Split Plot Mixture Process Variable Experiment on Steel Slag Concrete

Faula Arina1*, Aji Hamim Wigena2, I Made Sumertajaya2, Utami Syafitri 2

1 Department of Industrial Engineering, Sultan Ageng Tirtayasa University, Cilegon, Indonesia
2 Department of Statistics, Bogor Agricultural University, Bogor, 16680, Indonesia

*email : faula.arina71@gmail.com

Abstract. In many such mixture-process variable experiments, constraints such as cost or time prohibit the selection of treatments completely at random. This leads to a split-plot structure of the data and affects both the experimental design and the statistical modelling. The constraints of the mixture component affect the experimental region. The experimental region could be a regular or irregular shape. For the irregular shape, the design points are difficult to identify by hand. XVERT algorithm is one algorithm in mixture experiments to define the design points of mixture experiment. An algorithm used for selecting a subset of extreme vertices when the number of candidate vertices is large. This research a new split-plot designs of mixture experiments with process variables was developed. The study case was on cementitious mixture components for steelslag concrete. The five mixture components were cement, aggregate fine, aggregate coarse, percentage steel slag replaced the aggregate fine and water. The process variable was size of steel slag. The steel slag concrete experiment was run using a split plot mixture process variable design with 23 whole plots of three observations. The final design consisted of {8 7 8} and experiment design with 69 observations.

1. Introduction
A mixture experiment is an experiment in which the response depend on the proportions of the components not the total amount. There are two main constraints of mixture experiments. First, the proportion of a component is between 0 and 1. Second, the sum of proportions of all components is unity. Both constraint affect the experimental region. The experimental region becomes a \((q - 1)\) simplex. Furthermore, additional constraints on proportions, such as lower bounds, upper bounds, and linear constraint, will affect the shape of the experimental region. The experimental region becomes a regular or irregular shape. Design points of the irregular shape of the mixture experiment of more than three components are difficult to determine by hand. It is needed a computational approach. In practice, mixture experiments can be extended with independent process variables to model such systems [1]. This kind of experiment is called mixture process variable (MPV). Some process variables can be hard-to-change due to practical and economic considerations. These constraints prohibit complete randomization of the experimental runs. In contrary, combinations of proportions of components are easier to change then the level of the process variables. For this situation, complete randomization is difficult to be implemented. The easiest way is all proportions of combinations are run at a certain level of
the process variable. That systematic way which involves restricted randomization is called split-plot designs [2]. Hence, the split plot design is implemented in a mixture process variable experiment. To determine the design points of an irregular mixture experiment is needed a computational approach. XVERT algorithm is one algorithm that can be used for selecting a subset of extreme vertices when the number of candidate vertices is large [3]. A drawback of classical MPV is if the experiment consists of large \( q \) components, then the number of experiments will be larger. It affects the cost of experiments. In this paper, the aim of the study was to create a MPV design with split plot approach design. The consideration of building the design were the number of observations available, the split plot nature of the experiment, and the number of observations that can be performed within each whole plot. The study case in this paper was a steel slag concrete experiment.

2. The Steel Slag Concrete Experiment

The experiment involved five mixture components and a process variable. The five mixture components were water \((X_1)\), cement \((X_2)\), aggregate coarse \((X_3)\), aggregate fine \((X_4)\), percentage steel slag replaced the aggregate fine \((X_5)\). Mixture component have constrained components and their upper and lower bounds.

| Component                | Minimum | Maximum |
|--------------------------|---------|---------|
| \(X_1\) = water          | 0.14    | 0.21    |
| \(X_2\) = cement         | 0.07    | 0.15    |
| \(X_3\) = aggregate coarse | 0.36   | 0.48    |
| \(X_4\) = aggregate fine | 0.21    | 0.22    |
| \(X_5\) = percentage steel slag | 0.03 | 0.1 |

Process variable was size of steel slag with three level, \( z = -1, 0, 1 \). The value -1 represents 1.2 mm the value 0 represents 2.4 mm and the value 1 represents 4.8 mm. The experiment was carried out as a split plot experiment with the process variables as whole plot factors meanwhile the compositions of proportions as sub plot factors.

