The next non-Gaussianity frontier: what can a measurement with $\sigma(f_{\text{NL}}) \lesssim 1$ tell us about multifield inflation?

Roland de Putter, Jérôme Gleyzes, and Olivier Doré

1 California Institute of Technology, MC 367-17, Pasadena, CA 91125, USA
2 Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109, USA

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Future galaxy surveys promise to probe local primordial non-Gaussianity at unprecedented precision, $\sigma(f_{\text{NL}}) \lesssim 1$. We study the implications for multifield inflation by considering spectator models, where inflation is driven by the inflaton field, but the primordial perturbations are (partially) generated by a second, spectator field. We perform an MCMC likelihood analysis using Planck data to study quantitative predictions for $f_{\text{NL}}$ and other observables for a range of such spectator models. We show that models where the primordial perturbations are dominated by the spectator field, while fine-tuned within the broader parameter space, typically predict $f_{\text{NL}}$ of order unity. Therefore, upcoming galaxy clustering measurements will constitute a stringent test of whether or not the generation of primordial perturbations and the accelerated expansion in the inflationary universe are due to separate phenomena.

I. INTRODUCTION

Inflation is the leading theory describing the early universe [1–4], addressing both the standard problems of the hot big bang model (horizon problem, etc) and generating the primordial curvature fluctuations that are the seeds of the structure of the universe observed today. All this can be achieved, in a manner consistent with all current data, with the introduction of a single scalar field. However, there is no theoretical reason to expect only one field to be important in the early universe, and indeed fundamental physics models, such as those rooted in string theory, commonly predict multiple scalar fields (e.g. [5–7]). Moreover, multifield models provide an alternative important for falsifying the single-field paradigm.

A critical method for testing the single-vs. multifield nature of inflation is to look for local primordial non-Gaussianity (PNG), characterized by the parameter $f_{\text{NL}}$ [8] (since we exclusively consider PNG of the local type, we will often omit the qualifier “local”). While single-field models predict negligible local PNG [9, 10], $f_{\text{NL}} = -5/12(n_s - 1)$ (see e.g. [11, 12] for caveats to this rule), where $n_s$ is the scalar spectral index, multifield models can generate observably large $f_{\text{NL}}$ (see [13] for a review). The strongest current limits on $f_{\text{NL}}$ come from the bispectra of cosmic microwave background (CMB) fluctuations, observed by the Planck satellite, $f_{\text{NL}} = 0.8 \pm 5.0$ [14]. However, near-future galaxy surveys have the potential to improve on this significantly [15], taking advantage of an exciting new signal. In the presence of local PNG, the bias of galaxy density perturbations relative to the underlying matter density receives a scale-dependent correction, with scaling $\Delta b(k) \propto k^{-2}$ (where $k$ is the wave number), which becomes important on scales comparable to the Hubble scale [16–19]. By probing this characteristic signal on ultra-large scales, upcoming surveys such as SPHEREx [20–22], LSST [23] and EUCLID [24] are expected to improve on the current Planck constraint [1], eventually leading to order unity precision [26–28], $\sigma(f_{\text{NL}}) \lesssim 1$.

The motivation of this article is to address, in a quantitative way, what such a future $f_{\text{NL}}$ constraint can teach us about multifield inflation, and what signal we might expect to find.

While in single-field models the curvature perturbations are conserved from the time the modes of interest exit the horizon, in multifield inflation the perturbations can undergo super-horizon evolution after this time. This leads to a rich and complex phenomenology, where the final non-Gaussianity may strongly depend on physics both during and after inflation, including the reheating process. While large $f_{\text{NL}}$ is not generic in multifield inflation, often requiring significant fine-tuning [29], a number of interesting models have been identified where large (by large we mean $|f_{\text{NL}}| \gtrsim 1$) non-Gaussianity is generated. Examples include the curvaton [30–34], modulated reheating [35–39], models with an inhomogeneous end to inflation [40–43], the axion-quadratic model where an adiabatic limit is reached before reheating [44–46], two-field models with non-Gaussianity generated during slow-roll inflation [29], hybrid inflation [47, 48], N-flation [6, 49, 50], modulated trapping [51], and velocity modulation [52].

Although many previous studies have provided useful general, analytic insights into generation of non-Gaussianity in multifield inflation (see e.g. [29, 53–55]), an alternative approach is to use MCMC techniques to numerically sample the full parameter space of a number of multifield models (cf. [56–59]), given constraints from Planck on $f_{\text{NL}}$, $n_s$ and the tensor-to-scalar ratio $r$ (and implicitly on the scalar amplitude $A_s$). This is the

\[1\] Next-generation CMB missions may also significantly improve the $f_{\text{NL}}$ constraint relative to the Planck limit, see e.g. [25].
approach we will take here, paying specific attention to
the predictions for the distribution of $f_{\text{NL}}$ compared to
$s(f_{\text{NL}}) \lesssim 1$. To compute the evolution of perturbations,
and $f_{\text{NL}}$ in particular, we will use the $\delta N$ formalism [60–
63].

Instead of attempting to somehow sample the full
space of multifield models, we restrict ourselves to an
interesting subset, so-called “spectator models” (see also
e.g. [46, 64–66]), that capture key phenomenology of gen-
erating large $f_{\text{NL}}$. As a matter of definition, we assume
there are two fields during inflation, described by a separ-
able potential: the inflaton field, $\phi$, which at horizon exit
dominate the curvature perturbation and the back-
ground energy density, and a spectator field, $\chi$, which is
subdominant at this time, but the perturbations of which
(partially) determine the final curvature perturbation.
These models are a natural extension beyond single-field
inflation, with the inflaton still “driving” inflation. In
particular, in the “spectator-dominated” regime, where
the final curvature perturbations are dominated by the
spectator contribution, these models simply separate the
two main features of inflation: the inflaton drives the
background expansion, while the spectator generates the
primordial power spectrum. Our main interest will be
in this latter regime, as it is here that large $f_{\text{NL}}$ can be
produced.

In addition to simplifying calculations and thus allow-
ing for easier insights, requiring $\chi$ to be subdominant
during inflation (or at least at horizon exit) plays an im-
portant role in generating large $f_{\text{NL}}$. The reason is that,
quite commonly, if both fields individually have flat poten-
tials (small ratios of the first and second derivatives
relative to the potential itself, in Planck units), then $f_{\text{NL}}$
is typically suppressed. However, if the potential of $\chi$ is
small compared to the total energy density, it is allowed to
have a “non-flat” potential while still satisfying the
slow-roll conditions (since the latter depend on the ra-
tios of potential derivatives to the total energy density).
As we will see, evading this flat potential restriction is
what makes it easier to generate large non-Gaussianity
with the spectator field.

Since we wish to sample a concrete parameter space
and compare to current data, we will study three specific
spectator models (these models are not necessarily spec-
tator models, but we will restrict them to the parameter
space where they are), covering a range of mechanisms
for converting perturbations in the spectator field into
curvature perturbations: (A) a quadratic-axion poten-
tial, with conversion while both fields are rolling, (B) the
curvature, with conversion after the inflaton has decayed
into radiation, and (C) modulated reheating, with con-
version at the time of reheating.

By choosing specific models, our approach sacrifices
generality, but the benefit is that we will be able to de-
rive concrete, quantitative predictions for the probability
distribution of $f_{\text{NL}}$ and other quantities. In particular, it
is commonly claimed that various models naturally gen-
erate order unity $f_{\text{NL}}$ [15, 39, 55, 67, 68], which we will
back up here with a complete likelihood analysis.
Specific questions we will consider are:

- Do spectator-dominated models (which are the
ones interesting for $f_{\text{NL}}$) require fine-tuning of pa-
rameters?
- What is the posterior probability of finding $|f_{\text{NL}}| \gtrsim
1$ in spectator-dominated models?
- How does $f_{\text{NL}}$ relate to model parameters, and
what would measuring $f_{\text{NL}} \sim 1$ tell us about the
parameter space?
- What is the complementarity between primordial
non-Gaussianity and searches for primordial tensor
modes?

The article is organized as follows. In Section II, we
review general formalism describing inflation and the
primordial curvature perturbations. In Section III we intro-
duce spectator models in general and in particular the
three scenarios of interest. In Section IV, we discuss the
observational constraints that we will compare the mod-
els with. In Section V, we describe the results of our like-
lihood analysis of the three models and in Section VI, we
provide a final discussion and summarize our results.

II. GENERAL FORMALISM

In single-field inflation, the comoving curvature per-
turbation on uniform density hypersurfaces, $\zeta$ [69], is
non-linearly conserved after horizon exit [70, 71]. Thus,
the statistics of $\zeta$ remain frozen through the subsequent
evolution of the inflationary universe and the potentially
complicated phase of reheating. In particular, according
to the powerful single-field consistency condition [9, 10],
the level of local non-Gaussianity remains frozen at the
negligible value $f_{\text{NL}} = -5/12(n_s - 1)$.

The situation in multifield inflation is more compli-
cated. The additional field(s) lead to entropy pertur-
bations at the time of horizon exit, which can be trans-
sferred into the curvature perturbation through super-horizon
evolution, thus modifying the primordial power spectrum
and non-Gaussianity of $\zeta$. Eventually, inflation ends
and reheating takes place, giving rise to the radiation domi-
nated, conventional hot big bang phase. We will assume
that here the universe reaches a state of thermal equi-
librium without non-local conserved quantum numbers,
which implies that after reheating, perturbations are adi-
abatic, and $\zeta$ is conserved [72–74] until horizon re-entry
at a much later time. Thus, the statistics of $\zeta$ just
after reheating describe the standard adiabatic primordial
fluctuations and feed into the calculation of observational
phenomena in the late(r) universe, such as the cosmic mi-
crowave background anisotropies and cosmological large-
scale structure.

In the models considered in this work, entropy-to-
curvature conversion, and in particular the generation of
non-Gaussianity in $\zeta$, can take place both during inflation and/or during the period between the end of inflation and the time of reheating. By the latter we mean the time reheating/thermalization completes and the hot big bang phase with adiabatic perturbations is reached. In the following, we will give a brief overview of our treatment of evolution during both of these phases, and of how the perturbations are computed. We mostly follow standard methods and refer to the vast literature, e.g. the reviews [13, 75], for more details.

