Research Article

Mathematical Modeling of Carreau Fluid Flow and Heat Transfer Characteristics in the Renal Tubule

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Abstract

This study looks into the steady heat transfer issue of a flow of an incompressible Carreau fluid. Carreau fluid exhibits the shear thinning and thickening characteristics at low, moderate, and high shear rates. At the tubule wall, the fluid absorption is used as a function of pressure gradient and wall permeability through the tubule wall. Supposing the tubule radius considerably small in comparison to its length, the governing equations are considerably simplified. Significant quantities of interest are computed analytically by using perturbation, and the influence of emergent parameters is discussed through graphical results. The comparisons of results with existing data are set up to be in good agreement.

1. Introduction

The fluid flow problem in permeable tubule has significance in various physiological and industrial processes. The complete presentation of such flows can be found in mass transfer and filtration processes, such as reverse osmosis desalination and blood flow through a nonnatural kidney; in the human body, the process of lymphatic flow through lymphatic vessels network and the purification of blood occur in the nephron’s renal tubules. In direction to research the pressure fields and velocity in such circumstances, since there occurs a normal competent of velocity, the rule of Poissoile’s law cannot be applied directly on the tubule wall due to fluid reabsorption.

In renal tubule, the theoretic flow of study was first investigated by Macey [1, 2]. They supposed a crawling movement of a viscous fluid over a constricted porous tube. They foretold that an exponentially decomposing flow rate occurs along the tube. In addition, the work for the flow through porous walls’ duct for small Reynolds number is conferred by Kozinski [3]. The effects of the variable cross-section tube for the flow through tube were analyzed by Radhakrishnacharya et al. [4]. The exact closed form solution for a viscous flow over a permeable tubule was presented by Marshall et al. [5]. They have deserted the inertial terms by assuming a creeping flow situation. Palatt et al. [6] resolved the viscous flow through a permeable tube by imposing the supposition that fluid loss across the wall of tubule is a linear function of the pressure gradient through wall. Effects of variable wall permeability on the creeping flow of a viscous fluid through a tubule were examined by Chaturani and Ranganatha [7]. Siddiqui et al. [8] studied the effects of an external applied MHD on the theoretic model of the flow for renal tubule. Recently, Sajid et al. [9] investigated the Ellis fluid flow in renal tubule.

The literature survey designates that most of the theoretic studies of flow in renal tubules are examined for viscous fluids. It is now a well-known fact that many physiological
and industrial fluids do not obey Newton’s law of viscosity. On the basis of investigational studies, many relationships of the apparent viscosity are anticipated in the literature. These fluids are generally explained as generalized Newtonian fluids, and in such fluids, the fluid responses to an applied shear stress at an instant do not depend on the response at some previous instant. The generalized fluid models are widely applied to discuss the physiological flows such as peristaltic flows and blood flow. Ali et al. [10] used Newtonian fluid to examine peristaltic waves in a curved channel. In curved channel, the effect of MHD on peristaltic flow of non-Newtonian fluid was investigated by Hayat et al. [11]. Hina et al. [12] demonstrated peristaltic pseudoplastic fluid flow in a curved with compliant wall. Abbassi et al. [13] discussed in curved channel peristaltic transport of Eyring–Powell fluid. Narla et al. [14] studied the peristaltic flow in a curved channel peristaltic transport of Eyring–Powell fluid. Narla et al. [14] studied the peristaltic flow in a curved channel by discussing in–Powell fluid. Ali et al. [15] addressed heat transfer study of peristaltic flow in a curved channel by third-grade fluid. Canic and Kim [16] investigated quasi-linear properties mathematically in the hyperbolic blood flow model over compliant axisymmetric vessel. Zaman et al. looked at how unsteady and non-Newtonian rheology affected blood flow in a stuck time-variant stenotic artery. Unsteady and non-Newtonian blood flow over a tapered coinciding stenosed catheterized vessel was discussed by Ali et al. [18]. Zaman et al. [19] investigated unsteady blood flow over a vessel numerically by using the Sisko fluid model.

Keeping this fact in mind, we have revisited the hydrodynamical model of renal tubule using a Carreau fluid model. Carreau fluid is the general form of viscous fluid and power law fluid. At \( n = 1 \), it acts like a viscous fluid, while on strong shear rates, it behaves like a power law fluid. Akbar and Nadeem [20] looked into the Carreau fluid model for blood flow over a stenosis in a tapered artery. Salahuddin [21] used a Keller box and a shooting tool to study the Carreau fluid model for stretching a cylinder. Under the impacts of an induced and applied magnetic field, Bhatti and Abdelsalam [22] investigated the peristaltically driven motion of Carreau fluid in a symmetric channel. The hybrid nanofluid contains gold (Au) and tantalum (Ta) nanoparticles with thermal effects of radiation. However, there have been some developments in the realm of non-Newtonian fluids in recent years [23–26].

The main goal of this study looks into the steady heat transfer issue of a flow of an incompressible Carreau fluid. We have utilized the constitutive equations of a Carreau fluid model to discuss the characteristics of renal tubule flow. The mathematical formulation is investigated in Section 2. The analytical solutions are presented for the velocity distribution, temperature, pressure, shear stress, mean pressure drop, fractional reabsorption, and heat transfer rate and are investigated in Section 3. Section 4 is denoted for the analysis of obtained results in Section 3. On the basis of presented results, some conclusions are compiled in Section 5.

