Analysis of the QCD-improved factorization in $B \to J/\psi K$

Junegone Chay$^1$ and Chul Kim

Department of Physics, Korea University, Seoul 136-701, Korea

Abstract

We consider the exclusive decay $B \to J/\psi K$ using the QCD-improved factorization method in the heavy quark limit. It is shown that the decay amplitude is factorizable in this limit and nonfactorizable contributions are calculable from first principles in perturbation theory. Also the spectator contributions at order $\alpha_s$ are finite and suppressed in the heavy quark limit. We present the result at next-to-leading order in strong interaction, and leading order in $1/m_b$ in the heavy quark limit.

Exclusive nonleptonic decays of $B$ mesons have received a lot of attention since they are observed in experiments at CLEO, BABAR and BELLE [1]. They offer the opportunity to test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) triangle, the CP violation and to observe new physics effects. However, quantitative understanding of nonleptonic decays of $B$ mesons is difficult since there are final-state interactions which are intrinsically nonperturbative in nature. But when a $B$ meson decays into two light mesons, there is large momentum transfer to final-state mesons. In this case, we can systematically calculate the decay rates using perturbation theory in the heavy quark limit.

Beneke et al. [2] considered general two-body nonleptonic decays of $B$ mesons extensively including a light-light meson system as well as a heavy-light system in the final state. The general idea is that in the limit $m_b \gg \Lambda_{\text{QCD}}$, the hadronic matrix elements can be schematically represented as

$$
\langle M_1 M_2 | O | B \rangle = \langle M_1 | j_1 | B \rangle \langle M_2 | j_2 | 0 \rangle \times \left[ 1 + \sum r_n a^n + O(\Lambda_{\text{QCD}} / m_b) \right],
$$

(1)

1 e-mail address: chay@kupt4.korea.ac.kr
where $M_1, M_2$ are final-state mesons and $O$ is a local current-current operator in the weak effective Hamiltonian. If we neglect radiative corrections in $\alpha_s$ and power corrections in $\Lambda_{\text{QCD}}$, we get the factorized result with a form factor times a decay constant. At higher order in $\alpha_s$, this simple factorization is broken, but the corrections can be calculated systematically in terms of short-distance Wilson coefficients and meson light-cone distribution amplitudes. We call this the QCD-improved factorization.

When we use light-cone meson wave functions for exclusive decays, the transition amplitude of an operator $O_i$ in the weak effective Hamiltonian is given by

$$\langle J/\psi K | O_i | B \rangle = \sum_j F_j^{B \rightarrow K} (m_{\psi}^2) \int_0^1 dx T_{ij}^I (x) \phi_{\psi} (x) + \int_0^1 d\xi dx du T_{ij}^{II} (\xi, x, u) \phi_B (\xi) \phi_{\psi} (x) \phi_K (u),$$

(2)

where $F_j^{B \rightarrow K} (m_{\psi}^2)$ are the form factors for $B \rightarrow K$, and $\phi_M (x)$ is the light-cone wave function for the meson $M$. $T_{ij}^I (x)$ and $T_{ij}^{II} (\xi, x, u)$ are hard-scattering amplitudes, which are perturbatively calculable. The second term in Eq. (2) represents spectator contributions.

This method works well for the case with two light mesons like $\pi \pi$ or $\pi K$ [2,3], in which the final-state mesons carry large momenta. Interestingly enough, when there is a heavy quark in the final state such as $B \rightarrow D \pi^-$, this method still works when a spectator quark of the $B$ meson is absorbed by, say, a $D$ meson [2,4]. In Ref. [4], they calculated the ratio $\Gamma (B \rightarrow D^+ \pi^-)/\Gamma (B \rightarrow D^+ \gamma)$ since the Wilson coefficients were known at leading order at the time. The absolute branching ratios for $B \rightarrow D^{(*)} \pi^-$ were calculated recently since the Wilson coefficients are known at next-to-leading order accuracy [2,5]. However, when the spectator quark is absorbed by a light quark, say, in $B \rightarrow D\pi^0$, nonfactorizable contributions are infrared divergent, and the factorization breaks down.

