Indications for the new unitarity regime in the extensive air showers measurements

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Abstract

We note that the new unitarity regime when scattering amplitude goes beyond the black disc limit (antishadowing) could help in the explanation of the regularities such as knee in the energy spectrum, existence of penetrating and long-flying particles and other features observed in the measurements of the extensive air showers which originate from cosmic particles interactions with the atmosphere.
The experimental and theoretical studies of cosmic rays are the important source of astrophysical information (cf. e.g. [1]) and they simultaneously provide a window to the future results of accelerator studies of hadron interaction mechanism at the LHC.\footnote{It should be noted that the value of the total cross–section extracted from cosmic rays measurements significantly depend on the particular model for elastic scattering, because measurements of the extensive air showers provide information on inelastic scattering cross–section only\cite{2}.}

It can happen that the investigations of cosmic rays will give us a clue that the hadron interaction and mechanism of particle generation is changing in the region of $\sqrt{s} = 3 – 6$ TeV\footnote{\cite{3}}. Indeed, the energy spectrum which follows simple power–like law $F(E) = c E^{-\gamma}$ changes its slope in this energy region and becomes steeper: index $\gamma$ increases from 2.7 to 3.1. It is important that the knee in the energy spectrum appears in the same energy region where the penetrating and long–flying particles also start to appear in the extensive air showers (EAS): the absorption length is also changing from $\lambda = 90 \text{ g/cm}^2$ to $\lambda = 150 \text{ g/cm}^2$ (cf. \cite{3}). There is also specific feature of the events at the energies beyond knee such as alignment cf. \cite{5} and the references for the earlier papers therein. The above phenomena were interpreted as a result of appearance of the new particles which have a small inelastic cross–section and/or small inelasticity. These new particles can be associated with a manifestation of the supersymmetry, quark–gluon plasma

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{energy_spectrum.png}
\caption{Scaled energy spectrum of the cosmic rays, figure from \cite{2}.}
\end{figure}
formation and other new mechanisms. However, there is another possibility to treat those cosmic rays phenomena observed in EAS as the manifestations of the new unitarity regime (antishadow scattering mode) at such energies [6].

Unitarity of the scattering matrix \(SS^+ = 1\) implies, in principle, an existence at high energies \(s > s_0\), where \(s_0\) is a threshold\(^2\) of the new scattering mode — antishadow one. It has been revealed in [6] and described in some detail (cf. [8] and references therein) and the most important feature of this mode is the self-damping of the inelastic channels contributions at small values of impact parameter — antishadowing. The antishadowing leads to \(P(s, b = 0) \to 1\) at \(s \to \infty\), where \(P\) is a probability of the absence of the inelastic interactions, \(P(s, b) \equiv |S(s, b)|^2\), where \(S\) is the elastic scattering \(S\)-matrix.

Self-damping of the inelastic channels leads to asymptotically dominating role of elastic scattering. The cross-section of inelastic processes rises with energy as \(\ln s\), while elastic and total cross-sections behave asymptotically as \(\ln^2 s\). The antishadow scattering mode could definitely be observed at the LHC energies and studies of the extensive air showers originated from the cosmic particles interactions with the atmosphere provide evidence for it as we will argue in what follows. Starting at some threshold energy \(s_0\) (where amplitude reaches the black disk limit at \(b = 0\)), antishadowing can occur at higher energies in the limited region of impact parameters \(b < R(s)\) (while at large impact parameters only shadow scattering mode can be realized).

The inelastic overlap function \(\eta(s, b)\) becomes peripheral when energy goes beyond \(s = s_0\) (Fig.2). At such energies the inelastic overlap function reaches its maximum value at \(b = R(s)\), where \(R(s)\) is the interaction radius, while the elastic scattering occurs at smaller values of impact parameter, i.e. \(\langle b^2 \rangle_{el} < \langle b^2 \rangle_{inel}\). Note that

\[
\langle b^2 \rangle_i = \frac{1}{\sigma_i} \int_0^\infty b^2 \frac{d\sigma_i}{db^2},
\]

where \(i = tot, el, inel\) and

\[
\text{Im} f(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{tot}}{db^2}; \quad |f(s, b)|^2 \equiv \frac{1}{4\pi} \frac{d\sigma_{el}}{db^2}; \quad \eta(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{inel}}{db^2}
\]

and unitarity condition in the impact parameter space is the following

\[
\text{Im} f(s, b) = |f(s, b)|^2 + \eta(s, b),
\]

\(^2\)Model estimates show that new scattering mode starts to develop right beyond Tevatron energies, i.e. at \(\sqrt{s_0} \simeq 2\) TeV [9], which corresponds to the energy in the laboratory system \(E \simeq 2\) PeV.
where \( f(s, b) \) is the elastic scattering amplitude. The quantity \( \langle b^2 \rangle \) is a measure of the reaction peripherality. Despite that the asymptotics for \( \sigma_{el} \) and \( \sigma_{inel} \) are different, the quantities \( \langle b^2 \rangle_{el} \) and \( \langle b^2 \rangle_{inel} \) have the same asymptotical energy dependence, proportional to \( \ln^2 s \).

