Neural networks and logical reasoning systems. A translation table

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Abstract

A correspondence is established between the elements of logic reasoning systems (knowledge bases, rules, inference and queries) and the hardware and dynamical operations of neural networks. The correspondence is framed as a general translation dictionary which, hopefully, will allow to go back and forth between symbolic and network formulations, a desirable step in learning-oriented systems and multicomputer networks.

In the framework of Horn clause logics it is found that atomic propositions with \( n \) arguments correspond to nodes with \( n \)-th order synapses, rules to synaptic intensity constraints, forward chaining to synaptic dynamics and queries either to simple node activation or to a query tensor dynamics.

1 Introduction

Neural networks and logical reasoning systems, because they both rely on Turing models of computation, are equivalent in the sense that whatever is computable in one framework must also be computable in the other. How to establish the equivalence in each particular case is however a non-trivial and interesting issue, because problems that are addressed in a simple manner in one approach turn out to be intractable in the other and vice-versa. For example, combinatorial optimization problems are efficiently handled by neural networks, whereas logical systems succumb to combinatorial explosion. Conversely, for highly structured problems where appropriate heuristics are known, it is simpler and faster to use a logical reasoning system, the corresponding neural network system taking a long time to tune up because rule-based information is in general not so easy to build in the network architecture.

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This situation led a number of authors[1] [2] to propose the use of hybrid systems to solve complex problems, with a logic-like program for some parts of the process and a network for the rest. On the other hand, some authors have dealt with the identification of the network structures that might represent the logical structures of a symbolic system[3] [4] [5] [6] [7] [8] [13] [14] and conversely with the nature of the rule-based information that may be extracted from networks.

Extraction of rules from networks is an issue of practical importance in the construction of expert systems from example-trained networks[9] [10] [11] [12]. On the other hand when some prior rule-based information is known about a problem but nevertheless a network implementation seems appropriate, it would be useful to have some simple rules to implement the symbolic information on the architecture of the network. Neural networks take advantage of parallel VLSI hardware implementations, which largely improve the processing speed and it is not so clear how to take advantage of similar implementations for symbolic processing. Therefore even for problems that are naturally stated in logic terms, it might be useful to have a translation dictionary for hardware implementation purposes. The establishment of a concise way to go back and forth between symbolic and network formulations is also welcome in learning-oriented systems and in the design of multicomputer networks. Different architectures make different types of learning easier or harder to design and in multicomputer networks it is essential for algorithms and architectures to fit together as well as possible. Finally and independently of the practical issues, the establishing of a compact translation dictionary between the two paradigms might be a useful step in the development of a unified language for cognitive processes.

Most past attempts at establishing a logic-networks dictionary concern questions of the architecture required to represent some types of logical operations or the refining of the numerical part of the knowledge base. In some cases, an extension of the usual connectionist framework is required and a full correspondence is in general not established involving all the aspects of the systems (knowledge base, rules, inference, recursion and handling of queries). Here we describe an attempt to establish a correspondence involving all the basic elements that are present in logical and neural network systems. Recall that in a neural network one has a (distributed) memory on the connection strengths (synapses), a learning dynamics on the synapses and an activation dynamics of the node intensities. In a logical reasoning system there is some set of ground facts about the objects in the domain, a set of rules which are potential knowledge concerning relations between the objects and an inference mechanism (backward or forward chaining) allowing for the extraction of further information and the answering of queries.

It has been argued that trying to isolate, in a network, the structures that correspond to specific logical statements or operations is a waste of time because everything in a network (memory, rules and inference) is distributed everywhere and forever inseparable. This may well be true for some architectures and some
classes of concepts. However even the identification of the modular structures that correspond to the logic elements is already a useful step. For example one of our conclusions here is that a natural network representation of an atomic proposition is a node with $n^{th}$ order synapses. Because of the universal approximation properties of neural networks, that same proposition might as well be represented with first order synapses, the proposition corresponding then to a submodule of several neurons. However the identification of the kind of minimal submodule that corresponds to a specific logical element is already a useful step.

2 The network translation of a logic system

For definiteness the scope of the logical systems, that is considered is a subset of the Prolog logic programming language. Namely the logic system is specified by a set of Horn clauses which are constructed from terms which are either constants or variables or structures. A constant is sensed to represent some concrete object in the domain of the problem. It is represented in the logical system by an indecomposable elementary symbol (an atom). Structures are restricted to be atomic propositions of the general form

\[
\text{functor}(\text{parameter list})
\]

The functor is an atom and the parameter list is any list of atoms, variables or other atomic propositions. Finally a variable is an entity that can at any time be bound to any atom (constant or functor). Small letters will be used for the atoms and capitals for the variables.

