Constraints on electromagnetic form factors of sub-GeV dark matter from the Cosmic Microwave Background anisotropy

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Abstract

We consider dark matter which have non-zero electromagnetic form factors like electric/magnetic dipole moments and anapole moment for fermionic dark matter and Rayleigh form factor for scalar dark matter. We consider dark matter mass $m_\chi > O$(MeV) and put constraints on their mass and electromagnetic couplings from CMB and LSS observations. Fermionic dark matter with non-zero electromagnetic form factors can annihilate to $e^+e^-$ and scalar dark matter can annihilate to $2\gamma$ at the time of recombination and distort the CMB. We analyze dark matter with multipole moments with Planck and BAO observations. We find upper bounds on anapole moment $g_A < 7.163 \times 10^3\text{GeV}^{-2}$, electric dipole moment $D < 7.978 \times 10^{-9}\text{e-cm}$, magnetic dipole moment $\mu < 2.959 \times 10^{-7}\mu_B$, and the bound on Rayleigh form factor of dark matter is $g_4/\Lambda_4^2 < 1.085 \times 10^{-2}\text{GeV}^{-2}$ with 95%C.L.

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1. INTRODUCTION

It is well accepted that formation of large scale structures and the rotation curves of galaxies require an extra dark matter component beyond than the known particles of the standard model. The particle properties of this dark matter are, however, completely unknown. Direct detection experiments, which rely on nuclear scattering, have ruled out a large parameter space. But, these techniques are not efficient in measuring dark matter of sub-GeV mass [1, 2]. To measure sub-GeV mass dark matter a suitable method is scattering electrons from heavy atoms [3-8]. Dark matter with non-zero electric or magnetic dipole moments [9, 11] or anapole moment [12, 13] can be very effective in scattering electrons. The electromagnetic form factors can be viewed as effective operators [14-17], which arise by integrating out the heavy particles in a ultraviolet complete theory [18].

The electromagnetic couplings of dark matter can be constrained from cosmic microwave background and large scale structures observations. The electric and magnetic dipole moment vertex can give rise to dark matter-baryon coupling. For heavy dark matter (∼ 100 GeV) the baryon drag on the dark matter will show up in structure formation and CMB [10]. Light dark matter (O(MeV) will annihilate to radiation and lower the effective neutrino number ($N_{\text{eff}}$) [19-21].

In this paper we will analyze the effect of light dark matter with electromagnetic form factors on CMB anisotropy and polarization from dark matter annihilation to $e^+e^-$ or photons close to recombination era, $z \sim 1100$. The effects of annihilating dark matter on CMB are studied in [22-26]. Production of relativistic $e^+, e^-$ heats up the thermal gas and ionizes the neutral Hydrogen, which increases the free electron fraction. Due to this increased free electron fraction there is a broadening of the last scattering surface and suppression of CMB temperature anisotropy. The low-$l$ correlations between polarization fluctuations are also enhanced due to increased freeze-out value of the ionization fraction of the universe after recombination. These effects on CMB are significant and can be used to put constraints on thermal averaged annihilation cross-section $\langle \sigma v \rangle$. Planck-2018 reports $\langle \sigma v \rangle < (3.2 \times 10^{-28} / f) \times (M_{DM}/(\text{GeV}/c^2)) \text{cm}^3/s$ for velocity independent thermal average cross-section [27]. Here $f$ is the fraction of energy injected to the intergalactic medium (IGM) by annihilating dark matter. Forecasts for upcoming CMB experiments such as AdvACTPol, AliCPT, CLASS, Simons Array, Simons Observatory, and SPT-3G has been studied by [28] in detecting decaying/annihilating dark matter, and it is found that $\langle \sigma v \rangle \sim (10^{-29} / f) \times (M_{DM}/(\text{GeV}/c^2)) \text{cm}^3/s$.

The annihilation $\chi\chi \rightarrow e^+e^-$ occur with one dipole or anapole vertex and the annihilation cross sections are quadratic in dipole or anapole moments. For fermionic dark matter the $\chi\chi \rightarrow \gamma\gamma$ annihilation cross section are quartic in dipole moments and the bounds from this process are much weaker [29] than the ones we derive in this paper from CMB. For anapole dark matter the cross section for the process $\chi\chi \rightarrow \gamma\gamma$ is zero [12]. Scalar dark matter can have dimension-6 Rayleigh operator vertex with two-photons. For such Rayleigh dark matter the leading order contribution to annihilation will be from the $\phi\phi \rightarrow \gamma\gamma$ process which can distort the CMB near recombination and from this we put bounds on the dark-matter photon Rayleigh coupling.

This paper is organized as follows. In Section 2 we list the electromagnetic form factors of dark matter which we shall constrain from CMB data. In Section 3 we discuss the physics
of recombination and the effect of dark matter annihilation on the CMB. In Section 4, we compare the CMB analysis with data from Planck and BAO and using COSMOMC we put constraints on the dark matter form factors. In Section 5, we compare our bounds with earlier results and from other experiments, and in Section 6, we summarize our results and give our conclusions.

2. ELECTROMAGNETIC FORM FACTORS OF DARK MATTER

Spin-1/2 dark matter can have the following electromagnetic form factors. These are the magnetic moment described by the dimension-5 operators,

$$\mathcal{L}_{\text{magnetic}} = \frac{g_1}{\Lambda_1} \bar{\chi} \sigma^{\mu \nu} \chi F_{\mu \nu}$$

where $g_1$ is a dimensionless coupling and $\Lambda_1$ is the mass scale of the particles in the loop which generate the dipole moment. The magnetic moment of Dirac fermions is $\mu = 2g_1/\Lambda_1$ and the operator (1) is zero for Majorana fermions.

Similarly electric dipole operator is of dimension-5,

$$\mathcal{L}_{\text{electric}} = \frac{g_2}{\Lambda_2} i \bar{\chi} \sigma^{\mu \nu} \gamma_5 \chi F_{\mu \nu}$$

where the electric dipole moment of Dirac fermions is $D = 2g_2/\Lambda_2$ and the operator (2) is zero for Majorana fermions.

