The concept of effective field theory leads in a natural way to a construction principle for phenomenological sensible models known under the name of the Cheshire Cat Principle. We review its formulation in the chiral bag scenario and discuss its realization for the flavor singlet axial charge.

Quantum effects inside the chiral bag induce a color anomaly which requires a compensating surface term to prevent breakdown of color gauge invariance. The presence of this surface term allows one to derive in a gauge-invariant way a chiral-bag version of the Shore-Veneziano two-component formula for the flavor-singlet axial charge of the proton. We show that one can obtain a striking Cheshire-Cat phenomenon with a negligibly small singlet axial charge.

1 Introduction

The possibility of formulating a physical theory by means of equivalent field theories, defined in terms of different field variables, leads to a construction principle for phenomenological sensible models, which will be the subject of our presentation. Named the Cheshire Cat Principle (CCP), by Nielsen and collaborators, for reasons which will become apparent shortly, it allows to confront experimental data with a restricted parameter set obtained by imposing theoretical consistency within the model.

1 + 1-dimensional fermionic theories are bosonizable and thus an exact transformation relating the fermionic and bosonic fields may be defined, which allows the construction of the same $S$-matrix from apparently different bosonic and fermionic theories. The intricacies of describing fermions by bosonic fields results in the appearance of topologically non-trivial scenarios in field theory. Since both descriptions lead to the same physics, it is a matter of simplicity and/or taste which language to use. In terms of the 1+1-dimensional scenario, the Cheshire Cat Principle, as stated by Nielsen and collaborators, is exact and transparent, and therefore we shall discuss it in some detail in the next section.

In the real world, bosonization with a finite number of bosonic variables, is not exact. However the unproven theorem of Weinberg leading to the formulation of low energy effective theories supports the CCP. Moreover, the CCP can be used as a consistency check for effective theories, allowing to fix parameters, which would otherwise have to be determined from the data,
and therefore, the CCP, increases the predictive power of the theories.

At sufficiently low energies or long distances Quantum Chromodynamics (QCD) can be described accurately by an effective field theory written in terms of meson fields. In this regime, the color fermionic description of the theory is extremely complex due to confinement. The CCP is operative when one defines QCD in a two phase space-time scenario and matches appropriately two field theories. To be more specific, one defines a hypertube in four dimensions, the bag, whose interior is governed by conventional QCD described in terms of quark and gluon fields. In the exterior of the bag, one assumes a certain mesonic model satisfying the flavor symmetries of the theory and the basic postulates of field theory in accord with Weinberg’s theorem. The two descriptions are matched by using the appropriate boundary conditions which implement the symmetries and confinement. The CCP states that the physics should be approximately independent of the spatial size of the bag.

The CCP has been tested in many instances with notable success. There is one case, the flavor axial singlet charge (FSAC), whose implementation has not been successful beyond doubt and therefore it has merit our attention. From the phenomenological point of view the FSAC is associated with the $\eta'$ and therefore with the anomaly. This observable is relevant for what has been referred to as the proton spin problem. In the chiral bag model the formulation is very elaborate. Confinement induces through quantum effects a color anomaly, which leads to a surface coupling of the $\eta'$ with the gluon field. The latter induces a gauge non invariant Chern-Simons current, whose expectation value we need to calculate. We have shown, and we will review it here, that the presence of the surface term generated by the proper matching of the color anomaly with the surface gluon-$\eta'$ coupling allows us to derive in a gauge invariant way a chiral-bag version of the Shore-Veneziano two component formula for the FSAC of the proton.

2 Exact Cheshire Cat Principle in 1 + 1 dimensions

1 + 1-dimensional fermion theories are exactly bosonizable. The CCP arises when one implements bosonization in a bag. Let us consider a massless free single-flavored fermion $\psi$ confined to a region of volume $V$ (inside) coupled on the surface $\partial V$ to a massless free boson $\varphi$ living in a region of volume $V$ (outside). Of course in one space dimension, the volume is just a segment of a line but we will use the symbol in analogy

\footnote{We strongly advise to read the beautiful presentation of ref. where specially the graphics are extremely illuminating. One may also turn for completeness and further bibliography to ref.}
to higher dimensions. We will assume that the action is invariant under global chiral rotations and parity. The action contains three terms,

\[ S = S_V + S_V + S_{\partial V}, \]  

where

\[ S_V = \int_V d^2x \bar{\psi} \gamma^\mu \partial^\mu \psi + \ldots, \]  

\[ S_V = \int_V d^2x \frac{1}{2} (\partial^\mu \varphi)^2 + \ldots. \]  

Here the dots represent other possible contributions which we may add for phenomenological reasons satisfying the above restrictions. For the time being we will limit our discussion to the free theory. The boundary condition is essential in making the connection between the two regions. In 1+1 dimensions we are guided by the bosonization rules, which lead to

\[ S_{\partial V} = \int_{\partial V} d\Sigma_{\mu} \frac{1}{2} n^\mu \bar{\psi} e^{i\varphi} \psi, \]  

where the \( \varphi \) decay constant \( f = \frac{1}{4\pi} \), \( d\Sigma_{\mu} \) is an area element and \( n^\mu \) (\( n^2 = -1 \)) the normal vector.

