Non Fermi liquid renormalization of the conductivity of fermions
coupled to gauge fields

D.V.Khveshchenko\textsuperscript{1} and Michael Reizer\textsuperscript{2}

\textsuperscript{1} NORDITA, Blegdamsvej 17, Copenhagen DK-2100, Denmark
\textsuperscript{2} Department of Physics, The Ohio State University, Columbus,
\textit{OH 43210-1106}

Abstract

The method of the quantum kinetic equation is applied to the problem
of renormalization of the conductivity of normal metals by gauge electron-
electron interactions. It is shown that in the three-dimensional case the
relativistic electromagnetic interaction (vector interaction of electrons with
transverse photons) leads to an unusual temperature dependence, indicating
a deviation from the Fermi liquid theory at low temperatures. In two dimen-
sions such corrections are found to result from both the scalar (density-density
or Coulomb) and the vector (current-current) gauge interactions.
I. INTRODUCTION

The renormalization of different electronic properties due to the electron-electron interactions constitutes one of the major premises of the Fermi liquid theory (FLT). Although there exists lots of results about renormalizations of such equilibrium thermal quantities as specific heat or Pauli magnetic susceptibility [1,2], there are very few statements concerning renormalizations of electronic kinetic coefficients [3,4]. This issue was commonly believed to had been settled over 30 years ago in the work by Prange and Kadanoff [1] who considered the conventional electron-phonon problem and concluded that neither the electrical conductivity nor the thermoelectric power are affected by the electron-phonon interactions.

However, it was recently pointed out [5] that the analysis [1] was not complete. It was stated in [5] that in general there exists another type of renormalization corrections, which were not taken into account by the authors of [1] who only discussed the phonon renormalization of the electron-phonon scattering rate. The new term derived in [5] can be viewed as resulting from a quantum interference between electron-phonon and electron-impurity scattering processes. In the framework of the quantum kinetic equation, which we are going to use throughout this paper, it originates from corrections to the nonequilibrium electron density of states and from the nonlocal part of the electron-phonon collision integral.

Unfortunately, the final conclusions made in [5] in the context of the three-dimensional (3D) electron-phonon problem are incorrect. A more thorough analysis shows that the new renormalization effect proposed in [5] is negligible in all cases of 3D scalar (density-density) interactions, including the 3D electron-phonon, electron-paramagnon, and electron-electron Coulomb interactions. Nevertheless, as we will show below, the cancellation which occurs in the case of the 3D scalar coupling is not a generic property, but it is only due to some peculiarity of the relevant phase volume-type expression.

To this end, in the present paper we start out with an example of 3D vector (current-current) coupling provided by relativistic electron-electron interactions (the vector interaction of electrons with transverse photons), which is known to cause a quite unusual and
unexpected in FLT behavior of electron quantities at low enough temperatures. Namely, it was previously shown that the electron energy relaxation time $\tau_{\epsilon}^{-1}$ is proportional to $T \ln T$ \cite{6}, which obviously violates the FLT criterion of the existence of well-defined fermionic quasiparticles ($\tau_{\epsilon}^{-1} \ll T$). Moreover, the real part of the electron self-energy shows the "marginal" behavior $\text{Re}\Sigma(\epsilon) \propto \epsilon \ln \epsilon$ \cite{7,8}.

The breakdown of the FLT manifests itself in singular corrections to various thermodynamic quantities, such as the electronic specific heat: $\Delta C_p \propto -T \ln T$ \cite{9,7}.

Generically, the new renormalization conductivity correction derived in \cite{5} can be related to the derivative $\lambda = -\frac{\partial \text{Re}\Sigma(\epsilon)}{\partial \epsilon}$ (as opposed to the so-called kinetic terms, which can be expressed in terms of $\text{Im}\Sigma(\epsilon)$, provided a scattering is quasi-elastic).

In FLT $\lambda(\epsilon \to 0)$ is proportional to a coupling constant and independent of $\epsilon$, which yields a simple multiplicative reduction of the Drude conductivity via the effective mass enhancement: $m = m_0(1 + \lambda)$.

If, on the contrary, the function $\lambda(\epsilon)$ becomes singular at small $\epsilon$, one might expect that the corresponding renormalization correction to the conductivity will exhibit such non-Fermi liquid features as a non-analytic dependence on the coupling strength and/or temperature.

In the rest of the paper we demonstrate that in the presence of both scalar and vector gauge interactions this is indeed the case.

II. QUANTUM KINETIC EQUATION

We are going to study the renormalization of the classical impurity Drude conductivity in the framework of the method of quantum kinetic equation.

