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Self-Similar Hot Accretion onto a Spinning Neutron Star: Matching the Outer Boundary Conditions

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1 INTRODUCTION

Accretion flows around compact objects frequently radiate significant levels of hard X-rays. This has provided strong motivation for the study of hot accretion flows. Zeldovich & Shakura (1969) and Alme & Wilson (1973) considered spherically free-falling plasma impinging on the surface of a neutron star (NS). They calculated the penetration depth of the falling protons and made preliminary estimates of the radiated spectrum. Their ideas were followed up by a number of later authors (e.g., Turolla et al. 1994; Zampieri et al. 1994; Zane et al. 1993), who computed detailed spectra. Denefel, Dullemmond, & Sprintl (2001) modified the model of Zeldovich & Shakura (1969) by considering a rotating advection-dominated accretion flow around a NS. Their model represents an improvement on the earlier work since it includes angular momentum and viscosity.

A fluid approach to hot accretion onto a NS was pioneered by Shapiro & Salpeter (1975), who worked out the structure of the standing shock in a spherical flow, and computed the two-temperature structure and the resulting radiation spectrum of the post-shock gas. The equivalent problem for an accreting white dwarf was analyzed by Kylafis & Lamb (1982). In related work, Chakrabarti & Sahu (1997) described the hydrodynamics of spherical accretion onto black holes and NSs, but without including radiation processes.

The above studies involve flows in which the accreting matter crashes on the surface of the star, forming a discontinuity or a shock of some kind. Recently, Medvedev & Narayan (2001) discovered a rotating solution of the viscous fluid equations that corresponds to hot quasi-spherical accretion onto a spinning NS. Their solution is closely related to the two-temperature flow described by Shapiro, Lightman, & Eardley (1974). A feature of the MN01 solution is that the gas moves subsonically in the radial direction and merges with the accreting star without a shock. The flow essentially “settles” onto the rotating star; the solution may thus be referred to as a “hot settling flow” (hot because the gas is at the virial temperature and has a quasi-spherical morphology). MN01 showed that the accreting gas removes angular momentum from the central star and that this braking action dominates the energy equation of the accreting gas. The flow could thus also be called a “hot brake.” The hot settling flow should not be confused with the boundary layer which forms close the stellar surface (e.g., Narayan & Popham 1993), where the surface density is high and steep spatial gradients are present.

The hot settling flow forms outside the boundary layer and extends radially to a large distance, typically thousands of stellar radii or more (see MN01). Recently, Medvedev & Murray (2002), following up on earlier work by Davies & Pringle (1981), has recently described a subsonic hot accretion flow around a magnetized neutron star in the propeller state. The relation between his solution and our hot settling flow is discussed in §4.

The relevance of the hot settling flow to real systems is presently unclear, though several accreting white dwarf and black hole systems have been suggested as candidates for such a rotation-powered flow (Medvedev & Menou 2002; Medvedev & Murray 2002). While MN01 described the properties of the self-similar region of the flow, they did not discuss how to match the solution to realistic boundary conditions. The matching to an external medium at large radii is particularly problematic, since the self-similar settling flow solution has the remarkable property that the density, temperature and angular velocity of the gas at given radius depend only on the dimensionless spin of the central star, and are completely independent of the properties of the external medium. This leads to an apparently serious problem. If one tries to match the solution to the external medium by selecting the radius at which the pressure of the
solution matches that of the medium, then neither the density nor the temperature will agree. Does this mean that the solution is physically inconsistent? We argue otherwise in this paper.

We show that there exists a second self-similar solution at radii outside the MN01 hot settling flow, which acts as a bridge between the MN01 solution and the external medium. This bridging solution has an extra degree of freedom which allows it to match a general density and temperature in the external medium. Apart from the additional degree of freedom, the solution retains many of the features of the MN01 settling flow: (i) it is pressure supported, (ii) it resembles a static atmosphere at low mass accretion rates, and (iii) the angular momentum flux associated with the viscous braking of the central star dominates most of the physics.

The paper is organized as follows. In §2 we derive analytical expressions for various self-similar solutions. In §3 we present full numerical solutions and compare them with the analytical solutions. Finally, in §4 we conclude with a brief discussion.

