Gate-control of the current-flux relation of a Josephson quantum interferometer based on proximitized metallic nanojunctions

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We demonstrate an Al superconducting quantum interference device in which the Josephson junctions are implemented through gate-controlled proximitized Cu mesoscopic weak-links. The latter behave analogously to genuine superconducting metals in terms of the response to electrostatic gating, and provide a good performance in terms of current-modulation visibility. We show that, through the application of a static gate voltage, we are able to modify the interferometer current-flux relation in a fashion which seems compatible with the introduction of $\pi$-channels within the gated weak-link. Our results suggest that the microscopic mechanism at the origin of the suppression of the switching current in the interferometer is apparently phase coherent, resulting in an overall damping of the superconducting phase rigidity. We finally tackle the performance of the interferometer in terms of responsivity to magnetic flux variations in the dissipative regime, and discuss the practical relevance of gated proximity-based all-metallic SQUIDs for magnetometry at the nanoscale.

Keywords: Josephson Effect, SQUID, Superconducting Magnetometer, Gated Metallic Superconductor, Proximity Effect, SNS

All-metallic gated superconducting transistors (GSTs) are a class of mesoscopic quantum devices, entirely realized with Bardeen-Cooper-Schrieffer (BCS) metals, in which the critical supercurrent ($I_C$) can be largely regulated via electrostatic gating\textsuperscript{1–3}. Differently from proximitized semiconductors and low charge-density superconductors\textsuperscript{4–14} where the critical current is controlled via conventional field-effect-driven charge-density modulation, in GSTs the $I_C$ suppression is obtained, regardless of the sign of the gate voltage, without the carrier concentration being affected\textsuperscript{15}. The underlying physical mechanism has not been clearly identified yet, and a few hypotheses have been guessed to explain a plethora of experimental results, which cannot be comprehended in the bare framework of the BCS theory\textsuperscript{16}.

Recently, a high-energy electron injection due to cold electron field emission from the gate, has been claimed\textsuperscript{17–20} to have a major role in $I_C$ suppression. This picture does not rely on novel physics. Yet, it does not seem compatible with some of the observed phenomenology such as the absence of a sum rule between currents originating from different gates\textsuperscript{3}, the response of in-vacuum suspended gated superconducting nanowires\textsuperscript{21}, and the non-thermal character of superconductivity. While high-energy electron injection due to field emission is likely to be strongly detrimental for preserving phase coherence in the superconductor, the two latter models are supposed to preserve it up to a large extent, and both predict the occurrence of a rotation of $\pi$ in the macroscopic superconducting phase of the region affected by the gate voltage.

The information about the phase behavior of a superconductor subjected to the action of external stimuli can be experimentally accessed through a DC superconducting quantum interference device (SQUID)\textsuperscript{30}: a superconducting ring interrupted by two Josephson weak links in parallel. A magnetic field threading the loop controls the current vs. voltage ($I$-$V$) characteristics of the SQUID via magnetic flux quantization\textsuperscript{31,32} and the DC Josephson effect\textsuperscript{33}, thus resulting in a modulation of the amplitude of the critical supercurrent. The impact of the electrostatic gating on the superconducting phase of a BCS superconductor was investigated so far only in monolithic Ti interferometers based on gated Dayem bridges\textsuperscript{34}. Such systems allowed to retrieve a footprint of the action of the gating on the switching current ($I_s$) vs. flux ($\phi$) relation of the SQUID. Nonetheless, due to the large value of the SQUID inductance, the $I_s(\phi)$ of these interferometers exhibited poor modulation visibility, with a significant deviation from the ideal sinusoidal behavior\textsuperscript{30}. This limited the access to a detailed information on the dependence of the current vs. phase relation of gated metallic Josephson weak links on the applied voltage.

