SU(3) Predictions for Weak Decays of Doubly Heavy Baryons – including SU(3) breaking terms

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Abstract

We find expressions for the weak decay amplitudes of baryons containing two $b$ quarks (or one $b$ and one $c$ quark – many relationship are the same) in terms of unknown reduced matrix elements. This project was originally motivated by the request of the FNAL Run II $b$ Physics Workshop organizers for a guide to experimentalists in their search for as yet unobserved hadrons. We include an analysis of linear SU(3) breaking terms in addition to relationships generated by unbroken SU(3) symmetry, and relate these to expressions in terms of the complete set of possible reduced matrix elements.

I. INTRODUCTION

The $b$ quark sector is currently of great interest for a number of reasons. First, the $b$ quark is the heaviest quark which lives long enough to form hadrons. Second, its unsuppressed coupling to the top quark results in enhancements of a variety of decay modes. Within standard model physics, the $b$ quark sector provides an opportunity to probe the origin of $CP$
violation, and to judge whether it can be accommodated entirely within the standard model. Beyond the standard model, the $b$ sector is sensitive to extra Higgs particles, supersymmetric particles, and other types of new physics. Finally, the current experimental situation with respect to $b$ physics is very active at both electron and hadron colliders.

Studying the weak decays of doubly heavy baryons allows us to test our understanding both of different pieces of the weak effective Lagrange density and of heavy–to–heavy and heavy–to–light transitions [1]. Understanding the weak decay possibilities will aid in unraveling the spectroscopy of doubly heavy baryons – a rich field which tests the applicability of Nonrelativistic QCD [2], Heavy Quark Effective Theory [3], sum rules [4], and potential models to these systems [5]. On the experimental side, the first doubly heavy hadron containing a $b$ quark was seen at CDF in the form of $B_c$ [6]. A doubly charmed baryon may have been seen [7]. Run IIb of the Tevatron at Fermilab is expected to produce doubly heavy hadrons at the nanobarn level [8]. Consideration of their spectroscopy has been considerable [5,9], but only a few have looked at the baryon weak decay, especially for those containing a $b$ quark [10,11]. Here we study the weak nonleptonic decay of doubly heavy baryons using SU(3) symmetry.

In order to identify new heavy particles their weak decay modes should be analyzed to find the most promising discovery modes and those modes which will give the most physics insight. To this end, we apply the procedure of imposing SU(3) symmetries and the Wigner-Eckart theorem for the decays of a number of particles of interest. We also include the predictions which result from including linear SU(3) breaking terms. Finally, we point out how this analysis relates to the general group theory basis for describing the amplitudes.

This paper follows closely the procedures of, for example, Refs. [12,13] in identifying the relevant multiplets, forming the four-quark operators, decomposing them into irreducible SU(3) representations, and building all possible singlets. Extensive studies of the $B$–meson and its decay modes have been done in this manner [14], especially because of the potential for extracting CP–violating phases [15]. Baryons containing a single $b$ quark have been studied this way in Ref. [16], and those with two charmed quarks in Ref. [17]. Our expressions for decay amplitudes in terms of the most general group theoretic basis is done following the procedure of Grinstein and Lebed, Ref. [18]. Tables of the results are presented, and some of the relationships highlighted. The Clebsch-Gordon coefficients were found by writing a symbolic manipulation program which generated the latex files of the tables appearing here. The hope is that this will substantially reduce the typographical errors which tend to occur in a project such as this.

There are well-known examples of SU(3) violation in decays of heavy particles. The violation may come from the finite strange quark mass, from final state interactions, or from nearby resonances. SU(3) breaking has been studied in single–$b$ baryons in [13], and in $B$ mesons in [20], following treatments of lighter systems [21,22]. For the double heavy baryons, hints of SU(3) violation are seen in potential model treatments [23]. We hope that the analysis presented here will aid in the understanding of SU(3) breaking effects in doubly heavy baryons, and illuminate which types fall with increasing mass of the decaying particle.

The paper is organized as follows: in Section II the particles involved in our weak decays are introduced and defined, and their SU(3) transformation properties identified. In Section III we discuss the effective operators inducing the weak transitions. We categorize them based upon their degree of Cabbibo suppression, and then decompose them into irreducible
representations of SU(3). In the same section we include an SU(3) breaking effect for each operator and decompose those terms under SU(3) as well. Details of the decomposition are presented in Appendix D. The decay amplitudes themselves are found in Section IV, where we present tables of physical amplitudes in terms of reduced matrix elements. Three very long tables are relegated to Appendices A, B, and C. We include in Section IV a discussion of the relationship between the set of reduced amplitudes found from including a linear SU(3) breaking term, and those found when no assumptions are made. The details of how this is accomplished is given in Appendix F. The physical amplitudes expressed in terms of the group theoretic basis is given in Appendix E. Rate relationships, modulo phase space corrections, are also given in Section IV. In Appendix G we discuss the effect of phase space corrections. We conclude with a summary in Section V.

II. NOTATION AND PARTICLE MULTIPLETS

We first consider decays of those particles identified as belonging to the lowest mass SU(3) triplet we will call $3_{bb}$:

$$3_{bb} = \left( \Xi^0_{bb}, \Xi^-_{bb}, \Omega^-_{bb} \right)^T,$$

(2.1)

with valence quarks ($bbu$, $bbd$, $bbs$). The particle symbols are dictated by their isospin quantum number. These decay weakly predominantly when one of the $b$ quarks decays. The particle multiplets we will need for the final states are listed below.

1. The triplet of baryons with one $b$ and one $c$ quark are contained in

$$3_{bc} = \left( \Xi^+_bc, \Xi^0_{bc}, \Omega^0_{bc} \right)^T,$$

(2.2)

with the quark content ($bcu$, $bcd$, $bcs$).

2. The particles with one $b$ quark, which fall into the six representation of SU(3) and are therefore symmetric under interchange of indices, are:

$$[6_b]^{11} = \Sigma^+_b \sim buu, \quad [6_b]^{12} = \frac{1}{\sqrt{2}} \Sigma^0_b \sim bud,$$

$$[6_b]^{22} = \Sigma^-_b \sim bdd, \quad [6_b]^{13} = \frac{1}{\sqrt{2}} \Xi^0_{b2} \sim bus,$$

$$[6_b]^{33} = \Omega^-_b \sim bss, \quad [6_b]^{23} = \frac{1}{\sqrt{2}} \Xi^-_{b2} \sim bds.$$  

(2.3)

3. The particles with one $b$ quark which fall into an anti–triplet representation of SU(3) are:

$$\overline{3}_b = \left( \Xi^-_{b1}, -\Xi^0_{b1}, \Lambda^0_b \right),$$

(2.4)

with quark content ($bsd$, $bsu$, $bdu$).
4. The octet of lowest mass mesons are contained in
\[ M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & \pi^+ & K^+
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta_8 & K^0
K^- & \frac{1}{\sqrt{2}} \pi^0 - \frac{1}{\sqrt{6}} \eta_8 & K^0 \end{pmatrix}, \tag{2.5}
\]
with the quark content given in [24].

5. The $D$–meson antitriplet is
\[ D = (D^0, D^+, D_s^+) \]
with valence quarks ($c\bar{u}$, $c\bar{d}$, $c\bar{s}$).

6. $J/\Psi$ is a singlet under SU(3).

7. The $B$–meson antitriplet is
\[ B = (B^-, B_s^0, B_s^0) \]
with quark content ($b\bar{u}$, $b\bar{d}$, $b\bar{s}$).

8. The lowest mass octet of baryons is
\[ b = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Sigma^+ & p
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & n
\Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda^0 \end{pmatrix}, \tag{2.8}
\]
with quark content given in [24].

These multiplets will be used to build the decay amplitudes we will analyze under SU(3).

III. THE EFFECTIVE HAMILTONIAN AND WEAK FOUR-QUARK OPERATORS

We will group together decay processes which fall into the same multiplets of initial and final states. For instance, using the notation above, the decay processes $3_{bb} \to 3_{bc} + M$ specify fourteen physical amplitudes.

As Grinstein and Lebed [18] point out, it is instructive to re-write the decay amplitudes (the “physical basis”) in terms of the “group-theoretic” basis where the amplitudes are written in terms of reduced matrix elements of manifest SU(3) and isospin content. In the case $3_{bb} \to 3_{bc} + M$, there are fourteen such reduced matrix elements since no assumptions have been made about the Hamiltonian which induces the decay. From this group-theoretic basis it is easier to see how assumptions about the Hamiltonian (for instance, imposing SU(3) symmetry or including limited SU(3) breaking terms) can be used to make predictions about
relationships between decay amplitudes. In Appendix E we show this physical to group-theoretic basis change for each of the processes we consider. They are generated using Mathematica [25], using the necessary isoscalar factors from Kaeding [26]. Again, we follow the prescription of Ref. [18].

In order to obtain relationships between decay amplitudes within, for instance, the $3_{bb} \rightarrow 3_{bc} + M$ processes, we make assumptions about the Hamiltonian which induces the decay. The accuracy of these assumptions can then be tested experimentally.

The simplest choice is to match the Hamiltonian onto the four-quark operators generated by the standard model weak Lagrangian:

$$\mathcal{L} = \frac{g}{2\sqrt{2}} (\bar{u} \gamma \tau \bar{c}) \gamma^\mu (1 - \gamma_5) V \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\mu + \text{h.c.},$$

(3.1)

where $V$ is the CKM matrix. We will use the Wolfenstein parameterization [27] so that

$$V \sim \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3 (\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

(3.2)

Following the procedure of [13] we are interested in weak operators with the following flavor quantum numbers:

1. $(b\bar{c})(c\bar{s})$ and $(b\bar{c})(c\bar{d})$,
2. $(b\bar{c})(u\bar{s})$ and $(b\bar{c})(u\bar{d})$,
3. $(b\bar{u})(c\bar{s})$ and $(b\bar{u})(c\bar{d})$,
4. $(b\bar{u})(u\bar{s})$ and $(b\bar{u})(u\bar{d})$.

Since $(b\bar{c})(c\bar{d})$ and $(b\bar{u})(u\bar{s})$ are highly Cabibbo-suppressed, we will neglect them. We decompose the remaining operators into irreducible SU(3) multiplets which we will use, along with the multiplets of initial and final states, to form expressions for the terms in our Hamiltonian relevant to various decay processes. Constants (the reduced matrix elements of the Wigner-Eckart theorem) will remain undetermined in this procedure. Where an overall CKM parameter from the operators can be absorbed into reduced matrix elements, we do so. We give the SU(3) decomposition of the four-quark operators above and then include the effects of a term which transforms as $m_s(s\bar{s})$, the so-called “linear breaking term,” in an analysis similar to that in Ref. [21]. This will allow us to see, when data become available, if the dominant SU(3) breaking comes from the strange quark mass. Details of the decomposition, including normalization, are given in Appendix D.

1. The operators $(b\bar{c})(c\bar{s})$ and $(b\bar{c})(c\bar{d})$ transform as antitriplets and

$$H_{(1)(3)} \sim \begin{pmatrix} 0 \\ -\lambda \\ 1 - \lambda^2/2 \end{pmatrix} \approx \begin{pmatrix} 0 \\ -\lambda \\ 1 \end{pmatrix},$$

(3.3)
where the overall CKM constant $A\lambda^2$ has been dropped since it will not affect relationships between amplitudes induced by this operator.

Enlarging the group to incorporate the linear breaking terms $(s\bar{s}) = 8 \oplus 1$, we have $(b\bar{c})(c\bar{q})(s\bar{s}) \sim \overline{15} \oplus 6 \oplus \overline{3}_1 \oplus \overline{3}_2$, with $\bar{q} = \bar{d}$ or $\bar{s}$. This gives the following nonzero elements in addition to the $H_{(1)(3)}$ given above:

\[
H(\overline{15})_{33}^3 = 4, \\
H(\overline{15})_{32}^2 = H(\overline{15})_{23}^2 = H(\overline{15})_{13}^1 = H(\overline{15})_{31}^{1} = -2, \\
H(\overline{15})_{23}^3 = H(\overline{15})_{32}^3 = -3 \lambda, \\
H(\overline{15})_{21}^1 = H(\overline{15})_{12}^1 = \lambda, \\
H(\overline{15})_{22}^2 = 2 \lambda, \\
H(6)_{23}^3 = H(6)_{12}^1 = -H(6)_{21}^1 = -H(6)_{32}^3 = -\lambda, \\
H_{(2)(3)}^1 = 1, \\
H_{(2)(3)}^3 = 2, \\
(3.4)
\]

where again we keep only the CKM parameter $\lambda$ explicit. The elements of the 6 are given in terms of their Levi-Civita tensor contracted form.

2. The operators $(b\bar{c})(u\bar{d})$ and $(b\bar{c})(u\bar{s})$ transform as octets under SU(3):

\[
(b\bar{c})(u\bar{d}) = H_{(1)}(8)_{2} \sim V_{cb} V_{ud}^*, \\
(b\bar{c})(u\bar{s}) = H_{(1)}(8)_{3} \sim V_{cb} V_{us}^*, \\
H_{(1)}(8) \approx \begin{pmatrix} 0 & 1 - \frac{\lambda^2}{2} & \lambda \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & 1 & \lambda \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} . \\
(3.5)
\]

Including the linear breaking term we have

\[
(b\bar{c})(u\bar{q})(s\bar{s}) \sim 8_{(1)} \otimes (8 \oplus 1) = 27 \oplus 10 \oplus \overline{10} \oplus 8_{(1)} \oplus 8_{(2)} \oplus 8_{(3)}, \\
(3.6)
\]

where $\bar{q} = \bar{d}$ or $\bar{s}$. $8_{(3)}$ is already saturated by $8_{(1)}$ for our purposes, and furthermore there is no singlet for this particular operator. The nonzero operator terms in addition to those in Eqn. 3.5 above are:

\[
H(27)_{23}^{13} = H(27)_{23}^{31} = H(27)_{32}^{13} = H(27)_{32}^{31} = 2, \\
H(27)_{22}^{12} = H(27)_{22}^{21} = H(27)_{21}^{11} = H(27)_{12}^{11} = -1, \\
H(27)_{23}^{12} = H(27)_{32}^{12} = H(27)_{23}^{21} = H(27)_{32}^{21} = -\lambda, \\
H(27)_{13}^{11} = H(27)_{31}^{11} = -2 \lambda, \\
\]


\[ H(27)_{33}^{13} = H(27)_{33}^{31} = 3 \lambda, \]
\[ H(10)_{23}^{13} = -H(10)_{32}^{31} = -H(10)_{23}^{13} = 1, \]
\[ H(10)_{12}^{11} = -H(10)_{21}^{11} = 1, \]
\[ H\left(\mathbf{10}\right)_{23}^{13} = -H\left(\mathbf{10}\right)_{23}^{31} = H\left(\mathbf{10}\right)_{32}^{31} = 1, \]
\[ H\left(\mathbf{10}\right)_{22}^{21} = -H\left(\mathbf{10}\right)_{22}^{12} = 1, \]
\[ H\left(\mathbf{10}\right)_{32}^{21} = H\left(\mathbf{10}\right)_{23}^{12} = -H\left(\mathbf{10}\right)_{23}^{12} = \lambda, \]
\[ H\left(\mathbf{10}\right)_{33}^{13} = -H\left(\mathbf{10}\right)_{33}^{31} = \lambda, \]
\[ H\left(\mathbf{2}\right)_{3}^{1} = -1, \]
\[ H\left(\mathbf{2}\right)_{3}^{1} = 2 \lambda, \] (3.7)

where the elements of the 10 and \(\mathbf{10}\) are given in terms of their Levi-Civita tensor contracted form.

3. The operator \((b\bar{u})(c\bar{s})\) decomposes into a 3 and a \(\mathbf{6}\), with nonzero elements
\[ H'\left(\mathbf{6}\right)_{13} = H'\left(\mathbf{6}\right)_{31} = 1, \]
\[ H'(3)^2 = 1, \] (3.8)

where we now use \(H'\) to distinguish these operator elements from the ones appearing previously to guard against possible confusion. No overall CKM constants are necessary because this operator will not appear in the same process with any other operators.

Including a linear breaking term gives \((b\bar{u})(c\bar{s})(s\bar{s}) \sim \mathbf{24} \oplus \mathbf{15} \oplus \mathbf{6} \oplus \mathbf{3}\) with nonzero elements:
\[ H'\left(\mathbf{24}\right)_{321}^{2} = H'\left(\mathbf{24}\right)_{231}^{2} = H'\left(\mathbf{24}\right)_{123}^{2} = H'\left(\mathbf{24}\right)_{312}^{2} = H'\left(\mathbf{24}\right)_{132}^{2} = -1, \]
\[ H'\left(\mathbf{24}\right)_{113}^{1} = H'\left(\mathbf{24}\right)_{131}^{1} = H'\left(\mathbf{24}\right)_{311}^{1} = -2, \]
\[ H'\left(\mathbf{24}\right)_{331}^{3} = H'\left(\mathbf{24}\right)_{331}^{3} = H'\left(\mathbf{24}\right)_{331}^{3} = 3, \]
\[ H'\left(\mathbf{15}\right)_{3}^{32} = H'\left(\mathbf{15}\right)_{3}^{23} = 3, \]
\[ H'\left(\mathbf{15}\right)_{1}^{21} = H'\left(\mathbf{15}\right)_{1}^{12} = -1, \]
\[ H'\left(\mathbf{15}\right)_{2}^{22} = -2, \] (3.9)

and the \(\mathbf{6}\) and 3 the same as in Eqn. 3.8 above.

4. Finally we have the decomposition of the operator \((b\bar{u})(u\bar{d})\) into irreducible SU(3) representations. This operator induces the same decays as those in Eqn. 3.3 and so they will appear together in amplitude expressions. Therefore we need to include the
overall CKM factor $\lambda (\rho - i \eta)$. In the elements listed below we neglect CP violation and use only $\lambda \rho$; the $\eta$ dependence can be recaptured by substitution. We have

$$(bu\bar{d}) \sim T_{5(1)} \oplus 6 \oplus \overline{3},$$

(3.10)

where

$$H''_{(1)}(15)^{12} = H''_{(1)}(15)^{12} = 3 \lambda \rho,$$
$$H''_{(1)}(15)^{22} = -2 \lambda \rho,$$
$$H''_{(1)}(15)^{32} = H''_{(1)}(15)^{32} = -\lambda \rho,$$
$$H''(6)^{13} = H''(6)^{31} = \lambda \rho,$$
$$H''(3)^{2} = \lambda \rho.$$

(3.11)

We will also need the 6 in its Levi-Civita tensor contracted form:

$$H''(6)^{12} = -H''(6)^{21} = \lambda \rho; \ H''(6)^{23} = -H''(6)^{32} = \lambda \rho.$$

Including the linear breaking term gives

$$(bu\bar{d})(s\bar{s}) \sim (T_{5(1)} \oplus 6 \oplus \overline{3}) \otimes (8 \oplus 1)$$
$$= 12 \oplus 24_{(1)} \oplus 24_{(2)} \oplus 15' \oplus 15(1) \oplus 15(2) \oplus 15(3) \oplus 15(4) \oplus 6 \oplus \overline{3}. \quad (3.12)$$

There is no additional 6 or $\overline{3}$ compared to Eqn. [3.11]. The $15(1)$, the 6, and the $\overline{3}$ are given above, while the remaining nonzero elements are:

i) from the tensor product $\overline{3} \otimes 8$:

$$H''(15)^{12} = H''(15)^{12} = -\lambda \rho,$$
$$H''(15)^{22} = -2 \lambda \rho,$$
$$H''(15)^{32} = H''(15)^{32} = 3 \lambda \rho.$$

(3.13)

ii) from the tensor product $6 \otimes 8$:

$$H''_{(1)}(24)^{321} = H''_{(1)}(24)^{321} = H''_{(1)}(24)^{213} = -\lambda \rho,$$
$$H''_{(1)}(24)^{123} = H''_{(1)}(24)^{312} = H''_{(1)}(24)^{132} = -\lambda \rho,$$
$$H''_{(1)}(24)^{113} = H''_{(1)}(24)^{131} = H''_{(1)}(24)^{131} = -2 \lambda \rho,$$
$$H''_{(1)}(24)^{331} = H''_{(1)}(24)^{133} = H''_{(1)}(24)^{313} = 3 \lambda \rho.$$

(3.14)

iii) from the tensor product $15(1) \otimes 8$: 

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For our purposes, the $H''(\overline{12})$ are sufficient to provide a minimal set of unknowns, so we do not use $\overline{13}$. That is, the reduced amplitudes associated with $\overline{13}$ are linear combinations of the reduced amplitudes associated with $\overline{15}$. Notice, as suggested in Ref. [18], that this linear breaking term actually saturates more general SU(3) breaking than simply $(s\bar{s})$. If, for instance, we chose to include a term $(u\bar{u})$ in extending the operators from Eqn. (3.3) we would find that it is already mimicked in Eqn. (3.11) (albeit with some $\rho$ dependence now absorbed into the unknown constants). However, including the linear breaking $(s\bar{s})$ term does not saturate arbitrary SU(3) breaking, which is why we are still left with predictions.
IV. DECAY AMPLITUDES

A. $3_{bb} \rightarrow 3_{bc} + M$ (Final states with a b=-1, c=1 triplet baryon plus an octet meson)

This is the first of the decay amplitudes induced by Eqn. 3.3. From the product of $3_{bb}$, $3_{bc}$, $M$, and the operators in Eqn. 3.3 we can form three independent singlets, and so have expressions for all fourteen physical amplitudes in terms of only three unknown reduced matrix elements. We call them $A$, $B$, $C$, etc. We will use this notation for each type of decay, even though their values for one type of process are unrelated to those of another. Since the equations and tables will occur in different sections of the paper, this should not cause confusion.

