5-Dimensional Assisted Inflation and the Remedy of the Fine-Tuning Problem

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Abstract

We extend the idea of assisted inflation to the case of power-law potentials and demonstrate the simultaneous resolution of two major problems that plague chaotic inflation. The implementation of the same idea in the framework of a 5-dimensional, scalar field theory leads to a model of chaotic inflation free of fine-tuning.

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1 Introduction

The chaotic inflationary scenario \cite{1} is undoubtedly the simplest of all the available models of inflation. However, it suffers from two major shortcomings: the large initial conditions on the inflaton field ($\phi_0 > \text{few} M_P$) and the severe fine-tuning of the coupling parameter ($\lambda \sim 10^{-12}$). Here, we will resolve both of the above problems by implementing the idea of assisted inflation, first, in the framework of a 4-dimensional theory of multiple scalar fields, and secondly, in the context of a 5-dimensional theory of a single, self-interacting, scalar field \cite{2}.

2 Assisted Inflation in 4 Dimensions

We start by presenting, in its simplest form, the idea of assisted inflation as originally proposed by Liddle, Mazumdar and Schunck \cite{3} and studied further by Malik and Wands \cite{4}. We consider a massless, scalar field with a self-interacting potential of the form

$$V = V_0 e^{-\frac{\phi}{\sqrt{2} p}},$$

where $V_0$ is a constant, and $p$ the slope parameter of the potential. The above scalar theory leads to a power-law expansion of the universe, i.e. $R(t) \sim t^p$, and if $p > 1$, the expansion is fast enough to resolve the problems of the Standard Cosmological Model. On the other hand, we may assume the existence of multiple, scalar fields $\phi_i$, $i = 1, ..., N$, with the same type of potential and, by recognizing the late-time attractor of the system, i.e. the configuration of the fields that minimizes the potential energy, we can map the original theory of multiple fields to a theory of a single field with the same type of self-interaction. This effective theory leads once again to a power-law expansion of the universe with a power $\tilde{p} = N p$, which for large $N$ gives an adequate amount of inflation even for $1/3 < p < 1$.

Here, we will be mainly interested in power-law potentials which are easily applicable to chaotic inflation. Therefore, we consider the following theory of multiple fields

$$-\mathcal{L} = \sum_{i=1}^{N} \left\{ \frac{1}{2} \left( \frac{\partial \phi_i}{\partial t} \right)^2 + \frac{m^2}{2} \phi_i^2 + \frac{\lambda}{4!} \phi_i^4 \right\}.$$

By making use of the late-time attractor, which has all the fields equal, the above theory can be mapped to a theory of a single field – the inflaton – by making the redefinitions

$$\tilde{\phi} = \sqrt{N} \phi_i, \quad \tilde{\lambda} = \frac{\lambda}{N}.$$

Note, that the above theory can serve as a model for chaotic inflation which, quite remarkably, is free of its two major problems. For a large number of fields $N$, the inflaton field $\tilde{\phi}$
can easily reach the value of a few $M_P$ while the values of $\phi_i$ can be well below the above threshold. In addition, the quartic coupling parameter naturally acquires an extremely small value, thus, resolving the problem of fine tuning. However, for the assistance method to work, the assisted sector needs to be non-coupled, otherwise, the presence of multiple fields impedes inflation since it leads to $\tilde{p} = p/N$, in the former case $\frac{\lambda}{4! M_5}$ and $\tilde{\lambda} = N^2 \lambda$, in the latter case.

3 Assisted Inflation in 5 Dimensions

As a possible source of a theory with a large number of scalar fields, we consider the following 5-dimensional theory of a self-interacting, massless scalar field

$$S_5 = -\int d^4 x \sqrt{G_5} \left\{ \frac{M_5^3}{16\pi} R_5 + \frac{1}{2} G_5^{AB} \partial_A \hat{\phi} \partial_B \hat{\phi} + \frac{\lambda}{4! M_5} \hat{\phi}^4 \right\},$$  \hspace{1cm} (4)

where $\lambda$ is the 5-dimensional quartic coupling constant and $M_5$ the 5-dimensional Planck mass. We assume that the extra dimension is compactified along a circle with circumference $2L$ and we Fourier expand $\hat{\phi}$ as

$$\hat{\phi}(x, z) = \hat{\phi}_0(x) + \sum_{n=1}^{N} \left[ \hat{\phi}_n(x) e^{\frac{i n\pi}{L} z} + \hat{\phi}_n^*(x) e^{-\frac{i n\pi}{L} z} \right],$$  \hspace{1cm} (5)

where $N \sim 2LM_5 = (M_P/M_5)^2$ is the maximum number of Kaluza-Klein (KK) states. Then, the 5-dimensional, fundamental theory is reduced to a 4-dimensional, effective one which has the form

