Constraining early dark energy with gravitational waves before recombination

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We show that the nonperturbative decay of ultralight scalars into Abelian gauge bosons, recently proposed as a possible solution to the Hubble tension, produces a stochastic background of gravitational waves which is constrained by the cosmic microwave background. We simulate the full nonlinear dynamics of resonant dark photon production and the associated gravitational wave production, finding the signals to exceed constraints for the entire parameter space we consider. Our findings suggest that gravitational wave production from the decay of early dark energy may provide a unique probe of these models.

Measurements of the current expansion rate $H_0$ as inferred from the acoustic peaks in the cosmic microwave background radiation (CMB) are in tension with the value obtained from local measurements [1–3]. Rather than alter the expansion history at intermediate redshifts, new physics introduced to resolve the tension must alter the absolute scales of the low- or high-redshift anchors of the cosmic distance scale [4, 5]. Early-Universe resolutions thus focus on altering the high-redshift anchor, the CMB sound horizon at recombination. For a recent review, see Ref. [6].

In particular, an increased expansion rate before recombination decreases the sound horizon. However, simply adding more radiation [7, 8] also changes the damping scale in a way that is increasingly disfavored by high-precision measurements of the high-multipole damping tail [2, 9, 10]. To avoid this, one class of proposed solutions, so-called early dark energy (EDE) models, postulates an additional energy component that is only transiently important near recombination [11–18]. These proposed solutions supposedly relieve the tension between the early and late datasets; however, see Ref. [19].

In the simplest EDE implementations, a scalar field is initially frozen up its potential in a homogeneous configuration. The field’s mass is tuned such that it begins to evolve near matter-radiation equality, oscillating about the minimum of its potential. In order to redshift away fast enough (at least as fast as radiation), the potential must have no quadratic term about its minima. From a particle physics perspective, this requires an explanation; to avoid such extreme fine-tuning, Ref. [18] instead proposes a model of a decaying ultralight scalar (dULS). Instead of oscillating about a peculiar potential and redshifting away, the EDE scalar field decays resonantly to dark radiation during oscillations about a quadratic minimum.

Nonperturbative or resonant particle production is a common feature of early-Universe preheating after inflation (for reviews, see [20–22]). Substantial study has established that these violent processes can also lead to the copious production of gravitational waves [23–38]. The wavelength of the produced gravitational waves (GWs) is determined by the characteristic scale of particle production, which must be smaller than the horizon size. During preheating after inflation this restricts the production to frequencies from MHz to GHz, well beyond the reach of current or planned detectors [39–45].

By contrast, in order to resolve the Hubble tension, particle production due to the decay of the ultralight scalar must occur near the time of matter-radiation equality. Since resonant particle production occurs at scales near the horizon scale at that time, gravitational wave emission occurs at current-day frequencies near $10^{-16}$ Hz. CMB anisotropies constrain stochastic gravitational waves with peak sensitivity at present-day frequencies near $10^{-17}$ Hz [48–51]. In this paper, we confront models of ultralight scalar decay into dark photon with these constraints.

**Background.**—Following Ref. [18], we study the resonant decay of an ultralight axion $\phi$ into a (dark) Abelian gauge field $A_\mu$ described by the action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4j} \phi F_{\mu\nu} \tilde{F}^{\mu\nu} \right].$$

(1)

Here $f$ is the axion decay constant and $\alpha$ is a dimensionless coupling that parameterizes the rate and efficiency of energy transfer. The field strength tensor of the dark photon is $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, while its dual is $F^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}/2$, where $\epsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol with

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1 However, high-frequency gravitational waves contribute to the energy budget of the Universe as radiation. Constraints on the gravitational-wave energy density from $N_{\text{eff}}$ [46, 47] can be used to indirectly restrict preheating scenarios [36–38].
convention $\epsilon^{0123} = 1/\sqrt{-g}$. Following Ref. [18], we take the standard axion potential $V(\phi) = m_\phi^2 f^2 (1 - \cos \phi/f)$. We set $c = h = k_B = 1$ and use $M_{\text{pl}} = 1/\sqrt{8\pi G_N}$ to denote the reduced Planck mass. We work with the “mostly-plus,” conformal Friedmann-Lemaître-Robertson-Walker (FLRW) metric, $(\text{FLRW})$ metric, $g_{\mu\nu} = a^2 \eta_{\mu\nu} = a^2 \text{diag} \left(-1,1,1,1\right)$, using primes to denote derivatives with respect to conformal time $\tau$.

