Multiple scalar fields nonminimally interacting through pure affine gravity are considered to generate primordial perturbations during an inflationary phase. The couplings considered give rise to two distinct sources of entropy perturbations that may not be suppressed in the long wavelength limit. The first is merely induced by the presence of more than one scalar and arises even in the minimal coupling limit. The second source however is restricted to nonminimal interaction. Unlike the case of metric gravity, and due to the absence of anisotropic stresses, the second source disappears for single scalar, showing that nonminimal couplings become relevant to non-adiabatic perturbations only when more than one scalar field are considered. Hence the notion of adiabaticity is not affected by the transition to minimal coupling contrary to the metric gravity case where it becomes frame-dependent. Precise data that might be able to neatly track different sources of isocurvature modes, if any, must not only distinguish between different models of inflation but also determine the most viable approach to gravity which underlies the inflationary dynamics itself.

**Keywords:** Inflation; Entropy perturbation; Non-adiabatic pressure; Isocurvature; Affine gravity.
I. INTRODUCTION

One of the primary interest of the very early universe theories lies in understanding the origin of structure. Inflationary cosmology serves as a relevant mechanism in which the vacuum fluctuations, in an early phase of very rapid accelerated expansion, swept up to large scales and acts as seeds of structure formation later on. In its simplest realization by a slowly rolling single scalar field, inflation provides us with a nearly scale invariant spectrum of Gaussian, adiabatic density perturbations that fit observational constraints \cite{1}. However, since there are many models of inflation where predictions are relatively in agreement with observations, a specific model then is still yet to be determined with the help of a future more precise data \cite{2}.

Various possible theoretical realizations, such as incorporating more than one scalar fields to drive inflation have opened the question about the adiabatic character of the early cosmological perturbations. It turned out that multiple fields can generically lead to isocurvature (non-adiabatic) perturbations \cite{3-5}. While in single field models non-adiabatic (entropy) perturbation modes are merely suppressed in the long wavelength limits (super-horizon scales), in multiple fields models however, these modes can in principle amplify the curvature perturbations and alter its evolution even after they have crossed outside the horizon (Hubble radius).

Multiple fields can enter the gravitational action in various ways, the simplest is to minimally interact where all the fields enjoy a canonical kinetic term, and another way is by interacting directly with the spacetime curvature (nonminimal coupling). In both cases generating entropy perturbations that must not be suppressed on super horizon scales is inevitable \cite{6-8}. In fact, not only multiple fields but even one single field can source non-adiabatic perturbation if it is nonminimally coupled to gravity \cite{7,8}. This surely indicates that nonminimal couplings are relevant to isocurvature and thus play a role in affecting the evolution of the curvature perturbations (nevertheless, an attempt to exclude single-filed models with such a feature have been considered in \cite{9} by using different geometric formulation.) However, the later conclusion may contradict the fact that, at least for single field-case, nonminimal and minimal dynamics described in Jordan and Einstein frames respectively are physically equivalent, thus if the gauge invariant curvature perturbation is conserved in one of these frames it must be so in the other one. The two frames seem to be equivalent since one can easily switch from one to the other by performing a simple conformal transformation (of the metric tensor) followed by field redefinition (of scalar fields). Generally speaking, the conservation of the curvature perturbation, or the curvature perturbation itself is not invariant under conformal transformation, and then the notion of “adiabaticity” itself becomes a frame-dependent.

The above features are not only related to the presence of multiple fields but also are tightly related to the theory of gravity at hand: metric gravity. In fact, the conformal transformation which is at the heart of every possible frame-ambiguity is purely geometric and concerns the metric tensor itself. To that end, it maybe more viable to rather consider a metric-less gravity for inflation itself. Purely affine theory of gravity, based solely on affine connection with no notion of metric, supports already scalar fields with non-vanishing potentials and stands viable for inflationary dynamics \cite{10,11}. Dynamics of nonminimally and minimally coupled scalar fields are described by two pure affine invariant actions that could be transformed to each other using only simple field redefinition. Since the geometric part is not altered by this redefinition, important quantities such as the Hubble parameter, and then the curvature perturbations are not subjected to changing \cite{12}.

In the present paper we thoroughly study non-adiabatic perturbations during inflation in the context of affine gravity. The main goal is to track the possible sources of entropy perturbations that may not be suppressed in long wavelength limit. Our framework will be based on a primary affine action in which multiple scalars are nonminimally coupled to gravity but each enjoys a canonical kinetic term. The linearity of the curvature with respect to the affine connection results in a generalized energy-momentum free of any additional terms that represent anisotropic pressure, such that at first order in perturbation only isotropic components including momentum flow contribute to the dynamics of the perturbations. This compact form leads to significant simplifications compared to metric gravity, where the Bardeen potentials in Newtonian gauge are equal even in the case of nonminimal coupling. In this case, non-adiabatic pressure of the system which may not be suppressed on super-horizon scales will appear in terms of two distinct quantities where one is sourced by the presence of more than one scalar which is a generic source that holds even in metric gravity, but the second is related to nonminimal coupling. It turns out that the second source vanishes in the single field-case leaving us with the conclusion that nonminimal couplings do not contribute in entropy production unless multiple fields are present. It is only from future more precise data that one will be able to discriminate between theories with or without entropy perturbations by analyzing the power spectra of the cosmic microwave background anisotropies and polarization, and tracking any hints of isocurvature modes. The present paper is considered as a generic framework and model-independent formulation of entropy perturbations in (affine)

\footnote{There is still an other possibility in which the fields gain a non-canonical kinetic terms. This case would lead to models such as $k$-Inflation which is beyond the scope of the present paper.}
inflation, and a future work will be devoted to an application, with some specific models, that runs along the present results.

