Transient Stability of Power Systems Integrated with VSCs

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Abstract—The transient stability issue of power systems with high penetration of power electronic converters has raised growing concerns. In the literature, the issue was investigated mainly by numerical simulation, which, however, can neither give analytical insights into the stability mechanism nor provide accurate information on the stability boundary. This issue is investigated analytically in this letter. It is found that the converter output alters the power angle characteristics of generators; the converter’s synchronization requirement limits the maximum power angle of the generator, which proves to be the leading cause of a new type of transient instability. Also, a quantitative method is provided to accurately predict the stability boundary.

Index Terms—Converter, phase locked loop, synchronous generator, synchronization stability, transient stability.

I. INTRODUCTION

With the increasing use of voltage source converters (VSCs) in the power system, the power system stability has been significantly affected. When subjected to severe disturbances such as grid faults, VSCs need to implement the current-controlled mode to avoid overcurrent damage and provide reactive current support. The dynamic characteristics of a current-controlled converter are substantially different from those of a synchronous generator (SG). The latter is with the attribute of a voltage source. The power system dynamics and stability characteristics are inevitably affected by the use of a large number of current-controlled converters.

The methods used in the literature to investigate how VSCs affect the power system transient stability were mostly based on numerical simulation [1], [2]. Such methods, however, cannot reveal the physical mechanism behind the instability phenomenon and provide accurate information on the stability boundary. The conclusions drawn in such a case-by-case way are also not completely general. Few studies giving analytical insights into this topic have been reported. The analytical investigation can reveal the root causes of the instability phenomenon and provide a quantitative assessment of the stability region [3].

The transient stability of a parallel system of a SG and a VSC is studied analytically in this letter. By analyzing the converter synchronization requirement and the impact of the converter output on the power angle characteristics of the SG, a new transient instability cause is uncovered. The stability boundary is quantified by a reverse-time integral method, with which, the influence of the converter’s power penetration rate and connecting impedance on the stability region is analyzed.

II. SYSTEM OVERVIEW

The system under consideration is a simplified system, which is composed of a SG, a current-controlled VSC, and an infinite bus, as displayed in Fig. 1(a). Through a profound analysis towards such a simplified system, the fundamental mechanism of transient instability can be clearly revealed, and a basic stability analysis approach can be developed. These basic cognitions are of significant importance for the transient stability research on more complex systems. This way is just like the investigation on the transient stability of the well-known single-machine infinite bus (SMIB) system.

Several basic assumptions are highlighted before modeling the system. 1) The dynamics of current control loop (hundreds of hertz) and phase-locked loop (PLL) (tens of hertz) are overlooked as they act faster than the dynamics of rotor motion (0.1 ~ 2 hertz). 2) The electromagnetic transients of the circuit are neglected. 3) The voltage behind reactance of the SG remains constant; 4) The resistive component of the circuit is neglected. 5) The converter is power generation equipment, e.g., inverters of wind, PV, or VSC-HVDC systems. The modeling and analysis can be applied to power consumption equipment.

III. MODELING AND STABILITY ANALYSIS

The per-unit modeling is adopted, where $S_N$, $U_{N}$, and $\omega_N$ are the nominal capability, voltage, and frequency, respectively. Time $t$ is measured in seconds in the per-unit system.

A. Quasi-Steady-State Analysis of the PLL

Applying the superposition principle to Fig. 1(b), we can obtain the voltage equation in the synchronous reference frame,

$$ U_{g}e^{j\theta} = K_{g}e^{j\delta} + K_{u}U_{e}e^{j\omega t} + jX_{ge}I_{e}e^{j(\delta + \omega t)} \tag{1} $$

where $K_{g} = X_{ge}/(X_{ge} + X_{gd})$, $K_{u} = X_{ge}/(X_{ge} + X_{gd})$, and $X_{ge} = X_{gd}X_{gd}/(X_{gd} + X_{gd})$. Other variables are explained in Fig. 1(b). Given $\delta \in [0, \pi]$, we can obtain that $\delta_{\pi} \in [0, 3\pi/2]$ according to $i_{g} = \hat{l} \cos \theta \geq 0$.

The terminal voltage equation of the converter is

$$ (u_{g} + j\omega t) e^{j\theta} = K_{c}e^{j\omega t} + K_{u}U_{e}e^{j\omega t} + jX_{g}I_{e}e^{j(\delta + \omega t)} \tag{2} $$

where $X_{g} = X_{gd} + X_{ge}$. Neglecting the PLL dynamics implies

$$ I_{g} = I_{g} + I_{e} \tag{3} $$

Fig. 1. (a) The SG-VSC parallel system under consideration in this study. (b) Simplified circuit.)
\[ u_g = K_e E_s \sin(\delta_g - \delta_p) - K_i U_0 \sin \delta_p + X_{st} i_g = 0 \]  
(3)

which determines the relation between \( \delta_g \) and \( \delta_p \).

