Revisit the Rate of Tidal Disruption Events: The Role of the Partial Tidal Disruption Event

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Abstract

Tidal disruption of stars in dense nuclear star clusters containing supermassive central black holes (SMBH) is modeled by high-accuracy direct N-body simulation. Stars getting too close to the SMBH are tidally disrupted, and a tidal disruption event (TDE) happens. The TDEs probe the properties of SMBHs, their accretion disks, and the surrounding nuclear stellar cluster. In this paper, we compare the rates of full tidal disruption events (FTDEs) with partial tidal disruption events (PTDEs). Since a PTDE does not destroy the star, a leftover object emerges; we use the term “leftover star” for it. Two novel effects occur in the simulation: (1) variation of the leftover star’s mass and radius and (2) variation of the leftover star’s orbital energy. After switching on these two effects in our simulation, the number of PTDEs is reduced by roughly 28%, and the reduction is mostly due to the ejection of the leftover stars from PTDEs originally coming from a relatively large distance. The number of PTDEs is about 75% higher than the simple estimation given by Stone et al., and the enhancement is mainly due to the multiple PTDEs produced by the leftover stars residing in the diffusive regime. We compute the peak mass fallback rate for the PTDEs and FTDEs recorded in the simulation and find that 58% of the PTDEs have a peak mass fallback rate exceeding the Eddington limit, and the number of super-Eddington PTDEs is 2.3 times the number of super-Eddington FTDEs.

Unified Astronomy Thesaurus concepts: N-body simulations (1083); Supermassive black holes (1663); Stellar dynamics (1596); Galaxy nuclei (609); Tidal disruption (1696)

1. Introduction

If the tidal forces of a supermassive black hole (SMBH) overcome the self-gravity of a passing star, it is subject to tidal disruption. Tidal disruption events (TDEs) are bright flares that could last for months to years caused by the accretion of stellar debris from the event onto the SMBH (Rees 1988). If a star subject to tidal disruption is completely destroyed, we denote this as a full tidal disruption event (FTDE); if a leftover object remains, we use the term partial tidal disruption event (PTDE). The critical distance to the SMBH for such events to happen is denoted as the tidal radius, \( r_t \). An order-of-magnitude estimate leads to \( r_t = r_s (M_{\text{BH}}/m_*)^{1/3} \), where \( r_s \), \( m_* \), and \( M_{\text{BH}} \) are radius and mass of the disrupted star and the mass of the SMBH, respectively.

On a less violent tidal encounter with the SMBH, a star passing by the SMBH with a pericenter distance \( r_p \) slightly larger than \( r_t \) could cede only part of its mass to the SMBH via a PTDE. During a PTDE, the outer layers of a star are stripped by the tidal field of the SMBH, and the bound part of the stripped mass then falls back and is accreted onto the SMBH, powering a luminous flare, as in the case of an FTDE, though it might not be as luminous as an FTDE (Chen & Shen 2021). The amount of stripped mass \( \Delta m \) is computed from the competition between the tidal force, whose strength can be characterized by the penetration factor \( \beta \equiv r_t/r_p \), and the self-gravity of the star generated by its interior mass distribution. Guillochon & Ramirez-Ruiz (2013) studied the disruption of stars modeled as polytropes through grid-based hydrodynamic simulations and found that partial disruption starts at \( \beta_p = 0.6 \) and ends at \( \beta_p = 1.85 \) for stars modeled with \( \gamma = 4/3 \) polytropes (typical for a solar-type star at its zero-age main sequence). Beyond \( \beta_p \), the star is completely disrupted. Law-Smith et al. (2020) improved the work of Guillochon & Ramirez-Ruiz (2013) by using a more accurate stellar structure and providing tables of mass fallback rates. Ryu et al. (2020) performed full relativistic hydrodynamic simulations using the conservative grid-based relativistic code Harm3D with realistic stellar structure of the stars. They found greater tidal radius compared to the Newtonian simulations and also provided uniform expression for the remnant mass as a function of pericenter distance. As long as \( \Delta m \) remains smaller than \( m_* \), a remnant stellar core will survive and can continue its orbit inside the star cluster (we call it the “leftover star” in this paper).

The event rate of FTDEs is calculated under the framework of loss cone theory (Frank & Rees 1976) and can be worked out 7 using three different approaches: by solving the Fokker–Planck equation in phase space (Cohn & Kulsrud 1978; Magorrian & Tremaine 1999; Wang & Merritt 2004; Vasiliev 2017), by the gaseous model (Amaro-Seoane et al. 2004) and using Monte Carlo simulations (Shapiro & Marchant 1978; Marchant & Shapiro 1980; Duncan & Shapiro 1983). These results are also validated by N-body simulations (Baumgardt et al. 2004;
Brockamp et al. 2011; Zhong et al. 2014) that directly trace stellar orbits, in particular those of stars before tidal disruption. Recently, it has become possible to distinguish PTDEs from FTDEs by carefully analyzing their light curves (Guillochon et al. 2018; Mockler et al. 2019; Nicholl et al. 2019; Gomez et al. 2020). This progress in TDE observation raises the demand for knowledge of the event rate of PTDEs, which has not been studied in detail. Stone & Metzger (2016) estimated the rate of PTDEs by extrapolating the \( \beta \) distribution from the FTDE region \((\beta > \beta_0)\) to the PTDE region \((\beta_0 < \beta < \beta_0)\) by using the standard \( \beta \) distribution for an isotropic star cluster \((n(\beta) \propto \beta^{-2})\). With the limiting values of \( \beta_0 = 0.6 \) and \( \beta_0 = 1.85 \), it is straightforward to show that the event rate of PTDEs is roughly twice the event rate of FTDEs (Stone et al. 2020). The event rate of PTDEs reported by Chen & Shen (2021) is obtained in a similar way.

However, such a simple extrapolation is not sufficient. The leftover star is able to produce multiple PTDEs (or end its life in an FTDE) and in that way raise the event rate of PTDEs significantly. After a PTDE, the following questions need to be asked (Rossi et al. 2021).

1. Is the structure of the leftover star more tidally vulnerable after the mass stripping?
2. Does the leftover star remain near the SMBH for several more orbits without being scattered away by relaxation in the star cluster?
3. Is the leftover star retained in the vicinity of the SMBH even though it usually receives a velocity kick due to an asymmetric mass loss through the Lagrangian points L1 and L2 during the PTDE (Manukian et al. 2013; Gafon et al. 2015)?

If the answer to one or more of the above questions is no, the leftover star is not retained near the SMBH, and the event rates of both FTDEs and PTDEs may be reduced, because it is unable to cause any further PTDEs or FTDEs.

