I. INTRODUCTION

The success of the inflationary paradigm in providing a natural resolution for the flatness and homogeneity problems of Standard Big Bang cosmology has made inflation an essential part of early-universe cosmology. This success has led to intense efforts in realizing inflation within string theory, resulting in several new scenarios including brane-antibrane inflation [1], where the interbrane separation plays the role of the inflaton. These constructions provide new possibilities for constraining the parameters of string theory, within compactifications that could be compatible with the Standard Model.

A potential source of new phenomenological constraints, distinct from those arising in generic field theory models of inflation, is the reheating process at the end of inflation. If
inflation occurs in one warped throat, while the standard model (SM) is localized in another throat, there can be a difficulty in transferring the energy from brane-antibrane annihilation to the standard model degrees of freedom since the warp factor provides a gravitational potential barrier between the throats. If the barrier cannot be penetrated, “reheating” will be predominantly into invisible gravitons \cite{2}-\cite{4}, an unacceptable outcome. In ref. \cite{5} it was argued that the suppression due to the barrier can be counteracted by the enhanced coupling of the Kaluza-Klein (KK) modes to the deeply–warped SM throat; this scenario has been further studied in \cite{6}-\cite{8}. Another interesting possibility is that inflation could deform the SM throat in such a way that its oscillations at the end of inflation efficiently reheat the SM degrees of freedom \cite{12}.

A further challenge was recently pointed out in ref. \cite{13}, whose authors highlighted the possibility of long-lived, heavy KK modes, which could conflict with standard cosmology. In previous studies of reheating in a warped throat, the throat was modelled by a single extra dimension, leading to an AdS$_5$ geometry. Massive states are strongly peaked in the infrared (IR) region of the throat, so integrating them out by dimensional reduction (DR) resulted in large effective couplings, and hence efficient decay. Ref. \cite{13} emphasized that the actual background solution, the Klebanov-Strassler (KS) solution \cite{14}, contains an additional 5D internal space $\mathcal{M}_5$ with isometries along which nonradial KK excitations can occur. For realistic particle phenomenology $\mathcal{M}_5$ is usually taken as $T^{1,1}$, the Einstein-Sasak manifold for the group $SU(2) \times SU(2)/U(1)$. These isometries, as we shall review, result in approximately conserved angular momenta which constrain the possible decay channels and result in a long-lived relic corresponding to the lightest\footnote{As we will discuss, there are also zero-mass charged states in the KS background. Here we refer to the lightest massive KK states.} “charged” state, i.e., the lightest state with angular momentum in the $T^{1,1}$. We shall refer to this candidate relic as the lightest massive charged state (LMCS).

If the KS throat was the entire compactification manifold, the angular isometries would be exact and the LMCS would be stable. However it is necessary to cut off the throat in the ultraviolet (UV) region, joining it to a larger Calabi-Yau (CY) manifold which does not globally preserve the isometries. The process of gluing together the KS throat to the CY thus perturbs the KS geometry in the UV region, and this information propagates down the throat into the IR. We will assume that a mode of the metric which was zero due to the isometry will be sourced in the UV region, and that its radial profile decays exponentially toward the IR region so that the symmetry breaking is a weak effect in the IR. In the CFT description, via the AdS/CFT correspondence, this corresponds to turning on an irrelevant operator that breaks the symmetry. Of course, if the operator is relevant the symmetry-breaking is strong in the IR, and there is no problem of long-lived relics since in this case the throat geometry is not close to the KS solution.

Since the symmetry-breaking effect is suppressed in the IR, while the radial profile of the
LMCS is strongly peaked in the IR, the operators induced in the low-energy effective theory that describe the decay of the LMCS will be suppressed by powers of the warp factor $w$, which determines the hierarchy of scales between the bottom and top of the throat. If this suppression were too strong, the heavy KK relics could be long-lived and come to dominate the energy density of the universe at unacceptably high temperatures. In particular, they should decay before the era of big bang nucleosynthesis at $T \sim 1$ MeV at the very least, and most likely also before baryogenesis, since otherwise the entropy produced by their decays will greatly dilute the baryon asymmetry. Assuming that baryogenesis could not have happened later than the electroweak phase transition requires the KK relics to decay at temperatures greater than 100 GeV. We will show that to avoid this in the $T^{1,1}$ background, a SUSY breaking operator must be turned on at a scale greater than $6 \times 10^8$ GeV (more generally, $100 w^{-1.7}$ GeV for warp factor $w$). In addition, even if SUSY is broken at the Planck scale (or if the KK relics can decay without breaking SUSY in backgrounds other than KS), we derive general constraints which can be used to further exclude backgrounds with a very massive LMCS or a large warp factor.

In ref. [13], it was assumed that the suppression of the LMCS decay amplitude was of the form $w^p$, where $p+4$ was the dimension of the most relevant charge-violating (but 5D Lorentz and SUSY preserving) operator in the CFT. However, there was no detailed justification for this assumption, and it is not obvious that it should give the same answer as actually computing the decay rate from the effective theory. Our goal in this paper is to make an accurate estimate of the decay rate of the potentially dangerous relics in the $AdS_5 \times T^{1,1}$ type IIB supergravity background. We will find a parametrically different result than that of ref. [13]. Moreover, we will show that the CFT operator considered in ref. [13] is not sufficient to destabilize the LMCS in the KS background: one must turn on, in addition, an irrelevant SUSY-breaking operator for this purpose. We also show that decays are possible without invoking SUSY-breaking, but we did not find any example which gave a sufficiently large decay rate.

In ref. [9] it was found that the density of KK relics could be suppressed to an acceptable level through annihilations (rather than the decays we study in this paper) if the warp factor $w$ is sufficiently small, whereas larger values are needed for sufficiently fast decays. In section III E we will compare the two approaches, noting that the values of $w$ needed for annihilations to sufficiently deplete the KK relics are much smaller than one would like for getting the right inflationary scale in the throat.

In the remainder of the paper we examine the constraints on the warp factor and the mass of the LMCS resulting from considerations of BBN and baryogenesis, both for the KS background, and for more general warped compactifications. Section II gives a brief introduction to the problem at hand, including the origin of the long-lived relic and the means through which it may decay. For simplicity, section II employs a toy model that captures the key ingredients of the analysis, while section III analyzes the problem of the LMCS decay in the 10D supergravity background. This includes identifying the LMCS
(section III A) and its interactions (section III B), and deriving constraints on the scale of
SUSY breaking (section III C) as well as more general constraints in warped models where
the LMCS mass may take different values (section III E). In section III D we identify a
decay channel which does not require any SUSY-breaking background, but we find that it is
too suppressed to allow for sufficiently fast decay of the LMCS. In section III F we compare
the decay scenario to that where the KK relics annihilate with each other, showing that
annihilations of the LMCS are too inefficient unless the warp factor is \( w \sim 10^{-8} \), which is
far below the value needed for brane inflation. We give conclusions in section IV.

Some details are reserved for the appendices, including a comprehensive description of
the radial behaviours for 5D scalars, 1-forms and antisymmetric 2-forms (Appendix A), and
a discussion of the \( T^{1,1} \) harmonics (Appendix B).

II. BACKGROUND DEFORMATIONS AND KK MODE DECAY

In this section we will describe in greater detail the origin of the symmetry breaking for
the approximate angular isometries, and we will illustrate the approach we are going to take
using a simplified toy model.

The problem of relic angular KK modes is closely related to a moduli problem associated
with having an anti-D3 brane \( (\overline{D3}) \) placed at the bottom of the deformed conifold geometry,
as one might wish to do in order to uplift the AdS vacuum from Kähler modulus stabilization
to dS or Minkowski space, in the manner of KKLT [15]. The energy of the \( \overline{D3} \) is minimized
at the bottom of the throat, but, as pointed out in ref. [17] the base of the deformed conifold
has an \( S^2 \times S^3 \) topology, whose \( S^2 \) shrinks to vanishing size at the location of the \( \overline{D3} \). The
\( \overline{D3} \) can move freely inside the \( S^3 \), whose coordinates thus correspond to 3 massless moduli.
In order to stabilize these moduli one needs to break the isometries of the \( S^3 \).

In ref. [17] this was achieved by considering the dual field theory to the KS background
and turning on an irrelevant operator that gives a mass to the fields describing the \( \overline{D3} \) posi-
tion. The same mechanism might also destabilize the would-be angular KK relics since the
operator provides a background correction which perturbs the geometry and breaks the sym-
metries. To analyze this process we choose to work in the gravity side of the gauge/gravity
correspondence. Since a renormalization group (RG) flow in the field theory dual corre-
sponds to movement along the radial direction of the AdS space, turning on an operator in
the ultraviolet (UV) of the field theory and performing the RG flow corresponds to turning
on a source for the bulk classical field dual to that operator and following its effect along
the radial direction to the bottom of the throat.

In the AdS background geometry the fields have an exponential dependence on the ra-
dial direction, and the profile of the symmetry-breaking perturbation will be related to the
warping of the background geometry. This perturbation is sourced by the CY, which gener-
ically does not preserve the symmetries of the throat; hence we consider a source in the UV
which depends nontrivially on the angular coordinates of the $S^3$ cycle, and thus breaks the corresponding isometries of the $S^3$. As a consequence, the KK modes of fields that couple to the source will become unstable since the KK quantum numbers are no longer conserved quantities; this is just the gravitational dual of the mechanism that made the $D3$ position moduli massive in [17].

A. A simplified model

Our basic approach will be to compute the 4D effective Lagrangian for KK relics in the presence of a perturbation to the background geometry. The perturbation leads to symmetry-breaking terms in the effective Lagrangian, including vertices for the decay of the relic. It is useful to illustrate this procedure on a simpler model before tackling the full 10D supergravity theory.