A. Background Evolution

We will assume inflation is described by two scalar fields with a sum-separable potential,

$$W(\phi, \chi) \equiv U(\phi) + V(\chi). \quad (1)$$

The slow-roll parameters are then defined as,

$$\epsilon^\phi \equiv \frac{m_{\text{Pl}}^2}{2} \left( \frac{U_{\phi}}{W} \right)^2, \quad \eta^\phi \equiv m_{\text{Pl}}^2 \frac{U_{\phi\phi}}{W},$$

$$\epsilon^\chi \equiv \frac{m_{\text{Pl}}^2}{2} \left( \frac{V_{\chi}}{W} \right)^2, \quad \eta^\chi \equiv m_{\text{Pl}}^2 \frac{V_{\chi\chi}}{W}, \quad (2)$$

where a field subscript defines a derivative w.r.t. the field and $m_{\text{Pl}} = (8\pi G)^{-1/2}$ is the reduced Planck mass (the standard Planck mass is $M_{\text{Pl}} = \sqrt{8\pi m_{\text{Pl}}}$). We work in natural units, $c = \hbar = 1$. We also define a total slow-roll parameter $\epsilon \equiv \epsilon^\phi + \epsilon^\chi$, which to leading order in slow-roll approximation is commonly made in the literature [33, 76–78], but it must be noted that a more realistic treatment of reheating could non-negligibly alter the predictions for the primordial perturbations (see [79] for a review). After this transition, the energy density formerly in the field decays like $\rho \sim a^{-4}$. We will consider scenarios where the two fields are converted to radiation at different times. When used without specific context, we will reserve the term “reheating” for the final process leading to the hot big bang phase.

B. Perturbations

We compute the evolution of perturbations by taking advantage of the separate universe/δN formalism [60–63], which allows us to express the curvature perturbation in terms of field perturbations $\delta \phi, \delta \chi$ at the time of horizon exit, $t_\star$. In the large-scale limit, $k \to 0$, we can treat perturbed regions of the universe as separate FLRW universes obeying the background equations. The evolution of perturbations can then simply be obtained by considering the difference between background quantities in these separate universe patches. In particular, the curvature perturbation $\zeta$ can be computed in terms of the difference in the number of e-foldings of evolution between two patches. Specifically, at $t_\star$, on a spatially flat hypersurface, consider a patch of the universe at $x$. Then, the curvature perturbation $\zeta$ at some later time $t_c$ is simply the perturbation to the number of e-foldings of expansion needed to get from $t_\star$ to the constant energy density hypersurface, $\delta \rho = 0$, at time $t_c$. In terms of the initial field perturbations on a spatially flat hypersurface, $\delta \phi_\star = \phi(t_\star, x) - \phi(t_\star), \delta \chi_\star = \chi(t_\star, x) - \chi(t_\star)$ (where $\phi(t_\star)$ and $\chi_\star$ are the background values),

$$\zeta(t_c) = \delta N(t_c, t_\star) = N_{\phi_\star} \delta \phi_\star + N_{\chi_\star} \delta \chi_\star + \frac{1}{2} \frac{N_{\phi_\star}}{N_{\phi_\star}} \delta \phi_\star^2 + \frac{1}{2} \frac{N_{\chi_\star}}{N_{\chi_\star}} \delta \chi_\star^2 + N_{\phi_\star, \chi_\star} \delta \phi_\star \delta \chi_\star + \ldots, \quad (5)$$

where we have dropped the position coordinate $x$, and on the right hand side also the time dependence. Here,

$$N(t_c, t_\star) = \int_{t_\star}^{t_c} dt \langle H(t) \rangle, \quad (6)$$

and $N_{\phi_\star} = \partial N/\partial \phi_\star$, etc.

The $\delta N$ formalism thus allows us to compute perturbations purely in terms of the evolution of slightly different FLRW background universes. Note that by writing $\zeta$ in terms of $\delta \phi_\star$ and $\delta \chi_\star$ only, we have implicitly assumed the perturbations have reached the space of inflationary growing/attractor solutions. In general, a model with
two fields has four degrees of freedom, \( \delta \phi \) and \( \delta \chi \) in addition to \( \delta \phi \) and \( \delta \chi \) (defined on a hypersurface of zero spatial curvature). However, by assuming the slow-roll approximation, we have turned second order equations of motion into first order ones, thus reducing the effective number of degrees of freedom to two.

Note finally that at \( t_s \), we have to first order,

\[
\zeta_s = \frac{\sqrt{2} \epsilon_s}{2} \delta \phi_s + \frac{\sqrt{2} \epsilon_s}{2} \delta \chi_s,
\]

and the non-Gaussianity in \( \zeta_s \) is negligible [80].

### C. Connecting to Observation

In the \( \delta N \) formalism, the dimensionless power spectrum of curvature perturbations is given by

\[
P_\zeta = \frac{k^3 P_\zeta}{2\pi^2} = P_\zeta \left( N_{\phi}^2 + N_{\chi}^2 \right),
\]

where \( P_\zeta \) is the dimensionless power spectrum of the field perturbations \( \delta \phi \) and \( \delta \chi \) at horizon exit, and \( H_s \) is the Hubble parameter at \( t_s \). A useful quantity is the fraction of the primordial power spectrum generated by the field \( \chi \),

\[
R \equiv \frac{P_\zeta}{|\chi|} = \frac{N_{\chi}^2}{N_{\phi}^2 + N_{\chi}^2}.
\]

In addition to the amplitude, another important observational property of the primordial power spectrum is the spectral index \( n_s \). Taking the derivative of the power spectrum (8), one can express this quantity in terms of the slow-roll parameters at \( t_s \) [63, 81],

\[
n_s - 1 = -2 \epsilon_s + 2 R \eta = 2(1 - R) \eta_s^2 - \frac{2}{m_{\text{Pl}}^2} \left( N_{\phi}^2 + N_{\chi}^2 \right).
\]

Similarly, the tensor-to-scalar ratio is given by,

\[
r = \frac{8}{m_{\text{Pl}}^2 \left( N_{\phi}^2 + N_{\chi}^2 \right)}.
\]

Finally, the local non-Gaussianity parameter is\(^2\),

\[
f_{\text{NL}} = \frac{5}{6} \left[ (1 - R)^2 \frac{N_{\phi,\phi}}{N_{\phi}} + R^2 \frac{N_{\chi,\chi}}{N_{\chi}} \right. \\
\left. + 2R(1 - R) \frac{N_{\phi,\chi}}{N_{\phi}} \right].
\]

Thus, non-Gaussianity can be generated through non-linear evolution of initial field perturbations into the curvature perturbation. The expression above neglects a small, slow-roll suppressed contribution due to intrinsic non-Gaussianity in \( \delta \phi \) and \( \delta \chi \) [80].

### III. SPECTATOR MODELS

As motivated in the Introduction, in this paper we focus on spectator models. We define these here by requiring that the initial curvature perturbation at horizon exit, \( \zeta_s \), is dominated by the perturbation in the inflaton \( \phi \), so that \( \delta \phi_s \) is the initial adiabatic fluctuation. In these models, the perturbation \( \delta \chi_s \) therefore describes an entropy perturbation. In equations, this comes down to,

\[
\epsilon_s \gg \epsilon \chi \quad \text{&} \quad U_s \gg V_s.
\]

**Spectator Models**

In these models, the parameter \( R \) now distinguishes between the regimes where the final curvature power spectrum is dominated by inflaton (\( \phi \)) or spectator (\( \chi \)) fluctuations,

\[
R \approx 0: \text{ Inflaton-Dominated Regime} \\
R \approx 1: \text{ Spectator-Dominated Regime}
\]

The spectator-dominated regime is particularly interesting for the generation of observable levels of non-Gaussianity and is the main focus of this work.

In spectator models, we generally have

\[
N_{\phi} \approx \frac{1}{\sqrt{2} \epsilon_s m_{\text{Pl}}} \approx \text{const.},
\]

so that the final power spectrum is given by,

\[
P_\zeta \approx \frac{1}{1 - R} P_{\zeta, s} = \frac{1}{1 - R} \frac{1}{2 \epsilon_s m_{\text{Pl}}} \left( H_s \right)^2.
\]

Thus, the conversion of entropy to curvature can only increase the power spectrum, leading to a large boost in power in the spectator-dominated regime. The scalar spectral index, Eq. (10), for spectator models simplifies to,

\[
n_s - 1 = -2 \epsilon_s + 2 R \eta + 2(1 - R) (\eta^2 - 2 \epsilon_s),
\]

so that it varies between

\[
n_s - 1 = -6 \epsilon_s + 2 \eta^2 \quad \text{(inflaton-dominated regime)} \\
n_s - 1 = -2 \epsilon_s + 2 \eta^2 \quad \text{(spectator-dominated regime)}
\]

Note that both asymptotic values are of order slow-roll, and are entirely determined by the slow-roll parameters at horizon exit.

Since the tensor power spectrum remains constant after horizon exit, the tensor-to-scalar ratio for spectator models is suppressed for \( R > 0 \),

\[
r = 16(1 - R) \epsilon_s.
\]

\(^2\) Note that we use a sign convention consistent with Planck, but opposite to that of [9, 63].
The suppression of \( r \) in the spectator-dominated regime is both a feature and a bug. On the one hand, it becomes exceedingly difficult to detect primordial tensor modes observationally. On the other hand, large-field inflaton models that are currently ruled out because they predict a value of \( r \) above the observationally allowed range can be put in concordance with the data by adding a spectator field that seeds a large fraction of the primordial power spectrum (we will discuss this further in Section V D).

The non-Gaussianity in spectator models is approximately given by (cf. Eq. (12)),

\[
    f_{\text{NL}} = \frac{5}{6} \frac{N_\chi \chi^*}{N_\chi^2}.
\]

The behavior of \( \phi \) is very similar to that of single field inflation, so that the \( N_{\phi, \phi_*}/N_{\chi^2} \) term in Eq. (12) is slow-roll suppressed and thus negligible for our purposes (we remind the reader that we are focused on probing \( f_{\text{NL}} \) with order unity precision). Moreover, one can check that in the spectator-dominated regime that interests us, the cross-term proportional to \( N_{\phi, \chi^*} \) is suppressed compared to the \( N_{\chi, \chi^*} \) term. We will see that the above expression can give rise to large \( f_{\text{NL}} \) for \( R \sim 1 \).

We will discuss this more quantitatively in Section V D, but we see here already the complementarity between measuring primordial tensor fluctuations and non-Gaussianity (of the scalar fluctuations). For large-field potentials in the inflaton-dominated regime, \( N_{\phi, \chi^*} \) is strongly suppressed and difficult to detect, but \( f_{\text{NL}} \) can be within observational reach.

### A. Three Specific Spectator Models

In the following sections, we will introduce the three specific spectator models for which we will study observational predictions and constraints. For the spectator field potential, \( V(\chi) \), we will consider both an axion-like periodic potential and a quadratic potential, which we will discuss in more detail further below.

For the inflaton field, we consider simple, large-field (e.g. power law) potentials. Since our main interest is in the properties of the spectator field and how it generates \( f_{\text{NL}} \), the details of \( U(\phi) \) are less important for our study than the properties of \( \chi \) and the transfer of its perturbations. The spirit of our approach to the inflaton potential is that there is in principle ample freedom in its form to always be able to fit at least \( n_s \) and \( A_s \), and that \( f_{\text{NL}} \) is relatively insensitive to the details of \( U(\phi) \). In practice, to keep the calculations as simple as possible, our main implementation of the inflaton potential will be a quadratic potential,

\[
    U(\phi) = \frac{1}{2} m_\phi^2 \phi^2,
\]

where we treat the field value at horizon exit, \( \phi_* \), as a free parameter. We will briefly consider a more general setup, with varying power law index of the inflaton potential, in Section V D.

