Power System Dispatch with Electrochemical Energy Storage

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Abstract—Battery storage is essential for the future smart grid. The inevitable battery degradation renders the battery lifetime volatile and highly dependent on battery dispatch, and thus incurs opportunity cost. This paper rigorously derives the degradation cost function of battery storage for power system dispatch. We first derive the intertemporal optimality condition that guarantees the long-term dispatch optimality when short-term dispatch is made. Then we prove that the solution to a cost-form dispatch model implies this intertemporal optimality condition and thus is also optimal from the long-term perspective. The derived unit degradation cost is time-variant to reflect the time value of money and takes the form of a constant value (the marginal cost of usage) divided by a discount factor. In case studies, we demonstrate the existence of an optimal value of the marginal cost of usage that corresponds to the optimal long-term dispatch outcome. We also show that the optimal marginal cost of usage depends on system and battery parameters, such as the marginal cost of the system and the battery cycle life.

Index Terms—Electrochemical energy storage, power system dispatch, battery degradation cost, intertemporal decision.

NOMENCLATURE

Indices

\( t \) Indices for time, typically a day.
\( i \) Indices for control variables.
\( j \) Indices for inequality constraints.
\( k \) Indices for storage.
\( l \) Indices for storage control variables.
\( m \) Indices for thermal power plants.
\( n \) Indices for controllable loads.
\( \rho \) Indices for wind farms.
\( h \) Indices for time, typically an hour.
\( G \) Superscript for thermal power plant.
\( L \) Superscript for controllable loads.
\( S \) Superscript for storage.
\( S^+ \) Superscript for storage discharging.
\( S^- \) Superscript for storage charging.

Parameters and constants

\( U \) Total degradation/usage before the life of storage ends.
\( q_t \) Calendar degradation of storage during time \( t \).
\( \delta_t \) Discounting factor of storage for time \( t \).
\( a_{i,j}^{t,b} \) Coefficients in the cost functions for agents in the power system.
\( \Delta t \) Time horizon of short-term dispatch.
\( \Delta h \) Time interval of short-term dispatch.
\( W_{\rho,h} \) Forecasted available wind energy of the \( \rho \) th wind farm during time \( h \).
\( I_{n,h} \) Forecasted load level at node \( n \) during time \( h \).
\( \eta_k \) Charge/discharge efficiency of storage \( k \).
\( \sigma_k \) Self-discharge rate of storage \( k \).

Variables and Functions

\( f_t \) Objective function of the dispatch problem during time \( t \).
\( F_t \) The optimal value of \( f_t \).
\( g_t \) Inequality constraints of the dispatch problem during time \( t \).
\( x_t \) Control variables of the system during time \( t \).
\( x_s^t \) Control variables of storage during time \( t \), including both charging and discharging outputs.
\( u_t \) Degradation/usage of storage during time \( t \).
\( u_t^* \) The optimal usage of storage during time \( t \) for the long-term optimization problem (Problem A).
\( u_t^{\text{opt}} \) The optimal usage of storage during time \( t \) for Problem D.
\( d \) Degradation/usage function of storage.
\( \beta_t \) Binary variable that indicates whether the storage has reached its end of life.

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dispatch of battery storage, in general, because the capital/replacememt cost is a long-term cost and should not determine the marginal operational cost. In contrast, an intertemporal operational decision framework for battery storage that treats degradation cost as an opportunity cost is proposed in [23]. However, the proposed framework is for the single operation of battery storage rather than its dispatch in a power system. Also, the framework only presents degradation-constrained form dispatch model, whereas a justified cost function for battery storage is still absent.

In this paper, we rigorously derive the degradation cost of battery storage that guarantees the long-term dispatch optimality of the system. We first define the degradation-constrained dispatch model, the long-term objective, and the long-term dispatch optimization model. Then we derive the intertemporal optimality condition that maximizes the long-term dispatch objective. Furthermore, we prove that the degradation-constrained dispatch model combined with the intertemporal optimality condition is equivalent to a cost-form dispatch model, because the optimality condition of the cost-form dispatch model implies the intertemporal optimality condition. Mathematically, the derived unit degradation cost takes the form of a constant value (namely the marginal cost of usage, MCU) divided by a discount factor to reflect the time value of money. The cost-form model can be more conveniently applied to general power system dispatch and control problems. Finally, in case studies of microgrid economic dispatch, we demonstrate the existence of an optimal unit degradation cost that corresponds to the long-term optimality of the system and analyze the determinant factors of the optimal MCU. We find that the optimal MCU is positively dependent on the marginal cost of the system, while negatively dependent on the battery cycle life.

