On the Movement of Briquetted Mass in Extruder. Exact Solutions. 1

A. M. Bizhanov°, * and G. S. Podgorodetskii°, **
°J.C. Steele & Sons, Inc., Statesville, NC 28677 USA
°National University of Science and Technology MISIS (MISIS), Moscow, 119049 Russia
* e-mail: abizhanov@jcsteele.com
** e-mail: podgs@misis.ru

Received March 18, 2019; revised January 16, 2020; accepted January 17, 2020

Abstract—The increased interest in cold agglomeration in recent years has been largely due to the successful experience in operating briquette lines based on stiff vacuum extrusion (SVE). The high performance of SVE extruders and the satisfactory metallurgical properties of extrusion briquettes obtained in this way (brex) make it possible to consider this cold agglomeration technology as promising. SVE Extruders allow efficient briquetting of the materials with moisture contents values in the range of 12–16% and compacting pressure of 3.5–4.5 MPa, leading to the possibility of achieving high values of mechanical strength of raw briquettes and eliminates the need for drying briquetted charge and heat treatment of the green briquettes. The growing scale of practical use of extruders in the steel industry necessitated the development of simple and effective methods for determining their operating parameters. The briquetted mass is a moistened plastic mass, driven by the blades of a rotating auger and squeezed out further through the holes in the extruder die in the form of elongated briquettes, repeating in cross section the shape of the hole. In application to the optimization problems of extrusion briquette technology, the exact solution of the Navier–Stokes equations for a viscous incompressible medium shifted between coaxial cylinders along the common axis of symmetry and twisted around it by longitudinal displacement and axial rotation of the inner cylinder is given, respectively, under sticking conditions and given longitudinal pressure drop. In particular, it was found that the speed of transportation of the mixed mass cannot exceed the speed delivered by the supplied pressure, and the formula of the latter is transferred to the compressible medium as a special viscosity integral for a viscoplastic medium, where it serves as a generalization of known exact solutions. A similar solution for a compressible medium is being sought. The obtained analytical dependencies can be used to calculate the parameters of industrial briquette extruders operating in both the stiff extrusion mode and the semi-stiff and soft extrusion mode, differing in the moisture values of the briquetted mass and the values of the applied pressure.

Keywords: stiff vacuum extrusion, briquetting, brex, viscous incompressible medium, Navier–Stokes equations, coaxial cylinders, spiral Couette–Poiseuille flow

DOI: 10.3103/S0967091220010027

Stiff vacuum extrusion (SVE) is widely used for the production of metallurgical briquettes—components of the burden of blast furnaces and ferroalloy furnaces [1–8]. The mechanical and hot strength of extrusion briquettes (brex) is ensured by a smaller amount of binder materials than in alternative briquette technologies (roller briquetting and vibropressing), which, together with higher extruder productivity, makes it possible to consider that SVE fully meet the criteria for the best available technologies in ferrous metallurgy (BAT, [9]).

Unlike roller briquetting and vibropressing, SVE allows to agglomerate moistened materials with a moisture content in the range of 12–16% (maximum up to 20%) at pressures from 2.5 to 4.5 MPa. Varieties of extrusion agglomeration also used in metallurgy are soft extrusion (moisture content 10–27%, pressure 0.4–1.2 MPa) and semi-stiff extrusion (moisture content 15–22%, pressure 1.5–2.2 MPa). The most important criterion for the applicability of SVE for producing briquettes is the plasticity of the moldable mixture, which makes it possible to move such a mixture with the blades of a rotating screw and compact it when it pushed through holes in the extruder dies.

Due to the rotation of the auger blades in the working chamber of the extruder, the molded mass performs translational and rotational motion, which is slowed down by the walls of the housing (Fig. 1). The extruder can be represented in the form of two coaxial cylinders and a screw rotated by the inner cylinder.
around the axis \( z \) with speed \( v_\perp \) and transporting the moldable mass along the specified axis with speed \( w_\perp \) with the stationary outer cylinder \( (w_\perp = v_\perp = 0) \).

