Understanding Type Ia Supernova Distance Biases by Simulating Spectral Variations

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Abstract

In the next decade, transient searches from the Vera C. Rubin Observatory and the Nancy Grace Roman Space Telescope will increase the sample of known Type Ia supernovae (SNe Ia) from $\sim 10^3$ to $10^5$. With this reduction of statistical uncertainties on cosmological measurements, new methods are needed to reduce systematic uncertainties. Characterizing the underlying spectroscopic evolution of SN Ia remains a major systematic uncertainty in current cosmological analyses, motivating a new simulation tool for the next era of SN Ia cosmology: Build Your Own Spectral Energy Distribution (BYOSED). BYOSED is used within the SNANA framework to simulate light curves by applying spectral variations to model SEDs, enabling flexible testing of possible systematic shifts in SN Ia distance measurements. We test the framework by comparing a nominal Roman SN Ia survey simulation using a baseline SED model to simulations using SEDs perturbed with BYOSED, and investigating the impact of ignoring specific SED features in the analysis. These features include semiempirical models of two possible, predicted relationships: between SN ejecta velocity and light-curve observables, and a redshift-dependent relationship between SN Hubble residuals and host-galaxy mass. We analyze each BYOSED simulation using the SALT2 and BEAMS with Bias Corrections framework, and estimate changes in the measured value of the dark-energy equation-of-state parameter, w. We find a difference of $\Delta w = -0.023$ for SN velocity and $\Delta w = 0.021$ for redshift-evolving host mass when compared to simulations without these features. By using BYOSED for SN Ia cosmology simulations, future analyses (e.g., the Rubin and Roman SN Ia samples) will have greater flexibility to constrain or reduce such SN Ia modeling uncertainties.

Unified Astronomy Thesaurus concepts: Type Ia supernovae (1728); Cosmology (343); Astronomical simulations (1857)

1. Introduction

Type Ia supernovae (SNe Ia) are used for one of the fundamental observational tests of the ΛCDM “concordance cosmology.” The accelerating expansion of the universe was discovered with a sample of a few dozen SNe Ia extending to $z \sim 0.8$ (Riess et al. 1998; Perlmutter et al. 1999), and over the last 20 yr, this sample has grown to include $>1000$ SNe Ia reaching to $z \sim 2.3$ (Scolnic et al. 2018). SNe Ia are not standard candles, in that they do not all have the same absolute brightness. Rather, they are standardizable candles. Cosmological constraints depend upon SN Ia luminosity distance measurements, obtained by fitting the observed light curves with a carefully trained model (e.g., Guy et al. 2007; Jha et al. 2007). The model is used to standardize SN Ia absolute magnitudes by correcting for luminosity dependencies on model parameters such as shape and color (e.g., Tripp 1998). To test the SN Ia training and standardization process, and to evaluate robust systematic uncertainties, here we introduce Build Your Own Spectral Energy Distribution (BYOSED), a Python utility to generate realistic SN Ia SED models for simulated data samples.

Due to uncertainty in the physics of SN Ia progenitors, the standardization of SNe Ia is entirely empirical. For the last decade, the most commonly used and scrutinized model that implements shape and color corrections is the SALT2 model (Guy et al. 2007, 2010). SALT2 has been instrumental in SN Ia cosmology analyses due to its accuracy in reproducing observed SN Ia light curves and has become the standard choice for both large-scale SN Ia simulations and light-curve fitting for luminosity distance measurements (e.g., Betoule et al. 2014; Scolnic & Kessler 2016; Jones et al. 2018a; Scolnic et al. 2018; Abbott et al. 2019; Jones et al. 2019; Kessler et al. 2019a; Smith et al. 2020).

As more data have become available, additional observables that correlate with SN Ia properties or distance measurements have been identified. One of the earliest observations of such a relationship was based on the total stellar mass of the SN host galaxy (e.g., Hamuy et al. 1996; Ivanov et al. 2000; Kelly et al. 2010; Lampeitl et al. 2010; Sullivan et al. 2010). More recent studies suggest that corrections based on galaxy properties at the SN location could lead to more accurate SN Ia distance measurements (Neill et al. 2009; Rigault et al. 2013, 2020; Kim et al. 2014; Jones et al. 2018a; Roman et al. 2018; Jha et al. 2019). Today, it is generally accepted that the local environment of an SN Ia correlates with the properties of its spectra and light curve, but it is unclear if current corrections (e.g., the mass step; see Section 3.4) account for these relationships with sufficient precision to accommodate the much larger data sets that will be obtained over the next decade. Numerous studies have investigated the potential impact on SN Ia standardization.

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8 https://github.com/jpiere14/BYOSED, https://byosed.readthedocs.io/en/latest
of many variables beyond host mass including local specific star formation rate (sSFR; Rigaut et al. 2013; Kelly et al. 2015), intrinsic luminosity evolution with redshift (Howell et al. 2007), and metallicity (Childress et al. 2013; Hayden et al. 2013), as well as the ways in which SN properties such as ejecta velocity correlate with Type Ia standardization parameters and Hubble residuals (Wang et al. 2009; Foley et al. 2011; Siebert et al. 2020).

At present, simulations of SN Ia populations are used in cosmological analyses to correct for predicted or empirically measured biases (e.g., Betoule et al. 2014; Scolnic & Kessler 2016; Kessler & Scolnic 2017; Scolnic et al. 2018; Kessler et al. 2019a) and to plan for future surveys (e.g., Hounsell et al. 2018; Kessler et al. 2019b). In all cases, the simulations are created by drawing random stretch and color parameters from measured population distributions to obtain variations of the SALT2 model. Current simulations for bias corrections are capable of including relationships between SN Ia properties and the SN host environment at the population level but are not flexible enough to apply SED features and their correlations with stretch, color, and host-galaxy properties directly. This modeling limitation results in inadequate tests for biases in measurements of SN Ia distances and cosmological parameters.

