Affleck-Dine Leptogenesis with an Ultralight Neutrino

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Abstract

We perform a detailed analysis on Affleck-Dine leptogenesis taking into account the thermal effects on the dynamics of the flat direction field $\phi$. We find that an extremely small mass for the lightest neutrino $\nu_1$, $m_{\nu_1} \lesssim 10^{-8}$ eV, is required to produce enough lepton-number asymmetry to explain the baryon asymmetry in the present universe. We impose here the reheating temperature after inflation $T_R$ to be $T_R \lesssim 10^{8}$ GeV to solve the cosmological gravitino problem. The required value of neutrino mass seems to be very unlikely the case since the recent Superkamiokande experiments suggest the masses of heavier two neutrinos $\nu_2$ and $\nu_3$ to be in a range of $10^{-1}$–$10^{-3}$ eV. We also propose a model to avoid this difficulty based on the Peccei-Quinn symmetry, where the required neutrino mass can be as large as $m_{\nu_1} \simeq 10^{-4}$ eV.
1 Introduction

There have been growing interest in leptogenesis [1] to account for the baryon asymmetry in the present universe since the Superkamiokande collaboration presented convincing evidence of the atmospheric neutrino oscillation [2]. In fact, various mechanisms [3, 4, 5] for the leptogenesis have been proposed so far. Among them the leptogenesis by flat directions $\phi$ in supersymmetric (SUSY) theories, i.e., Affleck-Dine (AD) mechanism [6], is expected to work even with relatively lower reheating temperatures $T_R$ (e.g., $T_R \lesssim 10^8$ GeV) and hence it seems cosmologically safer than others, since the number density of gravitinos produced in the reheating processes is proportional to the reheating temperature $T_R$ of inflation [7]. However, it has been pointed out [9, 10] that the flat direction fields $\phi$ start their coherent oscillations at an earlier time than usually estimated due to interactions with thermal plasma of inflaton-decay products and that the resultant baryon asymmetry is strongly suppressed.

In this paper we perform a detailed numerical analysis on the AD leptogenesis taking into account the thermal plasma effects. It has been argued in Ref. [9] that a relatively low reheating temperature $T_R \lesssim 10^6$ GeV is necessary to escape from the thermal effects. We confirm that the baryon asymmetry is indeed suppressed for $T_R \gtrsim 10^5 - 10^6$ GeV due to the thermal effects, when the lightest neutrino mass is $m_{\nu_1} \gtrsim 10^{-9}$ eV. On the other hand, we find that the thermal effects become significant for a higher reheating temperature, $T_R \gtrsim 10^5 - 10^6$ GeV $\times (m_{\nu_1}/10^{-9}$ eV)$^{-1}$, if $m_{\nu_1} \lesssim 10^{-9}$ eV.

Furthermore, we estimate how much the resultant baryon asymmetry is suppressed for such a high reheating temperature by analytical and numerical calculations. What we have found is that an ultralight neutrino of mass $m_{\nu_1} \lesssim 10^{-8}$ eV is required to obtain the desired baryon asymmetry in the present universe, $n_B/s \simeq (0.1-1) \times 10^{-10}$, when we impose the reheating temperature $T_R$ to be $T_R \lesssim 10^8$ GeV to avoid the the overproduction of gravitinos [1]. Here, $n_B$ and $s$ are the present baryon-number and entropy densities, respectively.

We note, however, that the above neutrino mass is not necessarily the same as the mass to be observed today, if effective Majorana masses for right-handed neutrinos in the early universe are different from the values in the true vacuum. To demonstrate this point we show an explicit model based on the Peccei-Quinn symmetry [11], which is one of the most attractive solutions to the strong CP problem. In this model we find that the mass for the lightest neutrino in the true vacuum can be as large as $10^{-4}$ eV. This

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1 The gravitinos may be also produced nonthermally [8]. We neglect such an effect in this paper, since it depends on the details of inflation models.
is very close to the masses of heavier two neutrinos, $m_{\nu 2,3} \simeq 10^{-1} - 10^{-3}$ eV, suggested from the recent Superkamiokande experiments [2, 12].

## 2 Flat directions for the Leptogenesis

We adopt flat directions $H_u = \tilde{L}_i = \phi_i / \sqrt{2}$ (a family index $i = 1, 2, 3$) originally proposed by Murayama and one of the authors (T.Y.) [13] as the Affleck-Dine flat direction field for leptogenesis, where $\tilde{L}_i$ are scalar components of the chiral multiplets $L_i$ of $SU(2)_L$-doublet leptons. We have effective dimension-five operators in the superpotential,

\[ W = \frac{1}{2 M_i} (L_i H_u)^2 = \frac{m_{\nu_i}}{2 v_u^2} (L_i H_u)^2. \tag{1} \]

Here, $v_u \equiv \langle H_u \rangle = \sin \beta \times 174 \text{ GeV}$ and we have used the seesaw formula [14],

\[ m_{\nu_i} = \frac{\langle H_u \rangle^2}{M_i}. \tag{2} \]

Hereafter, we take $\sin \beta \simeq 1$, and we suppress the family index $i$ for simplicity. As we will see later, the relevant flat direction $\phi$ for the most effective leptogenesis corresponds to the first family (i.e., $\phi / \sqrt{2} \equiv \tilde{L}_1 = H_u$). The potential of the flat direction $\phi$ is given by

\[ V = m^2 |\phi|^2 + \frac{1}{8 M} \left( A \phi^4 + A^* \phi^{*4} \right) + \frac{1}{4 M^2} |\phi|^6, \tag{3} \]

where $m$ and $A$ are SUSY-breaking mass parameters and the last term is derived directly from the superpotential in Eq. (1). We choose $m \simeq |A| \simeq 1 \text{ TeV}$.

During inflation the energy density of the universe is dominated by the inflaton $\chi$. After the end of inflation the inflaton $\chi$ starts its coherent oscillation and the energy density of the universe is dominated by the inflaton $\chi$ also in this period. The non-zero vacuum energy of $\chi$ induces additional SUSY breaking effects. Therefore, the flat direction $\phi$ receives an additional SUSY breaking mass of the order of $H$ in the $\chi$-dominated era. Here $H$ is the Hubble parameter of the expanding universe. There is also an $A$-term proportional to the Hubble parameter, $V \sim (H/M) \phi^4$. Thus, we have the additional potential is given by

\[ \delta V = -c_H H^2 |\phi|^2 + \frac{H}{8 M} \left( a_H \phi^4 + a_H^{*} \phi^{*4} \right), \tag{4} \]

\[ ^2 \tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle, \] where $H_u$ and $H_d$ are Higgs supermultiplets which couple to “up”-type and “down”-type quarks, respectively.
where \( c_H \) and \( a_H \) are real and complex constants respectively, and we take \( c_H > 0 \). (See the next section.) Hereafter, we assume \( c_H \approx |a_H| \approx 1 \).

The crucial point in the following discussion is that the decay of the inflaton \( \chi \) occurs during the coherent oscillation of \( \chi \), while it completes much later than the beginning of the oscillation. Thus, even in the \( \chi \) oscillation period, there is a dilute plasma with a temperature \( T = (T_R^2 M_* H)^{1/4} \) (\( M_* = 2.4 \times 10^{18} \) GeV is the reduced Planck mass) arising from the inflaton decay \([10]\), although most of the energy density of the universe is carried by the coherent oscillation. Notice that in this dilute plasma the temperature decreases as \( T \propto H^{1/4} \) rather than as \( T \propto H^{1/2} \) in the usual radiation dominated universe.

