The twilight of the single field slow rolling inflaton

Jaume Amorós\textsuperscript{a,*} and Jaume de Haro\textsuperscript{a,†}

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\textsuperscript{a}Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya Diagonal 647, 08028 Barcelona, Spain

Abstract

We present a study of 49 single field, slow roll inflationary potentials in which we assess the likelyhood of these models fitting the spectral parameters of the CMB radiation, namely the spectral index $n_s$, its running $\dot{n}_s$, the running of the running $\ddot{\beta}_s$ and the ratio of tensor to scalar perturbations $r$, according to the currently most accurate determination of these spectral parameters given by the PLANCK collaboration. A double, partly redundant approach is followed: for most models we derive analytically bounds for the values of the spectral parameters that they can support, and for all of the models we check numerically with a MATLAB program the spectral parameters that each model can yield for a very broad, comprehensive list of possible parameter and field values.

The comparison of spectral parameter values supported by the models with their successive determinations by the PLANCK collaboration leads to contradictory conclusions: PLANCK2013+WP+BAO+$\Lambda$CDM+$r+\dot{n}_s$ disfavours all of the models with confidence at least 93%, conversely the data provided by PLANCK2015+TT+lowP without running of the running allows back most of the models, but taking into account the running of the running again disfavours 39 of the 49 models with confidence at least 92.8%, and 5 more because of the amount of expansion e-folds supported.

We identify a bias in the method of determination of the spectral parameters currently used to reconstruct the power spectrum of scalar perturbations that can explain these contradictory conclusions. The solution to this problem is likely to determine the fate of the inflaton.

1 Introduction

The PLANCK2013 data\cite{1} showed that the spectral index of scalar perturbations has an expected negative running, whose modulus was in general one order of magnitude greater than the theoretical values provided by slow roll inflation, which

\textsuperscript{*}E-mail: jaume.amoros@upc.edu
\textsuperscript{†}E-mail: jaime.haro@upc.edu
could be used to test some theoretical slow roll inflationary models. The aim of the present work is to present an analytic and numerical study, of the 49 theoretical models of single field, slow roll inflation that are described in [2] (see its Table 1), that shows that the constrains of the spectral index, its running and the ratio of tensor to scalar perturbations provided by PLANCK2013 combined with several data, disfavours all of these models, severely in most cases. The most familiar ones such as Hill-top, Natural, Plateau and Monomial potentials could be disregarded according to these PLANCK2013 data due to the small value, in modulus, of the theoretical value of the running provided by slow roll inflation. In fact, from PLANCK2013+WP+BAO: ΛCDM+r+αs data we show that the deviation from the theoretical value of the running, namely D, obtained from the majority of that models to its expected observational value is larger than 1.6σ, and in modulus all the theoretical values are smaller than the modulus of the expected observational one, which means that these models lies outside of the 94.5% C.L. Further numerical study has shown that for all of the models either αs or ns or r lie outside of the 93% C.L.

Our study, initially performed from PLANCK2013 data, could suggest that single scalar inflationary theories must be replaced by multiple fields theories, by other ones with a breakdown of the slow roll phase [3] or by reconstruction techniques [4]. Another completely different proposal is to abandon the inflationary paradigm in favour of the Matter Bounce Scenario [5, 6, 7] where the Big Bang singularity is replaced by a non-singular Big Bounce, which at this moment, constitutes a promising alternative to the slow roll inflationary paradigm, without the characteristic inflationary flaws, such as the initial singularity which or the fine-tuning of the degree of flatness required for the potential in order to achieve successful inflation [8].

Fortunately for single field slow roll inflation, the new PLANCK2015 observational data [9], reduce the modulus of the running one order, which allows the viability of the majority of single scalar field slow roll inflationary models. For example, using Planck2015+TT+lowP: ΛCDM+r+αs, where ns = 0.9667 ± 0.0066 and αs = −0.0126±0.0098 at 1σ C.L. (see table 4 of [9]), and the conservative constrain r ≤ 0.25 (see figure 6 of [9]), for the majority of tested potentials one has D ≃ 1.1σ, which means that these models are only disfavored at 86% C.L. or less. Moreover, if one introduces lensing, then D will be reduced to be lower than 1σ, and thus allowing single scalar field slow roll inflation.

However, dealing with the running of the running, namely βs, when one considers the PLANCK2015 TT+lowP (resp. PLANCK2015 TT,TE,EE+lowP) model βs = 0.029±0.016 (resp. βs = 0.025 ± 0.0013) (see (19) of [9]), the same methodology that we have applied in the previous cases show that the measured value of βs is incompatible with the theoretical value provided by single field slow roll, because its measured value is too large for a magnitude that depends on third order on the slow roll parameters, disfavouring almost all slow roll inflationary models studied, 39 out of 49 at 92.8 % C.L. or more.
The main lesson of this work, is the importance of the observational measures provided by PLANCK or other teams, in order to check the viability of single field slow roll inflation. Only after a precise determination of their expected observational value and the corresponding deviation we will able to determine which inflationary models could depict correctly our Universe.

The problem of the reliability of the observational measures of spectral parameters of the CMB starts with the above paradox of the the oscillating conclusions, from ruling out every model to allowing most of them and back, that the successive PLANCK2013 and PLANCK2015 determinations of spectral parameters support. We find that these oscillating conclusions can be explained by a bias in the method of fitting spectral values of the power spectrum to the observations. We believe that new data, and the addressing of this bias, will reduce significantly the observable value of the running of the running and may possibly allow back the viability of single slow roll inflation, in the same way as has happened with the early values of the running obtained by PLANCK2013 and its drastic decrease according to PLANCK2015.

The units used in the paper are: \( \hbar = c = 8\pi G = 1 \).

2 Slow-roll parameters

In slow roll inflation (see [10] for a review of inflation) the commonly used first order parameters are:

\[
\epsilon = -\frac{\dot{H}}{H^2} \cong \frac{1}{2} \left( \frac{V_{\phi}}{V} \right)^2 \quad \text{and} \quad \eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon} \cong \frac{V_{\phi\phi}}{V}. \tag{1}
\]

At the first slow roll order, the spectral index of scalar perturbations and its running are given by

\[
n_s - 1 = 2\eta - 6\epsilon \quad \text{and} \quad \alpha_s = 16\epsilon\eta - 24\epsilon^2 - 2\xi, \tag{2}
\]

where the second order slow roll parameter

\[
\xi \equiv \left( 2\epsilon - \frac{\dot{\eta}}{H\eta} \right) \eta \cong \frac{V_{\phi} V_{\phi\phi\phi}}{V^2}, \tag{3}
\]

has been introduced.

