C3IT-2012

Design and Simulation of Generalized Analytical Predictor with Adaptation Filter for an SOPDT MIMO system

T.Anushalalitha\(^a\), D.Sivakumar\(^b\)

\(^a\)BMS college of Engineering, Bull Temple Road, Bangalore - 560019, India
\(^b\)Annamalai University, Annamalai Nagar, Chidambaram - 608002, India

Abstract

This paper deals with the control of a multi-input, multi-output chemical process that involves significant time delay using Generalized Analytical Predictor. The Generalized Analytical Predictor (GAP) provides a truly generalized approach to dead time compensations and significant improvement has been achieved in the regulatory response as compared to the Internal Model Control (IMC). An adaptation filter based approach is applied to a higher order distillation column model and an improved performance is obtained with GAP compared to IMC.

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Keywords: Generalized analytical predictor; MIMO; Distillation column control; Adaptation filter

1. Introduction

Time delays are common characteristic of many industrial processes due to transportation delays associated with fluid flow and the time required to complete a composition analysis etc. The detrimental effects of time delays on closed loop stability and feedback control are well known. Two popular techniques to combat time delays are the Smith Predictor and the analytical predictor [1]. A generalized analytical predictor (GAP) that allows the standard Analytical Predictor approach to be used with any feedback controller is designed for a multi-input, multi-output system (MIMO).

2. Distillation column control

A simple control scheme is considered for the distillation system. The main objectives are to maintain product compositions, \(X_D\) (overhead distillate product composition) and \(X_B\) (bottom product composition) at their set points. The manipulated variables are steam rate \(Q_s\) and reflux rate \(Q_R\) and the controlled variables are \(X_B\) and \(X_D\) as in Fig 1(a).

2.1. The generalized analytical predictor
The general block diagram of the generalized analytical predictor is given in Fig.1(b). Wellons and Edgar have done an interesting comparative study showing the advantages of generalized analytical predictor[2]. In generalized analytical predictor, the disturbance predictor transfer function \( A^* (z) \) is included in the design of internal model control (IMC). The Wood and Berry methanol/water distillation column model reported in [3] is a typical MIMO plant with strong interaction between the overhead composition and the bottoms composition with significant time delays given by

\[
\begin{align*}
\begin{bmatrix}
y'_1(s) \\
y'_2(s)
\end{bmatrix}
=&
\begin{bmatrix}
12.8e^{-s} & -18.9e^{-3s} \\
16.7s + 1 & 14.9s + 1 \\
6.6e^{-7s} & -19.4e^{-3s} \\
10.9s + 1 & 14.4s + 1 \\
\end{bmatrix}
\begin{bmatrix}
u'_1(s) \\
u'_2(s)
\end{bmatrix}
+ 
\begin{bmatrix}
3.8e^{-3s} \\
14.9s + 1 \\
4.9e^{-3s} \\
13.2s + 1
\end{bmatrix}L(s)
\end{align*}
\]

(1)

The design of generalized analytical predictor for a multiple input multiple output system is done in the following series of steps, assuming there is no process modeling error, there is perfect load modeling and the load disturbance is a step input.

- Discretization of the process model transfer function matrix: The process model transfer function in discrete time domain is \( G_p(z) = Z \{ G_ho(s) * G_p(s) \} \)  (2)

- Discretization of the load transfer function matrix: The load transfer function is \( G_L(z) = Z \{ G_{ho}(s) * G_L(s) \} \)  (3)

- The filter transfer function is, \( F_L(z) = \frac{1 - \alpha}{1 - \alpha z^{-1}} \)  (4)

- Time delay Factorization: The process model is factored so that \( G_p(z) = G_v(z) * G_o(z) \) where \( G_p(z) \) contains the dead time and the non-minimum phase (non-invertible) factors and \( G_v(z) = \begin{bmatrix}
z^{-2} & 0 \\
0 & z^{-4}
\end{bmatrix} \)  (5)

- Design of the Controller Transfer Function matrix: The controller transfer function is \( G_c(z) = [G_o(z)]^{-1} \)  (6)

- Design of multivariable disturbance predictor transfer function matrix: The disturbance predictor which is not present in IMC design is \( A^*(z) = \frac{1 - N}{a} + \frac{1 - \bar{N}}{1 - \bar{a}} \cdot \frac{F_L(z)(1 - \bar{a}z^{-1})}{1 - \bar{a}} \)  (7)

- For GAP, the controller filter is \( F_c(z) = \frac{1 - \beta}{1 - \beta z^{-1}} \)  (8)
For deadbeat response, $\beta = 0$. For the purpose of detuning, the value of $\beta$ is varied.

2.2. Adaptation filter design for an SOPDT MIMO system

The design of generalized analytical predictor with adaptation filter is carried out for a second order plus dead time system (SOPDT). It is possible to have a straightforward adaptation filter formulation in the generalized analytical predictor for plants which can be modeled by polynomials of any order [4].

![Fig 2. Generalized analytical predictor with adaptation filter](image)

- The process model is taken to be $G_p(s) = \left[ \begin{array}{c} 1.6e^{-1.3s} & -1.2e^{-1.05s} \\ \frac{(13s+1)(3s+1)(15.5s+1)(3s+1)}{(-7.5e^{-2.3s})} & 23.1e^{-s} \\ \frac{(37.3s+1)(2s+1)(42s+1)(2s+1)}{(-7.5e^{-2.3s})} & 23.1e^{-s} \end{array} \right]$ (9)

- The transfer function of the second order model has the form

$$M(s) = \frac{K_m(1 + \tau_1 s)e^{-\theta_m s}}{(1 + \tau_{1m}s)(1 + \tau_{2m}s)} = M_0(s)e^{-\theta_m s}H$$

in which $\tau_1, \tau_{1m}, \tau_{2m}$ are time constants and $\theta_m$ is the delay. $\theta_m$ can be expressed in terms of the sampling interval $T$ as $\theta_m = \lambda T - \lambda_0 T$. Taking into account the sample and hold effect, the model of the process is taken to be the same as that of the actual process transfer function.

