Methods of Laser Cooling of Electron Beams in Storage Rings

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Methods of enhanced laser cooling of particle beams in storage rings and Robinson’s damping criterion are discussed. The dynamics of amplitudes of betatron oscillations and instantaneous orbits of electrons interacting with laser beams being displaced in the radial direction is investigated.

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I. INTRODUCTION

Different cooling methods were suggested to decrease the emittances and to compress the phase-space density of charged particle beams in storage rings. All of them, except the stochastic one, are based on a friction of particles in external electromagnetic fields or in media. Below we will consider laser methods of enhanced cooling of electron beams based on friction. The main results will be valid for cooling of other particles.

II. METHODS OF COOLING OF PARTICLE BEAMS IN STORAGE RINGS

In the ordinary three-dimensional radiative method of cooling of electron beams a laser beam overlaps an electron beam, its transverse position is motionless, all electrons interact with the laser beam independent of their energy and amplitude of betatron oscillations. A friction originating in the process of emission (scattering) of laser photons by electrons leads to a damping of amplitudes of both betatron and phase oscillations of electrons. The damping is because of the friction force is parallel to the electron velocity, and therefore the momentum losses include both the transverse and longitudinal ones. The longitudinal momentum losses are compensated by a radio frequency accelerating system of the storage ring. Meanwhile the longitudinal momentum of the electron tends to a certain equilibrium. The transverse vertical and radial momenta disappear irreversibly. The difference in rates of momentum loss of electrons having maximum and minimum energies in the beam is small, and that is why the cooling time of the electron beam is high.

When a cooling is produced in a dispersion-free straight section and the laser beam intensity is constant inside the area of the laser beam occupied by a being cooled electron beam, then the damping times of the horizontal vertical and phase oscillations are:

$$\tau_x = \tau_y = 2\tau_r = \frac{2\varepsilon}{P},$$

(1)

where $P$ is the average power of the radiation scattered by the electron of the energy $\varepsilon$.

The coupling arising in non-zero dispersion straight sections of the storage rings leads to a redistribution of the longitudinal and radial damping times when the radial gradient of the laser beam intensity is introduced.

The physics of three-dimensional radiative cooling both ion and electron beams is not differ from synchrotron radiation damping. We deal with the non-conservative system. Friction causes the appearance of a reaction force on the emitting particle which must be taken into account to describe particle dynamics. The Liouville’s theorem does not valid for such systems. At the same time in the case of non-selective interaction the Robinson’s damping criterion is valid: the sum of damping decrements (inverse damping times) is a constant ($\tau_{y}^{-1} + \tau_{r}^{-1} = 3P/2\varepsilon$). In a particular choice of lattice and laser or material target, damping rates can be shifted between different degrees of freedom. However the maximum decrements are limited by the condition $\tau_{y,s} \geq 0$ or by the value $\tau_{y,s} \geq 2\varepsilon/3P$.

The method of enhanced one-dimensional laser cooling of ion beams in the longitudinal plane is based on the resonance Rayleigh scattering of "monochromatic" laser photons by not fully stripped ion beams or by complicated nuclei. In this method the laser beam overlaps the ion beam and has a chirp of frequency. Ions interact with the laser beam at resonance energy, decrease their energy in the process of the laser frequency scanning until all of them reach the minimum energy of ions in the beam. At this frequency the laser beam is switched off. The higher energy of ions the longer the time of interaction of ions with the laser beam. Ions of minimum energy do not interact with the laser beam at all. In such a way, in this method, the selective interaction is realized. The damping time of the ion beam in the longitudinal plane is determined by the dispersion of its energy spread $\sigma_\varepsilon$

$$\tau_{\varepsilon} = \frac{2\sigma_\varepsilon}{P},$$

(2)

According to (1), the damping times of betatron and phase oscillations in the three-dimensional radiative method of cooling are equal to the time interval necessary for the electron energy loss, which is about the two-fold and four-fold initial energy of the electron, accordingly. The damping time (2) is $\varepsilon/\sigma_\varepsilon \sim 10^3$ times less than (1). It means, that the selectivity of interaction leads to the
violation of the Robinson’s damping criterion and open the possibility of enhanced cooling.

A one-dimensional method of ion cooling has been realized by now [3] - [6]. A three-dimensional radiative method of laser cooling of ion beams by broadband laser beams was suggested and developed in [7] - [11] by the analogy with the synchrotron radiation damping of amplitudes of betatron and phase oscillations of particles in storage rings1. The electron version of the three-dimensional radiative cooling was developed by Zh.Huang and R.D.Ruth [12]. They paid attention to the possibility to store laser wavepackets of picosecond duration and high intensity, \( I \simeq 10^{17} \text{ W/cm}^2 \) (magnetic field strength, \( B \sim 2 \cdot 10^7 \text{ Gs} \)) in the optical high-finesse resonator to interact repetitively with a circulating electron beam in a storage ring of the energy \( 10 \div 10^2 \text{ MeV} \) for the rapid cooling of the beam, counterbalancing of the intrabeam scattering, and x-ray generation. Below the enhanced methods of radiative laser cooling of electron beams in storage rings are discussed [13], [14].

