Supporting Information

High light outcoupling efficiency from periodically corrugated OLEDs.

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1. Theory of light emission from OLEDs

We develop a theoretical framework for the light emission from periodically corrugated OLEDs based on the rigorous scattering matrix approach. Our goal is to develop computational approaches to model enhanced light outcoupling ($\eta_{\text{out}}$), emission and losses from OLEDs fabricated on integrated, corrugated substrates. Given the inability to measure $\eta_{\text{out}}$ directly, modeling the losses and resulting $\eta_{\text{out}}$ is critical for determining $\eta_{\text{out}}$. This computational approach becomes more critical in assessing experimental parameters encountered with the integrated substrates.

We compute the power generated inside the OLED by the scattering matrix (SM) approach\textsuperscript{1} that has been extremely valuable in computing the reflection, transmission and absorption of photonic crystals and corrugated solar cells\textsuperscript{2}. However the scattering matrix approach and the transfer matrix approach utilized by Furno et al\textsuperscript{3} for simulating flat OLEDs have key differences. Within the emissive layer the dipole source emits with amplitude $a_{\text{inc}}^+$ and $a_{\text{inc}}^-$ in forward and backward directions. The total reflected electric fields amplitudes propagating within the OLED in the positive and negative direction ($b^+$ and $b^-$-Figure S1) are simulated by the SM. The front scattering matrices (F) for the substrate/ITO/HTL stack and the back matrix (B) for the ETL/Ag cathode stack already include multiple scattering effects, similar to the formalism\textsuperscript{4} by Egel and Lemmer. In contrast, the transfer matrix theory by Furno et al\textsuperscript{3} uses single-pass reflectance coefficients $a^+$ and $a^-$ from the top and bottom of the OLED stack. These reflectances are not directly calculated by the SM, and hence we cannot directly use the expressions of Appendix A in Furno et al\textsuperscript{3} for the power emitted by the OLED, within the present SM approach.

Our SM theory of OLED emission is guided by\textsuperscript{4}. The fields in the emissive layer are the sum of the incident field $a_{\text{inc}}$ and the reflected field: $b^+$, $b^-$, traveling in both directions\textsuperscript{4} (Figure S1). We first describe the emission from a flat OLED stack, and then generalize to the corrugated case.
The scattering matrix $S$ relates the emitted field $c$ and input field $a_{in}$ through the matrix relation 
\[
(S)_{out} = (S)(a_{in}).
\]

Thus fields in the air $c^+$ and reflected fields within the emissive layer are connected by the front scattering matrix $F$ according to 
\[
\begin{pmatrix}
  c^+ \\
  b^- 
\end{pmatrix} = 
\begin{pmatrix}
  F_{11} & F_{12} \\
  F_{21} & F_{22} 
\end{pmatrix} 
\begin{pmatrix}
  a_{in}^+ \\
  b^+ 
\end{pmatrix} (1)
\]

Although $F_{ij}$ are complex scalars for the planar case, they are $N_G \times N_G$ matrices for the corrugated case (where $N_G$ is the number of Fourier components). The emitted field is
\[
c^+ = F_{11}a_{in}^+ + F_{11}b^+ (2)
\]

The reflected field propagating in the $+z$ direction within the OLED stack is
\[
b^- = F_{21}a_{in}^+ + F_{21}b^+ (3)
\]

The fields from the cathode are connected by the rear scattering matrix $B$ according to
\[
\begin{pmatrix}
  b^+ \\
  d^- 
\end{pmatrix} = 
\begin{pmatrix}
  B_{11} & B_{12} \\
  B_{21} & B_{22} 
\end{pmatrix} 
\begin{pmatrix}
  a_{in}^- \\
  b^- 
\end{pmatrix} (4)
\]

The reflected field is
\[
b^+ = B_{12}a_{in}^- + B_{12}b^- (5)
\]

The emitted field through the cathode is theoretically given by
\[
d^- = B_{22}a_{in}^- + B_{22}b^- (6)
\]

**Figure. S1.** Schematic of the computational set-up for the flat OLED stack, that is generalized to corrugated OLEDs.
The emitted field \( d^- \) is negligible through a metal cathode (eqn. 4), and \( B_{22} \) is consequently a vanishing matrix.

