Isospin Analysis of pentaquark production

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Abstract

Significant signal-background interference effects can occur in experiments like $\gamma N \rightarrow \bar{K}\Theta^{+}$ which search for the $\Theta^{+}$ as a narrow $I = 0$ resonance in a definite final state against a nonresonant background, with an experimental resolution coarser than the expected resonance width. We show that when the signal and background have roughly the same magnitude, destructive interference can easily combine with a limited experimental resolution to completely destroy the resonance signal. Whether or not this actually occurs depends critically on the yet unknown relative phase of the $I = 0$ and $I = 1$ amplitudes. We discuss the implications for some specific experiments.

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1 Introduction - Some Isospin Considerations

The recent experimental discovery [1] of an exotic 5-quark $KN$ resonance $\Theta^+$ with positive strangeness, a mass of $\sim 1540$ MeV, a very small width $\lesssim 20$ MeV and a presumed quark configuration $uudd\bar{s}$ has given rise to a number of further experiments [2] and a new interest in theoretical models [3] for exotic hadrons including models with diquark structures [4]. But the controversy between experimental evidence for and against the existence [5, 6] of the $\Theta^+$ remains unresolved. There are also questions about isospin asymmetry [7].

We point out here one crucial factor which can explain why some experiments see the $\Theta^+$ and others do not. All experiments search for a narrow resonance against a nonresonant background, generally with an experimental resolution much coarser than the assumed $\Theta^+$ width. Some experiments lead to a definite final state; e.g.

$$\gamma p \rightarrow \bar{K}^0\Theta^+; \quad \gamma n \rightarrow K^-\Theta^+$$

The probability of observing the signal in such an experiment is very sensitive to the relative phase between the signal and the background. This is shown explicitly below in a toy model. Such interference effects are not expected in inclusive $\Theta^+$ production; e.g.

$$e^+e^- \rightarrow \Theta^+X$$

Here the incoherent sum over all inclusive final states $X$ destroys all phase information.

The $\Theta^+$ is believed to decay into a kaon and nucleon with isospin zero. But the observed decay modes $K^+n$ and $K^0p$ are equal mixtures of states with $I = 0$ and $I = 1$ with opposite relative phase. If the nonresonant background in a given experiment is mainly from an $I = 1$ amplitude, the relative phase between signal and background amplitudes will depend upon the particular decay mode observed. The phase when the $\Theta^+$ is detected in the $K^+n$ decay mode will be opposite to the phase in the same experiment where the $\Theta^+$ is detected in the $K^0p$ decay mode.

A serious isospin analysis may be necessary to understand the implications of any experiment where destructive interference between an $I = 0$ signal and $I = 1$ background can effectively destroy the signal. Simple isospin relations between similar reactions which consider only the signal and not the interference with the background can give very erroneous results. In particular one can expect apparent contradictions between negative and positive results which are connected by isospin when the interference with background is neglected. How this destruction can occur is illustrated in the toy model below.

2 A simple model for resonance and background

Consider a simple toy model for a resonance and background and write the amplitude for the production of this resonance as

$$A = b \cdot e^{i\phi} + \frac{1}{1 + ix}$$
where $x$ denotes the difference between the energy and the resonance energy, the strength of the resonance amplitude is normalized to unity and $b$ and $\phi$ denote the amplitude and phase of the nonresonant background. The background is assumed to be essentially constant over an energy region comparable to the width of a narrow resonance. The square of this amplitude is then

$$|A|^2 = \left| b \cdot e^{i\phi} + \frac{1}{1 + i x} \right|^2 = b^2 + 2 \text{Re} \left( b \cdot e^{i\phi} \right) + \frac{1}{1 + x^2}$$

$$= b^2 + \frac{1 + 2 b \cos \phi - 2 bx \sin \phi}{1 + x^2}$$

For a detector which integrates the cross section over a symmetric interval from $-X$ to $+X$ the term linear in $x$ drops out and

$$\int_{-X}^{X} |A|^2 dx = \int_{-X}^{X} dx \left( b^2 + \frac{1 + 2 b \cos \phi}{1 + x^2} \right) = 2 b^2 X + \int_{-\Theta}^{\Theta} d\theta (1 + 2 b \cos \phi)$$

$$= 2 b^2 X + 2(1 + 2 b \cos \phi) \Theta$$

where we have set $x = \tan \theta$ and $X = \tan \Theta$.

The ratio of signal to background is then

$$\frac{\text{signal}}{\text{background}} = \frac{1 + 2 b \cos \phi}{b^2} \cdot \frac{\Theta}{\tan \Theta}$$

We thus find that for the case where $b = 1$, i.e. the signal and background amplitudes are equal at the peak of the resonance, the ratio of integrated signal to integrated background varies from $-1$ to $3$, depending upon the relative phase.

Fig. 1 shows $|A|^2$ as function of $x$ for several representative values of of the relative phase: $\phi = 0, \pi, \frac{3\pi}{4}, \pi$ and $\pi$.

For illustration purposes we have taken $b = 1$, i.e. equal strength of the background and signal amplitudes at the peak of the resonance. To expound the effect experimental resolution being substantially coarser than the resonance width, $|A|^2$ was averaged over bins of width $\Delta x = 5$, i.e. five times wider than the resonance width. The resulting values are plotted at bin centres as red points, with additional 10% relative error bars.

The interplay of the limited experimental resolution with the relative phase of signal and background can lead to rather striking results, depending on the value of $\phi$. Thus e.g. for $\phi = \frac{3\pi}{4}$ the measured signal is almost completely washed out by this ‘conspiracy’ of the relative phase and the experimental resolution.

