Numerical investigation of the hydroelastic response in cavitating flow around a flexible hydrofoil

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Abstract. The objective of this paper is to investigate the hydroelastic response of cavitating flows around a flexible hydrofoil. The numerical simulations are performed by solving the incompressible and unsteady Reynolds Average Navier-Stokes (URANS) equations via the commercial CFD software ANSYS CFX. The k-ω SST turbulence model with the turbulence viscosity correction and the Kubota cavitation model are introduced to the present simulations. The results showed that the cavitation has significantly affected the foil deformation and the evolution of the transient cavity shape and the corresponding hydrodynamic response with time can be divided into three stages: during the development of the attached cavity, the cavity formed on the suction side of the flexible hydrofoil is much larger with a steeper slope of the cavity area, which is caused by the increase of the effective angle of attack due to the twist deformation. During the vortex-cavitation interaction process, the hydrodynamic loads for the rigid hydrofoil remain relatively flat, while that for the flexible hydrofoil fluctuates with high frequency because of the foil deformation, leading to a more complex cavitation pattern due to the interaction with the foil vibration. During the cavity shedding process, both the primary and the residual cavities shed downstream totally, together with the counter-rotational vortex structures, corresponding to a sharp drop in the hydrodynamic loads. The larger effective angle of attack leads to the advanced cavity inception of the next cavitation period.

1. Introduction
As the diameter of hydraulic machinery become larger and the material used to construct the blades become lighter, the composite material is becoming prevalent in a wide range of applications because of their reduced mass, increased strength to weight ratio and erosion resistance. Most earlier works on fluid-structure interaction problems have been on aerospace or wind engineering structures [1-2]. In addition to the dynamic response of flexible airfoils, recent works have also been conducted on the transient responses of flexible hydrofoils[3-4]. With the increasing importance of hydroelasticity in fluid structure interaction problems, the investigation of hydroelastic responses has attracted considerable interests, especially when the local pressure decreases to the saturated vapor pressure of the liquid, cavitation occurs, involving complex interaction between phase-change and vortex structures[5-7]. The unsteady breakdown and shedding of the cavities will induce strong transient loads and lead to further hydroelastic instabilities, even structure failures. Hence, it is important to study the effect of hydroelastic response on the cavitating flows to design safe and reliable hydraulic machinery.

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The objective of this paper is to investigate the hydroelastic responses of a flexible stationary hydrofoil in cavitating flows. The physical and numerical models are presented in Section 2. Detailed analysis of the hydroelastic responses of a flexible stationary hydrofoil in transient subcavitating and cavitating flow is presented in Section 3. Finally, the conclusions are summarized in Section 4.

2. Physical and numerical model

2.1. Fluid model
The Unsteady Reynolds Average Navier-Stokes (URANS) equations, in their conservative form, for a Newtonian fluid without body forces and heat transfers are presented in Ref. [8]. The current simulation solves the URANS equations using the $k-\omega$ SST turbulence model [9], which applies the $k-\varepsilon$ model away from the wall and the $k-\omega$ model near the wall. In order to avoid the over-prediction of the turbulent eddy viscosity resulting to the over-prediction of the turbulent stress and improve the numerical simulations by considering the local compressibility effect of multiphase mixtures on the turbulence model, the turbulent viscosity is revised according to Ref. [10] by replacing $\mu_t$ with $\mu_{t,\text{mod}}$:

$$\mu_{t,\text{mod}} = \mu_t f(n), \quad f(n) = \frac{\rho_t + (1 - \alpha) \rho_f (\rho_t - \rho_f)}{\rho_t + (1 - \alpha) (\rho_t - \rho_f)}$$

where $n=3$ is chosen since the validation studies made by Huang et al. [11] and Ducoin et al. [12] are performed in the same hydrofoil and show favorable agreements with the experimental results. In this work, the Kubota cavitation model [13] is used and the model constants are assumed as the default values in CFX [14].

