Vortical Structures in Wall-Bounded Turbulent Flow with Recirculation

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Abstract.
Hairpin or horse-shoe vortices are a widely-accepted feature of the wall-bounded flows. These vortical structures have mostly been studied in canonical flows. Relatively few studies have been conducted on the characteristics of the vortical structures in wall-bounded flows with adverse pressure gradient and still fewer on the detached flows with recirculation. In the present contribution, vortices have been educed using a DNS database of incompressible flow over a 2-dimensional surface bump in a converging-diverging channel at a Reynolds number $Re_\tau$ of 617, based on the friction velocity at inlet. Vortices have been educed from the instantaneous velocity field in streamwise/wall-normal and spanwise/wall-normal planes using the signed swirling strength criterion. Vortex validation is done through a fit of the vortex velocity field to the Oseen vortex model. The effects of a strong adverse pressure gradient and flow recirculation on the population density and sizes of the streamwise and spanwise-oriented vortices have been studied. It has been found that a strong adverse pressure gradient and flow recirculation leads to the generation of a new near-wall peak of small spanwise prograde vortex population. Furthermore, this peak of vortex density has been found to coincide and hence relate to the outward movement of the peak of streamwise rms velocity fluctuations typical of adverse pressure gradient wall-bounded turbulent flows.

1. Introduction

A turbulent flow consists of coherent and incoherent motions with varying scales. These coherent motions or structures were termed as “organised motion” in an effort to search for order in the apparent disorder of turbulence. Vortical structures play an important role in heat, mass and momentum transport in turbulent flows and this makes them important for combustion, chemical reactions as well as drag and aerodynamic noise. These structures have been observed in a variety of types according to size, shape, orientation, aspect ratio, convection velocities, strength, etc. Various kinds of vortical structures have been observed like vortex “worms” found in isotropic turbulence Jiménez et al. (1993), vortex “braids” in turbulent shear layers Rogers & Moser (1994), quasi-streamwise vortices Robinson (1991a) and “hairpin” vortices found in wall turbulence Adrian et al. (2000) etc.

The existence of hairpin vortices inclined to the streamwise direction, first proposed by Theodorsen Theodorsen (1952), was confirmed through flow visualization by Head and Bandyopadhyay Head & Bandyopadhyay (1981). They further proposed that this
structure is inclined at 45° to the streamwise direction. This three-dimensional vortical structure is composed of a pair of counter-rotating legs that are joined through a head segment. It has been observed by Adrian et al. Adrian et al. (2000) that these vortices travel together in groups called ‘hairpin packets’. However, a detailed description of their size, shape and dynamics is still not complete. The aspect ratio of these vortices changes with Reynolds number and a horse-shoe vortex becomes a hairpin vortex at higher Reynolds numbers Gad-El-Hak & Bandyopadhyay (1994). The properties like size, orientation, etc of the streamwise vortical structures very near to the wall were first investigated by Blackwelder and Eckelmann Blackwelder & Eckelmann (1979), who experimentally detected counter-rotating vortex pairs by means of conditional analysis. They found the expected vortex center at \( y^+ \approx 15 \), core radius \( r_o^+ \approx 15 \), streamwise length of around 200\( + \) and spanwise spacing of 50\( ^+ - 200^+ \), where ‘\(^+\)’ denotes the wall unit scaling and \( y \) is the wall-normal coordinate.

Carlier and Stanislas Carlier & Stanislas (2005) used Particle Image Velocimetry (PIV) to investigate the vortical structures in the log layer of a turbulent boundary layer at Reynolds numbers \( Re_\theta = 1900 \) and \( Re_\theta = 7500 \). Their results showed that vortical structures had an origin at \( y^+ \approx 25 \) and their radius was in the range 18-25. Using signed swirling strength criteria, they detected dense populations of prograde (vorticity \( \omega < 0 \)) and retrograde (\( \omega > 0 \)) vortices with prograde vortices mostly found near the wall. Retrograde vortices, on the other hand, were found to increase with wall-normal distance till the top of the log layer and then found to decrease further upwards.