The reflected fields \( b^- \) and \( b^+ \) represent two unknowns, given by equations (3) and (5). We solve for the reflected fields in terms of known fields. Eq. (3) becomes

\[
b^+ = B_{12}a^-_{in} + B_{12}b^- = B_{12}a^-_{in} + B_{12}F_{21}a^+_{in} + B_{12}F_{21}b^+ \quad (7)
\]

This leads to

\[
(I - B_{12}F_{21}) b^+ = B_{12}a^-_{in} + B_{12}F_{21}a^+_{in} \quad (8)
\]

We can then solve for the reflected field \( b^+ \) as

\[
b^+ = (I - B_{12}F_{21})^{-1}(B_{21}a^-_{in} + B_{12}F_{21}a^+_{in}) \quad (9)
\]

For the reflected field \( b^- \) within the OLED emissive layer, we similarly have from Eq. (3)

\[
b^+ = F_{21}a^+_{in} + F_{21}b^+ = F_{21}a^+_{in} + F_{21}B_{12}a^-_{in} + F_{21}B_{12}b^-
\]

\[
(1 - F_{21}B_{12})b^- = F_{21}a^+_{in} + F_{21}B_{12}a^-_{in} \quad (11)
\]

We solve for the reflected field \( b^- \) as

\[
b^- = (1 - F_{21}B_{12})^{-1}(F_{21}a^+_{in} + F_{21}B_{12}a^-_{in}) \quad (12)
\]

The emitted field in air (Eq. (2)) is calculated by substituting the solution for \( b^+ \)

\[
c^+ = F_{11}a^+_{in} + F_{11}(1 - F_{21}B_{12})^{-1}(F_{21}a^+_{in} + F_{21}B_{12}a^-_{in}) \quad (13)
\]

\[
c^+ = (F_{11} + M \cdot F_{21})a^+_{in} + F_{11} \cdot M \cdot B_{12}a^-_{in} \quad (14)
\]

With the matrix \( M \) being

\[
M = (1 - F_{21}B_{12})^{-1} \quad (15)
\]

The total field within the emissive layer are then

\[
l_{R}^e = A^{-e} \exp(-i k_{12}z_f) + A^{+e} \exp(i k_{12}z_f) \quad (16)
\]

with

\[
A^{-e} = a^-_{inc} + b^-_i \quad (17)
\]

\[
A^{+e} = a^+_{inc} + b^+_i \quad (18)
\]

These *include* multiple scattering effects. The power emitted within the OLED arises from the 3 polarizations of the dipole emitter corresponding to \( z, x, y \) orientations of the dipole (Figure S1) and derived below.

**a) Transverse magnetic vertical (TMv) polarization (z polarized dipole)**

As in \( 3 \), we introduce the dimensionless parallel component of the wavevector \( u \)

\[
u = \frac{k_{\parallel}}{m(\sigma r)k_o} \quad (18)
\]
where \( k_0 = \omega / c = 2\pi / \lambda \).

The power emitted by the vertical electric dipole (oriented in the z direction) or in the TMv polarization (P(TMv)), is given by the general OLED emission theory of Sullivan and Hall in terms of the total fields \( I_R^\varepsilon \) within the OLED emissive layer as

\[
P(TMv) = \frac{3}{2} \int_0^\infty du \frac{u^3}{\sqrt{1-u^2}} \left[ I_R^\varepsilon \right]^{TM} + 1
\]

This yields

\[
P(TMv) = \frac{3}{2} \int_0^\infty du \frac{u^3}{\sqrt{1-u^2}} \left\{ A^{-e} + A^{+e} \right\} = \frac{3}{2} \int_0^\infty du \frac{u^3}{\sqrt{1-u^2}} \left\{ 1 + b_{TM}^+ + b_{TM}^- \right\}
\]

The total field inside the OLED at the emitter location is a superposition of the waves travelling in the +z and –z directions, and includes the incident field, normalized to unity for waves propagating in both directions, thereby providing the unity term in the numerator.

