Vertex Operators for Super Yang-Mills and Multi D-branes in Green-Schwarz Superstring

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Abstract

We study vertex operators for super Yang-Mills and multi D-branes in covariant form using Green-Schwarz formalism. We introduce the contact terms naturally and prove space-time supersymmetry and gauge invariance. The nonlinear realization of broken supersymmetry in the presence of D-branes is also discussed. The shift of fermionic coordinate $\delta(-)\Psi = \eta$ becomes exact symmetry of D-brane in the static gauge, where $\eta$ is a constant spinor in $U(1)$ direction.

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1 Introduction

The D-brane description [1, 2, 3] of string solitons [4] has provided us with a powerful tool for studying world-volume theories of these extended objects in a perturbative formulation. Dp-branes are described in terms of open superstring theories that have Neumann boundary conditions in the $(p + 1)$ directions and Dirichlet boundary conditions in the remaining $9 - p$ directions which represent the locations of D-branes. The boundary relates left-moving and right-moving degrees of freedom so that it breaks half of space-time supersymmetry. Further, the Dirichlet boundary condition breaks the translational invariance of space-time. The Nambu-Goldstone modes associated with the broken symmetries are identified with collective coordinates of D-brane in the static gauge [5].

In this paper we construct vertex operators of multi D-branes in covariant form using Green-Schwarz (GS) superstring [6, 7, 8] and prove that the supersymmetry of multi D-branes is given by that of reduced ten dimensional super Yang-Mills theory [9]. Multi D-branes are described by attaching the Chan-Paton factors on vertex operators. To construct non-abelian vertex operators in covariant form, we must introduce the contact terms to ensure supersymmetry and gauge invariance. The importance of the contact terms is stressed in the work by Green and Seiberg [10]. Here we introduce the contact terms naturally and prove supersymmetry in the coordinate space.

This paper is organized as follows. In the next section we give a brief review of GS superstring. We then fix our notations and conventions. We first discuss the abelian case in Sect.3. We here give the vertex operators of D-brane in the coordinate space. We then generalize the argument to the non-abelian case in Sect.4 by replacing field strength and derivative with the covariant ones. This means that we introduce the contact terms like $[A_\mu, A_\nu]$ naturally. Naively, the addition of the contact terms would break the supersymmetry-transformation law of vertex operators at the order of coupling constant $g$. We here show, however, that the contact terms coming from the collision points of vertex operators exactly cancel the extra terms that break the transformation law and restore the supersymmetry. In Sect.5 we discuss the nonlinear realization [11] of the broken supersymmetry in the presence of D-branes. We here develop the argument of Green and Gutperle formulated in closed-string/cylinder frame [12]. The supersymmetry of D-branes becomes that of reduced super Yang-Mills theory, which is linearly
realized by the unbroken supersymmetry $Q^+$ which satisfies the boundary state condition $Q^+|B| > 0$. The nonlinear realization of broken supersymmetry satisfying $Q^-|B| \neq 0$ is given by a shift of fermionic collective coordinate, $\delta^{(-)}\Psi = \eta$, where $\eta$ is a constant spinor in $U(1)$ direction.

### 2 The Set Up

In this section we review the formulation of GS superstring and the procedure of light-cone gauge fixing. We then fix our notations and conventions. The covariant action of GS-superstring \cite{6,13} is

\[
S = -\frac{1}{\pi} \int d^2\xi \left[ \sqrt{-h} + i\epsilon^{ab}\partial_a X^\mu \left( \bar{\theta}^{(1)} \Gamma_\mu \partial_\nu \theta^{(1)} - \bar{\theta}^{(2)} \Gamma_\mu \partial_\nu \theta^{(2)} \right) \right],
\]

(2.1)

where $\xi^a = (\tau, \sigma)$, $\{\Gamma^\mu, \Gamma^\nu\} = -2\eta^{\mu\nu}$, $\eta^{\mu\nu} = (-1,1,\cdots,1)$ and $\bar{\theta} = \theta^0 \Gamma^0$.

$h_{ab} = \Pi^a \Pi^b \eta_{\mu\nu}$ and $\Pi^a_\mu$ is defined by

\[
\Pi^a_\mu = \partial_a X^\mu - i\bar{\theta}^{(1)} \Gamma_\mu \partial_a \theta^{(1)} - i\bar{\theta}^{(2)} \Gamma_\mu \partial_a \theta^{(2)}. \tag{2.2}
\]

The action is invariant under the global supersymmetry $\delta X^\mu = \sum_A i\bar{\epsilon}^{(A)} \Gamma^\mu \theta^{(A)}$, $\delta \theta^{(A)} = \epsilon^{(A)}$, $(A = 1, 2)$ and also under the local fermionic symmetry ($\kappa$-symmetry) $\delta X^\mu = \sum_A i\bar{\theta}^{(A)} \Gamma^\mu \alpha^{(A)}$, $\delta \theta^{(A)} = \alpha^{(A)}$. $\alpha^{(A)}$s are local fermionic fields defined by $\alpha^{(A)} = \left(1 - (-1)^A \Gamma^1\right) \kappa^{(A)}$, where $\Gamma = \frac{1}{2\sqrt{-h}} \epsilon^{ab} \tilde{\Gamma}_a \tilde{\Gamma}_b$ and $\tilde{\Gamma}_a = \Pi^a_\mu \Gamma^\mu$. The $\tilde{\Gamma}$-matrix satisfy the algebra $\{\tilde{\Gamma}_a, \tilde{\Gamma}_b\} = -2h_{ab}$ and $\tilde{\Gamma}^2 = 1$.