2 ANALYTICAL SELF-SIMILAR SOLUTIONS

We consider gas accreting viscously onto a compact spinning object. The central object has a radius $R_*$, a mass $M_*$, and an angular velocity $\Omega_*$, where $\Omega_*(R) = (GM/R^3)^{1/2}$ is the Keplerian angular velocity at radius $R$. We measure the accretion rate in Eddington units, $\dot{m} = \dot{M}/\dot{M}_{\text{Edd}}$, and the radius in Schwarzschild units, $r = R/R_\text{g}$, where $\dot{M}_{\text{Edd}} = 1.39 \times 10^{18} \text{m g s}^{-1}$ (corresponding to a radiative efficiency of 10%) and $R_\text{g} = 2GM/c^2$.

We assume that the flow is hot and quasi-spherical, which generally requires a low mass accretion rate (see Narayan et al. 1997). The accreting gas has nearly the virial temperature, i.e., $\delta \sim GM/R \sim (\Omega_*(R))^2$, and the local vertical scale height $H = c_s/\Omega_*$ is comparable to the local radius $R$. We may then use the height integrated hydrodynamic equations for a steady, rotating, axisymmetric flow, and for simplicity we may set $H = R$ (see MN01). (Note that, even when $H \approx R$, the stability properties of the flow may depend on whether we use $H$ or $R$ in the equations; this is briefly discussed in §4). In the following, we closely follow the analysis of MN01, with a few changes.

For simplicity, we set $\dot{m} = 0$ and omit the continuity equation. Thus, the gas configuration corresponds to a radially static “atmosphere.” The motivation for this approximation follows from the observation that the density $\rho$, temperature $T$ and the angular velocity $\Omega$ of the gas in the MN01 self-similar solution are completely independent of $\dot{m}$. Only the radial velocity $v$ depends on $\dot{m}$, and it is given trivially by the spherical continuity equation

$$v = \frac{\dot{M}}{4\pi R^2 \rho}. \quad (1)$$

Because of this, we do not lose any generality by setting $\dot{m} = v = 0$ in the analysis; we may always introduce a finite $\dot{m}$ and finite $v$ after the fact.

We assume that the flow is highly sub-Keplerian, $\Omega(R) \ll \Omega_*(R)$, so that the centrifugal support is negligible compared to the pressure support. The radial momentum equation then takes the following simple form,

$$\frac{GM}{R^2} = -\frac{1}{\rho} \frac{d(\rho v^2)}{dR}. \quad (2)$$

where we have used the fact that $v \sim 0$ and written the pressure as $p = \rho c_s^2$ where $c_s$ is the isothermal sound speed. MN01 present a more complete analysis in which they do not assume that the rotation is slow. They then obtain an extra factor of $(1 - s^2)$ in their equation, which propagates through to all the results. Since we ignore the factor, our analysis corresponds to the case of a slowly-spinning star: $s^2 \ll 1$. This approximation is made only to simplify the analysis, and all the results may be generalized for arbitrary $s$.

From the analysis in MN01, we know that the accreting gas in our problem acts as a brake on the central spinning star and transports angular momentum outward through the action of viscosity. We therefore write the angular momentum conservation equation for the gas as follows,

$$\dot{J} = 4\pi \nu R^4 \frac{d\Omega}{dR} = \text{constant}, \quad (3)$$

where $\dot{J}$ is the outward angular momentum flux, and $\nu$ is the kinematic coefficient of viscosity. This equation is exactly valid in steady state if $\dot{m} = 0$. When $\dot{m}$ is non-zero, there is an additional term, $\dot{M}\Omega R^2$, due to the flux of angular momentum carried in by the accreting gas. The key feature of the MN01 hot brake solution is that the latter flux is negligible compared to the outward flux from the star. Equation (3) is, therefore, valid even when $\dot{m} \neq 0$, so long as $\dot{m}$ is small enough for the term $\dot{M}\Omega R^2$ to be negligible.

We employ the usual $\alpha$ prescription for the kinematic coefficient of viscosity, which we write as

$$\nu = \alpha_c c_s H \approx \alpha_c R. \quad (4)$$

Often, in accretion problems, one makes use of the relation $H = c_s/\Omega_K$ and writes $\nu = \alpha c_s^2/\Omega_K$. This prescription is equivalent to equation (4) in the regime of the MN01 hot settling flow. However, in the outer regions of the flow, where the two new solutions described in §§2.2.2.3 appear, $H$ is much less than $c_s/\Omega_K$, and $\nu > \alpha c_s^2/\Omega_K$ is not a good approximation. Equation (4) is a superior prescription and is physically better motivated over a wide range of conditions (so long as the flow is quasi-spherical).

