Fragmentation versus Stability
in
Bimodal Coalitions

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Abstract

Competing bimodal coalitions among a group of actors are discussed. First, a
model from political sciences is revisited. Most of the model statements are found
not to be contained in the model. Second, a new coalition model is built. It ac-
counts for local versus global alignment with respect to the joining of a coalition.
The existence of two competing world coalitions is found to yield one unique sta-
ble distribution of actors. On the opposite a unique world leadership allows the
emergence of unstable relationships. In parallel to regular actors which have a clear
coalition choice, “neutral”, “frustrated” and “risky” actors are produced. The cold
war organisation after world war II is shown to be rather stable. The emergence
of a fragmentation process from eastern group disappearance is explained as well
as continuing western group stability. Some hints are obtained about possible poli-
cies to stabilize world nation relationships. European construction is analyzed with
respect to european stability. Chinese stability is also discussed.
1 Introduction

Mathematical tools and physical concepts might be a promising way to describe social collective phenomena. Several attempts along these lines have been made in past years, in particular to study strike process [1], political organisations [2], group decision making [3], social impact [4, 5], outbreak of cooperation [6], power genesis in groups [7, 8] and stock market [9].

However such an approach should be carefully controlled. A straightforward mapping of a physical theory built for a physical reality onto a social reality could be rather misleading. It could lead at best to a nice metaphore without predictability and at worst to a wrong social theory.

Physics has been successful in describing macroscopic behavior from microscopic properties. The task here, is to borrow from physics those techniques and concepts used to tackle the complexity of aggregations. In parallel the challenge is to build a collective theory of social behavior along similar lines, but within the specific constraints of the psycho-social reality. The contribution from physics should thus be restricted to qualitative guidelines for the mathematical modeling of complex social realities. Such a limitation does not make the program less ambitious.

Working at the edge of interdisciplinarity between social sciences and physical sciences has different inherent dangers for respectively the physicist and the social scientist. Coming from the physics side it is to stay in physics using a social terminology within a physical formalism. On the opposite, from the other side the danger is to dress subjective belief under a pseudo-scientific language.

In a recent work Axelrod and Bennett used the physical concept of minimum energy to build a landscape model (hereafter denoted as AB) of aggregation [10]. Possible coalitions and choices various entities can make among them are studied in this Statistical Physics based model.

In this paper we first position the AB model within the field of Statistical Physics. A change in variables and a gauge transformation are shown to map the AB model exactly to a $T = 0$ finite ferromagnetic Ising system. On this basis most AB statements are shown to be misleading with respect to their actual model. They are indeed confusing a Mattis spin glass and an Edwards-Anderson spin glass [11].

However along above analysis, a new coalition model can indeed be built to describe alignment and competition among a group of actors. The model is found to embody main properties claimed in the AB model. Temperature-like instabilities are introduced in the model using a concept of strange actor.

The following of the paper is organized as follows. The second part contains a review of the AB model within the field of Statistical Physics. A change in variables and a gauge transformation are shown to map the model exactly to a $T = 0$ finite ferromagnetic Ising system. On this basis most AB statements are shown to be misleading with respect to their actual model. They are indeed confusing a Mattis spin glass and an Edwards-Anderson spin glass [11].

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The following of the paper is organized as follows. The second part contains a review of the AB model. In Section 3 a new set of variables is shown to map the model onto a “zero temperature” finite Ising ferromagnet. Using this mapping in Section 4 most AB statements are found not compatible with the model. Their model turns out to be indeed a Mattis spin glass-like while their comments are drawn from the physics of an Edwards-Anderson spin glass. Section 5 deals with a presentation of a new model to describe alignment phenomena. Our model produces several new features in the dynamics of bimodal coalitions. Nato versus Warsaw Pact are analysed. Stability of respectively Europe and China is discussed. Hints to include instabilities in the model are presented.
in Section 6. Last Section contains some concluding remarks.

2 The Axelrod-Bennett (AB) model

Axelrod and Bennett (AB) address the problem of alignment between two competing coalitions within a group of \( n \) countries [10]. A set of positive variables \( \{ s_i \} \) accounts for various actor sizes where index \( i \) runs from 1 to \( n \). A pairwise propensity \( p_{ij} \) is also considered among each pairs of actors. It is positive for cases of cooperation and negative in cases of conflicts. Propensities are assumed to be symmetric, i.e., \( p_{ij} = p_{ji} \).

