Influence of compressive force in floating plate on its deflections at the unsteady body motion in liquid

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Abstract. The three-dimensional unsteady problem of the movement of a thin body in liquid under a floating plate is solved. The effect of lateral stress of the plate on its deflections and a slope of the curved surface is analyzed. It is received that compression (stretch) of the plate leads to the decreasing (increasing) of critical speeds. The wave height reaches the greatest values for the stretched plate. At supercritical compression of the plate the wave height grows rapidly and breaking occurs.

1. Introduction

For the first time the equation of balance of a floating plate under the influence of forces of compression (stretching), elasticity and external force was considered in [1]. Later many authors investigated influence of forces of compression (stretching) in a plate on parameters of flexural and gravitational waves [2, 3]. In [3] the combined effect of forces of compression and a current of liquid on propagation of surface gravity waves under the influence of the periodic pressure attached to a free surface was studied. The combined effect of compression and current speed on parameters of unsteady flexural-gravity waves in two-dimensional problem was considered [4].

The load movement on previously tense or compressed plate was considered in [5, 6, 7]. So, in work [7] it is received that presence of compressive force in the plate leads to increasing of the maximum depth of the deflection at the movement of pressure area on the floating plate. Despite a large number of works devoted to forces of compression in a plate, we can notice that influence of the compression on the plate deflections formed by the underwater body movement in liquid are studied still insufficiently fully.

Among works devoted to effect of compression on parameters of flexural-gravity waves in the presence of an underwater body it is possible to note works of Sturova I.V. The two-dimensional problem of small oscillations of a horizontal cylinder of arbitrary cross-section submerged in a linearly stratified fluid of finite depth was studied by Sturova [8]. The added-mass and damping coefficients were calculated as a function of the oscillation frequency for the case of an ice sheet and for three special cases: broken ice, free surface and rigid lid. The wave patterns generated by a steadily moving submerged sphere in deep water under ice cover were considered by Sturova [9]. It is shown that the wave resistance of the sphere in case of compressed plate wave is less than for a plate without compression. On the other hand, presence of the stretching in the plate leads to increase in wave resistance. The values of critical speed grow with increase in stretching forces and decrease with growth in compressing forces. In work [10] the velocity potential of a transient three-dimensional source of arbitrary strength and in arbitrary motion is considered. It is shown that forces of
compression (stretching) of the plate influence parameters of flexural and gravitational waves. The wave resistance increases with stretching of the ice cover.

The purpose of this work is to study theoretically influence of forces of internal compression (stretching) in the plate on its deflections at the movement of a thin body in liquid under the plate.

2. Mathematical statement
We consider an infinite elastic, originally stressed, homogeneous isotropic thin plate floating on the surface of fluid of depth $H$ and density $\rho_2$. At depth $d$, a slender blunt body moves rectilinearly with velocity $u(t)$ along the plate.

It is known [11], that the submerged body is simulated by a prescribed source–sink system. Therefore originally we will consider a task about a single source of strength $q=q_0 u(t)$ exposed to fluid flow which has a velocity $-u(t)$ at an infinity. The source is at the depth $d$ under the plate. The coordinate system $Oxyz$ attached to the source is arranged as follows: the plane $xOy$ coincides with the unperturbed plate–water interface, the $x$ direction coincides with the direction of the source, the $z$ axis is directed vertically up, and the coordinate origin coincides with the projection of the source onto the plate–water interface. It is assumed that the water is an ideal incompressible fluid of density $\rho_2$ and the fluid flow is potential.

