Non-BCS Superconductivity in Cuprates from Attraction of Spin Vortices

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Abstract. We propose a non-BCS mechanism for superconductivity (SC) in hole-underdoped cuprates based on a gauge approach to the t-J model. The gluing force is a long-range attraction between spin vortices centered on the empty sites of two opposite Néel sublattices, leading to pairing of charge carriers (spinless holons). In the presence of these pairs, a gauge force coming from the single occupancy constraint induces, in turn, a RVB pairing of the spin carriers (spinons), gapped by scattering against spin-vortices. This gives rise to a finite density of incoherent hole pairs, precursor to superconductivity, supporting a Nernst signal whose anticipated contour plot is qualitatively consistent with experiments. The true superconducting transition occurs at an even lower temperature via a planar XY-type transition and it involves a kinetic energy gain due to lowering of the spinon gap. Since the short-range antiferromagnetic (AF) order and the holon pairing originate from the same term of the t-J model, this approach incorporates a strong interplay between AF and SC, giving rise to a universal relation between the energy of the resonance mode (bound state of spinons) and $T_c$, as observed in neutron scattering experiments.

1. Introduction
In this Introductory Section we give just an outline of the paper: In Section 2 we explain the basic ideas underlying our approach [1] to the normal states of cuprates, in particular, motivating the appearance of spin vortices, that are, together with the composite structure of the hole, the key ingredients of the proposed mechanism for superconductivity. In Section 3 we present the mechanism for pairing [2] of charge and spin degrees of freedom (d.o.f.). In Section 4 we discuss the superconducting transition [3], while in the final Section we sketch some experimental consequences of the proposed scenario.

2. “Normal” state
We assume that the $t - J$ model, in case with $t' - t''$ extensions, captures the key low-energy physics of the CuO planes of the cuprates. We tackle the non-perturbative constraint of no-double occupation of the $t - J$ model by decomposing at each site $i$ the fermionic spin $1/2$ hole $c_{ai}$ into a product of a bosonic chargeless spin $1/2$ boson $b_{ai}$, the spinon, and a charged spinless fermion $h_i$, the holon, in the spirit of Anderson’s spin-charge separation: $c_{ai} = b_{ai} h_i$. With this
decomposition the constraint is automatically satisfied by the Pauli exclusion principle for the holon. However this decomposition involves an unphysical d.o.f., because one can multiply at each site the spinon and the holon by arbitrary opposite phase factors leaving the physical hole field unchanged. Therefore we need to implement this local $U(1)$ gauge invariance by introducing a slave-particle gauge field $A_\mu$.

The gauge field introduces an attraction between holon and spinon and the spin-charge decomposition is an efficient approach only if this attraction is not confining them again in a standard “point-like” quasi-particle excitation. In planar 2D (and in linear 1D) systems, we have an additional option to improve a Mean Field (MF) treatment: gauging a global symmetry of the system one can bind statistical fluxes to particle-excitations, resulting only in a change of the statistics. The introduction of these fluxes in the lagrangian formalism is materialized via statistical Chern-Simons gauge fields. In our case, holes carry both charge and spin degrees of freedoms, so we can couple them to two statistical gauge fields, a $U(1)$ gauge field $\mathbf{B}$ coupled to the holon field and related to the charge, an $SU(2)$ field $\mathbf{V}$ coupled to the spinon field and related to the spin. By carefully choosing the value of the coupling constants of the Chern-Simons terms, we can keep the original hole field still fermionic. If we introduce the $SU(2)$ field, this is only possible if the absolute value of the spin flux is $1/2$ and the charge flux is $-1/2$. With this choice the holon bound to the charge flux and the spinon bound to the spin flux obey semion statistics, i.e. they acquire under exchange a phase $\pm i$, intermediate between the bosonic $+1$ and the fermionic $-1$, hence the name “semion”. Notice that these semionic holons obey Haldane statistics of order 2 in momentum space, i.e. a maximum of two semions are allowed to have the same momenta, hence a gas of spinless semions of finite density should give a Fermi surface coinciding with that of spin $1/2$ fermions of the same density. If no approximations are made, all the slave-particle approaches to the $t-J$ model are strictly equivalent. However, as soon as MF approximations are made, this approach becomes distinct from e.g. the slave boson or slave fermion, becoming a new slave-semion approach. In the improved semionic mean field approximation (MFA), the spinon configurations around holons are first optimized leading to a new bosonic spinon, denoted by $z$, describing fluctuations w.r.t. the optimized spinon background. This is therefore different from the $b$-field in the standard slave fermion approach, but it still satisfies the constraint $z_\alpha^* z_\alpha = 1$. From now on it is this spinon that we refer to. In the adopted MFA we neglect the holon fluctuations in $\mathbf{B}$ and the spinon fluctuations in $\mathbf{V}$. This leads to a much simpler form of the two statistical gauge fields denoted by $\mathbf{B}$ and $\mathbf{V}$, respectively. The $\mathbf{B}$ field is actually a static one, without dynamics, and it provides a $\pi$-flux phase factor $e^{iB_x}$ per plaquette. For the $SU(2)$ gauge field only the $\sigma_z$-component survives:

$$V^z_\mu(x) \approx - \sum_l h^*_l h_l \frac{(-1)^{|l|}}{2} \partial_\mu \arg(x - \bar{l}), \quad (1)$$

and it can be viewed as the potential of spin vortices with the axis along the direction of the AF background. Note that there is the holon number operator in the right hand side of Eq.(1), which means that the spin vortex is always centered on the hole, and its topological charge (named chirality) is $(-1)^{|l|}$ depending on the parity of the site index, where $|l| = l_x + l_y$. The effect of the optimal spin flux is then to attach a spin-vortex to the holon, with opposite chirality on the two Néel sublattices. These vortices take into account the long-range quantum distortion of the AF background caused by the insertion of a dopant hole. The optimization of spinons discussed above involves also a spin-flip associated to every holon jump between different Néel sublattices. Therefore, in the $t$-term the spinons appear in the “ferromagnetic” Affleck-Marston form $AM_{ij} = (z_i^\alpha e^{iV^z(i)\sigma_x z_j})^\#(i)$, where $\#(i)$ denotes complex conjugation if $i$ belongs to the “odd” sublattice, whereas in the $J$-term it appears in the “AF” RVB form $RVB_{ij} = e^{\alpha\beta} z_\alpha (e^{iV^z(i)\sigma_x z_j})_\beta$. The above AM/RVB dichotomy is peculiar to the semion
approach involving the SU(2) spin rotation group even in 1D, where it can be rigorously derived. It does not appear in the standard U(1) slave fermion or boson approaches and it has the nice feature that in such formalism optimizing the hopping $|AM_{ij}| = 1$ we optimize also the Heisenberg term. In fact, neglecting $A$-fluctuations, the leading terms of the Hamiltonian can be written as:

$$H = \sum_{ij} (-t)AM_{ij}h_i^zh_j^z e^{iB_{ij}} + h.c. + J(1 - h_i^zh_i - h_j^zh_j)(1 - |AM_{ij}|^2) + Jh_i^zh_j^zh_iRVB_{ij}|^2,$$

where in the second term we used the identity holding for a bosonic spinon $|AM_{ij}|^2 + |RVB_{ij}|^2 = 1$. A long-wavelength treatment of the second term in Eq. (2) leads to a CP$^1$ spinon nonlinear $\sigma$-model with an additional term coming from the spin flux,

$$\tilde{J}(V^z)^2z^*z,$$

where $\tilde{J} = J(1 - 2\delta)$. In a quenched treatment of spin vortices we derive the MF expectation value $\langle (V^z)^2 \rangle = m^2 = 0.5|\log \delta|$, where $\delta$ is the doping concentration, which opens a mass gap for the spinon, consistent with AF correlation length at small $\delta$ extracted from the neutron experiments [4]. Thus, propagating in the gas of slowly moving spin vortices, the AF spinons, originally gapless in the undoped Heisenberg model, acquire a finite gap, leading to a short range AF order.

