Model independent method for
determination of the DIS structure of free neutron

Misak Sargsian

Department of Physics, Florida International University, Miami, FL 33199, USA

Mark Strikman

Department of Physics, Pennsylvania State University, University Park, PA 16802, USA

November 6, 2018

Abstract

We present a model independent procedure for extracting deep-inelastic structure function of "free" neutron from the electron - deuteron scattering with protons produced in the target fragmentation region of the reaction. This procedure is based on the extrapolation of \( t \), which describes the invariant momentum transfered to the proton, to the unphysical region corresponding to the mass of the struck neutron. We demonstrate that the impulse approximation diagram of the reaction has a pole at this limit with a residue being proportional to the "free" neutron structure function. The method is analogous to that of Chew and Low for extraction of the "free" pion-pion and neutron-neutron cross sections from \( p(\pi, p)X \) and \( d(n, n)pm \) reactions respectively. We demonstrate that in the extrapolation the final state interaction amplitudes are smooth functions of \( t \) and have negligible contribution in the extracted “free” nucleon structure function. We also estimate the range of the recoil nucleon momenta which could be used for successful extrapolation procedure.

1 Introduction

In spite of three decades of studies of the partonic structure of nucleons one still lacks a satisfactory knowledge of the relative \( d \) and \( u \) quark densities at large Bjorken \( x \) region.

To extract these quantities two major approaches have been considered to date: one is the neutrino scattering off the proton at large \( x \) which allows one to probe separately the \( u \) and \( d \) distributions. The second is the extraction of parton distributions from both protons and neutrons using inclusive scattering from the hydrogen and deuteron targets. (In the future it would be possible also to use \( W^\pm \) production at LHC.)
While deep inelastic neutrino-proton scatterings lack adequate statistics, the inclusive electron-deuteron measurements suffer from significant nuclear effects an estimation of which involves the consideration of specific models for the Fermi motion and the EMC effect [2, 3, 4, 5].

In our previous works [6, 7, 8] we outlined an alternative, model independent approach for extraction of “free” neutron deep inelastic structure function using semi-inclusive tagged neutron reactions, \( d(e, e', p)X \), in which slow protons are detected in the target fragmentation region of the reaction. These considerations were incorporated in the experimental proposal which was approved recently at Jefferson Lab (JLab) [9]. Preparations for the experiment are currently under way and hence it is timely to elaborate our approach quantitatively.

In our consideration of semi-inclusive deep inelastic scattering off the deuteron we focus in the region of relatively large \( x \geq 0.3 \), where coherence length is small and hence nuclear shadowing mechanism of the distortion of the nucleon spectrum discussed in [10] is negligible.

The extraction procedure is based on the observation that due to a weak binding in the deuteron the singularity of the amplitude in the \( t = (p_d - p_p)^2 \) -channel is much closer to the physical region of on-shell neutron than to all other singularities (the closest of which would be the pion production threshold). Hence one can in principle to continue analytically the scattering amplitude in \( t \) and find the residue at the pole of the struck neutron propagator at \( t = m_n^2 \). This is analogous to the Chew-Low procedure [11] for the extraction of the pion-pion scattering amplitude from the pion-nucleon data. It is worth emphasizing that in our case the analytic continuation is simpler since the elementary amplitude does not contain factors which go through zero at \( t \) close to the pole of the amplitude ( \( t = 0 \) as compared to \( t = m_\pi^2 \)). Since procedures of extrapolation work well for the pion case, we expect them to be even more effective for the case of the tagged nucleon processes.

The paper is organized as follows: In section II we outline the general framework of semi-inclusive deep inelastic scattering off nuclear targets, concentrating on the general properties of the scattering amplitude in the impulse approximation as well as beyond the impulse approximation. In section III we study the analytic properties of the impulse approximation and the final state interaction amplitudes at kinematics of recoil proton extrapolated to the pole values of the struck neutron propagator. First we study the singular behavior of the impulse approximation amplitude in the limit \( t \rightarrow m_n^2 \). Then we prove the loop theorem, which states that any additional loop in the scattering amplitude removes the singularity associated with the on-shellness of the struck neutron. In section IV we use a specific model to calculate the corrections to the impulse approximation due to the final state interactions. We observe that the deviations from the impulse approximation due to such reinteractions is mostly concentrated at recoil angles \( \leq 100-120^\circ \). Based on the considered final state interaction models we elaborate the extrapolation procedure and demonstrate that extracted ”free” neutron DIS structure function is insensitive to the strength of the rescattering amplitude. This explicitly demonstrates the model independence of the extrapolation procedure. We find that these procedure can be performed reliably if the tagged nucleons with momenta \( 50 \div 150 \text{ MeV}/c \) are detected.

1It is certainly of interest to study the reaction with tagged neutron as well which would allow a direct comparison of the scattering off a free and bound proton and hence identify the processes beyond the impulse approximation.
2 General Framework

We consider semiinclusive deep inelastic scattering (DIS) off the deuteron:

\[ e + d \rightarrow e' + N + X \]  

where nucleon N is detected in the target fragmentation region. We define \( E_e \) as the initial energy of the electron and \( E'_e, \theta_e \) as the energy and scattered angle of the final electron. \( q \equiv (\nu, \mathbf{q}) \) is the four momentum of the virtual photon, with \( \nu = E_e - E'_e \) and \( Q^2 = -q^2 \). The recoil nucleon is described by four-momentum \( p_s \equiv (E_s, \mathbf{p}_s) \). We identify the masses of recoil and struck nucleons by \( m_s \) and \( m_N \) respectively. \( t = (p_d - p_s)^2 \) determines the invariant momentum transferred to the recoil nucleon, where \( p_d \) is the four momentum of the deuteron.

