Revisiting the texture zero neutrino mass matrices

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Abstract

In the light of refined and large reactor mixing angle $\theta_{13}$, we have revisited the texture three and two zero neutrino mass matrices in the flavor basis. For Majorana neutrinos, it has been explicitly shown that all the texture three zero mass matrices remain ruled out. Further, for both normal and inverted mass ordering, for the texture two zero neutrino mass matrices one finds interesting constraints on the Dirac-like CP violating phase $\delta$ and Majorana phases $\rho$ and $\sigma$.

The recent measurement of reactor mixing angle $\theta_{13}$ and its subsequent refinements [1]-[4], along with the precision measurement of the solar and atmospheric mixing angles $\theta_{12}$ and $\theta_{23}$ as well as of the neutrino mass squared differences have given a new impetus to the neutrino oscillation phenomenology. The observation of non zero value of $\theta_{13}$, on the one hand, restored the parallelism between quark and lepton mixing, while on the other hand, it has triggered great deal of interest in the exploration of CP violation in the leptonic sector. It has also brought into focus the issue of constraining the CP violating phases of the mixing matrix from the textures of the mass matrices. This has further been strengthened by a very recent constraint on the sum of absolute neutrino masses provided by Planck experiment [5]. In particular, efforts have been put in to carry out fine grained analysis regarding the compatibility of texture specific mass matrices in the flavor [6] as well as non flavor basis [7, 8]. Specifically, in the flavor basis wherein the charged lepton mass matrix is considered to be diagonal, good deal of attempts have been made to explore the compatibility of texture zero mass matrices for Majorana neutrinos with the neutrino oscillation data.

Present refinements in the data warrant a re-look at the compatibility of the texture zero neutrino mass matrices. Therefore, the purpose of the present paper is to re-investigate all the possible structures of texture three and two zero neutrino mass matrices in the flavor basis. In particular, we have carried out an analysis, similar to the one carried out by Ref. [9], with an emphasis on obtaining useful constraints on the CP violating phases. Specifically, for the viable possibilities of
texture zero mass matrices, we have examined the implications of relatively large and refined $\theta_{13}$ on the parameter space of the Dirac-like CP violating phase $\delta$ and the Majorana phases $\rho$, $\sigma$.

The plan of the paper is as follows. To begin with, some essential details pertaining to the construction of the Pontecorvo Maki Nakagawa Sakata (PMNS) matrix [10] from the mass matrices have been presented in Section (1). Using the inputs given in Section (2) and keeping focus on the CP violating phases $\rho, \sigma, \delta$, the results pertaining to the analyses of texture three zero Majorana mass matrices have been presented in Section (3). For the texture two zero case, results of the analysis for Majorana neutrino mass matrices have been given in Section (4). Finally, Section (5) summarizes our conclusions.

1 Texture specific mass matrices and construction of the PMNS matrix

Before proceeding further, we briefly underline the methodology relating the elements of the mass matrices to those of the mixing matrix. In the flavor basis, wherein the charged lepton mass matrix $M_l$ is diagonal, the Majorana neutrino mass matrix $M_\nu$ can be expressed in terms of three neutrino masses $m_1, m_2, m_3$ and the flavor mixing matrix $V$ as

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T. \quad (1)$$

The mixing matrix $V$ can be written as $V = UP$, where $U$ denotes the Pontecorvo-Maki-Nakagawa Sakata (PMNS) [10] neutrino mixing matrix consisting of three flavor mixing angles and one Dirac-like CP violating phase, whereas, the matrix $P$ is a diagonal phase matrix, i.e., $P=\text{diag}(e^{i\rho}, e^{i\sigma}, 1)$ with $\rho$ and $\sigma$ being the two Majorana CP violating phases. The neutrino mass matrix $M_\nu$ can then be rewritten as

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T, \quad (2)$$

where $\lambda_1 = m_1 e^{2i\rho}, \lambda_2 = m_2 e^{2i\sigma}, \lambda_3 = m_3$.

