Coupled soliton solutions of modeled equations in a Noguchi electrical line with crosslink capacitor

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Abstract
An electrical line is one of the transmission media used worldwide for the transmission of signals. This is for the fact that it is less expensive and easy to manufacture than the other transmission media. In linear domain, the electrical lines transmit sinusoidal signals whose amplitude decreases exponentially and loss a lot of energy. It is therefore necessary to valorize electrical lines by enabling them to transmit solitary waves which are more stable and do not dissipate energy. This will be possible if the media of transmission line are dispersive and nonlinear. We define the analytical expressions that the charges of capacitors must obey so that the Noguchi electrical line with crosslink capacitor accepts to propagate a coupled solitary wave of the need type. The application of those definitions and Kirchhoff laws to the networks of a modified Noguchi electrical line with crosslink capacitor have enable to obtain new set of higher-order nonlinear partial differential equations which govern the dynamics of a coupled solitary wave in the said line. The construction of coupled solitary waves solutions of those set of equations by direct and effective mathematical methods notably that of Bogning-Djeumen Tchaho-Kofane has permitted to discover that, solitary waves of type (Kink; Kink) and Solitary wave solution of type (Pulse; Pulse) are easily propagated in that line when certain conditions we have elaborated are respected. The Noguchi electrical line with crosslink capacitor that we have studied is advantageous for the fact that it permits simultaneously the propagation of a set of two solitary waves contrary to a non-coupled Noguchi electrical line which only enables the propagation of one solitary wave when the signal considered is the voltage; the more we will multiply the crosslink in the line, the more we will multiply the simultaneous propagation of solitary wave in the line.

1. Introduction
An electrical line is a set of conductors comparable to the channel of identical electrical networks that permits the transmission of signals from a generator to a receiver. The signals can be numerical or analogical in the shape of plane wave or solitary wave. During the last decades, Solitary waves have evolved from a simple water wave to the propagation of Pulse solitons in optical fibers [1]. From the definition of solitary wave which is a wave capable to move on a long distance maintaining its shape and velocity; it has come to our mind that if such a signal is used in engineering of information through the Noguchi electrical line with crosslink capacitor, it will resist best on dissipation factors. We have in this regard decided to come up with definitions of nonlinear charges of capacitors constituting the two nonlinear parts linked through the capacitors in the said line, then we have applied them to model new set of two higher-order nonlinear partial differential equations which describe the dynamics of a coupled solitary wave in the given line. The construction of coupled solitary wave solutions of each set of modeled nonlinear partial differential equation by mathematical methods presented in [2–15] and new Bogning-Djeumen Tchaho-Kofane method presented in [16–22] has enabled us to obtain solitary wave solutions of type (Kink; Kink) and Solitary wave solution of type (Pulse; Pulse). The work we are presenting in this paper is partitioned as follows: in part 2, we present the general modeling of a modified Noguchi electrical
line with crosslink capacitor, in part 3, we construct the solitary wave solution of type (Kink; Kink) of the obtained set of differential equations, in part 4, we construct the solitary wave solution of type (Pulse; Pulse) of the obtained set of differential equations, in part 5, we present graphical results for certain values of electrical line parameters. We finally present the conclusion in part 6.

2. General modeling of a modified nonlinear Noguchi electrical line with crosslink capacitor

Electrical lines are generally presented by a periodical channel of LC circuit whose elements can be manufactured by nonlinear dielectric material for the capacitors or by nonlinear magnetic material for inductors. G Tiague Takongmo and J R Bogning have presented in [16] a non-coupled Noguchi electrical line and have established nonlinear laws that the capacitors must obey so that the line accepts to propagate certain types of solitary waves. In this work we consider two different Noguchi electrical line, then one couples them by a capacitor with constant capacitance to obtain figure 1.

Let us consider a nonlinear Noguchi electrical line shown in figure 1 as follows.

The line is constituted by a good number of identical networks numbered by the positive integer \( n \). The network number \( n \) is constituted by a capacitor with capacitance \( C_0 \) which link the two nonlinear capacitive parts; two capacitors in which each of the charge \( q^n_1 \) and \( q^n_2 \) changes respectively in nonlinear manner in terms of the voltage \( u^n_1 \) and \( u^n_2 \) across each capacitor; \( i^n_1 \) is the current flowing through the set of an inductor with inductance \( L_1 \) connected in parallel with a capacitor having a constant capacitance \( C_1 \); \( i^n_2 \) is the current flowing through the set of another inductor with inductance \( L_2 \) connected in parallel with another capacitor having a constant capacitance \( C_2 \). It should be noted that the electrical line presented in figure 1 is that of Noguchi with crosslink capacitor which was modified by the fact that, each inductor with constant inductance that used to be connected in parallel with each nonlinear capacitor was disconnected or has an elevated capacitance and constituting an open branch. Applying Kirchhoff’s laws to the circuit shown in figure 1, we obtain the following equations:

\[
\begin{align*}
    u^{n+1}_1 - u^n_1 &= -L_1 \frac{\partial i^{n+1}_1}{\partial t} + L_1 C_1 \frac{\partial (u^n_1 - u^{n+1}_1)}{\partial t} \\
    u^{n+1}_2 - u^n_2 &= -L_2 \frac{\partial i^{n+1}_2}{\partial t} + L_2 C_2 \frac{\partial (u^n_2 - u^{n+1}_2)}{\partial t}
\end{align*}
\]
\[ i_1^n - i_1^{n+1} = C_0 \frac{\partial(u_1^n - u_2^n)}{\partial t} + \frac{\partial q_1^n}{\partial t} \]  
(3)

\[ i_2^n - i_2^{n+1} = -C_0 \frac{\partial(u_1^n - u_2^n)}{\partial t} + \frac{\partial q_2^n}{\partial t} \]  
(4)

