Research Article

Mathematically forecasting for generalized business by using fuzzy trapezoidal numbers

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ABSTRACT

In this paper, we forecasted the future of a business by using fuzzy trapezoidal numbers and the fuzzy Delphi method. This result is compared with another result obtained using the fuzzy triangular numbers and Delphi method. At last, we see that our method is more general than others.

Introduction

The main theme of these Fuzzy trapezoidal numbers defined by Abbassbandy and Hajjari (2010) model construction is to predict the time duration for future forecasting of a business basis on the past historical observation, which is introduced by Ali et al. (2016) and Mutalib et al. (2018). To solve the future forecasting problem of a business, such a way is an appropriate way based on the fuzzy set theory defined by Zadeh (1996 and 1965). Based on the fuzzy set theory, many future forecasting models have been established for businesses using the fuzzy Delphi method introduced by Kuo and Chen (2008).

In this paper, we propose a trapezoidal model based on the fuzzy number and fuzzy Delphi method. Fuzzy numbers are introduced by Bellman and Zadeh (1970) and Kaufmann and Gupta (1985 and 1988). And Delphi method was developed by Roy and Garai (2012). California in the 1940s.

Preliminaries

2.1 Fuzzy set: A fuzzy set A is defined by a set, \( A = \{(x, \mu_A(x)) | x \in A, \mu_A(x) \in [0,1]\} \), where \( \mu_A(x) \) is a membership function belonging to \( [0,1] \) (Mohanpriya and Jeyanthi, 2016).

2.2 Fuzzy number: A fuzzy number is defined as a convex and normalized fuzzy set on the universe \( R \) (Mohanpriya and Jeyanthi, 2016).

2.3 Triangular fuzzy number: A triangular fuzzy number A with membership function \( \mu_A(x) \) is defined on \( R \) by (Gani & Assarudeen, 2012)

\[
A = \mu_A(x) = \begin{cases} \frac{x-a_1}{a_M-a_1} & \text{for} \ a_1 \leq x \leq a_M \\ \frac{x-a_2}{a_M-a_2} & \text{for} \ a_M \leq x \leq a_2 \\ 0 & \text{otherwise.} \end{cases}
\]

2.4 Trapezoidal fuzzy number: A trapezoidal fuzzy number A with membership function \( \mu_A(x) \) is defined on \( R \) by (Mohanpriya and Jeyanthi, 2016).

\[
A = \mu_A(x) = \begin{cases} \frac{x-a_1}{b_1-a_1} & \text{for} \ a_1 \leq x \leq b_1 \\ \frac{x-a_2}{b_2-a_2} & \text{for} \ b_1 \leq x \leq b_2 \\ 0 & \text{otherwise.} \end{cases}
\]

2.5 Fuzzy averaging: (i). Triangular fuzzy average formula (Bojadziev and Bojadziev, 2007). Consider n triangular numbers

\[
A_i = (a_1^{(i)}, a_M^{(i)}, a_2^{(i)}),
\]

where \( i = 1, 2, ..., n \).

The triangular average \( A_{\text{ave}} \),

\[
A_{\text{ave}} = (m_1, m_M, m_2) = \frac{A_1 + A_2 + \cdots + A_n}{n}
\]
\[ A_i = \left( a_1^{(i)}, a_M^{(i)}, a_2^{(i)} \right), \]

Where \( i = 1, 2, \ldots, n. \)  \hfill (3.1)

**Step 2:** First, the average (mean) \( A_{\text{ave}} = (m_1, m_M, m_2) \) of all \( A_i \) is computed (see 2.1). Then for each expert \( E_i \) the deviation between \( A_{\text{ave}} \) and \( A_i \) is computed. It is a triangular number defined by

\[ A_{\text{ave}} - A_i = (m_1 - a_1^{(i)}, m_M - a_M^{(i)}, m_2 - a_2^{(i)}) \]

\[ = \left( \frac{1}{n} \sum_{i=1}^{n} a_1^{(i)} - a_1^{(i)}, \frac{1}{n} \sum_{i=1}^{n} a_M^{(i)} - a_M^{(i)}, \frac{1}{n} \sum_{i=1}^{n} a_2^{(i)} - a_2^{(i)} \right). \]  \hfill (3.2)

The deviation \( A_{\text{ave}} - A_i \) is sent back to the expert \( E_i \) for reexamination.

