; sooner or later we will consider it also.

### 4 SUMMARY

(i) We have examined the VHE radiation of Crab-like pulsars via QLD intensified by the effects of corotation, which takes place in the zone near to the LC surface.

(ii) It has been emphasized that, due to the very strong magnetic field of pulsar magnetospheres, efficient energy losses lead to the damping of synchrotron process. Close to the LC surface the effects of corotation become significant and as well as the synchrotron radiative force, which decreases the pitch angles, the reaction force (responsible for corotation) also has to be taken into account. Generalizing the kinetic equation governing QLD, we have found the particle distribution with respect to \( \psi \) and estimated its average value.

(iii) Considering a Crab-like pulsar’s magnetospheric parameters, we have found that QLD becomes more efficient in the LC zone than for locations far from this area. It has been shown that the synchrotron emission process, via QLD, might provide VHE emission even in the TeV domain.

(iv) We have found that the VHE radiation is strongly connected with the cyclotron emission, having relatively low frequencies.

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### APPENDIX A: REACTION FORCE ESTIMATION

In this Appendix we estimate the reaction force close to the LC surface.
From equations (6)–(8) one can see that
\[ R = m\Omega \frac{d(\gamma_b r)}{dt} + m\gamma_b\nu\Omega = m\Omega r\frac{d\gamma_b}{dt} + 2m\gamma_b\nu\Omega. \]  \hfill (A1)

By applying equation (9) for the zone near to the LC area, \( r \approx r_k \), the aforementioned expression can be written as follows:
\[ R \approx 2m^{1/2}\nu\gamma_b^2 + 2m\gamma_b\nu\Omega, \]  \hfill (A2)

which by taking into account a natural relation \( m^{1/2}\gamma_b \gg 1 \) reduces to
\[ R \approx 2m^{1/2}\nu\gamma_b^2. \]  \hfill (A3)

Machabeli & Rogava (1994) showed (see their equation 10) that
\[ \nu = c \sqrt{\left(1 - \frac{r^2}{r_k^2}\right) \left[ 1 - \tilde{m} \left(1 - \frac{r^2}{r_k^2}\right)\right]}, \]  \hfill (A4)

which, applied to the LC surface by using equation (9), yields the following approximate form:
\[ \nu \approx \frac{c}{\tilde{m}^{1/4}\gamma_b^{1/2}}, \]  \hfill (A5)

and reduces equation (A3) to the final expression
\[ R \approx 2m^{1/4}c\Omega\gamma_b^{3/2}. \]  \hfill (A6)

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