Bulk viscosity-driven freeze-out in heavy ion collision

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Abstract

We give an review the HBT puzzle, and argue that its resolution requires the introduction of new physics close to the phase transition scale. We argue that a candidate for this new physics is bulk viscosity, recently postulated to peak, and even diverge, close to the phase transition temperature. We show that such a viscosity peak can force the system created in heavy ion collisions to become unstable, and fragment into fragments whose size is weakly dependent on the global size of the system, thereby triggering freeze-out.

1 The HBT puzzle

One of the most unexpected, and as yet unexplained, experimental results found at the Relativistic Heavy Ion Collider (RHIC) concerns the description of particle interference observables \[1\]. It was originally expected that deconfined matter would be a highly viscous, weakly interacting quark gluon plasma \[2\]. Thus, ideal hydrodynamics would not provide a good description of our observables sensitive to the early stages of the collision, such as azimuthal anisotropy. The signature of choice of a phase transition from hydrodynamics models, one less sensitive to viscosity, was to be an increase of the \texttt{out} to \texttt{side} emission radius ratio \(R_o\) and \(R_s\), see Fig. 1 center panel) due to longer lifetime of the system, caused by the softening of the equation of state in the transition/crossover region \[3\].

The data, however, exhibited an opposite behaviour. Hydrodynamic simulations provided a good description of transverse momentum spectra and their azimuthal anisotropy. The same simulations, however, failed to describe HBT data \[4\]. Perhaps the most surprising aspect of the problem is the way in which the data does not: Measured parameters \(R_o\) and \(R_s\) are nearly identical. (Fig. 1 left and right panel) Their (positive) difference \(R_o^2 - R_s^2\) is thought to depend on the duration of particle emission. Hence, it looks like the reballing of its particles almost instantaneously and does not show any sign of phase transition or crossover. Hydrodynamics, with a \texttt{reasonable} freeze-out condition (such as a freeze-out temperature of 100 MeV or so) cannot describe this even qualitatively. This behaviour, when compared to lower energy data, exhibits remarkably good scaling with multiplicity (Fig. 1 left panel). The scaling’s existence, however, is by itself surprising since the QCD equation of state, with its critical density for a phase transition, should break it.
Figure 1: Dependence of HBT radii on multiplicity, across different energies [1] (left panel), an expectation from hydrodynamics [3] (center panel), and a with hydrodynamic model [6] (right panel).

There could be three possible approaches to the HBT puzzle. It could be that the system is simply too complicated, and that once we include all possible ingredients (full 3D calculation, viscosity, hadronic kinetic afterburner, in-medium hadron modulations, etc.), everything will fall into place. It could also be that we are drastically misunderstanding the data, and the HBT puzzle is a symptom of inapplicability of hydrodynamics to heavy ion collisions. Finally, it could be that the hydrodynamic approach is basically correct, but just one element of relevant physics is missing [5].

The second possibility is unlikely because, in some ways, hydrodynamic prescription does not fit with HBT data. Scaling of HBT radii with the multiplicity rapidity density (dN/dy) allows for a large range of energies [1] is typical for an isentropically expanding uid that suddenly breaks apart. In addition, the good description, with parameters compatible with what is needed to describe data, of the azimuthal dependence of HBT radii [2], also suggests that the hydrodynamic framework is a good ansatz for describing the matter produced in heavy ion collisions up to freeze-out.

The rest possibility also appears problematic: successful models and/or parametrisations of the freeze-out which describe HBT radii are found in the literature [6], and they could provide a way to gain insights into what is missing. However, we feel that successful description involves a dynamical description of initial conditions plus a freeze-out criterion, rather than a fit to data with assumptions put in by hand. Such a description is far from lacking. Furthermore, the most plausible reason is strongly due to hydrodynamics, namely in the computation of fully 3D models and the addition of a kinetic theory afterburner [7] to do not do anything to solve the HBT discrepancy, suggesting that the problem is not a mere but rather a large missing physical effect.

