Comparison of the Reliability Methods for Steel Trusses Subjected to Fire

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Abstract. The paper concerns different methods of the reliability analysis of steel structures exposed to fire. Two types of trusses (statically determinate and statically indeterminate) were considered. The fire analysis was carried out in MES3D program, it was assumed that structures were insulated with the spray-applied fibre and were exposed to the fire described by the standard time-temperature curve. The calculation was made according to Eurocode rules. The reliability analysis was carried out by approximation-simulation methods (in Numpress Explore program) and with using system reliability analysis.

1. Introduction
The following paper combine the method of fire analysis of steel structures with two types of reliability methods. The first group are approximation - simulation reliability method: FORM, SORM, Importance Sampling Monte Carlo. The second group is method of structural systems analysis. Both methods can be alternative to 1st level of reliability analysis, what is the base for calibration of partial safety factors. The first group of involved method enable to estimate the reliability index for single element of the structure, using the method of structural system the structure as the whole is taken into consideration. The fire analysis was carried out according to Eurocode rules [1, 2].

2. Methods
2.1. Fire analysis methods
The steel structures are extremely low resistance to high temperatures. The most important mechanical properties of steel are yield strength and modulus of elasticity. A decrease in the values of mechanical parameters leads directly to a reduction in the load bearing capacity, which finally results in the ultimate limit state being exceeded.

In the present study, the fire analysis was performed using the standard fire curve. In trusses, which are analysed in the present study, only axial forces are generated. The bearing capacity of tension and compression elements was calculated according to formulas from Eurocodes [1, 2]. The whole analysis was carried out using MES3D program, developed by Szaniec [3]. The methodology of fire analysis is well described not only in Eurocodes but also in numerous handbooks [4,5].
2.2. The reliability analysis methods

2.2.1. The approximation and simulation methods

The reliability of a structure is its ability to fulfil design purposes for some specified reference period. In the fundamental case the loading is described by a single random variable $E$ and the strength by a single random variable $N$. The probability of failure $P_f$ is then defined as

$$P_f = \int_{\Omega_f} f_{N,E}(N,E) dNdE$$

where: $f_{N,E}(N,E)$ is the joint probability density function.

Generalizing the structural reliability analysis is formulated based on two fundamental assumptions: the state of the structure is defined in the outcome space of a vector of basic random variables; the structure can be in one of two states, the safe state or the failure state. The boundary between the two states in the outcome space is known as the limit state surface.

Let the vector $X$ denote the set of basic random variables pertaining to a structure. In the paper basic random variables include parameters defining loads, material properties and structural geometry. The criterion of structural failure is expressed by the condition of non-exceeding the bearing capacity. In the current paper only the time independent component reliability analysis problems are considered. The Hasofer-Lind index in conjunction with transformation method in the FORM is used as a reliability measure. In reliability analysis, it is convenient to transform the variables $X$ into the standard normal space through a probability transformation: $Y = Y(X)$. Elements of vector $Y$ are statistically independent and have the standard normal density. Der Kiureghian and Liu [6] suggested a probability transformation which is particularly useful in the finite element reliability methods. In this method, a joint distribution model, originally introduced by Nataf [7], with prescribed marginal distributions and correlation matrix was proposed. The limit state surface is specified as $G(Y)$.

Approximation of the limit state function at the design point by the function of the first or second degree leads to the methods of first-order reliability analysis (FORM) or second-order reliability analysis (SORM). In the method FORM nonlinear limit state function $G(Y)$ is approximated by a tangent hyperplane $l(Y)$ at the design point $Y^*$ of the failure surface closed to the origin. This procedure leads to determining the Hasofer-Lind index $\beta$.

$$l(Y) = -\alpha^T \cdot Y + \beta$$

$$\alpha = -\frac{\nabla G(Y)}{||\nabla G(Y)||_{Y=Y^*}}$$

$$\beta = \text{sign}[l(0)] \delta^*$$

Figure 1. Approximation of the limit state function $G(Y)$ at the design point $Y^*$ by the linear function $l(Y)$
In the FORM and SORM methods, one searches for the nearest point on the limit state surface to the origin in the standard normal space by solving the constrained optimization problem. The First Order Reliability Method (FORM) yields the best results when only a single design point exists, the limit function is not strongly linear and it is differentiable. Application of this method to the strength analysis of steel structures in basic design situation has already been tested [8, 9, 10].