In zone 1, the mixture is supplied to the working blades of the auger and is moved without compaction. In zone 2, the mixture is compacted. As the mixture moves toward the die, its rotation slows down, while the peripheral layers move at a faster speed. In zone 3, the density inhomogeneities arising in zone 2 are equalized due to the uneven movement of the moldable mass. Alignment is achieved by the special geometry of the blades of the point auger and their relative position. In zone 4, further alignment of the motion inhomogeneities occurs.

Mathematical modeling of the process of movement of the moldable mass in the extruder in full formulation remains a difficult task, requiring consideration of the rheological properties of the briquetted mass. Most of the known works use a simplified approach combining mathematical and physical modeling of the motion of the moldable mass. A review of modeling methods for the motion of moldable masses in an extruder is given in [10].

Consider the movement of the briquetted mass in the working chamber of the extruder from the standpoint of the basic laws of continuum mechanics and with the intention of obtaining simplified qualitative relationships that can serve as a basis for approximate calculations of the main parameters of the extrusion agglomeration process.

We assume that the mass of a wet and continuous medium rotated by the auger has an isotropic molecular pressure field \( p \) and obeys the general laws of conservation of mass, momentum and energy with proper rheology for the dynamic viscosity coefficient of the medium \( \mu [11–16] \).

Particle motion is considered in cylindrical coordinates. As in [13, 14], the formalism of direct products of unit vectors and representations of matrices by double vectors (divecctors) is used.

We consider the briquetted mass as a continuous medium of particles \( r \) with velocities \( u \), with a density \( \rho > 0 \) described by the continuity equation

\[
\rho_t = \nabla \rho u = 0
\]

and pressure \( p \) (uniform Pascal stress \( p \hat{e} \)). Adding to the rate of change of the bulk density of the pulse \( \rho u \) (\( \rho u_t \)) its acceleration in the medium

\[
\rho u \nabla u = \nabla \rho u u - u \nabla \rho u = \rho_t + \nabla \rho uu,
\]

and to the density \( \rho g \) of a given field of a specific external force \( g = g(t, r) \)—the volume density of the Archimedes force \( -\nabla p = -\nabla p \hat{e} \)

pushing the volume \( V \) on the area \( dS = \sqrt{dS \cdot dS} \) of its boundary \( \partial V \) in the direction of the unit external normal \( n = \frac{dS}{dS} \), we obtain the Euler hydrodynamic equations:

\[
\rho u_t + \rho u \nabla u = \rho g - \nabla p, \quad \text{or} \quad (\rho u_t)_t + \nabla \tilde{P} = \rho g
\]

at \( \tilde{P} = \rho uu + p \hat{e} \).

where arising as a force

\[
\int dS \tilde{P} = \int (\nabla \tilde{P}) dV,
\]

tension

\[
\tilde{P} = \rho uu + p \hat{e} - \mu \tilde{b},
\]
ON THE MOVEMENT OF BRIQUETTED MASS IN EXTRUDER

is refined by the coefficients of dynamic and bulk viscosities $\mu = \mu(t, r) > 0$ and $\zeta \mu$, and the matrix

$$b = \xi - \left(\frac{2}{3} - \zeta\right) (\nabla u b), \quad \zeta = \text{const} \geq 0.$$  

