Application of Adaptive Kalman Filtering Algorithm in IMU/GPS Integrated Navigation System

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Abstract  The IMU (inertial measurement unit) error equations in the earth fixed coordinates are introduced firstly. A fading Kalman filtering is simply introduced and its shortcomings are analyzed, then an adaptive filtering is applied in IMU/GPS integrated navigation system, in which the adaptive factor is replaced by the fading factor. A practical example is given. The results prove that the adaptive filter combined with the fading factor is valid and reliable when applied in IMU/GPS integrated navigation system.

Keywords  integrated navigation; adaptive filtering; fading filtering; the earth fixed coordinates

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Introduction

IMU is a kind of sensor sensing angle rate and acceleration based on MEMS with lower accuracy compared to routine inertial units. IMU errors come from the units selves, and the errors accumulate through time. This is the reason why IMU can not meet the requirement of navigation and positioning in accuracy. IMU/GPS, however, benefit with their own advantages, perform higher efficiency with the mutual benefit. It is well known that there are two kinds of IMU/GPS, i.e., loosely and tightly integrated systems[1,2], the former has lower accuracy, but is simple and easy to realize.

In order to control the influences of outdated state on the current state parameters, the fading filtering algorithm was presented in statistics in the early 1960’s and 1970’s[3,4], i.e., using factors limiting Kalman filter memory length, for the purpose of exploiting the current observation data sufficiently. On the basis of robust estimation principle, Yang Yuanxi, et al. presented the whole state information robust filtering[5]. Adaptive factors were adaptively determined based on the discrepancy between observation and model information with the whole robustness and reliability. After the careful analysis[5-7] and sufficient application, the new whole state information filtering advantages over the current fading filtering and other adaptive filtering. But if the number of the measurements is not enough in some epochs, it’s difficult to construct adaptive factors based on the state discrepancy or the variance ratio. Therefore, it is necessary to estimate the adaptive factors without estimating the state parameters at those epochs.

This paper investigates the loosely integrated navigation system with IMU and GPS positions and velocity as the observations, presents constructing the
adaptive factors with the predicted residuals statistic, which is similar to the fading factor of fading filtering, and designs adaptive Kalman filter of integrated navigation system. In order to verify the performance of the integrated navigation system, field data set is investigated for the system. The results show that the performance of the system is greatly improved comparing with the classical Kalman filter.

1 Error equations in Earth fixed coordinates

In theory, IMU mechanization algorithm can be performed in the inertial coordinates, the earth fixed coordinates or the local level coordinates. Because the earth fixed coordinates do not change as the body moves, the items related to $\omega_{el}$ (the rotating angular velocity of the local level coordinates relative to the earth fixed coordinates) do not exist, and both calculation formula and procedure are simple. In addition, the three dimension coordinates of the vehicle in the earth center right-angle coordinates, such as, CTF (WGS84) three dimension coordinates of the vehicle in the earth fixed coordinates (earth fixed coordinates) do not exist, and both calculation formula and procedure are simple. In addition, the earth rotation angular velocity, gravitational constant multiplied by the earth quality; $\Omega_e$ is the earth rotation angular velocity; $f^e$ is the force in the earth fixed coordinates.

3) Position error $\delta r$.

$$\delta r = \delta v, \quad \delta r = (\delta x, \delta y, \delta z)$$ (3)

4) The gyro and acceleration stochastic drift error $\delta g$ and $\delta a$. After analyzing the gyro and acceleration, it’s shown that the error model is a Markev stochastic process of one order separately as follows:

$$\delta g = -1/T_g \delta \dot{g} + w_g, \quad \delta a = -1/T_a \delta \dot{a} + w_a$$ (4)

where $T_g, T_a$ and $w_g, w_a$ are one order related time and Gauss noise of the gyro and acceleration.

2 IMU/GPS integrated navigation adaptive filtering

IMU/GPS exploits the loosely integrated navigation model with 15 dimension system state parameters, positions, velocity, attitude error, 3-axis gyro and accelerometer drift. All items are as follows:

$$\dot{X} = \left[ \delta x, \delta y, \delta z, \delta v_x, \delta v_y, \delta v_z, \delta \phi, \delta \theta, \delta \psi, \delta g, \delta a \right]$$

According to the Eqs.(1)-(4), the continued system state equations are as follows:

$$\dot{X}(t) = F(t)X(t) + G(t)W(t)$$ (6)

where $W(t) = \left[ w_g, w_a \right]^T$ is the system noise; $F(t)$ is the dynamic matrix.

$$F(t) = \begin{bmatrix}
0 & I_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
N & -2\Omega^e & F^e & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & T_g & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & T_a & 0 \\
\end{bmatrix}$$ (7)

where the signs have been shown in Eqs.(1)-(4).