It is quite plausible that the fields \( \psi_k \) which couple to \( \phi \) are produced by the inflaton decay and/or by thermal scatterings if their effective masses \( f_k |\phi| \) are smaller than the temperature \( T \),

\[
f_k |\phi| < T, \tag{5}
\]

and hence they must be included in the plasma. Here, \( f_k \) correspond to their Yukawa or gauge coupling constants of the flat direction \( \phi \) (we take \( f_k \) real and positive).

As stressed in Refs. [9, 10] the thermal effects from the dilute plasma may affect the dynamics of the \( \phi \) field. In fact, the flat direction \( \phi \) receives a thermal mass of order \( \sim f_k T \) if the condition Eq. (5) is satisfied. Then the total effective potential for \( \phi \) including the Hubble-induced terms and the thermal mass term is given by \([10]\)

\[
V_{\text{total}} = \left( m^2 - H^2 + \sum_{f_k |\phi| < T} c_k f_k^2 T^2 \right) |\phi|^2 \\
+ \frac{m}{8M} (a \phi^4 + \text{h.c.}) + \frac{H}{8M} (a_H \phi^4 + \text{h.c.}) + \frac{1}{4M^2} |\phi|^6, \tag{6}
\]

where \( c_k \) are real positive constants of order unity and \( a = A/m \) is a complex constant of order unity. The summation of \( f_k \) means that only the fields in thermal plasma [i.e., their couplings \( f_k \) satisfy the condition Eq. (5)] can induce the thermal mass for \( \phi \).

We include all the couplings relevant for the flat direction \( H_u = \bar{L}_1 = \phi/\sqrt{2} \) in the SUSY standard model, i.e., the gauge couplings for the \( SU(2)_L \) and \( U(1)_Y \) gauge groups, and Yukawa couplings for up-type quarks and charged leptons. The couplings \( f_k \), which are redefined so that the masses for the fields \( \psi_k \) couple to \( \phi \) become \( f_k |\phi| \), are listed in Table.3 with the coefficients \( c_k \) of the thermal mass for \( \phi \). Among the gauge supermulti-

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\[3\] The SUSY breaking effects due to the finite potential energy of the inflaton \( \chi \) depend on the details of the Kähler potential. In fact, the coefficients of the Hubble-induced terms, \( c_H \) and \( a_H \) in Eqs. (4) and (6), can be much smaller than order unity. However, we simply assume here \( c_H \approx |a_H| \approx 1 \) since the conclusion does not depend much on these parameters unless \( c_H \ll -1 \). (See, for example, Ref. [13].)
Table 1: The couplings of the flat direction field $\phi$ to other fields and the coefficients $c_k$ of the thermal mass of $\phi$ induced by these fields. $g_1$ and $g_2$ denote the gauge couplings for the $U(1)_Y$ and $SU(2)_L$ gauge group respectively, and $y_a$ ($a = t, c, u$) and $y_{l1}$ are the Yukawa couplings for up-type quarks and charged lepton, respectively.

The flat direction $\phi$ does not couple to one $U(1)$ group which remains unbroken by the condensate of $\phi$. Thus, there are one $Z$-like and two $W$-like massive gauge multiplets as in the SUSY standard model. For the couplings of the charged leptons, we should be careful. The flat direction $\phi$ receives its thermal mass from only one linear combination of charged leptons. The effective Yukawa coupling $y_{l1}$ is determined by the Yukawa couplings for the charged leptons and the mixing matrix among gauge eigenstates of the neutrinos. (Note that the relevant $\phi$ is the flat direction which corresponds to the first family $\nu_1$ of mass eigenstates.) Taking large mixing angles both for atmospheric and solar neutrino oscillations indicated by the recent Superkamiokande data \cite{2,12}, we find $y_{l1} \simeq \mathcal{O}(0.1-1) \times y_\tau$. Thus, $f_k$ are roughly $10^{-5}$–1 in the SUSY standard model. We will see in Sec. 4 that the Yukawa coupling for the up quark induces relevant thermal effects, while other couplings turn out to be too large to satisfy the condition Eq. (5) in most of the cases.

3 Evolution of the flat direction

In this section we discuss the evolution of the flat direction $\phi$, which is described by the equation of motion with the potential $V_{\text{total}}$ in Eq. (3),

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_{\text{total}}}{\partial \phi^*} = 0,$$

where the dot denotes the derivative with time.

During inflation the Hubble parameter $H_{\text{inf}}$ is much larger than the soft SUSY breaking mass, $H_{\text{inf}} \gg m$. We assume that the additional mass squared is negative ($c_H > 0$) and causes an instability of $\phi$ at the origin, that is, $m_{\text{eff}}^2 = m^2 - c_H(H_{\text{inf}})^2 < 0$. Then, the minima of the potential are given by

$$|\phi| \simeq \sqrt{MH_{\text{inf}}},$$
\[ \arg(\phi) \simeq -\arg(a_H) + \frac{(2n + 1)\pi}{4} \quad n = 0 \cdots 3. \]  
\hfill (9)

Note that there is no plasma and hence no thermal mass term during the inflation. Because curvatures of the potential around the minimum along both the radius and phase directions are of the order of the Hubble parameter \( H_{\text{inf}} \), the flat direction \( \phi \) runs to one of the four minima \((n = n_0)\) from any given initial value and is settled down it.

After the inflation ends, the inflaton \( \chi \) starts to oscillate, and there appears a dilute plasma with a temperature \( T = (T_R^2M_*H)^{1/4} \). Then, the dynamics of the flat direction \( \phi \) is determined by the total potential \( V_{\text{total}} \) in Eq. (E).

In the period of the inflaton oscillation, the minimum of the \( \phi \) potential moves toward the origin \( \phi = 0 \) as the Hubble parameter \( H \) decreases. As long as the potential for \( \phi \) is dominated by Hubble-induced (mass and \( A \)) terms and also \( |\phi|^6 \) term, the flat direction field \( \phi \) always tracks the following instantaneous minimum of the potential \[ |\phi| \simeq \sqrt{MH}, \]  
\hfill (10)
[\arg(\phi) \simeq -\arg(a_H) + \frac{(2n_0 + 1)\pi}{4}. \]  
\hfill (11)

This is because the curvatures around the minimum along both radius and phase directions are of the order of \( H \) also in this period, and hence the \( \phi \) always catches up the instantaneous minimum.