Relations (1), (2), (3) and (4) permit us to obtain the following system:

\[
\begin{cases}
L_1 C_1 \frac{\partial(u_1^{n+1} - 2u_1^n + u_2^n)}{\partial t} + u_1^{n+1} - 2u_1^n + u_1^{n-1} = L_1 C_0 \frac{\partial^2(u_1^n - u_2^n)}{\partial t^2} + L_1 \frac{\partial^2 q_1^n}{\partial t^2} \\
L_2 C_2 \frac{\partial(u_2^{n+1} - 2u_2^n + u_1^n)}{\partial t} + u_2^{n+1} - 2u_2^n + u_2^{n-1} = -L_2 C_0 \frac{\partial^2(u_1^n - u_2^n)}{\partial t^2} + L_2 \frac{\partial^2 q_2^n}{\partial t^2}
\end{cases}
\]  
(5)

To obtain the continuum model, the left hand side of each equation in (5) has to be approximated to a spatial partial derivative with respect to \( x = nh \) which represents the distance measured from the beginning of the line. \( h \) represents the distance that separates two consecutive nodes and which is equivalent to the spatial sampling derivatives period. We obtain as such spatial partial derivatives using Taylor expansion of \( u_1^n \) and \( u_1^{n-1} \) closely to \( u_1^n \) by considering the terms till fourth order in the following manner:

\[ u_1^{n+1} = u_1^n + \frac{h}{1!} \frac{\partial u_1^n}{\partial x} + \frac{h^2}{2!} \frac{\partial^2 u_1^n}{\partial x^2} + \frac{h^3}{3!} \frac{\partial^3 u_1^n}{\partial x^3} + \frac{h^4}{4!} \frac{\partial^4 u_1^n}{\partial x^4} \]
(6)

\[ u_1^{n-1} = u_1^n - \frac{h}{1!} \frac{\partial u_1^n}{\partial x} + \frac{h^2}{2!} \frac{\partial^2 u_1^n}{\partial x^2} - \frac{h^3}{3!} \frac{\partial^3 u_1^n}{\partial x^3} + \frac{h^4}{4!} \frac{\partial^4 u_1^n}{\partial x^4} \]
(7)

\[ u_1^{n+1} - 2u_1^n + u_1^{n-1} = h^2 \frac{\partial^2 u_1^n}{\partial x^2} + \frac{h^4}{12} \frac{\partial^4 u_1^n}{\partial x^4} \]
(8)

In the same light using Taylor expansion of \( u_2^{n+1} \) and \( u_2^{n-1} \) closely to \( u_2^n \) by considering the terms till fourth order we obtain the equation below:

\[ u_2^{n+1} - 2u_2^n + u_2^{n-1} = h^2 \frac{\partial^2 u_2^n}{\partial x^2} + \frac{h^4}{12} \frac{\partial^4 u_2^n}{\partial x^4} \]
(9)

Equations (8) and (9) permits us to rewrite the set of two differential equation (5) as follows:

\[
\begin{cases}
\frac{L_1 C_1 h^4}{12} \frac{\partial (\partial^4 u_1^n)}{\partial x^4} + \frac{L_1 C_1 h^4}{12} \frac{\partial (\partial^2 u_1^n)}{\partial x^2} + \frac{h^4}{12} \frac{\partial^4 u_1^n}{\partial x^4} + \frac{h^4}{12} \frac{\partial^2 u_1^n}{\partial x^2} = L_1 C_0 \frac{\partial^2(u_1^n - u_2^n)}{\partial t^2} + L_1 \frac{\partial^2 q_1^n}{\partial t^2} \\
\frac{L_2 C_2 h^4}{12} \frac{\partial (\partial^4 u_2^n)}{\partial x^4} + \frac{L_2 C_2 h^4}{12} \frac{\partial (\partial^2 u_2^n)}{\partial x^2} + \frac{h^4}{12} \frac{\partial^4 u_2^n}{\partial x^4} + \frac{h^4}{12} \frac{\partial^2 u_2^n}{\partial x^2} = -L_2 C_0 \frac{\partial^2(u_1^n - u_2^n)}{\partial t^2} + L_2 \frac{\partial^2 q_2^n}{\partial t^2}
\end{cases}
\]  
(10)

