Finding a Shortest Even Hole in Polynomial Time

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Abstract

An even (respectively, odd) hole in a graph is an induced cycle with even (respectively, odd) length that is at least four. Bienstock [DM 1991 and 1992] proved that detecting an even (respectively, odd) hole containing a given vertex is NP-complete. Conforti, Cornuéjols, Kappor, and Vuškovič [FOCS 1997] gave the first known polynomial-time algorithm to determine whether a graph contains even holes. Chudnovsky, Kawarabayashi, and Seymour [JGT 2005] estimated that Conforti et al.’s algorithm runs in \(O(n^{40})\) time on an \(n\)-vertex graph and reduced the required time to \(O(n^{31})\). Subsequently, da Silva and Vuškovič [JCTB 2013], Chang and Lu [JCTB 2017], and Lai, Lu, and Thorup [STOC 2020] improved the time to \(O(n^{19})\) and \(O(n^{11})\), respectively. The tractability of determining whether a graph contains odd holes has been open for decades until the algorithm of Chudnovsky, Scott, Seymour, and Spirkl [JACM 2020] that runs in \(O(n^{9})\) time, which Lai et al. also reduced to \(O(n^{8})\). By extending Chudnovsky et al.’s techniques for detecting odd holes, Chudnovsky, Scott, and Seymour [Combinatorica 2020 to appear] (respectively, [arXiv 2020]) ensured the tractability of finding a long (respectively, shortest) odd hole. They also ensured the NP-hardness of finding a longest odd hole, whose reduction also works for finding a longest even hole. Recently, Cook and Seymour ensured the tractability of finding a long even hole. An intriguing missing piece is the tractability of finding a shortest even hole, left open for at least 15 years by, e.g., Chudnovsky et al. [JGT 2005] and Johnson [TALG 2005]. We resolve this long-standing open problem by giving the first known polynomial-time algorithm, running in \(O(n^{31})\) time, for finding a shortest even hole in an \(n\)-vertex graph that contains even holes.

1 Introduction

An even (respectively, odd) hole in a graph is an induced cycle with even (respectively, odd) length that is at least four. Detecting induced subgraphs are fundamentally important problems [8, 10, 13, 14, 19, 27, 28, 32, 33, 35, 36, 38]. A most prominent example regarding detecting induced cycles is the seminal strong perfect graph theorem of Chudnovsky, Robertson, Seymour, and Thomas [15], conjectured by Berge in 1960 [3, 4, 5], ensuring that the perfection of a graph can be determined by detecting odd holes in the graph and its complement. Chudnovsky, Cornuéjols, Liu, Seymour, and Vuškovič [9] gave the first known polynomial-time algorithm, running in \(O(n^{9})\) time, for recognizing \(n\)-vertex perfect graphs. Bienstock [6, 7] proved that detecting an odd hole containing a prespecified vertex is NP-hard in the early 1990s. The tractability of detecting an odd hole remained unknown until the recent \(O(n^{9})\)-time algorithm of Chudnovsky, Scott, Seymour, and Spirkl [18]. Lai, Lu, and Thorup [37] implemented Chudnovsky et al.’s algorithm to run in \(O(n^{8})\) time, also leading to an \(O(n^{8})\)-time algorithm for recognizing perfect graphs. Chudnovsky, Scott, and Seymour [17] showed that it takes \(O(n^{20(k+3)})\) time to detect an odd hole with length at least \(\ell\). Chudnovsky, Scott,

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and Seymour [16] ensured the NP-hardness of finding a longest odd hole and gave an $O(n^{14})$-time algorithm to obtain a shortest odd hole, if one exists.