This paper is organized as follows. Section II introduces the generic power system dispatch problem. Section III presents the defines degradation-constrained dispatch and long-term optimization problem and derives the intertemporal optimality condition. Section IV proves the optimality of a cost-form dispatch model. Case study results are presented and discussed in Section V. Section VI draws the conclusion.

II. GENERIC POWER SYSTEM DISPATCH PROBLEM

The steady-state dispatch problem of power system is usually formulated as an optimization problem to find the best control of the system with respect to an objective function $f$ (such as total social welfare maximization, total cost minimization, and so on), subject to a set of operating constraints (such as power balance, line flow limit, voltage limit, and so on) and given parameters (such as load demand, equipment status, cost metrics, and so on), as in (1):

$$
\min_{x} \quad f(x, v, p) \\
s.t. \quad g(x, v, p) \leq 0 \\
\quad \quad h(x, v, p) = 0
$$

where $x$ are the control variables that may include all quantities controllable for system operator, such as power outputs of generators, demand response, and so on; $v$ represent the state
variables such as phasor angles, voltage magnitudes, and so on; represent the given parameters; \( g \) represent the inequality constraints, such as line flow limits, voltage limits, generation limits, and so on; \( h \) represent the equality constraints, which typically includes power balance constraint.

Depending on application scenarios, the dispatch models of power system take different mathematical forms. For example, the direct-current (DC) optimal power flow (OPF) model ignore the reactive power and assumes a flat voltage magnitude of all buses to achieve linearization; the economic dispatch (ED) model, as the simplest one, ignores transmission constraints and only implements steady-state power balance constraints. In transient stability assessment, differential equations that represent the dynamics of the system are also added to the model but are beyond the focus of this paper.

Many elements in \( p \), such as load and renewable generation, are uncertain. To take advantage of the more accurate forecasting information that is available when the real-time operation is approaching, the dispatch decision is usually made day-ahead, intra-day, intra-hour or rollingly across time. Because battery storage has energy constraints that are time-coupling, the dispatch model with battery storage needs to include multiple time steps. We denote the time index by \( t \), the time horizon by \( \Delta t \), and the optimal value of the objective function by \( F_t \). For conciseness, we hide the state variables and parameters and merge the inequality and equality constraints (the latter can be represented by the former), and then the dispatch model for time \( t \) becomes (2):

\[
F_t = \min_{x_t} f_t(x_t)
\]

s.t. \( g_t(x_t) \leq 0 \)

\( t \in \mathbb{N} \)

III. INTERTEMPORAL OPTIMALITY CONDITION OF DISPATCH WITH STORAGE

When battery storage is employed, degradation is inevitably incurred. As battery degrades, both of its energy capacity and charge/discharge efficiency will decrease. The end of battery life is usually defined as when its energy capacity drops to a critical percentage of the initial capacity, for example, 80% or 70%, because the capacity and efficiency will fade much more drastically afterwards. Here we denote the degradation/usage of storage in a grid during time \( t \) by \( d(x^t_T) \), which is a function of storage control \( x^t_T \) (charging and discharging). Further, we denote the upper limit on the degradation/usage of storage during time \( t \) by \( u_t \), as (3), and denote the total degradation/usage before the storage life ends by \( U \). The life-cycle usage constraint on the storage can be expressed as (4) and (5), where \( \beta_t \) is a binary variable that indicates whether the storage has reached its end of life (\( \beta_t = 1 \) indicates the storage is within its life at time \( t \), while \( \beta_t = 0 \) indicates the storage should be retired or replaced at time \( t \)). \( N \) is a large number that is greater than the lifetime of the storage.