The paper is organized as follows, the next section will be devoted to an overview of pure affine gravity with multiple scalars. Since it may not be familiar to the reader, we will bring detailed calculation for the derivation of the gravitational equations and the evolution of the scalar fields. In Sec III we tackle the scalar perturbations and study their evolution and see how they look like compared to the case of metric gravity. Finally we derive the non-adiabatic pressures sources responsible for entropy perturbations. In Sec IV we summarize the main results and conclude.

II. MULTIPLE FIELDS IN AFFINE GRAVITY: AN OVERVIEW

A. Nonminimal coupling and field equations

In purely metric theories of gravity (general relativity and its modifications) the interaction of matter fields with gravity is trivially performed by generalising related field theory Lagrangian densities in flat space such that the flat Minkowski metric is replaced by a curved spacetime metric tensor. The later is essential in contracting any matter or geometric tensor fields that allows finally for the construction of covariant actions. In the absence of any source of matter fields as well as vacuum energy, the gravitational field equations in free space is easily derived from Einstein-Hilbert action. However, it has been known that there is no fundamental principle that stands against extending spacetime structure itself by an ingredient more fundamental than the metric tensor. In fact, the major achievement of relativistic gravity is to introduce the concept of the connection that enables defining infinitesimal displacement of tensor fields in the curved background. Gravitational strengths then, would be measured by the curvature of this connection which in turn has no a priori relation with the metric. To that end, one would alternatively consider two possible approaches: (i) metric-affine [13, 14] where both metric and affine connection are introduced independently, (ii) purely affine [10–12, 15–21] in which no metric tensor is considered a priori but only an affine connection as a fundamental field. In this paper we are considering the second approach particularly the approach with nonminimal coupling provided in [10–12].

In the absence of metric tensor, the number of quantities that one could consider are less than that of the metric case, recalling that scalars formed by contractions (using metric) are not allowed in the first place. One could however consider scalar fields \( \phi^1, \ldots, \phi^N \) and their derivatives as well as associated potential \( V(\phi^1, \ldots, \phi^N) \). In the geometric sector we mainly have a symmetric connection \( \Gamma^\lambda_{\mu\nu} \) and the associated curvature or Ricci tensor \( R^\mu_{\nu\rho\sigma}(\Gamma) \) which for simplicity will be taken symmetric too (only symmetric part is taken.) Despite its simple structure, forming a familiar polynomial gravitational action is not trivial; this clearly stems from the absence of some essential covariant ingredients such as field kinetic terms which requires a metric tensor (see [22, 23] for attempts to construct polynomial affine actions).

Nevertheless, pure affine actions can still be constructed not as polynomials but in terms of volume measures. In fact, one can form the following diffeomorphism invariant action [12]

\[
S[\Gamma, \phi^1, \ldots, \phi^N] = \int d^4x \sqrt{|f(\phi^1, \ldots, \phi^N)|} R^\mu_{\nu\rho\sigma}(\Gamma) - \delta_{ab} \nabla_\mu \phi^a \nabla_\nu \phi^b],
\]

where the matter fields indices run as \( a, b = 1, \ldots, N \).

The spacetime coordinate dependent function \( f(\phi^1, \ldots, \phi^N) \) represents the nonminimal coupling function to the curvature and it can be considered as a varying mass, which reduces to the Planck mass in the case of minimal coupling to gravity. For a single field interacting nonminimally, this function would generically take the form \( f(\phi) = M_p^2 + \xi \phi^2 \) (see [10] concerning the affine approach of this case.) An interesting feature of the last action that metric gravity theories do not enjoy is the appearance of the potential energy in a denominator rather than in a separate term. If the total potential vanishes the action suffers from singularity, a feature that shows how potentials are crucial in these type of models and this is what inflation requires already at the first place. This would imply that when the scalar field’s potential enjoys symmetry-breaking solutions with some nonzero vacuum expectation values, one should certainly improve the potential with a nonzero constant term which will describe a possible cosmological constant that prevents the action from becoming singular in the vacuum [11].

The gravitational equations are derived by varying action (1) with respect to the affine connection. The later appears only in the curvature in terms of its first covariant derivative and quadratic forms that renders variation of curvature more compact as

\[
\delta R^\mu_{\nu\rho\sigma}(\Gamma) = \nabla_\lambda (\delta \Gamma^\lambda_{\mu\nu}) - \nabla_\nu (\delta \Gamma^\lambda_{\lambda\mu}).
\]
This would easily lead to the following infinitesimal variation of our action
\[ \delta_{\Gamma} S = \frac{1}{2} \int d^4x \left\{ \nabla_\mu \left( f \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\mu} \right) \delta^\beta_\lambda - \nabla_\lambda \left( f \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\beta} \right) \right\} \delta_{\alpha\beta}, \]

(3)

where we have used for brevity the following “Kinetic” part of the gravitational and scalar field sectors that enters the action as a tensor
\[ K_{\mu\nu}(\Gamma, \phi^1, \ldots, \phi^N) = f(\phi^1, \ldots, \phi^N) R_{\mu\nu}(\Gamma) - \delta_{ab} \nabla_\mu \phi^a \nabla_\nu \phi^b. \]

(4)
The action stays stationary under variation when the following dynamical equations are satisfied
\[ \nabla_\mu \left( f \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\mu} \right) \delta^\beta_\lambda - \nabla_\lambda \left( f \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\beta} \right) = 0, \]

(5)

which takes the simple final form
\[ \nabla_\lambda \left( f(\phi^1, \ldots, \phi^N) \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\beta} \right) = 0. \]

(6)

This is the main equation that will basically govern the affine dynamics. As we shall see later, two important consequences will arise from this equation, the first and most crucial is generating a metric tensor, whereas the second is the gravitational field equations in terms of this metric. For the moment, it is important to notice that like the primary action \[ \text{(1)} \] the last dynamical equation does not involve any and refer to any metric tensor.

