Local sunspot oscillations and umbral dots

Yuzef Zhugzhda\textsuperscript{1} and Robert Sych\textsuperscript{2}

\textsuperscript{1}IZMIRAN, Moscow 142092, Russia; YZhugzhda@mail.ru
\textsuperscript{2}Institute of Solar-Terrestrial Physics SB RAN, Irkutsk, Russia

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Abstract Data analysis of sunspot oscillations based on a 6-hr SDO run of an observation showed that low frequency (0.2 < \(\omega\) < 1 mHz) oscillations are locally similar to three and five minute oscillations. The oscillations in the sunspot are concentrated in cells of a few arcsec, each of which has its own oscillation spectrum. The analysis of two scenarios for sunspot oscillations leads to a conclusion that local sunspot oscillations occur due to a subphotospheric resonator for slow MHD waves. Empirical models of a sunspot atmosphere and the theory of slow waves in thin magnetic flux tubes are applied to modeling the subphotospheric resonator. The spectrum of local oscillations consists of a great number of lines. This kind of spectrum can occur only if the subphotospheric resonator is a magnetic tube with a rather weak magnetic field. Magnetic tubes of this sort are umbral dots that appear due to the convective tongues in monolithic sunspots. The interrelation of local oscillations with umbral dots and wavefronts of traveling waves in sunspots is discussed.

Key words: Sun: sunspot — methods: numerical — waves

1 INTRODUCTION

Sunspot oscillations were discovered by Beckers & Schultz (1972). They were so-called three minute oscillations. Sunspot oscillations are observed in the photosphere, chromosphere and corona. Periods of sunspot oscillations are from minutes to hours. Enormous amounts of related observational data have been stored. Ground-based and space observations have been used to investigate sunspot oscillations. Until recently, those were mainly ground-based observations (see reviews Bogdan & Judge (2006) and Khomenko & Collados (2015)). Ground-based observations allow us to apply many different methods for investigating oscillations, but typically suffer from a lack of frequency resolution due to the short time series of observations. Following launch of the Solar Dynamics Observatory (SDO), hours-long observations of sunspot oscillations became available. These observations, along with the Nobeyama radioheliograph data, augmented the stock of knowledge on oscillation properties (Sych 2016).

The sunspot oscillation spectrum is very wide. It comprises a great number of lines. The spectrum of the so-called three-minute oscillations involves tens of lines (Reznikova et al. 2012; Zhugzhda & Sych 2014). Besides, there are five-minute sunspot oscillations, as well as long-period (0.2 < \(\omega\) < 1 mHz) and super long-period (\(\omega\) < 0.2 mHz) oscillations (Bakunina et al. 2013). Another relevant feature of sunspot oscillations is their spatial non-uniformity. Centeno et al. (2005) revealed that the waves propagate only inside channels of subarcsecond width whereas the rest of the umbra remains nearly at rest. Numerous observations show that oscillations are localized in small spots in the umbra (Jess et al. 2012; Zhugzhda & Sych 2014; Krishna Prasad et al. 2015; Chae et al. 2017; Sych 2016), which we call local oscillations. In addition, there are non-local oscillations in sunspots. Namely, there are traveling waves that propagate through the sunspot umbra to penumbra (Zirin & Stein 1972; Giovanelli 1972; Sych & Nakariakov 2014; Su et al. 2016). Jess et al. (2017) claim that there are non-local three-minute oscillations with the \(m = 1\) slow magneto-acoustic mode.
All this challenges the theory to explain both local and nonlocal oscillations of sunspots. Indeed, there are two approaches to solve this problem. One approach is related to the assumption about the occurrence of resonance phenomena in the sunspot atmosphere. Within this approach, the oscillation spectrum is considered to be the spectrum of some resonators in the sunspot atmosphere. Within the resonance approach, the chromospheric resonator had been addressed for years (Zhugzhda 1981, 2008; Botha et al. 2011), because it predicted a rather complex spectrum of three-minute oscillations. However, the real spectrum appeared to be much more complex (Reznikova et al. 2012; Zhugzhda & Sych 2014) than could be explained within the chromospheric resonator model. The other approach is based on the fact that a sunspot is surrounded by a quiet solar atmosphere, in which there are p-mode oscillations of different frequencies. These oscillations should penetrate into a sunspot and convert into magnetogravity waves. In this case, the sunspot oscillation spectrum is determined by the spectrum of the arriving oscillations. To study the effect of p-mode oscillations on sunspot oscillations, great efforts have been undertaken in this research area (Khomenko & Collados 2015; Felipe & Khomenko 2017). Additionally, there are super-low frequency sunspot oscillations (Chorley et al. 2010; Bakunina et al. 2013) having absolutely another nature than that discussed above (Solov’ev & Kirichek 2014).

