Phase control of squeezing in fluorescence radiation

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Received 14 July 2014, revised 29 September 2014
Accepted for publication 6 October 2014
Published 19 November 2014

Abstract
We study squeezing properties of the fluorescence radiation emitted by a driven Λ-type atom in which the metastable lower energy levels are coupled by an additional field. We find that the relative phase of the applied fields can significantly modify the squeezing characteristics of radiation. It is shown that the additional field connecting the lower levels in the system can induce spectral squeezing in a parameter regime for which the squeezing is absent without the additional field. Moreover, the squeezing can be shifted from inner- to outer-sidebands of the spectrum by simply changing the relative phase. A dressed-state description is presented to explain these numerical results. The phase control of squeezing in the total variance of quadrature components is also examined. We show that the squeezing in total variance attains its maximal value when the system reduces to an effective two-level system.

Keywords: resonance fluorescence, squeezing, three-level atoms

1. Introduction

Squeezing of the radiation emitted in resonance fluorescence of driven atoms has been extensively investigated over the last couple of decades [1]. Squeezed states of light have a reduced variance in quadrature components of the electric field below its shot-noise limit [2]. Theoretical studies considered either the total variance of phase quadratures or the squeezing spectrum of fluorescence radiation to demonstrate squeezing [3–12]. Walls and Zöller first predicted total variance squeezing in the fluorescent light of driven two-level systems [3]. The calculations on the squeezing spectrum demonstrated single- and two-mode squeezing in the weak- and strong-excitation regimes [4, 5]. The studies on squeezing have been extended to three-level systems in Λ [6, 7], V [8] and ladder [9] configurations. Dalton et al [10] examined the role of atomic coherence on the squeezing in fluorescence from three-level systems. It was shown that maximal squeezing is obtained when the atomic system evolves into a pure state [10]. Recently, Grünwald and Vogel [11] have proposed an ingenious scheme using cavity-assisted purification to achieve near-maximal squeezing in fluorescence. A detailed study by Gao et al [12] has shown that the squeezing spectrum of three-level atoms may exhibit ultranarrow peaks.

In all these publications, the squeezing properties of the fluorescence radiation are obtained independent of the phases of applied lasers. Recently, much attention has been paid on the control of medium properties by the phases of applied fields [13–24]. One way to achieve phase control is by vacuum induced coherences which arise due to the atomic transitions coupled by same vacuum modes [13, 14]. However, this method requires the existence of non-orthogonal dipole transitions which is rarely met in atomic systems. An alternative way of phase control is to use a closed-loop scheme of transitions in atoms [15–24]. In this scheme, phase-dependent behavior has been reported in both the dynamics and steady-state properties of driven systems [15]. Many interesting effects have been studied on the phase control of population dynamics [15], photoionization [16], preparation of microwave-spin dressed states [17], quantum interferences in probe absorption [18], electromagnetically induced transparency (EIT) [19–21], fast and slow light propagation [22], fluorescence quenching and line narrowing [23]. In Λ-type systems controlled by a microwave field coupling the ground states, it has been shown experimentally that the trapping state evolves into a microwave-spin dressed state for an appropriate choice of laser phases [17]. Further, phase-dependent effects on probe light absorption [18], EIT in double-Λ system [19], splitting of EIT transparencies [20], and probe transmission in EIT [21] have also been experimentally demonstrated. A recent theoretical study [24] has discussed the
rigorous dark state conditions required to establish EIT in a $\Lambda$-type system with closed-loop transitions.

In this paper, we consider a closed $\Lambda$-type system interacting with two coherent fields (as shown in figure 1). It is assumed that the excited atomic state decays spontaneously to the ground states which are metastable. We further assume that an additional field couples the metastable ground states as in earlier publications [17, 21, 23, 24]. This additional field could be a microwave, infrared or a rf field depending upon the level spacings. The role of the additional field and its phase control was investigated in the fluorescence spectrum of this system in [23]. In the present work, we study the squeezing properties of the fluorescence field and examine how the relative phase of applied fields can modify squeezing aspects. The outline of the paper is as follows: section 2 establishes the model and basic dynamical equations which govern the atom-field interaction including decay processes. In section 3, we present the numerical results and study the effects of additional field on the squeezing spectrum. Section 4 is devoted to analyze the squeezing in total variance optimized with respect to parameters of the system. Finally, the main results are summarized in section 5.