Finally the anapole moment is a dimension-6 operator

$$\mathcal{L}_{\text{anapole}} = \frac{g_3}{\Lambda_3} i \bar{\chi} \gamma^\mu \gamma_5 \chi \partial^\nu F_{\mu \nu}$$

This operator is non-zero for Dirac as well as Majorana fermions and the coefficient $g_A = g_3/(\Lambda_3^2)$ is called the anapole moment of $\chi$.

Stringent bounds on sub-GeV mass dark matter are put from the observation of $\chi e^- \rightarrow \chi e^-$ scattering [3] in direct detection experiments like Xenon-10 [4], DarkSide [5] and Xenon-1T [6].

Using the experimental limits on dark matter electron scattering from Xenon-10, Xenon-1T and DarkSide bound on the electric dipole, magnetic dipole and anapole form factors of dark matter have been put in ref. [7, 8].

Real and complex scalar dark matter can have interaction with 2-photons by dimension-6 Rayleigh operator

$$\mathcal{L}_{2\phi 2\gamma} = \frac{g_4}{\Lambda_4} \phi^* \phi F_{\mu \nu} F^{\mu \nu}$$

These will contribute to $\phi \phi \rightarrow \gamma \gamma$ annihilations which can be constrained from CMB [17]. In the absence of CP violation the $\bar{F}F$ operator does not arise. The annihilation $\phi \phi \rightarrow \gamma \gamma$ takes place via s-wave in the leading order and cross section $\sigma(\phi \phi \rightarrow \gamma \gamma) \nu \simeq (g_4^2 m_\phi^2 / \Lambda_4^4)$ [17]. Bounds on the operator (4) from Xenon1T [6] and gamma ray searches from dwarf spheroidal satellites (dSphs) [30] and halo of the Milky way [31] by Fermi-LAT are obtained in ref. [17] for dark matter with mass larger than $\mathcal{O}$(GeV).
3. THERMAL HISTORY OF THE UNIVERSE WITH ANNIHILATING DARK MATTER

Recombination occurs around $z = 1100$ when electrons and protons combine together to form neutral hydrogen. If the annihilation cross-section of dark matter particles is sufficiently large, it can modify the history of recombination and hence can leave a clear imprint on CMB power spectrum. The shower of particles produced due to annihilation can interact with the thermal gas in three different ways. (i) The annihilation products can ionize the thermal gas, (ii) can induce Lyman-$\alpha$ excitation of the hydrogen that will cause more electrons in $n = 2$ state and hence increase the ionization rate and (iii) can heat the plasma. Due to the first two effects the evolution of free electron fraction $\chi_e$ changes and the last effect changes the temperature of baryons. The equation governing the evolution of ionization fraction in the presence of annihilating particles is given as

$$\frac{d\chi_e}{dt} = \frac{1}{(1 + z) H(z)} [R_s(z) - I_s(z) - I_X(z)].$$ \tag{5}

Here $R_s$ is the standard recombination rate, $I_s$ is the ionization rate due to standard sources and $I_X$ is the ionization rate due to annihilating dark matter particles. The computation of standard recombination rate was done in [32–34] and it is described as

$$[R_s(z) - I_s(z)] = C \times \left[ \chi_e^2 n_H \alpha_B - \beta_B (1 - \chi_e) e^{-h_\nu_2s/k_B T_B} \right].$$ \tag{6}

Here $n_H$ is the number density of hydrogen nuclei, $\alpha_B$ and $\beta_B$ are the effective recombination and photo-ionization rates for principle quantum numbers $\geq 2$ in Case B recombination, $\nu_2s$ is the frequency of the 2$s$ level from the ground state and $T_B$ is the temperature of the baryon gas. The factor $C$ appearing in Eqn. (6) is given by:

$$C = \frac{[1 + K\Lambda_2s1s n_H (1 - \chi_e)]}{[1 + K\Lambda_2s1s n_H (1 - \chi_e) + K\beta_B n_H (1 - \chi_e)]}.$$ \tag{7}

Here $\Lambda_1s2s$ is the decay rate of the metastable 2$s$ level, $n_H (1 - \chi_e)$ is the number of neutral ground state $H$ atoms and $K = \frac{\lambda_\alpha^3}{8\pi H(z)}$, where $H(z)$ is the Hubble expansion rate at redshift $z$ and $\lambda_\alpha$ is the wavelength of the $Ly - \alpha$ transition from the 2$p$ level to the 1$s$ level.

The term $I_X$ appearing in Eq. (5) represents the evolution of free electron density due to nonstandard sources. In our case it is due to annihilation of dark matter during recombination. which increases the ionization rate in two ways. (i) By direction ionization from the ground state and (ii) by additional $Ly - \alpha$ photons, which boosts the population at $n = 2$ increasing the the rate of photoionization by CMB. Hence, the ionization rate $I_X$ due to dark matter annihilation is expressed as

$$I_X(z) = I_{Xi}(z) + I_{X\alpha}(z).$$ \tag{8}

Here $I_{Xi}(z)$ represents the ionization rate due to ionizing photons and $I_{X\alpha}$ represents the ionization rate due to $Ly - \alpha$ photons.

The rate of energy release $\frac{dE}{dt}$ per unit volume by a relic self-annihilating dark matter particle can be expressed in terms of its thermally averaged annihilation cross-section $\langle \sigma v \rangle$
and mass $m_{\chi}$ as

$$ \frac{dE}{dt} = 2g\rho_c^2c^2\Omega_{DM}^2 (1+z)^6 f(z) \frac{\langle \sigma v \rangle}{m_{\chi}}, $$

(9)

where $\Omega_{DM}$ is the dark matter density parameter, $\rho_c$ is the critical density today, $g$ is degeneracy factor 1/2 for Majorana fermions and 1/4 for Dirac fermions, and $f(z)$ is the fraction of energy absorbed by the CMB plasma, which is $O(1)$ factor and depends on redshift. A detailed calculation of redshift dependence of $f(z)$ for various annihilation channels is done in [25, 35–38] using generalized parameterizations or principle components. It is shown in [26, 39, 40] that the redshift dependence of $f(z)$ can be ignored up to a first approximation, since current CMB data are sensitive to energy injection over a relatively narrow range of redshift, typically $z \sim 1000 - 600$. Hence $f(z)$ can be replace with a constant $f$, which we take as 1 for our analysis. Here we use 'on-the-spot' approximation, which assumes that the energy released due to dark matter annihilation is absorbed by IGM locally [24, 41, 42].