Finally, we obtain the continuum model of the nonlinear Noguchi electrical line with crosslink capacitor presented in figure 1 by the set of two nonlinear partial differential equation below:

\[
\begin{cases}
\frac{L_1 C_1 h^4}{12} \frac{\partial (\partial^4 u_1^n)}{\partial x^4} + \frac{L_1 C_1 h^4}{12} \frac{\partial (\partial^2 u_1^n)}{\partial x^2} + \frac{h^4}{12} \frac{\partial^4 u_1^n}{\partial x^4} + \frac{h^4}{12} \frac{\partial^2 u_1^n}{\partial x^2} + \frac{h^2}{12} \frac{\partial^2 u_1^n}{\partial x^2} = L_1 C_0 \frac{\partial^2(u_1^n - u_2^n)}{\partial t^2} + L_1 \frac{\partial^2 q_1^n}{\partial t^2} \\
\frac{L_2 C_2 h^4}{12} \frac{\partial (\partial^4 u_2^n)}{\partial x^4} + \frac{L_2 C_2 h^4}{12} \frac{\partial (\partial^2 u_2^n)}{\partial x^2} + \frac{h^4}{12} \frac{\partial^4 u_2^n}{\partial x^4} + \frac{h^4}{12} \frac{\partial^2 u_2^n}{\partial x^2} + \frac{h^2}{12} \frac{\partial^2 u_2^n}{\partial x^2} = -L_2 C_0 \frac{\partial^2(u_1^n - u_2^n)}{\partial t^2} + L_2 \frac{\partial^2 q_2^n}{\partial t^2}
\end{cases}
\]  
(11)

3. Construction of solitary wave solution in the shape (Kink; Kink) of the set of two partial differential equation (11)

We define each nonlinear charge of the two capacitors constituting each of the two parts cross-linked on the analytical shape as follows:

\[ i_1^n - i_1^{n+1} = C_0 \frac{\partial(u_1^n - u_2^n)}{\partial t} + \frac{\partial q_1^n}{\partial t} \]  
(3)

\[ i_2^n - i_2^{n+1} = -C_0 \frac{\partial(u_1^n - u_2^n)}{\partial t} + \frac{\partial q_2^n}{\partial t} \]  
(4)
\begin{align*}
\left\{ \begin{array}{l}
qu_t(u_t(x,t)) = A_1u_t(x,t) + A_2u_t^2(x,t) + A_3u_t^3(x,t) + A_4u_t^4(x,t) + A_5\ln(u_t(x,t) - A_0) + A_6\ln(u_t(x,t) + A_0) \\
q_{uu}(u_t(x,t)) = B_1u_t(x,t) + B_2u_t^2(x,t) + B_3u_t^3(x,t) + B_4u_t^4(x,t) + B_5\ln(u_t(x,t) - B_0) + B_6\ln(u_t(x,t) + B_0)
\end{array} \right.
\end{align*}

(12)

With \(|u_t(x,t)| > |A_0|, |u_t(x,t)| > |B_0|, A_1, A_2, A_3, A_4, A_5, A_6, B_1, B_2, B_3, B_4, B_5, B_6\) and \(A_0, B_0\) are no zero real numbers which will be chosen conveniently. Let us note that as physical interpretation; \(A_0, B_0\) stand for voltage; \(A_1, B_1\) stand for capacitance; \(A_2, B_2\) stand for capacitance per unit voltage; \(A_3, B_3\) stand for capacitance per unit voltage of power two; \(A_4, B_4\) stand for capacitance per unit voltage of power three; \(A_5 = -A_6, B_5 = -B_6\) stand for charges.

By substituting each of the nonlinear charge \(q_t(u_t(x,t))\) and \(q_{uu}(u_t(x,t))\) of (12) in (11) we obtain the set of two nonlinear partial differential equations written as:

\begin{align*}
\left\{ \begin{array}{l}
h^4\frac{\partial^4u_t(x,t)}{\partial x^4} + h^2\frac{\partial^2u_t(x,t)}{\partial x^2} + \frac{L_1C_1h^4}{12}\frac{\partial^4u_t(x,t)}{\partial x^4\partial t} + \frac{L_1C_1h^2}{12}\frac{\partial^2u_t(x,t)}{\partial x^2\partial t} + \frac{L_1C_0h}{12}\frac{\partial^2u_t(x,t)}{\partial t^2} \\
+ \left( -L_1C_0 - L_1A_1 - \frac{L_1A_0}{u_t(x,t) + A_0} - 2L_1A_2u_t(x,t) - \frac{L_1A_3}{u_t(x,t) - A_0} - 3L_1A_3u_t^2(x,t) - 4L_1A_4u_t^3(x,t) \right)\frac{\partial^2u_t(x,t)}{\partial t^2} \\
- L_2\left( 2A_1 + 6A_2u_t(x,t) - A_5 \frac{u_t(x,t) - A_0}{(u_t(x,t) + A_0)^2} + 12A_1u_t^2(x,t) - A_6 \frac{u_t(x,t) + A_0}{(u_t(x,t) - A_0)^2} \right) \frac{\partial^2u_t(x,t)}{\partial t^2} = 0
\end{array} \right.
\end{align*}

(13)

