Electric S-brane solutions corresponding to rank-2 Lie algebras: acceleration and small variation of G

V.D. Ivashchuk\textsuperscript{1, a, b}, S.A. Kononogov\textsuperscript{2, a} and V.N. Melnikov\textsuperscript{3, a, b}

\textsuperscript{a} Center for Gravitation and Fundamental Metrology, VNIIMS, 46 Ozyornaya St., Moscow 119361, Russia
\textsuperscript{b} Institute of Gravitation and Cosmology, Peoples’ Friendship University of Russia, 6 Miklukho-Maklaya St., Moscow 117198, Russia

Abstract

Electric S-brane solutions with two non-composite electric branes and a set of $l$ scalar fields are considered. The intersection rules for branes correspond to Lie algebras $A_2$, $C_2$ and $G_2$. The solutions contain five factor spaces. One of them, $M_0$, is interpreted as our 3-dimensional space. It is shown that there exists a time interval where accelerated expansion of our 3-dimensional space is compatible with a small enough variation of the effective gravitational constant $G(\tau)$. This interval contains $\tau_0$, a point of minimum of the function $G(\tau)$. A special solution with two phantom scalar fields is analyzed and it is shown that in the vicinity of the point $\tau_0$ the time variation of $G(\tau)$ (calculated in the linear approximation) decreases in the sequence of Lie algebras $A_2$, $C_2$ and $G_2$. 

\textsuperscript{1}ivashchuk@mail.ru
\textsuperscript{2}kononogov@vniims.ru
\textsuperscript{3}melnikov@phys.msu.ru
1 Introduction

Multidimensional cosmological models with diverse matter sources are at present widely used for describing possible time variations of fundamental physical constants, e.g. gravitational constant $G$, see [1]-[16] and references therein.

It has been shown in [10] that, in the pure gravitational model with two non-zero curvatures, there exists an interval of synchronous variable $\tau$ where accelerated expansion of “our” 3-dimensional space co-exists with a small enough value of $\dot{G}$. This result was compared with our exact $(1+3+6)$-dimensional solution [17] obtained earlier.

Recently, in [16], we suggested a similar mechanism for a model with two form fields and several scalar fields (e.g., phantom ones). The main problem here was to find an interval of the synchronous time $\tau$ where the scale factor of our 3-dimensional space exhibits an accelerated expansion according to the observational data [18, 19] while the relative variation of the effective 4-dimensional gravitational constant is small enough as compared with the Hubble parameter, see [20, 21, 11] and references therein. As it was shown in [16] such interval did exist, it contained $\tau_0$ – the point of minimum of the function $G(\tau)$. The analysis carried out in [16] was based on an exact $S$-brane solution with the intersection rules for branes corresponding to the Lie algebra $A_2$.

In this paper, we extend the results of our previous work to exact $S$-brane solutions with the intersection rules corresponding to other simple rank 2 Lie algebras: $C_2$ and $G_2$. Thus, here we generalize the results of ref. [16] to the sequence of Lie algebras $A_2$, $C_2$ and $G_2$.

The paper is organized as follows. In Section 2, the setup for the model is done and exact $S$-brane solutions are presented. In Section 3, solutions with acceleration and small $G$-dot are singled out. In Section 4, a special configuration with two phantom field is considered. Here we compare the $G$-dot calculated (in the linear approximation near the point of minimum of $G(\tau)$) for these three algebras and show that the variation of $G$ decreases in the sequence of Lie algebras $A_2$, $C_2$ and $G_2$. In the Appendix, we give a derivation of relation (3.17) for an approximate value of the dimensionless parameter of relative $G$ variation.
2 The model

We consider $S$-brane solutions describing two electric branes and a set of $l$ scalar fields.

