Multi-Hop Relay Selection Based on Fade Durations

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Abstract—The selection of a relay in a cooperative network is usually based on signal-to-noise ratio (SNR). In a situation where the SNR is low due to Rayleigh fading, the receiver signal may show a sudden loss. Such signal loss may result in significant loss of information at the receiver and it is the duration of such a loss period that will most significantly cause channel and source coding to fail. To avoid such a phenomenon in relay selection, it is important to compute second-order statistics and select relays based on fade duration (FD), both average fade duration (AFD) and the probability that a fade duration will exceed a threshold. This paper provides algorithms for AFD-based and FD threshold-based relay selection in multiple and single hop cooperative wireless networks. It also shows how to increase the reliability of the network by minimizing the average fade duration.

Keywords—Cooperative relay wireless network, decode and forward (DF), average fade duration, level crossing rate (LCR), relay selection

I. INTRODUCTION

The rapid growth and development of mobile applications and the significant increase of wireless users imposes a high reliability requirement on wireless services, especially for emergency and public safety applications. However, due to the presence of fading, it is impossible to maintain the dependable signal quality and channel characteristics. To deal with such uncertainty and improve the reliability of wireless service, it is important to implement a cooperative relay network.

The issue of relay selection in cooperative relay networks has been studied thoroughly for some time. Several types of selection criteria have been proposed. For instance in [6], a relay is selected to transfer a source signal to the destination only if it successfully decodes the source signal. Each relay stores information about the successfully decoded source signal. In [5], the best harmonic mean relay selection has been proposed. In this scheme the best harmonic mean of the channel gain from the source-relay link and relay-destination link has been considered as a relay selection criteria. In [9], spectral efficiency improvement based best relay selection has been proposed. The selection of best relay based on the maximum SNR has been proposed in [7], [8]. In [10] the analysis of time varying fading and its direct impact on relay selection was studied. In this method, the relay switching rate for each link (source-relay, Relay-Destination) derived as a function of channel gains and maximum Doppler shift. The threshold based relay selection to minimize system outage probability, energy consumption proposed in [11]. In This method relay whose instantaneous SNR value more than the threshold value are allowed to transmit.

The unpredictable behavior of the channel gain due to Rayleigh fading and its implications on the performance of the cooperative relay network needs to be more fully exploited. Previous works on selection of relays have placed little emphasis on the fade duration as a relay selection criteria. This is unfortunate because fade durations have the biggest effect on user experience and the performance of channel and source coding. Therefore, this paper focuses on this issue to make use of fade duration to find best relay selection decisions.

In this paper, the average fade duration (AFD) based relay selection for two hop and three hop cooperative wireless relay networks over Raleigh faded channels is presented. The selection method is based on the analysis of the second order statistics for LCR and AFD over multiple relay hops. We derive the closed form expression of AFD over a decode and forward (DF) cooperative network. We also consider the selection of relays and a relay path based on the probability that a fade duration would exceed a given threshold.

The article is presented to four parts. In Part II, the system model is presented. Part III discusses threshold-based relay selection for two and three hop cooperative relay networks; Part IV provides numerical analysis, optimization results and final conclusions.

II. SYSTEM MODEL

The system model is composed of a two hop and three hop cooperative relay system that consists of the source (S), relay (R), and the destination (D), as shown in Fig. 1. The model is based on the DF protocol over the half-duplex sub-channel of S-R-D for a two hop relay network, and three half-duplex sub-channels S-Rx-Ry-D for a three hop relay cooperative network. There is an assumption that the direct link between the source and the destination is weak and each sub-channel experiences independent Rayleigh flat fading. The fading is assumed to be the have the same stochastic characteristics within each time slot.

For a two hop network, the time slot is subdivided into two sub time slots. In the first time sub-slot, S transmits while R and D listen, and in the second sub-slot R transmits and D listens. In the three hop relay network the time slot is subdivided into three sub time slots. In the first sub-slot S transmits while Rx, Ry, D listen, and in the second sub-slot...
RLR transmits and D listens. We assume that only the links $S$-$R_1$-$R_2$-$D$ contribute significant signal strength to the destination. In this paper the signal transfer can be chosen to either use a single relay or multiple relays, based on the path that is found to have the minimum AFD or lowest probability of exceeding FD thresholds.