Each actor has then the choice to be in either one of two coalitions. A distance \( d_{ij} \) is then introduced between each pair of actors \( i \) and \( j \). It is \( d_{ij} = 0 \) if \( i \) and \( j \) belong to the same coalition while \( d_{ij} = 1 \) when they belong to different coalitions.

Given a configuration \( X \) of actors, a quantity called “frustration” (not to be confused with the spin glass frustration),

\[
F_i = \sum_{j=1}^{n} s_j p_{ij} d_{ij}(X),
\]

(1)

is defined for each nation, where the summation is taken over all other countries including \( i \) itself with \( p_{ii} \equiv 0 \). Given a configuration \( X \), all country frustrations sum up to an “energy”,

\[
E(X) = \sum_i s_i F_i.
\]

(2)

This “energy” which measures the level of propensity satisfactions is rewritten,

\[
E(X) = \sum_{i > j}^{n} s_i s_j p_{ij} d_{ij}(X),
\]

(3)

where the sum runs over the \( n(n-1)/2 \) distinct pairs \((i, j)\). Eq.(3) is the central formula of AB model.

It is then postulated that actual configuration is the one which minimizes the energy \( E(X) \). There exist by symmetry \( 2^n/2 \) distinct sets of alliances since each country has 2 choices for coalition. Starting from some initial configuration, Axelrod and Bennett treats the problem numerically. A dynamics of the system is implemented by single actor coalition flips. An actor turns to the competing coalition only if the flip decreases its local energy. The system has reached its stable state once no more flip occurs. Given \( \{ s_i, p_{ij} \} \), the \( \{ d_{ij} \} \) are thus obtained minimizing Eq. (3). Axelrod and Bennett made following statements about their model.

(a) Eq. (1) shows “that the source of conflict with a small country is not as important for determining alignment as an equivalent source of conflict with a large country” [10, p. 214].

(b) The physical concept of frustration [11] is said to be embodied in their model with “For example, if there are three nations that mutually dislike each other (such as Israel, Syria and Iraq), then any possible bipolar configuration will leave someone frustrated” [10, p. 217].

3
It is stated that alignment can be predicted in real cases with \textquotedblleft Landscape theory begins with sizes and pairwise propensities ... to make predictions about the dynamics of the system\textquotedblright{} [10, p. 217].

We will show below (Section 4) that indeed these three statements are misleading with respect to AB model content.

3 The AB model is a \textit{``$T = 0$''} problem

The introduction of a new set of variables shows AB model to map onto a \textit{``$T = 0$''} finite Ising ferromagnet. First the two coalitions are denoted respectively by A and B. Then a variable \( \eta_i \) is associated to each actor. It is \( \eta_i = +1 \) if actor \( i \) belongs to alliance A while \( \eta_i = -1 \) in case it is part of alliance B. From symmetry all A-members can turn to coalition B with a simultaneous flip of all B-members to coalition A.

Given a pair of actors \((i, j)\) their respective alignment is readily expressed through the product \( \eta_i \eta_j \). The product is +1 when \( i \) and \( j \) belong to the same coalition and –1 otherwise. Using variables \( \{ \eta_k \} \), distance \( d_{ij} \) can be recast exactly under the form,

\[
d_{ij} = \frac{1}{2} (1 - \eta_i \eta_j) ,
\]

and the configuration energy becomes,

\[
E(X) = E_0 - \frac{1}{2} \sum_{i>j} J_{ij} \eta_i \eta_j ,
\]

where

\[
J_{ij} \equiv s_i s_j p_{ij} ,
\]

with \( J_{ii} = 0 \) and

\[
E_0 = \frac{1}{2} \sum_{i>j} J_{ij} ,
\]

is a constant which dependents on initial propensities and sizes of involved actors (countries, firms etc). However this constant is independent of actual coalition actor distribution. As such it has no effect over the dynamics of shifting coalitions in the stable state searching. Dynamics operates trough the expression,

\[
H = - \frac{1}{2} \sum_{i>j} J_{ij} \eta_i \eta_j ,
\]

which has to be minimized with respect to \( \{ \eta_k \} \) given \( \{ J_{ij} \} \). Eq. (8) turns out to be the Ising model Hamiltonian with competing interactions [11]. Cooperation occurs for \( J_{ij} > 0 \) while \( J_{ij} < 0 \) produces conflict.

