A note on bulk locality and covariance in AdS/CFT

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Abstract

This paper studies a recently proposed relation between the emergence of bulk locality in AdS/CFT and the theory of quantum error correction. We show that if this relation is indeed realized in AdS/CFT, then bulk covariance is broken in the semi-classical limit.

Keywords: locality, covariance, effective quantum field theory, AdS/CFT correspondence

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I. INTRODUCTION

According to the “extrapolate” dictionary of the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, one can construct unbounded bulk CFT operators from the boundary ones, i.e.

$$\hat{\phi}(x) = \int dX K(x, X) \hat{\mathcal{O}}(X),$$

where $K(x, X)$ is the so-called “smearing function”, $x \in \mathcal{M}$ and $X \in \partial \mathcal{M}$. One can further define an algebra $\mathcal{O}(\mathcal{M})$ generated by polynomials of the bulk CFT operators smeared over test functions $\{f(x)\}$, i.e. functions which are smooth and compactly supported. Thus, one may consider the bulk CFT operator algebra $\mathcal{O}(\mathcal{M})$ instead of the boundary CFT algebra $\mathcal{O}(\partial \mathcal{M})$ inside of AdS space $\mathcal{M}$.\(^1\)

On the other hand, one may consider a semi-classical quantum field theory in AdS space $\mathcal{M}$. For simplicity, we choose a linear scalar field model characterized by an unbounded field operator $\hat{\Phi}(x)$, where $x \in \mathcal{M}$. One can define another algebra, i.e. $\mathcal{A}(\mathcal{M})$, generated by polynomials of smeared (over $\{f(x)\}$) scalar field operators. This algebra is supposed to satisfy the basic principles (axioms) of quantum field theory, among which are locality, i.e. $[\hat{\Phi}(x), \hat{\Phi}(y)] = 0$, whenever $x$ and $y$ are space-like separated, and covariance roughly meaning that the algebraic structure of $\mathcal{A}(\mathcal{M})$ does not change under diffeomorphisms (see, for instance, [2, 3]).

In the semi-classical limit of quantum gravity, the bulk CFT theory should reduce to an effective quantum field theory on a classical geometric background. Hence, one may expect the bulk CFT operator algebra $\mathcal{O}(\mathcal{M})$ satisfies the standard axioms of local quantum field theory. However, it has been recently argued that $\mathcal{O}(\mathcal{M})$ is not local even at the semi-classical approximation [4, 5]. The argument is based on the following assumptions:

1. the time-slice axiom holds for $\mathcal{O}(\mathcal{M})$;
2. $\mathcal{O}(\mathcal{M})$ is complete (or irreducible).

This argument is further employed in [4, 5] to suggest a relation between the quantum error correction theory and the reproduction of the bulk in AdS/CFT. However, if this is more than an analogy, then it is problematic to have covariance in the bulk. This is our motivation to revisit the argument of [4, 5].

The outline of the paper is as follows. In Sec. III we show that the algebra $\mathcal{O}(\mathcal{M})$ could be local in the semi-classical limit. We clarify then the meaning of the assumptions made in [4, 5] and explain why the argument of [4, 5] cannot lead to bulk non-locality at the level of the operator algebra. In Sec. III we discuss bulk covariance and point out that if the

\(^1\) Note that the bulk operators may also be treated as elements of an enlarged CFT algebra composed of $\mathcal{O}(\partial \mathcal{M})$ and $\mathcal{O}(\mathcal{M})$. This is not of a fundamental importance for our discussion below.
quantum error correction theory is indeed encoded in AdS/CFT as envisaged by \cite{4,5}, then bulk covariance must be broken. In Sec. [IV] we provide final concluding remarks.

II. BULK LOCALITY

A. Bulk locality of CFT operators

It has been recently argued \cite{4,5} that bulk locality of the bulk CFT operators is not respected at the level of the operator algebra (i.e. in the strong sense), but only at the level of certain matrix elements (i.e. in the weak sense). The statement is made in the semi-classical limit. If it is correct, then a certain $\mathcal{O}(\mathcal{M})$ theory does not reduce to a certain local quantum field theory $\mathcal{A}(\mathcal{M})$ in the bulk, which is supposed to be an effective field theory in the low-energy limit.

