Neutrinos masses in a supersymmetric model with exotic right-handed neutrinos in Global \( (B - L) \) Symmetry

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Abstract

We build a supersymmetric version with \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) gauge symmetry. We impose, also, the \( \mathbb{Z}_3 \) symmetry to keep the proton stable at least at tree level. The model has three right-handed neutrinos with non identical \( (B - L) \) charges and some extra Higgs fields. There are, also, a global \( (B - L) \), where \( (B) \) and \( (L) \) are the usual baryonic and leptonic numbers respectively, symmetry. We will show only right handed neutrinos and one left-handed neutrino get mass at tree level while the others two left handed neutrinos get their masses at 1-loop level. We will also explain the mixing angle in the neutrino sector in agreement with the experimental data.

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1 Introduction

Today we know that neutrinos have mass and we notice oscillation at neutrinos sector, for example in the case of solar neutrinos problem is explained as \( \nu_e \) disappearance while to explain atmospheric neutrino data via \( \nu_\mu \rightarrow \nu_\tau \) oscillation and the experimental data has two large angles, they are \( \theta_{solar} \) and \( \theta_{atm} \) and such mixing is termed as “tribimaximal” mixing scheme.

There is an interesting model with the same gauge symmetry as in the Standard Model (SM), it means \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) gauge symmetries
and we introduce two right-handed neutrinos having \((B - L) = -4\) and the third one having \((B - L) = 5\) and besides the usual scalar \(S\) we introduce two extra doublets (they are inert under \(Z_3\) symmetry), this model was presented at [1], in this reference the authors also presented a first short analyse about the supersymmetric version of this model. Our goal here is present this model in more detail.

We will present the supersymmetric version of the model described at [1], where the particle spectra is enlarged with four inert doublets, some scalars as singlets under \(SU(2)\) symmetry and three right-handed neutrinos with non-standard assignment of global \((B - L)\) symmetry which make the model anomaly free [2], the SUSY versions of these models were done at [3, 4], and we will also impose \(Z_3\) symmetry to avoid the proton decay at least at tree level.

The outline of this paper is as follows: In Sec.(2) we present the particle content of this model in superfield formalism, in Sec.(3) we present the lagrangian of this model. In Sec.(4) we calculate the masses to fermions at tree level. We present, in a short way, the scalar potential at Sec.(5). Some left-handed neutrinos get mass their masses at to 1-loop level in Sec.(6) in similar way as showed at [1]. In the end of this article, we present our conclusion.

2 The Model

Now we will review the non-SUSY version of the model, where our symmetry is

\[
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y,
\]

as the gauge symmetry of this model is the same as in the Standard Model (SM), we have the same gauge bosons of SM. They are the gluons fields \(g^a\), and the bosons \(W^i\) of the group \(SU(2)\) and \(b'\) of \(U(1)\) and we have omitted the Lorentz indeces. We will, also, impose \((B - L)\) as global symmetry.

The representation content of the non-SUSY model is the following: under \(SU(2)_L\) we have the lepton doublets \(L_{iL} = (\nu_i, l_i)_L^T \sim (1, 2, -1)^1\), and \((i = 1, 2, 3)\) denote fermion generations with \((B - L) = -1\); charged singlets

\[\text{The parentheses are the transformation properties under the respective representation of } (SU(3)_C, SU(2)_L, U(1)_Y).\]
$E_{iR} \sim (1, 1, -2)$ with $(B - L) = -1$; three sterile neutrinos, one of them
$N_{1R} \sim (1, 1, 0)$ with $(B - L) = -5$ and the others two $N_{\alpha R} \sim (1, 1, 0), (\alpha = 2, 3)$ with $(B - L) = +4$; the quarks sector is exactly the same as in the SM
with $(B - L) = (+1/3)$; we also introduce the SM Higgs $S \sim (1, 2, 1)$ with
$(B - L) = 0$, and two scalar inert doublet $D_{1,2} \sim (1, 2, 1)$ and $(B - L) =
+6, -3$, respectively, more details about this model can be find at [1].

Now we will start to construct the supersymmetric version of this model,
all the fields listed above we will put, as usual in supersymmetric models,
in chiral superfield, the gauge sector of this model is identical of SM are
introduced at vector superfield [5, 6, 7, 8, 9, 10].

### 2.1 The Superfields

The usual fermions as done in Minimal Supersymmetric Standard Model
(MSSM) are $\hat{L}_{iL}, \hat{E}_{iR}, \hat{Q}_{iL}, \hat{u}_{iR}$ and $\hat{d}_{iR}$ and the news one are $\hat{N}_{1R}$ and $\hat{N}_{\beta R}$ are
put in chiral superfields and their quantum numbers are shown at Tabs.(1,2).

| Superfield | $\hat{L}_{iL}$ | $\hat{E}_{iR}$ | $\hat{N}_{1R}$ | $\hat{N}_{\beta R}$ |
|------------|---------------|---------------|--------------|---------------|
| $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ | $(1, 2, -1)$ | $(1, 1, 2)$ | $(1, 1, 0)$ | $(1, 1, 0)$ |
| $(B - L)$ | $(-1)$ | $(+1)$ | $(+5)$ | $(-4)$ |

Table 1: Transformation properties of the lepton under $(SU(3)_C, SU(2)_L, U(1)_Y)$ and $(B - L)$.

| Superfield | $\hat{Q}_{iL}$ | $\hat{u}_{iR}$ | $\hat{d}_{iR}$ |
|------------|---------------|---------------|--------------|
| $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ | $(3, 2, \frac{4}{3})$ | $(3^*, 1, -\frac{4}{3})$ | $(3^*, 1, \frac{2}{3})$ |
| $(B - L)$ | $(+\frac{1}{3})$ | $(-\frac{1}{3})$ | $(-\frac{1}{3})$ |

Table 2: Transformation properties of the quark under $(SU(3)_C, SU(2)_L, U(1)_Y)$ and $(B - L)$. 
We add also three right-chiral neutrinos superfields, in similar way as done at MSSM with three right-handed neutrinos (MSSM3RHN) \([6]\), we will represent them as \(\hat{N}_{iR}\) \(^2\), see \([6, 10]\), in the following way
\[
\hat{N}_{iR}(y, \theta) = \tilde{N}_{iR}(y) + \sqrt{2} (\theta N_{iR}(y)) + (\theta \theta) F_{N_{iR}}(y), \; (i = 1, 2, 3), \tag{2}
\]
where the fields \(N_{iR} \equiv \nu_{iL}^{c}\), as we presented in some preliminar studies presented at \([5, 6]\), are the right-handed neutrinos known as sterile neutrinos \([11]\) and we should also introduce three right-handed sneutrinos \(\tilde{N}_{iR}\), and we will call them as sterile sneutrinos, as we have discussed at \([6]\).

The new scalars, the left-handed sneutrinos \(\tilde{\nu}_{iL}\) and the right-handed sneutrinos \(\tilde{N}_{iR}\), are inner due \((B - L)\) symmetry, therefore, we can write the following constraints in their vacuum expectation values (vev)
\[
\langle \tilde{\nu}_{iL} \rangle = \langle \tilde{N}_{1R} \rangle = \langle \tilde{N}_{3R} \rangle = 0. \tag{3}
\]

An interesting feature of this model is that the right-handed neutrino \(N_{1}\) do not mix with the right-handed neutrinos \(N_{2R}, N_{3R}\) \(^3\) because they have different \((B - L)\) quantum numbers, we will present it at Sec.(4.2).

In the non-SUSY model, as we mentioned in our Introduction, the right-handed neutrinos \(N_{1R}, N_{3R}\) has \((B - L)\) charge as \(-5, +4\), see \([1]\), respectively. However in the supersymmetric model they are introduced as antiparticles, see Eq.(2), due this fact they have opposite \((B - L)\) charge, as we presented at Tab.(1), more details about it can be found at \([5, 6]\).