For Eq.(6), the Taylor series can be dispersed[6], the discrete state equation is:

$$\overline{X}_k = \Phi_{k,k-1}X_{k-1} + w_k$$ (8)

where $X_k$ is the state parameter vector at $k$ time; $X_{k-1}$ is the state vector at $(k-1)$ time, its esti-
mated value is \( \hat{X}_{k+1} \); \( w_k \) is the dynamic model error vector, its covariance matrix is \( \Sigma_{w_k} \), \( \Phi_{k,k-1} \) is the state transition matrix.

The position and velocity differences of GPS and IMU outputs are taken as the observation (loosely integrated), in order to construct the observation equations. Designing \( x_{gps} \), \( v_{gps} \), \( r_{gps} \), and \( v_{imu} \), \( r_{imu} \), \( r_{imu} \) and \( v_{imu} \) as the outputs of GPS in WGS84, \( x_{imu}, v_{imu}, r_{imu} \) and \( v_{imu} \) as the outputs of IMU, where GPS outputs can control the influences of the observation outliers and the vehicle disturbances\(^{[5-7]} \) efficiently.

Assuming that:

\[
L_{k+1} = \begin{bmatrix} r_{gps} - r_{imu} & v_{gps} - v_{imu} \end{bmatrix}^T
\]

(9)

The error equations are as follows:

\[
V_k = A_k \hat{X}_k - L_k
\]

(10)

where \( A_{k,k+1} = \begin{bmatrix} 1 \times 3 & 0 \times 3 & 0 \times 9 \\ 0 \times 3 & 0 \times 9 \end{bmatrix} \) is the observation matrix; \( L_k \) is the observation vector, its covariance matrix is \( \Sigma_{k} \); \( V_k \) is the residual vector; \( X_k \) is the state parameter vector.

Taking the adaptive filtering risk function as:

\[
V_k^T \Sigma_{k}^{-1} V_k + \alpha_k (\hat{X}_k - \tilde{X}_k)^T \Sigma_{k}^{-1} (\hat{X}_k - \tilde{X}_k) = \min
\]

(11)

The recursive estimators are:

\[
\hat{X}_k = \Phi_{k,k-1} \hat{X}_{k-1} + \Sigma_{x_k}
\]

(12)

\[
\Sigma_{x_k} = (\Phi_{k,k-1} \Sigma_{x_{k-1}} \Phi_{k-1,k-1} + \Sigma_{w_{k}}) / \alpha_k
\]

(13)

\[
\hat{X}_k = \Sigma_{x_k} + K_k [L_k - A_k \hat{X}_k]
\]

(14)

\[
K_k = \Sigma_{x_k} A_k^T [A_k \Sigma_{x_k} A_k + \Sigma_{e_k}]^{-1}
\]

(15)

\[
\Sigma_{x_k} = [I - K_k A_k] \Sigma_{x_k}
\]

(16)

where \( \Sigma_{x_k} \) and \( \Sigma_{x_{k-1}} \) are the covariance matrix of \( L_k \) and \( \tilde{X}_k \); \( \alpha_k \) is the adaptive factors, with values of \( 0 < \alpha_k \leq 1 \). If \( \alpha_k = 1 \), the adaptive filter changes into the classical Kalman filter. When the dynamics model error increases or the vehicle movement is in an unstable state, \( \alpha_k \) should be smaller than 1 and the influence of predicted state information on the filtering results will be decreased. When the vehicle is in an obvious disturbance state, \( \alpha_k \) should be nearly equal to 0. Obviously \( \alpha_k \) plays an important role in balancing the dynamic model information and the measurements.

Unlike Eq.(11), the fading filtering mainly controls the state estimating error at \( k-1 \), and the fading factors act on \( \hat{X}_{k+1} \) of the Eq.(13) as follows:

\[
\Sigma_{x_{k+1}} = \lambda_k \Phi_{k,k-1} \Sigma_{x_k} \Phi_{k-1,k-1} + \Sigma_{w_k}
\]

(17)

The rest formulas are the same as the Eqs.(12)-(16), \( \lambda_k \) is the fading factor. When the filtering disperses or the state bias increases, it is difficult to judge that it is the state estimating errors or the disturbances errors.

