POYNTING JETS FROM ACCRECTION DISKS

R.V.E. Lovelace 1, H. Li 2, A.V. Koldoba 3, G.V. Ustyugova 4, M.M. Romanova 1

Abstract

We give further considerations on the problem of the evolution of a coronal, force-free magnetic field which threads a differentially rotating, conducting Keplerian disk, extending the work of Li et al. (2001). This situation is described by the force-free Grad-Shafranov (GS) equation for the flux function Ψ(r, z) which labels the poloidal field lines (in cylindrical coordinates). The GS equation involves a function $H(Ψ)$ describing the distribution of poloidal current which is determined by the differential rotation or twist of the disk which increases linearly with time. We numerically solve the GS equation in a sequence of volumes of increasing size corresponding to the expansion of the outer perfectly conducting boundaries at $(R_m, Z_m)$. The outer boundaries model the influence of an external non-magnetized plasma. The sequence of GS solutions provides a model for the dynamical evolution of the magnetic field in response to (1) the increasing twist of the disk and (2) the pressure of external plasma. We find solutions with magnetically collimated Poynting jets where there is a continuous outflow of energy, angular momentum, and toroidal magnetic flux from the disk into the external space. This behavior contradicts the commonly accepted “theorem” of Solar plasma physics that the motion of the footpoints of a magnetic loop structure leads to a stationary magnetic field configuration with zero power, angular momentum, and flux outflows.

In addition we discuss magnetohydrodynamic (MHD) simulations which show quasi-stationary collimated Poynting jets similar to our Grad-Shafranov solutions. In contrast with the Grad-Shafranov solutions, the simulations show a steady uncollimated hydromagnetic (non-force-free) outflow from the outer part of the disk. The Poynting jets are of interest for the understanding of the jets from active galactic nuclei, microquasars, and possibly gamma ray burst sources.

1. INTRODUCTION

Highly-collimated, oppositely directed jets are observed in active galaxies and quasars (see for example Bridle & Eilek 1984), and in old compact stars in binaries (Mirble & Rodriguez 1994; Eikenberry et al. 1998). Further, well-collimated emission line jets are seen in young stellar objects (Mundt 1985; Bührke, Mundt, & Ray 1988). Recent work favors models where twisting of an ordered magnetic field threading an accretion disk acts to magnetically accelerate the jets (Meier, Koide, & Uchida 2001). There are two regimes: (1) the hydromagnetic regime, where energy and angular momentum are carried by both the electromagnetic field and the kinetic flux of matter, which is relevant to the jets from young stellar objects; and (2) the Poynting flux regime, where energy and angular from the disk are carried predominantly by the electromagnetic field, which is relevant to extra-galactic and microquasar jets, and possibly to gamma ray burst sources.

Stationary Poynting flux dominated jets have been found in axisymmetric MHD simulations of the opening of magnetic loops threading a Keplerian disk (Romanova et al. 1998; Ustyugova et al. 2000). Theoretical studies have developed models for Poynting outflows from accretion disks (Lovelace, Wang, & Sulkaneen 1987; Colgate & Li 1998). The present work represents a continuation of the study of Li et al. (2001) where self-consistent, axisymmetric, non-relativistic solutions of the Grad-Shafranov equation are calculated inside given conducting boundaries. The solutions are self-consistent in the respect that the twist of each field line is that due to the differential rotation of a Keplerian disk.

In §2 we summarize the theory, and in §3 we discuss the different types of solutions found. In §4 we develop an analytic model for Poynting jets. In §5, we discuss the consequences of expanding the boundaries. Sections 6 - 9 discuss the relevant conservation laws. In §10 we compare the Poynting outflows with centrifugally launched winds. In §11 we discuss the collapse of the inner part of the disk due to angular momentum outflow to Poynting jets. In §12 we discuss conditions for occurrence of Poynting jets including the influence of the kink instability. In §13, we give results of MHD simulations which give Poynting jets similar to our Grad-Shafranov solutions. Section 14 gives conclusions from this work.

2. THEORY OF POYNTING OUTFLOWS

Here, we consider further the theory of Poynting outflows (Li et al. 2001). We assume that magnetic field loops threads a differentially rotating highly conducting Keplerian accretion disk at some initial time $t = 0$. Above the disk we assume a “coronal” or force-free magnetic field in the non-relativistic limit. This situation is described by the force-free Grad-Shafranov (GS) equation for the flux function $Ψ(r, z)$ which labels the poloidal field lines (in

1Department of Astronomy, Cornell University, Ithaca, NY 14853-6801; RVL1@cornell.edu; Romanova@astrosun.tn.cornell.edu
2Theoretical Astrophysics, T-6, MS B288, Los Alamos National Laboratory, Los Alamos, NM 87545; HLI@lanl.gov
3Institute of Mathematical Modelling, Russian Academy of Sciences, Moscow, Russia, 125047; Koldoba@spp.keldysh.ru
4Keldysh Institute of Applied Mathematics, Russian Academy of Sciences, Moscow, Russia, 125047, Ustyug@apk.keldysh.ru
cylindrical coordinates). The GS equation involves a function $H(\Psi)$ describing the distribution of poloidal current which is determined by the differential rotation or twist of the disk which increases linearly with time. We numerically solve the GS equation in a sequence of volumes of increasing size corresponding to the expansion of the outer perfectly conducting boundaries at $(R_m, Z_m)$. The outer boundaries model the influence of an external non-magnetized plasma. The sequence of GS solutions provides a model for the dynamical evolution of the magnetic field in response to (1) the increasing twist of the disk and (2) the pressure of external plasma.

