Global Symmetries in Duals of Supersymmetric $SU(N) \times SU(M)$ and Application to Composite Axion

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Abstract

In this paper, we construct a supersymmetric composite axion ($c$-axion) model based on the gauge group $SU(N) \times SU(M)$, which is one possible physical application of the N=1 duality. The dual of SU(M) is interpreted as the color gauge group. We illustrate the existence of $c$-axion for the case of one dual quark in the dual gauge group $SU(\tilde{M})$.

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1. Introduction

Sometime ago the composite axion idea was suggested \[1\] so that it is the minimal extension of the standard model at low energy, i.e. the addition of just one pseudoscalar field $a$. In this composite axion (“c-axion”) model, one introduces an additional confining gauge group at the scale $\Lambda_h \sim 10^{12}$ GeV. In supersymmetric theories, this confining gauge group can be the hidden sector gauge group needed for supersymmetry breaking.

Generally, the supersymmetric extension of c-axion is very much constrained because one needs a superfield, transforming nontrivially both in SU(3)$_c$ and in the hidden sector gauge group SU(N), which will be called the $Q$ field. $Q$ contributes positively to the QCD $\beta$-function, which can lead to phenomenological difficulties such as in $\sin^2 \theta_w$ and $\alpha_c$. But the most urgent requirement for $Q$ is that it does not destroy the asymptotic freedom of QCD below the scale $\Lambda_h$. Here, we impose this asymptotic freedom condition for QCD. Indeed, c-axion in a supersymmetric preon model has been considered in the literature \[2\]. Dynamical symmetry breaking with strong gauge groups is an old idea under the name of “moose” \[3\], but it was not possible to draw exact results without supersymmetry. Supersymmetric gauge theories offer more accurate predictions.

In this paper, we study a possibility that the supersymmetric QCD is a dual theory of a supersymmetric SU(M) gauge theory. It shown that a c-axion is generated in this dual description, but our c-axion is different from that considered before in Ref. \[2\].

For the number $(N_f)$ of flavors in SU(M) between $M + 1$ and $3M$, there can be a dual theory \[4,5\]. We consider the possibility that the dual description is more appropriate below a scale $\Lambda$. For $N_f$ quarks $Q_{\alpha I}$ and $N_f$ anti-quarks $\bar{Q}_{\bar{\alpha}}$ with $I = 1, 2, \cdots, N_f$, the standard global charge assignment is

$$
\begin{array}{ccc}
V & A & R \\
Q_{\alpha I} & 1 & 1 & 1 \\
\bar{Q}_{\bar{\alpha}} & -1 & 1 & 1
\end{array}
$$

From which SU($N_f$)$_L \times$SU($N_f$)$_R$ and two anomaly free U(1)’s, U(1)$_V$ and U(1)$_\bar{R}$, survive below $\Lambda$, where

$$
\bar{R} = R - \frac{M}{N_f} A.
$$

We will use this observation throughout the paper.

We assume that QCD, the dual description of SU(M), has a perturbative coupling constant below the SU(M) scale $\Lambda_2$ and also it is asymptotically free below the SU(N) confinement scale $\Lambda_1$. For c-axion, one must study the dual of SU(N)$\times$SU(M) where we interpret SU(N) as the confining force at the hidden sector scale and SU(M) as the source of QCD. The general properties of SU(N)$\times$SU(M) has been studied by Poppitz, Shadmi and Trivedi (PST) \[6\]. The PST study, however, does not include the quarks of the standard model, and does not attempt to apply to particle interactions.

2. Fundamental fields in SU(N)$\times$SU(M)

There are several scales involved in an SU(N)$\times$SU(M) gauge theory. Let us denote the SU(N) scale as $\Lambda_1$ and the SU(M) scale as $\Lambda_2$. Following PST, we consider the dual SU($\hat{M}$)
of SU(M) gauge group. We take the viewpoint that it is more appropriate to describe in the
dual theory SU(\tilde{M}) below the SU(M) scale \( \Lambda_2 \). Thus below \( \Lambda_2 \), the spectrum of the theory
is SU(\tilde{M}) quarks \( q \) and \( \bar{q} \), and SU(M) singlet mesons \( M \) and leptons \( \bar{L} \). The scale defining
SU(M) singlet mesons \( M \) will be described as \( \mu \). In the dual description, the SU(\tilde{M}) scale
\( \bar{\Lambda}_2 \) is related to \( \mu \) and \( \Lambda_2 \) by
\[
\bar{\Lambda}_2^{b_M^0} \Lambda_2^{b_{\tilde{M}}^0} = \mu^{b_M^0 + b_{\tilde{M}}^0} \tag{3}
\]
where \( b_M^0 = 3M - N_f \) and \( b_{\tilde{M}}^0 = 3\tilde{M} - N_f \) are the coefficient of the \( \beta \) functions of the SU(M)
and SU(\tilde{M}), respectively. \( N_f \) is the number of flavors in SU(M) and SU(\tilde{M}). Below \( \Lambda_2 \), the
number of fields transforming nontrivially under SU(N) is changed from that of above \( \Lambda_2 \)
and hence the SU(N) scale must be redefined as \( \bar{\Lambda}_1 \). The \( \bar{\Lambda}_1 \) must be calculable in terms of
\( \Lambda_1, \Lambda_2 \) and \( \mu \). These scales will be given after presenting the model.