For simplicity, we assume that the gas is one-temperature; it is straightforward to generalize to the two-temperature regime, as done in MN01, but the one-temperature analysis suffices for the present paper. Hence do not need to treat electrons and protons separately.

Viscous braking heats the accreting gas, and we assume a steady state in which this heating is balanced by cooling. The energy conservation equation thus becomes

$$q^+ = q^-, \quad (5)$$

where the viscous heating rate per unit volume $q^+$ and the radiative cooling per unit volume via bremsstrahlung $q^-$ are given by (MN01)

$$q^+ = \nu_p R^2 \left( \frac{d\Omega}{dR} \right)^2, \quad q^- = Q_{\text{h, NR}} \rho^2 \left( \frac{k T}{m_e c^2} \right)^{1/2}, \quad Q_{\text{h, NR}} = 5\sqrt{2} \pi^{-3/2} \alpha f \sigma_T m_e c^7/m_p^2$$

where $\alpha_f$ is the fine structure constant, $\sigma_T$ is the Thomson cross-section, and $m_e$ and $m_p$ are the mass of the electron.
and the proton. Equation (5) is exactly valid for a steady state flow with $\dot{m} = v = 0$. When $v \neq 0$, the energy equation has another term corresponding to the advection of energy. In advection-dominated accretion flows, for instance, this term dominates over the cooling term $q$ [Narayan et al. 1997]. In the present case, however, we consider a situation in which the advection term is negligible (which corresponds to low $\dot{m}$).

Finally, we assume that the spinning star is immersed in a uniform external medium with a density $p_{\text{ext}}$, temperature $T_{\text{ext}}$ and pressure $p_{\text{ext}}$. We seek an accretion flow solution that extends from the spinning star on the inside to the external medium on the outside. As we show below, the solution consists of two distinct self-similar regimes, plus a third asymptotic regime inside the external medium.

### 2.1 Inner self-similar solution

We first consider the inner regions of the flow, where the pressure $p \gg p_{\text{ext}}$. This is the regime of the MN01 solution, where the variables have the following radial dependences:

$$
\rho = \rho r^{-2}, \quad T = T_1 r^{-1}, \quad \Omega = \Omega_1 r^{-3/2}.
$$

(6)

The subscript “1” in the coefficients is to indicate that this is the first solution, to distinguish it from the second and third solutions described below. By substituting the above solution in equations (2), (4) and (5), we see that it satisfies the basic conservation laws. We may also solve for the numerical constants:

$$
\rho_1 = \frac{\alpha s^2 R_g^2}{2t^{2/3}} \left( \frac{m_e}{m_p} \right)^{1/2} c^3 \frac{Q_{\text{f, NR}}}{e},
$$

$$
kT_1 = \frac{m_p \rho_1^2}{12 \sqrt{2} R_g},
$$

$$
\Omega_1 = \frac{s}{\sqrt{2} R_g} = s \Omega_K(R_g).
$$

(7)

We note that if $\dot{m} \neq 0$ then the flow has a small constant radial velocity:

$$
v \propto r^0,
$$

(8)

as follows from equation (1).

The angular momentum flux in the solution is given by

$$
\dot{J} = -\alpha s^2 R_g^2 \frac{3t^{2/3}}{2t^{1/2}} \left( \frac{m_e}{m_p} \right)^{1/2} c^5 \frac{Q_{\text{f, NR}}}{e}.
$$

(9)

By assumption, this flux is much greater than the angular momentum flux due to accretion, which sets an upper limit on the mass accretion rate for the solution to be valid (see MN01). The pressure is given by

$$
p = \rho r^{-2}, \quad \rho = \rho_1 c_2^5 r^{-3} \equiv \rho_1 r^{-3},
$$

where $c_2^5 = 2kT_1/m_p$.

The above self-similar solution describes the flow at radii $r \ll (\rho_1/p_{\text{ext}})^{1/3}$, where the pressure $p \gg p_{\text{ext}}$. As mentioned in §1, the solution has the remarkable property that all the quantities are uniquely determined by a single parameter $s$ — the dimensionless spin of the central object — specified on the inner boundary. The fact that the solution does not depend on the outer boundary condition in any way means that there is no simple way to match it to the external medium. Clearly, there has to be a second solution to bridge the gap between this solution and the external medium. We derive the bridging solution in the next subsection.

