The QCD analysis of the CCFR data for $x F_3$: higher twists and $\alpha_s(M_Z)$ extractions at the NNLO and beyond

A.L. Kataev
Institute for Nuclear Research of the Academy of Sciences of Russia,
117312 Moscow, Russia,

G. Parente
Department of Particle Physics, University of Santiago de Compostela,
15706 Santiago de Compostela, Spain

A.V. Sidorov
Bogoliubov Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research,
141980 Dubna, Russia

Abstract

The improved next-to-next-to-leading order (NNLO) QCD analysis of the experimental data of the CCFR collaboration for the $x F_3$ structure function is made. Theoretical ambiguities of the NNLO fits are estimated by means of the Padé resummation technique, which was applied both in the expanded and non-expanded forms. The NNLO and the new N$^3$LO $\alpha_s(Q^2)$ $\overline{MS}$-matching conditions are used. In the process of the fits we are taking into account the target mass corrections and the twist-4 $1/Q^2$-terms. Our NNLO results for $\alpha_s(M_Z)$ values, extracted from the CCFR $x F_3$ data, are $\alpha_s(M_Z) = 0.118 \pm 0.002(stat) \pm 0.005(syst) \pm 0.003(theory)$ provided the twist-4 contributions are fixed through the infrared renormalon model and $\alpha_s(M_Z) = 0.116^{\pm 0.009(stat)} \pm 0.005(syst) \pm 0.004(theory)$ provided the twist-4 terms are considered as the free parameters. It is shown that the extracted at the NNLO order $x$-shape of the twist-4 correction is almost unchanged after application of the Padé resummation approximant.
1. Introduction.

Deep-inelastic lepton-nucleon scattering (DIS) belongs to the classical and continuously studying processes in the modern particle physics. The traditionally measurable characteristics of $\nu N$ DIS are the SFs $F_2$ and $xF_3$. It should be stressed that the program of getting the information about the behavior of the SFs of $\nu N$ DIS is among the aims of the experimental program of Fermilab Tevatron and CCFR/NuTeV collaboration in particular. The CCFRR collaboration started to study the $\nu N$ scattering process over 1980 year [1]. The data for the SFs of $\nu N$ DIS, obtained by the follower of the CCFRR collaboration, namely CCFR group, was distributed among the potential users in the beginning of 1997 [3], while the final results of the original CCFR DGLAP NLO analysis of this data was presented in the journal publication of Ref.[2].

This experimental information was already used in the process of different NLO analysis, performed by CITEQ, MRST and GRV groups (see Refs.[4, 5, 6] correspondingly). The subsequent steps of performing NLO and first NNLO analysis of the CCFR data with the help of the Jacobi polynomial - Mellin moments version of the DGLAP method were made in Refs.[7]-[12] (the definite stages in the development of this formalism are described in Refs.[13]-[15]).

In the process of the analysis of Refs.[9]-[11] the authors used the information about the NNLO corrections to the coefficient functions [14] and the available analytical expressions for the NNLO corrections to the anomalous dimensions of the NS moments with $n = 2, 4, 6, 8, 10$ [17], supplemented with the given in Ref.[1] $n = 3, 5, 7, 9$ similar numbers, obtained using the smooth interpolation procedure, which was previously proposed in Ref.[18]. Moreover, the attempts to extract the shape of the twist-4 contributions and study the predictive abilities of the IR-renormalon (IRR) model of Ref.[19] were performed (for the definite details of modeling the effects of the power-suppressed contributions to measurable physical quantities using the IRR language see Ref.[20] and the review of Ref.[21]).

However, the important question of the estimation of theoretical uncertainties of the NNLO analysis of the CCFR data of Ref.[11] was still non-analyzed in detail. These uncertainties are determined by

1) the differences in the definitions of $\alpha_s(Q^2)$ matching conditions (see e.g. [22, 23, 24]) which are responsible for penetrating into the energy region, characteristic for $f = 5$ numbers of flavours, where the pole of the $Z^0$-boson is manifesting itself;

2) the incorporation into the condition of Ref.[22] recalculated NNLO QCD corrections [25] and the newly calculated N$^3$LO corrections, namely the 4-loop coefficient of the QCD $\beta$-function [26] and the N$^3$LO-term [27] in the matching condition of Ref.[22];

3) the consideration of theoretical uncertainties due to other non-calculated N$^3$LO contributions into the coefficient functions and the anomalous dimensions function.

This work is devoted to the analysis of the important problems outlined above and to the more detailed extraction of the values of $\alpha_s(M_Z)$ and the $x$-shape of the twist-4 power-suppressed term at available orders of perturbative QCD with taking into account the effects enumerated above. We are supplementing the NNLO fits of Ref.[11] by the N$^3$LO analysis, which is based on the application of the Padé resummation technique (for the review see Ref.[28]), developed in QCD in the definite form in Refs.[29, 30] and considered previously as the possible method of fixing theoretical uncertainties in the analysis of DIS data in Ref.[31]. It should be stressed that a posteriori this technique gives the results similar to those, obtained with the the help of the application of different methods of fixing scale-scheme dependence ambiguities.
(compare the results of Ref. 32 with the results of Refs. 29, 30 obtained using the Padé resummation technique). Thus, our analysis could be considered as the attempt to estimate perturbative QCD uncertainties beyond the NNLO level. Moreover, it could give us the hint whether the outcomes of the NNLO fits, related to perturbative and non-perturbative sectors, stay stable after the inclusion of the explicitly calculated and estimated N^3LO QCD corrections.

2. The theoretical background of the QCD analysis.

Let us define the Mellin moments for the NS SF $x F_3(x, Q^2)$:

$$M_{NS}^n(Q^2) = \int_0^1 x^{n-1} F_3(x, Q^2) dx$$

where $n = 2, 3, 4, \ldots$. The theoretical expression for these moments obey the following renormalization group equation

$$\left( \mu \frac{\partial}{\partial \mu} + \beta(A_s) \frac{\partial}{\partial A_s} + \gamma_{NS}(A_s) \right) M_{NS}^n(Q^2/\mu^2, A_s(\mu^2)) = 0$$

where $A_s = \alpha_s/(4\pi)$. The renormalization group functions are defined as

$$\mu \frac{\partial A_s}{\partial \mu} = \beta(A_s) = -2 \sum_{i \geq 0} \beta_i A_s^{i+2}$$

$$\mu \frac{\partial \ln Z_{NS}^n}{\partial \mu} = \gamma_{NS}(A_s) = \sum_{i \geq 0} \gamma^{(i)}_{NS}(n) A_s^{i+1}$$

where $Z_{NS}^n$ are the renormalization constants of the corresponding NS operators. The solution of the renormalization group equation can be presented in the following form

$$M_{NS}^n(Q^2) = M_{NS}^n(Q_0^2) \exp \left[ - \int_{A_s(Q_0^2)}^{A_s(Q^2)} \frac{\gamma_{NS}(x)}{\beta(x)} dx \right] \frac{C_{NS}^{(n)}(A_s(Q^2))}{C_{NS}^{(n)}(A_s(Q_0^2))}$$

where $M_{NS}^n(Q_0^2)$ is the phenomenological quantity related to the factorization scale dependent factor. It can be parametrized through the parton distributions at fixed momentum transfer $Q_0^2$ as

$$M_{NS}^n(Q_0^2) = \int_0^1 x^{n-2} A_s(Q_0^2) x^{b(Q_0^2)} (1-x)^{c(Q_0^2)} (1+\gamma(Q_0^2)x) dx$$

with $\gamma \neq 0$ or $\gamma = 0$. In principle, following the models of parton distributions, used in Refs. 3, 4, one can add in the used model for the SF the term, proportional to $\sqrt{x}$. However, since this term is important in the region of rather small $x$, we will neglect it in this our analysis.