The model is governed by the action

$$S = \int d^D x \sqrt{|g|} \left\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a=1,2} \frac{1}{N_a!} \exp[2\lambda_a(\varphi)](F^a)^2 \right\}, \tag{2.1}$$

where $g = g_{MN}(x) dx^M \otimes dx^N$ is a $D$-dimensional metric of the pseudo-Euclidean signature $(-,+,\ldots,+)$, $F^a = dA^a$ is a form of rank $N_a$, $(h_{\alpha\beta})$ is a non-degenerate symmetric matrix, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of $l$ scalar fields, $\lambda_a(\varphi) = \lambda_{a\alpha} \varphi^\alpha$ is a linear function, with $a = 1, 2$ and $\alpha, \beta = 1, \ldots, l$ and $|g| = |\det(g_{MN})|$. We consider the manifold

$$M = (0, +\infty) \times M_0 \times M_1 \times M_2 \times M_3 \times M_4. \tag{2.2}$$

where $M_i$ are oriented Riemannian Ricci-flat spaces of dimensions $d_i$, $i = 0, \ldots, 4$, and $d_1 = 1$.

Let two electric branes be defined by the sets $I_1 = \{1,2,3\}$ and $I_2 = \{1,2,4\}$. They intersect on $M_1 \times M_2$. The first brane also covers $M_3$ while the second one covers $M_4$. The first brane corresponds to the form $F^1$ and the second one to the form $F^2$.

For the world-volume dimensions of branes we get

$$d(I_s) = N_s - 1 = 1 + d_2 + d_{2+s}, \tag{2.3}$$

$s = 1, 2$, and

$$d(I_1 \cap I_2) = 1 + d_2 \tag{2.4}$$

is the brane intersection dimension.

We consider an $S$-brane solution governed by the function

$$\dot{H} = 1 + Pt^2, \tag{2.5}$$

where $t$ is a time variable and

$$P = \frac{1}{4\pi_s} K_s Q_s^2 \tag{2.6}$$
is a parameter. Here, $Q_s$ are charge density parameters,

$$K_s = d(I_s)(1 + \frac{d(I_s)}{2 - D}) + \lambda_{s\alpha}\lambda_{s\beta}h^{\alpha\beta},$$  \hspace{1cm} (2.7)

$s = 1, 2$, $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}$, and

$$(n_1, n_2) = (2, 2), (3, 4), (6, 10)$$  \hspace{1cm} (2.8)

for the Lie algebras $A_2$, $C_2$, and $G_2$, respectively. The parameters $K_s$ and $Q_s$ are supposed to be nonzero.

The intersection rules read:

$$d(I_1 \cap I_2) = \frac{d(I_1)d(I_2)}{D - 2} - \lambda_{1\alpha}\lambda_{2\beta}h^{\alpha\beta} - \frac{1}{2}K_2.$$  \hspace{1cm} (2.9)

These relations correspond to the simple Lie algebras of rank 2 [22, 23].

Recall that $K_s = (U_s, U_s)$, $s = 1, 2$, where the “electric” $U^s$-vectors and the scalar products were defined in [24, 25, 22]. Relations (2.9) follow just from the formula

$$(A_{ss'}) = (2(U^s, U^{s'})/(U^{s'}, U^{s'})),$$  \hspace{1cm} (2.10)

where $(A_{ss'})$ is the Cartan matrix for Lie algebra of rank 2 with $A_{12} = -1, A_{21} = -k$, where here and in what follows

$$k = 1, 2, 3,$$  \hspace{1cm} (2.11)

for the Lie algebras $A_2$, $C_2$ and $G_2$, respectively. We remind the reader that (see [25])

$$(U^1, U^2) = d(I_1 \cap I_2) - \frac{d(I_1)d(I_2)}{D - 2} + \lambda_{1\alpha}\lambda_{2\beta}h^{\alpha\beta}.$$  \hspace{1cm} (2.12)

Due to (2.10) and relations $A_{12} = -1, A_{21} = -k$, we get

$$K_2 = K_1k.$$  \hspace{1cm} (2.13)