The classical equations of motion,

$$A''_\nu = \partial_\nu \partial_\mu A_{\mu} + \eta_{\rho\sigma} \alpha_f \partial_\mu \phi \left( \frac{1}{2} \sqrt{-g} \epsilon^{\alpha\beta\rho\sigma} F_{\alpha\beta} \right) \tag{2}$$

$$\phi'' = \partial_\mu \partial_\nu \phi - 2\mathcal{H} \phi' - a^2 \frac{dV}{d\phi} - a^2 \frac{\alpha}{4f} F_{\mu\nu} \tilde{F}^{\mu\nu}, \tag{3}$$

permit solutions in which fluctuations of the gauge fields are exponentially enhanced via a tachyonic instability sourced by a homogeneous, rolling axion. Initially, a cosmological axion has some static homogeneous component $\langle \phi \rangle = \theta f$, expressed in terms of the initial misalignment angle $\theta$. As a result, the axion’s energy is dominated by its potential, acting as a source of early dark energy which could alleviate the Hubble tension [13]. On the other hand, to linear order the helical polarizations of $A_i$ obey [52]

$$A_{\pm}''(\mathbf{k}) + k \left( k^2 \mp \frac{\alpha}{f} \langle \phi' \rangle \right) A_{\pm}(\mathbf{k}) = 0. \tag{4}$$

Thus, once the axion begins to oscillate (when the Hubble parameter drops below $\sim m_\phi/3$ [53]), its non-negligible background velocity causes one of the two polarizations to undergo tachyonic resonance, i.e., to be amplified exponentially for modes $k < \alpha/f \times \langle \phi' \rangle$.

As the axion crosses zero, $\langle \phi' \rangle$ changes sign and so amplifies the other polarization. Eventually, the gauge field fluctuations become so large that nonlinear effects begin to fragment the axion background, ending the phase of tachyonic resonance. Both the initial exponential gauge field production and subsequent nonlinear dynamics can source a significant gravitational wave background. Gravitational waves correspond to the tensor part of perturbations to the spatial part of the spacetime metric,

$$h''_{ij} - \partial_k \partial_k h_{ij} + 2\mathcal{H} h'_{ij} = \frac{2}{M_{\text{pl}}^2} T_{ij}^{TT}, \tag{5}$$

where $T_{ij}^{TT}$ is the transverse and traceless part of the stress-energy tensor $T_{ij}$.

We employ numerical simulations in order to fully capture resonance, the nonlinear dynamics which terminate energy transfer, and the resulting production of gravitational waves. We solve the classical equations of motion Eqs. (2), (3), and (5) in a homogeneous $\Lambda$CDM cosmology, self-consistently including the contribution of the dULS sector to the expansion rate. We discretize these equations onto a three-dimensional, periodic, regularly-spaced grid, computing spatial derivatives via fourth-order centered differencing and utilizing a fourth-order Runge-Kutta method for time integration. All results presented use grids with $N^3 = 768^3$ points, side-length $L = 10/m_\phi$, and a timestep $\Delta \tau = \Delta z/10 = L/10N$. We implement simulations using pystella [37, 38] and provide details on our algorithm, initial conditions, and convergence tests in the Appendices.