When we consider the decay $\bar{B} \rightarrow J/\psi K$, it looks ambiguous at first sight whether we can apply the same method used in $\bar{B} \rightarrow \pi \pi$, or $\bar{B} \rightarrow D^+ \pi^-$, since the spectator quark in the $B$ meson goes into a light $K$ meson. However, what is special about $J/\psi$ is that the size of the charmonium is so small ($\sim 1/\alpha_s m_\psi$) that the charmonium has a negligible overlap with the $(\bar{B}, K)$ system, hence enabling the same improved factorization method in the decay $\bar{B} \rightarrow J/\psi K$. As the explicit calculation shows below, the nonfactorizable

---

2 In Ref.[2], Beneke et al. correctly pointed out the typos in Ref.[5].
contribution is infrared safe and the spectator contribution is suppressed in the heavy quark limit. These facts indirectly justify the use of the improved factorization formula Eq. (2) in $\bar{B} \to J/\psi K$.

When the mass of the $J/\psi$ meson is not negligible, the light-cone wave function of the $J/\psi$ meson should include higher-twist contributions. The light-cone wave functions are obtained in powers of $m_\psi/E$ or $\Lambda_{QCD}/E$ where $E \sim m_\psi$ is the energy of the $J/\psi$ meson. For $B$ decays into two light mesons, the higher-twist contributions are negligible since they are of order $\Lambda_{QCD}/E$. However, for $\bar{B} \to J/\psi K$, higher-twist contributions are important. Therefore we expect that the decay rate using only the leading, asymptotic wave function of $J/\psi$ will be smaller than the experimental result.

Due to the nonzero mass of $J/\psi$, we can think of several approximations with different limits. For example, we can take the infinite mass limit of the $b$ quark in which $m_b$ goes to infinity while $m_\psi$ is fixed ($m_\psi/m_b \to 0$). In this case the result simply reduces to the case of $B \to \pi\pi$. But this limit is hardly realized in nature and it is not reasonable to compare the theoretical prediction in this limit with experimental data. We can consider another limit in which $m_b$ goes to infinity, while $m_\psi/m_b$ is held fixed. In this case, if $m_\psi$ is heavy enough, it would be a better idea to employ a nonrelativistic wave function for $J/\psi$. However, we can still use the light-cone wave function for $J/\psi$. But we have to include the effects of higher-twist wave functions in order to compare with experimental data reasonably since these effects can be appreciable.

From now on, we assume that, in the limit in which $m_b$ goes to infinity, $m_\psi$ is heavy enough to regard the size of the $J/\psi$ meson as small, but light enough to employ the leading-twist light-cone wave function for $J/\psi$. It is difficult to satisfy this interesting, but hypothetical limit in reality, but we will not go further into the detail about this point. In this paper, we present the decay amplitude at next-to-leading order in the strong interaction, neglecting $O(\Lambda_{QCD}/m_b)$ corrections, employing the same technique to $\bar{B} \to J/\psi K$ as in $\bar{B} \to \pi\pi$.

One technical point is that we will keep the mass $m_\psi$ of the $J/\psi$ meson in the hard scattering amplitude. In order for this to be theoretically consistent, we also have to consider the higher-twist wave functions for $J/\psi$ since higher-twist wave functions contain corrections of order $m_\psi/m_b$. However, the main issue is whether the hard-scattering amplitudes in the heavy quark limit are infrared finite at each order in $\alpha_s$ and $1/m_b$. The effects of the higher-twist wave functions correspond to the expansion in $1/m_b$, and each wave function is, in general, an independent function. Therefore we have to verify that the hard-scattering amplitude convoluted with each independent wave function is finite. What we pursue in this paper is whether the hard-scattering amplitude convoluted with the leading-twist wave function for $J/\psi$ can be reliably cal-
culated in the heavy quark limit. The inclusion of higher-twist wave functions to compare with experimental data will be considered elsewhere.

