Holographic Nuclear Matter in AdS/QCD

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(February 1, 2008)

Abstract

We study the physics with finite nuclear density in the framework of AdS/QCD with holographic baryon field included. Based on a mean field type approach, we introduce the nucleon density as a bi-fermion condensate of the lowest mode of the baryon field and calculate the density dependence of the chiral condensate and the nucleon mass. We observe that the chiral condensate as well as the mass of nucleon decrease with increasing nuclear density. We also consider the mass splitting of charged vector mesons in iso-spin asymmetric nuclear matter.

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1 Introduction

Inspired by AdS/CFT correspondence\(^1\), several attempts to construct a holographic model of QCD appeared recently, in both bottom-up\(^2, 3, 4\) and top-down\(^5\) approaches. The morale is to introduce a dynamically generated additional space, which roughly corresponds to the energy scale of the field theory, and try to construct a holographic dual model that captures important aspects of the original 4D field theory, such as large N expansion, confinement, and chiral symmetry breaking et cetera. The amazing observation is that these non-perturbative aspects of strongly coupled field theory, which are highly non-trivial to analyze, can be described in simple terms in the 5D holographic dual model, sometimes called AdS/QCD model\(^4\). In a sense, the holographic model may be viewed as a relevant effective theory for strongly coupled, non-perturbative field theory. The counter-intuitive aspect at first sight is that this effective theory necessitates a new space dimension generated dynamically. Note that this does not necessarily mean that the additional dimension is physical; in view of the 4D field theory, it is simply a derived notion that facilitates a magically simple description of important aspects of strongly coupled field theory.

The origin of the simplicity in the 5D dual model is traced back to the large N limit as was the case in the original AdS/CFT correspondence in string theory. We should thus keep in mind that a usual simple, tree-level calculation in the holographic dual model is capturing the leading N contributions, and we are bound to suffer from sub-leading corrections.\(^5\) Within this limitation, the AdS/QCD model may be taken as a new alternative effective theory of low energy QCD.

The simplest bottom-up construction for AdS/QCD\(^2, 3\), sometimes called the hard-wall model, takes a slice of 5D AdS spacetime

\[
d s^2 = \frac{1}{z^2} \left( -d z^2 + d x^\mu d x^\nu \eta_{\mu\nu} \right),
\]

with \( 0 \leq z \leq z_m \), where \( z_m \) is the IR cut-off corresponding to the confinement scale, which must be fitted to physical observables later. In this paper, we will concern only with \( N_F = 2, (u,d) \) sector of QCD, whose extension to \( N_F = 3, (u,d,s) \) sector will be left open to a future work. Corresponding to the global chiral symmetry \( SU(2)_L \times SU(2)_R \) of QCD, we introduce local gauge fields \( A_L \) and \( A_R \) in the 5D AdS slice, whose values at \( z = 0 \) act as external sources for \( SU(2)_L \) and \( SU(2)_R \) currents respectively. More precisely, \( (A_L)^{ij}_\mu |_{z=0} \) couples to \( (j^\mu)^{ij}_L = \bar{q}^i_L \gamma^\mu q^j_L \), and similarly for \( SU(2)_R \) sector. The chiral symmetry breaking, explicitly by the small current quark mass \( (M_q)_{ij} \bar{q}^i_L q^j_R + \text{h.c.} \), as well as spontaneously by the chiral condensate \( \Sigma^{ij} = \langle \bar{q}^i_R q^j_L \rangle \neq 0 \) is holographically realized by a VEV of a scalar field \( X \) which is bi-fundamental with respect to the gauge

\(^4\)We will later refer AdS/QCD model for phenomenological bottom-up constructions only, but for now we mean both top-down and bottom-up.

\(^5\)There is no framework in phenomenological AdS/QCD models yet to do a systematic \( \frac{1}{N} \) expansion.

\(^6\)Our metric convention is \( \eta_{MN} = \text{diag}(+ − − − −) \), and we denote 5D coordinates by capital letters while 4D coordinates are written with Greek letters. We will follow mostly the convention in Ref.\(^2\).
group $SU(2)_L \times SU(2)_R$, 

$$\langle X(z) \rangle = \frac{1}{2} M_q z + \frac{1}{2} \Sigma z^3 = \left( \frac{1}{2} m_q z + \frac{1}{2} \sigma z^3 \right) \mathbf{1},$$ 

(2) 

where in the last equality, we assume iso-spin symmetry for simplicity.\footnote{Iso-spin violating effects from $m_u \neq m_d$ are neglected in this work.} From the standard 5D action for the field $X$, 

$$S_X = \int dz \int dx^4 \sqrt{G_5} \, \text{Tr} \left( |\nabla X|^2 + 3|X|^2 \right),$$ 