\[
d(x^t_T) \leq u_t \quad (3)
\]

\[
\sum_{t \in N} \beta_t u_t \leq U \quad (4)
\]

\[
\beta_t \geq \beta_{t+1} \quad (5)
\]

Because of the existence of calendar degradation that is independent of battery cycling or usage and is only dependent on temperature and the state of charge (SOC) of battery, the degradation is positive even if the battery stops operation. This phenomenon is expressed as (6) and (7), where \( q_t \) denotes the calendar degradation of the storage during time \( t \). If \( u_t = q_t \), there is only calendar degradation and no cycling degradation, which implies that the battery is staying idle and \( x^t_T = 0 \). To represent that the storage stops operation after its end of life (\( u_t = q_t \) when \( \beta_t = 0 \)), we reformulate (7) to (8).

\[
\min [d(x^t_T)] = d(0) = q_t \quad (6)
\]

\[
u_t \geq q_t \quad (7)
\]

\[
q_t + \beta U \geq u_t \quad (8)
\]

To analyze the optimality of dispatch subject to the life-cycle degradation limit of battery storage, here we propose the degradation-constrained dispatch that implements battery storage degradation into dispatch problem.

**Definition I:** The degradation-constrained dispatch is formulated as (9):

\[
F_t(u_t) = \min_{x_t} f_t(x_t)
\]

s.t. \( g_t(x_t) \leq 0 \)

\[
d(x^t_T) \leq u_t \quad (9)
\]

\[
x^t_T \geq 0
\]

In the degradation-constrained dispatch above, the optimal objective for time \( t \) is a function of the degradation/usage constraint set for time \( t \). \( u_t \) denotes the degradation/usage limit of battery storage and is exogeneous for the degradation-constrained dispatch. \( F_t(u_t) \) is generally a monotonically decreasing function, as the optimal solution when \( u_t = u \) is always feasible for the problem above when \( u_t = u + \Delta u \), where \( \Delta u > 0 \). One sufficient condition for \( F_t(u_t) \) being convex is that \( f_t(x_t) \), \( g_t(x_t) \), and \( d(x^t_T) \) are convex [24]. General optimal power flow problems consisting of \( f_t(x_t) \) and \( g_t(x_t) \) are non-convex but can be linearized (DC OPF) or relaxed (convex relaxation) to convex problems [25],[26]. \( d(x^t_T) \) is also convex for common battery chemistries [27],[28].

Whereas the constraint \( x^t_T \geq 0 \) is not necessary for general storage dispatch problems, this domain limit makes \( d(x^t_T) \) differentiable. \( d(x^t_T) \) is only piece-wise differentiable when \( x^t_T \) represent the storage output including both charging and discharging (for example, the element of \( x^t_T \) is positive when discharging and negative when charging). We will also discuss the case when \( d(x^t_T) \) is only piece-wise differentiable and this constraint can be relaxed. To satisfy the domain limit, the discharging and charging outputs can be expressed as different
non-negative elements in $x^s$ (for example, $x^s = [x_1^s, x_2^s]_+$, where $x_1^s$ is for discharging and $x_2^s$ is for charging). The optimal solution cannot have positive discharging and charging outputs at the same time, which only increases degradation without any additional contribution to the system.

**Definition II:** The long-term objective of dispatch with storage is defined as the sum of the present values of all optimal objectives from degradation-constrained dispatches over the storage lifetime, as (10):

$$ Y = \sum_{t=0}^T \delta_t F_t(u_t) $$

where $Y$ is the long-term objective, and $\delta_t$ is the discounting factor for time $t$.

**Definition III:** The long-term optimization problem of dispatch with storage (Problem A) is defined as (11) [23]:

Problem A:

$$ \min_{u_t, \beta_t} Y = \min_{u_t} \sum_{t=0}^T \beta_t u_t \leq U $$

$$ \text{s.t.} \quad \sum_{t=0}^T \beta_t u_t \leq U $$

$$ q_t \geq 0, \quad \forall t \leq N $$

where $\delta_t$ is the discounting factor for time $t$, and $\Omega_q$ is the set of battery storage units in the power system. For feasibility, $U \geq q_t$.

