Security SMC for networked fuzzy singular systems with semi-Markov switching parameters

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ABSTRACT In this paper, the issue of security sliding mode control (SMC) is addressed for networked nonlinear singular systems with semi-Markov switching parameters via the Takagi-Sugeno (T-S) fuzzy strategy. The goal is to guarantee a good steady performance of dynamical systems under the framework of deception attacks. First, the common sliding surface is proposed to avoid the potential instability caused by repeated jumps of sliding surface. Then, based on the partly unknown transition rate, sufficient conditions are constructed to realize the stochastic admissibility for the corresponding system. Moreover, a fuzzy security SMC law is designed to drive the state trajectory onto the specified sliding region. Finally, a practical example is shown to validate the proposed algorithm.

INDEX TERMS singular systems; deception attacks; sliding surface; stochastic admissibility.

I. INTRODUCTION

WITH the rapid development of modern control technology, communication technology, and computer technology, the traditional point to point control approach cannot satisfy the requirement of actual industrial systems. Networked control systems (NCSs) connect the multiple distributed components through a shared communication network to form the closed-loop control system [1]. Different from the traditional control systems, the shared communication network brings many advantages, such as low cost and easy maintenance. Under the technology wave of industry 4.0, NCSs provide a key technical support for the stable and optimal operation of real-time system, and have been widely applied to automobile manufacturing, robot control, smart home, smart grid, mobile sensor network, and automatic vehicle formation [2]. Although the shared communication network brings a great deal of convenience, the malicious attacks make that NCSs face a severe security challenge. Therefore, it is of great significance to minimize the impact of cyber attacks on NCSs and realize system security.

It is noted that the typical cyber-attacks mainly cover denial-of-service (DoS) attacks [3], deception attacks [4], and replay attacks [5]. For the deception attacks, the attacker injects false data into the communication packet by tampering with the sensor transmission data or control commands, so as to degrade the system performance. In fact, the false data injection attacks are recognized as a typical representative of deception attacks. Recently, a wonderful amount of significant research works have been developed for the corresponding security control system with deception attacks [6-9].

It is well-known that Markov jump systems (MJSs), as a special kind of dynamical systems [10-14], are well suitable to model physical systems subject to sudden environment disturbances, random failures, and abrupt variation of the operating point, which have extensive applications in the fields of power systems, financial systems, and aerospace [15-19]. For MJSs, the transition rate (TR) between system modes is sojourn-time-independent, in which the sojourn time (ST) obeys an exponential distribution. However, it is
difficult for many industrial processes to satisfy this strict condition. More generally, the ST may follow Gaussian distribution, Weibull distribution, and phase distribution. Due to the extended probability distribution, the TR of semi-Markov jump systems (S-MJSs) is ST-dependent. Obviously, S-MJSs are less conservative in modeling practical systems than MJSs. Very recently, lots of remarkable results have been proposed for S-MJSs, such as stability and stabilization [20-25], SMC [26-34], event-triggered scheme [35,36], time delay [37], and disturbance rejection [38]. As a description form of differential algebraic (difference algebraic) equations, singular systems can better maintain the physical property of dynamical systems [39]. Along with the successful application of singular systems in practical problems, singular systems have become a hot research topic in the control field [40,41]. Combining stochastic switching systems, many significant results have been proposed for singular MJSs and singular S-MJSs [31,34,42,43].

Meantime, many practical systems always exhibit the high nonlinear property, such as economics, systems engineering, medicine, psychology, and welding process, so it is difficult to model and control these systems directly. Takagi-Sugeno (T-S) approach provides an effective tool to characterize complex nonlinear systems [44], in which the main characteristic is that the nonlinear dynamic is represented by a set of local linear systems through the nonlinear fuzzy membership functions. Also, the fuzzy systems can approximate the nonlinear systems with arbitrary accuracy, which is convenient for system analysis. Recently, combining with stochastic switching systems, the research of T-S fuzzy systems has attracted great interest in the control community [34,28,34,45-48].

SMC has been recognized as a classical robust control strategy owing to its robustness against modeling error and parameter variations [49-56]. Considering the advantages of simple structure, fast response, and strong robustness, SMC law can realize high-performance control of dynamical systems. Very recently, a review of literature shows that [53,55,63-66], more specifically, the main contributions are summarized as: (i) Different from the sliding surface dependent of system mode [20,30-32,35,42,43,47,61], a common sliding surface is adopted to avoid the instability owing to the repeated jumping of the sliding surface. (ii) To get rid of the constraint condition of completely known TR or bounded TR [20-23,25-38], partly unknown TR is adopted for stochastic admissibility of the corresponding system. (iii) The proposed security SMC law can guarantee that the specified sliding region is reachable within a finite-time level.

**Notations:** $\mathcal{P} > 0$ means the positive definite matrix. $\Xi \{ \cdot \}$ is the mathematical expectation. $He(\mathcal{P})$ represents $\mathcal{P} + \mathcal{P}^T$.

