A new method for calculating single-point productivity equation coefficient $\alpha$ in heterogeneous gas reservoir

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Abstract. Single-point test is mainly based on quantitative statistics of productivity well testing data to obtain average $\alpha$ value of gas fields or blocks, then corresponding single-point productivity equation can be obtained. However, for heterogeneous gas reservoir in Ordos Basin, because of the influences of reservoir physical property and test data, there are relatively big error between calculated average $\alpha$ value of gas field and actual $\alpha$ value of single gas well, which results in application of single-point empirical formula is limited. Through derivation of binomial productivity equation in this paper, it is found that there are good linear relationship between $\alpha$ and reciprocal of the square of pressure $1/p_R^2$. Combined with actual test data of Jingbian gas field, the empirical formula between $\alpha$ and $1/p_R^2$ for gas wells is got through regression analysis. The results indicate that new method greatly improves computational accuracy for absolute open flow of gas wells, which has good application prospect in similar gas field.

1. Introduction

Regular productivity well testing include systematic well test, isochronal test, modified isochronal test and single-point test[1-4]. Due to low permeability of lithologic gas reservoir, strong heterogeneity, difference of single well production capacity and large amount of gas wells, regular multipoint production well test are restricted in Ordos Basin. Thus, single-point test is applied in most of gas wells[5-7].

Empirical formula of single-point test is based on large quantity of steady well test data. In another word, single-point productivity equation is derived from average $\alpha$ which is based on reliable productivity equation and corresponding absolute open flow. Generally, the more stable well testing data of gas field, the more representative single-point productivity equation will be[8-10]. However, because of strong heterogeneity, there are big errors between average $\alpha$ which is obtained from a gas block by statistics data and single wells. Considering above problems, in order to accurately forecast gas well productivity, it is essential to find a new method to improve computational accuracy.
2. Derivation of basic theory

2.1 Single-point productivity equation

Making production of gas wells reach steady state under single working system, single-point test can obtain absolute open flow through substituting reservoir pressure, gas production and corresponding steady flowing bottom pressure into empirical productivity formula.

Binomial productivity equation is defined as follow:

\[
P_R^2 - P_{wf}^2 = Aq_g + Bq_g^2
\]

(1)

where, the coefficient A expresses viscous resistance and coefficient B expresses inertia resistance along flow path, which can all be got through high pressure physical parameters of reservoir and fluid[11-12]. When \( p_{wf} = 0.101\) MPa (1 atm), the maximum potential production capacity is absolute absolute open flow:

\[
p_R^2 - (0.101)^2 = Aq_{AOF} + Bq_{AOF}^2
\]

(2)

Combination Eq. (1) and Eq. (2), this correlation is given in Eq. (3)

\[
\frac{p_R^2 - P_{wf}^2}{P_R^2} = \frac{Aq_g + Bq_g^2}{Aq_{AOF} + Bq_{AOF}^2}
\]

(3)

where

\[
q_D = \frac{q_g}{q_{AOF}}
\]

(4)

\[
p_D = \frac{P_R^2 - P_{wf}^2}{P_R^2}
\]

(5)

\[
\alpha = \frac{A}{A + Bq_{AOF}}
\]

(6)

Through derivation, single-point productivity equation can be written as:

\[
q_{AOF} = \frac{2(1 - \alpha)q_g}{\alpha \sqrt{1 + 4 \left(1 - \frac{\alpha}{\alpha^2} \left(\frac{P_R^2 - P_{wf}^2}{P_R^2}\right) - 1\right)}}
\]

(7)

2.2 Analysis of influence factor

Binomial productivity equation also can be given as[13-15]:

\[
p_R^2 - P_{wf}^2 = \frac{1.291 \times 10^{-3} q_{sw} T \mu z}{kh} \ln (\frac{0.472 r_T + S'}{r_w})
\]

(8)

where

\[
S' = S + Dq_{sw}
\]

(9)

\[
D = 2.191 \times 10^{-16} \frac{\beta r_g K}{\mu h r_w}
\]

(10)

\[
\beta = 7.644 \times 10^{10} \frac{1.5}{k^{1.5}}
\]

(11)

Combination Eq. (8) , Eq. (9) , Eq. (10) and Eq. (11), Eq. (12) can be written as:

\[
p_R^2 - P_{wf}^2 = \frac{1.291 \times 10^{-3} q_{sw} T \mu z}{kh} \ln (\frac{0.472 r_T + S'}{r_w}) + \frac{2.828 \times 10^{-21} \beta r_g z T q_{sw}^2}{r_w h^2}
\]

