Beyond six parameters: extending ΛCDM

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Cosmological constraints are usually derived under the assumption of a 6 parameters Λ-CDM theoretical framework or simple one-parameter extensions. In this paper we present, for the first time, cosmological constraints in a significantly extended scenario, varying up to 12 cosmological parameters simultaneously, including the sum of neutrino masses, the neutrino effective number, the dark energy equation of state, the gravitational waves background and the running of the spectral index of primordial perturbations. Using the latest Planck 2015 data release (with polarization) we found no significant indication for extensions to the standard Λ-CDM scenario, with the notable exception of the angular power spectrum lensing amplitude, $A_{\text{lens}}$, that is larger than the expected value at more than two standard deviations even when combining the Planck data with BAO and supernovae type Ia external datasets. In our extended cosmological framework, we find that a combined Planck+BAO analysis constrains the value of the r.m.s. density fluctuation parameter to $\sigma_8 = 0.781^{+0.065}_{-0.063}$ at 95% c.l., helping to relax the possible tensions with the CFHTLenS cosmic shear survey. We also find a lower value for the reionization optical depth $\tau = 0.058^{+0.044}_{-0.043}$ at 95% c.l. with respect to the one derived under the assumption of Λ-CDM. The scalar spectral index $n_S$ is now compatible with a Harrison-Zeldovich spectrum to within 2.5 standard deviations. Combining the Planck dataset with the HST prior on the Hubble constant provides a value for the equation of state $w < -1$ at more than two standard deviations while the neutrino effective number is fully compatible with the expectations of the standard three neutrino framework.

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I. INTRODUCTION

In the past twenty years, measurements of the Cosmic Microwave Background (CMB, hereafter) anisotropy angular power spectrum have witnessed one of the most impressive technological advances in experimental physics. Following the first detection of CMB temperature anisotropies at large angular scales by the COBE satellite in 1992 [1], the angular power spectrum has been measured with increasing precision by balloon-borne experiments such as BOOMERanG [2], MAXIMA [3] and by ground-based experiments as DASI [4], showing the unambiguous presence of a “first peak” and subsequent oscillations in the angular power spectrum at intermediate angular scales ($\theta \sim 0.2^\circ$). The spectacular measurements obtained by the WMAP satellite mission [5] have not only confirmed the presence of these acoustic oscillations but also provided the first precise measurement of the cross temperature-polarization angular spectrum and the first constraints on the epoch of reionization. The very small-scale part of the angular temperature power spectrum, and especially the damping tail, has been accurately determined by experiments as ACT [6] and SPT [7]. This impressive progress in the measurement of the CMB anisotropies temperature angular spectrum has culminated with the cosmic-variance limited measurements of the Planck experiment that has now also provided exquisite results on the polarization and cross temperature-polarization spectra.

Despite this impressive progress on the experimental side, it is interesting to note that the constraints on cosmological parameters are still presented (as in the latest Planck 2015 data release, [8]) under the assumption of a simple Λ-CDM model, based on the variation of just 6 cosmological parameters. While this model still provides a good fit to the data, it is the same model used, for example, in the analysis of the BOOMERanG 1998 data (see [2]), i.e. more than fifteen years ago. While this "minimal" approach is justified by the good fit to the data that the Λ-CDM provides we believe that it does not do adequate justice to the high quality of the most recent datasets. In light of the new precise data, some of the assumptions or simplifications made in the 6 parameters approach are indeed not anymore fully justified. For example, fixing the total neutrino mass to zero or to some small value is completely arbitrary since we know that neutrinos must have masses and that current cosmological datasets are sensitive to variations in the absolute neutrino mass scale of order $\sim 100$ meV. At the same time, considering that a cosmological constant offers difficulties in any theoretical interpretation, it seems reasonable to incorporate in the analysis a possible dynamical dark energy component. This is certainly plausible (and even preferred if one wants to address the "Why Now?" problem), and indeed fixing the dark energy equation of state to $-1$ is not favoured by any theoretical argument. Most inflationary models predict a sizable contribution of gravitational waves. Given the progress made in the search for B-mode polarization, especially by the recent
combined BICEP2+Planck analysis [20], it is an opportune moment to allow any such contribution to be directly constrained by the data, without assuming a null contribution as in the 6-parameter model. A similar argument can be made for the running of the scalar spectral index $dn_s/dlnk$. Moreover, the neutrino effective number, $N_{\text{eff}}$, could be easily different from the standard expected value of 3.046. Even assuming the standard three neutrino framework, non-standard decoupling, inflationary reheating, dark matter decay and many other physical processes could alter its value. Finally, the Planck 2015 release still hints for an anomalous value for the lensing amplitude $A_{\text{lens}}$ [21]. While this parameter is purely phenomenological, one should clearly consider it and check if the cosmology obtained is consistent with other datasets.

