Resource-saving steel hopper for loading of railway cars

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Abstract. The article is devoted to the study of geometric characteristics of corrugated sheets of railway grain hopper. In particular, the problem of finding the optimal parameters of a corrugated sheet of a given length, such as the height and width of the corrugation shelves, is solved. The optimality condition in this problem is determined by the minimum mass of the corrugated sheet. Because of analytical calculations, it was found that, regardless of the angle of tip of the corrugation wall, there will always be such a value of optimum height at which the corrugation area will be the smallest. For the analysis of this parameter, a series of curves of dependence of the corrugation cross-section area on the height for different angles of tip were constructed. The study also provided concise and engineering-friendly equations to determine the optimum height and width of corrugation shelves for wall angles up to 60°. These expressions can be applied to the case of corrugated sheet with a limited number of erection joints. For other cases of jointing of the corrugated sheets with each other (complete or partial overlapping of the lower part), additional studies should be performed similar to the procedure presented.

1. Introduction
Grains are the main export products of our country. The length of transportation routes for transporting grain from different regions is quite large. Hence, the rational choice of delivery increases the overall level of profitability for this product. The freight rate of grain trucks is always high. This causes an increase in energy consumption for grain production, particularly when transporting over long distances. Consequently, it reduces the competitiveness of Ukrainian farmers, compared to foreign markets [1, 2]. In addition, road transport increases the burden on roads and their maintenance cost.

Reducing the cost of the finished product is possible by reducing energy consumption and use of rail transportation. The effectiveness of grain terminals increases significantly when realizing the possibility of fast and uniform loading of cars (figure 1). Initially, it depends on the equipment and technological constructive solutions of constructions for loading. They have to meet the requirements for strength and rigidity and have an appropriate performance for temporary storage of grain. Loading structures with a loading capacity of 100 to 500 m³ made of galvanized sheet steel are normally used as such constructions (Figure 2).
Figure 1. Loading of railway cars.

Figure 2. Steel hoppers with corrugated sheet body (ZEO Sokil LLC).

In order to decrease energy and financial losses, in the production of steel railway grain hopper, it is advisable to use standard elements in the form of corrugated steel panels. Corrugated steel panels have increased bending stiffness, therefore they less sensitive to local buckling due wind load and own local defects. The geometric characteristics of corrugated sheets of the hopper body (corrugation height and step) in different manufacturers acquire their own values because of production aspects. Quite often, these indicators are interpreted as advantages for some design solutions and are constantly compared to competitive ones. The method of simple analytical calculation of these characteristics for cylindrical silo was developed in the previous work of the authors [3].
It is worth noting that, as a whole, the cost of the hoppers is estimated by metal consumption of panels. Therefore, it is quite interesting to consider the problem of determining the minimum mass of grain hopper corrugated sheet. This will allow to obtain a resource-saving design solution for the loading hopper and increase the economic effect by reducing the metal content without losing loading-bearing capacity.

The outlined direction of rational construction with corrugated sheets can be successfully applied also to other types of square hoppers [4-7], as well as to other structures, since the corrugated elements became considerable widespread in various areas of construction. They are used as part of load-bearing elements of building structures and as enclosing structures, and can combine these functions. Corrugated metal sheets are used for walls and roofs of buildings, culverts and overpasses, bridges, protruding elements of engineering structures, etc. [8-12].

2. Formulation of the problem
Corrugated sheet, which schematizes the wall of the hopper for the storage of bulk material, was loaded with pressure $p_0$. The cross-sectional dimensions of the sheet are determined by four independent parameters such as the height of corrugation $h_p$, the width of the corrugation shelves $a_p$, the distance between the corrugations $b_p$ and the angle of tip of the corrugation walls $\beta_p$ (figure 3).

![Figure 3. Geometric layout of the corrugated sheet.](image)

Suppose that the sheet is fixed at the edges. It is necessary to find the size of corrugation $h_p$, $a_p = b_p$ and $\beta_p$, at which the mass of the sheet will be minimal.

