The model for estimating the measurement error in geometric parameters of complex surfaces

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Abstract. The measurement results for the geometric parameters of complex surfaces largely depend on the measurement parameters. The different processing algorithms for measurement results implemented in the coordinate measuring machine software gives different values for the mentioned parameters. In this paper, a model for estimating the measurement errors in the geometric parameters of complex surfaces caused by the use of different result processing tools is considered. The represented work consists of several stages: development of surfaces with a known deviation of form and location, simulation of the coordinate measurement process, calculation of target parameters in software of coordinate measuring machines and analysis of simulation results. The model was tested through the example of determining the measurement error in the involute surfaces of tooth wheels. The represented model may be used to determine the optimal measurement parameters for geometric parameters of complex surfaces.

1. Introduction

The development of tools for designing and processing of engineering products resulted in the extensive use of parts of a complex shape. Such parts include blades of compressors and turbines of aircraft engines [1], impressions of dies and moulds, surfaces of optical systems [2], tooth wheels of gears. The surfaces of such parts may not be accurately represented using the former widely used surfaces – a plane, a cone, a sphere, etc., so the previously developed control tools are not suitable for instrumental inspection of complex-surface parts.

To solve the geometrical parameter control problem for complex-surface parts, coordinate measuring machines have become widely used. Coordinate measuring machines (CMM) are universal machines capable of measuring various products. The CMM software manages the hardware portion of measuring machines and ensures processing of measurement results. The flexibility of coordinate measuring machines results in the fact that the CMM software implements a wide range of algorithms for processing of measurement results.

A lot of algorithms for processing the measurement results lead to the problem of determining the methods that must be used when measuring specific types of surfaces. The use of various algorithms gives different values of the geometric parameters of the measured surfaces [3]. Therefore, the topical problem is to determine the error in measuring the geometric parameters of complex surfaces, which arises from the use of various information processing methods.

The measurement error can be estimated using two approaches: based on practical experiments and on numerical simulation of the measurement process [4]. The first approach is based on measuring of complex surfaces of a pre-known shape and comparing the calculated geometric parameters of the surface with the actual parameters set during manufacturing the surface. This approach is difficult to
implement, since it requires the development of accurate models of complex surfaces and significant
time for conducting a large number of measurement experiments.

In this paper the second approach is considered – the use of numerical simulation for the measuring
process of complex surfaces, determining their geometric parameters and comparing the calculations
results with actual parameters. Numerical simulation is widely used in solving production problems,
including improving the accuracy of manufacturing various parts [5].

2. The model for estimating the measurement error in geometric parameters
Let us consider a model for estimating the error in measuring geometric parameters of complex
surfaces using the example of involute surfaces. These surfaces are widely used to describe the profile
of the tooth wheel – the main part of the gears. Gears are widely used in agricultural, transport and
chemical engineering. A mandatory reliability and durability condition of these gears is the
manufacturing accuracy of involute tooth wheel profiles applied in these gears. To estimate the
accuracy in manufacturing of the involute profiles, a geometric parameter is used such as the form
deviation of the actual profile from its nominal value [5].

The structure of the developed model is shown in Fig.1.

![Model structure](image)

The nominal involute profile is described by the system of equations (1) [6]:

\[
\begin{align*}
    x(t) &= \frac{D_w}{2} \left( \cos(t) + t \cdot \sin(t) \right) \\
    y(t) &= \frac{D_w}{2} \left( \sin(t) - t \cdot \cos(t) \right)
\end{align*}
\]

(1)

where \(x, y\) – coordinates of the profile point; \(D_w\) – pitch circle diameter; \(t\) – parameter.