Including the extension to linear breaking embodied in Eqn. 3.7, we can form six more singlets with this additional freedom. We call the unknown constants associated with each of these $A_{LB}$, $B_{LB}$, etc., where $LB$ stands for “linear breaking” even though, as we have seen, it can be more general than this. With this looser assumption about the Hamiltonian, we have fourteen amplitudes expressed in terms of nine reduced matrix elements. The expressions are listed in Table I. In order to recapture the amplitudes in the limit of exact SU(3) symmetry, we can set all of the $LB$ coefficients to zero.

$$
\mathcal{H}_{LB}(3_{bb} \rightarrow 3_{bc} + M) = A \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(1)(8)_k^j \right) + B \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(8)_k^j \right) + C \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(10)_k^j \right) + A_{LB} \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(2)(8)_k^j \right) + B_{LB} \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(2)(8)_k^j \right) + C_{LB} \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(2)(8)_k^j \right) + D_{LB} \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(2)(8)_k^j \right) + E_{LB} \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(2)(8)_k^j \right) + F_{LB} \langle 3_{bb} | i \left( 3_{bc} | M_j^k H(2)(8)_k^j \right). \tag{4.1}
$$

Examples of relationships between Cabibbo suppressed and Cabibbo allowed decays which occur under exact SU(3) are (see table I)

$$
\Gamma \left( \Xi^0_{bb} \rightarrow \Xi^+_{bc} \pi^- \right) = \frac{1}{\lambda^2} \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^+_{bc} K^- \right),
$$

$$
\Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{bc} \pi^- \right) = \frac{1}{\lambda^2} \Gamma \left( \Omega^-_{bb} \rightarrow \Omega^0_{bc} K^- \right). \tag{4.2}
$$

From this and the branching ratio for the decay $\Xi^0_{bb} \rightarrow \Xi^+/0 \pi^-$ found using potential models (2.2%) [11], we would predict that $\text{BR} \left( \Xi^0_{bb} \rightarrow \Xi^+_{bc} K^- \right) \sim 0.1\%$. The extent to which such relationships do not hold experimentally is a measure of SU(3) symmetry breaking in these decays. Semileptonic decays can be treated in the same way. For $3_{bb} \rightarrow 3_{bc} l \bar{\nu}$ the results are trivial unless SU(3) breaking is included, in which case only isospin relationships survive.

The reduced matrix elements induced by arbitrary SU(3) breaking, and their relationships to the fourteen allowed processes in this category, are given in Appendix E1. Matching

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1In writing these rate relationships we are ignoring potential phase space issues. See Appendix G for a discussion.
these results to those where only linear breaking of SU(3) is imposed (Table I), according to the description in Appendix F, we find the following relationships for these matrix elements:

\[ \langle 6 | \bar{T}_{I=\frac{1}{2}} | 3 \rangle = 0, \]
\[ \langle 15 | \bar{T}_{I=\frac{1}{2}} | 3 \rangle = 0, \]
\[ \langle 15 | \bar{T}_{I=2} | 3 \rangle = 0, \]
\[ \langle 15 | \bar{T}_{I=\frac{3}{2}} | 3 \rangle = \lambda \langle 15 | \bar{T}_{I=1} | 3 \rangle, \]
\[ \langle 15 | \bar{T}_{I=\frac{5}{2}} | 3 \rangle = \sqrt{\frac{3}{2}} \lambda \langle 15 | \bar{T}_{I=1} | 3 \rangle. \] (4.3)

It is clear why the higher isospin reduced amplitudes are zero for linear breaking: extension by \((s\pi)\) cannot change the isospin of the underlying octet operators in Eqn. 3.3, which have \(I = 1/2\) for the Cabbibo suppressed decay and \(I = 1\) for the Cabbibo allowed decay.

Note that if the \(b\) quark decays first, a table with entries identical to Table I applies to the analogous decay \(3_{bc} \rightarrow 3_{cc} + M\), where \(3_{cc}\) are the triplet of doubly charmed baryons with quark quantum numbers \((ccu, ccd, ccs)\). This is obtained by making the \(b \rightarrow c\) substitution in the right-hand column (with the appropriate changes in charge and particle symbol). Pauli

| \(A + C - A_{LB} - C_{LB} + D_{LB} - F_{LB}\) | \(\Xi_{bb}^0 \rightarrow \Xi_{bc}^+ \pi^-\) |
| \(\frac{1}{\sqrt{2}} B - \frac{1}{\sqrt{2}} C - \frac{1}{\sqrt{2}} B_{LB} + \frac{1}{2} E_{LB} - \frac{1}{\sqrt{2}} D_{LB} + \frac{1}{\sqrt{2}} F_{LB}\) | \(\Xi_{bb}^0 \rightarrow \Xi_{bc}^0 \phi^0\) |
| \(\frac{1}{\sqrt{6}} B + \frac{1}{\sqrt{6}} C - \frac{1}{\sqrt{6}} B_{LB} - \frac{1}{\sqrt{6}} C_{LB} - \sqrt{\frac{3}{2}} D_{LB} - \sqrt{\frac{3}{2}} E_{LB} - \sqrt{\frac{3}{2}} F_{LB}\) | \(\Xi_{bb}^0 \rightarrow \Xi_{bc}^0 \eta_8\) |
| \(C - C_{LB} - D_{LB} + E_{LB} + 2 F_{LB}\) | \(\Omega_{bb}^0 \rightarrow \Omega_{bc}^0 K^0\) |
| \(A + B - A_{LB} - B_{LB} + E_{LB} - F_{LB}\) | \(\Xi_{bb}^- \rightarrow \Xi_{bc}^0 \pi^-\) |
| \(B - B_{LB} + D_{LB} - E_{LB} + 2 F_{LB}\) | \(\Omega_{bb}^- \rightarrow \Omega_{bc}^0 K^-\) |
| \(A - A_{LB} - B_{LB} - D_{LB} - E_{LB} + 2 F_{LB}\) | \(\Omega_{bb}^- \rightarrow \Omega_{bc}^0 \pi^-\) |

TABLE I. Matrix elements for the decay \(3_{bb} \rightarrow 3_{bc} + M\)
suppression factors are the same for each decay and so do not affect relationships among them.

**B.** $3_{bb} ightarrow 3_{bc} + M + M$ (**Final states with a $b=-1$, $c=1$ triplet baryon plus two octet mesons**)

The operators which induce the $3_{bb}$ decay into $3_{bc}$ plus two mesons are the same as those in the last section. Now however we have many more possible ways of creating a singlet using these tensors, resulting in a larger set of reduced matrix elements in which to express decay rates. Further, because we are dealing with two meson octets in the final state – which are identical under SU(3) – we must impose the appropriate symmetry. For mesons in a relative even angular momentum state $L$, our Hamiltonian is symmetric under interchange of SU(3) indices on the meson tensors. For mesons in a relative odd angular momentum state $L$, our Hamiltonian is antisymmetric under interchange of SU(3) indices on the meson tensors. Forty-six even–$L$ physical decays are expressed in terms of six reduced matrix elements when SU(3) symmetry is imposed. There are forty-two odd–$L$ physical amplitudes, where because of the symmetry requirement there are only five independent reduced matrix elements. Because of the large number of additional reduced matrix elements appearing when the SU(3) symmetry is broken by a linear term, we do not present tables including them. The SU(3) conserving predictions are found in Appendices A and B. These tables list the decay “amplitudes” by already taking into account the symmetry factors for identical mesons in the final state. So, excepting mass corrections in the phase space, rates are obtained simply by taking the absolute value squared of the expressions given in Appendices A and B. Note that no identical mesons are found in the final state of an $L$–odd decay because of symmetry restrictions. However, two final states occur for $L$–odd which do not appear for $L$–even: $\Omega_{bb}^- \rightarrow \Omega_{bc}^0 \pi^0 \pi^-$ for the Cabibbo allowed case, and $\Xi_{bb}^- \rightarrow \Omega_{bc}^0 \pi^0 \pi^-$ for the Cabibbo suppressed case. For the $L$–even case we have:

$$H_{LB}(3_{bb} \rightarrow 3_{bc} + M + M) =$$

$$+ A [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(1)(8)_j^i + B [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(1)(8)_j^i$$

$$+ C [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(1)(8)_j^i + D [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(1)(8)_j^i$$

$$+ E [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(1)(8)_j^i + F [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(1)(8)_j^i$$

$$+ A_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(2)(8)_j^i + B_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(2)(8)_j^i$$

$$+ C_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(2)(8)_j^i + D_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(2)(8)_j^i$$

$$+ E_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(2)(8)_j^i + F_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(2)(8)_j^i$$

$$+ G_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(10)_{jk}^{pq} + H_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(10)_{jk}^{pq}$$

$$+ I_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(10)_{jk}^{pq} + J_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(10)_{jk}^{pq}$$

$$+ K_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(10)_{jk}^{pq} + L_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(10)_{jk}^{pq}$$

$$+ M_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(27)_{jk}^{pq} + N_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(27)_{jk}^{pq}$$

$$+ O_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(27)_{jk}^{pq} + P_{LB} [3_{bb}]_i [3_{bc}]^j M_p^l M_{1j}^i H(27)_{jk}^{pq}. \quad (4.4)$$

For the unbroken SU(3) case there are many relationships. Below are some examples.
If there is no subscript on the meson states then the relationship holds independently of whether $L$ is even or odd.

$$2 \Gamma(\Xi_{bb}^0 \to \Xi_{bc}^0 \pi^- K^0) = \frac{1}{2} \Gamma(\Xi_{bb}^- \to \Xi_{bc}^+ (\pi^- \pi^-)_{L=even}) =$$

$$2 \Gamma(\Xi_{bb}^0 \to \Xi_{bc}^0 (\pi^- K^0)_{L=even}) = \Gamma(\Xi_{bb}^0 \to \Omega_{bc}^0 (\pi^- K^0)_{L=even}) =$$

$$\frac{1}{\lambda^2} \Gamma(\Omega_{bb}^- \to \Xi_{bc}^0 (K^- \bar{K}^0)_{L=even}) = \frac{1}{2\lambda^2} \Gamma(\Omega_{bb}^- \to \Xi_{bc}^0 (K^- K^-)_{L=even}). \quad (4.5)$$

Two pairs of the above decay rates also hold regardless of $L$ value, and odd values of $L$ add a decay rate to each:

$$\Gamma(\Xi_{bb}^- \to \Omega_{bc}^0 \pi^- K^0) = \frac{1}{\lambda^2} \Gamma(\Omega_{bb}^- \to \Xi_{bc}^0 K^- \bar{K}^0) = 6 \Gamma(\Xi_{bb}^- \to \Xi_{bc}^0 (\pi^- \eta_S)_{L=odd}), \quad (4.6)$$

$$\Gamma(\Omega_{bb}^- \to \Xi_{bc}^0 \pi^- K^-) = \frac{1}{\lambda^2} \Gamma(\Xi_{bb}^- \to \Xi_{bc}^0 \pi^- K^-) = 6 \Gamma(\Xi_{bb}^0 \to \Xi_{bc}^0 (\pi^- \eta_S)_{L=odd}). \quad (4.7)$$

Additional examples of relationships between Cabbibo-allowed and Cabbibo suppressed decays which hold regardless of $L$ value are:

$$\Gamma(\Xi_{bb}^0 \to \Xi_{bc}^0 \pi^+ \pi^-) = \frac{1}{\lambda^2} \Gamma(\Xi_{bb}^- \to \Omega_{bc}^0 K^+ K^-),$$

$$\Gamma(\Omega_{bb}^- \to \Xi_{bc}^0 K^0 K^-) = \frac{1}{\lambda^2} \Gamma(\Xi_{bb}^- \to \Xi_{bc}^0 \pi^- \bar{K}^0),$$

$$\Gamma(\Xi_{bb}^0 \to \Omega_{bc}^0 \pi^- K^+) = \frac{1}{\lambda^2} \Gamma(\Xi_{bb}^0 \to \Xi_{bc}^0 \pi^+ K^-). \quad (4.8)$$

Note, however, that these are unbroken SU(3) relationships. We have not treated explicit violations which may result, for instance, from final state interactions in the form of mixing with nearby resonances, which is a known issue for the two meson final states $13$.

Once again, analogous relationships hold for $3_{bc} \to 3_{cc} + M + M'$.

C. $3_{bb} \to 6_b + D$ (Final states with a $b=-1$ 6 baryon plus a D meson)

The $3_{bb}$ may also decay to a member of the $D$-meson antitriplet, Eqn. $2.6$, and a baryon from the 6 representation of SU(3), whose particles are identified in Eqn. $2.3$. There exist only two ways of constructing a singlet using these multiplets and the operator in Eqn. $3.3$, yielding only two reduced matrix elements in which ten amplitudes can be written when SU(3) symmetry is exact. To include linear breaking terms we also use the operators in Eqn. $3.7$, which increases the number of reduced matrix elements by four.

$$\mathcal{H}_{LB}(3_{bb} \to 6_b + D) = A \mathbb{[}\bar{\xi}_{bb}\mathbb{]}_i [6_b]_j^{ij} D_k H_{(1)(8)}^j_k + B \mathbb{[}\bar{\xi}_{bb}\mathbb{]}_i [6_b]_j^{ijk} D_k H_{(1)(8)}^j_k +$$

$$+ A_{LB} \mathbb{[}\bar{\xi}_{bb}\mathbb{]}_i [6_b]_j^{ij} D_k H_{(2)(8)}^j_k + B_{LB} \mathbb{[}\bar{\xi}_{bb}\mathbb{]}_i [6_b]_j^{ijk} D_k H_{(2)(8)}^j_k +$$

$$+ C_{LB} \mathbb{[}\bar{\xi}_{bb}\mathbb{]}_i [6_b]_j^{kl} D_j H_{(\Sigma)}^{ij}_{kl} + D_{LB} \mathbb{[}\bar{\xi}_{bb}\mathbb{]}_i [6_b]_j^{kl} D_j H_{(27)}^{ij}_{kl}. \quad (4.9)$$
Note that there is no $H(10)$ contribution to this decay because of the symmetry of the $[6_b]$. The amplitudes are shown in Table II. Example relationships under SU(3) are:

$$\Gamma(\Xi_{bb} \rightarrow \Sigma^+_b D^+) = 2 \Gamma(\Xi^0_{bb} \rightarrow \Xi^0_{b2} D^+_s) = \frac{1}{\lambda^2} \Gamma(\Xi^0_{bb} \rightarrow \Omega^-_b D^+_s).$$

\[4.10\]

| Reduced Matrix Elements | Matrix Elements |
|------------------------|-----------------|
| $\frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} B - \frac{1}{\sqrt{2}} A_{LB} - \frac{1}{\sqrt{2}} B_{LB} - \sqrt{2} D_{LB}$ | $\Xi^0_{bb} \rightarrow \Sigma^0_b D^0$ |
| $B - B_{LB} - C_{LB} - D_{LB}$ | $\Xi^0_{bb} \rightarrow \Sigma^+_b D^+$ |
| $\frac{1}{\sqrt{2}} B - \frac{1}{\sqrt{2}} A_{LB} + \sqrt{2} C_{LB} + 2\sqrt{2} D_{LB}$ | $\Xi^0_{bb} \rightarrow \Xi^0_{b2} D^+_s$ |
| $A - A_{LB} + C_{LB} - D_{LB}$ | $\Xi^-_{bb} \rightarrow \Sigma^- D^0$ |
| $\frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} A_{LB} + \sqrt{2} C_{LB} - 2\sqrt{2} D_{LB}$ | $\Omega^-_{bb} \rightarrow \Xi^-_{b2} D^0$ |

\[4.11\]

Including only linear breaking of SU(3) simplifies the ten processes, which are given in terms of the ten reduced matrix elements required by arbitrary SU(3) breaking in Appendix E.2. In that case, the reduced matrix elements behave as follows:

$$\langle 15||27_{I=2}||3 \rangle = 0,$$
$$\langle 15||27_{I=4}||3 \rangle = 0,$$
$$\langle 15||10_{I=1}||3 \rangle = \lambda \langle 15||10_{I=1}||3 \rangle,$$
$$\langle 15||27_{I=4}||3 \rangle = \lambda \sqrt{\frac{3}{2}} \langle 15||27_{I=1}||3 \rangle.$$ 

\[4.11\]

Note that the $I = 2$ and $I = \frac{3}{2}$ reduced matrix elements are again zero and that the same pattern of CKM parameters relating half-integer isospin to integer isospin reduced matrix elements occurs in Eqn. 4.3. The analogous relationships for $3_{bc}$ decays are to the $6_c$ and $D$. The members of the $6_c$ are $\Omega^0_c, \Xi^+_c, \Xi^0_{c2}, \Sigma^{++}_c, \Sigma^+_c, \Sigma^0_c$, and $\Sigma^0_c$.

**D. $3_{bc} \rightarrow 3_b + D$ (Final states with an antitriplet baryon plus a D meson)**

The last decay we will consider which takes place primarily through the operators given in Eqn. 4.3 (or its extension in Eqn. 4.7) is to a D meson plus one of the members of the antitriplet from Eqn. 2.4.
Including the linear SU(3) breaking terms the expression becomes:

$$\mathcal{H}_{LB}(3b\bar{b} \rightarrow \bar{3}_b + D) = A \left[ \bar{3}_b \right]_i \left[ \bar{3}_b \right]_l D_k H(1)(8)_j^k \epsilon^{lj} + B \left[ \bar{3}_b \right]_i \left[ \bar{3}_b \right]_l D_k H(1)(8)_j^i \epsilon^{lj} + C_{LB} \left[ \bar{3}_b \right]_i \left[ \bar{3}_b \right]_m D_j H(10)_{kl}^i \epsilon^{mkl}. \quad (4.12)$$

This decay does not admit of a term involving either the $\bar{u}u$ or the 27 because of the antisymmetry of the $[\bar{3}_b]_i \epsilon^{ijk}$ structure. The decay amplitudes are given in Table III. When SU(3) holds the amplitudes are related as follows:

$$\Gamma \left( \Xi^0_{bb} \rightarrow \Xi^+_{b1} D^+_s \right) = \frac{1}{\lambda^2} \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^-_{b1} D^+ \right),$$

$$\Gamma \left( \Xi^0_{bb} \rightarrow \Lambda^0_b D^0 \right) = \frac{1}{\lambda^2} \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{b1} D^0 \right),$$

$$\Gamma \left( \Omega^-_{bb} \rightarrow \Xi^-_{b1} D^0 \right) = \frac{1}{\lambda^2} \Gamma \left( \Xi^-_{bb} \rightarrow \Xi^-_{b1} D^0 \right). \quad (4.13)$$

| $B - B_{LB} + 2 C_{LB}$ | $\Xi^0_{bb} \rightarrow \Xi^-_{b1} D^+_s$ |
|---------------------------|----------------------------------|
| $A - B - A_{LB} + B_{LB} + 2 C_{LB}$ | $\Xi^0_{bb} \rightarrow \Lambda^0_b D^0$ |
| $- A + A_{LB} + 2 C_{LB}$ | $\Omega^-_{bb} \rightarrow \Xi^-_{b1} D^0$ |
| $\lambda \left( - B - 2 B_{LB} \right)$ | $\Xi^0_{bb} \rightarrow \Xi^-_{b1} D^+$ |
| $\lambda \left( A - B + 2 A_{LB} - 2 B_{LB} \right)$ | $\Xi^0_{bb} \rightarrow \Xi^0_{b1} D^0$ |
| $\lambda \left( A + 2 A_{LB} \right)$ | $\Xi^-_{bb} \rightarrow \Xi^-_{b1} D^0$ |

**TABLE III.** Matrix elements for the decay $3b\bar{b} \rightarrow \bar{3}_b + D$

Arbitrary SU(3) breaking yields six reduced matrix elements for the six amplitudes of this type, shown in Appendix E. When only linear breaking is included, we find:

$$\langle \bar{6} | \left[ \bar{10} \right]_{f=\frac{3}{2}} | 3 \rangle = 0, \quad (4.14)$$

because the Cabbibo suppressed operator $(\bar{b} \bar{c})(u \bar{c})(s \bar{s})$ has no nonzero 10 components. (Note that an operator employed using the tensor methods described in the body of this paper is expressed in terms of its barred components in the arbitrarily broken SU(3) reduced matrix elements used in Appendix E. Therefore, the lack of 10 components for $(\bar{b} \bar{c})(u \bar{c})(s \bar{s})$ corresponds to $\langle \bar{3} | \left[ \bar{10} \right]_{f=3/2} | 3 \rangle = 0.$) In this case, we have six amplitudes expressed in terms of five reduced matrix elements. Analogous relationships for the $3_{bc}$ decays are found from the table by making the appropriate $b \rightarrow c$ and charge replacement. The final state particles are $\bar{3}_c$ and $D$, where the members of the $3_c$ are $\Lambda^+_c$, $\Xi^+_c$, and $\Xi^0_c$. 

