Thermal ground state for pure SU(2) Yang-Mills thermodynamics

Francesco Giacosa
Institute for Theoretical Physics, Goethe University, Max-von-Laue-Str. 1, Frankfurt am Main, Germany
e-mail: giacosa@th.physik.uni-frankfurt.de

In this proceeding the emergence of a composite, adjoint-scalar field as an average over (trivial holonomy) calorons and anti-calorons is reviewed. This composite field acts as a background field to the dynamics of perturbative gluons, to which it is coupled via an effective, gauge invariant Lagrangian valid for temperatures above the deconfinement phase transition. Moreover a Higgs mechanism is induced by the composite field: two gluons acquire a quasi-particle thermal mass.

On the phenomenological side the composite field acts as a bag pressure which shows a linear dependence on the temperature. As a result the linear rise with temperature of the trace anomaly is obtained and is compared to recent lattice studies.

8th Conference Quark Confinement and the Hadron Spectrum
September 1-6 2008
Mainz, Germany

*Speaker.
1. Introduction

The SU(2) Yang-Mills (YM) theory at nonzero temperature is subject both of theoretical and numerical on-going efforts. The aim is a deeper understanding of its non-perturbative properties, which makes this theory complex and rich. This is also a necessary step toward the understanding of the quark gluon plasma (QGP), see for instance [1] for a review. In this work, based on Refs. [2, 3, 4, 5, 6], the non-perturbative sector of SU(2) is described by a composite, (adjoint-)scalar field \( \phi \) in the deconfined phase \( (T > T_c) \).

Topological objects named calorons (i.e. instantons at nonzero \( T \)) with zero and nonzero holonomy -the latter being a property of the fields at spatial infinity- have been described in [7]. A non-trivial holonomy caloron carries also monopole-antimonopole constituents. The ‘composite’ field \( \phi \), which we want to introduce, emerges as an ‘average’ over calorons and anticalorons with trivial holonomy, see [2, 3] for a microscopic derivation and [4] for a macroscopic one. It depends only on the temperature \( T \) and the YM-scale \( \Lambda \). On a length scale \( l > |\phi|^{-1} \) it is thermodynamically exhaustive to consider only the average field \( \phi \) and neglect the (unsolvable) microscopic dynamics of all YM-field configurations, such as calorons and monopoles. One can then build up an effective theory for YM-thermodynamics valid for \( T > T_c \), in which the scalar field \( \phi \) acts as background field coupled to the residual, perturbative gluons. It also acts as an Higgs-field in the thermal medium, implying that two gluons (out of three) acquire a non-vanishing quasi-particle thermal mass. On a phenomenological level it contributes to the energy and pressure as a temperature-dependent bag constant. Here we shall focus on one particular implication of this effective description: the linear growth with \( T \) of the stress-energy tensor, which has been obtained in some analyses of lattice data.

2. Linear growth of \( \theta = \rho - 3p \) and bag constant

Be \( \rho \) the energy density and \( p \) the pressure of a system at a given temperature \( T \). The quantity \( \theta = \rho - 3p \) is the trace of the stress-energy tensor and vanishes for a conformal theory (as, for instance, a gas of photons). In SU(2) this symmetry is broken by quantum effects (trace anomaly). In [8], based on the new lattice data of Ref. [9], it is found that \( \theta \) grows linearly with \( T \):

\[
\theta = aT, \quad 2T_c \lesssim T \lesssim 5T_c, \quad a \simeq 1.5 \text{ GeV}^3.
\]

(2.1)

(We notice that in the analysis of the same lattice data a quadratic rise of \( \theta \), rather than a linear one, has been found [10].) We also refer to [11], where a simple linear fit \( \theta = aT \) was found to reproduce old lattice results. Recently, in [12] the linear rise has been confirmed by studying the Lattice data of [13].

We now turn to a phenomenological description of a plasma of quasi-particles [14], where also a \( T \)-dependent bag is introduced to mimic a non-perturbative behavior. As an example let us consider only one scalar field with a \( T \)-dependent mass \( m = m(T) \) and a bag \( B = B(T) \). The energy density and the pressure read (see, for instance, [15]):

\[
\rho = \rho_p + B(T), \quad p = p_p - B(T),
\]

(2.2)
\[
p_p = \int_k \frac{\sqrt{k^2 + m^2(T)}}{\exp\left[\frac{\sqrt{k^2 + m^2(T)}}{T}\right] - 1}, \quad p_p = -T \int_k \log\left[1 - \exp\left(-\frac{\sqrt{k^2 + m^2(T)}}{T}\right)\right]
\]

where \( f_k = \int \frac{dk}{(2\pi)^4} \). Requiring the validity of the thermodynamical self-consistency \([13,16]\) \( \rho = T(d\rho/dT) - p \) (which is a consequence of the first principle of thermodynamics), we obtain the equation

\[
\frac{dB}{dT} = -D(m) \frac{dB}{dT} \quad D(m) = \int_k \frac{m}{\sqrt{k^2 + m^2}} \exp\left[\frac{1}{\sqrt{k^2 + m^2}}\right] - 1.
\]

(2.3)

Imposing that \( B(T) = cT \) and following the analytical steps of Ref. [5], we obtain:

\[
\theta = \rho - 3p = 4B + \rho_p - 3p_p \quad T_{\text{large}} = 6B(T) = 6cT.
\]

(2.4)

Thus, also the quasi-particle excitation contributes \textit{linearly} to \( \theta \) at high \( T \). The important result is that a linear rise of the bag constant implies also a linear rise of \( \theta = \rho - 3p \). If the linear rise shall be confirmed on the lattice, it means that the SU(2), nonperturbative bag shall be a linear rising function with \( T \).