For the purpose of calculations, we have adopted the parameterization of the mixing matrix $U$ considered by Ref. [9], e.g.,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (3)$$

where $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$ for i,j=1,2,3 and $\delta$ is the CP violating phase.
2 Inputs used in the present analysis

Before discussing the results of the analysis, we summarize the experimental information about various neutrino oscillation parameters. For both normal mass ordering (NO) and inverted mass ordering (IO), the best fit values and the latest experimental constraints on neutrino parameters at 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) confidence level (CL), following Ref. [11], are given in Table 1.

| Parameter   | Best Fit | 1\(\sigma\) | 2\(\sigma\) | 3\(\sigma\) |
|-------------|----------|-------------|-------------|-------------|
| \(\delta m^2\) [10^{-3} eV^2] | 7.60     | 7.42 - 7.79 | 7.26 - 7.99 | 7.11 - 8.18 |
| \(|\Delta m^2_{31}|\) [10^{-3} eV^2] (NO) | 2.48     | 2.41 - 2.53 | 2.35 - 2.59 | 2.30 - 2.65 |
| \(|\Delta m^2_{31}|\) [10^{-3} eV^2] (IO) | 2.38     | 2.32 - 2.43 | 2.26 - 2.48 | 2.20 - 2.54 |
| \(\theta_{12}\) (NO) | 34.6°    | 33.6° - 35.6° | 32.7° - 36.7° | 31.8° - 37.8° |
| \(\theta_{12}\) (IO) | 48.9°    | 41.7° - 50.7° | 40.0° - 52.1° | 38.8° - 53.3° |
| \(\theta_{23}\) (NO) | 49.2°    | 46.9° - 50.7° | 41.3° - 52.0° | 39.4° - 53.1° |
| \(\theta_{23}\) (IO) | 8.6°     | 8.4° - 9.8°  | 8.2° - 9.1°  | 7.9° - 9.3°  |
| \(\theta_{13}\) (NO) | 8.7°     | 8.5° - 9.8°  | 8.2° - 9.1°  | 8.0° - 9.4°  |
| \(\theta_{13}\) (IO) | 254°     | 182° - 353°  | 0° - 360°    | 0° - 360°    |
| \(\delta\) (NO) | 266°     | 210° - 322°  | 0° - 16° \(\oplus\) 155° - 360° | 0° - 360°    |
| \(\delta\) (IO) | 266°     | 210° - 322°  | 0° - 16° \(\oplus\) 155° - 360° | 0° - 360°    |

Table 1: 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) CL ranges of neutrino oscillation parameters. NO (IO) refers to normal (inverted) neutrino mass ordering.

3 Texture three zero Majorana neutrino mass matrices

In the flavor basis, there are 20 possible texture three zero patterns, the texture structures of these have been elaborated in Ref [12] and it has been shown that all these are found to be incompatible with the neutrino oscillation data. Our present analysis, carried out be the current refined data also reinforces this earlier conclusion. To elaborate this, we explicitly examine one of the possibilities and bring forward its incompatibility with the latest neutrino mixing data. In particular, we consider the case wherein \(M_{ee}=0\), \(M_{e\tau}=0\), \(M_{\mu\mu}=0\), i.e.,

\[
M_{\nu} = \begin{pmatrix}
0 & \times & 0 \\
\times & 0 & \times \\
0 & \times & \times
\end{pmatrix}.
\] (4)

The neutrino mass matrix \(M_{\nu}\) can be diagonalized by using flavor mixing matrix \(U\). For the present case, the constraints \(M_{ee}=0\) and \(M_{e\tau}=0\) yields the following expression for ratios of the complex neutrino mass eigenvalues

\[
\frac{\lambda_1}{\lambda_3} = -\frac{s_{13}}{c_{13}} \left( \frac{s_{12} c_{23}}{c_{12} s_{23}} e^{i\delta} + s_{13} \right); \quad \frac{\lambda_2}{\lambda_3} = +\frac{s_{13}}{c_{13}} \left( \frac{c_{12} c_{23}}{s_{12} s_{23}} e^{i\delta} - s_{13} \right).
\] (5)
For $M_{\mu\mu}=0$, we obtain

$$\lambda_3 \left( U_{\mu 1} U_{\mu 1} \frac{\lambda_1}{\lambda_3} + U_{\mu 2} U_{\mu 2} \frac{\lambda_2}{\lambda_3} + U_{\mu 3} U_{\mu 3} \right) = 0.$$  \hspace{1cm} (6)

From the real and imaginary parts of the above equations, we obtain the following constraints on $\theta_{12}$. From the real part, we get

$$\theta_{12} = \frac{1}{2} \text{acot} \left( \frac{s_{23} c_{23} s_{13} (2 + \cot 2\delta) - s_{23}^3 \cos 2\theta_{13}}{2c_{23} s_{13} \cos \delta} \right),$$

whereas, from the imaginary part, we obtain

$$\theta_{12} = \frac{1}{2} \text{acot} \left( \frac{s_{13} \cos (2\delta) s_{23}}{2 \sin \delta} \right).$$  \hspace{1cm} (8)

