On the possible experimental manifestations of
the torsion field at low energies

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Abstract. We construct the theoretical base for the search of the possible experimental manifestations of the torsion field at low energies. The weakrelativistic approximation to the Dirac equation in an external torsion field is considered. For the sake of generality we introduce the external electromagnetic field in parallel. The generalized (due to torsion dependent terms) Pauli equation contains new terms which have the different structure if compared with standard electromagnetic ones. Just the same takes place for the weakrelativistic equations for spin $\frac{1}{2}$ particle in an external torsion and electromagnetic field. It is given the brief description of the possible experiments.
1 Introduction

The modern gravitational theories are based on the geometrical description of the gravitational field. For instance in the framework of General Relativity the space-time is the Riemannian Manifold and the gravitational field occurs as a metric tensor field on this manifold.

It is known for a long time that the non-Riemannian Geometry gives the appropriate base for the new gravitational theories. In particular the theory where the torsion field is called for the description of the gravitational field along with metric is of special interest. In the framework of General Relativity only the Energy-Momentum Tensor of matter fields is the source of gravity. At the same time in the theories with torsion one can consider the Spin Tensor of matter as an additional source of the gravitational interaction [1,2]. The torsion field naturally arises within the gauge approach to gravity [3,4]. Thus this kind of theories possesses better conceptual features, and is interesting for investigation. One can find the review of gravity with torsion in Ref’s [1,2,5-8]. Some questions related with our subject, namely the equation for particle with spin $\frac{1}{2}$ have been discussed in the papers [15-17].

If the torsion really exist, the investigation of its coupling with matter fields is of crucial importance for the understanding of this phenomena. The interaction of free matter fields with external torsion field have been considered in a number of papers (see, for example, [3-6, 9-11,18,19] and references therein). Some aspects of the interacted matter fields theory in an external gravitational field with torsion have been discussed in [12,13] (see also [6]). As it was shown in [7,8], the requirement of the multiplicative renormalizability makes us to introduce the nonminimal interaction of torsion with spinor and scalar fields. The renormalization group analysis of GUT’s in an external gravitational field with torsion shows that the interaction of matter fields with torsion increase in a strong gravitational field. Therefore one can conclude that the torsion have more essential manifestations at high energy level. On the other hand the interaction with torsion is weakened at low energies. This fact gives the possible reason to the absence of torsion in a modern experimental data. Note that the low-energy manifestations of torsion are quite interesting. In particular, the investigation of the weakrelativistic limit for the spin $1/2$ field in an external torsion field gives some new predictions which may turn out to be the base for the experimental search of torsion [14].

Does the torsion field really exist? The definite answer can be obtained only on the experimental basis. The purpose of the present paper is to consider the theoretical grounds for the experimental tests which can detect the possible torsion effects.
Since the torsion field is the element of the gravitational interaction, this field must couple with matter in a universal way. Therefore we can suppose that torsion interact with all the particles which have the nontrivial spin. The investigation of the torsion-matter coupling is usually argued by the possible cosmological applications. Here we discuss the possible low-energy manifestations of torsion field, and show that the torsion field may lead to some phenomena in the microscopic physics. Of course we do not claim the existance of torsion, but only consider the way to test this fact.

The paper is organized as following. In section 2 we shall write the action of spinor Dirac field in an external gravitational field with torsion. We introduce the nonminimal interaction of torsion with spinor field, that is the only way to obtain consistent quantum theory \[12,13\] (see also \[7\]). In section 3 the weakrelativistic approximation to the Dirac equation in an external torsion and electromagnetic fields is constructed. The generalized (due to torsion dependent terms) Pauly equation contains new terms which are different from standard electromagnetic ones. In section 4 the quasiclassical equations of motion for the weakrelativistic particle with spin $\frac{1}{2}$ in an external torsion and electromagnetic fields is derived. These equations contains the standard terms corresponding to the interaction with electromagnetic field and also some new terms related with torsion. In spite of usual point of view we find that this terms have the different structure if compared with electromagnetic ones. We use these new terms in section 5 where the brief description of the possible experiments is given.