If we consider SU(N)\times SU(3), only two extra flavors in addition to the three families of
quarks are admissible. This is very restrictive. Therefore, a better scheme is to have an
SU(M) gauge theory above \( \Lambda_2 \) and QCD is interpreted as the dual of SU(M) below \( \Lambda_2 \). The
minimal representation for \( c\text{-axion} \) needs a representation \((N, M)\) of SU(N)\times SU(M). In
addition to the PST fields, we introduce two more fields, the pre-up quark fields \( R_P \), and
the anti-pre-up quark fields \( \bar{R}_R \). Thus we introduced N+1 flavors in SU(M). These fields
are shown in Table 1.

Table 1. **Global charges of the fields, including the \( R \) charges of fermions.**

*dots in the group representation columns denote singlets.*

| Fields | SU(N) | SU(M) | L | C | V | A | R | \( \bar{R} \) | \( \bar{R}_{fermion} \) | \( Q_\mu \) |
|--------|-------|-------|---|---|---|---|---|---------|-------------|-------|
| \( Q_{a\bar{\alpha}} \) | \( N \) | \( M \) | 0 | -1 | 1 | 1 | 1 | \( -{M \over N_1} \) | \( -{M \over N_1} \) | . |
| \( R_{aP} \) | \cdot | \( M \) | 0 | \( N \) | 1 | 1 | 1 | \( -{M \over N_1} \) | \( -{M \over N_1} \) | . |
| \( \bar{R}_{\bar{\alpha}T} \) | \cdot | \( M^* \) | 0 | 0 | -1 | 1 | 1 | \( -{M \over N_1} \) | \( -{M \over N_1} \) | . |
| \( \bar{R}_{\bar{R}}^{\bar{\alpha}} \) | \cdot | \( M^* \) | 0 | 0 | -1 | 1 | 1 | \( -{M \over N_1} \) | \( -{M \over N_1} \) | . |
| \( \bar{L}_A^\alpha \) | \( N^* \) | \cdot | -1 | 0 | 0 | 0 | 1 | \( -{M \over N_1} \) | \( -{M \over N_1} \) | . |
| \( \lambda_N \) | \( N^2 - 1 \) | \cdot | 0 | 0 | 0 | 0 | \cdot | \cdot | 1 | . |
| \( \lambda_M \) | \cdot | \( M^2 - 1 \) | 0 | 0 | 0 | 0 | \cdot | \cdot | 1 | . |

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1 The SU(M) singlet leptons can transform nontrivially under SU(N).

2 Since we are interested in \( c\text{-axion} \) only, we do not pay attention to leptons.
Here the number of flavors $N_f$ is

$$N_1 = N + 1.$$  \hspace{1cm} (4)

In Table 1, we have written $Q_\mu$ which is not a relevant U(1) charge above the scale $\Lambda_2$. It will be introduced below $\Lambda_2$. Its entries are represented as irrelevant dots. There are four types of chiral fields plus gauginos. (Note that the SU(M) gauge interactions cannot distinguish $\tilde{R}_T^\alpha$ and $\tilde{R}_R^\alpha$.) Therefore, we considered five U(1) symmetries: L, C, V, A, and R. The V, A, and R charges are given as in Eq. (1). For SU(M) quarks, L charges are vanishing. The U(1)$_C$ is a subgroup of SU($N_1$)$_L$. Note that we have not introduced the weak SU(2). In this sense, this model is not realistic; but our objective here is to show the existence of c-axion.

3. Global symmetries in SU($\tilde{M}$)

Suppose that $\Lambda_2$ is larger than $\Lambda_1$. Below $\Lambda_2$, we can consider SU(M) singlet mesons $M$, baryons $B$ and anti-baryons $\bar{B}$. For $N_f = M + 1$ flavors of SU(M) theory, it has been shown that it confines, leading to $N_f$ baryons $B_i$ and anti-baryons $\bar{B}^i$ where $i = 1, 2, \ldots, N_f$. For $N_f > M + 1$, the SU(M) singlet baryon number increases very rapidly. Seiberg [4] has shown that in this case it can be described by a dual gauge theory with the mesons $\tilde{M}$, dual quarks $q$ and anti-dual-quarks $\bar{q}$ in the gauge group SU($N_f$–M). The baryons and anti-baryons are SU($N_f$–M) singlets formed with the dual-quarks and anti-dual-quarks. In our case the dual gauge group is SU($\tilde{M}$), where

$$\tilde{M} = N + 1 - M.$$  \hspace{1cm} (5)