At the N^3LO the expression for the coefficient function $C_{NS}^{(n)}$ can be presented as

$$C_{NS}^{(n)}(A_s) = 1 + C^{(1)}(n) A_s + C^{(2)}(n) A_s^2 + C^{(3)}(n) A_s^3,$$

while the corresponding expansion of the anomalous dimensions term is

$$\exp \left[ - \int_{A_s(Q_0^2)}^{A_s(Q^2)} \frac{\gamma_{NS}(x)}{\beta(x)} dx \right] = (A_s(Q^2))^{\gamma_{NS}^{(n)}(n)/2\beta_0} \times AD(n_s)$$
The coupling constant $L = \ln(\beta_{in})$ in Ref.\[27\].

The inverse-log expansion for $\beta_{in}$ was obtained in Ref.\[26\]. The inverse-log expansion for $\beta_{in}$ incorporates the information about the coefficient $\beta_3$, which was presented in Ref.\[27\].

Few words ought to be said about the used approximation for the anomalous dimension function $\gamma_{NS}(\beta_{in})$. The analytical expression for its one-loop coefficient is well-known: $\gamma_{NS}^{(0)}(\beta_{in}) = (8/3)[4 \sum_{j=1}^{n}(1/j) - 2/n(n + 1) - 3]$. In the cases of both $F_2$ and $xF_3$ SFs the numerical expressions for $\gamma_{NS}^{(1)}(\beta_{in})$-coefficients are given in Table 1.

$$AD(n_s) = [1 + p(n)A_s(Q^2) + q(n)A_s(Q^2)^2 + r(n)A_s(Q^2)^2]$$

and $p(n)$, $q(n)$ and $r(n)$ have the following form:

$$p(n) = \frac{1}{2}\left(\frac{\gamma_{NS}^{(1)}(n)}{\beta_1} - \frac{\gamma_{NS}^{(0)}(n)}{\beta_0}\right) \frac{\beta_1}{\beta_0}$$

$$q(n) = \frac{1}{4}\left(2p(n)^2 + \frac{\gamma_{NS}^{(2)}(n)}{\beta_0} + \gamma_{NS}^{(0)}(n)\left(\frac{\beta_2}{\beta_0} - \frac{\beta_3}{\beta_0} - \frac{\gamma_{NS}^{(1)}(n)}{\beta_0}\right)\right)$$

$$r(n) = \frac{1}{6}\left(p(n)^3 + 6p(n)q(n) + \frac{\gamma_{NS}^{(3)}(n)}{\beta_0} - \frac{\beta_1 \gamma_{NS}^{(2)}(n)}{\beta_0}\right)$$

$$- \frac{\beta_2 \gamma_{NS}^{(1)}(n)}{\beta_0} + \frac{\beta_3 \gamma_{NS}^{(0)}(n)}{\beta_0} - \frac{\beta_4 \gamma_{NS}^{(0)}(n)}{\beta_0} + \frac{2\beta_1 \beta_2 \gamma_{NS}^{(0)}(n)}{\beta_0}$$

The coupling constant $A_s(Q^2)$ can be expressed in terms of the inverse powers of $L = \ln(Q^2/A^2)$ as $A_s^{LO} = A_s^{NLO} + \Delta A_s^{NLO}$, $A_s^{NLO} = A_s^{LO} + \Delta A_s^{NLO}$ and $A_s^{N^3LO} = A_s^{NNLO} + \Delta A_s^{N^3LO}$, where

$$A_s^{LO} = \frac{1}{\beta_0 L}$$

$$\Delta A_s^{NLO} = -\frac{\beta_1 \ln(L)}{\beta_0 L^2}$$

$$\Delta A_s^{NNLO} = \frac{1}{\beta_0 L^3}\left[\beta_1^2 \ln^2(L) - \beta_2^2 \ln(L) + \beta_2 \beta_0 - \beta_1^2\right]$$

$$\Delta A_s^{N^3LO} = \frac{1}{\beta_0 L^4}\left[\beta_1^3 (-\ln^3(L) + \frac{5}{2} \ln^2(L) + 2 \ln(L) - \frac{1}{2})
- 3 \beta_0 \beta_1 \beta_2 \ln(L) + \frac{3 \beta_0^2 \beta_3}{2}\right].$$

Notice that in our normalization the numerical expressions for $\beta_0$, $\beta_1$, $\beta_2$ and $\beta_3$ read

$$\beta_0 = 11 - 0.6667 f$$
$$\beta_1 = 102 - 12.6667 f$$
$$\beta_2 = 1428.50 - 279.611 f + 6.01852 f^2$$
$$\beta_3 = 29243.0 - 6946.30 f + 405.089 f^2 + 1.49931 f^3$$

where the expression for $\beta_3$ was obtained in Ref.\[26\]. The inverse-log expansion for $\Delta A_s^{N^3LO}$, which incorporates the information about the coefficient $\beta_3$, was presented in Ref.\[27\].
\( \gamma_{NS,F_2}^{(1)}(n) \)
\( \gamma_{NS,F_3}^{(1)}(n) \)
\( \gamma_{NS}^{(2)}(n) \)
\( \gamma_{NS}^{(3)}[1/1] \)
\( \gamma_{NS}^{(3)}[0/2] \)

Table 1. The used numerical expressions for the NLO and NNLO coefficients of the anomalous dimensions of the moments of the NS SFs at \( f = 4 \) number of flavours and the N\(^3\)LO Padé estimates.

These results are normalized to the world with \( f = 4 \) numbers of active flavours. In the same Table the numerical expressions for \( \gamma_{NS}^{(2)}(n) \), used in the process of the fits, are presented. In the cases of \( n = 2, 4, 6, 8, 10 \) they are following from the explicit calculations of \( \gamma_{NS,F_2}^{(2)}(n) \)-terms \([17]\), normalized to \( f = 4 \), while the \( n = 3, 5, 7, 9 \) numbers were modeled using the smooth interpolation procedure, originally proposed in Ref.\([18]\). Note in advance, that since \( \gamma_{NS,F_2}^{(2)}(n) \)-coefficients differ from \( \gamma_{NS,F_3}^{(2)}(n) \)-terms, though by the small additional contributions (for discussions see Ref.\([9]\)), it would be interesting to verify the precision of the model for \( \gamma_{NS}^{(2)}(n) \), used in the process of our NNLO \( xF_3 \) fits, by the explicit analytical calculations.