The effective Hamiltonian $H_{\text{eff}}$ for $B \rightarrow J/\psi K$ is written as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( V_{cb} V_{cs}^* (C_1 O_1 + C_2 O_2) - V_{tb} V_{ts}^* \sum_{i=3}^{6} C_i O_i \right),$$

(3)

where $C_i$ are the Wilson coefficients at next-to-leading order evaluated at the renormalization scale $\mu$. The relevant operators in $H_{\text{eff}}$ are given by

$$O_1 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha \cdot \bar{c}_\beta \gamma_\mu (1 - \gamma_5) b_\beta,$$

$$O_2 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta \cdot \bar{c}_\beta \gamma_\mu (1 - \gamma_5) b_\alpha,$$

$$O_3 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha \cdot \sum_q \bar{q}_\beta \gamma_\mu (1 - \gamma_5) q_\beta,$$

$$O_4 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\beta \cdot \sum_q \bar{q}_\beta \gamma_\mu (1 - \gamma_5) q_\alpha,$$

$$O_5 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha \cdot \sum_q \bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\beta,$$

$$O_6 = \bar{s}_\alpha \gamma^\mu (1 - \gamma_5) b_\beta \cdot \sum_q \bar{q}_\beta \gamma_\mu (1 + \gamma_5) q_\alpha,$$

(4)

where $\alpha, \beta$ are the color indices and the sum over $q$ runs over $u, d, s, c$ and $b$.

In calculating the decay amplitude, we introduce the vector and the tensor decay constants as [6]

$$\langle J/\psi | \gamma_\mu c(0) | 0 \rangle = -i f_\psi m_\psi \epsilon_\mu^*, $$

$$\langle J/\psi | \gamma_\mu \gamma_\nu \gamma_\rho c(0) | 0 \rangle = i f_{\psi T}^T (\epsilon_\rho^* p_\nu - \epsilon_\nu^* p_\rho),$$

(5) (6)

where $f_\psi$ is the decay constant which can be determined from the leptonic decay of $J/\psi$. $f_{\psi T}^T$ is the tensor decay constant arising from the tensor current, which formally depends on the renormalization scale.

The distribution amplitudes on the light cone at leading-twist accuracy are expressed compactly as [7,8]

$$\langle J/\psi (p, c) | \bar{c}_\alpha (y) c_\beta (z) | 0 \rangle = - \frac{i}{4} \int_0^1 dx e^{i (x p - y + (1-x)p - z)}$$

$$\times \left[ f_\psi m_\psi (\epsilon^*_{\beta})_{\gamma_\alpha} \phi_\psi (x) + f_{\psi T}^T (\epsilon^*_{\beta})_{\gamma_\alpha} \phi_{\psi T}^T (x) \right].$$

(7)

In Eq. (7), $x$ is the momentum fraction of a $c$ quark inside the $J/\psi$ meson, and the asymptotic wave functions $\phi_\psi (x)$, $\phi_{\psi T}^T (x)$ for the $J/\psi$ meson are symmetric
functions under $x \leftrightarrow 1-x$. The asymptotic form of the distribution amplitudes $\phi_\psi(x)$ and $\phi^T_\psi(x)$ is the same, given as $\phi_\psi(x) = \phi^T_\psi(x) = 6x(1-x)$. In the numerical analysis, we also consider the wave function of the form $\phi_\psi(x) = \phi^T_\psi(x) = \delta(x - \frac{1}{2})$, which appeals to the intuitive expectation of the wave function.

If we neglect the radiative corrections and the power corrections in $\Lambda_{\text{QCD}}/m_b$, the factorized amplitude at leading order is written as

$$iM_0 = -2if_\psi m_\psi \epsilon^* \cdot p_B F_1(m^2_\psi) \frac{G_F}{\sqrt{2}} \times \left( V_{cb} V_{cs}^*(C_2 + \frac{C_1}{N}) - V_{tb} V_{ts}^*(C_3 + \frac{C_4}{N} + C_5 + \frac{C_6}{N}) \right).$$

We do not include the effects of the electroweak penguin operators since they are numerically small. Here $N$ is the number of colors, $m_\psi$ is the mass of $J/\psi$, and $\epsilon^*$ is the polarization vector of $J/\psi$. $F_i(q^2) \ (i = 0, 1)$ are the form factors for $B \to K$, which are given as

$$V_\mu = \langle K(p')|\bar{\sigma}^\gamma_{\mu} b|B(p_B)\rangle$$

$$= \left[ (p_B + p')_{\mu} - \frac{m^2_B - m^2_K}{p^2 p_\mu} \right] F_1(p^2) + \frac{m^2_B - m^2_K}{p^2} p_\mu F_0(p^2),$$

where $p = p_B - p'$ is the momentum of $J/\psi$ with $p^2 = m^2_\psi$. From now on, we will neglect the kaon mass for simplicity.

