TRITON-$^3$HE RELATIVE AND DIFFERENTIAL FLOWS AND THE
HIGH DENSITY BEHAVIOR OF NUCLEAR SYMMETRY ENERGY

GAO-CHAN YONG
Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, P.R. China
yonggaochan@impcas.ac.cn

BAO-AN LI
Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, TX 75429-3011, USA
Bao-An.Li@tamu-commerce.edu

LIE-WEN CHEN
Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China
Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou, 730000, China
lwchen@sjtu.edu.cn

Received (received date)
Revised (revised date)

Using a transport model coupled with a phase-space coalescence after-burner we study
the triton-$^3$He relative and differential transverse flows in semi-central $^{132}$Sn + $^{124}$Sn
reactions at a beam energy of 400 MeV/nucleon. We find that the triton-$^3$He pairs carry
interesting information about the density dependence of the nuclear symmetry energy.
The $^t$-3He relative flow can be used as a particularly powerful probe of the high-density
behavior of the nuclear symmetry energy.

1. Introduction

The density dependence of nuclear symmetry energy especially at supra-
saturation densities is among the most uncertain properties of neutron-rich nu-
clear matter. However, it is very important for nuclear structure, heavy-ion reactions,
and many phenomena/processes in astrophysics and cosmology. Heavy-ion reactions especially those induced by radioactive beams provide a unique opportunity to constrain the symmetry energy at supra-saturation densities in terrestrial laboratories. Various probes using heavy-ion reactions have been proposed in the literature, see, e.g., ref. [10] for the most recent review. It is particularly interesting to mention that, besides many significant results about the symmetry energy at sub-saturation densities, see, e.g., refs. [11][12][13][14][15][16][17][18][19][20], circumstantial evidence for a rather soft symmetry energy at supra-saturation densities has been reported very recently [21] based on the IBUU04 transport model [22] analysis...
of the $\pi^-/\pi^+$ data taken by the FOPI Collaboration at SIS/GSI\(^{23}\). To constrain tightly and reliably the nuclear symmetry energy especially at supra-saturation densities, much more efforts by both the nuclear physics and the astrophysics communities are still needed.

There is an urgent need to verify the conclusion about the soft symmetry energy at supra-saturation densities required to reproduce the FOPI $\pi^-/\pi^+$ data within transport model analyses\(^{21,23}\). It was predicted that the neutron-proton differential flow is another sensitive probe of the high-density behavior of the nuclear symmetry energy\(^{24}\). However, it is difficult to measure observables involving neutrons. One question often asked by some experimentalists is whether the triton-$^3$He pair may carry the same information as the neutron-proton one. We will try to answer this question quantitatively by coupling the IBUU04 calculations to a phase-space coalescence after-burner. Indeed, we found that, similar to the neutron-proton pair, the triton-$^3$He relative and differential transverse flows are sensitive to the high-density behavior of the nuclear symmetry energy. They can be used to test indications about the high-density behavior of the symmetry energy observed earlier from analyzing the $\pi^-/\pi^+$ data.

2. The theoretical models

Our study is carried out based on the IBUU04 version of an isospin and momentum dependent transport model and the simplest phase-space coalescence after-burner. The single nucleon potential is one of the most important inputs to BUU-like transport models for nuclear reactions. In the IBUU04 transport model, we use a single nucleon potential derived within the Hartree-Fock approach using a modified Gogny effective interaction (MDI)\(^{22,25}\). The corresponding MDI symmetry energy can be found in Ref.\(^{26}\).