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E. $3_{bb} \rightarrow 3_{bc} + \overline{D}$ (Final states with a $b=1$, $c=1$ triplet baryon plus a $\overline{D}$ meson)

Now we treat the decays which utilize the operators given in Eqn. 3.3 and Eqn. 3.11 and their linear breaking extensions found in Eqns. 3.4 and 3.13-3.15. The $b=2$ SU(3) triplet particles in Eqn. 2.1 can decay through these operators to the triplet $b=1$, $c=1$ particles in Eqn. 2.2, and an anti-$D$ meson. There are six reduced matrix elements for the full SU(3) case and two additional when linear breaking is included.

| $A - 2B_{LB}$ | $\Xi_{bb}^{0} \rightarrow \Xi_{bc}^{+}D_{s}^{-}$ |
|--------------|-----------------------------------|
| $B - 2B_{LB}$ | $\Xi_{bb}^{0} \rightarrow \Omega_{bc}^{0}\overline{D}^{0}$ |
| $A - 2B_{LB}$ | $\Xi_{bb}^{-} \rightarrow \Xi_{bc}^{0}D_{s}^{-}$ |
| $B - 2B_{LB}$ | $\Xi_{bb}^{-} \rightarrow \Omega_{bc}^{0}\overline{D}^{-}$ |
| $A + B + 4B_{LB}$ | $\Omega_{bb}^{-} \rightarrow \Omega_{bc}^{0}\overline{D}^{-}$ |

$\lambda (- A + \rho C + \rho E + 3\rho F - \rho A_{LB} + B_{LB})$  
$\lambda (- B + \rho D - \rho E + 3\rho F - \rho A_{LB} + B_{LB})$  
$\lambda (- A - B + \rho C + \rho D - 2\rho F - 2\rho A_{LB} + 2B_{LB})$  
$\lambda (- B + \rho D + \rho E - \rho F + 3\rho A_{LB} - 3B_{LB})$  
$\lambda (- A + \rho C - \rho E - \rho F + 3\rho A_{LB} - 3B_{LB})$

$\mathcal{H}_{LB}(3_{bb} \rightarrow 3_{bc} + \overline{D}) = A [\overline{3}_{bb}]_{i} [3_{bc}]_{i} \overline{D}^{j} H^{(1)}(\overline{3})_{j} + B [\overline{3}_{bb}]_{i} [3_{bc}]_{i} \overline{D}^{j} H^{(1)}(\overline{3})_{j}$  
$+ C [\overline{3}_{bb}]_{i} [3_{bc}]_{i} \overline{D}^{j} H''(\overline{3})_{j} + D [\overline{3}_{bb}]_{i} [3_{bc}]_{i} \overline{D}^{j} H''(\overline{3})_{j}$  
$+ E [\overline{3}_{bb}]_{i} [3_{bc}]_{i} \overline{D}^{j} H''(\overline{6})_{j} + F [\overline{3}_{bb}]_{i} [3_{bc}]_{i} \overline{D}^{j} H''(\overline{3})_{j}$  
$+ A_{LB} [\overline{3}_{bb}]_{i} [3_{bc}]_{j} \overline{D}^{k} H''(\overline{2})(\overline{15})_{jk} + B_{LB} [\overline{3}_{bb}]_{i} [3_{bc}]_{j} \overline{D}^{k} H'(\overline{15})_{jk}$. \quad (4.15)

The results are shown in Table IV. With a linear breaking term included we have not only the $\overline{15}$ from the $(b\overline{u})(u\overline{d})$ operator, $H''^{(1)}(\overline{15})$, but also $H''^{(2)}(\overline{15})$ as well as a different $\overline{15}$ from $(b\overline{c})(c\overline{s})$ of Eqn. 3.4 $H(\overline{15})$. Note that we do not need to include separate terms for $H(6)$ from Eqn. 3.4 because the $H''(6)$ operator of Eqn. 3.11 already saturates this one (with the appropriate absorption of $\rho$ factors). For a similar reason, we do not need $H(2)(\overline{3})$, $H'(\overline{3})(\overline{15})$, or $H''(4)(\overline{15})$; the most general terms are already contained in the operators we have included.

Table IV shows that the following relationships hold not only under unbroken SU(3) but also in the linear breaking case:

$$\Gamma \left( \Xi_{bb}^{0} \rightarrow \Xi_{bc}^{+}D_{s}^{-} \right) = \Gamma \left( \Xi_{bb}^{-} \rightarrow \Xi_{bc}^{0}D_{s}^{-} \right),$$

$$\Gamma \left( \Xi_{bb}^{0} \rightarrow \Omega_{bc}^{0}\overline{D}^{0} \right) = \Gamma \left( \Xi_{bb}^{-} \rightarrow \Omega_{bc}^{0}\overline{D}^{-} \right). \quad (4.16)$$
Comparing these results to Appendix E 4 where arbitrarily broken SU(3) gives the ten allowed processes in terms of the full 10 reduced matrix elements, the reduced matrix elements behave as follows: We have no isospin 1 elements because \((b\bar{c})(c\bar{s})\) of Eqn 3.3 contains only \(I = 0\), and linear breaking with \((s\bar{s})\) does not modify this. The \(I = 1\) terms correspond to the highly Cabbibo suppressed operator neglected in our treatment. Therefore

\[
\langle 3|\mathbf{5}_{I=1}|3\rangle = 0,
\langle 6|\mathbf{15}_{I=1}|3\rangle = 0,
\]

and we see how the ten unknown reduced matrix elements for arbitrarily broken SU(3) are reduced to the 8 in Eqn. 4.15. As in previous sections, the results for decays of the type \(3_{bc} \to 3_{cc} + D\) are easily obtained from Table IV.

\[
(4.17)
\]

**F. J/Ψ final states**

The most promising decay modes for detecting the presence and decay of doubly heavy baryons involves those with a \(J/Ψ\) particle in the final state since the subsequent \(J/Ψ \to \mu^+\mu^-\) decay provides a clean experimental signature.

The \(J/Ψ\) transforms as a singlet under SU(3). It can occur in a final state along with the antitriplet of Eqn. 2.4 via the operators in Eqn. 3.3 and in Eqn. 3.11 and their extensions in Eqns. 3.4 and 3.13-3.15. The most general SU(3) conserving form is unchanged by the addition of linear breaking terms:

\[
\mathcal{H}_{LB}(3_{bb} \to 3_b + J/Ψ) = A [\mathbf{3}_{bb}]_i [\mathbf{3}_b]_l H(1)(\mathbf{3})_j e^{ij} + B [\mathbf{3}_{bb}]_i [\mathbf{3}_b]_l H''(\mathbf{3})_j e^{ij} + C [\mathbf{3}_{bb}]_i [\mathbf{3}_b]_j H''(6)_{ij},
\]

(4.18)

because \(H(6)_{ij}\) is already saturated in \(H''(6)\) and \(H(2)(\mathbf{3})\) is contained in \(H(1)(\mathbf{3})\) and \(H''(\mathbf{3})\). The result is found in Table V and

\[
\Gamma \left( Ξ_{bb}^0 \to Ξ_{b1}^0 J/Ψ \right) = \Gamma \left( Ξ_{bb}^- \to Ξ_{b1}^- J/Ψ \right)
\]

(4.19)
is a robust prediction, albeit a result of isospin.

|   | \(A\) | \(Ξ_{bb}^0 \to Ξ_{b1}^0 J/Ψ\) |
|---|---|---|
|   | \(A\) | \(Ξ_{bb}^- \to Ξ_{b1}^- J/Ψ\) |
| \(λ(−A + ρB + ρC)\) | \(Ξ_{bb}^0 \to Λ_{b1}^0 J/Ψ\) |
| \(λ(A − ρB + ρC)\) | \(Ω_{bb}^- \to Ξ_{b1}^- J/Ψ\) |

**TABLE V.** Matrix elements for the decay \(3_{bb} \to 3_b + J/Ψ\)

When we compare this treatment with the result in Appendix E 3, which gives the decay amplitudes in terms of the most general SU(3) breaking expressions, we find that:
\[ \langle 3 | \bar{b}_{I=1} | 3 \rangle = 0. \quad (4.20) \]

This matrix element corresponds to a highly Cabbibo suppressed operator, which we have neglected. Now we have four amplitudes given in terms of three reduced matrix elements.

The \( J/\Psi \) can also occur in a decay with a 6 from Eqn. (2.3) in the final state. Three unknown reduced matrix elements come from the unbroken SU(3) case, and two additional from the linear breaking term.

\[
\mathcal{H}_{LB}(3_{bb} \rightarrow 6_b + J/\Psi) = A \langle \bar{3}_{bb} | i | 6_b \rangle \frac{ij}{H^{(1)}(\bar{3})} + B \langle \bar{3}_{bb} | i | 6_b \rangle H^{''}(\bar{3}) + C \langle \bar{3}_{bb} | i | 6_b \rangle H^{'''}(\bar{3}) + A_{\text{LB}} \langle \bar{3}_{bb} | i | 6_b \rangle H^{''''}(\bar{3}) + B_{LB} \langle \bar{3}_{bb} | i | 6_b \rangle H^{(T5)} , \quad (4.21)
\]

with results shown in Table VI. There is no term containing \( H^{(6)}_{jk} \), because of the antisymmetry in \( j \) and \( k \). And, as discussed before, the above expression already saturates terms involving \( H^{(2)}(\bar{3}), H^{''}(\bar{3}T5) \) and \( H^{'''}(\bar{3})T5) \).

| \( \frac{1}{\sqrt{2}} A - 2\sqrt{2} B_{LB} \) | \( \Xi_{bb} \rightarrow \Xi_{b2} J/\Psi \) |
| --- | --- |
| \( \frac{1}{\sqrt{2}} A - 2\sqrt{2} B_{LB} \) | \( \Xi_{bb} \rightarrow \Xi_{b2} J/\Psi \) |
| \( A + 4 B_{LB} \) | \( \Omega_{bb} \rightarrow \Omega_{b2} J/\Psi \) |

| \( \lambda (- \frac{1}{\sqrt{2}} A + \sqrt{2} \rho B + 3\sqrt{2} \rho C - \sqrt{2} \rho A_{\text{LB}} + \sqrt{2} B_{LB}) \) | \( \Xi_{bb} \rightarrow \Sigma_{b2} J/\Psi \) |
| --- | --- |
| \( \lambda (- A + \rho B - 2 \rho C - 2 \rho A_{\text{LB}} + 2 B_{LB}) \) | \( \Xi_{bb} \rightarrow \Sigma_{b2} J/\Psi \) |
| \( \lambda (- \frac{1}{\sqrt{2}} A + \sqrt{2} \rho B - \sqrt{2} \rho C + 3\sqrt{2} \rho A_{\text{LB}} - 3\sqrt{2} B_{LB}) \) | \( \Omega_{bb} \rightarrow \Omega_{b2} J/\Psi \) |

TABLE VI. Matrix elements for the decay \( 3_{bb} \rightarrow 6_b + J/\Psi \)

In the case of unbroken SU(3) we have

\[
\Gamma \left( \Xi_{bb} \rightarrow \Xi_{b2} J/\Psi \right) = \frac{1}{2} \Gamma \left( \Xi_{bb} \rightarrow \Xi_{b2} J/\Psi \right) = 2 \Gamma \left( \Omega_{bb} \rightarrow \Omega_{b2} J/\Psi \right).
\]

Only the first equality survives when linear breaking terms are included.

Comparing the arbitrarily broken SU(3) reduced matrix elements as given in Appendix E7 to the linear broken results we find

\[ \langle 6 | 15_{I=1} | 3 \rangle = 0, \quad (4.23) \]

which also follows from the fact that the integer isospin \( \bar{T5} \) comes from the operator \((b\bar{c})(c\bar{s})\) in Eqn. 3.3, which is purely isospin 0.

Analogous results are obtained, if the \( b \) quark decays first, for the decays \( 3_{bc} \rightarrow \bar{T5}(6_c) J/\Psi \).
G. $3_{bb} \rightarrow 3_b + M$ (Final states with a b=−1 antitriplet and an octet meson)

The operators shown in Eqn. 3.3 and Eqn. 3.11 also induce decays to a final antitriplet of b=−1 baryons and an octet meson. In the SU(3) symmetric case we have eight reduced matrix elements. Including linear SU(3) breaking terms gives six additional from the available $\bar{\Omega}$'s and 24's. And so we have

$$H_{LB}(3_{bb} \rightarrow 3_b + M) =$$

$$+ A \prod [3_{bb}]_i [3_b]_m M^k_i H_{(1)}(3)_k \epsilon^{ijm} + B [3_{bb}]_i [3_b]_m M^i_j H_{(1)}(3)_k \epsilon^{kjm}$$

$$+ C [3_{bb}]_i [3_b]_m M^k_j H''(3)_k \epsilon^{jim} + D [3_{bb}]_i [3_b]_m M^i_j H''(3)_k \epsilon^{kjm}$$

$$+ E [3_{bb}]_i [3_b]_j M^i_j H''(6)_l \epsilon^{jim} + F [3_{bb}]_i [3_b]_j M^i_j H''(6)_l \epsilon^{kjm}$$

$$+ G [3_{bb}]_i [3_b]_m M^k_i H_{(1)}(15)_{jk} \epsilon^{ijm} + H [3_{bb}]_i [3_b]_m M^i_j H_{(1)}(15)_{jk} \epsilon^{kjm}$$

$$+ A_{LB} [3_{bb}]_i [3_b]_m M^k_i H''(2)(15)_{jk} \epsilon^{ijm} + B_{LB} [3_{bb}]_i [3_b]_m M^i_j H''(2)(15)_{jk} \epsilon^{kjm}$$

$$+ C_{LB} [3_{bb}]_i [3_b]_m M^k_i H(15)_{jk} \epsilon^{jim} + D_{LB} [3_{bb}]_i [3_b]_m M^i_j H(15)_{jk} \epsilon^{kjm}$$

$$+ E_{LB} [3_{bb}]_i [3_b]_j M^i_j H_{(1)}(24)_l \epsilon^{jim} + F_{LB} [3_{bb}]_i [3_b]_j M^i_j H_{(2)}(24)_l \epsilon^{kjm}.$$ (4.24)

From Tables VII and VIII we see that the following relationships between decay amplitudes

| $-B - 2D_{LB}$ | $\Xi^0_{bb} \to \Xi^0_{b1} \pi^+$ |
|----------------|--------------------------------|
| $-\frac{1}{\sqrt{2}} B - \sqrt{2} D_{LB}$ | $\Xi^0_{bb} \to \Xi^0_{b1} \pi^0$ |
| $-\sqrt{\frac{2}{3}} A - \frac{1}{\sqrt{6}} B - 2\sqrt{6} C_{LB} - \sqrt{6} D_{LB}$ | $\Xi^0_{bb} \to \Xi^0_{b1} \eta_8$ |
| $A - 2C_{LB} + 2D_{LB}$ | $\Xi^0_{bb} \to \Lambda^0_{b1} K^0$ |
| $\frac{1}{\sqrt{2}} B + \sqrt{2} D_{LB}$ | $\Xi^0_{bb} \to \Xi^0_{b1} \pi^0$ |
| $-\sqrt{\frac{2}{3}} A - \frac{1}{\sqrt{6}} B - 2\sqrt{6} C_{LB} - \sqrt{6} D_{LB}$ | $\Xi^0_{bb} \to \Xi^0_{b1} \eta_8$ |
| $-B - 2D_{LB}$ | $\Xi^0_{bb} \to \Xi^0_{b1} \pi^0$ |
| $-A + 2C_{LB} - 2D_{LB}$ | $\Xi^0_{bb} \to \Lambda^0_{b1} K^0$ |
| $-A - B + 2C_{LB} + 4D_{LB}$ | $\Omega^0_{bb} \to \Xi^0_{b1} K^0$ |
| $-A - B + 2C_{LB} + 4D_{LB}$ | $\Omega^0_{bb} \to \Xi^0_{b1} K^0$ |

TABLE VII. Matrix elements for the decay $3_{bb} \rightarrow 3_b + M$; Cabibbo allowed decays

hold:

$$\Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{b1} \pi^+ \right) = 2 \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{b1} \pi^0 \right) = 2 \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{b1} \pi^0 \right) = \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{b1} \pi^0 \right),$$

$$\Gamma \left( \Xi^0_{bb} \rightarrow \Lambda^0_{b1} \bar{K}^0 \right) = \Gamma \left( \Xi^0_{bb} \rightarrow \Lambda^0_{b1} \bar{K}^0 \right),$$

$$\Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{b1} \eta_8 \right) = \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{b1} \eta_8 \right),$$

$$\Gamma \left( \Omega^0_{bb} \rightarrow \Xi^0_{b1} \bar{K}^0 \right) = \Gamma \left( \Omega^0_{bb} \rightarrow \Xi^0_{b1} \bar{K}^0 \right).$$ (4.25)
Comparing this to the results given in Appendix E, we see that restricting SU(3) breaking to a linear term gives us the following: All six non-zero integer isospin reduced matrix elements are zero, for the same reasons as given in previous sections. Therefore we see how the 20 reduced matrix elements are collapsed to 14 in the case of linear breaking.

