Local Chern marker of smoothly confined Hofstadter fermions

Urs Gebert, Bernhard Irsigler, and Walter Hofstetter
Institut für Theoretische Physik, Goethe-Universität Frankfurt am Main, Germany

The engineering of topological non-trivial states of matter, using cold atoms, has made great progress in the last decade. Driven by experimental successes, it has become of major interest in the cold atom community. In this work we investigate the time-reversal invariant Hofstadter model with an additional confining potential. By calculating a local spin Chern marker we find that topologically non-trivial phases can be observed in all considered trap geometries. This holds also for spin-orbit coupled fermions, where the model exhibits a quantum spin Hall regime at half filling. Using dynamical mean-field theory, we find that interactions compete against the confining potential and induce a topological phase transition depending on the filling of the system. Strong interactions furthermore yield a magnetic edge, which is localized through the interplay of the density distribution and the underlying topological band structure.

I. INTRODUCTION

Optical lattice experiments offer great possibilities in engineering model Hamiltonians in a clean and well-controlled environment. One focus of current experimental and theoretical interest lies in the investigation of topological states of matter such as the integer or fractional quantum Hall state. Realizations of the paradigmatic Harper-Hofstadter [1][2] and the Haldane models [3][4], which both feature topologically non-trivial states, are showing that these states are now experimentally accessible within cold-atom setups. The topologically non-trivial bulk of a quantum Hall state manifests itself in propagating robust edge states located at the boundary of the system. In cold-atom experiments these boundaries are usually defined by a smooth trapping potential. Recent studies report that this significantly changes the properties of the edge states, as it decreases their group velocity and results in the emerging and splitting of edge states [5][7]. For strong harmonic confinement the trap can even destroy the edge states and therefore the topological phase [8]. Two-particle interactions, on the other hand, can lead to an enhanced localization of the edge states even in harmonic confinement [9][10]. The steepness of the trap affects also the bulk of the system and can lead to shrinking [6] and localization [11] of the bulk. In this work we study the influence of smooth confinement on the topological properties of the bulk and show that these are preserved in different trap geometries. Time-reversal (TR) invariant topological insulators can be realized in cold atoms by engineering artificial spin-orbit coupling (SOC) [12]. This has been done experimentally in the absence of optical lattices [13][15], and proposals for the realization of fully tunable TR-invariant SOC in optical lattices exist [16][17].

This paper is structured as follows: In section II we introduce the underlying Harper-Hofstadter-Hamiltonian, including SOC and a staggered potential, and explain the geometry of the additional trap. To analyse the topological properties of our system we use a real-space marker for the Chern number, which we discuss in section III. The results are given in section IV where we proceed in the following way. In section IV.A we first discuss phase diagrams of a system with non interacting fermions to pick proper parameters for the calculations including the confining trap potential. We discuss the topological properties for absent SOC (IV.B) as well as strong SOC (IV.C) and find topologically non-trivial phases in all trap geometries. Last we extend our calculations to interacting spin-orbit coupled fermions in section IV.D. In section V we summarize our results.

II. MODEL

The model we consider is the well known Hofstadter model [13], which describes electrons in a two-dimensional square lattice with a strong perpendicular external magnetic field. Here, we use its spinful and TR-invariant version [19]

\[ \hat{H}_0 = -t \sum_{j} \left( \hat{c}^\dagger_{j+\hat{x}} e^{i\theta_j} \hat{c}_{j+\hat{x}} + \hat{c}^\dagger_{j+\hat{y}} e^{i\theta_j} \hat{c}_{j+\hat{y}} + \text{h.c.} \right). \]  

(1)

