General lepton textures and their implications

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Abstract

The present work attempts to provide an overview of texture specific lepton mass matrices. In particular, we summarize the findings of some recent analyses carried out within non flavor basis, wherein a parallel texture structure for the lepton and neutrino mass matrices is considered.

1 Introduction

In the last decade, we have almost reached ‘precision’ level for the measurement of neutrino oscillation parameters, including the recently measured mixing angle $\theta_{13}$. This has led to the need of a more intense activity towards understanding the pattern of neutrino masses and mixings which is quite different from the corresponding quark mixing case. In the absence of a theory providing a viable understanding of these issues, most of the phenomenological work is carried out within the general premises of ‘bottom-up’ approach. As an example of this approach, texture specific lepton mass matrices have been tried with a good deal of success. In particular, several attempts [1]-[8] have been made to understand the neutrino mixing data by formulating the phenomenological mass matrices with charged lepton matrix being diagonal, usually referred to as the flavor basis case. In addition, for both Majorana as well as Dirac neutrinos, some attempts [9]-[10] have also been made to explain the neutrino mixing data by considering texture specific structures for both the charged lepton and the neutrino mass matrices, referred to as the non flavor basis case. It may be noted that the non flavor basis enables quarks and leptons to be treated at the same footing and also to explore the possibility to arrive at a minimal set of fermion mass matrices which are compatible with the latest mixing data.

It is now well known, that, in the leptonic sector, the search for viable mass matrices is complicated by the ‘smallness’ of neutrino masses. The most popular explanation for this smallness is the see-saw mechanism [11]-[16] which requires the
neutrinos to be Majorana fermions. However, at present, neither the Majorana nature is established nor can we rule out the Dirac nature of neutrinos. On theoretical grounds, the existence of small Dirac masses requires the corresponding Yukawa couplings to be exceptionally small compared to their charged counterparts. The Dirac neutrino mass, although seemingly ‘unnatural’, can be explained by additional $U(1)_{B-L}$ symmetry of the Lagrangian which forbids Majorana mass term for the neutrinos. Apart from SM, this possibility can be realized in many of the models such as supersymmetry, superstring, supergravity and large extra dimensions. Keeping in mind that Dirac neutrinos are still not ruled out, in the present work we discuss texture specific mass matrices for both Dirac as well as Majorana neutrinos.

In particular, we have presented an overview of some recent analyses wherein texture specific lepton mass matrices have been considered in the non flavor basis for Dirac as well as Majorana neutrinos. In the following section, we first present the relation between lepton mass matrices and mixing matrix. The present experimental status of the neutrino mixing parameters have been give in Section 3. A brief summary of texture 6, 5 and 4 zero lepton mass matrices has been presented in Section 4. Finally, Section 5 summarizes our conclusions.

## 2 Lepton mass matrices and PMNS matrix

For the case of neutrinos, it is important to note that these may have either the Dirac masses or the more general Dirac-Majorana masses. A Dirac mass term can be generated by the Higgs mechanism with the standard Higgs doublet. In this case, the neutrino mass term can be written as

$$\mathbf{\nu}_l^c M_{\nu D} \nu_R + h.c.,$$

where $a = e, \mu, \tau$. $\nu_e, \nu_{\mu}, \nu_{\tau}$ are the flavor eigenstates and $M_{\nu D}$ is a complex $3 \times 3$ Dirac mass matrix. As mentioned earlier, in the non flavor basis, both the charged lepton and the neutrino mass matrices are considered having the same texture structure $\mathbf{9}$, e.g.,

$$M_l = \begin{pmatrix}
0 & A_l & 0 \\
A_l^* & D_l & B_l \\
0 & B_l^* & C_l
\end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix}
0 & A_\nu & 0 \\
A_{\nu}^* & D_\nu & B_\nu \\
0 & B_\nu^* & C_\nu
\end{pmatrix},$$

(2)

$M_l$ and $M_{\nu D}$ respectively corresponding to hermitian Dirac-like charged lepton and neutrino mass matrices. It may be noted that each of the above matrix is texture 2 zero type with $A_{l(\nu)} = |A_{l(\nu)}|e^{i\alpha_{l(\nu)}}$ and $B_{l(\nu)} = |B_{l(\nu)}|e^{i\beta_{l(\nu)}}$, in case these are symmetric then $A_{l(\nu)}^*$ and $B_{l(\nu)}^*$ should be replaced by $A_{l(\nu)}$ and $B_{l(\nu)}$, as well as $C_{l(\nu)}$ and $D_{l(\nu)}$ should respectively be defined as $C_{l(\nu)} = |C_{l(\nu)}|e^{i\gamma_{l(\nu)}}$ and $D_{l(\nu)} = |D_{l(\nu)}|e^{i\omega_{l(\nu)}}$.