**II. PRELIMINARY**

Consider the following networked fuzzy singular S-MJSs:

Plant Rule $\alpha$: IF $\varsigma_1(t)$ is $W_{\alpha_1}$, and $\varsigma_2(t)$ is $W_{\alpha_2}$, and ... and $\varsigma_p(t)$ is $W_{\alpha_p}$, THEN

$$
E \dot{y}(t) = (A_\alpha(\gamma_t) + \Delta A_\alpha(\gamma_t, t))y(t) + B_\alpha(\gamma_t)u^d(t),
$$

where $W_{\alpha_\beta}$ ($\alpha = 1, 2, ..., \hat{h}$, $\beta = 1, 2, ..., \hat{p}$) are the fuzzy sets with $\varsigma_1(t)$, $\varsigma_2(t)$, ..., $\varsigma_p(t)$ being the premise variables, $y(t)$ and $u^d(t)$ represent the state and the input. The singular matrix $E$ is considered as $\text{Rank}(E) < n$. $A_\alpha(\gamma_t)$ and $B_\alpha(\gamma_t)$ are the appropriately dimensional matrices. $\Delta A_\alpha(\gamma_t)$ are norm-bounded, i.e. $\Delta A_\alpha(\gamma_t, t) = \mathcal{L}_\alpha(\gamma_t)F_\alpha(\gamma_t, t)H_\alpha(\gamma_t)$ with known real-constant matrices $\mathcal{L}_\alpha(\gamma_t)$, $H_\alpha(\gamma_t)$ and unknown real-constant matrix $F_\alpha(\gamma_t, t)$ satisfying $F_\alpha(\gamma_t, t) F_\alpha(\gamma_t, t)^T \leq I$.

The stochastic process $\{ \gamma_t, t \} \geq 0 : = \{ \gamma_t, t \} \in \mathbb{N}_{\geq 1}$ means continuous-time and discrete-state homogeneous semi-Markov process (SMP) in $\Theta = \{ 1, 2, ..., J \}$ with the TR matrix $A(\tau) = \{ \lambda_{\varpi \tau}(\tau) \}$, where $\{ \gamma_t, t \} \in \mathbb{N}_{\geq 1}$ is the system mode index and $\{ \gamma_t, t \} \in \mathbb{N}_{\geq 1}$ is the ST of $\gamma_{t-1}$ between the $(l - 1)$th transition and $l$th transition with the probability transitions:

$$
\begin{align*}
Pr(\gamma_{t+1} = \tau, \gamma_{t+1} \leq t + 1 | \gamma_t = \varpi, \gamma_{t+1} > t) &= \lambda_{\varpi \tau}(\tau) \Delta \sigma(\Delta), \varpi \neq \tau, \\
Pr(\gamma_{t+1} = \tau, \gamma_{t+1} \leq t + 1 | \gamma_t = \varpi, \gamma_{t+1} > t) &= 1 + \lambda_{\varpi \tau}(\tau) \Delta \sigma(\Delta), \varpi = \tau,
\end{align*}
$$

where $\lambda_{\varpi \tau}(\tau) \geq 0$ denotes the TR for $\varpi \neq \tau$, and

$$
\sum_{\tau = 1, \varpi \neq \tau}^{J} \lambda_{\varpi \tau}(\tau) = -\lambda_{\varpi \tau}(\tau) < 0.
$$
In this paper, the TR is partly unknown in $\Lambda_i = \{\lambda_{\tau}(i)\}$. For $\forall \tau \in \emptyset$, $\emptyset^\tau = \emptyset^\tau_k \cup \emptyset^\tau_{\omega k}$ with

$$\emptyset^\tau_k \triangleq \{ \tau : \lambda_{\tau}(i) \text{ is known, for } \tau \in \emptyset \},$$

$$\emptyset^\tau_{\omega k} \triangleq \{ \tau : \lambda_{\tau}(i) \text{ is unknown, for } \tau \in \emptyset \}.$$  

If $\emptyset^\tau \neq \emptyset$, it is further given by

$$\emptyset^\tau_k \triangleq \{ k^\tau_1, k^\tau_2, ..., k^\tau_m \}, 1 \leq m \leq j,$$  

where $k^\tau_m$ means the $m$th known TR of $\emptyset^\tau_k$ in the $\omega$th row of $\Lambda_i$.

**Remark 1.** In practical systems, there exists the difficulty to obtain precise information of TR owing to some complicated factors. For example, the packet loss and delay always exist in NCSSs characterized by the SMP under the ideal framework of of completely known TR. Either the delay or the packet loss is vague owing to stochastic complexity of the network environment, that is, to get an expected TR matrix is seldom possible. Compared with completely unknown TR and bounded TR [20-23,25-38], partly unknown TR is adopted to model stochastic semi-Markov switching process to reduce some conservatism.

When $\gamma_i = \infty$, define $A_{\alpha}(\gamma_i) \triangleq A_{\alpha, \infty}$ and $B_{\alpha}(\gamma_i) \triangleq B_{\alpha, \infty}$. Here, it is assumed that $B_{\alpha, \infty} = B_{\infty}$, for $\alpha = 1, 2, ..., h$.

