Effect of polarization force on the propagation of dust acoustic solitary waves

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Abstract. We report the modifications in the propagation characteristics of dust acoustic solitary waves (DASWs) due to the polarization force acting on micron-size dust particles in a non-uniform plasma. In the small amplitude limit, we derive a K–dV-type equation and show that there is an increase in the amplitude and a reduction in the width of a solitary structure as the polarization force is enhanced for a given Mach number. For arbitrary amplitude waves we employ the Sagdeev potential method and find that the range of Mach numbers where solitary structures can exist becomes narrower in the presence of the polarization interaction. In both limits there exists a critical value of grain size beyond which the DASW cannot propagate.

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1. Introduction

Over the last two decades, there has been a great deal of interest in understanding the characteristics of different types of collective modes in complex (dusty) plasmas both in the laboratory environment as well as in space and astrophysical objects. In such plasmas the presence of massive, highly charged dust grains brings about a significant change in the collective properties of the medium including the creation of new eigenmodes. The dust acoustic wave (DAW) [1]–[15] is one such mode which has received a great deal of theoretical and experimental attention over the past two decades. The DAW is an extremely low-frequency mode that is analogous to the ion acoustic wave found in electron–ion plasmas, in which the dust particles provide the inertia, whereas the restoring force comes from electrons and ions. Some other novel excitations found in complex plasmas are dust ion acoustic waves (DIAWs) [16]–[18] and transverse shear waves (TSWs) [4], [19]–[27], which have also been observed experimentally. The linear characteristics of all these waves are now quite well established.

There has also been significant progress in the study of nonlinear aspects of the collective modes in dusty plasmas. It is well known that the presence of nonlinearity can lead to a number of interesting localized structures such as solitons, shock fronts, vortices, etc. Over the past few years, a number of theoretical studies have been carried out in this direction [1], [28]–[37]. In the limit of a small but finite wave amplitude it has been shown that the nonlinear wave dynamics of a DAW is governed by the Korteweg–de Vries (K–dV) equation [29]–[31], [36, 37]. The K–dV equation admits soliton solutions which are localized stable entities created by an exact balance between dispersion-induced broadening and nonlinear wave steepening. Solitons have very special properties that enable them to retain their shapes and identities even after undergoing collisions with each other. For the case of arbitrary wave amplitudes [29, 31, 32, 37], one cannot obtain a canonical equation like the K–dV, but it is still possible to obtain solitary pulse solutions by resorting to a Sagdeev potential-type analysis. Such dust acoustic solitary waves (DASWs) are found to exist only for negative Sagdeev potentials. There have also been some experimental studies [11, 18], [38]–[41] on the propagation of nonlinear DAW.

Recently, there has been an interesting development in the investigation of the propagation characteristics of DAWs in the presence of polarization force acting on the dust grains in a non-uniform plasma background of a complex plasma [42]. In the context of the present paper, non-uniformity is related to the non-zero gradient of the local electron and ion density in the plasma. The polarization force arises due to any kind of deformation of the Debye sheath around the particulates in the background of non-uniform plasmas, and as discussed by Hamaguchi and Farouki [43], is given by

\[ F_P = -Q^2 \nabla \lambda_d/2 \lambda_d^2, \]

where \( Q \) is the grain charge, \( \lambda_d = \lambda_{di}/\sqrt{1 + (\lambda_{di}/\lambda_{de})^2} \) is the linearized Debye radius with \( \lambda_{di(e)} \) being the ion (electron) Debye radius defined as

\[ \lambda_{di(e)} = (e_0 k_B T_{i(e)}/n_{i(e)} e^2)^{1/2}. \]