We consider the following exact solutions (containing three subcases corresponding to the Lie algebras $A_2$, $C_2$ and $G_2$)

$$g = \hat{H}^{2A}\left\{-dt \otimes dt + g^0 + \hat{H}^{-2(B_1 + B_2)}(t^2g^1 + g^2) + \hat{H}^{-2B_1}g^3 + \hat{H}^{-2B_2}g^4\right\},$$  \hspace{1cm} (2.14)

$$\exp(\varphi^\alpha) = \hat{H}^{B_1\lambda_0 + B_2\lambda_2},$$  \hspace{1cm} (2.15)

$$F^1 = -Q_1\hat{H}^{-2n_1+n_2}tdt \wedge \tau_1 \wedge \tau_2 \wedge \tau_3,$$  \hspace{1cm} (2.16)

$$F^2 = -Q_2\hat{H}^{-2n_2+k_n}tdt \wedge \tau_1 \wedge \tau_2 \wedge \tau_4,$$  \hspace{1cm} (2.17)
where
\[
A = \sum_{s=1,2} \frac{n_s K_s^{-1} d(I_s)}{D - 2},
\]
(2.18)
\[
B_s = n_s K_s^{-1},
\]
(2.19)
\[s = 1, 2.\] Here \(\tau_i\) denotes a volume form on \(M_i\) \((g_1 = dx \otimes dx, \tau_1 = dx)\).

These solutions are special cases of more general solutions from [26] corresponding to the Lie algebras \(A_2, C_2\) and \(G_2\). They can also be obtained as a special 1-block case of S-brane solutions from [28]. The \(A_2\)-case \((k = 1)\) was considered in [16].

We also note that the charge density parameters \(Q_s\) obey the following relation (see (2.6) and (2.13))
\[
\frac{Q_1^2}{Q_2^2} = \frac{n_1}{n_2} k = 1, \quad \frac{3}{2}, \quad \frac{9}{5}
\]
(2.20)
for the Lie algebras \(A_2, C_2\) and \(G_2\), respectively.

### 3 Solutions with acceleration and small \(\dot{G}\)

Let us introduce the synchronous time variable \(\tau = \tau(t)\) by the relation:
\[
\tau = \int_0^t d\bar{t}[\dot{H}(\bar{t})]^A
\]
(3.1)

We put \(P < 0\), and hence due to (2.6) all \(K_s < 0\) which implies \(A < 0\). Consider two intervals of the parameter \(A\):

(i) \(A < -1\),
(ii) \(-1 < A < 0\).

(3.2) \(3.3\)

In case (i), the function \(\tau = \tau(t)\) monotonically increases from 0 to \(+\infty\), for \(t \in (0, t_1)\), where \(t_1 = |P|^{-1/2}\), while in case (ii) it is monotonically increases from 0 to a finite value \(\tau_1 = \tau(t_1)\).

Let the space \(M_0\) be our 3-dimensional space with the scale factor
\[
a_0 = \dot{H}^A.
\]
(3.4)
For the first branch (i), we get the asymptotic relation
\[ a_0 \sim \tau^\nu, \quad (3.5) \]
for \( \tau \to +\infty \), where
\[ \nu = A/(A + 1) \quad (3.6) \]
and, due to (3.2), \( \nu > 1 \). For the second branch (ii) we obtain
\[ a_0 \sim \text{const}(\tau_1 - \tau)^\nu, \quad (3.7) \]
for \( \tau \to \tau_1 - 0 \), where \( \nu < 0 \) due to (3.3), see (3.6).

Thus, we get an asymptotic accelerated expansion of the 3-dimensional factor space \( M_0 \) in both cases (i) and (ii), and \( a_0 \to +\infty \).

Moreover, it may be readily verified that the accelerated expansion takes place for all \( \tau > 0 \), i.e.,
\[ \dot{a}_0 > 0, \quad \ddot{a}_0 > 0. \quad (3.8) \]
Here and in what follows we denote \( \dot{f} = df/d\tau \).

Indeed, using the relation \( d\tau/dt = \dot{H}A \) (see (3.1)), we get
\[ \dot{a}_0 = \frac{dt}{d\tau} \frac{da_0}{dt} = \frac{2|A||P|^t}{\dot{H}}, \quad (3.9) \]
and
\[ \ddot{a}_0 = \frac{dt}{d\tau} \frac{d}{dt} \frac{da_0}{d\tau} = \frac{2|A||P|^t}{H^2 + A}(1 + |P|^t^2), \quad (3.10) \]
which certainly implies the inequalities in (3.8).

