Coulomb blockade at a tunnel junction between two quantum wires with long–range interaction

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The non–linear current–voltage characteristic of a tunnel junction between two Luttinger systems is calculated for an interaction with finite range. Coulomb blockade features are found. The dissipative resistance, the capacitance and the external impedance, which were introduced ad hoc in earlier theories, are obtained in terms of the electron–electron interaction. The frequency dependence of the impedance is given by the excitation spectrum of the electrons.

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The Coulomb blockade effect in the non-linear current voltage (I–U) characteristics of mesoscopic tunnel junctions [1,2] has been the subject of many theoretical and experimental investigations [3,4] during the past decade. Basically, due to the repulsion between the electrons, tunneling is suppressed for voltages below $U_C = e/2C$, and temperatures smaller than $T_C \equiv E_C/k_B$ (K Boltzmann constant, $e$ elementary charge, $C$ capacitance). The quantity $E_C = eU_C$ is the charging energy.

In the semi-phenomenological theory of the phenomenon [5], the tunnel junction is modelled by a capacitance and a tunnel resistance $R_t$. An impedance $Z(\omega)$ is included into the circuit. It represents the coupling of the tunneling particles to a reservoir of Bosonic degrees of freedom. They guarantee incoherence between different tunneling processes. When $Z(0) \equiv R = 0$, the current voltage characteristic is linear, $I(U) = U/R_t$. For $R \neq 0$, $I(U) \propto U^{2R/R_k+1}$ when $U \ll U_C$ ($R_k$ von Klitzing constant), and $I(U) \approx (U - U_C)/R_t$ when $U_C \ll U \rightarrow \infty$. It must be emphasized that the shift of the linear behavior of $I(U)$ by $U_C$ is the important characteristic feature of the Coulomb blockade for $R \rightarrow \infty$.

In this paper, we present a microscopic theory of the effect for a one-dimensional (1D) tunnel junction. The parameters introduced in the above mentioned theory by ad hoc assumptions are deduced consistently, and in a natural way, from the interaction between the electrons.

Two semi-infinite (1D) systems of interacting electrons described within the Luttinger approximation [6] are coupled by a tunnel junction. The interaction potential between the electrons is assumed to have a finite, non-zero range. The tunneling current as a function of a voltage applied across the junction is obtained.

The charging energy is found to be the interaction potential at zero distance, and the dissipative resistance is given by the spatial average of the interaction potential. The spectrum of the elementary excitations determines the impedance of the circuit. In order to explain the latter no additional 'environmental modes' are needed. The crucial point is that the above mentioned asymptotic behavior of $I(U)$ for large $U$ appears to be directly related to the finite, non-zero range of the interaction. The latter implies that the dispersion relation of the elementary excitations of the electron system becomes $\omega(k) \approx v_F |k|$ in the short wavelength limit [7]. Our results show that in 1D a charging energy, and, in turn, a capacitance can be defined in the Luttinger model provided the interaction has non-zero range.

We consider the Hamiltonian $H = H_0 + H_i + H_U$. Here, $H_0 \equiv H_{el}^{(1)} + H_{el}^{(2)}$ consists of the Hamiltonians of the two disconnected electron systems, which extend from $-L$ to 0 and from 0 to $L$ ($L \rightarrow \infty$), respectively. The tunnel junction (at $x = 0$) is described by $H_t$, and $H_U$ is the energy contributed by the external voltage.

The electrons are described by the Bosonic Luttinger Hamiltonian [8], $(j = 1, 2)$

$$H_{el}^{(j)} = \sum_{q>0} \omega(q)\gamma_{q}^{(j)}\gamma_{q}^{(j)} + \frac{\pi v_F}{4L}(\Delta N^{(j)})^2. \quad (1)$$