3. XVERT Algorithm

Several algorithms are available to obtain the design to fit the linear model to constrained mixture experiment. The extreme vertices design for such type of mixture experiments when factors have constraints placed on them for mixture experiments [4]. McLean and Anderson’s algorithm starts with writing \( 2^{q-1} \) combinations of upper and lower bounds for all but one factor which is left blank. Do the same for all \( 2^{q-1} \) combinations. The procedure is repeated \( q \) times; this will consist of possible \( q 2^{q-1} \) combinations. This algorithm is simple to compute but produces large number of points having large number of inadmissible points and replications of vertices.

Overcome the drawback of listing large number of design points obtained by using McLean and Anderson’s method, Ronald Snee and Donald Marquardt proposed an algorithm named XVERT to find the extreme vertices to fit a linear model for mixture experiment. The extreme vertices also can be computed using the XVERT Algorithm describe below.

1. Rank the components in order of increasing ranges \( U_i - L_i \). \( x_1 \) has the smallest range and \( x_q \) has the largest range.

2. Consider first \( q - 1 \) components with the smallest ranges. Form a two level design from the lower - upper bounds of these \( q - 1 \) components. There are \( 2^{q-1} \) combinations.
3. Determine the level of the omitted component $x_q$ with each of the $2^{q-1}$ combination in step 2 using $t x_q = 1 - \sum_{i=1}^{q-1} x_i$.

4. If this computed value lie within the constraint limits it is an extreme vertex called as core point. If it falls outside the constraint limits of the corresponding component it is called as candidate point. For the points which are outside of the constraint limits, set $x_q$ equal to the upper or lower limit, whichever is closest to the computed value.

5. Additional points are generated from the candidate points. Find the difference between computed value and substituted upper or lower limit. Adjust this difference to one of the candidate point.

The design points obtained from the same candidate point after adjusting the component levels constitutes the candidate subgroup. Thus XVERT starts with listing $2^{q-1}$ design points. This results into candidate points and core points. Each candidate point is to be adjusted $q-1$ times. Thus in all $q*2^{q-1}$ design points are generated. Though the method is generating large number of design points it is computationally faster than that of McLean and Anderson’s method.

Snee and Marquardt prefer minimum trace $(X^TX)^{-1}$ criterion since it is the most appropriate criterion according to them, to fit linear mixture model. The designs obtained by using XVERT are near optimum in terms of trace $(X^TX)^{-1}$ criterion.

4. Mixture Process Variable (MPV)
The model of the MPV experiments, depends on the blending properties of the mixture components, the effects of the process variables, and any interactions between the mixture components and process variables. Process variables are factors that are not part of the mixture, although when the levels are variable, it can affect the mixture properties. The steel slag concrete experiment involved five mixture components and a process variable. It was obtained by combining the Scheff’e linear model of mixture model is [5]

$$\hat{y}(x) = \sum_{i=1}^{5} b_i x_i$$

and model for the process variable. Suppose there is a process variables $z$. A model from a three level factorial design in process variable

$$\hat{y}(z) = \alpha_o + \alpha_1 z + \alpha_{11} z^2$$

The usual way of combining these mixture and process models is to multiply them. Such crossed models often contain an undesirable high number of terms, and it is usual practice to reduce the models, e.g. by removing higher order interactions. MPV model [6]:

$$\hat{y}(x,z) = \sum_{i=1}^{5} \beta_i x_i + \sum_{i=1}^{5} \gamma_i x_i z + \alpha_{11} z^2$$

Eq. (3) contains 11 parameter which includes 5 parameter the linier mixture terms, 5 parameter mixture $\times$ process interactions and 1 parameter the process variable model.

5. Split-Plot Mixture Process Variable (SPMPV)
In general, a split plot experiment has whole plot variables (hard to change factors) and sub plot variables (easy to change factors). Sets of observations for which the whole plot variables are held fixed are called whole plots.