Since here the system stable configuration minimizes the energy, the AB model is indeed at the temperature \textit{``$T = 0$''}. Otherwise when \textit{``$T \neq 0$''} the free-energy has to be minimized. In practise for a finite system the theory can tell which coalitions are possible and how many of them exist. But when several coalitions have the same energy, it is not possible to predict which one will be the actual one.
4 The undressed AB model

Above three statements (end of Section 2) by Axelrod and Bennett [10] can be recast as, *Asymmetric size effect*, *Frustration effect*, and *Alignment prediction*. Unfortunately these statements are misleading within standing AB model. Respective proofs follow.

4.1 Asymmetric size effect

Though statement of asymmetric size effect sounds reasonable from Eq. (1) it is indeed not founded. Dynamics and stable minima are obtained from minimization of the Eq. (3) energy. Cost in “energy” for having two countries not aligning according to their propensity, is multiplicative of both country sizes. Therefore, in case of a pair of misaligned countries, respectively large and small, the energy cost is the same whatever country breaks proper alignment from associated propensity.

4.2 Frustration effect

The frustration statement is misleading with respect to both its physics counterpart and its meaning in alliances. The physical concept of frustration as introduced by Toulouse [11] can be defined precisely with the case of Israel, Syria and Iraq mentioned by Axelrod and Bennett.

We attach respectively the labels 1, 2, 3 to each one of the three countries. In case we have equal and negative exchange interactions $J_{12} = J_{13} = J_{23} = -J$ with $J > 0$, the associated minimum of the energy (Eq. (8)) is equal to $-J$. However this value of the minimum is realized for several possible and equivalent coalitions. Namely for countries (1, 2, 3) we can have respectively alignments (A, B, A), (B, A, A), (A, A, B), (B, A, B), (A, B, B), and (B, B, A). First 3 are identical to last 3 by symmetry since here what matters is which countries are together within the same coalition. The peculiar property is that the system never gets stable in just one configuration since it costs no energy to switch from one onto another. This case is an archetype of frustration. It means in particular the existence of several ground states with exactly the same energy.

Otherwise, for non equal interactions the system has one stable minimum and no frustration occurs within the physical meaning defined above. The fact that some interactions are not satisfied does not automatically imply frustration. Such a situation prevails in cases studied by Axelrod and Bennett, since minima exist and are stable. In other words, the fact that the AB model localizes well defined minima is the proof that no frustration is present in the model. Within the framework of physical models, the AB model with the associated numerical propensities used to determine their actual output is indeed a Mattis model, i.e., a random site spin glass without frustration. Since they eventually found one stable minimum, it means aposteriori that it is possible to find a set of site variables which will allow a factorisation of their initial $p_iJ$. However their discussion and statements are based on a random bond spin glass.

We now make this point more quantitative within the present formalism. Consider a
given site \( i \). Interactions with all others sites can be represented by a field,

\[
h_i = \sum_{j=1}^{n} J_{ij} \eta_j
\]

resulting in an energy contribution

\[
E_i = -\eta_i h_i
\]

To the Hamiltonian \( H = \frac{1}{2} \sum_{i=1}^{n} E_i \). Eq. (10) is minimum for \( \eta_i \) and \( h_i \) having the same sign. For a given \( h_i \) there exists always a well defined coalition except for \( h_i = 0 \). In this case site \( i \) is “neutral” since then both coalitions are identical with respect to its local “energy” which stays equal to zero. A neutral site will flip with probability \( \frac{1}{2} \). Such a situation is absent from AB results.

### 4.3 Alignment prediction

The prediction power of AB model is more subtle. Above formulation using Eqs. (9, 10) sheds light on what comes really out of AB model. Indeed the output reduces to the input. According to AB, the coupling \( \{J_{ij}\} \) are given and only one or two minima are found for the energy. To make the argument simple without losing in generality, we assume there exists only one minimum. Once the system reaches its stable equilibrium it gets trapped and the energy is minimum. At the minimum the field \( h_i \) can be calculated for each site \( i \) since \( \{J_{ij}\} \) are known as well as \( \{\eta_i\} \).

