Influence of vector interactions on the hadron-quark/gluon phase transition

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The hadron-quark/gluon phase transition is studied in the two-phase model. As a further study of our previous work, both the isoscalar and isovector vector interactions are included in the Polyakov loop modified Nambu–Jona-Lasinio model (PNJL) for the quark phase. The relevance of the exchange (Fock) terms is stressed and suitably accounted for. The calculation shows that the isovector vector interaction delays the phase transition to higher densities and the range of the mixed phase correspondingly shrinks. Meanwhile the asymmetry parameter of quark matter in the mixed phase decreases with the strengthening of this interaction channel. This leads to some possible observation signals being weakened, although still present. We show that these can be rather general effects of a repulsion in the quark phase due to the symmetry energy. This is also confirmed by a simpler calculation with the MIT–Bag model. However, the asymmetry parameter of quark matter is slightly enhanced with the inclusion of the isoscalar vector interaction, but the phase transition will be moved to higher densities. The largest uncertainty on the phase transition lies in the undetermined coupling constants of the vector interactions. In this respect new data on the mixed phase obtained from Heavy Ion Collisions at Intermediate Energies appear very important.

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I. INTRODUCTION

The phase transition from nuclear matter to quark-gluon matter is one of the most concerned topics in modern physics related to heavy-ion collision experiments and compact stars. As a principle tool, lattice QCD provides us a framework to investigate non-perturbative phenomena, such as confinement and quark-gluon plasma formation at finite temperature and vanishing (small) chemical potential [1–7]. However, lattice calculations suffer the sign problem at large chemical potential. To evade this problem several approximate methods have been proposed [8–12], but the validity of the result at \( \mu_q/T > 1 \) still should be taken with care [13]. On the other hand, to give a complete description of QCD phase diagram some phenomenological effective models [14–24] have been also developed. Among these models, the Nambu–Jona-Lasinio (NJL) type models [19–24], especially those coupled with Polyakov loop (PNJL) [28–35] are predominant, offering a simple illustration of chiral symmetry breaking and restoration, as well as (de)confinement effect.

The lattice QCD and (P)NJL type models are based on the degrees of freedom of quarks and gluons. Recently, the two-phase model with both hadron and quark degrees of freedom, widely used in the description of the phase transition in neutron star matter under the weak equilibrium (e.g., [36–45]), has also been taken to study the phase transition related to heavy-ion collisions [46–54], particularly the phase transition in asymmetric matter. The latter is possible to be probed in the planned facilities, such as FAIR at GSI-Darmstadt and NICA at JINR-Dubna [47, 49–54]. In these studies, only the scalar interacting channel was considered for quark matter, and this channel interaction is responsible for the dynamical masses of quarks. The isoscalar-vector channel interaction was also included to study the properties of quark matter [30, 55] or the hadron-quark phase transition in neutron star matter [56, 57].

However, up to date, the vector interacting channels (including both the isoscalar vector and isovector vector interaction) have not been considered in describing quark matter, in the context of heavy-ion collisions, in the two-phase model. The inclusion of these vector interactions will modify the quark pressure and chemical potentials. In particular, the isovector-vector interacting channel contributes to \( u, d \) quark flavors differently. Correspondingly, the onset densities of the quark phase are possibly modified and some observation signals of hadron-quark phase transition in asymmetric matter may be influenced. Therefore, it is important to study the effect of the vector channel interactions on the hadron-quark phase transition, and related information deduced from new data in heavy ion collisions at intermediate energies.

The paper is organized as follows. In Section II, we describe briefly the modified effective quark model and give the relevant formulas with the newly included isovec-
tor and isoscalar vector interactions. In Section III, we present the numerical results and discuss the influence of vector channel interactions on the hadron-quark phase transition of dense asymmetric matter. For the isovector part we include also some results with an Isospin-MIT-Bag model, just to stress the physics behind the symmetry terms in the quark sector, not dependent on the models. Finally, a summary is given in Section IV.