The processes of Eq. (1) with recoil proton can be used to extract the DIS structure function of the neutron. Since in this case the neutron is bound, it requires a careful treatment of off-shell effects in maximally model independent way. The approach we will discuss in this work is similar to one of Chew and Low \(^{11} \) who studied the issues of extracting \( \pi^+ + \pi^0 \) and \( n + n \rightarrow n + n \) cross sections from \( \pi^+ + p \rightarrow p + X \) and \( n + d \rightarrow p + n + n \) reactions respectively. In their analysis they observed that the analytical structure of the impulse approximation amplitude (as in Eq.(1)) is such that it has a pole in nonphysical region of \( t \) corresponding to the one-mass-shell kinematics of the bound particles involved in the interaction.

In the similar way we will analyze analytic properties of the scattering amplitude of the reaction (1) in the impulse approximation focusing on the issues related to the extraction of the on-shell DIS structure function of the neutron.

First, we summarize the general formulae for the cross section of process (1):

\[
\frac{d\sigma}{dxdQ^2dp_s/E_s} = \frac{4\pi\alpha_{em}^2}{xQ^4} (1 - y - \frac{x^2y^2m_N^2}{Q^2}) \times \left[ F^D_L + \left( \frac{Q^2}{2q^2} + \tan^2(\frac{\theta}{2}) \frac{\nu}{m_N} F^D_T \right) \right] F^D_{T/L},
\]

where four independent nuclear structure functions, \( F_L^{D,T},F_T^{D,T} \) depend on \( Q^2, x, \alpha_s, p_{st} \), with \( \alpha_s = 2 \frac{E_s - p^z_s}{m_D} \) being light cone momentum fraction of the deuteron carried out by recoil nucleon. The latter is normalized in such way that the sum of \( \alpha_s + \alpha = 2 \), where \( \alpha \) is the similar quantity for the interacting nucleon. The Bjorken \( x = \frac{Q^2}{2m_N\nu} \) and \( y = \frac{\nu}{E_e} \). The z axis aligned in the direction of \( \vec{q} \). In many practical considerations one integrates over the azimuthal angles \( \phi \) of the recoil nucleon, which yields:

\[
\frac{d\sigma}{dxdQ^2dp_s/E_s} = \frac{4\pi\alpha_{em}^2}{xQ^4} (1 - y - \frac{x^2y^2m_N^2}{Q^2}) \left[ F_{2D}^{SI} + 2\tan^2(\frac{\theta}{2}) \frac{\nu}{m_N} F_{1D}^{SI} \right],
\]

where: \( F_{2D}^{SI}(x, Q^2, \alpha_s, p_t) = F^D_L + \frac{Q^2}{2q^2m_N} \frac{\nu}{F^D_T} \) and \( F_{1D}^{SI}(x, Q^2, \alpha_s, p_t) = \frac{F^D_T}{2} \).

The theoretical description of the reaction (1) at not very large values of \( p_s < 700 \text{ MeV/c} \) is based on the assumption that it proceeds through the interaction of virtual photon off one of the bound nucleons in the deuteron, while produced particles can interact in the final state with
the other (spectator) nucleon. Since we are interested in \( x \geq 0.3 \) kinematics it is legitimate to neglect simultaneous interaction of \( \gamma^* \) with two nucleons. Two main diagrams will contribute to the cross section of reaction (1): impulse approximation (IA) (Fig.1a) and diagram representing a rescattering of the recoil nucleon off the products of DIS scattering (Fig.1b), which we will refer as final state interaction (FSI) diagram.

2.1 Impulse Approximation (IA)

In IA the recoil nucleon is a spectator of \( \gamma^* \) scattering off the bound nucleon \( N \). One can apply Feynman diagram rules to write down the IA amplitude in a formal form (see e.g. [12, 13]):

\[
A_{IA}^\mu = \langle X | J_{em}^\mu(Q^2, \nu, p_s) \frac{\hat{p}_d - \hat{p}_s + m}{m_N^2 - (p_d - p_s)^2} \bar{u}(p_s) \Gamma_d \rangle = \langle X | J_{em}^\mu(Q^2, \nu, p_s) \frac{\hat{p}_d - \hat{p}_s + m}{m_N^2 - t} \bar{u}(p_s) \Gamma_d \rangle
\]

(4)

where \( \Gamma_d \) is the covariant \( d \rightarrow pn \) transition vertex which is a smooth function at the pole of the struck nucleon propagator, and \( J_{em}^\mu(Q^2, \nu, x) \) represents the electromagnetic DIS operator of electron scattering off the bound nucleon. Here we suppressed the polarization indices of the deuteron and the nucleons.

Taking the recoil nucleon on mass shell in Fig.1(a) and using \( \hat{p}_d - \hat{p}_s + m \approx \sum_{\text{spins}} u(p_d - p_s) \bar{u}(p_d - p_s) \) one can factorize the IA amplitude into two parts, consisting of the DIS current of the bound nucleon, \( J_{X,N}^\mu = \langle X | J_{em}^\mu(Q^2, \nu, p_s) u(p_d - p_s) \rangle \) and the wave function of the deuteron. The latter is expressed through the \( \Gamma_d \) vertex function and the bound nucleon propagator. Since the deuteron wave function is not a Lorentz invariant quantity its determination depends on the reference frame in which the above factorization is performed.