The scalars in doublets representation of this model are presented at Tabs.(3,4)

The usual scalars \(H_{1,2}\) \(^4\) their vev, as usual, are given by:
\[
\langle H_{1} \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle H_{2} \rangle = \frac{v_2}{\sqrt{2}} \tag{4}
\]
while the new scalars \(D_{1,2}\) has \((B - L)\) charge as \(+6, -3\), in the non-SUSY model, respectively. Those quantum number came from the following Yukawa interactions
\[
G_i \left( \overline{L}_{iL} D_1 \right) N_{1R} + P_{i\beta} \left( \overline{L}_{iL} D_2 \right) N_{3R}, \tag{5}
\]

\(^2\)It means \(\hat{N}_{1R}, \hat{N}_{2R}\) and \(\hat{N}_{3R}\) and we will use this notation allways we do not says their \((B - L)\) charges.

\(^3\)The same is hold to right-handed sneutrinos.

\(^4\)In the non-SUSY model the scalar field was denoted as \(S\) \([1]\)
Table 3: Transformation properties of the scalars doublets introduced in the non-SUSY model under \((SU(3)_C, SU(2)_L, U(1)_Y)\) and \((B - L)\).

| Superfield | \(H_1\) | \(H_2\) | \(D_1\) | \(D_2\) |
|------------|--------|--------|--------|--------|
| \(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\) | \((1, 2, 1)\) | \((1, 2, -1)\) | \((1, 2, 1)\) | \((1, 2, 1)\) |
| \((B - L)\) | \((0)\) | \((0)\) | \((-4)\) | \((+5)\) |

Table 4: Transformation properties of the new scalars, introduced to avoid chiral anomalies in this model, under \((SU(3)_C, SU(2)_L, U(1)_Y)\) and \((B - L)\).

| Superfield | \(D'_1\) | \(D'_2\) |
|------------|--------|--------|
| \(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y\) | \((1, 2, -1)\) | \((1, 2, -1)\) |
| \((B - L)\) | \((+4)\) | \((-5)\) |

where we have, as done usually in supersymmetric models defined

\[
(AB) \equiv \epsilon_{\alpha\beta} A_\alpha B_\beta, \tag{6}
\]

where \(\alpha = 1, 2\) is a spinorial index and the Yukawa interactions are presented at [1]. However our right-handed neutrinos have different \((B - L)\) charge and it imply our new scalars \(D_1, D_2\) have others \((B - L)\) charges. The motivation to those values will be more clear when we presente our superpotential at Eq.\((??)\).

The new scalars \(D_1, D'_1, D_2, D'_2\) are inert due \(Z_3\) symmetry, therefore, we can write

\[
\langle D_1 \rangle = \langle D_2 \rangle = \langle D'_1 \rangle = \langle D'_2 \rangle = 0. \tag{7}
\]

As a consequence, this fact imply that our gauge bosons have the following masses \([5, 7, 8, 9, 10]\)

\[
M_W = \left(\frac{g v_2}{2}\right) \sqrt{1 + \tan^2 \beta}, \quad M_Z = \frac{M_W}{\cos \theta_W}, \tag{8}
\]

where

\[
\tan \beta = \left(\frac{v_1}{v_2}\right), \quad \tan \theta_W = \left(\frac{g'}{g}\right). \tag{9}
\]
The new parameter $\beta$ is a free parameter and $\theta_W$ is, as usual, the Weinberg angle.

In the non-SUSY model we can write the following Majorana mass term to our right-handed neutrinos [1]

$$M_1(N_{1R})^cN_{1R} + M_{\alpha\beta}(N_{\alpha R})^cN_{\beta R} + h.c., \quad (10)$$

this term in terms of superfield would be translated as

$$M_1\tilde{N}_{1R}\tilde{N}_{1R} + M_{\alpha\beta}\tilde{N}_{\alpha R}\tilde{N}_{\beta R} + h.c., \quad (11)$$

but their result is not a chiral superfield [5, 6] and as consequence this kind of term is not allowed in our superpotential, see Sec.(3.2).

Therefore, to obtain an arbitrary mass matrix for the neutrinos in this model we need to introduce some scalars in the singlet representation, as we present at Tab.(5). Their vev are

$$\langle \phi \rangle = \frac{u_1}{\sqrt{2}}, \quad \langle \bar{\phi} \rangle = \frac{u_2}{\sqrt{2}}. \quad (12)$$

The extra scalar field $S$ is introduced to solve the $\mu$-problem, see [5, 7, 8, 10].

As happen with neutrinos, the higgsinos $\tilde{H}_{1,2}$ do not mix with others Higgsinos, but they mix with the gauginos in the same way as in the MSSM, because they have different $(B-L)$ quantum numbers. The higgsinos $\tilde{D}_1$ and $\tilde{D}'_1$ can mix, the same happen to $\tilde{D}_2$ and $\tilde{D}'_2$. Therefore the higgsinos $\tilde{\varphi}, \bar{\phi}$ and $\tilde{S}$ are already mass eigenstates and they are very massive, due their masses came from soft terms as we will present at Sec.(3.3).

Concerning the gauge bosons and their superpartners, known as gauginos, are introduced in vector superfields [5, 6, 7, 8, 9, 10]. See Tab.(6) where we

| Superfield          | $\tilde{\varphi}$ | $\bar{\phi}$ |
|---------------------|-------------------|---------------|
| $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ | $(1,1,0)$        | $(1,1,0)$     |
| $(B-L)$             | $(-10)$           | $(+8)$        |

Table 5: Transformation properties of the scalars under $(SU(3)_C, SU(2)_L, U(1)_Y)$ and $(B-L)$. 

$$\langle \varphi \rangle = \frac{u_1}{\sqrt{2}}, \quad \langle \phi \rangle = \frac{u_2}{\sqrt{2}}. \quad (12)$$
Table 6: Information on fields contents of each vector superfield of this model. The Latin index \( m \) identify Lorentz index as [5, 6].

we present the particle content together with the gauge coupling constant of each group.

These are the minimal fields, we need to construct this supersymmetric model.

### 2.2 R-Parity

Let us begin defining the \( R \)-parity in the model with the particle content listed above. We define at Tabs.(7,8) the \( R \)-charge \( (n_\Phi) \) of each superfield in our model. Using these \( R \)-charges we can get the \( R \)-Parity of each fermion field contained in these chiral superfield, as shown at [5, 6], these results we shown at Tab.(9).

Table 7: Information about the \( R \)-charge \( (n_\Phi) \) of all the leptons and scalars at chiral superfields of this model, our notation here \( S = H_{1,2}, D_{1,2}, D'_{1,2}, \varphi, \phi \).

Therefore, the lightest supersymmetric particle (LSP) is stable and a possible candidate to Dark Matter, we will discuss about this subject latter in this article.

\(^5\)Here \( \Phi \) means chiral superfield as defined at [5, 6, 7, 8, 9, 10].
| Superfield | $Q_{iL}$ | $\bar{u}_{iR}$ | $\bar{d}_{iR}$ |
|------------|---------|----------------|----------------|
| $R$-charge | $n_L = (+1)$ | $n_u = (-1)$ | $n_d = (-1)$ |

Table 8: Information about the $R$-charge ($n_\Phi$) of quarks at chiral superfields of this model.

| Fermion | $N_1$ | $N_\beta$ | $D_1$ | $D'_1$ | $D_2$ | $D'_2$ | $\bar{\phi}$ | $\phi$ | $S$ |
|---------|-------|----------|-------|--------|-------|--------|-----------|-------|-----|
| $(B - L)$ | 5     | -4       | -4    | +4     | +5    | -5     | -10       | +8    | 0   |
| $R$-Parity | +1    | +1       | -1    | -1     | -1    | -1     | -1        | -1    | -1  |
| Scalar  | $N_1$ | $N_\beta$ | $D_1$ | $D'_1$ | $D_2$ | $D'_2$ | $\phi$    | $\phi$ | $S$ |
| $(B - L)$ | 5     | -4       | -4    | +4     | +5    | -5     | -10       | +8    | 0   |
| $R$-Parity | -1    | -1       | +1    | +1     | +1    | +1     | +1        | +1    | +1  |

Table 9: Information about the $(B - L)$ quantum number and $R$-Parity of new fields of this model.