Moreover, in inflationary cosmology, the tensor/scalar ratio, namely \( r \), is related with the slow roll parameter \( \epsilon \), via the following consistency relation \( r = 16\epsilon \).

The other important parameter that we will use in this work is the running of the running \( \beta_s = \frac{d\alpha_s}{d\ln k} \), given, in the slow roll approximation, by \([11][12]\)

\[
\beta_s = -192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 - 24\epsilon^2\xi + 2\eta\xi + 2\xi, \tag{4}
\]
where we have introduced the third order slow roll parameter

\[ \zeta \equiv \left( 4\epsilon - \eta - \frac{\dot{\zeta}}{H\xi} \right) \xi \approx \frac{V''''\varphi^5}{V^3}. \tag{5} \]

### 3 Analytical fitting of the parameters

#### 3.1 PLANCK2013 data: the running

In this section we will take into account the PLANCK2013 constrains on \( n_s, \alpha_s \) and \( r \) which are model dependent (see table 5 of [1]), and the fact that more than 50 e-folds are needed to solve the horizon and flatness problems of GR. If one does not consider the running and makes the analysis in the plane \((n_s, r)\), then PLANCK2013 data shrink the space of allowed standard inflationary models, preferring potentials with a concave shape \((V'' < 0)\) [1]. But, it is the combination of the three data \((n_s, \alpha_s, r)\) what rules out all the standard slow roll inflationary models [3].

Effectively, for instance, we will consider in detail the \( \Lambda \)CDM + \( r + \alpha_s \) model from PLANCK2013 combined with WP and BAO data, which gives the following results (see table 5 of [1]):

\[ n_s = 0.9607 \pm 0.0063, \quad r \leq 0.25 \quad \text{at} \quad 95\% \ C.L. \quad \text{and} \quad \alpha_s = -0.021^{+0.012}_{-0.010}. \]

In slow roll inflation, a simple calculation leads to the relation

\[ \alpha_s = \frac{1}{2}(n_s - 1)r + \frac{3}{32}r^2 - 2\xi. \tag{6} \]

And thus, assuming that the potentials we consider have spectral index \( n_s \) at less than 2\( \sigma \) deviations and satisfy the conservative bound \( r \leq 0.32 \) (see figure 4 of [1]), the minimum of the function \( \frac{1}{2}(n_s - 1)r + \frac{3}{32}r^2 \) reached at the point \((n_s = 0.9481, r = 0.1384)\) is greater than \(-0.0018\), what provides the bound

\[ \alpha_s \geq -0.0018 - 2\xi, \tag{7} \]

meaning that plateau potentials such as \( V(\varphi) = V_0 \left( 1 - \frac{\varphi^2}{\mu^2} \right) \) (Hill-Top Inflation (HTP)) [12], \( V(\varphi) = V_0 \left( 1 - \frac{\varphi^2}{\mu^2} \right)^2 \) with \(|\varphi| \leq \mu\) (Double-Well Inflation (DWI)) [13] or \( V(\varphi) = V_0 \left( 1 + \cos \left( \frac{\varphi}{\mu} \right) \right) \) (Natural Inflation (NI)) [14], when one considers values of the running at 1\( \sigma \) C.L., are disfavoured by PLANCK data because for all of them \( \xi \leq 0 \). In fact, the deviation from the theoretical value of the running to its expected observable value, namely \( D \), is larger than 1.6\( \sigma \).

A distance larger than 1.6\( \sigma \) is also obtained for the potential that leads to Exponential SUSY Inflation (ESI) \( V(\varphi) = V_0 (1 - e^{-p\varphi}) \) [15] and for Power Law...
Inflation (PLI) whose potential is given by \( V(\phi) = V_0 e^{-p\phi} \) \[16\], because in this case one has \( \xi = \eta^2 \), that is,

\[
\alpha_s = \frac{r}{8} \left( (n_s - 1) + \frac{3}{16} r \right) - \frac{1}{2} (n_s - 1)^2 = -\frac{3r^2}{32} \geq -0.0018, \tag{8}
\]

where we have evaluated \( \alpha_s \), as a function of \( n_s \) and \( r \) at the absolute minimum, namely \( n_s = 1 - \frac{3r}{8} \) with \( r = 0.1384 \), in the rectangle \([0.9481, 0.9733] \times [0, 0.32] \).

To be more precise, since power law inflation has no running, the distance to the main observational value is \( 1.77 \sigma \), because in this case one has the bound \( \alpha_s \geq -0.00045 \).

Dealing with the K"aller Moduli Inflation I (KMII) \[17\] given by the potential \( V(\phi) = V_0 \left(1 - \alpha \phi e^{-\phi} - \phi^2 \right) \) and for Open String Tachyonic Inflation (OSTI) \[19\] with potential \( V(\phi) = -V_0 \phi^p \ln(\phi) \phi^2 \), one always has \( \xi \leq \eta^2 \) and one obtains the same conclusions as in ESI, i.e., \( \mathcal{D} \geq 1.6\sigma \).

When one considers Large Field Inflation (LFI) given by the monomial potential \( V(\phi) = V_0 \phi^p \) with \( p \geq 1 \) \[20\], one obtains

\[
(n_s - 1) = \frac{p(p + 2)}{\phi^2} \quad \text{and} \quad \alpha_s = -\frac{2p^2(p + 2)}{\phi^4}, \tag{9}
\]

which means that \( p \) must be positive in order to have a spectral index with a red tilt and a negative running. As a first consequence, Inverse Monomial Inflation (IMI) \[21\] is disfavored. Second, from the relation

\[
\alpha_s = -\frac{2}{p + 2} (n_s - 1) \geq -\frac{2}{3} (n_s - 1)^2 \geq -0.0018, \tag{10}
\]

we conclude that \( \mathcal{D} \geq 1.6\sigma \). And for Radiation Gauge Inflation (RGI) \[22\], whose potential is given by \( V(\phi) = V_0 \phi^p \) one has \( \xi = \frac{12\phi^2(\phi^2 - \alpha \phi^2)}{(3\phi^2 - \alpha \phi^2)^2} \eta^2 \leq \frac{4}{3} \eta^2 \), because \( \frac{12\phi^2(\phi^2 - \alpha \phi^2)}{(3\phi^2 - \alpha \phi^2)^2} \) increases as a function of \( \phi \). Then, evaluating at \( n_s = 0.9481 \) one has

\[
\alpha_s = -\frac{2}{3} (n_s - 1)^2 \geq -0.0018, \tag{11}
\]

giving as a result \( \mathcal{D} \geq 1.6\sigma \).