Before writing the gravitational equations, let us first focus on the dynamics of the scalar fields. These are described and given by their equations of motion derived by varying the action with respect to the scalar fields themselves where in this case
\[ \delta_{\phi} S = \int d^4x \left\{ \frac{1}{2} \frac{\partial f}{\partial \phi^a} \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\beta} R_{\alpha\beta}(\Gamma) + \partial_\alpha \left( \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\beta} \partial_\beta \phi^a \right) - \frac{\sqrt{|K(\Gamma, \phi)|}}{V^2(\phi)} \frac{\partial V}{\partial \phi^a} \right\} \delta \phi^a \]

(7)

which leads to the equations of motion
\[ \partial_\alpha \left( \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\beta} \partial_\beta \phi^a \right) + \frac{1}{2} \frac{\partial f}{\partial \phi^a} \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi)} (K^{-1})^{\alpha\beta} R_{\alpha\beta}(\Gamma) - \frac{\sqrt{|K(\Gamma, \phi)|}}{V^2(\phi)} \frac{\partial V}{\partial \phi^a} = 0. \]

(8)

Though appears complicated, this is nothing but the equation of motion that govern the dynamics of the scalar field \( \phi^a \) in a curved affine background. Again, before generating the metric and writing this equation in a “familiar” form, one should notice its independence of the metric.

Let us now return to the gravitational sector and examine equation \[ \text{(6)}. \] The first result we can obtain from this equation is that the affine connection that has been taken arbitrary (but symmetric) in the action obeys now a constraint that reduces it to the Levi-Civita connection of an invertible tensor that we shall denote as \( g_{\mu\nu} \). In other words, this tensor is generated as a solution to equation \[ \text{(6)} \] by setting
\[ f(\phi^1, \ldots, \phi^N) \frac{\sqrt{|K(\Gamma, \phi)|}}{V(\phi^1, \ldots, \phi^N)} (K^{-1})^{\alpha\beta} = M_{Pl}^2 \sqrt{|g(\phi^1, \ldots, \phi^N)|} (g^{-1})^{\alpha\beta}, \]

(9)

where \( M_{Pl} \) is simply the Planck mass that balances the dimensionality in the last equation since the nonminimal coupling function \( f(\phi^1, \ldots, \phi^N) \) has a dimension of mass squared. It could have been taken as an arbitrary mass, however, consistency with Einstein field equations in vacuum implies that this mass must coincide with the Planck mass \[ \text{(10)}. \]

Now with the aid of the last equation, the dynamical equation \[ \text{(6)} \] becomes a gravitational field equations with a metric tensor \( g_{\mu\nu} \) compatible with the connection \[ \text{(2)} \] thus
\[ K_{\mu\nu}(g, \phi^1, \ldots, \phi^N) = M_{Pl}^2 V(\phi^1, \ldots, \phi^N) - g_{\mu\nu}, \]

(10)
\[ \nabla_\lambda g_{\mu\nu} = 0. \]

(11)

Footnote:

2 In general, these relations lead to \( \nabla_\lambda (\sqrt{|g(\phi^1, \ldots, \phi^N)|}) = 0 \), but since the affine connection is taken symmetric one easily obtains the compatibility condition \[ \text{(11)}. \] For the metric to be physical, only those configurations \( (\Gamma, \phi) \) for which \( K_{\mu\nu} \) has the signature \((-\), +, +, +\) are considered \[ \text{(12)}. \)
This couple of equations shows the interesting transition from the pure affine dynamics of the system to the metrical structure where the later arises only \textit{a posteriori} and not imposed from scratch. With this metric, lowering and raising indices as well as contractions become possible, and finally one can form the Einstein tensor, and equations (10) takes the form

\begin{equation}
    f(\phi^1, \ldots, \phi^N)G_{\mu\nu} = \delta_{ab} \nabla_\mu \phi^a \nabla^b - \frac{1}{2} \delta_{ab} \nabla^\lambda \phi^a \nabla_\lambda \phi^b g_{\mu\nu} - \frac{M_{Pl}^2 V(\phi^1, \ldots, \phi^N)}{f(\phi^1, \ldots, \phi^N)} g_{\mu\nu}. \tag{12}
\end{equation}

It is clear that for the minimal coupling limit where \( f = M_{Pl}^2 \), the last field equations get reduced to Einstein field equations with scalar fields in a familiar form. Thus, the pure affine action gives rise to Einstein equations where spacetime curvature is sourced by the generalised energy-momentum tensor of the form

\begin{equation}
    T_{\mu\nu} = \frac{1}{f(\phi^1, \ldots, \phi^N)} \left\{ \delta_{ab} \nabla_\mu \phi^a \nabla^b - \frac{1}{2} \delta_{ab} \nabla^\lambda \phi^a \nabla_\lambda \phi^b g_{\mu\nu} - \frac{M_{Pl}^2 V(\phi^1, \ldots, \phi^N)}{f(\phi^1, \ldots, \phi^N)} g_{\mu\nu} \right\}. \tag{13}
\end{equation}

One might easily notice the difference between this tensor and the energy-momentum tensor that arises from non-minimal coupling of pure metric gravity. The crucial difference relies on the absence of the terms proportional to \( \nabla_\mu \nabla_\nu f - g_{\mu\nu} \Box f \) which appear in metrical gravity due to the nonlinearity of Einstein-Hilbert action. These terms are the sources of the so called \textit{anisotropic pressure} and their presence certainly affects the scalar fields dynamics. Among its effects is the contribution to generating entropy perturbations through non-adiabatic pressure even for a single scalar field \cite{foot1,foot2}, this however is prevented as we shall see in Sec III when studying scalar perturbations.

