Poincare algebra in chiral $QCD_2$

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Abstract

For the chiral $QCD_2$ on a cylinder, we give a construction of a quantum theory consistent with anomaly. We construct the algebra of the Poincare generators and show that it differs from the Poincare one.
1. Chiral gauge models with right–left asymmetric coupling of the matter and gauge fields are anomalous. Some of the first–class constraints at the classical level become second–class ones at the quantum level. The anomaly raises some problems. First of all, the anomalous behaviour of the constraints should be taken into account when we quantize the anomalous model and construct its quantum theory. Secondly, there is a problem of relativistic invariance, namely whether the Poincare algebra is valid in the quantum theory consistent with the anomaly.

The chiral QED$_2$ (the chiral Schwinger model) [1] is the simplest example of the anomalous models. There are different ways of the consistent canonical quantization of this model [2], [3], [4]. In the physical sector, the corresponding quantum theory turns out to be relativistically non–invariant.

In this paper, we consider another anomalous model – the chiral QCD$_2$. We assume that space is a circle of length $L$, $-\frac{L}{2} \leq x < \frac{L}{2}$, so space–time manifold is a cylinder $S^1 \times R^1$. Our aim is to construct the quantum theory of the chiral QCD$_2$ and to derive the algebra of the Poincare generators.

We use the canonical Hamiltonian formalism. For the standard non–anomalous QCD$_2$, a construction of the quantum theory was given in [5]. In our case, to incorporate the anomaly into this construction we apply the Gupta–Bleuler method.

2. We consider the most general version of the chiral QCD$_2$, namely the model in which the right–handed and left–handed components of the massless Dirac field are coupled to two different Yang–Mills fields. With $A_{\mu}^\pm = A_{\mu}^{\pm,a} \frac{1}{2} \tau^a$ the Yang–Mills fields and $\psi^\pm = \frac{1}{2}(1 \pm \gamma^5)\psi$ the Dirac fields, the Lagrangian density for our model is

$$\mathcal{L} = \mathcal{L}_+ + \mathcal{L}_-,$$

$$\mathcal{L}_\pm = -\frac{1}{2} \text{tr}(F_{\mu\nu}^\pm F^{\mu\nu}_\pm) + i \bar{\psi}_\pm \gamma^\nu D^\pm_\nu \psi_\pm,$$  

(1)

where $D^\pm_\nu = (\partial_\nu - ie_\pm A_\nu,\pm)\psi_\pm$, $\bar{\psi}_\pm = \psi^\dagger_\pm \gamma^0$, $\tau_a$ ($a = 1, 3$) are the Pauli matrices, $e_\pm$ are the coupling constants, and $F_{\mu\nu}^\pm = \partial_\mu A_\nu^\pm - \partial_\nu A_\mu^\pm - ie_\pm [A_\mu^\pm, A_\nu^\mp]$ are the YM field strength tensors.

We choose the Dirac matrices as $\gamma^0 = \tau_1$, $\gamma^1 = -i\tau_2$, $\gamma^5 = \gamma^0\gamma^1 = \tau_3$. The structure group of the YM fields is $SU(2)$ and $(\frac{1}{2} \tau^a)$ are the generators of the corresponding Lie algebra in the fundamental representation of the group.

For $e_+ = e_- \equiv e\sqrt{2}$ and $A_{\mu}^+ = A_{\mu}^- \equiv \frac{1}{\sqrt{2}} A_{\mu}$, we get from [1] the Lagrangian density of the standard QCD$_2$ with the Dirac field $\psi$ coupled to the YM field $A_{\mu}$. For $e_+ = 0$, $A_{\mu}^- = 0$ ( or $e_- = 0, A_{\mu}^+ = 0$ ), we have the model in which the YM field is coupled only to one chiral component of the Dirac field.

