On the accuracy of HI observations in molecular clouds – More cold HI than thought?

D. Seifried,1* H. Beuther,2 S. Walch,1 J. Syed,2 J. D. Soler,2 P. Girichidis,3 R. Wünsch4

1 Universität zu Köln, I. Physikalisches Institut, Zülpicher Str. 77, 50937 Köln, Germany
2 Max Planck Institute for Astronomy, Königstuhl 17, 69117 Heidelberg, Germany
3 Universität Heidelberg, Zentrum für Astronomie, Institut für Theoretische Astrophysik, Albert-Ueberle-Str. 2, 69120 Heidelberg, Germany
4 Astronomical Institute of the Czech Academy of Sciences, Bocní II 1401/1, 141 00 Prague 4, Czech Republic

ABSTRACT

We present a study of the cold atomic hydrogen (HI) content of molecular clouds simulated within the SILCC-Zoom project for solar neighbourhood conditions. We produce synthetic observations of HI at 21 cm including HI self-absorption (HISA). We find that HI column densities, $N_{\text{HI}}$, of $\gtrsim 10^{22}$ cm$^{-2}$ are frequently reached in molecular clouds with HI temperatures as low as $\sim 10$ K. Hence, HISA observations assuming a fixed HI temperature tend to underestimate the amount of cold HI in molecular clouds by a factor of 3 – 10 and produce an artificial upper limit of $N_{\text{HI}}$ around $10^{21}$ cm$^{-2}$. We thus argue that the cold HI mass in molecular clouds could be a factor of a few higher than previously estimated. Also $N_{\text{HI}}$-PDFs obtained from HISA observations might be subject to observational biases and should be considered with caution. The underestimation of cold HI in HISA observations is due to both the large HI temperature variations and the effect of noise in regions of high optical depth. We find optical depths of cold HI around 1 – 10 making optical depth corrections essential. We show that the high HI column densities ($\gtrsim 10^{22}$ cm$^{-2}$) can in parts be attributed to the occurrence of up to 10 individual HI-H$_2$ transitions along the line of sight. This is also reflected in the spectra, necessitating Gaussian decomposition algorithms for their in-depth analysis. However, also for a single HI-H$_2$ transition, $N_{\text{HI}}$ frequently exceeds $10^{21}$ cm$^{-2}$, challenging 1-dimensional, semi-analytical models. This is due to non-equilibrium chemistry effects and the fact that HI-H$_2$ transition regions usually do not possess a 1-dimensional geometry. Finally, we show that the HI gas is moderately supersonic with Mach numbers of a few. The corresponding non-thermal velocity dispersion can be determined via HISA observations within a factor of $\sim 2$.

Key words: MHD – radiative transfer – methods: numerical – ISM: clouds – radio lines: ISM – ISM: atoms

1 INTRODUCTION

The chemical composition of molecular clouds (MCs) is still a field of active research. In particular the transition from atomic hydrogen (HI) to molecular hydrogen (H$_2$) is of interest. During the initial formation of MCs, HI is continuously transformed into H$_2$ as the cloud collapses and its density increases (e.g. Glover & Mac Low 2007b; Glover et al. 2010; Clark et al. 2012b; Mac Low & Glover 2012; Seifried et al. 2017, but see also the review by Dobbs et al. 2014). Knowledge about the exact content of HI and H$_2$ in MCs would also be of great value to assess the amount of CO-dark H$_2$ gas, i.e. molecular gas which is not traced by CO emission and thus affects the $X_{\text{CO}}$-factor (see e.g. the review by Bolatto et al. 2013).

Plenty of semi-analytical works studied the transition of HI- to H$_2$-dominated gas in MCs under various conditions with either plane-parallel or spherically symmetric models (e.g. van Dishoeck & Black 1986; Sternberg 1988; Röllig et al. 2007; Krumholz et al. 2008, 2009; Wolfire et al. 2010; Sternberg et al. 2014; Bialy & Sternberg 2016). For solar-neighbourhood conditions these models predict that the HI-H$_2$ transition occurs around column densities of $10^{20}$ – $10^{21}$ cm$^{-2}$. Furthermore, beside the column density where the transition occurs, Sternberg et al. (2014) and Bialy & Sternberg (2016) also find that the total HI column density, $N_{\text{HI}}$, should be limited to a maximum of about $10^{21}$ cm$^{-2}$. These models are, however, typically highly idealised as they assume chemical equilibrium and are limited to a 1D geometry, although attempts are made to extend them to turbulent environments (Bialy et al. 2017b).

Modelling of the HI-H$_2$ transition in 3D, magneto-hydrodynamical (MHD) simulations remains a highly challenging task. This is due to that fact that MCs are not necessarily in chemical equilibrium concerning their hydrogen content due to turbulent mixing of H$_2$ into the lower-density environment (Glover et al. 2010; Valdivia et al. 2016; Seifried et al. 2017). Therefore, any simulation trying to study the physics and chemistry of the HI-H$_2$ transition in a self-consistent manner requires the inclusion of an on-the-fly chemical network. Moreover, in order to achieve a converged H$_2$ (and thus HI) content in the simulations, a high spatial resolution of $\sim 0.1$ pc is required (Seifried et al. 2017), which was confirmed subsequently by means of semi-analytical considerations (Joshi et al. 2019). Despite these difficulties, there have been a great number of studies attempt-
ing to model the chemical composition of the (dense) interstellar medium (ISM) (e.g. Gnedin et al. 2009; Glover et al. 2010; Mac Low & Glover 2012; Valdivia et al. 2016; Bialy et al. 2017b; Clark et al. 2019; Joshi et al. 2019; Nickerson et al. 2019; Bellomi et al. 2020; Smith et al. 2020, and many more). However, not all of these studies fulfill the requirements of non-equilibrium chemistry and a sufficient spatial resolution. In agreement with semi-analytical results the HI–H$_2$ transition is found to occur around $\sim 10^{20}$–$10^{21}$ cm$^{-2}$ (Gnedin et al. 2009; Valdivia et al. 2016; Seifried et al. 2017; Bellomi et al. 2020).

From the observational perspective, the HI content in the ISM is often determined via observations of the HI 21 cm emission line, UV absorption measurements and far-infrared studies. A large number of observations on both Galactic (e.g. Savage et al. 1977; Kalberla et al. 2005; Gillmon et al. 2006; Rachford et al. 2009; Barriault et al. 2010; Stanimirović et al. 2014; Lee et al. 2012, 2015; Burkhart et al. 2015; Imara & Burkhart 2016) and extragalactic scales (e.g. Wong & Blitz 2002; Browning et al. 2003; Blitz & Rosolowski 2004, 2006; Bigiel et al. 2008; Wong et al. 2009; Schruba et al. 2011) find that the HI–H$_2$ transition occurs around $\sim 10^{20}$–$10^{21}$ cm$^{-2}$. Furthermore, beside the value for the transition, some of these observations (e.g. Wong & Blitz 2002; Bigiel et al. 2008; Barriault et al. 2010; Schruba et al. 2011; Lee et al. 2012; Stanimirović et al. 2014; Burkhart et al. 2015) also suggest an upper threshold of $N_{\text{HI}}$ around $10^{21}$ cm$^{-2}$, similar to the aforementioned semi-analytical models.

On smaller scales of individual MCs, the measurement of their cold HI content via the HI 21 cm line is challenging due to the simultaneous emission of HI in the warm neutral medium. This problem can be overcome by the study of HI self-absorption (HISA) first reported by Heeschen (1954, 1955). These HISA features arise from the more diffuse, atomic ISM. On similar scales, the HI absorption measurements and far-infrared studies. A large number of observations on both Galactic (e.g. Savage et al. 1977; Kalberla et al. 2005; Gillmon et al. 2006; Rachford et al. 2009; Barriault et al. 2010; Stanimirović et al. 2014; Lee et al. 2012, 2015; Burkhart et al. 2015; Imara & Burkhart 2016) and extragalactic scales (e.g. Wong & Blitz 2002; Browning et al. 2003; Blitz & Rosolowski 2004, 2006; Bigiel et al. 2008; Wong et al. 2009; Schruba et al. 2011) find that the HI–H$_2$ transition occurs around $\sim 10^{20}$–$10^{21}$ cm$^{-2}$. Furthermore, beside the value for the transition, some of these observations (e.g. Wong & Blitz 2002; Bigiel et al. 2008; Barriault et al. 2010; Schruba et al. 2011; Lee et al. 2012; Stanimirović et al. 2014; Burkhart et al. 2015) also suggest an upper threshold of $N_{\text{HI}}$ around $10^{21}$ cm$^{-2}$, similar to the aforementioned semi-analytical models.

The simulations are part of the SILCC-Zoom project (Seifried et al. 2017, 2020a), where we model the formation and evolution of MCs located in a part of a stratified galactic disk, which in turn is part of the SILCC project (Walch et al. 2015; Girichidis et al. 2016). The disk has an initial Gaussian density profile given by

$$\rho(z) = \rho_0 \times \exp \left(-\frac{1}{2} \left(\frac{z}{h_z}\right)^2\right),$$

(1)

with $h_z = 30$ pc and $\rho_0 = 9 \times 10^{-24}$ g cm$^{-3}$, resulting in a total gas surface density of $\Sigma_{\text{gas}} = 10 M_{\odot}$ pc$^{-2}$. We run two simulations, one without a magnetic field and one with. For the magnetized run we initialise the magnetic field as $B_z = B_{z,0} \sqrt{\rho(z)/\rho_0}$, $B_x = 0$, $B_z = 0$, $B_x = 3 \mu$G in accordance with observations (Beck & Wielebinski 2013). We emphasise that the magnetic field is dynamically

\begin{equation}
\begin{align*}
\rho(z) &= \rho_0 \times \exp \left(-\frac{1}{2} \left(\frac{z}{h_z}\right)^2\right), \\
B_z &= B_{z,0} \sqrt{\rho(z)/\rho_0}, \\
B_x &= 3 \mu G \text{ with observations (Beck & Wielebinski 2013). We emphasise that the magnetic field is dynamically}
\end{align*}
\end{equation}
important for the (chemical) evolution of the MCs (Seifried et al. 2020a,b). In addition to the gas self-gravity we include a background potential from the old stellar component modelled as an isothermal sheet with a scale height of 100 pc and $\Sigma_{\text{star}} = 30 \, \text{M}_\odot \, \text{pc}^{-2}$.

In the initial simulation phase, up to a time $t_0$, the spatial resolution is 4 pc and we drive turbulence by injecting supernovae (SNe) with a rate of 15 SNe Myr$^{-1}$ (see Walch et al. 2015; Girichidis et al. 2016; Gatto et al. 2017, for details). At $t_0$ we stop the SN injection to allow for the formation of MCs unaffected by nearby SN remnants, which could influence their evolution (Seifried et al. 2018). For both the unmagnetized and the magnetized run, we pick two regions, each with a typical size of $\sim (100 \, \text{pc})^3$, in which MCs are about to form. We thus have in total 4 MCs, henceforth denoted as MC1-HD, MC2-HD, MC1-MHD and MC2-MHD, where the first two are non-magnetised runs and the latter two include a dynamically relevant magnetic field. The typical $\text{H}_2$ masses of these clouds are around $20 - 50 \times 10^3 \, \text{M}_\odot$, and for the magnetized runs the volume-weighted magnetic field is around $4 \, \mu\text{G}$. Starting at $t_0$, we then progressively increase the spatial resolution in these zoom-in regions over 1.65 Myr reaching a maximum resolution of 0.06 pc. Afterwards, we evolve the clouds on this resolution for a few more Myr. We note that throughout the paper all times refer to the time elapsed since $t_0$, i.e. the start of the zoom-in procedure. We have $t_0 = 1.9$ and 16.0 Myr for the runs without and with magnetic fields, respectively.

