Optomechanical Cavity with a Buckled Mirror

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We study an optomechanical cavity, in which a buckled suspended beam serves as a mirror. The mechanical resonance frequency of the beam obtains a minimum value near the buckling temperature. Contrary to the common case, in which self-excited oscillations of the suspended mirror are optically induced by injecting blue detuned laser light, in our case self-excited oscillations are observed with red detuned light. These observations are attributed to a retarded thermal (i.e. bimetric) force acting on the buckled mirror in the inwards direction (i.e. towards other mirror). With relatively high laser power other interesting effects are observed including period doubling of self-excited oscillations and intermode coupling.

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I. INTRODUCTION

Systems combining mechanical elements in optical resonance cavities are currently a subject of intense interest [1][2]. Coupling of nanomechanical mirror resonators to optical modes of high-finesse cavities mediated by the radiation pressure has a promise of bringing the mechanical resonators into the quantum realm [2, 4, 7, 13] (see Ref. [14] for a recent review). In addition, the micro-optoelectromechanical systems (MOEMS) are expected to play an increasing role in optical communications [15] and other photonics applications [14, 18].

Besides the radiation pressure, another important force that contributes to the optomechanical coupling in MOEMS is the bolometric force [3, 19–24, 25]. The bolometric force is known as the thermal force. This force can be attributed to the heat induced deformations of the micromechanical mirrors [21, 23, 24]. In general, the thermal force plays an important role in relatively large mirrors, in which the thermal relaxation rate is comparable to the mechanical resonance frequency. Phenomena such as mode cooling and self-excited oscillations [6, 23, 24, 27–31] have been shown in systems in which this force is dominant [3, 19, 24, 26, 32, 33].

In this paper we investigate an optomechanical cavity having a suspended mirror in the shape of a trilayer metallic doubly clamped beam [34, 30]. The system experimentally exhibits some unusual behaviors. For example, contrary to other experiments, in which self-excited oscillations of the mechanical resonator are optically induced when the cavity is blue detuned [37], here the same effect occurs for the case of red-detuned cavity. Moreover, the dependence of the mechanical resonance frequency on laser power is found to be non-monotonic [38]. These findings are attributed to optically induced thermal strain in the beam, which gives rise to compressive stress that may result in buckling [39–44]. Bagheri et al. [45] have recently reported the utilization of the buckling phenomenon to develop a non-volatile mechanical memory element in a similar optomechanical system (see also Ref. [46]).

We generalized the theoretical model that has been developed in Ref. [32] to account for the effect of buckling [47, 48]. We find that close to buckling the effective thermal force acting on the beam can become very large, and consequently optomechanical coupling is greatly enhanced. Consequently, the threshold laser power needed for optically driving self-excited oscillations in the mirror can be significantly reduced. We show that the proposed theoretical model can account for some of the experimental results. On the other hand, some other experimental observations remain elusive. For example, self-modulation of self-excited oscillations is observed in some ranges of operation. Further theoretical study, which will reveal the complex structure of stability zones and bifurcations of the system [49], is needed in order to account for such findings.

II. EXPERIMENTAL SETUP

The experimental setup (see Fig. 1) is similar to the one employed in Ref. [33]. A fiber Bragg grating (FBG) [50, 52] and a microlens [53] are attached to the end of a single mode fiber. The fiber can be accurately positioned using piezomotors. The system, which consists of doubly clamped suspended multilayer micromechanical beam and the optical fiber monitoring assembly, is located inside a cryostat maintained under typical pressure of $10^{-3}$ mbar and temperature of 77 K. Optical cavity is formed between the freely suspended beam oscillating parallel to the optical axis of the cavity and between either the FBG or the glass-vacuum interface at the tip of the fiber, as shown in Fig. 1(a).

The multilayer beam is fabricated by using bulk micromachining process. A 200 nm thick Nb layer is coated on prefabricated silicon nitride membranes over silicon substrates using DC magnetron sputtering at working pressure of $5.2 \times 10^{-3}$ torr and argon atmosphere. Patternning on the coated substrates is done using photolithography followed by a liftoff process, in which a 10 nm thick layer of Cr and a 30 nm thick layer of gold-palladium (Au$_{85}$Pd$_{15}$) is patterned on top of the Nb layer. Front electron cyclotron resonance (ECR) plasma etching of the
The injected laser power of wavelength \( \lambda \) is controlled by attenuating the incident optical power. The ground plate is located at 500 \( \mu \)m below the suspended beam. Incident optical power is controlled by attenuating the injected laser power of wavelength \( \lambda \approx 1550 \) nm. The optical power reflected off the cavity is fed to a photodetector, which is monitored using a spectrum analyzer, as shown in Fig. (a).