2. Model system and density matrix equations

We consider a three-level atom of the $\Lambda$ configuration driven by three fields as shown in figure 1. The excited state $|1\rangle$ is driven to the ground states $|3\rangle$ and $|2\rangle$ by two coherent fields (frequencies $\omega_1$, $\omega_2$ and phases $\phi_1$, $\phi_2$) of Rabi frequencies $\Omega_1$ and $\Omega_2$, respectively. The ground states $|2\rangle$ and $|3\rangle$, being metastable states, are coupled by an additional field (frequency $\omega_3$ and phase $\phi_3$) of Rabi frequency $\Omega_3$. We assume that the atom decays by spontaneous emission along the channels $|1\rangle \rightarrow |3\rangle$ and $|1\rangle \rightarrow |2\rangle$ with rates $2\gamma_1$ and $2\gamma_2$, respectively. The atomic dynamics is studied in an appropriate rotating frame and by changing the phases of basis states as $|1\rangle \rightarrow e^{i\phi_1}|1\rangle$ and $|2\rangle \rightarrow e^{i(\phi_2-\phi_3)}|2\rangle$. In the rotating frame and new basis states, the Hamiltonian is given in the dipole approximation as

$$H = -\hbar \Delta_1 |1\rangle \langle 1| - \hbar (\Delta_1 - \Delta_2) |2\rangle \langle 2| - \hbar (\Omega_1 |1\rangle \langle 3| + \Omega_2 |1\rangle \langle 2| + \Omega_3 e^{i(\Delta_1+\phi_3)} |2\rangle \langle 3| + \text{ h. c.}).$$

Here, $\Delta_1$ ($\Delta_2$) denotes the detuning of the field driving the transition $|1\rangle \leftrightarrow |3\rangle$ ($|1\rangle \leftrightarrow |2\rangle$). Similarly, $\Delta_3$ corresponds to the detuning of the additional field coupling the transitions $|2\rangle \leftrightarrow |3\rangle$. The relative detuning $\Delta_4 = \Delta_1 - \Delta_2 - \Delta_3 = \omega_1 - \omega_2 - \omega_3$ gives the frequency difference and the relative phase $\Phi = \phi_1 - \phi_2 - \phi_3$ represents the phase difference of the applied fields.

We use the master equation framework to include spontaneous emission in the atomic dynamics. Under the assumption of a Markovian approximation, the density operator of the system obeys a master equation of Lindblad form

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] - \gamma_1 (A_{11} \rho - 2A_{33} \rho_{11} + \rho A_{11}) - \gamma_2 (A_{12} \rho - 2A_{22} \rho_{11} + \rho A_{11}),$$

where the operators $A_{mn} = \langle m| \langle n|$ represent the transition operators for $m \neq n$ and atomic population operators for $m = n$. The density matrix elements $\rho_{ij}$ obtained from equation (2) obey the following time-dependent equations