The steel slag concrete experiment was run using a SPMPV design. The process variable is the whole-plot variable, whereas the five mixture components are the sub-plot variables. The model SPMPV is given by

$$\hat{y}(x,z) = f^T(z,x) \beta = \sum_{i=1}^{5} \beta_i x_i + \sum_{i=1}^{5} \gamma_i x_i z + \alpha_{11} z^2$$

$$\sum_{i=1}^{5} x_i = 1$$
Where \( z = [z]^T, x = [x_1, x_2, x_3, x_4, x_5]^T \), where \( f_T(z, x) \) represents the model expansion of the whole plot variables \( z \) and the sub plot variables \( x_1, x_2, x_3, x_4 \) and \( x_5 \) and \( \beta \) contains all the unknown model parameters.

In matrix notation, the model corresponding to a split plot design can be written as [7]
\[
y = X\beta + Z\gamma + \epsilon
\]  
(5)

Where \( X \) represents the \( n \times p \) model matrix containing the setting of both the whole plot variables \( z \) and the sub plot variables \( x \). The matrix \( Z \) is a \( n \times b \) matrix of zeroes and ones assigning the \( n \) observations to the \( b \) whole plots. The random effects of the \( b \) whole plots are containing within the \( b \) dimensional vector \( y \) and the random errors are contained within the \( n \) dimensional vector \( \epsilon \). It is assumed that \( y \sim N(0, \sigma_y^2 I_b) \) and \( \sim N(0, \sigma_\epsilon^2 I_n) \).

Under these assumption the covariance matrix can be written as
\[
V = \sigma_y^2 ZZ' + \sigma_\epsilon^2 I_n = \sigma_\epsilon^2 (I_n + \eta ZZ')
\]  
(6)

Where \( V \) is a \( n \times n \) matrix. Split plot design has different the covariance matrix structure from completely randomized design. The covariance matrix in split plot design is not a diagonal matrix. Estimation matrix \( V \) used Bayesian estimation. The variance component \( \sigma_v^2 \) is obtained by finding a posterior distribution based on the prior information. After distribution \( \sigma_v^2 \) known, The value of empirical estimates \( \hat{\sigma}_v^2 \) is obtained MCMC (markov Chain Monte Carlo). The value of empirical estimates \( \hat{\sigma}_\epsilon^2 \) is obtained \( \frac{\hat{\sigma}_\epsilon^2}{\eta} \). Where \( \eta \) a measure for the extent to which observations within the same whole plot are correlated. The information matrix on the unknown model parameter vector \( \beta \) is given by
\[
M = X^T V^{-1} X = \sigma_\epsilon^{-2} X^T (I_n + \eta ZZ^T)^{-1} X
\]  
(7)

The final design for MPV split-plot experiments is as follows:

i) Specify the number of whole plots \( (b) \), ii) specify the number of split plots per whole plot \( (k_i) \) \( n \) total observations \( n = \sum_{i=1}^{b} k_i \), iii) Specify the respon model \( f_T(z_i, x_{ij}) \), iv) Specify estimate of the variance ratio \( (\eta) \), v) exam design use maximum \( |X^T V^{-1} X| \).

6. Result and Discussion

The objective of concrete study was to develop a blending model for a five component with the following component ranges based on Table 1. Here are the components ranked in order of increasing ranges.

| Component | Minimum | Maximum | Range |
|-----------|---------|---------|-------|
| \( m_1 = x_4 \) | 0.21 | 0.22 | 0.01 |
| \( m_2 = x_1 \) | 0.14 | 0.21 | 0.07 |
| \( m_3 = x_5 \) | 0.03 | 0.1 | 0.07 |
| \( m_4 = x_2 \) | 0.07 | 0.15 | 0.08 |
| \( m_5 = x_3 \) | 0.36 | 0.48 | 0.12 |