To check the compatibility of the above two solutions, in Fig.1(a) and (b), we have presented the correlation plots between $\theta_{12}$ and $\theta_{23}$ for the constraints obtained from the real and imaginary parts respectively. These graphs have been obtained by giving full allowed variation to phase $\delta$ and $3\sigma$ variation to $\theta_{13}$. From Fig.1(a),

![Correlation plots](image)

Figure 1: Correlation plots for (a) real and (b) imaginary part of $M_{\mu\mu}=0$ for three-zero texture with $M_{ee}=0, M_{e\tau}=0, M_{\mu\mu}=0$, where angles are measured in degrees.

we find that the range obtained for $\theta_{12}$ here has no overlap with its experimental range, i.e., $31.8^0$ - $37.8^0$, whereas, the plot given in Fig.1(b) shows that $\theta_{12}$ found here includes its experimental range. For the compatibility of the texture three zero possibility considered here, both the graphs should include the experimental range of $\theta_{12}$, therefore, ruling out the present case. A similar analysis corresponding to the other possibilities of texture three zero Majorana neutrino mass matrices also yield the same result, therefore ruling these out.
Texture two zero Majorana neutrino mass matrices

Coming to the case of texture two zero Majorana neutrino mass matrices, it is well known that there are 15 possible structures of neutrino mass matrix. Adopting the classification scheme of Ref. [9], these can be divided into six classes A, B, C, D, E and F. For the purpose of examining whether a possible pattern of texture two zero mass matrices is viable or not, one needs to examine the compatibility of the corresponding PMNS matrix constructed using these. For executing the present analysis, to begin with, we vary the input neutrino oscillation parameters, i.e., the three mixing angles \( \theta_{12} \), \( \theta_{23} \) and \( \theta_{13} \) as well as the mass squared differences \( \delta m^2 \) and \( \Delta m^2 \) within their 3\( \sigma \) CL ranges, summarized in Table 3.1. In the absence of any experimental constraint on Dirac-like CP violating phase \( \delta \), we give full variation of 0° to 360° to it. In order to obtain constraints on the Majorana phases \( \rho \) and \( \sigma \), these can be expressed in terms of the mixing matrix elements, e.g.,

\[
\rho = \frac{1}{2} \text{arg} \left( \frac{U_{a3}U_{b3}U_{l2}U_{m2} - U_{a2}U_{b2}U_{l3}U_{m3}}{U_{a2}U_{b2}U_{l1}U_{m1} - U_{a1}U_{b1}U_{l3}U_{m3}} \right),
\]

and

\[
\sigma = \frac{1}{2} \text{arg} \left( \frac{U_{a1}U_{b1}U_{l3}U_{m3} - U_{a3}U_{b3}U_{l1}U_{m1}}{U_{a2}U_{b2}U_{l1}U_{m1} - U_{a1}U_{b1}U_{l2}U_{m2}} \right).
\]

Coming to the results of the analysis, it may be mentioned that even with the present refinements of the mixing angle \( \theta_{13} \), out of the 15 possible texture two zero Majorana neutrino mass matrices, the texture patterns \( A_{1,2}, B_{1,2,3,4} \) and \( C \) remain viable for the normal mass ordering (NO). The possibilities \( D_{1,2} \) have already been ruled out by Ref. [9] due to mixing angle \( \theta_{12} \) being less than 38°, conclusions of our analysis in this regard remain the same. Similarly, patterns \( E_{1,2,3} \) and \( F_{1,2,3} \) remain phenomenologically disfavored. In the following, for NO, we present results for the patterns of categories A, B and C. In particular, we present the implications of the relatively large and refined \( \theta_{13} \) on the parameter space of three CP violating phases \( \delta, \rho \) and \( \sigma \).

4.1 Class A

This class consists of 2 possible texture two zero matrices \( A_1 \) and \( A_2 \). Interestingly, there exists a 2-3 permutation symmetry between these, therefore, the phenomenological implications of these are similar and thus we have discussed pattern \( A_1 \) only in detail. As a first step, to examine the implications of non zero and large \( \theta_{13} \) on phase \( \delta \), in Fig.2 we have given \( \theta_{13} \) versus \( \delta \) plot. From the graph, one finds that corresponding to the 3\( \sigma \) CL experimental range of angle \( \theta_{13} \) i.e. 7.9° to 9.3°, one does not obtain any constraint on phase \( \delta \), this being in agreement with the conclusions of Ref.[9].

As a next step, in Fig.3(a) and (b) we have given the correlation plots of Dirac-like CP violating phase \( \delta \) and Majorana phases \( \rho \) and \( \sigma \). Again, we find that one is
Figure 2: Pattern $A_1$: Mixing angle $\theta_{13}$ versus phase $\delta$. All the parameters are in degrees.

not able to obtain any useful constraints on these phases, in particular, both $\rho$ and $\sigma$ take values from $-90^0 - 90^0$.

