Rotational dynamics of piston diesel engine crankshaft with deactivated cylinders at idle

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Abstract. Diesel engines have been diagnosed on the basis of a dynamic method. To improve the informativity of the determination of general technical condition of each of the cylinders, it is proposed to apply a well-known test action i.e. termination of operation. In this case, analysis should be given to both the contribution of the cylinder in the operation and the acceleration imparted to the crankshaft by the cylinder in the absence of fuel supply. The paper presents a mathematical model that takes into account the relationship of the dynamics of rotation of the crankshaft with the pressure of gases in the cylinder, including when it is deactivated. The analysis of functional dependences has allowed to receive coupling equations between parameters of tightness of the cylinder and angular crankshaft acceleration. Analytical and experimental study of the crankshaft rotation dynamics with deactivated cylinders at idle speed allowed to conclude that parameters of the crankshaft rotation irregularity allow assessing the engine cylinders’ technical state both in the run mode and in the idle mode. The diagnostic parameter allows determining the leak tightness of the over-piston space at idle speed. It equals air pressure differential in the cylinder at 50 degrees before TDC and at 50 degrees after TDC over the pressure at 50 degrees before TDC. It was installed, that both extremas of angular accelerations and their average values during compression and expansion can be used to diagnose the technical condition of cylinders.

1. Introduction
The reliability of a piston diesel engine as well as its technical and economic characteristics fully depend on the technical condition of its cylinders. Currently, there is a wide range of methods and techniques that allow detecting their technical condition [1-4]. However, the issue of developing new diagnostic methods and improving the existing ones in order to increase information value and to decrease labour intensity remains relevant.

One of the most common techniques of diagnosing is the idle mode with the minimal crankshaft speed. In this case the whole indicated power produced in cylinders is spent only on overcoming the mechanical losses. The mode itself remains quite stable for definite types of diesel and it is standard for conducting some diagnostic tests. It is often used for detecting ignitions missed by the self-diagnostics system.

The identification of the dead cylinder is not usually complicated and it may be done, for example, with the help of the widely used activation test of “deactivating cylinders” (or power balance) in the
manual or automated modes, as well as of monitoring the cylinders balancing data, etc. However, it is often difficult or even impossible to define the reason of the service quality worsening. In the case of simultaneous occurrence of various malfunctions, the technical condition becomes indeterminate which causes the necessity of further testing [4-5]. The decreased number of diagnostic manipulations aimed at improving diagnosis accuracy allows lessening its labour intensity, though, it demands the scientific evidence of diagnostic and mode characteristics of diesel; within this context the research is topical and relevant.

2. Mathematical modelling

The uniformity of operation of diesel cylinders at idle speed does not always testify their correct operation; for example, a slight decrease in cylinder power caused by leak tightness reduction can be compensated by the increased fuel delivery. To eliminate the uncertainty in stating technical condition, the authors suggest an original testing manipulation – to stop operating processes in the checked cylinder. In this case two aspects are relevant and important – qualitative response to the testing manipulation and its quantitative values. Intracycle changes of speed and crankshaft acceleration are chosen as diagnostic parameters for this purpose.

The balance of torques is described by the equation of dynamical crankshaft rotation [6-9]:

$$\frac{d\omega_c}{dt} = \frac{1}{I_a} \left( T_i - T_f - T_l \right) = \frac{1}{I_a} \left( T_e - T_l \right)$$

(1)

where $I_a$ – mass moment of inertia for a crankshaft; $\frac{d\omega_c}{dt}$ – angular acceleration of a crankshaft, rad/s²; $\omega_c$ – angular speed of a crankshaft, rad/s; $T_i , T_f , T_l$ – respectively, indicated moment, mechanical losses moment and external (load) moment of an engine, N·m.

The engine torque is defined as [8-10]:

$$T_e = \sum_{i=1}^{7} \left( p - p_r - p_{mn} \right) \cdot F_p \cdot r \cdot \dot{r}(\varphi) - r^2 \omega^2 \sum_{j=1}^{9} m_{ij} \ddot{\varphi}(\varphi) \dot{\varphi}(\varphi)$$

(2)

where $m_{ij}$ – masses of a piston and a connecting rod converted to the piston pin axis that produce reciprocating movement along the cylinder axis, kg; $z$ – number of cylinders; $p_r$ – gas pressure in a crankcase, Pa; $p_{mn}$ – mechanical losses pressure, Pa.

