Effective neutrino number shift from keV-vacuum neutrino-phobic 2HDM

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If the Dirac neutrino masses are generated by a new scalar doublet with a vacuum at keV order, there would be rare hope to probe the framework in low-energy flavor physics, such as the lepton-flavor violating processes. However, the predicted neutrino Yukawa couplings being around the order of electronic Yukawa can realize a purely thermal Dirac leptogenesis, and thus render a more direct link between the high- and low-energy CP violation. Despite its inert property in the low-energy flavor processes, the keV-vacuum induced Dirac neutrino model can generate a significant shift of the effective neutrino number, which can in turn be probed by the big-bang nucleosynthesis and cosmic microwave background epochs. We show that such a keV-vacuum induced Dirac neutrino model is already constrained by the current observations and will be probed with forecast sensitivity, serving therefore as a complementary avenue to test the Dirac leptogenesis.

I. INTRODUCTION

The observations of neutrino oscillations have thus far prompted several puzzles about neutrinos in the Standard Model (SM), including their tiny mass origin, their Dirac or Majorana nature, their distinctive mixing pattern from the quark sector, etc. Any trial towards these problems has catalyzed a host of investigations from theoretical constructions to experimental searches, and from low-energy particle physics to high-temperature early Universe. Despite the leading interests in Majorana neutrinos, the Dirac neutrino scenarios are also welcome and can generate promising experimental signals. If neutrinos are of the Dirac type, they can trigger the Dirac leptogenesis [1, 2] to explain the observed baryon asymmetry of the Universe (BAU). Recently, we have demonstrated that a purely thermal Dirac leptogenesis (PTDL) mechanism [3, 4] can establish a more direct link between the low-energy leptonic CP violation and the high-scale lepton asymmetry, where the BAU is formulated by the neutrino mixing without invoking specific Yukawa textures. In addition to the intimate relation with the BAU mystery, the right-handed Dirac neutrinos can also have a significant impact on the evolution of the early Universe, e.g., via generating the effective neutrino number shift, $\Delta N_{\text{eff}}$, which is constrained by the big-band nucleosynthesis (BBN) [5–8] and cosmic microwave background (CMB) observations [9–13].

A simple and testable scenario for Dirac neutrino mass generation is by introducing a new Higgs doublet with a much smaller vacuum than the electroweak one, which is nowadays called neutrino-phobic two-Higgs-doublet model (2HDM) [14, 15] (for Majorana neutrinos based on the 2HDM, see, e.g., Refs. [16–20]). In previous investigations [21–23], an eV-scale Higgs vacuum was widely considered to embrace $\mathcal{O}(1)$ neutrino Yukawa couplings. Phenomenological analyses of such an eV-vacuum Dirac neutrino model have also been performed therein, pointing out especially that the lepton-flavor violating (LFV) transitions, such as $\mu \rightarrow e\gamma$, can reach the future MEG sensitivities [24, 25]. However, such $\mathcal{O}(1)$ neutrino Yukawa couplings can delay the decoupling of right-handed Dirac neutrinos in the early Universe via, e.g., effective four-fermion interactions mediated by the new scalar, and thus violate the bound of $\Delta N_{\text{eff}}$ extracted from the BBN and CMB measurements [9].

If the new scalar doublet has instead a keV-scale vacuum, the resulting LFV signals from $\ell_{\alpha} \rightarrow \ell_{\beta}\gamma$, $\ell_{\alpha} \rightarrow 3\ell_{\beta}$, $Z \rightarrow \ell_{\alpha}\ell_{\beta}$, $h \rightarrow \ell_{\alpha}\ell_{\beta}$, or $\mu - e$ conversion in nuclei would hardly reach the future sensitivities [23]. Nevertheless, such a flavor-physics inert model can readily satisfy the thermal conditions predicted by the PTDL mechanism [3, 4]. In this paper, we will show that, depending on the detailed setup, the model can also manifest itself via a contribution to $\Delta N_{\text{eff}}$, which is found to be compatible with the current data, and can be further probed by the forecast sensitivity from, e.g., the CMB Stage-4 (CMB-S4) [26] and its combination with the BBN [8]. Thus, $\Delta N_{\text{eff}}$ can serve as a promising observable to probe such a keV-vacuum induced Dirac neutrino model, and hence as a complementary avenue to test the PTDL mechanism.