The fundamental representation of the dual gauge group is $\tilde{M}$. The $\tilde{M}$ fields are indexed by $\nu$. So the indices introduced so far run up to

| $\alpha = 1, 2, \ldots, N$ |
| $\dot{\alpha} = 1, 2, \ldots, M$ |
| $\nu = 1, 2, \ldots, \tilde{M}$ |
| $A = 1, 2, \ldots, M$ |
| $T = 1, 2, \ldots, N$ |
| $P, R = 1$ |

Thus for $N_f = N_1 = N + 1$, greater than $M + 1$ but less than $3M$, we have a dual description in the gauge group SU($\tilde{M}$). In this dual theory, the quarks and anti-quarks, $q$ and $\bar{q}$, and SU(M) singlet mesons $M$ will appear. These mesons are defined as d=1 superfields, introducing a scale $\mu$. 

4
Table 2. Global charges of the composite and elementary fields below $\Lambda_2$.

Here $N_1 = N + 1$ and $M_{N_1} = M + N + 1$.

| Fields | $SU(N)$ | $SU(\tilde{M})$ | $L$ | $C$ | $V$ | $Q_\mu$ | $\tilde{R}$ | $C_N$ | $V_N$ | $A_N$ | $R_N$ |
|--------|---------|-----------------|-----|-----|-----|---------|---------|-------|-------|-------|-------|
| $\tilde{L}_A$ | $N^*$ | · | $-1$ | $0$ | $0$ | $0$ | $1 - \frac{M}{N_1}$ | $-1$ | $-1$ | $1$ | $1$ |
| $q_\nu^\alpha$ | $N^*$ | $\tilde{M}$ | $0$ | $\frac{-M}{M}$ | $\frac{M}{M}$ | $1$ | $\frac{M}{N_1}$ | $\frac{M}{M}$ | $-1$ | $1$ | $1$ |
| $M_{\alpha T}$ | $N$ | · | $0$ | $-1$ | $0$ | $-2$ | $\frac{M}{N_1}$ | $0$ | $1$ | $1$ | $1$ |
| $M_{\alpha R}$ | $N$ | · | $0$ | $-1$ | $0$ | $-2$ | $\frac{M}{N_1}$ | $0$ | $1$ | $1$ | $1$ |
| $r_\nu^P$ | · | $\tilde{M}$ | $0$ | $\frac{NM}{M}$ | $\frac{M}{M}$ | $1$ | $\frac{M}{N_1}$ | $\frac{M}{M}$ | $-1$ | $1$ | $1$ |
| $\tilde{r}^{\nu T}$ | · | $\tilde{M}^*$ | $0$ | $0$ | $\frac{-M}{M}$ | $1$ | $\frac{M}{N_1}$ | $-\frac{M}{N_1}$ | $0$ | $-2$ | $0$ |
| $\tilde{r}^{\nu R}$ | · | $\tilde{M}^*$ | $0$ | $0$ | $\frac{-M}{M}$ | $1$ | $\frac{M}{N_1}$ | $-\frac{M}{N_1}$ | $0$ | $-2$ | $0$ |
| $M_{\alpha TP}$ | · | · | $0$ | $N$ | $0$ | $-2$ | $\frac{M}{N_1}$ | $0$ | $1$ | $1$ | $1$ |
| $M_{\alpha RP}$ | · | · | $0$ | $N$ | $0$ | $-2$ | $\frac{M}{N_1}$ | $0$ | $1$ | $1$ | $1$ |
| $\lambda_N$ | $N^2 - 1$ | · | $0$ | $0$ | $0$ | $0$ | $1$ | $0$ | $0$ | $0$ | $1$ |
| $\lambda_{\tilde{M}}$ | · | $\tilde{M}^2 - 1$ | $0$ | $0$ | $0$ | $0$ | $1$ | $0$ | $0$ | $0$ | $1$ |
| $\mu$ | · | · | $0$ | $0$ | $0$ | $2$ | $0$ | · | · | · | · |
| $\Lambda_1^{2N-1}$ | · | · | $-M$ | $-M_{N1}$ | $M$ | $-M_{N1}$ | $2(N - \frac{M^2}{N_1})$ | $0$ | $0$ | $2N_1$ | $2N$ |
| $\Lambda_1^{3N-M}$ | · | · | $-M$ | $-M$ | $M$ | $0$ | $2(N - \frac{M^2}{N_1})$ | · | · | · | · |
| $\Lambda_2^{3M-N-1}$ | · | · | $0$ | $0$ | $0$ | $0$ | $0$ | · | · | · | · |
| $\Lambda_2^{2N-3M+2}$ | · | · | $0$ | $0$ | $0$ | $2N_1$ | $0$ | $0$ | $-N_1$ | $-N_1$ | $N_1 - 2M$ |
\[ M = \frac{1}{\mu} (Q_{\alpha \bar{a}} \text{ or } R_{\bar{a}, P}) \bar{R}_{T,R}^{e_{\bar{a}}}. \] (6)

These composite particles together with the lepton \( \bar{L} \) are shown in Table 2.