However, the bulk CFT algebra could be local in the strong sense. Indeed, in the semi-classical limit, we have

$$[\hat{O}(x), \hat{O}(y)] = i \int dX dY K(x, X) K(y, Y) \Delta_\mathcal{O}(X, Y),$$

where $[\hat{O}(X), \hat{O}(Y)] \equiv i \Delta_\mathcal{O}(X, Y)$. It is worth emphasizing that $\Delta_\mathcal{O}(X, Y)$ does not depend on a quantum state, i.e. it is state-independent. For simplicity, we consider the Poincaré patch of the three-dimensional AdS space, i.e. $\mathcal{M} = \text{PAdS}_3$. The commutator of the CFT operator at two boundary points is given by

$$[\hat{O}(T_x, X_x), \hat{O}(T_y, X_y)] \propto \frac{1}{((\Delta T - i\varepsilon)^2 - \Delta X^2)\Delta} - \frac{1}{((\Delta T + i\varepsilon)^2 - \Delta X^2)\Delta},$$

where $\Delta T \equiv T_x - T_y$, $\Delta X \equiv X_x - X_y$, and $\Delta$ is the conformal weight of the CFT operator. To further simplify computations, we set $y = (T_y, X_y, Z_y) = 0$ and $\Delta = 2$. Following \cite{1}, one can obtain

$$[\hat{O}(x), \hat{O}(0)] = \frac{Z_x^2}{2\pi} \left( \frac{1}{((T_x - i\varepsilon)^2 - X_x^2 - Z_x^2)^2} - \frac{1}{((T_x + i\varepsilon)^2 - X_x^2 - Z_x^2)^2} \right).$$

Thus, the operator $\hat{O}(x)$ at $T_x = 0$ commutes with $\hat{O}(0)$ which is $\hat{O}(0)$ up to the rescaling $1/Z_y^2$ in the limit $Z_y \to 0$.

This result can be reproduced from $[\hat{Φ}(x), \hat{Φ}(0)]$ if the scalar field satisfies the massless Klein-Gordon equation with minimal coupling to gravity. This is completely consistent with the conformal weight of the CFT operator and the dimension of the AdS geometry. Thus, $[\hat{φ}(x), \hat{φ}(y)]$ does commute for $x$ and $y$ space-like separated in the strong sense. This is also consistent with an expectation that the low-energy limit of quantum gravity corresponds to the semi-classical quantum field theory.\cite{2} Consequently, the argument of \cite{4,5} based on a combination of the time-slice axiom and the completeness axiom should be revisited.

\footnote{See also the last par. of sec. II.D in the third reference of \cite{1}}
B. Time-slice and completeness axiom

The time-slice axiom states that if $O$ is any fixed neighbourhood of a Cauchy surface $\Sigma$ of a globally hyperbolic space $M$, then $A(M)$ is generated by the scalar field operators having a non-vanishing support inside of $O$. In other words, $A(M) \cong A(O)$, whenever $\Sigma \subset O \subset M$.\[^3\]

The completeness axiom can be formulated in different ways.\[^3\] It is worth emphasizing that this axiom is related to a representation of the algebra $A(M)$ on a certain state $|\Omega\rangle$.\[^3\] In other words, this axiom imposes a certain constraint on a state. We will denote the algebra representation as $A_\pi(M)$ which is defined on a Hilbert space $H$. The representation $\pi$ of $A(M)$, i.e. $A_\pi(M)$, is said to be irreducible if there is no a non-trivial bounded operator commuting with all operators from $A_\pi(M)$. If the representation is irreducible, then the set of operators $A_\pi(M)$ is said to be complete.\[^4\]

The completeness axiom and time-slice axiom allow to uniquely determine a quantum state by measurements performed in a small time interval. If the former is violated, then one cannot uncover all properties of the state. If the latter is not fulfilled by a certain field theory, then one needs to observe the state at all times to determine its properties.\[^6\]

It is worth mentioning that a more profound property of a representation of the operator algebra is its cyclicity (see Note added in proof in \[^6\]). A representation $\pi$ is cyclic if $A_\pi(M)|\Omega\rangle$ is dense in $H$. A state generating $\pi$ is then said to be cyclic.