## 2 Spinor field in an external gravitational field with torsion

Let us start with the basic notations for the gravity with torsion. In the space-time with metric $g_{\mu\nu}$ and torsion $T_{\beta\gamma}^\alpha$ the connection $\bar{\Gamma}^\alpha_{\beta\gamma}$ is nonsymmetric, and

$$\bar{\Gamma}^\alpha_{\beta\gamma} - \bar{\Gamma}^\alpha_{\gamma\beta} = T^\alpha_{\beta\gamma}$$ \hspace{1cm} (1)

If one introduce the metricity condition $\nabla_\mu g_{\alpha\beta} = 0$ where the covariant derivative $\nabla_\mu$ is constructed on the base of $\bar{\Gamma}^\alpha_{\beta\gamma}$ then the following solution for connection $\bar{\Gamma}^\alpha_{\beta\gamma}$ can be easily found

$$\bar{\Gamma}^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} + K^\alpha_{\beta\gamma}$$ \hspace{1cm} (2)

where $\Gamma^\alpha_{\beta\gamma}$ is standard symmetric Christoffel symbol and $K^\alpha_{\beta\gamma}$ is contorsian tensor

$$K^\alpha_{\beta\gamma} = \frac{1}{2} \left( T^\alpha_{\beta\gamma} - T^\alpha_{\beta\gamma} - T^\alpha_{\gamma\beta} \right)$$ \hspace{1cm} (3)
It is convenient to divide the torsion field into three irreducible components that are: the trace \( T_\beta = T^\alpha_{\beta\alpha} \), the pseudotrace \( S^\nu = \varepsilon^{\alpha\beta\mu\nu} T_{\alpha\beta\mu} \) and the tensor \( q^\alpha_{\beta\gamma} \), which satisfy the conditions
\[
q^\alpha_{\beta\alpha} = 0, \quad \varepsilon^{\alpha\beta\mu\nu} q_{\alpha\beta\mu} = 0
\]

Then the torsion field can be written in the form
\[
T_{\alpha\beta\mu} = \frac{1}{3} (T_\beta g_{\alpha\mu} - T_\mu g_{\alpha\beta}) - \frac{1}{6} \varepsilon_{\alpha\beta\mu\nu} S^\nu + q_{\alpha\beta\mu}
\] (4)

Now we consider the Dirac field \( \psi \) in an external gravitational field with torsion. Various aspects of the interaction of the Dirac field with torsion have been discussed in the literature (see, for example, [1,2, 15 - 18]). It is well known that the standard way to introduce the minimal interaction with external fields require the substitution of the partial derivatives \( \partial_\mu \) by the covariant ones. The covariant derivatives of the spinor field \( \psi \) are defined as follows
\[
\bar{\nabla}_\mu \psi = \partial_\mu \psi + \frac{i}{2} w^{ab}_\mu \sigma_{ab} \psi
\]
\[
\bar{\nabla}_\mu \bar{\psi} = \partial_\mu \bar{\psi} - \frac{i}{2} w^{ab}_\mu \bar{\psi} \sigma_{ab}
\] (5)
where \( w^{ab}_\mu \) are the components of spinor connection. We use the standard representation for the Dirac matrices (see, for example, [20]).
\[
\beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]
\[
\bar{\alpha} = \gamma^0 \bar{\gamma} = \begin{pmatrix} 0 & \bar{\sigma} \\ \bar{\sigma} & 0 \end{pmatrix}
\]
\[
\gamma_5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3, \quad \sigma_{ab} = \frac{i}{2} (\gamma_a \gamma_b - \gamma_b \gamma_a)
\]
The verbein \( e^a_\mu \) obey the equations \( e^a_\mu e^\nu_a = g_{\mu\nu} \), \( e^a_\mu e^{ab} = \eta^{ab} \) and \( \eta^{ab} \) is the Minkowsky metric. The gamma matrices in curved space - time are introduced as \( \gamma^\mu = e^a_\mu \gamma^a \) and obviously satisfy the metricity condition \( \bar{\nabla}_\mu \gamma^\beta = 0 \). The condition of metricity enables us to find the explicit expression for spinor connection which agree with (2).
\[
w^{ab}_\mu = \frac{1}{4} (e^b_\mu \partial_\mu e^\nu_a - e^a_\mu \partial_\mu e^b_\nu) + \Gamma^a_{\nu\beta} (e^\nu_a e^b_\alpha - e^b_\nu e^a_\alpha)
\] (6)