Zhugzhda & Sych (2014) studied three-minute sunspot oscillations that are known to represent slow magnetohydrodynamic waves. Two scenarios were addressed for the origin of slow oscillations as a result of wave arrival from sunspot subphotospheric layers. Non-local sunspot oscillations appeared to be excited by p-modes, whereas local oscillations may be excited only by slow waves arriving from sunspot subphotospheric layers. Thereby, the assumption that local and non-local oscillations have different natures enables uniting the above two approaches in oscillation theory.

Zhugzhda & Sych (2014) arrived at a conclusion that a slow-wave subphotospheric resonator may be the source of the slow waves responsible for local three-minute oscillations. From this hypothesis, some conclusions follow that we attempt to analyze in this paper. The spectrum of local three-minute oscillations represents a set of a larger number of closely spaced lines. Such a spectrum may be produced, as long as the subphotospheric resonator’s fundamental frequency is sufficiently low. In this case, the spectrum of three-minute oscillations should comprise upper oscillation harmonics. Besides, one should observe not only three-minute local oscillations, but also low-frequency local oscillations. This paper intends to demonstrate the existence of such oscillations.

Another important aspect is building a simple numerical model that describes a slow-wave subphotospheric resonator. The presence of localized sunspot oscillations should be related to some fine-scale structures in the sunspot atmosphere. An obvious candidate for this role is umbral dots. A separate section of this paper compares the parameters of our simple model for the resonator with the parameters of convective tongues that, according to present-day ideas, are responsible for the origin of umbral dots.

In addition, we consider it necessary to briefly address the phenomenon of sunspot wavefronts that, by the proposed classification, are a typical representative of non-local oscillations which should be excited by p-modes. Indeed, this is corroborated by simulations (Felipe & Khomenko 2017). However, these simulations and observations (Chmielewski et al. 2016; Zhugzhda & Sych 2014; Sych et al. 2010) show that the sources of wavefronts should be local. Thereby, there begs a question, whether local oscillations may be the source of divergent sunspot wavefronts. We discuss this possibility in the final section of the paper.

The paper is organized as follows. First of all we show that the low-frequency and five-minute oscillations, like the three-minute case, are local oscillations. Then scenarios for three-minute oscillations are generalized to the case of low-frequency oscillations. The theory of slow waves in thin flux tubes is described. Empirical models of a sunspot atmosphere are used to develop a simple model of a subphotospheric resonator for slow waves. The connection of local oscillations with umbral dots and wavefronts is discussed.

2 OBSERVATIONS

For our analysis of local oscillations in sunspots, we selected a single, symmetric sunspot from NOAA Active Region 11131. This group was passing through the northern hemisphere of the solar disk in December 2010. We studied the time period 2010 December 8 00:00–06:00 UT, when the group was crossing the central meridian. A time cube of the sunspot images obtained with the SDO/AIA in ultraviolet (UV) at the upper chromosphere/transition region level (HeII line, 304Å, $T = 80\,000 \,K$) was used. The pixel size was 0.6″. The data were obtained at a 12-sec cadence. The observation du-
ration was 6 h, which allowed us to study oscillations within the period range 0.5–120 min.

Figure 1 shows the sunspot at 02:30 UT, with the umbra and penumbra boundaries in white light superimposed like contours. The umbra size is 25′′, with the penumbra being 50′′. The brightness is presented in a logarithmic scale. To obtain the images, we used the SDO/AIA \url{http://www.lmsal.com/get_aia_data/} resource that allows us to obtain Level 1 calibrated images for different wavelengths within the given time period. The source selection was manual by assigning the active region center coordinates and the width and height of the site in arcsec. Differential rotation of the given object during the observational time was removed by introducing an integer shift through the algorithm implemented at the website. The data consist of 1800 images with a time lapse of 12 seconds. We investigated the square site that covers 9 × 9 pixels, located in the center of the sunspot. The temporal variations in UV brightness were studied separately for each pixel in this square site.

3 PROPERTIES OF SUNSPOT OSCILLATIONS

Our analysis of the observational data is aimed at studying whether sunspot oscillations are local or not. These oscillation properties, in our view, are key to the theory of sunspot oscillation.

