A Novel Design and Evaluation Theory of 1-Bit Reconfigurable Reflectarray Antenna Elements Based on Radiation Viewpoint

Changhao Liu, You Wu, Songlin Zhou, Fan Yang, Fellow, IEEE, Shenheng Xu, Member, IEEE and Maokun Li, Senior Member, IEEE

Abstract—1-bit reconfigurable reflectarray antennas (RRAs) have been developing rapidly in recent years, and numerous prototypes with various functions are brought out in literatures. When designing wideband, multiband or high-frequency RRAs, current design methodologies face great challenges, such as the difficulty to optimize parameters due to deteriorated switch performance, and great simulation time waste due to the lack of clear design guidance. These problems originate from the scattering point of view of the RRA element, where antenna structures and switches are coupled in design process. In this articl, a novel theory is proposed to model, design and evaluate the single-switch 1-bit RRA elements. Based on two-port network equivalence, a radiation viewpoint of 1-bit RRA elements is presented, where the aim to design the RRA element is to match impedance that is pre-calculated at given switch parameters. Based on the radiation viewpoint, various advanced and systematic impedance matching techniques developed in radiation antennas can be used to design 1-bit RRA elements. Besides, the theoretical performance limit is pre-calculated at given switch parameters before designing specific antenna structures. Moreover, the constant loss curves are developed as an intuitive visualization tool of evaluating element performance in Smith chart. Finally, a standard design process is proposed and the theory is validated by two practical examples. The proposed theory can offer a systematic and effective guidance to design wideband, multiband, and high-frequency 1-bit RRAs.

Index Terms—Metasurfaces, microwave network theory, one-bit, reconfigurable intelligent surfaces, reconfigurable reflectarray antennas.

I. INTRODUCTION

As a new research trend of large-aperture antennas, reflectarrays are attracting increasing attentions in recent years, due to their low cost, low profile, light weight and high performance [1]. Reconfigurable reflectarray antennas (RRAs), loading tunable materials on reflectarray elements and controlling them independently, are capable of dynamic beam steering. RRAs have tremendous benefits compared with conventional phased arrays, such as low cost, low power dissipation, light weight and low loss. Therefore, RRAs have been experiencing intensive research nowadays, and are playing an important role in mobile communication, radar detection, remote sensing scenarios and so on [2], [3].

Analog RRAs can introduce continuous phase tuning range by loading functional materials on each element, such as liquid crystals, ferroelectric materials, phase changing materials and so on, which have lower cost than transmitreceive (T/R) modules in conventional phased arrays. To further reduce the cost and controlling complexity, digital RRAs are invented with discrete reflection phase states. As the simplest digital RRA architecture, 1-bit RRA element has only two states, and as low as only one RF switch is required to generate 1-bit phase change, such as PIN diodes and micro-electro-mechanical-systems (MEMSs). Although the performance is a little weaker than analog RRAs [4], [5], low-cost 1-bit RRAs still apply for most scenarios. On balance, 1-bit RRA is the mainstream RRA architecture, and numerous 1-bit RRA designs are presented in recent years.

1-bit RRAs with a single switch are investigated with various frequency designs [6]–[16]. In 2012, literature [6] designed and fabricated X-band 1-bit RRA element and array, which can realize dynamic beam control with three directions \(-5^\circ, 0^\circ, 5^\circ\). In [7], a 26.5-GHz RRA based on MEMS was proposed to switch the beam directions. In 2016, H. Yang et al. did a comprehensive investigation on a Ku-band 1-bit RRA, and a 10×10 RRA based on PIN diodes was fabricated to scan beams within ±50° range [8]. Later on, various single-switch 1-bit RRA prototypes are brought out, such as a 14×14 X-band RRA [9] and a 10240-element large-aperture RRA [10]. Furthermore, 1-bit RRAs can also support beam scanning in millimeter wave band. A 60-GHz RRA prototype based on single PIN diode on each element was fabricated with gain of 42 dBi [11]. A W-band 96-GHz RRA based on integrated PIN diodes was reported in recent years [12].

Sub-millimeter wave band 210-GHz RRA using high electron mobility transistor (HEMT) switch was designed in [13]. Apart from increasing operating frequency, wideband and multiband RRAs are also research highlights. Literature [14] and [15] proposed wideband RRAs with 1-dB bandwidth over 8.4% and 22.3% respectively. Besides, based on different resonant modes, a 1-bit RRA can support dual-frequency operation at X/Ku-band [16].