We therefore consider a massless scalar field $\phi$ in 6D, where one of the compact dimensions corresponds to the angular direction which is an isometry of the unperturbed throat (in this case the $U(1)$ symmetry along a circle of length $L$) and the other is the radial direction along the AdS. Its Lagrangian is

$$L = \frac{1}{2} \int d^4x \sqrt{-g} g^{AB} \partial_A \phi \partial_B \phi$$

and the original throat geometry is described by the line element

$$ds^2 = a^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + L^2 d\theta^2$$

between $r = 0$ and $r = r_0$. In Randall-Sundrum coordinates [16], the warp factor takes the form $a = e^{-kr}$, where $k$ is the AdS scale; so $r = 0$ corresponds to the top of the throat (the UV), where it joins to the CY, and $r = r_0$ is the bottom of the throat (the IR). To be a solution to Einstein’s equations, this geometry requires an exotic bulk stress energy, with $T^a_b = k^2 \text{diag}(6, 6, 6, 6, 6, 10)$, but we only use it as an illustrative toy model. To model the symmetry-breaking effect of the CY, there are two possible effects on the background. One is that the metric (2) gets perturbed by

$$\Delta ds^2 = \sum_n \sin(n\theta) \left[ a^2(r) \alpha_n(r) dx^2 + \beta_n(r) dr^2 + L^2 \gamma_n(r) d\theta^2 \right]$$

corresponding to KK excitations of the angular direction. (For simplicity we ignore the fluctuations proportional to $\cos(n\theta)$.) Another possibility is that angular KK modes of the scalar $\phi$ get sourced,

$$\Delta \phi = \sum_n \sin(n\theta) \varphi_n(r)$$

All these can be thought of as solutions to the vacuum Einstein or scalar field equations in the throat, sourced by some boundary conditions at the CY, $r = 0$. In the AdS background,
the solutions for the radial wave functions generically have the form
\[ \alpha_n(r) = \sum_{\pm} \alpha_{n,\pm} e^{-z_{n,\pm} kr} \]  
(similarly for \( \beta_n, \gamma_n, \varphi_n \)) where \( z_{n,+} \) and \( z_{n,-} \) are related by
\[ \Delta_n = -z_{n,-}, \quad 4 - \Delta_n = -z_{n,+}. \]  
In the AdS/CFT, \( \Delta_n \) is the dimension of an operator \( \mathcal{O}_n \) which corresponds to the deformation \( \mathcal{O} \) of the background geometry.

An important concept throughout this paper is the necessity for perturbations like (3) to remain small as one goes to the bottom of the throat; otherwise the KS throat is not a good approximation to the actual geometry. This requirement is fulfilled as long as we only turn on marginal or irrelevant operators in the CFT, with \( \Delta_n \geq 4 \). In that case, \( z_{n,+} < 0 \) and the corresponding solution decays toward the bottom of the throat. If the operator is relevant, then both solutions grow toward the IR, and it is impossible to keep them small without fine-tuning in the UV.

As explained in ref. [17], the coefficient of the \( z_{n,-} \) solution, which grows toward the IR, is not exactly zero, but depends on the boundary conditions at the bottom of the throat, \( r = r_0 \). Regardless of the details of these boundary conditions however, generically one expects both solutions to be of the same order of magnitude at \( r_0 \). As one moves toward the top of the throat, the \( z_{n,+} \) solution quickly comes to dominate. For the purposes of the kind of estimates we will be making, it is sufficient to approximate the background deformations by just the \( z_{n,+} \) part of the solution. Thus, for example,
\[ \alpha_n(r) \approx \alpha_n e^{(4-\Delta_n)kr} = \alpha_n e^{-z_{n,+} kr}, \]  
where \( \alpha_n \) characterizes the magnitude of the symmetry breaking in the UV, and thus could be \( \mathcal{O}(1) \).

Now we turn our attention to the propagating fluctuations of the scalar field, which include the would-be stable relic KK excitation. The radial \( (n) \) and angular \( (m) \) KK decomposition is
\[ \phi = \frac{1}{\sqrt{2\pi L}} \sum_{n,m} R_{nm}(r) e^{im\theta} \phi_{nm}(x^\mu). \]  
In the absence of the metric and scalar perturbations (3,4), the interactions of the angular excited states with \( n = 0, m \neq 0 \) conserve the total angular momentum, so there is no way for the massive states \( \phi_{0,\pm 1} \) to decay. These, then, represent the lightest massive charged states (LMCS) in the toy model.

In the perturbed metric, angular momentum is no longer conserved, and we can construct an interaction from the kinetic term for the decay \( \phi_{0,\pm 1} \to \phi_{00} h_{\mu\nu} \), where \( h_{\mu\nu} \) is a massless graviton which is a perturbation about the Minkowski metric factor in (2): \( \eta_{\mu\nu} \to \eta_{\mu\nu} + \).
This decay channel may come from the 6D kinetic term, and the 4D effective interaction is

\[ L_{\text{decay }1} = \frac{1}{M_p^2} \frac{\alpha_1}{4\pi} \int d\theta \sin \theta e^{\pm i\theta} \int dr e^{-(2z_1 + kr)} R_{01}(r) R_{00}(r) \]  

(9)

However, this particular decay channel is not illustrative of the more realistic SUGRA theory we are ultimately interested in, because it involves the massless scalar \( \phi_{00} \) as a decay product. In a realistic theory, massless scalars should be somehow projected out or given a mass, to avoid various phenomenological difficulties.

Another possibility, which does not require any massless scalar mode, is to use the scalar perturbation (4) to the background. Substituting such a deformation for one of the \( \phi \) factors in the kinetic term (1) allows us to generate a vertex for the \( \phi_{0\pm1} \) excitation to decay into massless gravitons, \( \phi_{0\pm1} \rightarrow hh \).

\[ L_{\text{decay }2} = \phi_{0\pm1} h^{\mu\nu} \frac{\varphi_1}{8M_p^2} \sqrt{\frac{L}{2\pi}} \left[ \int d\theta \sin \theta e^{\pm i\theta} \int dr e^{-4kr} \partial_r e^{-z_1, kr} \partial_r R_{01}(r) R_{00}^2(r) \right] \]  

(10)

where we note that \( R_{00}(r) \) is just a constant. This process is more analogous to the ones we will be interested in for the SUGRA model, so we will focus on it.

To evaluate the radial integrals in (10) we must solve for the radial wave functions, which obey the equation of motion with 4D mass \( m_{nm} \)

\[ e^{2kr} \partial_r \left( e^{-4kr} \partial_r R_{nm} \right) - \frac{m^2}{L^2} e^{-2kr} R_{nm} + m_{nm}^2 R_{nm} = 0 . \]  

(11)

(see eq. (A5) for the generalization to 10D). Defining the warp factor at the bottom of the throat

\[ w = e^{-kr_0} , \]  

(12)

the solutions have the form

\[ R_{nm}(r) \simeq \frac{w\sqrt{k}e^{2kr}}{J_{\nu_m}(x_{nm})} \left[ J_{\nu_m} \left( x_{nm} we^{kr} \right) + w^{2\nu_m} Y_{\nu_m} \left( x_{nm} we^{kr} \right) \right] \]  

\[ \nu_m = \sqrt{4 + (m/kL)^2} , \]  

(13)

where \( x_{nm} \sim 1 \) is determined by the boundary conditions, and the 4D mass of the excitation is given by \( m_{nm} = kw x_{nm} \), as shown in ref. [18]. The radial behaviour for excited modes is dominated by the \( J_\nu \) solution, which is strongly peaked in the IR. The zero-mode solution \( R_{00} \equiv \sqrt{2k} \) is just a constant, whose value is determined by the normalization condition

\[ \int dr e^{-2kr} R_{nm}^2 = 1 . \]  

(14)
The reader is referred to Appendix A for a detailed discussion of the radial behaviour. There, a discussion of vector fields and antisymmetric tensor fields is also included since they have a different radial behaviour that is important for the SUGRA theory we will discuss.

We can estimate the integrals determining the coefficient of the decay-mediating operator in (10) using the small- and large-$r$ asymptotics of the Bessel function. Near $r = 0$, the argument of $J_\nu$ is exponentially small and $J_\nu(x) \sim (x/2)^\nu/\Gamma(1+\nu)$, while near $r = r_0$, the argument is of order unity. Both behaviours are consistently approximated by $R_{01} \sim w^{1+\nu}\sqrt{k}e^{(2+\nu)kr}$, leading to the estimate

$$L_{\text{decay}} \sim \phi_0 \pm \frac{h^{\mu
u}h_{\mu
u}}{8M_p^2} \frac{\varphi_1}{2\pi} \sqrt{\frac{Lk}{2\pi}} \left[ \pm i(2 + \nu)z_{1,+}k^2 - L^{-2} \right] \int dr e^{(\nu-2-z_{1,+})kr}. \quad (15)$$

Our principal interest in the present work will be to track the parametric dependence of the decay amplitude on the warp factor $w$, since it can be the strongest source of suppression. To complete the evaluation of (15), it is necessary to know whether the combination $(\nu - 2 - z_{1,+})$ appearing in the exponent is positive or negative. In the former case, the radial integral grows toward the IR, and yields an inverse power of the warp factor, $k^{-1}w^{-(\nu-2-z_{1,+})}$. In the latter case, the integral converges in the IR, and yields only the mild dependence $k^{-1}|\nu - 2 - z_{1,+}|^{-1}$. In the present example, we have not explicitly determined the value of $z_{1,+}$ to determine which case is true, but in the SUGRA model we will do so.

Let us contrast this with the estimate made in ref. [13]. There it was assumed that the operator $O_+$ (corresponding to a background correction to the metric) directly mediates the decay in the 4D effective theory, and its degree of irrelevancy controlled the amount of warp factor suppression, so that

$$L_{\text{decay}} \sim w^{\Delta_+ - 4} = w^{z_{1,+}}. \quad (16)$$

On the other hand our explicit calculation indicates that the warp factor dependence in $L_{\text{decay}}$ is either $w^{1+\nu}$, or else $w^{3+z_{1,+}}$, depending on the sign of $(\nu - 2 - z_{1,+})$.

In the full 10D SUGRA model we want to consider, the simple decay process illustrated above will not be possible because the background deformation corresponding to $\varphi_1$ will turn out to be a relevant operator in the CFT. This contradicts our requirement that only deformations of the throat which decay toward the IR are allowed. We will be able to overcome this by turning on a different irrelevant deformation, which leads to mixing between $\phi_{0,\pm 1}$ and some massless field which carries trivial angular quantum numbers. The heavy $\phi_{0,\pm 1}$ will thus appear as an intermediate state in the decay process. But the method illustrated in the previous subsection will still apply for computing the mixing amplitude.