Because most BUU-type transport models including the IBUU04 are incapable of forming dynamically realistic nuclear fragments, some types of after-burners, such as statistical and coalescence models, are normally used as a remedy. This kind of hybrid models can be used to study reasonably well, for instance, nuclear multifragmentation, see, e.g., refs.\(^{27,28,29,30}\), collective flow of light fragments, see e.g., 31,32,33 and the formation of hypernuclei\(^{34}\). There are, however, some remaining issues, such as the freeze-out time of fragments that is related to the time of coupling the transport model with the after-burner, etc. There are also interesting work in using advanced coalescence models, see, e.g., refs.\(^{37,38}\). We notice here that, several advanced cluster recognition routines, such as, the Early Cluster Recognition Algorithm (ECRA)\(^{39}\), the Simulated Annealing Clusterization Algorithm (SACA)\(^{40}\), have been put forward in recent years. For the purposes of the present exploration, however, we use the simplest phase-space coalescence model, see, e.g., refs.\(^{52,53}\), where a physical fragment is formed as a cluster of nucleons with relative momenta smaller than $P_0$ and relative distances smaller than $R_0$. The results presented in the following are obtained with $P_0 = 263$ MeV/c and $R_0 = 3$ fm.
This simple choice may thus limit the scope and importance of our study here. For instance, we shall limit ourselves to studies of the relative/differential observables for $t-^3$He pairs without attempting to study pairs of the heavier mirror nuclei. An extended study including the heavy mirror nuclei using the advanced coalescence and/or earlier cluster recognition methods is planned.

3. Results and discussions

![Graph showing the flow of Triton and $^3$He as functions of reduced C.M. rapidity](image)

**Fig. 1.** Triton and $^3$He transverse flows (in unit of MeV) as functions of the reduced C.M. rapidity in the semi-central $^{132}$Sn + $^{124}$Sn reactions at a beam energy of 400 MeV/nucleon. Taken from Ref. 41.

We now investigate whether the transverse collective flows of triton and $^3$He can be used to probe the symmetry energy. Firstly, we examine in Fig. 1 their transverse flows individually. The average C.M. transverse momentum per nucleon $<p_x/A>$ in the reaction plane is defined as

$$<p_x/A>(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_x^i/A(y)$$

where $N(y)$ is the total number of fragments of mass $A$ in the rapidity bin at $y$. The correlation between the $<p_x/A>$ and rapidity $y$ reveals the transverse collective flow. It is seen that $^3$He clusters show a stronger flow than triton clusters. This is mainly due to the stronger Coulomb force experienced by the $^3$He clusters. More interestingly, the transverse flow of $^3$He clusters show appreciable sensitivity to the variation of the symmetry energy.

The transverse flow is a result of actions of several factors including the isoscalar, symmetry and Coulomb potentials and nucleon-nucleon scatterings. It is well known
that the transverse flow is sensitive to the isoscalar potential. Given the remaining uncertainties associated with the isoscalar potential and the small size of the symmetry energy effects, it would be very difficult to extract any reliable information about the symmetry energy from the individual flows of triton and $^3$He clusters. Thus techniques of reducing effects of the isoscalar potential while enhancing effects of the isovector potential are very helpful. We thus study in Fig. 2 the triton-$^3$He relative and differential flows. The relative flow is given as

$$
< p_{x}^{t}/A > - < p_{x}^{3He}/A > = \frac{1}{N_{t}} \sum_{i=1}^{N_{t}} p_{x}^{t}/A - \frac{1}{N_{3He}} \sum_{i=1}^{N_{3He}} p_{x}^{3He}/A.
$$

The triton-$^3$He differential flow reads

$$
< p_{x}^{t-3He}/A > = \frac{1}{N_{t} + N_{3He}} \left( \sum_{i=1}^{N_{t}} p_{x}^{t}/A - \sum_{i=1}^{N_{3He}} p_{x}^{3He}/A \right) = \frac{N_{t}}{N_{t} + N_{3He}} < p_{x}^{t}/A > - \frac{N_{3He}}{N_{t} + N_{3He}} < p_{x}^{3He}/A > ,
$$

