Control of the chaotic dynamics of a turbulent 3D wake

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In this study we focus on the control of the dynamics of 3D turbulent wake downstream a square-back Ahmed body ($Re_H = 3.9 \times 10^5$). The peculiar dynamics of such a wake is first characterized through the trajectories of the pressure barycenter over the rear part of the model as well as the recirculation barycenter in the wake. In particular it is shown that these dynamics allow the definition of three different states: the two so-called reflectional symmetry-breaking (RSB) modes and the transient symmetric (TS) mode. It was shown recently [1] that the time-fluctuations of the pressure barycenter could be characterized as a weak chaotic system with two well-defined attractors (the two RSB modes). We show that the dynamics of the bimodal wake can then be forced into a stable asymmetric or symmetric state in open loop control, using tangential continuous or pulsed blowing in three different regions along the upper edge of the rear part of the model. Finally, a simple closed-loop opposition control, based on real-time identification of the wake barycenter in the PIV fields, is used to force the chaotic dynamics of the wake into a regular oscillatory motion at a well-controlled frequency. Depending on the actuation parameters, the wake dynamics can also be switched from bimodal to a new multimodal behavior. We show that this new mode also exhibits a peculiar dynamics with an up-down instead of left-right chaotic oscillations. Interestingly, the recirculation area (size of the recirculation bubble) is much more reduced for the closed-loop experiments when the jets are pulsed rather than continuous. For the pulsed jets, the reduction is also increased when the proper frequency is chosen.

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INTRODUCTION

The turbulent wakes downstream of three-dimensional (3D) bluff bodies, like cylinders (circular or square), spheres or cubes, either wall-mounted or in the freestream, can be very multifaceted, exhibiting large-scale and small-scale coherent structures, either spanwise or streamwise, with strongly intermittent behaviors [2–8].

Passenger cars or trucks can be considered as 3D bluff-bodies interacting with a wall (underbody flow). The flow over a real vehicle is very complex but most of the aerodynamics forces are related to various flow separations over the front part (A-pillar vortices, wake of the rear-view mirrors) to the massive flow separation over the rear part, which is responsible of 50 to 70 % of the drag, depending on the geometry. Studying the structure and dynamics of the 3D wake downstream of the vehicle is then crucial to optimize and reduce the overall drag and related fuel consumption [9]. As a consequence, one can focus on simplified geometries that could mimic the wake of a real vehicle.

In response to the need of a simplified 3D model for automotive aerodynamics studies, the so-called Ahmed body has thus been designed obeying a reflectional symmetry [10, 11]. In this geometry, only the rear-slant angle $\alpha$ can be changed ($0^\circ \leq \alpha \leq 90^\circ$). This simple modification leads to drastic modifications of the overall structure of the wake, together with large variations of the aerodynamic drag coefficient. As an illustration, the slanted configuration ($\alpha = 25^\circ$) involves large-scale streamwise longitudinal vortices [12, 13], together with an unsteady recirculation bubble over the inclined surface [14, 15], a pair of a horseshoe vortices containing each a recirculation bubble [16], and spanwise Kelvin-Helmholtz vortices, together with smaller-scales turbulent structures.

The wake behind the square-back configuration ($\alpha = 0^\circ$) carries other topologies and dynamics. Indeed, time-averaged velocity and pressure fields reveal a toroidal vortex in the near wake [17, 18] and spectral analyses highlight a low-frequency mechanism, the bubble pumping [17, 19]. Recently, a right-left oscillation of the global wake has been observed, defining the so-called reflectional symmetry breaking (RSB) modes and leading to the bi-stable (or bimodal) wake [20, 21], which appears from the laminar regime [22, 23]. This behavior is sensitive to other geometric parameters like the aspect ratio of the bluff-body’s cross-section and the ground clearance [24], and to experimental conditions like the yaw angle [19, 25]. A transient symmetric state is even identified when a cluster-based reduced-order model of the wake is statistically computed [26]. Recently, it was shown that the large-scale slow-time oscillations behaves like a high dimensional chaotic system with a well-defined strange attractor [1]. This result was obtained through the analysis of the time-series of the coordinate of the barycenter of the wall-pressure field measured over the rear vertical part of the model. It was the first clear demonstration that the global dynamics of a turbulent 3D wake can be reduced to a simple dynamic system characterized by a set of two variables. The same kind of results was recently found into a turbulent swirling flow [27]. If such a simplification is possible, then it should also be possible to apply simple forcing to control the overall dynamics of the 3D wake. This is precisely the objective of the present study.

Because of the sensitive dependence on the initial conditions, chaos was believed for a long time to be neither predictable nor controllable. The OGY control method proposed by Ott, Grebogi and Yorke [28] changed completely this point of view. They showed that it is possible to use small time-dependent perturbations to force a chaotic attractors system in a time-periodic motion, defined from one of its unstable periodic orbits (UPOs) [29]. Since this pioneering work, control of chaotic systems has become a very active research field and many new control methods have been proposed such as linear-feedback [30], adaptive control [31], impulsive or intermittent control [32].

Controlling chaotic systems is a challenge encountered in various domains such as laser, plasma, electronics, communications, brain activity or fluid dynamics. In fluid dynamics, the turbulence problem is often associated to chaotic representation, although such modelling is not always justified [33]. For instance, the dynamics of a convective fluid confined in a vertical toroidal channel are well described by a Lorenz system, whose one of the two strange attractors can be removed through a feedback control [34]. In simulated fluid flows, the Lorenz model [35] is often used as a benchmark case for closed-loop control algorithms. Thereby, a control law is established from limited information on the full Lorenz system and can stabilize it in one of the foci [36]. Machine learning control is also applied to such system as a training exercise [37]. Finally, a first control of a chaotic open flow has been recently experimentally achieved. After slightly forcing the dynamics of an axisymmetric jet into a low-dimensional chaotic attractor [38], a closed-loop control is designed to sustain a selected UPO [39]. In this experiment, the chaotic dynamics correspond to the vortex ring formation downstream of the jet nozzle exit, which includes successive vortex pairings. The control strategy enables to stabilize the ring formation.

Our objective in the present study is to control the dynamics of the bimodal chaotic oscillations of a 3D turbulent wake, not the reduction of time-averaged quantities like the drag coefficient or the pressure coefficients. Nevertheless, in some cases, controlling the large-scale fluctuations of a turbulent wake is a key step toward the control of various global quantities like the drag coefficient. Actually, stabilization of the wake by suppressing the bimodal behavior is of interest to industrial applications since it can lead to drag reduction [25, 40]. Unlike the previous examples on the
control of the Lorenz system, this implies here to steady an unstable symmetric state.

Previous experimental studies show how different control strategies change the wake dynamics behind a square-back bluff body. For instance, such a wake has been stabilized toward a symmetric state by adding a well-designed cavity behind the rear surface [41]. Using pulsed jets along continuous slits at the trailing edges, the unsteady dynamics of the wake can be amplified by selecting either the vortex shedding frequency for an asymmetric forcing configuration, or its subharmonic for a symmetric forcing configuration [42]. Inversely, selecting higher actuating frequencies damps the dynamics [43]. Feedback controls achieve the suppression of the RSB modes, based either on a stochastic model of the complete wake dynamics using flaps [44], or on an opposition strategy using lateral slit jets [45]. In the present study, we define a similar control law as the latter one but the sensors being the velocity fields and the actuators being discontinuous jets only localized at the top trailing edge. To our knowledge, this is the first tentative to control a turbulent 3D wake considered as a chaotic system.

The rest of the paper is organized as follows. First, the experimental setup and methods are presented. Then, the natural, uncontrolled flow is detailed, especially through the chaotic dynamics of the pressure or velocity barycenters. The open-loop forcing of the wake is then presented, showing that the bimodal state can be forced into a single mode state, either symmetric or asymmetric. Finally, closed-loop experiments are presented showing it is possible to force the chaotic system into a regular time-periodic system.

**EXPERIMENTAL SETUP**

![Ahmed Body Diagram](image)

**Ahmed Body**

The bluff-body is a 0.7 scale of the original Ahmed body [11] and is $L = 0.731$ m long, $H = 0.202$ m high and $W = 0.272$ m wide, as described in Ref. [1]. The rear part of the model is a square-back geometry with sharp edges.
leading to a massive separation of the flow and to the creation of a large 3D recirculation bubble over the rear part of the model.

