Violations of a Leggett-Garg inequality without signalling for a photonic qudit probed with ambiguous measurements

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We realise a quantum three-level system with photons distributed among three different spatial and polarization modes. Ambiguous measurement of the state of the qudit are realised by blocking one out for the three modes at any one time. Using these measurements we construct a test of a Leggett-Garg inequality as well as tests of no-signalling-in-time for the measurements. We observe violations of the Leggett-Garg inequality that can not be accounted for in terms of signalling. Moreover, we tailor the qudit dynamics such that both ambiguous and unambiguous measurements are simultaneously non-signalling, which is an essential step for the justification of the use of ambiguous measurements in Leggett-Garg tests.

I. INTRODUCTION

Macrealism, as defined by Leggett and Garg [1], posits that a macroscopic system will exist in a well-defined state at all times, and that this state can be measured without disturbing it (the assumption of non-invasive measurability). From these assumptions follow the Leggett-Garg inequalities (LGIs) [1–3], which hold under macrorealism but can be violated by quantum mechanics [4–11]. The same assumptions also imply the non-signalling-in-time (NSIT) equalities, which demonstrate the absence, on the statistical level, of signalling between measurements [12–15]. Having NSIT hold completes the formal similarity between the temporal LGI and spatial Bell tests [16]. Violations of a LGI without NSIT, however, provides a convenient loophole for a macrorealist to explain the experiment in terms of the signalling of invasive measurements.

It has been shown theoretically that when unambiguous, projective measurements are used, violations of LGIs are always accompanied by violations of NSIT [17, 18], and thus the use of projective measurements is generally problematic in this context. In Ref. [17], however, George et al. realised LGI violations without signalling through use of measurements that were ambiguous, i.e. measurements where the individual results do not completely reveal the state of the system [17]. Quantum-mechanically, such measurements are sometimes described as “semi-weak”. LGI violations with ambiguous measurements were also discussed in Refs. [5, 20, 21]. In Ref. [18], a general framework for LGI tests with ambiguous measurements was discussed. There it was shown that the derivation of LGIs that use data from ambiguous measurements rely on an assumption that equates the invasive influence of the ambiguous measuring device to that of an unambiguous one acting on the same system. Whilst it is perhaps hard to see how this assumption might hold in general, it has the clear implication that an LGI test in which ambiguous measurements are observed to be non-signalling is only consistent with its own assumptions if the corresponding set of unambiguous measurements on the same system is also observed to be non-signalling.

In this paper, we report on LG experiments with single photons that implement a three-level quantum system measured with both ambiguous and unambiguous measurements. We test LGIs and NSIT equalities with both sets of measurements. In the case of unambiguous measurements, we confirm that all observed LGI violations can explained in terms of signalling. In the ambiguous case, however, we show that it is possible arrange the time-evolution of our three-level system such that the ambiguously-measured LGI is violated whilst at the same time NSIT is satisfied for both measurement types. In this case, we obtain an LGI violation that is consistent both with assumption of non-invasive measureability as well as the assumptions implicit in the usage of ambiguous measurements in this type of test.

This paper proceeds as follows. In Sec. II we describe what is meant here by ambiguous measurements and in Sec. III we describe their experimental realisation for our photonic qudit. In Sec. IV we consider the non-violations of the LGI with unambiguous measurements when signalling is taken into account. Section V contains our main results where we employ ambiguous measurements to violate a LGI whilst all no-signalling constraints are fulfilled. We conclude with discussions in Sec. VI.
II. AMBIGUOUS MEASUREMENTS

We begin by discussing the meaning of unambiguous and ambiguous measurements following Ref. 19, which establishes these concepts identically in both quantum and classical contexts.

In our three-level system, unambiguous measurements reveal one of three distinct results \( n \in \{ A, B, C \} \), and since these results are repeatable, we associate \( n \) with the “realistic” system state. Let us denote the probability that we measure result \( n \) as \( P(n) \).

On the other hand, ambiguous measurements do not reveal complete information about the state of the system and are non-repeatable. The particular scheme we will consider here is a set of three individual measurements, each of which serves to exclude one of the three system states. Thus, the measurement outcomes are \( \alpha \in \{ B \cup C, A \cup C, A \cup B \} \) and our experiments record probabilities such as \( P(B \cup C) \), etc.

The probabilities obtained with the two different measurements setups are clearly related. Elementary probability theory gives \( P(B \cup C) = P(B) + P(C) \), for example, which could easily be verified experimentally. Given the complete set of three ambiguous probabilities, a macrorealist would have no qualms inferring the probabilities that the system “really was in” such as \( A \), by calculating

\[
P'(A) = \frac{1}{2} P(A \cup B) + \frac{1}{2} P(A \cup C) - \frac{1}{2} P(B \cup C),
\]

and so on [22]. Here we maintain the notation \( P' \) for a probability inferred from ambiguous measurements, as opposed to one that is measured directly.

