Evolution of entanglement in quantum neural network

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Abstract. This study concerns with the evolution of entanglement in a quantum neural network (QNN) model that is locally in contact with data environments. As a valuable resource, duration of entanglement in quantum systems is extremely important. Therefore, the effect of various initial states on the occurrence or decay of entanglement are examined in the presence of information reservoirs. In this study, central spin model was investigated as a quantum version of neural networks inspired by biological models. The architecture of the model is based on a central spin system with two nodes where the nodes are coupled to independent spin baths. Numerical results show that initial state preparation has a profound effect on the fate of entanglement. The results show that the entanglement lifetime can be adjusted by engineering the reservoir states as well as the initial states of the system of interest. The results can be used to improve the performance of the formation or distribution of entanglement in realistic communication network states.

1. Introduction

In popular discussions on artificial intelligence, especially artificial neural networks (ANN) and machine learning have been the focus of interest. Machine learning, which is a computer science area followed by the beginning of the ANN, provides the ability to learn to computers without being programmed explicitly [1-3]. Artificial neural networks are interconnected computing structures composed of a set of layers that process and yield output for corresponding input information. The basic unit of a neural network is a perceptron [3] with a single output node connected to several inputs. The mathematical structure of a perceptron was developed by Rosenblatt inspired by the work of McCulloch and Pitts presents a model [2] mimics the functionality of a biological neuron. The Hebb's learning rule was also developed regarding the biological context in which specifies that how much weight of a connection between connected units should be adjusted in proportion to their activations [4]. Classical learning rules govern the dynamics in a statistical information environment defined in terms of probability density functions [5]. The formulations and constraints of learning laws therefore depend on the relationships between the global and local information environments of each transaction item.

Networks formed of quantum systems are known as quantum networks that are important for quantum communication and the distribution of quantum resources [6]. Therefore, the idea of the use of quantum systems as neural networks [7, 8] has triggered various studies [9-14], including attempts to implement QNN suggestions in the context of quantum computations [14-18]. In particular, quantum resources make the scheme more attractive over classical communication tasks. In this context, entanglement is one of the most interesting features of quantum mechanics [19-21].
Therefore, understanding their characteristics and dynamics is very important for a number of applications in quantum mechanics. In addition, quantum cryptography, quantum information processing, secure communication, the transmission of unknown quantum data and quantum measurement are the examples in which the entanglement has the central role [20-24]. Because of the fragile nature of the entanglement, it is a challenge to maintain and distribute entanglement for a sufficient amount of time.

When the system units carrying entanglement content get in contact to environmental degrees of freedom, entanglement decays to zero in finite time. This situation is generally known as entanglement sudden death (ESD) [20, 25-27] and has also been observed experimentally [28]. As we mentioned earlier [21], in the case of Markovian reservoirs, the dynamics of entanglement in the common and independent reservoirs varies dramatically depending on the system initial conditions. In the presence of a common reservoir, the entanglement can be lost for a finite time, and then it may revive again. This is because the common reservoirs tend to create entanglement rather than completely destroy it, since they indirectly combine the qubits. Even if qubits are initially prepared in a factorized situation, the correlation created by the environment may lead to a phenomenon known as entanglement sudden birth (ESB) [20, 29].

This study focuses on entanglement dynamics based on a small-scale QNN architecture [30]. The dynamic behaviour of quantum entanglement under certain assumptions in the QNN architecture should be reassessed for the sake of exploiting the quantum resources, although the dynamics of the entanglement is well studied for a few qubits scale. In addition, due to the rapid progress of technology, the production of entanglement on demand was achieved faster than the entanglement decay [31]. More specifically, we investigate the dynamics of a small QNN unit that contacts a quantum information reservoir. The concept of the information reservoir was originally introduced in a classical style and was studied for quantum systems [21]. We simulate the proposed quantum system up to three input nodes with different states and examine the open system dynamics. The present study is related to the numerical simulation of a dynamically evolution of an open QNN that is initially entangled and initially separable. The basic assumption of the manuscript is that the QNNs are open quantum systems and they exist in the information environments. Therefore, the aim of the study is to examine the dynamics of entanglement based on quantum data of QNNs. Open system dynamics are modelled through repeated interactions [32, 33] between the local nodes of the network and the units representing the reservoir. This type of repeated interactions has been shown to be dynamic maps equivalent to the Markovian master equation approach due to the divisibility of quantum channels [34]. We have found that the dynamics of entanglement is strongly related to the initial state of the quantum network. Both asymptotic decay and the decay of entanglement in finite time were observed due to the diversity in environmental states. The following sections are devoted to describe the model and the results of the dynamics.

