Electrical Conductivity of Hot QCD Matter

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We study the electric conductivity of hot QCD matter at various temperatures $T$ within the off-shell Parton-Hadron-String Dynamics (PHSD) transport approach for interacting partonic, hadronic or mixed systems in a finite box with periodic boundary conditions. The response of the strongly interacting system in equilibrium to an external electric field defines the electric conductivity $\sigma_0$. We find a sizable temperature dependence of the ratio $\sigma_0/T$ well in line with calculations in a relaxation time approach for $T_c < T < 2.5 T_c$. The ratio drops in the hadronic phase with $T$, shows a minimum close to $T_c$ and becomes approximately constant ($\sim 0.3$) above $\sim 5 T_c$. Our findings imply that the QCD matter even at $T \approx T_c$ is a much better electric conductor than Cu or Ag (at room temperature).

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High energy heavy-ion reactions are studied experimentally and theoretically to obtain information about the properties of nuclear matter under the extreme conditions of high baryon density and/or temperature. Ultra-relativistic heavy-ion collisions at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) at CERN have produced a new state of matter, the strongly interacting quark-gluon plasma (sQGP), for a couple of fm/c in volumes up to a few thousand $fm^3$ in central reactions. The produced QGP shows features of a strongly-interacting fluid unlike a weakly-interacting parton gas \cite{1} as had been expected from perturbative QCD (pQCD). Large values of the observed azimuthal asymmetry of charged particles in momentum space \cite{2} could quantitatively be well described by ideal hydrodynamics up to transverse momenta of 1.5 GeV/c \cite{3}. Recent studies of 'QCD matter' in equilibrium – using lattice QCD calculations \cite{4,5} or partonic transport models in a finite box with periodic boundary conditions \cite{6,7} – have demonstrated that the ratio of the shear viscosity to entropy density $\eta/s$ should have a minimum close to the critical temperature $T_c$, similar to atomic and molecular systems \cite{8,9}. On the other hand, the ratio of the bulk viscosity to the entropy density $\zeta/s$ should have a maximum close to $T_c$ or might even diverge at $T_c$ \cite{10}. Indeed, the minimum of $\eta/s$ at $T_c \approx 160$ MeV is close to the lower bound of a perfect fluid with $\eta/s = 1/(4\pi)$ \cite{11} for infinitely coupled supersymmetric Yang-Mills gauge theory (based on the AdS/CFT duality conjecture). This suggests the 'hot QCD matter' to be the 'most perfect fluid' \cite{1,12,13}. On the empirical side, relativistic viscous hydrodynamic calculations (using the Israel-Stewart framework) also require a very small $\eta/s$ of 0.08 – 0.24 in order to reproduce the RHIC elliptic flow $v_2$ data \cite{14}; these phenomenological findings thus are in accord with the theoretical studies for $\eta/s$ in \cite{7,15,16}.

Whereas shear and bulk viscosities of hot QCD matter at finite temperature $T$ presently are roughly known, the electric conductivity $\sigma_0$ is another macroscopic quantity of interest \cite{17}. The basic question is: Is the 'hot QCD matter' a good electric conductor? At first glance one might expect the deconfined QCD medium to be highly conductive, since color charges – and associated electric charges of the fermions – might move rather freely in the colored plasma. However, due to the actual high interaction rates in the plasma – reflected in a low ratio $\eta/s$ – this expectation is not so obvious. First results from lattice calculation on the electromagnetic correlator provide results that vary by more than an order of magnitude \cite{18,22}. Furthermore, the conductivity dependence on the temperature $T$ (at $T > T_c$) is widely unknown, too. The electric conductivity $\sigma_0$ is also important for the creation of electromagnetic fields in ultra-relativistic nucleus-nucleus collisions from partonic degrees-of-freedom, since $\sigma_0$ specifies the imaginary part of the electromagnetic (retarded) propagator and leads to an exponential decay of the propagator in time $\sim \exp(-\sigma_0(t-t')/(\hbar c))$ \cite{23}. High values of $\sigma_0$ would thus lead to the screening of external electromagnetic fields in the bulk of the highly-conducting quark-gluon plasma similar to the Meissner effect in super-conductors as well as the "skin-effect" for the electric current. Accordingly, a sufficient knowledge of $\sigma_0(T)$ is mandatory to explore a possible generation of the Chiral-Magnetic-Effect (CME) in predominantly peripheral heavy-ion reactions \cite{24}.