### 2. Mathematical Formulation

We assume the axial symmetric, incompressible creeping flow of Carreau fluid in a long thin permeable tubule of length \( L \) and radius \( a \) whose axis extend in the \( \z \)-axis. We supposed that hydrostatic pressure is the sole driving force for fluid association and that osmotic pressure and hydrostatic pressure outside the tubule are both constant along the length of the tubule. 

For velocity \( \mathbf{V} = (\tilde{v}, 0, \bar{u}) \), the equations that control the flow are as follows.

Continuity equation for incompressible fluid [19] is

\[
\text{div} (\mathbf{V}) = 0. \tag{1}
\]

Momentum equation is

\[
\rho \frac{d\mathbf{V}}{dt} = -\nabla P + \text{div} \mathbf{S}, \tag{2}
\]

in which

\[
\mathbf{S} = \mu_\infty \left( \mu_0 - \mu_\infty \right) \left( 1 + \left( \Gamma \right)^2 \right)^{n-1/2} \tilde{v}. \tag{3}
\]

From equations (1)–(3), we obtain

\[
\frac{\partial}{\partial \tau} (\bar{\mathbf{r}} \mathbf{V}) + \frac{\partial}{\partial \bar{z}} (\bar{\mathbf{r}} \mathbf{u}) = 0, \tag{4}
\]

\[
\frac{\partial \bar{p}}{\partial \tau} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left[ \bar{r} \left( \mu_\infty \left( \mu_0 - \mu_\infty \right) \left( 1 + \tau^2 \right) \left( 2 \bar{v}^2 + \left( \bar{v} + \bar{u} \right)^2 + 2 \left( \frac{\bar{v}}{\bar{u}} \right)^2 + 2 \bar{u}^2 \right) \right)^{\frac{n-1}{2}} \left( 2 \bar{r} \right) \right] +
\]

\[
\frac{\partial}{\partial \bar{z}} + \left( \mu_\infty \left( \mu_0 - \mu_\infty \right) \left( 1 + \tau^2 \right) \left( 2 \bar{v}^2 + \left( \bar{v} + \bar{u} \right)^2 + 2 \left( \frac{\bar{v}}{\bar{u}} \right)^2 + 2 \bar{u}^2 \right) \right)^{\frac{n-1}{2}} \left( \bar{v} + \bar{u} \right) \right] -
\]

\[
\frac{1}{\bar{r}} \left( \mu_\infty \left( \mu_0 - \mu_\infty \right) \left( 1 + \tau^2 \right) \left( 2 \bar{v}^2 + \left( \bar{v} + \bar{u} \right)^2 + 2 \left( \frac{\bar{v}}{\bar{u}} \right)^2 + 2 \bar{u}^2 \right) \right)^{\frac{n-1}{2}} \left( 2 \frac{\bar{v}}{\bar{u}} \right), \tag{5}
\]
\[ \frac{\partial \bar{p}}{\partial \bar{z}} = \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left[ r \left( \mu_{\infty} + (\mu_0 - \mu_{\infty}) \left( (1 + \bar{r}^2) \left( 2\bar{\nu}_r^2 + (\bar{v}_z + \bar{u}_r)^2 + 2\left(\frac{\nu}{\bar{r}}\right)^2 + 2\bar{u}_r^2 \right) \right) \right) \right] + \frac{\partial}{\partial \bar{z}} \left[ \frac{\mu_{\infty} + (\mu_0 - \mu_{\infty})}{2} \left( (1 + \bar{r}^2) \left( 2\bar{\nu}_r^2 + (\bar{v}_z + \bar{u}_r)^2 + 2\left(\frac{\nu}{\bar{r}}\right)^2 + 2\bar{u}_r^2 \right) \right) \right] \]

The heat transfer equation of Carreau fluid is

\[ p_c \bar{r} \left( \bar{v} \bar{T} + \bar{u} \bar{T}_z \right) = k \left( \frac{\bar{T}}{\nu} + \frac{1}{\bar{r}} \bar{T}_r + \bar{T}_{zz} \right) + \left( \mu_{\infty} + (\mu_0 - \mu_{\infty}) \right) \left[ \left( 1 + \bar{r}^2 \left( 2\bar{\nu}_r^2 + (\bar{v}_z + \bar{u}_r)^2 + 2\left(\frac{\nu}{\bar{r}}\right)^2 + 2\bar{u}_r^2 \right) \right) \right] \left( 2\bar{\nu}_r^2 + (\bar{v}_z + \bar{u}_r)^2 + 2\left(\frac{\nu}{\bar{r}}\right)^2 + 2\bar{u}_r^2 \right) \]

under boundary conditions,
\[ \bar{v}(0, \bar{z}) = 0, \]
\[ \frac{\partial \bar{u}}{\partial \bar{r}} (0, \bar{z}) = 0, \]
\[ \bar{v}(a, \bar{z}) = L_p \left( \bar{p}(a, \bar{z}) - p_m \right), \]
\[ \bar{u}(a, \bar{z}) = 0, \]
\[ \bar{p}(\bar{r}, 0) = \bar{p}_0, \]
\[ \bar{T}(\bar{r}, \bar{z}) = \bar{T}_0, \]
\[ \frac{\partial \bar{T}}{\partial \bar{r}} (0, \bar{z}) = 0, \]
\[ \bar{Q}_0 = 2\pi \int_0^a \bar{r} \bar{u}(\bar{r}, 0) d\bar{r}, \]

where \( \bar{u} \) and \( \bar{v} \) are the axial and radial components of velocity, respectively, \( \bar{p} \) is the intertubular hydrostatic pressure, \( c_p \) is the specific heat at constant pressure, \( L_p \) is the hydrodynamic permeability coefficient of the tubule wall, \( p_m = p_c - \pi \), \( p_c \) is hydrostatic pressure, the osmotic pressure \( \pi \) is osmotic pressure, and \( \bar{Q}_0 \) is the constant flow rate at the inlet of the tubule.