If the metric is flat, then from (6) follows
\[
w^{ab}_\mu = K^a_{\nu\beta} (e^\nu_a e^b_\alpha - e^b_\nu e^a_\alpha)
\] (7)

The action of spinor field minimally coupled with torsion have the form
\[
S = \int d^4x \, e \{ \frac{i}{2} \bar{\psi} \gamma^\mu \nabla_\mu \psi - \frac{i}{2} \bar{\nabla}_\mu \bar{\psi} \gamma^\mu \psi + m \bar{\psi} \psi \}
\] (8)
where $m$ is the mass of the Dirac field and $e = \det \| e^a_\mu \|$. Further we shall consider only the torsion effects and therefore restrict ourselves by the only special case of flat metric. So we put $g_{\mu\nu} = \eta_{\mu\nu}$ but keep $T^a_{\beta\gamma}$ arbitrary. The expression (8) can be rewritten in the form

$$S = \int d^4x \{ i\bar{\psi}\gamma^\mu (\partial_\mu + \frac{i}{8} \gamma_5 S_\mu) \psi + m\bar{\psi}\psi\} \quad (9)$$

One can see that the spinor field minimally interact only with the pseudovector $S_\mu$ part of the torsion tensor. The nonminimal interaction is more complicated. There are strong reasons to introduce the nonminimal coupling of the form

$$S = \int d^4x \{ i\bar{\psi}\gamma^\mu (\partial_\mu + \eta_1 \gamma_5 S_\mu + \eta_2 T_\mu) \psi + m\bar{\psi}\psi\} \quad (10)$$

Here $\eta_1, \eta_2$ are the dimensionless parameters of the nonminimal coupling of spinor fields with torsion. The minimal interaction corresponds to the values $\eta_1 = \frac{1}{8}, \eta_2 = 0$.

The introduction of the nonminimal interaction looks artificial. Within the classical theory one can explain the use of a nonminimal action only as an attempt to explore the more general case. However the situation is different in quantum region where the nonminimal interaction is the necessary condition of consistency of the theory. The reason is following. It is well-known that the interaction of quantum fields leads to the divergences and therefore the renormalization is needed. As it was shown in [12, 13], the requirement of the multiplicative renormalizability makes us to introduce the nonminimal interaction of torsion with spinor and scalar fields.

### 3 The equation of motion for spinor field in the weakrelativistic approximation

Let us consider the spin $\frac{1}{2}$ particle in an external torsion and electromagnetic fields. The equation of motion follows from (10) with the usual electromagnetic addition.

$$i\hbar \frac{\partial \psi}{\partial t} = \left\{ c\bar{\psi}\vec{p} - c\bar{\psi}\vec{A} - \eta_1 \bar{\psi}\vec{S}\gamma_5 - \eta_2 \bar{\psi}\vec{T} + c\Phi + \eta_1 \gamma_5 S_0 + \eta_2 T_0 + mc^2 \beta \right\}\psi$$

$$\quad (11)$$

Here the dimensional constants $\hbar$ and $c$ are taken into account, $A_\mu = (\Phi, \vec{A}), \ T_\mu = (T_0, \vec{T}), \ S_\mu = (S_0, \vec{S})$

Following the standard procedure we write (see, for example, [20])