Boundary conditions are assumed such that the original Fermion fields vanish at $x = 0, \pm L$. This introduces additional (quadratic) off-diagonal terms in the Hamiltonian [9]. We neglect them here for the sake of simplicity, since they do not affect the final results qualitatively. The dispersion law $\omega(q) = qv(q)$ with the wave number dependent velocity $v(q) = v_F[1 + \tilde{\tilde{V}}(q)/\pi v_F]^{1/2}$, reflects the Fourier transformed of the interaction potential $\tilde{\tilde{V}}(q)$. For the latter, we assume a 3D screened Coulomb
potential with the range $\alpha^{-1}$ projected onto a quantum wire of diameter $d$. Depending on whether $\alpha^{-1} \ll d$ or $\alpha^{-1} \gg d$ the interaction is exponentially (Luttinger limit) or algebraically ($x^{-1}$, Coulomb limit) decaying, respectively. For small $q (=n\pi/L, n$ integer) we have the charge-sound excitations with the renormalized velocity $v(0) \equiv v_F/g$ characteristic of the Luttinger system, with $g^{-1} \equiv [1 + \tilde{V}(0)/\pi v_F]^{1/2}$. For large $q$, we find the excitation spectrum of the non-interacting electrons, due to the finite range of the potential. The velocity associated to $\Delta N$ (see below) is $v(0)/g$.

The operators $\gamma^{(j)t}, \gamma^{(j)}$ are related via a Bogolubov transformation \[ b_{qj} \equiv \cosh(\varphi_{qj}) \gamma^{(j)} - \sinh(\varphi_{qj}) \gamma^{(j)t}, \] to the Fourier components of the phase fields, $b_{qj} \equiv \cosh(\varphi_{qj}) \gamma^{(j)} - \sinh(\varphi_{qj}) \gamma^{(j)t}$.

\[ \Phi^{(j)}(x) = \sum_{q \neq 0} \sqrt{\frac{\pi}{qL}} \left( e^{iqx} b^{(j)}_{q} + e^{-iqx} b^{(j)t}_{q} \right), \] (2)

They define (right moving) Fermion fields

\[ \Psi^{(j)}_{R}(x) = \frac{1}{\sqrt{2L}} e^{-i\vartheta^{(j)}_0} e^{i\pi x \Delta N^{(j)}/L} e^{i\Phi^{(j)}(x)}, \] (3)

with the density operators $\rho^{(j)}_{R}(x) = \Psi^{(j)t}_{R}(x)\Psi^{(j)}_{R}(x)$. The functions $\varphi_{q}$ contain the above dispersion relation, $\exp(-2\varphi_{q}) = \omega(q)/v_{F}q$.

The variables $\vartheta^{(j)}_0$, defined modulo $2\pi$, are conjugate to the number operators, $\left[ \vartheta^{(j)}_0, \Delta N^{(j)} \right] = i,$

\[ \Delta N^{(j)} \equiv 2L \left( \rho^{(j)}_{R} - \frac{\partial_x \Phi^{(j)}(x)}{2\pi} \right). \] (4)

The latter represent extra electrons in the systems on the left and the right hand sides of the junction.

The above boundary conditions imply that the corresponding left- and right moving parts are not independent but $\Psi^{(j)}_{R}(x) = -\Psi^{(j)}_{L}(-x)$, $\Psi^{(j)}_{R}(x + 2L) = \Psi^{(j)}_{R}(x)$, and either one of the two alone suffices to describe the system. The tunnel Hamiltonian in terms of the latter is

\[ H_t = L \Delta \left[ \Psi^{(j)t}_{R}(0)\Psi^{(j)}_{R}(0) + \Psi^{(j)}_{R}(0)\Psi^{(j)}_{R}(0) \right]. \] (5)

By inserting the above Bosonized form \[ \Psi^{(j)}_{R}(x) = \frac{1}{\sqrt{2L}} e^{-i\vartheta^{(j)}_0} e^{i\pi x \Delta N^{(j)}/L} e^{i\Phi^{(j)}(x)}, \] one obtains $H_t \equiv H^{+}_{t} + H^{-}_{t}$, with

\[ H^{\pm}_{t} \equiv \frac{\Delta}{2} \exp \left\{ \pm i \sum_{j=1,2} (-1)^{j} [\vartheta^{(j)}_0 - \Phi^{(j)}(0)] \right\}. \] (6)