II. THE MODEL

In the two-phase model hadron matter is described by the nonlinear Walecka model, and quark matter is described by the PNJL model with the newly added vector interactions. In the mixed phase, the pure hadronic phase and quark phase are connected to each other through the Gibbs conditions with the thermal, chemical and mechanical equilibria, based on baryon number and isospin conservation in the strong interacting process.

The Relativistic Mean Field (RMF) approach will be described to take the properties of hadronic matter. This model can provide an excellent description of nuclear matter and finite nuclei. The exchanged mesons in this model include the isoscalar-scalar meson ($\sigma$), isovector-vector meson ($\omega$), isovector-vector meson ($\rho$) and isovector-scalar meson ($\delta$). This is called the Non Linear-\(\rho, \delta (N\!L\!\rho\!\delta)\) effective interaction. For details, see Refs. [41, 47–50] and references therein.

For the quark phase, we adopt an extension of the two-flavor NJL model to include the Polyakov loop contribution[28, 29]. The Lagrangian is given by

$$L_q = \bar{q}(i\gamma^\mu D_\mu - \hat{m}_0)q + G_\sigma \left[ (\bar{q}q)^2 + (\bar{q}\gamma_5\tau q)^2 \right] + G_\delta \left[ (\bar{q}\tau q)^2 + (\bar{q}\gamma_5\tau q)^2 \right] - G_\omega \left[ (\bar{q}\gamma^\mu q)^2 + (\bar{q}\gamma_5\gamma^\mu q)^2 \right] - G_\rho \left[ (\bar{q}\gamma^\mu q)^2 + (\bar{q}\gamma_5\gamma^\mu q)^2 \right] - U(\Phi[A], \hat{\Phi}[A], T)$$

where $q$ denotes the quark fields with two flavors, $u$ and $d$, and three colors; $\hat{m}_0 = \text{diag}(m_u, m_d)$ in flavor space. The covariant derivative is defined by $D_\mu = \partial_\mu - iA_\mu$ with the background gluon field $A_\mu = \delta_{\mu,0} A_0$ supposed constant and uniform. The temperature-dependent Polyakov effective potential, $U(\Phi[A], \hat{\Phi}[A], T)$, is a function of the Polyakov loop $\Phi[A]$ and its hermitian conjugate $\hat{\Phi}[A]$. In some analogy with the nonlinear Walecka model, in this study we also try to include in the NJL term the isoscalar–vector and isovector–vector interaction channels as given in Ref. [57].

Here some considerations are in order. In our previous work, Refs. [53, 54], where only the scalar interaction channels were considered, calculations were performed within a relativistic mean field approximation which essentially corresponds to the Hartree approximation. However, the inclusion of the Fock (exchange) terms would be desirable. In fact, these terms originate from the correlations due to the Fermi–Dirac statistics, therefore they are related to a genuine quantum effect, which in general cannot be neglected when studying a many-body system. Including the Fock terms, the whole variety of processes in quark dynamics arising from the fermionic intrinsic degrees of freedom (spin, flavor and color), are automatically accounted for. Indeed, even starting from an effective Lagrangian containing only scalar channels, the exchange terms naturally yield contributions in the vector channels, as we will show in the following.

A quantity of interest in the study of quark dynamics is the statistical average of the canonical energy-momentum density tensor, $<T_{\mu\nu}(x):>$, from which thermodynamical quantities, such as the pressure, can be derived. For the considered Lagrangian, the interaction part of this quantity, $<T_{\mu\nu}(x):>$, depends on the statistical average of the product of four quark fields, that in the Hartree-Fock (HF) approximation, can be written as:

$$<\bar{q}\gamma_\alpha(x)q_\alpha(x)\bar{q}\gamma_\beta(x)q_\beta(x):>$$

$$= <\bar{q}\gamma_\alpha(x)q_\alpha(x):> <\bar{q}\gamma_\beta(x)q_\beta(x):>$$

$$- <\bar{q}\gamma_\alpha(x)q_\beta(x):> <\bar{q}\gamma_\beta(x)q_\alpha(x):>, \quad (2)$$

where the brackets denote statistical averaging and the colons denote normal ordering. It is useful to define the matrix $[\hat{F}(x)]_{\alpha\beta}$:

$$[\hat{F}(x)]_{\alpha\beta} = <\bar{q}\gamma_\alpha(x)q_\alpha(x):>$$

where $\alpha$ and $\beta$ are triple indices for spin, isospin (flavor) and color.

