Flux instabilities in type-II superconductors

N. A. Taylanov

National University of Uzbekistan

Abstract

The flux jump dynamics in the flux flow regime of type II superconductors is investigated, analytically. It is found that under some conditions flux jump avalanche may occur in a superconductor sample, which takes into account an inertial properties of the vortex matter.

Key words: flux jumps, vortex mass, critical state, flux flow.

Introduction

As we know, the flux jumps results in a large-scale flux avalanches in a superconductor and their origin are related to the magnetothermal instabilities [1-5]. Thermomagnetic instability or flux jump phenomena have been observed in conventional hard superconductor, as well as in high-temperature superconductors, recently [1-6]. The spatial and temporal development of this instability depends on the sample geometry, temperature, external magnetic field, its rate of change and orientation, initial and boundary conditions, etc. The critical state instabilities result in flux redistribution towards the equilibrium state and are accompanied by a significant heat release, which often leads to the superconductor-to-normal-transition. Recently, Chabanenko et al. [6] have reported an interesting phenomenon in their experiments - convergent oscillations of the magnetic flux arising from flux jump avalanches [6-11]. The authors argued that the observed oscillations due to flux avalanches can be interpreted as a result of the existence of a definite value of the effective vortex mass [12-21]. Thus, it is necessary to take into account collective modes, i.e., the inertial properties of the vortices in studying the dynamics of the flux avalanches. In the present work, we study the dynamics of the magnetic flux avalanche, which takes into account an inertial properties of the vortex matter.

1. Formulation

Bean [1] has proposed the critical state model which is successfully used to describe magnetic properties of type II superconductors. According to this model, the distribution of the magnetic flux density $\vec{B}$ and the transport current density $\vec{j}$ inside a superconductor is given by a solution of the equation

$$rot \vec{B} = \mu_0 \vec{j}. \quad (1)$$

When the penetrated magnetic flux changes with time, an electric field $\vec{E}(r,t)$ is generated inside the sample according to Faraday’s law

$$rot \vec{E} = \frac{1}{c} \frac{d \vec{B}}{dt}. \quad (2)$$

In the flux flow regime the electric field $\vec{E}(r,t)$ induced by the moving vortices is related with the local current density $\vec{j}(r,t)$ by the nonlinear Ohm’s law

$$\vec{E} = \nu \vec{B}. \quad (3)$$

To obtain qualitative results, we use a classical equation of motion of a vortex, which it can derived by integrating over the microscopic degrees of freedom, leaving only macroscopic forces [21]. Thus, the equation of the vortex motion under the action of the Lorentz, pinning, and viscosity forces can be presented as

$$m \frac{dV}{dt} + \eta V + F_L + F_p = 0. \quad (4)$$

Here $m$ is the vortex mass per unit length, $F_L = \frac{1}{c} \vec{E} \times \vec{v}$ is the Lorentz force, $F_p = \frac{1}{c^2} \xi_0 \eta \vec{B}$ is the pinning force, $\eta$ is the vortex viscosity, $\xi_0$ is the critical current density [4]. For simplicity we have neglected the Magnus force, assuming that it is much smaller than the viscous one (for example, for Nb see [6]). In the absence of external currents and fields, the Lorentz force results from currents associated with vortices trapped in the sample.

2. Basic equation

In combining the relation (3) with Maxwell’s equation (2), we obtain a nonlinear diffusion equation for the magnetic flux induction $\vec{B}(r,t)$ in the following form

$$m \frac{dV}{dt} + \eta V = -\frac{1}{c} \Phi_0 (\vec{j} - j_c), \quad (5)$$

$$\frac{d\vec{B}}{dt} = \nabla [\vec{v} \vec{B}], \quad (6)$$

The temperature distribution in superconductor is governed by the heat conduction diffusion equation

$$\nu(T) \frac{dT}{dt} = \nabla [\kappa(T) \nabla T] + j \vec{E}, \quad (7)$$

Here $\nu = \nu(T)$ and $\kappa = \kappa(T)$ are the specific heat and thermal conductivity, respectively. The above equations should be supplemented by a current-voltage characteristics of superconductors, which has the form

$$\vec{j} = j_c(T, \vec{B}, \vec{E}).$$

In order to obtain analytical results of equations (5)-(7), we suggest that $j_c$ is independent on magnetic field induction $\vec{B}$ and use the Bean critical state model $j_c = j_c(B_c,T)$, i.e., $j_c(T) = j_0 - a(T - T_0)$ [4]; where $B_c$ is the external applied magnetic field induction, $a = j_0/(T_c - T_0)$ and $T_c$ and $T_0$ are the equilibrium and critical temperatures of the sample, respectively, $j_0$ is the equilibrium current density. For the sake of simplifying of the calculations, we perform our calculations on the assumption of negligibly small heating and assume that the temperature profile is a constant within the across sample and thermal conductivity...
\( \kappa \) and heat capacity \( \nu \) are independent on the temperature profile \[5\].

