The $L_p$–$E_p$ relation is a well-known relation in gamma-ray bursts (GRBs). Its implication remains unclear. We propose to investigate the underlying mechanisms of this relation by considering the corresponding kinetic and dynamic mechanisms separately. In this way, one can tell how much the kinetic or dynamic mechanism contributes to the index of the relationship. Our analysis gives rise to several conclusions. (1) The index of the kinetic effect in the $L_p$–$E_p$ relation can simply be derived from the theory of special relativity, which is generally larger than 2, depending on the situation concerned. (2) The index of the dynamic effect in the relation can be deduced from observation once a model of jets is adopted. According to current GRB data, we find that the dynamic effect alone tends to give rise to an anti-correlation between $L_p$ and $E_p$; in terms of statistics, the dynamic effect is obviously smaller than the kinetic effect; in the situation of jets with moving discrete radio clouds that move directly toward the observer, the index of the dynamic effect is currently constrained within $(-1.6, -1)$, while in other situations of jets, the constraints are different; both internal and external shocks can account for the current data.

Key words: gamma-ray burst: general – gamma rays: general – gamma rays: stars – methods: analytical – methods: statistical – stars: kinematics and dynamics

1. INTRODUCTION

Gamma-ray bursts (GRBs) are among the most violent astrophysical events ever observed in the universe. The objects are detected mainly at gamma-ray bands (Klebesadel et al. 1973) and many of them are followed by afterglows which can be detected at X-ray or lower energy bands (see Costa et al. 1997; van Paradijs et al. 1997; Frail et al. 1997). Thanks to the observation of the afterglow, redshifts of many GRBs are available (see, Metzger et al. 1997; Bloom et al. 1998; Kulkarni et al. 1998; Djorgovski et al. 1998), favoring the previous proposal that they are of cosmological origin. The largest redshift of GRBs is found to be 9.4 to date (Cucchiara et al. 2011).

Statistically, GRBs can be divided into two classes, long-soft bursts and short-hard bursts (Kouveliotou et al. 1993; Fishman & Meegan 1995; Qin et al. 2000). It has been generally believed that many long-soft bursts are the event associated with the massive star collapsars while many short-hard bursts are produced in the event of binary neutron star or neutron star–black hole mergers (e.g., Eichler et al. 1989; MacFadyen & Woosley 1999; Paczynski 1986, 1998; Woosley 1993). The most important achievement after the successful launching of the Swift satellite is that the two-progenitor proposal for GRBs is favored by a large number of evidence. It was reported that short bursts were found in regions with lower star formation rates, and no evidence of supernovae to accompany them was detected (Barthelmy et al. 2005; Berger et al. 2005; Hjorth et al. 2005). Meanwhile, long bursts were found to have originated from star-forming regions in galaxies (Fruchter et al. 2006), and in some of these events, supernovae were detected accompanying the bursts (Hjorth et al. 2003; Stanek et al. 2003).

Several statistical relations between different parameters of GRBs have been established. Among them, two are very important since they might be associated with the origin of the explosion energy of GRBs or the mechanism of the GRB event. One is the $E_p$–$E_{\text{iso}}$ relation (Amati et al. 2002) and the other is the $L_p$–$E_p$ relation (Yonetoku et al. 2004). These relations were observed from long bursts, but as Tsutsui et al. (2012) reported, they hold for short bursts as well (generally, they are 1/10 and 1/100 dimmer than the ones of long GRBs, respectively). The two relations have obvious different behaviors when short and long GRBs are separately considered. It was found that while long bursts form a tight relation between $E_p$ and $E_{\text{iso}}$, short GRBs do not follow the same relation; in fact, short GRBs are likely to follow another comparable relation (Amati 2006, 2010; Ghirlanda et al. 2009; Tsutsui et al. 2012; Zhang et al. 2012a). Quite the contrary, long and short bursts were found to follow almost the same $L_p$–$E_p$ relation (Ghirlanda et al. 2009; Tsutsui et al. 2012; Zhang et al. 2012a, 2012b). This suggests that the two kinds of bursts might differ in explosion energy (which is likely to give rise to $E_{\text{iso}}$) which must be closely associated with their progenitors, though they might share the same mechanism of radiation (which might affect $L_p$ and $E_p$) such as internal or external shocks. Or, to plainly interpret, the fact that long and short GRBs do not follow the same $E_p$–$E_{\text{iso}}$ relation is likely due to their different progenitors; the fact that long and short GRBs do follow the same $L_p$–$E_p$ relation might be due to their radiation mechanisms being intrinsically the same.

The scenario that long and short bursts share almost the same radiation mechanism but correspond to different progenitors is a common consensus. It was favored by the recent investigation of time resolved behaviors of Fermi bursts (Ghirlanda et al. 2011a, 2011b).

For the radiation mechanism shared by both long and short GRBs, why and how does it give rise to a certain power-law relation between $L_p$ and $E_p$ as shown by current GRB data (see Yonetoku et al. 2004)? What parameters cause this relation? This motivates our analysis below.