Even holes in graphs have also been extensively studied in the literature [1, 21, 22, 25, 26, 29, 30, 34, 40, 42].Vušković [41] gave a comprehensive survey on even-hole-free graphs. According to Bienstock [6, 7], detecting an even hole containing a prespecified vertex is also NP-hard. Conforti, Cornuojols, Kapoor, and Vušković [20, 23] gave the first polynomial-time algorithm for detecting even holes in an $n$-vertex graph, running in $O(n^{40})$ time. Chudnovsky, Kawarabayashi, and Seymour [11] reduced the time to $O(n^{31})$. Chudnovsky et al. [11] also observed that the time of detecting even holes can be further reduced to $O(n^{15})$ as long as detecting prisms is not too expensive, but this turned out to be NP-hard [39].Chudnovsky and Kapadia [10] and Maffray and Trotignon [39, Algorithm 2] devised $O(n^{35})$-time and $O(n^3)$-time algorithms for detecting prisms in theta-free and pyramid-free graphs, respectively. Later, da Silva and Vušković [26] improved the time of detecting even holes in $G$ to $O(n^{19})$. Chang and Lu [8] further reduced the running time to $O(n^{11})$. The best currently known algorithm for detecting even holes, due to Lai, Lu, and Thorup [37], runs in $O(n^9)$ time. Very recently, Cook and Seymour [24] announced an $O(n^{5.4f+58})$-time algorithm for detecting even holes with length at least $f$. Following the approach of Chudnovsky et al. [16] for the NP-hardness of longest odd hole, one can verify that the longest even hole problem is NP-hard by reducing from the problem of determining whether the $n$-vertex graph $G$ admits a Hamiltonian $uw$-path for two given vertices $u$ and $w$: For the graph $H$ obtained from $G$ by subdividing each edge once and then adding a path $uvw$, a longest even hole of $H$ has $2n$ vertices if and only if $G$ admits a Hamiltonian $uw$-path. As displayed in Figure 1, the complexity of finding a shortest even hole, open for at least 15 years (see, e.g., [11, Page 86] and [31, page 166]), became the only missing piece. We resolve the long-standing open problem by presenting the first known polynomial-time algorithm, as summarized in the following theorem.

**Theorem 1.** For any $n$-vertex graph $G$, it takes $O(n^{31})$ time to either obtain a shortest even hole of $G$ or ensure that $G$ contains no even hole.

Our shortest-even-hole algorithm is based upon Chudnovsky et al.’s techniques [11] that lead to their $O(n^{31})$-time algorithm for detecting even holes. To our surprise, their techniques for detecting even holes suffice to resolve the problem that they left open in the first paragraph of their paper, writing “the complexity of finding the shortest even hole in a graph is still open as far as we know”. It is perhaps their more general settings of allowing for weighted graphs that caused them to overlook the possibility of further pushing for a shortest even hole, e.g., in their Lemma 5.3.

## 2 Proving Theorem 1

We first reduce Theorem 1 to Lemmas 4 and 5 via Lemmas 2 and 3 and then prove Lemmas 4 and 5 in §2.1 and §2.2.
Lemma 2 (Lai, Lu, and Thorup [37, Theorem 1.6]). For any n-vertex graph G, it takes $O(n^9)$ time to determine whether G contains even holes.

Let $[i, k]$ for integers i and k consist of the integers j with $i \leq j \leq k$. Let $|S|$ denote the cardinality of set S. Let $\|G\|$ denote the number of edges of graph G. Let $V(G)$ consist of the vertices of graph $G$. For any subgraph H of G, let $G[H]$ denote the subgraph of G induced by $V(H)$. Let $d_c(u, v)$ for vertices u and v of graph G denote the distance of u and v in G. A uv-path is a path with end-vertices u and v.

Let G be a graph containing even holes. Let C be a shortest even hole of G. Let P be a uv-path of G for distinct and nonadjacent vertices u and v of C. P is C-good if the union of P and a uv-path of C remains a shortest even hole of G. P is C-bad if P is not C-good. P is C-shallow if

$$\|P\| \geq d_c(u, v) - 1$$

and $G[P \cup C_2]$ for the uv-paths $C_1$ and $C_2$ of C with $\|C_1\| \leq \|C_2\|$ is a hole. P is a C-shallow if

$$2 \leq \|P\| \leq d_c(u, v) \quad \text{and} \quad \|P\| < \frac{\|C\|}{4}.$$  (2)

Observe that if P is a C-good C-shallow, then $\|P\| = d_c(u, v)$. C is good in G if each C-shortcut in G is C-good. C is bad in G if C is not good in G. P is a worst C-shortcut in G if P is a C-bad C-shortcut such that either (i) $\|P\| = \|P'\|$ and $d_c(u, v) \geq d_c(u', v')$ or (ii) $\|P\| < \|P'\|$ holds for each C-bad C-shortcut $u'v'$-path $P'$ in G. Observe that C is bad in G if and only if there is a worst C-shortcut in G. A graph G is shallow if there is a shortest even hole C of G such that each worst C-shortcut is C-shallow. A graph G is anti-shallow if each worst C-shortcut for each bad shortest even hole C of G is not C-shallow. Observe that if G is shallow and anti-shallow, then G contains a good shortest even hole.

Lemma 3 (Chudnovsky et al. [11, Lemma 4.5]). For any n-vertex graph G that contains even holes, it takes $O(n^{25})$ time to obtain induced subgraphs $G_1, \ldots, G_r$ of G with $r = O(n^{23})$ such that a G, with $i \in [1, r]$ is shallow and contains a shortest even hole of G.