BYOSED provides a new and general method for producing simulated SN Ia light-curve samples, by using real SN Ia spectra or a theoretical model to simulate these potential effects on an SN Ia population. BYOSED is uniquely poised to help identify inadequacies in SN Ia light-curve models such as SALT2 by independently simulating variations beyond the scope of the model used to fit the generated light curves. Previous simulated bias corrections for the JLA (Betoule et al. 2014), Pantheon (Scolnic et al. 2018), PS1 (Jones et al. 2018b), and DES3YR (Abbott et al. 2019) have used the SALT2 model and the Supernova ANAlysis (SNANA: Kessler et al. 2009, 2010, 2019a) software package. These analyses do not characterize the SED features correlating with observables such as host-galaxy mass or SN ejecta velocity, which could propagate through to distance measurements and cosmological inferences. BYOSED also generates SN Ia light curves within the SNANA framework for state-of-the-art bias correction and cosmological parameter estimation but is able to include these features directly into the source SED. There has been one previous effort to characterize SALT2 training uncertainties with a more general simulation (Mosher et al. 2014). This effort focused on warping SED templates from Hsiao et al. (2007) in order to avoid generating a model in which the SALT2 spline basis is already imprinted on the SEDs. However, Mosher et al. (2014) did not introduce correlations between the host galaxy and the SN SED, nor did they introduce redshift-dependent parameters.

Section 2 gives a description of the implementation of BYOSED, including the process needed to vary a baseline SED with observed spectra. Section 3 provides a simulation case study in which light curves are generated with BYOSED inside of SNANA and fit to obtain distance measurements. These measurements are propagated to Section 4 to identify resulting biases in the dark-energy equation-of-state parameter, \( \omega \), relative to a fiducial survey. We conclude with a discussion of the results in Section 5.
BYOSED is designed to precisely address these limitations in the cosmology analysis pipeline. Simulations using BYOSED can be used to both identify and quantify presently unaccounted for biases and determine the relative benefit of using one model over another for cosmological analyses. The two core components of the BYOSED framework are a "baseline SED" and one or more "perturbers," defined as follows:

1. **Baseline SED:** An SED model describing the evolution of an SN Ia as a function of wavelength and time, \( H(t, \lambda) \). Examples include the SALT2 \( M_0 \) component or the template from Hsiao et al. (2007, hereafter H07).

2. **Perturber:** A relationship \( P_i \) derived from theory or observations, which BYOSED uses to "perturb" the baseline SED model with added SED features that correlate with observables. Examples include spectra defining SNe Ia with high and low ejecta velocity or a correlation between host-galaxy metallicity and SN Ia luminosity.

The process used to create these perturbers is described below in Section 2.2. With a set of perturbers, BYOSED produces a rest-frame source SED, \( F(t, \lambda) \), in the following way:

\[
F(t, \lambda) = A \times H(t, \lambda)[1 + P_1(\theta_{SN}, \theta_{HOST}, t, \lambda)s_1 + \ldots + P_N(\theta_{SN}, \theta_{HOST}, t, \lambda)s_N] \times C_1(\theta_c_1)c_1 \times \ldots \times C_M(\theta_c_M)c_M, \tag{2}
\]

where \( \theta_{SN} \) is a set of SN parameters (e.g., ejecta velocity), \( \theta_{HOST} \) is a set of host-galaxy parameters (e.g., mass, metallicity, redshift, etc.), and \( s_i \) is a scale factor for \( P_i \), the \( i \)th SED perturber. Here \( C_j \) and \( c_j \) are the \( j \)th "global correction" and scale parameter, respectively. A global correction is simply a perturber that corrects the overall flux of the model, similar to the color law in Equation (1), which is dependent on parameters \( \theta_{C_j} \). Note that it is possible to create a SALT2-like simulation with BYOSED by choosing stretch and color perturbers (see Section 3) that mimic the observed variation in SN Ia light-curve shape and color described by the SALT2 \( M_1 \) and CL components (Equation (1)). In this case, the BYOSED framework would create a rest-frame SED, \( F(t, \lambda) \), in the following way:

\[
F(t, \lambda) = A \times H(t, \lambda)[1 + S(t, \lambda)s] \times C(\lambda)c, \tag{3}
\]

where \( A \) is the amplitude, \( H \) is the baseline SED flux, and \( s, c \) are scale parameters chosen from a distribution defined for the stretch and color perturbers (\( S, C \)).
Equations (1)–(2) highlight the similarity of BYOSED to a PCA-like model such as SALT2, except that BYOSED can vary the baseline SED far beyond the flexibility introduced by the $M_1$ and CL components. This more realistic source SED is used in the SNANA simulation as described at the beginning of Section 2 (and Figure 1) to produce SN Ia light curves with brightness dependencies beyond the SALT2 framework. By fitting the resulting light curves with SALT2 or other models like those of Saunders et al. (2018) and Léget et al. (2020), we are able to identify model-specific biases and quantify the benefits or drawbacks of a choice of model.