Here we tackle this relevant question by demonstrating a
Our gate-controllable superconducting interferometers (SNS SQUID) consist of a 100-nm-thick Al superconducting loop interrupted by two Al/Cu/Al planar gated junctions. The loop of the SQUID spans a surface of about 7.5 $\mu$m$^2$. Aluminum shows a strong proximization capability over copper, thanks to the good quality of the interfaces formed between these two metals\cite{39}. The Cu normal-metal wire was 120 nm wide, 630 nm long, and 20 nm thick. The weak links operate in the diffusive regime and within the long-junction limit, holding when the Thouless energy of the junction $E_{Th} = \frac{\hbar D}{L^2} \sim 13 \mu$eV $< \Delta_{Al} \approx 180 \mu$eV, where $D \approx 0.008$ $m^2/s$ is the Cu diffusion coefficient\cite{19}, $L$ the weak-link length, and $\Delta_{Al}$ the superconducting gap of the Al banks. Moreover, two 80-nm-wide Cu gate electrodes, labelled $G_L$ and $G_R$, were separated from the normal-metal wire by a distance of about 60 nm and 45 nm, respectively (in the representative device whose data are discussed in the following). Further details of the fabrication process are reported in the Methods section. A 3-dimensional representation of a typical SNS SQUID comprising the scheme of the 4-wire electrical setup is displayed in Fig. 1a, whereas a false color scanning electron micrograph of a representative device is shown in Fig. 1b.

Figure 1c shows the $I-V$ characteristics of a representative SNS SQUID collected at several temperatures ranging from 30 mK to 650 mK. The curves are horizontally offset for clarity. For temperatures smaller than 750mK, the $IV$s exhibit a clear Josephson effect with a switching current $I_S$ of $\sim 7 \mu$A at 30 mK, and a normal-state resistance $R_N \sim 50\Omega$. Due to electron heating in the the normal state$^{40,41}$, the usual hysteretic behavior is observed when the $IV$ is measured forward and backward with a retrapping current $I_R \sim 2 \mu$A at 30 mK. A plot of the switching and the retrapping current vs. temperature ($T$) is shown in Fig. 1d. The difference between $I_S$ and $I_R$ decreases by increasing $T$, as routinely observed in similar systems$^{40,41}$, and vanishes at $T \sim 350$ mK.

To study the $I_S(\phi)$ characteristics of the SNS interferometers, we measured their $IV$s as a function of the external magnetic field threading the SQUID loop. The device switching current was then extracted from the $IV$s to build the $I_S$ vs $\phi$ curves. The $I_S(\phi)$ of the device is reported in Fig. 2a for selected temperatures between 30 mK and 500 mK, where $\phi_0 \approx 2.067 \times 10^{-15}$ Wb is the magnetic flux quantum. For each temperature we plot both the positive ($I_{S+}$) and nega-
The modulation visibility is mainly determined by the difference between the critical currents of the two junctions. The latter can be extracted by fitting the $I_S(\phi)$ data against the static zero-temperature resistively-shunted junction (RSJ) model\textsuperscript{30}

$$i = I_0[(1 - \alpha) \sin(\delta_1) + (1 + \alpha) \sin(\delta_2)],$$

$$2j = I_0[(1 - \alpha) \sin(\delta_1) - (1 + \alpha) \sin(\delta_2)],$$

$$\delta_2 - \delta_1 = 2\pi\phi/\phi_0 + \pi\beta j,$$  \hspace{1cm} (3)

where $\delta_{1,2}$ are the phase differences across the weak links, $i$ and $j$ are the supercurrent passing through and circulating in the SQUID, respectively. Within this formalism, defining $\alpha = |I_0 - I_L|/|I_0 + I_L|$, the asymmetry between the critical currents of the two junctions is accounted for. At fixed magnetic flux, $I_{S+}$ and $I_{S-}$ are defined as proportional respectively to the maximum and minimum values of $i$ over all the values of $\delta_1$ and $\delta_2$ satisfying Eqs. 1, 2, and 3, with the coefficient $I_0 = |I_0 - I_L|$ corresponding to one half of the maximum supercurrent of the SQUID as function of $\phi$. This model accounts also for the inductance $L$ of the SQUID, through the screening coefficient $\beta = 2\omega L_0/\phi_0$. Although the RSJ model was conceived for tunnel-like Josephson junctions, it retains its validity also for SNS weak-links that, like ours, fall in the long junction limit. A detailed description of the fit procedure is reported in the Methods section. The fit curves are shown on top of experimental data in Fig. 2a (solid black lines). The good agreement between the RSJ model and experimental data is quantitatively confirmed by the coefficient of determination $R^2$ of the fits, which ranges from 0.996 (at 500 mK) to 0.97 (at 30 mK). The value for $2I_0$ determined through the fitting procedure is plotted against the temperature in Fig. 2b. Furthermore, we extracted the $\alpha$ parameter, which is reported in Fig. 2c. $\alpha$ reaches the maximum value of $\sim 0.2$ at 30 mK, and decays when the temperature is increased. From $\alpha$ it is also possible to deduce the value of the critical currents of the two weak-links, that are $I_{S+} \sim 6 \mu A$ and $I_{S-} \sim 3 \mu A$ for the junction with the higher and the lower critical supercurrent, respectively. A plot of $I_{S+}$ and $I_{S-}$ as a function of the temperature is reported in Fig. 2b. The value for $\beta$ derived from the fit is around 0.01 for every temperature, thereby confirming the negligible inductance contribution provided by the Al loop.