2 Bulk viscosity close to T_c

The bulk viscosity of high temperature strongly interacting matter has recently been calculated using perturbative QCD [8], and found to be negligible, both in comparison to shear viscosity and w.r.t. its effect on any reasonable collective evolution of the system. This is not surprising: The QCD Lagrangian, as long as no "heavy" quarks are present, is nearly conformally invariant [8]. Since, within a uid, the violation of conformal symmetry is linearly proportional to a bulk viscosity term, the near conformal invariance of the QCD Lagrangian should guarantee that bulk viscosity is nearly zero, in the perturbative regime. In the hadron gas phase, the numerous scales associated with hadrons render conformal invariance a bad symmetry, and hence it is natural that bulk viscosity not be negligible.
This is, again, rooted in a fundamental feature of QCD: the non-perturbative conformal anomaly, that manifests itself in the scale (usually called $q_{\text{CD}}$) at which the QCD coupling constant stops being small enough for the perturbative expansion to make sense. This scale coincides with the scale at which confining forces hold hadrons together. This violation of conformal invariance is not seen perturbatively, but should dominate over perturbatively calculated bulk viscosity as temperature drops close enough to the QCD phase transition. What happens to bulk viscosity in this regime, where hadrons are not yet formed, presumably the matter is still condensed, but conformal symmetry is badly broken? There is a quenched lattice calculation, and compelling physical arguments [9] that bulk viscosity rises sharply, or even diverges, close to the phase transition temperature.

Lattice simulations find that $T = 0$ for a conformally invariant system), increases rapidly close to $T_c$. Remembering that the shear and bulk viscosities roughly scale as $[10]$:

$$\eta_{\text{el}} = \frac{1}{3} v_s^2 \eta_{\text{in}} T^4,$$

where $(\text{inelastic})$ refers to the equilibration timescale of (inelastic) collisions. Finite temperature sum rules in conjunction with lattice data [9] give a sharp rise in bulk viscosity.

The rise is, in fact, likely to be considerably sharper than [9] suggests. The dependence of inelastic on temperature can be guessed from the fact that, at $T_c$, the quark condensate $\langle \bar{q}q \rangle$ acquires a finite value, and the gluon condensate $\langle G^2 \rangle$ sharply increases at the phase transition. Kinetically, therefore, timescales of processes that create extra $q\bar{q}$ and $GG$ pairs should diverge close to the phase transition temperature, by analogy with the divergence of the spin correlation length in the Ising model close to the phase transition. The sharp rise of bulk viscosity can also be understood within string kinematics: confinement is microscopically, can be thought of as a "string tension" appearing in the potential. Even in a regime where the momentum exchange of the average collision is more than enough to break the string, and the relevant degrees of freedom are still quarks, not mesons, string tension introduces a huge change in kinetics: what were elastic collisions without string tension, become inelastic once string tension appears. Even if the energy needed to break the string is low, over many collisions, the heat energy would be converted into creating more slightly colder, less pressing particles. That's exactly the kind of processes that contribute most to bulk viscosity [11].

These arguments give evidence to the conjecture that, close (from above) to $T_c$, bulk viscosity goes rapidly from a negligible value to a value capable of dominating the collective evolution of the system. For our analysis, we assume the bulk viscosity to be of the form $\zeta_{\text{QCD}} = \frac{\zeta_{\text{QCD}}}{2} \exp(\frac{\zeta_{\text{QCD}}}{2})$ where $t = T - T_c$ and $\zeta_{\text{QCD}} = 10^3$ [8]. At $T > T_c$, this ansatz provides a reasonable fit to the results of [8], considering the peak height and width of the distribution are still unknown.