2.2. Creating BYOSED Perturbers from Observables

The BYOSED toolkit accepts perturbers in one of two forms: SED format or a theoretical function. For example, a theoretical function that could be used to create a perturber is from Moreno-Raya et al. (2016):

$$M_B(Z) = M_{B,Z=0} - 0.25 \log_{10} \left[ 1 - 0.18 \frac{Z}{Z_\odot} \left( 1 - 0.10 \frac{Z}{Z_\odot} \right) \right] - 0.191 \text{ mag},$$ (4)

where $Z_\odot$ is the solar metallicity and $M_B$ is the absolute magnitude of the SN in the $B$ band. Equation (4) contains no wavelength dependence but will simply increase or decrease the brightness of the resulting SN depending on the chosen metallicity of the host galaxy. The equation is based on empirically measured correlations between SN Ia absolute magnitudes and oxygen abundances in their host galaxies by Moreno-Raya et al. (2016). When paired with a distribution of observed host-galaxy metallicities, an SNANA simulation using Equation (4) and BYOSED would produce an SN Ia sample with a theoretically motivated relationship between luminosity and host-galaxy metallicity.

BYOSED perturbers can also be created from observations. Here we use composite spectra generated from the KAEPORA database, an open-source relational database of SN Ia observations (Siebert et al. 2019). We use the Gini-weighting method to create the composite spectra, outlined in Siebert et al. (2019), which provides a representative spectrum that maximizes the signal-to-noise ratio ($S/N$) while mitigating the impact of high-$S/N$ outliers. We control for average properties of phase and $\Delta m_{15}(B)$ to produce sequences of composite spectra with desired properties (e.g., ejecta velocity, host-galaxy mass). We apply selection requirements on $\Delta m_{15}(B)$ to remove peculiar SN Ia spectra from the sample (requiring $0.7 \leq \Delta m_{15}(B) \leq 1.8$ mag) and to remove correlations between light-curve shape and host environment (see Sections 3.3.1 and 3.4.1).

The final SEDs from KAEPORA have 2 Å spectral resolution, and 3–5 day temporal resolution. We perform a linear interpolation in the wavelength dimension as the spacing is more than sufficient to ensure accuracy, but use a more robust Gaussian Process interpolation in the phase dimension to obtain a smoother phase-dependent SED surface. These interpolations are used to sample the composite spectra at the wavelength and phase grid values defined by the baseline SED and to transform them into BYOSED perturbers.

For a summary of how perturbers are created and implemented, see Figure 1 and Section 2.1. As an example, consider using KAEPORA to obtain two composite spectra representing high and low ejecta velocity SNe, respectively. BYOSED calculates the average of the two composites, $A(t, \lambda)$, and then the fractional difference between the high- or low-velocity spectra and $A(t, \lambda)$, with respect to the baseline SED:

$$P_{V,H}(t, \lambda) = \frac{V_H(t, \lambda) - A(t, \lambda)}{H(t, \lambda)},$$ (5)

$$P_{V,L}(t, \lambda) = \frac{V_L(t, \lambda) - A(t, \lambda)}{H(t, \lambda)},$$ (6)

where $P_{V,H}$ ($P_{V,L}$) contains fractional deviations of the composite spectrum $V_H$ ($V_L$) from $A$ for the high-velocity (low-velocity) composite spectrum, with respect to the baseline SED, $H(t, \lambda)$. The final “ejecta velocity perturber” is a function of ejecta velocity as well as phase and wavelength, $P_V(v, t, \lambda)$, defined in the following manner:

$$P_V(v, t, \lambda) = \begin{cases} P_{V,H}(t, \lambda) & v > v_0 \\ P_{V,L}(t, \lambda) & v < v_0, \end{cases}$$ (7)

where $v$ is a velocity chosen by BYOSED and $v_0$ is the break point between a high and low ejecta velocity SN. Given a large enough sample of spectra, one could produce composites for a range of ejecta velocities instead of just two, and $P_V$ would simply interpolate along the ejecta velocity dimension.

We use specific perturbers in the course of this paper to show the capabilities of BYOSED (see Sections 3 and 4), but of course, any choice of perturber is possible and easily included within the BYOSED framework. The perturbers in this work were chosen due to their necessity for an accurate SN Ia model (stretch, color), and as a useful starting point to study a subset of the potential biases for cosmology (host mass, SN velocity).

3. Simulating an SN Ia Survey with BYOSED and SNANA

In this work, we simulate Roman Space Telescope SN surveys to demonstrate the value of BYOSED and to test for potential biases in cosmological measurements (Section 4). We use the survey and telescope specifications outlined by Hounsell et al. (2018, hereafter H18) to produce realistic light curves and global survey features for the Roman Space Telescope mission. For the SED source model, we replace SALT2 with a variety of perturbed SEDs generated by BYOSED. We first describe the simulation process then present a series of four simulation case studies. These examples begin with a fiducial survey using a baseline SED model with minimal variations then progressively add more perturbations that modify the underlying SED. All simulations assume a FlatwCDM cosmology with $w = -1$ and $\Omega_m = 0.315$ (Planck Collaboration et al. 2020).

3.1. Simulations with SNANA

SNANA simulates SN light curves for an arbitrary set of survey properties while accounting for variations in noise, point-spread function (PSF), and cadence. Due to its speed, accuracy, and flexibility, SNANA has become the standard tool for simulating SN surveys in recent years (e.g., Betoule et al. 2014; Scolnic et al. 2018; Jones et al. 2019; Kessler et al. 2019a).
Following Figure 1 in Kessler et al. (2019a), a brief overview of the SNANA+BYOSED simulation is as follows:

1. **Source Model**
   (a) Generate source SED at each simulated epoch using baseline SED and BYOSED perturber choices (see Sections 3.2–3.4).
   (b) Apply cosmological dimming, Galactic extinction, weak lensing, peculiar velocity, and redshift to the SED.
   (c) Integrate redshifted SED over each filter transmission function to create the noise-free photometric light curve.

