The Indispensability of Ghost Fields in the Light-Cone Gauge Quantization of Gauge Fields

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Abstract

We continue McCartor and Robertson’s recent demonstration of the indispensability of ghost fields in the light-cone gauge quantization of gauge fields. It is shown that the ghost fields are indispensable in deriving well-defined antiderivatives and in regularizing the most singular component of gauge field propagator. To this end it is sufficient to confine ourselves to noninteracting abelian fields. Furthermore to circumvent dealing with constrained systems, we construct the temporal gauge canonical formulation of the free electromagnetic field in auxiliary coordinates \( x^\mu = (x^-, x^+, x^1, x^2) \) where \( x^- = x^0 \cos \theta - x^3 \sin \theta, \quad x^+ = x^0 \sin \theta + x^3 \cos \theta \) and \( x^- \) plays the role of time. In so doing we can quantize the fields canonically without any constraints, unambiguously introduce ”static ghost fields” as residual gauge degrees of freedom and construct the light-cone gauge solution in the light-cone representation by simply taking the light-cone limit (\( \theta \to \frac{\pi}{4} \)). As a by product we find that, with a suitable choice of vacuum the Mandelstam-Leibbrandt form of the propagator can be derived in the \( \theta = 0 \) case (the temporal gauge formulation in the equal-time representation).
§1. Introduction

Recently the search for nonperturbative solutions of QCD has led to an extensive explo-
ration of light-front field theory (LFFT), in which the infinite-momentum limit is incorpo-
rated by the change of variables
\[ x^+_i = \frac{x^0 + x^3}{\sqrt{2}}, \quad x^-_i = \frac{x^0 - x^3}{\sqrt{2}} \]
so that one is able to have vacuum state composed only of particles with nonnegative
longitudinal momentum and also to have relativistic bound-state equations of Schrödinger-type. For a good overview of LFFT see ref.2)

A fundamental problem is to specify the antiderivatives which arise in relating
constrained fields to the true degrees of freedom of the system. Quantization has tradi-
tionally been carried out in parallel with the axial gauge formulation of QED in the
ordinary space-time coordinates. Thus \( x^+_i \) and \( A^a_0 = A^a_3 = \) 0 have been chosen respectively to be the
evolution parameter and the gauge fixing condition and the Gauss’ law constraint has been
solved by using the operator \( (\partial^-_l)^{-1} \) to express Hamiltonian in terms of the physical
degrees of freedom.3) Unfortunately, the quantity \( ((\partial^-_l)^{-1})^2 \), which is needed for that
construction, turns out to be not well defined.4) In fact if one defines \( (\partial^-_l)^{-1} \) by
\[
(\partial^-_l)^{-1} f(x^-_i) = \frac{1}{2} \int_{-\infty}^{\infty} dy^-_i \epsilon(x^-_i - y^-_i) f(y^-_i), \tag{1.1}
\]
then \( ((\partial^-_l)^{-1})^2 \) is divergent.5) Thus one subtracts divergent terms by hand to define \( (\partial^-_l)^{-2} \) by
\[
(\partial^-_l)^{-2} f(x^-_i) = \frac{1}{2} \int_{-\infty}^{\infty} dy^-_i |x^-_i - y^-_i| f(y^-_i). \tag{1.2}
\]
Dirac’s canonical quantization procedure can not resolve this difficulty because one has to
make use of the same operator \( (\partial^-_l)^{-1} \) to define inverse of the constraints matrix.6)

Furthermore, if (1.1) is used to define the inverse derivative, the spurious singulari-
ty at \( n \cdot k = 0 \) of the gauge field propagator
\[
D^{ab}_{\mu\nu}(k) = \frac{i\delta^{ab}}{k^2 + i\epsilon} (-g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{n \cdot k}) \tag{1.3}
\]
is necessarily defined as the principal value (PV); but evaluating the singularity as the
PV generates extra contributions so that light-cone gauge calculations do not agree with
those performed in covariant gauges.7) To overcome the latter difficulty the Mandelstam-
Leibbrandt (ML) prescription has been proposed so as not to generate extra contributions
and give results consistent with Feynman propagators.8) Bassetto et al derived the ML
prescription in the light-cone gauge using a canonical operator formalism in the ordinary
space-time coordinates and found that ghost fields associated with Lagrange multiplier
fields are essential to the derivation.9)
Since there exists an operator solution which has the ML form of the propagator, one
expects that a consistent operator formulation of LFFT can also be constructed by intro-
ducing the ghost fields as residual gauge degrees of freedom. However, since the residual
gauge functions in the light-cone gauge formulation are ones depending on \( x_1^+ \), \( x^1 \) and \( x^2 \),
one cannot expect to calculate the dynamical operators by integrating densities over the
three dimensional hyperplanes \( x_1^+ \) = constant. Recently these problems were studied by one
of the authors of the present paper (McCartor) and Robertson in the light-cone formulation
of QED.\(^\text{10}\) They found that the ghost fields can be introduced in such a way that the trans-
lational generator \( P^l \) consists of physical degrees of freedom integrated over the hyperplane
\( x_1^+ \) = constant and ghost degrees of freedom integrated over the hyperplane \( x_1^- \) = constant.
They also found that the ghost fields have to be initialized along a hyperplane \( x_1^- \) = constant,
while physical fields evolve from the usual hyperplane \( x_1^+ \) = constant. The same problems
were considered by Morara and Soldati\(^\text{11}\), who constructed the light-cone temporal gauge
formulation, where all fields evolve from a single initial value surface and the ML form of
the propagator is realized.

In this paper we further investigate how the ghost fields fulfill roles as regulator fields
in the light-cone gauge formulation of gauge theories. To avoid inessential complications
and to circumvent dealing with constrained systems, we confine ourselves to noninteracting
abelian fields — the free electromagnetic fields — and construct the canonical operator
solution of them in the gauge \( A_- = A^0 \cos \theta + x^3 \sin \theta = 0 \) and in the auxiliary coordinates
\( x^\mu = (x^-, x^+, x^1, x^2) \) where
\[
- x^- = x^0 \cos \theta - x^3 \sin \theta, \quad x^+ = x^0 \sin \theta + x^3 \cos \theta. \quad (1.4)
\]
The same framework was also used by Hornbostel to analyze two-dimensional models.\(^\text{12}\) In
doing so we can choose \( x^- \) as the evolution parameter in the interval \( 0 \leq \theta < \frac{\pi}{4} \) and construct
the temporal gauge formulation, where canonical quantization conditions are to be imposed
without any constraints. Furthermore we can take \( x^+ \) to be the evolution parameter in
the interval \( \frac{\pi}{4} < \theta \leq \frac{\pi}{2} \) and thus construct the axial gauge formulation. Consequently we
can expect that taking the light-cone limit \( \theta \to \frac{\pi}{4} \) will enable us to find light-cone gauge
solutions and to compare the temporal light-front limit (\( \theta \to \frac{\pi}{4} - 0 \)) with the axial light-front
limit (\( \theta \to \frac{\pi}{4} + 0 \)).

In sect. 2 the temporal gauge canonical operator solution is constructed in the auxiliary
coordinates and it is shown that static ghost fields are present as residual gauge degrees
of freedom. We also encounter the problem that canonical commutation relations can not
distinguish the PV and ML prescriptions\(^\text{13}\) but that the ML prescription can always be
obtained if we employ an appropriate representation of the ghost fields. As a consequence
we find that the ML prescription can always be implemented even in the $\theta = 0$ case, namely in the temporal gauge formulation in the ordinary space-time coordinates. We also point out that the axial gauge formulation is not straightforward to construct, at least by these procedures.