### 2.2 Radiative transfer

The radiative transfer simulations are performed in a post-processing step with RADMC-3D (Dullemond et al. 2012) for the HI 21 cm emission line of atomic hydrogen. In order to calculate the (two-)level population, we apply the spin method to calculate the spin temperature of atomic hydrogen, $T_s$, described in Kim et al. (2014, their Eqs. 4 – 7). In particular, we include the Wouthuysen-Field (WF) effect (Wouthuysen 1952; Field 1958) assuming $T_s = T_{\text{gas}}$ for the effective temperature of the Ly$\alpha$ field (Field 1959) and $n_e = 10^6 \, \text{cm}^{-3}$ for the Ly$\alpha$ photon density (Lizt 2001). We emphasise that including or excluding the WF effect has only a marginal impact as for the temperatures of $\lesssim 10^5 \, \text{K}$ typical for MCs, the WF effect does barely affect $T_s$, which remains close to the actual gas temperature in both cases (fig. 2 of Kim et al. 2014). Furthermore, the Einstein coefficient of the HI line is $A_{\mu\gamma} = 2.8843 \times 10^{-15} \, \text{s}^{-1}$ (Gould 1994) and the spectral resolution is set to 200 m s$^{-1}$ over a range of $\pm 20 \, \text{km s}^{-1}$ resulting in 201 velocity channels.

We consider the emission of the four MCs in isolation, i.e. the emission stemming from the gas in the aforementioned zoom-in regions only. This allows us to focus on the HISA signal originating from the MCs themselves (and to a smaller extent from warm HI in the zoom-in region) avoiding any foreground contamination. We use a resolution of 0.06 pc, i.e. identical to the maximum resolution of the underlying simulations. We investigate the emission for two points in time, that is $t_{\text{evol}} = 2$ and 3 Myr. As the results for both times are qualitatively similar, for most of the plots we focus on $t_{\text{evol}} = 2$ Myr. In order to model the emission (and its absorption) of a diffuse HI background, we include a (spatially and spectrally) fixed background radiation field with a brightness temperature of 100 K in the radiative transfer calculation. This background temperature is motivated by results from recent HISA observations within the galactic plane (Syed et al. 2020; Wang et al. 2020b) and is also used in the numerical study presented in Soler et al. (2019). This makes our HISA observations sensitive to absorption of HI gas with $T_s$ below 100 K, warmer gas will be seen in emission. We note that we have also tested the usage of a background temperature of 200 K. The observed changes are, however, only very moderate which is why we focus on the case of 100 K here.

#### 2.2.1 Adding observational effects

In a final step we incorporate observational effects into our obtained (ideal) HI emission maps. For this purpose we (i) convolve our emission maps with a Gaussian beam of 80" (at the chosen distance, see below), (ii) reduce the spectral resolution from 200 m s$^{-1}$ to 1 km s$^{-1}$ by summing up the contribution of 5 neighbouring channels, i.e. the data cubes analysed in the following have only 40 velocity channels (one fifth of the original cubes), and (iii) finally add random Gaussian noise with a standard deviation of 3 K to the obtained emission maps. All the above stated values are average values from recent HISA observations towards MCs (e.g. Dénès et al. 2018; Beuther et al. 2020; Syed et al. 2020; Wang et al. 2020b). We choose two different distances for the observed clouds of 150 pc and 3 kpc, corresponding to a physical beam size of 0.06 pc and 1.2 pc, respectively. However, as the results for both distances are relatively similar, in the following we focus on the distance of 150 pc.

#### 2.3 HISA calculations

In order to investigate the properties of the absorbing atomic hydrogen gas, i.e. the HISA features, we follow the approach described in Wang et al. (2020b, see their section 2.2) to convert the absorption spectrum into an effective emission spectrum of the cold HI. In short, assuming a temperature of the absorbing, cold HI gas, $T_{\text{HISA}}$, and that both foreground and background emission are optically thin, one can relate the observed emission to the optical depth, $\tau_{\text{HISA}}$, of the absorption layer via

$$T_{\text{off--on}} = T_{\text{off}} - T_{\text{on}} = (p_{\text{off}} + T_{\text{cont}} - T_{\text{HISA}}) \times (1 - e^{-\tau_{\text{HISA}}}) \, (3)$$

For the sake of readability, we have omitted the dependence on the velocity channel in the above equation. Here, $T_{\text{on}}$ is the actually observed brightness temperature, and $T_{\text{off}}$ is the brightness temperature at an off-position, i.e. the brightness temperature which would be measured if no absorbing cold HI gas were present along the line of sight (LOS) towards the observer. Furthermore, $T_{\text{cont}}$ is the brightness temperature of the diffuse continuum background, and the dimensionless quantity $p$ parametrises the ratio of foreground to background emission (Feldt 1993; Gibson et al. 2000) and is usually estimated to be close to 1 (McClure-Griffths et al. 2006; Rebolo et al. 2017; Dénès et al. 2018; Wang et al. 2020b).

In Wang et al. (2020b), $T_{\text{off}}$ – which is not accessible via observations – is inferred by fitting a polynomial to the absorption-free channels, i.e. those channels where no HISA feature is present. As we use a fixed background brightness temperature of 100 K in our synthetic observations, which varies neither spatially nor spectrally. Therefore, we cannot test additional inaccuracies arising from the uncertainty to determine $T_{\text{off}}$ in actual observations – a caveat to keep in mind throughout the paper. However, due to this simplifying
assumption, in our case the spectrum in the absorption-free channels is practically flat as the emission of HI gas warmer than 100 K in the considered zoom-in region is almost negligible, similar to the findings of Soler et al. (2019). For this reason, we can set $T_{\text{off}} = 100$ K for our data which yields

$$T_{\text{off-on}}(v) = 100 \text{ K} - T_{\text{on}}(v).$$  \hspace{1cm} (4)

The quantity $T_{\text{off-on}}(v)$ is positive and gives the depth of the absorption feature seen in $T_{\text{on}}(v)$. In the following we refer to these kind of spectra as HISA spectra.

Furthermore, using a constant background temperature and assuming that the emission of HI warmer than 100 K is negligible is congruent with setting $p = 1$ and $pT_{\text{off}} + T_{\text{cont}} = 100$ K\(^3\). Hence, we can simplify Eq. 3 to yield

$$T_{\text{off-on}}(v) = (100 \text{ K} - T_{\text{HISA}}) \times (1 - e^{-T_{\text{HISA}}(v)}).$$  \hspace{1cm} (5)

We apply the spectral analysis tool BTS\(^4\) (Clarke et al. 2018) to $T_{\text{off-on}}(v)$ to find the location and properties of the HISA feature. BTS identifies and fits Gaussian peaks in a spectrum. Assuming that the HISA feature has approximately the shape of a single Gaussian, we restrict the fitting function used in BTS to a single Gaussian. We set the noise level to 3 K and the required signal-to-noise ratio to 3. Of the obtained fitting values, here only the line width, $\sigma_{\text{BTS}}$, will be used later in the paper. Next, for each pixel for which BTS identifies a (Gaussian) HISA feature, we calculate the column density of the cold HI responsible for the HISA feature.

For this purpose, for most of our paper we adopt a fixed value for $T_{\text{HISA}}$ for the entire map, identical to the approach applied in recent observations (e.g. Syed et al. 2020; Wang et al. 2020b, but see below and Section 3.5 for a different approach). We then solve Eq. 5 for the optical depth of the HISA feature, $\tau_{\text{HISA}}$, for each channel independently, which yields

$$\tau_{\text{HISA}}(v) = -\ln \left( 1 - \frac{T_{\text{off-on}}}{100 \text{ K} - T_{\text{HISA}}} \right).$$  \hspace{1cm} (6)

With this we can calculate the column density of the HISA feature via (Wilson et al. 2013)

$$N_{\text{H,I}} = 1.8224 \times 10^{18} \text{ cm}^{-2} \frac{T_\text{s}}{1 \text{ K}} \int \tau(v) \text{ d}v \left/ \text{ km s}^{-1} \right.,$$  \hspace{1cm} (7)

where we assume $T_s = T_{\text{HISA}}$ for the HI spin temperature and $\tau(v) = \tau_{\text{HISA}}(v)$ for the optical depth.

Though being straightforward to use, this method has the disadvantage that a fixed value of $T_{\text{HISA}}$ for every pixel of the map has to be assumed, which might not be the case. Furthermore, when choosing a fixed $T_{\text{HISA}}$, Eq. 3 might not yield a result for $\tau_{\text{HISA}}$ for every channel. The maximum useable value for each channel, $T_{\text{HISA}, \text{max}}(v)$, is obtained by assuming $\tau_{\text{HISA}} \rightarrow \infty$ (Wang et al. 2020b), which results for our setup in

$$T_{\text{HISA,max}}(v) = -T_{\text{off-on}}(v) + pT_{\text{off}}(v) + T_{\text{cont}} = T_{\text{on}}(v).$$  \hspace{1cm} (8)

Hence, if the assumed $T_{\text{HISA}}$ exceeds the observed brightness temperature $T_{\text{on}}(v)$ of a given channel, this channel has to be dropped and cannot be taken into account for the calculation of the column density (see Section 3.2).

An alternative way to determine the optical depth, which simultaneously leaves $T_{\text{HISA}}$ as a free parameter, is given by Knapp (1974)

$$T_{\text{HISA}}(v) = \tau_0 e^{-\frac{1}{2} \left( \frac{v-v_0}{\sigma_0} \right)^2}. $$  \hspace{1cm} (9)

Here, $\tau_0$, $v_0$, and $\sigma_0$ are free parameters (together with $T_{\text{HISA}}$) which are determined by fitting the observed spectrum $T_{\text{off-on}}(v)$ with Eq. 9 inserted in Eq. 5. As before, the fit is only performed for those pixels where we identify a Gaussian HISA feature with BTS. Beside the approach using a fixed $T_{\text{HISA}}$ we will also test this approach in the following.

3 RESULTS

In order to get a first impression about the HI and H\(_2\) content of the simulated MCs, in Fig. 1 we show the 2D-PDFs of $N_{\text{HI}}$, $N_{\text{HI}}$, and their ratio vs. $N_{\text{H, tot}}$ for a selected simulation snapshot. Here, $N_{\text{H, tot}} = N_{\text{HI}} + 2 N_{\text{H}_2} + N_{\text{H}_2}$ denotes the total hydrogen column density. We find that $N_{\text{H}_2}$ starts to form above $N_{\text{H, tot}} = 10^{20}$ cm\(^{-2}\).

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\(^{3}\) Effectively, we now do not differentiate any more between $pT_{\text{off}}$ and $T_{\text{cont}}$.

\(^{4}\) Acronym for “Behind The Spectrum”, https://github.com/SeamusClarke/BTS
The transition from atomic- to molecular-hydrogen dominated gas, however, occurs around $N_{\text{H}_2, \text{tot}} \approx 10^{21} \text{ cm}^{-2} \approx 8 \, M_\odot \text{ pc}^{-2}$, as discussed in detail in Seifried et al. (2020a). This is in agreement with other numerical and semi-analytical works (Krumholz et al. 2008, 2009; Gnedin et al. 2009; Sternberg et al. 2014; Bialy & Sternberg 2016; Valdivia et al. 2016; Bellomi et al. 2020). However, despite $H_2$ forming rapidly above $N_{\text{H}_2, \text{tot}} \approx 10^{20} \text{ cm}^{-2}$, also $N_{\text{HI}}$ continues to rise. Most of the HI has column densities around $10^{21} \text{ cm}^{-2} \approx 8 \, M_\odot \text{ pc}^{-2}$ similar to theoretical predictions (Krumholz et al. 2008, 2009; Sternberg et al. 2014; Bialy & Sternberg 2016). However, even significantly higher HI column densities up to a few $10^{22} \text{ cm}^{-2}$ are reached, i.e. $H_2$ and HI coexist on the projected maps. Hence, having the HI-$H_2$ transition around a certain value (e.g. $10^{21} \text{ cm}^{-2}$) does not exclude the occurrence of significantly higher HI column densities.

