Precision predictions for $W$-pair production at LEP2†

Wiesław Placzek

Institute of Computer Science, Jagellonian University, ul. Nawojki 11, 30-072 Cracow, Poland, and
CERN, TH Division, CH-1211 Geneva 23, Switzerland.

Abstract

Theoretical calculations for the $W$-pair production process at LEP2 in terms of Monte Carlo event generators RacoonWW and KorlawYFSWW3 are reviewed. The discussion concentrates on precision predictions for the main LEP2 $WW$ observables. The theoretical precision of the above programs is estimated to be $\sim 0.5\%$ for the total $WW$ cross section $\sigma_{WW}$, $\sim 5\text{ MeV}$ for the $W$-boson mass $M_W$, and $\sim 0.005\%$ for the triple-gauge-boson coupling $\lambda = \lambda_g = \lambda_Z$, which is sufficient for the final LEP2 data analyses.

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1 Introduction

The process of $W$-pair production in electron–positron colliders is very important for testing the Standard Model (SM) and searching for signals of possible “new physics”; see e.g. Refs. [1,2]. One of the main goals when investigating this process at present and future $e^+e^-$ experiments is to measure precisely the basic properties of the $W$ boson, such as its mass $M_W$ and width $\Gamma_W$. This process also allows for a study of the triple and quartic gauge-boson couplings at the tree level, where small deviations from the subtle SM gauge cancellations can lead to significant effects on physical observables – these can be signals of “new physics”. On the other hand, the process $W$-pair production and decay constitutes a significant background to other processes, in particular to Higgs-boson production [3].

During the 1996–2000 LEP run, the four LEP2 experiments collected about 10,000 $W$-pair events each. This allows a test of the $W$-boson physics at the precision level of $\sim 1\%$ [4]. The main $WW$ observables investigated at LEP2 are (1) the total $WW$ cross section $\sigma_{WW}$, (2) a distribution of the $W$ invariant mass, which is used to extract the $W$-boson mass $M_W$, and (3) a distribution of the $W$ production polar angle $\cos \Theta_W$, which is the most sensitive observable to the triple-gauge-boson couplings (TGCs). The expected experimental accuracies of the final LEP2 data analyses for $\sigma_{WW}$, $M_W$ and the C- and P-conserving TGC $\lambda = \lambda_\gamma = \lambda_Z$ are given in Table 1. In this table we also present requirements for the theoretical precision of these quantities. For the theoretical errors ($\delta_{th}$) it is commonly required that they should be smaller than the experimental ones ($\delta_{ex}$) by a factor of at least 2.

Table 1:

| Quantity | Final LEP2 $\delta_{ex}$ | Required $\delta_{th}$ |
|----------|--------------------------|------------------------|
| $\sigma_{WW}$ | $\sim 1\%$ | $\leq 0.5\%$ |
| $M_W$ | $\sim 30\text{ MeV}$ | $\leq 15\text{ MeV}$ |
| $\lambda$ | $\sim 0.01$ | $\leq 0.005$ |

The final LEP2 data analyses need theoretical predictions in terms of Monte Carlo event generator(s) (MCEG) that meet these precision requirements.

2 Theoretical description of $W$-pair production

The basic lowest-order process in which $W$ pairs were produced at LEP2 is:

$$e^+ + e^- \rightarrow W^+ + W^- \rightarrow f_1 + \bar{f}_2 + f_3 + \bar{f}_4. \quad (1)$$

This can be described by three Feynman diagrams, called CC03: two annihilation diagrams with $\gamma$ and $Z$ in the $s$-channel (this is where the TGCs appear), and a conversion diagram with an electron neutrino in the $t$-channel (this contribution dominates at LEP2 energies). And here the first theoretical problem appears. Namely, these three diagrams alone are not gauge-invariant. In order to obtain a gauge-invariant scattering amplitude,
one has to include a certain class of singly $W$-resonant diagrams. Therefore, the minimal gauge-invariant subset of Feynman diagrams containing $W$ pairs is the so-called CC11-class – for more details see e.g. [3]. This leads in practice to the necessity of considering the full $4f$-process, i.e. $e^+e^- \rightarrow 4f$, which considerably complicates the theoretical description.

