Towards a systematic characterization of the potential of demand side management

David Kleinhans

Abstract With an increasing share of electric energy produced from non-dispatchable renewable sources both energy storage and demand side management might gain tremendously in importance. While there has been significant progress in general properties and technologies of energy storage, the systematic characterization of features particular to demand side management such as its intermittent, time-dependent potential seems to be lagging behind. As a consequence, the development of efficient and sustainable strategies for demand side management and its integration into large-scale energy system models are impeded.

This work introduces a novel framework for a systematic time-resolved characterization of the potential for demand side management. It is based on the specification of individual devices both with respect to their scheduled demand and their potential of load shifting. On larger scales sector-specific profiles can straightforwardly be taken into account. The potential for demand side management is then specified in terms of size and capacity of an equivalent storage device. This intermediate layer of abstraction isolates the gross effects and opportunities of demand side management from questions concerning the operation of individual devices, which eases the integration of the results in existing schemes for the commitment of production units and system models.

The novel framework is applicable for conceptual or predictive approaches and for real-time optimization and operation of energy systems involving demand side management.

Keywords Demand response, Energy System Modeling, Energy storage
1 Introduction

Recent studies emphasize that the energy demand of future large scale power systems can be supplied from renewable sources \[1,2\]. The increased integration of energy converters based on renewable sources, however, exhibits a tremendous challenge for the operation of power systems, since the production from the most extensive resources solar and wind is intermittent and therefore not flexibly dispatchable \[3,4\]. Consequently their contributions to the balancing of the power systems and to ancillary services are limited in comparison to today’s power plants \[5,6\]. For a large-scale integration of conversion technologies based on renewable sources as planned in Europe until 2050 therefore alternative technologies for balancing and ancillary services need to be considered, installed, and integrated. From the current perspective the most promising options are energy storages and demand side management \[7–9\], since these technologies allow for a shifting of electricity provision or consumption in time and, hence, can contribute to a significant reduction in required balancing energy and power.

In the context of this contribution demand side management (DSM) is used as a proxy for technologies for the short term coordinated control and operation of loads at consumers’ side. Prominent examples for the application of DSM in the domestic sector are e.g. electric heating devices operated either via power line or radio communication \[9,11–13\]. Large-scale consumers from the industrial sector often sign individual agreements with the grid operators on their contribution to DSM. While DSM currently is mainly employed in cases of large unbalances in the grid or in order to increase the overall efficiency of unit commitment schemes \[14\], its role might change in future. For future systems based first and foremost on renewable energy sources, energy storage and DSM techniques are one option for the instantaneous adjustment of the residual load \[14\]. As a component of systemic importance which e.g. in so-called smart grids might even be controlled autonomously \[15\], a need for the quantitative characterization of the instantaneous potential for DSM arises. Although DSM already today is a substantial component of procedures for the optimization of power systems \[7,11,16–18\] both with respect to system stability and operating costs \[16\], its large-scale potential has not been characterized and investigated systematically.

In a number of recent contributions on energy system modeling general aspects of the integration of renewable sources were addressed from a conceptual perspective, with a focus on the gross interactions between energy storages, the extension of the transmission grids and strategies for the expansion of conversion capacities for renewable sources (see e.g. \[1,3,8,19,20\]). Techni-

---

1 Equivalent expressions e.g. would have been demand response or load management. Strategies for the more long term implementation of improvements in energy efficiency, which sometimes are also associated with demand side management \[10\], however are not considered.

2 A general formulation applicable to complex power systems, which also involves demand side management at the respective nodes, is e.g. provided in \[15\].
cally storage solutions and DSM technologies often are not differentiated in these approaches [5, 21, 22]. While there from a modeling perspective indeed might be analogies between energy storage and DSM [15], fundamental differences are evident. In contrast to traditional storage devices with DSM rather complicated preconditions have to be considered, since the restrictions of the individual loads with respect to their time frame of management and need to be taken into account. Generally this results in time dependent buffer sizes and capacities. How can these restriction be simplified, formalized and efficiently be integrated into energy system models?