Here, \( \hat{c}_j \) = (\( \hat{c}_{j\uparrow}, \hat{c}_{j\downarrow} \)) is the annihilation operator on spin-1/2 fermions for lattice site \( j = (x,y) \), \( t \) is the tunneling amplitude, which we set to 1, and \( \theta_j = 2\pi \alpha x \sigma_x \) denotes the spin-dependent Peierls phase in the Landau gauge. We choose the plaquette flux \( \alpha \) to be 1/6 for our calculations [20], which yields a six-band model. \( \theta_j = 2\pi \gamma \sigma_x \) is a TR-invariant Rashba SOC, where we focus on the cases without SOC (\( \gamma = 0 \)) and maximal SOC (\( \gamma = 1/4 \)). Furthermore, we add a staggered potential \( \hat{H}_\Lambda \), a trap potential \( \hat{H}_V \) and in Sec. IV.D a local Hubbard interaction \( \hat{H}_U \) to the Hamiltonian

\[ \hat{H} = \hat{H}_0 + \hat{H}_\Lambda + \hat{H}_V + \hat{H}_U, \]

(2)

with the staggered potential of amplitude \( \lambda_x \)

\[ \hat{H}_\Lambda = \sum_j (-1)^x \lambda_x \hat{c}^\dagger_j \hat{c}_j. \]

(3)

We choose our system to have cylinder geometry, i.e. periodic boundary conditions (PBC) in \( y \)-direction and...
open boundary conditions (OBC) with an additional confining trap potential in x-direction

\[ \hat{H}_V = \sum_j V(x) \hat{c}_j^\dagger \hat{c}_j, \quad V(x) = V_0 \left( x - \frac{N}{2} \right)^\delta, \quad (4) \]

with parameter \( \delta \) to tune the steepness of the trap and \( N \) the number of lattice sites in x-direction. \( V_0 \) is fixed such that the trapping potential has the value \( V(1) = V(N) = 10 \) at the boundaries of the system. We investigate the system for a harmonic \( (\delta = 2) \), a quartic \( (\delta = 4) \) and a box-shaped \( (\delta \to \infty) \) trap geometry. All calculations are done for a \( 48 \times 48 \) square lattice.

### III. LOCAL CHERN MARKER

The topological features of a TR-invariant spin-1/2 system are characterized by a topological \( \mathbb{Z}_2 \)-quantum number \([19, 21, 22]\), which takes the value 0 if the system is in a normal insulating (NI) phase and 1 if it is in a quantum spin Hall (QSH) phase. If the spins in the system are not coupled to each other, i.e. \( \gamma = 0 \), the \( \mathbb{Z}_2 \)-index is given by

\[ Z_2 = \frac{1}{2} (C_\uparrow - C_\downarrow) = C_\uparrow \mod 1, \quad (5) \]

where \( C_\sigma \) is the Chern number \([23]\) of the respective spin-\( \sigma \) subsystem and the second equality holds only in TR-invariant systems were \( C_\uparrow = -C_\downarrow \). The Chern number is expressed as a Brillouin-zone integral over the Berry curvature and formulated in \( k \)-space, which makes the computation impossible in finite and spatially inhomogeneous systems. It is therefore often obtained by counting the number of gapless edge states in the energy spectrum. However, in confined systems this can be problematic \([3] \), since there the edge and bulk states are hard to distinguish and one needs to differentiate between edge region and bulk. Another approach which yields a spatially resolved quantity to distinguish between topological phases is the local Chern marker (LCM), developed in Ref. \([24]\) by mapping the \( k \)-space Berry curvature to real-space

\[ \text{LCM}(x, y) = -2\pi i \langle x, y | [\hat{P} \hat{x} \hat{P}, \hat{P} \hat{y} \hat{P}] | x, y \rangle, \quad (6) \]

where \( \hat{P} \) is the projector onto the occupied states, i.e. onto the states with energies below the Fermi energy \( E_F \), and \( \hat{x} \) (\( \hat{y} \)) is the position operator for the \( x \) (\( y \)) direction. This approach was already used to topologically characterize regions in system with heterojunctions \([24]\), in quasicrystals \([25]\) and also in systems with interacting fermions \([26, 27]\). The LCM could be applied in the way that its average over multiple unit cells in the bulk area gives a good estimate of the Chern number. Since the position operator is ill-defined within PBC the LCM yields non-physical boundary effects at the edges, which one needs to cut off. The LCM averaged over the whole lattice \( \langle \text{LCM}\rangle_{\text{latt}} \) is always zero, since it corresponds to the trace over a commutator. Nevertheless its average over the bulk area \( \langle \text{LCM}\rangle_{\text{bulk}} \) gives the expected Chern number with good accuracy, where \( \langle \text{LCM}\rangle_{\text{bulk}} \) is the average over lattice sites with a distance of several sites to the edge of the lattice. In our calculations a distance of 12 sites turned out to be more than enough to minimize the error from boundary effects. The local Chern marker is accessible in experiments by measuring elements of the single-particle density matrix as proposed in \([28, 29]\) and recently performed in a system of photonic Landau levels \([30]\).