1-bit RRAs are emerging rapidly with various functions, and the research trend of RRA is evolving toward high-frequency, broadband and multiband designs. However, current design methodologies face great challenges when designing RRAs.
with high performance. For instance, in millimeter wave and terahertz band, the switch performances deteriorate, resulting in the huge reflection loss. Despite the loss, designers have no idea on whether the current element design is optimal at the given switch parameters. Therefore, qualitative evaluation method and performance limit theory are in need to help judging the element performance. Besides, without clear design targets or directions, designers lack guidance when designing and optimizing the 1-bit RRA structures with 1-bit phase difference at desired frequency bands, leading to great time waste in simulation and iteration. Furthermore, in broadband and multiband designs, designers will not only obtain the 1-bit phase shift at a single frequency point, but also consider a series of multi-frequency resonant modes in desired frequency bands simultaneously, which leads to great design complexity. Consequently, clear design target and useful design guideline are in demand.

In this article, a qualitative design and evaluation theory of single-switch 1-bit RRA element is presented, which can offer a guideline when designing 1-bit RRA elements with a single switch. In Section II, based on two-port network equivalence, the dynamic single-switch RRA element is viewed as a passive radiation antenna, and performance limit, 1-bit characteristic reflection coefficient as well as the constant loss curves are calculated at once when given switch parameters. Then a systematic 1-bit RRA design process based on our proposed theory is presented. In Section III, the proposed theory is validated by two practical design examples at microwave band and sub-millimeter wave band respectively. Finally, the conclusion is drawn in Section IV.

II. A THEORY OF 1-BIT RRA ELEMENT USING TWO-PORT NETWORK EQUIVALENCE

A. Modeling 1-Bit RRA Element Using Network Equivalence

A schematic diagram of an 1-bit RRA element with a single switch is shown in Fig. 1(a). The incident wave is from Floquet port, and after encountering the antenna structure with the lumped switch, most energy is reflected with no cross polarization. The ON/OFF states of the switch can generate two reflection coefficients with different amplitude and phase. Note that there is a reflective ground plane, so no energy is transmitted, and little energy is absorbed by the switch as well as the substrate. The element is surrounded by periodic boundary.

Here, suppose the size of the lumped switch is much smaller than the operating wavelength, so a lumped port can be defined at the location of the switch. Hence, the passive element structure can be modeled as a two-port $[S]$ parameter, and the switch is modeled as a variable impedance $Z$ which is connected to the lumped port 2 (switch port), as shown in Fig. 1(b).

Based on the definition of scattering matrix [17], outward wave $[b]$ is obtained by $[S]$ and the inward wave $[a]$ as follows

$$
\begin{bmatrix}
 b_1 \\ b_2
\end{bmatrix} =
\begin{bmatrix}
 S_{11} & S_{12} \\ S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
 a_1 \\ a_2
\end{bmatrix}.
$$

(1)

The reflection coefficient of the switch is defined as

$$
\gamma = \frac{Z - \eta}{Z + \eta} = \frac{a_2}{b_2},
$$

(2)

where $\eta$ is the reference characteristic impedance, i.e. $120\pi \Omega$ in free space. Note $Z$ is a two-state variable impedance, so $\gamma$ also has two states: $\gamma_{ON}$ and $\gamma_{OFF}$, which is only determined by the switch parameters.

Using the two-port network to model the RRA element with a single switch, the active element is decoupled. $[S]$ is used to model passive antenna structures generally, and $Z$ is used to model active lumped switches, which are applied to all RRA elements with a single switch under normal incidence. Under two-port equivalence, the antenna structure can be interpreted as an impedance transformation network, which can transform the impedance of the switch at port 2 into free space at port 1. The aim of the RRA element design is to design a proper impedance transformation structure, which can generate the desired $[S]$ matrix and then the optimal reflection coefficients from port 1, $\Gamma_{ON}$ and $\Gamma_{OFF}$.

B. Theoretical Analysis of 1-Bit RRA Element

In this part, a theoretical derivation is done to prove that the reflection coefficient $\Gamma$ is only related with the reflection coefficient from port 2 ($S_{22}$) under lossless assumption. Taking $a_2 = \gamma b_2$ into (1) and solving the equation, we finally get

$$
b_1 = S_{11} a_1 + \frac{S_{12} S_{21} \gamma}{1 - S_{22} \gamma} a_1.
$$

(3)
Suppose the passive structure is magnet-free and reciprocal, so \( S_{21} = S_{12} \). The reflection coefficient from port 1 is

\[
\Gamma = S_{11} + \frac{S_{12}^2 \gamma_{b}}{1 - S_{22}^2 \gamma_{b}},
\]

(4)

Here, suppose the passive structure is lossless, and thus the \([S]\) matrix is a unitary matrix:

\[
[S][S]^H = I,
\]

(5)

where \([S]^H\) is the conjugate transpose matrix of \([S]\). If the expansion of \([S]\) is written as

\[
[S] = \begin{bmatrix}
S_{11} e^{j \theta_{11}} & S_{12} e^{j \theta_{12}} \\
S_{21} e^{j \theta_{21}} & S_{22} e^{j \theta_{22}}
\end{bmatrix},
\]