**Step 3:** Each expert \( E_i \) presents a new triangular number

\[ B_i = (b_1^{(i)}, b_M^{(i)}, b_2^{(i)}), \; i = 1, \ldots, n. \]  \hfill (3.3)

This process starts with Step 2 is repeated. The triangular average \( B_{\text{ave}} \) is calculated according to formula (2.1) with the difference that now \( a_1^{(i)}, a_M^{(i)}, a_2^{(i)} \) are substituted correspondingly by \( b_1^{(i)}, b_M^{(i)}, b_2^{(i)}. \) If necessary, new triangular numbers \( C_i = (c_1^{(i)}, c_M^{(i)}, c_2^{(i)}) \) are generated, and their average \( C_i \) is calculated. The process could be repeated again and again until two successive means \( A_{\text{ave}}, B_{\text{ave}}, C_{\text{ave}}, \ldots \) become reasonably close.

**Step 4:** Later, the same process may be reexamine the forecasting if there is important information available due to new discoveries.

An Innovative Product Time Estimation for Technical Realization (Bojadziev and Bojadziev, 2007).

A group of 15 computer experts are asked to estimate using the Fuzzy Delphi method for the technical realization of a brand-new product, say a cognitive information processing computer. They are ranked equally; hence their opinions carry the same weight. The triangular numbers \( A_i, i = 1, \ldots, 15 \) (see (3.1)) presented by the experts are shown in Table 1.
Table 1. Triangular numbers $A_i$ presented by experts (first request) (Bojadziev and Bojadziev, 2007).

| $E_i$ | $A_i$ | Earliest date | Most plausible date | Latest date |
|-------|-------|---------------|---------------------|-------------|
| $E_1$ | $A_1$ | $a_1^{(1)}=1995$ | $a_M^{(1)}=2003$ | $a_2^{(1)}=2010$ |
| $E_2$ | $A_2$ | $a_1^{(2)}=1997$ | $a_M^{(2)}=2004$ | $a_2^{(2)}=2010$ |
| $E_3$ | $A_3$ | $a_1^{(3)}=2000$ | $a_M^{(3)}=2005$ | $a_2^{(3)}=2010$ |
| $E_4$ | $A_4$ | $a_1^{(4)}=1998$ | $a_M^{(4)}=2003$ | $a_2^{(4)}=2008$ |
| $E_5$ | $A_5$ | $a_1^{(5)}=2000$ | $a_M^{(5)}=2005$ | $a_2^{(5)}=2015$ |
| $E_6$ | $A_6$ | $a_1^{(6)}=1995$ | $a_M^{(6)}=2010$ | $a_2^{(6)}=2015$ |
| $E_7$ | $A_7$ | $a_1^{(7)}=2010$ | $a_M^{(7)}=2018$ | $a_2^{(7)}=2015$ |
| $E_8$ | $A_8$ | $a_1^{(8)}=1995$ | $a_M^{(8)}=2007$ | $a_2^{(8)}=2013$ |
| $E_9$ | $A_9$ | $a_1^{(9)}=1995$ | $a_M^{(9)}=2002$ | $a_2^{(9)}=2007$ |
| $E_{10}$ | $A_{10}$ | $a_1^{(10)}=2008$ | $a_M^{(10)}=2009$ | $a_2^{(10)}=2020$ |
| $E_{11}$ | $A_{11}$ | $a_1^{(11)}=2010$ | $a_M^{(11)}=2020$ | $a_2^{(11)}=2024$ |
| $E_{12}$ | $A_{12}$ | $a_1^{(12)}=1996$ | $a_M^{(12)}=2002$ | $a_2^{(12)}=2006$ |
| $E_{13}$ | $A_{13}$ | $a_1^{(13)}=1998$ | $a_M^{(13)}=2006$ | $a_2^{(13)}=2010$ |
| $E_{14}$ | $A_{14}$ | $a_1^{(14)}=1997$ | $a_M^{(14)}=2005$ | $a_2^{(14)}=2012$ |
| $E_{15}$ | $A_{15}$ | $a_1^{(15)}=2002$ | $a_M^{(15)}=2010$ | $a_2^{(15)}=2020$ |