As the universe evolves, the field value \( |\phi| \) decreases with time. When the field value \( |\phi| \) satisfies the condition Eq. (E), the fields \( \psi_k \) which couple to the \( \phi \) can enter in the dilute plasma so that they induce a thermal mass \( c_kf_k^2T^2 \) for \( \phi \). If this thermal mass exceeds the Hubble-induced mass, i.e.,
\[ H^2 < \sum_{f_k|\phi|<T} c_kf_k^2T^2, \]  
\hfill (12)

the dynamics of the \( \phi \) fields is drastically changed. The equation of motion for \( \phi \) at this epoch is approximately given by
\[ \ddot{\phi} + 3H\dot{\phi} + c_kf_k^2T^2\phi = 0. \]  
\hfill (13)

This equation can be solved analytically and the solution is given by
\[ \phi(t) = \phi_1 \left[ \frac{1}{z^{2/3}}J_{2/3}(z) \right] + \phi_2 \left[ \frac{1}{z^{2/3}}J_{-2/3}(z) \right] \]
\[ z = \frac{4}{3} \left( \frac{2}{3} c_k^2f_k^4M_*T_R^2 t^3 \right)^{1/4}, \]  
\hfill (14)
where $\phi_1$ and $\phi_2$ are constants and $J_\nu$ is the Bessel function. We see that the $\phi$ oscillates around the origin $\phi = 0$. The time scale of the oscillation is $\sim (f_4^4 M_s T_R^2)^{-1/3}$ and the amplitude of the oscillation is damped as $|\phi| \sim t^{-7/8} \sim H^{7/8}$.

Now let us estimate the cosmic time $t_{\text{osc}}^{\text{th}}$ when the flat direction $\phi$ starts its oscillation due to the thermal mass term, since the time $t_{\text{osc}}^{\text{th}}$ plays a crucial role in the production of the lepton-number asymmetry. First of all, we consider a simple case in which there is only one field $\psi_1$ with a coupling $f_1$. It is easily found from Eqs. (5) and (12) that the following two conditions must be satisfied in order to cause the $\phi$’s oscillation,

$$H < \frac{1}{f_1^3} \frac{M_s T_R^2}{M^2},$$

where we have used the relations $|\phi| \simeq \sqrt{M H}$ and $T = (T_R^2 M_s H)^{1/4}$. If the coupling $f_1$ is small enough, the $\psi_1$ can easily enter in the thermal plasma, but the induced thermal mass for $\phi$ is smaller than the Hubble-induced mass $H$ at the beginning. However, since the temperature decreases more slowly than the Hubble parameter $H$ ($T \propto H^{1/4}$) as the universe expands, the thermal mass eventually exceeds $H$. Then the $\phi$’s oscillation starts at the time when the condition Eq. (15) is satisfied. On the other hand, if $f_1$ is very large, the would-be thermal mass can be large enough to exceed the Hubble mass. However, the thermal mass does not appear until the $|\phi|$ becomes small enough to satisfy Eq. (5). In this case the $\phi$ starts to oscillate at the time when the condition Eq. (15) is satisfied. Therefore, we find the Hubble parameter $H_{\text{osc}}^{\text{th}}$ at $t = t_{\text{osc}}^{\text{th}}$ to be

$$H_{\text{osc}}^{\text{th}} \equiv \min \left[ \frac{1}{f_1^3} \frac{M_s T_R^2}{M^2}, \left( c_1^2 f_1^4 M_s T_R^2 \right)^{1/3} \right].$$

Notice that in order to have a significant thermal effect, as discussed in Ref. [10], a coupling $f_1$ with intermediate strength is necessary. This is because too large $f_1$ can not satisfy the condition Eq. (5), while too small $f_1$ can not give a large thermal mass for $\phi$ which satisfies the condition Eq. (15).

It is easy to apply the above discussion to the case of more than one couplings. If there is another coupling $f_i$ which can satisfy the both conditions Eqs. (15) and (16) earlier than $f_1$, the flat direction $\phi$ starts its oscillation earlier. Therefore, the Hubble parameter $H_{\text{osc}}^{\text{th}}$ is given by

$$H_{\text{osc}}^{\text{th}} = \max_i \left[ \min \left\{ \frac{1}{f_i^3} \frac{M_s T_R^2}{M^2}, \left( c_i^2 f_i^4 M_s T_R^2 \right)^{1/3} \right\} \right].$$

\footnote{We assume here hierarchical couplings and neglect effects of the summation.}
Thus, \( H_{\text{osc}}^{\text{th}} \) becomes larger (i.e., the oscillation begins earlier) as we take the reheating temperature \( T_R \) higher. One can see from Eq. (18) that it is nontrivial which coupling among \( f_i \) makes the \( \phi \)'s oscillation earliest, i.e., it is not always the coupling with strongest or weakest strength. If one takes the reheating temperature higher, a larger coupling makes the oscillation earlier and determines the time \( H_{\text{osc}}^{\text{th}} \).

As the universe becomes cooler, the thermal mass for the \( \phi \) becomes eventually smaller than the original mass \( m \). Then, the evolution of \( \phi \) is described by the equation of motion

\[
\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0. \tag{19}
\]

The flat direction oscillates around the origin with a time scale \( \simeq m^{-1} \), and the amplitude of the oscillation is damped as \( |\phi| \sim t^{-1} \sim H \) as the universe evolves.

We have, so far, assumed that the thermal mass term dominates the potential when the oscillation of \( \phi \) starts. However, the expression of \( H_{\text{osc}}^{\text{th}} \) in Eq. (18) could be smaller than the original mass \( m \), depending on the parameters \( M \) and \( T_R \), which conflicts with the above assumption. If the thermal mass does not exceed the Hubble mass \( H \) until \( H \simeq m \), the original mass term \( m^2|\phi|^2 \) dominates the potential for \( H \lesssim m \). Then, the thermal effects induced by \( \psi_k \) in the dilute plasma do not affect the evolution of \( \phi \). In this case the \( \phi \) field begins its oscillation at \( H \simeq m \).

In summary, the oscillation of \( \phi \) starts when the Hubble parameter of the universe becomes \( H \simeq H_{\text{osc}} \), where

1. \( H_{\text{osc}} = H_{\text{osc}}^{\text{th}} \), if the thermal mass term dominates the potential before \( H \simeq m \),
2. \( H_{\text{osc}} = m \), if otherwise.

We show a schematic behavior of \( H_{\text{osc}} \) in Fig. 1 for illustration. One can see from Eq. (18) that the Hubble parameter \( H_{\text{osc}} \) depends on the reheating temperature non-trivially. We consider three couplings \( f_1, f_{\Pi} \) and \( f_{\text{III}} \) assuming a hierarchy \( f_1 \ll f_{\Pi} \ll f_{\text{III}} \). The region of \( T_R \) can be divided into three regions I, II and III as in Fig. 1 where \( H_{\text{osc}}^{\text{th}} \) is determined by \( f_1, f_{\Pi} \) and \( f_{\text{III}} \), respectively. [See the discussion below Eq. (18).] The dependence of \( H_{\text{osc}}^{\text{th}} \) on \( T_R \) can be understood from Eq. (17) or (18). For example, if \( T_R \) lies in the region II, \( H_{\text{osc}} \) behaves as

\[
H_{\text{osc}} \propto \begin{cases} T_R^2 & \text{for } T_R < T_{C_{\Pi}} \\ T_R^{2/3} & \text{for } T_R > T_{C_{\Pi}} \end{cases}, \tag{20}
\]

where the critical temperature \( T_{C_{\Pi}} \) is

\[
T_{C_{\Pi}} = f_{\Pi}^4 \left( c_{\Pi} M^3 M_*^{-1} \right)^{1/2}. \tag{21}
\]
Figure 1: A schematic behavior of the Hubble parameter $H_{\text{osc}}$ when $\phi$ starts its oscillation. The oscillation of $\phi$ starts at $H_{\text{osc}} = m$ in region Ia, while the oscillation starts at an earlier time ($H_{\text{osc}} = H^\text{th}_{\text{osc}}$) in regions Ib - III.

