AN UPPER BOUND ON THE HIGGS BOSON MASS
FROM YUKAWA UNIFICATION
AND A COMMENT ON VACUUM STABILITY CONSTRAINTS

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Abstract

Only small regions in the \( m_t \), \( \tan \beta \) plane are allowed when considering simultaneously (assuming the MSSM) coupling constant unification and (minimal) GUT relations among Yukawa couplings (i.e., \( h_b = h_t \) at the unification point). In particular, if \( m_t < 175 \) GeV we find that only \( 1 < \tan \beta < 1.5 \) or \( \tan \beta > 40 \) is allowed. The former implies that the light Higgs boson is \( < 110 \) GeV and, in principle, visible to LEP II. The prediction for the Higgs boson mass in the \( \tan \beta = 1 \) scenario is discussed and uncertainties related to (i) vacuum stability constraints, (ii) different methods for calculating the Higgs boson mass, (iii) two-loop calculations and (iv) GUT corrections are briefly reviewed. It is shown that large left-right mixing between the t-scalars can significantly enhance the Higgs boson mass. That and an ambiguity in the size of the two-loop correction lead to our conservative upper bound of \( 110 \) GeV. Vacuum stability considerations constrain the t-scalars mixing and slightly diminish the upper bound (depending on the value of \( m_t \)). Improved two-loop calculations are also expected to strengthen the bound.

Realizing the minimal supersymmetric extension of the standard model (MSSM) within a grand-unified theory (GUT) and assuming minimal matter content, often imply specific relations between the different Yukawa couplings. When considering the simplest example, i.e., bottom-tau unification and \( h_b = h_t \), we find (in agreement with others) that the predicted ranges for \( s(M_Z) \) and for \( \tan \beta = \frac{u_p}{d_w} \) are strongly correlated with the weak angle \( \sin^2 2\theta_W \) and with \( m_t \), and are strongly constrained. Such relations provide a consistency test of simple GUT models, as well as a means to probe the GUT structure. Further on, they can be used to distinguish such models from generic string models that typically do not constrain the Yukawa couplings in this manner. Thus, exploration of all implications of the bottom-tau unification (and similar relations) is well motivated. Below, we study the implications for the mass of the light Higgs \( CP \) even Higgs boson \( m_{h^0} \). We will motivate the exploration of the \( \tan \beta = 1 \) limit and show that \( m_{h^0} \) in this region is heavier than naive expectations, but most probably not too heavy to be seen in LEP II. Exploring that region, where the SM minimum

1 Talk presented at SUSY '94, International Workshop on Supersymmetry and Unification of Fundamental Interactions, Ann Arbor, Michigan, May 14-17, 1994. Pennsylvania Report No. UPR-0595T.

2 For \( m_{h^0} \) \( \approx 143 \) GeV.

3 \( s, \tan \beta \) and \( m_t \) are the strong coupling, the ratio of the two Higgs doublet expectation values and t-quark mass, respectively.
/ \cos^2 2 \theta \text{ is shallow, requires us to pay special attention to vacuum stability constraints and we will briefly comment on that issue.}

The most impressive evidence for TeV scale supersymmetry is the well publicized coupling constant unification \cite{5}, however, it is important to note that [up to m matching functions induced by GUT-scale physics, which dominate the 0.008 theoretical error bar in \cite{4}] unification requires

\[ s(M_Z) \geq 0.12, \text{ i.e.,} \]

\[ s(M_Z) = 0.125 \quad 0.008 + H_s + 32 \quad 10^7 \text{ GeV}^2 \quad (m_{\tau}^{\text{pole}})^2 \quad (143 \text{ GeV})^2; \quad (1) \]

The heavier the t-quark, the more the extracted value for the weak-angle is diminished and the predicted value for \( s \) is increased. Two-loop contributions from Yukawa interactions \( H_s \) can diminish \( s \) by as much as 0.003 (25\%) if the Yukawa coupling is of order unity (i.e., the function \( H_s > 0.003 \)). The above observations act the region in parameter space which is consistent with the observational range for the b-quark mass and with bottom-tau unification. In particular, for \( m_{\tau}^{\text{pole}} < 200 \text{ GeV} \) only two regions are allowed: (i) \( \tan \beta \geq 1 \) (see Fig. 1) and (ii) \( \tan \beta \geq 15 \) or \( \tan \beta \geq 40 \). Hence, I will discuss region (i) only. An important implication of Yukawa unification, choosing the tan near unity region, is that the SM-like Higgs boson mass nearly vanishes at tree-level. Thus, a careful examination of the loop corrections is required.