For potentials such as: \( V(\phi) = V_0 \left(1 - \frac{\phi^2}{\mu^2} \right)^p \) (Brane Inflation (BI)) \[23\], and \( V(\phi) = V_0 \left(1 + \frac{\phi^2}{\mu^2} \right)^p \) (Dynamical Supersymmetric Inflation (DSI)) \[24\], since one has \( \xi = \frac{p+2}{p+1} \eta^2 \), one can obtain the following exact formula

\[
\alpha_s = \frac{(p - 2)r}{8(p + 1)} \left( (n_s - 1) + \frac{3r}{16} \right) - \frac{p + 2}{2(p + 1)} (n_s - 1)^2. \tag{12}
\]
The minimum of $\alpha_s$ is obtained at $n_s = 1 - \frac{3r^2}{2}$ with $r = 0.1384$, and thus, inserting this expression in (12), one gets

$$\alpha_s \geq -\frac{3r^2}{32} \geq -0.0018,$$

which is incompatible with the running provided by PLANCK at $1\sigma$ C.L., because $D \geq 1.6\sigma$.

For general hill-top potentials such as: $V(\varphi) = V_0 \left(1 - \left(\frac{\varphi}{\mu}\right)^p\right)$ (Small Field Inflation (SFI)) [12] and $V(\varphi) = V_0 \left(1 + \left(\frac{\varphi}{\mu}\right)^p\right)$ (Valley Hybrid Inflation (VHI)) [25] with $p \geq 3$, since one has $\xi = \frac{p-2}{p-1} \eta^2$, one can obtain the following exact formula

$$\alpha_s = \frac{(p + 2)r}{8(p - 1)} \left(n_s - 1\right) + \frac{3r}{16} - \frac{(p - 2)(n_s - 1)^2}{2(p - 1)}.$$

Since the minimum of $\alpha_s$ is obtained at $n_s = 1 - \frac{3r^2}{2}$ with $r = 0.1384$, one also has

$$\alpha_s \geq -\frac{3r^2}{32} \geq -0.0018,$$

giving $D \geq 1.6\sigma$.

However when one deals with Arctan Inflation (AI) [26] with potential $V(\varphi) = V_0 \left(1 - \frac{2}{\pi} \arctan \left(\frac{\varphi}{\mu}\right)\right)$, where one has $\xi \leq \frac{3}{2} \eta^2$. The absolute minimum is reached at the point $(n_s = 0.9481, r = 0.32)$, leading to the constrain

$$\alpha_s = \frac{(n_s - 1)r}{16} - \frac{3r^2}{256} - \frac{3(n_s - 1)^2}{4} \geq -0.0022,$$

which means that the deviation to the expected observational value is greater than $1.56\sigma$.

In the case of Loop Inflation (LI) [27] with potential $V(\varphi) = V_0(1 + \alpha \ln \varphi)$, one has $\xi = 2\eta^2$, leading to the constrain

$$\alpha_s = -\frac{1}{4} (n_s - 1)r - \frac{3r^2}{64} - (n_s - 1)^2 \geq -0.0034,$$

when one evaluates at the point where $\alpha_s$ reaches its absolute minimum, namely $(n_s = 0.9481, r = 0.32)$. Consequently, $D \geq 1.46\sigma$.

For Mixed Large Field Inflation (MLFI) [28] with potential $V(\varphi) = V_0 \varphi^2 \left(1 + \alpha \varphi^2\right)$, one has $\xi = \frac{3\alpha}{2(1+2\alpha\varphi^2)} r$. On the other hand, the relation

$$r = 16e = \frac{32}{\varphi^2} \left(\frac{1 + 2\alpha\varphi^2}{1 + \alpha\varphi^2}\right)^2 \geq \frac{32}{\varphi^2},$$

(18)
leads to the bound $\frac{1}{1 + 2\alpha^2} \leq \frac{r}{r^2 + 64\alpha}$ and thus, $\xi \leq \frac{3r^2}{128}$. Finally, evaluating at the absolute minimum ($n_s = 0.9481, r = 0.276$) we can conclude

$$\alpha_s \geq \frac{1}{2}(n_s - 1)r + \frac{3}{64}r^2 = -\frac{3r^2}{64} \geq -0.0036,$$

and thus, $D \geq 1.45\sigma$.

Finally, in Witten-O’Raifeartaigh Inflation (WRI) [29] with potential $V(\phi) = V_0 \ln^2\left(\frac{\phi}{\mu}\right)$ one has $\xi \leq \frac{r}{4} \eta^2$, leading to the constrain

$$\alpha_s \geq -\frac{11}{32}(n_s - 1)r - \frac{33}{512}r^2 - \frac{9}{8}(n_s - 1)^2 \geq -0.0040,$$

when one evaluates at the point where $\alpha_s$ reaches its absolute minimum, namely $(n_s = 0.9481, r = 0.32)$. Consequently, $D \geq 1.42\sigma$.

To end this section a remark is in order: First of all, it is important to realize that we have studied analytically only 25 of the 49 models provided by [2]. Secondly, our analytic results have been obtained bounding the minimum of $\alpha_s$ in the rectangle

$$R = \{(n_s, r) : 0.9481 \leq n_s \leq 0.9733, \quad 0 \leq r \leq 0.32\}.$$  

However, the running could be parametrized with only one independent variable, for instance, the scalar field $\phi$, which means that the bounds obtained in the rectangle could be improved, because one only needs to find a bound inside a curve inside the rectangle $R$. The problem to perform this calculation analytically is that this curve can only be obtained explicitly for a few potentials. The numerical calculations of Section 4 are needed to improve the analytical ones.