To investigate the impact of the gate bias on the SNS SQUID current-flux relation we measured $I_S(\phi)$ when several values of gate voltage were independently applied to either the left and right gate electrode. Figure 3a shows the modulation patterns of $I_{S+}$ and $I_{S-}$, for different positive values of gate voltage $V_R$ applied to $G_R$ measured at 30 mK, $G_L$ was left grounded. It is worth to discuss the several interesting gate-dependent features emerging from the data. $I_{S+}$ is constant up to about 12 V. Above this threshold it is suppressed by further increasing $V_R$, and exhibits the same reduction for positive and negative gate voltages as well as for positive and negative current bias. The same qualitative behavior was observed by polarizing the left gate electrode, which due to a
larger gate-junction distance was effective at higher voltages. The SQUID switching current as a function of gate voltage applied alternatively to $G_L$ or $G_R$ is shown in Fig. 3b against $\tilde{V}$, i.e., the voltage normalized to the values at which the switching current was suppressed by $\%10$. This equals 14 V and 46 V for $G_R$ and $G_L$, respectively.

In stark contrast to the conventional $T$-dependent case, in which both minima and maxima of the modulation pattern converge to 0 by enhancing the temperature, in the gate-dependent case the amplitude of the minima of $I_{S+}$ (and the maxima of $I_{S-}$) are apparently almost locked in the explored voltage range. We start our discussion on such an unconventional phenomenology by recalling that, following from Eq. 1,

$$I_{S+}(\phi) = I_0 \sqrt{(1-\alpha)^2 + (1+\alpha)^2 + 2(1-\alpha^2)\cos^2 \frac{2\pi\phi}{\phi_0}}.$$  (4)

This expression, which holds when $\beta$ is negligible, allows to derive the $I_{S}(\phi)$ extremal values $I_{MAX} = 2I_0 = I_L + I_R$ and $I_{MIN} = 2\alpha I_0 = |I_L - I_R|$. These relations imply that it is not possible to affect $I_{MAX}$ (which in our data is suppressed by a $\sim 50\%$ factor) keeping $I_{MIN}$ constant unless $I_L(V_L = 0) - I_L(V_R = 0) - I_R(V_R = 0) - I_R(V_L = 0) - I_R(V_L = 0) - I_L(V_L = 0) - I_L(V_R = 0)$ for each value of $V_R$ (and $V_L$). This condition is not only extremely unlikely to be satisfied, but it seems also incompatible with the typical length scale of the gating effect in metallic superconductors. Indeed, it was shown\cite{1,17} that the critical current suppression due the application of a gate voltage exponentially decays with the distance from the gate itself. In other words, gating is a local effect, which, acting on just one of the weak links, yet is able to affect non-locally the response of the whole SNS SQUID.

In order to further elaborate on the above question, we believe interesting to discuss the results of the RSJ fitting of the $I(\phi)$s obtained at different gate voltage values (see black lines in Fig. 3a). The fit was performed by exploiting the same technique of the temperature-dependent case, but now including an additional phase shift ($\delta \phi$) in Eq. 3 such that $\delta \phi = 2\pi \phi/\phi_0 + \pi \beta j + \delta \phi$. The introduction of the latter parameter was necessary to successfully fit the $I_{S\pm}(\phi)$ obtained for $V_R > 15$V (and for $V_L > 50$V). $\delta \phi$ is plotted as a function of the gate voltage applied to either $G_L$ or $G_R$ in Fig. 3d, while $\alpha(\tilde{V})$ is plotted in Fig. 3c. For $|\tilde{V}| \leq 1$, the agreement between fit and data is optimal, with $R^2$ ranging between 0.98 and 0.99. Above this threshold, however, the ability of the RSJ model to represent the current-flux characteristics progressively weakens: in particular, at $|\tilde{V}| = 1.3$.
II. PHASE FRUSTRATION THROUGH $\pi$-DOMAIN ACTIVATION