2. **Noise Model**
   (a) Use image zero point to convert each true light curve in magnitude to true flux in photoelectrons.
   (b) Compute flux uncertainty from zero point, PSF, and sky noise; apply random fluctuation to true flux.

3. **Trigger Model**
   (a) Check for detection (S/N > 3 σ in two or more bands)
   (b) Write selected events to data files.

The Roman SN survey will provide a unique probe for constraining the nature of dark energy, delivering near-IR observations for thousands of SNe Ia over a wide redshift range of 0.01 ≤ z ≤ 3.0. Because the SALT2 model is poorly constrained at UV (λ < 3000 Å) and near-IR (λ > 8500 Å) wavelengths, we restrict our simulated redshift range from Roman to 0.5 < z < 1.5, ensuring that our simulations are sampled mostly within the SALT2 wavelength range (∼3000–9000 Å). With an updated SN Ia light-curve template that extends training into the UV and near-IR, this process could be repeated to evaluate the impact over a wider redshift range.

**H18** explored 11 survey strategies including 4 variations on each of 3 primary strategies. Here we investigate only the “All-z” primary strategy. The All-z strategy is constructed as a series of “tiers” labeled as Shallow, Medium, and Deep. Each tier was simulated here over the redshift range of 0.5–1.5, using the RZYZ filters for the Shallow and Medium tiers, and YJHF for the Deep tier (see H18’s Figure 1 for filter transmission details). These tiers have single-visit depths equal to 22.0 mag (J band), 24.8 mag (J band), and 26.2 (H band) for the Shallow, Medium, and Deep tiers respectively.

The only difference between the **H18 All-z** simulation and the simulations produced for this work is the choice of rest-frame source SED model as BYOSED, and the slightly more limited redshift range. In order to effectively constrain cosmological parameters, we also simulate a low-z anchor sample of SNe Ia that mimics the Foundation survey (Foley et al. 2018; Jones et al. 2019). We proceed with a variety of identical survey simulations, varying only one of perturbers in BYOSED. In each case, we repeat the full simulation 50 times to create distinct random realizations of the same SN survey, helping us constrain the relevant statistical uncertainties on the biases measured for cosmology. The following sections describe each individual simulation, which are summarized in Table 1.

### 3.2. Fiducial Simulation

In this section, we create our fiducial simulated SN survey that mimics the **H18 All-z** simulation, which used SALT2 and its associated stretch and color parameters as the rest-frame source SED model. We add coherent scatter for simplicity across all simulations, which is a good approximation to a wavelength-dependent intrinsic scatter model like that of Kessler et al. (2013). The following sections describe the BYOSED perturbers used for the fiducial simulation, which are meant to produce a SALT2-like source SED (see Equation (3)), and the matching of stretch and color to the bias-correction sample (see Section 4.1 and Figure 8).

#### 3.2.1. Stretch Perturber

Any model that attempts to reproduce observed light curves of SNe Ia should include some variation of the SED with light-curve stretch. For this work, we have implemented stretch as a simple time dilation according to

\[
F(t, \lambda) = A \times H(t/f, \lambda),
\]

where \(F\) is the final flux, \(A\) is a scaling amplitude, \(H\) is the baseline SED flux, and \(f\) is the stretch factor (\(f = 1.1\) corresponds to \(x_1 \sim 0\) in the SALT2 framework). Because our analysis includes light-curve fitting with the SALT2 model (Section 4), differences between the **H07** and SALT2 SEDs result in large Hubble residuals that hide the underlying systematic effects from the BYOSED perturbers. To focus on the BYOSED perturbers, we have warped the **H07** template at each rest-frame epoch so that the synthetic photometry matches that of the SALT2-M0 template. The effects of Equation (8) and the **H07** warping are shown in Figure 3.

#### 3.2.2. Color Perturber

As with the stretch perturber in Section 3.2.1, **BYOSED** needs a color perturber in order to reproduce realistic SN Ia flux variation with wavelength. Again, we limit wavelength-dependent differences between models by adopting the SALT2 CL as the BYOSED color perturber, implemented as a global correction according to Equation (2). The effect of implementing this **BYOSED** perturber is shown in Figure 4.

#### 3.2.3. Matching Stretch and Color Distributions

Because the warped **H07** template and stretch perturbers are not the same as the **M0** and **M1** components of SALT2, we do not expect the simulated values of stretch and color to map precisely onto the measured SALT2 \(x_1\) and \(c\) parameters, even in the limit of infinite S/N. We generate stretch and color such that the “measured” distribution of \(x_1\), \(c\) match the measured \(x_1\), \(c\) distributions from Scolnic & Kessler (2016, hereafter SK16) and H18. This matching is done iteratively, varying the **BYOSED** stretch and color distributions until the fitted distributions of \(c\) and \(x_1\) match those of SK16. We determine that distributions are sufficiently “matched” when a Kolmogorov–Smirnov (KS) test

| Perturbers Label Section |
|--------------------------|
| Stretch, color, intrinsic scatter Fiducial 3.2 |
| Fiducial+SED dependence on Si II velocity Fiducial+V_{Si II} 3.3 |
| Fiducial+SED dependence on host stellar mass Fiducial+M_{stellar} 3.4 |
| Fiducial+SED z dependence on host stellar mass Fiducial+M_{stellar}(z) 3.4 |