The present work aims to develop a framework for the characterization of the instantaneous potential for DSM at intermediate complexity suitable for the integration in large scale simulations. Similarly to the characterization of energy storage solutions the DSM potential is determined by the properties of storage-equivalent energy buffers, but now with explicit time dependence. At this stage practical problems involving e.g. the control of individual devices and rebound effects by intention are avoided, since these questions need to be addressed and solved independently from the general concept of DSM integration. As a consequence the resulting potential of DSM much more straightforwardly can be interpreted and integrated into conceptual simulation of future power systems than the current state of the art, which treats DSM mainly from a unit-commitment-perspective (see e.g. [7, 11]).

For reasons of clarity, throughout the work the size of energy buffers refers to their volume in terms of energy, whereas their capacity characterizes the maximum power of production or consumption.

2 Novel framework for the quantitative characterization of demand side management

Generally two classes of DSM can be differentiated: (1) loads which can be shifted in time without a significant change in the gross energy consumption and (2) loads which can be reduced without replacement (curtailment). The first class e.g. involves a broad range of heating and cooling processes, where shifting the cycles in time only has a negligible impact on consumers. Also dispatchable devices such as e.g. domestic white goods belong to this class. DSM actions of the 2nd class such as e.g. requests for the temporal reduction of production without replacement generally have a huger impact on the electricity consumers’ interests, since the overall availability of energy is directly affected. For the core part of the present paper we will restrict to DSM class 1, where management and distribution of the energy demand is the most relevant issue. Aspects for the integration of class 2 DSM in the framework are discussed briefly in the context of Appendix A. For reasons of clarity we henceforth use the time $t$ as a continuous variable. For correspondent expression applicable in case of discrete time steps as often used in simulations we refer to Appendix B. Constants and variables used for parametrization are listed in Table 1.
Consumers or loads eligible for DSM class 1 can be characterized with respect to their potential for contributions to DSM. In order to ease the scalability of the procedure a number of $C$ categories $c = 1, \ldots, C$ is introduced, each specified by means of the maximum allowed time frame for management, $\Delta t_c$, in units of hours. The respective consumer loads then need to be apportioned among the categories in a sense, that time resolved profiles of scheduled loads $L_c(t)$ and maximum loads $\Lambda_c(t)$ can be aggregated for the each category, both in units of MW. Here $\Lambda_c(t)$ is defined as the load capacity (i.e. the maximal realizable load) available for DSM in category $c$ at time $t$. The data can e.g. stem from simulations, predictions, experience or standard load profiles. Examples for the application to different load profiles and capacities are provided and discussed in Section 3. From the individual loads $L_c(t)$ the cumulative scheduled load $L(t)$ can be obtained as

$$L(t) = \sum_{c=1}^{C} L_c(t) \quad .$$

With DSM deviations from the scheduled loads can be enforced resulting in new time series $R_c(t)$ of realized loads in the respective categories. For interpretation of DSM as an energy storage-equivalent action, the time-resolved energy balance of the DSM actions can be investigated. If the scheduled loads are considered as a reference, we can figuratively consider an energy buffer of category $c$ to be charged if $R_c(t) > L_c(t)$. On the contrary, the buffer is discharged if $R_c(t) < L_c(t)$. The utilization of DSM in terms of energy buffers in the respective categories can then be determined by the charge rate

$$P_c[R_c(t)](t) = R_c(t) - L_c(t) \quad .$$

If we define the corresponding energy buffers to be empty at $t = 0$, their energy content (or filling levels) can be calculated as integral of the charge rates,

