Possibility of Radiation Reaction Observation under Ultraintense Laser

Keita SETO,1 James KOGA,2 and Sen ZHANG3
1Institute of Laser Engineering, Osaka University, 2-6, Yamada-oka, Suita, Osaka 565-0871
2Quantum Beam Science Directorate, Japan Atomic Energy Agency, 8-1-7 Umenomura, Kizugawa, Kyoto 619-0215
3Okayama Institute for Quantum Physics, Kyoyama 1-9-1, Kita-ku, Okayama 700-0015

(Received October 29, 2013)

Radiation reaction is one of the remaining big problems in theoretical physics. When an electron has high energy, radiation from this electron might become significant. Since this regime involves laser intensities over $10^{19} \text{ W/cm}^2$, we need to consider it under next generation laser-electron interactions. Moreover, radiation reaction is considered to represent the electron model in classical physics. Therefore, research into ultrahigh intense laser-high energy electron interactions has the potential to take us to the center and essence of physics. However, the Lorentz-Abraham-Dirac theory, which is the standard model of radiation reaction, has the difficulty of the run-away solution. In this paper, the history of the research of radiation reaction, our recent studies and the experimental design of this process will be presented.

Key Words: Radiation reaction, Ultraintense laser, Nonlinear QED

1. Introduction

Ultrahigh intense lasers are being planned and constructed.1) What kind of physical processes can we observe in the regime of these laser intensities? We often discuss QED effects like pair creation/annihilation via extreme intensities in the order of $10^{14} \text{ W/cm}^2$.1) This regime represents a new horizon in laser physics. Before reaching this physical process, we need to pass through the region of $10^{15}$ to $10^{19} \text{ W/cm}^2$. In this regime, it is predicted that an electron will emit a significant amount of its energy as light. Therefore, the motion of the electron needs to be corrected by the radiation feedback,1,4) This is a basic physical process, named ‘radiation reaction’. Many readers may consider that this is very obvious and there is no room for discussion. However, this radiation reaction remains to be one of the difficult problems in physics. This standard theory was formulated by Lorentz,2) Abraham5) and Dirac.5) Therefore, the equation of motion with radiation reaction is called the Lorentz-Abraham-Dirac (LAD) equation.5) The easiest way to derive this equation is, from the radiation energy loss formula (Larmor’s formula)5) in the non-relativistic regime:5,10)

$$\frac{dE}{dt}_{\text{Larmor}} = -m_0 \tau_s \left( \frac{dv}{dt} \right)^2$$

(1)

Where, $m_0$ is the electron’s rest mass, $v \in \mathbb{R}^3$ is the velocity of the electron ($\mathbb{R}^3$ is a 3-dimensional Euclidian space), $c$ is denoted as the speed of light, $\tau_s = c^2/6\pi\epsilon_0 m_0 c^2 = O(10^{-24})$. Therefore, the energy change of the electron is as follows ($F_{\text{ext}} \in \mathbb{R}^3$ is an external field):

$$\frac{d}{dt} \left( \frac{m_0 v^2}{2} \right) = F_{\text{ext}} \cdot v - m_0 \tau_s \left( \frac{dv}{dt} \right)^2$$

(2)

From this equation, we can obtain the equation of motion named the Lorentz-Abraham (LA) equation.

$$m_0 \frac{dv}{dt} = F_{\text{ext}} + F_{\text{LA}},$$

(3)

$$F_{\text{LA}} = m_0 \tau_s \left( \frac{dv}{dt} \right)^2.$$  

(4)

Here, our dynamics is in $t \in [t_{\text{initial}}, t_{\text{final}}] \subset \mathbb{R}$, periodic conditions in which

$$v(t_{\text{initial}}) = v(t_{\text{final}}), \quad \frac{dv}{dt}(t_{\text{initial}}) = \frac{dv}{dt}(t_{\text{final}})$$  

