Abelian and Non–Abelian Dualities in String Backgrounds

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We present a brief discussion of recent work on duality symmetries in non–trivial string backgrounds. Duality is obtained from a gauged non–linear $\sigma$–model with vanishing gauge field strength. Standard results are reproduced for abelian gauge groups, whereas a new type of duality is identified for non–abelian gauge groups. Examples of duals of WZW models and 4–d black holes are given.

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1. Introduction

String Theory and Physics are at the moment two unrelated subjects which are expected to have an interesting connection. The only available approach towards this connection is to study the structure and physical properties of the semiclassical string vacua given by classes of conformal field theories (CFT). For applications to Particle Physics, it is enough to assume a flat four-dimensional spacetime and parametrize the different models by an internal CFT restricted by some consistency conditions such as worldsheet modular invariance. In this way many classes of models have been studied and some, close to the Standard Model of Particle Physics give hope that strings may actually be related to the real world. Furthermore, duality and mirror symmetries have been identified in these vacua and play an important role in determining the low energy properties of the theory. To ask questions about gravitation in the context of String Theory, we have to relax the flat spacetime assumption and substitute it by a general noncompact CFT. These questions are of fundamental importance since the main motivation for studying string theories is to provide a consistent way of quantizing gravity. In particular it is of prime importance to study singularities of cosmology and black hole-type of geometries in the context of String Theory, since it is in those regimes that the standard field theory methods of General Relativity fail to apply. Duality, being a property of all string vacua with abelian isometries [1], has also been found in these backgrounds and could provide a way to understand how string theory probes those singular geometries. A generalization of this duality symmetry to backgrounds with non–abelian isometries was recently discovered [2]. In this talk we will briefly discuss these developments.

2. Non–Compact String Backgrounds

We know several ways to construct non–compact string vacua. The straightforward approach is to look for solutions of the string background equations. Except for a few exceptions [3], these are solutions of only the lowest order in $\alpha'$ equations [4]:

\[ R_{MN} + D_M D_N \Phi - \frac{1}{4} H^L_M H_{NLP} = 0 \quad (2.1) \]

\[ D_L H^L_{MN} - (D_L \Phi) H^L_{MN} = 0 \quad (2.2) \]

\[ R - 2\Lambda - (D\Phi)^2 + 2D_M D^M \Phi - \frac{1}{12} H_{MNP} H^{MNP} = 0 , \quad (2.3) \]
where \( \Lambda \equiv (c - 26)/3 \) is the cosmological constant in the effective string action, \( c \) is the central charge and, as usual \( H_{MNP} \equiv \partial[M B_{NP}] \). For the heterotic string, these equations are modified in order to include the background gauge fields. From this we can extract the trivial but powerful conclusion that all solutions of Einstein’s equations in vacuum are also solutions of the leading order string background equations with constant dilaton and antisymmetric tensor field. Therefore we already have a large class of solutions to these equations \[5\]. In particular the 4–d Schwarzschild black hole geometry is a solution to the leading order string equations, when tensored with an appropriate CFT to provide the correct central charge. This is not the case however for the Maxwell–Einstein system which is generally modified by the dilaton coupling to the gauge fields and then the Reissner–Nordstrom geometry, for instance, does not lead to a string background. The corresponding stringy solution is the so–called charged dilatonic black hole of reference \[6\].

The second approach is to look for non–compact CFT’s directly. In this case non–compact cosets are the appropriate class of models to study. However, contrary to the compactification approach, we cannot only construct CFT’s and use their algebraic properties, in the non–compact case we have to provide a geometrical interpretation to those CFT’s.

In order to have a geometrical interpretation of these CFT’s, we need to have a Lagrangian formulation of them, which is naturally provided by the WZW construction. The WZW action for a group \( G \) with elements \( g(z, \bar{z}) \) in complex coordinates is \[7\],

\[
L(g) = \frac{k}{4\pi} \int d^2z \text{tr}(g^{-1} \partial g g^{-1} \bar{g}) - \frac{k}{12\pi} \int_B \text{tr}(g^{-1} dg \wedge g^{-1} dg \wedge g^{-1} dg), \tag{2.4}
\]