We can also see the same behavior in the regions I and III. Furthermore, when the reheating temperature $T_R$ is low enough (i.e., in the region Ia), the thermal effects become negligible and the oscillation of $\phi$ begins at $H_{\text{osc}} = m$. We see from Fig. 1 that the oscillation time $H_{\text{osc}}$, which is crucial for the estimation of the lepton asymmetry (see the next section), depends on the reheating temperature and also the spectrum of the couplings $f_k$ in a very complicated way.

4 Lepton asymmetry

Now we are at the point to calculate the lepton asymmetry produced by the $\phi$ field. Since the $\phi$ field carries the lepton charge, its number density is related to the lepton number density $n_L$ as

$$n_L = \frac{1}{2} i \left( \phi^* \phi - \phi^* \phi^* \right).$$  \hspace{1cm} (22)
From Eqs. (6) and (7), the evolution of $n_L$ is described by the equation,

$$\dot{n}_L + 3Hn_L = \frac{m}{2M} \text{Im}(a\phi^4) + \frac{H}{2M} \text{Im}(a_H\phi^4).$$  \tag{23}$$

Therefore, non-trivial motion of the $\phi$ field generates the asymmetry in lepton number \cite{6}.

First we consider the case $H_{\text{osc}} = H_{\text{osc}}^{\text{th}} > m$. Suppose that the original $A$-term would vanish, i.e., $A = ma = 0$. If this is the case, the flat direction $\phi$ is always trapped in one of the valleys induced by the Hubble $A$-term during the both periods $t < t_{\text{osc}}^{\text{th}}$ and $t > t_{\text{osc}}^{\text{th}}$, and the direction of the valley does not change with time. Therefore, there is no force which causes the motion of $\phi$ along the phase direction and no lepton-number asymmetry is produced. However, we have the original $A$-term, and the phase of $\phi$ is kicked by the relative phase difference between $a$ and $a_H$ in Eq. (6). The phase of $\phi$ changes during its rolling towards the origin because the Hubble parameter $H$ decreases, and the direction of the true valleys changes with time. Therefore, the original $A$-term [the first term in Eq. (23)] plays a role of the source of the lepton asymmetry.

One might wonder if the Hubble $A$-term [the second term in Eq. (23)] gives a larger contribution to the lepton asymmetry since $H \gg m$ in the present situation. However, since the $\phi$ almost traces one of the valleys determined mainly by the Hubble $A$-term, $\text{Im}(a_H\phi^4)$ in Eq. (23) is highly suppressed compared with $\text{Im}(a\phi^4)$. In fact, we have found numerically that the contribution of the second term in Eq. (23) is always comparable or less than that from the original $A$-term. Thus, we neglect the lepton asymmetry produced from the second term in Eq. (23) in our analytic calculation, for simplicity. By integrating Eq. (23), we obtain the resultant lepton number at the time $t$ as

$$\left[R^3n_L \right](t) \simeq \int^t dt' R^3 \frac{m}{2M} \text{Im} \left(a\phi^4 \right)$$

$$= \int^{t_{\text{osc}}^{\text{th}}} dt' R^3 \frac{m}{2M} \text{Im} \left(a\phi^4 \right) + \int^{t} dt' R^3 \frac{m}{2M} \text{Im} \left(a\phi^4 \right), \tag{24}$$

where $R$ denotes the scale factor of the expanding universe, which scales as $R \propto H^{-2/3} \propto t^{2/3}$ in the universe dominated by the oscillating inflaton $\chi$. Notice that the integrand of the first term in Eq. (23) is almost constant, since $|\phi| \simeq \sqrt{MH}$ and $R \propto H^{-2/3}$. On the other hand, the second term gives only small contribution to the total lepton asymmetry because of the following two reasons; (i) $\text{Im}(a\phi^4)$ changes its sign rapidly due to the oscillation of $\phi$, and (ii) even if the sign would remain unchanged, the amplitude of $\phi^4$ is damped as $|\phi|^4 \sim t^{-7/2}$ (see Sec. \cite{8}).

Therefore, the production of lepton asymmetry is suppressed for $t \gg t_{\text{osc}}^{\text{th}}$, and the
resultant lepton asymmetry at $t \simeq t_{\text{osc}}^{\text{th}}$ is given approximately by

$$n_L = \frac{m}{2M} \text{Im} \left[ a \phi^4(t_{\text{osc}}^{\text{th}}) \right] t_{\text{osc}}^{\text{th}}$$

$$= \frac{1}{3} m M H_{\text{osc}}^{\text{th}} \delta_{\text{eff}},$$

(26)

where $\delta_{\text{eff}} \equiv \sin(4 \arg \phi(t_{\text{osc}}^{\text{th}})+\arg a)$ represents an effective CP violating phase and we assume $\delta_{\text{eff}} \simeq O(1)$. Notice that we have used the fact $|\phi(t_{\text{osc}}^{\text{th}})| \simeq \sqrt{M H_{\text{osc}}^{\text{th}}}$. The lepton-to-entropy ratio when the reheating process of inflation completes ($T = T_R$) is estimated as

$$\frac{n_L}{s} = \frac{M T_R}{12 M_*^2} \left( \frac{m}{H_{\text{osc}}^{\text{th}}} \right) \delta_{\text{eff}}.$$  

(27)

This ratio takes a constant value as long as an extra entropy production does not take place at a later epoch, since both $n_L$ and $s$ decrease at the same rate $R^{-3}$ as the universe expands. In deriving Eq. (27) we have assumed that the $\phi$ oscillation starts before the reheating process of inflation completes.

Next we turn to the case without thermal effects, i.e., $H_{\text{osc}} = m$. Detailed analysis for this case is found in Refs. [9, 15]. Even in this case, the production of the lepton asymmetry becomes ineffective after the $\phi$ starts the oscillation (for $H \ll H_{\text{osc}} = m$). Thus, the lepton asymmetry can be estimated by replacing $H_{\text{osc}}^{\text{th}}$ with $m$ in Eq. (27) as

$$\frac{n_L}{s} = \frac{M T_R}{12 M_*^2} \delta_{\text{eff}}.$$  

(28)

The produced lepton asymmetry is partially converted [9] into the baryon asymmetry through the “sphaleron” effects [17], since it is produced before the electroweak phase transition at $T \simeq 10^2$ GeV. Then, the present baryon asymmetry is given by [18]  

$$\frac{n_B}{s} = \frac{8}{23} \frac{n_L}{s}.$$  

(30)

Now we have formulae of the lepton asymmetry Eqs. (27) and (28). Notice that an additional suppression factor $m/H_{\text{osc}}^{\text{th}}$ ($H_{\text{osc}}^{\text{th}} > m$) appears in Eq. (24), compared with

$$\frac{n_B}{s} = \frac{24 + 4 N_H}{66 + 13 N_H} \frac{n_L}{s},$$  

(29)

rather than Eq. (30). Here $N_H$ denotes the number of the Higgs doublets. Then, $n_B/s = -(8/23)n_L/s$ in the minimal SUSY standard model with $N_H = 2$.  

\[10\]
Figure 2: A schematic behavior of the produced lepton asymmetry $n_L/s$. Regions I, II, III correspond to those in Fig. 1.

