Distribution of shear to the columns of short multistorey building frames subjected to lateral loads

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ABSTRACT

Approximate solutions for multistorey buildings subjected to lateral loads are accepted in lieu of exact analysis by some Engineers. For short building frames in which panel distortion (shear mode) is predominant, the approximate method used is the Improved portal method. This method has been propounded for finding column shears based on a principle widely accepted on its intrinsic merit. However, this maxim has a minor defect, i.e., it gives rise to zero axial forces in the interior columns which is contrary to the actual physical lateral deformation behaviour of the building. In this paper, a new approach is put forward for the determination of shear in columns, which eliminates the flow of the Improved portal method and at the same time yields results close to that of the Improved portal method. The proposed method is scientific as it is founded on a principle depending on the principal flexibility influence co-efficients. It is a valid and useful addition to the Improved portal method.

Keywords: Approximate; Exact; Flexibility; Joint load; Improved; Nodal load; Portal method; Shear deformation.

1. Introduction

Construction of multistorey buildings is increasing year by year in all the metropolis. These buildings are mostly of 25 storeys or less than 25 storeys. The analysis and design of these buildings is primarily governed by the lateral forces due to wind, earthquake and blast. For lateral load analysis, the plane building frame is classified as (Selvam and Bindhu 2011a).

(a) Short

(b) Intermediate

(c) Tall

Even though there exists three classifications, no distinct definition is available to distinguish the same. In all frames, flexural and shearing deformation of the panels are present during loading. In short frame, panel distortion (shear mode) is predominant compared with flexural mode which produces axial loads in the columns (Smith and Coull 1991, Selvam and Bindhu 2010). The exterior columns bear most of the loads compared with the interior columns. In the intermediate frames, both panel distortion and bending modes are present almost in equal degrees. In tall multistorey frames, flexural mode is significant while shear distortion occurs to a lesser degree. Hence, the columns are subjected to axial forces to considerable degree.
Whatever may be the height of the frames, axial forces will be present in the columns in varying levels depending upon the height of the building. All these features are well delineated in reference (Selvam and Bindhu 2011a) and a method is suggested for application to the three types of frames.

The process of obtaining suitable dimensions for the members of a plane frame made up of linear material consists of two phases (Timoshenko and Young 1963, Schultz 1992):

**Phase I: Analysis**
- Equilibrium
- Compatibility

**Phase II: Design**
- Strength
- Serviceability
- Stability

For fulfilling the requirements of the two phases, a certain dimensions for the members are assumed and the forces are obtained satisfying both the conditions of phase I using software in the computer. Making use of these forces thus arrived, dimensions are reckoned checking the three design criteria in Phase II. The process is repeated till all the design conditions are satisfied. Assumption of member dimensions leads to errors. Instead of assuming the dimensions in order to avoid personal errors, to begin with, approximate lateral load analysis is performed which assists in getting a reasonable dimensions in the first trial itself. For preliminary lateral load analysis, the approximate methods are used. These methods satisfy only equilibrium conditions and not the compatibility. Further, these methods are independent of member dimensions and material properties for their execution. Because of this fact, in the published literature, a number of methods are available (Selvam, 1991). Among them, the most conspicuous for preliminary use in short frames are

1. a) Simplified portal method (SPM)
1. b) Improved portal method (IPM)

**1 a) Simplified portal method: (SPM)**

This method is discussed in reference Norris et al., 1976. In this method, two assumptions are made which are:

(a) Hinges occur in the middle of beams and columns

(b) In any plane passing through the hinges of the columns, the shear is distributed in the ratio among the columns as (Selvam and Bindhu 2011b)

\[ 1 : 2 : 2 : \ldots : 2 : 1 \]  \hspace{1cm} (1)

In the realm of structural analysis, no other method is as simple as this method. However, it possesses the following limitations:

(a) In any floor, the beam terminal moments in each bay is same irrespective of the bay width. This prediction is not correct.
(b) The two exterior columns take up the same shear force. Similarly, all the interior columns are subjected to the same magnitude of shear force. This assumption also is not fully true.

1b) Improved portal method : (IPM)

This method is described in reference Wang 1983. In this method also, two assumptions are made use of for the analysis. The first assumption is the same as the previous one. In the second assumption, improvement is effected. It is based on a well grounded proposition. For a storey having ‘n’ bays, the shear among the columns is distributed in the ratio

\[ l_1 : (l_1+I_2) : (l_{i-1}+I_i) + \ldots + l_n \]  

where \( l_i \) is the bay width of \( i^{th} \) bay.