Similar to [1-3, 5-10] the governing equations are written as follows:

$$\Delta \Phi = 0,$$

$$D \nabla^2 \zeta + \rho h \left( \frac{\partial^2 \zeta}{\partial t^2} - u \frac{\partial \zeta}{\partial x} - 2u \frac{\partial^2 \zeta}{\partial x \partial t} + u^2 \frac{\partial^3 \zeta}{\partial x^2 \partial t} \right) + \frac{N}{2} \nabla^2 \zeta = -\rho_2 g \zeta - \rho_2 \left( \frac{\partial \Phi}{\partial t} - u \frac{\partial \Phi}{\partial x} \right). \quad (z = 0),$$

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \zeta}{\partial t} - u \frac{\partial \zeta}{\partial x}, \quad (z = 0),$$

$$\frac{\partial \Phi}{\partial z} = 0, \quad (z = -H),$$

$$\Phi = \frac{q}{4\pi} \left( -\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_3} + \frac{1}{R_4} \right) + \phi \frac{1}{4\pi},$$

$$R_1 = \sqrt{x^2 + y^2 + (z + d)^2}, \quad R_2 = \sqrt{x^2 + y^2 + (z - d)^2},$$

$$R_3 = \sqrt{x^2 + y^2 + (z + 2H - d)^2}, \quad R_4 = \sqrt{x^2 + y^2 + (z + 2H + d)^2},$$

$$\left. \frac{\partial \Phi}{\partial z} \right|_{z=0} = 0, \quad \left. \left( \frac{\partial \Phi}{\partial t} + \rho_2 h \frac{\partial^2 \Phi}{\partial z^2} \right) \right|_{z=0} = 0.$$

Here $D$ is the flexural rigidity of plate, $D = Eh^3/(12(1-\nu^2))$; $E$, $h$, $\zeta$, $\nu$ and $\rho_1$ are respectively the elastic modulus, thickness, deflection, Poisson’s ratio and density of the plate; $N$ is its lateral stress (with compression at $N > 0$ and stretch at $N < 0$); The velocity potential of the fluid motion, $\Phi$, is represented as the sum of the potentials of the point source $Q_i(0,0,-d)$, the imaginary point sink $Q_i(0,0,d)$, the imaginary point source $Q_i(0,0,-2H+d)$, the imaginary point sink $Q_i(0,0,-2H-d)$, and the velocity potential of the fluid wave motion, $\phi$.

The speed of the source and the distance travelled by the source are calculated by the formulas:

$$u(t) = U \tanh \left( \frac{\mu t}{2} \right), \quad s(t) = U \ln \left( \cosh \left( \frac{\mu t}{2} \right) \right). \quad (1)$$

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The problem is solved using the Fourier and Laplace transforms similar to [11]. We work out the formula for calculating deflections of the originally stressed plate when the source moves in the liquid:

\[ \zeta(x, y, t) = \frac{L_0^2}{4\pi^2} \int_{-\pi}^{\pi} d\theta \int_{0}^{\infty} \exp\left(-k\chi_0 + ik\left(x\cos(\theta) + y\sin(\theta)\right)\right) \times \]

\[ \times \left[ \left(\ddot{u}'_\alpha \cdot \omega \cos(\sqrt{\rho\tau}) + \int_{0}^{\tau} f_2(\tau) \cos(\sqrt{\rho}(t' - \tau))d\tau \right) \exp(\sigma s') + \left(u'' - u'\right) \right] dk, \]

\[ f_2(\tau) = \exp(-\sigma s'(\tau))(\alpha u''(\tau)(1 + \kappa k^4 - nk^2) + \varepsilon \omega \left(u''(\tau) - 3u'(\tau)u'(\tau) - u'(\tau)\sigma + u''(\tau)\sigma^2\right)), \]

\[ \omega = \left[ 1 + \exp(k\chi_0 - 2k\gamma_0) \right] \sinh(k\chi_0 \left(1 - k^{-1}e^{-1}\right)), \quad \alpha_\gamma = \left[ 1 + \exp(k\chi_0 - 2k\gamma_0) \right] \sinh(k\chi_0) ], \]

\[ p = \frac{(1 + \kappa k^4 - nk^2) k \tan(k\gamma_0)}{1 + \kappa k \tan(k\gamma_0)}, \quad \varepsilon = \frac{\rho_2 h}{\rho_2 L_0}, \quad \kappa = \frac{D}{\rho_2 g L_0^4}, \quad n = \frac{N}{\rho_2 g L_0^2}, \]

\[ \gamma_0 = \frac{H}{L_0}, \quad \chi_0 = \frac{d}{L_0}, \quad x' = \frac{x}{L_0}, \quad y' = \frac{y}{L_0}, \quad s' = \frac{s}{L_0}, \quad u' = \frac{u}{\sqrt{g}L_0}, \quad t' = t \sqrt{\frac{g}{L_0}}, \quad L_0 = \sqrt{q_0}. \]

