Transient evolitional behaviours of double-control electromagnetically induced transparency

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Abstract. The evolitional optical behaviours (turn-on dynamics) of a four-level double-control tripod-configuration (electromagnetically induced transparency) system are considered based on the transient solution to the equation of motion of the probability amplitudes of the atomic levels. As the most remarkable property (quantum interference between the two control transitions) will arise in the present tripod-configuration system, the transient evolution of the permittivity in cases of both destructive and constructive quantum interferences is presented. It can be shown that the four-level double-control vapour can become a destructive-interference medium, (exhibiting a two-level resonant absorption) and a constructive-interference medium, (exhibiting transparency to the probe field), respectively, under certain conditions (related to the ratio of the two control field intensities). The present double-control scenario can be applicable to designs of some new photonic and quantum optical devices such as logic and functional devices (logic gate circuits) and the key component of the technology of quantum coherent information storage.
1. Introduction

During the past two decades, quantum coherence (atomic phase coherence) effects have exhibited many physically interesting phenomena such as electromagnetically induced transparency (EIT) [1], light amplification without inversion [2], spontaneous emission cancellation [3], multi-photon population trapping [4], coherent phase control [5, 6] and sensitive optical responses in waveguides [7]. Recently, quantum coherence has received increasingly more attention from physicists in the area of quantum information (communication), e.g. EIT-based coherent information storage [8, 9] and quantum logic gates [10]–[12]. Since transient evolution accompanies these physical processes (information storage and logic-gate operations) when switching on and off the control fields, the time-dependent behaviours of EIT media deserve consideration. In the literature, Li et al investigated the resonant transient properties induced by a quantum interference effect in a three-level EIT system [13]. These transient properties included the absorption for the probe field, transient gain without population inversion and enhancement of dispersion in the three-level EIT atomic medium when the coupling laser is switched on [13]. Greentree et al studied the resonant and off-resonant transient behaviours of EIT in a three-level medium [14]. Yao et al first studied the transient optical responses of a four-level $N$-configuration system under certain approximation conditions [15]. More recently, we presented a general formulation for the transient evolution of the susceptibility (and absorption) induced by the quantum interference and phase coherence in this $N$-configuration medium [16]. Apart from the four-level $N$-configuration system, there is another interesting four-level system (tripod configuration) that can exhibit nontrivial double-control destructive and constructive quantum interferences [17]–[20]. In this tripod-configuration system (see figure 1 for the energy-level diagram), one can manipulate the probe transitions (and hence the optical properties of the atomic vapour) via the double-control destructive and constructive quantum interferences [20]. Here, the three lower levels can form a three-level dark state [20] that can be viewed as a generalization of the two-level dark state (consisting of the probe and control levels) that appears in a conventional three-level EIT system [1].

The transient evolution is a very important physical process when one considers the mechanism of storage and readout of pulses in technology of quantum coherent information storage. The double-control EIT exhibits an interesting quantum coherent effect, in which the resonant probe laser beam will get absorbed if the destructive quantum interference between the two control beams arises, and the probe field (and hence the information) would be stored.
in the double-control atomic system; but it would be read out if the constructive quantum interference between the two control transitions occurs. We believe that the scheme of double-control EIT in vapours and quantum dot (QD) materials would be promising in developing the coherent storage technology. In this paper, the transient dynamics of a four-level tripod-configuration atomic system shortly after the control laser beams are switched on will be considered. We first obtain the steady probability amplitudes of the atomic levels and then derive the transient solution to the equation of motion of the probability amplitudes. Based on this, the evolitional behaviours of the electric permittivity of the double-control tripod-configuration vapour is presented. This will enable us to see how fast the optical evolution responds to the switching on of the control fields. It can be seen that the absorption of the vapour will be oscillatory damped and finally reach a steady-state value (the four-level steady value). As there are more peaks and valleys in the curve of the permittivity of the double-control medium than in that of a conventional three-level single-control medium, and the dispersion in a four-level double-control vapour is therefore more sensitive to the probe frequency than in a single-control vapour, the slowing down of the speed of light in the double-control medium also deserves discussion.

2. Equation of motion in a transient evolution process

In the present double-control scheme, the four-level atomic system is coupled to two control beams and one probe beam (see figure 1). Here, we assume that the intensity of the probe beam is sufficiently weak and therefore nearly all the atoms remain in the ground state, i.e. the atomic population in level \(|1\rangle\) is unity. Under this condition, the equation of motion of the probability

\[
\begin{align*}
\Delta c & = \Delta c' \Delta p \\
|1\rangle & \rightarrow |2\rangle \\
|2\rangle & \rightarrow |2'\rangle \\
|2'\rangle & \rightarrow |3\rangle \\
|3\rangle & \rightarrow |1\rangle
\end{align*}
\]