The analysis of the measurement results for reference and working tooth made it possible to reveal
that the form deviation is harmonic in its nature. To simulate the form deviation of the profile, a
function of the following form was applied (2):

\[
\Delta F = A \cdot \sin(k \cdot t)
\]

(2)

In the third stage, the deviation of the location with the displacement and rotation parameters was
added to the profile model with the form deviation:

\[
\begin{pmatrix}
    x_{fl} \\
    y_{fl}
\end{pmatrix} = MR^{-1} \begin{pmatrix}
    x_f \\
    y_f
\end{pmatrix} + MT,
\]

(3)

where \(x_{fl}, y_{fl}\) – coordinates of the profile point considering the deviation of form and location; \(x_f, y_f\)
– coordinates of the profile point considering the deviation of form; \(MR\) – rotation matrix of the
coordinate system; \(MT\) – translation vector of the coordinate system.

This surface modification model was implemented in the Matlab system and the result of its work
are CAD models of measured involute profile in the IGES format (Fig. 2). During the development of
the module, the correspondence of the resulting surfaces to the analytic expressions embedded in the
generation model of the measured involute surfaces was checked [7].

At the next stage, the numerical simulation of coordinate measurements through the application
“Virtual Coordinate Measuring Machine” is performed. The nominal and actual involute surfaces are
loaded in this application with specifying the measurement plan and simulation parameters of the
instrumental error. After indicating all the necessary parameters, the process of coordinate
measurements is simulated. The reliability of simulation results is confirmed by the compliance of this application with the requirements for metrological software verified by a certificate issued by the All-Russian Scientific Research Institute of Metrological Service.

Figure 2. Generated involute surfaces.

As a result of simulation, a set of measured points of the involute surface is formed. Using the measurement results processing module, this set was exported to the PC-DMIS application (Fig. 3).

Figure 3. The object model of the PC-DMIS/Matlab interaction module.

Special-purpose software PC-DMIS used together with coordinate measuring machines, allows calculating the form deviation being one of the main parameters that characterizes the involute surface manufacturing quality. To calculate this parameter, the measured points [8] are pre-aligned and then the maximum deviations of the points of the measured profile from the points of the nominal profile are calculated (Fig. 4).

Figure 4. Profile parameters calculation by PC-DMIS application.

At the final stage, the experiment parameters are recorded in the database. The following variables act as the specified parameters:

1. Parameters of generation of the measured profile (function, amplitude, frequency, etc.)
2. Parameters for simulation of the coordinate measurement process (measurement plan, magnitude of instrumental error)
3. The calculation results of the form deviation obtained by using different algorithms for calculating this parameter

The implemented model reproduces the measurement process for the involute surface taking into consideration the effects of the factors indicated above. Through selecting different values of the influencing factors and performing the measurement simulation, a database of involute surface measurements is formed. The obtained parameters of the surfaces can be processed using the statistical methods given in [10].
Following the results of analysis of the obtained database, the influence of various factors on the measuring uncertainty of the geometric parameters of involute surfaces is estimated.

3. Testing of the suggested model
In the course of the experiments, involute profile models having the form deviation shown in Fig. 5 were developed.

![Figure 5. Form deviation functions for involute profile simulation a) k=2, b) k=4, c) k=6.](image)

The parameters indicated in table 1 were used as the location deviation parameters. The values of these parameters were selected based on the practical measurements of involute teeth and industry documents for involute surfaces [9].

| № | dX (µm) | dY (µm) | dAlpha (rad) | № | dX (µm) | dY (µm) | dAlpha (rad) |
|---|---------|---------|--------------|---|---------|---------|--------------|
| 1 | 0       | 0       | 0.00000000   | 7 | 28      | 0       | -0.000066   |
| 2 | 28      | 0       | 0.00000000   | 8 | 28      | 0       | 0.000815    |
| 3 | 0       | 28      | 0.00000000   | 9 | 0       | 28      | -0.000066   |
| 4 | 0       | 0       | -0.000066    | 10| 0       | 28      | 0.000815    |
| 5 | 0       | 0       | 0.000815     | 11| 20      | 20      | -0.000066   |
| 6 | 20      | 20      | 0.00000000   | 12| 20      | 20      | 0.000815    |