By applying Bogning-Djeumen Tchaho-Kofane method \([16–22]\) as shown in appendix A, one come up with the solution of (13) as follow:

\begin{align*}
a &= A_0; \quad b = B_0; \quad k = \pm \left( \frac{2A_2^2A_4^2}{3h^4A_3L_1C_1^2} \right)^{\frac{1}{2}}; \quad \nu = \pm \frac{A_4A_0}{3A_3L_1C_1}; \quad A_3 > 0; \quad B_3 = \frac{h^4k^4}{6B_0^2L_2v^2}; \\
B_4 &= \frac{C_2h^4k^4}{2B_0^2v^2}; \\
A_1 &= \frac{C_0B_0}{A_0} - C_0 - \frac{h^4k^4}{6L_1v^2} + \frac{h^2k^2}{L_1v^2}; \quad A_2 = \frac{C_4h^2k^2}{A_0\nu} - \frac{2C_4h^4k^4}{A_0\nu^2}; \quad A_5 = \frac{C_0B_0}{2} - \frac{A_0C_0}{2}; \\
A_6 &= \frac{C_0A_0}{2} - \frac{B_0C_0}{2} - \frac{A_0h^2k^2}{2L_1v^2} + \frac{A_0h^4k^4}{12L_1v^2}; \quad B_1 = \frac{C_0A_0}{B_0} - C_0 - \frac{h^4k^4}{6L_2v^2} + \frac{h^2k^2}{L_2v^2}; \\
B_2 &= \frac{C_2h^2k^2}{B_0v} - \frac{2C_2h^4k^4}{3B_0v^2}; \\
B_3 &= \frac{C_0A_0}{2} - \frac{B_0C_0}{2} - \frac{B_0h^2k^2}{12L_2v^2} + \frac{B_0h^4k^4}{2L_2v^2}; \quad B_6 = \frac{C_0B_0}{2} - \frac{A_0C_0}{2} - \frac{B_0h^2k^2}{12L_2v^2} + \frac{B_0h^4k^4}{12L_2v^2}; \\
\left\{ \begin{array}{l}
u(x,t) = A_0\tanh \left( \frac{2A_2^2A_4^2}{3h^4A_3L_1C_1^2} \right)^{\frac{1}{2}} \pm \frac{A_4A_0}{3A_3L_1C_1} \left( x - \frac{A_4A_0}{3A_3L_1C_1} \right) \right. \\
u_{uu}(x,t) = B_0\tanh \left( \frac{2A_2^2A_4^2}{3h^4A_3L_1C_1^2} \right)^{\frac{1}{2}} \pm \frac{A_4A_0}{3A_3L_1C_1} \left( x - \frac{A_4A_0}{3A_3L_1C_1} \right)
\end{array} \right.
\end{align*}

(14)

Expressions in (14) present coupled soliton solution with conditions relative to the set of two non-linear partial differential equation (13). The obtained coupled soliton solution is solitary wave of type (Kink, Kink).
4. Construction of solitary wave solution in the shape (pulse; pulse) of the set of two partial differential equation (11)

We define each nonlinear charge of the two capacitors constituting each of the two parts cross-linked on the analytical shape as follows:

\[
\begin{align*}
q_1(u_1(x, t)) &= A_1u_1(x, t) + A_2u_1^3(x, t) + (A_3u_1(x, t) + A_4u_1^3(x, t))\sqrt{1 - \frac{u_1^2(x, t)}{A_0^2}}, \\
q_2(u_2(x, t)) &= B_1u_2(x, t) + B_2u_2^3(x, t) + (B_3u_2(x, t) + B_4u_2^3(x, t))\sqrt{1 - \frac{u_2^2(x, t)}{B_0^2}}.
\end{align*}
\]  

(15)

With \(|A_0| > |u_1(x, t)|; |B_0| > |u_2(x, t)|\). \(A_1; A_2; A_3; A_4; B_1; B_2; B_3\) and \(B_4\) are non-zero real numbers which will be chosen conveniently. Let us note that as physical interpretation; \(A_0, B_0\) stand for voltage; \(A_1, B_1, A_3, B_3\) stand for capacitance; \(A_2, B_2, A_4, B_4\) stand for capacitance per unit voltage of power two.

By substituting each of the nonlinear charge \(q_1(u_1(x, t))\) and \(q_2(u_2(x, t))\) of (15) in (11) we obtain the set of two nonlinear partial differential equation written as:

\[
\begin{align*}
\frac{h^4}{12} \frac{\partial^4u_1(x, t)}{\partial x^4} + & \frac{h^2}{12} \frac{\partial^2u_1(x, t)}{\partial x^2} + \frac{L_1C_0h^4}{12} \frac{\partial^4u_1(x, t)}{\partial x^4\partial t^2} + \frac{L_1C_0h^2}{12} \frac{\partial^2u_1(x, t)}{\partial x^2\partial t^2} + \frac{L_1C_0}{12} \frac{\partial^2u_1(x, t)}{\partial t^2} \\
&+ \frac{L_1(A_3u_1^2(x, t) + A_4u_1^4(x, t))}{A_0^2} \sqrt{1 - \frac{u_1^2(x, t)}{A_0^2}} \frac{\partial^2u_1(x, t)}{\partial t^2} \\
&- L_1 \left( \frac{6A_2u_1(x, t) - 2(A_3u_1(x, t) + 3A_4u_1^3(x, t))}{A_0^2} \sqrt{1 - \frac{u_1^2(x, t)}{A_0^2}} + A_3u_1^3(x, t) + A_4u_1^5(x, t) \right) \\
&- \frac{A_0}{12} \frac{\partial^2u_1(x, t)}{\partial t^2} \\
&- L_1 \left( \frac{2A_2 + 6A_3u_1(x, t)}{u_1(x, t) - A_0^2} + 12A_4u_1^3(x, t) - \frac{A_3}{u_1(x, t) + A_0^2} \right) \frac{\partial^2u_1(x, t)}{\partial t^2} = 0
\end{align*}
\]