So, beyond the transition energy range there are two regions in impact parameter space: the central region where self-damping of inelastic channels occurs (antishadow scattering at \( b < R(s) \)) and the peripheral region of shadow scattering at \( b > R(s) \).

![Figure 2: Impact parameter dependence of the inelastic overlap function in the framework of the unitarization scheme with antishadowing. Arrows indicate the directions of movement of minimum at \( b = 0 \) and maximum at \( b = R(s) \) with the energy increase. In the region of \( b = R(s) \) the complete absorption takes place, i.e. \( |S(s, b = R(s))|^2 = 0 \).](image)

At the energies \( s \gg s_0 \) small impact parameter scattering is almost elastic one.

Thus head–on colliding particles will provide appearance of penetrating long-flying component in the EAS and such particles will spend only small part of their energy for the production of secondaries. The head-on collisions will lead to smaller number of secondary particle and it will provide faster decrease of the energy spectrum of cosmic rays, i.e. it will result in the appearance of the
knee. This qualitative picture will be explained in more detail in what follows. It should be noted that this effect has a threshold in the energy dependence. It is also important to note that due to small probability of the sequential head-on collisions the number of events with penetrating particles also should be small. Nonetheless, such events have been observed in the experiments PAMIR [10].

Antishadowing leads to suppression of particle production at small impact parameters:

\[ \bar{n}(s) = \frac{1}{\sigma_{inel}(s)} \int_0^\infty \bar{n}(s, b) \frac{d\sigma_{inel}}{db^2} \, db^2, \]  
\[ (1) \]
i.e. multiplicity distribution

\[ P_n(s, b) \equiv \frac{1}{\sigma_{inel}(s)} \frac{d\sigma_n(s)}{db^2} \]

and mean multiplicity \( \bar{n}(s, b) \) in the impact parameter representation have no absorptive corrections, but peripherality of \( d\sigma_{inel}/db^2 \) leads to suppression of particle production at small impact parameters and the main contribution to the integral multiplicity \( \bar{n}(s) \) comes from the region of \( b \sim R(s) \) (Eq. (1)). This would lead to the events with alignment observed in EAS and also to the imbalance between orbital angular momentum in the initial and final states since particles in the final state will carry out large orbital angular momentum. To compensate this orbital momentum spins of secondary particles should become lined up, i.e. the spins of the produced particles should demonstrate significant correlations when the anti-shadow scattering mode appears [11]. Thus, the observed phenomena of alignment in EAS [5] and predicted spin correlations of final particles should have a common origin. The model estimate for the primary energy when these phenomena should appear is \( E_0 \approx 2 \, P eV \) — \( E_0 \) is the energy when the new unitarity regime starts to develop at small impact parameters.

The detected particle composition of the EAS is closely related to the quantity known as gap survival probability. Antishadowing leads to the nonmonotonous energy dependence of this quantity [12]. The gap survival probability, namely the probability to keep away inelastic interactions which can result in filling up by hadrons the large rapidity gaps, reaches its minimal values at the Tevatron highest energy and this is due to the fact that the scattering at this energy is very close to the black disk limit at \( b = 0 \) (Fig. 3). It is clear that its higher value means higher fraction for diffractive component and consequently the increasing of this component would result in the enhancement of the relative fraction of protons in the observed cosmic rays spectrum. Otherwise, decreasing of this quantity will lead to
increase of pionization component and consequently to the increasing number of muons observed as multi-muon events. Experiment reveals that relative fraction of protons in cosmic rays also shows nonmonotonous energy dependence (cf. Fig. 4). To explain such dependence an additional component is introduced \textit{ad hoc} at the energies above $3 \cdot 10^7$ GeV. It was shown that account of the antishadowing makes an introduction of this \textit{ad hoc} component unnecessary.

