Collective Cooper-pair transport in the insulating state of Josephson junction arrays

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We investigate collective Cooper-pair transport of one- and two-dimensional Josephson junction arrays in the insulating state. We derive an analytical expression for the current-voltage characteristic revealing thermally activated conductivity at small voltages and threshold voltage depinning. The activation energy and the related depinning voltage represent a dynamic Coulomb barrier for collective charge transfer over the whole system and scale with the system size. We show that both quantities are non-monotonic functions of magnetic field. We propose that formation of the dynamic Coulomb barrier as well as the size scaling of the activation energy and the depinning threshold voltage, are consequences of the mutual phase synchronization. We apply the results for interpretation of experimental data in disordered films near the superconductor-insulator transition.

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Recent experimental studies of the superconductor-insulator transition (SIT) (see [1] for a review) in thin disordered superconducting films proved formation of a collective insulating state exhibiting thermally activated Arrhenius-like conductivity at low biases [2, 3, 4] and the threshold voltage depinning [3, 5] behavior. Below the depinning voltage, \( V_T \), a film falls into a zero-conductivity phase and abruptly switches to a finite conductance regime when the bias achieves \( V_T \). The discovery of the novel phase existing in a narrow window of disorder strength near SIT poses a quest for a comprehensive theory.

Josephson junction arrays serve as a perfect testing ground for SIT studies (see, e.g., [3, 7, 8, 9, 10]). A salient similarity of the voltage threshold behavior in SC films [3, 5] and the voltage depinning in one-dimensional Josephson arrays [11] suggests an intimate relation between these systems. Further parallel appears from the striking observations of voltage threshold \( V_T \) dependence on the array length in [10], the sample size dependent activation energy, \( k_B T_0 \), observed in [4], and the connection between \( V_T \) and \( T_0 \) revealed in [3]. An advantage of a Josephson array as a model system is that it offers a straightforward theoretical description of the current-voltage characteristics, which is what measured in all the major experimental studies of SIT. In this Letter we develop a theory of the collective transport of large Josephson junction arrays in the insulating state and apply our results for interpretation of experimental data on SIT.

The current-voltage characteristics of Josephson systems in an insulating state were discussed in a single junction [11, 12, 13] and two junction [14, 15] systems. Each junction is characterized by the Josephson coupling energy, \( E_J = \hbar I_c/2e \), where \( I_c \) is the Josephson critical current, and by charging energies \( E_c \) related to inter-island capacitance and \( E_c \) associated with capacitance to ground, \( C_0 \) [10]. We consider an insulating state, where charging energies \( E_c, E_c \ll E_J \). We asume further the superconducting gap \( \Delta > E_c \). This implies that the transport is mediated by the thermally activated motion of the Cooper pairs [17]. We show that in the regular and disordered arrays a collective current state develops. This state is characterized by the energy gap, \( \Delta_c \), stemming from the coherent Coulomb blockade effect, i.e. the Coulomb blockade involving all junctions and extending over the whole system. We derive a low bias \( I-V \) dependence:

\[
I \propto \exp \left[ - \frac{(\Delta_c - eV)^2}{2 \Delta_c k_B T} \right].
\]  

Eq. (1) reveals that there are two dynamic regimes: first,
thermally activated charge transfer with the resistance
\[ R \propto \exp[\Delta_e/(2k_B T)] \] (2)
at \( eV \ll \Delta_e \) and, second, threshold behavior at \( V \approx V_T \approx \Delta_e \), where the activated conductivity turns to a finite non-activated transport. We find that in a regular 1D array \( \Delta_e \approx E_c L/d \), while in a 2D array \( \Delta_e \approx E_c \ln(L/d) \), where \( L \) is the array length and \( d \) is the size of the elemental cell of the array. We show that at the depinning voltage threshold in disordered 2D systems the current breaks through the system along the first percolative path connecting the leads. As a result, the depinning transition acquires an one-dimensional character, and the depinning voltage \( V_T \), scaling linearly with the sample length, much exceeds the corresponding low bias activation energy. We demonstrate, finally, that the magnetic field perpendicular to the film gives rise to a non-monotonic \( \Delta_e(B) \) dependence in an excellent agreement with the experimental data on both 1D artificial Josephson arrays \cite{11} and superconducting films near SIT \cite{3,33,34}.