Consider the following deception attacks model:

$$u^{\Delta}(t) = u(t) + \kappa(t)Q_{\omega}(t)R_{\omega}(y(t), t),$$  

where the nonlinearity $N_{\omega}(y(t), t)$ satisfies $\| N_{\omega}(y(t), t) \| \leq \psi_{\omega}(y(t), t)$ with known function $\psi_{\omega}(y(t), t) > 0$. The unknown weighting matrix $Q_{\omega}(t)$ denotes the attack satisfying $\| Q_{\omega}(t) \| \leq q_{\omega}$ with $q_{\omega} > 0$. $\kappa(t)$ is subject to the Bernoulli distribution with

$$P_r\{\kappa(t) = 1\} = \Xi(\kappa(t)) = \pi,$$

$$P_r\{\kappa(t) = 0\} = 1 - \Xi(\kappa(t)) = 1 - \pi, \forall \pi \in [0, 1].$$

By fuzzy blending, the networked fuzzy singular model is inferred as:

$$E\dot{y}(t) = \sum_{\alpha=1}^{h} L_{\alpha}(\varsigma(t))[(A_{\alpha, \infty} + \Delta A_{\alpha, \omega}(t))y(t) + B_{\omega}(u(t) + \kappa(t)Q_{\omega}(t)R_{\omega}(y(t), t))],$$  

where $\varsigma(t) = [\varsigma_1(t), \varsigma_2(t), ..., \varsigma_p(t)]^T$, and $L_{\alpha}(\varsigma(t))$ is the membership function given by

$$L_{\alpha}(\varsigma(t)) = \prod_{\beta=1}^{p} \frac{\varsigma_\alpha^{\beta}(\varsigma_\beta(t))}{\sum_{\alpha=1}^{h} \sum_{\beta=1}^{p} \varsigma_\alpha^{\beta}(\varsigma_\beta(t))},$$  

where $\varsigma_\alpha^{\beta}(\varsigma_\beta(t))$ is the grade of membership of $\varsigma_\beta(t)$ in $\varsigma_\alpha$. Besides, it is clear that $\sum_{\alpha=1}^{h} L_{\alpha}(\varsigma(t)) = 1$ with $L_{\alpha}(\varsigma) \geq 0$ for $t \geq 0$.

Consider the nominal singular system as

$$E\dot{y}(t) = A_{\infty}y(t).$$  

**Definition 1.** [43] System (6) is said to be: (i) regular if $\det(sE - A_{\infty})$ is not identically zero; (ii) impulse free if $\deg(sE - A_{\infty}) = \text{rank}(E)$; (iii) stochastically stable, if there holds $E\{q^2(y(s))ds\} |_{y(0), \tau_0} \leq T(y(0), \tau_0)$, for any initial condition $(y(0), \tau_0)$ with $T(y(0), \tau_0) > 0$; (iv) stochastically admissible, when (i), (ii), and (iii) are satisfied simultaneously.

**III. STOCHASTIC ADMISSIBILITY ANALYSIS**

As stated in [31], the repeated switching of sliding surface is inevitable. Meantime, the switching number of sliding surface will be increased by the jumps of semi-Markov switching mode. For some details, please see Ref. [31]. In order to reduce the frequent switching, a common sliding surface is designed as

$$s(t) = G\mathcal{E}y(t),$$  

where $G = \sum_{\tau=1}^{j} \alpha_{\tau}B_{\tau}^T$. In fact, $B_{\omega}$ does not need to satisfy full column rank. Design the security SMC law as:

$$u(t) = B_{\tau}^TP_{\omega}y(t) - (\pi q_{\omega}\psi_{\omega}(y(t), t)) + \chi_{\omega}\text{sgn}(\phi_{\omega}(\Psi\tilde{G}B_{\omega})^T s(t)),$$  

where the nonsingular matrix $P_{\omega}$ and the positive constant $\chi_{\omega}$ will be given later.

Therefore, based on the security SMC law (8), one has

$$E\dot{y}(t) = \sum_{\alpha=1}^{h} E\dot{\varsigma}(t) \bigg[ (A_{\alpha, \infty} + \Delta A_{\alpha, \omega}(t))y(t) + B_{\omega}(u(t) + \kappa(t)Q_{\omega}(t)R_{\omega}(y(t), t)) \bigg].$$  

**Theorem 1.** System (9) is stochastically admissible, if there exist $\Psi > 0$, nonsingular matrix $P_{\omega} > 0$ and scalar $\phi_{\omega} > 0$, such that for $\varpi \in \emptyset,$

$$E^TP_{\omega} = P_{\omega}E \geq 0,$$

$$P_{\omega}B_{\omega} = \phi_{\omega}E^T\tilde{G}^TP_{\omega} \Psi \tilde{G}B_{\omega},$$

$$\Theta_{\alpha, \omega}(t) < 0,$$  

where

$$\Theta_{\alpha, \omega}(t) = H \epsilon \{P_{\omega}^T\tilde{A}_{\alpha, \omega}(t) + P_{\omega}^T\tilde{B}_{\omega}B_{\omega}^TP_{\omega}\}$$

$$+ \sum_{\tau=1}^{j} \tilde{\gamma}_{\tau}E^TP_{\tau},$$

$$\tilde{A}_{\alpha, \omega}(t) = A_{\alpha, \infty} + \Delta A_{\alpha, \omega}(t).$$