The factorization procedure in the Lab frame of the deuteron in nonrelativistic limit (corresponding to the equal time quantization) yields the nonrelativistic deuteron wave function which is the solution of the Schroedinger equation:

\[
\Psi_{dNR}^\nu(p_s) = \frac{1}{2\sqrt{(2\pi)^3 E_s}} \frac{\bar{u}(p_s) \bar{u}(p_d - p_s) \Gamma_d}{m_n^2 - (p_d - p_s)^2}.
\]

(5)

It is normalized as \( \int |\Psi_d(p_s)|^2 d^3p_s = 1 \). This correspondence usually achieved in the virtual nucleon (VN) approximation in which the scattering is described in the LAB frame of the nucleus and electrons scatter off the virtual nucleon whose virtuality is defined by the kinematic parameters.
of the spectator nucleon. In this case the form of the wave function is defined through the evaluation of the IA amplitude at the one-mass shell pole of the spectator nucleon propagator in the Lab frame. This yields an off-energy-shell state of the bound nucleon.

In another approximation one can formally associate the $\Gamma_d$ vertex with Light Cone deuteron wave function by considering equal light cone time $\tau = t - z$ quantization. In this case:

$$\Psi_{d}^{LC}(\alpha_s, p_{st}) = \frac{\Gamma_d}{\sqrt{(2\pi)^3}} \frac{4(m^2 + p^2)}{(\alpha_s(2-\alpha_s) - M_d^2)}$$

(6)

normalized as $\int |\Psi_d(\alpha_s, p_{st})|^2 d^2p_{st} \frac{d\alpha_s}{\alpha_s} = 1$. In this approximation referred as the light cone (LC) approximation [17] the scattering is described in the light cone reference frame where the wave function is evaluated at the pole of the spectator nucleon in the LC reference frame. This yields an off-light cone energy $(E + p_z)$-shell state of the bound nucleon.

The above described factorization of the scattering amplitude into two parts in IA allows one to express the nuclear DIS structure functions through the convolution of bound nucleon DIS structure functions and the nuclear spectral function, $S$ as follows: [6,15]:

$$F_{2D}^{SI}(x, Q^2, \alpha_s, p_t) = \frac{S(\alpha_s, p_t) m_N \nu}{n pq} \times \left[ (1 + \cos \delta)^2 (\alpha + \frac{pq}{Q^2} \alpha \mu)^2 + \frac{1}{2} \sin^2 \delta \frac{p_t^2}{m_N^2} \right] F_{2N}^{eff}(\tilde{x}, Q^2, \alpha, p_t),$$

$$F_{1D}^{SI}(x, Q^2, \alpha_s, p_t) = \frac{S(\alpha_s, p_t)}{n} \left[ F_{1N}^{eff}(\tilde{x}, Q^2, \alpha, p_t) + \frac{p_t^2}{2pq} F_{2N}^{eff}(\tilde{x}, Q^2, \alpha, p_t) \right] ,$$

(7)

where $F_{1N}^{eff}$ and $F_{2N}^{eff}$ are the structure functions of the bound nucleon and $\sin^2 \delta = \frac{Q^2}{Q^2}$. The nuclear spectral function, $S$ describes the probability of finding an interacting nucleon in the target with momentum $(\alpha, p_t)$ and a recoil nucleon in the final state of the reaction with momentum $(\alpha_s, p_{st})$. Note that in IA $\alpha_s + \alpha = 2$ and $p_t = -p_{st}$. $S$ is a model dependent quantity whose form depends on the framework/formalism used to describe the interaction.

In VN approximation $n = \frac{M_d}{2(M_d-E_S)}$ and for the spectral function one obtains [15]:

$$S_{VN}^{IA}(\alpha_s, p_{st}) = E_n \Psi_d^2(p_s),$$

(8)

where $\int S_{VN}^{IA}(\alpha_s, p_{st}) \frac{d\alpha_s}{\alpha_s} d^2p_t = 1$. It can be shown [15] that $S_{VN}$ has a normalization such that in Eq.(7) it conserves the number of the baryons involved in the reaction. On the other hand the momentum sum rule is not satisfied. This reflects the fact that due to the virtuality of the interacting nucleon part of its momentum is shared by the non-nucleonic degrees of freedom in nuclei.

In LC approximation [17] $n = (2 - \alpha_s)$ and:

$$S_{LC}^{IA}(\alpha_s, p_{st}) = \Psi_{d}^{LC}(\alpha_s, p_{st}) = \frac{E_k}{2 - \alpha_s} \Psi_d^2(k), \quad \alpha_s = \frac{E_k - k_z}{E_k}, \quad p_{st} = k_t .$$

(9)

The remarkable property of the LC spectral function is that it fulfills both baryon number conservation ($\int S_{LC}^{IA}(\alpha_s, p_t) \frac{d\alpha_s}{\alpha_s} d^2p_t = 1$) and momentum sum rule ($\int \alpha S_{LC}^{IA}(\alpha_s, p_t) \frac{d\alpha_s}{\alpha_s} d^2p_t = 1$) (17) (for recent discussion see also [15]).
2.2 Final State Interaction

