\( \Xi_{bb} \) and \( \Omega_{bbb} \) molecular states

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Abstract

Using as a source of interaction the vector exchange of the local hidden gauge approach that in the light quark sector generates the chiral Lagrangians, and has produced realistic results for \( \Omega_c, \Xi_c, \Xi_b \) and hidden charm pentaquark states, we study the interaction of meson-baryon coupled channels that leads to \( \Xi_{bb} \) and \( \Omega_{bbb} \) excited states of molecular type. We obtain seven states of \( \Xi_{bb} \) type with energies between 10408 MeV and 10869 MeV and one \( \Omega_{bbb} \) state at 15212 MeV.

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I. INTRODUCTION

Doubly and triply heavy baryons are the subject of continuous theoretical attention [1–3] which has intensified with the recent finding of a $\Xi_{cc}^{++}$ state at LHCb [4]. $\Xi_{bb}$ and $\Omega_{bbb}$ states have not yet been found, but are likely to be observed in the future in LHCb or Belle II facilities. It is, thus, most opportune to make theoretical predictions before experiments are done. Concerning $\Xi_{bb}$ and $\Omega_{bbb}$ states, most of the theoretical work concentrates on quark model calculations with three quarks [5–9], but calculations with pentaquark configurations are already available in Refs. [10, 11], where one can also find a thorough survey of earlier work on quark models for doubly and triply heavy baryon states. Suggestions to observe these states looking at weak decay products have been done in Refs. [12, 13] and in $e^+e^-$ colliders in Ref. [14]. Yet, molecular states of these types from meson-baryon interaction have not been investigated so far, and this is the purpose of the present work.

Molecular states of meson-baryon type bound by the strong interaction of the mesons and baryons in coupled channels are peculiar. While there can be states bound by several tens of MeV, there are others which are very close to the thresholds of some meson-baryon states. Assume to begin with that we have just one meson-baryon channel bound with a small binding energy $B$. The coupling of this state to the meson-baryon component, $g$, is such that $-g^2 \frac{\partial G}{\partial E} \bigg|_{E_B} = 1$ ($E_B$, energy of the bound state), where $G$ is the meson-baryon loop function such that the scattering matrix is given by $T = V + VGT$. This function has a cusp at threshold of the meson-baryon channel, such that its derivative to the left is infinite at the threshold. Thus $g^2 \to 0$ as the binding $B$ goes to zero [15]. This can be derived from another perspective and is known as the Weinberg compositeness condition [16, 17], and $g^2 \sim \sqrt{B}$. What is less known is that when one has coupled channels if the bound state gets close to the threshold of one of the coupled channels, the couplings of the bound state to all the channels go to zero [15, 18]. As a consequence, the decay widths for the given channels, proportional to $g_i^2$, go to zero and one obtains automatically very narrow widths. This property, so naturally obtained with molecular states, is a source of permanent problems in three quarks or tight pentaquark models for these states [19].

A clear situation favoring molecular states is the recent finding of three narrow pentaquark states by the LHCb collaboration close to the $\Sigma_c \bar{D}, \Sigma_c \bar{D}^*$ thresholds [20], which have been interpreted in a large number of papers as molecular states [21–35]. This is also the case of
the work of Ref. [36] updating the earlier predictions made in Ref. [37]. The same molecular model has been successful predicting three of the narrow $\Omega_c$ states of LHCb [38] in Refs. [39–41] and some $\Xi_c$ states reported in the PDG [42] and the $\Xi_b(6227)$ observed by the LHCb collaboration [43] in Ref. [44]. Predictions for the yet not observed $\Xi_{bc}$ states are done in Ref. [45].

We use here the same source of interaction that has been tested successfully in former cases and make predictions for $\Xi_{bb}$ and $\Omega_{bbb}$ states.

**II. FORMALISM**

In order to understand the classification of the states of the meson-baryon channels that we consider, it is convenient to begin with the source of interaction that we use. Let us look at the $B^−\Lambda_b \to B^−\Lambda_b$ transition for example, which we depict in Fig. 1. By means of the mechanism of Fig. 1(b), one can exchange a $u\bar{u}$ state between the $B^−$ and the $\Lambda_b$. This could physically correspond to a $\pi$ or also $\rho, \omega$. One can equally exchange a $b\bar{b}$ pair which could correspond to $\eta_b$ or $\Upsilon$, but with this exchange corresponding to a meson propagator we can anticipate that $\eta_b$ or $\Upsilon$ exchange would be very much suppressed because of the large mass of the $b\bar{b}$ state compared to the $u\bar{u}$ one.

The next consideration is that, should we have a $K^−$ instead of a $B^−$ in Fig. 1, one can use chiral Lagrangians to obtain the strength of the exchange mechanism. After that we recall the observation in Ref. [46] that the chiral Lagrangians can be obtained from the local hidden gauge Lagrangians which rely upon the exchange of vector mesons [47–50]. In the $K^−\Lambda_b \to K^−\Lambda_b$ interaction we would have the same $u\bar{u}$ exchange and the $s$ quark would be a spectator, the same as in the diagram of Fig. 1(b) where the $b$ quark is a spectator. We
can make a mapping from the \( K^- \Lambda_b \rightarrow K^- \Lambda_b \) interaction to the \( B^- \Lambda_b \rightarrow B^- \Lambda_b \) interaction at the quark level, taking into account that when writing the \( S \) matrix at the meson level the normalization factors of the meson fields \( \frac{1}{\sqrt{2E_K}}, \frac{1}{\sqrt{2E_B}} \) are different. These considerations are done in Ref. [51].

