Sloshing in liquid tank with the density function method

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Abstract. The Great East Japan Earthquake of March 2011 was the huge earthquake with a magnitude of 9.0 exceeding the imagination, ranging from the offshore to Iwate Prefecture to the offshore of Ibaraki Prefecture became the focal region. It is easy to imagine that damage increases to long-period earthquake ground motion as the magnitude increases. Damage to dangerous facilities due to long-period earthquake ground motion is caused by liquid surface fluctuation (sloshing) of oil, LNG and liquid storage tanks. In this paper, the interface capturing method deals with complex water surface large deformation problems. We use the density function method which is one of the effective methods of interface capturing method for numerical analysis of two phase fluid fields of gas and liquid. As a result, the proposed method shows a good agreement with the experimental result of the Martin and Moyce dam break problem, and this method improved the suppression of effortless motion. Also, under the calculation condition of the sloshing phenomenon, this method does not generate numerical diffusion. Therefore, this model is more robust than the conventional model.

1. Introduction

Viscous soil ground does not cause sudden changes in mechanical properties like liquefaction of sandy soil. Therefore, the study of viscous soil has not been carried out as much as liquefaction of sandy soil. However, many structures are now built on the soft ground of the coastal area and the urban area. For these structures, the behaviour of the viscous soil ground at the time of an earthquake may have a big influence [1].

The Great East Japan Earthquake of March 2011 was the huge earthquake with a magnitude of 9.0 exceeding the imagination, ranging from the offshore to Iwate Prefecture to the offshore of Ibaraki Prefecture became the focal region. It is easy to imagine that damage increases to long-period earthquake ground motion as the magnitude increases. Damage to dangerous facilities due to long-period earthquake ground motion is caused by liquid surface fluctuation (sloshing) of oil, LNG and liquid storage tanks [2]. Sloshing of the liquid fuel storage tank increases amplitude due to resonance with long-period earthquake ground motion, leading to disasters such as fire and content leakage [3]. Although it is not a dangerous structure, water facilities, one of the important lifelines, should have facilities such as a storage tank and a piping network in a comprehensive water state [4]. It is necessary to consider these earthquake resistance focusing on the sloshing of the liquid resonating with the seismic motion. In this way, the sloshing phenomenon is an important issue common to fluid engineering and geotechnical engineering [5].

In this paper, the interface capturing method is performed with fixed calculation lattice, and it is a calculation method for analysing the large surface deformation problem. The VOF method [6] is one of the promising techniques in the interface capturing method. A method [7] for indirect solving two phase flows of gas and liquid has been developed such as the Level Set method [8], the density function method [9], and the like. The Level Set method expresses the gas phase and a liquid phase with the positive and...
negative of distance from gas and liquid interface, and the density function method defines the gas phase
and liquid phase of value of density function. Tsubogo et al. [10] combined the high-order differential
advective calculation scheme [11] with the volume correction method of the density function method.
In this method, numerical diffusion near the interface was suppressed and the liquid phase volume was
preserved. However, even with the method, we found that the velocity vector of fluid sometimes
dispersed. Therefore, this paper modifies the previous method [10] and develops a new method which
has the features as follows:
(i) The new method preserves the initial liquid phase volume during the calculation.
(ii) The original method completely suppresses the dispersion of the velocity vector of the fluid.
In addition, we investigate its applicability to slosh phenomenon.

2. The density function method

2.1. Fundamental Equations
With regards to the incompressible fluid, fundamental equations consist of the two continuous and
momentum equations:
\[ \nabla \cdot u = 0 \quad (1) \]
\[ \frac{\partial u}{\partial t} + \nabla (u \cdot u) = \frac{1}{\rho} (F - \nabla p + \mu \nabla^2 u) \quad (2) \]
where \( u, \rho, p, \mu, F \) are respectively the velocity vector, density, pressure, viscous coefficient and volume
power. The density \( \rho \) and the viscous coefficient \( \mu \) satisfy the following equations.
\[ \frac{\partial \rho}{\partial t} + \nabla (u \rho) = 0 \quad (3) \]
\[ \frac{\partial \mu}{\partial t} + \nabla (u \mu) = 0 \quad (4) \]