$$\begin{align*}
\rho_{11} &= -2(\gamma_1 + \gamma_2) \rho_{11} + i \Omega_1 \rho_{31} + i \Omega_2 \rho_{21} - i \Omega_3 e^{i(\Delta_1+\phi)} \rho_{32} - i \Omega_3 e^{i(\Delta_1+\phi)} \rho_{23}, \\
\rho_{22} &= 2\gamma_2 \rho_{11} + i \Omega_2 \rho_{12} + i \Omega_2 e^{i(\Delta_2+\phi)} \rho_{32} - i \Omega_3 e^{i(\Delta_2+\phi)} \rho_{23}, \\
\rho_{12} &= -(\gamma_1 + \gamma_2 - i \Delta_2) \rho_{12} + i \Omega_2 (\rho_{22} - \rho_{11}) + i \Omega_2 \rho_{32} - i \Omega_3 e^{i(\Delta_1+\phi)} \rho_{13}, \\
\rho_{13} &= -(\gamma_1 + \gamma_2 - i \Delta_1) \rho_{13} + i \Omega_1 (\rho_{33} - \rho_{11}) + i \Omega_2 \rho_{23} - i \Omega_3 e^{i(\Delta_1+\phi)} \rho_{12}, \\
\rho_{23} &= i(\Delta_1 - \Delta_2) \rho_{23} + i \Omega_2 e^{i(\Delta_1+\phi)} (\rho_{33} - \rho_{22}) - i \Omega_4 \rho_{23} + i \Omega_2 \rho_{13},
\end{align*}$$

with $\rho_{ij} = \rho_{ji}^*$ and $\rho_{11} + \rho_{22} + \rho_{33} = 1$. It is seen from equations (3)–(7) that the exponential phase terms $e^{i\phi}$ are always accompanied by the additional field Rabi frequency $\Omega_3$. This shows that the atomic dynamics becomes dependent on the relative phase $\Phi$ only when the additional field is applied on the system.

In what follows we assume the frequencies of the applied fields to satisfy the condition $\omega_1 = \omega_2 + \omega_3$ which implies the relative detuning $\Delta_4$ to be zero. The explicit time dependence in equations (3)–(7) is then removed and the equations can be easily solved in steady state. For convenience in the calculation of steady-state properties, we rewrite the density matrix equations (3)–(7) in a more compact matrix form by the definition

$$\dot{\tilde{\Psi}} = (\rho_{11}, \rho_{22}, \rho_{12}, \rho_{13}, \rho_{23}, \rho_{33}, \rho_{12})^T.$$

Substituting equation (8) into equations (3)–(7) with $\Delta_4 = 0$, the matrix equation for the variables $\tilde{\Psi}(t)$ obeys

$$\frac{d}{dt} \tilde{\Psi} = \tilde{L} \tilde{\Psi} + \tilde{I},$$

where $\tilde{\Psi}_j$ is the $j$th component of the column vector $\tilde{\Psi}$ and the
inhomogeneous term \( \hat{I} \) is also a column vector with non-zero components

\[
\hat{I}_x = i\Omega_1, \quad \hat{I}_y = -i\Omega_1, \quad \hat{I}_z = i\Omega_2 e^{i\varphi}, \quad \hat{I}_k = -i\Omega_2 e^{-i\varphi}.
\] (10)

In equation (9), \( \hat{L} \) is a 8 \times 8 matrix whose elements are time independent and can be found explicitly from equations (3)–(7). The steady-state values of the density matrix elements can be obtained by setting the time derivative equal to zero in equation (9):

\[
\hat{\Psi}(\infty) = -\hat{L}^{-1}\hat{I}.
\] (11)

3. Calculation of the squeezing spectrum

Since the atom is driven by two coherent fields, each field produces its own fluorescence field from the system. However, the fluorescence fields generated by the \( |11\rangle \leftrightarrow |3\rangle \) and \( |11\rangle \leftrightarrow |2\rangle \) transitions in the atom will have no correlations because the frequencies \( \omega_1 \) and \( \omega_2 \) of the applied fields driving the transitions are quite different. We consider squeezing in the fluorescent light exclusively emitted by the \( |11\rangle \leftrightarrow |3\rangle \) transitions in the atom. Assuming that the detection of fluorescence field is in a direction perpendicular to the atomic dipole moment, the positive and negative frequency parts of the electric field operator in the radiation zone can be written as

\[
\hat{E}^{(+)}(t) = f(r)\hat{\mu}_{31}\exp[-i(\omega_1t + \phi_1)],
\]

\[
\hat{E}^{(-)}(t) = \left[ \hat{E}^{(+)}(t) \right]^*.
\] (12)

where \( \hat{I} = t - r/c \), \( f(r) = \omega_1^2/c^2r \), and \( r \) is the distance of the detector from the atom. To calculate the squeezing spectrum, we introduce slowly varying quadrature components with phase \( \theta \) as

\[
\hat{E}(\theta, t) = \hat{E}^{(+)}(t)e^{i(\omega_1t+\theta)} + \hat{E}^{(-)}(t)e^{-i(\omega_1t+\theta)}.
\] (13)