(6)

we finally obtain three equations under unitary condition:

\[
\begin{cases}
|S_{11}| = |S_{22}| \\
|S_{12}| = \sqrt{1 - |S_{22}|^2} \\
2 \theta_{12} = \theta_{11} + \theta_{22} + (2k + 1) \pi
\end{cases}
\]

(7)

Taking (7) into (4), the reflection coefficient can derived as follows

\[
\Gamma = |S_{22}| e^{j \theta_{11}} + \frac{(1 - |S_{22}|^2) e^{j (\theta_{11} + \theta_{22} + \pi) \gamma_{b}}}{1 - |S_{22}| e^{j \theta_{22} \gamma_{b}}} = e^{j \theta_{11}} \left( \frac{|S_{22}|}{1 - |S_{22}| e^{j \theta_{22} \gamma_{b}}} - \frac{(1 - |S_{22}|^2) e^{j \theta_{22} \gamma_{b}}}{1 - |S_{22}| e^{j \theta_{22} \gamma_{b}}} \right).
\]

(8)

In 1-bit RRA design, the reflection phase difference and amplitudes are the main considerations. \( e^{j \theta_{11}} \) in (8) only influences the absolute reflection phase shift with normalized reflection amplitude, which has no affect on the element performance, so we can ignore \( e^{j \theta_{11}} \) and simplify (8) to

\[
\Gamma = \frac{|S_{22}| - e^{j \theta_{22} \gamma_{b}}}{1 - |S_{22}| e^{j \theta_{22} \gamma_{b}}}.
\]

(9)

In (9), it is obvious that the reflection coefficients from Floquet port 1 (\( \Gamma_{ON}, \Gamma_{OFF} \)) are influenced by the switch parameters (\( \gamma_{ON}, \gamma_{OFF} \)) and the structure parameter \( S_{22} \), which means only \( S_{22} \) determines the performance of a 1-bit RRA element at given switch parameters.

Fig. 2 illustrates two understandings of a 1-bit RRA element. Traditionally, the antenna structure with the two-state switch, are interpreted for the function of scattering the incident wave, as shown in Fig. 2(a), \( \Gamma_{ON} \) and \( \Gamma_{OFF} \) are simulated and derived with two switch states \( \gamma_{ON} \) and \( \gamma_{OFF} \) respectively.

Since the reflection coefficient \( \Gamma \) is only related with \( S_{22} \), we can propose a radiation understanding of the 1-bit RRA element, as shown in Fig. 2(b). Port 1 always matches with the characteristic impedance \( \eta \), and then \( a_1 = 0 \), where the reflection coefficient of port 2 is

\[
\Gamma_2 = \frac{b_2}{a_2} = S_{22}.
\]

(10)

The structure parameter \( S_{22} \) equals to the reflection coefficient \( \Gamma_2 \). So if a lumped port is applied at port 2 and the radiation boundary is connected to port 1, we only need to simulate the passive antenna structure once to get the response \( \Gamma_2 \), and then the two-state reflection coefficient \( \Gamma_{ON} \) and \( \Gamma_{OFF} \) can be calculated based on (9). It is worth mentioning that

Fig. 2(b) is exactly the equivalent model of a typical single-port radiation antenna. Therefore, the scattering 1-bit RRA structure can be interpreted as a classical passive radiation antenna structure, which means the advanced and systematic impedance matching art of radiation antenna design can be applied to 1-bit RRA element design.

It is also noted that the conclusion is drawn under the condition that the passive antenna structure is lossless. Actually, the substrate is lossy, so strictly speaking, \( \Gamma \) is related with three structure parameters as derived in (4). However, in most cases, the loss of RF substrates is less than the loss of switches. Hence, the loss in antenna structures has little effect on the final 1-bit reflection performance. So designers can ignore the substrate loss at first and design the antenna structure under lossless assumption using (9), and then verify the performance with lossy substrates using (4).

C. The Theoretical Performance Limit and Characteristic Reflection Coefficient of 1-Bit RRA Element

To obtain the design target and the theoretical limit of RRA element performance at given switch parameters, the equivalent reflection amplitude is introduced in this part.

The reflection coefficients from port 1 under two states are expressed as the following form:

\[
\begin{cases}
\Gamma_{ON} = A_{ON} e^{j \gamma_{ON}} \\
\Gamma_{OFF} = A_{OFF} e^{j \gamma_{OFF}}
\end{cases}
\]

(11)

where \( A \) is the reflection amplitude and \( \varphi \) is the reflection phase. Here, we define the equivalent reflection amplitude (ERA) as

\[
A_{eq}(S_{22}) = \frac{1}{4} \int_{0}^{2\pi} P_{\phi} d\phi,
\]

(12)

where

\[
P_{\phi} = \max\{A_{ON} \cos(\phi - \varphi_{ON}), A_{OFF} \cos(\phi - \varphi_{OFF})\}.
\]

(13)

According to (9), \( A_{eq} \) is a function of \( S_{22} \) at given switch parameters. (12) is the statistical average of reflection amplitude after quantization, and (13) is a certain quantization strategy. As shown in (12) and (13), ERA takes both reflection amplitude and phase difference into consideration, which can describe the reflection efficiency of a single 1-bit element comprehensively. For instance, under ideal condition, the reflection amplitudes under two cases are 1, and the reflection phase difference is 180°, so the ERA equals to \( A_{eq} = 1 \), which means the ideal 1-bit RRA element has no equivalent reflection...
amplitude loss. Actually, the expression of ERA is defined to maximize the gain of the array statistically, and details of the derivation are shown in Appendix A.