The next point in the evaluation of the diagrams is that instead of Lagrangians one can use operators at the quark level, both in the upper vertex \( BBV \) [52] and at the lower vertex \( \Lambda_b \Lambda_b V \) with \( V \) the vector meson exchanged [40], to get the same results as with chiral Lagrangians. The result of all these considerations is that we can use for practical reasons the Lagrangian for the upper vertex

\[
\mathcal{L} = -ig \langle [P, \partial_\mu P] V^\mu \rangle, \tag{1}
\]

with \( \langle \cdots \rangle \) standing for the matrix trace, \( g = \frac{M_V}{f_\pi} (M_V \sim 800 \text{MeV}, \text{a vector mass}, f_\pi = 93 \text{MeV}) \) and \( P, V \) the \( q\bar{q} \) matrices written in terms of pseudoscalar or vector mesons respectively, with the quark \( u,d,s,b \) that we have. Hence

\[
P = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{3}} \eta + \frac{1}{\sqrt{6}} \eta' & \pi^+ & K^+ & B^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{3}} \eta + \frac{1}{\sqrt{6}} \eta' & K^0 & B^0 \\
K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}} \eta + \sqrt{\frac{2}{3}} \eta' B_s^0 & \eta_b \\
B^- & B^0 & B_s^0 & \eta_b
\end{pmatrix}, \tag{2}
\]

\[
V = \begin{pmatrix}
\frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{2}} \omega & \rho^+ & K^{*+} & B^{*+} \\
\rho^- & -\frac{1}{\sqrt{2}} \rho^0 + \frac{1}{\sqrt{2}} \omega & K^{*0} & B^{*0} \\
K^{*-} & K^{*0} & \phi & B_s^{*0} \\
B^{*-} & \bar{B}^{*0} & \bar{B}_s^{*0} & \Upsilon
\end{pmatrix}, \tag{3}
\]

where we use the \( \eta-\eta' \) mixing of Ref. [53]. The lower vertex is of the type \( V_\nu \gamma^\nu \) and we make the approximation where the momenta of the particles are small compared to their masses and \( \gamma^\nu \rightarrow \gamma^0 \equiv 1 \), rendering the interaction spin independent. This means that upon contraction of \( V^\nu V^\nu \) only the \( \partial_0 \) component of \( \partial_\mu \) in Eq. (1) is operative. The lower vertex is still evaluated at the quark level and the Lagrangian is trivial in terms of operators,

\[
\mathcal{L} \rightarrow \begin{cases}
\frac{g}{\sqrt{2}} (u\bar{u} - d\bar{d}), & \text{for } \rho^0 \\
\frac{g}{\sqrt{2}} (u\bar{u} + d\bar{d}), & \text{for } \omega
\end{cases} \tag{4}
\]

and so on, which has to be sandwiched between the baryon wave functions. The next step to complete the program is to write the wave functions, and here we divert from using SU(4) or
other extensions of SU(5) because the heavy quarks are not identical particles to the \( u, d, s \) quarks. Then we single out the heavy quark and impose the symmetry of flavor-spin on the light quarks. If we instead have two or three \( b \) quarks, then we impose the flavor-spin symmetry on the \( b \) quarks. This said, if we look for meson-baryon states with two \( b \) quarks we have the possibilities:

1) \( bb \) in the baryon,

\[ \Xi_{bb}, \ 
\Omega_{bb}, \]

and the meson-baryon states are

\[ \pi \Xi_{bb}, \ 
\eta \Xi_{bb}, \ 
K \Omega_{bb}. \quad (5) \]

Since we have two identical \( b \) quarks, the spin wave function has to be symmetric in these quarks. We take them as number 1 and 2, and thus we must use the mixed symmetric spin wave function \( \chi_{MS} \) in the first two quarks.

2) One \( b \) quark in the baryon and one in the meson. We have the meson-baryon states

\[ \bar{B} \Lambda_b, \ 
\bar{B} \Sigma_b, \ 
\bar{B}_s \Xi_b, \ 
\bar{B}_s \Xi'_b. \quad (6) \]

In this case the flavor-spin symmetry is imposed on the second and third (light) quarks. The baryon states are classified as shown in Table I.

Table I. Wave functions for baryons with \( J^P = \frac{1}{2}^+ \) and \( I = 0, \frac{1}{2}, 1 \). \( MS \) and \( MA \) stand for mixed symmetric and mixed antisymmetric, respectively.

| States   | \( I, J \) | Flavor | Spin  |
|----------|-----------|--------|-------|
| \( \Xi^0_{bb} \) | \( \frac{1}{2}, \frac{1}{2} \) | \( bbu \) | \( \chi_{MS}(12) \) |
| \( \Omega^-_{bb} \) | 0, \( \frac{1}{2} \) | \( bbs \) | \( \chi_{MS}(12) \) |
| \( \Lambda^0_b \) | 0, \( \frac{1}{2} \) | \( b \frac{1}{\sqrt{2}}(ud - du) \) | \( \chi_{MA}(23) \) |
| \( \Sigma^0_b \) | 1, \( \frac{1}{2} \) | \( b \frac{1}{\sqrt{2}}(ud + du) \) | \( \chi_{MS}(23) \) |
| \( \Xi^0_b \) | \( \frac{1}{2}, \frac{1}{2} \) | \( b \frac{1}{\sqrt{2}}(us - su) \) | \( \chi_{MA}(23) \) |
| \( \Xi^{0'}_b \) | \( \frac{1}{2}, \frac{1}{2} \) | \( b \frac{1}{\sqrt{2}}(us + su) \) | \( \chi_{MS}(23) \) |
We need the $\chi_{MS}(12)$, $\chi_{MS}(23)$ and $\chi_{MA}(23)$ which are given by Ref. [54] for $s_3 = \frac{1}{2}$,

$$\chi_{MS}(12) = \frac{1}{\sqrt{6}}(\uparrow\uparrow\uparrow + \downarrow\uparrow\uparrow - 2 \uparrow\uparrow\downarrow), \quad (7)$$

$$\chi_{MS}(23) = \frac{1}{\sqrt{6}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow - 2 \downarrow\uparrow\uparrow), \quad (8)$$

$$\chi_{MA}(23) = \frac{1}{\sqrt{2}}(\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow). \quad (9)$$