With the U(1) charges, which will be given later, one can calculate the charge of \( \Lambda \) according to [8]
\[ \Lambda^{b_0} \propto e^{\frac{2\pi i}{\pi} \frac{\theta}{\pi} + \frac{i}{g^2} - \frac{4}{g^2}}. \] (7)

These are shown in the lower part of Table 2. Except for gauginos the \( \bar{R} \) charges are for the bosonic partners. We have not included the \( A \) and \( R \) charges in Table 2, because they are broken. The conserved U(1) combination is
\[ \tilde{R} = R - \frac{M}{N_1} A. \] (8)

The parameter \( \mu \) appears in the dual theory as given in Eqs. (3) and (5) [6]. It is treated as a holomorphic variable. Thus the meson fields \( M \) introduces an additional global symmetry U(1)\( \mu \). However, the conserved global symmetry above and below \( \Lambda_2 \) must be four. Thus, Seiberg introduced a superpotential of the form (dual quark)-(meson)-(dual quark) [4]. We will construct U(1) charges such that this kind of superpotential is generated at the scale \( \Lambda_2 \). To guarantee the ’t Hooft anomaly matching conditions [4], we will represent the U(1) charges in the form given in Eq. (1). Since we will be discussing SU(\( N \)) confinement at \( \Lambda_1 \), the U(1)’s must be of the form, U(1) subgroup of SU(\( N_1 \)), U(1)\( V \), U(1)\( A \), and U(1)\( R \). Because these charges are relevant for the SU(\( N \)) confinement, a subscript \( N \) is added. Then the four U(1) charges are
\[ C_N = L + V \] (9)
\[ V_N = L - \frac{M}{2M} V - \frac{1}{2} Q_{\mu} \] (10)
\[ A_N = -L + \frac{3M}{2M} V - \frac{1}{2} Q_{\mu} \] (11)
\[ R_N = \tilde{R} - \frac{M}{N_1} L + \frac{M}{2M} V + \left( -\frac{1}{2} + \frac{\tilde{M}}{N_1} \right) Q_{\mu} \] (12)

which are also shown in Table 2. Note that the following superpotential respects the four global symmetries and hence is generated at the scale \( \Lambda_2 \),
\[ W \sim q_{\rho} (M_{\alpha T} \bar{r}^{\nu T} + M_{\alpha R} \bar{r}^{\nu R}). \] (13)

The dual quarks \( q \) will form SU(\( \tilde{M} \)) singlet baryons which is identified as the baryons constructed from SU(\( M \)) quarks \( Q \) [4],
\[ \epsilon^{\nu_1 \nu_2 \cdots \nu_{\tilde{M}}} q_{\alpha_1} q_{\alpha_{M+1}} q_{\bar{a}_{M+2}} \cdots q_{\bar{a}_{\tilde{M}}} \sim \epsilon^{\alpha_1 \cdots \alpha_{M+1} \cdots \alpha_{\tilde{M}}} \epsilon^{\dot{\alpha}_1 \cdots \dot{\alpha}_{\tilde{M}}} Q_{\alpha_1 \bar{a}_1} \cdots Q_{\alpha_{\tilde{M}} \bar{a}_{\tilde{M}}} \] (14)

whence we can schematically write \( q \), for the purpose of obtaining the U(1) charges, as
\[ q \sim \frac{1}{\mu^{n_1} N_2^{n_2}} (Q Q \cdots Q)^{1/\tilde{M}} \] (15)
where the number of $Q$’s are $M$. We considered only $\Lambda_2$ and $\mu$ in the above equation since we consider strong SU(M). The powers $n_1$ and $n_2$ are determined to give dimension 1 superfield $q$ and appropriate U(1) charges. From one column of Table 2, we obtain

$$n_1 = -\frac{1}{2}, \quad \text{and} \quad n_2 = -\frac{1}{2} + \frac{M}{\tilde{M}}.$$  \hspace{1cm} (16)

With this definition of $q$ we obtained all entries of the upper part of Table 2. $\tilde{\Lambda}_2$ is calculable from the matching equation (3). $\tilde{\Lambda}_1$ must be calculable in terms of $\mu$, $\Lambda_1$, and $\Lambda_2$,

$$\tilde{\Lambda}_1^{2N-1} = \Lambda_1^{3N-M} \Lambda_2^{4(3M-N-1) \mu} - \frac{1}{4} (M+N+1).$$  \hspace{1cm} (17)

All columns of Table 2 without dots satisfy the identification of $\tilde{\Lambda}_1$ given in Eq. (17).

