A remark on the AdS/CFT correspondence and the renormalization group flow.

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Abstract

The correspondence between the four-dimensional $SU(N)$, $\mathcal{N} = 4$ SYM taken at large $N$ and the type II B SUGRA on the $AdS_5 \times S_5$ background is considered. We argue that the classical equations of motion in the SUGRA picture can be interpreted as that of the renormalization group on the SYM side. In fact, when the D3-brane is slightly excited higher derivative terms in the field theory on its world-volume deform it from the conformal $\mathcal{N} = 4$ SYM limit. We give arguments in favor of that the deformation goes in the way set by the SUGRA equations of motion. Concrete example of the $s$-wave dilaton is considered.

1. Recent developments in super-string theory [1, 2, 3] indicate that one can incorporate in it unusual non-perturbative excitations. The latter are sub-manifolds of the ten-dimensional space-time bulk on which strings can terminate [4, 5] – so called D-branes. They appear to be a new useful tool for studying low energy dynamics in different SUSY field theories [1]-[3]. Remarkably, the D-branes give a geometric description of different phenomena in the SYM theories which live on their world-volumes [6]-[9]. For example, movement in flat directions, i.e. the Higgs mechanism, is represented as splitting and joining of the D-branes [6]. Thus, masses of different fields and couplings in the SYM and SQCD theories are represented as distances between D-branes and angles of their respective orientation [7]-[9].

Having in mind those facts, in this note we try to give a geometric interpretation of the renormalization group flow in the SYM theory. Our work is based on the proposed duality [10] between the four-dimensional large $N$ SYM theory and type IIB SUGRA on a background which we describe below. Concretely, we argue that the classical equation of motion of the dilaton in the bulk SUGRA theory is nothing but the renormalization group equation for the SYM ”coupling constant” on the D-brane world-volume [11]. As we discuss below, this fact is sensible if the dilaton – ”coupling constant” – is excited and, hence, is a function of the four-dimensional coordinates. While if the dilaton is an arbitrary constant it remains to be the one at any energy scale. In the both cases the movement in the direction transversal to the D-brane is the renormalization group transformation [11, 12] in the field theory on its world-volume.

In fact, it is widely believed that super-strings suggest the regularization of field theory [13]. What is new in the D-brane case is that the regularization of their world-volume field theories can happen at much smaller energy scale than the Plank one, if some particular double scaling limit is taken [11, 14, 15]. The regularized theory should be considered as a vacuum in that of super-strings [10]. For after the regularization we are missing information

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about high frequency modes. In our case, from the point of view of a bulk observer they are flying away from the D-brane [10]. However, in the limit under consideration they do not escape to infinity. On the contrary, they stay inside the throat near the black brane horizon [17]. So that, from the point of view of a D-brane world-volume observer the following is happening. At low energy scale, smaller than the curvature of the black brane solution, we have the SYM theory. But as we deform to bigger energy scales higher derivative terms in the D-brane world-volume action becoming relevant. The latter are summing up and deform the theory from the conformal limit. At the same time if the dilaton is excited and becomes a function of the four-dimensional coordinates, during the deformation in question it acquires the dependence on the energy scale. Which, as we argue below, is defined by the corresponding SUGRA theory.

All that resembles the well established observations in old matrix models. The matrix models suggest the regularization of the two-dimensional conformal field theories with discreet target spaces. Moreover, their ”renormalization group” flows in the vicinity of the conformal points (continuum limits) are described by equations from various integrable hierarchies [18, 19].

2. As the beginning of our presentation, we describe here a few features of the D-brane physics and of the correspondence between SYM and SUGRA theories. This is done just to set the notations.