Note that $\Xi_{bb}$ and $\Omega_{bb}$ can have spin overlap with the other baryons components of Table I with our spin independent interaction, since

$$\langle \chi_{MS}(12)|\chi_{MS}(23) \rangle = -\frac{1}{2}, \quad (10)$$

$$\langle \chi_{MS}(12)|\chi_{MA}(23) \rangle = -\frac{\sqrt{3}}{2}. \quad (11)$$

We shall also consider vector-baryon states and pseudoscalar combinations with baryons of $J^P = \frac{3}{2}^+$, $\Xi_{bb}^*$, $\Omega_{bb}^*$, $\Sigma_b^*$, $\Xi_b^*$ which we describe in Table II. They all have the full symmetric spin wave function $\chi_S$,

$$\chi_S(s_3 = 1) = \uparrow\uparrow\uparrow. \quad (12)$$

| States | $I, J$ | Flavor | Spin |
|--------|--------|--------|------|
| $\Xi_{bb}^*$ | $\frac{1}{2}, \frac{3}{2}$ | $b_{bu}$ | $\chi_S$ |
| $\Omega_{bb}^*$ | $0, \frac{3}{2}$ | $b_{bs}$ | $\chi_S$ |
| $\Sigma_b^*$ | $1, \frac{3}{2}$ | $b_{\frac{1}{\sqrt{2}}(ud + du)}$ | $\chi_S$ |
| $\Xi_b^*$ | $\frac{1}{2}, \frac{3}{2}$ | $b_{\frac{1}{\sqrt{2}}(us + su)}$ | $\chi_S$ |

The combination of vector-baryon($\frac{3}{2}^+$) gives rise to states in a region difficult to identify experimentally [36] and we do not study them.

For the case that we have vector-baryon interaction, the upper vertex is evaluated using Eq. (1) substituting $[P, \partial_{\mu}P]$ by $[V_{\nu}, \partial_{\mu}V^{\nu}]$, and in the limit of small momenta $V^\mu$ of Eq. (1) is the exchanged vector. Hence, the interaction is calculated in the same way as for pseudoscalars except that there is an extra $\vec{e} \cdot \vec{e}'$ factor which states the spin independence of the upper vertex [40], hence all the interaction is spin independent. This feature of the interaction allows us to classify the channels into different blocks:
a) $\pi \Xi_{bb}$, $\eta \Xi_{bb}$ and $K \Omega_{bb}$ with $\chi_{MS}(12)$, $\bar{B} \Lambda_b$ and $\bar{B}_s \Xi_b$ with $\chi_{MA}(23)$;

b) $\pi \Xi_{bb}$, $\eta \Xi_{bb}$ and $K \Omega_{bb}$ with $\chi_{MS}(12)$, $\bar{B} \Sigma_b$ and $\bar{B}_s \Xi'_b$ with $\chi_{MS}(23)$;

c) $\rho \Xi_{bb}$, $\omega \Xi_{bb}$ and $K^* \Omega_{bb}$ with $\chi_{MS}(12)$, $\bar{B}^* \Lambda_b$ and $\bar{B}_s^* \Xi_b$ with $\chi_{MA}(23)$;

d) $\rho \Xi_{bb}$, $\omega \Xi_{bb}$ and $K^* \Omega_{bb}$ with $\chi_{MS}(12)$, $\bar{B}^* \Sigma_b$ and $\bar{B}_s^* \Xi_b$ with $\chi_{MS}(23)$;

e) $\pi \Xi_{bb}^*$, $\eta \Xi_{bb}^*$, $K \Omega_{bb}^*$, $\bar{B} \Sigma_b^*$ and $\bar{B}_s \Xi_b^*$ with all states in $\chi_{S}$.

Taking into account our isospin phase convention ($-\pi^+, \pi^0, \pi^-$), ($B^+, B^0$), ($\bar{B}^0, -B^-$), ($K^+, K^0$), ($\bar{K}^0, -K^-$), we can construct our isospin wave function for the former blocks to have isospin $I = \frac{1}{2}$ for the global “$\Xi_{bb}$” states and using the interaction of vector exchange discussed above, we obtain an interaction of the type

$$V_{ij} = D_{ij} \frac{1}{4f_{\pi}^2} (k^0 + k'^0),$$

(13)

where, $k^0, k'^0$ are the energies of the mesons and $D_{ij}$ are the coefficients which are shown in the tables below.

Note that since $\chi_{MS}(12)$ of the $\Xi_{bb}$, $\Omega_{bb}$ states has overlap with $\chi_{MS}(23)$ and $\chi_{MA}(23)$ the blocks a) and b) can mix and then have to be put together. The same can be said about the blocks c) and d) which also have to be put together. Then we obtain the $D_{ij}$ coefficients shown in Tables IV, VI, VIII (note that we change the order to baryon-meson in the tables, which must be taken into account when constructing the isospin wave functions). In Tables III, V, VII we show the thresholds of the channels considered. The masses not tabulated in the PDG [42] are taken from Ref. [55]. In the tables we have some terms that go with the parameter $\lambda$. They correspond to transitions that require $B^*$ exchange. Because of the large mass of the $B^*$ compared to the light vectors, these terms are very much suppressed.