(3) 

with $\nabla X = \partial X - i A_L X + i X A_R$, the above VEV breaks $SU(2)_L \times SU(2)_R$ to the diagonal iso-spin $SU(2)_I$ as expected. The tower of (pseudo)vector and scalar mesons, including pions, arise from the normalizable KK modes of the above 5D fields. In addition to the parameters appearing in the above, the 5D $SU(2)_L \times SU(2)_R$ gauge coupling constant in 

$$S_g = \int dz \int dx^4 \sqrt{G_5} \, \text{Tr} \left( -\frac{1}{4g_5^2} F_{MN} F^{MN} \right),$$ 

(4) 

specifies the model completely for the two-flavor meson sector. After fitting these parameters, the physical observables are predicted, which compare well with experiments\cite{2, 3}. 

We point out that this phenomenological model in fact encodes essential ingredients of the top-down model of Sakai-Sugimoto\cite{5}, in that the relevant 5D part of the 10D spacetime is approximated by an AdS slice and the world volume gauge fields on two branches of D8 branes are identified with $A_L$ and $A_R$ respectively, whose breaking to the diagonal $SU(N_F)_I$ is achieved by tachyon condensation $\langle X \rangle \neq 0$ that joins the two D8 branes. 

Since the nucleons, i.e. the protons and neutrons, are integral objects in QCD, it is pertinent to include them in the AdS/QCD model. An early attempt to see the high spin versus mass Regge trajectory was pioneered by Brodsky-Teramond\cite{6}, but recently Ref.\cite{7} focused on the lowest spin $\frac{1}{2}$ nucleons with chiral symmetry breaking effects carefully included, and provided us with a concrete AdS/QCD model with holographic nucleons. This will be our framework for studying the physics of finite nuclear density. 

We comment that the model in Ref.\cite{7} again mirrors the salient aspects of the top-down description of spin $\frac{1}{2}$ baryons in the Sakai-Sugimoto model \cite{8}. Spin $\frac{1}{2}$ baryons from quantized small instanton-solitons on the D8 brane \cite{9, 10, 11} are effectively described by a 5D Dirac spinor field of $SU(2)_F$ fundamental. As we identify two branches of D8 brane as representing separate $SU(2)_L$ and $SU(2)_R$ sectors in the phenomenological AdS/QCD model (which are combined to $SU(2)_I$ at the IR), the 5D Dirac spinor on the two branches of D8 brane will corresponds to two separate 5D Dirac spinors in the AdS/QCD model, which are fundamental under $SU(2)_L$ and $SU(2)_R$ respectively. Indeed, this is the field content of the model in Ref.\cite{7}. 

Chiral symmetry is believed to be restored at high density and/or temperature, such as in the core of neutron stars or in the early stage of fire balls in the relativistic heavy
ion collisions. Since the discovery of the pulsar [12], neutron stars have provided extreme environments to test the validity of general relativity, especially the gravitational wave radiation [13] and the emission of short hard gamma ray bursts [14] from neutron star mergers. Many observatories to detect the gravitational waves (such as LIGO, GEO, and Virgo) and gamma-ray bursts (such as Swift and HETE II) from neutron star mergers are in operation or under development. Neutron stars can also set-up nuclear matter of extreme density in which chiral symmetry is restored. Various phase transitions, such as kaon condensation [15], are suggested in the dense core of neutron stars [16, 17]. Even though the core of neutron star cannot be observed directly, the maximum mass of neutron star which can be observed strongly depends on the equation of state of neutron star [17]. The change in the maximum mass of neutron stars results in the change in the ratio of neutron star-black hole binaries to double neutron star binaries, which can be detected by the gravitational wave observation [18, 19]. In addition, the inner structure of neutron star will be revealed by a detailed pattern of the detected gravitational wave radiation from the gravitational wave observatories. Hence theoretical investigation on the physics of extreme dense matter is very important in order to understand the physics of the neutron stars.

At high temperature, in addition to the chiral phase transition, QCD deconfinement phase transition from hadronic matter to the perturbative (weakly interacting due to asymptotic freedom) quark gluon plasma (QGP) is expected. Such QGP states might have been formed at the early stages of the evolution of our Universe after Big Bang. However, experimental results of the recent relativistic heavy collision (RHIC) indicate that the matter after deconfinement (and chiral) phase transition is nothing like a weakly interacting perturbative QGP [20, 21]. Instead, the matter formed after phase transition is a strongly interacting quark gluon liquid. These results initiated very active research on the physics of hot and dense matter after chiral and deconfinement phase transition [22, 23, 24, 25].

