Further tests of special interactions of massive particles from the $Z$ polarization rate in $e^+e^- \rightarrow Zt\bar{t}$ and in $e^+e^- \rightarrow ZW^+W^-$. 

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Abstract

We propose further tests of the occurrence of scale dependent heavy particle masses $(Z,W,t)$ and of strong final state interactions by comparing $Z$ longitudinal polarization rates in different kinematical distributions of the $e^+e^- \rightarrow Zt\bar{t}$ and in $e^+e^- \rightarrow ZW^+W^-$ processes.
1 INTRODUCTION

In previous papers \cite{1, 2, 3, 4} we have shown that the rate of $Z_L$ polarization in several $Zt\bar{t}$ and $ZWW$ production processes is directly sensitive to the occurrence of scale dependent masses (see \cite{5, 6}) and of final state interactions between heavy particles (for example due to a substructure \cite{7, 8, 9, 10, 11} like in the hadronic case or to a dark matter environment \cite{12, 13, 14}).

We now want to improve these tests by looking at different kinematical distributions of the $Z_L$ rate and by comparing the effects in the $Zt\bar{t}$ and $ZWW$ production processes in order to identify the origin of the effects, pure $t$, pure $Z$, $W$ or both.

We will concentrate on the $e^+e^-\rightarrow ZWW^-$ and $e^+e^-\rightarrow Zt\bar{t}$ processes and illustrate the effects on the distributions of the $Z_L$ rates versus different final 2-body invariant energies. The SM properties (from the respective Born diagrams) have been recalled in \cite{1, 4} and illustrated for the $p_Z$ distribution. The sensitivity of the $Z_L$ rate to the concerned masses is natural in SM due to the Goldstone equivalence \cite{15}.

In this paper we will first compute the corresponding $s_{WW^+}$, $s_{ZW^+}$ and $s_{t\bar{t}}$, $s_{Zt}$, $s_{Zt\bar{t}}$ distributions of the $Z_L$ rate. Like in the previous papers we will then introduce the two different types of modifications, scale dependent top quark, $Z$, $W$ masses and 2-body possible final state interactions. Illustrations will be made with simple kinematical dependences but one can easily imagine what would give more elaborated forms like resonances with Breit-Wigner forms.

In Section 2 we consider the $e^+e^-\rightarrow ZWW^-$ process with scale dependent $Z$, $W$ masses (keeping $c_W$ at its SM value) and $WW$, $ZW^+$, $ZW^-$ final state interactions.

In Section 3 we consider the $e^+e^-\rightarrow Zt\bar{t}^-$ process with scale dependent masses for the $Z$ or the $t$ or both and $t\bar{t}$, $Zt$, $Zt\bar{t}$ final state interactions.

In Section 4 we will conclude by summarizing the informations that may be obtained from the comparison of the two processes, in particular about the simultaneous occurrence or not of the scale dependence of the top quark mass and of the $Z$, $W$ masses.

2 $e^+e^-\rightarrow ZWW^-$

The Born SM diagrams have been given in \cite{4} with illustration of the $p_Z$ distribution for the $Z_L$ rate

$$R_L = \frac{\sigma(Z_LWW)}{\sigma(Z_tWW) + \sigma(Z_LWW)} \quad (1)$$

The scale dependence of the $Z$, $W$ masses has been studied (assuming that the $m_W/m_Z$ ratio (i.e. $c_W$) is fixed) with the test form

$$m_W(s) = m_W \frac{(m_{th}^2 + m_0^2)}{(s + m_0^2)} \quad (2)$$

2
Effects of final state interactions were illustrated by multiplying the amplitudes by the 

\[(1 + C(s_{ZW}))(1 + C(s_{ZW^-}))(1 + C(s_{W^+W^-})) \]  

"test factor" with

\[ C(x) = 1 + \frac{m_Z^2}{m_0^2} \ln \frac{-x}{(m_Z + m_W)^2}, \]  \hspace{1cm} (3)

In Fig.1 (up) we plot the \(s_{WW}\) distribution of the \(Z_L\) rate for \(\sqrt{s} = 5\) TeV and \(\theta = \pi/2\). It is directly related to the \(p_Z\) distribution shown in [4] as \(s_{WW} = s + m_Z^2 - 2E_Z\sqrt{s}\).

In Fig.1 (down) we plot the \(s_{ZW^+}\) distribution for the same kinematical conditions; we do not show the \(s_{ZW^-}\) distribution which is very similar.

In both cases we can see the basic SM contributions and the effect of a modification of the \(Z, W\) masses according to eq.(2). The shapes of the distributions and of their modifications are typically different in the \(s_{WW}\) and in the \(s_{ZW^\pm}\) cases.

In Fig.2 we then show, with the same conditions, the effects of final state interactions according to eq.(3) and as in [4] from the addition of the \(Z\) and of the \(G^0\) intermediate contributions. We can also see the differences between the shapes of these distributions and between the ones due to scale dependent masses or final interactions.

With other types of "test forms" the differences could even be stronger and specific of the origin of these new interactions (for example with resonance contributions).