In sect. 3 we show that the free electromagnetic fields given as the temporal light-cone limit are identical with the ones given by McCartor and Robertson and by Morara and Soldati and equivalent to those given by Bassetto et al in the ordinary space-time coordinates. We also show in detail that in the light-cone gauge formulation the ghost fields are indispensable to well-defined antiderivatives and that linear divergences are eliminated in what is otherwise the most singular component of $x_+^+$ ordered propagator.

Sect. 4 is devoted to concluding remarks.

§2. Temporal Gauge Formulation in the Auxiliary Coordinates

We begin by fixing the metric of the auxiliary coordinates $x^\mu = (x^-, x^+, x^1, x^2)$ where $x^-$ and $x^+$ are defined by

$$x^- = x^0 \cos \theta - x^3 \sin \theta, \quad x^+ = x^0 \sin \theta + x^3 \cos \theta. \quad (2\cdot1)$$

Inverting these, we find that $x^0$ and $x^3$ are given by

$$x^0 = x^- \cos \theta + x^+ \sin \theta, \quad x^3 = -x^- \sin \theta + x^+ \cos \theta \quad (2\cdot2)$$

so that $(x^0)^2 - (x^3)^2$ is expressed in terms of $x^-$ and $x^+$ as

$$(x^0)^2 - (x^3)^2 = x^- (x^- \cos 2\theta + x^+ \sin 2\theta) + x^+ (x^- \sin 2\theta - x^+ \cos 2\theta). \quad (2\cdot3)$$

Rewriting this in the form $x^- x_- + x^+ x_+$ requires that $x_-$ and $x_+$ are defined respectively by

$$x_- = x^- \cos 2\theta + x^+ \sin 2\theta, \quad x_+ = x^- \sin 2\theta - x^+ \cos 2\theta. \quad (2\cdot4)$$

It follows from this that

$$g_-- = \cos 2\theta, \quad g_-+ = g_+ = \sin 2\theta, \quad g_++ = -\cos 2\theta. \quad (2\cdot5)$$

Upper components of metric tensor are obtained by inverting (2·4)

$$g^{--} = \cos 2\theta, \quad g^{-+} = g^{+--} = \sin 2\theta, \quad g^{++} = -\cos 2\theta. \quad (2\cdot6)$$

Substituting (2·1) into (2·4) enables us to express $x_-$ and $x_+$ in terms of $x^0$ and $x^3$ as follows

$$x_- = x^0 \cos \theta + x^3 \sin \theta, \quad x_+ = x^0 \sin \theta - x^3 \cos \theta. \quad (2\cdot7)$$
Now we notice from (2.5) and (2.6) that if we keep \( \theta \) in the interval \( 0 \leq \theta < \frac{\pi}{4} \), we can employ \( x^- \) as an evolution parameter, whereas in the interval \( \frac{\pi}{4} < \theta \leq \frac{\pi}{2} \) we can employ \( x^+ \) as an evolution parameter. Thus we expect that fixing \( \theta \) to be \( \frac{\pi}{4} \) will allow us to have either \( x^- \) or \( x^+ \) as evolution parameters.

2.1. Temporal Gauge Quantization

To construct the temporal gauge formulation in the auxiliary coordinates, we choose

\[
A^- = A_0 \cos \theta - A_3 \sin \theta = 0
\]

as the gauge fixing condition. Accordingly we consider the Lagrangian

\[
L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - B (n \cdot A)
\]

where \( F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) with \( \partial_\mu = (\frac{\partial}{\partial x^-}, \frac{\partial}{\partial x^+}, \frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}) \), \( n^\mu = (n^-, n^+, n^1, n^2) = (1, 0, 0, 0) \) and \( B \) is a Lagrange multiplier field. From (2.8) we can derive the field equations

\[
\partial_\mu F^{\mu \nu} = \Box A^\nu - \partial^\nu (\partial^\mu A_\mu) = n^\nu B
\]

and the gauge fixing condition

\[
A^- = 0.
\]

The field equation of \( B \),

\[
\partial_+ B = 0
\]

is obtained by multiplying (2.9) by \( \partial_\nu \). Lowering the indices of (2.9) and observing that \( n_\mu = (\cos 2\theta, \sin 2\theta, 0, 0) \) and \( A^- = 0 \) enable us to derive the field equation of \( A^- \) as

\[
\partial_-(\partial^\mu A_\mu) + \cos 2\theta B = 0.
\]

Then, differentiating this by \( \partial_- \) gives rise to the following equation for \( \partial^\mu A_\mu \)

\[
\partial_-^2 (\partial^\mu A_\mu) = 0.
\]

Consequently, upon multiplying (2.9) by \( \partial_-^2 \) we obtain the following equation for \( A_\mu \)

\[
\Box (\partial_-^2 A_\mu) = 0.
\]

From (2.8) we can also obtain the canonical conjugate momenta

\[
\pi^- = 0, \quad \pi^+ = -F^- + \frac{\sin 2\theta F_{+i}}{\cos 2\theta}, \quad \pi_i = -F^- - F^i, \quad \pi_B = 0.
\]

It should be noticed that we have three pairs of canonical variables, in contrast with one pair in the light-cone temporal gauge formulation. The fields \( B, \partial_- A_+ \) and \( \partial_- A_i \) \((i = 1, 2)\) are expressed in terms of canonical variables as follows

\[
B = \partial_+ \pi^+ + \sum_{i=1}^2 \partial_i \pi^i, \quad \partial_- A_+ = \pi^+, \quad \partial_- A_i = \frac{\pi^i - \sin 2\theta F_{+i}}{\cos 2\theta}.
\]
Thus we can impose the following equal-time canonical quantization conditions

\[ [A_r(x), A_s(y)] = 0, \quad [A_r(x), \pi^s(y)] = i\delta_{rs} \delta^{(3)}(x^+ - y^+), \quad [\pi^r(x), \pi^s(y)] = 0 \quad (2.17) \]

where \( r, s = +, 1, 2, \) \( \delta^{(3)}(x^+ - y^+) \equiv \delta(x^1 - y^1)\delta(x^2 - y^2)\delta(x^+ - y^+) \) and in all cases \( x^- = y^- \).

To obtain 4-dimensional commutation relations of \( A_\mu \) we express \( A_\mu \) in an integral form.\(^{14}\)

This is done most easily by making use of the commutator function of a free massless field

\[ D(x) = \frac{-i}{2(2\pi)^3} \int \frac{d^3k_+}{k^-} (e^{-ik\cdot x} - e^{ik\cdot x}) \quad (2.18) \]

and a commutator function \( E(x) \) satisfying the equation

\[ \Box E(x) = -x^- \delta^{(3)}(x^+) \equiv D_s(x), \quad \partial^- E(x) = D(x) \quad (2.19) \]

and the initial conditions

\[ E(x)|_{x^- = 0} = 0, \quad \partial_- E(x)|_{x^- = 0} = 0, \quad \partial_+^2 E(x)|_{x^- = 0} = 0, \quad \partial_- \partial_+^2 E(x)|_{x^- = 0} = -\delta^{(3)}(x^+). \quad (2.20) \]

Note that \( D(x) \) satisfies a free massless D’Alembert’s equation, which imposes the following on-shell condition on four momentum \( k_\mu \)

\[ 0 = k^2 = n^2k_\perp + 2\sin2\theta k_+ k_- - k_1^2 - k_2^2 - n^2k_+^2, \quad (n^2 = \cos2\theta). \quad (2.21) \]

We solve this in such a way that \( k_- \) is expressed in terms of \( k_1, k_2, k_+ \) as follows

\[ k_- = \frac{k_1^2 + n^2k_+^2}{k^- + \sin2\theta k_+} \quad (2.22) \]

where

\[ k_\perp = \sqrt{k_1^2 + k_2^2}, \quad k^- = \sqrt{k_+^2 + n^2k_\perp^2}. \quad (2.23) \]

Also note that \( d^3k_+ \) denotes \( dk_1dk_2dk_+ \) and the integration region of \( k_+ \) is \((-\infty, \infty)\).