An example of the reducibility of the Hilbert space or incompleteness of the operator algebra should clarify these. Suppose one chooses a factorized Hilbert space representation $H_L \otimes H_R$ of the total algebra $\mathcal{A}_T(M)$ in the eternal Schwarzschild black-hole geometry $M$. The “right” operator algebra $\mathcal{A}_R(M)$ (having a vanishing support in the “left” outside region of the hole) is incomplete in this representation, because the “left” operator algebra $\mathcal{A}_L(M)$ (having a vanishing support in the “right” outside region of the hole) is non-trivial. However, the physical vacuum, i.e. the Hartle-Hawking one, generates an irreducible representation $\mathcal{H}_T$ of the algebra, because the Hartle-Hawking state is cyclic with respect to it.\[^10\] Specifically, $\mathcal{A}_R(M)$ (or $\mathcal{A}_L(M)$) acting on the Hartle-Hawking state generates a space being dense in the Hilbert space $\mathcal{H}_T$. It is worth emphasizing at this point that the algebras $\mathcal{A}_L(M)$ and $\mathcal{A}_R(M)$ commute independent on the representation (cf. \[^4, 5\]).

Thus, we have two representations of local (causal) quantum field theory. One of the representations is reducible, while another is irreducible. If one wants the completeness axiom to be fulfilled by the quantum theory, one needs to choose the irreducible representation. As it should be evident this has nothing to do with the locality axiom.

\[^3\] If one specifies a state defined on $A(M)$, then one can construct a Hilbert space representation of $A(M)$ associated with this state. This is achieved through the so-called Gelfand-Naimark-Segal construction.\[^7\]

\[^4\] Note that the authors of \[^8\] refer to \[^4\] when they introduce a notion of irreducibility (completeness) of a set of operators, while the author of \[^9\] seems to refer to \[^6\].
It is worth noting that these two representations from our example are not unitarily equivalent (for more on this, see [11]). We discuss analogous (reducible and irreducible) representations from a different perspective in the eternal AdS black-hole geometries in [12].

It should be also evident that locality (causality) and the time-slice property of $\mathcal{A}(\mathcal{M})$ are state-independent, whereas completeness is state-dependent. These three axioms are a part of the postulates of local quantum field theory (see, for instance, [7]). They are not inconsistent with each other in the field model we consider in the bulk.\(^5\) As we have shown above, locality is also preserved at the level of the operator algebra $\mathcal{O}(\mathcal{M})$ in the semi-classical limit.

Nevertheless, there could be a certain CFT theory on the boundary, which is not local in the bulk. However, such a theory would not reduce to any semi-classical quantum field theory in the low-energy limit of quantum gravity.

C. Hilbert space representation and locality

In Sec. IIA, we have not used any Hilbert space representation of the operators. In other words, the equations (2) and (4) are operator equalities. To put it differently, this holds for any CFT state from the CFT Hilbert space. Note that if the backreaction in a certain CFT state is not negligible, then the whole geometry changes. This leads in particular to change of the smearing function as well as the commutator, because these depend on spacetime metric. In general, one cannot a priori say anything about locality in the new geometry. However, this is certainly beyond of what is considered in [4, 5].\(^6\)

In order to discuss the completeness or irreducibility axiom, one should introduce a quantum state $|\omega\rangle$. We assume that this state is the ordinary CFT vacuum. This state generates a Hilbert space $\mathcal{H}_\omega$ through the Gelfand-Naimark-Segal procedure (see, for instance, [7]). This is the ordinary CFT Hilbert space. A representation $\pi_\omega$ of $\mathcal{O}(\mathcal{M})$ is a set of linear operators acting on $\mathcal{H}_\omega$. This representation of the operator algebra will be denoted as $\mathcal{O}_\pi(\mathcal{M})$ which is $\pi_\omega(\mathcal{O}(\mathcal{M}))$. It is worth emphasizing that the representation $\pi_\omega$ of the algebra does not change the algebraic structure of it – $\pi_\omega$ is homomorphism. In other words, if $\mathcal{O}(\mathcal{M})$ is local (causal), then $\mathcal{O}_\pi(\mathcal{M})$ is automatically local as well.

The representation $\pi_\omega$ could be of various types: faithful, cyclic, irreducible and so on [7]. We are mainly interested in understanding the irreducibility/reducibility property of $\pi_\omega$ in light of the argument of [4, 5]. As noted above, the irreducibility can be defined in different

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\(^5\) Note, these axioms are not satisfied a priori in a certain quantum field theory. These have to be checked. We are guided by these principles in order to have a non-pathological quantum field theory.

\(^6\) Note that it is not excluded that there could be a certain asymptotically AdS space $\bar{\mathcal{M}}$ such that the bulk algebra $\mathcal{O}(\bar{\mathcal{M}})$ (or the enlarged algebra) is non-local in the strong sense in the low-energy limit. However, as shown above, this is not the case in AdS geometry, wherein the author of [4] has however argued that the algebra is local only in the weak sense.
ways [6, 7]. Following [7], the irreducibility of $\pi_\omega$ means that no non-trivial element $\hat{o}$ of $\mathcal{O}(\mathcal{M})$ is represented on $\mathcal{H}_\omega$ by a trivial operator. In other words, if the representation $\pi_\omega$ is irreducible, then $\pi_\omega(\hat{o}) \equiv \hat{o}_\omega \not\propto \hat{1}$ for any $\hat{o} \not\propto \hat{1}$ belonging to $\mathcal{O}(\mathcal{M})$.

