Generalized correlations for predicting heat transfer and friction factor of turbulent flow in tubes with inner helical ribs

I A Popov¹, A N Skrypnik¹ and A V Schelchkov¹,²

¹Kazan National Research Technical University named after A. N. Tupolev – KAI, Russia
²All-Russian Research Institute of Flow Metering, Kazan, Republic of Tatarstan, Russian Federation

E-mail: popov-igor-alex@yandex.ru

Abstract. Generalized correlations for predicting heat transfer and pressure drop for turbulent flow of Newtonian fluid through tubes with inner helical ribs are presented. A comparison with existing correlations in the literature shows that revealed correlations allow decreasing the data deviation and cover the wide ranges of working fluids, operational conditions, and different dimensions of helical ribs.

1. Introduction

Pipes with inner helical ribs could be used for heat transfer intensification purpose for the single-phase and evaporating flow as well. Therefore, helically-ribbed pipes are widely used in boilers, air conditioning systems, chemical industry, and food-processing. The literature indicates that pipes with the helical angle of ribs \( \theta = 18-30^\circ \) are widely used in industry. However, research studies show greater efficiency for the pipes with the higher helical angle of ribs \( \theta = 70-86^\circ \).

There are various methods of producing the helically-ribbed pipe surfaces: by wire inserts, surface corrugation, machining on a lathe etc. (fig. 1).

![Figure 1. Various helically-ribbed pipe geometries (a – f) and characteristic dimensions (g):
    a – multistart corrugation; b – inner thread; c – tape insert, d – wire insert,
    e – corrugation, f – extrusion.](image)
For the practical application, helically-ribbed pipe geometrical parameters must be justified in accordance with the demands.

2. Development of correlations for pressure drop and heat transfer for the single-phase flow in pipes with inner helical micro-ribs

The report considers results on heat transfer and friction of the single-phase turbulent flow in pipes with inner helical micro-ribs. In order to obtain a database of friction and heat transfer coefficients, numerous investigations were examined: E. Sams (1956), D. Wilkie (1966), N. Sheriff (1966), P. Kumar (1970), R. Webb (1971, 1972, 1980, 2000), H. Yoshitomi (1976), R. Gupta (1979), T. Carnavos (1979, 1980), J. Withers (1980), V. Migay (1980, 1981, 1989), H. M. Li (1982), Raja Rao (1982, 1983, 1985), W. Nakayama (1983), A. Bergles and T. Ravigururajan (1983, 1986, 1996), J. Chiou (1987), V. Zimparov (1991), Y. Nazmeev (1993), M. Jensen (1999), D. Yang (2001), S. Rainieri (2002), A. Garcia (2004, 2005, 2007, 2012, 2018), G. Efimov (2006) (30), G. Zdaniuk (2008), N.-H. Kim (2018) and authors [4] [5].

The greatest scope of the experimental data on heat transfer and friction factor for the flow in tubes with inner helical ribs was used in T. Ravigururajan, A. Bergles [1]. The database included 1650 and 1800 points for the friction factor and Nusselt number respectively for a wide range of tube parameters: e/d=0.01…0.2; p/d=0.1...7.0; α/90=0.3...1, and flow parameters: Re=5000...250000; Pr=0.66...37.6. However, the prediction of the experimental data for the friction factor values was found to be 50% and 69% for the heat transfer with the confidence level of P=95% [1].

In the view of high demands on the reliable prediction of thermal and hydraulic properties of heat exchange equipment (in the transport and energy sectors) mentioned experimental data deviation is not satisfactory.

In that regard, further research on the heat transfer and friction factor experimental data variation for the flow in helically-ribbed pipes were carried out. Based on the results obtained and analysis of the flow structure the experimental data were categorized by the flow parameters and ribs geometry.

Heat transfer and friction factor data were correlated by the assumed model:

\[
\frac{\Delta P}{[(L / D) \cdot ( \rho \cdot \omega^2 )]} \text{Nu} = f(Re_D, \frac{e}{D}, \frac{p}{D}, \frac{p}{e}, N, \frac{\theta}{90})
\]

(1)

The choice of factors included in the model was based on the similarity theory, dimensional analysis, and literature review. The reviewed parameters and their physical meaning are explained in Table 1.

| Parameter | Description | Parameter meaning |
|-----------|-------------|-------------------|
| Re_D      | Reynolds number | The ratio of inertial and viscous forces. Characterizes a flow regime. |
| p/e       | Relative rib spacing | Characterizes a separation and reattachment of the flow after the ribs. |
| e/D       | Relative rib height | The exposure of the rib height on the flow structure. The influence is subject to the boundary layer height (or roughness height e+). |
| p/D       | Axial pitch | Characterizes a flow separation and reattachment zone. |
| N         | Number of starts | Characterizes an integral roughness of the heat transfer surface. It unites geometrical parameters – p and \( \theta \). |
| \( \theta/90 \) | Normalized helix angle | Characterize the flow rotation degree at the near wall layers in helically-ribbed pipes. The helix angle value is normalized by the maximum angle of the helix (\( \theta=90^\circ \)). |