### IV. RESULTS

#### A. Phase diagrams for non interacting fermions

We use the LCM to obtain the phase diagrams for non interacting spin-1/2 fermions as a function of the parameter \( \lambda_x \). The LCM is calculated as described and shown in Fig. 1 for a system with OBC corresponding to a hard-wall box potential, which gives the same phase diagram as with PBC \([31]\), since the topological properties of the bulk are not affected by the choice of the boundary conditions.

#### B. Trapped fermions without SOC

We first discuss the case without SOC \( (\gamma = 0) \). Here the Hamiltonian \( \hat{H} \) is diagonal in spin-space and we can directly apply the LCM. The phase diagram shows a QSH phase for a Fermi energy within the lowest and highest gap for all values of \( \lambda_x \). At half filling the gap is closed for \( \lambda_x \lesssim 1.5 \) and opens to a NI phase for \( \lambda_x \gtrsim 1.5 \). We chose \( \lambda_x = 0 \) to study the influence of different trap potentials in the system. For this purpose we average the LCM in the translationally invariant \( y \)-direction. In Fig. 2 we
Figure 2. Spectral density (a-f) and LCM (g-l) for different traps (hard wall, harmonic, quartic) and vanishing staggered potential and SOC. For all cases we see a QSH phase in the lowest and highest gap. The colored lines in subfigures (d-i) show the results for three different values of the Fermi energy and correspond to the dashed lines in the contour plots (a-c,j-l). In the gapped regimes, one can see that a change of the LCM is correlated with a peak in the spectral density.

Figure 3. Spectral density (a-f) and LCM (g-l) for different traps (hard wall, harmonic, quartic) in the case of maximum SOC. The staggered potential is tuned so that the gap at $E_F = 0$ has its maximum size. We set $\lambda_x = 0.7$ and $\gamma = 0.25$.

Next we consider the case of maximum SOC ($\gamma = 1/4$). For this case the SOC term in the Hamiltonian (1) simplifies to a hopping in $x$-direction followed by a spin flip. To apply the LCM the Hamiltonian needs to be decoupled in spin space, which can be achieved by using the

C. Trapped fermions with SOC

compare the results for the LCM to the momentum integrated spectral density \[ \rho_x(E,k_y) = -2 \text{Im} \langle x,k_y,\sigma | \frac{1}{E - H + i0^+} | x,k_y,\sigma \rangle \] (7)