## 3 Bulk viscosity-triggered fragmenting

To study the effect of our conjectured behaviour of bulk viscosity on solutions to hydrodynamics, we perform a stability analysis [12] of a boost-invariant solution to the Navier-Stokes equations (both the 1D and 3D cases). The Navier-Stokes equations with boost-invariant symmetry [2,12,13] can be rewritten in terms of the Reynolds number $R$, the entropy $s$, the comoving time $t$, the total number $(N)$ of dimensions, and the dimensionality of the homogeneous expansion $(M)$:

$$\frac{d(M s)}{d} = \frac{M s}{R}$$

$M = 1$ $N = 3$ corresponds to Bjorken hydrodynamics [13], $M = N = 3$ to a homogeneous 3D expansion. The Reynolds number is a function of temperature $T$, bulk and shear viscosity and entropy density $s$: $R(T)$, with given expressions for $s(T)$, $(T)$, $(T)$, this set of
Equations become closed and solvable. For the equation of state, we use a parametrization of lattice data (we have checked that our results do not vary qualitatively if the ideal EoS is used).

We follow the stability analysis performed in [12]. The amplitude of a generic perturbation to the 1D Boost-invariant system is a vector $\mathbf{x}$ in the two-dimensional space of entropy perturbations and $\omega$ (rapidity $y$) perturbations, and its frequency in rapidity can be decomposed into Fourier components

$$\mathbf{x}(y) = \sum_k x(k)e^{iky}; \quad x_1 = \frac{\mathcal{S}^{-1}}{\mathcal{S}}; \quad x_2 = y \mathcal{Y}_{\text{spacetime}}$$

The equation of motion for $\mathbf{x}$ will then be given by

$$\frac{\partial}{\partial t} x_1 = A_{11} x_1 + A_{12} x_2; \quad \frac{\partial}{\partial t} x_2 = A_{21} x_1 + A_{22} x_2$$

where $A_{ij}$ is a real matrix function of $s; \mathcal{R}$ and $k$. We refer the reader to Eq. 4.23-4.26 of [12] for the full form of $A_{ij}$. The system's stability can be understood via the behavior of the modulus of $\mathbf{x}, X = x^T x$

$$\frac{\partial}{\partial t} X = x^T \dot{\mathbf{M}}_{ij} x; \quad \mathbf{M}_{ij} = \mathbf{A}_{ij} + \mathbf{A}_{ij}^T$$

Since $\mathbf{M}_{ij}$ is real and symmetric, it will always have two real eigenvalues, $\lambda_{\max}$ and $\lambda_{\min}$ (corresponding eigenvectors $x_{\max}$ and $x_{\min}$), as well as orthogonal matrices $\mathbf{B}_{ij}$ diagonalizing it. Defining $y_1 = \beta_{ij}^{-1} x_j$, we see that $\lambda_{\min} y_m^2 < \frac{\partial X}{\partial t} < \lambda_{\max} y_m^2$. Thus, if $\lambda_{\max} > 0$, the system is unstable, since perturbation in any direction will produce a positive growth rate. An instability will grow as a power-law with $\lambda = \lambda_{\max}$, where $\pi$ is the starting time of the perturbation $x(\pi) = \lambda_{\max} x(\pi)$. If $\lambda_{\max} < 0$, on the other hand, the system will be stable against perturbations, an instability will be suppressed as $X(\infty) = 0$.

If $\lambda_{\min} < 0$ and $\lambda_{\max} > 0$, some modes will be stable and some will be unstable. In the latter case, the time dependence of $\mathbf{A}_{ij}$ will in general continuously rotate $x_{\min}$ and $x_{\max}$ in time, stopping the growth of the instability. Since $\mathbf{A}_{ij}$ is time-dependent, $x_{\max}(\pi) = \mathbf{A}_{ij} x_{\max}(\pi)$ might be in the direction of $x_{\max}(\pi) > \lambda_{max}$ a short time later. Solving Eq. 4.2 will take these effects into account.

In what follows, we use $\mathcal{S}_0 = 0.01$ as $\mathcal{T}_c$ and evolve the system from an initial temperature $T = 0.3$ GeV and comoving time 0.6 fm. Note that the qualitative features of our study are independent of the details of the evolution before $T_c$, such as the initial temperature and time scale.