The squeezing spectrum is defined by the Fourier transformation of the normal and time-ordered correlation of the quadrature component \( \hat{E}(\theta, t) \):

\[
S(\omega, \theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{T} : \hat{E}(\theta, t) \hat{E}(\theta, t + \tau) : e^{i\omega\tau} d\tau,
\] (14)

where \( \langle \hat{A}, \hat{B} \rangle = \langle \hat{A} \rangle \langle \hat{B} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle \) and \( \hat{T} \) represents the time ordering operator.

In the steady-state limit \( t \rightarrow \infty \), the correlation function appearing in equation (14) can be easily obtained using the quantum regression theorem and the density matrix equation (9). For the purpose of calculations, we introduce column vectors of two-time averages

\[
\hat{U}^{mn}(t, \tau) = \left[ \begin{array}{c} \langle \Delta A_{11}(t + \tau) \Delta A_{mn}(t) \rangle \\
\langle \Delta A_{21}(t + \tau) \Delta A_{mn}(t) \rangle \\
\langle \Delta A_{31}(t + \tau) \Delta A_{mn}(t) \rangle \\
\langle \Delta A_{32}(t + \tau) \Delta A_{mn}(t) \rangle \end{array} \right],
\]

\[
m, n = 1, 2, 3.
\] (15)

Here, \( \Delta A_{mn}(t) = A_{mn}(t) - \langle A_{mn}(\infty) \rangle \) are the deviations of the atomic operators from its steady-state values (11). According to the quantum regression theorem [25], the column vectors of two-time correlation functions must satisfy

\[
\frac{d}{dt} \hat{U}^{mn}(t, \tau) = \hat{L} \hat{U}^{mn}(t, \tau).
\]

Now, applying the time ordering of operators [26] and quantum regression theorem as explained in [14], the...
squeezing spectrum can be obtained as

\[
S(\omega, \theta) = \left| \frac{\mu_3 m_3}{\pi} \right|^2 \text{Re} \left\{ \sum_{j=1}^{\infty} \lim_{\Delta \to 0} \left\{ \tilde{M}_{j,k} \tilde{U}_{j,k}^{(3)}(t, 0) \right\} \right\},
\]

where \( \tilde{M}_{j,k} \) denotes the \((j, k)\) element of the matrix \( \mathbf{M} = [i(\omega - \tilde{L})^{\dagger} + (-i\omega - \tilde{L})]^{-1} \).

We now proceed to present the numerical results of the squeezing spectrum and its interpretation. From the definition of squeezing [2], a fluorescence light exhibits spectral squeezing in a selected quadrature component \((\theta)\) if the squeezing spectrum is negative, \( S(\omega, \theta) < 0 \), at a certain frequency \( \omega \). To demonstrate this squeezing in spectral components, we analyze numerically the spectrum calculated according to equation (16) for a special parameter choice \( \Omega_1 = \Omega_2 \) and \( \Delta_1 = -\Delta_2 \). In the numerical calculation, all the frequency parameters such as decay rates, Rabi frequencies and detunings are scaled in units of \( \gamma_2 \). We also assume \( e^{2(i\omega - \phi + \omega_0 i/\gamma)} = 1 \) and scale the spectrum in units of \( \mu_3 m_3 (r^2)/(\pi \gamma_2) \). In figure 2, the numerical results\(^{1}\) are presented for the in-phase quadrature \((\theta)\) with two different values of the relative phase \((\phi = 0, \pi)\). The graphs show that the squeezing is absent in the spectrum (see dotted curves in figure 2) when the additional field is not applied on the system \((\Omega_2 = 0)\).\(^{2}\) Note that the squeezing spectrum is independent of the relative phase \(\phi\) of the applied fields without the additional field \((\Omega_2 = 0)\) as expected. Interesting features appear in the spectrum only when the additional field connects the lower metastable levels in the system. As seen in figure 2, the squeezing is induced in the spectrum depending on the relative phase \((\phi)\) for \(\Omega_2 \neq 0\). The spectral squeezing is shifted from inner- to outer-sidebands of the spectrum as the relative phase is changed from \(\phi = 0\) to \(\phi = \pi\) (solid curves in figure 2).