We find that the function \( E(x) \) is given by

\[ E(x) = \frac{1}{\partial_-^2} D(x) - \frac{1}{\partial_\perp^2 + n^2\partial_+^2} D_s(x) - \frac{\sin2\theta \partial_+ + \partial^-}{(\partial_\perp^2 + n^2\partial_+^2)^2} \partial_- D_s(x) \quad (2.24) \]

where

\[ \frac{1}{\partial_-} \equiv \frac{\sin2\theta \partial_+ + \partial^-}{\partial_\perp^2 + n^2\partial_+^2}, \quad \partial_\perp^2 \equiv \partial_1^2 + \partial_2^2. \quad (2.25) \]

Now we can express \( A_\mu \) in the following integral form

\[ A_\mu(x) = \int d^3z [\partial^- D(x-z)A_\mu(z) - D(x-z)\partial^- A_\mu(z) + \partial^2 E(x-z)\Box A_\mu(z) - E(x-z)\Box \partial_- A_\mu(z)] \quad (2.26) \]

(Continues on the next page)
where $d^3z^+$ denotes $dz^1dz^2dz^+$. It can easily be shown that the integral form satisfies the field equation $\Box(\partial_\mu^2 A_\mu) = 0$ and is independent of $z^-$ so that it satisfies the initial conditions at $z^- = x^-$. Furthermore, we can utilize the latter property to calculate its 4-dimensional commutation relations using only equal-time canonical commutation relations. It turns out that

$$[A_\mu(x), A_\nu(y)] = i\{-g_{\mu\nu}D(x-y) + (n_\mu \partial_\nu + n_\nu \partial_\mu)\partial_- E(x-y) - n^2 \partial_\mu \partial_\nu E(x-y)\}. \quad (2.27)$$

From (2.12), $B$ is expressed as

$$B = -\frac{\partial_- (\partial^\mu A_\mu)}{\cos 2\theta} \quad (2.28)$$

so the commutation relations of $B$ are obtained from (2.27) as follows

$$[B(x), A_\nu(y)] = -i\partial_\nu \delta^{(3)}(x^+ - y^+), \quad [B(x), B(y)] = 0. \quad (2.29)$$

2.2. Constituent fields in the Temporal Gauge Formulation

To obtain constituent fields in the temporal gauge formulation, we solve the field equations (2.9). We multiply (2.9) with $\nu = i$ ($i = 1, 2$) by $\partial_i$ and sum over $i = 1, 2$. Consequently we obtain

$$\partial^\mu A_\mu = \frac{\Box}{\partial^2} \sum_{i=1}^{2} \partial_i A_i \quad (2.30)$$

so that (2.9) is rewritten as

$$\Box(A_\mu - \frac{\partial^\mu}{\partial^2} \sum_{i=1}^{2} \partial_i A_i) = n_\mu B. \quad (2.31)$$

Because $B$ satisfies $\partial_- B = 0$, any solution of (2.31) is described as

$$A_\mu - \frac{\partial_\mu}{\partial^2} \sum_{i=1}^{2} \partial_i A_i = a_\mu - \frac{n_\mu}{\partial^2 + n^2 \partial^2} B \quad (2.32)$$

where $a_\mu$ is a homogeneous solution. As a condition that $A_\mu$ in (2.32) satisfies (2.30), we obtain

$$\partial^\mu a_\mu = 0. \quad (2.33)$$

Furthermore multiplying (2.32) with $\mu = i$ ($i = 1, 2$) by $\partial_i$ and summing over $i = 1, 2$ yields another consistency condition

$$\sum_{i=1}^{2} \partial_i a_i = 0. \quad (2.34)$$

To obtain $\sum_{i=1}^{2} \partial_i A_i$, we impose the gauge fixing condition on (2.32) with $\mu = -$ and integrate it with respect to $x^-$. It should be noted here that we can introduce a static field,
which we denote $C$, as an integration constant. For later convenience we introduce it in the following way
\[- \frac{1}{\partial_{\perp}^2} \sum_{i=1}^{2} \partial_i A_i = \frac{1}{\partial_{\perp}^-} a_- - \frac{n_- x^-}{\partial_{\perp}^2 + n^2 \partial_{\perp}^2} B + \frac{1}{\partial_{\perp}^2 + n^2 \partial_{\perp}^2} (C - n^2 \sin 2\theta \partial_{\perp}^+ B). \tag{2.35}\]
On substituting (2.35) into (2.32) we have
\[A_\mu = a_\mu - \frac{\partial_\mu}{\partial_-} a_- + \Gamma_\mu\tag{2.36}\]
where
\[\Gamma_\mu = -\frac{n_\mu}{\partial_{\perp}^2 + n^2 \partial_{\perp}^2} B - \frac{\partial_\mu}{\partial_{\perp}^2 + n^2 \partial_{\perp}^2} (C - n^2 x^- B - n^2 \sin 2\theta \partial_{\perp}^+ B). \tag{2.37}\]
Now that we have obtained the constituent fields $a_\mu$, $B$ and $C$, we proceed to derive the commutation relations they satisfy. It is straightforward to accomplish if we express them in terms of the canonical variables as follows
\[a_r = A_r - \frac{\partial_r}{\partial_{\perp}^2} (\sum_{i=1}^{2} \partial_i A_i) + \frac{n_r}{\partial_{\perp}^2 + n^2 \partial_{\perp}^2} B, \quad (r = +, 1, 2) \tag{2.38}\]
\[B = \partial_+ \pi^+ + \sum_{i=1}^{2} \partial_i \pi^i, \tag{2.39}\]
\[\partial^- a_i = \pi^i - \frac{\partial_i}{\partial_{\perp}^2} (\sum_{j=1}^{2} \partial_j \pi^j), \quad (i = 1, 2) \tag{2.40}\]
\[\partial^- a_+ = n_- \pi^+ - \frac{\partial_+}{\partial_{\perp}^2} (\sum_{i=1}^{2} \partial_i \pi^i) + \frac{n_+^2}{\partial_{\perp}^2 + n^2 \partial_{\perp}^2} \partial_+ B, \tag{2.41}\]
\[C = n_+ \pi^+ - n_\perp \partial_+ A_+ - \sum_{i=1}^{2} \partial_i A_i + n^2 x^- B - \frac{n_+^2 n_-}{\partial_{\perp}^2 + n^2 \partial_{\perp}^2} \partial_+ B. \tag{2.42}\]
We find that $a_r, \partial^- a_r, B, C$ are fundamental operators, satisfying the following commutation relations
\[[a_r(x), a_s(y)]_{x^- = y^-} = [\partial^- a_r(x), \partial^- a_s(y)]_{x^- = y^-} = 0, \quad (r, s = +, 1, 2) \tag{2.43}\]
\[[a_+(x), \partial^- a_+(y)]_{x^- = y^-} = i \frac{\partial_{\perp}^2 + n^2 \partial_{\perp}^2}{\partial_{\perp}^2} \delta^{(3)}(x^+ - y^+), \tag{2.44}\]
\[[a_i(x), \partial^- a_j(y)]_{x^- = y^-} = i (\delta_{ij} - \frac{\partial_i \delta_{ij}}{\partial_{\perp}^2}) \delta^{(3)}(x^+ - y^+), \quad (i, j = 1, 2) \tag{2.45}\]
\[[B(x), B(y)] = [C(x), C(y)] = 0, \tag{2.46}\]
\[[B(x), C(y)] = -[C(x), B(y)] = -i (\partial_{\perp}^2 + n^2 \partial_{\perp}^2) \delta^{(3)}(x^+ - y^+). \tag{2.47}\]
Any other commutators among $a_r, \partial^- a_r, B$ or $C$ are zero.
2.3. Expression of the Constituent fields and Implementation of the ML prescription

