Jet lag effect and leading hadron production

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Abstract

We propose a solution for the long standing puzzle of a too steeply falling fragmentation function for a quark fragmenting into a pion, calculated by Berger [1] in the Born approximation. Contrary to the simple anticipation that gluon resummation worsens the problem, we find good agreement with data. Higher quark Fock states slow down the quark, an effect which we call jet lag. It can be also expressed in terms of vacuum energy loss. As a result, the space-time development of the jet shrinks and the $z$-dependence becomes flatter than in the Born approximation. The space-time pattern is also of great importance for in-medium hadronization.

1 Leading hadrons in Born approximation

We are interested here in the production of leading pions which carry a major fraction of the momentum of a highly virtual quark originating from a hard reaction. The Born graph for the perturbative fragmentation $q \to \pi q$ is shown in Fig. 1a, and the corresponding fragmentation function was calculated in [1],

$$\frac{\partial D_{\pi/q}^{(\text{Born})}(z)}{\partial k^2} \propto \frac{(1-z)^2}{k^4}, \quad (1)$$

where $k$ and $z$ are the transverse and fractional longitudinal momenta of the pion. This expression is derived under the conditions $1-z \ll 1$ and $k^2 \ll Q^2$, where $Q^2$ is the scale of the hard reaction. We neglect higher twist terms [1, 2], which are specific for deep-inelastic scattering (DIS).
Figure 1: **a:** Berger mechanism [1] of leading pion production in Born approximation. **b:** A high Fock component of the quark emerging from a hard reaction and producing a pion with a higher momentum fraction $\tilde{z} > z$ than measured experimentally.

The fragmentation function (FF) Eq. (1) is in an apparent contradiction to data, since it falls towards $z = 1$ much steeper than is known from phenomenological fits (e.g. see [3]), and even the inclusion of higher order correction does not seem to fix the problem. Moreover, at first glance gluon radiation should worsen the situation, producing even more suppression at $z \to 1$ because of energy sharing.

Nevertheless, we demonstrate below that the effect of jet lag (JL), i.e. the effect that comes from the fact that higher Fock states retard the quark, substantially changes the space-time pattern of jet development. The JL cuts off contributions with long coherence time in pion production and makes the $z$-dependence less steep.

One can rewrite (1) in terms of the coherence length of pion radiation,

$$L_\pi^c = \frac{2Ez(1 - z)}{k^2 + z^2m_q^2 + (1 - z)m_\pi^2},$$

(2)

where $E$ is the jet energy, and $m_q$ is the quark mass which may be treated as an effective infrared cutoff. Then, the Born approximation takes the form,

$$\frac{\partial D_{\pi/q}^{(Born)}}{\partial L_\pi^c} \propto (1 - z).$$

(3)

Thus, the production of the leading pion is homogeneously distributed over distance, from the point of jet origin up to the maximal distance $(L_\pi^c)_{max} = 2E(1 - z)/zm_q^2$.

Integrating (3) over $L_\pi^c$ up to $(L_\pi^c)_{max}$ we recover the $(1 - z)^2$ dependence of Eq. (1).

Now we understand where the extra power of $(1 - z)$ comes from: it is generated by the shrinkage of the coherence pathlength for $z \to 1$. This is the source of the too steep fall off of the Born term Eq. (1) in the FF.

### 2 Jet lag effect and the fragmentation function

The color field of a quark originated from a hard reaction (high-$p_T$, DIS, $e^+e^-$, etc.) is stripped off, and gluon radiation from the initial state generates the scale dependence of the quark structure function of the incoming hadron (if any). Therefore the quark originated from such a hard process is bare, lacking a color field up to transverse frequencies $q \lesssim Q$. 
Then the quark starts regenerating its field by radiating gluons, i.e. forming a jet. This can be described by means of an expansion of the initial "bare" quark over Fock states containing a physical quark and different number of physical gluons with different momenta, as is illustrated in Fig. 1b. Originally this is a coherent wave packet equivalent to a single bare quark $|q\rangle$. However, different components have different invariant masses and start gaining relative phase shifts as function of time. As a result, the wave packet is losing coherence and gluons are radiated in accordance with their coherence times.

