QCD correlation functions and instantons

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QCD point-to-point correlation functions at distances 0.2–1 fm are very different for different channels, and they tell us a lot about inter-quark interactions. Recent studies based on experimental data, 'instanton liquid' approach and lattice measurements are reviewed. Agreement between all of them show that instanton-induced forces dominate the light quark physics, and new findings by Negele et al., that hadrons survive 'cooling', make this statement obvious. We also argue that chiral symmetry restoration is due to breaking of the 'instanton liquid' into polarized 'instanton molecules'.

1. INTRODUCTION

Although space is limited, let me start with a very general comments. There is no need to repeat at this conference how complicated is the QCD vacuum, and how basic it is for the whole domain of particle and nuclear physics. However, it seems worth repeating that lattice simulations should not only try to reproduce the Particle Data Table, with the best accuracy possible, they should lead to understanding of its structure.

How one can do it? Of course, there are multiple examples in other areas. Just the preceding talk gives an example of similar attempts: cosmologists try hard to find 'large scale structures' in positions of galaxies and inhomogeneous background radiation. Much more familiar example is meteorology: very complicated charts of wind velocity, pressure and temperature measured in many points can be translated to easily understandable forecasts, just by pointing out few main cyclons, their magnitude and direction of motion.

Similarly, one may look for 'structures' among various fluctuations of the gauge and quark fields, trying to single out the most important ones. I am going to argue below, that it is the 4-dimensional relatives of cyclons, the instantons, which one should look for first.

My second remark deals with the tools used, the point-to-point correlation functions. We need them because for light quark channels (unlike the heavy quarkonia) the knowledge of hadronic masses does not provide simple and unique picture of interquark interactions. (Note here analogy with nuclear physics: compare 30's, when only binding of nuclei were known, to the days after precise NN scattering experiments were done.)

Lattice simulations have an important role here: new mass measurements should be supplemented by new wide scale studies of details unavailable in 'real' experiments, such as correlators, wave functions and structure functions for channels with various quantum numbers.

Of particular interest is a transition region, $x \approx 0.2–0.5 \text{fm}$ where free propagation of quarks and gluons (at small distances) turns into complicated non-perturbative behaviour. Discussion of relevant physical questions and available phenomenological information on this region can be found in my recent review [1].

Tunneling phenomena in gauge theories, discovered in 2, fascinating semiclassical theory, explanation of chiral anomalies 3, first applications to QCD problems 4 etc. have attracted a lot of attention in late 70's. However, as no explanation for 'diluteness' and validity of semiclassical approximation were suggested from the first principles, optimism has soon died out and most people left the field.

Next difficult period has mainly focused on phenomenological manifestations of instanton-induced effects. The so called 'instanton liquid' model 5 has emerged as a qualitative picture. It suggested relative diluteness and large action per
instanton due to their relatively small size, but also emphasized significant interaction in the ensemble as the origin of density stabilization. It was shown, that this picture can explain several puzzles of the hadronic world (see e.g. \cite{7,1} for details).

Attempts to describe interacting instantons were initiated by the variational approach \cite{6}, which has qualitatively reproduced the 'instanton liquid' picture. Further numerical studies of this problem \cite{12} have allowed to get rid of many approximations and eventually included fermionic effects to all orders in 't Hooft effective Lagrangian \cite{3}. We return to discussion of this approach below, and now let me jump directly to several important steps made during the last year.

First of all, more than 40 correlation functions were calculated in the framework of the simplest ensemble of the kind, the Random Instanton Liquid Model (RILM) \cite{8}. Agreement with data is generally good, and in some cases (including $\pi, N$ etc) it is really surprising.

Next, some of those functions were calculated on the lattice \cite{15}, also with good agreement with the RILM results. That the agreement is not occasional but based on the same dynamics was clearly shown by recent findings, a kind of decisive experiment, reported at this conference \cite{16}. 'Cooled' lattice configurations, containing only instantons, not only has reproduced parameters of the 'instanton liquid', but they also lead to the same correlation functions!

I think these studies, taken together, has essentially answered many old questions like: why a nucleon is bound? Instanton-induced attraction rather than confinement or perturbative effects clearly play the major role here.

2. WHY INSTANTONS?

The main formal reason why instantons are so important for physics of light fermions is related to the famous 't Hooft 'zero modes', the localized solutions of the Dirac equation

\[ D_{\mu} \gamma_{\mu} \phi_0 (x) = 0 \]

in the instanton field. Evaluating the (Euclidean) quark propagator $S = -1/(iD_{\mu} \gamma_{\mu} + im)$ for $m \to 0$ one has to deal mainly with small eigenvalues.