Now we can express the constituent fields in terms of creation and annihilation operators. First of all we notice that the free massless field \(a_+\) satisfies the commutation relation (2.44) with the operator \(\partial^2 + n^2 \partial^2 \), which in effect multiplies the integrand of the Fourier expanded free massless fields by \((k^-)^2\). Thus we express \(a_+\) in the form

\[
a_+(x) = \frac{-1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \{a_1(k_+)e^{-ik\cdot x} + a_1^\dagger(k_+)e^{ik\cdot x}\}. \tag{2.48}
\]

Here the factor \(-1\) is included for later convenience and \(k_+\) denotes the three-component vector \((k^+, k_1, k_2)\). The expression of \(a_-\) is obtained by solving the constraint \(\partial^+a_+ + \partial^-a_- = 0\) and hence we have

\[
a_-(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \{a_1(k_+)e^{-ik\cdot x} + a_1^\dagger(k_+)e^{ik\cdot x}\}. \tag{2.49}
\]

Similarly, because \(a_i\) satisfies the constraint \(\sum_{i=1}^2 \partial_i a_i = 0\) and the commutation relation (2.45), we have

\[
a_i(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \{a_2(k_+)e^{-ik\cdot x} + a_2^\dagger(k_+)e^{ik\cdot x}\} \tag{2.50}
\]

where \(\epsilon^{(2)}_i(k)\) is a physical polarization vector given by

\[
\epsilon^{(2)}_\mu(k) = (0, 0, \frac{k_2}{k_\perp}, \frac{-k_1}{k_\perp}). \tag{2.51}
\]

The operators \(a_\lambda(k_+)\) and \(a_\lambda^\dagger(k_+)\) \((\lambda = 1, 2)\) are normalized so as to satisfy the usual commutation relations

\[
[a_\lambda(k_+), a_{\lambda'}(q_+)] = 0, \quad [a_\lambda(k_+), a_\lambda^\dagger(q_-)] = \delta_{\lambda\lambda'}\delta^{(3)}(k_+ - q_+). \tag{2.52}
\]

They are nothing but annihilation and creation operators of physical photons, as we see from the fact that, with the help of (2.51) and another physical polarization vector

\[
\epsilon^{(1)}_\mu(k) = (0, -\frac{k_1}{k_-}, -\frac{k_2}{k_-}, \frac{k^+}{k_-}), \tag{2.53}
\]

we can express \(a_\mu - \frac{\partial_\mu}{\partial^-}a_-\) in the following compact form

\[
a_\mu(x) - \frac{\partial_\mu}{\partial^-}a_-(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \sum_{\lambda=1}^2 \epsilon^{(\lambda)}_\mu(k) \{a_\lambda(k_+)e^{-ik\cdot x} + h.c.\}. \tag{2.54}
\]
Note that the polarization vectors satisfy

\[ k^\mu \epsilon_\mu^{(\lambda)}(k) = 0, \quad n^\mu \epsilon_\mu^{(\lambda)}(k) = 0, \quad (\lambda = 1, 2) \]  \hspace{1cm} (2.55)

\[ \sum_{\lambda=1}^{2} \epsilon_\mu^{(\lambda)}(k) \epsilon_\nu^{(\lambda)}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{k_-} - n_\mu k_\mu k_\nu. \]  \hspace{1cm} (2.56)

Let us next determine expressions for \( B \) and \( C \). We expand \( B \) in terms of conjugate zero-norm creation and annihilation operators \( b(k_+) \), \( b^\dagger(k_+) \). In so doing we can realize Gauss’ law in physical space specified below. Similarly, we expand \( C \) in terms of conjugate zero-norm creation and annihilation operators \( c(k_+) \), \( c^\dagger(k_+) \). It seems at first sight that there arises no problem in expressing \( B \) and \( C \) in terms of the zero-norm operators, because one can employ the following naive expressions for the static fields

\[ B(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{|k_+|} \{b(k_+)e^{-ik\cdot x} + b^\dagger(k_+)e^{ik\cdot x}\}_{x^-=0}, \]  \hspace{1cm} (2.57)

\[ C(x) = \frac{i}{\sqrt{2(2\pi)^3}} \int d^3k_+ \sqrt{|k_+|} \{c(k_+)e^{-ik\cdot x} - c^\dagger(k_+)e^{ik\cdot x}\}_{x^-=0}, \]  \hspace{1cm} (2.58)

where

\[ [b(k_+), c^\dagger(q_+)] = [c(k_+), b^\dagger(q_+)] = -\delta^{(3)}(k_+ - q_+). \]  \hspace{1cm} (2.59)

any other commutators being zero. However, we encounter the problem pointed out by Haller\(^{13}\), namely the problem that the canonical commutation relations can not distinguish the PV and ML prescriptions. In fact if we take the \( k_+ \)-integration region to be the whole interval \((-\infty, \infty)\), then we are obliged to have the PV form of propagator. Judging from the fact that the ML form of propagator has to be employed in the light-cone limit and from indications that the PV form of propagator does not lead to the correct behavior of the Wilson loop in perturbative calculations,\(^{15}\) but the ML form of propagator does,\(^{16}\) we conclude that the correct form of temporal gauge free theory from which to start perturbative calculations should perhaps have the ML prescription in the temporal gauge free propagator. So far extensions of the ML prescription outside the light-cone gauge formulation have been done by limiting the \( k_+ \)-integration region by hand to be \((0, \infty)\).\(^{17}\)

We solve this problem by introducing an inequivalent Fock space. We can rewrite \( B \) and \( C \) in such a way that \( k_+ \)-integrations are carried over the interval \((0, \infty)\) as in the following

\[ B(x) = \frac{1}{\sqrt{(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \theta(k_+) \{B(k_+)e^{-ik\cdot x} + B^\dagger(k_+)e^{ik\cdot x}\}_{x^-=0}, \]  \hspace{1cm} (2.60)

\[ C(x) = \frac{i}{\sqrt{(2\pi)^3}} \int d^3k_+ \sqrt{k_+} \{C(k_+)e^{-ik\cdot x} - C^\dagger(k_+)e^{ik\cdot x}\}_{x^-=0} \]  \hspace{1cm} (2.61)
where
\[ B(k_+) = \frac{b(k_+) + b^\dagger(-k_+)}{\sqrt{2}}, \quad B^\dagger(k_+) = \frac{b^\dagger(k_+) + b(-k_+)}{\sqrt{2}}, \]  
\[ C(k_+) = \frac{c(k_+) - c^\dagger(-k_+)}{\sqrt{2}}, \quad C^\dagger(k_+) = \frac{c^\dagger(k_+) - c(-k_+)}{\sqrt{2}}. \]  

Now we see that \( B(k_+) \) and \( C(k_+) \) are nothing but canonical transformations, which are generated as follows
\[ B(k_+) = e^{A}b(k_+)e^{-A}, \quad B^\dagger(k_+) = e^{A}b^\dagger(k_+)e^{-A}, \]  
\[ C(k_+) = e^{A}c(k_+)e^{-A}, \quad C^\dagger(k_+) = e^{A}c^\dagger(k_+)e^{-A}, \]  

where
\[ A = \frac{\pi}{4} \int d^3k_+ \theta(k_+) \{ b(k_+)c(-k_+) - b(-k_+)c(k_+) - b^\dagger(k_+)c^\dagger(-k_+) + b^\dagger(-k_+)c^\dagger(k_+) \}. \]  