The core points are generated by using levels of a \( 2^{q-1} = 2^4 \) design to determine the levels of the first \( q - 1 = 4 \) factors. The level of \( m_5 \) is given as follow \( m_5 = 1 - m_1 - m_2 - m_3 - m_4 \).
### Table 3 The Core Points

| Point | $m_1$  | $m_2$  | $m_3$  | $m_4$  | $m_5$  | Comment          |
|-------|--------|--------|--------|--------|--------|------------------|
| 1     | 0.21   | 0.14   | 0.03   | 0.07   | 0.55   | Out of limits    |
| 2     | 0.21   | 0.14   | 0.03   | 0.15   | 0.47   | Vertex           |
| 3     | 0.21   | 0.14   | 0.1    | 0.07   | 0.48   | Vertex           |
| 4     | 0.21   | 0.14   | 0.1    | 0.15   | 0.4    | Vertex           |
| 5     | 0.21   | 0.21   | 0.03   | 0.07   | 0.48   | Vertex           |
| 6     | 0.21   | 0.21   | 0.03   | 0.15   | 0.4    | Vertex           |
| 7     | 0.21   | 0.21   | 0.1    | 0.07   | 0.41   | Vertex           |
| 8     | 0.21   | 0.21   | 0.1    | 0.15   | 0.33   | Out of limits    |
| 9     | 0.22   | 0.14   | 0.03   | 0.07   | 0.54   | Out of limits    |
| 10    | 0.22   | 0.14   | 0.03   | 0.15   | 0.46   | Vertex           |
| 11    | 0.22   | 0.14   | 0.1    | 0.07   | 0.47   | Vertex           |
| 12    | 0.22   | 0.14   | 0.1    | 0.15   | 0.39   | Vertex           |
| 13    | 0.22   | 0.21   | 0.03   | 0.07   | 0.47   | Vertex           |
| 14    | 0.22   | 0.21   | 0.03   | 0.15   | 0.39   | Vertex           |
| 15    | 0.22   | 0.21   | 0.1    | 0.07   | 0.4    | Vertex           |
| 16    | 0.22   | 0.21   | 0.1    | 0.15   | 0.32   | Out of limits    |

### Table 4 The Candidate Sub Group Points

| Candidate sub group | $m_1$  | $m_2$  | $m_3$  | $m_4$  | $m_5$  |
|---------------------|--------|--------|--------|--------|--------|
| 1                   | 0.21   | 0.14   | 0.03   | 0.07   | 0.55   |
| 1a                  | 0.28   | 0.14   | 0.03   | 0.07   | 0.48   |
| 1b                  | 0.21   | 0.21   | 0.03   | 0.07   | 0.48   |
| 1c                  | 0.21   | 0.14   | 0.1    | 0.07   | 0.48   |
| 1d                  | 0.21   | 0.14   | 0.03   | 0.14   | 0.48   |
| 8                   | 0.21   | 0.21   | 0.1    | 0.15   | 0.33   |
| 8a                  | 0.54   | 0.21   | 0.1    | 0.15   | 0.36   |
| 8b                  | 0.21   | 0.18   | 0.1    | 0.15   | 0.36   |
| 8c                  | 0.21   | 0.21   | 0.07   | 0.15   | 0.36   |
| 8d                  | 0.21   | 0.21   | 0.1    | 0.12   | 0.36   |
| 9                   | 0.22   | 0.14   | 0.03   | 0.07   | 0.54   |
| 9a                  | 0.28   | 0.14   | 0.03   | 0.07   | 0.48   |
| 9b                  | 0.22   | 0.2   | 0.03   | 0.07   | 0.48   |
| 9c                  | 0.22   | 0.14   | 0.09   | 0.07   | 0.48   |
| 9d                  | 0.22   | 0.14   | 0.03   | 0.13   | 0.48   |
| 16                  | 0.22   | 0.21   | 0.1    | 0.15   | 0.32   |
| 16a                 | 0.18   | 0.21   | 0.1    | 0.15   | 0.36   |
| 16b                 | 0.22   | 0.17   | 0.1    | 0.15   | 0.36   |
| 16c                 | 0.22   | 0.21   | 0.06   | 0.15   | 0.36   |
| 16d                 | 0.22   | 0.21   | 0.1    | 0.11   | 0.36   |
Note that points 1-16 form a $2^4$ design in $m_1, m_2, m_3$, and $m_4$. This value is within the 0.36 to 0.48 range for $m_5$. The levels of $m_5$ points 1, 8, 9 and 16 are not in the range specified for $m_5$. Points 1, 8, 9 and 16 must be adjusted to meet the constraint on $m_5$.

