On extension of minimality principle in supersymmetric electrodynamics

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Deep geometric ideas of Eli Cartan inspired Dmitrij Vasiljevitch Volkov in his investigations of the phenomenological lagrangians method and in his work at the construction of supersymmetry and supergravity. The Cartan idea on the extension of the connection conception and the introduction of torsion was applied by Dmitrij Vasiljevitch Volkov in the formulation of minimality principle for the interactions of Goldstone particles with other fields. His profound intuition at once allowed to assume that the Pauli matrices \( \sigma^\mu_{\alpha\dot{\alpha}} \) play a fundamental role of the torsion tensor components in \( z^M = (x^\mu, \theta^\alpha_i, \bar{\theta}^\dot{\alpha}_i) \) superspace.

The Cartan’s geometry naturally emerged in early papers by Scherk and Schwarz on the string theory of gravitation where the strength \( H_{\mu\nu\rho} \) of the antisymmetric Kalb-Ramond field plays the role of torsion. These and some other impressive results show ties between superspace torsion and spin.

Here we wish to single out a possibility of an extension of the minimality principle for electromagnetic interactions of charged and neutral particles having spin \( 1/2 \). This possibility is also based on the Cartan’s idea of an extension of the connection conception.

I.

An extension of the minimality principle can be achieved by the addition of new terms to the standard gauge covariant and supersymmetric derivatives \( D_M \) of supersymmetric electrodynamics:

\[
D_M = D_M + \epsilon A_M \quad \rightarrow \quad \nabla_M = D_M + i \mu \tilde{W}_M \equiv D_M + \epsilon_{(q)} \tilde{A}_M^{(q)}, \]

where the additional superfield \( \tilde{W}_M(z) \) is an invariant of the gauge group \( U(1) \), \( \epsilon_{(q)} \tilde{A}_M^{(q)} \equiv \epsilon A_M + i \mu \tilde{W}_M \) is a new generalized connection, and the constant \( \mu \) with the dimension of length \( (\hbar = c = 1) \) has the physical meaning of anomalous magnetic moment (AMM).

The algebra of the doubly lengthened covariant derivatives \( \nabla_M = (\nabla_\mu, \nabla_{\alpha i}, \bar{\nabla}_{\dot{\alpha} i}) \) differs from the algebra of the standard \( N \)-extended supersymmetric derivatives \( D_M, D_N \) by the presence of the electromagnetic superfield strength \( F_{MN} \):

\[
[D_M, D_N] = T_{MN}^L D_L, \]

\[
T_{MN}^L = -2i \sigma^\mu_{\alpha\dot{\alpha}}, \quad \text{for } N = 1 \text{ SUSY} \]

\[
T_{MN}^L = 2i \sigma^\mu_{\alpha\dot{\alpha}} \delta^I_{\dot{J}}, \quad \text{for } N > 1 \text{ SUSY} \]

(2)

which takes into account AMM of superparticle and is defined as

\[
\epsilon_{(q)} F_{MN}^{(q)} \equiv \epsilon F_{MN} + i \mu \bar{F}_{MN}^{(\mu)}, \]

\[
F_{MN} \equiv D_M A_N - (-1)^{MN} D_N A_M - T_{MN}^R A_R, \]

\[
F_{MN}^{(\mu)} \equiv D_M \bar{W}_N - (-1)^{MN} D_N \bar{W}_M - T_{MN}^R \bar{W}_R. \]

(4)