for a hard wall ($\delta = \infty$), quartic ($\delta = 4$) and harmonic confined ($\delta = 2$) system. The spectral density shown in Fig. 2 (a-c) shows six bulk bands, but gapless edge states at the boundary of the system. For a Fermi energy $E_F = 0$ the system is semi-metallic due to touching bands. We compare the LCM in Fig. 2 (g-i) to the spectral density Fig. 2 (d-f) for Fermi energies within the two lowest gaps and $E_F = 0$. If the Fermi energy lies within a gap, the LCM shows a continuous transition from zero outside the trap to a plateau with constant value in the bulk region. The system is in a QSH phase if only the lowest band is filled, and in a NI phase if two bands are filled. If we compare the behaviour of the LCM to the momentum integrated spectral density, we can clearly identify changes in the LCM with peaks in the spectral density. Let us consider for example the blue line in Fig. 2 (e,h), which corresponds to a Fermi energy of $E_F = -2.25$. The spectral density shows two large peaks located at the position where the LCM changes from zero to one and back. In between these two edge states the system is gapped and in a QSH phase. Since we have spinful fermions, each peak in the spectral density corresponds to a counterpropagating pair of edge states and we see, as stated by the bulk-boundary correspondence, how such a counterpropagating pair of edge states connects topologically distinct regions. The LCM allows us to distinguish these regions in real-space even without looking at edge states. If the Fermi energy lies within the second lowest gap we can see two pairs of edge states in the spectral density and the LCM takes a value of 2 in the middle of the trap. The two edge states can scatter on each other and the system becomes a NI. For the gapless regime at half-filling the system is also topologically trivial as expected. In Fig. 2 (j-l) we plot the LCM for a large range of Fermi energies. Each horizontal line shows the values of the LCM along the $x$-direction for the corresponding Fermi energy. We can see that the different topological regimes of the system can be found in all trap geometries and that sharp boundaries are not necessarily needed for the realization of a topological insulator.
pseudospin basis
\[ \hat{d}_{x,y,\sigma} = \begin{cases} \hat{c}_{x,y,\sigma} & \text{if } x \text{ is even} \\ \hat{c}_{x,y,\bar{\sigma}} & \text{if } x \text{ is odd} \end{cases} \] (9)
where the Hamiltonian now reads
\[ \hat{H}_0 = -t \sum_j \left( \hat{d}^\dagger_{j+x} \hat{d}_j + \hat{d}^\dagger_{j+y} e^{(-1)^j 2\pi i x \sigma \hat{c}_j} \right) + \text{H.c.,} \] (10)
with spinors \( \hat{d}_j = (\hat{d}^\dagger, \hat{d}^\dagger) \). The other terms \( \hat{H}_\lambda \) and \( \hat{H}_V \) in the full Hamiltonian \( \hat{H} \) are invariant under the basis transformation \( \hat{c} \). The phase diagram (see Fig. 1) shows a topological non-trivial gap at half-filling, for appropriate values of the staggered potential. In this regime interaction effects are most pronounced, which makes it interesting for further studies on interacting fermions that will be discussed in Sec. III D. Therefore, we concentrate on half filling and set \( \lambda_x = 0.7 \), where the gap is maximal. Fig. 3 shows the results for the spectral density and the LCM. There we average the LCM also over the unit cell in \( x \) direction. The spectral density is very spiky over the whole energy range, but for all trap geometries a gap is visible at \( E = 0 \). Fig. 3 (g-i) show how the LCM behaves within this gap. For hard-wall and quartic confinement the LCM takes the value 1 even for Fermi energies close to the bulk-band. The LCM shows also a plateau for harmonic confinement, although it is less smooth. Nevertheless, the LCM is approximately 1 in the center of the trap and we could therefore expect the bulk of the system to be in a QSH phase. The relatively small size of the gapped area in harmonic confinement can also lead to finite size effects as the edge states may have a finite overlap.

D. Trapped interacting fermions

We now study the effect of finite Hubbard interactions, i.e., we add the following local term to the Hamiltonian in Eq. 2
\[ H_U = U \sum_j \hat{c}_{j,\uparrow}^\dagger \hat{c}_{j,\uparrow} \hat{c}_{j,\downarrow}^\dagger \hat{c}_{j,\downarrow} \] (11)
where \( U \) is the interaction strength. The unconfined Hofstadter-Hubbard model has been intensively studied \( \text{[20, 27, 31–33]} \) in many different aspects, however, the interplay of interactions and smooth confinement is still lacking. We restrict ourselves to the harmonically trapped case, since it is experimentally the most common one and yields a topologically non-trivial state as discussed in the previous sections.