The D-branes are charged with respect to the R-R fields and preserve a part of supersymmetry in the corresponding super-string theory [20]. The low energy action describing dynamics of one D-brane and its interaction with bulk modes looks, in the light-cone gauge, as follows [3 4 21]:

\[
S = T_p \int d^{p+1}\xi e^{-\varphi} \sqrt{-\det (g_{mn} + b_{mn} + 2\pi F_{mn})} + Q_p \int d^{p+1}V_{m_0...m_p}C_{m_0...m_p} + \\
\frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} e^{-2\varphi} \left[ R + 4(\nabla_\mu \varphi)^2 - \frac{1}{12} \left( \partial_\mu B_\nu \right)^2 \right] + \ldots + O(\alpha'), \\
F_{mn} = \partial_m A_n, \quad g_{mn} = G_{ij} \partial_m \phi_i \partial_n \phi_j + G_{in} \partial_m \phi_i + G_{mn}, \\
b_{mn} = B_{ij} \partial_m \phi_i \partial_n \phi_j + B_{in} \partial_m \phi_i + B_{mn} \quad i, j = p + 1, \ldots, 9 \quad m, n = 0, \ldots, p. \tag{1}
\]

Where dots stand for the R-R fields and fermionic terms. In this formula \(\kappa = 8\pi^2 g_s \alpha'^2\), where \(g_s\) and \(\alpha'\) are the string coupling constant and its inverse tension, respectively; \(p\) is the spatial dimensionality of the D-brane world-volume; \(T_p = \frac{\pi}{g_s} (4\pi^2 \alpha' )^{\frac{p+1}{2}}\) and \(Q_p\) are its tension and charge with respect to the R-R tensor field \(C_{m_0...m_p}\). Also \(G_{\mu\nu}, B_{\mu\nu}, \varphi\) \((\mu, \nu = 0, \ldots, 9)\) are NS-NS [3] closed string modes, living in the bulk. While \(\phi_i\) and \(A_m\) are the D-brane coordinates and the gauge field living on its world-volume [4], respectively. The action (1) also has a generalization to the case of \(N \geq 2\) coinciding D-branes [22]. Which describes the situation when the open strings carry \(U(N)\) Chan-Paton factors [8].

We are interested in degenerations of the theory (1), in which the bulk and D-brane modes seemingly decouple from each other. The first one happens at low energies and small enough \(g_s\), if the theory is considered from the point of view of an observer, placed at a big distance from the D-brane position. The observer does not see fluctuations of the D-brane modes \((A_m, \phi_i\) and their super-partners). In this case, one is left with the ten-dimensional SUGRA containing \(\delta\)-functional sources of the mass and R-R charge. The \(\delta\)-functions have supports on the \(p + 1\)-dimensional sub-manifolds. One can get reed of them via introduction of the classical black brane background [23 24] into the SUGRA action.

The second degeneration happens at low energies and small enough \(g_s\), if the theory is considered from the point of view of a small distance observer. This observer does not feel...
long wavelength fluctuations of the bulk modes \((G_{\mu\nu}, B_{\mu\nu}, \varphi, \text{R-R fields and their superpartners})\). Therefore, when all other fields are small, we get the maximally super-symmetric \(U(N)\) SYM theory living on the D-brane world-volume \([\bar{3}]\).

At first sight, the two limits in question describe different types of theories. It appears, though, that they coincide, if a particular double scaling limit (with \(N \to \infty\)) is taken \([10, 12, 14, 15]\). Thus, actually the decoupling between the bulk and D-brane modes does not happen \([17]\). Qualitatively, as an observer goes further from the D-brane position, one averages over fluctuations in the SYM theory. The result of the averaging is the classical SUGRA on a particular background as an effective theory for the SYM \([24]\).

An example of this situation is the correspondence between the four-dimensional \(\mathcal{N} = 4\), \(SU(N)\) SYM theory taken at large \(N\) and the type IIB SUGRA on the \(AdS_5 \times (S_5)_N\) background \([10]\). The latter contains \(N\) units of the elementary flux of the R-R field \(C_0\) through the \(S_5\), which is indicated by the subscript \(N\). The geometry in question \([10]\), being equal to:

\[
ds^2 = \frac{r^2}{R^2} \eta_{mn} dx_m dx_n + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5, \quad n, m = 1, \ldots, 3;
\]

where \(r = \sqrt{x_1^2 + \ldots + x_9^2}\), \(2\)

is valid near the D3-brane horizon \(r < R\). The horizon is at \(r = 0\). Also in this correspondence one takes \(g_{ym}^2 = \text{const} \cdot g_s\) and the radii of the \(AdS_5\) and \(S_5\) are defined as \(R^2 = 4\pi\alpha'^2 g_s N\). Below we are going to work with the Euclidean signature and to represent the \(AdS_5\) as: \(ds^2 = \frac{R^2}{z^2} [dz^2 + (d\vec{x})^2]\) with the boundary at \(z = 0\). Where \(z\) is related to \(r\) as \(z = \frac{R^2}{r}\).

