Ohmic Decay of Magnetic Fields due to non-spherical accretion in the Crusts of Neutron Stars
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ABSTRACT

We consider magnetic field evolution of neutron stars during polar-cap accretion. The size of the polar cap increases as the field decays, and is set by the last open field line before the accretion disk. Below the polar cap we find the temperature to be so high that electron-phonon scattering dominates the conductivity. Outside the polar cap region, the temperature is such the conductivity is dominated by temperature independent impurity scattering which can be a few orders of magnitude larger than the electron-phonon conductivity. The time-scale for field decay is therefore initially given by impurity scattering dominated conductivity. When the field strength has been reduced to $\sim 10^8$ gauss the accretion is spherical and the time scale for field decay is given by the smaller electron-phonon scattering conductivity. The field strength is now reduced rapidly compared to before and this could be a reason for there being no pulsars known with field strengths below $10^8$ gauss. We also investigate the evolution of multipoles at the neutron star surface. We find that contribution from higher-order multipoles are at most 30% to that of the dipole mode.

1. Introduction

Since the discovery of pulsars there has been much discussion of observational evidence for decay of magnetic fields in neutron stars, as well as much theoretical work. As the reviews by Lamb (1991), Chanmugam (1992), and Phinney & Kulkarni (1994) indicate, there is at present no consensus on the question of whether or not magnetic fields in isolated neutron stars can decay significantly. The general view has been that the electrical conductivity of matter in the cores of neutron stars is so high that the characteristic decay time for fields generated by electrical currents in the core is greater than the age of the Universe. Recently Pethick & Sahrling (1995) showed that even if the conductivity in the core was small the shortest possible decay time is some two orders of magnitude longer than the decay time for configurations where the magnetic field is confined to the crust. An incorporation of general relativistic effects, Sengupta (1997), further reduces the decay rate. Still, millisecond pulsars have typical field strengths in the range $10^8$ – $10^{10}$ gauss compared to isolated radio pulsars which have typically field strengths around $10^{12}$ gauss and this indicates that during the spin-up phase the accretion process is reducing the field strength somehow.
Millisecond pulsars are generally found in binary systems where the companion star is a white dwarf with a mass less than a solar mass, \( M_{\odot} \). This system is called a Low-Mass-Binary Pulsar (LMBP) referring to the mass of the companion star. The progenitor to this system is thought to be the Low-Mass-Xray Binaries (LMXB) where a neutron star is accreting matter from a companion having a mass less than about 2 \( M_{\odot} \). The accreting matter is spinning up the neutron star to millisecond periods. For details concerning this process see reviews by Phinney & Kulkarni (1994) and Bhattacharya & van den Heuvel (1991) among others. The evolution of the binary system after the neutron star is formed either by a supernovae or tidal capture, is assumed to occur on at least two time scales when the companion star is in radiative equilibrium. At first the companion star is evolving on a nuclear time scale slowly filling its Roche lobe. When it has been filled up the matter overflows and falls onto the companion neutron star on a thermal time scale \( \tau_{th} = GM^2/RL = 5 \times 10^7 (M_{\odot}/M)^2 \) yrs, see Bhattacharya & van den Heuvel (1991) for details. If LMXB’s are the only progenitors to LMBP’s the lifetime of the LMXB, or the accretion phase of the binary system, must be of order \( 10^7 \) yrs. However, by using the amount of mass needed to be accreted to spin up the neutron star to a spin period \( P_i \) one finds a time scale that ranges from \( 10^8 - 10^{10} \times (P_i/2 \text{ ms})^{-4/3} \) yrs. This discrepancy suggests that there might other progenitors to the LMBP’s. For details see the review by Phinney and Kulkarni (1994). We will in this paper assume the accretion phase of progenitors to millisecond pulsars to last between roughly \( 10^7 - 10^9 \) yrs.

Matter accreting onto a neutron star is expected to be disrupted by the magnetic field at some radius, \( r_A \), sometimes called the Alfvén radius. Beyond this bare description there is no generally accepted view on how or where the matter attaches to the field lines and flows to the neutron star’s surface, or on the interaction of the field with the matter outside \( r_A \), despite a large number of papers on the subject, see King (1995). In the case of disk accretion there are two main approaches to the problem. In one (Ghosh & Lamb 1978, 1979a,b; Kaburaki 1986 and Wang 1987) the field is assumed to thread a large fraction of the disk because of Kelvin-Helmholtz instabilities. The other approach (Aly 1980; Anzer & Börner 1980, 1983; Scharlemann 1978) assumes the disk is a perfect conductor, completely excluding the field. In both approaches the matter is often assumed to leave the disk in a narrow transition zone at the inner edge (near \( r_A \)), thereafter flowing along field lines to the neutron star.

