Mathematical representation of Wind Turbine Power reliability

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Abstract. Nowadays wind resource assessment utilize new advanced technologies and appropriate analytical methods which are applied to estimate how much wind as a fossil free fuel will be available from a wind farm over the period of its performance. The utmost important piece of information is determining the expected generation of energy from the power plant and ultimately how much cost effective it will be. In our study we have proposed a new method, the improved mixture Weibull distribution produced from the combination of two and three parameter Weibull distribution with six parameters which include shape, scale, location parameter and the weight component or the mixing parameter. The basic properties of the improved mixture Weibull distribution and the estimation of parameters using maximum likelihood method are discussed. The estimated parameters are used to derive a mathematical model to compute the capacity factor and wind power density.

1. Introduction

Swedish physicist applied Weibull distribution in the study of materials. But Weibull distribution has a very accurate and close approximation to the laws of probability [1]. The Weibull distribution gives a best fit to experimental data has been revealed by several authors. In literature there are a number of mixture distributions with a combination of two different distributions and two same distributions [2].

Weibull distribution with 2 & 3 parameters were used to describe all the wind sites experiencing unimodal frequency distribution (single peak). However using Weibull distribution in several regions of the world experiencing bimodal frequency gave an inaccurate result. To overcome this [3] discussed about the mixture bimodal Weibull distribution which is now often used in wind speed analysis.

In several wind data analysis zero percentage of null wind speed, that is minimum of wind speed series which is always higher than the central limit is not considered. The location parameter included in the proposed model is closely associated with the null wind speed. So the bimodal Weibull distribution is extended to improved mixture Weibull distribution \( l(2,3) \).

The improved mixture Weibull distribution is defined in 2nd and some of its basic properties are discussed. In the next section MLE method is discussed. In the fourth section we have the discussion about the wind speed experienced by a wind turbine, the capacity factor and the expression for computing wind power that can be expected from a turbine.

2. Weibull Distribution
2.1 Weibull distribution
The two parameter Weibull distribution with probability density function is defined by
\begin{equation}
 f(v; \alpha, \beta) = \frac{\alpha}{\beta} \left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right] \text{ for } v > 0
\end{equation}

where \( v \) is the wind speed, \( \alpha \) is the shape parameter and \( \beta \) is the scale parameter. The scale parameter has the same units as the wind speed (m/s) [4].

The cumulative distribution function (cdf) of the two parameter Weibull distribution function is given by [5]
\begin{equation}
 F(v) = 1 - \exp\left(-\left(\frac{v}{\beta}\right)^{\alpha}\right)
\end{equation}

2.2 Three parameter mixture Weibull distribution
The probability function is represented by
\begin{equation}
 f(v; \alpha, \beta, \gamma) = \frac{\alpha}{\beta} \left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right] \text{ for } v > 0
\end{equation}

The corresponding cumulative distribution function is
\begin{equation}
 F(v) = 1 - \exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right]
\end{equation}

2.3 Improved mixture Weibull distribution
This probability density function is a linear combination of two probability functions[7]. A R.V \( V \) that is independently distributed as \( V_i \) with mixing parameter \( \omega_i \) (i.e., \( \omega_i + \omega_j = 1 \)) has a improved mixture Weibull distribution \( MWbl(2,3) \) with probability density function
\begin{equation}
 f_{(2,3)}(v; \alpha_i, \beta_i, \alpha_2, \beta_2, \alpha_0) = \omega f(v; \alpha_1, \beta_1) + (1-\omega) f(v; \alpha_2, \beta_2, \gamma_0)
\end{equation}

where \( v > 0 , \alpha_1, \beta_1, \alpha_2, \beta_2 > 0 , 0 \leq \omega \leq 1 \), \( f_{(2,3)}(v; \alpha_i, \beta_i, \alpha_2, \beta_2, \gamma) = 0 \) for \( v < 0 \).