There are important theoretical uncertainties which constrain the predictive power of the model \cite{4}. These are taken into account (assuming no conspiracies), e.g., in \cite{4} and Fig. 1, but will not be discussed here. The following points, however, should be stressed:

\[ s_{\text{dependence}}: \text{We use the predicted value of } s(M_Z) = \left| m_{\tau}^{\text{pole}} \right| \quad \text{for } m_{\tau}^{\text{pole}} = 100 \text{ GeV and } s(M_Z) = 0.12 \text{ for } m_{\tau}^{\text{pole}} = 180 \text{ GeV}. \]

\[ \text{Using instead a } s_{\text{pole}} \text{ value of } 0.003 \text{ for } s(M_Z), \text{ the whole } m_{\tau}^{\text{pole}} \text{ range implies some hidden assumptions on GUT-scale corrections. The lower the value of } s \text{ and the heavier the t-quark, the larger and less likely are these corrections.} \]

\[ \text{Finite-loop corrections to } m_b M_{\text{SUSY}}: \text{The relevance of those corrections was pointed out in } \cite{4} \text{ recently. However, while being extremely important for large } \tan \beta \text{ may be even to the point of removing any predictive power, they are negligible in region (i), as can be seen in Fig. 2. The details of the low-energy spectrum are nearly irrelevant in determining the size of that region.} \]

Different aspects of region (i) were discussed by various authors. The possibility that the Higgs boson mass is induced only at the loop level was discussed by Diaz and Haber \cite{4}. The consistency of this scenario with bottom-tau unification recently led to greater attention to that region of parameter space \cite{4,6,7}. The consistency with Yukawa unification is easily understood: From perturbativity one has a \( m_{\tau}^{\text{pole}} \)-dependent lower bound on \( \tan \beta > 1 \). Thus, we are in a region of large top-Yukawa coupling \( h_t \). Slightly decreasing \( \tan \beta \) (i.e., decreasing sin \( \beta \) or increasing \( h_t \)) at \( M_Z \) would lead to divergences (i.e., \( h_t \backslash \text{ hits } \) its Landau pole) at higher scales \cite{4}: \( h_t \) has a quasi-stable point at \( \tan \beta = 1 \). This balances too large corrections to the \( h_t \) ratio, can explain the heavy t-quark, and makes region (i) relatively insensitive to theoretical uncertainties.

For \( \tan \beta = 1 \) there is a custodial symmetry SU(2)_L SU(2)_R \neq SU(2)_{L+R} \text{ in the Higgs sector} \cite{4}. The deviation of \( \tan \beta \) from unity measures the symmetry breaking, which is induced at the loop level due to the Yukawa interactions. Thus, the enhanced symmetry can be responsible for the smallness of the region. It also puts an upper bound on \( m_{\tau}^{\text{pole}} < 185 \text{ GeV} \) so that \( \tan \beta < 2 \).
As mentioned above, we will not discuss all possible signatures, but will focus on the loop-induced Higgs boson mass. (Another interesting signature is a possible light mixed t-scalar\cite{Barger}.) In particular, Barger et al. argued\cite{13}, using the leading-logarithm approximation, that $m_{h^0} < 85$ GeV for $m_t^{\text{pole}} < 160$ GeV. We will show that this is rather a typical mass range and that the upper bound, though still relevant for LEP II, is much higher. An important enhancement comes from the large left-right mixing in the t-scalar sector and from the large split between the two t-scalar mass eigenstates. That enhancement leads to the upper bound $m_{h^0} < 100$ (110) GeV for $m_t^{\text{pole}} < 160$ (175) GeV. The enhancement is sensitive to the type of vacuum stability constraints one imposes (see below).

From the minimization of the Higgs scalar potential one has for the supersymmetric Higgsino mass parameter $2 / 1 = \tan^2 \beta$, and $2$ diverges as $\tan \beta \rightarrow 1$, i.e., in the symmetric limit. (In fact, one expects finite loop corrections to be relevant near the limit, and we can assume, without loss of generality, $\tan \beta > 1$ \cite{14}.) The large parameter dictates the phenomenology of the scenario. In particular, the light CP even mass matrix is nearly degenerate: it has a very heavy and a nearly zero mass eigenvalue (this is a trivial consequence of the custodial symmetry).