The global symmetries are physical at the long distance scale, which is culminated by the so-called ’t Hooft’s anomaly matching condition \[9\] which states that whatever degrees of freedom one uses the global anomalies should match to render the same physics at the long distance scale. If the global anomalies do not match, the confinement is not obtained or the global symmetry in question is broken by the strong nonperturbative dynamics. The global symmetry\[\text{3}\] below the scale $\Lambda_2$ is $SU(N_1)_L \times SU(N_1)_R \times U(1)_V \times U(1)_R$. Since $U(1)_C$ is a subgroup of $SU(N_1)_L$, all the above $U(1)$’s satisfy ’t Hooft’s anomaly matching conditions.

For the generation of the composite axion, one needs a scale at $\Lambda_2$. Most plausible scale is the gaugino condensation scale of the SU(M) gauginos, $< \lambda_2 \cdot \lambda_2 > \neq 0$. \hspace{1cm} (18)

Here we simply assume the gaugino condensation \[10\].

We note from Table 2 that $\Lambda_2$ does not carry any global quantum number; thus a dynamical scale provided by the SU(M) confinement is $\Lambda_2$.

Since we want to interpret SU(N) as QCD, we may have the condition $N - M = 2$. The dual description is possible for $M + 1 < N + 1 < 3M$, which is satisfied for any $M$ and $N$ satisfying $N - M = 2$.

The scale $\mu$ must be comparable to $\Lambda_2$, since the mesons will form at around $\Lambda_2$. Even though we keep track of $\mu$ and $\Lambda_2$, the magnitude of these scales will be set equal in the end. So $\tilde{\Lambda}_2$ is also comparable to $\Lambda_2$. Similarly, $\tilde{\Lambda}_1$ is comparable to $\Lambda_1$.

4. QCD and composite axion

The largest scale below $\mu$ and $\Lambda_2$ is supposed to be $\tilde{\Lambda}_1$. At $\tilde{\Lambda}_1$, the SU(N) gauge group is assumed to become strong. For the SU(N) to become strong at $\tilde{\Lambda}_1$, the SU(N) $\beta$-function must be negative, which is always satisfied. The SU(N) gauge theory given in Table 2 has $N + 1$ flavors. Thus it confines, forming SU(N) singlet baryons $l^A, \bar{q}^\nu(\equiv \bar{u}^\nu), m^T, m^R$, and SU(N) singlet mesons $K_{AT}, K_{AR}, N_{\nu T}, N_{\nu R}$. These mesons are also defined to be d=1 superfields, introducing a scale $\mu_N$, similarly as we defined $M$’s in Eq. (6). Below the scale

\[\text{3}\] We can treat SU(N) as a global symmetry for a moment as far as SU(M) gauge force is concerned.
there survive three conserved U(1) charges. One anomalous U(1) is broken by the SU(N) instantons. These conserved charges are $C_N$, $V_N$ and $\tilde{R}_N$ which are shown in Table 3.

The superpotential given in Eq. (13) gives mass terms for $N\nu_T$, $N\nu_R$, $\bar{r}\nu_T$ and $\bar{r}\nu_R$ at order $\bar{\Lambda}_1$. There remains only one flavor of SU($\tilde{M}$) quark, $\bar{u}\nu$ ($\equiv \bar{q}\nu$), $r\nu_P$. Note that the surviving quark pair is not $\bar{r}\nu_R$ and $r\nu_P$.

The QCD $\beta$ function between $\bar{\Lambda}_2$ and $\bar{\Lambda}_1$ is positive for $N > 8$. But it is negative below $\bar{\Lambda}_1$ due to the removal of $N\nu_T$, $N\nu_R$ and their partners. We assume $N > 8$. We can equate $\alpha_{QCD}(1 \text{ GeV})$ and $\alpha_{\tilde{M}}(10^{12} \text{ GeV})$, i.e.

$$\frac{1}{\alpha_{QCD}(\bar{\Lambda}_{QCD})} = \frac{1}{\alpha_{\tilde{M}}(\bar{\Lambda}_2)} + \frac{N - 8}{2\pi} \log \frac{\bar{\Lambda}_2}{\bar{\Lambda}_1} - \frac{8}{2\pi} \log \frac{\bar{\Lambda}_1}{\Lambda_{QCD}},$$

implying

$$\Lambda_{QCD} = \bar{\Lambda}_2 \left( \frac{\bar{\Lambda}_1}{\bar{\Lambda}_2} \right)^{N/8}, \quad \text{for } N > 8.$$ 

