Recent Advances in Nonlinear Fracturing Characteristics of the Hydraulic Fracture in the Deep Reservoir

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Abstract. During hydraulic fracturing in the deep reservoir, the rock surrounding the hydraulic fracture (HF) presents three remarkable nonlinear mechanical behaviors: the fracture process zone (FPZ; microcrack zone) developing at the HF tip, the microcrack band surrounding the HF surface, and the plastic zone growing due to the opening compression of the HF. The above three mechanical behaviors constitute the HF nonlinear fracturing characteristics. However, understanding the HF nonlinear fracturing characteristics is challenging and crucial. In this work, we present our recent published advances in HF nonlinear fracturing. (1) We developed a characterization method for rock FPZ by utilizing and integrating fiber Bragg grating (FBG), digital imaging correlation, and acoustic emission (AE). The integrated measurements showed that FPZ softening followed a linear response. The relationship between FPZ length and crack tip opening displacement was linear, and the relationship between dissipated energy and FPZ length was quadratic. (2) Detailed analysis of the AE energy during cyclic fracturing tests of sandstone revealed that the FPZ propagation rate of subcritical fractures was related to the increasing rate of dissipated energy accumulated from all activated microcracks. Thus, a general law (Zhang’s law) of \( dl/dN = \left(1/2C_G\right)l^{-1}(dG_D/dN) \) was proposed and validated. (3) The HF microcrack band of high-temperature (120 °C) granite was identified using AE waveform analyses. The fracture energy was reduced by approximately 75% adjacent to the wellbore (approximately 40% of the fracture length) in the microcrack band. The effective width of the microcrack band was reduced by 40%–56.4% at 120 °C, indicating that high temperatures decreased the effective stimulated volume of HF. (4) We proposed a thermoplastic constitutive model for high-temperature rocks (such as shale) in deep reservoirs. The constitutive relation depends on hydrostatic pressure, stress deviator, and temperature. Parameters characterizing effects of temperature on thermoplasticity were also proposed. The above research advances can provide bases for HF in the deep reservoir.

Keywords: hydraulic fracturing, nonlinear fracture, fracture process zone, microcrack band, thermoplasticity

1. Introduction
With the wide application of hydraulic fracture (HF) in deep oil/gas/geothermal reservoirs [1-4] and mines [5,6], the fracture characteristics of HF is a major concern for the safe and efficient extraction of geo-energy.

To date, linear elastic fracture models are most widely used for delineating HF propagation. However, the elastic fracture theories assume that the sequential rupture of atomic bonds leads to fracture propagation [7], which does not comply with the real fracturing behavior of rock [8-12] during HF. On the basis of previous investigations on rock fracturing [12-21], during HF, massive microcracks first nucleate and coalesce in the fracture process zone (FPZ) ahead of the fracture tip. When the macro fracture extends across the FPZ, the microcrack band (i.e., the residual FPZ) generates on both side of HF. Subsequently, the opening of the HF will compress the surrounding rock, making the surrounding rock reach the yield state and generate the plastic zone. Therefore, the sequential development of FPZ, microcrack band, and plastic zone are the three dominant sources of nonlinear fracturing (Figure 1), which is more remarkable in the deep reservoir with high temperatures and pressures. However, the traditional fracturing theories are limited to the nonlinear fracturing of the HF.

![Figure 1. Diagram of HF nonlinear fracturing in the deep reservoir](image)

In this work, we reviewed our recent advances in nonlinear fracturing characteristics of HF on the basis of our published investigations. The new findings involve the integrated characterization of FPZ with acoustic emission (AE), digital imaging correlation (DIC), and fiber Bragg grating (FBG); the subcritical fracturing model of FPZ in sandstone; the temperature-dependent development of microcrack band surrounding the HF in high-temperature granite; and the thermoplastic constitutive model of shale bearing high-temperature compressive stress.

2. Comprehensive Fracturing Characterization: Integration of FBG, DIC, and AE Measurements

Rock fracturing characterization is the basis of a thorough understanding of nonlinear fracturing. In this section, we will present our published research advances in rock fracture characterization (Figure 2), which has been published in *Engineering Geology* (2018 245 45-56) [13].