$$E_c[R_c(t)](t) = \int_{0}^{t} dt' \; P_c[R_c(t)](t') \quad .$$

---

| Variable | Description | Unit | Constraints |
|----------|-------------|------|-------------|
| $C$      | Number of categories | —    | $\geq 1$    |
| $\Delta t_c$ | Time frame of management | [h]  | $\geq 0$    |
| $L_c(t)$ | Scheduled load for category $c$ | [MW] | $\geq 0$    |
| $\Lambda_c(t)$ | Maximum load / capacity for category $c$ | [MW] | $\geq 0$    |
| $R_c(t)$ | Realized load (after DSM) for category $c$ | [MW] | $\geq 0, \leq \Lambda_c(t)$ |
| $P_c[R_c(t)](t)$ | Charge rate DSM category $c$ for $R_c(t)$ | [MW] | see Eqn. 5 |
| $E_c[R_c(t)](t)$ | Energy content category $c$ for $R_c(t)$ | [MWh] | see Eqn. 5 |

*Table 1* Parameters and variables used for characterization of the DSM potential in Section 2 with their respective units and constraints.
The square brackets indicate, that the expressions depend explicitly on the
time series of realized loads in the respective categories.

In this terminology storage-equivalent buffers of DSM in the respective cat-
egories can now be specified by means of time-dependent envelopes of the sizes
and the capacities. Functions for the upper (max) and lower (min) envelopes
can be defined from the respective time series for the scheduled and maximal
loads by means of

\[ E_{\text{max}}(t) := E_c[R_c(t + \Delta t_c)](t) = \int_{t}^{t+\Delta t_c} \int_{t}^{t+\Delta t_c} L_c(t') \, dt' , \quad (4a) \]
\[ E_{\text{min}}(t) := E_c[R_c(t - \Delta t_c)](t) = -\int_{t}^{t-\Delta t_c} \int_{t}^{t-\Delta t_c} L_c(t') \, dt' , \quad (4b) \]
\[ P_{\text{max}}(t) := \Lambda_c(t) - L_c(t) , \quad (4c) \]
\[ P_{\text{min}}(t) := -L_c(t) . \quad (4d) \]

In (4a) it is assumed, that all loads in the respective categories are preponed
by their maximum time frame of management corresponding to the maximum
possible charge of the associated energy buffer. The time-dependent maximum
charge is, hence, determined by the energy of upcoming loads within the time
frame of management. The opposite case, i.e. loads delayed to their latest pos-
sible moment of realization, determines a time dependent lower limit for the
buffer size, \( E_{\text{min}}(t) \), as defined in Equation (4b). Equations (4c) and (4d) spec-
ify the upper and lower envelopes for charging and discharging the respective
buffers from time series of the scheduled and the maximum acceptable loads.

The resulting degrees of freedom in operation are the representations of
the realized loads \( R_c(t) \) in the respective categories. From the considerations
above the following requirements for the validity of these functions can be formu-
lated:

\[ E_{\text{min}}(t) \leq E_c[R_c(t)](t) \leq E_{\text{max}}(t) \quad \forall t, c \quad \text{and} \]
\[ P_{\text{min}}(t) \leq P_c[R_c(t)](t) \leq P_{\text{max}}(t) \quad \forall t, c . \quad (5a) \]

Equation (5a) guarantees, that the charge state of the storage-equivalent buffers
in the respective categories does not exceed its limits, whereas Equation (5b)
accounts for the limited charging capacities. Apart from the explicit dependence
on time these parameters correspond to the restrictions of storage de-
vice, which emphasizes a formal analogy to energy storage devices and eases
the practical implementation of DSM in relevant models and procedures.

Two examples for the application of this framework and the graphical
presentation and interpretation of the results are detailed in the subsequent
section.
Fig. 1 Application of the framework for characterization of the potential for demand side management to a single peak of a load eligible for demand side management with a time frame of 3 h. The maximum capacity just as the peak load was set to 1. The panels indicate the scheduled load (a) and the envelopes of the size (b) and the capacity (c) of a storage-equivalent buffer as defined in Equation 4. The blue line exemplifies the charge state $E_1[R_1](t)$ (panel b) and the charge rate $P_1[R_1](t)$ (panel c) for a specific representation of the realized load $R_1(t)$ plotted in panel (a).
3 Examples for application

3.1 Schematic example

As a first, schematic example a system with only one category (i.e. $C = 1$) and a single load pulse eligible for DSM is considered. Specifically we use $\Delta t_1 = 3$, $A_1(t) = 1$, and

$$L_1(t) = \begin{cases} 
1 & 0 < t < 0.25 \\
0 & \text{otherwise}
\end{cases},$$

reflecting a single load pulse of 15 min duration and amplitude 1 (see also panel (a) of Figure 1). The scheduled load and the corresponding envelopes of buffer sizes and capacities as calculated from Equations (4) are exhibited in Figure 1.