Cylindrical $(r, \phi, z)$ coordinates are used and axisymmetry is assumed. Thus the magnetic field has the form $B = B_r \hat{r} + B_\phi \hat{\phi}$, with $B_r = B_\phi = B_\phi(r, \phi)$. We can write $B_r = -(1/r)(\partial \Psi / \partial z), B_\phi = (1/r)(\partial \Psi / \partial \phi)$, where $\Psi(r, z) \equiv r A_\phi(r, z)$. In the force-free limit, the magnetic energy density $B^2/(8\pi)$ is much larger than the kinetic or thermal energy densities; that is, the flow speeds are sub-Alfvénic, $v^2 < v_A^2 = B^2/(4\pi \rho)$, where $v_A$ is the Alfvén velocity. In this limit, $0 \approx \nabla \cdot B$; therefore, $J = \lambda B$ (Gold & Hoyle 1960). Because $\nabla \cdot J = 0, (\nabla \cdot \nabla)\lambda = 0$ and consequently $\lambda = \lambda(\Psi)$, as well-known. Thus, Ampère’s equation becomes $\nabla \times B = 4\pi \lambda(\Psi)B/c$. The $r$ and $z$ components of Ampère’s equation imply

$$r B_\phi = H(\Psi), \quad dH(\Psi)/d\Psi = 4\pi \lambda(\Psi)/c,$$

where $H(\Psi)$ is another function of $\Psi$. Thus, $H(\Psi)$ = const are lines of constant poloidal current density; $J_p = (c/4\pi)(dH(\Psi)/B_\phi)$ so that $(J_p \cdot \nabla)H = 0$. The toroidal component of Ampère’s equation gives

$$\Delta^\ast \Psi = -H(\Psi) \frac{dH(\Psi)}{d\Psi}, \quad (1)$$

with $\Delta^\ast \equiv \partial^2/\partial r^2 - (1/r)(\partial / \partial r) + \partial^2/\partial z^2$, which is the Grad-Shafranov equation for $\Psi$ (see e.g., Lovelace et al. 1987; Li et al. 2001).

Ampère’s law gives $\oint \mathbf{A} \cdot d\mathbf{l} = (4\pi/c) \oint d\mathbf{S} \cdot \mathbf{J}$, so that $r B_\phi(r, z) = H(\Psi)$ is (2/c) times the current flowing through a circular area of radius $r$ (with normal $\hat{z}$) labeled by $\Psi(r, z) = \text{const}$. Equivalently, $-H(\Psi(r, 0))$ is (2/c) times the current flowing into the area of the disk $\leq r$. For all cases studied here, $-H(\Psi)$ has a maximum so that the total current flowing into the disk for $r \leq r_m$ is $I_{B, \text{tot}} = (2/c)(-H)_{\text{max}}$, where $r_m$ is such that $-H(\Psi[r_m, 0]) = (-H)_{\text{max}}$ so that $r_m$ is less than the radius of the $O$-point, $r_0$. The same total current $I_{B, \text{tot}}$ flows out of the region of the disk $r = r_m$ to $r_0$.

We consider an initial value problem where the disk at $t = 0$ is threaded by a dipole-like poloidal magnetic field. The form of $H(\Psi)$ is then determined by the differential rotation of the disk: The azimuthal twist of a given field line going from an inner footpoint at $r_1$ to an outer footpoint at $r_2$ is fixed by the differential rotation of the disk. The field line slippage speed through the disk due to the disk’s finite magnetic diffusivity is estimated to be negligible compared with the Keplerian velocity. For a given field line we have $rd\phi/B_p = ds_p/B_p$, where $ds_p = \sqrt{dr^2 + dz^2}$ is the poloidal arc length along the field line, and $B_p = \sqrt{B_r^2 + B_z^2}$. The total twist of a field line loop is

$$\Delta \phi(\Psi) = \int_1^2 ds_p \frac{B_p}{r B_r} = -H(\Psi) \int_1^2 \frac{ds_p}{r^2 B_r} \quad (2)$$

with the sign included to give $\Delta \phi > 0$. For a Keplerian disk around an object of mass $M$ the angular rotation is $\omega_K = \sqrt{GM/r^3}$ so that the field line twist after a time $t$ is

$$\Delta \phi(\Psi) = \omega_0 t \left[ \left( \frac{r_0}{r_1} \right)^{3/2} - \left( \frac{r_0}{r_2} \right)^{3/2} \right] = (\omega_0 t) F(\Psi/\Psi_0) \quad (3)$$

where $r_0$ is the radius of the $O$-point, $\omega_0 = \sqrt{GM/r_0^3}$, and $F$ is a dimensionless function (the quantity in the square brackets). The $O$-point is the point in the midplane of the disk encircled by the poloidal magnetic field lines; at this point $B_p = 0$. At sufficiently small $r_1$ one reaches the inner radius of the disk $r_1 \ll r_0$ where we assume $\omega_K$ saturates at the value $\omega_{K1} = \sqrt{GM/r_1^3}$. For the dipole-like field of Figure 1, $F \approx 3^{5/8}/(\Psi_0/\Psi)^{5/4}$ for $\Psi/\Psi_0 \ll 1$, while $F \approx 3.64/(1 - \Psi/\Psi_0)^{5/2}$ for $1 - \Psi/\Psi_0 \ll 1$.

An accretion disk around a massive black hole $M = M_\odot 10^{5-6} M_\odot$ in the nucleus of galaxy, the twist is $T = (t/3.17d)(r_0/10^{13} cm)^{3/2}/\sqrt{M_\odot}$. The inner radius of the disk is $r_1 \approx M_\odot 9 \times 10^{13} cm$ for a Schwarzschild black hole.

3. NUMERICAL SOLUTIONS OF GRAD-SHAFRANOV EQUATION

We solve equation (1) inside a cylindrical “box,” $r = 0$ to $R_m$ and $z = 0$ to $Z_m$ with $H(\Psi)$ determined by iteratively solving equations (1) - (3) (see Li et al. 2001; Finn & Chen 1990). The outer boundaries $(r = R_m$ and $z = Z_m)$ are treated as conducting surfaces representing the interface between the coronal B field and external, non-magnetized plasma. The external plasma will be pushed outward by the coronal field as it is twisted. Thus we consider the B field behavior as $R_m \rightarrow \infty$ and $Z_m \rightarrow \infty$.

For a dipole-like field threading the disk (Figure 1), it is natural to measure lengths in terms of the radius of the $O$-point in the disk, $r_0$. The natural value of the flux function $\Psi$ is its maximum value at the $O$-point, $\Psi_0$. In turn, a natural unit for the magnetic field strength is $\Psi_0/\sqrt{\mu_0}$. Notice that for the case of Figure 1, $\Psi_0/\sqrt{\mu_0} \approx B_z(0)/10.4$. 