In order to prevent the system from entering a topologically trivial magnetic phase, we adjust the staggered potential as
\[ \lambda_x = 1/2 + U/3 \] (12)
according to the phase diagram in Ref. \( \text{[32]} \). This ensures a non-trivial bulk topological phase in the center of the trap. For solving the many-body problem, we make use of dynamical mean-field theory (DMFT), using a local selfenergy \( \Sigma \). Since the systems in our context are highly inhomogeneous, we use the real-space version of DMFT \( \text{[35–37]} \). Within real-space DMFT, the full many-body problem on the lattice of \( L \) sites is transformed to \( L \) single-impurity problems due to the local selfenergy \( \Sigma_{\lambda \sigma}^\sigma(\omega) = \Sigma_{ii}^\sigma(\omega) \delta_{ij} \), where \( \omega \) denotes frequency and \( \delta_{ij} \) the Kronecker delta. Since we consider spin-orbit coupled situations, the selfenergy can have off-diagonal terms in spin space, i.e., it can be non-zero for \( \sigma \neq \sigma' \). For each single-impurity problem, we use a continuous-time quantum Monte Carlo solver in the auxiliary field expansion \( \text{[38]} \). Using spatial symmetries, the number of local selfenergies to be calculated can be reduced. In our case, only the selfenergies of one full row in \( x \) direction have to be computed since the system in cylinder geometry is translationally invariant in \( y \) direction. After all single-impurity problems have been solved, a lattice Green’s function \( G(\omega) \) is constructed from the local selfenergies using the Dyson equation
\[ [G^{-1}(\omega)]_{ij}^{\sigma \sigma'} = [G_0^{-1}(\omega)]_{ij}^{\sigma \sigma'} - \Sigma_{ii}^{\sigma \sigma'}(\omega) \delta_{ij}, \] (13)
where \( G_0(\omega) \) is the non-interacting lattice Green’s function. The new lattice Green’s function \( G(\omega) \) defines a new lattice problem which is again solved by reducing it to single-impurity problems. These DMFT iterations are repeated until changes in the selfenergy become sufficiently small and self consistency is reached.

DMFT is formulated in the grand-canonical ensemble which makes it difficult to solve problems with a fixed number of particles. However, since the latter is experimentally more feasible, we control the average number of particles \( \bar{N} \) by readjusting the chemical potential in each DMFT iteration during the self-consistency procedure. The number of particles cannot be perfectly fixed due to the uncertainty from the quantum Monte Carlo calculations, which increases with increasing interaction strength due to the auxiliary field expansion.

We show the density profiles of the harmonically trapped system as a function of the interaction strength for different average number of particles \( \bar{N} \).
We observe an interesting effect in the density profiles for strong interactions which is presented in Fig. 5. Here, we show examplarily the spin-resolved density and magnetization profiles for $N = 29.2$ and $U = 5$. We observe that in the center region $10 < x < 35$ as well as in the far edge regions $x < 5$ and $x > 40$ the two spins densities are equal and the magnetization vanishes. Locally, however, at $x \approx 9$ and $x \approx 37$ the occupancy between spin up and down particles differs and a finite magnetization emerges. We explain this effect in the following way: The bulk QSH phase is protected against magnetization since we control the staggered potential according to Eq. (12) as explained above. A phase transition to a magnetic phase would only be possible if the gap is closing. Due to the underlying band structure away from the trap center at $x \approx 9$ and $x \approx 37$, respectively, particles can enter a metallic phase. The gap is thus closed and a magnetic phase can emerge which vanishes again when going even further away from the center where the filling is too small.