As we can see easily in Eq. (8), this amplitude depends on the renormalization scale $\mu$ since the Wilson coefficients depend on $\mu$ while the matrix elements of the operators are replaced by decay constants and form factors which are independent of $\mu$. Therefore this amplitude is unphysical. However, if we include the $\alpha_s$ correction to the amplitudes, it turns out that the $\mu$ dependence of the Wilson coefficients is cancelled and the overall amplitude is insensitive to the renormalization scale.

Nonfactorizable contributions at order $\alpha_s$ come from the radiative corrections of the operators $O_1$, $O_4$ and $O_6$, and the relevant Feynman diagrams are shown in Fig. 1. The radiative corrections with a fermion loop do not contribute due
to the color structure. For each operator $O_1, O_4$ and $O_6$, if we add all the diagrams in Fig. 1 and symmetrize the result with respect to $x \leftrightarrow 1 - x$, the infrared divergence of each diagram cancels and the remaining amplitude is infrared finite. One thing to note is that there appear imaginary parts in the nonfactorizable contribution, which are due to the final-state interaction. The strong phase can be calculated in the QCD-improved factorization and it is important in exploring the CP violation in nonleptonic decays.

The decay amplitudes, in general, contain two terms proportional to $F_0$ and $F_1$, which come from the matrix elements for $B \to K$. However, if we use the equation of motion, we can find a relation between these terms, so that we can write the decay amplitude which depends only on $F_0$, or $F_1$. In the framework of the QCD-improved factorization with light-cone distribution functions for the mesons, we assume that all the quarks inside a meson are on their mass shell. Therefore we can safely use the equation of motion. For example, if we contract $p_B^\mu$ to the current $\bar{s} \gamma_\mu (1 - \gamma_5) b$, we get

$$p_B^\mu \bar{s} \gamma_\mu (1 - \gamma_5) b = \bar{s} p_B (1 - \gamma_5) b = m_b \bar{s} (1 + \gamma_5) b.$$  \hspace{1cm} (10)

Similarly, if we contract the kaon momentum $p'^\mu$ to the same current, we obtain

$$\bar{s} p'^\mu (1 - \gamma_5) b = 0.$$ \hspace{1cm} (11)

Here we neglect the mass of the strange quark, and accordingly the kaon mass, for simplicity. When we combine Eqs. (9) and (11), we obtain the relation between the two form factors $F_0$ and $F_1$ as

$$p' \cdot V = p_B \cdot p' \left[ \left( 1 - \frac{m_B^2}{p^2} \right) F_1 (p^2) + \frac{m_B^2}{p^2} F_0 (p^2) \right] = 0.$$ \hspace{1cm} (12)

From this relation, the ratio $F_0/F_1$ is given by

$$\frac{F_0 (p^2)}{F_1 (p^2)} = 1 - \frac{p^2}{m_B^2}.$$ \hspace{1cm} (13)

This relation was also observed in calculating heavy-to-light form factors in the large energy limit [9]. In parameterizing these form factors using the QCD sum rule, this ratio is also valid to first order in $p^2$ [10]. Note that, as $p^2 \to 0$, this ratio becomes 1 as it should be to remove the pole at $p^2 = 0$.

When we calculate the decay amplitude, some of the terms have the form
\begin{align}
(m_B - m_{\psi})^2 & \rightarrow \psi (1 - \gamma_5) b - 2m_B \epsilon^* \cdot p_B \overline{\psi} (1 + \gamma_5) b \\
= & 2 \left[ p \cdot p' \overline{\psi} (1 - \gamma_5) b - \epsilon^* \cdot p_B \overline{\psi} \rho (1 - \gamma_5) b \right] \\
= & 2 \left( p \cdot p' \epsilon^{\mu} - \epsilon^* \cdot p' p' \right) \overline{\psi} \gamma_\mu (1 - \gamma_5) b \equiv 2a^\mu \overline{\psi} \gamma_\mu (1 - \gamma_5) b. \tag{14}
\end{align}

