The Charge-Spin Separated Fermi Fluid in the High $T_c$ Cuprates: A Quantum Protectorate

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We find experimental evidence for spin-charge separation in all four relevant phases of the cuprates. It is argued that this phenomenon serves to protect the properties of the cuprates from the effects of impurities and phonons.

I. INTRODUCTION

Laughlin and Pines (1) have introduced the term “Quantum protectorate” as a general descriptor of the fact that certain states of quantum many-body systems exhibit properties which are unaffected by imperfections, impurities and thermal fluctuations. They instance the quantum Hall effect, which can be measured to $10^{-9}$ accuracy on samples with mean free paths comparable to the electron wavelength, and flux quantization in superconductors, equivalent to the Josephson frequency relation which again has mensuration accuracy and is independent of imperfections and scattering. An even simpler example is the rigidity and dimensional stability of crystalline solids evinced by the STM. Some of these examples exhibit broken symmetry but whether it is correct to ascribe broken symmetry to the quantum Hall effect is questionable. I would suggest that the source of quantum protection is a collective state of the quantum field involved such that the individual particles are sufficiently tightly coupled that elementary excitations no longer involve a few particles but are collective excitations of the whole system, and therefore, macroscopic behavior is mostly determined by overall conservation laws.

The purpose of this paper is, first, to present the overwhelming experimental evidence that the metallic states of the high $T_c$ cuprate superconductors are a quantum protectorate; and second, to propose that this particular collective state involves the phenomenon of charge-spin separation, and to give indications as to why such a state should act like a quantum protectorate.

II. EXPERIMENTAL EVIDENCE

We may define four regions of the generic phase diagram of the cuprates Fig (1): (I) the “normal” metallic state near optimal doping, widely assumed to be a non-Fermi liquid; (II) The “spin gap” or pseudogap state, separated from the above by the temperature $T^*$, probably a crossover region; (III) The $d$-wave superconducting phase; and (IV) the Mott insulating antiferromagnet. I shall assume that the “stripe” phase when encountered is merely an inhomogeneous mixture of (III) and (IV).

Phase IV, the Mott insulator, is on the face of it charge-spin separated. There is a charge gap of $\sim 2eV$, while the spin wave spectrum extends to zero energy. It is understood implicitly, but seldom stated, that the spin waves, which are Goldstone bosons of the broken symmetry, are weakly scattered by phonons and conventional impurities, and not scattered at all in the limit $\omega, Q \rightarrow 0$: they are in a quantum protectorate, because the spin and charge dynamics have become independent, and perturbations which interact primarily with charge do not affect spin.

It is the thesis of this paper that phases I, II and III all share this property, which is responsible for the anomalies of the high $T_c$ cuprates.

The transport properties of phase I have been particularly well studied in YBCO and to a lesser extent in BISCO and “214” $(La - Sr)CuO_4$, and in BISCO particularly very accurate ARPES gives us a window on the one-electron spectrum. The energy distribution curves have no features indicating phonon contributions to the self-energy of the electrons. But it is simply the scaling of the conductivity as a function of $T$ and $\omega$ which gives us the clearest indication.

$$\sigma = \omega F\left(\frac{T}{\omega}\right).$$

That is, there is no extraneous energy scale. In particular, phonon scattering would not show the striking linear rise of scattering rate $\frac{1}{\tau} \propto \omega$, above the Debye frequency, nor would any conventional electron-electron scattering. The same behavior of the one-electron self-energy is shown in ARPES, in marked contrast to conventional metals. This behavior is strikingly shown in Fig. 2, from Ref. (3) contrasting the phonon dominated self-energy of a $Mo$ surface state with that of a cuprate superconductor. Both observations show that there is little or no effect of phonon scattering. (Data are presented in Ref. 4)

A second peculiar result is the absence of resistivity saturation near the Mott limit, which (though not well understood) is seen universally in conventional poor metals, and assumed to be associated with strong phonon scattering.

I have elsewhere emphasized the striking observations of different relaxation rates for Hall angle and resistivity, which have been amply confirmed by measurements of $\theta_H(\omega)$. I have shown that this can be explained by
charge-spin separation. $\tau_H$, too, shows no evidence of phonon or impurity scattering.

The knowledgeable reader may object that Zn and Ni impurities, which substitute for Cu in the CuO$_2$ planes, do in fact act as strong scatterers. These impurities act as Kondo scatterers for the spin degrees of freedom: they trap a bound spinon, if you like. But it is easily shown that the Kondo effect is enhanced in a spin-charge separated system, so that the Kondo temperature may be above room temperature. Thus these impurities scatter spinons at the unitarity limit, as observed, but do not show magnetism except at high temperature. The observation of this strict dichotomy between these two scatterers and most others is very good evidence for the quantum protectorate and its explanation in terms of spin-charge separation. The idea that this dichotomy can be explained by a conventional quasiparticle theory is, frankly, not plausible.