Let us now describe the procedure of fixing other theoretical uncertainties. After the work of Ref.\([29]\) it became rather popular to model the effects of the higher order terms of perturbative series in QCD using the expanded Padé approximants. In the framework of this technique the values of the terms \( C^{(5)}(n) \) and \( r(n) \) could be expressed as

\[
Pade [1/1] : \quad C^{(3)}(n) = \frac{[C^{(2)}(n)]^2}{C^{(1)}(n)} \\
r(n) = q(n)^2/p(n)
\]

\[
Pade [0/2] : \quad C^{(3)}(n) = 2C^{(1)}(n)C^{(2)}(n) - [C^{(1)}(n)]^3 \\
r(n) = 2p(n)q(n) - [p(n)]^3
\]

The numerical expressions for \( p(n) \) and \( q(n) \), obtained from the results of Table 1 and definitions of Eqs.(9)-(11), together with the values of the coefficients \( C^{(1)}(n) \) and \( C^{(2)}(n) \) (which are coming from the calculations of Ref.\([16]\)), are presented in Table 2.
| n  | $p(n)$ | $q(n)$ | $r(n)|_{[1/1]}$ | $r(n)|_{[0/2]}$ | $C^{(1)}(n)$ | $C^{(2)}(n)$ | $C^{(3)}(n)|_{[1/1]}$ | $C^{(3)}(n)|_{[0/2]}$ |
|----|--------|--------|----------------|----------------|-------------|-------------|----------------|----------------|
| 2  | 1.646  | 4.232  | 10.829         | 9.476          | -1.778      | -47.472     | -1267.643      | 174.408         |
| 3  | 1.941  | 4.774  | 11.738         | 11.218         | 1.667       | -12.715     | 97.004         | -47.013         |
| 4  | 2.050  | 5.546  | 15.003         | 14.123         | 4.867       | 37.117      | 283.085        | 246.009         |
| 5  | 2.115  | 6.134  | 17.790         | 16.486         | 7.748       | 95.408      | 1174.834       | 1013.328        |
| 6  | 2.165  | 6.595  | 20.087         | 18.407         | 10.351      | 158.291     | 2420.569       | 2167.903        |
| 7  | 2.210  | 7.039  | 22.421         | 20.318         | 12.722      | 223.898     | 3940.284       | 3637.790        |
| 8  | 2.252  | 7.525  | 25.138         | 22.471         | 14.900      | 290.884     | 5678.657       | 5360.371        |
| 9  | 2.294  | 8.018  | 28.027         | 24.715         | 16.915      | 358.587     | 7601.720       | 7291.305        |
| 10 | 2.334  | 8.375  | 30.049         | 26.382         | 18.791      | 426.442     | 9677.390       | 9391.308        |

Table 2. The expressions for the NLO and NNLO QCD contributions, used in our fits, and the N$^3$LO Padé estimates.

In the same Table we also give the estimates for $r(n)$ and $C^{(3)}(n)$, obtained using the expanded $[1/1]$ and $[0/2]$ Padé approximants formulae of Eqs.(16)-(19). In the last two columns of Table 1 the estimates for the N$^3$LO contributions to the anomalous dimension function $\gamma^{(3)}_{NS}(A_s)$, obtained with the help of the expanded $[1/1]$ and $[0/2]$ Padé approximants are presented. One can see that the results of applications of $[1/1]$ and $[0/2]$ Padé approximants for $\gamma^{(3)}_{NS}(n)$ are almost identical to each other.

Using the numbers, presented in Table 1, one can construct Padé motivated expressions for $r(n)$ by substituting the estimates for $\gamma^{(3)}_{NS}(n)$ into Eq.(11). It should be stressed, that the obtained by this way estimates for $r(n)$ will qualitatively agree with the ones, presented in Table 2 within the “Padé world” only, namely only in the case of application in Eq.(11) of the $[1/1]$ or $[0/2]$ Padé estimate for the four-loop coefficient of the QCD $\beta$-function $\beta_3$. However, in the case of $f = 4$ the direct application of the $[1/1]$ and $[0/2]$ Padé approximants underestimates the calculated value of $\beta_3$ by over the factor 2.5 ($\beta_3|_{[1/1]} = 3216.66...; \beta_3|_{[0/2]} = 3058.38...$). In view of this the application of Eq.(11) with the Padé estimated values of $\gamma^{(3)}_{NS}(n)$ the explicit expression for $\beta_3$-coefficient are giving estimates of $r(n)$, drastically different from the ones presented in Table 2 (for example, for the case of application of $[0/2]$ Padé estimates it gives $r(2) \approx 18.83,...,r(10) \approx 55.70$). It is already known that the accuracy of the estimates of the N$^3$LO coefficient of the QCD $\beta$-function can be improved by some additional fits of polynomial dependence of $\beta_3$ on the number of flavours $f$ and applying the asymptotic Padé approximant (APAP) formula $[33]$. Therefore, it might be interesting to think about the possibility of putting bold guess Padé estimates of N$^3$LO contributions to $\gamma_{NS}(A_s)$ (see Table 1) on more solid background. The analogous steps were already done in Ref.$[34]$ in the case of the analysis of the status of N$^3$LO Padé estimates for the anomalous dimension function of quark mass and the agreement with the explicitly calculated at this level of QCD the results of Ref.$[35]$ turned out to be reasonable. One can hope, that the application of the similar procedure for the APAP estimates of $\gamma^{(3)}_{NS}$-terms and the substitution of the results obtained into Eq.(11) (together with the explicit expression for the $\beta_3$-term) might improve the agreement with the results, presented in Table 2, which at this stage we consider as the most stable results for the N$^3$LO fits.

It should be also stressed that the uncertainties of the values of $r(n)$ are not so important, since the results of our fits will be more sensitive to the form of the Padé approximations predictions for the contributions into the coefficient function (namely $C^{(3)}(n)$-terms).
From the results presented in Table 2 one can conclude, that the theoretical series for \( C_{NS}^{(n)} \) for large \( n (n \geq 4) \), which corresponds to the behavior of \( x F_3(x, Q^2) \) SF in the intermediate and large \( x \)-region, probably have sign constant structure with asymptotically increasing positive coefficients. Therefore, the applications of the expanded \([1/1]\) and \([0/2]\) Padé approximants are giving theoretical estimates for the terms \( C^{(3)}(n) \) \( (n \geq 4) \), which in both cases have the same positive sign and the same order of magnitude.

However, in the cases of \( n = 2, 3 \) our intuition does not give us the idea what might be the sign and order of magnitude of the third term in perturbative series \( C_{NS}^{(2)}(A_s) = 1 - 1.78 A_s - 47.47 A_s^2 \) and \( C_{NS}^{(3)}(A_s) = 1 + 1.67 A_s - 12.71 A_s^2 \). Indeed, in these two cases the manipulations with \([1/1]\) and \([0/2]\) Padé approximants are giving drastically different estimates for the terms \( C^{(3)}(n) \), which in the cases of \( n = 2, 3 \) differ both by sign and value (see Table 2). These facts, and the structure of the NNLO perturbative series for \( C_{NS}^{(3)}(A_s) \) especially, might indicate that this series is not yet in the asymptotic regime. Another possibility is that its coefficients do not have the \((+1)^n n!\) growth, but poses some zigzag structure, which is manifesting itself in the cases of definite perturbative series of quantum field theory models (for discussions see e.g. Ref.\[31\]). This might give the additional theoretical uncertainties of modeling higher-order perturbative QCD predictions for \( F_3(x, Q^2) \) in the region of small \( x \).

In view of the questionable asymptotic behavior of the NNLO series for the coefficient functions of NS moments with low \( n (n = 2, 3) \) we are also using the idea of Ref.\[31\] and consider the results of applications of non-expanded Padé approximants in the process of the analysis of the DIS data.

Let us remind that the corresponding non-expanded \([1/1]\) Padé approximants can be defined as

\[
AD(n)[[1/1]] = \frac{1 + a_1^{(n)} A_s}{1 + b_1^{(n)} A_s}
\]

\[
C_{NS}^{(n)}(A_s)[[1/1]] = \frac{1 + c_1^{(n)} A_s}{1 + d_1^{(n)} A_s}
\]

where \( a_1^{(n)} = \left( [p(n)]^2 - q(n) \right)/p(n) \), \( b_1^{(n)} = -q(n)/p(n) \) and \( C_1^{(n)} = \left( [C^{(1)}(n)]^2 - C^{(2)}(n) \right)/C^{(1)}(n) \), \( d_1^{(n)} = -C^{(2)}(n)/C^{(1)}(n) \).

