The effects of the cross-entropy stopping criterion and quadrature amplitude modulation on iterative turbo decoding performance

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ABSTRACT

One of the most often-used stopping criteria is the cross-entropy stopping criterion (CESC). The CESC can stop turbo decoder iterations early by calculating mutual information improvements while maintaining bit error rate (BER) performance. Most research on iterative turbo decoding stopping criteria has utilised low-modulation methods, such as binary phase-shift keying. However, a high-speed network requires high modulation to transfer data at high speeds. Hence, a high modulation technique needs to be integrated into the CESC to match its speed. Therefore, the present paper investigated and analysed the effects of the CESC and quadrature amplitude modulation (QAM) on iterative turbo decoding. Three thresholds were simulated and tested under four situations: different code rates, different QAM formats, different code generators, and different frame sizes. The results revealed that in most situations, the use of CESC is suitable only when the signal-to-noise ratio (SNR) is high. This is because the CESC significantly reduces the average iteration number (AIN) while maintaining the BER. The CESC can terminate early at a high SNR and save more than 40% AIN compared with the fixed stopping criterion. Meanwhile, at a low SNR, the CESC fails to terminate early, which results in maximum AIN.

Keywords:
Cross-entropy
Iterative decoding
Quadrature amplitude modulation
Stopping criteria
Turbo codes

1. INTRODUCTION

Error-correcting coding, also known as channel coding, is critical to achieving optimum efficiency in information transmission by lowering the likelihood of errors in digital communications systems [1], [2]. A channel encoder systematically adds redundant data and transforms bit sequences of information into encoded bit sequences. A channel decoder can correct a received signal’s redundant bits that have been distorted by the receiver’s noisy channel [3], [4]. This channel coding technique is called forward error correction (FEC).

Turbo codes are a leading candidate for FEC codes when superior output is required near the Shannon capacity limit [5], [6]. Turbo codes have been used widely in various areas, including digital video broadcasting, deep-space and satellite communications, to improve the chance of obtaining error-free data.
Researchers solve this problem by developing stopping criteria and increasing turbo codes’ performance [13]-[16]. Specifically, stopping criteria help reduce the iteration number in iterative turbo decoding while preserving turbo code efficiency [17], [18]. Most stopping criteria have been developed based on the frame level [7], [9], [14], and bit/window level [11], [12], [16]. Soft-decision-based stopping criteria have become a popular choice for output decisions at the frame level [10], [19], [20].

The average number of bits required to identify an event selected from the set is determined by the cross-entropy between two probability distributions A and B, over the same underlying set of events, according to information theory. This is especially true when the set’s coding method is optimised for an estimated probability distribution A rather than the genuine distribution B. Cross-entropy is used in many applications, such as for determining stopping criteria in iterative decoding scenarios [13]-[15], defining loss functions in machine learning [21], [22], and optimising classification models [23].

Fowdur et al. [14], [24] proposed the cross-entropy stopping criterion (CESC) to calculate improvements in mutual information to cease turbo decoder iterations. Most of the research on iterative turbo decoding stopping criteria have utilised the binary phase-shift keying (BPSK) modulation technique [6], [7], [11], [15], which is the simplest form of phase-shift keying. High-speed networks such as long-term evolution (LTE) networks and those based on fifth-generation (5G) technology require high modulation (For example, eight phase-shift keyings (8PSK), 16 phase-shift keyings (16PSK), 16-QAM, 32-QAM, and 64-QAM) to transfer data at a sufficient speed [14], [16], [25]. A higher modulation needs to be integrated into stopping criteria to meet the speed requirements of current LTE applications [14], [16].

This paper is intended to address the lack of information about the performance of CESC and high modulation via iterative turbo decoding [26]. Hence, this paper describes the effects of CESC and QAM on iterative turbo decoding performance. Three predefined thresholds of CESC were tested under four situations: different code rates, different QAM formats, different code generators, and different frame sizes. The CESC’s performance was assessed using the bit error rate (BER) and average iteration number (AIN).

The content of this paper is organised in the following manner. The CESC and its algorithm are discussed in Section 2. In Section 3, the simulation parameters and the methodology used in the research are described. In Section 4, the effects and performances of the CESC with QAM on iterative turbo decoding are analysed and compared. Finally, conclusions are drawn in Section 5.

