Mathematical Model of Spatial Heat Exchange of Rooms With Linearly Sources of Heat

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Abstract. An effective tool for solving the problems of optimizing the schemes of the organization of heating and air distribution in a closed volume of the premises of buildings for various purposes is the mathematical modeling of the processes of heat, and the mass of exchange taking place in it. The spatial nonlinear heat problem is solved, and the mass of exchange in the volume of the room with linear sources of heat located along the perimeter. Schemes of temperature distribution by area and height of the closed volume of a room with linear sources of heat located along its perimeter are presented. On the basis of the compiled mathematical model of a closed room with linear sources of heat around the perimeter, the dependences allowing to determine the thermal power of linear heat sources taking into account their design characteristics to ensure a given temperature distribution over the area and height of the room for the purpose of organizing energy-efficient heating and air distribution systems are obtained.

1. Introduction

An effective tool for solving the problems of optimizing the schemes of the organization of heating and air distribution in a closed volume of the premises of buildings for various purposes is the mathematical modeling of the processes of heat, and the mass of exchange taking place in it.

The tasks of heat supply, the solution of which is difficult by empirical analysis of various technological, design and construction solutions and proposals, are currently a very urgent problem [3-8]. Mathematical modeling of a complex of physical processes taking place in real systems that consume heat energy is an effective tool for solving the heat supply problem for buildings and structures for various purposes, followed by an experimental study of the most attractive solutions and schemes.

The aim of the work is to compile a mathematical model of spatial non-stationary heat transfer (heat transfer) in an object that is a closed volume with a linear source of heat release and inhomogeneous boundary conditions on the outer and inner boundaries of the solution domain of the problem.

2. Problem formulation

We consider the boundary-value problem of heat conductivity for the region represented in Fig. 1.
The solution area includes various structural elements having different sizes and thermophysical characteristics. On the boundaries between all the constructions and on the boundaries with the medium external to the object under consideration, the corresponding boundary conditions were set.

The model of the heat consumption object studied in this work (Figure 1) describes quite well the room heated by a centralized or local heating system in conditions of intense heat exchange with the external environment.

The problem is solved within the framework of the heat conductivity model without taking into account the possible heat transfer due to natural or forced convection.

In this setting, the heat transfer process in the analyzed object is described by a system of nonstationary heat conduction equations with nonlinear boundary conditions.

### 3. Theoretical part

The mathematical model of heat distribution in the region under consideration is described by the heat conduction equation [1].

\[ \rho_i c_i \frac{\partial T_i}{\partial t} = \lambda_i \left( \frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} + \frac{\partial^2 T_i}{\partial z^2} \right), \quad i = 1, 5 \]  

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**Figure 1.** The area of solution of the problem under consideration. 1 - wall construction, 2 - internal air; 3 - filling design of the light opening; 4 - the construction of the flooring; 5 - coating design; 6, 7, 8 - linear sources of heat generation
where \( T = T(t, x, y, z) \); \( T_i \) is the temperature; \( x, y, z \) are the Cartesian coordinates; \( c_i \) is the specific heat; \( \rho_i \) is the density; \( \lambda_i \) - thermal conductivity of the \( i \)-th material: \( i = 1 \), wall construction; \( i = 2 \), the air medium inside the volume; \( i = 3 \), the construction of the filling of the light opening; \( i = 4 \), the construction of the floor; \( i = 5 \), the construction of the coating; \( t \) is time.

The initial and boundary conditions for the formulated problem have the form:

The initial condition:

\[
T(t, x, y, z)|_{t=0} = T_0 = \text{const}. \tag{3}
\]

Border conditions:

- on the verge of separating the external environment and the volume under consideration

\[
-\lambda_i \frac{\partial T_i}{\partial y} = \alpha \cdot (T_n - T_i) + \varepsilon \cdot \sigma \cdot (T_n^4 - T_i^4), \tag{4}
\]

where \( i = 1,3,5 \); since on this face there are such constructive materials as \( i = 1 \) - the design of the outer wall; \( i = 3 \) - design of filling the light opening; \( i = 5 \) - Cover structure.

- at all external boundaries of the volume under consideration, except for the face on which heat exchange with the external medium takes place, the symmetry conditions

\[
-\lambda_i \frac{\partial T_i}{\partial x_k} = 0, \quad \text{where} \quad \begin{cases} i = 1,5 \\ k = 1,3 \end{cases} \tag{5}
\]

- Linear sources of heat release are given conditions of the first kind

\[
T = T_{\text{inc}} = \text{const}. \tag{6}
\]

Here \( \alpha \) is the coefficient of heat exchange between the external environment and the considered solution region; \( T_{\text{am}} \) - ambient temperature; \( \varepsilon \) is the reduced blackness; \( T_0 \) is the initial temperature of the object under consideration; \( T_{\text{sat}} \) - temperature on the surface of a linear heat source; \( \sigma \) is the Stefan-Boltzmann constant.

It was assumed that the sources of heat release have a constant temperature throughout the entire time, and boundary conditions of the first kind are satisfied on their boundaries.

The formulated boundary-value problem should be solved by the method of finite differences [2]. The peculiarity of the problem under consideration is that the solution region includes several elements with essentially different in magnitude thermophysical characteristics.

4. Application results

In the laboratory of SEC TGV NIU MGSU, a test was conducted to determine the nature of the temperature distribution in the room volume (the area of the solution of the problem under consideration), in the absence of external enclosing structures in the field of research, and in the case when all the enclosing structures are external, except for the opposite direction from the linear source of heat , walls. In Figure 2, a laboratory stand is presented for determining the character of the temperature distribution along the height and along the plane in the volume of the room under investigation with a linear source of heat.
Figure 2. Laboratory bench for determining the nature of the temperature distribution in the study area, in the presence of spatial heat transfer

\[ t^oC \]

\[ t=f(H) \]

a)

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad H, m \]

Figure 3. Temperature distribution in the volume of the study area: a) in the absence of external enclosing structures; b) in the presence of external enclosing structures
The nature of the change in temperature over the height of the volume under investigation with a linear source of heat release, in the presence of external fences, is more uniform. External heat shielding absorbs heat from a linear heat source, decreases the temperature of the indoor air (\(t_v\)), increases the radiation temperature of the room (\(t_R\)), and the resulting temperature (\(t_p\)) remains relatively constant. In the absence of external fences, there is practically no heat loss through the fences, hence the internal air temperature (\(t_v\)) also increases, and the radiation temperature (\(t_R\)) results in a higher value of the resulting temperature (\(t_p\)).

5. Conclusions

1. A mathematical model of spatial heat transfer for a closed volume with linear heat sources is presented.

2. The presented mathematical model allows to solve the spatial nonstationary heat conduction problem for a closed volume with linear sources of heat release.

3. The conducted laboratory studies of heat exchange in a closed volume with linear heat sources, showed the nature of temperature changes in height and area.

4. The presence of external enclosing structures in a closed volume with linear heat sources contributes to a uniform temperature distribution.

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