Closed String Amplitudes from Gauge Fixed String Field Theory

Nadav Drukker

Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100 Israel

(Dated: July 30, 2002)

Closed string diagrams are derived from cubic open string field theory using a gauge fixed kinetic operator. The basic idea is to use a string propagator that does not generate a boundary to the world sheet. Using this propagator and the closed string vertex, the moduli space of closed string surfaces is covered, so closed string scattering amplitudes should be reproduced. This kinetic operator could be a gauge fixed form of the string field theory action around the closed string vacuum.

*Electronic address: drukker@weizmann.ac.il
I. INTRODUCTION

Over the past few years there has been a resurgence of interest in string field theory as a tool in studying non-perturbative effects in string theory. It was proposed by Sen \cite{1} that condensing the open string tachyon in bosonic string theory will lead to the annihilation of the brane and to the closed string vacuum. In string field theory this should be described by a classical solution whose action is minus the tension of the annihilated brane.

This classical solution was studied numerically using level truncation techniques \cite{2, 3, 4} in Witten’s cubic string field theory \cite{5}, but to date the analytical solution was not found. Given a classical solution one can re-expand the action around the new vacuum which will result in a new kinetic term, but the same cubic interaction vertex. Instead of finding the classical solution one can guess a form for the kinetic operator in the closed string vacuum and start from there.

Gaiotto et al. \cite{6} proposed a pure ghost midpoint insertion as the new kinetic operator. That led to some beautiful results, but is rather singular. Therefore it makes sense to look for other forms of the kinetic operator in the closed string vacuum.

As a guide to finding this kinetic operator we look at the Feynman rules and find kinetic operators that reproduce closed string diagrams. The Feynman rules were derived from the singular ghost insertion in Ref. \cite{6} using some regularization. The propagator we propose here will be less singular and will not suffer some of the problems encountered there.

In Feynman-Siegel gauge the kinetic operator for the open string is $c_0(L_0 - 1)$, where $c_0$ is the ghost zero mode and $L_0 - 1$ the open string Hamiltonian. The propagator is

$$ G = g_s^2 b_0 \int dt \ e^{-t(L_0 - 1)} , $$

where $b_0$ is the antighost zero mode. The expression $\exp -t(L_0 - 1)$ can be visualized as generating an open worldsheet of length $t$. The variables $t$, which are integrated over, become the Feynman parameters of the string graphs and parameterize the moduli space of Riemann surfaces.

This propagator is clearly unsuited for closed strings, since it adds boundaries to the
world sheet. If instead of $L_0$ one uses\(^1\)
\[ \tilde{L}_0 = \frac{1}{\pi} \int d\sigma \sin \sigma (T + \bar{T}), \] (2)
this operator will generate a worldsheet without adding any boundary. The reason is that the energy momentum tensor will not act at $\sigma = 0$ and at $\sigma = \pi$. This is a realization of an idea presented in Ref. 10 that closed strings should arise from boundaries shrinking to points.

In the next section the Feynman rules are derived using this kinetic operator and it is demonstrated how they lead to all closed string diagrams.

In section 3 we propose a larger family of kinetic operators, which are all appropriate for generating closed surfaces. They are all related to each other by conformal transformations. In particular, we recover the singular propagator of Ref. 6.

We end with some comments.

II. FEYNMAN RULES

We wish to derive the Feynman rules from the cubic string field theory action with the kinetic operator replaced by (2). The action is
\[ S = \frac{1}{g_s^2} \int \frac{1}{2} \phi \star c_0 \tilde{L}_0 \Phi + \frac{1}{3} \phi \star \Phi \star \Phi. \] (3)
Here $\Phi$ is a string field, and the star and integral are defined as usual on string fields [5]. The only difference compared to the standard gauge fixed form is the kinetic operator, involving $c_0$ and $\tilde{L}_0$, whose mode expansion is
\[ \tilde{L}_0 = \frac{2}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{1 - 4n^2} L_{2n}. \] (4)
To avoid having too many fields in the theory, one must impose a gauge condition of the fields. The appropriate one seems to be $b_0 \Phi = 0$.