Since the properties of nuclear matter cannot be calculated from the fundamental QCD Lagrangian due to strong coupling, various effective theories, such as chiral perturbation theory, have been introduced to treat nuclear matter phenomenologically. Most of the parameters in these approaches were fixed by the experimental values at low densities (from vacuum to normal nuclear matter density), at which chiral symmetry is spontaneously broken. Hence, the validity of the extrapolation of the effective theories, which were built in the symmetry broken phase, to high densities where chiral symmetry is restored, may be questioned [16]. In this respect, recent holographic dual AdS/QCD approach which is based on the chiral symmetry at its construction provides a new tool to investigate the physics in the symmetry restored phase.

In this work, we introduce a new way of describing nuclear matter in the framework of AdS/QCD model with holographic baryon fields. We turn on the nuclear density in mean field approach by a non-zero 5D bi-nucleon condensate, and this subsequently affects various other holographic fields, such as $X$ and $U(1)_B$ gauge field $V$, to result in several physical consequences that we study in this paper. We point out the relation between our approach and the recent approaches by baryon chemical potential [26, 27, 28, 29].
spirit of AdS/CFT correspondence, the quark chemical potential in 4D QCD is encoded in the boundary value of the 5D $U(1)_B$ bulk gauge field, $\mu_q \bar{\psi} \psi \leftrightarrow V_0(x,z) = \mu_q + \cdots$. This is quite similar to a way of introducing various chemical potentials in chiral perturbation theory, where the global chiral symmetry is promoted to a local gauge one, and a chemical potential is introduced as the time component of the gauge potential $[32, 33]$. The physics behind this approach can be thought of as follows. When we introduce finite density of baryons as instanton-solitons, they are energetically pulled down towards IR boundary and deposit there. From the Chern-Simons coupling, these IR localized instanton-charge density will source the time-like component of the 5D $U(1)_B$ gauge field $V_0$, and the vector potential profile of non-zero baryon chemical potential results. A hidden assumption in this picture is that the IR-deposited baryons have a $\delta$-function localized wavefunction at the IR boundary, and we solve free EOM for $V_0$ in the bulk without describing IR-localized baryons explicitly. Although this is correct as a first approximation, our analysis based on the spread nucleon profile along $z$-direction can be viewed as a more precise analysis of the situation. We also comment that the interaction terms of $V_0$ and meson fields, say $\mathcal{H}$, feature the commutator structure, $\mathcal{L}_{\text{int}} \sim [V_0, \mathcal{H}]$, and therefore the chemical potential $\mu_q$ will be inoperative to neutral mesons, with an exception of Chern-Simons couplings studied in $[29]$.

After briefly reviewing some details about our AdS/QCD model with holographic baryon fields in the next section, we evaluate the density dependence of the chiral condensate and the nucleon mass. We also calculate the mass splitting of charged vector mesons in the iso-spin asymmetric nuclear matter. Finally, we re-analyze the in-medium nucleon mass in a more phenomenological approach, where we use the density dependence of the chiral condensate determined through Hellmann-Feynman theorem and the Gell-Mann-Oakes-Renner relation$[41]$. In this case, we find the nucleon mass drops drastically with increasing density up to the normal nuclear matter number density $\rho_0(\sim 0.16 \text{ fm}^{-3})$.

2 The model with holographic baryons

In this section, we briefly summarize the model in Ref.$[7]$ to set-up the stage. For simplicity, for $N_F = 2$ meson sector we take the simplest hard-wall AdS/QCD model that was explained in the Introduction. To describe spin $\frac{1}{2}$, iso-spin $\frac{1}{2}$ baryons in this set-up, with the nucleons (proton and neutron) as the lowest iso-spin doublet, we introduce 5D (Dirac) spinor fields $N$ corresponding to the 4D baryon operators $\mathcal{O}$ with the same spin and charges in the original QCD. Note that once we introduce a field in the 5D dual model, we will actually get a tower of 4D states as we can equivalently describe the system by Kaluza-Klein reduction along our compact extra dimension. The idea behind this is that these spin $\frac{1}{2}$, iso-spin $\frac{1}{2}$ 4D operators $\mathcal{O}$ in QCD can create/annihilate the whole spectrum of 4D baryons $B_n$ with the same quantum numbers,

$$\langle B_n | \mathcal{O} | 0 \rangle \neq 0 \quad , \quad n = 0, 1, 2, \cdots$$

(5)
and in the holographic dual model, these $B_n$'s are packed into a single 5D spinor field $N$ (of the same quantum numbers) as the normalizable Kaluza-Klein modes along the extra dimension. This is why holography maps a 5D field $N$ to a 4D operator $O$, not to a specific 4D state such as nucleons. Therefore, any holographic model is bound to include the excited spectrum as well, in addition to the lowest state proton and neutron we are interested in.