In practice, the light tree-level eigenvalue $m_{h^0}^2 < M_Z \cos 2 \beta$ is small but grows with $m_t^{\text{pole}}$ (i.e., with the lower bound on $\tan \beta$). At the one-loop level, one has $m_{h^0}^2 = m_{h^0}^2 + \frac{2}{h^0} \frac{j}{h^0}$. The upper bound on the loop correction can be estimated\cite{15} (assuming only t-scalar contributions):

$$m_{h^0}^2 = M_Z^2 \cos^2 2 \beta + \frac{3 m_t^4}{4 s^2(1 - s^2) M_Z^2} \ln \frac{m_t^2}{m_{h^0}^2} + \frac{t}{(1 - t)}$$  \hspace{1cm} (2)

where

$$t = m_t^2 \frac{m_{h^0}^2}{m_t^2} \frac{\sin^2 2 \beta}{2 m_t^2} \ln \frac{m_t^2}{m_{h^0}^2}$$

$$+ \frac{m_{h^0}^2}{m_t^2} \frac{2 \sin^2 2 \beta}{4 m_t^2} \ln \frac{m_{h^0}^2}{m_{h^0}^2}$$

Due to the large parameter the left-right mixing $\tau$ between the two t-scalars can be substantial and the leading-logarithm and the mixing terms in (1) can be equivalent. Thus, $m_{h^0}$ is enhanced by an additional factor of $\tau^2$. However, since $\tan \beta$ increases with $m_t$ (from perturbativity), $j$ and $t$ decrease and we obtain an interesting interplay between the overall factor of $m_t^4$ and $\tau$.

Eq. (1) is, of course, only an approximation. The results of a complete calculation for the Higgs boson mass are shown in Fig. 3 (for the details of the calculation, see Ref. 4). $\tan \beta$ is constrained as in Fig. 1, we assume the gaussian distribution $m_t^{\text{pole}} = 143 \pm 18$ GeV (from precision data)\cite{15} and all superpartner masses are constrained to be $< 1$ TeV. The Monte-Carlo generated histograms contain information on the upper bounds and on the distributions. The upper bound is a function of $m_t$:

- $m_{h^0} < 100$ GeV for $m_t^{\text{pole}} < 100$ GeV,
- $m_{h^0} < 110$ GeV for $m_t^{\text{pole}} < 175$ GeV,
- $m_{h^0} < 120$ GeV for $m_t^{\text{pole}} < 185$ GeV.

*Using the recent CDF range\cite{16} for $m_t^{\text{pole}}$ would only change the relative population in the different histograms and would not affect our discussion.
The distribution has two peaks, one which is enhanced by the t-scalar mixing term and which
determines the upper bound, and a peak at a much lighter mass from points with no enhancement.
Thus, the mass is typically much lighter than the upper bound, and, if the t-quark is
lighter than $170 \text{ GeV}$, may still be relevant for LEPI. (Note that we do not attempt to give
rigorous bounds.)

We compare the above bounds to those derived for any tan $\beta$ in Fig. 4. The 30 GeV
difference is due to the non-vanishing tree-level mass $< M_Z$ in the general case. (Loop corrections
are, however, typically smaller in the general case.) One can also compare either case with typical
upper bounds $m_{h^0} < 130$ 150 GeV derived from perturbativity considerations [2], and to the
lower bound given by Sher [17] for the non-supersymmetric case, $m_{h^0} > 132 \text{ GeV}$. However,
strong assumptions regarding vacuum stability are made in the latter case.

Complications in the calculations, which are explored elsewhere [2], suggest even stronger
upper bounds:

1. The magnitude of the two-loop terms is known to be small and probably negative [18].
   However, the exact magnitude is ambiguous given the complicated low-energy structure of the
   model. For example, in Fig. 3 we used the effective potential method [19] to extract $m_{h^0}$. In
   that method $h^0$ is given to a one-loop leading-logarithm order. It is straightforward to show
   that the renormalization-point dependence of the one-loop leading logarithm [e.g., in (2)] is
   equivalent to two-loop next-to-leading logarithm which are typically positive. Thus, one could
   overestimate $m_{h^0}$ when choosing the renormalization point $Q = M_Z$ (as in Fig. 3). In Fig. 5a
   $Q = 600 \text{ GeV}$, and indeed lower masses are suggested. In particular, if the scalar quarks are of
   the order of TeV or heavier, one would falsely get large Higgs boson masses using that method,
   unless $Q$ is properly adjusted (so that the loop expansion does not break down). Alternatively,
   we could use different methods, e.g., the renormalization group method [20] leads to typically
   lower one-loop upper bounds (in agreement with Fig. 5a). This is because one is implicitly
   including negative two-loop leading logarithm contributions in that method. In short, Fig. 3
   corresponds to a conservative estimate of the upper bounds. Once the non-trivial task of a
   complete two-loop calculation is carried out, we expect slightly stronger upper bounds.