Here \( n_{i(e)} \) and \( T_{i(e)} \) are the density and temperature of ions (electrons), respectively. As shown by Khrapak et al [42] for Boltzmannian electrons and ions (a good approximation for DAW) the polarization force can be written as

\[ F_P = \frac{1}{|e| n_{i(e)}} (|Q| e/\lambda_d k_B T_i) \times (1 - (T_i/T_e)) Q \nabla \phi, \]

which makes it clear that it acts in the direction opposite to the electrical force. Taking such a model form of the polarization force, it was demonstrated that the polarization effect can play an important role in the propagation characteristics of the linear low-frequency DAW and, in particular, it can lead to a decrease of the wave phase velocity [42]. The effect is particularly pronounced for bigger dust particles. Moreover, there exists a critical size beyond which DAW cannot propagate [42]. This happens...
when the polarization force exceeds the electrical force and the net force acting on the grains is no longer a restoring force. The threshold grain size depends on the plasma parameters. For typical conditions in gas discharges it was estimated to exceed $\approx 10 \, \mu m$ [42].

Our objective in the present paper is to investigate whether the polarization effect can lead to any significant changes in the nonlinear regime and how it can influence the propagation characteristics and existence criterion of nonlinear localized structures like solitons or solitary pulses of the DAW. We have therefore carried out a systematic analysis of the nonlinear DAW in the presence of the polarization force. Our main results are as follows. In the small amplitude limit, by employing the reductive perturbation method, we obtain a K–dV-type equation whose coefficients are modified due to the presence of the polarization interaction. We find that, with an increase of the polarization force, the amplitude of a solitary structure increases, whereas the width decreases for a given Mach number. For the arbitrary amplitude limit, we adopt the Sagdeev potential analysis and find a similar effect for the amplitude and width of the solitary pulse. Our analysis in this limit also shows that the Mach number regime where solitary structures can exist is reduced due to the presence of the polarization interaction. Further, similarly to the linear regime, in both limits we find that there is a critical value of grain size beyond which the DASWs cannot propagate.

The paper is organized as follows. In the next section, we discuss the model equations that include the effects of polarization force. In section 3, small but finite amplitude dust acoustic solitary structures are studied using the well-known reductive perturbation method. In section 4, the DASW is investigated for an arbitrary wave amplitude by using the pseudo-potential approach. A brief summary of our results and some concluding remarks are made in section 5.

2. Model equations

In the standard fluid description of dusty plasma for studying low-frequency ($\omega \ll k v_T_e, k v_T_i$) phenomena in the regime where dust dynamics is important, it is customary to treat the electrons and ions as light fluids that can be modeled by Boltzmann distributions and to use the full set of hydrodynamic equations to describe the dynamics of the dust component. The densities of electrons and ions at temperatures $T_e$ and $T_i$ are, respectively, given by

$$
n_e = n_{e0} \exp(\sigma_i \phi), \quad n_i = n_{i0} \exp(-\phi),$$

(1)

where $\sigma_i = T_i / T_e$ and $\phi = e\phi / k_B T_i$ is the normalized electrostatic potential.

The equilibrium electron density $n_{e0}$ and ion density $n_{i0}$ are related to the dust density $n_{d0}$ and the dust charge number $Z_d$ by the charge neutrality condition,

$$n_{i0} = n_{e0} + n_{d0} Z_d.$$

(2)

For the dust dynamics we use the following set of fluid equations including the polarization force term in the dust momentum equation,

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} (n_d v_d) = 0,$$

(3)

$$\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = \chi \frac{\partial \phi}{\partial x},$$

(4)

$$\frac{\partial^2 \phi}{\partial x^2} = n_d + \mu_e \exp(\sigma_i \phi) - \mu_i \exp(-\phi).$$