3 The Lagrangian

With the superfields we can build a supersymmetric invariant lagrangian. It has the following form

$$L^{(B-L)} = L_{SUSY} + L_{soft}.$$  \hspace{1cm} (13)

Here, as usual, $L_{SUSY}$ is the supersymmetric piece, while $L_{soft}$ explicitly breaks SUSY. Below we will write $L_{SUSY}$ in terms of the respective superfields. While in Sec.(3.3) we write $L_{soft}$ in terms of the fields.

3.1 The Supersymmetric terms

The supersymmetric term can be divided as follows

$$L_{SUSY} = L_{Quarks} + L_{Gauge} + L_{Lepton} + L_{Scalar},$$  \hspace{1cm} (14)

the terms $L_{Quarks}, L_{Gauge}$ are the same as in the MSSM and those terms in our notation is presented at [5, 6, 10].
The term $\mathcal{L}_{\text{Lepton}}$ is given by

$$
\mathcal{L}_{\text{Lepton}} = \mathcal{L}_{\text{charged}}^{\text{lepton}} + \mathcal{L}_{\text{neutral}}^{\text{lepton}},
$$

where

$$
\mathcal{L}_{\text{charged}}^{\text{lepton}} = \int d^4\theta \sum_{i=1}^{3} \left[ \hat{L}_{iL} e^{2iA_{W}^{+} + g' \left( -\frac{1}{2} \right) \hat{b}_{i}^{\dagger} \hat{L}_{iL} + \hat{E}_{iR} e^{2iA_{W}^{+} + g' \left( \frac{1}{2} \right) \hat{b}_{i}^{\dagger} \hat{E}_{iR}} \right].
$$

In the expressions above we have used $\hat{W} = T^{i} \hat{W}^{i}$ where $T^{i} = (\sigma^{i}/2)$ (with $i = 1, 2, 3$) are the generators of $SU(2)_{L}$ while $g'$ is the gauge constant of $U(1)_{Y}$ see Table 6. The second term in Eq.(15) is written as

$$
\mathcal{L}_{\text{neutral}}^{\text{lepton}} = \int d^4\theta \left[ \hat{N}_{1R} \hat{N}_{1R} + \sum_{\beta=2}^{3} \hat{N}_{\beta R} \hat{N}_{\beta R} \right].
$$

Therefore, our right handed neutrinos, and the right-handed sneutrinos, they do not interact with the usual gauge bosons then we will refer to them as “fully sterile” right handed neutrinos [11] and they can be Dark Matter candidate, as we will present at the end of Sec.(3.2).

Finally, the scalar part in (14) is

$$
\mathcal{L}_{\text{Scalar}} = \int d^4\theta \left[ \hat{H}_{1} e^{2iA_{W}^{+} + g' \left( \frac{1}{2} \right) \hat{b}_{1}^{\dagger} \hat{H}_{1} + \hat{H}_{2} e^{2iA_{W}^{+} + g' \left( -\frac{1}{2} \right) \hat{b}_{2}^{\dagger} \hat{H}_{2} + \hat{D}_{1} e^{2iA_{W}^{+} + g' \left( \frac{1}{2} \right) \hat{b}_{1}^{\dagger} \hat{D}_{1} + \hat{D}_{2} e^{2iA_{W}^{+} + g' \left( \frac{1}{2} \right) \hat{b}_{2}^{\dagger} \hat{D}_{2} + \hat{D}_{1}' e^{2iA_{W}^{+} + g' \left( -\frac{1}{2} \right) \hat{b}_{1}^{\dagger} \hat{D}_{1}' + \hat{D}_{2}' e^{2iA_{W}^{+} + g' \left( -\frac{1}{2} \right) \hat{b}_{2}^{\dagger} \hat{D}_{2}' + \hat{\varphi} \hat{\varphi} + \frac{\hat{\varphi}^{*} \hat{\varphi}}{2} + \hat{\varphi} \hat{\varphi} + \frac{\hat{\varphi}^{*} \hat{\varphi}}{2} + \int d^2\theta W + h.c \right),
$$

where $W$ is the superpotential, which we discuss in the Sec.(3.2).

### 3.2 The Superpotential

In the non-supersymmetric version of this model, only the doublets $L_{iL}, N_{1R}, N_{\beta R}, S$ of $SU(2)_{L}$ has $w$ as $Z_{3}$ charges; the new scalars $D_{1,2}$ has $w^{-1}$ as $Z_{3}$ charges and all the others fields $E_{iR}$ have this charge as identity [1].

The superpotential of our model is given by

$$
W = \frac{W_{2RC}}{2} + \frac{W_{3RC}}{3} + \frac{W_{2RV}}{2} + \frac{W_{3RV}}{3} + h.c,
$$

---

The right-handed neutrinos and the right-handed sneutrinos
where
\[
W_{2RC} = \mu_H \left( \hat{H}_1 \hat{H}_2 \right) + \mu_{D_1} \left( \hat{D}_1 \hat{D}_1^\dagger \right) + \mu_{D_2} \left( \hat{D}_2 \hat{D}_2^\dagger \right),
\]
\[
W_{3RC} = G^d \left( \hat{H}_2 \hat{Q}_{iL} \right) \hat{d}_{jR} + G^u \left( \hat{H}_1 \hat{Q}_{iL} \right) \hat{u}_{jR} + F_{ij} \left( \hat{H}_2 \hat{L}_{iL} \right) \hat{E}_{jR} + G_1 \left( \hat{D}_1 \hat{L}_{iL} \right) \hat{N}_{1R}
+ G_{i\beta} \left( \hat{D}_2 \hat{L}_{iL} \right) \hat{N}_{\beta R} + H_{11} \hat{\phi} \hat{N}_{1R} \hat{N}_{\beta R} + H_{\alpha\beta} \hat{\phi} \hat{N}_{\alpha R} \hat{N}_{\beta R}.
\] (20)

as usual we have defined
\[
\left( \hat{H}_1 \hat{H}_2 \right) \equiv \epsilon_{\alpha\beta} \hat{H}_1^\alpha \hat{H}_2^\beta.
\] (21)

In general all the parameters \(G^d,u\) and \(F\) are, in principle, complex numbers and they are symmetric in \(ij\) exchange and they are dimensionless parameters [8, 7].

Moreover, \(G^d\) and \(G^u\) can give account for the mixing between the quark current eigenstates as described by the Cabibbo-Kobayashi-Maskawa matrix (CKM matrix). In this model, we can also explain the mass hierarchy in the charged fermion masses as showed recently in [13, 14].

The couplings \(H_{11}\) and \(H_{\alpha\beta}\) will generate Majorana Mass terms to our right-handed neutrinos. Our superpotential is similar to Yukawa terms in the non-SUSY model and therefore three neutrinos get mass at tree level and we have three massless neutrinos, as we will show in Sec.(4.2).

