RADIATIVE TRANSFER PROBLEM IN DUSTY GALAXIES:
ITERATION SCALING APPROXIMATION

D. Semionov and V. Vansevičius
Institute of Physics, Savanorių 231, Vilnius LT-02300, Lithuania

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Abstract. We investigate the applicability and accuracy of the iteration
scaling approximation, proposed by Kylafis and Bahcall (1987). It is shown,
that while this method provides results sufficiently close to the exact solution
of radiative transfer problem, care must be taken in cases when the distribution
of interstellar dust significantly differs from that of light sources. The problem
is successfully circumvented by using the ratio of scattered light distributions
obtained in the first two iterations.

Key words: radiative transfer – ISM: dust, extinction

1. INTRODUCTION

The correct solution of the radiative transfer (RT) problem in the interstellar
medium is very important to a number of astrophysical topics (e.g., Li & Greenberg
2003). However, all algorithms, currently in use, while being similar in numerical
accuracy and applicability, suffer from a major drawback – they are quite slow
(e.g., Baes & Dejonghe 2001; Pascucci et al. 2004). Thus, while the algorithms and
hardware are being perfected, it is important to explore every approximation that
could produce the results with acceptable precision during computing time which
is shorter than that necessary to obtain the exact solution. If that approximation
introduces errors which are either below the observational error limit, or are lower
than the intrinsic numerical errors of a given method, then such a solution is
preferable. In this paper we will test the validity of one such an approximation for
iterative RT problem solving algorithms applied to spectrophotometric models of
dusty disk galaxies.

2. MODELS

Kylafis & Bahcall (1987, hereafter KB87) have proposed an approximate solution
for iterative algorithm, where each iteration accounts for a single scattering
event for all photons in the system. Their method (hereafter – iteration scaling
approximation, ISA), based on the fact that for the scattering phase function pa-
parameter $g_\lambda > 0.3$ (Henyey & Greenstein 1941) the scattering is directed mostly
forward, assumes that the ratio of global radiative energy distributions (as the
function of a position within the model and of the direction relative to the model
axes) $I_j/I_{j-1}$ for any two subsequent iterations $j - 1$ and $j$ is equal to the ratio
of global radiative energy distribution after the first scattering $I_1$ to the initial
distribution of star light $I_0$:
\[
\frac{I_j}{I_{j-1}} = \frac{I_1}{I_0}
\]

If this approximation produces results of acceptable accuracy, then the iterative method will have a decisive advantage over the Monte-Carlo method, since it will be possible to approximately solve the RT problem up to 10 times faster than is necessary to obtain the exact solution.

| \( \tau_V \) | S1 | S2 | S3 | S4 | S5 | S6 |
|----------------|----|----|----|----|----|----|
| \( z_{d}^{eff} \) | 0.5 | 1  | 2  | 0.5| 1  | 2  |

Table 1. Galaxy model parameters.

To verify the accuracy and applicability of ISA we use the same models and methods presented in our previous paper (Semionov & Vansevičius 2005b). The RT problem is solved using the Galactic Fog Engine (Semionov & Vansevičius 2002; Semionov 2003; Semionov & Vansevičius 2005a), a code for spectrophotometric modeling of dusty disk galaxies, employing a 2D iterative ray-tracing algorithm for six disk galaxy model groups S1–S6, having two values of the optical depth to the model center in V passband measured perpendicularly to the disk plane: \( \tau_V = 1 \) for model groups S1–S3 and 10 for model groups S4–S6. The stars and the dust follow a double exponential luminosity and mass distribution,

\[
\rho(r, z) = \rho_0 \exp \left( -\frac{r}{r_{eff}} - \frac{|z|}{z_{eff}} \right),
\]

with the same effective scale-length \( r_{eff} \) and scale-heights \( z_{eff} \) and \( z_{d eff} \) for the stellar and dust disks, respectively. The models represent three cases thought to approximate real disk galaxies - “dust within stellar disk” \( (z_{d eff} = 0.5z_{eff}, \text{ models S1 and S4}) \), “well-mixed dust and stars” \( (z_{d eff} = z_{eff}, \text{ models S2 and S5}) \) and “dust enveloping stellar disk” \( (z_{d eff} = 2z_{eff}, \text{ models S3 and S6}) \). Optical interstellar dust parameters were computed using the Laor & Draine (1993) model approximating the Milky Way galaxy extinction law.