The same for the evolution of the scalar fields, using the generated metric (9), the scalar field equations of motion \cite{foot3} take the form

\begin{equation}
    \Box \phi^a - \frac{\partial V}{\partial \phi^a} + \frac{1}{2} \frac{\partial f}{\partial \phi^a} R(g) + \psi(\phi^1, \ldots, \phi^N) = 0, \tag{14}
\end{equation}

where

\begin{equation}
    \psi(\phi^1, \ldots, \phi^N) = \left( 1 - \frac{M_{Pl}^2}{f} \right) \frac{\partial V}{\partial \phi^a} - \frac{1}{f} \frac{\partial f}{\partial \phi^a} \delta_{cd} \nabla^\lambda \phi^c \nabla_\lambda \phi^d. \tag{15}
\end{equation}

The function \( \psi(\phi^1, \ldots, \phi^N) \) is restricted to non-minimal coupling dynamics, hence it vanishes in the minimal coupling case where \( f = M_{Pl}^2 \), and it shows also the differences between metric and purely affine gravity.

Given this overview on the multiple scalar fields coupled to gravity in its affine picture, we then turn to an interesting part about how to perform the transition from nonminimal to minimal couplings.

### B. Transition to minimal coupling and flat field space

In general, the transition from nonminimal to minimal coupling is essential since it brings the gravitational sector to a canonical form in a frame where the observed quantities are generally calculated. In metric theories this is achieved by performing the so called conformal transformation where the metric tensor corresponding to a Jordan frame is mapped to a new one referred to as the Einstein frame. However, in the absence of any metric, our model does not reply on this, but rather, only a simple scalar field redefinition would bring the gravitational sector to a canonical form. In fact, action \( \Pi \) could be brought to a more compact form as

\begin{equation}
    S[\Gamma, \phi^1, \ldots, \phi^N] = \int d^4x \sqrt{\frac{M_{Pl}^2 R_{\mu\nu}(\Gamma) - G_{ab}(\phi^1, \ldots, \phi^N) \nabla_\mu \phi^a \nabla_\nu \phi^b}{V(\phi^1, \ldots, \phi^N)}}, \tag{16}
\end{equation}

where the original potential is rescaled as

\begin{equation}
    \tilde{V}(\phi^1, \ldots, \phi^N) = \left( \frac{M_{Pl}^2}{f(\phi^1, \ldots, \phi^N)} \right)^2 V(\phi^1, \ldots, \phi^N), \tag{17}
\end{equation}

and the matrix, or the new metric of the \( N \)-dimensional field space \( G_{ab} \) is given by

\begin{equation}
    G_{ab}(\phi^1, \ldots, \phi^N) = \frac{M_{Pl}^2}{f(\phi^1, \ldots, \phi^N)} \delta_{ab}. \tag{18}
\end{equation}
In general relativity, the conformal transformation which is necessary for bringing a canonical form of the gravitational sector would, however, bring the following extra terms to quantity (18) that make it, generally, impossible to be Euclidean \[24\]

\[ G_{ab}(\phi^1, \ldots \phi^N) = \frac{M_{Pl}^2}{2f(\phi^1, \ldots \phi^N)} \delta_{ab} + \frac{3M_{Pl}^2}{2f^2(\phi^1, \ldots \phi^N)} \frac{\partial f}{\partial \phi^a} \frac{\partial f}{\partial \phi^b}. \]  

(19)

The difficulty relies on the last term of the field derivatives and it translates somehow, the nonlinearity of general relativity with respect to the metric tensor. Here, a necessary condition for the existence of possible field transformations that would bring the field space metric (19) into \( \delta_{ab} \), thus recovering the canonical kinetic forms of the scalar fields, is if the curvature tensor constructed from the last metric vanishes identically. This has been shown to be impossible at least for field space dimensions \( N > 2 \) \[24\].

However in the present case, as one can realize from (18), although it breaks the canonical form of the field kinetic terms, one could always rescale the later such that unlike metric theories of gravity, the last field space metric could be made Euclidean by transforming only the scalar fields as

\[ \nabla_\mu \phi^a \nabla_\nu \phi^b \rightarrow \frac{f(\phi^1, \ldots \phi^N)}{M_{Pl}^2} \nabla_\mu \tilde{\phi}^a \nabla_\nu \tilde{\phi}^b. \]  

(20)

With this redefinition and the associated potential \[17\], action \[13\] would finally describe a minimal coupling case of multiple scalar fields in affine spacetime where both gravity and field kinetic terms enjoy a canonical form.

The conformal transformation of the metric tensor in metric theories of gravity which induce a complicated form of the field space metric (19) is the origin of the so called frame-ambiguities on which debates are not settled down (see \[12\] and references therein).

C. Background field dynamics

Before tackling the evolution of the perturbations, let us first examine the dynamics of the homogeneous parts of the scalar fields in a spatially flat Friedmann-Robertson-Walker (FRW) universe. It is from the homogeneous background fields that one imposes the slow roll conditions which finally provide us with solutions to the flatness and horizon problems.