The terms appearing on the right hand side of Eq. (8) are related to the rate of energy release as

$$ I_{X_i} = C\chi_i \frac{[dE/dt]}{n_H(z)E_i}, $$

(10)

$$ I_{X_\alpha} = (1-C)\chi_{\alpha} \frac{[dE/dt]}{n_H(z)E_\alpha}. $$

(11)

Here $E_i$ is the average ionization energy per baryon, $E_\alpha$ is the difference in binding energy between the $1s$ and $2p$ energy levels of a hydrogen atom, $n_H$ is the number density of Hydrogen Nuclei, and $\chi_i$ and $\chi_{\alpha}$ represent the fraction of energy going ionization and $Ly-\alpha$ photons respectively; which can be expressed in terms of free electron fraction as $\chi_i = \chi_{\alpha} = (1 - \chi_e)/3$ [22].

A fraction of energy released by annihilating dark matter particles also goes into heating the baryon gas, which modifies the evolution equation for the matter temperature $T_b$ by contributing one extra term $K_h$ as

$$ (1+z) \frac{dT_b}{dz} = \frac{8\sigma_T a_RT_{CMB}^4}{3m_e c H(z)} \frac{\chi_e}{1 + f_{He} + \chi_e} (T_b - T_{CMB}) - \frac{2}{3k_B H(z)} \frac{k_h}{1 + f_{He} + \chi_e} + 2T_b. $$

(12)

Here the nonstandard term $K_h$ arising due to annihilating dark matter is given in terms of rate of energy release as

$$ K_h = \chi_h \frac{[dE/dt]}{n_H(z)}, $$

(13)

with $\chi_h = (1 + 2\chi_e)/3$ being the fraction of energy going into heat.

In this work we consider annihilating dark matter with electromagnetic form factors. We will now obtain the energy deposition rate for dark matter with anapole moment, electric dipole moment and magnetic dipole moment. One can define a quantity $p_{ann}$ that depends on the properties of dark matter particles as

$$ p_{ann} = f \frac{\langle \sigma v \rangle}{m_{\chi}}. $$

(14)
The current constraint on $p_{\text{ann}}$ with velocity independent $\langle \sigma v \rangle$ is $1.795 \times 10^{-7}$ m$^3$s$^{-1}$kg$^{-1}$ 95% C.L from Planck-2018 [27]. In our analysis we will use various electromagnetic form factors and mass of the dark matter as our model parameters rather than $p_{\text{ann}}$. Hence we will express energy deposition rate in terms of these parameters for annihilating dark matter with anapole and dipole moments.

3.1. Dark matter with anapole moment

The annihilation cross-section for dark matter with anapole moment is given as [12],

$$\langle \sigma v \rangle_{\chi \chi \rightarrow e^+e^-} = \frac{2g_A^2\alpha m^2_\chi}{3} v_{\text{rel}}.$$  (15)

where $\alpha = e^2/(4\pi)$ and $v_{\text{rel}}$ is average relative velocity between the annihilating dark matter particles in the centre of mass frame. The thermally averaged velocity can be expressed in terms of temperature by $\frac{1}{2} \left( \frac{1}{2} m_\chi \right) \langle v^2_{\text{rel}} \rangle = \frac{3}{2} T$. Hence the cross-section (15) can be expressed in terms of temperature as

$$\langle \sigma v \rangle_{\chi \chi \rightarrow e^+e^-} = 4g^2_A\alpha m^2_\chi \left( \frac{T}{m_\chi} \right).$$  (16)

After decoupling the temperature of the dark matter behaves as $T \propto (1 + z)^2$. Assuming the decoupling temperature of the dark matter $T_d$ of the order of $m_\chi/10$ we get

$$T = T_d \left( \frac{1 + z_d}{1 + z} \right)^2 = T_d \frac{T^2_0}{T^2_d} (1 + z)^2$$

$$= \frac{10 T^2_0}{m^2_\chi} (1 + z)^2.$$  (17)

Here $z_d$ and $T_d$ are the redshift of dark matter decoupling and the temperature of photons at that redshift respectively, which is same as $T_d$. $T_0$ is the current temperature of CMB. Using Eq. (17) the annihilation cross-section (16) becomes

$$\langle \sigma v \rangle_{\chi \chi \rightarrow e^+e^-} = 40g^2_A\alpha T^2_0 (1 + z)^2.$$  (18)

Hence using Eqs. (18) and (14) we can obtain the expression for $p_{\text{ann}}$ as

$$p_{\text{ann}} = \frac{40g^2_A\alpha T^2_0}{m^2_\chi} (1 + z)^2.$$  (19)

As mentioned earlier we will choose $f \sim 1$ here. Since $p_{\text{ann}}$ is velocity dependent, the rate of energy release given by Eq. (9) will be

$$\frac{dE}{dt} = \rho c^2 \Omega^2_{DM} \frac{40g^2_A\alpha T^2_0}{m^2_\chi} (1 + z)^8.$$  (20)

Here the redshift dependence of the energy deposition rate is modified as compared to (9).
3.2. Dark matter with electric dipole moment

For DM with electric dipole moment the annihilation cross-section is given by \[^{[11]}\],

\[ \langle \sigma v \rangle_{\chi \chi \rightarrow e^+e^-} = \frac{\alpha D^2}{12} v_{rel}^2 \]  
(21)

where \(v_{rel}\) is the relative velocity of two annihilating WIMPS. For thermal averaged cross-section \(T = m_\chi \langle v_{rel}^2 \rangle / 3\). So the annihilation cross-section for dark matter can be expressed in terms of temperature as

\[ \langle \sigma v \rangle_{\chi \chi \rightarrow e^+e^-} = \frac{\alpha D^2}{4} \left( \frac{T}{m_\chi} \right). \]  
(22)

Assuming \(T_d \sim \frac{m_\chi}{10}\) and using Eq. (17) for temperature of the dark matter the annihilation cross-section for dark matter with electric dipole moment becomes

\[ \langle \sigma v \rangle_{\chi \chi \rightarrow e^+e^-} = \frac{5\alpha D^2 T_0^2}{2m_\chi^2} (1 + z)^2. \]  
(23)

Hence using (14) we get

\[ p_{ann} = \frac{5\alpha D^2 T_0^2}{2m_\chi^2} (1 + z)^2. \]  
(24)

In this case the energy deposition rate will be

\[ \frac{dE}{dt} = \frac{1}{2} \rho_c^2 c^2 \Omega_{DM}^2 \frac{5\alpha D^2 T_0^2}{2m_\chi^2} (1 + z)^6. \]  
(25)

Here also the redshift dependence is modified as compared to (9), since the thermally averaged cross-section is velocity dependent.

