Diagrammatic approach to coherent backscattering of laser light by cold atoms: double scattering revisited

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Abstract
We present a derivation of the coherent backscattering spectrum from two two-level atoms using the diagrammatic approach, wherein the multiple scattering signal is deduced from single-atom responses, and provide a physical interpretation of the single-atom building blocks.

1. Introduction

Coherent backscattering (CBS) of light emerges due to the constructive interference of multiply scattered counterpropagating waves surviving the disorder average [1]. With optical waves, CBS was successfully observed using classical—e.g., polystyrene particles [2]—and quantum—dilute cold atoms [3]—scatterers alike. A remarkable property of cold atoms is their nonlinear inelastic scattering induced by a powerful resonant laser field. Atomic saturation and inelastic scattering processes accompanying it were shown to affect the CBS interference from cold Sr (strontium) [4] and Rb (rubidium) [5] atoms, but a corresponding theory of CBS of intense laser light from cold atomic clouds is still lacking.

The main challenge one has to deal with when describing CBS from cold-saturated atoms can be briefly summarized as follows. A multiple scattering signal must be built on the basis of the accurately described responses of individual scatterers to a strong laser field. These responses can be found by solving the optical Bloch equations (OBEs) [6]. However, a standard generalization of the OBE to the N-atom case leads to a Lehmberg-type master equation governing the evolution of the reduced density operator of all atoms which are laser-driven and dipole–dipole interacting [7, 8], with the number of equations for the atomic averages growing exponentially with the number of scatterers. So far, such a master equation in the context of CBS has been solved only for \( N = 2 \) atoms [9, 10].

Recently, we have initiated an alternative method of generalizing the OBE to the many-atom case which we call the diagrammatic approach to CBS [11–13]. In its framework, the double scattering signal can be obtained from the solutions of the OBE for an atom subjected to a bichromatic classical driving. One component thereof represents the laser field and another one the field scattered from the second atom. Single-atom responses to a bichromatic field were evaluated nonperturbatively in the laser field and perturbatively—up to a second order—in the scattered field amplitude. Furthermore, by self-consistently combining single-atom responses (to which we will also refer as ‘building blocks’), we were able to derive analytical expressions for the background and interference spectra of double scattering. The spectra thus evaluated were rigorously shown to be equivalent to that deduced on the basis of the two-atom master equation [14].

The motivation of the present contribution is twofold. First, previously we focused on a discussion of the inelastic building blocks and their self-consistent combination into double scattering diagrams [12, 13]. These blocks determine the CBS signal only in the case of a very strong laser driving. In the present contribution we will consider a general case and present a diagrammatic derivation of the CBS spectra for arbitrary intensity of the laser field. Second, the single-atom building blocks have not been given a physical interpretation. Here, we furnish the single-atom building blocks with a physical interpretation and establish a close connection with the results of Mollow [15].

The rest of this paper is organized as follows. In the next section we present the building blocks contributing to the double scattering background and interference spectra. Thereafter it is shown how these blocks can be evaluated by solving the OBE under bichromatic driving. Thereby we establish the connection to the method used by Mollow in
scattered by the atoms (grey circles) into waves whose frequencies \( \omega', \omega'' \) and \( \omega_D \) may differ from \( \omega_2 \).

In section 3 we formulate rules for combining single-atom building blocks into double scattering diagrams, present the full set of diagrams contributing to the elastic and inelastic spectra and give the explicit expressions thereof. We conclude our work in section 4.

2. Single-atom building blocks

2.1. Graphical representation of the elastic and inelastic building blocks

To be self-contained, we will briefly outline here the main idea of the diagrammatic approach to CBS of laser light from two two-level atoms which was presented in detail in [12, 13].

The fundamental double scattering processes surviving the disorder averaging and contributing to the CBS background and interference signals are shown in figure 1. We assume that the laser field may be sufficiently strong to saturate the atomic transitions. Accordingly, the frequencies \( \omega', \omega'' \) and \( \omega_D \) of the waves scattered by the atoms towards each other and a detector can, but need not, differ from the laser frequency \( \omega_2 \), i.e. correspond to inelastic scattering. The co-propagating positive- and negative-frequency amplitudes (solid and dashed arrows, respectively) contribute to the background double scattering intensity, see figure 1(a), which is independent from the observation direction, whereas the counter-propagating ones contribute to the CBS interference, see figure 1(b). We will be interested in finding the frequency distributions of the background and interference spectra (intensity versus \( \omega_D \)) as a function of the driving field parameters (such as the Rabi frequency and the offset from the atomic transition frequency).