The theory we get at large \(N\) is the type II B non-linear \(\sigma\)-model on the \(AdS_5 \times (S_5)_N\) background \([20]\). As we have mentioned, it is expected \([10]\) that the theory has two degeneration limits. The first one happens when \(g_{ym}^2 N \sim g_s N \ll 1\) and leads to the weekly coupled SYM theory. While the second degeneration happens when \(\frac{R^4}{z^4} = g_s N \gg 1\) and leads to the II B SUGRA on the \(AdS_5 \times (S_5)_N\) background. Usually it is said that the strong coupling limit \((g_{ym}^2 N \to \infty)\) of the four-dimensional \(SU(N), \mathcal{N} = 4\) SYM theory taken at large \(N\) is described by the type II B SUGRA on the \(AdS_5 \times (S_5)_N\) \([10]\).

One might ask the following question: in what sense there is a correspondence between the SYM and the SUGRA theories? As an answer to this question, a more exact formulation of the statement was suggested in \([27, 28]\). Which establishes that as \(g_s N \to \infty\) and \(N \to \infty\) we have:

\[
< e^{-\sum J_j O^j dx} > \approx e^{-I_{\text{min}}(AdS_5 \times (S_5)_N)|_{J_j |_{b \sim J_0}^j}}.
\]

Where on the LHS the average is taken in the strongly coupled \(\mathcal{N} = 4\), \(SU(N)\) SYM theory and \([O^j]\) is a complete set of operators in it. While on the RHS \(I_{\text{min}}\) is the type IIB SUGRA action minimized on classical solutions, represented schematically as \(J_j\). These solutions have asymptotic values at the boundary \(J_j |_{b \sim J_0}^j\) in the sense explained in \([28]\). The latter serve as sources in the LHS.

The belief in this correspondence is partially based on the equivalence of two absorption probabilities of bulk modes by the D-branes \([14, 15]\). One of them is computed in the large \(N\) SYM picture while the other in that of the SUGRA.

To set the notations, we briefly describe the example of the \(s\)-wave dilaton which is independent from angles of the \(S_5\). From \([1]\) we get the vertex operator for the dilaton interaction with D-brane modes. It is equal to \(\varphi O_\varphi \sim \varphi \cdot tr (F_{mn}^2 + \ldots)\), where dots stand
for super-partners and higher derivative terms. Having this vertex at our disposal, we find
the absorption probability of the dilaton by the D-brane \[14, 15\].

At the same time, in the SUGRA picture the calculation goes as follows \[14, 15\]. For
the s-wave dilaton taken as \( \varphi(z, x) = (kz)^2 \chi(z) e^{ik\vec{x}} \), with \( k = |\vec{k}| \) we get the Laplace
equation in the black 3-brane background:

\[
(z \partial_z)^2 - \left( \frac{R^2}{z^2} + \frac{z^2}{R^2} \right) (kR)^2 - 4 \right] \chi(z) = 0. \tag{4}
\]

Solving the scattering problem for this equation, we find the absorption probability when
\((kR)^4 << 1 \) \[14, 15\]. The latter appears to be equivalent to the absorption probability found
in the SYM picture.

3. To proceed with the main topic of the note let us examine those facts. It is believed\[27, 28\] that the D-brane theory is the conformal \( \mathcal{N} = 4 \) SYM at an energy scale smaller
than \( R^{-1} \). At the same time, in the throat region \((z \geq R)\) when \((kR)^2 << 1\) the equation
\[(4)\] becomes \[27\]:

\[
(z \partial_z)^2 \chi(z) \approx 0. \tag{5}
\]

Let us take the following solution of this equation:

\[
\chi(z) = \frac{1}{2} K_2(kz), \quad \text{then} \quad \varphi(x, z) = e^{ik\vec{x}} \frac{1}{2} (kz)^2 K_2(kz). \tag{6}
\]

Here \( K_2 \) is the modified Bessel function and as \( kz \to 0 \) we get \( \varphi(x, z) \approx e^{ik\vec{x}} \). It is this
solution which is regular at the horizon \((z \to \infty)\). \[27\].

