Distributed Ledger Technology for IoT: Parasite Chain Attacks

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Abstract—Directed Acyclic Graph (DAG) based Distributed Ledgers can be useful in a number of applications in the IoT domain. A distributed ledger should serve as an immutable and irreversible record of transactions, however, a DAG structure is a more complicated mathematical object than its blockchain counterparts, and as a result, providing guarantees of immutability and irreversibility is more involved. In this paper, we analyse a commonly discussed attack scenario known as a parasite chain attack for the IOTA Foundation’s DAG based ledger. We analyse the efficacy of IOTA’s core MCMC algorithm using a matrix model and present an extension which improves the ledger’s resistance to these attacks.

I. INTRODUCTION

Distributed Ledger has attracted a great deal of attention in recent years, initially as a peer-to-peer electronic cash system [1], but more recently, distributed ledgers have been applied to problems in the IoT domain, as discussed in detail in [2]. Not all distributed ledgers are suited to IoT applications: often the necessity of transaction fees and the manner in which low value transactions are dealt with makes some DLTs unsuitable. We are particularly interested in DLTs based on Directed Acyclic Graphs (DAGs) in this work, and more specifically, one DAG based ledger known as the IOTA Tangle [3] which seems to possess properties that make it suitable for the use-cases of interest.

The security of distributed ledgers is still much disputed—the immutability and irreversibility of the ledger rely on cryptographic hashing and the consensus protocols built around this, but guarantees are generally probabilistic and subject to many network conditions being met [4]–[7]. In this paper, we analyse a commonly discussed attack scenario known as a parasite chain attack which aims to reverse a transaction that has been accepted by the network and spend the same coin again. We analyse this attack in the context of the IOTA Tangle and investigate the efficacy of IOTA’s core algorithm using an explicit model and present an extension of the algorithm which improves the ledger’s resistance to these attacks.

In Section II we describe the Tangle DAG in detail and introduce a double spending mechanism known as a parasite chain attack. Section III begins with the stochastic model for the Tangle presented in [2]. We extend this model by providing an explicit formulation for the Markov Chain Monte Carlo (MCMC) tip selection algorithm and use this model to simulate the algorithm’s resistance to parasite chain attacks. Section IV introduces a modification to the MCMC which makes use of the growth of the cumulative weight in the Tangle and we present the corresponding results for a parasite chain attack.

II. THE TANGLE

In this paper we expand upon the description of the Tangle given in [2]. Recall that the Tangle is a DAG-based DLT in which every new transaction validates $m$ previous transactions using a Proof of Work (PoW) mechanism. In what follows we use interchangeably the terms site, transaction, and vertex. All yet unapproved sites are called tips and the set of all unapproved transactions is called the tips set. The core metric we employ in this paper, to protect the Tangle against attacks, is the cumulative weight $\mathcal{H}(t)$ of a transaction: this value

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represents the total number of vertices that approve, directly or indirectly, a given site. Figure 2 shows an example of how the cumulative weight changes in time. In what follows, we assume that there is a simple way to verify whether the tips selected for approval by a new transaction are consistent with each other and with all the sites directly or indirectly approved by them (this verification occurs during the approval step). If verification fails, the selection process must be re-run until a set of consistent transactions is found. This consistency property is needed in order to prevent malicious users from tampering with the ledger by means of a double spending attack.

Let us consider an example to develop this idea further: Figure 3 shows a further instance of the Tangle. A malicious user sent a certain amount of money to a merchant. The corresponding transaction is the yellow block in the figure. The same user, afterwards, makes other transactions trying to double spend the same money that were sent to the merchant. These correspond to the green blocks. It is worth stressing, at this point, that there is no mechanism to force a user to select certain sites for confirmation. Any transaction can be selected as long as it is consistent with the sites that are approved (directly or indirectly) by it. Nevertheless it is reasonable to assume that the vast majority of nodes would have little interest in confirming specific transactions and would follow the tips selection algorithm proposed by the protocol. In this scenario, all the transactions that approve the original yellow site (the blue blocks) are incompatible with the green ones, therefore any new transactions can either approve the green/black sites or the blue/black ones. The green/blue combination would be considered invalid (as there is an inconsistency in the ledger) and a new selection would be made. The objective of an hypothetical attacker would be then to wait for the merchant to accept their payment, receive their goods, then create one or more double spending transactions that get approved by other honest sites.