In what follows, for simplicity we will take the Planck mass \( M_{Pl}^2 = 1 \) which can be easily recovered in practice.

Taking the scalar fields as homogeneous, \( \phi \sim \phi(t) \), the generalized energy momentum tensor \[13\] would simply split into a generalized energy density and pressure given as

\[ \rho = \frac{1}{f} \left( \frac{1}{2} \phi^a \phi^a + \frac{V}{f} \right) \quad \text{and} \quad \mathcal{P} = \frac{1}{f} \left( \frac{1}{2} \phi^a \phi^a - \frac{V}{f} \right) , \]  

(21)

where the nonminimal coupling function and the potential are evaluated at the background, \( f = f(\phi) \) and \( V = V(\phi) \). We will keep this notation when treading the field fluctuations later.

The gravitational field equations \[12\] are easily adapted for the flat FRW spacetime leading to

\[ 3H^2 = \frac{1}{f} \left( \frac{1}{2} \phi^a \phi^a + \frac{V}{f} \right) \]  

(22)

and

\[ \dot{H} + H^2 = - \frac{1}{3f} \left( \frac{1}{2} \phi^a \phi^a - \frac{V}{f} \right) \]  

(23)

The same for the evolution equation \[14\] which takes the form

\[ \ddot{\phi}^a + 3H \dot{\phi}^a + \frac{1}{f} V_{,a} - 3(\dot{H} + 2H^2)f_{,a} - \frac{1}{f} \phi^b \phi^b f_{,a} = 0. \]  

(24)

Here \( H = \dot{a}/a \) is the Hubble parameter in terms of the scale factor \( a(t) \). For the ease of notation we have used the sign “comma” to refer to derivatives with respect to the scalar fields. We have also omitted the \( \delta_{ab} \) symbol leaving only repeated indices for summation convention. Equation (24) can also be derived from the conservation of the total energy-momentum tensor \[13\] which takes the common form

\[ \dot{\rho} + 3H (\rho + \mathcal{P}) = 0 \]  

(25)
It is easy to notice the differences from the metric gravity in the case of both minimal and nonminimal couplings. First of all, the minimal coupling limit \( f = 1 \) is equivalent to that in metric gravity where the set of equations \([21, 24]\) are reduced to the standard cosmology equations in the presence of single field. In the nonminimal case where the function \( f \) is a field-dependent, the energy density and pressure as well as the potential are modified by a multiplicative factor \( f^{-1} \) compared to the minimal case. However, when compared to the nonminimal case of metric gravity we realize a crucial difference, in addition to the factor \( f^{-1} \) we notice here the absence of the first and second time-derivative of the function \( f \) in the energy density and pressure due to the absence of anisotropic terms in the energy-momentum tensor \([12]\). In other words, the differences between minimal and nonminimal couplings dynamics in affine gravity arises only through simple factors not in additional terms.

**III. FIELD FLUCTUATIONS AND ENTROPY PRODUCTION**

**A. Scalar perturbations and anisotropic stress-free dynamics**

In every model of inflation, inhomogeneities in the scalar fields are of great importance since they lead to curvature perturbations which in turn provides the measure of gauge invariant primordial perturbations acting as seeds for structure formation. In the following, we will follow the standard way of deriving the scalar perturbations dynamics from the equations of motion which are in this case summarized in \([12]-[15]\). First we expand the fields around a homogeneous backgrounds \( \phi^a \) as

\[
\phi^a = \phi^a(t) + \delta \phi^a(t, \vec{x}),
\]

where the first term satisfies the equations of motion of the last section, and the last term represents the multiple fields fluctuations.

We then impose deviations from FRW spacetime that would represent a perturbed metric in which \( g_{00} = -(1 + 2 \Psi) \) and \( g_{ij} = a^2 \delta_{ij}(1 - \Phi) \), where \( \Psi \) and \( \Phi \) are space and time dependent scalar potentials. Thus, up to first order, the generalized energy momentum tensor perturbations lead to a generalized energy density and pressure fluctuations as\(^3\)

\[
\delta \rho = \frac{1}{f} \left( \phi_\alpha \delta \phi^\alpha - \dot{\phi}_\alpha \phi^\alpha + \frac{1}{f} V_{,a} \delta \phi^a \right) - \frac{1}{f^2} \left( \frac{1}{2} \dot{\phi}_\beta \phi^\beta f_{,a} + \frac{2 V}{f} f_{,a} \right) \delta \phi^a,
\]
\[
\delta P = \frac{1}{f} \left( \phi_\alpha \delta \phi^\alpha + \dot{\phi}_\alpha \phi^\alpha + \frac{1}{f} V_{,a} \delta \phi^a \right) - \frac{1}{f^2} \left( \frac{1}{2} \dot{\phi}_\beta \phi^\beta f_{,a} - \frac{2 V}{f} f_{,a} \right) \delta \phi^a.
\]

Thus, the time-time part of the gravitational equations \([12]\) reads

\[
3H(\dot{\Psi} + H \Phi) + \frac{1}{a^2} \vec{\nabla}^2 \Psi = -\frac{1}{2f} \left( \phi_\alpha \delta \phi^\alpha - \dot{\phi}_\alpha \phi^\alpha + \frac{1}{f} V_{,a} \delta \phi^a \right) + \frac{1}{2f^2} \left( \frac{1}{2} \dot{\phi}_\beta \phi^\beta f_{,a} + \frac{2 V}{f} f_{,a} \right) \delta \phi^a,
\]

whereas the time-space part leads to

\[
\dot{\Psi} + H \Phi = \frac{1}{2f} \dot{\phi}_\alpha \delta \phi^\alpha.
\]

Note that these derivatives appear in the expansion of the potential and the nonminimal coupling function \( f \) around the background fields, i.e., \( f(\phi^a + \delta \phi^a) \approx f(\phi^a) + f_{,a}(\phi^a) \delta \phi^a + O(\delta \phi^a \delta \phi^b) \), and the same for the potential.