III. SELF-EXCITED OSCILLATIONS INDUCED BY RED-DETUNED CAVITY

As was pointed out above, the optomechanical cavity under study exhibits some unusual behaviors. As is demonstrated in Fig. 2, self-excited oscillations can be induced by red-detuned cavity. Subplot (a) shows the reflected optical power vs. the voltage \( V_z \), which is applied to the piezomotor that is used to position the optical fiber along the optical axis direction. The period of oscillation in the reflected power is \( \lambda/2 \). In this experiment the wavelength \( \lambda \) is not tuned to the reflective band formed by the FBG, and thus the optical cavity is formed between the freely suspended beam mirror and the glass-vacuum interface at the tip of the fiber. For this case the finesse of the cavity is much lower compared with values that can be achieved when the FBG is employed. Note that in addition to the oscillatory part, also the averaged (over one period of oscillation) measured reflected power exhibits dependence on cavity length [see subplot (a)].

The dependence on laser power and \( P_L \) is seen in Fig. 3. The threshold power of self-excited oscillations occurs at 0.092 mW. In subplot (a) the static position \( y_0 \) of the center of beam A is measured vs. laser power below the threshold power. This is done by measuring the the reflected optical power vs. the voltage \( V_z \). For each value of the laser power this yield a plot similar to the one seen in Fig. (a). However, the phase of oscillations is found to depend on laser power. The value of the static deflection \( y_0 \), which is extracted from that shift in the phase, is seen in subplot (b). The observed behavior indicates that the beam is deflected towards the optical fiber with increasing laser power. As will be seen below, this behavior indicates that the thermal force in the present case acts in the inwards direction, contrary to the more common situation (e.g. for the case of radiation pressure), where cavity optomechanical forces act in the outwards direction, and lead to elongation of the cavity (rather than shortening as in the current case).

In subplot (b) the mechanical resonance frequency \( f_0 \) is measured both below and above the threshold power of self-excited oscillations. While below the threshold of 0.092 mW the frequency \( f_0 \) is determined from forced
oscillations (FO) (i.e. from the peak in the measured frequency response), above threshold $f_0$ is determined from the peak in the spectrum of self-excited oscillations (SO). Note the non-monotonic dependence of $f_0$ on laser power. As will be argued below, this dependence suggests that the beam undergoes thermal buckling.

IV. MECHANICAL EQUATIONS OF MOTION

Most of the experimental findings that were presented in the previous section are not accountable by the theoretical model that has been developed in Ref. [32] (which, on the other hand, was very successful in accounting for previous measurements of Ref. [33]). We therefore generalize the model to account for the effect of buckling in the mirror. While the derivation of the system’s equations of motion is described in the appendices, the final equations are presented below in this section.

The height function $y(x,t)$ can be written as

$$y = \xi l \left(1 + \cos \frac{2\pi x}{l}\right),$$

where $\xi$ depends on time. The equation of motion for $\xi$, which is derived in appendix C, is given by

$$\ddot{\xi} + 2\gamma \dot{\xi} + \Omega^2 \xi = F_{\text{th}} + F_p e^{-\Omega_0 t},$$