From inspection of the figure the idea of the interpretation in terms of a storage-equivalent energy buffer can become clear. The reference is the scheduled load, $L_1(t)$. The envelope of the buffer size is determined by the curves $E_{1\text{max}}(t)$ and $E_{1\text{min}}(t)$ as defined in Equations (4). With the single peak considered here the energy balance of any valid realization with respect to the scheduled load by construction can be only positive at $t < 0$ and only negative at $t > 0.25$, since the scheduled load is non-zero only for $t \in [0, 0.25]$ implying a rather limited flexibility for DSM. This is reflected by the envelope in panel (b) of Figure 1. In the last panel the envelope of the charge and discharge rates is plotted. As a striking feature the buffer can only be discharged in the interval $0 < t < 0.25$, since the scheduled load is 0 otherwise.

Any realized load curve $R_1(t)$, that stays within the envelopes defined by the extremal sizes and capacities is a valid. Formally this criterion is provided by Equations (5). In the three panels of Figure 1 as an example the resulting charge state and rate for

$$R_1(t) = \begin{cases} 
N e^{-t^2} & |t| \leq 3 \\
0 & \text{otherwise}
\end{cases}$$

with the normalization constant

$$N := 0.25 \left( \int_{-3}^{3} dt \ e^{-t^2} \right)^{-1} \approx 0.141$$

are plotted. $R_1(t)$ is a valid representation of the realized load, since it meets the constraints defined in Equation (5) as also shown in panels (b) and (c) of the figure. From panel (b) it becomes evident, that the corresponding $E_1[R_1(t)]$ fits in the rather odd envelope of the buffer size. The charge rates have discontinuities at $t = 0$ and $t = 0.25$, which is due to discontinuities in the scheduled load $L_1(t)$. 
Fig. 2 Exemplified application of the framework for characterization of the potential for demand side management to the accumulated loads eligible for DSM in Germany during a winter week. The colors in the respective panes indicate contributions from the respective categories detailed in Table 2. The scheduled loads in panel (a) were obtained from [23, p. 154]. Panels (b) and (c) indicate the contributions of the respective categories to envelopes of the sizes and capacities of the respective buffers as defined in Equations 4. In winter, the potential for DSM is strongly dominated by electrical heating devices. Due to their limited shifting potential of only 8 h the size of the associated storage-equivalent buffer fluctuates significantly in time. Please note, that the conditions \( \text{eqn 5a} \) and \( \text{eqn 5b} \) need to be taken into account for each individual category and that a graphical validation of realized loads therefore is not possible from this accumulated plot. Individuals plots for the respective categories (just as e.g. presented in Figure 1) would be required for this purpose instead.
Figure 3 Exemplified application of the framework for characterization of the potential for demand side management to the accumulated loads eligible for DSM in Germany during a summer week. For details of the representation it is referred to the caption of Figure 2. In summer, the overall part of the demand available for DSM is much lower than in winter, since there is no significant contribution of electrical heating. Instead, air conditioning and warm water (category 2) gain in importance.
| c | Code  | Description                  | \( A \) [MW] | \( \Delta t \) [h] |
|---|-------|------------------------------|--------------|-----------------|
| 1 | HEAT  | Electric Heating            | 25000        | 8               |
| 2 | AC/W  | AC and hot water            | 10000        | 4-8             |
| 3 | CO-D  | Domestic cooling devices    | 2000         | 1               |
| 4 | WHIT  | Domestic white goods        | 1400         | 24              |
| 5 | VENT  | Ventilation                 | 900          | 4               |
| 6 | CO-L  | Cooling (low and intermediate power facilities) | 800 | 4 |
| 7 | CO-H  | Cooling (high power facilities) | 200 | 4 |
| 8 | IN-1  | Industry (cross-sectional technologies) | 800 | 4 |
| 9 | IN-2  | Industry (high demand, curtailment only) | 2200 | - |

* For analysis \( \Delta t = 6 \) h is considered.

