Unitarity constraints and role of geometrical effects in deep–inelastic scattering and vector–meson electroproduction

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Abstract

Deep–inelastic scattering at low $x$ and elastic vector meson electroproduction are analyzed on the basis of the $s$–channel unitarity extended to off–shell particle scattering. It appeared that the role of unitarity is important but contrary to the case of the on-shell scattering it does not rule out power–like behavior of the total cross–sections. We discuss behavior of the total cross–section of virtual photon–proton scattering in the geometrical approach and obtain that the exponent of the power–like energy dependence of $\sigma_{tot}^p$ is related to the constituent quark interaction radius. The mass effects and energy dependence of vector meson electroproduction are discussed alongside with the angular distributions at large momentum transfers in these processes.
Introduction

Rising dependence of the virtual–photon proton scattering total cross–section on the center of mass energy $W^2$ discovered at HERA \cite{1} led to the renewed interest in the mechanism of diffraction at high energies. Such behavior was in fact predicted in \cite{2} and expected in perturbative QCD \cite{3}. The total virtual photon–proton cross–section is related to the structure function $F_2$ at small $x$. The HERA effect is consistent with various $W^2$ – dependencies of $\sigma_{\text{tot}}$ and has been explained in the different ways, among them is a manifestation of hard BFKL Pomeron \cite{4}, an appearance of the DGLAP evolution in perturbative QCD \cite{3}, a transient phenomena, i.e. preasymptotic effects \cite{5} or a true asymptotical off–mass–shell scattering amplitude behavior \cite{6}. There is an extensive list of papers devoted to this subject and many interesting results are described in the review papers (cf. e.g. \cite{1,7}). The strong rise of the structure function $F_2$ at small $x$ which can be described by the power-like dependence $\frac{\sigma_{\text{tot}}}{W^2} \propto \frac{1}{Q^2}$ implies that

$$\frac{\sigma_{\text{tot}}}{W^2} \propto \frac{1}{Q^2} \quad (1)$$

with $Q^2$ rising with $Q^2$ from about 0±1 to about 0±5 and is regarded as a somewhat surprising fact. It is due to the fact that according to the experimental data an energy dependence of the total cross–sections in hadronic interactions, the total cross–section increase is rather slow (0±1). The mentioned variance may be regarded as not a fundamental one. First, there is no Froissart–Martin bound for the case off–shell particles \cite{2,8}. Additional assumptions are needed to reinstate this bound \cite{11,12}. Second, it cannot be granted that the preasymptotic effects and approach to the asymptotics are the same for the on–shell and off–shell scattering. It seems that for some reasons scattering of virtual particles reaches the asymptotics faster than the scattering of the real particles.

It is worth noting that the space–time structure of the low–$x$ scattering involves large distances $l=mx$ on the light–cone \cite{13}, and the region of $x$ 0 is sensitive to the nonperturbative contributions. Deep–inelastic scattering in this region turns out to be a coherent process where diffraction plays a major role and nonperturbative models such as Regge or vector dominance model can be competitive with perturbative QCD and successfully applied for description of the experimental data.

\footnote{The most recent results of H1 Collaboration \cite{8} confirmed an $x$-independence of the exponent at low $x$ by measuring the derivative $\frac{\partial \ln F_2}{\partial \ln x}$ as a function both of $Q^2$ and of $x$ for the first time. Some provisions against this independence were mentioned in \cite{8}, moreover there are also other parameterizations which describe the experimental data equally well (cf. e.g. \cite{10}). Despite that, it seems that the parameterization \cite{8} provides the most natural way to approximate the available experimental data.}
It is essential to obey the general principles in the nonperturbative region and, in particular, to satisfy unitarity. The most common form of unitarity solution – the eikonal one – was generalized for the off–shell scattering in [6]. In this paper we consider an off–shell extension of the U–matrix approach to the amplitude unitarization. It is shown that this approach along with the respective extension of the quark model for the U–matrix [14] leads to (1), where the exponent $(Q^2)$ is related to the $Q^2$–dependent interaction radius attributed to constituent quark. These results cannot be obtained in the eikonal unitarization which reproduces bare “Born” input form with subleading corrections for the output amplitude in the case of the off–shell scattering [6]. The fundamental distinction between the two forms of amplitude unitarization is in the analytical properties in the complex energy plane [15].

It is worth noting an importance of the effective interaction radius concept [16]. The study of the effective interaction radius dependence on the scattering variables appeared to be very useful for understanding of the dynamics of high energy hadronic reactions [17, 18]. It is widely known nowadays that the respective geometrical considerations provide a deep insight in hadron dynamics and deep–inelastic scattering (cf. [19]).

