Impact of Void Ratio and State Parameters on the Small Strain Shear Modulus of Unsaturated Soils

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ABSTRACT

The unsaturated small strain shear modulus, $G_{\text{max}}$, is a key reference value in predicting relationships between dynamic shear modulus and shear strain amplitude and is thus a key quantity to properly model the behavior of dynamically-loaded geotechnical systems such as pavements, rail beds, and machine foundations. From the interpretation of the experimental $G_{\text{max}}$ results for unsaturated soils, different definitions of trends between $G_{\text{max}}$ and the stress state of the unsaturated soils and material properties are proposed. However, in most of trends, the relationship between the stress state and void ratio is considered and the effect of void ratio on the unsaturated small strain shear modulus is not fully investigated. In the study presented herein, $G_{\text{max}}$ data published in the technical literature for two different types of unsaturated soils are critically reviewed with the goal of identifying trends with path-dependent stress state and void ratio. The literature data is also used to evaluate the reliability of an existing approach in predicting the small strains shear modulus of unsaturated soils under different loading conditions.

Keywords: Small strain shear modulus, unsaturated soil, void ratio, mean effective stress, double hardening, hydraulic hysteresis

1. INTRODUCTION

The progression of stress waves through the soil with time in the case of earthquake ground shaking or machine foundations can be predicted using solutions to the wave equations (Kramer 1996), and the corresponding strains in the soil can be estimated using constitutive modelling. In either case, the analysis depends on some representative material properties, which are the dynamic properties of soils (e.g. shear modulus and damping ratio) and the Poisson’s ratio. The dynamic properties of soils have been studied theoretically and experimentally for several years under both saturated (Hardin and Black 1968, 1969; Hardin and Drnevich 1972; Hardin 1978; Iwasaki et al. 1978; Stokoe et al. 1999) and unsaturated conditions (Cabarkapa et al. 1999; Mancuso et al. 2002; Inci et al. 2003; Marinho et al. 1995; Vassallo et al. 2007; Sawangsuriya et al. 2009; Khosravi and McCartney 2009; Ng et al. 2009; Khosravi et al. 2010; Khosravi and McCartney 2011; Khosravi and McCartney 2012). Of particular interest has been to understand the shear modulus of unsaturated soils at shear strain amplitudes less than $10^{-6}$ (elastic range of strain) which is defined as the small strain shear modulus, $G_{\text{max}}$. Based on the results presented in these studies, $G_{\text{max}}$ of unsaturated soils is dependent on different variables, such as state of stress, void ratio, soil grain characteristics (shape, size, mineralogy), and degree of saturation $S_r$. Based on the definition of stress state considered for analysis and from the interpretation of $G_{\text{max}}$ results, two general forms of predictive relationships for $G_{\text{max}}$ have been used in literature. In one form of equations (Inci et al. 2003; Sawangsuriya et al. 2009; Khosravi and McCartney 2009; Khosravi et al. 2010), the single-value mean effective stress definition proposed by Bishop (1959) for unsaturated soils is incorporated into the expression of $G_{\text{max}}$ proposed by Hardin and Black (1968) and a relationship for the small strain shear modulus of unsaturated soils was developed as follows:

$$G_{\text{max}} = A f(e) p^n$$  \hspace{1cm} (1)

where $f(e)$ is the void ratio function, $A$ and $n$ are fitting parameters that can be defined by fitting Eq. (1) to a set of $G_{\text{max}}$ and $p'$ is the unsaturated mean effective stress defined as:

$$p' = p_n + \chi \psi$$  \hspace{1cm} (2)

where $p_n$ is the mean net confining stress defined as the
difference between total mean stress and pore air pressure \((p_n = p - u_a)\), and \(\psi\) is the suction. For low suction magnitudes (less than 300 kPa), \(\psi\) is equal to the matric suction, which is the difference between the pore air pressure and pore water pressure \((\psi = u_a - u_w)\). For higher suction magnitudes, the total suction should be considered in this equation to incorporate the effects of osmotic suction. The term \(\chi\) is the effective stress parameter, which ranges from 0 (for dry soils) to 1 (for saturated soils) and has been defined using different approaches proposed in literature (Khalili and Khabbaz 1998; Wheeler et al. 2003; Sivakumar 1993; Gallipoli et al. 2003; Lu et al. 2010).