Fig. 1.—“Initial” dipole-like vacuum magnetic field. In this and subsequent plots, $r$ and $z$ are measured in units of the radius $r_0$ of the $O$-point in the disk plane. The solid lines are the magnetic field lines for the case where the flux function on the disk surface is $\Psi(r, z = 0) = a r^2 K/(a^2 + r^2)^{3/2}$ with $a = r_0/\sqrt{2}$. In this and subsequent plots $\Psi$ is measured in units of $\Psi_0 = r_0^2 K/3^{3/2}$. Note that $B_z(0) = 6\sqrt{2} \Psi_0/r_0^2 = 10.4 \Psi_0/r_0^2$. The dashed lines are the field lines for the case where the outer boundaries $(R_m = 10, Z_m = 10)$ are perfectly conducting; for this case $\Psi(r, z = 0) \rightarrow \Psi(r, 0)[1 - (r/r_0 - 1)^2/81]$ so that the $O$-point is still at $r_0$ and $\Psi_0$ is unchanged. Because of axisymmetry and reflection symmetry about the $z = 0$ plane, the field need be shown in only one quadrant.
Further, a natural unit for the total current (times c/2), \( c\omega t/2 = (-H)_{\text{max}} \) is \( \Psi_0/r_0 \). Also, we measure the total toroidal flux and the magnetic energy,

\[
\Phi_t = \int d\Omega B_\psi , \quad W_m = \frac{1}{4} \int r d\Omega B^2 ,
\]

in units of \( \Psi_0 \) and \( \Psi_0^2/r_0 \), respectively. For boundary conditions, on the disk surface, \( \Psi(r, 0) \) is fixed and equal to its “initial” value on the disk owing to the disk’s perfect conductivity. On the \( z \)-axis we can take \( \Psi(0, z) = 0 \), because \( B_r = -(1/\rho)(\partial \Psi/\partial z) = 0 \). On the conducting outer boundaries, \( \Psi(r, Z_m) = 0 \) and \( \Psi(R_m, z) = 0 \). Note however that in §14 we discuss simulation results for the case of open outer boundaries. A uniform \((r, z)\) grid of \( 200 \times 200 \) was used.

Figure 2 shows the behavior of a set of field solutions of equation (1) as the twist \( T = \omega t \) is increased. The nature of the solutions changes dramatically as the twist increases above a critical value \( T_c \approx 1.14 \text{ rad} \). For \( T < T_c \), the nature of the field solutions is shown in Figures 3 and 4. The twisting of the field by the differential rotation of the disk “pumps” magnetic flux and energy into the disk corona and the field tends to “inflates.” This behavior of coronal magnetic field loops of the Sun as a result of footpoint twisting is well known from the works of Aly (1984, 1991) and Sturrock (1991). The self-similar inflation of a force-free field threading a non-Keplerian disk without outer boundaries was studied Lynden-Bell and Boily (1994) and their solution for small twists is analogous to our low-twist solutions. The expansion of a force-free field into finite pressure external plasma has been studied by Lynden-Bell (1996) and Li et al. (2001) and the poloidal field is found to fairly uniformly fill the coronal space.

For \( T > T_c \), a new field configuration appears with a different topology. This is shown in Figure 5 where it is seen that a “plasmoid” consisting of toroidal flux has detached from the disk and is separated by the dashed field line which has an \( X \)-point above the \( O \)-point on the disk. Figure 6 shows a 3D view of two representative field lines for the same case. These high-twist equilibria consist of a region near the axis which is \textit{magnetically collimated} by the toroidal \( B_\psi \) field and a region far from the axis, on the outer radial boundary, which is \textit{anti-collimated} in the sense that it is pushed against the outer boundary. The field lines returning to the disk at \( r > r_0 \) are anti-collimated by the pressure of the toroidal magnetic field. The \textit{poloidal field} fills only a small part of the coronal space. In a purely analytical analysis, Heyvaerts (2001) has independently found MHD equilibria involving the simultaneous formation of a collimated axial jet and an uncollimated outflow. Figure 6 shows a three dimensional view of sample field lines.

As a test of our numerical solutions note that conservation of axial momentum can be written as

\[
\nabla \cdot (T \cdot \hat{z}) = 0 , \quad T \cdot \hat{z} \equiv B^2/8\pi - B_r B_r/4\pi .
\]

Integration of equation (5) over the “box” \((R_m, Z_m)\) gives

\[
\int_0^{R_m} r dr (B_r^2 + B_\psi^2 - B_z^2)_{z=0} = \int_0^{R_m} r dr (B_z^2)_{z=Z_m} ,
\]

where the integral on the left hand side represents the flux of momentum from the disk into the box and the other integral the flux of momentum out of the top of the box. This equation is accurately satisfied by our numerical solutions; the typical errors are \(< 0.1\%\).

4. **ANALYTIC SOLUTION FOR POYNTING JET**

Most of the twist \( \Delta \phi \) of a field line of a Poynting jet occurs along the jet from \( z = 0 \) to \( Z_m \). Because \(-r^2 d\phi/H(\Psi) = d\psi/B_z\), we have

\[
\Delta \phi(\Psi) = \frac{\omega_0 t}{-H(\Psi)} \approx \frac{Z_m}{r^2 B_z} ,
\]

where \( r^2 B_z(r, z) \) is evaluated on the straight part of the jet at \( r = r(\Psi) \). In the core of the jet \( \Psi \ll \Psi_0 \), we have \( F \approx 3^9/8(\Psi_0/\Psi)^3/4 \), and in this region we can take \( \Psi_0 = C_{1/2}(r/r_0)^2 \), and \( H = -K(\Psi_0/\rho_0) (\Psi(\Psi_0)^s \), where \( C, q, K \), and \( s \) are dimensionless constants. Equation (1) for the straight part of the jet implies \( q = 1/(1-s) \) and \( C_s(1-s) = s(1-s)^2 K^2/(1-2s) \). Thus we find \( s = 1/4 \) so that \( q = 4/3 \), \( C = [9/32]^{2/3} K^{4/3} \), and \( K = 3^{4/3}(4/3) (\rho_0/\rho)(\Psi/\Psi_0) \).