3. A thermal ground state

The SU(2) (euclidean) YM Lagrangian reads \( \mathcal{L}_{YM} = \frac{1}{4} Tr[G_{\mu\nu}G^{\mu\nu}] \), where \( G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \) and \( A_\mu = A_\mu^a t^a \) (\( t^a \) are the SU(2) generators). \( g \) is the fundamental coupling constant, which, upon renormalization, is function of the renormalization scale \( \mu \). The non-abelian nature of \( \mathcal{L}_{YM} \) is at the origin of its nonperturbative properties. A general field configuration can be decomposed as \( A_\mu = A_\mu^{\text{top}} + a_\mu \), where \( A_\mu^{\text{top}} \) refers to a topologically non-trivial function and \( a_\mu \) to the quantum fluctuations. It is very hard, if not impossible, to take into account at a microscopic level all the topological objects such as calorons and monopoles. As described in Refs. [3,4] we introduce an adjoint scalar gauge field \( \phi = \phi^a t^a \) as

\[
\phi = \text{`Spatial average over (anti-)calorons with trivial holonomy'}.
\]

(3.1)

The field \( \phi \) can be univocally determined by the three following conditions [3]: (i) Being a spatial average it depends (periodically) on \( \tau \) only. It transforms as \( \phi \rightarrow U \phi U^\dagger \) under (space-independent) gauge transformations. (ii) The corresponding Lagrangian \( \phi \) reads \( \mathcal{L}_\phi = Tr[(\partial_\tau \phi)^2] = V \); BPS saturation of (anti-)calorons implies also BPS-saturation for \( \phi \): \( \mathcal{H}_\phi = Tr[(\partial_\tau \phi)^2 - V] = 0 \). (iii) \( \phi \) represents (part of) the vacuum of the YM-system at nonzero \( T \) and a background field to the dynamics of the trivial quantum fluctuations \( a_\mu \). The gauge-invariant quantity \( |\phi| \) acts as a ‘condensate’ (strictly related to the gluon condensate, see [6]) and does not correspond to any new particle. As a consequence \( |\phi| \) shall not depend on \( \tau \).

The conditions (i), (ii) and (iii) imply that \( V(\phi) \propto 1/|\phi|^2 \) [3]. Introducing a scale \( \Lambda \), which is the only free parameter of the theory and is naturally identified with the YM-scale, we obtain \( V(\phi) = \frac{\Lambda^6}{|\phi|^2}. \) Solving the equation of motion one obtains as a unique solution (up to a phase) \( |\phi| = \sqrt{\Lambda^2/(2\pi T)} \). On the length scale \( l > |\phi|^{-1} \) (part of) the field configurations \( A_\mu^{\text{top}} \) are effectively described by \( \phi \).
A thermal ground state for SU(2) Yang-Mills theory and the trace anomaly

As a last step we couple $\phi$, which is a given background solution $|\phi| = \sqrt{\Lambda^3/(2\pi T)}$, to the quantum fluctuations $a_\mu$ in a gauge invariant manner and obtain the final form of the effective theory:

$$L_{\text{eff}} = \text{Tr} \left[ F_{\mu \nu} + (D_\mu \phi)^2 + \Lambda^6 \frac{|\phi|^2}{|\phi|^2} \right],$$

(3.2)

with $F_{\mu \nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - i e [a_\mu, a_\nu]$ and $D_\mu \phi = \partial_\mu \phi - i e [A_\mu, \phi]$. The new, $T$-dependent coupling constant $e = e(T)$ is univocally obtained by imposing thermodynamical self-consistency as described in Section 2. In Ref. [6] the fundamental coupling constant $g$ is related to $e(T)$ and it is found that also $g$ admits a Landau pole, which is just slightly shifted from the perturbative result. The effective coupling constant $e(\lambda = 2\pi T/\Lambda)$ together with some relevant thermodynamical quantities evaluated at tree-level in Fig. 1. Note that $e(\lambda)$ diverges at $\lambda_c = 13.86$, corresponding to a critical temperature of $T_c \sim 2\Lambda$. Thus the effective theory is valid for $T > T_c$. For $T >> T_c$ a plateau is reached for $e(\lambda) \sim \sqrt{8}\pi$. Moreover, as evident from eq. (3.2) a Higgs mechanism takes place: two gluons acquire a quasi-particle mass $m(T) = 2e(T)|\phi|$ which diverges at the phase boundary.

Finally, we summarize the effective theory in Fig. 2. Its tree-level equations are similar to those presented in Section 2. Corrections beyond tree-level are small (1%) [17].

