Acceleration Radiation for Orbiting Electrons.∗

W. G. Unruh
Program in Cosmology and Gravity of CIAR
Dept Physics and Astronomy
University of B.C.
Vancouver, Canada V6T 1Z1

Abstract

This paper presents an analysis of the radiation seen by an observer in circular acceleration, for a magnetic spin. This is applied to an electron in a storage ring, and the subtlety of the interaction of the spin with the spatial motion of the electron is explicated. This interaction is shown to be time dependent (in the radiating frame), which explains the strange results found for the electron’s residual polarisation in the literature. Finally, some brief comments about the radiation emitted by an accelerating detector are made where it is shown that the spectrum is correlated in that particles are emitted in pairs.

A uniformly accelerated objects sees the vacuum fluctuations as a thermal bath [1]. For an acceleration which is constant in amplitude and direction, this temperature is given by

$$T = \frac{a}{2\pi} \left( \frac{\hbar}{ck} \right)$$

(1)

Since it requires an acceleration of $2.6 \cdot 10^{22} \text{ cm/sec}^2$ to produce a temperature of 1K the experimental verification of this prediction is difficult, although recent advances in the laser acceleration of electrons promise to produce accelerations of over a 100 times this value [2]. These would give effective temperatures of the order of room temperature, but for very short times. Those brief intense accelerations make it difficult to measure the effects of the thermal bath on the object.

As was pointed out by Bell and Leinaas [4], the spin of an electron in a magnetic field is a possible candidate for a detector of acceleration radiation. Such an electron has two energy levels, and an examination of the population of those two energy levels after a long time can be used to measure the bath that the electron sees itself in at the Larmour precession frequency.

Since even room temperature corresponds to very low frequencies (compared with the Compton frequency of the electron), and since dipole radiation is suppressed by the third power of the frequency, the use of the electron spin as a detector of the acceleration bath requires a very long period of acceleration to produce an effect. Bell and Leinaas [4] therefore

*To be published in The Proceedings of the Quantum Aspects of Beam Physics, Monterey, Jan 1998, ed Pisin Chen,
suggested that circular accelerations (placing the electron into a circular orbit) be used instead. In an electron storage ring, the acceleration experienced by the electrons in the bending magnets corresponds to high temperature (about $10^5$ K for HERA), and relatively short decay times (about half an hour). The electron can be kept in the storage ring for that time and be expected to equilibrate with the thermal bath. They thus suggested that the residual polarisation of the electrons is therefore a measure of the temperature of the bath seen by the electrons. Jackson [3] however has raised serious doubts about the use of a thermal model to explain the residual polarisation of the electrons in such storage rings. In particular, he has argued that the details of the polarisation of the electron as a function of the $g$ factor found by Derbenev and Kondratenko [5] (hereafter DK) and himself [6] as plotted in Figure 1, made such a thermal explanation highly suspect. Zero polarisation does not occur at zero coupling with the field by the spin (ie, zero $g$ factor) nor at zero anomalous coupling ($g = 2$) but at about $g = 1.2$. Furthermore, there are details in the polarisation curve which do not resemble what one would expect from the equilibration with a thermal bath.

![Figure 1: Polarisation of an electron vs the g factor](image)

The primary purpose of this paper is to analyse the behaviour of an electron undergoing circular acceleration. I will restrict myself to the electron’s travelling at constant velocity in a constant magnetic field (ie, I assume that some tangential electric field is present to make up for the energy lost in synchrotron radiation). My conclusion is that the electron does respond to a thermal bath, but one with a frequency dependent temperature. However, the electron is actually a system with two field detectors, which are furthermore coupled to each other. In addition to the spin there are the vertical fluctuations in the orbit. A simple coupling should present no difficulties for understanding the system, but in this case that coupling is also time dependent (in the radiating frame of the spin). This time dependent coupling distorts the response of both detectors. (Almost all of the results here was already in the paper of Bell and Leinaas [4] (hereafter BL), at least implicitly. I hope that this different exposition may make some of the points clearer)

Finally, the emission of radiation by an accelerated detector (coupled to a scalar field in
this case) is very briefly analysed and it is shown that the emitted radiation comes out in correlated pairs of particles. This section is brief, and more detailed investigation of these correlations will be reported elsewhere.

Throughout I will use the convention that $\hbar = c = 1$.

I. POLE STRUCTURE OF THE FLUCTUATIONS

The fluctuations in the electromagnetic (and other massless fields) can be characterised by the two point correlation function. Because the vacuum state is a Gaussian state, the two point correlations completely characterise the field. These fluctuations typically have a term in the denominator of the form

$$D = (\Delta t)^2 - (\Delta x) \cdot (\Delta x)$$

where $\Delta t = t - t'$, the two times in the correlation function, and similarly for $\Delta x$. For circular acceleration, this term becomes

$$D = \gamma^2 (\tau - \tau') - 4R^2 \sin \left( \frac{1}{2} \gamma \Lambda (\tau - \tau') \right)^2$$

where $\gamma$ is the proper time along the circular path, $R$ the radius of the path, $\Lambda$ is the angular frequency (in the lab frame) of the orbit, and the relativistic gamma factor is given by

$$\gamma^2 = \frac{1}{(1 - R^2 \Lambda^2)}.$$  (4)

In calculating the response of the detector at its resonant frequency, the fourier transform of this correlation function will be important, and thus the zeros of $D$, which will become poles of the response function, determine the response \[\square\]. I will be interested in the case of extreme relativistic motion $\gamma >> 1$. The poles of the correlation function will correspond to zeros of $D$.

The most obvious zero is at $\tau = 0$, but others are scattered about the complex $\tau$ plane. There is another pole near $\tau = \tau'$. Expanding sin in a taylor series, I get

$$\gamma^2 \tau^2 - 4R^2 \sin \left( \frac{1}{2} \Lambda \gamma \tau \right)^2 \approx \tau^2 + \frac{8}{6} \frac{R^2 \gamma^4 \Lambda^4 \tau^4}{16} + ...$$

which has a zero at

$$\tau = \pm \tau_1 \approx \pm \frac{2\sqrt{3}}{\sqrt{\gamma^2 - 1} \Lambda}$$

where I have use eqn \[\square\] to eliminate $R$. Figures 2 and 3 graphically solve the equation

$$(x + iy)^2 = \frac{\gamma^2 - 1}{\gamma^2} \sin^2(x + iy)$$

or

3
\[ x = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \sin(x) \cosh(y) \]  
(8)

\[ y = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \cos(x) \sinh(y) \]  
(9)

and

\[ x = -\sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \sin(x) \cosh(y) \]  
(10)

\[ y = -\sqrt{\frac{\gamma^2 - 1}{\gamma^2}} \cos(x) \sinh(y) \]  
(11)

Figure 2
Real and imaginary parts of \( x = .99 \sin(x) \)
The other zeros of the equation (which give poles in the correlation function) all have a value of $x$ and $y$ of order unity or larger, which are of order $\gamma$ larger in magnitude than the pole at $x = 0$, $y \approx \frac{\gamma}{\sqrt{3}}$. Their contribution to the Fourier transform of the correlation function will be very small for large $\gamma$ unless $\omega$ is very near zero.