Cumulative Distribution function:

The cumulative distribution function of \( MWbl(2,3) \) is defined by
\begin{equation}
 F_{(2,3)}(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) = P(V \leq v)
\end{equation}
\begin{equation}
 = \omega F(v; \alpha_1, \beta_1) + (1-\omega) F(v; \alpha_2, \beta_2, \gamma_0)
\end{equation}

2.4 Properties of \( MWbl(2,3) \)
2.4.1 Proposition
Let \( V \sim MWbl(2,3) (v; \alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) \) then the first order moment of \( V \) is given by
\begin{equation}
 E[V] = \omega \beta \Gamma\left[1 + \frac{1}{\alpha_1}\right] + (1-\omega) \beta \Gamma\left[1 + \frac{1}{\alpha_2}\right] + (1-\omega) \gamma
\end{equation}

Proof:
\[
E[V] = \int_{0}^{\infty} v \cdot \omega \cdot \frac{\alpha_{1}}{\beta_{1}^{\alpha_{1}}} \cdot \exp\left[-\frac{\alpha_{1}}{\beta_{1}} v\right] \cdot \frac{\alpha_{1}}{\beta_{1}^{\alpha_{1}}} \cdot \exp\left[-\frac{\alpha_{1}}{\beta_{1}} v\right] \, dv \\
= \int_{0}^{\infty} \omega \cdot \frac{\alpha_{1}}{\beta_{1}^{\alpha_{1}}} \exp\left[-\frac{\alpha_{1}}{\beta_{1}} v\right] \, dv \\
+ \int_{0}^{\infty} \omega \cdot \frac{(1 - \omega)}{\beta_{2}^{\alpha_{2}}} \cdot \frac{\alpha_{2}}{\beta_{2}^{\alpha_{2}}} \exp\left[-\frac{\alpha_{2}}{\beta_{2}} \cdot \frac{\alpha_{2}}{\beta_{2}} v\right] \, dv \\
\]

Considering \( x = \left(\frac{v}{\beta_{1}}\right)^{\alpha_{1}} \) and \( y = \left(\frac{v - \gamma_{0}}{\beta_{2}}\right)^{\alpha_{2}} \) and integrating we get

\[
E[V] = \omega \beta_{1} \Gamma \left(1 + \frac{1}{\alpha_{1}}\right) + (1 - \omega) \beta_{2} \Gamma \left(1 + \frac{1}{\alpha_{2}}\right) + (1 - \omega) \gamma_{0} \quad (7)
\]

where the gamma function is defined as \( \Gamma = \int_{0}^{\infty} e^{-y} y^{\alpha-1} \, dy \)

### 2.4.2 Proposition

The variance of \( V \sim Wbl(2,3)(v; \alpha_{1}, \beta_{1}, \alpha_{2}, \beta_{2}, \omega, \gamma_{0}) \) is

\[
Var[V] = \beta_{1}^{2} \left[ \Gamma \left(1 + \frac{2}{\alpha_{1}}\right) - \left(\Gamma \left(1 + \frac{1}{\alpha_{1}}\right)\right)^{2} \right] + \beta_{2}^{2} \left[ \Gamma \left(1 + \frac{2}{\alpha_{2}}\right) - \left(\Gamma \left(1 + \frac{1}{\alpha_{2}}\right)\right)^{2} \right]
\]

**Proof:**

The second order moment is given by

\[
E[V^{2}] = \int_{0}^{\infty} v \cdot \omega \cdot \frac{\alpha_{1}}{\beta_{1}^{\alpha_{1}}} \cdot \exp\left[-\frac{\alpha_{1}}{\beta_{1}} v\right] \cdot \frac{\alpha_{1}}{\beta_{1}^{\alpha_{1}}} \cdot \exp\left[-\frac{\alpha_{1}}{\beta_{1}} v\right] \, dv \\
= \int_{0}^{\infty} \omega \cdot \frac{\alpha_{1}}{\beta_{1}^{\alpha_{1}}} \cdot \frac{\alpha_{1}}{\beta_{1}^{\alpha_{1}}} \cdot \exp\left[-\frac{\alpha_{1}}{\beta_{1}} v\right] \, dv + \int_{0}^{\infty} \omega \cdot \frac{(1 - \omega)}{\beta_{2}^{\alpha_{2}}} \cdot \frac{\alpha_{2}}{\beta_{2}^{\alpha_{2}}} \cdot \exp\left[-\frac{\alpha_{2}}{\beta_{2}} \cdot \frac{\alpha_{2}}{\beta_{2}} v\right] \, dv \\
\]