where the boundary of \( B \) is the 2D worldsheet. It is known that this action provides the conserved currents \( g^{-1} \partial g \) and \( \bar{g}g^{-1} \partial g \) to satisfy a chiral algebra and then giving a CFT for the group \( G \). For the coset \( G/H \) \[8\] the standard way in nonlinear sigma models \[9\] is to eliminate the \( H \) degrees of freedom by gauging the subgroup \( H \) as a symmetry of (2.4). To promote the global \( g \rightarrow h_L^{-1} g \) \( h_R \) invariance to a local \( g \rightarrow h_L^{-1}(z) g h_R(\bar{z}) \) invariance, we let \( \partial g \rightarrow \partial g + A g \), and \( \bar{g} \rightarrow \bar{g} - g \bar{A} \). The gauge fields transform as \( A \rightarrow h_L^{-1}(A + \partial)h_L \) and \( \bar{A} \rightarrow h_R^{-1}(\bar{A} + \bar{\partial})h_R \) (so that \( Dg \rightarrow h_L^{-1} D g h_R \) for \( D \) equal to either holomorphic or anti-holomorphic covariant derivative). Vector gauge transformations correspond to \( h_L = h_R \), and axial gauge transformations to \( h_L = h_R^{-1} \). For abelian groups \( H \), both vector and axial-vector gauging are anomaly free. In the non-abelian case, only the vector gauging is allowed. The gauged action may be written as

\[
L(g, A) = L(g) + \frac{k}{2\pi} \int d^2z \text{tr}(A \bar{g} g^{-1} \pm \bar{A} g^{-1} \partial g + A\bar{A} \mp g^{-1} A g \bar{A}), \tag{2.5}
\]
where the upper and lower signs represent respectively vector \((g \to hgh^{-1})\) and axial-vector 
\((g \to hgh)\) gauging.

We now consider some naive properties of the geometry described by (2.5) in the large \(k\) limit. Writing \(A = A^a \sigma_a\) in terms of the generators \(\sigma_a\) of \(H\), and integrating out the components \(A^a\) classically gives the effective action

\[
L = L(g) \pm \frac{k}{2\pi} \int d^2z \text{tr}(\sigma_b g^{-1} \partial g) \text{tr}(\sigma_a \partial gg^{-1}) \Lambda_{ab}^{-1},
\]

(2.6)

with \(\Lambda_{ab} \equiv \text{tr}(\sigma_a \sigma_b \mp \sigma_a g \sigma_b g^{-1})\). This action can be identified with a \(\sigma\)–model action of the form

\[
S = \int d^2z \left(G_{MN} + B_{MN}\right) \partial X^M \overline{\partial} X^N
\]

(2.7)

to read off the background metric and antisymmetric tensor field (torsion).

Notice that singularities of \(\Lambda\) occur at least at fixed points of the gauge transformation \(g \to hgh^\mp 1\). This is because for infinitesimal \(h \approx 1 + \alpha^a \sigma_a\), we see that a fixed point \(g\) satisfies \(\sigma_a g \mp g \sigma_a = 0\). Multiplying by \(g^{-1} \sigma_b\) and taking the trace, we see that \(\Lambda = 0\) at a fixed point.

It can be seen that, in the case of \(H\) abelian the ungauged axial or vector symmetry remains a global symmetry, i.e. an isometry of the spacetime geometry. In the non-abelian case, not even a global vestige of the ungauged symmetry remains (unless \(H\) commutes with a subgroup of \(G\)). In the abelian case, this implies that a fixed point of the ungauged symmetry corresponds to a point with vanishing Killing vector. For lorentzian signature, the surface carried into the fixed point by the isometry will be a null surface (the norm of the Killing vector is conserved), in general nonsingular and hence a horizon. We see that fixed points of symmetry transformations generically give rise to metric singularities when the symmetry is gauged and to horizons when ungauged.

The simplest class of coset models with a single timelike coordinate and any number of spacelike coordinates are the \(SL(2, \mathbb{R}) \otimes SO(1, 1)^{D-2}/SO(1, 1)\) models. In order to find the metric in the large \(k\) limit, we employ the standard procedure in nonlinear \(\sigma\)–models [3], i.e. find a parametrization of the \(G\) group elements, impose a unitary type gauge on the fields in the \(\sigma\)–model action and then solve for the (non-propagating) \(H\)-gauge fields to derive the \(G/H\) worldsheet action. From that action we can read off the corresponding background fields from (2.7). The results are that for the vector gauging, the metric is the
direct product of the 2–d black hole of [10] with metric $ds^2 = -da \, db/(1 - ab)$, (see fig. 1) times $\mathbb{R}^{D-2}$. That is a $D - 2$ black–brane.