Thus, if we define the physical vacuum state \(|\Omega>\) by
\[ |\Omega> = e^{A}|0> \]  
where \(|0>\) is the bare vacuum state satisfying
\[ b(k_+)|0> = c(k_+)|0> = 0, \]  
then it is easy to show that
\[ B(k_+)|\Omega> = C(k_+)|\Omega> = 0. \]  

Thus physical space \( V_P \) is defined by
\[ V_P = \{ \text{phys} | B(k_+)|\text{phys}>=0 \}. \]  

In this way we can always obtain the ML prescription in the physical space. To be complete we note that the \( x^- \)-ordered propagator
\[ D^-_{\mu\nu}(x-y) = \Omega|\{ \theta(x^- - y^-)A_\mu(x)A_\nu(y) + \theta(y^- - x^-)A_\nu(y)A_\mu(x) \}|\Omega> \]  
results in the ML form of propagator as follows
\[ D^-_{\mu\nu}(x-y) = \frac{1}{(2\pi)^4} \int d^4kD^-_{\mu\nu}(k)e^{-ik(x-y)} \]  
where
\[ D^-_{\mu\nu}(k) = \frac{i}{k^2 + i\epsilon} \{ -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{k_- + i\epsilon\text{sgn}(k_+)} - \frac{n^2 k_\mu k_\nu}{(k_- + i\epsilon\text{sgn}(k_+))^2} \}. \]
It is known that for interacting theories the ML form of propagator has to be employed in the light-cone limit $\left( \theta \to \frac{\pi}{4} \right)$. Therefore it is interesting to investigate whether it is also true of the $\theta = 0$ case, that is to say, the temporal gauge formulation in the ordinary space-time coordinates.

Translation generators $P_\mu = \int d^3x^+ T^-_\mu(x)$ are given in terms of the creation and annihilation operators of the constituent fields as follows

$$P_- = \int d^3k_+ \left\{ 2 \sum_{\lambda=1}^{2} k_- a_\lambda^\dagger(k_+) a_\lambda(k_+) + n^2 \frac{k_+^2}{k_+} \theta(k_+) B^\dagger(k_+) B(k_+) \right\}, \quad (2.74)$$

$$P_r = \int d^3k_+ k_r \left\{ 2 \sum_{\lambda=1}^{2} a_\lambda^\dagger(k_+) a_\lambda(k_+) - \theta(k_+) \left( B^\dagger(k_+) C(k_+) + C^\dagger(k_+) B(k_+) \right) \right\}, \quad (2.75)$$

where $r = +, 1, 2$.

We close this section by making two remarks. First, that the Heisenberg equation of $C$ is not satisfied, as is seen from

$$[P_-, C(x)] = i n^2 B(x) \quad (2.76)$$

but that this is necessary to assure the Heisenberg equations of $A_\mu$

$$[P_-, A_\mu(x)] = -i \partial_- A_\mu(x). \quad (2.77)$$

Second, that we have not attempted constructing the axial gauge formulation in the auxiliary coordinates. This is because we have not succeeded in finding any solutions other than (2.36). It seems to us that (2·36) is not an appropriate solution in the axial gauge formulation because $x^-$ is included explicitly and because the inverse Laplace operator $\frac{1}{\partial_{-}^2 + n^2 \partial_+}$ becomes singular.

§3. Light-Cone Gauge Quantizations in the Light-Front Coordinates

In this section we confine ourselves to the $\theta = \frac{\pi}{4}$ case, namely, to the formulation in the light-front coordinates $x^\mu = (x^-, x^+, x^1, x^2) = (\frac{\sqrt{2}}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{2}}; x^1, x^2)$ with metric tensors defined by

$$g_{11} = g_{22} = g^{11} = g^{22} = -1,$$

$$g_{-+} = g_{+-} = g^{-+} = g^{+-} = 1,$$

all other components = 0.

\(^a\) To avoid inessential complications, in this section we omit the suffix $l$ denoting quantities in the light-front coordinates and use the same notation as that used in sect.2.
First of all we recall that the Lagrangian \((2 \cdot 8)\) becomes singular in the light-front coordinates, where \(n_\mu = (0,1,0,0)\). In fact in the temporal gauge formulation with \(x^-\) being the evolution parameter, it happens that the canonical momenta \(\pi^i\) conjugate to \(A_i\) becomes noninvertible, as we seen from

\[
\pi^- = 0, \; \pi^+ = \partial_- A_+, \; \pi^i = \partial_+ A_i - \partial_i A_+, \; (i = 1, 2), \; \pi_B = 0. \tag{3.2}
\]

What is worse, in the axial gauge formulation with \(x^+\) being the evolution parameter, all momenta become noninvertible as in the following

\[
\pi^- = -\partial_- A_+, \; \pi^+ = 0, \; \pi^i = \partial_+ A_i, \; (i = 1, 2), \; \pi_B = 0. \tag{3.3}
\]

As to the temporal case, we can circumvent dealing with the constrained system because we can obtain canonically quantized free electromagnetic fields by simply taking the limit \(\theta \rightarrow \frac{\pi}{4} - 0\) of \((2 \cdot 36)\). As to the axial case, we can not follow the same approach because we have not succeeded in finding an appropriate axial gauge solution in the auxiliary coordinates. Nevertheless, we expect to get the axial light-front limit \((\theta \rightarrow \frac{\pi}{4} + 0)\) from the temporal light-front limit \((\theta \rightarrow \frac{\pi}{4} - 0)\). As a matter of fact, known light-cone gauge solutions are equivalent to each other, which is seen below.

In the temporal light-front limit \(n^2 = \cos 2\theta\) becomes zero and the mass-shell condition of the free massless fields is changed into \(2k_-k_+ - k_\perp^2 = 0\) so that the range of \(k_+\)-integration is reduced to \((0, \infty)\) in the Fourier expansions of the free massless fields. Consequently we obtain the following electromagnetic fields described in terms of the constituent fields

\[
A_\mu = a_\mu - \frac{\partial_\mu}{\partial_-}a_+ - \frac{n_\mu}{\partial_-}B - \frac{\partial_\mu}{\partial_-}C \tag{3.4}
\]

where

\[
a_+(x) = \frac{-1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \frac{k_\perp}{k_+} \theta(k_+) \{a_1(k_+)e^{-ik\cdot x} + a_1^\dagger(k_+)e^{ik\cdot x}\}, \tag{3.5}
\]

\[
a_-(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \frac{k_\perp}{k_+} \theta(k_+) \{a_1(k_+)e^{-ik\cdot x} + a_1^\dagger(k_+)e^{ik\cdot x}\}, \tag{3.6}
\]

\[
a_i(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \theta(k_+) \epsilon_1^{(2)}(k) \{a_2(k_+)e^{-ik\cdot x} + a_2^\dagger(k_+)e^{ik\cdot x}\}, \tag{3.7}
\]

\[
B(x) = \frac{1}{\sqrt{(2\pi)^3}} \int \frac{d^3k_+}{\sqrt{k_+}} \theta(k_+)k_\perp^2 \{B(k_+)e^{-ik\cdot x} + B^\dagger(k_+)e^{ik\cdot x}\}_{x^- = 0}, \tag{3.8}
\]

and

\[
C(x) = \frac{i}{\sqrt{(2\pi)^3}} \int d^3k_+ \theta(k_+) \sqrt{k_+} \{C(k_+)e^{-ik\cdot x} - C^\dagger(k_+)e^{ik\cdot x}\}_{x^- = 0}. \tag{3.9}
\]
We see immediately that (3.4) is identical with one given by McCartor and Robertson and by Morara and Soldati and is equivalent to the one given by Bassetto et al. Actually, the physical annihilation and creation operators \(a_\lambda(k)\) and \(a_\lambda^\dagger(k)\) in the ordinary space-time coordinates are identified with those in the light-front coordinates by

\[
a_\lambda(k) = \sqrt{\frac{k_+}{k_0}}a_\lambda(k_+), \quad a_\lambda^\dagger(k) = \sqrt{\frac{k_+}{k_0}}a_\lambda^\dagger(k_+), \quad (\lambda = 1, 2).
\]

Consequently our \(a_\mu - \frac{\partial_\mu}{\tau_\mu}a_-\) can be identified with \(T_\mu\) in the latter by changing the integration variable from \(k_+\) into \(k_3\). In addition our \(B\) and \(C\) can be identified with \(\lambda\) and \(U\) respectively in the latter if we identify our \(k_+\) with \(k_3\) in the latter.