### Table 1

**Summary of the Simulations Created in This Section**

| Perturbers Label Section |
|--------------------------|
| Stretch, color, intrinsic scatter Fiducial 3.2 |
| Fiducial+SED dependence on Si II velocity Fiducial+V_{Si II} 3.3 |
| Fiducial+SED dependence on host stellar mass Fiducial+M_{stellar} 3.4 |
| Fiducial+SED z dependence on host stellar mass Fiducial+M_{stellar}(z) 3.4 |

### Note

The **BYOSED** simulation is as follows:

- Use image zero point to convert each true light curve in magnitude to true flux in photoelectrons.
- Compute flux uncertainty from zero point, PSF, and sky noise; apply random fluctuation to true flux.
- Check for detection (S/N > 3 σ in two or more bands)
identifies the distributions as identical, with a confidence of >95%.

3.2.4. Matching Luminosity Parameters

In addition to the matching of stretch and color distributions in Section 3.2.3, we undergo a similar process for choosing the stretch and color luminosity coefficients for BYOSED. These coefficients ($\alpha'$ and $\beta'$) are analogs of $\alpha$ and $\beta$ in the SALT2 model and are used in the BYOSED simulation to adjust the SN magnitude by $\alpha' b_s - \beta' b_c$, with $b_s$ and $b_c$ being the BYOSED stretch and color parameters. Therefore, we vary $\alpha'$ and $\beta'$ in BYOSED until the $\alpha$ and $\beta$ measured in the analysis (Section 4.1) are within 3\sigma of the values used by H18 ($\alpha = 0.14$, $\beta = 3.1$). For the fiducial simulation, we find that input values of $\alpha' = 0.068$, $\beta' = 3.08$ yield average measurements of $\alpha = 0.142 \pm 0.001$, $\beta = 3.10 \pm 0.01$ across the 50 survey iterations. We note that $\beta' \sim \beta$ as expected because we are using the SALT2 CL as the BYOSED perturber, but $\alpha'$ and $\alpha$ differ by a factor of $\sim 2$ because of our choice of stretch perturber. While we could avoid the process outlined here and in Section 3.2.3 by simply using the SALT2 $M_0$ and $M_1$ templates as our baseline SED and stretch perturber, these transformations provide some valuable independence from SALT2 in our simulations. Past analyses assumed that the model used for light-curve fitting perfectly explains the variation introduced by simulations, and we wish to avoid that here as much as possible.

3.3. Fiducial + $V_{Si
II}$ Simulation

We next follow the process outlined in Section 3.2 and include an SN velocity perturber in addition to the color and stretch perturbers (Sections 3.2.1–3.2.2). Following the methodology outlined in Section 3.2.4, we find that BYOSED input values of $\alpha' = 0.07$, $\beta' = 3.32$ yield average BEAMS with Bias Corrections (BBC) measurements of $\alpha = 0.142 \pm 0.001$, $\beta = 3.11 \pm 0.01$. Here, we simulate each SN with a dependency (SN velocity) that has no corresponding model component in SALT2. Ejecta velocities are simulated according to the observed population from Siebert et al. (2020).

3.3.1. Ejecta Velocity Perturber

SN velocity is an active topic of investigation in the SN Ia community, with hints that high- and low-velocity SNe Ia vary in their intrinsic colors and intrinsic color scatter (e.g., Foley et al. 2011; Foley 2012; Mandel et al. 2014; Pan et al. 2015; Siebert et al. 2020; Dettman et al. 2021). This correlation motivated the inclusion of a velocity perturber in our simulations. We use the KAEPORA database to generate an SED time series (spectral sequence; see Section 2.2) for low- and high-velocity SNe Ia defined by $V_{\text{SSn}} > -11,000$ and $V_{\text{SSn}} < -11,000$, respectively. Each of these sequences comprises 20 composite spectra generated with phase bin sizes of 3 days ranging from $-14$ to $+47$ days and resulting in a median $\Delta m_{15}(B)$ of $1.10 \pm 0.01$ and $1.13 \pm 0.02$ for low and high velocity, respectively. By translating these composite spectra into a perturber, BYOSED is able to produce simulated SNe Ia with spectra and light curves matching those of low- and high-ejecta-velocity SNe (Figure 5). Figure 6 shows the spectral and rest-frame $B - V$ color difference between low- and high-velocity perturbers using BYOSED. The shape of the spectral features in Figure 5 is consistent with the variations due to velocity found by Siebert et al. (2020), with the most significant shifts seen in those associated with Ca II H&K, Si II, and the Ca II NIR triplet. BYOSED-simulated SNe Ia with lower
velocity are bluer than those with high velocity, consistent with previous observations (Foley et al. 2011; Siebert et al. 2020). We treat these color differences as “intrinsic” (i.e., not correlated with luminosity), as proposed by Foley et al. (2011).

3.4. Fiducial + M\textsubscript{Stellar} and Fiducial + M\textsubscript{Stellar}(z) Simulations

As a final example, we include host-galaxy mass correlations as a BYOSED perturber in our simulations, in addition to the stretch and color perturbers outlined in Section 3.2. Host-galaxy masses are simulated according to the observed population from Siebert et al. (2019). In this section, we create two sets of simulations: first, we include the host mass as a static BYOSED perturber with no redshift evolution. In the analysis, a standard static “host-step” will correct for the perturber. A small $\omega$ bias is expected because the host-mass perturber is based upon spectra from the KAEPORA database, and thus, we attempt to correct a wavelength-dependent perturber with a single wavelength-independent correction to the Hubble residuals. Following Section 3.2.4, we find that input BYOSED values of $\alpha = 0.07, \beta = 3.14$ yield BBC measurements of $\alpha = 0.143 \pm 0.001, \beta = 3.12 \pm 0.01$.