2. Model and system dynamics

Studies aimed at the application of advanced cognitive tasks revealed neural networks [5]. The simplest neural network unit is a perceptron which is a data classifier with \( x_1, x_2, \ldots, x_N \) input nodes connected to an output node with corresponding adjustable weights \( w_1, w_2, \ldots, w_N \). The output node experience the weighted linear summation of the input information as \( y = \sum_i x_i w_i \) modulated by an activation function \( f(y) \) which returns an output depending on the value of \( y \).

The QNN model we investigated is equivalent to the central spin model (also known as the spin-star network) with various applications such as of quantum coherence [35] or quantum communication [36]. Previously, we investigated the dynamic evolution of the central spin quantum coherence based on different spin coupling types under Markov Dynamics [37]. And also, we demonstrated that the non-equilibrium nature of the model may have advantages on a quantum scale for the harvesting of thermodynamic work [38]. In this study, central spin model was investigated as a quantum version of neural networks inspired by biological models (Figure 1 (a)). Figure 1 (b) summaries the basic ideas of the proposed model dynamics provided that the system is an interacting spin system coupled to two
independent spin baths locally. Therefore, the system of interest is an open quantum system and consists of three interacting qubits with a flip-flop type Hamiltonian. The time independent Hamiltonian representing the system dynamics as,

$$H = \frac{\omega}{2} \left( \sum_{i} \sigma_{i}^{z} + \sum_{i=1}^{N-1} \sigma_{u_{i}}^{z} \right) + \sum_{i=1}^{N-1} J_{i} \sigma_{i}^{+} \sigma_{-i}^{-} + \sum_{i=1}^{N-1} J_{i} \sigma_{u_{i}}^{+} \sigma_{u_{i}}^{-} + H.c.$$

(1)

which is plausible for quantum effects also in biological systems [39]. Here, $\sigma_{i}^{z}, \sigma_{-i}^{-}$ and $\sigma_{u_{i}}^{+}$ are the Pauli-z, lowering and raising operators for the qubits, respectively. Also, $\sigma_{out}$ is the respective Pauli operator for output qubit and $J_{i}$ is the coupling coefficient between input units and the output node, $\sigma_{i}$ is the Pauli operator representing an individual node in contact with the information reservoir and $\sigma_{u_{i}}$ is the Pauli operator of the individual unit representing the information reservoir. The Bohr frequency $\omega$ of each spin and single qubit information unit has been taken equal for simplicity. We employ density matrix formalism in our study to represent the quantum states of the system in question. The quantum neurons are initially assumed to be in a product state as $\rho(0) = \rho_{Res} \otimes \rho_{Sys}(0)$, where $\rho_{Res} = \rho_{Res1} \otimes \rho_{Res2}$ and $\rho_{Sys}(0) = \rho_{1}(0) \otimes \rho_{2}(0) \otimes \rho_{out}(0)$ are Reservoir units and System units, respectively. Individual qubit states were chosen as $\rho(0) = |\uparrow\rangle\langle\uparrow|$ in order to initially provide a blank memory. Here, $\rho_{up} = |\uparrow\rangle\langle\uparrow|$, $\rho_{dn} = |\downarrow\rangle\langle\downarrow|$, $|\mp\rangle = \frac{1}{\sqrt{2}}((|\uparrow\rangle \mp |\downarrow\rangle))$ are the cat states where $|\uparrow\rangle$, $|\downarrow\rangle$ are orthogonal spin states known as computational basis in quantum computing language. We define maximally entangled Bell states in the generic form as $|\psi^{\mp}\rangle = \frac{1}{\sqrt{2}}((|\uparrow\rangle \otimes |\downarrow\rangle) \mp (|\downarrow\rangle \otimes |\uparrow\rangle))$ or $|\phi^{\mp}\rangle = \frac{1}{\sqrt{2}}((|\uparrow\rangle \otimes |\uparrow\rangle) \mp (|\downarrow\rangle \otimes |\downarrow\rangle))$.