Considering \( x = \left(\frac{v}{\beta_{1}}\right)^{\alpha_{1}} \) and \( y = \left(\frac{v - \gamma_{0}}{\beta_{2}}\right)^{\alpha_{2}} \), integrating we get

\[
E[V^{2}] = \omega \beta_{1}^{2} \Gamma \left(1 + \frac{2}{\alpha_{1}}\right) + (1 - \omega) \beta_{2}^{2} \Gamma \left(1 + \frac{2}{\alpha_{2}}\right) + 2 \gamma_{0} \beta_{2} \Gamma \left(1 + \frac{1}{\alpha_{2}}\right) + \gamma_{0}^{2}
\]

We know

\[
Var(V) = E[V^{2}] - (E[V])^{2}
\]

\[
= \beta_{1}^{2} \left[ \Gamma \left(1 + \frac{2}{\alpha_{1}}\right) - \left(\Gamma \left(1 + \frac{1}{\alpha_{1}}\right)\right)^{2} \right] + \beta_{2}^{2} \left[ \Gamma \left(1 + \frac{2}{\alpha_{2}}\right) - \left(\Gamma \left(1 + \frac{1}{\alpha_{2}}\right)\right)^{2} \right] \quad (9)
\]

### 2.4.3 Proposition

For \( V \) following an unimodal distribution has only one value corresponding to one mode but for \( MWbl(2,3) \), i.e., for bimodal distribution it has two modes which is expressed by the value of \( V \) which satisfies the following equation

\[
\omega \cdot f_{2}(v) \left(\frac{\alpha_{2} - 1}{v} - \frac{\alpha_{1}}{\beta_{1}} \left(\frac{v}{\beta_{1}}\right)^{\alpha_{1}-1}\right) + (1 - \omega) \cdot f_{3}(v) \left(\frac{\alpha_{2} - 1}{v - \gamma_{0}} - \frac{\alpha_{2}}{\beta_{2}} \left(\frac{v - \gamma_{0}}{\beta_{2}}\right)^{\alpha_{2}-1}\right)
\]

where \( f_{2}(v) \) and \( f_{3}(v) \) are defined as two and three parameter Weibull probability density functions.
Proof:

\[ f_{(2,3)}(v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_0) = \omega f(v; \alpha_1, \beta_1) + (1 - \omega) f(v; \alpha_2, \beta_2, \gamma_0) \]

\[ = \omega \left[ \frac{\alpha_1}{\beta_1} \left( \frac{v}{\beta_1} \right)^{\alpha_1 - 1} e^{-v/\beta_1} \right] + (1 - \omega) \left[ \frac{\alpha_2}{\beta_2} \left( \frac{v - \gamma_0}{\beta_2} \right)^{\alpha_2 - 1} e^{- (v - \gamma_0)/\beta_2} \right] \]

Differentiate the above equation with respect to \( v \) we get the equation as

\[ \omega \left[ \frac{\alpha_1}{\beta_1} \left( \frac{v}{\beta_1} \right)^{\alpha_1 - 1} \right] \frac{d}{dv} \left[ \left( \frac{v}{\beta_1} \right)^{\alpha_1 - 1} \right] + (1 - \omega) \left[ \frac{\alpha_2}{\beta_2} \left( \frac{v - \gamma_0}{\beta_2} \right)^{\alpha_2 - 1} \right] \frac{d}{dv} \left[ \left( \frac{v - \gamma_0}{\beta_2} \right)^{\alpha_2 - 1} \right] = 0 \]

Equate the above equation to zero we get

\[ \omega f_1(v) \left[ \frac{\alpha_1 - 1}{v} - \frac{\alpha_1}{\beta_1} \right] + (1 - \omega) \frac{d}{dv} \left[ \left( \frac{v}{\beta_1} \right)^{\alpha_1 - 1} \right] = 0 \]

The value of \( v \) which satisfies the resultant equation is the representation of modes.