Figure 2 (upper panel) shows the temperature, entropy density and total entropy in the central rapidity unit as a function of time. As soon as $\mathcal{S}_0$ becomes non-negligible (i.e., viscous forces dominate around $T_c$), the kinematic evolution of the system "freezes". The system then stays at nearly constant temperature, through it keeps producing more and more entropy at the expense of advective energy. At first sight, large values of $\mathcal{S}_0$ are excluded by HBT data and multiplicity measurements. However, we will show that this long phase is unstable against all perturbations. Thus, its further evolution will not be given by the background solution, but by a rapid formation of local inhomogeneities.

Figure 2 (second panel) shows the $\lambda_{\min}$ and $\lambda_{\max}$ eigenvalues corresponding to representative $k = 2; 8$ (other values of $k$ were checked not to vary significantly with those presented here). As can be seen, the peak in bulk viscosity forces the growing/damping rates to increase rapidly. Thus, any initial perturbation in the unstable direction will rapidly grow to a value comparable with the background, unless the system's evolution will stop the growth by rotating the direction of the unstable modes.

Figure 2 (third panel) examines whether this occurs for larger values of $\mathcal{S}_0$. If the peak of viscosity is negligible, the unstable eigenvector keeps rotating throughout the evolution of the system. Thus, even an unstable mode's growth will very quickly stop growing since the dynamics will turn it into
Figure 2: Instability analysis of the boost-invariant solution. Top panel shows the evolution of the background temperature (solid), entropy density (dot-dashed) and total entropy (dashed). The two middle panels show the eigenvalues and eigenvectors of the $k = 2$ (solid) and $k = 8$ unstable mode. The bottom panel shows the full evolution of the linearized instability equation of motion. The three columns represent various appearances of the viscosity peak. See text for details.

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4 Fragments and the HBT puzzle

HBT radii are directly related to the system’s spacetime correlation tensor \([4,11]\):

\[
R_s^2(K) = \frac{D}{(x_o)^2} E_k^2 h(x_o) t + \frac{k_T}{k_0} (t)^2
\]

while \(R_o^2(K)\) is simply given by \(h(x_o)^2 t^2\). Here the \(K\) is the sum of the two four-momenta of the pion pair. As remarked in [4], the \(R_o\) \(R_s\) result is not easy to reconcile with naive hydrodynamics plus a straight-forward (critical temperature) emission for several reasons.

Firstly, the higher the energy, the longer the emission time, the larger is the expected discrepancy between \(R_o\) and \(R_s\). If the system starts close to the mixed phase, the time scale of freeze-out should be longer still due to the softest point in the equation of state. Hence, a generic prediction from Eq. [3] is that \(R_o=R_s > 1\), largely increases with energy, and exhibits a peak when the energy density is such that the system starts within, or slightly above the mixed phase. This is in direct contrast with experimental data, where \(R_o=R_s \approx 1\) is a feature at all reaction energies.

Secondly, generally \(h \times t < 0\), since particles on the outer layer freeze-out first. This increases \(R_o=R_s\) further (cf. eq [6]). Time dilation due to transverse flow does reverse this dependence [4].

Fragmentation of the bulk could help solving these problems. Firstly, fragment size, density and decay time scale is approximately independent of either reaction energy or centrality. Hence, the near energy independence of the (compared to short) emission time scale, and hence of \(R_o=R_s\), should be recovered. Secondly, if the decay products do not interact (or do not interact much) after fragment decay, it can also be seen that \(h \times t\) can indeed be positive: outward fragments are moving faster, resulting in time dilation. This effect can be obtained by fragment decay by increasing the temperature at which fragments form, or by increasing fragment size. Recovering the linear scaling of the radii with \((dN/dy)^{-3}(N_{\text{fragments}}) [1]\), while maintaining the correct \(R_o=R_s\) is also possible if the fragment decay when their distance w.r.t. each other is still comparable to their intrinsic size.