(5)

are required. Eq. (4) gives the effect of radiation reaction, called the radiation reaction force. Equation (3) with (4) was extended to the relativistic regime by Dirac. This is the LAD equation given by

$$m_0 \frac{d}{d\tau} w^\mu = -\epsilon F_{\alpha \mu} w^\alpha + f_{\text{LAD}}^\mu,$$

(6)

$$f_{\text{LAD}}^\mu = m_0 \tau_s \left( \frac{d^2 w^\mu}{dt^2} - w^\mu - \frac{d^2 w^\nu}{dt^2} w^\nu \right) w^\nu.$$  

(7)

Where, all of vectors in this equation belong to the 4-dimensional linear vector space $\mathbb{R}^4 \subset \mathbb{R}^5$ joining in Minkowski spacetime ($\mathbb{A}^4$, $g$) which is the $\mathbb{A}^4$ mathematical set of the 4-dimensional affine space $\mathbb{A}$ and the Lorentz metric $g$ with the signature of $(+, -, -, -)$. An affine space is a vector space that doesn’t have a set origin. This method is suitable for the theory of relativity. The force $f_{\text{LAD}} \in \mathbb{R}^4$ is the effect of the radiation feedback, denoted as the radiation reaction force. However, Eq. (6) doesn’t have stable solutions, all solutions go to infinity (run-away; see Eq. (10)).9,11) At first, the LAD theory was derived modelling an electron in classical physics. In 1938, Dirac considered how to avoid the infinity in QED. After that, it was solved via renormalization,12,13) but he tried to apply the Lorentz model (1906)5) [Fig. 1]. Different parts of the elec-
Fig. 1 The electron model of Lorentz and Abraham. The distribution of the charge is on a spherical surface, each charge element interacts with the others via the Liénard-Wiechert (retarded) field which has directivity due to the motion of the electron. Therefore, this field is not always symmetric like the Coulomb field. The integration of these interactions on an electron causes the effects of radiation reaction.

Since high energy electron interactions in experiments, which run-away and the concrete problem of treating ultraintense laser high energy electron interactions in experiments, which will be attempted to carry out.

2. Models of Radiation Reaction

The standard method for the avoidance of run-away was suggested by Ford-O’Connel and Landau-Lifshitz. They considered the radiation reaction force in terms of higher order corrections. By replacing \( m_0 \frac{d^2w}{dt^2} \) with \( d(-eF_{\mu\nu}w_\nu)/dt \) in Eq. (7),

\[
 f_{\mu0} = -\frac{\tau_0 e}{c^2} \left[ \frac{dF_{\mu0}^w}{dt} w_\nu - \frac{dF_{\mu\nu}^w}{dt} w_\nu \right] w_\nu , \tag{11}
\]

as derived by Ford and O’Connel.\(^{20}\) For PIC simulations, the external fields are a function of spacetime. Using the chain rule of the derivative in Eq. (11), we can obtain the force of Landau-Lifshitz:\(^{21}\)

\[
 f_{\mu L} = -\frac{\tau_0 e}{c^2} w^\nu \partial_\nu F_{\mu0}^w
 + \frac{\tau_0 e^2}{c^4} \left( F_{\mu0}^w F_{\nu0}^w w_\nu + F_{\mu0}^w F_{\nu0}^w w_\nu w_\nu \right) , \tag{12}
\]

Since this method doesn’t have a run-away, it is useful as the reference for simulations. A scheme similar to this was obtained by Rohrlich.\(^{20}\) Another method of the solution is pre-acceleration.\(^{21-22}\)

\[
 m_0 w^\mu(\tau) = \int^{\tau} \frac{d\tau'}{\tau_0} \frac{e}{c} \left( F_{\mu0}^w + \frac{m_0}{\tau_0} \frac{dw^\mu}{d\tau} + \frac{dw^\nu}{d\tau} \frac{dv^\mu}{d\tau} \right) (\tau') . \tag{13}
\]

This equation can be derived in a straightforward manner from the LAD Eq. (6,7). This form is correct and strict, however, it requires us to consider the field in the future. This is not for simulation, since this isn’t a normal time evolution. And there are other methods by Caldirola,\(^{23}\) Sokolov\(^{24}\) and Seto,\(^{26-27}\) For fluid plasmas, there are solutions by K. K. Tam and D. Kiang\(^{28}\) and Brezhianni.\(^{29,30}\) These methods have both strong points and weak points. These theoretical studies don’t have the confirmation of experiments. Finally, we need to converge toward the truth of nature. Until experiments on radiation reaction are carried out, theoreticians should suggest new models for proving their models. In this sense, I will explain our new model of radiation reaction in the next section.

