Electron channeling, de Broglie’s clock and the relativistic
dynamical time operator

M. Bauer

Instituto de Física, Universidad Nacional Autónoma de México and
A.P. 20-364, 01000 México, D.F., MEXICO

(Date textdate)

Abstract

Electron channeling in silicon crystals has brought forward the possibility of having detected a particle’s ”internal clock”, as an intrinsic oscillation with de Broglie’s frequency. The transmission probability along a major axial direction is reduced with respect to neighboring angles, except for a sharp peak at the atomic row direction. The pattern observed is a ”W” instead of a ”U”. This central peak is attributed to a process known as ”rosette motion”, in which the crossing of successive atoms would be related to the de Broglie frequency. A classical multiple scattering calculation found that, to represent the experimental results, the interaction frequency had to be about twice the de Broglie’s clock frequency; that is, the ”Zitterbewegung” frequency. In the present paper, the observed characteristics of this process are shown to be consistent with a free particle quantum mechanical motion described by Dirac’s Hamiltonian, albeit with an effective mass resulting from the interaction with the crystal atoms. The introduction of a self-adjoint dynamic time operator provides the connection with an internal ”system time”, the de Broglie clock.
I. INTRODUCTION

Electron channeling in silicon crystals\cite{1, 2} has brought forward the possibility of having detected a particle’s “internal clock”, as an intrinsic oscillation whose frequency is given by de Broglie’s daring association \( h\nu = m_0c^2 \), where \( h \) is Planck’s constant, \( m_0 \) is the particle rest mass and \( c \) the speed of light in vacuum.\cite{3} More recently, a clock linked to this relation has been demonstrated using an optical frequency to self-reference a Ramsey-Bordé atom interferometer.\cite{4}

The channeling experiments in which the electrons are aligned along a major axial direction of a thin single crystal, do exhibit a reduced transmission probability with respect to neighboring angles, except for a sharp peak at the atomic row direction. The pattern observed is a “W” instead of a “U”. This central peak is attributed to a process known as “rosette motion”, which results in a reduction of the multiple scattering effects for electrons moving parallel to a string of atoms, with a momentum such that they pass atoms with a frequency equal to the de Broglie frequency. The expected consequence is a higher transmittivity relative to closely nearby directions and momenta. A phenomenological calculation by a Montecarlo method was carried out, in which classical mechanics was used to describe the electron motion. It was found however that to represent the experimental results, the interaction frequency had to be about twice the de Broglie’s clock frequency; that is, closer to the “Zitterbewegung” frequency, which appears in Dirac’s relativistic formulation of quantum mechanics.

In the present paper, the observed characteristics of this process are shown to be consistent with a free particle motion described by a Dirac Hamiltonian, albeit with an effective mass resulting from the interaction with the crystal atoms. The introduction of a dynamic time operator\cite{5} provides the connection with an internal system time, the de Broglie clock.

II. THE FREE PARTICLE DIRAC HAMILTONIAN AS A SYMMETRY OPERATION

Consider the free particle Dirac Hamiltonian

\[ H = c\alpha \cdot \mathbf{p} + \beta m_0 c^2 \]
where $\alpha = (\alpha_x, \alpha_y, \alpha_z)$ and $\beta$ are the Dirac matrices. Recalling that the infinitesimal ($\epsilon \ll 1$) unitary operator $S(\epsilon) = e^{i\epsilon p/h}$ acting on a position eigenstate yields a displaced eigenstate, namely:

$$S(\epsilon)|x\rangle = e^{i\epsilon p/h}|x\rangle = [1 + (i\epsilon p/h) + \frac{1}{2}(i\epsilon p/h)^2 + ....]|x\rangle = |x + \epsilon\rangle,$$

it follows that the infinitesimal unitary operator ($\tau \ll 1$):

$$U(\tau) = e^{-i\tau H/h} = e^{-i\tau (c\alpha \cdot p + \beta m_0 c^2)/h} = [1 + (i\tau \alpha \cdot p / hc) + \frac{1}{2}(i\tau \alpha \cdot p / hc)^2 + ....]e^{i\tau m_0 c^2 / h}$$

induces, in configuration space, a position displacement by an amount $\delta r = \tau c\alpha$ and a phase shift $\delta \phi = \beta (\tau m_0 c^2 / h)$, i.e.:

$$\Phi(r) = \langle r | \Phi \rangle = e^{i\phi}\varphi(r) \rightarrow e^{i(\phi + \delta \phi)}\varphi(r + \tau c\alpha).$$

As $[S(\tau), H] = 0$, the displaced wave function satisfies the same Schrödinger equation. $S(\tau)$ is thus a symmetry operation.

Averaging over a general positive energy wave packet (i.e., with $\langle \beta \rangle = 1$ but including both positive and negative energy eigenstates) yields:

$$\langle \delta r \rangle = \tau < c\alpha >= \tau\{< c^2p/E > + oscillating terms \ (Zitterbewegung) \},$$

where $< c^2p/E >$ is the group velocity $v_{gp} = dE/dp$. Finite displacements are achieved by repeated applications. The phase shift is seen to be related to the reduced de Broglie frequency $(m_0 c^2 / h)$.