Wind-tunnel

Experiments have been carried out in the PRISME laboratory closed wind tunnel (Orléans, France). The test section of this wind tunnel is 5 m long, with a square cross-section 2 m wide, resulting in a blockage ratio lower than 3%. The model is mounted on a raised floor with a properly profiled leading edge and an adjustable trailing edge to avoid undesired flow separations. The ground clearance is set to \( C/H = 0.248 \). In the following, the free-stream velocity is \( U_\infty = 30 \text{ m.s}^{-1} \), which corresponds to a Reynolds number based on the height of the model \( Re_H = U_\infty H/\nu_{\text{air}} = 3.9 \times 10^5 \) where \( \nu_{\text{air}} \) is the air kinematic viscosity at ambient temperature. The origin of the \((x, y, z)\) coordinate axis (Fig. 1) is located on the rear of the model \((x = 0)\), in the vertical symmetry plane \((y = 0)\) and on the raised floor \((z = 0)\). Nondimensionalization is applied to spatial variables such as \( x^* = x/H, \ y^* = y/H, \ z^* = z/H \), while the convective time and the Strouhal number are respectively defined as \( t^* = t U_\infty / H \) and \( St_H = f H/U_\infty \).

Sensors

Wall pressure measurements

The wall-pressure distribution over the rear part of the model is studied using a set of 95 pressure probes defining a measurement area denoted \( S_p \) and covering 70% of the entire rear surface \( S_r \), as shown on Fig. 1. Each vinyl is 2 cm away from each of its neighbors and it is connected to a 32-channels microDAQ pressure scanner (Chell®), ensuring an accuracy of ±17 Pa and located inside the bluff-body. The maximum number of sampling points is \( 3 \times 10^4 \) for each time-series because of hardware limitations. As a consequence, the sampling frequency for the pressure acquisition \( f_P \) depends on the acquisition time \( T_P \): \( f_P = 3 \times 10^4 / T_P \), but can not exceed 500 Hz. From these measurements of the pressure \( p \), we compute the pressure coefficient:

\[
C_p(t) = \frac{p(t) - p_\infty}{\frac{1}{2} \rho_{\text{air}} U_\infty^2},
\]

where \( p_\infty \) is the free-stream static pressure, measured by a Prandtl tube located above the leading edge of the raised floor, and \( \rho_{\text{air}} \) is the air density. A typical instantaneous pressure field over the rear part of the model is shown on Fig. 2(a) after interpolation. One can see that the spatial organization of the instantaneous pressure field can be complex. It is necessary to define some global parameters that could characterize the state of the instantaneous wall-pressure field which can be seen as the footprint of the wake.

As we are interested in controlling the large-scale dynamics of the wake, a global indicator of the state of the wake can be inferred from the instantaneous pressure fields. Denoting the instantaneous spacial average of a quantity \( X(t) \) over an area \( S \) as \( \langle X \rangle_S(t) \), we can define the instantaneous wall pressure barycenter \( G_p(t) \) at \( x^* = 0 \) as

\[
\overrightarrow{OG_p(t)} = \left( \frac{y_p^*(t)}{z_p^*(t)} \right) = \left( \frac{\langle y^* p(t) \rangle_S}{\langle p(t) \rangle_S} \right) = \left( \frac{\langle z^* p(t) \rangle_S}{\langle p(t) \rangle_S} \right),
\]

where \( p(t) = p(y^*, z^*, t) \) is the local pressure measured at \((0, y^*, z^*)\) and at time \( t \). Thus the instantaneous barycenter of the depression can be tracked at each time step as illustrated on Fig. 2(a) (white circle). For convenience we call the instantaneous spatially-averaged pressure coefficient as the base coefficient \( C_b(t) \)

\[
C_b(t) = \langle C_p(t) \rangle_{S_p},
\]
Real-Time Velocity fields measurements

The velocity fields are obtained using a standard Particle Image Velocimetry (PIV) setup. A double-cavity pulsed YaG laser is used to generate the laser sheet which enlightens the seeding oil droplets which are injected in the wind-tunnel. A double-frame 4Mp TSI PowerView Plus camera is used to capture the instantaneous pairs of snapshots which are then streamed to a computer. The maximum acquisition frequency of the PIV setup is $f_{PIV} = 7 \text{ Hz}$. The investigated PIV planes are the horizontal planes at $z^* = 1.00$ and $z^* = 1.25$ as shown in Fig. 1.

The two-dimensional two-components (2D-2C) velocity fields are computed in real-time at the maximum frequency of the PIV setup $f_{PIV} = 7 \text{ Hz}$, using an optical flow algorithm implemented on a Nvidia Graphics Processor Unit (GPU) card GeForce GTX580. As long as the velocity fields and related quantities are computed and used in a closed-loop control, the PIV setup can be used at its maximum frequency. It would work at higher frequencies if the laser and camera were faster. When the data are stored instead of being used in a closed-loop, $f_{PIV}$ drops at 4 Hz because of the writing time on the hard-drive.

One can find more details about the principle of optical flow computations and a rigorous demonstration of its offline accuracy in Refs. [46–48]. Gautier and Aider [49–52] achieved the implementation of the algorithm on a GPU to enable high-frequency real-time computations of PIV fields in closed-loop flow control experiments of a backward-facing step flow in an hydrodynamic channel. One of the interest of this algorithm is its scalability which makes it very efficient on the GPU architecture. Another interest is its ability to handle regions with large velocity gradients and also to lead to a dense vector field (one vector per pixel).

In the present study, the interrogation window size is $16 \times 16$ pixels and the calculation is based on three iterations for each of the three pyramid reduction levels. An example of a 2D-2C instantaneous velocity field is shown in Fig. 2(b). One can see the complexity of the turbulent wake, with strongly fluctuating large-scale and small-scale vortices. Quantities derived from these fields, such as the instantaneous recirculation area $A_{rec}$, the swirling strength or the $Q$-criterion, can then be computed in real-time.

To characterize the state of the wake based on the velocity measurements, we define the instantaneous recirculation area $A_{rec}(t)$ as the region where the streamwise velocity is negative. A typical example is shown on Fig. 2(b) where the frontier of the instantaneous recirculation area - the separatrix - is visualized by a white line.

In the same manner as for the wall pressure fields, we define the instantaneous recirculation intensity barycenter $G_{rec}(t)$ from the instantaneous velocity fields measured in the $z^* = 1$ horizontal plane

$$
\overrightarrow{OG_{rec}(t)} = \left( \frac{x^*_{rec}(t) A_{rec}(t)}{u_{rec}(t) A_{rec}(t)}, \frac{y^*_{rec}(t) A_{rec}(t)}{v_{rec}(t) A_{rec}(t)} \right),
$$

(4)

where $u_{rec}(t) = u_{rec}(x^*, y^*, 1, t)$ is the local streamwise component of the recirculation velocity at time $t$. $G_{rec}(t)$ can be computed in real-time from the PIV fields and then be used as an input for the control law. A typical example is shown on Fig. 2(b) where the recirculation barycenter is visualized by a white diamond.
The micro-jets

To realize the present control, the rear part of the Ahmed body used in Ref. [1] has been re-designed to have several independent groups of actuators. The wake of the bluff-body is indeed forced using micro-jets which are distributed along the rear body width, 10 mm under the upper trailing edge ($z^* = 1.25$). The jets are either continuous or pulsed at the jet frequency $f_j$ using three solenoid valves (Matrix®) integrated inside the model just upstream the blowing jets. Each solenoid valve controls a group of jets, which corresponds to the three regions of actuation defined in Fig. 1. The pressurized air is supplied through pneumatic tubes passing inside a cylindrical pipe of diameter 32 mm at the center of the bottom face (see the gray part in Fig. 1). Even if this pipe does not break the reflexive symmetry of the body, it has certainly an influence on the near wake, which will be discussed.

The outlets of the micro-jets have a rectangular cross-section $l_j \times h_j = 5.5 \times 0.5 \text{ mm}^2$. The distance between each jet is $l_j = 12.5 \text{ mm}$. The jets angle $\theta$ relative to the horizontal axis can be varied but in the following it will be fixed to $\theta = 0^\circ$ (blowing parallel to the freestream velocity). The jet velocity $u_j$ can reach up to 75 m.s$^{-1}$, i.e. a velocity ratio $u_{jet}^* = u_j/U_\infty = 2.5$. Based on the Strouhal number, the normalized jet frequency is defined as

$$f_j^* = f_j H U_\infty.$$  \hspace{1cm} (5)

The momentum coefficient is the ratio between the momentum injected by the actuator and the one due to the bluff body

$$c_\mu = \frac{2l_j h_j u_j^2}{S_r U_\infty^2}. \hspace{1cm} (6)$$

It is evaluated at $c_\mu = 0.053\%$ (continuous) and $c_\mu = 0.013\%$ (pulsed) for each jet when $U_\infty = 30 \text{ m.s}^{-1}$.