Most studies concerned with the magnetic field evolution inside the neutron star have assumed matter to accrete onto the surface spherically, e.g. Fujimoto et al. (1984), Miralda-Escudé et al. (1990), hereafter Mir90, Urpin & Geppert (1995), and Konar & Bhattacharya (1997). In this paper I consider non-spherical accretion where matter is assumed to flow onto the polar caps in a column. The cap is consequently heated up and the conductivity in the crust below the polar cap we estimate to be much smaller than the conductivity outside the accretion column. The magnetic evolution in this scenario can be divided into two stages where in stage I the global decay rate is controlled by the conductivity outside the accretion column, \( \tau_B \sim 10^8 - 10^{10} \) yrs. When the field has reached a value of about \( 10^8 \) gauss the accretion is spherical and the evolution enters the second stage. Here, the whole crust is being heated up and the conductivity is dominated
by electron-phonon scattering. In this stage the magnetic field decay time is a few orders of magnitude shorter than in stage I and compared to the earlier evolution the field is dissipating rapidly. I argue that this could account for the fact that no binary pulsars have been found with a magnetic field less than $10^8$ gauss. A similar effect where the accreted flow is pushing the field lines has already been noticed by Romani (1993).

The paper is organized as follows: section 2 contains a brief description of the model and the basic equations. In section 3 we estimate the length scale for temperature change at the accretion column-normal crust boundary to be less than a crust thickness. The time scale for the temperature to reach a stationary state is also shown to be much smaller than the magnetic field decay time scale. Therefore, we do not solve the energy equation explicitly but assume the conductivity as a function of angle to be close to a top-hat function at all times. Section 4 presents the numerical results for some initial depths of the magnetic field.

2. Magnetic field evolution

The purpose of the present paper is to investigate the effect of asymmetric accretion on the magnetic evolution in the crust of a neutron star. We therefore ignore the complex interaction of matter and magnetic fields in the magnetosphere and for simplicity assume there is vacuum outside the star. The basic model we are considering is an axi-symmetric spherical shell of outer radius $R_2$ and inner radius $R_1$. Below $R_1$ there is a superconductor and outside the shell is vacuum. I assume the equation of state in the shell to be given by Baym et al. (1971). Furthermore, the magnetic field is assumed to vanish within the London penetration depth of the superconductor.

The magnetic field, $\mathbf{B}$, obeys the usual equation

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c^2}{4\pi} \nabla \times \left( \frac{1}{\sigma} \nabla \times \mathbf{B} \right),$$

(1)

where we only include ohmic dissipation. If one assumes the field to be initially poloidal, no toroidal field will appear and one can write the magnetic field in terms of the vector potential, $\mathbf{A} = A_\phi \mathbf{e}_\phi$ in spherical coordinates. One finds the following equation for $A_\phi$:

$$\frac{\partial^2 A_\phi}{\partial r^2} + \frac{2}{r} \frac{\partial A_\phi}{\partial r} + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial A_\phi \sin \theta}{\partial \theta} \right) = \frac{4\pi \sigma(r, \theta)}{c^2} \frac{\partial A_\phi}{\partial t}.$$

(2)

A detailed discussion of the boundary conditions used in this model can be found in Sahrling (1996). As a summary, the boundary conditions to be imposed are that, at the inner edge of the crust the field should be zero, no field diffuse into the core, and at the outer boundary of the crust the poloidal field should match onto the vacuum solution.