Let us now turn to holons. In the parameter region corresponding to the “pseudogap (PG) phase” of the cuprates, identified with the inflection point in resistivity, the optimal charge flux is $\pi$ per plaquette and via Hofstadter mechanism it converts the low-energy modes of the spinless holons $h$ into Dirac fermions with a small Fermi Surface (FS) $\epsilon_F \sim t\delta$. Their dispersion is defined in the Magnetic Brillouin Zone (MBZ), which we choose as a union of two square regions, denoted as R(ight) and L(eft) centered at $Q^R = (\pi/2, \pi/2)$ and $Q^L = (-\pi/2, \pi/2)$, respectively; these momenta are also the centers of the holon FS. Increasing doping or temperature one reaches the crossover line $T^* \approx t/(8\pi)|\log \delta|$ entering the “strange metal phase” (SM) of the cuprates, above the inflection point of in-plane resistivity, where the optimal charge flux per plaquette is 0 instead of $\pi$ and we recover a “large” FS for the charge excitations with $\epsilon_F \sim t(1 - \delta)$.

Finally we turn to the slave-particle gauge field. The dynamics of its transverse mode is dominated by the contribution of the gapless holons. Their Fermi surface produces an anomalous skin effect, with a momentum scale $Q \approx (Tk_F^2)^{1/3}$, the Reizer momentum, where $k_F$ is the holon Fermi momentum, and it destroys the logarithmic confinement that would arise from the massive spinons. As a consequence of the $T$-dependence of $Q$, the hole (holon-spinon) and the magnon (spinon-antispinon) resonances formed by the gauge attraction have a strongly $T$-dependent lifetime, exhibiting a less coherent behavior than what is expected from a standard Fermi-liquid scenario. The presence of a gap for spinons is crucial for the appearance of the Reizer skin effect, because gapless spinons would condense at low $T$ thus gapping the gauge field through Anderson-Higgs mechanism and destroying this effect.

### 3. Holon and spinon pairings

The gluing force of the proposed superconductivity mechanism is a long-range attraction between spin vortices centered on holons in two different Néel sublattices. Therefore its origin is magnetic, but it is not due to exchange of AF spin fluctuations. Explicitly the relevant term in the effective Hamiltonian is the same producing the spinon gap, Eq. (3), but it is obtained by averaging the spinons using Eq. (1), as:

$$J(1 - 2\delta)(z^*z)\sum_{i,j}(-1)^{|i|+|j|}\Delta^{-1}(i - j)h_i^zh_i^zh_j^zh_j,$$

where $\Delta$ is the doping concentration.
where $\Delta$ is the 2D lattice laplacian and $\langle z^* z \rangle \sim (\Lambda^2 + m_s^2)^{1/2} - m_s$, with $\Lambda \approx 1$ as a UV cutoff, due to the spinon gap. From Eq. (4) we see that the interaction mediated by spin vortices on holons is of 2D Coulomb type. From the study of planar Coulomb systems we know that below a crossover temperature $T_{ph} \approx J(1 - 2\delta)\langle z^* z \rangle$, a finite density of incoherent holon pairs appears. It turns out that temperature is located fully in the SM phase and a consistent treatment in this phase would require a next-nearest neighbor hopping term $t'$. For simplicity we thus discuss the SC mechanism only in the PG phase, where such complication is not needed. We already remarked that the low-energy holon modes in PG have two small FS in the MBZ, the 2D Coulomb interaction acts on these modes coupling opposites sites of each FS with an attractive potential with a range given by the screening length of the Coulomb gas. A BCS treatment provides p-wave pairings on the two FS, that glued in the entire BZ produce a d-wave order parameter for $h$, denoted $\Delta^h$, as first suggested in a different setting in Ref. [5]. We do not have however condensation of holon pairs because the fluctuations of the phase of the pairing field are too strong, restoring the $h/s$ gauge symmetry broken by the BCS approximation. Nevertheless the mechanism of holon pair formation is BCS-like in the sense of gaining potential energy from attraction and losing kinetic energy, as shown by the induced reduction of the spectral weight [6]. Since the hole is made of holon and spinon, to form a hole-pair we need also the pairing of spinons. It is the gauge attraction between holon and spinon, that, roughly speaking, using the holon-pairs as sources of attraction, induces in turn the formation of short-range spin-singlet (RVB) spinon pairs (see Fig.1).