### III. ENHANCED LASER COOLING OF ELECTRON BEAMS

Below the enhanced cooling of particle beams both in the longitudinal and transverse planes will be considered. A universal kind of selective interaction of particles with moving in the radial direction laser beams or media targets is suggested. Selective interaction means that the laser beam or media target at some moment interacts with one part of the being cooled beam and does not interact with the rest. At the other moments the interaction is switched on for another parts of the being cooled beam. Robinson’s damping criterion does not work in this case.

#### A. Cooling methods based on selective interaction of electron and laser beams and on dispersion coupling of the transverse and longitudinal motion of electrons in storage rings

For the sake of simplicity we will neglect the emission of the synchrotron radiation in the bending magnets of the storage rings, supposing that the RF system of a storage ring is switched off and the laser beams are homogeneous and have sharp edges in the radial directions. We suppose that the dispersion of the energy loss of electrons in the laser beam is small and the jump of their instantaneous orbits caused by the energy loss is less than the amplitude of betatron oscillations.

In a smooth approximation, the movement of an electron relative to its instantaneous orbit is described by the equation

\[
x_\beta = A \cos(\Omega t + \varphi).
\]

where \( x_\beta = x - x_n \) is the electron deviation from the instantaneous orbit \( x_n \); \( x \), its radial coordinate; \( A \) and \( \Omega \), the amplitude and the frequency of betatron oscillations.

If the coordinate \( x_{\beta 0} \) and transverse radial velocity of the electron \( \dot{x}_{\beta 0} = -A \Omega \sin(\Omega t_0 + \varphi) \) correspond to the moment \( t_0 \) of change of the electron energy in a laser beam then the amplitude of betatron oscillations of the electron before an interaction is \( A_0 = \sqrt{x_{\beta 0}^2 + \dot{x}_{\beta 0}^2 / \Omega^2} \). After the interaction, the position of the electron instantaneous orbit will be changed by a value \( \delta x_n \), the deviation of the electron relative to the new orbit will be \( x_{\beta 0} - \delta x_n \), and the change of its transverse velocity can be neglected2. The new amplitude of the electron betatron oscillations will be \( A_1 = \sqrt{(x_{\beta 0} - \delta x_n)^2 + \dot{x}_{\beta 0}^2 / \Omega^2} \) and the change of the square of the amplitude

\[
\delta(A)^2 = A_1^2 - A_0^2 = -2x_{\beta 0}\delta x_n + (\delta x_n)^2.
\]

When \( |\delta x_n| \ll |x_{\beta 0}| < A_0 \) then in the first approximation the value \( \delta\Delta = -(x_{\beta 0}/A)\delta x_n \). From this it follows that to produce the enhanced cooling of an electron beam in the transverse plane we must create such conditions when electrons interact with a laser beam under deviations from the instantaneous orbit \( x_{\beta 0} \) of one sign \( x_{\beta 0} < 0 \), when the dispersion function is positive \( \partial x_n / \partial \varepsilon > 0 \) or in the opposite case \( x_{\beta 0} > 0 \), \( \partial x_n / \partial \varepsilon < 0 \). 3 In this case the value \( \delta A \) has one sign and the rate of change of amplitudes of betatron oscillations of electrons \( \partial\Delta / \partial t \) is maximum. A selective interaction of electrons with the laser beam is necessary to realize this case.

In Fig.1, two schemes of a selective interaction of electron and laser beams are shown for cooling of electron beams in the transverse and longitudinal planes. For the transverse cooling, the laser beam \( T_1 \) is used. At the initial moment it overlaps a small external part of the electron beam in the radial direction in the straight section of the storage ring with non zero dispersion function. First electrons with largest initial amplitudes of betatron oscillations interact with the laser beam. Immediately after the interaction and loss of the energy the position and

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1 The difference in cooling of electron and ion beams is in the dependence of the average power of scattered radiation on the relative energy \( \sigma \sim \gamma^2 \), \( \sigma_{\text{ion}} \sim \gamma \) and in the finite decay time of ions excited in the process of the Rayleigh scattering. The latter may be neglected when length of the ion decay is much less than that of the period of ion betatron oscillations [8], [10].

2 It means that we neglect a week ordinary damping of amplitudes of betatron oscillations determined by a loss of the transverse electron momentum.

3 Friction in non-conservative system can lead to cooling in one plane and to heating in the other one.
direction of momentum of an electron remain the same, but the instantaneous orbit is displaced inward in the direction of the laser beam. The radial coordinate of the instantaneous orbit and the amplitude of betatron oscillations are decreased to the same value owing to the dispersion coupling. After every interaction the position of the instantaneous orbit approaches the laser beam more and more, and the amplitude of betatron oscillations is coming smaller. It will reach some small value when the instantaneous orbit reaches the edge of the laser beam. When the depth of dipping of the instantaneous orbit of the electron in the laser beam becomes greater than the amplitude of its betatron oscillations, the orbit will continue its movement in the laser beam with constant velocity. The amplitude of betatron oscillations will not be changed.

