A Method to Precise Determine the Young’s Modulus from Dynamic Measurements

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Abstract. Knowing the mechanical characteristics of materials is crucial for properly interpreting the results attained from experiments. In this paper, we present a method to experimentally determine the Young modulus of various materials by employing multi-modal analysis. The method, which implies finding the structures’ natural frequencies, is based on the resonance test. If the natural frequencies are known, the dynamic Young’s modulus can be extracted from the relation that expresses these frequencies in respect to the stiffness. An advanced signal processing algorithm is proposed for precise frequency evaluation. The algorithm constitutes the core of the method; herein we demonstrate how it is involved to extract the frequencies of several bending vibration modes. Tests were made by involving non-contact and impulsive excitation, respectively, the fist one providing best repeatability.

1. Introduction

The elastic properties for linearly elastic isotropic solids are described by two independent constants, namely Young’s modulus $E$ and the shear modulus $G$. These elastic constants of materials have great importance for the design of engineering structures. For that reason, numerous experimental techniques to find moduli $E$ and $G$ were developed. These techniques can be divided into two groups: (i) static methods, and (ii) dynamic methods.

Attempts reported in [1]-[3] are among the first made to find the elastic properties of isotropic materials basing on nondestructive vibration tests. The equations established in the first paper for computing moduli $E$ and $G$ from the fundamental flexural and torsional frequencies of prisms and cylinders were found from the Timoshenko beam theory. Advances in calculating the Young modulus from the proper fundamental frequencies along with an improvement of the experimental techniques is proposed in [2] and [3], respectively. In fact, these three papers have formed the base of the ASTM standard test method [4] for characterization of dynamic elastics properties based on the impulse excitation method [5].

Nowadays, there are also numerous methods to estimate the elastic properties of structures made of non-isotropic and/or inhomogeneous materials as concrete [6], timber [7]-[8] or laminated composites [9]-[14] involving vibration-based methods.

Some aspects of our prior research concerned with the effect of mass variation on the frequencies of beams with constant or variable cross-section [15]-[17]. We also studied the behavior of beam-like structures, to find out how the transition from the behavior of a beam to that of a plate is influenced by the width of the structural element [18]. For all this research precise frequency estimation was necessary [19]. In the present investigation, we show how signal processing algorithm applied for determination of the Young’s modulus of laminated composite plates.
2. The principle of deriving Yong’s modulus from vibration measurements

The dynamic method involved to calculate Young’s modulus base on the univocal relationship between it and the dimensions, weight and natural frequencies of the beam for a given support configuration. This method presents following advantages: it is rapid, nondestructive, implies the application of low loads and can be performed at extreme temperatures. Because the method is nondestructive, it enables monitoring the materials properties and makes possible observing their variation in time.

From the point of view of performing the experiments, the method consists of two stages: in the first stage, the excitation and signal acquisition and processing are performed to detect the natural frequencies; in the second one, mathematical relations and computational procedures are performed to obtain E from the natural frequencies.

Basically, there are three vibration mode types that we can use to find Young’s modulus of a prismatic beam with an asymmetric cross-section around one axis, namely out-of-plane and in-plane bending modes, respectively the longitudinal vibration mode. Unlike the other two modes of vibration, the out-of-plane modes can be easier excited, and the risk of occurring combined loads is minimal. Current methods used to excite the test specimen are impulsive-based [4]. From our research, we found that sweep-sine excitation or an excitation applied for short-time in a controlled frequency bandwidth provides better results [20]. This aspect is treated in next section.

In most cases of using bending vibration modes, a slender beam that respects the Euler-Bernoulli theory is chosen as the test specimen. To this aim, the length L per thickness h ratio must be greater than 200 [7]. Obviously, for in-plane vibration, the ratio considers the beam’s width b and length L. The specimen has, usually, free-free or fixed-free end conditions. These supports are preferred because almost perfect boundary condition can be ensured and therefore uncertainties due to supports are avoided. Henceforth fixed-free end conditions are considered.

