The generation of the \((k - 1)\)-dimensional defect objects and their topological quantization*

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Abstract

In the light of \(\phi\)-mapping method and topological current theory, the topological structure and the topological quantization of arbitrary dimensional topological defects are investigated. It is pointed out that the topological quantum numbers of the defects are described by the Winding numbers of \(\phi\)-mapping which are determined in terms of the Hopf indices and the Brouwer degrees of \(\phi\)-mapping. Furthermore, it is shown that all the topological defects are generated from where \(\vec{\phi} = 0\), i.e. from the zero points of the \(\phi\)-mapping.

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1 Introduction

The world of topological defects is amazingly rich and have been the focus of much attention in many areas of contemporary physics[1, 2, 3]. The importance of the role of defects in understanding a variety of problems in physics is clear[4, 5, 6, 7]. So, it is necessary for us to investigate the topological properties of the topological defects meticulously. Recently, some physicists noticed[8, 9] that the topological defects are closely related to the spontaneously broken of \(O(m)\) symmetry group to \(O(m - 1)\) by \(m\)-component order parameter field \(\vec{\phi}\) and pointed out that for \(m = 1\), one has domain walls, \(m = 2\), strings and \(m = 3\), monopoles, for

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$m = 4$, there are textures. But for the lack of a powerful method, the topological properties are not very clear yet.

In this paper, in the light of $\phi$–mapping topological current theory\textsuperscript{[10]}, a useful method which plays an important role in studying the topological invariants\textsuperscript{[11], [12]} and the topological structures of physical systems\textsuperscript{[13], [14], [15]}, we will investigate the topological quantization and the evolution of these topological defects.

2 The generalized topological current

As is well known, in our previous papers, only the topological current of point-like particles was discussed. In this paper, we will extend the concept to present an arbitrary dimensional generalized topological current. We consider the $\phi$–mapping as a map between two manifolds, while the dimensions of the two manifolds are arbitrary. It is an important generalization of our previous work on topological current and is of great usefulness to theoretical physics and differential geometry.

In $n$–dimensional Riemann manifold $G$ with metric tensor $g_{\mu\nu}$ and local coordinates $x^\mu$ ($\mu, \nu = 1, ..., n$), a $m$–component vector order parameter $\vec{\phi}(x)$ can be looked upon as a mapping between the Riemann manifold $G$ and a $m$–dimensional Euclidean space $R^m$

$$\phi: G \to R^m, \quad \phi^a = \phi^a(x), \quad a = 1, ..., m.$$  

The direction field of $\vec{\phi}(x)$ is generally determined by

$$n^a(x) = \frac{\phi^a(x)}{||\phi(x)||}, \quad ||\phi(x)|| = \sqrt{\phi^a(x)\phi^a(x)} \quad (1)$$

with

$$n^a(x)n^a(x) = 1. \quad (2)$$

It is obviously that $n^a(x)$ is a section of the sphere bundle $S(G)\textsuperscript{[10]}$. If $n^a(x)$ is a smooth unit vector field without singularities or it has singularities somewhere but at the point $\vec{\phi}(x) \neq 0$, from (2) we have

$$n^a\partial_\mu n^a = 0, \quad \mu = 1, ..., n, \quad (3)$$

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which can be looked upon as a system of \( n \) homogeneous linear equations of \( n^a (a = 1, ..., m) \) with coefficient matrix \([\partial_\mu n^a]\). The necessary and sufficient condition that (3) has non-trivial solution for \( n^a(x) \) is rank \([\partial_\mu n^a]\) < \( m \), i.e. the Jacobian determinants

\[
D^{\mu_1 \cdots \mu_k}(\partial n) = \frac{1}{m!} \epsilon^{\mu_1 \cdots \mu_k \mu_{k+1} \cdots \mu_n} \epsilon_{\alpha_1 \cdots \alpha_m} \partial_{\mu_{k+1}} n^{\alpha_1} \cdots \partial_{\mu_n} n^{\alpha_m}
\]

are equal to zero, where \( k = n - m \). While, at the point \( \vec{\phi} = 0 \), the above consequences are not held. In short, we have the following relations

\[
D^{\mu_1 \cdots \mu_k}(\partial n) \begin{cases} 0, & \text{for } \vec{\phi} \neq 0, \\ \neq 0, & \text{for } \vec{\phi} = 0, \end{cases}
\]