To find the average $A_{ave}$ the sums of the numbers in the last three columns are calculated

$$
\sum_{i=1}^{15} a_1^{(i)} = 29996, \quad \sum_{i=1}^{15} a_2^{(i)} = 30210
$$

and substituted into (2.1), which gives

$$
A_{ave} = \left( \frac{29996}{15}, \frac{30109}{15}, \frac{30210}{15} \right) = (1999.7, 2010.6, 2014)
$$

or approximately, $A_{ave}^a = (2000, 2007, 2014)$.

The deviations (3.2) between $A_{ave}^a$ and $A_i$ are presented in Table 2.

Table 2. Deviation $A_{ave} - A_i$.

| $E_i$ | $m_1 - a_1^{(i)}$ | $m_M - a_M^{(i)}$ | $m_2 - a_2^{(i)}$ |
|-------|-------------------|--------------------|-------------------|
| $E_1$ | 5                 | 4                  | -6                |
| $E_2$ | 3                 | 3                  | 4                 |
| $E_3$ | 0                 | 2                  | 4                 |
| $E_4$ | 2                 | 4                  | 6                 |
| $E_5$ | 0                 | 2                  | -1                |
| $E_6$ | 5                 | -3                 | -1                |
| $E_7$ | -10               | -11                | -6                |
| $E_8$ | 5                 | 0                  | 1                 |
| $E_9$ | 5                 | 5                  | 7                 |
| $E_{10}$ | -8               | -2                 | -6                |
| $E_{11}$ | -10              | -13                | -10               |
| $E_{12}$ | 4                | 5                  | 8                 |
| $E_{13}$ | 2                | 1                  | 4                 |
| $E_{14}$ | 3                | 2                  | 2                 |
| $E_{15}$ | -2               | -3                 | -6                |

Table 2. shows the divergence of each expert's opinion from the average. A quick glance gives that the experts $E_3, E_5, E_8, E_{13}, E_{14}$ are close to the average while $E_7, E_{11}$ is not.

Since the word close is fuzzy, a more detailed study requires some clarification. It can be based on distance $d_{ij}$ between two triangular numbers $A_i$ and $A_j$. If all $d_{ij}$ are calculated and recorded in a table (in our case consisting of 15 rows and columns), we will have a better grasp of how close various pairs of $A_i$ and $A_j$ are. Here we do not give a formula for calculating the distance $d_{ij}$ (there are several), 4 but refer to Kaufmann and Gupta (1988).

Suppose the manager is not satisfied with the average (2000, 2007, 2014). Then the deviation $(m_1 - a_1^{(i)}, m_M - a_M^{(i)}, m_2 - a_2^{(i)})$ is given to each expert $E_i$ for reconsideration. The experts
suggest new triangular numbers $B_i$ (see (3.3)) presented in Table 3.

Table 3. Triangular numbers presented by experts (second request) (Bojadziev and Bojadziev, 2007).