The paper is organized as follows. In section II, we recapitulate the neutrino-phobic 2HDM, categorize the two possible thermal conditions in realizing the PTDL mechanism, and then determine the favored vacuum ranges. In section III, depending on the thermal conditions, we calculate the respective evolution of the right-handed Dirac neutrinos in the early Universe. The resulting $\Delta N_{\text{eff}}$ phenomenology is analyzed in section IV. Finally, our conclusion is made in section V.

II. NEUTRINO-PHOBIC 2HDM WITH A KEV-SCALE VACUUM

The Dirac neutrino masses can be generated by coupling the right-handed Dirac neutrinos to a new Higgs doublet via

$$-\mathcal{L}_{\nu} = Y_{\nu} \tilde{\Phi} \nu_R + \text{H.c.}$$

(1)
in addition to the SM content. Here, a $Z_2$ (or $U(1)$) symmetry
can be imposed to forbid $\nu_R$ from interacting with the SM-like Higgs doublet $\Phi_{SM}$ that is responsible for all the charged fermion masses. In addition, the lepton-number symmetry must be present to forbid the Majorana mass term $\nu_R^\dagger \nu_R$, a condition also implemented in the Dirac lepto genesis. The neutrinoophilic scalar doublet $\Phi$ is usually assumed to interact with $\Phi_{SM}$ via a $Z_2$- or $U(1)$-symmetric potential, which has a soft-breaking source from $\mu^2 \Phi_{SM}^\dagger \Phi + \text{H.c.}$ This soft-breaking term also seeds a seesaw-like suppression of the scalar vacuum $\langle \Phi \rangle = (0, v_\Phi/\sqrt{2})^T$, with [14, 15] \[ v_\Phi = \frac{\mu^2 v_{\Phi SM}}{m_\Phi^2 + \lambda v_{\Phi SM}^2}, \] where $(v_{\Phi SM}^2 + v_\Phi^2)^{1/2} \approx 246 \text{ GeV}, m_\Phi$ is the bare mass of the neutrinoophilic scalar doublet, and $\lambda$ encodes the dimensionless potential parameters. Then, with an electroweak-scale $\Phi$ concerned throughout this paper, a keV-scale vacuum can be readily obtained by $\mu \approx O(10^{-2})$ GeV, and hence generates the tiny Dirac neutrino masses via Eq. (1). The resulting hierarchy $v_{\Phi SM} \gg v_\Phi$ suppresses the $\Phi_{SM} \Phi$ mass mixing at the order of $v_\Phi/v_{\Phi SM} \approx 10^{-8}$, making therefore $\Phi_{SM}$ the SM-like Higgs [15]. It is worth mentioning here that, if the potential exhibits a $U(1)$ symmetry that is only softly broken by the quadratic mixing term, there would be no quartic mixing term, $(\Phi_{SM}^\dagger \Phi)^2 + \text{H.c.}$, and the neutral components of $\Phi$ would be degenerate in mass [22], which is a distinguishing feature of the $U(1)$-symmetric 2HDM (see, e.g., also Refs. [27, 28]) and has been considered in the PTDL mechanism [4]. Furthermore, given that the constraints from electroweak precision tests already force small mass splitting between the neutral and charged scalars [29], a strongly first-order phase transition in such a degenerate neutrinoophilic 2HDM would be less feasible [30–32] to trigger the electroweak baryogenesis [33]. This leaves the PTDL mechanism [3, 4] as a natural candidate to address the BAU problem.

For the current neutrino masses inferred from oscillation measurements [34] and cosmological constraints [9], we expect that at least two neutrinos reside at $0.01 - 0.05$ eV, as can be clearly seen from Fig. 1, where the neutrino mass spectrum in light of current oscillation data and under the Planck mass bound $\sum_i m_{\nu_i} < 0.12$ eV is depicted. In the neutrinoophilic 2HDM framework, such a neutrino mass spectrum indicates a striking property when we consider the evolution of the right-handed Dirac neutrinos in the early Universe. In essence, regardless of the vacuum scales, the two heavier neutrinos ($\nu^R_h$) have a similar order of Yukawa couplings in both the NO and IO patterns. This implies that the two heavier $\nu^R_h$ would basically follow the same thermal evolution in the early Universe. On the other hand, given that the lightest neutrino has an upper rather than a lower mass bound, it can either be the case where the lightest neutrino ($\nu_1$) mass is comparable to the two heavier ones, $O(0.01)$ eV, so that the three $\nu_R$ would exhibit a similar history of evolution, or the case where $\nu_1$ is much lighter so that it would undergo a distinguishable evolution from those of the two heavier ones. Based on the above observations, we can conclude that there are only two possible thermal conditions that would be mostly feasible in realizing the PTDL mechanism in the neutrinoophilic 2HDM framework:

(i) all the three right-handed Dirac neutrinos never establish thermal equilibrium before the sphaleron decoupling, which is considered in Ref. [3].