Following the previous discussion, the radial integral $R$ appearing in the dimensional reduction from 10D to 4D of an interaction involving $N_0$ appearances of the inverse 4D metric, $N_m$ massive modes, $N_0$ massless modes, and $N_t$ tadpole insertions in the background,
with dimensions $\Delta_b > 4$ is

$$\mathcal{R} \simeq \int dr e^{-(4-2N_i)kr} \left[ \Pi_{i=1}^{N_m} R_{\nu_i}(r) \right] R_0(r)^{N_0} \left[ \Pi_{b=1}^{N_b} R_{tb}(r) \right]$$

$$\simeq w^{N_m} \int dr e^{-(4-2N_i)kr} \left[ \Pi_{i=1}^{N_m} e^{2kr} J_{\nu_i} \left( x_n w e^{-kr} \right) \right] \left[ \Pi_{b=1}^{N_b} e^{(4-\Delta_b)kr} \right]$$

$$\simeq w^{N_m+\sum_i \nu_i} \int dr e^{(-4+2N_i+N+\sum_i \nu_i-\sum_b (4-\Delta_b))kr}$$

$$\sim w^{N_m+\sum_i \nu_i} \left[ w^{4-2N_i+N_m-N+\sum_b (4-\Delta_b)} \right]. \quad (17)$$

where we have defined $N = 2N_s + N_v$ in terms of the number $N_s$ ($N_v$) of scalars (vectors) amongst the $N_m$ massive states. Thus $N_m - N = N_s - N_a$, where $N_a$ is the number of antisymmetric 2-form states within $N_m$. The relation between the values of $\nu_i$ for the external states and the 5D mass eigenvalues is given in appendix A and, generally, depends on the helicity of the state. The final estimate in (17) is found by determining which contribution dominates, the UV or IR; the correct answer is whichever is largest.

**III. KK DECAY IN TYPE IIB SUPERGRAVITY**

We now want to apply the methods illustrated in the 6D model to the KS geometry sourced by a stack of D3-branes in the 10D theory. The line element is

$$ds^2 = H^{-1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/2} \left( dR^2 + R^2 ds^2_{T^{1,1}} \right), \quad (18)$$

where

$$H(R) = \frac{27\pi}{4R^4} \alpha' g_s M \left[ K + g_s M \left( \frac{3}{8\pi} + \frac{3}{2\pi} \ln \left( \frac{R}{R_{\text{max}}} \right) \right) \right] \quad (19)$$

is in terms of the flux quantum numbers $K$ and $M$. Far from the tip $R < R_{\text{max}}$ we neglect the logarithmic contributions to $H(R)$ and further ignore the small contribution from the second term on the right to obtain the metric of an $AdS_5 \times T^{1,1}$ throat. Using the coordinate transformation $R = k^{-1} e^{-kr}$, the metric becomes

$$ds^2 = e^{-2kr} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + \frac{1}{k^2} ds^2_{T^{1,1}}. \quad (20)$$

The corresponding low-energy effective theory is type IIB supergravity on an approximate $AdS_5 \times T^{1,1}$ background [19]:

$$S_{IIB} = \frac{1}{2\kappa_0^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[ R + 4(\nabla \phi)^2 - \frac{1}{12} (H^{(3)})^2 \right] - \frac{1}{12} \left( F^{(3)} + A^{(0)} \wedge H^{(3)} \right)^2 - \frac{1}{2} \left( dA^{(0)} \right)^2 - \frac{1}{480} \left( F^{(5)} \right)^2 \right\}$$

$$+ \frac{1}{4\kappa_0^2} \int \left( A^{(4)} + \frac{1}{2} B^{(2)} \wedge A^{(2)} \right) \wedge F^{(3)} \wedge H^{(3)}. \quad (21)$$
As usual, $H^{(3)}$ is the field strength of the NS 2-form $B^{(2)}$, $F^{(n+1)}$ is the field strength for the RR $n$-form $A^{(n)}$, $2\kappa_0^2 = (2\pi)^7\alpha'^4$, $\sqrt{\alpha'} = l_s = M_s^{-1}$, where $M_s$ is the string scale. In our subsequent analysis, we will refer to several other mass scales. The AdS curvature scale $k$ is determined by the flux quantum numbers $M$ and $K$ through the relation $k^{-4} \equiv \frac{27\pi}{4\alpha'}^2 g_s M K$. The warped string scale, $wM_s$, is also determined by the fluxes, through $w = e^{-2\pi K/(3g_s M)}$. Finally, the Planck scale is given by $M^2_p = \frac{2V_6}{g_s^2 \kappa_0} \kappa_0^2$, where $V_6$ is the compactification volume.

From eq. (20) it is apparent $V_6 \approx R_{AdS}^6 = k^{-6}$, so with $g_s < 1$ and $k < M$ we find the 4D Planck scale is greater than the string scale.

### A. Identifying the LMCS

The first step is to discover which angular KK excitation, among the many fields that result from dimensionally reducing the action (21), is the potentially dangerous relic, the LMCS. Fortunately, the masses and quantum numbers of all the lowest-lying KK excitations in the KS background have been tabulated in refs. [17, 20]. The correspondence between 10D and 5D fields from integrating over the $T^{1,1}$ directions is indicated in Table I, taken from [20].

| Dim  | fields                     | harmonic |
|------|----------------------------|----------|
| 10D  | $h_{\mu\nu}(x,y)$          |          |
| 5D   | $H_{\mu\nu}(x)$            |          |
| 10D  | $h_{a\mu}(x,y)$            |          |
| 5D   | $B_a(x)$                   |          |
| 10D  | $A_{\mu\nu\alpha}(x,y)$   |          |
| 5D   | $b_{\mu\nu}(x)$            |          |
| 10D  | $h_{ab}(x,y)$              |          |
| 5D   | $\phi(x)$                  |          |
| 10D  | $\lambda(x,y)$             |          |
| 5D   | $\lambda(x)$               |          |
| 10D  | $\psi_a(x,y)$              |          |
| 5D   | $\psi^{(T)}(x)$            |          |
|      | $Y(y)$                     |          |
|      | $a_{\mu\nu}(x)$            |          |
|      | $Y_a(y)$                   |          |
|      | $Y_{(ab)}(x,y)$            |          |
|      | $\Xi(y)$                   |          |
|      | $\Xi_a(y)$                 |          |

**TABLE I:** The harmonic expansion of the 10D fields. $h_{MN}$ is the 10D metric, $A_{MNOP}$ the 10D four-form, $B$ the complex 0-form, and $A_{MN}$ the 10D complex 2-form. We have not included the NS 2-form. The different polarizations of the fields appear as 5D scalars, vectors, and tensors. (Adapted from ref. [21])

The corresponding expansions of the fields in terms of scalar ($Y^{(\nu)}$), vector ($Y_a^{(\nu)}$) and...
tensor \((Y^\nu_{ab})\) harmonics of the \(T^{1,1}\) are given by expressions like

\[
h_{\mu\nu}(x, y) = \sum_{\nu} H_{\mu\nu}^{(\nu)}(x) Y^{(\nu)}(y)
\]

(22)

\[
h_{\mu a}(x, y) = \sum_{\nu} B_{\mu}^{(\nu)}(x) Y^a_{(\nu)}(y)
\]

(23)

\[
h_{(ab)}(x, y) = \sum_{\nu} \phi_{(\nu)}(x) Y_{(ab)}^{(\nu)}(y)
\]

(24)

\[
h_{a}^{\nu}(x, y) = \sum_{\nu} \pi^{(\nu)}(x) Y_{a}^{(\nu)}(y)
\]

(25)

\[
A_{abcd}(x, y) = \sum_{\nu} b^{(\nu)}_{abcd} \epsilon_{de} Y^{(\nu)}
\]

(26)

\[
A_{\mu bcd}(x, y) = \sum_{\nu} \phi_{\mu}^{(nu)} \epsilon_{bcde} \epsilon_{def} Y_f^{(\nu)}
\]

(27)

where \(x = (x^\mu, r)\), and \(\{\nu\} = (j, l, r)\) are the quantum numbers identifying the \(T^{1,1} = SU(2) \times SU(2)/U(1)\) representation. \(j\) and \(l\) are the usual angular momentum quantum numbers corresponding to the two \(SU(2)\) factors. Higher-rank fields are given in terms of their dual representation. In general, \(r = j_3 - l_3\), where \(j_3, l_3\) are the respective eigenvalues of the \(T_3\) generators of the first and second \(SU(2)\)'s. For the scalar harmonics which will be of most interest to us, \(r = 2j_3 = -2l_3\), and so is restricted to the range \(|r| < \min(2j, 2l)\).

Ref. \[20\] has computed the 5D masses of all the states in the theory as functions of their \((j, l, r)\) quantum numbers, and organized them into supermultiplets of the \(N = 1\) 5D SUGRA theory. By going through these results and computing the masses of all particles which have nontrivial \((j, l, r)\) values, we find that the field \(b(x)\) in vector multiplet I is the LMCS, with \((j, l, r) \in \{(1, 0, 0), (0, 1, 0)\}\). Thus there are two species of LMCS, depending on whether \(j\) or \(l\) is nonzero. The 5D masses are defined in terms of the ubiquitously appearing function

\[
H_0(j, l, r) = 6 \left( j(j + 1) + l(l + 1) - \frac{r^2}{8} \right)
\]

(28)

which takes the value \(H_0(1, 0, 0) = 12\) for the LMCS. Its 5D mass is given by

\[
m_b^2 = H_0 + 16 - 8\sqrt{H_0 + 4} = -4
\]

(29)

in units of the AdS curvature, \(k^2\). As is well known \[21\], squared masses can be negative on an AdS background without leading to instabilities, down to the Breitenlohner-Freedman bound \(m^2 \geq -4\) \[22\]. This bound is saturated by (29). To find the corresponding mass in 4D, one must do the final dimensional reduction on \(r\) by solving for the radial wave functions. These are identical to the 6D case \[13\] (assuming that we approximate the throat geometry by AdS_5), except for the replacement of the 5D mass, \(m_b^2\), in the index of the Bessel function

\[
\nu = \sqrt{4 + m_b^2}.
\]

(30)
We have carried out this calculation to find the 4D mass as a function of the 5D one, using the boundary conditions of the RS model, as in ref. \[18\]. The result, shown in Fig. 1(a) shows that the 4D LMCS mass is approximately \(m_{4D} = 1.7wk\). Fig. 1(b) shows this value is quite insensitive to the details of the boundary conditions in the UV (whether they are Neumann, Dirichlet, or mixed), which represent how the throat is joined to the CY. Additionally, Fig 1 extends the result found in ref. \[18\] from \(m^2_{5D} \geq 0\) to include the Breitenlohner-Freedman range \(m^2_{5D} \geq -4\). It indicates the LMCS in 5D is the LMCS in 4D as well, i.e., \(m^2_{4D}\) is a monotonically increasing function of \(m^2_{5D}\). Notice that these results are for the lowest state in the radial KK tower.