At the classical level the fermion satisfies

\[ i\gamma^\mu \partial^\mu \psi = 0, \]  

inside, while outside the boson obeys

\[ \partial^\mu \partial^\mu \varphi = 0, \]  

subject to the boundary conditions

\[ i n_\mu \gamma^\mu \psi = -e^{\frac{2\pi\varphi}{f}} \psi, \]  

\[ n_\mu \partial^\mu \varphi = \frac{1}{2f} \bar{\psi} n_\mu \gamma^\mu \gamma^5 \psi. \]  

From the boundary condition, Eq. [7], one may prove

\[ n_\mu \bar{\psi} \gamma^\mu \psi = 0, \]  

at the surface, which states that no flux of isoscalar current flows through the surface. But we are suppose to be describing at this stage a free fermion
theory! The crucial observation that leads to the solution of this apparent contradiction is that this classical result is invalidated by quantum mechanical effects. The boundary condition generates a quantum anomaly in the fermion number current, i.e.,

$$dt \dot{Q} \sim \dot{\varphi} \sim \delta \Sigma \overline{\psi} \gamma_\mu \psi |_{\partial V} \sim t^\mu \partial_\mu,$$  

(10)

where $t_\mu$ is the tangential unit vector to the surface. In this way we recover the full bosonization equation,

$$\partial_\mu \varphi = \frac{1}{2f} \overline{\psi} \gamma_\mu \gamma_5 \psi.$$  

(11)

In two dimensions it turns out that

$$\varepsilon^{\mu \nu} \partial_\nu \varphi = \overline{\psi} \gamma^\mu \psi$$  

(12)

and this current is a topological current (trivially conserved), whose charges define the different solitonic sectors. In $V$ the fermions are described as solitons. The bag wall is transparent for the fermions.

2.1 Fermion charge leakage

The interaction of the pseudoscalar field $\varphi$ with the fermion field $\psi$ gives rise to an anomaly and therefore the fermion number is not conserved inside the bag. Physically what happens is that the amount of fermion number $dQ$ corresponding to $\dot{\varphi}$ is pushed into the Dirac sea at the bag boundary and so is lost from inside. This accumulated charge must be carried by something residing in the meson sector. The meson field can only carry fermion charge if it supports a soliton. In the present model we find that the fermion charge $Q = 1$ is partitioned as

$$Q = 1 = Q_V + Q_{\overline{V}},$$  

(13)

where

$$Q_V = 1 - \frac{\Theta}{\pi},$$  

(14)

$$Q_{\overline{V}} = \frac{\Theta}{\pi},$$  

(15)

with the chiral angle defined by

$$\Theta = \frac{\varphi}{f} |_{\partial V}$$  

(16)
We thus learn that the quark charge is partitioned into the bag and outside the bag, without however any dependence of the total on the size or location of the bag boundary. In the $1+1$-dimensional case, one can calculate other physical quantities such as the energy, response functions, partition functions, and show that the physics does not depend upon the presence of the bag. The complete independence of the physics on the bag-size or -location is the realization of the Cheshire Cat Principle, and in two dimensions it holds exactly because of the exact bosonization rule.

2.2 Color anomaly

Let us imitate the real world by incorporating a color charge. In the present case it will be sufficient with a $U(1)$ charge, since the corresponding model, i.e., the Schwinger model, is confining. When the anomaly at the boundary forces the quark to drawn into the Dirac sea, the color charge of the quark vanishes too,

$$\tag{17} \dot{Q}_c = \frac{e^2 \dot{\phi}}{2\pi f}.$$  

The charge becomes non-conserved and hence gauge invariance is broken. To avoid it, in a confining scenario (no gluons outside), we must introduce a compensating charge at the surface. Thus the surface action changes to

$$\tag{18} S_{\partial V} = \int_{\partial V} d\Sigma \left[ \frac{1}{2} \bar{\psi} \gamma_5 \phi f \psi - \frac{e^2}{2\pi} \varepsilon^{\mu\nu} n_\mu A_\nu (\phi) \right].$$

The boundary conditions change in order to respect the gauge symmetry. This consistency condition is crucial if the CCP is to be respected.

Let us conclude this section with some comments which capture the full scope of the CCP. The Schwinger model has a Coulomb confinement. The CCP tells us, that the bag does not confine! In the large bag limit, confinement is described by means of a linearly rising Coulomb potential between the fermions. In the small size limit, the boson acquires a mass

$$\tag{19} m^2 = \frac{e^2}{\pi}.$$  

It is this mass which confines them. In the bosonized Schwinger model the bosons represent the fractionally charged color fermions.

2.3 Approximate CCP in $3+1$-dimensions

There is no exact CCP in $3+1$-dimensions since fermion theories cannot be bosonized exactly. The strategy will thus change. We will implement effective
equivalent bosonic theories by means of Weinberg’s theorem, i.e., symmetry requirements and field theory principles, and will invoke the CCP as a consistency condition of the bosonized theory.

We consider quarks confined within a volume $V$ which we take spherical for definiteness, surrounded by a triplet of Goldstone bosons $\pi = \vec{\pi} \cdot \frac{\tau}{2}$ and the singlet $\eta'$ populating the outside volume $\nabla$. In such a scenario the action is given by the three terms

$$S = S_V + S_{\nabla V} + S_{\partial V},$$

where

$$S_V = S_{\text{QCD}},$$

$$S_{\nabla V} = S_{\text{effective}} = \frac{f^2}{4} \int_V d^4x \left[ \text{Tr} (\partial_\mu U^+) (\partial^\mu U) + \frac{m_\eta^2}{4N_f} \text{Tr} (\ln U - \ln U^+)^2 \right].$$

Here $U$ is given by

$$U = e^{i \eta_f f_0} e^{\frac{2i\pi}{f}}$$

and $f_0 = \sqrt{\frac{N_f}{2}} f$. We use $\eta$ to symbolize the $\eta'$ field.