The velocity of an electron instantaneous orbit \( \dot{x}_{T_1} \) depends on the distance \( x_{T_1,2} - x_\eta \) between the edge of the laser beam and the instantaneous orbit, and on the amplitude of betatron oscillations. When the orbit enters the laser beam at the depth higher than the amplitude of betatron oscillations then electrons interact with the laser beam every turn and theirs velocity reaches the maximum value \( \dot{x}_\eta \), which is given by the intensity and the length of the interaction region of the electron and laser beams. In the general case, the velocity \( \dot{x}_\eta \) can be presented in the form \( \dot{x}_\eta = W \cdot \dot{x}_{\eta\text{in}} \), where \( W \) is the probability of an electron crossing the laser beam. \( W \) is the ratio to a period of a part of the period of betatron oscillations of the electron determined by the condition \( |x_{T_2} - x_\eta| \leq |x_0| \leq \Delta \) when the deviation of the electron from the instantaneous orbit at azimuths of the laser beam is directed to the laser beam and is greater than the distance between the orbit and the laser beam. When the length of the laser beam is much less than the length of the period of electron betatron oscillations, the prob-

![Diagram](image.png)

**FIG. 1:** The scheme of the enhanced electron cooling. The axis "y" is the equilibrium orbit of the storage ring; \( T_1 \) and \( T_2 \) the laser beams. The transverse positions of laser beams are displaced with the velocity \( \dot{v}_{T_1,2} \) relative to the equilibrium orbit, 1-1, 2-2, ... the location of the instantaneous electron orbit, and 1,2,3, ... the electron trajectories after 1,2,3, ... events of the energy loss.

The degree of overlapping is changed by moving uniformly the laser beam position from inside in the direction of the being cooled electron beam with some velocity \( \dot{v}_{T_1} \). When the laser beam reaches the instantaneous orbit corresponding to electrons of maximum energies then the laser beam must be switched off and returned to a previous position. All electrons of the beam will have small amplitudes of betatron oscillations and increased energy spread. Electrons with high amplitudes of betatron oscillations will start to interact with a laser beam first, their duration of interaction and absolute decrease of amplitudes of betatron oscillations will be higher.

To realize the enhanced cooling of an electron beam in the longitudinal plane we can use a laser beam \( T_2 \) located in the straight section of a storage ring with non zero dispersion function (see Fig.1). The radial laser beam position is moving uniformly from outside in the direction of the being cooled electron beam with a velocity \( \dot{v}_{T_2} \) higher than maximum velocity of the electron instantaneous orbit deepened in the laser beam. At the initial moment, the laser beam overlaps only a small part of the electron beam. The degree of overlapping is changed in such a way that electrons of maximum energy, first and then electrons of lesser energy, come into interaction. When the laser beam reaches the orbit of electrons of minimum energy then it must be switched off and returned to the previous position. In this case, the rate of the energy loss of electrons in the beam will not be increased, but the difference in duration of interaction and hence in the energy losses of electrons having maximum and minimum energies will be increased essentially. As a result all electrons will be gathered at the minimum energy in a short time.

### B. Interaction of electron beams with transversely moving laser beams

In the methods of enhanced laser cooling of electron beams the internal and external laser beam positions are displaced in the transverse directions (see Fig.1). Below the evolution of amplitudes of betatron oscillations and positions of instantaneous orbits in the process of the energy loss of electrons in laser beams will be analyzed.

The velocity of an electron instantaneous orbit \( \dot{x}_\eta \) can be presented in the form \( \dot{x}_\eta = W \cdot \dot{x}_{\eta\text{in}} \), where \( W \) is the probability of an electron crossing the laser beam. \( W \) is the ratio to a period of a part of the period of betatron oscillations of the electron determined by the condition \( |x_{T_2} - x_\eta| \leq |x_0| \leq \Delta \) when the deviation of the electron from the instantaneous orbit at azimuths of the laser beam is directed to the laser beam and is greater than the distance between the orbit and the laser beam. When the length of the laser beam is much less than the length of the period of electron betatron oscillations, the prob-

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4 Instantaneous orbits can be moved in the direction of the laser beam, instead of moving of a laser beam. A kick, decreasing of the value of the magnetic field in bending magnets of the storage ring, a phase displacement or eddy electric fields can be used for this purpose.
ability can be presented in the form $W = \varphi_{1,2}/\pi$, where

$\varphi_1 = \pi - \arccos \xi_1, \varphi_2 = \arccos \xi_2, \xi_{1,2} = (x_{T_{1,2}} - x_\eta)/A$, indices 1,2 correspond to laser beams.

The behavior of the amplitudes of betatron oscillations of electrons, according to (4), is determined by the equation $\partial A/\partial x_\eta = < x_{\beta_0} > /A$, where $< x_{\beta_0} >$ is the electron deviation from the instantaneous orbit averaged through the range of phases $2\varphi_{1,2}$ of betatron oscillations where electrons cross the laser beam. The value $< x_{\beta_0} > \equiv \pm \text{sinc} \varphi_{1,2}$, where $\text{sinc} \varphi_{1,2} = \sin \varphi_{1,2}/\varphi_{1,2}$, signs + and − are related to the first and second laser beams. Thus the cooling processes are determined by the system of equations

$$\frac{\partial A}{\partial x_\eta} = \pm \text{sinc} \varphi_{1,2}, \quad \frac{\partial x_\eta}{\partial t} = \frac{\dot{x}_\eta}{\varphi_{1,2}}. \quad (5)$$

From equations (5) and the expression $\partial A/\partial x_\eta = [\partial A/\partial t]/[\partial x_\eta/\partial t]$ it follows:

$$\frac{\partial A}{\partial t} = \frac{\dot{x}_\eta}{\varphi_{1,2}} \sin \varphi_{1,2} + \frac{\dot{x}_\eta}{\varphi_{1,2}} \sqrt{1 - \xi_{1,2}^2}. \quad (6)$$

Let the initial instantaneous electron orbits be distributed in a region $\pm \sigma_{x,0}$ relative to the location of the middle instantaneous orbit $x_\eta$, and the initial amplitudes of electron radial betatron oscillations $A_0$ be distributed in a region $\sigma_{x,0}$ relative to their instantaneous orbits, where $\sigma_{x,0}$ and $\sigma_{x,0}$ are dispersions. The dispersion $\sigma_{x,0}$ is determined by the initial energy spread $\sigma_{x,0}$.