To simulate the instrumental error of the CMR measurement, random variables obtained from the normal distribution law and having a 95% confidence interval with the values of 1, 2, 3 and 4 µm were used. The instrumental error parameters are assumed based on the analysis of the accuracy characteristics obtained from the certificates of various CMMs. Each simulation experiment for measurement of the formed nominal profile and the profile with a deviation of form and location was repeated 30 times. For each given deviation of the profile form, studies were performed with different location options from table 1. Thus, in total, the number of experiments carried out was: 3 x 12 x (4 + 1) x 30 = 5400 numerical experiments.

The result of each numerical experiment is the estimation of the deviation of the calculated form deviation from the form deviation specified at the profile model formation stage.

$$\sigma = \Delta F_{calc} - \Delta F.$$  \hspace{1cm} (4)

This value and the simulation parameters of the surface model were stored in the database for further processing.

4. The results of experimental studies
In the course of experimental studies, a large information array was accumulated regarding the relationship between the deviation parameters of the profile form, the CMM instrumental error, and the calculated form deviation. Figure 6 shows the result of the experiment for a single value of the form deviation amplitude and the instrumental error value.
Figure 6. The deviation of the calculated form deviation from the form deviation specified at the profile model formation stage. А=0.001, confidence interval 1µm.

The distribution of the form deviation for this series of experiments is shown in Fig. 7.

Figure 7. The distribution of deviation for the calculated form deviation from the form deviation specified at the profile model formation stage. А=0.001, confidence interval 1µm.

Summarized information on the deviation measurement error of the involute profile is shown in table 2.

5. Conclusions
When checking the geometric parameters of complex-surface parts, special attention should be paid to the measurement accuracy of these parameters. The use of different measurement parameters or processing algorithms results in non-coincident values of geometric parameters for the same parts. Practical estimation of measurement error is difficult to be implemented, therefore, to this end, the mathematical simulation methods shall be employed.

| №  | The permissible profile deviation, µm (A factor in the formula 2) | Measurement instrument uncertainty, µm | Standard uncertainty, µm | Mathematical expectation, µm |
|----|---------------------------------------------------------------|---------------------------------------|--------------------------|-----------------------------|
| 1  | ±1                                                            | 1                                     | 0.4662                   | 0.4013                      |
| 2  | ±1                                                            | 2                                     | 0.5657                   | 1.1472                      |
| 3  | ±1                                                            | 3                                     | 0.6389                   | 2.0692                      |
| 4  | ±1                                                            | 4                                     | 0.8011                   | 3.0575                      |
| 5  | ±1                                                            | 2                                     | 1.5765                   | 0.4249                      |

The paper represents a model for estimating the measurement error of geometric parameters of complex surfaces based on numerical simulation of the measurement process using the involute surface analyzing example.

At the first stage, a model of the actual surface is formed with the specified parameters for the form deviation and profile location.
The actual and nominal models are sent to the module for numerical simulation of coordinate measurements. This module simulates the surface measurement using a contact measurement method, and also simulates the effect of random factors on the measurement results.

The results of numerical simulation of the profile measurement are sent to a special-purpose software application PC-DMIS, which is intended for interaction with coordinate measuring machines and processing of measurement results. At this stage, the key geometric parameters of the surface are determined, which are subsequently exported to the database. The parameters that characterize the actual surface simulation and measurement process, are exported to the database as well.

The experimental studies using the example of the analyzing involute surfaces have shown that the deviation of the measured parameters from their actual value can reach significant values, sometimes exceeding the permissible values.

The suggested technique can be used both for estimating the error in the measurement of the surface and for determining the optimum parameters of the measurement process in order to achieve the required measurement accuracy.

6. References

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Acknowledgements

This work was supported by the Ministry of Education and Science of the Russian Federation in the framework of the implementation of the Program State Assignment for 2018. The project code is 9.11560.2018 / 10.11.