Hadron mass corrections in semi-inclusive deep inelastic scattering

A. Accardi\textsuperscript{a,b}, T. Hobbs\textsuperscript{b}, W. Melnitchouk\textsuperscript{b}

\textsuperscript{a}Hampton University, Hampton, Virginia 23668, USA
\textsuperscript{b}Jefferson Lab, Newport News, Virginia 23606, USA

Abstract

We derive mass corrections for semi-inclusive deep inelastic scattering of leptons from nucleons using a collinear factorization framework which incorporates the initial state mass of the target nucleon and the final state mass of the produced hadron $h$. The hadron mass correction is made by introducing a generalized, finite-$Q^2$ scaling variable $\zeta_h$ for the hadron fragmentation function, which approaches the usual energy fraction $z_h = E_h/\nu$ in the Bjorken limit. We systematically examine the kinematic dependencies of the mass corrections to semi-inclusive cross sections, and find that these are even larger than for inclusive structure functions. The hadron mass corrections compete with the experimental uncertainties at kinematics typical of current facilities, $Q^2 \lesssim 10$ GeV$^2$ and intermediate $x_B \gtrsim 0.3$, and will be important to efforts at extracting parton distributions from semi-inclusive processes at intermediate energies.
I. INTRODUCTION

In recent years semi-inclusive deep inelastic scattering (SIDIS) has received much attention as a tool to investigate various aspects of hadron structure, such as the flavor dependence of the nucleon’s parton distribution functions (PDFs), both unpolarized and polarized, through flavor tagging of hadrons in the final state. Observation of the momentum distribution of produced hadrons also allows access to the largely unexplored transverse momentum dependent parton distributions, which reveal a much richer landscape of the spin and momentum distribution of quarks in the nucleon, and which are the subject of increasingly greater focus at modern facilities such as Jefferson Lab.

At high energies the scattering and hadronization components of the SIDIS process factorize, and the cross section can be represented as a product of PDFs and fragmentation functions. In practice, however, experiments are generally carried out at few-GeV energies with $Q^2$ as low as 1 GeV$^2$, suggesting that $1/Q^2$ power corrections must be controlled in order to determine the applicability of partonic analyses of data.

One of the standard finite-$Q^2$ corrections that must be applied in analyses of inclusive deep inelastic scattering data is target mass corrections (TMCs) [1]. Kinematical in origin, TMCs arise from leading twist operators in QCD, but enter as $1/Q^2$ corrections to structure functions. They are especially egregious at high values of the Bjorken scaling variable $x_B$, even at relatively large $Q^2$, and are crucial for reliable extractions of parton distributions at large parton fractional momentum $x$. To date, however, TMCs have not been considered in SIDIS; we do so in this paper.

Target mass corrections in inclusive DIS have usually been formulated within the operator product expansion (OPE), in which the subleading $1/Q^2$ corrections arise from twist-two operators involving derivative insertions into quark bilinears [2–8]. It is not clear, however, how this method can be rigorously extended to the production of hadrons in the final state. An alternative approach to computing TMCs makes use of the collinear factorization (CF) framework [9, 11], which has recently been applied to both unpolarized [12–14] and polarized [15] inclusive DIS. Because here one works directly in momentum space, the method can be readily extended to SIDIS.

In this work we use the CF framework to derive the mass corrections to the SIDIS cross section at finite $Q^2$. In Sec. II we review the collinear formalism and discuss its application
to semi-inclusive hadron production. To expose the origin of the corrections we work at leading order in $\alpha_s$; next-to-leading order effects can be included in future analyses.

Note that in contrast to inclusive DIS, where the only mass scale entering is that of the target hadron, in SIDIS finite-$Q^2$ corrections arise from both the target mass and the mass of the produced hadron. For generality we shall refer to the combined effects as “hadron mass corrections” (HMCs). In Sec. III we explore the relative importance of the HMCs numerically, and evaluate the size of the corrections in the cross sections and fragmentation functions at various kinematics. To assess their possible impact on data analyses, we also compare the magnitude of the HMCs at kinematics typical of modern facilities, such as Jefferson Lab and HERMES, with experimental errors from recent experiments. Finally, in Sec. IV we summarize our results and outline avenues for future developments of this work.