In this work, we carry out a series of direct \( N \)-body simulations, taking into account the changes of stellar mass and orbital energy caused by PTDEs, to assess the event rates of both FTDEs and PTDEs. In Section 2, we describe the details of the \( N \)-body simulation, as well as the implementation of the mass stripping and velocity kick imparted on the leftover stars. Simulation results are presented in Section 3. We find that the occurrence of PTDEs (and possible ejection of the leftover star thereafter) reduces the FTDE rate relative to a model in which only FTDEs are taken into account (Section 3.1). This result suggests that conclusions about TDE rates obtained by using only FTDEs should be treated with caution. We find the event rate of PTDEs to be higher than a prediction based simply on the \( n(\beta) \propto \beta^{-2} \) extrapolation, mainly due to multiple PTDEs produced by leftover stars that remain deeply inside the star cluster (Section 3.2). We also measure the distribution of the peak mass fallback rate, which could be used to infer the peak bolometric luminosity of PTDEs and FTDEs (Section 3.3). We draw our conclusions in Section 4.

### 2. Details of the \( N \)-body Simulation

#### 2.1. General Settings of the \( N \)-body Model

In \( N \)-body simulations, it is convenient to adopt the Hénon unit, in which the gravitational constant \( G \) and the total mass of the star cluster \( M_c \) are equal to 1, and the total energy of the star cluster is \(-1/4\) (Heggie 2014). With this unit system, the coordinate, velocity, mass, and time in the \( N \)-body model are dimensionless quantities, enabling us to scale up the computer models to real star clusters. However, the kick velocity imparted on the leftover star is given with physical units. In order to implement the velocity kick into the \( N \)-body simulation, we need to express the physical kick velocity with the Hénon unit. Now we check the relation between the physical unit and the Hénon unit.

From the definition of the Hénon unit that the total cluster mass equals 1, it is straightforward that the Hénon mass unit \([M]\) corresponds to \( M_c \). The total energy of \(-1/4\) results in the Hénon length unit \([L] = R_{\text{vir}}\), where \( R_{\text{vir}} \) is the virial radius of the star cluster. The Hénon velocity unit can be obtained by \( [V] = \sqrt{GM_c/R_{\text{vir}}} \), and the Hénon time unit \([T] = \sqrt{R_{\text{vir}}^3/(GM_c)} \). Thus, it is evident that \( M_c \) and \( R_{\text{vir}} \) are the key parameters that control the spatial and temporal scales of the star cluster.

For all of the model clusters, we choose \( M_{\text{BH}} = 10^6 M_c \), since this mass is typical for the SMBHs residing in galaxies similar to ours, and this BH mass has been used in many hydrodynamical simulations of FTDEs and PTDEs (Guillochon & Ramirez-Ruiz 2013; Manukian et al. 2013; Ryu et al. 2020). For the star cluster mass, we follow the relation of Antonini et al. (2015) between nuclear star cluster and central black hole mass. Adopting \( M_{\text{BH}} = 10^6 M_c \) in their Equation (40) provides \( M_{\text{BH}} = 0.075 M_c \); i.e., we get \( M_c = 1.33 \times 10^6 M_c \). The \( R_{\text{vir}} \) for the star cluster of this mass is estimated to be roughly 5 pc according to Figure 11 of Turner et al. (2012). With these choices of \( M_c \) and \( R_{\text{vir}} \), one Hénon velocity unit \([V]\) equals \( 107 \text{ km s}^{-1} \), and one Hénon time unit \([T]\) equals \( 4.57 \times 10^4 \text{ yr} \). The definitions of the Hénon units and the corresponding physical values are summarized in Table 1.

| Quantities | Hénon Unit | Definition | Physical Value |
|------------|------------|------------|---------------|
| Mass       | \([M]\)    | \(M_c\)    | \(1.33 \times 10^6 M_c\) |
| Length     | \([L]\)    | \(R_{\text{vir}}\) | 5 pc |
| Velocity   | \([V]\)    | \(\sqrt{GM_c/R_{\text{vir}}}\) | \(107 \text{ km s}^{-1}\) |
| Time       | \([T]\)    | \(\sqrt{R_{\text{vir}}^3/(GM_c)}\) | \(4.57 \times 10^4 \text{ yr}\) |

### Table 1

The Hénon Units and the Corresponding Physical Values

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angular momentum. The specific loss cone angular momentum can be approximated as \( J_{\text{sc}} = \sqrt{2GM_{\text{BH}}r_0} \). The specific angular momentum variation per orbit is computed based on the definition of the relaxation timescale, \( \langle \Delta J \rangle = J_{\text{sc}}/t_{\text{relax}} \), where \( J_{\text{sc}} = \sqrt{GM_{\text{BH}}r} \) is the specific circular angular momentum, and \( t_{\text{relax}} = r/\sigma(r) \) is the dynamical timescale measured at \( r \). The local relaxation timescale measured at \( r \) is given by (Spitzer 1987)

\[
    t_{\text{relax}}(r) = \frac{0.065 \sigma^3(r)}{G^2 m \rho(r) \ln(\Lambda N)},
\]

where \( m = M_*/N \) and \( \Lambda = 0.11 \) (Giersz & Spurzem 1994). The SMBH is embedded in a stellar cusp with a density profile \( \rho(r) = \rho_0(r_h/r)^s \). Applying the condition that the enclosed stellar mass within \( r_h \) equals \( M_{\text{BH}} \), we find \( \rho_0 = (3 - s)M_{\text{BH}}/(4\pi r_h^3) \). Inside the stellar cusp \((r < r_h)\), we assume that the gravitational potential is dominated by the SMBH; hence, the velocity dispersion of the stars follows \( \sigma^2(r) = GM_{\text{BH}}/r \). Following Frank & Rees (1976), we require \( J_{\text{sc}} = \langle \Delta J \rangle \) at the critical radius and obtain the relation between the critical radius and the initial tidal radius,

\[
    r_{\text{crit}} \propto \left[ \frac{\sqrt{N C_{\text{BH}}}}{\ln(0.11N)} \right]^{\frac{3-s}{s-3}} \left( \frac{M_{\text{BH}}}{M_*} \right)^\frac{1}{s-3} r_0^{\frac{1}{s-3}}.
\]

The influence radius is defined as \( r_h = GM_{\text{BH}}/\sigma^2 \), where \( \sigma \) is the stellar velocity dispersion outside of the stellar cusp. Combined with the \( M_{\text{BH}}-\sigma \) relation (Schulze & Gebhardt 2011), we obtain \( r_h = 1.09 \times M_\odot^{0.54} \text{pc} \), where \( M_\odot = M_{\text{BH}}/(10^9 M_\odot) \). On the other hand, the influence radius in the N-body model is often defined as the radius where the enclosed stellar mass equals the SMBH mass. From a test run of our model cluster, we find the influence radius based on the enclosed mass to be \( r_h \approx 0.22 [L] \), and it roughly equals the influence radius obtained based on the velocity dispersion argument.