To estimate conductivities we shall use the calculations of the electrical conductivity of matter in the crust of neutron star by Urpin & Yakovlev (1980) and Raikh & Yakovlev (1982). To my
knowledge, there exist no detailed studies of phonons in the inner crust of neutron stars, where nuclei are immersed in a neutron liquid. However, one would expect the phonon spectrum to be qualitatively similar to that for a lattice of ions, provided one replaces the mass of an ion by the total mass of all nucleons in a cell of matter containing one nucleus. At high temperatures, the dominant source of electrical resistivity is scattering of electrons by phonons. One gets from Urpin & Yakovlev (1980) the corresponding conductivity \( \sigma_{ph} \approx 1.5 \times 10^{20} \rho_6 / \mu_e (1 + (\rho_6 / \mu_e)^{2/3})^{-1/2} 1/T_8 \) s\(^{-1}\), where \( \rho_6 \) is the density in units of \( 10^6 \) g cm\(^{-3}\), \( \mu_e \) is the mean molecular weight per electron, \( x \) is the proton fraction of matter, and \( T_8 \) is the temperature in units of \( 10^8 \) K. For temperatures less than the Debye temperature, \( T_D \approx 3.4 \times 10^9 \rho_6^{1/2} / (\pi / 0.1) K \), electron scattering is due chiefly to Umklapp processes. The corresponding conductivity is \( \sigma_{Um} \approx 5.5 \times 10^{23} \rho_6^{7/6} / (\pi / 0.1)^{5/3} T_9^{2} \) s\(^{-1}\). Below a temperature \( T_U \approx 2.2 \times 10^8 \rho_6^{1/2} / (Z / 60)^{1/3} / (\pi / 0.1) K \), Umklapp processes are frozen out, and the conductivity rises very sharply, since only normal processes contribute, and the electrical conductivity due to phonon scattering is \( \sigma_{ph,N} \approx 2.1 \times 10^{28} \rho_6^{8/3} / (\pi / 0.1)^{14/3} T_9^{−5} \) s\(^{-1}\). However, at temperatures below \( T_U \), impurities are likely to be the dominant scatterers of electrons. One can see this from the estimate for the impurity contribution, \( \sigma_i \approx 5.5 \times 10^{25} (\rho_6 / 0.1)^{1/3} Z / (60 \, Q) \) s\(^{-1}\) (Urpin & Yakovlev 1980). Here \( Q \) is the mean square deviation of the atomic number from its average value. The impurity parameter \( Q \) is not well known, but the estimate of Flowers & Ruderman (1977) suggests \( \approx 10^{-3} \), to be a reasonable lower limit.

3. Asymmetric accretion

The problem of accretion onto neutron stars and the interaction with the magnetosphere is a major research area in modern astrophysics and many interesting and challenging problems remain to be solved, see for example Lamb (1991), Michel (1991), White et al. (1995) for interesting discussions of this and related phenomena. I assume here the accreting matter to fall upon the neutron star in an accretion column whose size is given by the last open field line before the accretion disk. For simplicity, I also assume there be no “slipping” between field lines as argued by Arons & Lea (1980). The basic idea (Lamb et al. 1973 and Shapiro & Teukolsky 1983) is that matter is captured by some instability mechanism at \( r_A \) where the kinetic energy density of the accreting matter is equal to the magnetic energy density, \( r_A = (B_s^4 R^{12} / (2GM \hat{M}^2))^ {1/7} \approx 3 \times 10^8 \hat{M}^{-2/7}_1 B_{s,12}^{4/7} R_6^{12/7} (M / M_\odot)^{-1/7} \) cm, where \( B_{s,12} \) is the surface magnetic field strength in units of \( 10^{12} \) gauss, \( \hat{M}_1 \) is the accretion rate in units of \( 10^{17} \) g s\(^{-1}\), \( R_6 \) is the stellar radius in units of \( 10^6 \) cm, and \( M \) is the mass of the neutron star. Matter is then channeled by the field lines onto the neutron star surface in a column with an opening angle \( \theta_{cap} \). The area under the accretion column is \( A_{cap} = 2 \pi R^2 (1 - \cos \theta_{cap}) \). The angle \( \theta_{cap} \) is given by those dipole field lines that would, in the absence of accretion, have penetrated beyond \( r_A \). The field lines for a dipole field are defined by \( \sin^2 \theta / r = \text{constant} \), so the base of the last undistorted field line which would close inside \( r_A \) lies at an angle

\[
\sin^2 \theta_{cap} = R / r_A = 3 \times 10^{-3} \left[ R_6^{-5/7} \hat{M}_1^{-2/7} B_{s,12}^{-4/7} (M / M_\odot)^{1/7} \right],
\] (3)
for a detailed derivation see e.g. Shapiro & Teukolsky (1983). As the field strength decreases due to ohmic losses the opening angle gets larger and larger. Eventually, the field strength will be so low the accretion is expected to be spherical since then $\theta_{cap} = \pi/2$. We have $B_{s,12}^{sph} = 5 \times 10^{-5} [R_6^{-5/4} M_{17}^{1/2} (M/M_\odot)^{1/4}]$ where $B_{s,12}^{sph}$ is the value of the magnetic field when the accretion is spherical.