Knowing the end conditions, we can express the mathematical relation between Young’s modulus E and the beam dimensions L, B respectively H, weight m and natural frequencies f<sub>i</sub>, as:

\[ E = \frac{mL^4}{T} \left( \frac{2\pi f_i}{\alpha_i^2} \right)^2, \]  

where \( I \) is the moment of inertia of plain area, derived as \( I = (B\cdot H^3)/12 \), and \( \alpha_i \) are coefficients related to the beam’s support condition and the mode numbers. Table 1 presents the first six coefficients derived for the beam with fixed-free end conditions (cantilever beam).

| Mode No. | 1     | 2     | 3     | 4     | 5     | 6     |
|----------|-------|-------|-------|-------|-------|-------|
| \( \alpha_i \) [-] | 1.87510 | 4.69409 | 7.85476 | 10.99554 | 14.13717 | 17.27876 |

All parameter values on the right side of relation (1) can be measured with accuracy, except for the natural frequency, for which an advanced estimation algorithm is required. The following sections present an algorithm developed by the authors, which allows high-precision natural frequency evaluation and how it is used in the experimental procedure to find Young’s modulus through the herein proposed dynamic method.

3. Precise frequency evaluation by post-processing the vibration signal

As already shown, vibration-based assessment of E modulus is supported by the precision of the frequency estimation method. Unfortunately, the vibration response to impulsive excitation is rapidly damped and offers, in standard frequency evaluation is performed, a spectrum with rare spectral bins [21]. This happens because the frequency resolution \( \Delta f \) depends on the acquisition or analysis time \( t_s \) in accordance to the relation \( \Delta f = 1/t_s \). Therefore, the estimated frequency \( f_e \) can differ from the true frequency \( f_\ell \) by an amount \( \varepsilon \), this error being limited to \( \varepsilon = \Delta f/2 \).
We propose, in this section, an algorithm to improve the frequency readability by increasing the number of the spectral bins, without being necessary to lengthen the acquisition time. This is made by superposing more spectra, each of them found from the original signal cropped with different time lengths. The original signal has the time length \( t_s \), the number of samples \( N_s \) and the constant sampling time \( \Delta t \), nominated also as time resolution. In this example, the individual spectra were obtained by calculating the Discrete Fourier Transform (DFT), but the algorithm also works well if the spectra are accomplished from the Power Spectrum (PS) or Power Spectral Density (PSD).

Due to leakage, the amplitude value indicated at any other position than the actual frequency \( f_E \) is less than the true amplitude value. Finding the real frequency-amplitude pair consists in shortening the original signal to which a DFT is afterward applied, until the amplitude achieves a clear maximum. This process is repeated for any targeted vibration mode. Applying the algorithm involves performing the following steps:

Step 1. Establish the coarse value of the targeted frequency, for example from a standard DFT or from numerical simulation, resulting the evaluated frequency \( f_E \):

Step 2. Find the coarse signal period \( T_E \) from the relation:

\[
T_E = \frac{1}{f_E};
\]  

(2)

Step 3. Establish the time length \( t_{S-\text{prim}} \) for the original signal. It must exceed a number of \( n \) complete cycles, each having the length of period \( T_E \). A good option is to choose:

\[
t_{S-\text{prim}} = (n + 0.45) \cdot T_E \quad \text{for} \quad 4 < n < 6 ;
\]  

(3)

Step 4. Find the length of the last signal used in the analysis from the relation:

\[
t_{S-\text{fin}} = (n - 0.45) \cdot T_E ;
\]  

(4)

Step 5. Calculate the number of samples that are contained in the original respectively last analyzed signal from the relations:

\[
N_{S-\text{prim}} = \frac{t_{S-\text{prim}}}{\Delta t} \quad \text{respectively} \quad N_{S-\text{fin}} = \frac{t_{S-\text{fin}}}{\Delta t} ;
\]  

(5)

Step 6. Calculate with the following relation the total number of samples to be reduced:

\[
N_{S-\text{red}} = N_{S-\text{prim}} - N_{S-\text{fin}} ;
\]  

(6)

Step 7. Establish the number of iterations \( k \);

Step 8. Calculate the number of samples to be reduced by iteration, from the relation:

\[
N_{S-\text{it}} = \frac{N_{S-\text{red}}}{k} ;
\]  

(7)

Step 9. Round down the result of relation (7) to attain \( N_{S-\text{it}} \) as an integer;

Step 10. Perform a DFT for the original signal and the \( k \) signals attained after iterative cropping with \( N_{S-\text{it}} \) samples.