where overdot denotes a derivative with respect to the dimensionless time $\tau$, which is related to the time $t$ by the relation $\tau = t/\sqrt{\rho A_{cs}/E}$, where $\rho$ is the mass density, $A_{cs}$ is the cross section area of the beam, $E$ is the Young’s modulus, $\gamma$ is the dimensionless damping constant, and $F_p$ and $\Omega_0$ are the dimensionless amplitude and frequency respectively of an externally applied force. The equation of motion (2) contains two thermo-optomechanical terms that depend on the temperature of the beam $T$. The first is the temperature dependent angular resonance frequency $\Omega$ and the second is the thermal force $F_{\text{th}}$. In terms of the dimensionless temperature $\theta = (T - T_0)/T_0$, where $T_0$ is the temperature of the supporting substrate (i.e. the base temperature) the frequency $\Omega$ is given by $\Omega = \Omega_0 \theta$, where $\Omega_0 = \sqrt{\rho A_{cs}/E\omega_0}$, $\omega_0$ is the mode’s angular resonance frequency and the temperature dependence is expressed in terms of the dimensionless function $\nu(\theta)$. The dimensionless thermal force is given by $F_{\text{th}} = \Omega^2 \xi C F_\nu (\theta)$, where $\xi C = \Omega_0 \sqrt{2\gamma} (\alpha - \alpha_s)/I^{1/2}$, $\alpha$ and $\alpha_s$ are the thermal expansion coefficients of the metallic beam and of the substrate respectively, $I$ is the moment of inertia corresponding to bending in the $xy$ plane, and the dimensionless function $F_\nu (\theta)$ represents the beam’s
temperature dependent deflection. For the case of a rectangular cross section having width $l_y$ in the $y$ direction and width $l_z$ in the $z$ direction, and for the case of bending in the $xy$ plane $I$ is given by $I = \frac{l_z}{12}$. When small asymmetry is taken into account [see Eqs. (3) and (4)] the functions $f_Y (\theta)$ and $\nu (\theta)$ can be expressed as (see Fig. 5)

$$f_Y (\theta) = \text{Re} \left( \sqrt{\theta - \theta_C - i\theta_t} \right),$$

$$f_Y^2 (\theta) = - (\theta - \theta_C) + 3 f_Y^2 (\theta),$$

where $\theta_C$ is the dimensionless buckling temperature, which is related to the buckling temperature $T_C$ by the relation $\theta_C = (T_C - T_0) / T_0$, and where the real dimensionless constant $\theta_t$ represents the effect of asymmetry.

V. OPTICAL CAVITY

The finesse of the optical cavity is limited by loss mechanisms that give rise to optical energy leaking out of the cavity. The main escape routes are through the on fiber mirror (FBG or glass-vacuum interface), through absorption by the metallic mirror, and through radiation; the corresponding transmission probabilities are respectively denoted by $T_B$, $T_A$ and $T_R$. Let $y_D$ be the displacement of the mirror relative to a point, at which the energy stored in the optical cavity in steady state obtains a local maximum. In the ideal case, all optical properties of the cavity are periodic in $y_D$ with a period of $\lambda/2$, where $\lambda$ is the optical wavelength (though, as can be seen in Fig. 2(a), deviation from periodic behavior is experimentally observed).

It is assumed that $y_D$ is related to $\xi$ by the following relation $4 \pi y_D / \lambda = \beta_Y (\xi + \xi_t)$, where both $\beta_Y$ and $\xi_t$ are constants, which depend on the position of the fiber. For a fixed $\xi$ the cavity reflection probability $R_C$, i.e. the ratio between the reflected (outgoing) and injected (incoming) optical powers in the fiber, is given by [33]

$$R_C = \frac{(T_0 - T_A - T_B)^2 + 2 \left[ 1 - \cos (\beta_Y (\xi + \xi_t)) \right]}{(T_0 + T_A + T_B)^2 + 2 \left[ 1 - \cos (\beta_Y (\xi + \xi_t)) \right]}.$$  

The heating power $Q$ due to optical absorption of the suspended micromechanical mirror can be expressed as $Q = I (\xi) P_L$, where $P_L$ is the power of the monochromatic laser light incident on the cavity and where the function $I (\xi)$ is given by [33]

$$I (\xi) = \frac{T_B T_A}{(T_0 + T_A + T_B)^2 + 2 \left[ 1 - \cos (\beta_Y (\xi + \xi_t)) \right]}.$$  

VI. THERMAL BALANCE EQUATION

The temperature $T$ evolves according to the following thermal balance equation

$$\frac{dT}{dt} = \frac{Q}{C} - \frac{H (T - T_0)}{C},$$

where $C$ is heat capacity, $H$ is thermal transfer coefficient and $Q$ is the heating power due to optical absorption. In terms of the dimensionless temperature $\theta$ and dimensionless time $\tau$ the thermal balance equation becomes

$$\dot{\theta} = \beta_p I (\xi) - \beta_{TR} \theta,$$

where $\beta_{TR} = \sqrt{Q/\mathcal{E} H/C}$ is the dimensionless thermal rate, and $\beta_p = \sqrt{Q/\mathcal{E} P_L C T_0}$ is the dimensionless injected laser power.