3.3. Dark matter with magnetic dipole moment

For dark matter with magnetic dipole moment the annihilation cross-section is given as \[^{[11]}\],

\[ \langle \sigma v \rangle_{\chi \chi \rightarrow e^+e^-} = \alpha \mu^2, \]  
(26)

and hence

\[ p_{ann} = \frac{\alpha \mu^2}{m_\chi}. \]  
(27)

Here the annihilation cross-section does not depend on the velocity of dark matter so the energy deposition rate will be

\[ \frac{dE}{dt} = \frac{1}{2} \rho_c^2 c^2 \Omega_{DM}^2 \frac{\alpha \mu^2}{m_\chi} (1 + z)^6, \]  
(28)

which has the same redshift dependence as in Eq. (9).
3.4. Rayleigh dark matter

For scalar dark matter with Rayleigh coupling (4) the annihilation cross-section is given by [17],
\[ \langle \sigma v \rangle_{\phi\phi \rightarrow \gamma\gamma} = \left( \frac{g_4^2}{4} \right) m_{\phi}^2 \Lambda_4^4 \] (29)
and hence
\[ p_{ann} = \frac{(g_4^2)m_{\phi}}{\Lambda_4^4}. \] (30)
Here again the annihilation cross-section is independent of the velocity of dark matter, so the energy deposition rate will be same as (9).
\[ \frac{dE}{dt} = \frac{1}{2} \rho_e^2 c^2 \Omega_{DM} \left( \frac{g_4^2}{4} \right) m_{\phi} \Lambda_4^4 (1 + z)^6. \] (31)

4. CMB CONSTRAINTS ON VARIOUS MULTIPOLe MOMENTS OF DARK MATTER

As mentioned earlier annihilating dark matter increases the ionization fraction during recombination and heats the plasma. Hence the evolution equations of free electron fraction and matter temperature get modified as given by Eq. (5) and Eq. (12) respectively. The non-standard ionization rate I_X to compute free electron fraction can be obtained using Eq. (8) along with Eqs. (10) and (11). We use these equations along with energy deposition rates (20), (25), (28) and (31) for dark matter with anapole moment, electric dipole moment and magnetic dipole moment, and Rayleigh coupling to modify RECFAST routine [34] in CAMB [43]. We have also checked our analysis using CosmoRec [44] and HyRec [40, 45] code instead of RECFAST, and we found similar results. With this we obtain modified theoretical angular power spectra, which can be used to compute the bounds on various electromagnetic form factors and mass of the dark matter from Planck-2018 data using COSMOMC [46]. The priors for the multipole moments and mass of the dark matter are given in Table I. All these priors are sampled logarithmically to cover a larger range for the new parameters. We also vary the other six parameters of \( \Lambda CDM \) model with priors given in [47]. We have imposed flat priors for all parameters.

| Type of dark matter coupling | Priors |
|-----------------------------|--------|
| Anapole                     | \( 5.0 < \ln \left( 10^9 \left( g_A/GeV^2 \right) \right) < 40 \)  
\( -3.0 < \log_{10} \left( m_\chi/GeV \right) < 2.0 \) |
| Electric dipole             | \( -5.0 < \ln \left( 10^{18} \left( D/(e - cm) \right) \right) < 40 \)  
\( -3.0 < \log_{10} \left( m_\chi/GeV \right) < 2.0 \) |
| Magnetic dipole             | \( -10.0 < \ln(10^9 \left( \mu/\mu_B \right) ) < 15.0 \)  
\( -3.0 < \log_{10} \left( m_\chi/GeV \right) < 2.0 \) |
| Rayleigh coupling           | \( -10.0 < \ln(10^9 g_4/(\Lambda_4^2 GeV^{-2}) ) < 20.0 \)  
\( -12.0 < \log_{10} \left( m_\chi/GeV \right) < 2.0 \) |

**TABLE I:** Priors on input parameters for annihilating dark matter.
We use the lower bound for the mass of dark matter with anapole moment, and electric and magnetic dipole moment as 1 MeV since the annihilation channel for this case is $\chi \chi \rightarrow e^+ e^-$. However, in case of scalar dark matter with Rayleigh coupling, [17], the dark matter annihilates to photons having energy around 1 eV during recombination, which is used as the lower bound for mass of Rayleigh dark matter. We also use BAO and Pantheon data along with Planck-2018 observations for our analysis. We perform MCMC convergence diagnostic tests on 4 chains using the Gelman and Rubin “variance of mean”/“mean of chain variance” $R^{-1}$ statistics for each parameter.