(12)

where
\[ A = \frac{1.291 \times 10^{3} T \mu z}{kh} \ln \frac{0.472 r_{w}}{r_{w}} + S \]  
\[ B = \frac{2.828 \times 10^{-21} \beta \gamma s \bar{Z} T}{r_{w} h^{2}} \]

For dry gas reservoir, following equation can be used to calculate production rate

\[ q_{sc} = \frac{774.6kh(p_{R}^{2} - p_{wf}^{2})}{T \mu z \ln \frac{0.472 r_{w}}{r_{w}} + s'} \]  

In fact, wellbore radius, gas viscosity, reservoir temperature, Z-factor and relative gas density is almost the same in the same gas reservoir if ignoring skin effect caused by turbulence flow. Combination Eq.(6), Eq.(13), Eq.(14) and Eq.(15), following simplified formula can be obtained:

\[ \alpha = \frac{1}{1 + C_{1}^{0.5} p_{R}^{2}} \]  

In low-permeability gas reservoir, permeability is related to formation pressure, especially under the condition of stress sensitive effect[16-17]. According to Eq. (16), there is some correlation between \( \alpha \) and permeability as well as reservoir pressure. Actually, coefficient \( \alpha \) is an empirical parameter, which is various in different gas wells or blocks along changes of reservoir characteristics[18-19]. For gas wells in Ordos Basin, it is found that relatively to reservoir pressure, the error of permeability \( k \) is big and the influence on result is small. Thus, Eq. (17) can be written as

\[ \alpha = \frac{1}{1 + C_{2} p_{R}^{2}} \]  

\( P_{R} \) and \( \alpha \) are determined by actual test data of gas field. Empirical formula about reciprocal of pressure square \( 1/p_{R}^{2} \) and \( \alpha \) can be obtained through regression analysis of series of \( P_{R} \) and \( \alpha \) value for every well.

3. Field examples

Based on test data of 20 gas wells in Jingbian gas field, the value of \( \alpha \) can be calculated from static pressure(\( p_{R} \)), \( p_{wf}, q_{sc}, q_{AOF} \). Range of \( \alpha \) value in low-permeability and high-permeability area are 0.548~0.853 and 0.218~0.781, with average value of 0.684 and 0.495 respectively (Table 1).

Calculation results show that \( \alpha \) is variable for each well. Average error in low-permeability and high-permeability area is 13.3% and 41.9%, with range of 3.4%~23.7 and 15.5%~127.1% respectively.

From Eq.(7), it is found that it is relevant between \( \alpha \) and reservoir pressure. Through regression analysis of \( \alpha \) value and \( p_{R} \) in 20 gas wells, it is shown that as decrease of \( 1/p_{R}^{2} \), value of \( \alpha \) will increase, with good linear relation.

Thus, \( \alpha \) can be calculated through \( 1/p_{R}^{2} \) which is proposed as follow.

For \( \alpha \) in low-permeability area (Figure 1): \( \alpha = 1820.3/P^{2} - 1.2645 \)  

For \( \alpha \) in high-permeability area (Figure 2): \( \alpha = 2260.9/P^{2} - 1.8488 \)
### Table 1. Statistic results of productivity well testing

| Reservoir types   | Wells | $q_e$ $(10^4 \text{m}^3/\text{d})$ | $P_{wf}$ (MPa) | $P_R$ (MPa) | $q_{AOF}$ $(10^4 \text{m}^3/\text{d})$ | $\alpha$ | Result | Absolute error(%) |
|-------------------|-------|-----------------------------------|----------------|-------------|--------------------------------------|---------|--------|-------------------|
| **Low permeability** | X1    | 1.49                              | 22.8           | 29.9        | 3.26                                 | 0.851   | 23.5   |                   |
|                   | X2    | 2.00                              | 28.7           | 31.3        | 8.40                                 | 0.548   | 18.8   |                   |
|                   | X3    | 2.00                              | 28.3           | 30.7        | 10.77                                | 0.768   | 15.2   |                   |
|                   | X4    | 4.00                              | 27.1           | 31.0        | 12.99                                | 0.674   | 3.4    |                   |
|                   | X5    | 4.00                              | 28.4           | 31.3        | 15.87                                | 0.601   | 8.4    |                   |
|                   | X6    | 2.50                              | 29.5           | 30.9        | 18.58                                | 0.585   | 11.3   |                   |
|                   | X7    | 3.00                              | 27.4           | 29.0        | 24.36                                | 0.853   | 23.7   |                   |
|                   | X8    | 2.80                              | 28.7           | 30.3        | 19.69                                | 0.677   | 3.8    |                   |
|                   | X9    | 3.50                              | 28.2           | 31.2        | 13.01                                | 0.563   | 15.6   |                   |
|                   | X10   | 4.50                              | 29.0           | 30.3        | 40.13                                | 0.717   | 9.2    |                   |
|                   | Average | 2.98                          | 27.8           | 30.6        | 16.71                                | 0.684   | 13.3   |                   |
| **High permeability** | X11   | 3.01                              | 29.5           | 31.1        | 23.15                                | 0.781   | 36.5   |                   |
|                   | X12   | 4.00                              | 30.0           | 30.8        | 34.98                                | 0.430   | 15.5   |                   |
|                   | X13   | 8.01                              | 26.7           | 29.5        | 36.46                                | 0.761   | 34.8   |                   |
|                   | X14   | 5.97                              | 31.0           | 32.1        | 37.50                                | 0.332   | 49.5   |                   |
|                   | X15   | 8.00                              | 28.3           | 30.6        | 43.00                                | 0.715   | 30.7   |                   |
|                   | X16   | 15.03                             | 30.8           | 32.6        | 58.19                                | 0.218   | 127.1  |                   |
|                   | X17   | 9.85                              | 29.3           | 30.5        | 65.17                                | 0.419   | 18.3   |                   |
|                   | X18   | 9.99                              | 31.2           | 32.0        | 78.32                                | 0.309   | 60.6   |                   |
|                   | X19   | 4.60                              | 30.5           | 31.8        | 27.93                                | 0.385   | 28.8   |                   |
|                   | X20   | 4.10                              | 29.3           | 30.0        | 55.71                                | 0.597   | 16.9   |                   |
|                   | Average | 7.26                          | 29.6           | 31.1        | 46.04                                | 0.495   | 41.9   |                   |

