Relativistic mean field study of the properties of \(Z=117\) nucleus and the decay chains of \(^{293,294}\)\(^{117}\) isotopes

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We have calculated the binding energy, root-mean-square radius and quadrupole deformation parameter for the recently synthesized superheavy element \(Z=117\), using the axially deformed relativistic mean field (RMF) model. The calculation is extended to various isotopes of \(Z=117\) element, starting from \(A=286\) till \(A=310\). We predict almost spherical structures in the ground state for almost all the isotopes. A shape transition appears at about \(A=292\) from prolate to a oblate shape structures of \(Z=117\) nucleus in our mean field approach. The most stable isotope (largest binding energy per nucleon) is found to be the \(^{294}\)\(^{117}\) nucleus. Also, the \(\alpha\)-decay properties of \(Z=117\) and the half-lives \(T_\alpha\) are calculated for the \(\alpha\)-decay chains of \(^{293}\)\(^{117}\) and \(^{294}\)\(^{117}\), supporting the magic numbers at \(N=172\) and/or \(184\).

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I. INTRODUCTION

Nuclei can survive beyond the macroscopic limit, far into the transuranium region, where the necessary balance between the nuclear and Coulomb force is achieved only through shell stabilisation. Superheavy element (SHE) are hypothesised to the nuclear and Coulomb force is achieved only through shell stabilisation. Superheavy element (SHE) are hypothesised to exist in this region. The next double shell closer, beyond \(^{208}\)\(\text{Pb}\), predicted at \(Z=114, N=184\), may have suprisingly long half-life, even of the order of a million year \(^{[1-5]}\).

Experimentally, till to-date, elements upto \(Z=118\) have been synthesised by heavy ion reactions \(^{[6, 7]}\), with half-lives ranging from a few minutes to about a milli-second. The more microscopic theoretical calculations have predicted the next region of stability, beyond \(Z=82, N=126\), as \(Z=120, N=172\) or 184 \(^{[8, 10]}\) and \(Z=124\) or 126, \(N=184\) \(^{[11, 12]}\). However, the recent experimental possibility of \(Z=122\) from natural \(^{211, 213, 217, 218}\)\(\text{Th}\)-isotopes, associated with long lived superdeformed (SD) and/or hyperdeformed (HD) isomeric states \(^{[13-15]}\), by 16 to 22 orders of magnitude longer than their corresponding ground-state, and more recently the synthesis of \(Z=117\) at Flerov Laboratory \(^{[16]}\) from the reaction \(^{20}\)\(\text{Ca}+^{249}\)\(\text{Bk}\)\(\rightarrow^{297}\)\(\text{Bk}\) \((Z=117, A=297)\), which decay simultaneously via three and four neutrons into two differnet isotopes \(^{293}\)\(^{117}\) and \(^{294}\)\(^{117}\), motivates us to focus on their properties, using a microscopic theoretical model with better predictive power. Such estimations of structure properties of nuclei in the superheavy mass region is a challenging area in nuclear physics and a fruitful path towards the understanding of “Island of stability” beyond the spherically doubly-magic nucleus \(^{208}\)\(\text{Pb}\).

The paper is organised as follows. Section II gives a brief description of the relativistic mean field formalism. The pairing effects for open shell nuclei, included in our calculations, are the same as discussed in \(^{[15]}\). The results of our calculation are presented in Section III, and Section IV includes the \(\alpha\)-decay modes of \(^{293}\)\(^{117}\) and \(^{294}\)\(^{117}\) isotopes. A summary of our results, together with the concluding remarks, are given in the last Section V.

II. THE RELATIVISTIC MEAN-FIELD (RMF) METHOD

The relativistic Lagrangian density for a nucleon-meson many-body system \(^{[17, 18]}\),

\[
\mathcal{L} = \overline{\psi}_i \left(i\gamma_\mu \partial_\mu - M\right) \psi_i + \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{2} g_3 \sigma^4 - g_s \overline{\psi}_i \psi_i \sigma - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_w^2 V_\mu V_\mu + \frac{1}{4} \mathcal{G}(V_\mu V_\mu)^2 - g_\omega \overline{\psi}_i \gamma_\mu \psi_i V_\mu \\
- \frac{1}{2} \vec{B}_{\mu\nu} \cdot \vec{B}_{\mu\nu} + \frac{1}{4} m_\rho^2 \vec{R}_\mu \cdot \vec{R}_\mu - g_\rho \overline{\psi}_i \gamma_\mu \vec{R}_\mu \psi_i \\
- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e \overline{\psi}_i \gamma_\mu \left(1 - \gamma_5 \right) \frac{1}{2} \psi_i A_\mu. \tag{1}
\]

