Mathematical description of the processes of synthesis of
digital multispectral images

M A Kalitov and N P Kornyshev
Yaroslav-the-Wise Novgorod State University, Veliky Novgorod, Russian Federation

E-mail: Nikolai.Kornishev@novsu.ru

Abstract. The article discusses the features of the formation of digital multispectral images corresponding to narrow registration zones. The processes of signal transformations are analyzed during the synthesis of such images from the original multispectral images with overlapping spectral flux registration zones. Questions of the mathematical description of the differential and multiplicative method for the synthesis of spectrozonal images are discussed. Analytical expressions are given that correspond to signal transformations of the original digital multispectral images in their differential and multiplicative synthesis.

1. Introduction
Methods of digital processing of spectrozonal images obtained in overlapping areas of radiant flux registration zones make it possible to extract additional visual information about these areas. A number of such methods are known [1-4], based, in particular, on subtraction (differential methods) [1-3] and multiplication (multiplicative methods) [4] of the original pair of digital television multispectral images. However, these methods are obtained, mainly, heuristically, and their theoretical justification, including the mathematical description, is not sufficiently highlighted in literature.

The purpose of this article is to get acquainted with the results of theoretical signaling processes in a television spectrozonal system with overlapping areas of radiant flux registration zones using differential and multiplicative methods of spectrozonal visualization.

2. Results and discussion
Let us consider the signal processes in a television spectrozonal system with overlapping sections of the zones of registration of radiant fluxes at the general range of spectral sensitivity of the photodetector $\Delta \lambda = \lambda_{\text{max}} - \lambda_{\text{min}}$. Let us use the expression for the total photocurrent of the photodetector element [5]:

$$i = \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \varepsilon_{A/W}(\lambda) P_{W}(\lambda) d\lambda ,$$

(1)

where $\varepsilon_{A/W}(\lambda)$ [A/W] is the characteristic of the spectral sensitivity of the photodetector, and $P_{W}(\lambda)$ [W] is the characteristic of the spectral power of the radiation source.
Taking into account the normalization of characteristics $0 \leq \varepsilon(\lambda) = \frac{\varepsilon_{A/\lambda}(\lambda)}{\varepsilon_{\text{max}}} \leq 1$ and $0 \leq P(\lambda) = \frac{P_{\lambda}(\lambda)}{P_{\text{max}}} \leq 1$, where $\varepsilon_{\text{max}}$ is the maximum photodetector sensitivity, $P_{\text{max}}$ is the maximum power of the radiation source, expression (1) has the form:

$$i = \varepsilon_{\text{max}} P_{\text{max}} \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \varepsilon(\lambda) P(\lambda) d\lambda .$$

Since the function $\varepsilon(\lambda)$ integrable on the interval $[\lambda_{\text{min}}, \lambda_{\text{max}}]$, and the function $P(\lambda) \geq 0$, then we use the well-known mean value theorem and rewrite (2) as follows:

$$i = i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P(\lambda) d\lambda ,$$

where $i_0 = \varepsilon_{\text{max}} P_{\text{max}} \epsilon_{\text{cp}}$.

Then the electrical signals (photocurrents) received in two overlapping areas of the zones of registration of radiant fluxes using the same photodetector can be expressed in the following form:

$$i_1 = i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) d\lambda$$

$$i_2 = i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_2(\lambda) d\lambda ,$$

where $P_1(\lambda)$ is the characteristic of the spectral power of the radiation source in the range from $\lambda_1$ to $\lambda_{\text{max}}$, $P_2(\lambda)$ is the characteristic of the spectral power of the radiation source in the range from $\lambda_2$ to $\lambda_{\text{max}}$, moreover, $\lambda_{\text{min}} \leq \lambda_1 \leq \lambda_2 \leq \lambda_{\text{max}}$, and $P_1(\lambda) \neq P_2(\lambda)$.

Consider the signaling processes of the differential method, in which we have:

$$i = i_1 - i_2 = i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) d\lambda - i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_2(\lambda) d\lambda .$$

Let us show that the inversion $\frac{i_2}{i_1}$ of signal $i_1$ corresponds to inversion $\frac{P_2(\lambda)}{P_1(\lambda)}$ of spectral characteristics $P_2(\lambda)$ radiant flux accurate to a constant value $C$. This implies that $\frac{i_2}{i_1} = i_W - i_2$, where $i_W = \text{const}$ is the level of “white” in the image signal, corresponding, in particular, to the value of 255 relative units for eight-bit digital coding.