First consider all sites which have the value -1. The existence of a unique non-degenerate minimum makes associated fields also negative. We then take one of these sites, e.g. \( k \), and shift its value from -1 to +1 by simultaneously changing the sign of all its interactions \( \{J_{kl}\} \) where \( l \) runs from 1 to \( n \) (\( J_{kk} = 0 \)). This transformation gives,

\[
\eta_k = +1 \text{ and } h_k > 0, \quad (11)
\]

instead of,

\[
\eta_k = -1 \text{ and } h_k < 0, \quad (12)
\]

which means that actor \( k \) has shifted from one coalition into the other one.

It is worth to emphasize that such systematic shift of propensities of actor \( k \) has no effect on the others actors. Taking for instance actor \( l \), its unique interaction with actor \( k \) is through \( J_{kl} \) which did change sign in the transformation. However as actor \( k \) has also turn to the other coalition, the associated contribution \( J_{kl}\eta_k \) to field \( h_l \) of actor \( l \) is unchanged.

The shift process is then repeated for each member of actor \( k \) former coalition. Once all shifts are completed there exits only one unique coalition. Everyone is cooperating with all others. The value of the energy minimum is unchanged in the process.

Above transformation demonstrates the \( \{J_{ij}\} \) determine the stable configuration. It shows in particular that given any site configuration, it always exists a set of \( \{J_{ij}\} \) which will give that configuration as the unique minimum of the associated energy. At this stage, what indeed matters is the calculation of propensities. To get the right output, i.e., the
right alignment is not a result of the model, but instead, a check of propensity calculation correctness. Therefore AB output (right alignment) reduces to AB input (propensities) making the statement (c) of Sec. II misleading.

On this basis we can conclude the input, i.e., propensity calculations and thus associated \( \{J_{ij}\} \) are the relevant and interesting results of Axelrod and Bennett.

However these data have to be handle with caution depending on the specific cases. Indeed above gauge transformation shows what matters is the sign of field \( \{h_i\} \) and not a given \( J_{ij} \) value. A given set of field signs, positive and negative, may be realized through an extremely large spectrum of \( \{J_{ij}\} \).

This very fact opens a way to explore some possible deviations from a national policy. For instance given the state of cooperation and conflict of a group of actors, it is possible to find out limits in which local pair propensities can be modified without inducing coalition shift. Some country can turn from cooperation to conflict or the opposite, without changing the belonging to a given alliance as long as the associated field sign is unchanged. It means that a given country could becomes hostile to some former allies, still staying in the same overall coalition. One illustration is given by German recognition of Croatia against the will of other european partners like France and England, without putting at stake its belonging to the European community. The Falklands war between England and Argentina is another example since both countries have strong american partnerships.

5 A new model

The idea to build an “energy”-like approach to describe alignment processes within a group of actors could be indeed rather powerful. However the proposed AB model was shown to have many setbacks. Nevertheless at this stage we are in a position to develop another Statistical Physics like model to address the problem of bimodal coalition phenomena. Following the AB model we start with a group of \( n \) actors and two competing coalitions A ans B. We keep above notations.

5.1 Setting the model

From historical, cultural and economic frames there exit bilateral propensities \( p_{i,j} \equiv G_{i,j} \) between any pair of countries \( i \) and \( j \) to either cooperation \( (G_{i,j} > 0) \), conflict \( (G_{i,j} < 0) \) or ignorance \( (G_{i,j} = 0) \). Each propensity \( G_{i,j} \) depends solely on the pair \( (i, j) \) itself and is positive, negative or zero. Here factorisation over \( i \) and \( j \) is not possible. Indeed we are dealing with competing given bonds or links. It is equivalent to random bond spin glasses as opposed to Mattis random site spin glasses [12].

Propensities \( G_{i,j} \) are somehow local since they don’t account for any global organization or net. However coalitions have been known to exist since long ago. To include such a macro-level of alignment we consider the case of two competing bimodal coalitions \( A \) and \( B \) like for instance western and eastern blocks during the so-called cold war.