In order to converted the problem in nondimensional form, we define the following parameter:
\[ r = \frac{\bar{r}}{a}, \]
\[ z = \frac{\bar{z}}{L}, \]
\[ u(r, \bar{z}) = \frac{\pi a^2}{Q_0} \bar{u}, \]
\[ v(r, \bar{z}) = \frac{\pi a L}{Q_0} \bar{v}, \]
\[ \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_0}, \]
\[ p_c = \frac{\rho c_p \bar{Q}_0}{\pi ka} p(r, \bar{z}) = \frac{(\bar{p}(\bar{r}, \bar{z}) - p_m)\pi a^4}{\mu_0 LQ_0}. \]

Using the above parameters, equations (4)–(6) are transformed into the following nondimensional form:

\[ \frac{\partial}{\partial r} (rv) + \frac{\partial}{\partial \bar{z}} (ru) = 0, \]
\[ \frac{\partial \bar{p}}{\partial \bar{r}} = \delta^2 \left[ \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( r (m + (1 - m))(1 + w e^2(u_r^2)) \right) \right] \frac{n-1}{2} (2vr) + \frac{\partial}{\partial \bar{z}} \left( (m + (1 - m))(1 + w e^2(u_r^2)) \right) \frac{n-1}{2} \left( \frac{\nu}{\bar{r}} \right) \left( \frac{\nu}{\bar{r}} \right), \]
\[
\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \left( m + (1 - m) \left( 1 + We^2(u_r^2) \right) \right) \left( \delta_z^2 v_z + u_r \right) \right) + \delta_z^2 \frac{\partial}{\partial z} \left( m + (1 - m) \left( 1 + We^2(u_r^2) \right) \right) \left( \frac{2 v_z}{r} \right) \right],
\]
\[
p_t \delta (v \theta_r + u \theta_z) = \left( \theta_{rr} + \frac{1}{r} \theta_r + \delta^2 \theta_{zz} \right) + B_r \left( m + (1 - m) \left( 1 + We^2(u_r^2) \right) \right) \cdot \left( 2 \delta^2 v_r^2 + \left( \delta_z^2 v_z + u_r \right)^2 + 2 \delta^2 u_r^2 + 2 \delta^2 \left( \frac{v_z}{r} \right)^2 \right).
\]

Along with boundary conditions,
\[
v(0, z) = 0,
\]
\[
\frac{\partial u}{\partial r} (0, z) = 0,
\]
\[
v(1, z) = K p(1, z),
\]
\[
u(1, z) = 0,
\]
\[
\frac{\partial \theta}{\partial z} (0, z) = 0,
\]
\[
\theta(1, z) = 1,
\]
\[
p(r, 0) = P_0,
\]
\[
2 \int_0^1 r u(r, 0) dr = 1,
\]
where \( m = \mu_2 / \mu_0, \ We = \tilde{Q}_0 \Gamma / \pi a^3, \ \delta = a / L, \ K = L_p \mu_0 L / a^2 \delta, \ B_r = \tilde{Q}_0 / \pi^2 T_0 k^a, \) and \( Q(z) = Q(0) / Q_0. \) The dimensionless number \( K \) is the permeability coefficient of the tubule wall.

### 3. Solution of the Problem

Let \( V_w \) be the magnitude of the outward radial velocity at the wall and \( U_m \) be the mean axial velocity; at any cross-section perpendicular to the \( z \) direction, then, in view of the physiological data \([6]\), we have \( \delta \ll 1 \) and \( V_w \ll U_m. \) This allows ignoring terms of order \( \delta, \delta^2 \) and higher order. Consequently, equations (10) and (11) can be written as
\[
\frac{\partial p}{\partial r} = 0,
\]
\[
\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( m + (1 - m) \left( 1 + We^2(u_r^2) \right) \right) \right) (u_r),
\]
\[
\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + B_r \left( m + (1 - m) \left( 1 + We^2(u_r^2) \right) \right) u_r^2 = 0.
\]

Using regular perturbation method to find the value of \( p(z), u(r, z), \) and \( v(r, z), \) solve equation (15) by using conditions \( \partial u / \partial r (0, z) = 0 \) and \( u(1, z) = 0 \) for finding the values of \( u_0(r, z) \) and \( u_1(r, z), \) and we obtain
\[
u_0(r, z) = \frac{\left( r^2 - 1 \right)}{4} \frac{dp_0}{dz} + \frac{(r^2 - 1)}{64} \left( \frac{dp_1}{dz} \right)^3 (1 - r^4),
\]
\[
u_1(r, z) = \frac{(r^2 - 1)}{4} \frac{dp_1}{dz} + \frac{(1 - m)(n - 1)}{64} \left( \frac{dp_0}{dz} \right)^3 (1 - r^4).
\]