Because we introduced only one quark, we will not draw any phenomenological consequence from this relation. But we note that a similar behavior of generating a small $\Lambda_{QCD}$ can be realized in a more realistic $c$-axion model. By construction, the global symmetries shown in Table 3 do not have SU(N) anomalies. Below $\bar{\Lambda}_1$, the SU(N) confines and only SU($\tilde{M}$) nonabelian groups survive. All the three global symmetries of Table 3 have SU($\tilde{M}$) anomalies. For the fields to return to the original value after $2\pi$ phase rotation, the U(1) quantum numbers must be integers. So the properly normalized $C_N$ charges are obtained from those given in Table 3 multiplied by $\tilde{M}$ and divided by the G.C.D. ($\equiv n_C$) of $M$ and $\tilde{M}$. Similarly, the properly normalized $\tilde{R}$ charges are obtained from the entries of Table 3 times $N_1$. Let the corresponding properly normalized currents are $J_{\mu C_N}^\nu$, $J_{\mu V_N}^\nu$, and $J_{\mu \tilde{R}_N}^\nu$, which satisfy

$$\partial_{\mu} J_{\mu C_N}^\nu = \frac{MN_1}{n_C} \{ F \tilde{F} \},$$

$$\partial_{\mu} J_{\mu V_N}^\nu = -N_1 \{ F \tilde{F} \},$$

$$\partial_{\mu} J_{\mu \tilde{R}_N}^\nu = N_1(2\tilde{M} - 1) \{ F \tilde{F} \}$$

where $\{ F \tilde{F} \}$ is the SU($\tilde{M}$) anomaly, $(1/2)\epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$. If the coefficient of this anomaly is greater than 2, there can be a potential domain wall problem [11]. However, the calculation of the domain wall number depends on the model [12]. Actually there exists only one anomalous U(1) symmetry which is a combination of the above three. The other two nonanomalous symmetries identify the different vacua. If there is no massless quark, the number of degenerate vacua is given by

$$N_{DW} = \text{G.C.D. of } \frac{MN_1}{n_C}, \ N_1, \ N_1(2\tilde{M} - 1)$$

which is equal to the G.C.D. of $N_1$ and $MN_1/n_C$. Probably, the inflation idea is necessary to remove the cosmologically dangerous domain walls if $N_{DW} > 1$. But the gluino condensation $< \lambda \lambda >$ carries $2N_1$ units of $N_1\tilde{R}_N$ charge, gives the real domain wall number of $N_{DW}/n_G$ with $n_G \equiv \text{G.C.D. of } N_{DW}$ and $2N_1$. Since $N_1$ has been already considered in Eq. (22), the real domain wall number is $N_{DW}$ for an odd $N_{DW}$ and $N_{DW}/2$ for an even $N_{DW}$.
Table 3. SU(N) singlets. In the definition column, the SU(N) quarks forming the baryons are also shown.

| Fields      | Definition                      | SU(\tilde{M}) | C_N | V_N | \tilde{R}_N | \tilde{R}_N(fermion) |
|-------------|---------------------------------|---------------|-----|-----|-------------|----------------------|
| \bar{u}^\nu (\equiv \bar{q}^\nu) | \bar{q}_\nu^A, \bar{L}_A^A      | M^*           | \frac{M_N}{M} | -N | \frac{N}{N_1} | -\frac{1}{N_1}      |
| \bar{l}^A  | \bar{q}_\nu^A, \bar{L}_A^A      | 1             | -N  | -N  | \frac{N}{N_1} | -\frac{1}{N_1}      |
| m^T        | M_{\alpha T}, M_{\alpha R}      | 1             | 0   | N   | \frac{N}{N_1} | -\frac{1}{N_1}      |
| m^R        | M_{\alpha T}, M_{\alpha R}      | 1             | 0   | N   | \frac{N}{N_1} | -\frac{1}{N_1}      |
| N_{\nu T}  | \frac{1}{\mu^N} M_{\alpha T} q^A_\nu | \tilde{M}    | \frac{M}{\tilde{M}} | 0 | \frac{2}{N_1} | -1 + \frac{2}{N_1} |
| N_{\nu R}  | \frac{1}{\mu^N} M_{\alpha R} q^A_\nu | \tilde{M}    | \frac{M}{\tilde{M}} | 0 | \frac{2}{N_1} | -1 + \frac{2}{N_1} |
| K_{AT}     | \frac{1}{\mu^N} M_{\alpha T} \bar{L}_A^A | \tilde{M}    | \frac{M}{\tilde{M}} | 0 | \frac{2}{N_1} | -1 + \frac{2}{N_1} |
| K_{AR}     | \frac{1}{\mu^N} M_{\alpha R} \bar{L}_A^A | \tilde{M}    | \frac{M}{\tilde{M}} | 0 | \frac{2}{N_1} | -1 + \frac{2}{N_1} |
| \bar{r}^\nu T | \cdot                           | \tilde{M}    | \frac{M}{\tilde{M}} | 0 | \frac{2N}{N_1} | -1 + \frac{2N}{N_1} |
| \bar{r}^\nu R | \cdot                           | \tilde{M}    | \frac{M}{\tilde{M}} | 0 | \frac{2N}{N_1} | -1 + \frac{2N}{N_1} |
| \bar{r}^P \nu | \cdot                           | \tilde{M}    | \frac{M}{\tilde{M}} | -1 | \frac{1}{N_1} | -1 + \frac{1}{N_1} |
| M_{TP}     | 1                              | \frac{M}{\tilde{M}} | 0 | \frac{1}{N_1} | -1 + \frac{1}{N_1} |
| M_{RP}     | 1                              | \frac{M}{\tilde{M}} | 0 | \frac{1}{N_1} | -1 + \frac{1}{N_1} |
| \Lambda^{3\tilde{M}−1} QCD | \cdot                           | \cdot         | \frac{M N_1}{M} | -N_1 | \cdot | 2\tilde{M}−1 |