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*Signal strength is defined as the difference between the measured (signal+background) and background alone. Relative signal strength of $-1$ corresponds to a dip, cf. $\phi = \pi$ entry in Fig. 1.*
Figure 1: $|A|^2$, eq. (4), as function of $x$ for $b = 1$ and for different values of the relative phase, $\phi = 0, \pi/4, \pi/2, 3\pi/4$ and $\pi$. Red points denote values averaged over bins of width $\Delta x = 5$, assuming an additional 10% relative error.
Mass difference in $K^+n$ and $K_{s}p$ modes

In experiments which did observe the $\Theta^+$ there seems to be a small but systematic and non-negligible mass difference between the mass observed in the $K^+n$ and $K_{s}p$ modes.

It is interesting to ask if this could have something to do with isospin effects in signal-background interference. As already mentioned, $K^+n$ and $K_{s}p$ are equal mixtures of states with $I = 0$ and $I = 1$ with opposite relative phase. If the nonresonant background in a given experiment is mainly from an $I = 1$ amplitude, the relative phase between signal and background amplitudes will depend upon the particular decay mode observed. The phase when the $\Theta^+$ is detected in the $K^+n$ decay mode will be opposite to the phase in the same experiment where the $\Theta^+$ is detected in the $K^0p$ decay mode.

We now consider the effect of this phase flip in our simple model of signal-background interference. A sign change of the relative phase in eq. (3) results in

$$\phi \rightarrow \phi + \pi; \quad A \rightarrow \tilde{A} = b \cdot e^{i\phi + \pi} + \frac{1}{1 + ix}$$

(7)

Since the observed signal is given by the absolute value of the amplitude, we can replace $A$ by its complex conjugate, i.e.

$$\phi \rightarrow \phi + \pi; \quad |A|^2 \rightarrow |A^*|^2 = \left| b \cdot e^{i(\pi - \phi)} + \frac{1}{1 - ix} \right|^2$$

(8)

So if Fig. 1 were to describe the $K^+n$ mode, the corresponding plots of $|A|^2$ for the $K_{s}p$ mode can be obtained by the transformation $\phi \rightarrow \phi - \pi, \ x \rightarrow -x$, possibly resulting in the shift in the peak location.

On the other hand, backgrounds differ from experiment to experiment, so it is not at all clear why this would result in a systematic shift between $K^+n$ and $K_{s}p$ modes. It would be very interesting if our experimental colleagues could redo this analysis with a realistic background parametrization.

3 Conclusion

Serious signal-background interference effects can occur in some experiments which search for the $\Theta^+$ as a narrow $I = 0$ resonance against a nonresonant background with an experimental resolution larger than the expected resonance width. The resonance signal can be completely destroyed by destructive interference with a background having a magnitude of the same order as the signal and a destructive phase.

Experiments detect $\Theta^+$ via the decay modes $K^+n$ and $K_{s}p$ which are equal mixtures of $I = 0$ and $I = 1$ states with opposite relative phase. Unless the experiment projects out the $I = 0$ component of the signal, interference can occur between the $I = 0$ state and an $I = 1$ background. Such interference can vary greatly between final states which are related by isospin in the absence of interference.

These considerations must be understood in considering the question of why some experiments see the $\Theta^+$ and others do not. The theory needed to understand apparent contradictions turns out to be more complicated than one naively expects. But experimenters
who do not see the $\Theta^+$ tend to ignore these questions and immediately conclude that the $\Theta^+$ does not exist.

To really clarify these issues it is important to have experimental data which can include both the neutral and charged kaons.

One example of an issue where neutral kaon data can help is to explain differences between $\Theta^+$ photoproduction on protons vs. neutrons. A photon which turns into $K^+K^-$ can make a $\Theta^+K^-$ directly on a neutron, but cannot make a $\Theta^+$ directly on a proton. A photon which turns into $K^0\bar{K}^0$ can make a $\Theta^+\bar{K}^0$ directly on a proton, but cannot make a $\Theta^+$ directly on a neutron. So how much of the photon appears as $K^+K^-$ and how much as $K^0\bar{K}^0$ is an experimental question that can be clarified by measuring the neutral kaons.

In the vector dominance picture the photon is a combination $\rho$, $\omega$ and $\phi$. The relative importance of the $\phi$ component is an open question. In $\Theta^+$ photoproduction the $\bar{s}$ strange antiquark is already present in the initial state in the isoscalar $\phi$ component. The $\rho$ and $\omega$ components contain no strangeness and can produce the $\Theta^+$ only via the production of an $s\bar{s}$ pair from QCD gluons. How much this extra strangeness production costs is still open. This cost does not appear in treatments using the Kroll-Ruderman theorem [7] which involves only pions and ignores the $\phi$ component of the photon.

One example of an experiment that projects out the $I = 0$ state of a $KN$ state is the photoproduction on a deuteron [8] of the final state $\Lambda(1520)K^+n$.

If the strangeness in the reaction comes from the isoscalar $\phi$ component of the photon, the final $KN$ state is required by isospin invariance to be isoscalar, and the signal is observed against a purely isoscalar background. This is not true for the other $K^-pK^+n$ final states observed in the same experiment where the $K^-p$ is not in the $\Lambda(1520)$ and the effects of a nonresonant $I = 1$ background can give very different results.*

Extensive isospin analyses that include the background are needed before conclusions can be drawn from apparent violations of isospin symmetry or from negative search results using a particular final state.

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*We recall old SLAC experiments [9, 10] which looked at photoproduction of $K^+$-hyperon from hydrogen and deuterium at 11 and 16 GeV. It would be interesting to re-examine these data in view of [8].
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