| $E_i$ | $B_i$       | Earliest date | Most plausible date | Latest date |
|------|-------------|---------------|---------------------|-------------|
| $E_1$ | $B_1$       | $b_1^{(1)} = 1996$ | $b_1^{(1)} = 2004$ | $b_1^{(1)} = 2018$ |
| $E_2$ | $B_2$       | $b_2^{(2)} = 1997$ | $b_2^{(2)} = 2004$ | $b_2^{(2)} = 2011$ |
| $E_3$ | $B_3$       | $b_3^{(3)} = 2000$ | $b_3^{(3)} = 2005$ | $b_3^{(3)} = 2011$ |
| $E_4$ | $B_4$       | $b_4^{(4)} = 1998$ | $b_4^{(4)} = 2003$ | $b_4^{(4)} = 2010$ |
| $E_5$ | $B_5$       | $b_5^{(5)} = 2000$ | $b_5^{(5)} = 2005$ | $b_5^{(5)} = 2015$ |
| $E_6$ | $B_6$       | $b_6^{(6)} = 1997$ | $b_6^{(6)} = 2009$ | $b_6^{(6)} = 2015$ |
| $E_7$ | $B_7$       | $b_7^{(7)} = 2005$ | $b_7^{(7)} = 2015$ | $b_7^{(7)} = 2016$ |
| $E_8$ | $B_8$       | $b_8^{(8)} = 1996$ | $b_8^{(8)} = 2007$ | $b_8^{(8)} = 2013$ |
| $E_9$ | $B_9$       | $b_9^{(9)} = 1997$ | $b_9^{(9)} = 2004$ | $b_9^{(9)} = 2010$ |
| $E_{10}$ | $B_{10}$   | $b_{10}^{(10)} = 2004$ | $b_{10}^{(10)} = 2009$ | $b_{10}^{(10)} = 2017$ |
| $E_{11}$ | $B_{11}$   | $b_{11}^{(11)} = 2004$ | $b_{11}^{(11)} = 2015$ | $b_{11}^{(11)} = 2016$ |
| $E_{12}$ | $B_{12}$   | $b_{12}^{(12)} = 1996$ | $b_{12}^{(12)} = 2004$ | $b_{12}^{(12)} = 2006$ |
| $E_{13}$ | $B_{13}$   | $b_{13}^{(13)} = 1998$ | $b_{13}^{(13)} = 2006$ | $b_{13}^{(13)} = 2010$ |
| $E_{14}$ | $B_{14}$   | $b_{14}^{(14)} = 1997$ | $b_{14}^{(14)} = 2004$ | $b_{14}^{(14)} = 2012$ |
| $E_{15}$ | $B_{15}$   | $b_{15}^{(15)} = 2001$ | $b_{15}^{(15)} = 2009$ | $b_{15}^{(15)} = 2015$ |

The manager is satisfied that $A_{ave}$ and $B_{ave}$, also $A_{ave}^a$ and $B_{ave}^a$, are very close (see Fig. 1), stop the fuzzy Delphi process, and accepts the triangular number $B_{ave}^a$ as a combined conclusion of experts’ opinions. The interpretation is that the realization of the invention will occur in the time interval [1999, 2013], the supporting interval of the triangular number $B_{ave}^a$ which is almost in central form.

Fig. 1. Average triangular numbers $A_{ave}^a$ and $B_{ave}^a$.

Materials and methods

The fuzzy Delphi method consists of the following parts for trapezoidal:

Step 1. Experts $E_i$, $i = 1, \ldots, n$, are asked to provide the possible realization dates of a particular event in science, technology, or business, namely: the earliest date $d_{1i}^{(i)}$, the earliest most plausible date $d_{M1}^{(i)}$, the latest most plausible date $d_{M2}^{(i)}$, and the latest date $d_{2i}^{(i)}$. The data given by the experts $E_i$ are presented in the form of trapezoidal numbers

$$A_i = (d_{1i}^{(i)}, d_{M1}^{(i)}, d_{M2}^{(i)}, d_{2i}^{(i)})$$

Where $i = 1, 2, \ldots, n$. (4.1)

Step 2. First, the average (mean) $A_{ave} = (m_1, m_{M1}, m_{M2}, m_2)$ of all $A_i$ is computed (see 2.2). Then for each expert $E_i$ the deviation

$$B_{ave} = (1999.07, 2006.9, 2013.2)$$

Which is approximately,

$$B_{ave} = (1999, 2007, 2013).$$
between $A_{\text{ave}}$ and $A_i$ is computed. It is a trapezoidal number defined by

$$A_{\text{ave}} - A_i = (m_1 - a_1^{(i)}, m_{M1} - a_{M1}^{(i)}, m_{M2} - a_{M2}^{(i)}, m_2 - a_2^{(i)})$$

$$= \frac{1}{n} \sum_{i=1}^{n} a_1^{(i)} - a_1^{(i)} - \frac{1}{n} \sum_{i=1}^{n} a_{M1}^{(i)} - a_{M1}^{(i)},$$

$$= \frac{1}{n} \sum_{i=1}^{n} a_2^{(i)} - a_2^{(i)} - \frac{1}{n} \sum_{i=1}^{n} a_{M2}^{(i)} - a_{M2}^{(i)}.$$  \hspace{1cm} (4.2)