4. Result for $\theta$ and conclusions

The last term of eq. (3.2) acts as a $T$-dependent bag term, $B(T) \sim \frac{\Lambda^6}{|\phi|^2} = 2\pi T \Lambda^3$. The important point is that $B(T)$ is linear in $T$! It then implies, as seen in Section 2, that $\theta$ of SU(2) YM-theory grows linearly in $T$ for high $T$. The precise result can be obtained analytically [5]:

$$\theta = \rho - 3p T \sim 24\pi \Lambda^3 T \simeq (1.7 \text{ GeV}^3) T.$$  (4.1)

Note that the coefficient 1.7 GeV$^3$ is similar to 1.5 GeV$^3$ found in Ref. [8] and is also not far from the theoretical result of Ref. [18]. We also note that the same approach can be applied to the SU(3) case, where one also finds $\theta \sim 24\pi \Lambda^3 T$. 

Figure 1: Left: effective coupling as function of $\lambda = 2\pi T/\Lambda$. Right: (scaled) thermodynamical relevant quantities as function of $\lambda/\lambda_c = T/T_c$. 

Figure 2: The effective theory in tree-level.
We conclude this brief report on an effective approach for the description of YM at nonzero $T$ by summarizing its main idea: the introduction of a simple, average background field over caloron and anticaloron. The aim is to avoid an impossible, microscopic evaluation of complicated field configurations, and to obtain a thermodynamically exhaustive description of a YM system. A simple consequence of this effective approach, namely the linear growth with $T$ of the trace anomaly, is in agreement with recent lattice simulations. Some applications of this approach can be found in Refs. [19].

Acknowledgments

The author thanks R. Hofmann and M. Schwarz for a fruitful collaboration.

References

[1] D. H. Rischke, Prog. Part. Nucl. Phys. 52 (2004) 197 [arXiv:nucl-th/0305030].
[2] R. Hofmann, Int. J. Mod. Phys. A 20 (2005) 4123 [Erratum-ibid. A 21 (2006) 6515] [arXiv:hep-th/0504064].
[3] U. Herbst and R. Hofmann, arXiv:hep-th/0411214.
[4] F. Giacosa and R. Hofmann, Prog. Theor. Phys. 118 (2007) 759 [arXiv:hep-th/0609172].
[5] F. Giacosa and R. Hofmann, Phys. Rev. D 76 (2007) 085022 [arXiv:hep-th/0703127].
[6] F. Giacosa and R. Hofmann, Phys. Rev. D 77 (2008) 065022 [arXiv:0704.2526 [hep-th]].
[7] W. Nahm, Phys. Lett. B 90 (1980) 413. T. C. Kraan and P. van Baal, Nucl. Phys. B 533 (1998) 627 [arXiv:hep-th/9805168]. K. M. Lee and C. h. Lu, Phys. Rev. D 58 (1998) 025011 [arXiv:hep-th/9802108].
[8] D. E. Miller, Phys. Rept. 443 (2007) 55 [arXiv:hep-ph/0608234]. D. E. Miller, Acta Phys. Polon. B 28 (1997) 2937 [arXiv:hep-ph/9807304].
[9] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier and B. Petersson, Nucl. Phys. B 469 (1996) 419 [arXiv:hep-lat/9602007].

[10] R. D. Pisarski, Prog. Theor. Phys. Suppl. 168 (2007) 276 [arXiv:hep-ph/0612191].

[11] C. G. Kallman, Phys. Lett. B 134 (1984) 363.

[12] K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, arXiv:0807.2391 [hep-ph].

[13] M. Cheng et al., Phys. Rev. D 77 (2008) 014511 [arXiv:0710.0354 [hep-lat]].

[14] P. Levai and U. W. Heinz, Phys. Rev. C 57 (1998) 1879 [arXiv:hep-ph/9710463].

[15] R. A. Schneider and W. Weise, Phys. Rev. C 64 (2001) 055201 [arXiv:hep-ph/0105242].

[16] D. H. Rischke, M. I. Gorenstein, A. Schafer, H. Stoecker and W. Greiner, Phys. Lett. B 278 (1992) 19.

[17] M. Schwarz, R. Hofmann and F. Giacosa, Int. J. Mod. Phys. A 22 (2007) 1213 [arXiv:hep-th/0603078]. R. Hofmann, arXiv:hep-th/0609033. D. Kaviani and R. Hofmann, Mod. Phys. Lett. A 22 (2007) 2343 [arXiv:0704.3326 [hep-th]].

[18] D. Zwanziger, Phys. Rev. Lett. 94 (2005) 182301 [arXiv:hep-ph/0407103].

[19] F. Giacosa and R. Hofmann, Eur. Phys. J. C 50 (2007) 635 [arXiv:hep-th/0512184]. M. Schwarz, R. Hofmann and F. Giacosa, JHEP 0702 (2007) 091 [arXiv:hep-ph/0603174]. J. Ludescher and R. Hofmann, arXiv:0806.0972 [hep-th]. M. Szopa and R. Hofmann, JCAP 0803 (2008) 001 [arXiv:hep-ph/0703119].