Calculation beyond the impulse approximation is a very complex task in DIS. This includes FSI contribution in which a reinteraction is taken place between spectator nucleon and the products of DIS scattering off the struck nucleon. The typical FSI amplitude can be represented as in Fig.1b. Calculation of this requires a detailed understanding of the dynamics of DIS as well as its hadronic structure at the final state of the reaction. However the local character of DIS scattering off one of the nucleons in the deuteron allows us to represent FSI diagram of Fig.1b in the most general form as follows:

$$A_{FSI} = \sum_{X'} \int \frac{d^4p_{s'}}{(2\pi)^4} \langle X, s | \hat{A}_{FSI} \cdot G(X') \cdot \hat{j}^{em}(Q^2, x) \frac{\hat{p}_d - \hat{p}_{s'} + m_N}{(p_d - p_{s'})^2 - m_N^2 + i\epsilon} \frac{\hat{p}_{s'} + m_{N_2}}{p^2_{s'} - m_{N_1}^2 + i\epsilon} \Gamma_d \rangle$$

where \( \hat{j}^{em}(Q^2, x) \) and \( \hat{A}_{FSI} \) represent operators of DIS and FSI scattering and \( G(X') \) is a notation for the propagation of intermediate state \( X' \). The amplitude in Eq.(10) is too general to allow any practical calculation. Our further discussion is based on the presence of at least one integration over the momentum of spectator nucleon in Eq.(10). Complexity of the intermediate state \( X' \) and its final state interaction may lead to additional integrals over momenta in Eq.(10). Obviously for purpose of demonstrating the lack of singularity it is sufficient to consider the case of one integration over the momentum of the spectator nucleon.

3 Analytic Properties at the Pole

Next we discuss the analytic properties of IA (Eq.(4)) and FSI (Eq.(10)) amplitudes at the pole of the struck nucleon propagator.

3.1 IA Amplitude

We first consider IA amplitude, in which the momentum of the struck nucleon is fixed by the kinematics of the spectator nucleon. It follows from Eq.(4) that IA amplitude has a pole at \( t = m_N^2 \) which corresponds to the struck-nucleon being on-mass shell. Using the following relation between \( t \) and kinetic energy of the recoil nucleon, \( T_s \):

$$t = -2M_dT_s + m_N^2 - |\epsilon_d|(M_d + m_N - m_s)$$

it is straightforward to represent the amplitude of Eq.(4) in the following form:

$$A_{IA} = \langle X|j^{em}(Q^2, x)|n > \bar{u}(p_d - p_s)\bar{u}(p_s)\frac{\Gamma_d}{|\epsilon_d|(M_d + m_N - m_s) + 2M_dT_s}.$$  

This equation shows that the pole is associated with the negative value of kinetic energy of the spectator nucleon, equal to the half of the magnitude of the deuteron binding energy \( \epsilon_d \):

$$T_{s pole} = -\frac{|\epsilon_B|}{2}(1 + \frac{m_n - m_p}{M_d}) \approx -\frac{|\epsilon_d|}{2}.$$


Despite being in nonphysical region of the reaction (1) the very fact of the existence of the pole indicates that the extrapolation of the measured semiinclusive cross section to the positive region of $t$ (negative kinetic energy region of the spectator nucleon) will allow us to isolate the “free” cross section of the electron-nucleon scattering. This can be true only if the singularity is unique to the IA amplitude.

### 3.2 Loop Theorem and non IA Amplitudes

Now we will prove that since all FSI type amplitudes contain at least one loop integral in the amplitude, they are regular at the one-shell pole of the struck nucleon propagator.

It is sufficient to prove that no singularities exist for the amplitude that contains at least one loop involving the spectator nucleon, to prove that all higher order diagrams will have a smooth behavior at $t \to m_N^2$ limit. In considering the FSI diagram, using Eq.(11), we will analyze the analytic behavior of $A_{FSI}$ with respect to $t$. Due to the relation in Eq. (11) all we need to demonstrate is that $A_{FSI}$ is a smooth function in $T_s \to -|\epsilon|^2$ limit.

In Eq.(11) we first evaluate the integral over $d^0 p_s' =$, then introducing $k = p_s - p_s'$ one obtains:

$$A_{FSI} = \int \frac{d^3 k}{(2\pi)^3} \left[ \langle X, s | \hat{A}_{FSI} G(X') \hat{J}_{em}(Q^2, x) | N \rangle \bar{u}(p_d - p_s - k) \bar{u}(p_s - k) \right] \frac{2(m_N + T_s - k_0)}{2m_N + T_s - k_0} \Gamma_d \frac{M_d}{-M_d^2 + 2M_d(m + T_s - k_0)}.$$

To estimate the contribution of this amplitude at the pole of the struck nucleon propagator it is enough now to estimate the integral

$$\int \frac{d^3 k}{M_d |\epsilon| + 2M_d T_s - 2M_d k_0}.$$  \hspace{1cm} (15)

For relatively small $k$, using the relation: $k_0 = \sqrt{m_N^2 + p_s^2} - \sqrt{m_N^2 + (p_s - k)^2} \approx \frac{\mu}{m_N} - \frac{k^2}{2m_N}$ we see that in the limit $T_s \to -|\epsilon|$, integrand in Eq.(15) is finite,

$$\frac{k^2 dk}{2M_d(\frac{k^2}{2m_N})} \to \frac{dk}{2}.$$  \hspace{1cm} (16)

This demonstrates that the integral in Eq.(14) is finite in $T_s \to -|\epsilon|$, limit. Note that the integral converges also in the large $k$ limit since in this case the integrand is suppressed due to the amplitude of rescattering of the system $X$ off the spectator nucleon. This result demonstrates that all contributions that contain at least one rescattering with spectator nucleon have a smooth behavior at the pole of the struck nucleon propagator as compared to the singular behavior of IA amplitude in Eq.(12).
4 Extracting “Free” Neutron DIS Structure Function

We demonstrate the procedure of extraction of “free” neutron structure functions using specific models for evaluation of the FSI. We then demonstrate that the result of the extrapolation is practically insensitive to the parameters which define the strength of FSI in these models. The latter indicates the model independence of the procedure.