Let us consider a variation of the effective constant \( G \). In Jordan’s frame the 4-dimensional gravitational “constant” is
\[ G = \text{const} \prod_{i=1}^{4} (a_i^{-d_i}) = \dot{H}^2A^{-1}, \quad (3.11) \]
where
\[ a_1 = \dot{H}^{A-B_1-B_2}, \quad a_2 = \dot{H}^{A-B_1-B_2}, \quad a_3 = \dot{H}^{A-B_1}, \quad a_4 = \dot{H}^{A-B_2} \quad (3.12) \]
are the scale factors of the “internal” spaces \( M_1, \ldots, M_4 \), respectively.

The dimensionless variation of \( G \) reads
\[ \delta = \dot{G}/(GH) = 2 + \frac{1 - |P|^t^2}{2A|P|^t^2}, \quad (3.13) \]
where

\[ H = \frac{\dot{a}_0}{a_0} \]  

(3.14)

is the Hubble parameter of our space. It follows from (3.13) that the function \( G(\tau) \) has a minimum at the point \( \tau_0 \) corresponding to \( t_0 \), where

\[ t_0^2 = \frac{|P|^{-1}}{1 + 4|A|}. \]  

(3.15)

At this point, \( \dot{G} \) is zero.

The function \( G(\tau) \) monotonically decreases from \(+\infty\) to \( G_0 = G(\tau_0) \) for \( \tau \in (0, \tau_0) \) and monotonically increases from \( G_0 \) to \(+\infty\) for \( \tau \in (\tau_0, \bar{\tau}_1) \). Here \( \bar{\tau}_1 = +\infty \) for the case (i) and \( \bar{\tau}_1 = \tau_1 \) for the case (ii).

We consider only solutions with accelerated expansion of our space and small enough variations of the gravitational constant obeying the experimental constraint \([20, 21]\)

\[ |\delta| < 0.1. \]  

(3.16)

Here, as in the model with two curvatures \([10]\), \( \tau \) is restricted to a certain range containing \( \tau_0 \). It follows from (3.13) that in the asymptotical regions (3.5) and (3.7) \( \delta \rightarrow 2 \), which is unacceptable due to the experimental bounds (3.16). This restriction is satisfied for a range containing the point \( \tau_0 \) where \( \delta = 0 \).

Calculating \( \dot{G} \), in the linear approximation near \( \tau_0 \), we get the following approximate relation for the dimensionless parameter of relative \( G \) variation:

\[ \delta \approx (8 + 2|A|^{-1})H_0(\tau - \tau_0), \]  

(3.17)

where \( H_0 = H(\tau_0) \) (compare with an analogous relation in \([10]\)). This relation gives approximate bounds for values of the time variable \( \tau \) allowed by the restriction on \( \dot{G} \). A derivation of this is given in Appendix.

The solutions under consideration with \( P < 0, d_1 = 1 \) and \( d_0 = 3 \) take place when the configuration of branes, the matrix \((h_{\alpha\beta})\) and the dilatonic coupling vectors \( \lambda_a \), obey the relations (2.7) and (2.9) with \( K_s < 0 \). This is possible when \((h_{\alpha\beta})\) is not positive-definite, otherwise all \( K_s > 0 \). Thus, there should be at least one scalar field with negative kinetic term (i.e., a phantom scalar field).
4 Example: a model with two phantom fields

Let us consider the following example: \( l = 2, (h_{\alpha \beta}) = -(\delta_{\alpha \beta}) \), i.e. there are two phantom scalar fields. Due to (2.3), \( d(I_1) = N_1 - 1 = 1 + d_2 + d_3 \) and \( d(I_2) = N_2 - 1 = 1 + d_2 + d_4 \).

Then the relations (2.7) and (2.9) read

\[
\vec{\lambda}_2^2 = (N_a - 1)(1 + \frac{N_a - 1}{2 - D}) - K_a > 0,
\]

\( a = 1, 2, \) and

\[
\vec{\lambda}_1 \vec{\lambda}_2 = 1 + d_2 - \frac{(N_1 - 1)(N_2 - 1)}{D - 2} + \frac{1}{2} K_2,
\]

where \( K_1 < 0 \) and \( K_2 = K_1 k, k = 1, 2, 3 \) for Lie algebras \( A_2, C_2 \) and \( G_2 \), respectively. Here we have used the relation for brane intersection: \( d(I_1 \cap I_2) = 1 + d_2 \).