The explicit expressions for the non-expanded \([0/2]\) Padé approximants read:

\[
AD(n)[[0/2]] = \frac{1}{1 + b_1^{(n)} A_s + b_2^{(n)} A_s^2}
\]

\[
C_{NS}^{(n)}(A_s)[[0/2]] = \frac{1}{1 + d_1^{(n)} A_s + d_2^{(n)} A_s^2}
\]

where \( b_1^{(n)} = -p(n), b_2^{(n)} = p(n)^2 - q(n), d_1^{(n)} = -C^{(1)}(n) \) and \( d_2^{(n)} = [C^{(1)}(n)]^2 - C^{(2)}(n) \). Since we consider the applications of both \([1/1]\) and \([0/2]\) Padé approximants as the attempts to model the behavior of the perturbative series for the NS Mellin moments beyond the NNLO level, we will use in Eqs.(20)-(23) the N³LO expression for the coupling constant \( A_s \), defined through Eqs.(12)-(14). It is worth to mention here, that quite recently the expanded and non-expanded Padé approximants were successfully used for the study of the N³LO approximation of the ground state energy.
in quantum mechanics \[37\] and of the behavior of the \(\beta\)-function for the quartic Higgs coupling in the Standard Electroweak Model \[38\].

The next step is the reconstruction of the structure function \(xF_3(x, Q^2)\) with taking into account both target mass corrections and twist-4 terms. The reconstructed SF can be expressed as:

\[
x F_3^{N_{\text{max}}}(x, Q^2) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{\text{max}}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta)M_{j+2,xF_3}(Q^2) \quad (24)
\]

where \(\Theta_n^{\alpha,\beta}\) are the Jacobi polynomials and \(\alpha, \beta\) are their parameters, fixed by the condition of the requirement of the minimization of the error of the reconstruction of the SF. In order to take into account the target mass corrections the Nachtamm moments

\[
M_{n,xF_3} \rightarrow M_{n,xF_3}^{TMC}(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^2} F_3(x, Q^2) \frac{1 + (n + 1)V}{(n + 2)} , \quad (25)
\]

can be used, where \(\xi = 2x/(1 + V)\), \(V = \sqrt{1 + 4M_{\text{nucl}}^2x^2/Q^2}\) and \(M_{\text{nucl}}\) is the mass of a nucleon. However, to simplify the analysis it is convenient to expand equation (25) into a series in powers of \(M_{\text{nucl}}^2/Q^2\) \[39\]. Taking into account the order \(O(M_{\text{nucl}}^4/Q^4)\) corrections, we get

\[
M_{n,xF_3}^{TMC}(Q^2) = M_{n,xF_3}^{NS}(Q^2) + \frac{n(n+1)M_{\text{nucl}}^2}{2(n+4)Q^2} M_{n+2,xF_3}^{NS}(Q^2) \quad (26)
\]

\[
+ \frac{(n+2)(n+1)nM_{\text{nucl}}^4}{2(n+4)Q^4} M_{n+4,xF_3}^{NS}(Q^2) + O\left(\frac{M_{\text{nucl}}^6}{Q^6}\right),
\]

We have checked that the influence of the order \(O(M_{\text{nucl}}^4/Q^4)\) terms in Eq.(26) to the outcomes of the concrete fits is very small. Therefore, in what follows we will use only the first two terms in the r.h.s. of Eq.(26).

The form of the twist-4 contributions \(h(x)\) in Eq.(19) was first fixed as

\[
h(x) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{\text{max}}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta)M_{j+2,xF_3}^{IRR}(Q^2) \quad (27)
\]

where \(c_j^{(n)}(\alpha, \beta)\) are the polynomials, which contain \(\alpha\) and \(\beta\)-dependent Euler \(\Gamma\)-functions and

\[
M_{n,xF_3}^{IRR}(Q^2) = \tilde{C}(n)N_{n,xF_3}^{NS}(Q^2)A_2 + O\left(\frac{1}{Q^2}\right) \quad (28)
\]

with \(A_2\) taken as the free parameter and \(\tilde{C}(n)\) defined following the IRR model estimates of Ref.\[1\] as \(\tilde{C}(n) = -n - 4 + 2/(n + 1) + 4/(n + 2) + 4S_1(n)\) \((S_1(n) = \sum_{j=1}^{n} 1/j)\). It should be stressed that the appearance of the multiplicative QCD expression \(M_{n,xF_3}^{NS}(Q^2)\) in Eq.(28), generally speaking different from the intrinsic coefficients function of the twist-4 contribution, is leading to definite theoretical uncertainties in the contributions of higher-order QCD corrections to the twist-4 part of \(xF_3(x, Q^2)\). This could provide the additional theoretical errors in the studies of
the status of the IRR-model predictions for the twist-4 terms at the NNLO and beyond.

In order to study this question at more definite theoretical level it is instructive to consider the function \( h(x) \) as the free parameters of the fits, not related to IRR-model estimates.

We will estimate the uncertainties of the values of \( \Lambda^{(4)}_{\overline{MS}} \alpha_s(M_Z) \) IRR-model parameter \( A'_2 \) and the twist-4 function \( h(x) \) due to the inclusion of the definite explicitly calculated \( N^3\)LO QCD corrections in the fits and modeling other ones with the help of the Padé resummation technique. Our aim will be also the verification of the stability of the results of Ref.\[11\] and the study of the possible reason of the peculiar behavior of \( h(x) \) for \( xF_3 \) SF, discovered in Ref.\[11\] at the NNLO level.

3 (a). The results of the analysis of the experimental data: the extraction of \( \Lambda^{(4)}_{\overline{MS}} \) vs \( \alpha_s \) value.

The definite results for our NLO and NNLO fits, made for the case of \( f = 4 \) number of active flavours, are presented in Table 1 of Ref.\[9\], where the values of the parton distribution parameters \( A, b, c, \gamma \neq 0 \) are also given. In the present Section we study the influence of the Padé-motivated estimates of the \( N^3\)LO expressions for anomalous dimensions and corresponding coefficient function (written down both in the expanded and non-expanded forms) to the results of the fits, which are resulting in the extractions of the parameter \( \Lambda^{(4)}_{\overline{MS}} \) (and thus \( \alpha(M_Z) \)), and of the common factor \( A'_2 \) of the IRR model.

It should be stressed that despite the general theoretical preference of applications of the diagonal Padé approximants (for the recent analysis see e.g. Ref.\[11\]) the \( N^3\)LO \([1/1]\) Padé approximant description of the CCFR'97 experimental data turned out to be not acceptable in our case, since it produces rather high value of \( \chi^2 \): \( \chi^2/nep > 2 \) (where \( nep = 86 \) is the number of the experimental points, taken into account in the case of the cut \( Q^2 > 5 \text{ GeV}^2 \)). However, the application of \([0/2]\) Padé approximants produced reasonable results. We think that the nonapplicability of the \([1/1]\) Padé method in the process of fitting CCFR \( xF_3 \) data using the Jacobi polynomial approach can be related to the manifestation of rather large value of the ratio \( [C^{(3)}(2)/C^{(2)}(2)]_{[1/1]} \) in the expression for NS moment \( M^{NS}_{2,xF_3} \).

The similar effect of the preference of the \([0/2]\) Padé approximant analysis over the \([1/1]\) one was found in Ref.\[30\] in the case of the comparison of the QCD theoretical predictions for the polarized Bjorken sum rule (which are closely related to the QCD predictions for the first moment of the \( xF_3 \) SF, namely for the Gross-Llewellyn Smith sum rule) with the available experimental data.