In the 3D case the retarded Green’s functions of scalar photons $V_{00}^R$ (the scalar Coulomb potential) and transverse vector photons $V_{11}^R$ in the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$ ($\mathbf{A}$ is the vector potential) are given by the standard formulae \cite{6,7}

$$V_{00}^R(Q) = \frac{4\pi e^2}{q^2 - 4\pi e^2 F_{00}^R(Q)}, \quad Q = (\mathbf{q}, \omega),$$

(1)
\[(V_{11}^{R}(Q))_{m,n} = V_{11}^{R}(Q)T_{m,n}, \quad T_{m,n} = \delta_{m,n} - \frac{q_m q_n}{q^2}, \quad (2)\]

and

\[V_{11}^{R}(Q) = \frac{4\pi e^2 c^2}{\omega^2 - c^2 q^2 - 4\pi e^2 c^2 P_{11}^{R}(Q)}; \quad (3)\]

\[m, n\] are the Cartesian indices and \(c\) is the speed of light. The scalar and the vector components of the vertex \(a_\mu\) describing the electron-photon interaction are:

\[a_0 = -1, \quad a = \frac{1}{mc} (p + q/2), \quad (4)\]

where \(p\) is the electron momentum and \(m\) is the electron mass. The polarization operators for \(q l >> 1\) and \(qv_F >> \omega\) are:

\[P_{00}^{R}(Q) = -\nu_0 (1 + i \frac{\pi \omega}{2qv_F}), \quad P_{11}^{R}(Q) = -i \frac{\pi \omega v_F v_F}{4qc^2}, \quad \nu_F = \frac{mp_F}{\pi^2}, \quad (5)\]

where \(v_F\) is the Fermi velocity, and \(\nu_F\) is the two-spin electron density of states at the Fermi surface. The bare retarded electron Green’s function in an impure metal is

\[G_{0}^{R}(P) = (\epsilon - \xi_p + i/2\tau)^{-1}, \quad P = (p, \epsilon) \quad \xi_p = (p^2 - p_F^2)/2m. \quad (6)\]

In the Keldysh formalism [10], in addition to the retarded and advanced Green’s functions, the more complicated electron \(G^C\) and photon \(V^C_{ii}\) Keldysh Green’s functions are introduced. Assuming that photons are in the thermodynamic equilibrium one can relate them as

\[V^C_{ii}(Q) = (2N(\omega) + 1)[V_{ii}^{R}(Q) - V_{ii}^{A}(Q)]. \quad (7)\]

where \(N(\omega)\) is the Bose distribution function.

The electron system is considered to be driven out of equilibrium by an external electric field. Deriving the quantum kinetic equation we make the conventional transformation from the coordinate to the momentum representation, the nonuniformity of the system being taken into account by means of the corrections to the Poisson brackets.

In the lowest order in nonuniformity \(G^C\) is given by the expression:

\[G^C(P) = S(P)[G^A(P) - G^R(P)] + \delta G^C(P), \quad \delta G^C(P) = \frac{i}{2}\{S_0(\epsilon), G^A(P) + G^R(P)\}. \quad (8)\]
where the Poisson brackets in the presence of an electric field $\vec{E}$ are

$$\{A, B\}_E = e\vec{E} \left( \frac{\partial A}{\partial \epsilon} \frac{\partial B}{\partial \vec{p}} - \frac{\partial B}{\partial \epsilon} \frac{\partial A}{\partial \vec{p}} \right).$$  

(9)

The function $S(P)$ plays a role of the electron distribution function given in the equilibrium by the formula $S = S_0 = -\tanh(\epsilon/2T)$. In the presence of the electric field $S$ is determined by the linearized quantum transport equation:

$$e(\vec{v} \cdot \vec{E}) \frac{\partial S_0}{\partial \epsilon} = I_{e-imp} + I_{e-e},$$  

(10)

where $I_{e-imp}$ and $I_{e-e}$ are the collision integrals which correspond to the electron-impurity and the electron-electron scattering. The collision integrals are expressed in terms of the corresponding self-energies by virtue of the equations:

$$I(S) = I^0(S) + \delta I(S), \quad I^0 = -i[\Sigma^C - S(\Sigma^A - \Sigma^R)],$$

$$\delta I = -i[\delta \Sigma^C - S_0(\delta \Sigma^A - \delta \Sigma^R)] + \frac{1}{2}\{\Sigma^A + \Sigma^R, S_0\},$$  

(11)

where $\delta \Sigma$ is the correction in the Poisson brackets form. In our case $\delta \Sigma$ is obtained by taking into account the correction $\delta G^C$ in the expressions for $\Sigma$. The collision integral $I_{e-imp}$ can be chosen in its simplest form:

$$I_{e-imp} = \frac{2}{\pi \nu \tau} \int \frac{d\vec{k}}{(2\pi)^3} [S(\vec{k}, \epsilon) - S(\vec{p}, \epsilon)] \text{Im} G^A_0(\vec{k}, \epsilon) = \frac{S_0(\epsilon) - S(\epsilon)}{\tau}. $$  

(12)

Constructing the electron-electron collision integral we need the retarded electron self-energy

$$\Sigma^R_{e-e}(P) = -\int \frac{dQ}{(2\pi)^4} \left[ a_\mu V^{A\mu}(Q) a_\nu \text{Im}[G^A(P + Q)] S(P + Q) \right. $$

$$\left. + a_\mu \text{Im}[V^{R\mu}(Q)] a_\nu G^A(P + Q)(2N(\omega) + 1) \right].$$  

(13)

Assuming that the electron-impurity scattering is a dominant momentum relaxation process, we solve the transport equation (10) by iterations: $S = S_0 + \phi_0 + \phi_1$, where $\phi_0$ is the first correction to the equilibrium distribution function $S_0$ which depends on the electron-impurity scattering but not on the electron-electron interactions

$$\phi_0(\vec{p}, \epsilon) = -\tau(\vec{v} \cdot \vec{E}) \frac{\partial S_0(\epsilon)}{\partial \epsilon}. $$  