The surface action is non trivial because it must contain the terms necessary to cancel the color anomaly induced by the $\eta'$ coupling. Its functional form is given by

$$\frac{1}{2} \int_{\partial V} d\Sigma^\mu \left[ n_\mu \bar{\psi} U_5 \psi + i \frac{g^2}{16\pi^2} K_5^\mu \text{Tr} (\ln U^+ - \ln U) \right],$$

where

$$U_5 = e^{i \frac{\eta}{m_\eta}} e^{\frac{2i\pi}{f}}$$

and $K_5^\mu$ is the so called Chern-Simons current

$$K_5^\mu = e^{\mu\nu\alpha\beta} \left( A_\nu^a F_{a\alpha}^\beta - \frac{2}{3} \delta^a_f A_\nu^a A_\alpha^b A_\beta^\beta \right).$$

The CCP has been observed at the level of topological quantities, i.e., baryon charge fractionation and approximately at the level of non topological observables, i.e., masses, magnetic moments, etc... The explicit manifestation of the CCP in the latter is through some type of minimum sensitivity principle in terms of the bag radius. Moreover the mean value about which observable are not sensitive to the radius corresponds to the confinement scale $R \sim \frac{1}{\Lambda_{\text{QCD}}}$.

\footnote{We shall just consider $u$ and $d$ flavors. If strangeness is included the above lagrangian has to be supplemented by the Wess-Zumino-Witten term.}
3 Anomaly and proton spin

The anomalous suppression of the first moment, $\Gamma_1^p$, of the polarised proton structure function $g_1^p$ has been the focus of intense theoretical and experimental activity for nearly a decade. While it is now generally accepted that the key to understanding this effect is the existence of the chiral $U(1)$ anomaly in the flavor singlet pseudovector channel there are several explanations reflecting different theoretical approaches to proton structure. In here we analyze the phenomenon from the point of view of the Cheshire Cat Principle.

The starting point is the sum rule for the first moment, i.e.,

$$\Gamma_1^p(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2) = \frac{1}{12} C_1^N S(\alpha_s(Q^2)) \left( a^3 + \frac{1}{3} a^8 \right) + \frac{1}{9} C_1^S(\alpha_S(Q^2)) a^0(Q^2). \quad (25)$$

Here $a^3$, $a^8$ and $a^0(Q^2)$ are the form factors in the forward proton matrix elements of the renormalised axial current, i.e.,

$$<p,s|A_\mu^3|p,s> = s_\mu a^3, \quad <p,s|A_\mu^8|p,s> = s_\mu \frac{1}{2\sqrt{3}} a^8,$$

and

$$<p,s|A_\mu^0|p,s> = s_\mu a^0,$$

where $p_\mu$ and $s_\mu$ are the momentum and the polarisation vector of the proton. $a^3$ and $a^8$ can be chosen $Q^2$ independent and may be determined from the $G_A$ and $F_D$ ratios. $a^0(Q^2)$ evolves due to the anomaly and its evolution can be described in the AB scheme \cite{21} by

$$a^0(Q^2) = \Delta \Sigma - N_F \frac{\alpha_s(Q^2)}{2\pi} \Delta g(Q^2). \quad (26)$$

Naive models or the OZI approximation to QCD lead at low energies to

$$a^0 \approx a^8 \approx 0.69 \pm 0.06. \quad (27)$$

Experiments \cite{22}

$$a^0(\infty) = 0.10 \pm 0.17. \quad (28)$$

The explanation of this unexpected behavior ranges from those authors attributing the whole effect to hadron structure \cite{23}, to those attributing it to evolution \cite{24}. We shall discuss the problem from the point of view of hadron structure and analyze if the Cheshire Cat Principle is realized within the formalism described in the previous section.
3.1 Chiral bag formulation

We next show that taking a proper account of the surface term, cf. Eq. 23, allows us to formulate a fully consistent gauge invariant treatment of the flavor-singlet axial current (FSAC) matrix element, which is related to the above observables.

\[ a^0(Q^2) = \frac{NF}{M} < p, s | O | p, s >, \]  

where

\[ O = \frac{\alpha_s}{8\pi} Tr(\bar{G}_{\mu\nu}G^{\mu\nu}) + \frac{1}{2NF} (1 - Z) \partial^\mu A^0_{\mu,\text{bare}}. \]  

\[ Z \] is the renormalization constant of the current.

The equations of motion for the gluon and quark fields inside and the \( \eta' \) field outside are the same as in Eq. 12. However the boundary conditions on the surface now read

\[ \hat{n} \cdot \vec{E}^a = -\frac{N_F g^2}{8\pi f} \hat{n} \cdot \vec{B}^a \eta \]  

\[ \hat{n} \times \vec{B}^a = \frac{N_F g^2}{8\pi f} \hat{n} \times \vec{E}^a \eta \]

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where \( C = \frac{NFg^2}{16\pi} \) and \( \vec{E}^a \) and \( \vec{B}^a \) are, respectively, the color electric and color magnetic fields. Here \( \psi \) is the QCD quark field.