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} \exp\left(\frac{imc^2t}{\hbar}\right) \quad (12)$$
Within the weakrelativistic approximation $\chi \ll \varphi$. From equations (11), (12) it follows that
\[
(i\hbar \frac{\partial}{\partial t} - \eta_1 \vec{\sigma} \vec{S} - e\Phi - \eta_2 T_0) \varphi = (c\vec{\sigma} \vec{p} - e\vec{\sigma} \vec{A} - \eta_1 S_0 - \eta_2 \vec{\sigma} \vec{T}) \chi \tag{13a}
\]
and
\[
(i\hbar \frac{\partial}{\partial t} - \eta_1 \vec{\sigma} \vec{S} - e\Phi - \eta_2 T_0 + 2mc^2) \chi = (c\vec{\sigma} \vec{p} - e\vec{\sigma} \vec{A} - \eta_1 S_0 - \eta_2 \vec{\sigma} \vec{T}) \varphi \tag{13b}
\]
Now we keep only the term $2mc^2\chi$ in the left side of (13b) and then it is possible to find $\chi$ from (13b). In the leading order in $\frac{1}{c}$ we meet the following equation for $\varphi$.
\[
(i\hbar \frac{\partial}{\partial t} = \{\eta_1 \vec{\sigma} \vec{S} + e\Phi + \eta_2 T_0 +
+ \frac{1}{2mc^2\chi}(c\vec{\sigma} \vec{p} - e\vec{\sigma} \vec{A} - \eta_1 S_0 - \eta_2 \vec{\sigma} \vec{T})^2\}) \varphi \tag{14}
\]
The last equation is easily rewritten in the Scrodinger form
\[
(i\hbar \frac{\partial}{\partial t} = \hat{H} \varphi \tag{15}
\]
where the Hamiltonian have the form
\[
\hat{H} = \frac{1}{2m} \vec{\pi}^2 + B_0 + \vec{\sigma} \vec{Q}
\]
\[
\vec{\pi} = \vec{P} - \frac{e}{c} \vec{A} - \frac{\eta_2}{c} \vec{T} - \frac{\eta_1}{c} \vec{\sigma} S_0
\]
\[
B_0 = \frac{e}{\Phi} + \eta_2 T_0 - \frac{1}{mc^2\eta_1^2 S_0^2}
\]
\[
\vec{Q} = \eta_1 \vec{S} + \frac{\hbar}{2mc}(e\vec{H} + \eta_2 \text{rot} \vec{T}) \tag{16}
\]
Here $\vec{H} = \text{rot} \vec{A}$ is the magnetic field strength. The equation of (15), (16) is the analog of the Pauly equation in the case of external torsion and electromagnetic field.

The expression for the Hamiltonian (16) indicate on the physical effects of the torsion field, that is especially clear if compared with the electromagnetic terms. For example, the quantity $T_0$ looks like scalar potential $\Phi$, $\vec{T}$ looks like vector potential $\vec{A}$. The quantities $\vec{S}$ and rot $\vec{T}$ may play the role of the magnetic field. However there is some difference between torsion and electromagnetic sectors. The term $-\frac{\hbar}{mc} \eta_1 S_0 \vec{\sigma}$ does not have the analogies in quantum electrodynamics.
The equation of motion for the particle with spin $\frac{1}{2}$ in an external torsion field.

If we consider (16) as the Hamiltonian operator of some quantum particle, then the corresponding classical energy have the form

$$H = \frac{1}{2m}\vec{\pi}^2 + B_0 + \vec{\sigma} \vec{Q}$$

(17)

where $\vec{\pi}, B_0, \vec{Q}$ are defined by (16) and $\vec{\pi} = m\vec{v}$. Here $\vec{v} = \dot{\vec{x}}$ is the velocity of the particle. From (17) it follows the the expression for the canonical conjugated momenta $\vec{p}$.

$$\vec{p} = m\vec{v} + \frac{e}{c} \vec{A} + \frac{\eta_2}{c} \vec{T} + \frac{\eta_1}{c} \vec{\sigma} S_0$$

(18)

One can consider $\vec{\sigma}$ as the coordinate of internal degrees of freedom, corresponding to spin.