2.2. Density Function Method
The density function method considers to solve the equation (5) instead of directly solving equations (3)
and (4).
\[ \frac{\partial \Phi}{\partial t} + \nabla (u \Phi) = 0 \quad (5) \]
where, the density function \( \Phi \) normalizes the fluid density with a value between 0 and 1. The fluid
represents the liquid phase at \( \Phi = 1 \), the interface phase at \( \Phi = 0.5 \), and the gas phase at \( \Phi = 0.0 \). The
computational grid through which the contour line \( \Phi = 0.5 \) passes is the interface phase.
Therefore, the density \( \rho \) and the viscous coefficient \( \mu \) are represented by using the density function \( \Phi \)
as follows:
\[ \rho = \Phi \rho_{Liq} + (1 - \Phi) \rho_{Gas} \quad (6) \]
\[ \mu = \Phi \mu_{Liq} + (1 - \Phi) \mu_{Gas} \quad (7) \]
where \( \rho_{Liq} \) and \( \rho_{Gas} \) are the liquid phase and gas phase density, \( \mu_{Liq} \) and \( \mu_{Gas} \) are the liquid phase and
gas phase viscosity coefficients. The density and the viscosity coefficient are assumed to be constant.
Below, the volume correction method is dealt with.

2.3. Volume Correction Method
For volume correction, it is important to obtain the liquid phase volume at the initial \( t = 0 \) and the current
time \( t \). Therefore, it is necessary to distinguish the calculation lattice in the calculation region according
to the attributes of liquid phase, interface phase and gas phase. The density function \( \Phi(x, y, t) \) is represented by the lattice value \( \{\Phi_{i,j}\} \) as shown in Figure 1.

If we pay attention to the density function \( \Phi \) (grey point), we need to consider the proximity point \( \Phi_k \) (white points).

We define the attributes of the cell the following discriminatory conditions.

\[
\begin{align*}
\text{if } \Phi &> 0.5, \text{then liquid cell} \\
\text{if } \Phi &= 0.5, \text{then interface cell} \\
\text{if } \Phi &< 0.5, \text{then gas cell}
\end{align*}
\]

It is important to find the contour line of \( \Phi(x, y, t) = 0.5 \) in order to search for the interface phase.

It is really important that the interface position to look for a cell through which the contour line passes. We present a method to correct the initial liquid phase volume. First, we search for cells close to \( \Phi = 0.5 \). Second, another \( \Phi \) value with a value closes to \( \Phi = 0.5 \) is established. Third, for each pair of cells, we obtain an internal division point and use this point to determine whether the contour line \( \Phi = 0.5 \) passes through the cell. However, the interface boundary we got is not perfect. Therefore, fourthly, consider the correction of the value of \( \Phi \) for volume correction. Then we get the volume correction value from equation (8).

\[
\begin{align*}
L_{\text{Err}} &= \frac{V_{\text{Err}}(t)}{A(t)} \\
\end{align*}
\]

where, \( V_{\text{Err}}(t) = V_{\text{Liq}}(t) - V_{\text{Liq}}(0) \) : The difference of the volumes (of liquid phase) at time \( t \) and zero.

\( A(t) \): sum of \( \Phi \) values at the grid points closest to the calculated interface phase. After that we modify \( \Phi \) values:

\[
\begin{align*}
\Phi(i,j,t) &\leftarrow \begin{cases} 
\Phi(i,j,t) + |L_{\text{Err}}(t)| & \text{if } \Phi(i,j,t) < 0.5 \\
\Phi(i,j,t) - |L_{\text{Err}}(t)| & \text{else}
\end{cases}
\end{align*}
\]

We can determine the new interface position with this procedure.

In addition, we move the interface position to \( V_{\text{Err}}(t)/V(0) < 10^{-4} \) so that the volume of the liquid phase can ensure conservation. The volume correction method moves the contour line \( \Phi = 0.5 \). But velocity vector of fluid remains to be dispersible.

Under the assumption \( |u_{\text{Gas}}| > |u_{\text{max}}| \), the dispersion is protected if we adopt the rule

\[
\begin{align*}
u &= \begin{cases} 
|u_{\text{max}}| & \text{if } u \geq 0 \\
-|u_{\text{max}}| & \text{else}
\end{cases}
\end{align*}
\]

Figure 1. Discrete model of \( \Phi(x,y,t) \).

Figure 2. Sectional view of the water surface.
where $u_{\text{max}}$ is the maximum value of the velocity vector over the liquid and interface phases. By doing so, our new method can protect the dispersion of the velocity vector, whereas our previous method could not. In addition, the distribution of the velocity vector of the gas phase takes a natural behaviour. This numerical calculation tool including the above contents is a CFD code that we develop independently.