### 3.2 Analytical results from PLANCK2013: $\Lambda$CDM+$r+\alpha_s$ model combined with other data

The same kind of results could be obtained from other models with running. In fact, assuming that the potentials we choose have a spectral index $n_s$ at less than $2\sigma$ deviations and also they satisfy the conservative constrain $r \leq 0.32$, we have summarized the results in four Tables: Table 1 contains the description of three models and Table 2 (WP+high-$\ell$), Table 3 (WP) and Table 4 (WP+lensing) contain the deviation of the running, for the potentials we have analytically studied, for each one of the models. Again, the computed likelihoods reflect that the values of the running $\alpha_s$ predicted by the potentials lie all in the same tail of the Gaussian distribution.
Planck2013+WP+ high-\(\ell\) ≤ 0.23 \(0.9570 \pm 0.0075\) \(-0.022^{+0.011}_{-0.010}\)
Planck2013 +WP ≤ 0.25 \(0.9583 \pm 0.0081\) \(-0.021 \pm 0.012\)
Planck2013 +WP+lensing ≤ 0.26 \(0.9633 \pm 0.0072\) \(-0.017 \pm 0.012\)

Table 1: Planck2013 estimations of spectral parameters, without running of the running ([1]).

| Potential                              | Running deviation | Disfavored at         |
|----------------------------------------|-------------------|-----------------------|
| PLI                                    | ≥ 2\(\sigma\)     | 97.75 % C.L. or more  |
| HTI, DWI, NI, ESI, KMII, HI, PSNI      | ≥ 1.8\(\sigma\)   | 96.4 % C.L. or more   |
| SFI, LFI, VHI, DSI, BI, OSTI, RGI     | ≥ 1.7\(\sigma\)   | 95.55 % C.L. or more  |
| AI                                     | ≥ 1.6\(\sigma\)   | 94.5 % C.L. or more   |
| LI, MLFI, WRI                          | ≥ 1.2\(\sigma\)   | 86.45 % C.L. or more  |

Table 2: Planck2013+WP+high-\(\ell\):\(\Lambda\)CDM+\(r\)+\(\alpha_s\) model.

### 3.3 Planck2015 data: the running almost vanishes

The new observational data provided by [9], reproduced in Table 5 reduces one order or more (depending on other data such as BAO, lensing and lowP), the modulus of the expected observational value of the running.

This drastic reduction of the running in modulus is what allows the viability of the potentials disregarded from Planck2013 data. Effectively, using for instance, the Planck2015 TT+ lowP+BAO:\(\Lambda\)CDM+\(r\)+\(\alpha_s\) data, and assuming that the potentials have spectral index \(n_s\) at less than 2\(\sigma\) deviations, and also satisfying the conservative bound \(r \leq 0.32\). Then, for the simplest potentials, analytically one can show:

1. For LFI, one gets the following bound

   \[ |\alpha_s| \leq \frac{2}{2+p}(n_s - 1)^2 \leq (n_s - 1)^2 \leq 0.0017, \]  

   which means \(D \leq 1.2\sigma\), and thus, disfavoring the potential less than the 86.45 % C.L..

2. For PLI, which has no-running, one obtains \(D \leq 1.4\sigma\), and thus, disfavoring the potential less than the 91.95 % C.L..

3. For HTP, ESI, BI, DSI, SFI and VHI the minimum and maximum of \(\alpha_s\) are obtained respectively at \(n_s = 1 - \frac{3r}{3\sqrt{\pi}}\) with \(r = 0.1101\) and \(r = 0.0642\). And
Potential | Running deviation | Disfavored at
--- | --- | ---
PLI | $\geq 1.7\sigma$ | 95.55 % C.L. or more
HTI, DWI, NI, ESI, KMII, HI, PSNI | $\geq 1.5\sigma$ | 93.93 % C.L. or more
SFI, LFI, VHI, DSI, BI, OSTI, RGI, AI | $\geq 1.5\sigma$ | 93.93 % C.L. or more
LI | $\geq 1.4\sigma$ | 91.95 % C.L. or more
MLFI, WRI | $\geq 1.3\sigma$ | 90.3 % C.L. or more

Table 3: PLANCK2013+WP: $\Lambda$CDM+$r+a_s$ model.

Potential | Running deviation | Disfavored at
--- | --- | ---
PLI | $\geq 1.4\sigma$ | 91.95 % C.L. or more
HTI, DWI, NI, ESI, KMII, HI, PSNI | $\geq 1.2\sigma$ | 88.5 % C.L. or more
SFI, LFI, VHI, DSI, BI, OSTI, RGI, AI | $\geq 1.2\sigma$ | 88.5 % C.L. or more
LI, MLFI, WRI | $\geq 1.1\sigma$ | 86.45 % C.L. or more

Table 4: For PLANCK2013+WP+lensing: $\Lambda$CDM+$r+a_s$ model.

thus, one gets $-0.0012 \leq a_s \leq -0.0003$, what implies $\mathcal{D} \leq 1.4\sigma$, and thus, disfavors the potential less than the 91.95 % C.L. of.

The rest of potentials in the examined list fit even better the values of the spectral parameters of Table 5 for some choice of parameter or field values.

### 3.4 PLANCK2015 data: the running of the running

The last PLANCK2015 data about the running $a_s$ and its running $\beta_s = \frac{da_s}{d\ln k}$ are reproduced in Table 6.

These values are similar to the ones of PLANCK2013, reproduced in Table 7.

These results contradict single field slow roll inflation, because in that case, the running $a_s$ is second order in the slow roll parameters, and its running is given by Eqs. (4), (5), which make $\beta_s$ a third order parameter, while the values determined by PLANCK place $\beta_s$ in a higher order of magnitude than the running $a_s$ itself. Moreover, disregarding the running of the running, the running is negative while taking into account it, the running becomes positive. This seems a signature of the problem that suffers the method used to reconstruct the power spectrum of scalar perturbations from observational data: the value of the coefficients in the Taylor series of the power spectrum logarithm function could suffer a bias. We will address this question in Section 5.
Table 5: PLANCK2015 estimations of spectral parameters, without running of the running ([9]).

| Determination                  | \( r \)   | \( n_s \)       | \( \alpha_s \)  |
|--------------------------------|-----------|-----------------|-----------------|
| PLANCK2015 TT+ lowP            | \( \leq 0.180 \) | \( 0.9667 \pm 0.0066 \) | \( -0.0126^{+0.0098}_{-0.0087} \) |
| PLANCK2015 TT+ lowP+lensing    | \( \leq 0.186 \) | \( 0.9690 \pm 0.0063 \) | \( -0.0076^{+0.0092}_{-0.0080} \) |
| PLANCK2015 TT+ lowP+BAO       | \( \leq 0.176 \) | \( 0.9673 \pm 0.0043 \) | \( -0.0125 \pm 0.0091 \) |
| PLANCK2015 TT, TE, EE+ lowP   | \( \leq 0.152 \) | \( 0.9644 \pm 0.0049 \) | \( -0.0085 \pm 0.0076 \) |