Therefore, the total axial velocity is
\[
u(r, z) = \frac{(r^2 - 1)}{4} \frac{dp_0}{dz} + We^2 \left( \frac{(r^2 - 1)}{4} \frac{dp_1}{dz} + \frac{(1 - m)(n - 1)}{64} \left( \frac{dp_0}{dz} \right)^3 (1 - r^4) \right).
\]

Substitute (18) in equation (10) and integrate from \( r = 0 \) to \( r = R: \)
\[ v_0(r, z) = \frac{(2r - r^3)}{16} \left( \frac{d^2 p_0}{dz^2} \right) \]

\[ v_1(r, z) = \frac{(2r - r^3)}{16} \left( \frac{d^2 p_1}{dz^2} \right) + \frac{(1 - m)(n - 1)}{128} (r - 2r^3 + r^5) \left( \frac{d p_0}{dz} \right)^2 \left( \frac{d^2 p_0}{dz^2} \right). \]

(19)

The total radial velocity is

\[ v(r, z) = \left( \frac{2r - r^3}{16} \right) \left( \frac{d^2 p_0}{dz^2} \right) + \text{We}^2 \left( \left( \frac{2r - r^3}{16} \right) \left( \frac{d^2 p_1}{dz^2} \right) + \frac{(1 - m)(n - 1)}{128} (r - 2r^3 + r^5) \left( \frac{d p_0}{dz} \right)^2 \left( \frac{d^2 p_0}{dz^2} \right) \right). \]

(20)

Solving equation (16) by using conditions \( \theta(0, z) = 0 \) and \( \theta(1, z) = 1 \) for finding the values of \( \theta_0(r, z) \) and \( \theta_1(r, z) \), we obtain

\[ \theta_0(r, z) = 1 - B_r \left( \frac{d p_0}{dz} \right)^2 (r^4 - 1) \]

(21)

\[ \theta_1(r, z) = 1 - B_r \left( \frac{d p_1}{dz} \right)^2 (r^4 - 1) - \frac{(1 - m)(n - 1)}{36} \left( \frac{d p_0}{dz} \right)^3 \left( \frac{1}{16} \frac{d p_1}{dz} - 8 \frac{d p_0}{dz} \right) (r^6 - 1) + \frac{(1 - m)^2 (n - 1)^2}{9984} \left( \frac{d p_0}{dz} \right)^6 (r^8 - 1). \]

(22)

Therefore, the total heat transfer is

\[ \theta(r, z) = 1 - B_r \left( \frac{d p_0}{dz} \right)^2 (r^4 - 1) + \text{We}^2 \left( 1 - B_r \left( \frac{d p_1}{dz} \right)^2 (r^4 - 1) - \frac{(1 - m)(n - 1)}{36} \left( \frac{d p_0}{dz} \right)^3 \left( \frac{1}{16} \frac{d p_1}{dz} - 8 \frac{d p_0}{dz} \right) (r^6 - 1) \right) + \frac{(1 - m)^2 (n - 1)^2}{9984} \left( \frac{d p_0}{dz} \right)^6 (r^8 - 1). \]

(23)

Solve equation (10) for \( p_0(z) \) and \( p_1(z) \) using the boundary condition \( v(1, z) = Kp(1, z) \):
(24) \begin{align*}
\frac{d^2 p_0}{dz^2} - \xi^2 p_0 &= 0,
\frac{d^2 p_1}{dz^2} - \xi^2 p_1 &= \frac{(1 - m)(n - 1)}{4} \left( \frac{dp_0}{dz} \right)^2 \frac{d^2 p_0}{dz^2},
\end{align*}
\end{equation}

where \( \xi^2 = 16K. \) Solve equations (25) and (26) parallel and use conditions \( p(r, 0) = P_0 \) and \( 2 \int_0^r u(r, 0) dr = 1, \) and we have

\[ p_0(z) = \left( P_0 + \frac{8}{\xi} \right) e^{-\xi z} + \left( P_0 - \frac{8}{\xi} \right) e^{\xi z}, \]  

(26)

\[ p_1(z) = \frac{(1 - m)(n - 1)}{32\xi} \left( (P_0 + 8) e^{-3\xi z} + (P_0 - 8) e^{3\xi z} + \left( (P_0 - 8)^3 + 2(P_0 - 8)^2 (P_0 + 8) + \frac{2048}{3} \right) e^{-\xi z} 
- 2(P_0 + 8)^2 (P_0 - 8) - 4\xi z (P_0 + 8)^2 (P_0 - 8) + \frac{2048}{3} \right) e^{-\xi z} \]  

(27)

From equations (26) and (27), we have

\[ p(z) = \left( P_0 \cosh \xi z - \frac{8}{\xi} \sinh \xi z \right) + \frac{2(1 - m)(n - 1)}{128} \left( P_0 + 8 \right) e^{-3\xi z} + \left( P_0 - 8 \right) e^{3\xi z} \]  

\begin{align*}
&+ \left( (P_0 - 8)^3 + 2(P_0 - 8)^2 (P_0 + 8) - 4\xi z (P_0 + 8)^2 (P_0 - 8) + \frac{2048}{3} \right)
&+ 4\xi z (P_0 - 8)^2 (P_0 + 8) - \frac{2048}{3} \right) e^{-\xi z} \]  

(28)