#### 2.2 A second self-similar solution

We consider next the gas that lies just outside the region of validity of the first self-similar solution described above. In this zone, the pressure is expected to be approximately equal to the external pressure $p_{\text{ext}}$:

$$
\rho c_2^2 = p_{\text{ext}} = \text{constant}.
$$

(10)

This condition replaces the hydrostatic equilibrium equation (2), while equations (3) and (4) continue to be valid. In this region, we find that there is a second self-similar solution of the form

$$
\rho = \rho_2 r^{-7/2}, \quad T = T_2 r^{-7/2}, \quad \Omega = \Omega_2 r^{-9/4},
$$

(11)

where the label “2” refers to the fact that this is our second solution.

To match the second and first solutions, we require that the fluxes of angular momentum in the two solutions must be equal; this yields the constraint $(3/2)\rho_1 \Omega_1 T_1^{1/2} = (9/4) \rho_2 \Omega_2 T_2^{1/2}$. Making use of this and the other equations, we solve for the numerical coefficients in equation (11):

$$
\rho_2 = \frac{\alpha s^2 R_g^2}{p_{\text{ext}} R_g^{3/2}} \frac{3t^{1/2}}{2t^{1/2}} \left( \frac{m_e}{m_p} \right)^{3/4} c_2^4 \frac{Q_{\text{f, NR}}}{e},
$$

$$
kT_2 = \frac{3t^{1/2}}{\alpha s^2 R_g^{3/2}} \frac{2t^{1/2}}{3t^{1/2}} \left( \frac{m_e}{m_p} \right)^{3/4} m_p Q_{\text{f, NR}}^{1/2},
$$

$$
\Omega_2 = \frac{\alpha s^2 R_g^{3/4}}{p_{\text{ext}} R_g^{5/4}} \frac{2t^{3/8} t^{1/8}}{3t^{1/4} 3t^{3/4}} \left( \frac{m_e}{m_p} \right)^{1/8} c_2^2 \frac{Q_{\text{f, NR}}}{e}.
$$

The pressure in this solution is constant and equal to the external pressure, $p_{\text{ext}}$, and the angular momentum flux is also constant and is equal to $\dot{J}$ in equation (11). If the flow has a small but nonzero accretion rate, $\dot{m} \neq 0$, then its radial velocity varies as $|\dot{J}|$:

$$
v \propto r^{3/2}.
$$

(12)

Whereas the original MN01 self-similar solution has a unique profile for a given choice of $s$, we see that the second solution derived here has an extra degree of freedom, namely the external pressure $p_{\text{ext}}$. This extra degree of freedom solves the problem discussed in §1. Thus, the full solution consists of two zones: an inner zone described by the first (MN01) solution (11) and an outer zone described by the second solution (11). The radius $r_{\text{match}}$ at which the two solutions match is obtained by equating the pressures:

$$
r_{\text{match}} = \frac{\alpha^{1/3} s t^{1/3}}{p_{\text{ext}} R_g^{1/3}} \frac{2t^{1/2}}{27 t^{1/6}} \left( \frac{m_e}{m_p} \right)^{1/6} c_2^5 Q_{\text{f, NR}}^{1/3}.
$$

(13)

The second solution matches the external medium at the radius $r_{\text{ext}}$ at which its temperature matches that of the medium. This gives

$$
r_{\text{ext}} = \frac{\alpha^{3/7} s t^{6/7}}{p_{\text{ext}} (kT_{\text{ext}})^{2/7} R_g} \frac{3 t^{2/7}}{25 t^{1/4}} \left( \frac{m_e}{m_p} \right)^{3/14} m_p Q_{\text{f, NR}}^{8/7}.
$$

(14)

If we wish we could also write this in terms of the external density by making the substitution $kT_{\text{ext}} = m_p p_{\text{ext}}/2p_{\text{ext}}$. But
2.3 Asymptotic solution in the external medium

For completeness, we present here the solution inside the external medium. By assumption, the external medium has a uniform temperature and density, and a uniform rate of cooling. To maintain equilibrium, there has to be some constant source of heat that exactly compensates for the cooling. We assume that such a source of heat exists (e.g., cosmic rays). The rotation Ω is non-zero, but it decays rapidly outward. The small amount of rotation helps to transport the angular momentum flux from the star out into the external medium.