Table 6: Determinations of the spectral parameter values, with running of the running by PLANCK2015 ((19) of [9]).

| Determination                  | \( n_s \)       | \( \alpha_s \)  | \( \beta_s \) |
|--------------------------------|-----------------|-----------------|----------------|
| PLANCK2015 TT+lowP             | \( 0.9569 \pm 0.0077 \) | \( 0.011^{+0.014}_{-0.013} \) | \( 0.029^{+0.015}_{-0.016} \) |
| PLANCK2015 TT,TE,EE+lowP      | \( 0.9586 \pm 0.0056 \) | \( 0.009 \pm 0.010 \) | \( 0.025 \pm 0.013 \) |

To show the improbability of the observed value of \( \beta_s \) analytically for all the potentials that appear in [2] is very involved due to the increasing complexity of the formulas (4), (5) for the new parameter \( \beta_s \). This work thus follows a double approach, combining analytical calculations for LFI, LI (with \( \alpha > 0 \)), VHI, SFI, BI and DSI (with \( p \) and even number or \( \varphi \geq 0 \)), HTI, ESI and PLI, with a numerical analysis to which we subject all the potentials in [2].

Let us start with the analytical approach. In the case of LFI one has

\[
ns - 1 = -\frac{p(p + 2)}{\varphi^2}, \quad \alpha_s = -\frac{2p^2(p + 2)}{\varphi^4} \quad \text{and} \quad \beta_s = -\frac{8p^3(p + 2)}{\varphi^6},
\]

and thus,

\[
\beta_s = -\frac{8}{(p + 2)^2}(n_s - 1)^3 \Rightarrow |\beta_s| \leq \frac{8}{9}|n_s - 1|^3.
\]

Then, for the PLANCK2015 TT,TE,EE+lowP data, considering \( n_s \) at 2\( \sigma \) C.L., and thus, after inserting \( n_s = 0.9474 \) in (23), one obtains the bound \( \beta_s \leq 0.00013 \), which means that the deviation from the theoretical value of the running of the running to its expected observational value is larger than 1.9\( \sigma \). A deviation larger than 1.9\( \sigma \) is also obtained from PLANCK2015 TT+lowP data, meaning that LFI is completely disfavored by PLANCK2015 data.

In the case of LI (with \( \alpha > 0 \)), VHI, SFI, BI and DSI (with \( p \) and even number or \( \varphi \geq 0 \)), HTI, ESI and PLI, a simple calculation shows that \( \beta_s \leq 0 \) which means that the deviation from the theoretical value of the running of the running to its expected observational value is also larger than 1.9\( \sigma \).
Table 7: Determinations of the spectral parameter values, model $\Lambda$CDM+$\alpha_s$+$\beta_s$
by PLANCK2013 (table 5 of [1])

| Determination                  | $n_s$         | $\alpha_s$    | $\beta_s$    |
|-------------------------------|---------------|---------------|--------------|
| PLANCK2013+WP+BAO             | $0.9568^{+0.068}_{-0.063}$ | $0.000^{+0.013}_{-0.016}$ | $0.017^{+0.016}_{-0.014}$ |
| PLANCK2013+WP+high-\ell       | $0.9476^{+0.086}_{-0.088}$  | $0.001^{+0.013}_{-0.014}$  | $0.029^{+0.016}_{-0.013}$  |
| PLANCK2013+WP+lensing         | $0.9573^{+0.077}_{-0.079}$  | $0.006^{+0.015}_{-0.014}$  | $0.019^{+0.018}_{-0.014}$  |
| PLANCK2013+WP                 | $0.9514^{+0.087}_{-0.090}$  | $0.001^{+0.016}_{-0.014}$  | $0.020^{+0.016}_{-0.015}$  |

4 Numerical fitting of the parameters

4.1 Methodology

Let us describe the numerical tests that the authors have applied to all single field inflationary models from the list of [2]. These tests have been built into a MATLAB program that takes as input a list of potentials $V(\phi)$ and values for spectral parameters in the list $r, n_s, \alpha_s, \beta_s$, and assesses the likelihood of each model in the list to fit the values of the spectral parameters using a 95% confidence limit for the value of $r$, and assuming Gaussian distribution for the values of $n_s, \alpha_s, \beta_s$.

For each cosmological model, a broad range of possible values for the parameters on which it depends has been determined following [2]. A test list of values for each parameter has been selected, covering in a dense, equispaced fashion finite intervals of possible values for the parameter, and approaching with log-equispaced values every finite or infinite limit value for the parameter.

After allowing simplifications induced by rescaling, if a model still depended on more than one parameter all possible combinations of values for each parameter were tested. Table 8 lists the selected parameter values for each model.

Also separately for each model, the range of possible values for the inflaton field was determined taking into account whether the model admitted values of the field with any sign, or only positive values, and further peculiarities such as periodicity of the model and the applied rescalings. The considered range $[\phi_0, \phi_f]$ for field values in each model is also listed in Table 8.

The MATLAB software developed by the authors, for each model $V(\phi)$ and choice of value of the parameter(s) on which it depends, takes an equispaced mesh of values in the range $[\phi_0, \phi_f]$ of possible values of the field in this model. This mesh is taken increasingly fine, currently up to step $\Delta \phi = 2 \cdot 10^{-4}$.

The subintervals in the range of field values for which the potential satisfies $V(\phi) > 0$ are numerically determined over the selected mesh, and each interval of positive values of the potential for the selected values of the model parameters is considered as a case, which thus consists of:

- a candidate theory with a given potential $V(\phi)$,
• a specific choice of parameter values for $V$,
• and a range of values $[\bar{\varphi}_0, \bar{\varphi}_f]$ of the inflaton field $\varphi$ such that $V > 0$ on them.

The numerical test for each case consists in meshing the interval of field values with a uniform step (of size $\Delta \varphi = 2 \cdot 10^{-4}$ for the results reported in this work), computing the spectral parameters $r, n_s, \alpha_s, \beta_s$ for each value of the field $\varphi$ in the mesh using the formulas of Section 2 and symbolic derivation of the potential $V$ to produce the derivatives $V, \ldots, V_{\varphi \varphi \varphi \varphi}$, and then applying successive filtering criteria to determine which values of the field $\varphi$ fulfill simultaneously all of them, thus allowing the model in this particular case to fit the spectral measured data for which the model is tested.