3 \(e^+e^- \rightarrow Ztt\bar{t}\)

The behaviour of the \(Z_L\) rate

\[ R_L = \frac{\sigma(Z_Ltt)}{\sigma(Ztt) + \sigma(Z_Ltt)} \]  \hspace{1cm} (4)

in this process has been studied in [1] where one can find the SM diagrams and the corresponding \(p_Z\) distributions.

In addition to the scale dependence of the \(Z, W\) masses one may now have a scale dependence of the top quark mass that we will similarly study with the test form

\[ m_t(s) = m_t \left( \frac{m_t^2 + m_0^2}{s + m_0^2} \right) \]  \hspace{1cm} (5)

Final state interactions may now appear differently between \((Zt)\) or \((Z\bar{t})\) and \((tt)\). So we will separately study their effects with the test factors affecting the amplitudes respectively:

\[(1 + C(s_{Zt}))\], \( (1 + C(s_{Zt}))\), \ and \ (1 + C(s_{tt}))\)

with

\[ C(x) = 1 + \frac{m_t^2}{m_0^2} \ln \frac{-x}{(m_Z + m_t)^2}, \]  \hspace{1cm} (6)
Results of scale dependent masses and of final state interactions are respectively illustrated in Fig.3 and 4.

As expected from the expression of the $Z_L$ polarization vector, a decrease of the $Z$ mass leads to an increase of the corresponding amplitudes. On another hand a decrease of the top quark mass leads to a decrease of the longitudinal amplitudes; this is expected, by Goldstone equivalence (15), from the couplings of the Goldstone boson to the top quark which is proportional to the top quark mass.

Consequently the presence of both $Z$ and $t$ scale dependent masses may cancel and lead to almost no visible effect if the forms of the dependences are similar. This is illustrated in Fig.3 for both $s_{tt}$ and $s_{Zt}$ (and similarly $s_{Zt}$). This is the remarkable feature of this process.

For comparison we then show, in Fig.4, the effects of specific final state interactions on the $s_{tt}$ and $s_{Zt}$ distributions. We separately illustrate the effects of $s_{tt}$ interactions (label $t$), of $s_{Zt}$ and $s_{Zt}$ interactions (label $Z$), and of all of them (label $Zt$) giving progressively stronger effects and again specific shapes as compared to the above ones.

4 Conclusion

In this paper we have made a comparative study of the longitudinal $Z$ polarization rate in the $e^+e^- \rightarrow Ztt$ and $e^+e^- \rightarrow ZWW^-$ processes; this has shown its remarkable richness. In $ZWW$ production this rate is directly controlled by the $W$ and $Z$ masses; the $W$ mass dependence occurs in the $ZGW$ couplings and both the $W$ and $Z$ masses in the respective polarization vector. We assumed that the SM structure is maintained ($m_W/m_Z = c_W$) even with scale dependent masses. This leads to an increase of the $Z_L$ rate as shown in Fig.1.

In $Ztt$ production the rate is controlled by both $Z$ and $t$ masses. Contrarily to the $ZWV$ case there is no obvious relation between them in SM. The $Z$ mass controls the $Z$ polarization vector and the $t$ mass the $Gtt$ couplings (with $Z_L - G$ equivalence). Their effects are opposite and almost cancel in the total $Z_L$ rate (Fig.3).

In addition we have shown that the shapes of the $s_{WW}$, $s_{ZW\pm}$, $s_{tt}$ and $s_{Zt,zt}$ are kinematically different and differently affected by masses and by specific final state interactions (Fig.2,4).

The illustrations were made with arbitrary choices of parameters controlling the scale dependence of the masses and the sizes and energy dependences of the final state interactions. Our figures just show that one may indeed suspect the presence of BSM effects and guess their type from the behaviours of the $Z_L$ rates, for example those originating from substructures or from special interactions with a dark matter environment.

For experimental possibilities relative to these processes see [16].

As already mentioned in [4] other production processes may be interesting for confirming possible indications coming from the present proposal, for example $\gamma - \gamma$, see [17], or
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Figure 1: $e^+e^- \rightarrow Z_LWW$ ratio in SM and with an effective $Z$ mass with parameter $m_0^2 = 20$ or 40 in eq. (2); invariant distributions for $s_{WW}$ (up) and $s_{ZW}$ (down).
Figure 2: $e^+e^- \rightarrow Z_L WW$ ratio in SM and in the cases of an effective final (WW and ZW) interaction (Z) and of an additional Goldstone contribution (ZG) contribution.
Figure 3: $e^+e^- \rightarrow Z_L t\bar{t}$ ratio in SM and in the cases of an effective top mass (t), of an effective Z mass (Z) and of both (Zt); invariant distributions for $s_{tt}$ (up) and $s_{Zt}$ (down).
Figure 4: $e^+e^- \rightarrow Z_L t\bar{t}$ ratio in SM and in the cases of an effective final ($t\bar{t}$) interaction ($t$), of an effective final ($Zt$ and $Z\bar{t}$) interaction ($Z$) and of both ($Zt$).