Based on the definition of ERA, the theoretical performance limit (PL) of 1-bit RRA element is defined to describe the maximum possible value of ERA at given switch parameters

$$\text{PL} = \max_{S_{22}} (A_{eq}).$$

Correspondingly, the 1-bit characteristic reflection coefficient \(S_{1\text{bit}}\) is the optimal solution of \(S_{22}\), which is defined to maximize the ERA

$$S_{1\text{bit}} = \arg \max_{S_{22}} (A_{eq}).$$

To maximize ERA in (12), numerical method is employed to solve \(S_{22}\) ergodically. Since \(S_{22}\) has only two degrees of freedom, namely \(|S_{22}| \in [0, 1]\) and \(\theta_{22} \in [0, 2\pi]\), the solving process takes within one second. Finally, the optimal solution \(S_{1\text{bit}}\) is plotted in Smith chart in Fig. 3 as an example, where the original close switch parameters \(\Gamma_{\text{ON}}, \Gamma_{\text{OFF}}\) are transformed by \(S_{1\text{bit}}\) into two distant states \(\Gamma_{\text{ON}}, \Gamma_{\text{OFF}}\) to maximize ERA. Correspondingly, the maximum value of ERA is the element performance limit.

In Fig. 3 it is observed that the optimal reflection coefficients are always in the opposite positions with \(\Gamma_{\text{ON}} = -\Gamma_{\text{OFF}}\). This observation agrees with the previous design experience that, if the reflection phase difference is near 180° and the reflection amplitudes are close, the element performance is usually optimal. Based on \(\Gamma_{\text{ON}} = -\Gamma_{\text{OFF}}\), the analytical solution of \(S_{1\text{bit}}\) can be derived, as shown in Appendix B.

Using ERA to evaluate the element performance quantitatively, the theoretical performance limit of 1-bit RRA element is pre-calculated once the switch parameters are given, which can help designers to predict the reflection loss and aperture gain before designing specific antenna structures. Besides, The 1-bit characteristic reflection coefficient \(S_{1\text{bit}}\) gives the quantitative design target of the antenna structure. When optimizing a specific element structure, if the actual reflection coefficient from port 2 \((S_{22}^{22})\) matches with \(S_{1\text{bit}}\), the 1-bit RRA element can reach the performance limit. Moreover, since the scattering antenna can be viewed as a radiation antenna, and design target is to match impedance with \(S_{1\text{bit}}\), the design and parameter tuning process of 1-bit RRA element can refer to the impedance matching techniques of radiation element.

D. The Quantitative Evaluation Method at Mismatched Points

In practical 1-bit RRA design process, \(S_{22}\) changes with frequency, which does not always match with the optimal reflection coefficient \(S_{1\text{bit}}\) within a certain frequency band, so the element will not always meet the performance limit over the band. Accordingly, it is worth developing a method to evaluate the performance quantitatively when the \(S_{22}\) does not match with the \(S_{1\text{bit}}\).

Actually, the ERA defined in (12) is used to evaluate the element performance for a given \(S_{22}\). So the equivalent reflection loss at mismatched points can be calculated using ERA. ERA at every point in Smith chart can be pre-calculated when enumerating \(S_{22}\) ergodically. Consequently, every \(S_{22}\) point of the same ERA forms a closed curve in Smith chart. If we choose different ERA levels, a series of curves can be generated, which are called constant loss curves (CLCurves), compared with the maximum performance limit value, as illustrated in Fig. 4 as an example. If the \(S_{22}\) varies inside a CLCurve of a certain loss level over the band, the equivalent reflection loss of this RRA element is smaller than that loss level within the band. Obviously, the outer curve has larger constant loss level, which means that the design restrictions of 1-bit RRA element is looser if tolerating the element performance deterioration. The introduction of CLCurves can offer an intuitive aid of designing and evaluating 1-bit RRA element over a frequency band.

The quantitative evaluation method at mismatched points extends the theory when considering reflection performance in a frequency band. It can help designers to design and evaluate wideband or multiband RRA elements quantitatively. Besides, CLCurves in Smith chart can provide an intuitive standard to evaluate the wideband performance of 1-bit RRA element, on analogy of -15 dB bandwidth, -10 dB bandwidth and -6 dB bandwidth standards in radiation antenna evaluation.