(14)
The next order correction $\phi_1$ includes the effects of the electron-electron interactions

$$
\phi_1 = \tau [\delta I_{e-e}(S_0)] = \frac{\tau}{2} \{ \Sigma_{e-e}^A + \Sigma_{e-e}^R, S_0 \}
$$

$$
= \frac{\partial S_0(\epsilon)}{\partial \epsilon} \int \frac{dQ}{(2\pi)^4} \tau^2 e \left( \mathbf{v} + \frac{\mathbf{q}}{m} \right) \cdot \mathbf{E}
$$

$$
\text{Re}[a_\mu V_{\mu\nu}(Q)a_\nu] S_0(\epsilon + \omega) \text{Im}\left( G_0^A(P + Q) \right)^2
$$

(15)

In (15) we took into account only the nonlocal part of the collision integral from (11) which is expressed in terms of the Poisson brackets. The reason is that only this part depends on the real part of the exchange potential $\text{Re}V_{ii}$ describing the interaction mediated by virtual photons. In what follows we show that the processes involving virtual photons have a dominant effect on the conductivity renormalization.

The electric current is given by the equation

$$
\mathbf{J}_e = e \int \frac{dP}{(2\pi)^4} \mathbf{v} \text{Im} G^C(P),
$$

(16)

We now treat corrections to the Drude current due to the electron-electron interactions as the corrections to the distribution function and the electron density of states

$$
\Delta J_e = \delta \sigma \mathbf{E} = 2e \int \frac{dP}{(2\pi)^4} \mathbf{v} \left[ \phi_0 \text{Im}\left( G_0^A(\Sigma_{e-e}^A(S_0)) \right) + \phi_1 \text{Im} G_0^A 
$$

$$
+ S_0 \text{Im}\left( (G_0^A)^2 \Sigma_{e-e}^A(\phi_0) + \phi_1 \text{Im} G_0^A \right) \right].
$$

(17)

The contributions of the first two terms in (17) cancel out. The contributions of the third and the fourth terms give

$$
\Delta \sigma = 2e^2 \tau \int \frac{dP}{(2\pi)^4} \int \frac{dQ}{(2\pi)^4} \mathbf{v} \cdot \mathbf{n} \left( \mathbf{v} + \frac{\mathbf{q}}{m} \right) \cdot \mathbf{n} \text{Re}[a_\mu V_{\mu\nu}(Q)a_\nu]
$$

$$
\left[ S_0(\epsilon) \frac{\partial S_0(\epsilon + \omega)}{\partial \epsilon} \text{Im}(G_0^A(P))^2 \text{Im} G_0^A(P + Q) + 
$$

$$
+S_0(\epsilon + \omega) \frac{\partial S_0(\epsilon)}{\partial \epsilon} \text{Im} G_0^A(P) \text{Im}\left( G_0^A(P + Q) \right)^2 \right].
$$

(18)

Changing variables $\mathbf{p} \rightarrow \mathbf{p} + \mathbf{q}$ and $\epsilon \rightarrow \epsilon + \omega$ and then $\mathbf{q} \rightarrow -\mathbf{q}$ and $\omega \rightarrow -\omega$, one can see that the two terms in square brackets in (18) are identical.
III. GAUGE INTERACTIONS IN 3D

Typically, in the case of gauge interactions relatively small momenta transfers \( q << p_F \) are important. On the other hand, it can be readily seen that the main contribution to the renormalization corrections comes from photon momenta \( q >> \frac{1}{l} \) and frequencies \( \omega < v_F q \). In this regime we first integrate in (18) the product of the electron Green's functions over \( \xi_p \) and obtain

\[
\int d\xi_p \text{Im} G^A_0(P) \text{Im}(G^A_0(P + Q))^2 = -\frac{\pi^2}{(qv)^2} \delta' \left( x - \frac{\omega}{qv F} + \frac{q}{2p_F} \right), \quad x = \frac{p \cdot q}{pq}.
\]  

Next we perform the angular integration

\[
\int \frac{d\Omega_p}{4\pi} \int \frac{d\Omega_q}{4\pi} \text{Re}[a_\mu V_{\mu\nu}(Q) a_\nu] \delta' \left( x - \frac{\omega}{qv F} + \frac{q}{2p_F} \right) v \cdot n \left( v + \frac{q}{m} \right) \cdot n =
\]

\[
= \frac{v_F^2}{3} \left( -\frac{q}{p_F} + \frac{\omega}{qv F} \right) \left( \frac{v_F}{c} \right)^2 \text{Re}V_{11}(Q) + \left( -\frac{q}{2p_F} \right) \text{Re}V_{00}(Q)
\]

At this point one can notice the difference between the vector and the scalar couplings. In the latter case the angular integral does not contain a \( \omega \)-term, and the absence of such a term dramatically reduces the overall renormalization effect from the Coulomb interaction \( V_{00}(Q) \) due to the parity reason (see later Eq.(22)).