### 3.1 Deriving $N_{\text{HI}}$ using a fixed $T_{\text{HISA}}$

In the following we will assess how accurately the cold HI content in MCs can be determined via HISA observations\(^5\). We define cold HI as all HI with temperatures below 100 K. This definition is motivated by the chosen background brightness temperature (Section 2.3), which makes our synthetic HISA observations sensitive to HI with temperatures below 100 K. Depending on the simulation, we find that around and in our MCs 22 – 43% of the HI is warmer than 100 K, i.e. in all cases the HISA observations are sensitive to more than 50% of the entire HI mass. In the top row of Fig. 2, we show the velocity-integrated intensity, $I_{\text{HI}}$, of our synthetic HI observations of MC1-HD from three different directions at $t_{\text{eval}} = 2$ Myr including observational effects for an assumed distance of 150 pc. However, even significantly higher HI column densities up to a few $10^{22} \text{ cm}^{-2}$ are reached, i.e. $H_2$ and HI coexist on the projected maps. Hence, having the HI-$H_2$ transition around a certain value (e.g. $10^{21} \text{ cm}^{-2}$) does not exclude the occurrence of significantly higher HI column densities.

\(^5\) In Seifried et al. (2020a) we have already discussed a new approach to determine the $H_2$ content by means of combined CO(1-0) and dust emission observations, which allows for an accurate determination of $H_2$ within a factor of 1.8.
Next, we investigate the HI column densities obtained from the HISA observations, denoted as \(N_{\text{HII,obs}}\) (Eq. 7), assuming a fixed HISA temperature, \(T_{\text{HISA}}\). In the top row of Fig. 4 we show \(N_{\text{HII,obs}}\) for \(T_{\text{HISA}} = 20, 40,\) and 60 K for MC1-HD at \(t_{\text{evol}} = 2\) Myr for one LOS at an assumed distance of 150 pc. In the bottom row we show the ratio of \(N_{\text{HII,obs}}\) and \(N_{\text{HII,real}}\). Most prominently, we find that for all three values of \(T_{\text{HISA}}\), the observed column densities \(N_{\text{HII,obs}}\) in the central and most dense regions are a factor of \(\geq 10\) too low. This trend becomes more pronounced with increasing \(T_{\text{HISA}}\). In the outer regions \(N_{\text{HII,obs}}\) could not be calculated for all pixels as here the absorption features are partly too weak and therefore the spectral analysis tool BTS does not identify any HISA feature. For the remaining pixels in the outer parts, \(N_{\text{HII,real}}\) is also typically underestimated ranging from a few 10% up to a factor of a few. However, increasing \(T_{\text{HISA}}\) pushes \(N_{\text{HII,obs}}\) in the outer regions closer to \(N_{\text{HII,real}}\). Overall, however, the match between the actual and observed HI column density is rather poor with a clear tendency to underestimate \(N_{\text{HII,real}}\) by factors up to \(\sim 10\) (see also Section 3.4 for the effect on total estimated HI mass). This does not change when considering different LOS, times, MCs or assuming a distance of 3 kpc. Reasons for this underestimation will be discussed in detail in Sections 3.2 and 3.3.

In Fig. 5 we show the mean value of \(N_{\text{HII,obs}}\) as a function of \(N_{\text{HII,real}}\) for the four different MCs placed at 150 pc at \(t_{\text{evol}} = 2\) Myr focusing on the case of \(T_{\text{HISA}} = 20, 40\), and 60 K (coloured lines). Overall, the underestimation shown in Fig. 4 is observed in all cases, the degree of underestimation becomes more pronounced with increasing \(N_{\text{HII,real}}\) and decreasing \(T_{\text{HISA}}\). Lowering or increasing \(T_{\text{HISA}}\) beyond the values shown only amplifies the trends, which is why we do not explicitly discuss them here. In the following we focus on the range of \(N_{\text{HII,real}} \geq 10^{19.5}\) cm\(^{-2}\).

First, we note that \(T_{\text{HISA}} = 20\) K (blue lines) is apparently a rather bad choice resulting in an underestimation by about one order of magnitude (and more). Considering \(T_{\text{HISA}} = 60\) K (red lines), we find that in the range \(10^{19.5}\) cm\(^{-2} \leq N_{\text{HII,real}} \leq 10^{21}\) cm\(^{-2}\), the actual values are underestimated by a factor of \(\sim 2 - 5\). In the same range, for \(T_{\text{HISA}} = 40\) K (green lines), the actual values are underestimated even more severely by a factor of \(\sim 3 - 10\). Moreover, for \(N_{\text{HII,real}} > 10^{21} \) cm\(^{-2}\), \(N_{\text{HII,obs}}\) seems to level off around \(\sim 10^{20.5 - 21}\) cm\(^{-2}\), for both \(T_{\text{HISA}} = 40\) and 60 K. This results in an increasing underestimation of the actual column density when going to denser and denser regions. We note that this artificial levelling-off around \(N_{\text{HII,obs}} \approx 10^{21}\) cm\(^{-2}\) matches well the maximum values reported in recent HISA observations (Kavara et al. 2003, 2005; Li & Goldsmith 2003; Goldsmith & Li 2005; Klaassen et al. 2005; Krčo et al. 2008; Barriault et al. 2010; Krčo & Goldsmith 2010; Syed et al. 2020; Wang et al. 2020b, but see also Section 4.1 for a further discussion). Finally, even for a single MC there is a large scatter of the measured \(N_{\text{HII,obs}}\) for a given \(N_{\text{HII,real}}\) (shown by the full distribution in grey scale in the background for MC1-HD for \(T_{\text{HISA}} = 40\) K). This further lowers the accuracy with which HISA observations seem to be able to constrain the actual HI column density. An additional uncertainty in observations arises from the unknown value of \(T_{\text{off}}\) (and thus \(T_{\text{off- on}}\), which in our case is chosen to be constant (\(T_{\text{off}} = 100\) K). Eq. 6 for the optical depth implies that this uncertainty increases even further the scatter found for individual MCs at a given \(N_{\text{HII,real}}\).

### 3.2 The HI temperature

Investigating Eq. 3 shows that there is a certain upper threshold for \(T_{\text{HISA}}\), above which the equation is not solvable for \(T_{\text{HISA}}\) for at least some of the velocity channels, which then would have to be omitted for the calculation of \(N_{\text{HII}}\). We denote this upper threshold as \(T_{\text{dip}}\), which is set by the minimum of \(T_{\text{HISA}, \text{max}}(v)\) (Eq. 8) over all velocity channels for a given pixel, i.e.

\[
T_{\text{dip}} = \min(T_{\text{HISA}, \text{max}}(v)) = \min(T_{\text{on}}(v)).
\]  

(10)

The denomination as \(T_{\text{dip}}\) is motivated by the fact that it corresponds to the temperature at the dip of the observed absorption spectrum \(T_{\text{on}}(v)\). For the sake of clarity, in Table 1 we give a short summary of the most relevant temperature definitions used in this paper.

In the left panel of Fig. 6 we show the map of \(T_{\text{dip}}\) for one snapshot. In addition, the right panel shows the actual mass-weighted, LOS-averaged HI temperature, \(T_{\text{HI, mw}}\). Both \(T_{\text{dip}}\) and \(T_{\text{HI, mw}}\) show a strong drop towards the central, high-column density areas (see also top panel of Fig. 8). Interestingly, we find that both temperature measures show an agreement within about 20 K (Fig. 7). Based on this, in Section 3.5 we investigate whether the usage of \(T_{\text{dip}}\) as an approximation for the temperature of the cold HI along the LOS (and thus for \(T_{\text{HISA}}\)) is suitable.

The strong variations of \(T_{\text{dip}}\) and \(T_{\text{HI, mw}}\) down to values as low as \(\sim 10\) K cause the significant underestimation of \(N_{\text{HII,real}}\) seen in the Figs. 4 and 5 by means of two effects explained in the following and sketched in Fig. 8:

(i) \(T_{\text{HISA}}\) chosen too high \((\geq T_{\text{dip}})\): These regions typically correspond to high \(N_{\text{HII,real}}\) \((\geq 10^{21}\) cm\(^{-2}\)) (top panel of Fig. 8). Here, Eq. 3 does not yield any results for at least some of the velocity channels. This happens for \(\leq 10\%\), 5 - 20\%, and 30 - 40\% of the pixels for \(T_{\text{HISA}} = 20, 40,\) and 60 K, respectively. Hence, for these pixels \(\text{some of the channels have to be neglected, which reduces}\) \(N_{\text{HII,obs}}\) significantly (Eq. 7). As with decreasing \(T_{\text{dip}},\) i.e. increasing \(N_{\text{HII,real}}\), more and more velocity channels have to be omitted (at a fixed \(T_{\text{HISA}}\)), this leads to the observed artificial levelling-off of \(N_{\text{HII,obs}}\) at \(\sim 10^{21}\) cm\(^{-2}\). In consequence, for central, high column density regions of MCs, \(N_{\text{HII,real}}\) is underestimated by a factor of about 10 and more (Fig. 5).

---

6 As the synthetic HI observations are only sensitive to HI with temperatures below 100 K, also for this average (as for \(N_{\text{HII,real}}\)) only HI gas with \(T \leq 100\) K is considered.
Figure 4. Top row: Observed HI column density assuming a fixed $T_{\text{HISA}}$ of 20, 40 and 60 K (from left to right) for MC1-HD at $t_{\text{evol}} = 2$ Myr at an assumed distance of 150 pc (beam size corresponds to 0.06 pc). Bottom row: Ratio of the observed HI column density shown in the top row to the actual HI column density (for $T < 100$ K). Overall, assuming a fixed $T_{\text{HISA}}$ underestimates $N_{\text{HI,real}}$ by almost about one order of magnitude in the central, high column density regions. Increasing $T_{\text{HISA}}$ improves the match only in the outer regions.

Table 1. List of the most relevant temperature definitions used in this paper including a short explanation and the nature of each temperature.

| $T$ | actual spin temperature of the HI gas used for the radiative transfer (Section 2.2), close to the actual gas temperature of the simulation | spin temperature, varies along the LOS |
|-----|-----------------------------------------------------------------------------------------------------------------------------------|--------------------------------------|
| $T_{\text{HISA}}$ | assumed spin temperature of the HISA features needed for the calculation of the HI optical depth (Eq. 6) and subsequently the column density (Eq. 7) | spin temperature |
| $T_{\text{HI,raw}}$ | mass-weighted, LOS-averaged HI temperature calculated from the simulation data for a given pixel, see right panel of Fig. 6 | kinetic gas temperature |
| $T_{\text{HI,min}}$ | minimum HI temperature calculated from the simulation data along the LOS for a given pixel | kinetic gas temperature |
| $T_{\text{on}(v)}$ | spectrum of the measured HI brightness temperature for a given pixel, see Fig. 3 | brightness temperature |
| $T_{\text{dip}}$ | lowest temperature of the $T_{\text{on}(v)}$-spectrum for a given pixel, see Eq. 10 and left panel of Fig. 6 | brightness temperature |

(ii) $T_{\text{HISA}}$ chosen too low ($< T_s$): This leads to an underestimation of the optical depth $\tau_{\text{HISA}}$ (Eq. 6) as well as $N_{\text{HI,obs}}$ (Eq. 7, both via $T_{\text{HISA}}$ and the assumption $T_s = T_{\text{HISA}}$). This effect is dominant mainly in the low to intermediate column density regions in the outer parts of the MCs ($N_{\text{HI,real}} \leq 10^{21}$ cm$^{-2}$). Here, $N_{\text{HI,obs}}$ underestimates $N_{\text{HI,real}}$ on average by a factor of 3–10. Due to the linear dependence of $N_{\text{HI,obs}}$ on $T_{\text{HISA}}$, the actual value of $N_{\text{HI,obs}}/N_{\text{HI,real}}$ increases with increasing $T_{\text{HISA}}$ at high $T_{\text{dip}}$ (coloured lines in the bottom panel of Fig. 8).

