Chern-Simons Theory for Quantum Hall Stripes

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Abstract. – We develop a Chern-Simons theory to describe a two-dimensional electron gas in intermediate magnetic fields. Within this approach, inhomogeneous states emerge in analogy to the intermediate state of a superconductor. At half filling of the highest Landau level we find unidirectional charge-density-wave (CDW) solutions. With a semiclassical calculation we give an intuitive explanation of the change of CDW orientation in the presence of an in-plane magnetic field. An anisotropy in the electron band mass is suggested as a possible source of the reproducible orientation of the CDW.

In two-dimensional electron systems both interactions and disorder have surprising and striking consequences. A strong perpendicular magnetic field quenches the kinetic energy and allows for the observation of both fractional quantum Hall effect and composite fermions in the lowest Landau level (LLL), where correlations among electrons are especially important. In higher Landau levels on the other hand, interest has mainly been devoted to reentrant localization-delocalization transitions causing the integer quantum Hall effect, with interactions thought of as modifying its non-universal features. The discovery of a pronounced resistivity anisotropy close to half filling of higher LLs with filling factor $\nu > 4$ has recently attracted attention to interaction effects in this regime. The transport anisotropy is consistent with the formation of a unidirectional CDW state which was predicted theoretically on the basis of Hartree-Fock calculations. Numerical exact diagonalization studies also support the idea of a CDW formation. The orientation of this CDW structure is related to the anisotropy observed in resistivity.

In this letter, we show that a CDW ground state arises naturally in the framework of a Chern-Simons (CS) theory. The CS approach successfully describes the fractional quantum Hall effect and composite fermions in the LLL. In intermediate LLs it is complementary to Hartree-Fock calculations which become exact in the limit of high Landau levels. Our bosonic CS theory has solutions similar to the intermediate state of superconductors and allows for an intuitive interpretation of the striped phase: in the highest, partially filled LL (HLL) domain walls separate incompressible “superconducting” stripes with local filling factor.
\( \nu_{\text{HLL}} = 1 \) from empty stripes (\( \nu_{\text{HLL}} = 0 \)) penetrated by flux. The CDW wave vector is determined by the competition of the domain-wall energy favoring a long wave length and the Hartree energy which generally increases with CDW wave length but has minima for special wave vectors due to the modulation of the electron charge density by the wave-function form factor. The CDW wave length and structure are calculated numerically within a mean-field approximation. Furthermore, we calculate the wave-function form factor in the presence of an in-plane magnetic field using a semiclassical approximation. We find that for a thin quantum well the in-plane component of the field is parallel to the axis of high resistivity, in agreement with experiments \([10, 11]\) and numerical calculations \([12, 13]\). The band mass anisotropy is shown to provide a mechanism determining the the experimentally reproducible orientation of the CDW wave vector for perpendicular fields.

Chern-Simons theory. — To employ the CS approach we first map the problem of a partially filled Nth LL onto an effective LLL problem. The scattering of electrons between different LLs can be neglected when the Bohr radius \( a_B = (4\pi\hbar^2/\alpha e)^2/(me^2) \) is much smaller than the magnetic length \( \ell = \sqrt{\hbar/eB} \), a condition satisfied in experimentally investigated systems. We describe screening by the completely filled LLs by a wave-vector dependent dielectric constant \( \epsilon(q) = \epsilon_0 \epsilon, \{1 + (2qAB)/[1 - J_0^2(qR_c)]\} \) \([13]\) with \( J_0 \) denoting the zeroth order Bessel function and \( R_c = \sqrt{2N + 1} \ell \) the cyclotron radius. The intra–LL matrix elements of the Coulomb interaction depend on the LL index \( N \) only via the Laguerre polynomial \( L_N(\epsilon) \) in the form factor of the single particle eigenstates \([15]\). One obtains the same matrix elements for states in the zeroth instead of the \( N \)-th LL but with the effective interaction \( V_{\text{eff}}(q) = e^2/2\epsilon_0q^2L_N^2(\ell^2q^2/2) \). The Laguerre polynomial can be interpreted as the quotient of the form factor of the \( N \)-th and the zeroth LL as \( L_N = F_N/F_0 \). Since all LLs span equivalent Hilbert spaces, the new interaction \( V_{\text{eff}} \) in the LLL gives a faithful representation of the original problem.

