Calculation and analysis of whiplash effect in multi-DOF system

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Abstract. The response of structural vibration depends on the degree of freedom, mass, stiffness, external effect of the structure. Natural frequency of structure is its inherent property, also it is related to its own factors, but not related to the external factors. Firstly the nature and characteristics of resonance effect and whiplash effect are analyzed. Secondly whiplash effect of multi-degree freedom system is analyzed, then orthogonality in main modes is used to verify the results of calculation. Then the improvements of whiplash effect are proposed, also the improved models are analyzed. Finally the conclusions are made.

1 Introduction
Resonance effect and whiplash effect in the structure are usually happened during vibration, and this is very harmful to the structure. We should avoid resonance effect and whiplash effect when the structure is designed, also we must take precaution to avoid the resonance effect and whiplash effect. Not only the contact, but also the difference are existed between resonance effect and whiplash effect. The contact is that the effects may cause strenuous exercise and large displacement in the structure. However, when the external frequency of structure is equal to or close to its natural frequency, resonance effect which cause strenuous exercise and large displacement in the structure may happen. When the mass and stiffness on top of the structure shrink suddenly, whiplash effect which cause strenuous exercise and large displacement in the structure may happen. The difference is that resonance effect may happen when the structure is forcing to vibrate, but whiplash effect may happen when the structure is forcing to vibrate or free to vibrate.

In our daily life, whiplash effect is a very common phenomenon. When we grip the root of whip and throw it off energetically, it will move accordingly with our hands. The tip of whip will move sharply, and the displacement and movement will increase. Whiplash effect has been explained in Stanrdard for Terminolog in Earthquake Engineering: “In the earthquake, the top of the elongate projection in tall buildings and other structures will vibrate intensely.” Since the mass and stiffness on top of the structure are both little, the first natural frequency is close to the vibration frequency of earth or natural frequency in mian structure, so whiplash effect may happen. That’s the nature of whiplash effect. In the earthquake, whiplash effect will cause damage easily in the parapet, tower building, attic, etc.
2 Calculation and analysis of whiplash effect in multi-DOF system

2.1 Multi-DOF System Model
A three-freedoms-system is analyzed in the model 1, and model 1 is the example. Model 1 is showed by Figure 1. In Figure 1, m is the concentrated mass of every frame, the stiffness of beam is supposed to unlimited; k is the lateral stiffness of floor in the frame, also it is meaning of the force required to be applied when the single displacement is taken place; n is the adjustment factor of mass and stiffness.

2.2 Calculation of Free Vibration in Multi-DOF System
When $n=1$, $n=0.1$, $n=10$, free vibrate of frame is analyzed, the conclusions is made in Table 1, Table 2, Table 3, Table 4. In the Table, $\omega_1$ is the first natural frequency of the vibrate system, $\omega_2$ is the second natural frequency of the vibrate system, $\omega_3$ is the third natural frequency of the vibrate system; $Y_1$ is the displacement amplitude of the first floor in the vibrate system, $Y_2$ is the displacement amplitude of the second floor in the vibrate system, $Y_3$ is the displacement amplitude of the third floor in the vibrate system.

2.3 Analysis of Whiplash Effect in Multi-DOF System
Firstly, the conclusion is made in Table 1: natural frequency of multi-degree of freedoms system is related with the adjustment factor of mass and stiffness, the step of vibrate system, mass factor, stiffness factor. Secondy, we can make conclusions according to the Table 2, Table 3, Table 4:

When $n=1$, mass and stiffness of structure distribute evently along vertical direction, also the displacement amplitude are changing evently.

When $n=0.1$, although the frontal two-step natural frequency distribute evently along vertical direction, the displacement amplitude of third-step natural frequency in the structure system increase significantly, when $Y_3=9.923$, the absolute value is approximately ten times as much displacement amplitude as the other floors below. It indicates that whiplash may happen when mass and stiffness of multi-degree of freedoms system reduce suddenly in third-step natural frequency.

When $n=10$, mass and stiffness at the top of structure is ten times as much as the other floors below, it is impossible in the engineering practice, but mass and stiffness at the top of structure change suddenly. Displacement amplitude on the second floor where mass and stiffness change suddenly increase obviously when it is at second natural frequency, at the same time, $Y_2=10.008$, the absolute value is approximately ten times as much displacement amplitude as the other floors below. It indicates that the whiplash effect may happen at the junction where changes suddenly, when mass and stiffness at the top of structure increase suddenly. Otherwise, when it is at third natural frequency, the displacement amplitude on the first floor is different from the displacement amplitude other floors as twenty times as much. The accident when mass and stiffness change suddenly cause displacement amplitude changes obviously.
Table 1 Relationship between adjustment factor and natural frequency

| n   | ω1     | ω2     | ω3     |
|-----|--------|--------|--------|
| n=1 | \(0.445\sqrt{\frac{k}{m}}\) | \(1.802\sqrt{\frac{k}{m}}\) | \(1.247\sqrt{\frac{k}{m}}\) |
| n=0.1 | \(0.586\sqrt{\frac{k}{m}}\) | \(1.632\sqrt{\frac{k}{m}}\) | \(1.045\sqrt{\frac{k}{m}}\) |
| n=10 | \(0.207\sqrt{\frac{k}{m}}\) | \(3.465\sqrt{\frac{k}{m}}\) | \(1.396\sqrt{\frac{k}{m}}\) |