The classical Hamiltonian density is

$$\mathcal{H} = \mathcal{H}_+ + \mathcal{H}_-,$$

$$\mathcal{H}_\pm = \mathcal{H}_{YM}^\pm + \mathcal{H}_{F}^\pm - A^{a\pm}_{\mu} G^{a}_\pm,$$  

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We use the canonical Hamiltonian formalism. For the standard non–anomalous QCD$_2$, a construction of the quantum theory was given in [5]. In our case, to incorporate the anomaly into this construction we apply the Gupta–Bleuler method.
where $\mathcal{H}_{\text{YM}}^\pm = \frac{1}{2}(\Pi_{1,\pm}^a)^2$, with $\Pi_{1,\pm}^a$ the momenta canonically conjugate to $A_{1,\pm}^a$,

$$\mathcal{H}_{\text{F}}^\pm = \mathcal{H}_0^\pm \mp e_{\pm} j_{\pm}^a A_{1,\pm}^a,$$

with $\mathcal{H}_0^\pm = \mp i \psi_{\pm}^1 \partial_1 \psi_{\pm}$ the free fermionic Hamiltonian densities,

$$j_{\pm}^a = \psi_{\pm}^1 \frac{1}{2} \tau^a \psi_{\pm}$$

are the fermionic currents, and

$$G_{\pm} = D_1 \Pi_{1,\pm} + e_{\pm} j_{\pm}$$

are the Gauss law generators, $(D_1 \Pi_{1,\pm})^a \equiv \partial_1 \Pi_{1,\pm}^a + e_{\pm} \varepsilon_{abc} A_{1,\pm}^b \Pi_{1,\pm}^c$.

Note that $\Pi_{0,\pm}^a = 0$ are the primary constraints which imply the secondary ones $G_{\pm}^a = 0$. In what follows we will use the temporal gauge $A_{0,\pm}^a = 0$.

Two other generators of the Poincare algebra, i.e. the momentum and the boost generator, are given by

$$\mathcal{P}_{\pm} = -i \psi_{\pm}^1 \partial_1 \psi_{\pm} - \Pi_{1,a}^\pm \partial_1 A_{1,\pm}^a,$$

$$\mathcal{K}_{\pm} = x \mathcal{H}_{\pm}.$$

On the constrained submanifold $G_{\pm}^a = 0$, we get

$$\mathcal{P}_{\pm} = \pm \mathcal{H}_{\text{F}}^\pm.$$

On the circle boundary conditions for the fields must be specified. We impose the periodic ones

$$A_{1,\pm}^a\left(-\frac{L}{2}\right) = A_{1,\pm}^a\left(\frac{L}{2}\right),$$

$$\psi_{\pm}\left(-\frac{L}{2}\right) = \psi_{\pm}\left(\frac{L}{2}\right).$$

We require also that $\mathcal{H}_{\text{YM}}^\pm$ and $\mathcal{H}_{\text{F}}^\pm$ be periodic. Without loss of generality, we can put

$$\mathcal{H}_{\text{YM}}^\pm\left(\frac{L}{2}\right) = \mathcal{H}_{\text{F}}^\pm\left(\frac{L}{2}\right) = 0. \quad (2)$$

Next we transform the fields to their momentum representation which on the circle is discrete. We get

$$A_{1,\pm}^a(x) = \sum_{n \in \mathbb{Z}} A_{1,\pm}^a(n)e^{i\frac{2\pi}{L}nx},$$

$$\psi_{\pm}(x) = \frac{1}{\sqrt{L}} \sum_{n \in \mathbb{Z}} \psi_{\pm}(n)e^{i\frac{2\pi}{L}nx},$$

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and in all other cases
\[ X(x) = \frac{1}{L} \sum_{n \in \mathbb{Z}} X(n) e^{i\frac{2\pi}{L}nx} \]
for \( X = \Pi_{1,\pm}, j_{\pm}, \mathcal{H}_0^{\pm}, \mathcal{P}^{\pm}, \mathcal{K}^{\pm} \).

3. At the quantum level the fields are represented by operators which act on a Hilbert space. The canonical commutation relations for the Fourier transformed field operators are

\[
[\hat{A}_{1,\pm}^a(n), \hat{\Pi}_{1,\pm}^b(m)]_- = i\delta^{ab}\delta_{n,-m},
\]
\[
[\hat{\psi}_+(n), \hat{\psi}_\pm^+(m)]_+ = \delta_{n,m}. \tag{3}
\]

We assume that the Hilbert space is a fermionic Fock space with vacuum \(|\text{vac}; A\rangle\) such that

\[
\hat{\psi}_+(n)|\text{vac}; A\rangle = 0 \quad \text{for} \quad n > 0,
\]
\[
\hat{\psi}_\pm^+(n)|\text{vac}; A\rangle = 0 \quad \text{for} \quad n \leq 0,
\]
and

\[
\hat{\psi}_-(n)|\text{vac}; A\rangle = 0 \quad \text{for} \quad n \leq 0,
\]
\[
\hat{\psi}_\pm^-(n)|\text{vac}; A\rangle = 0 \quad \text{for} \quad n > 0.
\]

At the same time, the fermionic Fock states are functionals of \( A_{1,\pm}^a(n) \) with \( \hat{\Pi}_{1,\pm}^a(n) = -i\partial/\partial A_{1,\pm}^a(-n) \).