Besides the studies of deep–inelastic scattering (DIS) at low $x$ the measurements of the characteristics of the elastic vector meson (VM) production were performed in the experiments H1 and ZEUS at HERA [21, 22]. It was shown that the integral cross section of the elastic vector meson production increases with energy in the way similar to the $t$ tot $p$ $(W^2; Q^2)$ dependence on $W^2$ [1]. It appeared that an increase of VM electroproduction cross–section with energy is steeper for heavier vector mesons as well as when the virtuality $Q^2$ is higher. Discussion of such a behavior in various model approaches based on the nonperturbative hadron physics or perturbative QCD can be found in (cf. e.g. [7]).

Application of approach based on the off–shell extension of the $s$–channel unitarity to elastic vector meson electroproduction $p \rightarrow V p$ allows to obtain angular dependence and predict interesting mass effects in these processes. It appears that the obtained mass and $Q^2$ dependencies do not contradict to the experimentally observed trends. It is also valid for the angular distributions at large momentum transfers.

1 Off–shell unitarity

Extension of the U–matrix unitarization scheme for the off–shell scattering was considered in brief in [11]. Here we give a more detailed treatment of this problem. We adopt a commonly accepted picture of DIS at small $x$, i.e. it is supposed that the virtual photon fluctuates into a quark–antiquark pair $q \overline{q}$ and this pair is
considered as an effective virtual vector meson state in the processes with small $x$. This effective virtual meson interacts then with a hadron. For simplicity we consider single effective vector meson field. We use for the amplitudes of the processes

$$V + h \rightarrow V + h; \quad V + h \rightarrow V + h \quad \text{and} \quad V + h \rightarrow V + h$$  \hspace{1cm} (2)

the notations $F (s; t; Q^2)$, $F (s; t; Q^2)$ and $F (s; t)$, respectively, i.e., we denoted in that way the amplitudes when both initial and final mesons are off mass shell, only initial meson is off mass shell and both mesons are on mass shell.

The unitarity relation for the amplitudes $F$ and $F$ has a similar structure as the unitarity equation for the on–shell amplitude $F$ but relates, in fact, different amplitudes. Therefore, the unitarity constraints in DIS are much less stringent than in hadron–hadron scattering. In impact parameter representation at high energies it relates the amplitudes $F$ and $F$ in the following way

$$\text{Im} F (s; b; Q^2) = F (s; b; Q^2) F (s; b; Q^2)$$ \hspace{1cm} (3)

where $(s; b; Q^2)$ is the contribution to the unitarity of many–particle intermediate on–shell states. The function $(s; b; Q^2)$ is the sum of the $n$–particle production cross–section in the process of the virtual meson interaction with a hadron $h$, i.e.

$$(s; b; Q^2) = \sum \limits_n (s; b; Q^2);$$

There is a similar relation for the functions $F$ and $F$,

$$\text{Im} F (s; b; Q^2) = F (s; b; Q^2) F (s; b; Q^2) + (s; b; Q^2);$$ \hspace{1cm} (4)

Contrary to $(s; b; Q^2)$ the function $(s; b; Q^2)$ has no simple physical meaning and it will be discussed later. The solution of the off–shell unitarity relations

![Diagram](image)

Figure 1: The solution of the off–shell unitarity relation for the amplitude $F$. 
Figure 2: The solution of the off–shell unitarity relation for the amplitude \( F \).

can be graphically represented for the amplitudes \( F \) and \( F^* \) in the Figs. I and 2 respectively and has a simple form in the impact parameter representation:

\[
\begin{align*}
F (s; b; Q^2) &= U (s; b; Q^2) + i U (s; b; Q^2) F (s; b; Q^2) \\
F (s; b; Q^2) &= U (s; b; Q^2) + i U (s; b; Q^2) F (s; b).
\end{align*}
\]

(5)

It is worth noting that the solution of the off–shell unitarity in the nonrelativistic case for a K –matrix representation was obtained for the first time in [23]. The solution of this system is the following

\[
F^* (s; b; Q^2) = U (s; b; Q^2) + i U (s; b; Q^2) F (s; b; Q^2)
\]

(6)

\[
F (s; b; Q^2) = \frac{U (s; b; Q^2) U (s; b; Q^2) F (s; b; Q^2)}{U (s; b; Q^2) U (s; b; Q^2)} F (s; b; Q^2)
\]

(7)

where the on–shell amplitude \( F (s; b) \) has the following representation

\[
F (s; b) = U (s; b) = [1 + i U (s; b)]
\]

