Measuring a piecewise constant axion field in classical electrodynamics

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In order to settle the problem of the “Post constraint” in material media, we consider the propagation of a plane electromagnetic wave in a medium with a piecewise constant axion field. Although a constant axion field does not affect the wave propagation in a homogeneous medium, we show that the reflection and transmission of a wave at an interface between the two media is sensitive to the difference of the axion values. This observation can be used to determine experimentally the axion piece in matter despite the fact that a constant axion value does not contribute to the Maxwell equations.

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I. LOCAL AND LINEAR MEDIA WITH AXION PIECE

Recently, we discussed local and linear media in classical electrodynamics [4]. In particular, we investigated possible magnetoelectric effects, which are related to the crossterms between the magnetic (electric) field strength and the electric (magnetic) excitation. Using the premetric formalism of electrodynamics, see [2,3,4], the most general local and linear constitutive relation can be written as

\[ D^a = (\varepsilon^{ab} - \epsilon^{abc} n_c) E_b + (\gamma_a^b + s_a^c - \delta_a^b s_c^c) B^b + \alpha B^a, \]

\[ H_a = (\mu_{ab} - \hat{\epsilon}_{abc} m^c) B^b + (-\gamma_b^a + s_b^c - \delta_b^c s_a^c) E_b + \alpha E_a. \]

Here \( D^a \) and \( H_a \) are the electric and magnetic excitations, respectively, and \( E_a \) and \( B^a \) the electric and the magnetic field strengths. The Kronecker symbol is denoted by \( \delta_a^b \), the totally

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antisymmetric Levi-Civita symbol by $\epsilon^{abc}$ and $\hat{\epsilon}_{abc}$, respectively. We have 36 constitutive functions or moduli: the permittivity matrix $\varepsilon^{ab} = \varepsilon^{ba}$ (6 independent components), the impermeability matrix $\mu^{-1}_{ab} = \mu^{-1}_{ba}$ (6 components), the tracefree principal magnetoelectric matrix $\gamma^a_b$ (8 components), the 15 skewon pieces $m^a, n_a, s^b_a$, and, eventually, 1 axion piece $\alpha$. Such a local and linear medium with 36 moduli — called sometimes bi-anisotropic — has been considered, amongst others, by Lindell, Sihvola, Tretyakov, and collaborators [9, 15, 22, 27].

In conventional materials the skewon and the axion pieces vanish and we are left with

$$D^a = \varepsilon^{ab} E_b + \gamma^a_b B^a,$$

$$H_a = \mu^{-1}_{ab} B^b - \gamma^b_a E_b.$$  

The principal magnetoelectric cross terms induced by $\gamma^a_b$ are known to exist in various media, see O’Dell [20]. Such media are described by $6 + 6 + 8 = 20$ constitutive functions. Eqs. (3) and (4) represent the most general material considered by Post [21], e.g..

The skewon pieces are not considered in this article, see, however, [19]. We address here the question on whether the axion piece is permitted — thereby possibly extending a material with 20 moduli to one with 21 moduli — and if so whether it can be determined experimentally by standard methods. In Post [21] it was argued that the axion piece has to vanish, i.e., $\alpha = 0$ (and even $d\alpha = 0$). For this reason, Lakhtakia [7, 8] (and references given there) called $\alpha = 0$ the Post constraint and advocated it as a condition each medium has to fulfill. We quoted in literature in which materials are described (Cr$_2$O$_3$ and Fe$_2$TeO$_6$) that carry an axion piece. Moreover, Lakhtakia, loc.cit., pointed out that a constant axion piece $\alpha$ should not be measurable since it drops out of the Maxwell equations. However, this is only true if we have an axion piece that is globally constant at all spatial points at all times. We will show explicitly that a material with a piecewise constant $\alpha$ can very well be investigated experimentally and thereby $\alpha$ measured uniquely. Thus, neither can the Post constraint be upheld nor poses the measurability of $\alpha$ a problem, as we will show below.