Suppose that the initial spread of amplitudes of betatron oscillations $\sigma_{x,0}$ is identical for all instantaneous orbits of the beam. The velocities of the instantaneous orbits in a laser beam $\dot{x}_\eta < 0$, the transverse velocities of the first laser beams $v_{T_1} > 0$, and $v_{T_2} < 0$. Below we will use the relative radial velocities of the laser beam displacement $k_{1,2} = v_{T_{1,2}}/\dot{x}_\eta$, where $v_{T_{1,2}} = dx_{T_{1,2}}/dt$. In our case $\dot{x}_\eta < 0, k_{1,2} > 0, k_{1,2} < 1$.

From the definition of $\xi_2$ we have a relation $x_\eta = x_{T_1} - \xi_2 A_1(\xi_2)$. The time derivative is $\partial x_\eta/\partial t = v_{T_1} - [A + \xi_2(\partial A/\partial \xi_2)]/\partial \xi_2/\partial t$. Equating this value to the second term in (5) we will receive the time derivative

$$\frac{\partial \xi_2}{\partial t} = \frac{\dot{x}_\eta}{\varphi_{1,2}} \frac{\pi k_{1,2} - \varphi_{1,2}}{A(\xi_2) + \xi_2(\partial A/\partial \xi_2)}. \quad (7)$$

Using this equation we can transform the first value in (5) to the form $\pm \text{sinc} \varphi_{1,2}(\xi_2) = (\partial A/\partial \xi_2)((\partial \xi_2/\partial t)/[\partial x_\eta/\partial t]) = (\pi k_{1,2} - \varphi_{1,2})(\partial A/\partial \xi_2)/[A + \xi_2(A/\partial \xi_2)], \varphi_2$ which can be transformed to $\partial \ln A/\partial \xi_2 = \pm \sin \varphi_{1,2}/\pi k_{1,2} - (\varphi_{1,2} + \xi_2 \sin \varphi_{1,2})$. The solution of this equation is

$$A = A_0 \exp \int_{\xi_{1,2}}^{\xi_{1,2}} \frac{\pm \sin \varphi_{1,2} d\xi_2}{\pi k_{1,2} - (\varphi_{1,2} + \xi_2 \sin \varphi_{1,2})}. \quad (8)$$

where the index 0 correspond to the initial time. Substituting the values $A$ and $\partial A/\partial \xi_2$ determined by (8) in (7) we find the relation between time of observation and parameter $\xi_2$

$$t - t_0 = \frac{\pi A_0}{|x_{\eta_{in}}|} \psi(k_{1,2}, \xi_{1,2}), \quad (9)$$

where $\psi(k_{1,2}, \xi_{1,2}) = - \int_{\xi_{1,2}}^{\xi_{1,2}} A(\xi_2)/\{A_0[\pi k_{1,2} - (\varphi_{1,2} + \xi_2 \sin \Delta \varphi_{1,2})] \} d\xi_2$.

The equations (9) determine the time dependence of the functions $\xi_{1,2}(t - t_0)$. The dependence of the amplitudes $A[\xi_{1,2}(t - t_0)]$ is determined by the equation (8) through the functions $\xi_{1,2}(t - t_0)$ in a parametric form. The dependence of the position of the instantaneous orbit follows from the definition of $\xi_{1,2}$

$$x_\eta(t - t_0) = x_{T_1} + v_{T_1}(t - t_0) - A_0((\xi_2(t - t_0)) \cdot (t - t_0). \quad (10)$$

The function $\psi(k_2, \xi_2)$ for the case $k_2 > 0$ according to (9) can be presented in the form

$$\psi(k_2, \xi_2) = \int_{\xi_2}^{1} dx \exp \int_{x}^{1} \sqrt{1 - t^2/(\pi k_{1,2} - \arccos t + \sqrt{1 - t^2})} \frac{1}{\pi k_{2,2} - \arccos x + \sqrt{1 - t^2}} d t. \quad (11)$$

The instantaneous orbits of electrons having initial amplitudes of betatron oscillations $A_0$ will be deepened into the laser beam to the depth greater than their final amplitudes of betatron oscillations $A_T$ at a moment $t_f$. According to (9), $t_f = t_0 + \pi A_0\psi(k_2, \xi_2)/|x_{\eta_{in}}|$, where $\xi_2 = \xi(t_f) = 1$. During the interval $t_f - t_0$ the laser beam will pass a way $l_f = \int_{|v_{T_2}|}(t_f - t_0) = \pi k_{2,2}\psi(k_2, \xi_2)/A_0$. The dependence $\psi(k_2, \xi_2)$ determined by (11) is presented in Table 1.

| $\xi_2$ | 1.0  | 1.02 | 1.03 | 1.05 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.7 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\psi | 6.38 | 9.90 | 6.52 | 3.71 | 2.10 | 1.51 | 1.18 | 0.98 | 0.735 | 0.538 |

