Continuous wave magnonic fractals in dynamic artificial crystal

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We report on the first observation of an exact fractal pattern in the time-domain arising spontaneously from an all-magnon process in a passive, un lithographed magnetic waveguide. A spatiotemporally periodic potential created by a standing spin wave of frequency $f_1$ excited a highly nonlinear region of waveguide. At certain applied magnetic fields ($H$), the interaction of travelling spin waves of frequency $f_2$ with the nonlinear region resulted in a series of new modes appearing in a comb with intervals $\Delta f^{(0)} = |f_1 - f_2|$. As $H$ was increased a 1st pre-fractal pattern was observed with frequency interval $\Delta f^{(1)} = \Delta f^{(0)}/2$. Finally, the onset of a 2nd pre-fractal was observed with frequency interval $\Delta f^{(2)} = \Delta f^{(1)}/2 = \Delta f^{(0)}/4$.

INTRODUCTION

The study of fractals began with Benoit Mandelbrot in the 1960s and has been of scientific interest ever since. The phenomenon of fractal behaviour, or self-similarity, exists in a remarkable range of observable physical systems. Indeed, fractal analysis may be applied to such varied systems including crystal growth, diffusion fronts, lung structure, moon crater distribution, stock market fluctuations and authenticating priceless artwork [1–3].

The ubiquity of fractals points to their fundamentality, and the simplicity with which they may be described is testament to their beauty.

Fractals may be subdivided into two main categories: exact and statistical. An exact fractal is an object that exhibits a perfect symmetry of scale: it replicates the structure of the whole upon varying degrees of magnification. The application of this general condition can generate some rather striking images; famous examples include the Sierpinski gasket and the Mandelbrot set.

A statistical fractal on the other hand, is an object in which the self-similarity is only present in the statistical properties of the system. Perhaps intuitively, it is these fractals that are vastly more common in Nature as they allow for random deviations from the exact regime, which in contrast is relatively rare.

Despite the rarity however, exact fractals have been observed in nonlinear dynamical systems [4]. Space-domain soliton fractals were first demonstrated numerically using nonlinear optics [5, 6] with experiments demonstrating similar results [7], while time-domain soliton fractals were experimentally realised within a magnetic medium [8]. Until recently [9], all observed exact time-domain fractals were exclusively soliton in nature. Richardson et al. have demonstrated however, that spontaneous continuous wave (CW) time-domain fractals can emerge in magnonic waveguides possessing highly nonlinear dispersion, as in a quasi-1D magnonic crystal. In their experiment, the artificial crystal had periodic etched grooves which created a rejection band for magnons of wavelengths associated with the periodicity of the grooves.

The altered transmission band corresponded to a sharp kink in the dispersion curve [10], and consequently a large spike in the dispersion coefficient resulting in the enhanced nonlinearity which facilitated the formation of fractals when driven at high power. In this case, the fractal behaviour originated from modulational instability (MI) [11] where the time-domain side bands represented the amplitude modulation. Significantly, these fractals appeared spontaneously out of the passive waveguide element, rather than being forced into existence [9].

In this Letter, we report on the first observation of CW fractals in a simple, isotropic thin film, free from patterning and lithography. This demonstration contrasts with the previous reports of fractals in magnetic media such that it neither involves solitons, nor does it utilise a permanent artificial crystal. While the precise mechanism for the presence of fractals is unclear, we posit an explanation not inconsistent with previous reports.

We create a dynamic artificial crystal (DAC) in a magnon waveguide [12, 13] consisting of the ferrimagnetic insulator, yttrium iron garnet (YIG). To create the DAC, we introduce a spatially periodic potential by creating a standing spin wave across the width of the waveguide [14]. Due to the well-documented nonlinearity of YIG [15–19], the local refractive index of the film depends on the amplitude of the magnons in the given region [20]. The periodic change in refractive index resulting from the standing wave may be considered as an effective-grating that, as such, alters the transmission efficiency of magnons with wavelength relating to that of the effective-grating spacing. In turn, this significantly modifies the dispersion at that given wavenumber, facilitating the creation of fractals.

