Electronic heat current fluctuations in a quantum dot

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The fluctuations of heat current in a quantum dot coupled to reservoirs of electrons are calculated at finite frequency, voltage and temperature using the non-equilibrium Green function technique. A generic expression of the non-symmetrized heat noise is obtained in the form of an integral on energy of four contributions, each of which including transmission amplitudes, electron-hole pair distribution functions and energy difference factors. Using this expression, the effect of the couplings between the quantum dot and the reservoirs, whether weak or strong, symmetrical or asymmetrical, as well as the effect of a temperature gradient between the reservoirs are numerically studied. New features of heat noise are highlighted and discussed for both a quantum dot in equilibrium and a quantum dot out-of-equilibrium.

In quantum devices the heat fluctuates in time for several reasons: the first one is related to the presence of thermal agitation at finite temperature, the second one to the fact that the device interacts with its electromagnetic environment emitting or absorbing energy via phonons or photons, and the third one to the probabilistic nature of particle transfer in quantum systems. The characterization of these heat fluctuations brings a variety of information on the energy dissipation, on the presence of coherence and entanglement in open quantum systems and on the higher-order cumulants of the charge counting statistics. Moreover, they reveal features that are not visible in the charge noise such as for example the signature of a crossover from Coulomb blockade to Kondo physics in the energy fluctuations.

In the case of on-demand single electron sources, the heat fluctuates whereas the charge emission is noiseless. Up to now only the temperature fluctuations, related to the energy fluctuations, have been measured but it exists several proposals for the measurement of heat fluctuations. With the fast progress in the heat measurement techniques in nanosystems, one can anticipate that this will be achievable in a foreseeable future. The heat transport in quantum devices in itself is well controlled with notably the experimental confirmation of the existence of a quantum of thermal conductance, and the highlighting of heat Coulomb blockade effect.

The issues raised by such studies are also of fundamental interest. The question of the generalization of the fluctuation-dissipation theorem to heat transport has been addressed as well as the verification of the fluctuation theorem which is a microscopic extension of the second law of thermodynamics. The statistics of heat exchange in a driven open quantum system have been studied as well as the statistics of work for a two-level system in the presence of dissipation.

Among the theoretical approaches used to study the heat fluctuations in quantum devices one can cite the Landauer-Büttiker formalism, the non-equilibrium Schwinger-Keldysh Green function technique, the circuit theory, the Boltzmann-Langevin approach and the inchworm quantum Monte Carlo method. The systems in question are either molecular junctions, quantum wires, mesoscopic constrictions, quantum dots, double quantum dots or qubits. In these works, the generating function for the heat full counting statistics has been determined as well as the symmetrized finite-frequency heat noise as well as for symmetrical couplings between the dot and the reservoirs of electrons. The objective of the present work is double, it aims first to generalize the calculation of the non-symmetrized finite-frequency heat noise to the case of asymmetrical couplings, looking to both auto-correlators and cross-correlators, and second to highlight the main features in the heat noise spectrum. Only the electronic contribution to the heat noise is considered in this work.

The hamiltonian describing a non-interacting QD connected to left (L) and right (R) reservoirs of electrons is given by the Anderson hamiltonian: 

\[ H = \sum_{\alpha=L,R} \sum_{k \epsilon \alpha} \varepsilon_{\alpha k} c_{\alpha k}^\dagger c_{\alpha k} + \varepsilon_0 d^d + \sum_{\alpha=L,R} \sum_{k \epsilon \alpha} (V_{\alpha k} c_{\alpha k}^\dagger d + h.c.), \]

where \( c_{\alpha k}^\dagger \) (\( c_{\alpha k} \)) are the creation (annihilation) operators associated to the reservoir \( \alpha \) (respectively the QD). The energies \( \varepsilon_{\alpha k}, \varepsilon_0 \) and \( V_{\alpha k} \) are respectively the energy of the electrons in the reservoir \( \alpha \), the discrete energy level of the QD, and the hopping energy between the reservoirs and the QD. The retarded Green function associated to the QD connected to the reservoirs is given in the flat wide-band limit by 

\[ G_r(\varepsilon) = \langle \varepsilon - \varepsilon_0 + i(\Gamma_L + \Gamma_R)/2 \rangle^{-1}, \]

where \( \Gamma_\alpha = 2\pi \rho_\alpha |V_\alpha|^2 \) is the coupling between the QD and the reservoir \( \alpha \) assuming that the density of states \( \rho_\alpha \) and \( V_\alpha \equiv V_{\alpha k} \) are energy independent.