In order to have a Poynting jet, we find that \( K \) must be larger than \( \approx 0.5 \). This corresponds directly to the condition \( T > T_c \) for the occurrence of the high-twist solutions. The condition arises from the fact that there is a competition between the build up of toroidal flux inside the box due to twisting by the disk which acts to increase \( B_\phi \) and the expansion of the boundaries which acts to decrease \( B_\phi \) (see §12). If the boundaries expand too rapidly \( B_\phi \) does not increase sufficiently to give a self-collimated Poynting jet. For the case of uniform expansion of the top boundary, \( Z_m = V_t t \), this condition is the same as \( V_t < 0.2(r_0\omega_0) \). For the case of Figure (5), \( K \approx 0.844 \). The field components in the straight part of the jet are

\[
B_\phi = -\sqrt{2} B_z = -\sqrt{2} \left( \frac{3}{16} \right)^{1/3} K^{4/3} \left( \frac{\Psi_0}{\rho_0} \right) \left( \frac{r_0}{r} \right)^{2/3} .
\]

These dependences agree approximately with those found in numerical simulations of Poynting jets (Ustyugova et al. 2000). On the disk, \( \Psi \approx 3^{1/3} \Psi_0(r/r_0)^2 \) for \( r < r_0/3^{1/4} \). Using this and the formula for \( \Psi(\rho) \) gives the relation between the radius of a field line in the disk, denoted \( r_d \),...
and its radius in the jet, \( r/r_0 = 6.5(r_d/r_0)^{3/2} K^{-1} \). Thus the power law for \( \Psi \) is applicable for \( r_1 < r < r_2 \), where \( r_1 = 6.5r_0(r_1/r_0)^{3/2}/K \) and \( r_2 = 1.9r_0/K \), with \( r_1 \) the inner radius of the disk. The outer edge of the Poynting jet has a transition layer where the axial field changes from \( B(z_2)/r_2 \) to zero while (minus) the toroidal field increases from \(-B_0/r_2\) to \((-H)_{max}/r_2\). Using equation (6), which is only approximate at \( r_2 \), gives \((-H)_{max} \approx 1.2K\Psi_0/r_0\). This dependence agrees approximately with our Grad-Shafranov solutions.

5. EXPANSION OF BOUNDARIES

The magnetic forces on the outer wall increase by a large factor in going from the low-twist to high-twist solutions. The radial force on the cylindrical wall and the axial force on the top wall are

\[
F_r = \frac{1}{4} R_m \int_0^{Z_m} d\tau B_z^2, \quad F_z = \frac{1}{4} \int_0^{R_m} r dr B_z^2.
\]

For the low-twist solution of Figure 3 \( (T = 1.1) \), \( (F_r, F_z) \approx (0.0061, 0.013) \), whereas for the high-twist solution of Figure 5 \( (T = 1.79) \), \( (F_r, F_z) \approx (0.26, 0.45) \) in units of \( (\Psi_0/r_0)^2 \).

Figure 7 shows the radial dependence of the magnetic pressure on the top boundary for the cases of low-twist and high-twist solutions.

Different behavior is exhibited by the low-twist and high-twist solutions as the conducting boundaries are moved outward. For the low-twist solutions, the poloidal field tends to expand outward to fairly uniformly fill the available space. As \( R_m \rightarrow \infty \) and \( Z_m \rightarrow \infty \), we find that these solutions are similar to those obtained by Lynden-Bell and Boily (1994) where there are no outer boundaries.

In contrast, for the high-twist solutions, the poloidal field near the axis maintains its collimation due to \( B_0 \), whereas the “return” poloidal field far from the axis is pushed against the outer cylindrical wall due to the \( B_0 \) field. As \( R_m \rightarrow \infty \) and \( Z_m \rightarrow \infty \), the collimated field near the axis will extend outward along the \( z \)-axis in the absence of instabilities. It is clear from Figure 7 that the magnetic pressure on the top boundary peaks near the axis. Thus, this region of the boundary should expand most rapidly in the physical case where the boundary is an interface with external plasma.

Estimation of the axial expansion of a Poynting jet can readily be made assuming a region of radius \( g r_2 \) \( (g \sim 2 - 3) \) of the jet expands with velocity \( V_z \) into a constant density external medium. For likely conditions \( V_z \) is much larger than the sound speed in the external medium so that the ram pressure due to the jet motion is \( \rho_{ext}V_z^2 \), assuming \( (V_z/c)^2 \ll 1 \) as required by our non-relativistic treatment. Balancing this pressure with the magnetic pressure of the jet gives

\[
\rho_{ext}V_z^2 = \frac{(-H)_{max}^2}{4\pi g r_2^2} \approx 0.14[B_z(0)/g]^2(r_0\omega_0/V_z)^4,
\]

or

\[
V_z \sim (r_0\omega_0)^{2/3}[B_z(0)/g]^{1/3}/\rho_{ext}^{1/6}.
\]

The condition for a Poynting jet \( K > 0.5 \) corresponds to \( V_z < 0.2(r_0\omega_0) \). For \( B_z(0) = 100G, r_0 = 10^{15}\)cm, and \( M = 10^8M_0 \), the external density \( \rho_{ext} \) must be larger than \( 2 \times 10^{-22}g/cm^3 \) in order to have \( V_z < c \). For larger magnetic fields, a much larger external density is needed to give \( V_z < c \). This points up the need for relativistic treatment of the Poynting jet expansion.

6. ANGULAR MOMENTUM CONSERVATION

Conservation of angular momentum about the \( z \)-axis can be written as

\[
\nabla \cdot \bar{f} = 0, \quad \bar{f} = -rB_\phi B_\rho/4\pi = -\hat{B}_\phi H/4\pi,
\]

where \( \bar{f} \) is the angular momentum flux-density vector. Integration of equation (11) over the “box” \( (R_m, Z_m) \) gives

\[
0 = -\frac{1}{2} \int_0^{R_m} (1/r_2) H(B_z)z_0.
\]

The subscript \( z = 0 \) here and subsequently indicates that the quantity is evaluated on the top surface of the disk. For a dipole-like field threading the disk where \( \Psi(R_m, 0) = 0 = \Psi(0, 0) \), equation (12) gives

\[
0 = -\frac{1}{2} \int_0^{\Psi_0} d\Psi H(\Psi) - \frac{1}{2} \int_{\Psi_0}^0 d\Psi H(\Psi).
\]

The first integral represents the outflow of angular momentum from the inner part of the disk (interior to the \( O \)-point), and this equals the angular momentum inflow into the outer part of the disk given by the second integral.