In order to simulate the motion of a slender body in the liquid, we place the point sink at the depth \(d\) and at the distance \(2L_0\), downstream behind the source. Necessary parameters for the “source-sink system” are determined by the given sizes of the slender body (length \(L\) and maximum cross-sectional area \(\Omega\)) by formulas [12]:

\[ q_0 \approx R^2 \left(1 + \frac{r^2}{2}(1 + C) + \ldots\right), \quad L_q = \frac{L}{2} - \Delta, \quad R = \sqrt{\frac{\Omega}{\pi}}, \quad r = \frac{R}{L/2}, \]

\[ \Delta \approx \frac{R}{2} \left(1 + \frac{r^2}{4} \left(\frac{7}{8} + C + \frac{1}{8} B\right) + \ldots\right), \quad C = -\frac{1}{(4\chi^2 + 1)^{3/2}} - \frac{1}{(4\gamma^2 + 1)^{3/2}} + \frac{1}{(4\chi^2 + 1)^{3/2}}, \]

\[ B = \frac{1}{(\chi^2 + 1)^{3/2}} + \frac{1}{(\gamma^2 + 1)^{3/2}} - \frac{1}{(\gamma^2 - \chi^2 + 1)^{3/2}}, \quad \chi = \frac{d}{L/2}, \quad \gamma = \frac{H}{L/2}. \]

The deflection, \(w\), of the floating elastic plate, when the slender body moves in the liquid, is given by the formula:

\[ w(x, y, t) = \zeta(x; y; t) - \zeta(x - 2L_q; y; t), \]

where \(\zeta(x; y; t)\) and \(\zeta(x - 2L_q; y; t)\) are provided by equation (2).

3. Theoretical results and discussions

In theoretical calculations of the deflection of the plate, equations (1)–(4) were used with the following parameters: \(L=100.5\ m, \quad \Omega=124\ m^2, \quad h=1\ \text{m}, \quad E=5.9\times10^3\ Pa, \quad \rho_i=900\ \text{kg/m}^3, \quad v=0.3; \quad H=1000\ m; \quad \rho_2=1000\ \text{kg/m}^3; \quad d=50m; \quad U=0\ \text{m/s} \pm 30\ m/s, \quad t_0=0\ s \pm 60\ s, \quad \mu=5\ \text{s}^{-1}.

Figure 1 presents the results of calculation of the plate deflection at \(t=60\ s, \quad y=0\ m, \quad h=2\ m.\) We note that according equation (1) the body approaches uniform motion with speed \(U\) at \(t=60\ s.\) In figure 1 it is evident that depending on speed \(U\) the compressive force can lead to an increase of deflection, or to
its reduction. We introduce the critical velocity $u_*$, which for $H=1000$ m and $N=0$ equals approximately to the minimum phase velocity of flexural-gravity waves in deep water

$$ u_{\text{min}} = 2 \left( \frac{Dg^3}{27 \rho g} \right)^{1/3}, \quad \text{here } u_{\text{min}} = 20.7 \text{m/s for } h=2 \text{ m. Therefore for } N=0, \ h=2 \text{ m the value } U=12 \text{m/s is subcritical speed, } U=21 \text{m/s is critical speed, and } U=27 \text{m/s is supercritical speed. It is seen that for } U<u_{\text{min}}, \text{ the presence of compression in the plate leads to increase in amplitude of deflection. For } U>u_{\text{min}} \text{ the maximum amplitudes of plate deflection are observed for the plate subject to force of stretching. For } U=u_{\text{min}} \text{ the plate without compression (stretching) gives the maximum deflections. Note that here } N < N_c = 2 \sqrt{\rho g D}, \text{ where } N_c \text{ is critical value of the compressive force, ensures the stability of the floating elastic plate [2].}$$