As it stands, the boundary condition for the \( \eta' \) field looks gauge non-invariant because of the presence of the normal component of the Chern-Simons current on the surface. However this is not so. As shown in Eq. 17, the term on the LHS of Eq. 33 is not well-defined without regularization and when properly regularized, say, by point-splitting, it can be written in terms of a well-defined term which we will write as \( \frac{1}{2} \) : \( \bar{\psi} \gamma \gamma_5 \psi \) : plus a gauge non-invariant term (see Eq.(2) of 17) which cancels exactly the second term on the RHS. The resulting boundary condition

\[ \frac{1}{2} \hat{n} \cdot (\bar{\psi} \gamma \gamma_5 \psi) = f \hat{n} \cdot \vec{E}^a \eta + C \hat{n} \cdot \vec{K} \]

is then perfectly well-defined and gauge-invariant. However it is useless as it stands since there is no simple way to evaluate the left-hand without resorting to a model. Our task in the chiral bag model is to express the well-defined operator : \( \bar{\psi} \gamma \gamma_5 \psi \) : in terms of the bagged quark field \( \Psi \). In doing this, our key strategy is to eliminate gauge-dependent surface terms by the NRWZ surface counter term.
3.2 Flavor-singlet axial current

Let us write the flavor-singlet axial current in the model as a sum of two terms, one from the bag and the other from the outside populated by the meson field $\eta'$ (we will ignore the Goldstone pion fields for the moment)

$$A^\mu = A_B^\mu \Theta_B + A_M^\mu \Theta_M.$$  \hspace{1cm} (35)

We shall use the short-hand notations $\Theta_B = \theta(R - r)$ and $\Theta_M = \theta(r - R)$ with $R$ being the radius of the bag which we shall take to be spherical in this paper. We demand that the $U_A(1)$ anomaly be given in this model by

$$\partial_\mu A^\mu = \frac{\alpha_s N_f}{2\pi} \sum a \vec{E}^a \cdot \vec{B}^a \Theta_B + f m_\eta^2 \eta \Theta_M.$$  \hspace{1cm} (36)

Our task is to construct the FSAC in the chiral bag model that is gauge-invariant and consistent with this anomaly equation. Our basic assumption is that in the nonperturbative sector outside of the bag, the only relevant $U_A(1)$ degree of freedom is the massive $\eta'$ field. (The possibility that there might figure additional degrees of freedom in the exterior of the bag co-existing with the $\eta'$ and/or inside the bag co-existing with the quarks and gluons will be discussed later.) This assumption allows us to write

$$A_M^\mu = f \partial^\mu \eta$$  \hspace{1cm} (37)

with the divergence

$$\partial_\mu A_M^\mu = f m_\eta^2 \eta.$$  \hspace{1cm} (38)

Now the question is: what is the gauge-invariant and regularized $A_B^\mu$ such that the anomaly (36) is satisfied? To address this question, we rewrite the current absorbing the theta functions as

$$A^\mu = A_1^\mu + A_2^\mu$$  \hspace{1cm} (39)

such that

$$\partial_\mu A_1^\mu = f m_\eta^2 \eta \Theta_M,$$  \hspace{1cm} (40)

$$\partial_\mu A_2^\mu = \frac{\alpha_s N_f}{2\pi} \sum a \vec{E}^a \cdot \vec{B}^a \Theta_B.$$  \hspace{1cm} (41)

We shall deduce the appropriate currents in the lowest order in the gauge coupling constant $\alpha_s$ and in the cavity approximation for the quarks inside the bag.
The “quark” current $A_1^\mu$

Let the bagged quark field be denoted $\Psi$. Then to the lowest order in the gauge coupling and ignoring possible additional degrees of freedom alluded above, the boundary condition (34) is

$$\frac{1}{2} \hat{n} \cdot (\nabla \gamma_5 \Psi) = f \hat{n} \cdot \partial \eta$$

and the corresponding current satisfying (40) is

$$A_1^\mu = A_{1q}^\mu + A_{1\eta}^\mu$$

with

$$A_{1q}^\mu = (\nabla \gamma_5 \Psi) \Theta_B,$$

$$A_{1\eta}^\mu = f \partial^\mu \eta \Theta_M.$$

We shall now proceed to obtain the explicit form of the bagged axial current operator. In momentum space, the quark contribution is

$$A_{1q}^\mu(q) = \frac{1}{2} \int d^3r e^{iq \cdot r} \langle N_{Bag} | \Psi^\dagger \sigma_j \Psi | N_{Bag} \rangle$$

$$= (a(q) \delta_{jk} + b(q)(3 \delta_{jk} \hat{q}_k - \delta_{j,k})) \left( \frac{1}{2} \sum_{\text{quarks}} \sigma^k \right)$$

where

$$a(q) = N^2 \int d\omega r^2 j_0^2(\omega r) - \frac{1}{3} j_1^2(\omega r) j_0(qr),$$

$$b(q) = \frac{2}{3} N^2 \int d\omega r^2 j_1^2(\omega r) j_2(qr)$$

where $N$ is the normalization constant of the (bagged) quark wave function. In the limit that $q \to 0$ which is what we want to take for the axial charge, both terms are non-singular and only the $a(0)$ term survives, giving

$$A_{1q}^\mu(0) = g_{A, quark}^0 \left( \frac{1}{2} \sum_{\text{quarks}} \sigma^j \right)$$

where $g_{A, quark}^0$ is the singlet axial charge of the bagged quark which can be extracted from (47). In the numerical estimate made below, we shall include the Casimir effects associated with the hedgehog pion configuration to which...
the quarks are coupled \[^{25,26}\], so the result will differ from the naive formula \(^{(47)}\).