Overall, our results demonstrate that finding an accurate value for $T_{\text{HISA}}$ is crucial but at the same time not possible when using a single value for the entire map. In addition, both choosing a too high or too low value for $T_{\text{HISA}}$ leads to an underestimation of the HI column density.
In general, the actual column density is underestimated significantly, and the three different 

Figure 5. Map of $N_{\text{HI,obs}}$ against $N_{\text{HI,real}}$ for the three different directions of all four MCs placed at a distance of 150 pc at $t_{\text{end}} = 2$ Myr and using three different $T_{\text{HISA}}$ (colored lines). In the background the full distribution for one snapshot (MC1-HD, $T_{\text{HISA}} = 40$ K) is shown in grey scale. The black lines show lines of constant ratio $N_{\text{HI,obs}}/N_{\text{HI,real}}$ to guide the reader’s eye. In general, the actual column density is underestimated significantly, and the maximum $N_{\text{HI,obs}}$ levels off around $10^{21}$ cm$^{-2}$.

Figure 6. Map of $T_{\text{dip}}$ (left, Eq. 10), which can be used such that Eq. 3 yields a result for every velocity channel, and the mass-weighted average of the temperature of the HI gas (right) for the same snapshot as shown in Fig. 4. Both quantities show a reasonable agreement within about 20 K. However, $T_{\text{dip}}$ is quite low in the central areas. This explains the poor match of the observed with the actual column density (Fig. 4), as here often $T_{\text{HISA}} > T_{\text{dip}}$ (depending on the actual choice of $T_{\text{HISA}}$).

3.2.1 HI temperature variations

Interestingly, even for regions where $T_{\text{HISA}} = T_{\text{dip}}$ (indicated by the peak of the coloured lines in the bottom panel of Fig. 8), a ratio of $N_{\text{HI,obs}}/N_{\text{HI,real}}$ close to 1 is barely reached. An additional source of uncertainty causing this are the significant variations of $T_{\text{HISA}}$ across the map (right panel of Fig. 6). Similar variations will also occur for each individual pixel along the LOS. Hence, even for an individual pixel the assumption of a constant $T_{\text{HISA}}$ presents an oversimplification, which in turn results in the observed underprediction of the HI column density particularly for the dense regions.

We emphasise that $T_{\text{HISA}}$ can be a few 10 K higher than the temperature of the coldest HI gas along each LOS, denoted as $T_{\text{HI, min}}$ (not shown), due to HI gas warmer than $T_{\text{HI, min}}$ along the LOS. This also explains why there are regions where $T_{\text{dip}} < T_{\text{HI, min}}$ (Fig. 7), whereas $T_{\text{HI, min}}$ is, as expected, always smaller than $T_{\text{dip}}$. This also

Figure 7. 2D-PDF of $T_{\text{dip}}$ vs. $T_{\text{HI, mw}}$ and its mean value (green line) for the same snapshot as shown in Fig. 6. The black line corresponds to $T_{\text{dip}} = T_{\text{HI, mw}}$. Overall there is a rough correspondence between both quantities with a typical scatter of $\sim 10$–20 K.

Figure 8. Top: Phase diagram of $N_{\text{HI,real}}$ vs. $T_{\text{dip}}$ for the same snapshot as in Fig. 4. The highest HI column densities are associated with low $T_{\text{dip}}$. Bottom: Phase diagram of the ratio of $N_{\text{HI,obs}}$ and $N_{\text{HI,real}}$ vs. $T_{\text{dip}}$ for the same snapshot as in the top panel using $T_{\text{HISA}} = 40$ K. The coloured lines show the mean value of the distribution for three different $T_{\text{HISA}}$. Overall the actual HI column density is underestimated by up to a factor of $\sim 10$. Changing $T_{\text{HISA}}$ increases the accuracy only locally, i.e. where that assumed $T_{\text{HISA}}$ roughly corresponds to $T_{\text{dip}}$, which represents the real temperature of the HISA feature. We indicated the reasons of the underestimation in the two temperature ranges above and below $T_{\text{dip}} = T_{\text{HISA}}$. 

Figure 5. Mean value of $N_{\text{HI,obs}}$ against $N_{\text{HI,real}}$ for the three different directions of all four MCs placed at a distance of 150 pc at $t_{\text{end}} = 2$ Myr and using three different $T_{\text{HISA}}$ (colored lines). In the background the full distribution for one snapshot (MC1-HD, $T_{\text{HISA}} = 40$ K) is shown in grey scale. The black lines show lines of constant ratio $N_{\text{HI,obs}}/N_{\text{HI,real}}$ to guide the reader’s eye.
indicates that $T_{\text{dip}}$ only gives an upper limit to the actual spin temperature $T_s$ of the absorbing HI layer. The non-isothermality of the cold HI gas is further emphasised by its temperature distribution in Fig. 9 showing the cumulative PDF of HI gas above a certain threshold temperature for all four MCs at $t_{\text{eval}} = 2$ Myr. The amount of HI gas is rising steadily with decreasing temperature, independent of the considered MC. This shows that no single temperature can be used to describe the HI content of MCs. As an example, for $T_{\text{HISA}} = 40$ K, the HISA observations would at (least) miss out 20 – 40% of the cold HI mass.

The above results explain why also in the absence of observational effects like noise and limited spectral or spatial resolution the poor match between the observed and actual HI column density remains (see Fig. A1 in Appendix A). Also for different assumed distances of 150 pc and 3 kpc we find little differences. This further supports our claim that the underestimation of $N_{\text{H,I,real}}$ can be attributed – at least in parts (see Section 3.3) – to the non-uniform HI temperatures present in the clouds.

Finally, we note that the rather low HI temperatures found in our simulations ($\leq 40$ K, Fig. 6) are in good agreement with a number of observations of Galactic MCs, which find typical HI temperatures between 10 K and 40 K (Gibson et al. 2000; Kavars et al. 2003, 2005; Klaassen et al. 2005; Fukui et al. 2014, 2015; Stanimirović et al. 2014; Dénes et al. 2018; Murray et al. 2018; Nguyen et al. 2019; Syed et al. 2020; Wang et al. 2020). Our results thus demonstrates that optical depth effects cannot be neglected in HI observations of MCs, also when calculating $N_{\text{HI}}$ from HI emission observations.

We note that at first view the maximum values of $\langle \tau \rangle > 10$ appear high in comparison with those found in the aforementioned observational works. However, as these measurements are limited by observational noise, $\Delta T$, the observationally reported values have to be taken as lower limits (see e.g. fig. 10 of Bihr et al. 2015).

In regions of high optical depth, the observed brightness temperature $T_{\text{obs}}$ will be close to the spin temperature $T_s$ of the absorbing, cold HI layer, i.e. $T_{\text{off-on}} \approx 100$ K - $T_s$. Hence, choosing $T_{\text{HISA}} \leq T_s$ will result in an underestimation of $N_{\text{H,I,real}}$ for optically thick regions as well, as discussed in Section 3.2. Moreover, for $T_{\text{HISA}} > T_s$ an additional source of error in such optically thick regions is caused by the observational noise $\Delta T$, as now $T_{\text{off-on}} \approx 100$ K - $T_{\text{HISA}} - \Delta T$ (from Eq. 5). Inserting this into Eq. 6 yields

$$
\tau_{\text{HISA,noise}} = -\ln \left( 1 - \frac{100 \text{ K} - T_{\text{HISA}} - \Delta T}{100 \text{ K} - T_{\text{HISA}}} \right) = -\ln \left( \frac{\Delta T}{100 \text{ K} - T_{\text{HISA}}} \right).
$$

Analysing Eq. 11 shows that the observational uncertainty $\Delta T$ in highly optically thick regions (if $T_{\text{HISA}} > T_s$) results in an underestimation of $N_{\text{HI,obs}}$ regardless of its sign:

(i) $\Delta T < 0$: If noise artificially lowers $T_{\text{on}}$, this increases $T_{\text{off-on}}$ beyond a value of $100$ K - $T_{\text{HISA}}$ in a highly optically thick region. Hence, Eq. 11 would contain a negative expression in the logarithm and the contribution from the corresponding velocity channel has to be omitted.

(ii) $\Delta T > 0$: If noise, but also the potential emission of warm and diffuse HI in the foreground, increases $T_{\text{on}}$ (and thus decreases $T_{\text{off-on}}$), this results in an underestimation of the true value of $\tau_{\text{HISA}}$ (which can be larger than $\tau_{\text{HISA,noise}}$) and thus also $N_{\text{HI,obs}}$ (Eq. 7). The effect of foreground emission is thus also related to the problem of identifying $T_{\text{HISA}}$ correctly.

3.3 The HI optical depth

In order to investigate the typical optical depths in our clouds, in Fig. 10 we plot a proxy for the HI optical depth, $\langle \tau \rangle$, for MC1-HD at 2 Myr. The definition of $\langle \tau \rangle$ is given in Appendix B. It represents a channel-averaged approximation to the real optical depth which is accurate within a few 10% above $\langle \tau \rangle = 1$, i.e. in the optically thick regions we are interested in here. For optically thin regions, the approximation is not applicable, which is why we do not show these regions here. The values of $\langle \tau \rangle$ span a wide range, from the moderately optically thick regime up to highly optically thick regions with $\langle \tau \rangle \sim 10$. In particular, the entire area of central cloud (compare with bottom middle panel of Fig. 2) has an average optical depth $\geq 1$, thus optical depth corrections cannot be neglected when calculating HI column densities.

Figure 9. Cumulative temperature PDF showing the amount of cold HI above a certain threshold temperature for all four MCs at $t_{\text{eval}} = 2$ Myr. The steady rise with decreasing temperature indicates that no single temperature choice for $T_{\text{HISA}}$ is suitable to accurately determine the amount of cold HI in the clouds. Note that gas above 100 K is not considered here.

Figure 10. Map of the HI optical depth proxy $\langle \tau \rangle$ (see Appendix B) for MC1-HD at 2 Myr. The dense cloud region (compare with bottom middle panel of Fig. 2) has an average optical depth $\geq 1$, thus optical depth corrections cannot be neglected when calculating HI column densities.
Hence, even if one were to choose the correct value of $T_{\text{HISA}}$, $N_{\text{HI,obs}}$ is in general underestimated in optically thick regions (see regions of high $N_{\text{HI,real}}$ in Fig. 5). The observational noise contributes to the fact that even at the peaks of the mean-value lines in the bottom panel of Fig. 8, where $T_{\text{HISA}} \approx T_{\text{dip}}$, the real HI column density is on average underestimated. We emphasise that this underestimation due to $\Delta T$ adds on top of the problem to determine a reasonable value of $T_{\text{HISA}}$. This effect also contributes to the more pronounced underestimation at low values of $T_{\text{dip}}$ (higher values of $N_{\text{HI,real}}$; bottom panel of Fig. 8): the lower $T_{\text{dip}}$, the higher is the HI column density and thus the optical depth, which amplifies the issue arising from this effect.