\(^1\) In Ref. it is shown how similar expressions arise from the gauge invariant kinetic terms of Ref. whose cohomology is trivial [9]. Using that as the starting point one finds that apart for the usual energy momentum tensor there is an extra ghost contribution appropriate for the twisted $b, c$ system with central charge $-2$.\footnote{In Ref. it is shown how similar expressions arise from the gauge invariant kinetic terms of Ref. whose cohomology is trivial [9]. Using that as the starting point one finds that apart for the usual energy momentum tensor there is an extra ghost contribution appropriate for the twisted $b, c$ system with central charge $-2$.}
The propagator derived from this action is
\[ G = g_s^2 b_0 \int dt \, e^{-t \hat{L}_0}. \] (5)

Ignoring the \( b \) insertion for now, the propagator for given \( t \) generates a worldsheet that is a segment of a sphere. The points \( \sigma = 0, \pi \) are the north and south pole, and are not moved by this propagator. The equator at \( \sigma = \pi/2 \) is moved forward a distance \( t \).

Closed strings do not exist as external states in open string field theory. Instead, one can use the open-to-closed string vertices of \([11, 12, 13]\), which can be regarded as the gauge invariant observables of the theory \([6, 14]\). For any closed string vertex operator \( V = c\bar{c}V_m \), where \( V_m \) is a dimension \((1, 1)\) primary in the matter CFT, one defines the gauge invariant operator
\[ O_V = \int V(\frac{\pi}{2}) \Phi. \] (6)

In Feynman diagrams the closed string vertex \( V \) will be inserted at the midpoint at the end of a propagator and the right and left parts of the string will be identified by the integral.

Closed string amplitudes are the correlators of any number of those gauge invariant operators. We will concentrate on the three-point function
\[ \langle O_1 O_2 O_3 \rangle. \] (7)

At tree level it is given by one cubic interaction vertex, three propagators extending from it, with a closed string vertex at the end of each. This is depicted schematically in Fig. 1.

To better visualize the worldsheet it is useful to cut the three propagators along their midpoints. Since the propagators were segments of a sphere, we find now six segments of a hemisphere glued to each other, forming one segment of the hemisphere. If the lengths of the three propagators are \( t_1, t_2, t_3 \), the total angle around the hemisphere is \( 2(t_1 + t_2 + t_3) \).

Since the two ends of the segment are identified, there could be a conical singularity at the pole. This is shown in Fig. 2.

If we fix the total length of the three propagators \( t = t_1 + t_2 + t_3 \), the integration over the relative lengths covers the moduli space of spheres with four marked points (the three vertices and the pole). This fact will be proven in the next subsection. The three \( b \) insertions can be represented as contour integrals on the Riemann surface. Two of them should be associated with the two moduli giving the correct measure over the moduli space. The
FIG. 1: A schematic depiction of the tree level Feynman diagram contributing to the three point function of closed string vertices. The three lines $op$ make up the interaction vertex, and attached to them are three propagators. At $a$, $b$ and $c$ closed string vertex operators are inserted, and the two halves of the strings labeled by $pa$, by $pb$ and by $pc$ are identified with each other.

FIG. 2: Another depiction of the same diagram as in Fig. 1. Here we cut the propagators along the midpoints $oa$, $ob$ and $oc$, and glued the ends of the propagators marked by $pbp$ and $pcp$. The result is a segment of the southern hemisphere, with a deficit angle (where we should glue the two lines marked $ap$).

third, associated with the integration over the total angle, should give some measure for that integral.

So at tree level the correlator of the three closed string vertices in string field theory will be given by a conformal field theory calculation on the sphere with the closed string vertices and one extra marked point. This marked point is the remnant of the shrunken
boundary, and we should take care to see what happens there. If, for example, we started
with all Neumann conditions we will find that no momentum flows through this marked
point. This looks like a zero momentum vertex insertion, and may be a soft dilaton as
proposed in [6].

Instead, if we had Dirichlet boundary conditions on the open strings, we will be left with
the constraint that this point on the worldsheet is mapped to a fixed point in space. To
remove this constraint we have to integrate over all locations of this D-instanton. Alterna-
tively, one may impose periodic boundary conditions [15], which will not restrict the position
of this point, or the momentum flowing through it, but this is not very natural for open
strings.