(8)

We assume the following relation to be valid at the level of the “Born” amplitudes in the impact parameter space

\[
\frac{U}{U} = \frac{U}{U}
\]

(9)

This relation is valid, e. g. in the Regge model with factorizable residues and the \( Q^2 \)–independent trajectory. It is also valid in the off–shell extension of the chiral
quark model for the $U$–matrix which we will consider further. Eq. (9) implies the following forms for the impact parameter dependent functions $U$ and $U^*$:

$$U(s; b; Q^2) = \frac{1}{iU(s; b)} (s; b; Q^2)U(s; b)$$

$$U^*(s; b; Q^2) = \frac{1}{iU(s; b)} (s; b; Q^2)U(s; b) (s; b; Q^2)$$

(10)

This factorization may be treated as a reflection of the universality of the initial and final state interactions responsible for the transitions between the on and off mass shell states. It seems to be a quite natural assumption. Note, that this factorization does not survive for the amplitudes $F(s; t)$, $F(s; t; Q^2)$ and $F^*(s; t; Q^2)$, i.e. after Fourier-Bessel transform is performed.

Thus, we have for the amplitudes $F$ and $F^*$ the following relations

$$F(s; b; Q^2) = \frac{1}{iU(s; b)} (s; b; Q^2)F(s; b)$$

$$F^*(s; b; Q^2) = \frac{1}{iU(s; b)} (s; b; Q^2)F(s; b) (s; b; Q^2)$$

(11)

and unitarity provides inequalities

$$|F(s; b; Q^2)|^2 \leq (s; b; Q^2)$$

$$|F^*(s; b; Q^2)|^2 \leq (s; b; Q^2)$$

(13)

It is worth noting that the above limitations are much less stringent than the limitation for the on–shell amplitude $F(s; b) \leq 1$. As a result, there is no Froissart–Martin bound in DIS at low $x$ and experimentally observed power-like energy dependence of the total cross–section can represent a true asymptotical dependence.

When the function $(s; b; Q^2)$ is real we can write down a simple expression for the inelastic overlap function $(s; b; Q^2)$:

$$(s; b; Q^2) = |(s; b; Q^2)\frac{\text{Im}U(s; b)}{iU(s; b)} (s; b; Q^2)|$$

(14)

The following relation is valid for the function $(s; b; Q^2)$:

$$(s; b; Q^2) = \left[ (s; b; Q^2) (s; b) \right]^{-2}$$

(15)

Eq. (15) allows one to connect the integral

$$\left. \frac{1}{Z} \right|_0^\infty (s; Q^2) \frac{8}{b b} (s; b; Q^2)$$

with the total inelastic cross–section:

$$(s; Q^2) \left. \right|_0^\infty = \text{inel}(s)$$
2 Off–shell scattering and the quark model for $U$–matrix

The above formulas are rather general and are not useful alone under analysis of the data. We need an explicit form for the functions $U$, $U$ and $V$ and therefore a phenomenological model is to be constructed. As a starting point we use a quark model for the hadron scattering described in [14]. In this section we list the main features and then construct an off–shell extension of the model. In fact it is based on the ideas of chiral quark models. The picture of hadron structure in the model with the valence constituent quarks located in the central part and the surrounding condensate implies that the overlapping of hadron structures and interaction of the condensates occur at the first stage of collision and results in generation of the quasiparticles, i.e. massive quarks (cf. Fig. 3).

Figure 3: Schematic view of initial stage of the hadron interaction and formation of the effective field.

These quarks play role of scatterers. To estimate number of such quarks one could assume that part of hadron energy carried by the outer condensate clouds is being released in the overlap region to generate massive quarks. Then their number can be estimated by the quantity:

$$N^\prime (s; b) / (1 - k_8 \frac{D_n^D}{m_0} D_c^D) D_c^D;$$

(16)

where $m_0$ – constituent quark mass, $k_8$ – fraction hadron energy carried by the constituent valence quarks. Function $D_n^D$ describes condensate distribution inside the hadron $h$, and $b$ is an impact parameter of the colliding hadron $h$ and meson $\psi$. Thus, $N^\prime (s; b)$ quarks appear in addition to $N = n_h + n_\psi$ valence quarks. Those quarks are transient ones: they are transformed back into the condensates of the final hadrons in elastic scattering. It should be noted that we use subscript $Q$ to...
refer the constituent quark \( Q \) and the same letter \( Q \) is used to denote a virtuality \( Q^2 \). However, they enter formulas in a way excluding confusion.