Elsinga et al. Elsinga et al. (2007) conducted tomographic PIV of a turbulent boundary layer at momentum thickness Reynolds number \( Re_\theta = 1900 \) and employed Q-criterion to educe the vortical structures. They observed mostly the asymmetric hairpin vortices in the lower part of the boundary layer (\( y/\delta < 0.5 \)), where \( \delta \) is the boundary layer thickness. Streamwise vortices were found to be located near the sweep events and to have an approximate length of 0.2\( \delta \). With increasing distance from the wall, the length was noted to increase and at \( y/\delta = 0.6 \), the largest structure was found to be approximately 0.5\( \delta \).

In addition to PIV, Direct Numerical Simulations (DNS) data has also been used to study vortical structures. Using the DNS database of Spalart Spalart (1988), Robinson Robinson (1991b) found that the number density of vortices is maximum at \( y^+ \approx 10 - 50 \), the vortex core radius \( r_o^+ \approx 5 - 20 \) and the circulation \( \Gamma^+ = (\Gamma/\nu) \approx 60 - 250 \). Del Alamo et al. del Álamo et al. (2006) performed a DNS of turbulent channel flow and classified the population of vortices into two groups of wall-detached eddies and attached eddies, in terms of their dependance on the distance from the wall. The typical size of the wall-detached eddy was found to be of order of the Kolmogorov length scale (\( \eta \)), and not to depend on the distance from the wall. The second group of the Townsend’s wall-attached eddies was found to closely resemble the hairpins as described by Adrian et al. Adrian et al. (2000). Das et al. Das et al. (2006) performed DNS of channel flow at \( Re_\tau \) upto 1270 and found out that the size of vortices ranges between 6\( \eta \) and 90\( \eta \).

Skote and Henningson Skote & Henningson (2002) performed DNS of a turbulent boundary layer undergoing adverse pressure gradient with a displacement thickness Reynolds number \( U_e \delta^+ / \nu \) of 400 at the starting position of the simulation (\( x=0 \)), where \( U_e \) is the freestream velocity. They found that near-wall streaks become weak due to adverse pressure gradient and the mean spacing in viscous units tends to increase. The streaks were noted to vanish at separation. The length in the streamwise direction was found to
Figure 1. Geometry of the bump. Solid vertical lines show the planes used for vortex detection. The dashed lines $x_s$ and $x_r$ show the mean separation and re-attachment points. Flow is from left to right.

be 3400 wall units. APG was found to have a damping effect on these structures. Spacing between structures was noted to increase from 100 (as in zero pressure gradient) to 130 viscous units towards the end of the computational domain.

Wu and Moin (2009) performed a DNS of zero-pressure gradient flat-plate boundary layer and took it from Blasius layer at $Re_{\theta} = \frac{U_e \theta}{\nu} = 80$ through transition to 1000, where $U_e$ is the freestream velocity, $\theta$ is the boundary layer momentum thickness and $\nu$ is the kinematic viscosity. They argued that none of the previous simulations were of genuinely spatially-developing turbulent boundary layers. They found that the instantaneous flow fields in both transitional and turbulent regions were populated by hairpin vortices. In contrast to the earlier studies, they found out that the hairpin vortices were mostly quasi-symmetric. A dense population of hairpin vortices in near-wall region, referred to as “hairpin forest” was found to exist close to the wall. In summary, there exists some concensus on the geometric features of vortical structures in the canonical wall-bounded flows, with some concensus on the shape and dynamical properties. Still experiments and simulations have mostly been performed on the canonical flows like zero pressure gradient boundary layers and fully-developed channels. Studies of vortical structures with additional strain rates like adverse pressure gradient or surface curvature are indeed rare. The purpose of the present paper is to study the effects of adverse pressure gradient and more importantly, effects of flow detachment on the vortical structures of the wall-bounded flow. This effect has been judged separately on each characteristic of the vortices like density and mean radius.

