The influence of the type of the test signal on the result of numerical optimization of regulators

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Abstract. This paper studies the effect of the choice of the test signal on the result of optimization of the regulator by the example of PID and PI\(^2\)D structures. It is shown that if the quality response to ramp signals is necessary, optimization of the system only by the simple signals is ineffective. In this case, the resulting system is characterized by excessively large overshoot when the step is supplied to the system input. The paper proposes a method for designing an effective system consisting in numerical optimization for two parallel operating control systems. Its effectiveness has been proved by simulation.

1. Introduction
Astatic control of an object is required in many applications [1–17]. It suppresses the influence of a disturbance with the help of the integrator in the regulator. Sometimes, a second-order astatic system is necessary. This means that the system suppresses not only step disturbance, but ramp disturbance too. This property is usually achieved with the use of the two integrators in the regulator.

It is believed that the second-order astatic system is more accurate than a system with a first-order one. Researches showed that the system which better suppress the ramp signal, works worse under the step signal. The converse is also true.

This paper reveals the contradictions between the requirements and offers a technique to achieve the required compromise by numerical optimization of regulators using appropriate objective (cost) functions.

2. Task Statement
Let the object transfer function be the following:

\[
W(s) = \frac{\exp(-\alpha)}{(T_1 s + 1)(T_2 s^2 + \xi T_2 s + 1)}. \tag{1}
\]

Here, \(s\) is an argument of Laplace transform, and the remaining values in the numerator and the denominator are constant coefficients. It is necessary to calculate the regulator, which provides a better suppression of the error, while input signal and (or) disturbances are step and (or) ramp functions.

3. The method of solving the problem
To control the object, the paper proposes PID, and if that is not enough, the regulator should have double integration, which is called a PI\(^2\)D-regulator. A numerical optimization using modeling and optimization program VisSim is proposed too. The simplest cost function for optimization is as follows [2–4]:
Here, $e(t)$ – is the control error, $t$ – is time of the beginning of the transition process, $T$ – is the time of the end of the simulation. Also, the cost function can contain the additional term of the following form [4, 5]:

$$
\Psi_2(T, e) = \int_0^T \max\{0, e(t) \frac{de(t)}{dt}\} dt.
$$

In summations (2) and (3), it is advisable to use a weighting factor. The result of optimization is estimated by transient processes.

4. A numerical example and results

Let define the object model with specific numerical values, for example

$$
W(s) = \exp(-10s)/[(20s + 1)(s^2 + 2s + 1)].
$$

The object model in program VisSim is shown in Figure 1-a, and the model of the entire system is shown in Figure 1-b. Optimization was performed using a step input signal, as well as using a ramp signal. In the first case, the following regulator coefficients were calculated: $K_p = 2.29; K_i = 0.0543; K_d = 9.5$. In the second case, the following coefficients were obtained: $K_p = 3.06; K_i = 0.496; K_d = 23.9$. These transients as a result of optimization are shown in Figure 2. With this, Figure 2-a gives the results of working under the ramp signal, and Figure 2-b is the same under step signal.

![Figure 1. A model of the object in program VisSim: a – object; b – total system.](image)

![Figure 2. Results of regulator optimization: a – responses to the ramp up input function; b – responses to the step jump.](image)

Analysis of the processes in Figure 2-a shows that the system optimized for the ramp signal is better for such task, a static error is relatively small. The system optimized for testing the step jump, work with ramp signal with significant static error (in Figure 2-a, it is 0.2 units). In addition, analysis
of the processes in Figure 2-b shows that the system optimized for the ramp signal is ineffective for the step disturbance: an overshoot is about 180%, the system is prone to oscillations, and the process duration exceeds 200 s. The system, optimized for the step signal, works quickly with it: the process duration is 100 s, overshoot is small (~25%).

**Conclusion 1:** The system optimized with the step disturbance, is ineffective for the ramp disturbance; and vice versa, the system optimized for the ramp disturbance, is ineffective for the step disturbance.

5. **Refined optimization technique and the results**

In order to find a compromise solution, the paper proposes to optimize the PID, which is used simultaneously in two identical systems, one of which works with the step signal and the other works with the ramp signal.

Figure 3 shows the structure to optimize the regulator on the basis of this principle. Here, the cost function is the sum of cost functions of form (2). The errors in the two simulated systems are used to calculate the total cost function. When summing without weight coefficients, the obtained result is the following set of PID coefficients: $K_P = 2.59; K_I = 0.211; K_D = 13.97$. The resulting processes are shown in Figure 4-a.

