Numerical study of the effect of gravitational sedimentation of large particles on the propagation of a shock wave from a homogeneous gas into a polydisperse gas suspension

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Abstract. The effect of the nonuniform concentration of one of the fractions of a two-fraction gas-suspension on the parameters of a shock wave moving from pure gas to a gas-suspension is numerically studied. The motion of a direct shock wave in a two-fraction gas suspension was simulated. The finely dispersed fraction of the gas-suspension had a uniform initial mass content, while larger particles had a nonuniform initial mass content along the y coordinate. It was revealed that in the case of a uniform concentration of all fractions, the shock wave in the channel propagates at a lower speed. It was also found that a nonuniform concentration of the larger particles fraction along the y coordinate affects the spatial distribution of the gas velocity.

1. Introduction
Many processes in nature and technology are associated with the flows of multiphase media [1]. For this reason, the dynamics of multiphase media is an important branch of fluid and gas mechanics [1-3]. The study of the shock waves propagation in dusty environments is important for mining, powder metallurgy, and aerospace technologies [2]. A practical problem is the screening of industrial explosions with a layer of solid or liquid suspensions. In this connection, the problem of studying the effect of the parameters of the dispersed phase on the velocity and profile of the shock wave in a gas suspension arises. In this case, mathematical modeling of these phenomena is essential

2. Mathematical model.
The fundamentals of the multiphase media mechanics are presented in the monograph [1]. Monographs [2-4] are devoted to the development of mathematical models of the gas suspensions dynamics. In the monograph [2], using the method of large particles numerical simulation, the shock waves propagation in gas-suspensions in one-dimensional formulation with an inviscid carrier medium was studied. In [5-9], shock-wave flows in dusty media are numerically modeled using a mathematical model of the monodisperse gas suspension dynamics, taking into account the viscosity of the carrier medium. Articles [10, 11] are devoted to the practical application of gas suspension dynamics.
In this work, using the mathematical model of polydisperse gas suspension dynamics [1], the effect of the dispersed inclusions parameters on the motion of a shock wave in the two-dimensional channel is numerically studied. The carrier phase is described as a viscous, compressible, heat-conducting gas [12–14].

Carrier phase motion is described by a two-dimensional system of equations for a viscous compressible heat-conducting gas taking into account the interphase force interaction and heat transfer:

\[
\frac{\partial \rho_i}{\partial t} + \frac{\partial (\rho_i u_i)}{\partial x} + \frac{\partial (\rho_i v_i)}{\partial y} = 0,
\]

\[
\frac{\partial (\rho_i u_i)}{\partial t} + \frac{\partial \left(\rho_i u_i^2 + p - \tau_{xx}\right)}{\partial x} + \frac{\partial \left(\rho_i u_i v_i - \tau_{xy}\right)}{\partial y} = -\sum_{i=2}^{n} F_{yi} + \sum_{i=2}^{n} \rho_i \frac{\partial p}{\partial x},
\]

\[
\frac{\partial (\rho_i v_i)}{\partial t} + \frac{\partial \left(\rho_i u_i v_i - \tau_{xy}\right)}{\partial x} + \frac{\partial \left(\rho_i v_i^2 + p - \tau_{yy}\right)}{\partial y} = -\sum_{i=2}^{n} F_{yi} + \sum_{i=2}^{n} \rho_i \frac{\partial p}{\partial y},
\]

\[
\frac{\partial (e_i)}{\partial t} + \frac{\partial \left(e_i + p - \tau_{xx}\right)}{\partial x} u_i - \tau_{xy} v_i + \lambda \frac{\partial T}{\partial x} + \frac{\partial \left(e_i + p - \tau_{yy}\right)}{\partial y} v_i - \tau_{xy} u_i + \lambda \frac{\partial T}{\partial y} = 0,
\]

\[
= \sum_{i=2}^{n} Q_i - \sum_{i=2}^{n} \left[\left|F_{yi}\right|(u_i - u) - \left|F_{yi}\right|(v_i - v_i)\right] + \sum_{i=2}^{n} \left[\frac{\partial (p u_i)}{\partial x} + \frac{\partial (p v_i)}{\partial y}\right],
\]

where

\[
\tau_{xy} = \mu \left(2 \frac{\partial u_i}{\partial x} - \frac{2}{3} D\right), \quad \tau_{yy} = \mu \left(2 \frac{\partial v_i}{\partial y} - \frac{2}{3} D\right), \quad \tau_{xy} = \mu \left(\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}\right), \quad D = \frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y}.
\]

In the above equations, the following notation is introduced: \(p, \rho_i, u_i, v_i\) are pressure, density, carrier medium components of the velocity; \(T_i, e_i\) are temperature and total energy of the gas. The temperature of the carrier medium is found from the equation

\[
T_i = (\gamma - 1)(e_i/\rho_i + 0.5(u_i^2 + v_i^2)) / R.
\]

In the presented expressions, \(R\) is the gas constant of the carrier phase, \(\gamma\) is heat capacity ratio, \(\mu\) is a coefficient of dynamic viscosity, \(\lambda\) is a coefficient of gas thermal conductivity, \(\tau\) is a viscous stress tensor.