Figure 1. The schematic diagram of a four-level double-control tripod-configuration system. The two control laser beams, \(\Omega_c\) and \(\Omega_{c'}\), drive the \(|2\rangle \rightarrow |3\rangle\) and \(|2'\rangle \rightarrow |3\rangle\) transitions, respectively. The probe transition \(|1\rangle \rightarrow |3\rangle\) can be manipulated controllably via the destructive and constructive quantum interferences between the \(|2\rangle \rightarrow |3\rangle\) and the \(|2'\rangle \rightarrow |3\rangle\) transitions. If levels \(|1\rangle\), \(|2\rangle\) and \(|2'\rangle\) form a three-level dark state, the atomic vapour is transparent to the probe field, while if levels \(|2\rangle\) and \(|2'\rangle\) form a two-level dark state, the vapour is opaque to the probe field.
amplitudes $a_2$, $a_3$, $a_4$ takes the following form (see appendix for the derivation procedure)

\[
\begin{align*}
\dot{a}_2 &= - \left[ \frac{\gamma_2}{2} + i (\Delta_p - \Delta_c) \right] a_2 + \frac{i}{2} \Omega_{c}^* a_3, \\
\dot{a}_2 &= - \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c') \right] a_2 + \frac{i}{2} \Omega_{c}'^* a_3, \\
\dot{a}_3 &= - \left( \frac{\Gamma_3}{2} + i \Delta_p \right) a_3 + \frac{i}{2} (\Omega_c a_2 + \Omega_c a_2') + \frac{i}{2} \Omega_p,
\end{align*}
\]

where the Rabi frequencies of the probe beam and the two control beams are defined through $\Omega_p = \varphi_{31} E_p / \hbar$, $\Omega_c = \varphi_{32} E_c / \hbar$, and $\Omega_c' = \varphi_{32}' E_c / \hbar$, respectively. Here, $E_p$, $E_c$ and $E_c'$ denote the probe and control field envelopes. The three frequency detunings $\Delta_c$, $\Delta_c'$ and $\Delta_p$ are defined as follows: $\Delta_c = \omega_{32} - \omega_c$, $\Delta_c' = \omega_{32}' - \omega_c$, $\Delta_p = \omega_{31} - \omega_p$, where $\omega_{32}$, $\omega_{32}'$ and $\omega_{31}$ stand for the atomic transition frequencies, and $\omega_c$, $\omega_c'$, $\omega_p$ represent the mode frequencies of the control and probe beams, respectively. The collisional dephasing rates (nonradiative decay rates) and the spontaneous emission decay rate are denoted $\gamma_2$, $\gamma_2'$ and $\Gamma_3$, respectively. In general, such a double-control tripod-configuration system can be found in alkali metallic atoms. For example, according to the selection rule ($\Delta L = \pm 1$, $\Delta J = 0, \pm 1$, $\Delta m_J = 0, \pm 1$) for the electric-dipole allowed transition, the system $\{|1\rangle, |2\rangle, |2'\rangle, |3\rangle\}$ can be chosen as $\{5^2S_{1/2}, 4^2D_{5/2}, 4^2S_{1/2}, 6^2P_{3/2}\}$ of the neutral rubidium atom. If the energy level of the ground state $5^2S_{1/2}$ is assumed to be zero, the energies of the other three atomic levels $4^2D_{5/2}, 6^2S_{1/2}, 6^2P_{3/2}$ are 19355.649, 20132.510 and 23715.081 cm$^{-1}$, respectively [21]. Such a tripod-configuration atomic system can also be found in the neutral lithium atom $\{2^2S_{1/2}, 3^2S_{1/2}, 3^2D_{3/2}, 4^2P_{1/2}\}$ with energy levels $\{0.000, 27206.066, 31283.018, 36469.714\}$ cm$^{-1}$ [22], and in the neutral sodium atom $\{3^2S_{1/2}, 4^2S_{1/2}, 3^2D_{3/2}, 4^2P_{1/2}\}$ with energy levels $\{0.000, 25739.991, 29172.889, 30266.99\}$ cm$^{-1}$ [23]. It can be readily verified that the steady state solution to equation (1) is given by

\[
\begin{align*}
a_2^{(s)} &= - \frac{1}{4\mathcal{D}} \Omega_p \Omega_c^* \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c) \right], \\
\dot{a}_2^{(s)} &= - \frac{1}{4\mathcal{D}} \Omega_p \Omega_c^* \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c) \right], \\
a_3^{(s)} &= \frac{i}{2\mathcal{D}} \Omega_p \left[ \frac{\gamma_2}{2} + i (\Delta_p - \Delta_c) \right] \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c') \right],
\end{align*}
\]

where the parameter $\mathcal{D}$ is expressed by

\[
\mathcal{D} = \left( \frac{\Gamma_3}{2} + i \Delta_p \right) \left[ \frac{\gamma_2}{2} + i (\Delta_p - \Delta_c) \right] \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c') \right] + \frac{1}{4} \Omega_c^* \Omega_c \left[ \frac{\gamma_2}{2} + i (\Delta_p - \Delta_c) \right] + \frac{1}{4} \Omega_c^* \Omega_c' \left[ \frac{\gamma_2'}{2} + i (\Delta_p - \Delta_c') \right].
\]

It can be concluded from the steady solution that the double-control atomic vapour is transparent to the probe field if levels $|1\rangle$, $|2\rangle$ and $|2'\rangle$ form a three-level dark state (satisfying $\Omega_c a_2 + \Omega_c a_2' + \Omega_p a_1 = 0$), while it is opaque to the probe field when levels $|2\rangle$ and $|2'\rangle$ form a two-level dark state (satisfying $\Omega_c a_2 + \Omega_c a_2' = 0$, where the destructive quantum interference between the two control transitions arises) [20]. Though the principal optical properties of
the double-control atomic vapour can be determined by the steady solution, yet the transient evolutional behaviours of the optical ‘constants’ really deserve consideration, particularly for the process of dynamical storing of light pulses in the EIT media in the technique of coherent information storage [8, 9], where the turn-on and turn-off dynamics should be established in order to check whether the information (probe field) is affected or not during the storage and readout processes (limits on optical pulse compression and delay bandwidth product are very important factors for analysing these processes [24]). In what follows, we discuss the transient case of the double-control optical responses and show how the electric permittivity of the vapour evolves when the control fields are switched on.