Numerical calculations of the dependence $\psi(k_2, \xi_2)$ on $\xi_2$ for the cases $k_2 = 1.0, k_2 = 1.1$ and $k_2 = 1.5$ are presented in Tables 2, 3, and 4, respectively. It can be presented in the next approximate form

$$\psi(k_2, \xi_2) \approx C_3(k_2)\psi\left(\frac{1 - \xi_2}{k_2 + \xi_2}\right), \quad (12)$$

where $C_3(k_2) \approx 0.492 - 0.680(k_2 - 1) + 0.484(k_2 - 1)^2 + ..., \psi\left((1 - \xi_2)/(k_2 + \xi_2)\right)|_{k_2=1} \approx (1 - \xi_2)/(1 + \xi_2)$.  

| $\xi_2$ | 1.0  | 1.02 | 1.03 | 1.05 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.7 | 2.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\psi | 6.38 | 9.90 | 6.52 | 3.71 | 2.10 | 1.51 | 1.18 | 0.98 | 0.735 | 0.538 |
C. The enhanced transverse laser cooling of electron beams

In the method of the enhanced transverse laser cooling of electron beams a laser beam $T_1$ is located in the region $(x_{T_1}, x_{T_1} - a)$, where $a$ is the laser beam width (Fig. 1). The degree of the transverse cooling of the electron beam is determined by (8). The final amplitude in this case can be presented in the form

$$A_f = A_0 \exp \int_{\xi_{1,0}}^{\xi_{1,f}} \frac{\sqrt{1 - \xi_1^2} d\xi_1}{\pi k_1 - \pi + \arccos \xi_1 - \xi_1 \sqrt{1 - \xi_1^2}}.$$  

(13)

The numerical calculations of the dependence of the ratio $A_f/A_0$ on the relative radial velocity $k_1$ of the laser beam displacement are presented in Fig. 2 and Table 5 for the case $\xi_{1,0} = -1$, $\xi_{1,f} = 1$. This dependence can be presented by the approximate expression

$$A_f \approx A_0 \sqrt{|k_1| / |k_1| + 1}.$$  

(14)

The time of the laser beam cooling and the final total radial dimension of the beam are equal to

$$\tau_{x,1} \simeq \frac{\sigma_{x,0}}{v_{T_1}}, \quad \sigma_{x,f} |_{|k_1| \ll 1} \simeq \frac{\sigma_{x,0}}{|k_1|} + \sigma_{x,\tau,0},$$  

(15)

where $\sigma_{x,0} = \sigma_{x,b,0} + \sigma_{x,\tau,0}$ is the total initial radial dimension of the electron beam. For the time $\tau_{x,1}$ the instantaneous orbits of electrons of a beam having minimum energy and maximum amplitudes of betatron oscillations at $|k_1| \ll 1$ pass the distance $\sim |x_{\tau,1}|$. According to (14) and (15) the enhanced transverse laser cooling can lead to an appreciable degree of cooling of electron beams in the transverse plane and a much greater degree of heating in the longitudinal one. From this it follows that the four- and six-dimensional emittances of the beam will be increased. Straight sections with low-beta and high dispersion functions have to use. In this case the radial beam dimension (15) will be lesser and lesser events of photon emission are required to cool the beam in the transverse direction, as the change of amplitudes of betatron oscillations of electrons is of the order of the change of positions of their instantaneous orbits. Meanwhile, the spread of amplitudes of betatron oscillations is small and the step between positions of instantaneous orbits is high.

Similar method of interaction of external and internal targets with proton beams was described in 1956 by O’Neil [13, 16]. However, O’Neil considered the question of damping of betatron oscillations of proton beams in the transverse direction by means of motionless solid wedge-shaped material targets. The targets in that case could not provide any enhanced cooling. They could be used for injection and capture of only one portion of protons. For the multi-cycle injection and storage of protons O’Neil suggested an ordinary three-dimensional ionization cooling based on a thin hydrogen jet target located in a working region of the storage ring.

An internal target could be rotated out of the medium plane only to prevent the proton beam losses when the positions of instantaneous orbits reached the edge of the storage ring.
D. The enhanced longitudinal laser cooling of electron beams

In the method of the enhanced longitudinal laser cooling of electron beams a laser beam $T_2$ is located in the region $(x_{T_2}, x_{T_2}+a)$ (see Fig. 1). Its radial position is displaced uniformly with the velocity $v_{T_2} < 0$, $|v_{T_2}| > |x_{q,n}|$ from outside of the working region of the storage ring in the direction of a being cooled electron beam. The instantaneous orbits of electrons will go in the same direction with a velocity $|x_{q,n}|$ beginning from the moment of their first interaction with the laser beam. When the laser beam reaches the instantaneous orbit of electrons having minimum initial energies it must be removed to the initial position.

