The Fate of Flat Directions in Higher Derivative Gravity

Nabamita Banerjee\textsuperscript{a,*}, Suvankar Dutta\textsuperscript{b†} and Ivano Lodato\textsuperscript{a‡}

\textsuperscript{a}Nikhef Theory Group, Science Park 105, 1098 XG Amsterdam, The Netherlands
\textsuperscript{b}Dept. of Physics, Indian Institute of Science Education and Research Bhopal, Bhopal 462 023, India

Abstract: We discuss the fate of flat directions in higher derivative gravity by studying two explicit examples, namely higher derivative gauged supergravity in five dimensions and higher derivative type IIB string theory in ten dimensions. In the first case, the supersymmetric spinning black hole solution in asymptotically $\text{AdS}$ spacetime, found by Gutowski and Reall, is analyzed. In this case we find that the flat direction at the two derivative level is not lifted after addition of higher derivative terms, and as it turns out, this result holds even for non-supersymmetric deformations of the higher derivative action. For the rotating D3-brane solutions in type IIB theory, the dilaton parametrizes a flat direction at leading order, but its fate changes upon including order $(\alpha')^3$ supersymmetric higher derivative corrections to the type IIB action, i.e. its leading value gets fixed.

\textsuperscript{*}nbaner@nikhef.nl
\textsuperscript{†}suvankar@iiserb.ac.in
\textsuperscript{‡}ilodato@nikhef.nl
1. Introduction and summary

According to the attractor mechanism, the near horizon field configuration of an extremal black hole is insensitive to the asymptotic data on scalar fields of the theory. Also, many moduli fields of the theory are fixed at the horizon, while others remain unfixed, meaning that the black hole entropy does not depend on them. The attractor mechanism has been observed for asymptotically flat as well as asymptotically $AdS$ black holes and in theories with higher derivative interactions, and there is a long list of papers where this subject has been studied widely [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. Here, we summarize the main points, that will be play a role in our analysis:

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1. Let us consider a general theory of gravity, coupled to abelian gauge fields, neutral scalar fields and p-form gauge fields, with a local Lagrangian density, which is invariant under gauge and general coordinate transformation. Suppose the theory admits a rotating or spherically symmetric extremal black hole solution. It has been proven \cite{22, 23} that the entropy of this black hole remains invariant under continuous deformation of the asymptotic data for the moduli fields\(^1\).

It is important to stress that this result does not depend on the supersymmetries the solution preserves but relies only on the existence of an \(AdS_2\) component in the near-horizon geometry. This allows us to define the “entropy function” \cite{22} of the theory, and by extremizing it, find the explicit values of the near-horizon parameters.

In presence of higher derivative terms, even finding a generic solution of the full theory is a non-trivial task. However we can restrict our attention on the subclass of extremal black hole solutions that are not destabilized by higher derivative terms, i.e. they still admit an \(AdS_2\) component in the near-horizon geometry. If that is the case, then we can expect the results for the attractors at the two derivative level to hold even for a covariant theory of higher derivative gravity.

2. **Flat direction:** As we already pointed out, the attractor equations fix some of the moduli fields at the horizon in terms of the black hole charges. On the other hand, it is possible that certain moduli fields cannot be fixed by extremization, meaning the entropy function has a series of degenerate stationary points. In that case, the entropy will be independent of the near-horizon values of these moduli, that we will refer to as flat directions.

The existence of flat directions is strictly related to the (super)symmetries preserved by the solution, and it’s likely that the same symmetries will completely constrain the behaviour of flat directions even when higher derivative terms are considered. It is however possible that the specific form of the higher derivative interactions, and, as a consequence, its symmetries, might influence the fate of flat directions in higher derivative gravity.

Generically, we expect that if the two derivative theory has a BPS black hole solution with a flat direction, then supersymmetry will protect the structure of the near horizon geometry and the flat direction will not be lifted, when supersymmetric higher derivative

\(^1\)It has been observed that there can be discrete jumps because of multi-centered black holes.
interactions are considered. On the other hand, nothing can be said a priori for non-BPS solutions. To confirm our expectations for the BPS case and obtain some knowledge about the non-BPS case, we study two concrete examples:

1. We consider five dimensional minimal gauged supergravity in presence of higher derivative corrections. There exists an asymptotically AdS, supersymmetric solution of two derivative gravity \[26\] that preserves 1/4 of the supersymmetries and the near-horizon geometry of this solution is specified in terms of a single parameter \(\Delta\). The five dimensional field content is given by the metric and one gauge field. We perform a dimensional reduction on this five dimensional geometry over a circle (\(\psi\) direction), obtaining a larger field content including the metric, two gauge fields and two scalars. We write down the entropy function and the attractor equations in four dimensions and find that the equations are satisfied without any knowledge of one scalar field and, moreover, the entropy is independent of its near-horizon value. This means that this scalar field is a flat direction of our theory. We explicitly check that the flat direction remains flat, even in presence of higher derivative terms in the Lagrangian. The details are studied in section 2.

2. Next, we consider rotating \(D3\) brane solution in type IIB string theory, which does not preserve any supersymmetry. The solution admits an extremal limit and the near-horizon geometry has an \(AdS_2\) part. The near-horizon value of dilaton does not appear in the entropy function, or in the entropy, therefore it is a flat direction of the theory. When higher derivative terms are considered in the action, the fate of dilaton changes, i.e. its near-horizon value gets fixed, but remains independent of physical charges. The details are studied in section 3.

Finally, we end the paper by outlining possible extensions and future projects in section 4. We hope to report on these questions in future. The paper also contains four appendices, where all the technical details are provided. Appendices A and B contain the details of five dimensional Noether charges and Kaluza-Klein reduction formulae. In appendix C we explain some issues concerning the higher derivative corrections of the five dimensional near-horizon geometry. In the last appendix D we analyze explicitly the solution of the dilaton equation of motion and give the value of the higher derivative invariant as a function of the near-horizon parameters.
2. Flat direction in five dimensional theory

Not long ago all possible purely bosonic supersymmetric solutions of minimal gauged supergravity in 5 dimensions were classified [25], using the properties of the Killing spinors. These solutions are known to preserve $1/4$ of supersymmetries\(^2\) (only 2 supercharges in the minimal theory) and of course, they solve the equations of motion arising from the minimal gauged supergravity action [24, 25]. Analyzing all possible near horizon geometries of these supersymmetric solutions, Gutowski and Reall [26] were able to find an one-parameter family of black hole solutions, which has a spatially compact horizon (squashed $S^3$) and is (globally) asymptotically $AdS_5$, in contrast with the ungauged case where the near horizon geometry of a BPS solution is always maximally symmetric. We work in a suitable coordinate system, where the $AdS_2$ part of the near horizon geometry is manifest [27]. The metric, the $U(1)$ gauge field and its field strength have the following form:

\[
\begin{align*}
\text{ds}^2 &= v_3 \left( B \cos \theta d\chi + \frac{\epsilon_0}{r} dr + \epsilon_0 r dt + d\psi \right)^2 + v_2 \left( d\theta^2 + \sin^2 \theta d\chi^2 \right) + v_1 \left( \frac{dr^2}{r^2} - r^2 dt^2 \right), \\
A &= (\epsilon_0 \varphi + \epsilon_1) r dt - (B \varphi + P) \cos \theta d\chi + \frac{\epsilon_0}{r} dr + \varphi d\psi, \\
F &= (\epsilon_0 \varphi + \epsilon_1) dr \wedge dt + (B \varphi + P) \sin \theta d\theta \wedge d\chi.
\end{align*}
\]

We consider, in the following, minimal gauged supergravity theory coupled to a single $U(1)$ gauge field, including some supersymmetric higher derivative terms. These higher derivative terms are all four derivatives and they are related to mixed gauge gravitational Chern-Simons term by supersymmetry. The supersymmetric completion of this term was first found in [28], using the superconformal formalism, which gives a complete off-shell result. An on-shell version of these higher derivative supersymmetric invariants was derived later in [29], by integrating out all the auxiliary fields. The action obtained in this method, however, can be reduced further, by means of partial integrations, field redefinitions and Bianchi identities [30]. Finally one can show that the action includes only five bosonic higher derivative terms

\[
S_5 = \int d^5x \sqrt{-g} \left[ \hat{R} + \frac{12}{L^2} - \frac{\hat{F}^2}{4} + \frac{\kappa}{3} e^{\mu \nu \rho \sigma \delta} A_\mu \hat{F}_\nu \hat{F}_\rho \hat{F}_\sigma \hat{F}_\delta + L^2 \left( c_1 \hat{R}_{\mu \nu \rho \sigma} \hat{R}^{\mu \nu \rho \sigma} \\
+ c_2 (\hat{F}^2)^2 + c_3 \hat{F}_\mu \hat{F}_\nu \hat{F}_\sigma \hat{F}_\rho \hat{F}_\sigma \hat{F}_\rho + c_4 \hat{R}_{\mu \nu \rho \sigma} \hat{F}^{\mu \nu} \hat{F}^{\rho \sigma} + c_5 e^{\mu \nu \rho \sigma \delta} \hat{A}_\mu \hat{R}_{\nu \rho \sigma \eta} \hat{R}_{\delta \gamma} \right) \right],
\]

\(^2AdS_5\) is the only maximally supersymmetric solution of the gauged theory.
where the supersymmetric values of the coefficients $c_2, c_3, c_4, c_5$ and $\kappa$ are given, in terms of $c_1$, by:

$$
\kappa = \frac{1}{4\sqrt{3}} (1 - 288c_1), \quad c_2 = \frac{c_1}{24}, \quad c_3 = -\frac{5c_1}{24}, \quad c_4 = -\frac{c_1}{2}, \quad c_5 = \frac{c_1}{2\sqrt{3}}.
$$

(2.3)

Notice that the action (2.2) is not gauge invariant, due to the presence of Chern-Simon terms. In the following sub-section we deal with this issue, obtaining a generalization of a result known at the two derivative level for asymptotically AdS black holes: a relation between the 5 dimensional and the reduced, 4 dimensional, black hole charges.

2.1 Black hole charges

In order to obtain any knowledge on the behaviour of the moduli fields of the theory under consideration, we want to make use of Sen’s entropy function formalism [10], which, however, is applicable only to gauge and diffeomorphism invariant theories, unlike (2.2). To circumvent this problem, we dimensionally reduce our five dimensional theory over a circle, obtaining a four dimensional gauge invariant action (details about the reduction are presented in sub-section 2.1.2 and appendix B). Even so, the entropy of the reduced 4D black hole solution will depend on the four dimensional charges, thus we must first find a relation linking the lower and higher dimensional charges. In this way we can still get the entropy of the “original” black hole solution as a function of the five dimensional charges. Now, as it turns out, at two derivative level it was proven that the 5D and 4D black hole charges are exactly equivalent (for 5-dimensional AdS solutions this result was first found in [31]). Obviously, in our case, we would like to find a relation between the black hole charges when higher derivative interactions are considered. While the answer is known for asymptotically flat BPS black hole [32, 33, 34], where a mismatch, due to the gravitational Chern-Simon term, was found among the charges, in the asymptotically AdS case, no one (to the best of our knowledge) has studied this relation. In the following three sub-sections we intend to fill this gap. The construction is generic and in particular does not assume the supersymmetric values of the various coefficients mentioned in (2.3).

2.1.1 5 dimensional charges - Noether potential

It is well known that for a given local invariance of the action there exists a current $J^\mu$, conserved on-shell:

$$
\partial_\mu J^\mu(\xi, \phi) = -E_i \delta \phi^i
$$

(2.4)

where $E_i$ indicates the equations of motion for the collection of fields $\phi^i$, and $\xi$ is the local parameter of the invariance transformation. Imposing on-shell conditions, the current is
conserved and it can be re-written as a total derivative of a second rank antisymmetric tensor $Q_{\mu\nu}$, called Noether potential

$$J^\mu = \partial_\mu Q^{\mu\nu}(\xi, \phi).$$

(2.5)

The integral of this object over the horizon of black hole will give us its associated charges.