The second important evolution equation which generically describes a constraint on the scalar potentials \( \Psi \) and \( \Phi \) could be derived easily from the spacial part of the gravitational equations \([12]\) and reads

\[
\nabla_i \nabla_j (\Psi - \Phi) = 0 \quad (\text{for} \quad i \neq j).
\]

This interestingly shows that even multiple fields, coupled nonminimally to gravity, do not contribute to anisotropic stress. In other words, the evolution equation \([31]\) which generically holds in the minimal coupling case is conserved when every possible nonminimal interaction is present. In metric gravity however, this is no longer the case. In fact, the nonlinearity of the actions would result in the presence of a generalized energy momentum tensor from which arises an anisotropic stress, and finally the right hand side of \([31]\) would not vanish \([7, 8]\). The appearance of the anisotropic term in the nonminimal coupling dynamics of GR means that the Bardeen potentials totally differ. This is ambiguous in the case of pure scalar fields as we shall argue below.

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\(^3\) Remember that we are taking \( M^2_{Pl} = 1 \) for brevity.
In the standard cosmological model, baryons and cold dark matter do not contribute anisotropic stress since they are successfully approximated to perfect fluids. Photons and neutrinos on the other hand could in principle contribute anisotropic stress when they have considerable quadrupole moments. While photons contribute less, collisionless neutrinos however have appreciable quadrupole moments during the radiation dominated era \[25\]. In the case of scalar fields and mainly minimally coupled ones, one can considerably simplify the equations of motion and show that to first order in perturbations the spatial part of the energy momentum tensor is proportional to \(\delta\), thus do not contribute any source to the right hand side of \(31\). This happens in both GR and pure affine gravity\[10\]. In case of anisotropic stress when they have considerable quadrupole moments. While photons contribute less, collisionless uniform-density hypersurfaces \[25\] do not contribute any source to the right hand side of \(31\). This happens in both GR and pure affine gravity\(31\). One then faces again a frame ambiguity in GR.

Condition \(31\) which is equivalent to \(k^2(\Psi - \Phi) = 0\) in Fourier space means simply that we end up with only one scalar potential \(\Psi = \Phi\) which finally simplifies the the above evolution equations.

### B. Sources of entropy perturbations

An important scalar quantity in cosmological perturbations is the gauge-invariant curvature perturbation on a uniform-density hypersurfaces \[25\]

\[
\zeta \equiv -\Psi - \frac{H}{\dot{\rho}} \delta \rho,
\]

where \(\rho\) is the energy density which is caused by scalar fields in the present case.

An interesting feature of this quantity is that it remains constant outside the horizon for adiabatic matter perturbations. In the case of single scalar field minimally coupled to gravity, it can easily be shown that the perturbation \(32\) does not evolve outside the horizon, i.e, when \(k \ll aH\). The reason is simply that minimally coupled slowly rolling single field does not contribute “non-adiabatic” pressure on super-horizon scales, i.e

\[
\delta P_{\text{nad}} = \delta P - \frac{\rho}{\dot{\rho}} \delta \rho = 0 \quad \text{for} \quad k \ll aH
\]

The quantity \(\delta P_{\text{nad}}\) refers to the non-adiabatic pressure, and any source that contribute a nonzero value to this quantity would imply an evolving curvature perturbation on cosmologically interesting length scales. This in turn ends up with producing considerable entropy perturbations. In the following, our goal is to examine these entropy perturbations sourced by multiple scalar fields nonminimally coupled to (affine) gravity. Hence, every source of entropy perturbation shall appear through every possible nonzero term that forms the non-adiabatic pressure calculated from the “generalised” energy density and pressure \(24\).

Energy density and pressure with their associated fluctuations \(27\) and \(28\) lead to

\[
\delta P_{\text{nad}} = -\frac{2\dot{\varphi} a V_a}{3H \dot{\varphi} \dot{\varphi} f} \delta \rho + \frac{4V a \phi_a}{3H \dot{\varphi} \dot{\varphi} f^2} \delta \rho - \frac{2V_a \delta \phi_a}{f^2} + \frac{4V f_a \delta \phi_a}{f^3},
\]

where we have used \(\dot{\rho} = -3H(\rho + P)\).

We then add and subtract the term \(3H \dot{\varphi} b \delta \phi^b / f\) to obtain

\[
\delta P_{\text{nad}} = -\frac{2\dot{\varphi} a V_a}{3H \dot{\varphi} \dot{\varphi} f} \left( \delta \rho + \frac{3H}{f} \dot{\varphi} b \delta \phi^b \right) + \frac{4V a \phi_a}{3H \dot{\varphi} \dot{\varphi} f^2} \left( \delta \rho + \frac{3H}{f} \dot{\varphi} b \delta \phi^b \right) + \frac{2\dot{\varphi} a V_a \phi_a b \delta \phi^b}{f^2 \dot{\varphi} \dot{\varphi} \dot{\varphi}} - \frac{2V_a \delta \phi_a}{f^2} - \frac{4V \dot{\varphi} a \phi_a \phi^b \delta \phi^b}{f^3 \dot{\varphi} \dot{\varphi} \dot{\varphi}} + \frac{4V f_a \delta \phi_a}{f^3}.
\]

The term in parenthesis is the generalized gauge-invariant comoving density perturbation and can be obtained easily by combining the evolution equations \(29\) and \(30\) which yield

\[
\delta \rho + \frac{3H}{f} \dot{\varphi} b \delta \phi^b = -\frac{k^2}{a^2} \Psi,
\]

\[4\text{ In the case of minimal coupling, affine gravity, though generally different, leads to the same dynamics as GR, however, deviations from GR become crucial when the fields are nonminimally coupled \[10\].}
where $k$ is the wave vector (momentum) that comes out of $\tilde{\nabla}^2 \Psi$ in Fourier space.