Each country has either a natural belonging to one of the two world level coalitions or not. A variable \( \epsilon_i \) is then attached to actor \( i \). It is \( \epsilon_i = +1 \) if actor should be in \( A \), \( \epsilon_i = -1 \) for \( B \) and \( \epsilon_i = 0 \) for no coalition belonging. Natural belonging is induced by cultural,
political and historical interests. Within the two world level coalition framework the 
benefit $J_{i,j}$ gained by exchanges between a pair of countries $(i, j)$ is always positive since 
sharing resources, informations, weapons is basically profitable. Nevertheless a pair $(i, j)$ 
propensity to cooperation, conflict or ignorance is $p_{i,j} \equiv \epsilon_i \epsilon_j J_{i,j}$ which can be positive, 
negative or zero. Now we do have a Mattis random site spin glasses [12].

Including both local and macro exchanges result in the pair propensity 
\[ p_{i,j} \equiv G_{i,j} + \epsilon_i \epsilon_j J_{i,j} \] 

between two countries $i$ and $j$ with always $J_{i,j} > 0$.

An additional variable $\beta_i = \pm 1$ is introduced to account for benefit from economic 
and military pressure attached to a given alignment. It is still $\beta_i = +1$ in favor of $A$, 
$\beta_i = -1$ for $B$ and $\beta_i = 0$ for no belonging. The amplitude of this economical and military 
interest is measured by a local positive field $b_i$ which also accounts for the country size 
and importance. At this stage, sets $\{\epsilon_i\}$ and $\{\beta_i\}$ are independent.

Actual actor choices to cooperate or to conflict result from the given set of above 
quantities. The associated energy is, 
\[ H = -\frac{1}{2} \sum_{i>j} \{G_{i,j} + \epsilon_i \epsilon_j J_{i,j}\} \eta_i \eta_j - \sum_i \beta_i b_i \eta_i, \] 

where $\{\eta_i = \pm 1\}$ are Ising variables which discriminate between the two coalition choice 
with $\eta_i = +1$ for $A$ and $\eta_i = -1$ for $B$.

5.2 Cold war scenario

The cold war scenario means that the two existing world level coalitions generate much 
stonger couplings than purely bilateral ones, i.e., $|G_{i,j}| < J_{i,j}$ since to belong to a world 
level coalition produces more advantages than purely local improper relationship. In 
others words local propensities were unactivated since overwhelmed by the two block 
trend. The overall system was very stable. We can thus take $G_{i,j} = 0$. Moreover each 
actor must belong to a coalition, i.e., $\epsilon_i \neq 0$ and $\beta_i \neq 0$. In that situation local 
propensities to cooperate or to conflict between two interacting countries result from 
their respective individual macro-level coalition belongings. the cold war energy is, 
\[ H_{CW} = -\frac{1}{2} \sum_{i>j} \epsilon_i \epsilon_j J_{i,j} \eta_i \eta_j - \sum_i \beta_i b_i \eta_i, \] 

5.2.1 Coherent tendencies

We consider first the coherent tendency case in which cultural and economical trends go 
along the same coalition, i.e., $\beta_i = \epsilon_i$. Then from Eq. (15) the minimum of $H_{CW}$ is unique 
with all country propensities satisfied. Each country chooses its coalition according to 
its natural belonging, i.e., $\eta_i = \epsilon_i$. This result is readily proven via the variable change 
$\tau \equiv \epsilon_i \eta_i$ which turns the energy to, 
\[ H_{CW1} = -\frac{1}{2} \sum_{i>j} J_{i,j} \tau_i \tau_j - \sum_i b_i \tau_i, \]
where $J_{i,j} > 0$ are positive constants. Eq. (16) is a ferromagnetic Ising Hamiltonian in positive symmetry breaking fields $b_i$. Indeed it has one unique minimum with all $\tau_i = +1$.

The remarkable result here is that the existence of two apriori world level coalitions is identical to the case of a unique coalition with every actor in it. It shed light on the stability of the Cold War situation where each actor satisfies its proper relationship. Differences and conflicts appear to be part of an overall cooperation within this scenario. Both dynamics are exactly the same since what matters is the existence of a well defined stable configuration. However there exists a difference which is not relevant at this stage of the model since we assumed $G_{i,j} = 0$. However in reality $G_{i,j} \neq 0$ making the existence of two coalitions to produce a lower “energy” than a unique coalition since then, more $G_{i,j}$ can be satisfied.