Thus, total axial velocity, radial velocity, and temperature become

\begin{align*}
u(r, z) &= \frac{(r^2 - 1)}{4} \left( P_0 \xi \sinh \xi z - 8 \cosh \xi z \right) + \frac{2(1 - m)(n - 1)}{128} u(r, z) 
&= \frac{(r^2 - 1)}{4} \left( P_0 \xi \sinh \xi z - 8 \cosh \xi z \right) + \frac{2(1 - m)(n - 1)}{128} e^{\xi z} (r^2 - 1) 
+ 2 \left( 3(P_0 - 8)^3 e^{-3\xi z} - (P_0 - 8)^3 e^{-3\xi z} - 3(P_0 - 8)^2 (P_0 + 8) e^{\xi z} + 3(P_0 + 8)^2 (P_0 - 8) e^{-\xi z} \right) 
\cdot \left( 1 - r^4 \right), \\
v(r, z) &= \frac{1}{16} \left( 2r - r^3 \right) \left( P_0 \xi^2 \cosh \xi z - 8 \xi \sinh \xi z \right) - \\
\frac{1}{512128} \left( 1 - m \right)(n - 1) \xi \frac{1}{\xi} 
\end{align*}
\[ \theta(r, z) = 1 - \frac{1}{16} B_r (P_0 \sinh \xi z + \cosh \xi z)^2 (r^4 - 1) + We^2 (1 - B_r) \left( \frac{(1 - m)(n - 1)}{256} \right) \]

\[ 3 (P_0 \xi - 8)^3 e^{3 \xi z} - 3 (P_0 \xi + 8)^3 e^{-3 \xi z} - (P_0 \xi - 8)^3 - 2 (P_0 \xi + 8)^3 \]
\[ + 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) - 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) - 4 \xi z (P_0 \xi + 8)^2 (P_0 \xi - 8) \]
\[ + \frac{2048}{3} e^{-\xi z} + (P_0 \xi + 8)^3 - 2 (P_0 \xi - 8)^3 + 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) + 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) + 4 \xi z (P_0 \xi - 8)^2 (P_0 \xi + 8) \]
\[ - \frac{2048}{3} e^{2 \xi z} \left( r^4 - 1 \right) - \frac{(1 - m)(n - 1)}{36} \left( (P_0 \xi - 8) e^{\xi z} - (P_0 \xi + 8) e^{-\xi z} \right)^3 \]
\[ + \frac{(1 - m)(n - 1)}{256} \left( 3 (P_0 \xi - 8)^3 e^{3 \xi z} - 3 (P_0 \xi + 8)^3 e^{-3 \xi z} \right) \]
\[ - 2 (P_0 \xi - 8)^3 + 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) + 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) + 4 \xi z (P_0 \xi - 8)^2 (P_0 \xi + 8) - \frac{2048}{3} e^{\xi z} \]
\[ - 128 \left( (P_0 \xi - 8) e^{\xi z} - (P_0 \xi + 8) e^{-\xi z} \right) \left( r^6 - 1 \right) + \frac{(1 - m)^2 (n - 1)^2}{9984} \left( (P_0 \xi - 8) e^{\xi z} - (P_0 \xi + 8) e^{-\xi z} \right)^6 \left( r^8 - 1 \right). \]  

(29)

3.1. Average Pressure. The difference between mean pressure drop and the inlet of the tubule at some point \( z \) is defined as

\[ \Delta p = \Delta p_0 + We^2 \Delta p_t, \]
\[ \Delta p = P_0 \left( 1 - \cosh \xi z \right) + \frac{8}{P_0 \xi^2} \sinh \xi z \]
\[ \cdot \left( (P_0 \xi + 8)^3 e^{-3 \xi z} + (P_0 \xi - 8)^3 e^{3 \xi z} + \left( P_0 \xi - 8 \right)^3 - 2 \left( P_0 \xi + 8 \right)^3 + 2 \left( P_0 \xi - 8 \right)^2 \left( P_0 \xi + 8 \right) - 2 \left( P_0 \xi + 8 \right)^2 \left( P_0 \xi - 8 \right) \left( 1 + 2 \xi z \right) + \frac{2048}{3} \right) \]
\[ e^{-\xi z} + \left( (P_0 \xi + 8)^3 - 2 \left( P_0 \xi - 8 \right)^3 + 2 \left( P_0 \xi + 8 \right)^2 \left( P_0 \xi - 8 \right) - 2 \left( P_0 \xi - 8 \right)^2 \left( P_0 \xi + 8 \right) \left( 1 - 2 \xi z \right) - \frac{2048}{3} \right) e^{\xi z} \].