![Figure 2. Sensor layouts: (a) a random pattern of white and black speckles on the front surface (detected by DIC); (b) FBG sensors fixed on the rear surface (detected by FBG) [13].](image)

The prominent characteristics of rock fracturing can be described by crack opening displacement (COD), the FPZ, and fracture energy. Given the absence of real-time comprehensive measurement techniques for characterizing the rock fracturing of HF, we utilized and integrated FBG, DIC, and AE.
Mode I fracturing test was performed on the rectangular sandstone specimen by three-point bending, and the fracturing process was simultaneously detected and recorded by FBG, DIC, and AE (Figure 2) [13]. Dimensions of the specimen were as follows: height H = 60 mm, span S = 100 mm, thickness B = 20 mm, and notch length a = 10 mm.

DIC-based measurements of horizontal displacements across the fracture presented localized discontinuity in accordance with FBG-based real-time measurement. At the onset of traction-free zone formation (peak load), DIC and FBG revealed that the critical COD (\(\delta_c\)) was 76 \(\mu\)m (Figure 3) [13].

Moreover, the dissipated energy distribution and cohesive crack profile in the FPZ characterized by the integrated measurements indicated that 70%–90% of AE energy (dissipated energy) occupied the FPZ. Minimal AE energy was concentrated in the traction-free zone, and the position of \(\delta_c\) delineated a boundary for specifying FPZ (Figure 4). The length of a fully developed FPZ was 20 mm (±2 mm) in length [13].

The integrated measurements delineated the cohesive crack, which was not available previously. The integrated measurements revealed that the softening curve followed a straight line; the relationship
between FPZ length and crack tip opening displacement (CTOD) was linear, whereas the relationship between dissipated energy and FPZ length was quadratic (Figure 5). The interrelations of cohesive crack characteristics identified by the integrated measurements demonstrated that real-time fracturing could be captured by one technique (e.g., AE or microseismic monitoring) in field application [13].

![Figure 5](image1.png)

**Figure 5.** Characterization of the cohesive crack model delineated with the integrated measurements: (a) the softening curve verified by AE energy and FBG-based CTOD; (b) the relation between FPZ (identified by DIC, AE, and FBG) length and CTOD (identified by DIC and FBG); (c) the relation between accumulated dissipated energy in FPZ (identified by AE) and FPZ length (identified by DIC, AE, and FBG) [13].

3. Subcritical Fracturing Characteristics of Sandstone with AE-based identification of FPZ

As mentioned above, FPZ characterization has been investigated comprehensively. In this section, with the detailed analyses of AE energy released from FPZ development, we introduce the subcritical fracturing characteristics of sandstone under cyclic loading, which has been published in *Rock Mechanics and Rock Engineering* (2019 52(7) 2459-2469) [14].

To delineate FPZ development during subcritical fracturing, we performed a set of tests on SCB sandstone specimens under cyclic loading with different stress ratios ($R$: the loading ratio of the minimum to the maximum in a loading cycle), and AE energy analysis was conducted to characterize the subcritical fracturing process (Figure 6). As the stress ratio $R$ varied from 0.15 to 0.25, the growth rate of the equivalent fracture length was divided into two stages, namely, a deceleration stage followed by an acceleration stage of fracture propagation, indicating that the Paris law was applicable only for the acceleration stage (Figure 7a) [14].

![Figure 6](image2.png)

**Figure 6.** Dimensions, sensor locations, and material properties of the SCB specimens: (a) photo of the SCB specimen; (b) dimensions of and sensor locations on the SCB specimen [14].

The propagation rate of subcritical fractures was found to be related to the increasing rate of dissipated energy accumulated from all activated microcracks. Thus, a general law (Eq. 1) was proposed. The general law indicated that the propagation rate of FPZ ahead of a subcritical fracture was physically related to the increasing rate of dissipated energy being accumulated from all active microcracks in the FPZ. We named this general law as Zhang’s law [14].
\[
dl/dN = \left(\frac{1}{2}C_G\right)l^{-1}(DG_D/dN)
\]

where \(l\) is the developing FPZ length, \(N\) is the loading cycle, \(G_D\) is the accumulated dissipated energy of the FPZ, and \(C_G\) is the combined modulus of material constants.

