The relationship between resonance scattering and the formation of an acoustojet under the interaction of ultrasound with a dielectric sphere immersed in water

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Abstract. We demonstrated for the first time the influence of the main parameters of dielectric spherical cavity, immersed in water, to transformation of whispering gallery mode into acoustojet (acoustic jets) by interaction of acoustic plane wave scatterer. It has been shown that the relative speed of sound in the material, the relative density of the material and the radius of particle significantly affect the condition for the formation of WGM resonance. However, the "more sensitive" parameter is the relative speed of sound.

1. Introduction

It is well known that whispering gallery mode (WGM) originally were introduced almost a century ago for sound wave propagating close to the cylindrical wall in St. Paul’s Cathedral, London [1-3], where the acoustical modes were partially confined due to the suppression of the wave diffraction by the sound reflection from the curved dome walls. Since WGM is a morphology-dependent phenomenon, the radius of the resonator determines the effective volumes and distribution of the modes [1-3].

The theory of open resonators yields an abnormally high radiative emission Q-factor for lossless microspheres. It must be of the order of 1073 at \( \lambda =600\text{nm} \) for a droplet of water with a 50-\( \mu \text{m} \) radius [4]. But Q-factor is more than 60 orders of magnitude less because of lights cattering and absorption.

The electromagnetic wave and laser beam interaction with dielectric microsphere immersed in water were considered in several papers [5-7].

The features of resonant scattering of an ultrasonic wave on an air spherical cavity in water were considered in [8].

In Ref. [9] and Ref. [10] studied the influence of longitudinal \( c_l = \sqrt{(\lambda + 2\mu)/\rho} \) and transverse \( c_t = \sqrt{\mu/\rho} \) velocities in an isotropic scatterer on the characteristics of peripheral waves. It was
found that the velocity \( c_l \) influences these waves weakly, while the velocity \( c_t \) influences them strongly. It was demonstrated that increasing the transverse velocity \( c_t \) strongly shifts the position of the resonances on the ka-axis to the right, but increasing the longitudinal velocity \( c_l \) slightly shifts the position of the resonances on the ka-axis to the right.

It also was shown that the incident plane wave generates WGM waves of transverse type in the spherically isotropic sphere [11].

In [12] the idea of acoustojet formation as an analog of photonic jet [13] was offered and discussed at the first time. In this paper we briefly consider the features of the transition from resonant scattering of an ultrasonic wave on the dielectric sphere to the onset of formation of an acoustojet.

2. Simulation results

To simulate the problem we examine multiply acoustic resonant wave scattering at penetrable spherical particle [13]. The relevance of penetrable scatterers [14] in acoustics is recovering a renewed interest, where special interest appears for the case of low contrast systems [15]. By penetrable [14], as in [16], we mean that pressure waves may travel through the particle, that is, the particle is not rigid. The transmission loss as the wave travels through the particle is neglected. The problem of scattering on homogeneous limited area, placed inside the other homogeneous area, in the first approximation in the harmonic case, can be solved based on the Helmholtz equation [17]:

\[
(\nabla^2 + k^2)u(x) = 0,
\]

where the pressure \( p \) and the velocity \( u \) are:

\[
p(x,t) = R[u(x)e^{i\omega t}]
\]

and the wave number \( k \) is \( k = 2\pi/\lambda = \omega/c \). Here \( (\nabla) \) is Laplace operator, \( R \) is a real part of \( |u| \). For permeable spherical particles represent the total field \( u \) on the boundary as the sum of three fields: the incident field of plane wave \( u_i \), the scattered field \( u_s \) and the field inside the particle \( u_d \). Approximately can be written:

\[
u_i + u_s = u_d.
\]

It is interesting to note that sound focusing devices have been known since the 17th century. So, Athanasius Kircher described the device for focusing the sound - an acoustic horn - back in 1673 [18]. The first experiments with a lens filled with carbon dioxide in an elastic shell, as in [16], were described by Sondhaus in 1852 [19]. In the works by John Tyndall [20], an acoustic lens is described that is a thin balloon filled with some gas heavier than air, for example, carbon dioxide. And the first detailed qualitative work on the sound lens was made and reported, simultaneously with the lens demonstration, at the meeting of the Russian Physical Society in 1889 by N.A. Gezeus [21].