With the same considerations as in Ref. [40], we can estimate $\lambda$ as

$$\lambda = \frac{-m_{\pi}^2}{(m_B - m_{\eta})^2 - m_{B^*}^2} \approx 0.1.$$  

(14)

We should note that when we exchange light vector mesons, the heavy quarks are spectators and, hence, automatically these terms fulfill the rules of heavy quark symmetry. The exchange of $B^*$ makes the $b$ quark active. This term goes barely as $O(\frac{1}{m_{Q}})$ (with $m_{Q}$ the heavy quark mass) and is not subject to the heavy quark spin symmetry rules. Note that these terms are very small in our approach, as expected.
TABLE III. Channels considered for sector $J^P = \frac{1}{2}^-$. 

| Channel | $\Xi_{bb} \pi$ | $\Xi_{bb} \eta$ | $\Omega_{bb} K$ | $\Lambda_b \bar{B}$ | $\Sigma_b \bar{B}$ | $\Xi_b \bar{B}_s$ | $\Xi'_b \bar{B}_s$ |
|---------|----------------|----------------|----------------|-------------------|-----------------|-----------------|-----------------|
| Threshold (MeV) | 10335 | 10745 | 10756 | 10899 | 11092 | 11160 | 11302 |

TABLE IV. Coefficients for sector $J^P = \frac{1}{2}^-$. 

| $J^P = \frac{1}{2}^-$ | $\Xi_{bb} \pi$ | $\Xi_{bb} \eta$ | $\Omega_{bb} K$ | $\Lambda_b \bar{B}$ | $\Sigma_b \bar{B}$ | $\Xi_b \bar{B}_s$ | $\Xi'_b \bar{B}_s$ |
|----------------------|----------------|----------------|-------------------|-------------------|-----------------|-----------------|-----------------|
| $\Xi_{bb} \pi$       | -2             | 0              | $\sqrt{\frac{2}{3}}$ | $\frac{3}{4} \lambda$ | $-\frac{1}{4} \lambda$ | 0               | 0               |
| $\Xi_{bb} \eta$      | 0              | -2 $\sqrt{\frac{2}{3}}$ | $\frac{1}{2} \sqrt{2} \lambda$ | $\frac{1}{2} \sqrt{2} \lambda$ | $-\frac{1}{2} \sqrt{2} \lambda$ | $\frac{1}{2} \sqrt{6} \lambda$ |
| $\Omega_{bb} K$      | -1             | 0              | 0                 | $\sqrt{\frac{3}{8}} \lambda$ | $-\frac{1}{2} \sqrt{2} \lambda$ | |
| $\Lambda_b \bar{B}$  | -1             | 0              | 0                 | -1                | 0               | 0               | 0               |
| $\Sigma_b \bar{B}$   | -3             | 0              | $\sqrt{3}$        | 0                 | 0               | 0               | 0               |
| $\Xi_b \bar{B}_s$    | -1             | 0              | 0                 | 0                 | 0               | 0               | 0               |
| $\Xi'_b \bar{B}_s$   | -1             | 0              | 0                 | 0                 | 0               | 0               | 0               |

TABLE V. Channels considered for sector $J^P = \frac{1}{2}^-, \frac{3}{2}^-$. 

| Channel | $\Lambda_b \bar{B}^*$ | $\Xi_{bb} \rho$ | $\Xi_{bb} \omega$ | $\Sigma_b \bar{B}^*$ | $\Omega_{bb} K^*$ | $\Xi_b \bar{B}_s^*$ | $\Xi'_b \bar{B}_s^*$ |
|---------|-------------------|-----------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Threshold (MeV) | 10945 | 10972 | 10980 | 11138 | 11156 | 11208 | 11216 | 11350 |

Next we turn our attention to the $\Omega_{bbb}$ states. Our coupled channels here are $\eta \Omega_{bbb}, B \Xi_{bb}, B_s \Omega_{bb}$. The baryon states are tabulated in Table IX. The $\eta \Omega_{bbb}, B \Xi_{bb}, B_s \Omega_{bb}$ states, with $J = \frac{3}{2}$, do not couple with the “$\Xi_{bb}$” states constructed before since they contain one more $b$ quark. We can also construct $V$-baryon ($\frac{1}{2}^+$) states coupling to “$\Omega_{bbb}$” and they are: $\omega \Omega_{bbb}, \phi \Omega_{bbb}, \bar{B}^* \Xi_{bb}, \bar{B}_s^* \Omega_{bb}$. In Table X we show the threshold masses of the pseudoscalar-baryon channels and in Table XI the $D_{ij}$ coefficients.

III. RESULTS

With the $V_{ij}$ potential of Eq. (13), we solve the Bethe-Salpeter equation in coupled channels

$$T = [1 - VG]^{-1}V,$$  

(15)
TABLE VI. Coefficients for sector $J^P = \frac{1}{2}^-, \frac{3}{2}^-$.