Using the DNS of Lee and Sung (2008), Lee et al. (2010) studied the coherent structures of an equilibrium turbulent boundary layer subjected to adverse pressure gradient. They scrutinized the population trends of the spanwise vortices and their distribution among prograde and retrograde vortices according to the normalized swirling strength being negative or positive respectively. They examined both a moderate and a strong pressure gradients with parameter $\beta = \frac{\delta^*}{\tau_w dP/dx} = 0.73$ and $\beta = 1.68$ respectively, where $\delta^*$ is the boundary layer displacement thickness and $\tau_w$ is the wall shear stress. They found that for adverse pressure gradient turbulent boundary layers, there are more prograde vortices as compared to retrograde ones.

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2. Direct Numerical Simulation

Direct Numerical Simulation of industrially-important flows is still unaffordable due to hardware constraints but it has been possible in the recent past to carry out DNS of academic or canonical flows like isotropic turbulence and channel flow albeit at low Reynold numbers. Still relatively few DNS have been carried out for wall-bounded flows with adverse pressure gradient.

Most of the Direct Numerical Simulations were performed with flows over flat surfaces. To investigate the turbulence statistics and coherent structures of wall-bounded flows under strong pressure gradient with curvature, Direct Numerical Simulation (DNS) of a converging-diverging channel flow was performed at $Re_\tau = u_\tau h/\nu = 617$, where $h$ is the half-channel height. Adverse pressure gradient was created through a surface bump on the lower wall of the channel. DNS of adverse pressure gradient flow was conducted to meet two main objectives. The first objective was to gather fully resolved three-dimensional data to study structures, statistics and scaling of wall-bounded turbulence. Secondly, it was meant to serve as a reference for the evaluation of RANS and LES models. Geometry of the bump used in simulation is shown in the figure 1.

DNS was performed on a domain, which is $4\pi$ long in streamwise direction, 2 in wall-normal direction and $\pi$ in spanwise direction. The spatial resolution is $2304 \times 385 \times 576$. The ratio of Kolmogorov scale to the maximum mesh size is of the order of 2 to 3 in most of the domain, goes up to 4 in the diverging part and up to a value of 5 very close to
the wall. However, in order to evaluate the resolution of the near wall region, the mesh size in wall units ($\Delta x^+,$ $\Delta y^+,$ $\Delta z^+$) are more relevant. The maximum values at the inlet are $\Delta x^+ = 5.1,$ $\Delta y_{min}^+ = 0.02$ and $\Delta z^+ = 3.4$. The global maximum values are reached in the converging part of the channel with $\Delta x^+ = 10.7,$ $\Delta y_{max}^+ = 0.03$ and $\Delta z^+ = 7.4$.

The resolution of the computational domain compares favorably with other contemporary simulations. Lee and Sung Lee & Sung (2009) used a resolution of $\Delta x^+ = 12.5,$ $\Delta y_{min}^+ = 0.17,$ $\Delta y_{max}^+ = 24$ and $\Delta z^+ = 5$ based on friction velocity at the inlet. The resolution in streamwise direction $\Delta x^+$ was 5.91 for Wu and Moin Wu & Moin (2009), $\approx 13$ for Skote et al. Skote et al. (1998) and $\approx 16.6$ for Na and Moin Na & Moin (1998). Thus the numerical resolution for the current DNS is fine enough to capture and accurately represent the vortical structures.

For spatial discretization in the streamwise direction, 4$^{th}$ order central finite differences are used for 2$^{nd}$ derivatives and 8$^{th}$ order finite differences are used for 1$^{st}$ derivatives. Chebyshev-collocation is used in the wall-normal direction. The spanwise direction $z$ is considered periodic and is discretized using a spectral Fourier expansion. For time-integration, implicit 2$^{nd}$ order backward Euler differencing is used. Thus, numerical code combines the advantage of good accuracy and spectral resolution with fast integration procedure for flow simulations over smooth surfaces. Further details of the code and flow characteristics can be found in Laval and Marquillie Laval & Marquillie (2010).

