Interference-Free and Interference-Dominated Photoionization: Synthesis of Ultrashort and Coherent Single-Electron Wave Packets

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Abstract. Ionization of hydrogen-like ions driven by intense, short, and circularly-polarized laser pulses is considered under the scope of the relativistic strong-field approximation. We show that the energy spectra of photoelectrons can exhibit two types of structures, i.e., interference-dominated or interference-free ones. These structures are analyzed in connection to the time-dependent ponderomotive energy of electrons in the laser field. A possibility of synthesis of ultrashort single-electron pulses from those structures is also investigated.

1. Introduction
In four-dimensional electron microscopy and diffraction (see, for instance, Refs. [1, 2, 3, 4] and the reviews [5, 6, 7, 8]), short electron wave packets are used to monitor dynamical changes occurring on the femtosecond and picometer scales. Furthermore, as pulses comprising of many electrons tend to spread fast in time, which is due to space-charge effects, a lot of effort has been devoted to the generation of single-electron pulse sources [3, 4, 5, 7, 9, 10, 11].

Recently, we have demonstrated that ultrashort and coherent single-electron wave packets can be obtained during ionization driven by circularly-polarized and intense laser pulses [12, 13, 14, 15]. We have shown that the shape of the temporal ponderomotive energy curve allows one to predict the polar and azimuthal angles at which the electron pulses can be synthesized. Also, the time-duration of the resulting electron wave packets can be significantly modified by changing the intensity of the laser pulse or its polarization [16, 17].

It is the aim of this paper to further analyze the relation between the probability distribution of photoionization and the time-dependent ponderomotive energy. This is done in Sec. 3, after the necessary theoretical formulation is given in Sec. 2. Furthermore, in Sec. 4, we demonstrate how different patterns in the energy spectra of photoelectrons may lead to the synthesis of very short and coherent single-electron wave packets. Finally, in Sec. 5, we present our conclusions and remarks.

In the following, we use units such that $\hbar = 1$. The Feynman slash notation $\not{a} = \gamma \cdot a = \gamma^\mu a_\mu$, where $a$ is an arbitrary four-vector and $\gamma$ are the Dirac gamma matrices, is also employed. In Sec. 2 we make use of the so-called light-cone variables, which are defined with respect to a unit vector $\mathbf{n}$. For instance, for the space-time four-vector $x = (ct, \mathbf{x}) \equiv (x^0, \mathbf{x})$, such variables are:
\[ x^\parallel = \mathbf{n} \cdot \mathbf{x}, \quad x^\perp = x^0 - x^\parallel, \quad x^+ = (x^0 + x^\parallel)/2, \quad \text{and} \quad x^- = x - x^\parallel \mathbf{n}. \]

Furthermore, the average of a function \( F(\phi) \) over the laser pulse duration is denoted with the angle brackets, i.e., \( \langle F \rangle \).

## 2. Theory

Consider a hydrogen-like ion interacting with an intense, short, and circularly-polarized laser pulse represented by the four-vector potential \( A(x) \). The latter, in the so-called plane-wave-fronted pulse approximation, can be written in terms of two arbitrary shape functions, \( f_1(\phi) \) and \( f_2(\phi) \), which depend on the laser phase, \( \phi = k \cdot x \), and vanish for \( \phi < 0 \) and \( \phi > 2\pi \). Namely,

\[ A(k \cdot x) = A_0 [\varepsilon_1 f_1(k \cdot x) + \varepsilon_2 f_2(k \cdot x)], \tag{1} \]

where \( A_0 \) is the amplitude of the vector potential and \( k = k^0(1, \mathbf{n}) = k^0 n \) is the wave four-vector. Here, \( \mathbf{n} \) is a unit vector which represents the direction of propagation of the laser field, \( k^0 = \omega/c \) is the wave-number, \( \omega = 2\pi / T_p \) is the fundamental frequency of laser field oscillations, and \( T_p \) is the pulse duration. Also, in (1), \( \varepsilon_i = (0, \varepsilon_i) \ i = 1,2 \) are two polarization four-vectors such that \( \varepsilon_i \cdot \varepsilon_j = -\delta_{ij} \) and \( k \cdot \varepsilon_i = 0 \). Due to the interaction with the laser field, the ion being initially in the bound state \( \Psi_i(x) \) of energy \( E_0 \) and the spin degree of freedom \( \lambda_i = \pm \), gets ionized. The probability amplitude of ionization, in the framework of the relativistic strong-field approximation (RSFA) \[12, 18, 19, 20\], which is based solely on the lowest-order Born expansion of the final scattering state with respect to the bounding potential, is given by