Phase II is the pseudogap state. Here the most striking evidence for spin charge separation is the pseudogap itself, which shows up as a gap in the one-electron spectrum along the “anti-nodal” directions in $k$ space, while there is no evidence for a gap for charge excitations (except in systems with static stripes.)

Because the phenomena are much complicated by the pseudogap, it is not possible to completely eliminate the possibility of impurity or phonon scattering, but there is certainly no evidence for either. Much attention has been given to the mysterious nature of the pseudogap; for instance it has been realized that the violation of the Luttinger theorem on the Fermi surface essentially excludes conventional renormalized Fermi liquid theory in this region.

The superconductor, phase III, shows, surprisingly, the clearest evidence of all for the quantum protectorate. Almost all of the superconductors are self-doped, presumably by non-stoichiometry at the level of 10–20%. The doping centers are only one layer away from the cuprate in an insulating region, and as demonstrated in my book they should scatter quite efficiently. If so they are necessarily pair-breaking for conventional $d$-wave superconductors. Surely this level of pair-breaking impurities would lower $T_c$ probably to zero. These is no evidence whatever that $T_c$ is even affected by purity level or by phonon scattering, which will also be pair-breaking for a $d$-wave. For instance, the optimum $T_c$ in YBCO is achieved not in $YBa_2Cu_3O_7$, which is almost the only stoichiometric cuprate, but in $YBa_2Cu_3O_6.93$, with 7% charged impurities. In a very true sense, the biggest mystery of high $T_c$ superconductivity is that $T_c$ is so high! It seems likely that such a $T_c$ can only appear in a quantum protectorate; certainly this is true of a $d$-wave superconductor.

The absence of pair-breaking effects is confirmed when we examine transport properties, especially the thermal conductivity of the superconductors in a magnetic field. The field-sensitive thermal conductivity for $T$ well below $T_c$ must be carried by quasiparticle excitations in the gap nodes. (It is electronic because it shows a Hall-like (Righi – LeDuc) effect and because is all cases it is eventually destroyed by fields $H<<H_{c2}$.)

A number of theorists have shown that the only possible interpretation of the data involves true Dirac Fermions at the gap nodes with effectively zero mass ($E \propto |k−k_0|$). The node is smeared out by impurity scattering, to any degree that can be measured, as it would have to be in conventional $d$-wave superconductors. This, to me, is the crucial evidence for a new kind of quantum protectorate.

### III. Spin-Charge Separation

The hypothesis which has been put forward for some of this behavior is charge-spin separation: that the elementary excitations in the normal state are not quasiparticles with the quantum numbers of electrons but are solitons which are fractionalized electrons, one carrying the spin quantum number and the other (or others) the charge. In particular, the crucial component of this idea is the spinon, a neutral excitation carrying only the spin quantum number of an electron. The spinon has a history dating back to work by Des Cloiseaux and Fadeev on the excitation spectrum of the Bethe solution of the 1D Heisenberg model, and was explicitly demonstrated by E. Lieb and F. Wu for the 1D Hubbard model. But aside from a remark by Landau, its possible validity as an excitation in higher dimensions dates to the RVB theories stimulated by high $T_c$.  

Spin-charge separation is a very natural phenomenon in interacting Fermi systems from a symmetry point of view. The Fermi liquid has an additional symmetry which is not contained in the underlying Hamiltonian, in that the two quasiparticles of opposite spins are exactly degenerate and have the same velocity at all points of the Fermi surface. This is symmetry $SO(4)$ for the conserved currents at each Fermi surface point since we have 4 degenerate real Majorana Fermions. But the interaction terms do not have full $SO(4)$ symmetry, since they change sign for improper rotations, so the true symmetry of the interacting Hamiltonian is $SO4/Z_2 = SU2 \times SU2$, i.e., charge times spin. A finite kinetic energy supplies a field along the $\uparrow$ direction of the charge $SU(2)$ and reduces it to $U(1)$, the conventional gauge symmetry of charged particles.

The reason why conventional Fermi liquid theory works is that $U$ renormalizes to irrelevance because of the ultraviolet divergence of the ladder diagrams in 3 dimensions or higher. The result is the “effective range” theory which allows us to approximate the interaction terms, for forward scattering, by a scattering length $a$, which leads only to irrelevant symmetry-breaking terms. In one dimension there is no ultraviolet divergence, this does not happen, and spin-charge separation always occurs. 2 is the critical dimension and I have shown that in fact there
Spin-charge separation tells us that the spectrum of exact elementary excitations does not consist of quasiparticles, which carry both charge and spin. In the Mott insulator antiferromagnet there is a large charge gap and the Goldstone boson excitations are spin waves. In the other phases, neither charge nor spin are gapped; but nonetheless, the spin spectrum remains distinct and reflects the symmetries of the spin system. In particular, in the absence of time-reversal breaking, the Kramers degeneracy of the electron states reverts to the spin spectrum.