which implies \( D^{\mu_1 \cdots \mu_k}(\partial n) \) behaves itself like a function \( \delta(\vec{\phi}) \). So we are focussed on the zeroes of \( \phi^a(x) \). Suppose that the vector field \( \vec{\phi}(x) \) possesses \( l \) zeroes, according to the implicit function theorem\([16]\), when the zeroes are regular points of \( \phi \)-mapping at which the rank of the Jacobian matrix \([\partial_\mu \phi^a]\) is \( m \), the solutions of \( \vec{\phi} = 0 \) can be expressed parameterizedly by

\[
x^\mu = z_i^\mu(u^1, \cdots, u^k), \quad i = 1, ..., l,
\]

where the subscript \( i \) represents the \( i \)-th solution and the parameters \( u^I (I = 1, ..., k) \) span a \( k \)-dimensional submanifold with the metric tensor \( g_{IJ} = g_{\mu\nu} \frac{\partial x^\mu}{\partial u^I} \frac{\partial x^\nu}{\partial u^J} \) which is called the \( i \)-th singular submanifold \( N_i \) in the Riemannian manifold \( G \) corresponding to the \( \phi \)-mapping.

For each singular manifold \( N_i \), we can define a normal submanifold \( M_i \) in \( G \) which is spanned by the parameters \( v^A \) with the metric tensor \( g_{AB} = g_{\mu\nu} \frac{\partial x^\mu}{\partial v^A} \frac{\partial x^\nu}{\partial v^B} \ (A, B = 1, ..., m) \), and the intersection point of \( M_i \) and \( N_i \) is denoted by \( p_i \). In fact, in the words of differential topology, \( M_i \) is transversal to \( N_i \) at the point \( p_i \). By virtue of the implicit function theorem, at the regular point \( p_i \), it should be hold true that the Jacobian matrix \( J(\frac{\phi}{v}) \) satisfies

\[
J(\frac{\phi}{v}) = \frac{D(\phi^1, \cdots, \phi^m)}{D(v^1, \cdots, v^m)} \neq 0.
\]

In the following, we will induce a rank-\( k \) topological current through the integration of \( D^{\mu_1 \cdots \mu_k}(\partial n) \) in (3) on \( M_i \). As is well known, the generalized Winding Number\([17]\) has been given by the Gauss map \( n : \partial \Sigma_i \rightarrow S^{m-1} \)

\[
W_i = \frac{1}{A(S^{m-1})(m-1)!} \int_{\partial \Sigma_i} n^*(\epsilon_{\alpha_1 \cdots \alpha_m} n^{a_1} dn^{a_2} \wedge \cdots \wedge dn^{a_m})
\]
where
\[ A(S^{m-1}) = \frac{2\pi^{m/2}}{\Gamma(m/2)} \]
is the area of \((m - 1)\)-dimensional unit sphere \(S^{m-1}\), \(n^a\) denotes the pull back of map \(n\) and \(\partial \Sigma_i\) the boundary of a neighborhood \(\Sigma_i\) of \(p_i\) on \(M_i\) with \(p_i \notin \partial \Sigma_i\), \(\Sigma_i \cap \Sigma_j = \emptyset\). The generalized Winding Numbers \(W_i\) can also be rewritten as
\[ W_i = \frac{1}{A(S^{m-1})(m - 1)!} \int_{\Omega[\partial \Sigma_i]} \epsilon_{a_1\cdots a_m} n^{a_1} n^{a_2} \cdots d\sigma_{\mu_1\cdots \mu_k} \]
which means that, when the point \(x^\mu\) or \(v^A\) covers \(\partial \Sigma_i\) once, the unit vector \(n^a\) will cover a region \(\Omega[\partial \Sigma_i]\) whose area is \(W_i\) times of \(A(S^{m-1})\), i.e. the unit vector \(n^a\) will cover the unit sphere \(S^{m-1}\) \(W_i\) times. From the above equation, one can deduce that
\[
W_i = \frac{1}{A(S^{m-1})(m - 1)!} \int_{\partial M_i} \epsilon_{a_1\cdots a_m} n^{a_1} \partial_{\mu_{k+2}} n^{a_2} \cdots \partial_{\mu_n} n^{a_m} dx^{\mu_{k+2}} \cdots dx^{\mu_n} = \frac{1}{A(S^{m-1})(m - 1)!} \int_{\partial M_i} \frac{1}{k!} \frac{1}{\sqrt{g_x}} \epsilon_{\mu_1\cdots \mu_{k+1}\cdots \mu_n} \epsilon_{a_1\cdots a_m} \partial_{\mu_{k+1}} n^{a_1} \partial_{\mu_{k+2}} n^{a_2} \cdots \partial_{\mu_n} n^{a_m} d\sigma_{\mu_1\cdots \mu_k}, \tag{9}
\]
where \(d\sigma_{\mu_1\cdots \mu_k}\) is the invariant surface element of \(M_i\) and \(g_x = \det(g_{\mu\nu})\). As mentioned above, the deduction \(\tag{9}\) shows that the Winding Numbers \(W_i\) can be expressed as the integration of \(D^{\mu_1\cdots \mu_k}(\partial n)\) on \(M_i\).

