Mathematical Modeling and Experimental Verification of Oil-Gas Spring

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Abstract. A method for studying the flexural deformation of the damping valve when subjected to a local uniform load was proposed, according to the physical model of valve plate and the related theory of thin plate mechanics, the differential equation of the deformation potential energy of the valve plate was solved, and the analytical formula of the flexural deformation under the local uniform load was derived. Further, combining the analytic formula of flexural deformation of the valve plate, the analytic formula of the annular gap throttling and the actual gas state equation, the mathematical model of the indicator characteristic of the hydro-pneumatic spring was derived. Through programming simulation, the relationship between the elastic force of the system and the total output force with displacement were analysed, compared with the test data, the correctness of the mathematical model of the hydro-pneumatic spring was verified, and the theoretical basis was provided by analytical formula of valve plate deformation.

1. Introduction
The damping valve is the core component in the oil-gas spring, which is mainly used to damp the body vibration to improve the ride and handling stability of the vehicle [1]. There are many ways to assemble damping valves, of which the annular throttle valve to restrict oil is widely used in engineering practice. This method is convenient to process and assemble, and the cost is low, but the existing technology can only approximate the solution of the uniform load deformation, such approximation method may cause large error, and it is difficult to make the actual damping force of the damper correspond to the theoretical value, which cannot meet the design requirements well[2]. Based on the self-developed oil-gas springs, the deformation analysis formula of the throttle plate under local uniform load is derived and the system modeling is studied through using the relevant theory of thin plate deformation.

2. Structure Diagram
Fig. 1 is structure diagram of single-air chamber oil-gas spring, the throttle plate is installed outside the piston end. When the excitation speed is small, the oil-gas spring mainly generates damping force for oil throttling through the orifice. When the excitation speed is large enough, the pressure difference across the damping valve will deform the valve plate, and the oil will flow through the deformed gap. High-pressure air chamber is located in the hollow cavity of the piston rod.

The simplified model of the throttle plate is shown in Fig.2, the inner ring of the valve plate is fully constrained, and the outer ring is in a freely deformed state. In the figure, $q$ is local uniform load, $h$ is valve thickness, $r_i$ is inner radius of valve plate, $r_o$ is outer radius of valve plate, $r_e$ is external radius under locally uniform load.
3. Analytic Deduction of Throttle Plate Deformation

According to the thin plate mechanics, the differential equation of the deformation potential energy of the throttle plate in polar coordinates is[3]:

\[
U = \frac{D}{2} \int \left[ r \left( \frac{d^2w}{dr^2} \right)^2 + \frac{1}{r} \left( \frac{dw}{dr} \right)^2 + 2u \frac{dw}{dr} \frac{d^2w}{dr^2} \right] dr d\theta
\]  

(1)

Where, \( D = \frac{Eh^3}{12(1-u^2)} \) is bending stiffness of the valve disc, \( r \) is Radius anywhere, \( \theta \) is circumferential angle of valve plate, \( w \) is the amount of deflection at the radius of the valve plate, \( u \) is poisson's ratio.

The total potential energy of the valve plate when deriving axisymmetric is as follows[4]:

\[
\Pi = \pi D \int \left[ r \left( \frac{d^2w}{dr^2} \right)^2 + \frac{1}{r} \left( \frac{dw}{dr} \right)^2 + 2u \frac{dw}{dr} \frac{d^2w}{dr^2} \right] dr - 2\pi \int q wr dr
\]

(2)

The boundary conditions of the throttle plate are given by Fig2:

\[
w|_{r=a} = 0; \quad \frac{dw}{dr}|_{r=a} = 0; \quad M|_{r=a} = 0; \quad V|_{r=a} = 0
\]  

(3)

Assume that the deflection function has the following expression:

\[
w = \left( 1 - \frac{r^2}{r_a^2} \right)^2 \left[ C_1 + C_2 \left( 1 - \frac{r^2}{r_b^2} \right) + C_3 \left( 1 - \frac{r^2}{r_c^2} \right)^2 \right]
\]  

(4)

Where \( C_1, C_2, C_3 \) are the coefficient to be determined.