The three regions of actuations are called “N”, “0” and “P” (see Fig. 1). “N” contains the eight jets located in the Negative part of the $y$-axis, whereas “P” contains the eight jets located in the Positive part and “0” contains the four central jets (for a total of 20 jets). Due to the temporal constraints imposed for experiments carried in wind tunnel, all recorded control runs lasted only $T^* = 8.9 \times 10^3$. Regarding the aerodynamics, only the changes in the pressure coefficient will be considered to analyze influences of the control on the wake to avoid the structural response visible in the aerodynamics measurements.

NATURAL FLOW

The unforced flow is investigated in Ref. [1]. It is shown that the dynamics of the large-scale structures can be tracked either by the pressure center or by the recirculation barycenter since both are highly anti-correlated: when the pressure barycenter is on one side, then the recirculation barycenter is on the other side.

Bimodality

An important feature of the wake dynamics is its bimodal behavior which is well illustrated in Figs. 3 by the evolution of the pressure barycenter $G_p(y^*_p,z^*_p)$ on the rear surface. The barycenter switches from one side to the other ($y^*_p$ switches from negative to positive values) along the spanwise direction while it fluctuates around a mean position along the vertical axis. The most probable location (red square plots) near which $G_p$ evolves during each stay alternates between two opposite spanwise positions, keeping the same vertical position [Figs. 3(a, c and e)].

The 2D probability density function (PDF) of the pressure barycenter, noted $P_{G_p}$, is displayed in Fig. 4. The barycenter moves from one side to the other, between two more probable locations, one located in the negative part of the $y^*$-axis and the other one in the positive part. This is the kind of sidewise bimodal configurations that is indeed expected for a square-back bluff body with such geometric parameters [24]. Therefore, albeit the central pipe across the under-body flow modifies clearly the near wake topology, particularly in the median longitudinal plan [53, 54], the bimodal behavior in the spanwise direction is observed like for wakes without the influence of such a pipe [19, 20].

To travel from left [Fig. 3(a)] to right [Fig. 3(c)], the pressure barycenter crosses a central region which can be considered as a third unstable fixed point [Fig. 3(b)]. Even if time spent in one of the sides randomly varies [20, 44, 54], the low-frequency dynamics of the pressure barycenter are weakly chaotic and the two more probable spanwise locations can indeed be considered as the focii of a strange attractor [1]. Since the complex dynamics of this...
FIG. 3. Successive trajectories of $G_p(y_p^*, z_p^*)$ during periods of (a) $\Delta t^* = 367$, (b) $\Delta t^* = 105$, (c) $\Delta t^* = 126$, (d) $\Delta t^* = 145$, (e) $\Delta t^* = 194$, (f) $\Delta t^* = 111$, (g) $\Delta t^* = 414$, and (h) $\Delta t^* = 96$. The solid blue line shows the barycenter trajectory and the grey dashed line displays its previous one. The green circle and the red square are respectively the last position and the most probable location in the corresponding sub-figure of $G_p$.

FIG. 4. Normalized 2D PDF of $G_p(y_p^*, z_p^*)$ for a 2-min run, together with the conditional-mean positions of $G_p$ (red squares) for each identified mode.

3D wake can be reduced to a 2D high-dimensional chaotic system, one should be able to force the system and control its dynamics using the same kind of state parameters.

**Spectral analysis**

For different Reynolds numbers, the power spectra density (PSD) of the sidewise position $y_p^*$ [Fig. 5(a)] and the vertical position $z_p^*$, noted $S_{y_p^* y_p^*}$ and $S_{z_p^* z_p^*}$ respectively, contain high amplitude at a very-low-frequency range ($St_H <$
FIG. 5. PSD of the fluctuations of (a) $y^*$ and (b) $z^*$ for five Reynolds number: $2.6 \times 10^5$ (circle), $3.2 \times 10^5$ (diamond), $3.9 \times 10^5$ (square), $4.5 \times 10^5$ (star) and $5.1 \times 10^5$ (cross). Curves are shifted for clarity. The frequency resolution is $\Delta St_H = 2 \times 10^{-3}$. The observation of high energy level in this part of the spectrum is common in measurements close to the rear body [19, 20]. Considering the average time spent in a RSB mode ($\sim 7 \times 10^2$), it may be associated to the lateral bimodal behavior. A decaying but still high level of energy is then observed in $S_{y^*y^*}$ for $St_H \in [0.02, 0.08]$. Refs. [19, 55, 56] linked 0.08 to the bubble pumping phenomenon. Then a plateau appears until $St_H \sim 0.14$. It contains the value 0.13 previously found with hot-wire measurements in the lateral shear layers, indicator of vortex shedding [19, 53, 57]. Recent frequency measurements in a simulated turbulent wake behind a circular disk confirm the presence of two close low frequencies: 0.02 and 0.03 [58]. The lowest one is associated to the asymmetric changes in the wake orientation at such low frequency, whereas the highest one is related to the bubble pumping. Similar results have been obtained for a turbulent axisymmetric body wake [59]. Through a POD analysis of the velocity fluctuations, they find very-low-frequency peaks, $St_H \sim 10^{-4}$ and $St_H = 10^{-3}$, in the dynamics of the modes associated respectively to the symmetry breaking process and to the bubble pumping. The PSD of the vertical position $z^*$, $S_{z^*z^*}$, alone reveals a peak at $St_H = 0.19$, independent from the Reynolds number, as displayed in Fig. 5(b). This Strouhal number is characteristic of the signature of vortex shedding from the top-bottom shear layers interaction [19, 42, 53, 60], meaning that $z^*$ is influenced by these far wake dynamics. This natural feedback inside the wake has been reported in Ref. [38] for a turbulent axisymmetric jet, where the spectral signature of vortex pairings downstream of a nozzle appears in the velocity fluctuations measured at the nozzle exit. For the turbulent axisymmetric body wake, the characteristic vortex shedding frequency also appears in the spectrum of the POD mode describing the switching behavior [59]. Physically, the feedback is due to upstream propagating pressure fluctuations.

Thus, the dynamics of the depression seem to be rather dominated by the spanwise bimodal behavior of the recirculation and the top-bottom shear layers interaction. The central pipe may partly intensify this interaction, as highlighted in Ref. [53], but most of the added structures in the underbody flow have higher frequency signatures like the Karman vortex street at $St_H = 1$ and $St_H = 1.6$ [57]. It shall be recalled that the switching process is linked to the turbulent perturbations growth [44].

**Transient symmetric state**

During most of the flipping processes, the pressure center goes around a central position $y^*_p \sim 0$, as shown in Figs. 3(b, d, f and h). In average, this step lasts for well shorter time than the RSB modes. It shall be noteworthy that, after being in this transient state, the wake may also go back in the previous RSB mode [Figs. 3(e-g)].

As mentioned hereinbefore, three states can be considered for the system: two corresponding to the RSB modes, and a third one being a transient mode. For a more quantitative analysis, two thresholds for $y^*_p(t)$ has consequently been defined. Above a given positive threshold $y^*_{p,P}$, the wake is in the P mode, and lower than a given negative
threshold $y_{p,N}^*$, it is in the N mode. Between these values, it is the transient state. The thresholds are computed as

$$y_{p,N}^* = w_{th} \times y_{p,max}^* \left( P_{y_p^* \mid y_p^* < y_{p,N}^*} \right)$$

and

$$y_{p,P}^* = w_{th} \times y_{p,max}^* \left( P_{y_p^* \mid y_p^* > y_{p,P}^*} \right),$$

where $P_X$ is the PDF of the variable $X$, $\bar{X}$ is the temporal mean of $X$, and $w_{th}$ is a positive weight to avoid too large thresholds ($w_{th} = 0.25$ is chosen). From now on, the state of the wake is defined as negative (N), positive (P) or transient symmetric (TS), where respectively $y_{p}^* < y_{p,N}^*$, $y_{p}^* > y_{p,P}^*$ and $y_{p,N}^* < y_{p}^* < y_{p,P}^*$.

Even if the switching process seems to randomly occur, some characteristic time scales can be statistically estimated. Hence, the time the wake spent on each mode is evaluated from all runs, giving $T_{RSB} \sim 7 \times 10^2$ for each asymmetric mode, whereas the switch itself lasts for $T_{switch} \sim 4 \times 10^1$. As expected, these results have a high dispersion caused by the turbulent noise, their respective standard deviations being $\sigma_{T_{RSB}} = 10^2$ and $\sigma_{T_{switch}} = 10^1$. A mean switching frequency is also computed by counting the switching occurrences during runs: $f_{switch} \sim 3 \times 10^{-3}$. In probability terms, the RSB modes exist 83.4% ($\pm$5.3) of the time (41.1% for the N state, and 42.3% for the P one), whereas the wake is in the unstable TS mode during the remaining 16.6% ($\pm$1.2). This is the reason why the present wake can also be considered thereafter exhibiting a trimodal dynamics.