Looking at the problem more carefully, one may be puzzled by a few things. In the AdS/QCD model side, we have two gauge fields $A_L$ and $A_R$ for $SU(2)_L \times SU(2)_R$ chiral symmetry which are broken to the iso-spin $SU(2)_I$ by a Higgs mechanism with $X$. Any field must have a specific representation under $SU(2)_L \times SU(2)_R$. In QCD side, nucleons (and their excitations) form a doublet under $SU(2)_I$ and since they are states in the broken chiral symmetry phase, it is not at all obvious to even talk about their $SU(2)_L \times SU(2)_R$ quantum numbers.

A guide to way-out may be seen in thinking about how the chiral symmetry breaking is realized in the AdS/QCD side. The symmetry $SU(2)_L \times SU(2)_R$ exists as a definition of the theory, and its breaking is achieved by putting a VEV of $X$; this allows a theoretical or fictitious limit of turning off the VEV of $X$ restoring the chiral symmetry. In QCD side, this would again be an imaginary limit of chiral symmetry restoration, and every operator will have a definite charge under $SU(2)_I$ and since they are states in the broken chiral symmetry phase, it is not at all obvious to even talk about their $SU(2)_L \times SU(2)_R$ quantum numbers.

Although whether or not the above presumptuous chiral symmetry restoration can actually be realized is unclear, it may be taken as simply an intermediate tool to identify 5D field contents and their couplings to the Higgs field $X$. This chiral symmetry restoration limit was in fact considered by 't Hooft in his arguing that the massless chiral nucleon doublet should exist in this limit to match UV anomalies of $SU(2)_L \times SU(2)_R$ with those in IR. More precisely, a left-handed chiral doublet $O_L = (p_L, n_L)^T$ of fundamental representation under $SU(2)_L$, and similarly a right-handed chiral doublet $O_R = (p_R, n_R)^T$ of $SU(2)_R$ exist in this limit, and the chiral symmetry breaking to the diagonal $SU(2)_I$ induces a mass coupling $O_L O_R + h.c.$ that results in the observed nucleon mass $m_N \sim 0.94$ GeV. Hence, we are led to introduce two 5D spinor fields $N_1$ and $N_2$ whose charges under $SU(2)_L \times SU(2)_R$ are $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$, corresponding to the 4D operator $O_L$ and $O_R$ respectively. Moreover, we have to ensure that in the chiral symmetry restoration limit $\langle X \rangle = 0$, we must have a massless left-handed zero mode from $N_1$ and a massless right-handed zero mode from $N_2$ in their Kaluza-Klein reduction. They are the massless chiral nucleon doublets to match chiral anomalies in the restoration limit. This requirement in fact unambiguously fixes the IR boundary conditions for $N_1$ and $N_2$.

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8By $O_{L,R}$, we mean the QCD operators $(q_L q_L q_L)$ and $(q_R q_R q_R)$ that create massless chiral nucleon states, with a slight abuse of notation compared with the above which means the massless states themselves.
The chiral symmetry breaking by $\langle X \rangle \neq 0$ must then introduce a mass coupling between $N_1$ and $N_2$. The relevant gauge invariant coupling of the lowest dimension is

$$L_m = -g \bar{N}_1 X N_2 + \text{h.c.},$$

(6)

with a strength $g$ that must be fitted to reproduce the nucleon mass $m_N = 0.94$ GeV as the lowest mass eigenvalue of the KK spectrum. Note that the would-be massless chiral zero modes from $N_1$ and $N_2$ are lifted to the single massive nucleon state with $m_N = 0.94$ GeV, while the mixing between already massive excitations from separate $N_1$ and $N_2$ will split them into a parity doubling pattern of massive excitations. Here, the definition of 4D parity involves the exchange of $N_1$ and $N_2$, so that the two nearly degenerate states split by mixing $N_1$ and $N_2$ have opposite 4D parity. The parity doubling for excited baryons is thus a prediction of the model.