In Section 2, we study the kinetic and dynamic effects in terms of statistics and discuss the implication of the dynamic index in some typical cases. In Section 3, we use current GRB data to provide constraints to the dynamic index. Three important issues are discussed in Sections 4–6, and conclusions are presented in Section 7.
2. KINETIC AND DYNAMIC EFFECTS IN TERMS OF STATISTICS

It is generally believed that radiation of GRBs is generated by internal or external shocks from the ejecta moving away from the central region of the burst with relativistic speeds (Rees & Meszaros 1992, 1994; Meszaros & Rees 1993, 1994; Katz 1994; Paczynski & Xu 1994; Sari et al. 1996). In this situation, the radiation observed must be highly influenced by the relativistic effect due to the very large speed of motion. In fact, there are two modes of analyzing the radiation of these objects. One is to directly consider the radiation in the observer rest frame (or, more precisely, in the cosmological rest frame), which we call mode A. The other is first to consider the radiation in the ejecta rest frame and then consider how this original radiation is changed by the motion of the ejecta with respect to the observer, which we call mode B. With mode A, one can display how the light curve as well as the spectrum of different radiation mechanisms evolve, where the kinetic and dynamic effects are not separately considered (see, e.g., Granot & Sari 2002). When applying mode A, one should know which particular mechanisms are involved. With mode B, one can clearly see how the kinetic effect works, regardless of the details of the radiation mechanisms. For example, with mode B, the kinetic effect can be revealed in detail even though one only knows a certain form of emission such as the Band function emission that the ejecta rest-frame radiation bears. This attempt succeeds in the investigation of the curvature effect, where characteristics of the kinetic effect arising from an expanding fireball shell can be well displayed without knowing the details of the ejecta rest-frame radiation (see, e.g., Qin et al. 2004; Qin & Lu 2005; Qin 2009). Here, we try to analyze the implication of the \( L_p - E_p \) relation with mode B.

2.1. The Role of the Kinetic Effect

Consider the radiation from an ejecta moving with a Lorentz factor \( \Gamma \). Let us assign

\[
D \equiv \frac{1}{\Gamma(1 - \beta \cos \theta)} \quad (1)
\]

to represent the Doppler effect factor, where \( \theta \) is the angle between the direction of motion of the ejecta and the line of sight. In the theory of special relativity, the observed specific intensity \( I_\nu \) (erg s\(^{-1}\) cm\(^{-2}\) str\(^{-1}\) Hz\(^{-1}\)) is related to the rest-frame specific intensity \( I_{\nu,0} \) by \( I_\nu = D^3 I_{\nu,0} \), indicating that the Doppler effect can boost the intensity to a much larger value. (Note that \( I_\nu / v^3 \) is Lorentz invariant, keeping the same form under the transformation between different inertial frames, and the relation \( I_\nu / v^3 = I_{\nu,0}/v_0^3 \) leads to \( I_\nu = D^3 I_{\nu,0} \) when the Doppler effect is applied.)

Generally, the observed peak luminosity of the radiation from the ejecta, \( L_p \), is proportional to the ejecta rest-frame peak luminosity, \( L_{p,0} \), and is a function of the Lorentz factor as well as the Doppler effect factor:

\[
L_p = L_{p,0}\Gamma^{\alpha_G} D^{\alpha_D} \quad (2)
\]

where the indexes \( \alpha_G \) and \( \alpha_D \) depend on the situation, e.g., the size of emitting material and the viewing angle.

Let the cosmological rest-frame \( v_F \) spectrum peak energy be \( E_{p,r} \) and the ejecta rest-frame peak energy be \( E_{p,0} \). According to the Doppler effect we get

\[
E_{p,r} = E_{p,0} D \quad (3)
\]

Combing Equations (2) and (3) gives

\[
L_p = L_{p,0}\Gamma^{\alpha_G} \left( \frac{E_{p,r}}{E_{p,0}} \right)^{\alpha_D}. \quad (4)
\]

Equation (4) describes the pure kinetic effect in the \( L_p - E_p \) relation. It depends on the Lorentz factor and would be applicable so long as indexes \( \alpha_G \) and \( \alpha_D \) are available.

Unlike in the case of mode A, this analysis does not refer to the details of radiation. Its result does not depend on any particular radiation mechanism. Equation (4) can be applied to the investigation of an individual GRB behavior as well as to the investigation of the collective behavior of any GRB sample (say, the investigation of the statistical properties of GRB samples).

2.2. The Role of the Dynamic Effect

Actually, the ejecta rest-frame peak luminosity \( L_{p,0} \) and the ejecta rest-frame peak energy \( E_{p,0} \) themselves might rely on the Lorentz factor of the ejecta if the radiation is produced by shocks. If so, they can be expressed as \( L_{p,0} = L_{p,0}(\Gamma) \) and \( E_{p,0} = E_{p,0}(\Gamma) \), respectively. These relations might depend on the strength of the shocks and the type of radiation as well as other physical conditions such as the environment of the ejecta.