The texture 6 zero mass matrices can be obtained from the above mentioned matrices by taking both $D_l$ and $D_{\nu}$ to be zero, which reduces the matrices $M_l$ and $M_{\nu D}$ each to texture 3 zero type. Texture 5 zero matrices can be obtained by taking
either $D_l = 0$ and $D_\nu \neq 0$ or $D_\nu = 0$ and $D_l \neq 0$, thereby, giving rise to two possible cases of texture 5 zero matrices, referred to as texture 5 zero $D_l = 0$ case pertaining to $M_l$ texture 3 zero type and $M_\nu D$ texture 2 zero type and texture 5 zero $D_\nu = 0$ case pertaining to $M_l$ texture 2 zero type and $M_\nu D$ texture 3 zero type.

It should be noted that in the case of texture 6 zero and texture 4 zero mass matrices we can have parallel structures for both the neutrino mass matrix and the charged lepton mass matrix, however, for the case of texture 5 zero mass matrices, one cannot have parallel structures. To consider all possible textures, we have considered only those possibilities which are compatible with the ‘Weak Basis’ transformations [19, 20]. To this end, in Table 1, we have presented all possible texture 2 zero mass matrices, from which we can derive texture 6 zero, 5 zero and 4 zero mass matrices for the discussion.

| Class | Class II | Class III | Class IV |
|-------|----------|-----------|----------|
| a     | \[
\begin{pmatrix}
0 & A e^{i\alpha} & 0 \\
A e^{-i\alpha} & D & B e^{i\beta} \\
0 & B e^{-i\beta} & C
\end{pmatrix}
\] | \[
\begin{pmatrix}
D & A e^{i\alpha} & 0 \\
A e^{-i\alpha} & 0 & B e^{i\beta} \\
0 & B e^{-i\beta} & C
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & A e^{i\alpha} & D e^{i\gamma} \\
A e^{-i\alpha} & 0 & B e^{i\beta} \\
D e^{-i\gamma} B e^{-i\beta} C & 0 & B e^{-i\beta}
\end{pmatrix}
\] |
| b     | \[
\begin{pmatrix}
0 & 0 & A e^{i\alpha} \\
0 & C & B e^{i\beta} \\
A e^{-i\alpha} B e^{-i\beta} & 0 & D
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & 0 & A e^{i\alpha} \\
0 & C & B e^{i\beta} \\
A e^{-i\alpha} B e^{-i\beta} & 0 & D
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & 0 & A e^{i\alpha} \\
0 & C & B e^{i\beta} \\
0 & A e^{i\alpha} & 0
\end{pmatrix}
\] |
| c     | \[
\begin{pmatrix}
D & A e^{i\alpha} B e^{i\beta} \\
A e^{-i\alpha} & 0 & 0 \\
B e^{-i\beta} & 0 & C
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & A e^{i\alpha} B e^{i\beta} \\
A e^{-i\alpha} & 0 & 0 \\
B e^{-i\beta} & 0 & C
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & A e^{i\alpha} B e^{i\beta} \\
A e^{-i\alpha} & 0 & 0 \\
B e^{-i\beta} & 0 & C
\end{pmatrix}
\] |
| d     | \[
\begin{pmatrix}
C & B e^{i\beta} & 0 \\
B e^{-i\beta} & D & A e^{i\alpha} \\
0 & A e^{-i\alpha} & 0
\end{pmatrix}
\] | \[
\begin{pmatrix}
C & B e^{i\beta} & 0 \\
B e^{-i\beta} & D & A e^{i\alpha} \\
0 & A e^{-i\alpha} & 0
\end{pmatrix}
\] | \[
\begin{pmatrix}
A & 0 & 0 \\
0 & C & B e^{i\beta} \\
0 & B e^{-i\beta} & D
\end{pmatrix}
\] |
| e     | \[
\begin{pmatrix}
D & B e^{i\beta} A e^{i\alpha} \\
B e^{-i\beta} & C & 0 \\
A e^{-i\alpha} & 0 & 0
\end{pmatrix}
\] | \[
\begin{pmatrix}
C & 0 & B e^{i\alpha} \\
0 & D & A e^{i\beta} \\
B e^{-i\alpha} A e^{-i\beta} & 0
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & C e^{-i\gamma} B e^{i\alpha} \\
C e^{-i\gamma} D & A e^{i\beta} \\
B e^{-i\alpha} A e^{-i\beta} & 0
\end{pmatrix}
\] |
| f     | \[
\begin{pmatrix}
C & 0 & B e^{i\beta} \\
0 & 0 & A e^{i\alpha} \\
B e^{-i\beta} A e^{-i\alpha} & D
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & B e^{i\alpha} A e^{i\beta} \\
B e^{-i\alpha} C & 0 \\
A e^{-i\beta} & 0
\end{pmatrix}
\] | \[
\begin{pmatrix}
0 & B e^{i\alpha} A e^{i\beta} \\
B e^{-i\alpha} C e^{-i\gamma} C & 0 \\
A e^{-i\beta} & 0
\end{pmatrix}
\] |