The relations (4.1) and (4.2) for \( \vec{\lambda}_1, \vec{\lambda}_2 \), belonging to the Euclidean space \( \mathbb{R}^2 \), are compatible for small enough \( K_1 \in (-\infty, K_0) \), \( K_0 \leq 0 \), since it may be verified that they imply (for \( K_1 \leq K_0 \))

\[
\frac{\vec{\lambda}_1 \vec{\lambda}_2}{|\vec{\lambda}_1||\vec{\lambda}_2|} \in (-1, +1)
\]

i.e., the vectors \( \vec{\lambda}_1, \vec{\lambda}_2 \), belonging to the Euclidean space \( \mathbb{R}^2 \), and obeying the relations (4.1) and (4.2), do exist. The left-hand side of (4.3) gives \( \cos \theta \), where \( \theta \) is the angle between these two vectors. For the special case \( k = 1, N_1 = N_2, d_3 = d_4 \) considered in [16], \( K_0 = 0 \).

Now we compare the \( A \) parameters corresponding to different Lie algebras \( A_2, C_2 \) and \( G_2 \), when the parameter \( K_1 \) and factor space dimensions \( d_2, d_3, d_4 \) are fixed. We get from the definition (2.18)

\[
A = A_{(k)} = \frac{1}{K_1(D - 2)}(n_1 d(I_1) + k^{-1} n_2 d(I_2)),
\]

or, more explicitly, (see (2.8))

\[
A_{(1)} = \frac{1}{K_1(D - 2)}(2d(I_1) + 2d(I_2)),
\]

\[
A_{(2)} = \frac{1}{K_1(D - 2)}(3d(I_1) + 2d(I_2)),
\]

\[
A_{(3)} = \frac{1}{K_1(D - 2)}(6d(I_1) + \frac{10}{3} d(I_2)),
\]

8
for Lie algebras $A_2$, $C_2$ and $G_2$, respectively. Here $K_1 < K_0 \leq 0$.

Hence,
\[ |A_{(1)}| < |A_{(2)}| < |A_{(3)}|. \] (4.8)

Due to relation (3.17) for dimensionless parameter of relative variation of $G$ calculating in the leading approximation when $(\tau - \tau_0)$ is small, we get for approximate values of $\delta$: $\delta_{(1)}^{ap} > \delta_{(2)}^{ap} > \delta_{(3)}^{ap}$ that means that the variation of $G$ (calculated near $\tau_0$) decreases in a sequence of Lie algebras $A_2$, $C_2$ and $G_2$, but the allowed interval $\Delta \tau = \tau - \tau_0$ (obeying $|\delta| < 0.1$) increases in a sequence of Lie algebras $A_2$, $C_2$ and $G_2$. This effect could be strengthen (even drastically) when $|K_1|$ becomes larger. We note that for $|K_1| \to +\infty$ we get a strong coupling limit $\bar{\lambda}_a^2 \to +\infty$, $a = 1, 2$.

## 5 Conclusions

We have considered $S$-brane solutions with two non-composite intersecting electric branes and a set of $l$ scalar fields. The solutions contain five factor spaces, and the first one, $M_0$, is interpreted as our 3-dimensional space. The intersection rules for branes correspond to the Lie algebras $A_2$, $C_2$ and $G_2$.

Here, as in the model with two nonzero curvatures [10], we have found that there exists a time interval where accelerated expansion of our 3-dimensional space is compatible with a small enough value of $G$ obeying the experimental bounds. This interval contains a point of minimum of the function $G(\tau)$ denoted as $\tau_0$.

We have analyzed special solutions with two phantom scalar fields. We have shown that in the vicinity of the point $\tau_0$ the time variation of $G(\tau)$ (calculated in the linear approximation) decreases in the sequence of Lie algebras $A_2$, $C_2$ and $G_2$. Thus, this treatment justifies the consideration of different non-simply laced Lie algebras (such as $C_2$ and $G_2$) that usually do not appear for stringy-inspired solutions [22].