Accordingly, we discuss here the case of two neighboring homogeneous media with different but constant axion pieces. They are separated by the plane $x = 0$, as shown in Fig.1. Permittivity and permeability are assumed to be isotropic. Thus, the constitutive relations
The field equations read
\[ \star \] Here \( \star \) is the 4-dimensional Hodge star operator defined in terms of the metric of spacetime. We use here the calculus of differential forms, see [3, 9].

FIG. 1: Two homogeneous media are separated by the plane \( x = 0 \). Here \((x, y, z)\) are Cartesian coordinates. Moreover: Permittivity \( \varepsilon \), permeability \( \mu \), and axion piece \( \alpha \), see Eqs. (4) and (6).

for the two half-spaces carry 3 constitutive constants \( \varepsilon, \mu^{-1}, \) and \( \alpha \), respectively,

\[ \mathcal{D} = (\varepsilon \varepsilon_0) \star E + \alpha B, \]
\[ \mathcal{H} = (\mu \mu_0)^{-1} \star B - \alpha E, \]  
with \( \star \) as the 3-dimensional Hodge star operator and \( \varepsilon_0 \) and \( \mu_0 \) as electric and magnetic constants (of the vacuum). We use here the calculus of differential forms, see [3, 9].

As sideremark let us remind ourselves that (4), (6), formulated 4-dimensionally with the excitation 2-form \( H = (\mathcal{H}, \mathcal{D}) \) and the field strength 2-form \( F = (E, B) \) and in vacuum, reduce to axion (Maxwell-Lorentz) electrodynamics (see Ni [16, 17, 18] and Wilczek [28]) with the constitutive relation

\[ H = \sqrt{\frac{\varepsilon_0}{\mu_0}} \star F + \alpha F. \]  

Here \( \star \) is the 4-dimensional Hodge star operator defined in terms of the metric of spacetime. The field equations read

\[ \sqrt{\frac{\varepsilon_0}{\mu_0}} d \star F + (d \alpha) \wedge F = J, \quad dF = 0. \]

It is as if the current 3-form \( J \) picked up an additional piece depending on the gradient of the axion field. For \( \sqrt{\varepsilon_0/\mu_0} = 0 \), this corresponds to the pure axion case, that is, to the
perfect electromagnetic conductor (PEMC) of Lindell and Sihvola, a structure that is equivalent to the Tellegen gyrator, see also. The real part of Kiehn’s chiral vacuum is a subcase, for \( \alpha = \text{const} \), of axion electrodynamics.

Thus we see that if the axion piece is globally constant, \( \alpha = \text{const} \), it does not contribute to the Maxwell equations, even though it emerges in the constitutive relation and in the boundary conditions to be discussed in the next section.

II. WAVE PROPAGATION IN TWO HOMOGENEOUS MEDIA

The Maxwell equations without charges and currents read

\[
\begin{align*}
\frac{d}{dt} \mathbf{D} &= 0, \\
\frac{d}{dt} \mathbf{H} - \mathbf{D} &= 0, \\
\frac{d}{dt} \mathbf{B} &= 0, \\
\frac{d}{dt} \mathbf{E} + \mathbf{B} &= 0.
\end{align*}
\]

The calculus of differential forms is used and the conventions of. In the absence of the surface charges and currents, the jump conditions on the boundary surface read (see, pp. 150,151):

\[
\begin{align*}
[D(2) - D(1)]_{S} \wedge \nu &= 0, \\
[\tau_{\alpha} (H(2) - H(1))]_{S} &= 0, \\
[B(2) - B(1)]_{S} \wedge \nu &= 0, \\
[\tau_{\alpha} (E(2) - E(1))]_{S} &= 0.
\end{align*}
\]

Here \( \nu \) is the 1-form density normal to the surface \( S \) and \( \tau_{\alpha} \), \( \alpha = 1, 2 \), are the two vectors tangential to \( S \). The constitutive relations were formulated in and .