Let’s start by calculating the current corresponding to abelian gauge invariance of the action. The equations of motions obtained from the Lagrangian (2.2) are as follows

$$E_A^\mu = \nabla_\nu \left( - \hat{F}^{\mu\nu} + 8c_2(\hat{F}^2)\hat{F}^{\mu\nu} - 8c_3 \hat{F}^{\mu\rho}\hat{F}^{\sigma\rho} + 4c_4 \hat{R}^{\mu\rho\sigma}\hat{F}_{\rho\sigma} \right) + \epsilon^{\mu\rho\sigma\tau} \left( \kappa \hat{F}_{\nu\rho} \hat{F}_{\sigma\tau} + c_5 \hat{R}_{\nu\rho\sigma\tau} \hat{R}_{\rho\sigma} \right)$$

$$E_g^{\alpha\beta} = \frac{1}{2}(\mathcal{L} - \frac{\kappa}{3} \epsilon^{\mu\rho\sigma\tau} \hat{A}_\mu \hat{F}_{\nu\rho} \hat{F}_{\sigma\tau} - c_5 \epsilon^{\mu\rho\sigma\tau} \hat{A}_\mu \hat{R}_{\nu\rho} \hat{R}_{\sigma\tau}) \hat{g}^{\alpha\beta}$$

$$- \hat{R}^{\alpha\beta} + \frac{1}{2} \hat{F}^{\alpha\gamma} \hat{F}_{\gamma}^{\beta} + c_1 \left( - 2 \hat{R}^{(\alpha\mu\rho\sigma)} \hat{R}^{\beta}_{\nu\rho\sigma} + 4 \nabla_\rho \nabla_\sigma \hat{R}^{(\alpha\sigma\beta)} \right)$$

$$- 4c_2 \hat{F}^{\alpha(\gamma} \hat{F}^{\beta)} - 4c_3 \hat{F}^{(\alpha} \hat{F}_\gamma \hat{F}^{\lambda\rho} \hat{F}^{\beta)} + c_4 (3\hat{R}_{\nu\rho\sigma}(\alpha \hat{F}^{\beta)} \hat{F}^{\mu\rho} + 2 \hat{\nabla}_\rho \hat{\nabla}_\sigma (\hat{F}^{(\alpha} \hat{F}^{\beta)})$$

$$+ 2c_5 \epsilon^{\mu\rho\sigma(\alpha} \left( \hat{\nabla}_\lambda \hat{F}_{\mu\nu} \hat{R}_{\rho\sigma}^{\lambda}\beta) + 2 \hat{F}_{\mu\nu} \hat{\nabla}_\rho \hat{F}_{\sigma}^{\beta) \right).$$

(2.6)

Next step is to evaluate the Noether potential $Q^{\mu\nu}$. For this, we would need to add suitable improvement terms, to cast the current in a form proportional to the symmetry variation $\delta \xi A_\mu = \partial_\mu \xi$ of the Lagrangian density. Due to this variation we get the following conserved gauge current $J^\mu$,

$$J^\mu = \sqrt{-g} \left( - \hat{F}^{\mu\nu} + \frac{4}{3} \kappa \epsilon^{\mu\rho\sigma\tau} \hat{A}_\rho \hat{F}_{\sigma\tau} + 8c_2(\hat{F}^2)\hat{F}^{\mu\nu} + 8c_3 \hat{F}^{\mu\rho}\hat{F}_{\rho\sigma} \hat{F}^{\sigma\mu} \right) \hat{\nabla}_\nu \xi + 4c_4 \hat{R}^{\mu\rho\sigma}\hat{F}_{\rho\sigma}$$

$$- \xi \epsilon^{\mu\rho\sigma\tau} \left( \frac{\kappa}{3} \hat{F}_{\nu\rho} \hat{F}_{\sigma\tau} + c_5 \hat{R}_{\nu\rho\sigma\tau} \hat{R}_{\rho\sigma} \right).$$

(2.6)

Now, consider a symmetry variation $\delta \xi A_\mu = \partial_\mu \xi$ of the Lagrangian density. Due to this variation we get the following conserved gauge current $J^\mu$,

$$E_A^\mu = \nabla_\nu Q^{\mu\nu}.$$

Now, it is obvious that $Q^{\mu\nu}$ is a conserved quantity when the equation of motion for the gauge field are imposed. Furthermore its integral over the horizon corresponds to the black hole electric charge associated to gauge symmetry, i.e. the electric charge:

$$Q_{5D} = \int d\Sigma_{\mu\nu} Q^{\mu\nu}$$

(2.7)
where \( d \Sigma_{\mu
u} = dS \sqrt{h} \epsilon_{\mu\nu} \), \( \sqrt{h} \) is the determinant of the induced metric on the null surface \( S \) of the horizon, and the tensor \( \epsilon_{\mu\nu} \) is the binormal on that surface, satisfying the normalization condition \( \epsilon_{\mu\nu} \epsilon^{\mu\nu} = -2 \), with only one non-zero component, \( \epsilon_{tv} = v_1 \). After some manipulation, using the Bianchi identities for the field strength and the Riemann tensor, we obtain:

\[
Q^{\mu\nu} = - \hat{F}^{\mu\nu} + 2K \epsilon^{\mu\nu\rho\sigma\tau} \hat{A}_\rho \hat{F}_{\sigma\tau} + 8c_2 (\hat{F}^2) F^{\mu\nu} - 8c_3 \hat{F}^{\mu\rho} F_{\rho\sigma} \hat{F}^{\sigma\nu} + 4c_4 \hat{R}^{\mu\nu\rho\sigma} F_{\rho\sigma}
- 4c_5 \epsilon^{\mu\nu\rho\sigma\tau} \left( \hat{\Gamma}_{\rho\beta}^\alpha \hat{\partial}_{\sigma} \hat{\Gamma}_{\tau\alpha}^\beta + \frac{2}{3} \hat{\Gamma}_{\rho\beta}^\alpha \hat{\Gamma}_{\sigma\gamma}^\beta \right).
\]

(2.8)

Now, using the above expression of Noether potential in (2.7), we can compute the five dimensional conserved electric charge for the background (2.4). The result is quite long: we present it in appendix A.

The evaluation of the angular momentum is completely analogous. The full action (2.2) is invariant under diffeomorphism and the current associated to this invariance reads:

\[
J^\mu (\xi, A, g) = \left( - \hat{F}^{\mu\nu} + 4K \epsilon^{\mu\nu\rho\sigma\tau} \hat{A}_\rho \hat{F}_{\sigma\tau} + 4c_4 \hat{R}^{\mu\nu\rho\sigma} \hat{F}_{\rho\sigma}
+ 8c_2 (\hat{F}^2) \hat{F}^{\mu\nu} + 8c_3 \hat{F}^{\mu\rho} \hat{F}_{\rho\sigma} \hat{F}^{\sigma\nu} \right) \left( \xi^\lambda \hat{F}_{\lambda\nu} + \nabla_\nu (\xi^\lambda \hat{A}_\lambda) \right)
- 2 \left( g^{[\nu}[\xi] g^{\rho]\mu] + 2c_5 \epsilon^{\mu\alpha\beta\sigma\tau} \hat{A}_\tau \hat{R}_{\alpha\beta\rho\sigma} + 2c_1 \hat{R}^{\mu\rho\sigma} + c_4 \hat{F}^{\mu\nu} \hat{F}^{\rho\sigma} \right) \nabla_\rho \left[ 2 \nabla_\nu (\xi_\nu) \right]
+ 2 \nabla_\rho \left( 2c_5 \epsilon^{\alpha\beta\rho\sigma\tau} \hat{A}_\tau \hat{R}_{\alpha\beta\mu\nu} + 2c_1 \hat{R}^{\mu\rho\sigma} + c_4 \hat{F}^{\mu\nu} \hat{F}^{\rho\sigma} \right) \left( 2 \nabla_\nu (\xi_\nu) \right)
- \xi^\mu \mathcal{L}^\prime \right)
\]

(2.9)

where, \( \xi^\mu \) is the rotational Killing vector of the black hole spacetime. Now, to extract a total derivative from the current, we add a linear combination of the equations of motion of \( E_A^\mu \) and \( E_g^{\mu\nu} \). As it can be easily understood, adding these term will not alter in any way the final physical result, since they vanish on-shell. The correct combination we use to extract the Noether potential for diffeomorphism is:

\[
J^\mu + 2E_g^{\mu\nu} \xi_\nu + (\xi \cdot A) E_A^\mu = \nabla_\nu \Theta^{\mu\nu}
\]

(2.10)

\(^3\)The notation we use for the binormal and the antisymmetric epsilon tensor are similar, but they are tensors of different ranks.
where
\[
\Theta^{\mu\nu} = \left( -\hat{F}^{\mu
u} + 4c_4 \hat{R}^{\mu\nu\rho\sigma} \hat{F}_{\rho\sigma} + 8c_2 (\hat{F}^2) \hat{F}^{\mu\nu} - 8c_3 \hat{F}^{\mu\rho} \hat{F}_{\rho\sigma} \hat{F}^{\sigma\nu} + 4\frac{\kappa}{3} e^{\mu\nu\rho\tau} \hat{A}_\rho \hat{F}_{\sigma\tau} \right) (\xi \cdot A) \\
+ c_5 \left( 4e^{\mu\rho\alpha\beta} \hat{A}_\rho \hat{R}_{\alpha\beta} \nu \tau \hat{\nabla}_\nu \xi_\sigma + 2e^{\mu\rho\alpha\beta} \hat{F}_{\rho\alpha} \hat{R}_{\nu\beta} \mu \tau \xi_\sigma + 4e^{\rho\sigma\alpha\beta} (\hat{F}_{\rho\sigma} \hat{F}_{\nu\tau}) \xi_\mu \right) \\
- 2\hat{g}^{\nu[\rho} \hat{g}^{\sigma\mu]} \hat{\nabla}_\rho \xi_\sigma + c_1 \left( - 4\hat{R}^{\mu\rho\sigma} \hat{\nabla}_\rho \xi_\sigma + 8\hat{\nabla}_\rho (\hat{R}^{\mu\rho\sigma}) \xi_\sigma \right) \\
+ c_4 \left( - 2\hat{F}^{\mu\rho} \hat{F}_{\rho\sigma} \hat{\nabla}_\rho \xi_\sigma + 2\hat{\nabla}_\rho (\hat{F}^{\mu\rho} \hat{F}^{\nu\sigma}) \xi_\sigma + 4\hat{\nabla}_\rho (\hat{F}^{\mu[\sigma} \hat{F}^{\rho\nu]}) \xi_\sigma \right). \tag{2.11}
\]

With the above expression of the Noether potential, the conserved angular momentum can be computed using:
\[
\Theta_{5D} = \int d\Sigma_{\mu\nu} \Theta^{\mu\nu}. \tag{2.12}
\]

Again, the result is presented in appendix A.

2.1.2 4 dimensional charges - Dimensional reduction and entropy function

In this subsection, we determine the four dimensional charges for the corresponding four dimensional system using the entropy function formalism. The derivation is, again, completely generic as it only depends on the form of the near horizon solution presented in (2.1) and does not assume any particular value for any near horizon parameters.

As explained above, to apply entropy function, we need gauge invariant Lagrangian and thus, we first need to perform a Kaluza-Klein reduction of the action (2.2). We take the following ansatz for the metric and gauge field for the reduction,
\[
ds^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = h_{ij} dx^i dx^j + \phi(r)(d\psi + A_{kk})^2, \\
\hat{A}_\mu = A_i dx^i + \sigma(r)(d\psi + A_{kk}) \tag{2.13}
\]

and compactify along $\psi$ direction. We denote $A_{kk}$ as the Kaluza-Klein (KK) gauge field and the corresponding field strength is given by $(F_{kk})_{ij} = \partial_i (A_{kk})_j - \partial_j (A_{kk})_i$. The four dimensional gauge field is $A$ and the corresponding field strength is $F$. All un-hatted curvature quantities are composed of the four dimensional metric $h_{ij}$. The reduction of gauge invariant terms in the action is straightforward (appendix B). On the other hand, the reduction of the two Chern-Simons terms, which are gauge non-invariant, is tricky, and requires the addition of some total derivatives terms. Here, we present only the reduced four dimensional action corresponding to these two gauge non-invariant terms.