To achieve the grid-synchronization of the converter, i.e., (3) must be solvable, the existence of the PLL equilibrium point raises the following requirement,

\[ \sqrt{[K_e E_s]^2 + [K_i U_0]^2} + 2K_e K_i E_s U_0 \cos \delta_g \geq X_{st} i_g. \]  
(4)

i. If \( i_g \leq K_i E_s - K_i U_0 \/X_{s} \), (4) always holds, \( \delta_{\text{max}} = \pi \).

ii. If \( K_i E_s - K_i U_0 \/X_{s} < i_g \leq (K_i E_s + K_i U_0) \/X_{s} \),

\[ \delta_{\text{max}} = \arccos \left( \frac{(X_{st} i_g)^2 - (K_i E_s)^2 - (K_i U_0)^2}{2K_i K_s E_s U_0} \right) \in [0, \pi). \]  
(5)

iii. If \( i_g > (K_i E_s + K_i U_0) \/X_{s} \), (4) (or (3)) never holds.

**Condition 1:** One of the necessary conditions for the system stability is \( i_g \leq K_i E_s - K_i U_0 \/X_{s} \), which is raised from the viewpoint of the PLL having an equilibrium point.

**Remark 1:** With Condition 1, only if \( \delta_g \in [0, \delta_{\text{max}}] \), (4) can hold and (3) is solvable. In other words, the PLL synchronization (faster than the SG) limits the maximum allowable angle (MAA) \( \delta_{\text{max}} \) less than the angle at the expected equilibrium point of the SG, because this will cause the SG to fail to reach the equilibrium point.

**Condition 2:** Given a specific \( \delta_g \), there are two solutions for \( \delta_p \) to meet (3). The stable one can be identified by the negative feedback condition of the PLL \( du/g/\delta < 0 \) [4],

\[ K_i E_s \cos(\delta_g - \delta_p) + K_i U_0 \cos \delta_p > 0. \]  
(6)

**B. Power Angle Characteristics Analysis of the SG**

The output power of the SG is

\[ P + jQ = E_s e^{j\phi} \left( \left[ E_s e^{j\phi} - U_s e^{j\phi} \right] / (jX_{s}) \right) \]  
(7)

Recalling (1), we can obtain the active power component,

\[ P(\delta_g) = P_1(\delta_g) = K_i E_s \cos(\delta_g - \delta_p) \]  
(8)

where \( P_1(\delta_g) = E_s U_0 \sin \phi \left( X_{st} + X_{s} \right) \) represents the generator power output when disconnecting the converter. The converter output current produces the last term in (8), which, therefore, affects the power angle characteristic curve and makes it no longer a standard sinusoidal curve.

**1) Pre-Fault or Post-Fault Condition:** It can be considered that the converter purely outputs active current in normal conditions, i.e., \( \phi = 0 \), \( P(\delta_g) = P_1(\delta_g) = K_i E_s \cos(\delta_g - \delta_p) \).

**Condition 2:** Another necessary condition for ensuring the system stability is \( P_1 = \max P(\delta_g) \), where \( P_1 \) represents the mechanical power input. This condition is raised from the viewpoint of the SG having an equilibrium point.

**Remark 2:** Conditions 1 and 2 raise stability requirements only from the static analysis point of view. It is noted that transient stability and other stability/operation constraints may raise more stringent stability requirements.

Regarding how the converter output affects the power angle characteristics of the SG, we have the proposition below to indicate the difference between \( P(\delta_g) \) and \( P_1(\delta_g) \).

**Proposition:** If \( K_i E_s + X_{st} i_g \leq K_i U_0 \), \( \exists \delta_g \in [0, \pi) \) meets \( P(\delta_g) > P_1(\delta_g) \).

**Proof:** Zeroing the last term in (8) and considering (3) and (6) lead to \( \cos \delta_p = \{-K_i E_s + X_{st} i_g\} / (K_i U_0) \). There exists a solution \( \delta_{\text{max}} \) within \( (0, \pi) \), if \( K_i E_s + X_{st} i_g < K_i U_0 \). We can further verify \( P(0) < P_1(0) \). Hence, the proof is completed by the continuity of the functions \( P(\delta_g) \) and \( P_1(\delta_g) \).

**Remark 3:** Generally, the power angle characteristic curve of the SG is lowered in normal conditions due to the impact of the converter output current. This is because the converter active current output drops the common bus voltage [see the last term in (1)]. Only if \( K_i E_s + X_{st} i_g < K_i U_0 \), the power angle curve is elevated only within the range \( \delta_{\text{max}}, \pi \).