As pointed out above, this property of the representation is independent on the locality axiom and vice verse, because, for instance, locality is an operator statement. In other words, one cannot use one of these axioms to prove or disprove another. The confusion apparently appears when one employs an alternative formulation of what the irreducibility/completeness axiom means. Specifically, if the representation $\pi_\omega$ is irreducible, then a bounded operator $\hat{B}$ commuting with all (unbounded) operators of $\mathcal{O}_\pi(\mathcal{M})$ is a multiple of the identity. Can one use this to prove non-locality of $\mathcal{O}(\mathcal{M})$? No, otherwise one is coming dangerously close to proving the inconsistency of the Wightman axioms. First, the operator $\hat{\phi}_\omega(x)$ is unbounded. Second, when one asks whether $[\hat{\phi}_\omega(x), \hat{\mathcal{O}}_\omega(X)]$ is vanishing or not for space-like separated $x$ and $X$, one should bear in mind that $\hat{\phi}_\omega(x)$ is defined through (11). Hence, one should instead ask whether

$$\int dY K(x, Y) [\hat{\mathcal{O}}_\omega(Y), \hat{\mathcal{O}}_\omega(X)]$$

vanishes whenever $x$ and $X$ are space-like separated. The commutator of the CFT operators does not depend on a state in the semi-classical limit. Therefore, one can omit the index $\omega$ which refers to the representation. The answer depends thus on the smearing function, rather than on the completeness/irreducibility axiom. As shown above, it does vanish at the level of the operator algebra, at least in the case we have considered in Sec. II A.

To sum it up, we disagree with the general statement made in [4] that bulk locality cannot be respected within the CFT at the level of the algebra of operators (i.e. in the strong sense). Our argument is based on two observations: First, one cannot prove non-locality of the theory employing the completeness and time-slice axioms. Second, the bulk operator algebra is local in the strong sense in the semi-classical limit at least in AdS geometry.

III. BULK COVARIANCE

A. Covariance

The three-dimensional AdS geometry is a hyperboloid embedded in space $\mathbb{R}^{2,2}$ with the line element $ds^2 = \eta_{ab}dx^a dx^b$, where $a, b$ run from 0 to 3 and $\eta_{ab} = \text{diag}(+,-,-,+)$. One may choose various coordinates which can cover the whole hyperboloid or merely a certain part of it. We will consider the Poincaré patch $\mathcal{M}$ mentioned above and the AdS-Rindler patch, which is denoted as $\mathcal{N}$, i.e. $\mathcal{N} = \text{RAdS}$, in the following. The RAdS patch

7 It is worth noting that the quantum field $\hat{\phi}(x)$ smeared out over a test function $f(x)$ is still unbounded operator, although the test function is bounded as this follows from its definition and the Weierstrass theorem.
is mapped to the Poincaré one by a hyperbolic embedding $\psi$, namely $\psi: \mathcal{N} \to \mathcal{M}$. In other words, this map is specified via expressing the Rindler-AdS coordinates through the Poincaré ones (see, for instance, [13]).

We have defined the algebra $\mathcal{A}(\mathcal{M})$ above. We denote the scalar field operator belonging to this algebra as $\hat{\Phi}_\mathcal{M}(x)$, where $x \in \mathcal{M}$ as before. One can also define an operator algebra $\mathcal{A}(\mathcal{N})$, which is associated with the Rindler patch of AdS space. The field operator $\hat{\Phi}_\mathcal{N}(\tilde{x})$ refers to $\mathcal{A}(\mathcal{N})$, where $\tilde{x} \in \mathcal{N}$. By covariance one understands

$$\alpha_\psi \circ \hat{\Phi}_\mathcal{N} = \hat{\Phi}_\mathcal{M} \circ \psi_*,$$

where $\psi_*$ is a push-forward of $\psi$ which maps test functions $\{f(\tilde{x})\}$ to $\{\psi_* f(x)\}$, where $x = \psi(\tilde{x})$. The map $\alpha_\psi$ is an injective homomorphism from $\mathcal{A}(\mathcal{N})$ to $\mathcal{A}(\mathcal{M})$ [2, 3].