The electrostatic energy of the external voltage that is assumed to drop only at the tunnel junction, $U(x) = U [\Theta(x) - \Theta(-x)]/2$ ($\Theta(x$) Heavyside function), is

\[ H_U \equiv -e \int_{-L}^{L} U(x)\rho(x), \] (7)

with $\rho(x) = \rho^{(1)}(x)\Theta(-x) + \rho^{(2)}(x)\Theta(x)$. In the dc limit, it has been shown that the current-voltage relation is independent of how the voltage drops \[ 12-13 \]. Only the voltage drop between $x \to -\infty$ and $x \to \infty$, which is assumed to be fixed by an external "battery", is important. Therefore, interaction induced rearrangement of charges in the presence of the impurity is unimportant for the present calculation. By inserting the above relation \[ (\theta) \] between $\rho^{(j)}_{R}$ and $\Delta N^{(j)}$, noting that $\rho^{(j)}_{R}(x) \equiv \rho^{(j)}_{R}(x) + \rho^{(j)}_{L}(x) = \rho^{(j)}_{R}(x) + \rho^{(j)}_{R}(-x)$, and using $\Phi^{(j)}_{R}(L) = \Phi^{(j)}_{R}(-L)$ one obtains

\[ H_U = \frac{eU}{2} \left( \Delta N^{(1)} - \Delta N^{(2)} \right). \] (8)

The current operator is defined as in previous works $I \equiv ie[H_{t} - H^{+}_{t}]$ \[ 13 \]. Fermi’s golden rule with $H_t$ as a perturbation \[ 10 \] yields the average current

\[ I(U) = \frac{e\Delta^2}{4} \left[ 1 - e^{-\beta eU} \right] \int_{-\infty}^{\infty} dt e^{iUt} e^{-W_{g}(t)} \] (9)

($\beta$ inverse temperature). This equation is independent of the boundary conditions applied. The thermal equilibrium correlation function

\[ W_{g}(t) \equiv \sum_{j=1,2} \left\langle \left( \Phi^{(j)}_{R}(0) - \Phi^{(j)}_{R}(t) \right) \Phi^{(j)}_{R}(0) \right\rangle \] (10)

is evaluated with respect to $H_{0}$. The $\Phi^{(j)}_{R}(t)$ evolve in the interaction picture with respect to $H_{0} + H_{U}$. This gives

\[ W_{g}(t) = \int_{0}^{\infty} d\omega \frac{J(\omega)}{\omega^2} \times \left[ 1 - \cos(\omega t) \right] \coth \left( \frac{\beta \omega}{2} \right) + i \sin(\omega t) \] (11)

The key quantity is the spectral function $J(\omega)$ which is directly given in terms of the dispersion law of the charge excitations of the system

\[ J(\omega) = \frac{2\omega^2 q^2(\omega)}{v_{F}q^{2}(\omega)}, \] (12)

where $q(\omega)$ is the inverse of the dispersion function $\omega(q)$. Equation \[ (12) \] establishes the main result of this work in the sense that the quantity on the left hand side, which was already present in the former theory \[ 3 \], is now directly related to spectrum of the elementary excitations of the electron system. \[ 3 \]

The impedance function $Z(\omega) \equiv J(\omega)/\omega - 2$ for the interaction potential of the Luttinger limit is shown in Figure 1. Qualitatively similar curves are obtained for the Coulomb limit. For $\omega \to 0$, $Z(0) = 2(g^{-1} - 1) \equiv R/R_K$.

For large $\omega$, $Z(\omega)$ tends to zero, i.e., $J(\omega) \to 2\omega$, the limit of non-interacting electrons. The peak in $Z(\omega)$,
which appears for smaller values of the interaction parameter $g$, is a consequence of the inflection point in the spectrum of the elementary excitations at $\omega_p$ (the frequency of the maximum in $q'(\omega)$). When the screening length decreases, $\omega_p$, and consequently the frequency beyond which $Z(\omega)$ vanishes, increase proportional to $\alpha$ (Fig. 1), for $\alpha \to \infty$, $Z(\omega) = 2(g^{-1} - 1)$.

In order to calculate the current voltage characteristic we have to Fourier transform $\exp\{-W_g(t)\}$. This is done by using the relation between $W_1(t)$, and the Fermi distribution $f(E)$\[15\],

$$e^{-\frac{W_1(t)}{\omega_e}} = \frac{1}{\omega_e} \int_{-\infty}^{\infty} dE f(E)e^{iEt}.$$  \(13\)

The result is

$$I(U) = \frac{e\Delta^2}{2\pi}(1 - e^{-\beta eU})$$

$$\times \int_{-\infty}^{\infty} dE f(E)[1 - f(E + eU)]$$

$$\times D(E)D(E + eU)$$  \(14\)

with the tunneling density of states

$$D(E) = \int_{-\infty}^{\infty} dt \cos(Et)e^{-W_0(t)/2}.$$  \(15\)

It is nothing but the local density of states at the position of the tunnel junction\[10,13,14\].

$$P(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{iEt} e^{-[W_0(t) - W_0(t)]}.$$  \(17\)

The tunnel resistance is $R_t \equiv 2\omega_p^2/e^2\Delta^2\pi$\[13\].