Let \( S \) be the plane \( x = 0 \) in Fig.1 that divides the two parts of space that are filled with two different homogeneous material media. For the left half-space \( x < 0 \), we assume homogeneous matter characterized by constant values \( \varepsilon_1, \mu_1, \alpha_1 \). Similarly, for the right half-space \( x > 0 \), we have the constant values \( \varepsilon_2, \mu_2, \alpha_2 \). Somewhat related situations with reflected and scattered waves were discussed by Lindell and Sihvola.

Consider a plane electromagnetic wave travelling along the \( x \) axis in the left half-space. At the interface \( S \), such an incident wave will be partly reflected and partly refracted into the right half-space. Accordingly, the ansatz for the electromagnetic field configuration in the first medium will be a superposition of the right- and left-moving plane waves. Let \( W_y = W_y(\xi_1) \) and \( W_z = W_z(\xi_1) \) be the components of the incident wave with the argument
\[ \xi_1 := \omega t - k_1 x, \] whereas \( R_y = R_y(\eta) \) and \( R_z = R_z(\eta) \) are those of the reflected wave with the argument \( \eta := \omega t + k_1 x \). Then, with the 1-forms \( dx, dy, \) and \( dz, \)

\[
E = (W_y + R_y) \, dy + (W_z + R_z) \, dz, 
\]

\[
B = \frac{k_1}{\omega} \, dx \wedge [(W_y - R_y) \, dy + (W_z - R_z) \, dz], 
\]

\[
\mathcal{D} = \varepsilon_1 \varepsilon_0 [(W_y + R_y) \, dz - (W_z + R_z) \, dy] \wedge dx + \alpha_1 \frac{k_1}{\omega} \, dx \wedge [(W_y - R_y) \, dy + (W_z - R_z) \, dz], 
\]

\[
\mathcal{H} = \varepsilon_1 \varepsilon_0 \frac{\omega}{k_1} [(W_y - R_y) \, dz - (W_z - R_z) \, dy] - \alpha_1 [(W_y + R_y) \, dy + (W_z + R_z) \, dz]. 
\]

A direct check shows that the Maxwell equations (9),(10), together with the constitutive relations (5),(6), are satisfied provided the only nonvanishing component of the wave covector has the value

\[
k_1 = \frac{\omega n_1}{c}, \quad \text{with} \quad n_1 := \sqrt{\varepsilon_1 \mu_1}. \]

Analogously, the electromagnetic field configuration in the second half-space is represented by the right-moving transmitted wave:

\[
E = T_y \, dy + T_z \, dz, 
\]

\[
B = \frac{k_2}{\omega} \, dx \wedge (T_y \, dy + T_z \, dz), 
\]

\[
\mathcal{D} = \varepsilon_2 \varepsilon_0 (T_y \, dz - T_z \, dy) \wedge dx + \alpha_2 \frac{k_2}{\omega} \, dx \wedge (T_y \, dy + T_z \, dz), 
\]

\[
\mathcal{H} = \varepsilon_2 \varepsilon_0 \frac{\omega}{k_2} (T_y \, dz - T_z \, dy) - \alpha_2 (T_y \, dy + T_z \, dz). 
\]

Here the functions \( T_y = T_y(\xi_2) \) and \( T_z = T_z(\xi_2) \), with the argument \( \xi_2 := \omega t - k_2 x \), describe the transmitted wave in the second medium. Analogously to (17) we have

\[
k_2 = \frac{\omega n_2}{c}, \quad \text{with} \quad n_2 := \sqrt{\varepsilon_2 \mu_2}. \]