where $N_{t}$, $N_{3He}$ are the number of triton and $^3$He in the rapidity bin at $y$. From the upper panel of Fig. 2 it is seen that the triton-$^3$He relative flow is very sensitive to the symmetry energy. Because of the larger slope of the $^3$He flow, the triton-$^3$He relative flow shows a negative slope at mid-rapidity. Effects of the symmetry
energy on the differential flow shown in the lower panel, however, is relatively small. Although the $^3\text{He}$ flow is more sensitive to the symmetry energy, the small number of $^3\text{He}$ clusters makes the $^3\text{He}$ flow contributes less to the triton-$^3\text{He}$ differential flow (as indicated in Eq. (3)). The triton-$^3\text{He}$ differential flow is therefore dominated by triton clusters. Consequently, it is less sensitive to the symmetry energy than the triton-$^3\text{He}$ relative flow. The slope $F(x) \equiv d < p_x/A > /d(\gamma/\gamma_{beam})$ of the transverse flow at mid-rapidity can be used to characterize more quantitative the symmetry energy effects. We found that for the t-$^3\text{He}$ relative flow, $F(x = 1) \approx -74$ MeV/c and $F(x = -1) \approx -22$ MeV/c, respectively. For the t-$^3\text{He}$ differential flow, $F(x = 1) \approx 21$ MeV/c and $F(x = -1) \approx 42$ MeV/c, respectively. The t-$^3\text{He}$ relative flow thus can be used as a very useful and independent tool to test the soft symmetry energy at supra-saturation densities extracted from studying the $\pi^-/\pi^+$ ratio.

4. Summary

In summary, using a hybrid approach coupling the transport model IBUU04 to a phase-space coalescence after-burner we studied the t-$^3\text{He}$ relative and differential flows in semi-central $^{132}\text{Sn} + ^{124}\text{Sn}$ reactions at an incident energy of 400 MeV/nucleon. We found that the nuclear symmetry energy affects strongly the t-$^3\text{He}$ relative and differential flows. The t-$^3\text{He}$ relative flow can be used as a particular powerful probe of the high-density behavior of the nuclear symmetry energy.

5. Acknowledgements

This work was supported in part by the US National Science Foundation Awards PHY-0652548 and PHY-0757839, the Research Corporation under Award No.7123 and the Texas Coordinating Board of Higher Education Award No.003565-0004-2007, the National Natural Science Foundation of China under grants 10710172, 10575119, 10675082 and 10975097 and MOE of China under project NCET-05-0392, Shanghai Rising-Star Program under Grant No.06QA14024, the SRF for ROCS, SEM of China, and the National Basic Research Program of China (973 Program) under Contract No.2007CB815004.