**Figure 1.** $\alpha$ vs. $1/P_R^2$ in low-permeability area
The error of new method is obviously smaller than traditional method, where error decline from 13.3% to 7.0% and 41.9% to 18.2% in low and high permeability area respectively (Table 2, Figure 3 and Figure 4). Contrasting with traditional method, there are high accuracy for calculating absolute open flow of gas well with new method, which has good application prospects in similar gas field.

Figure 2. $\alpha$ vs. $1/pR^2$ in high-permeability area

Figure 3. Bar graph for error of $\alpha$ in low-permeability area

Figure 4. Bar graph for error of $\alpha$ in high-permeability area
Table 2. Contrast results of traditional and new methods

| Reservoir types | Wells | Traditional method ($\alpha$) | New method ($\alpha$) |
|-----------------|-------|-------------------------------|-----------------------|
|                 |       | Results | Absolute error(%) | Results | Absolute error(%) |
| Low permeability| X1    | 0.851   | 23.5       | 0.769   | 9.6         |
|                 | X2    | 0.548   | 18.8       | 0.599   | 9.4         |
|                 | X3    | 0.768   | 15.2       | 0.666   | 13.2        |
|                 | X4    | 0.674   | 3.4        | 0.626   | 7.1         |
|                 | X5    | 0.601   | 8.4        | 0.591   | 1.6         |
|                 | X6    | 0.585   | 11.3       | 0.643   | 9.9         |
|                 | X7    | 0.853   | 23.7       | 0.900   | 5.5         |
|                 | X8    | 0.677   | 3.8        | 0.718   | 6.1         |
|                 | X9    | 0.563   | 15.6       | 0.605   | 7.5         |
|                 | X10   | 0.717   | 9.2        | 0.718   | 0.2         |
|                 | Average | 0.684 | 13.3       | 0.684   | 7.0         |
| High permeability| X11  | 0.781   | 36.5       | 0.482   | 38.3        |
|                 | X12  | 0.430   | 15.5       | 0.527   | 22.7        |
|                 | X13  | 0.761   | 34.8       | 0.755   | 0.8         |
|                 | X14  | 0.332   | 49.5       | 0.345   | 4.1         |
|                 | X15  | 0.715   | 30.7       | 0.564   | 21.2        |
|                 | X16  | 0.218   | 127.1      | 0.278   | 27.1        |
|                 | X17  | 0.419   | 18.3       | 0.589   | 40.4        |
|                 | X18  | 0.309   | 60.6       | 0.358   | 15.8        |
|                 | X19  | 0.385   | 28.8       | 0.387   | 0.5         |
|                 | X20  | 0.597   | 16.9       | 0.663   | 11.1        |
|                 | Average | 0.495 | 41.9       | 0.495   | 18.2        |

4. Conclusions

(1) Because of influences of reservoir property and systematic well testing data, value of $\alpha$ in traditional single-point productivity equation is different from actual $\alpha$ value of every gas well, which results in error generation of absolute open flow.

(2) There are some correlation between $\alpha$ value and formation pressure. Empirical formula calculating $\alpha$ value in high and low permeability area in Jingbian gas field are established.