Let us now perform the canonical quantization of the theory. To make this we introduce the operators of coordinate $\hat{x}_i$, momenta $\hat{p}_i$ and spin $\hat{\sigma}_i$ and input the equal - time commutation relations of the following form:

$$[\hat{x}_i, \hat{p}_j] = i\hbar \delta_{ij}, \quad [\hat{x}_i, \hat{\sigma}_j] = [\hat{p}_i, \hat{\sigma}_j] = 0,$$

$$[\hat{\sigma}_i, \hat{\sigma}_j] = 2i\varepsilon_{ijk} \hat{\sigma}_k$$

(19)

The Hamiltonian operator $\hat{H}$ which corresponds to the energy (17) is easily constructed in terms of the operators $\hat{x}_i, \hat{p}_i, \hat{\sigma}_i$ and then these operators yield the equations of motion

$$i\hbar \frac{d\hat{x}_i}{dt} = [\hat{x}_i, \hat{H}],$$

$$i\hbar \frac{d\hat{p}_i}{dt} = [\hat{p}_i, \hat{H}],$$

$$i\hbar \frac{d\hat{\sigma}_i}{dt} = [\hat{\sigma}_i, \hat{H}],$$

(20)

After the computation of the commutators in (20) we obtain the explicit form of the operators equations of motion. Now we can omit all the terms which vanish when $\hbar \to 0$. Then the classical equations arise which can be interpreted as the (quasi)classical equations of motion for the particle in external torsion and electromagnetic fields. Note that the operator arrangement problem is irrelevant because of the use of $\hbar \to 0$ limit. The straightforward calculations lead to the equations

$$\frac{d\vec{x}}{dt} = \frac{1}{m} \left( \vec{p} - \frac{e}{c} \vec{A} - \frac{\eta_2}{c} \vec{T} - \frac{\eta_1}{c} \vec{\sigma} S_0 \right) = \vec{v},$$

(21a)
\[
\frac{d\vec{\sigma}}{dt} = e\vec{E} + \frac{e}{c} \left[ \vec{v} \times \vec{H} \right] + \frac{\eta_2}{c} \left[ \vec{v} \times \text{rot} \vec{T} \right] - \eta_2 \text{grad} T_0 - \frac{\eta_2}{c} \frac{\partial \vec{T}}{\partial t} - \eta_1 \left( \vec{\sigma} \cdot \nabla \right) \vec{S} - \eta_1 \left[ \vec{\sigma} \times \text{rot} \vec{S} \right] - \frac{\eta_1}{c} \frac{\partial S_0}{\partial t} + \frac{\eta_1}{c} \left\{ (\vec{v} \cdot \sigma) \text{grad} S_0 - (\vec{v} \cdot \text{grad} S_0) \vec{\sigma} \right\} + \frac{1}{mc^2} \eta_1^2 \text{grad} (S_0^2) - \frac{\eta_1}{c} S_0 \frac{d\vec{\sigma}}{dt}, \quad (21b)
\]

\[
\frac{d\vec{\sigma}}{dt} = \left[ \vec{R} \times \vec{\sigma} \right]
\]

\[
\vec{R} = \frac{2\eta_1}{\hbar} \left[ \vec{S} - \frac{1}{c} \vec{v} S_0 \right] + \frac{e}{mc} \vec{H} + \frac{\eta_2}{mc} \text{rot} \vec{T} \quad (21c)
\]

Here $\vec{E}$ is the strength of the external electric field. Equations (21) contain the torsion-dependent terms which have the same symmetries as the usual electromagnetic terms. Really, the $T_\mu$ dependent terms are in a perfect analogy with the $A_\mu$ dependent terms. However the equations (21) contains some terms which have a qualitatively new structures. All this terms contains $S_\mu$, that is more relevant part (with respect to the interaction with the matter fields) of the torsion tensor. Thus we see that standard claim concerning magnetic field analogy of torsion effects is not completely correct, and there exist serious difference between magnetic field and torsion.