The quantization of GS-superstring has been carried out in the light-cone gauge

\[
X^+ = p^+ \tau, \quad \Gamma^+ \theta^{(A)} = 0, \tag{2.3}
\]

where $X^\pm = \frac{1}{\sqrt{2}} \left( X^0 \pm X^9 \right)$ and $\Gamma^\pm = \frac{1}{\sqrt{2}} \left( \Gamma^0 \pm \Gamma^9 \right)$. To preserve the gauge we have to take $\delta X^+ = 0$ and $\Gamma^+ \delta \theta = 0$. This is carried out by combining the global supersymmetry and $\kappa$-symmetry with $\kappa^{(A)} = (-1)^{A-1} \frac{1}{2p^+} \partial_\sigma X^I \Gamma^+ \Gamma^I \epsilon^{(A)}$. We then get

\[
\delta \theta^{(1)} = -\frac{1}{2p^+} \Gamma^+ \left\{ \left( \partial_\tau - \partial_\sigma \right) X^I \Gamma^I - p^+ \Gamma^- \right\} \epsilon^{(1)},
\]

\[
\delta \theta^{(2)} = -\frac{1}{2p^+} \Gamma^+ \left\{ \left( \partial_\tau + \partial_\sigma \right) X^I \Gamma^I - p^+ \Gamma^- \right\} \epsilon^{(2)}, \tag{2.4}
\]

\[
\delta X^\mu = 2i\bar{\epsilon}^{(1)} \Gamma^\mu \theta^{(1)} + 2i\bar{\epsilon}^{(2)} \Gamma^\mu \theta^{(2)} + i\bar{\theta}^{(1)} \Gamma^\mu \delta \theta^{(1)} + i\bar{\theta}^{(2)} \Gamma^\mu \delta \theta^{(2)}.
\]
These just satisfy the conditions $\hat{\delta}X^+ = \Gamma^+ \delta\theta^{(A)} = 0$ because of $(\Gamma^+)^2 = 0$. Since $\hat{\delta}^{(A)} \Gamma I \partial_\alpha \theta^{(A)} = 0$ in the light-cone gauge, the action reduces to the simple form

$$S_{\text{LC}} = -\frac{1}{2\pi} \int d^2\xi \left[ -(\partial_\tau X^I)^2 + (\partial_\sigma X^I)^2 - 2ip^+ \hat{\theta}^{(1)} \Gamma^- (\partial_\tau + \partial_\sigma) \theta^{(1)} \
\nonumber\n- 2ip^+ \hat{\theta}^{(2)} \Gamma^- (\partial_\tau - \partial_\sigma) \theta^{(2)} \right]. \quad (2.5)$$

In the following we only consider chiral theory with negative chirality, $\Gamma^{11} \theta^{(A)} = -\theta^{(A)}$. The extension to non-chiral theory is straightforward. Let us introduce the variables

$$S^{(A)} = 2\frac{i}{2} \sqrt{p^+} \Gamma^- \theta^{(A)}, \quad \text{or} \quad \theta^{(A)} = \frac{1}{2} \frac{1}{2\pi i} \Gamma^+ S^{(A)} \quad (2.6)$$

such that $\Gamma^{11} S^{(A)} = S^{(A)}$ and $\Gamma^- S^{(A)} = 0$. The action is then written in the form

$$S_{\text{LC}} = -\frac{1}{2\pi} \int d^2\xi \left[ -(\partial_\tau X^I)^2 + (\partial_\sigma X^I)^2 - i(S^{(1)})^l (\partial_\tau + \partial_\sigma) S^{(1)} \right.
\nonumber\n\left. - i(S^{(2)})^l (\partial_\tau - \partial_\sigma) S^{(2)} \right]. \quad (2.7)$$

In this paper we use the following $\Gamma$-matrix; $\Gamma^0 = i \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$, $\Gamma^9 = i \begin{pmatrix} 0 & J \\ J & 0 \end{pmatrix}$, $\Gamma^I = i \begin{pmatrix} 0 & \hat{\gamma}^I \\ \hat{\gamma}^I & 0 \end{pmatrix}$ and $\Gamma^{11} = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, where $I$ is $16 \times 16$ identity matrix, $J = \begin{pmatrix} I_8 & 0 \\ 0 & -I_8 \end{pmatrix}$ and $\hat{\gamma}^I = \begin{pmatrix} 0 & \gamma^I \\ (\gamma^I)^t & 0 \end{pmatrix}$. $I_8$ is $8 \times 8$ identity and $\gamma^I$ is $8 \times 8$ $\gamma$-matrix familiar in the light-cone gauge formulation of GS-superstring [8, 6, 7]. Hence $(S^{(A)})^l = (S^{(A)})^a, 0, 0, 0, \; a = 1, \ldots, 8$.

In $SO(8)$ $\gamma$-matrix notation the supersymmetry transformation becomes

$$\delta^{(A)}_\epsilon X^I = 2i\epsilon^{(A)} \Gamma I \theta^{(A)} = -\frac{1}{2} \frac{i}{\sqrt{p^+}} \epsilon^{(A)} a \gamma^I \gamma^{(a)} S^{(A)} a, \quad (2.8)$$

$$\delta^{(A)}_\epsilon S^{(A)} = 2\frac{i}{2} \sqrt{2p^+} \epsilon^{(A)} a + 2\frac{i}{2} \frac{1}{\sqrt{p^+}} \epsilon^{(A)} a \gamma^I \gamma^{(a)} \partial_\tau X^I, \quad (2.9)$$

where $\Gamma^{11} \epsilon^{(A)} = -\epsilon^{(A)}$ and $\delta^{(A)}_\epsilon f = [\epsilon^{(A)} a \pi Q^{(A)} a + \epsilon^{(A)} a \hat{Q}^{(A)} a, f]$. The transformation law of $\hat{\delta}_\epsilon X^-$ can read out from eq.(2.4). The supercharges are defined
\[
\begin{align*}
Q^{(1)a} &= 2^{\frac{1}{2}} \sqrt{2p^+} \int_0^\pi \frac{d\sigma}{\pi} S^{(1)a}(\sigma), \\
Q^{(2)a} &= 2^{\frac{1}{2}} \sqrt{2p^+} \int_0^\pi \frac{d\sigma}{\pi} S^{(2)a}(\sigma), \\
\dot{Q}^{(1)a} &= 2^{\frac{3}{2}} \frac{2}{\sqrt{p^+}} \int_0^\pi \frac{d\sigma}{\pi} \partial X^I(\sigma) S^{(1)a}(\sigma) \gamma^{I}_{\dot{a}a}, \\
\dot{Q}^{(2)a} &= 2^{\frac{3}{2}} \frac{2}{\sqrt{p^+}} \int_0^\pi \frac{d\sigma}{\pi} \partial X^I(\sigma) S^{(2)a}(\sigma) \gamma^{I}_{\dot{a}a}.
\end{align*}
\]

where \(\partial = \frac{1}{2}(\partial_r - \partial_\sigma)\) and \(\bar{\partial} = \frac{1}{2}(\partial_r + \partial_\sigma)\). These satisfy the superalgebra \(\{Q^{(A)}, Q^{(B)}\} = 2p^+ \Gamma_{\mu} \delta^{AB}\), where \(Q^{(A)} = (Q^{(A)a}, Q^{(A)\dot{a}}, 0, 0)\) and \(C = -\Gamma^0\) is the charge conjugation matrix.