In this work we extract the electric conductivity $\sigma_0(T)$ for 'infinite parton/hadron matter' employing the Parton-Hadron-String Dynamics (PHSD) transport approach \cite{25}, which is based on generalized transport equations derived from the off-shell Kadanoff-Baym equations \cite{26} for Green’s functions in phase-space representation (beyond the quasiparticle approximation). This approach describes the full evolution of a relativistic heavy-ion collision from the initial hard scatterings and

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string formation through the dynamical deconfinement phase transition to the strongly-interacting quark-gluon plasma (sQGP) as well as hadronization and the subsequent interactions in the expanding hadronic phase. In the hadronic sector PHSD is equivalent to the Hadron-String-Dynamics (HSD) transport approach \cite{30} – a covariant extension of the Boltzmann-Uehling-Uhlenbeck (BUU) approach \cite{31} – that has been used for the description of pA and AA collisions from lower SIS to RHIC energies in the past. On the other hand, the partonic dynamics in PHSD is based on the Dynamical Quasi-Particle Model (DQPM) \cite{29}, which describes QCD properties in terms of single-particle Green’s functions (in the sense of a two-particle irreducible (2 PI) approach) and reproduces lattice QCD results – including the partonic equation of state – in thermodynamic equilibrium. For further details on the PHSD off-shell transport approach and hadronization we refer the reader to \cite{6, 25, 30, 31}.

A note of caution has to be given, since due to an external field, such that the energy density (temperature) in time. Therefore we estimate the hadronic sector PHSD is equivalent to the Hadron-String-Dynamics (HSD) transport approach \cite{30} – a covariant extension of the Boltzmann-Uehling-Uhlenbeck (BUU) approach \cite{31} – that has been used for the description of pA and AA collisions from lower SIS to RHIC energies in the past. On the other hand, the partonic dynamics in PHSD is based on the Dynamical Quasi-Particle Model (DQPM) \cite{29}, which describes QCD properties in terms of single-particle Green’s functions (in the sense of a two-particle irreducible (2 PI) approach) and reproduces lattice QCD results – including the partonic equation of state – in thermodynamic equilibrium. For further details on the PHSD off-shell transport approach and hadronization we refer the reader to \cite{6, 25, 30, 31}.

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where \( n \) denotes the density of non-localized charges, \( \tau \) is the relaxation time of the charge carriers in the medium and \( m^*_e \) their effective mass. This expression can be directly computed for partonic degrees-of-freedom within the DQPM, which was used to match in PHSD the quasiparticles properties to lattice QCD results in equilibrium for the equation-of-state (EoS) as well as various correlators \([29]\). We note that the electromagnetic correlator from IQCD calculations \([18]\) appears to match rather well the back-to-back dilepton rate from PHSD at \( T = 1.45 T_c \) (cf. Fig. 2 in \([33]\)), which suggests that the results of our calculations for \( \sigma_0 \) should also be close to the IQCD extrapolations from \([18]\).