VII. SMALL AMPLITUDE LIMIT

The coupling between the equation of motion for $\xi$ [Eq. (2)] and the one for $\theta$ [Eq. (3)] originates by three terms, the $\theta$ dependent frequency $\Omega$, the $\theta$ dependent force $F_{th}$ (i.e. the thermal force) [see Eq. (2)], and the $\xi$ dependent optical heating power [the term $I (\xi)$ in Eq. (3)]. The case where the first two coupling terms are linearized (i.e. approximated by a linear function of $\theta$) is identical to the case that was studied in Ref. [32], in which slow envelope evolution equations for the system were derived, and the amplitudes and the corresponding oscillation frequencies of different limit cycles were analyzed. By employing the same analysis (and the same simplifying assumptions) for the present case one finds that the coupling leads to renormalization of the mechanical damping rate, which effectively becomes

$$\gamma_{eff} = \gamma + \frac{f_Y' I' \xi C \beta_p}{2 (1 + \beta_{TR}^2)},$$

where $I' = dI / d\xi$ and where $f_Y' = df_Y / d\theta$. The corresponding effective mechanical resonance frequency $\Omega_{eff}$ is given by

$$\Omega_{eff} = \Omega - \frac{(\gamma_{eff} - \gamma) \beta_{TR}}{\Omega_p}.$$  

The white dotted line in Fig. 2(b) shows the calculated value of $\Omega_{eff}$ [see Eq. (10)] for the regions in $V_2$ for which $\gamma_{eff} < 0$ [see Eq. (9)]. The system’s parameters that were used for the calculation are listed in the caption of Fig. 2. Note that Eqs. (9) and (10), which were derived by assuming the limit of small amplitudes, become inapplicable in most of the region, in which self-excited oscillations are observed in Fig. 2(b), and consequently, relatively large deviation between data and the prediction of Eq. (10) is expected.

The threshold of instability, i.e. Hopf bifurcation, occurs when $\gamma_{eff}$ vanishes. For the case where the dominant coupling mechanism between the mechanical resonator and the optical field arises due to radiation pressure, instability can occur only for the so-called case of blue detuning, i.e. the case where $I' < 0$. However, contrary to the case of radiation pressure [37, 34], which always acts in the outwards direction, the thermal force due to buckling can act in both directions. We refer to the case where
FIG. 4: (Color Online) In addition to the harmonics in the measured spectrum due to self-excited oscillations of mode 1 of beam B, subharmonics of order 1/2 and of order 1/5 are observed.

$\beta_Y > 0$ as the case of outwards buckling, in which due to buckling the optical cavity becomes longer, whereas the opposite case of inwards buckling occurs when $\beta_Y < 0$. As can be seen from Eq. (9), the threshold laser power $\beta_P$ at which Hopf bifurcation occurs is inversely proportional to $|f_Y'|$. Close to the buckling temperature $\theta_C$ and when asymmetry is small (i.e. when $|\theta_t| \ll 1$) the factor $|f_Y'|$ can become very large [see Eq. (3) and Fig. 6], and consequently, the threshold laser power in that region can become very small.

VIII. HIGH LASER POWER

Other interesting effects have been observed with relatively high laser power. Two examples are presented below. The multilayer beam mirror that was used for the experiments that are introduced in this section (see Figs. 4 and 5) has the same layer structure as beam A [see Fig. 1(b)], however its length is $l = 120 \mu m$ and its width is $b = 20 \mu m$. We will refer to this beam hereafter as beam B. The same fabrication process has been employed for both beams. The two lowest lying modes of beam B, which are hereafter referred to as mode 1 and mode 2, have resonance frequencies $f_1$ and $f_2$ of about 510 kHz and 1200 kHz respectively (note that these values vary with laser power and cavity length).

Self-excited oscillations of mode 1 are seen in Fig. 4. In this measurement the sample is kept grounded and the reflected optical power is measured using a spectrum analyzer as a function of input optical power. In the frequency window that has been employed for this measurement the first 5 harmonics of the self-oscillating mode 1 are seen. In addition, for some values of input power the spectrum contains subharmonics. Close to input power of 11 mW the subharmonics indicate that period doubling occurs, whereas in the range 16.2 – 18.6 mW the period is multiplied by a factor of 5. While the period doubling effect has been predicted for a similar system by Blocher et al. [49], the underlying mechanism responsible for the period multiplication by a factor of 5 is yet undetermined.