Figure 3: Pattern $A_1$: Correlation plots of phases $\rho$ and $\sigma$ versus $\delta$. All the parameters are in degrees.

4.2 Class B

The class B consists of 4 possible texture two zero matrices $B_{1,2,3,4}$. Similar to the matrices of class A, there exists a 2-3 permutation symmetry between the matrices $B_1$ and $B_2$ as well as between $B_3$ and $B_4$, therefore, the phenomenological implications of these are similar and thus we have discussed the patterns $B_1$ and $B_3$ only in detail. In order to obtain constraints on Dirac-like CP violating phase $\delta$ due to the refined measurement of mixing angle $\theta_{13}$, in Fig. 4 we have given $\theta_{13}$ versus $\delta$ plot for patterns $B_1$ and $B_3$. From the graph one finds that unlike the case of matrices belonging to class A, the $3\sigma$ CL experimental range of angle $\theta_{13}$ considerably shrinks the parameter space of $\delta$ near $90^0$ and $270^0$. Therefore, further refinements in the measurement of phase $\delta$ would have implications for these matrices considered here.
Figure 4: Patterns $B_1$ (in green) and $B_3$ (in red): Mixing angle $\theta_{13}$ versus phase $\delta$. All the parameters are in degrees.

For the matrices $B_1$ and $B_3$, the correlation plots of Dirac-like CP violating phase $\delta$ and Majorana phases phases $\rho$ and $\sigma$ have been shown in Fig. 5(a) and (b). The plots indicate that the Majorana phases $\rho$ and $\sigma$ now take quite small values, in particular, phase $\rho$ lies between $-1.5^0 - 1.5^0$, whereas, $\sigma$ take values from $-7^0 - 7^0$.

Figure 5: Patterns $B_1$ (in green) and $B_3$ (in red): Correlation plots of phases $\rho$ and $\sigma$ versus $\delta$. All the parameters are in degrees.

4.3 Class C

Similar to the earlier classes, for the matrix belonging to class C, as a first step, we have examined the implications of non zero and large $\theta_{13}$ on phase $\delta$, in Fig. 6 we have given $\theta_{13}$ versus $\delta$ plot. From the graph, one finds that corresponding to the $3\sigma$ CL experimental range of angle $\theta_{13}$, the phase $\delta$ takes values from $0^0 - 60^0$, $120^0 - 230^0$ and $300^0 - 350^0$.

In Fig. 7(a) and (b) we have given the correlation plots of Dirac-like CP violating phase $\delta$ and Majorana phases phases $\rho$ and $\sigma$. For both these phases, we do not obtain any useful constraint, i.e., these takes values from $-90^0 - 90^0$.  

Figure 6: Pattern C: Mixing angle $\theta_{13}$ versus phase $\delta$. All the parameters are in degrees.

Figure 7: Pattern C: Correlation plots of phases $\rho$ and $\sigma$ versus $\delta$. All the parameters are in degrees.

4.4 Comparing the results for normal and inverted mass ordering

After having discussed the implications of recent refinements of neutrino oscillation parameters on the Dirac-like CP violating phase $\delta$ as well as on Majorana phases $\rho$ and $\sigma$, we now briefly discuss and compare the results of the analysis for the normal (NO) and inverted (IO) mass orderings.
| Class | Normal mass ordering (NO) | Inverted mass ordering (IO) |
|-------|---------------------------|----------------------------|
|       | \(\delta = 0^\circ - 360^\circ\) | \(\delta = 90.05^\circ - 91.03^\circ\) |
|       | \(\rho = -90^\circ - 90^\circ\) | \(\rho = 0.09^\circ - 3.00^\circ\) |
|       | \(\sigma = -90^\circ - 90^\circ\) | \(\sigma = -1.3^\circ - 0.2^\circ\) |
|       | \(|\delta| = 0.021 - 0.0113\) | \(|\delta| = 0.0540 - 0.278\) |
|       | \(|\rho| = 0.0087 - 0.0144\) | \(|\rho| = 0.0536 - 0.0811\) |
|       | \(|\sigma| = 0.0447 - 0.0565\) | \(|\sigma| = 0.0290 - 0.0274\) |

Table 2: The allowed ranges of Dirac-like CP violating phase \(\delta\), the Majorana phases \(\rho, \sigma\) and three neutrino masses \(m_1, m_2, m_3\) for the experimentally allowed classes. Masses are in eV.
To this end, in Table 3.3 we have summarized the allowed ranges of Dirac-like CP violating phase $\delta$, the Majorana phases $\rho, \sigma$ and three neutrino masses $m_1, m_2, m_3$ for the experimentally allowed classes. A few interesting points are as follows:

- As is obvious from the table, the matrices belonging to class A are ruled out for IO, whereas for NO, although both $A_1$ and $A_2$ are compatible, however, one is not able to obtain any useful constraints for either of the CP violating phases.