The kinematic functions $\pi(\varphi), \dot{\pi}(\varphi)$ and $\sigma(\varphi)$ can be expressed as an approximate answer [9]:

$$\pi(\varphi) = 1 - \cos(\varphi + \Delta \varphi) + \frac{\lambda}{2} \sin^2(\varphi + \Delta \varphi)$$

(3)

$$\dot{\pi}(\varphi) = \sin(\varphi + \Delta \varphi) + \frac{\lambda}{2} \sin 2(\varphi + \Delta \varphi)$$

$$\sigma(\varphi) = \cos(\varphi + \Delta \varphi) + \lambda \cos 2(\varphi + \Delta \varphi)$$

(4)

where $\lambda$ – invariable of the crank-and-rod mechanism; $\Delta \varphi$ – angle of explosions alteration in cylinders.

To determine the mechanical losses pressure, the following formula is used [10]:

$$p_{mn} = a + b |v_p| = a + b r \omega \left| \sin(\varphi + \Delta \varphi) + \frac{\lambda}{2} \sin 2(\varphi + \Delta \varphi) \right|$$

(5)

where $v_p$ – piston velocity, m/s; $a, b$ – coefficients.

The value of pressure in the engine cylinder at the start time is regarded as constant [7].

The external (load) moment taken without the bench-test loading is considered as $T_i=0$; taking this into account the equation of dynamic mode can be presented in general as [7,10]:

$$J_e \frac{d\omega_c}{dt} = z_p T_i - T_f$$

(6)

where $z_p$ – number of operating cylinders.

Both the improper balance of the torque depending on the rotation angle of the crankshaft and the design characteristics of the crank-and-rod mechanism cause the fact that instantaneous angular speed
and acceleration of the crankshaft are not constant; they change according to the time and angle of the diesel engine crankshaft. The increase of angular speed is caused by the operating cycle in the cylinder, whereas its reduction occurs due to losses conditioned by compression, friction, auxiliary drive [7-10].

The analysis of dynamics of changes in angular speed and acceleration of the crankshaft according to its rotation angle is performed based on the assumption of the infinite torsional rigidity of the crankshaft [10].

The reduced moment of inertia of the crank-and-rod mechanism moving parts is presented as follows [6,8,9]:

\[ J_e = J_M + \tau^2 \sum_{i=1}^{s} \left[ \sum_{j=1}^{n} m_p \dot{\pi}^2(\varphi) + \sum_{k=1}^{p} m_c \left( k_{rot} + k_{rec} \dot{\pi}^2(\varphi) \right) \right] \]  

(7)

where \( J_M \) – moment of inertia of the crank with the flywheel; \( m_p \) – piston mass; \( m_c \) – connecting rod mass; \( k_{rot} \) and \( k_{rec} \) – parts of connecting rod mass referring to rotating and reciprocating moving parts, respectively; \( \dot{\pi}(\varphi) = \sin(\varphi + \Delta \varphi) + \frac{2}{\lambda} \sin 2(\varphi + \Delta \varphi) \) – dynamic-response kinematic function of the crank-and-rod mechanism (approximately defined).

The parts of rod mass referring to rotating and reciprocating moving parts are calculated as follows [11]:

\[ k_{rot} = \frac{J_c}{m_{cr} \tau} \left( \sin(\varphi + \Delta \varphi) + \frac{2}{\lambda} \sin 2(\varphi + \Delta \varphi) \right) \]  

(8)

\[ k_{rec} = \left( \frac{\nu_c}{\nu_p} \right)^2 \]  

(9)

where \( J_c \) – moment of connecting rod inertia; \( m_p \) – piston mass; \( \nu_c \) – connecting rod velocity; \( \nu_p \) – piston velocity.

The current capacity of the engine cylinder is defined as follows [9]:

\[ V = V_c + F_p r \pi(\varphi) \]  

(10)

where \( V_c \) – combustion chamber volume; \( F_p \) – piston surface area; \( r \) – crank radius; \( \pi(\varphi) \) – kinematic function of the crank-and-rod mechanism depending on the rotation angle of the crankshaft.