The heat noise is defined as the Fourier transform of the
non-symmetrized correlator of the heat currents at two different times: $S_{\alpha\beta}^{\text{heat}}(\omega) = \int_{-\infty}^{\infty} \langle \Delta J_\alpha(t) \Delta J_\beta(0) \rangle e^{-i\omega t} dt$, where $\Delta J_\alpha(t) = J_\alpha(t) - \langle J_\alpha \rangle$. The heat current operator is given by $J_\alpha(t) = -\mathcal{H}_\alpha + \mu_\alpha \mathcal{N}_\alpha$, where $\mathcal{H}_\alpha = \sum_{k\in\alpha} \varepsilon_{ak} c_{\alpha k}^\dagger c_{\alpha k}$ is the hamiltonian of the uncoupled reservoir $\alpha$ and $N_\alpha = \sum_{k\in\alpha} c_{\alpha k}^\dagger c_{\alpha k}$ is the operator number of electrons in the reservoir $\alpha$. The calculation of the non-symmetrized finite-frequency heat noise is performed in the framework of the non-equilibrium Green function technique. It yields to

$$S_{\alpha\beta}^{\text{heat}}(\omega) = \frac{1}{\hbar} \sum_{\gamma,\delta} \int_{-\infty}^{\infty} d\varepsilon M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \omega) f_\gamma^\ast(\varepsilon) f_\delta^\ast(\varepsilon - \hbar \omega) \quad (1)$$

where $f_\gamma^\ast(\varepsilon) = (1 + \exp((\varepsilon - \mu_\gamma)/k_B T_\gamma))^{-1}$ and $f_\delta^\ast(\varepsilon) = 1 - f_\gamma^\ast(\varepsilon)$ are the Fermi-Dirac distributions for the electrons in the reservoir $\gamma$ and the holes in the reservoir $\delta$. $\mu_\gamma$ and $T_\gamma$ are respectively the chemical potential and the temperature in the reservoir $\gamma$. The matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \omega)$ are listed in Table I. The result of Eq. (1) is new and is applicable at any frequency $\omega$, temperatures $T_{L,R}$, voltage $V$ and couplings $\Gamma_{L,R}$. It generalizes the results of Ref. 40 to arbitrary couplings between the QD and the reservoirs. The expressions of the elements for the matrix $M$ reduce to the ones of the matrix entering in the expression of the charge noise $S_{\alpha\beta}^{\text{charge}}(\omega)$ of Refs. 49, 50 providing that the factor $\mathcal{E}_{\alpha}(\varepsilon)$ is replaced by the value 1. One remarks that three of such factor enter in the expression of the heat noise: $\mathcal{E}_{\alpha}(\varepsilon) = \varepsilon - \mu_\alpha$, the energy of the electron in the reservoir $\alpha$, $\mathcal{E}_{\alpha}(\varepsilon - \hbar \omega) = \varepsilon - \mu_\alpha - \hbar \omega$, the energy of the hole in the reservoir $\alpha$, and $\mathcal{E}_{\alpha}(\varepsilon - \hbar \omega/2) = \varepsilon - \mu_\alpha - \hbar \omega/2$, the average energy of the electron-hole pair in the reservoir $\alpha$. There factors are related to the energy exchanged with the electromagnetic environment surrounding the QD during the various processes contributing to the heat noise which involve either an electron, a hole, or an electron-hole pair.