(16)
Solving the set of two non-linear partial differential equation (16) as shown in appendix B, one come up with the coupled solution below:

\[
\begin{align*}
    a &= A_0; \quad b = B_0; \quad k = \pm \left( -\frac{2A_0^2A_1^2}{3h^4A_1^3C_1^2} \right)^{\frac{1}{4}}; \quad \nu = \frac{A_4}{3C_1A_2L_1}; \quad A_2 < 0; \quad B_2 = -\frac{h^4k^4}{6B_0^2L_2v^2}; \\
    B_4 &= \frac{C_2h^4k^4}{2B_0^2v}; \quad A_1 = \frac{h^4k^4}{L_1v^2} + \frac{C_0B_0}{A_0} + \frac{h^4k^4}{12L_1v^2} - C_0; \quad B_1 = \frac{h^4k^4}{L_2v^2} + \frac{C_0A_0}{B_0} + \frac{h^4k^4}{12L_2v^2} - C_0; \\
    A_3 &= \frac{C_1h^4k^4}{12v} + \frac{C_0h^2k^2}{\nu}; \quad B_3 = \frac{C_2h^4k^4}{12v} + \frac{C_1h^2k^2}{\nu}; \\
    u_1(x, t) &= A_0\text{sech}\left( \pm \frac{2A_0^2A_1^2}{3h^4A_1^3C_1^2} \right)^{\frac{1}{4}} x - \frac{A_4}{3C_1A_2L_1} t \\
    u_2(x, t) &= B_0\text{sech}\left( \pm \frac{2A_0^2A_1^2}{3h^4A_1^3C_1^2} \right)^{\frac{1}{4}} x - \frac{A_4}{3C_1A_2L_1} t \end{align*}
\]

(17)

Expressions in (17) present coupled soliton solution with conditions relative to the set of two non-linear partial differential equation (16). The obtained coupled soliton solution is solitary wave of type (Pulse, Pulse).

5. Real representations of obtained soliton solutions

Considering the values of the following parameters

\[
\begin{align*}
    A_0 &= 10 \text{ V}, \quad B_0 = -10 \text{ V}, \quad h = 10^{-4} \text{ m}, \\
    C_0 &= 37 \times 10^{-10} \text{ F}, \quad C_2 = 37 \times 10^{-13} \text{ F}, \quad C_4 = 370 \times 10^{-12} \text{ F}, \quad L_1 = 470 \times 10^{-5} \text{ H}, \quad L_2 = 470 \times 10^{-7} \text{ H}, \\
    A_3 &= 7, 28 \times 10^{-11} \text{ F/V}^2, \quad A_4 = 7, 28 \times 10^{-15} \text{ F/V}^3, \\
\end{align*}
\]

the expression of coupled Kink soliton (14) takes the shape

\[
\begin{align*}
    u_1(x, t) &= -10 \tanh(1.65 \times 10^5x + 1, 91 \times 10^4t) \\
    u_2(x, t) &= 10 \tanh(-1.65 \times 10^5x + 1, 91 \times 10^4t) \\
\end{align*}
\]

This permits to obtain in figure 2 the representation of real profile of that coupled Kink soliton.

The profiles shown in figure 2 are topological solitons since the properties of their media are not the same at infinity.

Considering the values of the following parameters

\[
\begin{align*}
    A_0 &= 10 \text{ V}, \quad B_0 = -10 \text{ V}, \quad h = 10^{-4} \text{ m}, \\
    C_0 &= 37 \times 10^{-10} \text{ F}, \quad C_2 = 37 \times 10^{-13} \text{ F}, \quad C_4 = 370 \times 10^{-12} \text{ F}, \quad L_1 = 470 \times 10^{-5} \text{ H}, \quad L_2 = 470 \times 10^{-7} \text{ H}, \\
    A_3 &= -7, 28 \times 10^{-11} \text{ F/V}^2, \quad A_4 = 7, 28 \times 10^{-14} \text{ F/V}^3, \\
\end{align*}
\]

the expression of coupled Pulse soliton (17) takes the shape

\[
\begin{align*}
    u_1(x, t) &= 10 \text{sech}(93193, 3x + 1, 91 \times 10^4t) \\
    u_2(x, t) &= -10 \text{sech}(93193, 3x + 1, 91 \times 10^4t) \\
\end{align*}
\]

This permits to obtain in figure 3 the representation of real profile of that coupled Pulse soliton.
The profiles shown in figure 3 are non-topological solitons since the properties of their media are the same at infinity.

