Application of cohesive zone model to the fracture process of freshwater polycrystalline ice under flexural loading

I Gribanov, R Taylor and R Sarracino
Memorial University of Newfoundland, St.John’s, NL, Canada
E-mail: ig1453@mun.ca

Abstract. Cohesive zone modeling is a promising technique for modeling fracture processes in polycrystalline solids. The Park-Paulino-Roesler (PPR) formulation for cohesive zones is a flexible method that allows one to reproduce behaviours of a wide range of materials. In this work, an implicit finite element simulation is performed to capture the fracture of freshwater, polycrystalline ice beams in 4-point bending tests. The distribution of the damaged cohesive zones and the indentation force are recorded as a function of time. Several simulations are performed to establish the relationship between the grain size and the flexural strength of the beams.

1. Introduction
While ice is a beautiful natural material, it can pose a risk to ships and structures in the marine environment, as well as to people who may use winter ice cover as a transportation route. During the formation process, prior to establishing a level ice cover, sea ice may partially form, break up and refreeze multiple times. Its microstructural composition depends largely on its age and growth conditions. In general, sea ice contains inclusions of brine and air, often having a granular microstructure in the uppermost layer, which transitions to a columnar microstructure within a few centimeters below the top surface [1]. In dynamic ocean environments, deformation processes in the sea ice can result in more complex microstructures as the sea ice is repeatedly broken, reoriented and refrozen. Icebergs, on the other hand, calve off glacial ice masses formed from compacted snow and generally are nearly free of solid particles and soluble substances [2]. Iceberg ice typically has a polycrystalline structure that can be characterized by grain size. Over large volumes, significant variation in grain size may be expected, as well as the occurrence of pre-existing cracks which may remain in the ice or may heal over time, resulting in large veins of ice with a different microstructure in the healed crack regions [3].

In the vicinity of a large iceberg (Figure 1), one can occasionally hear cracking sounds – a sign of gradual fracture processes that can culminate in sudden catastrophic splitting. For interactions between ice and structures, fracture is highly important both in limiting loads and in localizing contact into zones of intense local pressures through which the majority of loads are transmitted to a structure [4]. Consequently, understanding and modelling fracture mechanisms in such ice features is highly important. While full-scale ice structure interactions are highly complex, insights into certain aspects of ice fracture behaviour can be studied through
the measurement of ice strength under controlled conditions. Such tests are performed and experimental data is made available for compressive indentation conditions [5, 6, 7, 8, 9, 10, 11], as well as uniaxial compression tests on cylinders [12, 13] and flexural tests on beams [3, 14]. In the present paper, emphasis is placed on modelling freshwater polycrystalline (e.g. glacial) ice subjected to flexural loading conditions.

1.1. Related work on modeling
Finite element approaches to modeling damage can be roughly classified into two groups. In the first group, the damage is treated as continuous and expressed as a function of material coordinates. The crack surfaces are not explicitly tracked, and the material is not fragmented [15]. In the second group, the grains, cracks and other individual constituents are explicitly resolved. Concepts that are specific to the material, such as wing cracks, can be introduced into latter models [16]. Such approaches usually require more computational resources, because the shapes of the individual constituents are recorded and processed.

Cohesive zone models (CZM), where individual grains in the material are held together by cohesive tractions (Figure 2), fall into the second category of methods. Several applications of this method to the mechanics of ice have been proposed [17, 18, 19].

![Figure 1](image1.png)

**Figure 1.** An iceberg sighting on May 28, 2017, near Topsail Beach, Conception Bay South, NL. Photo courtesy of http://mynewfoundlandkayakexperience.blogspot.ca.

![Figure 2](image2.png)

**Figure 2.** Polyhedral grains connected with cohesive zones (CZ). The rest volume of CZs can be zero or non-zero. Grains may be deformable or rigid.

2. Description of the method
To capture the dynamic behaviour of the modeled sample we utilize the implicit finite element method (FEM) with Newmark-beta integration. The discretized non-linear equation of motion is solved via Newton-Raphson iterations. The modeled geometry consists of separate grains which are connected by cohesive zones at the beginning of the simulation. The grains have linear elastic response, with the rotational component eliminated from the deformation gradient tensor to allow arbitrary rotations of the elements [20].