The results for \( \Lambda^{(4)}_{\overline{MS}} \), obtained at the LO, NLO, NNLO and \( N^3\)LO, modeled by the expanded and non-expanded Padé approximants, are presented in Table 3 in the cases of both \( \gamma \neq 0 \) and \( \gamma = 0 \).

Looking carefully on Table 3 we arrive to the following conclusions:

• Our fits demonstrate that the effects of the NNLO perturbative QCD contributions are important in the analysis of the CCFR data. Indeed, for different \( Q^2\)-cuts they are diminishing the values of the QCD scale parameter \( \Lambda^{(4)}_{\overline{MS}} \) by the contribution, which is varying in the range over \( 50 - 120 \text{ MeV} \), provided twist-4 corrections are taken into account through the IRR model of Ref.\[19\];

• The values of \( \Lambda^{(4)}_{\overline{MS}} \) which are coming from the fits with taking into account the \([0/2]\) Padé estimates (both in the expanded and non-expanded variants) turn out
| $Q^2 > 5 \, GeV^2$ | $\gamma$ - free | $\gamma = 0$ - fixed |
|-----------------|----------------|------------------|
|                 | $\Lambda^{(4)}_{\overline{MS}}$ (MeV) | $A'_2$(HT) | $\chi^2$/points | $\Lambda^{(4)}_{\overline{MS}}$ (MeV) | $A'_2$(HT) ($GeV^2$) | $\chi^2$/points |
| LO             | 266±37 | - | 113.2/86 | 231±37 | - | 126.8/86 |
| NLO            | 436±56 | -0.33±0.06 | 82.8/86 | 367±59 | -0.30±0.06 | 102.0/86 |
| NNLO           | 341±41 | - | 87.1/86 | 278±33 | - | 109.3/86 |
| NNLO           | 371±31 | -0.12±0.05 | 81.8/86 | 301±36 | -0.10±0.06 | 105.7/86 |
| N$^3$LO        | 293±29 | - | 78.4/86 | 258±28 | - | 100.6/86 |
| N$^3$LO (n.e.) | 305±32 | -0.01±0.05 | 78.4/86 | 258±29 | -0.01±0.05 | 100.5/86 |
| N$^3$LO        | 293±29 | - | 78.4/86 | 258±29 | -0.01±0.05 | 100.5/86 |
|                 | 299±32 | -0.02±0.05 | 78.8/86 | 297±29 | -0.03±0.05 | 86.8/86 |
| LO             | 287±37 | - | 77.7/63 | 279±40 | - | 79.6/63 |
| NLO            | 531±74 | -0.52±0.11 | 58.0/63 | 503±97 | -0.49±0.15 | 61.7/63 |
| NNLO           | 350±55 | -0.24±0.10 | 58.6/63 | 392±173 | -0.19±0.27 | 65.3/63 |
| NNLO           | 308±34 | - | 58.7/63 | 285±34 | - | 68.1/63 |
| N$^3$LO        | 313±37 | -0.03±0.09 | 58.6/63 | 285±38 | -0.01±0.09 | 68.1/63 |
| N$^3$LO (n.e.) | 303±33 | - | 60.4/63 | 303±33 | - | 63.0/63 |
| N$^3$LO        | 309±36 | -0.03±0.09 | 60.3/63 | 310±37 | -0.04±0.09 | 62.8/63 |
| N$^3$LO        | 295±30 | - | 59.9/63 | 296±31 | - | 62.2/63 |
|                 | 297±33 | -0.01±0.09 | 59.8/63 | 298±34 | -0.02±0.09 | 62.2/63 |
| LO             | 319±41 | - | 58.8/50 | 318±41 | - | 58.9/50 |
| NLO            | 531±57 | -0.57±0.13 | 50.2/50 | 517±112 | -0.54±0.27 | 50.9/50 |
| NNLO           | 365±40 | - | 53.0/50 | 355±44 | - | 54.1/50 |
| NNLO           | 441±44 | -0.25±0.13 | 50.9/50 | 408±43 | -0.19±0.13 | 52.8/50 |
| NNLO           | 314±37 | - | 50.9/50 | 294±36 | - | 56.7/50 |
| N$^3$LO        | 308±45 | 0.03±0.14 | 50.8/50 | 279±44 | 0.09±0.15 | 56.3/50 |
| N$^3$LO (n.e.) | 304±36 | - | 52.8/50 | 303±36 | - | 54.3/50 |
| N$^3$LO        | 297±43 | 0.05±0.14 | 52.7/50 | 294±43 | 0.05±0.14 | 54.1/50 |
| N$^3$LO        | 296±33 | - | 52.3/50 | 295±33 | - | 53.8/50 |
|                 | 286±38 | 0.07±0.14 | 52.0/50 | 283±39 | 0.08±0.14 | 53.4/50 |

**Table 3.** The results of the extractions of the parameter $\Lambda^{(4)}_{\overline{MS}}$ and the IRR coefficient $A'_2$, (in $GeV^2$) defined in Eq.(23), from LO, NLO, NNLO and N$^3$LO non-expanded (n.e.) and expanded Padé fits of CCFR’97 data.
to be really nonsensitive to choosing the $Q^2$-cut of the data, fixation of the value of $\gamma$ and thus incorporation of $(1 + \gamma x)$-factor in the parton distribution model. This in its turn can indicate that the change of the used by us model $xF_3(x, Q_0^2) = A(Q_0^2) x^{b(Q_0^2)} (1 + \gamma(Q_0^2)x) + \epsilon(Q_0^2) \sqrt{x}$, used in the MRST and GRV fits, should not affect significantly the results obtained;

- The large errors in the definite LO and NLO results for $\Lambda^{(4)}_{\overline{MS}}$, presented in Table 3, are reflecting the correlations of these values with the errors of parton distribution parameters, which will be not considered in this paper and presented by us elsewhere;

- For all $Q^2$-cuts and values of the parameter $\gamma$ the applications of the $N^3$LO fits performed with the help of the expanded [0/2] Padé approximants technique result in the smaller value of $\Lambda^{(4)}_{\overline{MS}}$ and slightly smaller $\chi^2$ value than in the case of application of the non-expanded [0/2] Padé approximants. In our future studies we will consider the results of application of both expanded and non-expanded Padé approximations.

- For all $Q^2$-cuts the expanded $N^3$LO results for $\Lambda^{(4)}_{\overline{MS}}$ is smaller then the similar results for non-expanded Padé fits by 10-15 MeV. This effect is coming from the expansion of Padé approximants in Taylor series. The difference in the results of the expanded and non-expanded Padé approximants can be considered as the estimate of the part of theoretical error of the $N^3$LO results.

