Vortex Solutions of Nonrelativistic Fermion and Scalar Field Theories Coupled to Maxwell-Chern-Simons Theories

Bom Soo Kim,∗ Hyuk-jae Lee,† and Jae Hyung Yee,‡
Department of Physics and Natural Science Research Institute
Yonsei University, Seoul, 151-742, Korea

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Abstract

We have constructed nonrelativistic fermion and scalar field theories coupled to a Maxwell-Chern-Simons gauge field which admit static multi-vortex solutions. This is achieved by introducing a magnetic coupling term in addition to the usual minimal coupling.
1 Introduction

Since the introduction of the Chern-Simons action as a new possible gauge field theory in (2 + 1)-dimensional space-time \([1]\), it has been successfully applied to explain various (2 + 1)-dimensional phenomena including the high \(T_c\) superconductivity and the integral and fractional quantum Hall effects. The Chern-Simons term has also made it possible to construct various field theoretic models which possess classical vortex solutions with various physically interesting properties. They include the relativistic \([2]\), nonrelativistic \([3]\) scalar field theories, and relativistic fermion field theory \([4]\) interacting with Abelian Chern-Simons fields, which admit static multi-vortex solutions saturating the Bogomol’nyi bound \([5]\) that reduces the second-order field equations to the first-order ones. In (2 + 1)-dimensions, there are two possibilities for the kinetic term of gauge field, Maxwell and Chern-Simons terms. However, the simple theories with both kinetic terms failed to have vortex solutions. In order for such theories to have vortex solutions one need to add new fields or extend the models to supersymmetric theories \([6]\).

Recently, Kogan and Stern\([7]\) introduced a new coupling of the electromagnetic field to the matter field interpreted as a generalization of the familiar Pauli magnetic moment coupling in (2 + 1)-dimensions. The tensor structure in (2 + 1)-dimensions allows magnetic moment couplings independent of spin representation. This coupling gives rise to consistent vortex solutions for the relativistic matter field theories interacting with Abelian and non-Abelian gauge field with both Maxwell(Yang-Mills) and Chern-Simons kinetic terms\([8, 9]\).

Such vortex solutions of (2 + 1)-dimensional field theories have attracted much attention recently in the hope of understanding the dynamics of magnetic vortices appearing in high temperature superconductors \([10]\) in the field theoretic approach \([11]\). The phenomena of magnetic vortices in high temperature superconductors are determined by electrons confined to move in 2-dimensional surfaces. To understand such phenomena in the field theoretic approach, therefore, one is better to study the nonrelativistic fermion field theory models which possess static magnetic vortex solutions. As a first step to understand the dynamics of such realistic magnetic vortices in the field theoretic approach, we construct nonrelativistic fermion field theory models that possess static vortex solutions in this paper.

In section II we construct nonrelativistic spinor field models where a single two-component spinor field couples to a Maxwell-Chern-Simons gauge field, and show that they possess static magnetic vortex solutions. In section III, we consider a non-relativistic scalar field theory coupled to a Maxwell-Chern-Simons field that supports the static vortex solutions. We conclude with some discussions in the last section.
2 Nonrelativistic Fermion Models

There exist many fermionic field theory models that possess static vortex solutions. Recently Duval, Horváthy and Palla [13] and Németh [14] have constructed nonrelativistic spinor field models that support static vortex solutions, where a couple of two-component spinor fields couples to a Chern-Simons gauge field. In this section we are interested in constructing nonrelativistic spinor field models where a single two-component spinor field couples to a Maxwell-Chern-Simons gauge field. For such models to support static vortex solutions one need to introduce a magnetic coupling as explained in the last section.

For this purpose we follow the procedure of Levy-Leblond [15], which enables one to write the Schrödinger equation as a first order differential equation. The nonrelativistic fermion fields must satisfy the Schrödinger equation,

\[ S\Psi = \left( i \frac{\partial}{\partial t} + \frac{1}{2m} \vec{\nabla}^2 \right) \psi = \left( E - \frac{\vec{p}^2}{2m} \right) \psi = 0. \]  

(1)

To write this second-order differential equation as a first-order one we introduce an operator \( \Theta \) :

\[ \Theta \psi \equiv \left( A E + \vec{B} \cdot \vec{p} + C \right) \psi = 0, \]  

(2)

where \( A, \vec{B}, C \) are matrices to be determined.