The first step is to find a network representation of the basic entities of the logical system. Each atom will correspond to a node. Therefore not only constants but also functors will be assigned a node in the network.

The network nodes have output zero or one and threshold one. The nodes are connected in a network, with the inputs equipped with connection strengths (synapses) $W_{f,i_1,\ldots,i_n}$ of order $n$ if $n$ is the number of parameters in the parameter list of the functor. In particular a simple constant has no input. The node activation is

\[
x_f = \theta \left( \sum_{i_1,\ldots,i_n} W_{f,i_1,\ldots,i_n} x_1 \cdots x_n + y_f \right)
\]  

where the polarization $y_f$ is zero unless there is a query involving this node. Fig. 1 displays the neural network representation of a constant node and Fig. 2 that of a functor node.

Whenever in the knowledge base, the relation

\[
f(i_1,\cdots,i_n).
\]
is stated as a true fact, the corresponding synaptic strength is

\[ W_{f,i_1,\cdots,i_n} = 1 \]  

(3)

Figure 1: Network representation of a constant

Otherwise, if the relation \( f \) does not hold for these particular set of parameters the connection strength vanishes. This is the straightforward way to represent in the network the ground facts of the knowledge base, that is, the relations that are explicitly expressed among the atoms. On the other hand relations and clauses that involve variables correspond to constraints on the remaining connection strengths. This is better explained through an example. Let a knowledge base be

\[
\begin{align*}
  f(b, c). \\
  m(a, c). \\
  m(d, a). \\
  p(X, Y) &\Leftarrow m(X, Y). \\
  p(X, Y) &\Leftarrow f(X, Y). \\
  g(X, Z) &\Leftarrow p(X, Y) \land p(Y, Z). 
\end{align*}
\]

(4)

The ground facts (the first three headless clauses) imply

\[ W_{fbc} = 1; W_{mac} = 1; W_{mda} = 1 \]  

(5)

all other \( W_{fXY} \) and \( W_{mXY} \) being set to zero. Let the connection strengths take only the values zero and one. Then the last three clauses in (4) translate
into the following constraints on the remaining $W_{pXY}$ and $W_{gXY}$ connection strengths

$$
W_{pXY} \geq W_{mXY} \\
W_{pXY} \geq W_{fXY} \\
W_{gXZ} \geq W_{pXY}W_{pYZ}
$$

(6)

These constraints may be realized in the network by a parallel dynamical law. Define the potential function

$$
V_1 = \frac{1}{2} \sum_{XY} \chi (W_{mXY} - W_{pXY}) + \frac{1}{2} \sum_{XY} \chi (W_{fXY} - W_{pXY}) + \frac{1}{2} \sum_{XYZ} \chi (W_{pXY}W_{pYZ} - W_{gXZ})
$$

(7)

where $\chi(x) = x^2 \theta(x)$. Now the strengths (6) fixed by the ground statements are considered as frozen and the others evolve in parallel according to a gradient descent dynamics

$$
\frac{dW_{pXY}}{dt} = - \frac{\partial V_1}{\partial W_{pXY}}; \quad \frac{dW_{gXY}}{dt} = - \frac{\partial V_1}{\partial W_{gXY}}
$$

(8)

This dynamics drives the connection strengths to values zero or one satisfying the constraints (6). Fig. 3 shows the dynamical evolution of the connection strengths for the potential (7).

The method used in this example holds in general and for each rule of the form

$$
g (X_1, ..., X_n) \Leftarrow \prod_i \land f_i (Y_1, ..., Y_m)
$$

(9)
a term

\[ \frac{1}{2} \sum \chi \left( \prod W_{f,Y_1 \ldots Y_{\rho_i}} - W_{g,X_1 \ldots X_n} \right) \]  

(10)

is added to the potential \( V_1 \) that drives the connection strengths, the sum being over all the variable terms.

We have therefore established a correspondence where atomic propositions with \( n \) parameters correspond to \( n^{th} \) order nodes and the rules are implemented by a parallel dynamics of the synapses. Because the synapse dynamics is equivalent to a forward chaining step, the network now contains on its connection strengths all the available information about the problem. Therefore information may be retrieved by simply looking at the values of the connection strengths. Alternatively an explicit scheme to answer queries may be established in the following way.

We may consider two different types of queries, simple queries without variables and queries with variables. In the first case, for example

\[ ? \Leftarrow g(a, b). \]  

(11)

the question is to check whether the relation \( g \) holds for the atoms \( a \) and \( b \). In this case it suffices to excite the polarizations \( y_a \) and \( y_b \) of the nodes \( a \) and \( b \) and see whether the node \( g \) lights up. The same happens for more complex queries like, for example

\[ ? \Leftarrow g(h(a, b), c) \land f(c, d). \]  

(12)
where now the polarizations to be excited are $y_a$, $y_b$, $y_c$ and $y_d$ and the answer to the query is positive if nodes $g$ and $f$ light up simultaneously.