(5)
In equation (4), \( \chi = 1 - \Re \), where \( \Re (= \frac{1}{16\varepsilon_0} \epsilon t (1 - (T_e/T_d)) / \lambda_D k_B T_d) \) represents the effects of plasma–particle polarization interaction [42]. For typical complex plasmas with grain radius \( = 1 \mu m, Q \sim 10^3 e, \lambda_D \sim 10^{-4} m \) and \( T_d = 0.03 \) eV, we have \( \Re = 0.12 \). For larger sized grains that can retain higher values of \( Q \) the value of \( \Re \) can get larger and approach unity. For \( \Re > 1 \) however, the polarization force exceeds the electrostatic force and the net force acting on the particles is no longer a restoring force, which results in a growing unstable perturbation. At \( T_e \gg T_d, \Re = 1/4 \beta_r \), where \( \beta_r \) is the ratio of the Coulomb interaction radius between the ions and dust grains to the linear Debye radius. Further, \( n_d \) and \( v_d \) are the dust density and dust mean velocity normalized by initial dust density \( n_{d0} \) and dust acoustic velocity \( C_d = (Z_d k_B T_d / m_d)^{1/2} \). The time and space variables are normalized by dust plasma period \( \omega_{pd}^{-1} = (\epsilon_0 m_d / n_{d0} Z_d^2 e^2)^{1/2} \) and the dust Debye length \( \lambda_{DM} = (\epsilon_0 k_B T_d / n_{d0} Z_d^2 e^2)^{1/2} \). Here, \( Z_d e = Q \) and \( m_d \) are the charge and mass of each dust grain, respectively. \( \mu_e = n_{d0} / Z_d n_{d0} = 1 / (\rho - 1) \) and \( \mu_i = n_{d0} / Z_d n_{d0} = \rho / (\rho - 1) \) and \( \rho = n_{d0} / n_{e0} = 1 / \beta \). Systematic or stochastic dust charge variations are not taken into account.

3. Small amplitude limit: K–dV solitons

To study the dynamics of small but finite amplitude DASWs, we have derived a single nonlinear evolution equation from equations (3)–(5) by employing the reductive perturbation technique and with stretched coordinates given by \( \xi = \epsilon^{1/2} (x - v_0 t) \) and \( \tau = \epsilon^{3/2} t \). Here \( \epsilon \) is a smallness parameter measuring the weakness of the amplitude or dispersion and \( v_0 \) is the wave phase velocity normalized by the dust acoustic velocity \( C_d \). We expand the dynamical variables \( n_d, v_d \) and \( \phi \) about the unperturbed states in a power series of \( \epsilon \) given by

\[
\begin{align*}
    n_d &= 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots, \\
    v_d &= \epsilon v_d^{(1)} + \epsilon^2 v_d^{(2)} + \cdots, \\
    \phi &= \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots.
\end{align*}
\]

Using the above equations (6)–(8) in equations (3)–(5), we get the following relations in the lowest order of \( \epsilon \):

\[
\begin{align*}
    n_d^{(1)} &= \frac{\chi}{v_0} \phi^{(1)}, \quad \quad \quad v_d^{(1)} = \frac{\chi}{v_0} \phi^{(1)} \quad \quad \text{and} \quad \quad \quad v_0 = \sqrt{\frac{\mu_e \sigma_i + \mu_i}{\mu_e \sigma_i}}.
\end{align*}
\]

To the second order of \( \epsilon \), we get

\[
- v_0 \frac{\partial n_d^{(2)}}{\partial \xi} + \frac{\partial n_d^{(1)}}{\partial \tau} + \frac{\partial v_d^{(2)}}{\partial \xi} + \frac{\partial v_d^{(1)}}{\partial \tau} + (n_d^{(1)} v_d^{(1)}) = 0,
\]

\[
- v_0 \frac{\partial v_d^{(2)}}{\partial \xi} + \frac{\partial v_d^{(1)}}{\partial \tau} + v_d^{(1)} \frac{\partial v_d^{(1)}}{\partial \xi} = \chi \frac{\partial \phi^{(2)}}{\partial \xi} ,
\]

\[
\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \frac{\chi}{v_0^2} \phi^{(2)} + n_d^{(2)} + \frac{\mu_e \sigma_i^2 - \mu_i}{2} [\phi^{(1)}]^2.
\]

Now, with the help of equations (9) and (10), we obtain a single evolution equation in the form of a K–dV-type equation [31, 37] given by

\[
\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0
\]

