Dynamic analysis of vibrating flip-flow screens equipped with support and shear rubber springs

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Abstract. Vibrating flip-flow screens provide an effective means of screening highly viscous or fine materials, and the dynamic characteristics of the main and the floating screen frames are largely responsible for a flip-flow screen’s screen performance and its processing capacity. An accurate dynamic model of the rubber shear springs used within the frame of the screen is critical for its dynamic analysis – to understand deficiencies and improve its performance. In this paper, the Sjöberg model is used to predict the frequency-and amplitude-dependent behaviour of the rubber shear springs. A friction model represents the amplitude dependency of the rubber shear springs. The fractional derivative model is used to describe its frequency dependency with its elasticity being represented by a linear spring. This model is further validated by cyclic tests of the rubber shear springs. Furthermore, dynamic response of the VFFS have been analysed using the Sjöberg model and the Kelvin-Voigt model, respectively. Experimental results indicate that dynamic response of VFFS can be better predict using the Sjöberg model than Kelvin-Voigt model in time region as well as in the frequency domain.

1. Introduction
Screening of materials is an on-going issue in processing minerals such as gold deposited rocks, iron or coal ore. During traditional screening equipment frequently encounters the problems of blockage of the screen’s grid with debris, which is associated with lower efficiency [1-2]. Vibrating flip-flow screens (VFFS) are composed of elastic screen panels which are stretched and relaxed, thereby creating changing tension motion to efficiently sort loose materials, which makes VFFS are widely used in screening processes of moist, viscous and fine materials where it is superior to conventional screening methods [3-4]. In order to
achieve the best screen performance, an optimized design and control is required, which fundamentally requires an in-depth understanding of the dynamics of the machinery [5].

Many researchers investigated and analysed the dynamics of VFFS using linear rubber shear springs even though at least the shear spring and the support spring are made of elastomer material [6]. However, as the key component of a VFFS the rubber shear spring exhibits frequency- and amplitude-dependent behaviours. These dependencies largely affect the vibration characteristics of VFFS and are responsible for a screen’s performance and its processing capacity. To date, many researchers have investigated the influence of factors such as frequency and amplitude of the rubber spring’s motion on rubber material properties. The frequency dependence is often foreseen, for example, an increase in frequency is a response of an increasing stiffness prior to viscous effects within the rubber material, as discussed by Knothe and Grassie [7]. The Kelvin-Voigt (KV) model - a linear spring in parallel with a viscous dashpot - is most widely used to study frequency-dependent behaviour, and this model are also extensively used for rubber shear spring involved in the dynamic analysis of VFFS [6]. However, owing to the viscous dashpot, the KV model overestimates damping in the higher frequency regimes [8-9]. Placing a spring in series with the viscous dashpot, a three- parameter Maxwell model can be obtained [10], which better approximates the dynamic stiffness at higher frequencies yet underestimates the damping. To reduce the required number of parameters and yet improve the frequency-dependent dynamic behaviour by predicting viscous forcing, a fractional derivatives model has been used by Sjöberg, Sedlaczek and Shi [11-13]. For an enhanced description of the vibration amplitude dependent behaviour of rubber properties, Berg presented a smooth friction model which uses smoother and more realistic friction curves than other models, such as the stick-slip component model [10]. Due to this smoothness and ease of use Berg’s friction model with two constant parameters is nowadays often applied; the two parameters are generally validated on measured curves by using large amplitude harmonic excitation at low frequencies [14].

The rubber shear spring model presented in this paper uses the Sjöberg model [15] composed of not only an elastic but also of an amplitude- and frequency-dependent model. The elastic model is described by a linear spring, the friction force uses the classic Berg’s friction model, and the viscoelastic model is described by a fractional derivative model with only two parameters. The support spring model we used in this paper is the KV model, which uses a linear spring in parallel with a viscous dashpot [6]. We compare the experimental results of dynamic characteristics with those of Sjöberg’s nonlinear model applied to the rubber shear spring for the first time. Then, the dynamic response of a VFFS analytical model has been calculated using the Sjöberg and the KV model, respectively, and experimental results of dynamic response of an industrial VFFS was used to compare the dynamic results derived from these two models in both the time and the frequency domain.