Figure 7. Paris law and Zhang’s law: (a) Paris law-based characterization of the subcritical fracture growth rate; (b) Zhang law-based characterization of the subcritical fracture growth rate [14].

AE energy analyses revealed that 70%–90% of the AE energy was concentrated in a small zone, which represented the FPZ. The AE sources were located over a large zone for different cycles (Figure 8). The physical explanation of Zhang’s law was consistent with the propagation of the FPZ characterized by the AE energy, and the fitting results of the new law reached 84%. The proposed law was found to be strongly applicable for the entire process of subcritical fracture growth, and it was insensitive to the \(R\) ratio. Thus, the proposed law overcomes the limitations of the Paris law for rock-like materials and supplies a significant theoretical basis for HF design [14].

Figure 8. FPZ propagation of subcritical fracturing (\(R\) ratio = 0.15) characterized by the AE energy [14].

4. Characterization of microcrack band surrounding HF in high-temperature granite

Section 3 has revealed the relationship between the FPZ growth rate and the dissipated energy in the FPZ. Once the macro fracture extends across the FPZ, the microcrack band (i.e., the residual FPZ at the fracture tip) will be generated on both sides of HF. Recently, limited success of HF was reported in enhanced geothermal systems (EGSs). Therefore, thorough understanding of the HF characteristics in high-temperature granites remains crucial. In this section, the characteristics of microcrack band
surrounding HF in high-temperature granite will be discussed based on our published research in the Journal of Petroleum Science and Engineering (2019 178 475-484) [15].

In our published investigation [15], four groups of cube granite specimens (dimensions: 300/300/300 mm) were tested to investigate the characteristics of HF for two confining pressures (0.1/0.1/0.1 MPa and 10/25/30 MPa) and temperatures (20 °C and 120 °C). AE was employed to characterize the HF processes. For different temperatures, the fracture geometry was almost unchanged (Figure 9), whereas the spatial distribution of AE energy and AE-based source mechanisms significantly changed. The spatial distributions of AE energy delineated a microcrack band (Figure 9) distributed along the HF. At the high temperature (120 °C), the effective width of the microcrack band was reduced by 40%-56.4%, and the fracture energy was reduced by approximately 75% adjacent to the wellbore (approximately 40% of the fracture length) in the microcrack band. AE source analysis of the fracture mechanisms in the microcrack band indicated that the high temperature (120 °C) reduced the proportion of shear microcracks by 6%-12%. The characteristics of high temperatures reduced the effectiveness of EGS HF due to the change in granite microstructures from temperature induction and the transient temperature differential (ΔT) between granite and fracturing fluid.

**Figure 9.** Spatial distributions of AE energy and source mechanisms affected by temperatures and confining pressures: (a) the experiment at 20 °C and under 0.1/0.1/0.1 MPa; (b) the experiment at 120 °C and under 0.1/0.1/0.1 MPa; (c) the experiment at 20 °C and under 10/25/30 MPa; and (d) the experiment at 120 °C and under 10/25/30 MPa [15].

The above findings provide three suggestions for the EGS HF: (1) the net pressure should be enhanced in real-time with HF propagation to avoid fracturing arrest; (2) ΔT between high-temperature granite and fracturing fluid should be lowered to enlarge the stimulated reservoir volume, and (3) the proppant is suggested to be appropriately placed to prevent the further reduction of the fracturing effectiveness from fracture closures.

5. Thermoplastic Constitutive Modeling of Rock Based on Temperature-Dependent Drucker–Prager Plasticity

With HF propagation, the continuous opening of HF will significantly compress the surrounding rock, thereby making the surrounding rock reach the yielding (plastic) state. In the deep reservoir, such as
deep shale reservoir and hot dry rock reservoir, high temperatures and pressures strongly affect rock plastic behavior, i.e., the rock in the deep high-temperature reservoir presents thermoplastic characteristics due to HF opening compression. Therefore, characterizing the thermoplastic behavior of rock is fundamental to understanding HF nonlinear fracturing in the deep reservoir. In this section, we proposed a new thermoplastic constitutive model of shale based on temperature-dependent Drucker–Prager plasticity, published in the International Journal of Rock Mechanics and Mining Sciences (2020 130 104305) [16].

