Cosmological Moduli Problem from Thermal Effects

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Abstract

We estimate the cosmological abundance of a modulus field that has dilatonic couplings to gauge fields, paying particular attention to thermal corrections on the modulus potential. We find that a certain amount of the modulus coherent oscillations is necessarily induced by a linear thermal effect. We argue that such an estimate provides the smallest possible modulus abundance for a given thermal history of the Universe. As an example we apply our results to a saxion, a bosonic supersymmetric partner of an axion, and derive a tight bound on the reheating temperature. We emphasize that the problem cannot be avoided by fine-tuning the initial deviation of the modulus field, since the minimal amount of the modulus is induced by the dynamics of the scalar potential.
I. INTRODUCTION

Many scalar fields are expected to be present in Nature, and some of them may play important roles in cosmology. In supergravity and string theories, there are modulus fields, some of which remain light and acquire masses from supersymmetry (SUSY) breaking and non-perturbative effects. They have interactions with the standard-model (SM) particles typically suppressed by a high energy scale such as the grand unification theory (GUT) scale or the Planck scale, and it is known that those modulus fields induce a notorious cosmological moduli problem [1]. Recently the problem turned out to be much more acute than previously thought, since the modulus decay generically produces too many gravitinos which will significantly affect the standard cosmology [2, 3].

After reheating of the inflation, the universe will be filled with a hot thermal plasma of the SM particles. If the modulus field couples to the SM particles (or any particles in thermal plasma), the potential gets generically modified, which may affect the cosmological evolution of the modulus field. In fact, it was pointed out in Refs. [4, 5, 6] that the modulus potential can be significantly modified especially if the modulus possesses a dilatonic coupling, which induces a correction that is linear in the modulus field. In particular, such a thermal effect may destabilize the modulus after inflation, setting a tight bound on the highest temperature of the universe [6].

In this paper we rigorously estimate the modulus abundance when such a linear thermal correction is present, and find that the modulus abundance is bounded below by a non-vanishing value. Although it was known to some people that the linear thermal correction induces a certain amount of the modulus [5], it has not been studied how serious the resultant cosmological moduli problem would be. Since such an estimate provides an absolute minimum of the modulus abundance for a given thermal history of the Universe, our discussion is conservative and generic. In particular, one cannot avoid the problem by tuning the initial deviation of the modulus field (e.g., based on an anthropic argument). As an example we will apply our result to the saxion [7], a bosonic supersymmetric partner of an axion [9], and derive a tight upper bound on the reheating temperature. For the Peccei-Quinn (PQ) scale $f_a = \mathcal{O}(10^{16})$ GeV and the saxion mass $m_s = \mathcal{O}(10)$ eV, we will see that the reheating temperature should be lower than $10^3$ GeV, which is in conflict with the thermal leptogenesis scenario [8].
II. THERMAL EFFECTS ON THE MODULUS POTENTIAL AND MINIMUM MODULUS ABUNDANCE

As pointed out in Refs. [4, 5, 6], a modulus potential receives corrections from thermal effects. The free energy of the SUSY $SU(N_c)$ QCD with $N_f$ flavors in the fundamental representation, assuming that the temperature is high enough to thermalize all these species, reads

$$\mathcal{F}(T, \phi) = -\frac{\pi^2 T^4}{24} \left[ a - bg_s(\phi)^2 + \mathcal{O}(g_s(\phi)^3) \right],$$

where $a = 2N_cN_f + N_c^2 - 1$ and $b = (3/8\pi^2)(N_c^2 - 1)(N_c + 3N_f)$, and $g_s$ denotes the QCD gauge coupling constant, which is in general depends on the modulus field value ($\phi$). This yields finite-temperature effective potential for the modulus. In particular, we focus on the the following linear term,

$$V_T(\phi) = -\kappa \frac{T^4}{M_P}\phi,$$

where the coefficient $\kappa$ depends on the model. Since the modulus potential is time-dependent, some amount of the modulus condensate will be necessarily produced, which is the main concern of this letter. Possible effects of the thermal mass term will be discussed later.

As a toy model, we consider a modulus field whose potential is given by

$$V(\phi) = \frac{1}{2}m^2 \phi^2 - \kappa \frac{T^4}{M_P}\phi.$$  \hspace{1cm} (3)

In the absence of the temperature-dependent linear term, one can tune the initial position of the modulus field as $\phi \sim 0$ to suppress the modulus abundance, possibly based on anthropic arguments. However, the linear term shifts the position of the potential minimum in a time-dependent way, and this dynamically induces a coherent motion of the modulus field. The typical amplitude of the motion induced by this effect is of the order $\delta\phi \sim \kappa T^4/(m^2 M_P)$, and hence the minimum modulus abundance is estimated as

$$\rho_{\phi} \sim \begin{cases} \frac{45\kappa^2}{4\pi^2 g_s} \frac{T_{os}^5}{m^2 M_P^2} & \text{for } T_{os} < T_R, \\ \frac{45\kappa^2}{4\pi^2 g_s} \frac{T_{os}^5}{m^2 M_P^2} & \text{for } T_{os} > T_R, \end{cases}$$  \hspace{1cm} (4)