For this aim, we should first derive the transient solution to equation (1). Apparently, the general solution to the homogeneous equation corresponding to equation (1) is

$$\begin{pmatrix} a_2(t) \\ a_2(0) \\ a_3(t) \end{pmatrix} = \begin{pmatrix} a_2(0) \\ a_2(0) \\ a_3(0) \end{pmatrix} e^{\lambda t}. \tag{4}$$

Then, substitution of (4) into the homogeneous equation yields

$$\det \begin{pmatrix} \frac{\gamma}{2} + i (\Delta_p - \Delta_c) & 0 & \frac{\gamma}{2} \Omega^* \\ 0 & -\left(\frac{\gamma'}{2} + i (\Delta_p - \Delta_c)\right) - \lambda & \frac{\gamma}{2} \Omega^* \\ \frac{\gamma}{2} \Omega & \frac{\gamma}{2} \Omega & -\left(\frac{\gamma'}{2} + i \Delta_c - \lambda\right) \end{pmatrix} = 0. \tag{5}$$

Thus the eigenvalue, $\lambda$, satisfies the cubic equation

$$\lambda^3 + (a + b + c)\lambda^2 + (ab + bc + ca + d + e)\lambda + (abc + ad + eb) = 0, \tag{6}$$

where the parameters are defined through

$$a = \frac{\gamma}{2} + i (\Delta_p - \Delta_c), \quad b = \frac{\gamma'}{2} + i (\Delta_p - \Delta_c), \quad c = \frac{\Gamma}{2} + i \Delta_c,$$

$$d = \frac{1}{3} \Omega^* \Omega_c, \quad e = \frac{1}{4} \Omega^* \Omega_c. \tag{7}$$

It can then be readily verified that the three eigenvalues in equation (6) are as follows

$$\lambda_1 = u + v - \frac{a + b + c}{3}, \quad \lambda_2 = u w + v w^2 - \frac{a + b + c}{3}, \quad \lambda_3 = u w^2 + v w - \frac{a + b + c}{3}. \tag{8}$$

Here, $u = \left[(-q + \sqrt{\Delta})/2\right]^{1/3}$, $v = -p/u$, $w = (-1 + i \sqrt{3})/2$, $\Delta = 4p^3 + q^2$, where

$$p = \frac{ab + bc + ca + d + e}{3} - \left(\frac{a + b + c}{3}\right)^2,$$

$$q = abc + ad + eb - \frac{(a + b + c)(ab + bc + ca + d + e)}{3} + 2\left(\frac{a + b + c}{3}\right)^3. \tag{9}$$

Hence, the transient solution to equation (1) is given by

$$\begin{pmatrix} a_2(t) \\ a_2(t) \\ a_3(t) \end{pmatrix} = \sum_{i=1}^{3} \begin{pmatrix} a_2(i)(0) \\ a_2(i)(0) \\ a_3(i)(0) \end{pmatrix} e^{\lambda_i t} + \begin{pmatrix} a_2(s) \\ a_2(s) \\ a_3(s) \end{pmatrix}, \tag{10}$$
with
\[
a_2^{(i)}(0) = \left[\frac{i}{2}\Omega_c^* + i\left(\Delta_p - \Delta_c\right)\right] + \lambda_i a_j^{(i)}(0), \quad a_2^{(i)}(0) = \left[\frac{i}{2}\Omega_c^* + i\left(\Delta_p - \Delta_c\right)\right] + \lambda_i a_j^{(i)}(0).
\]

Here, the subscript \( j \) and the superscript \( i \) in \( a_j^{(i)}(0) \) (with \( j = 2, 2' \)) denote the atomic level \( |j\rangle \) and the eigenvalue \( \lambda_i \), respectively. It follows from expressions (10) and (11) that the only parameters to be determined are \( a_3^{(i)}(0) \) (with \( i = 1, 2, 3 \)), which will be obtained by using the initial condition (the initial probability amplitude values \( a_2(0), a_2'(0), a_3(0) \) of the three lower levels).

### 3. Transient permittivity of the double-control EIT

In the preceding section, we have derived the general solution to the equation of motion of the probability amplitudes. Here, we obtain the particular solution by using the initial condition. By substituting the initial values \( a_2(0), a_2'(0), a_3(0) \) of the three lower levels into solution (10), one can show that the three coefficients \( a_3^{(i)}(0) \) \( (i = 1, 2, 3) \) agree with the following matrix equation

\[
\begin{pmatrix}
a_2(0) - a_2^{(s)} \\
a_2'(0) - a_2'^{(s)} \\
a_3(0) - a_3^{(s)}
\end{pmatrix} =
\begin{pmatrix}
A_1 & A_2 & A_3 \\
B_1 & B_2 & B_3 \\
1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
a_3^{(1)}(0) \\
a_3^{(2)}(0) \\
a_3^{(3)}(0)
\end{pmatrix},
\]

where the matrix elements \( A_i, B_i \) \((i = 1, 2, 3)\) are defined by
\[
A_i = \left[\frac{i}{2}\Omega_c^* + i\left(\Delta_p - \Delta_c\right)\right] + \lambda_i, \quad B_i = \left[\frac{i}{2}\Omega_c^* + i\left(\Delta_p - \Delta_c\right)\right] + \lambda_i.
\]