![Figure 3](image)

**Figure 3.** A structure for optimization of the regulator for two similar systems with different inputs.

It is seen from Figure 4 that the result reaches some compromise. Overshooting, while the step is used, decreased by half in comparison with the system, optimized for the ramp earlier, so it became 90%. The static error when the ramp signal is supplied is less than 0.05 units, that is 4 times less than in the earlier system optimized for the step signal.

![Figure 4](image)

**Figure 4.** Output responses in the system with different weight coefficients: a $- K_W = 1$; b $- K_W = 4$.

To reduce the overshoot, it is useful to introduce weighting coefficient $K_W$. Hence, $\Psi = K_W \Psi_1 + \Psi_2$. Figure 7 illustrates the transient process in the system, obtained with
$K_W = 4$. Overshooting, while the step is used, decreases to 60%; the static error, when the ramp is used, has grown twice. The further increase of the $K_W$ allows further reducing the overshooting, while the step is used at the cost of a static error rising when the ramp input is acting.

Conclusion 2: The system with an PID regulator, optimized for a compromise between the quality of working out step jump and the high quality of the ramp signal provides the demanded compromise, but the resulting quality is not enough.

6. Solution of the problem with the help of the PI²D regulator

The second-order astatic system should comprise two integrators in the regulator. Therefore, it is proposed to use the structure, as shown in Figure 4-a, to optimize the PI²D regulator. This regulator contains a double integration link in addition to the traditional proportional, integrating and differentiating links. Therefore, four coefficients should be calculated in the structure. Also in this structure, an additional term of form (3) is introduced into the cost function. In this system, when step input acts, the overshooting is about 85%. Both processes’ duration is about 60 s.

The introduced term with chosen weight $K_W$ allows for a compromise search for coefficients that would ensure an acceptable quality of transient processes when every type of input signals acts. However, even keeping weight value $K_W = 1000$ does not significantly reduce the overshoot. The result of such an optimization is illustrated as transient responses in Figure 4-b.

For example, when $K_W = 10000$, the overshoot with the step is reduced to 40%. The duration of the transient process when the ramp signal acts is about 100 s. These processes are illustrated in Figure 5-a.

![Figure 4](image)

**Figure 4.** The resulting structure (a) and output responses (b) in the system with a PI²D-regulator.

![Figure 5](image)

**Figure 5.** Resulting processes with different weight coefficients: a – $K_W = 10000$; b – $K_W = 20000$. 
Similar results with $K_W = 20,000$ are the following: the overshoot is reduced to 25%. The process duration for both types of signals is about 160 s. A further increase of the weighting factor seems impractical. It allows decreasing the overshoot, but it increases the duration of the transient process.

**Conclusion 3:** The system with the PI$^2$D regulator, optimized for a compromise between the quality of working out with the step signal and with the ramp signal, provides a high quality of the system.

7. **An object prone to fluctuations**

Let us change the object model coefficients as follows:

$$W(s) = \exp(-10s) /[s + 1] / \left[ s^2 + 0.01s + 1 \right].$$  \hspace{1cm} (5)

This object is prone to oscillations. Figure 6-a shows the response of the object to the step jump. The proposed structure for optimizing successfully solves even the task of control of such an object. Indeed, the optimization result is shown in Figure 6-b. The system response to a step has a 30% overshoot, and the processing time is about 130 s.

![Figure 6. Responses of object (5): a – without feedback; b – with feedback, $K_W = 20000$.](image)

**Conclusion 4:** The proposed method works successfully with the object, prone to fluctuations.

8. **An alternative method**

Since the studied system is linear, it can be assumed that the sum of responses of the systems with the two types of values is equal to the response of the system to the sum of these values. The simulation showed that this alternative technique is successful only if the used values are with opposite signs (see Figure 7). If their signs are the same, then the resulting regulator is much worse. In each case, the resulting regulators differs.

![Figure 7. Responses of the system, optimized with the sum of step and ramp inputs: a – with the same signs; b – with opposite sings.](image)
9. Conclusion
This paper shows that the effect of the choice of the test signal on the result of optimization of the regulator is significant. The best system for ramp disturbance is not the best one for the step jump disturbance. The proposed method allows getting a compromised optimal result.

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