| $M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \omega)$ | $\gamma = \delta = L$ | $\gamma = \delta = R$ | $\gamma = L, \delta = R$ | $\gamma = R, \delta = L$ |
|---|---|---|---|---|
| $\alpha = L, \beta = L$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha) t_{LL}(\varepsilon)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ |
| $\alpha = L, \beta = R$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha) t_{RL}(\varepsilon)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ |
| $\alpha = R, \beta = L$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha) t_{LR}(\varepsilon)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ |
| $\alpha = R, \beta = R$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha) t_{RR}(\varepsilon)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ | $\mathcal{E}_{\alpha}(\varepsilon - \mu_\alpha)$ |

TABLE I: Expressions of the matrix elements $M_{\alpha\beta}^{\gamma\delta}(\varepsilon, \omega)$ appearing in the finite-frequency heat noise of Eq. (1), setting $\hbar = 1$, where $t_{\alpha\beta}(\varepsilon) = i\sqrt{\Gamma_{\alpha}\Gamma_{\beta}} G^\ast(\varepsilon)$ is the transmission amplitude, $T_{\alpha\beta}(\varepsilon) = |t_{\alpha\beta}(\varepsilon)|^2$, the transmission coefficient, and $\mathcal{E}_{\alpha}(\varepsilon) = \varepsilon - \mu_\alpha$, the difference between the energy $\varepsilon$ of the particle and the chemical potential in the reservoir $\alpha$.

Before exploiting the result of Eq. (1), one checks that it gives the expected behavior for heat noise within known limits. At zero-frequency $\omega = 0$, symmetrical couplings $\Gamma_{L,R} = \Gamma$ with $T(\varepsilon) = \Gamma^2 G^\ast(\varepsilon) G^\ast(\varepsilon)$, and using the optical theorem which holds for a non-interacting QD, meaning that one has $t(\varepsilon) + t^\ast(\varepsilon) = 2T(\varepsilon)$, Eq. (1) leads
for the auto-correlators ($\alpha = \beta$) to the expression\[48\]

$$S_{\alpha\alpha}^{\text{heat}}(0) = \frac{1}{h} \int_{-\infty}^{\infty} d\varepsilon (\varepsilon - \mu_\alpha)^2$$

$$\times \left[ T(\varepsilon)(1 - T(\varepsilon))(f^{\text{eq}}_\alpha(\varepsilon) - f^{\text{eq}}_\alpha(\varepsilon))^2$$

$$+ T(\varepsilon)(f^{\text{eq}}_\alpha(\varepsilon)f^{\text{eq}}_\alpha(\varepsilon) - f^{\text{eq}}_\alpha(\varepsilon)) \right]$$

(2)

in agreement with the results of Refs. [4, 35, 41]. The index $\alpha$ takes the value $R$ for $\alpha = L$ and the value $L$ for $\alpha = R$. The last line in Eq. (2) corresponds to the equilibrium heat noise $S_{\alpha\alpha}^{\text{heat}}$ (Johnson-Nyquist) which can be thought of as a function of the thermal conductance $K_\alpha = \partial (J_\alpha)/\partial T_\alpha$ through the relation $S_{\alpha\alpha}^{\text{heat}} = k_B T_\alpha^2 K_\alpha + k_B T_\alpha^2 K_R$, in perfect agreement with Refs. [35, 45, 51].

