Stochastic modeling of transient neutral and stably-stratified Ekman boundary layers

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Neutral and stably-stratified Ekman boundary layers (EBLs) are numerically investigated with a stochastic one-dimensional turbulence (ODT) model. EBLs achieve the bulk-surface coupling in Earth’s atmosphere. They are numerically challenging due to transient and non-universal turbulence properties even at small scales. ODT addresses this problem by distinguishing turbulent-adveective from molecular-diffusive transport processes for a vertical column along which all relevant scales of the flow are resolved. We demonstrate the model’s capabilities for economical, accurate, and stratification regime independent simulation of EBLs for the wind-turning angle. ODT reproduces and extrapolates reference direct numerical simulation results consistent with observations. We conclude that ODT may be useful for modeling of atmospheric surface layers.

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1 Introduction

Near-surface transport processes in atmospheric boundary layers are intricate and occur down to a scale of meters (e.g. [1]). Therefore, economical and accurate numerical modeling is crucial correct for the bulk-surface coupling in weather and climate prediction (e.g. [2]). From the perspective of modeling additional challenges arise from the complex flow physics that encompass counter-gradient fluxes (e.g. [3]), non-uniform turbulence properties (e.g. [4]), and transients due to diurnal forcing (e.g. [5]). Numerical simulations of night-time atmospheric boundary layers need to address all of these aspects since the bulk-surface coupling is sensitive to an accurate representation of the relevant (vertical) transport processes. In the present study, neutral and stably-stratified Ekman boundary layers (EBLs) are numerically investigated as canonical problem for night-time atmospheric boundary layers over flat terrain. We address both numerical efficiency and accuracy by utilizing a lower-order stochastic approach for regime independent numerical modeling and simulation.

2 Model formulation and application to Ekman boundary layers

Kerstein’s [6] one-dimensional turbulence (ODT) is a self-contained lower-order turbulence model that aims to resolve vertical (wall-normal) transport processes on all relevant scales of the flow. The key idea is to distinguish molecular-diffusive from turbulent-adveective transport processes in a dimensionally reduced setting, for which map-based advection is used to model the effects of overturning turbulent motions.

The model application to atmospheric boundary layers was previously described in [7], but here we utilize a fully-adaptive numerical implementation [8] that has recently been validated for transient flat-plate boundary layers [9]. Numerical ODT simulations of EBLs are carried out for a one-dimensional columnar domain (‘ODT line’) that is aligned with the mean horizontal no-slip wall in a with angular velocity $\Omega = \Omega \hat{e}_z$ rotating frame of reference subject to the background gravity $g = -g \hat{e}_z$. A typical viscous Ekman layer length scale (thickness) is $D = \sqrt{\nu/\Omega}$. The ODT $f$-plane equations applicable to the EBL and Ekman flow in general are given by [7]

$$\frac{\partial u_i}{\partial t} + \mathcal{E}_i(u, v, w, \rho) = \nu \frac{\partial^2 u_i}{\partial z^2} - f \epsilon_{ijk} (u_k - G \delta_{1k}) , \quad \frac{\partial T}{\partial t} + \mathcal{E}_T(u, v, w, \rho) = \kappa \frac{\partial^2 T}{\partial z^2} ,$$

(1a, b)

where $z$ denotes the wall-normal coordinate, $t$ the time, $(u_i) = (u, v, w)^T$ with $i \in \{1, 2, 3\}$ the Cartesian components of the velocity vector, $T$ the (potential) temperature and $\rho(T) = \rho_0 [1 - (T - T_0)/T_0]$ the mass density in the Oberbeck–Boussinesq approximation (subscript 0 indicates reference values), $\nu$ and $\kappa$ the fluid’s kinematic viscosity and thermal diffusivity, respectively, $f = 2\Omega$ the inertial frequency of horizontal (wall-normal) Coriolis forces, $G$ the bulk flow velocity, $\epsilon_{ijk}$ the Levi–Civita tensor, and $\mathcal{E}$ a stochastic process that models the effects of turbulent advection as detailed in [6–8]. Note that $\mathcal{E}_i$ also accounts for pressure fluctuations in the turbulent EBL [7]. Constant-temperature no-slip wall-boundary conditions are prescribed at $z = 0$ and homogeneous Neumann boundary conditions are prescribed at $z = H$, which is well in the geostrophically balanced bulk flow that acts as momentum reservoir. Stratified stability is imposed analogous to [4] by a sudden cooling of the surface for a fully-developed turbulent but initially neutral EBL.

