Nuclear Magnetic Resonance Implementations of Remote State Preparation of Arbitrary Longitudinal Qubit and Remote State Measurement of a Qubit

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Abstract

A qubit chosen from equatorial or polar great circles on a Bloch sphere can be remotely prepared with an Einstain-Podolsky-Rosen (EPR) state shared and a cbit communication. We generalize this protocol into an arbitrary longitudinal qubit on the Bloch sphere in which the azimuthal angle $\phi$ can be an arbitrary value instead of only being zero. The generalized scheme was experimentally realized using liquid-state nuclear magnetic resonance (NMR) techniques. Also, we have experimentally demonstrated remote state measurement (RSM) on an arbitrary qubit proposed by Pati.

PACS numbers: 03.67.Hk, 03.65.Ud

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I. INTRODUCTION

Quantum entanglement is a novel attribute in quantum world that has no classical counterpart. It is viewed not just as a tool for exposing the weirdness of quantum mechanics, but as a potentially essential and valuable resource for quantum communication. By exploiting entangled quantum states, one can perform tasks that are otherwise difficult or even impossible, e.g., transmitted quantum states using local operations and classical communication (LOCC) in absence of a quantum communications channel linking the sender of the quantum state to the recipient. Among them are quantum teleportation [1] and remote state preparation (RSP) [2–4], which relies on the peculiar and apparently non-local correlations inherent in entangled states.

Quantum teleportation [1,5–7] of an unknown state is achieved from Alice to Bob by a shared EPR state (one ebit) and two classical bits (cbits) communication, whereas remote state preparation [2–4] of a quantum state known to Alice but unknown to Bob can be completed by one ebit shared and only one cbit communication. In previous literatures [2–4], the qubit in RSP can only be chosen from a special ensemble, i.e., a qubit chosen from equatorial or polar great circles on a Bloch sphere, which had been experimentally demonstrated by liquid-state nuclear magnetic resonance techniques [8]. While RSP of an arbitrary qubit can succeed only half of the time. Though Bob cannot entirely get an arbitrary qubit in remote state preparation, Pati [2] also showed that it is possible for Bob to simulate with 100% efficiency any single-particle measurement on an arbitrary qubit known to Alice but unknown to him, in virtue of an EPR pair and one cbit communication.

In this paper, based on, we generalize Pati's scheme [2] for RSP of a special ensemble of qubits into qubits from arbitrary longitudinal lines on the Bloch sphere whose azimuthal angle $\phi$ can be an arbitrary value instead of only being zero. And the generalized scheme of RSP has been experimentally implemented via hydrogen and carbon nuclei in molecules of carbon-13 labeled chloroform $^{13}$CHCl$_3$ over interatomic distances using liquid-state nuclear magnetic resonance techniques. Moreover, we have experimentally demonstrated remote
state measurement on an arbitrary qubit proposed by Pati [2]. These studies will be useful to understand the foundations of quantum theory and perform effectively quantum information processing with less resources.

II. THE GENERALIZED SCHEME OF REMOTE STATE PREPARATION AND REMOTE STATE MEASUREMENT

Let’s begin with Pati’s scheme of RSP [2] whose schematic diagram is shown in Fig. 1a. The goal of RSP is for Alice to help Bob in a distant laboratory to prepare a qubit state $|\psi\rangle$ known to her but unknown to Bob with LOCC. An arbitrary single qubit can be represented by a point on a Bloch sphere with two real parameters $\theta$ and $\phi$, i.e.,

$$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\phi}|1\rangle,$$  \hspace{1cm} (1)

where $\theta$ and $\phi$ are the polar and azimuthal angles on the Bloch sphere, respectively. The scheme requires that Alice and Bob share an EPR pair at first,

$$|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B),$$  \hspace{1cm} (2)

which can be rewritten in the qubit orthogonal basis $\{|\psi\rangle, |\psi\perp\rangle\}$,

$$|\psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|\psi\rangle_A|\psi\perp\rangle_B - |\psi\perp\rangle_A|\psi\rangle_B),$$  \hspace{1cm} (3)

where $|\psi\perp\rangle = \cos(\theta/2)|1\rangle - \sin(\theta/2)e^{-i\phi}|0\rangle$, the complement qubit with $|\psi\rangle$. Due to her knowledge of $|\psi\rangle$, Alice can carry a single particle von Neumann measurement by projecting onto the basis $\{|\psi\rangle, |\psi\perp\rangle\}$ and then sends her measurement result to Bob. Depending on the received classical information, Bob can recover the desired state $|\psi\rangle$ by deciding to do nothing $E$ or an operation $U$ which converts $|\psi\perp\rangle$ into $|\psi\rangle$. Finally Bob’s qubit is prepared in the unknown state $|\psi\rangle$. However, converting an unknown $|\psi\perp\rangle$ into $|\psi\rangle$ is an antiunitary operation and thus RSP of an arbitrary qubit can succeed only half of the time [2]. Fortunately, Pati [2] found a definite unitary operator $U$ for a qubit of a special ensemble. For
instance, for a qubit chosen from equatorial \( \theta = \frac{\pi}{2} \) and polar great circles \((\phi = 0)\) on a Bloch sphere, \( U = \sigma_z \) and \( U = i\sigma_y \), respectively.