b) Transverse electric horizontal (TEh) polarization modes (y polarized dipole)
The TEh polarization also involves an even integrand with the displacement field \( D \) given by

\[
\left[ \hat{\mathbf{r}} \cdot D_R^{TE} \right]_{HED} = \frac{3}{4} \varepsilon_1 E_S \int_0^\infty du \frac{u}{\sqrt{1-u^2}} \left[ I_R^\varepsilon \right]^{TE}
\]

\[
= \frac{3}{4} \varepsilon_1 E_S \int_0^\infty du \frac{u}{\sqrt{1-u^2}} \left[ A^{-e,TE} \exp(-ik_{1z}z_s) + A^{+e,TE} \exp(i k_{1z}z_s) \right]
\]

The power emitted in TEh modes is:

\[
P(TEh) = \frac{3}{4} \int_0^\infty du \frac{u}{\sqrt{1-u^2}} \left\{ 1 + (b_{TE}^+ + b_{TE}^-) \right\}
\]

(22)

c) Transverse magnetic horizontal polarization (TMh) modes (x polarized dipole)
The TMh modes have an odd integrand (Eq. 46 in) with

\[
\left[ \hat{\mathbf{r}} \cdot D_R^{TM} \right]_{HED} = -\frac{3}{4} \varepsilon_1 E_S \int_0^\infty du \frac{u(1-u^2)}{\sqrt{1-u^2}} \left[ I_R^\varepsilon \right]^{TM}
\]

\[
= -\frac{3}{4} \varepsilon_1 E_S \int_0^\infty du \frac{u(1-u^2)}{\sqrt{1-u^2}} \left[ A^{-e,TM} \exp(-ik_{1z}z_s) - A^{+e,TM} \exp(i k_{1z}z_s) \right]
\]

(23)

The power emitted in TMh modes is

\[
P(TMh) = \frac{3}{4} \int_0^\infty du \frac{u(1-u^2)}{\sqrt{1-u^2}} \left\{ 1 + b_{TM}^+ + b_{TM}^- \right\}
\]

(24)

Although the factor for the field amplitudes \( \left\{ b_{TM}^+ + b_{TM}^- \right\} \) is the same for the TMv and TMh modes, the prefactors for the two polarizations are completely different, leading to different results for the emission profiles for the vertical and horizontal polarizations. We utilize equations 20, 22, and 24 for the numerical results, for each polarization (Figure 2).
\[ P(tot) = \left( \frac{1}{3} \right) P(TMv) + \left( \frac{2}{3} \right) P(TMh) + \left( \frac{2}{3} \right) P(TEh) = \int_0^\infty du \, P(u) \quad (25) \]

It is worth noting that the limit of the free dipole radiating in free space can be obtained by neglecting the back reflected fields so that \( b^+, b^- \to 0 \), for both TM and TE modes. Also the denominator \( 1-u^2 \) is replaced by 1 in equations 19-24 when there is no back-reflection, for the radiated power:

\[ P(TMv, \text{free dipole}) = \frac{3}{2} \int_0^\infty du \, u \, u^2 \quad (26) \]

The integrand is the expected \( \sin^2 \theta = u^2 \) dependence of the radiated power. Similarly the TE\(_h\) polarization is analogous to the H-plane of the free dipole and the radiated power is constant represented by the constant integrand:

\[ P(TE \_h, \text{free dipole}) = \frac{3}{2} \int_0^\infty du \, u \quad (27) \]

Similarly since \( (1-u^2) = 1-\sin^2 \theta = \cos^2 \theta \), the integrand is the expected \( \cos^2 \theta \) dependence of the TM\(_h\) power where the dipole is along the x axis.

\[ P(TM \_h, \text{free dipole}) = \frac{3}{2} \int_0^\infty du \, u(1-u^2) \quad (28) \]

**Corrugated OLEDs**

We describe light emission from a periodically corrugated OLED with pitch \( a \), and corrugation height \( h \) (Figure 3), where the reciprocal lattice vectors \( G \) describe the periodic corrugations in the \((x,y)\) plane. The \( G \) vectors of a triangular lattice are:

\[ G_1 = \frac{2\pi}{a} \left( 1, -\frac{1}{\sqrt{3}} \right); \quad G_2 = \frac{2\pi}{a} \left( 0, \frac{2}{\sqrt{3}} \right). \quad (29) \]

A general reciprocal lattice vector is \( G(m, n) = m G_1 + n G_2 \).