It worth to notice that field terms $b_i \epsilon_i \eta_i$ account for the difference in energy cost in breaking a pair proper relationship for respectively a large and a small country. Consider for instance two countries $i$ and $j$ with $b_i = 2b_j = 2b_0$. Associated pair energy is

$$H_{ij} \equiv -J_{ij} \epsilon_i \eta_i \epsilon_j \eta_j - 2b_0 \epsilon_i \eta_i - b_0 \epsilon_j \eta_j.$$  \hspace{1cm} (17)

Conditions $\eta_i = \epsilon_i$ and $\eta_j = \epsilon_j$ give the minimum energy,

$$H_{ij}^m = -J_{ij} - 2b_0 - b_0.$$  \hspace{1cm} (18)

From Eq. (18) it is easily seen that in case $j$ breaks proper alignment shifting to $\eta_j = -\epsilon_j$ the cost in energy is $2J_{ij} + 2b_0$. In parallel when $i$ shifts to $\eta_i = -\epsilon_i$ the cost is higher with $2J_{ij} + 4b_0$. Therefore the cost in energy is lower for a breaking from proper alignment by the small country ($b_j = b_0$) than by the large country ($b_j = 2b_0$). In the real world, it is clearly not the same for instance for the US to be against Argentina than to Argentina to be against the US.

5.2.2 Uncoherent tendencies

We now consider the uncoherent tendency case in which cultural and economical trends may go along opposite coalitions, i.e., $\beta_i \neq \epsilon_i$. Using above variable change $\tau \equiv \epsilon_i \eta_i$, the Hamiltonian becomes,

$$H_{CW2} = -\frac{1}{2} \sum_{i>j}^n J_{ij} \tau_i \tau_j - \sum_{i}^n \delta_i b_i \tau_i,$$  \hspace{1cm} (19)

where $\delta_i \equiv \beta_i \epsilon_i$ is given and equal to $\pm 1$. $H_{CW2}$ is formally identical to the ferromagnetic Ising Hamiltonian in random fields $\pm b_i$. However, here the fields are not random.

The local field term $\delta_i b_i \tau_i$ modifies the country field $h_i$ in Eq. (9) to $h_i + \delta_i b_i$ which now can happen to be zero. This change is qualitative since now there exists the possibility to have “neutrality”, i.e., zero local effective field coupled to the individual choice. Switzerland attitude during World war II may result from such a situation. Moreover countries which have opposite cultural and economical trends may now follow their economical interest against their cultural interest or vice versa. Two qualitatively different situations may occur.

- Unbalanced economical power: in this case we have $\sum_i^n \delta_i b_i \neq 0$. 

The symmetry is now broken in favor of one of the coalition. But still there exists only one minimum.

- Balanced economical power: in this case we have $\sum_i^n \delta_i b_i = 0$.

Symmetry is preserved and $H_{CW2}$ is identical to the ferromagnetic Ising Hamiltonian in random fields which has one unique minimum.

### 5.3 Unique world leader scenario

Now we consider current world situation where the eastern block has disappeared. However it is worth to emphasize the western block is still active as before in this model. Within our notations, denoting $A$ the western alignment, we have still $\epsilon_i = +1$ for countries which had $\epsilon_i = +1$. On the opposite, countries which had $\epsilon_i = -1$ now turned to either $\epsilon_i = +1$ or to $\epsilon_i = 0$.

Therefore above $G_{i,j} = 0$ assumption based on inequality $|G_{i,j}| < |\epsilon_i \epsilon_j| J_{i,j}$ no longer holds for each pair of countries. In particular propensity $p_{i,j}$ becomes equal to $G_{i,j}$ in respective cases where $\epsilon_i = 0$, $\epsilon_j = 0$ and $\epsilon_i = \epsilon_j = 0$.

A new distribution of actors results from the collapse of one block. On the one hand $A$ coalition countries still determine their actual choices according to $J_{i,j}$. On the other hand former $B$ coalition countries are now found to determine their choices according to competing links $G_{i,j}$ which did not automatically agree with former $J_{i,j}$. This subset of countries has turned from a Mattis random site spin glasses without frustration into a random bond spin glasses with frustration. In others world the former $B$ coalition subset has jumped from one stable minimum to a highly degenerated unstable landscape with many local minima. This property could be related to the fragmentation process where ethnic minorities and states shift rapidly allegiances back and forth while they were part of a stable structure just few years ago.