The last two terms \(W_{2RV}, W_{3RV}\), defined at Eq.(19), breaks the \((B-L)\) symmetry and they are written as
\[
W_{2RV} = \mu_{0i} \left( \hat{H}_1 \hat{L}_{iL} \right)
W_{3RV} = \lambda_{ijk} \left( \hat{L}_{iL} \hat{L}_{jL} \right) \hat{E}_{kR},
\] (22)

the first term above allow the mixing between the Higgsinos with the usual leptons. The invariance under \(SU(2)_L\) symmetry of the SM requires the antisymmetry of coupling \(\lambda_{ijk}\) in \(i,j\) and this parameter is, in principle, complex number. Therefore, we can expect to generate the phases of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, responsible for describe neutrino oscillation.

In the MSSM model in \(W_{RV}\) are include the terms \(\lambda'_{ijk} \left( \hat{L}_{iL} \hat{Q}_{jL} \right) \hat{d}_{kR}\) and \(\lambda''_{ijk} \hat{u}_{iR} \hat{d}_{jR} \hat{d}_{kR}\) they are eliminated imposing the following \(Z_3\) symmetry [15]
\[
\hat{L}, \hat{E}, \hat{\bar{N}}, \hat{H}_{1,2}, \hat{D}_{1,2}, \hat{D}'_{1,2}, \hat{\phi}, \hat{\phi} \to \hat{L}, \hat{E}, \hat{\bar{N}}, \hat{H}_{1,2}, \hat{D}_{1,2}, \hat{D}'_{1,2}, \hat{\phi}, \hat{\phi}
\hat{Q} \to w \hat{Q}, \hat{u}, \hat{d} \to w^{-1} \hat{u}, w^{-1} \hat{d}.
\] (23)
where \( w = e^{(2\pi/3)} \), forbids the \( B \)-violating terms then we can avoid the proton decay, neutron-antineutron oscillations at tree level.

Therefore our symmetry are

\[
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L} \otimes Z_3, \tag{24}
\]

where the symmetries \( (B-L) \) and \( Z_3 \) are global ones no local as the symmetry of the SM.

The interactions came at Eq.(22), which appears in the diagram to generate masses to left-handed neutrinos at 1-loop level, is given by

\[
\lambda_{ijk} \left[ \tilde{\nu}_L^i \tilde{R}^k_L \tilde{l}_L^j + \tilde{l}_L^j \tilde{R}^k_L \tilde{\nu}_L^i + (\tilde{l}_R^k)^*(\tilde{\nu}_L^i)^c \tilde{l}_L^j - (i \leftrightarrow j) + h.c. \right]. \tag{25}
\]

The coupling \( \lambda \) can contribute to various (low-energy) process: charged current universality, bound on masses of \( \nu_e, \mu, \tau \) and etc, for more details about this subject see [7, 8].

The interactions in (22) induce neutrino masses, in the MSSM, at 1-loop can be written as [7]:

\[
\delta m_{\nu_k} \propto (\lambda_{kii})^2 \frac{M_S m_i^2}{m_i}, \tag{26}
\]

where \( m_i \) is the mass of exchanged fermion and the factor \( m_i M_S \) comes from the left-right mixing of the sfermion. The interactions in (22) also induce flavour changing neutral currents, for instance, will generater the following decay:

\[
\Gamma \left( \tilde{\nu}_i \rightarrow l_j^{+} l_k^{-} \right) = \frac{1}{16 \pi} (\lambda_{ijk})^2 m_{\tilde{\nu}_i}, \tag{27}
\]

where \( m_{\tilde{\nu}_i} \) is the mass of sneutrinos and this decay violate lepton number conservation and it can generate Leptogenesis in not the same way as in the MSSM\(3RHN \) and we think it can be interesting to perform this analyses in this model. We have another interesting LSP decay, in the case of MSSM with \( R \)-parity conservation, given by

\[
\tilde{\chi}_1^0 \rightarrow \tilde{l}_i l_j \nu_k, \tag{28}
\]

and the decay from the lighest neutralino, the Dark Matter candidate at MSSM with \( R \)-parity conservation, produce missing energy in its decay. This
decay can be observed if the appropriate $\lambda$ coupling satisfy the following relation

$$|\lambda| > 5 \times 10^{-7} \left( \frac{m_{\tilde{l}}}{100\text{GeV}} \right)^2 \left( \frac{100\text{GeV}}{M_{\text{LSP}}} \right)^{5/2},$$  \hspace{1cm} (29)

where $m_{\tilde{l}}$ is the mass of charged slepton exchanged and $M_{\text{LSP}}$ is the mass of LSP, but the neutrinos will escape detection, leading to a missing transverse energy, but this decay will not be a problem in our case because the lightest neutralino is no more our Dark Matter candidate. In this model, the right-handed neutrinos $N_{iR}$ and the right-handed sneutrinos $\tilde{N}_{iR}$ are possible candidates to Dark Matter because they do not interact with the gauge bosons, see Eq.(17), and they interact only with the new scalars, their higgsinos, the leptons and sleptons, via the superpotential defined at Eq.(??), in a similar way as done at [1, 6] but we will not consider this issue here.

### 3.3 Soft terms

Now we can write the soft terms as

$$\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{GMT}} + \mathcal{L}_{\text{SMT}} + \mathcal{L}_{\text{INT}}.$$  \hspace{1cm} (30)

The first term $\mathcal{L}_{\text{GMT}}$ give masses to all the gauginos in this model and it is written as

$$\mathcal{L}_{\text{GMT}}^{\text{MSSM}} = -\frac{1}{2} \left[ \left( M_{3} \sum_{a=1}^{8} \lambda_{C}^{a} \lambda_{C}^{a} + M \sum_{i=1}^{3} \lambda_{i} \lambda_{i} + M' \lambda \lambda \right) + h_{c} \right].$$  \hspace{1cm} (31)

The term $\mathcal{L}_{\text{SMT}}$, known as scalars mass term, is given by:

$$\mathcal{L}_{\text{SMT}} = - \left( \sum_{i=1}^{3} \left[ M_{L}^2 |\bar{L}_{iL}|^2 + M_{L}^2 |\bar{E}_{iR}|^2 \right] + M_{N_1}^2 |\bar{N}_{iR}|^2 + \sum_{\beta=2}^{3} M_{\tilde{N}_{\beta}}^2 |\bar{N}_{\beta R}|^2 + M_{H_1}^2 |H_1|^2 \right.$$  
$$+ \left. M_{H_2}^2 |H_2|^2 + M_{D_1}^2 |D_1|^2 + M_{D_1}^2 |D'_1|^2 + M_{D_2}^2 |D_2|^2 + M_{D_2}^2 |D'_2|^2 + M_{\tilde{\phi}}^2 |\tilde{\phi}|^2 \right.$$  
$$+ \left. M_{\phi}^2 |\phi|^2 \right) + [\beta_{H_1} (H_1 H_2) + \beta_{D_1} (D_1 D'_1) + \beta_{D_2} (D_2 D'_2) + h_{c}].$$  \hspace{1cm} (32)
The last term is given by

\[ \mathcal{L}_{\text{Int}} = \sum_{i,j,k=1}^{3} A_{ij} G_{ij} \left( H_2 \tilde{L}_{iL} \right) \tilde{E}_{jR} + A_{i1} G_{i1} \left( D'_1 \tilde{L}_{iL} \right) \tilde{N}_{1R} + \sum_{\beta=2}^{3} A_{i\beta} P_{i\beta} \left( D'_2 \tilde{L}_{iL} \right) \tilde{N}_{\beta R} \]

\[ + A_{11} M_{11} H_{11} \phi \tilde{N}_{1R} \tilde{N}_{1R} + \sum_{\alpha=2}^{3} \sum_{\beta=2}^{3} A_{\alpha\beta} H_{\alpha\beta} \tilde{N}_{\alpha R} \tilde{N}_{\beta R} + A_{ij} d_{ij} \left( \tilde{H}_2 \tilde{Q}_{iL} \right) \tilde{d}_{jR} \]

\[ + A_{ij} G_{ij} \left( H_1 \tilde{Q}_{iL} \right) \tilde{u}_{jR} + A_{ijk} \left( \tilde{L}_{iL} \tilde{L}_{jL} \right) \tilde{E}_{kR} + hc \right\}. \]  

(33)

The terms \( A^\nu, A^M \) can generate one physical CP violating phase at sneutrinos mass matrix [16, 17, 18, 19].