E. The Standard Design Process

Based on the proposed methods to model, design and evaluate the 1-bit RRA element, a flow chart of the novel method is generalized, which can provide a step-by-step methodology to design and evaluate the 1-bit RRA elements with a single switch, as shown in Fig. 5(a) For comparison, the design process based on conventional method is shown in Fig. 5(b) As shown in two flow charts, the proposed method
makes more preparation before designing a specific element, which can help designers to estimate the element performance and set clear goals before designing. When designing and tuning parameters, the impedance matching art can refer to the classical radiation antenna design art. Conventional method is a more straightforward process, but it will simulate the same structure twice to get the ON/OFF performance, which is time-costly. Besides, the optimizing art of conventional method relies on the designers’ experience, which is qualitative and not intuitive.

F. Summary and Discussions

In this section, two-port network is employed to model 1-bit RRA elements with single switch, and a novel method is proposed to design and evaluate the 1-bit RRA element, which can provide a systematic design process, clear design targets and a quantitative evaluation method. Compared with conventional design process, the novel model of 1-bit RRA element can give the limit of the element performance and clear design targets once the switch parameters are given before designing a specific structure. Besides, by viewing the scattering structure as a single-port radiation antenna, the art of designing 1-bit scattering structure can refer to the impedance matching art of designing radiation antennas. Specifically, based on conventional method, it is hard to obtain the 1-bit performance in desired frequency bands. But using the novel method, the wideband or multiband impedance matching techniques can be employed to design structures with desired 1-bit response over the bands. A comparison between conventional method and proposed novel method is summarized in Table I.

| Comparison | Conventional method | Novel method |
|------------|---------------------|-------------|
| Point of view | Scattering structure | Radiation antenna |
| Performance limit | Uncertain | Explicit |
| Design target | Ambiguous | Clear |
| Multiband design technique | Not intuitive | Intuitive |
| Visualization approach | Amplitude and phase diagram | Smith chart |
| Number of simulations | 2 | 1 |
| Design period | Long | Short |

III. Theory Validations

In this section, two practical examples in microwave band and sub-millimeter wave band are used to demonstrate the feasibility of the proposed theory respectively.

A. Validation of The Theory in Mircowave Band

1) The Basic Setups: Literature [14] reported a wideband 1-bit $12 \times 12$ RRA using a single switch at 5-GHz band. Here, results are reproduced using our proposed theory. The simulation structure of the proposed element is shown in Fig. 6, which is built in Ansys HFSS software. For conciseness, biasing circuit is ignored in the simulation model. The substrate is F4B with thickness of 2.2 mm, and the element is backed by a reflective ground plane. The lumped switch is modeled in the gap between the two patches. In our method, lumped switch is replaced by lumped port to excite the patch and get the reflection coefficient. The element is surrounded by periodic boundary.

2) The Calculated Parameters: Before simulation, the auxiliary parameters can be pre-calculated only based on the parameters of the switch. In this literature, the PIN diode is used at 5 GHz, with $R_{on} = 1 \ \Omega$, $L_{on} = 0.45 \ \text{nH}$; $R_{off} = 10 \ \Omega$, $L_{off} = 0.45 \ \text{nH}$, $C_{off} = 0.16 \ \text{pF}$. Based on the methods introduced in Section II-C and Section II-D the 1-bit characteristic reflection coefficient $S_{1\text{bit}}$ and CLCurves are obtained and plotted in Fig. 7. The performance limit is -0.28 dB.
3) Element Simulation and Theory Validations: The element is simulated using Ansys HFSS and the reflection coefficient from lumped port $S_{22}^a(f)$ is obtained from 4 GHz to 6 GHz. The curve of $S_{22}^a(f)$ is plotted in Smith chart with the pre-calculated auxiliary parameters in Fig. 8. As it illustrates, the $S_{22}^a$ at central frequency 5 GHz is close to the target $S_{1\text{bit}}$, which means the element can generate 1-bit response at 5 GHz approaching the performance limit. Since the $S_{22}^a(f)$ curve over the band is around $S_{1\text{bit}}$, element can generate wide bandwidth. $S_{22}^a(f)$ curve in Smith chart validates it is a successful design.

The results are validated by conventional simulation process with lumped RLC series replacing lumped port. As shown in Fig. 9, the calculated reflection amplitude and phase difference from $S_{22}^a(f)$ using (9) match with the simulated curves. The simulation loss is about 0.5 dB, and the extra loss is due to the substrate loss. Despite the substrate loss, the calculated resonant frequencies are at 4.6 GHz (OFF) and 5.4 GHz (ON), which match with the simulated results. The proposed method can still work in spite of lossy structures. Besides, the reflection amplitudes do not break the theoretical limit, which validates the correctness of the theory.