Using the LO, NLO, NNLO and $N^3$LO variants of the rigorous $\overline{MS}$-scheme matching conditions, derived in Ref. [22] following the lines of Ref. [22], we transform $\Lambda^{(4)}_{\overline{MS}}$ values through the threshold of the production of the fifth flavour $M_5 = m_b$ (where $m_b$ is the $b$-quark pole mass) and obtain the related values of $\Lambda^{(5)}_{\overline{MS}}$ with the help of the following equation:

$$
\beta_0^{f+1} \ln \frac{\Lambda^{(f+1)}_{\overline{MS}}}{\Lambda^{(f)}_{\overline{MS}}} = (\beta_0^{f+1} - \beta_0^f) L_h + \delta_{NLO} + \delta_{NNLO} + \delta_{N^3LO}
$$

$$
\delta_{NLO} = \left( \frac{\beta_0^{f+1}}{\beta_0^{f+1}} - \frac{\beta_0^f}{\beta_0^f} \right) \ln L_h - \frac{\beta_0^{f+1}}{\beta_0^{f+1}} \ln \frac{\beta_0^{f+1}}{\beta_0^f} - \frac{\beta_0^{f+1}}{\beta_0^{f+1}} - \frac{\beta_0^f}{\beta_0^f}
$$

$$
\delta_{NNLO} = \frac{1}{\beta_0^{f+1}} L_h \left[ \frac{\beta_0^f}{\beta_0^f} \left( \frac{\beta_0^{f+1}}{\beta_0^{f+1}} - \frac{\beta_0^f}{\beta_0^f} \right) \ln L_h + \left( \frac{\beta_0^f}{\beta_0^{f+1}} \right)^2 - \left( \frac{\beta_0^f}{\beta_0^f} \right)^2 - \frac{\beta_0^{f+1}}{\beta_0^{f+1}} + \frac{\beta_0^f}{\beta_0^f} - C_2 \right]
$$
\[ \delta_{N^3LO} = \frac{1}{(\beta_0^f L_h)^2} \left[ -\frac{1}{2} \left( \frac{\beta_1^f}{\beta_0^f} \right)^2 \left( \frac{\beta_2^f}{\beta_0^f} - \frac{\beta_3^f}{\beta_0^f} \right) \ln^2 L_h + \frac{\beta_2^f}{\beta_0^f} - \frac{\beta_3^f}{\beta_0^f} + C_2 \right] \ln L_h \]

\[ + \frac{1}{2} \left( - \left( \frac{\beta_2^f}{\beta_0^f} \right)^3 - \left( \frac{\beta_1^f}{\beta_0^f} \right)^3 \right) + \frac{\beta_2^f}{\beta_0^f} + \frac{\beta_3^f}{\beta_0^f} + C_2 - C_3 \]

where \( C_2 = -7/24 \) was calculated in Ref. [23] and the analytic expression for \( C_3 \), namely \( C_3 = -(80507/27648)\zeta(3) - (2/3)\zeta(2) + (1/3)\ln(2+1) - 58933/124416 + (f/9)[\zeta(2) + 2479/3456] \) was recently found in Ref. [27]. Here \( \beta_i^f \) \( (\beta_i^f)^{1+} \) are the coefficients of the \( \beta \)-function with \( f \) \( (f+1) \) numbers of active flavours, \( L_h = \ln(M_{f+1}/\Lambda(\beta_f^f)^2) \) and \( M_{f+1} \) is the threshold of the production of the quark of \( (f+1) \)-th flavour. In our analysis we should take \( f = 4 \) and \( m_h \approx 4.6 \) GeV.

In the case of the non-zero values of the twist-4 function \( h(x) \neq 0 \) the results of the LO, NLO, NNLO and N3LO fits, made both in the expanded and non-expanded versions of the \([0/2]\) Padé motivated approach are presented in Table 4.

It should be stressed that we are considering the outcomes of our N3LO approximated fits as the rate of theoretical uncertainties of the NNLO results in the same manner like the results of the NNLO analysis will be considered as the measure of theoretical uncertainties of the NLO results. In particular, we will introduce the characteristic deviations \( \Delta_{NNLO} = |(\Lambda_{MS}^{(4)})_{N^3LO} - (\Lambda_{MS}^{(4)})_{NNLO}| \), \( \Delta_{NLO} = |(\Lambda_{MS}^{(4)})_{N^3LO} - (\Lambda_{MS}^{(4)})_{NLO}| \).

It is worth to emphasize that the results for the Padé motivated N3LO results for \( \Lambda_{MS}^{(4)} \) and thus \( \alpha_s(M_Z) \) became closer to the NNLO ones (provided the statistical error bars are taken into account, see Table 1). Moreover, the difference \( \Delta_{NNLO} = |(\Lambda_{MS}^{(4)})_{N^3LO} - (\Lambda_{MS}^{(4)})_{NNLO}| \), is drastically smaller then the NLO correction term \( \Delta_{NLO} = |(\Lambda_{MS}^{(4)})_{NNLO} - (\Lambda_{MS}^{(4)})_{NLO}| \). The similar tendency \( \Delta_{NNLO} << \Delta_{NLO} \) is taking place in the case of the fits without twist-4 corrections. These observed properties indicate the reduction of the theoretical errors due to cutting the analyzed perturbation series at the different orders.

It is known that the inclusion of the higher-order perturbative QCD corrections into the comparison with the experimental data is decreasing the scale-scheme theoretical errors of the results for \( \Lambda_{MS}^{(4)} \) and thus \( \alpha_s(M_Z) \) (see e.g. Refs.[11, 13, 20]). Among the ways of probing the scale-scheme uncertainties are the scheme-invariant methods, namely the principle of minimal sensitivity, the effective charges approach, (which is known to be identical to the scheme-invariant perturbation theory) and the BLM approach (for the review of these methods see e.g. Ref.[12]). The scheme-invariant methods were already used to estimate the effects of the unknown higher order corrections in SFs (see Ref. [13], where a strong decrease of the value of the QCD scale parameter was found in the process of the scheme-invariant fit of the experimental data for \( F_2 \) SF ) and to try to predict the unknown at present N3LO corrections to the definite physical quantities [22], and DIS sum rules among others. Note that the
predictions of Ref.\[32\] are in agreement with the results of applications of the Padé resummation technique (see Ref.\[29\]). Therefore, we can conclude that the application of the methods of the Padé approximants should lead to the reduction of the scale-scheme dependence uncertainties of the values of $\alpha_s(M_Z)$ in the analysis of the CCFR data.

We are presenting now the values of $\alpha_s(M_Z)$, extracted from the fits of the CCFR’97 experimental data for the $xF_3$ SF, obtained with fixing twist-4 contribution through the IRR model of Ref.\[19\]:

\[ NLO HT \; of\; [19] \; \alpha_s(M_Z) = 0.121 \pm 0.002(stat) \pm 0.005(syst) \pm 0.005(theory) \] (33)

\[ NNLO HT \; of\; [19] \; \alpha_s(M_Z) = 0.118 \pm 0.002(stat) \pm 0.005(syst) \pm 0.003(theory) \] (34)

The results of the extractions of $\alpha_s(M_Z)$, with twist-4 contribution, considered as the additional free parameters of the fit, have the following form:

\[ NLO \; HT \; free \; \alpha_s(M_Z) = 0.124^{+0.007}_{-0.006}(stat) \pm 0.005(syst) \pm 0.008(theory) \] (35)

\[ NNLO \; HT \; free \; \alpha_s(M_Z) = 0.116^{+0.006}_{-0.007}(stat) \pm 0.005(syst) \pm 0.004(theory) \] (36)

where the systematic uncertainties are taken from the CCFR experimental analysis, presented in the first work of Ref.\[3\], and the theoretical uncertainties in the results of Eqs.(33),(35) [Eqs.(34),(36)] are estimated by the differences between the central values of the outcomes of the NNLO and NLO [N$^3$LO and NNLO] fits, presented in Table 1, plus the arbitrariness in the application of smoothing procedure of the $MS$-scheme matching condition (which following the considerations of Ref.\[24\] was estimated as $\Delta \alpha_s(M_Z) = \pm 0.001$) and the uncertainties in $\alpha_s(M_Z)$ due to nuclear effects on iron target (see discussions in Sec.4). In estimating the theoretical errors of the inclusion of the N$^3$LO corrections we take into account the differences between the applications of the expanded and non-expanded Padé approximants.