A. Two Hop Relay Network

In a single hop system, a relay is selected to transfer the received signal to the destination. The mutual information of the received signal is denoted by

$$I(t) = \frac{1}{2} \log_2 (1 + SNR|H|^2)$$

(1)

where SNR and $H$ represent signal to noise ratio and channel gain respectively. Due to Rayleigh fading, the mutual information $I(t)$ may drop below a certain level. Assuming the minimum acceptable spectral efficiency to be $I_o$ then

$$I(t) = \frac{1}{2} \log_2 (1 + SNR|H|^2) \geq I_o$$

(2)

the received signal will be decoded successfully as the mutual information exceeds $I_o$. The channel amplitude can be represented in terms of $I_o(R_o)$ as.

$$|H| = \sqrt{\frac{2I_o - 1}{SNR}}$$

(3)

An outage occurs with probability $P_r(R \leq R_o)$. During such event the received signal amplitude $R$ crosses the threshold value $R_o$ and the number of times the crossing occurs per unit time determines the level crossing rate (LCR). The average fade duration depends on the LCR value. The channel gain $H(t)$ is a random process and the threshold value of $R_o$ determines the second order statistics of the LCR and the AFD. The AFD determines the average time the received signal remains below $R_o$ and it is defined as

$$AFD = \frac{P_r(R \leq R_o)}{LCR}$$

(4)

where LCR as a function of $R_o$ is determined by the Rice equation as given by [2, equation 5.80]:

$$LCR = L(R_o) = \int_0^{\infty} \hat{r} p(R_o, \hat{r}) d\hat{r} = \int_0^{\infty} \hat{r} K_0 \exp(-\frac{r^2}{\Omega_o}) d\hat{r}$$

(5)

Now we derive the fade duration equation for multiple links. The two links between source to relay and relay to destination are represented as $Q(t)$ and $S(t)$ respectively are distributed according to the Rayleigh PDF as:

$$f_r(r) = \frac{2r}{\Omega_o} \exp\left(-\frac{r^2}{\Omega_o}\right).$$

(6)

The cumulative distribution function CDF for $P_r(a \leq r)$ is obtained as

$$\Omega_s = E[q^2] = \int_0^{\infty} r^2 dr = 2\sigma_q^2$$

(7)

and $\Omega_s = E[s^2]$ = where $\Omega_a$ denoted as average squared amplitude. In terms of $R_o$, the probability that one of the links is below the threshold value (hence the two-hop amplitude $R$ is below $R_o$) is given by

$$P_r(R \leq R_o) = 1 - P_r(Q \geq R_o)P_r(S \geq R_o)$$

(8)

From the Rayleigh fading calculation

$$P_r(Q \geq R_o) = \exp\left(-\frac{R_o^2}{\Omega_Q}\right)$$

(9)

and also

$$P_r(S \geq R_o) = \exp\left(-\frac{R_o^2}{\Omega_S}\right)$$

(10)

then from (8)

$$P_r(R \leq R_o) = 1 - \exp\left(-\frac{R_o^2}{\Omega_Q} + \frac{R_o^2}{\Omega_S}\right)$$

(11)

Assuming $\Omega_Q = \Omega_S$, (11) can be simplified as

$$P_r(R \leq R_o) = 1 - \exp\left(-\frac{2R_o^2}{\Omega_o}\right)$$

(12)

From the Rice Equation, the joint probability of R and $\dot{r}$ is expressed as

$$K_{R_o,\dot{r}}(R_o, \dot{r}) = K_{Q_o,\dot{q}}P_r(Q(t) \geq R_o) + K_{S_o,\dot{s}}P_r(S(t) \geq R_o)$$

(13)

where $K_{R_o,\dot{r}}$ denotes the PDF of $\dot{r}$ when $\dot{r}$ is a time differentiation of R when it crosses the threshold value of $R_o$. Then applying (13) into the Rice equation for the entire two hop relay network we obtain

$$L(R_o) = L_Q(R_o)P_r(s(t) \geq R_o) + L_S(R_o)P_r(q(t) \geq R_o)$$

(14)
as given by [2, equation 5.80]. The LCR $L_N$ for individual links in terms of $R_o = Q_o = S_o$ as their respective thresholds is represented as

$$L(Q_o) = 2\pi f_o \rho \exp(-\rho^2)$$

(15)

$$L(S_o) = 2\pi f_o \rho \exp(-\rho^2)$$

(16)

where $f_o$ and $f_r$ are denoted the Doppler shift and $\rho$ represents the value of $R$ normalized to the RMS value (see Fig. 4 for LCR plot). $L_Q(R_o)$ is expressed in terms of the received signal as

$$L_Q(R_o) = 2\pi f_m \rho e^{-\rho^2}$$

(17)

Similarly, $L_S(R_o)$ can be expressed as

$$L_S(R_o) = 2\pi f_m \rho e^{-\rho^2}$$

(18)

rearranging (14) using the above two equations yields

$$L(R_o) = \frac{R_o}{\Omega_Q} \exp(-2\frac{R_o^2}{\Omega_Q})$$

(19)

Finally, the AFD can be simplified as

$$AFD_k = \frac{2\pi Q_o \exp(\frac{2R_o^2}{3\Omega_Q})}{R_o} - 1$$

(20)

representing the closed form of average fade duration for path $k$ in a two hop cooperative relay network. The selection of the best relay easily relies on the value of the minimum AFD value observed during the selection period.