We now turn to the computation of the LCM for the interacting system. To this end, we make use of the topological Hamiltonian approach [49]. Here, the interacting Green’s function is smoothly transformed to the non-interacting Green’s function. If no singularity of this Green’s function occurs during this transformation then there is also no gap closing and the topological invariant cannot change. This simplifies computations of topological invariants tremendously. Instead of computing topological invariants of the interacting problem, the topology of the system is determined from an effective, non-interacting Hamiltonian $[H_{\text{top}}]_{ij}^\sigma = [H_0]_{ij}^\sigma + \Sigma_{ii}^\sigma(0)\delta_{ij}$, written here in matrix representation, where we have used that the selfenergy is local. Continuous-time quantum Monte Carlo output is generally expressed in imaginary time, which leaves us with the selfenergy as function of the fermionic Matsubara frequencies $\omega_n = (2n+1)\pi/\beta$. We determine the zero-frequency selfenergy $\Sigma_{ii}^\sigma(0)$ by polynomial fitting of $\Sigma_{ii}^\sigma(i\omega_n)$ around 0. The combination of the topological Hamiltonian approach and the LCM has been successfully applied in Refs. [26, 27]. We show the interacting LCM in Fig. 6 as a function of the interaction strength for different $\bar{N}$. Regions with a topologically non-trivial phase, where LCM = -1, are depicted in blue. Outside these regions the LCM assumes arbitrary values, as we have seen in the non-interacting case in Fig. 3 (i, l). We observe that the topologically non-trivial region is shifted to higher interaction strengths as the number of particles in the system is increased. This is due to the fact that interactions push the particles out of the center and then reach half filling in the trap center such that a topologically non-trivial band gap exists. This is a type of interaction-induced topological phase transition [32, 40, 42], however, here the phase transition is not induced through the competition of interaction strength and staggered potential but rather of interaction strength and trapping potential which completely breaks translational invariance, in contrast to a staggered potential which only increases the size of the unit cell.

We apply the local Chern marker to the trapped Hofstadter model and find distinct topologically non-trivial phases even in a smooth confinement. This is complementary to Ref. [6]. We generalize the treatment to the spin-mixed case, which features a quantum spin Hall gap at half filling. Also here, the local Chern marker indicates topologically non-trivial phases in different trap geometries. In addition, we use dynamical mean-field theory to study the effect of finite on-site interactions. Here, we find an interesting effect of a localized, magnetic edge but non-magnetic bulk which we explained with topological protection. By using the topological Hamiltonian approach we compute the local Chern marker for the interacting, trapped system and find an interaction-induced topological phase transition depending on the filling.

Based on recent works, we think that our findings can be observed in experiments with tomography methods including a quantum gas microscope. Furthermore, these ideas should be straightforwardly extendable to three dimensions [45] featuring the strong topological insulator phase [44].

**V. CONCLUSION**

![Figure 5. Spin-resolved density and magnetization profiles for strong interactions: Emergent magnetic strongly localized edge.](image_url)
ACKNOWLEDGMENTS

The authors would like to thank Jun-Hui Zheng for enlightening discussions. This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Project No. 277974659 via Research Unit FOR 2414. This work was also supported by the Deutsche Forschungsgemeinschaft (DFG) via the high-performance computing center LOEWE-CSC.