Now we perform the integration over the photon momentum \( q \) assuming that the \( \omega \)-dependent pole of the function \( V_{11}(Q) \) is located at \( q > \max(1/l, \frac{\omega}{v_F}) \) and end up with

\[
\int \frac{dq q^2}{2\pi^2} \frac{\pi^2 \omega}{(qv F)^2 qv F} \left( \frac{v_F}{c} \right)^2 \text{Re}V_{11}(Q) = -\left( \frac{v_F}{c} \right)^2 \frac{2\pi e^2 \omega}{v_F^4} \frac{\pi}{3^{3/2}(b|\omega|)^{2/3}}, \quad b = \frac{\pi^2 e^2 v_F v_F}{c^2}.
\]

The above assumption resulting in (21) requires \( \omega \) to lie in the interval between \( T_1 = \frac{1}{\pi(\kappa^2 c^2)} \) and \( T_3 = \frac{v_F^2}{c} \) (where \( \kappa^2 = 4\pi e^2 v_F \)), which is fairly broad in a clean metal. For instance, for \( E_F \sim 10eV, \) \( v_F/c \sim 10^{-2}, \) and \( E_F \tau \sim 10^4 \) we have \( T_1 \sim 10^{-3}K, \) and \( T_3 \sim 10^3K, \) so the condition \( T_1 < \omega < T_3 \) is easy to satisfy in a wide range of \( \omega. \)

If, on the contrary, neither of the above conditions is met then the \( q \) integral is determined by its lower limit, which can be roughly estimated as either \( q \sim 1/l \) or \( q \sim \omega/v_F. \) Instead of (21) we then obtain in the r.h.s. \( \propto min(l^2, (\frac{\omega}{\omega})^2) \left( \frac{\omega}{\omega} \right)^2 \frac{\pi^2 \omega}{v_F}. \)
Now we are ready to carry out the frequency integrations where we have to distinguish between three different regimes $\omega < T_1$, $T_1 < \omega < T_3$, and $\omega < T_3$ (the intermediate regime exists only in the case of a pure metal $T_3 >> T_1$ which is equivalent to $E_F \tau >> (\frac{e^2}{\epsilon F})^{1/2} \sim 10^2$):

$$\frac{\Delta_{e-v_F}}{\sigma_0} = -\frac{4e^2}{\pi e^2 v_F} \left[ \frac{\pi}{3^{3/2}2^{2/3}} \int_{T_1}^{T_3} d\omega \omega^{1/3} f(\omega/T) + \frac{l^2}{2} \int_0^{T_1} d\omega \omega f(\omega/T) + \frac{v_F^2}{2} \int_{T_3}^{E_F} d\omega \frac{f(\omega/T)}{\omega} \right]$$

(22)

where

$$f(\omega/T) = \frac{1}{2} \int d\epsilon S_0(\epsilon, \omega) \frac{\partial S_0(\epsilon)}{\partial \epsilon} = -\frac{1}{2} \int d\epsilon S_0(\epsilon) \frac{\partial S_0(\epsilon + \omega)}{\partial \epsilon} = \frac{\partial}{\partial \omega} [\omega \coth(\omega/2T)].$$

(23)

Note that $f(\omega)$ is an odd function, thus the finite result (22) stems only from the term in Eq.(20) linear in $\omega$ which, in turn, originates from the vector coupling to transverse gauge bosons.

For temperatures in the interval $T_1 < T < T_3$ the leading $T$-dependent correction results from the first term inside the brackets in Eq.(22) after a subtraction of the $T = 0$ counterpart and an extension of the $\omega$-integration from zero to infinity. Thus we finally obtain

$$\frac{\Delta_{e-v_F}}{\sigma_0} = -\frac{e^2 v_F}{\pi e^2} \ln \left( \frac{e^2}{\epsilon F} \right) + \frac{2\pi^{1/3}}{3^{5/2}} \Gamma \left( \frac{4}{3} \right) \left( \frac{k}{mc} \right)^{2/3} \left( \frac{T}{E_F} \right)^{4/3}, \quad \sigma_0 = e^2 v_F^2 \frac{\tau}{3 \nu_F}.\quad (24)$$

Notice that the overall correction to the conductivity remains negative whereas its variation with temperature is always positive (the correction monotonically decreases in magnitude as $T$ increases). Notably, both the constant and the $\sim T^{4/3}$ terms in Eq.(24) demonstrate non-analytic dependences on the dimensionless 3D vector coupling strength $(\kappa/mc)$.

At $T << T_1$ the $T$-dependent contribution to $\frac{\Delta_{e-v_F}}{\sigma_0}$ results from the second term in Eq.(22). It can be estimated by order of magnitude as $\sim \frac{e^2 v_F^2 T^2}{\epsilon F^2}$ whereas at high $T >> T_3$ it is due to the third term in Eq.(22) which yields $\sim \frac{e^2 v_F^2}{\epsilon F} \ln \left( \frac{T}{E_F} \right)$.

In a pure metal these latter regimes are essentially irrelevant, although they do develop and become more and more important as the amount of disorder increases.

At last, in the dirty limit $E_F \tau < 10^2$ the first term in Eq.(22) and, correspondently, the temperature correction $\sim T^{4/3}$ disappear. In this case the $T$-dependent contribution varies
as $\sim \frac{e^2 T^2}{v_F c}$ at $T << 1/\tau$ and as $\sim \frac{e^2 \rho}{c^2} \ln \frac{T}{E_F}$ at $1/\tau << T << E_F$.