\[ A_{\lambda,\lambda}(p) = -i \int d^4 x \ e^{-i(E_0/c)x^0}\sqrt{\omega^2 + \mathbf{k}^2}\Psi_{\lambda}(x)e^{iA(x)}\Psi_i(x), \tag{2} \]

where \( \psi_{\lambda}(x) \) is the Volkov state \[21, 22, 23\]. Such state describes the quantum-mechanical behavior of a free electron of asymptotic momentum \( p \), energy \( E_p = \sqrt{(mc^2)^2 + (cp)^2} \), and the spin degree of freedom \( \lambda = \pm \) in the laser pulse (1).

The total probability of photoionization, with fixed \( \lambda \) and \( \lambda_i \), can be calculated by integrating the modulus squared of the amplitude (2) over the density of final electron states. This allows us to define the spin resolved triply-differential probability distribution which, in atomic units, is represented as \( \mathcal{P}_{\lambda,\lambda}(p) \). Also, by summing the values corresponding to the final spin states while averaging over the initial ones, we obtain the spin-independent probability distribution \( \mathcal{P}(p) \), i.e.,

\[ \mathcal{P}(p) = \frac{1}{2} \sum_{\lambda,\lambda = \pm} \mathcal{P}_{\lambda,\lambda}(p). \tag{3} \]

Even though we do not present here the explicit formula defining this quantity, it can be found in Eqs. (18) and (21) of Ref. [14].

It is worth mentioning that, if the lowest-order Born expansion is used, the application of the RSFA is justified provided that the photoelectron kinetic energy is much larger than the ionization potential of the field-free ionic system, i.e.,

\[ \sqrt{(mc^2)^2 + (cp)^2} - mc^2 \gg mc^2 - E_0. \tag{4} \]

Thus, the analysis of the low-energy portion of the probability distributions using the RSFA could be only qualitative. For this reason, in this paper we shall concentrate on the high-energy ionization, in which Eq. (4) holds.

For our further considerations, we introduce here the time-dependent ponderomotive energy of the electron in the laser field. In terms of the vector potential (1), it is given by

\[ U_p(\phi) = \frac{e^2 A^2(\phi)}{2mc} = \frac{e^2 A_0^2}{2mc} [f_1^2(\phi) + f_2^2(\phi)]. \tag{5} \]
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2.1. Laser field

For our numerical illustrations, we consider a laser pulse consisting of \(N_{\text{osc}}\) field cycles within a \(\sin^2\) envelope interacting with a hydrogen-like ion of atomic number \(Z = 2\) (He\(^+\)). The electric field characterizing such pulse, in the plane-wave-fronted pulse approximation, is determined by the shape functions

\[
F_j(\phi) = N_0 \sin^2\left(\frac{\phi}{2}\right) \sin(N_{\text{osc}} \phi + \delta_j + \chi) \cos(\delta + \delta_j), \quad \text{for } j = 1, 2, \tag{6}
\]

As mentioned before, these functions are defined in the interval \(0 \leq \phi \leq 2\pi\) and are zero otherwise. Here, \(\delta\) and \(\delta_j\) define the polarization of the laser field, \(\chi\) is the carrier-envelope phase, and \(N_0 = \sqrt{8/3N_{\text{osc}}}\) is a normalization constant \([12]\). In the following, we assume a circular polarization \([\delta = \pi/4, \delta_j = (j - 1)\pi/2]\) for a laser pulse propagating along the \(z\)-direction \((\epsilon_1 = e_x, \epsilon_2 = e_y, n = e_z)\). Also, the carrier-envelope phase is \(\chi = \pi\).