For the axial gauging the geometry is a 3–d black string times $\mathbb{R}^{D-3}$ [11],[12] with two horizons and a singularity as shown in fig. 2.

We should remark at this point that these solutions are only valid for a very large $k$ limit. There has been some progress towards inferring the exact form of the metric both from the operator approach and from an exact treatment of the gauged WZW models. We refer the reader to [3] for a discussion of these methods.

3. Abelian Duality

We will briefly review here the standard duality corresponding to backgrounds with abelian isometries. The worldsheet action for the bosonic string in a background with $N$ commuting isometries, can be written as

$$S = \frac{1}{4\pi\alpha'} \int d^2 z \left( Q_{\mu\nu}(X_\alpha) \partial X^\mu \overline{\partial} X^\nu + Q_{mn}(X_\alpha) \partial X^m \overline{\partial} X^n + Q_{n\mu}(X_\alpha) \partial X^n \overline{\partial} X^\mu + Q_{mn}(X_\alpha) \partial X^m \overline{\partial} X^n + \frac{\alpha'}{2} R^{(2)} \phi(X_\alpha) \right),$$

(3.1)

where $Q_{MN} \equiv G_{MN} + B_{MN}$ and lower case latin indices $m, n$ label the isometry directions. Since the action (3.1) depends on the $X^m$ only through their derivatives, we can write it in first order form by introducing variables $A^m$ and adding an extra term to the action
Fig. 2: A two dimensional slice of the three dimensional black string metric. In addition to the regions of fig. 1, the regions VII, VIII lie between the singularities and inner horizons.

Λₘ(∂Aᵐ − ∂⁻¹Aᵐ) which imposes the constraint Aᵐ = ∂Xᵐ. Integrating over the Lagrange multipliers Λₘ returns us to the original action (3.1). On the other hand performing partial integration and solving for Aᵐ and A⁻¹ᵐ, we find the dual action S' which has an identical form to S but with the dual background given by (3.12)

\[
\begin{align*}
Q'_{mn} &= (Q^{-1})_{mn} \\
Q'_{μν} &= Q_{μν} - Q_{μm} (Q^{-1})^{mn} Q_{nν} \\
Q'_{nμ} &= (Q^{-1})^m_n Q_{mμ} \\
Q'_{μn} &= -Q_{μm} (Q^{-1})^m_n .
\end{align*}
\]

(3.2)

To preserve conformal invariance, it can be seen (3.13) that the dilaton field has to transform as Φ' = Φ − log det Gₘₙ. Equations (3.2) reduce to the usual duality transformations for the toroidal compactifications in the case Qₘμ = Qₜₜ = 0 and can map a space with no torsion (Qₘμ = Qₜₜ) to a space with torsion (Q'ₘμ = −Q'ₜₜ).

An equivalent interpretation of the duality process just described is given by gauging the symmetry, ∂Xᵐ with DXᵐ = ∂Xᵐ + Aᵐ and the term \( ∫ d^2z \ Λₘ(∂Aᵐ - ∂⁻¹Aᵐ) \) is added to the action. This extra term imposes the vanishing of the field strength F of the gauge fields after integration over the Lagrange multipliers Λₘ. This implies that locally the gauge field must be pure gauge, Aᵐ = ∂Xᵐ. The gauge fixing can be done either by choosing the gauge fields to vanish or by taking Xᵐ = 0 (a unitary gauge). In both cases this reproduces the original action. The dual theory is obtained by instead integrating out the gauge fields and then fixing the gauge.
In [14] it was shown that the axial and vector gaugings of WZW models are related by transformations like (3.2) and therefore lead to dual geometries. In order to make a connection between duality in this formulation and the vector–axial duality in $G/H$ WZW models, we just have to identify the correct action of the vector (axial) isometry when gauging the axial (vector) transformation and apply (3.2). It is straightforward to see that for the $SL(2,\mathbb{R}) \otimes SO(1,1)^{D-2}/SO(1,1)$ models for which we see that regions II of both geometries are interchanged. Region V of fig. 1 is mapped to region I of fig. 2 and in particular the singularity of the first is mapped to one horizon in the second. Also, region I of the vector gauging black hole gets mapped to regions V and VII together of the axial gauging black hole. This has the interesting implication that a surface which in one geometry is perfectly regular ($ab = \rho^2$) is mapped to the singularity in the other geometry ($uv = 1 + \rho^2$). In this case it can be seen explicitly that string theory can deal with spacetimes that have singularities at the classical level, in the sense that there still exists a description of interactions, etc. for that region of spacetime by going to the dual geometry!