To explain the origin of the new features in the squeezing spectrum, we go to the dressed-state description of the atom-field interaction. The dressed states \( |\Phi\rangle (i = \alpha, \beta, \kappa)\) defined as eigenstates \((H|\Phi\rangle = \hbar \omega |\Phi\rangle\) of the Hamiltonian (1) can be expanded in terms of the bare atomic states as

\[
|\Phi\rangle = a_1 |1\rangle + a_2 |2\rangle + a_3 |3\rangle,
\]

where the expansion coefficients are explicitly given by

\[
\begin{align*}
\alpha_1 &= N \left[ \lambda_1 \Omega_2 - \Omega_1 \Omega_3 \exp(-i\phi) \right], \\
\alpha_2 &= N \left[ \Omega_1^2 - \lambda_2 (\Delta_1 + \lambda_1) \right], \\
\alpha_3 &= N \left[ (\Delta_1 + \lambda_1) \Omega_2 \exp(-i\phi) - \Omega_1 \Omega_2 \right].
\end{align*}
\]

Here, the overall constant factor \( N \) is appropriately chosen to satisfy the normalization condition \( |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1 \).\(^{3}\)

The eigenvalues \( \lambda_i (i = \alpha, \beta, \kappa) \) can be obtained numerically by solving the characteristic equation of the Hamiltonian (1) in the basis of bare atomic states. In order to understand the spectral features, one has to consider the allowed transitions between the dressed states including decay processes. The central peak in the spectrum comes from transitions of the dressed states \( |\Phi\rangle (i = \alpha, \beta, \kappa) \) at the frequency of the driving field. The sideband peaks in the squeezing spectrum occur at the frequencies \( \omega_0 \neq \lambda_i - \lambda_j (i \neq j) \) due to the dressed-state transitions \( |\Phi\rangle (i, j = \alpha, \beta, \kappa) \).

In the high field limit \((\Omega_1, \Omega_2, \Omega_3 \gg \gamma_1, \gamma_2)\), the spectrum (16) at the sidebands can be worked out in the dressed-state basis. The contribution to the spectrum by the dressed-state transitions \( |\Phi\rangle (\Rightarrow |\Phi\rangle) \) can be given as

\[
S(\omega, 0) = \frac{\Gamma_{\alpha\beta}}{\Gamma_{\alpha\beta} - \omega} \left[ \frac{a_1 a_2 a_3 - a_3 a_2 a_1}{\Gamma_{\alpha\beta}} + \frac{a_3 a_2 a_1}{\Gamma_{\alpha\beta}} \right] \cdot \left[ \frac{a_1 a_2 a_3 - a_3 a_2 a_1}{\Gamma_{\alpha\beta}} + \frac{a_3 a_2 a_1}{\Gamma_{\alpha\beta}} \right],
\]