Table 2: Categorization of loads and their corresponding potential for demand side management as compiled and estimated by Klobasa [23, p. 133] (categories numbers were redefined for reasons of clarity). The corresponding loads \( L_c(t) \) are available from model simulations for different weekdays and seasons [23, p. 154]. This data is used for estimation of the potential for demand side management in Germany as shown in Figures 2 and 3.

3.2 Characterization of the national potential of a European country for demand side management

As a second example the framework introduced in Section 2 is used for the conceptual characterization of the potential for DSM in Germany. An adequate categorization of loads with respect to their DSM characteristics in \( C = 9 \) categories and corresponding scheduled loads \( L_c(t) \) is obtained from [23], where an in-depth analysis of the current German electricity sector and results from model simulations are presented. For this purpose Klobasa analyzed relevant industrial processes and domestic consumers with respect to their potential for the temporal shifting of loads. In combination with information on the dissemination of the respective technologies characteristic factors for DSM potential of different sectors of German electricity consumption could be estimated [23, p. 133]. Klobasa furthermore provides simulation data of the time-resolved loads of the respective sectors, both for a summer and a winter week [23, p. 154]. Details of the categories relevant for this work are compiled in Table 2. Simulation results for sector-specific load curves are available for weekdays, Saturdays and Sundays both in winter and summer. For the scope of this analysis the data was rearranged to cover one week (i.e. 5 weekdays, Saturday and Sunday) with periodic boundary conditions in winter and summer, respectively. The maximum capacities in the respective categories were set to the constant values listed in Table 2.

The data and results for the analysis with the framework introduced in Section 2 are exhibited in Figures 2 (winter week) and 3 (summer week). From inspection of the figures in particular the high potential of electric heating systems in winter for DSM actions becomes evident.

To our knowledge this time-dependent size, which is highly relevant for the characterization of the potential of DSM as energy storage-equivalent technol-

---

4 Periodic boundary conditions were implemented by investigating a sequence of three weeks in the respective seasons. Data for the innermost week was then used for analysis.
ology in energy system models, was not investigated before. As far as the capacities for charge and discharge are concerned the peak values are in line with the estimates available from other contributions (see e.g. \[7, 14\]). Generally first and foremost categories 1, 2 and 4 seem to be appropriate for short-time management purposes, since only these categories come with significant buffer sizes and capacities. From inspection of panel (c) of Figures 2 and 3, however, in particular for categories 1 and 4 limitations in the provision of positive control power (which is equivalent with a discharge of the buffer) become evident, which originate from the intermittent operation of the corresponding devices.

In winter, electrical heating provides an enormous potential for DSM. With existing technologies size and capacity of the associated storage could be increased straightforwardly by either increasing the number of electrical heating devices or by increasing the size of attached heat storages. The latter would result in an increase of $\Delta t_1$, which in particular would imply less fluctuations in the buffer size in time. If this flexibility would already be considered with scheduling the loads, increased time frames for management formally could even result in increased opportunities for the provision of positive control power. These effects would significantly ease the applicability of DSM as storage-equivalent technology.

In summer the potential for DSM is significantly lower (see Figure 3), with the highest share in category 2 (air conditioning and warm water). Both applications, however, in principle could be attached to larger reservoirs, which significantly would increase the size of the associated energy buffer.