According to Pati’s scheme, an preagreed convention about the prepared qubit, such as the qubit chosen from equatorial or polar great circles on a Bloch sphere, is made between Alice and Bob before performing RSP. If one want to get a successful RSP all the time, the key is to find a universal transformation \( U \) to convert \( |\psi_\perp\rangle \) into \( |\psi\rangle \). Then, does a universal transformation \( U \) for the states expect for those studied by Pati, exist?

Suppose that Alice and Bob, promise in advance that Alice would like to help Bob remotely prepare a qubit chosen from the longitude with \( \phi = \phi_0 \) (an arbitrary value in the range of \([0, 2\pi]\)) on the Bloch sphere, i.e., \( |\psi(\phi_0)\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\phi_0}|1\rangle \). The entangled-pair sharing by both and a von Neumann measurement by Alice are the same as in the Pati’s scheme. Then Bob performs operations \( E \) or \( U \) according to the received one cbit information from Alice after her measurement. For the qubit \( |\psi(\phi_0)\rangle \), we find a unitary operation

\[
U(\phi_0) = \begin{bmatrix}
0 & -e^{-i\phi_0} \\
e^{i\phi_0} & 0
\end{bmatrix} = i(\sin \phi_0 \sigma_x - \cos \phi_0 \sigma_y)
\]

(4)

to correct \( |\psi_\perp(\phi_0)\rangle \) into \( |\psi(\phi_0)\rangle \), that is, \( |\psi(\phi_0)\rangle = U(\phi_0) |\psi_\perp(\phi_0)\rangle \). Through the above procedure, RSP is successfully completed. Therefore, a qubit chosen from any longitude can be remotely prepared by our protocol. Two special cases can be deduced from Eq. (4), i.e.,

\[
\phi = \begin{cases}
0, & U = i\sigma_y; \\
\frac{\pi}{2}, & U = i\sigma_x.
\end{cases}
\]

(5)

It can be seen from Eq. (5) that the case with \( \phi = 0 \) contained in our generalized scheme is consistent with the Pati’s scheme. Our scheme includes all points on the Bloch sphere.

The process of RSP stated above can succeed only for a preagreed ensemble of qubits, however, Pati \[2\] showed that any single-particle measurement on an arbitrary qubit can be remotely simulated with one qubit and one cbit communication. This is attributed to Wigner’s theorem \[3\] on symmetry transformations that shows the invariance of the
quantum-mechanical probabilities and transition probabilities under unitary and antiunitary operations. Due to \( \rho = |\psi\rangle \langle \psi| = \frac{1}{2} (1 + \vec{n} \cdot \vec{\sigma}) \) and \( \rho_\perp = |\psi_\perp\rangle \langle \psi_\perp| = \frac{1}{2} (1 - \vec{n} \cdot \vec{\sigma}) \) for an arbitrary qubit, the probabilities of measurement outcomes in the states \( \rho \) and \( \rho_\perp \) for an observable \( \langle \vec{b} \cdot \vec{\sigma} \rangle \) are given by

\[
P_\pm (\rho) = \text{tr} \left[ P_\pm (\vec{b}) \rho \right] = \frac{1}{2} \left( 1 \pm \vec{b} \cdot \vec{n} \right),
\]

\[
P_\pm (\rho_\perp) = \text{tr} \left[ P_\pm (\vec{b}) \rho_\perp \right] = \frac{1}{2} \left( 1 \mp \vec{b} \cdot \vec{n} \right),
\]

where \( \vec{n} \) represents the direction vector, the Pauli operator \( \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \), \( \vec{b} \) is the direction of measurement that Bob wants to choose and the projection operator \( P_\pm (\vec{b}) = \frac{1}{2} \left( 1 \pm \vec{b} \cdot \vec{\sigma} \right) \). From Eq. (6), Pati suggested the RSM scheme can be always achieved by reversing the direction of \( \vec{b} \) to make \( P_\pm (\rho_\perp) = P_\pm (\rho) \) when Bob gets the state \( \rho_\perp \).