Let us consider \( N \times M \) superconducting islands comprising a two-dimensional array closed by a small (as compared to the quantum resistance for Cooper pairs \( R_{QP} = h/4e^2 \approx 6.45 \, k\Omega \) external resistance, \( R_{ext} \), see Fig. 1. We assign the fluctuating order parameter phase \( \chi_{ij}(t) \) to the \( \{i,j\} \)-th superconducting island (see Fig. 1). The phases of the left- and right leads, \( \chi_L(t) \) and \( \chi_R(t) \), respectively, are fixed by the dc voltage \( V \) across the array:
\[ \chi_R - \chi_L = 2eV t/h + \psi(t) \], (3)
where \( \psi(t) \) describes fluctuations in the leads. We single out the leftmost, \( i = 1 \), and rightmost, \( i = N \), columns of islands directly coupled to leads and represent the array Hamiltonian in a form:
\[ H = H_0 + H_{int} + \hbar^2 \sum_{j=1}^{M} \left[\left( \chi_{ij}(t) + \chi_{Nj}(t) \right)^2 - 2E_J \sum_{j=1}^{M} \cos \left( \frac{\chi_{ij}(t) + \chi_{Nj}(t)}{2} \right) \times \right] \]
\[ \times \cos \left[ \frac{2eV t/h + \psi(t) + \chi_{ij}(t) - \chi_{Nj}(t)}{2} \right] \] (4)
Here
\[ H_0 = \sum_{(ij,kl)} \left[ \hbar^2 \chi_{ij} - \chi_{kl} \right]^2 - E_J \cos(\chi_{ij} - \chi_{kl}) \]
\[ + \sum_{ij} \hbar^2 \frac{1}{4E_c} \chi_{ij}^2 \] (5)
the brackets \( (ij,kl) \) denote summation over the pairs of adjacent junctions, and the last term in (5) represents the self-charge energies of superconducting islands. The \( H_{int} \) term in (4) describes coupling of phases on the leads to the thermal heat bath \cite{11}.

The dc Josephson current through the array is
\[ I_c(V) = I_c \lim_{\tau \to \infty} \frac{1}{\tau} \int_0^\tau dt \sum_{j=1}^{M} \left[ \cos \left( \frac{\chi_{1j}(t) + \chi_{Nj}(t)}{2} \right) \times \right] \sin \left[ \frac{2eV t/h + \psi(t) + \chi_{1j}(t) - \chi_{Nj}(t)}{2} \right] \] (6)
where the brackets \( \langle \cdots \rangle \) stand for an averaging over thermal fluctuations in the leads and quantum mechanical averaging over phases of internal junctions \( \langle \chi_{ij}(t) \rangle \) and over the variable \( \phi_j = (\chi_{ij} + \chi_{Nj})/2 \). We construct the time-dependent perturbation theory with respect to small parameter \( E_J/E_c \), similarly to the case of the Cooper pair two-junctions transistor \cite{14,15}, omitting the last term in (5) since in most experiments \( C \gg C_0 \), and thus \( E_c \ll E_c \). In the first order one finds:
\[ \langle \cos \phi_j \rangle = \frac{E_J}{2E_c} \cos \left( \frac{2eV t/h + \psi(t) + \chi_{1j}(t) - \chi_{Nj}(t)}{2} \right) \] (7)
In the second order, using the approach developed in \cite{11}, one arrives at:
\[ I_c(V) = MI_c \left( \frac{E_J}{2E_c} \right)^2 \mathbb{E}_m \int_0^\infty dt e^{-t/\tau} K(t) e^{2i\omega_0 t} \] (8)
where \( \delta = 4e^2 R_{ext} k_B T \) reflects the Gaussian character of the current noise in the leads due to thermal fluctuations \cite{11,18}. The correlation function of internal phases is defined as
\[ K(t) = \langle \exp(i[\chi_{1j}(t) + \chi_{Nj}(0) - \chi_{Nj}(t) + \chi_{Nj}(0)]) \rangle_{H_0} \] (9)
In the two-junction system (single Cooper pair transistor), \( \chi_{1j} \equiv \chi_{Nj}, K(t) \equiv 1 \), and we recover the results of \cite{15}. In the zero-approximation one neglects the Josephson coupling inside the array, and \( K(t) \) can be found in a closed form as an analytical continuation of \( K(\tau) \), where \( \tau \) is the imaginary time:
\[ K(\tau) = \int D[\chi_{ij}] \exp(i[\chi_{1j}(\tau) - \chi_{Nj}(0) - \chi_{Nj}(\tau) + \chi_{Nj}(0)]) \exp \left( -\frac{h}{4} \int_0^\infty d\tau \sum_{(ij,kl)} \left[ \frac{\chi_{ij}(\tau) - \chi_{kl}(\tau)}{E_c} e^{-t/\tau} \right]^2 \right) \] (10)
Expanding phases \( \chi_{ij}(\tau) \) over the Matsubara frequencies \( \omega_m = 2\pi k_B T m/h \) as \( \chi_{ij}(\tau) = \sum \exp(i\omega_m \tau) \chi_{ij}(\omega_m) \), and going over to charge representation, \( \chi_{ij}(\omega_m) = n_{ij}(2E_c k_B T/(h\omega_m^2)) \exp(-i\omega_m \tau) - 1 \), with \( n_{ij} \) being the number of Cooper pairs localized on the \( \{ij\} \)-th island, one eventually obtains the correlation function \( K(t) \):
\[ K(t) = \exp(-2\Delta_c k_B T \tau^2/h^2 - 2i\Delta_c t/h) \] (11)
where $\Delta_c$ is the barrier for the Cooper pair propagation through the whole system defined by the relation