Table 2 Relationship between displacement amplitude and natural frequency when n=1

| Y   | ω1     | ω2     | ω3     |
|-----|--------|--------|--------|
| \(Y_1\) | 1      | 1      | 1      |
| \(Y_2\) | 1.802  | -1.247 | 0.445  |
| \(Y_3\) | 2.247  | 0.555  | -0.802 |

Table 3 Relationship between displacement amplitude and natural frequency when n=0.1

| Y   | ω1     | ω2     | ω3     |
|-----|--------|--------|--------|
| \(Y_1\) | 1      | 1      | 1      |
| \(Y_2\) | 1.656  | -0.665 | 0.908  |
| \(Y_3\) | 2.524  | 0.399  | -9.923 |
Table 4 Relationship between displacement amplitude and natural frequency when \( n=10 \)

| \( Y \) | \( \omega \) | \( \omega_1 \) | \( \omega_2 \) | \( \omega_3 \) |
|-------|-------|-------|-------|-------|
| \( Y_1 \) | 1     | 1     | 1     |       |
| \( Y_2 \) | 1.957 | -10.008 | 0.051 |       |
| \( Y_3 \) | 2.045 | 0.909  | -0.054 |       |

2.4 Verification of Orthogonal Property in Main Model

It is important that the models are orthogonal when they are vibrating in multi-degree of freedoms system, which is orthogonal of models. The Formulas are showed by Formula 1, Formula 2, Formula 3, Formula 4, Formula 5, Formula 6, Formula 7, Formula 8:

\[
Y^{(l)T}MY^{(k)} = 0 \quad (1)
\]

\[
Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} \quad (2)
\]

\[
M = \begin{bmatrix} m_1 \\ \vdots \\ m_n \end{bmatrix} \quad (3)
\]

In Formula 1, Formula 2, Formula 3, \( Y \) is displacement amplitude vector of multi-degree of freedoms system, where \( l \neq k \), \( \omega_l \neq \omega_k \). \( M \) is mass array of multi-degree of freedoms system. Any displacement amplitude vector and mass array are satisfied with orthogonal property. For example, when \( n=0.1 \), orthogonal property of main model in multi-degree of freedoms system is verified below:

\[
M = \begin{bmatrix} m \\ 0.1m \end{bmatrix} \quad (4)
\]

\[
Y^{(\alpha)} = \begin{bmatrix} 1 \\ 0.908 \\ -9.923 \end{bmatrix} \quad (5)
\]

\[
Y^{(\alpha)T}MY^{(\alpha)} \approx 0 \quad (7)
\]

The orthogonal property of other model can be verified, and be satisfied with orthogonal property. Similarly, the orthogonal property of second model can be deduced below:

\[
Y^{(l)T}KY^{(k)} = 0 \quad (8)
\]
3 Improvement measures
To prevent whiplash effect in the structure system from damaging it, the layout of structure should keep regular shape, also the facade stiffness of structure should keep consistency or change uniformly. When $n=0.1$, the facade stiffness of structure is improved to change uniformly along the facade, the improved model is showed by Figure 2. The free vibration of improved frame is analyzed, the conclusions is made to be showed by Table 5 and Table 6. When $n=0.1$, the natural frequency of the original model and improved model is nearly same, but displacement amplitude during floors of improved model is less than displacement amplitude during floors of original model. Otherwise, displacement amplitude of third natural frequency at the top is less than original model. It indicates that whiplash effect of improved model is reducing because of the facade stiffness which has no changing, and which is reducing from top to bottom.

![Figure 2 Model 2](image)

### Table 5 Natural frequency of improved model

| $\omega_1$ | $\omega_2$ | $\omega_3$ |
|------------|------------|------------|
| $0.637 \sqrt{\frac{k}{m}}$ | $1.465 \sqrt{\frac{k}{m}}$ | $1.071 \sqrt{\frac{k}{m}}$ |

### Table 6 Relationship between displacement amplitude and natural frequency in the improved model

| $Y$ | $\omega_1$ | $\omega_2$ | $\omega_3$ |
|-----|------------|------------|------------|
| $Y_1$ | 1          | 1          | 1          |
| $Y_2$ | 2.188      | -1.294     | 0.706      |
| $Y_3$ | 3.684      | 1.128      | -4.812     |

4 Conclusion
In summary, in order to prevent whiplash effect from happening in the structure system, the designers should pay attention to the regular in the plane, facade and vertical cross-section, which would affect seismic performance and economic rationality. The lateral stiffness and bearing capacity should not change suddenly, the unanimous and similar lateral stiffness of structuer system should be preferred. Since
earthquake causes the large forcing at the bottom of structure, damping and seismic isolation should be adopted in the structure.

Acknowledgments
This research is financially supported by the 2015 Research Project of Chengdu Textile College (Grant No.2015fzlkc06).

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