As in previous sections, results for the decay $3_{bc} \rightarrow \overline{3}_c + M$ are obtained directly.

\[
\begin{array}{ll}
\lambda (-B + \rho D + \rho E - 3\rho G - 3\rho H + 3\rho A_{LB} + \rho B_{LB}) & \Xi^{0}_{bb} \rightarrow \Xi^+_{b1} K^+ \\
\lambda (-A + \rho C - \rho F - 3\rho G + 3\rho H + 3\rho A_{LB} + \rho B_{LB}) & \Xi^{0}_{bb} \rightarrow \Xi^0_{b1} K^0 \\
\lambda \left( \frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} B - \frac{1}{\sqrt{2}} \rho C - \frac{1}{\sqrt{2}} \rho D + \frac{1}{\sqrt{2}} \rho E + 5 \frac{1}{\sqrt{2}} \rho G + 3 \sqrt{2} \rho H \\
+ \frac{1}{\sqrt{2}} \rho A_{LB} - \sqrt{2} \rho B_{LB} - \frac{1}{\sqrt{2}} C_{LB} + \sqrt{2} D_{LB} - \frac{1}{\sqrt{2}} \rho E_{LB} + 7 \frac{1}{\sqrt{2}} \rho F_{LB} \right) & \Xi^{0}_{bb} \rightarrow \Lambda^{0}_b \phi^0 \\
\lambda \left( - \frac{1}{\sqrt{6}} A + \frac{1}{\sqrt{6}} B + \frac{1}{\sqrt{6}} \rho C - \frac{1}{\sqrt{6}} \rho D + \frac{1}{\sqrt{6}} \rho E - \sqrt{2} \rho F + \sqrt{2} \rho G \\
- 3 \rho A_{LB} + 3 \sqrt{2} C_{LB} - 3 \sqrt{2} \rho E_{LB} + \sqrt{2} \rho F_{LB} \right) & \Xi^-_{bb} \rightarrow \Xi^-_{b1} K^0 \\
\lambda (A - B + \rho C - \rho D + \rho E - 3\rho G - 2\rho H) & \Xi^-_{bb} \rightarrow \Lambda^{0}_b \pi^- \\
+ \rho A_{LB} - 2 \rho B_{LB} - C_{LB} + 2 D_{LB} - \rho E_{LB} - 3 \rho F_{LB} & \\
\lambda \left( - \frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} B + \frac{1}{\sqrt{2}} C_{LB} + \sqrt{2} D_{LB} - \frac{1}{\sqrt{2}} \rho E_{LB} + 7 \frac{1}{\sqrt{2}} \rho F_{LB} \right) & \Omega^-_{bb} \rightarrow \Xi^-_{b1} \pi^0 \\
\lambda \left( \frac{1}{\sqrt{6}} A + \sqrt{3} B - \frac{1}{\sqrt{6}} \rho C - \sqrt{3} \rho D - \sqrt{3} \rho E + \frac{1}{\sqrt{6}} \rho F - \sqrt{3} \rho G - \sqrt{3} \rho H \\
+ 3 \sqrt{2} \rho A_{LB} + 3 \sqrt{2} \rho B_{LB} - 3 \sqrt{2} C_{LB} - 3 \sqrt{2} D_{LB} - 3 \sqrt{2} \rho E_{LB} + \sqrt{2} \rho F_{LB} \right) & \Omega^-_{bb} \rightarrow \Xi^-_{b1} \eta^8 \\
\lambda (A - \rho C - \rho F - 3\rho G - 3\rho H + \rho A_{LB} + 3 \rho B_{LB}) & \Omega^-_{bb} \rightarrow \Xi^-_{b1} \pi^- \\
- C_{LB} - 3 D_{LB} + \rho E_{LB} + 3 \rho F_{LB} & \\
\lambda \left( B - \rho D + \rho E + \rho F - \rho H + 3 \rho B_{LB} - 3 \rho D_{LB} + 3 \rho E_{LB} - \rho F_{LB} \right) & \Omega^-_{bb} \rightarrow \Lambda^{0}_b K^- \\
\end{array}
\]

TABLE VIII. Matrix elements for the decay $3_{bb} \rightarrow \overline{3}_b + M$; Cabbibo suppressed decays

**H. 3$_{bb}$ → 6$_b$ + M (Final states with a b=−1 6 baryon plus an octet meson)**

The operators shown in Eqn. 3.3 and Eqn. 3.11 also induce decays to a final 6 of b=−1 baryons and an octet meson. In the SU(3) symmetric case we have nine reduced matrix
SU(3): nonzero integer isospin reduced matrix elements are zero. That eliminates ten reduced matrix elements. Including linear breaking terms gives us ten additional ones from contraction with \(H''(15), H'(15), H''(15), H'_1(24), H''_1(24)\) and \(H''(15)\).

\[
\mathcal{H}_{LB}(3_{bb} \rightarrow 6_b + M) = 
\]
\[
+ A \left[ 3_{bb} \right]_i [6_b]^{ij} M^k_j H_{(1)}(3)_k + B \left[ 3_{bb} \right]_i [6_b]^{jk} M^i_j H_{(1)}(3)_k 
+ C \left[ 3_{bb} \right]_i [6_b]^{ij} M^k_j H''(3)_k + D \left[ 3_{bb} \right]_i [6_b]^{jk} M^i_j H''(3)_k 
+ E \left[ 3_{bb} \right]_i [6_b]^{ij} M^k_j H''(6)_j + F \left[ 3_{bb} \right]_i [6_b]^{jk} M^i_j H''(6)_j 
+ G \left[ 3_{bb} \right]_i [6_b]^{ij} M^k_j H''''(15)_j + H \left[ 3_{bb} \right]_i [6_b]^{jk} M^i_j H''''(15)_j 
+ I \left[ 3_{bb} \right]_i [6_b]^{ij} M^k_j H''''(15)_j + J \left[ 3_{bb} \right]_i [6_b]^{jk} M^i_j H''''(15)_j.
\]

The results are shown in Appendix C. The following relationships hold for unbroken SU(3):

\[
\Gamma (\Xi^{0}_{bb} \rightarrow \Sigma^{0}_b K^-) = 2 \Gamma (\Xi^{0}_{bb} \rightarrow \Sigma^{0}_b \overline{K}^0) = 2 \Gamma (\Xi^{0}_{bb} \rightarrow \Sigma^{0}_b K^-) = \Gamma (\Xi^{0}_{bb} \rightarrow \Sigma^{0}_b \overline{K}^0), 
\Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \pi^0) = \frac{1}{2} \Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \pi^+) = \frac{1}{2} \Gamma (\Xi^{0}_{bb} \rightarrow \Omega_0^- K^+) = \frac{1}{2} \Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \pi^-) = \Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \pi^0) = \frac{1}{4} \Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \pi^0) = \frac{1}{4} \Gamma (\Xi^{0}_{bb} \rightarrow \Omega_0^- K^0), 
\Gamma (\Xi^{0}_{bb} \rightarrow \Sigma_{bb}^- \pi^0) = 2 \Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} K^+), 
\Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \eta_b) = \Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \eta_b), 
\Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} K^-) = \frac{3}{4} \Gamma (\Xi^{0}_{bb} \rightarrow \Xi^{0}_{bb} \overline{K}^0). 
\]

(4.27)

For linear breaking, the first line of relationships survives; the second (wrapping to third) breaks up into pairs of related processes; the fourth no longer holds; the fifth is obeyed, and in the six line only the first equality holds.

Comparing to Appendix C, of the 32 reduced matrix elements in that table only nineteen survive after the following restrictions from simple linear breaking: As before, all nonzero integer isospin reduced matrix elements are zero. That eliminates 10 reduced matrix elements. There is no isospin \(\frac{5}{2}\) component from Eqn 3.3 or 1.11 so \(\langle 24||42_{I=\frac{3}{2}}||3\rangle\) is also eliminated. Eqn. 3.4 does not contain a 42, so \(\langle 24||42_{I=0}||3\rangle\) is zero. Finally, we find

\[
\frac{5}{2\sqrt{2}} \langle 24||42_{I=\frac{3}{2}}||3\rangle = \langle 24||42_{I=\frac{3}{2}}||3\rangle. 
\]

(4.28)

If the \(b\) quark decays first, we obtain similar results for the processes \(3_{bc} \rightarrow 6_c + M\).
The last decay types we will consider from the operators in Eqns. 3.3, 3.4, 3.11, and 3.13 are those with $B$ mesons in the final state. Using the multiplets given in Eqn. 2.7 and 2.8 we have

$$
\mathcal{H}_{LB}(3_{bb} \to B + b) = \\
+ A \left[ \overline{3}_{bb} \right]_i B_j b^k H(1)(\overline{3})_k \epsilon^{ijl} + B \left[ \overline{3}_{bb} \right]_i B_j b^l H(1)(\overline{3})_k \epsilon^{ikl} \\
+ C \left[ \overline{3}_{bb} \right]_i B_j b^k H''(6)_k \epsilon^{ijl} + D \left[ \overline{3}_{bb} \right]_i B_j b^l H''(6)_k \epsilon^{ikl} \\
+ E \left[ \overline{3}_{bb} \right]_i B_j b^k H''(6)_k + F \left[ \overline{3}_{bb} \right]_i B_j b^l H''(6)_k. 
$$

$$
B - 2 C_{LB} \\
- \frac{1}{\sqrt{2}} B + \sqrt{2} C_{LB} \\
- \sqrt{\frac{2}{3}} A - \frac{1}{\sqrt{6}} B - \sqrt{6} C_{LB} - 2 \sqrt{6} D_{LB} \\
- A + 4 C_{LB} + 2 D_{LB} \\
- \frac{1}{\sqrt{2}} B + \sqrt{2} C_{LB} \\
\sqrt{\frac{2}{3}} A + \frac{1}{\sqrt{6}} B + \sqrt{6} C_{LB} + 2 \sqrt{6} D_{LB} \\
- B + 2 C_{LB} \\
- A - 4 C_{LB} - 2 D_{LB} \\
A + B + 2 C_{LB} - 2 D_{LB} \\
A - B - 2 C_{LB} + 2 D_{LB} \\
\Omega_{bb} \to B^+ \Xi^0
$$

| $B - 2 C_{LB}$ | $\Xi^0_{bb} \to B^- \Sigma^+$ |
| $- \frac{1}{\sqrt{2}} B + \sqrt{2} C_{LB}$ | $\Xi^0_{bb} \to B^0 \Sigma^0$ |
| $- \sqrt{\frac{2}{3}} A - \frac{1}{\sqrt{6}} B - \sqrt{6} C_{LB} - 2 \sqrt{6} D_{LB}$ | $\Xi^0_{bb} \to B^0 \Lambda^0$ |
| $- A + 4 C_{LB} + 2 D_{LB}$ | $\Xi^0_{bb} \to B^0 \Xi^0$ |
| $- \frac{1}{\sqrt{2}} B + \sqrt{2} C_{LB}$ | $\Xi^0_{bb} \to B^- \Sigma^0$ |
| $\sqrt{\frac{2}{3}} A + \frac{1}{\sqrt{6}} B + \sqrt{6} C_{LB} + 2 \sqrt{6} D_{LB}$ | $\Xi^-_{bb} \to B^- \Lambda^0$ |
| $- B + 2 C_{LB}$ | $\Xi^-_{bb} \to B^0 \Sigma^-$ |
| $- A - 4 C_{LB} - 2 D_{LB}$ | $\Xi^-_{bb} \to B^0 \Xi^-$ |
| $A + B + 2 C_{LB} - 2 D_{LB}$ | $\Omega^0_{bb} \to B^- \Xi^0$ |
| $A - B - 2 C_{LB} + 2 D_{LB}$ | $\Omega^-_{bb} \to B^0 \Xi^-$ |

TABLE IX. Matrix elements for the decay $3_{bb} \to B + b$; Cabbibo allowed decays

Relationships are to be found in Tables IX and X. Linearly broken SU(3) yields:

$$
\Gamma \left( \Xi_{bb}^0 \to B^- \Sigma^+ \right) = 2 \Gamma \left( \Xi_{bb}^0 \to B^0 \Sigma^0 \right) = 2 \Gamma \left( \Xi_{bb}^- \to B^- \Sigma^0 \right) = \Gamma \left( \Xi_{bb}^- \to B^0 \Sigma^- \right), \\
\Gamma \left( \Xi_{bb}^0 \to B^0 \Lambda^0 \right) = \Gamma \left( \Xi_{bb}^- \to B^- \Lambda^0 \right), \\
\Gamma \left( \Xi_{bb}^0 \to B^0 \Xi^0 \right) = \Gamma \left( \Xi_{bb}^- \to B^0 \Xi^- \right), \\
\Gamma \left( \Omega_{bb}^0 \to B^- \Xi^0 \right) = \Gamma \left( \Omega_{bb}^- \to B^0 \Xi^- \right). 
$$
In terms of the twenty arbitrarily broken SU(3) reduced matrix elements from Appendix E, we see that the six nonzero integer isospin reduced matrix elements are zero.

In this case, analogous relationships for $3_{bc}$ decay will be found to final states $D$ plus octet baryon.

\[ \lambda \left( B - \rho D + \rho E + \rho F - 3 \rho G + \rho A_{LB} - C_{LB} - 2 \rho E_{LB} + 4 \rho F_{LB} \right) \]
\[ \Xi_{bb}^0 \rightarrow B^- \rho \]

\[ \lambda \left( - A + \rho C + \rho F + 2 \rho G - \rho H + 2 \rho A_{LB} + 3 \rho B_{LB} \right) \]
\[ -2 C_{LB} - 3 D_{LB} - \rho E_{LB} - 3 \rho F_{LB} \]
\[ \Xi_{bb}^0 \rightarrow \bar{B}_n^0 \]

\begin{align*}
\lambda & \left( - \frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} B \right. \\
& + \frac{1}{\sqrt{2}} \rho C + \frac{1}{\sqrt{2}} \rho D + \frac{1}{\sqrt{2}} \rho E + \frac{1}{\sqrt{2}} \rho G - 5 \frac{1}{\sqrt{2}} \rho H \\
& - 3 \frac{1}{\sqrt{2}} \rho A_{LB} - \frac{1}{\sqrt{2}} \rho B_{LB} + 3 \frac{1}{\sqrt{2}} C_{LB} + \frac{1}{\sqrt{2}} D_{LB} - \frac{1}{\sqrt{2}} \rho E_{LB} + 7 \frac{1}{\sqrt{2}} \rho F_{LB} \\
& + & 3 \frac{\sqrt{2}}{2} \rho A_{LB} + 3 \frac{\sqrt{2}}{2} \rho B_{LB} - 3 \frac{\sqrt{2}}{2} C_{LB} - 3 \frac{\sqrt{2}}{2} D_{LB} - 3 \frac{\sqrt{2}}{2} \rho E_{LB} + \frac{\sqrt{2}}{2} \rho F_{LB} \bigg) \\
\lambda & \left( A + B - \rho C - \rho D + \rho E + 2 \rho G + \rho H \right) \\
& - \rho A_{LB} - 3 \rho B_{LB} + C_{LB} + 3 D_{LB} - \rho E_{LB} - 3 \rho F_{LB} \bigg) \\
\Xi_{bb}^- & \rightarrow B^- n \]

\[ \lambda \left( - A - B + \rho C + \rho D + \rho E + \rho G + 3 \rho H \right) \\
-3 \rho A_{LB} - \rho B_{LB} + 3 C_{LB} + D_{LB} - \rho E_{LB} - 3 \rho F_{LB} \bigg) \\
\Xi_{bb}^- & \rightarrow \bar{B}_n^0 \Sigma^- \]

\begin{align*}
\lambda & \left( \frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} B \right. \\
& + \frac{1}{\sqrt{2}} \rho C + \frac{1}{\sqrt{2}} \rho F + 3 \sqrt{2} \rho G + 5 \frac{1}{\sqrt{2}} \rho H - \sqrt{2} \rho A_{LB} + \frac{1}{\sqrt{2}} \rho B_{LB} \\
& + \sqrt{2} C_{LB} - \frac{1}{\sqrt{2}} D_{LB} - \frac{1}{\sqrt{2}} \rho E_{LB} + 7 \frac{1}{\sqrt{2}} \rho F_{LB} \bigg) \\
\lambda & \left( - \frac{1}{\sqrt{2}} A - \sqrt{2} B + \frac{1}{\sqrt{2}} \rho C + \sqrt{2} \rho D - \sqrt{2} \rho E + \frac{1}{\sqrt{2}} \rho F + \sqrt{2} \rho H \right. \\
& -3 \sqrt{2} \rho B_{LB} + 3 \sqrt{2} D_{LB} - 3 \sqrt{2} \rho E_{LB} + \frac{\sqrt{2}}{2} \rho F_{LB} \bigg) \\
\lambda & \left( A - \rho C + \rho F - 2 \rho G - 3 \rho H - 2 \rho A_{LB} + \rho B_{LB} \\
& + 2 C_{LB} - D_{LB} - \rho E_{LB} - 3 \rho F_{LB} \bigg) \\
\Omega_{bb}^- & \rightarrow B^- \Sigma^0 \]

\[ \lambda \left( - B + \rho D + \rho E + \rho F - \rho G + 3 \rho A_{LB} - 3 C_{LB} + 3 \rho E_{LB} - \rho F_{LB} \right) \\
\Omega_{bb}^- & \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \lambda \left( - B + \rho D + \rho E + \rho F - \rho G + 3 \rho A_{LB} - 3 C_{LB} + 3 \rho E_{LB} - \rho F_{LB} \right) \\
\Omega_{bb}^- \rightarrow B^- \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow B^- \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow B^- \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow B^- \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow B^- \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow B^- \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow B^- \Xi^- \]

\[ \Omega_{bb}^- \rightarrow \bar{B}_n^0 \Xi^- \]

\[ \Omega_{bb}^- \rightarrow B^- \Xi^- \]
J. $3_{bb} \rightarrow \overline{3}_b + \overline{D}$ (Final states with an antitriplet $b$ baryon and a $\overline{D}$ in the final state)

The decay processes in these last two sections use the operators in Eqns. 3.8 and 3.9.

$$\mathcal{H}_{LB}(3_{bb} \rightarrow \overline{3}_b + \overline{D}) = A \, [3_{bb}]_i [\overline{3}_b]_j \overline{D}^i H'(3)^j + B \, [3_{bb}]_i [\overline{3}_b]_j \overline{D}^j H'(3)^i + C \, [3_{bb}]_i [\overline{3}_b]_l \overline{D}^k H'(6)^j_{jk} \epsilon^{ijl} + A_{LB} \, [3_{bb}]_i [\overline{3}_b]_j \overline{D}^k H'(15)^i_{jk}.$$ \hspace{1cm} (4.31)

The result is found in Table XI. Barring assumptions made about phases, there are no obvious relationships even for unbroken SU(3).

| $- A + C + A_{LB}$ | $\Xi^0_{bb} \rightarrow \Xi^0_{bb} \overline{D}^0$ |
| $B + C - A_{LB}$ | $\Xi^-_{bb} \rightarrow \Xi^-_{bb} \overline{D}^0$ |
| $- A - B + 2 A_{LB}$ | $\Xi^-_{bb} \rightarrow \Xi^0_{bb} D^-$ |
| $B + C + 3 A_{LB}$ | $\Xi^-_{bb} \rightarrow \Lambda^0_{bb} D^-$ |
| $- A - C - 3 A_{LB}$ | $\Omega^-_{bb} \rightarrow \Xi^0_{bb} D^-$ |

TABLE XI. Matrix elements for the decay $3_{bb} \rightarrow \overline{3}_b + \overline{D}$

Comparing this to arbitrarily broken SU(3), from Appendix E 10, we find that, if restricted to simply the linear breaking case, only $\langle 8 || 15 \rangle = 3$ is eliminated (since $(b\pi)(c\pi)$ is purely $I = \frac{1}{2}$).

A similar analysis applies to the process $3_{bc} \rightarrow \overline{3}_c + \overline{D}$.

K. $3_{bb} \rightarrow 6_b + \overline{D}$ (Final states with a b=-1 6 baryon and a $\overline{D}$ in the final state)

$$\mathcal{H}_{LB}(3_{bb} \rightarrow 6_b + \overline{D}) = A \, [3_{bb}]_i [6_b]^i \overline{D}^k H'(3)^j \epsilon_{ijk} + B \, [3_{bb}]_i [6_b]^i \overline{D}^k H'(\overline{6})_{jk} + C \, [3_{bb}]_i [6_b]^i \overline{D}^j H'(\overline{6})^i_{jk} \epsilon_{ijk} + A_{LB} \, [3_{bb}]_i [6_b]^i \overline{D}^j H'(15)^i_{jk} \epsilon^{ijkl} + B_{LB} \, [3_{bb}]_i [6_b]^i \overline{D}^j H'(\overline{24})^i_{jk}.$$ \hspace{1cm} (4.32)

Results are given in Table XII. According to Table XII the following relationships hold in the case of an exact $SU(3)$

$$\Gamma \left( \Xi^0_{bb} \rightarrow \Sigma^+_b D^-_s \right) = 2 \Gamma \left( \Xi^-_{bb} \rightarrow \Sigma^0_b D^-_s \right),$$
$$2 \Gamma \left( \Xi^0_{bb} \rightarrow \Xi^0_{bb} \overline{D}^0 \right) = \Gamma \left( \Omega^-_{bb} \rightarrow \Omega^-_s \overline{D}^0 \right).$$ \hspace{1cm} (4.33)

where the first relation remains true in the linear broken case.

Comparing this to arbitrarily broken SU(3), from Appendix E 11, we find that $\langle 8 || 15 \rangle = 0$ and $\langle 10 || 24 \rangle = 0$.