B. Three Hop Relay Network

Referring to Fig.1, the three hop relay path has three links in order to reach the final destination. If two relays are involved, the time slot to get from the source to the destination is divided into three sub time slots. In the first sub time slot, the source sends the data to the relay and destination, and in the remaining two sub time slots the selected two relays use those sub time slots to reach to the destination. Therefore the mutual information is expressed as $|H| = \sqrt{\frac{2\pi f_o}{\Omega_Q}}$. The probability that $R$ will be less than the threshold value, $R_o$, is given by

$$P_r(R \leq R_o) = 1 - P_r(P \geq R_o)P_r(Q \geq R_o)P_r(S \geq R_o)$$

(21)

Assuming $\Omega_Q = \Omega_S = \Omega_P$ as shown in (10), the equation (16) simplified as

$$P_r(R \leq R_o) = 1 - \exp(-\frac{3R_o^2}{\Omega_o})$$

(22)

The LCR for $L_Q(R_o)$, $L_R(R_o)$, $L_S(R_o)$ is expressed as

$$L_P(R_o) = 2\pi f_m \rho e^{-\rho^2}$$

(23)

Applying (14) for three hop relay using the above three equations yields

$$L(R_o) = \frac{R_o}{\Omega_Q} \exp(-2\frac{R_o^2}{\Omega_Q})$$

(26)

$$\Omega_Q = \int_0^\infty r^2 P(r)dr = (\sigma_Q^2)2$$

(27)

applying (22) and (26) into (4), finally yielding the AFD in the closed form as

$$AFD_k = \frac{\sqrt{\Omega_Q \pi}}{3R_o} \exp(\frac{2R_o^2}{3\Omega_Q}) - 1$$

(28)

To select the best relay for three hop cooperative network, the closed-form of equation (28) could be applied.

C. Fade Duration Probability

Due to Rayleigh fading, the received signal is usually affected by how long it remains in a dip. While the received signal remains in a dip, an outage occurs. To select a relay based on the frequency of dips and to minimize outages, it is important to compute $P_r(dip > T_{thr})$. To ensure a condition where $P_r(dip > T_{thr})$ is acceptable, the following conditions need to be satisfied.

Let $D_t$ be the time in a dip, and $T_{thr}$ the threshold time beyond which performance is adversely affected.

$$P_r(D_t > T_{thr}) = P_r(dip)P_r(D_d > T)$$

(29)

$$P_r(D_t > T_{thr}) = P_r(R \leq R_o)P_r(rate < \frac{1}{T_{thr}})$$

(30)

Putting both together $P_r(R \leq R_o)$ and $P_r(rate < \frac{1}{T_{thr}})$ yielding

$$P_r(D_t > T_{thr}) = (1 - e^{-\rho^2})(2\pi e^{-\rho^2})(\frac{1}{2} - Q(\frac{1}{T_{thr}\sqrt{\Omega_o}}))$$

(31)

The above equation represents the fade duration probability where the dip persists more than a certain threshold time. Selection of a relay, based on this criteria, minimizes the outage rate and increases the performance of the cooperative relay network.
III. AFD AND THRESHOLD BASED RELAY SELECTION

The traffic demand and the maximum allowable data loss that can be tolerable during wireless communication differs from one application to the other. For instance, a video signal will lose the picture synchronization if the data loss is more than a certain number of frames. During deep Rayleigh fading, as the length of the fade duration becomes longer and deeper, the more data gets lost before reaching the BS. Such loss will be hard to recover. To avoid such issues, relay selection based on the AFD value is proposed. The selection process needs to determine which relay performs better in order to satisfy the selection criteria. The method is based on the least AFD value from the source to the destination. The AFD value for the selected relay path also needs to be less than the AFD threshold value.

The cooperative relay selection problem to facilitate the best relay selection among N available relays for S number of sources has been formulated. To determine the best relay, two different methods are formulated.

A. Optimization method 1

To determine the AFD value, we apply equation (4) and use the derived closed forms (20) and (28). To select the best relay path, the optimization method utilizes (20),(28). Let $x_{kij}$ indicate choice of source k, relay i, and relay j. If all three values are nonzero then the three hop path is used and the path value equals 1. If j is zero and the others nonzero then the path value equals 1.