For the solution of Eq.(2) to obey the Schrödinger equation (1), there must exist some operator \( \Theta = A' E + \vec{B}' \cdot \vec{p}' + C' \), such that operating \( \Theta \) on Eq.(2) we recover the Schrödinger equation:

\[ \Theta' \Theta = 2mS. \]  

(3)

By identifying the various monomials in \( E \) and \( \vec{p} \), we obtain the following set of conditions:

\[ \begin{align*}
A' A & = 0 \\
A' C + C' A & = 2m \\
C' C & = 0
\end{align*} \]  

(4)

\[ \begin{align*}
A' B_i + B'_i A & = 0 \\
B'_i B_j + B'_j B_i & = -2\delta_{ij} \\
C' B_i + B'_i C & = 0,
\end{align*} \]

where \( i, j = 1, 2 \). Defining the new matrices as

\[ \begin{align*}
B_3 = i \left( A + \frac{1}{2m} C \right), & \quad B'_3 = i \left( A' + \frac{1}{2m} C' \right), \\
B_4 = A - \frac{1}{2m} C, & \quad B'_4 = A' - \frac{1}{2m} C',
\end{align*} \]  

(5)
the conditions (4) can be written as

$$B'_{\alpha}B_{\beta} + B'_{\beta}B_{\alpha} = -2\delta_{\alpha\beta}, \quad (\alpha, \beta = 1 \text{ to } 4).$$

(6)

All the representation of such an algebra can be obtained from those of a Clifford algebra with dimension 2:

$$\sigma_\mu \sigma_\nu + \sigma_\nu \sigma_\mu = 2\delta_{\mu\nu}, \quad (\mu, \nu = 1, 2, 3).$$

(7)

One can easily show that

$$B_\mu = B\sigma_\mu, \quad B'_\mu = -\sigma_\mu B^{-1}, \quad B_4 = -i\beta, \quad B'_4 = -i\beta^{-1},$$

(8)

satisfy the condition (6), where $B$ is an arbitrary nonvanishing constant and $\beta$ is an arbitrary nonsingular matrix.

Since we are only interested in the irreducible representations, we may use the standard results for the Pauli matrix algebra, Eq.(7). Thus we take $B = 1$, $B_\mu = \sigma_\mu (\mu = 1, 2, 3)$, and the nonsingular matrix $\beta$ to be

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{or} \quad \beta = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \quad \text{(9)}$$

We now consider the first possibility of Eq.(9), $\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, so that $B_4 = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$.

From this choice we obtain the matrices $A$ and $C$:

$$A = \begin{pmatrix} -i & 0 \\ 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \\ 0 & 2im \end{pmatrix}. \quad \text{(10)}$$

By using these results the first-order Schrödinger equation (2) can be written as

$$\left(\frac{1}{2}(1 + \sigma_3)E + i\vec{\sigma} \cdot \vec{p} - m(1 - \sigma_3)\right)\psi = 0. \quad \text{(11)}$$

In coordinate space representation Eq.(11) reads

$$\left(\frac{i}{2}(1 + \sigma_3)\partial_t + \vec{\sigma} \cdot \vec{\partial} - m(1 - \sigma_3)\right)\psi = 0, \quad \text{(12)}$$

where $\partial_t = \frac{\partial}{\partial t}$ and $\partial_i = \frac{\partial}{\partial x_i}$.

The other choice of nonsingular matrix $\beta$ of Eq.(7) enables one to construct another first-order differential equation which is independent of Eq.(12). We will discuss the difference between the two choices later.
We can easily construct a Lagrangian which leads to the equation of motion (12). Introducing a coupling to the Maxwell-Chern-Simons field and a four-fermion self interaction term, the Lagrangian for the nonrelativistic spinor field can be written as

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \kappa \frac{1}{4} \epsilon^{\mu
u\rho\sigma} A_{\mu} F_{\nu\rho} + \psi^\dagger \left( \frac{i}{2} (1 + \sigma_3) D_t + \vec{\sigma} \cdot \vec{D} - m (1 - \sigma_3) \right) \psi + g \left( \psi^\dagger \frac{1}{2} (1 + \sigma_3) \psi \right)^2 ,
\]

(13)

where \( \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \) is a two-component Pauli-spinor field, \( \sigma_i \)'s denote the Pauli matrices, and \( \kappa \) and \( g \) stand for the Chern-Simons coupling constant and non-linear self-interaction constant, respectively. In Eq.(13) \( D_\mu \) represent the covariant derivatives,

\[
D_t = \partial_0 + ieA_0 + \frac{i}{2} \epsilon^{ij} F_{ij},
\]

(14)

\[
D_i = \partial_i + ieA_i,
\]

(15)

where the last term of Eq.(14) represents the anomalous magnetic coupling, \( i, j = 1, 2 \) and we have used the Minkowski metric notation with signature \((+,-,-)\).