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with the coefficients

$$A = -\frac{v_0^3}{2} \left[ \frac{1}{\chi} \left( \mu e \sigma_1^2 - \mu_i \right) + \frac{3 \chi}{v_d^2} \right]$$

$$= -\frac{v_0^3}{\chi (1 - \beta)^2} \left[ 1 + \beta \sigma_i (3 + \beta \sigma_1) + \frac{\beta}{2} (1 + \sigma_i^2) \right]$$  \hspace{1cm} (12)

and

$$B = \frac{v_0^3}{2 \chi}.$$  \hspace{1cm} (13)

A stationary solution of the K–dV equation (equation (11)) can be obtained by transforming the space variable $\xi$ to $\eta$ such that $\eta = \xi - \delta M t$, where $M = 1 + \delta M$ is the Mach number normalized by the dust acoustic velocity. To solve the K–dV equation, we have used the boundary condition such that $\phi \to 0$, $d\phi/d\eta \to 0$, $d^2\phi/d\eta^2 \to 0$ at $\eta \to \pm \infty$. The stationary solution of the K–dV equation (equation (11)) has the form \cite{31,37}

$$\phi(\xi) = \phi_0 \text{sech}^2 \left( (\xi - \delta M t)/\Delta_s \right),$$  \hspace{1cm} (14)

where $\phi_{0d} = 3\delta M/\Lambda$ is the amplitude and $\Delta_s = \sqrt{4B/\delta M}$ is the width of the solitary wave. It is to be noted that all the constants in the expression for $A$ (see equation (12)) are positive, which makes $A$ negative. This indicates that dusty plasmas support DASWs with a negative potential. Figure 1 shows the time evolution of potential profiles of a solitary structure for two different $R$ values ($R = 0$ and $R = 0.5$). The $R = 0$ case corresponds to the standard soliton solution in the absence of polarization effects, as has, for example, been considered in \cite{29}. For a finite value of $R$ one clearly sees that the amplitude increases, whereas the width decreases due to the presence of polarization effects.

To quantitatively assess the influence of the polarization term on the amplitude and width of the soliton, we have plotted the amplitude and width as a function of the Mach number for different values of $R$ in figures 2 and 3, respectively. It is to be noted that at a particular Mach number, the amplitudes increase and the widths decrease with the increase of the polarization effect keeping the product of the amplitude and square of the width constant (as required by the K–dV solution). As the charges on dust particles depend on their size, the effect is more pronounced for bigger particles. This is seen clearly in figures 2 and 3. However, there is a limit to this effect. For particle sizes such that $R > 1$, the net force on the grain is no longer a restoring force and it generates a growing unstable perturbation.

4. Arbitrary amplitude limit: Sagdeev potential

We next look at the evolution of arbitrary amplitude pulses for which the reductive perturbation technique is not useful, but the pseudo-potential technique is a convenient analysis tool. We define the space and time variables in the following way: $\xi = x - Mt$ (where $\xi$ is normalized by $\lambda_{DM}$ and $M$ is the Mach number defined as the ratio of soliton speed to the dust acoustic speed). In the stationary frame, using equations (3) and (4), we obtain the dust density as

$$n_d = \frac{M}{(M^2 + 2\chi \phi)^{1/2}},$$  \hspace{1cm} (15)

where we have imposed appropriate boundary conditions for the localized disturbances, namely $v_d \to 0, n_d \to 1, \phi \to 0$ at $\xi \to \infty$. 

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Figure 1. Time evolution of the potential profiles of a solitary wave for two different values of \( \Re \) with \( \delta M = 0.1, \sigma_i = 0.01 \) and \( \rho = 1.11 \). The pulses drawn with a solid line have \( \Re = 0 \), whereas those with the dashed lines are for \( \Re = 0.5 \). Every normalized potential profile has been given a shift of \( \phi = -0.072 \) with respect to the previous one to avoid overlapping. The amplitude is seen to increase and the width to decrease with increasing \( \Re \).