Let us enumerate the properties of \(A_\mu\) in (3.4).

(1) It satisfies the following 4-dimensional commutation relations

\[
\{A_\mu(x), A_\nu(y)\} = i\{-g_{\mu\nu}D(x-y) + (n_\mu \partial_\nu + n_\nu \partial_\mu)\partial_- E(x-y)\},
\]

where

\[
D(x) = i\frac{-i}{2(2\pi)^3} \int \frac{d^3k}{k_+} \theta(k_+) \{e^{-ik_+x} - e^{ik_+x}\}
\]

and

\[
\partial_- E(x) = 2\partial_1 D(x) + \frac{1}{\partial^2_1} \delta^{(3)}(x^+).
\]

(2) It satisfies the following light-cone temporal gauge quantization conditions:

\[
\{A_+^i(x), A_+^j(y)\}|_{x^--y^-} = 0, \quad (i = 1, 2) \tag{3.14}
\]

\[
{\mathcal{A}}^i(x), A^j(y)\}|_{x^--y^-} = -\frac{i}{2} \delta_{ij} (\partial_\perp)^{-1} \delta^{(3)}(x^+ - y^+), \quad (i, j = 1, 2) \tag{3.15}
\]

\[
\{A^i_+(x), \pi^+(y)\}|_{x^--y^-} = i \delta^{(3)}(x^+ - y^+), \tag{3.16}
\]

\[
\{A^i_-(x), \pi^+(y)\}|_{x^--y^-} = i \frac{1}{2} \partial_\perp (\partial_\perp)^{-1} \delta^{(3)}(x^+ - y^+), \quad (i = 1, 2) \tag{3.17}
\]

\[
\{\pi^+(x), \pi^+(y)\}|_{x^--y^-} = i \frac{1}{2} \partial_\perp (\partial_\perp)^{-1} \delta^{(3)}(x^+ - y^+). \tag{3.18}
\]

(3) The \(x^-\)-ordered propagator results in the ML form of propagator

\[
\langle 0|\{\theta(x^- - y^-) A_\mu(x) A_\nu(y) + \theta(y^- - x^-) A_\nu(y) A_\mu(x)\}|0\rangle
\]

\[
= \frac{1}{(2\pi)^3} \int \frac{d^4k}{k^2 + i\epsilon} \{\frac{\partial_\mu}{\tau_\mu} \partial_- + n_\mu k_\nu + n_\nu k_\mu}{k_0 - k_3 + i\epsilon\text{sgn}(k_3)} e^{-ik_\cdot(x-y)}. \tag{3.19}
\]

(4) It satisfies the light-cone gauge quantization conditions in the ordinary space-time coordinates and the \(x^0\)-ordered propagator results in the ML form of propagator

\[
\langle 0|\{\theta(x^0 - y^0) A_\mu(x) A_\nu(y) + \theta(y^0 - x^0) A_\nu(y) A_\mu(x)\}|0\rangle
\]

\[
= \frac{1}{(2\pi)^3} \int \frac{d^4k}{k^2 + i\epsilon} \{\frac{\partial_\mu}{\tau_\mu} \partial_- + n_\mu k_\nu + n_\nu k_\mu}{k_0 - k_3 + i\epsilon\text{sgn}(k_3)} e^{-ik_\cdot(x-y)}. \tag{3.20}
\]

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(5) Translational generators can be given by integrating densities of the canonical energy-momentum tensor over the 3-dimensional hyperplane $x^- = \text{constant}$.

Let us next investigate whether the properties of the light-cone axial gauge formulation are satisfied by (3.4) or not. First we examine whether or not the light-cone axial gauge quantization conditions are satisfied by carefully evaluating the commutation relations (3.11) at $x^+ = y^+$. It will suffice to evaluate the commutator function $\partial_- E(x)$, which is rewritten as

$$\partial_- E(x) = -\frac{2}{(2\pi)^3} \int \frac{dk_1dk_2}{k_+^2} \exp[-i(k_1x^1 + k_2x^2)]I(x^-, x^+)$$  \hspace{1cm} (3.21)

where

$$I(x^-, x^+) = \int_0^\infty dk_+ \{ \cos k_+ x^+ - \cos(k_+ x^+ + \frac{k_+^2}{2k_+} x^-) \}. \hspace{1cm} (3.22)$$

Note that the first and the second terms result from the ghost and physical fields respectively and that they are ill-defined individually. It is shown in Appendix A that $I(x^-, x^+)$ is given by

$$I(x^-, x^+) = \frac{\pi}{2} \{ 1 + (x^-)e(x^+) \} \sqrt{\frac{|x^-|}{2|x^+|}} k_+ J_1(\sqrt{2|x^+x^-|} k_+) \hspace{1cm} (3.23)$$

where $J_1(x)$ is the Bessel function of order 1. From (3.23) we obtain

$$I(x^-, x^+)|_{x^+=0} = \frac{\pi}{4} |x^-| k_+^2, \hspace{1cm} \partial_+ I(x^-, x^+)|_{x^+=0} = \frac{\pi}{2} x^- \delta(x^+ k_+^2 - \frac{\pi}{16} x^- |x^-| k_+^4. \hspace{1cm} (3.24)$$

It follows that

$$[A_+(x), A_i(y)]|_{x^+=y^+} = i\partial_i \partial_- E(x-y)|_{x^+=y^+} = -\frac{i}{4} |x^- - y^-| \partial_i \delta(2)(x_- - y_-), \hspace{1cm} (3.25)$$

$$[A_+(x), A_+(y)]|_{x^+=y^+} = 2i\partial_+ \partial_- E(x-y)|_{x^+=y^+} = -i(x^- - y^-) \delta(3)(x^+ - y^+)$$

$$-\frac{i}{8} \epsilon(x^- - y^-)(x^- - y^-)^2 \partial_+ \delta(2)(x_- - y_-). \hspace{1cm} (3.26)$$

Furthermore the commutator $[A_i(x), A_j(y)]|_{x^+=y^+}$ is evaluated to be

$$[A_i(x), A_j(y)]|_{x^+=y^+} = i\delta_{ij} D(x-y)|_{x^+=y^+} = -\frac{i}{2} \delta_{ij} (\partial_-)^{-1} \delta(3)(x^- - y^-). \hspace{1cm} (i, j = 1, 2) \hspace{1cm} (3.27)$$

We see that these commutation relations agree with those given by Dirac’s canonical quantization procedure except the first term of (3.26), which results from the fact that $I(x^-, x^+)$ has a finite discontinuity at $x^+ = 0$.