With both kinetic terms, Maxwell and Chern-Simons, the field equation for the gauge field becomes

\[
- \partial_\nu F^{\mu\nu} + \frac{\kappa}{2} \epsilon^{\mu\nu\rho\sigma} F_{\nu\rho} = e J^\mu ,
\]

(16)

where the components of \( J^\mu = (J^0, J^1, J^2) \) are given by

\[
J^0 = \rho = |\psi_1|^2 ,
\]

(17)

\[
e \epsilon_{ij} J^i = -ie \epsilon_{ij} \psi^\dagger \sigma_j \psi - i \partial_\rho \rho .
\]

(18)

Eq.(16) can be decomposed into three equations:

\[
\partial_i E^i - \kappa B = e \rho ,
\]

(19)

\[
\epsilon_{ij} \partial_0 E^j + \partial_i B - \kappa E^i = -e \epsilon_{ij} J^j ,
\]

(20)

where \( E^i = -\partial_0 A^i - \partial_i A^0 \), and \( B = \nabla \times \vec{A} = \epsilon_{ij} \partial_i A^j \). Eq.(16) is the modified Gauss’ law. In the static limit, Eqs.(19) and (20) reduce to

\[
\partial_i \partial^i A^0 - \kappa B = e \rho ,
\]

(21)

\[
\partial_i B + \kappa \partial_\rho A^0 = -e \epsilon_{ij} J^j .
\]

(22)

Equations of motion for the nonrelativistic matter field can be written as

\[
i D_t \psi_1 + (D_1 - iD_2) \psi_2 + 2g(\psi_1^\dagger \psi_1) \psi_1 = 0 ,
\]

(23)

\[
(D_1 + iD_2) \psi_1 - 2m \psi_2 = 0 .
\]

(24)
Substituting Eq. (24) into (23), we obtain

\[
\frac{1}{2m}(D_1 - iD_2)(D_1 + iD_2)\psi_1 + iD_t\psi_1 + 2g(\psi_1^\dagger\psi_1)\psi_1 = 0,
\]

which is the second-order Schrödinger equation modified by the introduction of the gauge coupling and the self-interaction term. In the static limit, \(D_t\) reduces to \(D_t^s = ieA^0 - ilB\).

In order to find the static vortex solutions we choose the spinor field as

\[
\psi = \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix}, \quad \psi_2 = 0.
\]

Then Eq. (24) reduces to

\[
(D_1 + iD_2)\psi_1 = 0,
\]

which is the self-dual equation, and Eq. (23) reduces to an algebraic equation,

\[
eA^0 - lB - 2g\rho = 0.
\]

Note that, due to this choice of the spinor field, \(\psi^\dagger\sigma_i\psi = 0\) in Eq. (18).

By comparing equations (21) and (22) with the definition of the current, Eqs. (17) and (18), we obtain the field equations for the gauge field in the static limit,

\[
\partial_i\partial^iA^0 - \kappa B = e\rho, \tag{29}
\]

\[
\partial_iB + \kappa\partial_iA^0 = l\partial_\rho\rho. \tag{30}
\]

Eq. (30) can be written as,

\[
B + \kappa A^0 = l\rho. \tag{31}
\]

If we choose the gauge fixing condition, \(A^0 = 0\), then Eq. (29) also reduces to an algebraic equation. For this system to have consistent static solutions, the algebraic equations (28), (29) and (31) must be identical. Thus the coupling constants \(l, e\) and \(g\) must satisfy the conditions,

\[
l = -\frac{e}{\kappa}, \tag{32}
\]

\[
g = -\frac{e^2}{2\kappa^2}. \tag{33}
\]

The definition of the current, Eq. (18), can be written as

\[
eJ^i = -ie\psi^\dagger\sigma_i\psi + \epsilon^{ij0}l\partial_jJ^0. \tag{34}
\]
This implies that the current $J^i$ includes both the current generated by the matter field, $-i\psi^\dagger \sigma_i \psi$, and the induced current $G^i_{\text{ind}}$ from the magnetic coupling,

$$G^i_{\text{ind}} = \frac{l}{e} \epsilon^{i0j} \partial_j J^0 = -\frac{1}{\kappa} \epsilon^{i0j} \partial_j J^0. \quad (35)$$

There is no induced charge density because we have coupled the anomalous magnetic terms only through the time component of the covariant derivative. Due to the choice of the matter field, Eq.(26), the matter field current, $-i\psi^\dagger \sigma_i \psi$, vanishes and the charge density $\rho = J^0$ remains stationary in the static limit.