The analyzed data set does not make use of the entire flexibility of the framework, since time-dependent maximum loads $\Lambda_c$ (differing e.g. between summer and winter) are not provided for the respective categories. Instead the constant values listed in Table 2 are used. For this reason the lower panels of Figures 2 and 3 might overestimate the maximum charging rates e.g. for electrical heating in summer. In this particular example the effect seems to be insignificant, since the associated energy buffer for category 1 anyway is very small during summer and the overall effect of electrical heating therefore is not relevant. Generally the flexibility of considering temporal (or seasonal) fluctuations in the availability of devices, however, should be taken into account.

We would like to draw the attention to the fact, that in Figures 2 and 3 buffer sizes and capacities for all categories are accumulated in single plots, since they are relevant for a discussion of the potential for contributions to DSM from the individual categories. A graphical validation of realized loads as shown in Figure 1 is, however, not feasible in this representation, since the constraints \([9]\) separately need to be fulfilled for each of the nine categories. For this purpose instead individual plots for the respective categories – just as shown in Figure 1 – would be required.
4 Conclusions

In the scope of this contribution a novel, consistent framework for the characterization of DSM as storage-equivalent technology was introduced with the aim to ease the integration of DSM in energy system models and unit commitment schemes.

The framework first and foremost addresses the modeling of loads, which without significant reservation can be shifted in time. Traditionally DSM for these application is implemented in terms of the optimization of a redistribution of individual loads. However, differing requirements for individuals loads impede a graphical illustration of the potential for DSM, its integration into large-scale system models and the optimization of complex commitment schemes.

With the novel framework developed in Section 2 an intermediate degree of complexity is introduced instead: DSM is treated both with respect to energy and capacity balances. From scheduled load curves and the corresponding capacities in Equations (4) envelopes of storage-equivalent energy buffers and their respective capacities for charge and discharge are defined. At this level of abstraction the operation of individual devices and their individual time frames of management do not need to be considered. Instead it is sufficient to consider conditions summarized in Equations (5) in conceptual energy system models and large-scale unit commitment strategies. Technical aspects concerning the small-scale dispatch of individual devices then can be addressed later independently from the desired gross effects at large-scale. For reasons of clarity and generality the Equations in Section 2 treat the time $t$ as a continuous variable. A time-discrete formulation is, however, included in Appendix B.

In Section 3 the application of the framework was discussed based on two examples. The first example, a single load pulse, is mainly interesting for conceptual reasons. On this example the graphical validation of Equations (5) is demonstrated. In the second example the current DSM potential in Germany is investigated. The approach applies the framework developed in the scope of this work to simulation data compiled and published previously by Klobasa [23]. From the application of this framework the accumulated current potential for DSM in Germany can be estimated, which is likely to increase even further with an increased dissemination of electric vehicles not considered yet. From inspection of Figure 2 in particular the transient size of the buffer associated with electrical heating systems becomes evident. The data used for analysis gives a first indication, that the storage-equivalent size of the DSM buffer for electrical heating at times can accumulate to more than 100 GWh, which is in the order of magnitude of the demand for short-term storage in fully-renewable energy systems. The transient behavior is due to the rather limited management time frame of only $\Delta t_1 = 8$ h, which stems from the

---

5 The integration of loads suitable for curtailment is addressed in Appendix A.
6 Rasmussen et al. estimated the demand for short-term energy storage in a European energy system based on renewable sources to 2 TWh [5]. Scaled down to Germany this would imply a demand of approximately 400 GWh.
time when DSM was used for economic operation and commitment of huge power plants. From a technological point of view it would be straightforward to extend this time frame, e.g. by installation of larger heat storages or by investments in the energy efficiency of facilities. From a grid perspective this might be worthwhile if DSM is operated as alternatives to energy storages and obtains systemic importance.

Concluding, the novel framework developed in the scope of this work is applicable for the characterization of the potential of DSM. The resulting buffer sizes and capacities significantly ease the investigation of conceptional problems in energy system modeling due to the reduced level of detail required during simulation and optimization. Finally the storage-equivalent character of DSM actions becomes clear with sizes and capacities directly comparable to the storage properties of alternative technologies.