2. Model of rubber shear spring

Based on the specific experimental results, a phenomenological model of a rubber shear spring should be able to represent the properties of linear stiffness, hysteresis and characteristics of amplitude- and frequency- dependency, cf. Figure 1. The elastic model represents the static linear stiffness characteristics, the friction model accounts for the hysteresis and amplitude dependency, and the viscous model is responsible for the frequency-dependent dynamics. Hence, the relationship between the forces and the
displacements is based on the superposition of the three forces

\[ F = F_e + F_f + F_v \]  \hspace{1cm} (1)

with \( F_e, F_f \) and \( F_v \) representing the elastic, friction and viscous forces of the rubber shear spring, respectively.

2.1 Elastic model
The elastic model consists of a linear algebraic relationship between the instantaneous displacement \( x = x_0 \sin(\omega t) \) and the resulting elastic force \( F_e = K_e x \) based on measurements. Where \( \omega \) and \( t \) are the angular velocity and the time, respectively, both \( x_0 \) and \( K_e \), the excitation amplitude and stiffness are obtained through experiments.

2.2 Friction model
Experimental results show that increasing the amplitude give rise to a decreasing stiffness, which is indicative for non-linear behaviour; modelling of this phenomenon demands for a friction model. The amplitude effect can be modelled by adding a displacement friction force, \( F_f \). The model used here to present static friction is not based on the non-smooth stick-slip, Coulomb type model [11]. It rather uses a mathematical shape function based on Berg’s approach, which introduced smoothing characteristics and enable a good fit to measured curves [11]. The purely algebraic relationship between displacement \( x \) and corresponding friction force \( F_f \) can be described as

For \( x = x_s \):

\[ F_f = F_{fs} \]  \hspace{1cm} (2a)

For \( x > x_s \):

\[ F_f = F_{fs} + \frac{x-x_s}{a_2(1-\varepsilon)+(x-x_s)}(F_{f\max} - F_{fs}) \]  \hspace{1cm} (2b)

For \( x < x_s \):

\[ F_f = F_{fs} + \frac{x-x_s}{a_2(1+\varepsilon)+(x-x_s)}(F_{f\max} + F_{fs}) \]  \hspace{1cm} (2c)

Here, the reference displacement \( x_s \) and force \( F_{fs} \) are starting points for each loop and are updated during the motion, the maximum friction shear force \( F_{f\max} \) and the displacement \( a_2 \) (at half of \( F_{f\max} \)) are model parameters when given both the \( x_s \) and \( F_{fs} \) as zero. The instantaneous friction coefficient \( \varepsilon = F_{fs}/F_{f\max} \) ranges from -1 to 1.

2.3 Viscous model
There are several ways to model the viscous properties of rubber elements. One method of modelling the viscous properties is to introduce a linear viscous damper with damping
parameter $c$ and viscous force $cx$. However, this model can neither represent the nonlinear behavior of rubber springs, nor the frequency dependency of the energy loss mechanism. A better approximation of the real viscous force can be described using a Maxwell module which is represented by a linear viscous damper, which is placed in series with a linear spring. Due to the phase decay of the Maxwell module at higher frequencies, the fractional derivative model has been proved to be a powerful tool in describing the viscoelastic property of a rubber spring over a large frequency range. The function of the fractional derivative generalizing the operation of differentiation to non-integer orders, which defines the viscoelastic force as

$$F_v = b \frac{d^\alpha x}{dt^\alpha}.$$  \hfill (3)

Here non-integer $\alpha$ ($0 < \alpha < 1$) and $b$ ($b > 0$) are model constants, which represent the order of the time derivative and the constant coefficient of the viscoelastic force, respectively. For $\alpha = 0$ the model becomes that of a linear spring and for $\alpha = 1$ the model becomes that of a viscous damper. The Riemann-Liouville fractional derivative can be simplified as

$$\frac{d^\alpha x(t)}{dt^\alpha} = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{x(t)}{(t-\tau)^\alpha} d\tau$$  \hfill (4)

with the Gamma function being defined as

$$\Gamma(n) = \int_0^\infty t^{n-1} \exp(-t) dt.$$  \hfill (5)

Here, $n$ represents the truncation number, and $\exp$ is the exponential function.

For numerical approximation of the fractional derivative, a more convenient form can be given using the Grünwald-Letnikov definition, which is applied in the following by us using

$$F_{vf}(t_n) = bD^\alpha x_n \approx b \left( \frac{\Delta t}{\Gamma(1-\alpha)} \right)^\alpha \sum_{j=0}^{\eta-1} \frac{\Gamma(j+\alpha)}{\Gamma(j+1)} x_{n-j}$$  \hfill (6)

Here, $t_n = n\Delta t$, $x_n = x(t_n)$; $\Delta t$ being a constant time step.

