Collective nature of $0^+$- states in deformed rare-earth nuclei.

Vladimir P. Garistov
Institute for Nuclear Research and Nuclear Energy,
Sofia, Bulgaria.

March 30, 2022

Abstract

The description of the energy spectra of $0^+$ states for rare-earth nuclei has been done involving the degree of the collectivity of corresponding $0^+$-state as a systematics parameter. Holshtein-Primakoff representation leads to very good agreement with experiment. Within the framework of this approach the parameter of the collectivity is mainly determined by pairs of particles constructed on single ”effective” level. The results may be helpful both for nuclear structure experimentalists and theorists in their investigations of low-lying states structure and transition probabilities.

The great amount of experimental data for energy spectra and transition probabilities evokes the necessity of simplified description that can be easily used by experimentalists in explaining the collective properties of states and their systematics. For example the peculiarities of the $0^+$ spectra in nuclei and E0 transitions provoke great interest. In particular, the experimental data of the E0 transitions from second $0^+$- state to the ground state in even-even Sn and Cd isotopes indicate that they are very weak. The transitions from second $0^+$ to the first $0^+$ state in Cd are weak too, but in Sn isotopes the corresponding transition probability is very strong. Also the transitions from first to the ground $0^+$ states in Sn are weak, but in Cd are exceptionally strong. There are many investigations dedicated to
the \( E_0 \) and \( E_2 \) transition probabilities and analyzing the \( 0^+ \) spectra in different nuclei \[2\]. For instance in the rare-earth region the values of the \( B(E2; 2^+_K=0^+_2 \rightarrow 0^+_g.s.) \) and \( B(E2; 2^+_K=0^+_4 \rightarrow 4^+_g.s.) \) transitions as a function of neutron number change drastically for different isotopes \[3\]. In this paper we study the low-energy \( 0^+ \) spectra within the framework of simplified pairing vibrational model in Holstein-Primakoff representation and investigate the systematics of the \( 0^+ \) states for large amount of rare-earth nuclei. We analyze the behavior of their energies using the notion \( k \) - degree of the collectivity as a systematics parameter for the corresponding \( 0^+ \) states.

By minimizing the Chi-square values of the energies for different permutation of the possible values of \( k \) we obtain the spectra as a function of this parameter \( k \) for a large amount of \( 0^+ \) states of deformed rare earth nuclei. These energies can be explained by simple formula:

\[
E_n = Ak - Bk^2
\]

As the description of the structure of the \( 0^+ \) nuclear states in terms of pair configurations continues to be of great interest both for theorists and experimentalists we define the parameter of the degree of collectivity \( k \) as related to the monopole phonon operators

\[
R_j^+ = \frac{1}{2} \sum_m (-1)^{j-m} \alpha_m^+ \alpha_j^+;
R_j^- = \frac{1}{2} \sum_m (-1)^{j-m} \alpha_j^- \alpha_m^-;
R_j^0 = \frac{1}{4} \sum_m (\alpha_j^+ \alpha_m^- - \alpha_j^- \alpha_m^+) ,
\]

(2)

where \( \alpha_j^+, \alpha_j^- \) are the nucleons creation and annihilation operators.

Let us consider the Hamiltonian for \( N \) particles placed on "effective" single level \( j \) in terms of the operators \( R_j^+ , R_j^- , R_j^0 \):

\[
H = \alpha R_j^+ R_j^- + \beta R_j^0 R_j^0 + \frac{\beta \Omega}{2} R_j^0
\]

(3)

\[
\Omega = \frac{2j+1}{2}
\]

The operators (2) satisfy the commutation relations :

\[
[R_j^0, R_j^\pm] = \pm R_j^\pm, \quad [R_j^+, R_j^-] = 2R_j^0
\]

(4)
In order to simplify the notations further we will omit the indices $j$.

Let us now present this Hamiltonian in terms of "ideal" boson creation and annihilation operators $b, b^\dagger = 1$ ; $[b, b^\dagger] = 0$ , using the Holstein-Primakoff transformation \[^5\] for the operators $R_+, R_-$ and $R_0$ :

$$R_- = \sqrt{\Omega - b^\dagger b} b; \quad R_+ = b^\dagger \sqrt{\Omega - b^\dagger b}; \quad R_0 = b^+ b - \Omega/2 \quad (5)$$

The transformations (5) conserve the commutation relations (4) between $R_+, R_-$ and $R_0$ operators. Thus for the Hamiltonian (3) in terms of the new boson creation and annihilation operators $b^+, b$ we have:

$$H = Ab^\dagger b - Bb^\dagger bb^\dagger b \quad (6)$$

where :

$$A = \alpha(\Omega + 1) - \beta \Omega \quad B = \alpha - \beta \quad (7)$$

The energy of any monopole excited state $|n\rangle = \frac{1}{\sqrt{n!}}(b^+)^n |0\rangle$; where $b |0\rangle = 0$ can be written as:

$$E_n = An - Bn^2 \quad (8)$$

We see that we have the same energy spectrum as spectrum (4) ($k \to n$)