All the quantities have their usual well known meanings. From the above Lagrangian we obtain the field equations for the nucleons and mesons. These equations are solved by expanding the upper and lower components of the Dirac spinors and the boson fields in an axially deformed harmonic oscillator basis, with an initial deformation \(\beta_0\). The set of coupled equations is solved numerically by a self-consistent iteration method. The centre-of-mass motion energy correction is estimated by the usual harmonic oscillator formula \(E_{c.m.} = \frac{A}{2}(41A^{-1/3})\). The quadrupole deformation parameter \(\beta_2\) is evaluated from the resulting proton and neutron quadrupole moments, as \(Q = Q_n + Q_p = \int \frac{4\pi}{3} (A R^2 \beta_2)\). The root mean square (rms) matter radius is defined as \(\left\langle r_m^2 \right\rangle = \frac{1}{A} \int \rho(r, z) r^2 dr\), where \(A\) is the mass number, and \(\rho(r, z)\) is the deformed density. The total binding energy and other observables are also obtained by using the standard relations, given in \(^{[18]}\). We use the well known NL3 parameter set \(^{[19]}\). This set reproduces the properties of not only the stable nuclei but also well predicts for those far from the \(\beta\)-stability valley. As outputs, we obtain different potentials, densities, single-particle energy levels, radii, deformations and the binding energies. For a given nucleus, the maximum binding energy corresponds to the ground state and other solutions are obtained.
as various excited intrinsic states.

The constant gap, BCS-pairing approach is reasonably valid for nuclei in the valley of $\beta$-stability line. However, this method breaks down when the coupling of the continuum becomes important. In the present study, we deal with nuclei on or near the valley of stability line since the superheavy elements, though very exotic in nature, lie on the $\beta$-stability line. In order to take care of the pairing effects in the present study, we use the constant gap for proton and neutron, as given in [20], which are valid for nuclei both on or away from the stability line (for more details, see, e.g., Ref. [15], where $E_{\text{pair}}$, the pairing energy, is also defined).

### III. RESULTS AND DISCUSSION

In many of our previous works and of other authors [10, 18, 19, 21-23], the ground state properties, like the binding energies (BE), pairing energies $E_{\text{pair}}$, quadrupole deformation parameters $\beta_2$, charge radii ($r_{\text{ch}}$), and other bulk properties, are evaluated by using the above stated relativistic Langragian for different forces. From these predictions, it is found that, generally speaking, most of the recent parameter sets reproduce well the ground state properties, not only of stable normal nuclei but also of exotic nuclei far away from the valley of $\beta$-stability. This means to say that if one uses a reasonably well accepted parameter set, the predictions of the model will remain nearly force independent. In this paper we have used the successful NL3 parameter sets for our calculation.

#### A. Binding energy and two-neutron separation energy

The calculated binding energy per nucleon BE/A and binding energy BE, obtained from the RMF(NL3) formalism, are compared, respectively, in Figs. 1 and 2 and in Table I, with the Finite Range Droplet Model (FRDM) results [24]. We notice that the BE/A obtained in the RMF(NL3) model overestimate the FRDM result. In general, the BE/A value starts increasing with the increase of mass number A, reaching a peak value at A=288 for RMF(NL3) and at A=290 for the FRDM formalism. This means to say that $^{288}_{\text{117}}$ is the most stable isotope from the RMF(NL3) result and $^{290}_{\text{117}}$ from the FRDM predictions. Interestingly, $^{288}_{\text{117}}$ (with N=171) and $^{290}_{\text{117}}$ (with N=173) are both closer to the predicted closed shell at N=172 than at N=184. Note that the isotopes $^{300}_{\text{117}}$, next to the magic number N=184, are also included in this study. For the total binding energy BE of the isotopic chain in Table I and Fig. 2, we notice that the microscopic RMF binding energies agree well with the macroscopic FRDM calculations, their differences decreasing