Simultaneously (3) and (4) derive the expression of \( w \) with respect to \( C_3 \) and find the first derivative of the radius of the valve[5]:

\[
\frac{dw}{dr} = -\frac{8C_1r^2(r^2 - r_a^2)}{r_b^6 A_1} (A_2 + A_1)
\]  

(5)

\[A_1 = r_b^2 (1+u)(r_a^2 - 3r_b^2) + 2r_b^4 (2+u)\]
\[ A_2 = 2r_a^4 (u + 2)(r_a^2 + r^2) - r_b^6 (11 + 5u)(r_a^2 + r^2) \]

\[ A_3 = r_b^6 (u + 1)(r_b^2 r^4 - 3r_a^2 r_b^2 r^2 - 3r_b^6 r^2) + 2r_a^4 r_a^2 r^2 (4u + 5) + 3r_b^8 (3 + u) \]

And the second derivative of \( w \) against \( r \):

\[ \frac{d^2 w}{dr^2} = -\frac{8C_3}{r_a^6 A_1} [B_1 + B_2] \]  \hspace{1cm} (6)

\[ B_1 = 2r_a^4 (u + 2)(r_a^2 - 7r_b^6) + r_b^6 (5u + 11)(5r^4 - r_b^4) + 3r_b^8 (u + 3)(r_a^2 - 3r^2) \]

\[ B_2 = r_b^3 r_a^2 (1 + u)[3r_a^2 (7r^4 - 3r_b^4) + r_b^6 (18r_a^4 - 7r^4) + 5r^2 (r_a^4 - 6r_b^6)] \]

Bring equations (5) and (6) into the integral term of formula (1) to sort out:

\[ U = -\frac{128\pi D C_3^2 (r_a^2 - r_b^2)^3}{35r_a^{10} B_3^2} (B_4 + B_5 + B_6 + B_7) \]  \hspace{1cm} (7)

\[ B_4 = r_a^4 (1 + u) + 2r_b^6 (2 + u) - 3r_a^2 r_b^6 (1 + u) \]

\[ B_5 = 3r_a^{12} (1 + u^2 + 2u) - 24r_b^{10} r_a^2 (u^2 + i + 2u) \]

\[ B_6 = 8r_a^6 r_b^4 (13 + 8u^2 + 21u) - 2r_b^{10} r_a^4 (31u^2 + 117 + 148u) \]

\[ B_7 = r_b^4 r_a^6 (313 + 25u^2 + 158u) - 2u^2 r_b^{10} (9u - 39) \]

\[ B_8 = 6r_b^{12} (15 + 2u^2 + 15u) \]

From the above, the function equation of the external force on the valve plate is derived as follows:

\[ W = -\frac{\pi q (r_a^2 - r_b^2)^3 C_3}{15r_a^6 A_1} (W_1 + W_2 + W_3) \]  \hspace{1cm} (8)

\[ W_1 = r_a^4 (u + 1)[3r_b^6 - 24r_a^4 r_b^2 + 4r_a^4 r_b^2 - 27r_a^2 r_b^2 r_a^2 + 3r_a^2 r_b^4 - 9r_b^6 r_a^4] \]

\[ W_2 = 2r_a^4 r_b^4 (51 + 33u) + 6r_a^6 r_b^2 r_a^2 (8u + 11) - 5r_b^{10} r_a^2 (33 - 25u) \]

\[ W_3 = 6r_b^{12} r_a^4 (u + 2) + 30r_b^8 (u + 3) - 5r_b^{10} r_a^2 (5u + 11) \]

Eq. (7) and (8) respectively find the first derivative of \( C_3 \) and sort out the expression of \( C_3 \):

\[ C_3 = \frac{7r_b^8 q (r_a^2 - r_b^2)^3 A_1}{768D (r_a^2 - r_b^2)^3} \left[ C_{31} + C_{32} + C_{33} \right] \]  \hspace{1cm} (9)

\[ C_{31} = r_b^2 (u + 1)[3r_a^6 - 24r_a^4 r_b^2 + 4r_a^4 r_b^2 - 27r_a^2 r_b^2 r_a^2 + 3r_a^2 r_b^4 - 9r_b^6 r_a^4] \]
Bring equation (9) into the surface function expression and extract the common factor:

$$w(r) = G_L(r) \frac{q}{Eh^3} \quad (10)$$

Where $G_L$ is the deformation coefficient of the local uniform load, and $E$ is the elastic modulus of the valve disc.