### 3.3.1 Opacity correction in MCs

Motivated by the large extent of optically thick regions in MCs (Fig. 10), we suggest a method to improve the accuracy of $N_{\text{HI,obs}}$. A significant underestimation occurs in the high-$N_{\text{HI}}$/high-optical depth regions of the MCs, where Eq. 6 yields no result for $\tau_{\text{HISA}}$ any more and velocity channels have to be omitted (see Fig. 8). Hence, for these velocity channels we use an optical depth set by the typical rms noise ($\Delta T$) of the observation, which is given by $\tau_{\text{HISA,noise}}$ (Eq. 11), e.g. for $T_{\text{HISA}} \approx 40$ K and the adopted noise of 3 K (Section 2.2.1), we obtain $\tau_{\text{HISA,noise}} = 3.0$. We emphasise that this estimate is still a conservative estimate as the actual optical depth is likely to be higher. A similar approach is also followed by Bihr et al. (2015) for HI emission maps. We note that this approach has to be considered under the premise that, as shown before, a constant value of $T_{\text{HISA}}$ over the entire map is an oversimplification, which in addition does also not account for the temperature variations along the LOS (see Section 3.2).

The obtained column density maps are shown in Fig. A2 in Appendix A. As can be seen, $N_{\text{HI,obs}}$ in the denser parts of the MCs is represented better than before (compare with Fig. 4). In the very densest parts, however, $N_{\text{HI,real}}$ is still significantly underestimated. This is also visible in Fig. 11, where we show the mean values of $N_{\text{HI,obs}}$ for all MCs and directions at 2 Myr using this correction. There is an improvement in all areas compared to the case without any correction (see Fig. 5). However, $N_{\text{HI,real}}$ can still be underestimated by a factor of a few to ~10. Hence, the suggested method has only a moderate impact on increasing the accuracy, both due to the non-isothermality of the HI gas and the fact that $\tau_{\text{HISA,noise}}$ is most likely lower than the real optical depth.

### 3.4 The cold HI budget of molecular clouds

Summarizing the findings of the previous sections we find that the uncertainty in determining the HI column density is due to (i) the assumption of a fixed temperature $T_{\text{HISA}}$, for the calculation of the HISA column densities and (ii) noise in the temperature brightness measurement. As a consequence, either the optical depth and the true $T_{\text{HISA}}$ are underestimated (mainly in the outer parts of MCs) or velocity channels have to be omitted for the calculation of the column density (mainly in the densest parts of MCs). Overall, this results in a significant underestimation of the actual HI column densities by a factor of 3 - 10 (and even more in the densest regions of clouds).

This is also reflected in the total mass of cold HI in MCs inferred from HISA observations, $M_{\text{HI,HISA-obs}}$, shown in Fig. 12. Here, we add up the observed HI mass of all pixels for which a HISA feature is identified (i.e. pixels outside the coloured regions in Fig. 4 are ignored) using $T_{\text{HISA}} = 40$ K and 60 K. For the actual mass, $M_{\text{HI,real}}$, we only take into account HI gas with $T < 100$ K, to which our HISA observations are sensitive to. As for the column densities, also the total, cold HI mass is typically underestimated by a factor of 3 - 10, when no correction in the optically thick regions is applied (filled symbols). The correction (Section 3.3.1), however, improves the accuracy only moderately by a factor of ~1.5 - 2 (open symbols). We emphasise that increasing $T_{\text{HISA}}$ to obtain apparently more accurate mass estimates should be considered with caution. This merely leads to an overestimation of $N_{\text{HI}}$ at low column densities compensating the underestimation at high column densities (see Fig. 5 and also Section 3.5).

We emphasise that our results do not change significantly among the different MCs considered, i.e. whether or not dynamically important magnetic fields are present. This indicates that HISA observations in general tend to significantly underestimate the cold HI budget in MCs. This is markedly different to a complementary study for the more diffuse ISM (on scales $\geq 1$ pc) of Kim et al. (2014) and Murray et al. (2015, 2017), who find that in this regime HI absorption observations can trace the HI mass with an accuracy of a few 10%.

We tentatively attribute this to the fact that the authors probe...
several and more diffuse HI clouds along significantly longer LOS of several 100 pc length. These clouds might have lower optical depths and are thus less prone to the measurement uncertainties mentioned in Section 3.3. Furthermore, HI masses obtained from emission observations (e.g. Bihr et al. 2015), which correct for the optical depth and which also implicitly take into account warm HI (\(T > 100\) K, in our case 22 – 47\%), might achieve more accurate HI masses, a topic not investigated in this study.

### 3.5 Deriving \(N_{\text{HI}}\) with a variable \(T_{\text{HISA}}\)

#### 3.5.1 \(T_{\text{HISA}}\) as a free fit parameter

Following the results of Section 3.2, we next leave \(T_{\text{HISA}}\) as a free parameter. We determine its value and the optical depth by assuming a Gaussian optical depth profile, i.e. inserting Eq. 9 in Eq. 5 and fitting the observed HI spectrum. In Fig. 13 we show the various quantities obtained by the approach for MC1-HD at 2 Myr and an assumed distance of 150 pc. For other directions, times, clouds, and the 3 kpc-distance case, we find qualitatively and quantitatively similar results.

Overall, we find a poor match between the observed and actual \(N_{\text{HI}}\) (second panel from the left in Fig. 13 and top panel of Fig. 14): Although the mean values of the distribution (orange lines in the top panel of Fig. 14) show a reasonable match for \(N_{\text{HI,obs}}\) and \(N_{\text{HI,real}}\) below \(10^{21.5}\) cm\(^{-2}\), there is a significant scatter of more than 1 dex.

We attribute this rather poor match mainly to (i) the occurrence of multiple Gaussian absorption components in the spectra (see Fig. 3), which are not accounted for in our simplistic model, and – to a lesser extent – to (ii) the lack of spectral resolution (1 km s\(^{-1}\)) and (iii) observational noise. In consequence, it is not possible to reliably determine the optical depth with our fitting approach, which would require accurate spectral information, also about the wings of the spectrum. This is visible in the obtained values of \(\tau_0\) (right-most panel in Fig. 13), which show no clear correlation with the underlying column density distribution \(N_{\text{HI,real}}\) as opposed to the optical depth proxy (\(\tau\)) shown in Fig. 10.

We emphasise that when repeating the method for the noiseless, high-resolution spectra (200 m s\(^{-1}\)), we obtain a similar poor match between \(N_{\text{HI,obs}}\) and \(N_{\text{HI,real}}\). This further supports our assumption that the poor match is in parts due to the occurrence of multiple Gaussian absorption components not accounted for here and not due to a generic problem of this approach. We therefore suggest that multiple Gaussian components with individual temperatures have to be taken into account (e.g. Heiles & Troland 2003; Stanimirović et al. 2014; Murray et al. 2015; Dènes et al. 2018) to get a better match with \(N_{\text{HI,real}}\). This might, to some extent also remedy the temperature problem discussed in Section 3.2 as each component can be assigned an individual temperature. We will, however, postpone this investigation to future work.

Finally, we note that the fitted values of \(T_{\text{HISA}}\) (second panel from the right in Fig. 13) appear roughly comparable to the mass-weighted mean temperatures (right panel of Fig. 6). However, also here strong temperature variations along the LOS (Section 3.2) can affect the fit value of \(T_{\text{HISA}}\). In consequence, as \(\tau_0\) and \(T_{\text{HISA}}\) are degenerate, overestimating (underestimating) \(T_{\text{HISA}}\) requires a higher (lower) \(\tau_0\) to match the observed \(T_{\text{off--on}}\) at the dip of the absorption spectrum (Eq. 5). Following Eq. 7, this directly leads to a too high (low) value of \(N_{\text{HI,obs}}\).

#### 3.5.2 \(T_{\text{HISA}}\) given by \(T_{\text{dip}}\)

Given the similarity of \(T_{\text{dip}}\) and the mass-weighted HI temperature \(T_{\text{HI,mw}}\) (see Fig. 7), as well as the fact that the highest accuracy for \(N_{\text{HI,obs}}\) was found where \(T_{\text{HISA}} = T_{\text{dip}}\) (bottom panel of Fig. 8), we also try an alternative approach by setting \(T_{\text{HISA}}\) close to, but slightly below \(T_{\text{dip}}\). In detail, we set

\[
T_{\text{HISA}} = T_{\text{dip}} - \Delta T
\]

with \(\Delta T = 3\) K being the noise level of the synthesis observations and test the approach for MC1 at \(t_{\text{evol}} = 2\) Myr.

As for the method from Knapp (1974), we find a qualitatively poor match between \(N_{\text{HI,obs}}\) and \(N_{\text{HI,real}}\) (bottom panel of Fig. 14). The method gives a rather flat distribution of \(N_{\text{HI,obs}}\) with too high values at low \(N_{\text{HI,real}}\) and drops towards higher \(N_{\text{HI,real}}\). Overall, the reasons for this poor match are again the temperature variations along the LOS (Section 3.2), the partly high optical depths (for the high-\(N_{\text{HI}}\) regions, Section 3.3) as well as the degeneracy of \(T_{\text{HISA}}\) and \(\tau_{\text{HISA}}\) (for the low \(N_{\text{HI}}\) regions, Section 3.5.1). We emphasise that also the usage of \(T_{\text{HI,mw}}\) – which is not accessible to an observer – for \(T_{\text{HISA}}\) does not improve the situation but gives qualitatively similar results as using Eq. 12. We attribute this to the fact that \(T_{\text{HI,mw}}\) and \(T_{\text{dip}}\) are similar within a scatter of 10-20 K (see Fig. 7).

To summarise, even when choosing \(T_{\text{HISA}}\) by a physically motivated approach, the quality of the obtained HI column density maps does not increase, but partly even decreases. Contrary to the approach of a fixed \(T_{\text{HISA}}\), for these approaches not only the quantitative agreement but also the qualitative agreement between \(N_{\text{HI,obs}}\) and \(N_{\text{HI,real}}\) is lost.

#### 3.6 The HI velocity dispersion

Finally, we consider the accuracy of the HI velocity dispersion inferred from HISA observations. For this purpose, we compare the non-thermal velocity dispersion \(\sigma_{\text{obs}}\) identified via the BTS tool (Section 2.3) with the actual HI velocity dispersion along each LOS directly inferred from the simulation data, \(\sigma_{\text{real}}\). For \(\sigma_{\text{real}}\) we only take into account the velocity component along the LOS and HI with a temperature below 100 K and then calculate for each pixel the HI mass-weighted LOS average. For \(\sigma_{\text{obs}}\) we correct the value obtained by BTS, \(\sigma_{\text{BTS}}\), for the contribution from the limited channel width of 1 km s\(^{-1}\) and thermal motions, i.e.

\[
\sigma_{\text{obs}} = \left(\sigma_{\text{BTS}}^2 - \frac{1}{\sqrt{8 \ln 2}} \left(\frac{1 \text{ km s}^{-1}}{\sqrt{\ln 2}}\right)^2 - c_s^2\right)^{1/2}, \tag{13}
\]

with \(c_s\) being the sound speed for HI gas, where, for simplicity, we assume an average temperature of 40 K (right panel of Fig. 6). The factor \(\sqrt{8 \ln 2}\) accounts for the conversion of channel width into the standard deviation. In the following we only consider pixels where \(\sigma_{\text{obs}} > 0\).

