Axial and off-axial dynamic transitions in uniaxially anisotropic Heisenberg ferromagnet: A comparison

Muktish Acharyya

Department of Physics, Krishnanagar Government College
P.O.-Krishnanagar, PIN-741101, Dist.- Nadia, West-Bengal, India
E-mail: muktish@vsnl.net

Uniaxially anisotropic Heisenberg ferromagnet, in presence of a magnetic field varying sinusoidally in time, is studied by Monte Carlo simulation. The axial (field applied only along the direction of anisotropy) and off-axial (field applied only along the direction which is perpendicular to the direction of anisotropy) dynamic transitions are studied. By studying the distribution of dynamic order parameter component it is observed that the axial transition is discontinuous for low anisotropy and becomes continuous in high anisotropy. The off-axial transition is found to be continuous for all values of anisotropy. In the infinite anisotropy limit, both types of transitions are compared with that observed in an Ising ferromagnet for the same value of the field and frequency. The infinitely anisotropic axial transition and the dynamic transition in the Ising ferromagnet occur at different temperatures whereas the infinitely anisotropic off-axial transition and the equilibrium ferro-para transition in the Ising model occur at same temperature.

Keywords: Monte Carlo simulation, Dynamical phase transition, Heisenberg ferromagnet, Uniaxial anisotropy

1. Introduction

The nonequilibrium dynamical phase transition in magnetic model systems has become an interesting field of research in last decades [1, 2]. Extensive Monte Carlo simulation yields some interesting new results in the Ising model [1]. These studies [1, 2, 3] were able to establish the significant features of nonequilibrium phase transitions having similarities with well known equilibrium phase transitions.

However, the Ising model is a special case of general magnetic model, for example, the Heisenberg model. The Heisenberg model (with ferromagnetic interactions) having uniaxial anisotropy has some general properties which cannot be found in Ising model. But in the limit of infinite anisotropy, the Heisenberg model can be mapped into Ising model [4]. So, the natural expectation is, the Heisenberg model with uniaxial anisotropy can be studied to have the detailed and general microscopic view and the results can be checked in the limit of infinite anisotropy (which will give the results in Ising model). In this case of dynamic transitions, mainly in the magnetic model system in presence of a magnetic
field oscillating sinusoidally in time, the Heisenberg model can serve a better role than an Ising model. It would be quite interesting to know the dynamic response of uniaxially anisotropic Heisenberg model in presence of a magnetic field applied in different directions. On the other hand, there is another advantage. The results obtained in the Ising model is well established [1]. These results can be used to check the results obtained in Heisenberg model by approaching the limit of infinite anisotropy. This prompted to study the dynamic transition in Heisenberg model with uniaxial and single-site anisotropy. Recently, the dynamic transition was studied [5] in the uniaxially anisotropic ferromagnetic Heisenberg model and very interestingly it was observed that the dynamic symmetry of the order parameter component (along the anisotropy direction) can be broken in presence of a magnetic field applied along the direction which is perpendicular to the direction of anisotropy. This transition was named as off-axial transition. The transition is found to be continuous and the transition temperature increases as the strength of anisotropy increases.

So, the questions naturally arise what would be the difference in the dynamic transitions in presence of a field applied only along the direction of anisotropy? How the symmetry breaking takes place? What would be the nature (continuous or discontinuous) of the transition? More interestingly, what would happen in infinite anisotropic case and in the Ising case? To get the answers of these questions, the dynamic transitions in presence of the axial field (i.e., the magnetic field applied only along the direction of anisotropy) and the off-axial field (i.e., the magnetic field applied only along the direction which is perpendicular to the direction of anisotropy) are studied in this paper by Monte Carlo simulation using Metropolis rate. Also, a comparison between axial and off-axial transitions has been made and the results (in the limit of infinite anisotropy) for both cases are compared with that observed in the Ising model.

The uniaxially anisotropic Heisenberg model (with ferromagnetic interaction) is introduced and explained in section 2. The Monte Carlo simulation technique is discussed in the section 3. The numerical results are reported in the next section and the paper ends with a concluding remarks in section 5.