One reminds that the equilibrium charge noise $S_{\alpha\alpha}^{\text{charge}}$ is related to the electrical conductance through the relation $S_{\alpha\alpha}^{\text{charge}} = k_B T_\alpha G_\alpha + k_B T_\alpha G_R$ with $G_\alpha = \varepsilon \partial (I_\alpha)/\partial \mu_\alpha$, $(I_\alpha)$ being the electrical current associated to the reservoir $\alpha$. The Johnson-Nyquist heat and charge noises are displayed in Figs. [1](a) and (b) as a function of the dot energy level $\varepsilon_0$ of the QD. In a certain interval of values for the couplings $\Gamma_L$ and $\Gamma_R$, $S_{\alpha\alpha}^{\text{heat}}$ shows a double peak whereas a single one is observed in $S_{\alpha\alpha}^{\text{charge}}$. Indeed, at equilibrium the fluctuations of charge are maximal when the energy level of the QD are aligned with the chemical potentials, i.e. for $\varepsilon_0 = 0$ when $\mu_{L,R} = 0$, since the transfer of charges costs no energy. It leads to a local minimum in the heat noise at $\varepsilon_0 = 0$. For increasing values of $|\varepsilon_0|$, the heat noise starts to increase because the transfer of charge costs energy in that case. Then it finally decreases and converges to zero due to the fact that the probability for the charge to be transferred through the dot vanishes at high $|\varepsilon_0|$. When the two peaks in $S_{\alpha\alpha}^{\text{heat}}$ are present, their positions are at most $\varepsilon_0 \approx \pm 2.5 k_B T$ in line with Ref. [52] where such a double peak structure has been predicted in the thermal conductance of a QD. To highlight the condition for having such a double peak in $S_{\alpha\alpha}^{\text{heat}}$, one plots in Figs. [1](c) and (d) the distance $\Delta \varepsilon_0$ separating them as a function of temperature $T$ and coupling $\Gamma_L$ for both symmetrical and asymmetrical couplings: it shows unequivocally that the condition is $\Gamma_L + \Gamma_R \lesssim 8 k_B T$. These results could be checked experimentally since at equilibrium the heat noise is proportional to the thermal conductance.

The heat noise is sensitive to the fact that the system is driven out-of-equilibrium either by applying a
voltage bias or a temperature gradient, or by considering the noise at finite frequency. In Fig. 3 are plotted the heat and charge noises as a function of frequency for both negative (absorption noise) and positive (emission noise) frequencies. Both the heat noise and the charge noise show a symmetrical spectrum. The auto-correlators $S_{\alpha \alpha}^{\text{heat}}(\omega)$ and $S_{\alpha \alpha}^{\text{charge}}(\omega)$ are positive as required, whereas the real part of the cross-correlators $S_{LR}^{\text{heat}}(\omega)$ and $S_{LR}^{\text{charge}}(\omega)$ can change theirs signs under the influence of frequency or voltage. At zero-frequency, the cross-correlators take real values and the equality $S_{LR}^{\text{charge}}(0) = -S_{LR}^{\text{heat}}(0)$ is guaranteed due to charge conservation. Since the heat noise in a quantum system measures the exchange of energy with the electromagnetic environment, the heat is not a conserved quantity, accordingly one has $S_{LL}^{\text{heat}}(0) \neq -S_{LR}^{\text{heat}}(0)$ as observed in Figs. 3(a) and (c). It implies that the total heat noise summed over the reservoirs is non-zero: $\sum_{\alpha, \beta} S_{\alpha \beta}^{\text{heat}}(0) \neq 0$, contrary to what one has for the total charge noise where $\sum_{\alpha, \beta} S_{\alpha \beta}^{\text{charge}}(0) = 0$. The explicit expression for the total heat noise can be determined from Eq. (1) and Tab. I in the limit of zero temperature and energy-independent transmission coefficient $T$. One gets $\sum_{\alpha, \beta} S_{\alpha \beta}^{\text{heat}}(0) = |eV|^3 T(1 - T)/\hbar$ at zero-frequency but finite-voltage, and $\sum_{\alpha, \beta} S_{\alpha \beta}^{\text{heat}}(\omega) = 4|eV|^3 \Theta(-\omega) T(1 - T)/h$ at zero-voltage but finite-frequency, where $\Theta$ is the Heaviside function. Thus, the heat noise varies as a power law with an exponent three of the highest characteristic energy of system, in agreement with Ref. 31. For energy-dependent transmission amplitude $t_{\alpha \beta}(\epsilon)$ and coefficient $\epsilon_{\alpha \beta}(\epsilon)$, such a power law behavior is reminiscent as one can see in Fig. 3(a) (blue curve) with a heat noise proportional to $|\omega|^3$ at negative frequency and a vanishing heat noise at positive frequency. When the voltage $V$ increases a dip appears in $S_{LL}^{\text{heat}}(\omega)$ and $S_{LR}^{\text{charge}}(\omega)$ at zero-frequency whereas a peak appears in $\text{Re}\{S_{LR}^{\text{heat}}(\omega)\}$.
At higher voltage (red curve in Fig. 2(b)), the charge noise $S_{\text{charge}}^{\text{heat/charge}}(\omega)$ becomes constant with the frequency excepted close to the central dip.

