Storage of light: A useful concept?

M.G. Payne

Georgia Southern University, Statesboro, GA 30460-8031

L. Deng

Electron and Optical Physics Division, NIST, Gaithersburg, MD 20899-8410
(Dated: November 1, 2018)

We show both analytically and numerically that photons from a probe pulse are not stored in several recent experiments. Rather, they are absorbed to produce a two-photon excitation. More importantly, when an identical coupling pulse is re-injected into the medium, we show that the regenerated optical field has a pulse width that is very different from the original probe field. It is therefore, not a faithful copy of the original probe pulse.

Recent experiments [1,2] on the extremely slow propagation of a pair of optical pulses in a resonant medium and the subsequent coherent regeneration of the “probe” pulse have generated much excitement in the fields of optical computing and quantum optics. The physical explanation offered to these extraordinary phenomena rests on the adiabatic theory developed a decade ago by Eberly and coworkers[3]. To a large degree, the “dark-state polariton” theory [4] and all other recent works formulated around it are basically a field theoretical reformulation of the adiabatic approximation of pulse pair propagation in resonant media originally studied reasonably complete in the early 90’s. These new treatments have provided some further understanding of the process, especially from the viewpoint of atomic spin wave excitation and atomic coherence. Two main results of these works, however, do not, in our view, accurately describe the physical picture of such a highly nonlinear excitation process. In this letter, we show, based on the adiabatic theory, that the claim of the “revival of the probe pulse” in recent experiments [1,2,5] is not accurate. Indeed, we show that under the conditions of these experiments the original probe photons are neither “stopped” or “stored”. Rather, they are absorbed to produce two-photon excitation. Furthermore, we show that the “retrieval of the probe pulse” is not achieved without restrictions being placed on the characteristics of the second coupling pulse used to regenerate the “probe” field. In fact, we show that the regenerated field has time-spectra characteristics that are very different from the original probe field. Therefore, “storage of light” is not an accurate description of the process. In addition, we show that all experimental observables can be well predicted with the usual treatment of combining classical electrodynamics and the three-state model without invoking a full field theoretical methodology. This is not surprising since when a detailed handling of spontaneous emission is not critical and the fields are not extremely weak, a mean-field approximation to the quantum treatment of the electromagnetic field is valid. Our analysis and results on the physical picture are unique and provide a clear understandings of the physical process. To the best of our knowledge the subtle understandings presented here on the “tapped light” problem are not contained in either the original studies or any of the subsequent studies, including all recent theoretical works, on the subject.

We start with a Λ system (Fig. 1) with the assumption that a probe laser \( E_p \), frequency \( \omega_p \) is tuned on or near resonance with the \( |1\rightarrow 2>| \) transition. In addition, a coupling laser \( E_c \), frequency \( \omega_c \) is tuned so that exact two-photon resonance between states \( |1\> \) and \( |3\> \) is achieved. We therefore have the following atomic equations of motion

\[
\frac{\partial A_1}{\partial t_r} = i\Omega_p A_2, \tag{1a}
\]

\[
\frac{\partial A_2}{\partial t_r} = i\Omega_p^* A_1 + i\Omega_c^* A_3 + i\left(\delta + i\frac{\gamma_2}{2}\right) A_2, \tag{1b}
\]

\[
\frac{\partial A_3}{\partial t_r} = i\Omega_c A_2, \tag{1c}
\]

where \( A_j \) and \( \gamma_j \) are the \( j \)th amplitude of the atomic wave function and decay rate, respectively, and we have made phase transformation to remove \( z \) dependent phase factors. Also, \( \Omega_{p(c)} = D_{21(23)}E_{p(c)}/(2\hbar) \), \( \delta = \omega_p - \omega_{21} \), \( \omega_p - \omega_{c} = \omega_{31} \), and \( t_r = t - z/c \) is the retarded time which, in vacuum, is the combination of \( t \) and \( z \) that the laser field amplitudes depend on. Notice that we have included non-vanishing one-photon detuning \( 0 \leq \delta < \Omega_c \), a feature that is not included in both the original adiabatic theory and those developed recently around the “dark-state polariton” theory.