In the second simulation, we introduce a redshift-dependence using a linear approximation of the mass step vs. redshift relationship proposed by Childress et al. (2014). In the analysis, we use the same host-mass step correction that is applied to the first simulation. Here, input BYOSED parameters of $\alpha = 0.068, \beta = 3.14$ yield BBC measurements of $\alpha = 0.142 \pm 0.001, \beta = 3.12 \pm 0.01$.

3.4.1. Host-galaxy-mass Perturber

Previous work has found that high-mass galaxies hosting SNe Ia tend to have negative Hubble residuals, motivating the simulations produced in this section (e.g., Kelly et al. 2010; Lampeitl et al. 2010; Sullivan et al. 2010). We again use KAEPORA to generate low- and high-mass host-galaxy spectral sequences using spectra from SNe Ia with log $M\textsubscript{Stellar} < 10.7$ and log $M\textsubscript{Stellar} > 10.7$, respectively. This choice of mass-step maximizes the coverage of KAEPORA spectra in each mass range. Each of these sequences comprises 15 composite spectra ranging from $-10$ to $+62$ days. In order to include a statistically significant sample in each bin, phase bin sizes range from 4 days at early times to 7 days at later times. Given the correlation between SN Ia luminosity and host-galaxy environment (Hamuy et al. 2000; Howell 2001), we select $\Delta m_{15}(B)$ bins such that each composite spectrum has a median $\Delta m_{15}(B)$ of $1.15 \pm 0.06$ mag and $1.16 \pm 0.06$ mag for low- and high-mass, respectively. Thus, the remaining differences in the composite spectra (if any) should be related to host-galaxy mass and not light-curve shape.

We translate the composite spectra into a BYOSED host-mass perturber in the manner described in Section 2.2. Figure 7 shows an SN Ia light curve and fractional difference spectrum (relative to H07) from a low- and high-mass host galaxy using BYOSED. The similar $B$-band light curves reflect the selections based on $\Delta m_{15}$ above, and the resulting mass step should be due to variations in SED features alone (Siebert et al. 2019). Because the profile shape of the fractional differences is different from that of the fractional differences seen in the velocity case of Section 3.3.1, it is unlikely that there is a large velocity difference between the low- and high-mass samples. Given that there are still significant differences in Si II 6355 and the Ca II NIR triplet, it is possible that these features differ in their equivalent width. This topic is investigated briefly in Siebert et al. (2019) but likely warrants attention in future work.

We do observe some residual differences in recovered $x_1$ even after controlling for $\Delta m_{15}(B)$ that are not present in the case of the velocity composites (Section 3.3.1). It is possible that controlling for $x_1$ instead of $\Delta m_{15}(B)$ would reduce these differences, but we have chosen not to control for SALT2 parameters at the KAEPORA stage so that the perturbers remain as general as possible. We cannot be sure that our perturber exactly mirrors the effect of the mass step in nature, but regardless, our implementation of the $z$ dependence of this step faithfully follows the theoretical prediction from Childress et al. (2014) and the low-significance measurement from Scolnic et al. (2018).

4. Identifying Potential Sources of Bias for Roman

For each choice of BYOSED perturber(s), we have created 50 simulated Roman Space Telescope SN Ia surveys, as well as the anchor data sets of low-z SNe Ia. All simulated light curves are fit with the SALT2 model. The fitted SALT2 parameters for the simulations in Section 3 and the bias-correction sample (see...
Section 4.1) are used by the BBC method to determine bias-corrected distance moduli in 30 redshift bins. In this work, we define measured cosmological biases with respect to variations in the \( w \) parameter in the FlatwCDM model, whereby the dark-energy equation of state is defined as \( p = w \rho c^2 \). This process is outlined in Figure 8, and details of the final two stages are given in Section 4.1. The cosmological measurements are presented in Section 4.2.

4.1. Analysis

We transform fitted SALT2 light-curve parameters into distances by way of a modified Tripp formula (Tripp 1998):

\[
\mu = m_B - M + \alpha x_1 - \beta c + \gamma G_{\text{Host}} + \Delta \mu_{\text{Bias}},
\]

where \( \mu \) is the distance modulus, \( \alpha (\beta) \) is the coefficient of the relation between SN Ia luminosity and stretch (color), and \( M \) is the peak absolute magnitude of an \( x_1 = c = 0 \) SN Ia assuming some nominal value of \( H_0 \). \( \gamma \) corrects for a dependence on host-galaxy stellar mass, with \( G_{\text{Host}} = 1/2 \) or \(-1/2\) if \( M_{\text{Stellar}} > 10^{10.7} M_\odot \) or \( M_{\text{Stellar}} < 10^{10.7} M_\odot \), respectively. Finally, the \( \Delta \mu_{\text{Bias}} \) term is a selection bias correction determined by BBC, described below. Recall that in this work, the mass step location is set at \( 10^{10.7} M_\odot \) instead of the typical value of \( 10^{10.3} M_\odot \) to balance the number of KAEPORA spectra corresponding to high- and low-mass host galaxies (Section 3.4.1). For the fiducial and fiducial +\( V_{\text{Si II}} \) simulations, where we do not associate a simulated host-galaxy mass to each SN, we simply set the value of \( \gamma \) to 0 at the BBC stage.