The constraints obtained for anapole moment and mass of the dark matter along with other six parameters of ΛCDM model are shown in Table II. Fig. 1 represents the marginalized constraints on anapole moment and mass of the dark matter along with joint 68% CL and 95% CL constraints on both the parameters from Planck-2018 and BAO data.

| Parameter | 68% limits | 95% limits | 99% limits |
|-----------|------------|------------|------------|
| $\Omega_b h^2$ | $0.02243 \pm 0.00013$ | $0.02243^{+0.00034}_{-0.00034}$ | $0.02243^{+0.00026}_{-0.00026}$ |
| $\Omega_c h^2$ | $0.11917 \pm 0.00090$ | $0.1192^{+0.0023}_{-0.0018}$ | $0.1192^{+0.0024}_{-0.0018}$ |
| $\tau$ | $0.0567 \pm 0.0072$ | $0.057^{+0.015}_{-0.014}$ | $0.057^{+0.020}_{-0.018}$ |
| $\ln(10^{9}(g_A/GeV^{-2}))$ | $< 22.6$ | $< 29.6$ | $< 31.9$ |
| $\log_{10}(m_{\chi}/GeV)$ | $- - -$ | $- - -$ | $- - -$ |
| $\ln(10^{10} A_s)$ | $3.048 \pm 0.014$ | $3.048^{+0.039}_{-0.028}$ | $3.048^{+0.039}_{-0.036}$ |
| $n_s$ | $0.9670 \pm 0.0037$ | $0.9670^{+0.0097}_{-0.0073}$ | $0.9670^{+0.0096}_{-0.0073}$ |
| $H_0$ | $67.73 \pm 0.41$ | $67.73^{+0.81}_{-0.80}$ | $67.7^{+1.1}_{-1.1}$ |

TABLE II: Planck-2018 and BAO constraints on anapole momentum and mass of the dark matter with other 6 parameters of ΛCDM

![Marginalized constraints](image1.png)

![Joint 68% CL and 95%CL constraints](image2.png)

FIG. 1: Constraints for anapole moment and mass of the dark matter using Planck-2018 and BAO data
It can be seen from Table III that

\[ g_A < 7.163 \times 10^9 \text{GeV}^{-2} \quad 95\% \text{C.L.} \quad (32) \]

The constraints obtained using Planck-2018 and BAO data on electric dipole moment and the mass of the dark matter along with the other six parameters of ΛCDM model are listed in Table III. Fig. 2 depicts the marginalized constraints on electric dipole moment and mass of the dark matter and with joint 68% CL and 95%CL constraints on both the parameters from Planck-2018 and BAO data.

| Parameter                  | 68% limits          | 95% limits          | 99% limits          |
|----------------------------|---------------------|---------------------|---------------------|
| \( \Omega_b h^2 \)        | 0.02244 ± 0.00014   | 0.02244±0.00027     | 0.02244±0.00036     |
| \( \Omega_c h^2 \)        | 0.11921 ± 0.00091   | 0.1192±0.0018       | 0.1192±0.0024       |
| \( \tau \)                | 0.0565 ± 0.0073     | 0.056±0.015         | 0.056±0.020         |
| \( \ln(10^{18}(D/(e - \text{cm})) \) | < 12.9             | < 22.8              | < 26.7              |
| \( \log_{10}(m_X/\text{GeV}) \) | > −0.963          | − − −               | − − −               |
| \( \ln(10^{10} A_s) \)    | 3.048 ± 0.014      | 3.048±0.029         | 3.048±0.039         |
| \( n_s \)                 | 0.9670 ± 0.0037    | 0.9670±0.0071       | 0.9670±0.0093       |
| \( H_0 \)                 | 67.72 ± 0.41       | 67.72+0.82−0.79     | 67.7+1.1−1.0        |

TABLE III: Planck-2018 and BAO constraints on electric dipole momentum and mass of the dark matter with other 6 parameters of ΛCDM

![Marginalized constraints](image1.png)

(a)Marginalized constraints

![Joint 68% CL and 95%CL constraints](image2.png)

(b)Joint 68% CL and 95%CL constraints

FIG. 2: Constraints for electric dipole moment and mass of the dark matter using Planck-2018 and BAO data.

We can see from Table III that

\[ D < 7.978 \times 10^{-9} \text{e-cm} \quad 95\% \text{C.L.} \quad (33) \]
Table. IV represents the constraints on magnetic dipole moment and mass of the dark matter obtained from Planck-2018 and BAO data. Here also we have quoted the constraints on other six parameters of $\Lambda$CDM. Fig. 3 represents the marginalized constraints on magnetic dipole moment and mass of the dark matter along with joint 68% CL and 95%CL constraints on both the parameters.

| Parameter       | 68% limits       | 95% limits       | 99% limits       |
|-----------------|------------------|------------------|------------------|
| $\Omega_b h^2$  | 0.02244 ± 0.00013| 0.02244 ±0.00026 | 0.02244 ±0.00034|
| $\Omega_c h^2$  | 0.11920 ± 0.00091| 0.1192 ±0.0018   | 0.1192 ±0.0023   |
| $\tau$          | 0.0564 ± 0.0073  | 0.056 ±0.015     | 0.056 ±0.020     |
| $\ln(10^9(\mu/\mu_B))$ | −2.0±4.0       | < 5.69           | < 7.36           |
| $\log_{10}(m_\chi/GeV)$ | −−−             | −−−              | −−−              |
| $\ln(10^{10}A_s)$ | 3.047 ± 0.014   | 3.047 ±0.029     | 3.047 ±0.039     |
| $n_s$           | 0.9671 ± 0.0037  | 0.9671 ±0.0073   | 0.9671 ±0.0096   |
| $H_0$           | 67.72 ± 0.41     | 67.72 ±0.82      | 67.7 ±1.1        |

TABLE IV: Planck-2018 and BAO constraints on magnetic dipole momentum and mass of Majorana fermion dark matter with other 6 parameters of $\Lambda$CDM

Again we can read from Table IV that

\[ \mu < 2.959 \times 10^{-7}\mu_B \quad 95\% \text{C.L.} \]  \hspace{1cm} (34)

Similarly Table. IV lists the constraints on Rayleigh coupling and mass of the scalar dark matter. Fig. 3 represents the marginalized constraints on both of these parameters along with joint 68% CL and 95%CL constraints from Planck-2018 and BAO data. Again we have mentioned constraints on other six parameters of $\Lambda$CDM.
We can see from the Table V that

$$\frac{g_4}{A_4^2} < 1.085 \times 10^{-2} \text{ GeV}^{-2} \quad 95\% C.L. \quad (35)$$

The upper bounds given by Eqns. (32), (33), (34) and (35) are obtained after marginalizing over all other parameters.