In the left and middle panel of Fig. 15 we plot the distribution of \(\sigma_{\text{real}}\) and \(\sigma_{\text{obs}}\) and its mean value (black line) as a function of \(N_{\text{HI,real}}\) for MC1-HD at \(t_{\text{evol}} = 2\) Myr for one direction assuming a distance of 150 pc for the beam size. We note that the following results also hold for the other clouds and times. First, we find that the scatter for \(\sigma_{\text{real}}\) appears to be somewhat larger than for \(\sigma_{\text{obs}}\). Second, for \(\sigma_{\text{real}}\) there is only a moderate increase with \(N_{\text{HI,real}}\) in particular for \(N_{\text{HI,real}} > 10^{21}\) cm\(^{-2}\), whereas for \(\sigma_{\text{obs}}\) the increase is more pronounced. The latter could be attributed to opacity broadening occurring for high \(N_{\text{HI,real}}\) (see black line in Fig. 3), which we expect to happen frequently, given the high optical depths found in
our MCs (Fig. 10). The presence of multiple Gaussian components in the spectrum (red line in Fig. 3), however, cannot explain the somewhat larger values of $\sigma_{\text{obs}}$ compared to $\sigma_{\text{real}}$: Although the result of the single-component fit for $\sigma_{\text{BTS}}$ will be broader than the velocity dispersion of the individual HI components, multiple components will also increase $\sigma_{\text{real}}$. We also note that the non-thermal velocity dispersions of HI of a few 1 km s$^{-1}$ reported here are in general in agreement with the velocity dispersion of dense gas ($n > 100$ cm$^{-3}$) in these clouds (Seifried et al. 2017).

The black lines in the right panel of Fig. 15 show the mean of log($\sigma_{\text{obs}}/\sigma_{\text{real}}$) for all clouds and projection directions at 2 Myr and an assumed distance of 150 pc. Despite the differences seen in the left and middle panel, $\sigma_{\text{obs}}$ appears to trace the actual velocity dispersion with a reasonable accuracy. For $N_{\text{HI,real}} \leq 10^{22}$ cm$^{-2}$, the mean of log($\sigma_{\text{obs}}/\sigma_{\text{real}}$) is one average within $\pm 0.3$ dex around a value of 0, which would indicate a perfect agreement. Also the standard deviations of the various distributions (grey lines) are about 0.2 – 0.3 dex. Hence, we argue that for typical HI column densities between $\sim 10^{20}$ cm$^{-2}$ and $\sim 10^{22}$ cm$^{-2}$, HISA observations are able to probe the non-thermal velocity dispersion of HI with an accuracy of a factor of $\sim 2$, even in the case of a limited spectral resolution of $\sim 1$ km s$^{-1}$. Only for very high HI column densities ($N_{\text{HI,real}} > 10^{22}$ cm$^{-2}$) the non-thermal velocity dispersion might be somewhat underestimated, which we tentatively attribute to the aforementioned opacity broadening of the absorption feature. We note that these results also hold for $t_{\text{evol}} = 3$ Myr and an assumed distance of 3 kpc.

Finally, in Fig. 16 we show the distribution of the Mach numbers $\sqrt{3} \sigma/c_s$ (assuming $T = 40$ K) for MC1-HD at 2 Myr. The values are inferred directly from the simulation using $\sigma_{\text{real}}$ (black lines) and from the HISA observation using $\sigma_{\text{obs}}$ (red lines). For other clouds, we find similar results. Overall, the HI gas is moderately supersonic with the distribution peaking around Mach numbers of a few although also values up to $\sim 10$ are reached. The observationally determined Mach numbers somewhat underrepresent the lowest values. This could be related to the aforementioned opacity broadening or a broadening due to the limited spectral resolution. The overall similarity between both distributions, however, confirms the accuracy of a factor of $\sim 2$ between $\sigma_{\text{real}}$ and $\sigma_{\text{obs}}$ reported before. Furthermore, the Mach numbers found are also in good agreement with recent HISA observations (Burkhart et al. 2015; Nguyen et al. 2019; Syed et al. 2020; Wang et al. 2020b).

4 DISCUSSION

4.1 The $N_{\text{HI}}$-PDF: Comparison with observations

As discussed in Section 3.1, we find that HISA measurements tend to underestimate the actual column density of cold HI by a factor of 3 – 10 in the outer parts of MCs and potentially by an even higher factor in the central, high density parts. We attribute this to (i) a large temperature variation of the HI gas and the assumption of a fixed $T_{\text{HISA}}$ and (ii) the effect of noise in the brightness temperature measurements in particular for regions of high optical depth.

This underestimation is again demonstrated in Fig. 17 where we show the area-weighted PDFs of $N_{\text{HI,real}}$ and $N_{\text{HI,obs}}$ for MC1-HD at $t_{\text{evol}} = 2$ Myr for one direction at an assumed distance of 150 pc. The latter is derived for $T_{\text{HISA}} = 40$ and 60 K (coloured lines). As expected, the peak of the $N_{\text{HI,obs}}$-PDF is shifted by a factor of 3 – 10 towards lower column densities with respect to that of the $N_{\text{HI,real}}$-PDF (black solid line). Moreover, also the shapes of the two PDFs are very different with important implications. First, the $N_{\text{HI,real}}$-PDF is significantly broader than the $N_{\text{HI,obs}}$-PDF. This indicates that the width of the $N_{\text{HI,obs}}$-PDF obtained from HISA observations might not be a good quantity to assess turbulence statistics (see e.g. Burkhart & Lazarian 2012, for an application to the $N_{\text{H,I,obs}}$-PDF). Second, the $N_{\text{HI,real}}$-PDF shows signs of a power-law tail at column densities above a few $10^{21}$ cm$^{-2}$, which is roughly proportional to $N^{-1.5}$, similar to that of the $N_{\text{HI,obs}}$-PDF (black dotted line, see also Kainulainen et al. 2009; Kritsuk et al. 2011; Girichidis et al. 2014; Syed et al. 2020).

We note that the integrated area under the curves is not necessarily unity as many pixels have no observed HI column density (Fig. 4). Hence, the relative area under the curves gives the reader a direct indication of how many pixels are omitted (i.e. have $N_{\text{HI,obs}} = 0$) compared to the PDF of all HI.
power-law tail indicates that also the dense HI gas is undergoing gravitational collapse. We emphasise, however, that – as the dense gas ($N \geq 10^{21} \text{ cm}^{-2}$) is predominantly molecular (bottom panel of Fig. 1) – the gravitational force in this range is dominated by gas in form of H$_2$ with which the HI is mixed.

Our $N_{\text{HI,real}}$-PDFs are markedly different from PDFs found in recent HISA observations (Burkhart et al. 2015; Imara & Burkhart 2016; Syed et al. 2020; Wang et al. 2020b) which find a log-normal shape indicating that the cold HI is not gravitationally unstable. This apparent contradiction could have its origin in possible observational biases, indicated by some striking similarities between our synthetic HI observations and that of the aforementioned authors: Our synthetic and actually observed $N_{\text{HI,obs}}$-PDFs are of roughly lognormal shape, are in a similar range ($N_{\text{HI}} = 10^{20}$–$21 \text{ cm}^{-2}$), and are at significantly lower column densities than that of either H$_2$ and the total HI observed in emission (Syed et al. 2020; Wang et al. 2020b) or that of $N_{\text{HI,obs}}$ measured via dust emission (Burkhart et al. 2015; Imara & Burkhart 2016). Taking these similarities into account, we suggest that there indeed is an observation bias in the shape of observed $N_{\text{HI}}$-PDFs. This could be particularly pronounced at the high end of the PDF, which is often characterised by a power-law. In addition, the $N_{\text{HI}}$ values obtained should rather be considered as lower thresholds. We note, however, that an MC at an very early evolutionary stage might not have undergone gravitational collapse, i.e. might not yet have developed high column densities ($\geq 10^{21} \text{ cm}^{-2}$) and the associated power-law tail in the $N$-PDF. Therefore, for such an MC, the assessment of the HI column densities and masses via HISA observations might still be somewhat better compared to the findings presented here.

### 4.2 Multiple HI-H$_2$ transitions: Comparison with analytical results

As stated before, recent semi-analytical works predict that, for ISM conditions comparable to that of the solar neighbourhood, the transition from HI to H$_2$ occurs at column densities of $\lesssim 10^{21} \text{ cm}^{-2}$ (Krumholz et al. 2008, 2009; Sternberg et al. 2014; Bialy & Sternberg 2016). These models also suggest an upper limit of $N_{\text{HI}}$ around this value. Contrary to that, for the clouds simulated in this work, we find HI column densities partly well above this value (see Figs. 1 and 2). Also recent observations of W43 (Motte et al. 2014; Bihr et al. 2015) and indirect estimates towards Perseus (Okamoto et al. 2017) and clouds outside the Galactic plane (Fukui et al. 2014, 2015) have revealed HI column densities of up to a few $10^{22} \text{ cm}^{-2}$, thus well comparable to our findings, but in apparent contradiction to the theoretical predictions. Also observations of the Magellanic Clouds by Welty et al. (2012) seem to challenge the prediction for the value of $N_{\text{HI,real}}$, where the transition to H$_2$ dominated gas is supposed to occur (Krumholz et al. 2008, 2009; McKee & Krumholz 2010).

However, strictly speaking the suggested, upper limit around $10^{21} \text{ cm}^{-2}$ only applies to a single HI-H$_2$ transition. As pointed out by Motte et al. (2014), a possible solution for this contradiction could thus be the presence of several transitions along the LOS (see also Bialy et al. 2017a). Indeed, the occurrence of multiple absorption components (see Fig. 3) and the highly complex structure of our simulated MCs (see Fig. 2) and of real MCs indicates that the assumption of a single HI-H$_2$ transition might be an oversimplification and that rather several transitions are present.

In the following we test this hypothesis in physical space (i.e. considering spatial distances) as opposed to parts of the analysis done before in velocity space (Section 3). For this purpose, we identify the number of HI-density peaks along rays which intersect the entire length of the zoom-in regions and which are distributed uniformly over the entire area of the emission maps with a spacing of 0.24 pc. For this purpose, we first determine the profile of the logarithm of the HI density, $\log(n_{\text{HI}})$, along each ray, i.e. along the LOS of each pixel. A few selected profiles are shown in Fig. A4, demonstrating their variability for different pixels, which necessitates a more systematic study. Second, we identify the positions of the peaks of $\log(n_{\text{HI}})$ along the LOS, $l_{\text{peak},k}$, as well as the position of their left and right base, $l_{\text{base,left},k}$ and $l_{\text{base,right},k}$ respectively (see Fig. 18 for a schematic view). The base of a peak (green squares) is the minimum of the density profile between this and the neighbouring peak. In order to account for the influence of small-scale density fluctuations, we discard peaks which (i) have a prominence (blue arrow) of less than 0.5 (in log-space), or (ii) have a peak height $\log(n_{\text{HI,peak}})$ of less than 0.5 ($n_{\text{HI,peak}} \approx 3 \text{ cm}^{-3}$), or (iii) are separated from the next (and higher) peak by less then 2 grid cells along the LOS.

Furthermore, we note that $n_{\text{HI}}$ comes from the chemical network...
implemented in the simulations, a low \( n_{\text{HI}} \) could indicated either a low total gas density or a high gas density, where hydrogen is already predominantly in form of \( \text{H}_2 \).

Next, we calculate the column density of each HI-density peak as

\[
N_{\text{HI,peak},k} = \int_{b_{\text{base, left},k}}^{b_{\text{base, right},k}} n_{\text{HI}} \, dl,
\]

and determine the average column density, \( \langle N_{\text{HI,peak}} \rangle \), of all peaks along a given LOS. In addition, we identify the most massive peak, i.e. the peak which accumulates most of the HI along the given LOS, and determine its column density, \( N_{\text{HI,peak, max}} \). In Fig. 19, we plot the number of peaks along each LOS, \( \langle N_{\text{HI,peak}} \rangle \), \( N_{\text{HI,peak, max}} \) and the ratio \( N_{\text{HI,peak, max}}/N_{\text{HI,real}} \) for MC1-HD at 2 Myr along one direction. The last value shows how much the most massive peak contributes to the overall HI column density.