III. HARMONIC WAVES

For concreteness, we confine our attention to harmonic waves. Then,

\[
W_y(\xi_1) = a_1 \cos \xi_1 + a_2 \sin \xi_1, \quad W_z(\xi_1) = b_1 \cos \xi_1 + b_2 \sin \xi_1, 
\]

\[
R_y(\eta) = c_1 \cos \eta + c_2 \sin \eta, \quad R_z(\eta) = d_1 \cos \eta + d_2 \sin \eta, 
\]

\[
T_y(\xi_2) = p_1 \cos \xi_2 + p_2 \sin \xi_2, \quad T_z(\xi_2) = q_1 \cos \xi_2 + q_2 \sin \xi_2. 
\]
In order to construct the complete solution in the two regions, we have to match the configurations (13)-(16) and (18)-(21) on the interface $S$. Using the jump conditions (11) and (12) with $\nu = dx$ and $\tau_A = (\partial_y, \partial_z)$, we find

\[
(W_y + R_y)\big|_{x=0} = T_y\big|_{x=0},
\]

\[
(W_z + R_z)\big|_{x=0} = T_z\big|_{x=0},
\]

\[
\varepsilon_0 c \sqrt{\varepsilon_1/\mu_1} (W_y - R_y)\big|_{x=0} = \varepsilon_0 c \sqrt{\varepsilon_2/\mu_2} T_y\big|_{x=0} - [\alpha] T_z\big|_{x=0},
\]

\[
\varepsilon_0 c \sqrt{\varepsilon_1/\mu_1} (W_z - R_z)\big|_{x=0} = \varepsilon_0 c \sqrt{\varepsilon_2/\mu_2} T_z\big|_{x=0} + [\alpha] T_y\big|_{x=0}.
\]

Here $[\alpha] := \alpha_2 - \alpha_1$ is the \textit{jump} of the axion field on the interface $S$.

The algebraic system (26) to (29) can be straightforwardly solved. It yields the coefficients of the reflected ($c_{1,2}$ and $d_{1,2}$) and transmitted ($p_{1,2}$ and $q_{1,2}$) waves as combinations of those of the incident wave:

\[
c_{1,2} = \frac{1}{\Delta_{\perp}} \left[ \left( \frac{\varepsilon_1}{\mu_1} - \frac{\varepsilon_2}{\mu_2} - \frac{[\alpha]^2}{\lambda_0^2} \right) a_{1,2} + 2 \frac{[\alpha]}{\lambda_0} \sqrt{\varepsilon_1/\mu_1} b_{1,2} \right],
\]

\[
d_{1,2} = \frac{1}{\Delta_{\perp}} \left[ -2 \frac{[\alpha]}{\lambda_0} \sqrt{\varepsilon_1/\mu_1} a_{1,2} + \left( \frac{\varepsilon_1}{\mu_1} - \frac{\varepsilon_2}{\mu_2} - \frac{[\alpha]^2}{\lambda_0^2} \right) b_{1,2} \right],
\]

\[
p_{1,2} = \frac{2}{\Delta_{\perp}} \sqrt{\varepsilon_1/\mu_1} \left[ \left( \sqrt{\varepsilon_1/\mu_1} + \sqrt{\varepsilon_2/\mu_2} \right) a_{1,2} + \frac{[\alpha]}{\lambda_0} b_{1,2} \right],
\]

\[
q_{1,2} = \frac{2}{\Delta_{\perp}} \sqrt{\varepsilon_1/\mu_1} \left[ -\frac{[\alpha]}{\lambda_0} a_{1,2} + \left( \sqrt{\varepsilon_1/\mu_1} + \sqrt{\varepsilon_2/\mu_2} \right) b_{1,2} \right].
\]

Here $\Delta_{\perp} := \left( \sqrt{\varepsilon_1/\mu_1} + \sqrt{\varepsilon_2/\mu_2} \right)^2 + [\alpha]^2/\lambda_0^2$. As a consistency check, we can easily see that the above formulas, for $[\alpha] = 0$ (that is, either the axion is trivial everywhere or it has equal values for both material media), reduce to the well known expressions of the corresponding reflection and transmission coefficients for a plane wave, see Born and Wolf [1], Sec.1.5.