The next problem that appears in the theoretical description of this process is the inclusion of a finite $W$-boson width $\Gamma_W$. This inclusion is necessary to avoid singularities in the scattering amplitude coming from $W$-boson propagators. Up to now there is no fully satisfactory approach to deal with this in the general $4f$-process. Some simple schemes, such as the “fixed-width scheme” and the “running-width scheme” are known to violate gauge invariance. The schemes that do not violate gauge invariance, such as the “fermion-loop scheme” or the “complex-mass scheme” possess other drawbacks, see e.g. [5, 6]. In practice one commonly uses the “fixed-width scheme”, for which gauge-violating effects are numerically negligible at LEP2 energies [3].

All the above problems increase dramatically when one goes to $\mathcal{O}(\alpha)$ calculations. At the one-loop level, the number of Feynman diagrams increases to several thousands per $4f$-channel (final state) [3] and the inclusion of the finite $W$-boson width gets much more difficult.

One may ask the question: Do we need to go beyond the Born level? So let us briefly discuss the various radiative corrections and their effects on the main LEP2 $WW$ measurements.

**Pure QED correction:**

QED corrections can be divided into a few classes: (1) initial-state radiation (ISR), i.e. photon radiation from incoming beams; (2) Coulomb correction – electromagnetic interaction of slowly moving $W^+W^-$; (3) final-state radiation (FSR), i.e. photon radiation in $W$ decays; and (4) non-factorizable (NF) corrections corresponding to interconnections of various stages of the process. They affect the $WW$ observables in different ways. The most sizeable numerical effects come from the ISR. For $\sigma_{WW}$ they amount to between about $-20\%$ near the $WW$ threshold and about $-5\%$ at $E_{CM} = 200\text{ GeV}$ (they depend strongly on $E_{CM}$). $M_W$ is affected at $\sim 10\text{ MeV}$, but this may be enhanced by experimental reconstruction effects (kinematic fits, etc.). The TGC $\lambda$ is shifted by the ISR by $\sim 0.07$. The Coulomb correction modifies mainly $\sigma_{WW}$ at the level of $\sim 6\%$ near the $WW$ threshold. The FSR influences considerably the $M_W$ measurement, at the level of $\sim 10-80\text{ MeV}$, which strongly depends on experimental acceptances [7]. The NF corrections affect only $M_W$ at the $\sim 1-5\text{ MeV}$ level, when treated inclusively [8] (which is a good approximation for LEP2 experimental acceptances).

**Electroweak (EW) corrections:**

The leading EW corrections connected with an effective scale of the $W$-pair production and decay process can be taken into account by using the so-called $G\mu$ scheme. This corresponds to parametrizing the cross section by the Fermi constant $G\mu$ instead of the fine-structure constant $\alpha$. Numerically, this changes the overall normalization, i.e. $\sigma_{WW}$ by $\sim 15\%$. Then the $\mathcal{O}(\alpha)$ non-leading (NL) EW corrections amount to 1–2$\%$ for $\sigma_{WW}$.
at LEP2 energies. They also affect the TGCs, e.g. $\lambda$ at the level of $\sim 0.01$–$0.02$.

**QCD corrections:**

QCD corrections affect normalizations as well as events shapes of the hadronic $WW$ channels. They are usually accounted for by including the so-called naive QCD correction, i.e. the normalization factor $\alpha_s(M_W^2)/\pi \approx 3.8\%$ for each final-state quark pair and $2\alpha_s(M_W^2)/3\pi$ for $\Gamma_W$, in the parton-level calculation, while leaving the exclusive QCD effects to be modelled by dedicated MC packages, such as PYTHIA, etc.

All the above effects are necessary in a MCEG for $W$-pair production at LEP2. The most difficult are $\mathcal{O}(\alpha)$ NL EW corrections. To date, there exist no complete one-loop calculations, even for the simplest CC11-type channels. Even if such calculations existed, they would probably be very complex and slow in numerical computation. Therefore they would not be useful for MC event generation. These are the reasons why some efficient approximations for the $WW$ process are necessary, at least for LEP2.

