Adaptation Parameters of Time Series Models for Forecasting

S I Klevtsov and A V Maksimov

1First Institute of Radio Engineering Systems and Control, Southern Federal University, SFEDU, 44, Nekrasovsky str., Taganrog 347928, Russia

E-mail: sergkmps@mail.ru

Abstract. Prospects for using time series to predict changes in technical parameters in real time are considered. The task is to assess the trend dynamics of the parameter. Adaptive polynomial models of the first and second order, based on the method of multiple exponential smoothing, were selected for forecasting. The models have been modified to adapt to the peculiarities of the computing process in the microcontroller. The initial data, the acceleration values in three axes, were obtained using a three-axis accelerometer installed on the vehicle. Comparison of the forecasting results showed that the second-order adaptive polynomial model is generally more preferable from the point of view of the reduced error. Both models can be used to estimate the change in a parameter for an arbitrary number of prediction intervals. The efficiency of using the models for the forecasting problem largely depends on the determination of the adaptation parameters, such as the smoothing constant and the initial estimates of the coefficients of the time series model. The paper considers the features of the behavior of the models and defines the rules for the selection of adaptation parameters depending on the nature of the change in the technical parameter over time.

1. Introduction

The relevance of the task of assessing a technical parameter in real time is associated with the possibility of early detection of abnormal situations during the operation of a technical object and timely response to unacceptable changes. To predict the values of a technical parameter in real time in a microcontroller system using time series, it is necessary to take into account the features of information retrieval associated with digital signal processing [1, 2, 3]. The sampling step when taking data can be set small [4, 5]. Thus, the change in the parameter during several successive steps will be insignificant [5]. In this case, it is possible to use first and second order polynomial models [6, 7, 8] when constructing a time series model. A small discretization step will improve the forecasting accuracy due to the high density of the initial data on a limited time interval, which is important for the implementation of the adaptation mechanisms of the models. A decrease in the error is also associated with an increase in the accuracy of signal conversion in a microprocessor sensor [5].

Polynomial models of the first and second order form simple algorithms and are characterized by low computational costs for implementation [6, 7, 8]. Therefore, it is advisable to use these models to perform a predictive estimate of the values of the parameters of a technical object using a microcontroller system. In this case, the predictive assessment of the parameter change is carried out in the background and does not interfere with the performance of the main functions of the microcontroller system. In this regard, it is also advisable that the data on the parameter values coming from the sensor are not subjected to preliminary processing in the microcontroller, with the exception...
of rejection of emissions [5]. Deeper processing may require additional implementation time. The predicted values of parameters obtained as a result of the procedure can be used for an early preliminary assessment of trends in the state of a technical object.

This article discusses the features of adapting time series models and the results of their use to determine the predicted values of parameters.

2. Adaptive polynomial time series models

Let the values of the controlled parameter of the technical object \( y(t) \) be measured at discrete times with a constant step \( h \). As a result, we get an array \( Y = \{y_i\}_{i=0}^n \) of measured values of parameter \( y_i = y(t_i) \) at points \( t_0, t_1, \ldots, t_n \). \( t_i = t_{i-1} + h \). It is necessary to determine the value of the parameter \( y \) at points \( T + kh, k = 1, K \), where \( T \) is the current value of time, relative to which a forecast is made for \( k \)-steps or for a time interval \( \tau = kh \), called the lead time or forecasting horizon [7], and \( K \) is a number that determines the range forecasting.

We will consider the forecasting horizon for models starting from a value equal to the information retrieval step \( \tau = h \).

We will use adaptive polynomial models of the first and second order [6, 7] as models based on smoothing time series to approximate the change in the parameter \( y \) in time.

First-order polynomial model (hereinafter model 1) [7]:

\[
y(t) = a_1 + a_2(t)
\]  

Then the predicted value \( y \) at points \( \tau \):

\[
\bar{y}(T + \tau) = (2 + \frac{\alpha}{\beta})S_T - (1 + \frac{\alpha}{\beta})S_T^{[2]}
\]

where \( S_T \) and \( S_T^{[2]} \) are exponential averages, which are determined at time \( T \) by the formulas:

\[
S_T = \alpha y_T + \beta S_{T-1}, \quad S_T^{[2]} = \alpha S_T + \beta S_{T-1}^{[2]}
\]

\( \alpha \) - smoothing constant, which must be selected, \( \beta = 1 - \alpha \). In order to start the calculation process, you must set the initial values \( S_0 \) and \( S_0^{[2]} \):

\[
S_0 = \bar{a}_{1,0} - \frac{\beta}{\alpha} \cdot \bar{a}_{2,0}, \quad S_0^{[2]} = \bar{a}_{1,0} - \frac{2\beta}{\alpha} \cdot \bar{a}_{2,0}
\]

Here \( \bar{a}_{1,0} \) and \( \bar{a}_{2,0} \) are the initial estimates \( a_1 \) and \( a_2 \) in (1).