3. Experimental cyclic tests of shear rubber springs

Two experimental parts for validation are conducted, which are referred to as a quasi-static test at a low frequency and a harmonic dynamic test using frequencies ranging from 1 to 6 Hz and an amplitude of 6 mm. The quasi-static test we obtained the following parameters: $K_e$ for the elastic model, $F_{f,max}$ and $\alpha_2$ for the friction model. The parameters $\alpha$ and $b$ for the spring-pot are obtained from the harmonic dynamic experiment. The quasi-static and harmonic dynamic tests were also used to verify the proposed model [13].

In this study, the rubber shear spring was tested on a dynamic testing machine (Instron, Type E10000, accuracy $\pm 0.5\%$, Massachusetts, USA) (Figure 2). The upper part of the rubber shear spring was fixed with a fixture, which was used to connect the actuator bearing system. The bottom was fixed with another fixture which was install on the load cell (Instron, type 2527, capacity 10 kN, accuracy $\pm 0.005\%$, Massachusetts, USA). The force and the displacement signals were recorded using the Instron testing software. A thermal imager was used to keep every test at a constant temperature.

The amplitude of this rubber shear spring has been assumed to be commonly less than 8 mm. Hence in the quasi-static tests, harmonic displacement excitations of 5 mm, 5.5 mm, 6 mm, 6.5 mm, 7 mm amplitude at 0.01 Hz were applied to the actuator, respectively. The low frequency aimed to eliminate the effect of the rubber damping; the different amplitudes are
used to investigate friction effect. In addition, dynamic tests were carried out, which included different frequencies between one and six Hz were also carried applying an amplitude of 6mm to the rubber shear spring.

![Image of experimental setup]

**Figure 2.** Dynamic stiffness and damping tests of the rubber shear spring

4. Dynamic analysis of VFFS equipped with the presented rubber shear spring model

4.1 Analytical model of VFFS equipped with the presented rubber shear spring model

![Diagram of analytical model]

**Figure 3.** (a) Schematic and the (b) mathematical model of a vibrating flip-flow screen
The investigated VFFS is shown schematically in Figure 3(a). The main screen frame is supported by support springs. The inserted floating screen frame is connected to the main screen frame by the rubber shear springs. Moreover, the VFFS is excited by an exciter mounted to the main frame. The rotation of the eccentric blocks in the exciter causes a relative movement between the main and the floating frame, which periodically slackens and stretches the screen panel.

To investigate the dynamics of the VFFS, a corresponding dynamic model is established using the presented rubber shear spring model and the linear support spring model, as shown in Fig. 3(b). Here, \( m \) is the eccentric block mass of vibration exciter; \( m_1 \) and \( m_2 \) are the mass of main and floating screen frames respectively, \( r \) is the eccentricity of exciter; \( \omega \) the angular velocity of \( m \); \( k_1 \) and \( c_1 \) – elastic stiffness and damping of the support springs, \( F_c \), \( F_f \) and \( F_v \) – elastic, friction and viscous force of a rubber shear spring. The \( x \)-axis is along the screen panel, the \( y \)-axis is perpendicular to the screen panel. The co-ordinate \( x_1 \) represents the displacement of the main screen frame’s mass \( m_1 \) with respect to its foundation, while \( x_2 \) stands for the relative displacement of the floating screen frame’s mass \( m_2 \) with respect to the mass \( m_1 \). In this paper, the dynamics of the VFFS is studied only considering \( x \) direction, because that the main and floating screen frames will vibrate synchronously in \( y \) direction when VFFS works, which cause little effect on the performance of the VFFS. The machine is excited by a harmonic forcing of the form \( F = F_0 \cos \omega t \) \( (F_0 = mr \omega^2) \). Therefore, the governing equations of the investigated system can be written as:

\[
\begin{align*}
(m_1 + m_2)\ddot{x}_1 + m_2\ddot{x}_2 + c_1\dot{x}_1 + k_1x_1 &= F_0 \cos \omega t \\
m_2\ddot{x}_1 + m_2\ddot{x}_2 + F_e + F_f + F_v &= 0
\end{align*}
\]

Where \( F_e = 24F_e \), \( F_f = 24F_f \), \( F_v = 24F_v \) as the investigated VFFS is equipped with twenty-four rubber shear springs, and \( F_e \), \( F_f \) and \( F_v \) have been discussed in section 2.