In the framework of the diagrammatic approach [11–13], these frequency distributions can be derived on the basis of single-atom building blocks. The latter represent spectral responses of an atom subjected to a classical bichromatic driving field. The first component thereof corresponds to a laser field of arbitrary strength, whereas the second, weak, component describes the far field scattered by the other atom. Following the nomenclature used in laser spectroscopy [6, 16], we will refer to the laser and weak field components as the pump and probe fields, respectively. While the classical description of the laser field is common, applying the same description to the atomic radiation exhibiting photon antibunching [17] is, in general, wrong. However, in the dilute regime, when the double scattering originates from the exchange of a single photon, the validity of the semiclassical treatment of the atom probe field interaction does not contradict the nonclassical character of the scattered field [12, 13], which manifests itself only in correlations between at least two photons. Furthermore, the validity of the classical ansatz for the probe field was proven analytically by establishing the equivalence between the results of the diagrammatic and master equation calculations [14].

The classical ansatz for the fields exchanged between the atoms allows us to consider two atoms with their incoming and outgoing fields separately from each other. The left-hand sides in figures 2(a) and (b) show the decomposition of the background and interference contributions to CBS from figures 1(a) and (b), respectively, into single-atom blocks. In order not to overburden diagrams, in figure 2 only fields scattered by the atoms are depicted, but here and henceforth one should remember that the atoms are also laser driven and, furthermore, the effect of the laser field on atoms is accounted for non-perturbatively. Note that the arrows are not labelled by their frequencies in figure 2. The latter will be defined in the course of the subsequent analysis.

In general, for any finite value of the saturation parameter, there are non-zero cross-sections for elastic and inelastic scattering of photons by a single laser-driven atom [6]. The same also holds in the presence of additional probe field(s). In fact, one of the purposes of the diagrammatic approach is to calculate elastic and inelastic responses of a single laser-driven atom subject to probe fields.

Accordingly, the right-hand sides of figure 2 represent decompositions of the total single-atom responses into elastic (blank circles) and inelastic (hatched squares) building blocks. Processes contributing to (a) background, and (b) interference intensities. Frequencies of the probe and scattered fields will be defined after the expressions for the building blocks on the right-hand side, as well as the rules for combining the building blocks are known.

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Accordingly, the right-hand sides of figure 2 represent decompositions of the total single-atom responses into elastic (blank circles) and inelastic (hatched squares) building blocks. Different shapes of the blocks (i.e. circles or squares) emphasize that the corresponding expressions result from different equations of motion (for the atomic dipole averages and temporal correlation functions, see appendix A) describing the elastic and inelastic scattering, respectively. Different colours refer to photons emitted at the laser frequency (blank), or at the frequency different from the laser frequency (hatched), as will be explained in section 2.2.2.
There is an important difference between the diagrams in figures 1 and 2. While the former diagrams depict general (background and interference) double scattering processes for a random configuration of atoms, the latter ones show single-atom blocks which, by construction, only produce those background and interference contributions which automatically survive the disorder average. In accordance with this, all arrows in figure 2 are for convenience oriented along the horizontal line.

Prior to presenting the explicit expressions for the building blocks on the right-hand side of figure 2, we will show that the elastic blocks can be decomposed further.

### 2.1.1. Elementary ‘elastic’ building blocks.

Each of the building blocks in figure 2 contains two outgoing arrows. We will now express all elastic blocks (circles) as combinations of building blocks in figure 2(a), the outgoing arrows correspond to the elastic blocks on the right-hand side of figure 2, we will show that the elastic scattering of positive- and negative-frequency amplitudes of the atom driven only by the laser field (we will supply the expectation values in this case by the superscript (0)).