The law of change of the amplitudes of electron betatron oscillations is determined by (8), which can be presented in the form

$$A = A_0 \exp \left( \frac{t}{\xi_2} - \frac{\sqrt{1 - \xi_2^2}}{\pi k_2 - \arccos \xi_2 + \xi_2 \sqrt{1 - \xi_2^2}} \right). \tag{16}$$

The dependence of the ratio of a final amplitude of electron betatron oscillations $A_f = A(x_2 = 1)$ to the initial one on the relative velocity $k_2$ of the second laser beam is presented in Table 6 and Fig.3. This ratio can be presented by the next approximate expression

$$A_f \approx A_0 \sqrt{\frac{k_2}{k_2 - 1}}. \tag{17}$$

Table 6

| $k_2$     | 1.0001 | 1.001 | 1.01  | 1.1   | 1.5   | 2.0   |
|-----------|--------|-------|-------|-------|-------|-------|
| $A_f/A_0$ | 100.005| 31.64 | 10.04 | 3.32  | 1.73  | 1.414 |

The evolution of instantaneous orbits of electrons interacting with the laser beam depends on the initial amplitudes of betatron oscillations $A_0 = \sigma_{x,b,0}$ and the highest energies. The instantaneous orbit of these electrons, according to (8) - (10), is changed by the law $x_{q_1} = x_{T_2,0} + v_{T_2} (t-t_0_1) - \xi \sigma_{x,b} (x_2)$ up to the time $t = t_f$, where $t_0_1$ is the initial time of interaction of electrons with the laser beam. At the same time instantaneous orbits $x_{q_2}$ of electrons having the same maximum energy but zero amplitudes of betatron oscillations are at rest up to the moment $t_0_2 = t_0_1 + \sigma_{x,b,0}/|v_{T_2}|$. The orbit $x_{q_2}$ is displaced relative to the orbit $x_{q_1}$ by the distance $\Delta x_{q_1,q_2} = (x_{q_1} - x_{q_2})$. At the moment $t_0_2$, when $x_{q_2} = x_{T_2}$, this distance reaches the minimum

$$\Delta x_{q_{1,-2}}(t_0_2) = -\xi_2(t_0_2) \cdot \sigma_{x,b}(t_0_2) < 0, \tag{18}$$

where the parameter $\xi_2(t_0_2)$, according to (9) and the condition $|v_{T_2}|(t_0_2 - t_0_1) = \sigma_{x,b,0}$, will be determined by the equation $\psi[k_2, \xi_2(t_0_2)] = 1/\pi k_2$. The value $\psi[k_2, \xi_2(t_0_2)]|_{k_2=1} \approx 1/\pi, \xi_2(k_2, t_0_2)|_{k_2=1} \approx 0.22$ (see Tables 2-4), $\sigma_{x,b}(t_0_2) = 1.26\sigma_{x,b,0}$ and the distance $\Delta x_{q_{1,-2}}(t_0_2)$ $\approx 0.28\sigma_{x,b,0}$. This distance is decreased with increasing $k_2$.

![FIG. 3: The dependence of the ratio $A_f/A_0$ on $k_2$.](image)

The instantaneous orbit of particles $x_{q_2}$ inside the interval $t_0_2 < t \leq t_f$ is changed by the law $x_{q_2} = x_{T_2,0} - \sigma_{x,b} + \bar{x}_{q,n}(t - t_0_1 + \sigma_{x,b,0}/v_{T_2})$ and the distance

$$\Delta x_{q_{1,-2}} = \left( \frac{k_2 - 1}{k_2} \sigma_{x,b,0} + v_{T_2} (t - t_0_1) \right) - \xi_2 \sigma_{x,b} \xi_2 \sigma_{x,b,0} =$$

$$= \frac{k_2 - 1}{k_2} \left( \frac{l_{T_2}}{\sigma_{x,b,0}} - 1 \right) + \xi_2 D_2 \sigma_{x,b,0}, \tag{19}$$

where $D_2 = D_2(k_2, \xi_2) = \sigma_{x,b}/\sigma_{x,b,0}$, $l_{T_2} = x_{T_2} - x_{T_2,0} = \pi k_2 \psi(k_2, \xi_2)\sigma_{x,b,0} \leq l_f$ is the displacement of the laser beam. The typical dependence $D_2$ defined by (19) is presented in Fig.4.

When $t > t_f$ then the value $\xi_2 = \xi_2, l = -1, l_{T_2} = l_f$, $D_2 = \sqrt{k_2/(k_2 - 1)}$ and (19) have the maximum

$$\Delta x_{q_{1,-2}|_{t=t_f}} = \left( \frac{k_2 - 1}{k_2} \right) + \sqrt{\frac{k_2}{k_2 - 1}}$$

$$\pi(k_2 - 1) \psi(k_2, \xi_2, l_f) \sigma_{x,b,0}. \tag{20}$$

The instantaneous orbit $x_{q_2}$ will be at a distance $x_{q_{1,-2}} = [(k_2 - 1)/k_2] \sigma_{x,b,0}$ from the motionless instantaneous orbit $x_{q_3}$ of electrons having minimum energy and zero amplitudes of betatron oscillations when the laser beam is stopped at the position $x_{q_3}$.
If we take into account that the instantaneous orbits of electrons having maximum amplitudes of betatron oscillations and the minimum energy are below the instantaneous orbits of electrons having zero amplitudes of betatron oscillations and minimum energy, by the value 0.28σx,b,0, at the moment of the laser beam stopping then the total radial dispersion of the instantaneous orbits of the beam can be presented in the form

$$\sigma_{x,e,f} \leq \frac{k_2 - 1}{k_2} \sigma_{x,0} + \left[ \frac{1}{k_2 - 1} - \pi (k_2 - 1) \psi (k_2, \xi_2, f) \right] + 0.28 \sigma_{x,b,0}, \quad A_{T_2} > I_f, \sigma_{x,0}. \quad (21)$$

According to (21) the efficiency of the enhanced longitudinal laser cooling is the higher the less the ratio of the spread of the initial amplitudes of betatron oscillations to the spread of the instantaneous orbits of the being cooled electron beam.