2.4.4 Proposition

If the observed wind data are arranged in rank order and \( n \) denotes the size of the sample wind data, the median is the middle value. The median of MWbl(2,3) is represented as the value \( v \) which satisfies the following equation equated to 0.5.

\[ \omega \left[ 1 - \exp \left( - \frac{v}{\beta_1} \right)^{\alpha_1} \right] + (1 - \omega) \left[ 1 - \exp \left( - \frac{v - \gamma_0}{\beta_2} \right)^{\alpha_2} \right] = 0.5 \]  

(11)

2.4.5 Proposition

Let \( c > 0 \) and \( V \sim MWbl(2,3) \ (v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_0) \) then \( cV \sim MWbl(2,3) \)

(\( v; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_0 \))

Proof:

Consider the cumulative distribution function of MWbl(2,3) which implies

\[ FF_{(2,3)}(v) = P(cV \leq v) \]

\[ = P \left( \frac{V}{c} \leq \frac{v}{c} \right) = \omega \left[ 1 - \exp \left( - \frac{v}{c \beta_1} \right)^{\alpha_1} \right] + (1 - \omega) \left[ 1 - \exp \left( - \frac{v - \gamma_0}{c \beta_2} \right)^{\alpha_2} \right] \]

(12)

which is the cdf of Wbl(2,3)\((c \alpha_1, \beta_1, c \alpha_2, \beta_2, \gamma_0)\). Since the distribution function characterizes the law of the random variable, the conclusion follows instantly.

3. Estimation of Parameters

3.1 Maximum Likelihood Method

Fisher popularized the MLE techniques in 1920s. Since 1920 this technique was used by several authors in literature to estimate the parameters[7].

Definition:

Let \( v_1, v_2, \ldots, v_n \) be a sample of wind speed observations taken to the corresponding random variable \( V_1, V_2, \ldots, V_n \) which is continuous in nature then the likelihood \( L \) will be \( L(v_1, v_2, \ldots, v_n) \) is defined as the joint density evaluated at \( v_1, v_2, \ldots, v_n \) given by \( L = f(v_1) f(v_2) \ldots f(v_n) \)[8].
\[ L = \prod_{i=1}^{n} f(v_i) \] where \( f(v_i) \) is the probability density function associated with the observation of the wind speed data \( v_i \).

The log likelihood function is

\[
l(\alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_0) = \prod_{i=1}^{n} f(v_i; \alpha_1, \beta_1, \alpha_2, \beta_2, \omega, \gamma_0) = \prod_{i=1}^{n} \omega \left[ \frac{\alpha_1}{\beta_1} \right]^{v_i-1} \exp \left[ - \left( \frac{v_i}{\beta_1} \right)^\alpha_1 \right] + (1-\omega) \left[ \frac{\alpha_2}{\beta_2} \right] \exp \left[ - \left( \frac{v_i - \gamma_0}{\beta_2} \right)^\alpha_2 \right] \]

\[
= \omega \left( \frac{\alpha_1}{\beta_1} \right)^\alpha \exp \left[ - \frac{1}{\beta_1 \alpha_1} \sum_{i=1}^{n} v_i^{\alpha_1} \right] + (1-\omega) \left( \frac{\alpha_2}{\beta_2} \right)^\alpha \exp \left[ - \frac{1}{\beta_2 \alpha_2} \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2} \right]
\]

Taking the natural log on both the sides, we obtain that the log-likelihood function is

\[
l(\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) = n \log \alpha_1 - n \alpha_1 \log \beta_1 - \frac{1}{\beta_1 \alpha_1} \sum_{i=1}^{n} v_i^{\alpha_1} + \sum_{i=1}^{n} (\alpha_1 - 1) \log v_i
\]

\[
+ n \log \alpha_2 - n \log \beta_2 - \frac{1}{\beta_2 \alpha_2} \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2} + \sum_{i=1}^{n} (\alpha_2 - 1) \log (v_i - \gamma_0)
\]