As before, the processes in the multiplet structure $3_{bc} \rightarrow 6_c + \overline{D}$ give analogous relationships.
In conclusion, we have examined various decay modes of the lowest lying hadrons containing two $b$ quarks, with obvious extensions to hadrons containing one $b$ and one $c$ quark. We have provided the predictions resulting from imposing exact SU(3) flavor symmetry, and those arising from inclusion of a linear breaking piece in the form of a strange quark mass. The decomposition into the group-theoretic basis, and the relationship between this basis and the linear breaking case, should aid in further analyses involving diagrammatic, factorization, or large $N_c$ treatments. If the SU(3) relationships are measured to hold in the $b$ hadrons, this may suggest that the violations found in the charm system are due to the influence of nearby resonances unique to that system. If the linear breaking relationships hold, this may suggest that the breaking caused by the strange quark mass can be treated perturbatively. We hope that these results will not only aid the experimentalists in finding new doubly heavy hadrons, but will serve as a useful comparison to those studying decay modes using other methods. Together these efforts will help to illuminate the properties of heavy quark systems.

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APPENDIX A: MATRIX ELEMENTS FOR THE DECAY $3_{bb} \rightarrow 3_{bc} + M + M$, EVEN ANGULAR MOMENTUM $L$ ($SU(3)$ EXACT RESULTS).

| Expression | Decay | Expression | Decay |
|------------|-------|------------|-------|
| $\frac{1}{\sqrt{2}} B + \frac{1}{\sqrt{2}} C$ | $\Xi_{bb}^0 \rightarrow \Xi_{bb}^{+} \pi^0 \pi^-$ | $\lambda \left( \frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} B \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \pi^0 K^-$ |
| $\sqrt{\frac{2}{3}} A + \frac{1}{\sqrt{3}} B$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \pi^- \eta_8$ | $\lambda (A + E)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \eta_8 K^-$ |
| $+ \frac{1}{\sqrt{6}} C + \sqrt{\frac{2}{3}} E$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \eta_8 \pi^0$ | $\lambda \left( - \frac{1}{\sqrt{3}} A + 2 \sqrt{\frac{3}{9}} B \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \eta_8 K^-$ |
| $A + E$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} K^0 K^-$ | $\lambda \left( \frac{1}{\sqrt{2}} C - \frac{1}{\sqrt{2}} E \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \eta_8 K^-$ |
| $\frac{1}{\sqrt{2}} D - \frac{1}{\sqrt{2}} E$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \pi^0 \eta_8$ | $\lambda (B + E)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta_8 K^-$ |
| $+ \frac{1}{\sqrt{6}} E + \sqrt{\frac{2}{3}} F$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta_8 \eta_8$ | $\lambda \left( \frac{1}{\sqrt{2}} D + \sqrt{2} E \right)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \eta^0 \pi^0$ |
| $\lambda \left( \frac{1}{\sqrt{2}} C + \frac{1}{\sqrt{2}} D \right)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \pi^0 \eta_8$ | $\lambda \left( - \frac{1}{\sqrt{2}} A + 2 \sqrt{\frac{3}{9}} D \right)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \pi^0 \eta_8$ |
| $\lambda \left( \frac{1}{\sqrt{2}} C - \frac{1}{\sqrt{2}} E \right)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \pi^0 \eta_8$ | $\lambda (D + 2 F)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \eta_8 \eta_8$ |
| $\lambda \left( \frac{1}{\sqrt{2}} D + \sqrt{2} E \right)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \eta^0 \pi^0$ | $\lambda (E + 2 F)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \eta^0 \pi^0$ |
| $\lambda (B + C)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \pi^- K^-$ | $\lambda (B + C)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{+} \eta_8 K^-$ |
| $\lambda \left( \frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} E \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta^0 \pi^0$ | $\lambda \left( \frac{1}{\sqrt{2}} C - \frac{1}{\sqrt{2}} E \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta_8 K^-$ |
| $\lambda (A + C)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} K^0 K^-$ | $\lambda \left( \frac{1}{\sqrt{2}} A + 2 \sqrt{\frac{3}{9}} B \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta_8 K^-$ |
| $\lambda (B + D)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \pi^- \pi^-$ | $\lambda \left( \frac{1}{\sqrt{2}} C - \frac{1}{\sqrt{2}} E \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta_8 K^-$ |
| $\lambda (B + D)$ | $\Xi_{bb}^0 \rightarrow \Omega_{bc}^{0} \pi^0 \pi^0$ | $\lambda (B + D)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta_8 K^-$ |
| $\lambda (B + C)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \pi^- \pi^-$ | $\lambda \left( \frac{1}{\sqrt{2}} C - \frac{1}{\sqrt{2}} E \right)$ | $\Xi_{bb}^0 \rightarrow \Xi_{bc}^{0} \eta_8 K^-$ |
### Appendix B: Matrix Elements for the Decay $3_{bb} \rightarrow 3_{bc} + M + M$, Odd Angular Momentum $L$ (SU(3) Exact Results)

| $\sqrt{2} A - \frac{1}{\sqrt{2}} B$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^0 \pi^-$ |
|-----------------------------------|-----------------------------------------------|
| $- \frac{1}{\sqrt{2}} C - \sqrt{2} E$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^0 \pi^-$ |
| $\frac{1}{\sqrt{2}} B + \frac{1}{\sqrt{2}} C$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- \eta_8$ |
| $\frac{1}{\sqrt{2}} C - \sqrt{2} E$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- \eta_8$ |
| $\frac{1}{\sqrt{2}} B - \frac{1}{\sqrt{2}} C$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^0 K^-$ |
| $\frac{1}{\sqrt{2}} C + \sqrt{2} E$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^0 K^-$ |
| $\frac{1}{\sqrt{2}} B - \frac{1}{\sqrt{2}} C$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\frac{1}{\sqrt{2}} C + \sqrt{2} E$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( \frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} B \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^0 K^-$ |
| $- \frac{1}{\sqrt{2}} C - \sqrt{2} E$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( A - E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( \sqrt{2} A - \frac{1}{\sqrt{2}} B \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $- \frac{1}{\sqrt{2}} C - \sqrt{2} E$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( - \frac{1}{\sqrt{2}} C + \sqrt{2} E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( - B - E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( - \frac{1}{\sqrt{2}} C + \sqrt{2} E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( \frac{1}{\sqrt{2}} C - \sqrt{2} E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( - B - E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( - \frac{1}{\sqrt{2}} C - \sqrt{2} E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^+ \pi^- K^0$ |
| $\lambda \left( B + D + E \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^0 \eta_8$ |
| $\lambda \left( - B - C \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^0 \eta_8$ |
| $\lambda \left( \frac{1}{\sqrt{2}} A + \sqrt{2} B \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^0 K^-$ |
| $\lambda \left( A - C \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^0 K^-$ |
| $\lambda \left( \sqrt{2} C \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^- \eta_8$ |
| $\lambda \left( - B - D \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^- \eta_8$ |
| $\lambda \left( \sqrt{2} D \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^- \eta_8$ |
| $\lambda \left( B - C \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( \frac{1}{\sqrt{2}} A + \sqrt{2} B \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( A + D \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^- K^0$ |
| $\lambda \left( \sqrt{2} A \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^- K^0$ |
| $- \sqrt{2} B$ | $\Omega_{3bb}^- \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( \frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} B \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( A + D \right)$ | $\Xi_{3bb}^0 \rightarrow \Omega_{3bc}^0 \pi^- K^0$ |
| $\lambda \left( \frac{1}{\sqrt{2}} A + \sqrt{2} B \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( \sqrt{2} A \right)$ | $\Omega_{3bb}^- \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( - \frac{1}{\sqrt{2}} C - \sqrt{2} E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( - B - E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( \frac{1}{\sqrt{2}} C - \sqrt{2} E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
| $\lambda \left( - B - E \right)$ | $\Xi_{3bb}^0 \rightarrow \Xi_{3bc}^0 K^- K^+$ |
### APPENDIX C: MATRIX ELEMENTS FOR THE DECAY $3_{bb} \to 6_b + M$

| Term                                                                 | Symbol  | Notes |
|----------------------------------------------------------------------|---------|-------|
| $A - 2 D_{LB} - 2 E_{LB}$                                           | $\Xi_{bb}^0 \to \Sigma_0^0 K^0$  |
| $\frac{1}{\sqrt{2}} A - \sqrt{2} D_{LB} - \sqrt{2} E_{LB}$         | $\Xi_{bb}^0 \to \Sigma_0^0 K^0$  |
| $\frac{1}{2} B - E_{LB} - 2 F_{LB}$                                 | $\Xi_{bb}^0 \to \Xi_{bb}^0 \pi^0$ |
| $-\frac{1}{\sqrt{3}} A + \frac{1}{\sqrt{3}} B - 2 \sqrt{3} D_{LB} + \frac{1}{\sqrt{3}} E_{LB} - 2 \frac{1}{\sqrt{3}} F_{LB}$ | $\Xi_{bb}^0 \to \Xi_{bb}^0 \pi^0$ |
| $\frac{1}{\sqrt{2}} B - \sqrt{2} E_{LB} - 2 \sqrt{2} F_{LB}$       | $\Xi_{bb}^- \to \Xi_{bb}^0 \pi^0$ |
| $B - 2 E_{LB} + 4 F_{LB}$                                           | $\Xi_{bb}^- \to \Xi_{bb}^0 \pi^0$ |
| $\frac{1}{\sqrt{2}} A - \sqrt{2} D_{LB} - \sqrt{2} E_{LB}$         | $\Xi_{bb}^- \to \Xi_{bb}^0 K^-$  |
| $A - 2 D_{LB} - 2 E_{LB}$                                           | $\Xi_{bb}^- \to \Xi_{bb}^0 K^-$  |
| $\frac{1}{\sqrt{2}} B - \sqrt{2} E_{LB} - 2 \sqrt{2} F_{LB}$       | $\Xi_{bb}^- \to \Xi_{bb}^0 \pi^-$ |
| $-\frac{1}{2} B + E_{LB} + 2 F_{LB}$                                | $\Xi_{bb}^- \to \Xi_{bb}^0 \pi^0$ |
| $\frac{1}{\sqrt{3}} A + \frac{1}{\sqrt{3}} B - 2 \sqrt{3} D_{LB} + \frac{1}{\sqrt{3}} E_{LB} - 2 \frac{1}{\sqrt{3}} F_{LB}$ | $\Xi_{bb}^- \to \Xi_{bb}^0 \pi^0$ |
| $B - 2 E_{LB} + 4 F_{LB}$                                           | $\Xi_{bb}^- \to \Xi_{bb}^0 \pi^0$ |
| $\frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} B - \sqrt{2} D_{LB} + \sqrt{2} E_{LB} - 2 \sqrt{2} F_{LB}$ | $\Xi_{bb}^- \to \Xi_{bb}^0 K^-$  |
| $\frac{1}{\sqrt{2}} A - \sqrt{2} D_{LB} - \sqrt{2} E_{LB}$         | $\Xi_{bb}^- \to \Xi_{bb}^0 K^-$  |
| $\frac{1}{\sqrt{2}} A - \sqrt{2} B - 2 \sqrt{6} D_{LB} - 4 \sqrt{2} E_{LB} - 4 \sqrt{2} F_{LB}$ | $\Omega_{bb}^0 \to \Omega_{bb}^0 \eta_8$ |
| $\lambda \left( - A + \rho C + \rho E + \rho F + 3 \rho G + 3 \rho H - \rho A_{LB} \right)$ | $\Xi_{bb}^0 \to \Sigma_0^0 \pi^0$ |
| $- \rho B_{LB} + D_{LB} + E_{LB} - 2 \rho H_{LB} + 4 \rho I_{LB} + 8 \rho J_{LB}$ | $\Xi_{bb}^0 \to \Sigma_0^0 \pi^0$ |
| $\lambda \left( \frac{1}{2} A - \frac{1}{2} B - \frac{1}{2} D_{LB} + \frac{1}{2} E_{LB} - \frac{1}{2} F_{LB} + \frac{1}{2} G_{LB} + \frac{1}{2} H_{LB} + \frac{1}{2} I_{LB} + \frac{1}{2} J_{LB} \right)$ | $\Xi_{bb}^0 \to \Sigma_0^0 \pi^0$ |
| $+ 3 \rho I + \frac{1}{2} \rho A_{LB} - \rho C_{LB} - \frac{1}{2} D_{LB} + F_{LB} - \rho G_{LB} + \frac{1}{2} H_{LB} - \frac{1}{2} I_{LB} + 5 \rho J_{LB}$ | $\Xi_{bb}^0 \to \Sigma_0^0 \pi^0$ |
| $\lambda \left( - \frac{1}{2} \frac{1}{\sqrt{3}} A - \frac{1}{2} \frac{1}{\sqrt{3}} B + \frac{1}{2} \frac{1}{\sqrt{3}} C + \frac{1}{2} \frac{1}{\sqrt{3}} D - \frac{1}{2} \frac{1}{\sqrt{3}} E + \frac{1}{2} \frac{1}{\sqrt{3}} F + \sqrt{3} \rho G + 3 \sqrt{3} \rho H \right)$ | $\Xi_{bb}^0 \to \Sigma_0^0 \eta_8$ |
| $+ \sqrt{3} \rho I - \frac{1}{2} \sqrt{3} \rho A_{LB} - \frac{1}{2} \sqrt{3} \rho B_{LB} - \frac{1}{2} \sqrt{3} \rho C_{LB} + \frac{1}{2} \sqrt{3} \rho D_{LB} + \frac{1}{2} \sqrt{3} \rho E_{LB} + \frac{1}{2} \sqrt{3} \rho F_{LB}$ | $\Xi_{bb}^0 \to \Sigma_0^0 \eta_8$ |
| $+ \sqrt{3} \rho G_{LB} + \frac{1}{2} \sqrt{3} \rho H_{LB} - \frac{1}{2} \sqrt{3} \rho I_{LB} + 11 \sqrt{3} \rho J_{LB}$ | $\Xi_{bb}^0 \to \Sigma_0^0 \eta_8$ |
| $\lambda \left( - B + \rho D - \rho F + 3 \rho H - 2 \rho I - \rho B_{LB} - 2 \rho C_{LB} \right)$ | $\Xi_{bb}^0 \to \Sigma_0^0 \eta_8$ |
| $+ E_{LB} + 2 F_{LB} + \rho G_{LB} + \rho H_{LB} + 3 \rho I_{LB} + 3 \rho J_{LB}$ | $\Xi_{bb}^0 \to \Sigma_0^0 \eta_8$ |
| $\lambda \left( - \frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} B + \frac{1}{\sqrt{2}} C - \frac{1}{\sqrt{2}} D - \frac{1}{\sqrt{2}} E + \frac{1}{\sqrt{2}} F - \frac{1}{\sqrt{2}} G + 3 \frac{1}{\sqrt{2}} H + 3 \frac{1}{\sqrt{2}} A_{LB} \right)$ | $\Xi_{bb}^0 \to \Xi_{bb}^0 K^0$ |
| $- \frac{1}{\sqrt{2}} \rho B_{LB} - 3 \frac{1}{\sqrt{2}} D_{LB} + \frac{1}{\sqrt{2}} E_{LB} - \sqrt{2} \rho G_{LB} + 5 \frac{1}{\sqrt{2}} \rho H_{LB} - 5 \frac{1}{\sqrt{2}} \rho I_{LB} - 11 \sqrt{2} \rho J_{LB}$ | $\Xi_{bb}^0 \to \Xi_{bb}^0 K^0$ |
| Equation | Transformation |
|----------|----------------|
| \(\lambda \left( -\frac{1}{\sqrt{2}} B + \frac{i}{\sqrt{2}} \rho D - \frac{1}{\sqrt{2}} \rho F + 3 \frac{1}{2} \rho H - \sqrt{2} \rho I - \frac{1}{\sqrt{2}} \rho B_{LB} + 3 \frac{1}{2} \rho C_{LB} \right) + \frac{1}{\sqrt{2}} E_{LB} - 3 \sqrt{2} F_{LB} - \sqrt{2} \rho G_{LB} - 2 \sqrt{2} \rho H_{LB} - \sqrt{2} \rho I_{LB} - 11 \sqrt{2} \rho J_{LB})\) | \(\Xi_{bb}^0 \rightarrow \Xi_{02}^+ K^+\) |
| \(\lambda \left( -\frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} B + \frac{1}{\sqrt{2}} \rho C + \frac{1}{\sqrt{2}} \rho D + \frac{1}{\sqrt{2}} \rho E + 3 \frac{1}{2} \rho G - \sqrt{2} \rho H \right) + 3 \sqrt{2} \rho I - \frac{1}{\sqrt{2}} \rho A_{LB} - \sqrt{2} \rho B_{LB} - \sqrt{2} \rho C_{LB} + \frac{1}{\sqrt{2}} D_{LB} + \sqrt{2} E_{LB} + \sqrt{2} F_{LB} - \sqrt{2} \rho G_{LB} - \frac{3}{\sqrt{2}} \rho H_{LB} - 3 \frac{1}{\sqrt{2}} \rho I_{LB} + 3 \sqrt{2} \rho J_{LB})\) | \(\Xi_{bb}^- \rightarrow \Sigma_{b}^0\pi^-\) |
| \(\lambda \left( \frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} B - \frac{1}{\sqrt{2}} \rho C - \frac{1}{\sqrt{2}} \rho D - \frac{1}{\sqrt{2}} \rho E + 5 \frac{1}{\sqrt{2}} \rho G + \sqrt{2} \rho H \right) + \sqrt{2} \rho I + \frac{1}{\sqrt{2}} \rho A_{LB} + \sqrt{2} \rho B_{LB} + \sqrt{2} \rho C_{LB} - \frac{1}{\sqrt{2}} D_{LB} - \sqrt{2} E_{LB} - \sqrt{2} F_{LB} - \frac{1}{\sqrt{2}} \rho G_{LB} + \frac{1}{\sqrt{2}} \rho H_{LB} + 3 \frac{1}{\sqrt{2}} \rho I_{LB} + 9 \frac{1}{\sqrt{2}} \rho J_{LB})\) | \(\Xi_{bb}^- \rightarrow \Sigma_{b}^0\pi^0\) |
| \(\lambda \left( -\frac{1}{\sqrt{6}} A - \frac{1}{\sqrt{6}} B + \frac{1}{\sqrt{6}} \rho C + \frac{1}{\sqrt{6}} \rho D - \frac{1}{\sqrt{6}} \rho E - \frac{1}{\sqrt{6}} \rho G - \sqrt{2} \rho H \right) - \sqrt{2} \rho I - 3 \sqrt{2} \rho A_{LB} - \sqrt{2} \rho B_{LB} - \sqrt{2} \rho C_{LB} + 3 \frac{1}{\sqrt{2}} D_{LB} + \sqrt{2} E_{LB} + \sqrt{2} F_{LB} - \sqrt{3} \rho G_{LB} + \sqrt{3} \rho H_{LB} + 3 \sqrt{3} \rho I_{LB} - 3 \sqrt{3} \rho J_{LB})\) | \(\Xi_{bb}^- \rightarrow \Sigma_{b}^0\pi^0\) |
| \(\lambda \left( -\frac{1}{\sqrt{2}} A - \frac{1}{\sqrt{2}} B + \frac{1}{\sqrt{2}} \rho C + \frac{1}{\sqrt{2}} \rho D - \frac{1}{\sqrt{2}} \rho E - \frac{1}{\sqrt{2}} \rho G - \sqrt{2} \rho H \right) - \sqrt{2} \rho I + 3 \frac{1}{\sqrt{2}} \rho A_{LB} - \sqrt{2} \rho B_{LB} + 3 \sqrt{2} \rho C_{LB} - 3 \frac{1}{\sqrt{2}} D_{LB} + \sqrt{2} E_{LB} - 3 \sqrt{2} F_{LB} + \sqrt{2} \rho G_{LB} + \frac{1}{\sqrt{2}} \rho H_{LB} + 3 \frac{1}{\sqrt{2}} \rho I_{LB} + 3 \sqrt{2} \rho J_{LB})\) | \(\Xi_{bb}^- \rightarrow \Xi_{02}^0 K^0\) |
| \(\lambda \left( -\frac{1}{\sqrt{2}} B + \frac{1}{\sqrt{2}} \rho D + \frac{1}{\sqrt{2}} \rho F - \frac{1}{\sqrt{2}} \rho H + 3 \sqrt{2} \rho I + 3 \frac{1}{\sqrt{2}} \rho B_{LB} - \sqrt{2} \rho C_{LB} - 3 \frac{1}{\sqrt{2}} E_{LB} + \sqrt{2} F_{LB} - \sqrt{2} G_{LB} - \frac{1}{\sqrt{2}} \rho H_{LB} + 7 \frac{1}{\sqrt{2}} \rho I_{LB} - 11 \sqrt{2} \rho J_{LB})\) | \(\Omega_{bb}^- \rightarrow \Sigma_{b}^0\pi^- K^-\) |
| \(\lambda \left( - B + \rho D - \rho F - \rho H - 2 \rho I + 3 \rho B_{LB} - 2 \rho C_{LB} - 3 E_{LB} + 2 F_{LB} - \rho G_{LB} - \rho H_{LB} - 3 \rho I_{LB} + 3 \rho J_{LB})\) | \(\Omega_{bb}^- \rightarrow \Sigma_{b}^0\pi^- K^0\) |
| \(\lambda \left( -\frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} \rho C + \frac{1}{\sqrt{2}} \rho E - \frac{1}{\sqrt{2}} \rho F + 3 \frac{1}{\sqrt{2}} \rho G - \frac{1}{\sqrt{2}} \rho H + \frac{1}{\sqrt{2}} \rho A_{LB} + 3 \frac{1}{\sqrt{2}} D_{LB} - 3 \frac{1}{\sqrt{2}} E_{LB} + \sqrt{2} \rho G_{LB} + 5 \frac{1}{\sqrt{2}} \rho H_{LB} - 5 \frac{1}{\sqrt{2}} \rho I_{LB} - 11 \sqrt{2} \rho J_{LB})\) | \(\Omega_{bb}^- \rightarrow \Xi_{02}^0\pi^0\) |
| \(\lambda \left( \frac{3}{4} A - \frac{1}{4} \rho C - \frac{1}{4} \rho E + \frac{1}{4} \rho F + \frac{3}{4} \rho G + \frac{1}{4} \rho H + \frac{1}{4} \rho A_{LB} - \frac{3}{4} \rho B_{LB} + \frac{1}{4} \rho C_{LB} - \frac{3}{4} \rho D_{LB} - \rho E_{LB} + \frac{1}{4} \rho H_{LB} - \frac{3}{4} \rho I_{LB} - 14 \rho J_{LB})\) | \(\Omega_{bb}^- \rightarrow \Xi_{02}^0\eta^8\) |
| \(\lambda \left( -\frac{1}{\sqrt{2}} A + \frac{1}{\sqrt{2}} \rho C - \frac{1}{\sqrt{2}} \rho D - \frac{1}{\sqrt{2}} \rho E - \frac{1}{\sqrt{2}} \rho F + \frac{1}{2} \sqrt{3} \rho G + \frac{1}{2} \sqrt{3} \rho H + 2 \frac{1}{2} \sqrt{3} \rho A_{LB} - 2 \frac{1}{2} \sqrt{3} \rho B_{LB} + 2 \frac{1}{2} \sqrt{3} \rho C_{LB} + \frac{3}{2} \sqrt{3} \rho D_{LB} + \frac{1}{2} \sqrt{3} \rho E_{LB} + 2 \frac{1}{2} \sqrt{3} \rho H_{LB} + \frac{1}{2} \sqrt{3} \rho I_{LB} - 8 \frac{1}{2} \sqrt{3} \rho J_{LB})\) | \(\Omega_{bb}^- \rightarrow \Xi_{02}^0 K^0\) |
APPENDIX D: TENSOR DECOMPOSITION