For queries involving variables, for example

$$？ \leftarrow m(d, X) \land p(X, c). \quad (13)$$

the question is to find the instantiations of $X$, if any, that make this a true statement. The product of the connection strengths $W_{mdX}W_{pXc} = \alpha_X$ defines a vector $\alpha_X$ in atom space and the non-zero entries of the vector are the instantiations that positively satisfy the query. A similar scheme holds for queries involving several variables. For example, for

$$？ \leftarrow g(h(X, c), a) \land f(c, Y). \quad (14)$$

ones defines a tensor $\beta_{XY} = W_{gha}W_{hXc}W_{fY} = \alpha_X$ defines a tensor $\alpha_X$ in atom space and the non-zero entries of which are the positive answers to the query. With all the connection strengths established in the network by the (forward chaining-like) dynamics of Eq.(8) it might be an easy matter (depending on the type of implementation used) to look up the non-zero entries of $\alpha_X$ or $\beta_{XY}$. Alternatively one may regard the network connection strengths as potential knowledge and generate dynamically the non-zero entries of the tensors

$$\frac{d\alpha_X}{dt} = -\frac{\partial V_2}{\partial \alpha_X}; \quad \frac{d\beta_{XY}}{dt} = -\frac{\partial V_3}{\partial \beta_{XY}}. \quad (15)$$

with

$$V_2 = \sum_X (W_{mdX}W_{pXc} - \alpha_X)^2$$
$$V_3 = \sum_{XY} (W_{gha}W_{hXc}W_{fY} - \beta_{XY})^2 \quad (16)$$

Regarding the network structure and connection strengths as potential, as opposite to explicit knowledge, the dynamics of the query tensors defined above plays a role similar to backward chaining in logical programming.

As in (8)-(10) the generalization, of this dynamics in the space of query tensors, to general queries is straightforward.

The whole construction is summarized in Table 1.

| Logical System          | Network                          |
|-------------------------|----------------------------------|
| Constant                | Single node                      |
| Atomic proposition ($n$ arguments) | Node with $n^{th}$-order synapses |
| Rules                   | Synaptic intensity constrains    |
| Rule application (forward chaining) | Synaptic dynamics              |
| Queries                 | Node activation / Query tensor dynamics |

Table 1: Logical system - Neural network translation table

As a simple example we discuss problem 148 of Ref. [22] (see Appendix). Let us consider the following ground facts:
The clauses in this example define the following potential

\[ V_4 = \frac{1}{2} \sum_{X,Y,Z} \chi (W_{visits_XY}W_{atYZ} - W_{atXZ}) + \frac{1}{2} \sum_{U,V,N} \chi (W_{atUV}W_{phoneVN} - W_{numberUN}) \]

(17)

The evolution of the connection strengths, according to the gradient descendent dynamics, leads to the neural network in Fig. 4, where only the non-zero connections strengths are shown. The continuous lines represent the connection strengths of the ground facts and the dashed ones the connection strengths obtained from the clauses used to construct the potential \( V_4 \).

### 3 Conclusion and remarks

The construction in the previous section emphasized the establishment of a correspondence between the pure logic elements of logic programming and both the hardware and dynamical operations of neural networks. Logic programming systems however, in addition to its pure logic elements, involve some non-logical features like the handling of arithmetics and procedural prescriptions for computing efficiency. These features need not concern us here because numerical capabilities are native to networks and procedural rules are implementation dependent.

A more serious limitation concerns the restriction to Horn clauses. Horn clause logic is quite expressive and of wide computing power. However it cannot handle negation in a satisfactory way, especially for non-exhaustive databases. In some cases it is acceptable to reduce, by renaming[18], the set of non-Horn to a Horn clause set. However, depending on the nature of the database, it may be more convenient to retain a certain degree of non-Horness and to establish an inference procedure through case analysis[19] [20] [21]. At a formal level, and to some extent, our formalism might be extended to non-Horn clauses. For example, if a non-Horn clause is

\[ h(X, Z) \lor g(X, Z) \iff p(X, Y) \land p(Y, Z). \]

(18)

the relation for the connection strengths that extends [18] is

\[ W_{hXZ} + W_{gXZ} \geq W_{pXY}W_{pYZ} \]

(19)
However it is not clear, at this time, how to implement dynamically, in an
elegant manner, the case analysis mechanism.