**Proof.** First, we prove that the system (9) is regular and impulse free. Since $\text{rank}(E) = r < n$, there are two nonsingular matrices $D$ and $F$ satisfying the following conditions

$$D\mathcal{E}F = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},$$

$$D\tilde{A}_{\alpha, \omega}(t)F = \begin{bmatrix} A_{\alpha, \infty} & \tilde{A}_{\alpha, \omega} \\ \tilde{A}_{\alpha, \omega} & \tilde{A}_{\alpha, \omega} \end{bmatrix},$$

$$D^TP_{\omega}F = \begin{bmatrix} P_{1, \omega} & P_{2, \omega} \\ P_{3, \omega} & P_{4, \omega} \end{bmatrix}. $$  

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According to (10), one has \( \mathcal{P}_{2i} = 0 \). Pre- and post-multiplying (11) by \( \mathcal{F}^T \) and \( \mathcal{F} \) gives
\[
\begin{bmatrix}
\mathcal{F}^T \mathcal{A}_{4a, \omega} \mathcal{P}_{4a, \omega} + \mathcal{A}_{4a, \omega}^T \mathcal{F} \mathcal{P}_{4a, \omega}
\end{bmatrix} < 0,
\]
where * will not be used in the following development. Then, it is got that \( \mathcal{A}_{4a, \omega} \mathcal{P}_{4a, \omega} + \mathcal{A}_{4a, \omega}^T \mathcal{P}_{4a, \omega} < 0 \), which means that \( \mathcal{A}_{4a, \omega} \) is nonsingular. System (9) is therefore regular and impulse-free.

Next, prove the stochastic stability. Choosing Lyapunov function \( \Gamma(y(t), \omega) = y^T(t) \mathcal{P} y(t) \) results in
\[
\varphi \Gamma(y(t), \omega) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ \Gamma(y(t + \Delta), \tau + \Delta)|\bar{\tau} = \omega, \tau + \Delta > t \right\} - \Gamma(y(t), \omega)
\]
\[
\varphi \Gamma(y(t), \omega) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ \sum_{\tau=1, \tau \neq \omega} \mathcal{P}_{\tau n + 1} = \tau, \tau + 1 < \tau + \Delta | y^T(t + \Delta) \mathcal{P} y(t + \Delta) + \mathcal{P}_{\tau n + 1} = \tau, \tau + 1 > i \right\}
\]
\[
\varphi \Gamma(y(t), \omega) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ \sum_{\tau=1, \tau \neq \omega} q_{\omega, \tau} \right\} \left( y^T(t + \Delta) \mathcal{P} y(t + \Delta) - y^T(t) \mathcal{P} y(t) \right)
\]
where \( q_{\omega, \tau} \) is the probability intensity from \( \omega \) to \( \tau \) and \( \mathbb{E}_{\omega}(i) \) is the cumulative distribution functions of ST. According to
\[
\lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ y^T(t + \Delta) - y^T(t) \right\} = \lambda_{\omega}(i),
\]
and Taylor-series formula
\[
\mathbb{E} y(t + \Delta) = \mathbb{E} y(t) + \mathbb{E} \dot{y}(t) \Delta + o(\Delta), \Delta \to 0,
\]
one has
\[
\varphi \Gamma(y(t), \omega) = \sum_{\omega=1}^h \mathbb{E}_{\alpha}(\xi(t)) \left( y^T(t) \mathcal{P}_{\omega, \omega} y(t) + \mathcal{P}_{\omega, \omega}^T \mathcal{F} \mathcal{P} y(t) \right)
\]
where \( \mathcal{L}_{\omega, \tau} = \mathbb{E} \{ \lambda_{\omega, \tau}(i) \} = \int_0^\infty \lambda_{\omega, \tau}(i) F_{\omega}(i) di \) with the probability density function \( F_{\omega}(i) \) of ST.

Based on the condition (11), it is got that
\[
-2y^T(t) \mathcal{P} y(t) \mathcal{R}_{\omega, \omega} \mathcal{R}_{\omega, \omega} y(t) sgn(\mathcal{G} \mathcal{P} y(t)) s(t) + 2y^T(t) \mathcal{P} \mathcal{R}_{\omega, \omega} \mathcal{R}_{\omega, \omega} \mathcal{G} \mathcal{P} y(t) s(t) + 2y^T(t) \mathcal{P} \mathcal{R}_{\omega, \omega} \mathcal{R}_{\omega, \omega} \mathcal{G} \mathcal{P} y(t) s(t) \leq -2\mathbb{E} \{ \mathcal{P} y(t) \mathcal{P} y(t) \} = 0.
\]

It follows from (15)-(17) that
\[
\varphi \Gamma(y(t), \omega) \leq \sum_{\omega=1}^h \mathbb{E}_{\alpha}(\xi(t)) y^T(t) \Theta_{\omega, \omega}(t) y(t),
\]
which means that
\[
\varphi \Gamma(y(t), \omega) < 0,
\]
under the condition (12).

Therefore, system (9) is stochastically admissible.