## 3 Vertex Operators for D-branes

We first discuss the vertex operators for a single Dp-brane. The Dp-brane configuration is described by the boundary conditions

\[
\begin{align*}
\left(\partial X^I(\tau, \sigma) - N_I J \bar{\partial} X^J(\tau, \sigma)\right)|_{\sigma = 0, \pi} &= 0, \\
\left(S^{(1)a}(\tau, \sigma) - N_{ab} S^{(2)b}(\tau, \sigma)\right)|_{\sigma = 0, \pi} &= 0.
\end{align*}
\]

Here we take \(N_{IJ} = (\delta_{\alpha\beta}, -\delta_{ij})\) so that \(\alpha, \beta = 1, \cdots, p-1\) are the Neumann directions and \(i, j = p, \cdots, 8\) are the Dirichlet directions. \(N_{ab}\) satisfies the orthogonal condition \(N_{ac} N_{bc} = \delta_{ab}\). \(p\) is odd because we now consider the chiral theory. When we consider the non-chiral theory, \(p\) must be even. \(p = 9\) is the usual open superstring. Since \(X^{\pm}\)'s satisfy Neumann boundary conditions, the value of \(p\) is restricted to \(p \geq 1\). To discuss the case of \(p < 1\), we must go to the cylinder frame which is discussed in the Sect.5.

The boundary conditions break half of the supersymmetry. The left-moving and the right-moving modes are related by the boundary conditions so that \(\partial X^I(\tau, \sigma) - N_{IJ} \bar{\partial} X^J(\tau, -\sigma) = 0\) and \(S^{(1)a}(\tau, \sigma) - N_{ab} S^{(2)b}(\tau, -\sigma) = 0\) are satisfied. Therefore only the supercharges

\[
\begin{align*}
Q^a = \frac{1}{\sqrt{2}} (Q^{(1)a} + N_{ab} Q^{(2)b}) , \\
\dot{Q}^a = \frac{1}{\sqrt{2}} (Q^{(1)a} + N_{\dot{a}b} Q^{(2)b}) ,
\end{align*}
\]

survive, where \(N_{ab}\) satisfies the orthogonal condition and

\[
\gamma^{I}_{\dot{a}a} N_{IJ} - N_{ab} N_{\dot{a}b} \gamma^{J}_{\dot{b}b} = 0.
\]
The solution is given by $N_{ab} = (\gamma^p \cdots \gamma^8)_{ab}$ and $N_{\dot{a}\dot{b}} = (\gamma^p \cdots \gamma^8)_{\dot{a}\dot{b}}$. The supercharges satisfy the N=1 superalgebra $\{Q, Q^t\} = 2p^\mu \Gamma^a \sigma$ or

$$\{Q^a, Q^b\} = 2\sqrt{2}p^+ \delta^{ab}, \quad \{Q^\dot{a}, Q^\dot{b}\} = 2\gamma_a \Gamma_\alpha \delta^{\dot{a}\dot{b}}, \quad \{Q^a, Q^\dot{b}\} = 2\gamma^a \Gamma_\alpha \delta^{\dot{a}\dot{b}},$$

(3.5)

where

$$p^- = \frac{1}{2p^+} \int_0^\pi \frac{d\sigma}{\pi} \left[ (\dot{X}^I)^2 + (X'^I)^2 - iS^{(1)a}_\sigma S^{(1)a} + iS^{(2)a}_\sigma S^{(2)a} \right].$$

(3.6)

The supersymmetry transformation at the boundary is now given in the form

$$\hat{\delta}_\epsilon X^\alpha = -2\frac{1}{2} i \sqrt{p^+} \epsilon^b \gamma_{\dot{a}\dot{a}} S^a, \quad \hat{\delta}_\epsilon X^i = 0,$$

$$\hat{\delta}_\epsilon S^a = 2\frac{1}{2} \sqrt{2}p^+ \epsilon^a + 2\frac{1}{2} \frac{1}{\sqrt{p^+}} \epsilon^b \gamma_{\dot{a}\dot{a}} \partial_B X^I,$$

(3.7)

where $S^a = \frac{1}{\sqrt{2}}(S^{(1)a} + N_{ab}S^{(2)b})$ and $\partial_B X^I = \partial X^I + N_{IJ} \partial X^J = (\partial_\tau X^\alpha, -\partial_\sigma X^i)$.

Let us introduce the 10D superfield $A_\mu(X, \theta)$ defined by

$$A_\mu(X, \theta) = e^{\theta G} A_\mu(X) e^{-\theta G} = A_\mu(X) + \delta_\theta A_\mu(X) + \frac{1}{2!} \delta_\theta^2 A_\mu(X) + \cdots,$$

(3.8)

where $\theta = (2\frac{1}{2} + 2\sqrt{p^+})^{-1} \Gamma^+ S$. $G$ is a generator of space-time supersymmetry. The vertex operator $V(A_\mu(X, \theta))$ should satisfy the following relation;

$$\hat{\delta}_\epsilon V(A_\mu(X, \theta)) = V(\delta_\epsilon A_\mu(X, \theta)).$$

(3.9)

We will show that this equation is satisfied when the vertex operator is given by

$$V(A(X, \theta)) = \int d\tau A_\mu(X, \theta) \partial_B X^\mu,$$

(3.10)

where $\partial_B X^\mu = (\partial_\tau X^+, \partial_\tau X^-, \partial_B X^I)$ and the space-time supersymmetry is given by

$$\delta_\epsilon A^\mu(X) = -i \bar{\Psi}(X) \Gamma^\mu \epsilon, \quad \delta_\epsilon \Psi(X) = -\frac{1}{2} F_{\mu\nu}(X) \Gamma^{\mu\nu} \epsilon$$

(3.11)
and

\[ A^\mu = (A^{\bar{\alpha}}(X^{\bar{\alpha}}), \phi^i(X^{\bar{\alpha}})) \],
\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \],

(3.12)