Table 1: Table showing various ‘Weak Basis’ transformation compatible texture 2 zero possibilities categorized into four distinct classes and their permutations given by a,b,c,d,e.f.

Coming to the diagonalization of lepton mass matrices, similar to the quark
sector, these can also be diagonalized by bi-unitary transformations, e.g.,

$$M_{\nu_D}^{\text{diag}} = U_{\nu_L}^\dagger M_{\nu_D} U_{\nu_R} = \text{Diag}(m_1, m_2, m_3),$$  \hspace{0.5cm} (3)$$

where $U_{\nu_L}$ and $U_{\nu_R}$ are unitary matrices and $M_{\nu_D}^{\text{diag}}$ is a diagonal matrix. The corresponding mixing matrix obtained, known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) or lepton mixing matrix $V_{PMNS}$, is given as

$$V_{PMNS} = V_{\nu_L}^\dagger V_{l_L},$$  \hspace{0.5cm} (4)$$

where $V_{\nu_L}^\dagger$ and $V_{l_L}$ correspond to the diagonalization transformations of lepton and neutrino mass matrices respectively. The $V_{PMNS}$ expresses the relationship between the neutrino mass eigenstates and the flavor eigenstates, e.g.,

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau 
\end{pmatrix} =
\begin{pmatrix}
V_{e1} & V_{e2} & V_{e3} \\
V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\
V_{\tau 1} & V_{\tau 2} & V_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3 
\end{pmatrix},$$  \hspace{0.5cm} (5)$$

where $\nu_{e}, \nu_{\mu}, \nu_{\tau}$ are the flavor eigenstates; $\nu_1, \nu_2, \nu_3$ are the mass eigenstates and the $3 \times 3$ mixing matrix is leptonic mixing matrix. For the case of three Dirac neutrinos, in the Particle Data Group (PDG) parameterization, involving three angles $\theta_{12}, \theta_{23}, \theta_{13}$ and the Dirac-like CP violating phase $\delta_l$ the mixing matrix has the form

$$V_{PMNS} =
\begin{pmatrix}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta_l} \\
-s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i \delta_l} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i \delta_l} & -s_{13} c_{23} \\
s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i \delta_l} & -c_{12} s_{23} s_{13} e^{i \delta_l} & c_{23} c_{13}
\end{pmatrix},$$  \hspace{0.5cm} (6)$$

with $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$.

The neutrino might be a Majorana particle which is defined as is its own anti particle and is characterized by only two independent particle states of the same mass ($\nu_L$ and $\bar{\nu}_R$ or $\nu_R$ and $\bar{\nu}_L$). A Majorana mass term which violates both the law of total lepton number conservation and that of individual lepton flavor conservation can be written either as

$$\frac{1}{2} \overline{\nu}_{aL}^c M_L \nu_{aR}^c + h.c.,$$  \hspace{0.5cm} (7)$$

or as

$$\frac{1}{2} \overline{\nu}_{aL}^c M_R \nu_{aR}^c + h.c.,$$  \hspace{0.5cm} (8)$$

where $M_L$ and $M_R$ are complex symmetric matrices leading to the famous see-saw mechanism \cite{11}-\cite{16}, given by

$$M_\nu = -M_{\nu_D}^T (M_R)^{-1} M_{\nu_D},$$  \hspace{0.5cm} (9)$$

where $M_{\nu_D}$ and $M_R$ are respectively the Dirac neutrino mass matrix and the right-handed Majorana neutrino mass matrix. This mechanism requires the inclusion of right-handed neutrinos with very large Majorana masses, therefore inducing a very small mass for the left-handed neutrinos. Thus, the generation of masses in neutrinos is not straight-forward as they may have either the Dirac masses or the
more general Dirac-Majorana masses. Further, when discussing texture possibilities textures are imposed on $M_{\nu_D}$, unlike many other attempts [1]-[8] in the literature where texture is imposed on $M_{\nu}$.