The success probability of such an attack depends on the particular tip selection algorithm employed. So far two Selection algorithms have been proposed:

- **Uniform Random Tip Selection:** The Uniform Random Tip Selection (URTS) algorithm selects \( m \) (generally two) tips randomly from the pool of all possible tips. This algorithm, due to its simplicity, makes the Tangle vulnerable to double spending attacks. The upper panel of Figure 4 shows an example of the random selection procedure. The interested reader can refer to [3] for a detailed discussion on this topic.

- **Markov Chain Monte Carlo Algorithm:** The Markov Chain Monte Carlo (MCMC) tip selection algorithm works in a slightly more elaborate way than its random counterpart. In the MCMC algorithm \( m \) (generally two) independent random walks are created on the tangle; the walks start at the genesis transaction and move along the edges of the graph. The jumping probability from transaction \( j \) to transaction \( k \) is proportional to \( f(-\alpha(H_j - H_k)) \), where \( f(\cdot) \) is a monotonic increasing function (generally an exponential), \( \alpha \) is a positive constant and \( H_i \) represents the Cumulative Weight of transaction \( i \). The jumping process stops when the particle reaches a tip, which is then selected for approval. The lower panel of Figure 4 shows an example of the paths of two walks in this selection procedure. The main difference with the RS algorithm lies in the use of the graph structure: an attacker would need to create enough transactions, with cumulative weights equal to or larger than the cumulative weight of the main DAG, in order to make the double spending successful. This, in turn, would require the malicious user to possess an amount of computational power comparable to the network of honest users. The interested reader can find more details on this topic in [3].

The goal of the MCMC algorithm is to increase (and ultimately ensure) the security of the Tangle.
against double spending attacks performed by malicious users. The algorithm and its security from attacks can be tuned using the parameter $\alpha$: high values of $\alpha$ increase the probability that the particle will jump to the transactions with the largest available cumulative weight (as $\alpha$ approaches $\infty$ the particles will move in a deterministic way), whereas lower values of $\alpha$ make the algorithm and its output more unpredictable (as the cumulative weights matter less and the jumping probability tends to become uniform as $\alpha$ approaches zero). A good way of picturing the effects of this parameter is by comparing it to the inverse temperature of a gas: the smaller the value of $\alpha$ the warmer the gas and vice-versa.

A. The Parasite Chain Attack

To see how in practice an hypothetical attacker could carry out a double spending in the Tangle, consider the attack scenario known as Parasite Chain Attack. A Simple Parasite Chain (SPC) is illustrated in Figure 5 we refer to this as $k^{th}$-order SPC because the first $k$ transactions in the chain reference the main Tangle. The attacker publishes the green transaction in the Tangle and simultaneously, in secret, creates a second spend of the same coin with the yellow transaction followed by a chain of transactions which validate it—the parasite chain. The attacker waits for the green transaction to be confirmed, and the goods to be delivered, and then immediately broadcasts the parasite chain to the network and continues publishing transactions which validate it.
An SPC can be characterized by two parameters:

1) $T_{DS}$ is the time between the first spend of the coin on the Tangle and the SPC being broadcast;

2) $k$ is the ‘order’ of the SPC i.e. the number of transactions referencing the main Tangle.

Note that it is possible to create parasite chains with more complex structure to attack the Tangle. Due to the complexity of the analysis, to the best of our knowledge, it is not yet clear how a chain for an optimal attack should be designed. Therefore, for simplicity, we shall only consider the SPC in this paper.

III. THE MONTE CARLO MARKOV CHAIN ALGORITHM

In this section we present a matrix model for the MCMC algorithm which allows us to compute the probability that an MCMC random walk will terminate on a given tip of a given instance of the Tangle. We then use this model to simulate the effect of the MCMC parameters in the event of a parasite.
We have the relations

\[ \text{A. Matrix Model} \]

The Tangle is a growing family \( G(t) = \{V(t), E(t), t \geq 0\} \) with \( V(t) \) and \( E(t) \) being, respectively, the set of vertices and the set of edges of the graph. The evolution in time of the Tangle is described by the following variables:

1. \( L_i(t) \) is the number of tips of type \( i \) at time \( t \), so \( L(t) = \sum_{i=1}^{d} L_i(t) \)
2. \( W_i(t) \) is the number of ‘pending’ tips of type \( i \) at time \( t \)
3. \( X_i(t) = L_i(t) - W_i(t) \) is the number of ‘free’ tips of type \( i \) at time \( t \)
4. \( T_a \) is the time when transaction \( a \) is created
5. \( \tau_a \in \{1, \ldots, d\} \) is the type of the transaction \( a \), after a valid selection has been made
6. \( N_i(t) \) is the number of transactions of type \( i \) created up to time \( t \)
7. \( U(T_a) \in \{0, 1, 2\} \) is the number of free tips selected for approval by transaction \( a \) at time \( T_a \)