This is of course a very simplified picture of the accretion process, but it will nevertheless provide some insight as to how the crust affects the surface magnetic field.

### 3.1. Crude Temperature Structure

This section discusses some relevant order of magnitude estimates. Although there are many heat sources in an accreting neutron star we will use hydrogen burning in a thin shell at $\rho_{shell} \approx 10^6$ g cm$^{-3}$, see Mir90, as the major one. Mir90 calculated models suppressing all other heat sources and came up with temperature differences of a about a factor 2. The density where the accreted material starts to spread laterally, $\rho_{spread}$, is shown to be about an order of magnitude higher than the base of the hydrogen burning shell, $\rho_{shell}$. We also estimate typical thermal length scales to be shorter than the crust thickness. These estimates indicate the heat source in our model to have a size of about $l_{cap} \sim \sqrt{A_{cap}}$ and located at $\rho \approx 10^6$ g cm$^{-3}$. Further, it is shown that the time for the temperature distribution to reach a steady state is much shorter than typical magnetic field decay time scales.

The kinetic energy of the accreting matter is turned into heat in the upper parts of the atmosphere. After being thermalized the matter continues to flow downwards at sub sonic speed. It will follow the magnetic field lines down to a depth where the local magnetic force equals the local pressure gradient. At this point the flow is presumed to spread laterally, see Bildsten & Brown (1996, 1997). They found $P_{spread} = B^2 l_{cap}/(4\pi h)$, where $l_{cap} \sim \sqrt{A_{cap}}$ and $h$ is a pressure scale height. If we assume the pressure to be dominated by relativistic, degenerate electrons we find $\rho_{spread} = 2 \times 10^7 A_{cap,11}^{3/8} B_{12}^{3/2} x_{0.5}^{-1} h_4^{3/4}$ g cm$^{-3}$. As the accreted material flows down along the field lines its density will increase which at certain thin shells will cause nuclear reactions to occur which releases heat. For a thorough discussion of these matters see for example Brown & Bildsten (1997), Bildsten et al. (1993), and Mir90. The base of the hydrogen burning shell $\rho_{shell} \approx 10^6$ g cm$^{-3}$ for the spherical accretion case, see Mir90. In our model, the size of the polar cap $l_{cap} \gg h$ so one can expect $\rho_{shell}$ to be similar to the spherical calculation. We see for typical values of the relevant parameters $\rho_{spread} > \rho_{shell}$. However, it is also clear that as the field strength is reduced $\rho_{spread}$ is decreasing and at some some value of $B_s$, $\rho_{spread}$ may be less than $\rho_{shell}$ and the flow might start to spread before hydrogen ignites. The resulting temperature and density structure for that case must ultimately be answered by a thorough magneto-hydrodynamical calculation, and in this paper I will use a simple parametrization.

Assuming the energy generation in the accretion column to be dominated by hydrogen
burning at the $\beta$-decay limited value of the “hot” CNO cycle we get from Mir90 a luminosity
$L_{nuc} = 10^{33}(Z/0.001)^2 \dot{M}_{17}$ erg s$^{-1}$, where $Z$ is the heavy element content, or a flux
$F_{nuc} = L_{nuc}/A_{cap}$. To find a typical length scale for temperature variations, which we
denote by $l_T$, one can equate this flux with the thermal heat flux $F_{heat} = -K\Delta T/l_T$, where
$K = 2 \times 10^{18} \rho^{2/3} (A/Z)^{2/3}$ erg cm$^{-1}$ s$^{-1}$ K$^{-1}$ is the thermal conductivity calculated by Urpin &
Yakovlev (1980) for the case where the $T > T_D$. The radiative contribution to the heat flux is
negligible for the temperatures and densities of interest here, Mir90 and Mészáros (1992). We get
$l_T = K\Delta TA_{cap}/L_{nuc} = 10^2 \rho^{2/3} A_{cap,11} Z_{0.001}^{-2} \Delta T \dot{M}_{17}^{-1}$ cm , where I have normalized the density to
10$^6$ g cm$^{-3}$ which is roughly the value of $\rho_{shell}$. $A_{cap,11}$ is the area of the accretion cap in units
of 10$^{11}$ cm$^2$ and $Z_{0.001} = Z/0.001$. We see that $l_T$ is typically smaller than a crust thickness,
$\Delta R \approx 10^5$ cm and for some parameter values it can be much smaller. When $l_T \ll \Delta R$ or $l_T \gg \Delta R$
the temperature structure in the crust is spherical which has been studied elsewhere, see Mir90.
Here, I will focus on the intermediate case, where $l_T \leq$ a few $\Delta R$.