One last thing to mention is the chirality of the 4D QCD operators $O_L$ and $O_R$ and its relation to the 5D Dirac spinors $N_1$ and $N_2$. One may be puzzled about this since 5D spinor doesn’t have a chirality. The resolution lies in the fact that in 5D the signature of the Dirac mass term flips its sign under parity. The magnitude of the 5D Dirac mass $m_5$ for $N_1$ and $N_2$ is fixed by an AdS/CFT relation

$$(m_5)^2 = (\Delta - 2)^2,$$

(7)

with $\Delta$ being the scaling dimension of $O_{L,R}$, which we take the free theory value $\Delta = \frac{9}{2}$ for simplicity. With a positive $m_5$ for a 5D spinor $N$, it can be seen that only the right-handed component $N_R$ of $N$ survives kinematically near the AdS boundary $z = 0$, while the left-handed component decays fast enough to become a normalizable mode. The non-normalizable $N_R$ near the boundary then couples to a left-handed chiral operator $O_L$ in QCD side by $\tilde{N}_R O_L + \text{h.c.}$. Note that this is consistent with the fact that $N$ and $O_L$ have the same quantum number, and the possible normalizable chiral zero mode from $N$ is left-handed matching to the chirality of $O_L$. With a negative $m_5$, the story is simply reversed. Therefore, the right choice for our model is $m_5 = \frac{5}{2}$ for $N_1$ and $m_5 = -\frac{5}{2}$ for $N_2$ respectively.

In summary, the AdS/QCD model of spin $\frac{1}{2}$, iso-spin $\frac{1}{2}$ baryons is given by the action

$$S_{\text{kin}} = \int dz \int d^4 \sqrt{G_5} \left[ i \tilde{N}_1 \Gamma^M D_M N_1 + i \tilde{N}_2 \Gamma^M D_M N_2 - \frac{5}{2} \tilde{N}_1 N_1 + \frac{5}{2} \tilde{N}_2 N_2 \right],$$

$$S_m = \int dz \int d^4 \sqrt{G_5} \left[ -g \tilde{N}_1 X N_2 - g \tilde{N}_R X^\dagger N_1 \right],$$

(8)

where the covariant derivatives for $N_1$ and $N_2$ include the gauge group $SU(2)_L \times SU(2)_R$ as well as the metric connection, and a single parameter $g$ should be fixed to reproduce

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9 We believe considering anomalous dimension to $\Delta$ would be an important refinement from the analysis in Ref.[7].

10 Our convention is $i\gamma^5 \psi_L = +\psi_L$ and the mass term is $L_m = -m_5 \tilde{N} N$.

11 See also Ref.[31] for the case when both chiral components are normalizable with a specific range of $m_5$. 
the lowest mass nucleon mass \( m_N = 0.94 \text{ GeV} \). By expanding \( N_1 \) and \( N_2 \) in terms of KK eigenmodes, it is easy to find the eigenmode equations that must be solved to find the mass spectrum of 4D spin \( \frac{1}{2} \) baryons. Writing \( N_1(x, z) = f_{1L}(z)B_L(x) + f_{1R}(z)B_R(x) \) and similarly for \( N_2(x, z) = f_{2L}(z)B_L(x) + f_{2R}(z)B_R(x) \), where \( B_{L,R} \) are the components of the 4D eigenmode spinor \( B = (B_L, B_R)^T \) with mass \( m_N \) to be determined, we have

\[
\begin{pmatrix}
  \partial_z - \Delta & -g(X) \\
  -\frac{g(X)}{z} & \partial_z - \Delta
\end{pmatrix}
\begin{pmatrix}
  f_{1L} \\
  f_{2L}
\end{pmatrix} = -m_N \begin{pmatrix}
  f_{1R} \\
  f_{2R}
\end{pmatrix},
\]

with \( \Delta = \frac{9}{2} \) in our case) in general. As mentioned before, the existence of the chiral zero modes when \( \langle X \rangle = 0 \) requires us the IR boundary condition \( f_{1R}(z_m) = f_{2L}(z_m) = 0 \). From the meson sector, Ref.\[2, 3\] found the best fit for \( \langle X \rangle = \frac{1}{2}(m_q z + \sigma z^3) \) with \( m_q = 2.34 \) MeV and \( \sigma = (311 \text{ MeV})^3 \), as well as the IR cut-off \( z_m = (330 \text{ MeV})^{-1} \). Then, the only remaining parameter of the theory is the dimensionless coupling \( g \), which was found to be \( g = 9.3 \) in Ref.\[7\] to reproduce \( m_N = 0.94 \text{ GeV} \) as a lowest mass eigenvalue.