The result obtained clearly shows that despite the fact the constant axion drops out from the Maxwell field equation, the electromagnetic wave “feels” the presence of the axion by experiencing specific reflection and transmission effects.

### IV. VACUUM WITH AND WITHOUT AN AXION PIECE

In order to discuss the measureability of $\alpha$, let us consider the case when the first region of the space is vacuum, with $\varepsilon_1 = \mu_1 = 1$ and $\alpha_1 = 0$, whereas the second half-space is occupied
by a material substance that has trivial dielectric and magnetic properties, \( \varepsilon_2 = \mu_2 = 1 \),
but has a constant axion piece with \( \alpha_2 = \alpha \neq 0 \), see Fig.2. Let us assume, for simplicity,
that the incident wave is linearly polarized with the electric vector directed along the \( y \) axis,
which is achieved by putting \( b_1 = b_2 = 0 \):

\[
E^{\text{incident}} = W_y(\xi) \, dy = (a_1 \cos \xi + a_2 \sin \xi) \, dy,
\quad \xi = \omega(t - x/c).
\]  

(34)

Then we find that the reflected wave is also a linearly polarized wave but with the electric
vector tilted with respect to the \( y \)-axis [\( \eta = \omega(t + x/c) \)],

\[
E^{\text{reflected}} = R_y(\eta) \, dy + R_z(\eta) \, dz = - \frac{\alpha/\lambda_0}{4 + \alpha^2/\lambda_0^2} (a_1 \cos \eta + a_2 \sin \eta) \left[ (\alpha/\lambda_0) \, dy + 2dz \right],
\]  

(35)

whereas the transmitted wave is linearly polarized with the electric vector also tilted with
respect to both \( y \) and \( z \) axes,

\[
E^{\text{transmitted}} = T_y \, dy + T_z \, dz = \frac{2}{4 + \alpha^2/\lambda_0^2} (a_1 \cos \xi + a_2 \sin \xi) \left[ 2dy - (\alpha/\lambda_0) \, dz \right].
\]  

(36)

As we see, when \( \alpha = 0 \) the reflected wave is absent and the incident wave propagates from
vacuum into vacuum without being distorted, \( E^{\text{incident}} = E^{\text{transmitted}} \). However, when \( \alpha \neq 0 \)
the reflection takes place! Its presence is direct observational evidence for the axion piece. It
can be used for the experimental determination of the value of \( \alpha \). A qualitative check of the
axionic nature of the substance is provided by the fact that the polarization of the reflected
wave should be rotated with respect to the polarization of the incident wave. Furthermore,
the quantitative estimate of \( \alpha \) can then be extracted from the measurement of the intensity
and the angle of rotation of the reflected wave that explicitly depend on the value of the
axion.
V. THE GENERAL CASE: OBLIQUE INCIDENCE

The above analysis can be generalized to the case when the wave is not normally incident on the interface between the two media, see Fig.3. Then, for an arbitrarily moving wave, we have in the first medium (left half-space) a superposition of the incident and the reflected waves:

\[
E = (W_y + R_y) dy + W_z dz(i) + R_z dz(r),
\]

\[
B = \frac{k_1}{\omega} \left[ dx(i) \wedge (W_y dy + W_z dz(i)) - dx(r) \wedge (R_y dy + R_z dz(r)) \right],
\]

\[
\mathcal{D} = \varepsilon_1 \varepsilon_0 \left[ (W_y dz(i) - W_z dz(r)) \wedge dx(i) + (R_y dz(r) - R_z dz(r)) \wedge dx(r) \right]
\]

\[+ \alpha_1 \frac{k_1}{\omega} \left[ dx(i) \wedge (W_y dy + W_z dz(i)) - dx(r) \wedge (R_y dy + R_z dz(r)) \right],
\]

\[
\mathcal{H} = \varepsilon_1 \varepsilon_0 \frac{\omega}{k_1} \left[ (W_y dz(i) - R_y dz(r) - (W_z - R_z) dy \right]
\]

\[+ \alpha_1 \left[ (W_y + R_y) dy + W_z dz(i) + R_z dz(r) \right].
\]