We then employ the BBC method (Kunz et al. 2007, 2013; Kessler & Scolnic 2017), which estimates the correction term in Equation (9), \( \Delta \mu_{\text{Bias}} \), based on a large simulated sample of SNe Ia in a 5D space of \( \{ \mathcal{z}, x_1, c, \alpha, \beta \} \). BBC produces a

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**Figure 6.** (Top) The fractional difference between the KAEPORA composite spectrum for high and low ejecta velocity at peak brightness, relative to the warped H07 template. (Bottom) The rest-frame \( B - V \) color curve for low (blue dashed) and high (red solid) ejecta velocity.

**Figure 7.** (Top) The fractional difference between the KAEPORA composite spectrum for high and low host-galaxy mass as peak brightness, relative to the warped H07 template. (Bottom) Broadband light curves based on the SED described by BYOSED using the warped H07 template with low (green dashed) and high (magenta solid) host-galaxy mass perturbers.
bias-corrected Hubble diagram, from which we measure \( w \), and requires two primary inputs:

1. **Data**: The “observed” light curves, here simulated by SNANA using BYOSED as the source SED (see Sections 2–3). These light curves are fit with the SALT2 model.

2. **Bias-correction sample**: A large sample of SN Ia light curves simulated and fit with the SALT2 model. Note that the “data” and bias-correction sample are both fit with the SALT2 model, but simulated with independent models.

One must ensure that the properties of the SNe in the “data” are well matched by large simulations, which determine the nuisance parameters \( \alpha \) and \( \beta \) and simultaneously identify bias corrections for the remaining parameters in the Tripp formula: \( x_1, c \). The iterative process outlined in Section 3.2.3 ensures that the fitted light-curve parameter distributions for the simulated SNe (i.e., \( x_1, c \)) match the distributions of our large simulations for BBC, defined in SK16. Here we are proceeding in the same manner as previous cosmological analyses (e.g., Scolnic et al. 2018), except that here, the “data” are generated by SNANA with BYOSED as the source model. The large bias-correction simulations (~10× larger than those with BYOSED) are generated with a nominal SALT2 simulation in line with Kessler & Scolnic (2017). While we use the SALT2 model for our bias-correction simulations to mimic previous work and identify biases, we note that replacing the SALT2 model with BYOSED removes the biases reported in Section 4.2, as expected.

For the four choices of BYOSED perturbers, we implement the BBC method for each of the 50 distinct simulations created in Section 3 (a total of 200 survey simulations). This method creates a set of BBC distance modulus measurements, \( \mu_{\text{BBC}, z} \), which are bias-corrected. We use the BBC redshift–distance pairs in 30 redshift bins to fit a cosmological model using flat priors of \( \Omega_M = 0.315 \pm 0.007 \) (Planck Collaboration et al. 2020) and \( w = -1.0 \pm 1.0 \), minimizing the difference \( \mu_{\text{BBC}, z} - \mu_{\text{model}, z} \).

### 4.2. Results

The analysis of each simulation gives us a unique constraint on \( w \), which are combined with a weighted average to obtain final measurements for the fiducial, fiducial + \( V_{\text{SiII}} \), fiducial + \( M_{\text{Stellar}} \), and fiducial + \( M_{\text{Stellar}}(z) \) simulations. Biases in \( w \) relative to the fiducial simulation (\( \Delta w_{\text{bias}} = w - w_{\text{fiducial}} \)) are reported in Table 2. We note that the fiducial simulation does not precisely recover the input cosmology of \( w = -1 \) due to the wavelength-dependent differences between our baseline warped H07 template combined with a stretch perturber and the SALT2 model. These differences introduce small variations in distance measurement accuracy as a function of redshift that reveals themselves as a biased measurement on \( w \). By comparing the subsequent measurements of \( w \) to this fiducial survey, we are directly quantifying the impact of the additional perturbers. Figure 9 shows the difference in Hubble residuals between the fiducial survey and each respective survey.

For the fiducial + \( V_{\text{SiII}} \) case, we find a bias on \( w \) of \( -0.023 \pm 0.006 \) with respect to the fiducial survey cosmology, which is 1.11 times the statistical uncertainty on \( w \) for the fiducial survey. We also observe a Hubble residual step of \( \sim 0.07 \) mag consistent with Siebert et al. (2020; 0.091 ± 0.035 mag), which is unsurprising as both studies used the same KAEPORA spectra (Figure 10). For the host-mass simulations, we have broken the results into the simulations that do not include redshift evolution and those that include redshift evolution. For these cases, as we are simulating meaningful host-galaxy masses associated with
each SN, we measure the $\gamma$ parameter for each simulation. We find that without redshift evolution (fiducial $+$ $M_{\text{Stellar}}$), a host-mass step is observed (Figure 11), and a simple luminosity correction with $\gamma = 0.043 \pm 0.003$ results in a very small bias on $w$ ($0.006 \pm 0.007$) that is 0.28 times the fiducial statistical uncertainty. When redshift evolution is included for the host-mass perturber (fiducial $+$ $M_{\text{Stellar}(z)}$), we find a bias in $w$ of $0.021 \pm 0.006$, or $\sim 1.02$ times the fiducial statistical uncertainty. The measured host-mass step correction term ($\gamma = 0.043 \pm 0.003$) undercorrects for the effect at low redshift, and overcorrects at high redshift due to the choice of redshift evolution (Figure 12).