The data in Fig. 5 which was taken with a modified cavity length, exhibits self-excited oscillations of mode 2 in the laser input power range $6 - 16.5$ mW. Above that range self-excited oscillations of mode 1 are observed. Near input power of 11.9 mW the ratio between $f_2$ and $f_1$ is 3, i.e. $f_2 = 3f_1$. This gives rise to strong intermode coupling, which leads to excitation of mode 1 by the self-oscillating mode 2, as can be seen from the subharmonics of order 1/3 that are measured in that range.

Note that for other cavity lengths the measured spectrum of reflected power can exhibit wide band response when the input laser power is sufficiently large. This possibly indicates that the dynamics becomes chaotic [49].

IX. SUMMARY

In summary, an optomechanical cavity having a buckled mirror may exhibit some unusual behaviors when the operating temperature is close to the buckling temperature. In that range the instability threshold can be reached with relatively low laser power. Moreover, contrary to the more common case, the instability is obtained with red detuned laser light for the case of inwards buckling. With relatively high laser power other interesting effects are observed in the unstable region including period doubling and intermode coupling. Further theoretical work is needed to account for such effects.
Both first authors (D. Y. and M. B. K.) have equally contributed to the paper.

Appendix A: Lagrangian

Consider a beam made of a material having mass density $\rho$ and Young’s modulus $E$. In the absence of tension the length of the beam is $l_0$. The beam is doubly clamped to a substrate at the points $(x, y) = (\pm l/2, 0)$ and the motion of the beam’s axis, which is described by the height function $y(x, t)$, is assumed to be exclusively in the $xy$ plane. The corresponding moment of inertia is denoted by $I$. The Lagrangian functional $\mathcal{L}$ is given by

$$\mathcal{L} = \int_{-l/2}^{l/2} dx \left[ \frac{1}{2} \rho A_x \frac{\partial y}{\partial t}^2 + \frac{1}{2} N \left( \frac{\partial y}{\partial x} \right)^2 + \frac{1}{2} \frac{E I}{l} \left( \frac{\partial^2 y}{\partial x^2} \right)^2 - f y \right],$$

where

$$L = \frac{A_{cs} \rho}{2} \left( \frac{\partial y}{\partial t} \right)^2 - N \left( \frac{\partial y}{\partial x} \right)^2 - \frac{E I}{2l} \left( \frac{\partial^2 y}{\partial x^2} \right)^2 + f y.$$

is the Lagrangian density, $A_{cs}$ is the cross section area of the beam and

$$N = EA_{cs} \frac{l - l_0}{l_0}$$

is the tension in the beam for the straight case where $y = 0$. The beam’s equation of motion is found from the principle of least action to be given by

$$\frac{\partial^2 y}{\partial t^2} = \left( N + \frac{A_{cs} E}{2l} \int_{-l/2}^{l/2} dx \left( \frac{\partial y}{\partial x} \right)^2 \right) \frac{\partial^2 y}{\partial x^2} - \frac{E I}{l} \frac{\partial^4 y}{\partial x^4} + f,$$

where $\gamma = \rho A_{cs}$ is the mass density per unit length.

Appendix B: First Buckling Configuration

Consider the case where the deflection $y(x,t)$ has the shape of the first buckling configuration $\frac{47}{47}$, i.e. $y = Y(1 + \cos \frac{2\pi x}{l})$, where $Y$ is a dimensionless time dependent amplitude. In order to account for a possible asymmetry, the force $f$ is allowed to be a nonzero constant $\frac{44}{48}$. The Lagrangian $\frac{44}{48}$ for the present case becomes

$$\mathcal{L}_0 = \frac{3 \beta_A \rho k^5}{4} \left( \frac{4Y^2}{\pi^2} \right)^2 - \pi^2 N Y^2 - \frac{4\pi^4 \beta_I E l^3 Y^2}{2} + f l^2 Y - \frac{\pi^4 \beta_A E l^3 Y^4}{2},$$

where $\beta_A = A_{cs}/l^2$ and where $\beta_I = I/l^4$. Alternatively, $\mathcal{L}_0$ can be expressed as

$$\mathcal{L}_0 = T_0 - U_0,$$

where the kinetic energy $T_0$ is given by

$$T_0 = \frac{m_0 \rho l^2}{2} \left( \frac{\partial y}{\partial t} \right)^2,$$

where $m_0 = 3\beta_A \rho l^3/2$ and where the potential energy $U_0$ is given by