To obtain the \(\eta'\) contribution, we take the \(\eta'\) field valid for a static source

\[
\eta(\vec{r}) = -\frac{g}{4\pi M} \int d^3 r' \chi^\dagger \vec{S} \chi \cdot \nabla \frac{e^{-m_\eta |\vec{r} - \vec{r}'|}}{|\vec{r} - \vec{r}'|}
\]  

(50)

where \(g\) is the short-hand for the \(\eta'NN\) coupling constant, \(M\) is the nucleon mass, \(\chi\) the Pauli spinor for the nucleon and \(\vec{S}\) the spin operator. The contribution to the FSAC is

\[
A_{1\eta}(q) = \int_{V_B} d^3r e^{i \vec{q} \cdot \vec{r}} \hat{f} \delta_j \eta,
\]

\[
= (c(q) \delta_{j,k} + d(q)(3\hat{q}_j \hat{q}_k - \delta_{j,k}))(\frac{1}{2} \sum_{\text{quarks}} \sigma^k)
\]  

(51)

with

\[
c(q) = \frac{fg}{2M} \int_R^\infty dr r^2 e^{-m_\eta r} m_\eta^2 j_0(qr),
\]

(52)

\[
d(q) = -\frac{fg}{2M} \int_R^\infty dr r^2 e^{-m_\eta r} [r^2 m_\eta^2 + 3(m_\eta r + 1)] j_2(qr).
\]

(53)

In the zero momentum transfer limit\(^c\), we have

\[
A_{1\eta}(0) = \frac{gf}{2M} [(y_\eta^2 + 2(y_\eta + 1)) \delta_{j,k} - y_\eta^2 \hat{q}_j \hat{q}_k] e^{-y_\eta} \langle S^k \rangle
\]

(54)

where \(y_\eta = m_\eta R\).

The boundary condition \(^{[12]}\) provides the relation between the quark and \(\eta'\) contributions. In the integrated form, \(^{(42)}\) is

\[
\int d\Sigma f x_3 \hat{r} \cdot \nabla \eta = \int_{V_B} d^3r \frac{1}{2} \psi \gamma_3 \gamma_5 \psi
\]

(55)

from which follows

\[
\frac{gf}{M} = 3 \frac{e^{y_\eta}}{y_\eta^2 + 2(y_\eta + 1)} g_{A,\text{quark}}.
\]

(56)

\(^c\)With however \(m_\eta \neq 0\). The limiting processes \(q \to 0\) and \(m_\eta \to 0\) do not commute as we will see shortly.
This is a Goldberger-Treiman-like formula relating the asymptotic pseudoscalar coupling to the quark singlet axial charge. From (49), (54) and (56), we obtain

\[ A_1^j = g^0_{A_1} \langle S^j \rangle \]  
(57)

with

\[ g^0_{A_1} = \frac{g f}{3 M} \left( \frac{y_q^2 + 2(y_q + 1)}{e^{y_q}} \right) = \frac{3}{2} g^0_{A_{\text{quark}}}. \]  
(58)

This is completely analogous to the isovector axial charge \( g^3_A \) coming from the bagged quarks inside the bag plus the perturbative pion fields outside the bag. Note that the singlet charge \( g^0_{A_1} \) goes to zero when the bag is shrunk to zero, implying that the coupling constant \( g \) goes to zero as \( R \to 0 \) as one can see from eq. (56). This is in contrast to \( g^3_A \) where the axial charge from the bag “leaks” into the hedgehog pion outside the bag and hence even when the bag shrinks to zero, the isovector axial charge remains more or less constant in agreement with the Cheshire Cat.

An interesting check of our calculation of \( \vec{A}_1 \) can be made by looking at the \( m_\eta \to 0 \) limit. From (46) and (54), we find that our current satisfies

\[ \hat{q} \cdot \vec{A}_1(0) = \frac{g f}{M} (y_q + 1) e^{-y_q} \langle \hat{q} \cdot \vec{S} \rangle \]  
(59)

which corresponds to eq. (40). Now eq. (40) is an operator equation so one can take the limit \( m_\eta \to 0 \) and expect the right-hand side to vanish, obtaining \( \hat{q} \cdot \vec{A}_1 \to 0 \). Equation (59) fails to satisfy this. The reason for this failure is that the \( q \to 0 \) and \( m_\eta \to 0 \) limits do not commute. To obtain the massless limit, one should take the \( \eta' \) mass to go to zero first.

Before taking the zero-momentum limit, the expression for \( c(q) \) for the \( \eta \) field, (52), is

\[ c(q) = \frac{f g}{3 M} \left( \frac{m_\eta^2 e^{-y_\eta} (\cos(qR) + \frac{m_\eta}{q} \sin(qR))}{q^2} \right) \]  
(60)

which vanishes in the \( m_\eta \to 0 \) limit. On the other hand, the \( d(q) \), (53), which before taking the zero-momentum limit, is of the form

\[ d(q) = -\frac{f g}{2 M} \left( \frac{e^{-y_\eta} (y_q^2 + 3y_q + 3)}{qR} \right) j_1(qR) \]

\[ \left[ -\frac{m_\eta^2}{q^2} e^{-y_\eta} (y_q + 1) j_0(qR) + \frac{m_\eta^2 e^{-y_\eta} (\cos(qR) + \frac{m_\eta}{q} \sin(qR))}{q^4} \right] \]  
(61)
becomes in the $m_\eta \to 0$ limit
\[- \frac{f q}{2M} j_1(qR) \to 0 \]  
(62)

Adding the quark current in the $q \to 0$ limit, we get
\[ A_I^j(0) = \frac{f g}{M} (\delta^{jk} - \hat{q}^j \hat{q}^k) S_k \]  
(63)

which satisfies the conservation relation. This shows that our formulas are correct.