II. SEMI-INCLUSIVE SCATTERING AT FINITE $Q^2$

We begin the discussion of SIDIS at finite values of the photon virtuality $Q^2$ by defining in a collinear frame the relevant kinematics and momentum variables, and introduce the hadronic tensor in terms of factorized quark momentum distribution and fragmentation functions. We then restrict ourselves to the leading order approximation, in which the produced hadron is effectively also collinear with the scattered parton, in order to clearly expose the effects of hadron masses on the cross sections and fragmentation functions.

The four-momenta of the target nucleon ($p$), virtual photon ($q$) and produced hadron $h$ ($h$) can in general be decomposed in terms of light-cone unit vectors $n$ and $\bar{n}$ as [9, 10]

$$p^\mu = p^+ n^\mu + \frac{M^2}{2p^+} n^\mu,$$

$$q^\mu = -\xi p^+ n^\mu + \frac{Q^2}{2\xi p^+} n^\mu,$$

$$p_{h}^\mu = \frac{\xi m_{h}^2}{\xi_h Q^2} p^+ n^\mu + \frac{\xi_h Q^2}{2\xi p^+} n^\mu + p_{h}^\perp,$$

where $M$ is the target nucleon mass, $Q^2 = -q^2$, and the light-cone vectors satisfy $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 1$. The light-cone components of any four-vector $v$ are defined by $v^+ = v \cdot \bar{n} = (v_0 + v_z) / \sqrt{2}$ and $v^- = v \cdot n = (v_0 - v_z) / \sqrt{2}$. The three-momenta $\vec{p}$ and $\vec{q}$ are chosen to lie in the same plane as $n$ and $\bar{n}$, as for applications in inclusive deep inelastic scattering (DIS). The nucleon plus-momentum $p^+$ can be interpreted as a parameter for boosts along
the z-axis, connecting the target rest frame to the infinite-momentum frame. The transverse momentum four-vector of the produced hadron $p_{h\perp}^\mu$ satisfies $p_{h\perp} \cdot n = p_{h\perp} \cdot \bar{n} = 0$, and we define the transverse mass squared as $m_{h\perp}^2 \equiv m_h^2 - p_{h\perp}^2$, where $p_{h\perp}^2 = -\bar{p}_{h\perp}^2$.

The finite-$Q^2$ scaling variable $\xi$ for the quark distribution function is defined as \[ \xi = -\frac{q^+}{p^+} = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 M^2/Q^2}}, \tag{2} \]
which in the Bjorken limit ($Q^2 \to \infty$ at fixed $x_B$) reduces to the Bjorken scaling variable $x_B = Q^2/2p \cdot q$. In analogy, the scaling fragmentation variable $\zeta_h$ is defined by

\[ \zeta_h = \frac{\bar{p}_{h\perp}^\mu}{q^-} = \frac{z_h}{2x_B} \left( 1 + \sqrt{1 - \frac{4x_B^2 M^2 m_{h\perp}^2}{z_h^2 Q^4}} \right), \tag{3} \]
so that in the Bjorken limit $\zeta_h \to z_h = p_h \cdot p/q \cdot p$. Note that in the target rest frame $z_h = E_h/\nu$ is the usual ratio of the produced hadron to virtual photon energies. The positivity of the argument of the square root in Eq. (3) is ensured by the condition $E_h \geq m_{h\perp}$. A particular choice of kinematics will therefore impose a lower limit upon the physically accessible range of $z_h$:

\[ z_h \geq z_h^{\text{min}} = 2x_B \frac{M^2 m_h^2}{Q^4}. \tag{4} \]