4.1 Model for FSI

We discuss a model in which the FSI of the spectator nucleon occurs coherently with the system produced in the deep inelastic electron- bound nucleon scattering (see also [16]). Such approach is rather well founded for energies relevant to Jefferson Lab [9], in which the multiplicity of produced system is restricted to few hadrons. Due to the same reason a nonspectator contribution in which a nucleon with small momentum is produced in the process \( \gamma^* + N \rightarrow N + X \) is strongly suppressed at \( x \geq 0.3 \) as it requires scattering off a nucleon with a very large momentum \( \alpha > 1 + x \). Therefore contributions to the cross section associated with the incoherent rescattering will not contain any terms singular at \( t \rightarrow m_N^2 \). Thus we expect that the main contribution to DIS cross section will come from the diagrams of type of Fig.1b in which FSI amplitude could be estimated in analogy to the processes of \( e + d \rightarrow e + p + n \) considered in Ref.[12, 13, 18]. In this approach one arrives at the similar eikonal form of the rescattering amplitude treating the effective cross section and the \( t \)-dependence of the rescattering as free parameters. In the calculation we neglect a small effect of modification of Fermi momentum of the struck nucleon in the \( \gamma^* N \) amplitude due to FSI. This is justified due to the fact that the dominant process of rescattering corresponds to the conserved light-cone fraction of the interacting nucleon[12, 13] (see also below). Within this approximation we can factorize \( \gamma^* N \) DIS amplitude from the FSI. This allows us to use the whole theoretical framework described in Sec.2.1 by replacing \( S^{IA} \) with the distorted spectral functions, \( S^{DWIA} \) which include both IA and FSI effects.

Using the procedure of the generalized eikonal approximation of Ref.[12, 13] within virtual nucleon approximation for the distorted spectral function one obtains:

\[
S_{VN}^{DWIA} = E_s \left[ \Psi_d^2(p_s) - \frac{1}{2} \sqrt{\frac{M_d - E_s}{E_s}} Im \int \frac{d^2k_t}{(2\pi)^2} f(k_t) \left[ \Psi_d^j(p_s) \Psi_d(\tilde{p}_s) - i \Psi'_d(p_s) \Psi'_d(\tilde{p}_s) \right] \right] + \left( \frac{1}{4} \sqrt{\frac{M_d - E_s}{E_s}} \right)^2 \int \frac{d^2k_t}{(2\pi)^2} f(k_t) \left[ \Psi_d(\tilde{p}_s) - i \Psi'_d(\tilde{p}_s) \right]^2 \right], \tag{17}
\]

where \( \tilde{p}_s \equiv (\tilde{p}_{sz}, \tilde{p}_{st}) = (p_{sz} - \Delta_{VN}, p_{st} - k_t) \) and \( \Delta_{VN} = \frac{(M_d + \nu)}{q}(E_s - m) + \frac{W^2 - W_0^2}{2q} \). Here \( W^2 = (q + M_d - p_s)^2 \) and \( W_0^2 = (q + m)^2 \). \( \Psi'_d \) results from the non-pole contribution in the integration over the longitudinal component of the transferred momentum (for details and for the expression of \( \Psi'_q \) see Appendix B of Ref.[13]). Similar to the IA case an averaging over the polarizations of the deuteron is assumed.
In LC approximation for distorted spectral function we obtain:

\[
S_{DWIA}^{LC} = \frac{E_k}{\alpha} \left[ \Psi_d^2(k) - \frac{1}{2} \sqrt{m E_k} \text{Im} \int \frac{d^2l_t}{(2\pi)^2} f(l_t) \left[ \Psi_d^\dagger(k) \Psi_d(k) - i \Psi_d^\dagger(k) \Psi_d'(k) \right] + \frac{1}{16 \sqrt{m E_k}} \left| \int \frac{d^2l_t}{(2\pi)^2} f(l_t) [\Psi_d(k) - i \Psi_d'(k)] \right|^2 \right]
\]

(18)

where \( \tilde{k} \equiv (\tilde{k}_z, \tilde{k}_t) = (k_z \frac{m}{E_k} - \Delta_{lc}, k_t - l_t) \) and \( \Delta_{lc} = \frac{M_d + q - q^-}{q^+} + \frac{W^2 - W_0^2}{q^+} \). Here \( \tilde{k} \) represents the relative momentum of the target nucleons in the light cone reference frame. They are expressed through the light cone momentum fraction and the transverse momentum of the target nucleons according the relations given at the RHS of Eq.(9).

It follows from Eqs.(17) and (18) that in both cases the distorted spectral function has three distinctive terms. The first term represents the IA term discussed already in Sec. 2.1, the second term represents the interference between IA and FSI amplitudes which screens the \( \gamma^*d \) cross section, while the third term is the pure rescattering contribution.

Note that one of the important differences between VN and LC approximations, is that at high energy limit (\( q, q_0 \gg m \)), \( \Delta_{VN} \) is finite approaching to \( (E_s - m) \), while \( \Delta_{LC} \) vanishes. This reflects the fact that at high energy limit the light cone momentum fraction of the interacting nucleons (such us \( \alpha_s \)) conserves during final state interaction.