From the above discussions, especially the expressions \(\tag{4}\), \(\tag{5}\) and \(\tag{9}\), we can induce a generalized topological current \(j^{\mu_1\cdots \mu_k}\) which does not vanish only at the zeroes of the order parameter field \(\tilde{\phi}(x)\), and is exactly corresponding to the generalized Winding Number,
\[
j^{\mu_1\cdots \mu_k} = \frac{1}{A(S^{m-1})(m - 1)!} \frac{1}{\sqrt{g_x}} \epsilon_{\mu_1\cdots \mu_{k+1}\cdots \mu_n} \epsilon_{a_1\cdots a_m} \partial_{\mu_{k+1}} n^{a_1} \partial_{\mu_{k+2}} n^{a_2} \cdots \partial_{\mu_n} n^{a_m}. \tag{10}
\]
Obviously this tensor current is identically conserved, i.e.
\[
\nabla_{\mu_i} j^{\mu_1\cdots \mu_k} = 0, \quad i = 1, \ldots, k.
\]
The dual tensor of \(j^{\mu_1\cdots \mu_k}\) is
\[
\tilde{j}^{\nu_1\cdots \nu_m} = \frac{1}{A(S^{m-1})(m - 1)!} \epsilon_{a_1\cdots a_m} \partial_{\nu_1} n^{a_1} \partial_{\nu_2} n^{a_2} \cdots \partial_{\nu_m} n^{a_m}
\]
with
\[ j^{\mu_1 \cdots \mu_k} = \frac{1}{\sqrt{g_x}} \epsilon^{\mu_1 \cdots \mu_k \nu_1 \cdots \nu_m} j_{\nu_1 \cdots \nu_m}. \] (11)

It is easy to see that both \( j^{\mu_1 \cdots \mu_k} \) and \( \tilde{j}_{\nu_1 \cdots \nu_m} \) are completely antisymmetric tensors.

3 Topological quantization of defect objects

By making use of the \( \phi \)-mapping theory, we will study the global property of the generalized topological current \( j^{\mu_1 \cdots \mu_k} \) on the whole manifold \( G \) and conclude that \( j^{\mu_1 \cdots \mu_k} \) behaves itself like the generalized function \( \delta(\vec{\phi}) \) and the integration of \( \tilde{j} \) is the Winding Numbers at singularities \( z(u) \) of \( n^a(x) \). From (1) we have
\[
\partial_\mu n^a = \frac{1}{||\phi||} \partial_\mu \phi^a + \phi^a \partial_\mu \left( \frac{1}{||\phi||} \right) = -\phi^a \partial_\phi \frac{1}{||\phi||^3}
\]
which should be looked upon as generalized functions[18]. Using these expressions the generalized topological current (10) can be rewritten as
\[
j^{\mu_1 \cdots \mu_k} = C_m \frac{1}{\sqrt{g_x}} \epsilon^{\mu_1 \cdots \mu_k \mu_{k+1} \cdots \mu_n} \epsilon_{a_1 \cdots a_m}
\]
\[
\cdot \partial_{\mu_{k+1}} \phi^a \partial_{\mu_{k+2}} \phi^{a_2} \cdots \partial_{\mu_n} \phi^{a_m} \partial_{\phi^{a_1}} \partial_{\phi n} (G_m(||\phi||)), \quad m > 2.
\]
where \( C_m \) is a constant
\[
C_m = \begin{cases} 
- \frac{A(S^{m-1})(m-2)(m-1)!}{2\pi}, & m > 2 \\
\frac{1}{2\pi}, & m = 2
\end{cases}
\]
and \( G_m(\|\phi\|) \) is a generalized function
\[
G_m(\|\phi\|) = \begin{cases} 
\|\phi\|^{m-2}, & m > 2 \\
\ln \|\phi\|, & m = 2
\end{cases}
\]
Defining general Jacobians \( J^{\mu_1 \cdots \mu_k}(\frac{\phi}{\|\phi\|}) \) as following
\[
\epsilon^{a_1 \cdots a_m} J^{\mu_1 \cdots \mu_k}(\frac{\phi}{\|\phi\|}) = \epsilon^{\mu_1 \cdots \mu_k \mu_{k+1} \cdots \mu_n} \partial_{\mu_{k+1}} \phi^{a_1} \partial_{\mu_{k+2}} \phi^{a_2} \cdots \partial_{\mu_n} \phi^{a_m}
\]
and by making use of the \( m \)-dimensional Laplacian Green function relation[10]
\[
\Delta_\phi \left( \frac{1}{\|\phi\|^{m-2}} \right) = -\frac{4\pi^{m/2}}{\Gamma\left(\frac{m}{2} - 1\right)} \delta(\vec{\phi})
\]
where $\Delta_\phi = \left( \frac{\partial^2}{\partial \phi \partial \phi} \right)$ is the $m$–dimensional Laplacian operator in $\phi$–space, we do obtain the $\delta$–function like topological current rigorously