where the subindex \((\Rightarrow)\) stands for the positive \((\omega > 0)\) (negative \((\omega < 0)\)) part of the spectrum and \( \Gamma_{\alpha\beta} \) represents the population of the dressed state \(|\Phi\rangle (|\Phi\rangle)\). Equation (19) shows that the spectrum is a pair of Lorentzian curves centered at \( \omega = \pm \omega_{\alpha\beta} \) with its width proportional to the decay rate \( \Gamma_{\alpha\beta} \) of dressed-state coherence. The explicit form of the decay rate \( \Gamma_{\alpha\beta} \) is given in appendix. By using the numerical values of the expansion coefficients (18), the formula (19) reproduces well the squeezing peaks shown in figure 2. Specifically in the presence of additional field \((\Omega_2 \neq 0)\), the numerical values of eigenvalues (in units of \( \gamma_2 \)) for the parameters of figure 2 are \( \lambda_\alpha = 26.07 \), \( \lambda_\beta = -6.21 \), \( \lambda_\kappa = -64.86 \) (\( \phi = 0 \)) and \( \lambda_\alpha = -56.07 \), \( \lambda_\beta = -23.79 \), \( \lambda_\kappa = 34.86 \) (\( \phi = \pi \)). Thus, the inner- and outer-sidebands in the spectrum (solid curves in figure 2) can be seen as arising from the dressed-state transitions \( |\Phi\rangle (\Rightarrow |\Phi\rangle) \) and \( |\Phi\rangle (\Rightarrow |\Phi\rangle) \), respectively. In this case, only a single dressed-state transitions \( |\Phi\rangle (\Rightarrow |\Phi\rangle) \) contribute to each of the peaks in the squeezing spectrum. However, the situation differs significantly when there is no additional field acting on the system. For \( \Omega_2 = 0 \), the numerical values of eigenvalues for the case shown as dotted curves in figure 2 are \( \lambda_\alpha = -60 \), \( \lambda_\beta = -15 \), and \( \lambda_\kappa = 30 \). It is seen that the outer-sidebands originate from the transitions \( |\Phi\rangle (\Rightarrow |\Phi\rangle) \) of the dressed states. In the case of inner-sidebands peaked at \( \omega = \pm 45 \), both the dressed-state transitions \( |\Phi\rangle (\Rightarrow |\Phi\rangle) \) and \( |\Phi\rangle (\Rightarrow |\Phi\rangle) \) contribute to the spectrum. The inner-sideband spectrum is then a sum of two different Lorentzians of the form (19) with different widths. The net effect is that the spectral squeezing is absent in the fluorescence field.

4. Squeezing in total variance

A light field \( \mathcal{E} (\theta, t) \) in a selected quadrature \((\theta)\) is said to be squeezed if the variance \( \langle \Delta \mathcal{E}^2 (\theta, t) \rangle \) is below its value in vacuum state. An equivalent criterion for squeezing [2] is that

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1. In all figures, we assume \( \Delta_1 = \Delta_1 - \Delta_2 \) to satisfy the condition \( \Delta_4 = 0 \).
2. For \( \Omega_2 = 0 \), the spectral squeezing is absent for all values of Rabi frequencies and detunings provided the condition \( \Omega_4 = \Omega_2 \) and \( \Delta_1 = -\Delta_2 \).
3. For the parameters \((\phi = 0, \pi)\) considered in this paper, the expansion coefficients \((a_1, a_2, a_3)\) are real. So, the normalization condition implies \( a_1^2 + a_2^2 + a_3^2 = 1 \).
the normal ordered variance of the field $<\Delta\hat{E}(\theta, t)^2>$ is negative. Using the expression (13) of the quadrature component, the normal ordered variance is defined by

$$\langle [\Delta\hat{E}(\theta, t)]^2 \rangle = \langle (\Delta\hat{E}(\theta))^2 \rangle \exp [2i(\omega_1 t + \theta)]$$

$$+ \langle (\Delta\hat{E}(\theta))^2 \rangle \exp [-2i(\omega_1 t + \theta)]$$

$$+ 2 \langle \Delta\hat{E}(\theta) \cdot \Delta\hat{E}(\theta) \rangle,$$  \hspace{1cm} (20)

where $\Delta\hat{E}(\theta) = \hat{E}(\theta) - \langle \hat{E}(\theta) \rangle$. For the case of a single atom emitting fluorescence radiation, the operators $\hat{E}(\theta)$ in equation (20) are replaced by the source-field operators (12).