In the following we present the tensor decompositions needed to provide the elements given in Eqs. 3.3–3.15 of Sec. III. While the normalization does not play any role in Sec. III we will keep it explicit here.

1. The operator \( (bc)(c\bar{q})(s\bar{s})_8 \) decomposes via \( 3 \otimes 8 = \bar{15} \oplus 6 \oplus 3 \). Tensor methods yield

\[
3i8^j_k = \frac{1}{2} \left( 3i\delta^j_k + 3k8^j_i - \frac{1}{4} \delta^j_k\delta^i_l - \frac{1}{4} \delta^j_k\delta^i_l \right) + \epsilon_{ikl} \frac{1}{4} \left( \epsilon^{lmn}3m8^i_n + \epsilon^{mn3}n \right) \]

\[
+ \frac{1}{8} \left( 3i\delta^j_k8^i_l - \delta^j_k\delta^i_l \right). \tag{D1}
\]

Together with Eqs. 3.3 and an explicit expression for \( (s\bar{s})_8 \) we find (up to normalization) the numbers given in Eq. 3.4. \( (s\bar{s})_8 \) is found from

\[
3i3^j = \left( 3i3^j - \frac{1}{3} \delta^j_i \right) + \frac{1}{3} \delta^j_i \Rightarrow (s\bar{s})_8 = \begin{pmatrix}
-\frac{1}{3} & 0 & 0 \\
0 & -\frac{1}{3} & 0 \\
0 & 0 & \frac{2}{3}
\end{pmatrix}. \tag{D2}
\]

2. The operator \((bc)(u\bar{q})\) is contained in an 8-dimensional representation of \( SU(3) \). Including linear breaking, \((s\bar{s})\), requires that \( 8 \otimes 8' \) be decomposed.

\[
8^i_s8^j_l = \frac{1}{4} \left( 8^i_s8^j_l + \mathcal{P} - \frac{1}{5} \left( \delta^i_s8^m_k8^j_l + \delta^j_l8^m_k8^i_s + \mathcal{P} \right) + \frac{1}{10} \left( \delta^i_s\delta^j_k + \delta^j_l\delta^i_k \right) 8^m_n8^m_n \right)
\]

\[
+ \epsilon_{klm} \frac{1}{12} \left( \epsilon^{mno}8^i_s8^j_l + \mathcal{P} \right) + \epsilon_{ijm} \frac{1}{12} \left( \epsilon^{mno}8^i_s8^j_l + \mathcal{P} \right)
\]

\[
- \delta^i_s8_{(sym)}^j + \frac{3}{2} \delta^i_s8_{(sym)}^j + \frac{3}{2} \delta^j_l8_{(sym)}^i - \delta^i_s8_{(sym)}^j
\]

\[
- \frac{5}{6} \delta^i_s8_{(asym)}^j + \frac{5}{6} \delta^i_s8_{(asym)}^j + \frac{1}{8} \left( \delta^i_s\delta^j_k - \frac{1}{3} \delta^i_s\delta^j_k \right) 8^m_n8^m_n. \tag{D3}
\]

\( \mathcal{P} \) stands for all possible permutations of the free upper and free lower indices. We have defined the following abbreviations

\[
8_{(sym)}^i = \frac{1}{5} \left( 8^i_s8^j_l + 8^j_l8^i_s - \frac{2}{3} \delta^i_s8_p8_p \right),
\]

\[
8_{(asym)}^i = \frac{1}{5} \left( 8^i_s8^j_l - 8^j_l8^i_s \right). \tag{D4}
\]
3. The operator \((b\bar{u})(c\bar{s})\) decomposes into a 3 and a \(\bar{6}\). The numbers given in Eq. 3.8 are found using

\[
\bar{3}, \bar{3} = \frac{1}{2} (3, 3, \bar{3}) + \epsilon_{ijk} \frac{1}{2} \epsilon_{klm} \bar{3}, \bar{3}.
\]  
(D5)

Including a linear breaking term gives \((b\bar{u})(c\bar{s})(s\bar{s})\), or \((6 \oplus 3) \otimes (8 \oplus 1)\). We need the following tensor decompositions, where now \(\mathcal{P}\) does not include permutations over the (symmetric) indices of \(\bar{6}\).

\[
\bar{6}_{ij} s_m^l = \frac{1}{3} \left( 6_{ij} s_m^l + \mathcal{P} - \frac{1}{5} \left\{ \delta_i^l \bar{6}_{nj} s_m^o + \mathcal{P} \right\} \right)
\]

\[
\quad + \frac{1}{6} \epsilon_{inm} \left( 6_{lj} s_p^o \epsilon^{npq} + \bar{6}_{lj} s_p^o \epsilon^{npq} \right) - \frac{1}{4} \left( \delta_j^l \bar{6}_{in} s_p^o \epsilon^{npq} + \delta_j^l \bar{6}_{in} s_p^o \epsilon^{npq} \right) + i \leftrightarrow j
\]

\[
\quad + \frac{3}{20} \delta_i^l \left( 6_{mj} s_p^o \bar{6}_{jn} s_m^l + 6_{jn} s_m^l + \bar{6}_{jn} s_m^l \right) - \frac{1}{10} \delta_m^l \left( 6_{in} s_j^o + \bar{6}_{in} s_j^o \right)
\]

\[
\quad + \frac{1}{8} \left( \delta_{mio}^l + \delta_{nio}^l \right) 6_{qr} s_p^o \epsilon^{pro}
\]  
(D6)

and

\[
3^i s_k^j = \frac{1}{2} \left( 3^i s_k^j + 3^i s_k^j - \frac{1}{4} (3^l k^3 s_j^i - 3^l k^3 s_j^i) + \epsilon^{ijkl} \frac{1}{4} \epsilon_{mn}^l 3^m s_k^o + \epsilon_{knn}^l 3^m s_k^o \right)
\]

\[
\quad + \frac{1}{8} \left( 3^l k^3 s_j^i - 3^l k^3 s_j^i \right). 
\]  
(D7)

4. The tensor decomposition of the operator \((b\bar{u})(u\bar{d})\) is found in Eq. D1. Including a linear SU(3) breaking term, \((s\bar{s})\), we have \((\mathbf{15}_r \oplus \bar{3} \oplus 3) \otimes (8 \oplus 1)\). Below we provide \(\mathbf{15} \otimes 8\). The others are already given. \(6 \otimes 8\) can be obtained from Eqn. D8 by moving all down indices up and all up indices down. Below, \(\mathcal{P}\) includes permutations of all free upper or lower indices within the parenthesis in which it is found, but does not include permutations over the (symmetric) lower two indices of the \(\mathbf{15}\).

\[
\mathbf{15}_{jk} s_m^l = \frac{1}{6} \left( \mathbf{15}_{jk} s_m^l + \mathcal{P} - \frac{1}{6} \left\{ \delta_j^l \mathbf{15}_{jk} s_o^p + \delta_j^l \mathbf{15}_{jk} s_o^p + \mathcal{P} \right\} \right)
\]

\[
\quad + \frac{1}{18} \epsilon_{lm} \left( \mathbf{15}_{jk} s_p^l \epsilon^{qpo} + \mathcal{P} - \frac{1}{5} \left\{ \delta_j^l \mathbf{15}_{jk} s_p^o \epsilon^{qpo} + \mathcal{P} \right\} \right) + k \leftrightarrow j
\]
posed into reduced amplitudes through $R$ complete $SU(3)$ theoretical basis is orthogonal. The notation for the transformation matrix between the physical and group basis is $R$. For purposes of creating a self-contained article we describe briefly the method of the $(3)$ flavor decomposition, from Ref. [18]. The physical amplitudes are decomposed into reduced amplitudes through

$$\begin{align}
&+ \epsilon^{ilo} \frac{1}{24} \left( \overline{T^{jk}_{ik}} S^q_m \epsilon_{pqo} + \mathcal{P} \right) \\
&+ 37 \delta^{l}_{m} \overline{T^{(3)j}_{jk}} - 11 \delta^{l}_{j} \overline{T^{(3)j}_{mk}} - 11 \delta^{l}_{k} \overline{T^{(3)l}_{jm}} - 17 \delta^{l}_{m} \overline{T^{(3)l}_{jk}} + 7 \delta^{l}_{j} \overline{T^{(3)l}_{mk}} + 7 \delta^{l}_{k} \overline{T^{(3)l}_{jm}} \\
&+ 13 \delta^{i}_{j} \overline{T^{(4)i}_{jk}} + 13 \delta^{i}_{k} \overline{T^{(4)i}_{jm}} - 11 \delta^{i}_{m} \overline{T^{(4)i}_{kj}} - 5 \delta^{j}_{m} \overline{T^{(4)i}_{kj}} - 5 \delta^{k}_{m} \overline{T^{(4)i}_{kn}} + 7 \delta^{i}_{j} \overline{T^{(4)i}_{kj}} + 7 \delta^{i}_{k} \overline{T^{(4)i}_{jm}} \\
&+ \frac{1}{4} \epsilon_{mko} \delta^{i}_{j} \epsilon_{l} + \frac{1}{4} \epsilon_{mjo} \delta^{i}_{k} \epsilon_{l} - \epsilon_{mko} \delta^{i}_{j} \epsilon_{l} - \epsilon_{mko} \delta^{i}_{j} \epsilon_{l} \\
&+ \frac{1}{10} \delta^{l}_{m} \left( \delta^{j}_{i} \overline{T^{(3)}_{k}} S^{p}_{o} + k \leftrightarrow j \right) - \frac{1}{40} \delta^{j}_{k} \left( \delta^{i}_{i} \overline{T^{(3)}_{mp}} S^{p}_{o} + k \leftrightarrow m \right) \\
&- \frac{1}{40} \delta^{k}_{l} \left( \delta^{j}_{i} \overline{T^{(3)}_{mp}} S^{p}_{o} + j \leftrightarrow m \right).
\end{align}$$

(D8)

where we have used the following abbreviations

$$\begin{align}
\overline{T^{(3)}_{jk}} &= \frac{1}{72} \left( \overline{T^{(3)j}_{jk}} S^{l}_{o} - \frac{1}{4} \left\{ \delta^{j}_{i} \overline{T^{(3)}_{p}} S^{p}_{o} + \delta^{k}_{i} \overline{T^{(3)}_{j}} S^{p}_{o} \right\} \right), \\
\overline{T^{(4)i}_{km}} &= \frac{1}{72} \left( \overline{T^{(4)i}_{ok}} S^{o}_{m} + \overline{T^{(4)i}_{om}} S^{o}_{k} - \frac{1}{4} \left\{ \delta^{i}_{m} \overline{T^{(4)i}} S^{o}_{p} + \delta^{i}_{k} \overline{T^{(4)i}} S^{o}_{p} \right\} \right), \\
6^{(2)i}_{a} &= \frac{1}{15} \left( \overline{T^{(3)}_{pq}} S^{q}_{i} \epsilon^{qr} + \overline{T^{(3)}_{pq}} S^{r} \epsilon^{qr} \right).
\end{align}$$

(D9)

**APPENDIX E: COMPLETE $SU(3)$ FLAVOR DECOMPOSITION**

For purposes of creating a self-contained article we describe briefly the method of the complete $SU(3)$ flavor decomposition, from Ref. [28]. The physical amplitudes are decomposed into reduced amplitudes through

$$\begin{align}
A(i^{Rc}_{\nu} \rightarrow f^{R_{a}}_{\nu_{a}} f^{R_{b}}_{\nu_{b}}) = \frac{1}{15} \left( I_{3} + \frac{2}{3} + \frac{4}{3} \right) \sum_{R'_{c}, \nu'} \left( \begin{array}{ccc}
R_{a} & R_{b} & R'_{c} \\
\nu_{a} & \nu_{b} & \nu'
\end{array} \right) \left( \begin{array}{ccc}
R'_{c} & R_{c} & R \\
\nu' & -\nu_{c} & \nu
\end{array} \right) \left\langle R' || R_{c} || R_{c} \right\rangle.
\end{align}$$

(E1)

This convention ensures that the transformation matrix between the physical and group theoretical basis is orthogonal. The notation for the $SU(3)$ Clebsch-Gordan coefficients is from de Swart [24]. The representations of the initial state and the two final states are named $R_{c}$ and $R_{a}, R_{b}$, respectively; $\nu_{i}$ stands for the quantum numbers $I_{i}, I_{3i}, Y_{i}; T$ is the triality of the representation $R_{c}$, defined as follows (using Dynkin labels $n$ and $m$) (see, e.g., Ref. [28]):
\[ T = (n - m) \mod 3 \text{ for example } \begin{cases} 
1 \text{ for } (1, 0), \\
-1 \text{ for } (0, 1), \\
0 \text{ for } (0, 0), \\
0 \text{ for } (1, 1). 
\end{cases} \]  \hspace{1cm} \text{(E2)}

The coefficient coupling the representations \( R_a \otimes R_b \rightarrow R_c \) is given by \cite{18}:

\[
\left( \begin{array}{ccc}
R_a & R_b & R_c \\
\nu_a & \nu_b & \nu_c 
\end{array} \right) \equiv \left( \begin{array}{ccc}
R_a & R_b & R_c \\
I_a I_a^3 Y_a & I_b I_b^3 Y_b & I_c I_c^3 Y_c 
\end{array} \right) = F(R_c, Y_c, I_c; R_a, Y_a, I_a; R_b, Y_b, I_b) \times \langle I_c I_c^3 I_a I_a^3 I_b I_b^3 \rangle. \hspace{1cm} \text{(E3)}
\]

Tables of isoscalar factors are presented in \cite{26}. The necessary SU(2) Clebsch Gordan coefficients are given for example in \cite{29,30} or can be easily computed using Mathematica. For the treatment of final-state particles in the same representation or identical particles in the final state, see Ref. \cite{18}.

Below we present the transformation from the physical to the group-theoretical basis (Eqn. \text{(E1)}) for a class of processes \( \text{in} \rightarrow \text{out}_1 \text{ out}_2 \) as follows:

- Cabbibo allowed processes \( a_{\text{out}_1, \text{out}_2}^{\text{in}} = A_{\text{out}_1, \text{out}_2}^{\text{in}} \cdot \alpha_{\text{out}_1, \text{out}_2}^{\text{in}} \);
- Cabbibo suppressed processes \( s_{\text{out}_1, \text{out}_2}^{\text{in}} = S_{\text{out}_1, \text{out}_2}^{\text{in}} \cdot \sigma_{\text{out}_1, \text{out}_2}^{\text{in}} \).

1. \( 3_{bb} \rightarrow 3_{bc} + M \)

\[
a_{3_{bc}, M}^{3_{bb}} = \begin{pmatrix}
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^+ \pi^-) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^0 \pi^0) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^0 \eta_8) \\
A(\Xi_{bb}^0 \rightarrow \Omega_{bc}^0 K^0) \\
A(\Xi_{bb}^0 \rightarrow \Omega_{bc}^0 \pi^-) \\
A(\Omega_{bb}^0 \rightarrow \Xi_{bc}^0 \pi^-) \\
A(\Omega_{bb}^0 \rightarrow \Xi_{bc}^0 \pi^-) \\
A(\Omega_{bb}^0 \rightarrow \Omega_{bc}^0 K^-) \\
A(\Omega_{bb}^0 \rightarrow \Omega_{bc}^0 \pi^-)
\end{pmatrix}, \quad \alpha_{3_{bc}, M}^{3_{bb}} = \begin{pmatrix}
\langle 3|8_I=1||3 \rangle \\
\langle 6|8_I=1||3 \rangle \\
\langle 15|8_I=1||3 \rangle \\
\langle 15|10_I=1||3 \rangle \\
\langle 6|10_I=1||3 \rangle \\
\langle 15|27_I=1||3 \rangle \\
\langle 15|27_I=2||3 \rangle 
\end{pmatrix}. \hspace{1cm} \text{(E4)}
\]

\(^2\)Note the following typo: The replacement \( 24 \leftrightarrow 24 \) has to be made in the tables 11, 16, 20, 35, 36 and 39 of Ref. \cite{24}.  

