The reduction of computation load in SBAS system during the polynomial approximation of observations

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Abstract. Processing of the observations from GNSS augmentation system on an example of SBAS system is considered in the study. To compute the corrections the SBAS control center collects and processes enormous amount of data, some of the applied matrix algorithms of estimation of the polynomial coefficients are computationally expensive. Instead of the estimation of the polynomial coefficients with the matrix operations the representation of polynomial value by means of a linear combination of previous observations is suggested in the study. Significant advantage in terms of computation time (up to 250 times) for cycle-slip detection procedure is demonstrated for presented method which is important for aviation because of strict integrity requirements in avia-space sector.

1. Introduction

Global Navigation Satellite Systems (GNSS) augmentation systems have become an important tool in the multiple military and civil applications. Particularly, Satellite Based Augmentation Systems (SBAS) are widely being developed in a world: Wide Area Augmentation System (WAAS), European Geostationary Navigation Overlay Service (EGNOS), GPS-aided GEO augmented navigation system (GAGAN), etc. For instance, in [1] comparative analysis of performance of five available SBAS services is provided with details. Similar research is described in [2]. In [3,4] SBAS corrections are applied to precise point positioning methods, in [3] the results for WAAS are compared with results from European and Japanese augmentation systems. Investigation of GAGAN performance compared with local differential satellite corrections over Sri Lanka is described in [5]. In addition, the enormous number of GNSS measurements on multiple frequencies is currently available for GLONASS, GPS, GALILEO, BeiDou. As a result, progressive growth of accuracy is shown in many GNSS point positioning methods (precise point positioning). In spite of the fact that SBAS systems now do not broadcast satellite corrections for all above mentioned navigation systems this huge amount of measurements can be used, for instance, to estimate the ionosphere delays. Under such conditions it is critical for any control center to process data from the network of ground stations with the greatest possible computational efficiency. Very strict requirements of International Civil Aviation Organization (ICAO) for integrity and reliability of SBAS corrections for aviation make the efficiency of their computation at the network side a crucial issue.
Processing of the observations from the network of SBAS ground stations on an example of a future Russian SBAS structure is considered in the study. We consider future shape of Russian GNSS augmentation system for GLONASS and GPS. Satellite corrections should include satellite orbit corrections, satellite clock corrections and ionospheric corrections which provide an increased point positioning accuracy. Along with the positioning accuracy improvement of a single frequency user SBAS should provide messages with integrity information. By integrity information the data about a failure operation and the incorrect observations are meant. Basic procedure of cycle-slip detection for all available phase observations creates a very serious computation load in case of a large network. There are different methods to reduce computation load in GNSS network processing. In [6] the processing of large GNSS network datasets is considered in terms of parallel computing methods. In [7] enhanced filtration methods are applied, namely comparative analysis of legacy Extended Kalman Filter and suboptimal Particle Filter for combination of GNSS observable measurements is provided. In this study we are focused on so called enhanced computational methods, particularly, on the use of polynomial approximation for the phase measurements instead of the traditional least squares estimation during the routine cycle-slip detection procedure. This procedure provides a very serious computational load in a large network. Suggested method provides significant reduction of time required for such calculations.

2. Methodology

2.1. Analyzed structure of SBAS system

Russian SBAS system is currently under construction, so we arbitrary model the network structure according to the announced perspectives. Future shape of the system is considered here as the network of about 50-60 stations (figure 1), each one consisting of three high quality dual frequency geodetic receiver. Triple receiver scheme for every station can be applied to reduce multipath which is known as a painful error source.

![Figure 1. Modeled structure of future SBAS system for the territory of Russia.](image_url)

To compute the SBAS satellite corrections control center collects and processes enormous amount of data from the network of ground reference stations: 1sec GLONASS, GPS, BeiDou, GALILEO, QZSS observations from the tens of stations which are all equipped with three GNSS receivers.
2.2. Cycle-slip detection algorithm

Among the first basic routines several computationally expensive algorithms for cycle-slip detection are applied. We use the following phase observations and their linear combinations for this purpose:

1. Multiple frequency observations for GPS (frequency bands L1, L2, L5), GLONASS (frequency bands L1, L2, L3), GALILEO (frequency bands E1, E5), BeiDou (frequency bands B1, B2, B3), QZSS (frequency bands L1, L2, L5).