In fact, the functions of \( L_{p,0}(\Gamma) \) and \( E_{p,0}(\Gamma) \) might be complex rather than simple. However, if the dependence of \( L_{p,0} \) or \( E_{p,0} \) on \( \Gamma \) is certain or stable, in a wide range of \( \Gamma \) concerned, both \( L_{p,0}(\Gamma) \) and \( E_{p,0}(\Gamma) \) can generally be expressed as a power-law function of \( \Gamma \), especially when they are studied statistically. For a mechanism that depends on \( \Gamma \) within a narrow range but does not show a dependence on \( \Gamma \) in a wide range (such as in the case of a periodicity function), then the index of the power law will be zero in terms of statistics. In this situation, the narrow range dependence will show a scatter of data in the \( L_{p,0}(\Gamma) \) or \( E_{p,0}(\Gamma) \) plane when the data are concerned in a wide range of \( \Gamma \). In the case of a mechanism that depends strongly on \( \Gamma \) within a narrow range but depends weakly on \( \Gamma \) in a wide range, the index of the power law will be small, and in this situation, the narrow range dependence will show a scatter of data in the power-law relation of \( L_{p,0}(\Gamma) \) or \( E_{p,0}(\Gamma) \) in a wide range of \( \Gamma \). These functions might not fit the actual relations of \( L_{p,0}(\Gamma) \) and \( E_{p,0}(\Gamma) \) in a narrow range of \( \Gamma \), but they would fit the relations in a wide range, and this in turn would impose constraints to the dynamic mechanisms if the power-law indexes are available from statistical analysis.

Let us assume that

\[
L_{p,0} = L_{p,00}\Gamma^{\alpha_L} \quad (5)
\]

and

\[
E_{p,0} = E_{p,00}\Gamma^{\alpha_E} \quad (6)
\]

be valid in a wide range of \( \Gamma \), where \( \alpha_L \) and \( \alpha_E \) are constants, and \( L_{p,00} \) and \( E_{p,00} \) are free of \( \Gamma \) but they can vary from source to source.

Applying Equations (5) and (6), we get from Equation (4) that

\[
L_p = L_{p,00}\Gamma^{\alpha_L + \alpha_G} \left( \frac{E_{p,r}}{E_{p,00}} \right)^{\alpha_D}. \quad (7)
\]

This is a general form of the relation that combines the kinetic and the statistical dynamic effects. According to Equations (3) and (6), \( \Gamma \) and \( E_{p,r} \) are related by

\[
E_{p,r} = E_{p,00}\Gamma^{\alpha_E} D. \quad (8)
\]
Once the moving direction is known (say, when $\theta$ is provided), $\Gamma$ would merely be a function of $E_{p,r}$, and this would make Equation (7) applicable to explore the $L_p - E_p$ relation.

Generally, GRBs are believed to be the radiation arising from the ejecta that moves nearly close to the line of sight with a relativistic speed. Let us consider the situation that the ejecta moves with $\Gamma \gg 1$, directly toward the observer. In this case, one finds $D = 2\Gamma$. We get from Equation (8) that

$$E_{p,r} = 2E_{p,00}^{1/2}a_E.$$  

Inserting it into Equation (7) yields

$$L_p = 2^{a_D}L_{p,00} \left( \frac{E_{p,r}}{2E_{p,00}} \right)^{a_{\text{kin}}} \alpha_{\text{dyn}},$$  

with

$$a_{\text{kin}} = a_L + a_E$$  

and

$$a_{\text{dyn}} = \frac{a_L - a_{\text{kin}}a_E}{1 + a_E}.$$  

This is the relation that combines the kinetic and the statistical dynamic effects under the condition that the ejecta moves with $\Gamma \gg 1$, directly toward the observer. Revealed in Equation (10), while index $a_{\text{kin}}$ describes the pure kinetic effect, index $a_{\text{dyn}}$ corresponds to the statistical dynamic effect. We call $a_{\text{kin}}$ and $a_{\text{dyn}}$ the kinetic and dynamic indexes in the $L_p - E_p$ relation, respectively. The value of $a_{\text{dyn}}$ measured from statistical analysis of observational data would strongly constrain the dynamic mechanisms that are involved.

### 2.3. Implication of Cases with Typical Dynamic Indexes

As components of the dynamic index, $a_L$ reflects how the radiation strength of the shock in the ejecta rest frame is connected to the Lorentz factor of the ejecta and $a_E$ reflects how the hardness of the radiation spectrum that is produced by the shock is influenced by the Lorentz factor. Due to the vast difference of the physical conditions that the ejecta would encounter, it might be possible that, in some cases, the effects associated with $a_L$ and $a_E$ are similar, while in other cases, they are entirely different.

As shown in Equation (9), if $a_E = -1$, then $E_{p,r}$ would not depend on $\Gamma$. This very special situation is not discussed below. In the following we assume that $a_E \neq -1$. As far as we know, $a_G \geq 0$ and $a_D \geq 0$ always hold in the situation concerned, therefore we also assume $a_{\text{kin}} \geq 0$ in the following analysis.

It has been generally believed that the emission of GRBs comes from internal or external shocks which are produced when an inner ejecta catches up with the outer ejecta or a fast moving ejecta hits the external medium. Theoretically, we refer to the so-called ejecta rest frame as the frame relative to which there is no particular collective motion of all the radiation seeds (such as chaotic moving electrons). For the sake of simplicity, here we regard the final speed, relative to the cosmological rest frame, of the ejecta after the shock as the speed of the so-called ejecta rest frame.