2. The description of the model

The Hamiltonian of a classical anisotropic (uniaxial and single-site) Heisenberg model with nearest neighbour ferromagnetic interaction in the presence of a magnetic field can be written as

$$H = -J \sum_{<ij>} \vec{S}_i \cdot \vec{S}_j - D \sum_i (S_{iz})^2 - \vec{h} \cdot \sum_i \vec{S}_i,$$

(1)

where $\vec{S}_i[S_{ix}, S_{iy}, S_{iz}]$ represents a classical spin vector of magnitude unity situated at the $i$-th lattice site. So, $S_{ix}^2 + S_{iy}^2 + S_{iz}^2 = 1$ is an equation of a unit sphere. Classical spin means, this spin vector can be oriented in any direction in the vector spin space. $J(>0)$ is the uniform nearest neighbour strength of the ferromagnetic interaction. The factor $D$ in the second term is the strength of uniaxial ($z$ here) anisotropy favouring the spin
to be aligned along the $z$-axis. The last term is the spin-field interaction term, where $\vec{h}[h_x, h_y, h_z]$ is the externally applied magnetic field (uniform over the space). When the magnetic field is applied only along the $\alpha$-direction, the magnetic field component $h_\alpha$ (may be any one of $x$, $y$ and $z$) is oscillating sinusoidally in time and can be written as $h_\alpha(t) = h_0^\alpha \cos(\omega t)$, where $h_0^\alpha$ and $\omega$ are the amplitude and angular frequency ($\omega = 2\pi f$; $f$ is frequency) of the oscillating field respectively. Magnetic field $|\vec{h}|$ and strength of anisotropy $D$ are measured in the unit of $J$. The model is defined in a simple cubic lattice of linear size $L$ with periodic boundary conditions applied in all the three directions.

3. The Simulation technique

The model, described above, has been studied extensively by Monte Carlo simulation using the following algorithm [6]. Initial configuration is a random spin configuration. Here, the algorithm used, can be described as follows. Two different random numbers $r_1$ and $r_2$ (uniformly distributed between -1 and 1) are chosen in such a way that $R^2 = (r_1^2 + r_2^2)$ becomes less than or equal to unity. The set of values of $r_1$ and $r_2$, for which $R^2 > 1$, are rejected. Now, $u = \sqrt{1 - R^2}$. Then, $S_{ix} = 2ur_1$, $S_{iy} = 2ur_2$ and $S_{iz} = 1 - 2R^2$.

Starting from an initial random spin configuration (corresponding to high temperature configuration) the system is slowly cooled down. At any fixed temperature $T$ (measured in the unit of $J/K_B$) and field amplitude $h_0^\alpha$ (measured in the unit of $J$) a lattice site $i$ has been chosen randomly (random updating). The value of the spin vector at this randomly chosen site is $\vec{S}_i$ (say). The energy of the system is given by the Hamiltonian (equation 1) given above. Now, a test spin vector $\vec{S}'_i$ is chosen randomly (described by the algorithm above). For this choice of spin vector at site $i$ the energy will be $H' = -J \sum_{<ij>} \vec{S}_i \cdot \vec{S}_j - D \sum_i (S'_{iz})^2 - \vec{h} \cdot \sum_i \vec{S}_i$. The change in energy, associated to this change in direction of spin vector from $\vec{S}_i$ to $\vec{S}'_i$, is $\Delta H = H' - H$. Now, the Monte Carlo method [7, 8] will decide how far this change is acceptable. The probability of the change is given by Metropolis rate [7, 8] (used here) $W(\vec{S}_i \rightarrow \vec{S}'_i) = \text{Min}[1, \exp(-\Delta H/K_BT)]$. This probability will be compared with a random number $R_p$ (say) between 0 and 1. If $R_p$ does not exceed $W$, the move (the change $\vec{S}_i \rightarrow \vec{S}'_i$) is accepted. In this way the spin vector $\vec{S}_i$ is updated. $L^3$ such random updates of spins, defines one Monte Carlo step per site (MCSS) and this is considered as the unit of time in this simulation. The linear frequency ($f = \omega/2\pi$) of the oscillating field is taken equal to 0.001 and kept constant throughout this simulational study. So, 1000 MCSS is required to get one complete cycle of the oscillating field and consequently 1000 MCSS becomes the time period ($\tau$) of the applied oscillating magnetic field. To calculate any macroscopic quantity, like instantaneous magnetisation components, the following method was employed. Starting from an initially random configuration (which corresponds to a high temperature phase) the system is allowed to be stabilised (dynamically) up to $4 \times 10^4$ MCSS (i.e., 40 complete cycles of the oscillating field) and the averages of various physical quantities are calculated from further $4 \times 10^4$ MCSS (i.e., averaged over further 40 cycles of the oscillating field). This is quite important to get stable hysteresis loop and it is checked that the number of MCSS mentioned above
is sufficient to get stable dynamic phase. Here the total length of this simulation for one fixed temperature $T$ is $8 \times 10^4$ MCSS (which produces 80 complete cycles of the oscillating field). Then the system is slowly cooled down (the value of the temperature $T$ has been reduced by small interval) to get the values of the statistical quantities in the low temperature ordered phase. Here, the last spin configuration obtained at the previous temperature is used as the initial configuration for the new temperature. The CPU time required for $8 \times 10^4$ MCSS is approximately 22 minutes on an Intel-Pentium-III processor.