The fluctuations are Gaussian. If they are also stationary (depending only on $\tau - \tau'$), then the fluctuations can be regarded as thermal, with a possibly frequency dependent temperature. As BL showed, for a detector of massless scalar radiation in circular orbit, the temperature of the fluctuations is given by figure 3. The temperature is frequency dependent, and is scattered around the value expected from equation 1 for linear acceleration. However, just because the temperature is not frequency independent, does not make it act any the less like a temperature especially if the detector responds only to a narrow range of frequencies. The more important aspects are the lack of correlation between the fluctuations at the various frequencies (which comes from the stationarity of the correlation function) and the gaussian nature of those fluctuations.

For a scalar detector in circular orbit, the effective temperature of the scalar field fluctuations turns out to be

$$T(\omega) = \omega \frac{\omega}{\ln \left( 1 + \frac{4\sqrt{3}}{a_c} e^{\frac{4\sqrt{3}}{a_c}} \right)}$$  \hspace{1cm} (12)$$

Thus, for low $\omega$, the temperature is

$$T \approx \frac{a}{4\sqrt{3}}$$ \hspace{1cm} (13)$$

while for high $\omega$, it is approximately

$$T \approx \frac{a_c}{2\sqrt{3}}$$ \hspace{1cm} (14)$$

Figure 4 gives a plot of $T(\omega)/a$ vs $\omega/a_c$. The straight line is the temperature for the same acceleration but with linear rather than circular acceleration.
This was calculated on the basis of using the residue of only the three poles closest to zero. The contribution of the neglected poles is small and I estimate it to be less than about 2% for γ of 10.

For a scalar detector the estimation of the response is now simple. The detector is in contact with a thermal bath, and at long times, one would expect it to come into equilibrium with that bath with the same temperature T describing its internal state. This is born out for a scalar detector (where for example the response of a harmonic oscillator like internal detector can be solved exactly). If the coupling becomes too strong, then of course the internal state is no longer at the naive temperature due to correlation effects between the bath and the detector. However all situations of interest to us will be well within the weak coupling regime.

II. ELECTROMAGNETISM

Jackson [3] has criticized the use of the equilibrium polarisation of the spin of an electron in a storage ring as a detector of the thermal nature of the field as seen by an accelerated observer. Even Bell and Leinaas in their second paper raise doubts about regarding the residual polarisation as a measure of the temperature of the thermal bath seen by a circularly accelerating electron. The distribution amongst the states of the spin simply does not look thermal, given the interaction of the spin with the field. However they do not explain why the detector response differs from the naive explanation. The expectation that the spin polarisation should simply have a distribution which corresponds to the thermal distribution of the field is too naive. One must understand both the interaction of the spin with the bath and with other degrees of freedom of the system itself. After all, the snow seen on the screen of a TV set has a temperature vastly higher than the room temperature fluctuations which cause it, due to the interaction with other components (eg, amplifiers, etc). The fact that the observed temperature is so much higher than the temperature which causes the snow is not a failure of the TV set to act as a detector of thermal fluctuations, but rather a failure to understand how the TV set works. As we shall see, the electron also has similar complex behaviour including time dependent couplings which act as “amplifiers”.

The first question to ask is whether or not the electromagnetic field appears to be a thermal field to the circularly accelerating observer. Clearly the fluctuations are Gaussian, since they are simply the vacuum fluctuations of the electromagnetic field, which are Gaussian. Thus the key question is whether or not they are stationary. Ie, is the two-point correlation function of the fields a function only of the difference in times, or does it also depend on the absolute time?

For the scalar field this question is easily answered in the affirmative, but the electromagnetic field, being a vector field, has the additional complication of needing to have the components one is correlating be specified first. Ie, even for a static observer in the vacuum, the correlation function \(< e(t)_k B(t) \cdot e(t')_l B(t') \cdot >\) is not stationary for arbitrary time dependent unit vector \(e_k\). In the case of the circular acceleration case as there are numerous frames in which one could calculate the correlation function between the components of the
electromagnetic field. Three in particular spring to mind, namely the inertial system of basis vectors (ie, parallel transported along the curve), the rotating system (which rotates at the same rate as the particle revolves around the circle) or the Fermi-Walker transported system. Since each of these sets of basis vectors rotates with respect to the other, one would expect that only one of them will be the one (if any) in which the fluctuations are stationary. The answer is that it is the basis set which co-rotates with the particle. One can define a Killing vector field, a sum of time translation and rotation
\[ \zeta^\mu = (1, -\Lambda y, \Lambda x, 0) \] (15)
such that the particle follows one of the integral curves of this Killing vector field. It will then be the components of the electromagnetic field as Lie transported along this field which will be stationary. (Since the vacuum state is invariant both under time translations and rotations, it is also invariant under this Killing transformation). However, the electron effectively couples to only a few of these components. It will thus turn out for the electron that it is the correlation functions in the FW frame which are important, and the particular components of importance will again be stationary in that frame.