#### 1. Case A: The Quadratic-Axion in the Horizon Crossing Approximation

We first consider a model where the spectator field is governed by a periodic, axion-like potential,

\[
    V(\chi) = \frac{1}{2} V_0 \left[ 1 + \cos \left( \frac{2\pi \chi}{f} \right) \right],
\]

where \( f \) is a “decay constant”, and \( V_0 \) gives the normalization of the potential. In the case under consideration where the inflaton is described by a quadratic potential, this is the quadratic-axion model [44–46]. This is a known, simple example of a model capable of generating large non-Gaussianity (\( |f_{\text{NL}}| \gtrsim 1 \)) during or slightly after slow-roll inflation without necessarily relying on mechanisms during the reheating phase. We explain below why this is, after introducing the approximation we will use to compute perturbations in this model. The assumption of a quadratic potential for the inflaton is not crucial so that the quadratic-axion model is merely a specific example of a broader class of “inflaton-axion” models.

During inflation, while both fields obey the slow-roll conditions, the number of \( e \)-foldings between \( t_* \) and some later time \( t_c \) is given by

\[
    N = -\frac{1}{m_{\text{Pl}}^2} \int_{\phi_*}^{\phi_c} \frac{d\phi}{U(\phi)} - \frac{1}{m_{\text{Pl}}^2} \int_{\chi_*}^{\chi_c} \frac{V(\chi)}{V_\chi} d\chi.
\]

Assuming a zero spatial curvature hypersurface at \( t_* \) and a constant energy density surface at \( t_c \), the \( \delta N \) formalism allows us to write the curvature perturbation at \( t_c \) as,

\[
    \zeta(t_c) = \delta N = \left[ \frac{1}{m_{\text{Pl}}^2} \left( \frac{U}{U_\phi} \right)_s \delta \phi_s + \frac{1}{m_{\text{Pl}}^2} \left( \frac{V}{V_\chi} \right)_s \delta \chi_s \right] - \left[ \frac{1}{m_{\text{Pl}}^2} \left( \frac{U}{U_\phi} \right)_c \delta \phi_c + \frac{1}{m_{\text{Pl}}^2} \left( \frac{V}{V_\chi} \right)_c \delta \chi_c \right],
\]

which is straightforwardly extended to higher orders. Since we have fixed the gauge at \( t_c \), the perturbations \( \delta \phi_c \) and \( \delta \chi_c \) are fully specified in terms of \( \delta \phi_s \) and \( \delta \chi_s \). This enabled [63] to derive analytic expressions for \( \zeta \) in terms of \( \delta \phi_s \) and \( \delta \chi_s \) only. The \( \delta \phi_c \) and \( \delta \chi_c \) contributions make these expressions rather complicated.

\(^3\) The case of a sum-separable potential during slow-roll is special in the sense that \( N \) can be written as a path-independent integral through field space. In other words, there exists some function defined for all \( \phi \) and \( \chi \) and \( N \) is simply the difference of that function between the end point \( (\phi_c, \chi_c) \) and the starting point \( (\phi_*, \chi_*) \). It is this property that allows the derivation of closed analytic expressions for the curvature perturbation as in [63].
The Horizon Crossing Approximation (HCA) - If, however, before $t_\ast$, an adiabatic limit is reached where, independently of the initial perturbation, the fields always end up on the same field trajectory, the contributions from the perturbations at $t_\ast$ can be neglected. In this limit, the perturbations are well described by the so-called Horizon Crossing Approximation (HCA) [44, 49, 82] and are fully expressed in terms of the field perturbations at horizon exit (we note that, in single-field inflation, this assumption is generally satisfied at all times after horizon exit under the standard assumption of being on the single-field attractor solution, thus explaining why $\zeta$ is conserved in single-field inflation). The HCA simplifies the expressions for the perturbations and their non-Gaussianity considerably, giving easy insights in the multifield phenomenology and allowing us to straightforwardly identify models with the potential for generating large non-Gaussianity.

Before explicitly writing the HCA expressions to second order, it is useful to define slow-roll parameters for the individual potentials (cf. Eq. (2)),

\begin{equation}
\tilde{\epsilon}_\phi \equiv \frac{m_{\text{pl}}^2}{2} \left( \frac{U_\phi}{V} \right)^2, \quad \tilde{\eta}_\phi \equiv \frac{m_{\text{pl}}^2}{2} \left( \frac{U_{\phi \phi}}{V} \right) \tag{22}
\end{equation}

\begin{equation}
\tilde{\epsilon}_\chi \equiv \frac{m_{\text{pl}}^2}{2} \left( \frac{V}{\chi} \right)^2, \quad \tilde{\eta}_\chi \equiv \frac{m_{\text{pl}}^2}{2} \left( \frac{V_{\chi \chi}}{V} \right). \tag{23}
\end{equation}

While the true slow-roll parameters, normalized by the total energy density $W$, are required to be small for the slow-roll approximations to hold, the individual slow-roll parameters can in principle be larger than unity. In particular, for spectator models, $\tilde{\epsilon}_\phi \approx \tilde{\epsilon}_\phi \approx \epsilon_\star$, $\tilde{\eta}_\phi \approx \eta_\phi$, but $\tilde{\epsilon}_\chi = (W_\chi / V_\chi)^2 \epsilon_\chi \gg \epsilon_\chi$, $\tilde{\eta}_\chi = (W_\chi / V_\chi) \eta_\chi \gg \eta_\chi$.

In terms of these, the HCA gives,

\begin{equation}
m_{\text{pl}} N_{\phi,\chi} = \frac{1}{\sqrt{2\tilde{\epsilon}_\phi}}, \quad m_{\text{pl}} N_{\chi,\chi} = \frac{1}{\sqrt{2\tilde{\epsilon}_\chi}}, \tag{24}
\end{equation}

\begin{equation}
m_{\text{pl}}^2 N_{\phi,\phi} = 1 - \frac{\tilde{\eta}_\phi}{2\epsilon_\phi}, \quad m_{\text{pl}}^2 N_{\chi,\chi,\chi} = 1 - \frac{\tilde{\eta}_\chi}{2\epsilon_\chi}, \tag{25}
\end{equation}

and $N_{\phi,\chi} = 0$. Thus, assuming a spectator model, to be in the spectator-dominated regime, say $N_{\chi,\chi}^2 > N_{\phi,\phi}^2$, one requires

$$\epsilon_\chi < \epsilon_\phi \approx \epsilon_\ast.$$ \tag{26}

This means one needs a very small value of $\epsilon_\chi$. Next, assuming the spectator domination requirement is fully satisfied ($R \approx 1$), the non-Gaussianity is given by,

$$f_{\text{NL}} \sim \frac{5}{6} \frac{N_{\chi,\chi,\chi}}{N_{\chi,\chi}^2} = \frac{5}{6} (2\epsilon_\chi - \eta_\chi) \approx -\frac{5}{6} \tilde{\eta}_\chi.$$ \tag{27}

Therefore, for a spectator dominated model to generate large non-Gaussianity, one needs a large individual slow-roll parameter $|\tilde{\eta}_\chi| \gg 1$. This is not inconsistent with slow-roll inflation because $\eta_\chi$ is suppressed relative to $\tilde{\eta}_\chi$. This argument (based on the simple HCA assumption), nicely illustrates the more general point that, in multifield inflation, if both fields contribute significantly to the energy density of the universe, the slow-roll conditions typically restrict $f_{\text{NL}}$ to be small, and that this limitation can be evade by considering spectator fields, which may have very non-flat potentials without violating slow-roll because their energy density contribution is small.

![FIG. 1: Illustration of the behavior of the background densities in the quadratic-axion scenario (Case A), as a function of the number of e-folds N. We show two scenarios, depending on whether the spectator field/axion, $\chi$, starts rolling down its potential before (dark blue) or after (light blue) the end of inflation. As illustrated, in the latter case, $\chi$ could generate a brief second phase of inflation before it starts oscillating. We assume that reheating only occurs after the phase is reached where both energy densities decay like $\rho \sim a^{-3}$ and that the reheating process does not alter the curvature perturbations. After reheating, the universe is filled with radiation (thick red and pink for scenario 1 and 2 respectively) with adiabatic perturbations.](image)

The requirements of ultra-small $\tilde{\epsilon}_\chi$ and large $\tilde{\eta}_\chi$ are naturally incorporated in the axion model, Eq. (19), if the initial field $\chi_\ast$ is placed near the top of the cosine potential. In the limit $\chi_\ast / f \rightarrow 0$, the slope of the potential asymptotes to zero, while the curvature approaches a constant, thus satisfying the two conditions. By having the amplitude $V_0$ low, the slow-roll conditions are satisfied as well. This ability to produce large $f_{\text{NL}}$, together with the fact that axion potentials can be realized in a technically natural way from a more complete Lagrangian, makes the inflaton-axion model theoretically appealing.

To illustrate the background evolution of the fields in this model, we schematically plot the energy densities as a function of the number of e-folds $N$ in Figure 1. We show two scenarios. In both, the energy density of $\chi$ is subdominant throughout the inflationary period driven by the inflaton $\phi$. In the first scenario, $\chi$ starts rolling with its energy density decaying according to $\rho_\chi \propto a^{-3}$ slightly before the end of inflation (dark blue curve). After this, inflation ends, and both components decay like matter. Later, reheating takes place, after which we assume the total energy density of the universe to exist in the form of radiation (thick red curve). In the alternative
scenario, χ starts rolling/oscillating after the end of inflation. While not always the case, in the scenario shown, this happens after χ has come to dominate the energy budget of the universe, thus leading to a second phase of inflation of modest duration. Again, after both fields end up decaying proportional to $a^{-3}$, reheating takes place, and the universe is filled with radiation (thick pink).

Which scenario takes place depends on the model parameters in a relatively straightforward manner. The time that χ starts rolling (exits slow-roll) is partially determined by comparing the Hubble rate to the mass associated with the axion potential,

$$m_{\chi}^2 = \frac{2\pi^2}{f^2} V_0.$$  \hspace{1cm} (26)

Tuning the initial field value to be close to the hilltop, $\chi_i/f \ll 1$, however, will delay this moment. For $m_\chi > m_\phi$ and $\chi_i/f$ not too small, χ can thus start rolling before the end of inflation, as shown in scenario 1. If $V_0$ is sufficiently large, χ can also come to dominate the universe before the end of inflation, thus lengthening the duration of inflation (not shown). In most cases relevant to our likelihood analysis, χ starts oscillating well after the end of inflation. For large f and small $\chi_i/f$, χ first drives a second phase of inflation (scenario 2), but in a large fraction of parameter space, this is not the case, i.e. χ starts oscillating when its energy density is smaller than or comparable to $\rho_\phi$ (not shown).