While the $B$ coalition world organization has disappeared, the $A$ coalition world organization did not change and is still active. It makes $|G_{i,j}| < J_{i,j}$ still valid for $A$ countries with $\epsilon_i \epsilon_j = +1$. Associated countries thus maintain a stable relationship and avoid a fragmentation process. This result supports a posteriori arguments against the dissolution of Nato once Warsaw Pact was dissolved.

Above situation could also shed some light on the european debate. It would mean european stability is a result in particular of the existence of european structures with economical reality. These structures produce associated propensities $J_{i,j}$ much stronger than local competing propensities $G_{i,j}$ which are still there. In other words european stability would indeed result from $J_{i,j} > |G_{i,j}|$ and not from either all $G_{i,j} > 0$ or all $G_{i,j} = 0$. An eventual setback of the european construction ($\epsilon_i \epsilon_j J_{i,j} = 0$) would then automatically yield a fragmentation process with activation of ancestral bilateral oppositions.

In this model, once a unique economical as well as military world level organisation exists, each country interest becomes to be part of it. We thus have $\beta_i = +1$ for each actor. There may be some exception like Cuba staying almost alone in former $B$ alignment, but this case will not be considered here. Associated Hamiltonian for the $\epsilon_i = 0$ subset actor
which is formally equivalent to a random bond Hamiltonian in a field. At this stage 
\( \eta_i = +1 \) means to be part of a coalition which is an international structure. On the opposite \( \eta_i = -1 \) is to be in a non-existing coalition which really means to be outside of a.

For small field with respect to interaction the system may still exhibit physical-like frustration depending on the various \( G_{i,j} \). In this case the system has many minima with the same energy. Perpetual instabilities thus occur in a desperate search for an impossible stability. Actors will flip continuously from one local alliance to the other. The dynamics we are referring to is an individual flip each time it decreases the energy. We also allow a flip with probability \( \frac{1}{2} \) when local energy is unchanged.

It is worth to point out that only strong local fields may lift fragmentation by putting every actor in a coalition. It can be achieved through economical help like for instance in Ukraine. Another way is military enforcement like for instance in former Yugoslavia.

Our results point out that current debate over integrating former eastern countries within Nato is indeed relevant to oppose current fragmentation processes. Moreover it indicated that an integration would suppress actual instabilities by lifting frustration.

5.4 The case of China

China is an extremely huge country built up from several very large states. These state typical sizes are of the order or much larger than most other countries in the world. It is therefore interesting to analyse China stability within our model since it represents a case of simultaneous Cold war scenario and Unique world leader scenario.

There exists states which are all part of a unique coalition which is the Chinese central state. Then all \( \epsilon_i = +1 \) but \( \beta_i = \pm 1 \) since some states keep economical and military interest in the "union" \( \beta_i = +1 \) while capitalistic advanced rich states contribute more than their share to the "union" \( \beta_i = -1 \). Associated Hamiltonian is,

\[
H = -\frac{1}{2} \sum_{i>j} \{G_{i,j} + J_{i,j}\} \eta_i \eta_j - \sum_i \beta_i b_i \eta_i ,
\]  

where \( J_{i,j} > 0 \) and \( G_{ij} \) is positive or negative depending on each pair of state \((i, j)\).

At this point China is one unified country which means in particular that \( J_{i,j} > |G_{ij}| \) for all pair of states with negative \( G_{ij} \). Therefore \( \eta_i = +1 \) for each state. Moreover it also implies \( b_i < q_i J_{i,j} \) where \( q_i \) is the number of states state \( i \) interacts with. Within this model, three possible scenarios can be outlined with respect to China stability.

1. China unity is preserved.

Rich states will go along their actual economic growth with the central power turning to a capitalistic oriented federative like structure. It means turning all \( \epsilon_i \) to \(-1\) with then \( \eta_i = \epsilon_i \). In parallel additional development of poor states is required in order to maintain condition \( J_{i,j} > |G_{ij}| \) where some \( G_{ij} \) are negative.
2. Some rich states break unity.