The separate group of reliability analysis methods are simulation methods. The classical Monte Carlo method is the best known. The 1980s marked the development of the so-called importance sampling method. Through a proper selection of probability density function, according to which random variables are generated, one can significantly decrease sampling area and the number of simulations.

2.2.2. The structural systems method

To carry out reliability analysis according to structural system method it is essential to define properly all possible failure modes. In order to determine the reliability of this approach it is necessary to set KAFM (kinematically admissible failure mechanism). The KAFM by spectral analysis of stiffness matrix are specified in [11], [12]. Reliability issues are important not only in static analysis, but also in stability analysis or dynamic analysis [13].

Defining KAFM is easy for statically determinate structure, where it is only one failure mode, but the real structures are usually indeterminate and then more than one failure mode is possible. There are two basic types of system: series and parallel. A system is a series system if it is in a state of failure whenever any of its elements fails. Such a system is also called a weakest –link system. A typical example of a series system is a statically determinate structure. Failure in a single element in a structural system will not always result in failure of the total system, because the remaining elements may be able to sustain the external loads by redistribution of the loads. This situation is characteristic of statically indeterminate structures. Failure of such structures will always require that more than one element fails. This set of elements is called a parallel system.

The reliability of structures, the static scheme of which is compliant with the serial system, is computed according to the formula [14]:

\[ R = \prod_{i=1}^{n} R_i = R_1R_2...R_n \]  

In the parallel system, the structure remains reliable as long as at least one element is reliable. The reliability of the parallel system is computed as follows [14]:

\[ R = I - \prod_{j=1}^{m}(I - R_j) \]  

In equation 5 and 6, \( R_i \) and \( R_j \) are reliabilities of single elements of a structure.

In the paper reliability of a single element \( R(\omega) \) of considered system is defined as the probability that the bearing capacity of an element \( N(\omega) \) will be greater than the effect of actions \( E(\omega) \):

\[ R(\omega) = \Pr\{N(\omega) > E(\omega)\} \]

The bearing capacity of an element and the effect of actions have normal distribution and are characterised by standard deviation \((\sigma_E, \sigma_N)\) and the expected value \((\mu_E, \mu_N)\). Thus, the expected value \((\mu_Z)\) and standard deviation \((\sigma_Z)\) of the safety margin \( Z(\omega) = N(\omega) - E(\omega) \) and the reliability index for the \( i \)-th element can be expressed as follows:

\[ \beta_i = \frac{\mu_{Zi}}{\sigma_{Zi}} \]

If the reliability index \( \beta_i \) is known, it is possible to compute the probability of the element failure \((P_{fi})\) and the reliability for a single element \((R_i)\): \[ P_{fi} = \Phi(-\beta_i), \quad R_i = 1 - P_{fi} \]

where \( \Phi(\cdot) \) - the Laplace function.
3. Example

Fire analysis for two types of trusses (statically determinate and indeterminate), shown in the figure 2 was carried out using MES3D program. In the table, the profiles of elements and the effect of actions are shown, where (-) means compression. All elements were assumed to be made from S275 steel. The only load was the dead load p=3kN/m applied to the top flange, and it was converted to concentrated forces in the nodes. It was assumed that all elements were heated from each side and were under influence of standard fire curve [2]. Spray-applied mineral fibre with the thickness of 1.5cm was assumed as an insulation. This material is characterized by the following parameters: density $\rho_p=800\text{kg/m}^3$, specific heat $c_p=1700\text{J/(kgK)}$, thermal conductivity $\lambda_p=0.2\ \text{W/(mK)}$.

For presented structures (Figure 2) effect of action is constant during increase of temperatures, but with fire duration the bearing capacity for single elements decrease [1]. Knowing this values, it is possible to carry out reliability analysis in each minutes of fire duration. In the presented paper two methods were involved:

1. Analysis with NumpressExplore [15] program using approximation (FORM, SORM) and simulation methods (Importance Sampling, Monte Carlo).
2. System reliability analysis.

