A scalable, tunable qubit, based on a clean DND or grain boundary D-D junction

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Unique properties of a ballistic DND or grain boundary D-D junction, including doubly degenerate ground state with tunable potential barrier between the "up" and "down" states and non-quantized spontaneous magnetic flux, make it a good candidate for a solid state qubit. The role of quantum "spin" variable is played by the sign of equilibrium superconducting phase difference on the junction, which is revealed in the direction of spontaneous supercurrent flow in equilibrium.

Possibilities of design-specific simultaneous operations with several integrated qubits are discussed.

The pronounced shift from "software" to "hardware" in theoretical research on quantum computing (QC) is a good measure of growing confidence that QC can be realized on practically interesting scale (of at least 10 qubits). Though first experimental realizations of QC used such technologies as NMR and ion trapping, the problem of scalability for such approaches still looks formidable. Therefore much effort is directed at search for a practical solid state qubit (SSQ), with natural candidates being such mesoscopic devices as quantum dots, mesoscopic Josephson junctions, and superconducting single-electron transistors (parity switches). The evident advantage of a SSQ is scalability, where all the potential of existing solid state technologies could be used, while among the problems the main are (1) to achieve quantum beatings between distinguishable states of a single qubit, (2) to prevent loss of coherence during calculations, and (3) to minimize statistical dispersion of the properties of individual qubits.

The problems (1) and (2) pose specific difficulties in an SSQ due to huge number of degrees of freedom coupled to it, and to necessity to fine-tune two states of a mesoscopic system chosen as working ones to a resonance.

A possibility to circumvent these obstacles is presented by Josephson systems with d-wave cuprates, which violate time-reversal symmetry and as a result have doubly degenerate groundstate with a potential for quantum beatings (macroscopic quantum tunneling), or at least quantum noise. Ioffe et al. recently incorporated this property in their "quiet qubit" design, which uses tunneling (SID) or dirty SND junctions with equilibrium phase difference \( \phi_0 = \pm \pi/2 \).

Let us consider a device shown in Fig.1a. Its main part is a clean mesoscopic D-D junction (i.e. ballistic DND or D-(grain boundary)-D junction). Though we will concentrate on the DND qubit design, the same considerations are applicable, mutatis mutandis, to high quality grain-boundary D-D junctions, where recently an analogous current-phase dependence was observed. The grain boundary region where the superconducting gap is suppressed, is naturally modeled by a normal conductor with the same lattice parameters and chemical potential as in the superconducting banks, which only enhances the amplitude of purely Andreev scattering in the system.

The terminal B of the junction is formed by a massive d-wave superconductor; in a multiple-qubits system, they will all use it as a common "bus" bar. The terminal A is small enough to allow - when isolated - quantum phase fluctuations. It is essentially the sign of the superconducting phase difference \( \phi \) between the terminals A and B that plays the role of "spin variable" of quantum computing. The collapse of the wave function is achieved by connecting the terminal A with the external source of electrons ("ground") , thus blocking the phase fluctuations due to phase-number uncertainty relation \( \phi = \pi \). So far, the best way to do this is presented by using a "parity key", PK (superconducting single-electron transistor), which only passes Cooper pairs, and only at a certain gate voltage \( V_g \). Other parity keys, with different parameters, are used to link adjacent qubits, allowing for controllable entanglement (Fig.1b).

Such an architecture allows reasonably easy way to integrate a large number of qubits in a 1D or 2D matrix. We will see that it also provides a natural way of preparing all qubits on the same bus in the same initial quantum state, thus facilitating the implementation of error correction algorithms.

The readout of the state of a qubit is simplified by the presence of small spontaneous, non-dissipative currents and spontaneous fluxes (of order \( 10^{-6} \Phi_0 \) depending on the setup) concentrated in the central part of the DND junction, which have opposite directions in two degenerate equilibrium states. While too small to lead to unwanted inductive coupling between the qubits or decoherence, they can be still used to read out the state of the qubit once it was collapsed in one of the states with \( \pm \phi_0 \), e.g. using the magnetic force microscope tip M (which is removed during the computations). Collapsing and reading processes are thus time separated, and the computation results will be automatically preserved for the time limited only by the thermal fluctuations.

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A necessary condition for a qubit to work is \( t_{\text{tunneling}} < t_{\text{gate application}} < t_{\text{d(ecoherence)}} \). Here a clean DND junction has a very important advantage following from the fact that the absolute value of equilibrium phase difference, \( |\varphi_0| \), can now vary from 0 to \( \pi \) depending on the angle \( \Omega \) between the crystal axes of the d-wave superconductors and the ND boundary \([12]\). In practice, since the lattice structure of the d-wave superconductors allows only a limited set of easy cleavage directions, e.g., (010) and (100), the equilibrium phase can be varied by preparing a steplike ND interface, with the equilibrium phase determined by the relative weight of Andreev zero- and \( \pi \)-levels (coupled to the lobes of d-wave order parameter in A and B with the same or opposite sign respectively), produced by such an arrangement. Since the shape of the effective potential barrier between states with \( \pm \varphi_0 \) depends on \( \varphi_0 \), it can now be varied. Therefore the tunneling rate can be chosen in exponentially wide limits to achieve the optimal performance. (In an SND junction the equilibrium phase can also be varied, but it cannot be made less than \( \pi(\sqrt{2} - 1)/\sqrt{2} \) \([2]\).) Besides, this allows to fix the working interval of the device to \( |\varphi| \leq \pi \), since due to exponential dependence of the tunneling amplitude on the barrier action, for \( |\varphi_0| < \pi/2 \) the tunneling to the states in the next cell, \( \varphi_0 \rightarrow \varphi_0 \pm 2\pi \), can be completely neglected, whatever the inductance of the system.