**Remark 2.** In comparison with ST subject to the traditional exponential distribution in SMC for MSSs [18,42,43,61], it is required to recompute the weak infinitesimal operator in line with SMP constraint. Under the framework of Taylor-series formula, it is got that
\[
\lim_{\Delta \to 0^+} \frac{1}{\Delta} \mathbb{E} \left\{ y^T(t + \Delta) - y^T(t) \right\} = \lambda_{\omega}(i),
\]

**Theorem 2.** System (9) is stochastically admissible, if there exist symmetric matrix \( \mathcal{P}_{1, 0} > 0 \), matrices \( \Psi > 0, \mathcal{Q}_{1, \omega}, \mathcal{P}_{\omega, \omega} \) and scalars \( \varepsilon_{1, \omega} > 0, \delta > 0, \phi_{1, \omega} > 0 \), such that the following conditions are satisfied for all \( \omega \in \emptyset \),
\[
\begin{bmatrix}
-\delta I & \mathcal{P} \mathcal{B} - \phi \mathcal{G} \mathcal{P} \mathcal{B} \mathcal{P}^T \mathcal{G}^T \\
\mathcal{P}^T \mathcal{B} & -\mathcal{I}
\end{bmatrix} \leq 0,
\]
\[
\Theta_{1,0, \omega} < 0,
\]
\[
\mathcal{E}^T \mathcal{P} \mathcal{P} - \mathcal{P}_{1, \omega} \leq 0, \tau \in \mathbb{N}_{\omega}, \tau \neq \omega,
\]
\[
\mathcal{E}^T \mathcal{P} \mathcal{P} \mathcal{P} - \mathcal{P}_{1, \omega} \geq 0, \tau \in \mathbb{N}_{\omega}, \tau \neq \omega,
\]
\[
\Theta_{1,0, \omega} = \left[ \begin{array}{c}
\Theta_{1,0, \omega}^{11} \\
\Theta_{1,0, \omega}^{12}
\end{array} \right],
\]
\[
\Theta_{1,0, \omega}^{11} \mathcal{P} = \mathcal{E}^T \mathcal{W} \mathcal{E} + \mathcal{W} \mathcal{Q}_{1, \omega} \mathcal{E}^T \mathcal{W} = 0.
\]
**Proof.** Based on \( \mathcal{P}_{1, \omega} = \mathcal{U}_{\omega} \mathcal{E} + \mathcal{W} \mathcal{Q}_{1, \omega} \mathcal{E} \) and \( \mathcal{E}^T \mathcal{W} = 0 \), the condition (10) holds.
For the uncertainty in (12), one has
\[
2y^T(t)P_{\infty}^T \Delta A_{\infty\alpha}(t)y(t) \\
\leq \varepsilon_{\infty}^{-1}y^T(t)P_{\infty}^T L_{\alpha\infty}L_{\alpha\infty}^T P_{\infty} y(t) \\
+ \varepsilon_{\infty}y^T(t)H_{\alpha\infty}^T H_{\alpha\infty} y(t).
\] (23)

Regarding \(\sum_{t=1}^{s_{\infty}} P_{\infty} = 0\), results in
\[
\varphi \Gamma(y(t), \infty) \leq \sum_{t=1}^{s_{\infty}} L_{\alpha}(\varsigma(t))y^T(t)\Theta_{2\infty\alpha}(t)y(t) \\
= \sum_{t=1}^{s_{\infty}} L_{\alpha}(\varsigma(t))y^T(t)\Theta_{3\infty\alpha}(t)y(t), (24)
\]
where
\[
\Theta_{2\infty\alpha} = H e^{[P_{\infty}^T A_{\infty\alpha} + P_{\infty}^T B_{\infty} B_{\infty}^T P_{\infty}]} \\
+ \varepsilon_{\infty}^{-1}P_{\infty}^T L_{\alpha\infty}L_{\alpha\infty}^T P_{\infty} + \varepsilon_{\infty}H_{\alpha\infty}^T H_{\alpha\infty} \\
+ \sum_{t=1}^{s_{\infty}} \lambda_{\infty}(t)e^{T} P_{\infty} \lambda_{\infty}(t)e^{T}, \\
\Theta_{3\infty\alpha} = H e^{[P_{\infty}^T A_{\infty\alpha} + P_{\infty}^T B_{\infty} B_{\infty}^T P_{\infty}]} \\
+ \varepsilon_{\infty}^{-1}P_{\infty}^T L_{\alpha\infty}L_{\alpha\infty}^T P_{\infty} + \varepsilon_{\infty}H_{\alpha\infty}^T H_{\alpha\infty} \\
+ \sum_{t \in A_{\infty\alpha}} \lambda_{\infty}(t)e^{T} P_{\infty} \lambda_{\infty}(t)e^{T} \\
+ \sum_{t \in B_{\infty\alpha}} \lambda_{\infty}(t)e^{T} P_{\infty} \lambda_{\infty}(t)e^{T}.
\]

According to \(\lambda_{\infty} < 0\) (\(\forall \infty, \tau \in \emptyset, \infty \neq \tau\)) and \(\lambda_{\infty} \geq 0\) (\(\forall \infty, \tau \in \emptyset, \infty = \tau\)), if \(\infty \in A_{\infty\alpha}\), there holds
\[
\varphi \Gamma(y(t), \infty) < 0,
\] (25)
under (20) and (21). If \(\infty \in B_{\infty\alpha}\), there also holds (25) under (20)-(22).