As illustrated in Fig. 1, at the partonic level the SIDIS process is described through scattering from a quark carrying a light-cone momentum fraction $x = k^+/p^+$, which then fragments to hadron $h$ carrying a light-cone momentum fraction $z = p_{h\perp}^-/l^-$, where $k^\mu$ and $l^\mu$ are the four-momenta of the initial and scattered quarks, respectively. As for the external momenta, these can be parametrized in terms of the light-cone vectors $n$ and $\bar{n}$ as

\[ k^\mu = xp^+ \bar{n}^\mu + \frac{k^2 + k_{\perp}^2}{2xp^+} n^\mu + k_{\perp}^\mu, \tag{5a} \]
\[ l^\mu = \frac{l^2 + l_{\perp}^2}{2p_{h\perp}^- / z} \bar{n}^\mu + \frac{p_{h\perp}^-}{2z} n^\mu + l_{\perp}^\mu, \tag{5b} \]
with the parton transverse momenta $k_{\perp}^\mu$ and $l_{\perp}^\mu$ orthogonal to $n^\mu$ and $\bar{n}^\mu$.

At leading order in $\alpha_s$ the hadron tensor can be written as

\[ 2MW^{\mu\nu}(p, q, p_h) = \sum_q e_q^2 \int d^4k \ d^4l \ \delta^{(4)}(k + q - l) \ Tr[\Phi_q(p, k) \gamma^\mu \Delta_q^h(l, p_h) \gamma^\nu], \tag{6} \]
where the operators $\Phi_q$ and $\Delta_q^h$ encode the relevant quark distribution and fragmentation functions, respectively, and the sum is taken over quark flavors $q$. The $\delta$-function fixes the
FIG. 1: Kinematics of the semi-inclusive deep inelastic lepton–nucleon scattering, producing a final state hadron \( h \).

four-momentum of the outgoing parton to be \( l = k + q \). To this order the parton fractional momentum \( x \) also coincides with the Bjorken scaling variable \( x_B \).

In the CF framework the parton momenta \( k^\mu \) and \( l^\mu \) in the hard scattering amplitude are expanded around their on-shell \((k^2 \rightarrow 0, l^2 \rightarrow 0)\) and collinear \((k_\perp \rightarrow 0, l_\perp \rightarrow 0)\) components, so that effectively \( k^\mu \rightarrow xp^+\bar{n}^\mu \) and \( l^\mu \rightarrow (p^-_h/z) n^\mu \). This restricts the produced hadron momentum to be purely longitudinal, \( p_{h\perp}^2 = 0 \), and \( m_{h\perp}^2 = m_h^2 \). Hadrons with non-zero transverse momentum can be generated from higher order perturbative QCD processes, or from intrinsic transverse momentum in the parton distribution functions, as in the case of transverse momentum dependent (TMD) distributions [18], but are not considered in this work.

For the soft part of the amplitude, one defines integrated quark correlators by integrating over the transverse and minus (or plus) components of the momenta,

\[
\Phi_q(x) = \int dk^- d^2k_\perp \Phi_q(p, k) \bigg|_{k^+ = xp^+} ,
\]

\[
\Delta_h^\nu(z) = \frac{1}{2z_h} \int \frac{dl^+}{q^-} d^2l_\perp \Delta_q^\nu(l, p^-_h) \bigg|_{l^- = p^-_h/z} .
\]

Here one implicitly assumes that \( k^-, k_\perp \ll \Lambda \) and \( l^+, l_\perp \ll \Lambda \), where \( \Lambda \) is a hard momentum scale, which in SIDIS can be taken to be \( Q \). From Eq. (6) the energy-momentum conserving
\(\delta\)-function can be decomposed along the plus, minus, and transverse components of the light-cone. The plus and minus components of the momenta yield a product of \(\delta\)-functions that isolate the \(\xi\) and \(\zeta_h\) dependence of the hadronic tensor, while the transverse component constrains the transverse momentum of the scattered quark (in a frame where \(q_\perp = 0\)) to vanish, \(\delta^{(4)}(k + q - l) \sim \delta(x - \xi) \delta(z - \zeta_h) \delta^{(2)}(l_\perp)\). The resulting hadron tensor reads

\[
2MW^{\mu\nu}(p, q, p_h) = \frac{2\zeta_h}{z_h} \sum_q e^2_q Tr \left[ \Phi_q(\xi) \gamma^\mu \Delta_q(\zeta_h) \gamma^\nu \right].
\]