## 3 Finite nuclear density in AdS/QCD

The baryon chemical potential, introduced as a background for the time component of the bulk \( U(1)_B \) gauge field \[26, 27, 28, 29\], is of limited use for some observables due to a specific commutator structure of the interaction terms. An example would be its effect to the chiral condensate which is encoded in the VEV of the scalar \( X \) because \( DX = \partial X - i[V, X] \). In the present work, we introduce the nuclear density through the mean field of the 4D nucleon-bilinear, \( i.e., \rho_s = \langle \bar{\psi}(x)\psi(x) \rangle \), where \( \psi(x) \) is the 4D nucleon field, and \( \rho_s \) is the iso-scalar baryon number density.\[12\] From the term \( \mathcal{L}_m = -gN_1XN_2 + \text{h.c.} \), we can discuss its effects on the chiral condensate, et cetera.

### 3.1 In-medium chiral condensate

Without nuclear density, the Yukawa coupling \( \mathcal{L}_m \) in Eq. \[8\] is a purely cubic interaction term, and it does not enter the equation of motion for the bulk scalar field \( X \) or its VEV, \( \langle X \rangle \equiv X_0 \). As is well-known, \( X_0 \) has the following profile

\[
X_0(z) = \left( \frac{1}{2}m_q z + \frac{1}{2}\sigma_0 z^3 \right)1,
\]

where \( m_q \) is the current quark mass and \( \sigma_0 \sim (0.330 \text{ GeV})^3 \) is the chiral condensate in the vacuum. For simplicity, we take the chiral limit, \( m_q = 0 \), as our result is expected

\[12\] Note that the iso-scalar density \( \rho_s \) is roughly equivalent to the baryon number density \( \rho_B = \langle \bar{\psi}(x)\psi(x) \rangle \) at low density \[15\]: \( \rho_s \approx \rho_B - \langle \frac{p^2}{m_q^2} \bar{\psi}(x)\psi(x) \rangle \), where \( p \) is the baryon momentum.
not to change much from a small quark mass. As we turn on a finite nuclear matter and introduce the non-zero mean field $\langle \bar{N}_1 N_2 + \bar{N}_2 N_1 \rangle$ of the baryon bi-linear,

$$\langle \bar{N}_2 N_1 + \bar{N}_1 N_2 \rangle = (f_{2R}^2 - f_{1R}^2) \langle \bar{\psi}(x) \psi(x) \rangle \implies (f_{2R}^2 - f_{1R}^2) \rho_s,$$

where $N_1 = f_{1L}(z) \psi_L(x) + f_{1R}(z) \psi_R(x)$, $N_2 = f_{2L}(z) \psi_L(x) + f_{2R}(z) \psi_R(x)$, and we used the important parity relation

$$\begin{pmatrix} f_{1L} \\ f_{1R} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} f_{2L} \\ f_{2R} \end{pmatrix},$$

which is a consequence of even parity of the nucleons, the interaction \[8\] modifies the equation of motion for $X$ and hence the chiral condensate. We assume that only the lowest lying baryons, proton and neutron, have non-zero mean field value of their bi-linear, since the higher baryon resonances cost too much energy to form a Fermi sea. At very high density, of course, we will have to consider dense matter of higher resonances, too. The equation of motion for $X$ is now

$$\left( \partial_z^2 - \frac{3}{z} \partial_z + \frac{3}{z^2} \right) X = \frac{1}{4} g z \left( f_{2R}^2 - f_{1R}^2 \right) \rho_s,$$

and we solve it numerically with $\rho_s \sim \rho_B$ as a varying parameter.

It is an important question to specify a physically sensible IR boundary condition for $X$ at $z = z_m$, while our chiral condensate $\sigma$ sits in the UV asymptotic $X(z) \sim \frac{1}{2} \sigma z^3$ as $z \to 0$ in the massless limit.\[13\] Physically, what drives the chiral condensate in the vacuum or the VEV of $X$ would be a strong QCD IR dynamics, which may be attributed to some unspecified dynamics on the physical IR brane at $z = z_m$ in the present AdS/QCD model. We naturally expect that this IR-localized dynamics is not very much affected by the presence of our bulk bi-fermion condensate, as we imposed the IR boundary condition on our baryons such that their bi-linear vanishes at $z = z_m$. Therefore, we are led to identify $X(z)$ at $z = z_m$ with its value in the vacuum, $X(z_m) = \frac{1}{2} \sigma_0 z_m^3$, as our IR boundary condition. We show our numerical result for the ratio of the density dependent chiral condensate to that in the vacuum, $\frac{X}{\sigma_0}$, in fig.\[1\] where $c = \rho_B / \rho_0$ with $\rho_0$ being the normal nuclear matter density. We see that the chiral condensate is a decreasing function of the nuclear density.