In the DQPM, the relaxation time for quarks/antiquarks is given by \( \tau = 1/\Gamma_q(T) \) \([32]\), where \( \Gamma_q(T) \) is the width of the quasiparticle spectral function (cf. \([28, 30]\)). Furthermore, the spectral distribution for the mass of the quasiparticle has a finite pole mass \( M_q(T) \) that is also fixed in the DQPM, as well as the density of \( (u, \bar{u}, d, \bar{d}, s, \bar{s}) \) quarks/antiquarks as a function of temperature (cf. \([28, 30]\)). Thus, we obtain for the dimensionless ratio \( \xi \) the expression \([32]\):

\[
\frac{\sigma_0(T)}{T} \approx \frac{2}{9} \frac{e^2 n_q(T)}{M_q(T)\Gamma_q(T)/T},
\]

where \( n_q(T) \) denotes the total density of quarks and antiquarks and the prefactor \( 2/9 \) reflects the flavor averaged fractional quark charge squared \( \langle \sum_f q_f^2 \rangle/3 \). The result for the ratio \( \xi \) is displayed in Fig. 3 (dash-dot line) and does not involve any new parameters. Apparently, the PHSD results in equilibrium and the relaxation-time estimates match well up to \( \sim 2T_c \), which demonstrates again that PHSD in equilibrium is a proper transport realization of the DQPM \([3]\).

Our results from the DQPM suggest that above \( T \sim 5T_c \) the dimensionless ratio \( \xi \) becomes approximately constant (\( \approx 0.3 \)). This comes about as follows: At high temperature \( T \) the parton density scales as \( \sim T^3 \), while \( M_q(T) \sim T \) and \( \Gamma_q(T) \sim T \). Accordingly the ratio \( \xi \) is approximately constant. Note, however, that energy densities corresponding to \( T > 5T_c \) are not reached in present experiments with heavy-ions at RHIC or LHC! On the other hand, \( \sigma_0/T \) rises with decreasing temperature below \( T_c \) (in the dominantly hadronic phase), because at lower temperatures the system merges to a moderately interacting system of pions, which, in view of Eq. (5) has a larger charge (squared) to mass ratio than in the partonic phase as well as a longer relaxation time.

In summary, we have evaluated the electric conductivity \( \sigma_0(T) \) of the quark-gluon plasma as well as the hadronic phase as a function of temperature \( T \) by employing the Parton-Hadron-String Dynamics (PHSD) off-shell transport model in a finite box for the simulation of dynamical partonic, hadronic or mixed systems in equilibrium. The PHSD approach in the partonic sector is based on the lattice QCD equation of state of \([33]\); accordingly, it describes the QGP entropy density \( s(T) \), the energy density \( \varepsilon(T) \) and the pressure \( p(T) \) from IQCD \([6, 25, 30]\) very well. Studies of the QCD matter within PHSD have previously given reasonable results also for the shear and bulk viscosities \( \eta \) and \( \xi \) versus \( T \) \([7]\).
We find in the present study that the dimensionless ratio $\sigma_0/T$ rises above $T_c$, approximately linearly with $T$ up to $T = 2.5 T_c$, but approaches a constant above $5 T_c$, as expected from pQCD. This finding is naturally explained within the relaxation-time approach using the DQPM spectral functions. Below $T_c$ the ratio $\sigma_0/T$ rises with decreasing temperature because the system merges to a moderately interacting gas of pions with a larger charge to mass ratio than in the partonic phase and a longer relaxation time.

The actual values for the electric conductivity $\sigma_0(T)$ show that the sQGP even at its minimum (at $T \approx T_c$) is a better electric conductor than Cu or Ag (at room temperature) by about a factor of 500 (using $2\pi e^2 = 2.58 \times 10^4 \Omega^{-1} m^{-1}$). Furthermore, the damping of the electromagnetic propagator in the partonic medium $\tau_\epsilon(T) = \langle hc/\sigma_0(T) \rangle$ is moderate for temperatures $T < 2 T_c$, yet becomes shorter than 2 fm/$c$ for $T > 2.8 T_c$. This suggests the potential importance of the 'skin-effect' in the response of the partonic medium as created in the mid-rapidity region of heavy-ion collisions at the LHC to the electromagnetic fields generated by the spectators.

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$\Omega^{-1} m^{-1} = 7.39 \cdot 10^{-5} \text{eV} \frac{e^2}{h c/(4\pi\alpha)}$. 