Applying the equality (10) yields
\[
Trace[(P_{\infty}^T B_{\infty} - \phi_{\infty} e^{T} G^T \Psi GB_{\infty})^T \\
(P_{\infty}^T B_{\infty} - \phi_{\infty} e^{T} G^T \Psi GB_{\infty})] = 0,
\]
which means that
\[
(P_{\infty}^T B_{\infty} - \phi_{\infty} e^{T} G^T \Psi GB_{\infty})^T \\
(P_{\infty}^T B_{\infty} - \phi_{\infty} e^{T} G^T \Psi GB_{\infty}) < \delta T,
\] (26)
with positive small scalar \(\delta\).

Therefore, (19) holds.

**Remark 3.** Different from the existing literature about SMC for stochastic switching systems with completely known TR or bounded TR [18,20-23,25-34,42,43,61], this paper is based on the partly unknown TR. By means of the sum of every row being equal to zero, the TR matrix is divided into known part and unknown part. Under the framework of free weight matrix, a sufficient condition for the stochastic admissibility of the system is established in Theorem 2.

**IV. REACHABILITY OF SLIDING SURFACE**

An appropriate security SMC mechanism is designed to guarantee finite-time reachability of the sliding surface.

**Theorem 3.** Consider the solvable parameters \(P_{\infty}\) and \(\Psi\) in Theorem 2. The states of system (1) will be driven onto the sliding region under the condition of security SMC law (8) with \(\chi_{\infty}\) satisfying
\[
\chi_{\infty} \parallel \Psi GB_{\infty} \parallel - \varrho > 0,
\] (27)
where \(\varrho > 0\).

**Proof.** Choose Lyapunov function \(\Gamma(s(t)) = \frac{1}{2} s^T(t)\Psi s(t)\). By the expressions (7) and (9), one has
\[
\dot{s} \Gamma(s(t)) = s^T(t)\Psi GB \dot{y}(t) \\
= \sum_{t=1}^{s_{\infty}} L_{\infty}(\varsigma(t)) s^T(t)\Psi G[(A_{\infty\infty} + \Delta A_{\infty\infty})(t)y(t) + B_{\infty}\tilde{u}_{\infty}(t) + \tilde{u}_{\infty}(t)] \\
- \tilde{q}_{\infty}(\varsigma(t),t) s^T(t)\Psi G \tilde{u}_{\infty}(t) + \kappa(t) \| Q_{\infty}(y(t),t) \| \\
\leq \| s(t) \| \| \Psi G A_{\infty\infty} \| + \| \Psi G L_{\infty\infty} \| \| H_{\infty\infty} \| + \| \Psi G B_{\infty} \| s(t) \| - \chi_{\infty} \| s(t) \| \| \Psi G B_{\infty} \| - \varrho \| s(t) \|,
\] (28)
where
\[
a_{\infty} = max_{\alpha=1,2,..h\alpha\infty\infty} \{ \| \Psi G A_{\infty\infty} \| + \| \Psi G L_{\infty\infty} \| \| H_{\infty\infty} \| + \| \Psi G B_{\infty} \| \}
\]
For the following domain
\[
\sum_{t=1}^{s_{\infty}} \{ y(t) : a_{\infty} \parallel y(t) \parallel \leq \chi_{\infty} \parallel \Psi G B_{\infty} \parallel - \varrho \},
\]
it is got that
\[
\| s(t) \| \| a_{\infty} \parallel y(t) \parallel - (\chi_{\infty} \parallel \Psi G B_{\infty} \parallel - \varrho) \| \leq 0,
\]
which means \(\varphi \Gamma(s(t)) \leq -\varrho \| s(t) \| < 0\) for \(s(t) \neq 0\).

Therefore, the system states can be driven onto the sliding region and kept there in subsequent time.

**Remark 4.** The sliding region \(s_{\infty}\) is shown for the corresponding proof of reachability analysis. Under the unavoidable chattering effect, the ideal sliding motion will be broken on the sliding surface. In this instance, the sliding motion is set to be a local neighborhood around the predefined sliding surface.

**Remark 5.** For the difficulty \(Q\), a common sliding surface is chosen to reduce the frequent switching of sliding dynamics.

Next, the designed SMC law (8) is related to the corresponding parameters \(\pi, q_{\infty}\), and \(\psi_{\infty}(y(t),t)\) of deception attacks, the specified matrix \(P_{\infty}\), and the positive constant \(\chi_{\infty}\). Moreover, a reasonable sliding region \(\sum_{s}\) is adopted to characterize the reachability analysis. Therefore, under the deception attacks and the partly unknown TR, one has better dynamic performance by the designed SMC mechanism.

**V. SIMULATION**

Consider the single-link robot arm [45] described by
\[
\dot{\xi}(t) = -\frac{M(t)q_{\infty}}{J(t)} \sin(\xi(t)) - \frac{D(t)}{J(t)} \dot{\xi}(t) \\
- \frac{1}{J(t)} u^A(t),
\] (29)
where $\xi(t)$ is angle position of the arm, $u(t)$ is control input, and $M(\omega_0^2), J(\omega_0^2), D(t), g$, and $L$ mean mass of the payload, moment of inertia, coefficient of viscous friction, gravity acceleration, and arm length with $D(t) = D_0 = 2, g = 9.81$, and $L = 0.5$. $M(\omega_0^2)$ and $J(\omega_0^2)$ are with three different modes subject to the SMP in $\{1, 2, 3\}$ as $M_1 = 1, M_2 = 1.5, M_3 = 2.0, J_1 = 1, J_2 = 2.0, J_3 = 2.5$.