Another form of equations used the concept of independent stress state variables to define the value of \(G_{\text{max}}\) along the drying and wetting paths of the SWRC in by considering the effects of the mean net stress and suction independently (Mancuso et al. 2002; Mendoza et al. 2005; Oh and Vanapalli 2009; Ng et al. 2009; Sawangsuriya et al. 2009), as follows:

\[
G_{\text{max}} = Af(e) p_n^n + B f(e) \psi
\]  
(3)

where \(A\) and \(B\) are the model parameters describing the rate of change of \(G_{\text{max}}\) with respect to the mean net stress and matric suction, respectively, \(n\) is a material dependent fitting parameter and \(f(e)\) is a void ratio function. When fitting Eqs. (1) and (3) to experimental \(G_{\text{max}}\) data, the relationship for \(f(e)\) is typically incorporated empirically by considering that \(e\) and \(p'\) are uncoupled or defined by Hardin and Black (1969) and Hardin (1978) for saturated soils, as shown in Fig. (1).

\[
G_{\text{max}} = AP \left[ \frac{P_n}{P_s} \exp \left( \frac{\Delta e}{\lambda - \kappa} \right) \right]^\lambda \left[ \frac{P_n}{P_s} \exp \left( \kappa \left( S_{ao} - S_e \right) \right) \right]^\kappa \left( \frac{p'}{P_{a}} \right)^n
\]  
(4)

where \(P_s\) is the atmospheric pressure, \(A\) and \(n\) are stress dependency parameters, \(p_{c'}\) is the mean apparent preconsolidation stress (i.e., the mean yield stress), \(p'\) is the mean effective stress, \(p_n\) is the net stress, \(\Delta e\) is a plastic change in void ratio, \(\lambda\) and \(\kappa\) are the slopes of the virgin compression and the elastic rebound curves, respectively, \(K'\) and \(K\) are hardening constant, \(p_{c'}^0\) is the initial mean apparent preconsolidation stress, \(b\) is referred to as the double-hardening parameter which governs the rate of change in \(p_{c'}\) caused by changes in soil saturation, \(S_e\) is the effective saturation which is defined as:

\[
S_e = \frac{S_r - S_{r,\text{res}}}{1 - S_{r,\text{res}}}
\]  
(5)

and \(S_{r,\text{res}}\) is the initial effective saturation. In Eq. (5), \(S_r\) and \(S_{r,\text{res}}\) are the values of \(S_r\) at current and residual saturation conditions.

Khosravi and McCartney (2012) validated their model against experimental data under different values of mean net stress and matric suction, and the model was found to fit well with the experimental data. However, the specimens mostly stayed on the elastic unloading-reloading curve of \(e-p'\) throughout the tests so the effect of void ratio on the measured SWRC and \(G_{\text{max}}\) relationship was not fully investigated. In the study presented herein, the contribution of void ratio to the small strain shear modulus at unsaturated state is further investigated by re-interpreting experimental results of two soils reported in the literature. The literature data is also used to examine the validity of Eq. (4) for a wider spectrum of soil types and effective stress.

3. EXPERIMENTAL RESULTS

In this study, two soils including quartz silt
(Cabarkapa et al. 1999), and clayey silt (Ng et al. 2008; Ng et al. 2009) were identified from the literature for which data on the SWRC, void ratio and Gmax were available. The SWRC measurements of corresponding soils are presented in Fig. (2) and some of their characteristics are summarized in Table 1.

Table 1. Soil Properties

| Experimental data | Soil type   | Gs (%) | wI (%) | wp (%) | D50 (mm) | Cu | Cc |
|-------------------|-------------|--------|--------|--------|---------|----|----|
| Cabarkapa et al. 1999 | quartz silt | 2.67   | 31     | 0      | 0.02    | -  | -  |
| Ng et al. (2008, 2009) | clayey silt | 2.73   | 43     | 14     | 4.55    | 0.61 | -  |

Fig. 2. The SWRC measurements of soils analyzed in this study

Figs. (3) and (4) describe the effect of void ratio and various state parameters on the small strain shear modulus of different unsaturated soils. The state parameters are the matric suction and mean effective stress, p'. The mean effective stress employed in this study was defined using an approach similar to that used by Lu et al. (2010), Khosravi and McCartney (2012), and Haeri et al. (2014) as follows:

\[ p' = p_n + S_e \times \psi \]

(6)

This equation is similar to Bishop’s (1959) single-value effective stress variable where the effective stress parameter \( \chi \) is equal to \( S_e \). The value of \( S_e \) is obtained from Eq. (5) and using the SWRCs for each soil presented in Fig. 2. The normalized void ratio in Figs. (3) and (4) is defined as \( e/e_0 \), where \( e_0 \) is the void ratio corresponding to a mean net stress of 25 kPa for quartz silt and 110 kPa for clayey silt and the mean effective stress was defined using.

Fig. 3. Variation in (a) void ratio and (b) Gmax with \( p' \) for quartz silt (Cabarkapa et al. 1999)

Evaluation of the isotropic compression curves in Figs. (2a) and (3a) indicate that all of the soil specimens exhibit a nonlinear decrease in volume with increasing mean effective stress, as expected. However, the void ratio measurements of the soil specimens subjected to higher levels of suction were consistently higher for the same values of effective stress than those in low suction testing. This observation indicates a hardening response in the specimens as a result of suction increase. This hardening response leads to an increase in Gmax increase during drying.