\#1 Although there may be a Hubble mass term $\sim c^2 H^2 (\phi - \phi_0)^2$ with some arbitrary value of $\phi_0$, inclusion of this term does not modify the following argument unless the coefficient $c$ is much larger than one.
where $T_{os}$ is the temperature at which the modulus begins to oscillate, and $T_R$ is the reheating temperature after inflation. This can be evaluated as

$$\frac{\rho_\phi}{s} \sim \begin{cases} 
1.4 \times 10^6 \text{ GeV} \kappa^2 \left( \frac{g_*}{228.75} \right)^{-9/4} \left( \frac{m_\phi}{100 \text{ GeV}} \right)^{1/2} 
& \text{for } T_{os} < T_R \\
8.4 \times 10^{-4} \text{ GeV} \kappa^2 \left( \frac{g_*}{228.75} \right)^{-1} \left( \frac{m_\phi}{100 \text{ GeV}} \right)^{-2} \left( \frac{T_R}{10^9 \text{ GeV}} \right)^5 
& \text{for } T_{os} > T_R
\end{cases}$$

(5)

Here we have assumed that the temperature of the dilute plasma before the reheating completes is given by $T^4 \sim T_R^2 H M_P$, where $H$ is the Hubble parameter [11]. Note that in order for the above analysis to be valid, $\dot{\phi}$ must be much smaller than $\kappa^{-1} M_P$, and this sets an upper bound on the temperature as $T < T_c \sim (m M_P / \kappa)^{1/2}$. For $\kappa \lesssim 1$, the critical temperature $T_c$ is always higher than $T_{os}$, and so, the above estimate on the modulus abundance is valid. It is model-dependent what the dynamics of the modulus would be at an temperature above $T_c$. For instance, the modulus potential may be destabilized and the modulus field may start rolling toward infinity [6]. Since our concern here is the modulus dynamics at a temperature below $T_{os}$, the minimal modulus abundance [3] is a conservative one. We have numerically checked that Eq. (4) provides the minimum modulus abundance, which cannot be reduced further by tuning the initial value of the modulus field.#2 As we will see, even this minimum abundance causes a cosmological disaster in general.

III. EXAMPLES

We have seen that there exists a strict “minimum” abundance of moduli, dynamically induced by the thermal effects. In order to see that such an effect makes cosmological moduli problems worse than previously thought, let us consider the SUSY axion model [7]. We denote an axion supermultiplet by $A$ and the PQ scale by $f_a$. Here we assume $f_a \gtrsim 10^{16}$ GeV, motivated by string axion models [12]. The interaction of the axion multiplet $A$ to the $SU(3)_C$ gluon gauge multiplet is given by

$$\mathcal{L} = \int d^2 \theta \frac{A}{32 \pi^2 f_a} W^\alpha W_\alpha + \text{h.c.}$$

(6)

#2 If we allow ourselves to choose an arbitrary initial velocity $\dot{\phi}$ as well, the modulus abundance can be significantly reduced. However, the $\dot{\phi}$ is normally dependent on the potential and the initial position, and therefore the modulus dynamics would not allow us to do so. We thank K. Hamaguchi for useful comments on this issue.
The imaginary lowest component of $\mathcal{A}$ is the axion $a$ which is to solve the strong CP problem, whereas the real component is the saxion $\sigma$:

$$\mathcal{A} \equiv \frac{\sigma + ia}{\sqrt{2}}.$$  \hspace{2cm} (7)

Let us consider the thermal effects on the saxion potential. The QCD gauge coupling constant $g_s$ in Eq. (1) depends on $\sigma$ as

$$\frac{1}{g_s(\sigma)^2} = \frac{1}{g_s(0)^2} + \frac{\sigma}{8\pi^2 f_a}. \hspace{2cm} (8)$$

Thus this leads to the finite-temperature effective potential of the form (2) for the saxion. Note that the second term must be regarded as a small correction in order for the analysis to be valid. Substituting $N_c = 3$ and $N_f = 6$ in the SUSY SM, $\kappa$ in Eq. (2) is given by:

$$\kappa = \frac{21}{64\pi^2} g_s^4 \left( \frac{M_P}{f_a} \right) \sim 0.02 \left( \frac{M_P}{f_a} \right). \hspace{2cm} (9)$$