The pressure coefficient $C_p = (P - P_o)/(\frac{1}{2}\rho U_{max}^2)$ over bump is shown in figure 3, where $P_o$ is the reference pressure and $U_{max}$ is the maximum velocity. The kink in the $C_p$ curve is due to the flow separation on the lower wall of the channel. The friction coefficient $C_f = \tau_w/(\frac{1}{2}\rho U_{max}^2)$ for the DNS is also shown in figure 3, where $\tau_w$ is the wall shear stress. The flow over bump separates just after the maximum height or summit to form a small recirculation region. The thickness of the recirculation region is shown in the figure 2 and is about 20 wall units (based on inlet quantities) and its streamwise extent is almost from $x = 0.54$ to 1.38 with $x = 1.38$ as the flow reattachment point. The points of mean flow separation and reattachment have been shown in figures 1 and 2 as $x_s$ and $x_r$ respectively. The skin friction over the upper-wall of channel decreases to a very small value ($C_f \approx 1.55E-03$) and is close to separation. For wall-unit scaling, values at $x = 0$ have been chosen as reference values to avoid the issue of friction velocity $u_\tau$ going to zero at flow separation. The reference wall unit scaling is represented as $*^*$ in the superscript. The streamwise coordinate is represented by $x,$ the wall-normal coordinate by $y$ and spanwise coordinate is represented by $z$.

Figures 4 and 5 show the mean velocity and streamwise rms velocity fluctuation profiles over bump respectively. The velocity profile at $x = 1$ clearly shows the negative near-wall velocity and recirculation while the streamwise velocity fluctuations at $x = 1$ show the development of a peak which is typical of adverse pressure gradient flows, i.e., increasing in amplitude and moving away from the wall.

3. Vortex Detection

The different methods for identifying vortex cores in velocity vectors fields obtained from PIV or DNS have been reviewed in detail by Jeong and Hussain Jeong & Hussain (1995) and Chakraborty et al. Chakraborty et al. (2005). In the present study, an approach close to Das et al. Das et al. (2006) is followed. The detection of vortical structures from DNS velocity fields is performed through the computation of two-dimensional swirling strength from the available components of the velocity gradient tensor $\frac{\partial u_i}{\partial x_j}$.
Swirling strength identifies vortices using the imaginary part of the complex eigenvalue ($\lambda_{ci}$, where c stands for ‘complex’ and i for ‘imaginary’) of the local velocity gradient tensor and is an unambiguous measure of rotation Zhou et al. (1999). Swirling strength is a measure of the local swirling rate inside a vortex. Unlike vorticity, swirling strength does not show regions of intense shear and it has been shown to be an effective identifier of true vortex cores Adrian & Liu (2000). Signed swirling strength algorithm allows one to distinguish between positive and negative rotating vortices, which is used to collect statistics on prograde and retrograde vortices. Signed swirling strength is given as,

$$\Lambda_{ci} = \lambda_{ci} \times \frac{\omega'}{|\omega'|}$$  \hspace{1cm} (1)$$

where $\omega'$ is the fluctuating vorticity. The vortex validation is done through a fit of the Oseen model to the detected vortices, providing qualitative information on the detected and validated vortices. Because the database used in the present contribution is planar, the detection technique employed is based on the 2D velocity gradient tensor. This tensor is computed using a second order least squares derivative scheme Foucaut & Stanislas (2002). The swirling strength function is first normalized by the wall-normal profile of its standard deviation (as suggested by DelAlamo et al. (2006) and Wu & Christensen (2006)), and
then smoothed using a sliding average. The threshold on the detection function is set as 1. Extrema exceeding the threshold ($\lambda_{ci}(x, y) > \lambda_{ci,rms}(y)$) are retained as center of vortex cores. The velocity fields surrounding extrema of the detection function are then fitted to a model vortex with a non-linear least squares algorithm (Levenberg-Marquardt). The model is an Oseen vortex, defined in equation 2. This fit procedure determines a regression coefficient (maximum = 1) which is a measure of how closely the detected vortex resembles the Oseen model. The threshold on the regression coefficient was fixed as 0.8 in order to get vortices as close to the Oseen model as possible without discarding too much vortices for the convergence of statistics.