**Results.**—In our simulations of the decaying ultralight scalar model, we consider benchmark scenarios from Ref. [18], taking $m_\phi = 10^{-27} - 10^{-26}$ eV so that the dULS sector transitions from dark energy to matter-like behavior around the favored redshift $z_c \approx 15600$, and we set $f = 1.5 \times 10^{17}$ GeV so that the dULS sector makes up $\sim 3 - 4\%$ of the Universe’s energy budget at its peak. We fix an initial misalignment angle $\theta = 2$ for convenience. We comment later on the dependence of the gravitational wave signal to slight changes in these choices.

We first verify that the dynamics of the dULS sector qualitatively reproduce the effective fluid description employed in Ref. [18] (which we evaluate in more detail in the Appendices). In Fig. 1 we display the energy in the gauge fields $\rho_A$ and the fractional energy in the dULS sector, $\Omega_{\text{dULS}} = (\rho_A + \rho_\phi)/\rho$, as a function of redshift. We vary the coupling $\alpha$ from 50 to 70, spanning values large

![Fig. 1](image-url)
The analysis of Ref. [18] determined that the dULS sector should begin to decay like radiation by a redshift between $\sim 11000$ and $5000$; as a result, if this scenario is to alleviate the Hubble tension, the associated gravitational wave signal will be produced before recombination, $z \approx 1400 - 1100$ [49]. While the couplings we are able to study here only probe transitions to radiation-like behavior at the later end of this interval, our findings offer no reason to expect the signal from models with larger couplings to evade CMB constraints.

We now consider the dependence of gravitational wave production on other model parameters. The scaling of the signal with the axion mass $m_\phi$ is relatively simple. From the transfer function Eq. (A10), the present-day frequency scales as $f \sim k/\sqrt{H M_{pl}} \sim \sqrt{m_\phi/M_{pl}}$. While the axion mass scales out of the dynamics, it has an effect on the initial amplitude of gauge field vacuum fluctuations. A lower mass sets initial fluctuations with lower amplitude, requiring a longer period of resonance to fully deplete the axion’s energy. However, this effect is relatively unimportant and is easily compensated for by a slight increase in the coupling $\alpha$. At smaller masses, resonance begins later; therefore, marginally larger couplings are required in order for the process to complete before recombination (so that the signals are detectable). As can be seen in the bottom panel of Fig. 2, this condition is easily met with $\alpha = 70$ and a variety of masses between $10^{-26}$ and $10^{-27}$ eV. Furthermore, the shape and amplitude of the GW signal itself is qualitatively independent of the axion mass $m_\phi$.

The value of the axion decay constant $f$ is set by the requirement that the dULS sector comprises a fraction of the Universe’s energy between $3 - 4\%$, leaving little room for variation. Namely, at the onset of oscillations, the fractional energy in the axion scales as $\rho_{\phi}/\rho \sim m_\phi^2 (\theta f)^2 / H^2 \sim (\theta f)^2$. In turn, the amplitude of the resulting gravitational wave signal is directly proportional to the square of the fraction of the Universe’s energy residing in its source [56]. Since the signals we find here exceed current constraints by an order of magnitude, we do not expect the constraining power of the GW signal to be sensitive to any uncertainty in the best-fit $\rho_{\phi}/\rho$.

Since $\theta$ sets the amplitude of axion oscillations (and so $\langle \phi' \rangle$), its effect on the dynamics is, to linear order, degenerate with the coupling. However, nonlinear effects (e.g., rescattering of power to higher momenta) become more important with larger couplings $\alpha$, and so our choice of $\theta = 2$ allows reliable simulations with smaller couplings that still transition the dULS sector to a radiation-like state on the required timescales.

As a final investigation, we study whether the gravitational wave signal is significantly polarized. The same axial coupling of gauge fields to the inflaton generates a helical gravitational wave background during inflation [57–61], and can also imprint on the spectrum of gravitational waves produced during preheating [36]. However, in the

![Figure 2](image-url)
The polarization components of the present-day gravitational wave spectrum emitted by recombination (i.e., evaluated at $z \approx 1100$) for $\alpha = 50$ (top) and 70 (bottom). The plus and minus polarizations are in blue and red, respectively, while the total signal is portrayed in dashed black. Both panels fix $m_\phi = 10^{-26}$ eV, $f = 1.5 \times 10^{17}$ GeV, and $\theta = 2$. In solid black is the upper bound as constrained by the CMB, computed by Ref. [50].