Specifically the two derivative gauge Chern-Simon term takes the following form
\[
\frac{\kappa}{3} \int \sqrt{-g} e^{\mu\nu\rho\sigma} \hat{A}_\mu \hat{F}_{\nu\rho} \hat{F}_{\sigma\tau} = 4\pi \kappa \int d^4 x \sqrt{-h} e^{abcd} \left[ \frac{1}{3} \sigma^3 (F_{kk})_{ab} (F_{kk})_{cd} + \sigma^2 (F_{kk})_{ab} F_{cd} + \sigma F_{ab} F_{cd} \right]. \tag{2.14}
\]
while the four derivative mixed Chern-Simon term reads:

\[ c_5 \int \sqrt{-g} \epsilon^{\mu
u\rho\delta} \hat{A}_\mu \hat{R}_\nu \alpha^\beta \hat{R}_{\sigma\delta\alpha\beta} = c_5(T_1 - 4T_2), \]  

(2.15)

where, \( T_1 \) and \( T_2 \) are given by,

\[
T_1 = 4\pi \int d^4x \sqrt{-\epsilon} \epsilon^{ijkl} \left[ \sigma \left( R^{mn}_{\ ij}(R_{klmn} - \Phi((F_{kk})_{km}(F_{kk})_{ln} + (F_{kk})_{kl}(F_{kk})_{mn})) \right) \\
\quad + \frac{\Phi^2}{4} \left( (F_{kk})_{ij}(F_{kk})_{kl}(F_{kk})^2 - 2(F_{kk})_{ij}(F_{kk})_{km}(F_{kk})^{mn}(F_{kk})_{nl} \right) \\
\quad + \frac{\Phi}{2} \left( \nabla_m(F_{kk})_{ij} \nabla^n(F_{kk})_{kl} \right) \right] 
\]

\[
T_2 = 2\pi \int d^4x \sqrt{-\epsilon} \epsilon^{ijkl} F_{kl} \left[ \frac{\Phi}{2} R^{mn}_{\ ij}(F_{kk})_{mn} + \frac{\Phi^2}{4} (F_{kk})_{im}(F_{kk})_{jn}(F_{kk})^{mn} - \frac{\Phi^2}{8} (F_{kk})_{ij}(F_{kk})^2 \right]
\]

and \((F_{kk})^2 = (F_{kk})_{ab}(F_{kk})^{ab}\). The periodicity of the compact direction is \(4\pi\).

Now, the four dimensional action obtained is gauge invariant so that we can apply the entropy function formalism. Our ansatz for near-horizon metric and gauge field is (2.1).

From the four dimensional point of view we have two gauge fields, one coming from usual five dimensional gauge field and the other coming from the metric components \(g_{\psi\mu}\) (Kaluza-Klein gauge field): to each of those gauge fields corresponds a charge, respectively \(Q\) and \(\Theta\), to which we associate a charge parameter, \(e_1\) and \(e_0\) respectively.

The entropy function is, then, defined as follows:

\[ E = 2\pi(Qe_1 + \Theta e_0 - \bar{L}) \]  

(2.16)

where, \(\bar{L}\) is given by,

\[ \bar{L} = \frac{8\pi^2}{16\pi G_5} \int d\theta d\chi \mathcal{L}_4 \]  

(2.17)

and \(\mathcal{L}_4\) is the reduced four dimensional Lagrangian, including the higher derivative terms.

The attractor equations are obtained by minimizing the entropy function with respect to the near-horizon parameters, while the four dimensional physical charges are given by:

\[ Q = \frac{\partial \bar{L}}{\partial e_1}, \quad \Theta = \frac{\partial \bar{L}}{\partial e_0}. \]  

(2.18)

Calculating \(\bar{L}\) over the near-horizon geometry (2.1), we find the expression for the four dimensional physical charges. The proper four dimensional charges are rescaled as in [10] to \(\tilde{Q} = 2Q\), \(\tilde{\Theta} = 2\Theta\).
2.1.3 Relation Between 5D-4D Charge

Comparing the five dimensional charges with the corresponding four dimensional charges, we find a complete match at two derivative level. However, as already seen for asymptotically flat black hole solutions, they differ at the four derivative level and the difference is proportional to $c_5$ only (gravitational Chern-Simon term). We also find that the relation between five-dimensional and four-dimensional charge remains exactly the same as in the case of asymptotically flat black holes. For the angular momentum $\Theta_{5D}$, our result is more generic than the one in [35], as in our case the black hole can carry an extra parameter $P$.\footnote{\(P\) can be thought of as a magnetic field.} Thus, we see that the difference between five and four dimensional charges is purely a topological effect due to the Chern-Simon term which is not gauge invariant. The details can be found in [35]. The bottom line of our analysis is that the asymptotic geometry of the space time is not relevant to account for the differences in the physical charges. The relations between 4D and 5D charges for asymptotically $AdS$ black holes reads:

\[
\tilde{Q} = -Q_{5D} + \frac{8\pi B v_3}{G v_2} c_5, \quad \tilde{\Theta} = -\Theta_{5D} + \frac{4\pi P v_3^2(c_0^2 v_2^2 - B^2 v_2^2)}{G v_1^2 v_2^2} c_5 \tag{2.19}
\]

These expressions are one of the main results of our paper and constitute a generalization of previous work [31] to higher-derivative gravity theory.

2.2 Flat directions in higher derivative gravity

So far we have studied the generic relation between four dimensional and five dimensional charges. Now we concentrate on a particular class of supersymmetric solutions in five dimensions and find that it exhibits a flat direction. Our goal is to study the fate of this flat direction when higher derivative interactions are taken into account.

We consider, in the following, the supersymmetric asymptotically $AdS_5$ black hole solution presented by Gutowski and Reall [26]. This solution is 1/4 BPS (preserves 2 supercharges) and its near-horizon geometry has an $AdS_2$ component. We use the coordinates that make the $AdS_2$ part of the near horizon geometry manifest [27],

\[
d s^2 = \frac{1}{\Delta^2 + 9L^2} \left( \frac{dr^2}{r^2} - dt^2 r^2 \right) + \frac{1}{\Delta^2 - 3L^2} \left( d\theta^2 + d\chi^2 \sin^2(\theta) \right) + \left( \frac{\Delta}{\Delta^2 - 3L^2} \right)^2 \left( d\psi + \cos \theta d\chi - \frac{3r}{L\Delta} \frac{\Delta^2 - 3L^2}{\Delta^2 + 9L^2} \left( dt + \frac{dr}{r^2} \right) \right)^2
\]

\[
F = \frac{\sqrt{3}\Delta}{\Delta^2 + 9L^2} dr \wedge dt + \frac{\sqrt{3} \sin \theta}{L(\Delta^2 - 3L^2)} d\theta \wedge d\chi \tag{2.20}
\]
Comparing this solution with near-horizon ansatz given in (2.1) we find the following values of the near-horizon parameters

\[ v_1 = \frac{1}{\Delta^2 + 9L^{-2}}, \quad v_2 = \frac{1}{\Delta^2 - 3L^{-2}}, \quad v_3 = \left( \frac{\Delta}{\Delta^2 - 3L^{-2}} \right)^2, \]
\[ e_0 = -\frac{3}{L} \frac{\Delta^2 - 3L^{-2}}{\Delta^2 + 9L^{-2}}, \quad B = 1, \quad e_5 = e_1 + e_0\varphi = \frac{\sqrt{3}\Delta}{\Delta^2 + 9L^{-2}}, \]
\[ A_\chi = P + B\varphi = \frac{\sqrt{3}}{L(\Delta^2 - 3L^{-2})}. \] (2.21)

The leading attractor equations (i.e. derived from the two derivative action) are given by,

\[ v_1 \text{ equation : } v_1^2 \left( B^2 (\varphi^2 + v_3^2) - 2\Lambda v_2^2 - 4v_2 \right) + 2BP\varphi v_1^2 + v_2^2 (e_0^2 (\varphi^2 + v_3^2) + 2e_1 e_0\varphi + e_1^2) + P^2 v_1^2 = 0 \]
\[ v_2 \text{ equation : } B^2 \varphi^2 v_1^2 + B^2 v_3^2 v_1^2 + 2BP\varphi v_1^2 + e_0^2 \varphi^2 v_2^2 - 4v_2 v_1 + 2e_0 e_1 \varphi v_1^2 + e_0^2 v_2^2 + e_1^2 v_2^2 + P^2 v_1^2 + 2\Lambda v_2^2 v_1^2 = 0 \]
\[ v_3 \text{ equation : } v_1^2 \left( B^2 (\varphi^2 + 3v_3^2) - 2\Lambda v_2^2 - 4v_2 \right) + 2BP\varphi v_1^2 - v_2^2 (e_0^2 (\varphi^2 + 3v_3^2) + 2e_1 e_0\varphi + e_1^2) + P^2 v_1^2 + 4v_2^2 v_1 = 0 \]
\[ \varphi \text{ equation : } B^2 \varphi v_3 v_1^2 + Bv_1 (2\varphi v_2 (e_0\varphi + e_1)\kappa + P v_1 v_3) - v_2 (e_0\varphi + e_1) (e_0 v_2 v_3 - 24P v_1 \kappa) = 0 \] (2.22)

Other two attractor equations (for \( e_0 \) and \( e_1 \)) define the four dimensional charges \( \Theta_{4D} \) and \( Q_{4D} \) in terms of near-horizon geometry.

Substituting the leading values of near-horizon geometry (2.22) in the attractor equations one can check that the first three equations (corresponding to \( v_1, v_2 \) and \( v_3 \)) vanish. Furthermore the \( \varphi \) equation vanishes for a particular value of \( \kappa = \frac{1}{4\sqrt{3}} \), which is the supersymmetric value. Thus one does not need any specific \( \varphi \) to solve the attractor equations. It is also easy to check that the entropy of the black hole does not depend on the near-horizon value of this scalar field and it is given by:

\[ S = \frac{2\pi \Delta L^4}{G(\Delta^2 L^2 - 3)^2}. \] (2.23)

Therefore we conclude that, at two derivative level, \( \varphi \) is a flat direction, as it was already observed in [31].
2.2.1 Higher derivative correction to the extremal near-horizon geometry and fate of the flat direction

To check whether $\varphi$ remains flat in presence of higher derivative terms we follow the same procedure as we did at leading order. We first find the corrections to the five dimensional near-horizon geometry due to higher derivative interactions. Then we study the attractor equations derived from the entropy function in presence of higher derivative terms on this corrected solution.

We consider the following higher derivative correction to the five dimensional near-horizon geometry:\[5\]

\[v_1 = \frac{1}{\Delta^2 + 9L^{-2}} + \gamma V_1, \quad v_2 = \frac{1}{\Delta^2 - 3L^{-2}} + \gamma V_2, \quad v_3 = \frac{\Delta}{\Delta^2 - 3L^{-2}}, \]
\[e_0 = -\frac{3}{L} \frac{\Delta^2 - 3L^{-2}}{\Delta^2 + 9L^{-2}} + \gamma E_0, \quad e_5 = \frac{\sqrt{3} \Delta}{\Delta^2 + 9L^{-2}} + \gamma E_5, \]
\[A_\chi = \frac{\sqrt{3}}{L(\Delta^2 - 3L^{-2})}, \quad B = 1.\] (2.24)

We can solve for these higher derivative corrections ($V_1, V_2, E_0$ and $E_5$) using the five dimensional equations of motion. The solution is given in appendix C. One can easily check that the fifth component of five dimensional gauge field ($\varphi$) never appeared in any Einstein’s equation. Therefore, we can not fix this scalar or its higher derivative correction in five dimensions. However, it is important to check whether the entropy depends on this scalar field or not. To this end, we first verify that the corrected five dimensional solution solves the higher derivative attractor equations and then compute the entropy function on this solution. As it turns out, imposing on-shell conditions on the entropy function will make its dependence on $\varphi$ disappear. In fact, the entropy reads:

\[S = \frac{2\pi \Delta L^4}{G(\Delta^2 L^2 - 3)^2} \left[ 1 + \frac{\gamma}{(\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} \left\{ 72 \left( 2c_2(7\Delta^4 L^4 - 10\Delta^2 L^2 + 3) \right. \right. \right. \]
\[\left. + c_3(9\Delta^4 L^4 + 2\Delta^2 L^2 + 3) + 2\sqrt{3}c_5(\Delta^4 L^4 - 7\Delta^2 L^2 - 5) \right) \left. \right. \right. \]
\[\left. + c_1(-16\Delta^6 L^6 + 503\Delta^4 L^4 + 246\Delta^2 L^2 + 585) - 24c_4(\Delta^6 L^6 - 24\Delta^4 L^4 - 53\Delta^2 L^2 + 15) \right\} + \mathcal{O}(\gamma^2) \] (2.25)

\[5\]We do not consider any correction for $v_3, A_\chi$ and $B$ as one can use redundancies in the leading solution to choose the corrections to the above three parameters to be zero. We address this issue in appendix C.
As expected, the flat direction at the two derivative level is not lifted, if the solution preserves some supersymmetries. What strikes as a surprise, however, is that throughout the whole analysis we never specified the supersymmetric values for the coefficients $c'_i$'s of the higher derivative interactions in (2.3), meaning that the flat direction will remain flat even for non supersymmetric deformations of the higher derivative action (2.2). Unexpectedly, the symmetries of the leading order black hole solution seem to protect the flat directions from being lifted independently of the symmetries of the full action.