Solving the angular momentum conservation law (3), we obtain the following solution

\[ \rho = \rho_{\text{ext}}, \quad T = T_{\text{ext}}, \quad \Omega = \Omega_2 r^{-4}, \]

where

\[ \Omega_3 = \alpha s \left( \frac{3^{5/2}}{2\pi} \frac{m_1^{1/2} c_5}{R_5 Q_{\text{ff, NR}}} \right) p_{\text{ext}}^{-1} (kT_{\text{ext}})^{-1/2}, \]

and \( p_{\text{ext}} = 2kT_{\text{ext}} \rho_{\text{ext}}/m_p \). For \( \dot{m} \neq 0 \), the velocity scales as \( r^{-2} \).

3 NUMERICAL RESULTS

The three self-similar solutions written above are special solutions of the basic differential equations (2), (3) and (5), which are valid under specific conditions. To check the validity of these analytical solutions, we have computed numerical solutions of the basic differential equations. We use the same code as in MN01, with two changes. First, we switched to the viscosity prescription given in equation (6), rather than the prescription \( \nu = \alpha c_s^2/\Omega_2 \) used in MN01. Second, in addition to viscous heating, we included a constant heating rate which we adjusted so as to balance the radiative cooling in the homogeneous external medium (see §2.3). The code uses a relaxation method to solve the one-dimensional hydrodynamic equations with specified inner and outer boundary conditions. Although it employs the full equations of a hydrodynamic equations with specified inner and outer boundary conditions. Although it employs the full equations of a two-temperature plasma, the results are essentially equivalent to those of a one-temperature plasma in the region of interest for this paper, namely the region at large radius where the flow matches onto the external medium.

In the calculations, the flow was taken to extend from an inner radius \( R_{\text{in}} = 3 R_p \) to \( R_{\text{out}} = 10^5 R_p \). The mass accretion rate was taken to be low, \( \dot{m} = 2 \times 10^{-5} \), in order that the flow should correspond to the regime of the hot settling flow solution. We took the viscosity parameter to be \( \alpha = 0.1 \) and set the spin of the star to be \( s = 0.3 \) (i.e., 30% of the Keplerian rotation at the stellar surface). We took the other inner boundary conditions to be the same as in MN01. At the outer boundary, we specified the temperature and density of the external medium. Figure 1 shows four solutions. The external temperature is kept fixed at \( T(R_{\text{ext}}) = 10^6 \) K in all the solutions, but the external density varies by a decade and a half\(, \rho(R_{\text{ext}}) = 2.5 \times 10^8, 8.1 \times 10^7, 2.5 \times 10^7, 8.1 \times 10^6 \text{ cm}^{-3} \). We have also done other calculations in which we kept \( \rho_{\text{ext}} \) fixed and varied \( T_{\text{ext}} \). These give very similar results.

Fig. 1 shows that, right next to the star, there is a boundary layer, where the density rises sharply as one goes into the star and the temperature drops suddenly. We do not analyze this region. Once we are outside the boundary layer, the gas behaves very much according to the analytical solutions discussed in §2. Starting just outside the boundary layer and extending over a wide range of radius, the numerical solution exhibits a self-similar behavior with power-law dependences of the density, temperature and angular velocity. This region corresponds to the self-similar solution of MN01. There are, in fact, two zones, an inner two-temperature zone, and an outer one-temperature zone (MN01). The latter corresponds to solution 1 (eq. 4) discussed in §2.1. The most notable feature of this region is that the density, temperature and angular velocity of the numerical solutions are completely independent of the outer temperature and density, as predicted by the analytical solution. The slopes of the numerical curves also agree well with the analytical scalings.

At a radius \( R_{\text{match}} \sim 5 \times 10^4 \) to \( 2 \times 10^5 R_p \) depending on the outer pressure, see eq. (10), solution 1 merges with solution 2 (eq. (11)) as described in §2.2. Here, the solution does depend on the outer boundary conditions, and it scales roughly according to the slopes derived analytically. At even larger radii \( R > R_{\text{ext}} \sim 3 \times 10^7 \) to \( 2 \times 10^8 R_p \) (see eq. (13)), the flow matches onto the ambient external medium. In this region we have solution 3 (eq. (14)) as described in §2.3. As expected, out here only the angular velocity and the radial velocity vary with radius. Both have the scalings predicted for solution 3.