The 2D PDF of the pressure barycenter is computed and shown in Fig. 4. It gives a global overview of its most frequent positions on the rear. As revealed by the time series, three main areas corresponding to the wakes states are highlighted, confirming the trimodal behavior. We calculate then the positions of the identified centers of these areas, denoted $A_N(y_{A_N}^*, z_{A_N}^*)$, $A_P(y_{A_P}^*, z_{A_P}^*)$ and $A_{TS}(y_{A_{TS}}^*, z_{A_{TS}}^*)$ for the N, P and TS states respectively, such as

$$\overrightarrow{OA_N} = \left( y_{p\mid y_p^* < y_{p,N}^*}^*, z_{p\mid y_p^* < y_{p,N}^*}^* \right)$$

and

$$\overrightarrow{OA_P} = \left( y_{p\mid y_p^* > y_{p,P}^*}^*, z_{p\mid y_p^* > y_{p,P}^*}^* \right)$$

and

$$\overrightarrow{OA_{TS}} = \left( y_{p\mid y_p^* < y_{p,N}^* < y_{p,P}^*}^*, z_{p\mid y_p^* < y_{p,N}^* < y_{p,P}^*}^* \right).$$

For each area a quasi-attractive center can be identified, such as $y_{C_1}^* \sim -y_{C_2}^*$ and $z_{C_1}^* \sim z_{C_1}^*$. During the switch between these two positions, the pressure barycenter follows preferentially a trajectory along a well-defined path.

FIG. 6. Conditional averages of the pressure coefficient at the rear body and the velocity field at $z^* = 1$ for the modes (a-d) N, (b-e) TS and (c-f) P. The average position of the pressure center $G_p$ (white circle) and of the recirculation barycenter $G_{rec}$ (white diamond) are displayed. $y^* = 0$ is also displayed (dash-dotted grey lines). Some streamlines are plotted over the streamwise component of the velocity to localize the coherent large-scale structures.
It is then interesting to visualize the corresponding time-averaged pressure and velocity fields using conditional averaging. Figs. 6(a-c) display the conditional time-averaged pressure fields of the three modes, while Figs. 6(d-f) correspond to their conditional time-averaged velocity fields. The RSB modes presented here have identical pressure and velocity topologies as the ones without considering the transient state [19]: the depression is localized on the same side as the vortex being the closest to the rear surface. In the TS state, the wake appears to be a perfect mean of the two RSB modes. These results are in good agreement with the transient state revealed in the cluster-based modelling in Ref. [26]. Once again, the phase opposition between the pressure and velocity barycenters clearly appears.

The effects of the modified ground clearance on the wake reversal behind a square-back bluff body have been studied in Ref. [54]. It was reported in particular that the presence of a circular cylinder, very similar to the central pipe in our experiments but closer to the rear part, changes the vertical position of the modes but does not cancel the lateral bimodality.

OPEN-LOOP FORCING OF THE WAKE

Three different open-loop (OL) control strategies are investigated to evaluate the forcing on the state of the wake for 1-min runs. The three open-loop forcings are defined as "OL-P" when only the "P" solenoids group is activated, "OL 0" when only the "0" solenoids group is activated and "OL-P0" when both the "P" and "0" solenoids groups are activated (Fig. 1). Due to the reflection symmetry the results obtained with the "P" group are expected to be the same as for the "N" configuration. First we used continuous jets, which means only the forcing region is changed. Then pulsed jets are investigated, especially at the normalized frequency $f_j^* = 0.2$.

Continuous blowing in different regions

Influence on the trajectory

FIG. 7. Excerpts of $y_p^*$ and $y_p^*$ time series for the continuous forcing (a-d) OL-P, (b-e) OL-0, and (c-f) OL-P0 (solid red), all compared to a reference time series (dashed blue). Data are smoothed over $t^* = 30$ for clarity.

The influence of the location of the continuous blowing on the wake dynamics is studied by analysing directly the times series of the pressure center obtained for the three open-loop cases. On one hand, as displayed in Fig. 7(a) and Fig. 7(c), blowing from one side drives the depression to remain localized in this side. On the other hand, Fig. 7(b) reveals that the symmetric forcing seems not to affect the spanwise bimodal behavior. For all the forcing cases, the vertical position of the depression becomes closer to the upper trailing edge. The averaged $z_p^*$ indeed rises 2.1% for OL-P [Fig. 7(d)], 1.2% for OL-0 [Fig. 7(e)] and 2.8% for OL-P0 [Fig. 7(f)].

The previous observations are also confirmed by the 2D PDFs. Figure 8(a) shows that forcing with continuous jets the upper shear layer on one side of the model (OL-P) strongly stabilizes the wake oscillations. The pressure
barycenter is now kept on the side of the blowing ($y_p^* > 0$) and oscillates around a single attractor. If the forcing is extended to the central region (OL-P0), the pressure barycenter is still confined on one side of the model but explores a larger region around the attractor [see Fig. 8(c)]. Finally, forcing the wake only in the central region (OL-0) leads to the same kind of PDF as the natural case [see Fig. 8(b)]. Nevertheless, the probability to remain in the TS state is reduced by half compared to the natural state. The TS mode is thus clearly minimized by the central symmetric continuous blowing.

Chaotic behavior

Concerning the dynamics of the last case, random switches occur less often but the weak chaotic behavior in the low-frequency range remains ($St_H < 2 \times 10^{-3}$). First, Fig. 9(a) displays the second order-structure functions of $y_p^*$ and its derivative, denoted $S_2$ and $S_{2,d}$ respectively, measuring the self-affinity of the signal [61]. $S_2$ is defined as

$$S_2(n) = \langle |y_p^*(i+n) - y_p^*(i)|^2 \rangle_i,$$

where $n$ is the lag and $\langle \rangle_i$ stands for the average over $N - n$ points. For small $n$, both curves follow a scaling law $n^2$, specific to chaotic dynamics [62]. Secondly, Fig. 9(b) shows the computation of the embedding dimension $m$.
In brief, after building specific state vectors from $y_p^*$ time series for phase space reconstruction, based on the Takens’s time-delay embedding method [64], the evolution of the mean distance between them for different tested embedding dimensions $m$ is evaluated through a function $E_1$. Then, a minimum embedding dimension $m$ exists if $\forall k > m, E_1(k) = E_1(k + 1)$, leading to $m = 15$, of the same order as the reference flow [1]. The same computation is implemented directly with $y_p^*$ and the function is denoted $E_2$. It enables to verify if $y_p^*$ is a deterministic (chaotic) signal if $\exists k | E_2(k) \neq 1$, which is well the case when OL-0 is activated. More informations about the computations can be found in Ref. [1]. Unfortunately, according to Ref. [65], the recorded time for these data (1 min) is too short regarding the number of switching events to properly characterize their chaotic dynamics through the calculations of the correlation dimension and the largest Lyapunov exponent.

Trajectories in phase space

Instead of looking at the trajectories of the pressure barycenter over the rear of the model in the physical space, it is interesting to look at the trajectory in the phase space. Indeed the state of the wake can be characterized not only by the right-left oscillations (bimodality), through the state parameter $y_p^*$, but also by the normalized fluctuations of the instantaneous base pressure coefficient $\Delta C_b(t)$

$$\Delta C_b(t) = \frac{C_b(t) - C_{b,ref}}{C_{b,ref}}, \quad (10)$$

where $C_{b,ref}$ is the time-averaged value of $C_b$ for the reference flow. In Fig. 10(a) and (c), forcing the wake into one of the RSB modes increases roughly by 15% the depression. These results are well coherent with the sensibility analysis done in Ref. [40]. On the contrary, acting on the symmetric mode of the wake leads to similar value for $C_b$ than the reference case [see Fig. 10(b)].

Pulsed blowing in different regions

Finally, the pulsed jets are used with the normalized frequency $f_j^* = 0.2$ for the three previous configurations, since this corresponds to the vortex shedding frequency found in the up-down oscillations of the depression position for the reference case. At this actuating frequency, the depression behaves almost as previously for the OL-P and OL-P0 cases, but with more regular oscillations in the $z$-direction and a reduction of the spanwise fluctuations, amplifying the decrease in the base pressure. On the contrary, the OL-0 case is strongly modified: the average spanwise position of the depression is now centred [Fig. 11(b)], the amplitude of the vertical fluctuations doubled while the mean vertical position increases much more than for the continuous blowing [Fig. 11(e)]. For these reasons, the symmetric pulsed forcing will be more detailed in the following.