The deviation $A_{\text{ave}} - A_i$ is sent back to the expert $E_i$ for reexamination.

**Step 3.** Each expert $E_i$ presents a new trapezoidal number

$$B_i = (b_1^{(i)}, b_{M1}^{(i)}, b_{M2}^{(i)}, b_2^{(i)}), i = 1, \ldots, n. \hspace{1cm} (4.3)$$

This process starts with Step 2 is repeated. The trapezoidal average $B_{\text{ave}}$ is calculated according to formula (2.2) with the difference that now $a_1^{(i)}, a_{M1}^{(i)}, a_2^{(i)}, a_{M2}^{(i)}$ are substituted correspondingly by $b_1^{(i)}, b_{M1}^{(i)}, b_{M2}^{(i)}, b_2^{(i)}$. If necessary, new trapezoidal numbers $C_i = (c_1^{(i)}, c_{M1}^{(i)}, c_{M2}^{(i)}, c_2^{(i)})$ are generated, and their average $C_{\text{ave}}$ is calculated. The process could be repeated again and again until two successive means $A_{\text{ave}}, B_{\text{ave}}, C_{\text{ave}} \ldots$ become reasonably close.

**Step 4.** Later, the forecasting may be reexamined by the same process if there is important information available due to new discoveries.

An Innovative Product Time Estimation for Technical Realization.

A group of 15 computer experts are asked to estimate using the Fuzzy Delphi method for the technical realization of a brand-new product, say a cognitive information processing computer. They are ranked equally, hence their opinions carry the same weight. The trapezoidal numbers, $A_i$, $i = 1, \ldots, 15$ (see (4.1)) presented by the experts are shown in Table 4.