The shocks involved might belong to various kinds, depending on the actual physical conditions. We pay attention to the following two typical types of shocks.

One is the kind of shock that is associated with

$$a_L > 0$$  

and

$$a_E \geq 0,$$  

which we call an external-like shock. This kind of shock can be a real external shock: a fast moving ejecta hits or sweeps external medium and that gives rise to shocks. For the same kind of external medium and the same rest mass of the ejecta, the larger the $\Gamma$ value, the stronger the shock and possibly the harder of the intrinsic radiation spectrum. This kind of shock can also be an inner ejecta dominant internal shock: the rest mass of the outer ejecta is much smaller than that of the inner ejecta, and then the final value of $\Gamma$ would mainly depend on the motion of the inner ejecta. Under this condition, for the same kind of outer ejecta and the same rest mass of the inner ejecta, the larger the final value of $\Gamma$, the stronger the shocks and possibly the harder the intrinsic radiation spectrum.

The other is the kind of shock that is associated with

$$a_L < 0$$  

and

$$a_E \leq 0,$$  

which we call an internal-like shock. This kind of shock can be an outer ejecta dominant internal shock, where the rest mass of the outer ejecta is assumed to be much larger than that of the inner ejecta. For the same kind of inner ejecta (say, when the Lorentz factors and rest masses of all the inner ejecta concerned are almost the same) and the same rest mass of the outer ejecta, it is expected that a larger final speed of the ejecta would produce a weaker internal shock and possibly a softer intrinsic radiation spectrum.

Listed in Table 1 are some cases associated with typical values of the dynamic index.

| Case | $(a_E)$ | $(a_L)$ | $(a_{\text{dyn}})$ |
|------|----------|----------|------------------|
| 1a   | $a_E > -1$ | $a_L > a_{\text{kin}}a_E$ | $a_{\text{dyn}} > 0$ |
| 1b   | $a_E < -1$ | $a_L < a_{\text{kin}}a_E$ | $a_{\text{dyn}} > 0$ |
| 2a   | $a_E > 0$  | $a_L = a_{\text{kin}}a_E$ | $a_{\text{dyn}} = 0$ |
| 2b   | $a_E < 0$  | $a_L = a_{\text{kin}}a_E$ | $a_{\text{dyn}} = 0$ |
| 3a   | $a_E > -1$ | $a_L > -a_{\text{kin}}$ | $-a_{\text{kin}} \leq a_{\text{dyn}} < 0$ |
| 3b   | $a_E < -1$ | $a_L < -a_{\text{kin}}$ | $-a_{\text{kin}} \leq a_{\text{dyn}} < 0$ |
| 4a   | $a_E > 0$  | $a_L = -a_{\text{kin}}$ | $a_{\text{dyn}} = -a_{\text{kin}}$ |
| 4b   | $a_E < 0$  | $a_L = -a_{\text{kin}}$ | $a_{\text{dyn}} = -a_{\text{kin}}$ |
| 5a   | $a_E > -1$ | $a_L < -a_{\text{kin}}$ | $-a_{\text{kin}} < a_{\text{dyn}} \leq a_{\text{kin}}$ |
| 5b   | $a_E < -1$ | $a_L > -a_{\text{kin}}$ | $-a_{\text{kin}} \geq a_{\text{dyn}} < -a_{\text{kin}}$ |

3. CONSTRAINTS TO THE DYNAMIC INDEX

BY CURRENT GRB DATA

Spectral forms of GRBs vary significantly. While many of them can be well fitted by the Band function form (Band et al. 1993), some of them might be accounted for by other forms such as bremsstrahlung, Comptonized, and synchrotron radiations (Schafer et al. 1994). Even for those with an obvious Band function form, both the low- and high-energy indexes vary from...
source to source (see, e.g., Prececk et al. 2000). As the kinetic effect does not affect the spectral form, it is accordingly probable that emissions of GRBs are produced by various radiation processes. In this way, it is hard to interpret their statistical effect does not affect the spectral form, it is accordingly probable that mechanisms will be able to figure out even though the actual mechanisms themselves are unaware. This in turn can constrain the mechanisms and possibly provide clues to reveal them. Here, with the $L_p-E_p$ relation derived above, we try to draw out some statistical properties of the dynamic mechanism from current GRB data.

Many authors have explored the $L_p-E_p$ relationship from various GRB samples. The relation of $L_p \propto E_p^{\alpha_L}$ was confirmed by different groups of researchers in their statistical investigations. The index, $\nu$, was found to range from 1.4 to 2.0 (see Yonetoku et al. 2004, 2010; Ghirlanda et al. 2005; Wang et al. 2011; Zhang et al. 2012b). Compared with Equation (10), we find that, in our model, the dynamic index in the $L_p-E_p$ relation is confined within $\alpha_{\text{dyn}} = (1.4 - \alpha_{\text{kin}}, 2.0 - \alpha_{\text{dyn}})$ by current samples.