The correlation function for the electromagnetic field in Minkowski coordinates is
\[ < F(t, x)_{\mu\nu} F(t', x')_{\rho\sigma} > = 4 \partial_{[\mu} \eta_{\nu]} \partial_{[\rho} \eta_{\sigma]} \frac{1}{\pi ((t - t')^2 - (x - x')^2)} \]
\[ = 32 \frac{\Delta x_{[\mu} \eta_{\nu]} \Delta x_{\rho]} \eta_{[\sigma]}}{\pi (\Delta x_\alpha \Delta x^\alpha)^3} - 8 \frac{\eta_{[\mu] \eta_{\nu]} \eta_{[\rho]}}{\pi (\Delta x_\alpha \Delta x^\alpha)^2} \] (16)
where the square brackets around indices indicates anti-symmetrisation. \( \Delta x^\mu \) is the four-vector \([t - t', x - x']\]. Define a set of co-rotating basis vectors appropriate to the uniformly circling particle
\[ e^\mu_\tau = \gamma[1, -R\Lambda \sin(\Lambda \gamma \tau), R\Lambda \cos(\Lambda \gamma \tau), 0] \]
\[ e^\rho_\tau = [0, \cos(\Lambda \gamma \tau), \sin(\Lambda \gamma \tau), 0] \]
\[ e^\phi_\tau = \gamma[R\Lambda, -\sin(\Lambda \gamma \tau), \cos(\Lambda \gamma \tau), 0] \]
\[ e^z_\tau = [0, 0, 0, 1] \] (17)
where \( e^\mu_\tau \) is the tangent vector to the curve the particle is following, I get for the correlation functions of interest
\[ < B_\rho(\tau) B_\rho(0) > = < E_\rho(\tau) E_\rho(0) > \]
\[ = \frac{4\gamma^2}{\pi D^3} \left( \cos(\gamma \Lambda \tau)(\gamma^2 \tau^2 - 2R^2(1 + \lambda^2 R^2)) - 4\Lambda R^2 \gamma \tau \sin(\Lambda \gamma \tau) + R^2(2 + 2\Lambda^2 R^2 + \Lambda^2 \gamma^2 \tau^2) \right) \] (18)
\[ < B_\rho(\tau) B_\phi(0) > = < E_\rho(\tau) E_\phi(0) > \]
\[ = \frac{4\gamma^2 \tau}{\pi D^3} \left( -2\Lambda R^2 + 2\Lambda R^2 \cos(\Lambda \gamma \tau) + \gamma \tau \sin(\Lambda \gamma \tau) \right) \] (19)
\[ < B_\phi(\tau) B_\phi(0) > = < E_\phi(\tau) E_\phi(0) > = \frac{4}{\pi D^3} \left( \cos(\Lambda \gamma \tau)(\gamma^2 \tau^2 + 2R^2) - 2R^2 \right) \] (20)
\[ < B_z(\tau) B_z(0) > = < E_z(\tau) E_z(0) > \]
\[ < B_z(\tau) B_z(0) > = < E_z(\tau) E_z(0) > = \frac{4}{\pi D^3} \left( \cos(\Lambda \gamma \tau)(\gamma^2 \tau^2 + 2R^2) - 2R^2 \right) \] (21)
\[
\gamma^2 \pi D^3 \left( \cos(\Lambda \gamma \tau) \left( \gamma^2 \tau^2 \Lambda^2 R^2 - 2R^2 - 2\Lambda^2 R^4 \right) \\
-4 \sin(\Lambda \gamma \tau) \gamma \tau \Lambda R^2 + \gamma^2 \tau^2 + 2\Lambda^2 R^4 + 2R^2 \right)
\]

\[
< B_\rho(\tau) E_z(0) > = -< B_z(\tau) E_\mu(0) >= -\frac{4R\gamma^2}{\pi D^3} \left( \cos(\Lambda \gamma \tau)(\Lambda \gamma^2 \tau^2 - 4\Lambda R^2) \\
-2\gamma \tau \sin(\Lambda \gamma \tau)(1 + \Lambda^2 R^2) + \Lambda(\gamma^2 \tau^2 + 4R^2) \right)
\]

\[
< B_\phi(\tau) E_z(0) > = < B_z(\tau) E_\phi(0) > = \frac{4R\gamma^2}{\pi D^3} \left( 2\cos(\Lambda \gamma \tau) + \sin(\Lambda \gamma \tau) \Lambda \gamma \tau - 2 \right)
\]

All correlations are zero, except those which can be derived from these by the symmetry, \(< A(\tau) B(0) >= < B(0) A(\tau) >\). These correlations functions are thus much more complex than those for the uniformly accelerating observer, or a stationary observer with the field in a thermal state, where all cross correlations are zero, and all diagonal correlations are the same as each other.

Let us now examine the equation of motion of the electron, and its interaction with the field fluctuations along the path of the particle. I will attack the problem in three steps. Firstly I will examine the equations of motion of the spin on its own, assuming that the electron follows the circular orbit exactly (ie, all fluctuations from this path will be ignored.) Then I will include the effects of fluctuations in the path in the vertical (z) direction. This will introduce another internal system, a harmonic oscillator in the z component of the motion. Oscillations in the other two directions turn out not to couple to the spin system to lowest order, and will be ignored. I will calculate the the harmonic oscillations in the z direction driven by the electromagnetic field and the effects of such oscillations on the spin. Those effects arise because of the changes in the Thomas precession of the spin induced by this extra motion.

The motion of the spin of the electron has been extensively studied. The electron spin, in the absence of a magnetic field in the rest frame of the spin, is Fermi-Walker transported along the path of the particle. Fermi-Walker transport for a vector which is spatial in the rest frame of the particle is parallel transport continually projected back to the spatial subspace (ie orthogonal to the velocity vector). (In general parallel transport along a non-geodesic will take a vector orthogonal to the velocity vector and create components parallel to the velocity. Fermi-Walker transport continuously subtracts that velocity-parallel component from the motion of the particle.) Formally it is defined for a vector \(w^\mu\) which is orthogonal to the velocity of the particle \(V^\mu\) as

\[
\frac{D_{FW} w^\mu}{D\tau} = \frac{Dw^\nu}{D\tau} (\delta^\mu_\nu - V^\nu V^\mu) = \frac{Dw^\mu}{D\tau} + V^\mu a^\nu w^\nu
\]

(24)

where \(V^\mu\) is the tangent vector to the curve the particle follows, and \(a^\mu\) is the acceleration vector \(\frac{D^2 V^\mu}{D\tau^2}\). Note that this equation maintains the relation \(V^\mu w^\mu = 0\), ie the vector \(w^\mu\) remains spatial in the rest frame of the particle. The presence of a magnetic field in the rest frame of the particle exerts a torque on the spin. In the circular accelerating system, the spin equation is simply

\[
\frac{D_{FW} s^\mu}{D\tau} = g_{\mu 0} F^{\rho \nu} s^\nu (\delta^\rho_\mu - V^\rho V^\mu)
\]

(25)
or, explicitly expanding the Fermi-Walker derivative for the circularly accelerating electron, and the definition of the components $s_i$ in the co-rotating frame

$$\frac{ds_i}{d\tau} = \epsilon_{ijk} (g\mu_0 B^j - \gamma^2 \Lambda \epsilon^j_z) s^k$$  \hspace{1cm} (26)

where $g$ is the “g” factor for the spin, and $\mu_0$ classical dipole moment $\frac{e^2}{2m}$, and $\epsilon_{ijk}$ the antisymmetric tensor for the spatial indices, such that $\epsilon_{123} = 1$. Spatial indices are raised and lowered by the 4-metric and thus an extra minus sign tends to appear in the formulas due to my sign convention for the 4-metric. The external field portion of $B_j$ is the constant field (in the rest frame of the electron) $\gamma B_0 \epsilon_{zj}$ plus the vacuum fluctuations $B_j(\tau)$ whose correlations are calculated above. Note that I have chosen the constant $B_0$ field (in the inertial frame) to lie along the $z$ axis. If we assume, as I will, that the same magnetic field is responsible for the circular trajectory of the particle, then

$$\Lambda = \frac{eB_0}{m\gamma} = 2\mu_0 B_0/\gamma$$ \hspace{1cm} (27)