Note that the massless limit leads to
\[ g^0_A = \frac{g f}{2M} \]  
(64)

which is the $U(1)$ G-T relation of Shore and Veneziano in the OZI limit. In this formula one can recover the target independent factor which is determined by $f \sim \sqrt{\chi'(0)}$ while $g \sim \Gamma_{\varphi,NN}$ carries the target dependent contribution.

**The gluon current $A_2^\mu$**

The current $A_2^\mu$ involving the color gauge field is very intricate because it is not possible in general to write a gauge-invariant dimension-3 local operator corresponding to the singlet channel. We will see however that it is possible to obtain a consistent axial charge within the model. Here we shall calculate it to the lowest nontrivial order in the gauge coupling constant. In this limit, the right-hand sides of the boundary conditions (31) and (32) can be dropped, reducing to the original MIT boundary conditions. Furthermore the gauge field decouples from the other degrees of freedom precisely because of the color anomaly condition that prevents the color leakage, namely, the condition (34). In its absence, this decoupling could not take place in a consistent way.

We start with the divergence relation
\[ \partial_\mu A_2^\mu = \frac{\alpha_s N_f}{2\pi} \sum_a \bar{E}^a \cdot \bar{B}^a \Theta_{VB}. \]  
(65)

In the lowest-mode approximation, the color electric and magnetic fields are given by
\[ \bar{E}^a = \frac{\lambda^a}{4\pi} \frac{\hat{r}}{r^2} \rho(r) \]  
(66)

$^d$Here $\chi$ represents the topological susceptibility of the QCD vacuum.

$^e$To higher order in the gauge coupling, the situation would be a lot more complicated. A full Casimir calculation will be required to assure the consistency of the procedure. This problem will be addressed in a future publication.
\[ \vec{B}^a = g_s \frac{\lambda^a}{4\pi} \left( \frac{\mu(r)}{r^3} (3\hat{r} \vec{\sigma} \cdot \hat{r} - \vec{\sigma}) + \left( \frac{\mu(R)}{R^3} + 2M(r) \right) \bar{\sigma} \right) \]  

(67)

where \( \rho \) is related to the quark scalar density \( \rho' \) as

\[ \rho(r) = \int_\Gamma ds \rho'(s) \]  

(68)

and \( \mu, M \) to the vector current density

\[ \mu(r) = \int_0^r ds \mu'(s), \]

\[ M(r) = \int_r^R ds \mu'(s) s. \]

The lower limit \( \Gamma \) usually taken to be zero in the MIT bag model will be fixed later on. It will turn out that what one takes for \( \Gamma \) has a qualitatively different consequence on the Cheshire-Cat property of the singlet axial current.

Substituting these fields into the RHS of eq. (65) leads to

\[ \vec{q} \cdot \vec{A}^2 = 8 \alpha_s \frac{\lambda^a}{9\pi} \vec{q} \int_0^R dr \rho(r) \left( \frac{2\mu(r)}{r^3} + \frac{\mu(R)}{R^3} + 2M(r) \right) j_1(qr) \]  

(69)

where \( \alpha_s = \frac{g^2}{4\pi} \) and we have used \( \sum_{i \neq j} \sum_a \lambda^a_i \lambda^a_j = -\frac{8}{3} \) for the baryons.

In order to calculate the axial charge, we take the zero momentum limit and obtain

\[ \lim_{q \to 0} \vec{A}^2_2(\vec{q}) = \frac{8 \alpha_s^2 N_f}{9\pi} \hat{A}^2_2(R) \vec{S} \]  

(70)

where

\[ \hat{A}^2_2(R) = \int_0^R rdr \rho(r) \left( 2M(r) + \frac{\mu(R)}{R^3} + \frac{2\mu(r)}{r^3} \right) = 2 \int_0^R dr r \rho(r) \alpha(r). \]  

(71)

The quantity \( \alpha(r) \) is defined for later purposes. It is easy to convince oneself that (70) is gauge-invariant, i.e., it is \( \propto \int_{V_B} d^3r \vec{a} \cdot \vec{B}^a \) which is manifestly gauge-invariant. The result (70) was previously obtained in [30].

\[ \text{Here we are making the usual assumption as in ref. [29] that the } i = j \text{ terms in the color factor are to be excluded from the contribution on the ground that most of them go into renormalizing the single-quark axial charge. If one were to evaluate the color factor without excluding the diagonal terms using only the lowest mode, the anomaly term would vanish, which of course is incorrect. As emphasized in ref. [29], there may be residual finite contribution with } i = j \text{ but no one knows how to compute this and so we shall ignore it here. It may have to be carefully considered in a full Casimir calculation yet to be worked out.} \]
The two-component formula

The main result of this paper can be summarized in terms of the two component-formula for the singlet axial charge (with \( c_1 = 0 \)),

\[
g_A^0 = g_A^0 + g_A^0 = \frac{3}{2} g_A^{0, \text{quarks}} + \frac{8a_s^2 N_f}{9\pi} A_2(R). \tag{72}
\]

The first term is the “matter” contribution \( \frac{58}{70} \) and the second the gauge-field contribution \( \frac{72}{33} \). This is the chiral-bag version of Shore-Veneziano formula \( \frac{15}{33} \) relating the singlet axial charge to a sum of an \( \eta' \) contribution and a glueball contribution,

\[
g_A^0 = \frac{f g_{\eta NN}}{2M} + \frac{f^2 m_N^2}{2N_f} g_{GNN}(0) \tag{73}
\]

It is immediate to realize that there exists a one to one correspondence between the two expressions. The first term establishes a microscopic description of the \( \eta' \) coupling in a quarkish scenario \( \frac{13}{21} \). The second term reformulates the glueball contribution in terms of the axial anomaly inside the bag.