![Figure 1](image1.png)

Figure 1. Comparison of a biological neuron (a) and the open quantum system we investigate (b). Dendrites of a cell receive signals from the connected cells to the cell body. On the other hand, in our model, the open quantum system receives information from the reservoirs in which it is connected. The open quantum system is composed of three qubits with two inputs nodes ($Node_{1}$ and $Node_{2}$) and an output node ($Node_{0}$). For simplicity, each input node is coupled to the corresponding reservoir and to the output node with equal coupling strengths $J_{1}$ and $J_{2}$. All the nodes have an initial quantum state with an entanglement between any two of them and the evaluation of entanglement in time domain is the goal of the current study.

As mentioned above, the open system dynamics are discussed by adopting a repetitive interaction process, also known as a collision model [40]. Figure 1 (b) simulates the open dynamics of the network to illustrate the specific use of the collisional model. In our model, the initially and identically arranged qubit states interact in turn with the nodes of the neural network. The dynamical evolution of the QNN would be obtained by tracing out the reservoir (environment) degrees of freedom as

$$\rho_{s}(t + \tau) = Tr_{u_{i}}[U(\tau)\rho_{s}(t) \otimes \rho_{u_{i}}U(\tau)^{\dagger}]$$

(2)
Here, $U(\tau) = e^{-i H \tau}$ is the unitary operator representing the system plus environment dynamics for time independent Hamiltonians, \( \tau \) is the duration of each unit-node interaction and $\text{Tr}_{u_i}$ stands for partial trace over environmental degrees of freedom. Thus, the open quantum evolution was simulated by the discrete steps of the quantum channels represented by Equation 2. The calculations were performed by exact diagonalization. In this model, $\rho_{\text{Res}} = \rho_{u_i}$ represents the quantum state of each single qubit environment unit and $\rho_\gamma$ the state of the system of interest (QNN). Also, the state vector and the density matrix formalism were used interchangeably to represent the quantum states of the respective systems.

The aim of our study is to examine the fate of entanglement for various initial states. Therefore, we investigate two situations. The first is that there is no initial entangled between the readout node and one of the input nodes in contact with the environment. In the second case, the system starts in one of the maximally entangled Bell states.

After the system is interacted with the information reservoir, the environment is expected to send information to the system, i.e. the system is equilibrated by the environment carrying quantum information content.

The concurrence is a scalar function that is used to measure the entanglement of density matrices that initially define the mixed states of a system [41]. In this study concurrence is accepted as a measure of entanglement and calculated as $C = \max \{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4 \}$ where $\lambda_i$ are the square roots of non-Hermitian matrix $\rho \tilde{\rho}$, $\rho$ is the density matrix to be calculated and $\tilde{\rho}$ is its spin flipped from such as $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^*(\sigma_y \otimes \sigma_y)$ where $\rho^*$ is the complex conjugated from of $\rho$, the descending order [42].

### 3. Results

To understand the field of open quantum systems, quantum control [43,44] is important to protect the system from unwanted environmental noise [45] or engineering system environmental interactions [46]. Especially small quantum systems may be monitored by any quantum observable, given the mild assumptions on initial conditions of the system that relaxes to a steady value in the long term limit [47,48]. In this section, we analyze the QNN as a system that interacts weakly with the information reservoir in the Markov approach. In these interactions, the quantum state of the system is lost irreversibly through reservoir degrees of freedom, and the temporal evolution of the system depends only on the current state of the system.

In our case, a quantum information reservoir interacts weakly with the local subsystems (input nodes) of the QNN unit. Particularly, time-dependent entanglement dynamics of the discussed QNN is investigated in an information environment. Information environments can be represented by $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ that are single qubit pure states.

As mentioned above, the initial quantum state of the environment plus system is represented by $\rho(0) = \rho_{\text{env}} \otimes \rho_{\text{sys}}(0)$ where $\rho_{\text{sys}}(0) = \rho_1(0) \otimes \rho_2(0) \otimes \rho_0(0)$. Here, $\rho_1$, $\rho_2$ are the quantum states of the input nodes ($\text{Node}_1$, $\text{Node}_2$) and $\rho_0$ stands for the output node ($\text{Node}_0$). On the other hand, quantum state of the environmental qubit states are $\rho_{\text{env}} = \rho_{\text{Res}1} \otimes \rho_{\text{Res}2}$ and fixed. In the following, the time evolution of concurrence between two specific qubits of the three-qubit QNN system will be analysed for various initial states.