- All the matrices belonging to class B are viable for both NO and IO. For both the mass orderings, phase $\delta$ takes values close to $90^0$ and $270^0$. The Majorana phases $\rho$ and $\sigma$ are both quite small, in particular for IO these take values smaller to the corresponding values for NO. For NO, these conclusions have already been shown graphically in Figs. 4 and 5(a), (b). The corresponding graphs pertaining to IO are similar and hence not shown here.

- Again the matrix belonging to class C is viable for both NO and IO. Unlike the matrices of class B, now one finds significantly different constraints on CP violating phase $\delta$ for NO and IO. In particular, for NO, $\delta$ takes values from $0^0 - 62.4^0, 114^0 - 244^0$ and $294^0 - 360^0$. However for IO, the lower limit of $\delta$ becomes considerable higher, in particular $\delta$ takes values from $40.5^0 - 86.8^0, 93.2^0 - 266.5^0$ and $273.2^0 - 325^0$. Further, regarding the Majorana phases, for NO, the phases $\sigma$ as well as $\rho$ lie between $-90^0$ to $-27.3^0$ and $27.3^0 - 90^0$. However, for IO, although phase $\sigma$ remains unconstrained, the phase $\rho$ gets considerably constrained to $-18.4^0 - 18.2^0$.

For NO, these conclusions have already been shown graphically in Figs. 6 and 7(a), (b). For IO, mixing angle $\theta_{13}$ versus phase $\delta$ graph is shown in Fig. 8. From the graph, one finds that corresponding to the $3\sigma$ CL experimental range of angle $\theta_{13}$ i.e. $7.9^0$ to $9.3^0$, one obtains a slight constraint on phase $\delta$, in particular it largely lies between $40^0 - 325^0$.

![Figure 8: Pattern C: Mixing angle $\theta_{13}$ versus phase $\delta$ for IO. All the parameters are in degrees.](image)

Again for IO, in Fig. 9(a) and (b), we have shown the correlation plots of Dirac-like CP violating phase $\delta$ and Majorana phases $\rho$ and $\sigma$. Interestingly,
we find that although for phase $\sigma$ we do not obtain any useful constraint, i.e., it takes values from $-90^0 - 90^0$, however, phase $\rho$ gets narrowed down considerably, i.e., it lies between $-18^0 - 18^0$. Also, one finds that phase $\rho$ shows considerable dependence on Dirac-like CP violating phase $\delta$.

![Figure 9: Pattern C: Correlation plots of phases $\rho$ and $\sigma$ versus $\delta$ for IO. All the parameters are in degrees.](image)

## 5 Summary and conclusions

To summarize, in the light of recent refinements of mixing angle $\theta_{13}$, we have re-investigated all the possible texture structures of three zero and two zero neutrino mass matrices for both Majorana and Dirac neutrinos in the flavor basis. In particular, we have examined implications of angle $\theta_{13}$ on the parameter space of three CP violating phases $\rho$, $\sigma$, $\delta$. For the case of texture three zero mass matrices, in confirmation with the earlier results, we find that all possible structures remain ruled out even with the recent data. For a particular texture structure, this has been shown using correlation plots between angles $\theta_{12}$ and $\theta_{23}$ for the constraints obtained from real and imaginary parts of ratios of complex neutrino mass eigenvalues. These graphs show that $\theta_{12}$ obtained here has no overlap with its experimental range, therefore, ruling out the present texture specific case.

For the case of texture two zero mass matrices, out of the 15 possible structures, the matrices belonging to classes D, E and F remain ruled out even with the latest data. For the classes A, B and C, analysis has been carried out for both NO and IO. The analysis shows that the matrices of class A are ruled out for IO, whereas, for NO, one does not obtain any useful constraints on the CP violating phases. For the class B matrices, all these are viable for both NO and IO. For both the mass orderings, phase $\delta$ takes values close to $90^0$ and $270^0$, however, the Majorana phases $\rho$ and $\sigma$ both acquire quite small values. The matrix belonging to class C is again viable for both NO and IO. For NO, phase $\delta$ has lower limit $0^0$, whereas, for IO the lower limit is $40.5^0$. The Majorana phases also acquire different constraints for different mass orderings.
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