We have the relations

\[ N_i(t) = \sum_{a:T_a \leq t, \tau_a = i} 1 \]  
\[ W_i(t) = \sum_{a:t-h<T_a \leq t, \tau_a = i} U(T_a) \]  
\[ X_i(t) = N_i(t-h) - \sum_{a:T_a \leq t, \tau_a = i} U(T_a) \]  
\[ L_i(t) = N_i(t-h) - \sum_{a:T_a \leq t-h, \tau_a = i} U(T_a) \]

The value of the variable \( U(t) \) depends on the selection algorithm employed: while for the random selection algorithm, \( U(t) \) depends only on the values of \( X_i, W_i \) and \( L_i \), for all \( i \) just prior to time \( T_a \), for the Markov Chain Monte Carlo (MCMC) algorithm the distribution of \( U(t) \) depends on the whole graph (or a subset of it) \( G(t) \). Roughly speaking the MCMC algorithm works as follows: when a new transaction wants to approve a tip, it generates a particle that starts somewhere deep in the graph. This particle jumps randomly from one transaction to another with a probability that is proportional to the difference in cumulative weight between transactions. The process stops when the particle reaches a tip which is going to be selected for approval. Thus, to model the MCMC algorithm and characterize the distribution of \( U(t) \), we need to define explicitly the cumulative weight of each transaction and find an expression to describe the behaviour of the particle.

The cumulative weight \( H_i(t) \) of transaction \( i \) is defined as the number of transactions that directly or indirectly reference it, at time \( t \),

\[ H_i(t) = \# \{ z \in V(t) : z \ references \ i \} . \]  

The weight \( H_i(t) \) can be computed using the adjacency matrix \( M(t) \) of the DAG, and noting that \( [M^k(t)]_{ij} \) represents the number of paths that connect transaction \( i \) and transaction \( j \) in \( k \) steps. In what follows, we assume that if transaction \( i \) arrived at time \( t_i > t_j \), then \( j > i \). Furthermore, define \( D(t) \) as the diameter of the Tangle at time \( t \) (notice that \( D(t) \) represents the longest path that connects two vertices) and define

\[ P(t) = \sum_{k=1}^{D(t)} M^k(t) . \]

Then \( H_i(t) \) is equal to

\[ H_i(t) = \mu_i + \sum_{j=i+1}^{N(t)} \min \{ e_i^T P(t) e_j, 1 \} \mu_j , \]

where \( \mu_i \) is the individual weight of transaction \( i \) and \( e_i \) is the \( i \)-th vector of the canonical orthonormal basis. Notice that we use \( \min \{ \cdot, 1 \} \) in order to avoid counting the same transaction more than once (since there could be several paths with a different number of steps connecting two transactions).

Given \( \mu \), we can go back to the MCMC algorithm: in the most general case, the particle starts in a random site and we can define \( \pi \) as the vector, whose \( i \)-th entry represents the probability that the particle starts at transaction \( i \). Furthermore define the difference in cumulative weight between transaction \( j \) and transaction \( k \) to be \( \varphi_{jk}(t) = H_j(t) - H_k(t) \).
Then the transition matrix $T(t)$ whose $jk$ entry characterizes the probability of transitioning from site $j$ to site $k$ is defined as follows:

$$
[T]_{jk}(t) = \begin{cases} 
q/m & \text{if } k \in O_j \\
(1 - q) \frac{e^{-\alpha \theta_{jk}(t)}}{\sum_{z \in I_j} e^{-\alpha \theta_{jz}(t)}} & \text{if } k \in I_j \\
0 & \text{otherwise}
\end{cases} (8)
$$

where $q \in [0, 1/2]$ represents the probability of going backwards, $m$ is the number of sites directly referenced by $j$, $O_j \subseteq V$ is the set of all sites that are directly referenced by $j$, $I_j \subseteq V$ is the set of all sites that directly reference $j$ and $\alpha$ is a tuning parameter. Clearly, when the particle hits a tip, it remains there indefinitely, therefore we can assume that if transaction $j$ is a tip,

$$
[T]_{jk}(t) = \begin{cases} 
1 & \text{if } j = k \\
0 & \text{otherwise}.
\end{cases} (9)
$$