Barci and Oxmen [16] have shown that, at every point where we have a charge $Q$ and a magnetic dipole moment $\mu \hat{z}$ ($\hat{z}$ is a unit vector perpendicular to the plane in consideration), the magnetic dipole moment density is given by

$$\vec{m}(\vec{x}) = \frac{\mu Q}{J^0} \hat{z}, \quad (36)$$

in the case where the charge density is stationary and current density vanishes. This dipole moment density induces an electric current (in (2+1)-dimensional notation);

$$G^i_{\text{ind}} = \frac{\mu}{Q} \epsilon^{i0j} \partial_j J^0. \quad (37)$$

By comparing this induced electric current with that of our model, we find

$$\frac{l}{e} = -\frac{1}{\kappa} = \frac{\mu}{Q}. \quad (38)$$

We thus find that there exists a magnetic dipole moment, $\mu = -\frac{Q}{\kappa}$, in this system. This anomalous magnetic coupling can be thought of the (2+1)-dimensional reduction of the familiar Pauli magnetic moment coupling in (3+1)-dimensions [7].

The solutions of the self-dual equation (27) is well-known and given by

$$B = \epsilon_{ij} \partial_i A^j = -\frac{1}{2e} \nabla^2 \ln \rho. \quad (39)$$

Substituting this into the Gauss’ law constraint, Eq.(21), we obtain the Liouville equation for the charge density:

$$\nabla^2 \ln \rho = \frac{2e^2}{\kappa} \rho, \quad (40)$$

where $\kappa$ must be negative for the regularity of the matter density. The solutions of the Liouville equation is well-known [2], and this shows that the nonrelativistic fermion model described by the Lagrangian (13) supports the static vortex solutions.
We now consider the second possibility of choosing the nonsingular matrix $\beta$ of Eq. (41). If we take

$$
\beta = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},
$$

(41)

$B_4$ becomes $B_4 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$. From this choice, we obtain the matrices $A$ and $C$ as,

$$
A = \begin{pmatrix} 0 & 0 \\ 0 & i \end{pmatrix}, \quad C = \begin{pmatrix} -2im & 0 \\ 0 & 0 \end{pmatrix}.
$$

(42)

Then the first-order Schrödinger equation becomes

$$
\left( \frac{i}{2}(1 - \sigma_3)\partial_t - \vec{\sigma} \cdot \vec{\partial} - m(1 + \sigma_3) \right)\psi = 0.
$$

(43)

We can also construct the Lagrangian for the choice Eq. (41) of $\beta$ with self-interaction term $g(\psi^\dagger \frac{1}{2}(1 - \sigma_3)\psi)^2$:

$$
L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}e^{\mu
u\rho}A_\mu F_{\nu\rho} + \psi^\dagger \left( \frac{i}{2}D_4(1 - \sigma_3) - \vec{\sigma} \cdot \vec{D} - m(1 + \sigma_3) \right)\psi + \frac{g}{2}(\psi^\dagger \frac{1}{2}(1 - \sigma_3)\psi)^2.
$$

(44)

In order to find the static vortex solutions of this system, we choose the spinor field as

$$
\psi = \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix}, \quad \psi_1 = 0.
$$

(45)

Then the spinor field equation reduces to an antiself-dual equation,

$$
(\mathcal{D}_1 - i\mathcal{D}_2)\psi_2 = 0.
$$

(46)

The gauge field satisfies the same field equations (28), (29), and (31) as before, so that the coupling constants $l$, $e$ and $g$ satisfy the same conditions (32) and (33). Combining these field equations we finally obtain the Liouville equation,

$$
\nabla^2 \ln \rho = -\frac{2e^2}{\kappa} \rho,
$$

(47)

for the charge density $\rho = |\psi_2|^2$, where the minus sign comes from the different self-dual equation and $\kappa$ must be positive in this case for regular vortex solutions.