The \( A \)-terms are known to play an important role in Affleck-Dine baryogenesis [3, 10], as well as in the inflation models based on supersymmetry [20, 21, 22].

4 Fermion masses

We will present the masses of the usual fermions, it means the charged fermions, charginos, right-handed neutrinos, left-handed neutrinos and neutralinos at tree level.

4.1 Charged Lepton masses

In this case, it is possible to give mass to all charged fermions. Denoting

\[ \phi^+ = (e^+, \mu^+, \tau^+, -i\tilde{W}^+, \tilde{h}_1^+)^T, \]

\[ \phi^- = (e^-, \mu^-, \tau^-, -i\tilde{W}^-, \tilde{h}_2^-)^T, \]  

(34)

where the winos, the supersymmetric \( W^\pm \)-boson partner, are defined as

\[ \tilde{W}^\pm = \frac{1}{\sqrt{2}} \left( \tilde{W}^1 \mp i\tilde{W}^2 \right), \]  

(35)

all the fermionic fields are still Weyl spinors, we can define \( \Psi^\pm = (\phi^+, \phi^-)^T \), and the mass term \(-(1/2)[\Psi^T Y^\pm \Psi^\pm + hc]\) where \( Y^\pm \) is the mass matrix given by:

\[ Y^\pm = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}, \]  

(36)
with

\[
X = \begin{pmatrix}
-F_{ee} v_2 & -F_{e\mu} v_2 & -F_{e\tau} v_2 & 0 & 0 \\
-F_{e\mu} v_2 & -F_{\mu\mu} v_2 & -F_{\mu\tau} v_2 & 0 & 0 \\
-F_{e\tau} v_2 & -F_{\mu\tau} v_2 & -F_{\tau\tau} v_2 & 0 & 0 \\
\mu_0 e & \mu_{0\mu} & \mu_{0\tau} & \sqrt{2} M_W c_\beta & \mu_H
\end{pmatrix},
\]

(37)

with \( s_\beta = \sin \beta, c_\beta = \cos \beta \) is defined the parameter \( \beta \) is defined at Eq.(9), as we have presented at [23].

The charginos mass matrix, is in similar way as done in the MSSM, diagonalized by two unitaries matrix \( U \) and \( V \) [5, 6, 7, 8, 9, 10]

\[
\tilde{\chi}_i^+ = \sum_{j=1}^{5} U_{ij} \phi_j^+ ,
\]

\[
\tilde{\chi}_i^- = \sum_{j=1}^{5} V_{ij} \phi_j^- .
\]

(38)

In the Eq.(37), we have two sector. The first sector is very light and we will associate it with the usual charged lepton. The second sector has heavier masses than the first one, the usual charginos at MSSM, for more details see [23].

The masses of the quarks are the same as in the MSSM [5, 6, 7, 8, 9, 10].

4.2 Mixing Neutrinos with Gauginos at Tree level

Our right-handed neutrinos get Majorana mass term, due the following terms

\[
H_{11} \varphi N_{1R} N_{1R} + H_{a\beta} \phi N_{aR} N_{\beta R},
\]

(39)

in our superpotential defined at Eq.(20). As the new scalars \( D_1, D_2 \) are innert due \( (B - L) \) symmetry, see Eq.(7), the Yukawa couplings

\[
G_i (D_1 L_iL) N_{1R} + P_{i\beta} (D_2 L_iL) N_{\beta R} + hc,
\]

(40)

coming from our superpotential, defined at Eq.(20), does not generate mixing between left-handed neutrinos with the right-handed neutrinos, in similar way as happen in the non-SUSY model [1].
Defining the basis $\Psi^0 = (N_1, N_2, N_3, \nu_e, \nu_\mu, \nu_\tau, -i\tilde{W}^3, -i\tilde{b}', \tilde{h}_0, \tilde{h}_1)^T$, the mass term is $-(1/2)[\Psi^{0T}Y^0\Psi^0 + h.c]$, where $Y^0$ is the mass matrix given by

$$Y^0 = \begin{pmatrix} M_M & 0_{3\times3} & 0_{4\times3} \\ 0_{3\times3} & 0_{3\times3} & m_{4\times3} \\ 0_{3\times4} & (m_{4\times3})^T & M_{\text{MSSM}}^{\text{neu}} \end{pmatrix}.$$  \hfill (41)

We see, we have two sector. The first ones is given by the right-handed neutrinos and the second one are, similar, to the neutralinos from the MSSM.

In order to write $M_M$, first we define $\Gamma$ through the ratio

$$\tan \Gamma = \frac{u_1}{u_2},$$ \hfill (42)

and $M_M$ reads

$$M_M = \frac{2u_2}{\sqrt{2}} \begin{pmatrix} f_{11}^M & 0 & 0 \\ 0 & f_{22}^M \tan \Gamma & f_{23}^M \tan \Gamma \\ 0 & f_{32}^M \tan \Gamma & f_{33}^M \tan \Gamma \end{pmatrix}.$$ \hfill (43)

This sector is the same as the right-handed neutrinos at non-SUSY model presented at [1].

In the neutralinos sector, we can define, the following mass matrices

$$m_{4\times3} = \begin{pmatrix} 0 & 0 & 0 & -\mu_{0e} \\ 0 & 0 & 0 & -\mu_{0\mu} \\ 0 & 0 & 0 & -\mu_{0\tau} \end{pmatrix},$$

$$M^{neu}_{\text{MSSM}} = \begin{pmatrix} \begin{pmatrix} M \\ 0 \end{pmatrix} & \begin{pmatrix} M_z s_\beta c_W & -M_z c_\beta c_W \\ M_z s_\beta s_W & -M_z c_\beta s_W \end{pmatrix} \\ \begin{pmatrix} -M_z c_\beta c_W \\ -M_z c_\beta s_W \end{pmatrix} & \begin{pmatrix} \mu_H \\ \mu_H \end{pmatrix} \end{pmatrix}.$$ \hfill (44)

with $s_W = \sin \theta_W$, $c_W = \cos \theta_W$ where $\theta_W$ is the weak mixing. This mass matrix is diagonalized by an unitary $7 \times 7$ matrix $N$, defined as [5, 6, 7, 8, 9, 10]

$$\tilde{\chi}_i^0 = \sum_{j=1}^{7} N_{ij} \phi_j^0,$$ \hfill (45)

where we have defined $\phi^0 = (\nu_e, \nu_\mu, \nu_\tau, -i\tilde{W}^3, -i\tilde{b}', \tilde{h}_0, \tilde{h}_1)^T$. 

We obtain besides the two massless neutrinos a massive one with, we can choose the parameters in such way that \( m_{\nu_H} = 5 \times 10^{-2} \text{ eV} \), to explain the atmospheric neutrino, in similar way as done at [23].