A determination of a set of values for the spectral parameters provides their expected values $<n_s>, <\alpha_s>, <\beta_s>$, and their standard deviations $\sigma_{n_s}, \sigma_{\alpha_s}, \sigma_{\beta_s}$. In the case of spectral parameters without running of the running $\beta_s$, this is replaced by the tensor/scalar ratio $r$, which is conservatively estimated to have a value $r < 0.32$ (see [1],[9]).

The applied filters in the case of spectral parameters without running of the running consist in looking for the values of the field $\varphi$ such that:

1. $r < 0.32$,
2. $|n_s(\varphi) - < n_s >| < 2 \sigma_{n_s}$,
3. $|\alpha_s(\varphi) - < \alpha_s >| < 1.6 \sigma_{\alpha_s}$,

A model not passing the first filter is ruled out with 95% C.L., a model not passing the second filter is ruled out with 95.5% C.L., and a model not passing the third filter is ruled out with 94.5% C.L., because the values of $\alpha_s$ provided by the models are located uniformly in the same tail of the Gaussian distribution (namely, from the negative expected value towards zero) for the spectral valuations to which we have applied our test.

The applied filters in the case of spectral parameters with running of the running consist in looking for the values of the field $\varphi$ such that:

1. $\epsilon(\varphi) \leq 1$,
2. $|n_s(\varphi) - < n_s >| < 2 \sigma_{n_s}$,
3. $|\alpha_s(\varphi) - < \alpha_s >| < 2 \sigma_{\alpha_s}$,
4. $|\beta_s(\varphi) - < \beta_s >| < 1.8 \sigma_{\beta_s}$,

A model not passing the first filter (equivalent to asking for $r < 16$) is ruled out with 95% C.L., a model not passing the second or third filter is ruled out with 95.5% C.L., and a model not passing the fourth filter is ruled out with 92.8% C.L.
In either situation with or without running of the running, a case (choice of model, parameter values, and interval of values for the field) is considered possible only if there exists some value of the field $\varphi$ in its range such that it satisfies simultaneously all of the filtering conditions. The filters are not independent, thus if a case does not satisfy all of the filtering criteria simultaneously for any field value $\varphi$ in its range, it is disproved with the confidence level of the strongest filter it fails. A model such that it is disproved in any case (i.e., for any choice of parameters and range of field values) is regarded as disproved with the lowest confidence level with which any of its cases is disproved.

Conversely, if a case has values of the field $\varphi$ satisfying all of the filtering criteria, the minimal values among them of the distances $n_s(\varphi) - < n_s >, \alpha_s(\varphi) - < \alpha_s >, \beta_s(\varphi) - < \beta_s >$, expressed in standard deviations, are logged. The maximum among these distances provides the confidence level to which the particular case has been disproved. In the case of computations without running of the running such that all of the values of $\alpha_s$ provided by the models are found consistently in the same tail of the Gaussian distribution their likelihood is assessed taking into account only this tail.

In these cases satisfying all filters, i.e. that are not disproved with C.L. at least 92.8% the testing software also looks for values of the field $\varphi_e$ such that $\epsilon(\varphi_e) \equiv 1$, and using them as endpoints of the inflationary phase, computes the number of e-folds of inflation for any choice of $\varphi$ in the case, by integrating numerically with a trapezoidal rule

$$N(\varphi) = \left| \int_{\varphi_e}^\varphi \frac{V}{V_\varphi} d\varphi \right|$$  \hspace{1cm} (24)

The result $N(\varphi)$ ranges over the number of e-folds of expansion that the case supports for the field values $\varphi$ satisfying all of the filtering criteria. Its minimal and maximal values are logged, as they are the limit values for the number of e-folds of expansion that the case can support.

### 4.2 Numerical results

Single-field inflaton models were exhaustively studied in [2], from which we take the list of models and parameters to be numerically tested. Table 8, adapted from Table 1 of [2], presents each model’s potential, the range of values of the parameters for which it has been tested, and the range of values of the inflaton field over which it has been tested.

| Name | $V(\varphi)$ | Parameter values | Field values |
|------|--------------|------------------|--------------|
| HI | $V_0 \left(1 - e^{-\sqrt{2/3} \varphi}\right)^2$ | $\alpha_s$: linspace(-100,100,120), $\beta_s$: linspace(-3,3,200) | $[-40,40]$ |
| RCHI | $V_0 \left(1 - 2e^{-\sqrt{2/3} \varphi} + \frac{1}{2\text{ln}(\sqrt{6})} \right)$ | $A_I$: linspace(50,100,120), $A_{\varphi}$: linspace(5,20,60) | $[-10,20]$ |
| LFI | $V_0 (\varphi)^p$ | $p$: linspace(5,20,60) | $[-10,20]$ |
| MLFI | $V_0 e^{\varphi^2 \left(1 + \alpha \varphi^2\right)}$ | $\alpha$: linspace(-10,100,61), $\lambda$: linspace(0.1,0.1,120) | $[-10,10]$ |
| Function | Equation | Parameters | Domain |
|----------|----------|------------|--------|
| MSSMI | \( V_0 \left[ \left( \frac{\phi}{\mu} \right)^2 - \frac{\alpha}{\mu} \ln \left( \frac{\phi}{\mu} \right) \right] \) | \( \alpha \in [\text{im}-\text{impact}(1e-4,1.5,30), 10. \land \text{impact}(4,5,20)] \) | |
The models, choice of parameter values and range of field values of Table 8 have been subjected to the numerical test described in subsection 4.1 for the several determinations of the spectral parameters $n_s, \alpha_s, \beta_s$ listed in Section 3. Let us sum up the conclusions of the most relevant cases:

For the determination of spectral parameters PLANCK2013 combined with WP and BAO data using the $\Lambda CDM + r + \alpha_s$ model (i.e. without running of the running) depicted in Table 1, the computation has concluded that all the models in Table 8 are disproved within a confidence limit 93%. The reason in all cases is that the parameter values that match the bound for $r$ and the expected value of $n_s$ within 2 standard deviations can only provide values of the running $\alpha_s$ that are too close to 0, far above the value $\alpha_s = -0.021^{+0.012}_{-0.016}$ of this determination, and

| Parameter | Value Range |
|-----------|-------------|
| $n_s$     | $10^{-4}, 40$ |
| $\alpha_s$ | $-0.021^{+0.012}_{-0.016}$ |
| $\beta_s$ | $-10^{10}$ |

Table 8: Parameter values expressed in Matlab code: `linspace(a,b,n)` means $n$ equispaced values between $a$ and $b$; $10^a$ means $a$ log-equispaced values between $10^a$ and $10^{a+1}$. Parameter $V_0$ and the reduced Planck mass $M_{Pl}$ always set to 1. The range of studied field values is $\varphi \in [10^{-4}, 40]$ unless otherwise indicated.