Figure 2 represents the evolution of concurrence for the absence and the presence of the initial entangled of the quantum neuron depending on the number of collisions (nc). Figure 2 a) depicts the evolution of concurrence for separable initial states. More specifically, the initial system plus environment state is $\rho(0) = \rho_{\text{up}} \otimes \rho_{\text{up}}(0) \otimes \rho_{\text{dn}}(0) \otimes \rho_{\text{dn}}(0) \otimes \rho_{\text{up}}(0)$ for Figure 2 a). Despite there is no initial entangled between the nodes of the QNN, during the early evolution of concurrence a rapid formation of entanglement between the nodes is evident. The observed entanglement rapidly decays as the system evolution continues. More interestingly in the later evolution, a sudden birth of entanglement between $\text{Node}_1$ and $\text{Node}_2$ was observed where the nodes do not directly interact.
Conversely, the evaluation in Figure 2 b) corresponds to the initial \( \rho_{\text{sys}}(0) = \rho_{\Psi} \otimes \rho_2 \) where \( \rho_{\Psi} \) is the initially entangled state between Node_1 and Node_0. Here, \( \rho_2(0) = |+\rangle\langle+| \) and both the reservoir states are given \( \rho_1, \rho_2 = |1\rangle\langle1| \). As obvious in Figure 2 b), initial entangled decays exponentially and oscillatory. Also formation of entanglement between Node_1 and Node_2 was observed from the beginning of the evolution this time. Plots with smaller timescales can be found as the insets of Figure 2 a)-b). In Figure 2 c) the initial states of the system is given as \( \rho_{\text{sys}}(0) = \rho_{\phi^+} \otimes \rho_2(0) \) where \( \rho_2(0) = |\downarrow\rangle\langle\downarrow| \) and \( \rho_{\phi^+} \) is one of the maximally entangled states described in the introduction. The reservoir states in this case are gives as \( \rho_{\text{up}} \) and \( \rho_{\text{dn}} \) respectively. In this case, initial entangled decays fast and end up in finite time [49].

**Figure 2.** Evolution of concurrence for absence and presence of the initial entangled of the quantum neuron depending on the number of collisions (nc). a) The quantum state of the quantum neuron initially separable as \( \text{Node}_1 = \rho_{\text{dn}}, \text{Node}_2 = \rho_{\text{dn}} \) and \( \text{Node}_0 = \rho_{|+\rangle} \) states. The state of the units representing the environment were set as \( \rho_{\text{up}} \) for both of them. b) The quantum state of the quantum neuron was \( \rho_{|+\rangle} \) state for \( \text{Node}_2 \) while \( \text{Node}_1 \) and \( \text{Node}_0 \) were initially entangled as \( \rho_{\Psi} \) state. The state of the units representing the environment were set as \( \rho_{\text{up}} \) for both of them. c) The quantum state of the quantum neuron was \( \rho_{\text{dn}} \) state for \( \text{Node}_2 \) while \( \text{Node}_1 \) and \( \text{Node}_0 \) were initially entangled as \( \rho_{\phi^+} \) state. The state of the units representing the environment were set as \( \text{Res}_1 = \rho_{\text{dn}} \) and \( \text{Res}_2 = \rho_{\text{up}} \). The coupling between the environment unit and the input node is equal to the coupling between the input and the readout node \( J_1 = J_2 = J = 0.1 \). The duration of the each unit interaction \( \tau \) between the units and the input node is \( \tau = 5 \times 10^{-2}/J \).

As in Figure 3, this time we present the disentanglement dynamics of the proposed model for initially maximally entangled \( |\phi^+\rangle \) and \( |\Psi^+\rangle \) states. This time the only examined reservoir states are \( \rho_{\text{up}} \) pure states for both of the reservoir states in contact with the local nodes of the system. With the same parameters used in Figure 2, entanglement lifetime is longer so that it is not negligible for the \( \Psi^+ \) case compared with the \( \phi^+ \) case even though the initial states are the same. A rapid formation of entanglement in the very beginning of the evolution between \( \text{Node}_1 \) and \( \text{Node}_2 \) (C12) in which they are initially disentangled is evident. This formation of entanglement rapidly decays while the decay of
entanglement between Node\(_2\) and Node\(_0\) last longer compared to \(C\)\(_{12}\). Again a revival scheme is observed in the later evolution for \(C\)\(_{12}\) but this is much weaker than observed in Figure 2. The most interesting result of the model for the Figure 3 parameters is that a dynamical entanglement swapping occurs between Nodes\(_{1-0}\) and Nodes\(_{2-0}\). As is clear in Figures 3 b) and d), a swap mechanism between three nodes of the neural network is apparent while the decay of entanglement occurs.