We represent this equation graphically in figure 3(a). There, a circle with the outgoing solid (dashed) arrow corresponds to \( \sigma^+ (\sigma^-) \), respectively, whereas the product of these two amplitudes is depicted by the symbol ‘x’.

It is easy to generalize this result to the case when there are incoming probe-field amplitudes. In this case, the elastic building blocks can be expanded into sums of products of two elementary amplitudes, with the number of terms in this expansion being equal to the number of ways in which the incoming arrows can be distributed among the two circles (see figures 3(b)–(d)).

### 2.1.2. ‘Inelastic’ building blocks.

Concerning the inelastic building blocks of figure 2 (hatched squares), they result from the non-factorizable part of the atomic response, i.e. \( \langle \sigma^+ \sigma^- \rangle = \langle \sigma^+ \rangle \langle \sigma^- \rangle \); see equation (6). Unlike the elastic blocks, these blocks, which exhibit two outgoing arrows corresponding to \( \sigma^+ \) and \( \sigma^- \), respectively, cannot be factorized into a product of blocks with only one outgoing arrow.

### 2.2. Calculating building blocks

In section 2.1 we presented the disorder averaged elastic and inelastic single-atom building blocks which contribute to the double scattering background and interference spectra of CBS. All these blocks represent a single, two-level, laser-driven atom which additionally receives none, one or two classical probe-field amplitudes. We will next show how to evaluate these blocks using the OBE under bichromatic driving.

### 2.2.1. Single-atom OBEs under bichromatic driving.

A classical bichromatic driving field reads

\[
E(t) = \varepsilon_L e^{i\omega_L t} + \varepsilon_p e^{i\omega_p t} + \varepsilon^* e^{-i\omega_p t} + \varepsilon^* e^{-i\omega_L t},
\]

where \( \varepsilon_L, \varepsilon_p \) are complex amplitudes, and \( \omega_L, \omega_p \) the frequencies of the pump and probe fields, respectively. It is easy to show that the optical Bloch vector \( \langle \vec{\sigma} \rangle = (\langle \sigma^- \rangle, \langle \sigma^+ \rangle, \langle \sigma^\dagger \rangle) \), where \( \sigma^- = |0\rangle\langle 1|, \sigma^+ = |1\rangle\langle 0|, \sigma^\dagger = |1\rangle\langle 1| - |0\rangle\langle 0| \) and \( |0\rangle, |1\rangle \) being the ground and excited states of the atom, obeys the following equation of motion written in the frame rotating at the laser frequency:

\[
\frac{d\langle \vec{\sigma} \rangle}{dt} = M(\langle \vec{\sigma} \rangle) + \vec{L} + v e^{i\omega_p t} \Delta(+) \langle \vec{\sigma} \rangle + v^* e^{-i\omega_p t} \Delta(-) \langle \vec{\sigma} \rangle,
\]

where \( M \) denotes the optical Bloch matrix for an atom driven by the laser field:

\[
M = \begin{pmatrix}
-\gamma + i\delta & 0 & -i\Omega/2 \\
0 & -\gamma - i\delta & i\Omega^*/2 \\
-i\Omega^* & i\Omega & -2\gamma
\end{pmatrix},
\]

with \( \delta = \omega_L - \omega_0 \) being the laser detuning from the atomic transition frequency \( \omega_0 \), and \( \gamma \) being half the spontaneous decay rate. \( \Omega = 2\varepsilon_L/h \) and \( v = 2\varepsilon_p/h \) are the Rabi frequencies of the pump and probe fields, respectively, with \( \varepsilon_p \) being the (real) matrix element of the atomic dipole transition.
$\omega = \omega_p - \omega_L$ is the detuning between the probe and pump frequencies. Finally, the matrices

$$\Delta_(-) = \begin{pmatrix} 0 & 0 & -i/2 \\ 0 & 0 & 0 \\ 0 & i & 0 \end{pmatrix}, \quad \Delta_+ = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i/2 \\ -i & 0 & 0 \end{pmatrix}$$

(4)

describe the coupling of the atom to the positive and negative frequency components of the scattered field, respectively, and

$$\vec{L} = (0, 0, -2\gamma)^T$$

(5)
is a constant vector.

