2D full-wave simulation of ordinary mode reflectometry

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Abstract. A 2D full-wave simulation of ordinary mode propagation has been developed in an effort to model effects seen in reflectometry experiments but not properly explained by 1D analysis. The geometric fall-off of the fields, together with the effects of both refraction and diffraction, considerably modify the results obtained. The now commonly seen experimental observations of large amplitude and phase variations of the echo signal and occasional ramping of the phase can be explained by these 2D effects in the presence of fluctuations.

1. Introduction

During the last few years, reflectometry has proved to be a very useful diagnostic on several large toroidal plasma experiments (Simonet 1985, Hubbard et al 1987, Millot et al 1990, Doyle et al 1990). Not only have density profiles been measured, but large amounts of qualitative information have been gathered about fluctuations on these machines (TFR Group 1985, Doyle et al 1990, Hanson et al 1990). One of the most intriguing aspects of these results, however, is the extreme apparent sensitivity of the signals to fluctuations. Large, rapid variations in return signal amplitude and phase are often seen (Hubbard et al 1987, Hanson et al 1989). In addition, several experiments report that the phase of the echo can in fact begin to ramp as a function of time, indicating a Doppler shift in the return signal relative to the transmitted one (Hanson et al 1990, Bulanin, 1992, Sanchez et al 1992). These effects cannot be easily explained with 1D analysis, but even very simplified 2D analysis, in which the critical surface is modelled as a grating, qualitatively explains some of the experimental results (Irby and Stek 1990). The 2D, full-wave, cold-plasma analysis presented in this paper not only explains some of the experimental observations, but also modifies what one might otherwise surmise about scattering, the response to fluctuations, and localization of the reflection to the critical surface, where only 1D results are considered.

Previous 1D calculations (Bretz 1992, Cripwell 1992, Hutchinson 1992, Mazzucato and Nazikian 1991, Zou et al 1990, 1991) have typically been accomplished with Born approximation or shooting method solutions. The first 2D calculations (Mazzucato and Nazikian 1991, Zou et al 1990) have assumed plane-wave illumination of the plasma with arbitrary angles of incidence and reflection, and Born approximation solutions. Such calculations have greatly improved our understanding of the basic processes at work in reflectometry. We extend these results here to include geometrically complex incident and reflected waves with proper scaling of field amplitudes with distance, proper modification of the wave trajectories caused by the 2D model density profiles, multiple scattering, and scattering from large amplitude fluctuations.
2. Code description

A 2D solution to Maxwell's equations for the propagation of ordinary electromagnetic waves in a cold plasma is needed. The first two equations of interest for this problem are

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \]

where all plasma effects will be included in the response of the current density \( \mathbf{J} \) to the electric field \( \mathbf{E} \) (Hutchinson 1987). Restricting ourselves now to two dimensions, with ordinary mode propagation in the \( x-y \) plane, and no gradients allowed in the \( z \) direction, a simplified set of equations result for the wavefields:

\[ \frac{\partial B_z}{\partial t} = -\frac{\partial E_z}{\partial y}, \quad \frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x}, \quad \mu_0 \epsilon_0 \frac{\partial E_z}{\partial t} = -\mu_0 J_z + \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}. \]  

The large static magnetic field normally found in plasma experiments has been assumed to be in the \( z \) direction and much larger than the wavefields, so that current flow is restricted to that direction.

In addition to the field equations, an equation describing the plasma response to the waves is needed. Normally, in the cold plasma approximation, all electrons are assumed to move harmonically in response to the wavefields oscillating at a frequency \( \omega \). In such cases, a Fourier analysis of the equation of motion for the electron leads to a simple expression for the current density

\[ \mathbf{J} = \mathbf{\sigma} \cdot \mathbf{E} \]  

where \( \mathbf{\sigma} \) is the conductivity tensor and depends primarily on the electron density, magnetic field, and wave frequency (Hutchinson 1987). However, since a solution to the time-dependent problem in which Doppler-shifted waves are present at frequencies other than \( \omega \) is desired, the equation of motion for the electrons is solved directly. The equation for the current density is

\[ \frac{\partial J_z}{\partial t} = \epsilon_0 \omega_p^2 E_z \]  

where \( \omega_p \) is the electron plasma frequency given by \( \sqrt{n_e e^2/\epsilon_0 m_e} \), \( n_e \) is the electron density and a function of \( x \) and \( y \) only, \( e \) is the electron charge, and \( m_e \) the electron mass.