2.2. Fluid model
The cantilevered hydrofoil with the spanwise bending and twisting flexibility is modeled as a chordwise rigid, two degree of freedom system in this work, as shown in Figure 1. Both the bending deformation $h$ and the twisting deformation $\theta$ are defined about the elastic axis, which is located at the mid-chord of the foil. In Figure 1, $b$ is the semi-chord and the chord length $c=2b$, $U$ is the flow velocity.

![Figure 1](image-url) Figure 1. A two degrees-of-freedom model describing the bending $h$ and the twisting $\theta$ of foil

The governing equation of the 2-DOF system motion can be written as:

$$\begin{bmatrix} m & S_{\theta} & h \dot{h} \\ S_{\theta} & I_{\theta} & \dot{h} \dot{\theta} \end{bmatrix} + \begin{bmatrix} C_{h} & 0 \\ 0 & C_{\theta} \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} K_{h} & 0 \\ 0 & K_{\theta} \end{bmatrix} \begin{bmatrix} h \\ \theta \end{bmatrix} = \begin{bmatrix} F_{ex} \\ M_{ex} \end{bmatrix} + \begin{bmatrix} L \\ M \end{bmatrix}$$

where $m$, $S_{\theta}$, $I_{\theta}$ are the structural mass, the static imbalance and the moment of inertia respectively, $C_{h}$ and $C_{\theta}$ are the structural damping values for the bending and twisting motions respectively, $K_{h}$ and $K_{\theta}$ are the structural bending and torsional stiffness values respectively. $h$, $\dot{h}$ and $\ddot{h}$ are the bending displacement, velocity, and acceleration, while $\theta$, $\dot{\theta}$, $\ddot{\theta}$ are the twisting displacement, velocity, and acceleration. $F_{ex}$ and $M_{ex}$ are the external excitation force and moment and set as zero in this work. $L$ and $M$ are the fluid lift, which is defined positive along the Y-axis, and moment, which is defined positive counter-clockwise, acting on the hydrofoil.

2.3. Hybrid coupled fluid structure interaction model
The hybrid coupled fluid structure interaction model discretizes the equation of motion, Equation (2), in the time-domain as:

\[
\begin{pmatrix}
m & S \rho & I \rho & 1 \\
S \rho & 1 & 0 & 0 \\
C \rho & 0 & K & 0 \\
0 & 0 & K & 0 \\
\end{pmatrix}
\begin{pmatrix}
\hat{\theta}^{n+1}_i \\
\hat{\theta}^{n+1}_i \\
\hat{h}^{n+1}_i \\
\hat{h}^{n+1}_i \\
\end{pmatrix}
+
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
K_{\theta} & K & 0 & 0 \\
K & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\hat{\theta}^{n+1}_i \\
\hat{\theta}^{n+1}_i \\
\hat{h}^{n+1}_i \\
\hat{h}^{n+1}_i \\
\end{pmatrix}
-
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
L^{n+1}_f \\
L^{n+1}_f \\
M^{n+1}_f \\
M^{n+1}_f \\
\end{pmatrix}
-
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
L^{n+1}_f \\
L^{n+1}_f \\
M^{n+1}_f \\
M^{n+1}_f \\
\end{pmatrix}
= \begin{pmatrix}
F^{n+1}_c \\
F^{n+1}_c \\
L^{n+1}_c \\
M^{n+1}_c \\
\end{pmatrix}
\tag{3}
\end{equation}

where \( i \) is the sub-iteration number within each time step \( n \), \( L^{n+1}_f \) and \( M^{n+1}_f \) subtracted from the left and right sides of the equation are the potential flow estimate of the hydroelastic force and momentum, which is proposed by Theodorsen [15] and developed by Young [16] to account for the effects of fluid inertial, damping and restoring forces in each time step and sub-iteration, avoiding the over-estimation of the structural displacement.