The simulation self-developed program was coded in FORTRAN following the recommendations of the works [16, 22]. It has to be noted that the synthesized a microsphere approximated the ideal condition for a water solvent (the most important solvent for biosensing applications). Preliminary simulation results of resonant scalar acoustic wave scattering at penetrable sphere are as follows.

For investigation of resonant scattering of ultrasound in dielectric spherical cavity immersed in water we have selected Rexolite\textsuperscript{©} that closely match the impedance of water [23, 24], very small water absorption (24 hrs less than 0.08\%) and dissipation factor of 0.00012 at 1 MHz.

The main characteristics some of interesting materials are shown in the Table 1.

| Material   | Sound velocity | Density | Acoustic impedance |
|------------|----------------|---------|--------------------|
| Water      | 1490           | 0.998   | 1.487              |
| Rexolite   | 2337           | 1.04    | 2.430              |
| Ebonite    | 2400           | 1.2     | 2.880              |
| Plexiglas  | 2670           | 1.18    | 3.150              |
Let's consider briefly the influence of such parameters as the speed of sound in the particle material, density, acoustic impedance and radius of particle to the transition from resonant scattering to acoustojet formation.

In the Figure 1 below the results of simulation are shown. In the Figure 1a the resonant scattering with WGM is shown for the dielectric particle with parameters: radius of particle $R=4\lambda$, (at frequency of 1 MHz in water), relative density contrast is 1.0402 and the speed of sound contrast is 1.570. The WGM mode is clear visible. When we change the radius of the particle to 0.0005 then the WGM modes are destroyed, the intensity falls catastrophically and an acoustojet begins to form (Figure 1b).

![Penetrable Sphere Near Field, a=4.0](image)

(a) ![Penetrable Sphere Near Field, a=4.0](image)

(b) ![Penetrable Sphere Near Field, a=4.0](image)

(c) ![Penetrable Sphere Near Field, a=4.0](image)

(d)

**Figure 1.** Resonant scattering of acoustic wave on Rexolite© sphere immersed in water. The parameters of sphere materials are (relative density-relative speed of sound-radius): 1.0402-1.570-4.00 (a, acoustic resonance), 1.0402 - 1.570 - 4.0005 (b), 1.0405 - 1.570 – 4.0 (c), 1.0402 - 1.57001 – 4.0 (d).

On the other hand let’s fixed the radius of particle and speed of sound in particle material, but change a little the density contrast. In the Fig.1c the transformation of WGM mode and acoustojet formation is shown for this situation. Now let’s fixed the radius of particle and the density of particle material, but change a little speed of sound contrast. For this variant the transformation of WGM mode and acoustojet formation is shown in the figure 1d.

More detailed information according maximal intensity changes with variation of density and speed of sound contrast are shown in the figure 2.
Figure 2a. Dependences of maximal intensity vs relative density of sphere material for relative speed of sound as 1.570 and radius of sphere 4.00.

Figure 2b. Dependences of maximal intensity vs speed of sound in sphere material for relative density of 1.0402 and radius of sphere 4.0.

The analysis of simulation results shown that the spatial structure of the full acoustic field is defined by the type of the elastic wave, excited in the surface of particle.

The change in the relative speed of sound by \(10^{-5}\) leads to a decrease in intensity by 86 times, and the contrast of the density of the material of the particle and the medium by \(10^{-4}\) - by a factor of 7. Also the change in radius of particle by \(5 \times 10^{-5}\) leads to a decrease in intensity by 190 times.

Thus, all of these parameters - the relative speed of sound in the material, the relative density of the material and the radius - significantly affect the condition for the formation of WGM resonance. However, the "more sensitive" parameter is the relative speed of sound.

3. Conclusion

The transition of WGM into a regime for the formation of a acoustojet for the dielectric sphere immersed in water by interaction with an acoustic wave is studied. In the resonant mode of scattering of an ultrasonic wave on the dielectric sphere in water (with the formation of WGM), the formation of an acoustojet does not occur.
All three main characteristics - the contrasts of the speed of sound in the material of the particle and its density, as well as the acoustic impedance and the particle radius, affect the stability of the formation of WGM. However, the most critical parameter for the formation of an acoustojet is the contrast of the speed of sound.

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