### 3.2 In-medium masses of nucleons and charged vector mesons

In this subsection, we calculate the density dependence of the nucleon mass as well as the mass splitting of the charged vector mesons. We first consider the nucleon mass. As seen in the previous subsection, the profile of $X$ depends on the iso-scaler density, and plugging it back to our 5D mass coupling \[8\], the nucleon wave function equation \[9\] will also get a repercussion resulting a shift in its eigenvalue nucleon mass $m_N$, not to mention its wave

\[13\] This means that the UV boundary condition is $\frac{X(z)}{z} \to 0$ as $z \to 0$. 

8
functions. In fact, for a fully consistent analysis, we should solve the equation (13) for $X$ and the equation (9) for the nucleon wave functions $f_{(1,2)(L,R)}$ simultaneously, as we have a cubic interaction (8) between them. For this purpose, we performed an iterative analysis until we get a stabilized solution. Therefore our result is numerically reliable, and the result is shown also in Fig. 1. We observe that the nucleon mass decreases in the finite nuclear density.

In light of the results from other conventional QCD effective theories, for example Ref. [36, 39, 40, 41], both the in-medium nucleon mass and the chiral condensate have been argued to drop by $\sim 20\%$ at $\rho_B = \rho_0$ ($c = 1$). Our results for the density dependence from Fig. 1 seem a little smaller than these previous estimates. This might be attributed to the fact that our analysis is based on various crude assumptions in the AdS/QCD model, but it is also worth exploring possible improvements of the model. One thing is that we have ignored a back-reaction on the metric due to the finite nuclear density, for example, see [34, 35]. Suppose we incorporated the back-reaction due to the nucleon density, then a modified metric would depend on the nuclear density, and would introduce additional density dependence into the game other than Fig. 1. Changing the metric would correspond to setting-up a new effective theory around the Fermi surface studied in [36, 37, 38], similar to the spirit of quasi-particles. Instead of delving into this endeavor, one possible phenomenological way of incorporating this new density dependence is to simply accept the density dependent chiral condensate determined from other low energy effective theories, say in Ref. [41], and to focus on solving the nucleon mass (9) only. This will be discussed in the next section.

We now move on to the mass of vector mesons at finite density. The equation of

![Figure 1: Density dependence of the nucleon mass and the chiral condensate normalized with respect to those in the vacuum. Here $c = \rho_B/\rho_0$.](image)
motion for the time component of the bulk vector field turns out to be
\[
\left( \partial_z^2 - \frac{1}{z} \partial_z \right) V_0^3(z) = -g_5^2 \frac{1}{z^3} (f_{1L}^2 + f_{1R}^2) (\rho_p - \rho_n),
\]
where \( \rho_p(\rho_n) \) is the proton (neutron) number density, and the index 3 represents the iso-spin quantum number. Here we assume an iso-spin asymmetric environment \( \rho_p \neq \rho_n \), which in turn induces an iso-spin chemical potential. This means that once the iso-spin density is turned on, \( V_0^3 \) will develop the following classical profile, \( V_0^3 = c_1 + c_2 z^2 \) as \( z \to 0 \), as a consequence of solving (14), where \( c_1 \) is the iso-spin chemical potential \( \mu_I = \mu_p - \mu_n \) and \( c_2 \) corresponds to the iso-spin density \( \rho_I (= \rho_p - \rho_n) \). Let us denote \( \rho_p = x \rho_B \) and \( \rho_n = (1 - x) \rho_B \). In solving the above equation (14), one may take two different approaches. One would simply take the density as an input in the UV boundary condition, and impose the Neumann boundary condition \( \partial_z V_0^3 = 0 \) at IR \( z = z_m \) without breaking the symmetry by boundary condition. This is in accord with our motivation that a nuclear density is introduced as a bulk effect, without resorting to an unspecified IR-localized baryon dynamics to generate \( V_0^3 \) profiles. Another approach would be to treat both \( \mu_I \) and \( \rho_I \) as inputs, as we normally know the one when we are given the other. For example, \( \mu_{p,n} = \sqrt{m_N^2 + (k_F^p)^2} \) and \( \rho_{p,n} = (k_F^p)^2 / (3 \pi^2) \) relate the two by \( k_F^p \). In this case, we start from the given UV \( z \to 0 \) asymptotic by \( \mu_I \) and \( \rho_I \), and solve (14) into the IR region. In the present work, we adopt the latter scheme.