Quantum-mechanically, unambiguous and ambiguous measurement are realised respectively as a complete set of projection operators and a more general POVM that implements a “semi-weak” measurement [22]. In the case of a measurement of the systems state at a single time, calculating either with quantum-mechanics or classically, \( P'(A) \) and \( P(A) \) will clearly agree. However, when sequential measurements are made on the same system, the difference in the quantum case between directly-measured probabilities \( (P) \) and those that are inferred \( (P') \) becomes critical.

III. EXPERIMENTAL SET-UP

Our experiment realises a quantum three-level system, or qutrit, with single photons travelling through the apparatus depicted in Fig. 1. As the general set-up is similar to previous work [10], we refer the reader to these for more details on the implementation of the various components discussed below.

The basis states of the qutrit, \( |A\rangle = (1, 0, 0)^T \), \( |B\rangle = (0, 1, 0)^T \), and \( |C\rangle = (0, 0, 1)^T \), are respectively encoded by the horizontal polarization of the heralded single photons in the upper mode \( |HU\rangle \), the vertical polarization of the photons in the upper mode \( |VU\rangle \), and the horizontal polarization of the photons in the lower mode \( |HD\rangle \). For this experiment, the photons are prepared in the initial state \( |C\rangle \). The unitary evolution of the qutrit state is realised by a sequence of half-wave plates (HWP) and subsequent birefringent calcite beam displacers (BDs) that realise two unitary operators \( U_{21}(\theta_1, \chi_1, \phi_1) \) and \( U_{32}(\theta_2, \chi_2, \phi_2) \) that are nominally identical and can be
decomposed as \([24, 23]\)

\[
U(\theta, \chi, \phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \chi & 0 & \sin \chi \\ 0 & 1 & 0 \\ -\sin \chi & 0 & \cos \chi \end{pmatrix} \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]

Throughout the experiment, measurement of the photon state at \(t_3\) is always performed projectively. This is accomplished by BDq that maps the basis states of qutrit into three spatial modes followed by single-photon avalanche photodiodes (APDs), in coincidence with the trigger photons. The probability of the photons being measured in \(|A\rangle\), \(|B\rangle\) or \(|C\rangle\) is obtained by normalizing photon counts in the certain spatial mode to total photon counts. The count rates are corrected for differences in detector efficiencies and losses before the detectors. We assume that the lost photons would have behaved the same as the registered ones (fair sampling) \([23]\). Experimentally this trigger-signal photon pair is registered by a coincidence count at APD with 3ns time window. Total coincidence counts are about 14,000 over a collection time of 7s.

In the forms we consider them here, the Leggett-Garg and NSIT tests require two different types of measurement of time \(t_2\), i.e. between the two unitary evolution operations. The unambiguous measurement is realized by placing blocking elements into the optical paths \([10, 24]\). With, for example, the channels \(B\) and \(C\) blocked, the joint probabilities \(P(n_3, n_2 = A)\) is obtained without the measurement apparatus having interacting with the photon. In our experiment, this blocking is realized by a polarizing beam splitter (PBS) following by beam stoppers. The PBS is used to map the basis states of qutrit to three spatial modes and the beam stoppers are used to block photons in two of the three spatial modes and let the photons in the rest one pass through. By inserting the HWPs before and after the PBS, we can block any two of the channels and let the photons in the rest one pass through for the next evolution.

The ambiguous measurement is realized in a similar fashion but this time we block just one mode and let photons propagate forwards from the remaining two. With channel \(C\) blocked, for example, and with projective measurements at \(t_3\), we obtain the joint probability \(P(n_3, n_2 = A \cup B)\), where the inference that the photon must have occupied either state \(A\) or \(B\) at time \(t_2\) being the essential ambiguity in this scheme.

**IV. LGI WITH UNAMBIGUOUS MEASUREMENTS**

We first consider an LGI test with unambiguous measurements. In the case where the state preparation is elected to coincide with the first measurement \([8, 10, 28, 24]\), the LGI correlator reads

\[
K = \langle Q_2 \rangle + \langle Q_3 Q_2 \rangle - \langle Q_3 \rangle.
\]
of the usual LGI macrorealistic bound of $K \leq 1$. For these parameters, we obtain the righthand side of the LGI as $1 + \Delta$ = 2 analytically, which is constant as a function of $\theta_2$. This behaviour is recovered by experiment and for $\theta_2 = \pi/2$ we obtain $1 + \Delta = 1.995 \pm 0.011$. Thus, whilst the observed value of $K$ is clearly in excess of the standard bound, when the observed degree of signalling taken into account, we find that the modified LGI, Eq. (4), still holds. This is line with the theoretical results of Refs. [17, 18] which forbid violations of Eq. (4) with projective measurements.