The Fermi-Dirac statistics of electrons is mimicked by bosons carrying one flux quantum \( \Phi_0 = \hbar/e \) with them. The flux quanta are attached by a singular gauge transformation \( \Phi = \Phi_0 + \phi(r) \) with a statistical gauge field \( a(r) \) tied to the density of bosons \( \rho(r) = \phi^\dagger(r)\phi(r) \) via the condition \( \nabla \times a(r) = \Phi_0 \rho(r) \). The bosonic CS Hamiltonian is given by

\[
\mathcal{H} = \int d^2r \ \phi^\dagger(r) \left[ \frac{1}{2m} (-i\hbar \nabla + e\delta a)^2 \phi(r) + \frac{i}{2} \right] \int d^2r d^2r' \ \delta \rho(r) V_{\text{eff}}(r-r') \delta \rho(r') ,
\]

(1)

with \( \delta a = A + a, \delta \rho = \phi^\dagger \Phi - \mathbf{\tau}_{\text{HLL}} B/\Phi_0 \), and \( e \) the charge of the proton.

Mean-field analysis. — We perform a mean-field (MF) analysis of the bosonic CS problem and replace the operators by a complex field. The kinetic energy (of the bosons, not to be confused with the quenched kinetic energy of the electrons) alone would be minimized by the homogeneous saddle points \( \phi = \sqrt{B}/\Phi_0, \delta a = 0 \) and \( \phi = 0, \delta a = A \) corresponding to a completely filled or empty LL, respectively. Without the interaction potential the ground state of a half filled LL would clearly be one domain with filling factor \( \nu_{\text{HLL}} = 1 \) and another domain with \( \nu_{\text{HLL}} = 0 \). However, due to the long-range nature of the Coulomb interaction, these large domains break up into smaller ones in such a way that the sum of domain-wall and Coulomb energy is minimal, and a charge-density wave is formed in analogy to the intermediate state of a superconductor. The effective field \( \nabla \times \delta a \) is expelled from the “superconducting” regions with \( \nu_{\text{HLL}} = 1 \). For average fillings \( \nu_{\text{HLL}} = 1 - \epsilon \) close to one on the other hand, the solution with lowest energy is given by a flux-tube array in analogy to phases of type-I superconductors in the intermediate state, where each flux tube can carry a single or multiple flux quanta. For the electron problem this is equivalent to the formation of a Wigner crystal.

To make the physics of the mean-field solution transparent, we express the wave function by amplitude and phase, introduce a gauge invariant “velocity” \( Q \), and express the boson current
density $j$ in these new variables

$$\phi(r) = \sqrt{\frac{B}{\Phi_0}} f(r) e^{i \theta(r)}, \quad Q(r) = \ell^2 \left( \nabla \theta + \frac{2\pi}{\Phi_0} \delta a \right), \quad j(r) = -\frac{e \omega_c}{2\pi \ell^2} f^2 Q(r). \quad (2)$$