Clearly, one should be careful in neglecting these extensions following a motivation based simply on an Occam razor argument. Indeed having more parameters does not necessary means having a more complicated theory. For example, forcing the dark energy component to a cosmological constant means accepting an extreme fine tuning and a physical theory probably vastly more "complicated" as those arising from a multiverse scenario.

The goal of this Letter is to constrain cosmological parameters in this extended parameter space.

II. METHOD

As discussed in the introduction, besides the six parameters of the "standard" Λ-CDM model, i.e. the Hubble constant $H_0$, the baryon $\Omega_b h^2$ and CDM energy densities $\Omega_c h^2$, the primordial amplitude and spectral index of scalar perturbations $A_s$ and $n_s$ (at pivot scale $k_0 = 0.05h\,\text{Mpc}^{-1}$), and the reionization optical depth $\tau$, we also consider variations in 6 additional parameters: the total mass for the 3 standard neutrinos, $\Sigma m_\nu$, the dark energy equation of state $w$ assumed constant with redshift, the tensor/scalar ratio of amplitude $r$ at pivot scale $k_0 = 0.05h\,\text{Mpc}^{-1}$, the running of the scalar spectral index $dn_s/dlnk$, at pivot scale $k_0 = 0.05h\,\text{Mpc}^{-1}$, the amplitude of the lensing signal in the CMB angular spectra, $A_{\text{lens}}$ as defined in [21], the effective number of relativistic neutrinos, $N_{\text{eff}}$.

In what follows, we refer to this model as eCDM (extended Cold Dark Matter).

We let all these parameters vary freely simultaneously in a range of external, conservative, priors listed in Table I.

TABLE I: External priors on the cosmological parameters assumed in this paper.

| Parameter | Prior |
|-----------|-------|
| $\Omega_b h^2$ | 0.005, 0.1 |
| $\Omega_c h^2$ | 0.001, 0.99 |
| $\Theta_s$ | 0.5, 10 |
| $\tau$ | 0.01, 0.8 |
| $n_s$ | 0.8, 1.1 |
| $\log(10^{10} A_s)$ | 2.4 |
| $\sum m_\nu (\text{eV})$ | 0.3 |
| $\frac{dn_s}{dlnk}$ | -3.5, 0.5 |
| $r$ | 0.5 |
| $N_{\text{eff}}$ | 0.05, 10 |
| $A_{\text{lens}}$ | 0.10 |

We produce constraints on these cosmological parameters by making use of several, recent, datasets. Firstly, we use the full Planck 2015 release on temperature and polarization CMB angular power spectra. Since unresolved systematics may be present in the Planck 2015 polarization data at small angular scales (see discussion in [11]) we denote as Planck TT+lowP the Planck temperature data plus the low angular scale polarization, while with Planck the dataset that also includes the small-scale polarization data measured by Planck HFI.

We also include information on CMB lensing from Planck trispectrum detection (see [9]). We refer to this dataset as lensing. We add baryonic acoustic oscillation data from 6dFGS [12], SDSS-MGS [13], BOSSLOWZ [14] and CMASS-DR11 [14] surveys as in [8]. We refer to this dataset as BAO. We impose a constraint on the Hubble constant from the Hubble Space Telescope [15] dataset. Recently this constraint has been criticized in [16] where a more conservative value was suggested, a choice adopted in the recent Planck analysis [8]. Since the HST prior has been shown to be in tension with the Planck constraints on the Hubble constant obtained under Λ-CDM, we choose to use the less conservative determination in order to investigate its compatibility in our extended scenario. We refer to this dataset as HST. We use luminosity distances of supernovae type Ia from the "Joint Light-curve Analysis" derived from the SNLS and SDSS catalogs [17]. We refer to this dataset as JLA.
Finally, we include upper limits on CMB polarization from Planck+BAO-RSD [22]. We use the publicly available Monte Carlo Markov Chain package cosmomc [23] with a convergence diagnostic based on the Gelman and Rubin statistic. We use the July 2015 version which includes support for the Planck data release 2015 Likelihood [11] (see http://cosmologist.info/cosmomc/) and implements an efficient sampling using the fast/slow parameter decorrelations [24]. While in this paper we will focus the attention on cosmological parameters, we also vary foreground parameters using the same technique and parametrization described in [11] and [8].