| $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ | $\Lambda_b \bar{B}^*$ | $\Xi_{bb} \rho$ | $\Xi_{bb} \omega$ | $\Sigma_b \bar{B}^*$ | $\Omega_{bb} K^*$ | $\Xi_b \bar{B}_s^*$ | $\Xi_{bb} \phi$ | $\Xi_b^* \bar{B}_s^*$ |
|---|---|---|---|---|---|---|---|---|
| $\Lambda_b \bar{B}^*$ | $-1$ | $\frac{3}{4} \lambda$ | $\frac{\sqrt{3}}{4} \lambda$ | $0$ | $0$ | $-1$ | $0$ | $0$ |
| $\Xi_{bb} \rho$ | $-2$ | $0$ | $-\frac{1}{4} \lambda$ | $\frac{\sqrt{3}}{2}$ | $0$ | $0$ | $0$ |
| $\Xi_{bb} \omega$ | $0$ | $\frac{\sqrt{3}}{4} \lambda$ | $-\frac{1}{\sqrt{2}}$ | $0$ | $0$ | $0$ |
| $\Sigma_b \bar{B}^*$ | $-3$ | $0$ | $0$ | $0$ | $\sqrt{3}$ |
| $\Omega_{bb} K^*$ | $-1$ | $\frac{1}{\sqrt{2}} \lambda$ | $1$ | $-\frac{1}{2\sqrt{2}} \lambda$ |
| $\Xi_b \bar{B}_s^*$ | $-1$ | $\frac{3}{4} \lambda$ | $0$ |
| $\Xi_{bb} \phi$ | $0$ | $-\frac{1}{2\sqrt{2}} \lambda$ |
| $\Xi_b^* \bar{B}_s^*$ | $-1$ |

TABLE VII. Channels considered for sector $J^P = \frac{3}{2}^-$.  

| Channel | $\Xi_{bb}^* \pi$ | $\Xi_{bb}^* \eta$ | $\Omega_{bb}^* K$ | $\Sigma_b^* \bar{B}$ | $\Xi_b^* \bar{B}_s$ |
|---|---|---|---|---|---|
| Threshold (MeV) | 10374 | 10784 | 10793 | 11113 | 11320 |

TABLE VIII. Coefficients for sector $J^P = \frac{3}{2}^-$.  

| $J^P = \frac{3}{2}^-$ | $\Xi_{bb}^* \pi$ | $\Xi_{bb}^* \eta$ | $\Omega_{bb}^* K$ | $\Sigma_b^* \bar{B}$ | $\Xi_b^* \bar{B}_s$ |
|---|---|---|---|---|---|
| $\Xi_{bb}^* \pi$ | $-2$ | $0$ | $\sqrt{2}$ | $\frac{1}{2} \lambda$ | $0$ |
| $\Xi_{bb}^* \eta$ | $0$ | $-\frac{2}{\sqrt{3}}$ | $-\frac{1}{\sqrt{2}} \lambda$ | $-\frac{1}{\sqrt{6}} \lambda$ |
| $\Omega_{bb}^* K$ | $0$ | $-3$ | $\sqrt{3}$ |
| $\Sigma_b^* \bar{B}$ | $-1$ | $0$ | $\frac{1}{\sqrt{2}} \lambda$ |
| $\Xi_b^* \bar{B}_s$ | $-1$ |

TABLE IX. Wave functions for baryons with $J^P = \frac{3}{2}^+$ and $I = 0, \frac{1}{2}$.  

| States | $I, J$ | Flavor | Spin |
|---|---|---|---|
| $\Omega_{bb}^-$ | $0, \frac{3}{2}^-$ | $bbb$ | $xs$ |
| $\Xi_{bb}^{*0}$ | $\frac{1}{2}, \frac{3}{2}^-$ | $bbu$ | $xs$ |
| $\Omega_{bb}^-$ | $0, \frac{3}{2}^-$ | $bbs$ | $xs$ |
TABLE X. Channels considered for sector $J^P = \frac{3}{2}^-$.

| Channel          | $\Omega_{bb} \eta$ | $\Xi_{bb} \bar{B}$ | $\Omega_{bb}^* \bar{B}_s$ |
|------------------|--------------------|--------------------|-----------------------------|
| Threshold (MeV)  | 15382              | 15515              | 15664                       |

TABLE XI. $D_{ij}$ coefficients for sector $J^P = \frac{3}{2}^-$.

| $J^P = \frac{3}{2}^-$ | $\Omega_{bb} \eta$ | $\Xi_{bb} \bar{B}$ | $\Omega_{bb}^* \bar{B}_s$ |
|------------------------|--------------------|--------------------|-----------------------------|
| $\Omega_{bb} \eta$    | 0                  | $-\frac{2}{\sqrt{6}} \lambda$ | $-\frac{1}{\sqrt{3}} \lambda$ |
| $\Xi_{bb} \bar{B}$    | $-2$               | $\sqrt{2}$         |                             |
| $\Omega_{bb}^* \bar{B}_s$ |                     |                    | $-1$                        |

where $G$ is the diagonal meson-baryon loop function given by

$$G_l = i \int \frac{d^4 q}{(2\pi)^4} \frac{M_l}{E_l(q)} \frac{1}{k^0 + p^0 - q^0 - E_l(q)} \frac{1}{q^2 - m_l^2 + i\epsilon},$$

where $\omega_l$, $E_l$ are the energies of the meson and baryon respectively, $\omega_l = \sqrt{m_l^2 + q^2}$, $E_l = \sqrt{M_l^2 + \bar{q}^2}$, $m_l$, $M_l$ the meson and baryon masses and $p^0$ is the energy of the incoming baryon in the meson-baryon rest frame. As in former studies [52, 56, 57], we use $q_{\text{max}} = 650$ MeV to regularize the loop function. The poles are searched for in the second Riemann sheet as done in Refs. [52, 56, 57] and the couplings of the states to the different channels are obtained from the residues of the $T_{ij}$ matrix at the pole, $z_R$, knowing that close to the pole one has

$$T_{ij}(s) = \frac{g_i g_j}{\sqrt{s - z_R}}.$$  \hspace{1cm} (17)