Therefore, the ratio of \( r_{\text{crit}}/r_h \) is written as

\[
    \frac{r_{\text{crit}}}{r_h} \propto \left[ \frac{\sqrt{NC}}{\ln(0.11N)} \right]^{\frac{1}{2}} \left( \frac{M_{\text{BH}}}{M_*} \right)^\frac{1}{2} \left( \frac{m}{s_0} \frac{r_s}{r_0} \right)^{-1/3},
\]

where \( C = r_{s,0}/r_0 \). The equality of \( r_{\text{crit}}/r_h \) in both the N-body model and the real star cluster is translated into

\[
    \frac{N_m C_m}{\ln(0.11N_m)} = \frac{N_r C_r}{\ln(0.11N_r)},
\]

where the subscripts \( m \) and \( r \) indicate that the quantities are taken from the N-body model and real star cluster, respectively. In the star cluster with \( M_{\text{BH}} = 10^6 M_\odot \), the number of stars is \( N_r = 1.33 \times 10^7 \), assuming that the star cluster consists of solar-type stars, and the ratio \( C_r = 4 \times 10^{-6} \). The number of particles in our N-body model is \( N_m = 131,072 \). Inserting the values of \( N_m, N_r, \) and \( C_r \) into Equation (4), we obtain \( C_m = 2.7 \times 10^{-4} \). Hence, the initial tidal radius in the N-body model is \( r_{s,0} = C_m r_0 \approx 5.94 \times 10^{-5} [L] \). With this tidal radius, the critical radius in the N-body model is \( 0.14 [L] \), and the corresponding critical energy \( E_{\text{crit}} = \phi(r_{\text{crit}}) \approx -2.5 [V^2] \), where \( \phi(r) \) is the combined gravitational potential generated by the SMBH and the star cluster.

Note that in the above derivation, the loss cone angular momentum is computed from the assumption that the star is completely disrupted at \( r_p = r_{s,0} \). In this work, complete disruption should occur at \( r_p = r_{s,0}/\beta_3 \) and partial disruption begins at \( r_p = r_{s,0}/\beta_4 \). Substituting \( r_{s,0} \) with \( r_{s,0}/\beta_3 \) in Equation (2) and adopting \( s = 1.1 \) (presented in Section 3), we obtain the corrected critical radius for the FTDEs, \( r_{\text{crit,d}} = 0.81 r_{\text{crit}} (=0.11[L]) \). The critical radius for the PTDEs can be obtained in the same way, \( r_{\text{crit,p}} = 1.19 r_{\text{crit}} (=0.17[L]) \). Accordingly, we could use the expressions of \( r_{\text{crit,d}}/r_h \) and \( r_{\text{crit,p}}/r_h \) to derive Equation (4), but the results are the same. The fractional difference between \( r_{\text{crit,d}} \) and \( r_{\text{crit,p}} \) is within 20%, and the corresponding critical energies are close to each other; in the rest of this paper, we will just use \( r_{\text{crit}} \) and \( E_{\text{crit}} \) to separate the diffusive and pinhole regimes.

During the course of simulation, the mass and size of the leftover star vary after every PTDE; thus, the corresponding tidal radius for disrupting the leftover star should vary according to

\[
    n_t = n_{s,0} \times \left( \frac{r_s}{r_0} \right) \left( \frac{m_s}{m_{s,0}} \right)^{-1/3},
\]

where \( r_s \) and \( r_{s,0} \) are the current and initial stellar radius, and \( m_s \) and \( m_{s,0} \) are the current and initial stellar mass.

Whether a star is partially or completely disrupted by the SMBH depends on the penetration factor \( \beta \). In the simulation, we monitor \( \beta \) for all the stars at their pericenter passage. Once the condition \( 0.6 < \beta < 1.85 \) is satisfied, a PTDE ensues, and the position and velocity of the star at its pericenter are recorded. After that, the leftover star continues its orbit in the star cluster with a new stellar mass and velocity vector (introduced in Section 2.2). If the penetration factor goes beyond 1.85, an FTDE ensues, and the star is removed from the system (if a leftover star is completely disrupted, such an event is also classified as an FTDE). The SMBH does not gain mass from the partial and complete TDEs in order to avoid the artificial fast mass growth due to the low particle resolution (see, for example, the fast growth of \( M_{\text{BH}} \) recorded by Zhong et al. 2014). If we allow \( M_{\text{BH}} \) to grow and assume that all of the stripped stellar mass is accreted by the SMBH, then, in the \( N = 128 \) K model, by the end of the simulation, \( M_{\text{BH}} \) would increase by at least 30%. This lower limit is obtained based on the current simulation data by summing together the masses stripped in FTDEs and PTDEs and then dividing by the initial SMBH mass. In the simulations where the SMBH could gain mass from the disrupted stars, the maximum mass enhancement of the SMBH could be a factor of a few, as reported by Zhong et al. (2014) and models 11–15 of Hayasaki et al. (2018). However, such a large mass enhancement is not realistic. In the scaled system \( (M_{\text{BH}} = 10^6 M_\odot, M_* = 1.33 \times 10^7 M_\odot) \), assuming a constant FTDE rate \( (10^{-4} \text{ yr}^{-1}) \) over one half-mass relaxation time (2.8 Gyr) and 100% accretion of the stellar debris onto the SMBH, the mass of the SMBH would increase by at most \( 2.8 \times 10^5 M_\odot (28\%) \). We also note that, in reality, only a fraction (10%–50%) of the stripped mass should be added to the SMBH mass. This fact would suppress the mass enhancement of the SMBH. In conclusion, our treatment of fixing the SMBH mass during the simulation is justified.

We use NBODY6++GPU (Wang et al. 2015; Huang et al. 2016) to model the dynamical evolution of star clusters by direct N-body simulation. NBODY6++GPU is based on the earlier N-body codes NBODY6 (Aarseth 1999) and NBODY6++
(Spurzem 1999) and uses the GPU acceleration first described by Nitadori & Aarseth (2012) but parallelized on many nodes with many GPUs. Note also extensions of the code to model star accretion on SMBHs in galactic nuclei (Panamarev et al. 2019) and recent updates of stellar evolution (Kamlah et al. 2022).

In order to assess the impact of PTDEs on the event rates, the model clusters are simulated with two code configurations: the first one switches on the PTDE-related routines (introduced in Section 2.2), and the second one switches them off. The models simulated with the first configuration are referred to as the fiducial models, while those simulated with the second configuration are called control models. Both the fiducial and control models contain five realizations of the model clusters that are initialized with five different random seeds. All of the clusters have the same values of the particle number N and the initial tidal radius $r_t,0$. They are simulated for 1000 $T$, which is roughly one half-mass relaxation time in the $N = 128 \text{ K}$ model clusters.