B. Validation of The Theory in Sub-Millimeter Wave Band

1) The Basic Setups: A sub-millimeter wave band 1-bit RRA is reported in [13]. As reported in the literature, the RRA using HEMT switch can operate over 200 GHz with element loss about 8 dB. Here, a simulation model is built in CST Studio, shown in Fig. 10, the same as the setups described in [13]. The substrate is 100-um-thick sapphire backed by reflective ground plane. A lumped port is in the middle of element to excite the patches. The top surface is Floquet port with characteristic impedance of $120\pi\Omega$, and the element is surrounded by periodic boundary.

2) The Calculated Parameters: The parameters of switches are important in our method. The home-made HEMT switch is modeled as lumped RC series in [13], with $R_{\text{on}} = 210\Omega$; $R_{\text{off}} = 192.5\Omega$, $C_{\text{off}} = 2\text{ fF}$. Pre-calculation is made at given switch parameters, as shown in Fig. 11. Unlike high-performance switches in microwave band, the switch
parameters deteriorate in sub-millimeter wave band, leading to the performance limit of only -7.9 dB.

3) Element Simulation and Theory Validations: The element is simulated using CST Studio from 190 GHz to 220 GHz. The reflection coefficient from lumped port $S_{22}(f)$ is obtained and plotted in Smith chart in Fig. 12. The $S_{22}(f)$ curve passes through the $S_{\text{1bit}}$ point at 207 GHz, which means the optimal 1-bit RRA element can nearly reach the performance limit at 207 GHz. Despite the great loss using the HEMT switch, the structure is designed to reach the performance limit, which verifies it is a successful design at sub-millimeter wave band.

The simulation results based on the conventional method is obtained to compare with the calculated results. As plotted in Fig. 13, the amplitude and phase difference of both methods match, where the phase difference is $180^\circ$ at 207 GHz. The simulated reflection amplitudes are about -8.3 dB at 207 GHz, and the measured loss is about 8.5 dB according to [13], while the performance limit is -7.9 dB. The simulated and measured reflection amplitudes verify the correctness of the theory. Besides, the measured loss is near the performance limit, which also demonstrates the effectiveness of the performance limit theory. The performance limit theory can help to estimate the element performance before designing the antenna structure.

IV. CONCLUSIONS

This article proposes a novel method to design and evaluate the 1-bit RRA element with a single switch. Based on the idea that the active element is decoupled and modeled as passive antenna structure with active lumped switch, the two-port network can be employed to describe the antenna structure. Under lossless assumption, the reflection coefficients are calculated by the switch parameters and reflection coefficient from switch port. To evaluate the element performance, equivalent reflection coefficient is defined, and the element performance limit, 1-bit characteristic reflection coefficient, as well as the constant loss curves are pre-generated once the switch parameters are given, which can provide a guidance when designing antennas. Finally, the proposed novel design procedure is concluded, and the novel method is validated by two practical design examples.

The proposed method can predict the element performance and generate the design target at given switch parameters before designing a specific 1-bit element, which can help designers to evaluate and select proper switches to meet the performance requirements before carrying out element design. Besides, using the radiation point of view, the art of designing 1-bit RRA elements can refer to the mature impedance matching art of optimizing single-port radiation antennas, which saves much time during simulation and iteration. In particular, wideband and multiband 1-bit RRA elements can be designed intuitively with a clear design target, and millimeter and
terahertz 1-bit RRA elements can be designed and optimized despite the unavoidable loss due to the deterioration of switch parameters at such high frequency.

The idea of element decoupling, network equivalence and radiation viewpoint can be applied to other active metasurfaces besides single-switch 1-bit RRA elements, such as 2-bit RRA elements, energy-amplifying reflectarray antennas, dual-polarized RRA, reconfigurable transmitarray antennas (RTAs), active frequency selective surfaces (AFSSs), and so on.

APPENDIX A
DERIVATION OF THE EQUIVALENT REFLECTION AMPLITUDE

Based on the findings in literature [5], the incident phase on each RRA element is pseudorandom. To derive a expression of $A_{eq}$ theoretically, it is supposed that the each element has random incident phase $\phi$ with uniform incident amplitude. Besides, suppose the size of array is infinite with element number $N \to \infty$. The overall reflection amplitude is the summation of each element

$$E = \sum_{n=1}^{N} A_n e^{-j(\phi_n - \phi)},$$  \hspace{1cm} (16)

where $\phi_n$ is the additional phase of the $n$th element, and $A_n$ is the $n$th reflection amplitude. In 1-bit case, the reflection coefficients have two states, so the additional phase has two discrete states. To maximize the gain, the additional phase is selected to match with the incident phase $\phi$ and form in-phase addition.

