ABSTRACT. One of principal tasks of celestial mechanics is study of motion of natural satellites of planets. In this work, the authors have examined resonances in system of satellites and rings of Saturn. They have showed presence of resonances between all the regular satellites and satellites – rings.

Key words: Saturn, satellites, rings, resonance.

1. Introduction

Saturn is the sixth planet of Solar system and second biggest gas giant. 62 satellites and system of bright rings of the planet are known today.

Galileo was the first one who saw Saturn rings and Christian Huygens in 1655 has discovered the biggest one in Saturn system and second in Solar system satellite Titan (Lorenz et al., 2002). Actually, Titan is unique because it is the only planetary satellite with own dense atmosphere and was proved that only it and Earth have stable presence of liquid on its surface in all Solar system (Lorenz, 1994). After Huygens, Jean-Dominique, comte de Cassini has discovered 4 satellites: “two-faced” Iapetus (1671) with one light and second dark sides (Denk et al., 2010), Rhea (1672) that has own rings (Tiscareno et al, 2010), Dione and Tethys (1984), both of these have two co-orbital satellites moving in Lagrange points (Fulchignoni, 1986; Mourão et al., 2010). William Hershel has contributed to the study of the system by discovering of Mimas and Enceladus in 1789. This is the two of the smallest inner satellites (orbits of inner satellites limited by F ring). Satellites were capture in resonance by relatively massive Tethys and Dione respectively. At the same time they are different, Mimas has a large crater and tectonically unactive, but Enceladus has more plane surface and source of inner heat, geysers (Cuk et al., 2015). Eighth satellite Hyperion was found much later in 1848 (Memories of the Royal Astronomical Society, 1850). Later in 1899 was found the first irregular satellite Phoebe (Pickering, 1899). It is worth to say that regular satellites move by orbits with small eccentricity and their orbit planes lie in equator plane of the planet. Orbits of irregular satellites have big eccentricity between 0.11 and 0.58 and inclination of orbit plane between 34º and 179º, that is why most of them are retrograde (regress orbital motion) (Jewitt, 2007).

Saturn ring system causes serious difficulties in searching of satellites, because of that reason next satellite Janus was discovered by Audouin Dollfus in 1966 (Dollfus et al, 1981). Three days later, 18th of December 1966, had found that another satellite moves by very similar orbit. That way Epimetheus was discovered by Richard Walker (Gingerich, 1967). Satellite discovering has continued in 1980 with help of Voyager-1, Voyager-2 and Cassini-Huygens (Morrison, 1982; Russell, 2003).

Among all the satellites there are 24 regular and 38 irregular, which probably are captured asteroids. Lastly discovered 62nd satellite named S/2009 S1 is poorly researched now.

In the text were mentioned famous Saturn rings. They are composed of icy and dust particles and moving in accordance to the Kepler’s laws. There are highlighted main rings in all rings which were named with English letters as highlighted. For example: A and B rings, which are separated by Cassini division, C ring in the brightest B ring. At the same time, results of the named missions show that main rings are composed of great number of thin rings. (Pollack, 1975).

2. Orbital resonances between satellites

There are large numbers of a resonance phenomenon in Saturn satellite system. Let’s add that mean motion is the mean orbital angular speed (n):

\[ n = \frac{360}{T} \]  

where \( T \) – orbital period of celestial body in days (Murray & Dermot, 2010). System of two satellites, which move around main body, is in resonance, if their mean motions are related as two small natural numbers (Peale, 1976).

This topic is often examines in works about natural satellites (Peale, 1976; Dermott et al, 1988; Callegari et al, 2008; Fuller et al, 2016; Troianskyi, 2016; Luan et al, 2017). So orbital resonances among regular Saturn satellites and other bodies of Solar system are well known. Examining combinations of satellites in pairs shows that
all satellites have pare in resonance. As a result, the authors first wrote a formula:

\[-2 \left( P_{\text{Pan}} + P_{\text{Daphnis}} + P_{\text{Atlas}} + \right. \]
\[ + P_{\text{Prometheus}} + P_{\text{Pandora}} + P_{\text{Epimetheus}} + P_{\text{Janus}} + \]
\[ \left. + P_{\text{Deucalion}} + P_{\text{Athe}} + P_{\text{Telesto}} + P_{\text{Hyperion}} \right) \]
\[-1 \left( P_{\text{Mimas}} + P_{\text{Methone}} + P_{\text{Pallene}} + P_{\text{Tethys}} + \right. \]
\[ + P_{\text{Calypso}} + P_{\text{Tethyan}} + P_{\text{Rhea}} + P_{\text{Titan}} \]
\[ + \right) + \]
\[ + 1 \left( P_{\text{Enceladus}} + P_{\text{Dione}} + P_{\text{Rhea}} + \right. \]
\[ + 2 \times P_{\text{Polus}} = 0.00 \text{(days)} \]

There were considered orbital periods (Sheppard, 2017) for 23 regular satellites in the formula (2). There is an ability to search orbital resonances of irregular satellites only in their group, because relations about 1:100 with regular satellites, gaps and rings contradict orbital resonance definition (Murray & Dermot, 2010). But among pairs of irregular satellites no resonances. It confirms that they wasn’t formed with the other bodies of the system, but captured (Gladman et al, 2001).

3. Orbital resonances between satellites and rings

This topic was studied in already published works (Lissauer et al, 1982; Gordon et al, 2006; Colwell et al, 2009; Crida et al, 2016).

Orbital resonances with rings and gaps specify shepherd satellites in rings. For example: Pan that forms Encke gap or Prometheus and Pandora both form ring F. For wide rings like main A, B, C, D orbital resonances have poor accuracy. It is difficult to highlight a thin ring that is why the authors used distances between two edges of neighbor gaps. Orbital periods of ring particles were calculated in accordance to third Kepler’s law, where ring mass wasn’t taken in account. Actually, shepherd satellites have 1:1 relations with ring material or middle of gap.

4. Results

The authors studied all possible combinations: 252 pairs among regular, 703 pairs among irregular satellites, 529 pairs among rings and regular satellites, 414 pairs among gaps and regular satellites.

As a result first earned formula (2) that describes orbital resonances between all the regular satellites.

Authors shown orbital resonances between regular satellites and rings. Orbital resonances show conformity between gaps and satellites that form it.

Orbital resonances between irregular satellites and satellites – rings, authors wasn’t found.

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