4 DISCUSSION

In this paper, we have removed one piece of mystery surrounding the self-similar “hot settling flow” or “hot brake” solution discovered by MN01. Specifically, we have shown that the remarkable insensitivity of the MN01 solution to external boundary conditions is a consequence of the fact that the solution is insulated from the outer boundary by the presence of a second solution, which bridges the gap between the first solution and the external medium. We derived the form of the second solution analytically in §2.2 and showed via numerical computations (§3, Fig. 1) that the two solutions together are able to match a wide range of outer boundary conditions. This solves one of the mysteries associated with the hot settling flow solution.

There are, however, two other problems that still need to be addressed. First, the solution we have derived treats the mass accretion rate \( \dot{m} \) as a free parameter. (Indeed, the analytical solutions were obtained for the limit \( \dot{m} \to 0 \), i.e., for a hot atmosphere.) What determines \( \dot{m} \)? It is certainly not the outer boundary, since we have obtained the complete outer solution. The accretion rate must therefore be determined by an inner boundary condition. This is not unexpected. In the case of spherical accretion, one recalls that, while the accretion rate for the transonic solution is determined by the outer boundary conditions, the accretion rate for the subsonic settling solution is determined by an inner boundary condition. In that problem, a whole family of settling solutions exists. Each member of the family has a different mass accretion rate, and it is the manner in which the gas cools and condenses on the accreting star that determines which particular solution, i.e., which \( \dot{m} \), is selected. We expect the same situation to apply to our problem. Unfortunately, this means that in order to estimate \( \dot{m} \) we have to solve the full coupled hydrodynamic and radiative trans-
fer equations in the boundary layer region next to the neutron star, with proper boundary conditions. We have not yet succeeded in this difficult exercise.

The second problem that needs to be addressed is the stability of the hot settling flow solution. Because the solution is hot, optically thin and satisfies the energy balance condition \( q^+ \propto T \) while \( q^- \propto \sqrt{T} \). Hence, a local increase of temperature results in an increase of the net heating rate \( q^+ - q^- \) which leads to a further temperature increase. However, for the viscosity prescription given in equation (4), we have \( q^+ \propto \sqrt{T} \propto q^- \), so that the flow is marginally stable. A more detailed analysis of the problem is beyond the scope of the present paper.

We finally comment on the relationship of the hot settling flow discussed in this paper to the subsonic propeller flow described by Ikhsanov (2001, 2003). Both flows describe the braking action of hot gas on a spinning neutron star. The propeller flow has been discussed in connection with a strongly magnetized neutron star, while the hot settling flow was developed to model accretion onto an unmagnetized neutron star, but this is not a large distinction. The main difference between the two solutions is in the treatment of the energy equation. In Ikhsanov’s subsonic propeller flow, the heating rate of the accreting gas through viscous dissipation is much larger than the radiative cooling rate. The gas becomes convective and isentropic, with density falling as \( r^{-3/2} \). The solution is in some sense related to an advection-dominated accretion flow (Narayan et al. 1997) or a convection-dominated accretion flow (Narayan, Igumenshchev & Abramowicz 2000; Quataert & Gruzinov 2000). In contrast, the hot settling flow, as well as the two other solutions described in this paper, satisfy detailed energy balance at each radius. The viscous heating at each point is exactly balanced by local radiative cooling via optically thin bremsstrahlung, as indicated in equation (5) of the present paper. Furthermore, the density falls off as \( r^{-2} \) and \( r^{-7/2} \) for solutions 1 and 2 rather than \( r^{-3/2} \), the entropy of the gas increases outward, and the gas is convectively stable (MN01).

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Figure 1. Profiles of density (top left panel), temperature (top right panel, the electron temperature is the lower curve on the left and the proton temperature is the higher curve), angular velocity (bottom left panel), and radial velocity (bottom right panel), for four numerical solutions of the full height-integrated differential equations. The four solutions correspond to different values of the density of the external medium: $\rho_{\text{ext}} = 2.5 \times 10^9$, $8.1 \times 10^8$, $2.5 \times 10^8$, $8.1 \times 10^7$ cm$^{-3}$. The first and fourth solutions are labeled 1 and 4, respectively. The temperature of the external medium and the accretion rate are kept fixed in all the solutions: $T_{p, \text{ext}} = T_{e, \text{ext}} = 10^8$ K, $\dot{m} = 2 \times 10^{-5}$. The analytical slopes of the three self-similar solutions described in §§2.1–2.3 are shown for comparison.