Consider the partial derivatives w.r.t. \( \alpha_1, \beta_1, \alpha_2, \beta_2 \) we get

\[
\frac{\partial}{\partial \beta_1} \log l(\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) = -\frac{n \alpha_1}{\beta_1} + \frac{\alpha_1}{\beta_1 \alpha_1} \sum_{i=1}^{n} v_i^{\alpha_1}
\]

\[
\frac{\partial}{\partial \beta_2} \log l(\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) = \frac{n \alpha_2}{\beta_2} + \frac{\alpha_2}{\beta_2 \alpha_2} \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2}
\]

\[
\frac{\partial}{\partial \alpha_1} \log l(\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) = -n \log \beta_1 + \frac{\log \beta_1}{\beta_1} \sum_{i=1}^{n} v_i^{\alpha_1} - \frac{1}{\beta_1 \alpha_1} \left( \sum_{i=1}^{n} \log (v_i) \right)^{\alpha_1} + \sum_{i=1}^{n} \log (v_i)
\]

\[
\frac{\partial}{\partial \alpha_2} \log l(\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) = -n \log \beta_2 + \frac{\log \beta_2}{\beta_2} \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2} + \sum_{i=1}^{n} \log (v_i - \gamma_0)
\]

\[
\frac{\partial}{\partial \gamma_0} \log l(\alpha_1, \beta_1, \alpha_2, \beta_2, \gamma_0) = \frac{1}{\beta_2 \alpha_2} \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2} - \sum_{i=1}^{n} \frac{\overline{\alpha_2 - 1}}{(v_i - \gamma_0)^{\alpha_2}}
\]

Set the proceeding equations to zero, we get

\[
0 = -\frac{n \alpha_1}{\beta_1} + \frac{\alpha_1}{\beta_1 \alpha_1} \sum_{i=1}^{n} v_i^{\alpha_1} \rightarrow \beta_1 = \sum_{i=1}^{n} \left( \frac{v_i^{\alpha_1}}{n} \right)^{\alpha_1} \quad \text{or} \quad \beta_1^{-\alpha_1} = \sum_{i=1}^{n} \left( \frac{v_i^{\alpha_1}}{n} \right)
\]

\[
0 = -\frac{n \alpha_2}{\beta_2} + \frac{\alpha_2}{\beta_2 \alpha_2} \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2} \rightarrow \beta_2 = \sum_{i=1}^{n} \left( \frac{(v_i - \gamma_0)^{\alpha_2}}{n} \right)^{\alpha_2} \quad \text{or} \quad \beta_2^{-\alpha_2} = \sum_{i=1}^{n} \left( \frac{(v_i - \gamma_0)^{\alpha_2}}{n} \right)
\]

\[
0 = -n \log \beta_1 + \frac{\log \beta_1}{\beta_1} \sum_{i=1}^{n} v_i^{\alpha_1} - \frac{1}{\beta_1 \alpha_1} \left( \sum_{i=1}^{n} \log (v_i) \right)^{\alpha_1} + \sum_{i=1}^{n} \log (v_i)
\]
\[ \Rightarrow \sum_{i=1}^{n} v_i^{o_i} \log (v_i) - \frac{1}{\alpha_2} = \sum_{i=1}^{n} \log (v_i) \]  

\[ 0 = -\frac{n}{\alpha_2} n \log \beta_2 + \frac{\log \beta_2}{\beta_2^{o_2}} \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2} - \frac{1}{\beta_2^{o_2}} \left( \sum_{i=1}^{n} \log (v_i - \gamma_0) (v_i - \gamma_0)^{\alpha_2} \right) + \sum_{i=1}^{n} \log (v_i - \gamma_0) \]  

\[ \frac{\sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2} \log (v_i - \gamma_0)}{\sum_{i=1}^{n} (v_i - \gamma_0)} - \frac{1}{\alpha_2} = \frac{\sum_{i=1}^{n} \log (v_i - \gamma_0)}{n} \]  

\[ 0 = \frac{1}{\beta_2^{o_2}} \alpha_2 \sum_{i=1}^{n} (v_i - \gamma_0)^{\alpha_2 - 1} - \sum_{i=1}^{n} \frac{\alpha_2 - 1}{(v_i - \gamma_0)} \]  