We now discuss a possible phenomenological model based on the assumption that the gate voltage is able to affect only the phase of each superconducting domain composing the weak-link, and rotating it by a factor of $\pi$ (see Fig. 4a). This hypothesis assumes the existence of a fully coherent mechanism that is able to account for all the main features observed in gate-controlled $I_\gamma(\phi)$s. Due to the polycrystalline nature of the copper wire forming our weak-links, we describe each domain through an order parameter $\Delta e^{i\theta} \gamma$, where $\Delta_0$ and $\theta_\gamma$ are the amplitude of the gap and phase of the $\gamma$th domain, respectively. In this framework, when a supercurrent is injected through the weak-link, the phase drop $\delta$ built across the latter results from the accumulation of the phasor rotation of $\pi$ (see Fig. 4c) with respect to the unperturbed value. The phase drop over the weak-link in this configuration is, therefore, overall frustrated due the counter-rotation acquired by the phasor in the $\pi$-domain (green blocks in Fig. 4b). This physical intuition finds a mathematical representation by modifying the weak-link current phase relation as follows:

$$I = I_1 \left[ \sum_{n=1}^{N} \gamma_\alpha \sin(n\delta) + \gamma_\pi \sin(n\delta + \pi) \right] = I_1 \sum_{n=1}^{N} (\gamma_\alpha - \gamma_\pi) \sin(n\delta),$$

where we recover the most general functional form by including an arbitrary number $N$ of 0-phased and $\pi$-phased harmonics with weight $\gamma_\alpha$ and $\gamma_\pi$, respectively, determined by the contribution of the $\pi$ domains to the resulting phase drop. Following from this assumption, the RSJ current-flux relation of the SQUID modifies into

$$i = I_1 \sum_{n=1}^{N} (\gamma_\alpha - \gamma_\pi) \sin(n\delta) + I_2 \sin \left( \frac{2\pi \phi}{\phi_0} \right),$$

where $I_1$ and $I_2$ account for the amplitude of the critical current of gated and non-gated weak-link, respectively. Figure 4d shows the $I_2(\phi/\phi_0)$ calculated through this model with just two harmonics ($N = 2$), and with $I_1 = 1.18$ and $I_2 = 0.82$. The latter values correspond to an asymmetry parameter $\alpha = 0.18$, i. e., compatible to that of our SNS SQUIDs. The amplitude of 0-phase harmonics $\gamma_\alpha$ and $\gamma_\pi$ were set respectively to 1 and 0 in order to recover the conventional sinusoidal monochromatic behaviour when no gate voltage is applied. We show curves obtained for $\gamma_\pi = 0.05\gamma_\alpha$, and $\gamma_\pi$ ranging between 0 (blue curve in Fig. 4d), and 0.7 (light-green curve). The former case corresponds to a vanishing gate voltage. By increasing $\gamma_\pi$, we mimic the action of the gate voltage, which amplifies the weight of the $\pi$ terms for both the harmonics, thereby resulting in a suppression of the maxima of the current-flux relation (star plot in Fig. 4d). The latter reaches a value of $\sim 50\%$ for $\gamma_\pi = 0.7$. Besides, $I_2(\phi)$ minima undergo a non-monotonic and much more limited variation. We wish to emphasize that, by introducing just one additional harmonic, we obtained a significant deviation from the sinusoidal behavior, which resembles that of the experimental data. Furthermore, the shift of the maxima of $I_2(\phi)$ (dots in Fig. 4d) are consistent with the result of the RSJ fit procedure for the parameter $\delta$, reaching a value of $\sim 4\%$ (see Fig. 3d for a comparison).