According to (17) and (21) the enhanced longitudinal laser cooling can lead to a high degree of cooling of electron beams in the longitudinal plane and a much lesser degree of heating in the transverse one. From this it follows that the four- and six-dimensional emittances of the beam will be decreased.

The laser beam width \( a \) must be higher then \( \sigma_{x,e,f} \) in the methods of the enhanced laser cooling.

$$\tau_x = \frac{2\sigma_{x,0}}{v_{x,0}} = \frac{2\sigma_{x,0}\varepsilon}{k_1 D_x P},$$

$$\tau_s = \frac{2\sigma_{x,0}}{P} (1 + \frac{\sigma_{x,b,0}}{\sigma_{x,e,0}}), \quad (22)$$

where \( D_x = (\partial x_{\eta}/\partial p)|_{\gamma > 1} \approx \varepsilon (\partial x_{\eta}/\partial \varepsilon) \) is the local dispersion function of the storage ring at the laser beam azimuth; \( p = Mc\beta\gamma \), the momentum of the electron; \( \dot{x}_{\eta, in} \approx D_x P/\varepsilon, |k_1| \approx 0.1 \).

IV. DISCUSSION

The dynamics of instantaneous orbits and amplitudes of betatron oscillations of electrons depends on the depth of deepening of their instantaneous orbits in the laser beam and on the amplitudes. Moreover, the being displaced laser beam begins to interact with electrons of the beam located at different instantaneous orbits at different moments of time and interacts for different periods of time. These features of selective interaction of the moving laser beams lead to the enhanced cooling of electron beams either in the transverse or longitudinal planes.

In the method of the enhanced transverse laser cooling of electron beams, according to (14) and (15), the degree of transverse compression is \( C_1 = A_0/A = \sqrt{1 + |k_1|}/|k_1| \) and the increase in the spread of the instantaneous orbits of the beam (decompression), \( D_1 \approx C_1^2 \). At the same time in the method of the enhanced longitudinal laser cooling, according to (17) and (21), there is a significant decrease in the spread of instantaneous orbits of electrons defined by the compression coefficient \( C_{2,l} = \sigma_{x,e,0}/\sigma_{x,e,f} \), and a lesser value of increase in the amplitudes of betatron oscillations \( D_2 \approx \sqrt{C_2} \). From this it follows that cooling of electron beams both in the transverse and longitudinal planes, in turn or simultaneously, does not lead to their total cooling in these planes. We can cool electron beams either in the transverse or longitudinal planes or look for combinations of these methods of cooling with other methods.

In the method of enhanced longitudinal laser cooling, contrary to the transverse one, the degree of longitudinal cooling is greater than the degree of heating in the transverse plane. That is why we can use the emittance exchange between longitudinal and transverse planes, e.g., using a synchro-betatron resonance \([17, 23, 24, 25]\) or dispersion coupling by additional motionless wedgeshaped targets \([13, 16, 21]\) together with the moving target \( T_2 \) and in such a way to realize the enhanced two-dimensional cooling of the electron beam based on the longitudinal laser cooling only."
When the synchrotron radiation damping of electron beams in guiding magnetic fields of lattices of storage rings is high and the radio frequency accelerating system is switched on then we can do an additional enhanced laser cooling of such beams in the radio frequency buckets. Such cooling in the longitudinal plane can be produced by using of a being displaced laser beam $T_2$ at the condition when losses of energy by electrons in the laser beam are higher than synchrotron radiation losses. Cooling of the electron beams in the RF buckets is another problem to be considered elsewhere.

Notice that in the case of the three dimensional laser cooling of electron beams considered in [12] the spread of amplitudes of betatron oscillations is small and the energy spread and the spread of instantaneous orbits of electrons is high in another straight section if it has a high dispersion and low $\beta$-function. That is why we can locate the laser beam $T_2$ at this section and produce an additional longitudinal cooling one, two or more times. Then we can use the beam with low transverse and longitudinal emittances for generation of spontaneous or stimulated radiation in the storage ring or extract it for injection to a linear collider.

The enhanced transverse method of laser cooling together with the enhanced longitudinal one (see section 2) can be used for cooling of ion beams. Broad-band laser beam must be used in the first case and monochromatic one in the second case. A heating of the ion beam in the longitudinal plane in the process of the enhanced transverse cooling will be compensated completely by its following cooling in the longitudinal plane. If the laser beam in the transverse method of cooling has a broad spectrum with sharp frequency edge corresponding to an energy lesser than the minimal energy of the being cooled ion beam then the one-dimensional method can be omitted. Ion beam will be cooled in the transverse plane and all ions will be gathered at some energy lesser than minimal after several cycles of transverse cooling. In this case the enhanced transverse method of laser cooling is, at the same time, the enhanced longitudinal method for ion beams.

Laser beams for transverse and longitudinal cooling described above can be used for enhanced cooling of ion beams in both planes in radio frequency buckets.