Actually, the control with pulsed jets at $f_j^* = 0.2$ leads to a symmetric PDF with a single central attractor for the trajectory of the pressure barycenter, as illustrated by Fig. 12(a), exhibiting only one central peak. The probability
of being in the TS mode is multiplied by two. This has to be compared to the two attractors trajectories obtained with a constant blowing in the same region [Fig. 8(b)].

Figure 13 stresses the peculiarity of the actuation frequency at $f_j^* = 0.2$, since no other frequency enables such a symmetrization of the wake. This dynamic response of the wake to a specific perturbation reminds the one obtained for a forced axisymmetric jet resulting to UPOs [38]. As the pressure center of the turbulent wake acts like a strange attractor in the low-frequency range, its space phase shall embed UPOs according to Ref. [66]. Therefore, the OL-0 forcing at $f_j^* = 0.2$ may have enhanced large UPOs [29], such as the TS mode dominates. Further investigations to detect and map the UPOs from $y_p^*$ [67] are needed to verify this dynamical interpretation which remains an open question.

The sensitivity analysis made in Ref. [40] shows that targeting the center of the top or of the bottom shear layer with a control cylinder stabilizes the wake in a symmetric mode. In our case, this wake topology is obtained by forcing the top shear layer at the vortex shedding frequency. As highlighted recently, using this frequency for the actuation frequency leads to a strong increase of the pressure drag (over +25%) by enhancing the entrainment rate of the wake [42, 43]. Thereby, forcing the wake in its center at one of its main resonant frequency ($St_H = 0.2$) induces two opposite effects on the base pressure, resulting finally to its very slight increase (+0.75% in average), as shown by Fig. ??(b). The impact of a low pulse frequency previously observed may be attenuated in our case as the exit slit of the jets is discontinuous and only localized in the middle of the upper rear part, unlike the forcing in Refs. [42, 43]. It
FIG. 13. Normalized PDF for the reference flow (Ref), the continuous blow \( f_j^* = 0 \) and different actuating frequencies \( f_j^* > 0 \) for OL-0. Curves are shifted according to the respective cases. Forcing the wake with the \( f_j^* = 0.2 \) pulsing frequency clearly leads to a very strong modification of the dynamics of the wake. This frequency may correspond to a hidden UPO of the chaotic system.

It is noteworthy that base pressure recovery was also obtained at similar and lower normalized jet frequencies for other bluff body wakes [68, 69], axisymmetric wakes [70], and 2D wakes [71].

CLOSED-LOOP CONTROL OF THE WAKE DYNAMICS

Control law

FIG. 14. (a) Sketch of the opposition closed-loop control law, based on the detection of the spanwise position of the recirculation barycenter (white diamond), computed from the instantaneous velocity fields. (b) Illustration of the opposition control with the blowing regions chosen as a function of the position of the recirculation barycenter computed in real-time. See Supplemental Material at [URL will be inserted by publisher] for a short movie illustrating the opposition closed-loop control law.
The principle of the closed-loop experiments is based on the computation in real-time of the recirculation barycenter in the horizontal XY-plane at \( z^* = 1 \), defining \( y_{\text{rec}}^* \) as the control parameter. This tracking is then limited by the low acquisition frequency of the PIV setup (\( f_{\text{PIV}}^* = 2.8 \times 10^{-2} \)). It means that we will act on the low-frequency large-scale dynamics of the wake. We then choose an opposition algorithm for the closed-loop experiments. It consists in blowing in the region where the recirculation barycenter is detected, as summarized in Fig. 14. This is consistent with the three modes defining the state of the wake: the two RSB modes and the TS mode. To these three modes correspond three blowing regions that are activated when the recirculation barycenter is detected in the related region. For each solenoids group we define a range for the spanwise location of the recirculation intensity barycenter \( y_{\text{rec}}^* \), for which the group is activated. The instantaneous velocity fields and \( y_{\text{rec}}^* \) are computed in real-time. The control variable is updated with a period \( T_{\text{act}}^* \), the minimum being \( 1/f_{\text{PIV}}^* \).

To make things more simple, we focus on the detection - blowing algorithm with all other actuation parameters fixed. The parameters for a closed-loop experiment are chosen based on the best or more interesting parameters found in the open-loop experiments. The actuation frequency \( f_j^* \) and the mean jet velocity \( \overline{u_{\text{jet}}} \) are then chosen and set to fixed values before turning the control on.

In the following, two types of actuation are used in the closed-loop experiments: a continuous blowing, denoted “CL-CONT” (“CL” stands for closed-loop.), or a pulsed blowing with different actuation frequencies \( f_j^* \). We will focus on two actuation frequencies: \( f_j^* = 0.2 \) and \( f_j^* = 0.8 \), respectively denoted “CL-LF” (for low frequency) and “CL-HF” (for high frequency).

**Stabilization of the wake**

*Closed-loop with continuous blowing*

![Graphs](image)

FIG. 15. Time series of the \( y^* \)-position of the recirculation barycenter and the pressure center for the reference case (a-c), compared to the closed-loop controlled case with continuous blowing jets (b-d). Thresholds are displayed in dash-dotted red.

The objective of these experiments is to control the wake dynamics which are characterized by the fluctuations of the spanwise location of the recirculation barycenter \( y_{\text{rec}}^*(t) \). Typical time-series of the pressure and recirculation barycenters fluctuations for the reference case are displayed respectively in Fig. 15(a) and Fig. 15(c). When the closed-loop actuation CL-CONT is triggered, the dynamics of the barycenters [Figs. 15(b) and (d)] are completely changed. Instead of random large and small fluctuations, the oscillations of the barycenters become very regular and periodic. The right-left switching is now imposed by the characteristic time scale of the closed-loop control law \( T_{\text{act}}^* \).

This observation holds also for the pulsed actuations.

The trajectories of the pressure barycenter for CL-CONT during four successive control cycles are illustrated in Figs. 16(a-d). It reveals that the flipping process occurs at the defined period \( T_{\text{act}}^* \), only enforcing the presence of the
FIG. 16. Successive displacements of the pressure center $G_p$ for CL-CONT for (a) $[t^*_0 : t^*_0 + T^*_act]$, (b) $[t^*_0 + T^*_act : t^*_0 + 2T^*_act]$, (c) $[t^*_0 + 2T^*_act : t^*_0 + 3T^*_act]$, and (d) $[t^*_0 + 3T^*_act : t^*_0 + 4T^*_act]$. The red circle is the last position in the corresponding sub-figure, the solid blue line shows the barycenter trajectory whereas the grey dashed line displays its previous one. See Supplemental Material at [URL will be inserted by publisher] for a short movie detailing the top-bottom oscillations of the pressure center for CL-CONT at $T^*_act = 149$.

two asymmetric modes, whereas the transient state becomes very short. The bimodal behavior is thus now completely driven by the opposition control law.

Closed-loop with pulsed blowing

Introducing again pulsed jets instead of continuous blowing modifies the dynamics, depending on the jet frequency. On one hand, the dynamics obtained for CL-HF [Figs. 17(c) and (f)] are very close to the ones for CL-CONT [Figs. 17(a) and (d)]. On the other hand, Fig. 17(b) shows for CL-LF slightly lower spanwise fluctuations, of the same order as the reference, whereas Fig. 17(e) reveals a higher mean vertical position of the depression. This is probably due to the central jets which tend to symmetrize the wake and to attract the depression, as seen for the OL-0 case at this jet frequency.

It is then interesting to analyze how the pressure at the rear end of the model and the wake size are modified together, depending on the type of actuation used in the closed-loop experiments. Another state parameter for the wake is thus defined: the variation of the recirculation area $A_{rec}$ compared to the mean value of the reference case.
\[ \Delta A_{rec}(t) = \frac{A_{rec}(t) - A_{rec,ref}}{A_{rec,ref}}. \]

Instead of following the physical trajectories of the pressure barycenter over the rear part of the model, it is now possible to follow the trajectory of the wake in the state parameter \((\Delta C_b, \Delta A_{rec})\). Unfortunately, the acquisition frequency of the PIV fields is much smaller than the ones of the pressure fields, so that it is not possible to correlate all mean pressure coefficients to a corresponding recirculation area. Nevertheless, it is possible to correlate all PIV fields to a corresponding mean pressure coefficient and then plot the two state parameters.