The analysis of the reciprocal influence between the factors mentioned above revealed the usage of the full range overdetermination of the flow over the helically-ribbed pipe surface. Thus, through an increase in the number of helical starts N for the constant axial pitch p the helix angle \( \theta \) is changing.
Experimental results were obtained in the wide range of the helix angle values ($\theta=14-87^\circ$). Thus, the number of starts $N$ varied in the range $N=1-54$ for the fixed $\theta$ value.

According to the analysis, geometric simplices which include $e$, $p$ and $\theta$ were chosen. However, the axial pitch $p$ could be included in two different dimensionless simplices: $p/e$ and $p/D$. Therefore, $p/e$ simplex was excluded since the rib height $e$ is considered by the factor $e/D$. In this regard, the generalization model (1) transforms to:

$$\xi, Nu = f(R_e, e/D, p/D, \theta/90)$$

(2)

The model has been justified by the accuracy of the obtained correlations (described below).

The correlation equations were developed by polynomial regression method by means of the least square method. The database for the friction factor and heat transfer coefficient contains more than 2200 points.

One of the key parameters for the data characterization is the relative distance between the ribs $p/e$. As it is explained in table 1, the parameter $p/e$ characterizes the flow separation for the flow over the ribs as shown in Webb (1971) [2].

As it is shown in Webb, for the pipes with repeated rib roughness ($\theta=90^\circ$) the flow structure could be categorized by the flow patterns downstream the rib as a function of the relative rib spacing $p/e$. Edwards and Sheriff [1961] have shown that the heat transfer coefficient reaches the peak at the reattachment point downstream the rib at the relative distance between the rib equal $p/e\approx10$. Thus, for the rib spacing $p/e\leq10$ most of the rib spacing distance is filled by the recirculation zone and therefore lower values of the heat transfer coefficients are obtained. With an increase in the relative rib spacing $p/e>10$ after the reattachment point downstream the rib the boundary layer growth begins (if the rib height is comparable to the boundary layer thickness).

In the scope of this work it had been assumed that such patterns downstream the transverse ribs ($\theta=90^\circ$) are valid for the helical ribs ($\theta<90^\circ$). This assumption is supported by the analysis of the flow structure carried out in Bergles (1994), Garcia (2007).

Taking this into account, as the threshold value of $p/e$ at which the flow pattern change occurs was chosen $p/e=10$. Accordingly, the database for the friction factor and heat transfer coefficient was divided into two groups: with $p/e<10$ (predominance of the recirculation zone) and $p/e>10$ (with the declining impact of the rib on the flow structure). However, it must be noted that the reattachment point varies $p/e=6-15$ according to the mutual influence of geometrical and flow parameters.

Further, the experimental data were categorized by the value of the helix angle $\theta$ or by the normalized helix angle $\theta/90$, relatively to the maximum angle of the helix ($\theta=90^\circ$-transverse ribs). This parameter is a characteristic of the flow rotation in helically-ribbed pipes near the wall region. The helix angle increase leads to a decrease in the flow rotation intensity. For the value of the helix angle of $\theta>50^\circ$ the flow separation dominates over the flow rotation (Olimpiev 1992, Brodov 1987). The flow structure is similar to the flow over transverse ribs. Thus, the greatest contribution to heat transfer augmentation is made by flow separation.

A similar finding was observed in the work of Zimparov and Vulchanov [1990]. Results show that flow rotation prevails for the helix angle values of $\theta<45^\circ$. For the helix angle values over 60 degrees the flow regime and heat transfer processes are governed by the cross-flow. In the range of $45^\circ<\theta<60^\circ$ the flow rotation and cross-flow with the flow separation make comparable contributions.

Consequently, according to the database analysis and the flow patterns the threshold value was found to be: for $p/e<10$, $\theta=50^\circ$ and for $p/e>10$, $\theta=45^\circ$.

The friction factor $\xi$ (7) and Nusselt number $Nu$ (8) correlations for the turbulent flow of single phase Newtonian flow obtained by the authors are listed below:

$$\xi = A \cdot Re_\theta^\gamma \cdot (e/D)^m \cdot (p/D)^\delta \cdot (\theta/90)^i$$

(3)
date, however, there are no robust methods for accounting the rib shape influence on the flow structure.

where correlation with the confidence interval of characteristics was stated in the studies of Hijikata [1987], Dreizer [1990], Tarasevich et al. [2015]. To had not been taken into consideration in the scope of this work. The rib shape impact on flow view, might be associated with the fact that the rib shape influence on the friction factor augmentation for correlations obtained in A. Bergles and T. Ravigururajan [1]. The remained deviations, in the authors

\[ \text{Nu} = B \cdot \text{Re}_D^n \cdot (e/D)^m \cdot (p/D)^k \cdot (\theta/90)^l \cdot \Pr^{0.4} \]  

(4)

where \( A, B, n, m, k, l \) are coefficients of the model; \( P \) is the deviation of the experimental data from the correlation with the confidence interval of \( P=0.95 \), and \( R^2 \) is the coefficient of determination.