Note that, in our derivation of (2) and (4), terms involving a time-dependent dielectric were dropped. On the other hand, in what follows, we will perturb the dielectric as a function of time in investigating fluctuations and density pulse propagation. These perturbations will be done adiabatically, however, on time scales much longer than either the propagating wave period or the plasma response time.

Equations (2) and (4) together comprise a set of equations to be solved self-consistently. To do so, a finite-difference, time-domain scheme developed by Blaschak and Kriegsmann (1987) which is second order in both space and time is adopted. The equations are solved on a rectangular grid, typically with a source of radiation near the left boundary, and the plasma critical surface near the right boundary. To ensure stability of the code, several conditions must be met, including the Courant condition, \( \delta x, \delta y \geq 2c\delta t \). Here \( \delta x, \delta y, \) and \( \delta t \) are the spatial and temporal increments used for the finite-difference time-dependent integration. In addition, \( \delta t \leq 2/\omega_p \) assures that the current density equation is integrated
Finally, both $\delta x$ and $\delta y$ should be small compared to the vacuum wavelength of the propagating radiation. In all of the results quoted in this paper, $\delta t = \tau_0/20$, and $\delta x = \delta y = \lambda_0/10$, where $\tau_0$ is the source period, and $\lambda_0$ is the source vacuum wavelength.

It is essential to emphasize the importance of radiative (vanishing reflection coefficient) boundary conditions for the problem discussed here. Without radiative boundaries, large standing wave patterns grow on the grid and make it very difficult to draw even qualitative conclusions from the code results. Simply making the grid very large, as is sometimes done, will not work for this problem since one must also keep the grid spacing small compared to the vacuum wavelength if the code is to remain stable. Radiative boundary conditions are applied on the left, upper, and lower boundaries, while $E_z = 0$ is applied on the right boundary. The boundary on the right is well beyond the critical surface and is therefore well insulated from waves propagating to the critical surface from the left, so the fields can be safely assumed to be very small there. Radiative boundary conditions on the other three boundaries would imply that any waves propagating up to a boundary would continue through unimpeded and off the grid. Though a perfect radiative boundary cannot be generated for an arbitrary incident wave, a method developed by Higdon (Higdon 1986, Givoli 1991), in which radiative conditions are found for plane waves incident at several specific angles, can be used. Higdon’s second-order expression in which the boundary is perfectly absorbing for two angles, $\alpha_1$ and $\alpha_2$ is used. The boundary condition at $x = 0$, for example, is given by

$$\left[ \prod_{j=1}^{2} \left( \cos \alpha_j \frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right) \right] E_z = 0.$$  

These conditions are sufficient to reduce the reflection coefficient at the boundaries to the 5% level, for all angles. Angles of 0.0° and 45.0° were chosen for the left boundary, while angles of 22.5° and 45.0° were used for the upper and lower boundaries.

The unperturbed plasma is modelled as

$$\frac{\omega_0^2}{\omega^2} = \left[ \beta \ e^{-\psi} \right]^\gamma$$  

where

$$\psi = \left[ \frac{x - x_0}{w} \right]^2 + \left[ \frac{y - y_0}{\kappa w} \right]^2.$$  

$\psi$ represents an elliptical surface with elongation $\kappa$ on which the density is a constant. The plasma centre is determined by $x_0$ and $y_0$, while $w$ and $\kappa$ determine the relative scaling of the density in the $x$ and $y$ directions, and $\gamma$ allows the density gradient to be scaled without changing the location of the critical surface. The critical surface may be scanned toward and away from the plasma centre by adjusting $\beta$. Typical values for the above parameters were $\beta = 2.7183$, $\gamma = 1$, $x_0 = 18.75\lambda_0$, $y_0 = 0$, $w = 7.5\lambda_0$, and $\kappa = 4$.

Finally, waveguides and horns are modelled by setting $E_z = 0$ at grid points located inside the computational grid. Referring to figure 1, the upper structure contains the transmitting horn and guide, the lower the receiving horn and guide. Note that the upper waveguide is closed on its left boundary, so that all power generated in the guide propagates to the right and does not couple to the rear of the receiving guide. The receiving guide is left open so that standing waves will not be generated.