III. EFFECT OF GATING IN THE DISSIPATIVE REGIME

Among available magnetic field sensors, SQUIDs are the devices of choice for those applications requiring ultra-high sensitivity at the nanoscale. SQUIDs have progressively become an essential tool for probing several systems, such as magnetic molecules and nanoparticles, single electrons, and cold atom clouds. Beyond the detection of magnetic moments (down to the single spin resolution), SQUIDs play in a front row role in a vast field of applications ranging from microbolometry and spintronics to drug delivery and cancer treatment. In this last section we discuss the performance of our SNS SQUID in view of its possible exploitation as a gate-tuned magnetic flux sensors operating in the dissipative...
FIG. 4. Gate-driven phase frustration through $\pi$-rotation. a: Pictorial representation of the $\pi$ rotation mechanism induced by the gate voltage. The superconductor is represented through a one-dimensional chain of domains (blue blocks); each of them can be described by a generic complex order parameter $\Delta_r e^{i\theta_r}$, where $r$ is a domain index, and $\theta_r$ is the superconducting phase in each domain. Under the action of the gate voltage we assume the phase of some of the domains to be rotated by $\pi$ (green block). b: When a supercurrent is injected through the weak-link, the phase drop ($\delta$) built across the latter results from the accumulation of phasor rotations acquired at each domain. c: When a gate-voltage is applied, a fraction of the domains proportional to the intensity of the electric field acquire a phase-rotation of $\pi$ with respect to 0 V case. The resulting phase drop over the weak-link turns out to be frustrated due the counter-rotation acquired by the phasor in the $\pi$-domains. d: $I_S(\phi)$ calculated through Eq. 6 with $N=2$, $I_1=1.18$, and $I_2=0.82$. The latter values correspond to an asymmetry parameter $\alpha=0.18$. e: $\Delta S/I_0$ as function of both the current bias and the voltage applied to the SQUID. The impact of the gate voltage on the dissipative regime. The 0-voltage-drop flux interval was obtained at 30 mK, and vanishes around 300 mK. Such a performance is on par with that of interferometers of similar typology. 6.

The latter is conventionally obtained by current biasing the interferometer above its critical current. Variations of the magnetic field threading the loop translate into variations of the voltage drop ($V$) developed across the Josephson junctions. Figure 5a shows the $V(\phi)$ curves measured at 30 mK on a representative device by 4-wire lock-in technique for selected amplitudes of the 17 Hz sinusoidal current-bias signal $I$. Below $I \sim 6 \mu A$, the curves exhibit a zero voltage-drop for magnetic fluxes such that $I < I_s(\phi)$. A finite $V$ value is, instead, measured when the interferometer switches into the dissipative regime, for the biasing current being higher than the flux-dependent switching current. This results in a strongly nonlinear behaviour at the switching points, corresponding to a high value for the flux-to-voltage transfer function, $f_i = |\partial V/\partial \phi|$. The latter characteristic, nonetheless, cannot be easily exploited for highly-sensitive operation due to the stochastic nature of the switching, which results in an unstable working point and in a vanishing dynamic range.

The transfer function, calculated through numerical differentiation of the $V(\phi)$ curves, is shown in Fig. 5b for selected bias current values. The current provides an useful knob to select the flux values at which the interferometer responsivity is maximized. The maximum value of $f_i (f_{\text{max}})$ is plotted vs $T$ in Fig. 5c. $f_{\text{max}}$ decreases with the temperature almost linearly from the value of 400 $\mu V/\phi_0$ obtained at 30 mK, and vanishes around 300 mK. Such a performance is on par with that of interferometers of similar typology.

The impact of the gate voltage on the $V(\phi)$ was explored by repeating the acquisition of such characteristics at 30 mK as function of both the current bias and the voltage applied to either $G_L$ or $G_R$. Figure 5d shows the $V(\phi)$ curves obtained for $I = 3 \mu A$ (solid lines) and $I = 6 \mu A$ (dashed lines) for $V_R$ ranging between 0 and 18 V. The first family of curves (3 $\mu A$) corresponds to a position in the parameters space where, at null gate voltage, the interferometer is not fully operated in the dissipative regime. The 0-voltage-drop flux interval was observed to shrink by increasing the intensity of the gate voltage, until it completely disappears due to the gate-driven suppression of the critical current of the SQUID. At $V_R = 18$ V the device operates in a fully-dissipative regime. The second family of curves (6 $\mu A$) falls entirely in the dissipative regime. We note that the result of the action of the gate is rather dif-
different from the behavior obtained by increasing the biasing current. Indeed, in the latter case both the minimum and the maximum of the modulation pattern increases by increasing the bias current. In the gate-driven regime, instead, the maximum of the modulation turns out to be locked, whereas the minimum can be controlled through the gate. These characteristics can be exploited to adapt to specific tasks the transfer function of the interferometer at the switching points through an additional knob, the gate voltage. By shrinking the width of the non-dissipative region through the gate action, e.g., it is possible to magnify the flux dynamic-range at the switching point without reducing the overall voltage-drop swing and the resulting device sensitivity. The plot of $f_L$ vs. $\phi$ at $I = 3 \mu A$ and $T = 30\ mK$ is shown in Fig. 5e for several values of $V_R$. We note that $f_{\text{out}}$ remains almost constant in a wide gate-voltage range, as shown in Fig. 5f for selected bias current values.