Putting the value of \( n_d \) in the Poisson equation (equation (5)) and multiplying both sides of the resulting equation by \( \partial \phi / \partial \xi \), and then integrating once with respect to \( \xi \) and taking into account the above boundary conditions, we obtain the equation

\[
\frac{1}{2} \left( \frac{d\phi}{d\xi} \right)^2 + V(\phi) = 0, \tag{16}
\]

where the Sagdeev potential \( V(\phi) \) is defined as

\[
V(\phi) = \frac{M^2}{\chi} \left[ 1 - \left( 1 + \frac{2\chi \phi}{M^2} \right)^{1/2} \right] + \frac{\mu_e}{\sigma_i} [1 - \exp(\sigma_i \phi)] + \mu_i [1 - \exp(-\phi)]. \tag{17}
\]

As can be seen from equation (17), \( V(\phi) = dV(\phi)/d\phi = 0 \) for \( \phi = 0 \). Therefore, a solitonic solution of equation (16) can exist if and only if (i) \( (d^2 V(\phi)/d\phi^2)_{\phi=0} < 0 \), implying that the fixed point at the origin is unstable and (ii) \( (d^3 V(\phi)/d\phi^3)_{\phi=0} > (\leq) 0 \) for solitary waves with...
The nature of the solitary wave, i.e. whether it has a positive or a negative amplitude, depends on how the amplitude goes to zero as the Mach number ($M$) tends to its critical value $M_{\text{cr}}$. The critical Mach number is defined as the value of $M$ at which $\phi$ vanishes. $M_{\text{cr}}$ can be obtained by expanding the Sagdeev potential $V(\phi)$ in Taylor series to third order in $\phi$. The potential well will be located on the negative (positive) side of the $\phi$ axis if the cubic term is negative (positive).
The critical Mach number, at which the second derivative changes its sign, can be obtained as

\[ M_{cr} = \left( \frac{\chi(\rho - 1)}{\sigma_i + \rho} \right)^{1/2} = \left( \frac{\chi(1 - \beta)}{1 + \sigma_i \beta} \right)^{1/2}. \] (18)

The variation of critical value of \( M \) with \( \Re \) is shown in figure 4 for different values of \( \beta \) at a particular value of \( \sigma_i = 0.01 \). It is clear from the figure that the critical Mach number \( M_{cr} \) decreases when the polarization effects at a particular \( \beta \) and \( \sigma_i \) increase. At this critical value of \( M \), the cubic term in the expansion of \( V(\phi) \) can be expressed as

\[ -\frac{\rho^2 + \sigma_i(\sigma_i + 3\rho) + \rho(1 + \sigma_i^2)/2}{3(\rho - 1)^2}, \] (19)

which clearly shows that the cubic term of \( V(\phi) \) is totally independent of \( \chi \) (or \( \Re \)). It is also to be noted that the cubic term is always negative for any value of \( \sigma_i \) and \( \rho \), i.e. DASWs exist only with negative potential \( (\phi < 0) \). Thus, equation (16) is solved only for negative potentials to get the necessary information about the amplitudes and the widths.

One can also determine the upper limit of \( M \) for which DASW survives. The upper limit of \( M \) can be obtained from the condition \( V(\phi_c) \geq 0 \), where \( \phi_c = -M^2/2\chi \) is the minimum value of \( \phi \) for which the dust number density \( n_d \) is real. Thus, the upper limit of \( M \) is the maximum value of \( M \) for which \( S_m \geq 0 \), where \( S_m = \mu_i + \mu_e/\sigma_i + M^2/\chi - \mu_i \exp(M^2/2\chi) - (\mu_e/\sigma_i) \exp(-\sigma_i M^2/2\chi) \). The upper and lower limits of \( M \) are shown in figure 5 as a function of \( \Re \) for typical experimental parameters, \( \rho = 1.11 \) and \( \sigma_i = 0.01 \). The shaded region shows the existence of solitary structures for negative \( \phi \). It is clear from figure 5 that the domain of existence of the solitary waves becomes narrower with the increase of the polarization force. As in the case of the small but finite amplitude limit (the K–dV soliton), the arbitrary
amplitude DASW will not be able to propagate when the dust grain sizes are large enough so that $\mathcal{R} > 1$.