3. Flow analysis

3.1. Dam Break Problem

We compare the old method (Tsubogo et al [10]) and the new method (proposed method) with the experimental results of the dam break problem studied by Martin and Moyce [12] (This experimental result is used as a benchmark test of numerical analysis results). We use a lattice point of square type with lattice spacing of 0.015 m in Figure 2, and set the shape of a cube with a length of 0.15 m as the initial setting. We apply the property value $\rho_{\text{Gas}} = 1.25 \text{kg/m}^3$, $\rho_{\text{Liq}} = 1000 \text{kg/m}^3$, $\mu_{\text{Gas}} = 1.0 \times 10^{-3} \text{Pa}\cdot\text{s}$, $\mu_{\text{Liq}} = 1.5 \times 10^{-15} \text{Pa}\cdot\text{s}$, and $g$ (gravitational acceleration) = 9.8 m/s$^2$. In this paper, we use the 5th order conservation 6 point scheme (hereinafter referred to as 5C6P) developed by Tsubogo [11] as the advection term calculation method of the equation of motion and the transport equation for the density function.

For comparison, we used the 3rd order TVD-MUSCL scheme (3TM) and the 3rd order conservation 6 point scheme (3C6P below) for the advection term calculation scheme of the equation of motion and the transport equation of the density function in the previous method. 3TM approved the 2nd order Adams-Bashforth method for time integration. In addition, 3C6P and 5C6P introduce a universal limiter to eliminate numerical vibration.

3.2. Results of Dam Break Problem

Figure 3 displays the temporal change in the water surface tip distance on the bottom side after collapse of the water column. In the previous method, it is consistent with the experimental results up to a dimensionless time 1. However, the water surfaces moving speed gradually decreases and departs from the experimental value. On the other hand, the proposed method coincides very well with experimental results and numerical analysis results.
Figure 4 shows the temporal change of the left side water level. The previous method shows little value close to the experimental value of the dimensionless time 1. However, with the progress of time, the previous method has departed from the experimental value as the water surface moving speed decreases. The proposed method has a value close to the experimental result and is superior to the previous method.

As a result of applying the proposed method, the left and bottom sides of the water surface by the proposed method are in good agreement with the experiment. On the other hand, the previous method has not been (see Figures 3 to 4). Relative error between liquid phase volume and the total mass is within 10^{-4}\% up to 60 seconds.

3.3. Sloshing Phenomenon

In considering the effectiveness of the method proposed in Chapter 2, we address the two-dimensional sloshing phenomenon problem as showed in Figure 5. The excitation force gives the horizontal acceleration expressed by equation (11).

\[ a = -\frac{4\pi^2}{T^2} A \sin \frac{2\pi}{T} t \]  

(11)

In this model, the amplitude \( A \) is set to 0.1 m. In addition, the period \( T \) is 1.087 second. The calculation lattice interval is set to \( \Delta x = \Delta y = 0.010 \) m as the calculation condition, and the time step is made constant at \( \Delta t = 0.0001 \) sec. The computation time is 20 second, while continuing to give the excitation force. The advection term calculation scheme used in this section is the same as the method proposed in the previous section.
Figure 6 shows an example of the calculation result of period $T = 1.087$ second. Sloshing phenomenon accompanied by interfacial large deformation such as fusion/division of gas-liquid interface is reproduced. In addition, Figure 7 shows the change in the water level of the right and left walls at period $T = 1.087$ second. From these results, it is confirmed that the present method can calculate stable when the period $T = 1.087$ second, where the interface deformation is most severe.

Maximum water-surface displacement can be easily calculated from the theoretical formula (Housner Method [13]) in 2-D Tank Sloshing phenomena. Table 1 shows the displacement of the maximum water-surface with a period $T=1.087$ second. And the calculation result of Table 1 is the maximum water-surface displacement within the calculation time. The numerical solution shows a value about twice as high as the theoretical solution of Housner. We corrected flow velocity to suppress numerical diffusion. However, the numerical analysis of sloshing is thought to have resulted higher than the theoretical solution due to insufficient correction of the flow velocity.

![Graphs showing water level changes](image)

**Figure 7.** Time change of water level ($T=1.087$ second)

**Table 1.** Maximum water-surface displacement.

| Method       | Housner | Calculation |
|--------------|---------|-------------|
| Maximum water-surface displacement (m) | 0.212   | 0.500       |
4. Conclusions
The density function method is one of powerful tools for solving complex free surface flow problems. However, higher order difference method cannot obtain sufficient analysis result for free surface flow. Therefore, this paper has developed a new method. The proposed method conserves the initial liquid volume through the numerical analysis process. It summarizes the results obtained in this paper below.
In the case of using the previous volume correction method, this method suppresses the fluid motion and is insufficient as a reproduction of the fluid phenomenon. Furthermore, this method has stopped calculation halfway through numerical diffusion. On the other hand, Analysis results by the proposed method has good agreement with the experimental results by Martin and Moyce (1952). We develop the volume correction method to make it possible to perform calculations stable in the sloshing phenomenon accompanying the large interfacial deformation.
In the future, we will try to improve analytical accuracy by comparing with experiments.

5. References
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