As far as one is concerned with the equilibrium properties of isotropic and non–colored quark matter it is sufficient to consider only the scalar and vector channels (isoscalar and isovector). Then the matrix $\hat{F}(x, p)$ can be decomposed as:

$$\hat{F}(x) = F(x) + \gamma_\mu F^\mu(x)$$

$$+ \tau \cdot \hat{B}(x) + \gamma_\mu \tau \cdot \hat{B}^\mu(x) \quad (3)$$

It should be noticed that this matrix is related to the various densities characterizing the system. Indeed the scalar and current isoscalar densities are given by:

$$\rho_S(x) = <\bar{q}\gamma_\alpha(x)q_\alpha(x):> TrF(x) = 8 N_c F(x),$$

$$j^{\mu_\alpha}(x) = <\bar{q}\gamma^{\mu\alpha}(x)q_\alpha(x):> Tr\gamma^{\mu\alpha}F(x) = 8 N_c F^{\mu\alpha}(x),$$

while the isovector counterparts are given by:

$$\rho_3(x) = <\bar{q}\tau_\alpha x_\beta q_\alpha(x):> Tr\tau_\alpha x_\beta F(x) = 8 N_c B_3(x),$$

$$j^{\mu_\alpha}_3(x) = <\bar{q}\tau_\alpha x_\beta q_\alpha(x):> Tr\gamma^{\mu\alpha}_3 x_\beta F(x) = 8 N_c B^{\mu}_3(x),$$

where the traces are taken over spin, flavor and color indices. Then the interaction part of the energy–momentum density tensor, in the HF approximation, reads:
\[ \langle T^{(j)}_{\mu\nu} (x) \rangle = T^{(j)}_{\mu\nu} (x)_{\text{Hartree}} - \left[ 2G_\sigma T r(\tilde{F}(x)\tilde{F}(x) + (i\gamma_5)\tilde{\sigma}\tilde{F}(x) \cdot (i\gamma_5)\tilde{\sigma}\tilde{F}(x)) \right. \\
- 2G_\omega T r(\gamma_\lambda \tilde{F}(x)\gamma^\lambda \tilde{F}(x) + \gamma_5 \gamma_\lambda \tilde{F}(x)\gamma^\lambda \tilde{F}(x)) + 2G_\delta T r(\tilde{\tau}\tilde{F}(x) \cdot \tilde{\tau}\tilde{F}(x) + (i\gamma_5)\tilde{F}(x)(i\gamma_5)\tilde{F}(x)) \\
- 2G_\rho T r(\tilde{\tau}\gamma_\lambda \tilde{F}(x) \cdot \tilde{\tau}\gamma^\lambda \tilde{F}(x) + \tilde{\tau}\gamma_5 \gamma_\lambda \tilde{F}(x) \cdot \tilde{\tau}\gamma_5 \gamma^\lambda \tilde{F}(x)) \right] g_{\mu\nu}. \tag{4} \]

The exchange terms that appear in the energy–momentum tensor can be evaluated exploiting the decomposition given in Eq. (3). After some algebra one realizes that the effect of the Fock system is equivalent to redefine the coupling constants, as written below:

\[
\tilde{G}_\sigma = G_\sigma + (G_\sigma - G_\delta)/12 \\
\tilde{G}_\rho = G_\rho - (G_\rho - G_\delta)/12 \\
\tilde{G}_\omega = G_\omega(1 + 1/6) + (G_\sigma + G_\delta)/6 + G_\rho/2 \\
\tilde{G}_\rho = G_\rho(1 - 1/6) + G_\omega/6. \tag{5} 
\]

Then, with the effective coupling constants given above, calculations can be performed as in the Hartree approximation.