![Figure 1. Pictorial representation of hole pairs: holons (red circles) surrounded by spin-vortices (in blue), spinons (blue circles with spin-arrow), spin-vortex attraction (black line), gauge attraction (wavy line).](image)

This phenomenon occurs, however, only when the density of holon-pairs is sufficiently high, since this attraction has to overcome the original AF-repulsion of spinons caused by the Heisenberg $J$-term which is positive in our approach (see Eq. (2)), in contrast with the more standard RVB and slave-boson approaches.

Summarizing, at an intermediate crossover temperature, denoted as $T_{ps}$, lower than $T_{ph}$ in agreement with previous remarks, a finite density of incoherent spinon RVB pairs are formed, which, combined with the holon pairs, gives rise to a gas of incoherent preformed hole pairs. The lowering of free energy allowing the formation of spinon pairs is due to the fact that a short-range vortex-antivortex pair essentially do not contribute to the spinon gap, which originates from screening due to unpaired vortices. Therefore the spinon gap lowers proportionally to the density of spinon pairs, thus lowering the kinetic energy of spinons, a mechanism definitely not BCS-like. More concretely for a finite density of spinon pairs there are two (positive energy) spinon excitations, with different energies, but the same spin and momenta. They are given, e.g., by creating a spinon up and destructing a spinon down in one of the RVB pairs. The corresponding dispersion relation, thus exhibits two (positive) branches:

$$\omega(\vec{k}) = \sqrt{(m_s^2 - |\Delta^s|^2) + (|\vec{k}| \pm |\Delta^s|)^2},$$

(5)

where $\Delta^s$ is the RVB spinon pair order parameter multiplied by $|\Delta^h|^2$. The lower branch exhibits a minimum with an energy lower than $m_s$, it implies a backflow of the gas of spinon-pairs dressing the “bare” spinon.
As soon as we have a finite density of hole pairs, the RVB-singlet hole-pair field $\Delta_{c(ij)} = \langle c^{\dagger}_{i\alpha} c_{j\beta} \rangle$ is non-vanishing and the gradient of its phase, $\chi$, describes magnetic vortices (not to be confused with spin vortices). Hence below $T_{ps}$ a gas of magnetic vortex-loops in space-time appears, in the plasma phase, because the incoherence of the hole pairs leads to a vanishing expectation value of $\Delta_{c}$. Therefore, we propose to identify $T_{ps}$ with the experimental crossover corresponding to the appearance of the diamagnetic and (vortex) Nernst signal [7]. This interpretation is reinforced by the computation of the contour-plot of the spinon pair density in the $T_\chi$ plane [2], resembling the contour-plot of the diamagnetic signal. The presence of holon pairs is required in advance to have RVB pairs, and the holon density “enters the game” through two factors: the density of holon pairs itself and via the strength of the attraction, $\mathcal{J}$ from eq.(4). These two effects act in an opposite way increasing doping, thus yielding a finite range of doping for a non-vanishing expectation value of $|\Delta^c|$, starting from a non-zero doping concentration, producing a “dome” shape of $T_{ps}$ and of the contour-plot.

4. Superconductivity
We have already seen that below $T_{ps}$ the condensation of the hole-pairs is contrasted by magnetic vortices associated with the phase of the hole-pair field. Therefore to understand how these pairs gain coherence one should look to the low-energy effective action of these phases. One calculates the gauge effective action, obtained by integrating out the spinon. The result up to quartic terms for the lagrangian density is

$$L_{eff}(a,\partial \chi) = \frac{1}{2\sqrt{(m_s^2-2|\Delta|^2)}} \left[ (\partial_\mu a_\nu - \partial_\nu a_\mu)^2 + |\Delta|^2 (2(2a_0 - \partial_\nu \chi)^2 + (2\bar{\alpha} - \nabla \chi)^2) \right]$$ (6)

with $a_\mu = A_\mu - \partial_\mu \arg \Delta$. In the gauged XY-type model (See Eq. (6)) if the coefficient, $|\Delta^c|^2$, of the Anderson-Higgs mass term for $a$ is sufficiently small the angular field $\chi$ fluctuates so strongly that it does not produce a mass gap for $a_\mu$ and $\langle e^{i\chi} \rangle = 0$, which corresponds to the region in the phase diagram characterized by a non-SC Nernst signal discussed above.