2. In some cases large t-scalar mixing corresponds to the SM model populating only a local
   minimum of the full scalar potential. That is, when considering all scalar fields on equal footing,
   the global minimum may be in a direction in which the t-scalar has a vacuum expectation value
   and, thus, the physical vacuum is not stable. Eliminating cases in which a global color and
   charge breaking minimum exists slightly reduces the upper bound and also affects the mixing
   peak population. The effect is reduced with increasing $m_t$ and never diminishes the mixing
   peak completely. One could try to identify dangerous directions in the multidimensional scalar-
   field space and characterize those directions by analytic constraints. However, such constraints
   entail assumptions that are not always useful for identifying the deepest minimum (i.e., the
   constraints are too weak). In addition, analytic constraints typically do not distinguish a local
   minimum from a global one and can exclude legitimate points (i.e., are too strong). Thus,
   analytic treatments of the problem are often not satisfactory. Furthermore, a point in parameter space which
   is consistent with electroweak symmetry breaking contains at least a local minimum which does
   not respect color and charge symmetries. This is shown in Ref. 4 where we also compare
   numerical and analytic treatments of the problem. Recent progress was also reported by Carazza
   and Wagner [21].

Lastly, the prediction of $m_{h^0}$ is relatively stable when considering model-dependent evolution
of the soft parameters between the Planck and GUT scales [22]. This is shown in Fig. 5 and is
due to the cancellations between corrections to the relevant soft parameters and to $m_t$. (However,
the t-scalar mixing and mass are affected.) Thus, our discussion is only slightly affected by that
uncertainty.

To conclude, we studied the one-loop Higgs boson mass in the region of parameter space which is well motivated by Yukawa unification. We treat all theoretical uncertainties in a reasonable manner and the upper bounds given, e.g., $m_{h^0}^2 < 110$ GeV (for $m_{t_{\text{pole}}} < 175$ GeV), correspond to the most conservative estimate. The Higgs boson in this scenario is most likely in a range that is, in principle, visible to LEP II unless the t-quark is heavier than 180 GeV, in which case there is not much motivation to consider region (i). Two-loop calculations and vacuum stability constraints slightly strengthen the upper bounds.

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Figure 1: The allowed region for \( \tan \beta \). The t-quark pole mass \( m_{t}^{\text{pole}} \) is in GeV. The lower bound on \( \tan \beta \) is from \( \delta \mu < 3 \) at the GUT scale, and the upper bound from bottom-tau unification (including theoretical uncertainties). The one std. range for \( m_{t}^{\text{pole}} \), suggested by precision data, and \( \tan \beta = 1 \) are indicated for comparison. Recent CDF analysis suggests \( m_{t}^{\text{pole}} \approx 174 \pm 16 \) GeV. For \( \tan \beta < 1 \) no model with universal boundary conditions is possible.

Figure 2: The correction to the b-quark mass \( m_{b}^{\text{MS}} \) (in GeV) from nine superpartner-loops. \( m_{t}^{\text{pole}} = 174 \) GeV and universal boundary conditions for the soft parameters at the unification scale are chosen at random. All points included are consistent with electroweak breaking. \( \tan \beta = 20 \) is indicated for comparison. For \( \tan \beta < 20 \) corrections are less than 10%.

Figure 3: The Higgs boson mass \( m_{h}^{0} \) (in GeV) distribution in a sample of Monte Carlo calculations with random universal boundary conditions at the unification scale and using the effective potential method with subtraction scale \( M_{Z} \). \( \tan \beta \) is constrained as in Fig. 1. \( m_{t}^{\text{pole}} \) is in the range (a) [155, 165], (b) [165, 175], (c) [175, 185] GeV and has the gaussian distribution 143 \pm 18 GeV.

Figure 4: The upper bound on the Higgs boson mass \( m_{h}^{0} \) (in GeV) as a function of \( m_{t}^{\text{pole}} \) (all masses are in GeV). In the constrained case \( \tan \beta \) is constrained by bottom-tau unification as in Fig. 1. The general case is for all \( \tan \beta \) (consistent with electroweak breaking). Note the local minimum in the constrained curve which is due to the interplay between the mixing enhancement and the \( m_{t}^{4} \) factor. The upper bound curves are derived using Monte Carlo methods and are not rigorous. Generic upper bounds in the MSSM and in non-minimal models (NMSSM) derived from perturbativity considerations are shown for comparison. Also shown is a suggestive lower bound in the non-supersymmetric case (we use \( a = 0.25 \) in the formula of Ref. 17).

Figure 5: The Higgs boson \( m_{h}^{0} \) vs. Higgsino \( \chi_{1}^{0} \) mass plane (in GeV) for \( m_{t}^{\text{pole}} = 174 \) GeV, \( \tan \beta = 152 \), and with random universal boundary conditions at (a) the GUT scale \( 10^{3}G_{eV} \), (b) at the (reduced) Planck scale \( 10^{18} \) GeV. In case (b) we assume minimal SU(5) model above the GUT scale (for details, see Ref. 22). The effective potential method with a subtraction scale \( Q = 600 \) GeV is employed, which leads to lower values of \( m_{h}^{0} \) than for \( Q = M_{Z} \).
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ABSTRACT