Doing so is equivalent to having
a well terminated detector at the end of the guide. The received power is monitored by recording, as a function of time, the signal at a grid point located near the left end of this guide at $x = \lambda_0$ and $y = -1.1 \lambda_0$. In addition, as will be discussed below, several other grid points near the left boundary are monitored. Power is added to the system by sinusoidally driving current at the frequency $\omega$ at a grid point located near the left end of the upper waveguide. The current is allowed to rise slowly, over a four-cycle period of time, since a rapid turn-on results in the generation of high-frequency field components. To the left of this source, and filling the source guide to its left boundary, is an absorber. The absorber damps waves that would normally re-enter the source waveguide after reflecting off of the plasma. In cases where the critical surface is very close to the horns, these waves can actually be detected at the receiving horn after a second reflection from the plasma, and they can therefore complicate the analysis. In the cases discussed in this paper, the critical surface was kept well away from the horns, reducing the need for the absorber.

3. General features of the solution

Referring again to figure 1, power leaving the upper horn propagates toward the plasma critical surface and is reflected. Displayed here are contours of constant positive electric field, together with the more standard ray-tracing results. Note that the ray-trace trajectories are perpendicular to the wavefronts as the waves move away from the plasma. Near the critical surface, a very complex field structure exists, consisting of both standing and propagating waves. Unlike the 1D case, in which incident and reflected waves must have
equal amplitudes, the geometric fall-off of the fields, together with the curved geometry of the critical surface, result in a very rich field pattern, and corresponding plasma current distribution. Even well away from the critical surface, interference effects are evident, as are diffraction effects near the horns. Finally, note that the waves propagate smoothly off the grid as a result of the radiative boundary conditions.

4. Density pulse propagation

In a manner similar to that explored by Cripwell (1992) for a 1D code, a Gaussian density pulse is now launched outward from near the centre of the plasma toward the source. Such a simple perturbation should allow easy comparisons to be made between the 2D and 1D results. Specifically of interest are phase and amplitude responses and their associated localizations.

The density profile may be described as

\[
\frac{\omega_p^2}{\omega_0^2} = \frac{\omega_0^2}{\omega_f^2} \left[ 1 + a_f \exp \left( -\frac{(x - x_f)^2}{w_f^2} \right) \right]
\]

where \(x_f\) and \(w_f\) are the \(x\) position and \(1/e\) half-width of the pulse, respectively, and \(a_f\) is the fluctuation amplitude. The pulse moves at constant velocity from \(x_f = 16\lambda_0\) to \(x_f = 0\) during the run.

Figure 2 shows the amplitude and phase as a function of time in the receiver waveguide for two pulse amplitudes. The temporal position of the peak phase change in both cases matches well with the location of the pulse, once the propagation time (group delay) from the critical surface is considered. The phase changes very quickly as the pulse arrives from the high-density side of the critical surface, but then decays slowly as the pulse moves away, since the pulse is now continuously in the beam path. The ripples evident primarily on the amplitude occur as the pulse moves an incremental distance \(\lambda_0/2 < \Delta x < \lambda_0\), and are the result of interference between the waves scattering from the critical surface and the propagating density pulse. A comparison of the two cases indicates that the peak amplitude and phase response are linear functions of the pulse amplitude. Finally, note that even the relatively small density perturbations modelled here result in large changes in signal amplitude. Much higher edge-plasma fluctuation levels are found during many tokamak discharges.

An expanded timescale version of figure 2 is shown in figure 3. Also shown is the signal detected 4\(\lambda_0\) above the midplane, together with 1D full-wave results calculated by the method of Hutchinson (1992). The 1D result has been time-shifted by the group delay \((32\tau_0)\) to the critical surface. The peak phase responses agree well in all cases. However, the waveguide results show more amplitude variation than the above midplane results. Since the waveguide and horn modelling the reflectometer receiver will produce both amplitude and phase variations as a function of entrance angle to the horn, some additional variation should be expected as the pulse modifies the reflected wave trajectory.

5. Radially propagating oscillatory modes

The propagation of radial oscillatory modes is now investigated \((k_\theta = 0, k_r \neq 0)\). Such modes have been extensively studied with 1D codes (Zou et al 1990, Mazzucato and Nazikian 1991, Zou et al 1991, Hutchinson 1992, Bretz 1992), and several predictions...
Figure 2. Time histories of (a) the electric field amplitude, and (b) electric field phase, are shown. Results with pulse amplitudes of 0.10 (full), and 0.01 (dotted), with \( n_f = 0.5 \lambda_0 \) are shown. Note that the upper scale in each plot represents the pulse position relative to the critical surface at the midplane \( (x_{\text{cutoff}} - x_i) \).

from these codes should still apply in the 2D case. For orientation, we briefly summarize these expectations.