The traction-separation relations for the cohesive zones are as formulated by Park-Paulino-Roesler (PPR) [21]. All cohesive zones are triangular, with zero rest volumes. The normal separation is distinguished from the tangential separation; both normal and tangential forces
are found as the gradient of the common potential $\Psi$:

$$\Psi(\Delta_n, \Delta_t) = \min(\phi_n, \phi_t) +$$

$$\left[ \Gamma_n \left( 1 - \frac{\Delta_n}{\delta_n} \right)^{\alpha} \left( \frac{m}{\alpha} + \frac{\Delta_n}{\delta_n} \right)^{mn} + \langle \phi_n - \phi_t \rangle \right] \times$$

$$\left[ \Gamma_t \left( 1 - \frac{|\Delta_t|}{\delta_t} \right)^{\beta} \left( \frac{n}{\beta} + \frac{|\Delta_t|}{\delta_t} \right)^{nn} + \langle \phi_t - \phi_n \rangle \right],$$

(1)

where $\langle \cdot \rangle$ is the Macaulay bracket, i.e.

$$\langle x \rangle = \begin{cases} 0 & x < 0, \\ x & x \geq 0. \end{cases}$$

Separation parameters in the normal direction $\Delta_n$ and tangential direction $\Delta_t$ are independent, allowing material properties in these directions to be set separately. The parameters $(\phi_n, \phi_t; \delta_n, \delta_t; \alpha, \beta; \lambda_n, \lambda_t)$ determine the properties of the material, such as fracture energy, cohesive strength and brittleness. Energy constants $\Gamma_n$ and $\Gamma_t$ are defined by the expressions

$$\Gamma_n = (-\phi_n)^{\langle \phi_n - \phi_t \rangle / \langle \phi_t - \phi_n \rangle} \left( \frac{\alpha}{m} \right)^{mn}, \quad \Gamma_t = (-\phi_t)^{\langle \phi_t - \phi_n \rangle / \langle \phi_t - \phi_n \rangle} \left( \frac{\beta}{n} \right)^{nn}$$

when the normal and tangential fracture energies are not equal ($\phi_n \neq \phi_t$), and

$$\Gamma_n = -\phi_n \left( \frac{\alpha}{m} \right)^{mn}, \quad \Gamma_t = \left( \frac{\beta}{n} \right)^{nn}$$

when they are equal ($\phi_n = \phi_t$). Non-dimensional exponents $m$ and $n$ are related to initial slope indicators $\lambda_n$ and $\lambda_t$ by the following expressions

$$m = \frac{\alpha(\alpha - 1)\lambda_n^2}{(1 - \alpha\lambda_n^2)}, \quad n = \frac{\beta(\beta - 1)\lambda_t^2}{(1 - \beta\lambda_t^2)}.$$ 

A typical traction-separation law is illustrated on Figure 3.

Because of the exponential terms, the gradient of $\Psi(\Delta_n, \Delta_t)$ takes a substantial amount of time to compute, especially for a large number of CZs. For best performance, these functions are evaluated in parallel on a GPU. The matrix in the resulting linear system is indefinite symmetric and cannot be resolved by GPU-optimized algorithms, therefore a CPU-based linear solver is used. Additionally, unloading/reloading relations are implemented as described by Spring [22], as in the implicit method they perform better than those described by Park et al. [21].

To account for the interaction of fragmented material, the contact detection and response is implemented, which is based on the penetration distance. The implementation is distributed as open-source.

3. Results and discussion

The flexibility of the PPR traction-separation laws allows to tune the model to a wide range of materials, including ice. A set of parameters (Table 1) was selected to match the experimental data for uniaxial tensile and compressive tests [19]. Some of the parameters have physical meaning, i.e. fracture energy and cohesive strength. Other parameters, such as $\alpha$, $\beta$, $\lambda_n$ and $\lambda_t$, determine whether the fracture will be brittle or ductile.
Figure 3. Normal component of the traction-separation relations as a function of normal and tangential separations $\Delta_n, \Delta_t$. Tangential component may be similar or different, depending on the selected parameters.