**IV. CONCLUSIONS**

The physics of electrostatic gating on metallic superconductors is, to date, one of the latest unanswered questions in condensed matter physics. Despite a few theoretical interpretations have been proposed, a model able to account for the totality of the phenomenology observed so far, and to provide quantitative prediction has not been developed yet. Our experiments on gated all-metallic SNS SQUIDs show that the microscopic mechanism at the origin of the critical current suppression of gated weak-links is apparently phase coherent and produces a softening of the phase rigidity of the Josephson junctions. This latter observation provides a valuable reason to exclude any thermal-assimilated origin of gate-driven effects, on the one side. On the other side, we claim that, among the models aiming at the description of electrostatic gating in metallic superconductors, those in which it will be possible to take into account phase coherent effects should be preferred. Here, we interpreted our data through a phenomenological model based on the sole assumption that the gate induces a phase rotation of $\pi$ in the superconducting domains of the weak-link subjected to the action of the electric field. Although rather simplified, our model successfully captures the main features observed in gated all-metallic SNS SQUIDs, such as the suppression of the maximum switching current, the blocking of the minimum switching current, and the deviation from the monochromatic behavior of the interferometer current-flux relation. We conclude by emphasizing the practical relevance of gated all-metallic SNS SQUIDs for
magnetometers at the nanoscale. Indeed, the gate voltage provides an additional control on the transfer function of the interferometer, which can be exploited to tailor the response of the device on specific needs such as, for instance, the amplification of the flux dynamic range around the switching points for applications requiring higher sensitivity.

METHODS

A. Device nanofabrication

The SNS-SQUIDs were fabricated by a single-step electron-beam lithography (EBL) and two-angle shadow-mask metal deposition through a suspended resist-mask onto an intrinsic Si(111) wafer covered with 300 nm of thermal SiO2. The metal-to-metal clean interfaces were realized at room temperature in an ultra-high vacuum (UHV) chamber (base pressure $\sim 5 \times 10^{-11}$ Torr) of an electron-beam evaporator equipped with a tiltable sample holder. A 5-nm-thick Ti adhesion-film was deposited at an angle of 0°. Subsequently, 25 nm of Cu were evaporated to realize the SQUID nanowires and gates. Finally, the sample holder was tilted at 13° for the deposition of a 100-nm-thick layer of Al to realize the superconducting loop.

B. Cryogenic electrical characterization

The electrical characterization of our devices was performed by four-wire technique in a filtered cryogen-free 3He-4He dilution fridge equipped with a superconducting electromagnet, used to apply the external magnetic flux. Current-voltage ($IV$) measurement were carried out by setting a low-noise current bias and measuring the voltage drop across the weak-links with a room temperature pre-amplifier. Switching current average values were calculated over the switching points extracted from 15 repetitions of the same $IV$. The voltage-flux characterization was performed through a standard lock-in technique: the sinusoidal reference signal of the magnet, used to apply the external magnetic flux. Current-gate voltage was applied through a room-temperature low-temperature output voltage signal was pre-amplified at room temperature. The devices were also characterized in gate-weak-link leakage current, which was found to be always lower than 1 pA.

C. RSJ fit of experimental data

The fitting procedure was based on Eqs. (1)-(3) together with the maximum condition $i_{\phi}^r = \max(i)$, $i_{\phi}^l = \min(i)$. Substituting Eqs. (2),(3) in Eq. (1) we obtain a function for the current through the loop, depending on the flux $\phi$, with $\alpha$, $\beta$, $I_0$ and $\delta_{1,2}$ as parameters. The code used for the fit minimizes the distance of the function from the experimental points.

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AUTHOR CONTRIBUTIONS

N.L. fabricated the devices, S.B and G.D.S performed the experiment with input from F.G.. S.B. analyzed the data with input from G.D.S and F.G. G.D.S implemented the numerical model with inputs from M.T.M, M.C., and F.G. G.D.S. wrote the manuscript with input from all the authors. F.G. conceived the experiment. All of the authors discussed the results and their implications.
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