As far as we know, in common cases of jets, $\alpha_{\text{kin}} \geq 2$ always holds (see, e.g., the analysis below). The constraint of $\alpha_{\text{dyn}} = (1.4 - \alpha_{\text{kin}}, 2.0 - \alpha_{\text{dyn}})$ suggests that, in terms of statistics, the dynamic effect of GRBs does not boost up the kinetic effect, but instead, it partially cancels the latter. Or, the dynamic effect alone tends to give rise to an anti-correlation between $L_p$ and $E_p$, which is entirely different from what the kinetic effect does. In addition, the data reveal that the kinetic effect would be much larger than the dynamic effect.

In the following, we show how the dynamic index is constrained by data in a typical case of jets.

The effect of special relativity on the observed flux of jets in active galactic nuclei and GRBs have been discussed by various authors (e.g., Rybicki & Lightman 1979; Lind & Blandford 1985; Atoyan & Aharonian 1997; Sikora et al. 1997; Granot et al. 1999; Woods & Loeb 1999). Known as the so-called Doppler boosting effect, the ratio of observed flux density $S_{\text{obs}}$ (erg s$^{-1}$ cm$^{-2}$ Hz$^{-1}$) to emitted flux density $S_0$ from an optically thin, isotropically emitting jet is $S_{\text{obs}} = D^k S_0$, where $\alpha$ is the spectral index of the emission ($S_{\nu} \propto \nu^{-\alpha}$), and $k$ is a parameter that accounts for the geometry of the ejecta, with $k = 2$ for a continuous jet and $k = 3$ for a jet of moving discrete radio clouds (see, e.g., Mirabel & Rodriguez 1999). Since the luminosity we are concerned with in this paper covers the entire energy range, to consider the ejecta rest-frame peak luminosity, $L_p$, one should ignore the effect arising from the Doppler shifting of the spectrum of the source. Therefore, we prefer to use $S_\nu = D^k S_0$ instead of $S_{\text{obs}} = D^{k-\alpha} S_0$, where $\nu$ and $\nu_0$ are related by the Doppler effect. For luminosities corresponding to the sum of the flux over the entire energy range, $L_p = L_{p,0} D^3$ is applicable in the two situations of jets concerned above. In such cases, one finds $\alpha_G = 0$ and $\alpha_D = 2$ for a continuous jet, and $\alpha_G = 0$ and $\alpha_D = 3$ for a jet of moving discrete radio clouds. They correspond to $\alpha_{\text{kin}} = 2$ and $\alpha_{\text{kin}} = 3$, respectively.

Let us consider the case of jets with moving discrete radio clouds. In this situation, $\alpha_{\text{kin}} = 3$. Thus, the constraint of the dynamic index becomes $\alpha_{\text{dyn}} = (-1.6, -1)$, and Equation (12) can be expressed as

$$\alpha_{\text{dyn}} = \frac{-3\alpha_L}{1 + \alpha_E}.$$  \hfill (15)

and the indexes in Table 1 can be presented in a simpler way (see Table 2).

According to the vast variance of the observational characters and according to the many differences of the physical conditions that one can imagine (environments, masses, etc.), it is expected that GRB events might arise from different kinds of shock and might have different radiation mechanisms. Perhaps one can divide GRBs into several subsets merely according to their types of shock and/or their radiation mechanisms. We suspect that it is these radiation mechanisms and shocks and the distributions of these subsets that give rise to the observed $L_p-E_p$ relation. Thus, assuming a single mechanism to account for the $L_p-E_p$ relation of the whole sample seems unreasonable. However, this should not prevent us from exploring what mechanism could be expected when it is supposed to account for the relation. Perhaps this approach can provide constraints on mechanisms or provide clues to find them.

According to Equation (15), once $\alpha_{\text{dyn}}$ is known, the relation between $\alpha_L$ and $\alpha_E$ will be well established. Once a range of $\alpha_{\text{dyn}}$ is provided, the available regions of $\alpha_L$ and $\alpha_E$ will be well constrained. With $\alpha_{\text{dyn}} = (-1.6, -1)$, the constraints to the ranges of $\alpha_L$ and $\alpha_E$ in the situation of jets with moving discrete radio clouds are now available. Displayed in Figure 1 are the two marginal curves in the $\alpha_L-\alpha_E$ plane: the $\alpha_{\text{dyn}} = -1.6$ and $\alpha_{\text{dyn}} = -1$ curves. Four areas are confined by these curves and the axes. They are A1, A2, A3, and A4.