Eq. (28). This suppression clearly represents the effect of the early oscillation due to the thermal effects of the dilute plasma pointed out in Ref. [9, 10]. It should be also noted that the produced lepton asymmetry becomes larger as the scale $M$ increases, i.e., as the neutrino mass $m_\nu$ becomes smaller [See Eqs. (18), (27) and (28)]. Therefore, the leptogenesis is most effective for the flat direction corresponding to the lightest neutrino $\nu_1$, as mentioned in Sec. 2.

The produced lepton asymmetry depends on the reheating temperature $T_R$. We show again a schematic behavior of the lepton asymmetry in Fig. 2 where regions I, II, and III correspond to those in Fig. 1. If the reheating temperature $T_R$ is low enough, i.e., there is no thermal effect ($H_{osc} = m$), the lepton asymmetry $n_L/s$ is just proportional to $T_R$ as in Eq. (28),

$$\frac{n_L}{s} \propto T_R^{+1}. \quad (31)$$

However, it is not the case when the reheating temperature becomes higher (i.e., $H_{osc} = H_{osc}^{th}$), since $H_{osc}^{th}$ depends on $T_R$ (see Fig. 1). Depending on the condition which deter-
mines $H_{\text{osc}}^\text{th}$, the lepton asymmetry behaves as

$$\frac{n_L}{s} \propto T_R^{-1} \quad \text{or} \quad T_R^{1/3}. \quad (32)$$

The explicit formulae of the produced lepton asymmetry are easily derived from Eqs. (18) and (27). For example, if the reheating temperature lies in the region II, the resultant lepton asymmetry is given by

$$\frac{n_L}{s} = \frac{\delta_{\text{eff}} f_{\Pi}^4 m M^3}{12 M^2_{\ast} T_R} \quad \text{for} \quad T_R < T_{CII},$$

$$\frac{n_L}{s} = \frac{\delta_{\text{eff}} m M T_R^{1/3}}{12 c_{\Pi}^2 f_{\Pi}^{1/3} M^6_{\ast}} \quad \text{for} \quad T_R > T_{CII}, \quad (33)$$

where $T_{CII}$ is given by Eq. (21).

Now, we discuss how much lepton asymmetry is generated in the SUSY standard model. We have performed a numerical calculation to follow the evolution of the flat direction $\phi$ for given $M$ and $T_R$, including all the terms in the potential in Eq. (6). As for the thermal mass term, we have included all the couplings listed in Table 1. In Fig. 3 we show the produced lepton asymmetry for the lightest neutrino mass $m_{\nu 1} = 10^{-6}$ eV. Here, we take $m = 1$ TeV, $\tan \beta = 3$ and $\arg(a/a_H) = \pi/3$ (we take this parameter set, hereafter). As shown by the dotted line in Fig. 3, the produced lepton asymmetry is proportional to the reheating temperature $T_R$, if one neglects the thermal effects (i.e., takes all the coupling $f_k = 0$). In this case, we can derive from Eq. (28) the formula for the lepton asymmetry

$$\frac{n_L}{s} = 0.4 \times 10^{-10} \delta_{\text{eff}} \left( \frac{T_R}{10^8 \, \text{GeV}} \right) \left( \frac{10^{-6} \, \text{eV}}{m_{\nu 1}} \right), \quad (34)$$

which is in a very good agreement with the result of the numerical calculation in Fig. 3. The above expression Eq. (34) is also consistent with that obtained in the previous works [9, 15]. Notice that the produced lepton asymmetry is proportional to the reheating temperature and inversely to the lightest neutrino mass. Therefore, in order to explain the baryon asymmetry in the present universe a high reheating temperature of $T_R \simeq 10^8$ GeV is required for $m_{\nu 1} \simeq 10^{-6}$ eV. However, for such a high reheating temperature the effects of the thermal mass on the dynamics of $\phi$ is significant, and hence the $\phi$'s oscillation in fact begins at $H_{\text{osc}} = H_{\text{osc}}^\text{th}$ rather than $H_{\text{osc}} = m$. Thus, the formula Eq. (34) can not be applied to the case of $m_{\nu 1} \simeq 10^{-6}$ eV for the high reheating temperature.

The solid line in Fig. 3 represents the lepton asymmetry with the thermal effects. Even in this case, the produced lepton asymmetry is linear in $T_R$ for a low reheating
temperature region of $T_R \lesssim 4 \times 10^5$ GeV as in Eq. (34). However, we can see that the thermal effects become significant and the lepton asymmetry is suppressed for $T_R \gtrsim 10^5$–$10^6$ GeV, which is consistent with the argument in Ref. [9]. This is because the thermal mass term makes the $\phi$’s oscillation earlier ($H_{\text{osc}} = H_{\text{th osc}} \gg m$), which reduces the lepton asymmetry. In fact, the behavior of the produced lepton asymmetry shown in Fig. 3 can be understood by our analytic formula Eq. (27) and Fig. 2.

Fig. 4 shows the contour plot of the produced lepton asymmetry $n_L/s$ in the $M$-$T_R$ plane. We show both results by the numerical calculation and also from the analytic formulae Eqs. (27) and (28). We confirm that the analytic estimation discussed above reproduces very well the result obtained by the numerical calculation.

In the lower reheating temperature region where $H_{\text{osc}} = m$, the lepton asymmetry $n_L/s$ is proportional to $MT_R$ [see Eq. (28)]. On the other hand, in the region where $H_{\text{osc}} = H_{\text{th osc}} \gg m$, $n_L/s$ is proportional to $M^3T_R^{-1}$ or $MT_R^{1/3}$ [see Eqs. (18), (27) and (33)], depending on the condition which determines the cosmic time $t_{\text{osc}}$ when the early
oscillation of $\phi$ starts.

In most of the region in Fig. 4 the lepton asymmetry behaves as $n_L/s \propto M T_R^{1/3}$ and the thermal effects are controlled by only the Yukawa coupling constant for the up quark, which is roughly of order $10^{-5}$. Only in the parameter region $M < \sim 10^{21}$ GeV and $T_R > \sim 10^8$ GeV, larger couplings (say, the Yukawa coupling for the charm quark) become effective. Note that the result shown in Fig. 4 is almost independent of $\tan \beta$ as long as $\sin \beta \simeq 1$ since the Yukawa coupling for the up quark is $y_u = m_u/\langle H_u \rangle = m_u/(174 \text{GeV} \times \sin \beta)$, where $m_u$ is the mass for the up quark.