4. Wind Power distribution

Wind turbine acts as a tool to convert kinetic energy of the wind into mechanical energy. The power is defined as [9]

\[ P = \frac{1}{2} m|v|^2 \]  

We know \( m = \rho A \) where \( \rho \) is the air density measured in \( (kg/m^2) \) which is assumed to be 1.293 \( kg/m^2 \) [13]

\[ \Rightarrow P = \frac{1}{2} \rho A |v|^3 \]  

Power coefficient \( C_p \), considered as 0.593 [10] and \( \eta \in (0,1) \) [12].

The power output of the turbine can be written as

\[ P = \frac{1}{2} C_p \eta \rho A |v|^3 \]  

4.1 Proposition

The CDF of power is defined by

\[ F_{F_{\rho,(2,3)}} (v) = \omega \{ 1 - \exp \left[ -\frac{1}{\beta_1^{o_1}} \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \} + (1 - \omega) \{ 1 - \exp \left[ -\frac{1}{\beta_2^{o_2}} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \} \]  

And PDF is

\[ f_{F_{\rho,(2,3)}} (v) = \omega \left\{ \exp \left[ -\frac{1}{\beta_1^{o_1}} \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \right] \left\{ \frac{1}{\beta_1^{o_1}} \frac{\alpha_1 v^{\frac{\alpha_1 - 1}{3}}}{3} \left( \frac{2}{\rho A} \right)^{\frac{\alpha_1}{3}} \right\} \right\} + (1 - \omega) \left\{ 1 - \exp \left[ -\frac{1}{\beta_2^{o_2}} \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_2}{3}} \right] \left\{ \frac{1}{\beta_2^{o_2}} \frac{\alpha_2 (v - \gamma_0)^{\frac{\alpha_2 - 1}{3}}}{3} \left( \frac{2}{\rho A} \right)^{\frac{\alpha_2}{3}} \right\} \right\} \]
Proof:

\[ FF_{p(2,3)}(v) = P(P \leq v) = P \left( V \leq \left( \frac{2v}{\rho A} \right)^{\frac{1}{3}} \right) \]

\[ = \omega \left( \frac{1}{\beta_1^{\alpha_1}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}}} - \left( \frac{1}{\beta_2^{\alpha_2}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}}} \left( 1 - \omega \right) \left( \frac{1}{\beta_0^{\alpha_0}} \right) \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_0}{3}} \right] \]

Diff w.r.t. \( v \),

\[ f_{p(2,3)}(v) = \omega \left( \frac{1}{\beta_1^{\alpha_1}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}}} - \left( \frac{1}{\beta_2^{\alpha_2}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}}} \left( 1 - \omega \right) \left( \frac{1}{\beta_0^{\alpha_0}} \right) \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_0}{3}} \right] \]

\[ f_{p(2,3)}(v) = \omega \left( \frac{1}{\beta_1^{\alpha_1}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}}} - \left( \frac{1}{\beta_2^{\alpha_2}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}}} \left( 1 - \omega \right) \left( \frac{1}{\beta_0^{\alpha_0}} \right) \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_0}{3}} \right] \]

\[ f_{p(2,3)}(v) = \omega \left( \frac{1}{\beta_1^{\alpha_1}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_1}{3}}} - \left( \frac{1}{\beta_2^{\alpha_2}} \right) \left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}} \int_{0}^{\left( \frac{2v}{\rho A} \right)^{\frac{\alpha_2}{3}}} \left( 1 - \omega \right) \left( \frac{1}{\beta_0^{\alpha_0}} \right) \left( \frac{2(v - \gamma_0)}{\rho A} \right)^{\frac{\alpha_0}{3}} \right] \]