As the electric field and the vector potential obey the relation \(\mathbf{E}(\phi) = -\partial_t \mathbf{A}(\phi)\), the shape functions defining the latter [see, Eq. (1)] can be obtained from (6). Namely,

\[
f_j(\phi) = -\int_0^\phi d\phi' F_j(\phi'), \tag{7}
\]

and they also vanish outside the interval \(0 \leq \phi \leq 2\pi\).

In Fig. 1, we show the parametric plots of the tips of the vector potential (left panel) and electric field \(\mathbf{E}(\phi)\) (middle panel) for the laser pulse defined by (6). The corresponding time-dependent ponderomotive energy [Eq. (5)] is also shown (right panel). The laser field parameters are \(I = 10^{17} \text{ W/cm}^2, N_{\text{osc}} = 5,\) and \(\omega_L = N_{\text{osc}} \omega = 1.5498 \text{ eV}\) (see the text). All figures are presented in relativistic units in the \(xy\)-plane, when changing the laser field phase \(\phi\) from 0 to \(2\pi\).

As it will become clear shortly, this expression will be helpful in understanding the general features of the energy spectra of photoelectrons (see, Sec. 3.1).

Figure 1. Evolution of the tips of the vector potential \(\mathbf{A}(\phi)\) (left panel) and electric field \(\mathbf{E}(\phi)\) (middle panel) for the laser pulse defined by (6). The corresponding time-dependent ponderomotive energy [Eq. (5)] is also shown (right panel). The laser field parameters are \(I = 10^{17} \text{ W/cm}^2, N_{\text{osc}} = 5,\) and \(\omega_L = N_{\text{osc}} \omega = 1.5498 \text{ eV}\) (see the text). All figures are presented in relativistic units in the \(xy\)-plane, when changing the laser field phase \(\phi\) from 0 to \(2\pi\).

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In Fig. 1, we show the parametric plots of the tips of the vector potential (left panel) and electric field (middle panel) describing the laser pulse (6). The parameter used to obtain those curves, which is the phase \(\phi\), changes from 0 to \(2\pi\). Also, in the right panel of the same figure, the time-dependent ponderomotive energy \(U_p(\phi)\) [Eq. (5)] is plotted. Note that, in Fig. 1 the carrier frequency corresponds to \(\omega_L = N_{\text{osc}} \omega = 1.5498 \text{ eV}\), the number of field oscillations is \(N_{\text{osc}} = 5\), and the time-averaged intensity is \(I = 10^{17} \text{ W/cm}^2\). All curves are presented in relativistic units, they start at the origin of coordinates and evolve counterclockwise while increasing \(\phi\).

In the following, we define the azimuthal angle \(\varphi\) in terms of the vector potential. Namely,

\[
\varphi \equiv \varphi(\phi) = \arg(e\mathbf{A}(\phi) \cdot \epsilon_1 + ie\mathbf{A}(\phi) \cdot \epsilon_2), \tag{8}
\]
where \(0 \leq \arg(z) < 2\pi\) represents the argument of the complex number \(z\). According to this definition, \(\varphi\) is not a one-to-one function of the phase \(\phi\). For this reason, all curves presented in Fig. 1 intersect themselves at different phases. That will result in interference patterns observed in the ionization distributions (see, the next Section).

### 3. Probability distribution in strong-field photoionization

To calculate the probability distribution of ionization directly from Eq. (2) requires a considerable computational effort. For this reason, in recent publications (see, e.g., Refs. [14, 15]), the saddle-point approximation has been applied in (2), which greatly simplifies and speeds up numerical calculations. With this method, certain properties of the probability distributions, when calculated in a given space direction, were determined. The latter is defined by the polar and azimuthal angles of electron detection \(\theta_p\) and \(\varphi_p\), respectively. For the purpose of this paper, we shall only refer to the aforementioned properties and their consequences. Their more detailed description can be found in Refs. [12, 14, 15, 16, 17].