In order to correctly predict the propagation of the probe and coupling laser pulses in a resonant medium, Eq.(1)
must be solved simultaneously with Maxwell equations describing the propagation of the probe and coupling fields. For plane waves in the slowly varying amplitude approximation the Maxwell equations resume the form

\[
\begin{align*}
\frac{\partial \Omega_{p(c)}^*}{\partial z}_{tr} &= i\kappa_{12} A_1^* A_2, \\
\frac{\partial \Omega_{p(c)}^*}{\partial z}_{tr} &= i\kappa_{32} A_3^* A_2,
\end{align*}
\]

where \(\kappa_{12(32)} = 2\pi N_0\omega_p(1)|D_{12(32)}|^2/(hc)\). The process involves first seeking, from Eq.(1), an adiabatic solution for the atomic response, and applying the result to Eq.(2) to self-consistently describe the propagation of the pulse pair within the adiabatic limit. Following this procedure, we thus obtain adiabatic solution to the atomic response \[3\]

\[
\begin{align*}
A_1(z, t_r) &= \frac{\Omega_p(z, t_r)}{\Omega(z, t_r)}, \\
A_2(z, t_r) &= -\frac{i}{\Omega_p} \frac{\partial A_1}{\partial t_r} = -\frac{i}{\Omega_c} \frac{\partial A_3}{\partial t_r}, \\
A_3(z, t_r) &= -\frac{\Omega_p(z, t_r)}{\Omega(z, t_r)},
\end{align*}
\]

where \(\Omega(z, t_r) = \sqrt{|\Omega_p(z, t_r)|^2 + |\Omega_c(z, t_r)|^2}\). As usual, with the adiabatic approximation used in a resonance situation, we must have \(\Omega_c\) already strong when \(\Omega_p\) starts to build up. One scenario where the adiabatic approximation would hold through the whole laser pulse is if the two lasers peak at the same time, but the pulse length of the coupling laser is much longer. This feature of the pulse length and the requirement \(|\Omega_c, \tau| >> 1\) through the pulse length, \(\tau\), of the probe laser will make results based on the adiabatic approximation quite accurate.

With Eq.(3) as atomic response, we now solve Eq.(2) that now resumes the form of

\[
\frac{\partial \Omega_{p(c)}^*}{\partial z}_{tr} = -\frac{\kappa_{12(32)} \tau}{\sqrt{|\Omega_c|^2 + |\tau \Omega_p|^2}} \frac{\partial}{\partial t_r/\tau} \left( \frac{\tau \Omega_{p(c)}^*}{\sqrt{|\Omega_c|^2 + |\tau \Omega_p|^2}} \right).
\]

We first note that the quantity

\[
F(z, t_r) = \frac{|\Omega_p|^2}{\kappa_{12}} + \frac{|\Omega_c|^2}{\kappa_{32}}
\]

represents the sum of the photon fluxes at \(\omega_p\) and \(\omega_c\) divided by the concentration of the medium through which the waves propagate. Differentiate \(F\) with respect to \(z\) while holding \(t_r\) fixed and apply Eq.(4), we immediately reach the conclusion that \(F\) depends only on \(t_r\). This permits one to evaluate \(F\) by evaluating it at \(z = 0\), the entrance to the atomic vapor cell, e.g.

\[
F(z = 0, t_r) = \frac{|\Omega_c(0, t_r)|^2}{\kappa_{32}} + \frac{|\Omega_p(0, t_r)|^2}{\kappa_{12}}.
\]

Therefore, whenever \(F(z, t_r)\) occurs it can be replaced by \(F(0, t_r)\), as determined in Eq.(6)[6]. The lack of dependence of this quantity on \(z\) when the full set of equations is solved numerically is an important test for the validity of the adiabatic approximation.

Let us consider, for simplicity, the case where \(\kappa_{12} = \kappa_{32}\). Since \(\sqrt{|\Omega_c|^2 + |\Omega_p|^2}\) does not depend on \(z\) when \(t_r\) is held fixed, Eq.(4) can be recast into

\[
\frac{\partial}{\partial z} \left( \frac{\Omega_{p(c)}^* \tau}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}} \right) = -\frac{\kappa_{12(32)} \tau}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}} \frac{\partial}{\partial t/\tau} \left( \frac{\Omega_{p(c)}^* \tau}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}} \right).
\]