Hereafter, the volume of cylinder change can be presented as [8,9]:

\[ \frac{dV}{dt} = \omega F_p r \dot{\pi}(\varphi) \]  

(11)

where \( \dot{\pi}(\varphi) \) – kinematic function of the crank-and-rod mechanism depending on the rotation angle of the crankshaft.

When the engine is idling the angular speed of the crankshaft (as well as the piston) is 3-4 times higher than the speed in the starter mode. Besides, the maximum compression pressure is achieved in the area of UDC, though the process of air leakage through leakiness of the cylinder-piston group continues. This means the sensitiveness of the diagnostic parameter of compression to the cylinder leakiness reduces. That is the reason why the authors suggested one more parameter of evaluating cylinder leak tightness – conventional cylinder leakage logically following the equations of gas, heat and mass exchange.

Mass charge in the deactivated cylinder depends on the current cylinder volume and air contained in it at the moment of intake valves closure as well as on the air density. According to the perfect gas equation [7], pressure inside the cylinder changes in a proportionate manner as well.

To calculate the mass charge of the working medium \( m_a \) the perfect gas equation is applied:

\[ m_a = \frac{p_s V_{c, in}}{T_s R_{cm}} \]  

(12)

where \( V_{c, in} \) – cylinder volume at the moment of intake valves closure, m³; \( R_{cm} \) – gaseous invariable of the working medium (J/(kg K)).
Thus, if the cylinder volumes are equal at the definite moment at the start of compression before TDC and they are symmetrically equal depending on the crankshaft rotational angle after TDC, the differential pressure conditions the loss of air mass charge or conventional cylinder leakage. The calculation of this parameter can be performed as follows:

\[
\delta = \frac{(p_{-\varphi} - p_{+\varphi})}{p_{-\varphi}} \cdot 100\% \tag{13}
\]

where \(p_{-\varphi}\) – air pressure in the cylinder after input valve closure at the compression stroke in the crankshaft position – \(\varphi\) before TDC; \(p_{+\varphi}\) – air pressure in the cylinder after input valve closure at the expansion stroke in the crankshaft position + \(\varphi\) before TDC.

3. Materials and methods

| No. | Parameter | Value |
|-----|-----------|-------|
| 1   | Number of cylinders / placement | 4 / in-line |
| 2   | Bore / stroke, [mm] | 96 / 92 |
| 3   | Compression ratio | 21.8 |
| 4   | Displacement \(V_l\), [sm³] | 2663 |

Cylinder leakiness was simulated by changing the rate of the decompressor valve opening with the simultaneous recording of signals from the sensor of angle marks, current, voltage and actual pressure inside the cylinder.

**Figure 1.** Measuring system loop diagram: 1 – diesel engine; 2 – starting device; 3 – sensor of angle marks; 4 – accumulator battery; 5 – clamp meter; 6 – voltage meter; 7 – A/D converter; 8 – heating plug; 9 – pressure sensor; 10 – decompression valve; 11 – pulse former; 12 – PC.

Binding to the rotation angle of the crankshaft was done with the help of processing data from the angle marks sensor, that is, from the inductive sensor positioned opposite the flywheel ring gear [4,12,13]. In this case software processing of oscilloscope charts was used (Figure 2).
Software processing of signal from the inductive sensor involved follows the given algorithm.
1. Search for average (zero) signal value in the data set.
2. Finding moments of intersection by the signal with average (zero) value through the channel at the back edge.
3. Determining angular $\Delta \phi$ and time $\Delta t$ intervals, where $\Delta \phi = \text{const}$, $\Delta t = \text{var}$.

The angle $\Delta \phi$ was calculated as follows:

$$\Delta \phi = \frac{360}{z}$$  \hspace{1cm} (14)

where $z$ – number of marks per turn.

At the next stage signals were differentiated. The first derivative means the angle speed of the crankshaft, whereas the second one – the angle acceleration.

$$\omega = \frac{d \phi}{dt}, \quad \varepsilon = \frac{d \omega}{dt}$$  \hspace{1cm} (15)

The described material is presented in Figure 3.