To obtain the in-medium mass of the charged vector mesons, \( V_{\mu}^\pm \), which are linear combinations of \( V_{\mu}^1 \) and \( V_{\mu}^2 \), we consider their equation of motion in the presence of finite nuclear density. With a Kaluza-Klein mode expansion, \( V_{\mu}^\pm (x, z) = g_5 \sum_n f_n^\pm(z) V_{\mu}^{(n)}(x) \), it is not difficult to arrive at
\[
\left( \partial_z^2 - \frac{1}{z} \partial_z + m_n^2 \pm 2F(z) m_n^2 + F(z)^2 \right) f_n^\pm(z) = 0,
\]
where \( F(z) \) is the solution of the previous equation (14) for \( V_0^3 \). Here we impose the following boundary conditions: \( f_n^\pm (0) = 0 \), \( \partial_z f_n^\pm (z_m) = 0 \) for a normalizable eigen-function. We focus on the \( n = 1 \) mode, which are the charged \( \rho \)-mesons, and the results are shown in Fig. 2. Note that we are considering negative iso-spin chemical potential, which is relevant for the neutron star. As \( |\mu_I| \) increases, the energy cost of putting additional particle with a negative iso-spin quantum number into the system will decrease, and the mass of the particle will be reduced by \( |\mu_I| \) [33]. Our results in Fig. 2 confirm this physics. We remark, however, that as \( x \to 0 \), i.e., as \( |\mu_I| \) increases, it is known that the pion condensation comes into the story [33], and the ground-state of the system needs to be redefined, and therefore our results in Fig. 2 are no longer valid at small \( x \).

4 More phenomenological approach

As mentioned in the previous section, density dependence of chiral condensate in our work is rather weak, and this leads to the corresponding weak dependence of the nucleon
mass on the nuclear density. If one is simply interested in the nucleon mass in the finite nuclear density, it is logically meaningful to simply take the chiral condensate from other effective theories that one might trust more, and to just focus on its effects to the nucleon mass through the 5D mass coupling (8). More explicitly, we take $X = c_2 z^3$ with $c_2 = \sigma(\rho_B)/2$, where $\sigma(\rho_B)$ is the density dependent chiral condensate from the results of previous estimates, and we solve (9) to re-calculate the in-medium nucleon mass. Another possibility may be to use the in-medium $\rho$-meson mass, instead of the vacuum $\rho$-meson mass $\sim 770$ MeV, to fix the density dependent IR cutoff $z_m$, but we will not pursue this in the present work.

We adopt the model-independent $\sigma(\rho_B)$ in Ref.[41], which is valid at low density.\footnote{Higher order corrections to the model independent in-medium chiral condensate are studied in Ref.[42], and it is shown that there are at most 15% corrections up to two times the normal nuclear matter density.}

$$\sigma(\rho_B) \approx \sigma(\rho_B = 0) \left( 1 - 0.37 \frac{\rho_B}{\rho_0} \right). \quad (16)$$

and plugging (16) into the KK mode equation (9) for the nucleons, we obtain the in-medium nucleon mass as shown in Fig.3. At normal nuclear matter density, it drops about 30%, which seems to agree with the results from other low energy effective theories of QCD \cite{36,39,40,41}. This indicates that the nucleon mass is rather closely related to the chiral condensate.
5 Summary

In this paper, we study the physics of finite density of nuclear matter in the framework of AdS/QCD with holographic baryon fields. Based on a mean field type approach, we turn on the nucleon density by bi-nucleon condensates, and calculate the density dependence of the chiral condensate and the nucleon mass. Our result shows that the chiral condensate as well as the nucleon mass drop with an increasing nucleon density. We also investigate a mass splitting of the charged $\rho$-mesons in an iso-spin asymmetric nuclear matter, and the result is compatible with our expectation from a simple physical reasoning. We finally study the in-medium nucleon mass in a more phenomenological approach and find that the decrease of the nucleon mass in the nuclear matter is strongly correlated to the density dependence of the chiral condensate.

It would be quite interesting to extend our work to the three flavor case including strange quarks. Especially, the scaling of the kaon effective mass at high density is very important for the neutron star equation of state, because kaon is the least massive boson with a strange quantum number. This is left to a future work.

Acknowledgments

We thank Mannque Rho for useful comments, and Shin Nakamura, Piljin Yi for helpful discussions. C.H.L. is supported by Grant No. R01-2005-000-10334-0 (2005) from the Basic Research Program of the Korea Science & Engineering Foundation. H.U.Y. is partially supported by the Korea Research Foundation Grant (KRF-2005-070-c00030), and thanks Kimyeong Lee for a financial support from his fund.
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