Only small regions in the $m_t - \tan \beta$ plane are allowed when considering simultaneously (assuming the MSSM) coupling constant unification and (minimal) GUT relations among Yukawa couplings (i.e., $h_b = h_{\tau}$ at the unification point). In particular, if $m_t \lesssim 175$ GeV we find that only $1 \lesssim \tan \beta \lesssim 1.5$ or $\tan \beta \gtrsim 40 \pm 10$ is allowed. The former implies that the light Higgs boson is $\lesssim 110$ GeV and, in principle, visible to LEPII. The prediction for the Higgs boson mass in the $\tan \beta \approx 1$ scenario is discussed and uncertainties related to (i) vacuum stability constraints, (ii) different methods for calculating the Higgs boson mass, (iii) two-loop calculations and (iv) GUT corrections are briefly reviewed. It is shown that large left-right mixing between the $t$-scalars can significantly enhance the Higgs boson mass. That and an ambiguity in the size of the two-loop correction lead to our conservative upper bound of 110 GeV. Vacuum stability considerations constrain the $t$-scalar mixing and slightly diminish the upper bound (depending on the value of $m_t$). Improved two-loop calculations are also expected to strengthen the bound.

Realizing the minimal supersymmetric extension of the standard model (MSSM) within a grand-unified theory (GUT) and assuming minimal matter content, often imply specific relations between the different Yukawa couplings. When considering the simplest example, i.e., “bottom-tau unification” $h_b = h_{\tau}$, we find (in agreement with others) that the predicted ranges for $\alpha_s(M_Z)$ and for $\tan \beta = v_{up}/v_{down}$ are strongly correlated with the weak angle $s^2(M_Z) \approx 0.2324^*$ and with $m_t$, and are strongly constrained. Such relations provide a consistency test of simple GUT models, as well as a means to probe the GUT structure. Furthermore, they can be used to distinguish such models from generic string models that typically do not constrain the Yukawa couplings in this manner. Thus, exploration of all implications of the bottom-tau unification (and similar relations) is well motivated. Below, we study the implications for the mass of the light MSSM (CP even) Higgs boson $m_{h_{0}}$. We will motivate the exploration of the $\tan \beta \rightarrow 1$ limit and show that $m_{h_{0}}$ in this region is heavier than naive expectations, but most probably not too heavy to be seen in LEPII. Exploring that region, where the SM minimum $\propto \cos^2 2\beta$ is shallow, requires us to pay special attention to vacuum stability constraints and we will briefly comment on that issue.

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$^*$For $m_{t_{pole}} \approx 143$ GeV.

$^\dagger$ $\alpha_s$, $\tan \beta$ and $m_t$ are the strong coupling, the ratio of the two Higgs doublet expectation values and $t$-quark mass, respectively.
The allowed region for $\tan \beta \approx 1$. The $t$-quark pole mass $m_t^{\text{pole}}$ is in GeV. The lower bound on $\tan \beta$ is from $h_t < 3$ at the GUT scale, and the upper bound from bottom-tau unification (including theoretical uncertainties). The one s.d. range for $m_t^{\text{pole}}$, suggested by precision data, and $\tan \beta = 1$ are indicated for comparison. Recent CDF analysis suggests $m_t^{\text{pole}} \approx 174 \pm 16$ GeV. For $\tan \beta < 1$ no model with universal boundary conditions is possible.

The most impressive evidence for TeV scale supersymmetry is the well publicized coupling constant unification. However, it is important to note that [up to matching functions induced by GUT-scale physics, which dominate the 0.008 theoretical error bar in (1)] unification requires $\alpha_s(M_Z) \gtrsim 0.12$, i.e.,

$$\alpha_s(M_Z) = 0.125 \pm 0.001 \pm 0.008 + H_{\alpha_s} + 3.2 \times 10^{-7} \text{GeV}^{-2}(m_t^{\text{pole}})^2 - (143 \text{ GeV})^2. \quad (1)$$

The heavier the $t$-quark, the more the extracted value for the weak-angle is diminished and the predicted value for $\alpha_s$ is increased. Two-loop contributions from Yukawa interactions $H_{\alpha_s}$ can diminish $\alpha_s$ by as much as 0.003 ($\sim 2.5\%$) if the Yukawa coupling is of order unity (i.e., the function $H_{\alpha_s} \gtrsim -0.003$). The above observations affect the region in parameter space which is consistent with the observational range for the $b$-quark mass and with bottom-tau unification. In particular, for $m_t^{\text{pole}} \lesssim 200$ GeV only two regions are allowed: $(i)$ $\tan \beta \approx 1$ (see Fig. 1) and $(ii)$ large $\tan \beta$. For example, if $m_t^{\text{pole}} \approx 174$ GeV then $\tan \beta \approx 1.5$ or $\tan \beta \approx 40 \pm 10$. Hereafter, I will discuss region $(i)$ only. An important implication of Yukawa unification, choosing the $\tan \beta$ near unity region, is that the SM-like Higgs boson mass nearly vanishes at tree-level. Thus, a careful examination of the loop corrections is required.