5. Discussion and Conclusions

In this work, we have presented a new open-source tool, BYOSED, that will enable simple and rigorous tests of potential biases in SN Ia luminosity distance measurements. BYOSED is already included in the commonly used SNANA software package and can be employed immediately to investigate properties of the SN Ia cosmological measurement system that could impact cosmological analyses. BYOSED is capable of producing simulations using perturbers based on theoretical models or observed spectra.

As a proof of concept, we have applied BYOSED to simulate a representative Roman Space Telescope SN Ia survey. We created a fiducial population of SNe Ia analogous to simulations in the literature that use the SALT2 model exclusively and then compared the dark-energy equation-of-state parameter ($w$) inferred by the fiducial study to three further cases. The results are summarized as follows:

1. Fiducial $+$ $V_{\text{Si II}}$

(a) The addition of an SN velocity perturber causes a bias in $w$ relative to the fiducial case, likely due to the wavelength dependence of the velocity perturber causing redshift-dependent variation in measured color.
(b) The measured bias on $w$, relative to the fiducial simulation, is $-0.023 \pm 0.006$. This is 1.11 times the statistical uncertainty on $w$ for the fiducial survey, indicating that this could be a concerning source of potential systematic uncertainty for the Roman survey.

2. **Fiducial +$M_{\text{stellar}}$**
   (a) SN properties are strongly correlated with host-galaxy mass; while we have attempted to control for shape differences between SNe in high- and low-mass hosts, there could be residual correlations causing the step to be driven by shape and color differences in the perturbers that are treated as intrinsic. Therefore it is unclear if the bias in this case is modeled in a way representative of the true host-mass step, but serves as a useful test of a possible effect of host-mass-based SED variations.

   (b) The BBC method is able to correct for this effect, and we are left with a small bias on $w$ of $0.006 \pm 0.007$ relative to the fiducial survey. This is 0.28 times the statistical uncertainty on $w$ for the fiducial survey, indicating this would be a subdominant source of systematic uncertainty.

3. **Fiducial +$M_{\text{stellar}}(z)$**
   (a) The addition of redshift evolution causes the simple host-mass step correction to be insufficient, with SNe Ia undercorrected at low $z$ but overcorrected at high-$z$. 

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**Figure 10.** Example of binned Hubble residuals as a function of ejecta velocity. The input BYOSED simulation used $-11 \times 10^3 \text{ km s}^{-1}$ as the cutoff between “high-” and “low-” velocity SN, and the resulting “step” is $-0.07$ mag, consistent with previous work (Siebert et al. 2020, $0.091 \pm 0.035$ mag). Error bars show the standard error on the mean, and dashed red lines show the separate means for high- and low-mass hosts.

**Figure 11.** Example of binned Hubble residuals as a function of global host-galaxy mass. The input BYOSED simulation used 10.7 as the cutoff between “high-” and “low-” mass hosts, and the resulting “step” is $-0.071$ mag, consistent with previous work. Error bars show the standard error on the mean, and dashed red lines show the separate means for high- and low-mass hosts.
BYOSED can provide the next step forward in probing systematic steps as a function of redshift from this work are shown as black points with error bars, while the measured host-mass step (independent of redshift) is shown as a dotted magenta line. A scenario with no host-mass step would be represented by the dashed blue line.

(b) We observe a bias of $0.021 \pm 0.006$ on $w$ relative to the fiducial simulation or $\sim 1.02$ times the fiducial statistical uncertainty. This suggests a redshift-evolving host-mass correlation could contribute a systematic uncertainty on $w$ if a linear trend were not constrained by the cosmological analysis.

Based on these results, there could be an impact from these possible investigated relationships on the Roman SN Ia survey. These new potential systematics are roughly equal to the projected statistical uncertainty on measurements of $w$ with the Roman Space Telescope (Hounsell et al. 2018), making them important considerations when outlining uncertainty budgets for cosmology. Future work should investigate potential changes to the Roman SN Ia survey strategy that could mitigate, or help us understand, such effects. For example, it is plausible that a survey optimized for low-$z$ discoveries will be impacted quite differently than one optimized for high-$z$ discoveries when considering perturbers that vary with redshift. Alternatively, a well-balanced survey covering a wide redshift range or that includes high-S/N spectroscopy of many SNe Ia might be the most effective at identifying and understanding distance biases, as well as the extent to which SN Ia distance measurement systematics vary with redshift. Spectroscopic measurements could also help delineate between the effects that truly impact cosmological measurements and those that do not manifest themselves in observed data.

BYOSED can be used to investigate potentially redshift-dependent effects such as host-galaxy mass, metallicity, and star formation rate that could impact the results from different survey strategies in unpredictable ways. This work represents the first tool capable of flexibly producing light-curve simulations containing observed correlations between SED features and the SN progenitor or host environment. In much the same way that large-scale simulations with a model like SALT2 have improved our knowledge of SN Ia systematics over the last decade, BYOSED can provide the next step forward in probing systematic uncertainties that we have thus far been unable to accurately simulate at the SED level. BYOSED can be integrated into existing cosmology pipelines to more accurately represent these systematics for cosmological measurements, and test the relative accuracy of different SN Ia light-curve models.

The BYOSED tool will enable many interesting investigations into any perturber that has been proposed to impact SN Ia distance measurements. Even from this preliminary study into the effect of SN velocity and host-galaxy mass, we have gleaned useful information about the importance of acknowledging and accounting for these factors. More realistic simulations with BYOSED will also lead to more accurately trained light-curve models by identifying systematics and help leverage the next generation of SN Ia surveys as one of our most important cosmological probes.

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