Here \(x(i) = x \cos \theta_1 + z \sin \theta_1, \ z(i) = -x \sin \theta_1 + z \cos \theta_1, \ x(r) = x \cos \theta_2 - z \sin \theta_2, \ z(r) = x \sin \theta_2 + z \cos \theta_2\) describe the local coordinates adapted to the incident and the reflected wave, respectively, whereas

\[
W_y(\xi_1) = a_1 \cos \xi_1 + a_2 \sin \xi_1, \quad W_z(\xi_1) = b_1 \cos \xi_1 + b_2 \sin \xi_1,
\]

\[
R_y(\eta) = c_1 \cos \eta + c_2 \sin \eta, \quad R_z(\eta) = d_1 \cos \eta + d_2 \sin \eta,
\]

with \(\xi_1 = \omega t - k_1 x(i)\) and \(\eta = \omega t + k_1 x(r)\). The angles \(\theta_1\) and \(\theta_2\) give, as usual, the angles of the incident and the reflected waves with respect to the normal of the interface \(S\).

The transmitted wave in the right half-space has a form similar to (18)-(21):

\[
E = T_y dy + T_z dz(t),
\]

\[
B = \frac{k_2}{\omega} dx(t) \wedge (T_y dy + T_z dz(t)),
\]

\[
\mathcal{D} = \varepsilon_2 \varepsilon_0 \left[ (T_y dz(t) - T_z dy) \wedge dx(t) + \alpha_2 \frac{k_2}{\omega} dx(t) \wedge (T_y dy + T_z dz(t)) \right],
\]

\[
\mathcal{H} = \varepsilon_2 \varepsilon_0 \frac{\omega}{k_2} \left[ (T_y dz(t) - T_z dy) - \alpha_2 (T_y dy + T_z dz(t)) \right].
\]

Here we denote \(x(t) = x \cos \theta_3 + z \sin \theta_3, \ z(t) = -x \sin \theta_3 + z \cos \theta_3,\) where \(\theta_3\) describes the refraction angle. Similarly to (25), we have

\[
T_y(\xi_2) = p_1 \cos \xi_2 + p_2 \sin \xi_2, \quad T_z(\xi_2) = q_1 \cos \xi_2 + q_2 \sin \xi_2,
\]
FIG. 3: Oblique incidence. The wave \( W \) is Reflected and Transmitted.

with the argument \( \xi_2 = \omega t - k_2 x(t) \).

Now, the jump conditions (11) and (12) yield

\[
(W_y + R_y)_{x=z=0} = T_y|_{x=z=0}, \tag{48}
\]

\[
(W_z \cos \theta_1 + R_z \cos \theta_2)_{x=z=0} = T_z \cos \theta_3|_{x=z=0}, \tag{49}
\]

\[
\varepsilon_0 c \sqrt{\varepsilon_1 \mu_1} (W_y \cos \theta_1 - R_y \cos \theta_2)_{x=z=0} = \left( \varepsilon_0 c \sqrt{\varepsilon_2 \mu_2} T_y - [\alpha] T_z \right) \cos \theta_3|_{x=z=0}, \tag{50}
\]

\[
\varepsilon_0 c \sqrt{\varepsilon_1 \mu_1} (W_z - R_z)_{x=z=0} = \varepsilon_0 c \sqrt{\varepsilon_2 \mu_2} T_z|_{x=z=0} + [\alpha] T_y|_{x=z=0}, \tag{51}
\]

\[
k_1 (W_y \sin \theta_1 + R_y \sin \theta_2)_{x=z=0} = k_2 T_y \sin \theta_3|_{x=z=0}, \tag{52}
\]

\[
\varepsilon_1 \varepsilon_0 (W_z \sin \theta_1 - R_z \sin \theta_2)_{x=z=0} = \left( \varepsilon_1 \varepsilon_0 \frac{\alpha k_2}{\omega} T_y \right) \sin \theta_3|_{x=z=0}. \tag{53}
\]