For supersymmetric values given in (2.3) the entropy has the following form (remember that the AdS radius $L$ also picks up a correction):

$$S_{\text{susy}} = \frac{2\pi \Delta K L^4}{G (\Delta^2 L^2 - 3)^2} - \frac{\pi \gamma c_1 \Delta K L^4 (2\Delta^8 L^8 - 103\Delta^6 L^6 - 2633\Delta^4 L^4 + 8463\Delta^2 L^2 - 33)}{G (\Delta^2 L^2 - 3)^3 (\Delta^4 L^4 - 6\Delta^2 L^2 - 15)}$$

3. Flat direction in ten dimensional theory

Supersymmetric, asymptotically AdS$_5$ black hole solution, like the one analyzed in the previous section, has been certainly used for a huge number of applications in the AdS/CFT correspondence. However, these are not the only solutions used to obtain some knowledge of the dual field theory.

The first attempt of finding BPS black holes in 5 dimensional minimal gauged supergravity dates back few years [36], but all the solutions suffer from not having regular horizons or naked singularity. Later, they were found as a special limit of a more general class of non supersymmetric black hole solutions [37], which contain a non-extremality parameter $\mu$ linking solutions of the ungauged theory with supersymmetric solution of the gauged theory ($\mu = 0$ is the BPS-saturated limit). Such non supersymmetric black hole solutions of the minimal five dimensional gauged $U(1)^3$ supergravity, which are asymptotically AdS$_5 \times S^5$, can have a regular extreme limit with zero Hawking temperature and finite entropy[37]. It is also possible to embed such solutions in type IIB supergravity [38]. The full class of solutions\textsuperscript{6}, which was shown to satisfy the 10 dimensional equations of motion coming from the two derivative type IIB supergravity action, includes the 10 dimensional black hole metric, a self-dual five form $F_5$, three gauge fields $a_i$, coming from the 5 dimensional $U(1)^3$ gauged theory lifted in 10 dimensions and physical charges $\tilde{q}_i$:

$$ds_{10}^2 = \sqrt{\Delta} \left[ -(H_1 H_2 H_3)^{-1} f dt^2 + \left( f^{-1} dr^2 + r^2 (dM_3)^2 \right) \right] + \frac{1}{\sqrt{\Delta}} \sum_{i=1}^{3} L^2 H_i \left( d\mu_i^2 + \mu_i^2 [d\phi_i + a_i \ dt]^2 \right),$$

\textsuperscript{6}See also [39].
where $\mathcal{M}_3 = \{R^3, S^3\}$ is a spatial manifold corresponding to curvatures $\kappa = \{0, 1\}$,

$$
a_i = \frac{\bar{q}_i}{q_i} L^{-1} (H_i^{-1} - 1), \quad H_i = 1 + \frac{q_i}{r^2},
$$

$$
\triangle = H_1 H_2 H_3 \sum_{i=1}^3 \frac{\mu_i^2}{H_i}, \quad f = \kappa - \frac{\mu}{r^2} + \frac{r^2}{L^2} H_1 H_2 H_3,
$$

(3.2)

and

$$
\mu_1 = \cos \theta_1, \quad \mu_2 = \sin \theta_1 \cos \theta_2, \quad \mu_3 = \sin \theta_1 \sin \theta_2.
$$

(3.3)

$\kappa = 0$ corresponds to flat horizon. For $\kappa = 1$ horizon topology is $S^3$ and $\kappa = -1$ gives negatively curved horizon. The physical charges $\bar{q}_i$ are related to charge parameters $q_i$ in the following way,

$$
\bar{q}_i = \sqrt{q_i (\mu + \kappa q_i)}.
$$

(3.4)

The five form field strength is given by:

$$
F_5 = \mathcal{F}_5 + \ast \mathcal{F}_5, \quad \mathcal{F}_5 = dB_4,
$$

(3.5)

where,

$$
B_4 = -\frac{r^4}{L} \triangle dt \wedge d\text{vol}_{\mathcal{M}_3} - L \sum_{i=1}^3 \bar{q}_i \mu_i^2 \left( L d\phi_i - \frac{q_i}{\bar{q}_i} dt \right) \wedge d\text{vol}_{\mathcal{M}_3},
$$

(3.6)

where $d\text{vol}_{\mathcal{M}_3}$ is a volume form on $\mathcal{M}_3$. Note that the 10 dimensional Bianchi identity on the five form $\nabla_a F_{abced} = 0$ gives rise to the five dimensional equations of motion for the scalars and the gauge fields. Finally, the dilaton equation of motion admit a general solution of the form $\phi(r) = c_0 + c_1 h(r)$ where the function $h(r)$ is singular at the horizon. To circumvent this problem we can set $c_1 = 0$, so that the dilaton is just an arbitrary constant.

Once again, we remind the readers that the above solution does not preserve any supersymmetry. Nevertheless we will focus our analysis on it, since we are not aware of any asymptotically $AdS$ black hole solution in 10-dimensions that preserves some supersymmetry.

It is important to stress that the dilaton is constant and it is not possible to find its value by solving Einstein’s equations of motion. The entropy of this black hole solution does not depend on it. Therefore the dilation is a flat direction at the two derivatives level. We would like to see the fate of this flat direction when we add higher derivative terms in the action. However, for our purposes it’s easier to consider, without loss of generality, the extremal limit of this black hole solution and, once again, consider only the near-horizon geometry. This procedure is discussed in the following section.
3.1 Extremal near-horizon geometry

The extremal limit corresponds to (taking Hawking temperature to zero),

\[ 2r_0^6 + r_0^4 (\kappa + q_1 + q_2 + q_3) - q_1 q_2 q_3 = 0 \] (3.7)

where \( r_0 \) solves the above equation given the charges.

The mass parameter \( \mu \), in the near-horizon geometry is fixed to be:

\[ \mu = \kappa r_0^2 + \frac{r_0^4}{L^2} \prod (1 + \frac{q_i}{r_0}). \] (3.8)

3.1.1 Three equal charges

For simplicity we consider only three equal charge solution: \( q_1 = q_2 = q_3 = q \). In that case we see,

\[ H_1 = H_2 = H_3 = H = 1 + \frac{q}{r^2} \]

\[ \triangle = H^2, \] (3.9)

which leads to \( q = 2 r_0^2 \) and \( \mu = 27 r_0^4 \). For convenience we take \( L = 1 \) throughout this section. Therefore the three equal charge black hole metric becomes,

\[ ds_{10}^2 = \sqrt{\triangle} \left[ -(H)^{-3} f dt^2 + (f^{-1} dr^2 + r^2 (dM_3)^2) \right] + (d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2) + \sum_{i=1}^{3} \mu_i^2 \left[ d\phi_i + a_i \, dt \right]^2. \] (3.10)

We consider \( \kappa = 0 \) case, i.e. flat horizon. The analysis can be repeated for the \( \kappa = 1 \) case, and it’s completely analogous.

The extremal near-horizon metric is given by,

\[ ds^2 = \frac{1}{12} \left( -r^2 dt^2 + \frac{d\phi^2}{r^2} \right) + 3r_0^2 (dx^2 + dy^2 + dz^2) + (d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2) \]

\[ + \mu_1^2 \left( d\phi_1 + \frac{r}{3\sqrt{2}} dt \right)^2 + \mu_2^2 \left( d\phi_2 + \frac{r}{3\sqrt{2}} dt \right)^2 + \mu_3^2 \left( d\phi_3 + \frac{r}{3\sqrt{2}} dt \right)^2, \] (3.11)

and the four form field \( B \) reads,

\[ B_4 = -\sqrt{3} r_0^3 \left( r dt + \cos^2 \theta_1 d\phi_1 + \sin^2 \theta_2 (d\phi_2 \cos^2 \theta_2 + d\phi_3 \sin^2 \theta_2) \right) \wedge dx \wedge dy \wedge dz. \] (3.12)

Thus we see that at the extremal limit the near-horizon geometry admits an \( AdS_2 \) part. However, we shall determine this near-horizon geometry using again the entropy function analysis and discuss the fate of the flat direction \( \phi \).
3.2 The entropy function

We consider \( x, y \) and \( z \) direction to be compactified on a three torus. Therefore from the seven dimensional point of view the four form R-R field \( B_4 \) appears to be an one form field \( A_\mu = (B_4)_{\mu xyz} \), where \( \mu \) runs over all indices except \( x, y \) and \( z \) (the \( D3 \) brane is a point-like object and \( A_1 \) is electrically coupled to it).

Now we would like to compactify over \( \phi_i \) directions. Therefore from the four dimensional (\( \{t, r, \theta_1, \theta_2\} \)) point of view there are three KK gauge fields \( a_i = z_1 r dt \) (all of them are equal for the three equal charges case) and six scalars: three of them coming from the metric and three of them from \( B_4 \). Given the symmetry of the problem, the scalars coming from the metric are equal and will be denoted by \( w_1 \). Analogously, the scalars coming from the 4-form can be all denoted by \( b \). In fact even starting with different values for the scalars, they will be constrained to be equal. Therefore we can write down the following near-horizon ansatz for the metric and the gauge field\(^7\):

\[
ds^2 = v_1 \left( -\rho^2 dt^2 + \frac{d\rho^2}{\rho^2} \right) + v_2 \left( dx^2 + dy^2 + dz^2 \right) + w_1 \left( d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sum_{i=1}^{3} \mu_i^2 [d\phi_i + z_1 r dt]^2 \right)
\]

We can decompose the seven dimensional field \( A_1 \) in terms of four dimensional field \( A^{(4)} \) and KK fields as follows

\[
A = \frac{b}{2} \sum_i \mu_i^2 (d\phi_i + z_1 r dt) + A^{(4)}
\]

with

\[
A^{(4)} = e_0 r dt \wedge dz
\]

Explicitely, \( dA_1 \), which can be thought of as a field strength in four dimensions, reads:

\[
dA = \left[ q_5 \ dr \wedge dt + b \ \sin(\theta_1) \ \cos(\theta_1) \ (-d\phi_3 \sin^2(\theta_2) - d\phi_2 \cos^2(\theta_2) + d\phi_1) \wedge d\theta_1 \\
- b \ \ d\theta_2 \wedge (d\phi_2 - d\phi_3) \ \sin(\theta_2) \ \sin^2(\theta_1) \ \cos(\theta_2) \right]
\]

\(^7\)In fact the derivation is more involved. Once we break the \( SO(6) \) symmetry, the lower dimensional scalars and gauge fields depend on the angular direction of lower dimensional spacetime, in this case on \( \theta_1 \) and \( \theta_2 \). One has to solve the attractor equations to find the angular dependence of the lower dimensional fields. See \[40\] for the details. However in this case, as the leading near-horizon geometry is known, we substitute the angular dependence of the fields from the beginning. Therefore the scalars and the different components of the gauge fields are determined by \( AdS_2 \) symmetry only.
where
\[ q_5 = e_0 + \frac{z_1 b}{2} \] (3.17)