In $P(E)$ the role of the 'electromagnetic environment' is now played by the excitations of the interacting electrons. The result for zero temperature,

$$I(U) = \frac{1}{eR_t} \int_{0}^{eU} dE(eU - E)P(E),$$  \(18\)

is shown in Fig. 2 for different interaction strengths.

For small voltages $I(U) \approx (1/R_t)U^2/g^{-1}$. By comparison with the corresponding limit of reference\[3\], we recover the dissipative resistance in terms of the interaction\[10,16\],

$$\frac{R}{2R_K} = \frac{1}{g} - 1.$$  \(19\)

For $U$ much larger than the range of $P(E)$ eq. \(18\) becomes

$$I(U) = \frac{1}{R_t}\left(U - \frac{E_C}{e}\right),$$  \(20\)

from which the charging energy is found by using our microscopic expression \(12\) for $J(\omega)$

$$E_C = \int_{0}^{\infty} d\omega Z(\omega) = 2V(x = 0).$$  \(21\)

Taking into account the above mentioned off-diagonal terms induced by the boundary conditions does not change the dependence of $E_C$ on the interaction potential. Only the prefactor is altered\[3\].

FIG. 1. The impedance function $J(\omega)/\omega - 2$ of the Luttinger limit for different interaction parameters $g$.

The connection with the earlier results\[3\] is established by observing that the $I(U)$ can also be written as

$$I(U) = \frac{1 - e^{-\beta eU}}{eR_t} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' f(E)[1 - f(E')]$$

$$\times P(E + eU - E'),$$  \(16\)

with the probability density for a bulk excitation of energy $E$.

FIG. 2. The current voltage characteristic of a tunnel junction between 1D interacting electron systems for different strengths of the interaction $g$.

The capacitance found from $E_C$, $C = e^2/4V(0)$, scales in the same way as the one, $C_q$, found previously by considering the frequency dependent conductance of a pure quantum wire of interacting electrons\[3\]. This
was obtained by a completely different philosophy starting from the classical theory of antennas. In fact, in the Coulomb limit, \( C/C_q = O(1) \). Following this approach, one could consider the present problem as being the quantum equivalent of two only very weakly coupled wires. From the comparison of their AC properties with those of the quantum system considered here, one would get the same scaling law for the capacitance.

The capacitance responsible for the Coulomb blockade is naturally given here by the total capacitance of the circuit, i.e. the two wires and the junction. Apparently, in our ideal 1D model, the junction as such does not contribute significantly. It is only needed for detecting the Coulomb blockade induced by the latter. In the Coulomb limit, \( C = \pi \epsilon \epsilon_0 d \approx 0.03 \) F, for a quantum wire fabricated in a semiconductor heterostructure, AlGaAs/GaAs (\( \epsilon \approx 10, d \approx 100 \) nm), for instance, with only one subband occupied. The charging energy is \( E_C \approx 2.3 \text{ meV} \), which corresponds to a critical temperature of \( T_C = 30 \) K. By decreasing the width of the wire, which seems achievable with present day’s technology, one should be able to increase \( T_C \) close to room temperature. Thus, a one-mode narrow quantum channel with a weak link appears to be the ‘ultimate device’ for observing Coulomb blockade effects. [7]

From the AC properties [7], we find also an inductance \( L \) of the electron system, due to the presence of the resonance in the AC conductance at \( \omega_p \), for strong interaction. It is determined here by the ratio \( E_C/\omega_p = e^2 \sqrt{L/C} \), and is reflected by the resonant behavior of the impedance \( Z(\omega) \) (Fig. ). This is, however, of negligible influence on the behavior of \( I(U) \). Since the integral weight of the resonance is small compared with the charging energy, the total integral over \( Z(\omega) \), and also \( \omega_p \ll E_C \) (\( E_C \approx 500 \omega_p \) for the above single mode wire), the steps in the derivative of the DC current characteristic predicted earlier as a signature of a \( \delta \)-function like resonance in \( Z(\omega) \) cannot be obtained by using the present model. This prevents the inductance to be detected directly here.