The fermionic currents, the Hamiltonian densities and other fermionic bilinears should be normal ordered : \( \cdots : \) with respect to the vacuum \(|\text{vac}; A\rangle\). This modifies their naive commutation relations following from 3 as Schwinger terms show up. In the momentum representation, we have

\[
[j_{\pm}^a(n), j_{\pm}^b(m)]_- = i\varepsilon_{abc}j_{\pm}^c(n + m) \pm n\delta_{n,-m}\delta_{ab}, \tag{4}
\]
\[
[\hat{\mathcal{H}}_0^\pm(n), \hat{\mathcal{H}}_0^\pm(m)]_- = \pm \frac{2\pi}{L}(n - m)\hat{\mathcal{H}}_0^\pm(n + m) \pm \frac{1}{3}(\frac{2\pi}{L})^2n(n^2 - 1)\delta_{n,-m}, \tag{5}
\]
where the second term on the r.h.s. of 4 and 5 is the Kac–Moody and Virasoro cocycles, respectively, and

\[
[\hat{\mathcal{H}}_0^\pm(n), j_{\pm}^a(m)]_- = \mp \frac{2\pi}{L}m j_{\pm}^a(n + m),
\]
with no Schwinger term arising here.

The Fourier transformed Gauss law generators are

\[
\hat{G}_{\pm}^a(n) = i\frac{2\pi}{L}n\hat{\Pi}_{1,\pm}^a(n) + \varepsilon_{abc}\sum_{p \in \mathbb{Z}} A_{1,\pm}^b(n + p)\hat{\Pi}_{1,\pm}^c(-p) + \varepsilon_{\pm}j_{\pm}^a(n).
\]
The anomaly appears as a central charge for the commutation algebra of the generators $\hat{G}_a^\pm(n)$. Indeed, we have

\[ [\hat{G}_a^\pm(n), \hat{G}_b^\pm(m)] = i\epsilon_{abc} \hat{G}_c^\pm(n + m) \pm e_n^2 n\delta_{n,-m}\delta_{ab}, \tag{6} \]

i.e. the generators $\hat{G}_a^\pm(n)$ and $\hat{G}_a^\pm(n)$ form a Kac–Moody algebra with positive and negative central charge correspondingly. This central charge destroys the first–class nature of the constraints and all constraints with non–zero Fourier index become second–class ones.

For the standard QCD$_2$ with $e_+ = e_-$, the commutation algebra of the total generators $\hat{G}^a(n) = \hat{G}_+^a(n) + \hat{G}_-^a(n)$ has vanishing central charge and so no anomaly.

In terms of states in Hilbert space, the nonvanishing central charge in (6) means that the local gauge symmetry is realized projectively [8] and that we can not define physical states as those which are annihilated by the Gauss law generators.

For the chiral Schwinger model, the gauge symmetry is abelian and the Gauss law generators are therefore scalars. This makes the anomalous behaviour of the model trivial in the sense that the Schwinger term in the commutator of the Gauss law generators is removed by a redefinition of the generators. Indeed, the Fourier transformed abelian Gauss law generators

\[ \hat{G}_\pm(n) \equiv i\frac{2\pi}{L} n \hat{\Pi}_{1,\pm}(n) + e_\pm \hat{j}_\pm(n) \]

fulfil the algebra

\[ [\hat{G}_\pm(n), \hat{G}_\pm(m)] = \pm e_n^2 n\delta_{n,-m}. \]

If we modify the generators as

\[ \hat{G}_\pm(n) \rightarrow \hat{\tilde{G}}_\pm(n) = \hat{G}_\pm(n) \mp e_\pm \frac{L}{4\pi} A_{1,\pm}(n), \]

then the modified generators commute

\[ [\hat{\tilde{G}}_\pm(n), \hat{\tilde{G}}_\pm(m)] = 0. \]

This allows us to define physical states as those which are annihilated by the modified Gauss law generators, $\hat{\tilde{G}}_\pm(n)|\text{phys}\rangle = 0 \tag{4}.$

The chiral Schwinger model is an exceptional case of models with anomaly. In contrast with the chiral Schwinger model, the anomalous behaviour of the chiral QCD$_2$ as well as other models with anomaly is non–trivial, i.e. the Schwinger term in (6) can not be removed.