As we will see how fast the double-control vapour responds to the switching on of the control fields, the initial condition is chosen as: the two control fields are not present and the atomic vapour before \( t = 0 \) is driven by the probe field only. From equation (12), one can then obtain

\[
a_3^{(1)}(0) = \frac{\det \begin{pmatrix}
a_2(0) - a_2^{(s)} & A_2 & A_3 \\
a_2'(0) - a_2'^{(s)} & B_2 & B_3 \\
a_3(0) - a_3^{(s)} & 1 & 1
\end{pmatrix}}{\det \begin{pmatrix}
A_1 & A_2 & A_3 \\
B_1 & B_2 & B_3 \\
1 & 1 & 1
\end{pmatrix}}, \quad a_3^{(2)}(0) = \frac{\det \begin{pmatrix}
A_1 & a_2(0) - a_2^{(s)} & A_3 \\
B_1 & a_2'(0) - a_2'^{(s)} & B_3 \\
1 & a_3(0) - a_3^{(s)} & 1
\end{pmatrix}}{\det \begin{pmatrix}
A_1 & A_2 & A_3 \\
B_1 & B_2 & B_3 \\
1 & 1 & 1
\end{pmatrix}}, \quad a_3^{(3)}(0) = \frac{\det \begin{pmatrix}
A_1 & A_2 & a_2(0) - a_2^{(s)} \\
B_1 & B_2 & a_2'(0) - a_2'^{(s)} \\
1 & 1 & a_3(0) - a_3^{(s)}
\end{pmatrix}}{\det \begin{pmatrix}
A_1 & A_2 & A_3 \\
B_1 & B_2 & B_3 \\
1 & 1 & 1
\end{pmatrix}}.
\]

New Journal of Physics 9 (2007) 374 (http://www.njp.org/)
Therefore, the explicit expressions for the transient probability amplitudes \( a_2(t) \), \( a_2'(t) \) and \( a_3(t) \) in an evolutional process, during which the system is driven by both two control fields and one probe field, are given by

\[
\begin{pmatrix}
  a_2(t) \\
  a_2'(t) \\
  a_3(t)
\end{pmatrix} = \sum_{i=1}^{3} \begin{pmatrix}
  \frac{i}{2} \Omega_i^e \\
  \frac{i}{2} \Omega_i^e \\
  \frac{1}{2} \Omega_i^e
\end{pmatrix} \begin{pmatrix}
  \frac{(\gamma_2/2) + i(\Delta_p - \Delta_i)}{2} + \lambda_i \\
  \frac{(\gamma_2'/2) + i(\Delta_p - \Delta_i)}{2} + \lambda_i \\
  1
\end{pmatrix} a_3^s(0)e^{\lambda_i t} + \begin{pmatrix}
  a_2^s(t) \\
  a_2'^s(t) \\
  a_3^s(t)
\end{pmatrix}. \tag{15}
\]

The atomic electric polarizability of the probe transition \(|1⟩ - |3⟩\) is

\[\beta(\Delta_p) = 2\varphi_{13}\rho_{31}/(\epsilon_0 E_p)\]

with the density matrix element \(\rho_{31} = a_1^*a_3 \simeq a_3\). We should take into account the local field correction, i.e. the dramatic modification to the optical properties due to the dipole–dipole interaction between neighbouring atoms [25]. According to the Clausius–Mossotti relation [25], the relative electric permittivity is given by

\[\varepsilon_r(\Delta_p) = 1 + \frac{N\beta(\Delta_p)}{1 - (N\beta(\Delta_p)/3)}, \tag{16}\]

where \(N\) denotes the atomic concentration of the EIT vapour.

It should be noted that the phrase ‘transient permittivity’ under consideration is the permittivity of the atomic vapour that experiences a relaxation process once the control laser fields are switched on or off. As is well known, the optical properties (such as transmission, absorption and dispersion) of the atomic medium (when we turn on or off the applied control fields) would evolve from one steady case to another steady case (in this process, the relaxation time depends on the decay rates of the atomic system). The influence of the phase coherence between the two control transitions on the probe transition in the transient evolution processes can be studied based on the above formulation. Note that how fast the transient evolution processes respond to the switching on and off of the control fields depends on the decay terms (e.g. the spontaneous emission decay) in the equation of motion. Though the spontaneous emission decay rate \(\Gamma_3\) of level \(|3⟩\) takes various values for the above three alkali metallic atomic systems (see section 2), yet the typical order of magnitude of the spontaneous decay rate \(\Gamma_3\) is about \(10^7\) s\(^{-1}\) [26]. In the numerical example of this paper, \(\Gamma_3\) is taken to be \(2.0 \times 10^7\) s\(^{-1}\). The dephasing rates (nonradiative decay rates) \(\gamma_2, \gamma_2'\) depends on the atomic concentration: specifically, they are closely related to the pressure and temperature of the atomic vapour. As a result, the dephasing rates are linear functions of the collision rate, which itself is a linear function of pressure (at least for the relatively low pressures, e.g. less than 30 bar) [27]. In general, the typical values for the dephasing rates in an atomic vapour can be chosen as \(10^4\)–\(10^5\) s\(^{-1}\). In the section that follows, we give some numerical examples to show the optical responses of the double-control atomic vapour in the transient processes.