\begin{align*}
M(t) &= \max_{x \in (-\infty; +\infty)} w(t) - \min_{x \in (-\infty; +\infty)} w(t), \\
\alpha(t) &= \max_{x \in (-\infty; +\infty)} \left| \frac{\partial w}{\partial x} \right|.
\end{align*}

Figure 1. Plate deflections for different values of $U$; solid curves refer to $N=0$, dotted curves to $N = 1.5 \sqrt{\rho g D}$; dashed curves to $N = -1.5 \sqrt{\rho g D}$. Earlier, Bukatov and Zharkov obtained [7], that uniform compression increases the maximum depth of the deflection for subcritical speeds. Thus, our conclusion that presence of compression in the plate leads to increasing of the plate amplitude for subcritical speeds is confirmed by results of work ([7], Figure 5).
It is known [11], if the speed mode has form (1) and \( U \neq u_{\text{min}} \), then the wave height and coefficient \( \alpha \) tend with time to some constant values dependent on the speed. Figure 2 shows curves of the coefficient \( \alpha \) versus velocity \( U \) at \( t=60 \) s. We assume that behavior of coefficient \( \alpha \) is analogous to wave resistance of the underwater body. It is obvious that increasing the coefficient \( \alpha \) leads to an increase in the wave resistance of body. For steady moving load on the plate both the wave resistance, and coefficient \( \alpha \) tend to infinity for speed \( U = u_{\text{min}} \). We note that \( u_{\text{min}}=15.9 \) m/s for \( h=1 \) m. From Figure 2, it is clear that in the neighborhood of \( u_{\text{min}} \) the coefficient \( \alpha \) begins to increase rapidly for \( N=0 \). The coefficient \( \alpha \) reaches the greatest values for the stretched plate \( N<0 \). At the same time, the area of the maximum values of coefficient \( \alpha \) moves to the area of large speeds \( U > u_{\text{min}} \). On the other hand, for the compressed plate \( (N>0) \) the coefficient \( \alpha \) begins to grow quickly for speeds \( U < u_{\text{min}} \), but does not reach the maximum values, which are characteristic of the stretched plate or for unstressed plate. Results of Figures 2 are in agreement with the conclusion [9], that the wave resistance of the sphere increases with growth of stretching forces and decreases with growth of compression forces at the steady movement of the sphere with supercritical speeds.

![Figure 2](image_url)

**Figure 2.** Coefficient \( \alpha \) versus \( U \): solid curves refer to \( N=0 \), dotted curves to \( N = 1.5 \sqrt{\rho g D} \); dashed curves to \( N = -1.5 \sqrt{\rho g D} \).

From an analysis of Figure 2, it follows that in the case \( h=2 \) m and \( N = 1.5 \sqrt{\rho g D} \) (compressed plate) the value \( U = 12 \) m/s is the critical speed, corresponding to maximum of coefficient \( \alpha \). In the case \( N = -1.5 \sqrt{\rho g D} \) (stretched plate) the value \( U = 27 \) m/s is the critical speed. And for \( N=0 \) the value \( U = u_{\text{min}} = 21 \) m/s is critical speed. These critical speeds are compared with the values of [6, 9]. Good agreement is found. Our values it is a little more than numerical values [9] and a little less than the values received on an approximate formula \( u' = u_{\text{min}} \left( 1 - \frac{3}{4} \arcsinh \frac{N}{\sqrt{12 \rho g D}} \right) \) [6].