2. CROSS-ENTROPY STOPPING CRITERION

Figure 1 illustrates the process for the early termination of an iterative turbo decoder using the CESC. The CESC is between decoder two and the deinterleaver. The outputs from the first turbo encoder are decoded by decoder 1, which are the systematic input bit of the decoder, and the initial parity input bit of the CESC. The outputs from the first turbo encoder are input into the CESC. In CESC, the ith iteration is compared to the maximum number of iterations (i_max). If the condition is not fulfilled, the online CE threshold (T(i)) is calculated.

\[
T(i) \approx \sum_b \frac{\left| \Delta L_{i,2}(x_b) \right|^2}{L_{i,1}(x_b)}
\]
where $\Delta L_{x,2}(x_b)$,

$$\Delta L_{x,2}(x_b) = L_{x,2}(x_b) - L_{x,1}(x_b) = L_{x,2}(x_b) - L_{x,2}^{i-1}(x_b)$$ (4)

the iteration is stopped if $T(i)$ is in the thresholds predefined in (5),

$$T(i) < (10^{-2} - 10^{-4})T(l)$$ (5)

if the condition is not fulfilled, then $L_{x,2}^{i}(x_b)$ is returned to decoder 1, and the process is repeated until $i_{\text{max}}$ is achieved. Otherwise, when $i_{\text{max}}$ is met, the deinterleaver/final output ($x_b$) can be estimated from $L_{x,2}^{i}(x_b)$, as shown by (6),

$$x_b = \text{sign}(L_{x,2}^{i}(x_b)) = \begin{cases} 
0 & \text{if } L_{x,2}^{i}(x_b) \leq 0 \\
1 & \text{if } L_{x,2}^{i}(x_b) > 0
\end{cases}$$ (6)

the final turbo decoder output can be written in the following sequence,

$$x = \{x_b\}_{b=1}^{N} = x_1, x_2, \ldots, x_N$$ (7)

![Figure 1. Iterative turbo decoder with a CESC](image)

3. RESEARCH METHOD

Table 1 shows the parameters used in the present simulation. The thresholds for CESC were set at 0.1, 0.01, and 0.001. One million random binary data with three different modulations (4-QAM, 8-QAM, and 16-QAM) were applied in the simulation. Meanwhile, the frame size ($N$) was set to either 1000, 5000, or 10,000 for each data transmission. The turbo code generator (in octal) and code rate values were (7,5), (17,15), (37,21), and (1/2, 1/3). The codes were simulated under the AWGN channel, and the maximum iteration number ($I_{\text{max}}$) was set to 8.

The flowchart in Figure 2 outlines the simulation. The simulation was started with setting up the turbo decoder parameters, such as code generator (g), code rate (R), and frame size (N) as shown in Table 1. Next, threshold and QAM formats were selected based on the data in Table 1. Then, the turbo decoder with QAM was integrated and simulated and repeated for other turbo code parameters and thresholds for all CESC and QAM formats. Finally, the performances of the turbo decoders with QAM were analysed.
Table 1. Simulation parameters

| Parameters              | Name/ Value              |
|-------------------------|--------------------------|
| CESC threshold          | 0.1, 0.01, 0.001         |
| Modulation              | 4-QAM, 16-QAM, 32-QAM    |
| Frame size              | 1000 (1K), 5000 (5K), 10000 (10K) |
| Turbo code generator    | (7,5), (17,15), (37,21) |
| Turbo code rate         | 1/2, 1/3                |
| Channel model           | AWGN                     |
| Maximum iteration number| 8                        |

![Figure 2. Flowchart for the simulation](image)

4. RESULTS AND DISCUSSION

The effects and performances of CESC and QAM with three predefined thresholds were compared and analysed under four situations: different code rates, different QAM formats, different code generators, and different frame sizes as shown in Table 1. Each threshold’s performance was analysed based on the BER and AIN.

4.1. Performance of the CESC with different turbo code rates

The CESC’s AIN and BER performance for coding rates is shown in Figure 3. For the 1/3 code rate, the AIN for threshold values of 0.01, 0.001, and 0.0001 started to decrease at 2 dB. At 4 dB Eb/No, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at 2.8, 2.9, and 3 AIN, respectively. These values represent decreases of around 65%, 63.75%, and 62.5% compared to the fixed stopping criterion (FISC). Moreover, the BER output of the CESC and FISC are nearly the same for threshold values of 0.01, 0.001, and 0.0001.

For the 1/2 code rate, the AIN for threshold values of 0.01, 0.001, and 0.0001 started to decrease at 3 dB. At 4 dB, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at 3.7, 3.9, and 4.1, respectively. These values represent decreases of 53.75%, 51.25%, and 48.75% compared to FISC. Regarding BER performance, it can be observed that for 0.01, there is a slight difference in BER starting at 3.5 dB, which increases until 4 dB at BER = 10^{-4.5}. For thresholds of 0.001 and 0.0001, there are slight differences in BER, starting at 3.5 dB and increasing until 4 dB at BER = 10^{-4.7}. 

Indonesian J Elec Eng & Comp Sci, Vol. 23, No. 3, September 2021: 1557 - 1564
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4.2. Performance of the CESC with different QAM formats

The AIN and BER for CESC for QAM formats of 4-QAM, 16-QAM, and 32-QAM are shown in Figure 4. The 4-QAM graph shows that the AIN start to reduce at 0 dB for all three threshold values. At 4 dB Eb/No, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at two iterations, around 75% less than FISC. Moreover, slight differences in BER occurred for all three threshold values, starting at 0 dB and increasing until 0.5 dB at BER = 10^{-5}.