## Appendix

Here we give a derivation of the relation (3.17) for an approximate value of the dimensionless parameter of relative variation of $G$. 

9
We start with the relation (3.13) written in the following form

\[ \delta = \frac{\dot{G}}{GH} = \frac{t^2 - t_0^2}{|AP|t_0^3}, \]  

(A.1)

where \( H = \dot{a}_0/a_0 \) is the Hubble parameter and \( t_0 \) is defined in (3.15). (We recall that here and in what follows \( \dot{f} = df/d\tau \).) In the vicinity of the point \( t_0 \) we get in linear approximation

\[ \delta \approx \frac{\Delta \tau}{|AP|t_0^3}. \]  

(A.2)

Using the synchronous time variable \( \tau = \tau(t) \) we get

\[ \delta \approx \left( \frac{dt}{d\tau} \right)_0 \frac{\Delta \tau}{|AP|t_0^3} = \frac{\Delta \tau}{H_0^A|AP|t_0^3} \]  

(A.3)

\( (dt/d\tau = \dot{H}A) \). Here the subscript ”0” refers to \( t_0 \). For the Hubble parameter we get from (3.4) and (3.9)

\[ H_0 = \left( \frac{\dot{a}_0}{a_0} \right)_0 = \frac{2|A||P|t_0}{\dot{H}_0A^2+1}. \]  

(A.4)

Then, it follows from (A.3) and (A.4) that

\[ \delta \approx \frac{\dot{H}_0}{2A^2P^2t_0^4} H_0 \Delta \tau. \]  

(A.5)

Since (see (2.5) and (3.15))

\[ \dot{H}_0 = 1 - \frac{1}{1 + 4|A|} = \frac{4|A|}{1 + 4|A|} \]  

(A.6)

the pre-factor in (A.5) reads (see (3.15)):

\[ \Pi = \frac{\dot{H}_0}{2A^2P^2t_0^4} = 8 + 2|A|^{-1} \]  

(A.7)

and we are led to the relation

\[ \delta \approx \Pi H_0(\tau - \tau_0), \]  

(A.8)

coinciding with (3.17).

**Acknowledgments**

This work was supported in part by the Russian Foundation for Basic Research grant Nr. 07 – 02 – 13624 – of these.
References

[1] V.N. Melnikov, Multidimensional Classical and Quantum Cosmology and Gravitation. Exact Solutions and Variations of Constants. CBPF-NF-051/93, Rio de Janeiro, 1993;
V.N. Melnikov, in: “Cosmology and Gravitation”, ed. M. Novello, Editions Frontieres, Singapore, 1994, p. 147.

[2] V.N. Melnikov, Multidimensional Cosmology and Gravitation, CBPF-MO-002/95, Rio de Janeiro, 1995, 210 p.;
V.N. Melnikov. In: “Cosmology and Gravitation. II”, ed. M. Novello, Editions Frontieres, Singapore, 1996, p. 465.

[3] V.N. Melnikov, Exact Solutions in Multidimensional Gravity and Cosmology III. CBPF-MO-03/02, Rio de Janeiro, 2002, 297 pp.

[4] V.D. Ivashchuk and V.N. Melnikov, Nuovo Cim. B 102, 131 (1988).

[5] K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, Nuovo Cim. B 102, 209 (1988).

[6] V.N. Melnikov, Gravity as a Key Problem of the Millennium. Proc. 2000 NASA/JPL Conference on Fundamental Physics in Microgravity, NASA Document D-21522, 2001, p. 4.1-4.17, Solvang, CA, USA.

[7] V.N. Melnikov, Gravity and cosmology as key problems of the millennium. In: “Albert Einstein Century Int. Conf.”, eds. J.-M. Alimi and A. Fuzfa, AIP Conf. Proc. Melville-NY, 2006, v. 861, pp. 109-126.

[8] V.N. Melnikov, Variations of constants as a test of gravity, cosmology and unified models, Grav. Cosmol. 13, No. 2 (50), 81-100 (2007).