![Figure 1](image)

**FIG. 1:** (a) Left: 4D mass of the first KK state as a function of the 5D mass (squared). The result is presented for both 5d scalars and vectors. (b) Right: dependence of the 4D mass on the UV boundary conditions, for two different values of the warp factor.

Of course the KS geometry is not exactly the same as the Randall-Sundrum model, and the boundary conditions (b.c.’s) which should be imposed at the bottom of the throat may not coincide exactly with the \(Z_2\) orbifold b.c.’s imposed in the RS model. However, this detail is not important for our estimates, and for the proper identification of the LMCS, we need only rely on the fact that regardless of the exact b.c.’s in the IR region, the relation between the 4D mass and the 5D \(m^2\) eigenvalue is a monotonic one. This insures that the particle with the lowest \(m^2_{5D}\) will also be the lightest one in the 4D effective theory. We have verified the monotonic property by for the entire range of mixed b.c.’s which interpolate between Dirichlet and Neumann, as illustrated in fig. 2.

Table II lists the lowest-mass states in the spectrum. Several saturate the Breitenlohner-Freedman bound but most of these are uncharged; additionally, we identify the field \(b(x)\) from the 4-form as the lightest charged state. Interestingly, most of the lightest charged states come from the 4-form, or the 10D graviton. This table is useful for identification of the LMCS and possible irrelevant operators that may be turned on in the background to
FIG. 2: 4D mass of the first KK state as a function of the 5D mass (squared), for a wide range of b.c.’s of the form $R(r_0)\delta + R'(r_0) = 0$ at the IR end of the throat, showing that $m_{4D}$ is monotonically related to $m_{5D}^2$ regardless of the value of $\delta$.

States corresponding to relevant operators:

| Field | Multiplet | (5d) Mass | $\Delta$ | QN’s       |
|-------|-----------|-----------|----------|------------|
| $b$   | VM I      | -4        | 2        | (0,1,0), (1,0,0) |
| $\phi_\mu$ | VM I    | 0         | 3        | (1,0,0), (0,1,0) |
| $b$   | VM I      | -3        | 3        | (1,1,±2)   |
| $\phi_1, \phi_2$ | VM I | -3        | 3        | (1,0,0), (0,1,0) |
| $b$   | VM I      | -2.33     | 3.29     | (1,1,0)    |

States of marginal and irrelevant operators:

| Field | Multiplet | (5d) Mass | $\Delta$ | QN’s       |
|-------|-----------|-----------|----------|------------|
| $\phi_1/\phi_2$ | VM I | 0         | 4        | (1,1,±2)   |
| $\phi_3$   | VM I      | 0         | 4        | (1,0,0), (0,1,0) |
| $\phi_1/\phi_2$ | VM I | 1.25      | 4.29     | (1,1,0)    |
| $b$   | VM I      | 1.40      | 4.32     | (2,0,0), (0,2,0) |
| $a_1$  | VM III/IV | 2.79      | 4.61     | (1,0,0), (0,1,0) |

TABLE II: The lightest states charged in the KS background: we list the 5D field, its supermultiplet, bulk mass, conformal dimension, and $T^{1,1}$ quantum numbers. Most of the light, charged states correspond to the 4-form polarized along the $T^{1,1}$, $A_{abcd}^{(4)}$. The reader is referred to the appendices of [20] for the multiplet and mass listings.

accommodate the decay.

B. Interactions of the LMCS

We have seen that in terms of the 10D fields, the LMCS is contained in the RR 4-form $A_{abcd}^{(4)}$ polarized along the internal $T^{1,1}$ directions, resulting in a massive scalar field
\( b(x^\mu, r) \) from the 5D point of view.\(^2\) Therefore the relevant terms in the action for type IIB supergravity (21) which provide interactions for the 4-form are

\[
S_{\text{IIB}}(A^{(4)}) = \frac{1}{2\kappa_0^2} \int d^{10} x \sqrt{-G} \left[ -\frac{1}{240} (F^{(5)})^2 \right] + \frac{1}{2\kappa_0^2} \int A^{(4)} \wedge F^{(3)} \wedge H^{(3)}.
\]

(31)

Let us now try to follow the example of subsection II A by turning on the CFT operator used by ref. [17] to stabilize the \( D_3 \) moduli. As shown there, this operator corresponds to a KK mode of the warped metric on the \( T^{1,1} \) with quantum numbers \( (1,1,0) \), call it \( \delta g^{(1,1,0)}_{ab} \).

In combining this operator with the LMCS in the kinetic term for \( A_{abcd} \), the way to make a \( T^{1,1} \) singlet combination which is closest to the 6D example is to choose the two different species of LMCS for the \( A_{abcd} \) factors:

\[
\mathcal{L}_{\text{int}} = \int dr d^5 y \sqrt{g_6} \delta g^{aa'}_{(1,1,0)} h^{\mu\nu} \partial_\mu A_{abcd}^{(0,1,0)} \partial_\nu A_{a'b'c'd'}^{(1,0,0)} \gamma^{bb'} \gamma^{cc'} \gamma^{dd'}
\]

(32)

where \( y^a \) are the coordinates of \( T^{1,1} \) and \( h^{\mu\nu} \) is a massless graviton. Notice that the \( T^{1,1} \) quantum numbers of the various factors are such that their product contains a singlet (since \( j, l \) are angular momentum quantum numbers for the \( SU(2) \) factors, \( 1 \otimes 1 = 0 \oplus 1 \oplus 2 \)), so the integral over \( y^a \) should not vanish. However, this vertex involves the two different LMCS species, rather than just one of them and a massless mode of \( A_{abcd} \), so it does not provide phase space for the decay of the LMCS. One could possibly form a \( T^{1,1} \) singlet with other states in the vertex, but these terms are also ruled out on kinematic grounds.

It is natural, then, to turn to the Chern-Simons part of the action (31) since it contains a single \( A^{(4)} \) factor. However, the Chern-Simons action contains terms of the form

\[
\int d^5 x d^5 y \varepsilon^{abcd} \varepsilon^{e\gamma\delta\epsilon} A_{abcd}^{(4)} \partial_\epsilon A_{a'b'c'd'}^{(2)} \partial_\gamma B_{\delta\epsilon}^{(2)},
\]

(33)

with no appearance at all of the metric. Therefore the operator \( \delta g^{(1,1,0)}_{ab} \) cannot induce decay of the LMCS through this term. These two observations prove that we must consider other operators in addition to the one invoked by ref. [17] for the decay of the LMCS to proceed.

There is no \textit{a priori} reason to expect that just one operator should both give masses to the \( D_3 \) moduli \textit{and} mediate the decay of the LMCS. We are free to turn on \textit{any} irrelevant operator which breaks the symmetries of \( T^{1,1} \) (while preserving other desirable symmetries) since the CY will generically not respect the isometries of \( T^{1,1} \).

In the following subsections we will describe two decay channels resulting from the insertion of two operators. Before doing so, let us mention some of the constraints which led us

\(^2\) Our conventions for indices are: capital Latin letters \( \{M, N, \ldots\} \) run over all directions, small Latin letters \( \{a, b, \ldots\} \) run over internal directions, small Greek letters \( \{\mu, \nu, \ldots\} \) run over the four noncompact dimensions
to these examples. Many possibilities were ruled out due to insufficient phase space for the decay, 4D Lorentz violation (see [23] for some recent constraints), or the inability to form a $T^{1,1}$ singlet operator without employing relevant operators which strongly distort the KS background in the IR region of the throat. All the other possibilities which we found were similar to or subdominant to those which we shall describe next.

C. A SUSY-violating decay

Ideally, one might like to preserve 5D supersymmetry in the process of modeling the effects of the CY. This was a criterion that was used by ref. [17] in identifying $\delta g_{ab}^{(1,1,0)}$ as the most relevant (though still irrelevant) $T^{1,1}$-breaking operator. However, the quantum numbers of this operator show that by itself it is not sufficient to induce decays of the LMCS, $b^{(1,0,0)}$, since $(1,1,0) \otimes (1,0,0)$ does not contain a singlet. One way to overcome this problem is to break SUSY at a high scale and to solve the hierarchy problem by putting the standard model in some other more deeply warped throat.