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$$s_{3_{bc},M}^{3_{bb}} = \begin{pmatrix}
\mathcal{A}(\Xi_{bc}^0 \to \Xi_{bc}^+ K^-) \\
\mathcal{A}(\Xi_{bc}^0 \to \Xi_{bc}^0 K^0) \\
\mathcal{A}(\Xi_{bc}^0 \to \Omega_{bc}^0 \pi^0) \\
\mathcal{A}(\Xi_{bc}^0 \to \Omega_{bc}^0 \eta_8) \\
\mathcal{A}(\Xi_{bc}^0 \to \Omega_{bc}^0 K^-) \\
\mathcal{A}(\Xi_{bc}^0 \to \Omega_{bc}^0 \pi^-) \\
\mathcal{A}(\Omega_{bc}^0 \to \Omega_{bc}^0 K^-)
\end{pmatrix}, \quad \mathcal{A}_{3_{bc},M}^{3_{bb}} = \begin{pmatrix}
-\frac{1}{2} \sqrt{\frac{3}{2}} & -\frac{1}{2} \sqrt{\frac{1}{3}} & -\frac{1}{2} \sqrt{\frac{3}{10}} & 0 & \frac{1}{\sqrt{6}} & -\frac{1}{2} \sqrt{\frac{3}{5}} & -\frac{1}{2} \\
\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \sqrt{\frac{2}{3}} & -\frac{1}{2} \sqrt{\frac{3}{5}} & -\frac{1}{2} \sqrt{\frac{3}{10}} & 0 & -\frac{1}{\sqrt{2}} \\
-\frac{1}{2} \sqrt{\frac{2}{3}} & -\frac{1}{2} \sqrt{\frac{1}{3}} & -\frac{1}{2} \sqrt{\frac{2}{30}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{5}} & 0 \\
0 & 0 & -2 \sqrt{\frac{2}{15}} & -\sqrt{\frac{2}{15}} & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{2} \\
0 & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{15}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{5}} & 0 \\
0 & -\frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{15}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{5}} & 0
\end{pmatrix}, \quad \mathcal{S}_{3_{bc},M}^{3_{bb}} = \begin{pmatrix}
-\frac{1}{2} \sqrt{\frac{3}{2}} & -\frac{1}{2} \sqrt{\frac{1}{3}} & -\frac{1}{2} \sqrt{\frac{3}{10}} & 0 & -\frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{15}} & -\frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \sqrt{\frac{2}{3}} & -\frac{1}{2} \sqrt{\frac{3}{5}} & -\frac{1}{2} \sqrt{\frac{3}{10}} & 0 & -\frac{1}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{6}} & -\sqrt{\frac{3}{15}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\
0 & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{30}} & 0 \\
\frac{1}{2} & 0 & -\frac{1}{\sqrt{6}} & \frac{1}{2} & 0 & -\frac{3}{2} \sqrt{\frac{1}{5}} & 0 \\
0 & -\frac{1}{\sqrt{3}} & -\sqrt{\frac{3}{15}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{30}} & \frac{1}{5} \\
0 & \frac{1}{\sqrt{3}} & -\sqrt{\frac{3}{15}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{30}} & \frac{1}{5} \\
0 & 0 & -2 \sqrt{\frac{2}{15}} & \frac{1}{\sqrt{6}} & 0 & \sqrt{\frac{3}{10}} & 0
\end{pmatrix}. \quad (E5, E6, E7)

2. $3_{bb} \to 6_b + D$

$$\alpha^{3_{bb}}_{6_b,D} = \begin{pmatrix}
\mathcal{A}(\Xi_{bc}^0 \to \Sigma_{bc}^0 D^0) \\
\mathcal{A}(\Xi_{bc}^0 \to \Sigma_{bc}^0 D^+) \\
\mathcal{A}(\Xi_{bc}^0 \to \Xi_{bc} D^+) \\
\mathcal{A}(\Xi_{bc}^0 \to \Sigma_{bc}^0 D^0) \\
\mathcal{A}(\Omega_{bc}^0 \to \Xi_{bc} D^0)
\end{pmatrix}, \quad \alpha^{3_{bb}}_{6_b,D} = \begin{pmatrix}
\langle 3||8_{I=1}||3 \rangle \\
\langle 15||8_{I=1}||3 \rangle \\
\langle 15||10_{I=1}||3 \rangle \\
\langle 15||27_{I=1}||3 \rangle \\
\langle 15||27_{I=2}||3 \rangle
\end{pmatrix}. \quad (E8)
\[ s_{6b,D}^{3b} = \begin{pmatrix} a(\Xi_{bb}^0 \rightarrow \Xi_{bb}^0 D^0) \\ a(\Xi_{bb}^0 \rightarrow \Xi_{bb}^- D^+) \\ a(\Xi_{bb}^0 \rightarrow \Omega_{b}^- D_{s}^+) \\ a(\Xi_{bb}^- \rightarrow \Xi_{bb}^- D^0) \\ a(\Omega_{bb}^- \rightarrow \Omega_{b}^- D^0) \end{pmatrix}, \quad \sigma_{6b,D}^{3b} = \begin{pmatrix} \langle 3 | 8_{I=\frac{3}{2}} | 3 \rangle \\ \langle 15 | 8_{I=\frac{3}{2}} | 3 \rangle \\ \langle 15 | 10_{I=\frac{3}{2}} | 3 \rangle \\ \langle 15 | 27_{I=\frac{3}{2}} | 3 \rangle \\ \langle 15 | 27_{I=\frac{3}{2}} | 3 \rangle \end{pmatrix}, \quad (E9) \]

\[ A_{6b,D}^{3b} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \sqrt{\frac{2}{5}} & 0 & -\frac{1}{\sqrt{15}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{15}} & \frac{1}{2} \sqrt{\frac{2}{5}} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{15}} & \frac{1}{\sqrt{3}} & -\sqrt{\frac{2}{5}} & 0 \\ 0 & -2 \sqrt{\frac{2}{15}} & -\frac{1}{\sqrt{6}} & \frac{1}{2} \sqrt{\frac{2}{5}} & \frac{1}{2} \\ 0 & -2 \sqrt{\frac{2}{15}} & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{5}} & 0 \end{pmatrix}, \quad (E10) \]

\[ S_{6b,D}^{3b} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \sqrt{\frac{7}{5}} & 0 & -2 \frac{1}{\sqrt{15}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{15}} & \frac{1}{\sqrt{6}} & -\sqrt{\frac{7}{5}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{30}} & \frac{1}{\sqrt{6}} & -\sqrt{\frac{7}{10}} & 0 \\ 0 & -2 \frac{1}{\sqrt{15}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{15}} & \frac{1}{\sqrt{3}} \\ 0 & -2 \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{6}} & \sqrt{\frac{7}{10}} & 0 \end{pmatrix}. \quad (E11) \]

3. \(3_{bb} \rightarrow 3_{b} + D\)

\[ a_{3b,D}^{3b} = \begin{pmatrix} a(\Xi_{bb}^0 \rightarrow \Xi_{bb}^- D_{s}^+) \\ a(\Xi_{bb}^0 \rightarrow \Lambda_{b}^0 D^0) \\ a(\Omega_{bb}^- \rightarrow \Xi_{bb}^- D^0) \end{pmatrix}, \quad \alpha_{3b,D}^{3b} = \begin{pmatrix} \langle 3 | 8_{I=1} | 3 \rangle \\ \langle 6 | 8_{I=1} | 3 \rangle \end{pmatrix}, \quad (E12) \]

\[ s_{3b,D}^{3b} = \begin{pmatrix} a(\Xi_{bb}^0 \rightarrow \Xi_{bb}^- D^+) \\ a(\Xi_{bb}^0 \rightarrow \Xi_{bb}^0 D^0) \\ a(\Xi_{bb}^- \rightarrow \Xi_{bb}^- D^0) \end{pmatrix}, \quad \sigma_{3b,D}^{3b} = \begin{pmatrix} \langle 3 | 8_{I=\frac{1}{2}} | 3 \rangle \\ \langle 6 | 8_{I=\frac{1}{2}} | 3 \rangle \end{pmatrix}, \quad (E13) \]

\[ A_{3b,D}^{3b} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \quad S_{3b,D}^{3b} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \quad (E14) \]
4. $3_{bb} \rightarrow 3_{bc} + \bar{D}$

\[
a^{3_{bb},D}_{3_{bc}} = \begin{pmatrix}
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^+ D_s^-) \\
A(\Xi_{bb}^0 \rightarrow \Omega_{bc}^0 \bar{D}^0) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^0 D_s^-) \\
A(\Omega_{bb}^0 \rightarrow \Omega_{bc}^0 D_s^-)
\end{pmatrix}, \quad \alpha^{3_{bb},D}_{3_{bc}} = \begin{pmatrix}
\langle 3||3_{I=0}||3 \rangle \\
\langle 6||3_{I=0}||3 \rangle \\
\langle 3||6_{I=1}||3 \rangle \\
\langle 6||15_{I=0}||3 \rangle
\end{pmatrix}, \quad \text{(E15)}
\]

\[
s^{3_{bb},D}_{3_{bc}} = \begin{pmatrix}
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^+ D_s^-) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^0 \bar{D}^0) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{bc}^0 D_s^-) \\
A(\Omega_{bb}^0 \rightarrow \Xi_{bc}^0 D_s^-)
\end{pmatrix}, \quad \sigma^{3_{bb},D}_{3_{bc}} = \begin{pmatrix}
\langle 3||3_{I=\frac{1}{2}}||3 \rangle \\
\langle 6||3_{I=\frac{1}{2}}||3 \rangle \\
\langle 3||6_{I=\frac{1}{2}}||3 \rangle \\
\langle 6||15_{I=\frac{1}{2}}||3 \rangle
\end{pmatrix}, \quad \text{(E16)}
\]

\[
A^{3_{bb},D}_{3_{bc}} = \begin{pmatrix}
-\frac{1}{2} & -\frac{1}{2} & \frac{3}{ \sqrt{2}} & \frac{3}{ \sqrt{2}} & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & \frac{3}{ \sqrt{2}} & \frac{3}{ \sqrt{2}} & -\frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{2} & \frac{3}{ \sqrt{2}} & \frac{3}{ \sqrt{2}} & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & \frac{3}{ \sqrt{2}} & \frac{3}{ \sqrt{2}} & -\frac{1}{2} \\
0 & -\frac{1}{ \sqrt{2}} & 0 & -\frac{1}{ \sqrt{2}} & 0
\end{pmatrix}, \quad \text{(E17)}
\]

\[
s^{3_{bb},D}_{3_{bc}} = \begin{pmatrix}
-\frac{1}{2} & -\frac{1}{2} & \frac{1}{ \sqrt{2}} & \frac{1}{ \sqrt{2}} & -\frac{1}{2} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{1}{ \sqrt{6}} & -\frac{1}{ \sqrt{6}} & \frac{1}{2} \\
0 & -\frac{1}{ \sqrt{2}} & 0 & -\frac{1}{ \sqrt{6}} & -\frac{1}{ \sqrt{6}} \\
\frac{1}{2} & -\frac{1}{2} & -\frac{1}{ \sqrt{2}} & -\frac{1}{ \sqrt{2}} & \frac{1}{2} \\
-\frac{1}{2} & -\frac{1}{2} & -\frac{1}{ \sqrt{2}} & -\frac{1}{ \sqrt{2}} & 0
\end{pmatrix}, \quad \text{(E18)}
\]

5. $3_{bb} \rightarrow \bar{3}_b + J/\Psi$

\[
a^{3_{bb},J/\Psi}_{3_b} = \begin{pmatrix}
A(\Xi_{bb}^0 \rightarrow \Xi_{bb}^0 J/\Psi) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{bb}^+ J/\Psi)
\end{pmatrix}, \quad \alpha^{3_{bb},J/\Psi}_{3_b} = \begin{pmatrix}
\langle 3||3_{I=0}||3 \rangle \\
\langle 3||3_{I=1}||3 \rangle
\end{pmatrix}, \quad \text{(E19)}
\]

\[
s^{3_{bb},J/\Psi}_{3_b} = \begin{pmatrix}
A(\Xi_{bb}^0 \rightarrow \Lambda_b^0 J/\Psi) \\
A(\Omega_{bb}^0 \rightarrow \Xi_{bb}^- J/\Psi)
\end{pmatrix}, \quad \sigma^{3_{bb},J/\Psi}_{3_b} = \begin{pmatrix}
\langle 3||3_{I=\frac{1}{2}}||3 \rangle \\
\langle 3||3_{I=\frac{1}{2}}||3 \rangle
\end{pmatrix}, \quad \text{(E20)}
\]

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\[ A_{3_{bb}, J/\Psi}^{3_{bb}} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad S_{3_{bb}, J/\Psi}^{3_{bb}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \] (E21)

6. \(3_{bb} \rightarrow 6_b + J/\Psi\)

\[ a_{6_b, J/\Psi}^{3_{bb}} = \begin{pmatrix} A(\Xi_{bb}^0 \rightarrow \Xi_{b2}^0 J/\Psi) \\ A(\Xi_{bb}^0 \rightarrow \Xi_{b2}^- J/\Psi) \\ A(\Omega_{bb}^- \rightarrow \Omega_b^- J/\Psi) \end{pmatrix}, \quad \alpha_{6_b, J/\Psi}^{3_{bb}} = \begin{pmatrix} \langle 6||3_{I=0}||3 \rangle \\ \langle 6||15_{I=0}||3 \rangle \\ \langle 6||15_{I=1}||3 \rangle \end{pmatrix}, \] (E22)

\[ s_{6_b, J/\Psi}^{3_{bb}} = \begin{pmatrix} A(\Xi_{bb}^0 \rightarrow \Sigma_b^0 J/\Psi) \\ A(\Xi_{bb}^0 \rightarrow \Sigma_b^- J/\Psi) \\ A(\Omega_{bb}^- \rightarrow \Xi_{b2}^- J/\Psi) \end{pmatrix}, \quad \sigma_{6_b, J/\Psi}^{3_{bb}} = \begin{pmatrix} \langle 6||3_{I=\frac{1}{2}}||3 \rangle \\ \langle 6||15_{I=\frac{3}{2}}||3 \rangle \\ \langle 6||15_{I=\frac{5}{2}}||3 \rangle \end{pmatrix}, \] (E23)

\[ A_{6_b, J/\Psi}^{3_{bb}} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \quad S_{6_b, J/\Psi}^{3_{bb}} = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & -\sqrt{\frac{2}{3}} \\ -\frac{1}{2} & -\frac{1}{2} & \sqrt{\frac{2}{3}} \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}. \] (E24)

7. \(3_{bb} \rightarrow 3_b + M\)

\[ a_{3_b, M}^{3_{bb}} = \begin{pmatrix} A(\Xi_{bb}^0 \rightarrow \Xi_{b1}^- \pi^+) \\ A(\Xi_{bb}^0 \rightarrow \Xi_{b1}^0) \\ A(\Xi_{bb}^0 \rightarrow \Xi_{b1}^- \eta_b) \\ A(\Xi_{bb}^0 \rightarrow \Lambda_b^0 \eta_b) \\ A(\Xi_{bb}^- \rightarrow \Xi_{b1}^- \pi^0) \\ A(\Xi_{bb}^- \rightarrow \Xi_{b1}^- \eta_b) \\ A(\Xi_{bb}^- \rightarrow \Xi_{b1}^- \pi^-) \\ A(\Xi_{bb}^- \rightarrow \Lambda_b^0 K^-) \\ A(\Omega_{bb}^- \rightarrow \Xi_{b1}^- K^0) \\ A(\Omega_{bb}^- \rightarrow \Xi_{b1}^- K^-) \end{pmatrix}, \quad \alpha_{3_b, M}^{3_{bb}} = \begin{pmatrix} \langle \Xi||3_{I=0}||3 \rangle \\ \langle 6||3_{I=0}||3 \rangle \\ \langle \Xi||5_{I=1}||3 \rangle \\ \langle 15||6_{I=1}||3 \rangle \\ \langle 6||15_{I=0}||3 \rangle \\ \langle 6||15_{I=1}||3 \rangle \\ \langle 15||15_{I=0}||3 \rangle \\ \langle 15||15_{I=1}||3 \rangle \\ \langle 15||24_{I=0}||3 \rangle \\ \langle 15||24_{I=1}||3 \rangle \end{pmatrix}. \] (E25)

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\[
\begin{align*}
\mathbf{s}_{3b,\, M}^{3b} &= 
\begin{pmatrix}
\mathcal{A}(\Xi_{bb}^0 \to \Xi_{b1}^0 K^+) \\
\mathcal{A}(\Xi_{bb}^0 \to \Xi_{b1}^0 K^0) \\
\mathcal{A}(\Xi_{bb}^0 \to \Lambda_b^0 \pi^0) \\
\mathcal{A}(\Xi_{bb}^0 \to \Lambda_b^0 \eta_b) \\
\mathcal{A}(\Xi_{bb}^- \to \Xi_{b1}^- K^0) \\
\mathcal{A}(\Xi_{bb}^- \to \Lambda_b^0 \pi^-) \\
\mathcal{A}(\Omega_{bb}^- \to \Xi_{b1}^- \pi^0) \\
\mathcal{A}(\Omega_{bb}^- \to \Xi_{b1}^- \eta_b) \\
\mathcal{A}(\Omega_{bb}^- \to \Xi_{b1}^- \pi^-) \\
\mathcal{A}(\Omega_{bb}^- \to \Lambda_b^0 K^-)
\end{pmatrix}, \\
\mathbf{\sigma}_{3b,\, M}^{3b} &= 
\begin{pmatrix}
\langle 3 | 3_{J=\frac{1}{2}} | 3 \rangle \\
\langle 6 | 3_{J=\frac{1}{2}} | 3 \rangle \\
\langle 3 | 6_{J=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 6_{J=\frac{1}{2}} | 3 \rangle \\
\langle 6 | 15_{J=\frac{1}{2}} | 3 \rangle \\
\langle 6 | 15_{J=\frac{3}{2}} | 3 \rangle \\
\langle 15 | 15_{J=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 15_{J=\frac{3}{2}} | 3 \rangle \\
\langle 15 | 24_{J=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 24_{J=\frac{3}{2}} | 3 \rangle
\end{pmatrix},
\end{align*}
\]
(E26)
\[
\mathbf{A}_{3b,\, M}^{3b} = 
\begin{pmatrix}
\frac{1}{4}\sqrt{3} & \frac{1}{2} & \frac{1}{2}\sqrt{3} & -\frac{3}{4}\sqrt{6} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{3} \\
-\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{3}{4}\sqrt{6} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{3} \\
\frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{2} & \frac{1}{4}\sqrt{3} & \frac{3}{4}\sqrt{6} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{3} \\
-\frac{1}{4}\sqrt{3} & \frac{1}{2} & \frac{1}{2}\sqrt{3} & -\frac{3}{4}\sqrt{6} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{6} & \frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{6} & \frac{1}{4}\sqrt{3} \\
-\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{2} & -\frac{1}{4}\sqrt{3} & \frac{3}{4}\sqrt{6} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{6} & \frac{1}{4}\sqrt{3} & \frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{6} & \frac{1}{4}\sqrt{3} \\
0 & \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{6} & \frac{1}{2} & 0 & 0 & \frac{1}{2}\sqrt{6} & \frac{1}{2} & 0 & 0 & \frac{1}{2}\sqrt{6} & \frac{1}{2} & 0
\end{pmatrix},
\]
(E27)
\[
S_{3b, M}^{3bb} = \begin{pmatrix}
-\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{3}{2}\sqrt{3} & -\frac{1}{6}\sqrt{3} & -\frac{1}{6} & -\frac{1}{6} & -\frac{2}{3}\sqrt{3} & -\frac{1}{3} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{6}\sqrt{3} & -\frac{1}{6} & 0 & \frac{1}{12} & -\frac{1}{3} \\
0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{6}\sqrt{3} & \frac{1}{3}\sqrt{3} & -\frac{1}{3} & -\frac{1}{3}\sqrt{3} \\
\frac{1}{2} & 0 & -\frac{1}{2} & 0 & -\frac{1}{6}\sqrt{3} & \frac{1}{3}\sqrt{3} & -\frac{1}{3} & -\frac{1}{3}\sqrt{3} & -\frac{1}{3} \\
-\frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & 0 & \frac{1}{12} & -\frac{1}{3} & -\frac{1}{3} \\
-\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & 0 & \frac{1}{12} & -\frac{1}{3} & -\frac{1}{3} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & 0 & \frac{1}{12} & -\frac{1}{3} & -\frac{1}{3} \\
\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & 0 & \frac{1}{12} & -\frac{1}{3} & -\frac{1}{3} \\
\end{pmatrix}
\]
(E28)

\[8. \ 3_{bb} \rightarrow 6_b + M\]