First, it has been shown that the energy spectra of photoelectrons can exhibit two types of structures; namely, interference-dominated and interference-free ones [12, 14, 15, 17]. While the former are characterized by dense oscillations, the latter appear as broad lobes spanning over hundreds of single-photon energies. Furthermore, when the driving pulse is linearly polarized or lasts for a very long time, only interference-dominated structures are expected [17]. In contrast, when the pulse is short and its polarization is circular or, in general, elliptical, interference-free structures appear [12, 14, 15, 16, 17]. The interference-free structures are also known as supercontinua in photoionization.

Second, if the laser pulse is not linearly-polarized, the target ion is sufficiently light \((Z\alpha \ll 1)\), and the photoelectrons are fast enough \((E_p - m_e c^2 \gg m_e c^2 - E_0)\), there exist a real phase \(\phi_p\) which determines several properties of the probability distribution of photoelectrons. Such phase, which typically is associated with the time when the electron appears in the continuum, relates to the azimuthal angle of electron detection \(\varphi_p\) according to [16, 17]

\[
\varphi_p = \arg(eA(\phi_p) \cdot \varepsilon_1 + ieA(\phi_p) \cdot \varepsilon_2). \tag{9}
\]

As the vector potential \(A(\phi)\) is not a uniquely-defined function of the azimuthal angle \(\varphi\) [cf. Eq. (8)], it is possible to find different phases \(\phi_p\) which, for a given \(\varphi_p\), satisfy Eq. (9).

Third, it has been shown that there is a close relation between the central energy of the supercontinuum and the ponderomotive energy of the electron in the laser field at its time of birth. Namely, the supercontinua are approximately located at the kinetic energies given by the relation [16, 17]

\[
E_p - m_e c^2 \approx U_p(\phi_p). \tag{10}
\]

However, as the \(U_p(\varphi)\)-spiral may intersect itself at several points (see, the right panel of Fig. 1), two or more phases \(\phi_p\) can contribute to the probability of electron being detected in the given direction and with the given kinetic energy. Hence, the self-intersections of the ponderomotive energy curve lead to the formation of interference-dominated structures in the energy spectra of photoelectrons. In contrast, for azimuthal angles for which the arcs of the spiral do not intersect, supercontinua will be observed.

Finally, it is worth mentioning that, for the geometry considered in this paper, the RSFA predicts a shift of the direction of photoelectron detection towards polar angles smaller than \(\pi/2\). This shift depends on the final kinetic energy of the electrons, as the most energetic ones absorb a larger number of laser photons and, therefore, experience stronger radiation pressure [24]. Moreover, according to [16], it can be estimated that the maximum value of the probability
Figure 2. Spin-independent energy spectra of photoelectrons $P(p)$ calculated for the same laser field parameters as in Fig. 1. The polar and azimuthal angles of detection are specified in each panel.

A distribution is observed at the polar angle,

$$\theta_p \approx \arccos \sqrt{\frac{E_p - m_e c^2}{E_p + m_e c^2}}.$$  \hspace{1cm} (11)

As before, such expression is valid provided that the target ions are light enough and the photoelectrons are fast.

Note that, by setting the azimuthal angle of detection and determining the phase (or phases) $\varphi_p$ which satisfy (9), several features of the probability distribution can be predicted. For instance, Eq. (10) determines the most probable final kinetic energy of photoelectrons. Also, it can be inferred from the $U_p(\varphi)$-spiral whether or not interference effects would take place. Finally, Eq. (11) determines the direction in space where photoelectrons with a given energy will be most likely observed. In the remaining part of this Section, we shall make use of those observations to analyze the probability distributions.