In further analogy with the small amplitude K–dV soliton case, we can also assess the influence of the polarization interaction on the amplitude and width of the arbitrary amplitude solitary wave. The amplitude and width can be easily determined from a numerical solution of equation (16) or a numerical analysis of equation (17). The variations of the amplitude and width of an arbitrary solitary wave with Mach number for three different values of $\mathcal{R}$ are shown in figures 6 and 7, respectively. As before, the $\mathcal{R} = 0$ case corresponds to solitary
Figure 7. Dependence of the soliton width on the Mach number at \( \rho = 1.11 \) and \( \sigma_i = 0.01 \). Different curves correspond to \( \Re = 0 \) (solid), \( \Re = 0.15 \) (dotted), \( \Re = 0.45 \) (dashed), \( \Re = 0.75 \) (vertical-dashed) and \( \Re = 0.95 \) (dash-dotted), respectively.

solutions obtained from Sagdeev potential analyses in the absence of polarization effects (see, for example, [29]) and provides a benchmark against which to compare the finite \( \Re \) solutions. The dependence of the amplitude and the width on Mach number is similar to those shown in figures 2 and 3. In addition we note that, as in the case of the small amplitude solitons, for a fixed value of Mach number the amplitude increases and the width decreases with the increase of polarization effects.

5. Conclusion

We have studied theoretically the modifications arising in the propagation of DASWs due to the presence of polarization forces acting on the dust grains in an inhomogeneous plasma. In the case of small but finite wave amplitudes, we find that the DASWs obey a K–dV-type equation whose coefficients are modified by the polarization interaction. This leads to an increase of the soliton amplitude and a decrease of its width, compared to the case when the polarization effect is not taken into account. The changes of amplitude and width are more significant for bigger dust particles. For an arbitrary amplitude wave, we carry out a pseudo-potential-type analysis and show that solitary waves can exist only with a negative potential amplitude. We also determine the minimum and maximum limits of the Mach number for which a solitary solution can exist and find that this existence region decreases in width with an increase of the polarization force. When the polarization force dominates over the electrical one \( \Re > 1 \), solitary waves cannot propagate. Such a cut-off can happen for sufficiently large grain. It would be interesting to experimentally verify the existence of polarization-induced changes in both the linear and nonlinear propagation characteristics of the DAW as discussed in [42] and our present work, respectively.
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References