2.4. Numerical setup and boundary conditions

The simulation is conducted with a 2D fluid solver and the computational domain and boundary conditions are given according to the experimental setup in Beijing Institute of Technology [17], which is shown in Fig. 2. The results shown in this paper correspond to the stationary hydrofoil with initial angle of attack of \( \alpha_0 = 8^\circ \), which is subjected to a nominal free stream velocity of \( U_\infty = 10 \text{m/s} \), yielding a moderate Reynolds number of \( Re = U_\infty c/\nu = 7.5 \times 10^5 \), where \( c \) is the chord length and \( \nu \) is the dynamic viscosity of the liquid (water at 25°C). The outlet pressure is set according to the cavitation number \( \sigma = (p_\infty - p_v)/(0.5 \rho \mu U_\infty^2) = 1.4 \), where \( p_\infty \) is the tunnel pressure, \( p_v \) is the saturated vapor pressure.

![Figure 2. Computational domain and boundary conditions](image)

The numerical mesh is shown in Fig. 3, composed of 110,000 structured elements. The mesh is refined near the leading and trailing edge of the foil and in the wake region to satisfy \( y^+ = \frac{y u_c}{\nu} \approx 1 \), where \( y \) is the thickness of the first cell from the foil surface, and \( u_c \) is the wall frictional velocity. The time step chosen for the simulation is \( \Delta t = 1 \times 10^{-3} \text{s} \), which ensures that the averaged CFL = \( U_\infty \times \Delta t/\Delta x \approx 1 \).

![Figure 3. Fluid mesh details](image)

3. Results and discussion

Figure 4 shows the predicted bending and twisting deformations of the flexible hydrofoil in cavitating flow (\( \sigma = 1.4 \)). From Figure 4, it is shown that the hydrofoil bends and twists with the same period time of the lift coefficient. The hydrodynamic responses within one cycle
could be characterized by three stages: (I) the bending and twisting deformation increases as the evolution of the lift coefficient, while the twist angle fluctuates within a small range, which is corresponding to the development of the attached cavity, (II) the twist angle fluctuates in a large extent as the cavities break up and intensively interact with the vortex structures, (III) there is a sudden decrease of the bending and twisting deformation which is in phase with the lift coefficient, when the cavities collapse and totally shed downstream together with the vortex structures. Overall, the transient development of cavitation has affected the unsteady characteristics of the hydroelastic response, which is also closely related to the instability of the vortex structures and will be discussed following.

Figure 5 shows the evolution of the predicted lift, drag and effective angle of attack for the rigid and flexible hydrofoils with time. It is shown that the hydrodynamic load coefficients of the flexible hydrofoil fluctuate more significantly due to the disturbance caused by the deformation of the foil. The average of the lift and drag coefficients for the flexible hydrofoil are 1.067 and 0.141 respectively, which is higher than that for the rigid hydrofoil. This is because the twist deformation increases the effective angle and results to the larger hydrodynamic loads imposed on the foil.

![Figure 5](image-url) Comparisons of the predicted lift and drag coefficients for the rigid and flexible hydrofoils

In order to further investigate the influence of hydroelastic response on the cavitation development process and the vortex-cavitation interactions, Figures 6-7 show the evolution of vapor fraction and vorticity contours at representative times for both the rigid and flexible hydrofoils, along with the observed cavitation patterns of the flexible hydrofoil from the experimental visualizations, from which the cavity area is extracted and shown in Figure 8. The evolution of the transient cavity shapes and the corresponding hydrodynamic responses could be analyzed as follows:

3.1. development of the attached cavity (from \( t_1 \) to \( t_5 \))

During the development of the attached cavity, reasonable agreements of the cavity shapes can be obtained between the numerical results and experimental observations. As shown in Figure 5, the predicted lift and drag coefficients for both the rigid and flexible hydrofoils increase with the time, while the slope of the lift and drag coefficients for the flexible hydrofoil is larger. This is mainly because the larger effective angle of attack caused by the twist deformation between \( t_1 \) and \( t_5 \). At \( t_1=0.94s \), the partial sheet cavity is formed at the leading edge of the foil on the suction side. Between \( t_2=0.95s \) and \( t_4=0.97s \), the development of the small leading edge partial sheet cavity can be observed for both the rigid and flexible hydrofoil, as shown in Figures 6(b)-(c), with the cavity area increasing, as shown in Figure 8, which is corresponding to the increase of the lift and drag coefficients shown in Figure 5. Meanwhile, the re-entrant jet is formed at the rear region of the cavity, and a clockwise vortex is forming and developing on the suction side of the hydrofoil. Between \( t_4=0.97s \) and \( t_5=0.98s \), the re-entrant jet moves toward the leading edge of the foil and the clockwise leading edge vortex develops to its maximum extent, which has an effect on the lifted cavity and leads to the decrease of the lift coefficient (shown in Figure 5(a)). Compared with the rigid hydrofoil, the cavities formed on the suction side of the flexible hydrofoil is much larger with a steeper slope of the cavity area, which is caused by the increase of the effective angle of attack due to the twist deformation.
3.2. vortex-cavitation interaction (from $t_5$ to $t_{10}$)

