Initial studies of a linear motor for generating quantum turbulence in He-II with a drawn grid

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Abstract. Studies of quantum turbulence (QT) require a means of generating well-characterised QT. For systems containing negligible normal fluid density this must be achieved without simultaneously introducing extraneous heating due, for example, to friction between mechanical components. Methods relying on oscillating objects produce negligible extraneous heating, but suffer from the disadvantages of both spatially and temporally nonuniform velocity. In addition, the vibrating grid goes back and forth through the QT it has itself created. Hence the characteristic length scale is subject to uncertainty. We are therefore developing a one-dimensional linear motor to draw a grid steadily, once, through He-II at mK temperatures. Frictionless magnetic bearings are used to levitate the moving elements in contrast to other designs, precluding mechanical dissipation. Details of the design and of the initial tests, and a quantitative comparison of theoretical and actual cryogenic performance, will be presented and discussed.

1. Background

Oscillating grids \cite{1, 2} and wire loops \cite{3} may be used to excite quantum turbulence (QT) in the zero temperature limit where the normal fluid density is negligible. Although they do not introduce extraneous heating, these devices fail to achieve the idealised objective of spatially and temporarily uniform velocity. A further complication is that the grid moves repeatedly through the excited QT. Together these issues result in the characteristic length scale being ill-defined.

Our objective is therefore to develop a system capable of moving a grid through the fluid volume once at an almost constant velocity over the entire stroke. Performance specifications defined acceleration of the grid from rest to the desired velocity (up to 1 ms\textsuperscript{-1}) within 1 mm and deceleration in a similar distance. This will be achieved by a magnetic linear motor with a grid and supporting armature system within the fluid, driven by coil winding(s) outside the fluid.

The possibilities of either a moving-coil system with a fixed magnetic field distribution, or a moving magnetic field generated by spatially fixed field coil(s) with time varying current(s), were considered. Both are reliant upon the Meissner effect for a superconductor, attached to the armature, in a magnetic field. The moving-coil option was rejected given the high moving mass and acceleration/deceleration required (up to 500 ms\textsuperscript{-2}) and the resultant forces on the cryostat insert.

Non-contact magnetic quadrupole bearings exerting forces on further superconductors attached to the armature were included to preclude the possibility of frictional heating whilst
still supporting the armature.

2. Experimental details and design
The drive motor and bearing system is shown schematically in figure 1. The upper and lower magnetic quadrupole bearings enclose a 6-coil drive motor in one module. The magnetic fields of the upper and lower quadrupole bearings act upon a pair of 5 mm long, 2 mm diameter, superconducting Nb sleeves with 0.2 mm wall thickness attached concentrically to an armature tube open at both ends and along its length. The 114 mm spacing of the bearing sleeves is commensurate with the spacing of the planes of the upper and lower bearing coils. Each bearing coil comprises 965 turns of superconducting NbTi wire wound on a 20 mm long × 6.5 mm diameter Araldite coil former. These coils and their yokes define a bore of 11.5 mm through which the inner vacuum can (IVC) tail of the dilution refrigerator passes.

In contrast to other designs [4] a 6-coil motor is used with coil lengths of 20 mm (coils 1 and 6) and 5 mm (coils 2-5). Wound from superconducting NbTi wire on a common Araldite coil former with a 13.5 mm diameter winding surface, coils 1 and 6 have 1540 turns and coils 2-5 6160 turns. First this is believed to give better control of the spatial and temporal dependences of the magnetic fields in the drive section, and secondly it is required due to the employment of magnetic bearings in contrast to physical contact bearings in other designs [4]. A third Nb sleeve is attached to the armature within the drive motor section.

The drive motor and bearing coils are outside the IVC in the helium bath in contrast to other designs [4, 5] facilitating better heatsinking. This requires that the tails of the helium test cell, the IVC and the radiation shield contain negligible metallic components to minimise induced eddy currents. For the purposes of initial testing in a helium immersion cryostat at 4.2K these were omitted. The armature assembly of the tube and the three Nb sleeves was within the liquid and for the initial tests no grid was attached to the armature rod.