We have constructed two possible $(2 + 1)$-dimensional nonrelativistic fermionic theories coupled to a Maxwell-Chern-Simons gauge field that support static vortex solutions. The static matter field in these models satisfy self-dual or antiself-dual equations depending on the choice of the matrix $\beta$, which determines the sign of
the mass and energy terms in the first-order Schrödinger equations, which in turn determines the sign of the Chern-Simons coupling constant for regular static solutions.

In ref.[9] it was also shown that two types of static solutions exist for the relativistic four-fermion theory coupled to a Maxwell-Chern-Simons field, depending on the sign of the Chern-Simons coupling constant $\kappa$. This implies that the nonrelativistic fermion field equations (12) and (13) are somehow related to the nonrelativistic limits of the relativistic field equations. To see this we consider the free relativistic field equation,

$$i\gamma^{\mu}D_{\mu}\Psi - m\Psi = 0,$$

where $\gamma^0 = \sigma^3$, $\gamma^1 = i\sigma^1$, and $\gamma^2 = i\sigma^2$. To obtain the nonrelativistic limit of this equation, we write $\Psi = e^{\pm imt}\psi$ and use the fact that in the nonrelativistic limit the rest mass is the the largest energy [17]. If we take $\Psi = e^{-imt}\psi$ and use the fact that there exists an ambiguity in choosing the sign of the matrices $\sigma_i$ in our model, we can obtain the nonrelativistic equation (12). If one takes the other choice, $\Psi = e^{imt}\psi$, on the other hand, one obtains the field equation (13). Thus one can think of two possibility of the matrix $\beta$ for the nonrelativistic fermion field equation, as different nonrelativistic limits of the relativistic Dirac equation (18). It is worthwhile to compare the field equations we have derived and those of the ref.[13]. Duval et al.[13] have derived their field equations from 4-dimensional 4-component nonrelativistic Dirac equation by dimensional reduction, while we have constructed the $(2+1)$-dimensional non-relativistic field equations directly. By integrating out one of the two component spinor fields ($\chi$, $\Phi$), they obtained the $(2+1)$-dimensional nonlinear Schrödinger equation, thus deriving the self-interaction term automatically. We have, however, introduced the self-interaction term in order to cancel the effects of the newly introduced Maxwell and magnetic interaction terms to give consistent static vortex solutions.

3 Nonrelativistic Scalar Field Model

It is often the case in condensed matter phenomena that spin degrees of freedom in electron system are frozen and electrons are effectively described as excitations of scalar fields. For such systems it is meaningful to consider the nonrelativistic scalar field theories that possess static vortex solutions, and to use them in studying vortex dynamics. For this purpose we consider a nonlinear Schrödinger field theory coupled to a Maxwell-Chern-Simons gauge field:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}F_{\mu\nu}A_\mu F^{\nu\rho} + i\phi^* D_t\phi - \frac{1}{2m}|D_t\phi|^2 + \frac{g}{2}(\phi^*\phi)^2,$$  (49)
where

\[ D_t = \partial_0 + ieA_0 + \frac{1}{2} \epsilon^{ij} F_{ij}, \]  
\[ D_i = \partial_i + ieA_i, \]  

(50)

(51)

and \( \kappa, g \) and \( l \) are the coupling constants. Note that we have also introduced the anomalous magnetic coupling as in the previous case.

The gauge field equations are

\[ \partial_i E^i - \kappa B = e\rho, \]  
\[ \epsilon_{ij} \partial_0 E^j + \partial_i B - \kappa E^i = -\epsilon_{ij}(eJ^j + l\epsilon_{jl}\partial_l \rho), \]  

(52)

(53)

where \( E^i = -\partial_0 A^i - \partial_i A^0, B = \epsilon_{ij} \partial_i A^j, \rho = |\phi|^2 \) and \( J^j = -\frac{i}{2m}(\phi^* D^j \phi - (D^j \phi)^*) \). The right hand side of Eq.(53) includes the matter field current \( J^j \) and the induced one from the magnetic coupling. Equations of motion for a nonrelativistic matter field is given by

\[ iD_t \phi = -\frac{1}{2m} \vec{D}^2 \phi - g|\phi|^2 \phi, \]  

(54)

which is the nonlinear Schrödinger equation.