$$\vec{u}(r, \theta) = \vec{u}_c + \frac{\Gamma}{2\pi r} \left( 1 - e^\left( -\left( \frac{r}{r_o} \right)^2 \right) \right) e_\theta$$

where $\Gamma$ stands for circulation, $u_c$ the convection velocity, $r_o$ is the vortex core radius and $e_\theta$ is the tangential unit vector in polar coordinates. This procedure validates that the structure detected is indeed a vortex, and allows the retrieval of the vortex characteristics (radius, circulation, convection velocity, sub-grid position of the center) through the fitted parameters of the model. An example of an accepted vortex with a regression coefficient of 0.84 is shown in figure 6 with a field of view equal to the vortex diameter and in a reference frame moving with the vortex convection velocity. Figure 7 shows a vortex with a regression coefficient of 0.9, a radius of 23.8+ and a wall-normal location of $n^+ = 792$ in reference wall-units (with reference $u_\tau$ at $x = 0$).

**Figure 6.** Vortex velocity field with regression coefficient of 0.84 **Figure 7.** Vortex velocity field with regression coefficient of 0.9

### 4. Results and Discussion

Data and statistics on the vortical structures are important for the understanding of turbulent wall-bounded flows and the subsequent development of theoretical models for the calculation of such flows. Many experimental and numerical studies support the existence of hairpin-like structures in the near-wall flow region (Theodorsen (1952), Zhou *et al.* (1999), Adrian *et al.* (2000), Elsinga *et al.* (2007) among others). It has also been proposed...
that these structures align in the streamwise direction to form hairpin packets (Elsinga et al. (2007), Adrian et al. (2000), etc).

If sliced in the streamwise/wall-normal plane (XY), the heads of hairpins appear as spanwise vortex cores. If sliced in the spanwise/wall-normal plane (NZ), the legs of hairpins appear as streamwise vortex cores. To study both the streamwise and spanwise vortices, the detection of vortices has been carried out separately for streamwise/wall-normal and spanwise/wall-normal planes. The locations of spanwise/wall-normal planes are shown by solid vertical lines in the figure 1.

In the following subsections, statistical results on vortex density, mean vortex radius and mean vorticity for both streamwise/wall-normal and spanwise/wall-normal planes are presented and described. The output data of the detection code has been used to get these statistical results of the eddied vortices through a post-processing code. For both spanwise/wall-normal and streamwise/wall-normal planes, histograms were made with respect to wall-normal distance $n$ in order to get the variation of different quantities with respect to wall-normal distance $n$. The maximum wall-normal distance for the present DNS data is the half-channel height. This half-channel height has been divided into 50 wall-normal layers or bins and each layer has a thickness of $8.8^+$ based on the reference friction velocity $u_{τ,o}$. For streamwise/wall-normal planes, the vortex characteristics like radius, vorticity, etc. depend not only on the wall-normal distance $n$ but also vary with streamwise coordinate $x$. Thus for streamwise/wall-normal planes, small regions $Δs$ were defined to get vortex statistics. $Δs$ was chosen not to be more than ±0.1 of the corresponding spanwise/wall-normal streamwise stations. The streamwise stations for spanwise/wall-normal planes were taken at eight positions of $x = 0, 0.5, 1, 1.5, 2, 2.5, 3$ and 4. The corresponding sub-streamwise/wall-normal planes are $0±0.2, 0.5±0.1, 1±0.1, 1.5±0.1, 2±0.1, 2.5±0.1, 3±0.1$ and $4±0.1$ respectively. To enable comparison with flat plate quasi zero pressure gradient case, the data of Carlier and Stanislas (2005) at $Re_θ = 7500$ and the data of Herpin et al. (2009) at $Re_θ = 10140$ has been used.