(3.13)

where \( \bar{\alpha} = (+, -, \alpha) \). \( \phi^i \) denotes the collective coordinate of D-brane in the “static gauge”. The fields depend only on the coordinates \( X^{\bar{\alpha}} \) such that \( \partial^\alpha A^\mu = \partial^\alpha \Psi = 0 \) and then \( F^{\bar{\alpha}}_\alpha = -F^\alpha_{\bar{\alpha}} = \partial_\alpha \phi^i \) and \( F_{ij} = 0 \). These fields satisfy the equations of motion \( \partial^{\bar{\alpha}} F^{\alpha_\mu} = 0 \) and \( \Gamma^{\bar{\alpha}} \partial_\mu \Psi = 0 \). \( \Psi \) has the negative chirality, \( \Gamma^{11} \Psi = -\Psi \). The explicit form of the vertex operator is now given in the form

\[ V(A) = V_B(A) + V_F(\Psi) \],
\[ V_B(A) = \int d\tau \left[ A_\mu(X) - \frac{i}{2} \frac{1}{2!} F_{\nu\lambda}(X)(\tilde{\partial} \Gamma^\mu_{\nu\lambda} \theta) 
- \frac{1}{2} \frac{4!}{4!} \partial_\mu F_{\nu\lambda}(X)(\tilde{\partial} \Gamma^\mu_{\nu\lambda} \theta)(\tilde{\partial} \Gamma^\mu_{\nu\lambda} \theta) \right] \partial_B X^\mu \],
\[ V_F(\Psi) = \int d\tau \left[ -i \tilde{\Psi}(X) \Gamma^\mu_{\nu\lambda} \theta 
- \frac{1}{3!} \partial_\mu \tilde{\Psi}(X) \Gamma^\mu_{\nu\lambda} \theta(\tilde{\partial} \Gamma^\mu_{\nu\lambda} \theta) 
+ \frac{i}{5!} \partial_\mu \partial_\nu \tilde{\Psi}(X) \Gamma^\mu_{\nu\lambda} \theta(\tilde{\partial} \Gamma^\mu_{\nu\lambda} \theta)(\tilde{\partial} \Gamma^\mu_{\nu\lambda} \theta) \right] \partial_B X^\mu \].

(3.14)

(3.15)

(3.16)

In the familiar \( SO(8) \) \( \gamma \)-matrix notation, these are rewritten in the form

\[ V_B(A) = \int d\tau \left[ A^I \partial_B X^I - A^+ \partial_\tau X^- - A^- p^+ - \frac{i}{8} F_{IJ}(S\gamma^I S) 
- \frac{i}{4p^+} F^{+I}(S\gamma^I S)\partial_B X^J 
- \frac{1}{96p^+} \partial^+ + F^{+I}(S\gamma^I S)(S\gamma^J K S)(S\gamma^K S) 
+ \frac{1}{96p^+} \partial^+ F^{+I}(S\gamma^I S)(S\gamma^J K S)\partial_B X^J \right] \],
\[ V_F(\Psi) = i \left( \frac{1}{2} \right)^{\frac{1}{2}} \int d\tau \left[ \sqrt{p^+} (\psi S) + \frac{1}{\sqrt{2p^+}} (\psi \gamma^I S) \partial_B X^I 
- \frac{i}{12} \sqrt{2p^+} (\partial^I \psi \gamma^J S)(S\gamma^I J S) 
+ \frac{i}{12p^+ \sqrt{2p^+}} (\partial^I \psi \gamma^J S)(S\gamma^I J S)\partial_B X^I 
- \frac{1}{240p^+ \sqrt{2p^+}} (\partial^I \partial^+ \psi \gamma^J S)(S\gamma^I K S)(S\gamma^J K S) \right] \].

(3.17)

(3.18)
\[
\frac{1}{240p+2\sqrt{2p^+}}(\partial^+ \partial^+ \psi \gamma^I S)(S\gamma^{JK} S)(S\gamma^{JK} S)\partial_B X^J
\]

where we use the notations \( \Psi = (0, 0, \psi^a, \dot{\psi}^a) \) and \( S\gamma^{IJ} S = S^a \gamma^{IJ}_{ab} S^b \), \( \psi S^a \) and \( \psi \gamma^I S = \psi^a \gamma^I_{ab} S^b \).

The proof of supersymmetry in general case is rather complicated. In this section we work in the case \( \partial^+ A^\mu = \partial^+ \Psi = 0 \), but \( A^\pm \neq 0 \). Using the supersymmetry transformation (3.7) and noting \( \hat{\delta}_\epsilon f(X) = \partial_\mu f(X) \hat{\delta}_\epsilon X^\mu = \partial^a f(X) \delta X^\alpha \), where \( \partial^+ f = \partial^i f = \hat{\delta}_\epsilon X^+ = 0 \) are used, and also

\[
\hat{\delta}_\epsilon (\partial_B X^I) = \partial_\tau (\hat{\Delta}_\epsilon X^I) = -2\frac{i}{\sqrt{p^+}} \epsilon^{\alpha} \gamma^I_{ab} \partial_\tau S^a ,
\]

we obtain the following equations;

\[
\hat{\delta}_\epsilon V_B(A) = \mathcal{V}_F(\delta_\epsilon \Psi) + \partial_\tau (A_\mu \hat{\Delta}_\epsilon X^\mu) ,
\]

\[
\hat{\delta}_\epsilon V_F(\Psi) = \mathcal{V}_B(\delta_\epsilon A) + \partial_\tau \left( -\frac{1}{8\sqrt{2}p^+} \frac{1}{16} (\psi^a \gamma^{IJ}_{ab} \dot{\psi}^b) (S\gamma^{IJ} S) \right) ,
\]

where

\[
V_{B,F} = \int d\tau \mathcal{V}_{B,F}
\]

and \( A_\mu \hat{\Delta}_\epsilon X^\mu = A^a \hat{\delta}_\epsilon X^a - A^+ \hat{\delta}_\epsilon X^- + \hat{\phi}^i \hat{\Delta}_\epsilon X^i \). Note that \( \hat{\Delta}_\epsilon X^a = \hat{\delta}_\epsilon X^a \) but \( \hat{\Delta}_\epsilon X^i \neq \hat{\delta}_\epsilon X^i \ (= 0) \). The \( \tau \)-derivative terms play an important role when we discuss the non-abelian gauge field [10].