\[
\begin{align*}
\text{Figure 4: Schematic view of the virtual constituent quark } Q \text{ scattering in the} \\
\text{effective field generated by } N_{sc} (s; b) \text{ scatterers, where } N_{sc} (s; b) = N (s; b) + N 1.
\end{align*}
\]

In the model the valence quarks located in the central part of a hadron are supposed to scatter in a quasi-independent way by the effective field. Due to quasi–independence of the valence quarks scatterings the basic dynamical quantity (the function \( U \) ) can be factorized. When one of the hadrons (vector meson in our case) is off mass shell, the corresponding function \( U (s; b; Q^2) \) is represented as the following product

\[
U (s; b; Q^2) = \prod_{i=1}^{n} \prod_{j=1}^{h_f Q_i (s; b)} \prod_{j=1}^{h_f Q_j (s; b; Q^2)} i:
\] (17)

The factors \( h_f Q_i (s; b) \) and \( h_f Q_j (s; b; Q^2) \) correspond to the individual quark scattering amplitudes smeared over constituent quark transverse position and the fraction of longitudinal momentum carried by this quark. Under the virtual constituent quarks \( Q \) we mean the ones composing the virtual meson. Factorization (17) reflects the coherence in the valence quark scattering, i.e. all valence quarks are scattered in the effective field simultaneously and there are no spectator valence quarks. This factorization might be considered as an effective implementation of constituent quarks’ confinement. The averaged amplitudes \( h\xi Q_i (s; b) \) and \( h\xi Q_j (s; b; Q^2) \) describe elastic scattering of a single valence on-shell or off–shell quarks \( Q \) and \( Q \), respectively, off the effective field (cf. Fig. 4). We use for the function \( h\xi Q_i (s; b) \) the following expression

\[
\xi Q_i (s; b) = [N (s; b) + N 1] V_Q (b)
\] (18)

where \( V_Q (b) \) has a simple form

\[
V_Q (b) / g \exp (m_0 b )
\]
which corresponds to quark interaction radius

\[ r_Q = \frac{m_Q}{Q^2} : \]

The function \( h_{f_Q} (s; \gamma Q^2) \) is to be written as

\[ h_{f_Q} (s; \gamma Q^2) = \left[ N (s; b) + (s) \right] V_Q (\gamma Q^2) : \tag{19} \]

In the above equation

\[ V_Q (\gamma Q^2) / g (Q^2) \exp (m_Q b = (Q^2)) \tag{20} \]

and this form corresponds to the virtual constituent quark interaction radius

\[ r_Q = (Q^2) = m_Q : \tag{21} \]

The functions \( V_Q (\gamma) \) and \( V_Q (\gamma Q^2) \) in the model are associated with the "matter" distribution inside constituent quarks and can be considered as strong formfactors.

Equations (18,19) imply that each valence quark is being scattered by all other \( N \) valence quarks belonging to the same hadron as well as to the other hadron and by \( N^* (s; b) \) quarks produced by the excitation of the chiral condensates. Due to the different radii the \( b \)-dependence of \( N (s; b) \) being weak compared to the \( b \)-dependence of \( V_Q \) or \( V_Q \) and this function can be approximately taken to be independent on the impact parameter \( b \). Dependence on virtuality \( Q^2 \) comes through dependence of the intensity of the virtual constituent quark interaction \( g (Q^2) \) and the \( (Q^2) \), which determines the quark interaction radius (in the on-shell limit \( g (Q^2) \) and \( (Q^2) \)).

Introduction of the \( Q^2 \) dependence into the interaction radius of a constituent quark which in the present approach consists of a current quark and the cloud of quark–antiquark pairs of the different flavors is the main issue of the off–shell extension of the model and the origin of this dependence and its possible physical interpretation will be discussed in Section 6.

According to these considerations the explicit functional forms for the generalized reaction matrices \( U \) and \( U \) can easily be written in the form of (10) with

\[ (s; b^2 Q^2) = \frac{h_{f_Q} (s; b^2 Q^2)}{h_{f_Q} (s; b)^2} : \tag{22} \]