Because we are only interested in the optimal operational decisions of storage before its end of life (when $\beta_t = 1$), for each solution $\beta_t$, we can reformulate Problem A as (12):

$$ \min_{u_t} Y = \min_{u_t} \sum_{t=0}^T \delta_t F_t(u_t) $$

$$ \text{s.t.} \quad \sum_{t=0}^T u_t \leq U $$

$$ u_t \geq q_t, \quad \forall t \leq T $$

where $T$ is the lifetime of the storage ($\beta_t = 1, \forall t \leq T$), to be determined by $\beta_t$.

The following result shows the necessary condition to guarantee the long-term objective is optimized when the short-term dispatch decisions for each time $t$ are made [23]. First, we present a basic condition $C.1$ for Theorem I:

$C.1$: $F_t$ is differentiable with respect to $u_t$.

**Theorem I:** If $C.1$ holds, the optimal solution $u^*_t$ to Problem A (12) should satisfy either of (13) and (14):

$$ u^*_t = q_t $$

$$ \frac{\partial F_t(u^*_t)}{\partial u_t} = -c_t^* \frac{-c^*}{\delta_t} $$

where $c^*$ is a non-negative constant to be determined by the system conditions in the long run.

**Proof of Theorem I:** The Lagrangian function of the long-term optimization problem (11) is:

$$ L = \sum_{t=0}^T \delta_t F_t(u_t) + c \left( \sum_{t=0}^T u_t - U \right) + \sum_{t=0}^T \alpha_t (q_t - u_t) $$

where $c$ and $\alpha_t$ are Lagrangian multipliers. The first-order Karush–Kuhn–Tucker (KKT) conditions are (16)-(20):

$$ \frac{\partial L}{\partial u_t} = \delta_t \frac{\partial F_t(u_t)}{\partial u_t} + c^* - \alpha_t = 0 $$

$$ \frac{\partial F_t(u^*_t)}{\partial u_t} = -c^* + \alpha_t $$

$$ \alpha_t (q_t - u^*_t) = 0 $$

$$ c^* \left( \sum_{t=0}^T u^*_t - U \right) = 0 $$

$$ \alpha_t^* \geq 0 $$

$$ c^* \geq 0 $$

$$ \sum_{t=0}^T u^*_t \leq U $$

$$ u^*_t \geq q_t $$

If $u^*_t > q_t$, according to (17), we have:

$$ \alpha_t = 0 $$

Then substituting (23) into (16), we get (14).

**Remark I:** Theorem I says that the marginal objective improvement per unit of storage usage should be set to a discounting factor adjusted constant across the storage lifetime, if the storage operates ($u^*_t \neq q_t$). The value of the constant $c^*$ can be numerically obtained by solving (11). We name $c^*$ as the marginal cost of usage (MCU) and $c^*_t$ as the discounted MCU (DMCU). The discounting factor $\delta_t$ usually takes the form of $\frac{1}{(1+r)^t}$, where $r$ is the discount rate, and $y(t)$ is the year number for time $t$ from the begin of the storage project. Therefore, DMCU increases as time goes by.

Theorem I can be generalized to multi-storage case, and the differentiable assumption $C.1$ can be relaxed to a piece-wise differentiable assumption by applying the concept of subgradient, which gives Corollary I.

$C.1$: For $F_t$ is piece-wise differentiable with respect to $u_t$, which denotes the usage of all storages in the system, and $C.1$ holds for the $k$th storage, the optimal solution $u^*_{k,t}$ to Problem A (12) should satisfy either of (24) and (25):

$$ u^*_{k,t} = q_t $$

$$ \theta^*_{k,t} = -c_{k,t} \frac{-c_{t}}{\delta_t} $$

If $F_t(u_t)$ is differentiable at $u_{k,t}$:

$$ \theta^*_{k,t} = \frac{\partial F_t(u_{k,t})}{\partial u_{k,t}} $$

For cases when $F_t(u_t)$ is non-differentiable, let $u_{-k,t}$ be the vector that includes all elements in $u_t$ but $u_{k,t}$. If the right-hand derivative is greater than the left-hand derivative at $u_{k,t}$, then for all $v$ in the neighborhood of $u_{k,t}$, $\theta_{k,t}$ can take any values that satisfy:

$$ F_t(v, u_{-k,t}) \geq F_t(u_{k,t}, u_{-k,t}) + \theta_{k,t} \left( v - u_{k,t} \right) $$