$$S_4 = -\int d^4 x \sqrt{g} \left\{ \frac{M_5^2}{16\pi} R + \frac{(\partial \phi_0)^2}{2} + \frac{N}{2} \left( |\partial \phi_n|^2 + \frac{n^2 \pi^2}{L^2} |\phi_n|^2 \right) + \frac{\lambda}{4!} \left[ \phi_0^4 + 12 \frac{\phi_0^2}{\lambda} \sum_{n=1}^{N} \phi_n \phi_n^* \right. \right.$$

$$\left. + 12 \phi_0 \sum_{n,k=1}^{N} \left( \phi_n \phi_k \phi_{n+k}^* + h.c. \right) + \sum_{n,k,l=1}^{N} \left( 4 \phi_n \phi_k \phi_l \phi_{n+k+l}^* + h.c. + 6 \phi_n \phi_k \phi_l^* \phi_{n+k-l}^* \right) \right\},$$

and where the following redefinitions have been used $\|$ \[\|

$$M_5^2 = 2L M_5^2, \hspace{1cm} \phi_i = \sqrt{2L} \hat{\phi}_i, \hspace{1cm} \lambda = \frac{\lambda}{2LM_5} \simeq \frac{\hat{\lambda}}{N}$$  \hspace{1cm} (6)

Note that, for large $N$, the 4-dimensional fields $\phi_i$ are considerably less strongly coupled than the original 5-dimensional one.

$^2$A remedy, in the case of coupled scalar fields with exponential potentials, has been proposed by Copeland, Mazumdar and Nunes $\|$.

$^3$We assume that $G_{55} = e^{2\gamma}$ is fixed and we ignore the KK gauge field $G_{\mu 5} = e^{2\gamma} A_\mu$. 

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In order to map the above system of multiple, heavily-coupled scalar fields to a theory of a single, self-interacting field, which is known to lead to chaotic inflation, we need to determine the late-time attractor of the system. For that purpose, we make the following simplifications and assumptions:

1. We substitute the $N$ complex KK fields with $2N$ real KK fields.
2. We impose the periodic condition $\phi_{2N+n} = \phi_n$ in order to resolve the asymmetry with which the two “boundary fields” $\phi_1$ and $\phi_{2N}$ enter the Lagrangian.
3. We assume that $m_n^2 << \lambda \phi_0^2/2$, i.e. that the KK masses can been considered negligible.

Then, the late-time attractor is easily determined and is found to be the unique one and to have all the fields equal, thus, reducing the theory of multiple fields to the following one:

$$- \mathcal{L}_{4D} = \frac{1}{2} (\partial \tilde{\phi})^2 + A(N, q) \frac{\lambda}{4!} \tilde{\phi}^4,$$

(7)

where, now, the inflaton field is defined as

$$\tilde{\phi} = \sqrt{1 + \frac{2N}{q^2} \phi_0}, \quad \phi_0 = q \phi_n.$$

(8)

Note that, apart from the coefficient $A(N, q)$, the inflaton field is a self-interacting field with a quartic potential. The system of equations of motion accepts two solutions for the proportionality coefficient which give $q \sim N$ and lead to $A(N, q) \sim 1$ and, thus, to a model of chaotic inflation where the inflaton field is significantly more weakly coupled than the original, 5-dimensional one \[^4\]. This is exactly the type of solution we were searching for. If we assume that $\hat{V}(\hat{\phi}) \sim M_5^2$, we obtain the 4-dimensional constraint $\tilde{V}(\tilde{\phi}) \sim \lambda \tilde{\phi}^4 \sim M_5^2 M_5^2$. Substituting the values $\lambda \sim 10^{-12}$ and $\tilde{\phi} \sim M_P$, which are necessary for the occurrence of chaotic inflation, we are led to the constraint $M_5 \geq 10^{-6} M_P$ and subsequently to $\lambda \geq 10^{-12} \hat{\lambda}$.

We may, then, conclude that by starting with a 5-dimensional, scalar field theory with a coupling parameter of $O(1)$, we obtain a 4-dimensional, effective theory of a single, self-interacting scalar field which may have a coupling parameter as small as $10^{-12}$ without the need of any fine-tuning. Unfortunately, the above model does not lead to the relaxation of the large initial conditions on the inflaton field, however, a similar theory based on a 5-dimensional, non-interacting, massive field \[^5\] can provide a remedy for this problem, too.

\[^4\]There is another solution for $q$, namely $q = const$, which leads to $A(N, q) \sim N$ and, thus, to an inflaton field which is equally strongly coupled compared to the 5-dimensional field $\hat{\phi}$.
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