It is convenient to look at the instanton as a 'trap' for quarks, something like a 'receptor' atom in a semiconductor, creating a new state with the energy value being forbidden otherwise. At finite density of such atoms, an electron can propagate far, just by hopping from one atom to another. The same is true for finite instanton density, leading to a 'zero mode zone' of collectivized quark states. That is the mechanism leading to the non-zero quark condensate. If more than one quark is travelling in the QCD vacuum ($\bar{q} q$ for mesons and $qqq$ for baryons), they 'hop' over the same instantons. It implies an attractive interaction, which is in fact the one binding quarks together.

Why instantons and not any other fluctuation of the gauge field? Here one has to look more specifically into the chiral and flavor structure of the instanton-induced interaction. As shown by 't Hooft, at tunneling quarks with one chirality 'dive into the Dirac sea' while those with the opposite chirality 'emerge' from it. Therefore instanton-induced forces should be stronger in scalar and pseudoscalar channels compared to vector or axial ones. Looking at phenomenological correlators at small distances, one finds that it is exactly right.

Consider corrections to correlation functions are provided by instantons to the free propagation at small distances. (If those corrections are relatively small, and this is the main point, one may hope to use the 't Hooft interaction in the lowest order, which makes consideration simple.) There are 4 such channels for 2 flavors and it is a simple matter to see that correction is positive (or attractive) for $\pi, \sigma$ channels and negative (or repulsive for $\delta, \eta'$ ones. Thus the same mechanism leads to both light pion and heavy $\eta'$! Both splitting from 'typical mesons' are large, which is a very strong hint.

3. CORRELATORS IN THE INSTANTON VACUUM

Now we proceed from qualitative hints to quantitative calculations. We have to evaluate a quark propagator in the multi-instanton field configura-
tion, which can be done as follows:

\[ S(x,y) = \Sigma_{ZMZ} \frac{\phi_{\lambda}(x) \phi_{\lambda}^+(y)}{\lambda - im} + iS_{NZM}(x,y) \]

where the first term is the sum over states belonging to the 'zero mode zone'. The non-zero modes (analogos of 'scattering states') are taken into account by the last term, see details in [8].

We have first calculated correlators for the simplest ensemble possible, the 'random instanton liquid model' (RILM), in which: (i) all instantons have the same size \( \rho_0 = 0.35 \text{ fm} \); (ii) they have random positions and orientations; (iii) instanton and anti-instanton densities are equal, and in sum it is \( n_0 = 1 \text{ fm}^{-4} \). These are the parameters suggested a decade ago in [5]; the density comes from the gluon condensate and size from various other things, say from the quark condensate value. 'Cooling' of lattice configurations provide a way to see instantons and check them, see [16] for the latest results and earlier references. Let me only state that these numbers are essentially confirmed.

The main step in the calculation is inversion of the Dirac operator, written in the zero-mode subspace. (We typically use in sum 256 instantons and anti-instantons, which tells the dimension of this matrix and the volume of the box.)

Although the quark propagators are gauge dependent, we have looked at them first in order to see whether they can be reproduced by any simple model, say by 'constituent quarks' with a constant mass. We have found that chirality-non-flipping part of the propagator indeed looks as if quark get a mass about 300-400 MeV, but the chirality-flipping part does not look like that at all. None of many correlators calculated behave like 'constituent quark model'.

Our results for \( \pi, \rho, N, \Delta \) channels are shown in Fig.1 and Fig.2. All correlators are plotted in a normalized way, divided by those correponding to free quark propagation: that is why all of them converge to 1 at small distances. Solid lines correspond to experiment [1], while the long-dashed and short-dashed curves correspond to QCD sum rule predictions [13] and [14], respectively.

The agreement for the pion curve is as perfect as it can be: both the mass \((142 \pm 12 \text{ MeV})\) and the (pseudoscalar) coupling are reproduced correctly, inside the error bars! Note large deviations from perturbative behavior happens at very small distances for the \( \pi \) channel, while exactly the opposite is observed in the \( \rho \) case, the plotted ratio remains close to 1 up to a very large \( x \). Both RILM and lattice has reproduced that non-trivial observation.

Proceeding to baryonic channels, let me mention that we have actually measured all 6 nucleon correlators and 4 delta ones, and have fitted them all. Again, agreement between RILM and lattice results is surprisingly good, literally inside the error bars. Both display a qualitative difference between the nucleon and the delta correlators: this can be traced to attractive instanton-induces forces for the spin-isospin-zero diquarks.
Without any one-gluon exchange the RILM predicts the $N - \Delta$ splitting (actually, we have found that in RILM $m_N = 960 \pm 30$ MeV and $m_\Delta = 1440 \pm 70$ MeV, so the splitting is in fact somewhat too large).

There is no place here to discuss many other channels. The most difficult case proved to be the isosinglet scalar $\sigma$, for which one should not only evaluate the 'double quark loop' term, but also subtract the disconnected $| < \bar{q}q > |^2$ part. Curiously enough, we have found dominance of a light state, with $m \sim 500$ MeV, reminiscent of the 'sigma meson' of 60's. For several reasons such measurements are now beyond the reach of lattice calculations, although existence of attractive interaction at $x \sim 1/2 fm$ can probably be seen.