(ii) the lightest $\nu_R$ is out of equilibrium while the other two have already established the left-right equilibrium (LRE) prior to the sphaleron freezing out, which is realized and called $\nu_1$-leptogenesis in Ref. [4].

Starting firstly with the case (i), we can infer that the late-time LRE requires the decay rate $\Gamma_{\Phi \rightarrow L^\nu_R}$ to be smaller than the Hubble expansion at the sphaleron decoupling temperature $T_{sph} \approx 132$ GeV [35], which gives \[ \Gamma_{\Phi \rightarrow L^\nu_R} \lesssim 3H(T) \big|_{T = T_{sph}}, \] where $3H(T)$ comes from the friction term in the Boltzmann equation for the particle-number evolution of $\nu_R$. The Hubble expansion is given by \[ H(T) \approx 1.66 \sqrt{g_*^e T^2}/M_{Pl}, \] with $g_*^e$ the effective number of degrees of freedom (d.o.f) and $M_{Pl} \approx 1.2 \times 10^{19}$ GeV the Planck mass. Note that the decay rate in Eq. (3) is given as a sum of three channels, i.e., one charged and two neutral decay channels for the $\nu_R$ production. Neglecting the charged-lepton and neutrino masses in the final states, the decay rate is given by \[ \Gamma_{\Phi \rightarrow L^\nu_R} \approx \frac{m_i^2}{4\pi v_\Phi^2} m_\Phi, \] where we have replaced the Yukawa coupling $y_{\nu,i}$ by the neutrino mass $m_i$ via $y_{\nu,i} = \sqrt{2} m_i/v_\Phi$, and parametrized the scalar mass at this temperature regime by the free parameter $m_\Phi$. Note here that the thermal mass correction of $\Phi$ around the sphaleron decoupling temperature is at $O(10)$ GeV for small potential parameters $\lambda \approx O(0.1)$ [36], which ensures that the $m_\Phi$-parametrization is a good approximation when $m_\Phi \gtrsim v_{\Phi SM}$. Since Eq. (3) is applied to three generations of $\nu_R$, requiring the heaviest neutrino ($m_h \approx 0.05$ eV for both the NO and IO patterns) to satisfy Eq. (3) would render a lower bound of the vacuum: \[ \left(\frac{v_\Phi}{\text{keV}}\right)\gtrsim 52 \left(\frac{m_\Phi}{\text{GeV}}\right)^{1/2}, \] with $g_*^e \approx 106.75$ has been used. Thus, for an electroweak-scale scalar $\Phi$ concerned here, the case (i) necessitates a minimal vacuum around sub-MeV.

For the case (ii), the production for the two heavier $\nu^R_h$ should have a rate satisfying \[ \Gamma_{\Phi \rightarrow L^\nu_{h,R}} \gtrsim 3H(T) \big|_{T = T_{sph}}. \]
In this case, it is the second lightest neutrino that determines an upper bound of \( v_\Phi \). For the IO pattern, since \( m_{1,2} \sim 0.05 \text{ eV} \), we can immediately obtain as in deriving Eq. (6) that

\[
\left( \frac{v_\Phi}{\text{keV}} \right) \lesssim 52 \left( \frac{m_\Phi}{\text{GeV}} \right)^{1/2}.
\]

From the NO pattern, however, \( m_2 \simeq 0.01 \text{ eV} \) requires the vacuum to be

\[
\left( \frac{v_\Phi}{\text{keV}} \right) \lesssim 10 \left( \frac{m_\Phi}{\text{GeV}} \right)^{1/2}. \tag{9}
\]

Regarding the lightest Dirac neutrino, on the other hand, Eq. (3) imposes the bound

\[
\left( \frac{v_\Phi}{\text{keV}} \right) \gtrsim 1.0 \left( \frac{m_l}{\text{meV}} \right) \left( \frac{m_\Phi}{\text{GeV}} \right)^{1/2}. \tag{10}
\]