4.1. Density of Vortices
4.1.1. Vortex Density For the density of vortices, the number of eddy structures having their center in each layer or bin is divided by the layer surface ($\text{binsize} \times Δz$ for spanwise/wall-normal plane and $\text{binsize} \times Δs$ for streamwise/wall-normal plane) in wall units, where $Δs = 0.2$ and $Δz = 2π/3$. The value obtained is called density $N^*$ of the vortices. For spanwise/wall-normal and streamwise/wall-normal planes, vortex density has been calculated as:

$$N^*_{NZ} = \frac{(\text{Vortex Count in the Bin})}{\text{Binsize} \times Δz} \frac{\nu^2}{u_{τ,o}^2}$$

$$N^*_{XY} = \frac{(\text{Vortex Count in the Bin})}{\text{Binsize} \times Δs} \frac{\nu^2}{u_{τ,o}^2}$$

where $u_{τ,o}$ is friction velocity taken at $x = 0$ (bump summit). Figures 8 and 9 show the wall-normal and streamwise evolution of the vortex density $N^*$ for the spanwise/wall-normal planes and streamwise/wall-normal planes respectively and binsize represents the wall-normal distance. For both planes, the vortex density is maximum for $n^* ≤ 100$ and then decreases with the wall-normal distance. In the first spanwise/wall-normal plane at $x = 0$, the peak of vortex density is located at approximately $n^* = 80$ and this peak moves
out to higher $n^*$ downstream. Due to the favorable pressure gradient upstream of $x = 0$, the vortex density is globally lower at this station, as compared to downstream locations. At $x = 0$, the profiles are compared to the flat plate experimental results of Herpin et al. (2009) because at this station, the $n^*$ scaling is equal to local wall unit scaling. Due to favorable pressure gradient upstream of $x = 0$, the vortex population density is globally lower at this station as compared to flat plate but the shape of profile is comparable with roughly the same wall-normal location of the peak.

The population density at $x = 0$ is also comparable in magnitude to the experimental flat plate data of Carlier and Stanislas (2005). The peak value of experimental population density reaches 3.5E-05 in the spanwise/wall-normal plane and in the current simulation the peak value at $x = 0$ is 5E-05. The analogy with flat plate data does not hold further downstream because the vortex density increases almost an order of magnitude in the region of adverse pressure gradient.

At $x = 1.0$, which is the region of flow detachment, a sharp near-wall peak of vortices appears at almost $n^* = 25$, indicating the generation of a new population of vortices triggered by adverse pressure gradient and flow detachment between $x = 0.5$ and $x = 1$ (as shown by $C_f$ diagram ) at a wall-normal distance of almost $n^* = 25$. This result is also supported by the studies of Zhou and Lu (1997), who observed that the adverse pressure gradient enhances the growth of coherent structures whereas favorable pressure gradient has the opposite effect. After $x = 1.5$, the near-wall peak goes down progressively, while it widens rapidly to nearly the boundary layer thickness at $x = 3$, which is the relaxation region at the trailing edge of the bump. Figure 10 shows the ratio of population density of spanwise vortices to the streamwise vortices. The near-wall peak suggests that vortices are created initially more as spanwise than as streamwise. Away from the wall, ratio remains nearly constant.

**Figure 8.** Vortex density $N^*$ for the spanwise/wall-normal planes

**Figure 9.** Vortex density $N^*$ for the streamwise/wall-normal planes
4.1.2. Prograde and Retrograde Vortex Densities

Spanwise or streamwise vortices that rotate in the same sense as the mean shear are termed prograde vortices whereas the vortices that rotate in the opposite sense to the mean shear are termed retrograde vortices. Carlier and Stanislas Carlier & Stanislas (2005), Natrajan et al. Natrajan et al. (2007) and Wu and Christensen Wu & Christensen (2006) used negative vorticity $\omega < 0$ as definition for the prograde vortices and positive vorticity $\omega > 0$ for the retrograde vortices. These definitions can not be used in the current study because of the flow recirculation and change in the sign of mean shear. Thus the definition of Schecter and Dubin Schecter & Dubin (2001) is used here, according to which $\Gamma / A > 0$ for retrograde vortices and $\Gamma / A < 0$ for prograde vortices, where $A$ is the local shear rate and $\Gamma$ is the total circulation. Further, for vortices of positive circulation and negative circulation, they adopted the terminology of “clumps” and “holes” respectively. A vortex is a “clump” if its total circulation $\Gamma$ is positive (counter-clockwise rotation), whereas a vortex is a “hole” if its total circulation is negative (clockwise rotation); that is, $\Gamma > 0$ for clumps and $\Gamma < 0$ for holes. For the flows with recirculation or backflow, both clumps and holes can be prograde or retrograde, depending on the sign of the local shear rate $A$.