3. Theoretical modelling of the magnetic fields and forces
The magnetic fields in the bearings and the drive motor, the force on the armature and its resulting velocity-time dependence were modelled in MATLAB. For a single loop of wire of radius $a$, carrying current $I$, orientated perpendicular to the $z$ axis in the $r$-plane at $(r=0, z=0)$, the radial and axial magnetic field components, $B_r$ and $B_z$, at a position $(r, z)$ are given by [6]:

$$ B_r = \frac{\mu_0 I}{2\pi} \frac{z}{r \sqrt{(a+r)^2+z^2}} (e^{-K(k)} + \frac{a^2 + r^2 + z^2}{(a-r)^2 + z^2} E(k)) $$  \hspace{1cm} (1)
\[
B_z = \frac{\mu_0 I}{2\pi} \frac{z}{\sqrt{(a + r)^2 + z^2}} \left( K(k) + \frac{a^2 - r^2 - z^2}{(a - r)^2 + z^2} E(k) \right) \tag{2}
\]

where \( K(k) \) and \( E(k) \) are complete elliptic integrals of the first and second kind respectively and

\[
k^2 = \frac{4ar}{(a + r)^2 + z^2}. \tag{3}
\]

After summing contributions from all the relevant coils the resultant magnetic field magnitude at \((r, z)\) is therefore

\[
B = \sqrt{\text{total}B_r^2 + \text{total}B_z^2}.
\]

Assuming that the superconductor is perfectly diamagnetic, the radial and axial components of the force, \( F_r \) and \( F_z \), exerted upon a superconductor by the magnetic field are given by:

\[
F_{r, z} = -\frac{V}{\mu_0 B} \frac{\partial B}{\partial r, z} \tag{4}
\]

where \( V \) is the closed volume of the superconductor and the partial differential is with respect to the radial \((r-)\) or axial \((z-)\) direction as appropriate.

When modelling variations with armature position of the axial drive motor force exerted upon the armature, account was taken of both the Nb sleeves in the drive section and the bearings and net forces predicted. Time evolution of the forces resulting from singly- and multiply- enabled drive coils with independently varying currents have been calculated: the principle is a logical extension of that for the single drive coil, static field measurements shown here, and therefore only the simplistic case of one drive coil enabled with a constant current will be presented.

4. Comparison of the modelled and measured magnetic field distributions and drive motor forces

The measured variation of the magnetic field magnitude for one quadrupole bearing enabled with a 5 A coil current in the direction along the common axis of one pair of coils is shown (figure 2). Values measured using AC inductance techniques and with a proprietary Hall effect probe at 300 K and 4.2 K agree to within 0.2 percent. Similar consistency was observed for the 6-coil linear motor system.

The forces generated by the drive motor have been measured at 4.2 K. In all cases the enabled quadrupole bearings successfully restricted the horizontal displacement of the armature assembly. The local magnetic field magnitude was maintained below the superconductor’s critical value at all times. Forces independently measured against a tension spring in tethered tests and by levitating the unrestrained armature assembly against gravity at various altitudes.

**Figure 2.** The dependence of the magnetic field magnitude of the axial field of one quadrupole bearing, measured along the axis of an opposing pair of bearing coils. The coils begin at \( \pm 6.75 \text{ mm.} \)
in the drive motor in free-floating conditions were commensurate; therefore only the latter are presented.

The three curves in Figure 3 indicate the theoretically predicted drive motor axial force variations for currents in the lowest drive coil of 0.61 A (lower), 1.01 A (centre) and 1.31 A (upper), with all others not energised. The open circles show, in order, the corresponding measured positions of the central Nb sleeve on the armature assembly at the levitated weight of $3.4 \times 10^{-3}$ N. The intercepts agree qualitatively, although there is a systematic and reproducible deviation from unity, highlighted in figure 4, where the ratio of the magnetically generated force to the armature assembly weight, corrected for Archimedeans upthrust, is shown.

This discrepancy may be due the inappropriate assumption of perfect diamagnetism for the hollow, open superconducting tube. Studies conducted previously elsewhere found that a disc aligned perpendicular to the magnetic field has a diamagnetic constant closest to -1, whereas a long tube of equal volume aligned parallel to the magnetic field has the lowest magnitude values. Furthermore, these measurements were conducted on closed structures in uniform magnetic fields, and therefore these data may indicate that the dimensions of the material relative to the length scales of variations in the magnetic field may be important.

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