Now, if \( z \approx R \) the equation \[(3)\] becomes:

\[
(z \partial_z)^2 \chi(z) \approx 4 \chi(z), \quad \text{then} \quad z \partial_z \chi(z) \approx -2 \chi(z). \tag{7}
\]

One can recognize here the renormalization group equation for the dilaton\[4\] \( \varphi \sim (kz)^2 \chi(z) \sim 1 \) in the conformal SYM theory\[4, 11\]. As we mentioned the latter is valid on the energy
scale smaller than \( R^{-1} \). Thus, the \( z \) coordinate resembles the normalization point in the
renormalization group \[10\]. Below we argue that it is really the case. Concretely, we show
that a few leading non-trivial terms in the expansion of \[(3)\] over \((kz)^2\) can be recovered from
the renormalization group in the ”SYM picture”.

To begin with, we examine the relation \[(3)\] in more details. It is rather obscure because
both sides in it are divergent. For example, the SUGRA action on the \( AdS_5 \) background

\[
I(\varphi) = \frac{\pi^3 R^8}{4k^2} \int d^4xdz \frac{1}{z^3} \left[ (\partial_z \varphi)^2 + (\partial_m \varphi)^2 \right] \tag{8}
\]

has the IR divergence for the solution \[(3)\] \[27, 28\].

As was argued in \[27, 28, 29\] the IR regularization on the RHS of the \[(3)\] is related to
the UV one on the LHS. Let us use this observation. There is a natural IR regularization of
the \[(3)\] \[27, 28\]. In fact, one can shift the boundary of the \( AdS_5 \) from \( z = 0 \) to \( z = \epsilon \geq R \).

The regularized action \[(8)\] in this case is given by \[27, 28\]:

\[3\] The equation \( z \partial_z \chi'(z) = 2 \chi'(z) \) probably corresponds to the unity operator which couples to the volume
element \( O_u \sim \sqrt{\det g_{mn}} \) on the D-brane. The unity operator behaves as \( \sim z^4 \), when \( kz \to 0 \). Which perfectly
cancels the scale dependence of the volume element as it should be in the conformal field theory.

\[2\] Which defines the SYM coupling constant: \( \frac{1}{g_{\text{ym}}^2} \sim e^{\varphi_{\infty}} \). Here \( \varphi_{\infty} \) is the vev of the dilaton.
\[ I^\text{min}_\epsilon (\varphi_0) \sim N^2 \int d^4x \int d^4y \ \varphi_0(x)\varphi_0(y) \frac{1}{(\epsilon^2 + |\vec{x} - \vec{y}|^2)^{\frac{5}{2}}} - \\
-2N^2\epsilon^2 \int d^4x \int d^4y \ \varphi_0(x)\varphi_0(y) \frac{1}{(\epsilon^2 + |\vec{x} - \vec{y}|^2)^3}, \]  
(9)

with \( \varphi_0(x) = e^{i\vec{k}\cdot\vec{x}} \).

Now consider the generating functional in the SYM picture:

\[ Z(\varphi_0) = \int \mathcal{D}A_m \ldots \times \exp \left\{ -\frac{1}{g^2} \int d^4x \ \text{tr} \left( F_{mn}^2 + \ldots \right) + \right. \\
\left. + \frac{1}{g^2} \int d^4x \ \varphi_0(x) \cdot \text{tr} \left( F_{mn}^2 + \ldots \right) \right\}. \]  
(10)

From now on dots stand for the super-partners.

Integrating over the SYM fields in (10) at one loop, we get:

\[ Z_\epsilon(\varphi_0) = \text{const} \cdot \exp \left\{ -\text{const} \cdot \int d^4x \int d^4y \ \varphi_0(x) \cdot \varphi_0(y) \times \\
\times < \text{tr} \left( F_{lm}^2(x) + \ldots \right) \cdot \text{tr} \left( F_{np}^2(y) + \ldots \right) >_\epsilon \right\}, \]  
(11)

up to the quadratic order of the dilaton. This expression contributes to the renormalization of the unity operator which couples to the volume element on the D-brane.