Fig. 4. Variation in (a) void ratio and (b) Gmax with \( p' \) for clayey silt (Ng et al. 2008)
During the loading process, the $G_{\text{max}}$ measurements follow an increasing path with $p'$ increase. However, the rate of changes in $G_{\text{max}}$ varied depending on the magnitude of the applied mean effective stress and suction. The rate of changes in $G_{\text{max}}$ was lower at $p'$ less than the mean apparent preconsolidation stress where the specimens experienced smaller volume change during loading. However, after the mean effective stress exceeded the mean apparent preconsolidation stress during loading, $G_{\text{max}}$ increased at a greater rate with increasing $p'$. Therefore, it may be concluded that both $e$-log($p'$) and $G_{\text{max}}$-log($p'$) curves are almost composed of two linear sections with the intersection near the mean apparent preconsolidation stress. During unloading, the small strain shear modulus followed a decreasing path with $p'$ decrease. However, the rate of changes in $G_{\text{max}}$ during unloading was different from that during loading and a greater shear modulus was measured along the unloading path. It was also noted that the value of $G_{\text{max}}$ was not fully recovered once the initial applied effective stress was reached.

In this study, the effect of hydraulic hysteresis on $G_{\text{max}}$ of unsaturated soils was also investigated using the results of bender element tests which were conducted by Ng et al. (2008) at different stress state conditions (Fig. 5). The SWRCs of the tested specimens are shown in Figure 5(a) and the variations of $G_{\text{max}}$ with suction are presented in Figure 5(b).

As observed in Figure 5, and also as noted by other researchers (Ng et al. 2009; Khosravi and McCartney 2012), $G_{\text{max}}$ follows an increasing path with increasing matric suction. However, the rate of changes in $G_{\text{max}}$ for the different soils was lower at suctions below the air entry value. During wetting, $G_{\text{max}}$ decreased as matric suction decreased, with the greatest reduction between the water-entry value and the air-expulsion value, where the soil started to absorb greater amount of water.

4. ANALYSIS

The measurements of unsaturated small strain shear modulus reported in the literature were also used to assess the validity of the proposed relationship by Khosravi and McCartney (2012). Experimental $G_{\text{max}}$ data is shown in Fig. (6) in the effective stress space along with associated predictive relationships for $G_{\text{max}}$ following different paths of hydro-mechanical loadings. The model parameters required to predict the variation of $G_{\text{max}}$ were obtained following the methodology proposed by Khosravi and McCartney (2012).

The value of $K$ for each soil was defined using guidance from Hardin (1978), and the fitting parameters, $A$ and $n$, were determined from fitting a curve to $G_{\text{max}}$ data at zero matric suction (saturation condition) under different mean net stresses. The hardening parameters $K'$ and $b$ were determined from the results of $G_{\text{max}}$ tests along the drying path of the SWRC at a constant $p_n$ using least squares minimization. Table 2 summarizes the model parameters for different soils which were used in this study. The data in this figure indicate that the model shows a good fit with the data for the particular fitting values presented in Table 2. Evaluation of the results in Fig. 6 indicated that there are still some discrepancies between the data and the model and additional tests under different stress state conditions are recommended to further evaluate the reliability of the proposed approach.

Table 2. Model parameters required to solve the evolution of $G_{\text{max}}$ for different soils

| Experimental data | K   | K'  | A   | n   | b  |
|-------------------|-----|-----|-----|-----|----|
| Cabarkapa et al.  | 0.252 | 0.13 | 0.314 | 0.96 | 2.3 |
| (1999)            |     |     |     |     |    |
| Ng et al.         | 0.138 | 0.743 | 0.0879 | 0.72 | 1.37 |
| (2008, 2009)      |     |     |     |     |    |

5. CONCLUSION

$G_{\text{max}}$ is an important parameter to properly model the behavior of geotechnical systems under dynamic loading. This study aimed to improve our understanding of the trend between $G_{\text{max}}$ of unsaturated soils with void ratio and state parameters using the data from literature. The results presented in this study reflected the relative impacts of $e$, $S_r$, and $p'$ on $G_{\text{max}}$. Similar to $e$ vs. log($p'$)
The small strain shear modulus data of unsaturated soils obtained from the literature was also used to assess the reliability of a semi empirical approach in predicting $G_{\text{max}}$ of unsaturated soils during hydro-mechanical loading. The model was observed to provide adequate prediction of the $G_{\text{max}}$ data upon different stress paths.

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