As shown in Figs. 18, all closed-loop experiments lead to clear reduction of the recirculation area, i.e. of the size of the wake. The type of actuation has also a strong influence: pulsed jets are more efficient (−40% and −32% in average when \(f_j^* = 0.2\) and \(f_j^* = 0.8\) respectively) than continuous blowing (−18%). On the contrary, the depression over the rear end increases in all cases, with the most important increase for \(f_j^* = 0.2\) (+19%). The portion of the phase space explored by the wake is much larger for the CL-LF case [Fig. 18(b)] than for the two other configurations but there is no clear relation between the changes in \(\Delta A_{rec}\) and the ones in \(\Delta C_b\). It also confirms the influence of the actuation frequency on the efficiency of the closed-loop control. The opposition algorithm has a different impact on the dynamics of the wake depending on the actuation frequency. It can be seen has a hint of the specific response of a chaotic system. Indeed, Ref. [29] demonstrated that controlling a chaotic system may be more efficient if targeting the hidden frequency of the chaotic system, the UPO. If one can find the UPO of the system, then it should react to a forcing with small perturbations and the frequency of the UPO. The specific response of our system may be explained this way. Unfortunately our time series were not long enough to evaluate the UPO of the system.

**Modification of the time-averaged wake**

Regarding the RSB modes in particular, the more the CL controls reduce the recirculation area, the more asymmetric the wake is, as highlighted in Fig. 19(a). Even if the TS state existence is reduced, the separatrices computed for this state [Fig. 19(b)] have the shape similar to the mean profiles [Fig. 19(c)]. This is due to the enhanced equiprobability of the BSR modes. It is noteworthy that the recirculation barycenter comes also closer to the bluff-body for all types of actuations.

The conditional averages of the pressure and the velocity are computed and displayed in Figs. 20 for CL-LF. The modes organization is affected by the forcing: the BSR modes appear more asymmetric and the TS state concentrates the higher pressures in the center. The mean rear pressure of the TS state is higher than the BSR one (−0.227 and −0.234).
FIG. 19. Contour of the recirculation area for the reference (blue solid), CL-CONT (black dotted), CL-LF (red dash-dotted) and CL-HF (green dashed) for (a) the N state, (b) the TS state and (c) the mean velocity field.

FIG. 20. Conditional averages of the pressure coefficient at the rear body and the velocity field at $y^* = 1$ for the modes (a-d) N, (b-e) TS and (c-f) P in the CL-LF case. The average position of the pressure center $G_p$ (white circle) and of the recirculation barycenter $G_{rec}$ (white diamond) are displayed. $y^* = 0$ is also displayed (dash-dotted grey lines). Some streamlines are plotted over the streamwise component of the velocity to localize the coherent large-scale structures.

Creation of a multistable state

Another way to characterize the dynamics of the wake is to look again at the full trajectories of the pressure barycenter over the rear part of the model. For the three actuation types, the corresponding 2D PDF are shown in Figs. 21, where the previously highlighted strange attractors are displayed in blue.

When the recirculation barycenter is tracked and forced by the closed-loop algorithm with a continuous blowing,
the PDF is also completely changed and exhibits two strong foci, as shown in Fig. 21(a). The transient mode is nearly suppressed and the wake switches quickly from one attractor to the other. This modification is also observed with CL-HF [Fig. 21(c)]. In both cases, the dynamics are marked by a decrease of 50% in the TS mode existence. Interestingly, the PDF is also strongly modified by the lower pulse frequency (CL-LF). In this case, one can see in Fig. 21(b) a butterfly shape with four foci instead of two for all other cases. These results suggest the occurrence of multi-modal state with a right-left oscillation and a top-bottom oscillation, which are undoubtedly enlarged due to the actuating frequency corresponding to the resonant frequency found in the spectral analysis of the $z^*_p(t)$ fluctuations [Fig. 5(b)]. This lower frequency may be the one associated to the hidden UPOs of the chaotic system. It may explain the specific effect observed with this type of actuation, in a way similar to the work on a turbulent jet [39]. This harmonic resonance is also highlighted in Ref. [42], through an antisymmetric open-loop forcing, close to the one resulting from our closed-loop control law.

Figure 22 displays successive displacements of the pressure center, which oscillates at the constant period $T_j^* = 1/f_j^* = 5$ with a doubled amplitude compared to the natural dynamics.

The multi-modality of the controlled wake dynamics can also be analyzed using the proper orthogonal decomposition (POD) of the wall-pressure spatial distribution. The POD modes enables the detection of coherent structures in turbulent flows [72, 73] by quantifying the spatiotemporal organization of a characteristic field of the flow, like the
FIG. 23. First, second and third POD modes of the natural flow (a,e,i), CL-CONT (b,f,j), CL-LF (c,g,k) and CL-HF (d,h,l).
The respective energy is 42%, 17% and 8.8% for the natural flow; 58%, 11% and 5.6% for the CL-CONT; 34%, 19% and 12% for the CL-LF; 60%, 9.4% and 4.3% for the CL-HF.

rear pressure coefficient fluctuations \( \tilde{c}_p \) [1]

\[
\tilde{c}_p(t) \sim \sum_{i=1}^{k} a_i(t) \Phi_i, \quad (12)
\]

where \( k \) is the number of POD modes \( \Phi_i \) carrying most of the coherent structures energy and \( a_i \) are the corresponding temporal coefficients.

For the three controlled cases, the spatial organization of their first mode \( \Phi_1 \) remains strictly the same as the natural flow [see Figs. 23(a-d)]. The asymmetry in \( \Phi_1 \) is characteristic of the RSB modes. Figures 23(f) and 23(h) show that the second mode \( \Phi_2 \) spatial organization of CL-CONT and CL-HF are very similar, and are close to the reference flow [see Fig. 23(e)]. On the contrary, the spatial structure of the second POD mode of the CL-LF displayed in Fig. 23(g) is very different from the natural case, as we switch from a left-right to top-bottom symmetry. The same observation stands for the third POD mode \( \Phi_3 \), as shown in Figs. 23(i-k), except for CL-HF, whose \( \Phi_3 \) mode has also a spatial organization different from the reference case [see Fig. 23(l)] but keeps the right-left symmetry.

Comparing the PSD of the POD modes with the ones of \( y_p^* \) and \( z_p^* \) also explains how these modes are related to the dynamics of the wake. As expected, Fig. 24 shows that \( \Phi_1 \) is clearly linked to the \( y_p^* \) activity only, especially for the low frequencies. The \( St_H = 0.19 \)-peak exclusively present in \( S_{z_p^*z_p^*} \) appears in the spectra of the three following POD modes for each control case. Indeed, for CL-CONT and CL-HF, \( \Phi_2 \) shares some peaks with \( S_{y_p^*y_p^*} \) in the high-frequency range, but it contains particularly a similar low frequencies spectrum as \( S_{z_p^*z_p^*} \) and a low \( St_H = 0.19 \)-peak. For CL-LF, this peak is clearly stronger in the \( \Phi_2 \) and \( \Phi_4 \) spectra. \( S_{z_p^*z_p^*} \) and \( S_{z_p^*z_p^*} \) are quasi-identical, meaning \( \Phi_4 \) is rather only linked to \( z_p^* \) dynamics for this case.

The spatial organization and the dynamics of the POD mode enable thus to evaluate the impact of the closed-loop controls on the energy part associated to the large-scale fluctuations tracked by the barycenters. Among the four most energetic POD modes, \( \Phi_1 \) solely describes a left-right behavior. Energy associated to the RSB rises by 37.9% and 42.1% for CL-CONT and CL-HF respectively, whereas it decreases by 20.2% for CL-LF. The other POD modes are rather related to the TS state and slightly to the vortex shedding. The top-bottom variations of the wake fall by 30.2% and 37.9% for CL-CONT and CL-HF with 17.9% (−37.9%). So, these controls enforce vigorously the lateral motion of the wake. Concerning the CL-LF case, the top-bottom characterization is carried by \( \Phi_2 \) and \( \Phi_3 \), which contain
FIG. 24. Normalized PSD of the first (diamond), the second (star), the third (square) and the fourth (circle) POD coefficients, together with the PSD of $y_p^*$ (blue solid line) and $z_p^*$ (black dashed line) for the (a) reference flow, (b) CL-CONT, (c) CL-LF, and (d) CL-HF. Curves are shifted for clarity.

31.2% (+8.3%) of the total energy. This control tends thus to create a multistable state, making the top-bottom dynamics visible as the right-left one.