The Navier–Stokes equations in the case under consideration are as follows:

$$(\rho u)_t + \nabla \vec{P} = \rho g \text{ for } \vec{P} = \rho uu + P \vec{e} - \mu \xi$$

and $P = p + \mu \left[\frac{2}{3} - \zeta\right] (\nabla u)$ (1)

or

$$(\rho u)_t + \left[\left(\rho u u\right)_r + \left(\rho r u u\right)_r + \frac{\rho V^2}{r} + \rho r \left[u(u_z + w_r)\right]_r - \mu (u_z + w_r)\right] \right) + \nabla P = \rho g;$$

$$P = p + \mu \left[\frac{2}{3} - \zeta\right] \left[\frac{1}{r} (ru)_r + w_z\right],$$

$$\rho + \frac{1}{r} (\rho u)_r + (\rho w)_r = 0.$$  

Consider the stationary motion of a continuous medium with a constant density:

$$\rho = \text{const} > 0 \quad (\nabla u = 0)$$

and $\mu = \text{const} > 0$, in the limited section

$$0 < z < l, \quad l = \text{const} > 0,$$

of the spaces between two infinite coaxial cylinders,

$$\varepsilon a = r_{\min} < r < r_{\max} = a, \quad a = \text{const} > 0 \quad (\varepsilon < 1)$$

at $-\infty < z < \infty$, internal $(r = \varepsilon a)$, with the index “−”), external $(r = a)$, with the index “+”), moved along the common axis $r = 0$ and rotated around it with constant speeds $w_z$ and $v_+$, accordingly, under adhesion conditions

$$w|_{r=\varepsilon a,a} = w_-, w_+, v|_{r=\varepsilon a,a} = v_-, v_+; w_z, v_+ = \text{const}$$  

and constant pressure

$$-p_z = \frac{p_+ - p_-}{l} = \text{const} > 0, \quad 0 < z < l,$$

$$p_+ = p|_{z=0}, \quad p_- = p|_{z=l},$$

with specified pressure values $p_z$ at the ends $z = 0, l$ of the section $0 < z < l$.

In the absence of mass forces and radial displacements of the rotational and longitudinally shifted medium, under the assumption that

$$u = 0, \quad v = v(r), \quad w = w(r),$$

and $g^x = g^z = 0$, its dynamic equilibrium (1)–(4) reduces to the relations:

$$-\frac{1}{r} (rw_r)_r = \frac{p_z}{\mu}, \quad (rv_r)_r = v, \quad p_r = \frac{\rho V^2}{r},$$

$$-p_z = \frac{p_+ - p_-}{l}, \quad \varepsilon a < r < a, \quad 0 < z < l,$$

$$w (\varepsilon a) = w_-, \quad w(a) = w_+ = 0,$$

$$v(\varepsilon a) = v_-, \quad v(a) = v_+ = 0.$$  

Dynamic equilibrium can also be reduced to a spiral flow, i.e., to the combined Hagen–Poiseuille and Couette flow [11, 12, 17],

$$w(r) = \frac{p_+ - p_-}{4\mu} \frac{a^2 - r^2}{r},$$

and pressure

$$p(r, z) = p_+ - \frac{z}{l} (p_+ - p_-) - \int \frac{\rho V^2(r)}{r} dr,$$

$$p_r = \frac{\rho V^2}{r} > 0, \quad \varepsilon a < r < a, \quad 0 < z < l.$$  

Consider the case of a fixed external cylinder:

$$w_+ = v_+ = 0, \quad w_- > 0, \quad v_- < 0.$$  

Figure 2 shows dimensionless profiles of azimuthal and axial velocities

$$\frac{\nu(r)}{v_-} = Y = \frac{\varepsilon}{1 - \varepsilon} \left[\frac{1}{X} - X\right],$$

$$\varepsilon = \frac{r_{\min}}{r_{\max}} < X = \frac{r}{r_{\max}}, \quad 1, \quad r_{\max} = a,$$

$$w = Z = 1 - X^2 + \frac{1 - \varepsilon^2}{\varepsilon} \delta \ln X,$$

$$w_p = -\frac{p_+ a^2}{4\mu}, \quad \delta = \frac{w}{w_p},$$

accordingly, with dimensionless factors $\varepsilon, \delta$ and dimensional velocity of viscous pressure $w_p$. The former lead to a critical parabola $\delta = 1 - \varepsilon^2$, and to the following (i) subcritical, $\delta < 1 - \varepsilon^2$, and (ii) supercritical, $\delta \geq 1 - \varepsilon^2$, spiral flow regimes.
In the subcritical mode $\delta < 1 - \varepsilon^2$, the maximum possible speed of transportation, i.e. $w(r)$, longitudinal displacement, is