3. Radiation Reaction and Quantum Vacuum

When we discuss about next generation lasers, QED effects appear, like pair creation and annihilation. In this regime, a photon can interact with another photon via pair creation and annihilation. In this regime, a photon can interact with another photon via pair creation and annihilation. This process is a correction due to the fluctuation of quantum vacuum (photon-photon scattering). In the low energy photon limit these dynamics are described by the Heisenberg-Euler Lagrangian (non-linear QED).\(^{31,32}\)

\[
 L = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu}
 + \frac{e^2}{8} \left[ F_{\mu\nu} F^{\mu\nu} \right]^2 + \frac{7}{4} \left( F_{\mu\nu} F^{\mu\nu} \right) \tag{14}
\]

where, \( \eta = 4\alpha \hbar c/e/45m_0^2c^4 \) and the second term is the nonlinear effect of the quantum vacuum. When the Langrangian is given, the dynamics is defined uniquely. Now, when we calculate the variational of the action integral of this Heisenberg-Euler Lagrangian, we can obtain Maxwell’s equations with additional terms.
This represents the full Maxwell’s equations in the quantum vacuum. In the limit of $\hbar \to 0$, this equation converges to the equation in classical physics. Moreover, the formalism of Eq. (15) is the covariant Maxwell’s equations in a medium. For this reason, the tensor $M^\mu_4 \otimes V^s_4$ refers to the electric and magnetic polarization. In this formulation, the fluctuation of the quantum vacuum is introduced by $M^\mu_4 \otimes V^s_4$. This relation shows us that if certain electromagnetic fields $F^\mu_4 \otimes V^s_4$ exist at some location, then an additional field $M^\mu_4 \otimes V^s_4$ also exists there. In this paper, we will use the term “quantum polarization” as having the same meaning as “fluctuation of quantum vacuum”.

However, this vacuum fluctuation behaves like a dress of charge [Fig. 2]. It is essential that an electron always dresses additional charge via quantum polarization. The LAD theory is the model of a “bare” electron. But the vacuum description is corrected by Eqs. (15, 16). Therefore, it is necessary to consider radiation reaction with the corrected model of an electron. This fact connects to the newest model of radiation reaction. By applying Eq. (15) for the LAD field, the radiation reaction field is corrected as follows by using the standard perturbation:

$$F^{\mu_4} = \frac{1}{1-\eta(F_{\text{LAD}} \otimes a_F F_{\text{LAD}} \otimes a_F)} F_{\text{LAD}}^{\mu_4}$$

This equation is stable, avoiding run-away. The new equation conforms to the fact that an equation of motion with radiation reaction of any theory converges to

$$m_0 \frac{d}{dt} w^\mu = -e F^{\mu_4}_a w^\nu + m_0 \tau_a \frac{dv^\mu}{ct} \frac{dw^\nu}{dt} w^\mu,$$

when the normalized energy of an electron $\gamma$ is larger than 10. In deriving the LAD equation, we put a corrective term on the R. H. S. of Eq. (20), considering the relativistic relation $w_{\text{LAD}} = 0$. The model of radiation reaction with vacuum fluctuation is satisfied with this limitation of Eq. (20). If this is limited to only ultraintense laser-high energy electron interactions (the Schott term is relatively small compared to other terms), we don’t need to worry about the problem of run-away. For this reason, we can estimate the radiation reaction by using Eq. (19), in ultraintense laser - high energy electron interactions.