The crucial observation is that the resultant lepton asymmetry is suppressed by the thermal effects when the reheating temperature is high enough. The critical value of the reheating temperature, $T_{Rcr}$, is estimated analytically from Eq. (18) as

$$T_{Rcr} = \frac{1}{c_u f_u^2 M_*^{1/2}} \frac{m^{3/2}}{M_*^{1/2}} = 1 \times 10^5 \text{ GeV} \left( \frac{m}{1 \text{ TeV}} \right)^{3/2},$$

(35)
for the region of $M \leq M_{cr}$, and
\begin{equation}
T_{Rcr} = f_u^2 m m^{1/2} M_s^{1/2} = 4 \times 10^5 \text{ GeV} \left( \frac{M}{10^{23} \text{ GeV}} \right) \left( \frac{m}{1 \text{ TeV}} \right)^{1/2},
\end{equation}
for the region of $M \geq M_{cr}$, where $M_{cr}$ is given by
\begin{equation}
M_{cr} \equiv \frac{1}{c_u f_u^4} m = 4 \times 10^{22} \text{ GeV} \left( \frac{m}{1 \text{ TeV}} \right). \tag{37}
\end{equation}
This critical value for the scale $M$ corresponds to the lightest neutrino mass of $m_{\nu_1} = 0.9 \times 10^{-9} \text{ eV}$ for $m = 1 \text{ TeV}$. Here we have used the fact that only the up-quark Yukawa coupling is effective for the thermal effects. Therefore, we find that the thermal effects from the dilute plasma is suppressed in the region $T_R \lesssim 10^5 – 10^6 \text{ GeV}$ for $m_{\nu_1} \lesssim 10^{-9} \text{ eV}$. This is consistent with the result obtained in Ref. [9]. However, for the lighter neutrino mass region $m_{\nu_1} \lesssim 10^{-9} \text{ eV}$, this upper bound on the reheating temperature in order to neglect the thermal effects increases as $M$ (i.e., inversely proportional to the lightest neutrino mass $m_{\nu_1}$). In fact, it helps us a lot with the discussion in the next section. We also confirm the result by a numerical calculation.

In this section, we show that the thermal effects on the AD leptogenesis is very significant. In fact, when we require the reheating temperature should be $T_R \lesssim 10^8 \text{ GeV}$ to avoid the gravitino problem [7], we need very a large value of $M$, say, $M \gtrsim 3 \times 10^{21} \text{ GeV}$, in order to obtain the desired lepton asymmetry $n_L / s \simeq 10^{-10} – 10^{-9}$. This large value $M$ corresponds to an extremely small neutrino mass $m_{\nu_1} \lesssim 10^{-8} \text{ eV}$.

5 Large effective masses for right-handed neutrinos

Let us discuss the origin of the operators in Eq. (1) in the presence of the heavy right-handed Majorana neutrinos $N_I$ ($I = 1, 2, 3$). The relevant superpotential is
\begin{equation}
W = h_{\nu_{iI}} N_I L_i H_u + \frac{1}{2} M_{RI} N_I N_I , \tag{38}
\end{equation}
where we have used a base where the mass matrix for the heavy neutrinos $N_I$ is diagonal. Then, through the see-saw mechanism the heavy Majorana neutrinos $N_I$ induce the effective operators in Eq. (1) at low energies,
\begin{equation}
W = \frac{1}{2} h_{\nu_{iI}} M_{RI} L_i H_u L_j H_u . \tag{39}
\end{equation}
These operators gives light neutrino masses as
\begin{equation}
(m_{\nu})_{i,j} = v_u^2 h_{\nu_{iI}} h_{\nu_{jI}} M_{RI} . \tag{40}
\end{equation}
It is quite natural to consider that the Majorana masses $M_{RI}$ for $N_I$ are given by a vacuum expectation value (vev) of some field $X$, which is a singlet under the standard-model gauge groups, with the superpotential

$$W = \frac{1}{2} h_{N_I} X N_I N_I. \quad (41)$$

If it is the case, masses of the heavy Majorana neutrinos $N_I$ in the early universe may be different from those in the present universe. Namely, the extremely small neutrino mass discussed in the previous section ($m_{\nu_1} \lesssim 10^{-8}$ eV) may not be the same as the observed mass. In particular, when the field $X$ is responsible to the Peccei-Quinn symmetry breaking \[11\], we have necessarily a flat direction containing the $X$ field. Thus, the $X$ field may have a large value as $X \simeq M_*$ during the inflation whereas it has a vev of $\langle X \rangle \simeq F_a$ in the true vacuum. Here, the Peccei-Quinn breaking scale $F_a$ is constrained by laboratory experiments, astrophysics, and cosmology as $F_a \simeq 10^{10} - 10^{12}$ GeV \[12, 13\].

In this case, neutrino masses in the early universe are much smaller than those observed today (they are suppressed by the factor $X/F_a \simeq M_*/F_a \simeq 10^6 - 10^8$). Therefore, the extremely small neutrino mass required for a successful leptogenesis may be naturally explained in this scenario.

To demonstrate our point, we consider the following superpotential for the Peccei-Quinn symmetry breaking sector,

$$W = \lambda Y (X \overline{X} - F_a^2), \quad (42)$$

where $\lambda$ is a coupling constant, and $Y$, $X$, and $\overline{X}$ are supermultiples which are singlets under the standard-model gauge groups and have 0, +1, and −1 Peccei-Quinn charges, respectively. From Eq. (42) we see that there exists a flat direction $X \overline{X} = F_a^2$. We parameterize this direction by the scalar field $\sigma$ called “saxion”. This saxion $\sigma$ receives a soft SUSY breaking mass of $m_\sigma \simeq m_{3/2}$ ($m_{3/2}$ is the gravitino mass), and the $X$ and $\overline{X}$ have vevs of the order of $F_a$ if the soft SUSY breaking masses for $X$ and $\overline{X}$ are almost the same. ($\mathcal{L} = m_X^2 |X|^2 + m_{\overline{X}}^2 |\overline{X}|^2$ with $m_X \simeq m_{\overline{X}} \simeq m_{3/2}$).

Along this flat direction the $X$ field may have a large value during the inflation, if the additional SUSY breaking effects induce a negative mass squared for $X$. In this case the supergravity effects prevent the $X$ field from running over $M_*$ and give an initial value $X \simeq M_* \quad [21]$. We here assume $X \simeq M_*$ and hence $\overline{X} \simeq F_a^2/M_*$ at the end of inflation.\[6\]\[7\]

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^6 The upper bound on $F_a$ is relaxed as $F_a \lesssim 10^{16}$ GeV, when there is an extra entropy production after the QCD phase transition \[4\].

^7 In this case, the isocurvature fluctuation associated with the axion is very suppressed \[21\].
We turn to calculate the produced lepton asymmetry in the presence of the $X$ field. We see from Eq. (41) that the masses for $N_I$ are

$$M_{RI} = h_{NI}X.$$  \hspace{1cm} (43)

Then, we find

$$ (m_\nu)_{i,j} = \frac{v_\nu^2}{X} h_{\nu_i}^T h_{NI}^{-1} h_{\nu_{I,j}}.$$ \hspace{1cm} (44)

The vev of $X$ in the true vacuum, $\langle X \rangle \simeq F_a$, is supposed to give the neutrino masses observed today. As discussed in the previous sections, the lightest neutrino $\nu_1$ ($m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$) is relevant for the AD leptogenesis.

As for the neutrino masses we adopt the recent data from the Superkamiokande experiments. The atmospheric neutrino oscillation is well explained by the $\nu_\mu - \nu_\tau$ oscillation with $m_{\nu_3} \simeq 5 \times 10^{-2}$ eV \cite{2}. On the other hand, the large MSW solution to the solar neutrino problem is favored by the recent Superkamiokande data \cite{12}, which indicates that $m_{\nu_2} \simeq 5 \times 10^{-3}$ eV. However, it is very difficult to determine the lightest neutrino mass $m_{\nu_1}$. Here, we simply take $m_{\nu_1} = 1 \times 10^{-4}$ eV for a representative value.