The motion of the dispersed phase fractions is described by the conservation equation of the average density of the fraction, the conservation equations of momentum components and the conservation equation of energy are written taking into account heat transfer, momentum exchange with the carrier phase:

\[
\frac{\partial \rho_i}{\partial t} + \frac{\partial (\rho_i u_i)}{\partial x} + \frac{\partial (\rho_i v_i)}{\partial y} = 0,
\]

\[
\frac{\partial (\rho_i u_i)}{\partial t} + \frac{\partial \left(\rho_i u_i u_i + p - \tau_{xx}\right)}{\partial x} + \frac{\partial \left(\rho_i u_i v_i - \tau_{xy}\right)}{\partial y} = F_{yi} - \alpha_i \frac{\partial p}{\partial x},
\]

\[
\frac{\partial (\rho_i v_i)}{\partial t} + \frac{\partial \left(\rho_i u_i v_i - \tau_{xy}\right)}{\partial x} + \frac{\partial \left(\rho_i v_i^2 + p - \tau_{yy}\right)}{\partial y} = F_{yi} - \alpha_i \frac{\partial p}{\partial y},
\]

\[
\frac{\partial (e_i)}{\partial t} + \frac{\partial \left(e_i + p - \tau_{xx}\right)}{\partial x} u_i - \tau_{xy} v_i + \lambda \frac{\partial T}{\partial x} + \frac{\partial \left(e_i + p - \tau_{yy}\right)}{\partial y} v_i - \tau_{xy} u_i + \lambda \frac{\partial T}{\partial y} = -Q_i,
\]

\[
\rho_i = \rho_i, \quad e_i = \rho_i C_i T_i,
\]

\[
F_{yi} = \frac{3}{4} \frac{\alpha_i}{(2 \pi r)^3} C_{21} \rho_i \sqrt{(u_i - u)^2 + (v_i - v)^2},
\]

\[
\rho_i \left(\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} - u_i \frac{\partial v_i}{\partial x} - v_i \frac{\partial v_i}{\partial y}\right)
\]

\[
+ 0.5 \rho_i \left(\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + v_i \frac{\partial u_i}{\partial y} - u_i \frac{\partial v_i}{\partial x} - v_i \frac{\partial v_i}{\partial y}\right)
\]
The thermal energy of solid fractions suspended in gas is determined by the expression:

\[ e_i = \rho_i C_p T_i. \]

Here \( \rho_i \) and \( \rho_{0i} \) are average and true density of the dispersed phase fraction, \( m_i = \alpha_i \rho_{0i} \rho_i \) is a mass fraction of the dispersed phase, \( u_i, v_i \) are velocity vector components; \( T_i, c_i \) are temperature and energy of the dispersed phase fraction; \( C_p \) is the heat capacity of the material of dispersed inclusions \( \alpha_i \) – the volume content of the fraction, \( Q_i \) is interfacial heat flow, \( F_{si}, F_{ni} \) are components of the interphase force interaction between the \( i \)-th fraction of the dispersed phase and the carrier medium.

When modeling the dynamics of a viscous gas moving in a limited region in accordance with the methodology described in [8,13,15], the Dirichlet boundary conditions for the velocity components and the Neumann boundary conditions for the remaining functions were set at the boundaries of the computational domain:

\[ u_i(t,1,k) = 0, \quad v_i(t,1,k) = 0, \quad u_i(t,N_x,k) = 0, \quad v_i(t,N_x,k) = 0, \]
\[ u_i(t,j,1) = 0, \quad v_i(t,j,1) = 0, \quad u_i(t,j,N_y) = 0, \quad v_i(t,j,N_y) = 0, \]
\[ \rho_i(t,1,k) = \rho_i(t,2,k), \quad \rho_i(t,N_x,k) = \rho_i(t,N_x-1,k), \]
\[ \rho_i(t,j,1) = \rho_i(t,j,2), \quad \rho_i(t,j,N_y) = \rho_i(t,j,N_y-1), \]
\[ e_i(t,1,k) = e_i(t,2,k), \quad e_i(t,N_x,k) = e_i(t,N_x-1,k), \]
\[ e_i(t,j,1) = e_i(t,j,2), \quad e_i(t,j,N_y) = e_i(t,j,N_y-1). \]

Here \( N_x \) is the number of nodes along the \( x \)-axis, \( N_y \) is the number of nodes along the \( y \)-axis.