4. Numerical results

The linear size of the system $L$ has been taken equal to 20. The instantaneous magnetisation components (per lattice site) $m_x = \sum_i S_i^x / L^3$, $m_y = \sum_i S_i^y / L^3$, $m_z = \sum_i S_i^z / L^3$ are calculated at each time in presence of magnetic field. The time averaged (over a full cycle of the oscillating field) magnetisation components (the dynamic order parameter components) $Q_x = \frac{1}{T} \oint m_x dt$, $Q_y = \frac{1}{T} \oint m_y dt$ and $Q_z = \frac{1}{T} \oint m_z dt$ are calculated by integrating (over the complete cycle of the oscillating field) the instantaneous magnetisation components. The total (vector) dynamic order parameter is expressed as $\vec{Q} = \hat{x}Q_x + \hat{y}Q_y + \hat{z}Q_z$.

In this paper, two kinds of dynamic transitions were studied and compared. The axial transition means the dynamic order parameter component $Q_z$ becomes zero from a nonzero value at a finite temperature (the transition temperature) in presence of a magnetic field $\vec{h}[0,0,h_z]$ applied only along the direction which is parallel to the direction of anisotropy. Since the uniaxial anisotropy has been taken along the $z$-direction, in this case, the direction of magnetic field has only nonzero $z$-component. The off-axial transition [5] is the transition in presence of a magnetic field $\vec{h}[h_x,0,0]$ applied only along the direction which is perpendicular to the direction of anisotropy. In this case, the direction of the magnetic field has only nonzero $x$-component.

In the case of axial transition, the instantaneous magnetisation components are calculated at any fixed temperature $T$, strength of anisotropy $D$ and amplitude of axial magnetic field $h_z^0$. The time eliminated plot of $m_z - h_z$ gives the axial hysteresis loop. It was observed that at high temperature ($T = 2.2$) the axial hysteresis loop $m_z - h_z$ is symmetric (symmetric means the loop is distributed about $h_z$ axis in such a way that the total $z$-component of magnetization, over a complete cycle of field, vanishes) (fig.1a). As a result $Q_z = 0$. And at low temperature ($T = 1.0$) the $m_z - h_z$ loop becomes asymmetric ($Q_z \neq 0$) (fig.1b). In both cases, the $m_x - h_z$ and $m_y - h_z$ loops lie almost along $h_z$ axis, resulting $Q_x$ and $Q_y$ equal to zero respectively. Thus a dynamic transition occurs (as the temperature decreases) at a certain temperature from a symmetric ($Q_z = 0$; $\vec{Q} = 0$) to an asymmetric ($Q_z \neq 0$; $\vec{Q} \neq 0$) dynamic phase in presence of an axial magnetic field.
Fig. 1. Symmetry breaking in axial and off-axial transitions. The plot of instantaneous magnetization components against the instantaneous field components. (a) $m_x(t) - h_x(t)$ and $m_z(t) - h_z(t)$ loops for $D = 2.5$, $h_0^z = 0.5$ and $T = 2.2$, (b) $m_x(t) - h_x(t)$ and $m_z(t) - h_z(t)$ loops for $D = 2.5$, $h_0^z = 0.5$ and $T = 1.0$, (c) $m_x(t) - h_x(t)$ and $m_z(t) - h_z(t)$ loops for $D = 0.5$, $h_0^z = 0.5$ and $T = 1.8$ and (d) $m_x(t) - h_x(t)$ and $m_z(t) - h_z(t)$ loops for $D = 0.5$, $h_0^x = 0.5$ and $T = 0.6$.