$$
\exp \left( -\frac{\Delta_c}{k_B T} \right) = \int D[n_{ij}] \exp \left[ \frac{E_c}{k_B} \left( i(n_{ij} - n_{Nij}) - \sum_{(i,j,k)} \frac{1}{2}(n_{ij} - n_{kl})^2 - \sum_{ij} \frac{E_c n_{ij}^2}{2 Ec_0} \right) \right].
$$

Importantly, when deriving Eqs. (10-12), the non-zero windings numbers $W_{ij} = [\chi_{ij}(\hbar/k_B T) - \chi_{ij}(0)]/(2\pi)$ were neglected. This holds only at not very low temperatures, $T > E_c/(\pi k_B)$. At $T = E_c/(\pi k_B)$ a Berezinskii-Kosterlitz-Thouless-like transition into a super-insulating state with the resistivity $R \propto \exp \{(-\Delta_c/E_c)\exp[E_c/(\pi k_B T)]\}$ occurs [19]. In what follows we consider the interval of moderately low temperatures, $E_c/(\pi k_B) < T < \Delta_c/(2k_B)$.

Plugging (11) into (8), one finally derives the current-voltage characteristic of the insulating state as given by Eq. (1). At low biases, $V \ll \Delta_c$, it gives an Arrhenius activation temperature dependence [2] for the resistance, with the activation energy, $\Delta_c$, defined by Eq. (12). Carrying out calculations one finds an upper limit for a regular array:

$$
\Delta_c = \begin{cases} 
\frac{E_c}{2} \min \{\lambda_c, L/d\}, & \text{for 1D arrays}, \\
\frac{E_c}{\pi} \ln(\min \{\lambda_c, L/d\}), & \text{for 2D arrays},
\end{cases}
$$

where $\lambda_c \simeq d\sqrt{E_c}/E_c$ is the screening length related to capacitance to ground [20]. Usually, in experiments $E_c$ is so high that $\lambda_c$ well exceeds the sample size $L$.

Dependence of the activation energy on the sample size was recently observed in InO superconducting films [3]. Plotted in Fig. 2a are activation energies, $T_0 \equiv \Delta_c/k_B$, extracted from Fig. 4a of [3], vs. log $L$. One clearly sees logarithmic scaling of $T_0$ as Eq. (12) predicts.

The nature of the Coulomb barrier $\Delta_c$ and its size scaling can be understood in terms of the mutual phase locking or phase synchronization in the Josephson junction array. In the Coulomb blockade regime the charge at each junction is fixed, and, therefore conjugated phases fluctuate freely. Yet, the exponentially small de Josephson current couples phases of the adjacent junctions to provide a minimal power dissipation in the array. This establishes a global phase-coherent state, and transport occurs as a co-tunneling of Cooper pairs through the whole array. The probability of such a process in an 1D array is proportional to $\exp(-E_c/k_B T)^N$, giving the total Coulomb barrier as $\Delta_c \simeq E_c N$. Another way of thinking is to say that synchronization builds on the large screening length $\lambda_c$ which allows for the small charge fluctuations at each junction to interact over the whole system. In this sense the linear (logarithmic) scaling of $\Delta_c$ reflects linear (logarithmic) growth of the Coulomb energy in 1D (2D, respectively) systems. As a result, synchronization is rigid with respect to disorder: even large (of the order of the quantity itself, but Gaussian) fluctuations in $E_c$, $E_{c0}$, and $E_j$, as well as the effect of the offset charges is negligible as compared to the huge magnitude of $\Delta_c$.

That is why this scaling of $\Delta_c$ holds even in the amorphous superconducting films [4], where the granularity is of a self-induced nature [3, 23] and the variations in Josephson coupling strength are small.