In this case, the ratio should be met:

$$i = i_1 - i_2 = i_1 + \frac{i_2}{i_1} = i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} \frac{P_1(\lambda) + P_2(\lambda)}{\lambda_{\text{min}}} d\lambda + C .$$

(3)
Note that, on the one hand:

\[ i = i_1 - i_2 = i_1 + i_2 - i_W \]

but, on the other hand:

\[ P_2(\lambda) = 1 - P_2(\lambda) \]

Then the integral on the right-hand side of expression (3) can be rewritten as follows:

\[
i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} [P_1(\lambda) + (1 - P_2(\lambda))] d\lambda = i_0 \left\{ \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) d\lambda + \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} (1 - P_2(\lambda)) d\lambda \right\}
\]

or

\[
i_0 \left[ \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) d\lambda - \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_2(\lambda) d\lambda + \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} d\lambda \right] = i_1 - i_2 + (\lambda_{\text{max}} - \lambda_{\text{min}}) = i_1 + i_2 + C,
\]

where \( C = (\lambda_{\text{max}} - \lambda_{\text{min}}) - i_W \).

Thus, the right-hand side of expression (3) corresponds to its left-hand side, as required.

Let us consider in a similar way the signaling processes for the multiplicative method, in which the spectrozonal signal is formed according to the formula:

\[
i = i_1 \times i_2 = \left[ i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) d\lambda \right] \times \left[ i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_2(\lambda) d\lambda \right]
\]

It is known that, in the general case, it is true:

\[
\left[ i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) d\lambda \right] \times \left[ i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_2(\lambda) d\lambda \right] \neq \left[ i_0 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) P_2(\lambda) d\lambda \right]
\]

Consider separately the integral on the right-hand side of this inequality. Both functions \( P_2(\lambda) \) and \( P_1(\lambda) \) integrable on the interval \( [\lambda_{\text{min}}, \lambda_{\text{max}}] \), moreover, \( 0 \leq P_2(\lambda) \leq 1 \) and \( 0 \leq P_1(\lambda) \leq 1 \).

Thus, it is possible to use the mean value theorem, according to which in this case, on the one hand:

\[
\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) P_2(\lambda) d\lambda = \mu_1 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) d\lambda,
\]

\[
\int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_1(\lambda) P_2(\lambda) d\lambda = \mu_2 \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} P_2(\lambda) d\lambda,
\]

where \( 0 \leq \mu_1 \leq 1 \) and \( 0 \leq \mu_2 \leq 1 \).

Multiplying, respectively, the left and right sides of equalities (4) and (5), we obtain:
Thus, for the multiplicative method, it is true:

\[ i = i_1 \times i_2 = A i_0 \left( \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \right) P_1(\lambda)P_2(\lambda) d\lambda, \quad \text{где} \quad A = \frac{1}{\mu_1\mu_2}. \]  

(6)

However, the electrical signal from the radiant flux, obtained, for example, when combining light filters with spectral characteristics corresponding to \( P_1(\lambda) \) and \( P_2(\lambda) \), and, is determined by the expression:

\[ i_{1,2} = i_0 \left( \frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} \right) P_1(\lambda)P_2(\lambda) d\lambda, \]

which is based on the fact that the total transmittance of sequentially located optical media is equal to the product of the transmittances of these media.

Therefore, to obtain the correspondence of the electric signal to the real radiant flux, the following correction of expression (6) is necessary:

\[ i_{\text{out}} = N\sqrt{i}, \quad \text{where} \quad N \leq 1 \quad \text{is the normalizing factor that provides the required dynamic range of the output signal, and to invert the output signal, it is necessary to additionally perform} \quad i_{\text{out}} = i_W - i_{\text{out}}. \]

### 3. Conclusion

Mathematical description of signal processes in a television spectrozonal system with overlapping sections of the zones of registration of radiant fluxes in the general range of spectral sensitivity of the photodetector \( \Delta \lambda = \lambda_{\text{max}} - \lambda_{\text{min}} \), shows that the inversion of the spectral zone signal corresponds to the inversion of the spectral characteristic of the corresponding radiant flux with an accuracy of up to a constant value. The analytical expression (3) obtained in this case describes the differential synthesis method, and the analytical expression (6) describes the multiplicative method for synthesizing digital multispectral images from the original images with overlapping radiant flux registration zones.

### References

[1] Zubarev Yu B, Sagdullaev Yu S and Sagdullaev T Yu 2009 Spectrozonal methods and systems in space television Questions of radio electronics ser. Television technology 1 47–64

[2] Sagdullaev Yu S and Sagdullaev T Yu 2011 On the issue of the choice of registration zones in multispectral television Questions of radio electronics ser. Television technology 2 20

[3] Sagdullaev Yu S and Sagdullaev T Yu 2018 Fundamentals of building information-measuring systems for spectrozonal television Questions of radio electronics ser. Television technology 459–67

[4] Borisov DI, Erganzhiev NA, Kalitov M A and Kornyshev NP 2020 Method of forming digital multispectral television signals Patent RU 2731880

[5] Khalfin A M 1955 Fundamentals of television technology (Moscow: Soviet radio) p 86