Conclusions.—The rapid production of inhomogeneities from resonant particle production can induce a significant GW background. The amplitude of the GW signal is largest (and so offers the most constraining potential) when the GW source comprises a significant fraction of the Universe’s energy budget and occurs close to the horizon scale at the time of emission [56]. The model considered here, which exhibits a tachyonic instability via an axion coupled to dark photons, is especially efficient, as has been shown in the context of preheating after inflation [37, 38] in which case up to the entire energy budget of the Universe may source gravitational waves. New physics which relies on the same mechanism later in cosmological history is subject to constraints from direct probes of stochastic backgrounds of gravitational waves [48, 49]. Models of early dark energy proposed to alleviate the Hubble tension are a prime example, as their success hinges on the new sector making up a substantial ($\mathcal{O}(1\%)$) fraction of the Universe’s energy. Furthermore, the source of early dark energy must soon decay one way or another before recombination, pinning the relevant length scales to those probed by the cosmic microwave background.

In this work, we demonstrate that the decaying ultralight scalar model, as motivated in Ref. [18], produces a background of gravitational waves with a peak spectral energy fraction exceeding $\mathcal{O}(10^{-11})$ at its peak, with a power-law tail extending into the region that is already constrained by the CMB [50]. While we have not studied the entire available parameter space, we show that the requirements for the model to successfully alleviate the Hubble tension generally coincide with those for its gravitational wave signature to be constrained by the CMB. For the parameter space we considered, we find the signal exceeds constraints (on adiabatic modes) by an order of magnitude. Because GWs are actively sourced in this scenario, these constraints are not directly applicable, and we leave a detailed computation of the CMB signatures of these models to future work. We also point out that resonant particle production is not a unique feature to this model. The original single-field models of EDE exhibit similar parametric instabilities which may also emit significant GW backgrounds [15]. More broadly, our findings suggest that stochastic backgrounds of gravitational waves could provide an orthogonal probe with which to constrain models of early dark energy.

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used the Extreme Science and Engineering Discovery Environment (XSEDE) [65], which is supported by National Science Foundation grant number ACI-1548562; simulations were run on the Comet cluster at the San Diego Supercomputer Center through allocation TG-PHY180049. This work made use of the Illinois Campus Cluster, a computing resource that is operated by the Illinois Campus Cluster Program (ICCP) in conjunction with the National Center for Supercomputing Applications (NCSA) and which is supported by funds from the University of Illinois at Urbana-Champaign.

Simulations in this work were implemented with pystella, which is available at github.com/zachjweiner/pystella and makes use of the Python packages PyOpenCL [66], Loo.py [67], mpi4py [68, 69], mpi4py-fft [70], NumPy [71], and SciPy [72].

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Appendix A: Equations and numerical implementation

The dynamical system, our conventions, and our numerical implementation match those of Refs. [37, 38] (see the appendices of Ref. [37] for full details), with relatively minimal differences we describe here.

The energy density and pressure which source FLRW expansion comprise contributions from the standard background ΛCDM model, the axion, and the gauge fields. The spatially-averaged energy density and pressure are

\[ \rho(\tau) = \frac{\phi^2}{2a^2} + \frac{(\partial_\tau \phi)^2}{2a^2} + V(\phi), \]

\[ p(\tau) = \frac{\phi^2}{2a^2} - \frac{(\partial_\tau \phi)^2}{6a^2} - V(\phi), \]

for the axion and

\[ \rho_A(\tau) = \frac{1}{2} \langle E^2 + B^2 \rangle, \]

\[ p_A(\tau) = \frac{1}{6} \langle E^2 + B^2 \rangle \]

for the gauge fields. Above we defined the electric and magnetic fields

\[ E_i = \frac{1}{a^2} (A_i' - \partial_i A_0), \quad B_i = \frac{1}{a^2} \epsilon_{ijk} \partial_j A_k. \]

In our simulations, we consistently include the contributions of Eqs. (A1) to (A4) to the expansion rate alongside the background ΛCDM model.