Note that (9) and (10) imply that the amplitude of the process \( Q \rightarrow Q \) is the following

\[ h_{f_Q} (s; b^2 Q^2) = \left[ h_{f_Q} (s; b^2 Q^2) i h_{f_Q} (s; b) i \right]^{1/2} : \]
We consider the high–energy limit and for the simplicity assume here that all the constituent quarks have equal masses and parameters $g$ and $b$ as well as $g(Q^2)$ and $b(Q^2)$ do not depend on quark flavor. We also assume pure imaginary amplitudes. Then the functions $U$, $U'$ and $U''$ are

$$U(s; b) = ig^N \frac{s}{m_Q^2} \exp \left( \frac{m_Q b}{s} \right)$$

$$U'(s; b) = g(Q^2)U(s; b); \quad U''(s; b) = b^2 g(Q^2)U(s; b)$$

where the function $!$ is an energy-independent one and has the following dependence on $b$ and $Q^2$

$$! (b; Q^2) = \frac{g(Q^2)}{g} \exp \left( \frac{m_Q b}{s} \right)$$

with

$$\langle Q^2 \rangle = \langle \frac{Q^2}{s} \rangle$$

3 Total cross–sections of $p$ interactions

With explicit forms of the functions $U$, $U'$ and $U''$ the corresponding scattering amplitudes can be calculated. The most simple case is the forward scattering at large energies. For the on–shell scattering when $! = 1$ at large $W^2$, the total photoproduction cross–section has a Froissart-like asymptotic behavior

$$\frac{\text{tot}}{p} (W^2) / \frac{2}{m_Q^2} \ln^2 \frac{W^2}{m_Q^2};$$

where the usual for DIS notation $W^2$ instead of $s$ is used. Similar result is valid also for the off mass shell particles if the interaction radius of virtual quark does not depend on $Q^2$ and is equal to the interaction radius of the on–shell quark, i.e. $\langle Q^2 \rangle$. The behavior of the total cross–section at large $W^2$

$$\frac{\text{tot}}{p} (W^2) / \frac{g(Q^2)}{g m_Q} \frac{2}{m_Q^2} \ln^2 \frac{W^2}{m_Q^2};$$

We consider next the off-shell scattering and suppose now that $\langle Q^2 \rangle \not= \langle Q^2 \rangle$. It should be noted first that for the case when $\langle Q^2 \rangle < \langle Q^2 \rangle$ the total cross–section would be energy-independent

$$\frac{\text{tot}}{p} (W^2) / \frac{g(Q^2)}{g m_Q} \frac{2}{m_Q^2} \ln^2 \frac{W^2}{m_Q^2};$$

10
in the asymptotic region. This scenario would mean that the experimentally observed rise of $t_{p\text{tot}}$ is transient presymptotic phenomenon. It can be realized when we replace the mass $m_0$ by the quantity $m_0 = m_0^2 + Q^2$ in order to obtain the interaction radius of the off-shell constituent quark and write it down as $r_Q = m_0$, or equivalently replace by $(Q^2) = m_0 = m_0^2 + Q^2$. The above option cannot be excluded in principle, however, it is a self-consistent choice in the framework of the model only at large $Q^2$, since it was originally supposed that the function is universal for the different quark flavors.

However, when $(Q^2) >$ the situation is different and we have at large $W^2$

\[
tot p(W^2; Q^2) / G(Q^2) \sim \left( \frac{W^2}{m_0^2} \right) \ln \left( \frac{W^2}{m_0^2} \right); \tag{29}
\]

where

\[
(Q^2) = \frac{(Q^2)}{(Q^2)}; \tag{30}
\]

We shall further concentrate on this self-consistent for any $Q^2$ values and the most interesting case.

All the above expressions for $t_{p\text{tot}}(W^2)$ can be rewritten as the corresponding dependencies of $F_2(x; Q^2)$ at small $x$ according to the relation

\[
F_2(x; Q^2) = \frac{Q^2}{4} \frac{t_{p\text{tot}}(W^2)}{G(Q^2)}; \tag{29'}
\]

where $x = Q^2/W^2$.

In particular, (29) will appear in the form

\[
F_2(x; Q^2) / G(Q^2) \sim \left( \frac{1}{x} \right) \ln (1/x); \tag{31}
\]

It is interesting that the value and $Q^2$ dependence of the exponent $(Q^2)$ is related to the interaction radius of the virtual constituent quark. The value of parameter in the model is determined by the slope of the differential cross-section of elastic scattering at large $t$ [26], i.e.