Equations (8) and (9) provide us with useful information on the MCMC algorithm: the Markov chain has in fact $N(t) - L(t)$ transition states and $L(t)$ absorbing states, where $N(t) = |V(t)|$ is the number of nodes in the tangle at time $t$. Therefore, by re-labelling the states at each step the transition matrix can be displayed as follows:

$$
T(t) = \begin{pmatrix} 
Q(t) & R(t) \\
0 & I
\end{pmatrix}
$$

where $Q(t)$ is the transition matrix between transient states, $R(t)$ is the transition matrix from transient states to absorbing states, and $I$ is the identity matrix for the absorbing states of the Markov Chain. Furthermore, we define $L_i$, $X_i$ to be the sets containing the indices of the tips and free tips of type $i$ respectively, and define the absorbing probability matrix $B = (I - Q)^{-1}R$.

Using standard results for absorbing Markov chains we can express the probability for the random walk to terminate at a tip of type $i$ (i.e., to be absorbed by the subset $L_i$ of the absorbing states) as

$$
p_i(t) = \frac{\sum_{j \in L_i(t)} \pi^T Be_j}{\sum_{k=1}^d \sum_{l \in L_k(t)} \pi^T Be_l} (10)
$$

and then the probability for a new transaction $a$ to join the tip set of type $i$ is

$$
P(\tau_a = i) = p_i(t)^m \left( \sum_{j=1}^d p_j(t)^m \right)^{-1} (11)
$$

(this the second factor on the right side of (11) accounts for the requirement that all $m$ selections must have the same type). Note that this model assumes that $m$ independent random walks will be repeated until $m$ tips of the same type are chosen, which differs from the IOTA Reference Implementation at the time of writing, in which the first walk selects a reference tip and determines $\tau_a$ i.e. $P(\tau_a = i) = p_i(t)$.

This model was validated by generating a random tangle with two branches, one of type 1, and the other of type 2. We see that when we run a large number of monte carlo simulations, counting the number of times the MCMC walk terminates at each type of tip, the calculated probability distribution converges to the value obtained by (10). This is illustrated in Figure 6.

For $m = 2$ the conditional distribution of $U(t)$ can be characterized as follows:

https://github.com/iotaledger/iri
This provides all the necessary equations to characterize the behaviour of the Tangle with the MCMC algorithm.

B. Resistance to Parasite Chain Attacks

We now investigate how the parameters of an SPC attack and the MCMC algorithm will affect the likelihood of an MCMC walk terminating on an SPC tip. For an SPC attack to succeed, a majority of newly arriving transactions must validate tips on the SPC—we do not seek to calculate the probability of this event: for now, we compute the probability of a single MCMC walk selecting a tip of the SPC.

We denote tips on the main Tangle (which reference the first of the double spend transactions) as type 1, and tips on the SPC as type 2. Then (11) gives us:

$$P(U(T_a) = 2 | \tau_a = i) = \sum_{j \in X_i(t)} \frac{\pi^T B_{ej} (\sum_{l \in X_i(t), l \neq j} \pi^T B_{el})}{(\sum_{k \in L_i(t)} \pi^T B_{ek})^2}$$

$$P(U(T_a) = 0 | \tau_a = i) = \left(1 - \frac{\sum_{j \in X_i(t)} \pi^T B_{ej}}{\sum_{k \in L_i} \pi^T B_{ek}}\right)^2$$

$$P(U(T_a) = 1) = 1 - P(U(T_a) = 0 | \tau_a = i) - P(U(T_a) = 2 | \tau_a = i)$$

Nex, we investigate the effect of the double spend time, \(T_{DS}\). The attacker must wait at least until the first transaction is confirmed and accepted by the merchant before broadcasting the SPC containing the second spend. We assume here that this confirmation will require at least 2 minutes, corresponding to the results in Figure 7, but Figure 8 illustrates the effect of waiting longer to broadcast the SPC. The reason for the evident decrease in probability of selecting an SPC tip is related to how the cumulative weight of transactions grow in the Tangle. Because of the width of the Tangle, there is an adaptation period before all new arriving transactions will indirectly approve a site and increase its cumulative weight. However, the parasite chain is only one tip wide and hence, the cumulative weight of transactions on the chain will immediately grow at the rate of arrival of the attacker’s transactions, \(\mu\). This phenomenon is illustrated in Figure 9 which plots the average cumulative weight trajectory of 100 transactions. The intersection point on these plots indicates that the probability of selecting an SPC tip should be very high if the SPC is broadcast after only a minute, and this is confirmed by simulation as illustrated in Figure 10 (note that the first transaction would most likely not be confirmed by this time so this may be pointless for the attacker).