The mixing PMNS matrix is defined as

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}_L = U_{PMNS}
\begin{pmatrix}
\nu'_e \\
\nu'_0 \\
\nu_H
\end{pmatrix}_L
\] (46)

when we have one massive neutrino massive \( \nu_H \) and two massles \( \nu'_e \) and \( \nu'_0 \) we can define \( U_{PMNS} \) in the following way [24]

\[
U_{PMNS} = \begin{pmatrix}
1 & \left( \frac{c_{23}}{s_{23}} \right) \theta_1 & \theta_1 \\
-\left( \frac{s_{23}}{c_{23}} \right) c_{23} \left( 1 + \sin^2 \theta_{23} \frac{m_{\nu_e}}{m_{\nu_H}} \right) & s_{23} \left( 1 - \cos^2 \theta_{23} \frac{m_{\nu_e}}{m_{\nu_H}} \right) & s_{23} \left( 1 + \sin^2 \theta_{23} \frac{m_{\nu_e}}{m_{\nu_H}} \right) \\
0 & -s_{23} \left( 1 - \cos^2 \theta_{23} \frac{m_{\nu_e}}{m_{\nu_H}} \right) c_{23} \left( 1 + \sin^2 \theta_{23} \frac{m_{\nu_e}}{m_{\nu_H}} \right) & c_{23} \left( 1 + \sin^2 \theta_{23} \frac{m_{\nu_e}}{m_{\nu_H}} \right)
\end{pmatrix}
\] (47)

where \( s_{23} = \sin \theta_{23} \), \( c_{23} = \cos \theta_{23} \) and \( (m_{\nu_e}/2) \sim 3 \times 10^{-3} \text{ eV} \) is the mass of neutrino of tau and it is an order of magnitude smaller then \( m_{\nu_H} \).

It is natural, we choose \( \mu_{0e} \ll \mu_{0\mu} \approx \mu_{0\tau} \), because they break softly our \( B - L \) global symmetry. In this case, we can define a small angle \( \theta_1 \) defined as

\[
\theta_1 \approx \frac{\mu_{0e}}{\sqrt{\left| \mu_{0e} \right|^2 + \left| \mu_{0\mu} \right|^2 + \left| \mu_{0\tau} \right|^2}}
\] (48)

we can solve the solar neutrino problem imposing the following constraints \( \sin^2 \theta_1 \sim 10^{-2} \). There is a big mixing angle \( \theta_{23} \) in the following way

\[
\sin \theta_{23} \approx \frac{\mu_{0\mu}}{\sqrt{\left| \mu_{0e} \right|^2 + \left| \mu_{0\mu} \right|^2 + \left| \mu_{0\tau} \right|^2}}, \\
\cos \theta_{23} \approx \frac{\mu_{0\tau}}{\sqrt{\left| \mu_{0e} \right|^2 + \left| \mu_{0\mu} \right|^2 + \left| \mu_{0\tau} \right|^2}}
\] (49)

To explain the atmospheric neutrinos data we can choose \( \theta_{23} \sim (\pi/4) \).

Therefore only the right handed neutrinos get masses at tree level as happen at non-SUSY model presented at [1]. Therefore, the two left-handed neutrinos are massless and they will get masses at 1-loop level, as we will discuss at Sec.(6).
5 Scalar Potential

The Higgs potential of our model has the following form

\[ V_{\text{scalar}} = V_F + V_{\text{soft}} + V_D. \]  \hspace{1cm} (50)

where, we are only writing the terms of scalars in doublets because we want to compare our scalar potential with ones presented at non-SUSY model presented at [1]

\[ V_F = \frac{\mu_H^2}{4} (|H_1|^2 + |H_2|^2) + \frac{\mu_{D_1}^2}{4} (|D_1|^2 + |D_1'|^2) + \frac{\mu_{D_2}^2}{4} (|D_2|^2 + |D_2'|^2), \]

\[ V_{\text{soft}} = M_H^2 |H_1|^2 + M_{D_1}^2 |D_1|^2 + M_{D_1}'^2 |D_1'|^2 + M_D^2 |D_2|^2 + M_{D_2}'^2 |D_2'|^2 \]

\[ - [\beta_H (H_1 H_2) + \beta_{D_1} (D_1 D_1') + \beta_{D_2} (D_2 D_2') + h_c], \]

\[ V_D = \left( \frac{(g')^2}{8} + \frac{g^2}{8} \right) (|H_1|^2 + |H_2|^2) + \left( \frac{(g')^2}{8} + \frac{g^2}{8} \right) (|D_1|^2 + |D_1'|^2) \]

\[ + \left( \frac{(g')^2}{8} + \frac{g^2}{8} \right) (|D_2|^2 + |D_2'|^2) \]

\[ + \frac{g^2}{4} (|H_1|^2 + |H_2|^2) + |H_1| |D_1| + |H_1| |D_1'| + |H_2| |D_2| + |H_2| |D_2'| \]

\[ + |D_1|^2 + |D_1'|^2 + |D_2|^2 + |D_2'|^2 \]

\[ - \left( \frac{(g')^2}{4} + \frac{g^2}{8} \right) (|H_1|^2 |H_2|^2 + |H_1|^2 |D_1|^2 + |H_1|^2 |D_1'|^2 + |H_2|^2 |D_2|^2 + |H_2|^2 |D_2'|^2) \]

\[ + |D_1|^2 |D_1'|^2 + |D_1^2| |D_2|^2 + |D_2|^2 |D_2'|^2 \]

\[ + \left( \frac{(g')^2}{4} - \frac{g^2}{8} \right) (|H_1|^2 |D_1|^2 + |H_1|^2 |D_1'|^2 + |H_2|^2 |D_2|^2 + |H_2|^2 |D_2'|^2 + |D_1|^2 |D_2|^2 \]

\[ + |D_1|^2 |D_2'|^2), \]  \hspace{1cm} (51)

and we see we can these scalar potential are very similar to ones defined in the non-SUSY model presented at [1]. As it is done at MSSM when compare with the Two Higgs Double Model, we can write the following relations

\[ \mu_{SM}^2 = \frac{\mu_H^2}{4} + M_H^2, \]

\[ \mu_{D_{11}}^2 = \frac{\mu_{D_1}^2}{4} + M_{D_1}'^2, \]
\[ \mu_{d_2}^2 = \frac{\mu_{D_2}^2}{4} + M_{D_2}^2, \]
\[ \lambda_1 = \lambda_2 = \lambda_3 = \left( \frac{(g')^2}{8} + \frac{g^2}{8} \right), \]
\[ \lambda_4 = \lambda_5 = \lambda_6 = \left( \frac{(g')^2}{8} - \frac{g^2}{8} \right), \]
\[ \lambda_7 = \lambda_8 = \frac{g^2}{4}, \]

we can conclude we can not get the terms \( \mu_{12}^2, \lambda_{9,10} \) defined in the non-SUSY model and therefore we can not have scotogenic mechanism, but we will still have the 1-loop correction drawn at their Figure 1, as we will discuss in the next section.

In order to generate the term \( \mu_{12}^2 \), as it is quadratic in the usual scalars we would have to introduce term like
\[ \bar{D}_1 D_2, \]

in our superpotential defined at Eq.(19), but it is not chiral and therefore we can not introduce it. As we can not introduce this term we can not have
\[ \mu_{12}^2 D_1 D_2, \]

at soft terms, see Eq.(33), and therefore due SUSY algebra we can not generate this term in this supersymmetric model.

We can not generate the terms proportional to \( \lambda_{9,10} \) and it is due the fact to generate them we need to introduce term like
\[ \tilde{H} e^{2i(g\bar{W})} \bar{D}_{1,2}, \]

this term would give the following contribution to \( D \)-term
\[ g D^i H^i T^i D_{1,2} \]
and
\[ (H^i \sigma^i D_{1,2})(H^i \sigma^i D_{1,2}) = 2(H^i D_{1,2})(H^i D_{1,2}) - (H^i D_{1,2})(H^i D_{1,2}) = (H^i D_{1,2})^2, \]

but this term is not invariant under our global \( (B - L) \) symmetry.
6 1-loop mechanism to generate masses to the left-handed neutrinos

The left-handed neutrinos get masses at 1-loop level due the following interaction defined at Eq.(40). As we have the couplings $\lambda_4$ and $\lambda_5$, see Eq.(52), we can get the first diagram to generate neutrinos masses at 1-loop level drawn at Figure 1 presented at [1].