(27)
If the right-hand derivative is smaller than the left-hand derivative at \( u_{k,t} \), then for all \( v \) in the neighborhood of \( u_{k,t} \), \( \theta_{k,t} \) can take any values that satisfy:

\[
F_j(v, u_{k,t}) \leq F_j(u_{k,t}, u_{k,t}) + \theta_{k,t} \left( v - u_{k,t} \right)
\]

(28)

### IV. DEGRADATION COST OF STORAGE

In this section, we derive the degradation cost of storage from Theorem I. The derived storage cost is an opportunity cost from the loss of future benefit opportunity due to degradation and is independent of the initial capital cost of storage.

For preparation, we first present a definition for problem equivalence and two conditions C.2 and C.3 for Theorem II:

**Definition IV:** If the optimal solution sets of two optimization problems are identical, these two problems are equivalent.

**C.2:** \( u_i^* \) and \( c_i^* \) are the optimal solutions to Problem A.

**C.3:** \( F_j(u_i) \) is convex.

**Theorem II:** If C.1-C.3 hold, then Problem B and Problem C are equivalent.

Problem B:

\[
F_j(u_i^*) = \min_{u_i} f_j(x_i)
\]

s.t. \( g_j(x_i) \leq 0 \)

\( d(x_i^0) \leq u_i^* \)

\( x_i^* \geq 0 \)

Problem C:

\[
\min_{x_i} \left[ f_j(x_i) + c_i^* d(x_i^0) \right]
\]

s.t. \( g_j(x_i) \leq 0 \)

\( x_i^* \geq 0 \)

(29)

Problem B is the degradation-constrained dispatch, while Problem C is the dispatch that incorporates degradation cost. Theorem II says that solving the degradation-constrained dispatch is equivalent to solving the dispatch problem with MCU as the marginal degradation cost, under certain conditions. For Problem B and C, \( x_i^* \) represent the optimal solutions to the corresponding optimization problems. \( u_i^* \) is exogeneous for Problem B, and is equal to \( d(x_i^{v^*}) \) in Problem C.

**Proof of Theorem II:** Let us first introduce Problem D:

\[
\min_{u_i} \min_{x_i} \left[ f_j(x_i) + c_i^* u_i \right]
\]

s.t. \( g_j(x_i) \leq 0 \)

\( d(x_i^0) \leq u_i \)

\( x_i^* \geq 0 \)

(31)

We can rewrite Problem D as below:

Problem D:

\[
\min_{u_i} D_j(u_i)
\]

(32)

Sub-Problem D:

\[
D_j(u_i) = \min_{x_i,v} \left[ f_j(x_i) + c_i^* u_i \right]
\]

\( \Psi = \{ x_i \mid g_j(x_i) \leq 0, d(x_i^0) \leq u_i, x_i^* \geq 0 \} \)

where \( D_j(u_i) \) is the optimal value of sub-problem D. According to (6) and (33), \( u_i \geq q_i \) is also an implied constraint.

We can observe that \( d(x_i^0) = u_i \) is always a necessary optimal condition to Problem D, as \( D_j(d(x_i^0)) \leq D_j(u_i) \). Therefore, the optimal solution set to Problem C is the same as the optimal solution set to Problem D, considering that Problem C and D have the same feasible region and objective function. Each optimal solution to problem C is a feasible solution to Problem D, and vice versa.

**Problem D:**

\[
D_j(u_i) = F_j(u_i) + c_i^* u_i
\]

(34)

The property expressed in (34) is obvious as when \( u_i \) is fixed, \( c_i^* u_i \) is a constant, and \( \min_{x_i,v} \left[ f_j(x_i) + c_i^* u_i \right] \) has the same optimal solution set with \( \min_{x_i,v} f_j(x_i) \). Implementing (34) into Problem D:

\[
\min_{u_i} F_j(u_i) + c_i^* u_i
\]

s.t. \( u_i \geq q_i \)

(35)