To put it differently, we now consider points $p$, which lie in both $\mathcal{M}$ and $\mathcal{N}$, i.e. $p \in \mathcal{N} \cap \mathcal{M}$. For this set of points we have field operators $\hat{\Phi}_\mathcal{M}(x)$ and $\hat{\Phi}_\mathcal{N}(\tilde{x})$, such that $p$ is parametrized by both $\{x\}$ and $\{\tilde{x}\}$. The covariance principle implies

$$\hat{\Phi}_\mathcal{N}(\tilde{x}) = \hat{\Phi}_\mathcal{M}(\psi(\tilde{x})).$$

### B. Bulk CFT operator reconstructions and covariance

One may construct the bulk CFT operator for points $p$ either in $\mathcal{N}$ or $\mathcal{M}$ [1]. Thus, one can have two bulk CFT operators $\hat{\phi}_\mathcal{M}(x)$ and $\hat{\phi}_\mathcal{N}(\tilde{x})$ at the same bulk points $p$. The bulk operator $\hat{\phi}_\mathcal{M}(x)$ corresponds to the Poincaré or global reconstruction, while $\hat{\phi}_\mathcal{N}(\tilde{x})$ to the AdS-Rindler reconstruction.

A counter-intuitive observation has been made in [4, 5] based on different reconstructions of the bulk CFT operators and the non-locality of $\mathcal{O}(\mathcal{M})$. Specifically, those two bulk CFT reconstructions should not be equivalent as operators, i.e.

$$\hat{\phi}_\mathcal{N}(\tilde{x}) \neq \hat{\phi}_\mathcal{M}(\psi(\tilde{x})).$$

This observation was related to the gauge invariance of the boundary theory in [14] (see also [5]). However, the argument employed in [4, 5] in favor of this inequivalence is generally invalid as we have shown above. Nevertheless, direct computations may show that they are indeed inequivalent.10

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8 A hyperbolic embedding $\psi$ is an isometry, which preserves time and space orientation. An isometry is a diffeomorphism such that $\psi_* g_N = g_M|_{\psi(\mathcal{N})}$, where, for instance, $g_N$ is a metric tensor in $\mathcal{N}$.

9 A homomorphism is, roughly speaking, a map which respects the algebraic structure.

10 It is worth noting that the Bogolyubov transformation is canonical, i.e. it does not change the commutator of the field operators. In other words, if $\hat{\phi}_\mathcal{N}(\tilde{x})$ and $\hat{\phi}_\mathcal{M}(x)$ are related through the Bogolyubov transformation in $\mathcal{N} \cap \mathcal{M}$, then $\hat{\phi}_\mathcal{N}(\tilde{x}) = \hat{\phi}_\mathcal{M}(\psi(\tilde{x}))$ automatically.
Suppose that $\hat{\phi}_N(\tilde{x})$ and $\hat{\phi}_M(x)$ are inequivalent as operators in $\mathcal{N} \cap \mathcal{M}$. This implies then that covariance is broken in the bulk. This is unacceptable as it would be a pathological modification of effective quantum field theory.

Moreover, string theory which is supposed to be dual to the CFT boundary theory is covariant. Its semi-classical limit provides still with a covariant effective field theory. Thus, the equation

$$\hat{\phi}_N(\tilde{x}) = \hat{\phi}_M(\psi(\tilde{x}))$$

must hold at that limit.

**IV. CONCLUDING REMARKS**

The argument of [4, 5] based on the time-slice and completeness axiom cannot judge whether a bulk CFT algebra composed of smeared bulk CFT operators (or the enlarged CFT algebra) taken at a certain time slice satisfies or does not satisfy the locality (causality) axiom at the level of algebra of operators. We have explicitly shown an example of the bulk CFT operators which demonstrates this as well as an example when the completeness axiom is not fulfilled in a local (causal) theory.

The covariance principle is one of the basic axioms of general relativity which celebrates its 100th birthday this year. Thus, different reconstructions of bulk CFT operators should give the same operator at a given bulk point, at least in the low-energy limit. The contrary is employed in [4] to make a contact of inequivalent bulk reconstructions with the quantum error correction theory. However, we would like to remind here that one should be careful when one applies the logic of quantum mechanics to quantum field theory. If one does not take into account differences between quantum mechanics and quantum field theory, then one can obtain the well-known paradoxical consequences. One of the paradoxes is posed in [15] and resolved in [16] (see also [17]).

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