Table 1. Parameters of the material.

| Parameter                                                       | Notation | Value       |
|-----------------------------------------------------------------|----------|-------------|
| Young’s modulus                                                 | $E$      | 10 GPa      |
| Poisson’s ratio                                                 | $\nu$    | 0.3         |
| Density                                                         | $\rho$   | 916.2 kg/m³ |
| Mode I and II fracture energies                                 | $\phi_n, \phi_t$ | 30 J m⁻²     |
| Normal cohesive strength                                         | $\sigma_{max}$ | 1.0 MPa     |
| Tangential cohesive strength                                    | $\tau_{max}$ | 1.5 MPa     |
| Shape of traction-separation curve                              | $\alpha, \beta$ | 3           |
| Non-dimensional slope indicators in PPR model                   | $\lambda_n, \lambda_t$ | 0.02        |
| Magnitude coefficient for potential-based collision response    | $k$      | 5000        |

The four-point bending simulations are performed on 28 different beam models. The geometries with random arrangements of grains were generated in Neper [23], and the dimensions are summarized on Figure 4. The beams measure $10 \times 10 \times 50$ cm and consist of 500, 750, 1000, 1250, 1500 and 2000 grains. Each grain is subdivided into tetrahedral elements, having 100 elements per grain on average. The indenters measure 5 cm in diameter, are rigid, and move with the speed of 0.23 cm/s. The initial time step is 0.025 s. All initial setups are available along with the open-source implementation of this project.

Nodal displacements and the forces acting on the indenters are recorded at each step. Simulations perform 300 steps, passing the point of beam failure. One of the tests is illustrated in Figures. 5-7, where the initiation and propagation of the crack is shown in a beam with 500 grains. The corresponding force-time curve is nonlinear (Figure 8), with the slope gradually increasing. Displacements that occur during the loading stage are small and not visible on the images. However, the strain energy is high and the accumulation of damage in the cohesive zones is visible. In this test, the force on the indenters reaches 5349 N at the moment of fracture.

After the fracture occurs, the newly formed fragments acquire substantial velocity due to

1 https://github.com/igorg520b/icFlow
Figure 4. Setup of the four-point beam bending. Dimensions are shown in centimeters.

Figure 5. Simulation step 193; $t = 4.2797$ s. Damaged CZs shown in green, fracture surface shown in red.

Figure 6. Simulation step 221; $t = 4.5106$ s.

Figure 7. Simulation step 256; $t = 4.5216$ s.

Figure 8. Indentation force vs time.

the release of elastic strain energy. Before they start moving in opposite directions, an impact occurs along the formed boundary near the indenters. In the current model, only intergranular fracture and elastic deformation are considered. In nature, however, ice would be expected to also experience local crushing at the contact points. The interplay between fracture and crushing during full-scale compressive ice-structure interactions is highly important for modelling loads of interest for engineering design, but is beyond the scope of the present paper. Accounting for such mechanisms to better model interactions involving progressive failure, which occurs during processes such as indentation, is the direction of the future work.

In each performed test the maximum force is determined before the beam fails. The corresponding flexural strength is computed as

$$
\sigma = \frac{3F(L - L_i)}{2bd^2},
$$
where $F$ is the load force at the fracture point, $L$ is the length of the support (outer) span between the indenters, $L_i$ is the length of the loading (inner) span, $b$ is the width of the beam and $d$ is the thickness of the beam. The results are summarized in Figure 9 as a plot of flexural strength vs. grain size. As may be observed, the strength of the blocks increases with decreasing grain size, as expected from Hall-Petch relationship. The flexural strengths obtained from these simulations are in accord with several experimental results [24, 25, 26], however experimental measurements of the flexural strength of ice vary significantly [27] and the influence of scale effects is highly important [28]. Such effects will be explored further in future work.

4. Conclusion
The complexities of ice-structure interactions and the inherent variability in ice strength, coupled with the need for risk-based approaches in engineering design, result in a continued strong emphasis on empirical approaches in many branches of ice mechanics. Nevertheless, numerical modelling is an important tool for studying certain aspects of ice behavior, and the importance of numerical modelling will undoubtedly increase in the future as newer methods which capture the unique processes that take place in ice continue to be developed. In particular, detailed accurate modelling of ice fracture would be highly valuable in assessing load limiting processes and potential strategies for improving design.