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Conservation of supersymmetry and $U(1)$ gauge symmetry demands that the superfield $\tilde{W}_M(z)$ should be an invariant of these symmetries. Let us look for these invariants in the set of functions which are a linear combination of the strength components $F_{MN}(z)$ with constant coefficients. The dimensionality reasons, i.e. $|W_\mu| = L^{-2}$, $|\tilde{W}_\alpha| = L^{-3/2}$, together with the constraints imposed on $F_{MN}$ sharply restrict the form of the invariant $\tilde{W}_M$. As a result, the desired spinorial components of $\tilde{W}_M$ are taken in the form of the product of $F_{\mu\alpha}^{-1}$ by the torsion of the flat supertorsion $\tilde{W}_\alpha \sim \tilde{F}_{\mu\alpha}^{-1}, \tilde{\sigma}_\alpha^{-1}$. In particular, for $N = 2$ case the expression $\tilde{W}_\mu \sim \partial_\mu \tilde{F}_{\alpha\beta}^{-1} \tilde{\sigma}_\alpha^{-1} \tilde{\sigma}_\beta^{-1} \varepsilon$ may be taken for the vector component of $W_\mu$. Due to the symmetry of $F_{\alpha\beta}$ this expression equals zero for $N = 1$ case. The desired representation for the superfield $\tilde{W}_M$ in $N = 1$ case has the following form

$$\tilde{W}_M = W_M \equiv \frac{i}{4}(0, -\sigma_{\mu\alpha} F^{\mu\alpha}, \bar{\sigma}^{\mu\alpha} F_{\mu\alpha}). \quad (5)$$

The uniqueness of the representation arises when taking account of the standard $F_{MN}$ constraints

$$F_{\alpha\beta} = F_{\alpha\bar{\beta}} = 0, \quad (6)$$

$$F_{\alpha\bar{\beta}} = 0. \quad (7)$$

and the Bianchi identities

$$\mathcal{Cycl}_{MNR}(-1)^{(MNR)} (D_M F_{NR} - T_{MN} L A_R) = 0 \quad (8)$$

equivalent to the Jacobi identities for the covariant derivatives $D_M$ of supersymmetric electrodynamics (SUSY ED)

$$\mathcal{Cycl}_{MNR}(-1)^{(MNR)} [D_M, [D_N, D_R]] = 0. \quad (9)$$

Eqs. (5,7) require that the superfield $F_{MN}$ of $N = 1$ SUSY ED should be expressed in terms of the spinor chiral superfields $W_\alpha$ and $\tilde{W}_\alpha$ [10]

$$W^\alpha = \frac{i}{4} F_{\mu\alpha} \bar{\sigma}^{\mu\alpha}, \quad \tilde{W}^{\bar{\alpha}} = (W^\alpha)^* . \quad (10)$$

Therefore $\tilde{W}_\mu$ must be constructed of $W_\alpha$ and $\tilde{W}_\bar{\alpha}$. Using these superfields one cannot construct the vector superfield $\tilde{W}_\mu$ having the dimension $L^{-2}$. As a result, the desired extension of the minimality principle for $N = 1$ SUSY ED is defined by the superfield $\tilde{W}_M$ [10] having the form

$$\tilde{W}_M(z) = (0, W_\alpha, \tilde{W}_{\bar{\alpha}}) \quad (11)$$

with $W_\alpha$ and $\tilde{W}_{\bar{\alpha}}$ restricted by the chirality and reality conditions [11]

$$D_\alpha \tilde{W}_{\bar{\alpha}} = \tilde{D}_{\bar{\alpha}} W_\alpha = D^\alpha W_\alpha - \tilde{D}_{\bar{\alpha}} \tilde{W}^{\bar{\alpha}} = 0. \quad (12)$$

The proof of the uniqueness is completed by the analysis of the Jacobi identities

$$\mathcal{Cycl}_{MNR}(-1)^{(MNR)} [\nabla_M, [\nabla_N, \nabla_R]] = 0, \quad (13)$$

which take the form

$$e \mathcal{Cycl}_{MNR}(-1)^{(MNR)} (D_M F_{NR} - T_{MN} L F_{LR}) + i \mu \mathcal{Cycl}_{MNR}(-1)^{(MNR)} (D_M F_{NR}^{(\mu)} - T_{MN} L F_{LR}^{(\mu)}) = 0 \quad (14)$$

after the substitution of Eqs.(5,7) and (8) into Eq. (13). The first and the second term in Eq. (14) equal zero by construction and due to sufficient arbitrariness in the definition of $W_\alpha$ and $\tilde{W}_{\bar{\alpha}}$, respectively

Therefore the extended derivatives satisfy the Bianchi identities and give a solution of the desired problem of selfconsistent extension of the minimality principle for $N = 1$ SUSY ED particles with spin 1/2.