Then, the scale $M$ in the previous sections is estimated as

$$M = 3 \times 10^{17} \text{ GeV} \left( \frac{10^{-4} \text{ eV}}{m_{\nu_1}} \right) \left( \frac{\langle X \rangle}{F_a} \right).$$ \hspace{1cm} (45)

Notice that as shown in Fig. 4 such a scale $M$ for $\langle X \rangle \simeq F_a$ can not produce sufficient lepton asymmetry to explain the baryon asymmetry in the present universe. However, the large initial value of $X$ ($X_0 \simeq M_*$) gives

$$M = 7 \times 10^{25} \text{ GeV} \left( \frac{10^{-4} \text{ eV}}{m_{\nu_1}} \right) \left( \frac{10^{10} \text{ GeV}}{F_a} \right) \left( \frac{X_0}{M_*} \right),$$ \hspace{1cm} (46)

which is translated into the lightest neutrino mass in the false vacuum

$$m_{\nu_1}' = 4 \times 10^{-13} \text{ eV} \left( \frac{m_{\nu_1}}{10^{-4} \text{ eV}} \right) \left( \frac{F_a}{10^{10} \text{ GeV}} \right) \left( \frac{M_*}{X_0} \right).$$ \hspace{1cm} (47)

Thus, such a high scale $M$ in Eq. (46) can give a neutrino mass much smaller than that observed today. It should be noted that the flat direction saxion $\sigma$ stays at $X_0 \simeq M_*$ until $H \simeq m_\sigma \simeq m_{3/2}$ due to the friction of the expansion of the universe. Furthermore, we find from Eqs. (35) and (36) that for such a high scale of $M \simeq 10^{26}$ GeV we can estimate the lepton asymmetry without the thermal effects even if the reheating temperature is as high as $T_R \simeq 10^8$ GeV.
Therefore, for the reheating temperature of $T_R \lesssim 10^8$ GeV, the produced lepton asymmetry is estimated by using Eq. (28) as

$$
\frac{n_L}{s} = \frac{T_R}{12 M_*^2} \left( \frac{v_u^2 X_0}{m_{\nu_1} F_a} \right) \delta_{\text{eff}}
= 1 \times 10^{-4} \delta_{\text{eff}} \left( \frac{T_R}{10^8 \text{ GeV}} \right) \left( \frac{10^{-4} \text{ eV}}{m_{\nu_1}} \right) \left( \frac{10^{10} \text{ GeV}}{F_a} \right) \left( \frac{X_0}{M_*} \right).
$$

(48)

This shows that too large amount of the lepton asymmetry is produced through the AD mechanism. However, this asymmetry is sufficiently diluted, since there exists substantial entropy production at low energies. The saxion $\sigma$ is produced in the coherent oscillation after the lepton asymmetry is frozen, and the oscillation energy dominates the universe.\(^8\)

Then, the late decay of the saxion increases the entropy of the universe and dilutes the lepton (baryon) asymmetry substantially.

The saxion begins the coherent oscillation at $H \simeq m_{\sigma} \simeq m_{3/2}$ with the initial amplitude $X_0 \simeq M_*$. The oscillation energy at that time is given by $\rho_{\sigma} \simeq m_{\sigma}^2 X_0^2/2$. Note that the saxion oscillation starts before the reheating process of the inflation takes place.\(^9\)

Then, when $T = T_R$, the ratio of $\rho_{\sigma}$ to the entropy density of the universe is estimated as

$$
\frac{\rho_{\sigma}}{s} = \frac{1}{8} T_R \left( \frac{X_0}{M_*} \right)^2.
$$

(49)

Notice that $\rho_{\sigma}$ decreases at the rate $R^{-3}$ as the universe expands, while the radiation energy density decreases as $R^{-4}$. Therefore, the oscillation energy of the saxion dominates the energy of the universe soon after the reheating process completes. This energy is transferred into thermal bath by the saxion decay. The saxion decays into two gluons with the partial decay rate

$$
\Gamma(\sigma \to 2g) = \frac{\alpha_s^2}{32 \pi^3} \frac{m_{\sigma}^3}{F_a^2}.
$$

(50)

Through this decay the universe is reheated again, and its reheating temperature $T_{\sigma}$ is estimated as

$$
T_{\sigma} = 10 \text{ GeV} \left( \frac{m_{\sigma}}{1 \text{ TeV}} \right)^{3/2} \left( \frac{10^{10} \text{ GeV}}{F_a} \right).
$$

(51)

\(^8\) The saxions are also produced by the thermal scatterings in the reheating period. However, the energy density of the saxions produced thermally is much smaller than that produced in the coherent oscillations, and hence we neglect it.

\(^9\) It is the case for $T_R \lesssim 2 \times 10^{10}$ GeV$(m_{\sigma}/1 \text{ TeV})^{1/2}$. 

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The saxion decay takes place far before the beginning of the big-bang nucleosynthesis (BBN), and hence is cosmologically harmless. It should be noted that the saxion might decay dominately into two axions. If it is the case, the extra energy of the axion at the BBN epoch raises the Hubble expansion of the universe, which leads to overproduction of $^4\text{He}$. To avoid this difficulty the branching ratio of the saxion decay into two axion should be smaller than about 0.1. Here, we simply assume that the dominant decay process of saxion is $\sigma \rightarrow 2g$. Then the saxion decay increases the entropy of the universe by the rate

$$\Delta = \frac{T_R}{6T_\sigma} \left( \frac{X_0}{M_*} \right)^2 = 2 \times 10^6 \left( \frac{T_R}{10^8 \text{ GeV}} \right) \left( \frac{1 \text{ TeV}}{m_\sigma} \right)^{3/2} \left( \frac{F_a}{10^{10} \text{ GeV}} \right) \left( \frac{X_0}{M_*} \right)^2 . \quad (52)$$

Because of this entropy production by the saxion decay, the primordial lepton asymmetry shown in Eq. (48) is also diluted by the rate $\Delta$. Then the present baryon asymmetry is given by

$$\frac{n_B}{s} = 4 \times 10^{-4} \delta_{\text{eff}} \frac{v_u^2 m_\nu_1^3}{f_a^2 F_a^2 M_*^{1/2}} \left( \frac{M_*}{X_0} \right) = 0.2 \times 10^{-10} \delta_{\text{eff}} \left( \frac{10^{-4} \text{ eV}}{m_\nu_1} \right) \left( \frac{10^{10} \text{ GeV}}{F_a} \right)^2 \left( \frac{m_\sigma}{1 \text{ TeV}} \right)^{3/2} \left( \frac{M_*}{X_0} \right) . \quad (53)$$

Notice that the present baryon asymmetry is independent on the reheating temperature $T_R$. We see that the desired baryon asymmetry is just given by the lightest neutrino mass of $m_\nu_1 \simeq 10^{-4} \text{ eV}$ for $F_a \simeq 10^{10} \text{ GeV}$.