The outflow of angular momentum from the inner part of the disk causes enhanced accretion of this part of the disk, whereas the inflow of angular momentum to the outer disk reduces the accretion rate. Because the Poynting outflows carry negligible matter, the continuity equation for the disk is

\[
\partial\Sigma/\partial t + \nabla \cdot (\Sigma v, \hat{r}) = 0,
\]
where \( \Sigma(r,t) \) is the surface mass density of the disk. The continuity equation for the disk angular momentum is

\[
\frac{\partial (\Sigma \ell)}{\partial t} + \nabla \cdot (\Sigma \ell r \hat{r} + \mathcal{T}^{vis}_{r\ell}) = r(\dot{B}_\phi B_z)_{z=0}/2\pi ,
\]

where \( \mathcal{T}^{vis}_{r\ell} \) represents the viscous transport of angular momentum in the disk, the term on the right-hand side the outflow or inflow of angular momentum (from the two sides of the disk) due to the magnetic field, and \( \ell \) is the specific angular momentum of the disk matter. The disk is assumed almost Keplerian so that \( \ell = \sqrt{GM_r r} = rv_K \) and consequently the two continuity equations give the mass accretion rate \( \dot{M} = \dot{M}_B + \dot{M}_{vis} \), where

\[
\dot{M}_B(r) = -2\pi r \Sigma v_r = -2\pi (r^2 / v_K) (\dot{B}_\phi B_z)_{z=0} ,
\]

is the “magnetically driven” mass accretion rate, and \( \dot{M}_{vis} = 6\pi \sqrt{\ell} d(\nu \sqrt{\ell} \Sigma) / dr \) is the viscous accretion rate with \( \nu \) the kinematic viscosity (Lovelace, Newman, & Romanova 1997). We have \( \dot{M}_B > 0 \) (or < 0) for \( r < r_0 \) (or \( r > r_0 \)). The accretion speed is \( u = -v_r = u_B + u_{vis} \) with \( u_B = \dot{M}_B / (2\pi r \Sigma) \) and \( u_{vis} = \dot{M}_{vis} / (2\pi r \Sigma) \).

Figure 5 shows the radial dependence of \( \dot{M}_B \) for a high-twist case. That \( \dot{M}_B \) due to the Poynting outflow is a function of \( r \) emphasizes the fact that disk is not stationary.

7. MAGNETIC FIELD TRANSPORT IN THE DISK

Poloidal magnetic field threading the disk tends to be advected inward with the accretion flow, but at the same time it may diffuse through the disk owing to a finite magnetic diffusivity \( \eta_m \) of the disk. The continuity equation for poloidal flux through the disk \( B_z(r,0,t) \) is

\[
\dot{B}_z/r + \nabla \cdot [v_r B_z \hat{r} + U_c B_z \hat{r}] = 0 ,
\]

where \( U_c = (\eta_m / h) \tan(\theta) \) is the outward drift speed, \( h \) the half-thickness of the disk, \( \tan(\theta) \equiv (B_r / B_z)_{z=0} \), and a smaller, second order diffusion term \( (\eta_m / h^2 B_z / \partial z^2) \) has been omitted (see Lovelace et al. 1997). For cases where the diffusivity is of the order of the viscosity and where the viscosity is given the Shakura and Sunyaev (1973) prescription \( \nu = \alpha c_s h \) (with \( \alpha < 1 \) and \( c_s \) the midplane sound speed), the diffusive drift speed is \( U_c \approx \alpha c_s \tan(\theta) \). For a strong magnetic field threading the disk, the accretion speed \( u \) in the inner part of the disk \( (r < r_0) \) may be large with \( u_B \gg u_{vis} \) and \( u_B < U_c \) so that the disk flow advects the \( B_z(r,0) \) field inward. On the other hand at large radii, the magnitude of \( u \) is probably much smaller so that the \( B_z(r,0) \) field tends to drift outward, \( U_c < u \).

8. ENERGY CONSERVATION

We assume the coronal plasma is perfectly conducting so that \( \mathbf{E} = -\mathbf{v} \times \mathbf{B} / c \). For quasi-stationary and axisymmetric conditions, \( \nabla \times \mathbf{E} = 0 \), and thus \( E_\phi = 0 \), and \( E_p = -\nabla \Phi \), so that \( \mathbf{v} \propto \pm \mathbf{B} = \hat{\mathbf{B}}_p \), and the electrostatic potential \( \Phi = \Phi(\Psi) \). Thus \( \mathbf{E} = -\Omega \nabla \Psi / c \) with \( \Omega(\Psi) = d \Phi(\Psi) / d\Psi \). For the situations considered here, all field lines pass through the disk. At the disk surface, \( E_r(r,0) = -(v_\phi)_{disk} B_z(r,0)/c \) since \( v_z = 0 \). Therefore, \( \Omega(\Psi(r,0)) = (v_\phi)_{disk}/r \).

For a force-free plasma we have

\[
\frac{\partial}{\partial t} \left( \frac{\mathbf{B}^2}{8\pi} \right) + \nabla \cdot \left( \frac{c^2}{4\pi} \mathbf{E} \times \mathbf{B} \right) = 0 .
\]

Integration of equation (18) over the “box” \( (r = 0 \to R_m, z = 0 \to Z_m) \) gives

\[
\frac{dW_m}{dt} = -\frac{1}{2} \int_0^{R_m} rdr \ (v_\phi B_\phi B_z)_{z=0} - \frac{1}{4} R_{max} R_m \int_0^{Z_m} dz \ B^2(R_m,z) - \frac{1}{4} Z_m \int_0^{R_m} rdr \ B^2(r,Z_m) ,
\]

where \( W_m \) is the magnetic field energy in the box (equation 4). The second and third integrals in equation (19) represent the energy expended in “pushing” the boundaries outward (the pdV work). The first integral represents the outflow of energy from the inner part of the disk surface (inside the O—point) and the inflow into the outer part of the disk. For a dipole-like field threading the disk where \( \Psi(R_m,0) = 0 = \Psi(0,0) \), the first integral for the power output from the disk can be rewritten as

\[
\frac{1}{2} \int_0^{\Psi_0} \ d\Psi \Delta \phi \ (\Psi) \ H(\Psi) = \frac{\omega_0 \Psi_0^2}{2r_0} \int_0^1 d\Psi \bar{F} [-\dot{H}(\Psi)] ,
\]

where \( \omega_0 \equiv \sqrt{GM / r_0^3} \), and the tildes indicate dimensionless variables. For the low-twist solution \( (T = 1.1) \) of
Fig. 8.— The top panel shows the radial dependence of the mass accretion rate of the disk \( \dot{M}_B \) due to the Poynting outflow from the disk, and \((0.1 \times)\) the Poynting power outflow per unit radius \( d\dot{E}_B/dr \) for quasi-stationary conditions. The lower panel shows the radial dependence of the angle between poloidal field lines and the \( z \)-axis at the disk surface, \( \theta = \tan^{-1}(|B_r/B_z|) \). Both panels are for the high-twist case of Figures 5-7. As mentioned, for \( \Psi > 0.95 \) or \( 0.764 < r < 1.33 \) the field lines are closed. In the interval \( 1 < r < 1.5 \), \( B_r/B_z \) is less than zero.