The concept employed in the IPM for the determination of shear in the columns is surpassingly grand, lucid and efficacious. This method removes the deficiencies of the SPM. However, it possesses a minor flaw. That is, it gives rise to the fact that the axial force in the interior columns is zero. This method is widely preferred by many Engineers for its conceptual grandeur regarding shear distribution. Further, a few Engineers deem the solution as the final one because of the errors involved in the random nature of the lateral loads, dynamic characteristics which are unpredictable and uncertainty of the true magnitude of the load. Furthermore, for these loads, there is no standard is available correctly even though for wind 1.5kN/m\(^2\) is suggested in the various codes. These are well expatiated in reference Selvam 2010. It may be noted that the IPM is nothing but the same as the one related to loading R100P described in reference Selvam and Bindhu (2011a).

In this paper, a new procedure is put forward similar to IPM for the distribution of shear among the columns. It eliminates the limitation of IPM. In short, it is a well formulated adjunct to IPM.

2. Proposed Method

![Figure 1: A General Plane Frame with Joint Loads P](image)
Figure 1 shows a perfectly rectangular frame with reticulated elements carrying lateral loads $P$. Every joint carries a load $P$. It is known as joint load and is designated by the letter $P$. In Figure 2, the frame is split into two single bay frames carrying lateral loads. In these split frames, the various joints are known as nodes. Every node carries a nodal load designated as $R_i$. It is assumed that the split frames are symmetric and hinges occur in the middle of columns only. The two columns in each split frame and the beams are assumed to be of the same dimensions and hence the flexural rigidity $EI$ is same for all the members. To find $R_i$, each frame is assumed to carry only one joint load $P$ at a time. In Figure 3 (a), a single bay single storey frame above the hinges of the split frame is shown. It is symmetric in geometry. This frame is known as “Reference Frame”. Using this frame, the distribution of nodal loads $R_i$ in any floor due to single joint load $P$ is accomplished by means of two postulations.

In Figure 3 (b), the Reference Frame is subjected to unit load alone and the force analysis is shown in the same figure. Using the system forces in conjunction with Dummy unit load method, the displacement principal flexibility influence coefficient $\delta$ is found.

Similarly, in Figure 3 (b), the Reference Frame is subjected to a unit moment at the top left node and the force analysis is shown in the figure. Using the same, the rotational principal flexibility influence coefficient $\theta$ is found. Using $\delta$ and $\theta$, a postulation is propounded for finding the distribution law of $R_i$. The two postulations advanced are as follows:

**Postulation No.1:** The nodal load $R_i$ in any split frame at any floor level due to joint load $P$ is directly proportional to the displacement principal flexibility influence coefficient $\delta$. That is $R_i \propto \delta$

**Postulation No.2:** The nodal load $R_i$ in any split frame at any floor level due to joint load $P$ is directly proportional to the rotational principal flexibility influence coefficient $\theta$. That is $R_i \propto \theta$

Now $\delta$ and $\theta$ are determined as follows:
Figure 3: Reference Frame Subjected to Unit Load and Unit Moment Separately

(a) Determination of $\delta$: Referring to Figure 3(b) and using Dummy unit load method, $\delta$ is found as

$$\delta = \frac{1}{E I} \int_0^h \left( \frac{h}{4} - \frac{h x}{2l_1} \right)^2 dx + \frac{2}{E I} \int_0^h \left( \frac{1}{2} x \right)^2 dx$$

Upon integrating and simplifying

$$\delta = -\frac{h^2}{48EI} (h + l_1)$$

Or

$$\delta = k(h + l_1)$$

Where $k = a \text{ constant} = \frac{h^2}{48EI}$

Similarly for the other split frames

$$\delta = kl_2, \delta = kl_3, ..., \delta = kl_n$$

Now according to Postulation No.1
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\[ \begin{align*}
  k(h+l_1) : k(h+l_2) : k(h+l_n) \\
  \downarrow & \quad \downarrow & \quad \downarrow \\
  (h+l_1) : (h+l_2) : (h+l_n)
\end{align*} \]

or

\[ (h+l_1) : (h+l_2) : (h+l_n) \quad (7) \]

(b) **Determination of \( \theta \)**: As before, referring to Figure 3(c),

\[ \theta = \frac{1}{EI} \int_0^{l_1} \left( \frac{x}{l_1} \right)^2 \, dx = \frac{l_1}{EI} = t \]