In recent experimental work\(^6\) it was found that OLEDs fabricated on corrugated substrates grow nearly conformally – i.e. every layer of the OLED has the corrugation pitch and similar height. Although the conical protrusions are rounded in the fabricated substrates\(^6\), we approximated them in simulations as slanted cones with flat tops (Figure 3). The emissive dipoles form circular contours around the conical substrate corrugations (Figure 3(b)) leading to a complex emissive zone that follows the profile of corrugations.

The emitted intensity in air is described by the amplitudes \( c^+(u, G) \), whereas the fields inside the OLED have amplitudes \( b^+(u, G) \), and \( b^-(u, G) \), where \( G \) indexes the Fourier components of the field amplitudes. The spatially varying electric/magnetic fields in corrugated OLED are described with \( N_G \) Fourier components to describe the corrugation. Simulations tests indicate \( N_G \sim 60-70 \) Fourier components offer good convergence.

Following the SM formalism, reflected fields \( b^+(u, G) \) and \( b^-(u, G) \) in the emissive layer are functions of \( G \) and \( u \) are in the forward and backward directions:

\[ b^+(u, G) = (1 - B21F21)^{-1}(B21a_{in}^+ + B21F21a_{in}^-) \quad (30) \]
\[ b^-(u, G) = (1 - F21B12)^{-1}(F21a_{in}^+ + F21B12a_{in}^-) \quad (31) \]

From a generalization of (13) the amplitude of the emitted fields in air are
Here the scattering matrices \((F, B)\) are \(N_G \times N_G\) matrices. In contrast to the flat case, the emissive dipole layer has a corrugated profile that is conformal with the substrate. The dipole emission rate \(\Gamma_s\) is a function of lateral position \((x=(x,y))\) in the layer. \(H(x)\) are the locations of the dipole in each (constant \(z\)) slice of the OLED. We integrate to obtain the total emission \((\Gamma_{tot})\) from the dipole positions:

\[
\Gamma_{tot} = \int dx \Gamma_s(x)H(x) \tag{33}
\]

\[
\Gamma_{tot} = \int dx \int du \ (u)f(x)H(x) \tag{34}
\]

g(u) are the \(u\)-dependent functions from Furno et al\(^3\) for the 3 polarizations, \(f\) are the polarization-dependent fields. Thus

\[
\Gamma_{tot} = \int du \ g(u)j(u) \tag{35}
\]

and

\[
j(u) = \sum_G f(u, G)H(G) \tag{36}
\]

This expression introduces the Fourier transform of the field components \(f\). We describe the Fourier transform \(H(G)\) of the dipole positions to be

\[
H(x) = \sum_G \exp(iG \cdot x) H(G) \quad \text{or} \quad H(G) = \int dx \exp(-iG \cdot x) H(x) \tag{37}
\]

The power inside the corrugated OLED for the three polarizations is then:

\[
P(TM \ v) = \frac{3}{2} \int_0^\infty du \ \frac{u^3}{\sqrt{1-u^2}} \left\{ 1 + \sum_G \left[ b^+(u, G) + b^- (u, G) \right] H(G) \right\}
\]

\[
P(TM \ h) = \frac{3}{4} \int_0^\infty du \ \frac{u(1-u^2)}{\sqrt{1-u^2}} \left\{ 1 + \sum_G \left[ b^- (u, G) + b^+ (u, G) \right] H(G) \right\}
\]

\[
P(TE \ h) = \frac{3}{4} \int_0^\infty du \ \frac{u}{\sqrt{1-u^2}} \left\{ 1 + \sum_G \left[ b^+(u, G) + b^- (u, G) \right] H(G) \right\} \tag{38}
\]