The application of a temperature gradient $\Delta T = T_L - T_R$ between the reservoirs strongly affects the noise signal. The derivative according to the voltage $V$ of the heat and charge noises are plotted in Figs. 3 and 4 as a function of $V$ for several values of $\Delta T$. The choice to focus on the noise derivative rather the noise itself is made in order to remove the non-relevant contributions like the equilibrium noise already studied in Fig. 1. The curves are displayed at positive voltage only since the noise derivatives are odd functions with $V$. They show the curves for the derivatives of the auto-correlators and of the real part of the cross-correlators, $S_{\text{heat/charge}}^{\text{heat/charge}}(\omega)$ and $S_{\text{heat/charge}}^{\text{heat/charge}}(\omega)$ being complex conjugate. At $\Delta T = 0$ and symmetrical couplings ($\Gamma_L = \Gamma_R$), the curves for $dS_{LL}^{\text{heat}}(\omega)/dV$ and $dS_{LR}^{\text{heat}}(\omega)/dV$ coincide (see the blue curves in Figs. 3(a) and (c)) but they differ at $\Delta T \neq 0$. At $\Delta T = 0$, a discontinuity appears in the heat and charge noises when the voltage is equal to the frequency: $\epsilon V = \hbar \omega$. It is related to the fact that at low temperature a QD can not emit energy at voltage smaller than the frequency, knowing that one takes $\hbar \omega = 78$ GHz $= 0.32$ meV here. This discontinuity disappears when $\Delta T$ increases. At large $\Delta T$, when $T_L$ is close to the room temperature, $dS_{LL}^{\text{heat}}(\omega)/dV$ becomes linear in $V$ (see the red curve in Fig. 2(b)) meaning that $S_{\text{heat}}^{\text{heat}}(\omega)$ is a quadratic function in $V$ in that limit. Moreover, at increasing $\Delta T$ the curve for the real part of the heat cross-correlator derivative $\text{Re}[dS_{LR}^{\text{heat}}(\omega)/dV]$ converge towards the curve for $dS_{LR}^{\text{heat}}(\omega)/dV$ (compare the red curves in Figs. 3(c) and (e)), a behavior not observed for charge correlators. For asymmetrical couplings ($\Gamma_L \neq \Gamma_R$), one observes a reduction in the dispersion of the curves $dS_{LR}^{\text{heat}}(\omega)/dV$ and $\text{Re}[dS_{LR}^{\text{heat}}(\omega)/dV]$ obtained at various $\Delta T$ (compare Figs. 3(c) and (e)) to Figs. 3(c) and (e)) and the quadratic variation with $V$ is still apparent at high $\Delta T$.

In summary, the study of the electronic heat noise reveals several features that are not visible in the charge noise. At equilibrium, the Johnson-Nyquist heat noise plotted as a function of the QD energy level $\epsilon_0$ shows a double peak structure provided that $\sum \Gamma_\alpha \leq 8k_B T$. The maximum distance between the two peaks is given by $\Delta \epsilon_0 \approx 5k_B T$. At finite frequency, low voltage and temperature, the heat noise varies as $|\omega|^3$ at negative frequency and vanishes at positive frequencies for both energy-independent or energy-dependent transmission coefficient. In the former case, at finite voltage, low frequency and temperature, the heat noise varies as $|V|^3$. In the latter case, for a large temperature gradient $\Delta T$ between the reservoirs, the heat auto-correlator associated to the cold reservoir and the real part of the heat cross-correlators become quadratic in $V$, while it is not the case for the heat auto-correlator associated to the hot reservoir. A direct extension of this work is the determination of the heat noise in an interacting QD using the theory developed in Ref. 50 to calculate the charge noise.

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