To get an estimate of the temperature structure below and around the accretion cap we can
simply note that as the conductivity increases inwards the thermal lengthscale also increases so
for a given temperature change, $\Delta T$, we need to go a distance $l_{T,r} \sim K\Delta T$ radially which is
greater than the corresponding horizontal distance $l_{T,\theta} \sim \Delta T K$ where $K = (\int_r^{r+\Delta r} K^{-1} dr)^{-1} l_{T,r}$.
In other words, the hydrogen burning shell will produce a temperature structure that has an
ellipsoidal or, as I will assume in this paper, columnar shape. In the estimate of $l_T$ we have simply
assumed the relevant area to be $A_{cap}$ for all densities, and so ignored the heat flowing through
the sides of the column. However, since the size of the polar cap is of the same order as the crust
thickness this correction is likely to be small. Given the uncertainty of the horizontal lengthscale
and the size of the hot accretion cap I choose to parameterize $l_T$ in this paper, see section 4.

The time scale for reaching a steady state is now given by, $\tau_T = c_p h^2 \rho/K = 8 \times 10^7 \rho^{1/3} x_{0.5}^{-2/3} T_{14}^{-2} A_{11}/A$ s where we have assumed $T_{cap} > T_D$ which holds below
$\rho \approx 3 \times 10^9 T_{cap,8}^{-2} x_{0.5}^{-2} g$ cm$^{-3}$. $c_p$ is the specific heat capacity of the crust, see e.g. Shapiro
& Teukolsky (1983) and A is the atomic number. The magnetic field decay time is given by
$\tau_B = 4\pi\sigma l_T^2/\rho c^2$ with which $\sigma = \sigma_i$ is $\tau_{B,i} = 5.5 \times 10^{17} \delta R_5^2 (\rho_{14} x/0.1)^{1/3} Z/(60 (Q/0.01))$ s.
When the accretion is spherical $\sigma = \sigma_{ph}$ and $\tau_{B,ph} = 5.5 \times 10^{15} \delta R_5^2 \rho_{14}^{7/6} (x/0.1)^{5/3} T_8^{-2}$ s. We see that $\tau_T \ll \tau_B$ when the conductivity is dominated by either phonon or impurity scattering. In
the estimates of the field decay times we have normalized the values of the parameters to those of
the inner crust. The temperature will thus reach a steady state long before the magnetic field has
changed significantly.

Given these estimates it is likely that the temperature structure in the crust of a neutron
star, accreting in a column, is such that inside the accretion column the temperature resembles
the calculation by Mir90 and by simply extrapolating their result we find,

$$T_{cap} \approx 10^8 \left( \frac{\dot{M}}{10^{17} \text{ g s}^{-1}} \frac{A_{cap}}{A_{cap}} \right)^{1/4} K.$$ (4)
Outside the column the temperature is much lower. Precisely how much lower remains to be seen by detailed calculations but motivated by the previous estimates of $l_T$, we will in this paper assume it to be low enough for impurity scattering to be the dominant source of electrical conductivity $\sigma = \sigma_i$.

I use for the conductivity,

$$\sigma(\theta) = \frac{\sigma_i - \sigma_{\text{cap}}}{1 + \exp\{(C_{\text{orr}} + |\theta - \pi/2| - (\pi/2 - \theta_{\text{cap}}))/\beta\}} + \sigma_{\text{cap}},$$

(5)

where $\sigma_{\text{cap}} = \sigma_{Um}, T < T_D, \sigma_{\text{cap}} = \sigma_{\text{ph}}, T > T_D$ and $C_{\text{orr}} = (\theta_{\text{cap}}/(\pi/2))^{20}$ is a correction used to ensure that $\sigma_\theta = \sigma_{\text{cap}}$ when the accretion is uniform over the whole sphere $\theta_{\text{cap}} = \pi/2$. To see how the size of the horizontal thermal length scale affect the solution I use a simple parameterization of $l_T = R/\beta$ where $\beta$ is assumed to be independent of $r$ and in this paper range from $0.02 - 0.2$.