The positive $\sim T^{4/3}$ term from Eq.(24) has to be compared with the other known conductivity corrections. First, we compare it to the negative kinetic term $\Delta'_{e-v\gamma} \sigma/\sigma_0 \propto -\left(\frac{\kappa}{mc}\right)^{10/3} \frac{T^{5/3}}{E_F^{2/3}}$, which represents the relativistic electromagnetic interaction correction to the transport scattering rate and accounts for the exchange of real transverse photons [6]. The kinetic contribution to $\Delta'_{e-v\gamma} \sigma(T)$ vanishes at zero temperature and is proportional to $\sigma_0^2$ while the renormalization term (24) is only of the first order in $\sigma_0$. However, the high power of the small parameter $(\kappa/mc)$ present in the kinetic term guarantees that the temperature dependence of the measured conductivity is dominated by the renormalization term at all $T < \left(E_F^2 \tau^3 (\kappa/mc)^8\right)^{-1}$. The latter condition is easy to satisfy unless the system is in the extremely clean limit $E_F \tau > 10^5 - 10^6$.

The next conductivity correction, which Eq.(24) has to be compared with, is the well-known Altshuler-Aronov term originating from Coulomb exchange processes in the diffusive regime $ql < 1$ [11]. This quantum interference correction, which is only relevant at $T < 1/\tau$, has the same negative sign as Eq.(24) while it increases with temperature: $\Delta_{AA} \sigma/\sigma_0 = -c_1 (E_F \tau)^{-2} + c_2 (E_F \tau)^{-3/2} (T/E_F)^{1/2}$, where $c_{1,2}$ are positive constants.

In fact, at all $T < E_F (E_F \tau)^{-9/5} (\kappa/mc)^{-4/5} \sim 10^{-1} K$ the temperature dependent part of $\Delta_{AA} \sigma$ exceeds the new $\sim T^{4/3}$ term from the interference correction (24) resulting from processes with momenta transfers $ql > 1$. Nevertheless, at $T = 0$ the conductivity correction is controlled by the new term (24) rather than by the Altshuler-Aronov term unless $E_F \tau < 10^2$.

Since both $\Delta_{AA} \sigma$ and $\Delta_{e-v\gamma} \sigma$ given by Eq.(24) increase as a function of temperature, one could assume that at all feasible temperatures the overall conductivity of a system of 3D fermions weakly coupled to gauge bosons increases as well!

However, in a real metal the (negative) kinetic electron-phonon interaction correction $\Delta_{e-ph} \sigma/\sigma_0 \propto -\frac{T^2}{E_F^2}$ [9] dominates over the Altshuler-Aronov term at all $T > E_F (E_F \tau)^{-5/3} \sim 10^{-2} K$ and over the new term (24) at all $T > E_F (\frac{\Delta}{mc})(E_F \tau)^{-3/2} \sim 10^{-4} K$ for the typical
parameter values.

For the sake of completeness we note that in a metal $\Delta \sigma$ receives another contribution coming from interference between the electron-impurity and the electron-phonon scattering, which was estimated in [12] as $\Delta_{e-ph-imp} \sigma / \sigma_0 \propto -\frac{T^2}{E_{FPFW}u_l}$, where $u_l$ is the longitudinal sound velocity. Although formally this term receives contributions of both signs, its actual value is negative, given the fact that the longitudinal sound has higher velocity than the transverse one ($u_l > u_t$).

It is also worthwhile mentioning that in a ferromagnetic metal with a high magnetic permeability the electron interaction with transverse vector photons is strongly amplified [13], which makes the above renormalization effects essentially more pronounced.

As the last remark, we presume that our results can be also used in the analysis of transport properties of a hot relativistic quark-gluon plasma [14].

**IV. GAUGE INTERACTIONS IN 2D**

Next we consider the $q_l > 1$ conductivity renormalization in the case of 2D fermions coupled to scalar (longitudinal) and vector (transverse) gauge bosons whose propagators are given by the formulae (1-5) where instead of $e^2$ we now use the notation $g^2$ for the coupling constant with the dimension of energy, and the two-spin density of states is $\nu_{2D} = \frac{m}{\pi}$. The velocity of gauge bosons $c$ is another parameter which may well be comparable with $v_F$, especially if the above gauge theory serves as some sort of an effective description of an underlying non-relativistic microscopic Hamiltonian and the role of the gauge boson is played by one of the collective modes. To avoid a possible confusion we stress that the dynamics of gauge bosons is purely two-dimensional, so that the 2D scalar interaction is completely screened in the quasi-static limit $\omega << v_F q$: $V_{00}(Q) \approx \frac{4\pi g^2}{q^2 + \kappa^2}$, where $\kappa^2 = 4\pi g^2 v_F^2$. The 2D conductivity which we are going to compute, is the response to an ordinary ”in-plane” electric field coupled in the usual way to our 2D fermions carrying the electric charge $e$ in addition to the 2D gauge coupling $g$. 