V. CONCLUSION

In this paper we have presented methods of enhanced laser cooling of particle beams in transverse and longitudinal planes. These methods are based on an universal kind of selective interaction of particles with moving in the radial direction laser beams or media targets when the Robinson damping criterion is not valid. We hope that these methods can be used for cooling of ion and muon beams in storage rings intended for the elementary particle physics. The development and adoption of these methods can lead to new generations of light sources of spontaneous incoherent and stimulated radiation from optical to X-ray and $\gamma$-ray regions based on electron and ion storage rings [11, 12, 22]. Using of circular polarized laser beams for cooling can lead to a longitudinal polarization of stored $e^\pm$ beams in storage rings with more complicated lattices [28 - 29]. Hard circular polarized powerful photon beams produced in the process of the Backward Compton Scattering of laser photons by electrons in storage rings can be used for a production in material targets of longitudinally polarized positron beams for linear colliders [26, 27].

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7 We can use a "monochromatic" laser beam with a fast periodical modulation of frequency as well.
[1] H. Wiedemann, Particle Accelerator Physics I and II (Springer-Verlag, New York, 1993).
[2] E.G. Bessonov, \textit{physics/0202040}.
[3] P.J. Channel, Journal of Applied Physics, v. 52(6), p.3791 (1981).
[4] P.J. Channel, L.D. Selvo, R. Bonifacio, W. Barletta, Optics Communications, v.116, (1995), p.374.
[5] S. Schröder, R. Klein, N. Boos, M. Gerhard, R. Grieser, G. Huber, A. Karafilidis, M. Krieg, N. Schmidt, T. Kuhl, R. Numann, V. Balykin, M. Grieser, D. Habs, E. Jaeschke, D. Kramer, M. Kristensen, M. Musik, W. Petrich, D. Schwalm, D. Sigray, M. Steck, B. Wanner, A. Wolf, Phys. Rev. Lett., v. 64, No 24, p.2901 (1990).
[6] J.S. Hangst, M. Kristensen, J.S. Nielsen, O. Poulsen, J.P. Schiffer, P. Shi, Phys. Rev. Lett., v. 67, 1238 (1991).
[7] J.S. Hangst, K. Berg-Sorensen, P.S. Jessen, M. Kristensen, K. Molmer, J.S. Nielsen, O. Poulsen, J. P. Schiffer, P. Shi, Proc. IEEE Part. Accel. Conf., San Francisco, May 6-9, NY, 1991, v.3, p.1764.
[8] E.G. Bessonov, Proc. of the Internat. Linear Accel. Conf. LINAC94, Tsukuba, KEK, August 21-26, 1994, Vol.2, pp.786-788; Journal of Russian Laser Research, 15, No 5, (1994), p.403.
[9] E.G. Bessonov, Nucl. Instr. Meth. v.A358, (1995), pp. 204-207.
[10] E.G. Bessonov and Kwang-Je Kim, Preprint LBL-37458 UC-414, June 1995; Phys. Rev. Lett., 1996, vol.76, No 3, p.431.
[11] E.G. Bessonov, K.-J. Kim, Proc. of the 1995 Part. Accel. Conf. and Int. Conf. on High-Energy Accelerators, p.2895; Proc. 5th European Particle Accelerator Conference, Sitges, Barcelona, 10-14 June 1996, v.2, p. 1196.
[12] Zh. Huang, R.D. Ruth, Phys. Rev. Lett., v.80, No 5, 1998, p. 976.
[13] E.G. Bessonov, K.-J. Kim, F. Willeke, \textit{physics/9812043}.
[14] E.G. Bessonov, \textit{physics/0001067}.
[15] O’Neil G., Phys. Rev., 102, 1418 (1956);
[16] A. Shoch, Nucl. Instr. Meth, v.11, p.40 (1961).
[17] K.W. Robinson, Phys. Rev., 1958, v.111, No 2, p.373.
[18] A. Hoffman, R. Little, J.M. Peterson, Proc. VI Int. Conf. High Energy Accel. Cambridge (Mass.), 1967, p.123.
[19] H. Okamoto, A.M. Sessler, and D. Möhl, Phys. Rev. Lett. 72, 3977 (1994).
[20] T. Kihara, H. Okamoto, Y. Iwashita, K. Oide, G. Lamanna, J. Wei, Phys. Rev. E, v.59, No 3, p. 3594, (1999).
[21] D.V. Neuffer, Nucl. Instr. Methods, 1994, v.A350, p.24.
[22] E.G. Bessonov, Proc. of the ESRF-ICFA WS ”4th generation light sources”, 1996, ESRF, Grenoble, France, p.WG2-108.
[23] Yu. Bashmakov, E. Bessonov, Ya. Vazdik, Sov. Phys. Tech. Letters, v.1, p.239 (1975); Proc. 5th All-Union meeting on charged particle accel., ”NAUKA”, Moscow, 1977, v.1, p. 277.
[24] Ya. S. Derbenev, A. M. Kondratenko, E. L. Saldin, Nucl. Instr. Meth., v. 165 (1979), p. 201.
[25] J.E. Clendenin, SLAC-PUB-8465, 20 July 2000; 9th WS on Advanced Accelerator Concepts AAC2000, Santa FE, Hilton, June 10-16, 2000, Edited by P.L. Colestock, S. Kelley, p.563.
[26] V. Balakin, A. Mikhailichenko, Preprint BINP 79-85, Novosibirsk, 1979.
[27] E.G. Bessonov, Proc. 15th Int. Accelerator Conference on High Energy Accelerators (HEACC92), 1992, Hamburg, v.1, p. 138; Proc. 6th Int. Workshop on Linear Colliders, March 27-31, 1995, Tsukuba, Japan, v.1. p. 594.