\[
\begin{pmatrix}
A(\Xi_{bb}^0 \rightarrow \Sigma_6^+ K^-) \\
A(\Xi_{bb}^0 \rightarrow \Sigma_6^- \bar{K}^0) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{02}^0 \pi^0) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{02}^0 \eta_b) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{02}^0 \pi^+) \\
A(\Xi_{bb}^0 \rightarrow \Omega_6^- K^+) \\
A(\Xi_{bb}^0 \rightarrow \Sigma_6^0 K^-) \\
A(\Xi_{bb}^0 \rightarrow \Sigma_6^- \bar{K}^0) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{02}^0 \pi^-) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{02}^0 \pi^0) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{02}^0 \eta_b) \\
A(\Xi_{bb}^0 \rightarrow \Omega_6^- \bar{K}^0) \\
A(\Omega_6^- \rightarrow \Xi_{02}^- \bar{K}^0) \\
A(\Omega_6^- \rightarrow \Xi_{02}^0 K^-) \\
A(\Omega_6^- \rightarrow \Xi_{02}^0 \pi^0) \\
A(\Omega_6^- \rightarrow \Omega_6^- \eta_b)
\end{pmatrix} \cdot
\begin{pmatrix}
\langle 3 \parallel 3_{I=0} \parallel 3 \rangle \\
\langle 6 \parallel 3_{I=0} \parallel 3 \rangle \\
\langle 3 \parallel 6_{I=1} \parallel 3 \rangle \\
\langle 15 \parallel 6_{I=1} \parallel 15 \rangle \\
\langle 6 \parallel 15_{I=0} \parallel 3 \rangle \\
\langle 6 \parallel 15_{I=1} \parallel 3 \rangle \\
\langle 15 \parallel 15_{I=0} \parallel 15 \rangle \\
\langle 15 \parallel 15_{I=1} \parallel 15 \rangle \\
\langle 24 \parallel 15_{I=0} \parallel 3 \rangle \\
\langle 24 \parallel 15_{I=1} \parallel 3 \rangle \\
\langle 24 \parallel 15_{I=1} \parallel 3 \rangle \\
\langle 15 \parallel 24_{I=1} \parallel 3 \rangle \\
\langle 15 \parallel 24_{I=2} \parallel 3 \rangle \\
\langle 24 \parallel 42_{I=0} \parallel 3 \rangle \\
\langle 24 \parallel 42_{I=1} \parallel 3 \rangle \\
\langle 24 \parallel 42_{I=2} \parallel 3 \rangle
\end{pmatrix}
\]
(E29)
\begin{align}
\mathcal{A} & (\Xi_{bb}^0 \rightarrow \Sigma_b^+ \pi^-) \\
\mathcal{A} & (\Xi_{bb}^0 \rightarrow \Sigma_b^0 \pi^0) \\
\mathcal{A} & (\Xi_{bb}^0 \rightarrow \Sigma_b^0 \eta_8) \\
\mathcal{A} & (\Xi_{bb}^0 \rightarrow \Sigma_b^- \pi^+) \\
\mathcal{A} & (\Xi_{bb}^0 \rightarrow \Xi_{b2}^0 K^0) \\
\mathcal{A} & (\Xi_{bb}^- \rightarrow \Xi_{b2}^- K^+) \\
\mathcal{A} & (\Xi_{bb}^- \rightarrow \Sigma_b^- \pi^-) \\
\mathcal{A} & (\Xi_{bb}^- \rightarrow \Sigma_b^- \pi^0) \\
\mathcal{A} & (\Xi_{bb}^- \rightarrow \Sigma_b^- \eta_8) \\
\mathcal{A} & (\Omega_{bb}^- \rightarrow \Xi_{b2}^0 K^-) \\
\mathcal{A} & (\Omega_{bb}^- \rightarrow \Sigma_b^- \bar{K}^0) \\
\mathcal{A} & (\Omega_{bb}^- \rightarrow \Xi_{b2}^0 \pi^-) \\
\mathcal{A} & (\Omega_{bb}^- \rightarrow \Xi_{b2}^0 \pi^0) \\
\mathcal{A} & (\Omega_{bb}^- \rightarrow \Xi_{b2}^- \eta_8) \\
\mathcal{A} & (\Omega_{bb}^- \rightarrow \Omega_b^- K^0)
\end{align}

\[ s_{6b, M}^{3b} = \begin{pmatrix}
\mathcal{A}(\Xi_{bb}^0 \rightarrow \Sigma_b^+ \pi^-) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow \Sigma_b^0 \pi^0) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow \Sigma_b^0 \eta_8) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow \Sigma_b^- \pi^+) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow \Xi_{b2}^0 K^0) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow \Xi_{b2}^- K^+) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow \Sigma_b^- \pi^-) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow \Sigma_b^- \pi^0) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow \Sigma_b^- \eta_8) \\
\mathcal{A}(\Omega_{bb}^- \rightarrow \Xi_{b2}^0 \eta_8)
\end{pmatrix}\]

\begin{align}
\sigma_{6b, M}^{3b} = \begin{pmatrix}
\langle 3 | 3 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 6 | 3 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 3 | 6 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 6 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 6 | 15 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 6 | 15 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 15 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 15 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 24 | 15 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 24 | 15 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 24 | 15 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 24 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 15 | 24 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 24 | 24 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 24 | 42 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 24 | 42 \rangle_{f=\frac{1}{2}} | 3 \rangle \\
\langle 24 | 42 \rangle_{f=\frac{1}{2}} | 3 \rangle
\end{pmatrix}
\end{align}

(E30)
\[
A_{\text{6a, 4d}} = \begin{pmatrix}
-\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & 0 & -\frac{3}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}}
\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{10}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{10}} & \frac{\sqrt{3}}{\sqrt{10}} & \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{10}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}
\]
9. $3_{bb} \rightarrow B + b$

\[
3_{bb} = \begin{pmatrix}
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^- \Sigma^+) & (3|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Sigma^0) & (6|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Lambda^0) & (3|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Xi^0) & (15|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Sigma^-) & (6|15_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Xi^-) & (15|15_{I=0}||3) \\
\mathcal{A}(\Omega_{bb} \rightarrow B^- \Xi^-) & (15|24_{I=1}||3) \\
\mathcal{A}(\Omega_{bb}^- \rightarrow B^0 \Xi^-) & (15|24_{I=2}||3)
\end{pmatrix}
\]

\[
\alpha_{B,b}^{3_{bb}} = \begin{pmatrix}
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^- \Sigma^+) & (3|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Sigma^0) & (6|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Lambda^0) & (3|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Xi^0) & (15|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Sigma^-) & (6|15_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Xi^-) & (15|15_{I=0}||3) \\
\mathcal{A}(\Omega_{bb} \rightarrow B^- \Xi^-) & (15|24_{I=1}||3) \\
\mathcal{A}(\Omega_{bb}^- \rightarrow B^0 \Xi^-) & (15|24_{I=2}||3)
\end{pmatrix}
\]

\[
\alpha_{B,b}^{3_{bb}} = \begin{pmatrix}
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^- \Sigma^+) & (3|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Sigma^0) & (6|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Lambda^0) & (3|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Xi^0) & (15|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Sigma^-) & (6|15_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Xi^-) & (15|15_{I=0}||3) \\
\mathcal{A}(\Omega_{bb} \rightarrow B^- \Xi^-) & (15|24_{I=1}||3) \\
\mathcal{A}(\Omega_{bb}^- \rightarrow B^0 \Xi^-) & (15|24_{I=2}||3)
\end{pmatrix}
\]

\[
\sigma_{B,b}^{3_{bb}} = \begin{pmatrix}
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^- \Sigma^+) & (3|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Sigma^0) & (6|3_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Lambda^0) & (3|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^0 \rightarrow B^0 \Xi^0) & (15|6_{I=1}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Sigma^-) & (6|15_{I=0}||3) \\
\mathcal{A}(\Xi_{bb}^- \rightarrow B^- \Xi^-) & (15|15_{I=0}||3) \\
\mathcal{A}(\Omega_{bb} \rightarrow B^- \Xi^-) & (15|24_{I=1}||3) \\
\mathcal{A}(\Omega_{bb}^- \rightarrow B^0 \Xi^-) & (15|24_{I=2}||3)
\end{pmatrix}
\]
$$A^{{3b}_{b,b}} = \begin{pmatrix}
\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
-\frac{1}{4}\sqrt{3} & -\frac{1}{4} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4} & -\frac{1}{4}\sqrt{3} & 0 & -\frac{1}{4}\sqrt{3} \\
\frac{1}{4} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
-\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{3}{4}\sqrt{3} & \frac{3}{4}\sqrt{3} & -\frac{1}{4} & -\frac{1}{4}\sqrt{6} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
-\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & \frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
0 & \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
0 & -\frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
0 & \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
0 & -\frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
0 & \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
0 & -\frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
0 & -\frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
0 & \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & \frac{1}{4}\sqrt{10} & 0 & 0 & \frac{1}{2} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{2}\sqrt{2} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} \\
\end{pmatrix},\\
(E35)
$$

$$S^{{3b}_{b,b}} = \begin{pmatrix}
-\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{3}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{6} & \frac{1}{12} & \frac{1}{4}\sqrt{2} & \frac{2}{3}\sqrt{3} & -\frac{1}{3} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & -\frac{1}{4}\sqrt{3} & -\frac{3}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{6} & \frac{1}{12} & \frac{1}{4}\sqrt{2} & \frac{2}{3}\sqrt{3} & -\frac{1}{3} \\
0 & \frac{1}{2}\sqrt{2} & 0 & -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\
0 & -\frac{1}{2} & 0 & -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{2} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{2} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4}\sqrt{3} & -\frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{2} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{3} & -\frac{1}{4}\sqrt{3} & \frac{1}{4} & \frac{1}{4}\sqrt{2} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\
\end{pmatrix},\\
(E36)
$$

10. $3_{bb} \rightarrow 3_b + D$

$$a^{{3b}_{b,b}}_{3b,B} = \begin{pmatrix}
A(\Xi^0_{bb} \rightarrow \Xi^0_{b1} \ D^0) \\
A(\Xi^0_{bb} \rightarrow \Xi^0_{b1} \ D^-) \\
A(\Xi^0_{bb} \rightarrow \Lambda^0_b \ D^-) \\
A(\Omega^0_{bb} \rightarrow \Xi^0_{b1} \ D^-) \\
\end{pmatrix},
\quad
\alpha^{{3b}_{b,b}}_{3b,B} = \begin{pmatrix}
\langle 1|3_{f=\frac{3}{2}}|3 \rangle \\
\langle 8|3_{f=\frac{3}{2}}|3 \rangle \\
\langle 8|6_{f=\frac{3}{2}}|3 \rangle \\
\langle 8|15_{f=\frac{3}{2}}|3 \rangle \\
\end{pmatrix},
(E37)$$

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\[
A^{3_{bb}, D}_{3_{bb}, D} = \begin{pmatrix}
0 & -\frac{1}{2} \sqrt{\frac{3}{2}} & -\frac{1}{2} & -\frac{1}{2} \sqrt{\frac{3}{6}} & -\frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{3}} & -\frac{1}{2} \sqrt{\frac{3}{2}} & -\frac{1}{2} & \frac{1}{2} \sqrt{\frac{3}{6}} & \frac{1}{\sqrt{3}} \\
\frac{1}{\sqrt{3}} & -\frac{1}{2} \sqrt{\frac{3}{2}} & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{3}} & \frac{1}{2} \sqrt{\frac{3}{2}} & -\frac{1}{2} & \frac{1}{2} \sqrt{\frac{3}{2}} & 0 \\
0 & -\frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} & \frac{1}{2} \sqrt{\frac{3}{2}} & 0
\end{pmatrix}.
\]

\[\text{(E38)}\]

11. \(3_{bb} \rightarrow 6_b + D\)

\[
a^{3_{bb}}_{6_b, D} = 
\begin{pmatrix}
A(\Xi_{bb}^0 \rightarrow \Sigma_b^+ D_s^-) \\
A(\Xi_{bb}^0 \rightarrow \Xi_{bb}^0 D^0) \\
A(\Xi_{bb}^- \rightarrow \Sigma_b^0 D_s^-) \\
A(\Xi_{bb}^- \rightarrow \Xi_{bb}^0 D^-) \\
A(\Omega_{bb}^0 \rightarrow \Xi_{bb}^0 D^0) \\
A(\Omega_{bb}^- \rightarrow \Omega_{bb}^- D^0)
\end{pmatrix}
\]

\[
A^{3_{bb}}_{6_b, D} = 
\begin{pmatrix}
\langle 8|\bar{I}_{1\frac{1}{2}}|3 \rangle \\
\langle 8|6|\bar{I}_{1\frac{1}{2}}|3 \rangle \\
\langle 10|6|\bar{I}_{1\frac{1}{2}}|3 \rangle \\
\langle 8|15|\bar{I}_{1\frac{3}{2}}|3 \rangle \\
\langle 8|15|12|\bar{I}_{1\frac{3}{2}}|3 \rangle \\
\langle 10|24|\bar{I}_{1\frac{3}{2}}|3 \rangle
\end{pmatrix}.
\]

\[\text{(E39)}\]

\[
\begin{pmatrix}
-\frac{1}{2} & -\frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{15} & -\frac{1}{6} & -\frac{1}{3} \sqrt{2} & \frac{\sqrt{3}}{2} & -\frac{1}{3} \sqrt{3} \\
\frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} \sqrt{\frac{3}{2}} & -\frac{1}{2} \sqrt{\frac{3}{5}} & \frac{1}{6} \sqrt{2} & \frac{1}{3} & -\frac{2}{3} \sqrt{\frac{2}{5}} & -\frac{1}{3} \sqrt{2} \\
\frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} \sqrt{\frac{3}{5}} & -\frac{1}{6} \sqrt{2} & \frac{1}{3} & \frac{1}{3} \sqrt{\frac{2}{5}} & -\frac{1}{3} \sqrt{2} \\
0 & \frac{1}{\sqrt{3}} & 0 & -\frac{1}{3} \sqrt{2} & \frac{1}{3} & -\frac{1}{3} \sqrt{\frac{2}{5}} & -\frac{1}{3} \sqrt{2} \\
\frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} \sqrt{\frac{3}{5}} & -\frac{5}{6} \sqrt{2} & \frac{1}{3} & -\frac{1}{3} \sqrt{\frac{2}{5}} & -\frac{1}{3} \sqrt{2} \\
\frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} \sqrt{\frac{3}{2}} & \frac{1}{2} \sqrt{\frac{3}{5}} & 0 & \frac{1}{3} \sqrt{\frac{3}{5}} & 0 & 0
\end{pmatrix}.
\]

\[\text{(E40)}\]

**APPENDIX F: MATCHING ARBITRARY SU(3) BREAKING ONTO LINEAR SU(3) BREAKING**

In Sec. [IV] we presented SU(3) decompositions of various decays using tensor methods and assuming linear SU(3) breaking. In Appendix E we gave a full SU(3) decomposition. In this section we show how to match these results in order to find relations for the reduced amplitudes given in Appendix E. We write the unknowns given in Sec. [IV] and the reduced amplitudes from Appendix E as \(U_i = (A, B, \ldots, A_{\text{LB}}, B_{\text{LB}}, \ldots)\) and \(R_i = (\langle R' || R_{\nu} || R_{\text{out}} \rangle, \ldots)\), respectively. We then have
process \( i = T_{ij} U_j \), \( i_{\text{max}} \geq j_{\text{max}} \),
process \( i = S_{ij} R_j \), \( i_{\text{max}} = j_{\text{max}} \).  \( \text{(F1)} \)

The matrices \( S \) and \( T \) are the transformation matrices. \( S \) is orthogonal. After imposing the same phase convention for both methods (see \[18\]), we require

\[
S_{ij} R_j = T_{ik} U_k \implies R_l = S_{il} T_{ik} U_k,
\]

\( \text{(F2)} \)

which automatically produces relationships between different reduced amplitudes, holding in the case of linear breaking. For the sign convention see the discussion in Sec. II of Ref. \[18\]. The amplitude expressions have an overall sign ambiguity. In order to find a consistent matching a sign change in the physical amplitudes considered with the tensor approach is necessary every time one of the following particles appears: \( \pi^0, \pi^-, \eta_8, K^-, D^0, B^-, \Lambda_b^0, \Sigma^0, \Lambda^0, \Sigma^- \) and \( \Xi^- \).

Note that an operator employed using the tensor methods described in the body of this paper is expressed in terms of its barred components in the arbitrarily broken SU(3) reduced matrix elements used in Appendix E.

**APPENDIX G: PHASE SPACE CORRECTIONS**

In those \( b \)-decays where the final-state masses are not small compared to the energy release, we need to include phase space effects in going from amplitude relationships to rate relationships. In \( B \)-meson decays these corrections are normally small, but for some \( b \) baryon decays we can potentially find significant effects if the angular momentum \( l \) of the decay channel is different from zero. Here we present the equations needed to include phase space corrections. As new particle masses are measured, the phase space “SU(3) breaking effects” can be separated from the dynamical ones.

For two-body decays where the decaying particle is in the rest frame we can use \[30\]

\[
d\Gamma(a \to bc) = \frac{1}{32\pi^2} |\mathcal{M}(a \to bc)|^2 \frac{|\vec{p}_b|}{m_a^2} d\Omega,  \tag{G1}
\]

where \( |\vec{p}_b| = |\vec{p}_c| \) and

\[
|\vec{p}_b| = \frac{[(m_a^2 - (m_b + m_c)^2)(m_a^2 - (m_b - m_c)^2)]^{1/2}}{2m_a}.
\tag{G2}
\]

The above equation leads to the following phase space correction factor \[31\]

\[
\frac{\Gamma_l(a \to bc)}{\Gamma_l(d \to ef)} = \left[ \frac{1 - \left( \frac{m_b + m_c}{m_a} \right)^2}{1 - \left( \frac{m_e + m_f}{m_d} \right)^2} \right]^{l+1/2} \left[ \frac{1 - \left( \frac{m_b - m_c}{m_a} \right)^2}{1 - \left( \frac{m_e - m_f}{m_d} \right)^2} \right] |\mathcal{M}(a \to bc)|^2 \left| \mathcal{M}(d \to ef) \right|^2.  \tag{G3}
\]

We use Particle Data Book \[30\] masses where they are measured, and theoretical calculations \[5,8,9\] for the masses of doubly heavy baryons.
As an example, consider Eqn. 4.2. From Ref. [8,32] we have 
\[m_{\Xi^0_{bb}} \approx m_{\Xi^0_{bc}} \approx 10235 \text{ MeV}, \]
\[m_{\Omega^-_{bb}} \approx 10385 \text{ MeV}, \]
\[m_{\Xi^+_{bb}} \approx m_{\Xi^+_{bc}} \approx 6938 \text{ MeV} \]
and 
\[m_{\Xi^+_{bb}} \approx m_{\Xi^+_{bc}} \approx 7095 \text{ MeV}. \]
This gives the following corrected relationships:

\[\frac{\Gamma (\Xi^0_{bb} \rightarrow \Xi^+_bc \pi^-)}{\Gamma (\Xi^0_{bb} \rightarrow \Xi^+_bc K^-)} = \frac{1}{\lambda^2} \left(1.02\right)^{l+1/2}, \]
\[\frac{\Gamma (\Xi^-_{bb} \rightarrow \Xi^0_{bc} \pi^-)}{\Gamma (\Omega^-_{bb} \rightarrow \Omega^0_{bc} K^-)} = \frac{1}{\lambda^2} \left(1.01\right)^{l+1/2}. \]

(G4)

These are not significant corrections. We might expect larger corrections for final states containing a b and two charm quarks. In Eqn. 4.16, the relationships are from isospin and so we do not expect corrections. However, consider relationships from Eqn. 4.22, where we use the estimate of masses of the 6_b by supposing that the splitting between \(\Sigma_b\) and \(\Lambda_b\), for instance, tracks that of the splitting between the \(\Sigma_c\) and the \(\Lambda_c\) [16]:

\[\frac{\Gamma (\Xi^0_{bb} \rightarrow \Sigma^0_{b2} J/\Psi)}{\Gamma (\Omega^0_{bb} \rightarrow \Omega^0_{b} J/\Psi)} = \frac{1}{2} \left(0.98\right)^{l+1/2}, \]
\[\frac{\Gamma (\Xi^-_{bb} \rightarrow \Sigma^-_{b} J/\Psi)}{\Gamma (\Omega^-_{bb} \rightarrow \Xi^0_{b2} J/\Psi)} = 2 \left(0.98\right)^{l+1/2}. \]

(G5)

For more than two particles in the external state the situation becomes harder to analyze. However, to the level we are working (expanding about perfect SU(3) symmetry), we do not expect phase space issues to be a significant source of error. Only if we include mass splittings on the order of 300 MeV for both isospin breaking and strange quark mass differences do we begin to see large phase space corrections.