In the case of the Majorana neutrinos, there are extra phases which cannot be removed, therefore, the above matrix $V_{PMNS}$ takes the following form

$$
\begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_l} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_l} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_l} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_l} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_l} & c_{23}c_{13}
\end{pmatrix}
\begin{pmatrix}
e^{i\alpha_1/2} & 0 & 0 \\
0 & e^{i\alpha_2/2} & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(10)

where $\delta_l$ is the Dirac-like CP violating phase in the leptonic sector and $\alpha_1$ and $\alpha_2$ are the Majorana phases which do not play any role in neutrino oscillations.

3 Experimental status of neutrino masses and mixing parameters

While carrying out an analysis regarding exploring the compatibility of neutrino mass matrices with the recent data, one needs to keep in mind the experimental constraints imposed by the relationship between mass matrices and their corresponding mixing matrices. To facilitate our discussion in this regard, we present the status of relevant data in the lepton sector. The $3\sigma$ confidence level ranges of the neutrino oscillation parameters obtained in a latest global three neutrino oscillation analysis carried out by Fogli et al. [21] have been presented in Table (2).

| Parameter | $3\sigma$ range |
|-----------|----------------|
| $\Delta m^2_{sol}$ [10$^{-5}$eV$^2$] | (6.99-8.18) |
| $\Delta m^2_{atm}$ [10$^{-3}$eV$^2$] | (2.19-2.62) (NH); (2.17-2.61) (IH) |
| $\sin^2\theta_{13}$ | (1.69-3.13) (NH); (1.71-3.15) (IH) |
| $\sin^2\theta_{12}$ | (2.59-3.59) |
| $\sin^2\theta_{23}$ | (3.31-6.37) (NH); (3.35-6.63) (IH) |

Table 2: Current data for neutrino mixing parameters from global fits [21].

While carrying out the analysis, the magnitudes of atmospheric and solar neutrino mass square differences, defined as $m_2^2 - m_1^2$ and $m_3^2 - \frac{(m_1^2+m_2^2)}{2}$ respectively, are allowed full variation within their $3\sigma$ ranges. The lightest neutrino mass, $m_1$ for the case of normal hierarchy (NH) and $m_3$ for the case of inverted hierarchy (IH), is considered as the free parameter while the other two masses are obtained using the following relations,

$$
NH: \quad m_2^2 = \Delta m^2_{sol} + m_1^2, \quad m_3^2 = \Delta m^2_{atm} + \frac{(m_1^2+m_2^2)}{2},
$$

(11)

$$
IH: \quad m_2^2 = \frac{2(m_3^2+\Delta m^2_{atm})+\Delta m^2_{sol}}{2}, \quad m_1^2 = \frac{2(m_3^2+\Delta m^2_{atm})-\Delta m^2_{sol}}{2}.
$$

(12)
It should be noted that while carrying out analyses of different texture specific mass matrices, we have also imposed the condition of ‘naturalness’ [9] so as to keep the quark-lepton similarity in this regards. Further, the phases $\phi_1 = \alpha_{\nu D} - \alpha_l$, $\phi_2 = \beta_{\nu D} - \beta_l$ and the elements $D_{l,\nu}$, $C_{l,\nu}$ are considered to be free parameters. In the absence of any constraint on the phases, $\phi_1$ and $\phi_2$ have been given full variation from 0 to $2\pi$. Although $D_{l,\nu}$ and $C_{l,\nu}$ are free parameters, however, they have been constrained such that diagonalizing transformations $O_l$ and $O_\nu$ always remain real.

Before presenting the results, we would like to mention that unlike the quark case, wherein it has been shown that texture 4 zero Fritzsch like matrices are perhaps the only compatible matrices with data [9, 10], [22]-[25], in the case of leptons, we cannot arrive at this kind of conclusion. In the sequel, we present an overview of the viability of different textures for Dirac as well as Majorana nature of neutrinos.