Figure 2 demonstrates that, for \( h=1 \) m the value \( U = 21 \) m/s is the critical speed for stretched plate, and value \( U = 17 \) m/s is the critical speed for unstressed plate. These results are in agreement with the results of [9] and [6]. It is very difficult to determine the critical speed for the compressed plate from Figure 2. But according to works [6, 9], the critical speed for \( h=1 \) and \( N = 1.5 \sqrt{\rho g D} \) approximately equals to 8–10.5 m/s. These values correspond to speed range in Figure 2 in which the coefficient \( \alpha \) grows rapidly.

Figures 3 and 4 show curves of the maximum wave height \( M \) versus velocity \( U \) at \( t=60 \) s, \( h=2 \) m. It is seen that the rapid growth of the wave height takes place at values of critical speeds of Figure 2. Compression of the plate \( (N = 1.5 \sqrt{\rho g D}) \) leads to growth of the wave height for subcritical speeds.
in comparison with the unstressed plate. Stretching of the plate \( (N = -1.5\sqrt{\rho g D}) \) leads to the fact that for subcritical and critical speeds the wave height is smaller than for the unstressed plate. For supercritical speeds the wave height of the stretched plate begins to increase rapidly, and exceeds the wave height of unstressed plate. If the compressive force exceeds its critical value (Figure 4), the wave height reaches unreal great values. This case corresponds to conclusion of [1] about an unstable condition of the plate. It was pointed out in [1] that, for \( N > N_c \) the wave amplitude grows and breaking occurs. From an analysis of Figures 3 and 4, it follows that for \( N = 1.5\sqrt{\rho g D} \) the critical speed equals to 12 m/s, and for \( N = 2.5\sqrt{\rho g D} \) it equals to 5 m/s.

\[ gDN = 25.1 \rho - \gamma \]

**Figure 3.** Maximum wave height \( M \) versus speed \( U \) for different values of compressive force: solid curve refers to \( N=0 \); dotted curve to \( N = 1.5\sqrt{\rho g D} \); dashed curve to \( N = -1.5\sqrt{\rho g D} \).

**Figure 4.** Maximum wave height \( M \) versus speed \( U \) for \( N = 2.5\sqrt{\rho g D} \).

Figures 5 and 6 show three-dimensional surface of plate for \( U = 12 \text{m/s} \) and \( U = 5 \text{m/s} \), respectively, at \( h = 2 \text{m}, t = 60 \text{s} \). It is interesting to note here that both figures correspond to the compressed plate and critical speed. However, in Figure 5 crests of flexural-gravity waves move in the positive direction of an axis \( Ox \) and are located perpendicularly to the motion direction. For the case of \( N > N_c \) the crests of waves move in positive and in negative directions of an axis \( Ox \) (Figure 6). Perhaps, this fact corresponds to the case of existence of the negative and positive values of group velocity characteristic of the plate with supercritical compression noticed in work [13]. The group velocity becomes discontinuous, and wave breaking will occur. This phenomenon of wave breaking is referred to as buckling of the floating structure [13].

**Figure 5.** Plate deflections for \( h = 2 \text{m}, U = 12 \text{m/s}, t = 60 \text{s}, N = 1.5\sqrt{\rho g D} \).

**Figure 6.** Plate deflections for \( h = 2 \text{m}, U = 5 \text{m/s}, t = 60 \text{s}, N = 2.5\sqrt{\rho g D} \).
4. Conclusions
In this study, the characteristics of flexural gravity waves (maximum wave height and the maximum absolute value of the slope of the tangent to the curved surface of the plate) were analyzed in the presence of compression. It was observed that compression of the plate leads to growth of the wave height and plate slope for subcritical speeds in comparison with the unstressed plate. Stretching of the plate leads to the fact that for subcritical and critical speeds the wave height is smaller than for the unstressed plate. For supercritical speeds the wave height of the stretched plate begins to increase rapidly, and exceeds the wave height of unstressed plate.

Supercritical compression leads to plate destruction. At the same time there are waves having negative group velocity.

Acknowledgments
The reported study was funded by RSF (Russian Science Foundation) according to the research project No. 16-19-10097.

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