4. Illustrative examples and discussions

First we present an illustrative example to demonstrate how the transient optical behaviour of the double-control EIT vapour evolves once the two control fields are switched on (see figure 2). In this process, the three lower levels (including the probe level and two control
levels) of the four-level system form a three-level dark state will finally exhibit EIT to the probe field. The permittivity (in steady cases) is traditionally not a time-dependent but a frequency-dependent quantity. But what we consider here is a transient process, where the optical ‘constants’ will evolve due to switching on of the control fields. Here, the typical atomic and optical parameters for the system are as follows: \( \gamma_{13} = 1.0 \times 10^{-29} \text{ C m}, \ \Gamma_3 = 2.0 \times 10^7 \text{ s}^{-1}, \ \gamma_2 = 0.01 \Gamma_3, \ \gamma_2' = 0.02 \Gamma_3, \ N = 1.0 \times 10^{20} \text{ m}^{-3}, \ \Delta_p = 0.1 \Gamma_3, \ \Delta_c = 0.3 \Gamma_3, \ \Delta_c' = 0.2 \Gamma_3, \ \Omega_c = 5.0 \Gamma_3, \ \Omega_c' = 3.0 \Gamma_3, \ \Omega_p = 1.0 \times 10^4 \text{ s}^{-1}. \) The present four-level system is interacting only with the probe field at \( t < 0 \) (before the switching on of the other two control laser beams), so that the initial condition of the present problem can be regarded as the steady probability amplitudes of the reduced two-level system \( \{|1\rangle, |3\rangle\} \). Thus the initial condition of the system is: \( a_3(0) = 9.6 \times 10^{-5} + 4.8 \times 10^{-4}i \), which is excited by the weak probe field before \( t = 0 \) (and the two control fields are absent). By using this initial condition, one can obtain a transient solution of the four-level coherent system shortly after the switching on of the two control beams. It follows from figure 2 that the vapour medium is absorptive to the probe field at \( t = 0 \) since there is only one optical field (probe field) interacts with the atomic system and the system will unavoidably exhibit a strong resonant absorption \( (|1\rangle \rightarrow |3\rangle) \). But once the two control fields are switched on at \( t = 0 \), the three lower levels \( |1\rangle, |2\rangle, |2'\rangle \) will go into a three-level dark state, which experiences no net coupling to level \( |3\rangle \). Thus, the time-dependent imaginary part of the relative permittivity \( \varepsilon_r \) is oscillating (with a damped oscillating amplitude) during the transient process and finally reach a steady-state value (zero absorption). This, therefore, means that the quantum destructive interference occurs among the three optical fields. Therefore, the population cannot be excited from the three-level dark state to the upper level \( (|3\rangle) \), and the four-level system will exhibit an EIT effect.

**Figure 2.** The transient behaviour of the electric permittivity of the double-control EIT at a certain probe frequency detuning \( (\Delta_p = 0.1 \Gamma_3) \) once the two control fields are simultaneously switched on. The real and imaginary parts of the relative permittivity oscillatorily increases and decreases, respectively, and approaches the value of vacuum permittivity \( (\text{Re}(\varepsilon_r) = 1, \text{Im}(\varepsilon_r) = 0, \text{respectively}) \).
Figure 3. The transient evolutional behaviours of the relative electric permittivity of a DIM and a CIM. The atomic parameters $\varphi_{13}$, $\Gamma_3$, $\gamma_2$, $\gamma_2'$, the atomic concentration $N$, and the initial condition (the values of $\Omega_p$ and $a_3(0)$) are chosen the same as in figure 2. The permittivity of DIM and CIM evolves from the same initial condition and experiences different paths because of the different quantum interference condition (i.e. the different ratios $\Omega_c/\Omega_c'$). In the case of DIM, the four-level system exhibits a two-level resonant absorption because the two control levels form a dark state ($\Omega_c a_2 + \Omega_c' a_2' = 0$) and the destructive quantum interference occurs between the two control transitions.

We have shown that the present four-level atomic vapour can exhibit a double-control EIT effect, in which the vapour is transparent to the probe field. But under certain conditions, the four-level vapour can also become opaque to the probe field. In the preceding example, the three lower levels form a three-level dark state and the destructive quantum interference arises among the two control fields and one probe field. In other words, a constructive quantum interference takes place between the two control fields. Then one can conclude that if the two control fields experience a destructive quantum interference, the three lower levels would not form a dark state, and the ground state ($|1\rangle$) will be coupled to the excited level ($|3\rangle$) by the probe field. Under this circumstance, the four-level system can be regarded as being equivalent to a two-level system ($|1\rangle$, $|3\rangle$) that can exhibit a two-level resonant absorption to the probe field. We study both the constructive and destructive quantum interferences between the two control transitions driven by the control fields, and show that such quantum interferences lead to the transparency and the absorption, respectively, to the probe field. In figure 3, we plot the transient behaviours of the relative permittivity of a destructive-interference medium (DIM) and a constructive-interference medium (CIM). The frequency detunings of the three optical fields are chosen as $\Delta_p = 0.1 \Gamma_3$, $\Delta_c = 0.3 \Gamma_3$, $\Delta_c' = 0.05 \Gamma_3$. In the destructive-interference vapour medium, the Rabi frequencies of the two control fields are chosen as $\Omega_c = 1.0 \Gamma_3$, $\Omega_c' = 0.5 \Gamma_3$ (this can enable the two control fields to exhibit a destructive quantum interference between themselves), and the vapour medium will be opaque to the probe field due to the two-level absorption ($|1\rangle \rightarrow |3\rangle$) when $t \rightarrow \infty$. If, however, one chooses $\Omega_c = 0.5 \Gamma_3$ and $\Omega_c' = 1.0 \Gamma_3$
Figure 4. The difference between the electric permittivities of the double-control EIT and the single-control EIT. The curves correspond to the permittivity (including the real and imaginary parts) of the steady cases (i.e. time $t \to +\infty$). The atomic parameters $\wp_{13}$, $\Gamma_3$, $\gamma_2$, $\gamma_2'$ and the atomic number density $N$ of the present vapour are chosen exactly the same as figures 2 and 3. As there are more peaks and valleys in the curves of $\varepsilon_4$ than in $\varepsilon_3$, the dispersion in the four-level double-control vapour is in general more sensitive to the probe frequency than in the single-control three-level vapour.