During this stage, there is a discrepancy between the numerical results for the rigid and the flexible hydrofoil. As shown in Figure 5, the predicted lift and drag coefficients for the rigid hydrofoil remain relatively flat, while that for the flexible hydrofoil fluctuates with high frequency because the development of cavity is disturbed by the foil deformation, which leads to a more complex cavitation pattern due to the interaction with the foil vibration. At $t_6=0.992s$, the cloud cavity reaches to the trailing edge of the foil and is partially attached on the suction side of the foil, while the leading edge vortex is interacting with the counter-clockwise trailing edge vortex due to the reverse pressure gradient, resulting that two counter-rotating vortices with cavitating cores shed, as shown in Figure 7(b). The shedding cloud cavity moves downstream, and the cavity area reaches to its minimum value, as shown in Figure 5 and Figure 8, with the relatively lower lift coefficient and the highest drag coefficient achieved at $t_7$ due to the primary cavity shedding. Between $t_6=1.014s$ and $t_9=1.02s$, governed by the main flow, the shedding cavitating vortex structures are transported towards the trailing edge of the foil, as shown in Figure 7(c), with the centers of the shedding vortex structures moving downstream along the suction side of the hydrofoil and interacting with the downstream counter-clockwise vortex again, which is corresponding to the fluctuating lift coefficients and effective angle of attack, which are shown in Figure 5.

3.3. cavity shedding (from $t_{10}$ to $t_{12}$)

Between $t_{10}=1.02s$ and $t_{12}=1.032s$, both the primary and the residual cavities shed downstream totally and move downstream together with the primary cavity, as well as the clockwise and counter-clockwise vortex structures, which is due to the interaction between those counter-rotating vortex structures and the cavitation cores, corresponding to a sharp drop in the lift coefficient and followed by the formation of the partial sheet cavity in the next period.

![Figure 6](image_url) Predicted vapor fraction contours for the flexible and rigid hydrofoils during the development of the attached cavity, along with experimental observations of the cavitation patterns for the flexible hydrofoil.
Figure 7. Predicted vapor fraction contours for the flexible and rigid hydrofoils during the vortex-cavitation interaction, along with experimental observations of the cavitation patterns for the flexible hydrofoil.

Figure 8. The cavity area for the rigid and flexible hydrofoil

4. Conclusions
In this paper, physical and numerical analysis are presented for a rigid and a flexible NACA66 hydrofoil with the angle of attack $\alpha=8^\circ$ at $Re=750,000$ for subcavitating and cavitating conditions. The hydroelastic response has significant effect on the cavitation development process and the vortex-cavitation interactions. The evolution of the transient cavity shape and the corresponding hydrodynamic response with time can be divided into three stages: development of the attached cavity,
vortex-cavitation interaction and cavity shedding. During the development of the attached cavity, the partial sheet cavity is formed with the predicted lift and drag coefficients increasing, while the cavity formed on the suction side of the flexible hydrofoil is much larger with a steeper slope of the cavity area, which is caused by the increase of the effective angle of attack due to the twist deformation. During the vortex-cavitation interaction process, the hydrodynamic loads for the rigid hydrofoil remain relatively flat, while that for the flexible hydrofoil fluctuates with high frequency because of the foil deformation, leading to a more complex cavitation pattern due to the interaction with the foil vibration. During the cavity shedding process, both the primary and the residual cavities shed downstream totally, together with the counter-rotational vortex structures, which is corresponding to a sharp drop in the hydrodynamic loads.

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