### 4 Vertex Operators for Multi D-branes

In this section we study the extension of the previous argument to the non-abelian case. It is now straightforward. We replace the gauge field and the fermionic field with \( A_\mu = A^a_\mu \lambda^a \) and \( \Psi = \Psi^a \lambda^a \), where \( \lambda^a \) is the generator of gauge group. We also replace the field strength (3.13) with

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig' [A_\mu, A_\nu]
\]

and the space-time derivative \( \partial_\mu \) with the covariant one

\[
D_\mu = \partial_\mu + ig' [A_\mu, \ ] .
\]
The fields satisfy the equations of motion $D^\mu F_{\mu\nu} = 0$ and $\Gamma^\mu D_\mu \Psi = 0$. More explicitly,

\begin{equation}
D^{\dot{\alpha}} F_{\dot{\alpha}\beta} + ig' [\phi^i, F_{i\beta}] = 0, \quad D^{\dot{\alpha}} F_{\dot{\alpha}j} + ig' [\phi^i, [\phi_i, \phi_j]] = 0 \tag{4.3}
\end{equation}

\begin{equation}
\Gamma^{\dot{\alpha}} D_\alpha \Psi + ig' \Gamma^i [\phi_i, \Psi] = 0. \tag{4.4}
\end{equation}

where $F_{\dot{\alpha}i} = \partial_\alpha \phi_i + ig'[A_{\dot{\alpha}} , \phi_i]$. This means that the Chan-Paton factor is attached at the boundary. The thing we must take care here is the treatment of the contact term $[A_\mu, A_\nu]$. We here define the contact term, in the momentum space, by

$$
[A_\mu(X), A_\nu(X)] = [\lambda^s, \lambda^t] \int dk_1 \int dk_2 A^s_\mu(k_1) A^t_\nu(k_2) e^{ik_1 + k_2 - X}, \tag{4.5}
$$

where $k \cdot X = k_{\dot{\alpha}} X^\dot{\alpha}$ and the integrals are restricted to $k_1 \cdot k_2 = 0$. In the below we will show that the following supersymmetry transformation is realized when $g' = g$:

$$
P \int d\tau \left( \delta_\epsilon \mathcal{V}_B(A) + \hat{\delta}_\epsilon \mathcal{V}_F(\Psi) \right) \exp \left\{ -ig \int d\tau \left( \mathcal{V}_B(A) + \mathcal{V}_F(\Psi) \right) \right\}, \tag{4.6}
$$

where $\delta_\epsilon$ is the non-abelian supersymmetry defined in the form (3.11) with the field strength (4.1). $P$ is the $\tau$-ordering of vertex operators.

We first discuss the transformation property of $V_B(A)$. For simplicity we will show the supersymmetry in the background $A^+ = 0$ and $\partial^+ = 0$. In this case the vertex operator reduces to the simple form

$$
V_B(A) = \int d\tau \left[ A^I \partial_B X^I - A^- \partial^+ + \frac{i}{8} F^{IJ} (S \gamma^{IJ} S) \right]. \tag{4.7}
$$

As in the abelian case, noting that eq.(3.19) and $\hat{\delta}_\epsilon A^I = \partial^\alpha A^I \hat{\delta}_\epsilon X^\alpha = \partial^J A^I \hat{\Delta}_\epsilon X^J$ etc, the variation $\hat{\delta}_\epsilon \mathcal{V}_B(A)$ is calculated as

\begin{align}
\hat{\delta}_\epsilon \mathcal{V}_B(A) &= \left( \partial^I A^I - \partial^J A^I \right) \partial_B X^J \hat{\Delta}_\epsilon X^I - \left( \partial^I A^- - \partial^- A^I \right) \partial^+ \hat{\Delta}_\epsilon X^I \\
&\quad - \frac{i}{8} \partial^K F^{IJ} \hat{\Delta}_\epsilon X^K - \frac{i}{4} F^{IJ} (\hat{\delta}_\epsilon S \gamma^{IJ} S) + \partial_\tau \left( A^I \hat{\Delta}_\epsilon X^I \right) \\
&= F^{IJ} \partial_B X^J \hat{\Delta}_\epsilon X^I - F^- \partial^+ \hat{\Delta}_\epsilon X^I - \frac{i}{8} D^K F^{IJ} (S \gamma^{IJ} S) \hat{\Delta}_\epsilon X^K \\
&\quad - \frac{i}{4} F^{IJ} (\hat{\delta}_\epsilon S \gamma^{IJ} S) - ig' Y + \partial_\tau \left( A^I \hat{\Delta}_\epsilon X^I \right), \tag{4.8}
\end{align}
where

\[ Y = \left( [A^i, A^j] \partial_B X^j - [A^i, A^-] p^+ \right) \hat{\Delta}_\epsilon X^i - \frac{i}{8} [A^K, F^{I,J}] (S \gamma^{IJ} S) \hat{\Delta}_\epsilon X^K \]  

(4.9)

Applying the Fierz transformation and using the equation of motion \( D^I F^{I,J} = 0 \) and Bianchi identity \( D^{[K} F^{I,J]} = 0 \), we obtain the following equation;

\[ \hat{\delta} V_B(A) = V_F(\hat{\delta} \Psi) - ig' Y + \partial_\tau \left( A^I \hat{\Delta}_\epsilon X^I \right) \]  

(4.10)

Using this we evaluate

\[ \mathcal{P} \int d\tau \hat{\delta} V_B(A) \exp \left( -ig \int d\tau V_B(A) \right) = \int d\tau \hat{\delta} V_B(A) - ig \mathcal{P} \int d\tau \int d\tau' \hat{\delta} V_B(A(\tau')) V_B(A(\tau)) + o(g^2) . \]  

(4.11)

We will show that the extra term \( Y \) which appears in the first term of r.h.s. cancels the contact term coming from the collision point of operators in the second term when \( g' = g \). Noting the \( \tau \)-ordering of vertex operators, the contact term is evaluated in the following. The quantity which contributes to the contact term is

\[ -ig \left( \int d\tau \int_{\tau+\epsilon}^{\infty} d\tau' \partial_\tau' (A^I(\tau') \hat{\Delta}_\epsilon X^I) V_B(A(\tau)) \right. \]

\[ \left. + \int d\tau \int_{-\infty}^{\tau-\epsilon} d\tau' V_B(A(\tau)) \partial_\tau' (A^I(\tau') \hat{\Delta}_\epsilon X^I) \right) \]

\[ = ig \int d\tau \int dk dk' \varepsilon^{k' \cdot k} [A^I(k'), A^j(k) \partial_B X^j - A^-(k) p^+] - \frac{i}{8} F^{JK}(k)(S \gamma^{JK} S) \varepsilon^{i(k' + k) \cdot X(\tau)} \hat{\Delta}_\epsilon X^I(\tau) \]  