The KKT conditions of Problem D are (36)-(39):

\[
\frac{\partial L_j(u_i^{op})}{\partial u_i} = \frac{\partial}{\partial u_i} \left[ F_j(u_i^{op}) + c_i^* u_i^{op} + \nu_i (q_i - u_i^{op}) \right] = 0
\]

\( \Leftrightarrow \frac{\partial F_j(u_i^{op})}{\partial u_i} = -c_i^* + \nu_i 
\]

\( \nu_i (q_i - u_i^{op}) = 0 \)

(36)

\( u_i^{op} \geq q_i \)

\( \nu_i \geq 0 \)

(37)

where \( u_i^{op} \) is the optimal solution to Problem D, and \( \nu_i \) is Lagrangian multiplier.

As C.2 holds, \( u_i^* \) and \( c_i^* \) are the optimal solutions to Problem A. If \( u_i^* = q_i \), according to (16) and (17), we have:

\[
\alpha_i > 0 \quad \frac{\partial F_j(q_i)}{\partial u_i} = \frac{-c_i^* + \alpha_i}{\delta_i} > \frac{-c_i^*}{\delta_i} = c_i^*
\]

(40)

Given C.3 that \( F_j(u_i) \) is convex, if \( u_i^{op} \neq q_i \), then:

\[
\frac{\partial F_j(u_i^{op})}{\partial u_i} \geq \frac{\partial F_j(q_i)}{\partial u_i} > \frac{-c_i^*}{\delta_i}
\]

(41)

Combining (36),(37), and (42), we get:

\[
\nu_i > 0 \quad u_i^{op} = q_i
\]

(43)

So \( u_i^{op} = u_i^* \). If \( u_i^* > q_i \), according to Theorem I, we have:

\[
\frac{\partial F_j(u_i^*)}{\partial u_i} = -c_i^*
\]

(44)

Then \( u_i^{op} = u_i^* \) and \( \nu_i = 0 \) satisfy the KKT conditions of Problem D (36)-(39). Given C.3 that \( F_j(u_i) \) is convex, Problem D is also convex (as observed from (35)). Thus, \( u_i^* \) is the
unique optimal solution to Problem D. When implementing \( u_t = u^*_t \) into (31), Problem D becomes:
\[
\min_{x_t^*} \left[ f_t(x_t) + c_t^* u_t^* \right] \\
\text{s.t. } g_t(x_t) \leq 0 \\
d(x_t^*) \leq u_t^* \\
x_t^* \geq 0
\]
which has the same solution set with Problem B, because the only difference between (29) and (46) is a constant in the objective function.

Finally, as Problem D has the same solution set with both Problem B and C, Theorem II is proved. \( \blacksquare \)

Remark II: In contrast with Theorem I, Theorem II gives the cost/penalty form of storage dispatch model (Problem C). This form can be more easily applied to many other applications such as dynamic control, distributed control, and so on. C.2 brings long-term information to the short-term scheduling and guarantees the optimal use of storage and is non-trivial because many studies are applying the levelized initial capital cost [29] or future replacement cost [30] as the degradation cost, which could lead to great economic loss compared to using \( c_t^* \) [23].

Similar with Theorem I, Theorem II can also be easily generalized to multi-storage case.

The following corollary gives a more general statement for Theorem II:

Corollary II: If \( F_t(u_t) \) is piece-wise differentiable and convex, and C.2 hold, then Problem B is equivalent to Problem C.

Given Theorem II, we can prove that for each subsets of where \( F_t(u_t) \) is piece-wise differentiable and convex, Problem B and C are equivalent, and so does the universal set of \( x_t \). The piece-wise convex assumption is a relatively weak assumption; most concave functions can be approximated by piece-wise convex/linear functions.

V. CASE STUDY ON ECONOMIC DISPATCH

In this section, we present a model and simulation results for the economic dispatch of a sample power system to validate the analysis in previous sections. The sample power system consists of one wind farm, one thermal power plant, loads, and one battery storage. In the results, we first demonstrate the optimality of the dispatch model proposed in Theorem II, and then, reveal the relation between the MCU of storage and the marginal costs of units in the grid.