**Figure 2.** Processing of signal from the inductive sensors of angle marks.

**Figure 3.** Chart of developing graphs of angular speed and acceleration of the crankshaft: 1 – angular speed sensor (encoder); 2 – signal from angular speed sensors; 3 – first-order filter of differentiation; 4 – linear smoothing filter; 5 – second-order filter of differentiation; 6 – interface of data acquisition software.
Data set of acquired values of angular speed and angular acceleration was filtered by the moving-
average method [14,15]. For example, the angular speed values were filtered as follows:

\[ \omega_i = \frac{1}{3} \sum_{k=1}^{3} \omega_{i,k} \]  

(16)

where \( i \) – angular interval of the crankshaft rotation; \( k \) – number of moving-average points.

The further research was aimed at searching for a functional link between the cylinder acceleration
(average and maximal) in phase of expansion and compression of air in the deactivated cylinder and the
diagnostic parameter directly related to the cylinder leak tightness. The most common and generally
accepted parameter is compression, that is, the maximal pressure at the end of the compression stroke
when the engine is motored by the starting device. Moreover, the authors studied the relationship
between the parameters of irregularity of the crankshaft rotation speed and the suggested parameter –
conventional cylinder leakage.

4. Results and discussion

Depending on the value of cylinder leakage calculated in percentage terms according to the methodology
described above and on the basis of calculating gas parameters by the equations of dynamics, gas, heat
and mass exchange the pressure changes inside the cylinder of TD 27 diesel engine were determined
(Figure 4).

![Figure 4. Graph of pressure change inside TD 27 diesel engine cylinder with deactivated cylinder
(calculation) 1 – Cylinder leakage 10%; 2 – Cylinder leakage 20%;3 – Cylinder leakage 30%.
](image)

Simulation of leakages in the deactivated cylinders allowed stating that the cylinder leak tightness
influences both crankshaft angular speed irregularity (Figure 5) and its angular acceleration irregularity
(Figure 6).

![Figure 5. Graph of crankshaft angular speed changes depending on its rotation angle when simulating
leakage in deactivated cylinder (first one).](image)
Figure 6. Graph of crankshaft angular accelerations changes depending on its rotation angle when simulating leakage in deactivated cylinder (the case of the first one).

When simulating leakages in the other deactivated cylinders, the pattern of angular speed and acceleration changes is the same which testifies the functional link between the parameters of rotation irregularity and cylinder leak tightness.

Figures 7 and 8 present the comparison of angular accelerations and angular speed depending on the crankshaft rotation angle for TD 27 engine; the values were calculated with the help of GT-SUITE software and experimentally. The following parameters are chosen for the arithmetic model: cylinder bore of 96 mm; piston bore and stroke of 96/92 mm; rod mass is reduced to masses related to a crank and a piston pin; the cycle delivery is equal for all cylinders; rod length of 157 mm; piston mass of 1.08 kg; rod mass of 1.2 kg.

Figure 7. Graph of angular speed change depending on crankshaft rotation angle in TD 27 engine at idle speed: calculation and experiment (forth cylinder is deactivated).
Thus, the given process was studied in detail. Figures 9 and 10 present the results of the experiments carried out according to the methodology described above; they included comparison of air pressure change caused by leakage simulation inside the checked cylinder and angular speed (Figure 9), comparison of air pressure change caused by leakage simulation and angular acceleration (Figure 10) of the engine crankshaft in the intervals relevant to the strokes of compression and expansion [16].

**Figure 8.** Graph of angular acceleration change depending on crankshaft rotation angle in *TD 27* engine at idle speed: calculation and experiment (forth cylinder is deactivated).

**Figure 9.** Graph of pressure change in the combustion chamber of the checked deactivated cylinder (1) and angular speed irregularity (2) at idle speed (experiment).
Figure 10. Graph of pressure change in the combustion chamber of the checked deactivated cylinder (1) and angular acceleration (2) at idle speed (experiment).

Comparison of the results presented in Figures 7 and 8 shows that the relations of the angular acceleration are more obvious and reflect the qualitative vision of the influence of pressure inside the cylinder on the crankshaft rotation irregularity more clearly. Hereafter, during the diagnostics the angular accelerations (both maximal and average) are prior for the intervals related to the strokes of compression and expansion (Figure 11).