Notice that the Born expression (1) corresponds to the lowest Fock components relevant to this process, just a bare quark, $|q\rangle$, and a quark accompanied by a pion, $|q\pi\rangle$. In this case the initial quark momentum and the pion fractional momentum $z$ in (1) are the observables (at least in $e^+e^-$ or SIDIS).

An important observation is that the quark in higher Fock states carries only a fraction of the full momentum of the wave packet. At the same time, the pion momentum is an observable and is fixed. Therefore, one should redefine the fractional momentum of the pion convoluting the fragmentation function Eq. (1) with the quark momentum distribution within different Fock states,

$$\frac{\partial D_{q/\pi}(z)}{\partial L_c^\pi} = \sum_i C_{qi}^\pi \int_1^z \frac{dx}{z} \frac{\partial D_{q/\pi}(z/x)}{\partial L_c^\pi} F^i_q(x) \Theta(L_c^\pi - l_i^c). \tag{4}$$

Here $F^i_q(x)$ is the fractional momentum distribution function of a physical quark in the $i$-th Fock component of the initial bare quark. Such a component contributes to (4) only if it lost coherence with the rest of the wave packet. This is taken into account in (4) by means of the step function, where $l_i^c$ is the coherence length for this Fock state. We sum in (4) over different Fock states with proper weight factors $C_{qi}^\pi$.

Thus, the inclusion of higher Fock states results in a retarding of the quark, an effect which we call jet lag (JL). This effect plays a key role in shaping the quark fragmentation function for leading hadrons. Due to JL the variable of the Born FF in (1) increases, $z \rightarrow z/x$, causing a suppression. Then the convolution Eq. (1) leads to the following modification of the Born fragmentation function Eq. (3),

$$\frac{\partial D_{q/\pi}(z)}{\partial L_c^\pi} \propto 1 - \tilde{z}, \tag{5}$$

where

$$\tilde{z} = \frac{\langle z \rangle}{x} = z \left(1 + \frac{\Delta E}{E}\right) + O\left[z(1-z)^2\right]. \tag{6}$$

Here we made use of the limiting behavior at $1-z \ll 1$ we are interested in. The fractional energy loss of the quark is related to the energy carried by other partons within those Fock components which have lost coherence one the pathlength $L_c^\pi$,

$$\frac{\Delta E(L_c^\pi)}{E} = \langle 1 - x(L_c^\pi) \rangle = \sum_i C_{qi}^\pi \int_1^1 dx (1-x) F^i_q(x) \Theta(L_c^\pi - l_i^c) \tag{7}$$

Notice that in the above expressions we implicitly assume also integration on the other kinematic variables related to the participating partons.
2.1 Gluon bremsstrahlung

A part of energy loss related to radiation of gluons can be evaluated perturbatively. For this purpose we replace \( F_q(x) \) in (4) and (7) by the gluon number distribution [4],

\[
\frac{dn_g}{d\alpha dk^2} = \frac{2\alpha_s(k^2)}{3\pi} \frac{1 + (1 - \alpha)^2}{\alpha k^2},
\]

where \( \alpha = 1 - x \) is the fraction of the total energy carried by the radiated gluon, and the fractional momentum of the recoil quark is \( x \). In the numerator we added the splitting function of the DGLAP equations, although it is a small corrections, since \( \alpha \ll 1 \).

Then, the perturbative vacuum energy loss for gluon radiation reads [5–7],

\[
\Delta E_{\text{pert}}(L) = E \int \frac{Q^2}{\lambda^2} dk^2 \int \frac{1}{k/2E} d\alpha \alpha \frac{dn_g}{dk^2 d\alpha} \Theta(L - l_g^\gamma) \Theta \left(1 - z - \alpha - \frac{k^2}{4\alpha E^2}\right),
\]

Here the soft cutoff \( \lambda \) is fixed at \( \lambda = 0.7 \text{ GeV} \). The latter choice is dictated by data (see in [8,9]) demonstrating a rather large primordial transverse momentum of gluons.