The result of the conformal field theory calculation is multiplied by a power of the open
string coupling \( g_s^4 \) and by the integral over the extra parameter \( t \), call it \( T \) (which possibly
diverges). At higher order in perturbation theory there are graphs with more marked points.
Those come from open string diagrams with more than one boundary, each shrinking to a
point. The graph with \( m \) shrunken boundaries will have a factor of \( g_s^{2+2m} \), and the angle
around each marked point is still arbitrary, giving something like \( T^m \). Therefore we get the
final result

\[
\langle O_1 O_2 O_3 \rangle \sim g_s^2 \sum_m (g_s^2 T)^m \langle V_1 V_2 V_3 \rangle_{\text{sphere}, m},
\]

where the last correlation function is the conformal field theory result for the sphere with
three vertex operators and the \( m \) marked points (treated in one of the ways proposed above,
or differently).

Other graphs will give similar expressions for the torus and higher genus contributions
to this amplitudes. The structure will be very similar, with a series in \( g_s^2 T \) multiplying the
correct power of the coupling. The same is true for correlators of more closed string vertices.

A. Covering of moduli

To complete the discussion on the closed string diagrams it is necessary to prove that
integrating over the Feynman parameters in the string diagrams covers the moduli space of
Riemann surfaces. There are two standard ways of demonstrating this, one is by checking
that the limits of integration do not leave holes in the moduli space, so that all corners of
moduli space can be recovered from the Feynman parameters.
The other method is to find a minimal area problem that leads to the same surfaces as the string diagrams. For every point in moduli space there will be a minimal surface that will correspond to some string diagram \[16, 17, 18\].

One can use the first method directly. If we represent the surface like in Fig. 2, as a hemisphere with a conical singularity at the pole. This is similar to the open string diagram presented in Fig. 4 of Ref. 16, where instead of a hemisphere there is a cylinder. There too one has to glue segments of the top of the cylinder, and the moduli space is the same, except for the modulus associated with the size of the boundary, which now is the fixed angle \(2t\). The other two parameters represent the relative size of the six segments at the equator (they are pairwise equal). The corners of moduli space are when two vertex operators approach each other, which clearly can be done.

Like in the open string case, the same is true for arbitrary diagrams. At higher loops one can get disconnected diagrams with several hemispheres, and one has to sum over all different ways of gluing a fixed number of segments on them.

Instead of making this argument more rigorous, we can use the second method of minimal surfaces. This technique cannot be applied directly, since minimal surfaces have to be hyperbolic, or flat (except for singular points). Our surfaces have constant positive curvature, apart for the singular points. The trick is to map the hemisphere to a semi-infinite cylinder using \(u = \ln \tan(\sigma/2)\). This is a conformal transformation, so we can study all the surfaces with this flat metric, instead of the curved one. This can be seen in Fig. 3.

The original surfaces had in general \(n\) closed string punctures, and \(m\) shrunken boundaries with angles \(2t_i\). The new surfaces will be made up of \(m\) semi-infinite tubes of circumferences \(2t_i\) and with \(n\) punctures where they are glued. Those surfaces solve a minimal area problem very similar to the one discussed in Ref. 6, and the proof carries over.

Consider a Riemann surface with \(n\) punctures where vertex operators are inserted and \(m\) more punctures, for \(m \geq 1\), and positive numbers \(t_i\) with \(i = 1, \ldots, m\) associated to them. Find the minimal surface such that all curves homotopic to the puncture \(i\) are at least of length \(2t_i\). If one does not want to distinguish between the two sets of punctures, one just assigns \(t = 0\) to the punctures where the vertex operators are inserted.

It is not hard to find the solution to this problem. For any puncture at a finite distance on the surface there are arbitrarily small curves around it. Since all curves homotopic to the puncture \(i\) have a length \(2t_i > 0\), the puncture cannot be at a finite distance on the surface.
FIG. 3: By a conformal map it is possible to map the hemisphere in Fig. 2 to a semi-infinite cylinder of circumference $2t$. The point $p$, which is the remnant of the boundary was pushed to infinity. So it sits at the end of a semi-infinite cylinder.

The surface is therefore comprised of $m$ semi-infinite cylinders glued to each other. The $n$ other punctures are inserted along the line where the cylinders are glued. One should note that those surfaces are minimal in the sense that they solve the Euler Lagrange equation with the constraint. The area is infinite, so one cannot calculate the area of different surfaces and compare without some regularization. Still all the vertex operators should be along the same circles where the cylinders are identified. Otherwise there would be a finite piece of the cylinder that could be removed and the vertex operators brought closer.