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The four-level double-control EIT vapour has its own striking properties as compared with a three-level single-control EIT vapour. The difference between the double-control EIT and the single-control EIT is plotted in figure 4, where the atomic parameters $\wp_{13}$, $\Gamma_3$, $\gamma_2$, $\gamma_2'$ and the atomic concentration $N$ of the vapour are chosen exactly the same as figures 2 and 3. Here, the Rabi frequencies of the two control fields in the double-control EIT are $\Omega_c = 3.0 \Gamma_3$ and $\Omega_c' = 3.0 \Gamma_3$, and the frequency detunings are $\Delta_c = 1.0 \Gamma_3$ and $\Delta_c' = -2.0 \Gamma_3$, respectively. In the single-control EIT, the Rabi frequency of one of the control fields is $\Omega_c = 3.0 \Gamma_3$ (with the frequency detuning $\Delta_c = 1.0 \Gamma_3$), and the other control field, $\Omega_c'$, is switched off. When time $t \to +\infty$, the atomic system will evolve to a steady state, a typical case that also exhibits interesting optical behaviours and properties. In figure 4, we plot the steady permittivities of the single- and double-control EITs, and compare the dispersive behaviours between these two kinds of EITs. It follows from figure 4 that the curve of the real part of $\varepsilon_4$ (the relative permittivity of the four-level double-control EIT) has 3 peaks and 3 valleys, and the curve of the imaginary part of $\varepsilon_4$ has 3 peaks and 2 valleys (with the valley points corresponding to the two resonant frequencies, $\Delta_p = \Delta_c$ and $\Delta_p = \Delta_c'$). In the single-control three-level EIT, however, the curve of the real part of $\varepsilon_3$ has only 2 peaks and 2 valleys, and the curve of the imaginary
part of $\varepsilon_3$ has only 2 peaks and 1 valley (with the valley point corresponding to the resonant frequency, $\Delta_p = \Delta_c$). This means that the property (particularly the real part of the permittivity between the two EIT transparency windows) of the four-level double-control atomic vapour is more sensitive to the probe frequency compared with that of the single-control EIT. This may lead to a dramatic reduction in the speed of light in the double-control EIT medium. In the literature, the ultraslow light and superluminal propagation (negative group velocity) in three-level EIT media have attracted attention of many researchers ([28, 29] and references therein).

As the dispersion in both the real and imaginary parts of the optical ‘constants’ is more strong than that in a single-control EIT vapour, the ultraslow and superluminal propagation of light becomes physically interesting in the four-level double-control vapour. As we can easily expect, the curve of $\text{Re}(\varepsilon_n)$ of an $(n - 2)$-control $n$-level vapour [19] would have $n - 1$ peaks and $n - 1$ valleys, and the curve of $\text{Im}(\varepsilon_n)$ would have $n - 1$ peaks and $n - 2$ valleys (with the valley points corresponding to the $n - 2$ resonant frequencies). In general, the more is the number of the control fields, the more sensitive is the dispersion of the coherent vapour to the probe frequency. This, therefore, implies that one can dramatically slow down the group velocity of light in a multi-control coherent vapour.

We believe that the characteristic dynamics of a double-control quantum-interference medium can be expected to open up a new way to realize novel nanophotonic functional devices if we focus our attention on the double-control EIT effect in QD materials. Recently, ideas of realizing logic gates by using new optoelectronic materials have captured attention of some researchers [10]–[12]. It can be shown that the double-control interference effects can be used to realize some logic and functional operations, e.g. the operation of 2-input exclusive-OR (EXOR) gate. The EXOR gate is a circuit that gives a high output (‘1’) if either, but not both, of its two inputs are high and gives low output (‘0’) if both of its two inputs are simultaneously high or simultaneously low. Here we give an example to show how an 2-input EXOR gate works based on the double-control interference effects: choose the proper Rabi frequencies of the two control fields that satisfy the relation (i.e. $\Omega_c a_2 + \Omega_c a_2' = 0$) for the destructive quantum interference between the two control levels (2) and (2)). Once the control field $\Omega_c$ is switched off (logic operation IN$_b = 0$) and the control field $\Omega_c$ is present (logic operation IN$_b = 1$), the present scheme will exhibit a three-level EIT effect (logic operation OUT = 1). But when both $\Omega_c$ and $\Omega_c$ are switched on (logic operation IN$_a = \text{IN}_b = 1$), the present double-control scheme will exhibit a two-level resonant absorption to the probe field (logic operation OUT = 0) because of the destructive quantum interference between the two control transitions. Clearly, it will also be opaque to the probe field (leading to logic operation OUT = 0) when both $\Omega_c$ and $\Omega_c$ are switched off (logic operation IN$_a = \text{IN}_b = 0$). This is the working mechanism of a double-control 2-input EXOR gate (the schematic diagram is depicted in figure 5).