For experiments of radiation reaction, it is necessary to prepare a high energy electron. Therefore, ideally, a high energy electron is generated by an accelerator. So this experiment requires us to configure a laser and an accelerator. This is the first key.

The second key for the experiments of radiation reaction is the colliding angle between the laser and accelerator. To check this, it is necessary to consider the setup of Fig. 3. The effect of radiation reaction is simple, it is equivalent to the energy loss of an electron via light. Therefore, the most important result is the time evolution of the electron’s energy. By calculating Eq. (19) with Eq. (17), we got the typical dynamics including radiation reaction [Fig. 4]. This simulation is the case of $\theta = 0$ in Fig. 3. Initially, an electron has the energy of 600 MeV ($\gamma = 1200$), then it interacts with the laser and emits its energy as light. In this calculation, the laser field was given by

![Fig. 2 The electron model with quantum vacuum fluctuations, the “dressed electron”. In QED, pair creations appear in space everywhere, whether the point is vacuum or around the (bare) electron. The dress charge of an electron behaves like polarization via the Coulomb field or the Liénard-Wiechert field.](image1)

![Fig. 3 Set up of the simulation. This laser has an intensity of $5 \times 10^{12}$ W/cm², a wavelength of 1 µm, a pulse width of 23 fs and a spot size of 30 µm. The Gaussian-linearly polarized laser propagates in the x direction, E and B fields are in y and z directions, respectively. Initially, the electron is in the direction of $(\cos \theta, \sin \theta \cos \phi, \sin \theta \sin \phi)$ from the peak of the laser pulse and has an energy of 600 MeV ($\gamma = 1200$).](image2)
The possibility of radiation reaction observation under ultraintense laser is examined. In a strict sense, this laser field does not satisfy Maxwell’s equations, but is sufficient as a rough estimation of finite spot size effects. Therefore, this energy drop comes about from radiation reaction. In experiments, we need to observe this type of relation. However, the phase between electron and laser is important for experiments. In the first stage, it will be difficult to control ultra-intense lasers. For this reason, our first experiments should be good for checking only the fact whether strong radiation reaction is there or not at the laser intensity of $10^{22}$ W/cm$^2$. At this point, the best way of observation is the case without the dependence on $\varphi$. Fig. 5 shows relations of the initial colliding angle and final electron energy. The error bars in Fig. 5 represent the dependence on $\varphi$ and the difficulty of the experiments. Therefore, when these error bars are smaller, it will be easier to carry out. In this sense, the parameter $\theta$ should be smaller than $10^\circ$ for checking Fig. 4 (errors in the final energy of the electron are ~1%).

**4. Conclusion**

In this paper, we discussed the history of radiation reaction. This is the model of an electron with radiation, predicted in the regime where laser intensities are over $10^{22}$ W/cm$^2$. Many authors have considered various models to avoid one of the mathematical difficulties associated with the equation of motion, called run-away. Though approximations and ways of solving the original LAD equation exist, we considered a new physical model. An electron has a self-field (Coulomb or Liénard-Wiechert field), this self-field generates fluctuation of the quantum vacuum (quantum polarization). This stabilizes the run-away problem. The answer to the question, “can we observe radiation reaction?” is achieved in the following ways: At first, radiation reaction is the effect on an electron, therefore we need to observe Fig. 4. The laser is required to have the intensity of $10^{22}$ W/cm$^2$, but this might be difficult to control. From this point of view, since the electron beam’s property should be as perfectly well known as possible, we should use accelerators. Moreover, neglecting the phase ripples of the laser, the colliding has to be less than $10^\circ$ [Fig. 5]. This is a very rough design of experiments, but we need to clear this to investigate details of the radiation reaction. Finally, the idea of these experiments is shown in Fig. 6, the facility which has a possibility of carrying this out, is for example ELI-NP, since it will have 10 PW lasers and a 600 MeV accelerator.