| $E_i$ | $A_i$ | Earliest date | Earliest most plausible date | Latest most plausible date | Latest date |
|------|-------|---------------|----------------------------|--------------------------|-------------|
| $E_1$ | $A_1$ | $a_1^{(1)} = 1995$ | $a_{M1}^{(1)} = 2003$ | $a_{M2}^{(1)} = 2006$ | $a_2^{(1)} = 2020$ |
| $E_2$ | $A_2$ | $a_1^{(2)} = 1997$ | $a_{M1}^{(2)} = 2004$ | $a_{M2}^{(2)} = 2005$ | $a_2^{(2)} = 2010$ |
| $E_3$ | $A_3$ | $a_1^{(3)} = 2000$ | $a_{M1}^{(3)} = 2005$ | $a_{M2}^{(3)} = 2007$ | $a_2^{(3)} = 2010$ |
| $E_4$ | $A_4$ | $a_1^{(4)} = 1998$ | $a_{M1}^{(4)} = 2003$ | $a_{M2}^{(4)} = 2005$ | $a_2^{(4)} = 2008$ |
| $E_5$ | $A_5$ | $a_1^{(5)} = 2000$ | $a_{M1}^{(5)} = 2005$ | $a_{M2}^{(5)} = 2008$ | $a_2^{(5)} = 2015$ |
| $E_6$ | $A_6$ | $a_1^{(6)} = 1995$ | $a_{M1}^{(6)} = 2010$ | $a_{M2}^{(6)} = 2012$ | $a_2^{(6)} = 2015$ |
| $E_7$ | $A_7$ | $a_1^{(7)} = 2010$ | $a_{M1}^{(7)} = 2018$ | $a_{M2}^{(7)} = 2019$ | $a_2^{(7)} = 2015$ |
| $E_8$ | $A_8$ | $a_1^{(8)} = 1995$ | $a_{M1}^{(8)} = 2007$ | $a_{M2}^{(8)} = 2010$ | $a_2^{(8)} = 2013$ |
| $E_9$ | $A_9$ | $a_1^{(9)} = 1995$ | $a_{M1}^{(9)} = 2002$ | $a_{M2}^{(9)} = 2005$ | $a_2^{(9)} = 2007$ |
| $E_{10}$ | $A_{10}$ | $a_1^{(10)} = 2008$ | $a_{M1}^{(10)} = 2009$ | $a_{M2}^{(10)} = 2013$ | $a_2^{(10)} = 2020$ |
| $E_{11}$ | $A_{11}$ | $a_1^{(11)} = 2010$ | $a_{M1}^{(11)} = 2020$ | $a_{M2}^{(11)} = 2022$ | $a_2^{(11)} = 2024$ |
| $E_{12}$ | $A_{12}$ | $a_1^{(12)} = 1996$ | $a_{M1}^{(12)} = 2002$ | $a_{M2}^{(12)} = 2003$ | $a_2^{(12)} = 2006$ |
| $E_{13}$ | $A_{13}$ | $a_1^{(13)} = 1998$ | $a_{M1}^{(13)} = 2006$ | $a_{M2}^{(13)} = 2008$ | $a_2^{(13)} = 2010$ |
| $E_{14}$ | $A_{14}$ | $a_1^{(14)} = 1997$ | $a_{M1}^{(14)} = 2005$ | $a_{M2}^{(14)} = 2008$ | $a_2^{(14)} = 2012$ |
| $E_{15}$ | $A_{15}$ | $a_1^{(15)} = 2002$ | $a_{M1}^{(15)} = 2010$ | $a_{M2}^{(15)} = 2013$ | $a_2^{(15)} = 2020$ |
To find the average $A_{\text{ave}}$ the sums of the numbers in the last four columns are calculated

$$9996, \sum_{i=1}^{15} a_{M1}^{(i)} = 30109,$$

$$\sum_{i=1}^{15} a_{M2}^{(i)} = 30144, \sum_{i=1}^{15} a_{2}^{(i)} = 30210$$

and substituted into (2.2) which gives

$$A_{\text{ave}} = (\frac{9996}{15}, \frac{30109}{15}, \frac{30144}{15}, \frac{30210}{15})$$

= (1999.7, 2007.3, 2009.6, 2014)

or approximately,

$$A_{\text{ave}} = (2000, 2007, 2010, 2014).$$

The deviations (4.2) between $A_{\text{ave}}$ and $A_i$ are presented in Table 5.

**Table 5. Deviation $A_{\text{ave}} - A_i$**

| $E_i$ | $m_1 - a_1^{(i)}$ | $m_{M1} - a_{M1}^{(i)}$ | $m_{M2} - a_{M2}^{(i)}$ | $m_2 - a_2^{(i)}$ |
|-------|-------------------|--------------------------|--------------------------|-------------------|
| $E_1$ | 5                 | 4                        | 4                        | -6                |
| $E_2$ | 3                 | 3                        | 5                        | 4                 |
| $E_3$ | 0                 | 2                        | 3                        | 4                 |
| $E_4$ | 2                 | 4                        | 5                        | 6                 |
| $E_5$ | 0                 | 2                        | 2                        | -1                |
| $E_6$ | -10               | -3                       | -2                       | -1                |
| $E_7$ | 5                 | 0                        | 1                        |                   |
| $E_8$ | 5                 | 5                        | 5                        | 7                 |
| $E_{10}$ | -8              | -2                       | -3                       | -6                |
| $E_{11}$ | -10             | -13                      | -12                      | -10               |
| $E_{12}$ | 4               | 5                        | 7                        | 8                 |
| $E_{13}$ | 2               | 1                        | 2                        | 4                 |
| $E_{14}$ | 3               | 2                        | 2                        | 2                 |
| $E_{15}$ | -2              | -3                       | -3                       | -6                |