Figure 3, the dimensionless integral denoted \( \mathcal{I} \approx 1.75 \), whereas for the high-twist solution of Figure 5 \((T = 1.84)\) \( \mathcal{I} \approx 3.05 \). Thus, \( \dot{E} \approx 

\[
0.9 \times 10^{45} \text{erg s}^{-1} \left( \frac{\mathcal{I}}{3} \right) \left( \frac{B_z(0)}{3 \text{kG}} \right)^2 \left( \frac{r_0}{10^{15} \text{cm}} \right)^{3/2} \left( \frac{M}{10^8 M_\odot} \right)^{1/2}.
\]

(21)

for the power output from the two sides of the disk.

The Poynting power outflow per unit radius from the two sides of the disk is

\[
d\dot{E}_B/dr = -rv_K(B_\phi B_z)_{z=0} = (v^2_K/2r) \dot{M}_B,
\]

(22)

for quasi-stationary conditions. Figure 8 shows the radial dependence of \( d\dot{E}_B/dr \) which indicates that most of the power outflow occurs in the inner part of the disk.

9. GENERATION OF TOROIDAL FLUX

Perfect conductivity and Faraday’s law imply

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}),
\]

(23)

where \( \mathbf{v} \) is the plasma flow velocity. We apply this equation to a “box” extending from the disk at \( z = 0 \) to a non-rotating, perfectly conducting surface at \( z = Z_m \), and from the axis \( r = 0 \) to a cylindrical, non-rotating, perfectly conducting surface at \( r = R_m \). These outer surfaces are allowed to move so that \( R_m \) and \( Z_m \) are in general time-dependent. The toroidal flux \( \Phi_t \) (equation 4) obeys

\[
\frac{d\Phi_t}{dt} = \int_0^{R_m} dr (v_\phi B_z - v_z B_\phi) \bigg|_{Z_m}^{Z_m} - \int_0^{Z_m} dz (v_\phi B_z - v_z B_\phi) \bigg|_{R_m}^{R_m}.
\]

(24)

On the top boundary \( z = Z_m \), \( B_z = 0 \) and \( B_\phi = 0 \); on the outer cylindrical boundary \( r = R_m \), \( B_r = 0 \) and \( B_\phi = 0 \); on the \( z \)-axis \( B_\phi = 0 \) and \( B_r = 0 \); and on the disk \( v_z(r, z = 0) = 0 \). Consequently, equation (24) simplifies to

\[
\frac{d\Phi_t}{dt} = - \int_0^{R_m} dr v_\phi B_z(r, z = 0),
\]

(25)

where \( v_\phi = \sqrt{GM/r} \) is the azimuthal velocity of the disk. The integrand of equation (25) is independent of time in view of our assumption that \( \Psi(r, 0) \), and thus \( B_z(r, 0) = (1/r) \partial \Psi_t/(\partial r, 0) \), is time-independent.

For a dipole-like field threading the disk [where \( \Psi(r, 0) \) increases from zero to a maximum \( \Psi_0 \) and then decrease to zero at \( r = R_m \)], equation (25) can be rewritten as

\[
\frac{d\Phi_t}{dt} = - \int_0^{\Psi_0} d\Psi \Delta \dot{\phi} (\Psi),
\]

(26)

where \( \Delta \dot{\phi} (\Psi) \) is the time derivative of equation (3), which is independent of time. For the dipole-like field of Figure

1, evaluation of this integral gives \( d\Phi_t/dt \approx -12.1 \omega_0 \Psi_0 \approx -12.1 \omega_0 [B_z(0)/10^4] \). Thus, \( d\Phi_t/dt \approx -1.3 \times 10^{28} \text{Gcm}^2 \text{s}^{-1} \left( \frac{B_z(0)}{3 \text{kG}} \right) \left( \frac{r_0}{10^{15} \text{cm}} \right)^{3/2} \left( \frac{M}{10^8 M_\odot} \right)^{1/2}
\]

(27)

is the toroidal flux generation rate from one side of the disk.

10. POYNTING VERSUS CENTRIFUGAL OUTFLOWS

The centrifugal force near the surface of a Keplerian disk has a role in launching hydromagnetic outflows (Blandford & Payne 1982). The “centrifugal launching” requires that the field lines (projected into the poloidal plane) be tilted away from the \( z \)-axis by an angle \( \theta \) greater than \( 30^\circ \). The magnetic force, dominantly \( -\mathbf{B} \times \mathbf{v}_K/(8\pi)dz/dr \), is comparably important for launching hydromagnetic outflows (Lovelace, Berk, & Contopoulos 1991; Ustyugova et al. 1999); it also increases as \( \theta \) increases. Figure 8 shows the radial variation of \( \theta(r) = \tan^{-1}(|B_r/B_z|)_{z=0} \) for a high-twist case. For this case, \( \theta < 30^\circ \) for \( r > 0.3r_0 \), which includes the part of the disk giving the largest power output per unit radius, \( d\dot{E}_B/dr \), shown in the top panel of Figure 8. Thus there is a Poynting outflow under conditions where no centrifugal outflow occurs.

However, for the part of the disk where \( \theta > 30^\circ \) we predict a centrifugal hydromagnetic outflow with \( -(B_\phi B_z) > 0 \) and inward magnetically driven accretion, \( u_B > 0 \) (see Ustyugova et al. 1999). Hydromagnetic outflows from the region \( r > r_0 \) are found in the MHD simulations discussed by Ustyugova et al. (2000) and in §13.