### 2.2.2. Elastic building blocks.

It is straightforward to establish the correspondence between the elementary elastic building blocks of figure 3 and solutions of equation (2). We will start by noting that if $v = v^* = 0$ in equation (2), it reduces to the standard OBE having the steady-state solution $(\sigma^0)^{(0)}$. For non-zero probe fields, there appear corrections to the unperturbed components of the Bloch vector as well as oscillations at the frequencies $\pm \omega$ (see appendix A). Depending on whether a single incoming probe field represents a positive- or negative-frequency amplitude at frequency $\omega$, the corrections will be denoted as $[\sigma(\omega)]^{(\pm)}$ and $[\sigma^*(\omega)]^{(\pm)}$, respectively. A second-order correction, which corresponds to one solid and one dashed incoming arrow at the frequency $\omega$, will be denoted as $[\sigma_{\pm}^0]^{(0)}$. Finally, we recall that the emission of the positive- (negative-)frequency amplitudes is associated with the expectation values of the atomic lowering (raising) operators $[6]$. Summarizing the above, we express the elementary elastic building blocks in terms of the solutions of equation (2) as shown in figure 4. From now on we use the convention not to label the incoming (outgoing) waves which are exactly on resonance with the laser frequency. Furthermore, we will denote elastic building blocks with an outgoing wave at the laser frequency by a blank circle, whereas a hatched circle corresponds to an outgoing wave at a frequency $\omega \neq \omega_L$ (as shown below, the latter requires the presence of an incoming wave with frequency $\omega$ or $-\omega$).

Diagrams (a) and (b) of figure 4 describe positive- and negative-frequency amplitudes, respectively, of the elastic scattering by a laser-driven atom. Clearly, their product is negative-frequency amplitudes, respectively, of the elastic oscillations at the frequencies to the unperturbed components of the Bloch vector as well as oscillations at the frequencies $\pm \omega$. Finally, the matrices describe the coupling of the atom to the positive and negative frequency values as the incoming ones. In accord with the convention adopted above, these building blocks are hatched, since the frequency of the incoming wave—and therefore also of the elastically scattered outgoing wave—differs from the laser frequency. Further on, the processes (e) and (f) correspond to a nonlinear phase conjugation, whereupon the positive- (negative-)frequency amplitude at the frequency $\omega$ is scattered into a negative (positive) frequency, that is, conjugated, amplitude at the frequency $-\omega$, via a nonlinear mixing with the laser wave described by the nonlinear atomic susceptibility. Using the nonlinear optic terminology, one can regard $[\sigma^- (\omega)]^{(\pm)}$, $[\sigma^+ (\omega)]^{(\pm)}$ (and their complex conjugate quantities) as, respectively, the effective (that is, dependent on the laser pump) linear and nonlinear susceptibilities of a two-level system with respect to the probe field [18].

We note that the relevance of nonlinear susceptibilities for CBS from saturated atoms was mentioned before [19]. Finally, the processes (g) and (h) describe stimulated emission of the amplitudes at the pump field frequency in the presence of the probe field.

We conclude this part by a short remark. The diagrams of figure 4 describe all possible elementary building blocks that appear in figure 3. Consequently, solutions of equation (2) for the Bloch vector to second order in the probe-field amplitude allow us to evaluate completely the ‘elastic’ responses on the right-hand side of figure 2.

#### 2.2.3. Inelastic building blocks.

Let us now address the inelastic building blocks. They are deduced from the solutions of equation (2) for the atomic dipole temporal correlation function [13, 12] (see also appendix A),

$$g(t_1, t_2) = \langle \sigma^+ (t_1) \sigma^- (t_2) \rangle - \langle \sigma^+ (t_1) \rangle \langle \sigma^- (t_2) \rangle,$$

(6)
in the steady-state limit when $g(t_1, t_2) = g(t_1 - t_2)$, and subsequent Laplace transformation to the frequency domain.