Lemma 4. For any n-vertex graph G, it takes $O(n^6)$ time to obtain a subgraph C of G such that (i) C is a shortest even hole of G or (ii) G is anti-shallow.

A graph G is long if G does not contain any even hole with at most 22 vertices. A graph G is bad if G does not contain any good shortest even hole.

Lemma 5. For any n-vertex long graph G, it takes $O(n^6)$ time to obtain a subgraph C of G such that (i) C is a shortest even hole of G or (ii) G is bad.

Proof of Theorem 1. By Lemma 2, we may assume that G contains even holes. Spend $O(n^{22})$ time to either obtain a shortest even hole of G or ensure that G is long. If G is long, then apply Lemma 3 in $O(n^{25})$ time to obtain induced subgraphs $G_1, \ldots, G_r$ of G with $r = O(n^{23})$ such that for an (unknown) index $i \in [1, r]$ is shallow and contains a shortest even hole C of G. Since G is long, so is each $G_i$. For each j $\in [1, r]$, apply Lemmas 4 and 5 on $G_j$ in overall $O(n^6 + O(n^6)) \cdot O(n^{23}) = O(n^{31})$ time to obtain subgraphs $C_j$ and $D_j$ of $G[V_j]$ such that $C_j$ or $D_j$ is a shortest even hole of $G_j$ unless $G_j$ is anti-shallow and bad. Finally, we report a $C_j$ or $D_j$ that is an even hole of $G_j$ whose number of edges is minimized over all $j \in [1, r]$. Since $G_j$ is shallow, $G_j$ cannot be anti-shallow and bad. Thus, at least one of $C_i$ and $D_j$ is a shortest even hole of $G_i$, which has to be a shortest even hole of G by $C \subseteq G_i$.

It remains to prove Lemmas 4 and 5.
2.1 Proving Lemma 4

For any vertex subset $U$ of a graph $G$, let $G - U = G[V(G) \setminus U]$.

**Lemma 6** (Chudnovsky et al. [11, Lemma 5.1]). Let $C$ be a shortest even hole of $G$. Let $u$ and $v$ be distinct vertices of $C$. Let $uv$-path $P$ of $G$ be a $C$-shallow worst $C$-shortcut. Let $C_1$ and $C_2$ be the $uv$-paths of $C$ with $\|C_1\| < \|C_2\|$. If $x$ (respectively, $y$) is the neighbor of $u$ (respectively, $v$) in $C_1$ and $C_3$ is the $x y$-path of $C_1$, then the following statements hold:

1. If $P_{uv}$ is a $uv$-path of $G$, then $\|P\| \leq \|P_{uv}\|$.
2. If $P_{uv}$ is a $uv$-path of $G$ with $\|P\| = \|P_{uv}\|$, then $G[P_{uv} \cup C_2]$ is a hole of $G$.
3. If $P_{xy}$ is an $x y$-path of $G$, then $\|C_3\| \leq \|P_{xy}\|$.
4. If $P_{xy}$ is an $x y$-path of $G$ with $\|C_3\| = \|P_{xy}\|$, then $G[P_{xy} \cup C_2]$ is a hole of $G$.

**Proof of Lemma 4.** Let $G$ be connected without loss of generality. For any $U \subseteq V(G)$, let $N_G[U]$ denote the vertex subset of $G$ consisting of the vertices in $U$ and their neighbors in $G$. For any vertices $u$ and $v$, let $P_{uv}$ be an arbitrary shortest $uv$-path of $G$. For any vertex-disjoint edges $ux$ and $vy$ of $G$ with $\|P_{uv}\| = \|P_{xy}\| - 1$ such that $u$ and $v$ are connected in $H(u, v, x, y) = G[(V(G) \setminus N_G[V(P_{uv} \cup P_{xy}) \setminus \{u, v\}]) \cup \{u, v\}]$, let $Q(u, v, x, y)$ be a shortest $uv$-path of $H(u, v, x, y)$. If there are edges $ux$ and $vy$ of $G$ such that $Q(u, v, x, y)$ exists and $G[P_{xy} \cup Q(u, v, x, y)]$ is an even hole, then report a shortest such even hole; Otherwise, just report the empty graph. The procedure takes overall $O(n^6)$ time.