Meanwhile, for 16-QAM, the AIN for threshold values of 0.01, 0.001, and 0.0001 started to reduce at 2 dB. At 4 dB, the threshold value of 0.01, 0.001, and 0.0001 can terminate early at 2.8, 2.9, and 3 AIN, respectively, signifying reductions of around 65%, 63.75%, and 62.5% compared to FISC. The BER performance of the CESC was the same as FISC for all three threshold values.

The 32-QAM indicates that the AIN for threshold values of 0.01, 0.001, and 0.0001 started to reduce at 3 dB. At 4 dB, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at 4.5, 4.6, and 4.8, respectively—these values are 43.75%, 42.5%, and 40% lower than for FISC. The BER performance of the CESC is quite similar to the performance of the FISC at thresholds of 0.01, 0.001, and 0.0001.

4.3. Performance of the CESC with different turbo code generators

The AIN and BER performances of the CESC at different threshold values for different turbo code generators are depicted in Figure 5. The graphs for g = (7,5), (17,15) and (37,21) show that the AIN for threshold values of 0.01, 0.001 and 0.0001 started to reduce at 2 dB. At 4 dB Eb/No, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at 3 AIN, representing savings of around 62.5% from a FISC. Also, the BER performances of the CESC and FISC are the same for the threshold values of 0.01, 0.001, and 0.0001.

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For $g = (17, 15)$, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at 2.9, 3, and 3 AIN, respectively, representing reductions of around 65%, 65%, and 63.75% compared to FISC. The BER performances of the CESC and FISC were the same at thresholds of 0.01, 0.001, and 0.0001. Meanwhile, for $g = (37, 21)$, at 4 dB, the threshold value of 0.01, 0.001, and 0.0001 can terminate early at 2.8, 2.9, and 3 AIN, respectively, representing savings of around 65%, 63.75%, and 62.5% compared to the FISC. Once again, the CESC’s and FISC’s BER performances are the same for all three thresholds.

### 4.4. Performance of the CESC with different frame sizes

The CESC’s AIN and BER performances for frame sizes of 1K, 5K, and 10K are illustrated in Figure 6. The $N = 1K$ graph shows that the AIN for threshold values of 0.01, 0.001, and 0.0001 starts to decline at 1.5 dB. At 4 dB $E_b/N_0$, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at 2.2, 2.3, and 2.4 AIN, respectively, thus saving around 72.5%, 71.25%, and 70% compared to the FISC. For threshold values of 0.001 and 0.0001, the BER performances of the CESC and FISC are quite close. For threshold = 0.01, there is a slight indifference in BER it starts at 3 dB and increases until 4 dB at $BER = 10^{-4.8}$.

Meanwhile, for $N = 5K$, the AIN for threshold values of 0.01, 0.001, and 0.0001 starts to reduce at 2 dB. At 4 dB, the threshold values of 0.01, 0.001, and 0.0001 can terminate early at 2.6, 2.8, and 2.9, respectively, thereby saving around 67.5%, 65%, and 63.75% compared to the FISC. Further, the BER performances of the CESC and FISC are the same for thresholds of 0.01, 0.001, and 0.0001.
Finally, for N = 10K, the AIN for threshold values of 0.01, 0.001, and 0.0001 start to reduce at 2 dB. At 4 dB Eb/No, the threshold values of 0.01, 0.001 and 0.0001 can terminate early at 2.8, 2.9 and 3.0 AIN, respectively. This early termination represents reductions of around 65%, 63.75%, and 62.5% when compared to the FISC. The BER performances of the CESC and FISC are the same for all three threshold values.

5. CONCLUSION

In most situations, the use of the CESC is suitable only when Eb/No or SNR is high. In such cases, the CESC yields significant reductions in AIN while maintaining a desirable BER performance. The CESC can terminate early when Eb/No is high and can save more than 40% AIN compared with FISC. However, at a low Eb/No, the CESC fails to terminate early and increases AIN. The CESC’s BER performance remains unchanged at BER > 10^-4. However, for the cases of R = 1/2, 16-QAM and R = 1/3, 4-QAM, minor degradations (i.e. of less than 0.2 dB) occur in the high Eb/No region and increase to 0.5 dB at BER ≤ 10^-4. The same situation was apparent for R = 1/3, N = 1K, and threshold = 0.01, as the CESC’s BER performance started to decrease after BER = 10^-4. For future works, researchers can use SNR estimations to estimate the low Eb/No region for early CESC termination. In addition, suitable thresholds and enhanced stopping criteria should be calculated and implemented to improve the performance of CESC with QAM for iterative turbo decoding.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the Universiti Teknologi MARA (UiTM) for this study, which was funded by grant 600-RMC/MYRA 5/3/LESTARI (039/2020).

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