The bulk-viscosity-driven freeze-out adds another parameter to ab initio HBT calculations: in addition to critical temperature/energy density, we now have the fragment size. To see whether this

\[\text{Here } l (\text{long})\text{ is the } z\text{ direction (parallel to beam)}, o (\text{out})\text{ is the direction of the pair momentum, and } s (\text{side})\text{ is the cross product of the previous two. Averaging is done over the emission function } S(x;p), \text{ the probability of producing a particle of momentum } p\text{ at } x. \text{ See [1] for details.}\]
helps solving the HBT problem, output from hydrodynamics with a high ($T > T_c$) freeze-out temperature should be fragmented into pieces with a certain distribution in size, which then produce hadrons according to the prescription in the Appendix of [5].

In conclusion, we have introduced the HBT puzzle, and explained how bulk viscosity could help solve it by triggering the fragmentation of the system close to the critical temperature. We hope that, in the near future, this scenario will be developed to the point of experimental falsification.

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References

[1] M .A. Lisa, S. Pratt, R. Soltz and U. Wiedemann, Ann. Rev. Nucl. Part. Sci. 55, 357 (2005)

[2] P. Danielczuk and M. Gyulassy, Phys. Rev. D 31 (1985) 53.
   H. Heiselberg and A. M. Levy, Phys. Rev. C 59, 2716 (1999)

[3] D. H. Rischke and M. Gyulassy, Nucl. Phys. A 608, 479 (1996)

[4] P. F. Kolb and U. W. Heinz, arXiv:nucl-th/0305084.

[5] G. Torrieri, B. Tomaszik and I. Mishustin, Phys. Rev. C 77, 034903 (2008)
   G. Torrieri and I. Mishustin, Phys. Rev. C 78, 021901 (2008).

[6] A. Baran, W. Broniowski and W. Florkowski, Acta Phys. Polon. B 35, 779 (2004)
   S. V. Akkelin and Yu. M. Sinyukov, Nucl. Phys. A 774, 647 (2006)
   M. Csanad, T. Csorgo, B. Lorstad and A. Ster, Nucl. Phys. A 774, 535 (2006),
   J. G. Cramer, G. A. Miller, J. M. S. Wu and J. H. S. Yoon, Phys. Rev. Lett. 94, 102302 (2005),
   S. Pratt and J. Vredevoogd, arXiv:0809.0516 [nucl-th].

[7] T. Hiranai and K. Tsuchiya, Nucl. Phys. A 715, 821 (2003)
   D. Teaney, J. Lauret and E. V. Shuryak, arXiv:nucl-th/0110037.

[8] P. Arnold, C. Dogan and G. D. Moore, Phys. Rev. D 74, 085021 (2006)

[9] K. Paeck and S. Pratt, Phys. Rev. C 74, 014901 (2006)
   F. Karsch, D. Karzsev, K. Tuchin, arXiv:0711.0914 [hep-ph].
   H. B. Meyer, arXiv:0710.3717 [hep-lat].

[10] S. Weinberg, Astrophys. J. 168 (1971) 175.

[11] S. Jeon and L. G. Yae, Phys. Rev. D 53, 5799 (1996)

[12] H. Kuno, M. M aruyama, F. Takagi and K. Saito, Phys. Rev. D 41, 2903 (1990).

[13] J. D. Björken, Phys. Rev. D 27, 140 (1983).

[14] I. M. Mishustin, Phys. Rev. Lett. 82, 4779 (1999);
   J. Randrup, Phys. Rev. Lett. 92, 122301 (2004)
   L. P. Csernai and J. I. Kapusta, Phys. Rev. D 46, 1379 (1992).
[15] W. Israel and J. M. Stewart, Annals Phys. 118 (1979) 341.

[16] M. P. Heller and R. A. Janik, Phys. Rev. D 76, 025027 (2007)

[17] G. S. Denicol, T. Kodama, T. Koide and Ph. Mota, J. Phys. G 35, 115102 (2008) [arXiv:0807.3120 [hep-ph]].