Table 5 Coordinates of the 23 Design Point for the Constrained Steelslag Concrete

| No | Point | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|----|-------|-------|-------|-------|-------|-------|
| 1  | 2     | 0.14  | 0.15  | 0.47  | 0.21  | 0.03  |
| 2  | 3     | 0.14  | 0.07  | 0.48  | 0.21  | 0.1   |
| 3  | 4     | 0.14  | 0.15  | 0.4   | 0.21  | 0.1   |
| 4  | 5     | 0.21  | 0.07  | 0.48  | 0.21  | 0.03  |
| 5  | 6     | 0.21  | 0.15  | 0.4   | 0.21  | 0.03  |
| 6  | 7     | 0.21  | 0.07  | 0.41  | 0.21  | 0.1   |
| 7  | 10    | 0.14  | 0.15  | 0.46  | 0.22  | 0.03  |
| 8  | 11    | 0.14  | 0.07  | 0.47  | 0.22  | 0.1   |
| 9  | 12    | 0.14  | 0.15  | 0.39  | 0.22  | 0.1   |
| 10 | 13    | 0.21  | 0.07  | 0.47  | 0.22  | 0.03  |
| 11 | 14    | 0.21  | 0.15  | 0.39  | 0.22  | 0.03  |
| 12 | 15    | 0.21  | 0.07  | 0.4   | 0.22  | 0.1   |
| 13 | 1d    | 0.14  | 0.14  | 0.48  | 0.21  | 0.03  |
| 14 | 8b    | 0.18  | 0.15  | 0.36  | 0.21  | 0.1   |
| 15 | 8c    | 0.21  | 0.15  | 0.36  | 0.21  | 0.07  |
| 16 | 8d    | 0.21  | 0.12  | 0.36  | 0.21  | 0.1   |
| 17 | 9b    | 0.2   | 0.07  | 0.48  | 0.22  | 0.03  |
| 18 | 9c    | 0.14  | 0.07  | 0.48  | 0.22  | 0.09  |
| 19 | 9d    | 0.14  | 0.13  | 0.48  | 0.22  | 0.03  |
| 20 | 16b   | 0.17  | 0.15  | 0.36  | 0.22  | 0.1   |
| 21 | 16c   | 0.21  | 0.15  | 0.36  | 0.22  | 0.06  |
| 22 | 16d   | 0.21  | 0.11  | 0.36  | 0.22  | 0.1   |
| 23 | Average | 0.177727 | 0.116364 | 0.422727 | 0.215455 | 0.067727 |

The core points and candidate subgroup point for the XVERT design are shown in Table 3 and Table 4. In this case, all 22 extreme vertices are included in the core point and candidate subgroup points. The 4 candidate subgroup each contain four points, hence there are $4^4 = 256$ design to be evaluated. The designs obtained by using XVERT are near optimum in terms of minimum trace $(X'X)^{-1}$ criterion. In this case, linear model all 23 design points are 22 extreme vertices and 1 overall centroid, XVERT designs are shown in Table 5. The design approach presented here is also useful when constraints are imposed on the mixture components. The following constraints apply for the sub plot variables $x_1, x_2, x_3, x_4$, and $x_5$. Constructing final design for the constrained design region using the XVERT algorithm requires the specification of combination level sub plot factor. An appropriate level sub plot factor for this problem is obtained by combining the points of three level whole plot factor.

The steel slag concrete experiment was run using a SPMPV design with 23 whole plots of three observations. Estimation Matrix $V$ used Bayesian estimation. The variance component $\sigma^2_Y$ is obtained by finding a posterior distribution based on the prior information $\sigma^2_Y \sim inv \chi^2_{(n-1, s^2)}$. The value of empirical estimates $\hat{\sigma}^2_Y$ is obtained MCMC, $\hat{\sigma}^2_Y = 13.80601$. 