### 2.2. The Leftover Star

In this subsection, we describe the implementation of the mass stripping and velocity kick applied to the leftover stars.

The range of $\beta$ for PTDEs, the amount of stripped mass, and the velocity kick during the PTDE are obtained from hydrodynamic simulations in which the star is modeled as a polytrope (or realistic stellar model) and initially stays in hydrostatic equilibrium. This is the case for the star that has never experienced a tidal interaction with the SMBH (hereafter referred to as a “normal star”). However, it is not clear whether the leftover star could also be modeled as a polytrope and stay in hydrostatic equilibrium when it comes back to the vicinity of the SMBH, if possible. Hydrodynamic simulations of PTDEs show that strong perturbation may occur during a PTDE, depending on the penetration factor (Goicovic et al. 2019). As a result, the leftover star is substantially spun up, and the internal structure becomes different from the main-sequence star of the same mass (Ryu et al. 2020). After the tidal perturbation has ceased, the leftover star effectively rejoins the Hayashi track and will take a Kelvin–Helmholtz timescale ($10^5$–$10^6$ yr) to return to the main sequence (Manukian et al. 2013), which is longer than the typical orbital period for the leftover star. Gillochon & McCourt (2017) found a shorter timescale of $10^3\text{ yr}$, and Ryu et al. (2020) found a typical cooling timescale of $2 \times 10^4\text{ yr}$ in their fiducial model. The discrepancy of timescales among the different papers is largely caused by the methods adopted for timescale estimation. Furthermore, the abovementioned studies only simulated the partial disruption process for a few days, much less than the typical orbital period of the leftover star; thus, at the time of writing, the exact long-term evolution of the leftover star emerging from a PTDE is not clear.

For simplicity, we ignore the effects of the bulk rotation and nonpolytropic internal structure of the leftover star and just assume that the normal and leftover stars share the same mass–radius relation, $r_s \propto m_s^{0.3}$ (Kippenhahn & Weigert 1994), and the same recipes of mass stripping and velocity kick.

The fractional mass loss during the partial disruption depends primarily on $\beta$ and stellar structure (Gillochon & Ramirez-Ruiz 2013). In our model, the normal and leftover stars are modeled by $\gamma = 4/3$ polytropes; then the fractional mass loss $C_{4/3} = \Delta m/m_{s,\text{pre}}$ is computed through (Gillochon & Ramirez-Ruiz 2013)

$$C_{4/3} = \exp \left[ \frac{12.996 - 31.149\beta + 12.865\beta^2}{1 - 5.332\beta + 6.426\beta^2} \right].$$

where $\Delta m$ is the stripped mass, and $m_{s,\text{pre}}$ is the predisruption stellar mass. After the partial disruption, the new mass of the leftover star is $m_{s,\text{new}} = (1 - C_{4/3})m_{s,\text{pre}}$.

The kick velocity imparted on the leftover star depends on the penetration factor $\beta$ and escape velocity $v_{\text{esc}}$ at the surface of the star, namely, $v_{\text{kick}} = 0.0745 + 0.0571\beta^{2.539}v_{\text{esc}}$, which is nearly independent of $M_{\text{BH}}/m_s$ and never exceeds $v_{\text{esc}}$ (Manukian et al. 2013). Note that this relation has only been tested for the range of $1 < \beta < 1.8$, and we assume that it holds for the whole range of $0.6 < \beta < 1.85$. For a solar-type star, $v_{\text{esc}} = 617.7 \text{ km s}^{-1}$, but after the mass stripping, the escape velocity at the surface of the leftover star is reevaluated with the new stellar mass and radius. In the $N$-body simulation, we apply the velocity kick instantaneously at the pericenter of the orbit (Manukian et al. 2013). The specific orbital angular momentum of the leftover star is nearly invariant during the PTDE (Ryu et al. 2020); hence, the $v_{\text{kick}}$ is added to the radial velocity $v_r$ of the leftover star in the positive radial direction.

When the $v_{\text{kick}}$ is large enough, the leftover star could be ejected from the star cluster and hence reduce the event rate of both PTDEs and FTDEs. The value of $\beta$ above which the leftover star shall be ejected can be obtained by equating the orbital energy $E_{\text{tot}}$ before the PTDE to the specific energy gain $(v_{\text{kick}}^2/2)$ from the PTDE. The result is denoted as $\beta_3$, and we define $\beta_3 < \beta < \beta_4$ as an “ejection zone”; once a star enters this zone, will be ejected from the star cluster. Figure 1 shows the dependence of $\beta_3$ on the specific orbital energy before a PTDE. When $E_{\text{tot}}$ is small, a leftover star needs a lot of energy to escape from the cluster; however, $\beta_3$ is close to $\beta_4$, which results in the highest kick velocity. Note that the $\beta_3$ curve shall intersect with the boundary $\beta_4$ at the specific energy $E_{\text{tot}} = -v_{\text{esc}}^2/2$, and there is no ejection zone below this energy. In the $E_{\text{tot}} \approx 0$ region, a tiny energy increment could unbind the star from the cluster; therefore, $\beta_3$ is close to $\beta_4$ which only causes the minimum kick velocity. The $\beta_3$ curve does not touch the boundary of $\beta_4$ at $E_{\text{tot}} = 0$ because the formula of $v_{\text{kick}}$ given by Manukian et al. (2013) has a nonzero value at $\beta_4$, which is $v_{\text{kick}} = 0.08v_{\text{esc}}$. Since a star could lose mass in every PTDE, we also plot the $\beta_3$ curves for three different stellar masses: $m_s/m_0$ = 1, 0.5, and 0.1. With the adopted mass–radius relation ($r_s \propto m_s^{0.8}$) for the leftover stars, $v_{\text{esc}} \propto m_s^{0.1}$ only weakly depends on $m_s$. Hence, the kick velocity is not sensitive to the stellar mass and is mainly determined by $\beta$.

A leftover star could be ejected from the nuclear star cluster; however, the kinetic energy gained from the PTDE is not enough to unbind it from the host galaxy (Manukian et al. 2013). Though the ejected leftover stars are retained in the galaxy, the possibility of returning to the SMBH is negligible, since out there, they are more likely to be scattered away from the disruptive orbits.