To demonstrate this for the chiral QCD$_2$, let us modify the nonabelian Gauss law generators in the same way as before the abelian ones for the chiral Schwinger model:

\[ \hat{G}_\pm^a(n) \rightarrow \hat{\tilde{G}}_\pm^a(n) = \hat{G}_\pm^a(n) + \alpha_\pm e_n^2 A_{1,\pm}^a(n), \]

where $\alpha_\pm$ are arbitrary constants.
The commutator algebra for the modified generators is
\[ \{ \hat{G}_a^\pm (n), \hat{G}_b^\pm (m) \} = ie_{\pm} \varepsilon_{abc} \hat{G}_c^\pm (n + m) + i\alpha_{\pm} e_3^\pm \varepsilon_{abc} A_{1,\pm}^a (n + m) \pm e_{\pm}^2 n \delta_{n,-m} \delta^{ab} (1 \pm \frac{4\pi}{L} \alpha_\pm). \]

In the right-hand side of this equation, the second term is a new, additional Schwinger term. This term does not appear in the case of the chiral Schwinger model where the Gauss law generators are scalars. We can not choose \( \alpha_{\pm} \) in such a way that both old and new Schwinger terms vanish. For example, if we put, as before for the chiral Schwinger model, \( \alpha_{\pm} = \mp \frac{L}{4e} \), then the old Schwinger term vanishes, while the new one survives.

For the chiral QCD, to constrain physical states we act in another way. Let us note that the Gauss law constraints have a natural complex structure which relates the positive and negative Fourier modes:
\[ \hat{G}_a^\dagger (n) = \hat{G}_a^{-\pm} (-n). \]

In analogy with the Gupta–Bleuler quantization of ordinary electrodynamics we require that the physical states are annihilated only by 'half' of the Gauss law generators \( \hat{G}_a^\pm \) \( \hat{j}_b^\pm \). More precisely, we impose the constraints with positive Fourier index on the physical ket states
\[ \hat{G}_a^\pm (n) |_{\text{phys}} = 0 \quad \text{for} \quad n \geq 0. \]

Then for the constraints with negative Fourier index we have
\[ \langle_{\text{phys}} | \hat{G}_a^\pm (-n) = 0 \quad \text{for} \quad n \geq 0, \]
and therefore all expectation values of the constraints vanish on the physical states,
\[ \langle_{\text{phys}} | \hat{G}_a^\pm (n) |_{\text{phys}} = 0 \quad \text{for} \quad n \in \mathbb{Z}. \]

Eq. 4 implies also that
\[ \{ \hat{G}_a^\pm (n), \hat{j}_b^\pm (m) \} = ie_{\pm} \varepsilon_{abc} \hat{j}_c^\pm (n + m) \pm ne_{\pm} \delta^{ab} \delta_{n,-m}. \]
i.e. the fermionic currents no longer have the classical commutator relations with the Gauss law generators and therefore do not transform covariantly under gauge transformations.

However, the normal ordering : \( \cdots \) : is unique only up to finite terms. There are polynomials in \( A_{1,\pm}^a \) which can be added to the normal ordered fermionic bilinears to make them gauge–invariant (the so–called gauge covariant normal ordering \( \mathbb{F} \)). In particular, we can define the modified current operators
\[ \hat{j}_a^\pm (n) = \hat{j}_a^\pm (n) \mp \frac{L}{2\pi} e_{\pm} A_{1,\pm}^a (n) \]
which obey the desired relations
\[ \{ \hat{G}_a^\pm (n), \hat{j}_b^\pm (m) \} = ie_{\pm} \varepsilon_{abc} \hat{j}_c^\pm (n + m), \]
i.e. have canonical properties under gauge transformations.