In eq. (11) the correlator \(< \text{tr} \left( F_{lm}^2 + \ldots \right) \cdot \text{tr} \left( F_{np}^2 + \ldots \right) >_\epsilon \) is a regularized version of the < \text{tr} \left( F_{lm}^2 + \ldots \right) \cdot \text{tr} \left( F_{np}^2 + \ldots \right) >. From (3), (9) and (11) we get that \[27, 28\]:

\[ < \text{tr} \left( F_{lm}^2(x) + \ldots \right) \cdot \text{tr} \left( F_{np}^2(y) + \ldots \right) >_\epsilon \sim \frac{N^2}{(\epsilon^2 + |\vec{x} - \vec{y}|^2)^{\frac{5}{2}}} - \\
-2\epsilon^2 \frac{N^2}{(\epsilon^2 + |\vec{x} - \vec{y}|^2)^{\frac{7}{2}}}, \]  
(12)

which is natural if considered as the ”point splitting” in the extra (fifth) dimension. This regularization scheme can be formulated via an inclusion of non-local terms into the action (10), which can be expanded in powers of \( \epsilon^2 \).

Now we will use those considerations to compute counter-terms which renormalize the dilaton. Although within the theory from eq. (10) the dilaton does not get renormalized\footnote{Because fermionic loops perfectly cancel that of bosons in the \( \mathcal{N} = 4 \) SYM theory. This is true even if we consider the dilaton as some background non-constant field.}, in the LHS of (3) there are higher derivative terms included in \( \mathcal{O}^j \) operators. They can lead to the deformation of the dilaton.

It happens, though, that the next to leading term from the non-Abelian variant of the (1) – \( \text{tr} \left\{ F^4 - \frac{1}{4} (F^2)^2 \right\} \) – gives no contribution to the dilaton renormalization\footnote{It gives, however, non-zero contribution to the renormalization of the unity operator \[30\].} It is not a coincidence. All terms in (11) coming from the disc topology also should not give such a contribution. In fact, they are of the order of \( N^2 \), if \( N \to \infty \). Then, as one can estimate, if those terms could renormalize the dilaton, we would get contributions of the order of \( N^4 \).
Which contradicts to all our expectations from the large $N$ limit \[31\]. Hence, we can accept that there is no deformation of the dilaton coming from the disc topology and, in particular, from the leading action presented in \[1\].

There are, however, terms coming from higher topologies. The obvious leading term (in powers of derivatives) among them is given by:

$$S_2 = \frac{ae^4}{N^2} \left[ \text{tr} \left( F_{mn}^2 + \ldots \right) \right]^2. \quad (13)$$

Here $a$ is some constant irrelevant for our discussion below. The expression \(13\) is of the order $N^0$, as $N \to \infty$. Thus, it can give a proper counter-term ($\sim N^2$) which renormalizes the dilaton.

So, we add $S_2$ to the action in eq. \(10\). After that, we represent the super-gauge fields as $A_m = \bar{A}_m + \mathcal{A}_m$ and etc. for the super-partners. Where $\mathcal{A}_m$ are quantum fluctuations over the background $\bar{A}_m$. Then, we expand the action from \(10\), \(13\) in powers of the $\mathcal{A}_m$ and integrate over them at one loop. As we have already mentioned, from the action in \(10\) there do not appear terms renormalizing the dilaton. For there is a perfect cancellation of the fermionic and bosonic loops. However, because of the presence of the $S_2$ term, we get several counter-terms. Among them we are interested in the only one which contributes to the dilaton renormalization at the linear dilaton order. It is given by the second expression in the equation:

$$S_{\text{eff}}(\varphi_0, \bar{A}) = \frac{1}{g_{ym}^2} \int d^4 x \varphi_0 \cdot \text{tr} \left( \bar{F}_{mn}^2(x) + \ldots \right) + \frac{ae^4}{g_{ym}^2 N^2} \int d^4 x \int d^4 y \varphi_0(y) \cdot \text{tr} \left( \bar{F}_{mn}^2(x) + \ldots \right) < \text{tr} \left( \mathcal{F}_{lp}^2(x) + \ldots \right) \cdot \text{tr} \left( \mathcal{F}_{rt}^2(y) + \ldots \right) >_{\epsilon} \cdot (14)$$

Here $\bar{F}_{mn}$ and $\mathcal{F}_{mn}$ are the gauge field strengths of the vector potentials $\bar{A}_m$ and $\mathcal{A}_m$, respectively.