(8)

Finally, one defines the quark momentum distribution function in terms of the correlator \(\Phi_q(x)\) as

\[
f_q(x) = \frac{1}{2} Tr \left[ \gamma^+ \Phi_q(x) \right]
\]

(9a)

\[
\equiv \frac{1}{p^+} \int dw^- e^{i x p^+ w^-} \langle N|\bar{\psi}_q(0)\psi_q(w^- n)|N \rangle,
\]

(9b)

where \(w\) is a space-time integration variable, and “LC” denotes use of the light-cone gauge. Similarly, the quark \(\rightarrow\) hadron \(h\) fragmentation function is defined in terms of \(\Delta^h_q(z)\) by

\[
D^h_q(z) = \frac{1}{2} Tr \left[ \gamma^- \Delta^h_q(x) \right]
\]

(10a)

\[
\equiv \frac{1}{2z_h} \sum_X \int dw^+ e^{i (p_h^- / z) w^+} \langle 0|\bar{\psi}_q(w^+ n)|h, X\rangle \langle h, X|\bar{\psi}_q(0)|0 \rangle,
\]

(10b)

and is normalized according to \(\sum_h \int dz z D^h_q(z) = 1\). With these definitions, the unpolarized SIDIS cross section in the presence of hadron mass effects is factorized into a product of parton distribution and fragmentation functions evaluated at the finite-\(Q^2\) scaling variables \(\xi\) and \(\zeta_h\), instead of \(x_B\) and \(z_h\) as would be obtained in the massless limit:

\[
\sigma \equiv \frac{d\sigma}{dx_B dQ^2 dz_h} = \frac{2\pi\alpha_s^2}{Q^4} \frac{y^2}{1 - \varepsilon} \left(1 + \frac{\gamma^2}{2x}\right) \sum_q e^2_q f_q(\xi, Q^2) D^h_q(\zeta_h, Q^2),
\]

(11)

where in the target rest frame \(y = \nu/E\) is the fractional energy transfer to the target, \(\varepsilon = (1 - y - y^2\gamma^2/4)/(1 + y + y^2[1/2 + \gamma^2/4])\) is the ratio of longitudinal to transverse photon flux, and \(\gamma^2 = 4x_B^2 M^2/Q^2\), and we have kept the explicit dependence of the functions on the scale \(Q^2\). In the next section we shall examine the phenomenological consequences of the finite-\(Q^2\) rescaling of the SIDIS cross section.

### III. HADRON MASS CORRECTIONS

Using the hadron mass corrected expressions for the SIDIS cross section derived above, in this section we explore the dependence of the cross sections and fragmentation functions...
on the fragmentation variable $z_h$, for various $x_B$ and $Q^2$ values and for different final state hadron masses. We then compare the relative size of the HMCs with the experimental uncertainties from recent SIDIS experiments at HERMES and Jefferson Lab. For numerical computations we use the leading order CTEQ6 parton distributions [23] and KKP fragmentation functions [24], save for Fig. 5, which shows calculations performed using DSS fragmentation functions [25].

To illustrate most directly the effects of the HMCs, in Fig. 2 we consider charged pion production and plot the ratio of the full cross section $\sigma$ in Eq. (11) to the cross section $\sigma^{(0)}$, defined by taking the massless limit in the quark distribution and fragmentation functions ($\xi \rightarrow x$, $\zeta_h \rightarrow z_h$), as a function of $z_h$ for various values of $x_B$ and $Q^2$. The ratio at $x_B = 0.3$ in Fig. 2(a) is enhanced by around 20% at the lowest $Q^2$ value, $Q^2 = 2 \text{ GeV}^2$, for $z_h \lesssim 0.7$, but rises dramatically as $z_h \rightarrow 1$. The effect is naturally smaller at higher $Q^2$ values, but the dramatic rise at high $z_h$ is a common feature for all kinematics. The same ratios at $x_B = 0.8$ in Fig. 2(b) show approximately an order of magnitude larger overall effect (note the logarithmic scale!).