| Case | $\alpha_E$ | $\alpha_L$ | $\alpha_{\text{dyn}}$ |
|------|------------|------------|------------------|
| 1a   | $\alpha_E > -1$ | $\alpha_L > 3\alpha_E$ | $\alpha_{\text{dyn}} > 0$ |
| 1b   | $\alpha_E < -1$ | $\alpha_L < 3\alpha_E$ | $\alpha_{\text{dyn}} > 0$ |
| 2a   | $\alpha_E > 0$ | $\alpha_L = 3\alpha_E$ | $\alpha_{\text{dyn}} = 0$ |
| 2b   | $\alpha_E < 0$ | $\alpha_L = 3\alpha_E$ | $\alpha_{\text{dyn}} = 0$ |
| 3a   | $\alpha_E > -1$ | $\alpha_L > -3$ | $-3 < \alpha_{\text{dyn}} < 0$ |
| 3b   | $\alpha_E < -1$ | $\alpha_L < -3$ | $-3 < \alpha_{\text{dyn}} < 0$ |
| 4a   | $\alpha_E > 0$ | $\alpha_L = -3$ | $\alpha_{\text{dyn}} = -3$ |
| 4b   | $\alpha_E < 0$ | $\alpha_L = -3$ | $\alpha_{\text{dyn}} = -3$ |
| 5a   | $\alpha_E > -1$ | $\alpha_L < -3$ | $\alpha_{\text{dyn}} < -3$ |
| 5b   | $\alpha_E < -1$ | $\alpha_L > -3$ | $\alpha_{\text{dyn}} < -3$ |
In areas A1 and A2, one finds $\alpha_L < 0$ and $\alpha_E < 0$. This satisfies condition (14), suggesting that parameters confined within these areas would be associated with internal-like shocks such as the outer ejecta dominant internal shocks where the rest mass of the outer ejecta is assumed to be much larger than that of the inner ejecta.

In area A3, $\alpha_L < 0$ and $\alpha_E > 0$. Parameters within this area correspond to the situation where a larger value of $\Gamma$ would give rise to a weaker shock with a harder intrinsic spectrum. We do not know if any dynamical mechanisms can produce this kind of shock. If the answer is no, we would prefer to regard this area merely as a result contributed by different GRB subsets which are associated with different mechanisms.

In area A4, $\alpha_L > 0$ and $\alpha_E > 0$, satisfying condition (13). As mentioned above, parameters confined within this area would be associated with external-like shocks, including external shocks and inner ejecta dominant internal shocks where the rest mass of the inner ejecta is assumed to be much larger than that of the outer ejecta.

Figure 1 reveals that, in both areas A1 and A2, where $\alpha_L < 0$ and $\alpha_E < 0$, the relation between the indexes follows $\alpha_L < \alpha_E$. This indicates that, in the case of internal-like shocks, $L_{p,0}$ depends on $\Gamma$ more strongly than $E_{p,0}$ does. However, in the area A4, the lower left portion of A4, below the identity curve, where the values of the indexes are relatively smaller, the relation between the indexes is $\alpha_L > \alpha_E$, suggesting that, in some cases of external-like shocks, $L_{p,0}$ also depends on $\Gamma$ more strongly than $E_{p,0}$ does. However, in the rest of area A4, above the identity curve, where the values of the indexes are relatively larger, the relation between the indexes is $\alpha_L < \alpha_E$, showing that, in some cases of external-like shocks, $L_{p,0}$ depends on $\Gamma$ more mildly than $E_{p,0}$ does. This becomes a statistical constraint to the radiation mechanism.

4. DISCUSSION 1: WHAT WOULD HAPPEN WHEN VIEWING FROM THE BEAMING ANGLE

In the above analysis, we consider only the case where the ejecta moves directly toward the observer (say, $\theta = 0$). However, some GRBs might be the events of the emission from the ejecta which does not move along the direction of the line of sight.

It is generally believed that as the ejecta moves with relativistic speed, most of the detected GRB energy must be the radiation from the area confined within a very small solid angle relative to the explosion spot, around the line of sight. This is the so-called beaming effect. The corresponding angle is the beaming angle which satisfies $\cos \theta = \beta$. Is the emission from the beaming angle much different from that at $\theta = 0$?

Taking $\cos \theta = \beta$, one finds $D = \Gamma$, and then obtains from Equation (8) that

$$E_{p,r} = E_{p,00} \Gamma^{1+\alpha_E}.$$  \hspace{1cm} (16)

Inserting it into Equation (7) yields

$$L_p = L_{p,00} \left( \frac{E_{p,r}}{E_{p,00}} \right)^{\alpha_{\text{kin}}+\alpha_{\text{dyn}}},$$  \hspace{1cm} (17)

where $\alpha_{\text{kin}}$ and $\alpha_{\text{dyn}}$ are represented by Equations (11) and (12), respectively.

Comparing Equations (10) and (17) we find that the $L_p-E_p$ relation in the case of $\cos \theta = \beta$ is almost the same as that in the case of $\theta = 0$. The index is exactly the same, while the coefficient is slightly different. Therefore, the result of the discussion of the dynamic indexes presented above holds in the case of the beaming angle.

5. DISCUSSION 2: CONSTRAINTS TO THE DYNAMIC INDEX IN TWO GENERAL SITUATIONS

Let us consider two models of jets, which might be closer to real situations.

5.1. In the Case of Steady-state Jets with an Open Angle $\theta_j \sim 1/\Gamma_j$

Sikora et al. (1997) showed that, in the case of steady-state jets with an open angle $\theta_j \sim 1/\Gamma_j$, emitting isotropically in the jet frame, when viewing from the angle of $\theta_{\text{obs}} \sim 1/\Gamma_j$, the relation between the observed luminosity and the rest-frame luminosity may be $L_{\text{obs}} \sim 2\Gamma_j^2 L_{\text{em}}$, where $\Gamma_j$ is the bulk Lorentz factor of the jets.