There are important theoretical uncertainties which constrain the predictive power
Fig. 2. The correction to the $b$-quark $\overline{\text{MS}}$ mass $m_b(M_Z) \approx 3$ GeV (in GeV) from finite superpartner-loops. $m_t^{\text{pole}} = 174$ GeV and universal boundary conditions for the soft parameters at the unification scale are choosen at random. All points included are consistent with electroweak breaking, $\tan \beta = 20$ is indicated for comparison. For $\tan \beta < 20$ corrections are less than 10%.

of the model. These are taken into account (assuming no conspiracies), e.g., in (1) and Fig. 1, but will not be discussed here. The following points, however, should be stressed:

- $\alpha_s$ dependence: We use the predicted value of $\alpha_s(M_Z)[s^2, m_t, ...]$, e.g., $\alpha_s(M_Z) \approx 0.12$ for $m_t^{\text{pole}} \approx 100$ GeV and $\alpha_s(M_Z) \approx 0.13$ for $m_t^{\text{pole}} \approx 180$ GeV. Using instead a fixed value $\alpha_s^0(M_Z)$ for the whole $m_t$ range implies some hidden assumptions on GUT-scale corrections. The lower the value of $\alpha_s$ (and the heavier the $t$-quark) the larger and less likely are these corrections.

- Finite-loop corrections to $m_b(M_{\text{SUSY}})$: The relevance of those corrections was pointed out recently. However, while being extremely important for large $\tan \beta$ (maybe even to the point of removing any predictive power), they are negligible in region (i), as can be seen in Fig. 2. The details of the low-energy spectrum are nearly irrelevant in determining the size of that region.

Different aspects of region (i) were discussed by various authors. The possibility that the Higgs boson mass is induced only at the loop-level was discussed by Diaz and Haber. The consistency of that scenario with bottom-tau unification recently led to greater attention to that region of parameter space. The consistency with Yukawa
unification is easily understood: From perturbativity one has a $m_t$-dependent lower bound on $\tan \beta \gtrsim 1$. Thus, we are in a region of large top-Yukawa coupling $h_t \approx 1$. Slightly decreasing $\tan \beta$ (i.e., decreasing $\sin \beta$ or increasing $h_t$) at $M_Z$ would lead to divergences (i.e., $h_t$ “hits” its Landau pole) at higher scales\(^{11}\): $h_t$ has a quasi-fixed point at $\tan \beta \approx 1$. This balances too large $\alpha_s$ corrections to the $h_b/h_{\tau}$ ratio, which is induced at the loop level due to the Yukawa interactions. Thus, the enhanced $\tan \beta$ (i.e., decreasing $\sin \beta$) can be responsible for the smallness of the region. It also puts an upper bound on $m_{t,\text{pole}}^{\text{pole}} \lesssim 185$ GeV so that $\tan \beta < 2$.

As mentioned above, we will not discuss all possible signatures, but will focus on the loop-induced Higgs boson mass. (Another interesting signature is a possible light mixed $t$-scalar.\(^ {10} \)) In particular, Barger et al. argued\(^ {13} \) using the leading-logarithm approximation, that $m_{b,\text{pole}} \approx 85$ GeV for $m_{t,\text{pole}}^{\text{pole}} \approx 160$ GeV. We will show that this is rather a typical mass range and that the upper bound, though still relevant for LEP II, is much higher. An important enhancement comes from the large left-right mixing in the $t$-scalar sector and from the large split between the two $t$-scalar mass eigenstates. That enhancement leads to the upper bound $m_{b,\text{pole}} \approx 100$ (110) GeV for $m_{t,\text{pole}}^{\text{pole}} \approx 160$ (175) GeV. The enhancement is sensitive to the type of vacuum stability constraints one imposes (see below).

From the minimization of the Higgs scalar potential one has for the supersymmetric Higgsino mass parameter $\mu^2 \propto 1/|\tan^2 \beta - 1|$, and $\mu^2$ diverges as $\tan \beta \to 1$, i.e., in the symmetric limit. (In fact, one expects finite loop corrections to be relevant near the limit, and we can assume, without loss of generality, $\tan \beta \approx 1$.\(^ {14} \)) The large $\mu$-parameter dictates the phenomenology of the scenario. In particular, the tree-level CP even mass matrix is nearly degenerate: it has a very heavy and a nearly zero mass eigenvalue (this is a trivial consequence of the custodial symmetry). In practice, the light tree-level eigenvalue $m_{t,\text{pole}}^T < M_Z |\cos 2\beta|$ is small but grows with $m_{t,\text{pole}}^{\text{pole}}$ (i.e., with the lower bound on $\tan \beta$).