The larger absolute value of the ratio at high $x_B$ is due mostly to the Nachtmann rescaling in the quark distribution function, similarly to what happens in inclusive DIS. This is illustrated in Fig. 3, where we show the ratio of the inclusive $F_1$ structure functions with and without target mass corrections as a function of $x_B$ for several values of $Q^2$. In collinear factorization at leading order in $\alpha_s$, the target mass corrected structure function is given by $F_1(x_B) = F_1^{(0)}(\xi)$ [12–14], where $F_1^{(0)}$ is the massless limit function. At $Q^2 = 2 \text{ GeV}^2$ the
corrected structure function is $\sim 5$ times larger than the uncorrected one at $x_B = 0.8$; at $Q^2 = 5$ GeV$^2$ the ratio is $\sim 3$, and even at $Q^2 = 20$ GeV$^2$ the TMC is some 50%, with the effect increasing dramatically as $x_B \to 1$. In contrast the TMC effect is almost negligible at $x_B = 0.3$ for all the $Q^2$ considered. The additional hadron mass corrections in the fragmentation functions further augment the DIS correction to produce the full effect observed in the SIDIS cross section in Fig. 2.

The relative importance of HMCs for different produced hadron species is illustrated in Fig. 4(a), where the ratio $\sigma/\sigma^{(0)}$ is shown as a function of $z_h$ for $x_B = 0.3$ and $Q^2 = 5$ GeV$^2$. The corrections for the $\pi$ and $K$ are $\sim 10\%$ over the range $0.2 \lesssim z_h \lesssim 0.8$, and about 20–30\% for protons. At the extrema of the kinematical range, when $z_h \to 0$ and $z_h \to 1$, the effects increase dramatically. Note that we used the appropriate fragmentation function for each of the produced hadron species. This introduces a flavor dependence in the HMC because of the different slope in the different FFs. The second source of flavor dependence is the difference in the hadron masses themselves. Sensitivity to the hadron mass also reflects the role of the radical term in the definition of $\zeta_h$ in Eq. (3). Even though this term is suppressed by a factor $1/Q^4$, taking the scaling variable $\zeta_h \approx z_h \xi/x_B$ may not be a good approximation at all kinematics, especially at small $z_h$.

This can be seen more explicitly in Fig. 4(b), where the ratio of full cross sections $\sigma$ computed with pion fragmentation functions and $m_\pi = 0.139$ GeV, 0.5 GeV and 1.0 GeV is shown relative to the cross section with $m_\pi = 0$, which renders $\zeta_h = z_h \xi/x_B$. While the
FIG. 4: Dependence of the ratio $\sigma/\sigma^{(0)}$ of SIDIS cross sections with and without HMCs for different produced hadrons at $x_B = 0.3$ and $Q^2 = 5$ GeV$^2$: (a) for $h = \pi^+ + \pi^-$, $K^+ + K^-$ or $p + \bar{p}$, and (b) for $h = \pi^+ + \pi^-$, but varying the mass of the pion.

differences at the physical pion mass are very small, for larger hadron masses $\sim 1$ GeV the effects can be rather large at $z_h \lesssim 0.2$ and $z_h \sim 1$.

The up-turn in the ratios at small $z_h$ seen in Figs. 2 and 4 can be understood from the interplay between the finite-$Q^2$ kinematics and the shape of the fragmentation function. Assuming the fragmentation function is smooth, one can expand the ratio of corrected to uncorrected functions as

$$\frac{D(\zeta_h)}{D^{(0)}(z_h)} \approx 1 + \frac{1}{D^{(0)}(z_h)} \frac{dD}{dz_h} \left( \frac{d\zeta}{dz_h} \right)^{-1} |_{\zeta=z_h}(\zeta_h - z_h).$$