3.1 Properties of Oscillation Spectral Lines

Plots in Figure 2(a,b) present the amplitude spectrum for oscillations in the pixel under the site being studied with coordinates (3,4) where, within the investigated region, the largest number of amplitude oscillations is observed in the sunspot center. In order to obtain a spectrum of oscillations, Fourier transformations with and without zero padding were used. Blue squares on the graphs in Figure 2 are the amplitude spectrum obtained by applying fast Fourier transform to the original time series after zero padding up to $2^{11}$. Spectral lines in the spectrum appear so narrow that each line accounts for only two or three squares on plots of Figure 2, which makes it impossible to reproduce the line profile properly and to determine the position of the line maximum. To overcome this drawback, the time series was extended from 1800 points to $2^{15}$ by zero padding and the fast Fourier transform was applied to the extended time series. Results are shown by red lines in Figure 2(a,b) and by red circles in Figure 2(c,d).

The spectrum of oscillations comprises dozens of closely located spectral lines. This property of the spectrum became obvious due to a long time series, which provided high spectral resolution. Former ground-based observations about one-hour long enabled resolving only a few lines in the oscillation spectrum of sunspot oscillations.

Figure 2(c,d) displays two small segments of the amplitude spectrum, including a few spectral lines each. Figure 2(c,d) shows that the spectral lines are very narrow which is essential to the theory of oscillations. These plots allow us to determine the half-width of the spectral line with a high accuracy. Because the amplitude spectra are presented in these plots, the half-width should be determined by the line width at Level 1/$\sqrt{2}$. The half-width of the lines appears about $\sim 0.03 - 0.05$ mHz, which corresponds to the characteristic time of the oscillation damping, $\sim 9$ h, which is about the observational time (6h). In the case of three-minute oscillations, the half-width corresponds to very high $Q$-factor, $Q \geq 500$. This seriously constrains the theory related to the sunspot oscillation model. Only a resonator may be responsible for the occurrence of a spectrum with such narrow lines. Lines in the low-frequency part of the spectrum do not
correspond to such high $Q$-factor, but we expect that this is the result of insufficient spectral resolution.

### 3.2 Locality of Sunspot Oscillations

Another key issue for oscillation theory is whether sunspot oscillations are local or non-local, i.e., a sunspot oscillates as a whole, or it is broken down into individual cells which oscillate more or less independently of each other. Zhugzhda & Sych (2014) revealed that 3 min oscillations are local oscillations. The goal of the current investigation is to check whether the locality is a general property of sunspot oscillations.

Figure 3(a,b) exhibits the oscillation mean amplitude spectrum (blue line) for the entire site ($9 \times 9$) in the sunspot center. This mean spectrum was obtained by averaging 81 spectra of all the pixels in the square site shown on Figure 1. The same plot displays the amplitude spectrum (red line) of the site averaged oscillations that were obtained by averaging oscillations in all the pixels of the investigated site. These two spectra substantially differ from each other both in amplitude and in form. This provides evidence that the oscillations are not non-local. In case of non-local oscillations, these two spectra should agree with each other. Figure 3(c,d) shows (in different colors) the amplitude spectra of nine pixels with coordinates $x = 1, \ldots, 9, y = 4$. The averaged oscillation spectrum represented by the red curve in Figure 3(a,b) is exhibited by the black curve in Figure 3(c,d). The number of pixels whose spectra are shown in Figure 3(c,d) also includes the pixel with coordinates (3,4) where the largest number of amplitude oscillations is observed. In these two figures, one can see that the oscillation spectra in pixels dramatically differ both from one another, and from the averaged oscillation spectrum of the site. The listed peculiarities of spectra in Figures 3(a,b,c,d) are evidence that the sunspot oscillations are local oscillations, i.e., they represent a set of oscillation cells. To study the size of the oscillation cells, we built pixel amplitude histograms for fixed frequencies in the entire spectral range of sunspot oscillations. In total, 250 histograms were constructed for frequencies that encompass the whole sunspot oscillation spectrum, from 0.125 mHz to 10 mHz. We generated histograms for the frequencies that were a certain step from each other in each frequency band. Almost all the histograms indi-
Fig. 3 Amplitude spectra of sunspot oscillations. (a,b) Averaged amplitude spectrum of the site under study (blue curve) and the amplitude spectrum of averaged oscillations of the site (red curve). (c,d) Amplitude spectra of pixels with coordinates $x = 1, \ldots, 9$, $y = 4$ and $x = 1, \ldots, 9$, $y = 6$ are presented in different colors. The black line is the same as the red line on the graphs (a,b).

cated the presence of oscillation cells. In total, there were no oscillation cells in only several diagrams. But in these diagrams, there was a very low level of oscillation amplitudes in all the pixels.