It should be mentioned that the vacuum scale in case (ii) cannot be arbitrarily low by tuning \( m_l \), since otherwise the two heavier \( \nu_{h,R} \) will correspond to larger Yukawa couplings, and then the decoupling of \( \nu_{h,R} \) will be delayed to a sufficiently low temperature \( T_{\nu_{h,R}, \text{dec}} \lesssim m_\Phi \). This situation can readily have an impact on the synthesis of primordial elements and the CMB formation. Thus, there actually exists a lower bound of the decoupling temperature for the two heavier \( \nu_{h,R} \) [7, 10], which in turn sets a lower bound on the vacuum. To make this point clear, let us suppose that the late-time decoupling can occur at a lowest allowed temperature \( T_{\nu_{h,R}, \text{dec}} \lesssim m_\Phi \) when the annihilation rate is comparable to the Hubble expansion. Given that the thermal neutrino annihilation rate \( \Gamma_{\nu_{h,R}, \text{anni}} \) scales on naive dimensional grounds as

\[
\Gamma_{\nu_{h,R}, \text{anni}} \simeq \frac{m_{h}^{4} T^{5}}{v_{\Phi}^{2} m_{\Phi}^{4}}, \tag{11}
\]

a lower bound of the vacuum can be obtained as

\[
\left( \frac{v_\Phi}{\text{keV}} \right) \gtrsim 0.22 \left( \frac{m_h}{0.01 \text{ eV}} \right) \left[ \frac{T_{\nu_{h,R}, \text{dec}}/\text{GeV}}{m_\Phi/\text{GeV}} \right]^{3/4}. \tag{12}
\]

As \( m_h \simeq \mathcal{O}(0.01) \text{ eV} \) for both the NO and IO patterns, it can be seen that the lower bound from Eq. (10) would be tighter than from Eq. (12) if we focus on the case of \( m_l \gtrsim \mathcal{O}(1) \text{ meV} \).

Since the PTDL mechanism considered in Ref. [4] favors the NO pattern, the case (ii) would require a vacuum in the following range:

\[
1.0 \left( \frac{m_l}{\text{meV}} \right) \left( \frac{m_\Phi}{\text{GeV}} \right)^{1/2} \lesssim \left( \frac{v_\Phi}{\text{keV}} \right) \lesssim 10 \left( \frac{m_\Phi}{\text{GeV}} \right)^{1/2}, \tag{13}
\]

where \( m_l \gtrsim \mathcal{O}(1) \text{ meV} \) is considered. Together with Eq. (6), we can see that a keV-scale or higher vacuum is generically predicted for both cases (i) and (ii). Given that the LFV processes from first sensitivities can only probe a vacuum up to \( \mathcal{O}(10) \text{ eV} \) with an electroweak-scale \( \Phi \) [23], there would indeed be rare hope to see the LFV signals in neutrino-phobic 2HDM with an \( \mathcal{O}(\text{keV}) \) vacuum. Nevertheless, the expected sensitivity is dramatically different in cosmic regime, especially given the fact that the current precision of astrophysical and cosmological observations is now making the probe of feeble couplings and light species strikingly possible [37]. In the subsequent sections, we will firstly determine the corresponding evolution of \( \nu_R \) based on the two cases, and then show that the current limits of \( \Delta N_{\ell,ff} \) from the BBN, CMB, and their combinations are already available to constrain the \( \mathcal{O}(\text{keV}) \) vacuum, and the future forecast sensitivity can further test this keV-vacuum induced Dirac neutrino model.

### III. EVOLUTION OF RIGHT-HANDED DIRAC NEUTRINOS

To estimate the radiation contribution to the SM plasma from \( \nu_R \), we now proceed to determine the energy evolution for both cases. For the case (i), since the three \( \nu_R \) cannot establish thermalization throughout the Universe expansion, they are essentially produced via the freeze-in mecha-
nism [38, 39]. Let us consider the energy density defined by \( Y_{\nu_R, \rho} \equiv \rho_{\nu_R}/s_{SM}^{4/3} \), where the SM entropy density is given by

\[
s_{SM} = g^* \frac{2\pi^2}{45} T^3, \tag{14}\]

with \( g^* \) denoting the entropy d.o.f. The simplified Boltzmann equation reads