CONCLUSIONS

Using global state parameters of the wake as the instantaneous barycenters of the spatially-averaged mean pressure coefficient or of the instantaneous recirculation area, it was possible to capture the dynamics of a 3D turbulent wake. It was also shown in a previous study that the dynamics of the wake, based on the fluctuations of pressure barycenter, is weakly chaotic. Having reduced the dynamics of the 3D wake to simple state parameters, it was then possible to design a simple closed-loop control experiment based on the real-time computations of the location of the recirculation area barycenter and a simple opposition algorithm. To our knowledge, this is also the first implementation of a closed-loop flow control experiment in a wind-tunnel facility based on the velocity fields computations, using real-time PIV as a “visual sensor”.

The chaotic spanwise oscillations of the location of the recirculation (and pressure) barycenter is then forced into a well-controlled periodic oscillation at the time-scale imposed by the PIV setup and the algorithm. Moreover, a much larger reduction of the recirculation area is obtained when a pulsed actuation is used in the closed-loop control experiments, instead of a continuous blowing. When the frequency of the pulsed actuation used in the closed-loop experiments correspond to the natural shedding frequency the fluctuations in the phase space are strongly increased. Moreover, if the PDF of trajectories exhibits well-defined stable foci for most of the configurations, the low-frequency actuation case leads to a new four foci configuration, suggesting a multi-modal configuration with a right-left oscillation together with a top-bottom oscillation of the wake. These results show that once the turbulent wake dynamics has been reduced to a simple chaotic dynamics in physical space, it is indeed possible to design simple control laws to stabilize its dynamic around an unstable mode or to force its dynamics into simple harmonic oscillations. This work can find explanations in the framework of the theory of control of chaotic systems as proposed in Ref. [28]. From a general point of view, it shows that there are new opportunities to investigate turbulent flows in the framework of dynamic and chaotic systems. It also opens the way to different control strategy of turbulent flows, as long as their dynamics can be reduced to a few state parameters.

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E. Varon, Y. Eulalie, S. Edwige, P. Gilotte, and J.-L. Aider, “Chaotic dynamics of large-scale structures in a turbulent wake,” Phys. Fluids, vol. 64, no. 2, 034604 (2017).

C. H. K. Williamson, “Vortex dynamics in the cylinder wake,” Annu. rev. fluid mech. 28, 477–539 (1996).

I. P. Castro and A. G. Robins, “The flow around a surface-mounted cube in uniform and turbulent streams,” J. Fluid Mech. 79, 307–335 (1977).

S. C. C. Bailey, R. J. Martinuzzi, and G. A. Kopp, “The effects of wall proximity on vortex shedding from a square cylinder: Three-dimensional effects,” Phys. Fluids 14, 4160–4177 (2002).

K. Gumowski, J. Miedzik, S. Goujon-Durand, F. Jeniffer, and J.E. Wesfreid, “Transition to a time-dependent state of fluid flow in the wake of a sphere,” Phys. Rev. E 77, 055308 (2008).

G. Yun, D. Kim, and H. Choi, “Vortical structures behind a sphere at subcritical Reynolds numbers,” Phys. Fluids 18, 015102 (2006).

L. Klotz, S. Goujon-Durand, J. Rokicki, and J.E. Wesfreid, “Experimental investigation of flow behind a cube for moderate Reynolds numbers,” J. Fluid Mech. 750, 73–98 (2014).

M. Grandemange, M. Gohlke, and O. Cadot, “Statistical axisymmetry of the turbulent sphere wake,” Exp. Fluids 55, 1835 (2014).

W.-H. Huch, Aerodynamics of road vehicles: From fluid mechanics to vehicle engineering (Elsevier, 2013).

T. Morel, “The effect of base slant on the flow pattern and drag of three-dimensional bodies with blunt ends,” in Aerodynamic Drag Mechanisms of Bluff Bodies and Road Vehicles, edited by G. Sovran, T. Morel, and W. T. Mason (Springer US, Boston, MA, 1978) pp. 191–226.

S. R. Ahmed, G. Ramm, and G. Faltin, Some salient features of the time-averaged ground vehicle wake, Tech. Rep. (SAE Technical Paper 840300, 1984).

H. Lienhart, C. Stoots, and S. Becker, “Flow and turbulence structures in the wake of a simplified car model (Ahmed model),” in New Results in Numerical and Experimental Fluid Mechanics III: Contributions to the 12th STAB/DGLR Symposium Stuttgart, Germany 2000, edited by S. Wagner, U. Rist, H.-J. Heinemann, and R. Hilbig (Springer, Berlin, Heidelberg, 2002) pp. 323–330.

J. F. Beaudoin, O. Cadot, J.-L. Aider, K. Gosse, P. Paranthoen, B. Hamelin, M. Tissier, D. Allano, I. Mutabazi, M. Gonzales, et al., “Cavititation as a complementary tool for automotive aerodynamics,” Exp. Fluids 37, 763–768 (2004).

C. Hinterberger, M. García-Villalba, and W. Rodi, “Large eddy simulation of flow around the Ahmed body,” in The Aerodynamics of Heavy Vehicles: Trucks, Buses, and Trains, edited by R. McCallen, F. Browand, and J. Ross (Springer, Berlin, Heidelberg, 2004) pp. 77–87.

S. Krajnović and L. Davidson, “Flow around a simplified car, part 1: Large eddy simulation,” J. Fluid Eng. 127, 907-918 (2005).

J. Venning, D. Lo Jacono, D. Burton, M. C. Thompson, and J. Sheridan, “The nature of the vortical structures in the near wake of the Ahmed body,” in Proc. I.Mech.E. Part D: J. Automobile Engineering (2017).

E. G. Duell and A. R. George, “Experimental study of a ground vehicle body unsteady near wake,” in SAE Technical Paper (SAE International, 1999).

S. Krajnović and L. Davidson, “Numerical study of the flow around a bus-shaped body,” J. Fluid Eng. 125, 500-509 (2003).

R. Volpe, P. Devinant, and A. Kourta, “Experimental characterization of the unsteady natural wake of the full-scale square back Ahmed body: Flow bi-stability and spectral analysis,” Exp. Fluids 56, 99 (2015).

M. Grandemange, M. Gohlke, and O. Cadot, “Turbulent wake past a three-dimensional blunt body, part 1: Global modes and bi-stability,” J. Fluid Mech. 722, 51–84 (2013).

J. Östh, B. R. Noack, S. Krajnović, D. Barros, and J. Borée, “On the need for a nonlinear subscale turbulence term in pod models as exemplified for a high-reynolds-number flow over an ahmed body,” J. Fluid Mech. 747, 518–544 (2014).

M. Grandemange, O. Cadot, and M. Gohlke, “Reflectional symmetry breaking of the separated flow over three-dimensional bluff bodies,” Phys. Rev. E 86, 035302 (2012).

O. Evtufiyeva, A. S. Morgans, and L. Dalla Longa, “Simulation and feedback control of the Ahmed body flow exhibiting symmetry breaking behaviour,” J. Fluid Mech. 817, 10.1017/jfm.2017.118 (2017).

M. Grandemange, M. Gohlke, and O. Cadot, “Bi-stability in the turbulent wake past parallelepiped bodies with various aspect ratios and wall effects,” Phys. Fluids 25, 095103 (2013).

O. Cadot, A. Evrard, and L. Pastur, “Imperfect supercritical bifurcation in a three-dimensional turbulent wake,” Phys. Rev. E 91, 063005 (2015).

E. Kaiser, B. R. Noack, L. Cordier, A. Spohn, M. Segond, M. Abel, G. Daviller, J. sth, S. Krajnović, R. K. Niven, and et al., “Cluster-based reduced-order modelling of a mixing layer,” J. of Fluid Mech. 754, 365–414 (2014).

D. Faranda, Y. Sato, B. Saint-Michel, C. Wiertel, V. Padilla, B. Dubrulle, and F. Daviaud, “Stochastic chaos in a turbulent swirling flow,” Phys. Rev. Lett. 119, 014502 (2017).