**Acknowledgement**

This work is partly supported under the auspices of the Japanese Ministry of Education, Culture, Sports, Science and Technology (MEXT) project on “Promotion of relativistic nuclear physics with ultra-intense laser.”
References

1) G. A. Mourou, N. J. Fisch, V. M. Malkin, Z. Toroker, E. A. Khazanov, A. M. Sergeev, T. Tajima, and B. Le Garrec: Opt. Comm. 720 (2012) 285.
2) Report on the Grand Challenges Meeting, 27-28 April 2009, Paris.
3) J. Koga: Phys. Rev. E 70 (2004) 046502.
4) A. Zhidkov, J. Koga, A. Sasaki, and M. Uesaka: Phys. Rev. Lett. 88 (2001) 18.
5) H. A. Lorentz: The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat, A Course of Lectures Delivered in Columbia Univ., New York, in March and April 1906, 2nd edition (Teubner, Leipzig, 1916).
6) M. Abraham: Theorie der Elektrizität: Elektromagnetische Theorie der Strahlung (Teubner, Leipzig, 1905).
7) P. A. M. Dirac: Proc. Roy Soc. A 167 (1938) 148.
8) J. Larmor: Phil. Trans. Roy. Soc. London A 190 (1897) 205.
9) J. D. Jackson: Classical Electro-dynamics, 3rd Ed. (John Wiley & Sons, New York, 1998).
10) W. K. H. Panofski and M. Phillips: Classical Electricity and Magnetism, 1st edition (Addison-Wesley Publishing Inc., Cambridge, 1961).
11) L. D. Landau and E. M. Lifshitz: The Classical theory of fields (Pergamon, New York, 1994).
12) S. Tomonaga: Prog. Theor. Phys. 1 (1946) 27.
13) J. Schwinger: Phys. Rev. 74 (1948) 1439.
14) R. Feynman: Phys. Rev. 76 (1949) 769.
15) F. Dyson: Phys. Rev. 75 (1949) 486.
16) A. Liénard: L’Éclaiarge Électrique 16 (1898) 5, 53, 106.
17) E. Wiechert: Annalen der Physik 309 (1901) 667.
18) G. Farmelo: THE STRANDEST MAN The hidden Life of Paul Dirac. Quantum Genius, (Faber and Faber, London, 2009) Ch. 21.
19) G. W. Ford and R. F. O’Connell: Phys. Lett. A 174 (1993) 182.
20) F. A. Rohrlich: Phys. Lett. A 283 (2001) 276.
21) F. V. Hartmann: High-Field Electro-dynamics (CRC press, New York, 2002).
22) S. Zhang: arxiv: 1303.7120
23) P. Caldirola: Nuovo Cimento 3 (1956) 297-343.
24) I. V. Sokolov: J. Exp. Theo. Phys. 109 (2009) No.2.
25) K. Seto, H. Nagatomo, J. Koga, and K. Mima: Phys. Plasmas 18 (2011) 123101.
26) K. Seto, H. Nagatomo, J. Koga, and K. Mima: Plasma and Fusion Res. 7 (2012) 2404010.
27) K. Seto, H. Nagatomo, J. Koga, and K. Mima: Prog. Theor. Exp. Phys. 2013 (2013) 053A01.
28) K. K. Tam and D. Kiang: Prog. Theor. Phys. 62 (1979) 1245-1252.
29) V. I. Berezhiani, R. D. Hazeltine, and S. M. Mahajan: Phys. Rev. E 69 (2004) 056406.
30) V. I. Berezhiani, S. M. Mahajan, and Z. Yoshida: Phys. Rev. E 78 (2008) 066403.
31) W. Heisenberg and H. Euler: Z. Phys. 98 (1936) 714.
32) J. Schwinger: Phys. Rev. 82 (1951) 664.
33) K. Seto, S. Zhang, J. Koga, H. Nagatomo, M. Nakai, and K. Mima: arXiv: 1310.6646v2.
34) D. Habs, T. Tajima, and V. Zamfir: Nuclear Physics News 21 (2011) 23.