\[
dY_{\nu_R, \rho} \over dT = - \frac{C_{\nu_R, \rho}}{s_{SM}^{4/3} HT}, \tag{15}\]

where the collision term \( C_{\nu_R, \rho} \) is given by

\[
C_{\nu_R, \rho} = 2N_{\nu_R} \int \frac{d^3p_L}{(2\pi)^32\varepsilon_{\Phi}} f_\Phi \int \frac{d^3p_L}{(2\pi)^32\varepsilon_{L}} \frac{d^3p_{\nu_R}}{(2\pi)^32\varepsilon_{\nu_R}} \times (2\pi)^4 \delta^4(p_\Phi - p_L - p_{\nu_R})|\mathcal{M}_{\Phi \rightarrow L\nu_R}|^2. \tag{16}\]

Here \( N_{\nu_R} = 6 \) if the three \( \nu_R \) (and the three \( \bar{\nu}_R \)) have similar mass scales so that they have basically the same thermal history, or \( N_{\nu_R} = 4 \) if the lightest \( \nu_R \) has a much lower mass scale and hence a negligible effect on the energy budget of the early Universe. The amplitude squared \(|\mathcal{M}_{\Phi \rightarrow L\nu_R}|^2\) sums over the internal d.o.f without average. The factor 2 in front of the integral in Eq. (16) simply amounts to the two gauge components of \( \Phi \), i.e., here we treat \( \Phi \) (as well as \( L \)) as a single thermal species with two d.o.f. This is because the freeze-in production essentially occurs at the gauge symmetric phase for \( m_\Phi \approx O(100) \text{ GeV} \), and quickly shuts off due to Boltzmann suppression when temperature drops below \( m_\Phi \). Note that Eq. (16) is approximately obtained by neglecting the inverse decay and the Pauli-blocking effects, i.e., by assuming \( 1 - f_{\nu_R, L} \approx 1 \). With the Boltzmann distribution \( f_\Phi = e^{-\varepsilon/T} \), the energy density at decoupling is then given by

\[
\rho_{\nu_R, \text{dec}} = \frac{s_{SM}^{4/3}(T_{\nu_R, \text{dec}})}{\frac{4\pi}{3} \rho_{SM}} \int_0^\infty \frac{dY_{\nu_R, \rho}}{dT} \approx 0.09 N_{\nu_R} \left( \frac{106.75}{g_{\nu_R, \text{dec}}} \right)^{1/2} T_{\nu_R, \text{dec}}^4 \times \left( \frac{m_\nu}{0.01 \text{ eV}} \right)^2 \left( \frac{100 \text{ GeV}}{m_\Phi} \right) \left( \frac{500 \text{ keV}}{v_\Phi} \right)^2, \tag{17}\]

where we have taken the approximation \( g^*_\nu \approx g^*_\Phi \approx g_{\nu_R, \text{dec}} \) determined at the neutrino freeze-in temperature \( T_{\nu_R, \text{dec}} \). After the decoupling of \( \nu_R \), the energy exchange between the SM plasma and the right-handed Dirac neutrinos ceases, and the energy density \( \rho_{\nu_R, \text{ref}} \) at a late-time reference temperature \( T_{\text{ref}} \) can be readily rescaled by

\[
\frac{\rho_{\nu_R, \text{ref}}}{\rho_{\nu_R, \text{dec}}} = \frac{\rho_{SM, \text{ref}}}{\rho_{SM, \text{dec}}}, \tag{18}\]

where the SM energy density is given by

\[
\rho_{SM} = g^* \frac{2\pi^2}{45} T^4. \tag{19}\]

For the case (ii), on the other hand, the energy density from the thermalized \( \nu_{hR} \) after freezing out is given by Eq. (18), with a thermal energy spectrum

\[
\rho_{\nu_{hR}} = N_{\nu_{hR}} \frac{7\pi^2}{240} T_{\nu_{hR}}^4, \tag{20}\]

where \( N_{\nu_{hR}} = 4 \) takes into account the two heavier \( \nu_{hR} \) and their antiparticles, and \( T_{\nu_{hR}} \) denotes the \( \nu_{hR} \) temperature after freezing out. For the lightest neutrino \( \nu_{lR} \) in case (ii), the energy evolution follows Eqs. (15)–(17), with \( N_{\nu_R} = 2 \).