The second Riemann sheet is obtained using $G_{ll}^{II}(s)$ instead of $G(s)$ given by

$$G_{ll}^{II} = \begin{cases} 
G_l(s), & \text{for } \text{Re}(\sqrt{s}) < \sqrt{s_{th,l}} \\ 
G_l(s) + i \frac{2M_l q}{4\pi \sqrt{s}}, & \text{for } \text{Re}(\sqrt{s}) \geq \sqrt{s_{th,l}} \end{cases},$$  \hspace{1cm} (18)

where $\sqrt{s_{th,l}}$ is the threshold mass of the $l$-th channel, and

$$q = \frac{\lambda^{1/2}(s, m_l^2, M_l^2)}{2\sqrt{s}}, \quad \text{with } \text{Im}(q) > 0.$$  \hspace{1cm} (19)
In Tables XII, XIII we show the couplings and the wave function at the origin for two states with \( J = \frac{1}{2} \) that we obtain from the coupled channels of Table IV. In addition to the couplings \( g_i \), we show the values of \( g_i G_i^{II} \) at the pole which according to Ref. [15] provide the strength of the wave function of the origin.

### TABLE XII. The coupling constants to various channels and \( g_i G_i^{II} \) (in MeV).

| \( 10408.18 + i93.18 \) | \( \Xi_{bb} \pi \)  | \( \Xi_{bb} \eta \)  | \( \Omega_{bb} K \)  | \( \Lambda_b \bar{B} \) |
|----------------------------|---------------------|---------------------|---------------------|---------------------|
| \( g_i \)                  | 1.69 + i1.21        | -0.02 - i0.09       | -0.86 - i0.73       | -1.03 - i0.31       |
| \( g_i G_i^{II} \)         | -73.58 - i12.87     | 0.01 + i0.58        | 4.52 + i5.41        | 0.79 + i0.39        |
| \( \Sigma_b \bar{B} \)     | \( \Xi_b \bar{B}_s \) | \( \Xi'_b \bar{B}_s \) |
| \( g_i \)                  | 0.50 + i0.23        | -0.28 - i0.19       | -0.16 - i0.12       |
| \( g_i G_i^{II} \)         | -0.29 - i0.18       | 0.14 + i0.12        | 0.07 + i0.06        |

### TABLE XIII. The coupling constants to various channels and \( g_i G_i^{II} \) (in MeV).

| \( 10686.39 + i0.08 \) | \( \Xi_{bb} \pi \)  | \( \Xi_{bb} \eta \)  | \( \Omega_{bb} K \)  | \( \Lambda_b \bar{B} \) |
|----------------------------|---------------------|---------------------|---------------------|---------------------|
| \( g_i \)                  | 0.01 - i0.05        | -0.10 + i0.02       | -0.05 + i0.04       | 0.06 + i0.02        |
| \( g_i G_i^{II} \)         | 1.57 + i0.51        | 1.52 - i0.23        | 0.72 - i0.52        | -0.11 - i0.03       |
| \( \Sigma_b \bar{B} \)     | \( \Xi_b \bar{B}_s \) | \( \Xi'_b \bar{B}_s \) |
| \( g_i \)                  | 19.03               | 0.02 + i0.02        | -10.80             |
| \( g_i G_i^{II} \)         | -18.78              | -0.02 - i0.02       | 7.23               |

We find two states, one at 10408 MeV with about 186 MeV width, which couples mostly to the \( \Xi_{bb} \pi \) component, with a nonnegligible coupling to \( \Omega_{bb} K \) and \( \Lambda_b \bar{B} \). The large width of this state stems from the large coupling to the \( \Xi_{bb} \pi \) channel and the fact that this channel is open. The second state appears at 10686 MeV with a very small width. It couples mostly to the \( \Sigma_b \bar{B} \) channel, which is closed. The \( \Xi_{bb} \pi \) channel is open, but the coupling to this channel appears to be very small, which justifies the small width obtained.

Some of the components are quite bound and one may wonder that the sizes of these components would be very small. Yet, this is not the case, since, as shown in detail in Ref. [45] the size of the channels is not tied to the binding but is determined by the cut-off, and \( r^2|\psi(r)|^2 \) peaks around 0.7 fm, with still a sizeable strength around 1 fm.
Next we turn our attention to the states generated from the coupled channels of Table VI, from vector-baryon($\frac{1}{2}^+$) states. We find now three states with zero width, degenerate in $J^P = \frac{1}{2}^-, \frac{3}{2}^-$. We should note that the additional consideration of pion exchange would break this degeneracy but, as discussed in Ref. [45], their effects are largely incorporated in our approach through a suitable choice of $q_{\text{max}}$, and only a small remnant part remains to produce a small splitting between the $\frac{1}{2}^-$ and $\frac{3}{2}^-$ states. The small difference of masses between the hidden charm pentaquark states $P_c(4440)$ and $P_c(4452)$ of Ref. [20], assumed to be $\frac{1}{2}^-, \frac{3}{2}^-$ respectively, comes to corroborate this fact.