2. Geometric-free linear combinations for GPS (L1-L2, L1-L5, L2-L5), GLONASS (L1-L2, L1-L3, L2-L3), GALILEO (E1-E5), BeiDou (B1-B2, B1-B3, B2-B3), QZSS (L1-L2, L1-L5, L2-L5).

3. Code-phase Melbourne-Wubbena linear combination for all above mentioned GNSS systems.

4. Between satellites differences for all mentioned above frequency bands. These differences help to distinguish between real phase cycle-slips and periodical receiver clock steps.

During the cycle-slip detection processing we use following cubic polynomial

\[ G(t) = at^3 + bt^2 + ct + d \]  

for approximation of 8 previous values of phase observations \( \varphi_1, \varphi_2, \ldots, \varphi_8 \) to compute the predicted (extrapolated) observation value \( \varphi^\text{ext}(t) \) and to compare it with the real one \( \varphi(t) \). Previous observations and their ordering are shown in figure 2. This approximation is applied for mentioned above observation type 1, 2 and 4. Code-phase Melbourne-Wubbena linear combination is applied for cycle-slip detection in the processing but not considered here in the scope of the topic. The usual way to estimate polynomial (1) coefficients is the use of a least square solution (“LS method” hereinafter):

\[ X = (H^T H)^{-1} H^T y, \]

where \( X = [a \ b \ c \ d]^T \) – column-vector of estimated coefficients for polynomial (1), \( y = [\varphi_1 \ \varphi_2 \ \ldots \ \varphi_8]^T \) – column-vector of previous 8 phase observations for \( \varphi(t) \), \( H \) – corresponding design matrix for linear observation model \( y = HX \).

\[ \varphi_8 = \varphi(t-8) \]

\[ \varphi_7 = \varphi(t-7) \]

\[ \varphi_6 = \varphi(t-6) \]

\[ \varphi_5 = \varphi(t-5) \]

\[ \varphi_4 = \varphi(t-4) \]

\[ \varphi_3 = \varphi(t-3) \]

\[ \varphi_2 = \varphi(t-2) \]

\[ \varphi_1 = \varphi(t-1) \]

\[ \varphi(t) \]

\[ \text{time} \]

\[ t_8 = t-8 \]

\[ t_7 = t-7 \]

\[ t_6 = t-6 \]

\[ t_5 = t-5 \]

\[ t_4 = t-4 \]

\[ t_3 = t-3 \]

\[ t_2 = t-2 \]

\[ t_1 = t-1 \]

\[ \varphi(t) \]

\[ \text{Figure 2. Polynomial approximation scheme.} \]

SBAS network with 50 ground stations produces GPS, GLONASS, GALILEO, BeiDou measurement from 150 receivers. The total approximate number of least squares evaluations (2) for all considered types of observables is about 200000, and it should be computed on every observation epoch (typical time interval is 1sec for SBAS). According to ICAO requirements for time-to-alert value of SBAS corrections in aviation (6 sec for AVP-I, AVP-II [8]), computational load becomes very serious even for powerful server machines.

We tried to use a quadratic polynomial instead of a cubic one (1) but once faced with occurred cycle-slip which was not detected. The modeling showed that an error of a phase approximation with quadratic
polynomial is much bigger compared to cubic one. For this reason we recommend to use a cubic polynomial which is much more redundant.

2.3. Reduction of computation load

As a first simple method to reduce a computation load we consider the use of saved covariance matrix \((\mathbf{H}^\mathsf{T}\mathbf{H})^{-1}\) from equation (2) (“improved LS method” hereinafter). It is possible just during the time interval with constant observation scenario (constant set of used measurements). As soon as the new satellites become available (or some measurements are delayed or missed), matrix \((\mathbf{H}^\mathsf{T}\mathbf{H})^{-1}\) should be recalculated and saved again.

Instead of the least squares estimation of the cubic polynomial coefficients with matrix operations (2) we suggest in this study the representation of the predicted polynomial value by means of a linear combination of previous observations. By symbolic computing an equation (2) can be written in the following way to express estimated coefficients \(a, b, c, d\) as functions of measurements and times:

\[
\mathbf{X}(\varphi, t) = \begin{bmatrix}
    a(\varphi, t) \\
    b(\varphi, t) \\
    c(\varphi, t) \\
    d(\varphi, t)
\end{bmatrix} = \left((\mathbf{H}(t)^\mathsf{T}\mathbf{H}(t))^{-1}\right)^{\mathsf{T}} \mathbf{H}(t)^\mathsf{T} \mathbf{y}(\varphi).
\]