11. COLLAPSE OF INNER DISK

Quasi-stationary Poynting jets from the two sides of the disk within \( r_0 \) give an energy outflow per unit radius of the disk \( d\dot{E}_B/dr = rv_K(B_\phi B_z) \), where \( h \) subscript indicates evaluation at the top surface of the disk. This outflow is \( \sim r_0 d\dot{E}_B/dr \sim v_K(r_0)(\Psi_0/r_0)^2 \) is estimated in equation (22), which agrees approximately with the values derived from the simulations (see §14).

For long time-scales, the Poynting jet is of course time-dependent due to the angular momentum it extracts from the inner disk \( (r < r_0) \). This loss of angular momentum leads to a “global magnetic instability” and collapse of the inner disk (Lovelace et al. 1997). An approximate model of this collapse can be made if the inner disk mass \( M_\Delta \) is concentrated near the \( O \)-point radius \( r_0(t) \), if the field line slippage through the disk is negligible (see §8), \( \Psi_0 const \), and if \( -(r \Phi_t)_{max} \sim \Psi_0/r_0(t) \) (as found here). Then, \( M_\Delta d r_0/dt = -2\Psi_0^2 (GM/r_0)^{1/2} \). If \( t_i \) denotes the time at which \( r_0(t_i) = r_i \) (the inner radius of the disk), then \( r_0(t) = r_i[1 - (t - t_i)/t_{coll}]^{2/3} \), for \( t \leq t_i \), where \( t_{coll} = \sqrt{GM M_\Delta^{3/2}/(3\Psi_0^2)} \) is the time-scale for the collapse of the inner disk. (Note that the time-scale for \( r_0 \) to
decrease by a factor of 2 is \( \sim t_i (r_0/r_1)^{3/2} \gg t_i \) for \( r_0 \gg r_1 \).

The power output to the Poynting jets is

\[ \dot{E}(t) = \frac{2}{3} \frac{\Delta E_{\text{tot}}}{t_{\text{coll}}} \left(1 - \frac{t - t_i}{t_{\text{coll}}} \right)^{-5/3} , \]

where \( \Delta E_{\text{tot}} = GM \dot{M} / 2 r_1 \) is the total energy of the outburst. Roughly, \( t_{\text{coll}} \sim 2 \text{ day} M_2^2 (M_d/M_\odot) (6 \times 10^{32} \text{Gcm}^2/\Phi_0)^2 \) for a Schwarzschild black hole, where validity of the analysis requires \( t_{\text{coll}} \gg t_i \). Such outbursts may explain the flares of active galactic nuclei blazar sources (Romanova & Lovelace 1997; Romanova 1999; Levinson 1980) and the one-time outbursts of gamma ray burst sources (Katz 1997).

12. OCCURRENCE OF POYNTING JETS AND KINK INSTABILITY

The rate at which toroidal flux \( \Phi_t \) is created in the region above the disk is \( \partial \Phi_t / \partial t \sim -12 \dot{m}_t \dot{\Phi}_0 \) (see §10). If the area \( R_m Z_m \) of this region is fixed owing to a very dense external plasma, the average value of \( -B_\phi > 0 \) in it increases. On the other hand if the area \( R_m Z_m \) increases rapidly enough, \( -B_\phi \) will decrease. The Poynting jets occur under conditions where \( -B_\phi \) increases to a sufficient extent to cause pinching of the poloidal magnetic field. Because the most rapid expansion of the \( B \) field of occurs in the \( z \)-direction, a necessary condition for occurrence of a Poynting jet is that the rate of expansion of the boundary \( V_z = dZ_m/dt \) be bounded by some constant. In fact the condition obtained in §4 for occurrence of Poynting jets has this form, \( V_z < 9.2(\omega_0 r_0) \).

Note that there may be “self-regulation” in the respect that the field configuration which occurs is at the “boundary” between low and high-twist solutions. In view of Figure 7, the low-twist field gives a gradual expansion of the boundaries which allows build up of toroidal flux whereas the high-twist field gives a more rapid expansion which tends to give a slower increase of \( \Phi_t \).

The region of collimated field (see Figure 5) - the Poynting jet - has \( v_z^2 \ll v_A^2 \) and is kink unstable according to the standard non-relativistic analysis (e.g., Bateman 1980, ch. 6). The instability will lead to a helical distortion of the jet with the non-linear amplitude of shift of the helix \( \Delta r \sim v_A t \) and with the helix having the same twist about the \( z \)-axis as the axisymmetric \( B \) field. Note however that for the astrophysical conditions of interest \( v_A = |\mathbf{B}|/\sqrt{4\pi \rho} \) is likely to be larger than the speed of light. A relativistic perturbation analysis is then required including the displacement current. The physical Alfvén speed is \( V_A = c/\sqrt{(1 + c^2/v_A^2)^{1/2}} \ll c \). Thus the speed of lateral displacement of the helix is less than \( c \). The evolution of the Poynting jet evidently depends on both \( V_A \) and the velocity of propagation of the “head” of the jet \( V_z \) (§5) which may be relativistic. Relativistic propagation of the jet’s head may act to limit the amplitude of helical kink distortion of the jet. On the other hand a sub-relativistic propagation of the head may allow the helix amplitude to grow but this amplitude can be limited by flux conservation as discussed by Kadomtsev (1963).

13. MHD SIMULATIONS OF POYNTING JETS

For the MHD simulations described here, the initial magnetic field has a dipole-like form as shown in Figure 1. The computational region \( r = 0 \) to \( R_{\text{max}} \), \( z = 0 \) to \( Z_{\text{max}} \) is taken to have \( R_{\text{max}} = Z_{\text{max}} \approx 10r_0 \). Initially, the corona of the disk is in isothermal equilibrium without rotation.

At \( t = 0 \) the disk starts to rotate with Keplerian velocity \( v_{\phi}(r, 0) = r \Omega_K \), where \( \Omega_K = \sqrt{GM/(r^2 + r_0^2)^{3/4}} \), where the smoothing length \( r_0 = 0.2r_0 \) is interpreted as the inner radius of the disk. The smoothed gravitational potential is \( -GM/(r^2 + r_0^2)^{1/2} \).