- Filter transfer function:

$$F_2(z) = \frac{A_0^{l+1} - A_0^{l+1}}{A_0^{l+1} - A_0} + \frac{A_0^{l+1} - A_0^{l+1}}{A_0^{l+1} - A_0}z^{-1}$$

with $A_{01} = e^{-T/\tau_{1m}}, A_{02} = e^{-T/\tau_{2m}}$

- Design of Decoupling Controller Transfer Function: The controller can be expressed in terms of the model $M(z)$ as $M_+(z)M_-(z)$

$$M(z) = \begin{bmatrix} 1 - e^{-Tz} \\ s \end{bmatrix} M_0(s)e^{-\theta_m s} = \begin{bmatrix} B_0^2 z^2 + B_1^2 z + B_2^2 \\ A_0^2 z^2 + A_1^2 z + A_2^2 \end{bmatrix} z^{-\lambda}$$

$M_+(z)$ includes zeros outside the unit circle, $M_+(z)$ includes all the delays, $M_+(z)$ includes all the zeros whose real part belongs to the interval (-1, 0) and $M_-(z)$ contains the remaining terms of the model. $F_{rc}(z)$ is the second order adaptation filter and $f(z)$ is the filter that can be used for robustness.
The diagonal controller transfer matrix is obtained from
\[
G_c(z) = \frac{1}{f(z)F_{rc}(z)M(z)}
\] (13)

Design of second order adaptation filter with load disturbance gives
\[
D_2(z) = \frac{K_I(1-A_{01}z^{-1})(1-A_{02}z^{-2})}{1-[1-K_IK_m(1-A_{01})(1-A_{02})]z^{-1}}
\] (14)
\[
\gamma_2(z) = K_m(1-F_2(1))
\] (15)

3. Results of simulation

The simulations are done for generalized analytical predictor and internal model control using Simulink for a step load disturbance of +0.34 pound/minute in the feed flow rate for wood and Berry model of distillation column Fig.3(a). Responses for a change in set point are also obtained. The tuning factor \( \beta \) or \( k_I \) is varied and the Integral square error (ISE), integral absolute error (IAE) and Integral time absolute error (ITAE) are computed. The GAP response returns to its set point faster than that of IMC. The controller actions show the improvement obtained with GAP. The regulatory actions of IMC for \( \beta = 0.7 \) is slightly better for bottoms composition. But that is not quite high compared with the regulatory actions of GAP for overheads composition. The GAP output obtained for the servo response for a second order plus dead time model of a distillation column is shown in Fig .3(b). The results are indicative of the effect of adaptation filter in the structure of generalized analytical predictor.

Table 1. gives the performance comparison for Wood and Berry model. The filter constant \( \beta \) is varied from 0.1 to 0.999 and the errors decreased with increasing \( \beta \). The ITAE values show a marked difference in error between the GAP and the IMC. As the value of \( \beta \) is raised towards unity, the difference between the value of error narrows down. In order to eliminate the trade off between performance and robustness.
an optimum value of filter constant can be chosen. Tables 2, 3 are the performance comparisons for a higher order MIMO system with adaptation filter. The Generalized Analytical Predictor has been found to be more improved compared to the IMC.

Table 1. Performance comparison between GAP and IMC based on IAE

| β     | GAP  | IMC  | GAP  | IMC  |
|-------|------|------|------|------|
|       | Y₁  | Y₂   | Y₁  | Y₂   |
| 0.1   | 0.9074 | 1.678 | 0.908 | 1.678 |
| 0.5   | 0.8924 | 1.647 | 0.9022 | 1.666 |
| 0.95  | 0.4149 | 0.6585 | 0.6094 | 1.069 |
| 0.999 | 0.001149 | 0.00115 | 0.02774 | 0.02896 |

Table 2. Performance comparison between GAP and IMC based on ISE

| τ     | GAP  | IMC  | GAP  | IMC  |
|-------|------|------|------|------|
|       | Y₁  | Y₂   | Y₁  | Y₂   |
| 5     | 0.3269 | 1.012e055 | 0.3063 | 4.14e057 |
| 3     | 0.3726 | 2.22e060 | 0.9614 | 7.071e062 |
| 1     | 0.7055 | 1.52e077 | 2.312 | 2.2e079 |
| 0     | 0.2662 | 4.516e056 | 0.6136 | 3.566e059 |

Table 3. Performance comparison between GAP and IMC based on ITAE

| τ     | GAP  | IMC  | GAP  | IMC  |
|-------|------|------|------|------|
|       | Y₁  | Y₂   | Y₁  | Y₂   |
| 0     | 0.4525 | 17.65 | 0.1242 | 38.6 |
| 0.2   | 0.0003424 | 0.01032 | 5.669e-005 | 0.019 |
| 0.3   | 0.0008887 | 0.01032 | 0.019 | 0.0001441 |
| 0.4   | 0.001844 | 0.009681 | 0.0002926 | 0.01775 |

4. **Future enhancements**

The Generalized Analytical Predictor with adaptation filter can be applied to any higher order multiple input multiple output (MIMO) process. The stability and robustness analysis can be done.

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