Where \( t = \frac{1}{EI} \) = a constant

According to Postulation No.2

\[ \begin{align*}
  R_1 & \quad R_2 & \quad R_n \\
  \downarrow & \quad \downarrow & \quad \downarrow \\
  t_l_1 & : t_l_2 & : t(l_n) \\
  \downarrow & \quad \downarrow & \quad \downarrow \\
  l_1 & : l_2 & : l_n
\end{align*} \]

or

\[ l_1 : l_2 : l_n \quad (9) \]

It is seen that there is slight difference between Eq.(7) and Eq.(9). If both the postulations are true, they will result in the same nodal load \( R_i \). In this case, it has been found that there is difference in the two distribution laws in Eq. (7) and Eq. (9). To even out the difference average of Eq.(7) and Eq.(9) is taken after **normalising** these equations with respect to the first bay as indicated in the Illustrative example. Knowing the value of \( R_i \), shear in each column of any floor, any split frame is found by just halving the shear force at the level of column hinges in any floor. Before proceeding with the illustrative example, some information which will be useful in solving the problem is furnished below:

a) In any joint, sum of column moments = sum of beam moments

b) In any joint, the direction of column moments will be in opposite direction of the beam moments.

3. **Illustrative Example**

The frame shown in Fig.4 is taken from reference Norris et al., (1976).

The data are as follows:
Top storey height, \( h = 3.00 \) m
Bottom storey height, \( h = 4.00 \) m
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\[ l_1 = 4.00\text{m}, \quad l_2 = 5.00\text{m} \quad \text{and} \quad l_3 = 6.00\text{m} \]

Joint load, \( P = 10.00\text{kN} \)

\[
\begin{array}{cccc}
& I & J & K & L \\
p=10kN & 3m & & 3m & \\
p=10kN & & 4m & 4m & \\
& & 3m & & 3m & \\
& 4m & & 5m & 6m & \\
\end{array}
\]

**Figure 4**: Illustrative Example (Norris et al., 1976)

**Solution:**

(a) Top floor, \( P = 10.00\text{kN} \)

(i) Eq. (7):

\[
\begin{array}{c}
R_1 \\
(3+4) \\
7 \\
\end{array} \quad \quad \begin{array}{c}
R_2 \\
(3+5) \\
8 \\
\end{array} \quad \quad \begin{array}{c}
R_3 \\
(3+6) \\
9 \\
\end{array}
\]

Now normalising with respect to the first bay, gives

\[
\begin{array}{c}
R_1 \\
1.00 \\
\end{array} \quad \quad \begin{array}{c}
R_2 \\
1.142 \\
\end{array} \quad \quad \begin{array}{c}
R_3 \\
1.29 \\
\end{array}
\]

(ii) Eq. (9):

\[
\begin{array}{c}
R_1 \\
4.00 \\
\end{array} \quad \quad \begin{array}{c}
R_2 \\
5.00 \\
\end{array} \quad \quad \begin{array}{c}
R_3 \\
6.00 \\
\end{array}
\]

Normalising with respect to the first bay
Taking average of Eq. (10) and Eq.(11)

\[ R_1 : R_2 : R_3 = 1.00 : 1.191 : 1.400 \]  \quad (12)

Sum = 3.591

Now, \( R_1 = \frac{(P \times 1.00)}{3.591} = \frac{(10 \times 1)}{3.591} = 2.78 \text{ kN} \)
\( R_2 = \frac{(10 \times 1.191)}{3.591} = 3.33 \text{ kN} \)
Similarly \( R_3 = 3.90 \text{ kN} \)
Check = 2.78+3.33+3.90 = 10.01 = 10.00 \text{ kN}. \text{ OK}

(b) Bottom storey: \( P = 10.00 \text{kN} \)

(i) Eq. (7), (4+4) : (4+5) : (4+6)
\[ 8 : 9 : 10 \]
Normalising with respect to first bay

\[ 1 : 1.125 : 1.25 \]  \quad (13)

(ii) Eq. (9): 4 : 5 : 6
Normalising with respect to the first bay

\[ 1.00 : 1.25 : 1.50 \]  \quad (14)
Taking average of Eq. (13) and Eq. (14)

\[ R_1 : R_2 : R_3 \]
\[ 1.000 : 1.1875 : 1.375 \]  \quad (15)
Sum = 3.5625

As before : \( P = 10 \text{kN} \)
\( R_1 = \frac{(P \times 1.00)}{3.5625} = 2.80 \text{ kN} \)
\( R_2 = \frac{(10 \times 1.1875)}{3.5625} = 3.33 \text{ kN} \)
Similarly, \( R_3 = 3.85 \text{ kN} \)
Check = \( R_1 + R_2 + R_3 = 2.80 + 3.33 + 3.85 = 9.98 \approx 10.00 \text{ kN} \). \text{ OK}