References

1. M. Kutschera, Phys. Lett. B340, 1 (1994).
2. S. Kubis and M. Kutschera, Acta Phys. Pol. B30, 2747 (1999); Nucl. Phys. A720, 189 (2003).
3. B.A. Brown, Phys. Rev. Lett. 85, 5296 (2000).
4. C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. 86, 5647 (2001).
5. B.A. Li, C.M. Ko and W. Bauer, Int. Jour. Mod. Phys. E 7, 147 (1998).
6. Isospin Physics in Heavy-Ion Collisions at Intermediate Energies, Eds. Bao-An Li and W. Udo Schröder (Nova Science Publishers, Inc, New York, 2001).
7. P. Danielewicz, R. Lacey, W.G. Lynch, Science 298, 1592 (2002).
8. V. Baran, M. Colonna, V. Greco and M. Di Toro, Phys. Rep. 410, 335 (2005).
9. L.W. Chen, C.M. Ko, B.A. Li, and G.C. Yong, Front. Phys. China 2, 327 (2007).
10. B.A. Li, L.W. Chen and C.M. Ko, Phys. Rep. 464, 113 (2008).
11. K. Sumiyoshi and H. Toki, Astrophys. J. 422, 700 (1994).
12. J.M. Lattimer and M. Prakash, Science 304, 536 (2004).
13. A.W. Steiner et al., Phys. Rep. 411, 325 (2005).
14. L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. Lett. 94, 032701 (2005).
15. B.A. Li, L.W. Chen, and C.M. Ko, Phys. Rep. 464, 113 (2008).
16. K. Sumiyoshi and H. Toki, Astrophys. J. 422, 700 (1994).
17. D. Shetty, S.J. Yennello, and G.A. Souliotis, Phys. Rev. C75, 034602 (2007).
18. M. B. Tsang, Yingxun Zhang, P. Danielewicz, M. Famiano, Zhuxia Li, W. G. Lynch, and A. W. Steiner, Phys. Rev. Lett. 102, 122701 (2009).
19. M. Centelles, X. Roca-Maza, X. Vias, and M. Warda, Phys. Rev. Lett. 102, 122502 (2009).
20. G. Lehaut, F. Gulminelli, and O. Lopez, Phys. Rev. Lett. 102, 142503 (2009).
21. Z.G. Xiao, B.A. Li, L.W. Chen, G.C. Yong, and M. Zhang, Phys. Rev. Lett. 102, 062502 (2009); M. Zhang, Z.G. Xiao, B.A. Li, L.W. Chen, G.C. Yong, and S.J. Zhun, Phys. Rev. C80, 034616 (2009).
22. B.A. Li, C.B. Das, S. Das Gupta, and C. Gale, Nucl. Phys. A735, 563 (2004); Phys. Rev. C69, 064602 (2004).
23. W. Reisdorf et al. for the FOPI Collaboration, Nucl. Phys. A 781, 459 (2007).
24. B.A. Li, Phys. Rev. Lett. 85, 4221 (2000).
25. C. B. Das, S. Das Gupta, C. Gale, and B.A. Li, Phys. Rev. C67, 034611 (2003).
26. J. Xu, L. W. Chen, B. A. Li and H. R. Ma, Astrophys. J. 697, 1549 (2009).
27. H. Kruse, B.V. Jacak, J.J. Molitoris, G.D. Westfall, and H. Stöcker, Phys. Rev. C31 (1985) 1770.
28. B.A. Li, A.R. DeAngelis, and D.H.E. Gross, Phys. Lett. B303, 225 (1993).
29. K. Hagel et al., Phys. Rev. C62, 034607 (2000).
30. W.P. Tan et al., Phys. Rev. C64, R051901 (2001).
31. V. Koch et al., Phys. Lett. B241, 174 (1990).
32. L.W. Chen, F.S. Zhang, and G.M. Jin, Phys. Rev. C58, 2283 (1998).
33. F.S. Zhang, L.W. Chen, Z.Y. Ming, and Z.Y. Zhu, Phys. Rev. C60, 064604 (1999).
34. T. Gaitanos, H. Lenske, U. Mosel, Phys. Lett. B663, 197 (2008).
35. R. Mattiello et al., Phys. Rev. Lett. 74, 2180 (1995); R. Mattiello et al., Phys. Rev. C 55, 1443 (1997).
36. R. Scheibl, Ulrich Heinz, Phys. Rev. C59, 1585 (1999).
37. L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. C68, 017601 (2003); Nucl. Phys. A729, 809 (2003).
38. L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. C69, 054606 (2004).
39. A. Strachan and C. O. Dorso, Phys. Rev. C56, 995 (1997).
40. Rajeev K. Puri and Joerg Aichelin, J. Comput. Phys. 162, 245 (2000).
41. G.C. Yong, B.A. Li, L.W. Chen, and X.C. Zhang, Phys. Rev. C80, 044608 (2009).
42. P. Danielewicz and G. Odyniec, Phys. Lett. B157, 146 (1985).
43. G.C. Yong, B.A. Li, and L.W. Chen, Phys. Rev. C74, 064617 (2006); Phys. Lett. B650, 344 (2007).
44. G.C. Yong, B.A. Li, and L.W. Chen, Phys. Rev. C74, 064617 (2006); Phys. Lett. B650, 344 (2007).
45. B.A. Li, L.W. Chen, G. C. Yong, and W. Zuo, Phys. Lett. B634, 378 (2006).
46. V. Greco, V. Baran, M. Colonna, M. Di Toro, T. Gaitanos, and H.H. Wolter, Phys. Lett. B562, 215 (2003).
47. M.A. Famiano, et al., Phys. Rev. Lett. 97, 052701 (2006).