It should be remarked that all the relevant interaction channels in general can occur in the HF approximation, even if some channel is absent in the original Lagrangian. For instance, if the vector channels are not present, i.e., we take \( \tilde{G}_\omega = 0 \) and \( \tilde{G}_\rho = 0 \) as in our previous work, contributions to the \( \omega \) channel are naturally arising from the exchange terms associated with the scalar channels. On the other hand, the vector isovector \( \rho \) channel gets contributions (both direct and exchange) only from vector channels.

Since the critical end–point of the first order chiral transition appreciably depends on the strength of the vector channel interaction [58–60], the exchange contribution \( (G_\sigma + G_\delta)/6 \) to the effective value of \( \tilde{G}_\omega \) could represent a reference value for the vector channel interaction [61].

In the following we will adopt the choice \( \tilde{G}_\delta = 0 \) and the notation \( G \equiv \tilde{G}_\sigma = (G_\sigma + G_\delta) \). The coupling constants of the vector interactions, \( \tilde{G}_\omega \) and \( \tilde{G}_\rho \), will be taken as parameters, and different values will be used to investigate their influence on the phase transition. For convenience we define \( r_\omega = \tilde{G}_\omega/G \), \( r_\rho = \tilde{G}_\rho/G \). For the temperature dependent effective potential \( U(\Phi, \bar{\Phi}, T) \) we use the parametrization given in Ref. [64]

\[
\frac{U(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{a(T)}{2} \Phi \bar{\Phi} + b(T) \ln[1 - 6\Phi \bar{\Phi}] + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2, \tag{6} 
\]

where \( a(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2 \), \( b(T) = b_3 \left( \frac{T_0}{T} \right)^3 \). The parameters \( a_i \) and \( b_i \) are fitted to the lattice QCD results in pure gauge theory at finite temperature. In the equation above \( T_0 \) represents the temperature where the Polyakov potential gives a deconfinement phase transition in a pure gauge theory. The original value of \( T_0 \) fitted to pure gauge lattice QCD data is 270\,MeV [65]. When fermion fields are included, the temperature \( T_0 \) is usually rescaled to obtain a consistent result with the full lattice data, which give the value \( T^c = 173 \pm 8\,\text{MeV} \) for deconfinement transition temperature [1, 2, 4]. In this paper the value 210\,MeV for \( T_0 \) is adopted.

The PNJL model is not renormalizable, so a cut-off \( \Lambda \) is introduced to get finite results for three–momentum space integrations. For the model parameters we take the values \( \Lambda = 651\,\text{MeV} \), \( G = 5.04\,\text{GeV}^{-2} \), \( m_u, d = 5.5\,\text{MeV} \), determined by fitting the chiral condensate, \( f_\pi \) and \( M_\pi \) to their experimental values [29]. The coefficients in the Polyakov effective potential are listed in Table I.

| \( a_0 \) | \( a_1 \) | \( a_2 \) | \( a_3 \) |
|---|---|---|---|
| 3.51 | -2.47 | 15.2 | -1.75 |

The thermodynamical–potential density of quark matter in the mean field approximation reads

\[
\Omega = U(\Phi, \bar{\Phi}, T) + G(\phi_u + \phi_d)^2 - \tilde{G}_\omega(\rho_u + \rho_d)^2 - \tilde{G}_\rho(\rho_u - \rho_d)^2 - 2 \int_\Lambda \frac{d^3p}{(2\pi)^3} 3(E_u + E_d) \\
- 2T \sum_{u,d} \int \frac{d^3p}{(2\pi)^3} \left[ \ln[1 + 3\Phi e^{-(E_i - \mu_i^*)/T} + 3\bar{\Phi} e^{-2(E_i - \mu_i^*)/T} + e^{-3(E_i - \mu_i^*)/T}] \right].
\]
The baryon and isospin densities and the corresponding properties of quark matter can be obtained from $\Omega$. Partition functions in multicomponent systems can be found in Ref. [66] without the vector contributions.