4 Viable texture specific lepton mass matrices

In the context of quarks it is well known that texture 6 zero mass matrices are completely ruled out by the existing data. Interestingly, in case we consider Dirac like neutrinos, texture 6 zero or minimal texture is also ruled out for normal/ inverted hierarchy and degenerate scenario of neutrino masses. However, for Majorana neutrinos inverted hierarchy and degenerate scenario are ruled out whereas in the case of normal hierarchy, there are several compatible combinations with the current neutrino oscillation data. For a detailed discussion, we refer the readers to [10].

Coming to the cases of non-minimal textures, i.e., the texture 5 zero and texture 4 zero mass matrices, we present our conclusions from our recent analyses [17], [18]. To begin with, we first discuss the texture 5 zero and texture 4 zero lepton mass matrices for the case of Dirac neutrinos. Corresponding to this, a detailed and comprehensive analysis has been carried out for normal/ inverted hierarchy and degenerate scenario of neutrino masses. In this context, for texture 5 zero mass matrices, the analysis has been carried out for $D_l = 0$, $D_\nu \neq 0$ as well as $D_l \neq 0$, $D_\nu = 0$ cases, corresponding to all the viable classes. For class I, mentioned in Table I, inverted hierarchy is ruled out for both the cases, whereas normal hierarchy is viable for the $D_l = 0$, $D_\nu \neq 0$ case. For class II, normal hierarchy is viable for both the cases while the inverted hierarchy is ruled out for the case $D_l = 0$, $D_\nu \neq 0$. Finally, for class III we find that inverted hierarchy is viable for the case $D_l = 0$, $D_\nu \neq 0$, while the normal hierarchy is compatible with the $D_l \neq 0$, $D_\nu = 0$ case. It may be mentioned that Class IV is not phenomenologically viable due to de-coupling of one of the generations.

Coming to the texture 4 zero case, due to the availability of an additional parameter large number of viable possibilities emerge. Without getting into the details of these possibilities, we would like to mention only broad conclusions in this regard. Interestingly, unlike the case of texture 6 zero mass matrices, both inverted hierarchy and degenerate scenario are not ruled out for all the classes of texture specific mass matrices mentioned in Table 1. For the case of normal hierarchy, it seems mass matrices corresponding to all the classes are compatible with the data. However,
inverted hierarchy is ruled out for Class I but compatible with Class II and Class III. Similarly, degenerate scenario of neutrino masses is compatible only with Class III.

Coming to the case of texture 5 zero and texture 4 zero mass matrices for neutrinos being Majorana particles. To begin with, we consider texture 5 zero lepton mass matrices, for both the cases, viz. $D_l = 0, D_\nu \neq 0$ as well as $D_l \neq 0, D_\nu = 0$ for matrices mentioned in Class II and Class III of Table 1. For class II, normal hierarchy is viable for both the cases, while the inverted hierarchy seems to be ruled out for the case, $D_l = 0, D_\nu \neq 0$. Finally, for texture 5 zero mass matrices pertaining to class III, we find that inverted hierarchy is viable for the case $D_l \neq 0, D_\nu = 0$, while the normal hierarchy is compatible with the $D_l = 0, D_\nu \neq 0$ case.

It may be mentioned that the number of viable possibilities is understandably quite large. The analysis reveals that the Fritzsch like texture two zero lepton mass matrices are compatible with the recent lepton mixing data pertaining to normal as well as inverted neutrino mass hierarchies. Interestingly, one finds that both the normal as well as inverted neutrino mass hierarchies are compatible with texture four zero mass matrices pertaining to class II and III of Table 1 contrary to the case for texture four zero mass matrices pertaining to class I wherein inverted hierarchy seems to be ruled out. Interestingly for classes I and II, the degenerate neutrino mass scenario is incompatible, whereas it is compatible for mass matrices in Class III.

It is interesting to add that in the context of quarks, it has been recently shown [25] that texture 4 zero Fritzsch like mass matrices and their permutations, compatible with the Weak Basis transformations, provides a unique texture in agreement with the data. In case of leptons also, we have seen that texture 4 zero matrices are compatible with data for Dirac as well as Majorana neutrinos. Therefore, we can conclude that Fritzsch like texture 4 zero matrices may provide vital clues for the fundamental theories of flavor physics.

5 Summary and conclusions

A broad based survey of the texture specific lepton mass matrices has been presented. It seems that in the case of Dirac neutrinos, texture 6 zero mass matrices are ruled out. However, this is not true in the case of Majorana neutrinos. Lesser than texture 6 zeros, we find compatibility of the mass matrices with data for both the kind of neutrinos and for all kind of neutrino mass hierarchies.

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