First we compute the renormalization correction resulting from the transverse vector coupling. The angular integral analogous to Eq.(20) now reads as

\[
\int \frac{d\Omega_p}{2\pi} \int \frac{d\Omega_q}{2\pi} \text{Re}[a_n V_{\mu\nu}(Q)a_\nu] \delta' \left( x - \frac{\omega}{qv_F} + \frac{q}{2p} \right) v \cdot n \left( v + \frac{q}{m} \right) \cdot n = \frac{v_F^2}{\pi} \left[ -\frac{3q}{2p} + \frac{\omega}{qv_F} \right] \left( \frac{v_F}{c} \right)^2 \text{Re} V_{11}(Q) - \left( \frac{q}{2p} + \frac{\omega}{qv_F} \right) \text{Re} V_{00}(Q)\]

(25)

Instead of Eq.(22) we now obtain

\[
\frac{\Delta_{\text{vec}} \sigma^{2D}}{\sigma_0^{2D}} = -\frac{4g^2}{3\pi^2 v_F^2} \int_{T_1}^{T_3} d\omega f(\omega/T) + l^3 \int_0^{T_1} d\omega \omega f(\omega/T) + v^3 \int_{T_3}^{\infty} d\omega f(\omega/T) \omega^2 \]

(26)

where \( b_{2D} = \frac{2\pi g^2 v_F^2 c^2}{e^2} \) and \( \sigma_0^{2D} = \frac{1}{2} e^2 v_F^2 \tau v_F^2 \).

In the range of temperatures \( T_1 < T < T_3 \) which exists if coupling is strong enough \( \left( \frac{g^2}{mc^2} > (E_F \tau)^{-2} \right) \) the vector contribution is given by

\[
\frac{\Delta_{\text{vec}} \sigma^{2D}}{\sigma_0^{2D}} = -\frac{21/2}{3\pi} \left( \frac{\kappa}{mc} \right)(1 + \frac{\pi}{2}) + \frac{T}{6E_F} \]

(27)

Note, that the dependence on the coupling constant \( g \) is again non-analytic, in particular, the coefficient in front of the \( \sim T \) term does not contain \( g \) at all. Eq.(27) has to be contrasted with the 2D counterpart of the (negative) kinetic term \( \Delta_{\text{vec}} \sigma^{2D} / \sigma_0^{2D} \propto -\frac{T^{4/3}}{E_F^{4/3}} \left( \frac{\kappa}{mc} \right)^{8/3} \), which corresponds to the correction to the transport scattering rate.

At \( T << T_1 \) the r.h.s. of (26) varies with temperature as \( \sim \frac{g^2 v_F^2 \tau}{e^2} \) while at \( T >> T_3 \) it decays as \( \sim -\frac{g^2 v_F^2 \tau}{e^2} \).

It is also worthwhile mentioning that in the case of weak coupling \( \left( \frac{g^2}{mc^2} < (E_F \tau)^{-2} \right) \) the constant term is \( \sim -\frac{g^2 v_F^2 \tau}{e^2} \) and there are only two latter regimes for the \( T \)-dependent part which match together at \( T \sim 1/\tau \).

Remarkably, in the 2D case the scalar potential \( \text{Re} V_{00}(Q) \) also leads to a significant contribution associated with the \( \omega \)-odd term resulting from the 2D angular integral (25)

\[
\frac{\Delta_{\text{sc}} \sigma^{2D}}{\sigma_0^{2D}} = -\frac{1}{\pi^2 mc v_F^2} \left[ \int_{1/\tau}^{1/\tau} d\omega \omega f(\omega/T) \left( I - \frac{1}{\kappa} \tan^{-1} kl \right) + \int_{1/\tau}^{\infty} d\omega f(\omega/T) \left( v_F - \frac{\omega}{\kappa} \tan^{-1} \frac{kv_F}{\omega} \right) \right]

(28)
At $g^2 > 1/\tau (E_F \tau)$ there exists a range of temperatures $1/\tau < T < T_3 \sim (E_F g^2)^{1/2}$ where

$$\frac{\Delta_{sc}\sigma^{2D}}{\sigma_0^{2D}} = -\text{const} + \frac{T}{\pi E_F}$$

(29)

and the constant term behaves as $\sim (g^2/E_F)^{1/2}$. The term linear in $T$ comes without any smallness just as in the vector case. Notice that as compared to the vector case which we considered above the scalar vertices are free of the factor $(v_F/c)^2$.

At $T << 1/\tau$ the temperature dependent part is again quadratic $\sim T^2$ and independent of coupling whereas at $T >> T_3$ it now behaves as $\sim -g^2/T$.

In the weak coupling case $g^2 < 1/\tau (E_F \tau)$ the constant term in Eq.(28) is $\sim -g^2 \tau$ and there are only two different regimes exhibited by the $T$-dependent part of the r.h.s. of Eq.(28): $\sim g^2 \tau^3 T^2$ at $T < 1/\tau$ and $\sim -g^2/T$ in the opposite case.

The expressions (26) and (28) have to be compared with the Altshuler-Aronov-type 2D interference corrections resulting from the diffusive regime $q_l < 1$, which appear to be independent of the coupling strength at $T < 1/\tau$ and diverge logarithmically as $T$ tends to zero. It turns out that the 2D vector gauge interaction produces a negative term $\Delta_{AA}\sigma^{2D}/\sigma_0^{2D} \sim -(E_F \tau)^{-1}|\ln(E_F \tau)|^{-n}|\ln(T \tau)|^{1+n}$, where the exponent $n$ is either 0 or 1 depending on whether $T < 1/\tau (E_F \tau)^{-2}$ or $1/\tau (E_F \tau)^{-2} < T < 1/\tau$ (see also [15]).