TABLE V: Planck-2018 and BAO constraints on Rayleigh coupling and mass of dark matter with other 6 parameters of $\Lambda$CDM

| Parameter | 68% limits         | 95% limits         | 99% limits         |
|-----------|--------------------|--------------------|--------------------|
| $\Omega_b h^2$ | 0.02244 ± 0.00013 | 0.02244$^{+0.00026}_{-0.00026}$ | 0.02244$^{+0.00034}_{-0.00034}$ |
| $\Omega_c h^2$ | 0.11919 ± 0.00092 | 0.1192$^{+0.0018}_{-0.0024}$ | 0.1192$^{+0.0023}_{-0.0024}$ |
| $\tau$ | 0.0567 ± 0.0074 | 0.057$^{+0.015}_{-0.014}$ | 0.057$^{+0.020}_{-0.019}$ |
| $\ln(10^9 g_4/(A_4^2 \text{GeV}^{-2}))$ | < 6.43 | < 16.2 | < 19.7 |
| $\log_{10}(m_\chi/\text{GeV})$ | < −3.55 | — | — |
| $n_s$ | 0.9671 ± 0.0037 | 0.9671$^{+0.0073}_{-0.0073}$ | 0.9671$^{+0.0097}_{-0.0096}$ |
| $H_0$ | 67.73 ± 0.41 | 67.73$^{+0.81}_{-0.80}$ |

FIG. 4: Constraints on Rayleigh coupling and mass of scalar dark matter using Planck-2018 and BAO data

(a)Marginalized constraints

(b)Joint 68% CL and 95%CL constraints
5. COMPARISON WITH CONSTRAINTS FROM OTHER EXPERIMENTS

In order to obtain the correct relic density of sub-GeV dark matter we require mediators in the sub-GeV mass range \[2\]. We can describe dark matter interactions, where the energy transfer is so small, with effective operators like anapole, dipole and Rayleigh form factors, which are effective below the cutoff scale \( \Lambda \sim \text{GeV} \). These low energy effective form factors cannot be constrained from colliders, and the best bounds are obtained from the low energy processes like electron scattering in direct detection experiments or in searches of sub-GeV scale gamma rays from galaxies. The constraints which can be obtained on TeV scale effective theories from colliders is studied in \[48\]. We compare the bounds from CMB distortion by dark matter annihilation obtained in this paper with the bounds on the electromagnetic form factors from other experiments and astrophysical observations in this section.

5.1. Electric, Magnetic and Anapole moments

For sub-GeV mass fermionic dark matter the best bounds come from the ionization of atoms with electron scattering process \( \chi + e^- \rightarrow \chi + e^- \) in direct detection experiments like Xenon-10 \[4\], DarkSide \[5\] and Xenon-1T \[6\]. The most stringent bounds come from Xenon-10 \[4\] and Xenon-1T \[6\] which are dark matter mass dependent. The bound on anapole moment of Majorana dark matter is \( g_A < (10^2 - 0.5 \times 10^{-1}) \text{GeV}^{-2} \) in the mass range \( m_\chi = (0.2 \text{ MeV} - 1 \text{ GeV}) \) \[7\]. This is more stringent than the CMB bound \( g_A < 7.163 \times 10^3 \text{GeV}^{-2} \) (for \( m_\chi \geq 0.5 \text{ MeV} \)) since the CMB bound is based on annihilation process \( \chi \chi \rightarrow e^+e^- \), whose cross section is velocity suppressed \(15\).

The bound on electric dipole moment of Xenon-10 \[4\] and Xenon-1T \[6\] of Dirac dark matter with mass in the range \( m_\chi = (0.2 \text{ MeV} - 1 \text{ GeV}) \) is \( D < (6.6 \times 10^{-18} - 3.9 \times 10^{-20}) \text{e-cm} \). This is again more stringent than the CMB bound \( D < 7.978 \times 10^{-9} \text{e-cm} \) (for \( m_\chi \geq 0.5 \text{ MeV} \)) as the annihilation cross section for electric dipole annihilation is velocity suppressed \[21\].

Finally bound on magnetic dipole moment from Xenon-10 \[4\] and Xenon-1T \[6\] of Dirac dark matter with mass in the range \( m_\chi = (0.2 \text{ MeV} - 1 \text{ GeV}) \) is \( \mu < (1.2 \times 10^{-5} - 5.9 \times 10^{-8})\mu_B \). This is comparable to the CMB bound \( \mu < 2.959 \times 10^{-7} \mu_B \) (for \( m_\chi \geq 0.5 \text{ MeV} \)). The CMB bound is comparable with the direct detection bounds as the annihilation cross section for magnetic dipole annihilation is not velocity suppressed \[26\].

5.2. Rayleigh form factor of scalar dark matter

Constraints on the Rayleigh form factor \[4\] are obtained from electron ionisation by Xenon-1T \[6\] and by searches for \( \gamma \) ray line spectrum in the Milky Way center by Fermi-LAT \[31\]. For dark matter of mass \( \sim \text{GeV} \) the bound from Fermi-LAT search is \( g_4/A_4^2 < 10^{-3} \text{GeV}^{-2} \), and from Xenon-1T the bound is \( g_4/A_4^2 < 0.25 \text{GeV}^{-2} \) \[17\]. The CMB bound obtained in the paper \( g_4^2/A_4^2 < 1.1 \times 10^{-2} \text{GeV}^{-2} \) is weaker than the Fermi-LAT bound but is valid for larger range of dark matter masses up to \( m_\phi \geq \text{eV} \).
6. CONCLUSIONS

Electromagnetic form factors are an important class of interactions in the effective theories framework of classifying dark matter interactions. Dirac fermions dark matter with non-zero electric and magnetic dipole moments can give the correct relic density $\Omega_{\chi} h^2 = 0.11$ by the $\chi\chi \leftrightarrow f\bar{f}$ freeze-out process if $D = 2.5 \times 10^{-16}$ e-cm and $\mu = 8.2 \times 10^{-7} \mu_B$ respectively [11]. Majorana fermions with anapole moment of mass 10 MeV with anapole moment $g_A = 0.11 \text{Gev}^{-2}$ can be dark matter with the correct freeze-out relic density [12].