(30)

3.2. Wall Shear Stress. The dimensionless total wall shear stress is obtained as
\[ \tau_w(z) = \tau_{0w}(z) + We^2 \tau_{1w}(z), \]

\[ \tau_w(z) = \frac{1}{2} \left( P_0 \xi \sinh \xi z - 8 \cosh \xi z \right) - We^2 \frac{(1 - m)(n - 1)}{64} 3 (P_0 \xi + 8)^3 e^{-3 \xi z} \]

\[ - \left( (P_0 \xi - 8)^3 - 2 (P_0 \xi + 8)^3 + 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) - 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) (2 \xi z - 7) + \frac{2048}{3} \right) e^{-\xi z} + \left( (P_0 \xi + 8)^3 - 2 (P_0 \xi - 8)^3 + 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) - 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) (2 \xi z + 7) - \frac{2048}{3} \right) e^{\xi z}. \]

\[ (31) \]

3.3. Flow Rate. The total nondimensional volume flow rate is

\[ Q(z) = Q_0(z) + We^2 Q_1(z), \]

\[ Q(z) = \left( \cosh \xi z - \frac{P_0 \xi}{8} \sinh \xi z \right) + We^2 \frac{(1 - m)(n - 1)}{768} \left( 3 (P_0 \xi + 8)^3 e^{-3 \xi z} - 3 (P_0 \xi - 8)^3 e^{3 \xi z} + 3 (P_0 \xi - 8)^3 - 2 (P_0 \xi + 8)^3 \right) \]

\[ + 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) - 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) (2 \xi z - 5) + \frac{2048}{3} \right) e^{-\xi z} + 3 \left( (P_0 \xi + 8)^3 - 2 (P_0 \xi - 8)^3 + 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) - 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) (2 \xi z + 5) - \frac{2048}{3} \right) e^{\xi z}. \]

\[ (32) \]

3.4. Heat Transfer Rate. The total heat transfer rate is

\[ \hat{\theta}(z) = -\frac{1}{r} \frac{\partial (\hat{r} \theta)}{\partial r} \bigg|_{r=1}, \]

\[ \hat{\theta}(z) = \frac{1}{4} B_1 \left( P_0 \xi \sinh \xi z + \cosh \xi z \right)^2 - 1 + We^2 B_4 \left( \frac{(1 - m)(n - 1)}{256} \right) 3 (P_0 \xi - 8)^3 e^{3 \xi z} - 3 (P_0 \xi + 8)^3 e^{-3 \xi z} \]

\[ \left( (P_0 \xi - 8)^3 - 2 (P_0 \xi + 8)^3 + 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) - 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) - 4 \xi z (P_0 \xi + 8)^2 (P_0 \xi - 8) + \frac{2048}{3} \right) e^{-\xi z} + \left( (P_0 \xi + 8)^3 - 2 (P_0 \xi - 8)^3 + 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) - 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) (2 \xi z + 5) - \frac{2048}{3} \right) e^{\xi z} \]

\[ \left( e^{\xi z} \right)^2 \frac{(1 - m)(n - 1)}{6} ((P_0 \xi - 8) e^{\xi z} - (P_0 \xi + 8) e^{-\xi z})^3 \left( \frac{(1 - m)(n - 1)}{256} \right) 3 (P_0 \xi - 8)^3 e^{3 \xi z} - 3 (P_0 \xi + 8)^3 e^{-3 \xi z} \]

\[ - \left( (P_0 \xi - 8)^3 - 2 (P_0 \xi + 8)^3 + 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) - 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) - 4 \xi z (P_0 \xi + 8)^2 (P_0 \xi - 8) + \frac{2048}{3} \right) e^{-\xi z} + \left( (P_0 \xi + 8)^3 - 2 (P_0 \xi - 8)^3 + 2 (P_0 \xi + 8)^2 (P_0 \xi - 8) - 2 (P_0 \xi - 8)^2 (P_0 \xi + 8) (2 \xi z + 5) - \frac{2048}{3} \right) e^{\xi z} \]

\[ - 128 ((P_0 \xi - 8) e^{\xi z} - (P_0 \xi + 8) e^{-\xi z}) + \frac{(1 - m)(n - 1)^2}{1248} ((P_0 \xi - 8) e^{\xi z} - (P_0 \xi + 8) e^{-\xi z})^6 - 1. \]

\[ (34) \]
Table 1: Physiological data for the rat proximal convoluted tubule [6, 8].

| Quantity | Value |
|----------|-------|
| $a$      | 1.08 × 10^{-3} cm |
| $L$      | 0.67 cm |
| $L_p$    | 1.50 × 10^{-6} cm/sect cm H$_2$O |
| $P_0$    | 14.4 cm H$_2$O |
| $P_e$    | 10.3 cm H$_2$O |
| $\pi_z$  | 16.5 cm H$_2$O |
| $Q_0$    | 40.2 × 10^{-8} cm$^3$/sec |
| $\mu$    | 7.37 × 10^{-6} cm H$_2$O/sect cm |

Table 2: Physiological data for normal hydropenic rats [6, 8].

| Arterial pressure (mm - Hg) | $p_0$ (cm - H$_2$O) | $p_e$ (cm - H$_2$O) | $\pi_z$ (cm - H$_2$O) | $Q_0$ (cm$^3$/sec) |
|-----------------------------|---------------------|--------------------|-----------------------|-------------------|
| 122                         | 14.7                | 9.8                | 17.3                  | 44.7              |
| 100                         | 14.4                | 10.3               | 16.5                  | 40.2              |
| 79                          | 14.0                | 9.1                | 14.9                  | 31.0              |
| 55                          | 11.4                | 8.0                | 12.9                  | 15.0              |