In this paper we have been concerned with logical reasoning systems in its
sharp logics version. A natural concern is the extension to reasoning with un-
certainty. Given that the handling of real numbers is quite natural in networks,
an extension where all quantities (node activations and connection strengths),
instead of being 0 or 1, take a continuous range of values, does not seem to
raise any special difficulties. Notice however that the straightforward extension
of our network operations, as defined in Sect. 2, lead to represent the fuzzy
intersection by a product rather than by the min operation.

4 Appendix

Problem 184 in Ref. [22], originally suggested in "The Thinking Computer"
by Bertrand Raphael, considers the following functors:

\[
\begin{align*}
\text{number}(X,N) & \quad \text{means that person X can be reached by calling phone number N.} \\
\text{visits}(X,Y) & \quad \text{means that person X is visiting person Y.} \\
\text{at}(X,Y) & \quad \text{means that person X is at the residence of person Y.} \\
\text{phone}(X,N) & \quad \text{means that person X has phone number N.}
\end{align*}
\]

and the following clauses:

\[
\begin{align*}
at(X,Z) & \iff \text{visits}(X,Y) \land \text{at}(Y,Z) \\
\text{number}(U,N) & \iff \text{at}(U,V) \land \text{phone}(V,N)
\end{align*}
\]

The purpose is to reach a person, having a phone number data base and
knowing who visits whom and where the visited person is.

References

[1] J. A. Barnden and J. B. Pollack (Eds.); Advances in Connectionist and Neu-
ral Computation Theory, vol.1: High Level Connectionist Models, Ablex,
Norwood 1991.

[2] V. Honavar and L. Uhr; Artificial Intelligence and Neural Networks: Steps
toward principled integration, Academic Press, San Diego 1994 and refer-
ences therein.

[3] L. Shastri; Connectionism and the computational effectiveness of reasoning,
Theoretical Linguistics 16 (1990) 65.

[4] J. B. Pollack; Recursive distributed representations, Artificial Intelligence
46 (1990) 77.
[5] S. Pinker and A. Prince; On language and connectionism: Analysis of a parallel distributed processing model of language acquisition, Cognition 28 (1988) 73.

[6] J. A. Bardnen, Encoding complex symbolic data structures with some unusual connectionist techniques, in [1] pp. 180-240.

[7] R. Sun; Integrating Rules and Connectionism for Robust Reasoning, Wiley, New York 1993.

[8] R. Sun; On variable binding in connectionist networks, Connection Science 4 (1992) 93.

[9] S. I. Gallant; Connectionist expert systems, Commun. of the ACM 31 (1988) 152.

[10] S. I. Gallant; Neural Network Learning and Expert Systems, MIT Press, Cambridge, Massachusetts 1993.

[11] G. Towell and J. Shavlik; The extraction of refined rules from knowledge-based neural networks, Machine Learning 13 (1993) 71.

[12] LiMin Fu; Rule generation from neural networks, IEEE Trans. on Systems, Man and Cybernetics, 24 (1994) 1114.

[13] I. K. Sethi and J. H. Yoo; Symbolic mapping of neurons in feedforward networks, Pattern Recognition Lett. 17 (1996) 1035.

[14] M. Botta, A. Giordana and R. Piola; Refining first order theories with neural networks, in Foundations of Intelligent Systems, Z. W. Raś and A. Skowron (Eds.) pp. 84-93, Lecture Notes in Artificial Intelligence 1325, Springer, Berlin 1997.

[15] W. R. Softy and D. M. Kammen; Correlations in high dimensional or asymmetric data sets: Hebbian neuronal processing, Neural Networks 4 (1991) 337.

[16] J. G. Taylor and S. Coombes; Learning higher order correlations, Neural Networks 6 (1993) 423.

[17] J. W. Clark, K. A. Gernoith and M. L. Ristig; High-order probabilistic perceptrons as Bayesian inference engines, Trieste preprint Ic/94/236.

[18] X. Nie and Q. Guo; Renaming a set of non-Horn clauses, in Foundations of Intelligent Systems, Z. W. Raś and A. Skowron (Eds.) pp. 600-608, Lecture Notes in Artificial Intelligence 1325, Springer, Berlin 1997.

[19] D. W. Loveland; Near-Horn Prolog and beyond, Journal of Automated Reasoning 7 (1991) 1.
[20] D. A. Plaisted; *A simplified problem reduction format*, Artificial Intelligence 18 (1982) 227.

[21] D. A. Plaisted; *Non-Horn clause logic programming without contrapositives*, Journal of Automated Reasoning 4 (1988) 287.

[22] Helder Coelho and José C. Cotta; *Prolog by Example*, Springer-Verlag 1988.
Figure 4: Neural network representation of problem 148 in Ref. 22