For the first type of analysis the most stressed elements, which are marked by red line in the figure 2, were chosen. The limit state function was defined as:

$$g = 1 - \frac{E}{\chi_{fi} \cdot A \cdot f_y}$$

(7)

| Element | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---------|---|---|---|---|---|---|---|---|---|
| Profiles | I 80 | 1 100 | RO 44.5x5 | RO 20x2.3 |
| Effect of actions [kN] | 0 | -8 | -12 | 10 |

| Element | 1,3 | 2,4 | 5,6 |
|---------|-----|-----|-----|
| Profiles | RO 33.7x4 |
| Effect of actions [kN] | 1.00 | -5.25 | -1.25 |

Figure 2. The analysed trusses: a) statically determinate, b) statically indeterminate
where $\chi_f$ is the reduction factor for flexural buckling in the fire design situation and in following paper this value was assumed to be deterministic. Other characteristics were treated as the random variables according to table 1.

### Table 1. Probabilistic characteristics of random variables

| Type of truss                     | Random variable     | Expected value | Coefficient of variation | Standard deviation | Distribution type |
|-----------------------------------|---------------------|----------------|--------------------------|--------------------|-------------------|
| Statically determinate (Figure 2a) | Cross section area-A | 0.00106 m$^2$ | 6%                       | 0.0000636 m$^2$    |                   |
| Statically indeterminate (Figure 2b) |                       | 0.000373 m$^2$| 6%                       | 0.00002238 m$^2$   |                   |
| Statically determinate/indeterminate (Figure 2a,b) | Yield strength $f_y$ | 275 000 kPa  | 8%                       | 22 000 kPa         | normal            |
| Statically determinate (Figure 2a) | Effect of action $E$ | 8 kN           | 6%                       | 0.48 kN            |                   |
| Statically indeterminate (Figure 2b) |                       | 5.25 kN        | 6%                       | 0.315 kN           |                   |

To carry out the analysis according to second method the appropriate system must be defined. The truss in the figure 2a has statically determinate scheme and corresponds to the serial system, consequently, the failure of any element (1-9) results in the failure of the whole structure. For the statically indeterminate truss (Figure 2b) the appropriate kinematically admissible failure mechanisms have to be defined. They are shown in the figure 3. The whole method for calculating system reliability for this truss is presented in [17].

![Figure 3. Kinematically admissible failure mechanisms for the analyzed indeterminate truss](image)

### 4. Results and discussions

The reliability indices during fire duration received with using different method for both trusses are presented in table 2 and 3. The last column with element's reliability refers to most stressed elements of each structure and was calculated as the quotient of expected value and standard deviation of safety margin.
Table 2. Reliability index for statically determinate structure.

| Fire duration [min] | FORM | SORM Importance Sampling | Monte Carlo System reliability | Element's reliability |
|---------------------|------|---------------------------|------------------------------|----------------------|
| 0                   | 4.30 | 4.30                      | 4.11                         | 3.96                 |
| 5                   | 4.30 | 4.30                      | 4.11                         | 3.96                 |
| 10                  | 3.86 | 3.86                      | 3.72                         | 3.61                 |
| 15                  | 3.12 | 3.12                      | 3.06                         | 2.95                 |
| 20                  | 2.31 | 2.31                      | 2.27                         | 2.22                 |
| 25                  | 1.47 | 1.47                      | 1.48                         | 1.43                 |
| 30                  | 0.57 | 0.56                      | 0.60                         | <0                   |

Table 3. Reliability index for statically indeterminate structure.

| Fire duration [min] | FORM | SORM Importance Sampling | Monte Carlo System reliability | Element's reliability |
|---------------------|------|---------------------------|------------------------------|----------------------|
| 0                   | 3.44 | 3.37                      | 3.19                         | 3.23                 |
| 5                   | 3.44 | 3.37                      | 3.19                         | 3.23                 |
| 10                  | 3.10 | 3.02                      | 3.06                         | 2.93                 |
| 15                  | 2.40 | 2.34                      | 2.39                         | 2.30                 |
| 20                  | 1.67 | 1.63                      | 1.68                         | 1.62                 |
| 25                  | 0.92 | 0.90                      | 0.94                         | 0.90                 |
| 30                  | 0.08 | 0.15                      | 0.09                         | 0.08                 |

In the figure 4 the comparison of reliability indices gotten with using different methods is presented.