A. Model Formulation

1) Objective Function

The economic dispatch model is a cost-minimizing problem, with the power output schedules of thermal power plants, controllable loads, and battery storage units as its decision variables, as (47). The time horizon of each dispatch decision is usually a day, represented by \( \Delta t \), and \( t \) is the day index.
\[
\min_{x_t^*, x_t^+, x_t^{-}, x_t^d_t} f_t(x_t) = \sum_{m \in M} C^m_t + \sum_{k \in K} C^k_t + \sum_{h \in H} C^h_t \quad \forall t \in T
\]

The total system cost during day \( t \) is the sum of the costs of all thermal power plants, controllable loads, and battery storage units. The cost of the \( m \)th thermal power plant, \( C^m_t \), is assumed to be a quadratic function of its generation output at hour \( h \) denoted by \( x^m_{t,h} \), as in (48). \( a^m_t \) and \( b^m_t \) are the coefficients of the cost function of the \( m \)th thermal power plant, and \( \Omega^m_t \) is the set of thermal power plants in the system, whose cardinality is 1 in this case study.
\[
C^m_t = \sum_{h \in \Omega^m_t} \left[ a^m_t \left( x^m_{t,h} \right)^2 + b^m_t x^m_{t,h} \right] \quad \forall m \in \Omega^m_t \quad (48)
\]

The cost of the \( n \)th controllable load, \( C^L_n \), is also assumed to be a quadratic function of its load reduction at hour \( h \) denoted by \( x^n_{t,h} \), as in (49). \( a^n_t \) and \( b^n_t \) are the coefficients of the cost function of the \( n \)th controllable load, and \( \Omega^L_n \) is the set of controllable loads in the system, whose cardinality is 1 in this case study.
\[
C^L_n = \sum_{h \in \Omega^L_n} \left[ a^n_t \left( x^n_{t,h} \right)^2 + b^n_t x^n_{t,h} \right] \quad \forall n \in \Omega^L_n \quad (49)
\]

The degradation cost of the \( k \)th battery storage \( C^b_k \) is proportional to its degradation/usage, which is assumed to be linearly dependent on its charging and discharging power output, \( x^{+}_{k,t} \) and \( x^{-}_{k,t} \), as in (50). The degradation can be a more complicated convex non-linear function or piece-wise linear function of the charging and discharging schedules to fit the real degradation characteristics of a specific battery storage. \( c_{k,t} \) is the DMCU of the \( k \)th battery storage during \([t, t+\Delta t] \); \( \Delta h \) is the time interval of dispatch, assumed to be 1 hour in this paper; and \( q_{k,t} \) is the calendar degradation. The cardinality of \( \Omega^c_k \) is 1 in this case study.
\[
C^c_k = c_{k,t} \sum_{h \in \Omega^c_k} \left[ (x^{+}_{k,t} - x^{-}_{k,t}) \Delta h + q_{k,t} \right] \quad \forall k \in \Omega^c_k \quad (50)
\]

2) Constraints

The power balance constraint of the system is as (51). In (51) \( W_{\rho,h} \) is the forecasted available wind energy of the \( \rho \)th wind farm during time \( h \), and \( L_{n,h} \) is the forecasted load level at node \( n \). \( \Omega^L_n \) is the set of wind farms in the system with a cardinality of 1 in this case.
\[
\sum_{m \in M} x^m_{t,h} + \sum_{k \in K} \left( x^+_{k,h} - x^-_{k,h} \right) \Delta h + \sum_{\rho \in \Omega^W} W_{\rho,h} \geq \sum_{m \in M} L_{m,h} \quad \forall h \in [t, t+\Delta t] \quad (51)
\]

The control variables have physical upper limits as (52) to (54), where \( x^G_t \), \( x^L_t \), and \( x^b_t \) represent the limits of thermal generation plants, controllable loads, and battery storage units, respectively.
\[
0 \leq x^G_{t,h} \leq x^G_t \quad \forall m \in \Omega^G_t, h \in [t, t+\Delta t] \quad (52)
\]
\[
0 \leq x^L_{t,h} \leq x^L_t \quad \forall n \in \Omega^L_n, h \in [t, t+\Delta t] \quad (53)
\]
The energy constraints of battery storage are modelled in (55) to (57). The energy level of storage at time \( h+1 \), \( e_{k,h+1}^s \), is expressed as a function of the energy level at time \( h \), \( e_{k,h}^s \), and the charging/discharging output at time \( h \), as (55), where \( \sigma \) is the self-discharge rate, and \( \eta \) is the charge/discharge efficiency.