4 Results

In this section, we shall make a numerical estimate of \( \frac{58}{70} \) and \( \frac{70}{74} \) in the approximation that is detailed above. In evaluating \( \frac{58}{70} \), we shall take into account the Casimir effects due to the hedgehog pions but ignore the effect of the \( \eta' \) field on the quark spectrum. The interaction between the internal and external degrees of freedom occurs at the surface. Our approximation consists of neglecting in the expansion of the boundary condition in powers of \( \frac{1}{f} \) all \( \eta \) dependence, i.e.

\[
i' \hat{r} \cdot \gamma \Psi = e^{i \gamma_5 \hat{r} \cdot \hat{G}} e^{i \gamma_5 \eta \hat{f}} \Psi \sim e^{i \gamma_5 \hat{r} \cdot \hat{G}} \Psi \tag{74}
\]

This approximation is justified by the massiveness of the \( \eta' \) field in comparison to the Goldstone pion field that supports the hedgehog configuration. Within this approximation, we can simply take the numerical results from \( \frac{12}{20} \), changing only the overall constants in front.

The same is true with the gluon contribution. To the lowest order in \( \alpha_s \), the equation of motion for the gluon field is the same as in the MIT bag model. This is easy to see, since the modified boundary conditions eqs. \( \frac{31}{32} \) and \( \frac{32}{32} \) become

\[
i' \hat{r} G^{\mu} = -\frac{\alpha_s N_F}{2\pi} \hat{f} i' \hat{G}^{\mu} \sim 0. \tag{75}
\]
The only difference from the MIT model is that here the quark sources for the gluons are modified by the hedgehog pion field in (74). Again the results can be taken from 12,20 modulo an overall numerical factor.

In evaluating the anomaly contribution (71), we face the same problem with the monopole component of the $\vec{E}^a$ field as in 12. If we write

$$\vec{E}^a_i(r) = f(r)\hat{r}\lambda^a_i$$

(76)

where the subscript $i$ labels the $i$th quark and $a$ the color, the $f(r)$ satisfying the Maxwell equation is

$$f(r) = \frac{1}{4\pi r^2} \int_\Gamma ds' \rho'(s) \equiv \frac{1}{4\pi r^2} \rho(r).$$

(77)

If one takes only the valence quark orbit – which is our approximation, then $\rho'$ in the chiral bag takes the same form as in the MIT model. However the quark orbit is basically modified by the hedgehog boundary condition, so the result is of course not the same. The well-known difficulty here is that the bag boundary condition for the monopole component

$$\hat{r} \cdot \vec{E}^a_i = 0, \quad \text{at } r = R$$

(78)

is not satisfied for $\Gamma \neq R$. Thus as in 12,20, we shall consider both $\Gamma = 0$ and $\Gamma = R$.

The existence of a solution which satisfies explicitly and locally the boundary condition suggests an approach different from the one in the original MIT calculation 28, where the boundary condition of the electric field was imposed as an expectation value with respect to the physical hadron state. In 28, the

9 These choices describe two opposite scenarios for confinement. The standard MIT solution proposes a mechanism for satisfying the boundary condition of the electric field based on the color matrices which does not impose any restriction on $\Gamma$. The value $\Gamma = 0$ was chosen because the field becomes the typical field of a charged sphere in the abelianized theory. The spatial structure of the electric field is locally non-confining. The procedure introduces an asymmetry in the way confinement is realized between the electric and magnetic fields. This asymmetry leads to a peculiar treatment of the self-energy terms in the model. The $\Gamma = R$ solution describes the opposite scenario. Color electric screening occurs explicitly. As one moves away from the center the color charge decreases so that at the surface the color electric charge of the bag is zero. Abelianizing the theory, i.e., eliminating the color matrices, the mechanism can be visualized as a pointlike charge (-Q) at the origin superposed to the conventional spherical charge distribution, whose total charge is +Q. In this way one sees explicitly that the field lines for the quarks are moving towards the interior of the sphere, contrary to what happens in the other scenario. Now the deeper question arises: what is the microscopic mechanism producing this scenario? In our opinion this is a spherical modelization of the flux tube.
$E$ and $B$ field contributions to the spectrum were treated on a completely different footing. While in the former the contribution arising from the quark self-energies was included, thereby leading to the vanishing of the color electric energy, in the latter they were not. This gave the color magnetic energy for the source of the nucleon-$\Delta$ splitting. We have performed a calculation for the energy with the explicitly confined $E$ and $B$ field treated in a symmetric fashion\(^\text{20}\). Although in this calculation the contribution of the color-electric energy was non-vanishing, it was found not to affect the nucleon-$\Delta$ mass splitting, and therefore could be absorbed into a small change of the unknown parameters, i.e., zero point energy, bag radius, bag pressure etc. As we shall see shortly, the two ways of treating the confinement with $\Gamma = R$ and $\Gamma = 0$ give qualitatively different results for the role of the anomaly. One could consider therefore that the singlet axial charge offers a possibility of learning something about confinement within the scheme of the chiral bag. At present, only in heavy quarkonia\(^\text{32}\) does one have an additional handle on these operators.