In analogy to the toy model example described in section II A, an obvious choice would be to turn on a background for some KK excitation of $A_{abcd}$, since this field contains the LMCS, $b^{(1,0,0)}$. We cannot use $b^{(1,0,0)}$ as a background however, because its conformal dimension is $\Delta = 2$ (see Table II), and a relevant deformation would invalidate the KS background in the IR region. Instead, one can turn on a higher mode, namely $A_{abcd}^{(2,1,0)}$. This, together with $\delta g_{ab}^{(1,1,0)}$, has the right quantum numbers to neutralize the charge of the LMCS $b^{(1,0,0)}$. In the same way, $b^{(0,1,0)}$ gets its mixing from the metric perturbation $\delta g_{ab}^{(1,1,0)}$ combined with $A_{abcd}^{(1,2,0)}$. As shown in ref. [20], the conformal dimension of the operators corresponding to the KK modes of $A_{abcd}$ (in vector multiplet I) is given by

$$\Delta = E_0 = \sqrt{H_0 + 4 - 2}.$$  

(34)

Since the state $A_{abcd}^{(2,1,0)}$ has $H_0 = 48$, its dimension is

$$\Delta_{210} = 4.93 > 4,$$  

(35)

and therefore turning on a small background for this state results in an exponentially decaying perturbation in the throat. For reference, we note that the operator corresponding to the $\delta g_{ab}^{(1,1,0)}$ background used by [17] to give masses to the $D3$ moduli is the top component of vector multiplet I whose conformal dimension is $\Delta = E_0 + 2$, hence

$$\Delta_{110} = \sqrt{28} \approx 5.29.$$  

(36)

In this way, we obtain mixing between $b$ and the graviphoton $h_{\mu\nu}$. The zero-mode of the graviphoton is projected out in the presence of an orbifold plane, which is present in the compactification we consider, but we need only a massive virtual graviphoton since it can
decay into massless gravitons through the gravitational interaction
\[ \frac{1}{M_p} \partial_r h^{\mu \nu} \partial_\mu h_{\alpha \beta} h^{\alpha \beta}. \] (37)

The resulting decay process is shown in figure 3.

FIG. 3: SUSY-violating decay of the LMCS \( b \) into massless gravitons \( h_{\mu \nu} \) via mixing with the radial graviphoton \( h_{r \mu} \), in the background of a tadpole for the excited state \( \delta A_{abcd}^{(2,1,0)} \) and the metric perturbation \( \delta g_{ab}^{(1,1,0)} \) in the type IIB SUGRA model.

To compute the amplitude for the decay, we first evaluate the integrals that give rise to mixing between \( b = A_{a'b'c'd'}^{(1,0,0)} \) and \( h_{\mu \nu} \) in the combined backgrounds of \( \delta g_{aa'}^{(1,1,0)} \) and \( \partial_r \delta A_{abcd}^{(2,1,0)} \):

\[ \mathcal{L}_{\text{mixing}} = \int dr d^5 y \sqrt{g_6} \sqrt{g_4} \delta g_{a' b'}^{(1,1,0)} h^{\mu \nu} \partial_r \delta A_{abcd}^{(2,1,0)} \partial_\mu A_{a'b'c'd'}^{(1,0,0)} g^{b'b'} g^{c'c'} g^{d'd'} \] (38)

For the background deformations, we use \( \delta g_{a b}^{(1,1,0)} \approx e^{- (\Delta_{110} - 4) kr} \) and \( \delta A_{abcd}^{(2,1,0)} \approx M_{3/2} \sqrt{k} e^{-(\Delta_{210} - 4) kr} \), where in the latter we make a distinction between the size of a generic \( T^{1,1} \) symmetry-breaking tadpole, \( M_s \), and the scale \( M_{3/2} / 2 \) at which SUSY is broken. Moreover the radial wavefunction for the LMCS \( b \) is approximated by \( R_{(1,0,0)} \sim w^{1+\nu_b} \sqrt{k} e^{(1+\nu_b) kr} \) and \( \nu_b = \sqrt{4 + m_5^2/k^2} = 0 \) (see discussion above eq. [15]). For the vector \( h_{r \mu} \) on the other hand, the radial wave function is approximated by \( R_{(0,0,0)} \sim w^{1+\nu_h} \sqrt{k} e^{(1+\nu_h) kr} \) and \( \nu_h = \sqrt{1 + m_5^2/k^2} \), as shown in appendix A2. Recalling that \( B_\mu \) is the 4D field corresponding to \( h_{r \mu} \), to find the largest amplitude, we need the smallest 5D mass eigenvalue for \( B_\mu \) or \( \phi_\mu \) (as explained in ref. [20], the mass eigenstates are mixtures of \( \phi_\mu \) and \( B_\mu \)); this appears in gravitino multiplet I, with \( H_0^- (0, 0, 0) = -3/4 \) and \( m^2/k^2 = 7 + H_0^- - 4\sqrt{4 + H_0^-} \approx -0.96 \), giving \( \nu_h \approx 0.20 \).

Remembering \( \sqrt{g_4} = e^{-4kr} \) and \( h^{\mu \nu} = e^{2kr} h_{\mu \nu} \), the radial integral becomes

\[ w^{2+\nu_h} M_{3/2}^2 k^2 \int dr e^{(9+\nu_h - \Delta_{110} - \Delta_{210}) kr} \sim w^{2.2} M_{3/2} k. \] (39)

Combining this with the interaction (37), we get the effective decay vertex

\[ \mathcal{L}_{\text{decay}} \sim w^{2.2} \frac{M_{3/2} k}{M_p m_\eta^2} \partial_\mu h^{\alpha \beta} \partial^\mu h_{\alpha \beta}. \] (40)
where \( m_h \sim wk \) is the mass of the exchanged graviphoton. Taking \( m_h \) to be also of this order, we find that the decay rate is

\[
\Gamma \sim w^{3.4} \frac{M_{3/2}^2 k}{M_p^2}
\]  

(41)

and the decay temperature is of order \( T \sim w^{1.7} M_{3/2}/(k/M_p) \). Assuming the inflationary scale is of order \( 10^{14} \) GeV and \( k \sim M_p \), then \( w \sim 10^{-4} \), and \( T \sim 10^{-2} M_{3/2} \). For electroweak baryogenesis to work, the relics should decay before \( T \sim 100 \) GeV, which leads to a constraint on the SUSY-breaking scale,

\[
M_{3/2} > 100 w^{-1.7} \text{ GeV} \approx 6 \times 10^8 \text{ GeV}.
\]  

(42)

It was pointed out in ref. [24] that the gravitino mass is larger than this bound in the KKLMT construction [15] on which the warped brane-antibrane inflation scenario is based. Therefore we conclude that there is not a problem with heavy KK relics in the \( T^{1,1} \) background.

D. SUSY-preserving decay

It is also possible to find decay channels which do not require SUSY-breaking deformations of the background. Turning on the same SUSY background \( \delta g^{(1,1,0)} \) as previously, the kinetic term for the 4-form gauge field contains the terms

\[
\delta g^{(1,1,0)} h^{\mu d} \partial_T A^{(1,0,0)}_{abcd} \partial_T A^{(2,1,0)}_{abc \mu} \sim k^2 B^\mu b^{(1,0,0)} \phi^{(2,1,0)}.
\]  

(43)

The vector fields \( B^\mu \) and \( \phi_\mu \) are both taken as massive. The former can decay into massless gravitons similarly to the graviphoton \( h_\mu \), through the interaction

\[
\partial^\alpha h_{\mu \alpha} h_{\alpha \beta} \partial^\mu h^{\alpha \beta}
\]  

(44)

contained in the 10D Ricci scalar. Moreover, we can generate mixing between \( \phi_\mu \) and a massless particle by turning on an additional SUSY deformation of the 2-form gauge field, \( \delta A^{(2,1,0)}_{ab} \) and using it in the Chern-Simons (CS) action \( A_4 \wedge F_3 \wedge H_3 \),

\[
\epsilon^{abcdefgh \mu \nu} A^{(2,1,0)}_{abc \mu} \delta A^{(2,1,0)}_{de \sigma} B_{\nu \rho, \sigma} \sim \epsilon^{\mu \nu \rho \sigma} \phi^{(2,1,0)}_{\mu} B_{\nu \rho, \sigma}.
\]  

(45)

The massless mode of the Kalb-Ramond field is projected out of the spectrum similarly to the massless graviphoton, but if we assume there is a stack of D-branes in the throat, then \( B_{\nu \rho} \) mixes with a massless gauge field on the brane stack via the DBI action. The complete process is illustrated in figure 4. To estimate the amplitude, we consider the \( bB_\mu \phi^\mu \) vertex and the \( \phi_\mu B_{\mu \nu} \) mixing amplitudes separately. The effective coupling of the former is given by

\[
\mathcal{L}_{bB_\phi} \approx b B_\mu \phi^\mu k^2 w \sum_i (1+\nu_i) \int dr e^{-2kr+(4+\sum_i \nu_i)kr+(4-\Delta_{110})kr}
\]  

(46)
where $\nu_i$ refer to the fields in external lines, $i = b, B_\mu, \phi_\mu$ and the $e^{-2kr}$ factor comes from $\sqrt{g_4 g^{\mu\nu}}$. The quantities $\nu_i$ and the conformal dimension $\Delta_{110}$ can be deduced from ref. [20]. We have already noted that $\nu_b = 0$ and $\nu_B = 0.2$. Appendix C of ref. [20] gives $m_{5D}^2/k^2 = 60 - 6\sqrt{52} \approx 16.73$ for $\phi_\mu^{(2,1,0)}$ in vector multiplet I. Thus $\nu_\phi = \sqrt{1 + m_{5D}^2/k^2} \approx 4.21$. The conformal dimension $\Delta_{110} \approx 5.29$ of $\delta g^{(1,1,0)}$ was already given in (36). We then obtain

$$\mathcal{L}_{bb\phi} \approx k w^{\Delta_{110}-3} b B_\mu \phi^\mu$$

which is suppressed by $w^{2.29}$. This conclusion is completely insensitive to the values of $\nu_i$, because the integral is dominated by the IR contributions, and the factors of $w^{\nu_i}$ therefore cancel between the normalization and the integral.

FIG. 4: SUSY decay of the LMCS $b$ into massless gravitons $h_{\mu\nu}$ and gauge boson in the background $\delta A_{ab}^{(2,1,0)}$ and the metric perturbation $\delta g_{ab}^{(1,1,0)}$.