In the case of $W$-pair production, one is interested in the doubly $W$-resonant process. Therefore one can make use of another expansion parameter, which is $\Gamma_W/M_W \approx 2.5\%$, and apply the so-called pole expansion, i.e. an expansion about the complex pole corresponding to an unstable $W$. This is the expansion in increasing powers of $\Gamma_W/M_W$, which corresponds to decreasing powers of the resonance. Then, one usually applies the so-called leading-pole approximation (LPA) in which only the highest-pole (resonant) contributions are retained, i.e. terms $\sim (\Gamma_W/M_W)^0$. For the $W$-pair production process this means retaining only double-pole contributions, i.e. applying a double-pole approximation (DPA). In this context LPA means just DPA; however, LPA itself has a more general meaning. The resulting matrix element is gauge-invariant, and the imaginary part of the pole position corresponds to the finite $W$-boson width, see e.g. [3,6,9]. In general, the LPA is however not sufficient at the Born level, because the error introduced by this approximation can be here $\sim \Gamma_W/M_W$, i.e. $\sim 2.5\%$. It may also not be sufficient for the leading corrections because of big-log enhancements, i.e. $\sim (\Gamma_W/M_W)(\alpha/\pi)\log(Q^2/m^2)$, where $Q$ is some large momentum transfer and $m$ is a small fermion mass. Here lower-pole terms might be necessary for the desired experimental precision. In practice, it is usually simpler to take the whole $e^+e^- \rightarrow 4f$, including the leading corrections. Then, for the genuine $\mathcal{O}(\alpha)$ NL EW corrections the uncertainty introduced by the LPA is $\sim (\Gamma_W/M_W)(\alpha/\pi) < 10^{-4}$, therefore negligible for the LEP2 precision.

The above solutions have been implemented in two MC event generators: RacoonWW [10–13] and KoralW&YFSWW3 [7,14–22]. These programs are briefly described in the next two sections.

### 3 The MC program RacoonWW

The main features of the program RacoonWW are:

- Matrix elements for all $e^+e^- \rightarrow 4f$ and $e^+e^- \rightarrow 4f\gamma$ processes in massless-fermion approximation.
The ISR in the $\mathcal{O}(\alpha^2)$ leading-log (LL) approximation with soft-photon exponentiation through QED structure functions.

Coulomb correction for off-shell $W$’s.

The NF virtual corrections in the DPA.

The virtual $\mathcal{O}(\alpha)$ NL EW corrections in the DPA (for which one-loop calculations of on-shell $WW$ production and decay are used).

Two QCD-inspired methods of treating soft and collinear photon singularities: dipole subtraction and phase-space slicing (to get a proper matching between virtual and real-photon corrections).

Anomalous Triple and Quartic gauge-boson couplings.

Multichannel MC algorithm for integration and event generation.

More details can be found in Refs. [10–13].

4 The MC programs KoralW and YFSWW3

KoralW and YFSWW3 are actually two independent MCEGs, with some specific and some common features, which are listed below.

KoralW specific features:

- The fully massive matrix element for all $e^+e^- \rightarrow 4f$ processes (generated by the GRACE System [23]).
- Two independent efficient multichannel MC algorithms for the full $4f$ phase space.

YFSWW3 specific features:

- Multiphoton radiation in the $WW$ production stage in the Yennie–Frautschi–Suura (YFS) exponentiation scheme.
- The $\mathcal{O}(\alpha)$ NL EW corrections in the LPA (based on the $\mathcal{O}(\alpha)$ calculations for on-shell $WW$ production of Refs. [24, 25]).

Common features:

- The ISR in the YFS exponentiation scheme up to $\mathcal{O}(\alpha^3)$ LL with non-zero $p_T$ multiphoton radiation.
- Coulomb correction for off-shell $W$’s.
- Non-factorizable QED corrections in an inclusive approximation of the “screened-Coulomb” ansatz [8].
- Anomalous TGCs (three parametrizations).
• The FSR at $\mathcal{O}(\alpha^2)$ LL generated by PHOTOS \[26\] (with non-zero $p_T$ photons).