The dynamical quark masses and quark condensates are coupled with the following equations

\begin{align}
\mu_u^i = \mu_u - 2\tilde{G}_\omega (\rho_u + \rho_d) - 2\tilde{G}_\rho (\rho_u - \rho_d) \tag{9} \\
\mu_d^i = \mu_d - 2\tilde{G}_\omega (\rho_u + \rho_d) + 2\tilde{G}_\rho (\rho_u - \rho_d) \tag{10}
\end{align}

The values of $\phi_u, \phi_d, \Phi$ and $\bar{\Phi}$ are determined by minimizing the thermodynamical potential

\begin{equation}
\frac{\partial \Omega}{\partial \phi_u} = \frac{\partial \Omega}{\partial \phi_d} = \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \bar{\Phi}} = 0. \tag{15}
\end{equation}

All the thermodynamic quantities relevant to the bulk properties of quark matter can be obtained from $\Omega$. Particularly, we note that the pressure and energy density should be zero in the vacuum.

The number density of quarks of flavor $i$

\begin{equation}
\rho_i = 2 \times 3 \int \frac{d^3k}{(2\pi)^3} (n_i(k) - \bar{n}_i(k)) \tag{16}
\end{equation}

can be derived by means of the relation $\rho_i = -\partial \Omega / \partial \mu_i$.

The baryon and isospin densities in quark phase are defined by

\begin{align}
\rho_B^Q &= \frac{1}{3}(\rho_u + \rho_d), & \rho_3^Q &= \rho_u - \rho_d, \tag{17} \\
\mu_B^Q &= \frac{3}{2}(\mu_u + \mu_d), & \mu_3^Q &= \frac{1}{2}(\mu_u - \mu_d), \tag{18}
\end{align}

while the asymmetry parameter of quark matter is defined by

\begin{equation}
\alpha^Q \equiv -\frac{\rho_d^Q}{\rho_B^Q} = 3\frac{\rho_d - \rho_u}{\rho_u + \rho_d}. \tag{19}
\end{equation}

Analogous definitions hold for hadronic matter:

\begin{align}
\rho_H^B &= \rho_p + \rho_n, & \rho_3^H &= \rho_p - \rho_n, \tag{20} \\
\mu_B^H &= \frac{1}{2}(\mu_p + \mu_n), & \mu_3^H &= \frac{1}{2}(\mu_p - \mu_n), \tag{21} \\
\alpha^H &= \frac{\rho_p - \rho_n}{\rho_p + \rho_n}, \tag{22}
\end{align}

where $\rho_p$ and $\rho_n$ are the proton and the neutron densities, respectively. When a mixed phase of quarks and hadrons is considered, the Gibbs’ conditions (thermal, chemical and mechanical equilibrium)

\begin{align}
\mu_B^H (\rho_B, \rho_3, T) &= \mu_B^Q (\rho_B, \rho_3, T) \\
\mu_3^H (\rho_B, \rho_3, T) &= \mu_3^Q (\rho_B, \rho_3, T) \\
P^H (\rho_B, \rho_3, T) &= P^Q (\rho_B, \rho_3, T), \tag{23}
\end{align}

should be fulfilled (a general discussion of phase transitions in multicomponent systems can be found in Ref. [36]). In Eq. (23), $\rho_B = (1 - \chi)\rho_B^H + \chi\rho_B^Q$ is the total baryon density and $\rho_3 = (1 - \chi)\rho_3^H + \chi\rho_3^Q$ is the total isospin density, where $\chi$ is the quark fraction. In heavy-ion collisions, for a given initial charge asymmetry the global asymmetry parameter $\alpha$ of the mixed phase

\begin{equation}
\alpha \equiv -\frac{\rho_d}{\rho_B} = \frac{(1 - \chi)\rho_3^H + \chi\rho_3^Q}{(1 - \chi)\rho_3^H + \chi\rho_3^Q} \tag{24}
\end{equation}
should be constant according to the charge conservation, but the asymmetry parameters $a^H, a^Q$ in the separate phases can vary with $\chi$. For details, one can refer to Refs. [49, 53, 54].