The amplitude \( f \) in both Eqs.(17) and (18) describes the rescattering of the spectator nucleon off the products of deep inelastic scattering off struck nucleon. Since characteristic momentum transfer in FSI is relatively small (\( \ll m \)) we can model this amplitude in the form of: \( f(k) = \sigma_{eff}(i + \alpha)e^{-\frac{B}{2}k^2} \) with \( \sigma_{eff}, \alpha \) and \( B \) being free parameters. For our numerical estimates we fix \( B = 8 \text{ GeV}^2 \) and \( \alpha = -0.2 \) corresponding to characteristic values of hadronic interaction at small momentum transfer. We will allow variation of \( \sigma_{eff} \) between zero and 60–80 mb. The latter numbers represents that characteristic cross sections of scattering of the spectator nucleon off the “X” state containing one baryon and up to two mesons.

The distorted spectral functions of Eqs.(17) and (18) now can be used in Eq.(7) to estimate the \( F_{SI}^{2D} \) and \( F_{SI}^{1D} \) in the approximation which includes both IA and FSI effects. Such approximation we will refer as distorted wave impulse approximation (DWIA).

In Fig.2 we represent the predictions of VN and LC models for \( F_{2D}^{SI} \) calculated within PWIA and DWIA approximations. One can see from the figure that if the strength of \( XN \) interaction is varied within a reasonable range (\( \sigma_{XN} \leq 60 \text{ mb} \)) one finds a rather broad range of predictions. This highlights the necessity of the dedicated measurements of semiinclusive DIS reaction off the deuteron covering wide kinematic range of the recoil nucleon that will allow the focused comparison of the predictions of different theoretical approaches. It is important however that at \( \alpha = 1 \) the difference between numerical results of VN and LC approaches is negligible leaving the source of the main uncertainty to the strength of the FSI.
Figure 2: Angular dependence of $F_{2D}^{SI}$ for different values of recoil nucleon momenta. Dashed and solid curves correspond to PWIA and DWIA predictions within VN approximation. Curved labeled by circles corresponds to LC predictions. The total cross section of $XN$ state in the FSI amplitude is taken as 60 mb. Arrows identify the angles corresponding to $\alpha = 1$.

4.2 Extraction Procedure for $F_2$

We discuss now the procedure that allows us to extract the “free” $F_2$ for the struck nucleon. One starts with the assumption that we don’t know the strength of the FSI for which we can assume various values of effective reinteraction cross section from 0 to 80 mb (see discussion above).

To employ the extraction procedure based on the pole extrapolation method we introduce an extraction factor $I(p_s, t)$ defined as follows:

$$I(p_s, t) = \frac{1}{E_s} \frac{(m_N^2 - t)^2}{\left[\text{Res}(\Psi_d(T_pole))\right]^2} \cdot \frac{1}{m_N \nu_p \left[1 + \cos^2(\alpha + \frac{m_p}{m_N^2} \alpha_q)^2 + \frac{1}{2} \sin^2 \frac{\delta_p}{m_N^2} \right]^2},$$

where $\text{Res}(\Psi_d(T_pole)) = \frac{C}{\sqrt{\pi} 2m_N}$ GeV$^{-\frac{3}{2}}$ where $m_N$ corresponds to the mass of the struck nucleon. Here $C$ slightly depends on the particular potential used to calculate the deuteron wave function. For example $C = 0.3939$ for the wave function with Paris potential\cite{21} and $C = 0.3930$ for the Bonn potential\cite{22}.

Using $I(p_s, t)$ we define the extracted structure function as follows:

$$F_{2N}^{extr}(Q^2, x, t) = I(p_s, t) \cdot F_{2D}^{SI, EXP}(x, q^2, \alpha_s, p_t),$$

where $F_{2D}^{SI, EXP}$ is the experimentally measured value of the integrated structure function defined in Eq.(3).
If only PWIA contributions were important in the cross section of the reaction (11) then one can estimate \( F_{2D}^{SI,EXP}(x, q^2, \alpha_s, p_t) \) using Eq.(7) with spectral functions defined in Eqs.(8) and (9). In this case one observes that by extrapolating \( t \) to the value of \( m_N^2 \), \( F_{2N}^{extr} \) approaches to \( F_{2N}^{eff}(x, Q^2, \alpha = 1, p_t = 0) = F_{2N}^{free}(x, Q^2) \).

In the case when FSI effects are considered we evaluate \( F_{2D}^{SI,EXP}(x, q^2, \alpha_s, p_t) \) within DWIA framework, in which \( F_{2D}^{SI,EXP} \) is estimated using Eq.(7) with the distorted spectral functions defined in Eqs.(17) and (18). In this case we again extrapolate \( F_{2D}^{SI,EXP}(x, q^2, \alpha_s, p_t) \) to the values of \( t \rightarrow m_N^2 \).

To establish the appropriate extrapolation procedure one has to investigate the analytic properties of Eq.(20) with respect to the variable of \( t' \equiv t - m_N^2 \). Here one observes that \( F_{2N}^{extr} \), at small \( |t'| \ll m_N^2 \), is an explicit quadratic function of \( t' \) with IA-FSI interference term being proportional to \( t' \), while the double scattering term \( (|A_{FSI}|^2) \) is proportional to \( t'^2 \).