It is then interesting to investigate if these properties hold for physically more interesting objects such as 4–d black holes. In particular the Schwarzschild metric

$$ds^2 = -(1 - 2M/r)dt^2 + \frac{dr^2}{1 - 2M/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) , \quad (3.3)$$

times any CFT with $c = 22$ is a solution of (2.1)–(2.3). Direct application of the standard duality transformation to (3.3) for time translations, gives the dual metric

$$ds^2 = -\frac{dt^2}{1 - 2M/r} + \frac{dr^2}{1 - 2M/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) , \quad (3.4)$$

with the dilaton field now given by $\Phi' = \Phi - \log(1 - 2M/r)$. This metric defines a geometry with naked singularities at $r = 0$ and $r = 2M$, as it can be verified by computing the curvature scalar $R = \frac{4M^2}{(2M-r)^2}$. It is easy to check that equations (2.1)–(2.3) are satisfied by the dual metric and dilaton $\Phi'$. We have then found a spherically symmetric solution of the string background equations, which is not a black hole, but has naked singularities and is dual to the Schwarzschild solution.

A similar analysis can be done for the 4D charged dilatonic black holes of reference [6]. In this case the metric is

$$ds^2 = -(1 - 2M/r)/(1 - Q^2/Mr)dt^2 + \frac{dr^2}{(1 - 2M/r)(1 - Q^2/Mr)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) , \quad (3.5)$$
the dilaton field is $\Phi = -\log(1 - Q^2/Mr)$. It is very interesting to note that the dual of this solution with respect to time translations gives exactly the same solution except that the mass parameter $M$ changes into $Q^2/2M$, therefore it relates the black hole domain $Q^2 < 2M^2$ to the naked singularity domain $Q^2 > 2M^2$. In particular, the extremal solution $Q^2 = 2M^2$ is selfdual. Notice however that the isometry group of these 4–d geometries is given by time translations together with the $SO(3)$ space rotations, which is not abelian. It is then natural to inquire if there is a duality transformation associated to the existence of non–abelian isometries.

4. Non–Abelian Duality

Consider the $\sigma$–model action (3.1) and assume that the target space metric has a group $G$ of non–abelian isometries. In this case, $Q_{MN}$ does depend on $X^m$ and transforms accordingly under $X^m \rightarrow g^m_n X^n, g \in G$. We gauge the symmetry corresponding to a subgroup $H \subset G, \partial X^m \rightarrow DX^m = \partial X^m + A^\alpha(T_\alpha)^m_n X^n$, and add to the action the term $\int d^2z \tr(\Lambda F) = \int d^2z \Lambda^\alpha F^\alpha$, where in this case the gauge field strength is, in matrix notation, $F = \partial \overline{A} - \overline{\partial} A + [A, \overline{A}]$. The $N \times N$ matrices $T_\alpha$ form an adjoint representation of the group $H$. In the path integral we have then

$$= \int \frac{DX}{V_G} \int D\Lambda \, DA \, D\overline{A} \exp \left\{ -i \left( S_{gauged}[X, A, \overline{A}] + \int d^2z \tr(\Lambda F) \right) \right\} , \quad (4.1)$$

where $V_G$ is the “volume” of the group of isometries and $DX$ is the measure that gives the correct volume element $DX = DX \sqrt{G} e^{-\Phi}$. Similar to the abelian case, the original action is obtained by integrating out the Lagrange multiplier $\Lambda$. Locally, this forces the gauge field to be pure gauge $A = h^{-1}\partial h, \overline{A} = h^{-1}\overline{\partial} h, h \in H$. By fixing the gauge with the choice $A = 0, \overline{A} = 0$ we reproduce the original theory. The dual theory is obtained by integrating out the gauge fields in the path integral (4.1). Integrating over the gauge fields $A, \overline{A}$ we obtain

$$\mathcal{P} = \int DX \, D\Lambda \, \delta[\mathcal{F}] \det \frac{\delta \mathcal{F}}{\delta \omega} e^{-iS'[X, \Lambda]} \det(f^{-1}) , \quad (4.2)$$

where $\mathcal{F}$ is the gauge fixing function and $\omega$ are the parameters of the group of isometries. The matrix $f$ is the coefficient of the quadratic term in the gauge fields [2] and $S'$ is given by

$$S'[X, \Lambda] = S[X] - \frac{1}{4\pi \alpha'} \int d^2z \, h_\alpha(f^{-1})^{\alpha\beta} h_\beta . \quad (4.3)$$
Here $h$ and $\bar{h}$ are the currents coupled to $A$ and $A$ respectively [2]. After the gauge fixing, denoting the new coordinates in the dual manifold collectively by $Y$ we have

$$\mathcal{P} = \int \mathcal{D}Y \ e^{-iS'[Y]} \det(f(Y)^{-1}) \ .$$

(4.4)