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3 Wind turning in the neutral and stably-stratified Ekman boundary layer

The wind turning is generally sensitive to the vertical transport processes in the EBL [1, 2, 4] and is, therefore, of general interest when assessing ODT’s capabilities for application to atmospheric boundary layers. Lindvall and Svensson [2] point out that established general circulation models tend to underestimate wind-turning angles and their ranges such that midlatitude cross-isobaric mass transport is systematically underestimated. This introduces artificial transport barriers, which might be mitigated by stochastic surface-layer modeling.

Figure 2 shows ODT simulation results of the wind turning in the neutral EBL for laminar and turbulent flow conditions, respectively. The velocity vector is three-dimensional which is revealed by vertical profiles of the time-averaged horizontal velocity $\langle U(z,t) \rangle$, where $U(z,t) = \langle u(z,t) \rangle_t$ and $V(z,t) = \langle v(z,t) \rangle_t$. Some evenly spaced mean horizontal velocity vectors have been traced out for visualization (thin solid lines). The profile (thick solid line) of the turbulent solution has been obtained with ODT for the Reynolds number $Re = GD/\nu = 300$. The laminar profile (dotted line) is given by the well-known Ekman spiral (e.g. [5]) and is obtained from the momentum equations (1a) for $\varepsilon_t \equiv 0$. The wind turning is localized in the vicinity of the surface and generally reduces in magnitude with increasing $Re$.

Figure 3 shows parametric plots (hodographs) of the mean horizontal velocity vector $(U, V)$ for various stratification strengths one inertial period, $t = 2\pi f^{-1}$, after the cooling event. The Prandtl number is fixed at $Pr = \nu/k = 1$ and the Reynolds number at $Re = 500$. The flow is transient so that averaging over an $N$ ensemble of flow realizations was used to obtain $U(z,t) = \langle u(z,t) \rangle_N$ and $V(z,t) = \langle v(z,t) \rangle_N$. Stratification is weak for $Fr \gg 300$, moderate for $Fr \approx 300$, and strong for $Fr \ll 300$, where $Fr = G^2/(g D \Delta T/T_0)$ denotes the Froude number and $\Delta T = T_{\text{bulk}} - T_{\text{wall}}$ the prescribed bulk-wall temperature difference. ODT reasonably reproduces the wind-turning angle $\gamma_r \approx 25^\circ$ for weak stratification in agreement with reference DNS [4] as well as observations [2] (not shown explicitly). However, ODT is less sensitive to moderate stratification and it does not fully capture the maximum cross-stream velocity, $V_{\text{max}}$, for small but finite wall distances in the low $Re$ regime investigated.

4 Conclusion

Small-scale resolving numerical simulations of neutral and stably-stratified Ekman boundary layers (EBLs) have been performed using one-dimensional turbulence (ODT) [6]. The key idea of ODT consists in modeling turbulence by stochastically sampled mapping (‘eddy’) events. We have demonstrated that the model is able to reasonably capture the vertical EBL structure and the wind-turning angle for different stratification regimes. The agreement between ODT, available reference DNS [4] and observations [2] is better for weak and very strong stratification because the flow is uniformly turbulent or laminar, respectively. For moderate stratification, the one-dimensional model is unable to fully capture the lateral variability of the turbulence intensity [4], but it reasonably interpolates between flow regimes in an energetically consistent manner.

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