The first step-function in (9) restricts the radiation time of gluons,

\[
l_g^\gamma = \frac{2E\alpha(1 - \alpha)}{k^2},
\]

contributing to the quark energy loss along the pathlength \( L \). The second step-function in (9) takes care of energy conservation, namely, none of the gluons can have energy, \( \omega = \alpha E + k^2/4\alpha E \), larger than \( E(1 - z) \).

One can rewrite Eq. (8) at \( \alpha \ll 1 \) as a distribution of gluon number over the radiation length and fractional momentum,

\[
\frac{dn_g}{dl_g^\gamma d\alpha} = \frac{4\alpha_s(\mu^2)}{3\pi} \frac{1}{l_g^\gamma \alpha},
\]

where the scale in the running QCD coupling is \( \mu^2 = 2E\alpha(1 - \alpha)/l_g^\gamma \). Then, Eq. (9) takes the form,

\[
\Delta E_{\text{pert}}(L) = E \int_{1/Q}^{t_{\text{max}}} dl \int \frac{1}{(2E)^{-1}} d\alpha \alpha \frac{dn_g}{dl d\alpha} \Theta \left(1 - z - \alpha - \frac{1 - \alpha}{2lE}\right),
\]

where the upper limit of integration over \( l \) is given by the maximal value, \( t_{\text{max}} = \min\{L, E/2\lambda^2\} \).

An example of \( L \)-dependence of fractional energy loss calculated with Eq. (12) for \( E = Q = 20 \text{ GeV} \) is shown in Fig. 2 by dashed curve (left panel).

We see from Eq. (9) that the rate of the perturbative energy loss ceases at \( L > E/2\lambda^2 \), since no gluons are radiated any more. Of course propagation of a free quark is unphysical and the effects of confinement at a scale softer than \( \lambda \) must be introduced.
2.2 Sudakov suppression

As far as we imposed a ban for radiation of gluons with energy \( \omega > (1 - z)E \) in (9), (12), this restriction leads to a Sudakov type suppression factor,

\[
S(L, z) = \exp\left[-\langle n_g(L, z) \rangle \right], \tag{13}
\]

where \( \langle n_g(L, z) \rangle \) is the mean number of nonradiated gluons,

\[
\langle n_g(L, z) \rangle = \int_{1/Q}^{t_{\text{max}}} dl \int_{(2EL)^{-1}}^1 d\alpha \frac{dn_g}{dl d\alpha} \Theta \left( \alpha + \frac{1 - \alpha}{2lE} - 1 + z \right). \tag{14}
\]

The results are illustrated in Fig. 3 at \( E = Q = 20 \text{ GeV} \) and for different values of \( z \).

2.3 Higher twist nonperturbative effects also contribute

At long distances \( L > E/2\lambda_2 \), after completing restoring its field, the quark does not radiate any more, and then the energy loss may have only a nonperturbative origin. We assume that a soft quark develops a string (color flux tube [10]), which leads to a constant rate of energy loss [11, 12],

\[
\left. \frac{dE(L > E/2\lambda^2)}{dL} \right|_{\text{string}} = -\kappa, \tag{15}
\]

where the string tension is taken at its static value \( \kappa = 1 \text{ GeV/fm} \), given by the slope of Regge trajectories and by calculations on the lattice.
Figure 3: Sudakov suppression caused by a ban for radiation of gluons with fractional energy higher than $1 - z$. Calculations are done for a jet with $E = Q = 20 \text{ GeV}$

At shorter distances, $L < E/2\lambda^2$, the nonperturbative energy loss proceeds along with the perturbative one. The way how it is introduced is the most uncertain and model dependent part of the calculation, since we have no good knowledge of the relevant dynamics. Nevertheless, this is a higher twist effect, and the related uncertainties tend to vanish at high $Q^2$.

To see the range of this uncertainty we consider two models for nonperturbative energy loss.