Electromagnetic dark matter can be observed not only via electron scattering direct detection experiments [3–8] but can also be constrained from the CMB. In this paper we have considered anapole and dipolar dark matter matter with masses $m_{\chi} > \mathcal{O}(\text{MeV})$. We find that dark matter with electromagnetic dipole or anapole form factors will distort the CMB during recombination era by producing relativistic electron via the process $\chi\chi \rightarrow e^+e^-$. We find that the Planck data gives the bounds on electromagnetic form factors $D < 7.978 \times 10^{-9}$ e-cm and $\mu < 2.959 \times 10^{-7} \mu_B$, and $g_A < 7.163 \times 10^{3}\text{Gev}^{-2}$.

Dark matter with $\mathcal{O}(\text{MeV})$ mass in thermal equilibrium with radiation will be ruled out by BBN constraints on $N_{\text{eff}}$. These can only have be created after the BBN era by the freeze-in mechanism. Freeze-in requires very small couplings and our bounds on electric dipole and anapole moments also rules out the freeze-out mechanism for relic density. These may be produced by the freeze-in mechanism which requires smaller couplings [2, 49].

For scalar dark matter there is the dimension-six Rayleigh operator coupling with photons. We put the bound on the Rayleigh coupling as $\frac{g_4}{\Lambda^2} < 1.085 \times 10^{-2}\text{GeV}^{-2}$ (95% C.L). This bound is valid for dark matter mass as low as $\mathcal{O}(\text{eV})$. Such light dark matter can only be produced by the freeze-in mechanism to evade bounds from BBN.

Spectral distortion of the CMB can also arise from radiatively decaying dark matter [50]. The bounds derived on the radiative lifetime can be used for deriving bounds on dipolar couplings of Majorana dark matter which can have non-zero transition electric and magnetic moments dipole.

7. ACKNOWLEDGEMENTS

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[1] T. Marrodán Undagoitia and L. Rauch, “Dark matter direct-detection experiments,” J. Phys. G 43, no.1, 013001 (2016) [arXiv:1509.08767 [physics.ins-det]].
[2] S. Mohanty, “Astroparticle Physics and Cosmology: Perspectives in the Multimessenger Era,” Lect. Notes Phys. 975, 1-287 (2020) doi:10.1007/978-3-030-56201-4
[3] R. Essig, J. Mardon and T. Volansky, “Direct Detection of Sub-GeV Dark Matter,” Phys. Rev. D 85, 076007 (2012) [arXiv:1108.5383 [hep-ph]].
[4] R. Essig, A. Manalaysay, J. Mardon, P. Sorensen and T. Volansky, “First Direct Detection Limits on sub-GeV Dark Matter from XENON10,” Phys. Rev. Lett. 109, 021301 (2012) [arXiv:1206.2644 [astro-ph.CO]].

[5] P. Agnes et al. [DarkSide], “Constraints on Sub-GeV Dark-Matter–Electron Scattering from the DarkSide-50 Experiment,” Phys. Rev. Lett. 121, no.11, 111303 (2018) [arXiv:1802.06998 [astro-ph.CO]].

[6] E. Aprile et al. [XENON], “Light Dark Matter Search with Ionization Signals in XENON1T,” Phys. Rev. Lett. 123, no.11, 251801 (2019) [arXiv:1907.11485 [hep-ex]].

[7] R. Catena, T. Emken, N. A. Spaldin and W. Tarantino, “Atomic responses to general dark matter-electron interactions,” Phys. Rev. Res. 2, no.3, 033195 (2020) [arXiv:1912.08204 [hep-ph]].

[8] R. Catena, T. Emken and J. Ravanis, “Rejecting the Majorana nature of dark matter with electron scattering experiments,” JCAP 06, 056 (2020) [arXiv:2003.04039 [hep-ph]].

[9] M. Pospelov and T. ter Veldhuis, “Direct and indirect limits on the electromagnetic form-factors of WIMPs,” Phys. Lett. B 480, 181-186 (2000) [arXiv:hep-ph/0003010 [hep-ph]].

[10] K. Sigurdson, M. Doran, A. Kurylov, R. R. Caldwell and M. Kamionkowski, “Dark-matter electric and magnetic dipole moments,” Phys. Rev. D 70, 083501 (2004) [erratum: Phys. Rev. D 73, 089903 (2006)] [arXiv:astro-ph/0406355 [astro-ph]].

[11] E. Masso, S. Mohanty and S. Rao, “Dipolar Dark Matter,” Phys. Rev. D 80, 036009 (2009) [arXiv:0906.1979 [hep-ph]].

[12] C. M. Ho and R. J. Scherrer, “Anapole Dark Matter,” Phys. Lett. B 722, 341-346 (2013) [arXiv:1211.0503 [hep-ph]].

[13] E. De Simone and T. Jacques, “Simplified models vs. effective field theory approaches in dark matter searches,” Eur. Phys. J. C 76, no.7, 367 (2016) [arXiv:1603.08002 [hep-ph]].

[14] X. Chu, J. Pradler and L. Semmelrock, “Light dark states with electromagnetic form factors,” Phys. Rev. D 99, no.1, 015040 (2019) [arXiv:1811.04095 [hep-ph]].

[15] Kavanagh, B.J., Panci, P. and Ziegler, R. , Faint light from dark matter: classifying and constraining dark matter-photon effective operators. J. High Energ. Phys., 89 (2019).

[16] J. Kopp, L. Michaels and J. Smirnov, “Loopy Constraints on Leptophilic Dark Matter and Internal Bremsstrahlung,” JCAP 04, 022 (2014) [arXiv:1401.6457 [hep-ph]].

[17] C. M. Ho and R. J. Scherrer, “Sterile Neutrinos and Light Dark Matter Save Each Other,” Phys. Rev. D 87, no.6, 065016 (2013) [arXiv:1212.1689 [hep-ph]].

[18] C. Boehm, M. J. Dolan and C. McCabe, “A Lower Bound on the Mass of Cold Thermal Dark Matter from Planck,” JCAP 08, 041 (2013) [arXiv:1303.6270 [hep-ph]].

[19] C. Brust, D. E. Kaplan and M. T. Walters, “New Light Species and the CMB,” JHEP 12, 058 (2013) [arXiv:1303.5379 [hep-ph]].

[20] X. L. Chen and M. Kamionkowski, Phys. Rev. D 70, 043502 (2004) doi:10.1103/PhysRevD.70.043502 [arXiv:astro-ph/0310473 [astro-ph]].