The left panel of Fig. 19 shows that there are indeed up to \( \sim 10 \) HI-density peaks along the LOS as suggested by Bialy et al. (2017a) for the case of W43. There is a moderate tendency of a higher number of peaks towards the center of the MC, i.e. with increasing \( N_{\text{HI,real}} \), probably caused by the filamentary substructure of the MCs. This increase causes \( \langle N_{\text{HI,peak}} \rangle \) to remain below \( 10^{21} \, \text{cm}^{-2} \) for the vast majority of rays (second panel of the left), only for about 5 – 25% of all rays (depending on the cloud, direction, and time) it exceeds this value. Hence, on first view this appears to be in rough agreement with analytical predictions (Krumholz et al. 2008, 2009; Sternberg et al. 2014; Bialy & Sternberg 2016).

However, the column density of the dominant peak, \( N_{\text{HI,peak, max}} \), exceeds the value of \( 10^{21} \, \text{cm}^{-2} \) for a large number of rays (second panel from the right). We find that \( N_{\text{HI,peak, max}} \) exceeds \( 10^{21} \, \text{cm}^{-2} \) for 30 – 50% of the rays, i.e. more than twice as often as the corre-
sponding fraction for \( \langle N_{\text{HI,peak}} \rangle \). Again, the exact fraction depends on the considered MC, direction, and time. However, we do not see any dependence on the presence or absence of magnetic fields in the simulations, despite the fact that the field is dynamically important for the overall (chemical) evolution of the MCs (see Seifried et al. 2020a,b). Furthermore, the dominant peak accounts on average for about 40 – 60% and even more of \( N_{\text{HI,real}} \) (right panel) and is thus indeed dominating the overall HI budget of the clouds. Hence, these findings are in contrast to the theoretical models, and we will discuss their implications in detail in the following section.

5 IS THERE MORE COLD HI THAN THOUGHT?

5.1 The theoretical perspective

As discussed so far, our results indicate that semi-analytical models tend to underestimate the maximum column density of cold HI in MCs. This can be attributed to several reasons. First, as suggested by Motte et al. (2014) and Bialy et al. (2017a) and explicitly shown here for the first time, in realistic models of MCs there appear up to \( \sim 10 \) HI-H\(_2\) transitions along the LOS. Secondly, non-equilibrium effects can increase the HI content of MCs due to the limited time available for H\(_2\) to form (Glover & Mac Low 2007b; Glover et al. 2010; Mac Low & Glover 2012; Motte et al. 2014). We investigate this effect by artificially evolving the chemistry for a selected snapshot to chemical equilibrium (see Appendix A for details). Doing so, we find that this reduces the HI content in our MCs by a factor of 2 – 2.5 (see Fig. A3). Hence, the assumption of chemical equilibrium in semi-analytical models indeed results in too low HI abundances compared to the actual non-equilibrium HI present in dynamically evolving MCs.

In addition, we here suggest a third reason why even for a single HI-H\(_2\) transition \( N_{\text{HI}} \) could be higher (center-right panel of Fig. 19) than predicted by semi-analytical models with a 1-dimensional geometry (Krumholz et al. 2008, 2009; Sternberg et al. 2014; Bialy & Sternberg 2016). For this purpose, we consider the effect of assuming an idealised plane-parallel slab in more detail. In such a configuration the gas in the cloud is irradiated only from one side, i.e. a parcel of gas having a high column density in the slab direction will receive very little radiation. In other words, for a plane-parallel slab – at a given total hydrogen nuclei column density \( N_{\text{HI,tot}} \) towards the direction of the incident radiation – the extinction is maximal as radiation coming from other possible directions is neglected. In consequence, also the amount of HI is minimal due to the lack of H\(_2\) dissociation. The same chain of arguments can also be made for a spherically symmetric configuration.

In contrast, under realistic conditions in turbulent MCs, there may exist large low-density voids through which the radiation can propagate into the cloud (almost) unhindered. This is sketched in the left panel of Fig. 20: a dense region shielded completely against UV radiation from one direction (horizontal) can still be irradiated from another direction (vertical). Hence, if observed from the horizontal direction, this region would appear completely optically thick (i.e. a high value of \( A_V^{1D-\text{slab}} \)), corresponding to the assumption of a plane-parallel configuration. In reality, however, the region might still receive up to \( \sim 50\% \) of the radiation coming from the vertical direction, i.e. the real \( A_V \) might be significantly lower\(^9\). In consequence, H\(_2\) might still be dissociated by UV radiation even in the central regions of MCs thus increasing the amount of HI. Considering a 3-dimensional graphical representation of MCI1-HD at 2 Myr (right panel of Fig. 20) indeed shows that there exist these large low-density voids (bluish) through which radiation can travel (almost) unattenuated thereby dissociating H\(_2\) in the densest regions (reddish). Depending on the considered snapshot, 55 to 83% of the volume of the zoom-in regions is covered by gas with densities below 1 cm\(^{-3}\). We note that also for internal, stellar radiative feedback the actual cloud’s substructure and shielding parameters have a similar importance (Haid et al. 2019).

One could interpret this effect as either increasing the radiation strength or decreasing the effective shielding of UV radiation in the cloud. In the semi-analytical models of Sternberg et al. (2014), Bialy & Sternberg (2016) and Bialy et al. (2017b), this is parametrised by their parameter \( \alpha G \). There, \( \alpha \) indicates the radiation strength and \( G \) the shielding factor including H\(_2\) self-shielding and dust attenuation; the lower \( G \), the better the radiation is shielded. We speculate that large density voids can easily reduce the (self-)shielding of the surrounding dust and H\(_2\) by an order of magnitude. The corresponding increase of \( G \) (and thus \( \alpha G \)) in the models of the aforementioned authors would thus increase their \( N_{\text{HI}} \) (see e.g. equation 40 and figure 9 in Sternberg et al. 2014) by a factor of a few. This behaviour is thus in general accordance with the interpretation presented here and would bring their upper limits for \( N_{\text{HI}} \) closer to the values reported here.

5.2 The observational perspective

As observational works on Galactic (e.g. Savage et al. 1977; Kavars et al. 2003, 2005; Klaassen et al. 2005; Gillimon et al. 2006; Křečko et al. 2008; Barriault et al. 2010; Krčo & Goldsmith 2010; Lee et al. 2012, 2015; Stanimirović et al. 2014; Burkhart et al. 2015; Imara & Burkhardt 2016) and extragalactic scales (e.g. Wong & Blitz 2002; Browning et al. 2003; Blitz & Rosolowsky 2004, 2006; Bigiel et al. 2008; Wong et al. 2009) tend to find upper limits of \( N_{\text{HI}} \) around \( 10^{21} \) cm\(^{-2}\) (equivalent to 8 M\(_{\odot}\) pc\(^{-2}\)), it could be argued that the high HI column densities in MCs and the associated underestimation of HI by a factor of 3-10 suggested in this work are rather exceptional. Given

\(^9\) In order to calculate the average visual extinction in a point, one must not take the average \( \langle A_V \rangle \) over different directions \( i \), but must take the logarithm of \( \langle \exp(-yA_V) \rangle \), as the latter describes the amount of incident radiation. The latter averaging puts more emphasis on the low-\( A_V \) directions.
Figure 19. Maps of the number of HI-density peaks per LOS, the mean HI column density per HI-density peak, the HI column density of the most prominent HI-density peak and the ratio of this value to the total HI column density (from left to right). The left-most panel demonstrates the occurrence of up to 10 peaks, which corresponds to the same number of HI-H$_2$ transitions along the LOS. In consequence, the average HI column density per peak (center-left panel) mostly matches the theoretical predictions of $N_{\text{HI, peak}}$ ($\geq 10^{21}$ cm$^{-2}$). However, the most prominent HI-density peak (center-right panel) often has a column density that is considerably larger than this value and contributes significantly to $N_{\text{HI, real}}$ ($\geq 40\%$, right panel).

Figure 20. Left: Sketch of a situation in a MC (dense gas depicted with grey). Here, the observer’s assumption of a 1-dimensional, plane-parallel configuration as applied in semi-analytical models overestimates the shielding (black arrow, $A_V^{1D-slab}$) and thus underestimates the amount of HI in the cloud’s center. Under realistic conditions, radiation (red arrows) dissociating H$_2$ might be able to propagate through low-density voids (white areas) towards the center of the cloud. This results in a rather low actual visual extinction, $A_V^{\text{real}}$. Right: Volume rendering of MC1-HD at 2 Myr showing the highly complex and filamentary structure of the dense gas (reddish). Large low-density voids (bluish) are recognisable through which radiation can propagate into the cloud thus increasing the HI fraction. This resembles the simplified picture shown in the left panel.

the various reasons discussed in this work, however, we argue that the underestimation is indeed rather common. In addition, carefully investigating the observational literature we find further evidence that an upper $N_{\text{HI}}$ threshold of $10^{21}$ cm$^{-2}$ could be artificial:

- Some of the aforementioned HI measurements are emission observations, and for some of them the contribution of the cold HI might be omitted, e.g. Lee et al. (2012) report two absorption features in their HI spectra of the Perseus molecular clouds, which they do not consider in their $N_{\text{HI}}$ calculations. Furthermore, other highly-resolved HI emission observations indeed find HI column densities well above $10^{21}$ cm$^{-2}$ (Motte et al. 2014; Bihr et al. 2015; Syed et al. 2020; Wang et al. 2020b).

- Observations in emission often assume optically thin emission. However, as large parts of MCs have optical depths well above 1 (Fig. 10), optical depth corrections are crucial to infer the correct HI column densities. This is in line with observations of W43 by Bihr et al. (2015), who find an increase in HI by a factor of $\sim 2$ compared to the optically thin assumption (Motte et al. 2014). Similar correction factors were found for indirect HI measurements of off-Galactic plane gas (Fukui et al. 2015) and the Perseus molecular cloud (Okamoto et al. 2017), although for the latter Lee et al. (2012, 2015) argue for a correction of $\sim 20\%$ only. For the THOR survey Wang et al. (2020a) determined the correction factor to $\sim 31\%$. All these correction factors are lower limit as optical depth estimates have an upper limit set by the observational noise (Bihr et al. 2015, and our Eq. 11).

- Finally, extragalactic observations typically have spatial resolutions of a few 100 pc. Hence, they average over clouds and the surrounding diffuse ISM, which can lower the maximum value of
\(N_{\text{HI}}\) significantly. We show this by calculating the total HI column density (now including again HI with \(T > 100 \text{ K}\)) for our simulations using pixels with a side length of 31.5, 8 and 2 pc (black, magenta and green dots, respectively, in the top panel of Fig. 1). Overall, this reduces the maximum \(N_{\text{HI}}\) values to \(\sim 10^{22} \text{ cm}^{-2}\) for 2 pc pixels and even further to \(\sim 10^{21} \text{ cm}^{-2}\) for 31.5 pc pixels, i.e. in parts by more than one order of magnitude compared to the maximum around a few \(10^{22} \text{ cm}^{-2}\) for our highest resolution.

To summarize, we suggest that (i) HI column densities well beyond \(10^{21} \text{ cm}^{-2}\) \((\sim 8 M_{\odot} \text{ pc}^{-2})\) are significantly more common in MCs than thought (Fig. 5) and (ii) also the entire mass of cold HI gas in clouds could be a factor of a few higher \((\geq 3)\) than thought (Fig. 12). Vice versa, we argue that (iii) 1-dimensional PDR models might underestimate the amount of cold HI in a typical HI-H\(_2\) transition layer as those are in general not plane-parallel or spherically symmetric objects and (iv) HI observations might underestimate the HI content in MCs by a factor of a few due to the various systematic observational biases discussed in this work.