The models, choice of parameter values and range of field values of Table 8 have been subjected to the numerical test described in subsection 4.1 for the several determinations of the spectral parameters $n_s, \alpha_s, \beta_s$ listed in Section 3. Let us sum up the conclusions of the most relevant cases:

For the determination of spectral parameters PLANCK2013 combined with WP and BAO data using the $\Lambda CDM + r + \alpha_s$ model (i.e. without running of the running) depicted in Table 1, the computation has concluded that all the models in Table 8 are disproved within a confidence limit 93%. The reason in all cases is that the parameter values that match the bound for $r$ and the expected value of $n_s$ within 2 standard deviations can only provide values of the running $\alpha_s$ that are too close to 0, far above the value $\alpha_s = -0.021^{+0.012}_{-0.016}$ of this determination, and
consistently in the same tail of the Gaussian distribution.

For the determination of spectral parameters PLANCK2015 TT+lowP, without running of the running, of Table 5 the result changes drastically: the models

1. Natural Inflation (NI) [14]
2. Power Law Inflation (PLI) [16]
3. Constant $n_s$ A Inflation (CNAI) [30]
4. Constant $n_s$ B Inflation (CNBI) [30]
5. Open String Tachyonic Inflation (OSTI) [29]
6. Generalised MSSM Inflation (GMSSMI) [31]
7. Generalised Regularised Point Inflation (GRIPI) [32]
8. Constant Spectrum Inflation (CSI) [33]
9. Constant $n_s$ C Inflation (CNCI) [30]
10. Dynamical Supersymmetric Inflation (DSI) [34],

are disproved with confidence at least 94.5%, and asking for the number $N$ of e-folds of expansion that the model allows to be in the range [30,80] only rules out completely a further model, namely Minimal Super-symmetric Standard Model Inflation (MSSMI) [35]. The rest of the models in the table admit some choice(s) of parameter and field values that simultaneously satisfies all filtering conditions.

The inclusion of the running of the running in the numerical test results in another reversion of conclusions. For the determination of spectral parameters PLANCK2015 TT+lowP with running of the running of Table 6, the only models in Table 8 which are not disproved for any choice of parameter and field values with confidence at least 92.8% are:

1. Kâller Moduli Inflation I (KMII) [17].
2. Kâller Moduli Inflation II (KMIII) [17].
3. Logamediate Inflation (LMI) [36].
4. Twisted Inflation (TWI) [37].
5. Brane SUSY Breaking Inflation (BSUSYBI) [38].
6. Spontaneous Symmetry Breaking Inflation (SSBI) [39].
7. Running-mass Inflation (RMI) [40].

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8. Generalised Mixed Large Field Inflation (GMLFI) [34].

9. Constant $n_s$ D Inflation (CNDI) [33].

(albeit GMLFI is disproved with 92.1% confidence, and CNDI with 91.9% confidence). The most common reason for disproving the models is now that they only provide values of $\beta_s$ at more than 1.8 standard deviations of distance of the value 0.029 of this determination, which we point out that is a quite large value compared with those of $n_s - 1 = -0.043$ and $\alpha_s = 0.011$ in this determination. We will bring up this subject in the next section.

Of the models that are not disproved for this PLANCK2015 TT+lowP with running of the running determination of the spectral parameters, the e-fold test in our software has found only for the models KMIII,SBI,SSBI,GMLFI,CNDI choices of parameter and field values passing the filters and further allowing the number of expansion e-folds $N$ to lie in the range $[30, 80]$.

5 Accuracy and reliability of the spectral parameter values

The results of the analytical and numerical fitting of the studied single field inflationary models to the successive sets of values provided by the Planck collaboration for the spectral parameters $r, n_s, \alpha_s, \beta_s$ may be summed up as:

1. The spectral values provided by PLANCK2013 for $r, n_s, \alpha_s$, in models without running of the running, disfavour all of the studied models with a confidence limit in the ranges 93%-95.5%. The poor fit of the value of the running $\alpha_s$ supplied by the models to the measured one, which has a larger than expected size, is the most common reason for this invalidation.

2. The corrected values provided by PLANCK2015 for $r, n_s, \alpha_s$, in models without running of the running, may be fitted by most models in our study. The main reason for the change is the greatly diminished measured value of the running $\alpha_s$.

3. But if one uses the values provided by PLANCK2015, for models including running of the running, for the parameters $r, n_s, \alpha_s, \beta_s$ again most models are disfavoured with a confidence limit 92.8% or better. The most common reason is the poor fit these models provide for the running of the running $\beta_s$, which in this measurement has a larger than expected size.

The dramatic contrast in results leads to the question of the reliability of the successive determinations of the spectral parameters $r, n_s, \alpha_s, \beta_s$. The authors have found that the methodology of model fitting used in [1],[9] has a bias that often leads to the overestimation of the highest order parameter, which in turn triggers biases in the estimation of the lower order parameters.
The bias does not arise from the probabilistic algorithms, such as the Monte Carlo Markov Chain (MCMC) method, employed to determine the measured values of the spectral parameters. It comes from the procedure to judge the fit of the model.

It is another manifestation of the well known bias of the least square regression polynomial fit when the fitted polynomial \( p(x) \) has too small a degree: the highest order coefficient of \( p(x) \) is found to have a very large value, which is actually caused by the measured data growing faster than the degree of \( p(x) \) allows. The second highest order coefficient of \( p(x) \) typically has a bias of opposite sign to compensate for the overestimation of the highest degree term. Figure 1 illustrates this phenomenon with an elementary example.

This problem is a particular case of the more general difficulty in assessing the value of a Taylor polynomial, that approximates a function in the neighbourhood of a Taylor expansion point, from values of the function at points that lie further and further from the expansion point. Conceding any weight at all to the values of the function at points far from the expansion point in the assessment of best fit carries a great risk of introducing a bias such as that described in Figure 1.

The fitting of the model in every step of the MCMC algorithm of [1], [9], and in comparable algorithms, is probabilistic, but ultimately close to the classical regression fit: the spectral parameters on which the model depends are given values that maximize the likelihood of the observations that have to be described by the model. The coefficients \( C_l \) of the observed power spectrum are assumed to follow a Gaussian distribution. This means that the spectral parameters determining the theoretical power spectrum \( P_R(k) \) for scalar perturbations are selected in order to minimize the distance between the values \( C_l \) given by the model and the observed values \( \hat{C}_l \).