Validity of Horizon Crossing Approximation - We now come back to the question of the range of validity of the HCA. The approximation is exact if an adiabatic limit is reached while both fields are in the slow-roll regime. In practice, even if this is not the case, if after inflation a phase is reached where both fields oscillate around their minima (with energy densities decaying like $\rho \propto a^{-3}$), so that $\zeta = \text{const.}$, then the HCA still turns out to be a reasonable approximation in many cases (see e.g. [44]).

To test the range of validity of the HCA in the quadratic-axion model, we have numerically computed the perturbations into the $\zeta = \text{const.}$ phase using the exact $\delta N$ formalism and compared the results to the HCA predictions. We describe the details in Appendix B, but the main result is that, for the parameter space we will study here, the Horizon Crossing Approximation is a good estimator of $f_{\text{NL}}$ to within a factor of less than two. In addition, we introduce an $f$-dependent correction factor that brings the HCA prediction in much better agreement with the exact numerical calculation. We use both prescriptions separately in our likelihood analysis to bracket the possible range of $f_{\text{NL}}$ values. Both prescriptions give qualitatively similar results.

Finally, for Case A, we assume that reheating does not modify $\zeta$ after the $\zeta = \text{const.}$ phase described by the HCA. In the simplified instantaneous reheating picture, this would correspond to reheating taking place on a constant total energy density hypersurface.

2. Case B: The Curvaton

For Case B, we consider a simple quadratic potential for the spectator field,

$$V(\chi) = \frac{1}{2} m_{\chi}^2 \chi^2. \hspace{1cm} (27)$$

The curvaton scenario [30–34] relies on a post-inflationary phase where $\phi$ has already decayed into radiation and $\chi$ is oscillating around its minimum. Thus, the energy density of $\chi$ grows relative to that of $\phi$ and perturbations $\delta \chi$ are converted into curvature perturbations. It is known that in the limit where the curvature perturbations are dominated by $\delta \chi$, large non-Gaussianity ($|f_{\text{NL}}| \gtrsim 1$) can be generated [76].

Here, we consider the following specific curvaton scenario, with three main phases, as illustrated in Fig. 2. The first phase is the period of inflation, where both fields are slowly rolling. This phase ends when $H_{\text{end}} = m_\phi$, at $t_{\text{end}}$, after which the inflaton starts oscillating around its minimum, with energy density decaying like pressureless dust, $\rho_\phi \propto a^{-3}$. We assume that at some point during this phase, the inflaton decays into radiation, leading to $\rho_\phi \propto a^{-4}$ evolution (we keep using the subscript $\phi$ even though at this point the component consists of radiation). The second phase ends at $H_{\text{curv}} = m_\chi < m_\phi$, at $t_{\text{curv}}$, when the spectator field starts oscillating around its minimum, leading to $\rho_\chi = \frac{1}{2} m_{\chi}^2 \chi_{\text{curv}}^2 (a/a_{\text{curv}})^{-3}$. We refer to this third phase as the curvaton phase. It ends when also the curvaton decays into radiation at $H_{\text{reh}} = \Gamma_{\text{reh}} < m_\chi$. We assume all transitions take place on constant total energy density slices so that $\zeta$ is conserved across the transitions. While we assume throughout this paper that after reheating the perturbations are purely adiabatic, we refer to [57] for a recent study of the observational consequences of persisting isocurvature fluctuations.

For the curvaton scenario, we will make a slightly stronger assumption than the usual spectator requirements, namely that $\chi$ is subdominant not just at $t_*$, but until the beginning of the curvaton phase, $t_{\text{curv}}$, i.e. $\frac{1}{2} m_{\chi}^2 \chi_{\text{curv}}^2 \ll 3m_{\phi}^2$. This allows for simple analytic expressions for the spectator contributions to the final curvature perturbations [76] and for the evolution from $\chi_*$ to $\chi_{\text{curv}}$. At the linear level, we use

$$N_{\chi*} = \frac{2\chi_{\text{reh}}}{3\chi_*}, \hspace{1cm} (28)$$

---

4. When this is not the case, it is possible to modify $\zeta$ at the end of inflation through the dependence on $\delta \phi_\zeta$ and $\delta \chi_\zeta$, which is neglected in the HCA. In particular, models where the fields are on a turning trajectory at the end of inflation can generate large $f_{\text{NL}}$ in a way not captured by the HCA.

5. While $\zeta$ is constant in such a phase, this is not necessarily an adiabatic limit, as entropy perturbations may still exist. Only if these entropy perturbations are not converted to curvature through reheating at a later stage, will $\zeta$ remain constant into the hot big bang phase.
where $\chi_*$ is the initial field value and

$$r_\chi \equiv \frac{3\rho_\chi}{3\rho_\chi + 4\rho_\phi}$$  \hspace{1cm} (29)$$
gives the relative contribution of the curvaton to $\rho + 3p$ during the curvaton phase ($r_{\chi,\text{reh}} = r_\chi$ evaluated at $t_{\text{reh}}$). The non-Gaussianity parameter is given by

$$f_{\text{NL}} = \frac{5}{6} R^2 \left( -r_{\chi,\text{reh}} - 2 + \frac{3}{2r_{\chi,\text{reh}}} \right).$$  \hspace{1cm} (30)$$

From Eq. (28), one needs small $\chi_*$ ($\lesssim \sqrt{2} m_\phi$, $m_\text{Pl}$) to reach the spectator-dominated regime. Assuming $R \sim 1$ is indeed obtained during the curvaton phase, if this happens while $r_\chi$ is small, the non-Gaussianity can be very large $f_{\text{NL}} \sim 5/(4r_\chi)$. If and when the curvaton phase continues to the point where $\rho_\chi$ dominates ($r_\chi \to 1$), the asymptotic value $f_{\text{NL}} = -5/4$ is reached.

The results only minimally depend on exactly when during the second phase the inflaton decays into radiation\(^6\). Therefore, to keep the analysis minimal, instead of including a free parameter $\Gamma_\phi$ to describe this transition, we simply consider the extreme case, where $\phi$ decays immediately at $t_{\text{end}}$. We have checked that using the opposite extreme, where it decays at $t_{\text{curv}}$, leads to very similar results.

\(^6\) Varying the time of decay slightly changes the evolution of $\chi$ between $t_{\text{end}}$ and $t_{\text{curv}}$ and therefore affects the curvaton energy density at $t_{\text{curv}}$ and thus $r_\chi$.

3. Case C: Modulated Reheating

For Case C, we again consider a simple quadratic potential for the spectator field,

$$V(\chi) = \frac{1}{2} m_\chi^2 \chi^2.$$  \hspace{1cm} (31)$$

In the modulated reheating scenario [35–39] (see, e.g. [83, 84] for recent studies), the decay rate of the inflaton, which determines the time of reheating, depends on the spectator field $\chi$. Then, even if $\chi$ contributes negligibly to the energy density of the universe, the quantum fluctuations in $\chi$ at horizon exit can be transferred into curvature perturbations through the reheating process (the reheating hypersurface is not one of constant energy density, but is modulated by $\chi$). This is a well known scenario producing large $f_{\text{NL}}$ [39].

\[FIG. 2: Illustration of the behavior of the background densities in the curvaton scenario (Case B). After the inflaton has decayed into radiation (at $H = \Gamma_\phi$), $\rho_\phi \sim a^{-3}$, an until then subdominant spectator/curvaton field $\chi$ starts oscillating around its potential minimum. Since $\rho_\chi \sim a^{-3}$ in this phase, its energy density may become important and its perturbations can be converted into curvature perturbations. This curvaton phase ends at $H = \Gamma_{\text{reh}}$, when we assume $\chi$ decays into radiation with adiabatic fluctuations without further modifying the curvature perturbations.\]

\[FIG. 3: Illustration of the behavior of the background densities in the modulated reheating scenario (Case C). After inflation, the inflaton oscillates around its potential minimum and decays into radiation when $H = \Gamma_{\text{reh}}(\chi)$. The spectator field $\chi$, which has negligible energy density at the time of reheating, modulates the hypersurface of reheating and may thus convert its fluctuations into curvature perturbations. We assume that after reheating by $\phi$, the spectator field $\chi$ plays no further role (e.g. promptly decaying itself), leading to a universe composed of radiation with adiabatic perturbations.\]

The specific case we consider here, see Figure 3, consists of two phases: the standard inflationary phase, ending at $H_{\text{end}} = m_\phi$, and a subsequent phase where the inflaton oscillates around its minimum and $\chi$ is still slowly rolling. This phase ends at $t_{\text{reh}}$, when $H_{\text{reh}} = \Gamma_{\text{reh}}(\chi)$. We assume $\chi$ is subdominant all the way up to $t_{\text{reh}}$, $\frac{1}{2} m_\chi^2 \chi_{\text{reh}}^2 \ll 3\Gamma_{\text{reh}}^2(\chi_{\text{reh}})$, and that it plays no role in the generation of curvature perturbations other than through the reheating process. In particular, we assume that after $t_{\text{reh}}$, $\chi$ decays into radiation as well without further modifying $\zeta$.

For the reheating of the inflaton, we consider a toy model where decay to fermions ($q$) is the dominant process, through a coupling term of the form,

$$\mathcal{L} \supset -\lambda(\chi) \phi \bar{q} q.$$  \hspace{1cm} (32)$$
The decay rate is then [85]
\[ \Gamma_{\text{reh}} = \frac{m_\phi \lambda^2(\chi)}{8\pi}. \] (33)

For the dependence of the coupling constant on \( \chi \), we choose a simple expansion truncated at quadratic order (see also, e.g., [86, 87]),
\[ \lambda(\chi) = \lambda_0 + \lambda_1 \frac{\chi}{M_c} + \frac{1}{2} \lambda_2 \left( \frac{\chi}{M_c} \right)^2, \] (34)
where \( M_c \) is a cutoff scale in the effective field theory, \( M_c \gg H_s \), and the dimensionless parameters \( \lambda_0, \lambda_1, \lambda_2 \) are at most of order unity (additional bounds are described in Appendix A).

Following [83], we use,
\[ N_s = -\frac{1}{6} \frac{\Gamma'_{\text{reh}}}{\Gamma_{\text{reh}}} \frac{\partial \chi_{\text{reh}}}{\partial \chi_s}, \] (35)
and
\[ f_{\text{NL}} = 5 \left[ 1 - \frac{\Gamma''_{\text{reh}}}{(\Gamma'_{\text{reh}})^2} - \frac{\Gamma_{\text{reh}}}{\Gamma'_{\text{reh}}} \left( \frac{\partial \chi_{\text{reh}}}{\partial \chi_s} \right)^{-2} \frac{\partial^2 \chi_{\text{reh}}}{\partial \chi_s^2} \right], \] (36)
where primes denote derivatives w.r.t. \( \chi_{\text{reh}} \). We include the contributions due to the evolution of the spectator field between \( t_s \) and \( t_{\text{reh}} \) as in [83]. The expression for \( f_{\text{NL}} \) shows that, if the spectator-dominated regime is reached, one would naturally expect \( |f_{\text{NL}}| \sim 5 \). We will make this more quantitative in Section V.C.