Unlike the Neel-ordered Mott antiferromagnet, the normal phases (I) and (II) are based on a ground state with no broken symmetry, presumably a singlet spin liquid. The spin excitations in such a fluid are spinons, spin $1/2$, uncharged fermion-like objects with linear spectra and finite momenta, in the only two cases which have been studied formally. (1D, and relatively weakly interacting 2D.) The latter is our model for the “normal” phase (I), a spin liquid with a Luttinger Fermi surface at all points of which the spinon energy vanishes.

In Phase II we suppose the spin systems to be in a state related to the “π flux” phase of Laughlin, equivalent to Affleck and Marston’s “s+id” RVB. This has been extensively studied numerically and with Gutzwiller-projection based approximations for the half-filled, insulating case, but not in the doped insulator. In a subsequent paper we will demonstrate that the Fermi spin liquid is unstable via a BCS gap formation in the spin sector relative to “s+id”.

In both of these two phases the charge spectrum remains ungapped. In the ideal, weakly interacting, pure Fermi fluid it consists of “holons”, propagating, particle-like solitons which may have charge other than $e$, and anyon statistics. But in the actual substance the charge excitations are strongly scattered and their low-frequency, long-range dynamics is diffusive.

A lot of effort has gone into describing this phase and its spin excitations using gauge theory (Lee et al, Fisher et al [1]). These groups have indeed found a Fermionic field which appears to be equivalent to spinons, so at least in this sense there is a third formal treatment of spin-charge separation. These groups, too, seem to have less to say about charge excitations.

Formal theory for a charge-spin separated superconductor is even more rudimentary; the work of Fisher and Senthil may provide some structure, but their model is missing both impurity scattering and long-range Coulomb effects.

These are actually two sources for the “quantum protectorate” effect, not entirely independent but physically distinct. The first is that spinons are relatively weakly scattered because they are the “Goldstone Fermions” which express fundamental symmetries of the spin system. The spinon dynamics in low-frequency states is averaged over all configurations of the holes, hence effectively is the dynamics of a “squeezed”, smoothed Heisenberg-like model with a number of sites equal to the number of electrons. Impurities will lead merely to local variations of the effective exchange integrals, which are inefficient in scattering long-wavelength, low-frequency spin fluctuations.

A second view is more direct. In the charge-spin separated state, the electron is a composite particle whose Green’s function in space-time is the product of charge and spin factors. The resulting Fourier transfer $G(k, \omega)$ is the convolution of these and is in fact observed in ARPES measurements to have a broad, power-law shape with at best a cusp-like feature at the (presumed) spinon frequency. Taking either this argument, or the ARPES observations, one sees that the one-electron density of states vanishes at $\omega = 0$, as a power law.

$$N(\omega) \propto \omega^p, \quad 0 < p < 1$$

In the idealized models $p = 2\alpha$, twice the Fermi surface exponent, but the observations suggest $p \sim 1/2$. Any perturbation which couples to electrons, in particular any time-reversal invariant perturbation other than substitution in the copper sites, thus renormalizes to zero at low frequencies; it can not cause real scattering.

The above discussion holds for the “normal” phases I and II. For the superconducting phase III am going to make a rather radical proposal. This is that the charge excitations essentially remain separate and condense with “s-wave” symmetry: hence the insensitivity to scattering. The resulting condensate then automatically gives the spinons quasiparticle character, if, following Fisher et al, we suppose that the holons are boson-like rather than semions (thus returning to the original BZA hypothesis [2]). Admittedly, this hypothesis is speculative, but it is strongly supported by experimental fact.

IV. CONCLUSION AND QUESTIONS

The existence of a quantum protectorate effects seems to me to be amply justified by the striking experimental anomalies I have listed: absence of phonon scattering, absence of pair breaking effects, the unusual phenomenon of the spin gap. These anomalies are more strongly inescapable than many of the peculiarities often fastened on as the crucial key to understanding these very complex materials. Our version of the rather old phenomenon of charge-spin separation seems the most plausible source. We propose a new vision of CSS arising not from below, from the influence of a mysterious “Quantum Critical Point”, but as being a universal high-energy trait of electron systems only renormalized away in low temperature and high dimensional systems.
Many questions remain. Is the \( Z_2 \) symmetry which plays a key role in our view of CSS the same as the \( Z_2 \) which Senthil and Fisher find from a gauge theory analysis? It is very suggestive to say yes. If so, we presume that, contrary to their suggestion, even the superconducting state is one in which \( Z_2 \) symmetry is broken.

The precise mechanism for the final superconducting transition is of course still in question since we have made no specification of the holon dynamics; I suggest that its \( T_c \) is determined by the need to reduce the frustrated kinetic energy of the system, but do not here propose an explicit mechanism.

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FIG. 1. The Generalized Phase diagram. I-IV label the “protected” phases.
FIG. 2. Contrasting Self-Energies in a Fermi Liquid and a non-Fermi liquid (from Ref. (2))
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