The axion–dark-photon coupling violates parity and so the resonantly produced gauge bosons are helical (at least initially), which may in principle imprint on the resulting gravitational waves. For details on the decomposition of gravitational waves into the polarization basis, see Ref. [36].

The present-day spectrum \( \Omega_{gw}(a_0) \) is related to that at the time of emission, \( \Omega_{gw}(a_e) \), via

\[ \frac{\Omega_{gw}(a_0)}{\Omega_{gw}(a_e)} = \left( \frac{a_e}{a_0} \right)^4 \rho(a_e) / \rho(a_0), \]

since gravitational waves redshift like radiation. In turn, the present-day frequency of observation is determined from the physical momentum \( k_{phys} = k/a_0 \) via

\[ f = \frac{k_{phys}}{2\pi} = \frac{1}{2\pi} a_e a_0. \]

The entropy of the Standard Model (SM) provides a conserved quantity by which we may compute \( a_e/a_0 \) at any time after the SM thermalizes. The entropy density is

\[ s = 2\pi^2 g_* s(T) T^3 / 45, \]

where \( T \) is the SM temperature and \( g_* \) the number of effective degrees of freedom in entropy. Combining with \( \rho_{rad} = \pi^2 g_* (T)^4 / 30 \) (where \( g_* \) is the effective number of degrees of freedom in energy density) allows one to express \( a_e/a_0 \) in terms of the energy density in radiation, \( g_* \), and \( g_* \). Approximating \( g_* s = g_* \) allows us to evaluate Eq. (A6) at any point after thermalization as

\[ \Omega_{gw}(a_0) h^2 = \frac{\Omega_{rad}(a_0) h^2}{\Omega_{rad}(a_e)} \left( \frac{g_*}{g_* a_e} \right)^{1/3} \Omega_{gw}(a_e). \]

Likewise, Eq. (A7) may be expressed as

\[ f = \frac{k/2\pi a_e}{\sqrt{H(a_e) M_{pl}}} \left( \frac{\Omega_{rad}(a_0)}{\Omega_{rad}(a_e)} H^2 M_{pl}^2 \right)^{1/4} \left( \frac{g_*}{g_* a_e} \right)^{1/12}. \]

While in general \( g_* \) may be evaluated using, e.g., tabulated values from Ref. [73], the effective number of relativistic degrees of freedom is fixed to 3.36 since well before the time of our simulations. The present radiation fraction \( \Omega_{rad}(a_0) h^2 \approx 4.2 \times 10^{-5} \), and the radiation fraction at the time of emission is calculated during the simulation. Plugging in \( H_0 = h \cdot 3.2 \times 10^{-18} \text{ Hz} \) and \( M_{pl} = 3.7 \times 10^{12} \text{ Hz} \),

\[ f = \frac{k/a_e}{\sqrt{H(a_e) M_{pl}}} \Omega_{rad}(a_e) \times 4.33 \times 10^{10} \text{ Hz}. \]

To set initial conditions, we begin by solving for the dynamics of the linearized system. We evolve the Friedmann equations, the homogeneous and linear-order parts of the axion’s equation of motion,

\[ \langle \phi'' \rangle = -2\mathcal{H} \langle \phi' \rangle - a^2 \frac{dV}{d\langle \phi \rangle} - a^2 \frac{\alpha}{4f} \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle, \]

\[ \delta \phi''(k) = -2\mathcal{H} \delta \phi'(k) - \left( k^2 + a^2 \frac{d^2 V}{d\langle \phi' \rangle^2} \right) \delta \phi'(k), \]

and the linearized equation of motion for the gauge field polarizations,

\[ A_{\pm}''(k) = -k \left( k \mp \frac{\alpha}{f} \langle \phi' \rangle \right) A_{\pm}(k). \]

While solving the linearized system, we compute the gauge field contributions to the background dynamics by integrating over the evolved gauge field modes via numerical quadrature (as employed in Ref. [37]). We neglect the backreaction of the gauge fields onto the axion fluctuations in Eq. (A12), which is negligible even well after we initialize the full lattice simulation.