The current-voltage characteristic of Eq. (1) is valid as long as $(\Delta_c - eV)^2 > 2\Delta c k_B T$. At temperatures of interest, $T < \Delta_c/(2k_B)$, this gives an accurate estimate for the threshold voltage of the regular Josephson junction array as:

$$
e V_T \simeq \Delta_c,
$$

with $\Delta_c$ from (13). This result holds in disordered systems as well. However, in both 1D and 2D systems the threshold voltage scales linearly with the size of the sample, i.e. 1D systems well defined. The size dependence of $V_T$ on the sample size was observed in [10], where the chain of SQUIDs, schematically shown in the upper panel of Fig. 1, was studied. One can see from the Fig. 2d of [10] that for two largest samples indeed $V_T \propto L$. In 2D arrays with disorder the dielectric breakdown becomes of a percolative nature and occurs along the first “lowest resistance” path connecting the leads. This retains a 1D scaling of $V_T$ with $L$ (Fig. 2a). The experimental data from [4]: activation energy (circles, left axis) and voltage threshold (squares, right axis) as functions of the magnetic field $B$.

The lines are are according to Eq. (15): $E^{3D}_c(B)$, with values $\alpha E_{j0}/E_c = 0.8$ and $A_{coop} = 1.4 \cdot 10^{-3}$ $\mu$m$^2$ fits $T_0$ (solid line); $V_T(B)$ is the 1D quantity and is fitted by $E^{3D}_c$ with the same $A_{coop}$ and $\alpha E_{j0}/E_c = 0.96$ (dashed line) reflecting slightly different geometric factor.
\(V_T\). Consequently, one expects that in 2D films the corresponding energy \(eV_T\) is much larger than the activation energy determined from the low bias resistance behavior \(2\). Indeed observed in \(3\) was \(eV_T/k_BT_0 \approx 220\) at the magnetic field 0.7 T. The above percolative picture is identical to the threshold charge depinning in 2D arrays of metallic dots investigated numerically in \(21\).

Next, we discuss the effect of the magnetic field on the activation energy and voltage depinning threshold. The field modulates the effective Josephson coupling: in 1D SQUID chain one has \(E_J^D = E_J^0|\cos(\pi f)|\), while in the 2D array \(E_J^D = E_J^0[1-4f\sin^2(\pi(1-f)/4)]\) \(22\), where \(f = eBA_{loop}/\pi \hbar\), \(A_{loop}\) is the area of either the elemental SQUID or the plaquette in the 2D array. The correction to Coulomb barrier in the first order perturbation theory with respect to \(E_J/E_c\) follows from \(11,12\):

\[
\Delta_c(B) = \Delta_c[1-\alpha E_J(B)/E_c],
\]

where the parameter \(\alpha\) is of the order of unity, and depends on the geometry of the lattice. The field modulation of \(E_J(B)\) yield non-monotonic field behaviors of \(T_0\) and \(V_T(B)\). Shown in Fig.\(2b\) are fits to activation energy \(T_0\) and \(V_T\) vs \(B\) dependencies to the experimental data from \(3\). The quantity \(\alpha E_J^0/E_c = 0.8\) is chosen to match \(T_0(0)\) to \(B = 0\) experimental value and reflects that experiments were carried out in the vicinity of SIT (still allowing “borderline” estimates within the perturbation theory). The loop area is defined unambiguously by the position of the maximum in \(T_0\) (only the branch corresponding \(0 \leq f \leq 1/2\) should be taken in \(E_J^D\) \(22\)). Using the same \(A_{loop}\), the theoretical \(V_T(B)\) matches with the data of \(3\) (Fig.\(2b\); this procedure means the effective absence of fitting parameters). Fig. \(2\) confirms the 2D nature of activation energy and the 1D scenario of depinning threshold.

In conclusion, we have developed a theory of collective Cooper pair transport in the insulating state of one- and two-dimensional Josephson junction arrays. We have obtained the Arrhenius low-bias resistance and derived the corresponding activation energy. We have shown that both, the activation energy and the voltage depinning threshold, represent the dynamic Coulomb barrier \(\Delta_c\) controlling collective charge transfer in the insulating state. Josephson junction chains the activation energy and voltage threshold coincide and both scale linearly with the chain length. In two-dimensional arrays the activation energy scales logarithmically with the sample length, while threshold voltage, \(V_T\), exhibits the 1D linear scaling, since disorder sets the percolative dielectric breakdown mechanism of charge depinning. We have proposed that the physical origin of the energy gap and its scaling is the mutual phase-locking in junction arrays which maintains even in disordered systems. We expect that at temperatures above the energy gap \(\Delta_c/k_BT\) the coherent synchronized state breaks down and the collective activation transport transforms into a usual local variable range hopping as observed in \(3,23\). We have demonstrated that modulating Josephson coupling by the magnetic field leads to a peaked \(V_T(B)\)-dependence in agreement with the experimental findings for 1D Josephson arrays \(10\) and for superconducting films near SIT \(3,5\).

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