A Integration of class 2 DSM

In the context of this contribution the focus was on class 1 DSM, where DSM does not effect the long term energy balance. This excludes e.g. curtailment of loads. The resulting framework, however, straightforwardly can be generalized to also include class 2 DSM actions.

In the Equations introduced in Section 2 the constraint of the energy balance is reflected by Equation (5a). Formally this constraint can be relaxed for class 2 DSM. Equation (3) then may diverge in the course of time, reflecting the accumulated energy surplus or deficit due to class 2 DSM actions.

B Time-discrete formulation of the simulation equations

In Section 3 for reasons of clarity the functions $L_c(t)$ and $A_c(t)$ were assumed to be available at arbitrary, continuous $t$. In many examples of practical relevance, however, data instead is available at discrete time steps only. Here we re-formulate some equations from Section 2 with respect to data available at discrete time only.

We therefore assume that the scheduled load is available at discrete times $t^i = t^0 + i\tau$ with $i = 0, \ldots, I$. Then for each category $c$ scheduled loads $L^i_c$ with $i = 0, \ldots, I-1$ need to be specified, where $L^i_c$ is the load scheduled in the time period from $t^i$ to $t^{i+1}$ in units of MW. Accordingly the load capacities $A^i_c$ are defined. From this discrete data, in turn time continuous functions can be defined as

$$L_c(t) := \sum_{i=0}^{I-1} \begin{cases} L^i_c & t^i \leq t < t^{i+1} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad (9a)$$

$$A_c(t) := \sum_{i=0}^{I-1} \begin{cases} A^i_c & t^i \leq t < t^{i+1} \\ 0 & \text{otherwise} \end{cases} . \quad (9b)$$

In principle these functions now can be used with the Equations developed in the cope of Section 2. In fact, the integrals defined in Equations (4a) and (4b), however, can be evaluated for the case of stepwise constant data. Instead of these equations the envelope of the buffer
size can be estimated directly from the discrete data as

\[
E_{\text{max}}(t) := \sum_{i=0}^{t-1} \begin{cases} 
0 & \text{if } t + \Delta t_c \leq t' \\
(t + \Delta t_c - t') L_c^i & \text{if } t' \leq t' + \Delta t_c < t' + 1 \\
\tau L_c^i & \text{if } t' < t < t' + 1 \\
\min(t' + 1, t, \Delta t_c) L_c^i & \text{if } t' + 1 \leq t 
\end{cases}
\]

(10a)

\[
E_{\text{min}}(t) := \sum_{i=0}^{t-1} \begin{cases} 
0 & \text{if } t \leq t' \\
(t - t') L_c^i & \text{if } t' - \Delta t_c \leq t' \text{ and } t' < t' + 1 \\
\tau L_c^i & \text{if } t' < t < t' + 1 \\
\min(t' + 1, t, \Delta t_c) L_c^i & \text{if } t' + 1 \leq t - \Delta t_c 
\end{cases}
\]

(10b)

With these expressions replacing Equations (4a) and (4b), the evaluation of integrals is not necessary for the estimation of the envelopes for buffer sizes and capacities. The set of Equations for discrete data can be extended if also the realized loads \(R_c(t)\) are discrete, which, however, in practice generally is not the case.

Acknowledgments

The author kindly acknowledges discussions with and instructive comments on the manuscript by Konrad Meyer, Stefan Weitemeyer, Arjuna Nebel, Thomas Vogt, and three anonymous reviewers. Mitavachan Hiremath contributed to the preparation of the load profiles used in Section 3.2. Funding of the joint project RESTORE 2050 (funding code 03SF0439A) was kindly provided by the German Federal Ministry of Education and Research through the funding initiative Energy Storage.