### III. THE DYNAMICAL TIME OPERATOR AND ELECTRON CHANNELING

The introduction of the self-adjoint dynamical time operator:

$$T = \alpha \cdot r / c + \beta \tau_0$$

as a system observable, establishes that the phase velocity represents the change rate of the displacement with respect to the internal system time:

$$dr(t)/dT(t) = v_{ph},$$
whereas the group velocity yields the change rate with respect to the laboratory time \( t \), i.e.:

\[
\frac{dr(t)}{dt} = v_{gp}.
\]

In Eq.(5) and in Eq.(6), the oscillating terms (Zitterbewegung) have been omitted (they are not present in the case of wave packets of purely positive or purely negative energy eigenfunctions[6, 7]). Phase and group velocities satisfy the relation \( v_{ph}v_{gp} = c^2 \).

Consider now a crystal with \( d \) as the separation between atoms. The system time lapse needed for a fixed phase point to achieve such displacement will be, from Eq.(5):

\[
\Delta T = \frac{d}{v_{ph}}
\]

and the corresponding phase shift at \( d \) will be increased by:

\[
\Delta \varphi = (\Delta T)m_0c^2/h = (d/v_{ph})m_0c^2/h.
\]

An increment equal to \( n\pi \) \((n = 1, 2, \ldots)\) results in the same amplitude absolute value (odd \( n \) only changes the sign). The corresponding system time lapses are:

\[
(\Delta T)_{n\pi} = (n\pi)(h/m_0c^2) = (n/2)(h/m_0c^2),
\]

that is, \((n/2)\) times the de Broglie period. The corresponding phase velocity must then be:

\[
(v_{ph})_{n\pi} = (2/n)d(m_0c^2/h).
\]

As \( v_{ph} = E/p \), it follows that

\[
E_{n\pi}/c = p[(v_{ph})_{n\pi}/c] = (m_0\gamma c)[(v_{ph})_{n\pi}/c]
= (m_0c^2\gamma)[(v_{ph})_{n\pi}/c]/c,
\]

where \( \gamma \) is the relativistic Lorentz factor \([1 - (v_{gp}/c)^2]^{-1/2} = [1 - (c/v_{ph})^2]^{-1/2}\).

**IV. THE ELECTRON CHANNELING EXPERIMENT**

In the electron channeling experiment performed[1], \( d = 3.84 \text{ Å} = 3.84 \times 10^5 \text{ F} \) and therefore \([(v_{ph})_{n\pi}/c] = (2/n)158.265 \) and \([(v_{gp})_{n\pi}/c] = (n/2)(158.265)^{-1}. \) With this value of the
group velocity, \( \gamma \) is practically equal to 1 so no relativistic correction to the electron mass needs to be considered. Then:

\[
E_{n\pi}/c = (2/n)(0.511)(158.265)/c = (2/n) 80.873 \text{ MeV}/c. \tag{12}
\]

When \( n = 2 \), \( E_{2\pi}/c = 80.873 \text{ MeV}/c \), close to the observed rosette motion resonance at 81.1 MeV/c. This corresponds to a phase shift of 2\( \pi \) and, according to Eq.(10), to a phase velocity:

\[
v_{ph} = d(m_0c^2/h). \tag{13}
\]

i.e., the distance \( d \) between atoms multiplied by the de Broglie frequency.

To be noted from Eq.(13) is that the same phase velocity would result from an interatomic distance \( d/2 \) times the Zitterbewegung frequency \( 2m_0c^2/h \) without modifying the resonance energy \( E_{2\pi} \) from Eq.(12). These are the conditions under which the classical multiple scattering calculation of Ref.1 agrees with the experiment. This is also in agreement with the improvement found in Lindhard’s continuum model when one considers the transverse energy to be conserved only when measured on the transverse planes located midway between atoms in a string.\[8, 9\]

Also to be noted on the other hand is the following. The value \( n = 1 \) corresponds to a phase shift of \( \pi \). This changes the sign of the wave function but the absolute value remains the same at each crossing of a crystal atom. In this case the phase velocity, (Eq.(10)), would be given by the interatomic distance \( d \) times the Zitterbewegung frequency \( 2m_0c^2/h \). However, the rosetta motion resonance is then expected to be observed at twice the energy, i.e., 161.746 MeV (Eq.(12)), as noted in the earlier report of the experiment in Ref.2.

Finally, the difference between theoretical and experimental momenta could be accounted by considering that the electron’s motion in the crystal is well represented by the free particle Dirac Hamiltonian, albeit with an effective mass \( m^* \) instead of \( m_0 \) that takes into account the average interaction of the electron with the crystal atoms\[10, 11\]. Following this assumption through the above derivation yields \( (m^*/m_0)^2 = (81.1)/(80.873) = 1.0028 \) and \( m^* = 1.0014 \ m_0 \).
V. CONCLUSION

The resonant conditions observed in electron channeling experiments are shown to be consistent with the quantum mechanical description provided by Dirac’s free motion Hamiltonian (albeit with an effective mass), and the associated internal dynamical time operator. These conditions result from requiring: a) the same absolute value of the wave function at each atomic crossing; b) the association of the required phase velocity with the internal system time, as obtained from the system self-adjoint time operator introduced as an observable in a previous paper.[5]

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