**Power emitted in air for corrugated OLEDs**

In corrugated OLEDs the power inside cannot be simply subdivided into regions of waveguided and air modes, as in flat OLEDs. It is necessary to calculate the fields \(c^{e}(u, G)\) just above the
substrate in the air region from \( \) ). Only field components propagating in the +ve direction (i.e. outward to air) are present for these emitted modes. There is no incident field in the air so that the constant term in (19) is absent. \( c^+(u,G) \) is computed for TE, TM components \((y,x,z)\) to yield the emitted power as

\[
P_{\text{air}}(TM) = \frac{3}{2} \int_0^\infty du \frac{u^3}{\sqrt{1-u^2}} \{ A^+e \} = \frac{3}{2} \int_0^\infty du \frac{u^3}{\sqrt{1-u^2}} \{ \sum_{k_G^2>0} c_{TM}^+(u,G) \} = \int_0^\infty du P_{TM}^\text{air}(u) \tag{39}
\]

\[
P_{\text{air}}(TMh) = \frac{3}{4} \int_0^\infty du \frac{u(1-u^2)}{\sqrt{1-u^2}} \{ \sum_{k_G^2>0} c_{TM}^+(u,G) \} = \int_0^\infty du P_{TMh}^\text{air}(u) \tag{40}
\]

\[
P_{\text{air}}(TEh) = \frac{3}{4} \int_0^\infty du \frac{u}{\sqrt{1-u^2}} \{ \sum_{k_G^2>0} c_{TE}^+(u,G) \} = \int_0^\infty du P_{TEh}^\text{air}(u) \tag{41}
\]

The sum over Fourier components \( G \) is for propagating modes where \( k_z^2 > 0 \)

\[k_z^2 = \frac{(ab)^2}{c^2} - (u + G_y)^2 - G_y^2\]

The spectral power is

\[P_{\text{air}}^\text{spectral}(u) = P_{TMv}^\text{air}(u) + P_{TMh}^\text{air}(u) + P_{TE}^\text{air}(u) \tag{42}\]

To achieve numerical stability we utilize the condition \( P_{\text{air}}^\text{spectral}(u) \leq P_{\text{in}}^\text{spectral}(u) \).

2. Dielectric function

The dielectric function is computed in every slice of the OLED. In the overlapping slice (Figure 3) where nanocones of wavelength dependent dielectric function \( \varepsilon_1(\lambda) \), radius \( r \), reside in a background (organic material) of dielectric function \( \varepsilon_2(\lambda) \) with radius \( b \), enclosed by a metallic cathode with dielectric function \( \varepsilon_3(\lambda) \), the Fourier transform of the dielectric function is

\[\varepsilon(G = 0) = \varepsilon_2 + (\varepsilon_1 - \varepsilon_2) \frac{\pi r^2}{a_c} + (\varepsilon_3 - \varepsilon_2) \frac{3\pi b^2}{a_c} \tag{43}\]

for the uniform \( G=0 \) component.

When \( G \neq 0 \)

\[\varepsilon(G) = (\varepsilon_1 - \varepsilon_2) \frac{\pi a_c^2}{a_c} \frac{2J_1(Gr)}{Gr} + (\varepsilon_3 - \varepsilon_2) \frac{\pi b^2}{a_c} \frac{2J_1(Gb)}{Gb} \sum_{j=1}^6 e^{-ig \cdot R_j} \tag{44}\]

Here \( a_c \) is the area of the unit cell, \( a_c = (\sqrt{3}/2)a^2 \). The lattice vectors are

\[R_1 = \frac{a}{2} \left( \frac{1}{2}, 0 \right) ; \quad R_2 = \frac{a}{2} \left( 0, \frac{1}{2}, 0 \right) ; \quad R_3 = \frac{a}{2} \left( -1, \frac{1}{2}, 0 \right) ; \quad R_4 = \frac{a}{2} \left( 1, 0, 0 \right) ; \quad R_5 = \frac{a}{2} \left( -1, 0, 0 \right) ; \quad R_6 = \frac{a}{2} \left( 1, 0, 0 \right)\]
3. Spectral power for flat OLEDs

The spectral power \( P(u, \lambda) \) where \( u \) is the in-plane scaled parallel wave-vector allows division of emitted power into air modes, substrate waveguided modes, organic waveguided modes and plasmonic losses, for flat OLEDs. The regions \( u < 1/n(\text{org}) \) are outcoupled modes to air, \( 1/n(\text{org}) < u < 1/n(\text{subs}) \) are wave-guided substrate modes; whereas \( 1/n(\text{subs}) < u < 1 \) are wave-guided modes in the high index organic+ITO layers. \( u > 1 \) are evanescent modes that lead to plasmonic losses.