### 3. Results of the Simulations

We initialize our model star cluster as a Plummer sphere, which possesses a constant density core in the center. As the simulation proceeds in time, a density cusp will form around
the SMBH and quickly evolve into a slope around $s \approx 1.1$ (Figure 2). A similar evolutionary track from core to cusp was observed in Zhong et al. (2014), who also started with a Plummer sphere but allowed the SMBH mass to grow. We find a density cusp shallower than in Zhong et al. (2014) because (1) we do not grow the mass of the SMBH in time, and (2)
velocity kicks for leftover stars during the PTDE push them to higher energy, so the density in the innermost zones of the cusp is reduced. The velocity dispersion profile inside the cusp region follows very well the expected scaling $\sigma^2(r) \propto r^{-1}$. Hence, the assumptions of the density and velocity dispersion profiles used in the derivation of Equation (2) are justified.

To validate the scalability of the $N$-body models, we run two sets of models with $(N, r_{c0}) = (64, 1.14 \times 10^{-4})$ and $(256, 3.23 \times 10^{-5})$. The initial tidal radii for these two particle numbers are obtained by using Equation (4); thus, these models have the same value of $NC/\ln(0.11N)$. The time dependence of the event rates in these models is plotted in Figure 3. The event rates of PTDEs and FTDEs obtained in the three models with different $N$ and $r_{c0}$ are generally consistent.

At the beginning of the simulations, the star cluster only contains normal stars. As time goes on, the number of leftover stars retained in the cluster increases steadily at a pace of roughly $2[T]^{-1}$ (excluding the ejected ones and the ones destroyed in the FTDEs). These retained leftover stars, together with the normal stars, produce 10,180 TDEs throughout the whole simulation (averaged over the five realizations of the fiducial model cluster). Thus, the mean event rate (including PTDEs and FTDEs produced by both normal and leftover stars) is roughly $10.18[T]^{-1}$. Since we have properly chosen the tidal radius and particle number in the $N$-body simulation, the event rate keeps its value when scaling to the actual number of stars and the actual tidal radius (Section 2.1); i.e., the mean event rate in the scaled system is still $10.18[T]^{-1}$. Adopting $[T] = 4.57 \times 10^4$ yr given in Section 2.1, the mean event rate in the scaled system is $2.23 \times 10^{-4}$ yr$^{-1}$. The detailed event rates are listed in Table 2.

Figure 4 summarizes the pre-PTDE stellar mass $m_s$, orbital energy $E_{\text{orb}}$, and penetration factor $\beta$ (indicated by colors) for every PTDE. The left panel shows the PTDE after which the leftover star is ejected, while the right panel shows the PTDE after which the leftover star is retained in the star cluster. During every PTDE, the newly born leftover star will lose some mass and gain some orbital energy, so the general trend of the stars on this plane is moving toward the higher energy and lower stellar mass. A single star may appear many times in Figure 4 if it produces multiple PTDEs. For demonstrative purposes, we have chosen from the simulation data three stars that have produced multiple PTDEs before being ejected from the star cluster. Their trajectories are plotted in the left panel of Figure 4.

### 3.1. Reduction of the FTDEs

Previous theoretical (Cohn & Kulsrud 1978; Magorrian & Tremaine 1999; Wang & Merritt 2004; Vasiliev 2017) and numerical (Baumgardt et al. 2004; Brockamp et al. 2011; Zhong et al. 2014) works on the event rate focus solely on the FTDEs and have not taken into account the influences of PTDEs. Note that in the previous works, the criterion for FTDE is $\beta_d = 1$. In our fiducial and control models, the choice of $\beta_d = 1.85$ can also cause a lower FTDE rate than the $\beta_d = 1$ case. For instance, in the previous works, the FTDE rate can be estimated as $\Gamma(\beta_d = 1) = k_0^{1/g}$ (Baumgardt et al. 2004), while in the case of $\beta_d = 1.85$, the FTDE rate is $\Gamma(\beta_d = 1.85) = k(r_1/1.85)^{4/g}$; hence, we find $\Gamma(\beta_d = 1.85)/\Gamma(\beta_d = 1) \approx 0.76$. If we adopt the density cusp obtained from our simulation ($s = 1.1$), the

### Table 2

| Type         | Normal Star | Leftover Star | Total |
|--------------|-------------|---------------|-------|
| FTDE         | 3.33        | 1.68          | 5.01  |
| PTDE         | 8.73        | 8.53          | 17.26 |
| Total        | 12.06       | 10.21         | 22.28 |

Note. The values in this table are given in units of $10^{-5}$ yr$^{-1}$ and computed based on the results of the $N = 128$ K model.

$^7$ The power index of $r_1$ is $(9 - 4s)/(8 - 2s)$, and the equation of Baumgardt et al. (2004) has assumed a Bahcall–Wolf cusp ($s = 7/4$).
Figure 4. Distribution of stars that produce PTDEs in the parameter space spanned by the mass ($m_{\ast}$) and specific orbital energy ($E_{\text{cot}}$) of the stars. For every PTDE, the values of $m_{\ast}$ and $E_{\text{cot}}$ are measured at the moment immediately before the onset of the event. Colors indicate the penetration factor $\beta$. The vertical solid line marks the position of the critical energy, and the corresponding horizontal radius is roughly 0.7 pc. For clarity, the vertical axis shows the quantity $1 - m_{s}/m_{0}$ where $m_{0} = 1/N[M]$ is the initial mass of the star, and is set to log scale. However, with this configuration, the PTDEs produced by the normal stars are not visible in the figure because $1 - m_{s}/m_{0} = 0$. We have artificially reset the value of $1 - m_{s}/m_{0}$ to $10^{-5}$ for the PTDEs produced by normal stars. The left panel shows the PTDEs after which the leftover stars are ejected, while the right panel shows the PTDEs after which the leftover stars are retained in the star cluster. In the left panel, we also plot the historical PTDEs (diamonds connected by lines) for three individual stars. This figure is generated from the data of one realization of the fiducial model.

The resultant ratio of $\Gamma(\beta_{d} = 1.85)/\Gamma(\beta_{d} = 1)$ would be $(1/1.85)^{0.75} \approx 0.61$. However, this reduction of rates caused by the different values of $\beta_{d}$ is trivial, because the rates in the $\beta_{d} = 1$ and 1.85 cases are all estimated based on the classic loss cone theory, which has nothing to do with the effects of PTDEs. The FTDE rate obtained from our work is $5.01 \times 10^{-5}$ year$^{-1}$ (Table 2); if the factor of 61% correction caused by the value of $\beta_{d}$ and the 28% reduction caused by the effects of PTDEs (see below) are taken away, the corrected FTDE rate would be $5.01 \times 10^{-5}/0.61/(1 - 0.28) = 1.14 \times 10^{-4}$ year$^{-1}$, which is comparable to the rates reported by previous works for a $10^{6} M_{\odot}$ SMBH (Stone & Metzger 2016; Pfister et al. 2020).