In our experiment, the time domain fractals, shown in Fig. 11 manifest as follows: 1) there are two initial input signals which form the initiator, also known as the mother. 2) A comb of sidebands is excited which, in fractal parlance, represents the generator, or daughters. 3) At the mid-points between consecutive comb peaks, further granddaughter peaks appear. 4) Half-way between daughter and granddaughter peaks, great-granddaughter peaks were observed.
At this stage, a note on terminology should be made for the uninitiated - the granddaughters and great-granddaughters may be referred to as 1st and 2nd pre-fractals, respectively. The term pre-fractal simply refers to an exact fractal that is not infinitely scalable (as is necessarily the case for all exact fractals in the physical world). Despite this slightly imprecise language, we refer to the observed object as a fractal proper, as is convention [5, 6, 8, 9].

The time-domain fractal pattern may also be understood as an amplitude modulation, arising as a consequence of MI. For MI to occur, the Lighthill criterion must be satisfied [11], namely

\[
\left( \frac{\partial^2 \omega}{\partial k^2} \right) \left( \frac{\partial \omega}{\partial |a|^2} \right) < 0
\tag{1}
\]

where \(k\) is the magnon wavenumber, and \(a\) is the magnon amplitude. Our experiment utilises a type of dipolar magnon known as forward volume magnetostatic spin waves (FVMSWs) which have an isotropic dispersion (in contrast to the highly anisotropic dispersion of backward volume magnetostatic spin waves and surface spin waves [21, 22]). Furthermore, the dispersion of a FVMSW is such that the first term in eqn. 1 is always negative, while the second term is always positive [20]. This means that for FVMSW, the Lighthill criterion is always satisfied, allowing for the possibility of MI.

**EXPERIMENT**

A schematic of the experiment is shown in Fig. 2. We used a thin film of YIG that was 18 mm long, 2.1 mm wide, and 7.8 µm thick, with edges cut at 45°. On a printed circuit board (PCB) two parallel pump antennae were used to excite a standing spin wave across the film width. A transmitter-receiver antenna was placed approximately 3 mm away, oriented perpendicular to the former antennae. All antennae were designed in a meander pattern with three legs of length 4 mm and individual legs 40 µm wide, with a spacing of 50 µm between legs.

Figure 2 shows the two inputs of the experiment: the parallel antennae acting as a pump to create a standing wave of frequency \(f_1\), and the transmitter-receiver exciting a travelling FVMSW with \(f_2\).

The external magnetic field was applied normal to the film: a field configuration that is necessary to excite forward volume magnetostatic spin waves [23]. The sketch in Fig. 2 shows the two inputs of the experiment: the parallel antennae acting as a pump to create a standing wave of frequency \(f_1\), and the transmitter-receiver exciting a travelling FVMSW with \(f_2\).

Figure 4 shows a diagram of the experimental equipment used. The pump antennae were driven by a microwave source Hewlett Packard HP8672A with a frequency equal to \(f_1 = 3.915\) GHz and power approximately \(P_1 = 13\) dBm. The transmitter-receiver antenna was connected to a circulator allowing for excitation and detection of spin waves with a single antenna. This antenna was driven by a separate microwave source (HP8671A) at \(f_2 = 3.917\) GHz, with an input power of \(P_2 = 0\) dBm. Spin wave detection was performed by a spectrum analyser (Rhode & Schwarz ZVL). The magnetic field was applied to the sample using an electromagnet that was swept over the region of interest from 3070 Oe to 3015 Oe.

The experiment utilises magnons of two distinct frequencies, excited in separate regions of the waveguide. The signal is excited at \(f_2\) and propagates along the waveguide to the region flooded with magnons of frequency \(f_1\). A standing wave across the width is supported when the applied magnetic field strength shifts the

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**FIG. 1.** Sketch of the development of observed fractal pattern. Two inputs form the mother mode, comprising two frequencies detuned by \(\Delta f\) which are present at zero field. At certain field \(H_1\) a comb of peaks separated by \(\Delta f\), forming the daughter modes. At \(H_2\), a new comb of interval \(\Delta f/2\) appears. Then, at \(H_3\), a new comb of interval \(\Delta f/4\) is formed.

**FIG. 2.** Experimental configuration: YIG film with 45° edges placed on PCB with antennae. Counter-propagating pumps excite a standing-wave creating a grating due to intensity dependent refractive index. The transmitter-receiver antenna transmits excitation magnons and detects spin waves from the nonlinear region.