![Image](image-url)
Given the variance ratio ($\eta$) = 10, the value of empirical estimates $\hat{\sigma}_2^2 = 1.380601$. The 23 whole plots combining the points of three level whole plot factor $z = -1, 0, 1$, so three candidate designs are shown in Table 6.

| Table 6 Candidate Designs |
|---------------------------|
| Design | $|X^T V^{-1}X|$ |
| 8 8 7  | 1.72 e-16 |
| 8 7 8  | 2.93 e-16 |
| 7 7 8  | 2.15 e-16 |

When comparing various experimental designs, it is desirable to use some quantitative criterion. The useful criteria is maximum $|X^T V^{-1}X|$. It is clear that design {8 7 8} is the best design. The final design consisted of {8 7 8} for the steelslag concrete experiments are shown in Table 7.

| Table 7 SPMPV Design For The Steelslag Concrete Experiments |
|---------------------------------------------------------------|
| Whole plot | Sub Plot | x₁ | x₂ | x₃ | x₄ | x₅ | z |
|-------------|----------|----|----|----|----|----|---|
| 1           | 1        | 0.14 | 0.14 | 0.48 | 0.21 | 0.03 | -1 |
| 2           | 0.14 | 0.15 | 0.46 | 0.22 | 0.03 | -1 |
| 3           | 0.21 | 0.15 | 0.39 | 0.22 | 0.03 | -1 |
| 2           | 1      | 0.177727 | 0.116364 | 0.422727 | 0.215455 | 0.067727 | -1 |
| 2           | 0.18 | 0.15 | 0.36 | 0.21 | 0.1 | -1 |
| 3           | 0.14 | 0.07 | 0.48 | 0.21 | 0.1 | -1 |
| 3           | 0.21 | 0.07 | 0.47 | 0.22 | 0.03 | -1 |
| 3           | 0.14 | 0.15 | 0.39 | 0.22 | 0.1 | -1 |
| 3           | 0.21 | 0.15 | 0.36 | 0.21 | 0.07 | -1 |
| 4           | 1      | 0.21 | 0.11 | 0.36 | 0.22 | 0.1 | -1 |
| 2           | 0.17 | 0.15 | 0.36 | 0.22 | 0.1 | -1 |
| 3           | 0.2 | 0.07 | 0.48 | 0.22 | 0.03 | -1 |
| 5           | 1      | 0.21 | 0.11 | 0.36 | 0.22 | 0.1 | -1 |
| 2           | 0.21 | 0.07 | 0.4 | 0.22 | 0.1 | -1 |
| 3           | 0.14 | 0.07 | 0.47 | 0.22 | 0.1 | -1 |
| 6           | 1      | 0.21 | 0.07 | 0.41 | 0.21 | 0.1 | -1 |
| 2           | 0.14 | 0.15 | 0.47 | 0.21 | 0.03 | -1 |
| 3           | 0.14 | 0.15 | 0.4 | 0.21 | 0.1 | -1 |
| 7           | 1      | 0.14 | 0.13 | 0.48 | 0.22 | 0.03 | -1 |
| 2           | 0.14 | 0.07 | 0.48 | 0.22 | 0.09 | -1 |
| 3           | 0.21 | 0.12 | 0.36 | 0.21 | 0.1 | -1 |
| 8           | 1      | 0.21 | 0.15 | 0.36 | 0.22 | 0.06 | -1 |
| 2           | 0.21 | 0.15 | 0.4 | 0.21 | 0.03 | -1 |
| 3           | 0.21 | 0.07 | 0.48 | 0.21 | 0.03 | -1 |
| 9           | 1      | 0.21 | 0.07 | 0.48 | 0.21 | 0.03 | 0 |
| 2           | 0.21 | 0.07 | 0.4 | 0.22 | 0.1 | 0 |
| Whole plot | Sub Plot | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $z$ |
|-----------|----------|-------|-------|-------|-------|-------|-----|
| 3         |          | 0.14  | 0.07  | 0.47  | 0.22  | 0.1   | 0   |
| 10        | 1        | 0.21  | 0.15  | 0.36  | 0.22  | 0.06  | 0   |
|           | 2        | 0.21  | 0.15  | 0.39  | 0.22  | 0.03  | 0   |
|           | 3        | 0.14  | 0.13  | 0.48  | 0.22  | 0.03  | 0   |
| 11        | 1        | 0.14  | 0.15  | 0.4   | 0.21  | 0.1   | 0   |