Figure 4 provides examples of typical oscillation amplitude histograms within the investigated site for oscillations over the whole oscillation frequency range. In case of three-minute oscillations (top row of histograms in Figure 4), the histograms display frequencies that correspond to the mean amplitude spectrum maxima (Fig. 3a,b). On the histograms in rows 2 and 3 in Figure 4, one can see that the size of the cells appeared about several pixels over the entire investigated frequency range from $\nu = 0.125$ mHz to $\nu = 12$ mHz. For the lowest-frequency oscillations (bottom row of histograms in Figure 4), cells become more compact. Also, the ratio of the cell oscillation amplitude to the oscillation amplitudes in pixels surrounding the cell increases notably. The middle row of histograms in Figure 4 demonstrates that five-minute oscillations are also concentrated in small cells.

3.3 Discussion of the Sunspot Oscillation Analysis

An analysis of observations at high spectral and spatial resolution reveals local oscillations in a wide spectral
range \((0.1 - 10 \text{ mHz})\). These results provide a starting point for constructing a model of local sunspot oscillations. The model is based on two properties of sunspot oscillations. The first property is that fluctuations in the spot consist of individual cells whose size is at the limit of resolution. The second property is that the spectrum of local oscillations consists of a large number of narrow lines.

4 SCENARIOS OF LOCAL AND NONLOCAL SUNSPOT OSCILLATIONS

Zhugzhda & Sych (2014) analyzed two possible scenarios for the origin of three-minute oscillations in sunspots. Based on this analysis and observation data, the model of local three-minute oscillation has been proposed. We generalize the analysis of Zhugzhda & Sych (2014) to the case of low frequency oscillations.

According to the magnetoacoustic gravity (MAG) wave theory (Zhugzhda 1979; Zhugzhda & Dzhalilov 1982), a sunspot’s atmosphere consists of three layers. The weak field approximation is valid in the subphotospheric layers where the magnetic pressure appears much less than the gas one. Slow and fast waves propagate independently from each other and there is no coupling between them in this layer. In this case, fast waves are p-modes slightly modified by a weak magnetic field. The strong field approximation is valid in the chromosphere and corona where the gas pressure appears much less than the magnetic pressure. There is no coupling between slow and fast layers in these layers. Fast and low-frequency slow waves are evanescent while high-frequency waves are running under the sunspot. The coupling between slow and fast waves is possible in the photospheric layers of the sunspot where the magnetic and gas pressure are of the same order.

It is assumed that the source of sunspot oscillations observed in the photosphere, chromosphere and corona is located in the subphotospheric layers of the sunspot. Sunspot oscillations can appear due to p-modes or slow waves in subphotospheric layers. These two cases correspond to two scenarios of sunspot oscillations. Observations indicate that there are local sunspot oscillations. Zhugzhda & Sych (2014) demonstrate that local three-minute oscillations appear due to slow waves arriving from subphotospheric layers of sunspots. The results of data analysis on plots in Figure 4 show that local three-minute oscillations are accompanied by low-frequency local oscillations. Besides, there are nonlocal waves, which manifest themselves as wavefronts traveling through the umbra to the penumbra (Zirin & Stein 1972; Giovanelli 1972; Sych & Nakariakov 2014; Su et al. 2016).

Figure 5 displays two scenarios for both local and nonlocal in sunspots. These scenarios are an extension of scenarios developed by Zhugzhda & Sych (2014) for three-minute oscillations. It is shown by Zhugzhda & Sych (2014) that nonlocal three-minute oscillations of sunspots appear due to subphotospheric p-modes while local three-minute oscillations appear due to subphotospheric slow waves. Both scenarios include the chromospheric resonator and subphotospheric resonators for p-modes and slow modes, as was assumed by Zhugzhda (1981). A rich spectrum of local oscillations, including a large number of lines, is impossible to explain by the chromospheric resonator since its spectrum consists of just a few spectral lines (Settele et al. 2001). Moreover, the chromospheric resonator cannot explain the occurrence of sunspot oscillations with periods greater than three minutes. Only the subphotospheric resonator for slow waves can be considered as a candidate for explaining the entire spectrum of local sunspot oscillations.