$$w_\ast = \max_{r \in [0,a]} w(r) = w_p Z_\ast < w_p,$$

$$Z_\ast = \max_{r \in [0,a]} Z(X_\ast), \varepsilon < \frac{r_\ast}{a} = X_\ast < 1,$$

achieved within the flow region, $\varepsilon a < r_\ast < a$, and close to the velocity of viscous pressure $w_p$.

With a further increase in the factor $\delta$, the value $w_\ast$ reaches its maximum possible value $w_p$ at the inner boundary

$$w_\ast = w(r_\ast) = w_p Z_\ast = w_p, Z_\ast = 1,$$

$$\frac{r_\ast}{a} = X_\ast = \varepsilon, \text{ at } \delta \geq 1 - \varepsilon^2,$$

and remains the maximum possible speed of the transported mass $w_\ast = w_p$ at any supercritical factor $\delta$ (Fig. 2).

It can be shown that for the case of a stationary outer cylinder, there is a critical rotation speed of the inner cylinder,

$$w_c = (1 - \varepsilon^2) w_p \text{ (or } \delta = 1 - \varepsilon^2),$$

such that, with the appropriate subcritical or supercritical mode, the maximum transport speed is reached between the cylinders or on the inner cylinder and is less than or equal to the speed of the viscous pressure

$$w_p = -\frac{p_c a^2}{4\mu},$$

respectively:

$$w(r_\ast) < w_p \text{ and } \varepsilon a < r_\ast < a$$

at $w_\ast < w_c$ (or $\delta < 1 - \varepsilon^2$),

$$w(r_\ast) = w_p \text{ and } r_\ast = \varepsilon a$$

at $w_\ast \geq w_c$ (or $\delta \geq 1 - \varepsilon^2$).

Consider the case of compressibility of the moldable mass:

$$\rho = \rho(r) \neq \text{const} \ (\nabla u \neq 0) \text{ and } \mu = \text{const}.$$

Under the assumption that

$$u = u(r), \ v = v(r), \ w = w(r) \text{ and } g^r = g^z = 0,$$

the continuity equation in (1) leads to a constant

$$r \rho u = m = \text{const}, \text{ or } ru = m \varphi,$$

where $\varphi = \frac{1}{\rho}$—specific volume. Its relation to dynamic viscosity $\left(\alpha = \frac{m}{\mu}\right)$ is a dimensionless quantity. Putting in (1)

$$p_r = \frac{\rho v^2}{r} \text{ and } \frac{\alpha + 1 (ru)_r}{r^2},$$

$$-\left(\frac{1}{3} + \frac{1}{r^3 (ru)}\right) - \frac{1}{r} (ru)_r - \frac{\alpha ru}{r^3} = 0,$$

we find for given values $\varphi_+:$

$$\varphi = \frac{\varepsilon^{\lambda_+} \varphi_+ - \varphi (-\frac{r}{a})^{\lambda_+}}{\varepsilon^{\lambda_+} - \varepsilon^{-\lambda_+}} = \frac{\varepsilon^{\lambda_+} \varphi_+ - \varphi (-\frac{r}{a})^{\lambda_+}}{\varepsilon^{\lambda_+} - \varepsilon^{-\lambda_+}},$$

$$\lambda_+ = \frac{8 + 6 \zeta + 3 \alpha}{2 (4 + 3 \zeta)} \pm \sqrt{\frac{8 + 6 \zeta + 3 \alpha}{2 (4 + 3 \zeta)}^2 - \frac{3 \alpha}{4 + 3 \zeta}}$$

at

$$\left(\frac{2 + 6 \zeta + 3 \alpha}{2 (4 + 3 \zeta)}\right)^2 > \frac{3 \alpha}{4 + 3 \zeta},$$