V. LGI WITH AMBIGUOUS MEASUREMENTS

Following Ref. [18], an LGI constructed with the ambiguous measurements has exactly the same form as before

$$K_A \leq 1 + \Delta_A, \quad \Delta_A = \sum_{n_3} |\delta_A(n_3)|,$$

where the subscript A denotes quantities obtained from ambiguous measurements. These quantities have forms identical to those considered previously but with probabilities $P(n_3, n_2)$ replaced with those inferred from the ambiguous measurements. In particular, we obtain the joint probabilities $P'(n_3, A)$ in the same way as Eq. (4) and write

$$P'(n_3, A) = \frac{1}{2}P(n_3, A \cup B) + \frac{1}{2}P(n_3, A \cup C) - \frac{1}{2}P(n_3, B \cup C),$$

and similarly for the other two probabilities. The ambiguously-measured no-signaling quantities are then

$$\delta_A(n_3) \equiv P(n_3) - \sum_{n_2} P'(n_3, n_2),$$

and the correlation functions in $K_A$ are the same as before with the replacement $P \rightarrow P'$. The ambiguously-measured probabilities $P(n_3, A)$ are obtained experimentally in exactly the same way as before, but with ambiguous measurements replacing the unambiguous one at $t_2$. Theoretically, they are obtained with a POVM as outlined in Ref. [18]. Note that the algebraic bound of $K = 3$ is never violated, irrespective of measurement type [5, 30].

Results for $K_A$ and $1 + \Delta_A$ for the parameter set in Sec. IV are shown in Fig. 2. In this case $K_A \leq 1 + \Delta_A$ and no violations of the ambiguously-measured LGI are observed.

Figure 3, however, shows these quantities for a different set of evolution parameters, namely $\theta_1 = 0.831\pi, \chi_1 = \chi_2 = 0.688\pi, \phi_1 = \phi_2 = 0.423\pi$ and $0 \leq \theta_2 \leq \pi$. For a significant range of $\theta_2$ values, we obtain $K_A > 1$. Moreover, for $0.677\pi \leq \theta_2 \leq 0.983\pi$, we find that $K_A \geq 1 + \Delta_A$, and thus we find violations of the modified ambiguously-measured LGI. The maximum violation is found at $\theta_2 = 0.831\pi$ with values $K_A = 1.483 \pm 0.031$, in close agreement with the theoretical prediction 1.464.

![Figure 2](image1.png)

![Figure 3](image2.png)
VI. DISCUSSION

We have described here the experimental violation of the LGI using a realisation of a three-level system with single photons. We have shown that it is possible to obtain violations of the modified LGI, Eq. (7), that takes into account the observed degree of signalling. Violations of this inequality were observed for a range of our evolution parameter \(\theta_2\). Moreover, at the particular point \(\theta_2 = 0.83\pi\), both signalling quantities \(\Delta\) and \(\Delta_\Lambda\) were found to be zero. At this point, then, NSIT is obeyed by both the ambiguous and unambiguous measurements. This is particularly important because, according to Ref. [13], the derivation of Eq. (8) and hence Eq. (4) relies on the assumption that both unambiguous and ambiguous measurements are “equally invasive” and therefore must exhibit the same degree of signalling, i.e. \(\Delta = \Delta_\Lambda\) [39]. Only at the point \(\theta_2 = 0.83\pi\), are the dynamics of our three-level system such that we have \(\Delta = \Delta_\Lambda\). At this point then the use of the ambiguous measurements to construct the LGI for “macrorealistic” state \(n_i\) is justified.

Due to its use of photons, this is a proof-of-principle experiment and can not be viewed as a test of macroscopic realism, as originally envisaged by Leggett and Garg but rather of microscopic realism [40, 41] as has been famously tested in Bell-type experiments [42]. Nevertheless the general principle used for constructing ambiguous LGI tests without signalling could potentially be scaled up to larger, massive objects, perhaps most directly in molecular interference experiments [13].

Despite the enhanced no-signalling features of our experiment, and in common with all known Leggett-Garg-type tests, possible loopholes exist for a macrorealist determined to hold their position. The finding that some of the the inferred probabilities, \(P'(n_3, n_2)\) are negative would presumably lead the macrorealist to reject the possibility that it possible to learn anything about the unambiguous state of the system from ambiguous set-up. This position, however, would require a significant degree of contrivance given that both measurements are known to be individually non-signalling.

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