The superalgebra of the generalized derivatives $\nabla_M$ takes the form

$$\{\nabla_\mu, \nabla_\nu\} = -\frac{e}{2} (\tilde{D} \bar{\sigma}_{\mu\nu} W - D \sigma_{\mu\nu} W), \quad (15)$$

$$\{\nabla_\mu, \nabla_\alpha\} = i e W^\beta \bar{\sigma}_{\mu\beta\bar{\alpha}} + i \mu \partial_\mu W_\alpha, \quad (\nabla_\alpha, \nabla_\beta\} = i \mu (D_\alpha W_\beta + D_\beta W_\alpha), \quad (\nabla_\alpha, \nabla_{\bar{\alpha}}\} = -2 i \bar{\sigma}_{\bar{\alpha}\bar{\beta}} \nabla_\mu + i \mu (D_\alpha W_\beta + \tilde{D}_{\bar{\alpha}} \tilde{W}_{\bar{\beta}}),$$

$$\{\nabla_\alpha, \nabla_{\bar{\beta}}\} = -2 i \bar{\sigma}_{\bar{\alpha}\bar{\beta}} \nabla_\mu + i \mu (D_\alpha W_\beta + \tilde{D}_{\bar{\alpha}} \tilde{W}_{\bar{\beta}}),$$
and we observe the noncommutativity effect for the generalized spinor derivatives $\nabla_\alpha$.

Due to the considered extension procedure, the action for charged or neutral superparticle in an external electromagnetic field gets an additional term which describes the electromagnetic interaction of superparticle taking into account its AMM

$$S_{\text{int}} = i \int dz^M e_{(q)} A_M^{(q)} = S^{(e)} + S^{(\mu)},$$

$$S^{(e)} = i e \int dz^M A_M,$$

$$S^{(\mu)} = -\mu \int dz^M \ddot{W}_M = -\mu \int d\theta^\alpha W_\alpha - \mu \int d\bar{\theta}_\dot{\alpha} \bar{W}_{\dot{\alpha}}.$$

(16)

The component analysis of the action (14) carried out in [8,9] shows that $S^{(\mu)}$ term in Eq.(16) generates the nonminimal Pauli term with the constant $\mu$ which may be interpreted as the AMM of Dirac particle measured in values of Bohr magneton.

In the next section we consider a generalization of the extended minimality principle for the case $N = 2$.

II.

In the standard $N = 2$ SUSY ED [12,13] the strength components $F_{MN}$ obey the constraints

$$F_{\alpha \beta}^{\ i} \tilde{\alpha} \tilde{\beta} = F_{\tilde{\alpha} |\tilde{\beta}| \tilde{\gamma}} = 0,$$

(17)

$$F_{\alpha}^{\ i} \tilde{\beta} = 0.$$  

(18)

Together with the Bianchi identities, these constraints permit to express $F_{MN}$ in terms of the scalar chiral superfield $W(z)$ and $\bar{W}(z)$ having the dimension $L^{-1}$ and restricted by the conditions

$$\bar{D}_{\alpha} W = D_{\alpha}^i \bar{W} = 0.$$  

(19)