In the static limit, the gauge field equations become

\[ \kappa B = -e\rho, \]  
\[ \partial_i B = -\epsilon_{ij}(eJ^j + l\epsilon_{jl}\partial_l \rho), \]  

(55)

(56)

where we have chosen the gauge, \( A_0 = 0 \). Using the well known identity,

\[ \vec{D}^2 \phi = (D_1 \pm iD_2)(D_1 \mp iD_2)\phi \pm eB\phi, \]  

(57)

the nonlinear Schrödinger equation (54) can be written as,

\[ i\partial_0 \phi = -\frac{1}{2m}(D_1 \pm iD_2)(D_1 \mp iD_2)\phi + (\mp \frac{e}{2m}B - lB - g|\phi|^2)\phi. \]  

(58)

In the static limit, the Bogomol’nyi limit is saturated by the condition,

\[ \mp \frac{e}{2m}B - lB - g|\phi|^2 = 0, \]  

(59)

and Eq.(58) reduces to the self-dual equations,

\[ (D_1 \mp iD_2)\phi = 0. \]  

(60)

With \( \phi \) field satisfying Eq. (60) the current density simplifies to

\[ J^j = \pm \frac{1}{2m} \epsilon^{jk} \partial_k \rho. \]  

(61)
For static self-dual solutions, Eq. (56) becomes

\[ \partial_i (B - (\pm \frac{e}{2m} + l) \rho) = 0. \]  

(62)

For the three equations (55), (59) and (62) to be consistent the coupling constants must satisfy the conditions,

\[ l = \frac{-e}{k} \pm \frac{e}{2m}, \]  

(63)

\[ g = -\frac{e^2}{k^2}. \]  

(64)

Following the same procedure as in the fermionic case, i.e. Eqs. (34)−(37), we obtain the relation between \( l \) and \( \mu \),

\[ \frac{l}{e} = \frac{-1}{\kappa} \pm \frac{1}{2m} = \frac{\mu}{Q}. \]  

(65)

This implies that the magnetic dipole moment of the system is given by \( \mu = (\frac{-1}{\kappa} \pm \frac{1}{2m}) Q \). The dipole moment \( \mu \) is different from the one in the fermionic case by \( \pm \frac{1}{2m} Q \). This difference comes from the difference in the matter field structure. In the fermion models matter field currents vanish due to the static self-duality condition. In the scalar model, however, the matter field current survives after imposing the static self-duality condition. This survived matter field current induces the additional magnetic dipole moment \( \pm \frac{1}{2m} Q \).

The self-dual equations (60) can be written as

\[ B = \pm \frac{1}{2e} \nabla^2 \ln \rho. \]  

(66)

Substituting this into the Gauss’ law constraint (55), one obtains the well-known Liouville equation,

\[ \nabla^2 \ln \rho = \frac{2e^2}{\kappa} \rho, \]  

(67)

the regular solutions of which exist for \( \kappa > 0 (\kappa < 0) \) for the upper(lower) sign. This model is free of the singularity problem of the scalar Maxwell-Chern-Simons theory discussed by Németh, [12] due to the contribution from the anomalous magnetic coupling.
4 Conclusion

We have constructed two (2+1)-dimensional nonrelativistic fermion field theory models coupled to a Maxwell-Chern-Simons gauge field, that possess static vortex solutions. The regular vortex solutions of these models are possible due to the magnetic moment coupling introduced in addition to the usual minimal coupling to the gauge field. Each of those models supports the regular vortex solutions for a particular sign of the Chern-Simons coupling constant, similar to the case of the relativistic fermion Maxwell-Chern-Simons theory [9]. We have in fact shown that the first order Schrödinger equations satisfied by the fermion matter field are the nonrelativistic limits of the relativistic Dirac equation.

We have also constructed a nonrelativistic scalar field theory coupled to a Maxwell-Chern-Simons gauge field, that supports static vortex solutions. The regularity of the static solutions of this model is guaranteed by the magnetic moment coupling, which enables one to avoid the singularity problem pointed out by Neméth [12].

Although these models have the same static vortex solutions as those of the Jackiw-Pi model [3] as solutions of the Liouville equation, the moduli space dynamics of the solutions will be quite different due to the Maxwell term in the Lagrangian. The reason is that the Maxwell term is quadratic in the time derivatives of gauge fields, i.e., $\dot{A}_i \dot{A}_i$, which gives rise to a non-trivial contributions to the moduli space metric. It would be interesting to study how such a term modify the low energy dynamics of magnetic vortices.

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