At the one-loop level, one has $m_{b,\text{pole}}^2 = m_{t,\text{pole}}^{T,2} + \Delta_{b,\text{pole}}^2 \approx \Delta_{b,\text{pole}}^2$. The upper bound on the loop correction can be estimated\(^ {15} \) (assuming only $t$-scalar contributions):

$$m_{b,\text{pole}}^2 \leq M_Z^2 \cos 2\beta + \frac{3\alpha m_t^4}{4\pi s^2(1 - s^2) M_Z^2} \left\{ \ln \left( \frac{m_{t,\text{pole}}^2}{m_t^2} \right) + \Delta_{b,\text{pole}} \right\}$$

where

$$\Delta_{b,\text{pole}} = \left( m_{t,\text{pol}}^2 - m_{t,\text{pole}}^2 \right) \frac{\sin^2 \theta_{\text{w}}}{2m_t^2} \ln \left( \frac{m_{t,\text{pole}}^2}{m_{t,\text{pole}}^2} \right)$$

$$+ \left( m_{t,\text{pole}}^2 - m_{t,\text{pole}}^2 \right)^2 \left( \frac{\sin^2 \theta_{\text{w}}}{4m_t^2} \right)^2 \left[ 2 - \frac{m_{t,\text{pole}}^2 + m_{t,\text{pole}}^2}{m_{t,\text{pole}}^2} \ln \left( \frac{m_{t,\text{pole}}^2}{m_{t,\text{pole}}^2} \right) \right].$$

Due to the large $\mu$ parameter the left-right mixing $\theta_t$ between the two $t$-scalars can be substantial and the leading-logarithm and the mixing $\Delta_{b,\text{pole}}$ terms in (2) can be equivalent. Thus, $m_{b,\text{pole}}$ is enhanced by an additional factor of $\sim \sqrt{2}$. However, since $\tan \beta$ increases with $m_t$ (from perturbativity), $|\mu|$ and $\Delta_{b,\text{pole}}$ decrease and we obtain an interesting interplay between the overall factor of $m_t^2$ and $\Delta_{b,\text{pole}}$.

Eq. (2) is, of course, only an approximation. The results of a complete calculation for the Higgs boson mass are shown in Fig. 3 (for the details of the calculation, see Ref.
Fig. 3. The Higgs boson mass $m_{h^0}$ (in GeV) distribution in a sample of monte-carlo calculations with random universal boundary conditions at the unification scale and using the effective potential method with subtraction scale $M_Z$. $\tan \beta$ is constrained as in Fig. 1. $m_t^{\text{pole}}$ is in the range (a) $[155, 165]$, (b) $[165, 175]$, (c) $[175, 185]$ GeV and has the gaussian distribution $143 \pm 18$ GeV.

4). $\tan \beta$ is constrained as in Fig. 1, we assume the gaussian distribution $m_t^{\text{pole}} = 143 \pm 18$ GeV (from precision data\footnote{Using the recent CDF range\footnote{Using the recent CDF range} for $m_t^{\text{pole}}$ would only change the relative population in the different histograms and would not affect our discussion.}) and all superpartner masses are constrained to be $\lesssim 1$ TeV. The monte-carlo generated histograms contain information on the upper bounds and on the distributions. The upper bound is a function of $m_t$:

- $m_{h^0} \lesssim 100$ GeV for $m_t^{\text{pole}} \lesssim 160$ GeV,
- $m_{h^0} \lesssim 110$ GeV for $m_t^{\text{pole}} \lesssim 175$ GeV,
- $m_{h^0} \lesssim 120$ GeV for $m_t^{\text{pole}} \lesssim 185$ GeV.

The distribution has two peaks, one which is enhanced by the $t$-scalar mixing term and which determines the upper bound, and a peak at a much lighter mass from points with
The upper bound on the Higgs boson (MS) mass $m_{h^0}$ as a function of $m_{t}^{pole}$ (all masses are in GeV). In the constrained case $\tan \beta$ is constrained by bottom-tau unification as in Fig. 1. The general case is for all $\tan \beta$ (consistent with electroweak breaking). Note the local minimum in the constrained curve which is due to the interplay between the mixing enhancement and the $m_t^4$ factor. The upper bound curves are derived using Monte-Carlo methods and are not rigorous. Generic upper bounds in the MSSM and in non-minimal models (NMSSM) derived from perturbativity considerations are shown for comparison. Also shown a suggestive lower bound in the non-supersymmetric case (we use $\alpha_s = 0.125$ in the formula of Ref. 17).

no enhancement. Thus, the mass is typically much lighter than the upper bound, and, if the $t$-quark is lighter than $\sim 170$ GeV, may still be relevant for LEPI. (Note that we do not attempt to give rigorous bounds.)