4. Result and Discussion

4.1. Velocity Profile. This section shows how the Carreau fluid material parameters affects the pressure gradient, pressure distribution, flow rate, velocity profile, wall shear stress, fractional reabsorption, and mean pressure drop. The case $n = 1$ corresponds the Poiseuille flow for a nonporous tube. Equations (28) and (33), respectively, can be written as

$$ p(z) = \frac{P_0}{\pi} \left( \frac{z}{\beta} \right) \sinh \beta z - \frac{1}{\beta} \sinh \beta z, $$

$$ Q(z) = \frac{P_0}{\pi} \sinh \beta z - \beta \sinh \beta z, $$

where $\beta = P_0 \xi /8$. To investigate the volume flow rate $Q(z)$ in response to various values of $\beta$, Table 1 shows the physiological data for the rat proximal convoluted tubule and Table 2 shows the physiological data for normal hydropenic rats. Figure 1 is plotted for Carreau fluids and Newtonian fluid. There are important regions of interest for different $\beta$. For $-\infty < \beta < 0$, the flow rate monotonically increases from $Q = 1$ at $z = 0$ and $Q \rightarrow + \infty$ as $z \rightarrow \infty$. For $\beta = 0$, $Q = \cosh \beta z$ and flow rate also present the same properties. For $0 < \beta < 1$, the flow rate also decreases from $Q = 1$ at $z = 0$ and $Q$ approaches to $-\infty$ as $z \rightarrow \infty$. When $\beta = 1$, the flow rate monotonically decreases from $Q = 1$ at $z = 0$ and $Q \rightarrow 0$ as $z \rightarrow \infty$. For the case $1 < \beta < \infty$, $Q$ monotonically decreases from $Q = 0$ at $z = (1/2\beta) \ln (\beta + 1/\beta - 1)$. Negative flow rate is obtained after that point; it decreases monotonically and $Q \rightarrow -\infty$ as $z \rightarrow \infty$; this predicts the reverse flow phenomena that may not be admissible in many physically situations. Figure 2 is plotted for $p(z)/P_0$ with $z$, for different values of $\beta$ which present the same properties. Values of the axial velocity $u$ and radial velocity $v$ on the length of the tube can be deliberated from equations (22) and (23), respectively, for various values of $P_0$, $K$, and $z$. Figures 3 and 4 presented the variation of $u$ with $r$ for different values of $K$ and $We^2$, respectively, by keeping other parameters fixed.

From figures, we noted that the axial velocity reduces by increasing the value of permeability parameter $K$ and $We^2$. The variation of radial velocity $v$ is presented in Figures 5 and 6 with $r$ for different values of $K$ and $We^2$, respectively, by keeping other parameters fixed. It is observed that the velocity increases around the axis of tube by increments in $K$ and $We^2$. For the fixed value of $K$ and $We^2$, the radial velocity in the interval $r \in (0, r_m)$ is increased and reduced in the interval $r \in (r_m, 1)$, where $r_m$ is the root of the equation. Mean pressure drop is presented in Figures 7 and 8 for various values of $K$ and $We^2$, respectively. It is observed that mean pressure drop reduces by increasing $K$, which increases by raising the value of $We^2$. Figures 9 and 10...
Figure 2: Variation of pressure $p/P_0$ with $z$ for different values of $\beta$.

$\beta = 0.9, 1.0, 1.1, 1.4$

Figure 3: Variation of $u$ with $r$ for different values of $K$, where $z=0.1$, $P_0=43$, $n=-1$, $m=0.5$, and $We^2=0.0001$.

$K = 0.001$
$K = 0.003$
$K = 0.005$
$K = 0.007$

Figure 4: Variation of axial velocity $u$ with $r$ for different values of $We^2$, where $z=0.1$, $P_0=43$, $K=0.001$, $n=-1$, and $m=0.5$.

$We^2 = 0.0001$
$We^2 = 0.0002$
$We^2 = 0.0003$
$We^2 = 0.0004$

Figure 5: Variation of $v$ with $r$ for different values of $K$, where $z=0.1$, $P_0=43$, $n=-1$, $m=0.5$, and $We^2=0.0001$.

$K = 0.001$
$K = 0.002$
$K = 0.003$
$K = 0.004$

Figure 6: Variation of radial velocity $v$ with $r$ for different values of $We^2$, where $z=0.1$, $P_0=43$, $K=0.001$, $n=-1$, and $m=0.5$.

$We^2 = 0.00004$
$We^2 = 0.00003$
$We^2 = 0.00002$
$We^2 = 0.00001$

Figure 7: Variation of $\Delta p$ with $z$ for different values of $K$, where $P_0=43$, $We^2=0.01$, $n=-1$, and $m=0.5$.

$\Delta p$
$K = 0.001$
$K = 0.005$
$K = 0.003$
$K = 0.007$
3.5
4.0
3.0
2.5
2.0 $\tau_w$
0.0 0.2 0.4 0.6 0.8 1.0 $z$

$K = 0.001$
$K = 0.003$
$K = 0.005$
$K = 0.007$

Figure 8: Variation of $\tau_w$ with $z$ for different values of $K_2$, where $P_0 = 43$, $K = 0.001$, $n = -1$, and $m = 0.5$.

4.2
4.4
4.0
3.8 $\tau_w$
0.0 0.2 0.4 0.6 0.8 1.0 $z$

$We_2 = 0.01$
$We_2 = 0.02$
$We_2 = 0.03$
$We_2 = 0.04$

Figure 9: Variation of $\tau_w$ with $z$ for different values of $K_2$, where $P_0 = 43$, $K = 0.001$, $n = -1$, and $m = 0.5$.