Time discretisation of a fractional derivative of a variable thus requires the history of the variable to be saved and used for evaluating future values of its fractional derivative. The time discretized form of the governing equations of the investigated system can be given in matrix form.

\[
M\dddot{X}_n + C\dot{X}_n + KX_n = F_n
\]

Where \( M, K \) and \( C \) are matrices of the generalized mass, stiffness and damping coefficients of the VFFS system, respectively; \( F_n \) and \( X_n \) are the force and displacement vector of the VFFS system, respectively.

The general Newmark algorithm can be used to solve the equations numerically. A suitable choice of stable control parameter can make the algorithm nearly explicit resulting in the following equations:

\[
\begin{align*}
X_n &= X_{n-1} + \Delta t\dot{X}_{n-1} + \frac{1}{2}\Delta t^2\ddot{X}_{n-1} \\
\dot{X}_n &= \dot{X}_{n-1} + \Delta t \left( (1 - \beta)\ddot{X}_{n-1} + \beta\dddot{X}_n \right)
\end{align*}
\]

The algorithm is conditionally stable for the \( \beta \geq 1/2 \) with the critical time step \( \Delta t_{\text{crit}} = 1/(\omega_{\text{max}}\sqrt{\beta}/2) \), where \( \omega_{\text{max}} \) is the highest undamped natural frequency of the system. Let \( \beta = 1/2 \) will eliminate algorithmic damping which occurs for \( \beta > 1/2 \). A suitable and
A straightforward way of numerically calculating the dynamic response of the system can be achieved as illustrated in the following flowchart (Figure 4).

**Figure 4.** Flowchart for calculating the dynamic response of the system

### 4.2 Experimental tests of dynamic response of VFFS

In this section, the dynamics of the investigated VFFS was tested using dynamic testing instrument, as illustrated in figure 5. There are two acceleration sensors installed on the main and floating screen frame to measure the actual acceleration of the two frames, respectively. The acceleration signals were collected by the data acquisition system and recorded by the testing software. Between consecutive measurements we made sure that every test was carried out at a constant temperature, and that only steady-state data was used to analyse the VFFS’s dynamic response.

**Figure 5.** Dynamic tests of the investigated VFFS
5. Results

5.1 Comparison between experiment and simulation of rubber shear spring’s dynamics

Using the method discussed in [13], the following model parameters were obtained: 
\( K_e = 186.45 \text{ kN/m}, \quad F_{fmax} = 68.3 \text{ N}, \quad a_2 = 2.64 \text{ mm}, \quad \alpha = 0.16, \quad b = 9.9 \text{ kN·s/m}. \) Then the simulated results of the rubber shear spring using Sjöberg model [15] were compared with experimental data in quasi-static conditions, as illustrated in Figure 6 and 7.

![Figure 6. Hysteresis loops compared between experiment and simulation with frequency of 0.01Hz for different amplitudes: (a) 5mm, (b) 6mm](image1)

![Figure 7. Dynamic behaviour of the rubber shear spring compared for different amplitude with 0.01Hz frequency: (a) stiffness, (b) damping ratio](image2)

It can be found in Figure 6 that the hysteresis loops of the simulation results match well the experimental results; only small deviations for large compression became visible. In addition, a significant level of amplitude-dependence is observed in the experiments which overall matches the Sjöberg model (Figure 6). Both experiment and simulation results indicate decreasing stiffness and damping with increasing excitation amplitude. The largest deviations of stiffness and damping between experiment and simulation were 0.74% and 5.61%, respectively. The Kelvin-Voight model is left out here, since it would simply result in constant values for both, stiffness and damping in describing the amplitude dependence [13]. Frequency-dependent results of experiment and simulation in dynamic conditions are shown in Figure 8 and 9. Figure 8 shows that the hysteresis loops of experiments are again matched well with simulation results. The Kelvin-Voight model would result in a very poor fit of the frequency dependence of both measured stiffness and damping [15], while, here the Sjöberg
model approximates reasonably well the frequency-dependent behaviour of the rubber shear spring (Figure 9).