Our above results remain qualitatively true for a non-linear dispersion relation, beyond the Luttinger model, since the influence of the interaction is negligibly small for wave numbers much larger than the inverse of the range of the interaction. This is indicated by the large \( q \) behavior of the dispersion with interaction in the random phase approximation [8].

For many channels occupied some of the above conclusions are modified. Matveev and Glazman [10] treated the tunneling for a quasi-1D quantum wire with many \( (N) \) channels, but for zero range interaction. They find a crossover in the zero frequency impedance from eq. (19) to the non-interacting limit \( (R \to 0) \) when \( N \to \infty \). In that model, one cannot obtain the asymptotic linear behavior of \( I(U) \), even for finite \( N \). Therefore, we conclude that our above result – that using the Bosonization method the Coulomb blockade is closely related to the finite range of the interaction – remains also valid for many channels, though the quantitative behavior of the charging energy, and thus the capacitance, will be probably changed. We expect a contribution of the tunnel junction to the capacitance proportional to the channel number. This will be the subject of future studies [9].

In summary, we obtained the non-linear current voltage characteristic for a model of two 1D quantum wires of interacting electrons connected via a tunnel junction. The features of the Coulomb blockade phenomenon were found. The charging energy, the capacitance, and the inductance of the circuit were given in terms of the interaction potential. In order to obtain the Coulomb blockade in the Luttinger liquid model, it is necessary to assume that the interaction is of finite, non-zero range. The role of the modes of the environment is played by the elementary excitations of the interacting electrons.

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[1] D. V. Averin, K. K. Likharev, J. Low Temp. Phys. 62, 345 (1986); D. V. Averin, K. K. Likharev, in Mesoscopic Phenomena in Solids, edited by B. L. Altshuler, P. A. Lee, R. A. Webb (Elsevier, Amsterdam, 1991)
[2] L. J. Geerligs, V. F. Anderegg, P. A. M. Holweg, J. E. Mooij, H. Pothier, D. Estève, C. Urbina, M. Devoret, Phys. Rev. Lett. 64, 2691 (1990)
[3] Single Charge Tunneling. Coulomb Blockade Phenomena in Nanostructures, Volume 294 of NATO Advanced Study Institute Series B, edited by H. Grabert, M. H. Devoret (Plenum, New York 1992)
[4] Quantum Coherence in Mesoscopic Systems, Volume 254 of NATO Advanced Study Institute Series B, edited by B. Kramer (Plenum, New York 1991)
[5] G. L. Ingold, Yu. V. Nazarov, in [3], pp. 21–107; M. H. Devoret, et al., Phys. Rev. Lett. 64, 1824 (1990); S. M. Girvin et al., Phys. Rev. Lett. 64, 3183 (1990)
[6] J. M. Luttinger, J. Math. Phys. 4, 1154 (1963); J. Sólyom, Adv. Phys. 28, 201 (1979); F. D. M. Haldane, J. Phys. C14, 2585 (1981)
[7] G. Cuniberti, M. Sassetti, B. Kramer, J. Phys.: Cond. Matt. 8, L21 (1996)
[8] M. Fabrizio, A. O. Gogolin, Phys. Rev. B51, 17827 (1995); S. Eggert, H. Johannesson, A. Mattson, Phys. Rev. Lett. 76, 1505 (1996)
[9] M. Sassetti, G. Cuniberti, B. Kramer, unpublished results (1996)
[10] M. Sassetti, U. Weiss, Europhys. Lett. 27, 311 (1994)
Note that the usual high frequency cutoff $\omega_c (\to \infty)$ is included here. It does not affect the final results for the charging energy. It is only contained in the tunnel resistance $R_T$ in the usual way [5,10].

K. A. Matveev, L. I. Glazman, Phys. Rev. Lett. 70, 990 (1993)

K. Matsumoto, M. Ishii, K. Segawa, Y. Oka, 2nd Procs. Int. Workshop on Quantum Functional Devices, 90 (1995)

Q. P. Li, S. Das Sarma, Phys. Rev. B 40, 5860 (1989)