\[
e_{k,h+1}^s = (1-\sigma)e_{k,h}^s - \frac{x_{k,h}^g - x_{k,h}^l}{\eta_h} \Delta h + \frac{x_{k,h}^g - x_{k,h}^l}{\eta_h} \Delta h
\]

The energy level of the storage has to be kept within its capacity, as (56), where \( \overline{e}_k^s \) is the energy capacity of the \( k \) th battery storage unit.

\[
0 \leq e_{k,h}^s \leq \overline{e}_k^s \quad \forall k \in \Omega_k, h \in [t, t+\Delta t]
\]

To reserve flexibility, the initial and the final energy levels are set to be equal for the dispatch horizon \([t, t+\Delta t]\), as (57).

\[
e_{k,1}^s = e_{k,T}^s \quad \forall k \in \Omega_k
\]

### B. Data

The wind generation profile is presented in Fig. 1, which is produced based on the historical 100-m wind speed data in 2015 in selected locations in Texas. The wind capacity factor is approximately 63%. The load profile of the system is presented in Fig. 2, which is scaled from the annual total load profile in ERCOT in 2015. The average load is approximately 57 MW. Both the wind and load profiles are assumed to be the same for each year across the life cycle of the battery storage. The parameters of the thermal power plant, controllable load, and battery storage are summarized in Table 1 and 2. The life-cycle usage limit of the battery storage, measured by energy throughput, is 1.2 TWh, which is 3000 full cycles for a 200MWh battery. The discount rate is set to 7%.

![Annual wind generation profile](image1)

**Fig. 1.** Annual wind generation profile.

![Annual load profile](image2)

**Fig. 2.** Annual load profile.

### C. Results

By solving (11), which includes solving (47) to (57) repeatedly, we simulate the operation of the system and calculate the total contribution of the battery storage in terms of cost saving. The cost savings with different MCUs, or say, unit degradation costs, are presented in Fig. 3. The blue point indicates the optimal outcome and the degradation cost of battery storage we should set for this power system. The red point represents the case we do not consider battery degradation in the dispatch. In this power system, the cost saving of the optimal case considering degradation is approximately 10% more than that when not considering degradation. The optimal MCU, \( c_5 \), is approximately 6 $/MWh. We can conclude from Fig. 3 that if the degradation cost of storage is not set properly, there could be a great increase in the total system cost.

To find out how the degradation cost of the battery storage should vary as the system parameters change, we simulate the system operation with different values of the marginal cost of the thermal power plant, \( b^T \), and battery cycle life, \( U \). On the one hand, we can observe from Fig. 4 that the optimal MCU is linearly increasing as the marginal cost of the thermal power plant increases. This implies that the degradation cost of battery storage should be positively dependent on the marginal cost of the system—if the marginal cost of the system changes, the MCU should also change. On the other hand, the optimal MCU is negatively dependent on the cycle life of the battery storage, which implies that the degradation cost is lower with higher battery cycle life. These two factors have significant impacts on the value of the battery degradation cost.
Fig. 4. Different optimal life-cycle MCUs as the marginal cost of the thermal power plant and the cycle life of the battery storage vary.

VI. CONCLUSIONS

Dispatch model is the center of smart grid operation, and better dispatch model could imply significant cost savings. This paper derives the intertemporal condition and the degradation cost for a power system with electrochemical energy storage, with mathematical proofs of their optimality. We prove that there exists a value, named MCU, that should be discounted and implemented into the cost function of battery storage, as if it is the variable unit operational cost incurred by battery degradation. The MCU is independent of the capital cost of the battery storage.

Through case studies, we show the optimality of the derived MCU approach, and analyze the factors that determine the optimal value of MCU. We find that the optimal MCU is positively dependent on the marginal cost of the system, while negatively dependent on the battery cycle life.

The cost function of battery storage derived in this paper could serve as a foundation for dispatch and planning problems concerning electrochemical energy storage.

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