![Figure 8](image_url)  
**Figure 8.** Hysteresis loops compared between experiment and simulation with amplitude of 6mm for different frequencies: (a) 1Hz; (b) 6Hz;

![Figure 9](image_url)  
**Figure 9.** Dynamic behaviour of the rubber shear spring compared for different frequency with 6mm amplitude: (a) stiffness, (b) damping ratio

5.2 Comparison between experiments and simulation of the dynamic response of VFFS

This section focuses on compare the effect of the two models (Sjöberg model and KV model) on the dynamic response of the investigated VFFS based on experimental results. Parameters for the KV model can be seen in Table 1, the simulation parameters for the VFFS are listed in Table 2, and the parameters of Sjöberg model for the rubber shear spring have been reported in the previous section. Initially we choose $\mathbf{X}(0)$ and $\dot{\mathbf{X}}(0)$ to be zeros. As the VFFS works near the second resonance, an angular velocity $\omega$ of 67 rad·s\(^{-1}\) is used to simulate the dynamic response of the displacement of the VFFS computed by two different models of rubber shear springs.

| Symbol | Definition                  | Unit  | Value   |
|--------|-----------------------------|-------|---------|
| $k_2$  | Elastic stiffness           | N/m   | 215,000 |
| $c_2$  | Damping coefficient         | N·s/m | 171.2   |

Table 1. Parameters for the Kelvin-Voigt model of the investigated rubber shear spring
Table 2. Parameters for the investigated VFFS

| Symbol | Definition                          | Unit | Value    |
|--------|------------------------------------|------|----------|
| $m_1$  | Main screen frame mass            | kg   | 4130     |
| $m_2$  | Floating screen frame mass         | kg   | 1309     |
| $k_1$  | Elastic stiffness                  | N/m  | 3,606,700|
| $c_1$  | Floating screen frame mass         | N·s/m| 36,614   |
| $m$    | Eccentric block mass              | kg   | 242.24   |
| $r$    | Eccentricity of the exciter       | mm   | 25.48    |

In detail, for the KV model, the Sjöberg model and experimental results the maximum displacement of $x_1$ are 0.980 mm, 1.29 mm and 2.10 mm, and the maximum displacement of $x_2$ are 7.49 mm, 8.77 mm and 10.55 mm, respectively (Figure 10). Therefore, it can be easily seen that the dynamic response of VFFS in the time domain can be better predicted using the Sjöberg model than the KV model.

Figure 10. Dynamic response of the system computed by two different models of rubber shear spring in the time region: (a) $x_1$ and (b) $x_2$

Then, the steady-state responses of the system in the frequency domain are also studied. Figure 11 shows that the simulation results of these two models match well with experimental...
results when the VFFS works away from the resonance. However, the simulation results of the Sjöberg model for rubber shear spring match better with the experimental results than the KV model in the resonance zone. The KV model is known to overestimate both stiffness and damping in the higher frequency regimes [8-9], which could also be the reason why the Kelvin-Voigt model underestimates the dynamic response of the system in the time and frequency domain.

**Conclusion**

In this paper, an enhanced analytical model of a vibrating flip-flow screen (VFFS) with excitation frequency-and excitation amplitude-dependent rubber shear springs has been suggested; machine elements which are conventionally assumed to respond linearly in VFFS. Applying Sjöberg approach to the rubber shear spring, the elastic sub-model represents the static linear stiffness characteristics, the friction model accounts for the hysteresis and amplitude dependency, and the viscous model is responsible for the frequency-dependent dynamics. This model as validated by experimental results is capable to describe the frequency- and amplitude-dependent more accurately than KV model. Furthermore, the dynamic response of the VFFS has been analysed using the Sjöberg model and KV model, respectively, and experiment of dynamic response of VFFS was conducted to evaluate the performance of the Sjöberg model and KV model. However, how much better the response is in terms of the dynamics is not clear. To evaluate that it would be required to conduct statistical tests and also use other measures, preferably nonlinear invariant estimates, which could uniquely describe the dynamics and the strength of nonlinearity [16-17].

The amplitude dependency of the rubber shear spring demonstrates that the stiffness and the damping decrease with increasing excitation amplitude. However, the results show that the stiffness and damping grow with an increase of excitation frequency.

Further, the classic KV model shows lower peak displacements of $x_1$ and $x_2$ in the time domain and performs poorly in approximating the dynamic response of VFFS in the frequency domain when the VFFS work near the resonance region. However, the Sjöberg model is able to provide a more precise evaluation for dynamic response of the investigated VFFS in the time and frequency domain, which was validated experimentally on a real-life VFFS. The support spring, which is the other rubber element in the VFFS is assumed to be mostly influential at the first resonance frequency while the machine is designed to work at its second resonance. However, to confirm the model’s response sensitivity to all parameters, a variance-based sensitivity analysis should be conducted.

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