![FIG. 1: The binding energy per particle BE/A for the $^{284-310}_{\text{117}}$ isotopes, obtained in RMF(NL3) formalism and compared with the FRDM results [24], wherever available.](image1)

![FIG. 2: The total binding energy BE for $^{284-310}_{\text{117}}$ nuclei in RMF(NL3) compared with the FRDM results [24].](image2)

![FIG. 3: The two-neutron separation energy $S_{2n}$ for $^{286-310}_{\text{117}}$ nuclei, obtained from RMF(NL3) formalisms, and compared with the FRDM results [24], wherever available.](image3)
gradually towards the higher mass region (around \( A=298 \)), and then beyond this mass number the two curves again showing a similar behavior. Note that \(^{298}\text{117}\) (with \( N=181 \)) is in this case closer to \( N=184 \).

The two-neutron separation energy \( S_{2n}(N,Z) = \text{BE}(N,Z) - \text{BE}(N-2,Z) \) is also listed in Table I. From the table, we find that the microscopic RMF \( S_{2n} \) values also agree well with the macro-microscopic FRDM calculations. This comparison of \( S_{2n} \) for RMF with the FRDM result are further shown in Fig. 3, which clearly shows that the two \( S_{2n} \) values coincide remarkably well, except at masses \( A=290 \) and 296. Apparently, the \( S_{2n} \) decrease gradually with increase of neutron number, except for the noticeable kinks at \( A=290 \) (with \( N=173 \)) and \( A=300 \) (with \( N=183 \)) in RMF, and at \( A=296 \) (with \( N=179 \)) in FRDM. Interestingly, these neutron numbers for RMF(NL3) are close to the earlier predicted \([8–10]\) \( N=172 \) or 184 magic numbers.

**B. Shape co-existence**

We have also calculated, for the whole \( Z=117 \) isotopic chain, the solutions in both prolate and oblate deformed configurations. In many cases, we find low lying excited states. As a measure of the energy difference between the ground band-head and the first excited state, we have plotted in Fig. 4 (a) the binding energy difference \( \Delta E \) between the two solutions, noting that the maximum binding energy solution refers to the ground state (g.s.) and all other solutions to the intrinsic excited state (e.s.). From Fig. 4 (a), we notice that in RMF calculations, the energy difference \( \Delta E \) is small for the whole region of the considered isotopic series. This small difference in the binding energy for neutron-deficient isotopes is an indication of the presence of shape co-existence. In other words, the two solutions in these nuclei are almost degenerate for a small difference of output in energy. For example, in \(^{290}\text{117}\), the two solutions for \( \beta_2 = 0.017 \) and 0.123 are completely degenerate with the binding energies 2066.138 and 2065.778 MeV. This later result means to suggest that the ground state can be changed to the excited state, and vice-versa, by a small change in the input data, like the pairing strength, etc., in the calculations. In any case, such a phenomenon is known to exist in many other regions of the periodic table \([23]\).

Pairing is important for open shell nuclei whose value, for a given nucleus, depends only marginally on quadrupole deformation \( \beta_2 \). This means that for differing \( \beta_2 \)-values in a nucleus, the pairing energy \( E_{pair} \) changes only marginally (by \( \sim 5\%-6\% \)). On the other hand, even if the \( \beta_2 \) values for two nuclei are same, the \( E_{pair} \)'s could be different from one another, depending on the filling of the nucleons. This result is illustrated in Fig. 4 (b) for the RMF(NL3) calculation, where \( E_{pair} \) for both the g.s. and first-excited state (e.s.), referring to different \( \beta_2 \)-values, are plotted for the full isotopic chain. It is clear from Fig. 4(b) that \( E_{pair} \) decreases with increase in mass number \( A \), i.e., even if the \( \beta_2 \) values for two nuclei are the same, the \( E_{pair} \)'s are different from one another. This change of \( E_{pair} \) is \( \sim 15\% \) in going from, say, \( A=286 \) to 310.