IV. COMPARISON TO OBSERVATION

A. Current CMB Constraints

We derive constraints on the three spectator models discussed above using the most recent Planck cosmic microwave background measurements [88] of \( n_s \) and \( f_{\text{NL}} \). For \( r \), we use the joint analysis by Planck and BICEP2 of B-modes on the subset of the sky covered by BICEP2 [89] (which is why we will not include a correlation between \( n_s \) and \( r \) in our likelihood). Specifically, we model the measurements by Gaussian likelihoods (restricted to \( r \geq 0 \) for \( r \)) with mean and standard deviation,
\[ n_s = 0.9645 \pm 0.0049, \] (37)
\[ r = 0.0497 \pm 0.0383, \] (38)
where \( n_s \) and \( r \) are defined relative to a pivot scale \( k_s = 0.05 \text{ Mpc}^{-1} \), and
\[ f_{\text{NL}} = 0.8 \pm 5.0. \] (39)

We then apply standard Markov Chain Monte Carlo (MCMC) techniques using the python package emcee [90] to derive constraints on the spectator parameters.

We summarize the parameter space and physically motivated priors for each model in Appendix A.

Planck also provides a measurement of the amplitude of the primordial power spectrum, \( A_s \equiv P_\zeta(k_s) \), namely [88, 91]
\[ \ln(10^{10} A_s) = 3.094 \pm 0.034. \] (40)

We treat this measurement differently than the constraints on \( n_s, r \) and \( f_{\text{NL}} \). The reason is that we still have the freedom to take out an absolute energy scale from the equations describing our models by rescaling various model parameters. We can choose this energy scale to be the normalization of the inflaton potential, in this case \( m_\phi^2 \). Specifically, if we define rescaled quantities, \( V(\chi) \rightarrow \tilde{V}(\chi) \equiv V(\chi)/m_\phi^2 \), \( m \rightarrow \tilde{m} \equiv m/m_\phi \), \( \Gamma \rightarrow \tilde{\Gamma} \equiv \Gamma/m_\phi \), etc. (but leave the fields unchanged), the evolution equations retain the same form given previously, but in terms of the “tilded” quantities. The observables \( n_s, r \) and \( f_{\text{NL}} \) are also independent of the overall mass scale \( m_\phi \). The main quantity that does depends on \( m_\phi \) is \( A_s \). Therefore, we will in practice sample the rescaled parameters and, instead of also treating \( m_\phi \) as a free parameter, it is implicit that at each point in parameter space it is tuned in order to obey the \( A_s \) constraint. One subtlety in this approach is that physical constraints and priors (see Appendix A) sometimes are naturally given in terms of absolute, not rescaled, scales.

In order to translate these priors to the rescaled parameters, we will simply use a fiducial value for the overall mass scale, \( m_\phi^\text{fid} = 1.6 \cdot 10^{13} \text{GeV} \) (the mass scale required to reproduce the observed \( A_s \) for an inflaton-dominated model with \( \phi_s = 15 \text{ Mpc}^{-1} \)). This is a reasonable choice because the variation in \( m_\phi \) needed to fit \( A_s \) is relatively small compared to the very wide prior ranges considered here.

In principle, there is a constraint in addition to the measurements of \( n_s, r \) and \( f_{\text{NL}} \) and \( A_s \), namely on the number of e-folds of inflation between horizon exit of the mode of interest and the end of inflation, \( N_s \). Working backwards in time from the present, one can compute how far outside the horizon a given mode with wave vector \( k_* \) was at the time when inflation ends, which in turn specifies how many e-folds before the end of inflation that mode must have exited the horizon (see e.g. [88, 92]),
\[ N_s + \ln \left( \frac{H_{\text{end}}}{H_s} \right) = 61.7 - \ln \left( \frac{k_*}{0.05 \text{ Mpc}^{-1}} \right) \]
\[ - \frac{1}{12} \ln \left( g_s(T_{\text{reh}}) \right) + \frac{1}{4} \ln \left( \frac{H_{\text{end}}^2}{3m_{\text{Pl}}} \right) \]
\[ + \frac{1}{12(1 + w_{\text{reh}})} \ln \left( \frac{H_{\text{reh}}^2}{H_{\text{end}}^2} \right). \] (41)
Here, \( w_{\text{reh}} \) is the effective equation of state between the end of inflation and the finalization of the reheating phase, and \( g_s(T_{\text{reh}}) \) is the effective number of degrees of freedom at the temperature of reheating.
Therefore, in those models considered here that specify the reheating history (Cases B & C), once the model parameters are fixed, \(N_\star\) is fixed as well (modulo some variation due to uncertainty in the energy content of the universe after reheating). In this sense, the system is overconstrained because \(\phi_\star\) is not truly a free parameter. However, in principle, one could readily use the remaining freedom to tune the shape of the inflaton potential (beyond the quadratic form) to match \(N_\star\). To keep our treatment as straightforward as possible, instead of including this additional freedom explicitly and applying the mode matching to \(N_\star\), we simply do neither. Since, again, our main focus is the properties of the spectator field and \(f_{NL}\), this minimally affects our results. In particular, \(f_{NL}\) is rather insensitive to these choices. We will briefly consider a more general setup, with varying power law index of the inflaton potential, in Section V D.

B. Future Galaxy Clustering Constraints

Our MCMC runs exclusively include current CMB constraints. However, the motivation of this paper is to quantify the constraining power of next-generation measurements in the resulting space of multifield/spectator models allowed by current data. In particular, we are motivated by upcoming galaxy surveys, which, using scale-dependent halo bias, target order unity precision on local primordial non-Gaussianity, \(\sigma(f_{NL}) \lesssim 1\). Instead of modeling any specific survey, we will simply compare the posterior parameter and observable distributions from Planck data to this approximate level of constraint, \(\Delta f_{NL} \sim 1\).

V. RESULTS

A. The Quadratic-Axion in the Horizon Crossing Approximation

We first consider the quadratic-axion model using the (improved) Horizon Crossing Approximation, see Section III A 1 and Appendix B. The predictions of \(f_{NL}\) for this model will turn out to be very sensitive to the upper bound chosen for the “axion decay constant” \(f\). Since in a UV complete theory, it may be difficult to generate axion-like potentials with \(f\) larger than the Planck scale \(93, 94\), our default choice will be \(f < M_{pl} = \sqrt{8\pi M_{p}}\) (note that \(M_{pl}\) is not the reduced Planck mass here). To illustrate the dependence on this cutoff, we will also show results for the prior \(f < 3M_{pl}\) (see Appendix A 1 for the other parameter priors).

In order to gain insight on what the allowed parameter space looks like, let us highlight what imposing the spectator-dominated regime means. Within the HCA, \(R\) has a very simple form and \(R > 0.9\) translates into,

\[
\frac{\chi_\star}{f} < \frac{f}{3\pi^2 \phi_\star}.
\]

This behavior is clearly visible (specifically the contour edges at bottom-right) from Fig. 4, where we plotted the 2D 68% and 95% confidence level (C.L.) contours from our MCMC chains in the plane \((\chi_\star/f, f)\), in the spectator-dominated (Spec-Dom) regime. Note that because of the form of the expressions in Eq. (23), \(R\) is independent\(^7\) of \(V_0\). The upper bounds in the vertical direction in Fig. 4 come directly from the priors on \(f\). Throughout this paper, since bounds in the posterior parameter space are partially determined by (broad) priors, not just by the Planck measurements, the shapes of the posterior distributions commonly deviate from the narrow, Gaussian distributions one may find in a completely data dominated case with small error bars.

Since we envision \(\chi/f\) as an axion phase, our prior expectation is for \(\chi_\star\) to be uniformly distributed in the interval \([0, f/2]\). Therefore, the requirement of very small \(\chi_\star/f\) in Eq. (42) corresponds to significant fine-tuning of initial conditions. Indeed, if we do not explicitly impose \(R > 0.9\) in our MCMC runs, this condition is satisfied less than 1% of the time, while most of the points in the chains are concentrated in the inflaton-dominated \(R \ll 1\) region, with \(f_{NL} \approx 0\). However, this region corresponds precisely to the case that is very similar to single-field inflation. In order to explore the features that are specific to the presence of the extra field, we will now focus on the spectator-dominated regime \((R > 0.9)\), keeping in mind that this is a fine-tuned subset of models. In this regime, we are pushed towards large values of \(f\), close to the prior upper bound, because large \(f\) allows for a larger

\(^7\) In the HCA, the only place where \(V_0\) explicitly appears is in the spectral index \(n_s\), see Eq. (15).
range of initial field values satisfying Eq. (42).

The phenomenology of the background energy densities in the spectator-dominated parameter regime of \( f \lesssim M_{\text{pl}} \), \( \chi_s/f \lesssim f/(3\pi^2\phi_s) \) depends on the value of \( V_0/m_\phi^2 \). For the largest values of this quantity allowed by the spectator requirement and by the constraint on \( \eta^* \) coming from \( n_s \), \( \rho_\chi \) starts decaying like \( \alpha^{-3} \) slightly before the end of inflation. However, given our broad prior, most of the posterior volume corresponds to much smaller values of \( V_0/m_\phi^2 \). In that regime, \( \chi \) starts oscillating well after inflation, when \( \phi \) is already oscillating itself and \( \rho_\phi \) decays like matter. In particular, for \( f = M_{\text{pl}} \) and \( \chi_s/f = f/(3\pi^2\phi_s) \), \( \chi \) comes to dominate the total energy density of the universe before it starts oscillating, leading to a short second phase of inflation (the second scenario in Figure 1). Lowering \( f \) (still with \( \chi_s/f = f/(3\pi^2\phi_s) \) and still assuming the low \( V_0/m_\phi^2 \) regime), the ratio \( \rho_\chi/\rho_\phi \) at the time when \( \chi \) starts oscillating goes down, and there is no second phase of inflation once \( f \lesssim 0.1M_{\text{pl}} \).

We can understand the \( f_{\text{NL}} \) distribution better by noting that, in the fully spectator-dominated limit, cf. Eq. (25),

\[
 f_{\text{NL}} \approx \frac{5\pi^2}{3f^2}. \tag{43}
\]

The maximum value \( f = M_{\text{pl}} \) then corresponds to \( f_{\text{NL}} = -0.65 \), thus explaining the cutoff in the \( f_{\text{NL}} \) distribution. This cutoff is smoothed out because we consider the range \( R = 0.9 - 1 \) and the expression for \( f_{\text{NL}} \), above is to be multiplied by \( R^2 \). The relation between \( f \) and \( f_{\text{NL}} \) also makes clear that the posterior is dominated by a limited range of \( f \) just below and up to the cutoff. Therefore, it is the prior of a sub-Planckian decay constant that pushes us towards \( |f_{\text{NL}}| \gtrsim 1 \) (assuming the perturbations are dominated by the spectator field in the first place). For the more inclusive prior, \( f < 3M_{\text{pl}} \), the typical value of \( f_{\text{NL}} \) is significantly smaller. The low non-Gaussianity at large \( f \) can be understood by noting that in this limit, \( V(\chi) \) becomes more and more like a flat, slow-roll potential, which naturally has slow-roll suppressed \( f_{\text{NL}} \).