**IV. PROMISING SIGNALS FROM EFFECTIVE NEUTRINO NUMBER SHIFT**

The extra radiation contributing to the SM plasma in the early Universe can be parametrized by the shift of the effective neutrino number via

\[
\Delta N_{\text{eff}} \equiv \frac{\rho_{rad}}{\rho_{\nu_{lR}}}, \tag{21}\]

where \( \rho_{\nu_{lR}} \) is the energy density of a left-handed neutrino. In the SM, \( N_{\text{eff}}^{SM} = 3 \) just prior to the BBN and \( N_{\text{eff}}^{SM} = 3.044 - 3.045 \) [40–46] after taking into account the non-instantaneous decoupling of active neutrinos below \( T = O(1) \text{ MeV} \). The extra radiation contribution to \( \Delta N_{\text{eff}} \) can also be expressed in terms of the SM energy density via [13, 47]

\[
\Delta N_{\text{eff}} = \frac{4}{7} \left[ \frac{g^*_\nu(T_{\gamma\nu})}{g^*_\nu(T_{\text{ref}})} \right]^{4/3} \frac{7\pi^2}{240} \frac{\rho_{rad}(T_{\text{ref}})}{\rho_{\text{SM}}(T_{\text{ref}})}, \tag{22}\]

where \( g^*_\nu(T_{\gamma\nu}) = 10.75 \) corresponds to the epoch when the relativistic SM plasma contains photons, electrons, positrons, and neutrinos.

According to Eq. (22), we can calculate the shift \( \Delta N_{\text{eff}} \) at the decoupling temperature \( T_{\text{ref}} = T_{\nu_{lR}, \text{dec}} \) in case (i), giving

\[
\Delta N_{\text{eff}}^{(i)} \approx 7.345 \times 10^{-3} N_{\nu_R} \times \left( \frac{m_\nu}{0.01 \text{ eV}} \right)^2 \left( \frac{100 \text{ GeV}}{m_\Phi} \right) \left( \frac{500 \text{ keV}}{v_\Phi} \right)^2, \tag{23}\]

where \( g_{\nu_R, \text{dec}} \approx 106.75 \) has been used since we expect that \( T_{\nu_R, \text{dec}} \approx O(m_\Phi) \). For the case (ii), we can determine the shift \( \Delta N_{\text{eff}} \) from the two heavier \( \nu_{hR} \) by applying Eq. (20) to Eq. (22) at \( T_{\text{ref}} = T_{\nu_{hR}, \text{dec}} \), while the shift caused by \( \nu_{lR} \) is given by Eq. (23) with \( N_{\nu_R} = 2 \). The resulting total shift in case (ii) is then given by

\[
\Delta N_{\text{eff}}^{(ii)} \approx 0.0037 \left( \frac{m_\nu}{\text{ meV}} \right)^2 \left( \frac{100 \text{ GeV}}{m_\Phi} \right) \left( \frac{100 \text{ keV}}{v_\Phi} \right)^2 + 0.0937 \left( \frac{106.75}{g_{\nu_{hR}, \text{dec}}} \right)^{4/3}. \tag{24}\]

For an electroweak-scale decoupling temperature of the heavier neutrinos, we have \( g_{\nu_{hR}, \text{dec}} = 106.75 \), the maximum amount of entropy d.o.f available from SM particles.
Currently, a combined constraint from BBN (including primordial abundances of helium-4 and deuterium) and CMB, i.e., CMB+BBN+Y_p+D, gives $N_{\text{eff}} = N_{\text{eff}} - 3 < 0.151$ at the 2σ level [8], while the severest bound from the Planck 2018 results is given by $N_{\text{eff}} = 2.99 \pm 0.17$, limiting $\Delta N_{\text{eff}} = N_{\text{eff}} - 3.045 < 0.285$ at the 95% confidence level [9]. The future forecast sensitivity from CMB-S4 can reach $\Delta N_{\text{eff}} = 0(0.01)$, depending on the sky fraction $f_{\text{sky}}$ [26]. In addition, the forecast of $N_{\text{eff}}$ from BBN+CMB-S4 can reach a 1σ sensitivity, $\sigma_{\text{SR}}(N_{\text{eff}}|\text{BBN}) = 0.030$ [8]. On the other hand, the South Pole Telescope SPT-3G is expected to have a 2σ sensitivity of $\Delta N_{\text{eff}} < 0.116$ [48]. Given that the bound from CMB+BBN+Y_p+D is tighter than the Planck measurements, we will use the former as a constraint and apply the forecast sensitivities from SPT-3G and $\sigma_{\text{SR}}(N_{\text{eff}}|\text{BBN})$ to probe the vacuum scale. Noticeably, since $\Delta N_{\text{eff}} \gtrsim 0.0937$, which corresponds to an electroweak-scale $T_{\nu_{\text{R}},\text{dec}}$ and a negligible effect from the lightest $\nu_R$, the future BBN+CMB-S4 is able to exclude this second possibility of the PTDL mechanism, or the $\nu_1$-leptogenesis [4].