As $d\zeta/dz_h$ is largely independent of $z_h$, the $z_h$ dependence of the HMC is mostly determined by the magnitude of the rescaling (the difference $|\zeta_h - z_h|$) and the local rate of change over $z_h$ of the unmodified fragmentation function $D^{(0)}$. Since $\zeta_h < z_h$ at finite $Q^2$, and $D^{(0)}$ is a decreasing function of $z_h$, the ratio $D/D^{(0)}$ always exceeds unity. However, because fragmentation functions typically behave as $1/z_h$ at small $z_h$, the large negative slope of $D^{(0)}$ drives the ratio $D/D^{(0)}$ upward as $z_h \rightarrow z_h^{\text{min}}$, where $\zeta_h \rightarrow 0$. This effect is more pronounced at lower values of $Q^2$ and especially for larger final state masses $m_h$, which induce stronger deviations of $\zeta_h$ from $z_h$—see Eq. (3). The rise of the ratio as $z_h \rightarrow 1$ can be similarly understood.

One of the unique capabilities of SIDIS is the ability to tag individual quark flavors by selecting specific hadrons in the final state. Simply from its valence quark content, production of $\pi^+$, for example, is mostly sensitive to the $u$ quark, requiring only a single $q\bar{q}$ pair
FIG. 5: (a) Ratio of hadron mass corrected to uncorrected fragmentation functions $D/D^{(0)}$ for favored (solid) and unfavored (dotted) production of $\pi^+$, for $x_B = 0.8$ and $Q^2 = 2 \text{ GeV}^2$. (b) Comparison of the hadron mass correction to the ratio of unfavored to favored fragmentation functions $R = D^-/D^+$ with experimental errors on $R$ from the recent Jefferson Lab experiment E00-108 [19], normalized to the central values of the data points.

creation from the vacuum, while $\pi^-$ reflects the $d$ quark content of the target nucleon. This simple picture of primary fragmentation provides a good approximation to the production mechanism only at large $z_h$, however, and at low $z_h$ secondary fragmentation involving two or more $q\bar{q}$ pair productions dilutes the direct flavor tagging. The primary fragmentation process is parametrized by the “favored” fragmentation function $D^+$, describing $u \rightarrow \pi^+$ or $d \rightarrow \pi^-$ hadronization, while the secondary fragmentation is parametrized by the “unfavored” fragmentation function $D^-$, describing $u \rightarrow \pi^-$ or $d \rightarrow \pi^+$ hadronization.

Because they have rather different $z_h$ dependence, with unfavored fragmentation strongly suppressed at large $z_h$, the $D^+$ and $D^-$ functions will be affected differently by the hadron mass corrections. Naively one would expect enhanced HMCs for the unfavored process since the magnitude of the slope $|dD^{(0)}(z_h)/dz_h|$ in Eq. (12) is larger for $D^{-(0)}$ than for $D^{+(0)}$. In Fig. 5(a) one observes precisely this; here, we provide an upper limit (given the choice of $x_B = 0.8$ and $Q^2 = 2 \text{ GeV}^2$) to the relative size of the mass effect in $D/D^{(0)}$, which is universally larger in the unfavored fragmentation function. At lower $x_B$ the correction will be smaller, although the qualitative features of the effect will remain.

The relevance of the HMC to experimental data on the ratio $R = D^-/D^+$ is expressed in Fig. 5(b), which directly compares the difference $\delta^{\text{HMC}}R = (D^-/D^+) - (D^-/D^+)^{(0)}$ to the
FIG. 6: Comparison of the hadron mass correction to the SIDIS cross section, \( \delta_{\text{HMC}} \sigma = \sigma - \sigma^{(0)} \), relative to the experimental cross section, with the relative experimental uncertainty as a function of \( z_h \) for (a) HERMES kinematics [20] at \( Q^2 \approx 2.5 \text{ GeV}^2 \) and \( x_B = 0.082 \), and (b) Jefferson Lab experiment E00-108 [19] at a similar \( Q^2 \) but at \( x = 0.32 \).