Define

\[
\begin{align*}
W_p &= \frac{\Omega_p^* \tau}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}}, \\
W_c &= \frac{\Omega_c^* \tau}{\sqrt{|\Omega_c|^2 + |\Omega_p|^2}}, \\
v(t_r) &= \int_{-\infty}^{t_r/\tau} (|\Omega_c(0, t_r')|^2 + |\Omega_p(0, t_r')|^2) d \left( \frac{t_r'}{\tau} \right), \quad u(z) = \int_0^z \kappa_{12} \tau dz',
\end{align*}
\]

(8a)
Equation (7) now becomes
\[ \frac{\partial W_p}{\partial u} + \frac{\partial W_p}{\partial \nu} = 0, \quad (9a) \]
\[ \frac{\partial W_c}{\partial u} + \frac{\partial W_c}{\partial \nu} = 0, \quad (9b) \]
where general “travelling wave” type solutions are immediately obtained as
\[ W_p = F_p(v - u), \quad (10a) \]
\[ W_c = F_c(v - u). \quad (10b) \]
The functions \(F_p\) and \(F_c\) are easily determined by evaluating at \(z = 0\) (so \(u = 0\)). When the second coupling laser pulse is injected into the medium after a time delay, the predictions about the revival of the “probe” pulse are all contained in this solution. The tabulation of \(F_p\) remains the same as long as only a second coupling pulse is sent into the medium at a later time. Notice that if \(|\Omega_p(z, t_r)|^2 << |\Omega_c(z, t_r)|^2\), the population in state \(|3\rangle\) is always small and the coupling laser propagates as in vacuum. In this limit, with the replacement of \(\Omega_c(z, t_r) = \Omega_c(0, t_r)\), Eq. (10) can still be used to determine \(\Omega_p(z, t_r)\) even for \(\kappa_{12} \neq \kappa_{32}\). This is the essence of the approximations applied in the works by Ref.[4].

We now examine the wave propagation and “probe revival” with the field profiles [7]
\[ \Omega_p(0, t) = \Omega_{p0}e^{-\left(\frac{\tau}{\gamma_1}\right)^2}, \quad (11a) \]
\[ \Omega_c(0, t) = \Omega_{c0} \left( e^{-0.2\left(\frac{t}{\tau}\right)^2} + R e^{-0.2\left(\frac{t-x_0}{\tau}\right)^2} \right). \quad (11b) \]
Here, \(\Omega_{p0}\) and \(\Omega_{c0}\) are real constants characterizing the peak amplitudes of the two half-Rabi frequencies before the pulses enter the resonant medium. \(R\) is the ratio of the Rabi frequency at which the coupling laser recurs to its initial amplitude, and \(x_0 = t_\nu/\tau\) is the value of \(t_\nu/\tau\) at which the peak of the coupling laser recurs.