These equations of motion can be derived from a Hamiltonian. Taking $s^i = \frac{1}{2} \sigma^i$, the Pauli spin matrices, the Hamiltonian is

$$H = g\mu_0 \frac{1}{2} \sigma \cdot B - \gamma^2 \Lambda \sigma_z$$ \hspace{1cm} (28)

where $B$ is the total magnetic field as seen in the rest frame of the electron. This is composed of the fixed field $\gamma B_0$ in the $z$ direction, and the fluctuating vacuum field, $B^i$. To calculate the equilibrium population, I use the standard technique of time dependent perturbation theory to calculate the transition probabilities of the spin going from one state to the other. Defining,

$$\sigma_+ = \frac{1}{2} (\sigma_\rho + i \sigma_\phi)$$ \hspace{1cm} (29)

$$\sigma_- = \frac{1}{2} (\sigma_\rho + i \sigma_\phi)$$ \hspace{1cm} (30)

$$B_+(\tau) = \frac{1}{2} (B_\rho(\tau) + i B_\phi(\tau))$$ \hspace{1cm} (31)

$$B_-(\tau) = \frac{1}{2} (B_\rho(\tau) - i B_\phi(\tau))$$ \hspace{1cm} (32)

we have

$$H = \frac{1}{2} \left( (g\mu_0 (\gamma B_0 + B_z) - \gamma^2 \Lambda) \sigma_z + g\mu_0 (B_+ \sigma_- + B_- \sigma_+) \right)$$ \hspace{1cm} (33)

Defining

$$\Omega = g\mu_0 \gamma B_0 - \gamma^2 \Lambda = (g - 2) \mu_0 \gamma B_0$$ \hspace{1cm} (34)

and noticing that the component $B_z$ of the vacuum field in the $z$ direction will not (to lowest order) affect the transition probability between the two levels of the spin in the effective $\Omega$ field, the Hamiltonian becomes
\[ H = \frac{1}{2} \Omega \sigma_z + B_+ \sigma_- + B_- \sigma_+ \]  

(35)

Using first order time dependent perturbation theory, the transition probability per unit time to go from the lower to the upper state or the upper to the lower state is

\[ P_\uparrow = \lim_{T \to \infty} \frac{1}{2T} \left| \sum_{\sigma \in \{1,0\}} \int_{-T}^{T} < 1, \sigma \bigg| B_+(\tau) \bigg| 0, \sigma > d\tau \right|^2 \]  

(36)

\[ = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \int_{-T}^{T} < 0 \bigg| B_+(\tau) B_-(\tau') \bigg| 0 > e^{-i\Omega(\tau-\tau')} d\tau d\tau' \]  

(37)

\[ = \int < B_+(\tau - i0) B_-(0) > e^{-i\Omega \tau} d\tau \]  

(38)

where the last line has used the stationarity of the correlation function. Also the symmetry of the electromagnetic correlation function \(< B_\uparrow(\tau + i0) B_\downarrow(0) >= < B_\downarrow(0) B_\uparrow(t - i0) >= < B_\downarrow(-\tau + i0) B_\uparrow(0) > I also get \]

\[ P_\downarrow = \int < B_\downarrow(\tau + i0) B_\downarrow(0) > e^{-i\Omega \tau} d\tau \]  

(39)

Assuming that the up conversions, and the down conversions are uncorrelated (which the long time scale of the decay process may make a good approximation), the ratio of these is the ratio of the equilibrium populations of the two levels.

\[ \frac{N_\uparrow}{N_\downarrow} = \frac{P_\uparrow}{P_\downarrow} \]  

(40)

I get that

\[ < B_+(\tau) B_-(0) >= \frac{1}{4} \left( < B_\rho(\tau) B_\rho(0) + B_\phi(\tau) B_\phi(0) - i(B_\rho(\tau) B_\phi(0) - B_\rho(-\tau) B_\phi(0)) > \right) \]

\[ = \frac{\gamma^4}{\pi D^3} \left( \cos(\Lambda \gamma \tau) \left[ 2\tau^2 - \tau^2 \lambda^2 R^2 + 4\Lambda^4 R^6 - 4\Lambda^2 R^4 + i(4\Lambda^3 R^4 \tau - 4\Lambda R^2 \tau) \right] \right. \]

\[ \left. - \frac{2\tau}{\gamma} \sin(\Lambda \gamma \tau) \left[ \lambda^2 R^2 + i\tau + \frac{4\Lambda^2 R^4}{\gamma^2} + \lambda^2 R^2 \tau^2 + i\frac{4\Lambda R^2 \tau}{\gamma^2} \right] \right) \]  

(41)

in the co-rotating frame. Note that this particular correlation function, unlike the general electromagnetic correlation function, is stationary in any frame which rotates uniformly about the \( z \) axis. Under a transformation to another rotating frame (around the \( z \) axis) the \( B_+(\tau) \) field goes to \( e^{i\nu} B_+(\tau) \) while \( B_-(\tau) \) goes to \( e^{-i\nu} \). Thus the correlation function goes to \(< B_-(\tau) B_+(\tau') >= e^{-i\nu(\tau-\tau')} < B_-(\tau - \tau') B_+(0) > \) which is again stationary. One therefore expects to find a “thermal” distribution in all frames. However, this distribution will have a temperature which will be wildly varying except in one particular frame. This will play a role in the interpretation of the results presented below.

For large \( \gamma \), just as for the scalar case, the dominant poles in the denominator are again at \( \tau = 0 \) and at \( \tau = \pm \frac{2\gamma^3}{\gamma^2 R} \). However the order of these poles changes. Since the numerator is quadratic in \( \tau \) at \( \tau = 0 \), the former pole is a quadratic pole while the latter poles are triple poles. Evaluating the residue at these poles, and keeping terms only to lowest order in \( 1/\gamma \), I find
\[ P_\uparrow = 4(W + 1)^2 \mu_0^2 (\gamma^2 \Lambda)^3 \left( F_1(W) - \frac{|\Omega|}{\Omega} F_2(W) \right) e^{-2\sqrt{3}|W|} - \Theta(-W)F_0(W) \] (42)

\[ P_\downarrow = 4(W + 1)^2 \mu_0^2 (\gamma^2 \Lambda)^3 \left( \Theta(W)F_0(W) + \left[ F_1(W) - \frac{|W|}{W} F_2(W) \right] e^{-2\sqrt{3}|W|} \right) \]

where \( W = \Omega/\Lambda \gamma^2 = \Omega/a = g/2 - 1 \). The common factor of \( 4(W + 1)^2 \mu_0^2 \) term is just \((g\mu_0)^2\).