The number of sources be K and number of relays N. $AFD_{kij}$ represents AFD for source k, first relay i, and last relay j. $P_v = 1.5$ is a penalty value for using three hops, since it would use an extra sub time slot and more network resources. The penalty is applied only for a three hop relay path where two relays are in use otherwise, $P_v = 1.0$ . No penalty is imposed on two-hop paths. The penalty is based on the additional time slot required (three sub time slots) for a three hop relay path, compared to only the two sub time slot requirement for a two hop relay path.

The objective is to minimize the average fade duration from the source to the destination through the selected relay. The objective function is

$$\min(\sum_{k=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} AFD_{kij}x_{kij}P_j) \quad (32)$$

Subject to

$$\sum_{k=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} x_{kij} \leq 1 \quad \forall k = 1,2,3..n \quad (33)$$

$$\sum_{k=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} x_{kij} \leq 1 \quad \forall j = 1,2,3..n \quad (34)$$

$$AFD \leq AFD_{\text{threshold}} \quad (35)$$

This is a simple LP where constraint (33) states that one source can be associated with only one relay. Constraint (34) ensures one or two relays can be a helper for a source for a source node. The relay selected cannot be used by any other source in that particular time slot. (34) also states that the number of relays selected can either be 1 or 2. If only one relay is selected then the relay paths will be two hops, otherwise it will be three hops.

Constraint (35) ensures that the selected relay does not cause the minimum AFD value but also that the AFD needs to be less than the threshold value.

B. Optimization method 2

In this method the AFD value of each link of each source-to-relay and relay-to-destination are optimized and the min-max method applied to select the best relay.

- $y^R_i$ equals 1 if the source data is relayed by two relays, and 0 otherwise.
- $z^R_R$ equals 1 if the source data is relayed by a single relay R to reach the Destination.
- $P_v$ is the penalty value for using three hops (two relays). No penalty is imposed on two-hop paths. The penalty is based on the additional time slot required (total three sub time slots) for a three hop relay path, compared to only the two sub time slot requirement for a two hop relay path.

- $AFD(S,R_r)$ is the AFD value between the source i and relay r.
- $AFD(R_r,D)$ is the Fade duration value between the relay r and relay k.
- $AFD(R_k,D)$ is the value of the Fade duration between the relay k and destination D.

In each scenario, the AFD value of each of the source-relay and relay-destination links should be above the threshold value to be considered as a potential path. The AFD value for each link (source-relay,relay-destination) is computed as given by [2, equation 5.84].

The objective is first to select the link with the maximum average fade duration from the source to the destination through each two-hop or three-hop relay path. Then select the min of the selected values to find the best path. It is optimized as

$$\min(\max_{k=1}^{n}(\sum_{k=1}^{n} AFD(S,R_k)), \sum_{k=1}^{n} AFD(y^R_i R_i R_m),$$

Subject to

$$0 \leq y^R_i \leq 1 \quad (37)$$

$$0 \leq y^R_i \leq 1 \quad (38)$$

$$AFD \leq AFD_{\text{threshold}} \quad (39)$$

This is another simple LP where the constraints (37), (38)
IV. NUMERICAL ANALYSIS

Figs. 2 and 3 present path and relay selection optimization results from section III.A optimization where two or three hops could be chosen for each source to destination path. The values the Doppler shifts ($f_{SR}, f_{Rx,Ry}, f_{RD}$) are all assumed to be equal and the channel gain $|H|$ varies on each hop. Despite the similar SNR values, each source chooses different paths with respect to minimum AFD values, as shown in Fig. 2,3. Ref. Fig. 2, if the selection of the path is based on the SNR value, the path would have been either $S_1 - R_1 - D, S_2 - R_2 - D$ or $S_1 - R_2 - D, S_2 - R_1 - D$ without considering the relay link.

The probability $Pr(d_{ip} > T_{th})$ from sec.IIC for a single link is shown in Fig. 6. The standard AFD and LCR curves are given in Figs. 4 and 5 for comparison. Fig. 6 shows the dependency on the SNR, which is $\rho$ under equation(31). The highest probability of violating the threshold occurs at $\text{SNR} = \rho = 1$. 

state that each source will be served at least by one relay. Constraint (39) enforces the AFD value to remain below the threshold value.
V. CONCLUSION

In this paper we study the selection of relays in decode-and-forward cooperative relay networks. The closed-form expressions for average fade duration for two-hop and three-hop relay networks has been derived. Two optimization approaches have been developed from these formulations to facilitate best relay selection. The results have justified our argument that selection of relays based on average fade duration is a better choice than SNR-based selection for Rayleigh fading.

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