[9] V.D. Ivashchuk and V.N. Melnikov, Problems of $G$ and multidimensional models. In: Proc. JGRG11, Eds. J. Koga et al., Waseda Univ., Tokyo, 2002, pp. 405-409.

[10] H. Dehnen, V.D. Ivashchuk, S.A. Kononogov and V.N. Melnikov, On time variation of $G$ in multidimensional models with two curvatures, Grav. Cosmol. 11, 340 (2005).

[11] V. Baukh and A. Zhuk, Sp-brane accelerating cosmologies, Phys. Rev D 73, 104016 (2006).

[12] A.I. Zhuk, Conventional cosmology from multidimensional models, hep-th/0609126.
[13] J.-M. Alimi, V.D. Ivashchuk, S.A. Kononogov and V.N. Melnikov, Multidimensional cosmology with anisotropic fluid: acceleration and variation of $G$, *Grav. Cosmol.* **12**, 173-178 (2006); gr-qc/0611015.

[14] V.D. Ivashchuk, S.A. Kononogov, V.N. Melnikov and M. Novello, Non-singular solutions in multidimensional cosmology with perfect fluid: acceleration and variation of $G$, *Grav. Cosmol.* **12**, 273-278 (2006); hep-th/0610167.

[15] S. B. Fadeev and V. D. Ivashchuk, 5-dimensional solution with acceleration and small variation of $G$, In: Proc. of the Russian summer school-seminar Modern theoretical problems of gravitation and cosmology, September 9-16, 2007, TSHPU, Kazan-Yalcik, Russia, 4pp.; arXiv:0706.3988.

[16] J.-M. Alimi, V.D. Ivashchuk and V.N. Melnikov, An S-brane solution with acceleration and small enough variation of $G$, *Grav. Cosmol.* **13**, No. 2 (50), 137-141 (2007); arXiv:0711.3770.

[17] V.R. Gavrilov, V.D. Ivashchuk and V.N. Melnikov, Multidimensional Integrable Vacuum Cosmology with Two Curvatures, *Class. Quantum Grav.*, **13**, No 11, 3039-3056 (1996).

[18] A.G. Riess *et al*, *AJ* **116**, 1009 (1998).

[19] S. Perlmutter *et al*, *ApJ* **517**, 565 (1999).

[20] R. Hellings, *Phys. Rev. Lett.* **51**, 1609 (1983).

[21] J.O. Dickey *et al*, *Science* **265**, 482 (1994).

[22] V.D. Ivashchuk and V.N. Melnikov, Multidimensional classical and quantum cosmology with intersecting p-branes, *J. Math. Phys.* **39**, 2866-2889 (1998); hep-th/9708157.

[23] V.D. Ivashchuk and V.N. Melnikov, Exact solutions in multidimensional gravity with antisymmetric forms, topical review, *Class. Quantum Grav.* **18**, R82-R157 (2001); hep-th/0110274.

[24] V.D. Ivashchuk and V.N. Melnikov, Intersecting p-brane solutions in multidimensional gravity and M-theory, *Grav. Cosmol.* **2**, 297-305 (1996); hep-th/9612089.

[25] V.D. Ivashchuk and V.N. Melnikov, Sigma-model for the generalized composite p-branes, *Class. Quantum Grav.* **14** 3001-3029 (1997); Erratum: ibid., **15**, 3941-4942 (1998); hep-th/9705036.
[26] I.S. Goncharenko, V. D. Ivashchuk and V.N. Melnikov, Fluxbrane and S-brane solutions with polynomials related to rank-2 Lie algebras, *Grav. Cosmol.*** 13*, No. 4 (52), 262-266 (2007); [math-ph/0612079](http://arxiv.org/abs/math-ph/0612079).

[27] V.D. Ivashchuk, Composite fluxbranes with general intersections, *Class. Quantum Grav.* **19**, 3033-3048 (2002); [hep-th/0202022](http://arxiv.org/abs/hep-th/0202022).

[28] V.D. Ivashchuk, Composite S-brane solutions related to Toda-type systems, *Class. Quantum Grav.* **20**, 261-276 (2003); [hep-th/0208101](http://arxiv.org/abs/hep-th/0208101).