In Tables XIV, XV and XVI, we show the properties of these three states. The first state appears at 10732 MeV and couples mostly to $\Sigma_b \bar{B}^*$. The second state shows up at 10807 MeV and couples mostly to $\Lambda_b \bar{B}^*$. The third state appears at 10869 MeV and couples mostly to $\Xi_{bb} \rho$. Note that all the channels are closed and this is why we obtain a zero width.

| 10732.01 | $\Lambda_b \bar{B}^*$ | $\Xi_{bb} \rho$ | $\Xi_{bb} \omega$ | $\Sigma_b \bar{B}^*$ |
|-----------|-----------------|----------------|----------------|-----------------|
| $g_i$     | $-0.01$         | $0.15$         | $-0.14$        | $19.13$         |
| $g_i G^{II}_i$ | $0.02$ | $-1.01$ | $0.88$ | $-18.72$ |
| $\Omega_{bb} K^*$ | $\Xi_b \bar{B}^*_s$ | $\Xi_{bb} \phi$ | $\Xi'_b \bar{B}^*_s$ |
| $g_i$     | $-0.10$         | $0$            | $-0.03$        | $-10.86$        |
| $g_i G^{II}_i$ | $0.42$ | $0$ | $0.09$ | $7.18$ |

| 10807.41 | $\Lambda_b \bar{B}^*$ | $\Xi_{bb} \rho$ | $\Xi_{bb} \omega$ | $\Sigma_b \bar{B}^*$ |
|-----------|-----------------|----------------|----------------|-----------------|
| $g_i$     | $7.82$          | $-0.66$        | $-0.12$        | $0.06$          |
| $g_i G^{II}_i$ | $-18.77$ | $5.52$ | $0.97$ | $-0.07$ |
| $\Omega_{bb} K^*$ | $\Xi_b \bar{B}^*_s$ | $\Xi_{bb} \phi$ | $\Xi'_b \bar{B}^*_s$ |
| $g_i$     | $0.16$          | $7.57$         | $-0.10$        | $-0.04$         |
| $g_i G^{II}_i$ | $-0.76$ | $-7.37$ | $0.39$ | $0.03$ |

Next we turn our attention to the states formed from the pseudoscalar-baryon($\frac{3}{2}^+$) channels of Table VIII. We find two states that we show in Tables XVII and XVIII. The
first one appears at 10447 MeV with about 186 MeV width. This state couples mostly to \( \Xi_{bb}^* \pi \), which is open, justifying, thus, the large width. The second state appears at 10707 MeV and couples mostly to \( \Sigma_b^* \bar{B} \). The \( \Xi_{bb}^* \pi \) channel is open, but the small coupling to this channel makes the width of this state very small.

Finally we look at the only “\( \Omega_{bbb}^* \)” state that we find from the coupled channels of Table X. The state is found at 15212 MeV and couples mostly to \( \Xi_{bb}^* \bar{B} \), as shown in Table XIX. All coupled channels are closed and we obtain zero width for this state.

In summary we have obtained two excited \( \Xi_{bb} \) states with \( J^P = \frac{1}{2}^- \) coupled to pseudoscalar-baryon(\( \frac{1}{2}^+ \)) channels, three states with \( J^P = \frac{1}{2}^- , \frac{3}{2}^- \), degenerate in our approach, coupled to vector-baryon(\( \frac{1}{2}^+ \)) channels, two states of \( J^P = \frac{3}{2}^- \) coupled to pseudoscalar-baryon(\( \frac{3}{2}^+ \)) channels, and found only one state corresponding to an excited \( \Omega_{bbb} \) state, coupled to pseudoscalar-baryon(\( \frac{3}{2}^+ \)) channels.
TABLE XIX. The coupling constants to various channels and $g_i G_{i}^{II}$ (in MeV).

| $15212.04$ | $\Omega_{bb} \eta$ | $\Xi_{bb} B$ | $\Omega_{bb}^* B_s$ |
|------------|-------------------|--------------|-------------------|
| $g_i$      | 0.15              | 14.03        | -9.82             |
| $g_i G_{i}^{II}$ | -1.44          | -18.31       | 8.80              |

IV. CONCLUSIONS

We have carried out a study of the interaction of meson-baryon coupled channels that leads to the formation of some bound or resonant states, corresponding to excited $\Xi_{bb}$ and $\Omega_{bb}$ states. As in related studies of $\Xi_c$, $\Xi_b$, $\Xi_{bc}$ and hidden charm molecular states, we have used an interaction based on the exchange of vector mesons, which in the case of light quarks gives rise to the chiral Lagrangians. In particular, the exchange of light vectors, which produces the dominant part of the interaction, leaves the heavy quarks as spectators and fulfills the rules of heavy quark symmetry. We find seven states of $\Xi_{bb}$ nature and one state of $\Omega_{bb}$ nature. The success in describing the hidden charm pentaquark states, plus some $\Omega_c$, $\Xi_c$, $\Xi_b$ states using the same input for the interaction, makes us confident that the predictions made are realistic and it will be interesting to contrast them with future measurements likely to be done at LHCb and Belle II.

