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Published in:
JETP Letters

DOI:
10.1134/S0021364020070024

Published: 01/04/2020

Please cite the original version:
Volovik, G. E. (2020). On the Dimension of Tetrads in the Effective Gravity. JETP Letters, 111(7), 368-370. https://doi.org/10.1134/S0021364020070024

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On Dimension of Tetrads in Effective Gravity

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Submitted 29 February 2020
Resubmitted 10 March 2020
Accepted 11 March 2020

DOI: 1234567

1. Introduction. There are several scenarios of emergent gravity. Gravity may emerge in the vicinity of the topologically stable Weyl point [1–5]; the analog of curved spacetime emerges in hydrodynamics with the so-called acoustic metric for the propagating sound waves [6]; etc. Here we consider two very different scenarios, which however have unusual common property: the tetrad fields in these theories have dimension of inverse length. As a result all the physical quantities, which obey diffeomorphism invariance, such as the Newton constant, the scalar curvature, the cosmological constant, particle masses, fermionic and scalar bosonic fields, etc., are dimensionless.

Two different sources of emergent gravity lead to the inverse square of length dimension of metric field, \([g_{\mu\nu}] = 1/|l|^2\), as distinct from the conventional dimensionless metric, \([g_{\mu\nu}] = 1\), for \(c = 1\). In both scenarios all the physical quantities, which obey diffeomorphism invariance, such as the Newton constant, the scalar curvature, the cosmological constant, particle masses, fermionic and scalar bosonic fields, etc., are dimensionless.

\[ P_L = (r \rightarrow -r) \] and \( T_L = (t \rightarrow -t) \), and also the discrete symmetries in spin space, \( P_S \) and \( T_S \). The symmetry breaking scheme \( P_L \times P_S \rightarrow P \) and \( T_L \times T_S \rightarrow T \) leaves the combined symmetry \( P \) and the combined time reversal symmetry \( T \).

The VD symmetry breaking mechanism can be important for the consideration of the Big Bang scenario, in which the gravitational tetrads change sign across the singularity, \( e^A_\mu(r, x) = -e^A_\mu(-r, x) \) [13, 14]. The singularity can be avoided by formation of the bubble with a vanishing determinant of the metric [15, 16], which would correspond of the vacuum state with unbroken symmetry, i.e., with zero tetrad field, \( e^A_\mu = 0 \). On the other hand, the Big Bang can be considered as a symmetry breaking phase transition \( L_L \times L_S \rightarrow L \), at which the symmetry between the spacetime with \( e > 0 \) and anti-spacetime with \( e < 0 \) is spontaneously broken, where \( e \) is the tetrad determinant.

Correspondingly, in superfluid \(^3\)He the formation of the \( p \)-wave order parameter spontaneously breaks the symmetry under coordinate transformation \( r \rightarrow -r \). The VD scenario has also the connection to the chiral \(^3\)He-A phase: in both systems the topologically protected Weyl fermions emerge, which move in the effective tetrad field [5].

According to Eq. (1), the frame field \( e^A_\mu \) transforms as a derivative and thus has the dimension of inverse length, \([e^A_\mu] = 1/|l|\) (it is assumed that \( \psi \) is scalar under diffeomorphisms) [7, 8]. The dimension of the metric is \([g_{\mu\nu}] = 1/|l|^2\). For Weyl or massless Dirac fermions one has the conventional action:

\[ S = \int d^4x |e| e^{A_\mu} (\psi^\dagger \gamma^A \nabla_\mu \psi + \text{H.c.}) . \]
The action (2), when expressed in terms of the VD tetrads, is dimensionless, since \([e] = [l]^{-4} \), \([e^{A\mu}] = [l] \) and \([\psi] = 1 \). This suggests that the VD dimension of tetrads is natural, and the tetrads emerging for example in the chiral Weyl superfluid \(^3\)He-A and in Weyl semimetals may also have the dimension \(1/[l] \).