The flat OLED stack simulated (Figure S1) of substrate(1000 µm)/ITO(90 nm)/ HTL(60 nm)/EBL(20nm)/emissive layer 10 nm/ HBL(10 nm)/ ETL( \( d \) nm)/silver cathode (100 nm), is similar to that for red OLEDs. We chose ITO (rather than PEDOT:PSS) as the conductive anode, so that our results can be directly compared with previously published results. With these thicknesses the HTL+ITO thickness of 175 nm is very close to half a wavelength (\( \lambda/2n \))=174 nm for red wavelengths - the condition to obtain maximum power to the substrate.

Our SM simulations for flat OLEDs required somewhat different computational treatment of TM and TE modes, to achieve good convergence. For TMv and TMh modes we i) embed the emitter in an organic layer and ii) utilized infinitesimal organic slices between each OLED layer, and obtained the spectral power \( P_{\text{TMv}}(u, \lambda) \) and \( P_{\text{TMh}}(u, \lambda) \). We utilized the multiplicative factor of the dielectric function of the organic layer \( \varepsilon_1 \) for the TMh modes, from equation [22]. However for TE modes we i) embed the emitter in a thin air slice (thickness \( \delta \)), and ii) utilize infinitesimal slices of air between each OLED layer, to obtain the spectral power \( P_{\text{TEh}}(u, \lambda) \). We verified that the results were insensitive to the emissive layer thickness \( d \) (in the range 1-10 nm) Utilizing the same treatment or organic emitter layers and infinitesimal organic layers for TE modes results in an unphysical minimum in the spectral power \( P_{\text{TEh}}(u, \lambda) \) near \( u \sim 0.6 \), that disappears when the infinitesimal air slices are used.

The total spectral power \( P(u) \) is given by the sum of TMv, TMh, and TEh contributions from [25]. The polarization dependent spectral power yields valuable insight into loss mechanisms.

We computed the spectral power \( P(u) \) for a red OLED with \( \lambda=610 \) nm. The maximum emitted power is expected when the dipole is a quarter wavelength (\( \lambda/4n \)) ~ 87 nm corresponding to a maximum of the field at the emitter location. Assuming a penetration of 10-15 nm of the field into the cathode, we expect ~70 nm ETL thickness for the maximum power, as we verified later. For an ETL thickness of 70 nm the spectral power results are shown in Figure S2. Flat OLEDs utilize thick substrates where the reflections from the air-glass interface are expected to be incoherent, in contrast to the coherent reflections from the inner thin interfaces with the organic and ITO layers. A simple way to model the spectral power emitted inside the OLED (Figure S2) is to introduce a weak absorption in the substrate (e.g. Im \( \varepsilon \sim 0.005 \))< preventing unwanted reflections from the substrate-air interface, and reducing noise in the spectral power.
Figure S2. a) Spectral power for the three polarizations as a function of the in plane wavevector ($u_{||}$) for an ETL thicknesses of 70 nm corresponding to a ($\lambda/2n$) cavity. b) Total spectral power.

The results compare very favorably to published results\(^3\). For an ETL thickness of 70 nm, the emitting species is close to the cathode and the losses are dominated by the strong plasmonic peak at $u>1$, mostly for the TMv modes, and to a weaker extent for the TMh modes. There are two strong wave-guiding peaks ($u\sim 0.8-0.9$) in the organic layer for TM and TE modes. The power out-coupled to air is dominated by the horizontal polarization modes TMh and TEh. The spectral power $P(u)$ for the different polarizations agrees well with that published earlier for red OLEDs\(^3\).