The last two equations (when combined with the rest) yield the well known result that relates the angles of incidence, reflection, and refraction, namely, \( \sin \theta_1 = \sin \theta_2 \) (law of reflection) and \( n_1 \sin \theta_1 = n_2 \sin \theta_3 \) (law of refraction). In addition, from the algebraic system (48)-(51), we find the coefficients of the reflected \((c_{1,2} \text{ and } d_{1,2})\) and transmitted \((p_{1,2} \text{ and } q_{1,2})\) waves.
as combinations of those of the incident wave:

\[
c_{1,2} = \frac{\cos \theta_3}{\Delta} \left\{ -\frac{[\alpha]}{\lambda_0} \sqrt{\frac{\varepsilon_1}{\mu_1}} (\cos \theta_1 + \cos \theta_2) a_{1,2} + \left( \frac{\varepsilon_1 - \varepsilon_2}{\mu_2} - \frac{[\alpha]^2}{\lambda_0^2} \right) \cos \theta_1 + \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\mu_1 \mu_2}} (\cos \theta_3 - \cos \theta_1) \right\} b_{1,2},
\]

(54)

\[
d_{1,2} = \frac{\cos \theta_3}{\Delta} \left\{ -\frac{[\alpha]}{\lambda_0} \sqrt{\frac{\varepsilon_1}{\mu_1}} (\cos \theta_1 + \cos \theta_2) a_{1,2} + \left( \frac{\varepsilon_1 - \varepsilon_2}{\mu_2} - \frac{[\alpha]^2}{\lambda_0^2} \right) \cos \theta_1 + \sqrt{\frac{\varepsilon_1 \varepsilon_2}{\mu_1 \mu_2}} (\cos \theta_3 - \cos \theta_1) \right\} b_{1,2},
\]

(55)

\[
p_{1,2} = \sqrt{\frac{\varepsilon_1}{\mu_1}} (\cos \theta_1 + \cos \theta_2) \left[ \left( \sqrt{\frac{\varepsilon_1}{\mu_1}} \cos \theta_3 + \sqrt{\frac{\varepsilon_2}{\mu_2}} \cos \theta_2 \right) a_{1,2} + \frac{[\alpha]}{\lambda_0} \cos \theta_3 b_{1,2} \right],
\]

(56)

\[
q_{1,2} = \sqrt{\frac{\varepsilon_1}{\mu_1}} (\cos \theta_1 + \cos \theta_2) \left[ -\frac{[\alpha]}{\lambda_0} \cos \theta_2 a_{1,2} + \left( \sqrt{\frac{\varepsilon_1}{\mu_1}} \cos \theta_2 + \sqrt{\frac{\varepsilon_2}{\mu_2}} \cos \theta_3 \right) b_{1,2} \right].
\]

(57)

Here we denoted \( \Delta = \left( \cos \theta_2 \sqrt{\varepsilon_1/\mu_1} + \cos \theta_3 \sqrt{\varepsilon_2/\mu_2} \right) \left( \cos \theta_3 \sqrt{\varepsilon_1/\mu_1} + \cos \theta_2 \sqrt{\varepsilon_2/\mu_2} \right) \left( \cos \theta_2 \cos \theta_3 [\alpha]^2/\lambda_0^2 \right). \) For the special case \( \theta_2 = \theta_3 = 0, \) we recover \( \Delta_\perp. \)

VI. THE MEASUREABILITY OF THE AXION PIECE

As we saw, we can read off from the coefficients \( c_{1,2} \) and \( d_{1,2} \) of the reflected wave the jump \([\alpha]\) of the axion piece. In principle, one can also measure the axion piece by means of observing the properties of the transmitted wave inside the medium. However, the study of the reflected wave alone is clearly preferable as it provides a simple and elegant scheme of experimental determining the properties of a substance without destructing the latter.

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