Note however that the \( |A_{IA}|^2 \) term has a (hidden) weak polynomial dependence on \( t' \) due to higher mass singularities in the deuteron wave function \( \psi_d = \sum_i \frac{e_i}{\sqrt{t_i + M_i^2}} \) with \( M_i^2 = (i-1)4\gamma m_0 + 2(i-1)^2 m_0^0 \) (see e.g. Ref.[22]), where \( \gamma = 0.04568 \) GeV and \( m_0 = 0.17759 \) GeV. Since the second nearest pole corresponds to rather large positive values for \( t' \) \((\approx 0.1 \text{ Gev}^2)\), based on Eq.(11) on estimates that by restricting recoil nucleon momenta \( p_s \leq 250 \text{ MeV/c} \) the \( |A_{IA}|^2 \) term can be represented again as a quadratic function of \( t' \). Therefore if spectator nucleon kinetic energy (or \( t' \)) is much less than the energy scale corresponding to the higher mass singularity in the deuteron wave function, Eq.(20) will represent a quadratic function of \( t' \).

The above discussion suggests that the pole extrapolation can be achieved by the square fit of the experimental data measured at small and finite values of \( t' \). However such extrapolation can be meaningful only if we can exclude the variation of \( F_{2N}^{extr}(Q^2, x, \alpha, t) \) due to the change of the other kinematic variables involved in the reaction. One significant variation is the \( \alpha \) dependence of \( F^{extr} \) due to the combination of \( \frac{\alpha}{\alpha} \) entering in the bound nucleon structure function. This may significantly alter \( t' \) dependence at large \( x \) kinematics. Another important effect in high \( x \) kinematics is the higher twist effects due to sensitivity of structure functions on the produced final mass of the DIS scattering, \( W_N \) at intermediate \( Q^2 \). Since \( W_N^2 \sim \alpha W_{N0}^2 \) where \( W_{N0} \) is the final mass produced off the stationary nucleon, the \( W_N \) sensitivity will result in additional \( \alpha \) dependence of \( F^{extr} \). Thus, in the large \( x \) kinematics, especially at intermediate \( Q^2 \), the \( t' \)-dependence of \( F_{2N}^{extr} \) will change strongly with \( \alpha \).

Since
\[
\alpha \sim 1 + \frac{p_s \cdot \cos(\theta_s)}{m},
\]
the discussed above sensitivity will result in the sensitivity of the extrapolation procedure to the direction of the momentum of the recoil nucleon. To illustrate this point we calculate the ratio,
\[
R = \frac{F_{2N}^{extr}(Q^2, x, \alpha, t)}{F_{2N}^{free}(Q^2, t)}
\]
and study its variation with \( t' \) at different fixed values of \( \theta_s \). Figs.3(a) and (b) demonstrate such calculations within both DWIA and PWIA using VN approximation at \( x = 0.7 \) for \( Q^2 = 5 \text{ Gev}^2 \) (a) and \( Q^2 = 10 \text{ Gev}^2 \) (b) kinematics. These calculations reveal significant violation of quadratic dependence of \( F^{extr} \) at \( \theta_s > (\prec) 90^0 \) in which case the corresponding \( \alpha \) is away from unity. This
Figure 3: The $-(t - m_n^2)$ dependence of $R$ for different values of recoil nucleon angle. Dashed and solid curves correspond to PWIA and DWIA predictions within VN approximation. The total cross section of reinteraction of $XN$ state in the FSI amplitude is taken $60 \text{ mb}$.

reflects the fact that both $F_{\text{extr}}^{2N}(Q^2, x, \alpha, t) \sim F_{\text{bound}}^{2N}(Q^2, \frac{x}{2 - \alpha}, W_N, t)$ and $F_{\text{free}}^{2N}(Q^2, x, W_{N0})$ are sensitive functions of $x$ and $W$ and a little mismatch between $x$ and $\tilde{x} \approx \frac{1}{2 - \alpha}$ as well as $W_N$ and $W_{N0}$ can result to a substantial irregularity in $t'$ dependence of $R$. Note that $W$ sensitivity is especially strong at small $\theta_s$ (see $\theta_s = 30^0$ curves in Fig.3a) which corresponds to small (resonating) values of $W_N$. However we can see from Fig.3(b) that the $W$ dependence diminishes with an increase of $Q^2$ due to suppression of the higher twist effects.

Although both Fig.3a and 3.b demonstrate substantial angular ($\alpha$) dependencies, they also indicate the way to avoid these complications. Due to relation (21) these irregularities are gone at $\theta_s = 90^0$, since in this case $\alpha \approx 1$ and $x \approx \tilde{x}$ and $W_N^2 \approx W_{N0}^2$. Therefore one concludes that $\alpha_s = \alpha = 1$ represents the most suitable kinematical condition for $t'$ extrapolation of $F_{\text{extr}}^{2N}(Q^2, x, t)$. It is worth mentioning that according to Fig.2, $\alpha_s = 1$ also diminishes the discrepancy between VN and LC approaches.

Fig.4 represents one example of the extrapolating procedure for $\alpha_s = 1$ kinematics. In this case $R$ is calculated using both PWIA and DWIA models within VN approach for $x = 0.8$ and $x = 0.9$ at $Q^2 = 10 \text{ GeV}^2$ and for physically accessible values of $t' < 0$. Then we use three calculated points of $R$ to make a quadratic fit which is then extrapolated to the $t' \to 0$ values. For DWIA the FSI strength is set to $80 \text{ mb}$ which may be considered somewhat above the highest possible value for the cross section of the $X$-state scattering off the spectator nucleon. For the fitting points we choose three $R$ values calculated at $-t'$ (0.1, 0.2, 0.3) corresponding to (54MeV/c, 89MeV/c, 114MeV/c) values of recoil proton momenta. The choice of these three values is motivated by the kinematic acceptance of the experiment of Ref.[9].
Figure 4: Extrapolation of $R$ at $-(t - m_n^2) \to 0$ based on the square fit using first three points of calculated $R$. The triangles and open circles correspond to $R$ (Eq.(22) calculated within PWIA and DWIA using VN model.