In deriving Eq. (14), we use the equation of motion with \( p_B = p + p' \), \( \epsilon^* \cdot p = 0 \) and neglect the light quark masses and the light meson masses. The vector \( a^\mu \) satisfies the relation

\begin{equation}
a \cdot p' = 0, \quad a \cdot p = a \cdot p_B = -\epsilon^* \cdot p' m_{\psi}^2. \tag{15}\end{equation}

Therefore we obtain the relation

\begin{align}
a \cdot V = -\epsilon^* \cdot p_B m_{\psi}^2 \left[ \left( 1 - \frac{m_{\psi}^2}{p'^2} \right) F_1(p'^2) + \frac{m_{\psi}^2}{p'^2} F_0(p'^2) \right] = 0, \tag{16}\end{align}

as a result of Eq. (12). With this identity, in calculating the decay amplitude, those terms proportional to \( \epsilon^* \cdot p_B \overline{\psi} \gamma_\mu (1 + \gamma_5) b \) can be replaced by the terms proportional to \( \overline{\psi} \gamma^\mu (1 - \gamma_5) b \) or vice versa.

The full amplitude for \( B \rightarrow J/\psi K \) is written as

\begin{equation}
iM = -2 i f_{\psi} m_{\psi} \epsilon^*, p_B \frac{G_F}{\sqrt{2}} \left[ V_{cb} V_{cs}^{*} a_2 - V_{tb} V_{ts}^{*} (a_3 + a_5) \right] F_1(m_{\psi}^2), \tag{17}\end{equation}

where the coefficients \( a_i \) (\( i = 2, 3, 5 \)) in the NDR scheme are given as

\begin{align}
a_2 = C_2 + \frac{C_1}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_1 \left( -18 + 12 \ln \frac{m_{b}}{\mu} + f_I + f_{II} \right), \\
a_3 = C_3 + \frac{C_4}{N} + \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_4 \left( -18 + 12 \ln \frac{m_{b}}{\mu} + f_I + f_{II} \right), \\
a_5 = C_5 + \frac{C_6}{N} - \frac{\alpha_s}{4\pi} \frac{C_F}{N} C_6 \left( -6 + 12 \ln \frac{m_{b}}{\mu} + f_I + f_{II} \right). \tag{18}\end{align}

The function \( f_I \) is given by

\begin{equation}
f_I = \int_0^1 dx \phi_{\psi}(x) \left[ \frac{3(1 - 2x)}{1 - x} \ln x - 3i\pi + 3 \ln(1 - r^2) + \frac{2r^2(1 - x)}{1 - r^2} x \ln r^2 x + \frac{r^4 x^2 (\ln(1 - r^2) - i\pi)}{(1 - r^2(1 - x))^2} \right] \end{equation}
where \( r = m_\psi/m_B \). The function \( f_I \) is infrared finite even when \( r \) is nonzero. We organize the result such that it is proportional to \( F_1(m_\psi^2) \), and all the terms involving \( r \) go to zero as \( r \to 0 \). In this form, we can see clearly that the coefficients \( a_i \) in the limit \( r \to 0 \) is the same as the coefficients for, say, \( \bar{B} \to \pi\pi \) [2]. Note that the contribution from the tensor current vanishes as \( r \to 0 \). This is trivial since the polarization vector \( \epsilon^\mu \) is proportional to the momentum \( p^\mu \) in the massless limit and the tensor current does not contribute at all in this limit.

\( f_{II} \) is obtained from the spectator contribution to \( \bar{B} \to J/\psi K \), which is shown in Fig. 2. This corresponds to the second term in Eq. (2). When we symmetrize the amplitude with respect to \( x \leftrightarrow 1 - x \), it turns out that there is no infrared divergent part and the result is written as

\[
\int_0^1 dx \phi_\psi^T(x) 4r^2 \left[ \frac{1}{1 - r^2(1 - x)} - \frac{1}{1 - r^2x} \right] \ln x r^2 - \frac{\ln(1 - r^2) - i\pi}{1 - r^2(1 - x)},
\]