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[1] S. L. Olsen, T. Skwarnicki and D. Zieminska, Rev. Mod. Phys. 90, no. 1, 015003 (2018).
[2] M. Karliner, J. L. Rosner and T. Skwarnicki, Ann. Rev. Nucl. Part. Sci. 68, 17 (2018).
[3] Y. R. Liu, H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Prog. Part. Nucl. Phys. 107, 237 (2019).
[4] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 119, no. 11, 112001 (2017).
[5] T. D. Cohen and P. M. Hohler, Phys. Rev. D 74, 094003 (2006).
[6] Q. X. Yu and X. H. Guo, Nucl. Phys. B 947, 114727 (2019).
[7] J. M. Flynn, E. Hernandez and J. Nieves, Phys. Rev. D 85, 014012 (2012).
[8] X. Z. Weng, X. L. Chen and W. Z. Deng, Phys. Rev. D 97, no. 5, 054008 (2018).
[9] Z. Shah and A. K. Rai, Eur. Phys. J. C 77, no. 2, 129 (2017).
[10] Q. S. Zhou, K. Chen, X. Liu, Y. R. Liu and S. L. Zhu, Phys. Rev. C 98, no. 4, 045204 (2018).
[11] S. Y. Li, Y. R. Liu, Y. N. Liu, Z. G. Si and J. Wu, Eur. Phys. J. C 79, no. 1, 87 (2019).
[12] Y. J. Shi, W. Wang, Y. Xing and J. Xu, Eur. Phys. J. C 78, no. 1, 56 (2018).
[13] W. Wang and J. Xu, Phys. Rev. D 97, no. 9, 093007 (2018).
[14] X. C. Zheng, C. H. Chang and Z. Pan, Phys. Rev. D 93, no. 3, 034019 (2016).
[15] D. Gamermann, J. Nieves, E. Oset and E. Ruiz Arriola, Phys. Rev. D 81, 014029 (2010).
[16] S. Weinberg, Phys. Rev. 137, B672 (1965).
[17] V. Baru, J. Haidenbauer, C. Hanhart, Y. Kalashnikova and A. E. Kudryavtsev, Phys. Lett. B 586, 53 (2004).
[18] H. Toki, C. Garcia-Recio and J. Nieves, Phys. Rev. D 77, 034001 (2008).
[19] A. Pilloni, talk at the MIAPP Workshop, Munchen, October 2019. 
   http://www.munich-iapp.de/programmes-topical-workshops/2019/hadron-spectroscopy/daily-schedule-neu/?
[20] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 122, no. 22, 222001 (2019).
[21] H. X. Chen, W. Chen and S. L. Zhu, Phys. Rev. D 100, no. 5, 051501 (2019).
[22] M. Z. Liu, Y. W. Pan, F. Z. Peng, M. Sanchez Sanchez, L. S. Geng, A. Hosaka and M. Pavon Valderrama, Phys. Rev. Lett. 122, no. 24, 242001 (2019) doi:10.1103/PhysRevLett.122.242001  
   [arXiv:1903.11560 [hep-ph]].
[23] J. He, Eur. Phys. J. C 79, no. 5, 393 (2019).
[24] R. Chen, Z. F. Sun, X. Liu and S. L. Zhu, Phys. Rev. D 100, no. 1, 011502 (2019).
[25] J. R. Zhang, arXiv:1904.10711 [hep-ph].
[26] L. Meng, B. Wang, G. J. Wang and S. L. Zhu, Phys. Rev. D 100, no. 1, 014031 (2019).
[27] M. B. Voloshin, Phys. Rev. D 100, no. 3, 034020 (2019).
[28] Y. Yamaguchi, H. Garca-Tecocoatzi, A. Giachino, A. Hosaka, E. Santopinto, S. Takeuchi and M. Takizawa, arXiv:1907.04684 [hep-ph].
[29] Z. G. Wang and X. Wang, arXiv:1907.04582 [hep-ph].
[30] M. Pavon Valderrama, arXiv:1907.05294 [hep-ph].
[31] Y. J. Xu, C. Y. Cui, Y. L. Liu and M. Q. Huang, arXiv:1907.05097 [hep-ph].
[32] M. Z. Liu, T. W. Wu, M. Sanchez Sanchez, M. P. Valderrama, L. S. Geng and J. J. Xie, arXiv:1907.06093 [hep-ph].
[33] T. J. Burns and E. S. Swanson, arXiv:1908.03528 [hep-ph].
[34] Y. H. Lin and B. S. Zou, Phys. Rev. D 100, no. 5, 056005 (2019).
[35] Y. Yamaguchi, A. Hosaka, S. Takeuchi and M. Takizawa, arXiv:1908.08790 [hep-ph].
[36] C. W. Xiao, J. Nieves and E. Oset, Phys. Rev. D 100, no. 1, 014021 (2019).
[37] J. J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010).
[38] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 118, no. 18, 182001 (2017).
[39] G. Montana, A. Feijoo and A. Ramos, Eur. Phys. J. A 54, no. 4, 64 (2018).
[40] V. R. Debastiani, J. M. Dias, W. H. Liang and E. Oset, Phys. Rev. D 97, no. 9, 094035 (2018).
[41] J. Nieves, R. Pavao and L. Tolos, Eur. Phys. J. C 78, no. 2, 114 (2018).
[42] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no. 3, 030001 (2018).
[43] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 121, no. 7, 072002 (2018).
[44] Q. X. Yu, R. Pavao, V. R. Debastiani and E. Oset, Eur. Phys. J. C 79, no. 2, 167 (2019).
[45] Q. X. Yu, J. M. Dias, W. H. Liang and E. Oset, arXiv:1909.13449 [hep-ph].
[46] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B 223, 425 (1989).
[47] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. 54, 1215 (1985).
[48] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988).
[49] U. G. Meißner, Phys. Rept. 161, 213 (1988).
[50] H. Nagahiro, L. Roca, A. Hosaka and E. Oset, Phys. Rev. D 79, 014015 (2009).
[51] W. H. Liang, C. W. Xiao and E. Oset, Phys. Rev. D 89, no. 5, 054023 (2014).
[52] S. Sakai, L. Roca and E. Oset, Phys. Rev. D 96, no. 5, 054023 (2017).
[53] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B 283, 416 (1992).

[54] F. E. Close, “An Introduction to Quarks and Partons”, Academic Press, Cambridge, 1979.

[55] C. Albertus, E. Hernandez, J. Nieves and J. M. Verde-Velasco, Eur. Phys. J. A 32, 183 (2007), Erratum: [Eur. Phys. J. A 36, 119 (2008)].

[56] J. M. Dias, V. R. Debastiani, J.-J. Xie and E. Oset, Phys. Rev. D 98, no. 9, 094017 (2018).

[57] V. R. Debastiani, J. M. Dias, W. H. Liang and E. Oset, Phys. Rev. D 97, no. 9, 094035 (2018).