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**TABLE I: The RMF(NL3) results for binding energy \( \text{BE} \), two-neutron separation energy \( S_{2n} \), pairing energy \( E_{pair} \), the binding energy difference \( \Delta E \) between the ground- and first-excited state solutions, and the quadrupole deformation parameter \( \beta_2 \), compared with the corresponding Finite Range Droplet Model (FRDM) results \([24]\). The energy is in MeV.

| Nucleus | \( S_{2n} \) (MeV) | \( E_{pair} \) (MeV) | \( \Delta E \) (MeV) | \( \beta_2 \) | \( S_{2n} \) (MeV) | \( E_{pair} \) (MeV) | \( \Delta E \) (MeV) | \( \beta_2 \) |
|---|---|---|---|---|---|---|---|---|
| \( ^{288}\text{Fr} \) | 14.836 | 14.69 | 0.333 | 0.018 | 2047.09 | 15.16 | 0.080 |
| \( ^{290}\text{Fr} \) | 13.552 | 14.274 | 0.360 | 0.017 | 2061.65 | 14.56 | 0.080 |
| \( ^{292}\text{Fr} \) | 13.664 | 14.109 | 0.096 | -0.017 | 2075.72 | 14.07 | 0.072 |
| \( ^{294}\text{Fr} \) | 12.775 | 13.653 | 0.031 | 0.041 | 2089.22 | 13.50 | -0.087 |
| \( ^{296}\text{Fr} \) | 12.335 | 13.583 | 0.104 | 0.028 | 2102.66 | 13.45 | -0.035 |
| \( ^{298}\text{Fr} \) | 11.691 | 13.274 | 0.389 | 0.015 | 2114.79 | 12.13 | -0.008 |
| \( ^{300}\text{Fr} \) | 11.576 | 12.841 | 0.970 | 0.005 | 2126.14 | 11.34 | 0.000 |
| \( ^{302}\text{Fr} \) | 10.488 | 12.623 | 0.596 | 0.004 | 2136.25 | 10.11 | 0.000 |
| \( ^{304}\text{Fr} \) | 9.634 | 12.695 | 0.012 | 0.002 | 2145.71 | 9.46 | 0.000 |
| \( ^{306}\text{Fr} \) | 9.430 | 12.348 | 0.004 | 0.030 | 2154.84 | 9.13 | 0.000 |
| \( ^{308}\text{Fr} \) | 9.601 | 11.912 | 0.304 | 0.047 | 2163.93 | 9.09 | 0.001 |
| \( ^{310}\text{Fr} \) | 9.329 | 11.538 | 0.512 | 0.051 | 2172.61 | 8.68 | 0.000 |

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**FIG. 4:** (a) The energy difference between the ground-state and the first-excited state \( \Delta E \), and (b) the pairing energy \( E_{pair} \), for the relativistic RMF(NL3) calculation of \( Z=117 \) isotopic chain.
C. Quadrupole deformation parameter

The quadrupole deformation parameter $\beta_2$, for both the ground and first excited states, are also determined within the RMF formalism. In some of the earlier RMF and Skyrme Hartree-Fock (SHF) calculations, it was shown that the quadrupole moment obtained from these theories reproduce the experimental data pretty well [10, 17–19, 21, 26–29]. The g.s. quadrupole deformation parameter $\beta_2$ is plotted in Fig. 5 for RMF, and compared with the FRDM results [24]. It is clear from this figure that the FRDM results differ from the RMF(NL3) results for some mass regions. For example, the g.s. oblate solution appear for the nucleus $^{292}_{117}$ in RMF but is a prolate solution in FRDM. A more careful inspection shows that the solutions for the whole isotopic chain are prolate, except at $A=292$ for RMF and at $A=294-298$ for FRDM model. In other word, there is a shape change from prolate to oblate at $A=292$ for RMF and at $A=294$ for FRDM. Interestingly, most of the isotopes are almost spherical in their g.s. configurations.

D. Nuclear radii

The root-mean-square matter radius ($r_m$) and charge radius ($r_{ch}$) for the RMF(NL3) formalism are shown in Fig. 6. As expected, the matter distribution radius $r_m$ increases with increase of the neutron number. However, though the proton number $Z=117$ is constant for the isotopic series, the $r_{ch}$ value also increases with neutron number. A detailed inspection of Fig. 6 shows that, in the RMF calculations, both the radii show the monotonic increase of radii till $A=310$, with a jump to a lower value at $A=292$ (with $N=175$). There is no data or other calculation available for comparisons.