6 CONCLUSIONS

In this work we present the first fully self-consistent synthetic HI 21 cm observations including self-absorption (HISA) of MCs simulated within the SILCC-Zoom project. The synthetic observations are based on 3D MHD simulations including a non-equilibrium HI-H\(_2\) chemistry, detailed radiative transfer calculations, and realistic observational effects like noise and a limited spectral and spatial resolution adapted to actual observations. In addition, we analyse in detail the actual content of cold HI in the simulated clouds and compare it with the results obtained from the synthetic HISA observations. We summarize our main results in the following.

- We show that HISA observations, which assume a fixed HI temperature, typically tend to underestimate column densities of cold HI, \(N_{\text{HI}}\), and the total cold HI mass in molecular clouds by a factor of 3 – 10. This effect is particular pronounced towards the central regions, which frequently reach column densities up to \(\geq 10^{22} \text{ cm}^{-2}\). It occurs for MCs under various conditions, e.g. with and without dynamically important magnetic fields.
- We show that the underestimation of \(N_{\text{HI}}\) in HISA observations can be attributed to the following two effects. (i) The large temperature variations of cold HI \((-10 \text{ K up to } 100 \text{ K})\) make a reliable determination of \(T_{\text{HISA}}\) not possible. This leads to the fact that the real \(T_{\text{HISA}}\) and thus \(N_{\text{HI}}\) are underestimated and that velocity channels have to be omitted for the calculation of \(N_{\text{HI}}\). (ii) Observed noise and the emission of warm HI in the foreground either reduce the inferred optical depth or – as before – cause individual velocity channels to be omitted for the calculation of \(N_{\text{HI}}\). This effect is particularly pronounced in regions of high optical depth. In combination, both effects \((i + ii)\) can lead to an artificial upper limit in observation of \(N_{\text{HI,obs}}\) around \(10^{21} \text{ cm}^{-2}\).
- We suggest a method to correct for the aforementioned omission of high optical depth channels. This correction reduces underestimation of the HI mass budget by a factor of 1.5 – 2.
- We find that clouds typically have HI optical depths around 1 – 10. This implies that the optically thin HI assumption is usually not suitable and that optical depth corrections are essential when calculating \(N_{\text{HI}}\) from HI observations.
- We show that the high HI column densities \((\geq 10^{22} \text{ cm}^{-2})\) can (in parts) be attributed to the occurrence of up to 10 individual HI-H\(_2\) transitions along the LOS. This emphasises the necessity of Gaussian decomposition algorithms to fully analyse the individual components that constitute the HISA spectra.
- Also for individual HI-H\(_2\) transitions, \(N_{\text{HI}}\) frequently exceeds a value of \(10^{21} \text{ cm}^{-2}\), thus challenging 1-dimensional, semi-analytical models. This can be attributed to non-equilibrium chemistry effects, which are included in our models, and to the fact that HI-H\(_2\) transitions usually do not have a 1-dimensional geometry, i.e. to the fractal structure of MCs.
- We demonstrate that \(N_{\text{HI}}\)-PDFs obtained from HISA observations with a fixed temperature assumption should be considered with great caution both concerning the position of the peak and the width. Due to the underestimation of HI, the observed PDFs appear to lack the high-\(N_{\text{HI}}\) end, which in reality seems to be characterised by a power-law.
- Finally, we show that the cold HI gas in MCs is moderately supersonic with Mach numbers of up to a few. The corresponding non-thermal velocity dispersion can be determined via HISA observations with an accuracy of a factor of \(\sim 2\).

To summarize, our result indicate that measuring the HI content in MCs via HISA observations is a challenging task and that the amount of cold HI in MCs could be a factor of 3 – 10 higher than previously thought.

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DATA AVAILABILITY

The data underlying this article can be shared for selected scientific purposes after request to the corresponding author.

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In the left panel of Fig. A1 we show ratio of the observed and actual HI column density for the same snapshot as in Fig. 4, now for the case of no noise and a high spectral resolution (left) and an assumed distance of 3 kpc including observational effects like noise (right) inferred from the HISA observation assuming $\tau_{\text{HISA}} = 40$ K. The poor match is found also for the ideal observations (left) supporting the conclusion that strong temperature variation are (in parts) the cause for it.

In Fig. A2 we show the inferred HI column density for MC1 at 2 Myr along the $x$-direction used for the analysis in Section 4.2. The profiles show a large variability concerning the number of peaks, their widths and positions. Some of the peaks are discarded as they do not have either the required minimum prominence or the minimum peak height.

**APPENDIX A: SUPPLEMENTARY FIGURES**

In the left panel of Fig. A1 we show ratio of $N_{\text{HI,obs}}$ to $N_{\text{HI,real}}$ for MC1-HD at 2 Myr, where $N_{\text{HI,obs}}$ is calculated from the noiseless, high-spectral resolution (200 m s$^{-1}$) maps obtained directly from RADMC-3D using $T_{\text{HISA}} = 40$ K. We find a comparable poor match as for the case when observational effects are included. In the right panel we show the results obtained assuming a distance of 3 kpc (again including observational effects). Little differences are found compared to a distance of 150 pc (compare with the bottom middle panel of Fig. 4). This result holds also for the other snapshots considered in this work.

In Fig. A2 we show the inferred HI column density for MC1-HD at 2 Myr now including the correction in optically thick regions. The results are discussed in Section 3.3.1.

In Fig. A3 we show the effect of post-processing the chemical state of one of our simulation, i.e. pushing it towards a chemical equilibrium state. This is done exemplarily for MC1-HD taking two snapshots at $t_{\text{evol}} = 2$ and 3 Myr considered in this work. We stop the magneto-hydrodynamical evolution at these time, i.e. freeze the total density, velocity, etc., and only evolve the chemistry for additional 100 Myr. The chemical post-processing time (measured from $t_{\text{evol}}$ onwards) is denoted as $t_{\text{chem}}$. The HI content quickly drops to 40 – 50% of the the actual (non-equilibrium) HI content (at $t_{\text{chem}} = 0$), which indicates that equilibrium models generally underestimate the amount of HI in MCs.

In Fig. A4 we show the log($n_{\text{HI}}$)-profile for 5 selected pixels for MC1 at 2 Myr along the $x$-direction used for the analysis in Section 4.2. The profiles show a large variability concerning the number of peaks, their widths and positions. Some of the peaks are discarded as they do not have either the required minimum prominence or the minimum peak height.

**APPENDIX B: AN APPROXIMATION FOR THE OPTICAL DEPTH**

In order to estimate the optical depth of the HI gas in our simulations, we repeat one radiative transfer calculation for MC1-HD, however, now setting the background temperature to 0 K. Hence, the observed emission purely stems from the HI gas in the zoom-in region.

As noted in Eq. 7, but now written down for a single velocity channel, the HI column density is given by

$$dN_{\text{HI}} = 1.8224 \times 10^{18} \, \text{cm}^{-2} \, \frac{T_{\text{bg}}}{1 \, \text{K}} \, \frac{\nu}{1 \, \text{km s}^{-1}} \, dv.$$  \hfill (B1)

Next, we consider the expression

$$T_{\text{rad}} = \frac{h \nu_{\text{HI}}}{k_{\text{B}}} \left( f(T_{\text{bg}}) - f(T_{\text{bg}}) \right) \left( 1 - e^{-\tau} \right),$$  \hfill (B2)

for the observed brightness temperature $T_{\text{rad}}$. Here, $T_{\text{bg}}$ is the background temperature, $h$ is the Planck constant, $k_{\text{B}}$ the Boltzmann constant, $\nu_{\text{HI}} = 1420$ MHz the frequency of the HI 21-cm line, and

$$f(T) = \frac{1}{\exp \left( \frac{h \nu}{k_{\text{B}} T} \right) - 1}. $$  \hfill (B3)

Considering that $h \nu_{\text{HI}} / k_{\text{B}} = 0.068$ K $\ll T_{\text{bg}}$ and $f(T_{\text{bg}}) \approx f(T_{\text{bg}})$, and inserting Eq. B2 in Eq. B1 yields

$$dN_{\text{HI}} = 1.8224 \times 10^{18} \, \text{cm}^{-2} \, \frac{\tau}{1 - e^{-\tau}} \, \frac{T_{\text{rad}}}{1 \, \text{K}} \, \frac{dv}{1 \, \text{km s}^{-1}}.$$  \hfill (B4)

We now integrate over all velocity channels using a definition of a $T_{\text{rad}}$-weighted, channel-averaged approximation of the optical depth

$$N_{\text{HI}} = 1.8224 \times 10^{18} \, \text{cm}^{-2} \, \langle \tau \rangle \frac{T_{\text{rad}}}{1 \, \text{K}} \, \frac{dv}{1 \, \text{km s}^{-1}}.$$  \hfill (B5)

Here, we have defined

$$\langle \tau \rangle = \frac{\int \frac{\tau}{1 - e^{-\tau}} T_{\text{rad}} dv}{\int T_{\text{rad}} dv}.$$  \hfill (B6)

The interpretation of $\langle \tau \rangle$ as an approximation for a $T_{\text{rad}}$-weighted, channel-averaged optical depth can be understood, when considering the fact that $\frac{\tau}{1 - e^{-\tau}} \rightarrow \tau$ with $\tau \rightarrow \infty$. For $\tau = 1$, the expression $\frac{\tau}{1 - e^{-\tau}}$ is only $\sim 50\%$ larger than $\tau$, for $\tau = 2$ only $\sim 15\%$. For optically thin regions ($\tau < 1$), the approximation is not applicable. However, as in

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**Figure A1.** Map of the ratio of the observed and actual HI column density for the same snapshot as in Fig. 4, now for the case of no noise and a high spectral resolution (left) and an assumed distance of 3 kpc including observational effects like noise (right) inferred from the HISA observation assuming $T_{\text{HISA}} = 40$ K. The poor match is found also for the ideal observations (left) supporting the conclusion that strong temperature variation are (in parts) the cause for it.
Figure A2. Same as in Fig. 4, but now with the correction in optically thick regions where for channels, where Eq. 6 does not yield any result, we assume an optical depth given by $\tau_{\text{HI, noise}}$ (Eq. 11). Overall, the match in the moderately dense gas is improved, whereas in the most densest parts $N_{\text{HI, real}}$ is still significantly underestimated.

Figure A3. Evolution of the HI content of MC1-HD when post-processing the chemical abundances for a time of $t_{\text{chem}}$ relative to the actual (non-equilibrium) HI content (at $t_{\text{chem}} = 0$). Assuming chemical equilibrium would reduce the HI content by a factor of $2 - 2.5$.

Figure A4. Profiles of $\log(n_{\text{HI}})$ for 5 selected pixels for MC1 at 2 Myr along the $x$-direction. The profiles show a large variability. In addition we show the minimum prominence and minimum peak value (both in black), which a peak must have to be considered and not discarded.

Section 3.3 we are mainly interested in high optical depth regions, we consider our definition of $\langle \tau \rangle$ as a reasonable approximation for the typical optical depth of HI in our simulations.

Next, using the integrated intensity from the radiative transfer calculations without any background radiation field and the real HI column density from the simulation data, $N_{\text{HI, real}}$, we can now calculate $\langle \tau \rangle$ via

$$\langle \tau \rangle = \frac{N_{\text{HI, real}}}{1.8224 \times 10^{18} \text{cm}^{-2} \int \frac{T_{\text{rad}}}{\text{K}} \frac{dv}{1 \text{ km s}^{-1}}}.$$  \hspace{1cm} (B7)

where the denominator is describing the HI column density obtained from HI emission under the assumption of optically thin emission.

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