To estimate the maximal $\Delta N_{\text{eff}}^{(i)}$ that allows an observable imprint from the lightest neutrino, we assume that its mass $m_1$ reaches already the order of the Planck bound, i.e., $m_1 \simeq O(0.01)$ eV for both the NO and IO patterns (see also Fig. 1), so that all the three $\nu_R$ have comparable contributions to $\Delta N_{\text{eff}}$. Specifically, for the NO pattern, $\Delta N_{\text{eff}}^{(i)}$ is induced by one heavy $\nu_R$ with $m_3 \approx 0.05$ eV ($N_{\nu_R} = 2$) and two lighter $\nu_R$ with $m_{1,2} \approx 0.01$ eV ($N_{\nu_R} = 4$), while for the IO pattern, $\Delta N_{\text{eff}}^{(i)}$ comes from two heavier $\nu_R$ with $m_{1,2} \approx 0.05$ eV ($N_{\nu_R} = 4$) and the lightest $\nu_R$ with $m_3 \approx 0.01$ eV ($N_{\nu_R} = 2$). Confronting the current CMB+BBN+Y_p+D bound (red) as well as the forecast sensitivities from SPT-3G (blue) and BBN+CMB-S4 (green), all being at the 2σ level, to the resulting shift $\Delta N_{\text{eff}}^{(i)}$ from the three $\nu_R$ for both the NO (dashed curves) and IO (solid curves) patterns, we show in Fig. 2 the correlation between the electroweak-scale scalar mass $m_\Phi$ and the vacuum scale $v_\Phi$ in case (i), where the heavier neutrino mass is set at $m_\Phi \approx 0.05$ eV and the lighter one at $m_\Phi \approx 0.01$ eV, respectively.

As can be seen from Fig. 2, the vacuum in case (i) is generally pushed up to $O(1)$ MeV for $m_\Phi \simeq O(100)$ GeV. For example, a possible excess of the effective neutrino number around $2\sigma_{\text{SR}}(N_{\text{eff}}|\text{BBN}) = 0.06$ can be generated by a vacuum at 1 MeV with $m_\Phi \approx 300$ GeV, which can be tested by the future BBN+CMB-S4 sensitivity. For a much higher vacuum scale, however, the resulting shift $\Delta N_{\text{eff}}^{(i)}$ becomes negligible, as can be seen from Eq. (23). It should be mentioned that, just as the case (i) realized in Ref. [3] via a Yukawa texture-dependent BAU, we have shown there that the required vacuum is predicted to be $O(0.1)$ GeV in the trinophilic 2HDM. In this case, it would be hardly possible to test the PTDL mechanism if the Yukawa structures presumed in Ref. [3] are indeed responsible for the observed neutrino mixing and masses.

For an estimate of the shift $\Delta N_{\text{eff}}^{(ii)}$, it should be born in mind that the lightest neutrino in the NO pattern cannot reach $O(0.01)$ eV, because otherwise the out-of-equilibrium condition would be violated, as can also be seen from Eq. (13). If the lightest neutrino has a much smaller mass, on the other hand, the shift $\Delta N_{\text{eff}}^{(ii)}$ would primarily come from the two thermalized $\nu_{\nu_R}$, and hence depend on the decoupling temperature $T_{\nu_{\nu_R},\text{dec}}$. This has been recently analyzed in Ref. [10], pointing out that $T_{\nu_{\nu_R},\text{dec}}$ must be higher than the QCD phase transition temperature under the current limit from Planck 2018 release and will be pushed to $O(10)$ GeV by the future SPT-3G sensitivity. For our consideration here, we take $m_1 = 1$ MeV as a benchmark scale to include a non-negligible contribution from the freeze-in $\nu_R$. Note that, given the thermal condition presented in Eq. (13), we can determine the range of $m_\Phi$ in terms of $m_1$ and $v_\Phi$, which, after fixing $m_1 = 1$ MeV, would translate into an interval of $\Delta N_{\text{eff}}^{(ii)}$ in terms of the vacuum $v_\Phi$ and the d.o.f $g_{\nu_{\nu_R},\text{dec}}$.