Second order polynomial model (hereinafter model 2) [7]:

\[
y(t) = a_1 + a_2(t) + \frac{1}{2}a_3t^2
\]  

Then the predicted value \( y \) at points \( \tau \):

\[
\bar{y}(T + \tau) = (6\beta^2 + (6 - 5\alpha)\alpha \tau + \alpha^2 \tau^2)\frac{S_T}{2\beta^2} - (6\beta^2 + 2(5 - 4\alpha)\alpha \tau + 2\alpha^2 \tau^2)\frac{S_T^{[2]}}{2\beta^2} +
\]

\[
(2\beta^2 + (4 - 3\alpha)\alpha \tau + \alpha^2 \tau^2)\frac{S_T^{[3]}}{2\beta^2}
\]

where \( S_T \), \( S_T^{[2]} \) and \( S_T^{[3]} \) are exponential averages:

\[
S_T = \alpha y_T + \beta S_{T-1}, \quad S_T^{[2]} = \alpha S_T + \beta S_{T-1}^{[2]}, \quad S_T^{[3]} = \alpha S_T^2 + \beta S_{T-1}^{[3]},
\]
α - smoothing constant, which must be selected, β = 1 - α. In order to start the calculation process, you must set the initial values $S_0$, $S_0^{[2]}$, $S_0^{[3]}$:

$$
S_0 = \frac{\beta}{\alpha} a_{1,0} - 2\alpha a_{2,0} + \frac{\beta(2-\alpha)}{2\alpha^2} a_{3,0},
S_0^{[2]} = \frac{\beta}{\alpha} a_{1,0} - 2\alpha a_{2,0} + \frac{\beta(3-2\alpha)}{2\alpha^2} a_{3,0},
S_0^{[3]} = \frac{3\beta}{\alpha} a_{1,0} - 3\alpha a_{2,0} + \frac{3\beta(4-3\alpha)}{2\alpha^2} a_{3,0}.
$$

Here $a_{1,0}$, $a_{2,0}$, $a_{3,0}$ are the initial estimates $a_1$, $a_2$, $a_3$ in (2).

3. Setting up adaptive polynomial time series models

For effective use of models 1 and 2, it is necessary to determine the adaptation parameters, i.e. the smoothing constant and the initial estimates of the coefficients of the models in expressions (1) and (2) so that the adaptation period is insignificant for the forecasting procedure.

To determine the influence of adaptation parameters on the nature of the adaptation period, polynomials of the first and second degrees were considered as the initial dependencies:

$$
x_1(t) = b_1 + b_2 t,
$$

$$
x_2(t) = b_1 + b_2 t + \frac{1}{2} b_3 t^2.
$$

For the linear dependence (3), when the coefficients $a_{1,0}$, $a_{2,0}$ of models 1 and 2 were equal to the corresponding coefficients $b_1$, $b_2$, a significant adaptation period was recorded (Figure 1). Coefficient $a_{3,0} = 0.05$, $\alpha = 0.1$.

![Graph of the acceleration change and the predicted acceleration value for the initial dependence in the form of a first-order polynomial when the coefficients of the dependences and models.](image)

**Figure 1.** Graphs of the acceleration change and the predicted acceleration value for the initial dependence in the form of a first-order polynomial when the coefficients of the dependences and models.

The transient process in the case of a linear dependence in model 1 is greater than in model 2 at small $a_{3,0}$. A decrease in the coefficient $a_{3,0}$ does not lead to significant changes. When the coefficient $a_{3,0}$ ($a_{3,0} = 0.05$) of model 2 is increased, the result is reversed. There is a value of the coefficient $a_{3,0}$ such when models 1 and 2 practically merge. In this case, $a_{3,0} = 0.2$.

Changing the coefficient $a_{1,0}$ of models 1 and 2 does not make significant changes in the dynamics of predictive dependencies. The influence of the coefficient $a_{2,0}$ is more significant (Figure 2). With an
increase in coefficient $\bar{a}_{2,0}$ (Figure 2), the amplitude of model 2 within the transition period is greater than that of model 1. A decrease in coefficient leads to the inverse relationship, but the amplitudes of the models in this case are less.

![Figure 2](image)

**Figure 2.** Graphs of the acceleration change and the predicted acceleration value for the initial dependence in the form of a first order polynomial with a change in the coefficient $\bar{a}_{2,0}$.

For the quadratic dependence (4), when the coefficients $\bar{a}_{1,0}$, $\bar{a}_{2,0}$ of model 1 and $\bar{a}_{1,0}$, $\bar{a}_{2,0}$, $\bar{a}_{3,0}$ of model 2 were equal to the corresponding coefficients $b_1$, $b_2$, $b_3$, a significant adaptation period was also recorded. Coefficient $\alpha = 0.1$.

The transient process in the case of a quadratic dependence, in contrast to a linear dependence, is greater for model 2 than for model 1. A decrease in the coefficient $\bar{a}_{3,0}$ reduces the difference in amplitudes, at a certain value of $\bar{a}_{3,0}$ ($\bar{a}_{3,0} = 0.2$) the curves merge, and then the situation changes to the opposite, which does not lead to significant changes. When the coefficient $\bar{a}_{3,0}$ ($\bar{a}_{3,0} = 0.5$) of model 2 is increased, the result is reversed. There is a factor value of $\bar{a}_{3,0}$ such that models 1 and 2 practically merge. In this case $\bar{a}_{3,0} = 0.2$.