In other way, the fading factors in Reference [7] have its obvious advantages, i.e., the state estimating in current epoch need not to be estimated. So the fading factor is introduced to the adaptive Kalman filtering in Eq.(13), where the adaptive factor is the reciprocal of the fading factor as:

\[
\alpha_k = 1 / \lambda_k
\]

(18)

\[
M_k = A_k \Sigma_{x_k} A_k^T + \Sigma_{e_k}
\]

(19)

\[
C_{k+1} = \begin{cases} \hat{X}_k & k > 1 \\ \frac{1}{2} \hat{X}_k & k = 1 \end{cases}
\]

(20)

where \( \hat{X}_k = L_k - A_k \hat{X}_k \) is predicted residual vector.

Here the filtering results are the adaptive filtering results, and the significant difference between the adaptive Kalman filtering and the classical Kalman filtering is that the prior state covariance matrix and the state disturbances covariance matrix of the former are expanded to \( \frac{1}{\alpha_k} \) times at the same time, in order to reduce the application efficiency of the outdate state information and reuse current measurement information.

### 3 Calculations and comparisons

Data are collected in a gracier surveying campaign at Alps mountains. Trimble 4000 SSI GPS and NovAtel Millenium GPS receivers are mounted on the base station and mobile platform exploiting IMU Litton LN 200 to measure 3-axis angular acceleration and linear acceleration. IMU sampling frequency is 200 Hz, GPS sampling cycle is 1.0 s, and the integration cycle is 1.0 s. The initial longitude, latitude and altitude are 8.633°, 47.406° and 484.305 m respectively.
In the example, the following parameters are determined based on the experience, i.e., gyro and accelerometer correlation time are 100.0 s, 60.0 s respectively. The initial covariances of gyro and accelerometer bias are 1.0°/h and 50 μg; the initial position errors are 1.0 m, 1.0 m and 5.0 m; the initial velocity error is 0.01 ms⁻¹; the initial platform alignment errors are 100 s, 100 s and 500 s; the initial variances of GPS pseudorange and Doppler observation are 1.0 m² and 0.1 m²/s⁻². The position and velocity outputs from the kinematical positioning software: PosGPS3.0 (developed by the Institute of Navigation of University of Stuttgart) are taken as “true values”. Here three calculation schemes are performed: ① the classical Kalman filtering for IMU/GPS integrated navigation system (CKF); ② the fading Kalman filtering for IMU/GPS integrated navigation system (FKF); ③ the adaptive Kalman filtering for IMU/GPS integrated navigation system (AKF).

The differences between the results from above three schemes and “true values” are plotted out as in Figs.1-3. Because the errors in \( X, Y \) and \( Z \) axes are similar, only \( X \) axis error is given. The Comparison of RMS in Table 1.

After analyzing the above results, it’s shown as follows.

1) When the classical Kalman filtering is applied to IMU/GPS integrated navigation system, the output results are not ideal due to the system noise, observation noise covariance matrices are not accurate and there are the model errors in the state equation and observation equation. See Fig.1 and Table 1.

2) The Fading filtering can control the influences of state estimating error at the previous time through the fading factor \( \lambda \) and improve the accuracy, and the kinematic performance evidently over the classical Kalman filtering. See Fig.2 and Table 1.

3) The adaptive factor \( \alpha \) can control the influences of state estimating errors and state disturbances errors at the same time, and the accuracy of the adaptive
Kalman filtering is slightly better than the fading Kalman filtering. See Fig. 3 and Table 1.

Table 1  Comparison of RMS

|       | Position /m | Velocity /ms⁻¹ |
|-------|-------------|----------------|
|       | X axis      | Y axis         | Z axis      |
| CKF   | 3.382       | 3.350          | 3.405       | 0.782       | 0.325       | 0.754       |
| FKF   | 0.502       | 0.531          | 0.454       | 0.327       | 0.098       | 0.338       |
| AKF   | 0.500       | 0.528          | 0.453       | 0.326       | 0.097       | 0.338       |

Comparing the results of Fig. 3 and Table 1, we find that the adaptive Kalman filtering, by using adaptive factor $\alpha$, can control the influences of state estimating error and the disturbances of the dynamical model at the same time. In IMU/GPS integrated navigation system, adaptive factors should be selected according to the redundancy of the measurement vector $L$. When the observations are enough, adaptive factors can be constructed based on the discrepancy or the variance ratio theory\textsuperscript{[5-7]}, otherwise, constructed by using predicted residuals\textsuperscript{[7]}, when the adaptive factor $\alpha$ is constructed by the predicted residuals in IMU/GPS integrated navigation system, it’s necessary for the adaptive factor to efficiently control the influences of state estimating error and the disturbances errors at the same time.

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