5 LOCAL OSCILLATIONS AND ROBERTS EQUATION

Further analysis of local slow waves in a sunspot atmosphere is based on the equation for slow waves that was derived by Roberts (2006) in the limit \(k_z/k_\perp \rightarrow \infty\) for the case of a vertical uniform magnetic field. This equation is true for an arbitrary conductive atmosphere in a homogeneous magnetic field. This is a second-order equation, which means that in this limit there is not an interaction between slow and fast waves. The Roberts equation for slow waves reads

\[
C_T^2 \frac{d^2 \xi_z}{dz^2} + \gamma g \frac{C_T^4}{C_S^2} \frac{d \xi_z}{dz} \\
+ \left[ \omega^2 - \frac{C_T^2}{V_A^2} \left( N^2 + \frac{g}{H} \frac{C_T^2}{C_S^2} \right) \right] \xi_z = 0,
\]

\[
N^2 = g \left( \frac{1}{\rho_0(z)} - \frac{g}{C_T^2} \right) \\
= \frac{\gamma - 1}{\gamma} \frac{g}{C_T^2} \left( \frac{d \ln T}{dz} \right).
\]

where \(\xi_z\) is the vertical displacement and \(N\) is the Brunt-Väisälä frequency. The second alternative expression for the Brunt-Väisälä frequency is written in the form of the Schwarzschild criterion. The square of the Brunt-Väisälä frequency becomes negative when fulfilling the
Schwarzschild criterion for occurrence of convection. Thus, the square of the Brunt-Väisälä frequency is negative in the convection zone. Detailed analysis of the Roberts equation can be found in Zhugzhda & Sych (2014). For this analysis, we need only two formulas which are listed below.

The Roberts equation determines the cutoff frequency for the slow waves in a stratified atmosphere with an arbitrary vertical magnetic field. The cutoff frequency is a key parameter which is required in the analysis of wave propagation in a stratified atmosphere. To determine the cutoff frequency, one should transform from Equation (1) to an equation without a first derivative. This transformation is performed via introducing a new variable

\[ \xi_z = \xi(z) \exp \left( -\frac{\gamma g}{2} \int z \frac{C_T^2}{C_S^2} \mathrm{d}z \right). \]

In terms of this new variable, Equation (1) is now in the form

\[ C_T^2 \frac{d^2 \xi(z)}{dz^2} + \left( \omega^2 - \Omega_{\text{off}}^2 \right) \xi(z) = 0, \quad (3) \]

where the so-called cutoff frequency \( \Omega_{\text{off}} \) is equal to

\[ \Omega_{\text{off}}^2 = \frac{C_T^2}{V_A^2} \left( N^2 + \frac{g}{H} \frac{C_T^2}{C_S^2} + \frac{\gamma}{4} \frac{C_T^6}{C_S^6} \right) + \frac{\gamma g}{C_T^4} \frac{d \ln C_T}{dz} - \frac{2}{C_S^2} \frac{d \ln C_S}{dz}. \quad (4) \]

Slow waves with a frequency less than the cutoff frequency are evanescent waves. This formula is exact. The cutoff frequency for an isothermal atmosphere permeated by a uniform magnetic field is a very special case of Equation (4). The cutoff frequency for an isothermal atmosphere can be applied only to a treatment of the minimum temperature in the sunspot atmosphere.

Equation (1) contains coefficients depending on the depth \( z \), which makes the transition to a dispersion equation impossible. However, for qualitative analysis, one may use an approximation of local dispersion when a proper wave number \( k_z \) is introduced for each depth in the atmosphere. In this case, Equation (3) becomes a local dispersion equation, and it appears possible to determine a local phase and a group velocity for slow waves

\[ \omega(k_z) = \sqrt{C_T^2 k_z^2 + \Omega_{\text{off}}^2}, \]

\[ v_{ph} = C_T \sqrt{1 - \frac{\Omega_{\text{off}}^2}{\omega^2}}, \]

\[ v_{gr} = C_T \sqrt{1 - \frac{\Omega_{\text{off}}^2}{\omega^2}}. \quad (5) \]

These formulas for phase velocity are approximate. In fact, they are more exact than the oft-used formulas for the phase velocity of slow waves. Slow waves in a strong field are usually considered to propagate with the speed of sound because \( C_T \approx C_S \) for \( V_A^2 \gg C_S^2 \). In fact, the phase velocity of slow waves is only close to the speed of sound at the condition \( \omega^2 \gg \Omega_{\text{off}}^2 \), as follows from Equation (5). Just as the phase velocity of slow waves in a weak field is close to the Alfvén velocity only at the fulfillment of an additional condition, \( \omega^2 \approx \Omega_{\text{off}}^2 \approx N^2 \).

Having all the necessary formulas in hand, we try to develop a model of local sunspot oscillations.

### 6 OCCURRENCE OF SUBPHOTOSPHERIC RESONATOR FOR SLOW WAVES

To calculate the model of local sunspot oscillations, we used empirical models of a sunspot atmosphere developed by Staude (1981) and Maltby et al. (1986). Settele et al. (2001) updated these models and included sunspot subphotospheric layers in them. We also had to update these models a little to enable necessary calculations for the model of local sunspot oscillations. As far as we know (Moradi et al. 2010), there are no other models of a sunspot that would include the entire sunspot atmosphere, from the chromosphere to the subphotospheric layers, and would contain all the parameters necessary for our calculations.