3.1. Energy spectra of photoelectrons

We now illustrate our theory by calculating the spin-independent probability distribution in photoionization. Fig. 2 shows the energy spectra of photoelectrons $P(p)$ calculated at a given direction in space, as defined by the pair of angles $(\theta_p, \varphi_p)$, and for the laser pulse described in Fig. 1. While the azimuthal angle is chosen arbitrarily, the polar angle is set such that the distribution acquires its maximum values. From Fig. 2 one can see that, for $(\theta_p, \varphi_p) = (0.4645\pi, 0)$ (left panel), an interference-dominated structure within a Gaussian-like envelope appears. Its maximum is located at 14 keV whereas its energy width ($\Delta E_p$), calculated as the full-width at half maximum (FWHM), is roughly 1.25 keV. For this specific structure, the ratio $\Delta E_p/E_{\text{center}}$ (also known as the degree of monochromaticity [3, 11]), where $E_{\text{center}}$ represents its central kinetic energy, is approximately 0.09. Now, in the right panel of Fig. 2, where $(\theta_p, \varphi_p) = (0.46\pi, \pi)$, a broad supercontinuum is observed. In this case, the maximum of the distribution is located at 15.9 keV, its width is $\Delta E_p \approx 1.4$ keV, and the ratio $\Delta E_p/E_{\text{center}}$ is also approximately 0.09.

Let us now analyze the relation between the position of maxima in the high-energy portion of the photoelectron spectra and the shape of the ponderomotive energy curve (right panel of Fig. 1). The latter intersects itself at $\varphi_p = \varphi = 0$, which corresponds to the energy $0.0276m_e c^2 \approx 14.2$ keV. Hence, according to the observations mentioned above, at $\varphi_p = 0$,
Figure 3. The same as in Fig. 2 but for the time-averaged laser field intensity $I = 10^{18}$ W/cm$^2$. The polar and azimuthal angles of detection are shown in the upper part of both panels.

an interference-dominated structure with maxima at around 14 keV is expected. This is exactly the case, as shown in the left panel of Fig. 2. In contrast, the line $\varphi_p = \varphi = \pi$, crosses a single external arc of the $U_p(\varphi)$-curve at $0.0312 m_e c^2 \approx 16$ keV. This explains the formation of the supercontinuum observed in the right panel of Fig. 2 at roughly 16 keV.

Note that the predictions arising from Eq. (10) go even further. For instance, if we increase the laser field intensity by one order of magnitude, the maximum of the probability distribution of ionization will appear at ten times larger kinetic energies [see, also Eq. (5)]. As the ponderomotive energy curve scales linearly with the driving field intensity while conserving its general shape, it is also expected that the interference-free and interference-dominated structures will be located at the same azimuthal angles $\varphi_p$. In order to check that, in Fig. 3 we present the spin-independent energy spectra of photoelectrons $P(p)$ calculated at the same azimuthal angles $\varphi_p$ as in Fig. 2 but for the laser field intensity $I = 10^{18}$ W/cm$^2$. As expected, while at $\varphi_p = 0$ (left panel) we observe an interference-dominated structure with maximum at 140 keV, at $\varphi_p = \pi$ (right panel) the probability distribution exhibits a broad supercontinuum with maximum at 159 keV. This is in perfect agreement with the predictions arising from Eq. (10). Now, with respect to the bandwidth of each structure, it can be estimated that the interference-dominated pattern has the energy spread of roughly 5.8 keV and the ratio $\Delta E_p/E_{\text{center}} \approx 0.04$. On the other hand, while the supercontinuum exhibits a larger bandwidth, $\Delta E_p \approx 6.4$ keV, the ratio $\Delta E_p/E_{\text{center}}$ is also around 0.04. By comparing Figs. 2 and 3, one can conclude that the width of the high-probability structures increase with the laser pulse intensity. However, it does not change with intensity at the same rate as the central kinetic energy does. Moreover, one can expect that electron pulses with smaller degree of monochromaticity can be obtained in high-energy ionization by further increasing the intensity of the driving field. This is due to the fact that the ratio $\Delta E_p/E_{\text{center}}$ continuously decreases with this parameter.

In closing this Section, let us note that the polar angles at which we observe the maximum probability of ionization decrease from $\theta_p \approx 0.46 \pi$, for the intensity $I = 10^{17}$ W/cm$^2$, to $\theta_p \approx 0.38 \pi$, for $I = 10^{18}$ W/cm$^2$ (see, Figs. 2 and 3). Such values agree amazingly well with the predictions arising from Eq. (11).