In this supersymmetric case, we can get two more contribution to generate neutrinos masses at 1-loop level. The first contribution is due the sneutrinos masses and it is drawn at Figure 1. The coupling $\lambda_{ijk}$ defined at Eq.(22) generate the 1-loop diagrams drawn at Figure 2.

The quartic interactions between left-handed neutrino-left handed sneutrino-usual scalars is the same as appear at MSSM therefore it is given by [7]

$$ i \left( D[\phi, \phi', \tilde{f}, \tilde{f}] + D[\phi', \phi, \tilde{f}', \tilde{f}] \right), $$

for $\phi = \phi'$, only one of the two contributions is non zero, whereas for $\phi \neq \phi'$ both contributions are equal. The coefficients of various quartic interaction are:

$$
D[H, H, \tilde{\nu}_i, \tilde{\nu}_j] = -d_y[\tilde{\nu}_i] c_{2\alpha} \delta_{ij}, \quad D[H, h, \tilde{\nu}_i, \tilde{\nu}_j] = 2d_y[\tilde{\nu}_i] s_{2\alpha} \delta_{ij}, \\
D[h, h, \tilde{\nu}_i, \tilde{\nu}_j] = d_y[\tilde{\nu}_i] c_{2\beta} \delta_{ij}, \quad D[A, A, \tilde{\nu}_i, \tilde{\nu}_j] = d_y[\tilde{\nu}_i] c_{2\beta} \delta_{ij}, \\
D[H, H, \tilde{l}_s, \tilde{l}_t] = -d_Y[\tilde{l}_s, \tilde{l}_t] c_{2\alpha}^2 - d_y[\tilde{l}_s, \tilde{l}_t] c_{2\alpha}, \quad D[H, h, \tilde{l}_s, \tilde{l}_t] = d_Y[\tilde{l}_s, \tilde{l}_t] s_{2\alpha} + 2d_y[\tilde{l}_s, \tilde{l}_t] s_{2\alpha}, \\
D[h, h, \tilde{l}_s, \tilde{l}_t] = -d_Y[\tilde{l}_s, \tilde{l}_t] s_{2\beta}^2 + d_y[\tilde{l}_s, \tilde{l}_t] c_{2\alpha}, \quad D[A, A, \tilde{l}_s, \tilde{l}_t] = -d_Y[\tilde{l}_s, \tilde{l}_t] s_{2\beta}^2 + d_y[\tilde{l}_s, \tilde{l}_t] c_{2\beta},
$$

(59)

where the rotations angle $\alpha$ and $\beta$ seen to obey the relations [7, 8]

$$
\sin(2\alpha) = -\frac{M_{H^0}^2 + M_{H^0}^2}{M_{H^0}^2 - M_{\tilde{h}_0}^2} \sin(2\beta), \quad \cos(2\alpha) = -\frac{M_{\tilde{A}^0}^2 - M_{\tilde{Z}}^2}{M_{H^0}^2 - M_{\tilde{h}_0}^2} \cos(2\beta), \\
\tan(2\alpha) = \frac{M_{H^0}^2 + M_{H^0}^2}{M_{\tilde{A}^0}^2 - M_{\tilde{Z}}^2} \tan(2\beta),
$$

(60)

where $h$ is the lightest CP-even Higgs, $H$ the heavy CP-even Higgs and $A$ is the pseudo-scalar of MSSM. For any angle $\zeta$, we use $s_\zeta, c_\zeta, t_\zeta$ to mean $\sin\zeta$, $\cos\zeta$ and $\tan\zeta$ respectively.
We have, also, defined

\[ d_g[\tilde{\nu}_i] = \frac{g^2}{8} \left( 1 + t_W^2 \right), \quad d_g[\tilde{l}_s, \tilde{l}_t] = \frac{g^2}{4M_W^2 c_W^2} m_f^2 \cos (\theta_s - \theta_t), \]

\[ d_Y[\tilde{l}_s, \tilde{l}_t] = -\frac{g^2}{8} \left[ 2t_W^2 \sin \theta_s \sin \theta_t + \cos \theta_s \cos \theta_t \left( 1 - t_W^2 \right) \right], \]

where \( \theta_f \) is the mixing angle at charged slepton, \( m_f^2 \) their mass, while the symbols to Weinberg angle \( \theta_W \) are \( s_W, c_W, t_W \).

In the neutralinos sector, the mass eigenstates, see [5], are defined as

\[ \tilde{\chi}_0^i = Z_{ij} \psi_j^0, \quad i, j = 1 \ldots 4, \]

where

\[ \psi_j^0 = \left( -i \tilde{W}^3, -i \tilde{b}', \tilde{H}_1, \tilde{H}_2 \right)^T. \]

Therefore, the light sector of neutralinos of this model is exactly the same as in the MSSM and we can say that \( M_{\tilde{\chi}_0^i} \sim \mathcal{O}(100) \text{ MeV} \) as we hope at MSSM.

The mass matrix to the sneutrinos when we have three right-handed neutrinos can be write as [6]

\[ M_{\text{sneutrinos}}^2 = \begin{pmatrix} m_{\tilde{\nu}_e}^2 & A^\nu \nu^0 v_2 \\ A^\nu \nu^0 v_2 & m_{\tilde{\nu}_\tau}^2 \end{pmatrix}, \]

but \( m_\nu = A^\nu \nu^0 v_2 \) is the neutrinos masses at tree level and \( \tilde{\nu}_e, \tilde{\nu}_\nu \) and \( \tilde{\nu}_\tau \) are already mass eigenstates, as first approximation, and we will represent them as \( \tilde{\nu}_\alpha \). Therefore the left handed neutrino \( \nu_{\alpha} \) can couple only with its left-handed sneutrinos, this vertex came from \( \mathcal{L}_{\text{lepton}}^{\text{charged}} \) defined at Eq.(16).

In this model, the left-handed neutrinos do not get masses at tree level, and the coupling at soft supersymmetric terms

\[ A^\nu_{i1} G^\nu_1 \left( D_1 \bar{L}_{iL} \right) \bar{N}_{1R} + A^\nu_{i3} F^\nu_{i3} \left( D_2 \bar{L}_{iL} \right) \bar{N}_{\beta R} + h.c, \]

will induce an effective mixing between our left-handed sneutrinos with the right-handed sneutrinos and it is the responsible for the mixing in this sector.
The interactions between left-handed neutrino-left handed sneutrino-neutralino, came from \( L_{\text{lepton}}^{\text{charged}} \), see Eqs.(16,58), that appear in our 1-loop correction drawn at Figure 1, is written as

\[
-\frac{ig}{\sqrt{2}} \nu_\alpha \tilde{W}^3 \bar{\nu}_\alpha^\dagger - \frac{ig'}{\sqrt{2}} \nu_\alpha \tilde{b} \bar{\nu}_\alpha^\dagger + hc, \tag{66}
\]

now using Eqs.(62,63) at equation above we get

\[
-\frac{ig Z_{11}^*}{\sqrt{2}} \sin \theta_W \nu_\alpha \tilde{\chi}_0 \bar{\nu}_\alpha^\dagger - \frac{ig' Z_{12}^*}{\sqrt{2}} \cos \theta_W \nu_\alpha \tilde{\chi}_0 \bar{\nu}_\alpha^\dagger + hc, \tag{67}
\]

this is the vertices give contribution to generate masses to left-handed neutrinos at 1-loop level, its diagram is drawn at Figure 1, those kind of mechanism were discussed at [12, 25].

![Figure 1: The one loop correction to the masses of \( m_{\nu_\alpha} \) including (gauginos)neutralino-neutrino-sneutrino vertices, where \( \alpha = e, \mu, \tau \), this vertices came from Eq.(67), and is given by \((-i/2)(g Z_{11}^* \sin \theta_W + g' Z_{12}^* \cos \theta_W)\).](image)

Our right-handed sneutrinos \( \tilde{N}_{iR} \) are possible candidates to be the Dark Matter together with the right handed neutrinos \( N_{iR} \).