The rest of the proof shows that the subgraph reported by the above procedure is a shortest even hole of $G$ as long as $G$ is not anti-shallow, i.e., a $uv$-path $P$ is a $C$-shallow worst $C$-shortcut for a bad shortest even hole $C$ of $G$. Let $C_1$ and $C_2$ be the $uv$-paths of $C$ with $\|C_1\| \leq \|C_2\|$. (a) We first show that $G[P_{uv} \cup C_2]$ is a hole of $G$. By Lemma 6(2), $G[P \cup C_2]$ is a hole. By Equations (1) and (2), $\|P\| \leq \|C_1\| \leq \|P\| + 1$. We have

$$\|P\| = \|C_1\| - 1 \quad (3)$$

or else $\|P\| = \|C_1\|$ would imply that $G[P \cup C_2]$ is a shortest even hole of $G$, contradicting that $P$ is a $C$-bad $C$-shortcut. By Lemma 6(1) and the definition of $P_{uv}$,

$$\|P_{uv}\| = \|P\|, \quad (4)$$

implying that $G[P_{uv} \cup C_2]$ is a hole of $G$ by Lemma 6(2). (b) Let $x$ (respectively, $y$) be the neighbor of $u$ (respectively, $v$) in $C_1$. We next show that $G[P_{xy} \cup C_2]$ is a shortest even hole of $G$. By Lemma 6(3),

$$\|P_{xy}\| = \|C_2\| = \|C_1\| - 2 \quad (5)$$

By Lemma 6(4), $G[P_{xy} \cup C_2]$ is a shortest even hole of $G$. Since $G[P_{uv} \cup C_2]$ and $G[P_{xy} \cup C_2]$ are holes of $G$, the interior of $C_2$ is disjoint from $N_G[V(P_{uv} \cup P_{xy}) \setminus \{u, v\}]$, implying $C_2 \subseteq H(u, v, x, y)$ and that $u$ and $v$ are connected in $H(u, v, x, y)$. Therefore, $Q(u, v, x, y)$ exists with

$$\|Q(u, v, x, y)\| \leq \|C_2\|. \quad (6)$$

By Equations (3), (4), and (5), we have $\|P_{uv}\| = \|P_{xy}\| - 1$, implying that exactly one of $G[P_{uv} \cup Q(u, v, x, y)]$ and $G[P_{xy} \cup Q(u, v, x, y)]$ is an even hole of $G$. By Equations (3), (4), (5), and (6),

$$\|G[P_{uv} \cup Q(u, v, x, y)]\| \leq \|C\| - 1$$

$$\|G[P_{xy} \cup Q(u, v, x, y)]\| \leq \|C\|.$$ 

Therefore, $G[P_{xy} \cup Q(u, v, x, y)]$ is a shortest even hole of $G$. \qed

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2.2 Proving Lemma 5

Proof of Lemma 5. Let the long graph $G$ be connected without loss of generality. For each $i \in [0, 7]$, let $i^+ = (i + 1) \mod 8$ For each of the $O(n^8)$ choices of distinct vertices $v_0, \ldots, v_7$, let

$$C(v_0, \ldots, v_7) = P_0 \cup \cdots \cup P_7,$$

where each $P_i$ with $i \in [0, 7]$ is an arbitrary shortest $v_i v_{i^+}$-path of $G$. If one of these $O(n^8)$ subgraphs $C(v_0, \ldots, v_7)$ is an even hole of $G$ with $\|P_i\| \geq 3$ for each $i \in [0, 7]$, then report a shortest such one; Otherwise, just report the empty graph. A naive implementation of the algorithm takes $O(n^{10})$ time. The algorithm can be implemented to run in $O(n^4)$ time: Spend overall $O(n^4)$ time to obtain $d_C(u, v)$ and a shortest $uv$-path $P(u, v)$ of $G$ for any vertices $u$ and $v$. Spend overall $O(n^6)$ time to obtain a data structure from which it takes $O(1)$ time to determine (1) whether $G[P(u, v) \cup P(v, w)]$ is a path for any three vertices $u$, $v$, and $w$ and (2) whether $G[P(u, v) \cup P(x, y)]$ is disconnected for any four vertices $u$, $v$, $x$, and $y$. It then takes $O(1)$ time for any given vertices $v_0, \ldots, v_7$ to obtain $\|C(v_0, \ldots, v_7)\|$ and whether $C(v_0, \ldots, v_7)$ is an even hole with $\|P_i\| \geq 3$ for each $i \in [0, 7]$.