In Fig. 3, the solid (dashed) curve corresponds to the upper (lower) bound of $m_\Phi$ governed by the thermal condition of Eq. (13). Under this prescription, it can be seen that the vacuum in the range $30 \lesssim v_\Phi/\text{eV} \lesssim 110$ with a decoupling temperature at 1 GeV can generate a shift $\Delta N_{\text{eff}}$ of the current CMB+BBN+Y_p+D level. It is also found that, under the CMB+BBN+Y_p+D bound, the decoupling temperature $T_{\nu_{\nu_R},\text{dec}}$ must be larger than 0.5 GeV, independent of the eV-scale vacuum considered. In light of the future SPT-3G

![Figure 2. Correlation between the electroweak-scale scalar mass $m_\Phi$ and the vacuum scale $v_\Phi$ in case (i), by confronting the current CMB+BBN+Y_p+D bound (red) as well as the forecast sensitivities from SPT-3G (blue) and BBN+CMB-S4 (green), all being at the 2σ level, to the resulting shift $\Delta N_{\text{eff}}^{(i)}$ from the three $\nu_R$ for both the NO (dashed curves) and IO (solid curves) patterns. Noticeably, since $\Delta N_{\text{eff}}^{(i)} \approx 0.0937$, which corresponds to an electroweak-scale $T_{\nu_{\nu_R},\text{dec}}$ and a negligible effect from the lightest $\nu_R$, the future BBN+CMB-S4 is able to exclude this second possibility of the PTDL mechanism, or the $\nu_1$-leptogenesis.](image-url)
The two heavier ones have the electronic Yukawa of Dirac neutrino masses and the BAU problem does not dete-

In this respect, the neutrinophilic 2HDM that explains the future BBN+CMB-S4 can readily exclude the case

ing around $\Delta N_{\text{eff}} \approx 10$ to an electroweak freezing-out temperature of $g_{\nu_R,\text{dec}} = 106.75$ (blue) and will be probed by the future SPT-3G (green) sensitivities. The otherwise unknown scalar mass $m_4$ has been translated into an interval between the solid (upper bound) and dashed (lower bound) curves as governed by the thermal condition of Eq. (13), by fixing $m_l = 1$ meV.

sensitivity, if no excess at the level of $\Delta N_{\text{eff}} = 0.116$ is ob-

erved, the lower bound of the decoupling temperature would be pushed up to 20 GeV. For $g_{\nu_R,\text{dec}} = 106.75$ corresponding to an electroweak freezing-out temperature of $\nu_{h_R}$, a vacuum in the range $20 \lesssim \nu_{h_R}/\text{keV} \lesssim 65$ can generate a shift $\Delta N_{\text{eff}}$ of the future SPT-3G level. As mentioned before, the future BBN+CMB-S4 can readily exclude the case (ii), since a minimal value $\Delta N_{\text{eff}}^{(ii)} \approx 0.0937$ is expected in this case.

Finally, it is interesting to point out that, for a vacuum being around 100 keV, the lightest Dirac neutrino with a mass of $\mathcal{O}(1)$ meV will lead to a Yukawa coupling of $\mathcal{O}(10^{-8})$, and the two heavier ones have the electronic Yukawa of $\mathcal{O}(10^{-6})$. In this respect, the neutrino Yukawa regime, a similar pattern that already existed in the charged fermion Yukawa sector of the SM. We have shown in this paper that, while the neutrino Yukawa of a keV-scale vacuum is inert in low-energy flavor physics such that the observation of LFV processes cannot be expected in future experiments, the sensitivities from cosmic regime are able to probe such a flavor-physics inert model. Besides being distinguishable from the lighter-vacuum cases with observable LFV processes, the relativistic right-handed Dirac neutrinos contribute to the energy budget of the early SM plasma, prompting significant shift of the effective neutrino number. The current measurement from CMB+BBN+Y$_\nu$+D has already presented a restrictive regime, and the future forecast from SPT-3G and BBN+CMB-S4 is able to test or even fully exclude the keV-vacuum Dirac neutrino model, in which the BAU enigma could be successfully solved by the PTDL mechanism.

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