The relevance of the hadron mass corrections to experimental cross sections is examined in Fig. 6, where we compare the calculated difference \( \delta_{\text{HMC}} \sigma = \sigma - \sigma^{(0)} \) with the experimental uncertainties \( \delta_{\text{exp}} \sigma \), normalized to the central values of the cross sections from HERMES [20] and Jefferson Lab [19]. The mass corrections at the HERMES kinematics in Fig. 6(a), where \( Q^2 \approx 2.5 \text{ GeV}^2 \) and \( x_B = 0.082 \), are generally small compared with the size of the experimental uncertainties. On the other hand, for the Jefferson Lab experiment E00-108 [19] in Fig. 6(b) at a similar \( Q^2 \) but at \( x_B = 0.32 \), the mass effects grow to nearly an order of magnitude larger than the experimental statistical errors. This illustrates the potentially significant impact that HMCs can have on leading-twist analyses of SIDIS data at moderate to large \( x_B \) and low \( Q^2 \). To avoid these effects one would either need to go to smaller \( x_B \) or larger \( Q^2 \) values, for example afforded by the 12 GeV energy upgrade at Jefferson Lab; on the other hand, since the HMCs are calculated and parameter-free, lower-\( Q^2 \) and higher-\( x_B \) data still yield useful leading twist information provided the mass corrections are accounted
IV. CONCLUSION

In this work we have derived hadronic mass corrections to semi-inclusive deep inelastic cross sections at finite $Q^2$. Within the collinear factorization framework, the modifications to the SIDIS cross sections from initial and final state masses arise from a rescaling of the quark distribution and fragmentation functions in terms of the Nachtmann scaling variable $\xi$ and a finite-$Q^2$ fragmentation variable $\zeta_h$, respectively.

We have examined the effects of the hadron mass corrections numerically at kinematics relevant to recent experiments, finding sizable effects at both small and large values of $z_h$, as well as for increasing $x_B$ and $m_h$, and low $Q^2$. Our results emphasize the importance of controlling for such corrections in intermediate to high-$x_B$ experiments executed at low $Q^2$, of which measurements at Jefferson Lab are typical. This fact was illustrated in Figs. 5 and 6, which demonstrate that mass corrections can compete with (in the measurement of the ratio of unfavored to favored fragmentation functions $D^-/D^+$) or overwhelm (in the cross section) the quoted experimental errors.

The most direct use of the results presented here will be in leading twist analyses of SIDIS cross sections, where the HMCs must be included before extracting information on parton distribution and fragmentation functions, especially at large $x_B$ and $z_h$. Application of this work can also be found in studies of semi-inclusive data in the nucleon resonance region, which has been the focus of attention recently [21] in view of understanding the workings of quark-hadron duality [21, 22].

While the present analysis has been performed at leading order in $\alpha_s$, in future we plan to extend the formalism to next-to-leading order, which will permit a more quantitative treatment of transverse mass dependence of the produced hadrons, $p_{h\perp} \neq 0$. It will also allow contact with transverse momentum dependent parton distributions, in which nonzero parton momentum, $k_{\perp} \neq 0$, is an essential element. Finally, as in the inclusive DIS case, the fragmentation function corrected for hadron mass effects will exhibit a threshold mass effect which will render $D(z_h)$ nonzero at $z_h = 1$. As for inclusive DIS, where the nonzero value of $1 - \xi$ for $x_B = 1$ at any finite $Q^2$ forces the scaled distribution function to be nonzero at $x_B = 1$, the fragmentation function evaluated at the extreme point $\zeta_{h_{\text{max}}} \equiv \zeta_h(z_h = 1)$
\(1, x_B = 1, Q^2\) will be nonzero. On the other hand, the cross section at \(\zeta_{\text{max}}^h\) corresponds to exclusive hadron production, and should be strongly suppressed at large \(Q^2\) compared with SIDIS. Some resolutions of this “threshold problem” have been proposed in the literature for inclusive structure functions \([3, 8, 14]\), and extensions of these to the case of SIDIS will be the focus of future work.

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