The explicit form of the superfield $F_{MN}$ components is given by the following expressions

$$F_{\alpha}^{\ i} \tilde{\beta} = -\varepsilon_{\alpha \beta} \varepsilon^{ij} \bar{W}, \quad F_{\tilde{\alpha} \tilde{\beta}}^{\ i} = \varepsilon_{\alpha \tilde{\beta}} \varepsilon^{ij} W,$$

$$F_{\mu \alpha}^{\ i} = \frac{i}{4} (\sigma_{\mu} \bar{D}^i)_{\alpha} W, \quad F_{\mu \tilde{\alpha}} = -\frac{i}{4} (D_{\mu} \sigma_{\tilde{\alpha}})_{\bar{W}},$$

$$F_{\mu \nu} = \frac{1}{16} (D_{\mu} \sigma_{\nu} D^i) \bar{W} + \frac{1}{16} (D^i \sigma_{\mu \nu} D_{\nu}) W.$$  

(20)

In order to construct an extended connection for the $N = 2$ SUSY ED, we must again use the dimensionality reasonses and look for the required extension in terms of a linear combination of $W, \bar{W}$ and their covariant derivatives. Then the required representation for the additional superfield $T_M$ (which produces additional term to the connection $A_M$) may be written in the form

$$\bar{W}_M(z) = T_M = (\partial_\mu (bW + \bar{b}\bar{W}), aD_{\alpha}^i W, -\bar{a} \bar{D}^{\tilde{\alpha} i} \bar{W}).$$  

(21)

In a way analogous to that used in $N = 1$ case [3] it is convenient to introduce the two-component “charge” $e_{(q)} = (e, i \mu)$ and the superconnection $A_M^{(q)} = (A_M, T_M)$ in the extended “charged” space $e_{(q)}$. Then the doubly lengthened covariant derivative [3] can be more compactly presented as

$$\nabla_M = D_M + e_{(q)} A_M^{(q)}.$$  

(22)

Then the $U(1)$ gauge invariant superfield action for charged or neutral particles with AMM in an external superfield of the $N = 2$ Maxwell supermultiplet takes the form

$$S_{\text{int}} = S^{(e)}_{\text{int}} + S^{(\mu)}_{\text{int}} = i e_{(q)} \int_{\Gamma^*} \omega^M (dz) A_M^{(q)} = i \int_{\Gamma^*} (e \omega^M A_M + i \mu \omega^M T_M),$$  

(23)

where $N = 2$ Cartan’s differential forms $\omega^M (dz) = (\omega^\mu, \omega^\alpha_\tilde{\beta}, \omega^{\tilde{\alpha} i})$ in $N = 2$ superspace $z_{\alpha}$ are given by [3]:

3
\[
\omega^\mu = dx^\mu - i(d\theta^\mu d\bar{\theta}^\nu + i(\theta^\mu d\bar{\theta}^\nu),
\]
\[
\omega^{\alpha i} = d\theta^{\alpha i}, \quad \bar{\omega}_{\bar{\alpha} i} = d\bar{\theta}_{\bar{\alpha} i}. \tag{24}
\]

The use of the relation
\[
\frac{d}{d\tau} W = \omega_\tau^\mu \partial_\mu W + \theta^{\alpha i} D_\alpha D_i W + \bar{\theta}_{\bar{\alpha} i} \bar{D} \bar{\alpha} \bar{D} W
\]
allows to present \( S_{\text{Int}} \) in the equivalent form
\[
S_{\text{Int}} = ie \int d\tau \omega_\tau^M A_M + i\mu \int d\tau \left( \lambda_2 \theta^{\alpha i} D_\alpha D_i W + \bar{\lambda}_2 \bar{\theta}_{\bar{\alpha} i} \bar{D} \bar{\alpha} \bar{D} W \right). \tag{26}
\]

Following from Eq. (26) is the existence of two physically equivalent ways for the representation of the required superfield \( T_M \):
\[
T_M = (\partial_\mu (\lambda_1 W + \bar{\lambda}_1 \bar{W}), 0, 0), \tag{27}
\]
and
\[
T_M = (0, -i\lambda_2 D_\alpha i W, i\bar{\lambda}_2 \bar{D} \bar{\alpha} \bar{W}). \tag{28}
\]