|           | 2        | 0.14  | 0.07  | 0.48  | 0.22  | 0.09  | 0   |
|           | 3        | 0.177727 | 0.116364 | 0.422727 | 0.215455 | 0.067727 | 0   |
| 12        | 1        | 0.18  | 0.15  | 0.36  | 0.21  | 0.1   | 0   |
|           | 2        | 0.14  | 0.15  | 0.47  | 0.21  | 0.03  | 0   |
|           | 3        | 0.14  | 0.07  | 0.48  | 0.21  | 0.1   | 0   |
| 13        | 1        | 0.14  | 0.15  | 0.47  | 0.21  | 0.03  | 0   |
|           | 2        | 0.21  | 0.07  | 0.41  | 0.21  | 0.1   | 0   |
|           | 3        | 0.21  | 0.15  | 0.4   | 0.21  | 0.03  | 0   |
| 14        | 1        | 0.21  | 0.15  | 0.36  | 0.21  | 0.07  | 0   |
|           | 2        | 0.14  | 0.14  | 0.48  | 0.21  | 0.03  | 0   |
|           | 3        | 0.2   | 0.07  | 0.48  | 0.22  | 0.03  | 0   |
| 15        | 1        | 0.21  | 0.11  | 0.36  | 0.22  | 0.1   | 0   |
|           | 2        | 0.21  | 0.12  | 0.36  | 0.21  | 0.1   | 0   |
|           | 3        | 0.21  | 0.07  | 0.47  | 0.22  | 0.03  | 0   |
| 16        | 1        | 0.21  | 0.15  | 0.36  | 0.22  | 0.06  | 1   |
|           | 2        | 0.17  | 0.15  | 0.36  | 0.22  | 0.1   | 1   |
|           | 3        | 0.14  | 0.14  | 0.48  | 0.21  | 0.03  | 1   |
| 17        | 1        | 0.21  | 0.07  | 0.41  | 0.21  | 0.1   | 1   |
|           | 2        | 0.14  | 0.15  | 0.46  | 0.22  | 0.03  | 1   |
|           | 3        | 0.177727 | 0.116364 | 0.422727 | 0.215455 | 0.067727 | 1   |
| 18        | 1        | 0.21  | 0.15  | 0.39  | 0.22  | 0.03  | 1   |
|           | 2        | 0.21  | 0.15  | 0.36  | 0.21  | 0.07  | 1   |
|           | 3        | 0.21  | 0.11  | 0.36  | 0.22  | 0.1   | 1   |
| 19        | 1        | 0.14  | 0.13  | 0.48  | 0.22  | 0.03  | 1   |
|           | 2        | 0.2   | 0.07  | 0.48  | 0.22  | 0.03  | 1   |
|           | 3        | 0.14  | 0.15  | 0.4   | 0.21  | 0.1   | 1   |
| 20        | 1        | 0.21  | 0.12  | 0.36  | 0.21  | 0.1   | 1   |
|           | 2        | 0.18  | 0.15  | 0.36  | 0.21  | 0.1   | 1   |
|           | 3        | 0.14  | 0.07  | 0.47  | 0.22  | 0.1   | 1   |
| 21        | 1        | 0.14  | 0.07  | 0.48  | 0.21  | 0.1   | 1   |
|           | 2        | 0.14  | 0.15  | 0.47  | 0.21  | 0.03  | 1   |
|           | 3        | 0.21  | 0.07  | 0.4   | 0.22  | 0.1   | 1   |
| 22        | 1        | 0.14  | 0.15  | 0.39  | 0.22  | 0.1   | 1   |
|           | 2        | 0.21  | 0.07  | 0.48  | 0.21  | 0.03  | 1   |
|           | 3        | 0.14  | 0.07  | 0.48  | 0.22  | 0.09  | 1   |
| 23        | 1        | 0.14  | 0.07  | 0.48  | 0.21  | 0.1   | 1   |
### 7. Conclusion

The Steel Slag Concrete Experiment was run using a split plot mixture process variable design with 23 whole plots of three observations. The final design consisted of \{8 7 8\} and experiment design with 69 observations in total. The design of efficient small experiments involving MPV is a difficult problem because the number of observations tends to be larger as the number of process variables increases.

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