**STRING-I**: we assume the constant rate of energy loss Eq. (15) to be valid at all distance from the origin.

**STRING-II**: keeping the same rate of energy loss Eq. (15) at long distances $L > E/2\lambda^2$, we reduce and make time-dependent the rate of nonperturbative energy loss at $L < E/2\lambda^2$. Indeed, the original bare quark whose field has been stripped off, cannot produce any color flux at the origin, and starts developing a flux tube only during restoration of its field. We assume that the transverse area of the color flux follows the transverse size of the restored field of the quark, which receives contributions from all the gluons radiated during quark propagation through the pathlength $L$ ($l_g < L < E/2\lambda^2$),

$$\langle r^2 \rangle \sim \left\langle \frac{1}{k^2} \right\rangle \propto \frac{L}{2E}. \quad (16)$$

Therefore, in this scenario the transverse area of the color flux formed by a quark rises linearly with the pathlength of the quark.

In the MIT bag model the energy of a tube comes from two contributions, the bag term and the energy of the electric color field [10]. The first one is proportional to the transverse area times the bag constant, while the second contribution has inverse dependence on the tube area. Equilibrium corresponds to equal contributions of these two terms. If such a
tube fluctuates to a smaller transverse dimension \( r \), the second term rises as \( 1/r^2 \), and so does the string tension. Nevertheless, this is probably true for a stationary tube when the total flux of electric color field is independent of the transverse size of the tube and is equal to the color charge of the quark. However, a quark with a stripped field produces a color flux only at the transverse distances where the field is already restored. Therefore, both terms in the energy of a flux tube produced by a bare quark are reduced by the same factor \( \langle r^2 \rangle/a^2 \), where \( \langle r^2 \rangle \) is given by (17), and \( a \) is the transverse size of a stationary tube.

The parameter \( a \) in the stochastic vacuum model [13, 14], has the meaning of a gluon correlation radius, calculated on the lattice [15] as \( a = 0.3 - 0.35 \text{ fm} \). This value turns out to be in a good accord with our infrared cutoff in Eq. (9), \( a \approx 1/\lambda \).

Thus, the mean transverse dimension squared of the flux rises linearly with \( L \) from a tiny value \( r^2 \sim Q^{-2} \) up to the stationary value \( r^2 = a^2 \approx 1/\lambda^2 \). Correspondingly, the effective string tension rises \( \propto r^2(L) \) with a coefficient dependent on the QCD gluon condensate [14],

\[
\kappa_{\text{eff}}(L) = \frac{32\pi^2}{81} \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu}(x) G^{\mu\nu a}(0) \right\rangle r^2(L),
\]

where \( k = 0.74 \).

Summarizing, the nonperturbative energy loss rises linearly with pathlength during gluon radiation and restoration of the color field of the quark. The energy loss rate approaches its maximum value Eq. (15) at the maximal length available for radiation, \( L_{\text{max}} = E/2\lambda^2 \), so we can write,

\[
\frac{dE(L < E/2\lambda^2)}{dL}_{\text{string}} = -\frac{2\lambda^2}{E} L \kappa.
\]

Although we believe that this model String-II is more realistic than the previous one, in what follows we perform calculations with both models to see the range of theoretical uncertainty.

We add the two sources of energy loss, the perturbative gluon radiation and the string contribution. Fig. 2 shows an example of length dependence of the fractional energy loss, by a quark with \( E = Q = 20 \text{ GeV} \) and different fractional momenta \( z \) of produced pions (left panel), and at different energies, but fixed \( z = 0.7 \) (right panel). Notice that curves stop when all energy available for gluon radiation is exhausted, i.e. \( \tilde{z} \to 1 \). The smaller is \( (1-z) \), the earlier this happens.