Central power is unchanged with the same political and economical orientation making heavier limitations over rich state development. At some point the condition \( b_i > q_i J_{i,j} \) may be achieved for these states. These very states will then get a lower “energy” breaking down from chinese unity. They will shift to \( \eta_i = -1 \) in their alignment with the rest of China which has \( \eta_j = +1 \).

3. China unity is lost with a fragmentation phenomenon.

In this case, opposition among various states becomes stronger than the central organisational cooperation with now \( J_{i,j} < |G_{ij}| \) with some negative \( G_{ij} \). The situation would become spin glass-like and China would then undergo a fragmentation process. Former China would become a highly unstable part of the world.

6 The risky actor driven dynamics

In principle actors are expected to follow their proper relationship, i.e., to minimize their local “energy”. In other words, actors follow normal and usual patterns of decision. But it is well known that in real life these expectations are sometimes violated. Then new situations are created with reversal of ongoing policies.

To account for such situations we introduce the risky actor. It is an actor who goes against his well defined interest. It is different from the frustrated actor which does not have a well defined interest. Up to now everything was done at “\( T = 0 \)”. However a risky actor chooses coalition associated to \( \eta_i = -1 \), although its local field \( h_i \) is positive. Therefore the existence of risky actors requires a \( T \neq 0 \) situation. The case of Rumania, having its own independent foreign policy, in former Warsaw Pact may be an illustration of risky actor behavior. Greece and Turkey in the Cyprus conflict may be another example.

Once \( T \neq 0 \), it is not the energy which has to be minimized but the free energy,

\[
F = U - TS ,
\]

(22)

where \( U \) is the internal energy, now different from the Hamiltonian and equal to its thermal average and \( S \) is the entropy. To minimize the free energy means stability of a group of countries matters on respective size of each coalition but not, which actors are actually in these coalitions. At a fixed ”temperature” we thus can expect simultaneous shift of alliances from several countries as long as the size of the coalition is unchanged, without any modification in the relative strenghts. Egypt quitting soviet camp in the seventies and Afghanistan joining it may illustrate these non-destabilizing shifts.

Within the coalition frame temperature could be viewed as a way to account for some risky trend. It is not possible to know which particular actor will take a chance but it is reasonable to assume the existence of some number of risky actors. Temperature would thus be a way to account for some global level of risk taking.

Along ideas developed elsewhere [7, 8] we can assume that a level of risky behavior is profitable for the system as a whole. It produces surprises which induce to reconsider some aspect of coalitions themselves. Recent danish refusal to the signing of Maastricht agreement on closer european unity may be viewed as an illustration of a risky actor. The
net effect have been indeed to turn what seemed a trivial and apathetic administrative agreement into a deep and passionated debate among european countries with respect to european construction.

Above discussion shows implementation of $T \neq 0$ within the present approach of coalition should be rather fruitful. More elaboration is left for future work.

Last but not least, it is worth to mention two actual fields of research which could prove useful to our approach at $T \neq 0$. First, simulated annealing (slowly decreasing temperature) which helps to find better ground states. And the somehow similar recent “mutation works” [13]. For our dynamics at $T = 0$ studies [14] may turn useful.

7 Conclusion

The choice of alliances is probably one of the most crucial questions faced by social sytems such as individuals, nations, ethnic minorities, firms and so on. The Statistical Physics based approach opens up a fruitful way to tackle this basic problem. However we showed that partial use of Physics can be rather misleading. Building up a model requires to stick to what is contained in the equations used.

In this paper we have attempted to propose some new notions to construct a model of bimodal alliances. We have shown why the cold war organisation after world war II was rather stable. It was then found how the eastern group disappea rance has induced the emergence of a fragmentation process. Some hints were obtained about possible policies to stabilize world nation relationships. The importance of european construction was also pointed out.

We have outlined what could be a dynamics articulated by the presence of actors who can be either "risky" or "frustrated" or “neutral”. A "risky" actor acts against his well defined interest while a "frustrated" actor having no well defined interest, acts randomly. Associated effects are expected to be instrumental in the building up of alliances.

At this stage, even if the suggested dynamics can be illustrated by some examples or analogies, our model remains rather primitive. However we feel it opens up some possible new road to explore and to forecast international policies. A deeper investigation based on precise data is required to both check the validity of our model and to modify it to make it more realistic.

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