0.20
0.15
0.30
0.25 $FR$
20 25 30 35 40 45 $P_0$

$We_2 = 0.1$
$We_2 = 0.2$
$We_2 = 0.3$
$We_2 = 0.4$

Figure 10: Variation of $\tau_w$ with $z$ for different values of $We^2_2$, where $P_0 = 43$, $K = 0.001$, $n = -1$, and $m = 0.5$.

0.20
0.15
0.30
0.25 $FR$
20 25 30 35 40 45 $P_0$

$We_2 = 0.01$
$We_2 = 0.02$
$We_2 = 0.03$
$We_2 = 0.04$

Figure 11: Variation of $FR$ with $P_0$ for different values of $K$, where $We^2_2 = 0.001$.

0.20
0.15
0.30
0.25 $FR$
20 25 30 35 40 45 $P_0$

$We_2 = 0.01$
$We_2 = 0.02$
$We_2 = 0.03$
$We_2 = 0.04$

Figure 12: Variation of $FR$ with $P_0$ for different values of $We^2_2$, where $K = 0.001$.

3.5
3.0
4.5
4.0
2.5
2.0
1.5
1.0 $\theta$
0.0 0.2 0.4 0.6 0.8 1.0 $r$

$K = 0.001$
$K = 0.004$
$K = 0.007$

Figure 13: Variation of $\theta$ with $r$ for different values of $K$, where $P_0 = 43$, $z = 0.1$, $B_t = 1$, $We^2 = 0.0001$, $n = -1$, and $m = 0.5$. 
represent the deviations of wall shear stress \( \tau_w \) with \( z \) for various values of \( K \) and \( We^2 \), respectively. It is clear from figure that increasing the value of \( K \), the wall shear stress reduces and increases by increasing the value of \( We^2 \). The variation of fractional reabsorption \( FR \) with \( P_0 \) is presented in Figures 11 and 12 for different values of \( K \) and \( We^2 \), respectively. Fractional reabsorption increases by increasing the value of \( K \) and decreases by increasing \( We^2 \).

4.2. Temperature. The effect of \( K \) on temperature \( \theta \) at various locations is depicted in Figure 13. The effects of \( K \) on temperature are greatest in the middle and then decrease to zero at the surface. The temperature \( \theta \) of the tube’s entry area is founded to be higher than that of the tube’s middle and exit regions. The temperature at each axial position rises as \( K \) rises. It is self-evident that the presence of porous material generates greater fluid flow restriction, causing the fluid to decelerate. As a result, as the permeability parameter increases, the barrier to fluid motion increases, and hence, velocity fall that causes temperature rises. The effects of Brinkman number \( B_r \) on temperature \( \theta \) at various locations are depicted in Figure 14. The figure shows that the temperature increases as \( B_r \) increases. The effects of Weissenberg number \( We^2 \) against temperature \( \theta \) is plotted in Figure 15. The Weissenberg number and temperature have a direct relationship. The temperature profile increases as the Weissenberg number increases. As the Weissenberg number increases, the thermal boundary layer shrinks, causing the temperature profile to increase. Figure 16 depicts the variance of Nusselt number \( N_u \) in the axial direction as a function of \( K \). There is a maximum \( N_u \) at the entrance region for the same inlet velocity that decreases in the axial direction. In addition, \( K \) boosts \( N_u \) in the existing field.

5. Conclusion

(i) In this study, we examined the flow of the non-Newtonian Carreau fluid in insignificant diameter of permeable tubule with an application to the renal tubule

(ii) The flow rate and pressure can be controlled by increasing the value of \( \beta \)

(iii) The radial velocity \( v \), stress tensor \( \tau_w \), and fractional reabsorption \( FR \) increases by increasing the permeability coefficient \( K \); also, the radial velocity increases by increasing the value of \( We^2 \), where stress tensor and fractional reabsorption decrease by increasing \( We^2 \)

(iv) The axial velocity \( u \) decreases by increasing the permeability coefficient \( K \) and \( We^2 \), while the mean pressure drops decrease by increasing \( K \) and increase by increasing \( We^2 \)

(v) The temperature field increases at the tube’s centerline as the permeability coefficient decreases

(vi) The Nusselt number at the tube’s wall decreases as it travels down the tube, peaking at the tube’s exit region as the permeability coefficient increases
Nomenclature

\[ \mu_0^* \]: Viscosity at zero shear rate
\[ \mu_{\infty}^* \]: Viscosity at infinite shear rate
\[ \Gamma \]: Relaxation time
\[ n \]: Power index
\[ m \]: Ratio of viscosity at infinite and zero shear rate
\[ L_p^* \]: Hydrodynamics permeability coefficient
\[ p_c \]: Hydrostatic pressure
\[ \pi \]: Osmotic pressure
\[ p_m \]: Difference between hydrostatic pressure and osmotic pressure
\[ K \]: Permeability coefficient
\[ a \]: Radius of tubule
\[ L \]: Length of tubule
\[ U_m^* \]: Mean axial velocity
\[ V_w^* \]: Outward radial velocity
\[ \delta \]: Ratio between radius and length of tubule
\[ We^2 \]: Weissenberg number
\[ u, v \]: Velocity components
\[ r, z \]: Cylindrical coordinates
\[ \Delta p \]: Pressure drop
\[ \tau_w^* \]: Wall shear stress
\[ B_\eta \]: Brinkman number
\[ \zeta \]: Specific heat at constant pressure.

Data Availability

All data used to support the findings of the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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