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we display the numerical results and discuss the influence of isovector and isoscalar vector interactions on the phase diagram of the hadron-quark phase transition.

In actual calculations a value of 0.2 is chosen for the asymmetry parameter $\alpha$. We notice that in heavy–ion collision experiments the largest value of $\alpha$, $\alpha = 0.227$, is possibly reached in $^{238}$U + $^{238}$U collisions.

A. The role of the isovector vector interaction

Firstly, we only focus on the influence of the isovector vector interaction on the phase transition in the two-phase model, then we set $r_\omega = 0$, and perform calculations for various values of $r_\rho$, $r_\rho = 0.0, 0.25, 0.5, 1.0$.

![Diagram of the hadron–quark phase transition in the $T - \rho_B$ plane](image)

**FIG. 1:** (Color online) Diagram of the hadron–quark phase transition in the $T - \rho_B$ plane for symmetric ($\alpha = 0$) and asymmetric matter ($\alpha = 0.2$) with $r_\rho = 0.0, 0.25, 0.5$, and 1.0. The baryon density $\rho_B$ is expressed in units of the density of ordinary nuclear matter $\rho_0$. The lines in the left side corresponding to $\chi = 0$ represent the onset of the mixed phase, and those in the right side corresponding to $\chi = 1$ denote the beginning of the pure quark phase. The dot indicates the critical end point.

We display the phase diagram in $T - \rho_B$ plane in Fig. 1. In agreement with the results of Refs. [53, 54], this figure shows that the onset density of the mixed phase in asymmetric matter is smaller than that of symmetric matter. This means that in heavy–ion collision experiments the phase transition could be relatively easier to be reached for asymmetric matter with respect to the symmetric case. However, the onset density appreciably depends on the strength of the interaction in the isovector–vector channel. It is shifted to higher densities with increasing the coupling parameter $G_\rho$. Meanwhile the density range of the mixed phase shrinks. Indeed, similarly to what is observed in the hadron sector, the isovector vector channel yields a positive (repulsive) contribution to the pressure of the quark phase, connected to the corresponding symmetry energy.

![Diagram of the phase transition](image)

**FIG. 2:** (Color online) The same of Fig. 1 but for the $T - \mu_B$ plane.

Similar features are observed for the phase diagram in the $T - \mu_B$ plane as shown in Fig. 2. However, we notice that for symmetric matter, only one phase–transition line exists. This is due to the fact that in the symmetric case the curve $T - \mu_B$ does not depend on the quark fraction $\chi$, see also Refs. [53, 54]. Both figures 1 and 2 show that the isovector–vector channel of the quark interaction plays a more important role on the phase transition with increasing density and lowering temperature. In fact, at higher temperatures and/or lower densities the role of interactions in general becomes weaker with respect to the kinetic contributions. Finally, we still observe the occurrence of a critical end point (CEP) of the first order phase transition, analogously to what happens when only scalar channels are considered [53, 54].

In Fig. 3 the asymmetry parameters of hadronic and quark matter in the mixed phase are displayed as a function of the quark fraction $\chi$, for the temperature $T = 100$ MeV. We can observe a clear Isospin Distillation effect [47, 49], i.e. the asymmetry of quark matter is much larger than 0.2 at the beginning of the phase transition and decreases with increasing the quark fraction. Whereas the asymmetry of the hadronic matter
keeps below 0.2 and is a slowly decreasing function of $\chi$. These features of the local asymmetry may lead to some observable effects in the hadronization during the expansion phase of heavy ion collisions, such as an inversion in the trend of emission of neutron rich clusters, an enhancement of $\pi^-/\pi^+$, $K^0/K^+$ yield ratios in high-density regions, as well as an enhancement of the production of isospin-rich resonances and subsequent decays, for more details see Refs. [49, 53, 54]. These signals are possible to be probed in the newly planned facilities, such as FAIR at GSI-Darmstadt and NICA at JINR-Dubna.