We enforce the constraint on $a(r)$ with the help of a Lagrange multiplier $\lambda$ (which is the rescaled zero component of the CS field) and find the saddle-point equations

$$- f'' + \frac{1}{\ell^4} Q^2 f + 2(u_H - \lambda) f = 0, \quad \nabla \times Q = (v + f^2 - 1) \hat{z}, \quad \nabla \lambda = \frac{1}{\ell^4} f^2 Q \times \hat{z} \quad (3)$$

with the vortex density $v(r) = \ell^2 \nabla \times \nabla \theta(r)$. The Hartree potential $u_H$ is expressed in units of $\ell^2 \hbar \omega_c$. One immediate consequence of the saddle-point equations is the screening of an electrostatic potential by the CS superconductor in analogy to the screening of a vector potential by a conventional superconductor. A negative potential causes a reduction of $\nu_{\text{HLL}} = f^2$ from one and induces a current via the second part of Eq. (3). This argument can be made quantitative by describing the current response to an electric field with the help of the Hall conductivity $\sigma_{\text{H}} = e^2 / h$ as $j = \sigma_{\text{H}} E \times \hat{z}$. From the third part of Eq. (3) one finds $\nabla \lambda = (e / h \omega_c \ell^2) E$ leading to an exact cancellation of the external potential in the Schrödinger equation for the superfluid density $f^2$. Via this mechanism a Bose condensation in a spatially varying external potential is possible. Note that the bosonic wave functions are not restricted to the LLL in the variational approach. This is an valid approximation in the limit where the electron-electron interaction is weak compared to the inter-Landau-level spacing and a mixing of Landau levels is negligible. The same assumption is used also in the Hartree-Fock approaches in order to justify that the ground state does not involve single-particle states above the partially filled highest LL. Since on the scale $\ell$ of the typical particle distance the strength of the bosonic interaction $V_{\text{eff}}$ is comparable to that of the original fermionic interaction (for $N$ not too large as in experiments), the ground state of the bosonic problem will lie to a good approximation within the LLL, even if we search in an extended space of functions including higher LLs. Dropping the variational constraint in the bosonic problem therefore induces inaccuracies of the same order of magnitude as ignoring the Landau-level mixing in the fermionic problem. As can be seen from the variational result below (Fig. 1), the local filling factor overshoots only by a few percent over one, which indicates that the solution lies to a very good approximation within the LLL although the constraint is not imposed.

**1D numerical solution.** — In order to confirm the physical picture developed above and to provide a comparison of our CS approach with previous Hartree-Fock approaches, we have numerically determined the MF solution at half filling ($\nu_{\text{HLL}} = \frac{1}{2}$). This was done approximately by assuming a unidirectional modulation, i.e., that the charge density is modulated with a period $\Lambda$ in $x$ direction and is constant in $y$ direction. In this approximation, vortices cannot be taken into account as point-like objects. Instead, all vortices within a period have to be concentrated on lines, $u(r) = \sum_n \frac{1}{\Lambda} \delta(x - (n + 1/2)\Lambda)$. For symmetry reasons, currents then flow only in $y$ direction, $Q(r) = Q_y(x)e_y$. To obtain the MF solution, we use a variational ansatz $f(x) = \sum_k f_k \cos(2\pi kx / \Lambda)$ with the constraint $f((n + 1/2)\Lambda) = 0$ at the vortex positions. In a first step, we minimize $H$ for given $\Lambda$ by varying the coefficients $f_k$ with the constraints of half filling and $\partial_x Q_y(x) = v + f^2 - 1$. Then the resulting energy $H(\Lambda)$ is minimized as a function of $\Lambda$ to obtain the CDW period. The choice of $a_B = \ell / \sqrt{\pi}$ fixes the interaction strength and is representative for experiments. A full 2D numerical solution and the analysis of gauge field fluctuations is beyond the scope of the present analysis.