Even though storey heights differ, there is very little difference between the two sets of \( R_1, R_2 \) and \( R_3 \). In this paper, the first sets of values are considered for the bottom storey also because of negligible difference in values. Otherwise, the second set would have been retained. These values are registered in the three split frames shown in Fig.5. Now, the shear in any split frame along the column hinges is equally distributed between the two columns. The shear in the column multiplied by the lever arm gives the column terminal moments. For example, \( \text{MAE} = 2.78 \times 2.0 = 5.76 \text{ kNm} \). Here, \( \text{MAE} \) means moment at A in the member AE. Fig.5 is...
self explanatory. To obtain the terminal moments in the beams and columns, all the values of the split frames are added. In this process, the column moments get added up and the beam moments remain unaltered. For example

**Table 1:** Prediction by the proposed method and comparison of column moments

| No | Moments | Simplified portal method kNm | Slope-deflection method kNm | Improved portal method kNm | Error % | Proposed method kNm | Error % |
|----|---------|-------------------------------|----------------------------|----------------------------|---------|---------------------|---------|
| 1  | MAE     | 6.70                          | 5.90                       | 5.30                       | 10.2    | 5.56                | 5.8     |
| 2  | MEA     | 6.70                          | 5.00                       | 5.30                       | 6.0     | 5.56                | 11.2    |
| 3  | MBF     | 13.40                         | 12.20                      | 12.00                      | 1.6     | 12.22               | 0.0     |
| 4  | MFB     | 13.40                         | 10.70                      | 12.00                      | 12.1    | 12.22               | 14.2    |
| 5  | MCG     | 13.40                         | 15.40                      | 14.70                      | 4.5     | 14.47               | 6.0     |
| 6  | MGC     | 13.40                         | 13.70                      | 14.70                      | 7.3     | 14.47               | 5.6     |
| 7  | MDH     | 6.70                          | 9.10                       | 8.00                       | 12.0    | 7.80                | 14.3    |
| 8  | MHD     | 6.70                          | 8.20                       | 8.00                       | 2.4     | 7.80                | 4.9     |
| 9  | MEI     | 2.50                          | 1.30                       | 2.00                       | 53.8    | 2.09                | 60.8    |
| 10 | MIE     | 2.50                          | 2.10                       | 2.00                       | 4.8     | 2.09                | 0.5     |
| 11 | MFJ     | 5.00                          | 3.70                       | 4.50                       | 21.6    | 4.59                | 24.0    |
| 12 | MJF     | 5.00                          | 5.00                       | 4.50                       | 10.0    | 4.59                | 8.2     |
| 13 | MGK     | 5.00                          | 5.00                       | 5.50                       | 10.0    | 5.43                | 8.6     |
| 14 | MKG     | 5.00                          | 6.50                       | 5.50                       | 15.4    | 5.43                | 16.4    |
| 15 | MHL     | 2.50                          | 2.70                       | 3.00                       | 11.1    | 2.93                | 8.5     |
| 16 | MLH     | 2.50                          | 3.70                       | 3.00                       | 18.9    | 2.93                | 20.8    |

\[
\text{Error \%} = \left( \frac{\text{Exact} - \text{Approximate}}{\text{Exact}} \right) \times 100
\]

| No | Moments | Simplified portal method kNm | Slope-deflection method kNm | Improved portal method kNm | Error % | Proposed method kNm | Error % |
|----|---------|-------------------------------|----------------------------|----------------------------|---------|---------------------|---------|
| 1  | MEF     | 9.20                          | 6.30                       | 7.30                       | 15.8    | 7.65                | 21.4    |
| 2  | MFE     | 9.20                          | 5.90                       | 7.30                       | 23.7    | 7.65                | 29.6    |
| 3  | MFG     | 9.20                          | 8.50                       | 9.20                       | 8.2     | 9.16                | 7.8     |

MBF = MFB = MBF + MBF = 5.56+6.66 = 12.22 kNm

(MBF) (split) (split)

MCG = MGC = MCG + MCG = 6.66+7.80 = 14.46 kNm

(MCG) (split) (split)

Similarly MFJ = MJF and MGK = MKG are found. The various values are entered in Table 1 and Table 2.