6. Conclusion

At the end of this work where we have modeled a modified Noguchi electrical line with crosslink capacitor by two different set of higher-order nonlinear partial differential equations which have permitted us to construct the coupled solitary wave solutions; it is necessary to point out that the results obtained will first of all enable us in physics and engineering of telecommunication, the manufacturing of two new transmission lines notably those of Noguchi electrical line presented in figure 1 whose charge of nonlinear capacitors of its networks varies for one in nonlinear manner defined in (12) and the other in nonlinear manner defined in (15). In addition, these results will permit an amelioration of the quality of signals that will be propagated in the new lines. In fact, those signals are solitary waves of type (Kink; Kink) obtained in (14) and type (Pulse; Pulse) obtained in (17) which by their definition are propagated on longer distances without changing their shape and velocity by resisting best on different dissipation factors. Finally, in a typical mathematical domain, the results obtained has permitted to define in (13) and in (16) two new set of higher-order nonlinear partial differential equations which have respectively exact coupled solitary wave solutions (14) and (17); this by increasing the field of mathematical knowledge. It is necessary to recall that the Noguchi electrical line with crosslink capacitor that we have studied is advantageous for the fact that it permits simultaneously the propagation of a set of two solitary waves contrary to a non-coupled Noguchi electrical line which only enables the propagation of one solitary wave when the signal considered is the voltage; the more we will multiply the crosslink in the line, the more we will multiply the simultaneous propagation of solitary wave in the line. In order to bring new ideas on the stability of the obtained solitary waves; it is necessary for us to study next their modulational instability before carrying out a practical study where we will experiment the applicability and the perfection of those two new lines.

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Appendix A

Let us use Bognig-Djeumen Tchaho-Kofane method [16–22] to come out with the solution of (13) under the analytical shape below:
\[ \begin{align*}
    u_t(x, t) &= a \tanh(kx - vt) \\
    u_x(x, t) &= b \tanh(kx - vt)
\end{align*} \]

(A1.1)

Where \( a, b, k \) and \( v \) are non-zero real numbers to be determined in terms of modeled line parameters. Replacing \( u_t(x, t) \) and \( u_x(x, t) \) given by (A1.1) in (13) we yield the following set of two equations which are written in a simplified form when \( a = A_0 \) and \( b = B_0 \):

\[
\begin{align*}
    -60L_1v^2A_4A_0^4 + 30L_1C_1h^4A_0v^4 &+ (36L_1v^2A_4A_0^2 - 6h^4A_0k^4) \sinh(kx - vt) \cosh(kx - vt) \\
    + (-3L_1v^2A_6 - 12L_1v^2A_2A_0^2 + 12L_1C_1h^2A_0v^2 - 24L_1v^2A_4A_0^2 - 3L_1v^2A_3 + 4L_1C_1h^4A_0v^4) \cosh(kx - vt) &+ (18L_1v^2A_4A_0^2 + 2h^4A_0k^4 + 6L_1C_1h^2A_0v^2 - 6L_1v^2A_3A_0 - 6L_1v^2A_4A_0^2) \sinh(kx - vt) \cosh(kx - vt) \\
    + (84L_1v^2A_4A_0^2 + 18L_1v^2A_2A_0^2 - 30L_1C_1h^2A_0v^2 - 18L_1C_1h^4A_0v^4) \cosh^2(kx - vt) &= 0
\end{align*}
\]

Equation (A1.2) is valid if and only if each of its basic hyperbolic function coefficients is zero. This permits us to obtain the following set of ten equations:

\[
\begin{cases}
    -60L_1v^2A_4A_0^4 + 30L_1C_1h^4A_0v^4 = 0 \\
    36L_1v^2A_3A_0^2 - 6h^4A_0k^4 = 0 \\
    -3L_1v^2A_6 - 12L_1v^2A_2A_0^2 + 12L_1C_1h^2A_0v^2 - 24L_1v^2A_4A_0^2 - 3L_1v^2A_3 + 4L_1C_1h^4A_0v^4 = 0 \\
    -18L_1v^2A_4A_0^2 + 2h^4A_0k^4 + 6L_1C_1h^2A_0v^2 - 6L_1v^2A_3A_0 - 6L_1v^2A_4A_0^2 = 0 \\
    84L_1v^2A_4A_0^2 + 18L_1v^2A_2A_0^2 - 30L_1C_1h^4A_0v^4 - 18L_1C_1h^2A_0v^2 = 0 \\
    -60L_1v^2B_4B_0^4 + 30L_1C_2h^4B_0v^4 = 0 \\
    36L_1v^2B_3B_0^2 - 6h^4B_0k^4 = 0 \\
    -3L_1v^2B_6 - 12L_1v^2B_2B_0^2 + 12L_2C_2h^2B_0v^2 - 24L_1v^2B_4B_0^2 - 3L_1v^2B_3 + 4L_2C_2h^4B_0v^4 = 0 \\
    -18L_1v^2B_4B_0^2 + 2h^4B_0k^4 + 6L_2C_2h^2B_0v^2 - 6L_2v^2C_0B_0 - 6L_2v^2B_2B_0 = 0 \\
    84L_1v^2B_4B_0^2 + 18L_1v^2B_2B_0^2 - 30L_2C_2h^4B_0v^4 - 18L_2C_2h^2B_0v^2 = 0
\end{cases}
\]