Without restricting generality of considerations, we shall use the \( T_M \) representation in the form
\[
T_M = (0, \frac{1}{4} D_\alpha i W, -\frac{1}{4} \bar{D} \bar{\alpha} \bar{W}). \tag{29}
\]

Note that the equivalent representation for \( T_M \) is the following: \( T_M = (\frac{1}{4} \partial_\mu (W + \bar{W}), 0, 0) \). Eq. (29) creates the expressions for the components of \( N = 2 \) generalized superfield strength of the Maxwell supermultiplet
\[
e_{(q)} \mathcal{F}_{(q)}^{(q)} \sigma^i = -\epsilon_{\alpha \beta \gamma} \varepsilon^{ij} W,
\]
\[
e_{(q)} \mathcal{F}_{(q)}^{(q)} \sigma^i = -\epsilon_{\alpha \beta \gamma} \varepsilon^{ij} W,
\]
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\]
\[
e_{(q)} \mathcal{F}_{(q)}^{(q)} \sigma^i = -\epsilon_{\alpha \beta \gamma} \varepsilon^{ij} W.
\]

As a consequence of Eqs. (29), the algebra of the \( N = 2 \) doubly lengthened covariant derivatives \( \nabla_M \) takes the form
\[
\{ \nabla^\alpha_i, \nabla^\beta_j \} = -\epsilon_{\alpha \beta \gamma} \varepsilon^{ij} W,
\]
\[
\{ \nabla^\alpha_i, \nabla^\beta_j \} = -\epsilon_{\alpha \beta \gamma} \varepsilon^{ij} W,
\]
\[
\{ \nabla^\alpha_i, \nabla^\beta_j \} = -\epsilon_{\alpha \beta \gamma} \varepsilon^{ij} W,
\]
\[
\{ \nabla^\alpha_i, \nabla^\beta_j \} = -\epsilon_{\alpha \beta \gamma} \varepsilon^{ij} W.
\]

The motion equations of the charged \( N = 2 \) superparticle with AMM generated by the action (24) and \( S_0 \)
\[
S = S_0 + S_{\text{Int}} = \frac{1}{2} \int d\tau \left[ \omega_\tau^\mu \omega_\mu \right] + g_\tau m^2 \tag{32}
\]
take the form of the generalized Lorentz equations
\[
(g_\tau^{-1} \omega_\tau \partial_\mu \sigma^\mu) = -i \omega_\tau^M e_{(q)} \mathcal{F}_{(q)}^{(q)} M\mu,
\]
\[
(g_\tau^{-1} \omega_\tau \partial_\mu \sigma^\mu) = -i \omega_\tau^M e_{(q)} \mathcal{F}_{(q)}^{(q)} M\mu,
\]
\[
(g_\tau^{-1} \omega_\tau \partial_\mu \sigma^\mu) = -i \omega_\tau^M e_{(q)} \mathcal{F}_{(q)}^{(q)} M\mu,
\]
\[
(g_\tau^{-1} \omega_\tau \partial_\mu \sigma^\mu) = -i \omega_\tau^M e_{(q)} \mathcal{F}_{(q)}^{(q)} M\mu.
\]

where \( \mathcal{F}_{(q)}^{(q)} M\mu \) is defined by Eqs. (33).
The component expansion of the superfield $T_{\alpha}^i$ depending on the chiral variables has the form analogous to [11]

$$T_{\alpha}^i(z_L) = \frac{i}{4} \lambda_{\alpha}^i(x_L) + \frac{1}{8} \theta_{\alpha k} C^{ki}(x_L) - \frac{i}{8} \tilde{\theta}^i \gamma_\mu \partial_\mu \lambda_{\alpha}^i(x_L) - \frac{i}{2} \left( \tilde{\theta}^i \gamma_\alpha \partial_\beta \lambda_{\alpha}^i(x_L) \right)$$