Generally, the saxion has a mass of order of the gravitino and it is known that the coherent oscillations of the saxion cause cosmological problems for a wide range of the saxion mass $[13, 14, 15]$. Actually there exists a minimum abundance of the saxion induced by thermal effect given by Eq. (4), and even such a minimum abundance of the saxion has significant impacts on cosmology. In Fig. we show the minimum saxion abundance for $m_\sigma = 10$ eV, 1 MeV, 100 GeV as a function of the reheating temperature $T_R$. For $m_\sigma = 10$ eV, the constraint on the saxion abundance comes from the requirement that the saxion must not exceed the dark matter abundance, which restricts the saxion abundance as $\rho_\sigma/s \lesssim 4 \times 10^{-10}$ GeV. This translates into the bound on the reheating temperature, $T_R \lesssim 10^3$ GeV. Thus thermal leptogenesis scenario $[8]$ is incompatible with the cosmological saxion problem for the ultra-light gravitino $m_{3/2} \sim 10$ eV. For the intermediate mass scale $m_\sigma \sim \mathcal{O}$(keV)-$\mathcal{O}$(TeV), the diffuse X($\gamma$)-ray background or big-bang nucleosynthesis (BBN) sets strong bounds on the saxion abundance, and the reheating temperature cannot be as large as $T_R \sim 10^9$ GeV. On the other hand, if the saxion mass is heavy enough to decay before BBN, the constraint can be evaded. Thus we conclude that the thermal leptogenesis scenario is

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The value of $\kappa$ is of order unity for $f_a \gtrsim 10^{16}$ GeV, and so, $T_{\text{os}}$ can be comparable to $T_c$ if the reheating is completed before the commencement of the oscillations, i.e., $T_R > T_c$. On the other hand, we are more interested in a case of $T_R < T_{\text{os}}$, where $T_{\text{os}} \ll T_c$ is satisfied and therefore our analysis is valid.
FIG. 1: The minimum saxion abundances for $m_\sigma = 10$ eV, 1 MeV, and 100 GeV as a function of the reheating temperature. Here we take $\kappa = 0.1$, corresponding to $f_a \sim 5 \times 10^{17}$ GeV.

excluded in the SUSY axion model for $f_a \gtrsim 10^{16}$ GeV except for the heavy gravitino (saxion) case, as is realized in the anomaly-mediated SUSY breaking models [16].

On the other hand, if the PQ scale $f_a$ is in the ordinary axion window $10^{10}$ GeV $\lesssim f_a \lesssim 10^{12}$ GeV, and if the reheating temperature is high enough so that the thermal leptogenesis works, the saxion may be in thermal equilibrium. In this case the cosmological saxion abundance will be in conflict with either BBN or dark matter abundance except for $m \lesssim \mathcal{O}(10)$ eV or $m \gtrsim \mathcal{O}(10)$ TeV.

IV. DISCUSSION AND CONCLUSIONS

We have discussed that thermal effects generically produce a linear term in the modulus potential, which dynamically induces a coherent motion of the modulus field. This provides

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#4 The abundance of the axion ($a$) can be negligibly small by tuning the initial position of the axion. However, the axion always has isocurvature fluctuation [17] as well as possibly large non-Gaussianity [18, 19, 20] and an upper bound on the inflation scale is imposed in order to be consistent with cosmological observations.
the smallest possible modulus abundance among all the possible initial positions of the modulus field. Thus the cosmological constraints given in this letter is conservative. The same analysis applies to any moduli which have dilatonic coupling with gauge fields.

We comment on the effect of a thermal mass term such as \( \sim \lambda (T^4/f_a^2)\phi^2 \), where \( \lambda \ll 1 \) represents the one-loop factor as well as the gauge couplings. We have neglected the term assuming that the thermal mass is smaller than the Hubble parameter and therefore it does not affect the modulus dynamics for large \( f_a(\gtrsim 10^{16} \text{ GeV}) \). If the thermal mass term is effective, the modulus may be settled at the potential minimum due to the thermal mass.\(^5\) In this case, the modulus may adiabatically follow the temporal minimum during the subsequent cosmological evolution and the coherent oscillations of the modulus might not be induced as noted in Ref. \([10]\) in the context of a large Hubble mass term, although thermal production of saxion as well as axino \([21]\) may occur at an non-negligible rate.

Finally we comment on possible ways to relax the cosmological moduli problem. As already noted, if the moduli are heavy enough, the minimum abundance provided by Eq. (4) significantly decreases and also they decay well before BBN. For example, in the dynamical SUSY breaking models, the SUSY breaking field has mass of the order of the dynamical scale \( \Lambda \), which is much larger than the gravitino mass. Thus the upper bound on the reheating temperature is not so stringent in such a case. Also if there were additional entropy production processes in the early Universe \([22, 23]\), the moduli can be sufficiently diluted. However, one should note that the pre-existing baryon asymmetry is also diluted and only a few examples of baryogenesis mechanism are known to work \([24, 25, 26]\).

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\(^5\) The modulus dynamics is model-dependent when the thermal mass plays an important role.
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