Next we investigate $x^+$ ordered propagator

$$D^+_{\mu\nu}(x - y) = \Omega[\{ \theta(x^+ - y^+) A_{\mu}(x) A_{\nu}(y) + \theta(y^+ - x^+) A_{\nu}(y) A_{\mu}(x) \}] \Omega >$$

$$= \frac{1}{(2\pi)^4} \int d^4 q \ D^+_{\mu\nu}(q) e^{-iq(x-y)}. \hspace{1cm} (3.28)$$
Note that $B$ and $C$ are zero-norm fields so they have nonvanishing contributions for three cases, namely for $\mu = +, \nu = i; \mu = i, \nu = +$ and $\mu = \nu = +$. In case that $\mu = i$ and $\nu = j$ there arises no problem and we have

$$D_{ij}^+(q) = \frac{i\delta_{ij}}{q^2 + i\epsilon}. \quad (3.29)$$

When $\mu = +$ and $\nu = i$ or $\mu = i$ and $\nu = +$, we obtain

$$D_{ij}^+(q) = D_{ij}^+(q) = \frac{q_i}{q_-} \cdot \frac{i}{q^2 + i\epsilon} - \frac{iq_i}{q_+^2} (-i\pi) \text{sgn}(q_+) \delta(q_-)$$

$$= \frac{i}{q^2 + i\epsilon} \cdot \frac{q_i}{q_- + i\epsilon \text{sgn}(q_+)}. \quad (3.30)$$

We see that the second term, which is the contribution from the ghost fields, contributes the $\delta$ function part of the ML prescribed propagator.

In case that $\mu = \nu = +$, we obtain

$$D_{++}^+(q) = D_{++}^+(q) = \frac{q^2}{q_-^2} \cdot \frac{i}{q^2 + i\epsilon} - \frac{2\delta(q_-)}{q_-^2} \int_0^\infty dk_+ \left( \frac{ik_+}{q_- - k_+ + i\epsilon} - \frac{ik_+}{q_- + k_+ - i\epsilon} \right). \quad (3.31)$$

As was noticed by Morara and Soldati,\(^{1)}\) the second term diverges linearly. This implies that if the ghost fields really regularize $D_{++}^+(x - y)$, a linear divergence has to appear from the physical contribution so as to be canceled. As a matter of fact we see that if we rewrite the physical contribution as

$$\frac{2q_+}{q_-^2} \cdot \frac{i}{q^2 + i\epsilon} = \frac{2q_+}{q_-^2} \cdot \frac{i}{q^2 + i\epsilon} - \frac{i}{q_-^2}, \quad (3.32)$$

then the second term $\frac{i}{q_-^2}$ gives rise to a linear divergence as follows

$$\int_{-\infty}^\infty dq_- \frac{1}{q_-^2} e^{iq_+ - x_-} = 2 \int_0^\infty dq_- \frac{\cosq_+ - x_-}{q_-^2} = 2 \int_0^\infty dq_- \frac{1}{q_-^2} - \pi|x_-| \quad (3.33)$$

when $D_{++}^+(x)$ is restored by inverse Fourier transform. Furthermore by changing the integration variable from $k_+$ into $q_- = \frac{q^2}{2k_+}$, we see that it is canceled by the linear divergence due to the ghost fields as follows

$$\int_{-\infty}^\infty dq_- \frac{4i}{q_-^2} \delta(q_-)(\int_0^\infty dk_+ e^{-i q_- x_-} - 2i \int_0^\infty dq_- \frac{1}{q_-^2})$$

$$= 2i(\int_0^\infty \frac{2dk_+}{q_-^2} - \int_0^\infty dq_-) = 2i(\int_0^\infty \frac{dq_-}{q_-^2} - \int_0^\infty dq_-) = 0. \quad (3.34)$$

To the best of our knowledge this point has been overlooked so far. We verify in Appendix B that linear divergences do not appear from $D_{++}^+(x)$ if we carry out $k_+$-integrations of the
physical and ghost contributions simultaneously and in advance of Fourier transformation with respect $x^+$. This verifies that the linear divergences arising from the ghost and physical contributions cancel each other. Therefore, although changing the order of integrations prevents us from carrying out the Fourier transformations to obtain a closed form expression for $D_{++}^+(q)$, we are justified in neglecting the linear divergence arising from the ghost contribution as follows

$$D_{++}^+(q) = \frac{2q_+}{q_-} \cdot \frac{i}{q^2 + i\epsilon} - \frac{2iq_+}{q_+^2} (-i\pi)\varepsilon(q_+)\delta(q_-) - \frac{i}{2} \left\{ \frac{1}{(q_- + i\epsilon)^2} + \frac{1}{(q_- - i\epsilon)^2} \right\}$$

$$= \frac{i}{q^2 + i\epsilon} \cdot \frac{2q_+}{q_- + i\epsilon \varepsilon(q_+)} - \frac{i}{2} \left\{ \frac{1}{(q_- + i\epsilon)^2} + \frac{1}{(q_- - i\epsilon)^2} \right\}. \quad (3.35)$$

Here the finite contact term is obtained as a result of Fourier transformation of $|x^-|$. The appearance of contact terms implies the existence of Coulomb-like counter terms in the interaction Hamiltonian. We leave the specification of such terms for subsequent studies.

§4. Concluding Remarks

In this paper we have analyzed further the problem of the canonical derivation of the propagator in gauge theories, both in the light-cone representation and in the equal-time representation. We have emphasized the indispensability of ghost fields in the light-cone gauge formulation of gauge fields. In particular, we have found that the ghost fields are indispensable if we are to have well-defined antiderivatives and to properly regularize the most singular component of the naive gauge field propagator. We have also shown that the ML form of propagator is obtained in a consistent temporal gauge formulation of gauge fields in the ordinary space-time coordinates if a suitable vacuum is chosen.

What we have not completely understood is the discrepancy observed in the commutator $[A_+(x), A_+(y)]|_{x^+ = y^+}$, in which we find an extra $\delta(x^+ - y^+)$-type singularity. It seems that this discrepancy indicates that the light-cone limits are not the same and that the light-cone axial gauge formulation will require the elimination of such singularities. We have also left completing the Fourier transform of the (formally) most singular component of the gauge field propagator for subsequent studies.

Finally we point out that our approach may provide an easier way to construct perturbation theories of interacting gauge fields in the light-cone temporal gauge than that of Morara and Soldati\textsuperscript{11)}, because temporal gauge Lagrangians are regular in the auxiliary coordinates. We also leave this task for subsequent studies.
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Appendix A

Verification of (3·24)

With the help of the addition theorem of the trigonometric cosine function, we can decompose $I(x^-, x^+)$ as a sum of two well-defined integrals

$$I(x^-, x^+) = I^{(1)}(x^-, x^+) + I^{(2)}(x^-, x^+)$$

where

$$I^{(1)}(x^-, x^+) = \int_0^\infty dk_+ \sin k_+ x^+ \cdot \sin \frac{k_+^2 x^-}{2k_+},$$

$$I^{(2)}(x^-, x^+) = \int_0^\infty dk_+ \cos k_+ x^+ \cdot (1 - \cos \frac{k_+^2 x^-}{2k_+}).$$

We see that $I^{(1)}(x^-, x^+)$ is an odd function of $x^+$ and of $x^-$ and becomes trivially zero at $x^+ = 0$ and/or at $x^- = 0$. We also see that in the case that $x^+ > 0$ and $x^- > 0$, an explicit expression of $I^{(1)}(x^-, x^+)$ is known from an integral formula

$$\int_0^\infty dk \sin k \cdot \sin \frac{b}{k} = \frac{\pi}{2} \frac{b}{a} J_1(2\sqrt{ab}), \quad (a > 0, b > 0)$$

where $J_1$ stands for the Bessel function of order 1. Therefore we immediately obtain

$$I^{(1)}(x^-, x^+) = \frac{\pi}{2} \epsilon(x^-) \epsilon(x^+) k_\perp \sqrt{\frac{|x^-|}{2|x^+|}} J_1(k_\perp \sqrt{2|x^-x^+|}).$$

Because of the sign factor $\epsilon(x^+)$, $I^{(1)}(x^-, x^+)$ gives a finite discontinuity at $x^+ = 0$.