Each model group consists of M1, M2, M3, M5 and M8 models. For each \( M_i \) model eight anisotropic scattering iterations were performed, the first \( i \) iterations being computed exactly, and the remaining iterations – using the \( i \)th order ISA:

\[
\frac{I_j}{I_{j-1}} = \frac{I_n}{I_{n-1}}.
\]

The original ISA, as proposed by KB87 (Eq. 1), corresponds to model M1, while model M8 is computed exactly, without employing ISA. All models show the same total absorbed energy \( E_{\lambda}^{abs} \) values, differing less than the model numerical accuracy. The energy defect after the 8th iteration for all models does not exceed 1% of the total radiative energy \( E_{\lambda}^{tot} \).

3. RESULTS AND DISCUSSION

We compare synthetic multiaperture photometry of the models, computed using ISA, to the results obtained for exact solution as \( \Delta E_{B-V}^{M_i} = E_{B-V}^{M_8} - E_{B-V}^{M_i} \) (Figure 1) and \( \Delta E_{V-K}^{M_i} = E_{V-K}^{M_8} - E_{V-K}^{M_i} \) (Figure 2). Additionally, for all models
**Fig. 1.** Differences in $E_{B-V}$ photometric profile between the models computed with and without iteration scaling approximation (ISA). The panel rows correspond to the model groups S1–S6 (starting from top), panel columns – to the model galaxy inclinations 0°, 50°, 80° and 90° (left to right). Solid, dashed, dotted and dash-dotted lines correspond to differences between the M8 model on one side and the M1, M2, M3 and M5 models on other side. The aperture radius is given in the effective disk scale-length $r_{\text{eff}}$ units.
Fig. 2. The same as in Fig. 1, but for the $E_{V-K}$ photometric profile.
Fig. 3. Differences in the color excess $E_{V-K}$ minor axis cross-section between the models computed with and without ISA. Panel rows correspond to the model groups S1–S6 (starting from top), panel columns – to the model galaxy inclinations 0°, 50°, 80° and 90° (left to right). Solid, dashed, dotted and dash-dotted lines correspond to differences between the M8 model from one side and the M1, M2, M3 and M5 models from other side. The distance from the image center is given in the effective disk scale-height $z_{\text{eff}}$ units.
we have computed difference of color excess $E_{V-K}$ of the image minor axis cross-sections as $\Delta E^{C,M_1}_{V-K} = E^{C,M_8}_{V-K} - E^{C,M_1}_{V-K}$ (Figure 3).

It can be seen, that approximation-induced errors in all photometric profiles for the model M1 in groups S1, S2, S4 and S5, while having a complex radial dependence, remain tolerable for all inclination angles. In contrast, model groups S3 and S6 display large differences between models M1 and M8, with the error decrease at high inclinations. This behavior at high inclinations is also common for the errors shown by groups S2 and S5, which, on the average, also show the smallest errors in low (S1–S3) and high (S4–S6) opacity groups, respectively.

The first-order ISA-induced errors affect most of the minor axis cross-sections for all considered wavelengths in all models; the smallest deviation again is observed for groups S2 and S5. Except for groups S3 and S6, the deviations of the minor axis cross-section increase with the increasing inclination angle.

The ISA of higher order produce much better results, M2 being already sufficiently close to M8, however, in some cases the M3 approximation is necessary to achieve acceptable errors. It can also be noted, that the error distribution for photometric profiles of $A_V$ and $E_{V-K}$ is identical.

The original ISA method of KB87 seems to successfully reproduce photometric profiles of the exactly computed models only when the dust distribution geometrically coincides within the distribution of stellar population. The approximation is generally good at high inclinations even for large optical depths, however, the deviation becomes noticeable already at 80°. The minor axis cross-section of a model image is affected more severely and this might lead to erroneous determination of the geometric parameters of stellar populations for edge-on galaxies using a comparison with the spectrophotometric models. The success of the ISA of higher-orders does not exhibit any significant dependence on the values of $g_\lambda$ parameters or the model optical depth since similar results are obtained for all considered passbands.

When determining the required order of ISA it is of prime importance to fully account for the radiative energy redistribution in the course of scattering from the primary (stars) to the secondary (dust) sources. Therefore in most cases, except when the dust and stars have the same distribution, it is necessary to compute exactly at least the first two scattering iterations. Our tests show that the computation of the first three scattering iterations is generally enough to reach a numerical accuracy which is undistinguishable from the model computed exactly, and this decreases the computational time up to five times.

In extreme cases, when the primary and secondary light sources are fully separated (e.g., dusty shell surrounding dust-free stellar cluster), more iterations seem to be necessary. In these cases the applicability of ISA should be properly investigated. The usefulness of ISA would increase considerably if we were able to establish criteria, determining whether a given number of iterations is sufficient.

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