### 2.4 Jet lag modified fragmentation function

Now we are in a position to calculate the quark-to-pion fragmentation function, based on Berger’s result [1] obtained in Born approximation, and corrected for gluon resummation. Gluon radiation results in the JL effect, Eqs. (4), (7), since the pion momentum fraction should be redefined relative to the retarded quark Eq. (6). Then we arrive at the \( L \)-dependent fragmentation function,

\[
\frac{\partial D_{\pi/q}(z)}{\partial L^\pi_c} \propto (1-\tilde{z}) S(L^\pi_c, z).
\]

The JL effect and Sudakov factor suppress long distances in pion production. The \( L \)-distribution at \( E = Q = 20 \text{ GeV} \) is depicted in the left panel of Fig. 4 for \( z = 0.5, 0.7, 0.9 \).
The right panel of Fig. 4 shows the energy dependence of the $L$-distribution at $z = 0.7$.

Figure 4: The pion production rate as function of length calculated according to Eq. (19). Solid and thin curves correspond to the nonperturbative part calculated with models STRING-I and STRING-II respectively. left: $E = Q = 20$ GeV, $z = 0.5, 0.7, 0.9$. right: $z = 0.7$ and $E = Q = 10, 100$ GeV. The overall normalization is arbitrary.

Apparently the production length of leading pions is rather short, even at high energies.

Integrating the distribution function Eq. (19) over $L_c$ we arrive at the fragmentation function $D_{q/\pi}(z, Q^2)$, which is compared in Fig. 5 with two popular parametrizations fitted to data, KKP [3] and BKK [16]. Since our fragmentation function is valid only at large $z$ and is not normalized, we fix the normalization adjusting it to the KKP results at $z = 0.6−0.8$. We calculated the nonperturbative part with either constant (STRING-I) or rising (STRING-II) rates of energy loss.

We observe a rather good agreement between our calculated and the phenomenological fragmentation functions, with deviations which are similar to the differences between the two phenomenological FF depicted in the figure. Notice that according to [3] the results of fits are not trustable at $z > 0.8$ due to lack of data.

Thus, after inclusion of higher order corrections the quadratic $(1-z)^2$ behavior of the Born approximation Eq. (11) is replaced by a less steep dependence which complies well with data. This could happen only if the interval of accessible coherence lengths for pion production does not shrink $\propto (1-z)$ anymore. This seems to be in contradiction with the usually anticipated behavior [6,7,11,12,17,18],

$$L_c(z) \approx \frac{E(1-z)}{\langle|dE/dL|\rangle},$$

which is dictated by energy conservation. The rate of radiative energy loss is known to be constant [5] like in the string model [11,12], and then the coherence length Eq. (19) should be $\propto (1-z)$. 

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Figure 5: Comparison of our modeled FF (solid curves) with the phenomenological ones [3] (dashed) and [16] (dotted), at scales $E = Q = 5$, 20, 100, 200 GeV. Each curve is rescaled by factor 10 compared to the lower one. The nonperturbative part is calculated with either the model STRING-I (left), or STRING-II (right).

However, the energy conservation restrictions should be imposed to the rate of radiative energy loss as well [6, 7]. This was done above in Eq. (9) by introducing the second step function. The corresponding rate of energy loss at small $1 - z < LQ^2/2E$ reads,

$$\frac{dE}{dL} = -\frac{4\alpha_s E}{3\pi L} (1-z),$$

(21)

where we fix $\alpha_s$ (only here) at the scale $\mu^2 = 2EZ(1-z)/L$.

Thus, in the limit $z \to 1$ the rate of energy loss is small $\propto (1-z)$ and the interval of coherence length Eq. (20) does not shrink. A combination of radiation and nonperturbative sources of energy loss results in a $z$-dependence which lies in between of linear and quadratic $1-z$ behaviors, as is demonstrated in Fig. 5.

3 Summary and outlook

The Born approximation for leading pions result in a FF, Eq. (1), which drops too steeply at $z \to 1$. We complemented this result with a space-time evolution pattern, and found that pions are produced along a long path whose length rises with jet energy. Long distances turn out to be responsible for the extra power of $1-z$ in the FF.