What was observed in the case of off-axial transition? Recently studied [5] off-axial transition shows similar dynamic transition via breaking the symmetry of $m_z - h_x$ loop in presence of an off-axial field (along perpendicular to the anisotropy direction i.e., $x$-direction). Here, at high temperature ($T = 1.8$) the $m_z - h_x$ loop is symmetric (and $Q_z = 0$) and $m_x - h_x$ loop is also symmetric ($Q_x = 0$) (fig.1c). At some lower temperature ($T = 0.6$), the $m_z - h_x$ loop becomes asymmetric ($Q_z = 0$) and $m_x - h_x$ loop remains still symmetric ($Q_x = 0$) (fig.1d). In both temperatures $Q_y = 0$. So, here also a dynamic transition occurs (as the temperature decreases) at a certain temperature from a symmetric ($Q_z = 0$; $\vec{Q} = 0$) to an asymmetric ($Q_z \neq 0$; $\vec{Q} \neq 0$) dynamic phase in presence of an off-axial magnetic field. Interestingly, it may be noted here that in higher temperature the $m_z - h_x$ loop is ‘marginally symmetric’ (lies very close to $h_x$ axis) rather
than a symmetric loop (symmetrically distributed away from and about $h_x$ line). Strictly speaking, the dynamic transition occurs here from a ‘marginally symmetric’ (loop does not widen up) to an asymmetric phase. One can differentiate the symmetric phase from the ‘marginally symmetric’ phase by considering the loop area of that loop whose symmetry breaking is considered in the transition. In the symmetric phase loop is sufficiently widen up resulting nonzero loop area. In Fig. 1a, the $m_z - h_z$ loop area is 0.686 (symmetric loop; $Q_z = 0$). But the ‘marginally symmetric’ loops ($m_z - h_x$) have vanishingly small area (0.01)(see Fig. 1c) and $Q_z = 0$. It may be noted that, in the case of off-axial transition, if the magnetic field applied along the x-direction only (oscillating sinusoidally in time) the $m_x - h_x$ loop is always symmetric (consequently $Q_x = 0$) irrespective of the value of temperature and the strength of anisotropy $D$ (z-axis). Similarly, for any field applied along y-direction only, the $m_y - h_y$ loop is found to be always symmetric (i.e., $Q_y = 0$) irrespective of value of $T$ and $D$. But in both cases, whether the off-axial loops i.e., $m_z - h_x$ or $m_z - h_y$ will be symmetric (rather ‘marginally symmetric’) or asymmetric that depends upon the values of temperature $T$, anisotropy $D$ and the magnetic field amplitude $h_0^x$ (or $h_0^y$). These results signify that without anisotropy the dynamic transition (associated to the dynamic symmetry breaking) cannot be observed in the classical Heisenberg model.

![Fig. 2. The axial dynamic transitions. Temperature ($T$) variations of dynamic order parameter components $Q_z$ for different values of anisotropy strength ($D$) represented by different symbols. $D = 0.5(\Diamond)$, $D = 2.5(+)$, $D = 5.0(\square)$, $D = 15.0(\times)$ and $D = 400.0(\triangle)$. In all these cases for the axial transitions $h_0^z = 0.5$. The data for the temperature variation of dynamic order parameter in the Ising model (for $h_0^z = 0.5$ and $f = 0.001$) are represented by $\star$. Continuous lines in all cases are just connecting the data points.](image)

To investigate the dependence of transition temperature on the strength of anisotropy ($D$) in the case of axial transition, the temperature variation of dynamic order parameter
component $Q_z$ was studied for different values of $D$. Figure 2 shows the temperature variation of $Q_z$ for different values of $D$. Here, like the case of off-axial transition [5] the transition temperature increases as the strength of anisotropy increases. It is observed that the axial transition is discontinuous for lower values of anisotropy strength $D$ (i.e., 0.5, 2.5 etc.) and it becomes continuous for higher values of $D$ (i.e., 5.0 15.0 etc.). In the Ising limit ($D \to \infty$) the axial transition is also shown in the same figure for $D = 400$). This choice of the value of $D(= 400)$ is not arbitrary. In the case of equilibrium transition it was checked by MC simulation that the value of the magnetisation at any temperature (in the ferromagnetic region) becomes very close to that (at that temperature) obtained in the Ising model if the strength of anisotropy is chosen above 300.