We first consider the case where \(R = 0\) and the group velocity of the probe pulse is sufficiently small so that the coupling laser dies away before the probe pulse can propagate through the cell. When the coupling laser begins to die out, the intensity of the two lasers becomes proportional to each other, therefore the relation
\[ v(t_\nu) \simeq |\Omega_{c0}\tau|^2 \sqrt{5\pi/2} + |\Omega_{p0}\tau|^2 \sqrt{\pi/2} \quad (12) \]
is appropriate. Also, \(|\Omega_p(0, t_r)|^2\) has long been very small compared with \(|\Omega_c(0, t_r)|^2\). Thus, for \(t_\nu/\tau > 3.0\) we have, as a very good approximation
\[ \Omega^*_p(z, t_r) \simeq \Omega(0, t_r) F_p(|\Omega_{c0}\tau|^2 \sqrt{5\pi/2} + |\Omega_{p0}\tau|^2 \sqrt{\pi/2} - \kappa_{12} z). \quad (13) \]
At such a late time during the pulse, the argument of \(F_p\) depends only on the \(z\) coordinate so that the time dependence of \(\Omega^*_p\) is obviously exactly the same as that of \(\Omega(0, t_r)\). When \(|\Omega_c(0, t_r)|\) is several times larger than \(|\Omega_p(0, t_r)|\), as always is at such late times at \(z = 0\), this means that \(\Omega^*_p(z, t_r)\) has the same time dependence as \(\Omega^*_p(0, t_r)\) at such late times. Note that at late times \(\Omega^*_p(z, t_r)/\Omega(z, t_r) = A_3(z, t_r)\) is also independent of retarded time, indicating that as \(\Omega_p(z, t_r)\) approaches zero, the ratio of populations in \(|1\rangle\) and \(|3\rangle\) stays fixed. It is precisely this way of having two fields to go to zero with fixed ratio that preserves the the adiabatic approximation during the process. There must be a coherent superposition of states in \(|1\rangle\) and \(|3\rangle\), such that \(A_3(z, t_r)/A_1(z, t_r) = \Omega^*_p(z, t_r)/\Omega^*_c(z, t_r)\). A similar relation holds when the second coupling laser pulse starts to build up. In order for the behavior to remain adiabatic, the probe laser half-Rabi frequency must build up proportional to the coupling laser, with the ratio of the two being the local ratio of \(A_3/A_1\). This persists until the depletion of the population of \(|3\rangle\) forces the ratio to decrease at later \(t_\nu\). By choosing \(z\) such that \(2\kappa_{12} z = |\Omega_{p0}|^2 \sqrt{5\pi/2} + \sqrt{\pi/2} |\Omega_{c0}|^2\), we thus have the same argument of \(F_p\) at \(z = 0\) and \(t_\nu = 0\), resulting \(F_p = \Omega_{p0}/\sqrt{|\Omega_{p0}|^2 + |\Omega_{c0}|^2}\). This is the largest value that \(F_p\) takes on, and at this depth into the medium the value of \(|A_3(z, t_r)|\) matches its largest value at \(z = 0\). This population persists at large \(t_\nu\) until very slow collisional effects destroy the coherence left behind in states \(|1\rangle\) and \(|3\rangle\). It is this long persistence of a coherent mix of populations in states \(|1\rangle\) and \(|3\rangle\) that leads to the regeneration of an optical field when a delayed second coupling pulse is injected into the medium. Of course, the number of atoms left in \(|3\rangle\) is (within the adiabatic approximation) equal to the number of photons in the original probe pulse.

We now investigate the case where \(\Omega_{c0}\tau = 20\), \(\Omega_{p0}\tau = 5\), \(\gamma_2\tau = 0\), \(\kappa_{12}\tau = \kappa_{32}\tau = 2000 cm^{-1}\), and \(R = 4\). Based on our adiabatic theory, we predict a maximum population of \(|A_3| = 1/\sqrt{17}\) at a depth of 2.86 cm into the medium.
This is indicated by the line of constant color in a contour plot of $|A_3(z, t_r)|$ (see Fig. 2a) leading from $t_r = 0$ and $z = 0$ out to the horizontal path at $z \simeq 2.86$ cm. A corresponding surface plot for $\Omega_p \tau$ as functions of $t_r/\tau$ at $z = 3$ cm is given in Fig. 3a which shows the long asymmetric tail on $\Omega_p$, as described above. In the region between $2.5 \leq t_r/\tau \leq 5$ the ratio of the two half-Rabi frequencies is close to constant, averaging around 0.24. This ratio is also close to the adiabatic approximation for $A_3$ since $|\Omega_p|^2 << |\Omega_c|^2$. In the same region, however, there is no optical field left, indicating no photons are “stored” or “stopped”. Every probe photon is converted to the excitation of the state $|3 >$. We now come to an important point, e.g. to show that the regenerated field has characteristics that are very different from that of the original probe pulse. That is, the regenerated field is not a faithful copy of the original probe pulse unless careful restrictions are placed on characteristics of the second coupling laser pulse. Recall that $F_p$ was determined from the functional dependence of the probe and coupling laser pulses at $z = 0$, in particular, $W_p(v(t)) = -A_3(0, t)$, we thus have, during the recurring coupling laser pulse

$$v(t_r) = S + \frac{R^2}{2} |\Omega_{00}\tau|^2 \sqrt{5\pi/2} \left(1 + \text{erf} \left( \sqrt{\frac{5\pi}{2}} \frac{(t_r - t_d)}{\tau} \right) \right),$$  \hspace{1cm} (14)