Finally, the generalized non-adiabatic pressure reads

$$
\delta P_{\text{nad}} = \frac{4H\dot{\varphi}^a V_{,a}}{3\dot{\varphi}^c \dot{\varphi}_c f} \left( \frac{k}{aH} \right)^2 \Psi - \frac{8HV\dot{\varphi}^a f_{,a}}{3\dot{\varphi}^c \dot{\varphi}_c f^2} \left( \frac{k}{aH} \right)^2 \Psi
$$

The first two terms are suppressed on super-horizon scales the above non-adiabatic pressure remains nonzero due to the presence of two terms

$$
\delta P_{\text{nad}} = \delta P_{\text{nad}}^{\text{multiple}} + \delta P_{\text{nad}}^{\text{non-min}},
$$

which represent the two possible and distinct sources of entropy perturbations and they are as follows:

1. **Source from multiple fields**

   The first source is induced by multiple fields and is described by the first (not suppressed) term in (37)

   $$
   \delta P_{\text{nad}}^{\text{multiple}} = -\frac{2V_{,a}}{f^2} \left[ \delta \phi^a - \frac{\dot{\varphi}^a}{\dot{\varphi}^c \dot{\varphi}_c} \varphi^b \delta \phi^b \right].
   $$

   In fact, one can easily verify that this source vanishes for single (nonminimally or minimally) coupled scalar field, i.e, when $N = 1$ ($\phi^a \equiv \phi$) for both cases $f = \text{constant}$ (minimal) or $f \neq \text{constant}$ (nonminimal)

   $$
   \delta P_{\text{nad}}^{\text{multiple}} \quad \text{For single scalar field} \quad \to 0.
   $$

   However, for instance, two scalar fields $\phi^a = (\phi, \chi)$ would produce

   $$
   \delta P_{\text{nad}}^{\text{multiple}} = -\frac{2\dot{\chi}}{(\phi^2 + \chi^2) f(\phi, \chi)} \left( \chi V_{,\phi} - \dot{\phi} V_{,\chi} \right) \left( \frac{\delta \phi}{\phi} - \frac{\delta \chi}{\chi} \right),
   $$

   for a general coupling function $f(\phi, \chi)$ including a constant (minimal coupling).

   The second source of entropy perturbation in this framework is related to the nonminimal coupling and it is described by the last term in (37) or

   $$
   \delta P_{\text{nad}}^{\text{non-min}} = \frac{4V f_{,a}}{f^3} \left[ \delta \phi^a - \frac{\dot{\varphi}^a}{\dot{\varphi}^c \dot{\varphi}_c} \varphi^b \delta \phi^b \right].
   $$

   This quantity clearly vanishes for a constant $f$, which is the case of minimal coupling dynamics. Thus, the first remark is that when the coupling is minimal, only contributions from multifields (the previous source) induce entropy perturbations.

   Another interesting feature of this source is that like $\delta P_{\text{nad}}^{\text{multiple}}$, it also vanishes for single scalar. This interestingly means that nonminimal couplings have effects on the entropy perturbations only for the case of more than one scalar field. This does not hold in metric gravity where even a single field can source an entropy perturbation if it is nonminimally coupled.

   Unlike the present case, in metric gravity non-adiabatic pressure induced by nonminimal coupling appears as a contribution of several separate terms due to the complicated form of the energy-momentum tensor (see the discussion below equation (13)). In particular, terms that generate anisotropic pressure have their effects even when only one single field is considered, the fact that prevents the non-adiabatic pressure from vanishing for a
single field as well. This leads to serious ambiguity in metric gravity or general relativity. In fact, it is known that nonminimal coupling dynamics can be easily transformed to minimal coupling dynamics in Einstein frame at least in the case of single field. In Einstein frame however, single scalar cannot contribute to any source of non-adiabatic pressure that are not suppressed on super-horizon scales, hence, it does not generate any entropy perturbation. In other words, while entropy perturbation is suppressed in one frame (Einstein frame), it is generated in the other one (Jordan frame). This ambiguity arises from the conformal transformation of the metric which is necessary for switching from one to another frame, and this endanger the adiabaticity character.

In our framework, based on affine gravity, those conformal frames arising from conformal transformation are not present. As we have seen in Sec. III B the transition to minimal coupling is made by performing only scalar fields redefinition. The metric tensor in this sense is unique for both couplings (minimal and nonminimal), it has been generated dynamically from a pure affine action that does not refer to any metric to transform. This metric remains the same when switching to minimal coupling dynamics. Thus, in this picture the notion of adiabaticity is invariant under field redefinition. While multiple fields induce entropy perturbations in both nonminimal and minimal coupling cases, single scalar field does not in both cases as well.