The other key parameter of the SPC is the order, \(k\). There are two factors at play for an attacker when it comes to choosing \(k\). The higher the choice of \(k\), the more likely the MCMC walk is to jump on to the chain. However, more links to the main Tangle also means more opportunities to jump back on to the Tangle from the SPC. An interesting relationship arises between the probability of selecting an SPC tip, the attacker’s choice of \(k\), and the MCMC walk backstepping probability parameter \(q\). This relationship is illustrated in Figure 11.

IV. EXTENDING THE MCMC ALGORITHM

As described in the previous section, the MCMC algorithm is, at its core, a biased random walk whose transition probabilities depend on the cumulative weights of the graph vertices. The original motivation for using the MCMC selection was to incentivise network users to validate recent tips and defend the Tangle against attacks such
as the parasite-chain attack. The random walk is biased towards the 'heaviest' sub-DAG i.e. the sub-DAG with highest cumulative weight, and the aim of the attacker who wishes to double spend is to create a sub-DAG in secret quicker than the rest of the network’s honest users so that when they publish it, the MCMC walks will tend to choose their new, heavier sub-DAG.

Theoretically, if the MCMC parameter \( \alpha \) is high enough and if the attacker does not possess hashing power greater than the rest of the network combined, then the attack should never succeed. However an excessively high value of \( \alpha \) would essentially result in the Tangle becoming a chain, and many transactions would be orphaned. The design trade-off between security and liveness involved in choosing \( \alpha \) is discussed in [9], and solutions to the issue of transactions being orphaned are proposed in [8].

We now propose a modification to the MCMC algorithm which seeks to reduce the efficacy of double spending attacks whilst allowing us to maintain a low \( \alpha \), and hence a wide Tangle and low probability of orphaned transactions. The intuition for our modification stems from the phenomenon illustrated in Figure 9. Although the cumulative weight of a transaction on the main Tangle may lag behind due to the initial adaptation period it underwent, we can be quite sure that at the point of attachment of a parasite chain, deep in the Tangle, the rate of growth of the cumulative weight of transactions should be equal to \( \lambda \). Our modification
utilises the first order time derivative of the cumulative weight in calculating the jumping probabilities of the MCMC walk—we use a First Order MCMC. Define $\mathcal{H}_k'(t) = |\mathcal{H}_j'(t) - \mathcal{H}_k'(t)|$, where $\mathcal{H}_k'(t)$ is the time derivative of the cumulative weight, $\mathcal{H}_k(t)$:

$$[T]_{jk}(t) = \begin{cases} \frac{q}{m} & \text{if } k \in O_j \\ (1-q) \frac{e^{-\alpha \mathcal{H}_j'(t)} - \beta \mathcal{H}_j'(t)}{\sum_{z \in I_j} e^{-\alpha \mathcal{H}_j'(t)} - \beta \mathcal{H}_j'(t)} & \text{if } k \in I_j \\ 0 & \text{otherwise} \end{cases}$$

(13)

where $\beta$ is a positive tuning parameter.

A. Resistance to Parasite Chain Attacks

The cumulative weight of a transaction in a parasite chain grows linearly with rate equal to the computing power of the attacker, $\mu$, whilst the main Tangle will grow at the rate of the hashing power of the rest of the network, $\lambda$, as illustrated in Figure 9. Therefore, the parasite chain will be heavily penalized by the First Order MCMC.

Simulation results for the same instance of the Tangle used for examples in Section III are shown in Figure 12. These preliminary results suggest that the First Order MCMC achieves its goal. Of course, this algorithm performance will start to deteriorate when the particle approaches a tip, as at this height, the cumulative weight on the main Tangle grows at a different rate. Therefore, it could be useful to modify the derivative term adding a weight, $d_j$ that increases with each forward jump (and decreases with every backward jump). It would also be interesting to investigate the effects of such a modification on Nash equilibria and other game theoretic aspects in the Tangle as discussed in [10].

V. CONCLUSIONS

In this paper we present a matrix model for the MCMC tip selection algorithm. We use this model to simulate the resistance of this algorithm to parasite chains on a randomly generated Tangle for different parameters of the attack and of the algorithm. We then present an extension of the MCMC algorithm to utilise the first order time derivative of cumulative weight, and accordingly, we demonstrate its effectiveness at mitigating parasite chain attacks over its predecessor.

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