where $S = |\Omega_{00}\tau|^2 \sqrt{5\pi/2} + |\Omega_{00}\tau|^2 \sqrt{\pi/2}$. Equation (14) is of central importance in the following analysis on the characteristics of the regenerated field. We first note that $R$ must be large enough so that $v(t_r) - \kappa_{12}\tau z_m > 0$. That is, $R$ must be large enough so that the group velocity of the regenerated photons are large enough to exit the cell before the laser induced transparency ends. Three time markers, therefore, are important for describing the regenerated field when it reaches the end of cell where $z = z_m$. The first marker is the earliest time at which $F_p(0) = 0$. This time marker is determined by $v(t_{r1}) - \kappa_{12}\tau z_m = 0$. At a later time the value of $F_p(S/2)$ will be equal to $-A_3(0, 0)$. This second marker represents the time when $F_p$, for the given set of parameters, reaches it maximum value at $z_m$, and it is determined by $v(t_{rm}) - \kappa_{12}\tau z_m = S/2$. Finally, the third marker is the time at which the regenerated pulse completes its exit from the cell, i.e. the time at which $F_p(S) = 0$ and $v(t_{r2}) - \kappa_{12}\tau z_m = S$. When these relations are used in Eq.(14), we immediately obtain

$$\Omega_{p0}^*(z_m, t_{rm}) = R \Omega_{00} e^{-(t_{rm} - t_d)^2/(5\tau^2)} \frac{\Omega_{00}\tau}{\sqrt{|\Omega_{00}\tau|^2 + |\Omega_{00}\tau|^2}}.$$

(15)

Notice that if $R$ is chosen to make the argument of $F_p$ at the time $t_r = t_d$ exactly the same as its value at $z = 0$ and $t_r = t = 0$, we then have

$$|\Omega_{00}\tau|^2 \sqrt{\pi/8} + |\Omega_{00}\tau|^2 \sqrt{5\pi/8}(1 + R^2) = \kappa_{12}\tau z_m,$$

(16)

and

$$\Omega_{p0}^*(z_m, t_d) = R \omega_{00} \frac{\Omega_{00}\tau}{\sqrt{|\Omega_{00}\tau|^2 + |\Omega_{00}\tau|^2}}.$$  \hspace{1cm} (17)

By determining the value of $t_r$ such that $v(t_r) - \kappa_{12}\tau z_m = 0$ or $S$, a range of time over which the regenerated pulse rises from zero and returns to zero at the exit of the cell may be obtained. We thus estimate the FWHM pulse length, in the unit of the original probe pulse length $\tau$, to be

$$\Delta_{1/2} = \frac{\sqrt{5\pi/2}}{R^2} \left(1 + \frac{1}{\sqrt{5}} \frac{|\Omega_{00}|^2}{|\Omega_{00}|^2} \right).$$  \hspace{1cm} (18)

The key results shown in Eqs.(14-18) indicate that the regenerated field is not a replica of the original probe pulse. Therefore, the concept of “storage of light”, in the reported experimental studies [1,2], is not accurate. Furthermore, Eq.(15) clearly indicates that the instantaneous phase of the regenerated field is closely related to that of the second coupling pulse, in additional to the phase of the atomic coherence $\rho_{32}$, therefore, cannot be just that of the original probe pulse alone. In fact, if the probe laser has only penetrated a small fraction of the thickness of the vapor cell when the coupling laser pulse has passed by, then $R$ will turn out to be much larger than unity. This means that the width of the regenerated field is generally much smaller than the width of the initial probe pulse. Correspondingly, the bandwidth of the former will be much larger as can be seen from Fig. 3a. Both Fig. 2a and 3a show that in “reviving the probe pulse” every regenerated photon comes at the expense of flipping population from state $|3 >$ to $|1 >$. When the population of $|3 >$ has been exhausted, there can be no further photon generated.
Extensive numerical calculations carried out by simultaneously solving Eqs. (1-2) have shown very good agreement with the adiabatic calculation described above. In Figs. 2b and 3b we show a contour plot of $|A_3(z, t)|$ and a surface plot of $\Omega_c^*$ for the same parameters given in Figs. 2a and 3a. From Fig. 3b, we notice that there is no probe field between $t_2/\tau = 2$ and the arrival time of the second coupling pulse, i.e. there is no photon “left” or “stored” in the medium. All probe photons have been absorbed in producing the coherent excitation of the state $|3\rangle$ as can be seen from Fig. 2b. The case with non-vanishing one-photon detuning in our adiabatic theory also produces a result that is in very good agreement with numerical calculations. Numerical calculations have also been vigorously tested by making use of the fact that with $\gamma_2 \tau = 0$, the sum of the squares of the three state amplitudes should be unity. In all numerical examples described in this paper the condition of unity was preserved through at least seven significant figures if $\gamma_2 \tau = 0$.