In ten dimensions the five form RR field strength \( F_5 \) is given by,
\[ F_5 = dB_4 + \ast dB_4 \] (3.18)

where \( dB_4 = dA \wedge dx \wedge dy \wedge dz \). Therefore, the corresponding \( F_5^2 \) equals:
\[ \frac{1}{4} 5! F_5^2 = \frac{1}{2} v_1^3 \left( -\frac{q_5^2}{v_1^2} + \frac{2b^2}{w_1^2} \right) . \] (3.19)

Hence, the final result for the on-shell action reads,
\[
S = \frac{V_3}{16 \pi G_{10}} \int_0^{\pi/2} d\theta_1 \int_0^{\pi/2} d\theta_2 \sqrt{-g_{10}} \left[ R_{10} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4 \cdot 5!} (F_5)^2 \right] \\
= \frac{V_3}{128 \pi G_{10}} \frac{v_1 v_2^3 w_1^2}{2} \left( -\frac{2b^2}{w_1^2} - \frac{(b^2 + e_0)^2}{v_1^2} + \frac{w_1^2 z_1^2}{v_1^2} - 4v_1 + 40 \right) . \] (3.20)

We define the entropy function with respect to the lower dimensional charges,
\[ \mathcal{E} = 2 \pi A \left( Q e_0 + \Theta z_1 - \frac{S}{\mathcal{A}} \right) , \] (3.21)

where \( \mathcal{A} = \frac{V_3}{128 \pi G_{10}} \). Solving the attractor equations we find the following solution for the near horizon geometry:
\[ v_1 = \frac{\sqrt{-Q}}{24}, \quad v_2 = \frac{(2 \Theta/3)^{2/3}}{(-Q)^{5/6}}, \quad z_1 = \frac{1}{\sqrt{3} \sqrt{2}} \]
\[ b = \frac{4 \Theta}{3Q}, \quad w_1 = \frac{\sqrt{-Q}}{2} . \] (3.22)

Furthermore, the entropy is given by:
\[ S = \frac{\Theta}{192 \sqrt{2} G_{10}} . \] (3.23)

Since we know the extremal near-horizon geometry exactly (3.11, 3.12), we can solve the attractor equations and find the lower dimensional charges in terms of a single parameter \( r_0 \). We can then re-write the entropy as a function of \( r_0 \) and check it is in agreement.
with Bekenstein-Hawking law.

The near-horizon geometry reads:

\[
\begin{align*}
  v_1 &= \frac{1}{12}, & v_2 &= 3r_0^2, & z_1 &= \frac{1}{3\sqrt{2}} \\
  b &= -6\sqrt{6} r_0^3, & w_1 &= 1
\end{align*}
\]

Substituting these values in the attractor equations we get,

\[
Q = -4, \quad \Theta = 18\sqrt{6} r_0^3.
\] (3.25)

Therefore the entropy turns out to be

\[
S = \frac{3\sqrt{3} r_0^3}{32G_{10}} = \frac{\text{Area}}{4G_{10}}.
\] (3.26)

One should note that the near-horizon value of the dilaton does not appear in the entropy function, therefore it is a flat direction. Our goal is to check what happens to this flat direction when we add supersymmetric higher derivative terms which appear in type IIB string theory.

### 3.3 Higher derivative terms in type IIB string theory

For type IIB supergravity, which is a low-momentum expansion of type IIB superstring theory, the higher derivative corrections can be written as a series in \( \alpha' \). The series is of the following form:

\[
\alpha'^4 S_{IIB} = S^{(0)} + \alpha' S^{(1)} + \cdots + (\alpha')^n S^{(n)},
\] (3.27)

where the first non-zero term, expected to appear at tree-level or 1-loop in the string coupling, is of the order \( \alpha'^3 \). It is an eight derivative action, one of the terms being the well known \( R^4 \) term. Unfortunately, the standard superfield techniques ([11], [12]) can not be used for the construction of the full \( S^{(3)} \) contribution to the two derivative action, that corresponds to the supersymmetric completion of the \( R^4 \) term [13]. Nevertheless, if one considers only a subset of the full field content of type IIB theory, specifically the metric and the five-form, then a general formula for the supersymmetric higher derivative correction exists [12]. For the sake of completeness, we outline the steps taken to obtain such an invariant. First of all, \( U(1) \) gauge invariance of the theory allows us to separate all the higher derivative terms by their charge. We will then only look for terms that are
neutral under $U(1)$ and contain at most one fermion bilinear. These terms, schematically, look like:

\begin{align}
S^{(3)}_{0;B} &= \int d^{10}x f^{(0,0)}(\tau, \bar{\tau})(C^4 + (F_5)^8 + \cdots) \\
S^{(3)}_{0;BFF} &= \int d^{10}x f^{(0,0)}(\tau, \bar{\tau})(C^2 \bar{\psi}\psi + (F_5)^7 \bar{\psi}\psi \cdots)
\end{align}

where $C$ is the Weyl tensor, $\psi$ is the gravitino, $F_5$ is the self-dual five form and $f^{(0,0)}(\tau, \bar{\tau})$ is a modular function of the complex scalar fields $\tau$ and $\bar{\tau}$, which reads as in (3.32, 3.33). Note that the five form and the metric are the only bosonic fields neutral under $U(1)$. Now if one starts from this restricted set of fields, considering terms only linear in the fermions in the supersymmetry variations and setting $\partial\tau = \partial\bar{\tau} = \lambda = 0$ ($\lambda$ being the dilatino of the theory), it is possible to show that the supersymmetry variation of (3.28) cancels exactly against the supersymmetry variation of (3.29) (neglecting fermions trilinear). Of course, setting the derivative of the scalar field $\tau$ to zero we are effectively neglecting the variation of the modular form $f^{(0,0)}(\tau, \bar{\tau})$. Restricting our attention to these terms, it is pretty straightforward to show that the obstruction to the existence of the chiral measure, found in [41], is circumvented. As one would expect, then, the final result for the eight derivative action [42], turns out to be exact. This construction is, however, highly non-trivial and only few explicit calculations were carried out [44, 45]. Recently, a simplified, explicit expression for the eight derivative coupling between the metric and five-form was found in [46]. In the following we will make use of this general result, together with the solutions (3.1), (3.6). The full action reads:

\begin{equation}
I = \frac{1}{16\pi G_N} \int_{\mathcal{M}_{10}} d^{10}x \sqrt{-g} \left[ R_{10} - \frac{1}{2} (\partial \phi)^2 - \frac{1}{4 \cdot 5!} (F_5)^2 + \cdots + (\alpha')^3 \gamma(\phi) \mathcal{W} + \cdots \right]
\end{equation}

\begin{align}
\gamma(\phi) &= \frac{1}{16} f^{(0,0)}(\tau, \bar{\tau}), \\
G_N &\propto \alpha'^4
\end{align}

\begin{equation}
f^{(0,0)}(\tau, \bar{\tau}) = \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+n\tau|^3},
\end{equation}

where

\begin{equation}
\tau = \tau_1 + i\tau_2 = C^{(0)} + ie^{-\phi}.
\end{equation}

The above correction to the leading supergravity action is a complete quantum (i.e. $\alpha'$ as well as string-loop) correction. The string coupling $g_s \propto \exp \phi_\infty$. The first term in the expansion of $f^{(0,0)}(\tau, \bar{\tau})$ appears only as a supergravity ($\alpha'$) correction to the leading
two derivative Lagrangian, i.e. in finite $\alpha'$ and $N \to \infty$ limit and thus, in $g_s \to 0$ limit. For the following computation, we will keep the entire quantum correction to the leading supergravity action.

The higher derivative $W$ contribution is explicitly given by [46]:

$$W \equiv \frac{1}{80016} \sum_i n_i M_i$$  \hspace{1cm} (3.34)

| $n_i$  | $M_i$                                                                 |
|-------|----------------------------------------------------------------------|
| -43008| $C_{abcd} C_{abef} C_{cegh} C_{dgfh}$                                |
| 86016 | $C_{abcd} C_{aecf} C_{bgeh} C_{dqfh}$                                |
| 129024| $C_{abcd} C_{ae fg} C_{bf hi} T_{cd egh i}$                           |
| 30240 | $C_{abcd} C_{abef} T_{df ghi j} T_{ef hgi j}$                          |
| 7392  | $C_{abcd} C_{abef} T_{cdghi j} T_{ef hgi j}$                          |
| -4032 | $C_{abcd} C_{aecf} T_{beghi j} T_{df ghi j}$                          |
| -4032 | $C_{abcd} C_{aecf} T_{bghdi j} T_{eghi j}$                            |
| -118272| $C_{abcd} C_{ae fg} T_{be hi j} T_{df hgi j}$                         |
| -26880| $C_{abcd} C_{ae fg} T_{be hi j} T_{dhi fj g i}$                        |
| 112896| $C_{abcd} C_{ae fg} T_{be hi j} T_{de hi g j}$                         |
| -96768| $C_{abcd} C_{ae fg} T_{be hi j} T_{df hj g i}$                         |
| 1344  | $C_{abcd} T_{abefgh} T_{cdeijk} T_{fghijk}$                           |
| -12096| $C_{abcd} T_{abefgh} T_{cdefijk} T_{eghi j k}$                        |
| -48384| $C_{abcd} T_{abefgh} T_{cdefijk} T_{eghijk}$                          |
| 24192 | $C_{abcd} T_{abefgh} T_{ce-fijk} T_{dh gijk}$                         |
| 2386  | $T_{ab def} T_{ab egh} T_{gijkl} T_{fijhkl}$                          |
| -3669 | $T_{ab def} T_{ab egh} T_{ei jg k l} T_{f i k hjl}$                    |
| -1296 | $T_{ab def} T_{ab eghi} T_{de jk l} T_{fh kij l}$                     |
| 10368 | $T_{ab def} T_{ab eghi} T_{da jk l} T_{fh kij l}$                     |
| 2688  | $T_{ab def} T_{ab eghi} T_{egj k l} T_{f j khl}$                      |

The tensor $T$ is defined by

$$T_{abcdef} = P_{1050}^+ \left(i \nabla_a F_{bc def} + \frac{1}{8} F_{abcmn} F_{def}^{mn} \right).$$ \hspace{1cm} (3.35)

If we impose self-duality of the five-form, this reduces to

$$T_{abcdef} = i \nabla_a F_{bc def} + \frac{1}{16} \left(F_{abcmn} F_{def}^{mn} - 3 F_{abf mn} F_{dec}^{mn} \right),$$
where the RHS should be antisymmetrized in the triplets \([abc], [def]\) and symmetrized for their interchange. Here, we also note that the higher derivative correction has been given in the Einstein frame. We can also go to the string frame with proper transformation of the metric, but, obviously, the physical information of the system will not depend on the frame chosen.

### 3.4 The fate of the flat direction

As we already explained, the dilaton parametrizes a flat direction at two derivative level, that can be lifted or not by the presence of supersymmetric higher derivative interactions. To verify its fate, we first compute the entropy function in presence of these higher derivative terms (3.34) and then focus on the attractor equations corresponding to the two scalars, i.e. axion and dilaton. Since the higher derivative term \(W\) evaluated on the leading solution turns out to be constant\(^8\), the axion-dilaton equations take the following form,

\[
\frac{\partial f^{(0,0)}(\tau_1, \tau_2)}{\partial \tau_1} \bigg|_{\tau_1=(\tau_1)_h, \tau_2=(\tau_2)_h} = 0, \quad \frac{\partial f^{(0,0)}(\tau_1, \tau_2)}{\partial \tau_2} \bigg|_{\tau_1=(\tau_1)_h, \tau_2=(\tau_2)_h} = 0, \tag{3.36}
\]

The axion equation takes the following form:

\[
\sum_{(m,n)\neq(0,0)} \frac{n(m+n\tau_1)}{|m+n\tau|^5} = 0. \tag{3.37}
\]

and it’s easily solved by \(\tau_1 = 0\). On the other hand, the dilaton equation of motion is given by (setting \(\tau_1\) to zero):

\[
\sum_{(m,n)\neq(0,0)} \frac{\sqrt{\tau_2}(m^2 - n^2\tau_2^2)}{|m^2 + n^2\tau_2|^{5/2}} = 0. \tag{3.38}
\]

One solution of the above equation is \(\tau_2 = 0 \implies \phi_h \to \infty\), but this divergent behaviour destabilizes the near-horizon geometry, so we won’t take it into account. Another possible solution is \(\tau_2 = 1\), for which

\[
\sum_{(m,n)\neq(0,0)} \frac{(m^2 - n^2)}{|m^2 + n^2|^{5/2}} = 0, \tag{3.39}
\]

is identically satisfied. Therefore, the leading near-horizon value of the dilaton is \(\phi_h = 0\), so that the flat direction is lifted when we add higher derivative terms in the action.