(19)

\[ f_{II} = \frac{4\pi^2}{N} \frac{f_K f_B}{F_1(m_\psi^2)m_B^2} \frac{1}{1 - r^2} \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 dx \frac{\phi_\psi(x)}{x} \int_0^1 du \frac{\phi_K(u)}{u}, \]

(20)

where \( \phi_B, \phi_K \) are the light-cone wave functions for the \( B \) meson and the \( K \) meson respectively. The spectator contribution depends on the wave function \( \phi_B \) through the integral

\[
\int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \equiv \frac{m_B}{\lambda_B}.
\]

(21)

Since \( \phi_B(\xi) \) is appreciable only for \( \xi \) of order \( \Lambda_{\text{QCD}}/m_B \), \( \lambda_B \) is of order \( \Lambda_{\text{QCD}} \). Therefore \( f_{II} \) is given by

\[
f_{II} = \frac{4\pi^2}{N} \frac{f_K f_B}{F_1(m_\psi^2)m_B^2} \frac{1}{1 - r^2} \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 dx \frac{\phi_\psi(x)}{x} \int_0^1 du \frac{\phi_K(u)}{u}.
\]

(22)
The proof that the QCD-improved factorization method works for $B \to J/\psi K$ is sophisticated. There are two independent parts which depend on $f_\psi \phi_\psi(x)$ and $f_\psi^T \phi_\psi^T(x)$. And each hard-scattering amplitude amplitude proportional to $f_\psi$ or $f_\psi^T$ is infrared finite. In both limits of $r \to 0$ and $r \neq 0$, the decay amplitudes are infrared finite, justifying that the method of the QCD-improved factorization can be applied in $B \to J/\psi K$. This is the explicit proof of the QCD-improved factorization for $B \to J/\psi K$ at leading-twist accuracy, which was briefly mentioned in Ref. [2].

One important ingredient in the above argument is that it holds true only at leading-twist order. It means that we use only the leading-twist wave functions, and higher-twist wave functions are assumed to be suppressed. If $E$ is the energy of the $J/\psi$ meson, it corresponds to neglecting those terms suppressed by $(m_\psi, \Lambda_{QCD})/E \approx (m_\psi, \Lambda_{QCD})/m_b$. In reality, we expect that higher-twist effects are not negligible for appreciable values of $r$. When we consider higher-twist effects, we have to include the terms of order $\Lambda_{QCD}/m_b$ in the higher-twist wave functions as well as in the hard-scattering kernel. In order for the QCD-improved factorization to work, the higher-twist effects should also be infrared finite.

For numerical analysis, we use the following input parameters:

$$
\begin{align*}
    m_b &= 4.8 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \\
    m_B &= 5.28 \text{ GeV}, \quad m_\psi = 3.1 \text{ GeV}, \\
    f_\psi &= 405 \text{ MeV}, \quad f_B = 180 \text{ MeV}, \quad f_K = 160 \text{ MeV}.
\end{align*}
$$

(23)

We also choose $\Lambda_{\overline{MS}}^{(5)} = 225 \text{ MeV}$, and $\lambda_B \approx 300 \text{ MeV}$. For the form factors in $B \to K$, we use the values taken from Ref. [10].

$$
\begin{align*}
    F_1(m_\psi^2) &= 0.606, \quad F_0(m_\psi^2) = 0.418.
\end{align*}
$$

(24)

And the CKM matrix elements are expressed in terms of the Wolfenstein parameters [11] with $A = 0.81 \pm 0.06$ and $\lambda = \sin \theta_C = 0.2205 \pm 0.0018$, and we fix them to their central values. For the wave function of $J/\psi$, we use employ two kinds of the wave functions. One is the asymptotic function $\phi_\psi(x) = \phi_\psi^T(x) = 6x(1-x)$, and the other is $\phi_\psi(x) = \phi_\psi^T(x) = \delta(x - 1/2)$, which is more intuitive.