Gluon radiation leads to vacuum energy loss, which considerably reduces the pathlength of the quark. In terms of Fock state decomposition the effective value of the pion fractional momentum, Eq. (6), is larger, since the presence of other partons (gluons) is retarding the quark, an effect named JL.

Our central result, the final expression for the FF, given in Eq. (19), also includes the Sudakov suppression factor which causes more shrinkage of the pion production length, and
which is found to be rather short as is demonstrated in Fig. 4. Integrating over $L_\pi$ we arrived at a FF which agrees quite well with phenomenological ones fitted to data, as is shown in Fig. 5.

There is still much work to be done:

- Understanding the space-time pattern of hadron production is crucial for the calculation of medium modification of the FF. The present results can be applied to hadron attenuation in SIDIS off nuclei, and also to hadron quenching in heavy ion collisions [19].

- Transverse momentum distributions are always more difficult to calculate than the integrated FF. Nevertheless, the current approach has a predictive power for the $k_T$-distribution as well. The results will be published elsewhere.

- Quark fragmentation to leading heavier flavor mesons can be calculated as well. In this case a mass correction in the Born approximation energy denominator needs to be done. We also leave this for further study.

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References

[1] E.L. Berger, Phys. Lett. B 89 (1980) 241.
[2] H.J. Pirner and D. Grünwald, Nucl. Phys. A 782 (2007) 158.
[3] B.A. Kniehl, G. Kramer, B. Pötter, Nucl. Phys. B 597 (2001) 337.
[4] J.F. Gunion and G. Bertsch, Phys. Rev. D 25 (1982) 746.
[5] F. Niedermayer, Phys. Rev. D 34 (1986) 3494.
[6] B.Z. Kopeliovich, J. Nemchik and E. Predazzi, in Future Physics at HERA, Proceedings of the Workshop 1995/96, edited by G. Ingelman, A. De Roeck and R. Klanner, DESY, 1995/1996, vol.2, p. 1038 [nucl-th/9607036]; in Proceedings of the ELFE Summer School on Confinement Physics, edited by S.D. Bass and P.A.M. Guichon, Editions Frontieres, 1995, p. 391, Gif-sur-Yvette [hep-ph/9511214].
[7] B.Z. Kopeliovich, J. Nemchik and E. Predazzi, A. Hayashigaki, Nucl. Phys. A 740 (2004) 211.
[8] B.Z. Kopeliovich, A. Schäfer and A.V. Tarasov, Phys. Rev. D 62 (2000) 054022.
[9] B.Z. Kopeliovich and B. Povh, J. Phys. G30 (2004) S999; B.Z. Kopeliovich, B. Povh and I.A. Schmidt, Nucl. Phys. A 782 (2007) 24.
[10] A. Casher, H. Neubereger and S. Nussinov, *Phys. Rev.* **D20** (1979) 179.

[11] B.Z. Kopeliovich and F. Niedermayer, Sov. J. Nucl. Phys. **42** 504 (1985) 504 [Yad. Fiz. **42** (1985) 797].

[12] B.Z. Kopeliovich, Phys. Lett. B **243** (1990) 141.

[13] H.G. Dosch, Phys. Lett. B **190** (1987) 177.

[14] O. Nachtmann, High-energy collisions and nonperturbative QCD, e-Print: [hep-ph/9609365](http://arxiv.org/abs/hep-ph/9609365).

[15] A. DiGiacomo and H. Panagopoulos, Phys. Lett. B **285** (1992) 133.

[16] J. Binnewies, B.A. Kniehl and G. Kramer, Phys. Rev. D **52** (1995) 4947.

[17] A. Bialas and M. Gyulassy, Nucl. Phys. B **291** (1987) 793.

[18] A. Accardi and H.-J. Pirner, Nucl. Phys. A **711** (2002) 264.

[19] B.Z. Kopeliovich, I.K. Potashnikova and I. Schmidt, Quenching of high-$p_T$ hadrons: Alternative scenario in Proc. of the Workshop ”Heavy Ion Collisions at the LHC: Last Call for Predictions”, May 14 - June 8, 2007, CERN.