References

1. Jacobson, M.Z., Delucchi, M.A.: A path to sustainable energy by 2030. Scientific American 301, 58–65 (2009)
2. Delucchi, M.A., Jacobson, M.Z.: Providing all global energy with wind, water, and solar power, part ii: Reliability, system and transmission costs, and policies. Energy Policy 39, 1170–1190 (2011)
3. Georgilakis, P.S.: Technical challenges associated with the integration of wind power into power systems. Renewable and Sustainable Energy Reviews 12, 852–863 (2008)
4. Rodriguez, R.A., Becker, S., Andresen, G.B., Heide, D., Greiner, M.: Transmission needs across a fully renewable European power system. arXiv preprint [arXiv:1306.1079] (2013)
5. Rasmussen, M.G., Andresen, G.B., Greiner, M.: Storage and balancing synergies in a fully or highly renewable pan-European power system. Energy Policy 51, 642–651 (2012)
6. Genoese, F., Genoese, M.: Assessing the value of storage in a future energy system with a high share of renewable electricity generation. Energy Systems 1–26 (2013)
7. Stötzer, M., Gronstedt, P., Styczynski, Z.: Demand side management potential a case study for Germany. In CIRED 21st International Conference on Electricity Distribution, Paper, vol. 958 (2011)
8. Heide, D., Von Bremen, L., Greiner, M., Hoffmann, C., Speckmann, M., Bofinger, S.: Seasonal optimal mix of wind and solar power in a future, highly renewable Europe. Renewable Energy 35, 2483–2489 (2010)
9. Strbac, G.: Demand side management: Benefits and challenges. Energy Policy 36, 4419–4426 (2008)
10. Loughran, D., Kulick, J.: Demand-side management and energy efficiency in the united states. Energy Journal 25, 19–43 (2004)
11. Pollhammer, K., Kupzog, F., Gamauf, T., Kremen, M.: Modeling of demand side shifting potentials for smart power grids. In AFRICON, 2011, 1–5, IEEE (2011)
12. Arteconi, A., Hewitt, N., Polonara, F.: Domestic demand-side management (DSM): Role of heat pumps and thermal energy storage (TES) systems. Applied Thermal Engineering 51, 155–165 (2012)
13. Arteconi, A., Hewitt, N., Polonara, F.: State of the art of thermal storage for demand-side management. Applied Energy 93, 371–389 (2012)
14. Stadler, I.: Power grid balancing of energy systems with high renewable energy penetration by demand response. Utilities Policy 16, 90–98 (2008)
15. Palensky, P., Dietrich, D.: Demand side management: Demand response, intelligent energy systems, and smart loads. Industrial Informatics, IEEE Transactions on 7, 381–388 (2011)
16. Mathieu, J.L., Koch, S., Callaway, D.S.: State estimation and control of electric loads to manage real-time energy imbalance (2012)
17. Gayme, D., Topcu, U.: Optimal power flow with distributed energy storage dynamics. In American Control Conference (ACC), 2011, 1536–1542, IEEE (2011)
18. Heusen, K., Koch, S., Uhlig, A., Andersson, G.: Energy storage in power system operation: The power nodes modeling framework. In Innovative Smart Grid Technologies Conference Europe (ISGT Europe), 2010 IEEE PES, 1–8, IEEE (2010)
19. Heide, D., Greiner, M., Von Bremen, L., Hoffmann, C.: Reduced storage and balancing needs in a fully renewable European power system with excess wind and solar power generation. Renewable Energy 36, 2515–2523 (2011)
20. Murthy Balijepalli, V., Pradhan, V., Khaparde, S., Sheref, R.: Review of demand response under smart grid paradigm. In Innovative Smart Grid Technologies-India (ISGT India), 2011 IEEE PES, 236–243, IEEE (2011)
21. De Jonghe, C., Hobbs, B.F., Belmans, R.: Optimal generation mix with short-term demand response and wind penetration. Power Systems, IEEE Transactions on 27, 830–839 (2012)
22. Milano, F.: Power system modelling and scripting, Springer (2010)
23. Klobasa, M.: Dynamische Simulation eines Lastmanagements und Integration von Windenergie in ein Elektrizitätsnetz auf Landesebene unter regelungstechnischen und Kostengesichtspunkten. Ph.D. thesis, ETH Zürich (2007)