$$- \frac{1}{4} \xi^{(2,0)ij}(\tilde{\partial} \lambda_j(x_L))_{\alpha} - \frac{1}{4} \xi^{(2,0)\beta}(\tilde{\partial} \lambda^\gamma(x_L))_{\beta} + \frac{i}{8} \xi^{(2,0)jk}(\tilde{\partial} \tilde{\theta}^i)_{\alpha} C_{jk}(x_L)$$

$$- \frac{1}{4} \xi^{(2,0)j\gamma}(\tilde{\partial} \tilde{\theta}^i)_{\alpha} f_{\beta \gamma}(x_L) + \frac{1}{3} \xi^{(2,0)i} j \tilde{\partial} \tilde{\theta}^i \lambda_j(x_L))_{\gamma} + \frac{i}{6} \xi^{(4,0)}(\tilde{\partial} \tilde{\theta}^i)_{\alpha} \tilde{\partial} \tilde{w}(x_L).$$

The explicit form for the monomials $\xi^{(m,n)}$ is given in [4].

The physical content of the theory is convenient to analyse in the central base coordinates. In this base the part of the action (26) which describes the contribution of AMM particle has the form

$$S_{\text{int}}^{(\mu)} \bigg|_{\text{photon}} = -\mu \int_{\Gamma^*} \left( \omega_{\mu} M T_M (x^\alpha, \theta^\alpha, \bar{\theta}^\alpha) \right)_{\text{photon}} =$$

$$= \mu \int_{\Gamma^*} \left[ i(\tilde{\theta} \sigma^\mu \theta)^j \partial_\mu v_{ij} + \frac{1}{2} (\tilde{\theta} \sigma^\mu \bar{\theta}^i \gamma^\nu) \partial_\nu v_{ij} + \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta^k \gamma^\nu) \partial_\nu v_{ij} + \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta \gamma^\nu)^j \partial_\mu v_{ij}\right]$$

$$- \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta)^j \partial_\mu v_{ij} + \frac{1}{2} (\tilde{\theta} \sigma^\mu \bar{\theta}^i \gamma^\nu) \partial_\nu v_{ij} - \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta \gamma^\nu)^j \partial_\mu v_{ij}$$

$$+ \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta)^j \partial_\mu v_{ij} + \frac{1}{2} (\tilde{\theta} \sigma^\mu \bar{\theta}^i \gamma^\nu) \partial_\nu v_{ij} - \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta \gamma^\nu)^j \partial_\mu v_{ij}$$

$$+ \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta)^j \partial_\nu v_{ij} + \frac{1}{2} (\tilde{\theta} \sigma^\mu \bar{\theta}^i \gamma^\nu) \partial_\mu v_{ij} - \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta \gamma^\nu)^j \partial_\nu v_{ij}$$

$$+ \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta)^j \partial_\nu v_{ij} + \frac{1}{2} (\tilde{\theta} \sigma^\mu \bar{\theta}^i \gamma^\nu) \partial_\mu v_{ij} - \frac{1}{2} (\tilde{\theta} \sigma^\mu \theta \gamma^\nu)^j \partial_\nu v_{ij}$$

To elucidate the physical meaning of Eq. (34), note that it is a pseudoclassical limit $\hbar \to 0$ of field theory. Under such a consideration the Grassmann spinors $\theta_{\alpha}^i$ are treated as a limit of the fermionic Fock operators [15–17] $b_{\alpha}^i$ which describe spin degrees of freedom. Since $\theta_{\alpha}^i$ have dimensionality $L^{-1/2}$ and $b_{\alpha}^i$ are chosen dimensionless, they are connected by the relation

$$\sqrt{\frac{\hbar}{2mc}} b_{\alpha}^i \to \theta_{\alpha}^i \text{ when } \hbar \to 0,$$