We begin the linear evolution when \( H \gg m_\phi \) (so that all modes are far outside the horizon) and set \( \langle \phi' \rangle = \theta f \) and \( \langle \phi' \rangle = 0 \). The gauge fields are initialized in the Bunch-Davies vacuum,

\[ \langle |A_{\pm}(k)|^2 \rangle = \frac{1}{2\sqrt{k}}, \]

\[ \langle |A_{\pm}(k)|^2 \rangle = \frac{1}{2\sqrt{k}}. \]
The axion acquires a nearly scale-invariant spectrum of fluctuations during inflation,
\[ \langle |\phi(k)|^2 \rangle = \frac{1}{k^3} \frac{H_{\text{inf}}^2}{2\pi^2}, \tag{A16} \]
and \( \delta \phi' = 0 \). As a benchmark value, we take the inflationary Hubble scale \( H_{\text{inf}} = 2 \times 10^9 \) GeV, but our main results are insensitive to whether the axion is even initialized with any fluctuations.

We evolve the linear system until \( H = m_\phi \), at which point we initialize the lattice simulation. The subsequent dynamics are identical to when the lattice simulation is initialized when \( H = 10m_\phi \), and are relatively unchanged even if initialized at \( H = m_\phi / 10 \). Using the background values and power spectra obtained at this point \( (H = m_\phi) \), we initialize the lattice simulation with the procedure described in Ref. [37]. Our simulations use a grid with \( N^3 = 768^3 \) points and conformal box length \( L = 10/m_\phi \), with a timestep \( \Delta \tau = \Delta x / 10 = L/10N \).

### Appendix B: Convergence tests

The simulations we present in the main text require fairly large volumes to capture the IR tail of the gravitational wave spectrum which resides at CMB frequencies, while also sufficient resolution to capture both the initial tachyonic resonance band and the subsequent power transfer to higher momenta via nonlinear effects. As \( N^3 = 768^3 \) represents the largest grid size possible with our current resources, in lieu of an ideal convergence test (fixing the box length \( L \) while increasing \( N \)) we compare results for two box lengths, both with \( N = 768 \). We present this comparison for \( L = 10/m_\phi \) (as used for main results) and \( L = 5/m_\phi \) in Fig. 4, for \( \alpha = 70 \) (the largest coupling for which we present results, which should require the most resolution). Specifically, Fig. 4 depicts for each field the final dimensionless power spectra, i.e., for a field \( f \),
\[ \Delta_f(k)^2 = \frac{1}{2\pi^2} \frac{1}{V} \int \frac{d\Omega}{4\pi} k^3 |f(k)|^2. \tag{B1} \]
The results show excellent consistency, with expected discrepancy at the largest scales in the simulation (due to statistical variance) and negligible differences at the Nyquist frequency.

### Appendix C: Comparing to the two-fluid description

We now compare the results of the full numerical simulation, the numerical solution to the linearized system, and the two-fluid model detailed in Ref. [18]. The latter is described by the effective fluid equations
\[ \rho'_\phi + 3aH(1 + w_\phi)\rho_\phi = -a\Gamma(\tau)\rho_\phi, \tag{C1} \]
\[ \rho'_A + 4aH\rho_A = a\Gamma(\tau)\rho_\phi, \tag{C2} \]
where the axion equation of state is
\[ w_\phi(a) = -1 + \frac{1}{1 + (a_c/a(\tau))^3} \tag{C3} \]
and the decay rate is parameterized by
\[ \Gamma(\tau) = \frac{m_\phi}{1 + (a_r/a(\tau))^p} \tag{C4} \]
with \( p = 30 \). In this effective description, the scalar field begins to oscillate at \( a_c \), while the dULS sector transitions to radiation domination at \( a_r \). We display the evolution of \( \rho_\phi \) and \( \rho_A \) in Fig. 5. Qualitatively, the two-fluid model reproduces the energy transfer, but cannot