E. Ott, C. Grebogi, and J. A. Yorke, “Controlling chaos,” Phys. Rev. Lett. 64, 1196–1199 (1990).

Celso Grebogi, Edward Ott, and James A. Yorke, “Unstable periodic orbits and the dimensions of multifractal chaotic attractors,” Phys. Rev. A 37, 1711–1724 (1988).
[30] M.T. Yassen, “Controlling chaos and synchronization for new chaotic system using linear feedback control,” Chaos Solitons Fract. 26, 913 – 920 (2005).
[31] Y.-P. Tian and X. Yu, “Adaptive control of chaotic dynamical systems using invariant manifold approach,” IEEE T. Circuits-I 47, 1537–1542 (2000).
[32] C. Li, X. Liao, and T. Huang, “Exponential stabilization of chaotic systems with delay by periodically intermittent control,” Chaos 17, 013103 (2007).
[33] B. Noack and H. Eckelmann, “On chaos in wakes,” Physica D 56, 151 – 164 (1992).
[34] J. Singer, Y-Z. Wang, and Haim H. Bau, “Controlling a chaotic system,” Phys. Rev. Lett. 66, 1123–1125 (1991).
[35] E. Lorenz, “Deterministic nonperiodic flow,” J. Atmos. Sci. 20, 130–141 (1963).
[36] F. Gueniat, L. Mathelin, and M. Y. Hussaini, “A statistical learning strategy for closed-loop control of fluid flows,” Theor. Computat. Fluid Dyn. 30, 497–510 (2016).
[37] T. Duriez, S. L. Brunton, and B. R. Noack, Machine Learning Control ? Taming Nonlinear Dynamics and Turbulence (Springer, 2017).
[38] G. Broze and F. Hussain, “Nonlinear dynamics of forced transitional jets: Temporal attractors and transitions to chaos,” in Nonlinear Instability of Nonparallel Flows: IUTAM Symposium Potsdam, NY, USA July 26 – 31, 1993, edited by S. P. Lin, W. R. C. Phillips, and D. T. Valentine (Springer, Berlin, Heidelberg, 1994) pp. 459–473.
[39] S. Narayanan, G. H. Gunaratne, and F. Hussain, “A dynamical systems approach to the control of chaotic dynamics in a spatiotemporal jet flow,” Chaos 23, 033133 (2013), http://dx.doi.org/10.1063/1.4820819.
[40] M. Grandemange, M. Gohlke, and O. Cadot, “Turbulent wake past a three-dimensional blunt body, part 2: Experimental sensitivity analysis,” J. Fluid Mech. 752, 439 – 461 (2014).
[41] A. Evrard, O. Cadot, V. Herbert, D. Ricot, R. Vigneron, and J. Délery, “Fluid force and symmetry breaking modes of a 3d bluff body with a base cavity,” J. Fluid Mech. 61, 99 – 114 (2016).
[42] D. Barros, J. Borée, B. R. Noack, and A. Spohn, “Resonances in the forced turbulent wake past a 3d blunt body,” Phys. Fluids 28, 065104 (2016).
[43] D. Barros, J. Borée, B. R. Noack, A. Spohn, and T. Ruiz, “Bluff body drag manipulation using pulsed jets and coanda effect,” J. Fluid Mech. 805, 422–450 (2016).
[44] R. D. Brackston, J. M. Garca de la Cruz, A. Wynn, G. Rigas, and J. F. Morrison, “Stochastic modeling and feedback control of bistability in a turbulent bluff body wake,” J. Fluid Mech. 802, 726–749 (2016).
[45] R. Li, D. Barros, J. Borée, O. Cadot, B. R. Noack, and L. Cordier, “Feedback control of bimodal wake dynamics,” Exp. Fluids 57, 158 (2016).
[46] F. Champagnat, A. Plyer, G. Le Besnerais, B. Leclaire, S. Davoust, and Y. Le Saint, “Fast and accurate piv computation using highly parallel iterative correlation maximization,” Exp. Fluids 50, 1169–1182 (2011).
[47] C. Pan, D. Xue, Y. Xu, J. Wang, and R. Wei, “Evaluating the accuracy performance of Lucas-Kanade algorithm in the circumstance of PIV application,” Sci. China Phys. Mech. 58, 104704 (2015).
[48] A. Plyer, G. Le Besnerais, and F. Champagnat, “Massively parallel lucas kanade optical flow for real-time video processing applications,” J. Real-Time Image Proc. 11, 713–730 (2016).
[49] N. Gautier and J.-L. Aider, “Feed-forward control of a perturbed backward-facing step flow,” J. Fluid Mech. 759, 181–196 (2014).
[50] N. Gautier and J.-L. Aider, “Frequency-lock reactive control of a separated flow enabled by visual sensors,” Exp. Fluids 56, 1–10 (2015).
[51] N. Gautier, J.-L. Aider, T. Duriez, B. R. Noack, M. Segond, and M. Abel, “Closed-loop separation control using machine learning,” J. Fluid Mech. 770, 442–457 (2015).
[52] N. Gautier and J.-L. Aider, “Real-time planar flow velocity measurements using an optical flow algorithm implemented on GPU,” J. Vis. 18, 277–286 (2015).
[53] A. Lahaye, A. Leroy, and A. Kourta, “Aerodynamic characterisation of a square-back bluff body flow,” Int. J. of Aerodynamics 4, 43–60 (2014), pMID: 57804.
[54] D. Barros, J. Borée, O. Cadot, and B. R. Noack, “Forcing symmetry exchanges and flow reversals in turbulent wakes,” J. of Fluid Mech. 829 (2017), 10.1017/jfm.2017.590.
[55] B. Khalighi, K. Chen, and G. G. Iaccarino, “Unsteady aerodynamic flow investigation around a simplified square-back road vehicle with drag reduction devices,” J. Fluids Eng. 134, 061101 (2012).
[56] Damien McArthur, David Burton, Mark Thompson, and John Sheridan, “On the near wake of a simplified heavy vehicle,” J. Fluid Struct. 66, 293 – 314 (2016).
[57] Y. Eulalie, Aerodynamic analysis and drag reduction around an Ahmed bluff body, Ph.D. thesis, Université de Bordeaux, Bordeaux, France (2014).
[58] J. Yang, M. Liu, G. Wu, Q. Liu, and X. Zhang, “Low-frequency characteristics in the wake of a circular disk,” Phys. Fluids 27, 064101 (2015).
[59] V. Gentile, F. F. J. Schrijer, B. W. Van Oudheusden, and F. Scarano, “Low-frequency behavior of the turbulent axisymmetric near-wake,” Phys. Fluids 28, 065102 (2016), http://dx.doi.org/10.1063/1.4953150.
[60] V. Parezanovic and O. Cadot, “Experimental sensitivity analysis of the global properties of a two-dimensional turbulent wake,” J. of Fluid Mech. 693, 115–149 (2012).
[61] A. Provenzale, L.A. Smith, R. Vio, and G. Murante, “Distinguishing between low-dimensional dynamics and randomness in measured time series,” Physica D 58, 31 – 49 (1992).
[62] B. Mandelbrot, The fractal geometry of nature (Henry Holt and Company, New-York, 1982).
[63] L. Cao, “Practical method for determining the minimum embedding dimension of a scalar time series,” Phys. D 110, 43 – 50 (1997).

[64] F. Takens, “Detecting strange attractors in turbulence,” in Dynamical Systems and Turbulence, Warwick 1980: Proceedings of a Symposium Held at the University of Warwick 1979/80, edited by David Rand and Lai-Sang Young (Springer Berlin Heidelberg, Berlin, Heidelberg, 1981) pp. 366–381.

[65] J. P. Eckmann, S. Ollifson Kamphorst, D. Ruelle, and S. Ciliberto, “Liapunov exponents from time series,” Phys. Rev. A 34, 4971–4979 (1986).

[66] D. Auerbach, P. Cvitanović, J.-P. Eckmann, G. Gunaratne, and I. Procaccia, “Exploring chaotic motion through periodic orbits,” Phys. Rev. Lett. 58, 2387–2389 (1987).

[67] H. Ma, W. Lin, and Y.-C. Lai, “Detecting unstable periodic orbits in high-dimensional chaotic systems from time series: Reconstruction meeting with adaptation,” Phys. Rev. E 87, 050901 (2013).

[68] A. Brunn and W. Nitsche, “Active control of turbulent separated flows over slanted surfaces,” Int. J. Heat Fluid Fl. 27, 748 – 755 (2006), special issue of the 6th International Symposium on Engineering Turbulence Modelling and Measurements ETMM6.

[69] P. Joseph, X. Amandolèse, and J.-L. Aider, “Drag reduction on the 25° slant angle ahmed reference body using pulsed jets,” Exp. Fluids 52, 1169–1185 (2012).

[70] L. W. Sigurdson, “The structure and control of a turbulent reattaching flow,” J. of Fluid Mech. 298, 139 – 165 (1995).

[71] M. Pastoor, L. Henning, B. R. Noack, R. King, and G. Tadmor, “Feedback shear layer control for bluff body drag reduction,” J. of Fluid Mech. 608, 161196 (2008).

[72] J. L. Lumley, “The structure of inhomogeneous turbulent flows,” in Atmospheric turbulence and radio propagation, edited by A. M. Yaglom and V. I. Tatarski (Nauka, Moscow, 1967) pp. 166–178.

[73] L. Sirovich, “Turbulence and the dynamics of coherent structures,” Q. Appl. Math. 45, 561–571 (1987).