To find the deviation $m_{M2} - m_{M1}$, we substitute $a_{M1}^{(i)}$ and $a_{M2}^{(i)}$ from Table 5 into (4.2) and we get

$$\displaystyle \sum_{i=1}^{15} M_{1} = 30109, \sum_{i=1}^{15} M_{2} = 30144, \sum_{i=1}^{15} a_{2}^{(i)} = 30210$$

since the word close is fuzzy, a more detailed study requires some clarification. It can be based on distance $d_{ij}$ between two trapezoidal numbers $A_i$ and $A_j$. If all $d_{ij}$ are calculated and recorded in a table (in our case consisting of 15 rows and columns), we will have a better grasp of how close various pairs of $A_i$ and $A_j$ are. Here we do not give a formula for calculating the distance $d_{ij}$ (there are several), but refer to Kaufmann & Gupta (1988).

Suppose the manager is not satisfied with the average (2000, 2007, 2010, 2014). Then the deviation $(m_1 - a_1^{(i)}, m_{M1} - a_{M1}^{(i)}, m_{M2} - a_{M2}^{(i)}, m_2 - a_2^{(i)})$ is given to each expert $E_i$ for reconsideration. The experts suggest new trapezoidal numbers $B_i$ (see (4.3) presented in Table 6.

**Table 6. Trapezoidal numbers presented by experts (second request).**

| $E_i$ | Earliest date | Earliest plausible date | Latest date | Latest plausible date |
|-------|---------------|-------------------------|-------------|-----------------------|
| $E_1$ | $b_1^{(1)}$   | $b_{M1}^{(1)}$           | $b_{M2}^{(1)}$ | $b_2^{(1)}$           |
|       | =1996         | =2004                   | =2007       | =2018                 |
| $E_2$ | $b_2^{(1)}$   | $b_{M1}^{(2)}$           | $b_{M2}^{(2)}$ | $b_2^{(2)}$           |
|       | =1997         | =2004                   | =2006       | =2011                 |
| $E_3$ | $b_3^{(1)}$   | $b_{M1}^{(3)}$           | $b_{M2}^{(3)}$ | $b_2^{(3)}$           |
|       | =2000         | =2005                   | =2005       | =2011                 |
| $E_4$ | $b_4^{(4)}$   | $b_{M1}^{(4)}$           | $b_{M2}^{(4)}$ | $b_2^{(4)}$           |
|       | =1998         | =2003                   | =2005       | =2010                 |
| $E_5$ | $b_5^{(5)}$   | $b_{M1}^{(5)}$           | $b_{M2}^{(5)}$ | $b_2^{(5)}$           |
|       | =2000         | =2005                   | =2008       | =2015                 |
| $E_6$ | $b_6^{(6)}$   | $b_{M1}^{(6)}$           | $b_{M2}^{(6)}$ | $b_2^{(6)}$           |
|       | =1997         | =2009                   | =2011       | =2015                 |
| $E_7$ | $b_7^{(7)}$   | $b_{M1}^{(7)}$           | $b_{M2}^{(7)}$ | $b_2^{(7)}$           |
|       | =2005         | =2015                   | =2016       | =2016                 |

Table 5. shows the divergence of each expert’s opinion from the average. A quick glance gives that the experts $E_3, E_5, E_8, E_{13}, E_{14}$ are close to the average while $E_7, E_{11}$ is not.
### Results and Discussion

In this article, we use triangular and trapezoidal numbers. By comparing two of these numbers, we get,

(i) From the triangle, we get one peak point from where it’s not sure how many days it will run well. On the other hand, we get an interval of the peak points that define that the business will run well in this interval from trapezoidal numbers.

(ii) Also, we see from the trapezoidal numbers figure that a fast business will build up or fall. But there is no proper definition in the triangular numbers.

### Conclusion

Here we see that trapezoidal numbers give better results than triangular numbers for future business forecasting since the trapezoidal numbers are more generalized than triangular numbers. So, the results we have gotten using trapezoidal numbers will be better than triangular numbers.

### Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

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