We present our results for filling factors $\nu = 2N + \frac{1}{2}$ in the Landau levels $N = 8$ and $N = 2$. The structure of the MF solutions is depicted in Fig. 1. For $N = 8$, $f^2$ shows a clear separation between full ($f^2 \approx 1$) and empty ($f^2 \approx 0$) regions. The width of the transition...
between the full and empty region is of order $\ell$. Therefore, the separation between full and empty regions is blurred out for smaller $N$ (for $N=2$, see Fig. 1b). The actual charge and charge-current density, which are obtained after convoluting $f^2$ and $f^2 Q_y$ with the appropriate form factors are much smoother than $f^2$ and vary only on the scale of $R_c$. The charge density within this LL shows a relative variation of about 20% for $N=8$. This magnitude is roughly consistent with Hartree-Fock calculations \cite{4,5} for large $N$. For $N=2$ the charge density shows a stronger modulation of about 60%. Thus, although charge and charge-current density are modulated only on the scale $\Lambda$ for all $N$, the relative variation of charge density is found to increase with decreasing $N$.

The energy per electron as a function of the periodicity is displayed in Fig. 2. The energy scale in our saddle-point approximation is the cyclotron energy. We expect that gauge-field fluctuations will renormalize it to the Coulomb energy $e^2/4\pi\epsilon_0\epsilon_r \ell$. The optimal periodicity $\Lambda_N$, given by the global minimum of this function, is found at $\Lambda_8 \approx 3.1R_c$ and $\Lambda_2 \approx 3.9R_c$. Thus, the periodicity is larger than expected from the first zero of the form factor at $\Lambda \approx 2.6R_c$ in agreement with HF calculations \cite{4,5}. However, we find that $\Lambda_N/R_c$ increases notably with decreasing $N$. In our approach this effect can be understood from the increasing weight of the “kinetic” or domain-wall contribution within the CS energy.

**Tilted magnetic field.** — So far, we have studied the formation of a CDW by electrons in high Landau levels within the CS approach. In experiments \cite{1,10} the longitudinal resistivity appears to be systematically higher in [110] directions as compared to [110] directions. This raises the question of what mechanism breaks the fourfold crystal symmetry and orienting the CDW wave vector parallel to the [110] directions. For this reason resistivity was studied in **tilted** magnetic fields \cite{10,11} and a sufficiently strong in-plane component $B_\parallel$ of the magnetic field was found to orient the directions of large resistivity parallel to $B_\parallel$. We present a mechanism for the coupling between the CDW wave vector and $B_\parallel$ that involves the dynamics of the electrons in the $z$ direction perpendicular to the plane. We model the confinement of the electrons in this direction by a harmonic potential with eigenfrequency $\omega_0$. While similar models were studied recently in the framework of a Hartree-Fock approach \cite{12,13} and of Laughlin-like states \cite{16}, we use a simple semiclassical approach to obtain the anisotropic form factor of cyclotron orbits in tilted fields, which then can easily be combined with the theory of untitled fields.

The classical equations of motion for a single electron subject to a tilted magnetic field
Fourier transforms of the density frequency $\omega$ we consider only the projection of the orbits onto the $xy$ plane, which typically are small compared to the cyclotron frequencies associated with the in-plane and out-of-plane components of the axes of lengths $n$, choose the ground state, factorizes into two contributions from the normal modes, $T$, level, $n$, and energy. At small momenta, the zeros of $F$ have a larger radius. If one estimates the CDW periodicity from the location of the zeros of $F_-(\mathbf{q})$ one finds the periodicity proportional to the radius of the cyclotron ellipse in the direction of the CDW wave vector. The total energy of the CDW is then minimized if its wave