The system of equations supplemented with the boundary conditions was solved by the explicit finite-difference MacCormack method [13] using the spatial splitting scheme [16], as well as with the nonlinear correction of the grid function [14, 15].

3. Calculation results.

In numerical calculations, the propagation of a shock wave from a pure gas into a dusty medium was simulated. In conducted calculations, the length of the simulated canal was assumed to be \( L = 4 \) m, the canal height \( h = 0.2 \) m. At the initial time, the following initial distributions of the parameters of the two-phase medium were set: \( x < 0.75 \), \( p = 0.588 \) MPa, \( m_2 = m_3 = 0, x \geq 0.75 \), \( p = 0.098 \) MPa. The true density of the dispersed phase material is \( \rho_{20} = \rho_{30} = 1000 \) kg / m³. It was assumed that at the initial moment, the fraction of the dispersed phase with a particle diameter \( d = 2 \) \( \mu \)m in the low-pressure chamber \( (x \geq 0.75 \) L) has a mass content of \( m_2 = 0.17 \). For the fraction of particles with a diameter of \( d = 20 \) \( \mu \)m, two types of mass content were considered: uniform — \( m_3 = 0.65 \) and nonuniform: \( m_3 = 0, y > 0.5 \ h; m_3 = ay + b, y \leq 0.5h \), where the coefficients “\( a \)” and “\( b \)” are selected from the conditions

\[ \int_0^{h/2} ay + bdy = 0.65h, \quad a(h/2) + b = 0. \]
The monograph [3] describes that the deposition rate of spherical particles is proportional to the square of their linear size. Thus, the distribution of particles by fractions indicated in this article can be in the case of gravitational deposition of particles of a coarse fraction.

Figure 2 (a, b) shows the spatial distribution of the average density of particles fraction with a diameter of \( d = 20 \mu m \) for the initial and subsequent instants of time in a low-pressure chamber. During the shock wave propagation, a redistribution of the dispersed phase average density is observed both in the longitudinal and transverse directions. Calculations results of gas pressure along the longitudinal coordinate for various distributions of particle fraction concentration with a diameter of dispersed inclusions \( d = 20 \mu m \) are shown in fig. 3, a: uniform distribution – curve 1 and nonuniform distribution – curve 2. In the case of a uniform distribution of dusty medium fractions, the shock wave propagates at a lower velocity.

The results of calculations of the modulus of gas velocity \( V = \sqrt{u_1^2 + v_1^2} \) along the transverse coordinate for gas suspensions with a uniform and non-uniform distribution of the fraction of particles with a diameter \( d = 20 \mu m \) are presented in Fig. 3, b.

When a shock wave propagates through a gas suspension with a nonuniform distribution of the dispersed phase, a large magnitude of the gas velocity modulus is observed in the “upper” part of the canal — \( y > 0.5h \).

Figure 2. The spatial distribution of the fraction of the particles with a diameter of \( d = 20 \mu m \) at time \( t = 0 \) – Fig. 2 (a), \( t = 1.9 \) ms – Fig. 2 (b).
**Figure 3.** Distribution of pressure along the x-axis ($y = h / 2$) - Fig. 3 (a) and distribution of the gas velocity modulus along the y-axis ($x = 0.9 L$) - Fig. 3 (b) at time $t = 1.9$ ms.

### 4. Conclusion.
In this work, the propagation of a shock wave from pure gas into a two-fraction gas suspension consisting of particles with the same density and heat capacity of the material, but with different linear sizes of dispersed inclusions is studied. The shock wave motion was simulated over a gas suspension with a uniform distribution of all fractions of the dispersed phase and over a gas suspension with a nonuniform distribution of the concentration of one of the dispersed phase fractions. It was revealed that a nonuniform distribution of particle concentration along with the transverse coordinate forms a nonuniform distribution of gas velocity. Also, numerical calculations showed that, with a uniform distribution of the concentration of gas suspension fractions, the velocity of the shock wave in the gas suspension is lower.

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