(4.12)

where \( \varepsilon \) is a short distance cut-off. We here evaluate the singular part by going to the momentum space \( f(X(\tau)) = f \int df(k) e^{ik \cdot X(\tau)} \) as follows;

\[ f_1(X(\tau')) f_2(X(\tau)) = \int dk dk' f_1(k') f_2(k) e^{i(k' + k) \cdot X(\tau')} |\tau' - \tau|^{k' \cdot k} . \]  

(4.13)

When \( k' \cdot k = 0 \), the expression (4.12) gives a finite contribution at the limit \( \varepsilon \to 0 \). This is just the contact term we want, which becomes \( ig Y \) in the coordinate space. Thus, when \( g' = g \), the extra term \(-ig Y\) in (4.10) exactly cancels the contact term derived from the second term of r.h.s. of
When \( k' \cdot k > 0 \), (4.12) vanishes. It, however, becomes infinite when \( k' \cdot k < 0 \). In this case another kind of contact term is needed to subtract the infinity, which is not discussed in this paper. Resumming up in \( g \) we thus obtain

\[
\mathcal{P} \delta_\epsilon V_B(A) \exp \left(-ig V_B(A)\right) = \mathcal{P} V_F(\delta_\epsilon \Psi) \exp \left(-ig V_B(A)\right) .
\] (4.14)

In the following we set \( g' = g \).

Next we consider the transformation property of \( V_F(\Psi) \). To preserve the gauge \( A^+ = 0 \) we introduce the gauge parameter \( \Lambda = i \bar{\Psi} \Gamma^\mu \epsilon + D^\mu \Lambda \).

From \( \delta A^+ = 0 \), \( \chi \) is given by \( \bar{\Psi} \Gamma^+ \epsilon = \partial^+ \bar{\chi} \Gamma^+ \epsilon \). Since we now consider the case \( \partial^+ = 0 \), this means that we must take the background \( \Gamma^+ \Psi = \psi^a = 0 \). Therefore, in this case, the vertex operator \( V_F(\Psi) \) reduces to the simple form

\[
V_F(\Psi) = i \left(\frac{1}{2}\right)^{\frac{1}{4}} \int d\tau \sqrt{p^+ \psi^a S^a} .
\] (4.16)

In the following we carry out the calculation keeping the gauge parameter \( \chi \) arbitrary. This means that we consider the background \( A^+ = \psi^a = 0 \) and also \( \delta A^+ = 0 \) because of \( \partial^+ = 0 \), but \( \delta \psi^a \neq 0 \) and \( \delta^2 A^+ \neq 0 \). When we want to preserve the \( A^+ = 0 \) gauge exactly we must take \( \chi^a = \frac{1}{\partial^+} \psi^a (\neq 0) \).

The supersymmetry transformation is now

\[
\delta_\epsilon V_F(\Psi) = i(\psi^{Ia} \epsilon) \partial_B X^I + \frac{1}{4}(\partial^I \psi \gamma^J \epsilon)(S_{\gamma}^{IJ} S) + i \frac{g}{16} [A^I, \psi \gamma^J \gamma^K \epsilon](S_{\gamma}^{JK} S) ,
\] (4.17)

where we use the equation of motion \( D^I \psi \gamma^I = 0 \). Using the expression of \( V_B(A) \) (4.7) and \( \delta_\epsilon A^\mu \) (4.15), it is rewritten in the form

\[
\delta_\epsilon V_F(\Psi) = V_B(\delta_\epsilon A) + ig Z - \partial_\tau (i \sqrt{2} \chi \epsilon) ,
\] (4.18)

where \( \chi \epsilon = \chi^{a} \epsilon^{a} \). The extra term \( Z \) is given by

\[
Z = \frac{1}{16} [A^I, \psi \gamma^{JK} \gamma^I \epsilon](S_{\gamma}^{JK} S) - \frac{1}{4\sqrt{2}} [F^{IJ}, \chi \epsilon](S_{\gamma}^{IJ} S) - i \sqrt{2} \left( [A^I, \chi \epsilon] \partial_B X^I - [A^+, \chi \epsilon] p^+ \right) .
\] (4.19)
The \( \tau \)-derivative term needs to cancel that appears in the expression \( V_B(\delta\chi A) \) when we incorporate the gauge transformation into the supersymmetry transformation as in (4.15). Since \( \psi^A = 0 \), the \( \tau \)-derivative term appearing in the expression (3.21) vanishes now.

Let us calculate the following quantity:

\[
P\left( \hat{\delta}_\tau V_B(A) + \hat{\delta}_\tau V_F(\Psi) \right) \exp \left\{ -ig \left( V_B(A) + V_F(\Psi) \right) \right\}
\]
(4.20)

As in the previous calculation the contact terms coming from the expression

\[
- igP \int d\tau \partial_\tau (A^I \Delta \epsilon X^I) V_F(\Psi) - igP \int d\tau \partial_\tau (-i\sqrt{2}\chi\epsilon) V_B(A)
\]
(4.21)

just cancel \( igZ \). The contact term coming from \( -igP\hat{\delta}_\tau V_F(\Psi)V_F(\Psi) \) gives the extra term

\[
- ig \left( \frac{1}{2} \right)^{\frac{1}{4}} \sqrt{2p^+} \int d\tau [\chi \epsilon, \psi S].
\]
(4.22)

Note that

\[
V_F(\delta(\chi)\Psi) = V_F(\delta(\chi)\Psi) - ig \left( \frac{1}{2} \right)^{\frac{1}{4}} \sqrt{2p^+} \int d\tau [\chi \epsilon, \psi S],
\]
(4.23)

where \( \delta(\chi) \) is defined by \( \delta(\chi)\Psi = \delta(\chi)\Psi - ig[\Lambda, \Psi] \) with \( \Lambda = i\bar{\chi}\Gamma^+\epsilon \). Thus the extra contact term (4.22) and \( V_F(\delta(\chi)\Psi) \) in (4.14) are combined into \( V_F(\delta(\chi)\Psi) \) and we finally obtain

\[
P\left( \hat{\delta}_\tau V_B(A) + \hat{\delta}_\tau V_F(\Psi) \right) \exp \left\{ -ig \left( V_B(A) + V_F(\Psi) \right) \right\}
= P\left( V_B(\delta(\chi)A) + V_F(\delta(\chi)\Psi) \right) \exp \left\{ -ig \left( V_B(A) + V_F(\Psi) \right) \right\}.
\]
(4.24)

Until now we assumed the gauge invariance in the calculation. Since the equation is satisfied for the generic \( \chi \), however, this also gives the partial proof of the gauge invariance. The proof for the generic background is rather complicated, but straightforward.