We illustrate the relation between \( f \) and \( f_{\text{NL}} \) in Figure 6, which shows the joint posterior distribution of \( f \) and \( f_{\text{NL}} \) in the spectator-dominated regime. This figure also clearly illustrates the difference in the dependence of \( f_{\text{NL}} \) on \( f \) between the HCA and the rescaled/improved HCA. Note however that the main qualitative conclusions are not strongly affected by whether or not the correction factor is applied and are thus not sensitive to the exact details of the approximation used to compute \( f_{\text{NL}} \).

In Fig. 5 we plot the posteriors for \( f_{\text{NL}} \) for the two different choices of upper bound on \( f \). As discussed in Appendix B, we show results both assuming the standard Horizon Crossing Approximation, and the improved approximation calibrated on numerical calculations. The curves for the default prior \( f < M_{\text{pl}} \) are well within the Planck CMB bound, with typical values of \( f_{\text{NL}} \) of order unity. This is thus within range of future experiments, especially if \( \sigma(f_{\text{NL}}) \) could be pushed significantly below one.

![FIG. 5: Posterior distribution of \( f_{\text{NL}} \) in quadratic-axion scenario (Case A), assuming Planck constraints on \( f_{\text{NL}}, r_s \) and \( r \). The blue curve shows the general \( f_{\text{NL}} \) distribution in this model, dominated by the inflaton-dominated regime, where \( f_{\text{NL}} \approx 0 \). In red and green (see main text and Appendix for discussion of the two approaches), we impose the condition that the curvature perturbations are dominated by the spectator field \( \chi \) (\( R > 0.9 \)). In this regime, while the exact shape of the \( f_{\text{NL}} \) posterior is prior dependent (cf. purple curve), \( f_{\text{NL}} \) is generically of order unity (58% probability of \( |f_{\text{NL}}| > 1 \) for our default prior \( f < M_{\text{pl}} \), red curve).

![FIG. 6: Joint posterior distribution of \( f_{\text{NL}} \) and “axion decay constant” \( f \) in quadratic-axion model. The red (purple) contours show the posterior restricted to the spectator-dominated regime with a prior \( f < M_{\text{pl}} \) (\( f < 3M_{\text{pl}} \)). The green contour shows the same, but without the correction factor applied to the HCA prediction. The dashed horizontal lines indicate the current 1σ range from Planck, and the level of the constraints aimed for by future galaxy surveys, \( |f_{\text{NL}}| \sim 1 \).]
B. The Curvaton

For the curvaton, we find that the spectator-dominated regime is reached for low initial field values $\chi_*$ and low ratios $\Gamma_{\text{reh}}/m_\chi$, the latter corresponding to a long reheating phase. This is illustrated in Figure 7, which shows the posterior probability distributions (68 and 95% confidence level) in the spectator-dominated regime ($R > 0.9$). The blue regions include all CMB measurements discussed above, while the unfilled contours are derived without including the Planck $f_{\text{NL}}$ measurement.

The spectator-dominated parameter region can be understood as follows. The requirement $N_\chi \gg N_\phi$ can be phrased as (cf. Eqs. (13), (28)),

$$\frac{r_{\chi,\text{reh}}}{\chi_*} \approx \frac{\phi_*}{m_{\text{Pl}}}.$$  \hspace{1cm} (44)

Since by definition $r_{\chi,\text{reh}} \leq 1$, we clearly at least need $\chi_* \phi_* \ll m_{\text{Pl}}$. Assuming this is satisfied, there is in addition the requirement that $r_{\chi,\text{reh}}$ is not too small. As long as $r_{\chi,\text{reh}} \ll 1$, it is easy to show that $r_{\chi,\text{reh}} \sim \chi_*^2 \sqrt{m_\chi/\Gamma_{\text{reh}}/m_{\text{Pl}}}$, translating Eq. (44) into the requirement,

$$\frac{\Gamma_{\text{reh}}}{m_\chi} \ll \left(\frac{\chi_*}{\phi_*}\right)^2.$$  \hspace{1cm} (45)

Thus, the smaller the value of $\chi_*$, the more the ratio $\Gamma_{\text{reh}}/m_\chi$ has to be tuned to extremely small values. In physical terms, we are forced towards low initial field values, but the smaller $\chi_*$ is, the smaller the ratio of curvaton to radiation energy density is at the start of the curvaton phase, and thus the longer the curvaton phase needs to last to make the curvaton fraction $r_\chi$ non-negligible.

The above explains well the unfilled contours in Figure 7. The blue regions show that when the Planck $f_{\text{NL}}$ bound is added, an additional part of parameter space is excluded. Namely, in the curvaton-dominated regime, and for small $r_{\chi,\text{reh}}$, we have $f_{\text{NL}} \sim r_{\chi,\text{reh}}^{-1}$ so that the Planck bound forces

$$\frac{\Gamma_{\text{reh}}}{m_\chi} \lesssim \left(f_{\text{NL,max}}^{\text{Planck}}\right)^2 \chi_*^4$$  \hspace{1cm} (46)

(where the 2$\sigma$ Planck bound is $f_{\text{NL,max}}^{\text{Planck}} \sim 10$), thus explaining the steeper scaling of the maximum value of $\Gamma_{\text{reh}}/m_\chi$ with $\chi_*$ in the filled blue regions.

Is the spectator-dominated regime fine-tuned? We have seen above that to satisfy the condition of large $R$, one needs an extremely large hierarchy between the scales $m_\chi$ and $\Gamma_{\text{reh}}$, translating to a reheating scale many orders of magnitude below the inflation scale. In this sense, the regime where the spectator/curvaton is important is very fine-tuned. Moreover, we require small initial field values in Planck units. At the same time, we find the posterior probability for, say, $R > 0.5$ vs. $R < 0.5$, to be of the same order\(^8\). The reason for this is that we imposed logarithmic priors on $\Gamma_{\text{reh}}$, etc, with very small lower bounds, reflecting the huge hierarchy between the minimum allowed reheating scale (here chosen to be $H_{\text{reh}} \sim 10^{-13}$ GeV, corresponding to $T_{\text{reh}} \sim 1$ TeV, see Appendix A) and the Hubble scale at the end of inflation, $H_{\text{end}} \sim 10^{15}$ GeV. We also find that the posterior distribution of $R$ (not shown) is bimodal, with peaks at $R = 0$ and $R = 1$. This is again a prior driven effect. There is simply a large parameter volume in the regions where either the spectator or inflaton domination conditions are saturated, cf. e.g. Eq. (45), and only an order of magnitude of parameter range in the intermediate regime.

In Figure 8, we consider the posterior distribution of $f_{\text{NL}}$ both in the general model, and in the spectator dominated regime. In the latter case (red), the peak corresponds to the scenario where the curvaton stage lasts long enough for the curvaton to dominate the energy budget of the universe, $r_{\chi,\text{reh}} \to 1$ and $f_{\text{NL}} \to -5/4$. We find that

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8 The probability of being in the spectator-dominated regime is increased somewhat by the upper bound on the tensor-to-scalar ratio, but this is partially an artifact of our choice of a quadratic inflaton potential, which in the inflaton-dominated regime is in tension with the data (one can fit $n_s$ at the cost of too large a value of $r$). We have considered the more general case of a varying inflaton potential power law index, see Section V D, and find that in this case, the probability of being in the spectator-dominated regime is somewhat suppressed compared to the $\phi^2$ model.
79% of the posterior distribution has $|f_{NL}| > 1$, making future constraints at this level extremely interesting. In particular, $f_{NL} = -5/4$ is clearly an important target.

The dominance of the peak at $f_{NL} = -5/4$ reflects that our priors allow a large parameter volume where $r_{\chi,\text{reh}} = 1$ is saturated, i.e. once $\Gamma_{\text{reh}}$ is low enough for the curvaton to dominate the background energy, lowering $\Gamma_{\text{reh}}$ further by orders of magnitude will maintain $f_{NL} = -5/4$. If we had imposed priors that penalize a large hierarchy between $\Gamma_{\text{reh}}$ and $m_\chi$, the results would change, favoring the large negative $f_{NL}$ regime (low $r_{\chi,\text{reh}}$) relative to $f_{NL} = -5/4$. Of course, such a change in priors would also make satisfying the condition of large $R$ more manifestly fine-tuned. In the general case (blue curve), we see the aforementioned bimodality of the posterior of $R$, with the inflaton-dominated regime leading to the single-field value $f_{NL} \approx 0$ and the spectator-dominated case giving $f_{NL} = -5/4$.

Now focusing on the spectator-dominated regime, Figure 9 shows the joint posterior distribution of $f_{NL}$ with the parameter combination $\chi_\ast^2 \sqrt{m_\chi / \Gamma_{\text{reh}} / m_{\text{Pl}}^2}$, which, as explained above, is approximately equal to $r_{\chi,\text{reh}}$ for low values of $r_{\chi,\text{reh}}$. A future measurement of $f_{NL}$ with order unity precision thus may provide important information on the curvaton model, and in particular on this parameter combination.

C. Modulated Reheating

In the modulated reheating model, the spectator-dominated regime is reached if (cf. Eq. (35)),

$$\frac{\lambda'}{\lambda} \gg \frac{\phi_*}{m_{\text{Pl}}^2} \quad (47)$$

(the transfer function from $\chi$ to $\chi_{\text{reh}}$ generally has a small effect), corresponding to large $\lambda_1 / M_c$ and/or $\lambda_2 / M_c^2$ (most of the weight in the prior distribution of $\lambda_0$ lies around values of $O(10^{-1})$ because of the uniform prior). This region is shown in Figure 10. The filled contours show the usual confidence regions with the prior $R > 0.9$, including all Planck data discussed, while the solid empty contours represent the same region, but without the $f_{NL}$ bound.

We have also (dashed empty contours) included the posterior in the general model, i.e. without the spectator domination requirement on $R$ (and also without the $f_{NL}$ measurement included), to illustrate that the strong correlation between $\lambda_1 / M_c$ and $\lambda_2 / M_c^2$ is there regardless of the requirement on $R$. It is mostly prior driven, and comes from the fact that both quantities scale with the same cutoff mass $M_c$ (and that the dimensionless quantities $\lambda_1$ and $\lambda_2$ follow uniform priors). What the requirement of spectator domination does is to shift $\lambda_1 / M_c$ and...
\[ \lambda_2/M_c^2 \] to larger values along the correlation direction, as shown by the solid black contours and filled blue regions.