To characterize the temperature-dependent mechanical behavior of shale in the deep reservoir with high temperatures, we proposed a thermoplastic constitutive model that incorporated the temperature-dependent Drucker–Prager hardening rule (Eq. 2), thermal loading/unloading criteria (Eq. 3), and completely coupled stress–strain–temperature relation (Eq. 4). The thermoplastic properties in the proposed model were obtained by further processing of published thermomechanical test data on Tournemire shale by Masri et al. [22].

\[
\begin{align*}
S &= -\left[\frac{\partial \sigma_i(\theta)}{\partial \theta} + 2 \frac{\partial K_i(k_1, \theta)}{\partial \theta}\right] l_1 - \frac{\partial \sigma_i(\theta)}{\partial \theta} - 2 \frac{\partial K_i(k_2, \theta)}{\partial \theta} \Rightarrow \text{temperature sensitivity modulus}
\end{align*}
\]

where \(f\) is the yielding function, \(\theta\) is the temperature, \(\alpha_0\) and \(\beta_0\) are the initial yield parameters, \(K_m\) is the hardening parameter (\(m = 1, 2, 3 \ldots\)), \(K_m\) is an internal variable (\(m = 1, 2, 3 \ldots\)), \(S\) is the temperature-sensitive modulus, and \(l_1\) is the first stress invariant (i.e., \(\sigma_{ii}\)).

\[
\begin{align*}
f &= 0 \quad \text{and} \quad \frac{1}{A_1} \left[\left(-\alpha_0 + K_1\right)\delta_{ij}^{\sigma} + S \theta\right] > 0 \quad \Rightarrow \text{thermoplastic loading}
\end{align*}
\]

\[
\begin{align*}
f &= 0 \quad \text{and} \quad \frac{1}{A_1} \left[\left(-\alpha_0 + K_1\right)\delta_{ij}^{\sigma} + S \theta\right] = 0 \quad \Rightarrow \text{neutral loading}
\end{align*}
\]

\[
\begin{align*}
f &= 0 \quad \text{and} \quad \frac{1}{A_1} \left[\left(-\alpha_0 + K_1\right)\delta_{ij}^{\sigma} + S \theta\right] < 0 \quad \Rightarrow \text{thermoelastic unloading}
\end{align*}
\]

where \(J_2\) is the second invariant of the stress deviator tensor (i.e., \(s_{ij} s_{ji}/2\)).

\[
\begin{align*}
\sigma_{ij} &= D_{ijkl}^{ep} k_{ijkl} - \beta_{ijkl} \theta
\end{align*}
\]

\[
\begin{align*}
D_{ijkl}^{ep}(k_1, k_2, \theta) &= D_{ijkl}(\theta) - \frac{1}{A_2} D_{ijklmn}(\theta) \frac{3 s_{mn} s_{kl}}{4 l_2} D_{stkl}(\theta) - \delta_{mn} D_{ijklmn}(\theta) \left[\alpha_0(\theta) + K_1(\theta)\right]^2 D_{stkl}(\theta) \delta_{st} - \\
\beta_{ijkl}^{ep}(k_1, k_2, \theta) &= \frac{1}{A_1} \left[S[\alpha_0(\theta) + K_1(\theta, \theta)] + \alpha_0(\theta) + K_1(\theta, \theta)\right]^2 \delta_{mn} P_{mn} D_{ijkl}(\theta) \delta_{kl} - \\
&\frac{1}{A_2} D_{ijkl}(\theta) \left[\frac{\sqrt{3}}{2l_2} l_2 s_{kl} - \frac{3}{l_2} s_{kl} s_{mn} P_{mn}\right] - P_{ij} \\
S &= \frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial K_m} = -\left[\frac{\partial \alpha_0(\theta)}{\partial \theta} + 2 \frac{\partial K_1(k_1, \theta)}{\partial \theta}\right] l_1 - \frac{\partial \alpha_0(\theta)}{\partial \theta} - 2 \frac{\partial K_1(k_2, \theta)}{\partial \theta} \\
A_1 &= (\alpha_0 + K_1)^2 \delta_{ij} D_{ijkl} \delta_{kl} + H_1 \\
A_2 &= \left(\sqrt{3} s_{ij}/2 \sqrt{l_2}\right) D_{ijkl}(\sqrt{3} s_{kl}/2 \sqrt{l_2}) + H_2
\end{align*}
\]