A quantization strategy is adopted to maximize the gain of the array. As shown in Fig. 14 when deciding quantizing the random incident phase to which one of two states, the ON/OFF state vectors are first projected on the incident vector in the polar plot. If the ON-state projection value is larger than the OFF-state projection value, the incident phase is quantized to the ON state to maximize the projection value. Accordingly, the equivalent reflection coefficient is normalized as the following form

$$A_{eq} = \frac{1}{2\pi} \int_{0}^{2\pi} P_{\phi} d\phi,$$  \hspace{1cm} (18)

where

$$P_{\phi} = \max\{A_{ON} \cos(\phi - \varphi_{ON}), A_{OFF} \cos(\phi - \varphi_{OFF})\}.$$  \hspace{1cm} (19)

A$_{eq}$ takes both reflection amplitude and phase into consideration. For ideal case, if the reflection amplitudes of ON/OFF states are 1, and the phase difference is 180°, the equivalent reflection coefficient is

$$A_{eq} = \frac{1}{2\pi} \cdot 2 \int_{-\pi}^{\pi} \cos(\phi) d\phi = \frac{2}{\pi},$$  \hspace{1cm} (20)

or -3.9 dB, which is purely due to the 1-bit phase quantization effect. It is noted that the ideal phase quantization loss of near 3.9 dB is also observed in literature [4], which verifies the rationality of the quantization strategy and the definition of equivalent reflection coefficient.

The 1-bit phase quantization loss is often omitted when describing the amplitude performance of the 1-bit RRA element. Hence, the equivalent reflection coefficient is normalized as the following form

$$A_{eq} = \frac{1}{4} \int_{0}^{2\pi} P_{\phi} d\phi,$$  \hspace{1cm} (21)

where the ideal equivalent reflection coefficient is 1. (21) is the final definition of equivalent reflection coefficient (ERA).

APPENDIX B
ANALYTICAL SOLUTION OF 1-BIT CHARACTERISTIC REFLECTION COEFFICIENT

Based on the observations of the optimal reflection coefficients, it is found that $\Gamma_{ON} = -\Gamma_{OFF}$. Therefore, an analytical solution of $S_{22}$ can be derived by solving the following equation

$$\frac{|S_{22}| - e^{j\theta_{22}} \gamma_{ON}}{1 - |S_{22}| e^{j\theta_{22}} \gamma_{ON}} \gamma_{OFF} = \frac{|S_{22}| - e^{j\theta_{22}} \gamma_{OFF}}{1 - |S_{22}| e^{j\theta_{22}} \gamma_{OFF}},$$  \hspace{1cm} (22)

(22) is simplified to a binary quadratic equation

$$|S_{22}|^2 - 2x_{ON} x_{OFF} + |S_{22}| + 1 = 0,$$  \hspace{1cm} (23)

where $x_{ON} = e^{j\theta_{22}} \gamma_{ON}$ and $x_{OFF} = e^{j\theta_{22}} \gamma_{OFF}$. Note $|S_{22}|$ is a real number, so the conjugate form of (23) also satisfies

$$|S_{22}|^2 - 2x_{ON} x_{OFF} + |S_{22}| + 1 = 0.$$  \hspace{1cm} (24)

Subtracting (23) and (24), $|S_{22}|$ term is eliminated

$$\left(\frac{x_{ON} x_{OFF} + 1}{x_{ON} + x_{OFF}}\right) |S_{22}| = 0,$$  \hspace{1cm} (25)

with $|S_{22}| \neq 0$ in general case. So the following equation is obtained without $|S_{22}|$

$$\frac{x_{ON} x_{OFF} + 1}{x_{ON} + x_{OFF}} = \frac{x_{ON} x_{OFF} + 1}{x_{ON} + x_{OFF}},$$  \hspace{1cm} (26)
and deduced as
\[
(1 - |x_{\text{OFF}}|^2)x_{\text{ON}} + (1 - |x_{\text{ON}}|^2) x_{\text{OFF}} = (1 - |x_{\text{OFF}}|^2)x_{\text{ON}} + (1 - |x_{\text{ON}}|^2) x_{\text{OFF}}.
\] (27)
Taking \(x_{\text{ON}} = e^{j\theta_{22}} x_{\text{ON}}\) and \(x_{\text{OFF}} = e^{j\theta_{22}} x_{\text{OFF}}\) into (27), the solution of \(e^{j\theta_{22}}\) is
\[
e^{j\theta_{22}} = \pm \sqrt{\frac{(1 - |\gamma_{\text{ON}}|^2) x_{\text{OFF}} + (1 - |\gamma_{\text{OFF}}|^2) x_{\text{ON}}}{(1 - |\gamma_{\text{ON}}|^2) x_{\text{OFF}} + (1 - |\gamma_{\text{OFF}}|^2) x_{\text{ON}}}}.
\] (28)
Solving (23), we obtain
\[
|S_{22}| = \frac{x_{\text{ON}} x_{\text{OFF}} + 1 - \sqrt{(1 - x_{\text{OFF}}^2) (1 - x_{\text{ON}}^2)}}{x_{\text{ON}} + x_{\text{OFF}}}.
\] (29)
Taking \(e^{j\theta_{22}}\) in (28) into (29), the solution of \(|S_{22}|\) is
\[
|S_{22}| = \frac{1 - |\gamma_{\text{ON}}|^2 |\gamma_{\text{OFF}}|^2 - \sqrt{(1 - |\gamma_{\text{ON}}|^2) (1 - |\gamma_{\text{OFF}}|^2) (\gamma_{\text{ON}} |\gamma_{\text{OFF}} - 1) (|\gamma_{\text{OFF}}| |\gamma_{\text{ON}} - 1)}}{(1 - |\gamma_{\text{ON}}|^2) |\gamma_{\text{OFF}}|^2 + (1 - |\gamma_{\text{OFF}}|^2) |\gamma_{\text{ON}}|^2}.
\] (30)
(28) and (30) are the analytical solutions of \(S_{\text{1bit}}\), which coincide with the \(S_{\text{1bit}}\) found ergodically.