In order to extract the magnitude of the tensor decay constant $f_\psi^T$, we contract Eq. (6) with $p^\nu$, and use Eq. (5) and the equation of motion. Then we get the relation

$$
2m_c(\mu)f_\psi = m_\psi f_\psi^T(\mu),
$$

(25)
The coefficients $a_i^{1,0}$ at $\mu = m_b$ and $m_b/2$ with different wave functions of $J/\psi$. The values in the second and the third columns are the values with $\phi_\psi(x) = 6x(1-x)$ and the fourth and the fifth columns are the values with the delta function.

| $a_i$ | $\mu = m_b$ | $\mu = m_b/2$ | $\mu = m_b$, delta | $\mu = m_b/2$, delta |
|-------|--------------|---------------|---------------------|----------------------|
| $a_2 \times 10^3$ | $72.4-60.0i$ | $48.3-79.6i$ | $77.3 -60.1i$ | $54.9-79.8i$ |
| $a_3 \times 10^3$ | $4.01+1.39i$ | $5.37+2.45i$ | $3.90 +1.39i$ | $5.17+2.46i$ |
| $a_5 \times 10^3$ | $-4.95-1.71i$ | $-6.03-3.97i$ | $-4.80-1.71i$ | $-5.70-3.98i$ |

from which, we use

$$f_\psi^T m_c = \frac{2m_c^2 f_\psi}{m_\psi}. \quad (26)$$

The numerical result of the coefficients $a_i$ is summarized in Table 1. We can see that the coefficients $a_i$ do not depend sensitively on the choice of the wave functions since the numerical values with different choices of the wave functions for $J/\psi$ are not much different.

With these coefficients $a_i$, we can calculate the branching ratio. The branching ratio from experiment is given by [12]

$$\text{Br}(B \to J/\psi K) = (8.9 \pm 1.2) \times 10^{-4}. \quad (27)$$

Our result shows that, for $\phi_\psi(x) = 6x(1-x)$, the branching ratio is $1.03 \times 10^{-4}$ for $\mu = m_b$ and $1.05 \times 10^{-4}$ for $\mu = m_b/2$, which are about eight times smaller than the experimental result. Though the theoretical treatment of the QCD-improved factorization in $B \to J/\psi K$ is improved, the theoretical result still does not saturate the experimental result. This is expected since we do not include higher-twist effects which are supposed to be rather appreciable. In order to have a reasonable comparison, we need to include higher-twist effects, which is going to be pursued elsewhere.

In Fig. 3, the dependence of the branching ratio on the renormalization scale is shown. The horizontal short-dashed lines in Fig. 3 show the experimental branching ratio at $1\sigma$ level. We also show the factorized result without nonfactorizable contribution. Of course, this quantity is unphysical and very sensitive to the renormalization scale, as we can see clearly in Fig. 3. On the other hand, for the full branching ratio calculated at next-to-leading order, the dependence of the branching ratio on the renormalization scale is very mild.

Recent observation of CP asymmetry in $B \to J/\psi K S$ [16] attracted some
Fig. 3. Dependence of the branching ratio on the renormalization scale $\mu$. The solid line uses the wave function $\phi(x) = 6x(1-x)$, the long-dashed line uses $\phi(x) = \delta(x - \frac{1}{2})$. The dotted line represents the result without the $\alpha_s$ correction. The band with short dashes represents the experimental data at 1$\sigma$ level.

interests in the possibility of new physics [17]. In this scheme in the standard model, there is no contribution to CP asymmetry in the decay amplitude since the CKM matrix elements involved here are all real. The CP asymmetry totally comes from $B - \bar{B}$ mixing, and we have not considered this issue here.

In conclusion, we have considered the effect of nonfactorizable contributions in $\bar{B} \to J/\psi K$ in the heavy quark limit. In this limit, nonfactorizable contribution can be systematically calculated from first principles using perturbation theory. It is shown that the decay amplitude is factorizable at next-to-leading order in the strong interaction and at leading order in $\Lambda_{\text{QCD}}/m_b$. The nonfactorizable contribution and the spectator contribution are infrared finite.

In general, when the $B$ meson decays into two mesons in which one is heavy, if the spectator quark of the $B$ meson goes into the light meson, the decay amplitude is not factorizable. The overlap of the heavy meson with the remainder is sizable and the soft gluon exchange gives a significant effect. However, when the heavy meson is the charmonium, the factorization method still works at leading-twist order. This physically means that the size of the charmonium is so small that the overlap with other mesons is small. Therefore the QCD-improved factorization method holds in $\bar{B} \to J/\psi K$, in contrast to naive
expectations.