The three-mode TR matrix is given by

$$
\Lambda(t) = \begin{bmatrix}
-4t & ? & ? \\
3t^2 & -6t^2 & 3t^2 \\
? & ? & ?
\end{bmatrix}
$$

(30)

Consider that the ST obeys Weibull distribution with the probability density function given as $f_{\omega}(\omega) = \frac{\beta}{\alpha}t^{\beta-1}\exp[-(\frac{\omega}{\alpha})^\beta], \alpha \geq 0$. In particular, for $\omega = 1, \alpha = 1, \beta = 2, F(1) = 2e^{-t^2}$; for $\omega = 2, \alpha = 2, \beta = 3, F(2) = 3t^2e^{-t^4}$; for $\omega = 3, \alpha = 3, \beta = 4, F(3) = 4t^3e^{-t^4}$. Then, one has $\Xi\{\lambda(21)\} = \int_{0}^{\infty}3t^2F(2)dt = \int_{0}^{\infty}9t^2e^{-t^4}dt = 2.7082$. Through similar algebraic operation, it is got that

$$
\Xi\{\Lambda(t)\} = \begin{bmatrix}
-3.5449 & ? & ? \\
2.7082 & -5.4164 & 2.7082 \\
? & ? & ?
\end{bmatrix}
$$

It is assumed that $\xi(t) = \eta\xi(t)$ with the constant $\eta$. When $\omega = \omega_0$, (29) is rewritten as

$$
\ddot{\xi}(t) = \frac{-\eta M_2 gL}{J_2} \sin(\xi(t)) - \frac{D(t)}{J_2} \dot{\xi}(t) + \frac{\eta}{J_2} u(t),
$$

$0 = \xi(t) - \eta \dot{\xi}(t)$.

Define $y_1(t) = \xi(t)$ and $y_2(t) = \dot{\xi}(t)$. $\sin(y_1(t))$ is expressed as

$$
\sin(y_1(t)) = L_1(y_1(t))y_1(t) + \phi L_2(y_1(t))y_1(t),
$$

where $\phi = \frac{0.01}{\pi}$, $L_1(y_1(t)), L_2(y_1(t)) \in [0, 1], L_1(y_1(t)) + L_2(y_1(t)) = 1$.

Therefore, one has

$$
L_1(y_1(t)) = \begin{cases}
\frac{\sin(y_1(t)-\phi)}{y_1(t)(1-\phi)} , & \text{if } y_1(t) \neq 0, \\
1, & \text{if } y_1(t) = 0;
\end{cases}
$$

$$
L_2(y_1(t)) = \begin{cases}
\frac{y_1(t)-\sin(y_1(t))}{y_1(t)(1-\phi)} , & \text{if } y_1(t) \neq 0, \\
0, & \text{if } y_1(t) = 0,
\end{cases}
$$

in which if $y_1(t)$ is about 0 rad, then $L_1(y_1(t)) = 1$, and if $y_1(t)$ is about $\pi$ rad or $-\pi$ rad, then $L_2(y_1(t)) = 0$.

Thus, the uncertain fuzzy system with deception attacks is presented as

Plant Rule 1: If $y_1(t)$ is about 0 rad, then

$$
\dot{E}\dot{y}(t) = (A_{1,\omega} + \Delta A_{1,\omega}(t))y(t) + B_{1}(u(t) + Q_{1}(t)) R_{1}(y(t), t);
$$

Plant Rule 2: If $y_1(t)$ is about $\pi$ or $-\pi$ rad, then

$$
\dot{E}\dot{y}(t) = (A_{2,\omega} + \Delta A_{2,\omega}(t))y(t) + B_{2}(u(t) + Q_{1}(t)) R_{1}(y(t), t),
$$

where

$$
A_{1,1} = \begin{bmatrix}
-\eta gL - D_0 & 0 \\
1 & -\eta
\end{bmatrix},
B_1 = \begin{bmatrix}
0 \\
0
\end{bmatrix},
$$

$$
A_{1,2} = \begin{bmatrix}
-0.75\eta gL - 0.5D_0 & 0 \\
1 & -\eta
\end{bmatrix},
B_2 = \begin{bmatrix}
0.5\eta \\
0
\end{bmatrix},
$$

$$
A_{1,3} = \begin{bmatrix}
-0.8\eta gL - 0.4D_0 & 0 \\
1 & -\eta
\end{bmatrix},
B_3 = \begin{bmatrix}
0.4\eta \\
0
\end{bmatrix},
$$

$$
A_{2,1} = \begin{bmatrix}
-\eta gL - D_0 & 0 \\
1 & -\eta
\end{bmatrix},
B_2 = \begin{bmatrix}
0 \\
0
\end{bmatrix},
$$

$$
A_{2,2} = \begin{bmatrix}
-0.75\eta gL - 0.5D_0 & 0 \\
1 & -\eta
\end{bmatrix},
B_3 = \begin{bmatrix}
0 \\
0
\end{bmatrix},
$$

$$
A_{2,3} = \begin{bmatrix}
-0.8\eta gL - 0.4D_0 & 0 \\
1 & -\eta
\end{bmatrix},
B_3 = \begin{bmatrix}
0 \\
0
\end{bmatrix},
$$