Little response from the critical surface should be seen for fluctuations with wavelengths much shorter than the local wavelength of the incident radiation. The perturbed current distribution in this case should not effectively radiate. However, as the fluctuation wavelength is increased beyond \( \lambda_0/2 \), the Bragg condition will be satisfied somewhere in the plasma and the response away from the critical surface will be enhanced. In figure 4(b) the fluctuation wavelength required to meet the Bragg condition is shown as a function of position for our model profiles (figure 4(a)). These effects imply that only the response to
long-wavelength fluctuations will be localized near the critical surface. As the wavelength of the fluctuation becomes comparable to the size of the plasma, the phase and amplitude response will saturate (typically for \( \lambda_f / \lambda_0 \gg 1 \)). In this limit the entire density profile is shifted up and down as the mode propagates.

To model these radial modes, density waves are caused to propagate in the \(-x\) direction. The density profile is described by

\[
\frac{\omega_p^2}{\omega^2} = \frac{\omega_{p0}^2}{\omega_0^2} \left[ 1 + a_t \sin \left( \frac{2\pi}{\lambda_f} x + \omega_f t \right) \right]
\]

(9)

where \( \omega_f \) is the fluctuation frequency, and no variation in the \( y \) direction is allowed.
Signals detected both inside the receiver waveguide and at points above the midplane are compared with 1D full-wave results in figure 5. The general scaling with fluctuation wavelength is as expected. There is very little response for $\lambda_f/\lambda_0 < \frac{1}{2}$, and the response increases with increasing $\lambda_f$. No saturation of the response is observed, since for the size plasma modeled here the saturation does not occur until $\lambda_f/\lambda_0 > 30$. However, the waveguide phase response does not follow the 1D full-wave curve. The above-midplane results, on the other hand, agree with the 1D full-wave results to within 10%. As in the pulse propagation case, the fluctuations modify how the waves propagate into the horn and waveguide, and hence the detected phase and amplitude. In other runs, we have observed that above-midplane agreement with 1D results improves as one moves the horns further.
from the detector, and waves diffracted from the horns no longer interfere as strongly with those scattered directly from the plasma. We do not observe that the 2D full-wave phase response is always systematically higher than one would expect from 1D analysis. The 2D results are very sensitive to the transmitting and receiving horn geometry, and are often well above or below the 1D result, depending on the exact geometry modelled.

In order to gauge the localization of the radial modes, Gaussian wave packets were propagated radially outward from near the centre of the plasma. The density is modelled as

\[
\frac{\omega_p^2}{\omega^2} = \frac{\omega_p^2}{\omega^2} \left[ 1 + a_t \sin \left( k_t (x - x_t) \right) \exp \left( - \frac{(x - x_t)^2}{w_t^2} \right) \right]
\]

(10)
where, as in the pulse propagation case, the packet propagates from $x_f = 16\lambda_0$ to $x_f = 0$.

As the packet propagates, enhanced scattering should occur where the Bragg condition is satisfied. Again referring to figure 4(b), it is clear that shorter-wavelength fluctuations will be matched at smaller $x$. Our expectation is that the short-wavelength phase and amplitude response will be delayed relative to the long-wavelength case as the packet propagates from larger to smaller $x$.

In figure 6, the signal amplitude, phase, and density perturbation at the critical surface as a function of time are shown for two cases with different $\lambda_f$. The results show that the short-wavelength response is delayed relative to the long-wavelength case. Cross-correlations of the signals from the two cases with the density perturbation at the critical surface indicate that the response is localized to within approximately $\lambda_0/10$ of the expected position. That is, the maximum response occurs away from the critical surface, at a point close to where the Bragg condition is met. How well the echo signal generated by the packets appears to correlate with the packet wave shape depends on the packet wavelength. In the long-wavelength case, both the amplitude and phase correlate well with the perturbation. For the short-wavelength case, both amplitude and phase are more complicated, with temporally extended oscillations.

The degree to which scattering away from the critical surface occurs will play a large part in determining how well the reflectometer measurements can be localized. In the 1D case, in a lossless medium, localization is enhanced by the swelling of the electric field near the critical surface, which results in larger scattered fields there. In the 2D case, the fields actually decrease as one approaches the critical surface because of geometric effects and refraction (see figure 4(c)). Thus scattering away from the critical surface may play more of a role in a 2D simulation and indeed in the actual experiments. Scattering away from the critical surface will necessarily occur closer to the receiver and will thus be stronger than a similar signal propagating from the critical surface.