Figure 11 shows the density $N^*$ for prograde and retrograde vortices in the streamwise/wall-normal plane and it is evident that those newly generated spanwise vortices are mostly prograde. In the recirculation region, at streamwise location $x = 1.5$, prograde vortices are much more numerous than the retrograde ones. Hence a strong adverse pressure gradient and flow recirculation leads to the generation of a large number of spanwise prograde (hole) vortices. The experimental flat plate studies (nominally zero pressure gradient) of Wu and Christensen Wu & Christensen (2006) also showed the preponderance of the spanwise prograde vortices (holes) as compared to spanwise vortices.
retrograde vortices (clumps). Wu and Christensen Wu & Christensen (2006) showed that the prograde spanwise vortices outnumber the retrograde spanwise vortices by 2:1 in the wall-normal region $0.15\delta < y < 0.25\delta$. The current results are also supported by the findings of Lee et al. Lee et al. (2010), who showed that for the adverse pressure gradient turbulent wall-bounded flow, the spanwise vortices (detected in the streamwise/wall-normal plane) are mostly prograde. For the spanwise/wall-normal plane, density $N^*$ is almost the same for both prograde and retrograde vortices at all streamwise locations. This implies that among streamwise vortices of the current simulation, the prograde and retrograde vortices have almost the same population.

4.2. Mean Radius of the Educted Vortices

The evolution of mean vortex radius scaled with reference wall units ($u_\tau$ and $\nu$ at $x = 0$) is shown in figure 14 for the spanwise/wall-normal planes and figure 15 for the streamwise/wall-normal planes. In the spanwise/wall-normal plane, the mean vortex radius increases sharply with wall-normal distance. At $x = 0$, it increases from almost $15^*$ at $n^* = 10$ to $40^*$ at $n^* = 500$.

As a reference value of friction velocity $u_{\tau_0}$ has been used for scaling, figures 14 and 15 give in fact an idea of the evolution in physical units. In the region of negative skin friction at $x = 1$, mean vortex radius near the wall in the spanwise/wall-normal planes suddenly drops to almost $10^*$ but then starts to increases again further downstream. The sudden decrease of near-wall mean vortex radius at $x = 1$ is linked to the generation of a new population of very fine and small vortices between $x = 0.5$ and $1$. This population is

**Figure 12.** PDFs of the near-wall vortex radius in reference wall units for the vortex radius in Kolmogorov units for the spanwise/wall-normal planes

**Figure 13.** PDFs of the near-wall vortex radius in Kolmogorov units for the spanwise/wall-normal planes
linked to the sharp wall peak observed in the figure 8. This decrease in mean vortex radii is also found in the streamwise/wall-normal plane. In the near-wall region ($n^* = 10$) at $x = 1$, mean vortex radius drops from $27^*$ at $x = 0$ to nearly $10^*$ at $x = 1$. The mean vortex radius increases significantly with wall distance in both streamwise/wall-normal and spanwise/wall-normal planes (from $20^*$ to $35^*$ in both planes for $n^* = 100-500$ roughly).

In the flat plate experiment of Herpin et al. Herpin et al. (2009), although the vortices show the same increasing tendency in size with wall-normal distance, it is much less pronounced. Consequently, the size of vortices in the present simulation at $x = 0$ is slightly larger than the flat plate case near the wall and significantly larger in the outer part.

The maximum mean vortex radius in the flat plate data of Carlier and Stanislas Carlier & Stanislas (2005) is up to $25^+$ whereas for the current study, the mean vortex radius goes up to $30^+$ at the $x = 0$ in the streamwise/wall-normal plane. In the spanwise/wall-normal plane, the mean vortex radius goes up to $40^+$ at $x = 0$, which means that vortex sizes are greater at $x = 0$ as compared to the flat plate data. After $x = 0$, the mean vortex radius starts to decrease in the region of adverse pressure gradient and in the relaxation region at $x = 4$, it starts to reach the flat plate value of $25^+$.