Similarly, using the approximation for the $B_{\mu\nu}$ radial wave function $w^{1+\nu} \sqrt{4e^{4kr}}$ (see appendix A3 and above eq. (15)) the mixing amplitude is

$$\mathcal{L}_{B\phi} \approx e^{\mu\nu\rho\sigma} \phi_\mu B_{\nu\rho\sigma} w^{\sum_i (1+\nu_i)} \int dr e^{-4kr+3+\sum_i \nu_i)kr+(4-\Delta_{210})kr}$$

where $i = \phi_\mu, B_{\mu\nu}$, and $e^{-4kr} = \sqrt{g_4}$ is not accompanied by any inverse metric factors in the CS action. We already have $\nu_{\phi_\mu} = 4.21$ from above; for $\nu_{B_{\mu\nu}}$ we refer to the states $a_{\mu\nu}$ in ref. [20]. The lowest value of $m_{5D}/k$ for $a_{\mu\nu}^{(0,0,0)}$ is found to be $2 - \sqrt{4 - 3/4} = 0.197$ in gravitino multiplet I.\footnote{Masses of two-form states in appendix C of ref. [20] are given without being squared, and without regard to the overall sign} From section A3 we thus see that $\nu_{B_{\mu\nu}} = \sqrt{m_{5D}^2/k} \approx 0.2$. On the other hand, the lowest irrelevant value of $\Delta_{210}$ for the deformation $\delta A_{ab}^{(2,1,0)}$ comes from the
entry for $a$ in gravitino multiplet I or III with $m^2 = H_0^- + 4 - 4\sqrt{H_0^- + 4} = 22.6$, giving $\Delta_{210} = 2 + \sqrt{26.6} = 7.16$. Then we find that the $r$ integral is marginally dominated by the IR contribution, hence the result is insensitive to $\sum_i \nu_i$ as far as powers of the warp factor are concerned. We obtain $w^{\Delta_{210} - 1}$:

$$L_{B\phi} \simeq k w^{6.16} \epsilon^{\mu\nu\rho\sigma} \phi_{\mu} B_{\nu\rho\sigma}. \quad (49)$$

Finally, we can construct an effective vertex by integrating out the intermediate $\phi_{\mu}$, $B_{\mu}$ and $B_{\mu\nu}$ states. For the latter, we assume that there is a brane stack at $r = r_0$ in the bottom of the throat, with DBI action

$$\int d^4x \sqrt{\text{det}(g_{\mu\nu} + B_{\mu\nu} + F_{\mu\nu})} \quad (50)$$

which gives rise to the mixing term $\eta^{\alpha\mu} \eta^{\beta\nu} B_{\alpha\beta} F_{\mu\nu} = B \cdot F$. The important point is that $\sqrt{g^{\alpha\mu} g^{\beta\nu}}$ is independent of the warp factor, so no extra powers of $w$ arise due to the $B$-$F$ mixing. The effective interaction becomes

$$L_{bhhF} = w^{8.45} \frac{k^4}{M_p(wk)^6} b h_{\alpha\beta} \partial_{\mu} h^{\alpha\beta} \epsilon^{\mu\nu\rho\sigma} F_{\nu\rho\sigma}, \quad (51)$$

where the inverse powers of $wk$ come from the propagators for the intermediate states $B_{\mu}$, $\phi_{\mu}$ and $B_{\mu\nu}$, since all have masses of order $wk$. Furthermore each derivative (of which there are three) will be of order $m_b \sim wk$ in the decay amplitude, leading to $L_{bhhF} \sim w^{5.45}$. Squaring this, and taking account of the phase space going like $m_b \sim wk$, we obtain for the decay rate

$$\Gamma \simeq w^{11.9} \frac{k^3}{M_p^2} \quad (52)$$

The decay temperature is $T \sim w^6 \sqrt{k^4/M_p}$. For $w \simeq 10^{-4}$ and $k \sim M_p$, the relic will decay at $T \sim 10^4$ eV, which is too late to be consistent with nucleosynthesis. We have not been able to find a faster decay channel consistent with a SUSY-preserving background, hence the SUSY-violating decay of the previous section is the relevant one.

E. More general bounds

In other warped background geometries, for example using the manifold $Y^{p,q}$ instead of $T^{1,1}$ [25], the low-energy spectrum may be different, resulting in different masses and decay chains for the LMCS. Regardless of these details, however, we can expect some basic similarities to our previous analysis. The kinetic term will always enable the process shown in fig. 3 with the insertion of background corrections corresponding to an angularly excited state of the LMCS and of the internal metric absorbing the charge of the LMCS.
FIG. 5: Regions of allowed and excluded parameter space (see eq. (53)) from requiring that the LMCS decay before the onset of BBN (left) and before electroweak baryogenesis (right). The different contours correspond to different values of the string scale $M_s$. $m_{\text{eff}}^2$ is in units of $k^2$.

As an example of how our results might generalize to other backgrounds, we will consider the masses (conformal dimensions) of the LMCS and the lightest graviphoton to be free parameters, and let $\nu = \nu_b + \nu_h$. We define an effective 5D mass,

$$ m_{\text{eff}}^2 = k^2(\nu^2 - 4) $$

which approximately matches the mass of the LMCS in the $T^{1,1}$ background, since $\nu_h \approx 0.2$ is small in that background. Assuming that the radial integral for the mixing between $b$ and $h_{r_p}$ is still IR dominated (hence sensitive to $\nu$ rather than the $\Delta$'s of the background insertions), the decay rate for the LMCS takes the form

$$ \Gamma \simeq \frac{M_3^3}{M_p^2} w^{2\nu+3} $$

Here we ignore the distinction between the scales $M_{3/2}$ and $k$, assuming that $M_s$ is the only relevant scale aside from $M_p$. By demanding that the relics have decayed before nucleosynthesis, occurring at the temperature $T_{BBN} \sim 1$ MeV, we obtain the constraint

$$ \sqrt{4 + m_{\text{eff}}^2} \leq \left[ \frac{\log (M_p T_{BBN}^2/M_s^3)}{2 \log (w)} \right]^{3/2}. $$

A similar but more stringent constraint arises from requiring the relics to decay before baryogenesis, which can reasonably happen no later than the electroweak scale, $T_{EW} \sim 100$ GeV. Fig. 5 shows these bounds in the parameter space of $m_{\text{eff}}^2$ of the LMCS versus $w$ for several different values of $M_s$. A background falling within the excluded region of Fig. 5 will spoil BBN or baryogenesis by modifying the standard Hubble rate or though its late decays.
F. Decays versus annihilation

Complementary constraints may be obtained by considering annihilation processes of the LMCS instead of their decays. The former are efficient for small warp factors, while the latter dominate at larger values of $w$. From eq. (54), we see that the LMCS decays by a temperature $T_d \sim \sqrt{T_d/M_{pl}}$ provided the warping satisfies

$$w > \left( \frac{T_d^2 M_{pl}}{M_s^3} \right)^{1/(2\nu+3)}.$$  \hfill (56)

(where the inequality indicates that larger $w$ results in an earlier decay). On the other hand, the work of ref. [9] (in particular, eq. (5.11)) showed that in order to have fast enough annihilations, the warp factor must satisfy

$$w < \left[ \frac{g_{\text{eq}}}{M_p^4} \left( \frac{L}{R} \right)^{24} \right]^{1/8} \approx 10^{-8},$$  \hfill (57)

where $g \sim 100$ is the number of relativistic degrees of freedom, $L/R$ is the ratio of the bulk Calabi-Yau size to the AdS curvature scale, and $\rho_{\text{eq}} \simeq 1\text{eV}$ is the energy density at the time of radiation-matter equality. This constraint arises by demanding that KK relics do not dominate the energy density of the universe at this epoch. It implies an upper bound on the warp factor because the annihilation rate relies on the radial overlap of two massive modes; however, massive modes are peaked more in longer throats, so annihilation is most efficient in heavily warped throats.

To get $w$ as large as $10^{-8}$ in (57), it was necessary to take $L \sim 100R$. One would typically like $w \sim 10^{-4}$ to obtain the scale of inflation in the throat which is found from the COBE normalization for brane-antibrane inflation. This poses less of a challenge for the decay scenario. Assuming $M_s \sim M_p$ and $\nu = 0$, and demanding the LMCS to decay before the electroweak scale $T \simeq 100\text{GeV}$ gives the constraint

$$w > \left( \frac{T_{EW}^2}{M^2_{pl}} \right)^{1/3} \approx 10^{-11}.$$  \hfill (58)

We see that the single throat scenario has complementary constraints from decays and annihilations for avoiding the KK relic problem. A concern would arise if these constraints did not have an overlapping region. For instance, a background whose LMCS has $m_{sd}^2 = -2$ (in AdS units), or $\nu = \sqrt{2}$, requires $w > 10^{-5}$ in order to decay before the EW timescale. In this case, backgrounds whose warping lay in the range $10^{-8} < w < 10^{-5}$ will result in a relic density of massive particles, since neither annihilation nor decay is efficient enough.

IV. DISCUSSION

In this paper we have studied the problem of potentially long-lived KK relics localized in a KS throat following brane-antibrane inflation. We found that accidental symmetries
prevent the lightest mass charged state (LMCS) from decaying, even if the angular isometries giving rise to the conserved KK quantum numbers are broken by the Calabi-Yau manifold to which the throat is glued. In ref. [13], it was assumed the isometry-breaking effects of the operator $\delta g^{(1,1,0)}_{ab}$ provided a decay channel for the LMCS; however, this state either doesn’t produce a $T^{1,1}$ singlet and so the angular integral vanishes for the effective 4D interaction, or it doesn’t couple to states with light decay products and so there is no phase space for the process to proceed. Instead, for the KS background one must consider deformations of the throat geometry which break not only the isometries but also supersymmetry; additionally, we stress that any background correction used to mediate the decay must correspond to an irrelevant operator, otherwise the background solution is disrupted. A term satisfying these constraints was used to derive the viable interaction in eq. (40).

In that estimate, we omitted factors of order unity, e.g., $1/(\Delta_{110} + \Delta_{210} - 8)$, which contain the dependence on the dimension of the symmetry-breaking operators. The important point is that even if the $T^{1,1}$-breaking operators have a very high dimension, this only mildly suppresses the strength of the interaction since one integrates the radial profiles over the length of the throat. The 4D effective coupling receives UV and IR contributions, with suppression in the IR determined by the dimension of the symmetry breaking in the CY, and suppression in the UV controlled by the behaviour of the radial wave function of the LMCS (see Appendix A for more details). We find that the dependence of the decay amplitude on the warp factor is $w^{2+\nu_b+\nu_h}$, coming from the LMCS and graviphoton radial wave functions. It depends on the 5D masses of the particles, in units of the AdS curvature, through $\nu = \sqrt{4 + m^2_{5D}}$. This leads to a parametrically different rate of decay in terms of powers of the warp factor than the guess which was based upon the dimension of the symmetry-breaking operator.