**2) Fault-On Condition:** It can be considered that the converter purely outputs reactive current to support the common bus voltage during severe grid faults, i.e., \( \phi = -\pi/2 \). \( P(\delta_g) = P_1(\delta_g) + K_i U_0 \sin \delta_g \). We have \( 0 \leq \delta_g \leq \delta_{\text{max}} \) according to (3) and (6). Hence, the last term in \( P(\delta_g) \) is greater than zero except at \( 0 \) and \( \pi \) points. This suggests that the reactive current support improves the active power output capacity of the SG during the fault-on period.

**C. Transient Stability Analysis of the System**

The rotor motion equation of the SG is

\[ d\delta_g /dt = \omega_0 / \delta_g - \omega_0 / \phi \]  
(9)

The stability boundary of the well-known SMIB system is dominated by the unstable equilibrium point (UEP), and the stable manifold of the UEP forms the stability boundary [5]. Considering the possibility that there may be no UEP within \( [0, \delta_{\text{max}}] \), a separate discussion is needed.

**1) With UEP:** As shown in Fig. 2(a), in this case, the stability boundary is still dominated by the UEP. The exact stability region can be found using the method developed in [5], which is based on the backward integral starting from the UEP.

**2) Without UEP:** In this case, the stability boundary is taken over by the MAA \( \delta_{\text{max}} \), as shown in Fig. 2(b). After the fault clearance, the critical stable scenario is that: when the power angle reaches \( \delta_{\text{max}} \), the rotor speed decreases to zero exactly. To predict the stability boundary in this case, we can integrate (9) backward starting from the critical state \( (\delta_{\text{max}}, 0) \). The resultant state trajectory is exactly the stability boundary.

The power angle characteristic curve and stability boundary in the two cases are shown in Fig. 2. The system parameters used are given in Appendix. After the grid voltage dips, the mechanical power drives the rotor to accelerate. After the voltage-dip fault clearance, the rotor undergoes a decelerating and accelerating swing either to converge (stable) or to diverge (unstable). The critical clearing time (CCT) and stable/unstable trajectory are also shown in Fig. 2, where the correctness of the stability boundary is verified. It is noted that once the power angle crosses over \( \delta_{\text{max}} \), the system becomes unstable immediately because the PLL loses its equilibrium point.

**Remark 4:** When there is a UEP, the transient instability mechanism remains the same with the well-known UEP-related instability. When the penetration rate of the converter output power becomes higher, the PLL synchronization places a stricter limit on the maximum power angle. In this case, there is...
no UEP. The transient instability cause is linked to the maximum allowable power angle, creating a new type of transient instability phenomenon in the entire system. The new cause is intrinsically different from the UEP-related instability.

The changes in the stability boundary and CCT with the converter power penetration rate and the connecting impedance \( X_g \) are shown in Fig. 3. Both the stability region and the CCT become increasingly small with the increase of the penetration rate and \( X_g \).

![Fig. 2. Power angle characteristic curve, stability boundary and state trajectory. (a) \( I = 1.0 \text{ pu} \), there is a UEP. (b) \( I = 2.0 \text{ pu} \), there is no UEP.](image)

![Fig. 3. Both the stability region and the CCT become small (a) with the increase of the penetration rate of the converter power, and (b) with the increase of \( X_g \).](image)

IV. CONCLUSION

The transient stability of a SG-VSC parallel system is studied in this letter. Both the analytical insights into the stability mechanism and the method to quantify the stability region are provided. The major findings include that: 1) the power angle characteristics of the SG are altered by the converter output current. 2) The maximum power angle of the SG is limited by the fast PLL synchronization. In the context of high penetration rate of the converter output, the stability region is decide by the maximum power angle. The findings reveal a new type of transient instability in the SG-VSC parallel system and provide a clear understanding of the physical mechanism behind the transient instability. Based on the findings, future efforts will be devoted to the transient stability research on more complex generators-converters hybrid power systems.

APPENDIX

\[ X_{g1} = 0.2 \text{ p.u.}, \quad X_{g2} = 0.3 \text{ p.u.}, \quad X_{g3} = 0.2 \text{ p.u.}, \quad \phi_i = 0 \text{ or } -\pi/2, \quad E_g = 1.05 \text{ p.u.}, \quad U_0 = 1.0 \text{ or } 0.1 \text{ p.u.}, \quad T_f = 6 \text{ s}, \quad \beta = 10 \text{ p.u.}, \quad P_m = 0.9 \text{ p.u.}, \quad S_N = 10 \text{ MVA}, \quad U_N = 690 \text{ V}, \quad \omega_N = 100\pi \text{ rad/s}. \]

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