$$
E = \begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix},
L_{1,1} = \begin{bmatrix}
0.1 \\
0.1
\end{bmatrix},
L_{1,2} = \begin{bmatrix}
0.2 \\
0.1
\end{bmatrix},
$$

$$
L_{1,3} = \begin{bmatrix}
0.1 \\
0.1
\end{bmatrix},
L_{2,1} = \begin{bmatrix}
0.3 \\
0.1
\end{bmatrix},
L_{2,2} = \begin{bmatrix}
0.1 \\
0.2
\end{bmatrix},
$$

$$
L_{2,3} = \begin{bmatrix}
0.1 \\
0.1
\end{bmatrix},
H_{1,1} = \begin{bmatrix}
0.1 \\
0.1
\end{bmatrix},
H_{1,2} = \begin{bmatrix}
0 & 0.1
\end{bmatrix},
$$

$$
H_{2,1} = \begin{bmatrix}
0.2 \\
0.1
\end{bmatrix},
H_{2,2} = \begin{bmatrix}
0.2 & 0
\end{bmatrix},
$$

$$
H_{2,3} = \begin{bmatrix}
0.1 & 0.1
\end{bmatrix},
F_{\omega,\omega}(t) = \sin(t), \alpha = 1, \omega = 1, 2, 3, 
Q_1(t) = 0.5\cos(t), Q_1(y(t), t) = 0.5y_1^2\cos(t),
Q_2(t) = 0.5\sin(t), Q_2(y(t), t) = 0.5y_1^2\sin(t),
Q_3(t) = 0.5cost, Q_3(y(t), t) = 0.5y_1^2\cos(t),
\eta = 2, \tau = 0.5.
$$

For given $\rho = 0.2$ and $\phi = 0.07$, solving Theorem 2 gives rise to

$$
\Psi = 0.7027,
$$

$$
U_1 = \begin{bmatrix}
0.2065 & -0.0001 \\
-0.0001 & 3.2683
\end{bmatrix},
Q_1 = \begin{bmatrix}
0.0001 \\
1.2332
\end{bmatrix},
$$

$$
U_2 = \begin{bmatrix}
0.4828 & 0.0002 \\
0.0002 & 3.2683
\end{bmatrix},
Q_2 = \begin{bmatrix}
-0.0002 \\
-0.3861
\end{bmatrix},
$$

$$
U_3 = \begin{bmatrix}
0.5836 & -0.0015 \\
-0.0015 & 3.2683
\end{bmatrix},
Q_3 = \begin{bmatrix}
-0.2858 \\
-1.0838
\end{bmatrix}.
$$

Fig. 1 Mode evolution
For given initial state $y_0 = [2 \ 4]^T$, we get the simulation results in Figs. 1-4. Fig. 1 depicts the system mode. Fig. 2 shows the security SMC law $u(t)$. Fig. 3 plots the state $y(t)$ of the closed-loop system. Fig. 4 represents the sliding surface $s(t)$, which verifies the effectiveness of the proposed SMC law.

![Fig. 2 Control input](image1)

**Fig. 2 Control input**

![Fig. 3 State](image2)

**Fig. 3 State**

![Fig. 4 Sliding surface](image3)

**Fig. 4 Sliding surface**

**Remark 6.** For the SMC law design, the parameters selection plays an important role in characterizing system performance as well as its impacts on the state signals. First, for given single-link robot arm model (29), the parameters $\mathcal{M}(\tau_1)$, $\mathcal{L}$, $\mathcal{J}(\tau_1)$, and $\mathcal{D}(t)$ are chosen with appropriate values. Secondly, according to the statistical characteristics of SMP, the TR matrix is considered to be partly unknown in (30). Furthermore, to describe the dynamical behavior of single-link robot arm model, such as uncertainty and deception attacks, we choose the appropriate parameters $\mathcal{L}_\alpha, \mathcal{H}_\alpha, \mathcal{K}_\alpha, \mathcal{Q}_\alpha$, and $\psi_{\mathcal{W}}(y(t), t)$, for $\alpha = 1, 2, \mathcal{W} = 1, 2, 3$. Finally, for given constants $\phi_{\alpha} > 0$ and $\varrho > 0$, one has the corresponding parameter $P_\mathcal{W}$ by finding feasible solutions of positive-definite symmetric matrix $\mathcal{U}_\mathcal{W}$ and real matrix $\mathcal{Q}_\alpha$ in Theorem 2. In fact, the parameters selection lies in the appropriate values that are not too big or too small. It is not good for the feasible solutions that these parameters are chosen to be too small or too big. Therefore, it is of importance to choose a compromise value among them to get the feasible solutions at a reasonable computational cost.

**VI. CONCLUSION**

In this paper, a common sliding surface has been proposed for networked fuzzy singular S-MJSs with false date injection attacks. Based on the partly unknown TR, the feasible conditions have been derived to ensure the sliding dynamics stochastically admissible. Furthermore, by the synthesized security SMC law, the system states can be driven onto the predefined sliding region in a finite time. With the development of modern network communication technology, the corresponding cyber attacks also show a diversified trend. Malicious attackers may launch a variety of cyber attacks at the same time, resulting in the instability of control system performance. In the future, the security SMC will be extended to networked S-MJSs with hybrid cyber attacks.

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