![Figure 4](image)

Figure 4. The monitoring of reliability indices with fire duration according to different methods for a) statically determinate truss, b) statically indeterminate truss

The significant differences between reliability indices calculated according to system reliability analysis and the reliability analysis with using Numpress Explore are observed. The percentage differences for statically determinate structure, that are presented in table 4a, were calculated according to the following formula:

\[
\text{diff}_{\text{det}} = \frac{\text{Numpress} - \text{System}}{\text{System}} \times 100\%
\]  

(7)
The percentage differences for statically indeterminate structure, that are presented in table 4b, were calculated according to the following formula:

\[
\text{diff}_{\text{ind det}} = \frac{\text{System} - \text{Numpress}}{\text{Numpress}} \times 100\%
\]  \hspace{1cm} (8)

Table 4. Percentage differences of reliability indexes for:

| Fire duration [min] | FORM [%] | SORM [%] | Importance Sampling [%] | Monte Carlo [%] | FORM [%] | SORM [%] | Importance Sampling [%] | Monte Carlo [%] |
|--------------------|----------|----------|-------------------------|-----------------|----------|----------|-------------------------|-----------------|
| 0                  | 15       | 15       | 14                      | 10              | 44       | 44       | 47                      | 55              |
| 5                  | 15       | 15       | 14                      | 10              | 44       | 44       | 47                      | 55              |
| 10                 | 14       | 14       | 12                      | 10              | 46       | 46       | 50                      | 48              |
| 15                 | 16       | 15       | 13                      | 13              | 54       | 54       | 57                      | 54              |
| 20                 | 21       | 21       | 18                      | 19              | 67       | 67       | 71                      | 65              |
| 25                 | 44       | 44       | 38                      | 45              | 100      | 100      | 104                     | 96              |
| 30                 | -        | -        | -                       | -               | 860      | 880      | 417                     | 791             |

While using Numpress Explore the time of single analysis was monitored, the results are presented in the table 5.

Table 5. The time of reliability analysis for:

| Fire duration [min] | FORM [s] | SORM [s] | Importance Sampling [s] | Monte Carlo [s] | FORM [s] | SORM [s] | Importance Sampling [s] | Monte Carlo [s] |
|--------------------|----------|----------|-------------------------|-----------------|----------|----------|-------------------------|-----------------|
| 0                  | <1       | <1       | 3                       | 855             | 0        | <1       | 3                       | 19              |
| 5                  | <1       | <1       | 3                       | 855             | 5        | <1       | 3                       | 19              |
| 10                 | 1        | <1       | 3                       | 20              | 10       | <1       | 2                       | 19              |
| 15                 | 1        | <1       | 2                       | 19              | 15       | <1       | 2                       | 21              |
| 20                 | <1       | <1       | 2                       | 19              | 20       | <1       | 2                       | 6               |
| 25                 | 1        | <1       | 2                       | 5               | 25       | <1       | 2                       | 4               |
| 30                 | <1       | <1       | 2                       | 2               | 30       | <1       | 1                       | 2               |

5. Conclusions

The proposed paper concerns different methods of the reliability analysis of steel structures exposed to fire. Two types of trusses (statically determinate and statically indeterminate) were considered. The fire analysis was carried out in MES3D program, it was assumed that structures were insulated with the spray-applied fibre and were exposed to the fire described by the standard time-temperature curve. The calculation was made according to Eurocode rules.

To assess the reliability according to approximation (FORM, SORM) and simulation (Monte Carlo, Importance Sampling) methods the program NumPress Explore was used. In this way the Hasofer-Lind reliability index was calculated.

To carry out system analysis appropriate reliability systems were defined for each structure. For the statically determinate structure it was series system. For statically indeterminate truss it was need to build mixed (parallel - series) system. There were few model of structural failure (failure modes), so appropriate kinematically admissible failure mechanisms (KAFMs) had to be defined. According to comparison of results from system analysis and calculations made in NumpressExplore there were observed significant differences in the value of reliability index. In the case of the statically determinate truss higher reliability index was gotten as the results of calculation made by second level
method. The effect of statistical weakness of structure was observed (Table 2). For statically indeterminate structure opposite result- the effect of statistical reinforcement was noticed - the reliability index was much higher according to system analysis (Table 3). It means that using approximation or simulation methods, that seems to be easier, is safety for statically indeterminate structures. In the case of statically determinate structure it is recommended to support analysis with second level method by system analysis, what is not difficult task. Recognizing all failure modes for statically indeterminate structure is much more complicated and it is impossible to do this in reasonable time. For such structure approximation/ simulation methods are good alternative, because they gave answer very fast (Table 5), but the values of reliability indices are understated (Table 3, 4b). So the bearing capacity is not completely extracted, what may cause additional costs that could be avoided.

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