It must be mentioned that a recent experiment has found that the signals from fluctuations are far better localized to the critical surface than the above results would indicate (Rhodes et al 1992). The code is currently being modified to model more closely the experimental configuration used. Much shorter density scale-lengths and critical surface curvature appear in the experiment than are modelled here for example.

6. Poloidally propagating oscillatory modes

Poloidally propagating modes ($k_\theta \neq 0, k_r = 0$) are now investigated. However, some of our earlier conclusions should still be valid for the poloidal case. The response to short wavelength fluctuations near the critical surface should still be small since the perturbed current distribution will again not effectively radiate. As the fluctuations move away from the critical surface, one would again expect a Bragg matching condition to apply. Now, however, the incident waves can propagate at large angles relative to the direction of propagation of the fluctuations and the matching conditions are more complex. In addition, one would expect longer-wavelength fluctuations (e.g. $\lambda_f/\lambda_0 > 2$) near the critical surface to scatter waves very much as a diffraction grating, since the critical surface location will be modulated sinusoidally. For modes that are propagating, one should find Doppler-shifted scattering components. Still longer-wavelength fluctuations ($\lambda_f/\lambda_0 > 4$) near the critical surface should primarily cause changes in the trajectory of the echo, resulting in phase and amplitude changes not directly related to a change of the critical surface position.

To model poloidally propagating modes, the density profile is modulated in the $y$ direction. As the mode propagates, it remains centred at an $x$ position $x_f$, with a $1/e$
Figure 6. Time histories of the electric field amplitude, phase, and packet density perturbation at the critical surface are shown for two values of \( \lambda_f \). \( \lambda_f/\lambda_0 = 1.0 \) for (a)–(c), and \( \lambda_f/\lambda_0 = 0.75 \) for (d)–(f). Fluctuation parameters were \( a_t = 0.01 \) and \( w_f = \lambda_0 \).

Gaussian width of \( w_f \). The density profile is described by

\[
\frac{\omega_0^2}{\omega^2} = \frac{\omega_{p0}^2}{\omega^2} \left[ 1 + a_t \sin \left( \frac{2\pi}{\lambda_f} y + \omega t \right) \exp \left( -\frac{(x - x_f)^2}{w_f^2} \right) \right].
\] (11)

In figure 7, the amplitude and phase of the detected signal as a function of time for several fluctuation wavelengths are shown. As expected, the response to the fluctuations drops off rapidly as the wavelength decreases. For the longest-wavelength case, the signal amplitude is actually reduced to zero as the mode propagates. Phase changes of approximately \( \pm \pi \) occur when the signal amplitude approaches zero. Note that the signal
can be enhanced by almost a factor of three over the no-fluctuation level as the mode rotates, and the return beam is scanned across the receiving horn. In figure 8, signals observed away from the midplane are shown, for a long fluctuation wavelength and large \( w_f \). One finds that the phase now ramps. Note that the amplitude does not have to go to zero to generate this effect.

These results represent a complex combination of Doppler shifted return signals from the moving fluctuations and simple changes in the beam propagation trajectory. Such phase and amplitude changes result when not one, but at least two waves of similar amplitude interact in the receiver. Since a minimum in the total signal occurs when the two signals are out of phase, one would expect to see jumps of \( \pm \pi \) in the phase as their relative amplitudes

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**Figure 7.** Time histories of the electric field (a) amplitude, (b) and phase in the presence of poloidally propagating modes with \( \lambda_f / \lambda_0 = 4 \) (full), 3 (broken), 2 (dotted), and 1 (chain). One complete fluctuation period is displayed. Fluctuation parameters were \( w_f = 0.5 \lambda_0, \sigma_f = 0.10, \) and \( x_f = 10.75 \lambda_0. \)
Such phase changes have been seen in at least one experiment (Hanson et al 1990, 1992). Amplitude variations of up to a factor of four are also expected when the waves are in phase. Using a rotating grating as an analogue, one has both zeroth- and first-order components in the received signal, and the first-order component is Doppler-shifted relative to that of the zeroth order.

For such a scenario, one would also expect the zeroth-order component to be more localized to the midplane than the first-order one. As one moves away from the midplane, the Doppler-shifted component will become more dominant and the phase of the signal detected should begin to ramp. In actual experiments, misalignment of the horns or movement of the plasma off the midplane could result in such a situation. It should be mentioned that,
since only a few fluctuation periods are illuminated in this example, the grating modelled here is a very low-resolution, inefficient one, and the primary effect at the receiving horn results from shifts in alignment.