By this method one can get any Riemann surface of any genus, as long as $m \geq 1$. The main example we studied before was a sphere with $n = 3$ and $m = 1$. Another example is the torus, and to keep it simple we take $n = 0$ and $m = 1$, so it represents the one-loop vacuum amplitude. Since $m = 1$ this would be the disc with one handle if we used the regular propagator of open string field theory. That surface was constructed in Ref. [16]. It is made up of two three-point vertices connected by three propagators. One can connect the propagators in different ways, either giving a planar surface with three boundaries, or a surface with a single boundary, but a handle. We are interested in the second case, where $m = 1$.

Using the new propagator and slicing each propagator along the midpoint one ends up with the same geometry as depicted in Fig. 2. Again, six segments along the equator have
to be identified pairwise. But before, there were closed string vertices along the equator and the two segments around each of the vertices were identified (so we could label them as 112233). Now there are no closed string vertices, and neighboring segments should not be identified, so as to get the non-trivial topology of the torus (the arrangement is 123123).

By the conformal map, this is again a semi-infinite cylinder with the identifications at the end, and this is a solution of the minimal area problem presented above for the torus with $m = 1$ and $n = 0$. This surface has two moduli, the ratio of the lengths of the segments that are identified, and the torus degenerates as the ratio goes to zero. The total circumference is the parameter $t$ as before, which is integrated over, but is not a modulus for the Riemann surface, since it corresponds to a scaling. Indeed a torus with one marked point has two real moduli.

III. GENERALIZATIONS

The kinetic operator $b_0 \tilde{L}_0$ is not unique in generating closed surfaces. Instead of the function $\sin \sigma$ appearing in the definition (2) we can use any other function $f(\sigma)$ subject to the constraints that it is positive, except for a simple zero at $f(0) = f(\pi) = 0$, and that $f(\pi - \sigma) = f(\sigma)$. The first condition is required so the propagator will not generate a boundary and the second one so gluing the two halves of the string would not generate a line singularity.

The factor of $\sin \sigma$ in the definition of $\tilde{L}_0$ corresponds to the standard metric on the sphere $ds^2 = d\sigma^2 + \sin^2 \sigma dt^2$. For a general function the propagator will generate segments of a deformed sphere, with the metric $ds^2 = d\sigma^2 + f(\sigma)^2 dt^2$.

One special case is to take $f(\sigma) = \lim_{\epsilon \to 0} \epsilon$, which gives the sphere squeezed into a very thin cylinder. This is very close to the regularized propagator considered in Ref. 6. To be more rigorous, one should take a function that is equal to $\epsilon$ almost everywhere, and approaches zero at the boundaries. If one rescales the infinitesimal cylinder to finite size it will give the Feynman graphs considered at the end of the last section, and depicted in Fig. 3.

It is easy to prove the equivalence of all those propagators, a simple coordinate transfor-
information maps the sphere with the funny metric to the usual sphere
\[
\ln \tan \frac{\sigma}{2} = \int \frac{d\sigma'}{f(\sigma')}. \tag{9}
\]
This is a conformal transformation, so all those graphs are equivalent. Clearly the two simplest choices are the round sphere and the infinite cylinder.

IV. CONCLUSIONS

We have constructed a family of gauge fixed kinetic operators on open string fields that give closed string diagrams when used to calculate the correlators of closed string vertices.

This construction could serve as a guide to finding the gauge invariant kinetic operator at the vacuum of string field theory. This would be analogous to the way the BRST operator was identified as the gauge invariant kinetic operator for open string field theory \[19,20,21,22\].

It would be interesting to check these operators in a real calculation. One way to do that is doing the explicit conformal field theory calculations, mapping the surfaces to the sphere like was done for open strings in Ref. 23. One may also use an algebraic approach, like in Ref. 24, but in the standard oscillator basis the kinetic operators that vanish on the boundary \[22\] are very non-diagonal. The calculation might be simpler in a different basis.