5. Yukawa terms and quark masses

Let us discuss the possibility of generating the light quark mass. The superpotential generated at the SU(N) confining scale is \[ 1 \]

\[
W_N = -\frac{\mu^N_{N+1}}{A_1} \epsilon_1^{A_1} \cdots A_M \epsilon_1^{\nu M} \cdots \epsilon_1^{t_{N+1}} K_{A_{t_1}} \cdots K_{R_{t_{N+1}}} \cdot N_{\nu_{t_{M+2}}} \\
\cdots N_{\nu_{t_{N+1}}} N_{\nu_{t_1}} t_{N+1} + \bar{t}^{\nu} t_{N+1} m^{T_1} + \bar{q}^{\nu} N_{\nu_{T_1}} m^{T_1}
\]

(23)
where \( \{ T_i \} = \{ T, R \} = \{ 1, 2, \cdots, N; R \} \) and \( \{ t_i \} = \{ 1, 2, \cdots; N, R \} \). There remains a massless quark with the above superpotential.

The global symmetries we considered so far are respected by the gauge interactions. Any additional terms in the superpotential would violate some of the global symmetries. But these additional terms should be sufficiently small so that the exact results we derived are intact. One possible superpotential we can add is \( vR^\alpha \) where \( v \ll \mu \). Another possible superpotential is \( \epsilon (\mu / \mu) L^\alpha Q_{\alpha \bar{\alpha}} (Q^\alpha_{T_i}) \). For this not to interfere with the SU(3) strong force, we require \( \epsilon \ll 1 \). One can also introduce nonrenormalizable interactions of the form,

\[
\sum_{T_1} \frac{g}{M_{Pl}} R_{\alpha \mu} \bar{R}_{T_1}^\alpha, \quad \sum_{A} \sum_{T_1} \frac{g'}{M_{Pl}} L^\alpha Q_{\alpha \bar{\alpha}} Q^\alpha_{T_1} L^\beta Q_{\beta \bar{\beta}} Q^\beta_{T_1}, \cdots
\]

(24)

where \( M_{Pl} \) is the reduced Planck mass. Without the nonrenormalizable interactions, one cannot give a mass to \( \bar{u} \). Just for the mass generation the first term is enough. Then the mass matrix for \( (\bar{u}, \bar{u}^T) \) is

\[
W = \mu (\bar{u}^T N_{\nu T} + r^P M_{PT} \bar{r}^T) - (\text{Det. term}) + l^A K_{AT} m_{T_1} + \bar{u}^\nu N_{\nu T_1} m_{T_1} + \nu M_{RP} + \sum_{A} \sum_{T_1} \epsilon \mu^2_{A} K_{AT_1} + \frac{g^2}{M_{Pl}} M_{PT}^2 + \frac{g' \mu^2}{M_{Pl}} K_{AT_1}^2
\]

(25)

where the repeated indices imply summations. For supersymmetry, we require for example,

\[
\frac{\partial W}{\partial K_{AT}} = - \sum_{T} K_{AT} m^T - K_{AR} m^R = 0 \quad (26)
\]

\[
\frac{\partial W}{\partial M_{RP}} = - \sum_{A} l^A K_{AR} - \bar{u}^\nu N_{\nu R} = 0 \quad (27)
\]

\[
\frac{\partial W}{\partial K_{AR}} = - l^A m^R + \epsilon \mu^2_{N} + \frac{2g' \mu^2}{M_{Pl}^2} K_{AR} = 0 \quad (28)
\]

\[
\frac{\partial W}{\partial M_{RP}} = r^P \bar{r}^\nu + \nu \mu + 2g^2 \mu^2 \quad (29)
\]

\[
\frac{\partial W}{\partial M_{RP}} = \mu \nu N_{\nu R} + r^P M_{RP} = 0 \quad (30)
\]

Eqs. (29) and (30) determine

\[
I) \quad M_{RP} = - \frac{\nu M_{Pl}}{2g^2}, \quad \text{others} = 0, \quad \text{or}
\]

\[
(II) \quad N_{\nu R} = 0, \quad M_{RP} = 0, \quad < r^P \bar{r}^\nu >= - \nu \mu.
\]

Eqs. (26) – (28) determine

\[
(A) \quad K = 0, \quad < \bar{u}^\nu N_{\nu R} > = 0, \quad < l^A m^R > = \epsilon \mu^2_{N}, \quad \text{or}
\]

\[
(B) \quad m^R = 0, \quad l^A = 0, \quad K = - \frac{\epsilon \mu^2_{N}}{2g^2}
\]

where \( K \ll K_{AT} \ll K_{AR} \). Solution (II) is expected to break the color symmetry. Solution (B) has a huge vacuum expectation value for \( K_{AR} \), which may not be preferred in the inflationary scenario starting from the origin. Thus, we take Solution (IA).