Next we show that an explicit expression for $I^{(2)}(x^-, x^+)$, which is an even function of $x^-$ and of $x^+$, results from the same formula (A·4). In case that $x^+ \neq 0$, integrating by parts and then changing integration variable from $k_+$ to $k_- = \frac{k_+^2}{2k_+}$ results in

$$I^{(2)}(x^-, x^+) = \frac{x^-}{2x^+} k_\perp \int_0^\infty \frac{dk_+}{k_+^2} \sin k_+ x^+ \cdot \sin \frac{k_+^2 x^-}{2k_+} = \frac{x^-}{x^+} \int_0^\infty dk_- \sin k_- x^- \cdot \sin \frac{k_-^2 x^+}{2k_-}$$

$$= k_\perp \frac{\pi}{2} \sqrt{\frac{|x^-|}{2|x^+|}} J_1(k_\perp \sqrt{2|x^-x^+|}).$$
The value at \( x^+ = 0 \) is calculated as follows

\[
I^{(2)}(x^-, x^+)|_{x^+ = 0} = \int_0^\infty dk_+ (1 - \cos \frac{k_+^2 x^ -}{2k_+}) = \frac{k_+^2}{2} \int_0^\infty dk_- \frac{1 - \cos k_- x^ -}{k_-^2}
\]

\[
= \frac{k_+^2 x^-}{2} \int_0^\infty dk_- \frac{\sin k_- x^-}{k_-} = \frac{\pi}{4} k_+^2 |x^-|, \quad (A.7)
\]

which turns out to be the limit of \((A \cdot 6)\) as \( x^+ \to 0 \). Consequently \( I^{(2)}(x^-, x^+) \) is known to be a continuous function of \( x^- \) and \( x^+ \). Substituting \((A \cdot 5)\) and \((A \cdot 6)\) into \((A \cdot 1)\) yields \((3 \cdot 24)\).

**Appendix B**

—— Disappearance of the linear divergence from \( D^+_{++}(x) \) ——

To demonstrate that \( D^+_{++}(q) \) possesses no linear divergences owing to the ghost fields, we carry out the \( k_+ \)-integration in

\[
D^+_{++}(x) = \frac{2}{(2\pi)^3} \int d^3 k_+ \frac{k_+}{k_+^2} \theta(k_+) \{ \theta(x^+) (e^{-ik \cdot x} - e^{-ik \cdot x}|_{x^- = 0}) + \theta(-x^+) (e^{ik \cdot x} - e^{ik \cdot x}|_{x^- = 0}) \} \quad (B.1)
\]

in advance of Fourier integrations yielding \( D^+_{++}(q) \). We can then rewrite \( D^+_{++}(x) \) in the following form

\[
D^+_{++}(x) = \frac{2}{(2\pi)^3} \int \frac{dk_1 dk_2}{k_+^2} \exp[-i(k_1 x^1 + k_2 x^2)] \{ \partial_+ J(x^-, x^+) - i\epsilon(x^+) \partial_+ I(x^-, x^+) \} \quad (B.2)
\]

where

\[
J(x^-, x^+) = \int_0^\infty dk_+ \{ \sin(k_+ x^+ + \frac{k_+^2 x^-}{2k_+}) - \sin k_+ x^+ \} \quad (B.3)
\]

and \( I(x^-, x^+) \) is the function given in Appendix A. We see that owing to the second term, which comes from the ghost contribution, the integrand of \( J(x^-, x^+) \) behaves like \( \frac{\cos k_+ x^+}{k_+} \) as \( k_+ \to \infty \), which verifies that \( J(x^-, x^+) \) is well-defined when \( x^+ \neq 0 \).

It is useful to illustrate how the ghost contribution gives rise to a linear divergence. Applying the distribution procedure we obtain

\[
\int_0^\infty \sin k_+ x^+ = -\frac{i}{2} \int_{-\infty}^\infty \epsilon(k_+) e^{ik_+ x^+} = P \frac{1}{x^+} \quad (B.4)
\]

and hence

\[
\int_{-\infty}^\infty dx^+ e^{iq_+ x^+} \partial_+ (\int_0^\infty \sin k_+ x^+) = -\int_{-\infty}^\infty dx^+ \frac{1}{(x^+)^2} e^{iq_+ x^+} = -2 \int_0^\infty dx^+ \frac{\cos q_+ x^+}{(x^+)^2}
\]

\[
= -2 \int_0^\infty dx^+ \frac{1}{(x^+)^2} + \pi |q_+|. \quad (B.5)
\]
We see that the last term diverges linearly. Furthermore we notice that showing that $J(x^-, x^+)$ does not possess any terms tending to $\frac{1}{x^+}$ as $x^+ \to 0$ verifies the disappearance of linear divergences from $D^{++}_{++}(x)$. Note that $I(x^-, x^+)$ has no such terms, as is seen from (3·25).

We can find an explicit expression for $J(x^-, x^+)$ by making use of the $\mu \to 1 - 0$ limits of the following integral formulas

\[
\int_0^\infty d\kappa \kappa^{\mu-1} \sin(ak - \frac{b}{\kappa}) = 2 \left( \frac{b}{a} \right)^{\frac{\mu}{2}} \sin \frac{\mu \pi}{2} K_\mu(2\sqrt{ab}),
\]  
\[ (B.6) \]
\[
\int_0^\infty d\kappa \kappa^{\mu-1} \sin(ak + \frac{b}{\kappa}) = \pi \left( \frac{b}{a} \right)^{\frac{\mu}{2}} \left\{ \cos \frac{\mu \pi}{2} J_\mu(2\sqrt{ab}) - \sin \frac{\mu \pi}{2} N_\mu(2\sqrt{ab}) \right\}
\]  
\[ (B.7) \]

where $a > 0, b > 0, |\text{Re}\mu| < 1$ and $K_\mu, J_\mu$ and $N_\mu$ are Bessel functions. We are justified in using the limits to calculate $J(x^-, x^+)$, because $J(x^-, x^+)$ is decomposed as a difference of the two integrations and because the difference is well-defined so that it is independent of the regularizations. In the limit we have

\[
\int_0^\infty dk \sin(ak - \frac{b}{\kappa}) = 2 \sqrt{\frac{b}{a}} K_1(2\sqrt{ab}),
\]  
\[ (B.8) \]
\[
\int_0^\infty dk \sin(ak + \frac{b}{\kappa}) = -\pi \sqrt{\frac{b}{a}} N_1(2\sqrt{ab}).
\]  
\[ (B.9) \]

Furthermore, taking the $b \to 0+$ limit of (B·8) and (B·9) yields

\[
\int_0^\infty dk \sin \kappa \kappa a k = \frac{1}{a}
\]  
\[ (B.10) \]

which agrees with (B·4). It follows from (B·8)~(B·10) that

\[
J(x^-, x^+) = \frac{\epsilon(x^-) + \epsilon(x^-)}{2} k_\perp \sqrt{\frac{|x^-|}{2|x^+|}} \{ -\pi N_1(k_\perp \sqrt{2|x^-x^+|}) - \frac{1}{k_\perp} \sqrt{\frac{2}{|x^-x^+|}} \}
\]  
\[
+ \frac{\epsilon(x^+) - \epsilon(x^-)}{2} k_\perp \sqrt{\frac{|x^-|}{2|x^+|}} \{ 2K_1(k_\perp \sqrt{2|x^-x^+|}) - \frac{1}{k_\perp} \sqrt{\frac{2}{|x^-x^+|}} \}.
\]  
\[ (B.11) \]

This shows that the leading term for small $|x^+|$ is $-\frac{1}{2} x^- \log|x^+|$ so that no linear divergences arise in the Fourier transform of $\partial_+ J(x^-, x^+)$. 

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