(A1.3)

Haven solved the set of equation (A1.3), it has permitted to present in (A1.4) the coupled solution with conditions of the set of two nonlinear partial differential equations obtained in (13) which model the dynamics of solitary wave of type (Kink; Kink):

\[
\begin{align*}
    a &= A_0; \quad b = B_0; \quad k &= \pm \left( \frac{2A_0^2A_4^2}{3h^3A_3L_1C_1^2} \right)^{\frac{1}{2}}; \quad v = \pm \frac{A_4A_0}{3A_3L_1C_1}; \quad A_3 > 0 \quad B_3 = \frac{h^4k^4}{6B_0L_2v^2}; \\
    B_4 &= \frac{C_2h^4k^4}{2B_0^2v^2}; \\
    A_1 &= \frac{C_0B_0}{A_0} - C_0 - \frac{h^4k^4}{6L_1v^2} + \frac{h^2k^2}{L_1v^2}; \quad A_2 = \frac{C_1h^2k^2}{A_0v} - \frac{2C_2h^4k^4}{3A_0v}; \\
    A_5 &= \frac{C_0B_0}{2} - \frac{A_0C_0}{2} - \frac{A_0h^4k^4}{12L_1v^2} + \frac{A_0h^2k^2}{2L_1v^2}; \\
    A_6 &= \frac{C_0A_0}{2} - \frac{B_0C_0}{2} - \frac{A_0h^4k^4}{2L_1v^2} + \frac{A_0h^2k^2}{12L_1v^2}; \quad B_6 = \frac{C_0A_0}{B_0} - C_0 - \frac{h^4k^4}{6L_2v^2} + \frac{h^2k^2}{L_2v^2};
\end{align*}
\]
\[
B_2 = \frac{C_2 h^2 k^2}{B_0 v} - \frac{2C_2 h^4 k^4}{3B_0 v},
\]
\[
B_5 = \frac{C_0 A_0}{2} - \frac{B_0 C_0}{2} - \frac{B_0 h^4 k^4}{12L_2 v^2} + \frac{B_0 h^2 k^2}{2L_2 v^2}; \quad B_6 = \frac{C_0 B_0}{2} - \frac{A_0 C_0}{2} - \frac{B_0 h^2 k^2}{2L_2 v^2} + \frac{B_0 h^4 k^4}{12L_2 v^2};
\]
\[
\begin{align*}
  u_1(x, t) &= A_0 \tanh \left( \pm \frac{2A_0^3 A_1}{3h^4 A_3 L_1 C_1^2} \frac{1}{x} \pm \frac{A_4 A_0}{3A_3 L_1 C_1} t \right), \\
  u_2(x, t) &= B_0 \tanh \left( \pm \frac{2A_0^3 A_1^2}{3h^4 A_3 L_1 C_1^2} \frac{1}{x} \pm \frac{A_4 A_0}{3A_3 L_1 C_1} t \right),
\end{align*}
\]

(1.4)

Appendix B

By applying the same method, one come up with the solution of (16) under the analytical shape below:

\[
\begin{align*}
  u_1(x, t) &= \text{asech}(kx - vt) \\
  u_2(x, t) &= \text{bsech}(kx - vt)
\end{align*}
\]

(2.1)

Where \(a; b; k\) and \(v\) are non-zero real numbers to be determined in terms of modeled line parameters. Replacing \(u_1(x, t)\) and \(u_2(x, t)\) given by (2.1) in (16) we yield the following set of two equations which are written in a simplified form when \(a = A_0\) and \(b = B_0\):

\[
\begin{align*}
  & 240L_1 A_0^3 v^2 A_4 + 120L_1 C_1 h^4 A_0 k^4 v + (-144L_1 A_0^3 v^2 A_2 - 24h^4 A_0 k^4) \sinh(kx - vt) \cosh(kx - vt) \\
  & + (12L_1 C_0 B_0 v^2 + h^4 A_0 k^4 - 12L_1 A_0 v^2 C_0 - 12L_1 A_0 v^2 A_1 + 12h^2 A_0 k^2) \sinh(kx - vt) \cosh^3(kx - vt) \\
  & + (-36L_1 C_0 B_0 v^2 - 21h^4 A_0 k^4 - 36h^2 A_0 k^2 \\
  & + 36L_1 A_0 v^2 A_1 + 36L_1 A_0 v^2 C_0 - 108L_1 A_0^3 v^2 A_2) \sinh^2(kx - vt) \cosh^2(kx - vt) \\
  & + (-588L_1 A_0^3 v^2 A_4 - 72L_1 C_1 h^4 A_0 k^4 v - 300L_1 C_1 h^4 A_0 k^4 v + 72L_1 A_0 v^2 A_3) \sinh(kx - vt) \\
  & + (241L_1 C_1 h^4 A_0 k^4 v - 156L_1 A_0 v^2 A_3 + 456L_1 A_0^3 v^2 A_4 + 156L_1 C_1 h^4 A_0 k^4 v) \cosh^4(kx - vt) \\
  & + (96L_1 A_0 v^3 A_3 - 62L_1 C_1 h^4 A_0 k^4 v - 108L_1 A_0^3 v^2 A_4 - 96L_1 C_1 h^4 A_0 k^4 v) \cosh(kx - vt) \\
  & + \left(44h^4 A_0 k^4 + 24L_1 C_0 B_0 v^2 + 252L_1 A_0^3 v^2 A_2 \\
  & - 24L_1 A_0 v^2 C_0 - 24L_1 A_0 v^2 A_1 + 24h^2 A_0 k^2 \right) \sinh(kx - vt) \cosh^3(kx - vt) \\
  & + (-12L_1 A_0 v^2 A_3 + 12L_1 C_1 h^4 A_0 k^4 v + 12L_1 C_1 h^4 A_0 k^4 v) \cosh^4(kx - vt) = 0
\end{align*}
\]