We have shown analytically and numerically that when $0 \leq \delta < |\Omega_c|$, a coherent optical field with the frequency very close to that of the original probe can be regenerated by re-injecting a coupling pulse. Detailed analysis on the conditions and characteristics of this regenerated field, including the estimate of the pulse width, has shown that it is not the replica of the original probe pulse. We, therefore, caution the use of the concept of “storage of light” in the context of recently reported experiments, since there has no evidence that this is indeed the case [9]. In fact, one can couple the states $|1\rangle$ and $|3\rangle$ with a coherent magnetic pulse to create the coherence required. Under this circumstance, if an optical pulse that couples the states $|2\rangle$ and $|3\rangle$, commonly referred to as the coupling pulse, is injected into the system, an optical pulse that couples the states $|2\rangle$ and $|1\rangle$, commonly referred to as the probe pulse, will be generated. The latter is most certainly not the replica of the magnetic pulse used to create the coherence.

References

1. C. Liu et al., Nature (London) 409, 490 (2001).
2. D.F. Phillips et al., Phys. Rev. Lett. 86, 783 (2001).
3. J. Oreg, F.T. Hioe and J.H. Eberly, Phys. Rev. A29, 690 (1984); J.R. Kuklinski, U. Gaubatz, F.T. Hioe, and K. Bergmann, Phys. Rev. A 40 R6749 (1989). See also, R. Grobe and J.H. Eberly, Laser Physics 5, 542 (1995); J.H. Eberly, A. Rahman, and R. Grobe, Laser Physics 6, 69 (1996); J.R. Cseszegi and R. Grobe, Phys. Rev. Lett. 79, 3162 (1997).
4. M. Fleischhauer, Opt. Express 4, 107 (1999); M. Fleischhauer, S.F. Yelin, and M.D. Lukin, Opt. Communi. 179, 395 (1999); M. Fleischhauer and M.D. Lukin, Phys. Rev. Lett. 84, 5094 (2000).
5. E.A. Cornell, Nature (London) 409, 461 (2001).
6. Equation (7) can also be derived exactly from Eq.(3) for the case of slowly varying $|A_2|^2$ and not too large $\gamma_2$.
7. These field profiles are reasonable description of the experiments reported. In fact, as long as the adiabatic conditions are observed different profiles will lead to the similar conclusions. In addition, numerical simulations have suggested that even in the region where the adiabatic conditions are in valid, a similar conclusion can still be reached, provided certain non-adiabatic restriction are observed.
8. Figure 3a shows that all probe photons are converted to the excitation of the state $|3\rangle$. If one can regenerate, from the excitation of the state $|3\rangle$, photons with optical properties identical to that of the original probe field, then it would be correct to claim the revival of the probe field. As we will show later, however, this is not the case in recent reported experiments.
9. We emphasize that the purpose of the present study is to show that photons from the probe pulse are neither “stopped” or “stored” in the recently reported experiments [1,2]. Our intension is to stimulate further studies for truly storage of the quantum information.

Figure captions

Figure 1. Energy level diagram showing relevant laser excitations.

Figure 2. Contour plot of $A_3(z, t)$. a. Adiabatic solution. b. Full numerical solution. Parameters used: $\Omega_p \tau = 5$, $\Omega_c \tau = 20$, $\gamma_2 \tau = 0$, $\kappa_{12} \tau = \kappa_{32} \tau = 200 cm^{-1}$, $R = 4$, $t_d/\tau = 11$. 
Figure 3. Surface plot of $\Omega_p(z, t)$. a. Adiabatic solution. b. Full numerical solution. Parameters used: $\Omega_p \tau = 5$, $\Omega_c \tau = 20$, $\gamma_2 \tau = 0$, $\kappa_{12} \tau = \kappa_{32} \tau = 200 \text{cm}^{-1}$, $R = 4$, $t_d/\tau = 11$. 
This figure "f1.jpg" is available in "jpg" format from:

http://arxiv.org/ps/quant-ph/0203075v2
This figure "f2.jpg" is available in "jpg" format from:

http://arxiv.org/ps/quant-ph/0203075v2
This figure "f3.jpg" is available in "jpg" format from:

http://arxiv.org/ps/quant-ph/0203075v2