This distance is expressed in standard deviations for the probabilistic fitting that maximizes likelihood, and in natural units by the classical polynomial regression fit that minimizes residue. But the variation of the standard deviation for each spectral parameter changes little as the parameter varies its value, so minimizing the distance in natural units or in standard deviations ends up assigning very similar values to the spectral parameters. Indeed, this is the reason why probabilistic methods such as MCMC are often preferred to regression analyses that are more comprehensive but far more computationally costly and end up reaching a similar result.

A consequence of this equivalence of distances is that likelihood-maximizing Bayesian, MCMC ... methods, while computationally vastly more efficient than a regression fit, inherit the latter method’s bias, displayed in Figure 1: fitting a model for a function \( \ln P_R(k) \) with parameters up to an insufficient degree will result in an overestimation of the highest order parameter, which in turn triggers a cascade of estimation errors in the lower order parameters.

The order of the growth of the modelled function \( \ln P_R(k) \) (the logarithm of the power spectrum) is currently unknown, but the pattern of systematically over-
estimating the highest order spectral coefficient in its Taylor series, and drastically revising it down as higher coefficients are incorporated to the model suggest that this bias is indeed happening:

- The PLANCK2013 determinations of the value of the running \( \alpha_s \) in a \( \Lambda \)-CDM model without running of the running (table 5 of [1]) attribute to \( \alpha_s \) expected values ranging from -0.0149 to -0.0094.

- but the determinations by PLANCK2013 with the same methodology, adding a running of the running to the model put the expected value of \( \alpha_s \) in the range \([0, 0.006]\) (one order the magnitude smaller in absolute value than with only running; sign uncertain due to proximity to zero).

- The value of the running of the running \( \beta_s \) is estimated by PLANCK2013 (table 5 of [1]) to lie in the range \([0.017, 0.020]\), and by (19) in PLANCK2015 to lie in the range \([0.025, 0.029]\). These values are one order of magnitude greater than those attributed to the running in these estimations.

The values of the spectral parameters \( n_s - 1, \alpha_s, \beta_s \) will be reliably known only when higher order terms in the Taylor series of \( \ln P_R(k) \) are known, or a method without this bias is used to fit the models to the data.

6 Conclusions

Evidence mounts, both analytical and numerically, that single field slow rolling inflaton models do not fit well the observations of the CMB radiation. Sophistications such as multiple fields, or a breakdown in the slow roll regime, or a completely different paradigm such as the Matter Bounce Scenario ought to be contemplated.

But the value of the spectral parameters \( n_s - 1, \alpha_s, \beta_s \) of the CMB radiation is not yet well established, so we are witnessing the twilight, rather than the death, of this family of models.

In Section 5 of this work we have found analytically bounds for the values of the spectral parameters that most of the single field, slow roll inflation models support. In Section 4 we have described our MATLAB software package that finds, for each model given by its potential function \( V(\varphi) \), all possible values of the spectral parameters \( n_s - 1, \alpha_s, \beta_s \) that the model supports, for a comprehensive list of values of the field \( \varphi \) and of further parameters on which the potential \( V(\varphi) \) may depend. There is a deliberate redundancy in the two approaches, and both have reached the same conclusions for every experimental determination of the values of the spectral parameters. The numerical testing software described in this work can be applied to any single field slow roll model and determination of the spectral parameters beyond those studied here.

The conclusions reached by our analytic/numerical assessment of the likelihood of single field, slow roll inflation models vary strongly with the different
determinations of the values of the spectral parameters, but there is a clear pattern in this variation:

1. the most accurate determinations PLANCK2013 without running of the running (such as PLANCK2013+WP+BAO) assign to the running $\alpha_s$ a big value, that rules out most of the models because in them $\alpha_s$ is a second order expression in the slow roll parameters and can only be much smaller in magnitude,

2. the determinations PLANCK2015 give a much smaller value to the running $\alpha_s$, which can be fitted to most of the tested inflaton models,

3. but if the value of the running of the running $\beta_s$ of PLANCK2015 is included in the likelyhood test, it turns out that its determination by PLANCK2015 (both TT+lowP and TT,TE,EE+lowP) has such a big magnitude that again most of the tested inflaton models cannot furnish values within 1.8 standard deviations of the expected value.

The reason for this pattern of contradictory conclusions seems to be a mathematical bias in the method that has been used for the computation of the values of the spectral parameters, which results in a systematic overestimation of the highest order one. The bias is a migration of, and very close to, a classical bias of regression (i.e., minimization of residue) fitting: if one tries to fit a polynomial of too low degree to values of a function that actually grows faster, no matter how flawless the procedure for finding the better fit is, it will result in a polynomial with an exaggerate value for the magnitude of the leading term, which will result in a cascade of further errors for the lower coefficients as the table in Fig. 1 illustrates.

The probabilistic (Bayesian, MCMC, . . .) methods currently used to fit the value of spectral parameters following a Gaussian distribution in a model actually minimize the residue, expressed in standard deviations rather than in natural units, and reproduce this bias.

The pattern of disproving/validating/disproving of the single field models by successive determinations of the spectral parameters is not completely symmetrical, because the instances in which the models are validated are often extreme, narrow choices for the values of the parameters on which the model depends. Hence the authors’ suspicion that single field slow roll inflation models will ultimately have to be discarded.

Nevertheless, to rule out inflation models based on their fit to the measured spectral parameters $r$, $n_s$, $\alpha_s$, $\beta_s$ will not be possible until the value of these spectral parameters is reliably known, for which more terms in the Taylor series of $\ln P_R(k)$ or, even better, a determination procedure free of its current bias, will be required.

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Figure 1: Data points on curve $y = x^4$: the regression polynomials $\sum a_i x^i$ are

| degree | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|--------|-------|-------|-------|-------|-------|
| 4 (correct values) | 0 | 0 | 0 | 0 | 1 |
| 3 | -0.53 | 6.67 | -11.22 | 6 |
| 2 | 5.07 | -24.07 | 15.78 |
| 1 | -15.97 | 23.27 |
| 0 | 18.93 |

The leading order coefficient in each regression polynomial is systematically over-estimated (from its true value 0) to try fitting the growth of the function, which is actually of a higher order. This bias cascades down to the lower order coefficients, starting typically with an underestimation of the second to highest order coefficient.