Figure 3. Evolution of concurrence for presence of the initial entanglement of the quantum neuron depending on the number of collisions (nc). a) and b) The quantum state of the quantum neuron was \(\rho_{dn}\) state for Node\(_2\) while Node\(_1\) and Node\(_0\) were initially entangled as \(\rho_{\phi^-}\) states. c) and d) The quantum state of the quantum neuron was \(\rho_{dn}\) state for Node\(_2\) while Node\(_1\) and Node\(_0\) were initially entangled as \(\rho_{\psi^-}\) states. The state of the units representing the environment were set as \(\rho_{up}\) for both of them. The coupling between the environment unit and the input node is equal to the coupling between the input and the readout node \(f_1 = f_2 = J = 0.1\). The duration of the each unit interaction \(\tau\) between the units and the input node is \(\tau = 5 \times 10^{-2}/J\).

In conclusion, we have presented the dynamics of entanglement of a simple QNN, locally in contact with quantum data environments, in the time domain. We showed that initial state preparation plays a significant role in the time evolution of the entangled states as well as the reservoir states. Note that as depicted in Figure 2 c) entanglement lifetime is too short compared to the other cases. The reason for this is that two orthogonal reservoir states (spin-up and spin-down) are in contact with the corresponding nodes with equal coupling rates. In Ref. [40] it's shown that, in the steady state limit, a small quantum system in contact with orthogonal reservoir states results with a mixed quantum state causes entanglement sudden death. Though it's not surprising to observe the entanglement decay in finite time in the presence of the environments with mixed states, i.e., thermal environments, we showed that when the entangled nodes experience the linear combination of pure environmental states this affects equivalent to the mixed environmental states. For this reason Figure 2 c) exhibits the worst performance for the corresponding environmental states. We also analyse the dynamical entanglement swapping between the nodes of QNN and observe the generation of entanglement between the nodes.
in which they are initially not entangled. Figure 3 b)-d) clearly depict the exchange of entangled states during the dynamical evolution. As another result, we observed that a rapid entanglement generation occurs during the dynamical evolution where the nodes are initially separable (see Figure 2 a)-b)). Though this generated entanglement rapidly decays, a revival (or rebirth) of entanglement is evident (Figure 2 a)) only in the case of separable initial states. This finding is important to the cases in which initial entanglement generation is difficult or costly.

There have been proposals aiming efficient distribution of entanglement over quantum networks [50-53]. Optical implementations are particularly promising due to the already existing architecture built for efficient communication, i.e., optical fibers promote flying qubits [53]. Most recently, it's been experimentally shown that the generation of entanglement is possible faster than the decay of entanglement in quantum networks [54]. In that work, it's been pointed out that deterministic distribution of entanglement, generated between distinct NV centres, is possible due to their findings. At this point, our results could contribute deterministic generation of entanglement in terms of preparation initial entangled states as well as coupling or decoupling the system from the reservoirs. Our results could also be used to obtain rebirth of entanglement in case of non-entangled separable initial states.

4. Conclusions
This study, we examined a QNN unit for open quantum system dynamics. More specifically, we considered an interacting spin model as a quantum version of a neural network unit. We examined the QNN unit for open system dynamics and then connect to two reservoirs by a conventional flip-flop Hamiltonian. We examined the open system dynamics for Markov regimes for two input nodes, one output node and two different states of information reservoirs. We adopt a collisional model to simulate open system quantum dynamics. The study also examines the generation of entanglement due to initially separable states and initially entangled states. According to the results obtained from numerical calculations, the dynamic evolution of disentanglement varies dramatically according to the initial situation.

The obtained results show that disentanglement is slower in the presence of pure information reservoirs. We also show that entanglement revival occurs in the separable initial states. Another significant result is that there exist a commutative and periodic swapping of entanglement during the relaxation of the system through reservoir degrees of freedom. The results could be used to improve the schemes to distribute or connect the entanglement resource through quantum versions of artificial networks.

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