The transient behaviours of the EXOR gate can be studied by following the formulation presented in this paper. In an atomic vapour, the relaxation time of the transient evolution process, in which the two control fields experience turn-on or -off operations, depends on the spontaneous decay rate $\Gamma_3$. It thus takes the output operation about microsecond (retardation time) to respond to the input operation. It is, however, the dephasing effects (nonradiative decay) that dominate the relaxation process in a QD EIT material, since the nonradiative decay rates in a solid would be much larger than the spontaneous decay rates. For example, the nonradiative decay rates in some semiconductor-QD (SQD) materials are $10^{12}$ s$^{-1}$ [24]. This, therefore, means that the output operation in SQD EIT would respond very rapidly to the input operation (the delay time is picosecond), and that the SQD EIT is an ideal candidate to be utilized to
realize the double-control 2-input EXOR gates. Obviously, apart from the logic and functional devices, the double-control EIT can also be applied to some quantum optical or photonic devices such as microsecond and picosecond switches.

5. Concluding remarks

The transient optical properties of the coherent atomic media is of importance due to their potential applications such as the absorptive optical switch [13], in which the transmission of a highly absorptive medium is controlled dynamically by additional switching fields (control beams). Moreover, the transient evolution that is a necessary physical process receives increasingly more attention from physicists who are now investigating the mechanism of quantum coherent information storage [8, 9, 14]. This means that physicists should study the turn-on and turn-off dynamics of the quantum coherent media. We considered the transient evolution of a four-level double-control atomic vapour shortly after the control laser beams are switched on, and demonstrated that the quantum coherence effect in the present vapour can give rise to a stronger dispersion than in a single-control vapour. The most important feature of the turn-on dynamics is that once the two control fields are switched on, the imaginary part of the electric permittivity of the double-control vapour is oscillatorily damped from the case of large absorption to the case of transparency, and ultimately reaches a steady-state value (the four-level steady value). We studied the transient behaviours of both the constructive and destructive quantum interferences between the two control transitions (driven by the two control fields) and showed how such quantum interferences in the evolutional processes lead to the transparency and absorption, respectively, to the probe field.
In a conventional three-level EIT system, we need to change the (absolute) intensity of a control field in order that the optical responses of the atomic vapour can be controlled (controllably manipulated). However, the optical behaviours of the present four-level atomic vapour can be tunable just by adjusting the relative intensities (the ratio of the intensities) of the two control fields. This, therefore, implies that the double-control scheme would be more convenient and efficient for manipulating the wave propagation of the optical fields in coherent atomic media. For example, as there are more peaks and valleys in the dispersion curves, multi-control atomic media are promising to realize the wave propagations of the ultraslow and (negative group velocity NGV) light. The present double-control scenario can be applicable to designs of some new photonic, optoelectronic or quantum optical devices. For example, the double-control 2-input EXOR gate, one of the key components of logic gate circuits, can be realized based on the double-control quantum constructive and destructive interferences.

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Appendix. The derivation procedure for equations of motion of atomic-level probability amplitudes

The total Hamiltonian of the four-level double-control tripod-configuration system is $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_D$, where $\mathcal{H}_0$, $\mathcal{H}_1$ and $\mathcal{H}_D$ denote the free Hamiltonian, the interaction Hamiltonian and the decay term that results from the spontaneous emission decay and dephasing effect (nonradiative decay), respectively. The explicit expressions for the Hamiltonian (with rotating wave approximation RWA) are given by

\begin{align}
\mathcal{H}_0 &= \hbar \omega_1 |1\rangle \langle 1| + \hbar \omega_2 |2\rangle \langle 2| + \hbar \omega_2 |2'\rangle \langle 2'| + \hbar \omega_3 |3\rangle \langle 3|,
\mathcal{H}_1 &= -\hbar \left( \Omega_p e^{-i\omega_1 t} |3\rangle \langle 1| + \Omega_c e^{-i\omega_2 t} |3\rangle \langle 2| + \Omega_c e^{-i\omega_3 t} |3\rangle \langle 2'| \right) + \text{H.C.},
\mathcal{H}_D &= -i\hbar \frac{\gamma_2}{2} |2\rangle \langle 2| - i\hbar \frac{\gamma'_2}{2} |2'\rangle \langle 2'| - i\hbar \frac{\Gamma_3}{2} |3\rangle \langle 3|,
\end{align}