Thus, spectator domination requires an effective cutoff scale \( M_c \) not much larger than \( \sim 0.1 \, m_{\text{Pl}} \), allowing the effect of \( \chi \) on \( \lambda \) to be large enough. Since we do not want any contribution to \( \lambda \) to be larger than unity, the requirement of large \( \lambda_1/M_c \) and \( \lambda_2/M_c^2 \) in Planck units does again mean we need small initial field values, \( \chi_i \ll m_{\text{Pl}} \), which can be considered fine-tuning. For the same reasons discussed in the curvaton case, related to our choice of priors, our chains do give a bimodal distribution of \( R \) with peaks of comparable amplitude at \( R = 0 \) and \( R = 1 \) despite this fine-tuning.

Figure 11 depicts the posterior distribution of \( f_{NL} \) for both the general case and the spectator-dominated case. In the latter case, we see a relatively broad distribution of values (in contrast with the curvaton model), with typical values of order \( |f_{NL}| \sim 1 - 5 \) (we note that when we do not implement the current observational bound on \( f_{NL} \), the distribution is significantly broader (not shown), with typical values of order \( |f_{NL}| \sim 10 - 20 \)). We find that 72% of the parameter space in the spectator-dominated regime has \( |f_{NL}| > 1 \).

The distribution at the lower end has a relatively sharp cutoff. This follows from the specific form of the expression for \( f_{NL} \), Eq. (36). Ignoring the evolution of \( \chi \), it reduces to

\[ f_{NL} \approx 5 \left( 1 - \frac{\Gamma'_{\text{reh}} \Gamma_{\text{reh}}}{\Gamma''_{\text{reh}}^2} \right) = \frac{5}{2} \left( 1 - \frac{\lambda''(\chi_{\text{reh}}) \lambda(\chi_{\text{reh}})}{(\lambda'(\chi_{\text{reh}}))^2} \right), \]

(48)

Since we have chosen the coefficients in the expansion of the reheating coupling to all be positive, this gives an upper bound \( f_{NL} < 5/2 \). This cutoff gets smoothed out once the evolution of \( \chi \) is included (the partial derivatives in Eq. (36)), thus explaining the shape of the red curve at the high \( f_{NL} \) end.

In the general case (blue), the bimodal distribution of \( R \) again leads to a superposition of the inflaton-dominated regime’s \( f_{NL} \approx 0 \) and the broader distribution corresponding to the spectator-dominated regime.

Studying \( f_{NL} \) in the spectator-dominated scenario in more detail, Figure 12 shows the joint posterior of \( f_{NL} \) and \( \lambda_2 \lambda_0/\lambda_c^2 \). This parameter combination mostly determines \( f_{NL} \) in the spectator-dominated regime if \( \chi \) is dominated by the \( \lambda_1 \) contribution and \( \lambda \) by \( \lambda_0 \), cf. Eq. (48). A measurement of \( f_{NL} \) provides information on the modulated reheating parameter space and in particular on this combination of parameters describing the coupling of the inflaton to \( \chi \) and to the particles into which it reheats.

In summary, while the physics behind the mechanisms is very different, the modulated reheating has similar
phenomenology to the curvaton scenario. The main qualitative difference is that for spectator-dominated modulated reheating, the $f_{\text{NL}}$ distribution does not peak at a special value ($f_{\text{NL}} = -5/4$ for the curvaton). Instead, it has a broader distribution, with a “smooth” cutoff around $f_{\text{NL}} \sim 5/2$.

D. Observational Prospects

Spectator models are a relatively simple extension of single-field inflation, which itself can be seen as the inflaton-dominated corner of spectator model parameter space. Regarding the inflaton potential, $U(\phi)$, we have so far focused on the simple quadratic potential because predictions for $f_{\text{NL}}$ are rather robust against the details of the inflaton potential. Technically, however, to fit $n_s$ and $r$ well with realistic values of the number of $e$-folding before the end of inflation, more freedom in the shape of the inflaton potential is needed. In particular, let us consider the class of power-law models,

$$U(\phi) \propto \phi^n,$$

where we will allow non-integer values of $n$.

Figure 13 shows the predictions for such models in the $(n_s, r)$ plane, compared to the Planck constraint. The solid lines show the well known single-field/inflaton-dominated ($R = 0$) case, cf. e.g. Fig. 12 in [88]. The dots indicate the number of $e$-folds before the end of inflation, $N_s$. As is well known, the Planck data are already in significant tension with the inflaton-dominated quadratic model, but lower powers, e.g. $U \propto \phi^{2/3}$ are in reasonable agreement.

The effect of curvature perturbations more and more generated by the spectator field, i.e. increasing $R$, is indicated by the arrows, leading to the mostly spectator-dominated scenarios ($R = 0.95$) shown in dashed lines, cf. Eq. (15). Note that for a given $R$ and a given inflaton potential, $n_s$ does not generally have a fixed value because it still depends on $\eta^*$. However, we find that the regime with negligible $\eta^*$ contribution often dominates so that we chose $\eta^* = 0$ in this plot. To indicate the range of effects from non-zero $\eta^*$, the crosses show $(n_s, r)$ for the maximum (positive) $\eta^*$ consistent with the requirement that the spectator field is slowly rolling until after the end of inflation.

Figure 13 thus visualizes that, as the spectator field becomes more important, $r$ goes down, making it easier to evade the tensor-mode constraint, and $n_s$ shifts to larger values. This means that: (1) models that are currently a decent fit in the inflaton-dominated regime (low $n$ power laws) become poor fits in the spectator-dominated case and (2) models with larger power law indices, ruled out by Planck data in the single-field case, become viable again in the spectator-dominated scenario.

We illustrate this for the curvaton model in Figure 14 (top), which shows the same curves, but in a zoomed-in
VI. DISCUSSION & CONCLUSIONS

Upcoming galaxy surveys aim to significantly improve constraints on local primordial non-Gaussianity, from the current Planck bound $f_{\text{NL}} = 0.8 \pm 5.0$, to constraints with uncertainties $\sigma(f_{\text{NL}}) \lesssim 1$. Motivated by this prospect, we have here derived current constraints on a range of multifield inflation models given Planck CMB data and physically motivated parameter priors, and compared the resulting predicted values of $f_{\text{NL}}$ to the expected future constraints. Our goal was to obtain quantitative estimates, given an inflationary model, of the discovery potential of local non-Gaussianity with these future surveys, and to quantify what such a future $f_{\text{NL}}$ may teach us about the physics behind inflation.

We have specifically focused on so-called spectator models, where, while inflation is driven by the inflaton field, the primordial curvature perturbations are partially or fully generated by a second field, the “spectator”. At horizon exit, this spectator field does not contribute to the curvature perturbations, but its perturbations can be converted into curvature perturbations afterward through super-horizon evolution. We have considered three specific mechanisms for this process with the conversion occurring during different phases: (A) during or after inflation before either field has decayed into radiation, (B) after inflation while the inflaton has already decayed into radiation and the spectator (i.e. curvaton) oscillates around the minimum of its potential, and (C) after inflation during the reheating process itself.

If the relative contribution of the spectator field to the final primordial curvature power spectrum is close to one, significant non-Gaussianity can be generated, which is
why our main focus has been on this set of “spectator-dominated” models. While there are significant differences between the three scenarios (A)-(C), we will below discuss some of the main general conclusions.

Typically, to be in the spectator-dominated regime, some form of fine-tuning is required. For instance, in all three scenarios, small values of the initial spectator field value are required. Furthermore, in the curvaton scenario, the reheating scale needs to be tuned to be many orders of magnitude below the scale of inflation and the curvaton mass, corresponding to extremely late reheating (although not in clear tension with data). On the other hand, statements about fine-tuning are always strongly prior dependent. For example, since our MCMC analysis employed wide, logarithmic prior ranges on most dimensionful parameters, we found in both scenarios (B) and (C) that being in the spectator-dominated regime is approximately equally likely as the alternative. However, this does not remove the objection that large hierarchies between parameters may be unnatural from a model building perspective. Such theory-based prejudice could have been incorporated by modifying our priors, but we chose not to pursue this here.

Assuming spectator domination ($R > 0.9$), we have quantified the posterior distribution of $f_{NL}$ given current Planck data for each of the three scenarios. We have quantified the promise of next-generation $f_{NL}$ measurements by quoting the posterior probability of $|f_{NL}| > 1$, which we will summarize below. Assuming $|f_{NL}| > 1$ can be distinguished from zero at sufficient significance, this gives the probability of detection of non-Gaussianity. Conversely, if an upper bound $|f_{NL}| < 1$ is obtained from the data, the number above tells us what fraction of the currently allowed parameters space will be ruled out. However, the above quantity does not tell the full story so below we also quote what fraction of the posterior distribution obtained without including the Planck $f_{NL}$ bound has $f_{NL} > f_{NL,max} = 10$ (corresponding approximately to the 2σ Planck bound) and what fraction has $|f_{NL}| > 1$.

- **Case A - Quadratic-Axion**
  - With Planck $f_{NL}$: $P(|f_{NL}| > 1) = 58\%$
  - Without: $P(|f_{NL}| > 1(10)) = 63 (6)\%$

- **Case B - Curvaton**
  - With Planck $f_{NL}$: $P(|f_{NL}| > 1) = 79\%$
  - Without: $P(|f_{NL}| > 1(10)) = 83 (14)\%$

- **Case C - Modulated Reheating**
  - With Planck $f_{NL}$: $P(|f_{NL}| > 1) = 72\%$
  - Without: $P(|f_{NL}| > 1(10)) = 92 (60)\%$

We see that in the modulated reheating scenario, the Planck $f_{NL}$ constraint has already ruled out a significant fraction of the parameter space allowed without taking PNG into account, but that in the other cases we are only just starting to take advantage of $f_{NL}$. While, as we have discussed, the numbers above are prior dependent (especially in Case A, which relies on the maximum value of the decay constant $f$ being of order $M_{Pl}$), they suggest that, if inflation is described by one of these models where the curvature perturbations are generated by a field other than the inflaton, future $f_{NL}$ searches with $\sigma(f_{NL}) \lesssim 1$ have a good shot at a detection and will probe these models well beyond the current Planck $f_{NL}$ constraint.

If a detection of $f_{NL}$ is achieved, the most important implication would of course be the discovery of multifeild inflation (although there are caveats to the single-field consistency conditions that allow non-zero $f_{NL}$ in certain special single-field scenarios [11, 12]). In addition, we have shown that a measurement of $f_{NL}$ in the context of the models above also tells us about the values of certain parameter combinations, thus providing hints about the nature of the multifeild model describing the early universe. We have also highlighted the complementarity between B-mode searches constraining primordial tensor perturbations and measurements of galaxies and the CMB constraining primordial non-Gaussianity. It is such a multipronged approach that provides the best opportunity for improving our understanding of the physics of the extremely early universe.