With this substitution, the normal order variance (20) reduces further to [7]

$$\langle [\Delta\hat{E}(\theta, t)]^2 \rangle = \left| \mu_{13} \right|^2 f(r)^2 \cos^2 \left( \theta - \phi_3 - \psi_{13} \right).$$  

(21)

Here, $\phi_{31}$ denotes the phase of the matrix element $\rho_{31} = |\rho_{31}| \exp (i\psi_{31})$. Obviously, if the $\cos^2$ term in the above equation is unity, the field variance will be minimum. Considering the steady-state limit ($t \to \infty$) in equation (21), the phase-optimized (minimal) normal ordered field variance (denoted as squeezing parameter $F$) is now given by

$$F \equiv \langle [\Delta\hat{E}(\theta, t)]^2 \rangle = 2\rho_{11} - 4|\rho_{13}|^2,$$  \hspace{1cm} (22)

where $\rho_{11}$ and $\rho_{13}$ refer to the steady-state values (11) of the population and coherence, respectively.

The equation (22) can be used to study the squeezing in total field variance optimized with respect to the quadrature phase ($\theta$). In the absence of the additional field ($\Omega_3 = 0$), the driven $\Lambda$ system is known to exhibit fluorescence squeezing when one of the two transitions in the atom is detected [7]. It was shown [7] that the squeezing ($F < 0$) occurs in the fluorescence field only if the decay rate of the detected transition is greater than that of the neighboring transition, i.e., $\gamma_1 > \gamma_2$. Our numerical analysis shows that this is true even in the present case of the atom subject to an additional field coupling the ground states. Therefore, we focus only on the fast-decaying transitions in the atom ($\gamma_1 > \gamma_2$). For simplicity, we consider the case of resonant light fields ($\Delta_1 = \Delta_2 = 0$) for a parameter choice $\Omega_1 = \Omega_2 = \Omega$. In this case, the squeezing parameter $F$ can be obtained in an analytical form to be

$$F = \frac{4\Omega^2 \Omega_3^2 \sin^2(\Phi)}{M^2} \left[ G - H \right],$$  \hspace{1cm} (23)

where

$$G = \Omega^4 + \Omega_3^4 + \Omega_3^2 \left( \gamma_1 + \gamma_2 \right)^2 + 2\Omega^2,$$  \hspace{1cm} (24)

$$H = 2\Omega_3 \sin(\Phi) \left[ (\gamma_1 + \gamma_2)\Omega_3^2 - 2\gamma_2 \Omega^2 \right] + 3\Omega^2 \Omega_3 \cos(2\Phi),$$  \hspace{1cm} (25)

and

$$M = 2\Omega^4 - \Omega_3^2 \Omega_3^2 + 2\Omega_3^2 \left( \gamma_1 + \gamma_2 \right)^2 + \Omega_3 \Omega_3 \left[ 2(\gamma_1 - \gamma_2) \sin(\Phi) - 3\Omega_3 \cos(2\Phi) \right].$$  \hspace{1cm} (26)

Figure 3 displays the squeezing parameter $F$ calculated using (23) as a function of the Rabi frequency $\Omega_3$ for the relative phases $\Phi = -\pi/2$ and $\Phi = \pi/2$ (see footnote 1). From the graph it is seen that the squeezing parameter goes negative (positive) for $\Phi = -\pi/2$ ($\Phi = \pi/2$) as the Rabi frequency $\Omega_3$ is increased. This shows clearly that the squeezing ($F < 0$) induced in the fluorescence field for $\Phi = -\pi/2$ due to the additional field ($\Omega_3 \neq 0$) acting on the system. Note further that the squeezing is absent ($F = 0$) irrespective of the relative phase for $\Omega_3 = 0$. This is due to the well-known coherent population trapping effect which inhibits the atom from fluorescing. An important result in figure 3 is that the squeezing parameter exhibits a minimum value (maximum squeezing) as a function of $\Omega_3$ for $\Phi = -\pi/2$ (solid curve). We have found numerically that this behavior is generally present even in the case of nonzero detunings ($\Delta_1 \neq 0, \Delta_2 \neq 0$) of the driving fields. In order to understand this result, we plot in figure 4 the steady-state values (11) of the populations and coherences versus $\Omega_3$ for the same parameters of figure 3. On comparing the figures 3 and 4, it is observed that the population $\rho_{22}$ and the coherence $|\rho_{31}|$ along the (1) $\leftrightarrow$ (2) transitions are approximately zero near the value of $\Omega_3$ for maximal squeezing. This implies that the atomic system behaves much like a two-level system along the transitions (1) $\leftrightarrow$ (3). The population in state (2) is forced to return to state (3) rapidly by the action of the additional field. This feature may persist even if one replaces
the additional field with a relaxation between the ground atomic states [6].