\[ \mathbf{B} = (B_{||}, 0, B_\perp) \] and a harmonic confining potential are linear and possess two eigenmodes with frequencies $\omega_0^2 = \frac{1}{2}(\omega_x^2 + \omega_y^2 + \omega_z^2) \pm \frac{1}{2}\sqrt{(\omega_x^2 + \omega_y^2 + \omega_z^2)^2 - 4\omega_x^2\omega_y^2}$, where $\omega_x$ and $\omega_y$ are the cyclotron frequencies associated with the in-plane and out-of-plane components of the magnetic field, which typically are small compared to $\omega_0$. For vanishing $B_{||}$, the modes with frequency $\omega_0^2 \approx \omega_0^2[1 + \omega_0^2/(\omega_x^2 - \omega_y^2)]$ or $\omega_0^2 \approx \omega_0^2[1 - \omega_0^2/(\omega_x^2 - \omega_y^2)]$ become a pure oscillation in $z$ direction or a cyclotron orbit in the $xy$ plane, respectively. We determine the semiclassical orbits from the Bohr quantization condition $\oint p_\sigma \cdot r_\sigma = 2\pi \hbar (\frac{1}{2} + n_\sigma)$, where $\sigma = \pm$ and we choose the ground state, $n_+ = 0$, for the oscillator-like mode and the and the $N$th Landau level, $n_- = N$, for the cyclotron-like mode. This choice is unique for $\omega_0 > \omega_x$. Subsequently, we consider only the projection of the orbits onto the $xy$ plane, which are ellipses with half axes of lengths

\[
R_{x\sigma}^2 = (1 + 2N_\sigma) \frac{\hbar}{m\omega_\sigma} \frac{\omega_x^2}{\omega_x^2 + \omega_\sigma^2\omega_\sigma^2/(\omega_0^2 - \omega_\sigma^2)^2}, \quad R_{y\sigma}^2 = \frac{\hbar}{m\omega_\sigma} \frac{\omega_x^2}{\omega_x^2 + \omega_\sigma^2\omega_\sigma^2/(\omega_0^2 - \omega_\sigma^2)^2}.
\]

The cyclotron-like ellipse is larger than the oscillator-like ellipse ($R_{\alpha-} > R_{\alpha+}$). In addition, the cyclotron-like ellipse is longer in $x$ direction than in $y$ direction ($R_{x-} > R_{y-}$), whereas for the oscillator-like ellipse the opposite holds true (cf. Fig. 2a). Because of this contrary deformation the resulting anisotropy of the form factor is delicate. The form factor $F(\mathbf{q})$ is the Fourier transform of the density $\rho(\mathbf{r})$ of finding the electron at a position $\mathbf{r} = \mathbf{r}_-(t) + \mathbf{r}_+(t)$. It factorizes into two contributions from the normal modes, $F(\mathbf{q}) = F_-(\mathbf{q})F_+(\mathbf{q})$, which are the Fourier transforms of the density $\rho_\pm(\mathbf{r}) = \frac{1}{T_\pm} \int_{0}^{T_\pm} dt \delta(\mathbf{r} - \mathbf{r}_\pm(t))$ of the normal mode orbits, where $T_\pm = 2\pi/\omega_\pm$ is the orbit period.

In the presence of the tilted field the form factor is anisotropic and we now discuss the location of its first zero at small momenta, which essentially determines the CDW periodicity and energy. At small momenta, the zeros of $F$ are those of $F_-$ since the cyclotron-like orbits have a larger radius. If one estimates the CDW periodicity from the location of the zeros of $F_-(\mathbf{q})$ one finds the periodicity proportional to the radius of the cyclotron ellipse in the direction of the CDW wave vector. The total energy of the CDW is then minimized if its wave
in the absence of a tilt must lead to an energy gain of the same magnitude. We now examine two possible mechanisms.