The spectral power provides valuable physical insight into the loss mechanisms. As the ETL thickness decreases to 20 nm, the plasmonic losses increase rapidly corresponding to the large peak at $u>1$ (Figure S3 (a)), and are considerably larger that the waveguided power. Alternatively as the ETL thickness increases to 160 nm corresponding to ($\lambda/2n$) or a minimum of the power, the spectral power is dominated by a larger waveguiding loss from the TE modes, which exceeds the plasmonic loss peak at $u>1$. The spectral power changes shape at small $u$,that decreases outcoupled power to air (Figure S3 (b,c))

Finally when the ETL thickness becomes larger $d\sim (3\lambda/4n)\sim 260$ nm, corresponding to the next maximum of the outcoupled power, the waveguiding peak (near $u\sim 0.9$) increases even further, and clearly dominates the plasmonic loss peak, as expected.
**Figure S3.** Spectral power as a function of the in plane wave-vector (\(u||\)) for ETL thicknesses of 20 nm, 160 nm (\(\lambda/2\)), 240 nm (3\(\lambda/4\) cavity). The division of power into air, substrate, organics, and plasmonic modes is indicated in the labels. The emission wavelength is red (610 nm).

The power emitted inside the OLED is given by the sum of the TMv, TMh, and TEh contributions from equation (25) for each \(u\).

\[
K_{in}(u) = \left(\frac{1}{3}\right)[P_{TMv}(u) + 2P_{TMh}(u) + 2P_{TEh}(u)] \quad (45)
\]

The total power \(P_{in}\) emitted inside the OLED is then an integral over all wavevectors

\[
P_{in} = \int_{0}^{\infty} K_{in}(u) \, u \, du \quad (46)
\]

The effect of thick incoherent substrates in calculating the outcoupled power to air, is obtained by the treatment\(^3\), where the energy transmission to the substrate (\(T_{TM}, T_{TE}\)) is calculated for both polarizations, given by

\[
T_{TM} = |t_{TM}|^2 \left(\frac{n_{air}}{n_s}\right)^2 \frac{k_{zs}}{k_{ze}} \quad (46)
\]

\[
T_{TE} = |t_{TE}|^2 \frac{k_{ze}}{k_{zs}} \quad (47)
\]

\(T_{s,o}, R_{s,o}\) and \(R_s\) are the transmittance of the outer substrate-air interface, and reflectances of the outer substrate-air interface and the thin coherent layers respectively\(^3\). Here \(t_{TM}\) and \(t_{TE}\) are the calculated transmission coefficients, and \(k_{ze}\) and \(k_{zs}\) are the \(z\) components of the photon wavevector in the emitting layer and substrate respectively. Using these transmission coefficients we calculate the transmission of power to the thick incoherent substrate as

\[
K'_{TMv} = P_{TMv}(u)T_{TM}
\]

\[
K'_{TMh} = P_{TMh}(u)T_{TM}
\]
The fraction of power radiated to air in the far field $K_{\text{out}}$ is then given by

$$K_{\text{out}} = K'_{\text{out}} \frac{T_{s,0}}{1 - R_{s,0}R_c}$$

This formalism is applicable to flat interfaces, but is difficult to implement for corrugated interfaces.

The power emitted to air for parallel wavevectors within the air cone is,

$$P(\text{air}) = \int_0^{1/n(\text{org})} K_{\text{out}}(u) \, du$$

The outcoupling $\eta_{\text{out}}$ is then the ratio

$$\eta_{\text{out}} = \frac{P_{\text{air}}}{P_{\text{in}}}$$

The simulated $\eta_{\text{out}}$ or the external quantum efficiency (EQE), assuming an ideal OLED where the product $\gamma \cdot \eta_{\text{tot}} \cdot \eta_p = 1$ from equation (1) main text, as a function of the ETL thickness (Figure S4), has the first peak in outcoupling at $d(\text{ETL}) \sim (\lambda/4n) \sim 75 \text{ nm}$, where $\eta_{\text{out}} \sim 18\%$, very close to the classically expected value of $1/(2n^2) \sim 0.17$. The second peak is at $d(\text{ETL}) \sim (3\lambda/4n) \sim 250 \text{ nm}$, corresponding to the next maxima of the electric field at the emitter location. There is a deep minimum at $d(\text{ETL}) \sim (\lambda/2n) \sim 160 \text{ nm}$, where the electric field is near zero at the emitter position, similar to the well-known phenomena when an antenna is shorted out when located half a wavelength from a metallic back-plane. We utilized a cut-off of the power at $P_{\text{max}} \sim 5$, to limit contributions from the largest peaks in the spectral power, although the form $\eta_{\text{out}}$ was insensitive to the value of $P_{\text{max}}$. The calculated $\eta_{\text{out}}$ qualitatively agrees with the features in Furno et al with some differences due to the different wavelength dependent refractive indices utilized here from the previous work.