Similarly, the Fourier components of the fermionic Hamiltonian density
\[ \hat{\mathcal{H}}^\pm_F(n) = \hat{\mathcal{H}}^\pm_0(n) \mp e_\pm \sum_{m \in \mathbb{Z}} A^a_{1,\pm}(m + n) j^a_\pm(-m) \]
do not commute with the Gauss law generators, \([\hat{\mathcal{G}}^a_\pm(n), \hat{\mathcal{H}}^\pm_F(m)] = -e_\pm^2 n A^a_{1,\pm}(m + n)\), but the modified ones
\[ \hat{\mathcal{H}}^\pm_F(n) = \hat{\mathcal{H}}^\pm_0(n) + \hat{\mathcal{M}}^\pm(n), \]
where
\[ \hat{\mathcal{M}}^\pm(n) \equiv \frac{L}{4\pi} e^2 \sum_{m \in \mathbb{Z}} A^a_{1,\pm}(m + n) A^a_{1,\pm}(-m), \]
are gauge–invariant. We see that the gauge covariant normal ordering of the fermionic Hamiltonian produces the mass terms \(\hat{\mathcal{M}}^\pm \equiv \hat{\mathcal{M}}^\pm(0)\) for the YM fields.

As known, the operators \(\hat{\mathcal{H}}^\pm_F(n)\) and \(\hat{\mathcal{H}}^\pm_{YM}(n)\) do not have a common, dense invariant domain of definition in the Hilbert space. It is, however, possible to define the sum of these operators and therefore the total Hamiltonian density, if we impose on the vacuum \(|\text{vac}; A\rangle\) the condition
\[ (i\hat{\Pi}^a_{1,\pm}(n) + \frac{e_\pm L}{\sqrt{2\pi}} A^a_{1,\pm}(n)) |\text{vac}; A\rangle = 0 \quad \text{for} \quad n \in \mathbb{Z}, \]
and order also the YM Hamiltonian density and all other YM field operators with respect to this vacuum \(|\text{phys}\rangle\).

The Fourier components of the YM Hamiltonian density are
\[ \hat{\mathcal{H}}^\pm_{YM}(n) = \frac{1}{2L} \sum_{m \in \mathbb{Z}} \hat{\Pi}^a_{1,\pm}(m + n) \hat{\Pi}^a_{1,\pm}(-m). \]
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and order also the YM Hamiltonian density and all other YM field operators with respect to this vacuum \(|\text{phys}\rangle\).

The total gauge–invariant Hamiltonian densities become
\[ \hat{\mathcal{H}}^\pm(n) = \hat{\mathcal{H}}^\pm_0(n) \mp e_\pm \sum_{m \in \mathbb{Z}} A^a_{1,\pm}(m + n) j^a_\pm(-m) \]
\[ + \frac{1}{2L} \sum_{m \in \mathbb{Z}} (\hat{\Pi}^a_{1,\pm}(m + n) \hat{\Pi}^a_{1,\pm}(-m) + \frac{e^2 L^2}{2\pi} A^a_{1,\pm}(m + n) A^a_{1,\pm}(-m)) : \]
where \(: \cdots :\) denote the normal ordering for the YM field operators.

The quantum analogues of the conditions \(\mathcal{F}\) are
\[ \hat{\mathcal{H}}^\pm_F(\frac{L}{2}) |\text{phys}\rangle = \hat{\mathcal{H}}^\pm_{YM}(\frac{L}{2}) |\text{phys}\rangle = 0, \]
or, equivalently,
\[ \sum_{n \in \mathbb{Z}} (-1)^n \hat{\mathcal{H}}^\pm_F(n) |\text{phys}\rangle = 0, \]
\[ \sum_{n \in \mathbb{Z}} (-1)^n \hat{\mathcal{H}}^\pm_{YM}(n) |\text{phys}\rangle = 0. \]
The Sugawara construction [3] allows to write the free fermionic Hamiltonian densities \( \hat{H}_0^\pm \) in terms of the Kac–Moody currents \( \hat{j}_a^\pm \):

\[
\hat{H}_0^\pm (n) = \frac{\pi}{L} \sum_{n \in \mathbb{Z}} \times \hat{j}_a^\pm (n + m) \hat{j}_a^\mp (-m) \times
\]