The functions \( F_i \) are given by

\[ F_0 = \frac{2}{3}(W + 1)((W + 1)^2 + 1) \] (43)

\[ F_1 = \frac{\sqrt{3}}{288} \left( 84W^2 - 168W + 133 \right) \] (44)

\[ F_2 = \frac{1}{288} \left( 144W^2 - 294W + 192 \right) \] (45)

In figure 5 I have plotted the polarisation

\[ \mathcal{P} = \frac{P_\uparrow - P_\downarrow}{P_\uparrow + P_\downarrow} \] (47)

vs the normalised frequency \( W \), or the anomalous \( g \) factor \( g/2 - 1 \).

In which frame should one ask about the thermality of the spectrum of fluctuations? For an electromagnetic dipole moment, the damping is proportional to \( \frac{D_{FW}m}{D_{FW}m} - \alpha^2 \frac{D_{FW}m}{D_{FW}m} \) where \( D_{FW} \) refers to the Fermi-Walker derivative of the magnetic dipole moment \( m \). I.e., the damping is related to the rate of change of the magnetic moment in the Fermi-Walker frame. This thus makes the Fermi Walker Frame the natural frame to look at the fluctuations in. As argued above, the relevant correlation function for the spin polarisation is stationary.
in all frames, including the Fermi-Walker frame. This thus makes that frame the natural one to use to explain the results of the spin polarisation. Note that the $P_\uparrow - P_\downarrow$ is just proportional to $(W + 1)((W + 1)^2 + 1)\Omega_0^3$, where $\Omega_0$ is the precession frequency of the spin $\gamma^2\Lambda$, with $g$ factor 2 in a $B$ field of magnitude $B_0$. Since $g\Omega_0$ is the precession frequency in that $B$ field with g-factor of $g$, and since $\Omega_0$ is also the acceleration of the electron, this is just what we would expect from the radiation reaction term if we replace the Fermi-Walker derivative with the precession frequency $g\Omega_0$. But this is precisely what we would expect by the fluctuation-dissipation theorem, as the fluctuations from the pole at $t = 0$ in the Wightman function is just the terms required to maintain the commutation relations of the system against dissipation.

Thus in the Fermi-Walker frame, we can calculate the temperature of the radiation bath by

$$T = -\frac{g\Omega_0}{\ln \left( \frac{T_s}{T_F} \right)}$$  \hspace{1cm} (48)

This temperature is frequency dependent (or since the frequency here is proportional to $g$, is $g$ dependent), and is regular in this frame. Figure 6 is a plot of the fluctuation Temperature seen by the spin vs the $g$ factor, with the Temperature plotted in units of the acceleration $\gamma^2\Lambda$. Note that the temperature in this case is slightly more complex than it was in the scalar case, but has a very similar form, namely it varies slightly (by about 20%) across the frequency range. The spin polarisation is exactly what one would expect naively from the electron’s coming into equilibrium with this temperature.

![Figure 6](image_url)

*The effective Temperature of the EM field in the Fermi-Walker Frame*

Note that if we had chosen the wrong frame, like the co-rotating frame, to try to calculate the effective temperature, we would have obtained a very different result for the temperature. The new effective temperature $T_{corot} = T_{FW}W/(W + 1)$. Since $T_{FW}$ is regular with a temperature of about .2$\Lambda$, the co-rotating temperature will diverge at $W = -1$ and go to zero at $W = 0$. This effective temperature of the magnetic fluctuations in the co-rotating
frame would thus be a wildly varying function of frequency, going negative for $W$ between zero and one. This would be a key indication that one had chosen the wrong frame in which to calculate the effective temperature, even if the fluctuations are gaussian and stationary in that frame.

Were the electron fixed in its orbit (ie, were the particle not allowed to deviate from the given orbit), then the above analysis would be complete. However, for a real electron, the particle is not fixed in its orbit, but can deviate therefrom. These excursions themselves can be regarded as electric dipole moments around the fiducial circular path, and the will themselves couple to the electromagnetic fields. They will also couple to the spin. Let me begin by analysing these dipole fluctuations in the absence of the spin and show that they themselves also act as detectors for other components of the electromagnetic field.

Let me assume that there is a Harmonic restoring force which drives the particle back toward its fiducial orbit. This force will be assumed to have a frequency $K$ and a damping term $\kappa$. The equation of motion for these excursions in the various directions is therefore

$$\ddot{X}^i + K^2 X^i = \frac{e}{m} E^i$$

where $E^i$ has two components, the quantum fluctuations, and the radiation reaction term. As I showed elsewhere [10] (and as BL derived in a different manner), the radiation reaction term is

$$E_{RR}^i = \frac{2}{3} D_{FW} \left( -\frac{d^3}{d\tau^3} + a^2 \frac{d}{d\tau} \right) e X^i$$

since $e X^i$ is the dipole moment corresponding to this deviation from the fiducial path. I will be interested in the fluctuations in the $z$ direction since those are the ones which will couple to the spin. For the $z$ direction, the FW derivative is the same as the ordinary one. Assuming the radiation reaction force to be small, and that the time derivatives in the radiation reaction term can be approximated by the harmonic frequency, we have

$$\ddot{Z} + \frac{2}{3} \left( -\frac{d^3}{d\tau^3} + a^2 \frac{d}{d\tau} \right) Z + K^2 Z = \frac{e}{m} E_z$$

which has as solutions in the frequency domain for frequencies well below the classical frequency (the speed of light divided by the classical radius of the electron)

$$Z(\omega) = \frac{e}{m} \frac{E_z(\omega)}{-\omega^2 + i \frac{e^2}{3m} \omega (\omega^2 + a^2) + K^2}$$

where $Z(\omega) = \int e^{-i\omega \tau} Z(\tau) d\tau$. Note that this equation has a pole near $-i3m/2e^2$ which corresponds to the classical runaway solutions. Since the damping term is only of importance near the resonance ($\omega = K$), the $\omega^2 + a^2$ term can be replaced by $K^2 + a^2$, to give a traditional damped oscillator equation with damping coefficient

$$\kappa = \frac{2e^2}{3m} (K^2 + a^2)$$

Note that I have neglected the terms corresponding to the initial conditions for this $z$ displacement, assuming that we have waited long enough for these to damp out. For the oscillator, we can again define raising and lowering operators in the traditional way
\[ A = \sqrt{K}mZ + iPz \frac{1}{\sqrt{Km}} \]  

(54)

The transition probability for transitions to and from the \( n^{th} \) states are given by

\[
P_{\uparrow n} = \frac{1}{2T} (e)^2 \Sigma|1>| \int < n + 1|Z|n > < 1|E_z(\tau)|0 > d\tau^2
\]

\[
= \frac{e^2}{2mK} (n + 1) \int < E_x(\tau - i0)|E_z(0) > e^{-iK\tau} d\tau
\]

\[
= (n + 1) \frac{e^2}{2mK} (\Theta(-K)(\frac{4}{3}K(K^2 + a^2))
\]

\[
+ \frac{1}{i2} e^{-2|K|\sqrt{3}/a} (12\sqrt{3}K^2 + 30|K|a + 13\sqrt{3}a^2)) \]

\[
P_{\downarrow n} = P_{\uparrow n-1} + n \frac{e^2}{2mK} (\frac{4}{3}K(K^2 + a^2))
\]

(55)

Requiring detailed balance, so that the transitions between the states \(|n > \) and \(|n + 1 > \) balance requires that there be a population balance of

\[
\frac{N_{n+1}}{N_n} = \frac{P_{\uparrow n}}{P_{\downarrow n+1}}
\]

(58)

which is independent of \( n \). This gives an effective temperature of

\[
T = -\frac{K}{\ln(\frac{N_{n+1}}{N_n})}
\]

(59)

which I have plotted in figure 7 as a function of \( K \). Again the system sees a thermal spectrum with frequency dependent temperature which is similar, but not identical to both that seen by the spin and that seen by a scalar detector. (It is in fact more constant than that seen by a scalar detector).