When the velocity kick is activated in the PTDE, a fraction of the stars may gain enough energy to escape from the star cluster, while the retained leftover stars suffer from the mass stripping that reduces the tidal radius for the subsequent disruption. These two effects working together should result in a reduction of the number of FTDEs. In order to find the amount of reduction in FTDEs due to the effects of PTDEs, we compare the fiducial model with the control model. We choose $\beta_{d} = 1.85$ in the control model, so that the two models only differ in the inclusion/exclusion of mass stripping and velocity kick. Averaged over the five realizations, 2291 FTDEs are recorded in the fiducial model, while in the control model, the number of FTDE records is 3214. The two effects induced by PTDEs reduce the number of FTDEs by roughly 28%, but they contribute differently to the reduction of FTDEs.

The reduction of FTDEs in the fiducial model is mainly due to the ejection of the leftover stars. In the fiducial model, 875 leftover stars are ejected after they entering the ejection zone. Figure 4 shows that a significant fraction of the ejected stars (filled circles) are coming from the pinhole regime (i.e., $E_{\text{cot}} > E_{\text{crit}} \simeq -2.5|V|^{2}$). In the pinhole regime, the average change of orbital angular momentum per orbit caused by two-body scatters among the stars is larger than the loss cone angular momentum, which results in the $\Delta \beta$ between consecutive orbits being comparable to or larger than the width of the PTDE zone, $\beta_{d} - \beta_{p}$. If there was no velocity kick imparted on the leftover star, it could be scattered into the loss cone and completely disrupted or scattered out of the loss cone in the next orbit. If the latter happens, the leftover star still has a chance to come back to the loss cone as long as it is retained in the star cluster, although it may take a long time (possibly a few to hundreds of orbital periods, or even longer). If we “virtually” add the ejected stars to the category of FTDEs in the fiducial model, the number of FTDEs produced in the two models will come to the same level. The ejection zone occupies a sizable fraction of the PTDE zone at the orbital energies $E_{\text{cot}} > E_{\text{crit}}$ (see Figure 1); therefore, the PTDEs happening in the pinhole regime are very likely to cause ejections.

The changes of tidal radius after every PTDE could also influence the number of FTDEs because the event rate $\Gamma$ scales as $\Gamma \propto r_{t}^{-3/2}$ (Baumgardt et al. 2004). Using the relation between $r_{t}$ and $m_{s}$ (Equation 5) and adopting the mass–radius relation adopted in this work, we find $r_{t} \propto m_{s}^{-1/3} \propto m_{s}^{-0.47}$ and $\Gamma \propto m_{s}^{0.21}$. At the end of the simulation, the number fraction of leftover stars in the cluster is less than 2%, and most of them have an $m_{s}/m_{s,0}$ close to 1. Therefore, most of the leftover stars only experience a small reduction of the tidal radius $r_{t}$, which should not suppress the number of FTDEs noticeably. Another piece of evidence is that in the fiducial model, roughly one-third of the FTDEs are produced by the leftover stars (Table 2). Among these events, a few of them are fully disrupted, with $m_{s}/m_{s,0} < 0.2$, while the majority are produced by the leftover stars with $m_{s}/m_{s,0} \approx 1$ (Figure 5).

### 3.2. The Number of PTDEs

The number ratio of PTDEs to FTDEs obtained in the fiducial model turns out to be roughly 3.5, which is 75% larger than the simple estimation obtained by extrapolating the $n(\beta)$ from the FTDE region to the PTDE region (Section 1). Here we check the $\beta$ distribution of the PTDEs and seek the reason for
this enhancement. The PTDEs could be produced by either normal or leftover stars; however, the $\beta$ distributions of these two categories take different forms.

The $\beta$ distribution of the PTDEs (and FTDEs) produced by normal stars generally follows the $n(\beta) \propto \beta^{-2}$ power law (top left panel of Figure 6 and the red line in the residual plot), except for the bins of $\beta \approx \beta_p$. The measured $\beta$ distribution of the PTDEs produced by normal stars agrees with the theoretical $\beta$ distribution in the pinhole regime, though it was originally derived for the FTDEs (Stone & Metzger 2016).

The roughly 50% excess in the $\beta \approx \beta_p$ bins (Figure 6 residual plot) is contributed by the diffusive regime. This excess disappears when we exclude the PTDEs produced by the normal stars with $E_{tot} < E_{crit}$, as shown by the top left panel and the residual plot of Figure 7.

The $\beta$ distribution of the PTDEs produced by the leftover stars increases toward small $\beta$ faster than the $n(\beta) \propto \beta^{-2}$ law (top right panel of Figure 6). Actually, it is not well characterized by a power-law decline; the deviation from the $\beta^{-2}$ line becomes larger when approaching $\beta = \beta_p$ (also see the blue line in the residual plot of Figure 6). The excess of $n(\beta)$ is mainly contributed by the repeated PTDEs produced by leftover stars belonging to the diffusive population ($E_{tot} < E_{crit}$). We find that 40% of PTDEs are produced by the diffusive population, while the pinhole population is responsible for the remaining 60%. If the PTDEs produced by the diffusive population stars are excluded from the statistics, then the $\beta$ distribution restores the $n(\beta) \propto \beta^{-2}$ form, which is shown in the top right panel and the residual plot of Figure 7.

Figure 8 indicates that the diffusive population stars are more productive than the pinhole population stars. The diffusive population stars receive the least gravitational scattering from other stars and enter the PTDE zone with small $\beta$ steps, which in turn cause the least amount of mass stripping and velocity kick to the leftover star. Therefore, they could randomly walk in the PTDE zone for many orbits. The pinhole population stars, on the other hand, only stay in the PTDE zone for one or two orbits before they are scattered out of the PTDE zone or ejected from the star cluster. Note that a single leftover star could contribute to many energy bins in Figure 8 or even transfer from the diffusive to the pinhole regime due to the orbital energy increment caused by PTDEs (also see Figure 4).

### 3.3. The Detectability of the Disruptive Events

We have shown that the FTDEs could be reduced by the ejection of leftover stars, and the number of PTDEs is raised by the diffusive population of stars compared to previous papers (Stone et al. 2020). However, such results cannot be directly compared with observations because the observability of a TDE depends on its luminosity.

After the PTDE and FTDE, the stripped material bound to the SMBH will return to the pericenter at a rate of $\dot{m}_{fb}$, denoted as the mass fallback rate. If the material could rapidly dissipate the orbital kinetic energy and circularize into an accretion disk in which the viscous timescale is shorter than the fallback timescale (Cannizzo et al. 1990), then the accretion rate could be closely approximated by the mass fallback rate (this is true at least for the UV/optical TDEs reported by Mockler et al. 2019).