Satisfying conditions (2), (3) and equations (1),

$$\frac{m v}{r} + \frac{m v}{r^2} \frac{u}{r} (ru)_r + \frac{\mu}{r^2} = 0,$$

$$\frac{m w}{r} + \frac{m w}{r^2} \frac{u}{r} (rw)_r = -p_z = -p_c,$$

$$p = p + \mu \left(\frac{2}{3} \varepsilon - \frac{1}{r} (ru)\right), \varepsilon a < r < a,$$
the axial and azimuthal velocity components in this case have profiles:

\[ w = \frac{\varepsilon^r \nu_r - \nu_w}{\varepsilon^r - \varepsilon^w} \left( \frac{r}{a} \right)^{\nu_r} + \frac{\varepsilon^r \nu_r - \nu_w}{\varepsilon^r - \varepsilon^w} \left( \frac{r}{a} \right)^{\nu_r} \]

\[ + \frac{\rho_s a^2}{\mu (4 - \alpha)} \left[ \frac{\varepsilon^r - \nu_r}{\varepsilon^r - \nu_r} \left( \frac{r}{a} \right)^{\nu_r} \right] \]

\[ + \frac{\varepsilon^{\gamma_r} - \nu_r}{\varepsilon^{\gamma_r} - \nu_r} \left( \frac{r}{a} \right)^{\nu_r} - \left( \frac{r}{a} \right)^{2}, \]

\[ \gamma_r = \frac{\alpha}{2} + \left( \frac{3}{4} - 1 \right) \alpha \]

at \( \frac{\alpha}{4} - 1 \alpha > 0, \)

and

\[ v = \frac{\varepsilon^\beta \nu_r - \nu_w}{\varepsilon^\beta - \varepsilon^w} \left( \frac{r}{a} \right)^{\nu_r} + \frac{\varepsilon^\beta \nu_r - \nu_w}{\varepsilon^\beta - \varepsilon^w} \left( \frac{r}{a} \right)^{\nu_r} \]

\[ \beta_r = \frac{\alpha}{4} + \left( \frac{3}{4} - 1 \right) \alpha + 1. \]

Thus, the axial and angular (azimuthal) speeds of the spiral flow are determined respectively by the speeds of transportation and rotation by the auger of the mixed mass, \( w \) and \( v \), respectively. In contrast to dry bulk material (from small and solid particles) [18], this mass is a pasty mass of a viscous-plastic continuous medium, in which the force field of contact stresses \( \tau \) is beyond the plasticity limit, where the Bingham rheology [14, 15] reduces to constant viscosity, i.e. approaching Newtonian rheology, as in [19–21].

CONCLUSIONS

A stationary spiral flow with constant or variable density satisfactorily describes the movement of the briquetted mass in the extruder. There exists the highest possible speed of transportation (speed of viscous pressure \( w_r \)).

The obtained dependences are also applicable to the calculation of extruders “semi-stiff” and “soft extrusion”.

FUNDING

This work was carried out as part of the state assignment of the Federal State Institution Scientific Center for Research and Development of the Russian Academy of Sciences (General Scientific Research, GP 14) on theme 0065-2019-0005 “Mathematical modeling of dynamic processes in deformable and reacting media using multiprocessor computing systems” (no. AAAA-A19-119011590092-6).