Figure 11. Graph of dependence of maximal (1) and average (2) angular acceleration of the crankshaft for the intervals of compression (on the left) and expansion (on the right) during compression in the deactivated cylinder.

The analysis of the studied relations allows concluding that there is a functional link between compression and angular accelerations (both maximal and average) in the angular intervals coinciding with the strokes of compression and expansion of the checked cylinder and it is quite evident. The dependence of angular accelerations on compression becomes the most visible during compression, but the sensitiveness is higher during expansion.

Moreover, serious dispersion of indicators must be pointed out especially for the forth cylinder. In the authors’ opinion, it is conditioned by the specific character of the compression-expansion processes...
in the angular intervals coinciding with the strokes of compression and expansion at idle speed. That is why it is necessary to prove the significance of the crankshaft rotation angles for calculating conventional cylinder leakage.

The calculation and analysis of test results (Figures 4, 7 and 8) show that the best sensitiveness to cylinder leakages is achieved when comparing the pressure in the following crankshaft positions: approximately at 50 degrees before TDC and 50 degrees after TDC.

![Graph of dependence of maximal (1) and average (2) angular acceleration of the crankshaft for the intervals of compression (on the left) and expansion (on the right) during conventional leakage of the deactivated cylinder.](image)

**Figure 12.** Graph of dependence of maximal (1) and average (2) angular acceleration of the crankshaft for the intervals of compression (on the left) and expansion (on the right) during conventional leakage of the deactivated cylinder.

The given mathematical relations between the first-order parameters (compression and conventional leakage) and angular acceleration of the crankshaft (average and maximal) during compression and expansion (Figure 9) are presented in Table 2.

**Table 2.** Summary table of coupling equations of the diagnostic parameters of four-cylinder diesel engine

| No. | Diagnostic parameter | The parameter characterizing the leak tightness of the cylinder | Coupling equation | R² | Coupling equation | R² |
|-----|----------------------|-------------------------------------------------------------|------------------|----|------------------|----|
| 1   | Average compression angular crankshaft acceleration [rad/s²] | ε = -36.70p - 134.3 | 0.85 | ε = 1.121δ - 281.3 | 0.8 |
|     |                      | ε = -7.750p² + 0.167p - 171.8 | 0.86 |                |     |
| 2   | Average expansion angular crankshaft acceleration [rad/s²] | ε = 75.45p - 163.3 | 0.88 | ε = 0.014δ² - 3.906δ + 171 | 0.95 |
|     |                      | ε = -2.461δ + 145.3 | 0.93 |                |     |
| 3   | Maximal expansion angular crankshaft acceleration [rad/s²] | ε = 125.1p - 57.53 | 0.81 | ε = -4.159δ + 457.6 | 0.9 |
|     |                      | ε = 59.67δ⁰.⁵³⁴ | 0.88 | ε = 535.7e⁻⁰.⁰¹⁶ | 0.96 |
| 4   | Minimal compression angular crankshaft acceleration [rad/s²] | ε = -72.73p - 332.2 | 0.96 | ε = 2.276δ - 625.8 | 0.94 |

The analysis of the data presented in Table 2 allows concluding that the functional link between conventional cylinder leakage and angular accelerations of the crankshaft is a closer one. The most obvious link is seen in the angular interval of the crankshaft rotation coinciding with the expansion stroke in the checked deactivated cylinder.

5. Conclusion
Analytical and experimental study of the crankshaft rotation dynamics with deactivated cylinders at idle speed led to the following conclusions.
1. Parameters of the crankshaft rotation irregularity allow assessing the engine cylinders’ technical state both in the run mode and in the idle mode.
2. The suggested diagnostic parameter – “conventional cylinder leakage” that equals air pressure differential in the cylinder at 50 degrees before TDC and at 50 degrees after TDC over the pressure at 50 degrees before TDC – allows determining the leak tightness of the over-piston space at idle speed more accurately.
3. The values of the engine crankshaft angular acceleration with the deactivated cylinder in relation to the rotation angles coinciding with air compression and expansion can be used as a diagnostic parameter helping to assess the leak tightness of the over-piston space on a timely basis.
4. Both extremums of angular accelerations and their average values during compression and expansion can be used to diagnose the technical condition of cylinders.

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