In the case of the 2D scalar gauge interaction the diffusive regime yields a subdominant term of order $-(E_F \tau)^{-1}|\ln(T \tau)|$ at all $T < 1/\tau$ (a similar term arises due to the effects of weak localization, where the phase relaxation time due to 2D gauge interactions is given by $\tau_0^{-1} \sim T^{1/3}$ [16]). The low temperature divergence of the first order $q_l < 1$ interference and localization corrections requires a tedious account of higher order terms, which have not been done yet.

V. DISCUSSION

In the modern Condensed Matter Theory the effective description in terms of 2D gauge fields arises in a number of contexts. The well known examples are the gauge theory of
the doped Mott insulators, which is believed to be relevant for the problem of the high $T_c$ superconductivity [17], and the gauge theory of half filled Landau level [18].

Unfortunately, neither of these problems features an example of the weak gauge coupling regime in cases of physical interest. Therefore our results based on the perturbative solution of the quantum kinetic equation under the assumption of the dominant impurity scattering can not be used directly in these contexts.

Moreover, the present gauge theory of the normal state of high $T_c$ cuprates [17] involves two kinds of excitations (spinons and holons) coupled to the gauge field, the physical conductivity being governed by that of spinless charged bosons (holons). Given all these complications, we would like to warn against any attempt of a direct use of the formulae (27) and (29) in the context of linear resistivity of high $T_c$ materials, which was explained in [17] by the $\sim T$ behavior of the standard (kinetic) holon-gauge boson scattering rate.

Nevertheless, our analysis implies, for instance, that the gauge interaction of spinons in doped Mott insulators [17] strongly affects the classical impurity conductivity even at low temperatures. The $T = 0$ renormalization factor may well be of order unity, the fact to be kept in mind at an attempt to make any quantitative predictions.

In the case of half filled Landau level there is another reason why one can not straightforwardly apply the above results even in the artificial limit of small $\Phi$, the number of flux quanta attached to every electron (the physical case corresponds to $\Phi = 2$, of course). It was pointed out elsewhere [19] that despite the external magnetic field gets cancelled in average by the attached flux, the dynamics of new fermionic quasiparticles (named composite fermions [18]) remains diffusive up to transferred momenta $q \sim 1/l_B = B^{1/2}$. Therefore in the composite fermion theory there is no room for the ballistic regime $"ql > 1"$ which we discuss in the present paper.

To add to this point, we mention that if it were not the case, then the low-temperature conductivity, which is governed by the divergent (negative) diffusion correction, would manifest the $(\ln T\tau)^2$ behavior in the interval $(E_F^2\tau^3)^{-1} < T < 1/\tau$ [15]. Such a prediction would certainly contradict the experiment [20], which demonstrates the $\ln T$ behavior of the
conductivity $\sigma_{xx}(T)$ at filling factors $\nu = 1/2$ and $3/2$ at temperatures from $0.5K$ down to $15mK$, while the estimates based on the parameters of samples used in [20] yield $1/\tau \approx 0.5K$ and $(E_F^2 \tau^3)^{-1} \approx 10mK$.

On the other hand, assuming that the diffusion of composite fermions extends up to distances of order of the magnetic length $l_B \sim B^{-1/2}$, one finds the leading $\ln T$ behavior of $\sigma_{xx}(T)$ in the whole range of temperatures $(E_F^2 \tau^3)^{-1} < T < 1/\tau$ [19]. It is worthwhile to note that in the original electron picture the existence of diffusion at the magnetic length scale can be readily seen from the fact that the diffusive behavior results from electron hoppings between adjacent Landau orbitals, which are $l_B$ distance apart (this fact becomes much more obscure after a mapping of electrons onto composite fermions though).

As another implicit evidence supporting our arguments we mention a similar effect of the vector gauge interactions on the renormalization of thermopower, which was previously discussed in the contexts of the 3D electron-phonon [21] and the 3D electron-electron interaction [22] problems. In the case of thermopower the leading $ql > 1$ correction to the thermoelectric coefficient $\eta$ is given by the expression similar to Eq.(18) but with an extra factor $\epsilon/T$ in the integrand. Therefore one only needs the $\omega$-even part of (20) or (25), which is nonzero for both vector and scalar couplings.

Then in the (obviously 2D) case of half filled Landau level the $ql > 1$ correction to the Drude thermopower ($S_0 \approx \frac{\pi^2 T}{3eE_F}$) of electrons with the unscreened Coulomb potential would behave as $\Delta S \sim \frac{T\Phi^2 k_F}{e^2 m E_F} \log E_F/T$ (in the screened case it becomes even stronger $\Delta S_{xx} \sim \frac{1}{\epsilon} (\frac{T\Phi^2}{E_F})^{2/3}$ in accordance with the general expectations [18]). Although one can not simply interpolate these perturbative corrections into the physical case of $\Phi = 2$, they indicate a possible strong nonlinear $T$ dependence of the measured thermopower. However, the available experimental data for the diffusion thermopower $S(T)$ at even denominator fractions [23] do not seem to support the existence of any substantial non-linear terms. Moreover, they rather indicate that the corrections to $\sigma_0$ are relatively small (see also [24]). This observation is consistent with the absence of the ballistic regime (and the related $ql > 1$ renormalization effects) in the composite fermion 2D gauge theory.
Our analysis of the 2D scalar case also implies that similar renormalization effects due to the ordinary (non-gauge) electron-electron and electron-phonon interactions do occur, for example, in doped semiconductor heterostructures.