Accordingly, the bolometric luminosity of the event, $L_{bol}$, could be estimated as $L_{bol} = \eta \dot{m}_{fb} c^2$, where $\eta$ is the accretion efficiency and $c$ is the speed of light. When the stripped mass falls back to the vicinity of the SMBH at a super-Eddington rate, which happens for the FTDEs and some of the PTDEs, the energy released by the stream–stream collision (Jiang et al. 2016; Guillochon et al. 2014) or the circularization and accretion process could power a subrelativistic outflow. The outflowing material, together with the loosely bound debris that orbits at large radii and obscures the SMBH (Guillochon et al. 2014), could form a “reprocessing layer” that absorbs the high-energy photons from the accretion disk and reemits in the UV/optical band. Stone & Metzger (2016) found that the g-band peak luminosity derived from the reprocessing layer model matches with the observed TDEs, while the g-band peak luminosity derived from the outflow itself, the accretion disk, and the off-axis relativistic jet is substantially lower than the observations. In their reprocessing layer model, the bolometric luminosity is limited to $L_{gad}$ during the super-Eddington phase, while in the sub-Eddington phase, $L_{bol} \propto \dot{m}_{fb}$.

With the stellar mass and $\beta$ measured from the N-body simulation, we compute the peak mass fallback rate $\dot{m}_{peak}$ for every PTDE and FTDE. Adopting the mass–radius relation $r_s \propto M_0^{0.8}$, Equation (A1) of Guillochon & Ramirez-Ruiz (2013) becomes

$$\dot{m}_{peak} = A_0 (\beta) M_0^{1/2} (m_s/M_0)^{0.8} M_\odot \text{yr}^{-1},$$  \hspace{1cm} (7)
where $M_0 = M_{\text{BH}}/(10^6 \ M_\odot)$ and the coefficient $A_1(\beta)$ is computed by Equation (A6) of Guillochon & Ramirez-Ruiz (2013) because in this work, we assume that the normal and leftover stars are modeled by the $\gamma = 4/3$ polytrope. The value of $A_1(\beta)$ covers roughly 4 orders of magnitude in the $\beta$ range of our interest, while the stellar mass only modifies the $\dot{m}_{\text{peak}}$ within a factor of 10; thus, the value of $\dot{m}_{\text{peak}}$ is primarily determined by $\beta$. The peak mass fallback rate is then normalized to the Eddington accretion rate of the $10^6 \ M_\odot$ SMBH, $f_{\text{Edd,peak}} \equiv \dot{m}_{\text{peak}}/\dot{m}_{\text{Edd}}$, where $\dot{m}_{\text{Edd}} = 0.022 M_\odot \text{ yr}^{-1}$ is the Eddington accretion rate. Note that the fitting formula of Guillochon & Ramirez-Ruiz (2013) is only valid for $0.6 \leq \beta < 4$; in our calculation, the peak fallback rate of FTDEs beyond the upper limit is computed with $\beta = 4$. As a result, the peak fallback rates of 45.4% of the FTDEs have been affected. The coefficient $A_1(\beta)$ reaches its maximum value at $\beta \simeq 2.2$ (the corresponding $f_{\text{Edd,peak}} \approx 140$), then slowly declines beyond that $\beta$ (a similar trend was observed by Law-Smith et al. 2020). Our treatment for the $\beta > 4$ events will not change the upper boundary of the $f_{\text{Edd,peak}}$ distribution. Besides, $f_{\text{Edd,peak}}$ stays higher than 1 in the range of $4 < \beta < \beta_{\text{max}}$ ($\beta_{\text{max}}$ is defined below). The radius of the unstable circular orbit (UCO), $r_{\text{UCO}}$, sets the minimum $r_p$, below which the star will directly plunge onto the SMBH without being disrupted and hence be unable to release any photon emission. For an $e \simeq 1$ orbit around a Schwarzschild black hole, $r_{\text{UCO}} \simeq 4G M_{\text{BH}}/c^2$ (Gair et al. 2005). In our fiducial star cluster, the SMBH has a mass of $10^6 M_\odot$, and the initial mass of the member stars is 1 $M_\odot$, so the maximum $\beta$ for nonplunging TDEs is estimated as $\beta_{\text{max}} = r_{\text{UCO}}/r_{\text{Edd}} \approx 11.7(m_{\text{s}}/M_\odot)^{0.47}$. In the end, 338 FTDEs with $\beta > \beta_{\text{max}}$ are excluded from the statistics. In our model, the mass and radius of the leftover stars decrease after every PTDE, leading to a combined effect that, in turn, decreases $r_{\text{UCO}}$ and also the corresponding $\beta_{\text{max}}$ (see Section 3.1).

The results are plotted in Figure 9, from which we see that the $f_{\text{Edd,peak}}$ of the FTDEs are all distributed around $\sim 100$, while in the case of PTDEs, the distribution of $f_{\text{Edd,peak}}$ first decays with a $-1$ power-law decay in the $10^{-2} < f_{\text{Edd,peak}} < 10$ region and then turns to a shallower decline. The tail in the $f_{\text{Edd,peak}} < 10^{-2}$ region is due to the reduction of $\dot{m}_{\text{peak}}$ by the stellar mass.

We find in the simulation that roughly 58% of the PTDEs fall into the $f_{\text{Edd,peak}} > 1$ category, and the number of super-Eddington PTDEs is roughly 2.3 times the number of super-Eddington FTDEs.
Figure 7. Same as Figure 6 except that the PTDEs and FTDEs produced by the (normal and leftover) stars with $E_{\text{tot}} < E_{\text{crit}}$ are excluded.

Figure 8. Number of PTDEs produced by a single star in different orbital energy intervals, averaged over the five realizations (black line). The triangles are the corresponding values in each realization. The value is computed as $N_{\text{PTDE}}/N_{\text{star}}$, where $N_{\text{PTDE}}$ is the number of PTDEs in the orbital energy interval, and $N_{\text{star}}$ is the number of stars that produce these PTDEs. The vertical red line indicates the position of $E_{\text{crit}}$; the diffusive regime is on its left side, while the pinhole regime is on its right side.
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4. Summary and Discussion

We investigated the event rate of full and partial tidal disruption events (FTDEs and PTDEs) in nuclear star clusters with an embedded SMBH. For that, we have carried out a series of direct high-accuracy N-body simulations in which we follow in detail the orbits of stars coming close to the tidal disruption radius \( r_t \) near the SMBH. A PTDE happens if a star approaches the tidal radius, but not close enough for a full tidal disruption. We use the penetration factor \( \beta = r_t / r_p \) (where \( r_p \) is the pericenter distance of the star from the SMBH) as a parameter to distinguish the regimes of PTDEs and FTDEs. In the case of a PTDE, the star is not totally destroyed, but a leftover star emerges and introduces two novel effects that could modify the event rates but were not considered in previous papers on the subject. First, the leftover star will produce multiple PTDEs under certain conditions or end up in another final FTDE, although the tidal radius for disrupting the leftover star is reduced due to the mass stripping. Second, asymmetric mass loss during the PTDE will provide some additional kinetic energy to the leftover star, which could kick it from the diffusive regime to the pinhole regime or even eject it completely from the star cluster. Accordingly, we define an “ejection zone” in \( \beta \) space (\( \beta_2 < \beta < \beta_0 \)) in which a leftover star shall be ejected after the PTDE. Our main results are summarized as follows.