[1] Rao N N, Shukla P K and Yu M Y 1990 Planet. Space Sci. 38 543
[2] Melandso F, Aslaksen T K and Havnes O 1993 Planet. Space Sci. 41 321
[3] Rosenbarg M 1993 Planet. Space Sci. 41 229
[4] Kaw P K and Sen A 1998 Phys. Plasmas 5 3552
[5] D’Angelo N 1994 Planet. Space Sci. 42 507
[6] Barkan A, Merlino R L and D’Angelo N 1995 Phys. Plasmas 2 2363
[7] Merlino R L, Barkan A, Thompson C and D’Angelo N 1998 Phys. Plasmas 5 1607
[8] Thompson C, Barkan A, D’Angelo N and Merlino R L 1997 Phys. Plasmas 4 2331
[9] Fortov V E, Molotkov V I, Nefedov A P and Petrov O F 1999 Phys. Plasmas 6 1759
[10] Pieper J B and Goree J 1996 Phys. Rev. Lett. 77 3137
[11] Pramanik J, Veeresha V M, Prasad G, Sen A and Kaw P K 2003 Phys. Lett. A 312 84
[12] Bandyopadhyay P, Prasad G, Sen A and Kaw P K 2007 Phys. Lett. A 368 491
[13] Schwabe M, Rubin-Zuzic M, Zhdanov S, Thomas H M and Morfill G E 2007 Phys. Rev. Lett. 99 095002
[14] Khrapak S A et al 2003 Phys. Plasmas 10 1
[15] Yaroshenko V V et al 2004 Phys. Rev. E 69 066401
[16] Shukla P K and Silin V P 1992 Phys. Scr. 45 508
[17] Barkan A, D’Angelo N and Merlino R L 1996 Planet. Space Sci. 44 239
[18] Nakamura Y, Bailung H and Shukla P K 1999 Phys. Rev. Lett. 83 1602
[19] Mishra A, Kaw P K and Sen A 2000 Phys. Plasmas 7 3188
[20] Kalmann G, Rosenberg M and DeWitt H E 2000 Phys. Rev. Lett. 84 6030
[21] Ohta H and Hamaguchi S 2000 Phys. Rev. Lett. 84 6026
[22] Pramanik J, Prasad G, Sen A and Kaw P K 2002 Phys. Rev. Lett. 88 175001
[23] Bandyopadhyay P, Prasad G, Sen A and Kaw P K 2008 Phys. Lett. A 372 5467
[24] Misawa T, Ohno N, Asano K, Sawai M, Takamura S and Kaw P K 2001 Phys. Rev. Lett. 86 1219
[25] Nunomura S, Samsonov D and Goree J 2000 Phys. Rev. Lett. 84 5141
  Nunomura S, Goree J, Hu S, Wang X and Bhattacharjee A 2002 Phys. Rev. E 65 066402
[26] Nosenko V, Goree J and Ma Z W 2002 Phys. Rev. Lett. 88 135001
  Nosenko V and Goree J 2006 Phys. Rev. Lett. 97 115001
[27] Praburam G and Goree J 1996 Phys. Plasmas 3 1212
[28] Rao N N 1998 Phys. Scr. T75 1179
[29] Mamun A A 1999 Astrophys. Space Sci. 268 443
[30] Shukla P K and Mamun A A 2001 Introduction of Dusty Plasma Physics (Bristol: Institute of Physics Publishing)
[31] Mamun A A and Shukla P K 2002 Phys. Scr. T98 107
  Mamun A A and Shukla P K 2004 Phys. Scr. 47 A1–9
[32] Sagdeev R Z 1996 Rev. Plasma Phys. 4 23
[33] Bharutharam R and Shukla P K 1992 Planet. Space Sci. 40 973
[34] Popel S I et al 2003 Phys. Rev. E 67 056402
[35] Ma J X and Liu J 1997 Phys. Plasmas 4 253
[36] Li Y, Ma J X and Li J 2004 Phys. Plasmas 11 1366
[37] Mamun A A and Shukla P K 2005 Plasma Phys. Control. Fusion 47 A1
[38] Samsonov D, Ivlev A V, Quinn R A and Morfill G E 2002 Phys. Rev. Lett. 88 095004

New Journal of Physics 12 (2010) 073002 (http://www.njp.org/)
[39] Samsonov D, Zhdanov S K, Quinn R A, Popel S I and Morfill G E 2004 *Phys. Rev. Lett.* **92** 255004
[40] Bandyopadhyay P, Prasad G, Sen A and Kaw P K 2008 *Phys. Rev. Lett.* **101** 065006
[41] Heidemann R, Zhdanov S, Stterlin R, Thomas H M and Morfill G E 2009 *Phys. Rev. Lett.* **102** 135002
[42] Khrapak S A, Ivlev A V, Yaroshenko V V and Morfill G E 2009 *Phys. Rev. Lett.* **102** 245004
[43] Hamaguchi S and Farouki R T 1994 *Phys. Rev.* **E 49** 4430
  Hamaguchi S and Farouki R T 1994 *Phys. Plasmas* **1** 2110