The inelasticity parameter $K$, which is defined as ratio of the energy going to inelastic processes to the total energy, is important for the interpretation of the EAS cascades developments. Its energy dependence is not clear and number of models predict the decreasing energy dependence while other models insist on the increasing energy behaviour at high energies (cf. e.g. \cite{13}). Adopting simple ansatz of geometrical models where parameter of inelasticity is related to inelastic overlap function we can use the following equation for $\langle K \rangle$ \cite{14}

$$\langle K \rangle = 4 \frac{\sigma_{el}}{\sigma_{tot}} \left( 1 - \frac{\sigma_{el}}{\sigma_{tot}} \right)$$

to get a qualitative knowledge on the inelasticity energy dependence. The estimation of inelasticity based on the particular model with antishadowing \cite{9} leads to increasing dependence of inelasticity with energy till $E \simeq 4 \cdot 10^7$ GeV. In this region inelasticity reaches maximum value $\langle K \rangle = 1$, since $\sigma_{el}/\sigma_{tot} = 1/2$ and then starts to decrease at the energies where this ratio goes beyond the black disk limit $1/2$. Such qualitative nonmonotonous energy dependence of inelasticity is
the result of transition to the antishadowing scattering regime. The distribution on the inelasticity is related to the distribution on the effective mass number, i.e. changes of $A$ are equivalent to changes of $\langle K \rangle$, and, for example, high-inelasticity primary proton interaction produces the same result at the ground level as the low-inelasticity primary interaction of the heavy nuclei [15]. The available experimental data on the average logarithm of the effective nuclear mass number, extracted from the energy dependence of the depth of EAS maximum, have large error bars, but they also indicate a nonmonotonous energy dependence with the maximum in the region $E \simeq (4 - 5) \cdot 10^7$ GeV [7].

It is also worth to note that the maximum in inelasticity energy dependence, when the pionization component is maximal, is correlated with the minimum of the relative component of protons in the EAS, the following simple relation can be supposed

$$\Phi_p/\Phi_{all} \sim 1 - \langle K \rangle$$

i.e. the relative proton component in the detected EAS should have a non-monotonic energy dependence and this is in agreement with the experimental analysis represented in Fig. 4.

![Graph](image.png)

Figure 4: Relative fraction of protons in EAS, figure is taken from [7].

It should be noted that the behaviour of the ratio $\sigma_{el}/\sigma_{tot}$ when it goes to unity at $s \to \infty$ does not imply decreasing energy dependence of $\sigma_{inel}$. The
inelastic cross-section $\sigma_{\text{inel}}$ increases monotonically and it grows as $\ln s$ at $s \to \infty$. Such a dependence of $\sigma_{\text{inel}}$ is in good agreement with the experimental data and, in particular, with the observed falling slope of the depth of shower maximum distribution [16]. The predicted numerical value of the inelastic cross-section is $\sigma_{\text{inel}}(s) \simeq 76 \text{ mb}$ at the LHC energy $\sqrt{s} = 14 \text{ TeV}$. This value is also in a good agreement with the value for this quantity extracted from the proton-air inelastic cross-section [7]. This approach provides a reasonable description [17] of the energy dependence of mean multiplicity and leads to its power-like growth with a small exponent.

The relation of the knee and other effects observed in the EAS measurements with the modification of particle generation mechanism is under discussion since the time when they were discovered. We propose here one particular realization of this idea — an approach where the corresponding particle generation mechanism in EAS is strongly affected by the unitarity effects and the energy region between the knee and the ankle coincides with the transition region to the scattering mode where antishadowing develops at small and then at moderate values of impact parameter, i.e. the energy spectrum of the primary cosmic particles $F_0(E)$ is modulated by the significant variation of the scattering matrix $S$ in the energy region starting from about $E_1 \simeq 10^6 \text{ GeV}$ and finishing at about $E_2 \simeq 10^9 \text{ GeV}$ and this resulting in the regularities in the observed spectrum $F(E)$ measured in the EAS studies. Below the energy $E_1$ and beyond the energy $E_2$ variation of scattering matrix is slow and the primary energy spectrum $F_0$ is almost not affected. It seems to be a rather natural explanation of the observed regularities in the EAS measurements and has a close interrelation with the nonmonotonous energy dependence of gap survival probability and inelasticity. This hypothesis is based on the saturation of the unitarity and can be experimentally checked at the LHC [8]. The studies of the proton scattering in the forward region at the LHC will be very helpful for improving the interpretation of the results of the cosmic rays experiments.

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