We find nontrivial constraints on the parameters by requiring the heavy KK relics to decay before nucleosynthesis, and more stringently, before electroweak baryogenesis; the SUSY-breaking scale should exceed $\sim 10^9$ GeV if $w \sim 10^{-4}$ as needed for brane inflation to give the right amplitude of density perturbations. For other warped backgrounds, in which the states might have different masses, one can also constrain the effective LMCS mass versus the warp factor, as we have shown in figure 5.

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APPENDIX A: DIMENSIONAL REDUCTION

In this appendix we give details of the dimensional reduction from the 10D SUGRA action to 4D for scalar and vector fields, in the background of operators which perturb the KS solution for the warped throat.

1. Scalar fields

In the KS background the LMCS comes from the RR 4-form polarized along the $T^{1,1}$, which a 4D observer sees as a scalar field. We use this field as the eponymous example for scalar fields in the AdS; other 5D scalars will differ in their bulk mass spectrum, but their behaviour along the internal directions will be similar.

Rewriting the IIB supergravity action with terms involving $A_4$ gives

$$S_{IIB}(A_4) = \frac{1}{2\kappa_0^2} \int d^{10}x \sqrt{-G} \left[ -\frac{1}{240} (F^{(5)})^2 \right] + \frac{1}{2\kappa_0^2} \int A_4 \wedge F^{(3)} \wedge H^{(3)}. \quad (A1)$$

In this expression $\kappa_0$ is related to the 10D Newton constant, $F^{(n+1)} = dA^{(n)}$ is the field strength of the RR 4-form, and $H^{(3)}$ is the field strength of the NS 2-form $B^{(2)}$. To extract the conditions necessary to obtain a canonically normalized 4D scalar field for the LMCS we expand about background values of the metric, employ the harmonic expansion

$$A_{abcd}(x,y) = \sum_{m,\{\nu\}} \tilde{b}(x^\mu) R_{m\{\nu\}}(r) \epsilon_{abcd}^\text{e} D_e Y_{\{\nu\}}(y), \quad (A2)$$

and isolate the 4D kinetic term in (A1)

$$\Rightarrow S_{IIB}(A_4) = \frac{-1}{480\kappa_0^2} \int d^4x \, dr \, d^5y \sqrt{-|G_{AdS_5}|} \sqrt{|G_{T^{1,1}}|} G^{a_2b_2}G^{a_3b_3}G^{a_4b_4}G^{a_5b_5}
\times G^{\mu\nu} \partial_\mu A_{a_2a_3a_4a_5} \partial_\nu A_{b_2b_3b_4b_5}. \quad (A3)$$

To reduce this to a standard kinetic term in 4D for the LMCS, we require the following:

$$\frac{1}{\sqrt{|G_{T^{1,1}}|}} \partial_\alpha \left[ \sqrt{|G_{T^{1,1}}|} \epsilon_{bcdef}^\text{f} \epsilon_{bcdef}^\text{g} \left( \mathcal{D}_f \mathcal{D}_g Y_{\{\nu\}} \right) \right] = -H_{\{\nu\}} Y_{\{\nu\}} \quad (A4)$$

$$\square_r R_n - m_n^2 e^{-4kr} R_n = -m_n^2 e^{-2kr} R_n \quad (A5)$$

$$\frac{-1}{240} \int d^5y \sqrt{|G_{T^{1,1}}|} \epsilon_{bcdef}^\text{f} \epsilon_{bcdef}^\text{g} \mathcal{D}_f Y_{\{\nu\}} \mathcal{D}_g Y_{\{\mu\}} = \delta_{\{\mu\},\{\nu\}} \quad (A6)$$

$$\int dr \, e^{-2kr} R_m R_n = \delta_{m,n}. \quad (A7)$$

Eqs. (A5,A6) are the equations of motion for $T^{1,1}$ and radial wave functions respectively, with 4D mass $m_n$, while (A6,A7) are the orthonormality conditions. $H_{\{\nu\}}$ is the eigenvalue of the Laplacian on $T^{1,1}$ for the eigenfunction $Y_{\{\nu\}}$, given by

$$H_{\{j,l,r\}} = 6 \left[ j(j+1) + l(l+1) - r^2/8 \right] \quad (A8)$$
From Table C (Vector Multiplet I) of ref. \[20\], $H_{\{\nu\}}$ determines the 5D mass of the LMCS by

$$m_{5D}^2 = H_{\{\nu\}} + 16 - 8\sqrt{H_{\{\nu\}}} + 4.$$  \hspace{1cm} (A9)

$\{\nu\}$ is the set of quantum numbers specifying the representation of $SU(2) \times SU(2)/U(1)$ ($T^{1,1}$).

In analogy to the RS model \[16, 18\], the solutions for the radial wave functions have the form

$$R_n(r) = \frac{\sqrt{k we^{2kr}}}{J_\nu(x_n)} \left[ J_\nu (x_n we^{kr}) + b_{n\nu} Y_\nu (x_n we^{kr}) \right], \quad \nu = \sqrt{4 + H_{\{\nu\}}},$$ \hspace{1cm} (A10)

where $w = e^{-kr_c}$ is the warp factor and $x_n \equiv m_n e^{kr_c}/k \sim \mathcal{O}(1)$. Thus $m_n \sim wk$, which is of order the scale of inflation in the present application, while $k$ is of order the Planck scale. This solution assumes the bulk CY is not exponentially large compared to the length of the throat, $r_c$, so that the normalization factor is determined by integrating only over the throat itself.\(^4\)

It is worth noting the asymptotic limits of the Bessel functions, since these determine the UV and IR behaviour of the radial wavefunctions and play an important role in determining the 4D effective coupling. In the UV region, $r \ll 1$ and $x_n we^{kr} \simeq x_n w \ll 1$, and we employ the small-argument expansion

$$R_n(r) \simeq \frac{\sqrt{k we^{2kr}}}{J_\nu(x_n)} \left[ (x_n we^{kr})^\nu + b_{n\nu} (x_n we^{kr})^{-\nu} \right].$$ \hspace{1cm} (A11)

Taking $Z_2$ orbifold boundary conditions as in the RS model fixes the coefficient $b_{n\nu}$ to be

$$b_{n\nu} = -\frac{2J_\nu(x_n w) + x_n w J'_\nu(x_n w)}{2Y_\nu(x_n w) + x_n w Y'_\nu(x_n w)} \simeq -(x_n w)^{2\nu},$$ \hspace{1cm} (A12)

so that both solutions are of the same order ($R_n(0) \simeq \sqrt{k w^{\nu+1}}$) in the UV.

In the IR, $r \simeq r_c$, such that $x_n we^{kr} \simeq 1$ and

$$R_n(r) \simeq \frac{\sqrt{k we^{2kr}}}{J_\nu(x_n)} \left[ \sqrt{\frac{2}{x_n we^{kr}}} \cos \left( x_n we^{kr} - \left( \nu + \frac{1}{2} \right) \frac{\pi}{2} \right) \right] \sim \sqrt{k w e^{3/2kr}}.$$ \hspace{1cm} (A13)

In this case the coefficient $b_{n\nu}$ makes the $Y_\nu$ solution negligible, so the IR behaviour is dominated by the $J_\nu$ solution. At the bottom of the throat $r = r_c$ and $R_n(r_c) \sim \sqrt{\frac{k}{w}}$, so the wavefunction is exponentially peaked in the IR.

\(^4\) Ref. \[30\] examined the effect of a large bulk on the 4D mass spectrum and the transition to the ADD scenario \[32\] in the limit that the bulk becomes much larger than $(kw)^{-1}$. The effect of a large bulk to suppress tunneling between throats was also explored in ref. \[33\]. The work presented here is concerned mainly with reheating in one throat and envisions possible tunneling to a SM throat later on.
2. Vector fields

To give an example of DR for a vector field in a warped space, we consider the 4-form with one index polarized along the AdS, while the other three are along the $T^{1,1}$:

$$A_{\mu abc}(x, r) = \sum_{\nu} \phi_{\mu}^{(\nu)}(x) Y_{abc}^{(\nu)}(y). \quad (A14)$$

(in this section Greek indices include $r$ in addition to the noncompact directions). The kinetic term is

$$S_{\text{kin}}(\phi_r) = -\frac{1}{480 \kappa_5^2} \int d^{10}x \sqrt{-G} \partial_{[\alpha} A_{\beta]cde} \partial^{[\alpha} A_{\beta]cde} \quad (A15)$$

$$= -\frac{1}{480 \kappa_5^2} \sum_{\{\mu\}, \{\nu\}} \int d^5x \sqrt{-|G_{\text{AdS}}|} \partial_{\alpha} \phi_{\beta}^{(\mu)} \partial^{[\alpha} \phi_{\beta]}^{(\nu)} \int d^5y \sqrt{|G_{T^{1,1}}|} Y_{cde}^{(\mu)} Y_{cde}^{(\nu)}. \quad (A16)$$

We therefore normalize

$$\frac{1}{120} \int d^5y \sqrt{|G_{T^{1,1}}|} Y_{cde}^{(\mu)} Y_{cde}^{(\nu)} = \delta_{\{\mu\}, \{\nu\}} \quad (A17)$$

which gives the 5D action for the vector field $\phi_\gamma$:

$$S_{\text{kin}}(\phi_r) = -\frac{1}{4 \kappa_5^2} \int d^5x \sqrt{-|G_{\text{AdS}}|} (\partial_{[\alpha} \phi_{\beta]} + m^2 \phi). \quad (A18)$$

Here we have allowed for a 5D mass, which does not come from the terms in (A15), but which would generically arise from derivatives along the $T^{1,1}$ directions.