It can be seen that due to the large overall number of the fitted parameters the results of Eqs.(35),(36) for $\alpha_s(M_Z)$ have rather large statistical uncertainties. As can be seen from the results of Eq.(33),(34) for the QCD coupling constant it is possible to decrease their values by fixing the concrete form of the twist-4 parameter $h(x)$. However, if one is interested in the fitted form of the twist-4 parameter $h(x)$, one should take for granted these intrinsic theoretical uncertainties of the value of $\alpha_s(M_Z)$.

3 (b). The results of the analysis of the experimental data: the extraction of parameters of the twist-4 terms.

Apart of the perturbative QCD contributions the expressions for DIS structure functions should contain the power-suppressed high-twist terms, which reflect the possible non-perturbative QCD effects. The studies of these terms have rather long history. At the beginning of these studies it was realized that the twist-4 contributions to structure functions should have the pole-like behavior $\sim 1/((1-x)Q^2)$.
This behavior was used in the phenomenological investigations of the earlier less precise than CCFR DIS $\nu N$ data [46, 47, 48], which together with different other procedures on analyzing neutrino DIS data [49, 50] was considered as the source of the information about scaling violation parameters. The development of renormalon technique (Refs. [20, 19] and Ref. [21] for the detailed review) pushed ahead the more detailed phenomenological analysis of the possibility of detecting higher-twist components in the available at present most precise DIS data obtained by BCDMS, SLAC, CCFR and other collaborations. It turned out that despite the qualitative status of renormalon approach the satisfactory description of the results of QCD NLO $F_2$ analysis [51] in terms of IRR technique was achieved [19, 20]. The next step was to clarify the status of the predictions of Ref. [19] for the form and sign of the twist-4 contributions to $x F_3$ SF. The study of this problem was done in Ref. [11] (see also Ref. [10]). In this chapter we are discussing the results of more refined analysis of the behavior of the twist-4 contributions to $x F_3$ SF at the LO, NLO, NNLO and beyond.

In Table 3 we study the dependence of the extracted value of the parameter $A_2'$ from the different orders of perturbative QCD predictions, $Q^2$-cuts of the CCFR experimental data and the coefficient $\gamma$ of parton distributions model for $x F_3$. Note, that the parameter $A_2'$ was introduced in the IRR model of Eq. (28), taken from Ref. [14], and fixed there as $A_2' \approx -0.2 \text{ GeV}^2$, which is necessary for the description of the fitted twist-4 results of Ref. [51] for $F_2$ within IRR language. We found that the value of this parameter extracted at the LO and NLO, is negative, differ from zero for about one standard deviation and qualitatively in agreement with the IRR-motivated guess of Ref. [19]. Moreover, the results of our LO and NLO fits are also in agreement with the made in Ref. [48] earlier extraction from the old DIS neutrino data of the value $h = -0.38 \pm 0.06 \text{ GeV}^2$ of the different model of the twist-4 contribution to $x F_3$, namely $x F_3(x, Q^2) h/(1-x) Q^2$.

It is interesting to notice that the results of the multiloop extractions of the parameter $A_2'$ from the CCFR data are almost nonsensitive to the introduction of the additional parameter $\gamma$ in the parton distribution model.

Another observation is that the results of Table 3 reveal that for larger $Q^2$-cuts $10 - 15 \text{ GeV}^2$ the results of $A_2'$ in the LO and NLO fits are more stable to the inclusion of the concrete experimental data, than in the case of the low $Q^2$ cut ($5 \text{ GeV}^2$). This feature can be related to the logarithmic increase of the QCD coupling constant $\alpha_s$ at lower $Q^2$. However, since we are interested in the extraction of the power-suppressed twist-4 contribution, we shall concentrate on the discussion of the more informative from our point of view fits with low $Q^2$-cut $5 \text{ GeV}^2$, which contain more experimental points and thus are more statistically motivated.

We also observed that in the process of multiloop extraction of $A_2'$ the tendency $\chi^2_{LO} > \chi^2_{NLO} > \chi^2_{NNLO} \sim \chi^2_{N^3LO}$ takes place. It should be stressed that in opposite to the results of the LO and NLO extraction the NNLO value of $A_2'$ is compatible with zero and is stable to the inclusion of the N$^3$LO contribution to $A_s$ through the Padé approximants for the NS Mellin moments.

We are now turning to the pure phenomenological extraction of the twist-4 contribution $h(x)$ to $x F_3$ (see Eq. (24)), which is motivated by the work of Ref. [51] for $F_2$ data. In the framework of this approach the $x$-shape of $h(x)$ is parametrized by the additional parameters $h_i = h(x_i)$, where $x_i$ are the points of the experimental data bining. The results of the multiloop extractions of these parameters are presented in Table 4 and are illustrated by the curves of Fig. 1.
The results of the LO, NLO (with TMC) and non-expanded (n.e.) Padé QCD fit of the CCFR'97 xFHT contributions follow the general shape of IRR prediction [19] still survives; looking carefully on Table 4 and Fig.1 we observe the following features:

1. The x-shape of the twist-4 parameter is not inconsistent with the expected rise of h(x) for x → 1 [14, 45] in all orders of perturbation theory;

2. The values of the parameters h(xi) at the upper and lower points of kinematic region (x16=0.650 and x1=0.0125) are stable to the inclusion of the higher order perturbative QCD corrections and application of the Padé resummation technique. At large values of x this feature is in agreement with the previous statement;

3. The function h(x) seems to cross zero twice: at small x of order 0.03 and larger x about 0.4. It should be noted that the sign-alternating behavior of the twist-4 contributions to DIS structure functions was qualitatively predicted in Ref. [17];

4. In the LO and NLO our results are in qualitative agreement with the IRR prediction of Ref. [19] (for discussions see Ref. [21]);

5. In the NNLO this agreement is not so obvious, though the certain tendency of following the general shape of IRR prediction [13] still survives;

|            | LO     | NLO    | NNLO   | N^3LO  | N^3LO(n.e.) |
|------------|--------|--------|--------|--------|-------------|
| χ^2/points | 66.2/86| 65.6/86| 65.7/86| 65.6/86| 64.3/86     |
| A          | 5.44 ± 1.74 | 3.70 ± 1.56 | 4.54 ± 0.88 | 5.24 ± 0.92 | 5.16 ± 0.75 |
| b          | 0.74 ± 0.10  | 0.66 ± 0.11  | 0.69 ± 0.06  | 0.72 ± 0.06  | 0.73 ± 0.05  |
| c          | 4.00 ± 0.18  | 3.78 ± 0.21  | 3.72 ± 0.19  | 3.58 ± 0.26  | 3.48 ± 0.26  |
| γ          | 1.72 ± 1.25  | 2.86 ± 1.72  | 1.43 ± 0.69  | 0.72 ± 0.63  | 0.71 ± 0.51  |
| Λ_{MS}^{(4)} [MeV] | 338 ± 169 | 428 ± 158 | 264 ± 85 | 248 ± 76 | 310 ± 100 |

Looking carefully on Table 4 and Fig.1 we observe the following features:

- The x-shape of the twist-4 parameter is not inconsistent with the expected rise of h(x) for x → 1 in all orders of perturbation theory;
- The values of the parameters h(xi) at the upper and lower points of kinematic region (x16=0.650 and x1=0.0125) are stable to the inclusion of the higher order perturbative QCD corrections and application of the Padé resummation technique. At large values of x this feature is in agreement with the previous statement;
- The function h(x) seems to cross zero twice: at small x of order 0.03 and larger x about 0.4. It should be noted that the sign-alternating behavior of the twist-4 contributions to DIS structure functions was qualitatively predicted in Ref. [17];
- In the LO and NLO our results are in qualitative agreement with the IRR prediction of Ref. [19] (for discussions see Ref. [21]);
- In the NNLO this agreement is not so obvious, though the certain tendency of following the general shape of IRR prediction still survives;
Fig. 1 The results of the LO, NLO, NNLO and N$^3$LO [0/2] Padé extractions of the twist-4 contributions of $h(x)$. The solid line is the IRR-model prediction of Ref. [19].