The viewing angle $\theta = 1/\Gamma$ is equivalent to $\cos \theta = \beta$, which is the well-known beaming angle. Therefore, for $\theta_{\text{obs}} \sim 1/\Gamma_j$, one finds $D \sim \Gamma_j$. Then Equation (3) gives rise to $E_{p,r} \sim E_{p,00} \Gamma_j$. Applying Equations (5) and (6), Equation (10) should be replaced by

$$L_p \sim 2L_{p,00} \left( \frac{E_{p,r}}{E_{p,00}} \right)^{\alpha_{\text{kin}}+\alpha_{\text{dyn}}},$$  \hspace{1cm} (18)

with $\alpha_{\text{kin}} = 2$ and $\alpha_{\text{dyn}} = (\alpha_L - 2\alpha_E)/(1 + \alpha_E)$. In this situation, the constraint of the dynamic index by current GRB data becomes $\alpha_{\text{dyn}} = (-0.6, 0)$.

Shown in Figure 2 are the corresponding marginal curves in the $\alpha_L-\alpha_E$ plane: the $\alpha_{\text{dyn}} = -0.6$ and $\alpha_{\text{dyn}} = 0$ curves. As in the case of jets with moving discrete radio clouds, four areas, A1, A2, A3, and A4, are confined by these curves and the axes. Interpretations of these areas remain the same as those in Figure 1. Figure 2 reveals that the constraints to the dynamical indexes rely strongly on the model of jets. A model with a smaller value of $\alpha_{\text{kin}}$ would shift the constrained areas to the domain of larger $\alpha_L$.
that the normalization differs from model to model. It is the two marginal curves in Figure 1. Other symbols are the same as in Figure 1.

Constraints to the ranges of $\alpha_{\text{kin}}$ and $\alpha_{E}$ in the situation of jets with moving blobs radiating within a distance range, viewing from the angle $\theta_{\text{obs}} \sim 1/T_{j}$, by current GRB data, where the two thick solid lines stand for the $\alpha_{\text{dyn}} = -2$ and $\alpha_{\text{dyn}} = -2.6$ curves, respectively. The dashed lines represent the two marginal curves in Figure 1. Other symbols are the same as in Figure 1.

In the Case of Jets with Moving Blobs Radiating within a Distance Range

Sikora et al. (1997) also studied the situation of jets with moving blobs which radiate within a distance range. They assumed that (1) the blobs are injected into the “active zone” every $\Delta t_{\text{inj}}$; (2) all blobs are moving with the same Lorentz factor; (3) the blobs radiate isotropically in their rest frame, each at the same rate, $L_{\text{em},1}$; (4) each blob stops to radiate after passing a given distance range $\Delta r = r$. They also considered the case of viewing angle $\theta_{\text{obs}} \sim 1/T_{j}$. In this situation, they found the relation $L_{\text{obs}} = L_{\text{em}} D^{4}$, where $L_{\text{em}} = L'_{\text{em}} N_{\text{obs}} L_{\text{em},1}$, and $N_{\text{obs}}$ is the number of blobs counted by a distant observer within his observational time interval $\Delta t_{\text{obs}}$.

Repeating the above analysis we get

$$L_{p} = L_{p,00} \left( \frac{E_{p,r}}{E_{p,00}} \right)^{\alpha_{\text{kin}} - \alpha_{\text{dyn}}},$$

with $\alpha_{\text{kin}} = 4$ and $\alpha_{\text{dyn}} = (\alpha_{L} - 4\alpha_{E})/(1 + \alpha_{E})$. In this situation, the constraint of the dynamic index by current GRB data becomes $\alpha_{\text{dyn}} = (-2.6, -2)$.

Contrary to that hinted by Figures 2 and 3, with current GRB data, a model with a larger value of $\alpha_{\text{kin}}$ would shift the constrained areas to the domain of smaller $\alpha_{L}$.

6. DISCUSSION 3: NORMALIZATION OF THE RELATION

It should be pointed out that the $L_{p} - E_{p}$ relation has two parameters, i.e., index and normalization. In the analysis above, we only discuss the slope index in view of kinetic and dynamic effects. Here, we discuss how the kinetic and dynamic effects influence the normalization of the relation.

Comparing Equations (10), (17), (18), and (19), we find that the normalization differs from model to model. It is proportional to $L_{p,00}$ and $E_{p,00}^{\alpha_{\text{kin}} - \alpha_{\text{dyn}}}$. The latter strongly depends on both kinetic and dynamic indexes. The relation between the normalization and $E_{p,00}$ varies for different situations of jets. To interpret the normalization, one should note which jet model is concerned.