4. Synthesis of ultrashort and coherent single-electron wave packets

In this Section, we will show how the interference-dominated or interference-free structures can be used to synthesize very short and coherent single-electron pulses. Note that, according to the uncertainty principle, the short-in-time electron wave packets are characterized by large energy bandwidths. Therefore, for our current purpose, we shall concentrate on the structures located in the high-energy portion of the probability distributions, as they are characterized by the
Figure 4. Space-time profiles $P_{\lambda\lambda}(t,d)$ calculated from Eq. (13) for the pairs of angles $(\theta_0, \varphi_0) = (0.4645\pi, 0)$ (upper row) and $(0.46\pi, \pi)$ (lower row). Such distributions are obtained at a distance $d$ from the parent ion and correspond to the energy spectra of photoelectrons shown in the left and right panels of Fig. 2, respectively. The kinetic energy boundaries are $(E_{\text{min}}, E_{\text{max}}) = (11 \text{ keV}, 17 \text{ keV})$ for the upper panels and $(13 \text{ keV}, 19 \text{ keV})$ for the lower ones.

The spin-dependent space-time probability amplitude $A_{\lambda\lambda}(t,d)$, which defines the probability of finding a single-electron wave packet propagating along the direction $n_0$ and at a distance $d = x \cdot n_0$ from the parent ion, is given by [12, 14, 15, 16, 17]

$$A_{\lambda\lambda}(t,d) = N_A e^{-im_e c^2 t} \int_{E_{\text{min}} + m_e c^2}^{E_{\text{max}} + m_e c^2} dEp e^{-|E_p|t - |p|d} |p| p_{|n_0\lambda}\sqrt{E_p} |p\rangle A_{\lambda\lambda}(|p|n_0), \quad (12)$$

where $A_{\lambda\lambda}(p)$ is the probability amplitude of photoionization (2), $N_A$ is a normalization constant, and $E_{\text{min}}$ and $E_{\text{max}}$ select the kinetic energy range from which the wave packet is synthesized. Moreover, the spin-dependent space-time probability distribution is calculated from (12) as

$$P_{\lambda\lambda}(t,d) = [A_{\lambda\lambda}(t,d)]^* A_{\lambda\lambda}(t,d). \quad (13)$$

Here, the initial and final spin states are projected onto the laser field propagation direction, $e_z$.

In the upper and lower rows of Fig. 4, we present the temporal profiles $P_{\lambda\lambda}(t,d)$ [Eq. (13)] obtained from the energy spectra of photoelectrons shown in the left and right panels of Fig. 2, respectively. Here, the directions of propagation, defined by the pair of angles $(\theta_0, \varphi_0)$, are $(0.4645\pi, 0)$ (upper row) and $(0.46\pi, \pi)$ (lower row). While in the left column the distances from the parent ion are such that the photoelectron just left the laser pulse (i.e., $\phi = k \cdot x \gtrsim 2\pi$), the right column relates to a given distance of $1 \times 10^5 a_0 \approx 5.3 \mu m$, where $a_0$ is the Bohr radius. Note that, as the normalized space-time distributions behave in the same way independently of the initial and final spin states, only the case $(\lambda_0, \lambda) = (-, -)$ is considered.

From the upper-left panel of Fig. 4 it can be seen that, at $d = 1.2 \times 10^4 a_0 \approx 0.6 \mu m$, the interference-dominated structure leads to a double-hump temporal profile, characterized by two well-defined Gaussian-like lobes, each one of duration around 440 as. However, at larger distances from the parent ion, both lobes undergo a considerable broadening (upper-right panel). This leads to the formation of a single structure with a rich interference pattern, which is a signature...
Figure 5. The same as in Fig. 4 but calculated for the energy spectra of photoelectrons shown in Fig. 3 (i.e., for the time-averaged laser intensity $I = 10^{18}$W/cm$^2$). The directions of propagation are $(\theta_0, \varphi_0) = (0.3875\pi, 0)$ (upper row) and $(0.38\pi, \pi)$ (lower row). Here, the kinetic energy boundaries are $(E_{\text{min}}, E_{\text{max}}) = (130$ keV, 150 keV) for the upper row and $(145$ keV, 175 keV) for the lower one. In the right column we set $d = 10^6a_0 \approx 53$ µm.