### III. RESULTS

The results of our analysis are reported in Table II where we also include, for comparison, the constraints obtained assuming the standard, 6 parameters, $\Lambda$CDM model. The significant increase in the number of parameters produces, as expected, a relaxation in the constraints on the 6 $\Lambda$CDM parameters. Considering the allowable volume of the six-dimensional $\Lambda$CDM parameter space to be proportional to the square root of the determinant of the $6 \times 6$ parameter covariance, we find that moving from $\Lambda$CDM to $\epsilon$CDM expands this volume by a factor $\sim 63000$ in case of the Planck dataset. The parameters that are mostly affected are the Hubble constant and the r.m.s. amplitude of density fluctuations that are now practically undetermined from Planck measurements alone and have significantly larger errors with respect to $\Lambda$CDM even when external datasets such as BAO are included. The main reason for this relaxation is the inclusion in the analysis of the dark energy equation of state $w$, that introduces a geometrical degeneracy with the matter parameter $\epsilon$.

| Dataset | $\Omega_b h^2$ | $\Omega_c h^2$ | $H_0$ [km/s/Mpc] | $\tau$ | $n_s$ | $\sigma_8$ | $d_{\text{HIT}}$ | $r$ | $w$ | $\text{C}_{\text{m}} [\text{eV}]$ | $N_{\text{eff}}$ | $A_{\text{run}}$ |
|---------|----------------|----------------|----------------|------|------|----------|----------|-----|-----|-----------------|--------|----------|
| Planck TT+LowP | 0.02229±0.00041 | 0.11899±0.0042 | 67.3±1.9 | 0.077±0.038 | 0.969±0.012 | 0.829±0.028 | - | - | - | - | - |
| A CDM | 0.02229±0.00041 | 0.11899±0.0042 | 67.3±1.9 | 0.077±0.038 | 0.969±0.012 | 0.829±0.028 | - | - | - | - | - |
| Planck TT+LowP | 0.02229±0.00041 | 0.11899±0.0042 | 67.3±1.9 | 0.077±0.038 | 0.969±0.012 | 0.829±0.028 | - | - | - | - | - |

TABLE II: Constraints at 95% c.l. on the cosmological parameters assuming the standard 6-parameter $\Lambda$CDM model and the extended, 12-parameter, $\epsilon$CDM model.

We add weak lensing galaxy data from the CFHTLenS [18] survey with the priors and conservative cuts to the data as described in [8]. We refer to this dataset as WL. We consider redshift space distortions from [19] with the prescription give in [8]. We refer to this dataset as RSD.

Finally, we include upper limits on CMB polarization $B$ modes as recently placed by a common analysis of Planck, BICEP2 and Keck Array data [20]. We refer to this dataset as BKP.

We use the publicly available Monte Carlo Markov Chain package cosmomc [23] with a convergence diagnostic based on the Gelman and Rubin statistic. We use the July 2015 version which includes support for the Planck data release 2015 Likelihood [11] (see http://cosmologist.info/cosmomc/) and implements an efficient sampling using the fast/slow parameter decorrelations [24]. While in this paper we will focus the attention on cosmological parameters, we also vary foreground parameters using the same technique and parametrization described in [11] and [8].
density and the Hubble constant. Moreover, marginalizing over the lensing amplitude $A_{\text{lens}}$ removes the lensing information in the CMB spectra that could potentially break this geometrical degeneracy. In this respect, it is interesting to note that a Planck+HST analysis provides a value for the equation of state $w$ less than the cosmological constant value $-1$ at more than 95% c.l.. As mentioned, the HST prior we adopt is not conservative. This result shows that in case the HST measurement will be confirmed by future data, the only parameter between the extra 6 we consider that can be varied to accomodate this tension is the dark energy equation of state $w$. Increasing the neutrino number does not seem to offer a viable solution as was on the contrary the case with the previous Planck 2013 release.