• $\tau$ decays done by TAUOLA \[27\], quark fragmentation and hadronization managed by JETSET \[28\].

• The semi-analytical program KorWan for the $WW$ process including leading corrections (for test, fits, etc.).

As can be seen from the above description, KoralW is devoted to the full $e^+e^- \to 4f$ process and it includes the leading corrections, i.e. all except $O(\alpha)$ NL. YFSWW3, on the other hand, is dedicated to a precision description of the signal $WW$ process, i.e. it includes all numerically important radiative corrections to this process treated in the LPA, but does not include the $4f$-background contributions. However, thanks to the common features of the two programs, it is easy to correct the predictions of one of them, so that the final results include all the necessary contributions. In fact, the two programs implement special reweighting routines for this purpose. Moreover, we managed to combine the results of the two programs in a real-time execution (event-by-event) using the Unix/Linux named (FIFO) pipe mechanism. This resulted in the so-called Concurrent Monte Carlo (CMC) KoralW&YFSWW3. This CMC works effectively as a single MCEG and provides the description of the $W$-pair production process with all contributions/corrections needed for the LEP2 precision – for more details see Ref. \[18\].

5 Theoretical precision of the main LEP2 $WW$ observables

The precision requirements of the theoretical predictions for some of the main LEP2 observables has been shown in Table 1. Any MCEG to be used for the final LEP2 data analysis should satisfy these requirements. Estimating the precision of theoretical calculations is a difficult task, because it requires assessing the size of missing higher-order contributions. In practice, this is done by comparing different calculations (programs), switching on and off various contributions/corrections and looking at the effects for a given observable. In some cases this can be supplemented with a simplified scale-parameter analysis. All such investigations can give a hint on the size of the missing effects in the theoretical description.

The programs RacoonWW and YFSWW3 have been extensively cross-checked and compared for many observables, see e.g. \[6\]. It has been found that for $\sigma_{WW}$ the two programs agree within 0.3% over the LEP2 energy range and they describe the LEP2 data very well \[29\]. From the comparisons between these two programs, the comparisons with other calculations and investigations of various effects, the theoretical precision for $\sigma_{WW}$ at LEP2 has been estimated at $\sim 0.5\%$ \[3\]. Similar studies, although more involved, were also performed for the $W$ mass $M_W$ and the TGC $\lambda$. In the case of the $W$ mass, we performed $M_W$ fits using KorWan to the invariant-mass distributions from KoralW&YFSWW3 and RacoonWW, where various effects have been switched on and off. We found, for example, that results of the two programs differ in terms of the fitted $M_W$ by $\leq 3\,\text{MeV}$.
We also performed a scale-parameter estimation of the missing effects. From all these studies we estimated the theoretical precision of $M_W$ for the above programs at $\sim 5$ MeV. All this is described in detail in Ref. [30]. A similar analysis for the TGC $\lambda$ was done in Ref. [31]. In this case the fits of $\lambda$ to the $\cos \Theta_W$ distributions were performed using not only KorWan but also dedicated polynomial fitting functions. These fitting functions were constructed using MC-generated data from KoralW&YFSW3 and RacoonWW, not only at the parton level but also including the ALEPH detector-simulation effects. Based on these studies, we estimated the theoretical precision for $\lambda$ at $\sim 0.005$

6 Conclusions

There are two independent Monte Carlo programs for the precision predictions of $W$-pair production at LEP2: RacoonWW [10–13] and KoralW&YFSW3 [7,14–22]. They include the $4f$-background contributions as well as all the necessary radiative corrections at the precision level required by LEP2. The agreement between these programs for the main observables is within the required accuracy of the LEP2 experiments. From comparisons between these programs, comparisons with other calculations, and investigations of various effects, we have estimated the theoretical precision ($\delta_{th}$) for three of the main LEP2 $WW$ observables: $\delta_{th}\sigma_{WW} \sim 0.5\%, \delta_{th}M_W \sim 5$ MeV and $\delta_{th}\lambda \sim 0.005$. Comparing this with Table 1, it is clear that this satisfies the LEP2 precision requirements. However, for the future linear colliders (LC) this is not sufficient and further improvements in the theoretical calculations are necessary.

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