with normal ordering \( \times \). Note that \( \hat{j}_a^+(k) |\text{vac}; F\rangle = \hat{j}_a^-(k) |\text{vac}; F\rangle = 0 \) for \( k > 0 \). Combining this with 8, we finally get the Hamiltonian of the model in the following form

\[
\tilde{H} = \frac{\pi}{L} \sum_{n \in \mathbb{Z}} \{ \times \left( \hat{\tilde{z}}^a_j (n) \hat{\tilde{z}}^a_j (-n) + \hat{\tilde{z}}^a_j (-n) \hat{\tilde{z}}^a_j (-n) \right) \times
\]

\[+ \frac{1}{2\pi} (\hat{\Pi}_a^a(n) \hat{\Pi}_a^a(-n) + \hat{\Pi}_a^a(n) \hat{\Pi}_a^a(-n)) \}.
\]

4. The quantum momentum and boost generators are

\[
\hat{P}_\pm = \pm \hat{H}_F^\pm (0) + \sum_{n > 0} \hat{G}_a^a (n) A_{1,\pm}^a (n) + \sum_{n \leq 0} A_{1,\pm}^a (n) \hat{G}_a^a (-n),
\]

\[
\hat{K}_\pm = -i \frac{L}{2\pi} \sum_{n \in \mathbb{Z}} \frac{1}{n} (-1)^n \hat{H}_0^\pm (n).
\]

Commuting \( \hat{P}_\pm \) and \( \hat{G}_a^a (n) \), we get

\[
[\hat{G}_a^a (n), \hat{P}_\pm] = \frac{2\pi}{L} n \hat{G}_a^a (n),
\]
i.e. the quantum momentums are gauge invariant only in the sector of the physical states [7]. Moreover,

\[
\langle \text{phys}|\hat{P}_\pm|\text{phys}\rangle = \pm \langle \text{phys}|\hat{H}_F^\pm (0)|\text{phys}\rangle.
\]

Now, we construct the algebra of the quantum Hamiltonian \( \hat{H} \), the momentum \( \hat{P} = \hat{P}_+ + \hat{P}_- \) and the boost generator \( \hat{K} = \hat{K}_+ + \hat{K}_- \). With \[9\], it is straightforward to check that on the physical states

\[
[\hat{H}, \hat{P}]_\pm = 0,
\]

\[
[\hat{P}, \hat{K}]_\pm = -i \hat{H},
\]

and

\[
[\hat{H}, \hat{K}]_\pm = -i \hat{P} + i (\hat{M}^+ - \hat{M}^-).
\]

We see that these commutation relations differ from those of the Poincare algebra. The difference is in the mass terms in the last commutator. Only in the case of the standard QCD, \( \hat{M}^+ = \hat{M}^- \) and we get the Poincare algebra.

The mass terms \( \hat{M}^\pm \) can not be removed from the algebra by a redefinition of the Poincare generators, if the generators are required to be gauge invariant. We have added
these mass terms to the fermionic and total Hamiltonians just to make them gauge invariant.

Thus, for the chiral $QCD_2$ the Poincare algebra fails to close in the physical sector where the states satisfy the constraints $\mathcal{P}$ and the Poincare generators are gauge invariant. We have constructed the commutation relations of the new algebra explicitly in a compact form.

The failure of the Poincare algebra to close on the physical states implies that the model is not relativistically invariant. The physical Hamiltonian and momentum commute, so translational invariance is preserved. This situation is similar to that in the chiral Schwinger model (see, for example, [3], [4], [10]). The analysis performed in these references shows that when we construct a quantum theory consistent with the anomaly and use the Gauss law to constrain physical states, then relativistic invariance is lost. In other words, the Poincare algebra fails to close on the physical states for the chiral Schwinger model, too.

The origin of the breakdown of relativistic invariance is the same in both models and lies in the anomaly. Therefore, for the chiral $QCD_2$ as well as for the chiral Schwinger model the anomaly or, equivalently, the fact that the local gauge symmetry is realized projectively disturbs relativistic invariance. We believe that this is a fundamental feature characteristic for anomalous models. However, the question of whether relativistic invariance is broken for other models with the projective realization of a local gauge symmetry, especially in higher dimensions, remains open.
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