In spite of the theoretical improvement mentioned so far, the theoretical branching ratio is about eight times smaller than the experimental result. This is typical in class II decays in which there is color suppression in the lowest order contribution. And the $\alpha_s$ corrections give a significant change in magnitude and a significant strong phase compared to the leading-order result corresponding to naive factorization.

Another reason why the theoretical result is small compared to the experimental data is that we used only the leading-twist wave functions. In a realistic case, higher-twist effects are not negligible. Khodjamirian and Rückl [13] considered nonperturbative effects in $B \to J/\psi K$ using the QCD sum rules, and they found that nonperturbative effects including the higher-twist wave functions could indeed be large. Also in the inclusive production of $J/\psi$, the naive factorization does not explain the large production rate. It was suggested that the octet contribution would enhance the rate [14,15]. This contribution comes from the nonleading Fock state $|gcc\rangle$ state, which constitutes higher-twist effects, and it may not be negligible in the decay $B \to J/\psi K$ either. However, the method of the QCD-improved factorization can be safely applied to $B \to J/\psi K$ at leading-twist order and it is remarkable that we obtain a very significant correction to naive factorization.

Note added: While this paper has been written, Ref. [18] appeared, in which the decay amplitude for $B \to J/\psi K$ are computed including chirally enhanced contributions. Their result looks different from ours, but the definition of $f_I$ and $g_I$ are rather ambiguous since they can be switched back and forth using the equation of motion discussed in this paper. Furthermore, they put $f_T^2 \frac{m_c}{f_\psi m_\psi} = 2x^2$ using the equation of motion. That is why there is no $f_T^2$ in their result. However, when all is taken into account, both results are the same.

Acknowledgements

The authors were supported in part by the Ministry of Education grants KRF-99-042-D00034 D2002. C.K. is partially supported by the Korea University Research Fund. The authors would like to thank Pyungwon Ko and Hai-Yang Cheng for stimulating discussions.

References
[1] See, for example, CLEO Collaboration, Phys. Rev. D62 (2000) 051101; The BABAR Collaboration, hep-ex/0008050.

[2] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; hep-ph/0006124.

[3] T. Muta, A. Sugamoto, M.-Z. Yang, and Y.-D. Yang, hep-ph/0006022; D. Du, D. Yang, and G. Zhu, hep-ph/0008216.

[4] H. D. Politzer and M. B. Wise, Phys. Lett. B257 (1991) 399; G. D. Haas and M. Youssefmir, Phys. Lett. B272 (1991) 391.

[5] J. Chay, Phys. Lett. B476 (2000) 339.

[6] P. Ball, and V. M Braun, Phys. Rev. D54 (1996) 2182.

[7] P. Ball, and V. M. Braun, Nucl. Phys. B543 (1999) 201.

[8] M. Beneke, and T. Feldmann, hep-ph/0008253.

[9] J. Charles, A. Le Yaouanc, L. Oliver, P. Pène, and J.-C. Raynal, Phys. Rev. D60 (1999) 014001.

[10] P. Ball, JHEP 09 (1998) 005.

[11] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[12] Particle Data Group, Eur. Phys. J. C15 (2000) 1.

[13] A. Khodjamirian, and R. Rückl, Talk presented at the Third workshop “Continuous Advances in QCD”, Minneapolis (1998).

[14] P. Ko, J. Lee, and H. S. Song, Phys. Rev. D53 (1996) 1409.

[15] M. Beneke, F. Maltoni, and I. Z. Rothstein, Phys. Rev. D59 (1999) 054003.

[16] Plenary talks presented by D. Hitlin (BABAR Collaboration) and H. Aihara (BELLE Collaboration) at ICHEP 2000 Osaka, Japan, July 31, 2000 (to appear in the Proceedings).

[17] A. L. Kagan, and M. Neubert, hep-ph/0007360; J. P. Silva, and L. Wolfenstein, hep-ph/0008004; G. Eyal, Y. Nir, and G. Perez, hep-ph/0008009; Z. Xing, hep-ph/0008018.

[18] H.-Y. Cheng, and K.-C. Yang, hep-ph/0011173.