![Figure 7](image-url)
The effective Temperature of the EM field affecting the oscillations in the z direction

Finally we must take into account the coupling between these two “detectors”, namely the spin and the z oscillations. Each separately acts like a detector and sees a thermal spectrum of fluctuations of very similar temperatures. The z oscillations couple to the spin through two routes. The first is if the classical magnetic field strength varies in direction or amplitude with position in the rest frame of the particle. The particle will then see a different field depending on where it is located. The second is an interaction through the velocity of the particle. This will happen both because the particle will see the strong electric field (causing the acceleration) as having a magnetic component due to any velocity of the electron from the fiducial circular orbit. Furthermore, the particle will have a different FW frame because of any such velocity, causing an extra “Thomas precession”. Variations in the magnetic field off the fiducial orbit are typically present, and I will assume that the magnetic field is arranged in the “weak focusing” configuration such that an effective restoring force to vertical oscillations is provided by the presence of off-orbit radial magnetic fields.

Let us look at the Thomas precession terms first. The Fermi-Walker transport equations are

\[ \frac{D_{FW} s^\mu}{Ds} = \frac{Ds^\mu}{D\tau} - V^\mu V^\alpha \frac{Ds_\alpha}{D\tau} = \frac{Ds^\mu}{Ds} + V^\mu a_\alpha s^\alpha \] (60)

where \( s \) is the path length along the path of the particle. I will work only to lowest order in the deviation, \( Z \), from the path. We can define an orthonormal tetrad, based on the vectors \( e^\mu_\alpha \) defined above by

\[ e^\mu_1 = V^\mu = e^\mu + v_z e^\mu \]
\[ e^\mu_\rho = e^\mu_\rho \]
\[ e^\mu_\phi = e^\mu_\phi - \lambda v_z e^\mu_z \]
\[ e^\mu_z = e^\mu_z + v_z e^\mu_1 + \lambda v_z e^\mu_\phi \] (61) (62) (63) (64)

where \( v_z = dZ/d\tau \), and \( \lambda \) is an arbitrary constant. Defining the components of the spin by

\[ s_i = e^\mu_i s_\mu \] (65)

where \( i \) is one of \( \rho, \phi, z \), the derivatives of \( s_i \) are given by

\[ \frac{ds_i}{ds} = \frac{D_{FW} s_\mu e^\mu_i}{Ds} + s_\rho \frac{D_{FW} e^\mu_i}{Ds} \] (66)

where \( s \) is the path-length parameter along the path actually followed by the particle. But to lowest order in \( v_z \), \( ds = d\tau \). We then have

\[ \frac{ds_1}{d\tau} = \frac{D_{FW} s_\mu e^\mu_1}{d\tau} + \gamma^2 \Lambda [s_\phi + (R\Lambda + \lambda) v_z s_z] \] (67)
\[ \frac{ds_\rho}{d\tau} = \frac{D_{FW} s_\mu e^\mu_\rho}{d\tau} - \gamma^2 \Lambda s_\rho - \lambda v_z s_z \] (68)
\[ \frac{ds_\phi}{d\tau} = \frac{D_{FW} s_\mu e^\mu_\phi}{d\tau} - \gamma^2 \Lambda (R\Lambda + \lambda) s_\rho + \lambda v_z s_\phi \] (69)
But, $D_{FW}s_{\mu}/ds$ is just the local change of $s_{\mu}$ caused by the local $B$ field.

I will assume, as in weak focusing, that the magnetic field varies off the fiducial path, and in particular has a non-zero radial component which varies with height ($z$). To lowest order, in the rest frame of the ring, $B_\rho = -bz$. In the fiducial rest frame of the electron, it sees both a magnetic field of $\gamma bz$ in the $\rho$ direction and an electric field in the $z$ direction of $-\gamma R\Lambda bz \approx -\gamma bz$. This electric field will act as the harmonic restoring force for the electron’s excursions in the $z$ direction, giving a relation between the restoring force of the oscillator and this field of

$$K^2 = \frac{e}{m} \gamma b$$ (70)

Assuming that $Z$ and the quantum fields $E$, $B$ are of the same order, we have the local fields seen by the spin are

$$B_\rho = F_{\mu\nu} \tilde{e}^\mu_{\phi} \tilde{e}^\nu_{\rho} = B_\rho$$ (71)
$$B_\phi = F_{\mu\nu} \tilde{e}^\mu_{\zeta} \tilde{e}^\nu_{\phi} = -z\gamma(R\Lambda + \lambda) \gamma B_0 + B_\phi$$ (72)
$$B_z = F_{\mu\nu} \tilde{e}^\mu_{\rho} \tilde{e}^\nu_{z} = \gamma B_0$$ (73)

which drive the Fermi-Walker changes of the spin. The equations of motion can now be derived from the Hamiltonian

$$H = \left[ (g\mu_0\gamma(B_0 + B_z) - \gamma^2\Lambda)s_z + g\mu_0(B_\phi s_\phi + B_\rho s_\rho) \right]$$
$$+ \left\{ - \left( v_z(R\Lambda + \lambda)(g\mu_0\gamma B_0 - \gamma^2\Lambda) \right) s_\phi + (-\gamma bZ + \lambda \dot{v}_z) s_\rho \right\}$$ (74)

where the term in the square brackets is the term we have already used for the spin travelling along the fiducial path, and the terms in the curly brackets, $H_I$, are caused by the motion away from the fiducial path.

I will use this Hamiltonian to calculate the effects of the deviations from the path from the fiducial path. In doing so I will take $\lambda = 0$. As I will argue below, the results are independent of $\lambda$, at least to the order to which I am carrying out the calculation.