**Table 2:** Prediction by the proposed method and comparison of beam moments

| No | Moments | Simplified portal method kNm | Slope-deflection method kNm | Improved portal method kNm | Error % | Proposed method kNm | Error % |
|----|---------|-------------------------------|----------------------------|----------------------------|---------|---------------------|---------|
| 1  | MEF     | 9.20                          | 6.30                       | 7.30                       | 15.8    | 7.65                | 21.4    |
| 2  | MFE     | 9.20                          | 5.90                       | 7.30                       | 23.7    | 7.65                | 29.6    |
| 3  | MFG     | 9.20                          | 8.50                       | 9.20                       | 8.2     | 9.16                | 7.8     |
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| No | Column Designation | Simplified portal method kN | Improved portal method kN | Proposed method kN | Slope – deflection analysis |
|----|--------------------|-----------------------------|--------------------------|-------------------|-----------------------------|
| 1  | AE                 | 4.60                        | 4.65                     | 4.87              | 4.07                        |
| 2  | BF                 | 0.92                        | 0.00                     | 0.21              | 0.37                        |
| 3  | CG                 | 0.62                        | 0.00                     | 0.11              | 0.23                        |
| 4  | DH                 | 3.06                        | 4.61                     | 4.56              | 4.66                        |
| 5  | EI                 | 1.25                        | 1.00                     | 1.05              | 1.03                        |
| 6  | FJ                 | 0.25                        | 0.00                     | 0.05              | 0.06                        |
| 7  | GK                 | 0.17                        | 0.00                     | 0.02              | 0.14                        |
| 8  | HL                 | 0.83                        | 1.00                     | 0.98              | 1.22                        |

3.1 Axial forces in various columns

Using the beam terminal moments, the shears in the middle of all the beams are computed. From which using the free body diagrams, the axial force in the various columns are reckoned and are shown in Table 3. This completes the solution.

Table 3: Prediction of axial force in columns by the proposed method and comparison

| No | Column Designation | Simplified portal method kN | Improved portal method kN | Proposed method kN | Slope – deflection analysis |
|----|--------------------|-----------------------------|--------------------------|-------------------|-----------------------------|
| 1  | AE                 | 4.60                        | 4.65                     | 4.87              | 4.07                        |
| 2  | BF                 | 0.92                        | 0.00                     | 0.21              | 0.37                        |
| 3  | CG                 | 0.62                        | 0.00                     | 0.11              | 0.23                        |
| 4  | DH                 | 3.06                        | 4.61                     | 4.56              | 4.66                        |
| 5  | EI                 | 1.25                        | 1.00                     | 1.05              | 1.03                        |
| 6  | FJ                 | 0.25                        | 0.00                     | 0.05              | 0.06                        |
| 7  | GK                 | 0.17                        | 0.00                     | 0.02              | 0.14                        |
| 8  | HL                 | 0.83                        | 1.00                     | 0.98              | 1.22                        |

4. Discussion

It is seen from Table 1 and Table 2 that the prediction of the proposed method for moments in various members is very close to the solution of the IPM. It is a welcome feature. (It should not be the same as that of the IPM). Because of this fact, a small amount of axial force is induced in the interior columns as seen in Table 3. It is in agreement with the actual physical behaviour of the frame under the action of lateral loads. The slight blemish of the IPM is thus eliminated without foregoing accuracy. Since the accuracy of the proposed method is in close agreement with that of the IPM, it testifies the validity of the two proposed postulations. It is found that the method advanced is scientific and addresses the problem in a rational manner. Both the proposed and IP methods use simple mathematical operations to establish the shear in the various columns. The proposed method has an edge over the IPM in that it satisfies all the physical features of deformation.

Error % = \frac{\text{Exact} - \text{Approximate}}{\text{Exact}} \times 100

Mean 10.70 11.48
St.dev 7.50 9.67
5. Recapitulation

Multistorey building frames can be classified as short, intermediate and tall frames. For preliminary analysis of these frames subjected to lateral loads, a method has been already set forth in reference Selvam and Bindhu 2011a. The proposed method is a simple alternative for finding the stress resultants of short frames. The IPM is an efficient, powerful and explicit procedure which is used as final solution by some Engineers for reasons discussed in reference Selvam 2011. However, it has a minor fault, i.e., it gives rise to zero axial force in the interior columns which is contrary to the actual physical behaviour of the frame. (Whatever may be the height of the building, axial force in the interior columns will be present to some degree). This slight defect is rectified in the method proposed in this paper. In fine, the proposed method serves as an effective adjunct and, a well based and valid supplement to the IPM which is lucid, efficacious and realistic.

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