In nature, deformation, fracture and motion of ice are affected by numerous factors, making the simulation of full-scale processes extremely challenging. The work presented here, although employed for the specialized case of polycrystalline ice under flexural loading, provides an improved technique which can be extended to model the behavior of less homogeneous ice under more general conditions. Limitations of the approach at this stage of development include the absence of processes associated with crushing: specifically microcracking, dynamic recrystallization and local pressure melting. Neither does the model account for viscous deformation, intragranular fracture and variabilities in grain-grain cohesion. However, it does capture grain size dependence, accumulation of crack-related damage and well represents experimental force-time curves. It also captures discrete fracture events, fragmentation and allows for the integration of more complex models of ice continuum behaviour in future work. Being a new and flexible model, the PPR technique provides an improved approach for modeling ice using cohesive zones.
Acknowledgments
The authors gratefully acknowledge core funding for the Centre for Arctic Resource Development (CARD) program at C-CORE by Hibernia Management and Development Company, Ltd. (HMDC) and Terra Nova Development (Suncor Energy Inc. - Operator). Financial support from the Research and Development Corporation (RDC) of Newfoundland and the donation of Quadro M5000 GPU by NVIDIA are gratefully acknowledged. The authors also thank C-CORE for providing office space and creating a productive environment.

References
[1] Weeks W 2010 On Sea Ice (University of Alaska Press) ISBN 9781602231016
[2] Mellor M 1980 Physics and Mechanics of Ice ed Tryde P (Berlin, Heidelberg: Springer Berlin Heidelberg) pp 217–245 ISBN 978-3-642-81436-5 978-3-642-81434-1 dOI: 10.1007/978-3-642-81434-1_17
[3] Barrette P D and Jordaan I J 2002 Journal of Glaciology 48 587–591
[4] Taylor R and Jordaan I 2015 Engineering Fracture Mechanics 134 242–266 ISSN 00137944
[5] Browne T, Taylor R, Jordaan I and Gurtner A 2013 Cold Regions Science and Technology 88 2–9 ISSN 0165232X
[6] Habib K B, Taylor R S, Bruneau S and Jordaan I J 2015 (ASME) p V008T07A014 ISBN 978-0-7918-5656-7
[7] O’Rourke B J, Jordaan I J, Taylor R S and Gurtner A 2016 Cold Regions Science and Technology 124 25–39 ISSN 0165232X
[8] O’Rourke B J, Jordaan I J, Taylor R S and Gurtner A 2016 Cold Regions Science and Technology 124 11–24 ISSN 0165232X
[9] Birajdar P, Taylor R, Habib K and Hossain R 2016 (Offshore Technology Conference)
[10] Birajdar P, Taylor R and Hossain R 2017 (San Francisco, California, USA)
[11] Taylor R, Browne T, Jordaan I and Gurtner A 2013 (ASME) p V006T07A024 ISBN 978-0-7918-5540-9
[12] Lee R W and Schulson E M 1988 Journal of Offshore Mechanics and Arctic Engineering 110 187 ISSN 08997219
[13] Schulson E 1990 Acta Metallurgica et Materialia 38 1963–1976 ISSN 09567151
[14] Gagnon R and Gammon P 1995 Journal of Glaciology 41 103–111
[15] Moore P, Jordaan I and Taylor R 2013
[16] Kolari K 2017 International Journal of Solids and Structures 115 27–42
[17] Lu W, Lubbad R and Loset S 2014 Journal of Offshore Mechanics and Arctic Engineering 136 031501
[18] Wang F, Zou Z J, Zhou L, Ren Y Z and Wang S Q 2018 Cold Regions Science and Technology 149 1–15
[19] Gribanov I, Taylor R and Sarracino R 2018 Engineering Fracture Mechanics 196 142–156
[20] Gribanov I, Taylor R and Sarracino R 2018 International Journal for Numerical Methods in Engineering
[21] Park K, Paulino G H and Rosler J R 2009 Journal of the Mechanics and Physics of Solids 57 891–908 ISSN 0022-5096
[22] Spring D W, Giraldo-Londoo O and Paulino G H 2016 Mechanics Research Communications 78 100–109 ISSN 09936413
[23] Quey R, Dawson P R and Barbe F 2011 Computer Methods in Applied Mechanics and Engineering 200 1729–1745 ISSN 0045-7825
[24] Timco G W and Frederking R 1983 Cold Regions Science and Technology 8 35–41
[25] Tatniaux J C and Wu C Y 1978 Applied Techniques for Cold Environments (ASCE) pp 295–306
[26] Parsons B, Lal M, Williams F, Dempsey J, Snellen J, Everard J, Slade T and Williams J 1992 Philosophical Magazine A 66 1017–1036
[27] Lainey L and Tinawi R 1984 Canadian Journal of Civil Engineering 11 884–923
[28] Aly M, Taylor R S, Bailey E and Turbull I 2018 Proceedings of OMAE 2018