For the perturbations on the spin system, write the above equations in terms of $\sigma_+$ and $\sigma_-$. Using $\gamma \Lambda = 2\mu_0 B_0$ and $\Omega = \mu_0(g - 2)\gamma B_0$, and dropping the $B_z s_z$ term as not contributing to the spin flip transition rate, I define

$$Z_\pm = \frac{1}{2}(\pm i\dot{Z}\Omega + \frac{g}{2}K^2Z)$$ (75)

to give

$$H_I = -Z_- \sigma_+ - Z_+ \sigma_-$$ (76)

If we were to assume that the coupling to $B$ field fluctuations by the spin are zero, than these cross couplings from the motion of the harmonic oscillators would give transitions of the form

$$P_{I\uparrow} = \langle Z_+(\tau - i0)Z_- > e^{-i\Omega\tau} d\tau$$ (77)
$$P_{I\downarrow} = \langle Z_+(\tau + i0)Z_- > e^{-i\Omega\tau} d\tau$$ (78)
I get

\[ P_{T\downarrow} = \mu_0^2 \left( \frac{\Omega^2 - \frac{g}{2} K^2}{(\Omega^2 - K^2)^2 + \Omega^2 \kappa^2} \right) \int \langle E_z(\tau - i0)E_z(0) \rangle e^{-i\Omega \tau} d\tau \]  

\[ = \mu_0^2 (\gamma^6 \Lambda^3) \left( \frac{\Omega^2 - \frac{g}{2} K^2}{(\Omega^2 - K^2)^2 + \Omega^2 \kappa^2} \right) \left( -G_0 \Theta(-W) + e^{-2\sqrt{3}|W|} G_1 \right) \]  

\[ P_{I\downarrow} = P_{T\downarrow} + \mu_0^2 (\gamma^6 \Lambda^3) \left( \frac{\Omega^2 - \frac{g}{2} K^2}{(\Omega^2 - K^2)^2 + \Omega^2 \kappa^2} \right) \]  

where

\[ G_0 = \frac{4}{3} W(W^2 + 1) \]  

\[ G_1 = \frac{1}{72} \left( \sqrt{3}(12W^2 + 13) + 30|W| \right) \]  

However there are correlations between the components \( E_z \) which drive the vertical oscillations, and the \( B_{\pm} \) components which drive the spin flips. These add additional complications to the expressions. Let me define these correlation spin flip probabilities by

\[ P_{C\uparrow} = -g\mu_0 \left( \int \langle B_+(\tau - i0)Z_-(0) \rangle + \langle Z_+(\tau - i0)B_-(0) \rangle e^{-i\Omega \tau} d\tau \right) \]  

\[ = -g\mu_0^2 \left( \frac{\Omega^2}{K^2 - \Omega^2} \right) \int \left( \frac{\langle B_+(\tau - i0)E_z(0) \rangle}{(K^2 + i\Omega\kappa - \Omega^2)} + \frac{\langle E_z(\tau - i0)B_-(0) \rangle}{(K^2 - i\Omega\kappa - \Omega^2)} \right) e^{-i\Omega \tau} d\tau \]  

\[ \approx -g\mu_0^2 \left( \frac{\Omega^2}{K^2 - \Omega^2} \right) (\gamma^2 \Lambda^3) \left( -\Theta(-W) L_0 + e^{-2\sqrt{3}|W|} (L_1 + \frac{|W|}{W} L_2) \right) \]  

where I have have assumed \( \kappa \) is very small. Here we have

\[ L_0 = -\frac{1}{3}(2W + 1) \]  

\[ L_1 = -\frac{\sqrt{3}}{24}(4W^2 - 5W + 3) \]  

\[ L_2 = \frac{1}{24}(6W^2 - 10W + 4) \]  

The total interactive transition rate is then

\[ P_{T\uparrow} = P_{\uparrow} + P_{C\uparrow} + P_{I\uparrow} \]  

and similarly for \( P_{T\downarrow} \). These lead to the polarisation curve of figure 1.

There are a number of points which must be emphasised. The first is that it is the transverse oscillations of the electron which drive the extra terms in the transition. In the above I assumed that the only effects which drive those transverse oscillations are the vacuum fluctuations of the electric field in the rest frame of the particle. If there are other sources of noise which drive those transverse oscillations (field misalignments, interactions with other particles in the beam, etc) then those will all also contribute to the spin polarisation. (Note that the action of the spin system on the oscillator is not a source of significant noise. The
coupling between the spin and the Harmonic oscillator—reduced to dimensionless variables—is of order $\sqrt{\frac{\hbar K}{mc^2}} (K \pm \Omega)$ which is clearly tiny. The only reason the oscillator effects the spin is that its coupling to the electric field in the same units is enhanced by the factor $\sqrt{\frac{mc^2}{\hbar K}}$ over the direct coupling between the magnetic fluctuations and the spin, compensating for the weakness of the coupling between the two systems).

Secondly, for $g = 2$ the factor $\frac{gK^2}{K^2 + \kappa^2 - \Omega^2}$ is unity except at the resonance. However for $g$ not equal to 2, the system behaves very differently below the resonance than above.

In the co-rotating frame, the transition rates caused by the direct coupling to the local magnetic field, and the transition rate induced by the motion of the electron have very different forms. The direct transition rate is centered at $g=0$ (figure 5) while those for the oscillator are centered at $g=2$. The final polarisation curve is thus a mixture of the two, resulting in the shift to about $g = .8$ of figure 1. Also, the relative weights of the two terms shift with frequency, giving the fine structure in the figure 1.

I have argued that both detectors are immersed in essentially identical temperature heat baths. Why would the interaction not then not bring the joint system to the same joint temperature? The important point here is that the Fermi-Walker frame is the natural frame for the spin, and is the frame in which the fluctuations look thermal. However, the interaction is static in the co-rotating frame. The $z$ oscillations and $E_z$ are unaffected by the transformation to the FW frame, and thus in this frame both systems independently have the same distribution. However, in that frame, the interaction between the oscillator and the heat bath is no longer static, but is time dependent. (although $Z$ is unaffected by the transformation, the coupling is through $\sigma_\rho$ and $\sigma_\theta$ which do change under the transformation.)

Because of this time dependent coupling, the harmonic system tends to preferentially drive up-transitions in the spin system (and the spin system preferentially drives down-transitions in the oscillator). The coupling acts like an amplifier, driving the spin system to its higher energy state (at least for $g > 0$). The power for this amplifier clearly comes from the motion around the storage ring.

The electron system thus acts like the snow on the TV screen. Without understanding the details of the system, one cannot conclude that the lack of thermal output (the spin’s polarisation is not thermally distributed) in the detector demonstrates a lack of thermal input (the fluctuations in the relevant components of the EM field in the FW frame).