\[
\frac{d}{dt} / \exp \frac{2}{m_0 N} \frac{p - t}{t}; \tag{32}
\]

and from the pp-experimental data it follows that $x = 2$. Then from the data for $(Q^2)$ obtained at HERA [8] we can calculate the “experimental” $Q^2$–dependence
of the function $(Q^2)$:

$$(Q^2) = \frac{1}{\log(Q^2)}; \quad (33)$$

Evidently experiment indicates that $(Q^2)$ rises with $Q^2$. This rise is slow and consistent with $\ln Q^2$ extrapolation (Fig. 5):

$$(Q^2) = + a \ln 1 + \frac{Q^2}{Q_0^2};$$

where $a = 0.172$ and $Q_0^2 = 0.265 \text{ GeV}^2$. Assuming this dependence for higher values of $Q^2$ and using (30), we then predict saturation of $(Q^2)$ at large $Q^2$, i.e. the flattening should take place:

$$(Q^2) = a \ln 1 + \frac{Q^2}{Q_0^2} = + a \ln 1 + \frac{Q^2}{Q_0^2};$$

Increase of $(Q^2)$ corresponds to the increasing interaction radius of constituent quarks from the virtual vector meson which is illustrated on Fig. 6.
4 Elastic vector meson production

The calculation of the elastic and inelastic cross–sections can also be directly performed similar to the calculation of the total cross-sections using (23), (24) and (25) and integrating the functions $F(s; t; Q^2)$ and $(s; b; Q^2)$ over impact parameter. The following asymptotic dependencies for the cross–sections of elastic scattering and inelastic interactions are obtained in that way

$$\frac{\sigma_{el}}{\sigma_{el}}(W^2; Q^2) / G_e(Q^2) = \frac{W^2}{m_Q^2} \ln \frac{W^2}{m_Q^2}$$

(34)

and

$$\frac{\sigma_{inel}}{\sigma_{inel}}(W^2; Q^2) / G_i(Q^2) = \frac{W^2}{m_Q^2} \ln \frac{W^2}{m_Q^2}$$

(35)

with the universal exponent $\langle Q^2 \rangle$ given by Eq. (30).

The above relations imply that the ratios of elastic and inelastic cross–sections to the total one do not depend on energy. In order to confront these results with experimental data it is useful to keep in mind that as it was noted in [20], diffraction of the virtual photon includes both elastic and inelastic scattering of its fluctuations.

Now we consider elastic (exclusive) cross–sections both for the light and heavy vector mesons production. We assumed earlier that the virtual photon before the interaction with the proton fluctuates into the $Q\bar{Q}$ pair and for simplicity we limited ourselves with light quarks under discussion of the total cross–section. Therefore we need to get rid of the light quark limitation and extend the above approach in order to include the quarks with the different masses. The inclusion, in particular, of heavy vector meson production into this scheme is straightforward: the
virtual photon fluctuates before the interaction with proton into the heavy quark–antiquark pair which constitutes the virtual heavy vector meson state. After the interaction with a proton this state becomes a real heavy vector meson.

Integral exclusive (elastic) cross–section of vector meson production in the process $p \rightarrow V p$ when the final state vector meson contains not only the light quarks can be calculated directly according to the above scheme and formulas of Section 2:

$$\frac{V(p, W^2; Q^2)}{G_V(Q^2)} \propto \frac{W^2}{m_Q^2} \ln \frac{W^2}{m_Q^2};$$

where

$$V(Q^2) = \langle Q^2 \rangle \frac{\bar{m}_Q}{\bar{m}_Q};$$

In (37) $\bar{m}_Q$ denotes the mass of the constituent quarks from the vector meson and $\bar{m}_Q$ is the mean constituent quark mass of the vector meson and proton system. Evidently $V(Q^2) = \langle Q^2 \rangle$ for the light vector mesons. In the case when the vector meson is very heavy, i.e. $m_Q < \bar{m}_Q$ we have

$$V(Q^2) = \frac{5}{2} \langle Q^2 \rangle;$$

We conclude that the respective cross–section rises faster than the corresponding cross–section of the light vector meson production, e.g. (37) results in

$$J = \langle Q^2 \rangle \quad 2 \langle Q^2 \rangle;$$

The above results are in a qualitative agreement with the trends observed in the HERA experiments [21, 22].

5 Angular structure of elastic vector meson production

Now we turn to calculation of the scattering amplitudes at $t \in 0$. It will allow us to get a differential cross-sections and to confront the results with the first measurements of angular distributions at large $t$ in the light vector meson production [24]. It was found that the angular distribution in the proton–dissociative processes [25] is consistent with the power dependence $(t)^3$. Calculation of the differential cross–sections in elastic vector meson production can be performed using the analysis of the singularities of the amplitudes in the complex impact parameter plane which was applied for elastic hadron scattering in [26]. There are
different approaches to the vector meson production, e.g. recent application of the geometrical picture was given in [27]. Angular distributions can be described also in the approaches based on the perturbative QCD [28, 29, 30] which provides smooth power-like \( t \)-dependence. Brief review of the recent results of these approaches can be found in [31].