It is worth pointing out that if the different parameters are such that $l_T \ll \Delta R$ initially, $l_T$ will increase as the magnetic polar cap size increases and could eventually be such the whole crust is heated up. The spatial evolution of the magnetic field in the crust will then be different from this calculation. However, the temporal evolution, will be very similar, that is the timescale for the accretion to be completely spherical is the same as we find here, and also the subsequent rapid field decay.

4. Numerical Results

This section describes numerical solutions to equations (2) and (5) with two different initial conditions. First, I choose a configuration where the field is penetrating the whole crust, $\rho \leq 2.4 \times 10^{14}$ g cm$^{-3}$. In the second case the field was initially confined to $\rho \leq 10^{11}$ g cm$^{-3}$. In both cases the initial surface dipole strength was $10^{12}$ gauss.

The results are combined into three sections where in the first, section 4.1, the field strength is initially penetrating the whole crust. This is followed by section 4.2 where the field fills only the outer crust $\rho \leq 10^{11}$ g cm$^{-3}$ initially. The numerical calculations show the decay time scale can be divided into two stages where in stage I the decay time scale is given by $\tau_{B,i}$ and when the accretion becomes spherical it enters the second stage where the decay rate increases to $\tau_{B,ph}$. The calculations also show that starting from a dipole, the contribution from higher order surface multipoles is at the most 30 % to the dipole mode in agreement with Arons (1993). In section 4.3 I explore different values of the horizontal length scale by varying $\beta$.

Equations (2) and (5) was solved with the finite-difference code described elsewhere, Sahrling (1996). For the impurity parameter I chose four values $Q = 0.001, 0.01, 0.1,$ and 1 which span the likely range of values, see Flowers & Ruderman (1977), and is constant throughout the crust. The model I am considering here is an $M = 1.4M_\odot$ neutron star with the equation of state given by Wiringa et al. (1988) and Baym et al. (1971), with radius $R = 11.5$ km. The equation of state
for neutron stars is a debated issue but this choice represents an average stiffness of the range of models discussed.

At \( t = 0 \) the field was set to be dipolar, \( A_\varphi \sim \sin \theta \). Since the conductivity below the accretion cap \( \theta \leq \theta_{\text{cap}} \) and \( \theta \geq \pi - \theta_{\text{cap}} \) is smaller than the conductivity outside, the field will initially change more rapidly there producing higher-order multipoles.

Generally, it was found the dependence of the various runs on the impurity parameter \( Q \) is simple. Let us denote by \( \tau_Q \) the time scale for runs with a particular \( Q \). Then \( \tau_Q \approx \tau_Q = 1/Q \) which given our approximations is to be expected. From the observational constraints discussed in the introduction we can infer timescales for accretion lies roughly between \( 10^7 - 10^9 \) yrs depending on such things as mass of the companion star and the spin period \( P \). The numerical results of our model indicates that the value of \( Q \) has to be greater than about 0.1 to give a significant decay of the magnetic field.

4.1. Field initially penetrating the whole crust

The field strength was chosen to be \( 10^{12} \) gauss at the stellar surface initially. This high field is consistent with the pattern of High-Mass X-ray binaries (HMXBs), in which the X-rays originate in a small steady hot patch, see Lyne & Graham-Smith (1990). There is however one LMBP system known, PSR 0820+02, with a similar high field strength, so this initial condition can also be relevant for LMXB’s. To calculate \( \theta_{\text{cap}} \) from equation (3) we use a constant \( \dot{M} = 10^{17} \text{ g s}^{-1} \approx 10^{-9} M_{\odot} \text{ yr}^{-1} \). The parameter \( \beta = 0.02 \) for all runs discussed here.