With a decrease in the coefficient $\bar{a}_{2,0}$, the transient process is reduced, both models perform the prediction of the initial dependence with sufficient accuracy.

The forecasting accuracy and the adaptation period of the models also significantly depend on the parameter $\alpha$.

4. **Simulation results**

The study of the possibility of using models 1 and 2 to predict changes in the parameter of an object was carried out on the basis of data on the change in acceleration taken from a 3-axis accelerometer installed on a car [5]. The initial data are characterized by a significant scatter of values. The data acquisition step was $h = 0.015625\text{sec}$. Simulations were carried out at various time points.

Figure 3 and Figure 4 show the results of forecasting using adaptive polynomial time series models, consisting of the acceleration values measured by the accelerometer along the X axis and the prediction error. Results are shown for a portion with a sharp change in time series.
In the study of models 1 and 2, in order to exclude the adaptation period, the initial estimates of the approximation coefficients $\tilde{a}_{1,0}$, $\tilde{a}_{2,0}$, and $\tilde{a}_{3,0}$ were specified taking into account the above-defined tuning features. To minimize the prediction error, the smoothing parameter $\alpha$ was selected, since the prediction error outside the adaptation area is largely determined by the choice of smoothing constant $\alpha$ and the consistency between the real dependence of the parameter on time and the selected time series model [7].

Despite the strong fluctuations in the time series (Figure 3), both models in the forecasting process demonstrate a high degree of repeatability of the shape of the initial time series. However, the absence of preliminary data processing affects the forecasting error. The error in the area with high instability of the time series is quite high (Figure 4).

In the area with insignificant fluctuations, the degree of repeatability of the shape of the initial time series is generally higher, which was expected. Nevertheless, the forecast of the behavior of the initial time series in the case of using models 1 and 2 is delayed by approximately 3h. Sharp fluctuations in the time series are more accurately tracked by model 1.
The forecasting error in the case of insignificant fluctuations in the time series is more than two times less than for areas with abrupt changes. If we estimate the forecasting errors, then model 2 is characterized by a lower error in comparison with model 1.

The average value of the absolute forecasting error over the modeling area is \( \sim 0.003g \) for model 1 and \( \sim 0.002g \) for model 2, with the exception of a few areas where high instability was observed.

5. Conclusion
Analysis of the graphs shows that with increasing \( \alpha \), the predictive curve more accurately reproduces the shape of the experimental dependence. However, for each dependence there is a certain value of \( \alpha \), which determines the boundary of the model's capabilities in terms of forecasting accuracy.

According to the simulation results, in areas with a sharp increase in the parameter, a model with a high value of the smoothing constant gives a smaller prediction error and, conversely, in the case of small fluctuations in the parameter, a decrease in the prediction error is associated with a decrease in the smoothing constant.

The averaged value of the absolute prediction error over the entire modeling area was \( \sim 0.003g \) for model 1 and \( \sim 0.002g \) for model 2. In some areas of a jump-like change in the initial time series, the maximum absolute error was \( \Delta y = 0.03g \) for the first-order adaptive polynomial model and \( \Delta y = 0.025g \) for a second-order adaptive polynomial model.

Comparison of the forecasting results carried out on the same experimental data set for one forecasting step shows that the second-order model has a higher accuracy compared to the first-order model.

Thus, the presented adaptive polynomial time series models can be used to predict parameter changes in real time. The forecasting results are of interest for carrying out a preliminary assessment of the state of a technical object. Both models are practically equivalent in terms of ease of implementation. If a microprocessor monitoring system is implemented in a microcontroller, the forecasting process can be performed in the background. In this case, the data are analyzed without preliminary processing, except for the rejection of outliers.

6. References
[1] Sun Lihua, Guo Yingjun and Ran Haichao 2010 A New Method of Early Real-Time Fault Diagnosis for Technical Process in Proc. 2010 Int. Conf. Electrical and Control Engineering (ICECE) pp 4912 - 4915
[2] Vovk S, Ginis L 2012 Modeling and forecasting of transitions between levels of hierarchies in Difficult formalized systems European Researcher vol. 20 5-1 pp 541 – 545
[3] Darkhovsky B, Brodsky B 2013 Asymptotically Optimal Methods of Early Change-point Detection Sequential Analysis 32 pp 158-181
[4] Sergienko A 2002 Digital signal processing (SPb.: Peter)
[5] Klevtsov S 2010 Preliminary assessment of the state of a set of parameters of a technical object using an intelligent microprocessor module Izvestia SFedU. Technical science vol. 106 pp 43-48
[6] George E P Box, Gwilym M Jenkins, Gregory C Reinsel, Greta M Ljung 2015 Time series analysis: forecasting and control A JOHN WILEY & SONS, INC. (USA)
[7] Lukashin Yu P 2003 Adaptive methods of short-term forecasting of time series (M : Finance and statistics)
[8] Brillinger D R 1980 Time series Data processing and theory (M : Mir)