![Graphs showing normalized distributions of dynamic order parameter component $Q_z$ for different values of temperatures ($T$) in the case of axial transition. Here, $D = 2.5$ and $h_0 = 0.5$.](attachment:image.png)

To establish the discontinuous nature of the transition for lower values of $D$ the well known and widely used method [8] i.e., to check the distribution of order parameter component $Q_z$ at some temperatures very close to transition temperature, was employed here. It may be noted here that a similar method was successfully applied recently in the case of dynamic transition in the Ising model [9]. Figure 3 shows the normalized distribution ($\int P(Q_z)dQ_z = 1$) of dynamic order parameter component $Q_z$ for different temperatures. The distribution ($P(Q_z)$) at each temperature was found from 8000 different values of $Q_z$. For $D = 2.5$ the transition occurs around $T \approx 1.543$ (Fig. 2.). Below the transition
temperature (i.e., in ordered phase $T = 1.527$) the distribution shows two peaks (fig. 3a) and above the transition temperature (i.e., in disordered phase $T = 1.600$) this shows only one peak (fig.3c). However, very close to the transition temperature ($T \simeq 1.543$) the distribution has three peaks (fig.3b) indicating clearly the discontinuous nature of the transition. The transition becomes continuous for higher values of $D$. The temperature variations of $Q_z$ for infinitely anisotropic ($D = 400$) Heisenberg model and that in the Ising model were compared (for $h_z = 0.5$ and $f = 0.001$ in both cases). In both the cases, the transitions are found to be continuous. But, the transition temperatures were found to be different (see Fig. 2) although the anisotropic Heisenberg model maps into an Ising model in $D \to \infty$ limit.

Fig. 4. The off-axial dynamic transitions. Temperature ($T$) variations of dynamic order parameter components $Q_z$ for different values of anisotropy strength ($D$) represented by different symbols. $D = 0.5$, $D = 2.5$, $D = 5.0$, $D = 15.0$, and $D = 400$. In all these cases for off - axial transitions $h_x^0 = 0.5$. The data for the zero-field ferro-para equilibrium Ising transition are represented by *. Continuous lines in all cases are just connecting the data points.

The temperature variations of dynamic order parameter component $Q_z$ in the case of off-axial transition was also studied and shown in figure 4 for different values of $D$. This shows that the transition temperature increases as $D$ increases. Here, the transition is continuous for all values of strength of anisotropy $D$. The transition for $D = 400$ ($D \to \infty$ limit) was compared with that in the case of Ising model. This shows that both are continuous and occur at the same point ($T \approx 4.5$) which is very close to the Monte Carlo results of equilibrium ferro-para transition temperature ($T_c \approx 4.511$) [7] in 3-dimensional Ising model.
5. Concluding remarks:

The nonequilibrium dynamical phase transition in the uniaxially anisotropic Heisenberg model, in presence of magnetic field oscillating sinusoidally in time, is studied by Monte Carlo simulation. Two cases were studied in this paper. (i) magnetic field oscillating sinusoidally in time is applied only along the direction of anisotropy, (ii) magnetic field applied only along the direction perpendicular to the direction of anisotropy. The transition observed in the first case is named axial and that corresponding to the second case is called off-axial. A comparative study between axial and off-axial transition is reported in this paper. Three important aspects are considered here. (a) symmetry breaking, (b) the order of the transition and (c) the transition in infinite anisotropic limit.

A dynamic symmetry breaking is observed with this dynamic transition. In the case of axial transition the dynamic transition occurred as the temperature decreases from a symmetric to an asymmetric phase. Whereas, in off-axial case this symmetry breaking takes place from a 'marginally symmetric' to an asymmetric phase. The reason behind it is as follows: in the case of axial transition by the application of axial field (oscillating sinusoidally in time) there is a chance that the spin component along the z-direction may be reversed in opposite direction which would lead to sufficiently wide and symmetric \( m_z - h_z \) loop. But in the case of off-axial transition it is not possible to reverse the \( z \)-component of spin by applying a field (oscillating sinusoidally in time) perpendicular to the direction of uniaxial anisotropy. In this case the value of the \( z \)-component of magnetisation \( m_z \) is almost zero. As a result the \( m_z - h_x \) loop lies on \( h_x = 0 \) axis and hence the loop is marginally symmetric.