The Fadeev–Popov determinant in the path integral contributes to the measure such that the correct volume element for the dual manifold is obtained $\mathcal{D}Y = \mathcal{D}Y \sqrt{G'} e^{-\Phi'}$. The factor $\det(f^{-1})$ in the partition function can be computed using standard regularization techniques [1]. It generates a new local term in the action of the form $\frac{1}{8\pi\alpha'} \int d^2z \ \alpha' R^{(2)}(\Delta\Phi)$, which corresponds to the change in the dilaton due to the duality transformation

$$\Phi' = \Phi - \log \det f \ .$$

(4.5)

This change in the dilaton transformation is the shift necessary to retain the conformal invariance of the dual theory. The requirement that the correct volume element is obtained in the dual theory means that $e^{-\Phi'} = \left[ e^{-\Phi} \sqrt{G'} \det \frac{\delta F}{\delta \omega} \right]_{\mathcal{F}=0}$. This prescription reduces to the one for abelian duality because in that case the Fadeev–Popov determinant is trivial. A consistency check of the change in the dilaton is obtained by comparing both expressions. Notice that our assumption that the action of the group is linear on the coordinates was made only for simplicity and it is not necessary. We can actually rederive the duality transformations in a coordinate independent way, as it was done in [15] for any gauged sigma model.

In general, we cannot write explicitly the gauge fixed dual action. Therefore, we are not able to present the new metric and antisymmetric tensor fields in a closed form, as was done for the abelian case in equations (3.2). As an example, let us consider a theory for which the target space metric has a maximally symmetric subspace with $\mathcal{G} = SO(3)$ and no antisymmetric tensor. The coordinates $X^M, M = 1, ..., D$, can be decomposed into the two angular coordinates $(\theta, \phi)$ describing 2–dimensional spheres, and $D - 2$ extra coordinates $(v^\mu)$ specifying the different spheres in the $D$ dimensional spacetime. The action takes the form

$$S[v, \theta, \varphi] = S[v] + \int d^2z \ a^2\Omega(v) \left( \partial \theta \bar{\partial} \theta + \sin^2 \theta \partial \varphi \bar{\partial} \varphi \right) \ .$$

(4.6)

It is simpler to treat the coordinates $\theta$ and $\phi$ in terms of cartesian coordinates $X^m$ in 3–dimensional space on which $SO(3)$ can act linearly, so we write the $\sigma$ model action in the form

$$S[v, X] = S[v] + \int d^2z \ \Omega(v) \left\{ g_{mn} \partial X^m \bar{\partial} X^n + \frac{1}{2a \sqrt{\Omega}} \chi(g_{mn} X^m X^n - a^2) \right\}$$

$$+ \frac{1}{8\pi\alpha'} \int d^2z \ \alpha' R^{(2)} \Phi \ .$$

(4.7)
where \( S[v] = \int d^2 z g_{\mu\nu}(v) \partial v^\mu \bar{\partial} v^\nu \), the metric \( g_{mn} \) is diagonal and constant and the Lagrange multiplier term fixes the 3 dimensional space to be a sphere of radius \( a \). Gauging this action and fixing the gauge \( A = \bar{A} = 0 \) we obtain the original action.

A convenient choice of gauge is to set \( X^1 = X^2 = 0 \), \( X^3 = a \), \( A^1 = \partial \theta \), \( A^2 = -\sin \theta \partial \varphi \) and \( A^3 = \cos \theta \partial \varphi \) leading to the original action in spherical coordinates (4.6).