For the correctness, the rest of the proof shows that if $G$ is not bad, i.e., there is a good shortest even hole $C$ of $G$, then one of the $O(n^8)$ iterations yields a shortest even hole of $G$. Since $G$ is long, $\|C\| \geq 24$, implying integers $a \geq 3$ and $b \in [0, 7]$ with

$$\|C\| = 8a + b.$$

Let $v_0, \ldots, v_7$ be vertices of $C$ such that the shortest $v_i v_{i^+}$-paths $C_i$ of $C$ with $i \in [0, 7]$ are edge-disjoint and satisfy

$$\|C_i\| \in \{a, a + 1\} \quad \text{and} \quad \|C_i\| + \|C_{i^+}\| \leq 2a + \left\lceil \frac{b}{4} \right\rceil.$$

1. We first ensure for each $i \in [0, 7]$ that

$$\|P_i\| = \|C_i\|.$$

By $d_C(v_i, v_{i^+}) \geq a \geq 3$ and the fact that $C$ is a hole, we have $\|P_i\| \geq 2$. By $\|P_i\| = d_C(v_i, v_{i^+}) \leq d_C(v_i, v_{i^+}) = \|C_i\| \leq a + 1 < 2a \leq \frac{\|C\|}{4}$, $P_i$ is a $C$-shortcut. Since $C$ is a good shortest even hole of $G$, the $C$-shortcut $P_i$ is $C$-good, implying $\|P_i\| = d_C(v_i, v_{i^+}) = \|C_i\|$.

2. Assume for contradiction that a $G[P_i \cup P_{i^+}]$ with $i \in [0, 7]$ is not a path. Thus, $\|P\| < \|C_i\| + \|C_{i^+}\|$ holds for a shortest $v_i v_{i^+}$-path $P$ of $G[P_i \cup P_{i^+}]$. Since $C$ is good and the union of $P$ and the longer $v_i v_{i^+}$-path of $C$ is a hole of $G$, $P$ cannot be a $C$-shortcut. Hence,

$$2a + \frac{b}{4} = \frac{\|C\|}{4} < \|P\| < \|C_i\| + \|C_{i^+}\| \leq 2a + \frac{b}{4},$$

contradicting that $\|P\|$ is an integer. Therefore, each $G[P_i \cup P_{i^+}]$ with $i \in [0, 7]$ is a path.

3. To see that $G[P_0 \cup \cdots \cup P_7]$ is a hole, assume for contradiction integers $i \in [0, 7]$ and $d \in [1, 3]$ such that $G[P_i \cup P_j]$ with $j = (i^+ + d) \mod 8$ is connected. There are $v_i v_j$-path $Q$ and $v_i v_{j^+}$-path $R$ of $G[P_i \cup P_j]$ with $|V(Q) \cap V(R)| \leq 2$. Thus,

$$\|Q\| + \|R\| \leq \|P_i\| + \|P_j\| + 2 = \|C_i\| + \|C_j\| + 2 \leq 2a + 4.$$  (7)

We have

$$\|Q\| \geq a + 3$$  (8)

or else $\|Q\| \leq a + 2 < 2a \leq \min \{d_C(v_i, v_j), \frac{\|C\|}{4}\}$ and the fact that $C$ is good would imply that $Q$ is a $C$-good $C$-shortcut, contradicting $\|Q\| < d_C(v_i, v_j)$. Similarly, we have

$$\|R\| \geq a + 3$$  (9)
or else $\|R\| \leq a + 2 < 2a \leq \min \left\{ d_C(v_i, v_j), \frac{\|C\|}{2} \right\}$ and the fact that $C$ is good would imply that $R$ is a $C$-good $C$-shortcut, contradicting $\|R\| < d_C(v_i, v_j)$. Combining Equations (7), (8), and (9), we have $2a + 6 \leq \|Q\| + \|R\| \leq 2a + 4$, contradiction.

### 3 Concluding remarks

We resolve the long-standing open problem on the tractability of reporting a shortest even hole in an $n$-vertex graph by presenting an $O(n^{31})$-time algorithm. The complexity is much higher than that of the current $O(n^{10})$ time for reporting an arbitrary even hole, implied by the $O(n^8)$-time algorithm of Lai et al. \cite{37, Theorem 1.6} for detecting even holes. The current time for reporting an arbitrary odd hole is $O(n^6)$, implied by the $O(n^8)$-time algorithm of Lai et al. \cite{37, Theorem 1.4} for detecting odd holes. The shortest-odd-hole algorithm of Chudnovsky et al. \cite{16} runs in $O(n^{14})$ time. The $O(n^5)$ gap for odd holes is much smaller than the $O(n^{21})$ gap for even holes. It would be of interest to reduce either one of the gaps.

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