The quadratic function fitted to above points yields the following extrapolated values for $R$ at $-t' = 0$: (0.9923 and 0.9888) for (PWIA and DWIA) predictions at $x = 0.8$ and (0.9868, 0.9828) for $x = 0.9$. These numbers demonstrate that extrapolation procedure practically eliminates the uncertainty due to the FSI, making FSI effects less than 0.5%.

Similar estimations within LC approximation yield for extrapolated values of $R$ at $-t' = 0$: (0.9983, 0.9946) for (PWIA, DWIA) predictions at $x = 0.8$ and (1.0078, 1.0036) for $x = 0.9$. These numbers combined with the numbers obtained above in the VN approximation demonstrate that the overall uncertainty of extrapolation procedure using the fitting points consistent with the kinematics of experiment of Ref.[9] is of the order of 1%. This is an unprecedented accuracy that can be achieved in extraction of high $x$ DIS structure function of the nucleons.

Note that due to the higher order singularities in the deuteron wave function, moving the fitting points away from the $t' = 0$ limit will worsen the convergence of the quadratically fitted function of $R$ to the unity. In this case an additional improvement of the procedure can be achieved if we additionally normalize $R$ in Eq.(22) by the momentum distribution of the deuteron.

5 Conclusions

We have considered the pole extrapolation procedure aimed at model independent extraction of high $x$ DIS structure function of struck nucleon in semiinclusive DIS scattering off the deuteron. For analysis of the validity of this procedure we considered two theoretical approaches (virtual
nucleon and light cone) for description of semiinclusive deep inelastic scattering from the deuteron. Using these approaches we modeled the final state interaction based on distorted wave impulse approximation.

Within this framework we demonstrated that a simple quadratic fit is well suited for on-mass shell extrapolation of the bound nucleon structure function. Although the strength of FSI is varied in the wide range of the scattering cross sections $0 \div 80$ mb the overall uncertainty due to FSI and the choice of the particular (VN or LC) theoretical approximation is estimated to be on the level of 1%. This result indicates the model independent character of the mass-shell extrapolation procedure.

The numerical estimates are done for $t'$ values characteristic to the experiment of Ref.[9]. Thus one can expect the same level of uncertainty in the extraction of on-shell structure functions of nucleon if the pole extrapolation procedure is applied to the actual data.

Acknowledgments:
This work is supported by DOE grants under contract DE-FG02-01ER-41172 and DE-FG02-93ER40771 as well as by the Israel-USA Binational Science Foundation Grant.

References

[1] M. Strikman, AIP Conf. Proc. 747, 243 (2005).
[2] L. L. Frankfurt and M. I. Strikman Phys. Rept. 160, 235 (1988).
[3] A. Bodek, S. Dasu and S.E. Rock, in Intersection Between Particle and Nuclear Physics, edited by Willem T. H. van Oers, AIP Conf. Proc. No. 243 (AIP, New York, 1992), P.1151.
[4] M.I. Strikman, in Proceedings of XXVI International Conference on High Energy Physics, Dallas, Edited by J.R. Sanford AIP Conf. Proc. 272 (AIP, New York, 1993) Vol. 2, P. 806.
[5] W. Melnitchouk and A. W. Thomas, Phys. Lett. B 377, 11 (1996)
[6] W. Melnitchouk, M. Sargsian and M. I. Strikman Z. Phys. A 359, 99 (1997).
[7] M. M. Sargsian et al., J. Phys. G 29, R1 (2003).
[8] M. M. Sargsian, AIP Conf. Proc. 747, 191 (2005).
[9] H. Fenker, C. Keppel, S. Kuhn, and W. Melnitchouk (spokespersons), The Structure of the Free Neutron Via Spectator Tagging, JLab proposal, E-03-012, (2003).
[10] L. Frankfurt, V. Guzey and M. Strikman, Phys. Lett. B 586, 41 (2004).
[11] G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).
[12] L. L. Frankfurt, M. M. Sargsian and M. I. Strikman, Phys. Rev. C 56, 1124 (1997).
[13] M. M. Sargsian, Int. J. Mod. Phys. E 10, 405 (2001).
[14] M.M. Sargsian and M.I. Strikman in progress.

[15] M. M. Sargsian, S. Simula and M. I. Strikman, Phys. Rev. C 66, 024001 (2002).

[16] C. Ciofi degli Atti, L. P. Kaptari and B. Z. Kopeliovich, Eur. Phys. J. A 19, 145 (2004)

[17] L. L. Frankfurt and M. I. Strikman, Phys. Rept. 76, 215 (1981).

[18] L. L. Frankfurt, W. R. Greenberg, G. A. Miller, M. M. Sargsian and M. I. Strikman, Z. Phys. A 352, 97 (1995).

[19] K. Griffioen S. Kuhn (spokespersons), Electron Scattering from a High Momentum Nucleon in Deuterium, JLab proposal E-94-102, (1994).

[20] A. V. Klimenko et al. [CLAS Collaboration]. [arXiv:nucl-ex/0510032]

[21] M. Lacombe, B. Loiseau, R. Vinh Mau, J. Cote, P. Pires and R. de Tourreil, Phys. Lett. B 101, 139 (1981).

[22] R. Machleidt, Phys. Rev. C 63, 024001 (2001).