The gauge-invariant curvature perturbation \( \zeta \), a crucial quantity in primordial cosmology, represents a measure of the primordial perturbations which lead to fluctuations of the temperature in the cosmic microwave background and finally manifest as seeds for structure formation. Its conserved character, which is a generic but crucial feature in the most known models of inflation (particularly with single fields), turns out to be altered in the presence of multifields. In fact, one can show that the evolution (time dependence) of the curvature perturbation is generically proportional to the non-adiabatic pressure \( \delta P \).

\[
\dot{\zeta} = -\frac{H}{\rho + P} \delta P_{nad} + \text{terms suppressed by } \left( \frac{k}{aH} \right)^2. \tag{43}
\]

This relation can be derived for every conserved energy-momentum tensor including our present case in which it leads to

\[
\dot{\zeta} \supset -\frac{H f}{\phi_a \dot{\phi_a}} \left( \delta P_{nad}^{\text{multiple}} + \delta P_{nad}^{\text{non-min}} \right). \tag{44}
\]

From the conclusions drawn above, we may safely say that unlike in metric gravity, here it is sufficient to consider only one single scalar field and one then recovers the conservation of the curvature perturbation \( \dot{\zeta} = 0 \). In the single-field case then, nonminimal coupling to affine gravity does not break the conservation law of the curvature perturbation. This is a new and very important feature and it must be crucial in distinguishing between purely affine theory of gravity and metric theories of gravity when considering the inflationary dynamics in both contexts.

In the case of multiple fields, however, \( \dot{\zeta} \neq 0 \) and deviations from zero result from the contributions of the above sources of non-adiabatic pressure induced by the presence of more than one scalar. Entropy perturbation modes or as commonly called “isocurvature” perturbations can contribute to both power spectrum and bispectrum if they survive until the recombination era. Possible “cross-correlation” between adiabatic and isocurvature would also lead to production of mixed bispectra \([26,28]\). Hence on cosmological scales isocurvature perturbations, if any, may be constraints by observations, for instance by analyzing the angular power spectrum of the cosmic microwave background \([1]\).

We conclude by mentioning that the real perturbations which are constrained by observations must be adiabatic. It is the density perturbations at the era of primordial nucleosynthesis. Thus, the presence of isocurvature (non-adiabatic) perturbations during inflation does not necessarily imply its presence later \([29]\). The phase of reheating which is still yet to be understood may guarantee the transition to primordial adiabatic perturbations even when isocurvature perturbations are generated during inflation.

IV. SUMMARY AND CONCLUSION

The last few decades were remarkable for cosmology where even the physics of the early universe becomes accessible to high-precision observations. Lot of efforts have been devoted in parallel to different theoretical models for the early universe particularly those of inflation with the aim of coming up with one successful and convincing model that fits

\[^5\text{At first sight it seems that from (44), one can make } \dot{\zeta} = 0 \text{ (adiabatic perturbations) if } f \propto V^{1/2}. \text{ Unfortunately, this constrain affects at the first place the dynamics of the inflaton since it also clears away the effects of the potential in (25) leaving only kinetic terms of the inflaton that do not allow slow roll conditions.}\]
the accurate data. Among the inflationary models that gained much attentions recently are those of nonminimally interacting multiple fields \[7–9\] (see also \[30\] as example for attractor behavior in multifields-inflation). Indeed, while realistic models of elementary particles typically include many scalars, quantum field theory in curved spacetime generically requires nonminimal couplings for the scalar fields.

However when more than one scalar are present crucial changes arise (compared to the case of single field) not only in the dynamics but also in the perturbation itself that is generated during inflation. In fact, the fields may interact and cause significant non-adiabatic (entropy) perturbations that typically are not suppressed on super-horizon scales. Studies of these isocurvature perturbations and their possible detection through the Cosmic Microwave Background anisotropies and polarizations is at the heart of every serious work on multiple-fields inflation \[4, 5, 26–28, 31\].

In this paper we have presented a general framework in which multiple fields are considered to drive inflation but in the context of purely affine gravity rather than in general relativity. Indeed, we believe that not only the type of the fields are important in the very early universe but also the approach to gravity can play a crucial role. We have started with a metric-less action from which the metric tensor itself arises through the equation of motion. The primary goal was to investigate the possible sources of non-adiabatic pressure that cause entropy perturbations not only from the presence of multiple fields (which is a generic feature) but also due to the nonminimal interactions.

The scalar perturbations have shown that there must be two distinct sources of non-adiabaticity, one is the familiar source arising from multiple fields and the other one is related to nonminimal coupling. Although the two types of sources are expected as in metric gravity, here the source that arise from nonminimal couplings vanishes when only single scalar is considered. In other words, entropy perturbations are there simply because there are multiple fields. The later remark leads us to raise a serious issue encountered in metric gravity; frame-ambiguities. In fact, detailed calculations made in Jordan frame showed that even when only a single scalar is considered there will be still an entropy perturbations that survive when the field is nonminimally coupled \[4, 8\]. We know however that in Einstein frame where the inflaton is minimally coupled to gravity, non-adiabatic perturbations are suppressed in the long wavelength limit leading to adiabatic curvature perturbations. If we believe that the frames are equivalent (which must be the case) then one encounters a frame-ambiguity; \textit{while the curvature perturbation is adiabatic in one frame, it is non-adiabatic in the other one.} In the present framework it is clear that this frame issue is not encountered in affine gravity where the origin of the ambiguity which is the conformal transformation is not present \[12\].

It is only possible future more precise measurements, for instance of the power spectrum of Cosmic Microwave Background temperature and anisotropies will show whether there were really isocurvature (entropy) perturbations that are generated during inflation and survived until recombination. This must be investigated along with a specific model based on the present framework \[32\].

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