In both the cases (axial and off-axial) the transition temperature increases as the strength of anisotropy increases provided the amplitude of the applied field remains same. The strength of anisotropy tries to align the spin vector along the direction of anisotropy. So, as the strength increases it becomes harder to break the symmetry and consequently more thermal fluctuation is required to break the symmetry. As a result, the transition temperature increases as the strength of anisotropy increases. But the difference is the nature of transition. In the case of axial field the transition is discontinuous for lower values of anisotropy and it becomes continuous for higher values of anisotropy. The reason behind it is, the axial transition occurs in presence of axial field which reverses the \( z \)-component of magnetisation. So, in lower values of anisotropy the spin vector becomes comparatively more flexible and the transition occurs mechanically in presence of axial field at lower temperature and it is discontinuous. As the anisotropy increases the effect of axial field (of same value) becomes weak and the transition is driven by thermal fluctuations and the transition is continuous. In the case of off-axial transition, the off-axial field cannot reverse the \( z \)-component of magnetisation. But as the value of off-axial field increases, the value of \( x \)-component of magnetisation increases at the cost of \( z \)-component of magnetisation. The transition is driven by thermal fluctuations and continuous.

What will be the situation in the limit of infinite strength of anisotropy? In the case of axial transition it was observed that the transition temperature for infinitely anisotropic Heisenberg model differs from that obtained in an Ising model. Although the equilibrium
transitions in infinitely anisotropic Heisenberg model and that in the Ising model gives the same transition temperature, the nonequilibrium transition temperatures in those two cases are not same. Since the magnetic field applied in z-direction oscillating sinusoidally in time, keeps the system always away from the equilibrium, the system does not become an Ising system even in infinite anisotropy limit. As a result, the dynamic transition temperature in infinitely anisotropic Heisenberg model cannot be same for that obtained in the Ising model. But in the case of off-axial transition, the transition temperatures in infinitely anisotropic Heisenberg model and that in the Ising model becomes exactly equal. The reason behind it is as follows: in the case of off-axial transition the field is applied perpendicular to the direction of anisotropy. The effect of axial field oscillating sinusoidally in time has no effect in infinite anisotropic limit. Though the magnetic field applied in the x-direction oscillating sinusoidally in time, the infinite anisotropic Heisenberg model becomes an Ising model in statistical and thermal equilibrium. Hence, the infinitely anisotropic Heisenberg model in presence of off-axial field maps into the Ising model in zero field. That is why the nonequilibrium transition in infinitely anisotropic Heisenberg model in presence of off-axial field and the Ising model (in zero external field) give the same result.

One important point may be noted here regarding the dynamics chosen in this simulation. Since, the spin component does not commute with the Heisenberg Hamiltonian the spin component has an intrinsic dynamics. Considering this intrinsic dynamics there was a study [10] about structure factor and transport properties in XY- model. However, in this paper, the motivation is to study the nonequilibrium phase transition driven by thermal fluctuations. To study this, one should choose the dynamics which arises due to the interaction with thermal bath. Since the objective is different, in this paper, the dynamics chosen here (which arises solely due to the interaction with thermal bath), is Metropolis dynamics. The effect of intrinsic spin dynamics is not taken into account.

To find the phase boundaries of axial and off-axial transitions, and their dependence on the strength of anisotropy and the frequency of the oscillating field, is the plan of further study. It is a huge computational task and requires much computer time. The work is in progress and the details will be reported elsewhere.

**Acknowledgments:**  
The library facility provided by Saha Institute of Nuclear Physics, Calcutta, is gratefully acknowledged. Author would like to thank the referee for bringing Ref. [6] into his notice.
References:

1. B. K. Chakrabarti and M. Acharyya, Rev. Mod. Phys., 71, 847 (1999) and the references therein.

2. M. Acharyya and B. K. Chakrabarti, in Annual Reviews of Computational Physics, Vol. 1, ed. D. Stauffer (World Scientific, Singapore 1994), pp. 107

3. S. W. Sides et al., Phys. Rev. Lett., 81, 834 (1998); M. Acharyya, Phys. Rev. E 56, 2407 (1997)

4. D. C. Mattis, The theory of magnetism I: Statics and dynamics, Springer Series in Solid-State Science, Vol. 17 (Springer-Verlag, Berlin, 1988)

5. M. Acharyya, Int. J. Mod. Phys. C 12, 709 (2001)

6. D. P. Landau and K. Binder, in A Guide to Monte Carlo Simulations in Statistical Physics, Cambridge University Press, Cambridge, UK 2000, pp. 145.

7. D. Stauffer et al, Computer Simulation and Computer Algebra, (Springer-Verlag, Heidelberg, 1989).

8. K. Binder and D. W. Heermann, Monte Carlo Simulation in Statistical Physics, Springer Series in Solid-State Sciences, (Springer, 1997)

9. M. Acharyya, Phys. Rev. E 59, 218 (1999)

10. M. Krech and D. P. Landau, Phys. Rev. B 60, 3375 (1999)