Figure 6(c,d) presents the results for calculating the cutoff frequency \( \Omega_{\text{off}} \) (see Eq. (4)) for the Staude and Maltby models. For comparison, the same plots show calculations of the cutoff frequency for an isothermal atmosphere in the strong field. Certainly, the approximation of an isothermal atmosphere is not valid for the sunspot’s photosphere where a strong temperature gradient occurs. But for the temperature minimum, the cutoff frequencies provided by the two formulas are rather close due to a weak temperature gradient. The discussion of reasons for differences between the calculation results of isothermal and non-isothermal atmospheres can be found in Zhugzhda & Sych (2014). Plots of cutoff frequency show that the chromospheric resonator occurs. However, the chromospheric resonator cannot be responsible for three-minute oscillations because it cannot provide such a number of spectral lines over the frequency range of 6-10 MHz. Calculations by Settele et al. (2001) show that the chromospheric resonator leads to the emergence of only a few spectral lines (not tens of lines as in fact occurs) within this frequency range.
Fig. 4 Histograms of oscillation amplitudes for all pixels in the site under study are shown for a number of frequencies in the entire spectral range of sunspot oscillations.

Fig. 5 The schemes of two scenarios for MAG wave propagation in a magnetized atmosphere and diagnostic diagram for an isothermal atmosphere in terms of dimensionless frequency and horizontal wavenumber ($K = k_\perp H, \Omega = \omega C_S / g$) are shown. The boundaries between the p-mode region (I), g-mode region (II) and evanescent waves region are shown by yellow curves. Red represents slow waves and dark blue signifies fast waves. Arrows mark running waves and triangles stand for evanescent ones.

Fig. 6 The temperature as a function of the depth for the Staude (a) and Maltby (b) sunspot models is shown. The depth is counted from an upper boundary of the chromosphere models. The cutoff frequencies $\Omega_{\text{off}}$ as a function of depth for the Staude (c) and Maltby (d) sunspot models are calculated for different values of the sunspot magnetic field, $B = 3000, 2000, 1000$ G. In addition, the slow wave cutoff frequency for the isothermal atmosphere is shown in the plot.
Because Figure 6(c,d) presents real parts of the cutoff frequency, the cutoff frequency under a sunspot starting with a certain depth turns to zero. The cutoff frequency in the lower part of the photosphere of the sunspot decreases sharply due to the fall in Alfvén speed. It does mean that slow waves under photospheric layers of a sunspot are running waves. This is a confirmation of Zhugzhda’s prediction (Zhugzhda 1984) about the occurrence of a subphotospheric slow-wave resonator.

The occurrence of subphotospheric resonators may lead to the emergence of a spectrum with numerous lines. This is possible only in case of a depth-extended resonator with a sufficiently low fundamental resonance frequency. In this case, numerous spectral lines in the subphotospheric resonator are a result of high-harmonic excitation in the subphotospheric resonator.

The plots in Figure 7 represent profiles of the phase velocity for different frequencies and magnetic fields calculated for the Staude and Maltby models. These plots show that local slow waves have strong dispersion. For example, string oscillation eigenfrequencies obey the formula
\[ \nu_n = \nu_0 / n, \quad n = 1, 2, 3, \ldots, \]
where \( \nu_0 \) is the fundamental tone and \( \nu_n \) is an overtone. In the case of a resonator for waves with dispersion, the relation between the fundamental tone and overtones is non-linear.

There are two main hypotheses for the subsurface structure of sunspots: the monolithic model and the cluster model. Local helioseismology is unable to distinguish between these two models (Moradi et al. 2010). Local oscillations are a good tool to differentiate between models. If the sunspot is a cluster of magnetic flux tubes, some of these tubes may be responsible for local oscillations due to the resonances of slow waves in them. In the case of monolithic sunspots, convective tongues are candidates for slow mode resonance and local oscillations. The main difference between the magnetic flux tubes of the cluster model and the convective tongues in monolithic sunspots is the value of the magnetic field. The slow-wave cutoff frequency defined by Equation (4) and phase velocity given in Equation (5) depend on the magnetic field. Thus, modeling of the sub-photospheric resonator can help to reveal the structure of sunspots under the photosphere.