REFERENCES
[1] P. Nayeri, F. Yang, and A. Z. Elsherbini, Reflectarray Antennas: Theory, Designs and Applications. New York, NY, USA: Wiley, 2018.
[2] S. V. Hum and J. Perruisseau-Carrier, “Reconfigurable reflectarrays and array lenses for dynamic antenna beam control: A review,” IEEE Trans. Antennas Propag., vol. 62, no. 1, pp. 183–198, Jan. 2014.
[3] P. Nayeri, F. Yang, and A. Elsherbini, “Beam scanning reflectarray antennas: A technical overview and state of the art,” IEEE Antennas Propag. Mag., vol. 57, no. 4, pp. 32–47, Aug. 2015.
[4] B. Wu, A. Suntijo, M. E. Potter and M. Okoniewski, “On the selection of the number of bits to control a dynamic MEMS reflectarray,” IEEE Antenna Wireless Propag. Lett., vol. 7, pp. 183-186, Mar. 2008.
[5] H. Yang et al., “A study of phase quantization effects for reconfigurable reflectarray antennas,” IEEE Antennas Wireless Propag. Lett., vol. 16, no. 5, pp. 302-305, Mar. 2017.
[6] E. Carrasco, M. Barba, and J. A. Encinar, “X-band reflectarray antenna with switching-beam using pin diodes and gathered elements,” IEEE Trans. Antennas Propag., vol. 60, no. 12, pp. 5700–5708, Dec. 2012.
[7] O. Bayraktar, O. A. Civi, and T. Akin, “Beam switching reflectarray monolithically integrated with RF MEMS switches,” IEEE Trans. Antennas Propag., vol. 60, no. 2, pp. 854–862, Feb. 2012.
[8] H. Yang et al., “A 1-bit 10 × 10 reconfigurable reflectarray antenna: design, optimization, and experiment,” IEEE Trans. Antennas Propag., vol. 64, no. 6, pp. 2246-2254, June 2016.
[9] H. Zhang, X. Chen, Z. Wang, Y. Ge and J. Pu, “A 1-bit electronically reconfigurable reflectarray antenna in X band,” IEEE Access, vol. 7, pp. 66567-66575, 2019.
[10] X. Pan, F. Yang, S. Xu and M. Li, “A 10240-element reconfigurable reflectarray with fast steerable monopulse patterns,” IEEE Trans. Antennas Propag., vol. 69, no. 1, pp. 173-181, Jan. 2021.
[11] H. Kamoda, T. Iwasaki, J. Tsumochi, T. Kuki and O. Hashimoto, “60-GHz electronically reconfigurable large reflectarray using single-bit phase shifters,” IEEE Trans. Antennas Propag., vol. 59, no. 7, pp. 2524-2531, July 2011.
[12] X. Pan, S. Wang, G. Li, S. Xu and F. Yang, “On-chip reconfigurable reflectarray for 2-D beam-steering at W-band,” in 2018 IEEE MTT-S International Wireless Symposium (IWS), 2018, pp. 1-4.
[13] X. Pan, F. Yang and X. Nie, “Analysis and design of THz 1-bit RRA element with series inductance,” in 2021 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (APS/URSI), 2021.
[14] J. Han, L. Li, G. Liu, Z. Wu and Y. Shi, “A wideband 1 bit 12 × 12 reconfigurable beam-scanning reflectarray: Design fabrication and measurement,” IEEE Antennas Wireless Propag. Lett., vol. 18, no. 6, pp. 1268-1272, Jun. 2019.
[15] B. J. Xiang, X. Dai, and K. M. Luk, “A wideband low-cost reconfigurable reflectarray antenna with 1-bit resolution,” IEEE Trans. Antennas Propag., vol. 70, no. 9, pp. 7439-7447, Sep. 2022.
[16] H. Yang et al., “A 1600-element dual-frequency electronically reconfigurable reflectarray at X/Ku-band,” IEEE Trans. Antennas Propag., vol. 65, no. 6, pp. 3024-3032, June 2017.
[17] D. M. Pozar, Microwave Engineering, New York, NY, USA:Wiley, 2011.