## 5 Possible experimental investigations of the torsion field

Let us now consider the Schrodinger equation (15) with the Hamiltonian operator (16) with the vanishing electromagnetic field. How can the torsion field manifest itself? It is evident that the effect of torsion field can modify the particles spectrum. This modifications have the similar form to the ones which arise in the electromagnetic field. At the same time another modifications are possible due to the qualitatively new terms like $\frac{1}{mc} \eta_1 S_0 \vec{p} \vec{\sigma}$ in (16).

It is natural to suppose that the possible interaction with torsion is feeble enough and therefore one can consider it as some perturbation. This perturbation may leads to the splitting of the known spectral lines and hence one can hope to find the torsion display within the spectral analysis experiment. In particular one can expect the splitting of the spectral lines even for the more simple hydrogen atom. Now we consider the particular case of $T_\mu = 0, \ S_\mu = \text{const}$ and estimate the possible spectrum modifications. In this particular case Hamiltonian operator is

\[
\hat{H} = \frac{1}{2m} \hat{\vec{p}}^2 + \eta_1 \left( \hat{\vec{S}} - \frac{1}{2mc} \hat{S}_0 \hat{\vec{p}} - \frac{1}{2mc} \hat{\vec{p}} \hat{S}_0 \right) \cdot \hat{\vec{\sigma}}
\]
\[ \pi = \vec{p} - \frac{e}{c} \vec{A} \quad (22) \]

In the framework of the nonrelativistic approximation \(|\vec{p}| \ll mc\) and hence the second \(S_0\) dependent term in the brackets can be omitted. The remaining term \(\eta_1 \vec{S} \vec{\sigma}\) allows the standard interpretation and gives the contribution \(\pm \eta_1 S_3\) into the spectrum. Thus, if the \(S_3\) component of the torsion tensor is not equal to zero, the energy level is split into two sublevels with the difference \(2\eta_1 S_3\). If now the weak transversal magnetic field is switched on then the cross between the new levels will arise and energy absorption takes place at the magnetic field frequency \(w = \frac{2m}{\eta_3 S_3}\). Note that the situation is typical within the magnetic resonance experiments, however in the case this effect arise due to the torsion, but not magnetic field effects. It is natural this effect as the torsion resonance. Taking into account the previous consideration we arise at the conclusion that the described effect can be explored at different scales: torsion - induced spin resonance in atoms, the torsion electron resonance and the torsion nuclear resonance in a medium. Note that the experiments related with torsion induced splitting of the energy levels was recently considered in [23].

The next kind of the possible experiments related with torsion are the ones which deal with the particle equation of motion (21). Let the electromagnetic field is absent.

Then, according to (21) the interaction with torsion twists the particle trajectory and therefore any charged particles may be the source of the electromagnetic radiation. Then the structure of the radiation field enables one to look for the torsion effects.

Of course, some evident effects like the precession of spin in an external torsion field also follows from (21). This effect have been already described in Ref.'s [1,2,15,16]. It is interesting that from (21c) follows that the direction of precession of spin depends on the velocity of the particle. Therefore even the weak torsion field may violate the observable precession of spin in a magnetic field at high temperature.

To obtain the complete picture of the torsion influence to the energy spectrum modifications as well as to the radiation of charged particles it is necessary to make the detailed and systematic investigation of the equations (15), (21) solutions in the case of a various external field structures. The main difficulty is that there are no any experimental data for the values of coupling constants. That is why it is impossible to give the numerical estimate for the mentioned physical effects. Note that the inflationary cosmological model with torsion predict a tiny value of torsion field, which have to be very slowly varying in a modern epoch [21]. Thus there are some reasons to look for the evidence of some weak global (effectively) axial vector in the Universe, and try to give the upper bound for torsion field from any modern experiments.

From the results of Ref.'s [12,13,7] it follows that the interaction of torsion with matter
fields is essentially weakened at low energies due to quantum effects. That is why we have a very small hope to observe torsion in a low-energy experiments. On the other hand even the very weak interaction with torsion may be responsible for some symmetry violation because of the pseudovector nature of the vector $S_\mu$. Indeed such an effects are essentially related with high-energy physics and our consideration have to be extended. In any case the results of the above analysis may be useful in qualitative understanding of the structure of torsion-matter interaction.

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