of the coherence of the process. From the FWHM of its envelope, it can be estimated that at $d \approx 5.3$ µm such structure lasts for 4.4 fs. On the other hand, if the electron wave packets are obtained from the supercontinuum shown in the right panel of Fig. 2, we observe a single and smoothly-varying temporal profile. While at a distance $d = 2.1 \times 10^4a_0 \approx 1.1$ µm from the parent ion, the duration of the electron pulse is approximately 670 as (lower-left panel of Fig. 4), at $d \approx 5.3$ µm it increases up to 3.2 fs (lower-right panel of the same figure). However, even at the larger distance, the electron wave packet still exhibits a smooth, interference-free behavior.

In Fig. 5 we present the same as in Fig. 4 but for the laser field intensity $I = 10^{18}$W/cm$^2$. This time, we set $(\theta_0, \varphi_0) = (0.3875\pi, 0)$ (upper row) and $(0.38\pi, \pi)$ (lower row). Note that those temporal profiles relate to the energy spectra of photoelectrons shown in the left and right panels of Fig. 3, respectively. From the upper-left panel of Fig. 5 one can see that, at the distance $d = 7.1 \times 10^4a_0 \approx 3.8$ µm from the parent ion, a double-hump structure is also formed. As calculated from the FWHM of the Gaussian-like lobes, such structure would lead to an electron pulse lasting only for 307 as (each individual hump). However, at $d \approx 53$ µm a broad interference pattern, with duration of 4.6 fs, is formed (upper-right panel). In contrast, while an electron pulse synthesized from the supercontinuum lasts for 245 as at a distance of 3.4 µm (lower-left panel), it increases its duration up to 3.6 fs at $d = 53$ µm (lower-right panel).

Let us note that the analysis presented in this paper relates to photoelectrons with sub-relativistic kinetic energies, i.e., $E_p - m_e c^2 < m_e c^2$. Therefore, it is expected that the synthesized electron pulses spread fast in time. However, by increasing the intensity of the laser field, it is possible to create nearly-relativistic (or relativistic) wave packets of short duration [15]. In such case, the temporal spreading is negligible or very small. Moreover, it was recently demonstrated that it is also possible to obtain very energetic electron pulses by changing the polarization of the laser pulse from circular to almost linear, while maintaining a constant intensity of the laser field [17].

In closing this Section, let us note that the generation of attosecond electron wave packets
has also been investigated by Varró and Farkas [25]. However, the wave packets analyzed there have a different origin and occur under different circumstances. Namely, their train of pulses result from the interference of electron plane-waves and appear within both plane-wave laser fields and solids. In contrast, our solitary short electron pulses are built from free-electron states. Furthermore, by accounting for the interference effects, the broad structure present in Fig. 5 (upper-right panel) constitute in fact a train of attosecond (or even zeptoseconds) electron pulses observed at the mesoscopic distance $d \approx 53 \, \mu m$ from the parent ion.

5. Conclusions
In this paper we have analyzed the high-energy photoionization of hydrogen-like ions driven by short and circularly-polarized laser pulses. As it was shown in Refs. [12, 14, 15], the energy spectra of photoelectrons can present interference-dominated or interference-free structures. As we have checked here, those structures present maxima located at photoelectron kinetic energies given by Eq. (10), which is valid provided that the target ions are light enough and the electrons are fast [16, 17]. Furthermore, we have shown in this paper that the central kinetic energies of those structures increase linearly with the intensity of the driving field. Finally, we have demonstrated in Sec. 4 that the interference-dominated patterns lead to the creation of double-hump electron wave packets which appear, however, at very short distances from the parent ion. At mesoscopic distances, on the other hand, pronounced interferences are established. In contrast, the supercontinuum leads to the creation of solitary coherent and short electron pulses, the duration of which can be controlled by modifying the intensity of the laser field.

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