Since the integration in the Fourier-Bessel transform goes over the variable \( b^2 \) rather than \( b \) it is convenient to consider the complex plane of the variable \( b^2 \) and analyze singularities in \( \beta \)-plane. Using (11) we can write down the integral over the contour \( C \) around a positive axis in the \( \beta \)-plane:

\[
F \left( \Omega^2; t; Q^2 \right) = \int_{C} F \left( \Omega^2; \beta; Q^2 \right) K_0 \left( p \sqrt{t} \right) \beta \text{d}\beta;
\]

where \( K_0 \) is the modified Bessel function and the variable \( \Omega^2 \) was used instead of the variable \( s \). The contour \( C \) can be closed at infinity and the value of the integral will be then determined by the singularities of the function \( F \left( \Omega^2; \beta; Q^2 \right) \) in the complex \( \beta \)-plane (Fig. 7), where

\[
F \left( \Omega^2; \beta; Q^2 \right) = \frac{1}{\Omega^2} \frac{U(\Omega^2; Q^2)}{\Omega U(\Omega^2; \beta)};
\]

Figure 7: The singularities of the scattering amplitude in the complex \( \beta \)-plane.

With the explicit expressions for the functions \( U \) and \( ! \) we conclude that the positions of the poles which are determined by solutions of the equation

\[
1 \frac{\Omega^2}{\Omega U(\Omega^2; \beta)} = 0
\]

are located at

\[
n \Omega^2 = \left[ R(\Omega^2) + \frac{i}{M} \right] n^2; \quad n = 1; 3; \ldots
\]
where $M = m_0 n_v + m_0 n_h$ and

$$R(\tilde{W}^2) = \frac{n}{M} \ln \frac{\tilde{W}^2}{m_0^2}.$$  

The location of the poles does not depend on the virtuality $Q^2$.

Besides the poles the function $F(\tilde{W}^2; \Delta Q^2)$ has a branching point at $\Delta = 0$ and

$$\text{disc } F(\tilde{W}^2; \Delta Q^2) =$$

$$\frac{\text{disc } [\ln(\Delta Q^2) U(\tilde{W}^2; \Delta Q^2)]}{[1 - \text{disc } U(\tilde{W}^2; \Delta Q^2)]} =$$

i.e.

$$\text{disc } F(\tilde{W}^2; \Delta Q^2) = \text{disc } (\Delta Q^2)$$

since at $\tilde{W}^2 = 1$ the function $U(\tilde{W}^2; \Delta Q^2)$ is 1 at fixed $\Delta Q^2$. As a result the function $F(\tilde{W}^2; t; Q^2)$ can be represented as a sum of pole and cut contributions, i.e.

$$F(\tilde{W}^2; t; Q^2) = F_p(\tilde{W}^2; t; Q^2) + F_c(\tilde{W}^2; t; Q^2).$$

The pole and cut contributions are decoupled dynamically when $\tilde{W}^2 = 1$. Contribution of the poles determines the amplitude $F(\tilde{W}^2; t; Q^2)$ in the region $\tilde{W}^2 > 1$ and it can be written in a form of series:

$$F(\tilde{W}^2; t; Q^2) = \sum_{n=1}^{\infty} \exp \left[ \frac{i n}{N} \nu(Q^2) \right] \frac{p}{n} K_0 \left( \frac{p}{n} \right).$$

At moderate values of $t$ when $t \approx 1$ (GeV/\(c\)) the amplitude (39) leads to the Orear type behavior of the differential cross-section which is similar to the Eq.(32) for the on-shell amplitude, i.e.

$$\frac{d}{dt} \exp \left[ \frac{2 \tilde{W}^2}{M} \right] \approx 0.$$  

Note that at small $t$ the behavior of the differential cross-section is complicated. The oscillating factors $\exp \left[ \frac{i n}{N} \nu(Q^2) \right]$ absent in the on-shell scattering amplitude, play a role.
At large $t$ the poles contributions is negligible and contribution from the cut at $t = 0$ is a dominating one. It appears that the function $P_c (W^2, t, Q^2)$ does not depend on energy and differential cross section depends on $t$ in a power-like way

$$\frac{d\sigma}{dt} \propto Q^2 (Q^2)^3.$$ (41)

Therefore for large values of $t (t \approx m^2_0 = 2(Q^2))$ we have a simple $(t)^3$ dependence of the differential cross-section. This dependence is very distinct from the corresponding behavior of the differential cross-section of the on-shell scattering [14] which approximates the quark counting rule [32] because due to the off-shell unitarity effects. It is worth noting that the ratio of the two differential cross-sections for the production of the vector mesons $V_1$ and $V_2$ does not depend on the variables $W^2$ and $t$ at large values of $t$.