On the disk surface, the boundary conditions are as follows (ustyugova et al. 2000). Two of the boundary conditions come from the fact that the tangential electric field \( \mathbf{E}_t \) in the frame rotating with the disk (at the Keplerian velocity) is zero; \( B_z \) at the disk surface is time-independent whereas \( B_r \) and \( B_\phi \) at the surface vary with time. Two further boundary conditions fix the entropy of the plasma coming out of the disk to be \( s_d(r) \) and the density of the outflowing plasma to be \( \rho_d(r) \). If \( v_z \) at the disk surface, calculated by solving the MHD equations in the computational region, increases to the point where it is larger than the slow magnetosonic speed in the \( z \)-direction at the disk’s surface \( c_{\text{msz}} \), then we clamp it to be equal to \( c_{\text{msz}} \). This condition represents a limit on the mass efflux \( \dot{m}_e \) from the disk. For sub-magnetosonic outflow from the disk \( v_z < c_{\text{msz}} \), we have four boundary conditions, whereas when \( v_z = c_{\text{msz}} \) we have five boundary conditions.

For the outer boundaries, we first consider the case where these surfaces are perfect conductors. Secondly, we consider the case of “free” outer boundaries where \( \partial F_j/\partial n = 0 \) on all scalar variables except for the toroidal magnetic field. For this field component we take \( \mathbf{B}_p \cdot \nabla (r B_\phi) = 0 \) on the outer boundaries, which was shown by Ustyugova et al. (1999) to avoid artificial collimation which can come from using the “free” boundary condition on \( r B_\phi \). The free outer boundary conditions allow matter and Poynting flux to freely flow out through these surfaces.

For the cases we discuss, the strength of the poloidal magnetic field at the inner radius of the disk corresponds to \( (v_A / v_K)_i = 16.5 \) and \( (c_e / v_K)_i = 1 \), where \( v_A = |\mathbf{B}_p|/\sqrt{4\pi \rho} \). The \( i \)-subscript indicates evaluation at the inner radius of the disk \( r = r_1 \) on the disk surface. In the midplane of the disk \( (v_A / v_K)_z = 0 \) is less than or much less than unity. Different radial profiles of \( c_e \) on the disk surface have been used with similar results, including \( c_e / v_K = \text{const} \) and \( c_e = \text{const} \); the density profiles on the disk surface have been obtained as in Ustyugova et al. (1999).

Figure 9 shows the evolution of the coronal plasma for the case of fixed, conducting outer boundaries at \( (R_m, Z_m) \). After about 6 rotation periods of the inner disk, the outgoing poloidal field collimates along the \( z \)-axis, and the returning poloidal field is pressed outwards to the conducting walls. Most of the magnetic field is strongly field-dominated with flow speeds \( v^2 \ll v_A^2 \), where \( v_A \) is the Alfvén velocity. The field configuration is similar to that found for the high-twist Grad-Shafranov solutions as shown in Figure 5.

Figure 10 shows the long time evolution of the coronal plasma for the case where the outer boundaries are free (i.e., open) boundaries. These simulations evolve to a quasi-stationary final state where most of the region is strongly field-dominated. In the jet region along the \( z \)-axis, the poloidal field is collimated by the \( B_\phi \).
field. We find that the profiles of $B_\phi(r)$ and $B_z(r)$ from the simulations (Ustyugova et al. 2000) agree to a good approximation with equation (6) of our analytic model. The main respect in which the simulations differ from the Grad-Shafranov solutions is that region close to the disk where the magnetic field returns to the disk is not field-dominated. Instead this region has a hydromagnetic outflow from disk. This is predicted theoretically (Blandford & Payne 1982) and observed in simulations (Ustyugova et al. 1999) to be the case when the angle $\theta$ between the disk normal and the poloidal field lines is larger than 30°.

14. CONCLUSIONS

An ordered magnetic field threading an accretion disk can give powerful outflows or jets of matter, energy, and angular momentum. Most of the studies have been in the hydromagnetic regime where there is appreciable mass outflow and find asymptotic flow speeds of the order of the maximum Keplerian velocity of the disk, $v_K$. These flows are clearly relevant to the jets from protostellar systems which have flow speeds of the order of $v_K$. In contrast, observed VLBI jets in quasars and active galaxies point to bulk Lorentz factors $\Gamma \sim 10$ - much larger than the disk Lorentz factor. In the jets of gamma ray burst sources, $\Gamma \sim 100$. The large Lorentz factors as well as the small Faraday rotation measures suggest that these jets are in the Poynting flux regime. This work presents self-consistent solutions for the axisymmetric, non-relativistic plasma equilibria described by the force-free Grad-Shafranov equation.

We find solutions with magnetically collimated Poynting jets where there is a continuous outflow of energy, angular momentum, and toroidal magnetic flux from the disk into the external space. This behavior contradicts the commonly accepted “theorem” of Solar plasma physics that the motion of the footpoints of a magnetic loop structure leads to a stationary magnetic field configuration with zero power, angular momentum, and flux outflows (Aly 1984, 1991).

Important issues remain to be investigated - the relativistic expansion of the head of a Poynting jets into an external medium and the three-dimensional kink instability of the jet. Further, the magnetic extraction of energy from a rotating black hole may be important (Blandford & Znajek 1977; Livio, Ogilvie, & Pringle 1999).

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Fig. 10.— Time evolution of dipole-like field threading the disk from the initial configuration $t = 0$ (bottom panels) to the final quasi-stationary for the case of open outer boundaries at $r = R_m$ and $z = Z_m$. Here, $t_{out}$ is the rotation period of the disk at the outer radius $R_m$ of the simulation region; for the parameters used, $t_{out} \sim 200 t_i$. The initial field is shown by the dashed lines in Figure 1. The left-hand panels show the poloidal field lines which are the same as $\Psi(r, z) =$-const lines; $\Psi$ is normalized by $\Psi_0$ (the maximum value of $\Psi(r, z)$) and the spacing between lines is 0.1. The middle panels show the poloidal velocity vectors $v_p$. The right-hand panels show the constant lines of $-r B_\phi(r, z) > 0$ in units of $\Psi_0/r_0$ and the spacing between lines is 0.1. For this calculation a $100 \times 100$ inhomogeneous grid was used with $\Delta r_j$ and $\Delta z_k$ growing with distance $r$ and $z$ geometrically as $\Delta r_j = \Delta r_1 q^j$ and $\Delta z_k = \Delta z_1 q^k$, with $q = 1.03$ and $\Delta r_1 = \Delta z_1 = 0.05 r_0$ (Ustyugova et al. 2000).
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