We compare the above bounds to those derived for any $\tan \beta$ in Fig. 4. The $\sim 30$ GeV difference is due to the non-vanishing tree-level mass $\lesssim M_Z$ in the general case. (Loop corrections are, however, typically smaller in the general case.) One can also compare either case with typical upper bounds $m_{h^0} \lesssim 130 - 150$ GeV derived from perturbativity considerations, and to the lower bound given by Sher for the non-supersymmetric case, $m_{h^0} \gtrsim 132$ GeV. However, strong assumptions regarding vacuum stability are made in the latter case.

Complications in the calculations, which are explored elsewhere, suggest even stronger upper bounds:

1. The magnitude of the two-loop terms is known to be small and probably negative. However, the exact magnitude is ambiguous given the complicated low-energy structure of the model. For example, In Fig. 3 we used the effective potential method to extract $m_{h^0}$. In that method $\Delta_{h^0}$ is given to a one-loop leading-logarithm order. It is
straightforward to show that the renormalization-point dependence of the one-loop leading logarithm [e.g., in (2)] is equivalent to two-loop next-to-leading logarithms which are typically positive. Thus, one could overestimate $m_h^0$ when choosing the renormalization point $Q = M_Z$ (as in Fig. 3). In Fig. 5a $Q = 600$ GeV, and indeed lower masses are suggested. In particular, if the scalar quarks are of the order of TeV or heavier, one would falsely get large Higgs boson masses using that method, unless, $Q$ is properly adjusted (so that the loop expansion does not break down). Alternatively, we could use different methods, e.g., the renormalization group method leads to typically lower one-loop upper bounds (in agreement with Fig. 5a). This is because one is implicitly including negative two-loop leading logarithm contributions in that method. In short, Fig. 3 corresponds to a conservative estimate of the upper bounds. Once the non-trivial task of a complete two-loop calculation is carried out, we expect slightly stronger upper bounds.

2. In some cases large $t$-scalar mixing corresponds to the SM model populating only a local minimum of the full scalar potential. That is, when considering all scalar fields on equal footing, the global minimum may be in a direction in which the $t$-scalar has a vacuum expectation value and, thus, the physical vacuum is not stable. Eliminating cases in which a global color and charge breaking minimum exists slightly reduces the upper bound and also affects the mixing peak population. The effect is reduced with increasing $m_t$ and never diminishes the mixing peak completely. One could try to identify dangerous directions in the multidimensional scalar-field space and characterize those directions by analytic constraints. However, such constraints entail assumptions that are not always useful for identifying the deepest minimum (i.e., the constraints are too weak). In addition, analytic constraints typically do not distinguish a local minimum from a global one and can exclude legitimate points (i.e., are too strong). Thus, analytic treatments are often not satisfactory. Furthermore, a point in parameter space which is consistent with electroweak symmetry breaking contains at least a local minimum which does not respect color and charge symmetries. This is shown in Ref. 4 where we also compare numerical and analytic treatments of the problem. Recent progress was also reported by Carena and Wagner.

Lastly, the prediction of $m_{h,0}$ is relatively stable when considering model-dependent evolution of the soft parameters between the Planck and GUT scales. This is shown in Fig. 5 and is due to the cancellations between corrections to the relevant soft parameters and to $\mu^2$. (However, the $t$-scalar mixing and mass are affected.) Thus, our discussion is only slightly affected by that uncertainty.

To conclude, we studied the one-loop Higgs boson mass in the region of parameter space which is well motivated by Yukawa unification. We treat all theoretical uncertainties in a reasonable manner and the upper bounds given, e.g., $m_{h,s}^2 \lesssim 110$ GeV (for $m_t^{pole} \lesssim 175$ GeV), correspond to the most conservative estimate. The Higgs boson in this scenario is most likely in a range that is, in principle, visible to LEPII [unless the $t$-quark is heavier than $\sim 180$ GeV, in which case there is not much motivation to consider region (i)]. Two-loop calculations and vacuum stability constraints slightly strengthen the upper bounds.

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Fig. 5. The Higgs boson \((m_{h^0})\) vs. Higgsino \((\mu)\) mass plane (in GeV) for \(m_{\tilde{t}}^{\text{pole}} = 174\) GeV, \(\tan \beta = 1.52\), and with random universal boundary conditions at (a) the GUT scale \(\approx 10^{16}\) GeV, (b) at the (reduced) Planck scale \(\approx 10^{18}\) GeV. In case (b) we assume a minimal \(SU_5\) model above the GUT scale (for details, see Ref. 22). The effective potential method with a subtraction scale \(Q = 600\) GeV is employed, which leads to lower values of \(m_{h^0}\) than for \(Q = M_Z\).