1. We find in our fiducial simulations, which include the two new effects of PTDEs, as well as FTDEs, that the rate of FTDEs is reduced by 28% relative to control models that only use FTDEs. Roughly one-third of the FTDEs are produced by leftover stars. The reduction of FTDEs is mainly due to the ejection of the leftover stars. The ejection zone takes a sizable fraction of the PTDE zone in the pinhole regime; hence, the pinhole population stars are more likely to be ejected after the PTDE.

2. The number of PTDEs observed in our fiducial models is raised as compared to previous papers (Stone et al. 2020; Chen & Shen 2021), mainly due to multiple PTDEs produced by the diffusive population stars. Finally, the number ratio of PTDEs to FTDEs is about 75% larger than the previous estimations, which simply extrapolate the \( n(\beta) \) of the FTDEs to the PTDEs.

3. We calculated the peak mass fallback rate, normalized to the Eddington accretion rate, \( f_{\text{Edd,peak}} \), for the events recorded in the simulations. The \( f_{\text{Edd,peak}} \) of the PTDEs are distributed following a power law with a power index \(-1\) in the range of \( 10^{-2} < f_{\text{Edd,peak}} < 10 \), then turning to a shallower power law at \( f_{\text{Edd,peak}} > 10 \). As a result, 58% of the PTDEs shall experience super-Eddington mass fallback at their peaks, and the number of super-Eddington PTDEs is 2.3 times the number of super-Eddington FTDEs.

In our simulation, we have constructed the initial model with equal-mass stars and adopted the main-sequence mass–radius relation for all of the normal and leftover stars. Such an assumption is for the purpose of a pilot study of the effect of FTDEs and PTDEs in one simulation. However, this assumption is crude and might not be physically reasonable in reality. For example, in \( \beta < 0.8 \) events, the amount of stripped mass is less than \( 10^{-2} \) of the predisruption stellar mass, and the stripping is limited to the surface layers of the star, leaving the interior of the star untouched. This situation resembles the fast mass transfer on dynamic timescales between binary stars, which could be treated as an adiabatic process (Hjellming & Webbink 1987; Dai et al. 2013). For a \( \gamma = 4/3 \) star, upon removal of the surface layer, the stellar radius will become smaller than the value predicted by the standard relation \( r_e \propto m^{0.8} \). As a consequence, the tidal radius of the object will be reduced, and further PTDEs could follow. However, such a reduction of the tidal radius is less than 10% compared to our fiducial model and thus would not strongly affect our results of the event rates. On the other hand, for \( \beta > 0.8 \) events, the tidal force of the SMBH injects internal energy into the leftover star (see, for example, Figure 7 of Ryu et al. 2020), which would cause expansion of the star. A tidally heated star in a binary
system could expand by a factor of a few and keep that radius for $10^4$–$10^5$ yr (Podsiadlowski 1996). As a (not very precise) analog, the tidally heated leftover star should possess a larger tidal radius than that predicted by the main-sequence mass–radius relation, raising the possibility for the next full or partial tidal disruption. However, such a leftover star also receives a large velocity kick, causing an ejection or being kicked onto the pinhole orbit, where it is easily scattered away from the disruptive orbit. Thus, the expanding stellar radius of the leftover star emerging from $\beta > 0.8$ events is not likely to affect our results of the event rate significantly.

MacLeod et al. (2013) proposed a process of spoon-feeding gas to the SMBH via repeated partial disruption of giant stars. Although we have not implemented the giant star disruption, spoon-feeding of gas to the SMBH via partial disruption of main-sequence stars is observed in our simulation. The leftover stars are most likely to produce another PTDE in just one Keplerian period, though there are some cases where it takes more than one Keplerian period, as shown in Figure 10. The minimum time interval in this figure is roughly 350 yr. The instant PTDE rate based on this time interval could temporarily rise to roughly $10^{-3}$ yr$^{-1}$; however, the repeated PTDEs with $f_{\text{Edd,peak}} \ll 1$ should be difficult to detect. Mainetti et al. (2015) reported repeated flares with a period of 9.5 yr from the galactic center of IC 3599 and claimed that these flares are powered by repeated partial disruption of a star (Campana et al. 2015). Such short-period repeated PTDEs are not found in our model because the central density of our model cluster is not high enough to place a star in such a small orbit. Nevertheless, we speculate that in star clusters possessing the highest central density, repeated PTDEs of such short periods should be feasible. Another remark about the period is that, due to the increment of orbital energy after every PTDE, the time interval between consecutive PTDEs should increase as well. If the future flares of IC 3599 follow this manner, it will provide further support for the PTDE origin of these flares.

Mockler et al. (2019) identified three PTDEs out of 14 optically selected disruption events using the light-curve fitting package MOSFiT (Guillochon et al. 2018). Although the sample size of Mockler et al. (2019) is small, it seems that a significant fraction of PTDEs are missing in the observations. One possible reason is that the FTDEs generally have larger $f_{\text{Edd,peak}}$, which makes them intrinsically more likely to be detected. Based on our findings, we notice there might be another reason. In our simulations, roughly one-third of the FTDEs and half of the PTDEs are produced by the leftover stars (Table 2). These results suggest that a sizable fraction of the PTDEs and FTDEs may not be well characterized by the standard light-curve models derived from normal star disruptions, since the internal structure of the leftover stars should differ from the normal stars (Goicovic et al. 2019; Ryu et al. 2020) and hence misinterpret their nature. Currently, the internal structure of the leftover stars after long-term evolution is still unclear, which is an interdisciplinary problem that needs the efforts of both the hydrodynamic simulation and the stellar evolution (see, for example, the method proposed by Goicovic et al. 2019).

In future work, we will initialize the stellar system with a mass spectrum and turn on stellar evolution routines implemented in NBODY6++GPU, which contains more realistic mass–radius relations for stars of different masses and evolutionary stages. These new features may impact the overall rate of FTDEs and have a significant impact for PTDEs, as shown by the recent work of Bortolas (2022).

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