**Fig. 7** Low-frequency oscillation phase velocity as a function of depth for the Staude (a, c) and Maltby (b, d) models for three values of the magnetic field.
7 MODELING OF SUBPHOTOSPHERIC RESONATOR

Modeling of the subphotospheric resonator faces difficulties, since there are no models of magnetic tubes in the cluster sunspot model and convective tongues in the monolithic sunspot model. We were forced to use the Staude and Maltby models for a monolithic sunspot. The description of the subphotospheric layers by these models is quite arbitrary, because convection is neglected.

A subphotospheric resonator appears in the lower atmosphere of sunspots, where the slow waves are running ones. Phase velocities (Eq. (5)) of traveling slow waves in the lower layers of the sunspot’s atmosphere are calculated for Staude and Maltby models (Settele et al. 2001). The results are shown in the plots of Figure 7. At the upper boundary of the subphotospheric resonator, cutoff frequency drops to zero (see Fig. 6), and the phase velocity tends to infinity (see Fig. 7). The lower boundary of the resonator is determined by the models of Staude and Maltby. In fact, the lower boundary of the slow-wave resonator should be located at depths where the convection starts to work. At the convection, the temperature gradient is close to the adiabatic case, and the square of the Brunt-Väisälä frequency is close to zero. Thus, the slow wave phase velocity defined in Equation (5) should increase due to the onset of convection. Slow waves should experience a reflection at this level. This is most likely the lower boundary of a slow-wave resonator. Unfortunately, none of the existing models describes the transition to convection deep under the sunspot. There is also an increase in the phase velocity in the middle of the resonator, which should lead to the splitting of natural frequencies. This aspect leads to difficulties in applying standard procedures for the associated eigenproblem. In connection with all the above difficulties, only approximate calculations of the eigenfrequencies for the subphotospheric resonator were carried out to assess the possibility of its occurrence.

The resonance of slow waves in the flux tube in the sunspot atmosphere between depths \( z_1 \) and \( z_2 \) is considered. Zero boundary conditions are assumed. To derive the exact eigenfrequencies, Equation (1) has to be solved. To find out approximate eigenfrequencies, the time \( t_n \) as a function of \( \omega_n \) is calculated

\[
t_n = \int_{z_1}^{z_2} \frac{1}{v_{ph}(\omega_n)} \, dz,
\]

where \( v_{ph} \) is defined by Equation (5). If \( n\pi/t_n = \omega_n \), the frequency \( \omega_n \) is the \( n \)-th harmonic of the resonator.

After calculation of the integral, eigenfrequencies can be found by intersections on the plot. This method is sufficiently accurate for high harmonics, but rather rough for the fundamental one. However, this is not important because of the problems mentioned above, and due to the difficulties of correlation with the observations discussed below.

Figure 8 presents calculations of the subphotospheric resonator eigenfrequencies for the Staude model at different values of the magnetic field. We have to address low values of the magnetic field because only in this case does the number of harmonics over the three-minute oscillation range appear sufficiently great to correspond to observational data.

The natural next step after modeling the subphotospheric resonator should be a comparison of the model with the spectrum of oscillations in the sunspot. But, the spectra in Figures 2 and 3 are not at all similar to the spectrum of resonance oscillations consisting of equidistant lines. The spectrum of sunspot oscillations looks like a noise spectrum without signs of regularity. This is due to the fact that the amplitudes and phases of the resonator harmonics are functions of time because of changes in the size and magnetic field of the convective tongues. We face the problem of extracting signs of the subphotospheric resonator occurrence from the noisy spectrum of sunspots oscillations. There are signs of the presence of a resonance in the oscillation spectrum. It is well known that three-minute oscillations consist of a sequence of wave trains. If two lines dominate in the spectrum, then the wave trains will appear because of beating between the components. If the spectrum consists of equidistant...
lines due to the subphotospheric resonance, the wave trains must appear. Despite the fact that the spectrum of the oscillations is similar to the noise spectrum, this effect was revealed by Sych et al. (2012). The length of the wave train is determined by the distances between equidistant lines in the spectrum. In turn, the distances between equidistant lines are equal to the fundamental frequency of the resonance. Sych et al. (2012) found out that the characteristic duration of the wave trains is of the order of tens of minutes. This is in line with our idea of a low-frequency subphotospheric resonator. To clarify evidence for the existence of subphotospheric resonance, it is necessary to develop both special methods of spectral analysis and to consider different excitation mechanisms (Zhugzhda & Locans 2018). In addition to excitation of the subphotospheric resonator by turbulent convection, oscillatory convection in a magnetic field can be significant (Syrovatskii & Zhugzhda 1968). Oscillatory convection is the instability of the slow wave. However, addressing these issues is beyond the scope of the paper.