Equation (10) is the analysis formula for the flexural deformation of the annular throttle plate when it is subjected to a local uniform load.

### 4. Mathematical Model of Oil-Gas Spring

The mathematical model of the oil-gas spring is derived below. The output force of the oil-gas spring is:

$$F_i = F_d + F_j \quad (11)$$

Where $F_i$ is the total output force of the oil-gas spring, $F_d$ is the damping force, and $F_j$ is the elastic force.

The deformation of the outer edge of the valve plate can be solved according to formula (10):

$$f = G_L \frac{q}{Eh^3} \quad (12)$$

Where $f$ is the deformation amount of the outer edge of the throttle plate.

The pressure difference caused by the gap throttling can be obtained from the gap flow formula:

$$\Delta p_j = \frac{Q_j \mu \ln(r_i/r_f)}{\pi f^3} \quad (13)$$

Where $\mu$ is the kinematic viscosity of the oil, $Q_j$ is the flow rate through the gap.

The pressure difference produced by the orifice is:

$$\Delta p_o = \frac{\rho}{2} \left( \frac{Q_o}{A_o \epsilon} \right)^2 \quad (14)$$

Where $Q_o$ is the flow through the orifice, $\epsilon$ is the flow coefficient, and $A_o$ is the area of the through hole.

Oil-gas spring resistance value is:

$$F_d = (\Delta p_i + \Delta p_j) A_o \quad (15)$$

The actual gas state equation is used to push the pressure:
and the system elastic force:

\[ F_i = pA_{gw} \] (17)

5. Simulation and Experimental Research

The static friction test was used to determine the change of the elastic force of the oil-gas spring with displacement. The input incentive is \( f_i = 0.02 Hz \), \( A_i = 0.1 m \), it can be seen from Fig.3 that the simulated elastic force curve derived from the actual gas state equation is basically at the zero line of the test data, thereby verifying the correctness of the derivation process.

Then enter simple harmonic excitation \( f_i = 1 Hz \), \( A_i = 0.064 m \), to test and simulate the external characteristics of oil-gas springs, as shown in Fig.4, the two curves basically match, and the key point force values are shown in Table 1.

![Figure 3. Comparison of elastic characteristics test and simulation data](image1)

![Figure 4. Comparison of damping characteristic test and simulation data](image2)
Table 1. Comparison of force values at key points

| name                        | $F_{\text{z max}}$ | $F_{\text{z min}}$ | $F_{0y\text{z}}$ | $F_{0y\text{0}}$ |
|-----------------------------|---------------------|---------------------|------------------|------------------|
| Test data (KN)              | 24.40               | 5.67                | 17.71            | 7.87             |
| Simulation data (KN)        | 23.32               | 6.65                | 18.78            | 8.16             |
| Data bias (%)               | 4.4                 | 14.7                | 5.7              | 3.5              |

In the table, $F_{\text{z max}}$ is the maximum output force, $F_{\text{z min}}$ is the minimum output force, $F_{0y\text{z}}$ is the return stroke output force at zero, and $F_{0y\text{0}}$ is the compression stroke output force at zero.

The comparison results in Fig.4 and Tab.1 show that the mathematical model of the oil-gas spring established by using the analytical formula of local uniform load is correct and has high accuracy.

6. Conclusions

According to the self-developed damping valve form of oil-gas spring, the concept of local uniform load is proposed, and the analytical formula of the force and deformation of the annular throttle plate are derived based on the theory of thin plate mechanics. The mathematical model of single-chamber oil-gas spring is constructed, the damping and elastic force are derived respectively by using fluid mechanics and the actual gas state equation. The simulation results are compared with the characteristic test datas of the product to verify the correctness of the mathematical model.

7. References

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