In contrast to the above examples of 2D gauge theories the conventional Coulomb interaction $V_{00}(Q) = \frac{2\pi e^2}{\epsilon_0(q+\kappa)}$ (where $\kappa = 2\pi e^2\nu_2/\epsilon_0$ contains the dielectric constant $\epsilon_0$) may indeed feature a small parameter $\alpha = \kappa/2p_F$ at high enough sheet electron densities.

Repeating the calculations, which led to (28), we now obtain

$$\Delta_{\text{Coul}}^2 = \frac{1}{2\pi m v_F^2} \left[ \int_0^{1/\tau} d\omega f(\omega/T)(l - \frac{1}{\kappa} \ln(1 + \kappa l)) + \int_{1/\tau}^{\infty} d\omega f(\omega/T)(v_F - \frac{\omega}{\kappa} \ln(1 + \frac{\kappa v_F}{\omega})) \right]$$

(30)

Again, at $\kappa l >> 1$ there exists a range of temperatures $1/\tau < T < T_3 = \kappa v_F$ where

$$\Delta_{\text{Coul}}^2 = \frac{1}{2\pi} \int_0^{\Omega/E_F} dx \left[ 1 - \frac{x}{4} - \frac{x}{4\alpha} \ln\left(\frac{x + 4\alpha}{x(1 + \alpha)}\right) \right] + \frac{T}{\pi E_F}$$

(31)

The constant term in (31) yields zero temperature renormalization, which depends on both the upper frequency cutoff $\Omega$ and the coupling strength $\alpha$. We estimate the constant term at small $\alpha$ as $-\frac{\kappa}{2\pi p_F} \ln(\Omega/\kappa v_F)$ while in the strong screening limit ($\alpha >> 1$) it approaches the value $-\frac{\Omega}{2\pi E_F} (1 - \frac{\Omega}{2E_F})$.

Thus, despite of an uncertainty of the actual value of $\Omega \sim E_F$ we find that the renormalization effect might become quite substantial at low densities, which correspond to large values of $\alpha$. It is worthwhile mentioning that the situation $\alpha \sim 1$ arises, for example, in low-density ($n_e < 10^{11} cm^{-2}$) GaAs heterostructures characterized by the dielectric constant $\epsilon_0 = 13$ and the electron band mass $m = 0.067m_0$. In this system the renormalization correction (31) could remain greater than the 2D Altshuler-Aronov term or the localization correction (in the presence of Coulomb interactions the latter is governed by the 2D phase relaxation time $\tau_{\phi}^{-1} \sim T$ [24]), both given by the expression

$$\Delta_{\text{AA}}^2 = \frac{1}{\pi(E_F\tau)} |\ln(T\tau)|$$

except at extremely low temperatures.

At $T << 1/\tau$ the temperature dependent part of (30) is again quadratic $\sim \frac{\tau^2}{E_F}$ and independent of coupling whereas at $T >> \kappa v_F$ it now behaves as $\sim -\frac{\kappa}{p_F} \ln \frac{\Omega}{\tau}$.
In the strongly disordered case \( \kappa l < 1 \) (which is, however, not quite important for the analysis of typically clean GaAs samples) there are only two latter regimes left over.

Among other possible applications of the above results we mention the problem of transport properties in the vicinity of a quantum critical point corresponding to some charge transfer instability. In this scenario, which was argued to be relevant for the problem of high \( T_c \) cuprates [26], charged fermionic excitations are coupled in a scalar way to an overdamped critical mode described by the propagator \((i\omega/q^\alpha + q^\beta)^{-1}\). Our results suggest that at a crossover temperature, above which the critical fluctuations become effectively three-dimensional, the conductivity corrections get strongly suppressed as compared to those in the low temperature (effectively two-dimensional) regime.

VI. SUMMARY

To summarize, in the present paper we show that the Drude conductivity of fermions coupled to three- or two-dimensional gauge fields is strongly renormalized due to corrections to the nonequilibrium fermion density of states and the nonlocal part of the electron-electron collision integral. We also correct the earlier prediction [3] of a similar renormalization effect in the 3D electron-phonon problem. To this end, we demonstrate that it is a peculiarity of the relevant phase volume-type expression in the case of the 3D scalar (density-density) coupling which makes this new correction negligibly small. However, in the case of the vector (current-current) coupling such terms do appear in both 2D and 3D. At strong enough coupling which, however, still remains in the perturbative regime the temperature dependent part of the correction is found to behave as \( T^{4/3} \) in the 3D and as \( T \) in the 2D case (in the latter case the scalar coupling contributes as well, and the coefficient in front of the \( \sim T \) term appears to be independent of the coupling strength). The non-analytic dependences of these corrections resulting from large momenta transfers \( (ql > 1) \) on the coupling strength and/or temperature allow one to classify them as a non-Fermi liquid renormalization.

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