Constraints on the baryon and cold dark matter densities, the scalar spectral index $n_S$ and the optical depth $\tau$ are also much weaker, mainly due to degeneracies between these parameters and $A_{\text{lens}}$ and $N_{\text{eff}}$. Apart from the increase in the errors, it is interesting that parameters as $\sigma_8$ and the optical depth $\tau$ are shifted toward lower values respect to $\Lambda$CDM. These shifts are clear in Figures 1 and 2 where we plot the 68% and 95% c.l. contour plots in the $\sigma_8$ vs $\Omega_m$ and $\tau$ vs $n_S$ planes, respectively. This is mainly due to the anomalous value of $A_{\text{lens}}$ and persists when external datasets as BAO, JLA, WL and RSD are included. Looking at the results in Table II the value of $A_{\text{lens}}$ is always different from the standard value at more than 95% c.l. when the Planck CMB dataset is combined with external datasets, with the only notable exception of the lensing information from the Planck trispectrum that pushes the value of $A_{\text{lens}}$ back to agreement with unity. The nature of the Planck $A_{\text{lens}}$ anomaly could be different from the lensing determination but since it also persists in our extended $\epsilon$CDM scenario, this clearly deserves further investigation. Moreover, $A_{\text{lens}}$ is the only parameter that hints at a tension with standard $\Lambda$CDM. Again, by looking at Table II apart from $A_{\text{lens}}$, we see no evidence for “new physics”: we just have (weaker) upper limits on the neutrino mass, the running of the spectral index is compatible with zero, the dark energy equation of state is compatible with $w = -1$ (expect when we use the HST prior), and the neutrino effective number is remarkably close to the standard value $N_{\text{eff}} = 3.046$. It is impressive that even in a 12 parameter space, the neutrino effective number is still constrained with exquisite precision. This is mainly due to the inclusion of the Planck HFI small angular scale polarization data in the analysis. Removing this dataset, but keeping the low angular scale LFI polarization, we get a much weaker constraint from Planck+BAO of $N_{\text{eff}} = 4.35^{+1.3}_{-1.8}$ at 95% c.l.. The Planck+BAO constraint on neutrino mass of $\Sigma m_\nu < 0.534$ eV at 95% c.l. is significantly weaker with respect to the constraint $\Sigma m_\nu < 0.174$ eV at 95% c.l. obtained with the same dataset but assuming $\Lambda$CDM. The constraint on the tensor/scalar amplitude $r$ is about a factor two larger than in $\Lambda$CDM. However, when the BKP dataset is included, the 95% c.l. upper limit of $r < 0.108$ is recovered. This clearly shows how a measurement of primordial B modes is crucial to constrain the tensor amplitude in a model-independent way. The inclusion of the BKP dataset affects only the constraint on the tensor amplitude and leaves the other constraints virtually unchanged.

IV. CONCLUSIONS

In this Letter we have presented, for the first time, constraints on cosmological parameters in the framework of an “extended” cold dark matter model ($\epsilon$CDM) that is based on 12 parameters instead of the usual 6 assumed in the $\Lambda$CDM model. In this extension some of the parameters usually well constrained under $\Lambda$CDM such as the Hubble constant and the amplitude of matter density fluctuations $\sigma_8$ are now unconstrained by CMB observations. Combining the CMB data with several other datasets reveals no statistically significant evidence for any tensions. More specifically, we have found no evidence for ”new physics” beyond the standard $\Lambda$CDM model, i.e. there is no island of parameters in our extended theoretical framework that could be preferred to the standard $\Lambda$CDM territory. However $\epsilon$CDM prefers a lower value of $\sigma_8$ relative to that obtained for 6-parameter $\Lambda$CDM but still requires a slightly anomalous value of $A_{\text{lens}} > 1$. The lower value of $\sigma_8$ in $\epsilon$CDM brings the Planck data in more agreement with the results of the CFHTLenS survey. This result motivates further studies that could explain the physical nature of the $A_{\text{lens}} > 1$ anomaly.

The tension between the Planck and HST values of the Hubble parameter, in the $\epsilon$CDM scenario is solved by a value of the dark energy equation of state $w < -1$ while the neutrino effective number remains compatible with the standard value of 3.04.

Of course, the number of parameters can be further extended by considering, for example, non-zero curvature, isocurvature primordial perturbations, features in the primordial spectrum, a varying (with redshift) dark energy equation of state, non-standard Big Bang Nucleosynthesis and a change in the primordial helium abundance $Y_p$, and so on. Further extensions however may be premature because of degeneracies. For example, most effects of varying curvature are degenerate with a variation in $w$. CDM isocurvature modes have a spectrum similar to a GW background and the $A_{\text{lens}}$ parameter could account for undetected features in the angular spectrum. Moreover, a change in $N_{\text{eff}}$ could account for a change in $Y_p$.

We find it impressive that despite the increase in the number of the parameters, some of the constraints on key parameters are relaxed but not significantly altered. The cold dark matter ansatz remains robust, the baryon density is compatible with BBN predictions, and the neutrino effective number is compatible with standard expectations. The excellent quality of the new data moti-
vates our exploration beyond the overexploited territory of ΛCDM towards new and uncharted frontiers.

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