4.2 Power Distribution

The wind speed experienced by the turbine is [11]

\[
V_{\text{turbine}} = \begin{cases} 
0 & V_{\text{turbine}} < v_{\text{cut-in}} \\
V_{\text{cut-in}} & v_{\text{cut-in}} \leq V \leq v_{\text{rated}} \\
v_{\text{rated}} & v_{\text{rated}} < V < v_{\text{cut-off}} \\
0 & V \geq v_{\text{cut-off}}
\end{cases}
\]

(25)

Where \( v_{\text{cut-in}} \) is lowest speed at which the turbine starts rotating and \( v_{\text{cut-off}} \) is the max wind speed at which it switches off. [14].

\[
P_{\text{turbine}} = \begin{cases} 
0 & V_{\text{turbine}} < v_{\text{cut-in}} \\
\frac{1}{2} \rho C_P \eta AV^3 & v_{\text{cut-in}} \leq V \leq v_{\text{rated}} \\
\frac{1}{2} \rho C_P \eta AV_{\text{rated}}^3 & v_{\text{rated}} < V < v_{\text{cut-off}} \\
0 & V \geq v_{\text{cut-off}}
\end{cases}
\]

(26)

4.3 Proposition

\( P_{\text{turbine}} \) has CDF given by

\[
FF_{P_{\text{turbine}}(2,3)}(v) =
\]
\[
\left\{ \begin{array}{c}
0 \\
-\infty < v < 0 \\
1 - \alpha e^{\left( \frac{\nu - \nu_{c,\beta}}{\beta_1} \right)^2} - e^{\left( \frac{\nu - \nu_{c,\beta}}{\beta_2} \right)^2}
\end{array} \right. \\
1 - \alpha e^{\left( \frac{\nu - \nu_{c,\beta}}{\beta_2} \right)^2} - e^{\left( \frac{\nu - \nu_{c,\beta}}{\beta_1} \right)^2}
\]

\[
+ e^{\left( \frac{\nu - \nu_{c,\beta}}{\beta_2} \right)^2} - e^{\left( \frac{\nu - \nu_{c,\beta}}{\beta_1} \right)^2}
\]

\[
1 - \omega e^{\left( \frac{\nu_{c,\beta}^0 - \nu}{\beta_1} \right)^2} - e^{\left( \frac{\nu_{c,\beta}^0 - \nu}{\beta_2} \right)^2}
\]

\[
= \frac{1}{2} \rho C_p \nu V_{\text{cut-in}} \leq V < \frac{1}{2} \rho C_p \nu V_{\text{rated}}
\]

\[
(27)
\]

**Proof:**

\[
P(P_{\text{turbine}} \leq 0) \text{ is } P\left\{ \left[ V \leq v_{\text{cut-in}} \right] \cup \left[ V \leq v_{\text{cut-off}} \right] \right\}
\]

\[
= FF_{(2,3)}(v_{\text{cut-in}}) + (1 - FF_{(2,3)}(v_{\text{cut-off}}))
\]

\[
= \omega \left\{ 1 - \exp \left( - \left( \frac{v_{\text{cut-in}}}{\beta_1} \right)^2 \right) \right\} + (1 - \omega) \left\{ 1 - \exp \left( - \left( \frac{v_{\text{cut-in}} - \nu_0}{\beta_2} \right)^2 \right) \right\}
\]

\[
+ \left\{ 1 - \omega \right\} \left\{ 1 - \exp \left( - \left( \frac{v_{\text{cut-off}}}{\beta_1} \right)^2 \right) \right\} + (1 - \omega) \left\{ 1 - \exp \left( - \left( \frac{v_{\text{cut-off}} - \nu_0}{\beta_2} \right)^2 \right) \right\}
\]

\[
= \omega \left\{ 1 - e^{\left( \frac{\nu_{c,\beta}^0}{\beta_1} \right)^2} - e^{\left( \frac{\nu_{c,\beta}^0}{\beta_2} \right)^2} \right\} + e^{\left( \frac{\nu_{c,\beta}^0 - \nu}{\beta_1} \right)^2} - e^{\left( \frac{\nu_{c,\beta}^0 - \nu}{\beta_2} \right)^2}
\]

\[
(28)
\]