We now investigate the localization of these poloidal modes. In figure 9, the amplitude and phase response to a poloidally propagating mode as a function of distance from the critical surface for three values of \( \lambda_f \) are shown. Note that in the two longer-wavelength cases, very large amplitude variations occur, even well away from the critical surface. For the short-wavelength case, much smaller power fluctuations are seen.

The phase modulation also decreases dramatically as \( \lambda_f \) decreases, from a level that
somewhat exceeds the 1D result to a level that is smaller by about a factor of 10. The degree of localization of this response is somewhat greater for the 2D results than the 1D calculation indicates.

The large amplitude (and phase) effects can be attributed primarily to changes in the beam propagation trajectory, which occur dominantly near the reflection layer. This effect would help to explain the somewhat greater localization of the response than expected on the basis of 1D calculations. As the poloidal wavelength decreases, averaging over the poloidal direction reduces the net phase response to a small value.

Thus, the fluctuation response will be somewhat localized to the critical surface, but directly relating a fluctuation amplitude to a measured phase response will be very difficult. This effect, taken together with the fact that the response is highly wavelength-dependent, implies that one must take great care in drawing any conclusions from the phase response. Note that the 2D phase response peaks at the same location in the two longer-wavelength cases, approximately 0.5\lambda_0 in front of the critical surface. Zou et al (1990) have shown analytically that the selection rules governing 2D scattering of plane waves off poloidal fluctuations should result in a movement of the peak phase response away from the critical surface. This effect does indeed occur, as the shortest-wavelength case shows a clear movement of the peak response away from the critical surface. However, both the phase and amplitude response is at least a factor of 25 down from the longer-wavelength cases, and would tend not to be detected in competition with longer-wavelength fluctuations at the critical surface.

Finally, in figures 10 and 11 contour plots of the electric field for fluctuations
with two poloidal wavelengths and two density scale-lengths are shown. In figure 10, a very long fluctuation-wavelength case shows mainly the return beam scanning over the receiving horn as the fluctuation propagates. A point in time is shown at which a minimum in the the fluctuation is centred near the source horn, and the return beam has been made to diverge away from the midplane. At other times during a fluctuation period, the return beam can be made to converge on the midplane or scan above or below it. In figure 11, with a shorter fluctuation wavelength, and shorter density scale-length, we see that the return power is beginning to develop a mode structure reminiscent of an uncollimated reflection from a grating. The mode structure in this case does not change significantly during a fluctuation period.

7. Summary

In this paper some of the interesting fluctuation-induced effects possible in a reflectometry experiment have been simulated. Both radially and poloidally propagating modes can induce very large changes in amplitude and phase in the echo signal, even at low fluctuation amplitudes. Not only do fluctuations affect the signal by changing the propagation path, they also result in Doppler-shifted return signals that can interfere with the unshifted signal from the critical surface. One would surmise from these results that limiting the receiver bandwidth to some fraction of the fluctuation frequency
would improve the density profile measurements, and this has been shown to be the case experimentally (Prentice et al 1988). In addition, scattering away from the critical surface can become very important as the fluctuation amplitude is increased or the density profile broadened. Thus, the localization one can expect in the detection of radially propagating oscillatory modes is limited. This result is consistent with earlier 1D results, though the 2D geometry admits far more complex solutions. The 2D code results indicate that the response to poloidal modes can be somewhat localized near the critical surface. As the poloidal fluctuation wavelength is reduced and the response moves away from the critical surface, its amplitude is also greatly reduced. Phase ramping of the return signal can result from the propagation of poloidal fluctuations near the critical surface.

Certainly much more complex wave structures than those modelled in this paper exist in tokamaks. For example, drift wave turbulence is probably responsible for much of the amplitude and phase change seen in reflectometry data from tokamaks. However, as we have shown here, even the coherent mode results are not straightforward and easy to understand. Still, some attempt at modelling these modes should eventually be made.

Finally, it is important to point out that direct comparison with experimental results has not yet been attempted, but is very much needed to limit the parameter space over which the code is run. Large changes in signal level and phase can be obtained by making small changes in fluctuation scale-length, level, and location. Likewise, small changes in density profiles, together with the experimental geometry, can result in dramatic changes in the return signals. In the near future, the code will be used to model results from the C-Mod narrow-band reflectometer (Stek and Irby 1990).

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