**III. NMR EXPERIMENTS AND RESULTS**

Our implementation of the generalized RSP and RSM schemes stated above were performed using liquid-state NMR spectroscopy with carbon-13 labeled chloroform \(^{13}\text{CHCl}_3\) (Cambridge Isotope Laboratories, Inc.). The spin-spin coupling constant \( J \) between \(^{13}\text{C} \) and \(^1\text{H} \) is 214.95Hz. The relaxation times were measured to be \( T_1 = 4.8sec \) and \( T_2 = 0.2sec \) for the proton, and \( T_1 = 17.2sec \) and \( T_2 = 0.35sec \) for carbon nuclei. Hydrogen nucleus \(^1\text{H} \) was chosen as the sender (Alice) and the carbon nuclei \(^{13}\text{C} \) as the receiver (Bob) in the experiments, transmitting the state from \(^1\text{H} \) to \(^{13}\text{C} \). All experiments were performed on a BrukerARX500 spectrometer with a probe tuned at 125.77MHz for \(^{13}\text{C} \), and at 500.13MHz for the \(^1\text{H} \).

Using line-selective pulses and the gradient-pulse techniques \([10]\), we initialized the quantum ensemble in an effective pure state (EPS) \( \rho_0 \) from the thermal equilibrium. The shared EPR pair was experimentally achieved by the NMR pulse sequence \( Y_C (\frac{\pi}{2}) J_{CH} (\frac{\pi}{2}) Y_C (\frac{\pi}{2}) X_C (\frac{\pi}{2}) Y_H (\pi) X_H (\frac{\pi}{2}) \) to be applied from left to right, where \( X_H (\frac{\pi}{2}) \) and \( X_H (\frac{\pi}{2}) \) denote \( \frac{\pi}{2} \) and \( -\frac{\pi}{2} \) rotations about \( \hat{x} \) axis on spin \(^1\text{H} \) and so forth, and \( J_{CH} (\frac{\pi}{2}) \).
describes a time evolution of $1/2J_{CH}$ under the scalar coupling between $C$ and $H$. We followed our approach detailed in Ref. [8] to the generalized RSP scheme. In the experiment, the single particle von Neumann measurement in the qubit basis $\{|\psi\rangle, |\psi_\perp\rangle\}$ was simulated by using a two part procedure inspired by Brassard et al. [11]. Using Alice’s knowledge of the qubit, the transformation $R_H^+ (\phi_0)$ to rotate from the basis $\{|\psi (\phi_0)\rangle, |\psi_\perp (\phi_0)\rangle\}$ into the computational basis $\{|0\rangle, |1\rangle\}$,

$$ R_H^+ (\phi_0) = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2)e^{-i\phi_0} \\ -\sin(\theta/2)e^{i\phi_0} & \cos(\theta/2) \end{pmatrix}, \quad (7) $$

was realized by the NMR pulse sequence $X_H (\theta_1) Y_H (\theta_2) X_H (\theta_1)$ with $\theta_1 = \tan^{-1}(\tan(\theta/2)\sin\phi_0)$, and $\theta_2 = 2\sin^{-1}(\sin(\theta/2)\cos\phi_0)$. The projective measurement in the computational basis and the post-measurement operation was equivalent with the conditional unitary operation $S = U_C (\phi_0) E^H_+ + E^H_-$, where $E^H_{\pm} = \frac{1}{2} (1_2 \pm \sigma_z)$ and $1_2$ is a $2 \times 2$ unit matrix, which was implemented by the pulse sequence $X_C \left(\frac{\pi}{2}\right) J_{CH} \left(\frac{\pi}{2}\right) X_C \left(\frac{\pi}{2} - \phi_0\right) Y_C \left(\frac{\pi}{2}\right) J_{CH} (\phi_0) Y_H (\pi)$ up to an irrelevant overall phase factor, where $J_{CH} (\phi_0)$ describes a scalar coupling evolution of $\phi_0/\pi J_{AB}$. The measurement of the final state of spin $^{13}C$ is to record the spectra of $^{13}C$ nuclei and then integrate the entire multiplet after adjusting the right phase [12]. In each experimental run for a given value of $\phi_0$, a total of 13 qubit states with a $\theta$ increment of $\pi/12$ from 0 to $\pi$ were studied. The experimental procedure was repeated for different $\phi_0$ changed from 0 to $2\pi$ with the increment of $\pi/8$. Thus RSP of a qubit chosen from different longitude on the Bloch sphere were experimentally demonstrated. For each input state we recorded the total NMR spectra of $^{13}C$ without and with a reading-out pulse $Y_C \left(\frac{\pi}{2}\right)$, and then deduced the real ($I_x$), imaginary ($I_y$) components of the former and the real component ($I_z$) of the latter respectively. The measured data of the $I_x$, $I_y$ and $I_z$ components are plotted in Fig. 2, along with the theoretical expectations. The state of $^{13}C$ nuclei is shown in fair agreement with the expected form $|\psi\rangle$, which indicates the success of RSP.