5 Nonlinear Realized Supersymmetry of D-branes

In this section we study the D-brane in the cylinder/closed-string frame and then discuss the nonlinear realization of the broken supersymmetry. We
introduce the boundary states satisfying the boundary condition [12]

\[
\left( \partial X^I(\sigma) - M_{IJ} \bar{\partial} X^J(\sigma) \right) |B > = 0 , \quad (5.1)
\]

\[
\left( S^{(1)a}(\sigma) + i M_{ab} S^{(2)b}(\sigma) \right) |B > = 0 . \quad (5.2)
\]

at the boundary \( \tau = 0 \), where \( I = (\alpha = 1, \ldots, p + 1, \ i = p + 2, \ldots, 8) \) and \( M_{IJ} = (-\delta_{\alpha\beta}, \delta_{ij}) \) and \( M_{ac} M_{bc} = \delta_{ab} \). \( \alpha, \beta \) denote the Neumann boundary condition and \( i, j \) is Dirichlet ones. The boundary condition of coordinates \( X^\pm \) now become the Dirichlet ones so that the value of \( p \) is restricted to \(-1 \leq p \leq 7\) in the cylinder frame. This means that the world-volume has the Euclidean signature and one of the Dirichlet directions is time-like. So it is related to the D-brane by a double Wick rotation [12].

Let us define the supercharges

\[
Q^{\pm a} = \frac{1}{\sqrt{2}} (Q^{(1)a} \pm i M_{ab} Q^{(2)b}) , \quad Q^{\pm \dot{a}} = \frac{1}{\sqrt{2}} (Q^{(1)\dot{a}} \pm i M_{\dot{a}\dot{b}} Q^{(2)\dot{b}}) , \quad (5.3)
\]

where \( M_{ab} \) satisfy the orthogonal condition and

\[
\gamma^{I}_{aa} M_{IJ} - M_{ab} M_{\dot{a}\dot{b}} \gamma^{J}_{\dot{b}\dot{b}} = 0 . \quad (5.4)
\]

The solution is given by \( M_{ab} = (\gamma^1 \ldots \gamma^{p+1})_{ab} \) and \( M_{\dot{a}\dot{b}} = (\gamma^1 \ldots \gamma^{p+1})_{\dot{a}\dot{b}} \). From the boundary condition (5.2) \( Q^+ \) is the unbroken supersymmetry satisfying \( Q^+ |B > = 0 \) and \( Q^- \) is the broken one. These satisfy the following superalgebra;

\[
\{ Q^{+a} , Q^{-b} \} = 2 \sqrt{2} p^+ \delta^{ab} , \quad (5.5)
\]

\[
\{ Q^{+\dot{a}} , Q^{-\dot{b}} \} = \{ Q^{-\dot{a}} , Q^{+\dot{b}} \} = 2 \gamma_{ab} p^i , \quad (5.6)
\]

\[
\{ Q^{+\dot{a}} , Q^{-\dot{b}} \} = 2 \sqrt{2} p^- \delta^{\dot{a}\dot{b}} . \quad (5.7)
\]

All other types of anticommutators vanish. The supersymmetry transformation for the unbroken supercharge \( Q^+ \) at the boundary is given by

\[
\delta^{(+)}_\epsilon X^a = -2^{\frac{1}{4}} \frac{i}{\sqrt{p^+}} \epsilon^a \gamma^a_{aa} S^{-a} , \quad \delta^{(+)}_\epsilon X^\dot{a} = 0 ,
\]

\[
\delta^{(+)}_\epsilon S^{-a} = 2^{\frac{1}{2}} \sqrt{2p^+} \epsilon^a + 2^{\frac{1}{2}} \frac{1}{\sqrt{p^+}} \epsilon^a \gamma^I_{aa} \partial_C X^I . \quad (5.8)
\]
\[ \hat{\delta}_c^{(+)} (\partial_C X^I) = -\partial_\sigma (\hat{\Delta}^{(+)} X^I) , \quad \hat{\Delta}^{(+)} X^I = -2\frac{i}{\sqrt{p^+}} \epsilon^{a}_{aa} \gamma^I S^{-a} , \quad (5.9) \]

where \( \partial_C X^I = \partial X^I + M_{IJ} \partial X^J = (-\partial_\sigma X^\alpha , \partial_\tau X^i) \).

The vertex operators in this frame are defined at the boundary by

\[ V(A(X, \theta)) = V_B(A) + V_F(\Psi) = \int_0^\pi \frac{d\sigma}{i\pi} A_\mu(X, \theta) \partial_C X^\mu , \quad (5.10) \]

where \( \partial_C X^\mu = (\partial_\tau X^+, \partial_\tau X^-, \partial_C X^I) \). The fermionic coordinate \( \theta \) is now defined by

\[ \theta = \frac{1}{2} \frac{1}{\sqrt{p^+}} \Gamma^+ S^- , \quad (5.11) \]

where \( (S^-)^I = (S^{-a}, 0, 0) \) and \( S^{-a} = \frac{1}{\sqrt{2}} (S^{(1)a} - i M_{ab} S^{(2)b}) \). The superfield is given by (3.8) with the non-abelian generalization of supersymmetry transformation (3.11). The difference is the assignment of vector field

\[ A^\mu = \left( A^\alpha(X^\alpha) , \phi^{\tilde{i}}(X^\alpha) \right) , \quad (5.12) \]

where \( \tilde{i} = (+, -, i) \). The fields depend only on the coordinates \( X^\alpha \) such that \( \partial_\tau A^\mu = \partial_\tau \Psi = 0 \). Thus \( \partial^+ = 0 \) reduction used in the previous calculation now becomes essential.

The amplitude in the cylinder frame is defined by

\[ <0|V_1 \cdots V_n|B> = g^{n-2} \sum_{\text{perm}} Tr <0| \int \frac{d\sigma_1}{i\pi} V(\sigma_1) \cdots \int \frac{d\sigma_n}{i\pi} V(\sigma_n) |B> , \quad (5.13) \]

where \( <0| \) is the closed string vacuum. Although the vertex operators are all attached to the boundary, we need a sum over inequivalent permutations of the vertex operators described by \( \sum_{\text{perm}} \). This just corresponds to the \( \tau \)-ordering in the open-string frame.