For a sufficiently large coefficient $|\Delta^c|^2$, on the contrary, we are in the broken symmetry phase; the fluctuations of $\chi$ are exponentially suppressed and $\langle e^{i\chi} \rangle \neq 0$ at $T = 0$ or there is quasi-condensation at $T > 0$ and the gauge field is gapped. One can prove that in this model a gapless gauge field is inconsistent with the coherence of holon pairs, i.e., coherent holon pairs cannot coexist with incoherent spinon pairs. Thus as soon as $e^{i\chi}$ (quasi-)condenses the same occurs to the hole pairs so that SC emerges. It follows that $T_c < T_{ps}$. To find an approximate formula for the critical temperature we proceed as follows: an estimate of the critical value of $\Delta_0^c$ can be derived from a formula given in Ref.[8] for the critical value of the coefficient of Anderson-Higgs mass term in Eq.(6). If one rescales the gauge field $a_\mu$ to have the standard...
coefficient $1/2$ for the Maxwell term, denoting by $q$ the charge of the $\phi$ field w.r.t. the rescaled $a_q$ and denoting by $\beta$ the coefficient of the Anderson-Higgs mass term, then such formula reads: $\beta_c \approx (3 - \frac{q^2}{4})^{-1}$, where $\beta_c$ denotes the critical value. The value $q = 0$ corresponds to a pure XY model; in our case (see Eq.(6)) $q = 2\sqrt{3\pi M}$, with $M = \sqrt{(m_s^2 - 2|\Delta|^2)}$ and $\beta \approx |\Delta|^2/(12\pi M)$, obtaining $(|\Delta|^2)_c \approx \frac{m_s^2}{2} - O(m_s^4)$. For the critical value of $|\Delta|$ the value of $M$ is quite small, hence $q$ almost vanishes and within this approximation the SC transition is essentially of XY-type. This implies also that the gauge contributions of holons which have been neglected above would be indeed strongly suppressed in a self-consistent way. In general one can see from Eq.(6) that in our approach a reduction of $M$, and hence of spinon kinetic energy, implies a reduction of the gauge fluctuations.

5. Comparison with experiments

In this Section we outline further physical consequences of the superconducting mechanism sketched above that can be compared with experimental data. For some of them the details are still work in progress. The appearance of two positive branches in the spinon dispersion relation for a suitable spinon-antispinon attraction mediated by gauge fluctuations (in particular those corresponding to the $Z_2$ subgroup left unbroken by the condensation of the SC pairs) induces a similar structure for the magnon dispersion around the AF wave vector [6], reminiscent of the hour-glass shape of spectrum found in neutron experiments[9], with an energy at the AF wave vector approximately twice the spinon gap $Jm_s \sim J(1 - 2\delta)\delta \ln \delta^{1/2}$. A key feature of our approach is an intimate relation between short-range AF and SC attraction, both coming from the same term in the representation of the $t - J$ model Eq.(3). A consequence is an intrinsic approximately linear relation between the energy of the magnon resonance and $T_c$ [3]. This feature qualitatively agrees with experiments[10]. The gauge field binding spinon and holon into a hole resonance becomes massive in the superconducting state; the lifetime of such resonance then becomes $T$-independent, because the $T$-dependent anomalous skin effect appearing in the normal state is suppressed by the mass. Therefore holes become coherent at the superconducting transition. The XY nature of the superconducting transition should affect the superfluid density, causing a non-BCS behavior. For $T$ above the superconducting transition, the scattering of the phase of the holon pairing field against holons produces in the holon Green function, lowering $T$ and moving away from the diagonals of the BZ, a gradual reduction of the spectral weight at small frequency and simultaneously the formation and increase of two peaks of intensity at larger frequency corresponding to SC-like excitations, precursors of true superconductivity. This affects the spectral weight of physical holes [6] and their transport properties [11], in a way comparable with experimental data on tunneling, ARPES and conductivity.

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