6 Impact parameter picture

The results described above rely on the off–shell unitarity and the $Q^2$–dependence of constituent quark interaction radius. It is useful to consider an impact parameter picture to get insight into the physical origin of this $Q^2$–dependence. An impact parameter analysis of the experimental data was a particular tool for the detection of the unitarity effects in hadronic reactions [33] and, as it was proposed in [20], similar technique can be used in the diffractive DIS. Impact parameter profile of the amplitude is peripheral when $(Q^2)$ increases with $Q^2$ (Fig. 8). The dependence on virtuality of constituent quark interaction radius was assumed and this dependence appeared to be in a qualitative agreement with the experimental data. It was demonstrated then that the rising dependence of the constituent quark interaction radius with virtuality implies the rising $Q^2$-dependence of the exponent $(Q^2)$. The relation between $(Q^2)$ and $(Q^2)$ implies in its turn a saturation of the $Q^2$-dependence of $(Q^2)$ at large values of $Q^2$. The reason for the increase of the constituent quark interaction radius with virtuality should have a dynamical nature and it could originate from the emission of the additional $q\bar{q}$-pairs in the nonperturbative structure of a constituent quark. In the present approach constituent quark consists of a current quark and the cloud of quark–antiquark pairs of the different flavors [14]. It was shown that the available experimental data imply $ln Q^2$-dependence for the radius of this cloud.

The peripheral profile of the amplitude in its turn can result from the increasing role of the orbital angular momentum of the quark–antiquark cloud when the virtual particles are considered. The generation of $q\bar{q}$-pairs cloud could be considered in analogy with the theory of superconductivity. It was proposed [34] to push
further this analogy and consider an anisotropic extension of the theory of superconductivity which seems to match well with the above nonperturbative picture for a constituent quark. The studies \([35]\) of that theory show that the presence of anisotropy leads to axial symmetry of pairing correlations around the anisotropy direction \(\hat{\mathbf{l}}\) and to the particle currents induced by the pairing correlations. In another words it means that a particle of the condensed fluid is surrounded by a cloud of correlated particles ("hump") which rotate around it with the axis of rotation \(\hat{\mathbf{l}}\). Calculation of the orbital momentum shows that it is proportional to the density of the correlated particles. The value of the orbital momentum contribution into the spin of constituent quark can be estimated according to the relation between contributions of current quarks into a proton spin and corresponding contributions of current quarks into a spin of the constituent quarks and that of the constituent quarks into proton spin.

It is evident that the results of the impact parameter analysis are in favor of the increasing role of the orbital angular momenta with virtuality.

7 Conclusion

We considered limitations the unitarity provides for the \(p\)-total cross-sections and geometrical effects in the model dependence of \(\sigma_{\text{tot}}^p\). In particular, it was shown that the \(Q^2\)-dependent constituent quark interaction radius can lead to a nontrivial, asymptotical result: \(\sigma_{\text{tot}}^p(\mathbf{W}^2, Q^2)\), where \((Q^2)\) will be saturated at large values of \(Q^2\). This result is valid when the interaction radius of the virtual constituent quark is rising with the virtuality \(Q^2\). The data for the structure functions at low values of \(x\) continue to demonstrate the rising total cross-section of \(p\)-interactions and therefore we can consider it as a reflection of the rising
with virtuality interaction radius of a constituent quark. Thus, we have shown that the power-like parameterization of the experimental data \( \frac{\text{tot}}{p} (W^2) (Q^2) \) with \( Q^2 \)-dependent exponent can have a physical ground and should not be regarded merely as a convenient way to represent the data. Other scenarios which are consistent with unitarity have also been discussed. General conclusion is the following: unitarity itself does not lead to the saturation at \( x, 0 \), i.e. slow down of the power-like energy dependence of \( \frac{\text{tot}}{p} \) and transition to the energy behavior consistent with the Froissart–Martin bound valid for the on–shell scattering.

In elastic vector meson electroproduction processes the mass and \( Q^2 \) dependencies of the integral cross–section of vector meson production are related to the dependence of the interaction radius of the constituent quark \( Q \) on the respective quark mass \( m_Q \) and its virtuality \( Q^2 \). The behavior of the differential cross–sections at large \( t \) is in large extent determined by the off-shell unitarity effects. The smooth power-like dependence on \( t \) is predicted. New experimental data would have an essential meaning for discrimination of the model approaches and studies of the interplay between the non-perturbative and perturbative QCD regimes.

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