We study the evolution of the thermal and electromagnetic penetration process in a simple geometry - superconducting semi-infinite sample \( x \geq 0 \). We assume that the external magnetic field induction \( B_e \) is parallel to the \( z \)-axis and the magnetic field sweep rate \( B_e \) is constant. When the magnetic field with the flux \( \Phi \) is applied in the direction of the \( z \)-axis, the transport current \( j(x, t) \) and the electric field \( E(x, t) \) are induced inside the slab along the \( y \)-axis. For this geometry the spatial and temporal evolution of thermal and magnetic field perturbations

\[
T = T_0 + \Theta(x, t),
\]

\[
B = B_e + b(x, t),
\]

\[
V = V_0 + v(x, t)
\]

are described by the following system of differential equations \[8, 11\]

\[
\frac{d\Theta}{dt} = 2v - \beta \Theta,
\]

\[
\mu \frac{dv}{dx} + v = -\frac{db}{dx} + \beta \Theta,
\]

\[
\frac{db}{dt} = \left( \frac{db}{dx} + b \right) + \left( \frac{dv}{dx} + v \right),
\]

where \( T_0(x), B_e(x) \) and \( V_0(x) \) are solutions to the unperturbed equations, which can be obtained within a quasi-stationary approximation. Here we have introduced the following dimensionless variables

\[
b = \frac{B}{B_e} = \frac{B}{\mu_0 j L}, \quad \Theta = \frac{\nu \mu_0}{B_e^2}, \quad v = \frac{V_0 t_0}{L},
\]

and parameters

\[
\mu = \frac{\Phi_0 B_e}{\mu_0 \eta^2 L^2}, \quad \beta = \frac{\mu_0 j^2 L^2}{\nu (F_e - T_0)}.
\]

Here \( L = c B_e / \mu_0 j \) is the magnetic field penetration depth, which is determined from the following equation

\[
B(x, t) = B_e + \mu_0 j(x - L),
\]

with the appropriate boundary conditions

\[
dB(0, t) = B_e, \quad B(L, t) = 0.
\]

### 3. Dispersion relation

Assuming that the small thermal and magnetic perturbations has the form \( \Theta(x, t), b(x, t), v(x, t) \sim \exp[\gamma t] \), where \( \gamma \) is the eigenvalue of the problem to be determined, we obtain from equations (9)-(11) the following dispersion relations to determine an eigenvalues of the problem

\[
(\gamma + \beta) \frac{d^2 b}{dx^2} - [(\gamma + \beta) \mu - 2\beta] \frac{db}{dx} + [(\mu + 1) \gamma^2 + +[(\mu - 1) \beta - \mu - 1] \gamma - (\mu - 1) \beta] b = 0
\]

The instability of the flux front is defined by the positive value of the rate increase \( \Re \gamma > 0 \). An analysis of the dispersion relation shows that, the growth rate is positive \( \Re \gamma > 0 \), if \( \mu > \mu_c = 2 \) and any small perturbations will grow with time. For the case when \( \mu < \mu_c \), the growth rate is negative and the small perturbations will decay. At the critical value of \( \mu = \mu_c \), the growth rate is zero \( \gamma = 0 \). For the specific case, where \( \mu = 1 \) the growth rate is determined by a stability parameter \( \beta \). Thus, the stability criterion can be written as

\[
\beta > 1.
\]

For the case, where thermal effects is negligible \( (\beta = 1) \) we may obtain the following dispersion relation

\[
\frac{d^2 b}{dx^2} - \mu \frac{db}{dx} + (\gamma - 1)(\mu + 1)b = 0.
\]

Seeking for \( b \sim \exp(ikx) \) in dispersion relation, the growth rate \( \gamma \) dependence can be obtained as a functions of wave number \( k \). Figure 1. The dependence of the growth rate on the wave number for \( \mu = 0.1, 0.5, 0.8 \).

We analyze the growth rate \( \gamma \) of small perturbations as a function of wave number \( k \). When \( k < k_c = \mu \) the growth rate is positive and any small perturbations will grow with time. For wave number \( k > k_c \), the growth rate \( \gamma \) is negative. Consequently, the small perturbations always decay. It can be shown that, for wave number \( k = k_c \) the growth rate is zero \( \gamma = 0 \). As the wave number approaches zero \( k \rightarrow 0 \) or infinity \( k \rightarrow \infty \) the growth rate approaches \( \gamma = 1 \) and small perturbations grow with time. As the wave number approaches unity \( k = 1 \) the growth rate is determined by the value of \( \mu \)

\[
\gamma = \frac{2\mu}{\mu + 1}.
\]

For \( \mu = 0 \) the growth rate is zero \( \gamma = 0 \). For \( \mu = 1 \) the growth rate is unity \( \gamma = 1 \). Since the growth rate is zero at the critical wave number and approaches to unity in the limit of zero wave number, there must exist a wave number in between that maximizes the growth rate. Figs. (1-4) show the growing rate, \( \gamma \), as
a function of the wave number $k$, for various values of the vortex mass $\mu$. As the value of $\mu$ increases, the corresponding growth rate increases.

**Conclusion**

Thus, in the present work we show that under some conditions flux avalanche may occur in superconductor sample, which takes into account the inertial properties of the vortices. It has been noticed that a detailed theoretical study of this problem will be presented in our further papers.

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Fig. 2-4. The dependence of the growth rate on the wave number for $\mu = 0.1, 0.5, 0.8$. 