Figure 1 shows the evolution of the various surface magnetic multipoles, normalized to the initial value \( B_s \). It shows the evolution for a crust with \( Q = 1 \). This high impurity level results in very low conductivity and thus the most rapid field decay among the cases considered. Changing the value of \( Q \) resulted only in a change in time scale of a factor \( 1/Q \). As can be seen from the figure high-order multipoles are initially stronger than multipoles \( l = 3, 5 \) but as time progresses they disappear more and more and the lower ones start to dominate. The contribution of \( l = 3, 5 \) compared to the dipole is about 10% at the time the accretion is spherical, which happens when \( B_s \approx 10^8 \) gauss and the field evolution enters stage II. We saw earlier that in our model this value varies with the square root of the accretion rate. One has to conclude that if this mechanism is responsible for there being no pulsars with a magnetic field lower than \( 10^8 \) gauss the accretion rate for the majority of the millisecond pulsars during their recycling phase will have to have been, on average, greater than roughly \( 10^{17} \text{ g s}^{-1} \). Observationally it is hard to pin point the precise accretion rate but assuming the luminosity to be due solely to the infalling matter one finds accretion rates ranging from \( 10^{16} - 10^{17} \text{ g s}^{-1} \) Phinney & Kulkarni (1994).
4.2. Field initially buried in outer crust

If one instead starts out with a field buried below $\rho \leq 10^{11} \text{ g cm}^{-3}$ one finds the result presented in figure 2. There is an initially rapid decay because the field diffuses into the interior of the star on a time scale, $\tau_{B,i}$ calculated at $\rho = 10^{11} \text{ g cm}^{-3}$. The field starts to penetrate the whole crust at a time scale given by the conductivity of the inner crust. This is all what can be expected from dimensional analysis of the induction equation. The $l = 3,5$ multipole strengths are in this case also about 10% of the dipole when the accretion is spherical.

4.3. The dependence on the horizontal thermal length scale

To investigate the sensitivity of the solution to different horizontal thermal length scales we used three different values of the parameter $\beta = 0.02, 0.1, 0.2$. To avoid numerical problems I chose not to go below $\beta = 0.02$. The initial field was assumed to penetrate the whole crust and have an initial surface strength of $10^{12}$ gauss. The results are shown in figure 3 a),b). These plots should be compared to Fig. 1. As can be expected, the higher order multipoles are less significant when $\beta$ increases. Other than that the overall evolution is very similar for all values of $\beta$ considered.

5. Conclusion

We have examined the influence of asymmetric accretion on the magnetic field evolution of a neutron star. The temperature structure in the crust resulting from the accretion was roughly estimated and found to vary more rapidly than the crust thickness around the edge of the accretion cap. The time scale for reaching a steady state was shown to be shorter than the time scale for magnetic field decay. Therefore, instead of doing a full blown calculation of heat conduction coupled with the magnetic field evolution we used a simple smoothed top-hat function for the temperature structure and consequently for the conductivity. The global field decay time is roughly the one given by the largest conductivity in the crust, which occurs outside the accretion cap, $\tau_{B,i} = 5.5 \times 10^{17} (\delta R_5)^2 (\rho_{14} x/0.1)^{1/3} Z/(60 (Q/0.01))$ s. However, when the field strength is down to roughly $10^8$ gauss the accretion is spherically symmetric and we have $\tau_{B,ph} = 5.5 \times 10^{15} (\delta R_5)^2 \rho_{14}^{7/6} (x/0.1)^{5/3} T_{8}^{-2} s \ll \tau_{B,i}$ and so the decay rate is much faster than initially which could provide a clue why no binary pulsars are known with field strengths less than $10^8$ gauss. Romani (1993) discussed a similar effect and found a threshold close to ours. We also found the asymmetric accretion resulted in some higher order surface magnetic multipoles and the size of these were shown to be at most 30% of the surface dipole field.

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FIGURE CAPTIONS

Figure 1: The evolution of the various surface magnetic multipoles, normalized to the initial value $B_s = 10^{12}$ gauss with $Q = 1$ and $\beta = 0.02$. Initially the filed penetrated the whole crust. High-order multipoles are initially stronger than low-order ones but as time progresses they disappear more and more and the lower ones start to dominate. The dips in the curves corresponds to times when the multipoles changes sign. The contribution of $l = 3, 5$ compared to the dipole is about 10% at the time when the accretion is spherical. The early part of the evolution from $t = 0$ is not shown here since that describes the initial transients of the field due to the choice of initial conditions $A_\phi \sim \sin \theta$.

Figure 2: Same as figure 1 but with the initial field confined to $\rho \leq 10^{11} \text{ g cm}^{-3}$.

Figure 3: a) Same as figure 1 but with $\beta = 0.1$ initially. b) Same as figure 1 but with $\beta = 0.2$ initially.