The equation of motion arising from (A18) is

$$\partial^\alpha F_{\alpha\beta} - m^2 B_{\beta} = 0 \quad (A19)$$

where $F_{\beta\delta} = \partial_{[\beta} \phi_{\delta]}$. This leads to an equation for the radial wave function whose solution has the form

$$R_{1\text{-form}}(r) \simeq \sqrt{k} w e^{kr} \left[ J_\nu \left( x_n w e^{kr} \right) + b_n \nu \left( x_n w e^{kr} \right) \right], \nu = \sqrt{1 + m^2_5/k^2}. \quad (A20)$$

3. Antisymmetric tensor field

The equation of motion for the rank-2 antisymmetric tensor field is:

$$\frac{1}{\sqrt{-G}} \partial_A \left[ \sqrt{-G} F^{ABC} \right] + M^2 B^{BC} = 0 \quad (A21)$$

where $M$ is the 5-dimensional mass given by the non-trivial $T^{1,1}$ quantum numbers of the field. We want to write the equation in terms of $B_{\mu\nu}$ so we rewrite the above equation as:

$$\frac{1}{\sqrt{-G}} \partial_A \left[ \sqrt{-G} g^{AL} g^{BM} g^{CN} F_{LMN} \right] + M^2 g^{BM} g^{CN} B_{MN} = 0 \quad (A22)$$
where the metric is the RS metric:

$$ds^2 = e^{-2kr} \eta_{\mu \nu} dx^\mu dx^\nu + dr^2$$  \hfill (A23)

and the field strength is:

$$F_{LMN} = \frac{1}{6} \left[ \partial_L B_{MN} - \partial_M B_{LN} + \partial_N B_{LM} - \partial_L B_{NM} + \partial_M B_{LN} - \partial_N B_{ML} \right]$$  \hfill (A24)

For $B$ and $C$ corresponding to polarizations along the large 4 dimensions we get:

$$e^{4kr} \partial_\lambda \left[ e^{-4kr} e^{6kr} \left( 2 \partial^\lambda B_{\mu \nu} + 2 \partial_\mu B^\lambda_{\nu} + 2 \partial_\nu B^\lambda_{\mu} \right) \right] + e^{4kr} \partial_r \left[ e^{-4kr} e^{4kr} \left( 2 \partial^r B_{\mu \nu} + 2 \partial_\mu B^r_{\nu} + 2 \partial_\nu B^r_{\mu} \right) \right] + e^{4kr} M^2 B_{\mu \nu} = 0$$  \hfill (A25)

To proceed we will need to fix the gauge, i.e. we need a condition of the type:

$$g^{MN} \partial_M B_{N\nu} = e^{2kr} \partial_\lambda B^\lambda_{\nu} + \partial_r B^r_{\nu} = 0$$  \hfill (A26)

The equation of motion simplifies to:

$$e^{2kr} \Box_4 B_{\mu \nu} - \partial_\mu \partial_\nu B_{\mu \nu} + M^2 B_{\mu \nu} = 0$$  \hfill (A27)

If we now decompose:

$$B_{\mu \nu} (x, r) = B_{\mu \nu} (x) R (r)$$  \hfill (A28)

we obtain for $\chi (r)$ the equation:

$$- \partial_\mu^2 R (r) + M^2 R (r) - e^{2kr} m^2 R (r) = 0$$  \hfill (A29)

which has a solution of the type:

$$R (r) = J_\nu (x_n we^{kr}) + b_{\nu \nu} Y_\nu (x_n we^{kr}), \quad \nu = \frac{m_5}{k}$$  \hfill (A30)

This shows that for the rank-2 antisymmetric tensor, $\nu = m_5/k$.

### 4. Background deformation by source in UV

To model the effect of gluing a Calabi-Yau onto the throat, and the ensuing deformation, we can introduce a source term localized in the UV, which is the CY region. As an example we consider a background perturbation of the 4-form, with $T^{1,1}$ quantum numbers $\{\nu\}$, and coupled to a source of strength $S_{\{\nu\}} \sim M_s^4 \tilde{\phi}$, localized in the UV region, which we take to be at $r = 0$. The action for the source term is

$$S_{\{\nu\}} = \int d^{10} x \sqrt{-G} S_{\{\nu\}} A_{abcd}$$

$$= \int d^{10} x \sqrt{-G} \tilde{\phi} M_s^4 \delta (r) \epsilon^{abcd} \mathcal{D} Y_{\{\nu\}} \cdot \sum_{\{\nu'\}} b_{\{\nu'\}} (x) \epsilon_{abcd} \mathcal{D}_g Y_{\{\nu'\}} (y).$$  \hfill (A31)
Upon variation of the field, the source appears in the equation of motion for the radial wave function of the field,

$$\Box r R_{0,\{\nu\}} - m_{5D}^2 e^{-4kr} R_{0,\{\nu\}} = \tilde{\phi} \delta(r).$$

(A32)

where the subscript 0 indicates that the 4D mass is zero, since by Lorentz invariance this perturbation of the background cannot depend on the large dimensions. There are two particular solutions, of which one is subdominant over the entire throat, so we can approximate

$$R_{0,\{\nu\}}(r) \approx \sqrt{k} \tilde{\phi} e^{(2-\nu)kr} = \sqrt{k} \tilde{\phi} e^{(4-\Delta)kr}$$

(A33)

where $\nu = \sqrt{4 + m_{5D}^2}$ and $\Delta = \nu + 2$ is the conformal dimension of the operator. In order that the KS solution not be strongly deformed in the IR, we demand that $\Delta > 4.5$. In contrast to the radial wave functions of massive excitations which are peaked in the IR region, (A33) is peaked in the UV.

APPENDIX B: EVALUATING THE ANGULAR INTEGRALS

The existence of a particular decay channel depends upon having a nonvanishing overlap of $T^{1,1}$ harmonics for the various fields contributing to the effective 4D interaction. For reference we give some of the scalar harmonics (we do not know explicit expressions for vector and tensor harmonics). They have been calculated in refs. [34, 35]:

$$Y_L(\Psi) = J_{l_1,m_1,R}(\theta_1) J_{l_2,m_2,R}(\theta_2) e^{i m_1 \phi_1 + i m_2 \phi_2} e^{\frac{i}{2} L^R \phi},$$

(B1)

The functions $J$ are given by hypergeometric functions,

$$J^T_{l,m,R}(\theta) = N^T_L (\sin \theta)^m \left( \cot \frac{\theta}{2} \right)^{R \frac{R}{2}} \binom{-l + m,1 + l + m,1 + m - \frac{R}{2};\sin^2 \frac{\theta}{2}}{2F1},$$

$$J^\Omega_{l,m,R}(\theta) = N^\Omega_L (\sin \theta)^m \left( \cot \frac{\theta}{2} \right)^{R \frac{R}{2}} \binom{-l + \frac{R}{2},1 + l + \frac{R}{2},1 - m + \frac{R}{2};\sin^2 \frac{\theta}{2}}{2F1},$$

(B2)

where $J^T$ is nonsingular for $m \geq R/2$ and $J^\Omega$ is nonsingular for $m \leq R/2$. The particular scalar harmonics that come into play for the states we are interested in correspond to $l = 1$, $m = 0$, $R = 0$. The value of $m$ can be inferred following the calculation of ref. [20] where the parameters $r$ and $q$ were defined in terms of the $m_1$ and $m_2$ “magnetic” quantum numbers.

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5 The same procedure indicates that sources for AdS$_5$ vectors must satisfy $\Delta > 2$, and even higher-spin objects cannot be used at all. It is not unreasonable to contemplate such deformations, a priori, since sourcing only the $r$ polarizations of fields with indices in AdS$_5$ would still be consistent with 4D Lorentz invariance.
for each of the $SU(2)$ groups ($r = m_1 - m_2$, $q = m_1 + m_2$). $q = 0$ for scalars, so for the state $(j, l, r) = (1, 0, 0)$ we have $m_1 = m_2 = 0$ and the scalar harmonic

$$Y_{(1,0,0)}(y) = J_{1,0,0}(\theta_1) J_{0,0,0}(\theta_2) = N_L^2 \cos \theta_1$$  \hspace{1cm} (B3)

Some other common harmonics are given by

$$Y_{(0,1,0)}(y) = N_L^2 \cos \theta_2$$

$$Y_{(1,1,0)}(y) = N_L^2 \cos \theta_1 \cos \theta_2$$

$$Y_{(2,1,0)}(y) = \frac{1}{4} N_L^2 \left[ \cos \theta_2 + \frac{3}{2} \cos(2\theta_1 + \theta_2) + \frac{3}{2} \cos(2\theta_1 + \theta_2) \right]. \hspace{1cm} (B4)$$

To evaluate the various integrals we also require the expression of $\sqrt{g}$ where $g$ is the metric on the $T^{1,1}$. If each $SU(2)$ has Euler-angle coordinatization $(\theta_i, \phi_i, \gamma_i)$, and the left-coset acts to mod out the $\gamma_i$'s ($\psi = \frac{1}{\sqrt{2}} (\gamma_1 - \gamma_2)$), the metric in the basis $[\psi, \theta_1, \theta_2, \phi_1, \phi_2]$ is

$$g_{ab} = \begin{pmatrix}
\frac{1}{9} & 0 & 0 & \frac{\cos \theta_1}{9} & \frac{\cos \theta_2}{9} \\
0 & \frac{1}{9} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{9} & 0 & 0 \\
\frac{\cos \theta_1}{9} & 0 & 0 & \frac{1}{9} \left(1 + \sin^2 \theta_1 / 2 \right) & \frac{\cos \theta_1 \cos \theta_2}{9} \\
\frac{\cos \theta_2}{9} & 0 & 0 & \frac{\cos \theta_1 \cos \theta_2}{9} & \frac{1}{9} \left(1 + \sin^2 \theta_2 / 2 \right) 
\end{pmatrix} \hspace{1cm} (B5)$$

with the determinant

$$\sqrt{g} = \frac{|\sin \theta_1| |\sin \theta_2|}{\sqrt{11664}}, \hspace{1cm} (B6)$$

and coordinate ranges

$$\theta_i \in (0, 2\pi), \beta_i \in (0, \pi), \gamma_i \in (0, 4\pi). \hspace{1cm} (B7)$$

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