We can write a general expression for the dual action after fixing the coordinates \( X_m \) as above, but before fixing the remaining degree of freedom corresponding to one of the Lagrange multipliers \( \Lambda^\alpha \). We then have

\[
S^\text{dual}[v, \Lambda] = S[v] + \frac{1}{4\pi \alpha'} \int d^2 z \left( \partial \Lambda^\alpha (f^{-1})^{\alpha\beta} \bar{\partial} \Lambda^\beta \right) + \frac{1}{8\pi} \int d^2 z \ R^{(2)} \Phi'.
\]  

(4.8)

From this expression, we can in principle read off the new background fields as in (3.2). This is actually the general expression for any group. But we still have to complete the gauge fixing for the \( \Lambda^\alpha \). In our case, Choosing \( \Lambda^2 = 0 \) and defining \( x^2 = \Lambda^1 + \Lambda^3 \) and \( y = \Lambda_3 \), we obtain the dual theory action

\[
S^\text{dual}[v, x, y] = S[v] + \frac{1}{4\pi \alpha'} \int d^2 z \frac{1}{a^2 \Omega(v)} \left( a^4 \Omega(v)^2 \partial y \bar{\partial} y + x^2 \partial x \bar{\partial} x \right) + \frac{1}{8\pi} \int d^2 z \ R^{(2)} \Phi',
\]

(4.9)

where \( \Phi' = \Phi - \log[a^2 \Omega(v) (x^2 - y^2)] \).

We will now present, some 4D black hole backgrounds and their duals. Consider the dual geometry of (3.3) with respect to the \( SO(3) \) symmetry. We find

\[
ds^2 = -(1 - 2M/r)dt^2 + \frac{dr^2}{1 - 2M/r} + \frac{1}{r^2 (x^2 - y^2)} \ [r^4 dy^2 + x^2 dx^2],
\]

(4.10)

with the new dilaton \( \Phi' = \Phi - \log[r^2 (x^2 - y^2)] \). The regions \( x = y \) and \( r = 0 \) are real singularities whereas \( r = 2M \) is only a metric singularity corresponding to a horizon as in the original case. Notice that the metric (4.10) is not spherically symmetric, in fact its only isometry is time translations. Neither is it asymptotically flat. For large \( r \), the \( x \) dimension gets squeezed and the other dimensions behave like a 2 + 1 dimensional space–time. The surfaces \( x = \text{constant} \) are just 2 + 1 dimensional black holes away from the singularities \( \sin \theta = 0 \), \( (y = \pm x) \). Again, it is straightforward to check that this solution satisfies equations (2.1)–(2.3) thus providing new string vacua. To find new solutions, we can certainly combine both dualities above. We can also consider different coordinate systems. For instance, using the Eddington-Finkelstein instead of the Schwarzschild coordinate system, the dual metric with respect to time translations is identical to (3.4), but there is also torsion illustrating that duality does not commute with coordinate transformations.
5. Conclusions

We have shown some of the interesting properties that string theory backgrounds have due to the existence of duality symmetries. In particular we have presented a general non–abelian duality symmetry which reduces to the standard one when the gauged group is abelian and allows to find new string backgrounds at least to lowest order in $\alpha'$. It is not clear if this symmetry will survive beyond string tree–level though. We have shown examples where this symmetry generates new vacua but the list of possible applications is obviously very large. An interesting possibility is to study the dual geometries to Friedmann–Robertson–Walker cosmologies. Also, in 10–d Minkowski space there is a large group of isometries which we can use to find the dual geometries to flat spacetime. Furthermore, consider any of the ‘4–d strings’ with spacetime 4–d Minkowski space, for which billions of solutions are known. Again the isometry group of this space allows the existence of many new vacua, duals of 4–d strings. It would also be interesting to find the moduli space of the solutions connected by this new duality, analogous to the $SO(N, N)$ continuous transformations which generates the moduli space for the case of abelian duality [16], [17]. Let us finish with a curiosity. The process we followed for duality can be applied to any 2–d theory. In particular we can apply it to the free Dirac action which will have global symmetries. By gauging them and setting the field strength to zero, we can find the dual theory. In the abelian case, the gauge field equations yield to $\bar{\psi}\gamma^\mu\psi = \epsilon^{\mu\nu}\partial_\nu\Lambda$ (where $\Lambda$ is again the Lagrange multiplier). These are just the relations satisfied by the conserved currents in abelian bosonization. The explicit solution for the fermions requires the standard bosonization technology though. Also, for the non–abelian case the relation to the WZW model is not yet clear from this point of view but it is interesting to note that duality and bosonization in 2–d systems could be understood from the same underlying principle. We believe that other duality symmetries such as mirror symmetry should also be understood from this approach.

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