The Laplace-transformed solutions of equation (6), with the probe-field amplitudes $v$ and $v^*$ set equal to zero, together with the linear $(\alpha v, v^*)$ and quadratic $(\alpha [v^2])$ corrections, yield the inelastic building blocks shown in figure 5. The physical interpretation of the functions $p^{(0)}(\omega, v)$, $p^{(\pm)}(\omega, v)$ and $p^{(2)}(\omega, v)$ is analogous to the interpretation of the building

![Graphical representation of the elementary diagrams for elastic scattering.](image-url)
blocks of figure 4, and merely generalizes it to include the multi-photon scattering of the laser photons.

Diagram (a) in figure 5 represents the inelastic spectrum of a two-level atom subject to monochromatic driving. That is, the function $P^{(0)}(\omega)$ is the resonance fluorescence spectrum known, for $\Omega \gg \gamma$, as the Mollow triplet [20]. Diagram (b) represents the correction to the Mollow triplet $P^{(2)}(\omega, \nu)$ proportional to the intensity of the probe field at the frequency $\omega$. This diagram can be regarded as a generalization of diagrams (g) and (h) of figure 4, which accounts for the inelastic scattering. Finally, the functions $P^{(\pm)}(\omega, \nu)$ (see figures 5(c) and (d)) represent inelastic wave mixing processes resulting in the emission of two amplitudes which are off-resonant with either the laser or the probe fields. These diagrams should be contrasted with the diagrams (c), (d) in figure 3. Indeed, in the latter diagrams, the two outgoing arrows are on-resonant with the probe and laser fields.

### 3. Combining building blocks

#### 3.1. Rules

In the previous section we defined all single-atom building blocks. We recall that the elastic and inelastic building blocks from which the background and interference contributions are constructed are shown on the right-hand sides of figures 2(a) and (b), respectively. The four elastic blocks in figure 2 can be further decomposed according to figure 3, with the elementary diagrams defined in figure 4, whereas the inelastic blocks and their corresponding spectral response functions are shown in figure 5. So, all diagrams on the right-hand side of figure 2 are defined, and we need to introduce rules for combining them.

These rules can be formulated as follows:

- The ladder and crossed contributions are obtained by combining the building blocks of atom 1 with that of atom 2, on the right-hand side of figures 2(a) and (b); each combined two-atom diagram is proportional to the factor $g^2$, where $g = (kL/\ell)^{-1}$ scales as the far-field dipole–dipole coupling between the two atoms separated by the mean distance $\ell$.
- The elastic building blocks for atoms 1 and 2 are expanded according to figure 3.
- Each outgoing solid (dashed) arrow of atom 1 is merged with each incoming solid (dashed) arrow of atom 2. As a result, one obtains sums of diagrams containing two intermediate and two outgoing arrows. The intermediate arrows’ frequencies are defined self-consistently: the frequencies of the incoming and outgoing arrows must coincide and, moreover, respect the form of the response functions presented in figures 4 and 5. As a result, for example, there is only one independent inelastic frequency describing intermediate amplitudes.
- The response functions associated with the single-atom building blocks in the resulting double scattering diagrams must be multiplied. The symbolic expressions for these functions are given in figures 4 and 5.
- If an intermediate arrow representing the inelastic probe field (say, at frequency $\omega$) changes its frequency, then the expression corresponding to that diagram is integrated over this frequency as $\int_{-\infty}^{\infty} d\omega/(2\pi)$ (note that $\omega = \omega_2 - \omega_1$, and can take on negative values).
- Diagrams containing blocks connected by intermediate counter-propagating arrows with no outgoing arrows must be excluded. Such diagrams correspond to a photon cycling between the atoms without contributing to the detected signal. Exactly the same rule exists in the nonlinear diagrammatic theory of CBS in a Kerr medium [21].

Using the above rules, we can represent the background and the interference contributions as sums of the diagrams shown in figures 6 and 7, respectively. When writing down the explicit expressions for different contributions, we will omit the overall factor $g^2$. 

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**Figure 5.** Graphical representation of the inelastic building blocks and their mathematical expressions (see equation (A.4)). (a) $P^{(0)}(\omega)$—the Mollow triplet; (b) $P^{(2)}(\omega, \nu)$—correction to the Mollow triplet due to the probe field at the frequency $\omega$; (c) $P^{(\pm)}(\omega, \nu)$ and (d) $P^{(-)}(\omega, \nu)$—inelastic wave mixing of the pump and probe fields resulting in the scattering of the waves neither of which is in resonance with the laser or probe waves.