REFERENCES

1. Kurunov, I. and Bizhanov, A., Stiff Extrusion Briquetting in Metallurgy, New York: Springer, 2017.
2. Fernandez, M.O., Iglesias, J., Gonzales, D.F., et al., Cold agglomeration of ultrafine oxidized dust (UOD) from ferromanganese and silicomanganese industrial process, Metals, 2016, vol. 6, no. 9, art. ID 203.
3. Mohanty, M.K., Mishra, S., Mishra, B., Sarkar, S., and Sama, S.K., A novel technique for making cold briquettes for charging in blast furnace, IOP Conf. Ser.: Mater. Sci. Eng., 2016, vol. 115, no. 1, art. ID 012020.
4. Higuchi, K., Yokoyama, H., Sato, H., Chiba, M., and Nomura, S., Development of rapid curing process of reactive coke agglomerate, ISIJ Int., 2017, vol. 57, no. 1, pp. 55–61.
5. Rama Murthy, Y., Kapure, G.U., Tripathy, S.K., and Sahu, G.P., Recycling of ferromanganese gas cleaning plant (GCP) sludge by novel agglomeration, Waste Manage., 2018, vol. 80, pp. 457–465.
6. Mombeli, D., Cecca, C.D., Mapelli, C., Barella, S., and Bondi, E., Experimental analysis on the use of BF-sludge for the reduction of BOF-powders to direct reduced iron (DRI) production, Process Saf. Environ. Prot., 2016, vol. 120, pp. 410–420.
7. Kowitwarangkul, P., Behavior of Self-Reducing Pellets (SRP) for Use in a Low Height Blast Furnace, Herzogenrath: Shaker Verlag, 2014.
8. Xu, Q., Li, Z., Liu, Z., Wang, J., and Wang, H., The effect of pressurized decarbonization of CO on inhibiting the adhesion of fine iron ore particles, Metals, 2018, vol. 8, no. 7, p. 525.
9. Kurunov, I.F., Chizhikova, V.M., and Bizhanov, A.M., The best available technologies in production of agglomerated raw materials for blast furnaces, Chern. Metall., Byull. Nauchno-Tekh. Ekonom., 2018, no. 4, pp. 62–66.
10. Extrusion in Ceramics, Handle, F., Ed., Berlin: Springer, 2007.
11. Batchelor, G.K., An Introduction to Fluid Dynamics, Cambridge: Cambridge Univ. Press, 1967.
12. Loitsiansky, L.G., Mechanics of Liquid and Gases, Wallingford, NY: Begell-Haus, 1995.
13. Abramovich, G.N., Prikladnaya gazovaya dinamika (Applied Gas Dynamics), Moscow: Nauka, 1991, part 1.
14. Bingham, E.C., Fluidity and Plasticity, New York: McGraw-Hill, 1922.
15. Ishlinskii, A.Yu. and Ivlev, D.D., Matematicheskaya teoriya plasticnosti (Mathematical Theory of Plasticity), Moscow: Fizmatlit, 2001.
16. Laenger, K.-F., Laenger, F., and Geiger, K., Wall slip of ceramic extrusion bodies. Part 2, Process Eng., 2016, vol. 93, nos. 4–5, pp. 1–6.
17. Joseph, D.D., Stability of Fluid Motions, Berlin: Springer, 1976.
18. Landau, L.D. and Lifshitz, E.M., A Course of Theoretical Physics, Vol. 7: Theory of Elasticity; Oxford: Pergamon, 1970.
19. Belotserkovskii, O.M., Betelin, V.B., Borisievich, V.D., Denisenko, V.V., Erikhltsiev, I.V., Kozlov, S.A., Konukhov, A.V., Oparin, A.M., and Troshkin, O.V., On the theory of counterflow in a rotating viscous heat-conducting gas, Comput. Math. Math. Phys., 2011, vol. 51, no. 2, pp. 208–221.
20. Handle, F., Laenger, F., and Laenger, J., Determining the Forming pressures in the extrusion of ceramic bodies with the help of the Benbow–Bridgewater equation using the capillary check, Process Eng., 2015, vol. 92, nos. 10–11, pp. 1–7.
21. Troshkin, O.V., Elementy matematicheskoi gidrodinamiki i gidrodinamicheskoi ustoichivosti (Elements of Mathematical Hydrodynamics and Hydrodynamic Stability), Saarbrucken: LAP Lambert Academic, 2016.