where \(\varphi = [\varphi_1 \varphi_2 \ldots \varphi_N]^{\mathsf{T}}\), \(t = [t_1 \ t_2 \ldots \ t_N]^{\mathsf{T}}\), \(N\) – the number of previous phase measurements taken into account for cycle-slip detection. It is too formidable and sophisticated task to get \(a(\varphi, t), b(\varphi, t), c(\varphi, t), d(\varphi, t)\) for the general case. As an assumption we determined a constant time interval for \(t\) and received polynomial coefficients as functions of measurements for fixed time vector \(t\). By using such coefficients and \(N=8\) previous phase values for cubic polynomial (1) predicted phase observation can be expressed as

\[
\varphi^{pr,8}(t) = 2\varphi(t-1) - \frac{1}{7}\varphi(t-2) - \frac{6}{7}\varphi(t-3) - \frac{9}{14}\varphi(t-4) + \frac{4}{7}\varphi(t-5) + \frac{4}{7}\varphi(t-6) - \frac{1}{2}\varphi(t-7) - \frac{1}{2}\varphi(t-8)
\]

In [9] following high pass filter expression for predicted phase measurement value based on \(N=4\) previous measurements is considered:

\[
\varphi^{pr,4}(t) = 4 \cdot \varphi(t-1) - 6 \cdot \varphi(t-2) + 4 \cdot \varphi(t-3) - \varphi(t-4)
\]

For the case of polynomial (1) both expressions (4) and (5) may provide the same result for the predicted value (polynomial (1) is ideal mathematical model). Nevertheless, in case of noised previous polynomial values prediction by expression (5) is much coarser compared to the prediction by expression (4). Real phase measurements are noisy by their nature so it is a crucial aspect. In addition, our experience shows that \(N=4\) is not enough for cycle-slip detection, we recommend to use \(N=8\). In this study we consider the use of representation (4) and refer to it as “linear method” \((N=8)\).

The main limitation of suggested linear polynomial representation (4) is fixed time interval for the previous \(N=8\) observations. So in case of delayed or missed observations inside the last \(N=8\) time epochs it is necessary to come back to LS method (coefficients \(a(\varphi, t), b(\varphi, t), c(\varphi, t), d(\varphi, t)\) are changed and equation (4) is changed also). There is an option to calculate in advance all the possible variants of
matrix \( (H^TH)^{-1}H^T \) corresponding to all the possible variants of delays and missed observations in \( N=8 \) previous epochs and apply desired matrix according to the improved LS method.

3. Results and discussion

Figure 3 shows comparative results for reduction of computation load for improved LS method and linear method compared to the usual LS method. For the different number of receivers (25-250) and corresponding number of matrix evaluations \( (0.33 \cdot 10^5 – 3.33 \cdot 10^4) \) the ratio of LS method computation time to improved LS method and linear method computation times is shown in figure 3. Suggested in [9] expression (5) provides slight improvement in terms of computation time compared to applied expression (4). At the same time, the number of undetected cycle-slips with (5) is considerably higher. For this reason, we conclude that reliability of polynomial (5) [9] with \( N=4 \) observations currently is not enough. Shown results are averaged based on several days in September 2020. One can see from the figure that suggested linear method works about 250 times faster than usual LS method for described cycle-slip detection processing. It can significantly reduce computation load for SBAS network processing and for other perspective precise positioning methods based on network processing (PPP, precise point positioning [10,11]).

![Figure 3. Relative computation time benefit compared to LS method.](image)

Despite the modeled SBAS structure used in this study, common features of any SBAS system and main characteristics are well known in GNSS society from the publications [10]. So shown results are applicable for any GNSS augmentation system and other systems based on network processing.

4. Conclusion

In SBAS systems computational load for usual cycle-slip detection routines is very serious because of a huge number of observations and their linear combinations in current multi-GNSS conditions. Comparative results for the reducing of the computational load due to the suggested method are shown in the study for the different number of network reference stations. The suggested linear representation
of predicted phase value is rigorous (strict) and it significantly reduces the computational load in SBAS network for cycle-slip detection routines (advantage in terms of computation time is up to 250 times). It can be applied in other positioning techniques which require network processing. Because of strict ICAO requirements for SBAS systems in terms of integrity and reliability demonstrated significant reduction of computation time is of interest for avia-space sector.

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