The supercurrent through the normal part of the system is carried by a set of Andreev levels formed by reflections at the ND boundaries \([13,14]\). We will calculate it using the quasiclassical approach following from Eilenberger equations \([15]\) which is well suited to our problem. Here we can consider "Andreev tubes" along the quasiparticle trajectories in the normal part of the system \([12,16,17]\) each carrying the supercurrent density which in our case should be written as

\[
\mathbf{j}(\mathbf{r}, \mathbf{n}) = j_c \sum_{p=1}^{\infty} \frac{(-1)^p \mathcal{L}[\mathbf{r}, \mathbf{n}]}{l_T} \sin \frac{p \varphi_0[\mathcal{L}[\mathbf{r}, \mathbf{n}]]}{\sinh \frac{p \mathcal{L}[\mathbf{r}, \mathbf{n}]}{l_T}} e^{-\mathcal{L}[\mathbf{r}, \mathbf{n}]/l_{\text{imp}}},
\]

where \( \mathcal{L}[\mathbf{r}, \mathbf{n}] \) is the length of the quasiclassical trajectory linking two superconductors which passes through the point \( \mathbf{r} \) in the direction \( \mathbf{n} \); \( \varphi[\mathcal{L}[\mathbf{r}, \mathbf{n}]] \) is the phase gain along this trajectory (in this paper we do not concern with the effects of current-induced field, and therefore in the absence of external fields this is simply the phase difference between the superconductors on the ends of the trajectory, including the extra \( \pi \) if \( \mathcal{L}[\mathbf{r}, \mathbf{n}] \) connects the lobes of the d-wave order parameter with opposite signs \([12]\)). The normal metal coherence length \( l_T = v_F/2\pi k_B T \), and \( l_{\text{imp}} \) takes into account effects of weak elastic scattering by nonmagnetic impurities (in the ballistic regime, by definition, \( l_{\text{imp}} \gg S/\Pi \)). We have used standard approximation of steplike behaviour of the order parameter at the ND boundary, and neglected the own magnetic field of the supercurrent \([19]\).

The total supercurrent density at a point \( \mathbf{r} \) is thus given by

\[
\mathbf{j}(\mathbf{r}) = \int_0^{\pi} \frac{d\theta}{\pi} \mathbf{j}(\mathbf{r}, \mathbf{n}(\theta)).
\]

Calculating the total current flowing in \( \Lambda \), we find in the limit \( l_{\text{imp}}, l_T \rightarrow \infty \)

\[
I(\varphi) = \frac{2j_c W}{\pi} \left[ \frac{1 + Z(\Omega)}{2} F(\varphi) + \frac{1 - Z(\Omega)}{2} F(\varphi + \pi) \right],
\]

where \( F(\varphi) \) is the 2\( \pi \)-periodic sawtooth of unit amplitude, and the imbalance factor \( Z(\Omega) \) determines the equilibrium phase difference \([12]\)

\[
|\varphi_0| = \left| \frac{1 - Z(\Omega)}{2} \right| \pi.
\]

In the setup of Fig.1a

\[
|\varphi_0(\Omega)| = \frac{\sin |\Omega|}{\sqrt{2}} \pi.
\]

The current-phase dependence \([3]\) and corresponding Josephson energy \( E_J(\varphi) = \frac{\Phi_0}{\pi} \int_0^{\pi} d\varphi I(\varphi) \) are plotted in Fig.2, and current density distribution in the normal part of the system is shown in Fig.3. The vortex pattern is clearly seen. The spontaneous flux in the system is

\[
\Phi_s \sim \kappa(\Omega) \frac{j_c \Pi^2}{c} \sim \frac{\kappa(\Omega) e N_{\perp} v_F}{c} \sim \frac{1}{137} \frac{\kappa(\Omega) v_F}{c} N_{\perp} \Phi_0,
\]

\(2\)
where $\kappa < 1$ is a geometry-dependent attenuation factor (e.g., in SND junctions $\kappa = 0$ by symmetry if ND boundary is parallel to (100) or (010)); $N_{\perp}\sim \Pi/\lambda_F$ is the number of transport modes in the system.