\(^8\)The value of \(W\) for the near-horizon geometry (3.24) considered is 14580.
One important observation is that considering only the leading term in the modular function, that is, only the leading higher derivative correction \((\alpha'^3)\) to supergravity, setting all loop correction to zero, then the leading value of the dilaton is fixed to infinity. Thus the thermodynamics of the system (temperature, entropy) does not receive any correction due to this leading higher derivative term, but the system is destabilized.

However, considering the full quantum correction, then there is a possibility of a finite dilaton solution. This is a rather interesting phenomenon, as the full quantum correction stabilizes the system again. Not only that, it seems that supersymmetries, and not just extremality of a black hole solution is necessary to protect the flat directions from being lifted. Once again, it looks as if the symmetries of the higher derivative interactions do not play any role to decide the fate of flat directions.

### 3.5 Higher derivative correction to entropy

For completeness, we compute higher derivative correction to the entropy. We define the entropy function and the attractor equations as before. We computed the full supersymmetric higher derivative term (3.34) for the near-horizon geometry and the expression is presented in appendix D. Solving the corrected attractor equations we get the following corrections to the near-horizon geometry:

\[
\begin{align*}
v_1 &= \frac{\sqrt{-Q}}{24} - \frac{144155}{384Q} \hat{\gamma}, \\
v_2 &= \frac{(2\Theta/3)^{2/3}}{(-Q)^{5/6}} - \frac{25115(\Theta(-Q))^{2/3}}{8 \sqrt{2} 3^{2/3} Q^3 \hat{\gamma}}, \\
z_1 &= \frac{1}{3\sqrt{2}} - \frac{810 \sqrt{2}}{(-Q)^{3/2} \hat{\gamma}}, \\
b &= \frac{4\Theta}{3Q} + \frac{54115\Theta}{9(-Q)^{5/2} \hat{\gamma}}, \\
w_1 &= \frac{-Q}{2} + \frac{21085}{32Q} \hat{\gamma}
\end{align*}
\]  

(3.40)

where,

\[
\hat{\gamma} = \frac{\alpha'^3}{16} \sum_{(m,n) \neq (0,0)} \frac{1}{(m^2 + n^2)^{3/2}}.
\]  

(3.41)

The entropy is given by:

\[
S = \frac{\Theta}{192\sqrt{2}G_{10}} - \frac{405\Theta}{16\sqrt{2}G_{10}(-Q)^{3/2} \hat{\gamma}}.
\]  

(3.42)

### 4. Future Directions

We hope this work will pave the way for a number of possible applications and extensions. Here we will mention some of them and hope to report on them in future.

As of now, we are not aware of any asymptotically \(AdS\) black hole solutions of supergravity in ten dimensions that preserves some supersymmetries. Thus, one interesting
direction is to uplift the five dimensional spinning $AdS$ solution, analyzed in the first part of this paper, to ten dimensions. Knowing the correct uplift of this class of supersymmetric five dimensional black hole solutions, one can study the behavior of flat directions (if any) for the uplifted solution and compare the results obtained with the ones presented in this paper.

In our work, we have seen that, for the five dimensional case, the supersymmetry invariance of the higher derivative terms did not play any role. In fact, without specifying the correct supersymmetric coefficients of various higher derivative terms, the flat direction of the leading solution remains flat. As we stressed before, the supersymmetries preserves by the leading solution played an important role in the whole analysis. It would be interesting to find a supersymmetric black hole solution in the higher derivative theory as well although this would require an analysis of the complete off-shell formulation of minimal gauged supergravity in five dimensions. This analysis would certainly make use of the correct values of various coefficients of the higher derivative terms and give us the first supersymmetric asymptotic $AdS$ black hole solution away from supergravity limit\(^9\).

\[............................\]

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\(^9\)Other theories, e.g. type IIB in 6 dimensions, admit flat directions and extremal black hole solutions\(^{10}\); however for those theories the full, supersymmetric invariant, higher derivatives couplings are not known.
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A. 5D and 4D charges

In this appendix, we provide the explicit expressions of the 5D and the corresponding 4D charges for a \( \frac{1}{4} \) BPS solution in minimal gauged supergravity theory with supersymmetric higher derivative terms. The results are given for the generic form of the solution in (2.11) and for its supersymmetric form in (2.20). First, the expressions for the 5D charges, which are completely general, are obtained using the equations (2.7), (2.8), (2.11) and (2.12):

\[
q_{5D} = \frac{\pi}{G} \left[ \frac{2v_2 \sqrt{v_3}(e_1 + e_0 \varphi)}{v_1} - 8\kappa \varphi(2P + B \varphi) + 32c_2v_2\sqrt{v_3}(e_1 + e_0 \varphi) \left( \frac{(P + B \varphi)^2}{v_2^2 v_1} - \frac{(e_1 + e_0 \varphi)^2}{v_1^3} \right) \\
+ c_3 \frac{16v_2 \sqrt{v_3}(e_1 + e_0 \varphi)^3}{v_1^3} + 4c_4 \sqrt{v_3} v_2^2(e_1 + e_0 \varphi)(4v_1^2 - 3e_0^2 v_3) + 2B v_1^2 v_3 e_0(P + B \varphi) \\
+ c_5 \frac{4B v_3(e_0^2 v_2^2 v_3 + 2v_2^2(v_2 - B^2 v_3))}{v_1^2 v_2^2} \right] \tag{A.1}
\]

Of course, adding suitable improvement terms to the current would have led to the same exact result for the electric charge. Notice that \( \xi \) is just a constant, once we imposed the symmetric background condition \( \partial_\mu \xi = 0 \). Similarly, the angular momenta takes the following form:

\[
\Theta_{5D} = \frac{\pi}{G} \left[ \frac{2v_2 \sqrt{v_3}(e_0 v_3 + \varphi(e_1 + e_0 \varphi))}{v_1} - 8\kappa \varphi 3P + 2B \varphi}{3} + 16c_3 v_2 \sqrt{v_3} \varphi \frac{(e_1 + e_0 \varphi)^3}{v_1^3} \\
+ 2c_1 e_0 v_3^{3/2} \frac{v_2^2(11e_0^2 v_3 - 12v_1)}{v_1^3 v_2} - 6B^2 v_1^2 v_3 \\
+ 32c_2 \sqrt{v_3} \varphi(e_1 + e_0 \varphi) \frac{e_1 v_2^2(e_1 + 2e_0 \varphi) - P v_1^2(P + 2B \varphi) + \varphi(e_0^2 v_2^2 - B^2 v_1^2)}{v_1^3 v_2} \\
+ 4c_4 \sqrt{v_3} \frac{2v_1^2 v_3(e_1 + e_0 \varphi)(P + B \varphi) - v_2^2(e_1 + e_0 \varphi)(3e_0 v_3(e_1 + e_0 \varphi) - 4v_1 \varphi)}{v_1^3 v_2} \\
+ 4c_5 v_3 \frac{P v_2^2(5e_0^2 v_3 - 4v_1) + B(2e_1 e_0 v_2^3 v_3 + (9e_0^2 v_2^2 v_3 - 4v_1 v_2^2 + v_1^2(4v_2 - 3B^2 v_3))(\varphi)}{v_1^2 v_2^2} \right] \tag{A.2}
\]
Analogously, plugging in the specific values of the parameters of the Gutowski-Reall solution, one obtains, for the corrected black hole charges: 2.4

\[
q_5 = \frac{\sqrt{3}\Delta^2 L^2 \left( \frac{3L^2}{\Delta^2 L^2 - 3} + \varphi^2 \right) + 6L\varphi - 3\sqrt{3}\varphi^2}{12G(\Delta^2 L^2 - 3)} - \frac{\gamma\Delta^2 L^4}{48G(\Delta^2 L^2 - 3)^2(\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} \left( \sqrt{3}c_1(13\Delta^6 L^6 - 1179\Delta^4 L^4 - 7623\Delta^2 L^2 - 135) - 24(2c_5\Delta^6 L^6 + 65\sqrt{3}c_4\Delta^4 L^4 \\
- 6c_5\Delta^4 L^4 + 303\sqrt{3}c_4\Delta^2 L^2 + 60c_5\Delta^2 L^2 + 6\sqrt{3}c_2(\Delta^6 L^6 + 11\Delta^4 L^4 + 87\Delta^2 L^2 - 99) \\
+ 3\sqrt{3}c_3(\Delta^6 L^6 + 21\Delta^4 L^4 + 129\Delta^2 L^2 + 21) + 450\sqrt{3}c_4 - 900c_5) \right) 
\]

\[
\theta_5 = \frac{9\sqrt{3}\Delta^2 L^4 \varphi + \sqrt{3}\varphi^3 (\Delta^2 L^2 - 3)^2 + 9L\varphi^2 (\Delta^2 L^2 - 3) - \frac{27\Delta^2 L^5}{\Delta^2 L^2 - 3}}{36G(\Delta^2 L^2 - 3)^2} 
+ \frac{1}{48G(\Delta^2 L^2 - 3)^3(\Delta^4 L^4 - 6\Delta^2 L^2 - 15)}(\gamma\Delta^2 L^4(24(6\sqrt{3}c_2 + 3\sqrt{3}c_3 + 2c_5) \\
- 13\sqrt{3}c_1)\Delta^6 L^6 \varphi + \Delta^6 L^6(c_1(635L + 1218\sqrt{3}\varphi) + 24(6c_2(5L + 8\sqrt{3}\varphi) + 27c_3(L + 2\sqrt{3}\varphi) \\
+ 36c_4L - 4\sqrt{3}c_3L + 65\sqrt{3}c_4\varphi - 12c_5\varphi)) - 3\Delta^4 L^4((795c_1 + 816c_2 + 552c_3 + 1064c_4 \\
+ 96\sqrt{3}c_5)L - 6(227\sqrt{3}c_1 + 8(54\sqrt{3}c_2 + 33\sqrt{3}c_3 + 18\sqrt{3}c_4 + 13c_5))\varphi) - 9\Delta^2 L^2((449c_1 \\
- 816c_2 + 120c_3 + 920c_4 - 848\sqrt{3}c_5)L + 6(421\sqrt{3}c_1 + 960\sqrt{3}c_2 + 488\sqrt{3}c_3 + 204\sqrt{3}c_4 \\
+ 480c_5)\varphi) - 27(175c_1 + 8(26c_2 + 13c_3 - 20c_4 - 30\sqrt{3}c_5))L - 81(5\sqrt{3}c_1 + 8(-66\sqrt{3}c_2 \\
+ 7\sqrt{3}c_3 + 50\sqrt{3}c_4 - 100c_5))\varphi) 
\) 