3. Elasticity tetrads of superplastic vacuum. The elasticity tetrads describe elasticity theory \([10,11,17–19]\). In conventional crystals they are gradients of the three \(U(1)\) phase fields \(X^A\), \(A = 1,2,3\),

\[
e^A_\mu(x) = \partial_\mu X^A(x).
\]

The surfaces of constant phases, \(X^A(x) = 2\pi n^A\), describe the system of the deformed crystallographic planes. Being the derivatives, elasticity tetrads have also canonical dimensions of inverse length. This allows us to extend the application of the topological anomalies. For example, the Chern–Simons term describing the 3+1 intrinsic quantum Hall effect becomes dimensionless. As a result, the prefactor of this term is given by the integral of the even \(|\psi|^4\) over the whole system, which gives \([\sqrt{-g}] = [l]^{-3}\). This is the same for the effective metric for the propagating Goldstone modes of the coherent spin precession \([22]\), and actually for any Goldstone mode with linear spectrum. Note that when \(c\) and \(\hbar\) are incorporated into the tetrad fields and thus to the metric, these quantities do not enter explicitly into the diffeomorphism-invariant action \([23]\).

5. Conclusion. In both scenarios of emergent gravity, the dimensionless physics is supported by the invariance under diffeomorphisms. In the VD theory this invariance is assumed as fundamental. In the superplastic vacuum, the diffeomorphism invariance corresponds to the proposed invariance under deformations of the 4D crystal. All this suggests that the dimensionless physics can be the natural consequence of the diffeomorphism invariance, and thus can be the property of the gravity, which we have in our quantum vacuum.

Note the difference with the conventional expression of the physical parameters in terms of the Planck units, where the Newton constant \(G = 1\), and all the physical quantities also become dimensionless. In this approach the masses of particles are expressed in terms of the Planck energy, which is assumed to be the fundamental constant. However, in principle the Planck energy or the Newton constant may depend on the trans-Planckian physics, and thus can (and should) be space and coordinate dependent (see, e.g., the time-dependence of \(G\) in the so-called \(q\)-theory, when \(G\) approaches its asymptotic value in the Minkowski vacuum \([24]\)). On the contrary, in the VD approach the “fundamental constants” do not exist, and only dimensionless ratios and the topological quantum numbers make sense. Then, instead of the fundamental constants, the most stable physical quantities should be used. The masses of particles or the Newton and cosmological “constants” can be expressed for example in terms of the Bohr radius \([8]\). Note that in the modified gravity theories, such as the scalar-tensor and \(f(R)\) theories (see, e.g., \([25]\)), the effective Newton “constant” \(G\) can be space-time dependent, and thus is not fundamental.

The dimensionless physics emerging in the frame of the VD dimensionful tetrads leads in particular to the new topological terms in action, since some of the dimensionless parameters appear to be the integer valued quantum numbers, which characterize the topology of the quantum vacuum. This can be seen on example of the 3+1 dimensional quantum Hall effect in topo-
logical insulators [10, 11, 26]. When the Chern–Simons action is written in terms of the elasticity tetrads with \(e^A_\mu = 1/|l|\), its prefactor becomes dimensionless and universal, being expressed in terms of integer-valued momentum-space invariant.

The relativistic example of such phenomenon is the chiral anomaly in terms of the torsion fields suggested by Nieh and Yan [27–29]. For the conventional torsion and curvature in terms of the conventional dimensionless tetrads, the gravitational Nieh–Yan anomaly equation for the non-conservation of the axial current

\[
\partial_\mu j_5^\mu = \lambda^2 \left( T^A \wedge T_A - e^A \wedge e^B \wedge R_{AB} \right),
\]

contains the nonuniversal prefactor – the ultraviolet cut-off parameter \(\lambda\) with dimension of inverse length, \(|\lambda| = 1/|l|\). Because of such prefactor, the Nieh–Yan contribution to the anomaly is still contentious and subtle (see recent literature [30–35]). This is because the nonuniversal parameter may depend on the spacetime coordinates, which explicitly violates the topology. However, in terms of VD tetrads, the dimension of torsion becomes \(T_A = 1/|l|^2\), and as a result the prefactor \(\lambda\) becomes dimensionless, \(|\lambda| = 1\). The latter suggests that the prefactor is universal, and thus properly reflects the topology of the quantum vacuum.

This work has been supported by the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (Grant Agreement # 694248).

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