**Figure 6.** Background contributions to double scattering. (a) Fully elastic contribution originating from both atoms scattering elastically; (b) inelastic contribution due to second atom scattering inelastically; (c) mixed contribution including diagrams (c1), (c2) corresponding to elastic scattering, and (c3), (c4) corresponding to inelastic scattering; (d) fully inelastic contribution.
By definition, the elastic background

3.2 Background contributions

3.2.1 Elastic spectrum. By definition, the elastic background spectrum \( L_{el}(\nu) = L_{el} \delta(\nu) \) exhibits a single \( \delta \)-peak precisely at the laser frequency, corresponding to \( \nu = 0 \) in the rotating frame. Hence, it is given by the sum of all diagrams with unlabelled signal (outgoing) lines in figure 6, that is, diagrams (a), (c1) and (c2). Note that diagram (a) is to be unfolded into a sum of products of the following diagrams of figure 3: (a1) (b1), (a1)(b2), (a1)(b3) and (a1)(b4). Taking the expressions for the building blocks from figures 4 and 5, and applying the above rules for combining these building blocks, the elastic background intensity reads

\[
L_{el} = \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 (2) \\
+ \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 (2) \\
+ \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 (2) \\
+ \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 (2) \\
\int_{-\infty}^{\infty} \frac{d\nu}{2\pi} P_0(\omega) \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 (7a)
\]

where equations (7a)–(7c) correspond to diagrams (a), (c1) and (c2) in figure 6, respectively (see appendix A for the explicit evaluation of the right-hand side of equation (7)).

3.2.2 Inelastic spectrum. As evident from figure 6, its remaining diagrams describe the inelastic spectrum

\[
L_{in}(\nu) = \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 + \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 (8a)
\]

where equations (8a)–(8d) correspond to diagrams (b), (c3), (c4) and (d) of figure 6, and the (signal) frequency \( \omega \) was for convenience relabelled in lines 2 and 3 of equation (8) to \( \nu \).

3.3 Interference contributions

3.3.1 Elastic spectrum. By the same argument as in the case of the background diagrams, we identify diagrams (a), (b1) and (c1) of figure 7 as contributing to the elastic spectrum. In the present case, the fully elastic contribution represented by diagram (a) is explicitly decomposed into diagrams (a1)–(a3) corresponding to a combination of diagrams (c1)(d2), (c2)(d1), and (c2)(d2) of figure 3, respectively. The product of diagrams in figures 3(c1) and (d1) does not contribute to figure 7(a) because it corresponds to a photon cycling between the atoms (see section 3.1).

Note that the squares which are hatched in figures 7(b) and (c) become blank in diagrams (b1) and (c1). This is precisely the consequence of the self-consistent combination of diagrams: the connection of the phase conjugation diagram (figure 4(e)), for example, to figure 5(d) enforces the occurrence of two frequencies \( \omega \) and \( -\omega \), and therefore \( \nu = 0 \), in the building block (figure 5(d)). Thus, a blank square denotes an inelastic process, where the frequency of one of the outgoing arrows equals the laser frequency.

After these remarks, we present the explicit expression for the elastic interference spectrum:

\[
C_{el} = \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 \langle \sigma^+ \rangle^0 \langle \sigma^- \rangle^0 (9a)
\]

where the subsequent lines of equation (9) correspond to diagrams (a1)–(a3), (b1) and (c1) of figure 7, respectively.
3.3.2. Inelastic spectrum. The three contributions to the inelastic spectrum are depicted by diagrams (b2), (c2) and (d) in figure 7. In diagram (d), both the solid and dashed lines corresponding to intermediate amplitudes change their frequencies. To write down the expression for this diagram, we note that the outgoing inelastic photon frequency is $v$. Since the frequencies of the intermediate amplitudes are correlated, e.g., $\omega$ and $v - \omega$ in figure 7(d), there is only one independent probe-field frequency $\omega$ to be integrated over at fixed detected frequency $v$. Denoting the signal frequency $v$ in diagrams (b2) and (c2) of figure 7, finally we obtain the following result for the inelastic interference spectrum:

$$C_{\text{inel}}(v) = \langle \sigma^- \rangle \langle \sigma^+ \rangle P^{(-)}(0, v) + \langle \sigma^+ \rangle \langle \sigma^- \rangle P^{(-)}(v, v) + \int \frac{d\omega}{2\pi} P^{(+)}(\omega, v) P^{(-)}(v - \omega, v),$$

where equations (10a)-(10c) correspond to diagrams (b2), (c2) and (d) in figure 7.

4. Conclusion

We presented a diagrammatic derivation of the double scattering background and interference spectral distributions of coherent backscattering from two two-level atoms. Although these distributions were known before [11–13], here we re-derived them in an intuitive way which, we believe, makes the diagrammatic approach to CBS of intense laser light from cold atoms more accessible and attractive to the reader.

It is not only the diagrams that render our present approach intuitive, but also a physical interpretation of the building blocks which is given in this work. In particular, we established a connection of the ‘elastic’ building blocks with the response functions used to calculate the weak probe absorption spectra by a laser-driven atom [15]. Furthermore, the ‘inelastic’ building blocks include the Mollow triplet, a modification thereof proportional to the intensity of the weak probe fields and inelastic response functions which are linear in the probe fields, and which emerge due to mixing of the laser and probe fields.

Among the single-atom spectral response functions relevant to CBS, only the resonance fluorcence and probe absorption spectra were observed experimentally [22, 23]. It remains to be seen whether the other functions can also be measured directly in experiments with single quantum scatterers interacting with one strong and one weak laser field. Perhaps, strongly driven quantum dots [16] are the most suitable candidates for such observations.

The possibility of obtaining the double scattering signal diagrammatically from single-atom building blocks suggests two directions of future research in the field of CBS of intense laser light from cold atoms. First, the diagrammatic techniques presented in this work will be further developed to assess higher scattering orders of CBS. Second, a generalization of this method to realistic atoms having degenerate dipole transitions will be another useful application.

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Appendix A. Expressions for building blocks

Here we present expressions for the elementary elastic and inelastic building blocks (figures 4 and 5, respectively). A detailed derivation thereof can be found, for example, in [13].

A.1. Elastic blocks

The expressions associated with the building blocks of figure 4 follow from the perturbative (in the probe-field amplitude) solutions of the optical Bloch equation (2). They are given by the first or second entries of the following vectors:

$$\langle \tilde{\sigma} \rangle^{(0)} = G \tilde{L}, \quad \langle \tilde{\sigma} \rangle^{(1)} = G(\omega) \Delta^{(+)} \langle \tilde{\sigma} \rangle^{(0)}, \quad \langle \tilde{\sigma} \rangle^{(2)} = G(-\omega) \Delta^{(-)} \langle \tilde{\sigma} \rangle^{(0)},$$

where $G(x)$ is the free propagator given by

$$G(x) = (x - M)^{-1},$$

where $M$ is the Bloch matrix (see equation (3)), $G \equiv G(0)$ and the matrices $\Delta^{(\pm)}$ describe the coupling of the atomic dipole to the weak probe fields (see equation (4)). For the Bloch vector $\tilde{\sigma} = (\sigma^-, \sigma^+, \sigma^z)$, the zeroth-order elastic scattering amplitude $\langle \sigma^- \rangle^{(0)}$ is given by $(GL)^{1}$. Likewise, using (A.1), one finds all of the amplitudes of figure 4.