$$U_0 = \frac{m_0 \omega_0^2 l^2}{2} u(Y),$$

where

$$u(Y) = \eta_1 Y + \eta_2 Y^2 + \eta_3 Y^4.$$

Locally the potential $U_0$ is assumed to vanish. Since the substrate is much larger than the tension free temperature $T_{TF}$, the tension needed to keep the beam clamped to the substrate is thus given by $N = E A_{cs} (\alpha_s - \Delta_T)$. Thus, the dimensionless parameter $\beta_L$ can be expressed as a function of the dimensionless temperature

$$\theta = \frac{T - T_0}{T_0},$$

where

Appendix C: Mechanical equation of motion

Let $\alpha$ and $\alpha_s$ be the thermal expansion coefficients of the metallic beam and of the substrate respectively. At some given temperature $T_{TF}$ the tension $N$ is assumed to vanish. Since the substrate is much larger than the beam, one may assume that the substrate thermally contracts (or expands) as if the suspended beam was not attached to it, namely the distance between both clamping points at temperature $T = T_{TF} + \Delta_T$ becomes

$$\Delta_T = l_0 (1 + \alpha_s \Delta_T),$$

where $l_0$ is the distance at the tension free temperature $T_{TF}$. The tension $N$ needed to keep the beam clamped to the substrate is thus given by $N = E A_{cs} (\alpha_s - \Delta_T)$. Thus, the dimensionless parameter $\beta_L$ can be expressed as a function of the dimensionless temperature

$$\theta = \frac{T - T_0}{T_0},$$

where

$$\theta = \frac{T - T_0}{T_0}.$$
where $T_0$ is the temperature of the supporting substrate (i.e., the base temperature), as $\beta_L = \beta_A \beta_{T0} (\theta + \beta_{TF}) / 4\pi^2 \beta_1$ where $\beta_{T0} = T_0 (\alpha - \alpha_s)$ and $\beta_{TF} = (T_0 - T_{TF}) / T_0$. Alternatively, $\beta_L$ can be expressed as

$$\beta_L = 1 + \frac{\beta_A \beta_{T0} (\theta - \theta_C)}{4\pi^2 \beta_1}, \quad (C2)$$

where $\theta_C = 4\pi^2 \beta_1 / \beta_A \beta_{T0} - \beta_{TF}$ is the dimensionless temperature at which buckling occurs in the symmetric case (i.e., the case where $\eta_1 = 0$).

With the help of Eq. (C2), the parameters $\gamma_0$ and $\nu$ can be expressed as a function of the dimensionless temperature $\theta$. For the case of small asymmetry one has [see Eq. (B6)]

$$\gamma_0 = \xi C f_\nu (\theta), \quad (C3)$$

where the function $f_\nu (\theta)$ is given by Eq. $\nu_0 = 4\pi^2 \beta_1 (4\pi^2 \eta_4)^{1/3} / \beta_A \beta_{T0}$ and the coefficient $\xi_C$ is given by

$$\xi_C = \sqrt{\frac{\beta_A \beta_{T0}}{8\pi^2 \beta_1 \eta_4}} = \sqrt{\frac{3T_0 (\alpha - \alpha_s) A_{cs} \rho l^4 \omega_0^2}{16\pi^6 E I}}. \quad (C4)$$

The corresponding approximation for $\nu$ is given by

$$\nu = \nu_C f_\nu (\theta), \quad (C5)$$

where

$$\nu_C = \sqrt{\frac{\beta_A \beta_{T0}}{4\pi^2 \beta_1}} = \sqrt{\frac{A_{cs} l^2 T_0 (\alpha - \alpha_s)}{4\pi^2 I}}, \quad (C6)$$

and where the function $f_\nu (\theta)$ is given by Eq. (C5). The functions $f_\nu (\theta)$ and $f_\nu (\theta)$ are plotted in Fig. 6.

The temperature dependence of $\gamma_0$ can be taken into account by adding a thermal force term $F_{th}$ [40, 56, 57] to the equation of motion of the mechanical amplitude $\xi = \gamma - \gamma_0$. Similarly, the temperature dependence of $\nu$ can be taken into account by including a thermal frequency shift term. When in addition damping and external driving are taken into account the equation of motion for $\xi$ is obtained.

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![Graph](image_url)

**FIG. 6:** (Color Online) The functions $f_\nu (\theta)$ and $f_\nu (\theta)$ [see Eqs. (C3) and (C5)] plotted for 3 different values of the asymmetry parameter $\theta_1$ and for the case where the dimensionless buckling temperature is $\theta_C = 0.5$.

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