where $\hbar/nc$ is the Compton wavelength corresponding to the massive Dirac field quant. Further it is convenient to pass from $\theta_{\alpha}^i$ and $\bar{\theta}_{\alpha}^i$ having the dimension $L^{-1/2}$ to a Dirac bispinor $\Theta^i$

$$\Theta^i = \left( \begin{array}{c} \frac{1}{\sqrt{2}} \tilde{\theta}_{\alpha}^i \\ \frac{1}{m^2} \gamma^i \lambda_{\alpha}^i \end{array} \right), \quad \Sigma_{\mu \nu} = \frac{i}{4} [\gamma^i, \gamma^j],$$

where $\gamma^i$ are the Dirac matrices. The bispinor $\Theta^i$ has the dimension $L^{-1/2}$ and is an invariant under the proper time reparametrization due to the presence of the einbein $g_T$ [32], which has the dimension $L^{-2}$. In the terms of $\Theta^i$ (37) the action (26) is presented as

$$S_{\text{int}}^{(\mu)} \bigg|_{\text{photon}} = -\mu \int_{\Gamma^*} \left( \frac{g_{\alpha} dr}{m^2} \right) \left( \Theta^i \Sigma_{\mu \nu} \Theta^i \right) v^{\mu \nu} + \text{high order rel. corrections},$$

where $\left( \frac{g_{\alpha} dr}{m^2} \right)$ is a 1-dimensional reparametrization invariant “volume” having the dimension $L^4$. Due to the fact that the Grassmannian bispinor $\Theta^i$ are treated as a pseudoclassical limit of fermionic field operators $\Psi^i$ [15–17], the first term in (38) has a sense of a pseudoclassical limit of the Pauli term. The latter describes the electromagnetic
interaction of neutral particles possessing the AMM equal $\mu$. This observation explains the physical meaning of the constant $\mu$.

A possible field theory image of the action (38) can be restored by the substitution which conserves the reparametrization symmetry and all the dimensions

$$\Theta^i(\tau) \to \Psi^i(x), \quad \frac{g_\tau d\tau}{m^2} \to d^4x,$$

where $\Psi^i(x)$ has the sense of the field operator of a neutral particle with spin 1/2 possessing the canonical dimensionality $L^{-\frac{3}{2}}$.

Passing to the photino part of the action and preserving the leading term of the expansion in powers of $c^{-1}$, we find

$$S^{(\mu)}_{\text{int}} = -i \frac{\mu}{4} \int g_\tau d\tau \left[ \bar{\Lambda}_{Li} \Theta^i - \bar{\Theta}_{Li} \Lambda_{Ri} \right],$$

(40)

where $\Lambda^i_L = \frac{1 + \sqrt{2}}{2} \Lambda^i$, $\Lambda^i = \left( \Lambda^{\alpha i} \bar{\Lambda}_{\dot{\alpha}i} \right)$ is the photino field. The expression (40) is a pseudoclassical image of the field action

$$S^{(\mu)}_{\text{int}} = -i \frac{\mu m^2}{4} \int dx^4 \left[ \bar{\Lambda}_{Ri} \Psi^i_L - \bar{\Psi}^i_L \Lambda_{Ri} \right],$$

(41)

which can be interpreted as a hint for a possible conversion of the massive neutralino $\Psi^i_L$ into the photino $\Lambda_{Ri}$ with the coupling constant proportional to the AMM of neutralino.

Note also that supersymmetry, together with the proposed extension of the minimality principle, reproduces rather complicated, but controllable structure of high-order terms in the expansion with respect to $1/c$. This structure can be restored by the study of the component structure of the extended superfield and the interpretation of $\Theta^i$ as pseudoclassical images of the spinor fields $i$.

Thus, we conclude that the proposed here possibility of a generalization of the minimality principle can be found helpful for studying possible spin effects in the electromagnetic interaction of charged and neutral fermions predicted by supersymmetry.

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