IV. THE $Q_\alpha$ ENERGY AND THE DECAY HALF-LIFE $T_\alpha$

The $Q_\alpha$ energy is obtained from the relation [30]:

$$Q_\alpha(N, Z) = BE(N, Z) - BE(N - 2, Z - 2) - BE(2, 2).$$

Here, $BE(N, Z)$ is the binding energy of the parent nucleus with neutron number $N$ and proton number $Z$, $BE(2, 2)$ is the binding energy of the $\alpha$-particle ($^4$He), i.e., 28.296 MeV, and $BE(N - 2, Z - 2)$ is the binding energy of the daughter nucleus after the emission of an $\alpha$-particle.
TABLE II: The $Q_\alpha$ energy and $T_\alpha$ for $\alpha$-decay series of $^{293,117}$ nucleus, calculated on the RMF(NL3) model, and compared with the Finite Range Droplet Model (FRDM) results [24], the extrapolated data [32, 33] and the experimental data [16], wherever available. The energy is in MeV and the half-life time in second.

| Nucleus | Z | BE     | $Q_\alpha$ | $T_\alpha$ | FRDM Results | Extrapolated result | Experimental Results |
|---------|---|--------|------------|------------|---------------|---------------------|---------------------|
|         |   |        | $Q_\alpha$ | $T_\alpha$ |   |            |                      |
| 293     | 117| 2086.602| 11.480   | -2.71      | 2083.06       | 11.68              | -2.40              | 11.03              | -1.60              |
| 289     | 115| 2069.786| 10.552   | -0.96      | 2066.45       | 10.03              | 1.26               | 10.57              | -1.01              | 10.31              | -0.31              |
| 285     | 113| 2052.042| 9.765    | 0.60       | 2048.18       | 8.97               | 3.86               | 2045.45            | 9.99               | -0.04              | 9.74               | 0.67               |
| 281     | 111| 2033.511| 9.231    | 1.55       | 2028.85       | 9.37               | 1.90               | 2027.13            | 9.69               | 0.16               | 9.48               | 0.78               |

| Nucleus | Z | BE     | $Q_\alpha$ | $T_\alpha$ | FRDM Results | Extrapolated result | Experimental Results |
|---------|---|--------|------------|------------|---------------|---------------------|---------------------|
|         |   |        | $Q_\alpha$ | $T_\alpha$ |   |            |                      |
| 111     | 9.48| 9.69 | 2054.22   | 2015.32 | 9.93 | 2027.13  | 9.69 | 0.16 | 9.48 | 0.78 |

TABLE III: Same as for Table II, but for $^{294,117}$ nucleus.

| Nucleus | Z | BE     | $Q_\alpha$ | $T_\alpha$ | FRDM Results | Extrapolated result | Experimental Results |
|---------|---|--------|------------|------------|---------------|---------------------|---------------------|
|         |   |        | $Q_\alpha$ | $T_\alpha$ |   |            |                      |
| 294     | 117| 2092.578| 10.763    | -0.906     | 2089.22       | 11.66 | -3.13 | 10.81 | -1.03 |
| 290     | 115| 2075.045| 11.083    | -2.325     | 2072.59       | 9.93  | 0.77  | 2069.730 | -0.358 | 9.95 | 0.74 |
| 286     | 113| 2057.832| 9.612     | 1.055      | 2054.22       | 8.90  | 3.32 | 2051.764 | 9.752 | 0.639 | 9.63 | 1.002 |
| 282     | 111| 2039.148| 9.080     | 2.027      | 2034.82       | 8.80  | 2.95 | 2033.220 | 9.464 | 0.830 | 9.00 | 2.286 |
| 278     | 109| 2019.932| 8.775     | 2.316      | 2015.32       | 9.38  | 0.41 | 2014.388 | 9.176 | 1.028 | 9.55 | -0.099 |
| 274     | 107| 2000.411| 8.105     | 3.913      | 1996.40       | 8.71  | 1.81 | 1995.268 | 8.348 | 3.042 | 8.80 | 1.517 |

FIG. 8: Same as for Fig. 7, but for $^{294,117}$ nucleus.