The coupling \( \lambda_{ijk} \) defined at Eq.(22) and the mixing between the sleptons is defined as

\[
\begin{pmatrix}
\tilde{l}_1 \\
\tilde{l}_2
\end{pmatrix}
= 
\begin{pmatrix}
\cos \theta_{\tilde{l}} & \sin \theta_{\tilde{l}} \\
-\sin \theta_{\tilde{l}} & \cos \theta_{\tilde{l}}
\end{pmatrix}
\begin{pmatrix}
\tilde{l}_L \\
\tilde{l}_R
\end{pmatrix}, \tag{68}
\]

\footnote{We get Majorana Mass terms to our neutrinos}
Therefore, the charged sleptons sector of this model is exactly the same as in the MSSM. The charged sleptons generate the 1-loop diagrams drawn at Figure 2.

Figure 2: The one loop correction to the masses of $m_{\nu_\alpha}$ including charged lepton-neutrino-charged slepton vertices, this vertices came from $W_{RV}$, see Eq.(22), and the left vertices is proportional to $\lambda_{\alpha st}$ and the right ones is $\lambda_{\beta st}$.

The contribution to neutrinos masses at 1-loop correction is we get the following one loop correction to the left-handed neutrinos masses expressions to the $\nu_e$

$$
(m_{\nu})_{ij} = \frac{G_iG_jM_1}{32\pi^2} \left[ \frac{m_{R1}^2}{m_{R1}^2 - M_1^2} \ln \left( \frac{m_{R1}^2}{M_1^2} \right) - \frac{m_{I1}^2}{m_{I1}^2 - M_1^2} \ln \left( \frac{m_{I1}^2}{M_1^2} \right) \right] \\
+ \frac{F_{ik}F_{jk}M_K}{32\pi^2} \left[ \frac{m_{R2}^2}{m_{R2}^2 - M_k^2} \ln \left( \frac{m_{R2}^2}{M_k^2} \right) - \frac{m_{I2}^2}{m_{I2}^2 - M_k^2} \ln \left( \frac{m_{I2}^2}{M_k^2} \right) \right] \\
+ \frac{(gZ_{1i}^* \sin \theta_W + g'Z_{12}^* \cos \theta_W)^2}{64\pi^2} \left[ \frac{m_{\tilde{\nu}_i}^2}{m_{\tilde{\nu}_i}^2 - M_{\tilde{\chi}_j^0}^2} \ln \left( \frac{m_{\tilde{\nu}_i}^2}{M_{\tilde{\chi}_j^0}^2} \right) - \frac{m_{\tilde{\nu}_j}^2}{m_{\tilde{\nu}_j}^2 - M_{\tilde{\chi}_i^0}^2} \ln \left( \frac{m_{\tilde{\nu}_j}^2}{M_{\tilde{\chi}_i^0}^2} \right) \right] \\
+ \frac{\lambda_{\alpha st}^2 \lambda_{\beta st}^2}{16\pi^2} \sin^2 (\theta_s + \theta_t) \left[ \frac{m_s^2 M_s}{m_s^2 - M_s^2} \ln \left( \frac{m_s^2}{M_s^2} \right) - \frac{m_t^2 M_t}{m_t^2 - M_t^2} \ln \left( \frac{m_t^2}{M_t^2} \right) \right],
$$

(69)

where $Z_{lk}$ is the mixing in the neutralino sector, see Eq.(62), while $M_{\tilde{\nu}_j}$ is the mass of left-handed sneutrinos while $m_{\tilde{\chi}_i^0}$ is the mass of exchanged neutralinos.
In the last line, $m_{\tilde{\chi}_0}$ is the neutralinos exchanged. The parameter $\theta_{\tilde{s}}$ is the mixing in sleptons defined at Eq.(68), while $m_s$ is the mass of the exchanged lepton and $m_{\tilde{s}}$ is the mass of their respective slepton.

7 Conclusions

We presented the Supersymmetric version of the model presented at [1] in the superfield formalism and we present also briefly some phenomenological consequences of this model. We have shown we have similar scalar potential as in the non-SUSY model, however in this Supersymmetric model there are no the couplings $\mu_{12}, \lambda_{9,10}$ then we can not have scotogenic mechanism, our neutrinos are Majorana particles. We also studied the mechanism to generate Majorana mass term at tree level to the right-handed neutrinos. The left-handed neutrinos get their masses at 1-loop level in this model, we show we still have the contribution arise from Figure 1 from the non-SUSY model [1], but now we get two more contribution. The first one come from the neutralino-neutrino-sneutrino vertices drawn at Figure 1. The second arise from the charged lepton-neutrino-charged slepton interaction presented at Figure 2. We have shown a realistic radiative mechanism to generate masses to left-handed neutrinos with candidates for Dark Matter. In this model, the right-handed neutrinos and right-handed sneutrinos can be the Dark Matter candidate, it means this model has several possibilities for Dark Matter and we think it will be nice to study in more detail this subject.

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.1 Scalar Potential

The first term at Eq.(50) is

$$V_F = \sum_i F_i^\dagger F_i,$$

(70)
where \( l = H_{1,2}, D_{1,2}, D'_{1,2}, \varphi, \phi; \) the \( F \) terms are

\[
\begin{align*}
F_{H_1}^\dagger &= -\frac{\mu_H}{2} H_2, \quad F_{H_2}^\dagger = -\frac{\mu_H}{2} H_1 - \frac{G_\nu^l}{3} \bar{L}_i L \bar{E}_{jR}, \\
F_{D_1}^\dagger &= -\frac{\mu_{D_1}}{2} D_1', \quad F_{D_1'}^\dagger = -\frac{\mu_{D_1}}{2} D_1 - \frac{G_\nu^\nu}{3} \bar{L}_i L \bar{N}_{1R}, \\
F_{D_2}^\dagger &= -\frac{\mu_{D_2}}{2} D_2', \quad F_{D_2'}^\dagger = -\frac{\mu_{D_2}}{2} D_2 - \frac{F_{\gamma}^{\nu j}}{3} \bar{L}_i L \bar{N}_{\beta R}, \\
F_{\varphi}^\dagger &= -\frac{H_{\alpha \beta}}{3} \bar{N}_{1R} \bar{N}_{1R}, \quad F_{\phi}^\dagger = -\frac{H_{\alpha \beta}}{3} \bar{N}_{\alpha R} \bar{N}_{\beta R}.
\end{align*}
\]

The soft term that contribute to the scalar potential, see Eq.(33), is given by

\[
V_{soft} = -\mathcal{L}_{SMT} - \mathcal{L}_{int}.
\]

The third term at Eq.(50) is

\[
V_D = \frac{1}{2} \left[ D^i D^i + (D')^2 \right],
\]

where \( i = 1, 2, 3 \). There is one \( D \)-term came from \( \mathcal{L}_{Scalar} \) and from superpotential for each of the four gauge groups

\[
\begin{align*}
SU(2)_L : \quad D^i &= -\frac{g}{2} \left[ H_1^i \sigma^i H_1 + H_2^i \sigma^i H_2 + D_1^i \sigma^i D_1 + D_1'^i \sigma^i D_1' + D_2^i \sigma^i D_2 + D_2'^i \sigma^i D_2' \right], \\
U(1)_Y : \quad D' &= -\frac{g'}{2} \left[ |H_1|^2 - |H_2|^2 + |D_1|^2 - |D_1'|^2 + |D_2|^2 - |D_2'|^2 \right],
\end{align*}
\]

where \( |H_1|^2 \equiv H_1^\dagger H_1 \) as usual.

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