The numerical results for both cases are given in Table 1.

| $R$(fm) | $\theta/\pi$ | $g_A^\prime$ | $g_A^\prime(\Gamma = R)$ | $g_A^\prime(\Gamma = 0)$ | $g_A^\prime(\Gamma = R)$ | $g_A^\prime(\Gamma = 0)$ |
|------|-----------|------------|------------------|------------------|------------------|------------------|
| 0.2  | -0.742    | 0.033      | -0.015           | 0.009            | 0.018            | 0.042            |
| 0.4  | -0.531    | 0.164      | -0.087           | 0.046            | 0.077            | 0.210            |
| 0.6  | -0.383    | 0.321      | -0.236           | 0.123            | 0.085            | 0.444            |
| 0.8  | -0.277    | 0.494      | -0.434           | 0.232            | 0.060            | 0.726            |
| 1.0  | -0.194    | 0.675      | -0.635           | 0.352            | 0.040            | 1.027            |
| $\infty$ | 0.00      | 0.962      | -1.277           | 0.804            | -0.297           | 1.784            |

5 Discussion

The quantity we have computed here is relevant to two physical issues: the so-called “proton spin” issue and the Cheshire-Cat phenomenon in the baryon structure. A more accurate result awaits a full Casimir calculation which appears to be non-trivial. However we believe that the qualitative feature of the given model with the specified degrees of freedom will not be significantly modified by the full Casimir effects going beyond the lowest order in $\alpha_s$. 

17
In the current understanding of the polarized structure functions of the nucleon, the FSAC matrix element or the flavor-singlet axial charge of the proton is related to the singlet axial charge\(^{23,24}\):

\[ a_0(Q^2) = \Delta \Sigma(1, Q^2) - N_F \frac{\alpha_s(t)}{2\pi} \Delta g(1, Q^2) \]  

(79)

The presently available analyses give\(^{23}\)

\[ a_0(\infty) = 0.10 \pm 0.05 \text{(exp)} \pm 0.17 \text{(th)} = 0.10 \pm 0.11 \]  

(80)

Evolution to lower momenta will increase this value according to a multiplicative factor (see Fig. 5 in ref.\(^{22}\)), which is not large as long as one remains in the regime where the perturbative expansion is valid. Our predictions for \(g_A^0\) – which can be compared with \(a_0(\mu_0^2)\), where \(\mu_0^2\) is the hadronic scale at which the model is defined\(^{23}\) – differ drastically depending upon whether one takes \(\Gamma = 0\) for which the color electric monopole field satisfies only globally the boundary condition at the leading order (that is, as a matrix element between color-singlet states) as in the standard MIT bag-model phenomenology or \(\Gamma = R\) which makes the boundary condition satisfied locally. The former configuration severely breaks the Cheshire Cat with the bag radius \(R\) constrained to less than 0.5 fm (“little bag scenario”) to describe the empirical value (80). This is analogous to what Dreiner, Ellis and Flores\(^{34}\) obtained.

On the other hand, the configuration with \(\Gamma = R\) which we favor leads to a remarkably stable Cheshire Cat in consistency with other non-anomalous processes where the Cheshire Cat is seen to hold within, say, 30\%\(^{20,27}\). The resulting singlet axial charge \(g_A^0 < 0.1\) is entirely consistent with (80). One cannot however take the near zero value predicted here too literally since the value taken for \(\alpha_s\) is perhaps too large. Moreover other short-distance degrees of freedom not taken into account in the model (such as the light-quark vector mesons and other massive mesons) can make a non-negligible additional contribution\(^{31}\). What is noteworthy is that there is a large cancellation between the “matter” (quark and \(\eta'\)) contribution and the gauge field (gluon) contribution in agreement with the interpretation anchored on \(U_A(1)\) anomaly\(^{24}\).

As mentioned above – and also noted in\(^{12,20}\) – the electric monopole configuration with \(\Gamma = R\) is non-zero at the origin and hence is ill-defined there. This feature does not affect, however, other phenomenology as shown in\(^{23}\). We do not know yet if this ambiguity can be avoided if other multipoles and higher-order and Casimir effects are included in a consistent way. This caveat notwithstanding, it seems reasonable to conclude from the result that if one accepts that the singlet axial charge is small because of the cancellation in the two-component formula and if in addition one demands that the Cheshire Cat
hold in the $U_A(1)$ channel as in other non-anomalous sectors, we are led to (1)
adopt the singular monopole configuration that satisfies the boundary condi-
tion *locally* and (2) to the possibility that within the range of the bag radius
that we are considering, the $\eta'$ is primarily quarkish.

The fact that the CCP is realized in a specific dynamical way, requires fur-
ther investigations. It would seem natural that, if the effective theory chosen to
describe the low energy properties where close to reality, the CCP would arise
in a more natural fashion. Two have been our main simplifying assumptions.
In the exterior we have taken the minimal theory containing the required de-
grees of freedom. In the interior we have performed an approximate calculation
which obviates the full content of the coupling of gluons and $\eta'$. The CCP will
become a fundamental principle of hadron structure if it survives in a natural
way the complete calculation.

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Almost twenty years ago I was introduced to Mannque by Gerry Brown.
The *Little Bag* was being developed and I joined in. I recall those days with
pleasure: I was young and the physics was great!. Since then I have collabo-
rated with Mannque on some occasions, during which I was able to appreciate
his fascination for physics. I wish him well, in this very merited celebration,
and hope that our joint ventures continue for a long time.

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