(2.2)

\[
\begin{align*}
  & 240L_2 B_0^3 v^2 B_4 + 120L_2 C_1 h^4 B_0 k^4 v + (-144L_2 B_0^3 v^2 B_2 - 24h^4 B_0 k^4) \sinh(kx - vt) \cosh(kx - vt) \\
  & + (12L_2 C_0 A_0 v^2 + h^4 B_0 k^4 - 12L_2 B_0 v^2 C_0 - 12L_2 B_0 v^2 B_1 + 12h^2 B_0 k^2) \sinh(kx - vt) \cosh^3(kx - vt) \\
  & + (-36L_2 C_0 A_0 v^2 - 21h^4 B_0 k^4 - 36h^2 B_0 k^2 \\
  & + 36L_2 B_0 v^2 A_1 + 36L_2 B_0 v^2 C_0 - 108L_2 B_0^3 v^2 B_2) \sinh(kx - vt) \cosh^2(kx - vt) \\
  & + (-588L_2 B_0^3 v^2 B_4 - 72L_2 C_1 h^4 B_0 k^4 v - 300L_2 C_1 h^4 B_0 k^4 v + 72L_2 B_0 v^2 B_3) \sinh(kx - vt) \\
  & + (241L_2 C_1 h^4 B_0 k^4 v - 156L_2 B_0 v^2 B_3 + 456L_2 B_0^3 v^2 B_4 + 156L_2 C_1 h^4 B_0 k^4 v) \cosh^4(kx - vt) \\
  & + (96L_2 B_0 v^3 B_3 - 62L_2 C_1 h^4 B_0 k^4 v - 108L_2 B_0^3 v^2 B_4 - 96L_2 C_1 h^4 B_0 k^4 v) \cosh(kx - vt) \\
  & + \left(44h^4 B_0 k^4 + 24L_2 C_0 A_0 v^2 + 252L_2 B_0^3 v^2 B_2 \\
  & - 24L_2 B_0 v^2 C_0 - 24L_2 B_0 v^2 B_1 + 24h^2 B_0 k^2 \right) \sinh(kx - vt) \cosh^3(kx - vt) \\
  & + (-12L_2 B_0 v^2 B_3 + 12L_2 C_1 h^4 B_0 k^4 v + 12L_2 C_1 h^4 B_0 k^4 v) \cosh^4(kx - vt) = 0
\end{align*}
\]

Equation (2.2) is valid if and only if each of its basic hyperbolic function coefficients is zero. This permits to obtain the following set of eighteen equations:
Haven solved the set of equation (A2.3), it has permitted to present in (A2.4) the coupled solution with conditions of the set of two nonlinear partial differential equations obtained in (16) which model the dynamic of solitary wave of type (Pulse; Pulse):

\[
\begin{align*}
& a = A_0; \quad b = B_0; \quad k = \pm \left( -\frac{2A_0^2A_1^2}{3h^4A_2L_1} \right)^{1/4}; \quad \nu = \frac{A_4}{3C_1A_2L_1}; \quad A_2 < 0; \quad B_2 = -\frac{h^4k^4}{6B_0^2L_2^2}; \\
& B_4 = -\frac{C_4h^4k^4}{2B_0^2} \quad A_1 = \frac{h^4k^4}{L_1^2\nu^2} + \frac{C_0A_0}{A_0} + \frac{h^4k^4}{12L_2^2} - C_6; \quad B_1 = \frac{h^4k^4}{L_1^2\nu^2} + \frac{C_0A_0}{B_0} + \frac{h^4k^4}{12L_2^2} - C_6; \\
& B_3 = \frac{C_2h^4k^4}{12\nu} + \frac{C_2h^4k^2}{\nu}; \quad B_5 = \frac{C_2h^4k^4}{12\nu} + \frac{C_2h^4k^2}{\nu}; \\
& \begin{cases} u_0(x, t) = A_0 \operatorname{sech} \left( \pm \frac{2A_0^2A_1^2}{3h^4A_2L_1} \right)^{1/4} \left( x - \frac{A_4}{3C_1A_2L_1} t \right) \\
 u_1(x, t) = B_0 \operatorname{sech} \left( \pm \frac{2A_0^2A_1^2}{3h^4A_2L_1} \right)^{1/4} \left( x - \frac{A_4}{3C_1A_2L_1} t \right) \end{cases}
\end{align*}
\]

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