For \( 0 < v = \frac{1}{2} \rho C_p \nu V_{\text{cut-in}} \leq \frac{1}{2} \rho C_p \nu V_{\text{rated}} \), note that \( v \leq \frac{1}{2} \rho C_p \nu V_{\text{rated}} \iff v \leq \nu_r \) and \( \nu - \nu_0 \leq \nu_r \)

For \( \rightarrow \) \( FF_{\text{lower,}(2,3)}(v) = P(0 < P_{\text{turbine}} \leq v) + P(P_{\text{turbine}} = 0) \)

\[
= \frac{1}{2} \rho C_p \nu V_{\text{cut-in}}^3 \leq P_{\text{turbine}} \leq v
\]

\[
+ \left\{ 1 - \omega \right\} \left\{ 1 - e^{\left( \frac{\nu_{c,\beta}^0}{\beta_1} \right)^2} - e^{\left( \frac{\nu_{c,\beta}^0}{\beta_2} \right)^2} \right\} + e^{\left( \frac{\nu_{c,\beta}^0 - \nu}{\beta_1} \right)^2} - e^{\left( \frac{\nu_{c,\beta}^0 - \nu}{\beta_2} \right)^2}
\]

\[
(29)
\]

\[
v = \frac{1}{2} \rho C_p \nu V_{\text{cut-in}}^3 \Rightarrow V_{\text{cut-in}}^3 = \frac{2\nu}{C_p \rho \nu} \Rightarrow V_{\text{cut-in}} = \left( \frac{2\nu}{C_p \rho \nu} \right)^{\frac{1}{3}}
\]
\[ E[P_{turbine}] = \frac{1}{2} C_p \eta A \left( 1 - \frac{v}{v_{rated}} - \frac{v_{cut-off} - v}{v_{rated} - v} - \frac{v_{cut-off} - v_{in}}{v_{rated} - v_{in}} \right) \]

\[ + \frac{1}{2} C_p \eta A \left( 1 - \frac{v}{v_{rated}} \right) \left( \frac{v_{rated} - v}{v_{rated}} - \frac{v_{cut-off} - v}{v_{rated}} \right) \]

\[ + \frac{1}{2} C_p \eta A \left( 1 - \frac{v}{v_{rated}} \right) \left( \frac{v_{rated} - v_{in}}{v_{rated}} - \frac{v_{cut-off} - v_{in}}{v_{rated}} \right) \]

\[ + \frac{1}{2} C_p \eta A \left( 1 - \frac{v}{v_{rated}} \right) \left( \frac{v_{rated} - v}{v_{rated}} - \frac{v_{cut-off} - v}{v_{rated}} \right) \]

\[ + \frac{1}{2} C_p \eta A \left( 1 - \frac{v}{v_{rated}} \right) \left( \frac{v_{rated} - v}{v_{rated}} - \frac{v_{cut-off} - v}{v_{rated}} \right) \]

4.4 Proposition

The expected power from the turbine is

\[ E[P_{turbine}] = \frac{1}{2} C_p \eta A \left[ \beta_1^2 \omega \left( \frac{v_{rated}}{\alpha_1} - \frac{v_{cut-off}}{\beta_1} \right) \left( 1 + \frac{3}{\alpha_1} \frac{v_{rated} - v}{v_{rated}} - \frac{v_{cut-off} - v_{in}}{v_{rated} - v_{in}} \right) \right] \]

\[ + \frac{1}{2} \rho C_p \eta A v_{rated}^3 \leq v < \infty \Rightarrow FF_{\rho \eta A v_{rated}}(v) = 1. \]

Equation (30) is a combination of (27), (28), and (29).

5. Conclusion

The proposed distribution plays a vital role in wind resource assessment and in designing suitable wind turbines for installation. This method is different from the existing bimodal Weibull due to the influence of the parameter gamma. Mathematical representation has been provided to analyse the wind power using the improved distribution.

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