It should be emphasized that squeezing in the resonance fluorescence of \( \Lambda \) systems has been already investigated [6, 7]. In these publications, the fluorescence squeezing is shown to exist independent of the phases of applied lasers. However, the present paper has studied the dependence of squeezing in the fluorescence field on the relative phase of the applied fields (see figure 3). To demonstrate further the phase control of squeezing, the squeezing parameter \( F \) is plotted as a function of the relative phase \( \Phi \) in figure 5. The figure clearly indicates that the squeezing is present (absent) for \( \Phi < 0 \) (\( \Phi > 0 \)). Thus, one finds that the presence of an additional field induces phase-dependent squeezing of the radiation. The maximal squeezing occurs only, if \( \Phi = -\pi/2 \), in which case the system reduces to an effective two-level system as discussed above. Finally, we note that the squeezing in spectral components does not guarantee the occurrence of squeezing in total variance [2]. The spectral squeezing (see figure 2) may appear even if the detected transition decays slowly relative to the other transition (\( \gamma_1 < \gamma_2 \)). However, the squeezing in total variance exists only in the fluorescence field from the fast-decaying atomic transitions (\( \gamma_1 > \gamma_2 \)).

5. Conclusions

In this work we present a theoretical investigation of squeezing properties of the fluorescence radiation from a \( \Lambda \) system driven by two coherent fields and an additional field. In particular, we consider the case when each coherent field drives one transition and the additional field couples the metastable ground levels of the system. It is seen that the presence of the additional field induces spectral squeezing in the fluorescence. In contrast to the results of previous studies, the squeezing characteristics now exhibit a strong dependence on the relative phase of the applied fields. We show that the squeezing peaks in the spectrum can be shifted between the inner- and outer-sidebands just by changing the relative phase. A description based on dressed states has been given to explain these features. Further, we also investigate the influence of the additional field on squeezing in the total variance of the fluorescence field. The results show that the squeezing in total variance becomes phase-dependent and attains a maximal value for a particular relative phase. Moreover, when the maximal squeezing occurs, the system behaves like an effective two-level system with only the one-photon coherence contributing to the dynamics.

Appendix

In the secular approximation, the coherence term \( \rho_{\alpha\beta} (t) \) in the dressed-state basis obeys

\[
\frac{d\rho_{\alpha\beta}}{dt} = -(\Gamma_{\alpha\beta} + i\omega_{\alpha\beta})\rho_{\alpha\beta},
\]

with \( \omega_{\alpha\beta} = \lambda_\alpha - \lambda_\beta \). The decay rate \( \Gamma_{\alpha\beta} \) is given by (see footnote 3)

\[
\Gamma_{\alpha\beta} = \Gamma_1|\gamma_1| + \Gamma_2|\gamma_2|,
\]

where

\[
\Gamma_1 = a_{\alpha\beta}^2 + a_{\gamma\beta}^2 - 2a_{\alpha\alpha}a_{\beta\beta}a_{\gamma\gamma}a_{\delta\delta},
\]

\[
\Gamma_2 = a_{\alpha\alpha}^2 + a_{\beta\beta}^2 - 2a_{\alpha\beta}a_{\alpha\beta}a_{\gamma\gamma}a_{\delta\delta}.
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