FIG. 3: (Color online) Asymmetry of hadronic and quark matter in the mixed phase as a function of the quark concentration for various values of the coupling constant in the isovector–vector interaction channel.

However we also see that the strengthening of the coupling constant $G_\rho$ reduces the distillation effect and consequently the asymmetry parameter $\alpha^Q$, which may weaken the observational signals of the phase transition, although always present. The uncertainty is that the relevant $\rho$-coupling constant cannot be unambiguously determined from lattice–QCD calculations. So the observation or not of the related signals in experiments can provide some hints on this aspect.

In any case we note that both isospin effects, earlier density transition and isospin distillation are still there even with large values of $r_\rho$. They appear as general, quite robust effects of asymmetric matter, not much affected by the introduction of a symmetry interaction term in the quark sector. In order to confirm all that, in the next paragraph we present other results obtained in a much simpler Iso-MIT-Bag model.

**B. Few results with the MIT-Bag model**

As already remarked the isospin effects on the mixed phase (boundaries and asymmetries in the two phases) are naturally related to the presence of a symmetry repulsive term in the quark sector. In order to confirm that this is a general result, not depending on the different quark models, we present here few similar calculations of the hadron-quark transition, in a two-EoS approach, using the MIT-Bag model for the quark matter [47–50].

Now the results are very sensitive to the choice of the Bag-constant $B$, in particular at low temperatures and high baryon densities. In fact for low $B$-values we can even get a disappearing of the transition since the hadron pressure cannot match anymore the quark pressure, as discussed in detail in Ref. [50].

Here we choose a Bag-constant $B = (160 MeV)^4$, which gives for symmetric matter a high-density mixed phase structure very close to the one of Fig. 1, obtained with the $PNJL$ model. Of course the same $NL\rho\delta$ Relativistic Mean Field interaction is used for the hadron sector.

A quark isovector-vector ($\rho$-like) term is introduced by a naive application of a constituent quark model of the nucleons, i.e., just reducing of a factor 3 the Nucleon-$\rho$ coupling constant of the hadron part. As a consequence we get straightforward corrections to the quark pressure and chemical potentials with respect to the simple relativistic Fermi gas values of the MIT-Bag model ($P^Q(F), \mu_u(F), \mu_d(F)$):

$$P^Q = P^Q(F) + \frac{g_{p,q}^2}{2m^2_p}(\mu_u - \mu_d)^2,$$

$$\mu_u = \mu_u(F) + \frac{g_{p,q}^2}{m^2_p}(\mu_u - \mu_d),$$

$$\mu_d = \mu_d(F) - \frac{g_{p,q}^2}{m^2_p}(\mu_u - \mu_d),$$

with $g_{p,q} = g_{p,N}/3$ and $m_p$ is the $\rho$-meson mass. The coupling choice is fixed by the $NL\rho\delta$ parametrization of the hadron sector,

$$f_{\rho,N} = \frac{g_{p,N}^2}{m^2_p} = 3.15 fm^2,$$

see the detailed Appendix A of Ref. [50]. We note that such simple insertion of isovector-vector terms in quark phase (Iso-MIT-Bag model, in the following named $ISOE$ results) can be particularly justified at high baryon densities and chemical potentials, where we expect a more relevant role of the hadronic degrees of freedom. In any case here our aim is to show general quark symmetry energy effects on the hadron-quark transition at low temperatures in isospin asymmetric matter, that confirm the results of the previous section with the Hadron-PNJL approach.