Step 11. Superpose the \( k+1 \) spectra to achieve an overlapped spectrum;

Step 12. Extract the frequency that corresponds to the biggest amplitude in the bandwidth of interest from the overlapped spectrum. This frequency is very close or identical with the true frequency component in the analyzed signal.

An overlapped spectrum for a large \( N_{S-\text{it}} \) is presented in figure 1. Reducing the number of samples by iteration, i.e. small \( N_{S-\text{it}} \) results in a better frequency estimation. A detailed image of the top amplitudes for denser spectral lines in the overlapped spectrum is presented in figure 2. The total number of samples \( N_S \) does not influence the frequency resolution in one spectrum, but if large enough, it allows a fine tuning of the iteration process. The fines resolution in the overlapped spectrum is achieved if one sample is reduced by iteration, i.e. \( N_{S-\text{it}} = 1 \).
Figure 1. Overlapped spectrum with large number of samples extracted by iteration.

Figure 2. Top of an overlapped spectrum with few samples extracted by iteration.

Note that this procedure permits estimating the frequency for a selected vibration mode. If more frequencies are targeted, following steps 1 to 12 must be repeated for all selected modes independently.

4. Experimental setup and results

For the experimental tests, a stand developed in the “Eftimie Murgu” University of Resita was used. The stand allows an accurate measurement of the natural frequencies for beam-like structures. The test stand includes a rigid frame, on which a vise is mounted, allowing simulating cantilever beam conditions. The beam position is chosen to have the weak axis in horizontal plane and to avoid curving under the own weight. So, no transversal forces act in out-of-plane vibration. In figure 3, in order to highlight all the elements of the stand, the beam was drawn rotated by 90°.

For measuring the natural frequencies of the beam, the stand was equipped with a data acquisition and transmission system, which comprises a Kistler 8772A10 accelerometer, a NI9234 acquisition module and a ENET-9163 chassis. The accelerometer was fixed close to the beam end by wax bonding. The effect of the mass of the accelerometer can be eliminated by considering a concentrated mass placed at a known distance from the fixed end of the beam [15]. In this paper we didn’t reveal...
this aspect, the focus being set on the accuracy of the frequency determination. Due to the oscillations of the beam, the accelerometer generates electrical impulses proportional to the sensed acceleration. For reading and storing the natural frequencies, a computer connected to the data transmission system was used. The exact frequency extraction was done using a special developed application in LabView software. The free segment of the tested bar has the dimensions: \( L = 1000 \text{ mm}, \quad B = 50 \text{ mm}, \quad H = 5 \text{ mm} \) and the weight \( m = 1.9625 \text{ kg} \).

Figure 4. Response of a cantilever beam excited with acoustic pressure at 71Hz for one second.

Figure 5. Detail of the portion extracted from the free vibration response of the bar.

Excitation of the beam was made by two alternative methods: with an impulse hammer respectively by acoustic waves with harmonic frequency for a short duration (1 second). The acoustic waves were transmitted by a frequency generator, then amplified and emitted by a loudspeaker. The acoustic (non-contact) excitation has the advantage that energy can be transmitted to the beam in a controlled manner only in the frequency domain of interest, avoiding vibrations in other modes. Figure 4 exemplifies the beam response to an excitation produced by an acoustic wave with a frequency of 71 Hz for one second. As it can be observed, at the end of the excitation period, a significant amplitude increase of the response signal was noticed, while after stopping the excitation signal, the beam is damped vibrating, the amplitude of the response signal decreasing continually. Figure 5 shows a detail for a very short time (0.1 seconds) extracted from vibration response of the beam.

Figure 6. Spectrum of signal extracted from the free response of the acoustically excited beam.

Figure 7. Spectrum of signal extracted from the free response of the impulsively excited beam.