(A.1)

where H.C. denotes the Hermitian conjugate term, and $\hbar \omega_i$ represents the energy eigenvalues of the atomic level $|i\rangle$ ($i = 1, 2, 2', 3$). The parameters $\Omega_p$, $\Omega_c$, $\omega_p$, $\omega_c$, $\omega_2$, $\omega_3$ are the Rabi frequencies and the mode frequencies of the control and probe beams, respectively. The atomic state $|\Psi\rangle$ can be written by $|\Psi\rangle = \tilde{a}_1 |1\rangle + \tilde{a}_2 |2\rangle + \tilde{a}_2 |2'\rangle + \tilde{a}_3 |3\rangle$. With the help of the Schrödinger equation $i\hbar \partial |\psi\rangle / \partial t = \mathcal{H} |\psi\rangle$, one can arrive at

\begin{align}
\hbar \left[ \dot{\tilde{a}}_1 |1\rangle + \dot{\tilde{a}}_2 |2\rangle + \dot{\tilde{a}}_2 |2'\rangle + \dot{\tilde{a}}_3 |3\rangle \right] &= \hbar \omega_1 \tilde{a}_1 |1\rangle + \left( \hbar \omega_2 - i\hbar \frac{\gamma_2}{2} \right) \tilde{a}_2 |2\rangle + \left( \hbar \omega_2 - i\hbar \frac{\gamma'_2}{2} \right) \tilde{a}_2 |2'\rangle
+ \left( \hbar \omega_3 - i\hbar \frac{\Gamma_3}{2} \right) \tilde{a}_3 |3\rangle - \hbar \left( \Omega_p e^{-i\omega_1 t} \tilde{a}_1 |1\rangle + \Omega_c e^{-i\omega_2 t} \tilde{a}_2 |3\rangle + \Omega_c e^{-i\omega_3 t} \tilde{a}_2 |3\rangle \right)
- \hbar \left( \Omega_p e^{i\omega_1 t} \tilde{a}_3 |1\rangle + \Omega_c e^{i\omega_2 t} \tilde{a}_3 |2\rangle + \Omega_c e^{i\omega_3 t} \tilde{a}_3 |2'\rangle \right),
\end{align}

(A.2)
where dot denotes the time derivative. By using the orthogonality condition \( \langle i | j \rangle = \delta_{ij} \), one can obtain

\[
\begin{align*}
\dot{a}_1 &= \hbar \omega_1 a_1 - \frac{\hbar}{2} \Omega_p^* e^{i \omega_p t} a_3, \\
\dot{a}_2 &= \left( \hbar \omega_2 - \frac{\hbar}{2} \Omega_p^* e^{i \omega_p t} \right) a_2 - \frac{\hbar}{2} \Omega_c^* e^{i \omega_c t} a_3, \\
\dot{a}_2 &= \left( \hbar \omega_2 - i \frac{\hbar}{2} \gamma_c \right) a_2 - \frac{\hbar}{2} \Omega_p^* e^{i \omega_p t} a_3, \\
\dot{a}_3 &= \left( \hbar \omega_3 - i \frac{\hbar}{2} \Gamma_3 \right) a_3 - \frac{\hbar}{2} \left( \Omega_p^* e^{-i \omega_p t} a_1 + \Omega_c^* e^{-i \omega_c t} a_2 + \Omega_c^* e^{-i \omega_c t} a_2 \right).
\end{align*}
\]

As there are time-harmonic exponential factors in (A.3), a unitary transformation \( \tilde{a}_1 = a_1 e^{-i \omega_1 t} \), \( \tilde{a}_2 = a_2 e^{-i \omega_2 t} \), \( \tilde{a}_2 = a_2 e^{-i \omega_2 t} \), and \( \tilde{a}_3 = a_3 e^{-i \omega_3 t} \) can be utilized to eliminate all these exponential factors. Here, \( a_i \) is the slowly-varying probability amplitude of the atomic level \( |i \rangle \) (\( i = 1, 2, 2', 3 \)). Thus, the equations of motion of the slowly-varying probability amplitudes of the four-level tripod-configuration system take the following form

\[
\begin{align*}
\dot{a}_1 &= \frac{\hbar}{2} \Omega_p^* a_3, \\
\dot{a}_2 &= - \left[ \frac{\gamma_c}{2} + i \left( \Delta_p - \Delta_c \right) \right] a_2 + \frac{i}{2} \Omega_c^* a_3, \\
\dot{a}_2 &= - \left[ \frac{\gamma_c}{2} + i \left( \Delta_p - \Delta_c \right) \right] a_2 + \frac{i}{2} \Omega_c^* a_3, \\
\dot{a}_3 &= - \left[ \frac{\Gamma_3}{2} + i \Delta_p \right] a_3 + \frac{i}{2} \left( \Omega_p^* a_1 + \Omega_c^* a_2 + \Omega_c^* a_2 \right),
\end{align*}
\]

where the frequency detunings of the three laser fields are defined through \( \Delta_p = \omega_3 - \omega_1 - \omega_p \), \( \Delta_c = \omega_3 - \omega_2 - \omega_c \), \( \Delta_c = \omega_3 + \omega_2 + \omega_c \). In this paper, we solve equation (A.4) and study the transient behaviours of the atomic system. But for simplicity, we assume that the intensity of the probe beam is sufficiently weak (i.e. \( \Omega_p^* \) is very small, \( \Omega_p^* \ll \Delta_p \), \( \Omega_c^* \ll \Delta_c \) and \( \Omega_c^* \ll \Delta_c \)), and therefore nearly all the atoms remain in the ground state, i.e. the atomic population in level \( |1 \rangle \) is unity. On other hand, the term \( i \Omega_c^* a_3 / 2 \) of the first equation in (A.4) could be negligibly small, and the equation for \( \dot{a}_1 \) can therefore be deleted from equation (A.4). Under these conditions, the equations of motion of the probability amplitudes \( a_2, a_3, a_4 \) form a closed set of equation (1).

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