6. However, at the NNLO we observe the partial nullification of $h(x)$ within statistics error bars. Thus we conclude that the inclusion of the NNLO corrections into the game is shadowing the effects of the power suppressed terms, or that the effects of the twist-4 corrections are nondetectable at the NNLO. This property was previously observed at the LO as the result of the less precise DIS neutrino data in Ref. [46]. In the modern experimental situation, namely in the process of the analysis of the more precise DIS neutrino data of CCFR collaboration, we are observing this feature at the NNLO;

7. We checked the reliability of the foundation of partial nullification of $h(x)$ at the NNLO by going beyond this perturbative approximation using the methods of Padé approximants. The result of this analysis reveals the stability of the NNLO results for $h(x)$ and its partial nullification;

8. The property of partial nullification of $h(x)$ at the NNLO and N$^3$LO is identical to the effect of nullification of the IRR model parameter $A_2$ at the NNLO and N$^3$LO (see Table 3);

9. These observed properties clarifies why the results of the NNLO and N$^3$LO fits for $\Lambda^{(4)}_{\overline{MS}}$ presented in Tables 3, practically do not depend from the inclusion of the twist-4 contribution through the IRR model. Indeed, at this level the twist-4 terms have almost zero effect.
To our point of view the foundations (8)-(9) reflects the selfconsistency of the results of our different fits with twist-4 included by different ways.

4. The quest of the inclusion of the effects of nuclear corrections.

The effects of nuclear corrections are remaining the important source of the uncertainties of the analysis of the DIS data. This is especially important for the experiments on heavy targets and in the case of CCFR data on iron $^{56}$Fe.

The attempts to study these effects were done in Ref.[52] in the framework of Deutron-motivated model. The satisfactory QCD description of the CCFR data for $x_F^3$ was achieved due to the reason that in this case the nuclear effects do not exceed 5% effect. However, the more realistic description of nuclear effect for $x_F^3$ on $^{56}$Fe [53] revealed the appearance of new $1/Q^2$ and $1/M$ corrections for NS moments (where $M$ is the mass of the nucleon), which have the following form

$$M_n^A(Q^2)/A = \left(1 + \frac{\epsilon}{M}(n-1) + \frac{<p^2>}{6M^2}n(n-1) + O\left(\frac{1}{M^3}\right)\right)M_n^{NS}(Q^2)$$

$$+ \frac{<\Delta p^2>}{4M^2}n(n+1)M_n^{NS}(Q^2)$$

where for $^{56}$Fe the parameters of the nuclear model, adopted in Ref.[53] are $<\epsilon> \approx -56$ MeV, $<p^2>/(2M) \approx 35$ MeV, $<\Delta p^2>_{Fe} \approx -0.17$ GeV and the derivative $\partial p^2 M_n(Q^2)$ is taking into account that the target momentum $p$ can be generally off-shell. This effect is resulting in the following independent from the nuclear content contribution [53]

$$\partial p^2 M_n(Q^2) = \partial p^2 M_n^{as} + \frac{n}{Q^2}\left(M_n^{NS} + M^2\partial p^2 M_n^{as}\right)$$

where the numerical values of $\partial p^2 M_n^{as}$ were also presented in Ref.[53].

Note, that the effects of the nuclear corrections in DIS were also recently studied in Ref.[54] in the case of $xF_3$ SF and in Ref.[55],[56] in the case of $F_2$ SF (for the earlier related works see e.g. Ref.[57]). However, in our studies we will concentrate ourselves on the consideration of the results of Ref.[53].

We included the corrections of Eqs.(37)-(38) into our fits and observed the unacceptable increase of $\chi^2$ value. We think that this can be related to the manifestation of the possible asymptotic character of the $1/M$-expansion in Eq.(37), since the third term in the brackets of the r.h.s. of Eq.(32) becomes comparable with the first term (which is equal to unit) for the $n \sim 8$ used in our fits. Note that the moments with large $n$ are important in the reconstruction of the behavior of the $xF_3$ SF at $x \rightarrow 1$. This observed feature necessitates the derivation of the explicit expression for $M_n^A(Q^2)$, which is not expanded in powers of $1/M$-terms. It should be noted that the problem of the possible asymptotic nature of the power suppressed expansions was mentioned in the case of Ellis-Jaffe and Bjorken DIS sum rules in Ref.[58].

Another possibility of the nonconvergence of our fits with nuclear corrections of Eq.(37) taken into account might be related to the fact that the parton distribution model for the nuclear SF $xF_3^{56}$Fe can be different from the canonical model, used by us.

In any case we think that the problem of the taking into account of the heavy nuclear effects in the process of fits of $xF_3$ data is really on the agenda.
Conclusion

In this work we presented the results of the extractions of $\alpha_s(M_Z)$ and twist-4 terms from the QCD analysis of the CCFR data with taking into account definite QCD corrections at the NNLO and beyond. Within experimental and theoretical errors our results for $\alpha_s(M_Z)$ are in agreement with other extractions of this fundamental parameter, including its world average value $\alpha_s(M_Z) = 0.118 \pm 0.005$.

Our estimate of the NNLO theoretical uncertainties is based on application of the $[0/2]$ Padé approach at the N$^3$LO level. The uncertainties of our NNLO analysis can be decreased after explicit NNLO calculations of the NS Altarelli-Parisi kernel.

As to the twist-4 terms, we found that despite the qualitative agreement with the IRR model prediction, at the NNLO level they have the tendency to decrease and are stable to the application of the $[0/2]$ Padé motivated N$^3$LO analysis.

This feature can be related to the fact that the analysis of the CCFR data can not distinguish the twist-4 $1/Q^2$ terms from the NNLO perturbative QCD approximations of the Mellin moments. This possible explanation lies in the lines of the results of the LO analysis of the old less precise neutrino DIS data, made by the authors of Ref.[1], who were unable to distinguish between LO logarithmic and $1/Q^2$-behavior of the QCD contributions to Mellin moments of $xF_3$. The achieved in our days increase of experimental precision might move this effect to the NNLO.

Another related explanation is that the observed by us NNLO effect is manifesting itself in view of the fact that the detected by us twist-4 terms come from the partial summation of the definite terms of the asymptotic perturbative QCD series and thus the increase of the order of perturbative QCD analysis effectively suppresses the remaining sum of the perturbative QCD contribution. Unfortunately, at the NNLO level we can not detect the true twist-4 terms, which reflect the non-trivial non-perturbative nature of QCD vacuum. One can hope that the future experiments of NuTeV collaboration will allow to get the new experimental data at the precision level, necessary for extracting more detailed information about higher twist contributions to structure functions and will help to clarify the reason of the disagreement of the low $x$ CCFR data for $F_2$ SF with the ones, obtained by the BCDMS collaboration. We hope to return to the NNLO analysis of the experimental data of BCDMS collaboration in the nearest future.

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