Before we end this appendix, we summarize some of our conventions for the 5D analysis. The metric is always mostly positive, with signature \((-,-,+,+,+,-)\). The Levi-Civita tensor has the following form,

\[
\varepsilon^{\mu\nu\rho\sigma\delta} = -\frac{\varepsilon^{\mu\nu\rho\sigma\delta}}{\sqrt{-g}}, \quad \varepsilon_{\mu\nu\rho\sigma\delta} = \sqrt{-g}\varepsilon^{\mu\nu\rho\sigma\delta}, \quad (A.5)
\]

where, \(\varepsilon^{\mu\nu\rho\sigma\delta} = \varepsilon_{\mu\nu\rho\sigma\delta}\) is just \(\pm 1\) depending on the orientation of the space-time. In a local Lorentz frame, \(\varepsilon_{01234} = 1\). Also, the definition of the Riemann tensor is

\[
R_{\mu\nu\sigma\lambda} = 2\Gamma_{\nu[\sigma,\rho]} + 2\Gamma_{\lambda[\sigma,\rho]}\Gamma_{\mu,\rho\lambda}. \quad (A.6)
\]

**B. Kaluza-Klein Reduction of 5D Lagrangian**

In this appendix, we give the necessary details to perform the Kaluza-Klein reduction of the five dimensional Lagrangian. The reduction ansatz is given in (2.13). In the tangent space, the reduction of various components of the Riemann tensor reads:
\[ \hat{R}_{cd}^{ab} = R_{cd}^{ab} - \frac{1}{2} \phi \left[ F(B)_{c}^{[a} F(B)_{d}^{b]} + F(B)_{cd} F(B)^{ab} \right], \]
\[ \hat{R}_{cd}^{a5} = \phi \frac{1}{2} D_{c}^{(\omega)} F(B)_{a d} + \left[ \partial_{c} \sqrt{\phi} F(B)_{a d} - \partial^{a} \sqrt{\phi} F(B)_{cd} \right], \]
\[ \hat{R}_{c5}^{ab} = -\frac{1}{2} \phi D_{c}(\omega) F(B)^{ab} - \left[ F(B)^{ab} \partial_{c} \sqrt{\phi} - F(B)_{c}^{[a} \partial^{b]} \sqrt{\phi} \right], \]
\[ \hat{R}_{c5}^{a5} = -\phi^{-\frac{1}{2}} D_{c}(\omega) \sqrt{\phi} + \frac{1}{2} \phi F(B)_{c a} F(B)^{ab}. \]  \hfill (B.1)

and gauge invariant two derivative Lagrangian takes the following form:

\[ S_{0} = 4 \pi \int d^{4}x \sqrt{-h} \sqrt{\Phi} \left( R + \frac{12}{L^{2}} - \frac{1}{4} F_{ij} F^{ij} - \frac{1}{4} (\Phi + \sigma^{2}) (F_{kk})^{ij} (F_{kk})_{ij} \right. \]
\[ \left. - \frac{1}{2} \sigma F_{ij} (F_{kk})^{ij} + \frac{1}{2} \frac{1}{\phi^{2}} (\nabla \phi)^{2} - \frac{1}{\phi} \nabla^{2} \phi - \frac{1}{2} \phi (\nabla \sigma)^{2} \right) \]  \hfill (B.2)

The higher derivative action consists of different four-derivative terms. Here, we
present the dimensional reduction of the gauge invariant part:

\[
(S_{\text{HD}})^{GL} = 4\pi \int d^4 x \sqrt{-h} \left[ c_1 \left( \Phi^{1/2} R^{ijkl} R_{ijkl} + \frac{1}{4} \Phi^{-7/2} (\nabla_i \phi \nabla^i \phi)^2 + \Phi^{-3/2} \nabla^i \phi \nabla_i \nabla_j \phi \right)
\right.
\]

\[
- \frac{3}{2} \Phi^{3/2} R_{ijkl} (F_{kk})^{ij} (F_{kk})^{kl} + \Phi^{1/2} (F_{kk})^{ij} (F_{kk})^{jk} \nabla_k \nabla_i \phi - 2 \Phi^{-1/2} (F_{kk})^{ij} (F_{kk})_{ij} \nabla_k \phi \nabla^i \phi
\]

\[
+ \Phi^{3/2} \nabla_i (F_{kk})^{ij} (F_{kk})^{jk} \nabla^i \phi - 3 \Phi^{1/2} \nabla_i (F_{kk})^{jk} (F_{kk})_{ik} \nabla^i \phi - \Phi^{-5/2} \nabla_i \phi \nabla^i \phi \nabla_j \phi
\]

\[
+ \frac{5}{8} \Phi^{5/2} (F_{kk})^{ij} (F_{kk})_{jk} (F_{kk})^{kl} (F_{kk})_{li} - \frac{3}{2} \Phi^{-1/2} \nabla_i \phi \nabla^i \phi (F_{kk})_{ij} (F_{kk})^{ij}
\]

\[
+ c_2 \Phi^{1/2} \left( F_{ij} F^{ij} + \sigma^2 (F_{kk})^{ij} (F_{kk})_{ij} + 2 \sigma F_{ij} (F_{kk})^{ij} + \frac{2}{\Phi} (\nabla \sigma)^2 \right) + c_3 \left( \Phi^{1/2} F^{ij} F_{jk} F^{kl} F_{li} + \Phi^{1/2} \sigma^2 R^{ijkl} (F_{kk})^{ij} (F_{kk})^{kl} + 2 \Phi^{1/2} \sigma R_{ijkl} (F_{kk})^{ij} F_{kl}^{i} \right)
\]

\[
- \frac{1}{2} \Phi^{3/2} \left[ \sigma^2 (F_{kk})^{ij} (F_{kk})^{jk} (F_{kk})_{ik} + F_{ij} (F_{kk})_{jk} F_{kl}^{i} (F_{kk})^{kl} \right] + \Phi^{1/2} \nabla_i (F_{kk})^{ij} \nabla^i \nabla^k \sigma
\]

\[
+ 4 \Phi^{1/2} \nabla_i (F_{kk})^{ij} \nabla^i \nabla_i \nabla^j \nabla^k \phi - \Phi^{-1/2} \nabla_i \phi \nabla_j \phi \nabla^i \phi + \Phi^{-5/2} (\nabla_i \phi \nabla^i \phi)^2
\]

\[
2 \Phi^{-1/2} \nabla^i \phi (F_{ij} + \sigma (F_{kk})_{ij}) (F_{kk})^{jk} \nabla_k \sigma + \left( \frac{3 c_1}{8} \Phi^{5/2} + \frac{3 c_4}{4} \sigma^2 e^{3 \phi} \right) ((F_{kk})_{ij} (F_{kk})^{ij})^2 \right]
\]

(B.3)

The full Lagrangian contains the above gauge invariant parts as well as the reduction of the two gauge non-invariant Chern-Simons terms, that we have already presented in the main text. To performed the entropy function analysis we have made use of the full Lagrangian.

C. Redundancy of higher derivative corrections

As we already pointed out in the main text, the leading five dimensional solution (2.21) has certain redundancies. Keeping the geometry of the full solution fixed, we could impose
an ansatz for the corrections to the leading solution parameters:

\[
\begin{align*}
v_1 &= \frac{1}{\Delta^2 + 9L^{-2}} + \delta \tilde{V}_1, & v_2 &= \frac{1}{\Delta^2 - 3L^{-2}} + \delta \tilde{V}_2, & v_3 &= \frac{\Delta}{\Delta^2 - 3L^{-2}} + \delta \tilde{V}_3, \\
e_0 &= -\frac{3}{L\Delta} \frac{\Delta^2 - 3L^{-2}}{\Delta^2 + 9L^{-2}} + \delta \tilde{E}_0, & e_5 &= \frac{\sqrt{3}\Delta}{\Delta^2 + 9L^{-2}} + \delta \tilde{E}_5, \\
A_\chi &= \frac{\sqrt{3}\sin \theta}{L(\Delta^2 - 3L^{-2})} + \delta \tilde{\chi}, \\
\end{align*}
\]

Now, one can show that the leading Einstein’s equations of motion are satisfied up to order \(\delta\), if the following conditions are satisfied:

\[
\begin{align*}
\tilde{V}_1 &= \frac{(\Delta^2 L^2 - 3)^2 \left( 2\Delta^2 L^4 \tilde{B}_h + 2\sqrt{3}L \tilde{Y} (\Delta^2 L^2 + 1) + \tilde{V}_3 (\Delta^2 L^2 - 3)^2 \right)}{(\Delta^2 L^2 + 9)^2 (\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} \equiv f_1(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) \\
\tilde{V}_2 &= \frac{2\Delta^2 L^4 \tilde{B}_h + 2\sqrt{3}L \tilde{Y} (\Delta^2 L^2 + 1) + \tilde{V}_3 (\Delta^2 L^2 - 3)^2}{\Delta^4 L^4 - 6\Delta^2 L^2 - 15} \equiv f_2(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) \\
\tilde{E}_0 &= \frac{1}{6\Delta^3 L^5 (\Delta^2 L^2 + 9)^2 (\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} [(\Delta^2 L^2 - 3)^3 (3(16\Delta^4 L^6 (11\Delta^2 L^2 - 9) \tilde{B}_h + \tilde{V}_3 (\Delta^2 L^2 - 3)^2 (3\Delta^6 L^6 + 97\Delta^4 L^4 - 279\Delta^2 L^2 - 405)) + 2\sqrt{3}\Delta^2 L^3 \tilde{Y} (3\Delta^8 L^8 + 8\Delta^6 L^6 + 54\Delta^4 L^4 - 288\Delta^2 L^2 - 81))) \equiv f_3(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) \\
\tilde{E}_5 &= \frac{1}{2\Delta^2 L^2 (\Delta^2 L^2 + 9)^2 (\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} (\sqrt{3}(\Delta^6 L^6 - 5\Delta^4 L^4 + 147\Delta^2 L^2 - 135)(2\Delta^2 L^4 \tilde{B}_h + \tilde{V}_3 (\Delta^2 L^2 - 3)^2) + 16\Delta^2 L^3 \tilde{Y} (\Delta^6 L^6 + 9\Delta^2 L^2 - 54)) \equiv f_4(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) \\
\end{align*}
\]

The above solution is not supersymmetric and it is specified in terms of four parameters: \(\Delta, \tilde{V}_3, \tilde{Y}, \tilde{B}_h\).

Now, when we add the higher derivative corrections to the leading solution (2.24), we could in principle add correction terms to \(v_3, B\) and \(A_\chi\) also. Let the correction terms to these quantities be \(V_3, B_h\) and \(\Upsilon\) respectively. We can plug these corrected geometry to Einstein’s equations but solve for only four of them, because only four equations are linearly independent. We can choose to solve for \(V_1, V_2, E_0\) and \(E_5\) in terms of other three parameters. The final result is the following,

\[
\begin{align*}
v_1 &= v_1^{(0)} + \delta f_1(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) + \gamma f_1(V_3, B_h, \Upsilon) + \gamma g_1(c_i s) \\
v_2 &= v_2^{(0)} + \delta f_2(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) + \gamma f_2(V_3, B_h, \Upsilon) + \gamma g_2(c_i s) \\
e_0 &= e_0^{(0)} + \delta f_3(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) + \gamma f_3(V_3, B_h, \Upsilon) + \gamma g_3(c_i s) \\
e_5 &= e_5^{(0)} + \delta f_4(\tilde{V}_3, \tilde{B}_h, \tilde{Y}) + \gamma f_4(V_3, B_h, \Upsilon) + \gamma g_4(c_i s) \\
\end{align*}
\]
Now we can use the redundancy in the leading solutions to remove corrections terms from $V_3, B, \Upsilon$, i.e. we can choose $\delta$ to be $\gamma$ and $\tilde{V}_3 = -V_3$, $\tilde{B}_h = -B_h$ and $\tilde{\Upsilon} = -\Upsilon$. Then, the functions $g_i$’s read:

$$g_1 = \frac{1}{4(\Delta^2 L^2 + 9)^2(\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} \left( L^2(c_1(\Delta^8 (-L)^8) + 269\Delta^6 L^6 - 525\Delta^4 L^4 \\
+ 3861\Delta^2 L^2 + 1080) + 24(c_4 \Delta^4 L^8 + 13c_4 \Delta^6 L^6 + 2\sqrt{3}c_5 \Delta^6 L^6 - 40c_4 \Delta^4 L^4 \\
- 30\sqrt{3}c_5 \Delta^4 L^4 - 126c_4 \Delta^2 L^2 + 120\sqrt{3}c_5 \Delta^2 L^2 + 6c_2(\Delta^8 L^8 - \Delta^6 L^6 - 33\Delta^4 L^4 \\
+ 57\Delta^2 L^2 - 24) + 3c_3(\Delta^8 L^8 + 3\Delta^6 L^6 - 39\Delta^4 L^4 - 9\Delta^2 L^2 - 24) + 90c_4) \right) \quad (C.4)$$