\[ \begin{align*}
\Delta^2_{\phi, f} & \approx 0.1 \times 10^{-2} \left( \frac{f}{10^{-10}} \right)^2 \left( \frac{\tau}{10} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^9 \text{ GeV}} \right)^2 \left( \frac{m_\phi}{10^{-3} \text{ eV}} \right)^{-2/3} \\
\Delta^2_{\phi, k} & \approx 0.1 \times 10^{-2} \left( \frac{f}{10^{-10}} \right)^2 \left( \frac{\tau}{10} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^9 \text{ GeV}} \right)^2 \left( \frac{m_\phi}{10^{-3} \text{ eV}} \right)^{-2/3} \\
\Delta^2_{\phi, L} & \approx 0.1 \times 10^{-2} \left( \frac{f}{10^{-10}} \right)^2 \left( \frac{\tau}{10} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^9 \text{ GeV}} \right)^2 \left( \frac{m_\phi}{10^{-3} \text{ eV}} \right)^{-2/3}
\end{align*} \]

FIG. 4. The dimensionless power spectra of \( A_+ \) and \( \phi \) from top to bottom, evaluated at the end of the simulation (\( z = 1100 \)), defined by Eq. (B1). All simulations fix \( \alpha = 70 \), \( m_\phi = 10^{-26} \) eV, \( f = 1.5 \times 10^{17} \) GeV, and \( \theta = 2 \). We compare simulations with box lengths \( L = 10/m_\phi \) in solid, thin blue and \( L = 5/m_\phi \) in dashed red.

\[ \begin{align*}
\Delta^2_{\phi, f} & \approx 0.1 \times 10^{-2} \left( \frac{f}{10^{-10}} \right)^2 \left( \frac{\tau}{10} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^9 \text{ GeV}} \right)^2 \left( \frac{m_\phi}{10^{-3} \text{ eV}} \right)^{-2/3} \\
\Delta^2_{\phi, k} & \approx 0.1 \times 10^{-2} \left( \frac{f}{10^{-10}} \right)^2 \left( \frac{\tau}{10} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^9 \text{ GeV}} \right)^2 \left( \frac{m_\phi}{10^{-3} \text{ eV}} \right)^{-2/3} \\
\Delta^2_{\phi, L} & \approx 0.1 \times 10^{-2} \left( \frac{f}{10^{-10}} \right)^2 \left( \frac{\tau}{10} \right)^{3/2} \left( \frac{H_{\text{inf}}}{10^9 \text{ GeV}} \right)^2 \left( \frac{m_\phi}{10^{-3} \text{ eV}} \right)^{-2/3}
\end{align*} \]
FIG. 5. Comparing numerical solutions for the full nonlinear system of equations (red), the linearized system (thin black), and the effective two-fluid description of Ref. [18] (dotted blue). Vertical lines denote the redshift of oscillation $z_c = a_0/a_c - 1$ (dashed grey) and that of the transition to radiation domination $z_r = a_0/a_r - 1$ (dotted grey). For the fluid model, we choose $a_c = 6.5$, $a_r = 3.5a_c$ to be consistent with the dynamics of the simulation and linear solution, which take $m_\phi = 10^{-26} \text{eV}$, $\theta = 2$, $\alpha = 70$, and $f = 1.5 \times 10^{17} \text{GeV}$. account for the remnant energy remaining in the axion after resonance. This remaining energy is due to nonlinear effects preventing the axion condensate from being totally depleted as well as backreaction effects producing axion particles. In addition, Fig. 5 demonstrates the validity of the solution to the linearized system of equations in the early stages of resonance while also exhibiting the importance of nonlinear effects as resonance terminates.