The Fig. 4 is the $T - \rho_B$ phase diagram for symmetric and $\alpha = 0.2$ isospin asymmetric matter, corresponding to the previous Fig. 1 obtained with the PNJL model in the quark sector, with various weights of the $\rho$ coupling. We clearly see that at temperatures below 40 MeV and at high baryon densities the curves are very similar, in
particular in the choice in Fig. 1 of a \( r_\rho \) ratio equal to 0.5, close to the evaluation around 0.7 we use in the MIT-Bag model following the constituent quark picture.

The same comment is valid for the behavior of the asymmetry parameters in the mixed phase, shown in the ISOE calculation in the Fig. 5, to compare to the previous Fig. 3.

In general we can say that all the isospin influence on the hadron-quark transition is still present when we introduce vector-isovector terms in the quark phase, even with relatively large weights. Of course the interaction symmetry repulsion in the quark sector will reduce both effects, earlier transition densities and isospin distillation in the mixed phase, but the possibility of related observations appears still there.

C. The role of the isoscalar vector interaction

We discuss now the results obtained when the isoscalar–vector interaction channel in the quark sector is turned on. We choose the value of 0.2 for the ratio \( r_\omega = G_\omega / G \). This value is close to the contribution to this channel from the exchange terms of the scalar channels, see Eqs. (5).

In Figs. 6 and 7 are displayed the diagrams of the hadron–quark phase transition in the \( T - \rho_B \) and \( T - \mu_B \) planes respectively. Compared to Figs. 1 and 2 the phase–transition curves are significantly moved toward higher values of density/chemical potential. This can be explained in terms of the repulsive contribution of the isoscalar–vector channel to the quark energy and, as a consequence, to the chemical potential (see Eqs. 9 and 10). More specifically, the relevant quantity in the kinetic contribution to the thermodinamical potential, besides the temperature, is the effective chemical potential, \( \mu^*_L \). This quantity is appreciably smaller than the full chemical potential. Then, higher values of the latter quantity are necessary to fulfil the Gibb’s conditions. Moreover, we observe that for low values of temperature the quark chemical potential attains values of the same order of magnitude as the cut–off \( \Lambda \). However, this does not give rise to inconsistency, since the momentum scale...
is set by the Fermi momentum, which is determined by the effective chemical potential. Also, the CEP moves to higher values of the density/chemical potential although slightly, whereas the corresponding critical temperature almost keeps the same value. Moreover, in asymmetric matter the region of coexistence of the two phases is more extended when the isoscalar-vector interaction channel is included in the quark sector.

Finally, comparing Fig. 8 with Fig. 3 one can observe that the inclusion of the isoscalar–vector channel leads to a small increase of the asymmetry parameter of the quark matter in the mixed phase. The Isospin Distillation effect may be strengthened by this interaction channel, but the onset of the mixed phase is shifted toward higher densities.

IV. SUMMARY

We have studied the hadron-quark gluon phase transition in the two-phase model with the newly added isovector and isoscalar vector interactions for quark matter. We stress the presence of vector terms in the quark Equation of State just due to exchange contributions from scalar fields in the effective PNJL Lagrangian.

The consideration of the isovector interaction splits the effective quark chemical potential of u and d quarks. The calculations show that the phase transition densities are delayed to higher values and the ranges of the mixed phases shrink with the increase of the coupling constant $G_\rho$. Meanwhile the asymmetry parameter $\alpha^Q$ at small $\chi$ is reduced for a larger $G_\rho$.

Furthermore, with the inclusion of the isoscalar vector interaction, the whole phase diagrams move to higher baryon densities/chemical potentials, but the temperatures of the CEP almost keep unchanged.

In our previous study, we have proposed some possibly observable signals of the hadron-quark gluon phase transition for asymmetric matter in heavy-ion collision experiments. These signals are possibly weakened when the isovector vector interaction is included, whereas they are slightly strengthened by the isoscalar vector interaction. However, the main problem induced by the isoscalar vector interaction is that the onset densities of the mixed phase are moved to higher densities. Most uncertainty lies in the relevant vector couplings. The planned experiments with FAIR at GSI-Darmstadt and NICA at JINR-Dubna are expected to provide some hints on the related study.
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