Before closing this section, we should comment on the cosmological consequence of our model. The entropy production by the saxion in Eq. (52) ensures that we are free from the cosmological gravitino problem. The number density of gravitinos produced at the reheating process is diluted by the rate $\Delta$ and hence the radiative decay of gravitino does not disturb the BBN. Furthermore, we find that the model does not suffer from the cosmological problem of the axino which is a fermionic superpartner of the axion. The axinos are produced in the reheating process by the thermal scatterings and may lead to a cosmological difficulty [22]. However, in our model, the interaction of axino at the reheating epoch is suppressed by $X_0 \simeq M_*$, not by $F_a$, and hence the production of axino is less effective. Furthermore, the entropy production by the saxion decay dilutes the axino abundance and hence the axino becomes completely cosmologically harmless.

\footnote{This is realized when $m_X^2 \simeq m_\sigma^2$.}
6 Conclusions and discussion

In this paper we have performed a detailed analysis on the Affleck-Dine leptogenesis taking into account the thermal effects from the dilute plasma. We have first shown that the thermal effects change drastically the dynamics of the flat direction $\phi$ and suppress the lepton-number asymmetry produced by the flat direction $\phi$. In order to escape from the thermal effects, a relatively low reheating temperature of $T_R \lesssim 10^5$–$10^6$ GeV is found to be required when the lightest neutrino mass is $m_{\nu_1} \gtrsim 10^{-9}$ eV. This is consistent with the result obtained in Ref. [9]. On the other hand, we have found that this upper bound on $T_R$ is relaxed as $T_R \lesssim 10^6$ GeV $\times (m_{\nu_1}/10^{-9}$ eV)$^{-1}$ for the lighter neutrino mass region of $m_{\nu_1} \lesssim 10^{-9}$ eV. We have also estimated the resultant lepton asymmetry for the higher reheating temperature region where the thermal effects are important by both analytical and numerical calculations. We have found that an ultralight neutrino with a mass such as $m_{\nu_1} \simeq 10^{-8}$ eV is required to produce enough lepton asymmetry to account for the baryon asymmetry in the present universe. Here, we have assumed that the reheating temperature should be lower than about $10^8$ GeV to avoid the cosmological problem of the gravitino of mass $m_{3/2} \simeq 100$ GeV–1 TeV. Such an ultralight neutrino seems to be very unlikely the case, since the recent Superkamiokande experiments [2, 12] suggest the masses of heavier two neutrinos $\nu_2$ and $\nu_3$ to be in a range of $10^{-1}$–$10^{-3}$ eV.

However, in the second part of this paper, we have pointed out that the above neutrino mass $m_{\nu_1} \simeq 10^{-8}$ eV is not necessarily the mass to be observed today, if the heavy Majorana masses of the right-handed neutrinos are dynamical valuables in the early universe. We construct a model based on the Peccei-Quinn symmetry to demonstrate our point. We find in this model that the neutrino mass $m_{\nu_1}$ in the true vacuum can be as large as $m_{\nu_1} \simeq 10^{-5}$–$10^{-4}$ eV to obtain $n_B/s \simeq 10^{-10}$–$10^{-11}$ for the reheating temperature of $T_R \lesssim 10^8$ GeV. Therefore, we have shown that the AD leptogenesis works well as long as the masses for the right-handed neutrinos are dynamical valuables in the early universe.

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References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B174 (1986) 45.

[2] Y. Fukuda et al., Superkamiokande Collaboration, Phys. Lett. B433 (1998) 9; Phys. Lett. B436 (1998) 33; Phys. Rev. Lett. 81 (1998) 1562.

[3] See, for a recent review, W. Buchmüller and M. Plümacher, Phys. Rep. 320 (1999) 329; M. Plümacher, Nucl. Phys. B530 (1998) 207, and references there in.

[4] K. Kusukawa, T. Moroi, and T. Yanagida, Prog. Theor. Phys. 92 (1994) 437;
G. Lazarides, hep-ph/9904428 and reference therein;
G.F. Giudice, M. Peloso, A. Riotto, and I. Tkachev, JHEP 9908 (1999) 014;
T. Asaka, K. Hamaguchi, M. Kawasaki, and T. Yanagida, Phys. Lett. B464 (1999) 12; Phys. Rev. D61 (2000) 083512.

[5] B.A. Campbell, S. Davidson, and K.A. Olive, Nucl. Phys. B399 (1993) 111;
H. Murayama and T. Yanagida, Phys. Lett. B322 (1994) 349;
H. Murayama, H. Suzuki, Y. Yanagida, and J. Yokoyama, Phys. Rev. Lett. 70 (1993) 1912, Phys. Rev. D50 (1994) 2356;
T. Moroi and H. Murayama, JHEP 0007 (2000) 009.

[6] I. Affleck and M. Dine, Nucl. Phys. B249 (1985) 361.

[7] M.Y. Khlopov and A.D. Linde, Phys. Lett. B138 (1984) 265;
J. Ellis, J.E. Kim, and D.V. Nanopoulos, Phys. Lett. B145 (1984) 181;
M. Kawasaki and T. Moroi, Prog. Theor. Phys. 93 (1995) 879;
see also, for example, a recent analysis, E. Holtmann, M. Kawasaki, K. Kohri, and T. Moroi, Phys. Rev. D61 (1999) 023506.

[8] R. Kallosh, L. Kofman, A. Linde, and A.V. Proeyen, Phys. Rev. D61 (2000) 103503;
G.F. Giudice, A. Riotto, and I. Tkachev, JHEP 9908 (1999) 009.

[9] M. Dine, L. Randall, and S. Thomas, Nucl. Phys. B458 (1996) 291.

[10] R. Allahverdi, B.A. Campbell, and J. Ellis, Nucl. Phys. B579 (2000) 355.

[11] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440, Phys. Rev. D16 (1977) 1791.

[12] Y. Suzuki, talk presented at XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June, 2000

[13] H. Murayama and T. Yanagida in references 3.
[14] T. Yanagida, in Proc. Workshop on the unified theory and the baryon number in the universe, (Tsukuba, 1979), eds. O. Sawada and S. Sugamoto, Report KEK-79-18 (1979);
M. Gell-Mann, P. Ramond, and R. Slansky, in "Supergravity" (North-Holland, Amsterdam, 1979) eds. D.Z. Freedman and P. van Nieuwenhuizen.

[15] T. Moroi and H. Murayama in references [3].

[16] See, for example, E. Kolb and M. Turner, The Early Universe (Addison-Wisley, 1990).

[17] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, Phys. Lett. B155 (1985) 36.

[18] S.Y. Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. 308 (1988) 885,
J.A. Harvey and M.S. Turner, Phys. Rev. D42 (1990) 3344.

[19] See, for a review, J.E. Kim, Phys. Rep. 150 (1987) 1.

[20] G. Lazarides, R. Shaefer, D. Seckel, and Q. Shafi, Nucl. Phys. B346 (1990) 193;
M. Kawasaki, T. Moroi, and T. Yanagida, Phys. Lett. B383 (1996) 313.

[21] S. Kasuya, M. Kawasaki, and T. Yanagida, Phys. Lett. B409 (1997) 94; Phys. Lett. B415 (1997) 117.

[22] K. Rajagopal, M.S. Turner, and F. Wilczek, Nucl. Phys. B358 (1991) 447.