3. Solution
We introduce the notation $\alpha_p = a_p / h_p$ and obtain the equations for the basic geometry of the corrugation, such as area $A_p$, length of horizontal projection $\ell_p$, second moment of area $J_p$, and moment of resistance $W_p$. Given the symmetry of the profile relative to the vertical axis, the solution can be found for half the corrugation. Since $a_p = b_p$, the moments of resistance of the upper and lower shelves are equal. Considering previous studies [3] and skipping intermediate transformations, we write the result

$$A_p = h_p \ell_p \alpha_p^2,$$

$$\ell_p = h_p \psi, \quad (2)$$
\[ J_p = 0.333h_p^3\psi_J, \]  
\[ W_p = 0.667h_p^2\psi_J, \]

where \( \psi_A, \psi_\ell, \) and \( \psi_J \) are dimensionless factors whose values are given by equations

\[ \psi_A = 2\alpha_p + \sin^{-1} \beta_p, \]  
\[ \psi_\ell = 2\alpha_p + \cot \beta_p, \]

\[ \psi_J = (2\alpha_p + 3\alpha_p^2 \sin \beta_p + 0.25 \sin^{-1} \beta_p)(1 + 2\alpha_p \sin \beta_p)^{-1}. \]

To ensure the sheet strength, the equality should be maintained

\[ M_p R_y^{-1} = W_p, \]

where \( M_p \) is the bending moment in the design section, which is calculated as for a single beam from linear load \( q_0 = p_0 L_p \)

\[ M_p = p_0 L_p^2 \alpha_0^{-1} = h_p p_0 L_p^2 \alpha_0^{-1} \psi_\ell = h_p p_0 L_p^2 \alpha_0^{-1} (2\alpha_p + \cot \beta_p), \]

where \( \alpha_0 \) is the coefficient of the bending moment function. For example, with a hinged bearing \( \alpha_0 = 8 \), and with a fixed bearing \( \alpha_0 = 12 \).

Substituting (9) and (7) into equation (8) we obtain

\[ p_0 L_p^2 (2\alpha_p + \cot \beta_p)(\alpha_0 R_y) = 0.667h_p L_p \left( 2\alpha_p + 3\alpha_p^2 \sin \beta_p + 0.25 \sin^{-1} \beta_p \right)(1 + 2\alpha_p \sin \beta_p)^{-1}. \]

Given a number of simplifications and erection of similar ones, we get

\[ p_0 L_p^2 (\alpha_0 R_y h_p L_p) = \left( \alpha_p + 0.167 \sin^{-1} \beta_p \right) \left( 2\alpha_p + \cot \beta_p \right)^{-1}. \]

Let us express the parameter \( \alpha_p \) through the cross-section area

\[ \alpha_p = 0.5A_p \left( h_p L_p \right)^{-1} - 0.5 \sin^{-1} \beta_p. \]

Simplifying the result, we get

\[ p_0 L_p^2 (\alpha_0 R_y h_p L_p) = 0.5 \left( A_p \sin \beta_p - 0.667h_p L_p \left[ A_p \sin \beta_p - h_p L_p (1 - \cos \beta_p) \right] \right)^2. \]

Subsequently, we write the expression for the cross-section area of the corrugation

\[ A_p = 0.67h_p L_p \sin^{-1} \beta_p \left[ 3p_0 L_p^2 (1 - \cos \beta_p) - \alpha_0 R_y h_p L_p \right] \left( 2p_0 L_p^2 - \alpha_0 R_y h_p L_p \right). \]

The geometric interpretation of this equation is shown in figure 4 in the form of graphs of dependence \( A_p(h_p) \) at different angles of tip of the corrugation wall \( \beta_p \).

The analysis of a series of curves of equation (14) shows that irrespective of the angle of tip \( \beta_p \), there is always a value of height \( h_p \), at which the corrugation area will be minimal. The height that satisfies this criterion is called the optimum corrugation height \( h_{opt} \). Note that in this case the height
The function $A_p(h_p)$ in the vicinity of the abscissa $h_{opt}$ is shallow in nature and has small deviations of the actual corrugation height from the optimum. This has a rather insignificant effect on the minimum steel intensity of the profile.

![Figure 4. Geometric interpretation of equation (14).](image)

To find the optimum height of the corrugation $h_{opt}$ at a given angle $\beta_p$, we differentiate expression (14) by $h_{opt}$ and equate the result to zero. After the transformations, we get

$$
2 \frac{12 p_0 L_0^2 t_p \sin^2 \left(0.5 \beta_p \right) - \alpha_0 h_{opt} t_p^2 R_y \left(4 p_0 L_0^2 - \alpha_0 R_y h_{opt} t_p \right)}{3 \sin \beta_p} = 0.
$$

Solving this equation with respect to the required height $h_{opt}$, we obtain a sufficiently concise and engineering-friendly equation

$$
h_{opt} = p_0 L_0^2 \left(\alpha_0 R_y t_p \right)^{-1} \left[2 + \left(6 \cos \beta_p - 2\right)^{0.5}\right].
$$