Let us make some estimates. Taking the size of the system $\sim 10^3 \lambda$, $v_F \sim 10^7$ cm/s, we find that $\Phi_0 \sim \kappa \cdot 10^{-3} \Phi_0$. The magnetic moment of the spontaneous current will be of order $m_s \sim \kappa (N_{\perp} v_F)/c \sim \kappa \cdot 10^3 \mu_B$. The tunneling rate between the states is $\Gamma \sim \omega_0 \exp[-U(0)/\hbar \omega_0]$, where the frequency of oscillations near $\pm \varphi_0 = \omega_0 \sim \sqrt{N_{\perp} \epsilon_Q}/\hbar$ and the height of the potential barrier $U(0) \propto (\varphi_0/2\pi)^2$. Due to spatial Andreev quantization, there are no elementary excitations in the normal part with the system with energies below $\hat{\epsilon} \sim \hbar v_F/2\Pi \sim 10^{-15}$ erg. At temperatures below $T = \hat{\epsilon}/k_B$ 10K thermal excitations are frozen out, and dissipation can only be due to interlevel transitions generated by ac Josephson voltage generated by phase fluctuations. Therefore it will be absent if

$$2e < V_j >\sim \hbar \sqrt{\varphi^2} \sim \hbar \omega_0 < \hat{\epsilon},$$

which can be rewritten as $\epsilon_Q < \hat{\epsilon}/N_{\perp}$ (where the charging energy $\epsilon_Q = 2e^2/C$, and $C$ is the capacitance of the terminal A), or $\omega_0 < v_F/2\Pi$. The latter condition is a physically clear requirement that the quantum oscillations of superconducting phase allow time for readjustment of Andreev levels in the system (which is indeed $\sim 2\Pi/v_F$, the time necessary for the electron and Andreev reflected hole to travel across the system). Otherwise the coherent transport through the normal part of the system cannot be established, and dissipative currents flow instead. The maximum value of $\omega_0$ allowed by the above limitation is $\omega_{\max} \sim 10^{12}$ s$^{-1}$. (That is, the capacitance of the terminal A cannot be lower than $C_{\text{min}} = 2e^2N_{\perp}/\hat{\epsilon} \sim 10^{-11}$ F.) The corresponding tunneling rate is

$$\Gamma_{\max} \sim \omega_{\max} \exp[-N_{\perp} (\varphi_0/2\pi)^2].$$

Therefore we would require $\varphi_0 \sim 0.2\pi$ to have tunneling rate in the 100 MHz region.

For a system of integrated DND qubits of Fig.1b the bulk d-wave "bus" provides, the possibility for operations performed over all the qubits simultaneously by creating a supercurrent flow along the bus. In particular, one can easily prepare the whole register of qubits in the same (up or down) state. This is an attractive property, e.g., for implementation of a quantum correction algorithm. If take the size of a unit qubit with its periphery as $5 \cdot 10^3 \text{Å}$, a 2D 100$\times$100-qubit block will occupy only an area of 50×50 $\mu$m$^2$, which is realistic to keep below the dephasing length due to thermal excitations.

Application of quantum gates to individual qubits can be effectuated in various ways, lifting the degeneracy between up/down states either by directly applying localized magnetic field to a qubit (using a magnetic scanning tip), by creating local supercurrents in the bus, or by using laser beams with circular polarization. The entanglement of the states of adjacent qubits is achieved simply by opening a key between them for a certain time.

In conclusion, we have suggested a new design for a solid state superconducting, scalable qubit. Besides using the degeneracy of the ground state, common to all D-D junctions, it strongly relies on unique properties of clean DND or grain boundary junctions: tunability of the equilibrium phase difference across the junction, and spontaneous currents and fluxes in equilibrium. The former allows to optimize the design in order to achieve the fastest possible tunneling rate (which is vital in order to beat the dephasing processes) and thus makes it easier to integrate qubits in a computer. The latter presents an easier way to manipulate and read out the state of a qubit. Our estimates show that there is a real chance to create a working solid state qubit along these lines using existing experimental possibilities.

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FIG. 1. (a) Superconducting DND qubit: A, B are d-wave superconductors, N normal conductor, PK parity key, M scanning tip, Ω the mismatch angle between the lattices of A and B. The cut in B is here along (110) and (110) directions. Positive lobes of d-wave order parameter are shaded. (b) Multiqubit register. Terminal B plays the role of the bus bar. (c) Version of (b) using grain boundary (G) junctions.
FIG. 2. (a) Current-phase dependence in a DND junction of Fig. 1 at $l_T, l_i \to \infty$. The mismatch angle is $\Omega = \pi/8$. (b) Effective potential profile of the system. Minima at $\pm \phi_0$ correspond to "up" and "down" pseudospin states of a qubit.
FIG. 3. Current distribution in the normal part of the system in degenerate equilibrium states:
(a) $\Omega = \pi/8, \varphi = -\varphi_0 = -(\sin |\Omega|/2^{1/2})\pi \approx 0.27\pi$; (b) $\Omega = \pi/8, \varphi = \varphi_0 \approx 0.27\pi$. 