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dispersion such that the pump magnons have a wavenumber, \( k_1 \), that satisfies
\[
k_1(\omega, H) = \frac{\pi n}{w},
\]
where \( w \) is the width of the waveguide and \( n \) is an integer. When the incoming magnons with \( f_2 \) enters the region of periodic potential, the locally-altered dispersion facilitates highly nonlinear effects [24] including the formation of a time-domain fractal.

\[\text{FIG. 3. Illustration of experimental apparatus.}\]

**RESULTS AND DISCUSSION**

Figure 4 shows the different frequency spectra for various magnetic fields. The initiator spectrum measured at zero field is shown in Fig. 4(a), comprising the signals \( f_1 \) and \( f_2 \). The measured power \( P_1 \) was the direct coupling between the pump and the signal antenna, while the large \( P_2 \) signal was due to reflections at the transmitter-receiver antenna resulting from impedance mismatch. The magnetic field was increased to \( H = 3109 \text{ Oe} \) resulting in the creation of daughter peaks depicted in 4(b). This comb-like pattern appears with intervals of \( \Delta f^{(0)} = | f_1 - f_2 | = 2 \text{ MHz} \), forming the structure that is repeated on successively smaller scales. Also note that the amplitude of the peak at 3.9150 GHz has decreased upon generation of daughter modes and has been distributed to the surrounding comb. As the field was increased again to \( H = 3121 \text{ Oe} \) new granddaughter peaks appeared between the mother and daughter peaks. At this magnetic field strength, the frequency interval between peaks has been reduced to \( \Delta f^{(1)}/2 = \Delta f^{(0)} = 1 \text{ MHz} \). The magnetic field was increased further to \( H = 3134 \text{ Oe} \) the results of which are shown in Fig. 4(d). A great-granddaughter peak was observed with at 3.9155 GHz, between the mother and granddaughter peaks boxed in 4(c). Significantly, the great-granddaughter peak appears at a frequency interval of \( \Delta f^{(0)}/4 = \Delta f^{(1)}/2 = \Delta f^{(2)} = 0.5 \text{ MHz} \). For these data it is clear there is a fractal time-domain structure. While power limitations were set by the operating threshold of the equipment, we predict that higher powers would result in the observation of a 3rd iteration of pre-fractal. Indeed, the \( n \)th pre-fractal observed would be a frequency comb of interval \( \Delta f^{(n)} = \Delta f^{(0)}/2^n \).

A point of note is the relatively low input power compared to previous reports of CW fractal generation [9]. This is in part due to nature of the periodic potential. Given that the artificial crystal is itself a standing spin wave, the region is already flooded with magnons. Since the rate at which MI occurs generally scales with \( |a|^2 [25] \), this suggests the likelihood of FWM (and the associated side-bands) is vastly increased.

The observed peaks were extremely sensitive in response to magnetic field strength. The appearance and disappearance of the peaks was often abrupt, with 1st and 2nd iteration pre-fractals occurring only over a tight field range.

\[\text{FIG. 4. Measured frequency spectra for various magnetic fields. (a) Input frequencies constituting the mother mode. (b) Frequency comb representing the daughter modes. (c) Generation of granddaughter peaks which appear as intermediary peaks midway between daughters. (d) A zoomed in spectra showing a great-granddaughter mode between a daughter and granddaughter peak.}\]

The mechanism responsible for the generation of daughter peaks can be explained by four wave mixing (FWM) [24,26]. For the physical process to occur, the
following energy conservation condition must be satisfied:
\[ f_i + f_j = f_k + f_l. \]  

In the case of our experiment we may rewrite eqn. (3) as
\[ 2f_1 = f_2 + (f_1 + \Delta f^{(0)}), \]
where the final term in parentheses is the first daughter mode to the right of \( f_1 \). Similarly, we may obtain the first daughter mode to the left of \( f_2 \) by rewriting eqn. (3) as
\[ 2f_2 = f_1 + (f_2 - \Delta f^{(0)}). \]
Once we have the first set of sidebands the method may be repeated to obtain the entire comb of daughter peaks. This type of four magnon scattering is a physical process that is to be expected and is well understood and documented in the literature. While we also suggest that FWM is responsible for the generation of granddaughter and great-granddaughter peaks, the specific origin of the well-defined pattern is not completely understood.

**CONCLUSION**

In conclusion, we have observed experimentally for the first time, spontaneous fractals in simple un-patterned thin film. Specifically, we utilised an all-magnon dynamic artificial crystal (DAC) to enhance the nonlinearity of the film, resulting in a time-domain exact fractal pattern. We demonstrate successive levels of modulation with the each new set of side-bands appearing half-way between the previous. Extending this pattern, we would expect nth pre-fractal to have an interval of \( \Delta f^{(0)}/2^n \). Our observations differ from previous studies in that the film is not lithographed and the fractals are CW.

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