Substituting the result into equations (14) and (13), we obtain an expression for the width of the corrugation shelf $a_{opt}$, which satisfies the criterion of optimality

$$
a_{opt} = p_0 L_0^2 \left(3 \alpha_0 R_y t_p \right)^{-1} \sin^{-1} \left[3 \cos \beta_p + 0.71 \left(3 \cos \beta_p - 1\right)^{0.5} - 2\right].
$$

Thus, given the length of the corrugated sheet, its thickness and the material strength, the size of the optimal cross-section (in terms of minimum steel intensity) will be determined only by the angle of tip of the corrugation wall to the horizontal plane.
There is one limitation. Given that the width $a_{\text{opt}}$ cannot be negative, we find the limit value of the angle $\beta_p$ at which this condition will be satisfied. Equating the expression (17) to zero and performing a series of trigonometric transformations, we obtain

$$\beta_p = 60^\circ.$$  \hspace{1cm} (18)

Thus, all the provisions obtained will be valid for angles of the corrugation wall up to $60^\circ$, which is sufficient for engineering calculations.

The use of equations (16) and (17) is quite convenient. For example, we calculate the optimal dimensions of the corrugated sheet of length $L_0 = 5000$ mm, made of steel with a yield point $R_y = 240$ MPa for two thicknesses $t_p = 4.0$ mm and $t_p = 3.0$ mm; for five load levels $p_0$ and four angles of tip $\beta_p$.

The ends of the sheet will be considered rigidly fixed at the ends. The results of the calculations are given in table 1

| $p_0$ (kPa) | $\beta_p = 20^\circ$ | $\beta_p = 30^\circ$ | $\beta_p = 40^\circ$ | $\beta_p = 50^\circ$ |
|------------|----------------------|----------------------|----------------------|----------------------|
|            | $h_{\text{opt}}$ (mm) | $a_p$ (mm) | $h_{\text{opt}}$ (mm) | $a_p$ (mm) | $h_{\text{opt}}$ (mm) | $a_p$ (mm) | $h_{\text{opt}}$ (mm) | $a_p$ (mm) |
| 30         | 254.4                | 112.5               | 246.6                | 64.8       | 235.1                | 37.3       | 218.9                | 17.3       |
| 25         | 212.0                | 93.7                | 205.5                | 54.0       | 195.9                | 31.1       | 182.4                | 14.4       |
| 20         | 169.6                | 75.0                | 164.4                | 43.2       | 156.7                | 24.8       | 145.9                | 11.5       |
| 15         | 127.2                | 56.2                | 123.3                | 32.4       | 117.6                | 18.6       | 109.5                | 8.6        |
| 10         | 84.8                 | 37.5                | 82.2                 | 21.6       | 78.4                 | 12.4       | 73.0                 | 5.8        |
|            |                      |                     |                      |            | $t_p = 4$ mm         |            |                      |            |
| 30         | 339.2                | 150.0               | 328.8                | 86.3       | 313.5                | 49.7       | 291.9                | 23.0       |
| 25         | 282.7                | 125.0               | 274.0                | 72.0       | 261.2                | 41.4       | 243.2                | 19.2       |
| 20         | 226.1                | 100.0               | 219.2                | 57.6       | 209.0                | 33.1       | 194.6                | 15.4       |
| 15         | 169.6                | 75.0                | 164.4                | 43.2       | 156.7                | 24.8       | 145.9                | 11.5       |
| 10         | 113.1                | 50.0                | 109.6                | 28.8       | 104.5                | 16.6       | 97.3                 | 7.7        |

This problem is relevant for corrugated sheets with a limited number of erection joints, which will have little effect on its bending characteristics. In other cases, such as the relatively low sheets that face each corrugation overlapping, or the partial overlapping of the lower corrugation, the expressions for optimal performance will have a different representation, although the procedure for solving the problem will be the same.

4. Summary
1. In the framework of this study, the problem of determining the optimal geometric characteristics of corrugated sheet – the height, width and step of the corrugation shelves has been considered.
2. It has been found that regardless of the angle of the corrugation wall, there will always be a height at which the corrugation area will be minimal. This parameter will act as the optimum corrugation height.
3. Analysis of function of the corrugation cross-section area on the height showed that increasing the angle of tip of the wall would lead to a decrease in the optimum height.
4. We formulated concise and engineering-friendly equations to determine the optimum height and width of corrugation shelves for wall angles up to 60º. These expressions can be applied to the case of corrugated sheet with a limited number of erection joints. For other cases of jointing of the corrugated sheets with each other (complete or partial overlapping of the lower part), additional studies should be performed similar to the procedure presented.

5. The indicated calculation algorithm permits to implement optimal design solutions of resource-saving structures of rectangular steel hoppers for loading railway cars.

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