We now examine the collective pinning mechanism. Variations of the interface height in the normal state, the direction of the hard axis is not necessarily identical and therefore is determined by collective pinning, whereas it is determined by single-electron scattering in the normal state. This normal anisotropy, which has been reported for some of the samples studied in [1], can be ascribed to scattering processes caused by an interface roughness with anisotropic correlation lengths \( \xi_{[110]} > \xi_{[10]} \). The observed surface morphology in [18] is consistent with such an anisotropic interface roughness. Since the anisotropy in the CDW state is determined by collective pinning, whereas it is determined by single-electron scattering in the normal state, the direction of the hard axis is not necessarily identical and therefore we now examine the collective pinning mechanism. Variations of the interface height \( d \) result in an electrostatic potential \( \phi(q) = \frac{1}{2} n q n d(q) / \epsilon(q) \). Here \( n \) is the density of positive donors and the height fluctuations are assumed to be Gaussian correlated, \( \langle [d(q)]^2 \rangle_{av} = 2 \pi d_0^2 \xi_x \xi_y \exp[-(q_x^2 \xi_x^2 + q_y^2 \xi_y^2)/2] \) with correlation lengths \( \xi_x, \xi_y \gg \Lambda \). For a typical height \( d_0 \approx 2.8 \) Å of the order of a GaAs monolayer. Pinning leads to an energy gain via local displacements of the CDW profile which are governed by a smectic elastic response [6]. From the curvature of the CDW energy as a function of the wave length we estimate the compression modulus \( K_x \approx 10^{11} \) eV m\(^{-2} \) and the bending modulus \( K_y \approx (\Lambda/4\pi)^2 K_x \). For parameters representing the experiments [6, 2] we estimated the pinning energy density in a linear response calculation [5].

\[
E_{\text{rough}} \approx - \left( \frac{n \pi \epsilon}{2 \hbar \Lambda} \right)^2 \left( \frac{d^2}{2\pi \Lambda} \right)^2 \int \frac{d^2q}{(2\pi)^2} \left[ \langle \phi(q_x + 2\pi/\Lambda, q_y) \rangle^2 \right]_{av} \frac{K_x q_x^2 + K_y q_y^2}{K_x q_x^2 + K_y q_y^2}.
\]

Note that the pinning strength is given by the disorder correlator near the wave vector \( 2\pi/\Lambda \) and therefore it is exponentially small, \( \langle [\phi(q_x + 2\pi/\Lambda, q_y)]^2 \rangle_{av} \propto e^{-2(\pi \xi_x/\Lambda)^2} \). For \( \xi_x \gg \Lambda \) this exponential dependence suppresses the pinning energy per particle by orders of magnitude below 1mK per particle as determined above from the threshold tilt angle. Therefore interface roughness is irrelevant for the CDW orientation. In addition it would imply that the CDW wave vector is aligned in the direction of the shorter disorder correlation length, i.e., the [110] direction [1, 13] in contradiction to experiments.

As pointed out in Ref. [20] the electric field \( E = e n / 2 e_0 \xi_r \) in [001] direction between the electron system and the donor layer provides a different symmetry breaking mechanism. We now show that this field generates an anisotropic electronic band mass which induces the observed CDW orientation by an energy gain on the appropriate scale. Since Ga and As carry opposite partial charges they are displaced such that the GaAs bonds in [111] direction are stretched whereas the bonds in [111] direction are shortened. From pressure experiments [21] it is known that a shortening of bonds leads to an increase of the effective band mass \( m^* \). We estimate the effective mass changes \( m^*_{[110]} / m^*_{[110]} - 1 \approx \pm 10^{-4} \) by identifying the electric field with an effective pressure [22]. This anisotropic band mass affects the CDW
via the form factor. A semiclassical analysis implies elliptic cyclotron orbits with half axes \( R_{[110]/[110]} / R_x \sim 1 \sim -10^{-4} \) which leads to an increased CDW period \( \Delta_{[110]/[110]} / \Lambda \approx 10^{-4} \).

In analogy to the analysis of the in-plane field this implies that the mass anisotropy prefers the [110] orientation over the [110] orientation by about 1mK per electron in agreement with the experimentally observed orientation and the energy scale derived from the tilt experiments.

**Conclusion.** — We have developed a bosonic Chern-Simons theory to investigate the formation of a CDW state in intermediate Landau levels and the influence of an anisotropic interface potential on the CDW orientation. In a semiclassical calculation we have found that the CDW wave vector is parallel to an in-plane magnetic field for quantum wells with a hard confining potential. The band mass anisotropy is a possible origin for the CDW orientation in perpendicular fields.

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