In both DK and Jackson, the claim appears to be that the standard result is obtained for calculating the spin flip by the fluctuations with the electron assumed to be travelling along the fiducial circular path (see eqn 44-47 in Jackson and the first equation in section 5 for DK). However in both cases they assume that the fluctuating quantum fields cause a Thomas precession. They use the Thomas-BMT equations for the evolution of the spin, which assume that the fields drive the accelerations away from the fiducial path which drives the Thomas precession. However, this extra Thomas precession, caused by the quantum fields, is a kinematic effect, and arises solely because of deviations in the electron’s orbit. Neither Jackson nor DK explicitly examines the behaviour of the electron deviations from the fiducial orbit in computing the standard (figure 1) depolarisation for an electron. Their calculations thus effectively assume that $K = 0$ and that $\kappa = 0$.

Also, in my derivation above, I set $\lambda = 0$ which meant that the Thomas precession terms due to the $z$ motion depended only on the velocity in the $z$ direction. On the other hand, by
using the TBMT equations, Jackson and DK have an acceleration dependent term in their
equations (which they then set equal to the forces exerted on the particle by the vacuum
fields to get the TBMT equations). They were thus implicitly taking
\( \lambda = -R\Lambda \), which makes the Thomas precession terms depend only on the \( z \) acceleration. However, if one kept
the \( \lambda \) dependent terms in \( H_I \), this would add terms of the form
\( \lambda((\pm i\Omega \dot{Z} + \mp \dot{Z}) \) to \( Z_\pm \). But \( Z_\pm \) enters the equations for the spin flip probabilities integrated with \( e^{-i(\pm\Omega\tau)} \), which transforms \( \dot{Z} \) to \( i\Omega \dot{Z} \) and converts the \( \lambda \) dependent terms in \( Z_\pm \) to zero.

III. EMISSION OF RADIATION BY ACCELERATED PARTICLE

Unruh and Wald \[12\] examined the behaviour of a detector under uniform acceleration,
and in particular the emission of radiation by such a detector. We looked at two cases in
particular. In the one case we asked for the state of the field under the condition that a
two level detector was found at the end of the process to have been excited. In this case
they found that the field was in a single particle excited state. That single particle was
concentrated in the region which did not have causal contact with the detector. In the
case where the detector was found after the experiment to be unexcited, the field was in a
coherent superposition of an unexcited vacuum state, and a two particle excited state.

Here I will examine the question from a slightly different point of view. The detector is
taken to be a harmonic oscillator as above. I will ask what the state is of the radiation field
(the massless scalar field) without any measurement of the state of the detector.

The equation of motion for an internal oscillator coupled to a scalar field is
\[
\ddot{q} + 2\epsilon^2 q + \Omega^2 q = \epsilon \phi_0 \tag{85}
\]
\[
\partial_\tau^2 \phi - \nabla^2 \phi = \epsilon \int q(\tau) \delta^4(x^\mu - X^\mu(\tau)) d\tau \tag{86}
\]
where \( X^\mu(\tau) \) are the coordinates of the particle at proper time \( \tau \), and
\[
\phi = \phi_0 + \int G(x^\mu - X^\mu(\tau)) d\tau \tag{87}
\]
with \( G \) the retarded Green’s function. The solution for \( q \) is
\[
q(t) = \epsilon \int^\tau \frac{\sin(\Omega(\tau - \tau'))}{\Omega} e^{\lambda(\tau - \tau')} \phi_0(\tau') d\tau' \tag{88}
\]
while the solution for the field \( \phi \) is
\[
\phi(t, x) = \phi_0(t, x) + \epsilon \int q(\tau) \Theta(t - T(\tau)) \delta((t - T(\tau))^2 - (x - X(\tau))^2) d\tau \tag{89}
\]
Ie, the final outgoing field is a linear function of the ingoing field. This implies that the
annihilation and creation operators or the outgoing field are linear combinations of those for
the ingoing field. If the outgoing annihilation operators were functions only of the ingoing
annihilation operators, which is what happens when the particle is unaccelerated, then the
state of the outgoing field will be indistinguishable from the vacuum state. On the other
hand, if the outgoing field’s (\( \phi \)’s) annihilation operators are function of both the ingoing
field’s (\(\phi_0\)’s) annihilation and creation operators, then that outgoing field will in general by a “squeezed state” with respect to the outgoing vacuum state. A squeezed state has the property that it is a coherent sum of even numbers of particle states. Ie, the detector “scatters” the vacuum fluctuations in the field \(\phi_0\) so as to produce correlated pairs of particles in the outgoing state. Since this state is again a gaussian state, it is completely characterised by the pairwise correlation function 

\[ C((t, x), (t', x')) = \langle \phi(t, x)\phi(t', x') \rangle \]

where

\[ C((t, x), (t', x')) = \langle \phi_0(t, x)\phi_0(t', x') \rangle + \epsilon^2 \int G((t', x'), \tau') \langle \phi_0(t, x)\phi_0(\tau') \rangle \]

\[ + \epsilon^4 \int \int G((t', x'), \tau') G((t, x), \tau) \delta((t - T(\tau))^2 - (x - X(\tau))^2) d\tau d\tau' \]

ACKNOWLEDGEMENTS

I would like to thank P. Chen and David Cline for inviting me to their respective conferences and reviving my interest in this subject. I would like to thank David Jackson whose doubts about treating the circular acceleration as a thermal bath incited me to look more closely at that problem. Finally I thank the CIAR and NSERC for support while this work was being done.
REFERENCES

[1] W.G. Unruh Phys Rev D14, 870 (1976)
[2] See for example Pisin Chen, Toshi Tajima ”Testing Unruh Radiation with Ulitarintense Lasers” SLAC-PUB-7543, Mar 1998
[3] J.D. Jackson- Comment made at the Quantum Aspects of Beam Physics, Monterey, Jan. 1998. Also ”On Effective Temperatures and Electron Spin Polarisation in Storage Rings” to appear in Proceedings of Quantum Aspects of Beam Physics Conference- ed. Pisin Chen
[4] J.S. Bell, J.M. Leinaas, Nucl. Phys. B212 131 (1983), J.S. Bell, J.M. Leinaas, Nucl. Phys. B284,488 (1987).
[5] Ya.S. Derbenev, A.M. Kondratenko, Zh. Exp. Theor. Phys. 64,1918,(1973)
[6] J.D. Jackson Rev. Mod. Phys 48, 417 (1976)
[7] D.P. Barber, S.R. Mane Phys.Rev. A37, 456 (1988)
[8] See also S.K. Kim, K.S. Soh and J.H. Yee Phys. Rev D35 557 (1987)
[9] See for example J.D. Jackson Electrodynamics, 2nd ed. John Wiley and Sons (New York, 1975) eqn 11.162 .
[10] W.G. Unruh LANL XXX Preprint Library [physics/9802047] ”Radiation Reaction Fields for an accelerated dipole for scalar and electromagnetic Radiation”
[11] W.G. Unruh, W.H. Zurek Phys Rev D40 1071(1989)
[12] W.G. Unruh, R. Wald Phys Rev D29 1047(1984)