For RSM, we performed the similar procedure only with the conditional unitary operation
S omitted, i.e., measuring the NMR spectra of \( ^{13}\text{C} \) after the operation \( R^+_{\text{H}}(\theta, \phi) \). Two multiplet of \( ^{13}\text{C} \) can be labelled by two different subspaces of \( ^{1}\text{H} \) \([13]\), the spectrum on the high frequency for the \( |0\rangle_A \) subspace and the one on the low frequency for \( |1\rangle_A \), and the corresponding states of \( ^{13}\text{C} \) after the operation \( R^+_{\text{H}}(\theta, \phi) \) are \( |\psi\rangle_B \) and \( |\psi\rangle_B \), respectively. So the outcomes in the state \( \rho \) and \( \rho_{\perp} \) can be obtained from the measurements in different subspaces. In the experiments, we measured three observables \( \sigma_x \), \( \sigma_y \) and \( \sigma_z \), i.e., \( \vec{b} = (1, 0, 0), (0, 1, 0) \) and \( (0, 0, 1) \), respectively. As all NMR observables are traceless and the constant item has no effect on the NMR signal, we got the results of RSM merely by reversing the direction of the obtained \( ^{13}\text{C} \) signal in the \( |0\rangle_A \) subspace. We have studied a total of 221 points on the Bloch sphere with a 13 by 17 rectangular grid of a \( \theta \) spacing of \( \pi/12 \) and a \( \phi \) spacing of \( \pi/8 \). Fig. 3 showed the experimental results of RSM. A general observable \( (\vec{b} \cdot \vec{\sigma}) \) is the linear sum of the three basic observables \( \sigma_x \), \( \sigma_y \) and \( \sigma_z \). In this experiments, Bob didn’t use any information about the original qubit \( |\psi\rangle \), so RSM was carried on an arbitrary qubit state.

As any point on the Bloch sphere can be expressed as the product form \( I_z \sin \theta \cos \phi + I_y \sin \theta \sin \phi + I_z \cos \theta \) (apart from a constant unit matrix), the experimental results in Figs. 2 and 3 clearly shows the expected cosine and sine modulations, indicating that the generalized RSP and RSM schemes are effective for all these input states. However, it can be also seen that there are the discrepancies between the experimental data and theoretical expectations, which are caused by the static magnetic field and rf field inhomogeneities and the imperfect calibrations of rf pulses, etc..

**IV. CONCLUSION**

In summary, we have theoretically generalized Pati’s scheme into an arbitrary longitudinal qubit on the Bloch sphere with the azimuthal angle \( \phi \) being an arbitrary value instead of only being zero and experimentally implemented the generalized RSP protocol by using NMR quantum logic gates and circuits in quantum information. Furthermore,
RSM proposed by Pati’s were also experimentally investigated by NMR techniques, which will be another powerful demonstration and a direct verification of Wigner’s theorem on symmetry transformations. Though some arguments about the ensemble nature of NMR measurements and the short distances over which the quantum information communication between spins (normally confined to molecular dimensions in angstrom distance) happens, the concept and method of quantum information transmission should be useful for quantum computation and quantum communication.

ACKNOWLEDGMENTS

We thank A. K. Pati for bringing the topic of RSM to our attention and Hanzheng Yuan, Zhi Ren, Daxiu Wei and Guoyun Bai for help in the course of experiments.
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Figure Captions

Fig. 1 Schematic protocol for RSP proposed by Pati (a) and the corresponding network of implementing RSP (b). In (a), the meter represents measurement, and double lines coming out from it carry one classical bit $M$ (the measurement outcome), while single lines denote qubits. In (b) $H$ represents the Hadamard gate and the conditional operation on a spin being in the $|1\rangle$ state and the $|0\rangle$ state are represented by a filled circle and an empty circle, respectively. $\oplus$ denotes the module 2 addition. $U$ and $R^+$ is denoted in the text.

Fig. 2 Experimental results for RSP of qubits chosen from arbitrary longitudinal qubit. The left column represents the measured real (a) and imaginary (b) parts of the normalized NMR signals from $^{13}C$ without any reading-out pulse, and the real parts (c) of the same NMR signals after applying a reading-out pulse $Y_{C\left(\frac{\pi}{2}\right)}$, respectively. The theoretical expectations are plotted in the right column. All figures are shown as mesh and contour plots as a function of $\theta$ and $\phi$. The ordinate is in arbitrary units.

Fig. 3 Experimental data of remote state measurement for the observables (a) $I_x$, (b) $I_y$ and (c) $I_z$. The left and right columns represent Bob’s measurement outcome in the state $\rho$ and $\rho_\perp$ (by reversing the direction of the observable). The theoretical expectations are plotted in the middle column.
Fig. 1
Fig. 2
Fig. 3