[21] N. Padmanabhan and D. P. Finkbeiner, Phys. Rev. D 72, 023508 (2005) doi:10.1103/PhysRevD.72.023508 [arXiv:astro-ph/0503486 [astro-ph]].
[24] S. Galli, F. Iocco, G. Bertone and A. Melchiorri, Phys. Rev. D 80, 023505 (2009) doi:10.1103/PhysRevD.80.023505 [arXiv:0905.0003 [astro-ph.CO]].
[25] T. R. Slatyer, N. Padmanabhan and D. P. Finkbeiner, Phys. Rev. D 80, 043526 (2009) doi:10.1103/PhysRevD.80.043526 [arXiv:0906.1197 [astro-ph.CO]].
[26] D. P. Finkbeiner, S. Galli, T. Lin and T. R. Slatyer, Phys. Rev. D 85, 043522 (2012) doi:10.1103/PhysRevD.85.043522 [arXiv:1109.6322 [astro-ph.CO]].
[27] N. Aghanim et al. [Planck], Astron. Astrophys. 641, A6 (2020) doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].
[28] J. Cang, Y. Gao and Y. Z. Ma, Phys. Rev. D 102, no.10, 103005 (2020) doi:10.1103/PhysRevD.102.103005 [arXiv:2002.03380 [astro-ph.CO]].
[29] C. Arellano-Celiz, A. Avilez-López, J. E. Barradas-Guevara and O. Félix-Beltrán, “Annihilation of Dipolar Dark Matter to Photons,” [arXiv:1908.05695 [hep-ph]].
[30] A. Albert et al. [Fermi-LAT and DES], Astrophys. J. 834, no.2, 110 (2017) doi:10.3847/1538-4357/834/2/110 [arXiv:1611.03184 [astro-ph.HE]].
[31] M. Ackermann et al. [Fermi-LAT], Phys. Rev. D 91, no.12, 122002 (2015) doi:10.1103/PhysRevD.91.122002 [arXiv:1506.00013 [astro-ph.HE]].
[32] P. J. E. Peebles, “Recombination of the Primeval Plasma,” Astrophys. J. 153, 1 (1968). doi:10.1086/149628
[33] Y. B. Zeldovich, V. G. Kurt and R. A. Sunyaev, “Recombination of hydrogen in the hot model of the universe,” J. Exp. Theor. Phys. 28, 146 (1969) [Zh. Eksp. Teor. Fiz. 55, 278 (1968)].
[34] S. Seager, D. D. Sasselov and D. Scott, “How exactly did the universe become neutral?,” Astrophys. J. Suppl. 128, 407 (2000) [astro-ph/9912182].
[35] G. Huetsi, A. Hektor and M. Raidal, Astron. Astrophys. 505, 999-1005 (2009) doi:10.1051/0004-6361/200912760 [arXiv:0906.4550 [astro-ph.CO]].
[36] C. Evoli, S. Pandolfi and A. Ferrara, Mon. Not. Roy. Astron. Soc. 433, 1736 (2013) doi:10.1093/mnras/stt849 [arXiv:1210.6845 [astro-ph.CO]].
[37] S. Galli, T. R. Slatyer, M. Valdes and F. Iocco, Phys. Rev. D 88, 063502 (2013) doi:10.1103/PhysRevD.88.063502 [arXiv:1306.0563 [astro-ph.CO]].
[38] M. S. Madhavacheril, N. Sehgal and T. R. Slatyer, Phys. Rev. D 89, 103508 (2014) doi:10.1103/PhysRevD.89.103508 [arXiv:1310.3815 [astro-ph.CO]].
[39] S. Galli, F. Iocco, G. Bertone and A. Melchiorri, Phys. Rev. D 84, 027302 (2011) doi:10.1103/PhysRevD.84.027302 [arXiv:1106.1528 [astro-ph.CO]].
[40] G. Giesen, J. Lesgourgues, B. Audren and Y. Ali-Haimoud, JCAP 12, 008 (2012) doi:10.1088/1475-7516/2012/12/008 [arXiv:1209.0247 [astro-ph.CO]].
[41] L. Zhang, X. L. Chen, Y. A. Lei and Z. G. Si, Phys. Rev. D 74, 103519 (2006) doi:10.1103/PhysRevD.74.103519 [arXiv:astro-ph/0603425 [astro-ph]].
[42] L. Zhang, X. Chen, M. Kamionkowski, Z. g. Si and Z. Zheng, Phys. Rev. D 76, 061301 (2007) doi:10.1103/PhysRevD.76.061301 [arXiv:0704.2444 [astro-ph]].
[43] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. 538, 473-476 (2000) doi:10.1086/309179 [arXiv:astro-ph/9911177 [astro-ph]].
[44] J. Chluba and R. M. Thomas, Mon. Not. Roy. Astron. Soc. 412, 748 (2011) doi:10.1111/j.1365-2966.2010.17940.x [arXiv:1010.3631 [astro-ph.CO]].
[45] Y. Ali-Haimoud and C. M. Hirata, Phys. Rev. D 83, 043513 (2011) doi:10.1103/PhysRevD.83.043513 [arXiv:1011.3758 [astro-ph.CO]].
[46] A. Lewis and S. Bridle, Phys. Rev. D 66, 103511 (2002) doi:10.1103/PhysRevD.66.103511 [arXiv:astro-ph/0205436 [astro-ph]].

[47] P. A. R. Ade et al. [Planck], Astron. Astrophys. 571, A16 (2014) doi:10.1051/0004-6361/201321591 [arXiv:1303.5076 [astro-ph.CO]].

[48] C. Arina, A. Cheek, K. Mimasu and L. Pagani, “Light and Darkness: consistently coupling dark matter to photons via effective operators,” Eur. Phys. J. C 81, no.3, 223 (2021) [arXiv:2005.12789 [hep-ph]].

[49] L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, “Freeze-In Production of FIMP Dark Matter,” JHEP 03, 080 (2010) [arXiv:0911.1120 [hep-ph]].

[50] B. Bolliet, J. Chluba and R. Battye, “Spectral distortion constraints on photon injection from low-mass decaying particles,” [arXiv:2012.07292 [astro-ph.CO]].