Figures 6 and 7 illustrate comparatively the spectrum of the signals extracted from the free response of the beam excited with a concentrated energy, respectively by impulse. One can observe that, in the case of acoustic excitation, only the frequency corresponding to the analyzed mode is identified (figure 6). Therefore, excitation with low amplitude is sufficient and so nonlinearities are avoided. In the case of impulse excitation, several vibration modes can be observed (figure 7) and an important signal-to-noise ratio is reported.
Tables 2 and 3 present the measured frequencies, achieved by the two methods, for the first six out-of-plane vibration modes. For each vibration mode, the average was calculated for 5 measurements.

**Table 2.** Natural frequencies measured by impulse excitation.

| Measurement no. | $f_1$ [Hz] | $f_2$ [Hz] | $f_3$ [Hz] | $f_4$ [Hz] | $f_5$ [Hz] | $f_6$ [Hz] |
|----------------|------------|------------|------------|------------|------------|------------|
| 1              | 3.9995     | 24.9249    | 70.6922    | 138.0000   | 232.3350   | 341.5270   |
| 2              | 4.1099     | 24.6944    | 70.6964    | 138.3170   | 232.8470   | 341.8800   |
| 3              | 4.1099     | 24.8192    | 70.6469    | 138.0000   | 232.1430   | 341.5680   |
| 4              | 4.1199     | 24.6945    | 70.5982    | 137.9490   | 232.5880   | 341.9890   |
| 5              | 4.0960     | 24.9972    | 70.6469    | 138.0000   | 232.3370   | 341.5680   |
| Average        | 4.0871     | 24.8260    | 70.6561    | 138.0530   | 232.4500   | 341.7060   |

**Table 3.** Natural frequencies measured by acoustic excitation.

| Measurement no. | $f_1$ [Hz] | $f_2$ [Hz] | $f_3$ [Hz] | $f_4$ [Hz] | $f_5$ [Hz] | $f_6$ [Hz] |
|----------------|------------|------------|------------|------------|------------|------------|
| 1              | 4.0598     | 26.0621    | 72.0168    | 141.2410   | 234.0830   | 348.7590   |
| 2              | 4.0598     | 26.0531    | 72.0168    | 141.2410   | 234.0820   | 348.7590   |
| 3              | 4.0623     | 26.0729    | 72.0185    | 141.2410   | 234.0820   | 348.7590   |
| 4              | 4.0623     | 26.0919    | 72.0185    | 141.2410   | 234.0530   | 348.7590   |
| 5              | 4.0623     | 26.0984    | 72.0168    | 141.2410   | 234.0530   | 348.7590   |
| Average        | 4.0613     | 26.0757    | 72.0175    | 141.2410   | 234.0644   | 348.7590   |

From the tables above, one can see that the frequencies measurements have a better repeatability in the case of acoustic excitation. This may be explained by the fact that the acoustic excitation of the beam is done in the area of the natural frequency which will be measured. Furthermore, the differences between the two methods are in average at about 2%, the biggest difference (about 5%) being experienced at the frequencies of vibration mode no. 2.

**Table 4.** Determination of Young’s modulus from dynamic measurements

| Excitation method | Young’s modulus $E$ [GPa] | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ |
|-------------------|---------------------------|-------|-------|-------|-------|-------|-------|
| Impulse           | 201.0                     | 188.8 | 195.1 | 194.0 | 201.2 | 194.9 |
| Acoustic          | 198.5                     | 208.3 | 202.7 | 203.0 | 204.0 | 203.0 |

Based on the average frequencies experimentally determined by the two methods and the beam parameters, Young’s modulus $E$ was calculated using relation (1). The results are shown in table 4, the differences to the known value of Young’s modulus for structural steel, $E= 200$ GPa, being small.

5. Conclusion
We propose in this paper a method to identify Young’s modulus by involving the natural frequencies of the out-of-plane vibration modes, supported by a non-contact excitation method and an advanced frequency estimation algorithm, developed to this aim. The results achieved by involving these techniques ensure better repeatability as the case of using impulsive excitation, due to low signal-to-noise ratio and avoidance of nonlinearities. Also, it was found that higher modes provide more stable results if acoustic excitation is involved. As expected, more dispersed results for the impulsive excitation tests were observed. This qualify the acoustic excitation and the signal post-processing algorithm, introduced in this paper, to be used for experimental determination of Young’s modulus, with the remark that higher modes provide more stable results.
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