As in the previous section, we can prove that the supersymmetry transformation for the unbroken supercharge \( Q^+ \) satisfies the equation

\[ \sum_{\text{perm}} \hat{\delta}_c^{(+)} V(A) \exp(-igV(A)) = \sum_{\text{perm}} V(\delta_c^{(+)} A) \exp(-igV(A)) , \quad (5.14) \]
where we use the symbol $\delta_\varepsilon^{(\pm)}$ for the space-time supersymmetry transformation with the field contents (5.12). We here show that only in the case of $\hat{\delta}_\varepsilon^{(\pm)} V_B(A)$. The variation of $V_F(\Psi)$ is also calculated in the same way. For simplicity we discuss in the special background $\phi^+ = \phi^- = 0$. In this case the variation of $V_B(A)$ is given by

$$\hat{\delta}_\varepsilon^{(\pm)} V_B(A) = V_F(\delta_\varepsilon^{(\pm)} \Psi) - igY - \partial_\sigma \left( A_I \hat{\Delta}_\varepsilon^{(\pm)} X^I \right), \quad (5.15)$$

where $A_I = (A^a, \phi^i)$ and

$$Y = [A', A^j] \partial_C X^I \hat{\Delta}_\varepsilon^{(\pm)} X^I - \frac{i}{8} [A^K, F^{IJ}] (S^- g^{IJ} S^-) \hat{\Delta}_\varepsilon^{(\pm)} X^K. \quad (5.16)$$

Thus the variation of $V_B(A)$ in the boundary condensate becomes

$$\sum_{\text{perm}} \hat{\delta}_\varepsilon^{(\pm)} V_B(A) \exp \left(-igV_B(A)\right) = V_F(\delta_\varepsilon^{(\pm)} \Psi) - ig \int_0^\pi \frac{d\sigma}{i\pi} Y - ig \sum_{\text{perm}} V_F(\delta_\varepsilon^{(\pm)} \Psi) V_B(A)$$

$$+ ig \int_{|z'|>|z|} \frac{dz'}{i\pi} \partial_{z'} \left( A_I(\sigma') \hat{\Delta}_\varepsilon^{(\pm)} X^I \right) \int_0^\pi \frac{d\sigma}{i\pi} V_B(A(\sigma))$$

$$+ \int_0^\pi \frac{d\sigma}{i\pi} V_B(A(\sigma)) \int_{|z'|<|z|} \frac{dz'}{i\pi} \partial_{z'} \left( A_I(\sigma') \hat{\Delta}_\varepsilon^{(\pm)} X^I \right) + o(g^2), \quad (5.17)$$

where $z = e^{2i\sigma}$ and $z' = e^{2i\sigma'}$. The integral of $z'$ gives the contribution from the collision point $z' = z$. We here regularize the singularity by displacing the $\sigma$ contours by infinitesimal amounts along the cylinder axis, which is described by $|z'| > |z|$ and $|z'| < |z|$. Near the collision point $z \sim z'$, we take $z' - z = \varepsilon e^{i\theta}$ for $|z'| > |z|$ and $z' - z = \varepsilon e^{-i\theta}$ for $|z'| < |z|$, where $0 < \theta < \pi$. The singularity of the operator product is evaluated in the momentum space as

$$f_1(X(\sigma')) f_2(X(\sigma)) = \int dk' dk f_1(k') f_2(k) e^{i(k' + k) \cdot X(\sigma)} |z' - z|^{k' \cdot k}. \quad (5.18)$$

So we must evaluate the following integral

$$\oint_{|z'|>|z|} \frac{dz'}{i\pi} |z' - z|^{k' \cdot k} = \int_{|z'|>|z|} \frac{dz'}{i\pi} \frac{1}{z' - z} |z' - z|^{k' \cdot k} = \varepsilon^{k' \cdot k}. \quad (5.19)$$
In the case of $|z'| < |z|$ the integral gives the negative sign: $-\varepsilon^{k'k}$. When $k' \cdot k = 0$ the integral gives a finite contribution. In the coordinate space this just gives $i g \int_0^\pi \frac{d\sigma}{i\pi} Y$, which cancels the second term of r.h.s. in (5.17).

The space-time supersymmetry $\delta^{(+)}$ is now realized linearly as

$$\delta^{(+)}_\epsilon < 0|V_1 \cdots V_n|B >= < 0|\epsilon Q^+, V_1 \cdots V_n|B >= 0 ,$$  \hspace{1cm} (5.20)

where the equation (5.14) is used in the first equality and $< 0|Q^+ = Q^+|B > = 0$ in the second.

On the other hand the nonlinear realization of the broken supersymmetry $Q^-$ is given as follows. Let us consider a constant shift of fermionic collective coordinate

$$\delta^{(-)} \Psi = \eta.$$  \hspace{1cm} (5.21)

If the $\eta$ satisfies the equation

$$D^\mu \eta = 0 ,$$  \hspace{1cm} (5.22)

the vertex operator transforms under this transformation as

$$V_F(\delta^{(-)} \Psi) = -i \int_0^\pi \frac{d\sigma}{i\pi} \bar{\eta} \Gamma_\mu \theta \partial C X^\mu = \frac{1}{2} \left( \eta^a Q^{-a} + \eta^\dagger Q^{-\dagger} \right).$$  \hspace{1cm} (5.23)

Noting the commutator $\{Q^-, S^-\} = 0$ and $\tilde{\delta}^{(-)} X^\alpha = \tilde{\delta}^{(-)} (\partial C X^I) = 0$ such that $[Q^-, V] = 0$, we obtain

$$\delta^{(-)}_\eta < 0|V_1 \cdots V_n|B >= 0$$  \hspace{1cm} (5.24)

because of $< 0|Q^- = 0$. Thus this symmetry is exact even in the static gauge.

The presence of the supersymmetries $\delta^{(\pm)}$ reflects that D-branes can couple to closed superstrings. Since the condition (5.22) means that $[\phi^i, \eta] = 0$ for multi D-branes, if we take the Chan-Paton factor is $U(n)$, $\eta$ is a shift in $U(1)$ direction. This result seems to support the recent descriptions of supermembrane using multi D-particles [14] and superstring using multi D-instantons [15].

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