Another problem is the nature of the series in equation (8). This kind of sum should be expected in any formalism of closed strings in terms of open strings. Starting with a Riemann surface with no handles, but any number of boundaries and shrinking them to points will give the sphere with extra marked points. So many open string diagrams will lead to the same closed string diagram. In particular it would be nice if \[T \sim 1/g_s\], and the series converged to some number.

This kind of kinetic operator can be used also for the superstring, again generating closed surfaces. But finding the interaction terms in the action at the closed string vacuum might be a difficult problem.

Acknowledgments

I am grateful to Oren Bergman, Sunny Itzhaki, Harlan Robins, Adam Schwimmer, Joan Simón, Wati Taylor and Barton Zwiebach with whom I discussed this subject and who made
very helpful suggestions. I would also like to thank the organizers of the Cargèse summer school for providing a great environment that enabled me to complete this work.

[1] A. Sen, _Descent relations among bosonic D-branes_, Int. J. Mod. Phys. A 14, 4061 (1999) [hep-th/9902105].

[2] V. A. Kostelecky and S. Samuel, _On A Nonperturbative Vacuum For The Open Bosonic String_, Nucl. Phys. B 336, 263 (1990).

[3] A. Sen and B. Zwiebach, _Tachyon condensation in string field theory_, JHEP 0003, 002 (2000) [hep-th/9912249].

[4] N. Moeller and W. Taylor, _Level truncation and the tachyon in open bosonic string field theory_, Nucl. Phys. B 583, 105 (2000) [hep-th/0002237].

[5] E. Witten, _Noncommutative geometry and string field theory_, Nucl. Phys. B 268, 253 (1986).

[6] D. Gaiotto, L. Rastelli, A. Sen and B. Zwiebach, _Ghost structure and closed strings in vacuum string field theory_, [hep-th/0111129].

[7] N. Drukker, _On different actions for the vacuum of bosonic string field theory_, [hep-th/0301079].

[8] T. Takahashi and S. Tanimoto, _Marginal and scalar solutions in cubic open string field theory_, JHEP 0203, 033 (2002) [hep-th/0202133].

[9] I. Kishimoto and T. Takahashi, _Open string field theory around universal solutions_, Prog. Theor. Phys. 108, 591 (2002) [hep-th/0205257].

[10] S. L. Shatashvili, _On field theory of open strings, tachyon condensation and closed strings_, [hep-th/0105076].

[11] J. A. Shapiro and C. B. Thorn, _Closed string - open string transitions and Witten's string field theory_, Phys. Lett. B 194, 43 (1987).

[12] J. A. Shapiro and C. B. Thorn, _BRST invariant transitions between closed and open strings_, Phys. Rev. D 36, 432 (1987).

[13] B. Zwiebach, _Interpolating string field theories_, Mod. Phys. Lett. A 7, 1079 (1992) [hep-th/9202015].

[14] A. Hashimoto and N. Itzhaki, _Observables of string field theory_, JHEP 0201, 028 (2002) [hep-th/0111092].

[15] G. Moore and W. Taylor, _The singular geometry of the sliver_, JHEP 0201, 004 (2002).
[16] S. B. Giddings, E. J. Martinec and E. Witten, *Modular invariance in string field theory*, Phys. Lett. B 176, 362 (1986).

[17] B. Zwiebach, *A proof that Witten’s open string theory gives a single cover of moduli space*, Commun. Math. Phys. 142, 193 (1991).

[18] B. Zwiebach, *Minimal area problems and quantum open strings*, Commun. Math. Phys. 141, 577 (1991).

[19] W. Siegel, *Covariantly second quantized string. 3*, Phys. Lett. B 149, 162 (1984) [Phys. Lett. 151B, 396 (1985)].

[20] W. Siegel and B. Zwiebach, *Gauge string fields*, Nucl. Phys. B 263, 105 (1986).

[21] T. Banks and M. E. Peskin, *Gauge invariance of string fields*, Nucl. Phys. B 264, 513 (1986).

[22] K. Itoh, T. Kugo, H. Kunitomo and H. Ooguri, *Gauge invariant local action of string field from BRS formalism*, Prog. Theor. Phys. 75, 162 (1986).

[23] S. B. Giddings, *The Veneziano amplitude from interacting string field theory*, Nucl. Phys. B 278, 242 (1986).

[24] W. Taylor, *Perturbative diagrams in string field theory*, hep-th/0207132.