8 LOCAL OSCILLATIONS AND UMBRAL DOTS

The local oscillation cell size, according to some data (Centeno et al. 2005), is estimated as a fraction of an arcsec. In the analysis of our observation, the local oscillation cell size is at the limit of the instrument’s spatial resolution. Local sunspot oscillations should obviously be related to the fine structure of a sunspot. Umbral dots are the only candidate for this role. The size of umbral dots is of the same order as the size of local oscillation cells (Ebadi et al. 2017; Goodarzi et al. 2016). Besides, the umbral dots oscillate (Jess et al. 2012; Krishna Prasad et al. 2015; Chae et al. 2017; Ebadi et al. 2017). According to current ideas, umbral dots originate due to convection in the sunspot’s magnetic field. The best sunspot convection simulations performed by Schüssler & Vogler (2006) showed that umbral dots represent a convective tongue that originates as a result of convection in the magnetic field. In our opinion, convective tongues are candidates for the role of a slow-wave resonator.

Despite the obvious shortcomings of our subphotospheric resonator model, we have arrived at a magnetic field value and resonator size similar to the convective tongue parameters obtained in the simulation described in Schüssler & Vogler (2006). No doubt, one needs calculations for eigenfrequencies of convective tongues. Comparative analysis of natural frequencies of convective tongues and spectra of local oscillations in sunspots can be a test of the results of numerical experiments on convection in sunspots. Besides, it is crucial to check whether the locations of local oscillation cells and umbral dots are the same. It should be noted that according to simulations in Schüssler & Vogler (2006), the emergence of small-scale upflow tongues starts off like oscillatory convection columns below the solar surface but turns into narrow convective tongues driven by the strong radiative cooling around an optical depth of unity. This is very similar to the oscillatory convection (Syrovatskii & Zhugzhda 1968) that occurs due to the instability of slow waves in a stratified atmosphere permeated by a magnetic field.

9 LOCAL OSCILLATIONS AND WAVEFRONTS

Besides local oscillations, there are also traveling waves that propagate in the form of wavefronts through the umbra into the penumbra (Giovanelli 1972; Reznikova et al. 2012; Sych & Nakariakov 2014; Su et al. 2016). It is natural to consider these waves as non-local waves excited by p-modes. The simulation of this scenario was done by Felipe & Khomenko (2017). They were forced to postulate a localized source of p-modes at some depth under a sunspot. However, p-modes in the convection zone represent a distributed source.

Thereby, it makes sense to discuss a possibility of exciting the divergent wavefronts by local oscillations. When addressing the two scenarios for excitation of sunspot oscillation, we, indeed, postulated a non-interaction between local and non-local oscillations. In fact, this was based on the assumption that the approximation (Roberts 2006) is valid in all the sunspot layers.

Slow waves propagate along field lines in the regions with a relatively strong field (the chromosphere and the corona over a sunspot), and in the regions of a relatively weak field (sunspot subphotospheric layers) in accordance with the MAG-wave theory (Zhugzhda 1979; Zhugzhda & Dzhahilov 1982). This means that the propagations of slow waves in adjacent thin flux tubes are independent of each other. Thus, the Roberts approximation is valid in the upper layers (chromosphere and corona), and in the lower layers (subphotosphere of sunspot). However, the validity of the Roberts approximation (Roberts 2006) in the photosphere region and directly under the latter depends on the thickness of the magnetic tube. However, it is unknown how thin the magnetic tube must be for the Roberts approach to be valid. Nobody has investigated this issue. The emission of local oscillations from the magnetic flux tube is possible if it
is not thin enough at some level on the way through the photosphere. This emission may be responsible for running waves in the sunspots. This problem needs analysis that is beyond the scope of this paper.

10 CONCLUSIONS

Local waves propagate from the sunspot umbra into the corona through the chromosphere (Krishna Prasad et al. 2015; Sych et al. 2012). The amplitude of the local oscillation considerably exceeds the amplitude of nonlocal fluctuations. Moreover, the power of local oscillations is underestimated because the oscillation cell size is, obviously, beyond the spatial resolution of present-day instruments (Centeno et al. 2005). Consequently, local oscillations must be responsible for the dynamics of the chromosphere and be a manifestation of chromospheric oscillations in the corona.

It is argued that local oscillations can be explained under the assumption that a subphotospheric resonator exists. So far, no other interpretations have been proposed. Moreover, the proposed model manages to link up the local oscillations with umbral dots and running waves in sunspots. Undoubtedly, the proposed model of local oscillations should be improved and confirmed on the basis of observations.

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