$$g_2 = \frac{1}{4(\Delta^6 L^6 - 9\Delta^4 L^4 + 3\Delta^2 L^2 + 45)} \left( L^2(c_1(443\Delta^4 L^4 - 210\Delta^2 L^2 + 45) + 24(18c_4 \Delta^4 L^4 \\
+ 4\sqrt{3}c_5 \Delta^4 L^4 + 38c_4 \Delta^2 L^2 - 30\sqrt{3}c_5 \Delta^2 L^2 + 6c_2(7\Delta^4 L^4 - 10\Delta^2 L^2 + 3) \\
+ 3c_3(9\Delta^4 L^4 + 2\Delta^2 L^2 + 3) - 15c_4)) \right) \quad (C.5)$$

$$g_3 = -\frac{1}{3\Delta L(\Delta^2 L^2 + 9)^2(\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} \left( (\Delta^2 L^2 - 3)(c_1(\Delta^8 L^8 - 657\Delta^6 L^6 \\
- 14238\Delta^4 L^4 - 4347\Delta^2 L^2 + 1215) + 6(-6c_4 \Delta^8 L^8 + 3\sqrt{3}c_5 \Delta^8 L^8 - 173c_4 \Delta^6 L^6 \\
+ 9\sqrt{3}c_5 \Delta^6 L^6 - 2067c_4 \Delta^4 L^4 - 519\sqrt{3}c_5 \Delta^4 L^4 - 3987c_4 \Delta^2 L^2 + 1755\sqrt{3}c_5 \Delta^2 L^2 \\
- 12c_3(28\Delta^6 L^6 + 219\Delta^4 L^4 + 24\Delta^2 L^2 + 27) + 24c_2(\Delta^8 L^8 - 25\Delta^6 L^6 - 168\Delta^4 L^4 \\
+ 219\Delta^2 L^2 - 27) + 405c_4)) \right) \quad (C.6)$$

$$g_4 = -\frac{1}{\sqrt{3}(\Delta^2 L^2 + 9)^2(\Delta^4 L^4 - 6\Delta^2 L^2 - 15)} \left( (\Delta^2 L^2(c_1(4\Delta^8 L^8 - 135\Delta^6 L^6 - 1332\Delta^4 L^4 \\
- 21465\Delta^2 L^2 - 810) + 6(c_4 \Delta^8 L^8 - 23c_4 \Delta^6 L^6 + 3\sqrt{3}c_5 \Delta^6 L^6 - 399c_4 \Delta^4 L^4 \\
+ 57\sqrt{3}c_5 \Delta^4 L^4 - 3393c_4 \Delta^2 L^2 - 1071\sqrt{3}c_5 \Delta^2 L^2 - 36c_3(19\Delta^4 L^4 + 136\Delta^2 L^2 + 3) \\
+ 72c_2(\Delta^6 L^6 - 18\Delta^4 L^4 - 115\Delta^2 L^2 + 132) - 6750c_4 + 2835\sqrt{3}c_5)) \right) \quad (C.7)$$

**D. Dilaton equation and its solution**

We want to study the complete profile of the dilaton, when the supersymmetric higher derivative contribution to the action is considered. The dilaton equation of motion is
given by.

\[
\frac{d}{dr} \left( \sqrt{-g_{10}} \phi'(r) \right) + \frac{\gamma}{2} e^{-\frac{3}{2} \phi(r)} \sqrt{-g_{10}} W(r) = 0. \tag{D.1}
\]

One can solve this equation in the extremal, near-horizon limit, i.e. \( q = 2r_0^2 \) and \( \mu = \frac{27r^2}{4} \) \((L = 1)\). The leading solution is constant, and we denote it by \( \phi_h \). Corrections to the solution can be found perturbatively in \( \gamma \). Since the higher derivative part is already of the order \( \gamma \), we need to evaluate it only on the leading black hole geometry, obtaining:

\[
W(r) = \frac{180 \mu^4}{(r^2 + Q)^8} \tag{D.2}
\]

Now, we denote the correction term by \( \psi(r) \), so that the dilaton can be written as \( \phi(r) = \phi_h + \gamma \psi(r) \). Plugging this value in the dilaton equation, we get a second order differential equation for \( \psi(r) \), which is solved by:

\[
\psi(r) = \frac{1}{1024} \left( \frac{32 \left( 8C_1 + 243q^2 e^{-\frac{3\phi_h}{2}} \right)}{9q(q - 2r^2)} - \frac{32 \log(2r^2 - q) \left( 8C_1 + 4617q^2 e^{-\frac{3\phi_h}{2}} \right)}{81q^2} \right.
\]

\[
+ \frac{\log(4q + r^2)}{162q^2} \left( 512C_1 + 243q^2 e^{-\frac{3\phi_h}{2}} \right) - \frac{6561q^5 e^{-\frac{3\phi_h}{2}}}{10(q + r^2)^5} - \frac{6561q^4 e^{-\frac{3\phi_h}{2}}}{8(q + r^2)^4}
\]

\[
- \frac{2187q^3 e^{-\frac{3\phi_h}{2}}}{2(q + r^2)^3} - \frac{5589q^2 e^{-\frac{3\phi_h}{2}}}{4(q + r^2)^2} - \frac{4617q e^{-\frac{3\phi_h}{2}}}{2(q + r^2)} + \frac{3645}{2} e^{-\frac{3\phi_h}{2}} \log(q + r^2) \bigg) + C_2
\]

\[
\tag{D.3}
\]

This solution is singular at horizon \( r_0^2 = q \), in the extremal limit), specifically there are two terms which become singular at \( r = r_0 \), i.e. \( \log(2r^2 - q) \) and \( \frac{1}{2r^2 - q} \). We can choose \( C_1 \) to set one of these two terms to zero, but to remove the other singular term we need to set \( e^{-\frac{3}{2} \phi_h} = 0 \). Therefore the higher derivative term destabilizes the leading order solution (if we consider the term appearing in front of \( W \) to be \( \exp(-3/2\phi) \) only.). We found the same result from the entropy function calculation performed in section \([3.4]\). Finally, for sake of completeness we present the expression for \( W \) in terms of the near-horizon parameters. The result has been obtained using FORM, and it reads:
\[86016 \mathcal{W}_{NH} = -\frac{464}{9} v_1^{-8} v_2^6 w_1 w_2^4 z_4^4 + \frac{1341256}{27} v_1^{-8} v_2^{-3} w_1^3 z_1 q_5^2 + \frac{293461}{18} v_1^{-8} w_1^4 z_1^8 - \frac{3094952}{27} v_1^{-7} v_2^{-3} w_1^3 z_1^4 q_5^2 - \frac{640136}{9} v_1^{-7} w_1^3 z_1^6 + \frac{44608}{9} v_1^{-6} v_2^{-9} w_1^{-1} z_1^2 b^2 q_5^4 + \frac{327872}{9} v_1^{-6} v_2^{-6} z_1^3 b q_5^3 - 1848 v_1^{-6} v_2^{-6} z_1^3 b q_5^2 + \frac{1896832}{27} v_1^{-6} v_2^{-3} w_1^2 z_1^2 q_5^2 - \frac{1986880}{27} v_1^{-6} v_2^{-3} z_1^3 q_5 - \frac{3561656}{27} v_1^{-6} v_2^{-3} w_1 z_1^2 b q_5 + \frac{355936}{3} v_1^{-6} w_1^2 z_1^4 + \frac{2990288}{9} v_1^{-6} w_1^2 z_1^4 + 6720 v_1^{-5} w_1^{-1} w_2^{-2} b^2 q_5^2 - \frac{1410752}{27} v_1^{-5} v_2^{-3} z_1^2 q_5^2 - \frac{7592368}{27} v_1^{-5} v_2^{-3} z_1^3 b q_5 - \frac{815360}{9} v_1^{-5} w_1 z_1^2 - \frac{2389184}{3} v_1^{-5} w_1 z_1^4 + \frac{4032}{12} v_1^{-12} w_1^{-4} b^4 q_5 + \frac{122848}{9} v_1^{-4} v_2^{-9} w_1^{-3} z_1 b^3 q_5^3 + \frac{43192}{9} v_1^{-4} v_2^{-9} w_1^{-3} z_1^2 b^4 q_5^2 - \frac{5376}{9} v_1^{-4} v_2^{-6} w_1^{-2} b^2 q_5^2 + \frac{796736}{3} v_1^{-4} v_2^{-6} w_1^{-2} z_1^3 b q_5^2 + 43008 v_1^{-4} v_2^{-6} w_1^{-2} z_1^3 b q_5 - \frac{1764}{27} v_1^{-4} v_2^{-6} w_1^{-2} z_1^4 b q_5 + \frac{4046336}{27} v_1^{-4} v_2^{-3} w_1^{-1} z_4 q_5 + \frac{2308096}{27} v_1^{-4} v_2^{-3} w_1^{-1} z_1^2 q_5 + \frac{35321216}{27} v_1^{-4} v_2^{-3} z_1^3 b q_5^2 - \frac{4528160}{27} v_1^{-4} v_2^{-3} w_1^{-1} z_1^2 q_5^2 + \frac{250880}{9} v_1^{-4} w_1^4 z_1^4 + \frac{1637888}{3} v_1^{-4} z_1^2 + \frac{3236800}{3} v_1^{-4} z_1 - 13440 v_1^{-3} v_2^{-9} w_1^{-4} b^4 q_5^2 + \frac{10752}{3} v_1^{-3} v_2^{-6} w_1^{-3} b^3 q_5^2 - 43008 v_1^{-3} v_2^{-6} w_1^{-3} z_1 b q_5 + 6048 v_1^{-3} v_2^{-6} w_1^{-3} z_1^2 b^4 + \frac{10148992}{27} v_1^{-3} v_2^{-3} w_1^{-2} z_1 q_5 + \frac{9860032}{27} v_1^{-3} v_2^{-3} z_1^2 b q_5^2 - \frac{358400}{9} v_1^{-3} w_1^{-1} - \frac{1895936}{3} v_1^{-3} w_1^{-1} z_1 - 2016 v_1^{-2} v_2^{-12} w_1^{-6} b^6 q_5^2 + \frac{875392}{9} v_1^{-2} v_2^{-9} w_1^{-5} b^4 q_5^2 - \frac{204512}{9} v_1^{-2} v_2^{-9} w_1^{-5} z_1 b q_5 + 75264 v_1^{-2} v_2^{-6} w_1^{-4} b^2 q_5^2 - 8064 v_1^{-2} v_2^{-6} w_1^{-4} b^4 + \frac{759296}{9} v_1^{-2} v_2^{-6} w_1^{-4} z_1 b q_5 + 41664 v_1^{-2} v_2^{-6} w_1^{-4} z_1^2 b^4 - \frac{2551808}{27} v_1^{-2} v_2^{-3} w_1^{-3} b^2 + \frac{5594624}{27} v_1^{-2} v_2^{-3} z_1^3 b q_5 - \frac{16597504}{27} v_1^{-2} v_2^{-3} w_1^{-3} z_1^2 b^2 + \frac{501760}{3} v_1^{-2} w_1^{-2} + \frac{11992064}{9} v_1^{-2} w_1^{-2} z_1^2 - 5376 v_1^{-1} v_2^{-6} w_1^{-5} b^4 + \frac{555520}{27} v_1^{-1} v_2^{-3} w_1^{-4} b^2 - \frac{1003520}{9} v_1^{-1} w_1^{-3} + 1512 v_2^{-12} w_1^{-8} b^8 - \frac{260096}{9} v_2^{-9} w_1^{-7} b^6 - \frac{552704}{9} v_2^{-6} w_1^{-6} b^4 + \frac{17963008}{27} v_2^{-3} w_1^{-5} b^2 + \frac{2508800}{9} w_1^{-4} \]
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