Let us consider the simplest model of jets: jets with moving discrete radio clouds which move directly toward the observer. In this situation, Equation (10) becomes

$$L_{p} = 8L_{p,00} \left( \frac{E_{p,r}}{E_{p,00}} \right)^{3 + \alpha_{\text{dyn}}}. \quad (20)$$

The normalization is $2^{-\alpha_{\text{dyn}} - 3} L_{p,00} E_{p,00}^{3 - \alpha_{\text{dyn}}}$. According to the above analysis we know that, in this situation, $L_{p,00}$, $E_{p,00}$, and $\alpha_{\text{dyn}}$ are products of the dynamic effect, while the index 3 is the result of the kinetic effect. For the proposed model, while the kinetic effect is certain, the dynamic effect is not constrained. Let us assume that all sources concerned obey relation (20), where $\alpha_{\text{dyn}}$ is certain while $L_{p,00}$ and $E_{p,00}$ are allowed to vary from source to source. In this way, the dispersion of the normalization will only come from that of $L_{p,00}$ and $E_{p,00}$. Since $L_{p,00}$ and $E_{p,00}$ are the two basic parameters of the normalization, the dynamic effect is important to evaluate the quantity.

To our surprise, the kinetic effect influences the normalization as well. For example, if $E_{p,00}$ is 10 times smaller than that expected, then the kinetic effect alone will make the normalization 1000 times smaller than the expected one. Therefore, to study the normalization, the kinetic effect should not be ignored. In fact, the kinetic effect affects the normalization in two aspects: the kinetic index and the formula itself (compare Equations (10), (17), (18) and (19)).

As an example for discussion, let us study the normalization of the $L_{p} - E_{p}$ relation obtained by Yonetoku et al. (2004): $(2.34 \pm 2.07) \times 10^{-5}$, where $L_{p}$ is in units of $10^{52}$ erg s$^{-1}$ and $E_{p}$ is in units of keV. In their work, the index of $E_{p}$ is 2.

Here, we adopt the model of jets considered in this section (i.e., jets with moving discrete radio clouds that move directly toward the observer). In this way, Equation (20) is applicable. For this equation, the data suggest $\alpha_{\text{dyn}} = -1$. Therefore, the normalization of Equation (20) becomes $2L_{p,00} E_{p,00}^{-2}$. Hence, we approximate $2L_{p,00} E_{p,00}^{-2} = 2.34 \times 10^{-5}$.

If we assume that $E_{p,00} = 1$ keV, then we will get the rest-frame luminosity $L_{p,00} \sim 10^{37}$ erg s$^{-1}$. If we assume that $E_{p,00} = 0.1$ keV, then we will get $L_{p,00} \sim 10^{39}$ erg s$^{-1}$. If we believe that $E_{p,00}$ is around the range of 0.1–1 keV and the situation above can approximately represent the real situation of jets, then we will find that the rest-frame luminosity is around the range of $10^{37}$–$10^{39}$ erg s$^{-1}$.

Of course, this interpretation is not conclusive since we do not know the real range of $E_{p,00}$; neither do we know the real situation of jets. However, the above discussion suggests that, with our method, one can estimate the rest-frame luminosity of jets so long as the range of the rest-frame peak energy and the situation of jets are somehow known (perhaps from other independent investigations).

We suggest that, in a detailed discussion of the normalization of the $L_{p} - E_{p}$ relation in the near future, one should define and collect a neat subset of sources, assuming that all the sources belong to a single model. In this way, the formulae applied would be certain, and hence the index of the kinetic effect will be available, and this will make the form of the normalization more certain. (In fact, for such a subset, one might find a tighter correlation between the two quantities and that the index will be better established.) In this case, it might be expected that both $L_{p,00}$ and $E_{p,00}$ would not vary violently from source to source,
and this will certainly be helpful for a quantitative study of the normalization.

7. CONCLUSIONS

In this paper, we investigate the underlying mechanisms of the $L_p-E_p$ relation by separately considering the corresponding kinetic and dynamic mechanisms. In this way, one can tell how much the kinetic or dynamic mechanism contributes to the index of the relationship once a set of GRB data is available.

Based on the analysis above, we reach several conclusions. The first relies only on theoretical analysis and is quite robust.

1. The kinetic effect alone would give rise to an obvious correlation between $L_p$ and $E_p$, with the index of the correlation being as large as 2–4, depending on the situation concerned. This conclusion comes from the effect of the theory of special relativity, which is entirely independent of observational data.

2. The dynamic effect alone tends to give rise to an anti-correlation between $L_p$ and $E_p$, which partially cancels the kinetic effect.

3. In terms of statistics, the dynamic effect is much smaller than the kinetic effect.

4. In the case of jets with moving discrete radio clouds when they move directly toward the observer, the index of the dynamic effect in the $L_p-E_p$ relation is currently constrained within $\alpha_{\text{dyn}} = (-1.6, -1)$, while in the case of other models of jets, the constraints are different.

5. Both internal and external shocks can account for the current data.

From Figures 2 and 3 one finds that, while the constrained areas (A1, A2, A3, and A4) could become much narrower in the near future when much larger samples of data are available, uncertainties of the dynamic indexes would remain large if the model involved is uncertain. To get a more precise result, perhaps one should consider subsets of bursts by assuming that sources of each subset belong to a single model of jets.

This work was supported by the National Natural Science Foundation of China (No. 11073007) and the Guangzhou technological project (No. 11C62010685).

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