Not included in the preprints mentionioned are studies of the the so called 'wave functions' (known also as Bethe-Salpeter amplitudes). They have shown with greater clarity that RILM does lead to quark binding, without perturbative (Coulomb-like) and confining forces. Although the shape of the wave function is not the same as was found on the lattice, its width is only slightly larger. The main qualitative features (e.g.: $\pi, N$ are more compact than $\rho, \Delta$) are also reproduced.

4. INTERACTING INSTANTONS

Clearly, RILM discussed above cannot be but a crude approximation: at least, the very phenomenon studied above, 'hopping' of quarks from one instanton to another, should produce strong correlation between them. Another obvious source of interaction is the non-linear gluonic Lagrangian: a superposition of instantons and anti-instantons have the action different from the sum of the actions. Effective statistical system should be described by a partition function

$$Z = \int d\Omega \exp(-S_{glue}) | \det(i\hat{D} + im) |^{N_f}$$

(where $d\Omega$ is the measure in space of collective coordinates, 12 per instanton). It is a problem similar to those traditionally studied in statistical mechanics, the compication is the fermionic determinant which is non-local. However, it is still orders and orders of magnitude simpler than the lattice gauge theory. As shown in [6,12] this statistical sum describes a liquid, in which chiral symmetry is broken. Recent studies of correlation functions with interacting ensemble have shown other significant improvement over RILM. In particular, the global fluctuations of the topological charge are screened, and for $m \to 0$ the topological susceptibility vanishes, as it should. The distribution of Dirac eigenvalues $\lambda$ has a different shape, with a dip at $\lambda = 0$ instead of a peak, and satisfy several non-trivial general theorems, see [20].

Unfortunately, it is still not quite quantitative theory. The reason is we do not know how
to separate 'semiclassical' fluctuations with large action, from very close instanton-anti-instanton pair on the bottom of the 'valley' ([9]). Those configurations has small field, therefore they should be included in the perturbation theory. If we suppress them by some repulsive core, we get reasonable results, but a satisfactory solution of this problem is still missing.

5. CHIRAL SYMMETRY RESTORATION

Instantons at finite temperatures is a subject for separate talk, and here let me only discuss recent development related with critical phenomena near the chiral symmetry restoration point, $T \approx T_c$.

The main phenomenon in this region is a strong 'pairing' of instantons, leading to splitting of the 'instanton liquid' into the $\bar{\Pi}$ molecules. The first (strongly simplified) discussion of chiral restoration transition at this angle was made in [19]. One new finding is strong and rapid 'polarization' of these molecules in the critical region. $\bar{\Pi}$ interaction at finite $T$ were studied in details in [17], and the main anisotropy comes from the quark-induced interaction $|\sin(\pi T \tau)/\cosh(\pi T r)|^{2N_f}$, (where $\tau, r$ are distance between the two centers in time and space directions). Around $T - \epsilon$ the point $r = 0, \tau = 1/(2T)$ is strongly prefered, and, as shown in Fig.3, the degree of polarization rapidly frows.

This polarization leads to rapid growth of the gluonic energy density in the transition region, which was in fact observed in lattice simulations with dynamical fermions. Recall that, in terms of Minkowski field strengths, it is

$$\epsilon = \frac{1}{2}(E^2 + B^2) + g^2 \frac{(11/3)N_c - (2/3)N_f}{128\pi^2}(E^2 - B^2)$$

For 'unpolarized' objects the first term is zero, and only the second ('anomalous') one contributes. However, for the 'polarized' molecules one finds $E^2 = -8B^2$, and the first term works. It produces a jump in $\epsilon(T)$ of the order of 1 GeV/fm$^3$.

The second interesting findings are specific changes in the correlation functions below $T_c$. A simple 'cocktail model' [8] was used, containing both 'random' component and some 'molecule fraction' with weight $f_m = 2N_molecules/N_{all}$. We have first found that $<\bar{q}q>$, depends on $f_m$ in a way similar to its $T$-dependence, measured on the lattice: it changes little first, and then rapidly vanishes at $f_m \to 1$. Different correlation functions depend on $f_m$ quite differently. Two examples are shown in Fig.4 for $\pi, \rho$ channels. One can see, that while the vector correlator is not changed much, the pion on display remarkable stability for $f_m = 0.8$, with subsequent strong drop toward $f_m = 1$. At the last point complete chiral symmetry get restored, so the pion correlator coincides with its scalar partners $\sigma, \delta$.

More studies along these lines and their comparison with recent lattice works are in progress.

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Figure 4. $\pi, \rho$ correlation functions for the 'cocktail' model. Open points correspond to pure 'random' instanton ensemble, while the closed ones correspond to $f_m = 1$. The lines show intermediate cases $f_m = .25, .5, .75$.

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