Globally Convergent Coderivative-Based Generalized Newton Methods in Nonsmooth Optimization

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This talk proposes two globally convergent Newton-type methods to solve unconstrained and constrained problems of nonsmooth optimization by using tools of variational analysis and generalized differentiation. Both methods are coderivative-based and employ generalized Hessians (coderivatives of subgradient mappings) associated with objective functions, which are either of class $C^{1,1}$, or are represented in the form of convex composite optimization, where one of the terms may be extended-real-valued. The proposed globally convergent algorithms are of two types. The first one extends the damped Newton method and requires positive-definiteness of the generalized Hessians for its well-posedness and efficient performance, while the other algorithm is of the regularized Newton type being well-defined when the generalized Hessians are merely positive-semidefinite. The obtained convergence rates for both methods are at least linear, but becomes superlinear under the semismooth∗ property of subgradient mappings. Problems of convex composite optimization are investigated with and without the strong convexity assumption on smooth parts of objective functions by implementing the machinery of forward-backward envelopes. Numerical experiments are conducted for Lasso problems and for box constrained quadratic programs with providing performance comparisons of the new algorithms and some other first-order and second-order methods that are highly recognized in nonsmooth optimization.