where \(D_{ijkl}\) is the elastic-plastic stiffness matrix, \(D_{ijkl}^{ep}\) is the isotropic elastic stiffness matrix, \(\beta_{ijkl}^{ep}\) is the symmetric tensor converting temperature variations to stress, \(H_1\) is the hydrostatic pressure-dependent plastic modulus, and \(H_2\) is the stress deviator-dependent plastic modulus. The hydrostatic pressure-dependent (HPD) and stress deviator-dependent (SDD) initial/critical yield parameters were found to be quadratically and linearly dependent on temperatures. In the range of 20 °C–250 °C, the HPD and SDD hardening parameters varied linearly and quadratically with internal variables \(k_1\) and \(k_2\), respectively (Figure 9). All three coefficients in the correlative equations between hardening parameters and internal variables depend linearly on temperatures [16].
The temperature sensitivity modulus, as a function of hydrostatic pressure, hardening parameters, and initial temperature, was used to characterize the contraction or expansion of the yield surface with temperature. This proposed model was validated with thermo-mechanical measurements of Tournemire shale (Figure 10), fitting well with experimental data (correlation coefficient: 93%). During plastic loading, the temperature-sensitive modulus was found to be positively correlated with the expansion of the yield surface with temperature, and the SDD loading/unloading discriminant coefficient was positive. Thus, the proposed model may be applicable to other rock materials [16].

![Figure 9. Relationship between hardening parameters and internal variables: (a) $K_1 - \kappa_1$ under different temperatures; (b) $K_2 - \kappa_2$ under different temperatures [16].](image)

![Figure 10. Comparison between model-predicted results and measurements in the hardening process: (a) $P_c = 0$ MPa; (b) $P_c = 5$ MPa; (c) $P_c = 10$ MPa; and (d) $P_c = 20$ MPa [16].](image)

6. Conclusion
In this work, we introduced our recent advances in nonlinear fracturing characteristics of the HF in the deep reservoir, such as the sequential developing rule of FPZ, microcrack band, and plastic zone surrounding the HF. The main findings are summarized as follows:
We developed a characterization method for rock fracturing by integrating FBG, DIC, and AE measurements together. With the comprehensive sandstone fracturing characterization, FPZ softening was found to follow a linear response. The relationship between FPZ length and CTOD was linear, and the relationship between dissipated energy and FPZ length was quadratic.

Detailed analysis of the AE energy of sandstone cyclic fracturing tests demonstrated that the FPZ propagation rate of subcritical fractures was related to the increasing rate of dissipated energy accumulated from all activated microcracks. Thus, a general law, $dl/dN=(1/2C_{eff})^{1}(dG_{eff}/dN)$, delineating the FPZ propagation rate of subcritical fracture in sandstone was proposed and validated.

We characterized the HF microcrack band of high-temperature (120 °C) granite by AE waveform analyses. Along the microcrack band, the fracture energy was found to be reduced by approximately 75% adjacent to the wellbore (approximately 40% of the fracture length) in the microcrack band. The high temperature of 120 °C reduced the effective width of the microcrack band by 40%–56.4%, which indicated that the high temperature of 120 °C decreased the effective stimulated volume of hydraulic fracturing.

A thermoplastic constitutive model was proposed for high-temperature rocks (such as shale) in deep reservoirs. The constitutive relation depends on hydrostatic pressure, stress deviator, and temperature. Parameters characterizing effects of temperatures on thermoplasticity were also proposed.

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