Then the mass matrix for \( (\bar{u}, \bar{u}^T) \) is, by taking \( < l^A >= m^R >, \)

\[
\left( \begin{array}{cc}
0 & \epsilon^{1/2} \mu_N \\
-v M_{Pl}^2 & \mu_N
\end{array} \right).
\]

(31)

In the limit of \( m_u \ll \mu_N \), we obtain
Depending on the parameters, $m_u$ can be sufficiently small. Thus the $u$ quark can be made massive which is accomplished by the introduction of a small parameter $v$ which may be generated by a Higgs potential. Thus, the anomaly considered in Eq. (21) cannot be rotated away by the redefinition of a massless quark.

Introduction of the superpotential destroys some of the global symmetries. If all the global symmetries are broken by the superpotential, then the $c$-axion may be too massive.

In our case, the global charges carried by the new terms are multiples of those carried by $R_{\tilde{a}P} \tilde{R}_{\tilde{T}1}^a$ (or $M_{R T_1}$) and $\tilde{L}_a^\gamma Q_{\tilde{a} d} \tilde{Q}_{\tilde{T}1}^\alpha$ (or $\tilde{K}_{AT_1}$). Therefore, only two global symmetries are broken, leaving one anomalous global symmetry $Q_P Q = \tilde{R}_N + (2/N_1)C_N - (1/N_1)V_N$. For $M \geq 8$, the PQ symmetry violating operator $\epsilon_{\tilde{a}1 \cdots \tilde{a} M} \tilde{R}_{\tilde{T}1}^{\tilde{a} 1} \cdots \tilde{R}_{\tilde{T} M}^{\tilde{a} M}$ is sufficiently small, and the $c$-axion solves the strong CP problem.

We assumed that the Goldstone boson ($c$-axion) is created by breaking $\tilde{R}$ by $< \lambda_2 \cdot \lambda_2 >$. Then the axion scale $F_a$ is the $U(1)_{\tilde{R}}$ breaking scale,

$$F_a \sim \sqrt{< \lambda_2 \cdot \lambda_2 >^{2/3} + < \lambda_1 \cdot \lambda_1 >^{2/3}}.$$  

This gaugino condensation may be realized by introducing supergravity interactions. However, it is difficult to study the full supergravity Lagrangian. On the other hand, we pretend to reproduce the symmetry properties and its spontaneous breaking via an effective superpotential in terms of gauge singlet fields.

The relevant superpotential is taken as

$$W \sim (\lambda_1 \Phi \Upsilon + m^2)S + (\lambda_2 \Phi \Upsilon + m^2)S'$$

where $\Phi, S$ and $S'$ are gauge singlet fields and $\Upsilon$ is the effective chiral field for gaugino condensation. If the couplings $\lambda_{1,2}$ are tuned to small values (arising only through supergravity interaction), the discussion we presented so far is almost intact. This superpotential is chosen so that it preserves the $R$ symmetry with $S, S'$ and $\Phi$ carrying 2, 2, and –2 units of $R$ charges, respectively, and also it breaks supersymmetry through nonzero value of $\Upsilon$ and $\Phi$ for $\lambda_1 \neq \lambda_2$. So it has the phenomenologically anticipated properties of this paper. The supergravity interaction contains a term

$$\frac{1}{4} M_{Pt} e^{-G/2} G^l (G^{-1})^k f_{\alpha \beta \lambda} \lambda^{\alpha} \lambda^{\beta}.$$ 

With a gauge kinetic function $f \sim 1 + a \Phi / \Lambda_2$, we obtain the coupling $\sim \Phi \lambda \lambda$. This term can arise from the cross terms in $|\partial W / \partial S|^2 + |\partial W / \partial S'|^2$. Here, we have not shown that $W$ results from supergravity, but mimick some aspects of supergravity in terms of $W$. Of course, Eq. (35) is proportional to the gravitino mass, whence the overall coupling in $W$ is proportional to the gravitino mass. If we introduced $W$ independent from the gaugino

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4In a real application, it may be the $t$ quark mass.
condensation, i.e. Υ is not the gaugino condensation, then the $R$ symmetry breaking leads to a fundamental invisible axion \[\text{(10)}\].

6. Conclusion

In conclusion, we exploited an interesting class of $\text{SU}(N) \times \text{SU}(M)$ models, attempting to interpret the known quarks as the dual particles, and studied the global symmetries and their breaking by possible terms in the superpotential. There results a $c$-axion. For the extra factor group $\text{SU}(N)$, we motivated its rationale through generation of a $c$-axion which require a representation of the type $(N,M)$, implying a strong force at the intermediate scale. This intermediate scale can be the world where the duality idea is physically applicable.

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