A.2. Inelastic blocks

The expressions for the four elementary inelastic building blocks of figure 5 can be found by solving the equations of motions for the two-time correlation vectors:

$$\tilde{g}_1(t_1, t_2) = \langle \tilde{\sigma} (t_1) \sigma^- (t_2) \rangle - \langle \tilde{\sigma} (t_1) \rangle \langle \sigma^- (t_2) \rangle, \quad t_1 > t_2,$$

$$\tilde{g}_2(t_1, t_2) = \langle \sigma^+ (t_1) \tilde{\sigma} (t_2) \rangle - \langle \sigma^+ (t_1) \rangle \langle \tilde{\sigma} (t_2) \rangle, \quad t_1 < t_2.$$

By virtue of the quantum regression theorem the vectors $\tilde{g}_1(t_1, t_2)$ of equation (A.3) satisfy equations of motion similar to the OBE (2), which can be solved by the Laplace transformation. The resulting expressions for the inelastic building blocks are given as a sum of two terms corresponding to equations (A.3a) and (A.3b), respectively:

$$P^{(0)}(\omega) = \left[ \tilde{g}_2^{(0)} (-\omega) \right]_1 + \left[ \tilde{g}_1^{(0)} (\omega) \right]_2,$$

$$P^{(-)}(\omega, v) = \left[ \tilde{g}_2^{(-)} (\omega, \omega - iv) \right]_1 + \left[ \tilde{g}_1^{(-)} (\omega, iv) \right]_2.$$
derivation and definitions differ from ours, we here establish the relation between his and our results. Upon the replacements

\[ \kappa \rightarrow 2 \gamma, \ \kappa' \rightarrow \gamma, \ \Delta \omega \rightarrow \Delta, \]

\[ \Delta v \rightarrow \omega, \ 2 |\xi_0| \rightarrow \Omega, \ \lambda' \xi'_0 \rightarrow v, \]

where the left- and right-hand sides of each substitution correspond to the results of [15] and of our work, respectively, we obtain the following identities:

\[ \delta \alpha_+ \equiv v(\sigma^-(\omega))^{(-)}, \quad (\delta \alpha_+)^* \equiv v^*(\sigma^+(\omega))^{(+)}, \]

\[ (\delta \alpha_+)^* \equiv 4v^2(\sigma^-(\omega))(\sigma^+(\omega))^{(2)}. \]  

We note that the right-hand side of each identity is equal, up to a prefactor, to the elementary building block from figure 4. For completeness, we also provide the expressions on which the elementary building blocks depend implicitly:

\[ \bar{n} - \bar{m} \equiv \langle \sigma^-(\omega) \rangle, \quad \eta \equiv 2v(\sigma^-(\omega))^{(-)}. \]

The physical interpretation of the function \( \delta \alpha_- \) is defined by the sign of \( \delta \alpha_- \) and, consequently, \( \delta \alpha_- \) and, consequently, \( \delta \alpha_- \) and, consequently, \( \delta \alpha_- \)

\[ \delta \alpha_0 \equiv 4v^2(\sigma^-(\omega))(\sigma^+(\omega))^{(2)}, \quad \delta \alpha_0 \equiv 4v^2(\sigma^+(\omega))^{(2)}. \]

As shown in [15], the rate \( \mathcal{W}^\nu \) of absorption/stimulated emission of the probe field is given by

\[ \mathcal{W}^\nu = -i \mathcal{E}_0^\nu \delta \alpha_+ + i \mathcal{E}_0^\nu (\delta \alpha_+)^*. \]

Hence, the identities (B.2) imply our interpretation of the building blocks in figures 4(c) and (d) as describing a stimulated emission of the probe field in the presence of the pump field (whether the field is actually emitted or absorbed is defined by the sign of \( \mathcal{W}^\nu \) [15]). Analogously, the rate \( \delta \mathcal{W} \) of absorption/stimulated emission of the pump field reads

\[ \delta \mathcal{W} = -i \mathcal{E}_0^\nu \delta \alpha_0 + i \mathcal{E}_0^\nu (\delta \alpha_0)^*. \]  

Therefore, due to the identities (B.4), we interpret the amplitudes \( \langle \sigma^\pm(\omega) \rangle^{(2)} \) (see figures 4(g) and (h)) as describing stimulated emission of the pump field in the presence of the probe field.

The physical interpretation of the function \( \delta \alpha_- \) was not considered by Mollow. But, by analogy with the above results, and using the fact that it represents a correction to the negative- (positive-)frequency scattering amplitude for a pumped atom probed by a positive- (negative-)frequency field, we deduce that \( \delta \alpha_- \) has a phase conjugation due to nonlinear mixing of the pump and probe fields.
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