In conclusion, while large or order unity $f_{NL}$ is not a general prediction of multifeild inflation, it appears to be quite generic in spectator-dominated models. Arguably, these are the more interesting multifeild models regardless of $f_{NL}$, as the case where the primordial fluctuations are fully determined by the inflaton field is phenomenologically indistinguishable from single-field models. The appearance of order unity $f_{NL}$ in spectator-dominated and similar models has been highlighted many times in the literature, but here we have sampled the full parameter space of a range of models, taking into account observational constraints, leading to a more quantitative assessment of the typical prediction for $f_{NL}$.

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9 Since by default we included the Planck bound on $f_{NL}$ to compute the posterior, some caution is needed in interpreting the posterior probability of getting $|f_{NL}| > 1$. If the $f_{NL}$ posterior without including the $f_{NL}$ bound from Planck is very wide compared to $\sigma(f_{NL})$ from Planck, the default posterior with the Planck $f_{NL}$ bound included is essentially determined by $\sigma(f_{NL}) \sim 5$, and saying that a bound with $\sigma(f_{NL}) \lesssim 1$ rules out a large part of currently allowed parameter space is equivalent to the trivial statement that the future error bars are smaller than the current ones. In this scenario, it could be that the Planck $f_{NL}$ bound had already ruled out an overwhelming fraction of the previously allowed parameter space, and a future tighter bound will simply rule out a little bit more. Therefore, it is also important to quantify how much better a future $f_{NL}$ constraint does than the current CMB bound.
Appendix A: Parameters and Priors

In this Appendix, we consider parameter priors and other constraints assumed in the MCMC likelihood analysis. For each model, there is a set of basic parameter priors, given in Tables I-III. On top of these priors, various additional constraints, other than those from the data discussed in the main text, are imposed. There can be significant redundancy in these priors and constraints, i.e. they are not all independent. Let us first consider requirements that are imposed on all three models.

- First of all, we always demand the spectator definition given in Eq. (13) is satisfied. Secondly, we require all four slow-roll parameters at $t_\ast$ to be small,

$$e^\phi, e^\chi, |\eta^\phi|, |\eta^\chi| < 0.1.$$  \hspace{1cm} (A1)

- Moreover, for a classical treatment of the spectator field to be appropriate, we require that its initial value is much larger than the initial quantum fluctuations,

$$\chi_\ast > 10 \times \delta \chi_\ast = 10 \times \frac{H_\ast}{2\pi}$$  \hspace{1cm} (A2)

(we use a fixed value $H_\ast = 4 \times 10^{-5} m_{Pl}$).

- Unless otherwise noted, we apply logarithmic priors to dimensionful parameters (and parameters that were dimensionful before dividing out powers of $m_\phi$).

- We will define the spectator-dominated regime by the somewhat arbitrary threshold,

$$R > 0.9.$$  \hspace{1cm} (A3)

Let us now consider the specific parameters and priors/constraints for each model.

1. Priors Case A: Quadratic-Axion

- The parameters sampled in the MCMC and their default prior ranges are given in Table I.

- As already incorporated there, we assume by default that the decay constant is sub-Planckian

$$f < M_{Pl}$$  \hspace{1cm} (A4)

(note that this is the Planck mass, not the reduced Planck mass), although we explicitly study how the results depend on the upper bound.

2. Priors Case B: Curvaton

- The parameters sampled in the MCMC and their default prior ranges are given in Table II.

- The curvaton scenario under consideration requires

$$m_\phi > m_\chi > \Gamma_{reh}.$$  \hspace{1cm} (A5)

- As discussed in the main text, we require the curvaton to be subdominant up to $t_{curv}$.

$$\frac{1}{2} m_\chi^2 \chi_{curv}^2 \ll 3 m_\chi^2.$$  \hspace{1cm} (A6)

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10 We have confirmed the dependence of the time $\chi$ starts rolling on $V_0$ numerically.
| Param. | Description | Prior |
|--------|-------------|-------|
| $m_{\chi}/m_{\phi}$ | spectator mass | $[10^{-20}, 1]$ (log) |
| $\lambda_0$ | reheating coupling parameter | $[0, 1/2]$ (linear) |
| $\lambda_1$ | reheating coupling parameter | $[0, 1]$ (linear) |
| $\lambda_2$ | reheating coupling parameter | $[0, 1]$ (linear) |
| $M_c$ | reheating cutoff parameter | $[10 \times H_*, m_{Pl}]$ (log) |
| $\chi_{s}$ | spectator initial field | $[10 \times H_*/2\pi, m_{Pl}]$ (log) |
| $\phi_{*}$ | inflaton initial field | $[3m_{Pl}, 35m_{Pl}]$ (log) |

TABLE III: Parameters and default priors for Case C: the modulated reheating model. Additional constraints on the parameters are described in the text.

- Finally, we have determined the minimum value of $\Gamma_{\text{reh}}$ in Table I as in case A, requiring $\rho_{\text{reh}} > (10^3 \text{GeV})^4$. This leads to the prior,

$$\frac{\Gamma_{\text{reh}}}{m_{\phi}} > 10^{-26}. \quad (A7)$$

3. Priors Case C: Modulated Reheating

- The parameters sampled in the MCMC and their default prior ranges are given in Table III.

- The specific scenario under consideration corresponds to the requirement,

$$m_{\phi} > \Gamma_{\text{reh}}(\chi_{\text{reh}}) > m_{\chi}. \quad (A8)$$

- We also impose

$$\frac{m_{\chi}}{m_{\phi}} > 10^{-26}, \quad (A9)$$

which ensures reheating takes place before the electroweak phase transition (because $\Gamma_{\text{reh}} > m_{\chi}$).

- We require the coupling constant, Eq. (32), and its individual contributions to be significantly smaller than one$^{11}$ (to rule out cases where the individual terms are large but cancel due to opposite signs),

$$|\lambda| < 1/2, \quad |\lambda_1| \frac{\chi_{s}}{M_c} < 1/2, \quad \frac{1}{2} |\lambda_2| \left( \frac{\chi_{s}}{M_c} \right)^2 < 1/2. \quad (A10)$$

- Finally, as already included in Table III, we demand that the cutoff $M_c$ is significantly above the Hubble scale at $t_*$,

$$M_c > 10 \times H_* . \quad (A11)$$

In practice, we incorporate $M_c$ into redefinitions of $\lambda_1$ and $\lambda_2$ and marginalize $M_c$ out analytically.

Appendix B: Testing the Horizon Crossing Approximation

We discuss here the accuracy of the Horizon Crossing Approximation for computing the curvature perturbations and $f_{NL}$ in the quadratic-axion model. To do so, we numerically compute $f_{NL}$ in the $\delta N$ formalism using the full equations of motion, i.e.

$$\dot{\phi} + 3H\dot{\phi} + U_\phi = 0 \quad \ddot{\chi} + 3H\dot{\chi} + V_\chi = 0 \quad 3m_{h}^{2}H^{2} = \frac{1}{2}\dot{\phi}^{2} + \frac{1}{2}\dot{\chi}^{2} + U(\phi) + V(\chi). \quad (B1)$$

We compute the number of $e$-folds $N$ to a constant energy density hypersurface at a time when both fields have started oscillating around their minima, so that $\phi$ has become constant. From there, one can get the numerical derivatives of $N$ and compute $f_{NL}$ using Eq. (12).

As discussed in the main text, if we do not impose the spectator domination condition, the posterior is dominated by points in parameter space where $\chi_{s}/f$ is not tuned to be small, so that the inflaton dominates the final perturbations and $f_{NL} \approx 0$. Our main region of interest for testing the HCA is thus the regime where we explicitly impose the spectator domination condition,

$$\frac{\chi_{s}}{f} < \frac{f}{3\pi^{2}\phi_{s}}. \quad (B2)$$

This condition in turn favors larger values of $f$ as they leave a larger range of initial field values that satisfy the above requirement. We thus mainly want to test the HCA for models with $f$ within, say, an order of magnitude from the cutoff, i.e. $f$ close to the Planck scale $M_{Pl}$.

In Figure 15, we show $f_{NL}$ as a function of the amplitude of the axion potential, $V_{0}/m_{\phi}^2$, for various values of $f$ in the range motivated above. For each parameter choice, we find that $f_{NL}$ converges as $\chi_{s}/f \rightarrow 0$ and in the plot we have chosen values $\chi_{s}/f \ll f/(3\pi^{2}\phi_{s})$ such that convergence has been reached. The results are minimally sensitive to the choice of $\phi_{s}$.

For large values of $f$, the HCA (dashed lines) is a reasonably good approximation to the exact numerical results (plus signs). For smaller $f$ however, the HCA systematically overpredicts $f_{NL}$. For comparison, $|f_{NL}| < 5$ (cf. Figure 5) corresponds to $f \gtrsim 0.4M_{Pl}$, between the blue and light green results in Figure 15. In order to account for the difference between the HCA and exact result, we implemented a simple function of $f$ that rescales

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11 This does not ensure that non-perturbative effects are unimportant in the reheating process, but simply that the coupling constant is small enough for the leading-order perturbation theory expression for $\Gamma_{\text{reh}}$ to be valid. In addition to this, there may well be non-perturbative effects even for small $\lambda$, such as resonant particle production from the vacuum.
$f_{NL}$ to make it agree with the numerical results. We show the new $f_{NL}$ with this correction factor in solid lines. With the correction factor included, the agreement is quite good, except at high $V_0/m_\phi^2$ and low $f$ (again, the low $f$ regime is less relevant in our MCMC analysis).

We note that the results do depend also on $\chi_* / f$. While here we have shown the results in the low $\chi_* / f \rightarrow 0$ limit, for values of $\chi_* / f$ that marginally satisfy Eq. (B2), we find a deviation from the results plotted here. However, the results are always within the range set by the HCA approximation (dashed) and the HCA approximation modified with the correction factor (solid). In our likelihood analysis, to bracket the range of $f_{NL}$ values, we have considered results using either prescription.

We have also looked at how other quantities, such as $n_s$, $r$ and $R$ are different when using the full equations. Those differences are much smaller, and if the HCA values are within the Planck constraints, so are the ones from the full equations. Moreover, since our main focus is primordial non-Gaussianity, the specifics of those other parameters are of lesser importance to our analysis. Indeed, as we mentioned before, they could be adjusted by a different choice of potential for the inflaton.

![Graph](image)

**FIG. 15:** Comparison of different approaches to calculating $f_{NL}$ in the quadratic-axion model (Case A). We show $|f_{NL}|$ as a function of $V_0/m_\phi^2$ using the Horizon Crossing Approximation (dashed lines), the HCA with $f$-dependent correction factor (straight lines) and the full numerical calculation in the $\delta N$ formalism (+). Results are computed in the limit $\chi_* / f \rightarrow 0$ (see text) and we restrict the numerical computation to the range of $V_0/m_\phi^2$ values relevant for our likelihood analysis ($|\eta^2| < 0.1$). While accurate at large $f$, the HCA is only correct to order-of-magnitude level precision for $f$ significantly below $M_{Pl}$. We have used the numerical calculations to construct an $f$-dependent rescaling function that brings the HCA prediction in good agreement with the exact result.
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