The parameters of the approach which we have used are presented in figure 1 along with the experimental and calculated $0^+$ state distributions. The calculated energies are distributed in bell form because of the anharmonic terms in the Hamiltonian (3) and often the lowest $0^+$ states have much more collective structure (bigger $k$) than the states with higher energies. In the framework of this simple model we can predict that additional $0^+$ states should exist in the following cases: $^{194}Pt$ (one phonon state - 0.75 MeV, six phonon state - 2.38 MeV, and seven phonon state - 2.1 MeV), $^{196}Pt$ (one phonon state - 0.6 MeV, three phonon state - 1.5 MeV, twelve phonon state - 1.0 MeV ), $^{188}Os$ (one phonon state - 0.75 MeV and two phonon state - 1.3 MeV ) and $^{158}Er$ (one phonon state - 1.2 MeV). We indicate these predicted states by "?" in the figures 1a -> 1d. Thus it may be interesting to measure $E0$ transition probabilities in these nuclei and especially in $^{194}Pt$ nucleus.

3
from one phonon $0^+$ state with energy 0.75 MeV to the ground state, in $^{196}$Pt nucleus from one phonon $0^+$ state with energy 0.6 MeV to the ground state, in $^{188}$Os from one phonon $0^+$ state - 0.75 MeV to the ground state and in $^{158}$Er from one phonon $0^+$ state - 1.2 MeV to the ground state. We expect relatively intensive $E0$ transitions for these cases. Experimental data about the rotational bands in deformed nuclei show that the dependence of the energy on angular momentum $L$ is qualitatively similar for the ground band and the bands constructed on any excited $0^+$ state. So in the first approximation one may consider the rotational bands constructed on different excited $0^+$ states without including the band head structure. Nevertheless the influence of $0^+$ states structure on the rotational spectra must be included in order to explain the small quantitative differences in rotational bands with different $0^+$ band heads as well as transition probabilities, for instance the peculiarities in $B(E2; 2^+_{K=0^+_h} \rightarrow 0^+_\text{g.s.})$ [3]. This investigation is in successful progress. Furthermore the results of this paper may be helpful for more sophisticated analysis of the collective structure of the low-lying nuclear states. Having in mind the results of this paper one can estimate directly the degree of collectivity of any $0^+$ excited state.

I would like to thank professors S. Pittel, M. Stoitsov, Ani. Aprahamian, Ana Georgieva and P. Terziev for fruitful discussions and help.

This work has been supported in part by the Bulgarian National Foundation for Scientific Research under project Φ - 809.

References

[1] Julin R. et al. - Z. Physik A296, 1980, p. 315; Mheemeed A. et al. - Nucl. Phys. A412, 1984, p.113; Aprahamian A. et al. - Phys. Lett. 140B,1984, 1-2, p.22; Kantele J. et al. - Z. Physik A289, 1979, p.157.

[2] S. T. Belyaev Mat. Fys. Medd Dan. Vid Seelgk.31, N11,(1959); Wenes G. et al. - Phys. Rev. C23, 1981, p.2291; Van Hieven J. F. A. et al. - Nucl. Phys. A269, 1976, p.159; Sakahura M. et al. - Z. Physik A289, 1979, p. 163 ; Shikata Y. et al. - Z. Physik A300, 1981, p. 217; Lopak V and V.Paar - Nucl.Phys. A297, 1978, p.471; De Vries H. F., P. J. Brussard - Z. Physik A286, 1978, p.1; Ahalpara D. P. et al. - Nucl.
Phys. A371, 1981, p.210; Arima A., F. Iachello - Ann. Phys 99,1976, p, 253; 111, 1978, p. 201; 123, 1979, p. 468; Duval P. D. B. R. Barret - Phys. lett. 100B, 1981, p.223 and Nucl. Phys. A376, 1982, p. 213; Druce C.H. et al. - Nucl. Phys. 8, 1982, p.1565; V. P. Garistov Bulg. J. Phys.14, (1987), 4, 317; Tazaki S. et al. - Prog. Theor. Phys. 71,1981, ch.4; Cohen T. D. - Nucl. Phys. A436, 1985, p. 16; C. Volpe et al. Nucl. Phys A 647 (1999),246; A. K. Kerman Annals of Physics 12, (1961), 300; D. M. Brink, A.F.R. De Toledo Piza, A.K. Kerman - Phys. Lett. V.19, #5, (1965), 413, Kishimoto T., T. Tamura - Nucl. Phys. A192, 1972, p.246;D.R.Bes and R.A. Sorensen - Adv. in Nucl. Phys. 2 (1969), p. 129.

[3] Aprahamian A. private communication.

[4] T. Holstein, H. Primakoff Phys. Rev. 58, (1940), 1098; R. Marshalek Phys. Lett. b97 (1980), 337.

[5] Mitsuo Sacai, *Atomic Data and Nuclear Data Tables*, 31, 399-432 (1984).

Caption to the figure:

Figure 1. Comparison of calculated with Hamiltonian (3, 6) and experimental $0^+$ state energies for rare - earth nuclei $^{156}$Gd, $^{188}$Os, $^{194}$Pt, $^{196}$Pt.