[1] M. Aidelsburger, M. Atala, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch, Phys. Rev. Lett. 111, 185301 (2013).
[2] H. Miyake, G. A. Siviloglou, C. J. Kennedy, W. C. Burton, and W. Ketterle, Phys. Rev. Lett. 111, 185302 (2013).
[3] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature 515, 237 (2014).
[4] N. Flaschner, B. S. Rem, M. Tarnowski, D. Vogel, D.-S. Lühmann, K. Sengstock, and C. Weitenberg, Science 352, 1091 (2016).
[5] T. D. Stanscu, V. Galitski, and S. Das Sarma, Phys. Rev. A 82, 013608 (2010).
[6] M. Buchhold, D. Cocks, and W. Hofstetter, Phys. Rev. A 85, 063614 (2012).
[7] N. Goldman, J. Dalibard, A. Dauphin, F. Gerbier, M. Lewenstein, P. Zoller, and I. B. Spielman, P. Natl. Acad. Sci. USA 110, 6736 (2013).
[8] Z. Yan, B. Li, X. Yang, and S. Wan, Sci. Rep. 5 (2015).
[9] P. Nevado, S. Fernández-Lorenzo, and D. Porrás, Phys. Rev. Lett. 119, 210401 (2017).
[10] B. Galilo, D. K. K. Lee, and R. Barnett, Phys. Rev. Lett. 119, 203204 (2017).
[11] A. R. Kolovsky, F. Grusdt, and M. Fleischhauer, Phys. Rev. A 89, 033607 (2014).
[12] V. Galitski and I. B. Spielman, Nature 494, 49 (2013).
[13] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman, Nature 471, 83 (2011).
[14] P. Wang, Z.-Q. Yu, Z. Fu, J. Miao, L. Huang, S. Chai, H. Zhai, and J. Zhang, Phys. Rev. Lett. 109, 095301 (2012).
[15] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, Phys. Rev. Lett. 109, 095302 (2012).
[16] A. M. Dudarev, R. B. Diener, I. Carusotto, and Q. Niu, Phys. Rev. Lett. 92, 153005 (2004).
[17] F. Grusdt, T. Li, I. Bloch, and E. Demler, Phys. Rev. A 95, 063617 (2017).
[18] D. R. Hofstadter, Phys. Rev. 14, 2239 (1976).
[19] N. Goldman, I. Satija, P. Nikolic, A. Bermudez, M. A. Martin-Delgado, M. Lewenstein, and I. B. Spielman, Phys. Rev. Lett. 105, 255302 (2010).
[20] P. P. Orth, D. Cocks, S. Rachel, M. Buchhold, K. L. Hur, and W. Hofstetter, J. Phys. B 46, 134004 (2013).
[21] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005).
[22] C. Xu and J. E. Moore, Phys. Rev. B 73, 045322 (2006).
[23] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. 49, 405 (1982).
[24] R. Bianco and R. Resta, Phys. Rev. B 84, 241106 (2011).
[25] D.-T. Tran, A. Dauphin, N. Goldman, and P. Gaspard, Phys. Rev. B 91, 085125 (2015).
[26] A. Amaricci, L. Privitera, F. Petocchi, M. Capone, G. Sangiovanni, and B. Trauzettel, Phys. Rev. B 95, 205120 (2017).
[27] B. Isirigley, J.-H. Zheng, and W. Hofstetter, Phys. Rev. Lett. 122, 010406 (2019).
[28] B. Isirigley, J.-H. Zheng, and W. Hofstetter, arXiv:1904.03091 (2019).
[29] M. D. Caio, G. Möller, N. R. Cooper, and M. J. Bhaseen, Nat. Phys. 15, 257 (2019).
[30] N. Schine, M. Chalupnik, T. Can, A. Gromov, and J. Simon, Nature 565, 173 (2019).
[31] D. Cocks, P. P. Orth, S. Rachel, M. Buchhold, K. Le Hur, and W. Hofstetter, Phys. Rev. Lett. 109, 205303 (2012).
[32] P. Kumar, T. Mertz, and W. Hofstetter, Phys. Rev. B 94, 115161 (2016).
[33] B. Isirigley, J.-H. Zheng, M. Hafez-Torbati, and W. Hofstetter, Phys. Rev. A 99, 043628 (2019).
[34] A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, Rev. Mod. Phys. 68, 13 (1996).
[35] S. Okamoto and A. J. Millis, Phys. Rev. B 70, 241104 (2004).
[36] R. W. Helmes, T. A. Costi, and A. Rosch, Phys. Rev. Lett. 100, 056403 (2008).
[37] M. Snoek, I. Titvinidze, C. Töke, K. Byczuk, and W. Hofstetter, New J. Phys. 10, 093008 (2008).
[38] E. Gull, A. J. Millis, A. I. Lichtenstein, A. N. Rubtsov, M. Troyer, and P. Werner, Rev. Mod. Phys. 83, 349 (2011).
[39] Z. Wang and S.-C. Zhang, Phys. Rev. X 2, 031008 (2012).
[40] D. A. Abanin and D. A. Pesin, Phys. Rev. Lett. 109, 066802 (2012).
[41] T. I. Vanhala, T. Siro, L. Liang, M. Troyer, A. Harju, and P. Törnä, Phys. Rev. Lett. 116, 225305 (2016).
[42] J.-H. Zheng, B. Isirigley, L. Jiang, O. Weitenberg, and W. Hofstetter, arXiv:1812.01991 (2018).
[43] M. S. Scheurer, S. Rachel, and P. P. Orth, Sci. Rep. 5, 8386 (2015).
[44] L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. 98, 106803 (2007).