To find the counter-term in question, we take the expression \(12\) for the correlator $< \ldots >_{\epsilon}$. Now if the dilaton $\varphi_0$ is an arbitrary constant this counter-term is a trivial constant. While in the case when the dilaton is excited and equals to $\varphi_0 = e^{i\vec{k} \cdot \vec{x}}$ we get a renormalization. After the redefinition $\vec{y} - \vec{x} \to \vec{y}$ of the integral over $\vec{y}$ we obtain:

$$S_{\text{eff}}(\varphi_0, \bar{A}) = \frac{1}{g_{ym}^2} \int d^4 x \varphi_0(x) \cdot \text{tr} \left( \bar{F}_{mn}^2(x) + \ldots \right) + \frac{a}{g_{ym}^2 N^2} \int d^4 x \varphi_0(x) \cdot \text{tr} \left( \bar{F}_{mn}^2(x) + \ldots \right) \times \left[ e^4 \int d^4 y \frac{e^{i\vec{k} \cdot \vec{y}}}{(|\vec{x} - \vec{y}|^2 + \epsilon^2)^4} - 2e^6 \int d^4 y \frac{e^{i\vec{k} \cdot \vec{y}}}{(|\vec{x} - \vec{y}|^2 + \epsilon^2)^5} \right]. \quad (15)$$

It is easy to calculate the integrals in the second line of this formula because they are related to the $\mathcal{K}_2(k\epsilon)$ Bessel function and its derivatives.

The requirement of the renormalization group invariance:

$$\epsilon \partial_\epsilon \left\{ \varphi_0(\epsilon, x) [1 + \Phi(\epsilon)] \right\} = 0, \quad (16)$$

where $\Phi(\epsilon)$ schematically represents the counter-term in question, makes the dilaton to be dependent upon the scale $\epsilon$. Now, calculating the integrals \(13\), expanding them to the fourth order in the $k\epsilon$ and tuning the constant $a$, we get:
\[ \varphi_0(\epsilon, x) = e^{i\vec{k} \cdot \vec{x}} \left( 1 + a_1(k\epsilon)^2 + a_2(k\epsilon)^4 - \frac{1}{16}(k\epsilon)^4 \log \frac{\gamma k\epsilon}{2} + ... \right). \] (17)

Here \( \gamma \) is the Euler constant. Other terms in the expansion (17) receive contributions from the higher derivative corrections\[ to the action from eq. (14). This expression reproduces, up to the constants \( a_1 \) and \( a_2 \), the expansion of (3) over \((kz)^2\), when \( \epsilon = z \). Unfortunately, at present we are not able to recover from the renormalization group the exact values of the coefficients \( a_1 \) and \( a_2 \). In fact, they stand in front of the contact terms which get contributions from all higher corrections. Moreover, they can be altered via introduction of local boundary terms, which are yet undetermined, into the action on the RHS of (3). The boundary terms would change the value (12) of the correlator \< \text{tr} (F_{mn}^2(x) + ...) \cdot \text{tr} (F_{pq}^2(y) + ...) >_\epsilon \). But it is important that the universal term with logarithm does not receive any other contributions. Also, because of the SUSY we expect only linear logarithm corrections which is in agreement with the expansion of (3) over the \((kz)^2\).

4. We may conclude that after account of the higher derivative terms from the LHS of the eq. (3), the dilaton becomes dependent on the normalization scale. Very probably this dependence is governed by the classical SUGRA equations of motion.

At this point one may ask at least the following question: why the renormalization group equations are of the second order rather than of the first one? The answer possibly is as follows. We expect that the renormalization group equation for the dilaton depends only upon the unity operator and vice versa. If this is true, looking for the equation only in terms of the dilaton, one would find it to be of the second order. This possibility is supported by the fact that the second solution of the modified Bessel equation (5) has a good property to correspond to the unity operator. In the limit \( kz \to 0 \) the solution behaves as \( \sim z^4 \), which compensates the scale dependence of the D-brane space-time volume. As it should be in the conformal field theory [11].

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\[ \text{For example, there can be terms like } S_n \sim \frac{16^n}{N^3} \left[ \text{tr} (F_{mn}^2 + ...) \right]^n, \quad n \geq 3. \text{ Which are coming from the next to leading topology and behave as } N^0, \text{ when } N \to \infty. \]
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