(3) Contrasting with traditional method, calculation error of new method is obviously smaller, which declines from 13.3% to 7.0% in low permeability area, from 41.9% to 18.2 % in high permeability area respectively. New method can effectively improve calculation accuracy of $\alpha$ value and absolute open flow.

Nomenclature

$q_g$—wellhead production rate of gas well, $10^4 m^3/d$;
$q_{AOF}$—absolute open flow, $10^4 m^3/d$;
$k$—permeability, mD;
$h$—thickness of gas reservoir, m;
$p_R$—reservoir pressure, MPa;
$p_{wf}$—flow bottom hole pressure, MPa;
$T$—reservoir temperature, K;
$\mu$—gas viscosity, mPa·s;
References

[1] Wang Cai, Li Zhi-Pin, Lai Feng-Peng 2014 A novel binomial deliverability equation for fractured gas well considering non-Darcy effects. *Journal of Natural Gas Science and Engineering* 20(9) 27-37.

[2] Peng Chaoyang 2010 Discussion on equation application of deliverability with different form for gas well. *Natural Gas Geo-science* 21(1) 172-174.

[3] Li Zuyou, Yang Xiaobi, Luo Dongming 2008 Binomial deliverability equation of high pressure gas well. *Special Oil and gas Reservoirs* 15(3) 62-64.

[4] Yang Chengbo, Guo Jianchun, Yang Jian, et al 2014 Modification of horizontal well binomial deliverability equation of low permeability gas reservoir. *Journal of Southwest Petroleum University (Science & Technology Edition)* 36(4) 123-130.

[5] Lu Yiqun 2008 Application of one point gas testing method in Northeast of Sichuan area. *Well Testing* 17(5) 34-36.

[6] Han Huiling, Jiang Jianfang, YangYufeng, et al 2008 Research on application of one-point method about rapid gas production testing in shanbei gas fields. *Well Testing* 17(5) 17-19.

[7] Hu Jianguo, Zhang Zonglin, Zhang Zhenwen 2008 A new method on processing the data from one-point deliverability test in gas fields. *Natural Gas Industry* 28(2) 111-113.

[8] Hu Junkun, Li Xiaoping, Xiao Qiang, et al 2013 A new method of using dynamic data to determine deliverability equation of gas well. *Natural Gas Geoscience* 24(5) 1027-1031.

[9] Qiao Zhiguo, Shuai Jianjun, Dong Hai Feng, et al 2014 Deliverability equation with dynamic one-point method for Changxing formation, Yuanba gas field. *Natural Gas Technology and Economy* 8(5) 25-27.

[10] Deng Hui, Feng Xi, Yang Xuefeng, et al 2014 A new method of predicting gas wells deliverability in Longgang reef gas reservoir. *Natural Gas Geoscience* 25(9) 1451-1454.

[11] Yuan Yingzhong, Zhang Lianhui, Wang Jian, et al 2009 A binomial deliverability equation for horizontal gas wells in formations with nonlinear seepage flow features. *Oil & Gas Geology* 30(1) 122-126.

[12] Liu Yongliang, Yuan Yingzhong, Deng Lijing, et al 2016 A new method for binomial deliverability equation of horizontal gas well. *Natural Gas Geoscience* 27(2) 371-376.

[13] Tabatabaei M, Zhu D 2010 Generalized inflow performance relationships for horizontal gas wells. *Journal of Natural Gas Science and Engineering* 2(2-3) 132-142.

[14] Borsil, Fusi L, Rosso F, et al 2011 A well deliverability model for non-Darcian flow in geothermal reservoirs. *Computers & Geosciences* 37(10) 1555-1561.

[15] Kalantariasl A, Farhadi I, Nasriani H R 2013 On the accuracy of dimensionless inflow performance relationships for gas wells. *SPE paper* 164602.

[16] Zhang Hongxue, Liu Weiqun, Zhu Li 2014 A permeability model for changes during shale gas production. *Electronic Journal of Geotechnical Engineering* 19.X 3847-3860 Available at ejge.com.
[17] Hai Tang, Hongyuan Li, Dongliang Lv 2014 Relationship between stress sensitivity and productivity rate in low permeability reservoir. *Electronic Journal of Geotechnical Engineering* **19.V** 6619-6625 Available at ejge.com.

[18] Feng Xi, Zhong Fuxun, Wang Hao, et al 2005 Modified single point method to evaluate productivity of gas wells with big production for feixianguan group gas reservoirs in northeast Sichuan. *Natural Gas Industry* **25(supplement A)** 107-109.

[19] Sun Zhidao, Hu Yongle, Fang Xisheng, et al 2011 Applicable scope of the single-point test method for the evaluation of gas well deliverability. *Natural Gas Industry* **31(11)** 63-65.