Equation (5) statistics:

- for \( p/e \leq 10 \) and \( \theta \leq 50^\circ \): the database contains 341 experimental points in the following range \( \text{Re}_D=9.2 \times 10^3-8.5 \times 10^5; p/e=2.27-9.88; p/D=0.059-0.81; e/D=0.02-0.36; \theta=25-50^\circ; \theta_90=0.277-0.533; N=1-18; \)
- for \( p/e \leq 10 \) and \( \theta > 50^\circ \): the database contains 381 experimental points in the following range \( \text{Re}_D=2.7 \times 10^3-2.6 \times 10^5; p/e=1.95-10; p/D=0.156-2.17; e/D=0.02-0.236; \theta=50-90^\circ; \theta_90=0.53-1.0; N=1-54; \)
- for \( p/e > 10 \) and \( \theta < 45^\circ \): the database contains 459 experimental points in the following range \( \text{Re}_D=3.2 \times 10^3-2.5 \times 10^5; p/e=13.11-167.72; p/D=0.01-0.2; e/D=0.01-0.2; \theta=24-46^\circ; \theta_90=0.27-0.511; N=1-54; \)
- for \( p/e > 10 \) and \( \theta > 45^\circ \): the database contains 937 experimental points in the following range \( \text{Re}_D=3.1 \times 10^3-4.9 \times 10^5; p/e=10.59-76.61; p/D=0.143-3.11; e/D=0.01-0.22; \theta=45-90^\circ; \theta_90=0.5-1.0; N=1-54. \)

Equation (6) statistics:

- for \( p/e \leq 10 \) and \( \theta \leq 50^\circ \): the database contains 110 experimental points in the following range \( \text{Re}_D=1.2 \times 10^3-7.6 \times 10^5; p/e=0.7-8; p/e=0.35-9.71; p/D=0.059-0.389; e/D=0.02-0.105; \theta=29-48^\circ; \theta_90=0.33-0.53; N=1-18; \)
- for \( p/e \leq 10 \) and \( \theta > 50^\circ \): the database contains 434 experimental points in the following range \( \text{Re}_D=3.9 \times 10^3-4 \times 10^5; p/e=0.1-13.0; p/D=0.1-2.17; e/D=0.01-0.298; \theta=55.8-90^\circ; \theta_90=0.62-1.0; N=1-54; \)
- for \( p/e > 10 \) and \( \theta < 45^\circ \): the database contains 407 experimental points in the following range \( \text{Re}_D=3.1 \times 10^3-4.4 \times 10^5; p/e=13.85-167.72; p/D=0.54-17.81; e/D=0.01-0.218; \theta=24-46^\circ; \theta_90=0.11-0.51; N=1-54; \)
- for \( p/e > 10 \) and \( \theta > 45^\circ \): the database contains 1280 experimental points in the following range \( \text{Re}_D=3.1 \times 10^3-4.2 \times 10^5; p/e=10.72-81.71; p/D=0.14-4; e/D=0.01-0.21; \theta=46-90^\circ; \theta_90=0.51-1.0; N=1-54. \)

Consequently, the experimental data deviation from correlations (5) and (6) is considerably less than for correlations obtained in A. Bergles and T. Ravigururajan [1]. The remained deviations, in the authors view, might be associated with the fact that the rib shape influence on the friction factor augmentation had not been taken into consideration in the scope of this work. The rib shape impact on flow characteristics was stated in the studies of Hijikata [1987], Dreizer [1990], Tarasevich et al. [2015]. To date, however, there are no robust methods for accounting the rib shape influence on the flow structure.
The other key factor affecting the accuracy of the correlations is the presence of the data related to the flow in pipes with the wire coil inserts. The wire coil inserts is a frequently used heat transfer augmentation technique. However, at low axial pitch values, the contact between the wall surface and the wire may be interrupted. This leads to the friction value change due to the flow underneath the wire inserts. Flow structure downstream the rib changes what leads to deviation of the friction factor and Nusselt number at the related pipe geometry in the database.

Conclusion
General correlations for friction factor and Nusselt number for the flow in helical-ribbed pipes are presented. Reported correlations are useful in the heat exchanger design in a wide rib geometry range.

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