To estimate the distribution of vortex radii and the most probable vortex radius among the eddies and their variation with wall-normal and streamwise distances, Probability Density Functions (PDFs) have been computed for the spanwise/wall-normal and streamwise/wall-normal planes $^*$ units. To better understand the phenomenon of vortex generation, PDFs were constructed in the near-wall region of the wall-bounded flow. The maximum wall-normal distance used for these PDFs is $n^* = 400$. Figure 12 gives the PDFs of vortex radii in $^*$ units ($u_{r_0}, \nu$) for the different streamwise spanwise/wall-normal planes in the wall peak part. In the recirculation region, at $x = 1$, the distribution shows a shift towards the smaller values in agreement with the figure 14. The developing wall peak is already detectable at $x = 0.5$ and clearly visible at $x = 1$. This narrow peak centers at $r^* = 10$ and then moves towards higher values of radius downstream. Downstream at $x = 2.5$, it is not much different from the $x = 0$ distribution. Figure 13 shows PDFs of vortex radii in Kolmogorov length units for the near-wall region of spanwise/wall-normal planes. A rapid change is observed at $x = 1$ and 1.5, where the peak of mean vortex radius shifts to lower values of $r/\eta = 4$ but the distribution stabilizes rapidly from $x = 2$ with the most probable value of $r/\eta = 6$. The mean radius PDFs show that the newly created vortices have a mean radius of $10^*$ or almost $4\eta$. 

12
5. Conclusion

A viscous-incompressible fluid flow over a bump in a converging-diverging channel has been simulated with the use of a Direct Numerical Simulation code for the numerical integration of the Navier-Stokes equations at friction Reynolds number $Re_\tau$ of 617. Flow undergoes adverse pressure gradient in the divergent part with the formation of a separation bubble. A numerical database of the velocity field has been built and the vortical structures of the wall turbulence have been detected in the instantaneous velocity field using the swirling strength criteria followed by a fit to the Lamb-Oseen vortex model. Vortex densities, mean vortex radii and probability density functions show that the adverse pressure gradient and resultant flow detachment along the diverging part of the bump results in the generation of a new population of vortices close to the wall. The physical implication of this observation is that hairpin generation is significantly enhanced following separation. These newly-created vortices are evidenced by the development of a strong peak of vortex density near the wall in both spanwise/wall-normal and streamwise/wall-normal planes, which develops and spreads rapidly. These vortices have a mean radius of around 10 in reference wall units. These vortices were found to originate at a wall-normal distance of $n^* = 20-25$, in the recirculation region between flow detachment and re-attachment points. It has been observed that this new near-wall peak of vortex density coincides with the outward movement of the turbulence peak, providing a possible explanation for this non-canonical behaviour of the adverse pressure gradient turbulent flows.

In the spanwise/wall-normal direction, these newly-generated vortices have been found to be mostly prograde holes, in the terminology of Schecter and Dubin Schecter & Dubin (2001). The density of the spanwise prograde vortices is up to five times higher than that of retrograde vortices. The preponderance of the spanwise prograde vortices in adverse pressure gradient flows was also noted by Lee et al. Lee et al. (2010). On the other hand, the population of streamwise-oriented vortices is equally divided into prograde and retrograde vortices. It was also found that the density of spanwise vortices (detected in the streamwise/wall-normal plane) is slightly higher than the streamwise ones (detected in the spanwise/wall-normal plane). In agreement with the earlier studies, the mean radius of the spanwise vortices in reference wall units has been found to be greater than the mean radius of the streamwise vortices. Away from the wall, the mean vortex radius increases with wall normal distance in all streamwise planes.

In summary, in the region of flow detachment and recirculation, a new near-wall peak
of vortex density appears whereas mean vortex radius is significantly reduced, indicating the generation of newer smaller vortices. This set of results gives a strong indication of a different population of smaller vortices with different physical origin (developing after the summit of the bump). The existence of this new population of vortices certainly explains the difficulties encountered by researchers up till now to find a proper universal scaling for the adverse pressure gradient turbulent boundary layers.

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