With $Q_\alpha$ energy at hand, we estimate the half-life $log_{10}T_\alpha(s)$ by using the phenomenological formula of Viola and Seaborg [31]:

$$log_{10}T_\alpha(s) = \frac{aZ - b}{\sqrt{Q_\alpha}} - (cZ + d).$$

Here, $Z$ is the atomic number of parent nucleus, and $a=1.66175, b=8.5166, c=0.20228$ and $d=33.9069$.

A. The $\alpha$-decay series of $^{293,117}$ nucleus

The binding energies of the parent and daughter nuclei are obtained by using the RMF formalism. From these BE, we evaluate the $Q_\alpha$ energy and the half-life time $log_{10}T_\alpha(s)$, using the above formulae. Our predicted results for the decay chain of $^{293,117}$ are compared in Table II with the finite range droplet model (FRDM) calculation [24], the extrapolated [32, 33] and the experimental data [16], wherever possible. The same comparison is also carried out in Figs. 7 (a) and 7 (b), respectively, for $Q_\alpha$ energy and the half-life time $log_{10}T_\alpha(s)$.

From Figs. 7(a) and (b), and Table II, we notice that the calculated values for both $Q_\alpha$ and $T_\alpha(s)$ agree well with the known extrapolated as well as experimental data, but are overestimated with respect to the FRDM predictions. For example, the values of $T_\alpha$ from RMF coincide well for the whole mass region with the available experimental data, and with extrapolated values for $^{281}$Rg, $^{286}$Ni and $^{289}$Nb nuclei. Similarly, for $^{281}$Rg, the FRDM predictions match both the extrapolated and experimental results. Furthermore, the possible shell structure effects in $Q_\alpha$, as well as in $T_\alpha(s)$, are noticeable for the daughter nucleus $^{290}$Ni (with N=172) for both the RMF predictions and experimental data. Note that N=172 refers to the predicted magic number.

B. The $\alpha$-decay series of $^{294,117}$ nucleus

In this subsection, we present the $Q_\alpha$ and the $log_{10}T_\alpha(s)$ results for decay series of $^{294,117}$ nucleus, using the same procedure as in the previous subsection for $^{293,117}$. The results obtained are listed in Table III and plotted in Figs. 8(a) and 8(b), compared with the FRDM predictions [24], the extrapolated [32, 33] and experimental data [16], wherever possible.

From Fig. 8(a) and (b), and Table III, we found almost similar results as are predicted in the previous subsection for $^{293,117}$. Thus, the RMF(NL3) results for both $Q_\alpha$ and $T_\alpha(s)$ agree well with the known extrapolated and experimental data, but once again over-estimate the FRDM results. For example, the $T_\alpha$ values for RMF coincide well with the experimental data for the whole isotopic chain and with the extrapolated data for $^{290}115, 286$Ni, $^{282}$Rg, $^{278}$Mt and $^{274}$Bh nuclei. Similarly, FRDM predictions for $^{278}$Mt and $^{290}115$ match the ex-
trapped and for $^{274}$Bh and $^{278}$Mt with experimental results. The possible shell structure effects in $Q_\alpha$, as well as in $T_\alpha(s)$, are noticed for the daughter nucleus $^{286}$113 (with N=173) for RMF and $^{278}$Mt (with N=169) in experimental data, again coinciding with earlier predicted N=172 magic number.

V. SUMMARY

Summarizing, we have calculated the binding energy, the rms charge and matter radii, and quadrupole deformation parameter for the isotopic chain of recently synthesized $Z=117$ superheavy element for both the ground- as well as intrinsic first-excited states, using the RMF formalism. From the calculated binding energy, we have also estimated the two-neutron separation energy and the energy difference between ground- and first-excited state for studying the shape co-existence, for the isotopic chain. Also, we have estimated the pairing energy for the ground-state solution in the whole isotopic chain.

We found a shape change from oblate to prolate deformation, with increase of isotopic mass number, at $A=292$. Most of the ground-state structures are with spherical solutions, in agreement with the FRDM calculations. From the binding energy analysis, we found that the most stable isotope in the series is $^{288}$117, which is close to predicted magic number at N=172. Our predicted $\alpha$-decay energy $Q_\alpha$ and half-life time $T_\alpha$ match nicely with the available extrapolated and experimental data. Some shell structure is also observed in the calculated quantities at N=172 and/or 184 from RMF calculations of the various isotopes of $Z=117$ nucleus.

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