From the spectral power ($P(k_x)$) inside the OLED at different wavelengths in a two-dimensional contour plot (Figure S5), demarcating air, substrate, waveguided and plasmonic regions. The dominant loss mechanisms are the surface plasmon excitations and waveguided modes in the high index layers, along with a weak substrate mode in agreement with earlier flat OLED simulations.
Figure S4 The external quantum efficiency (EQE) as a function of the ETL thickness, for the red OLED stack described in the text. An ideal lossless OLED is assumed so that $\eta_{\text{out}} = \text{EQE}$.

Figure S5. Power inside a flat OLED as a function of wavelength and parallel wavevector $k_x$. (a) for a small ETL thickness (70 nm) corresponding to a maximum of the power emitted. A small absorption $\text{Im}(\varepsilon)=0.01$ was added to the polycarbonate substrate.

4. Outcoupling in corrugated OLEDs

The corrugated OLEDs show a simulated $\eta_{\text{out}} \approx 0.6-0.65$ for optimal pitch values of 1000-2500 nm. It is of great interest to calculate how the remaining emitted power can be assigned to trapped waveguided and plasmonic modes. We utilize the regions of $u_\parallel$ where $u_{\text{air}} = 1/n_{\text{org}} (=0.58$ for $\lambda=530$ nm), $u_{\text{subs}}=n_{\text{subs}}/n_{\text{org}} (=0.82$ for $\lambda=530$ nm), as for a flat OLED. Inside the corrugated OLED, the power emitted to the air cone $P_{\text{in}}(\text{air})$, substrate cone $P_{\text{in}}(\text{subs})$, and plasmonic regions $P_{\text{in}}(\text{pl})$ are then defined through the ranges of $u_\parallel$ in the spectral power:

$$p_{\text{in}}(\text{air}) = \sum_{u=0}^{u_{\text{air}}} K_{\text{th}}(u)$$
\[ p^{in}(subs) = \sum_{u=u_{air}}^{1} K_{in}(u) \]
\[ p^{in}(pl) = \sum_{u=1}^{\infty} K_{in}(u) \]

(53)

However in each of these three regions a portion of this power is already emitted to air due to diffraction. The corresponding spectral powers emitted to air are:

\[ p^{air}(air) = \sum_{u=0}^{u_{air}} p^{air}(u) \]
\[ p^{air}(subs) = \sum_{u=u_{air}}^{1} p^{air}(u) \]
\[ p^{air}(pl) = \sum_{u=1}^{\infty} p^{air}(u) \]

(54)

The fraction of the power still trapped inside the OLED in waveguided modes and plasmonic modes is

\[ p^{tr}(wg) = \left( p^{in}(subs) - p^{air}(subs) + p^{in}(air) - p^{air}(air) \right) / P_{tot} \]

(55)

\[ p^{tr}(pl) = \left( p^{in}(pl) - p^{air}(pl) \right) / P_{tot}. \]

(56)

We do not distinguish between modes trapped in the substrate and waveguided modes in the high index organic layer.

**DIELECTRIC FUNCTIONS**

The dielectric functions of the organic layers was taken from spectroscopic ellipsometric measurements of NPD- layers. ETL and HTL layers used the same wavelength dependent dielectric function which we plot in Figure S6. The dielectric functions utilized tabulated Johnson-Christy data for silver, were plotted in Figure S7. Tabulated data for ITO dielectric functions, are plotted in Figure S8.
Figure S6. Wavelength dependent dielectric function $\varepsilon(\lambda) = \varepsilon_1(\lambda) + i \varepsilon_2(\lambda)$ for organic layers taken from measurements.\(^8\)

Figure S7. Wavelength dependent dielectric function $\varepsilon(\lambda) = \varepsilon_1(\lambda) + i \varepsilon_2(\lambda)$ for silver.
Figure S8. Wavelength dependent complex dielectric function $n(\lambda) = n_1(\lambda) + in_2(\lambda)$ for ITO utilized for flat OLED simulations.

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