$$j^{\mu_1 \cdots \mu_k} = \frac{1}{\sqrt{g_x}} \delta(\vec{\phi}) J^{\mu_1 \cdots \mu_k}(\frac{\phi}{x}).$$

(12)

We find that $j^{\mu_1 \cdots \mu_k} \neq 0$ only when $\vec{\phi} = 0$ ( or when $x \in N_i$), which is just the singularity of $j^{\mu_1 \cdots \mu_k}$. In detail, the Kernel of the $\phi$–mapping is the singularities of the topological tensor current $j^{\mu_1 \cdots \mu_k}$ in $G$. We think that this is the essential of the topological tensor current theory and $\phi$–mapping is the key to study this theory.

As is well known[19], the definition of the $\delta$–function $\delta(N_i)$ in curved space-time on a submanifold $N_i$ is

$$\delta(N_i) = \int_{N_i} \frac{1}{\sqrt{g_x}} \delta^n(\vec{x} - \vec{z}_i(u^1, u^2)) \sqrt{g_u} d^k u, \quad g_u = \det(g_{IJ}).$$

(13)

Following this, by analogy with the procedure of deducing $\delta(f(x))$, since

$$\delta(\vec{\phi}) = \begin{cases} +\infty, & \text{for } \vec{\phi}(x) = 0 \\ 0, & \text{for } \vec{\phi}(x) \neq 0 \end{cases} = \begin{cases} +\infty, & \text{for } x \in N_i \\ 0, & \text{for } x \notin N_i \end{cases},$$

(14)

we can expand the $\delta$–function $\delta(\vec{\phi})$ as

$$\delta(\vec{\phi}) = \sum_{i=1}^{N} c_i \delta(N_i),$$

(15)

where the coefficients $c_i$ must be positive, i.e. $c_i = |c_i|$. From the definition of $W_i$ in (3), the Winding number can also be rewritten in terms of the parameters $v^A$ of $M_i$ as

$$W_i = \frac{1}{2\pi} \int_{\Sigma_i} \epsilon^{A_1 \cdots A_m} \epsilon_{a_1 \cdots a_m} \partial_{A_1} n^{a_1} \cdots \partial_{A_m} n^{a_m} d^m v.$$

Then, by duplicating the above process, we have

$$W_i = \int_{\Sigma_i} \delta(\vec{\phi}) J(\frac{\phi}{v}) d^m v.$$  

(16)

Substituting (13) into (16), and considering that only one $p_i \in \Sigma_i$, we can get

$$W_i = \int_{\Sigma_i} c_i \delta(N_i) J(\frac{\phi}{v}) d^m v = \int_{\Sigma_i} \int_{N_i} c_i \frac{1}{\sqrt{g_x} \sqrt{g_v}} \delta^n(\vec{x} - \vec{z}_i(u^1, u^2)) J(\frac{\phi}{v}) \sqrt{g_u} d^k u \sqrt{g_v} d^m v.$$

(17)
where \( g_v = \det(g_{AB}) \). Because \( \sqrt{g_u}\sqrt{g_v}d^kud^mv \) is the invariant volume element of the Product manifold \( M_i \times N_i \), so it can be rewritten as \( \sqrt{g_v}d^m x \). Thus, by calculating the integral and with positivity of \( c_i \), we get

\[
c_i = \frac{\beta_i \sqrt{g_v}}{|J(\frac{\phi}{v})_{p_i}|} = \frac{\beta_i \eta_i \sqrt{g_v}}{J(\frac{\phi}{v})_{p_i}},
\]

where \( \beta_i = |W_i| \) is a positive integer called the Hopf index of \( \phi \)-mapping on \( M_i \), it means that when the point \( v \) covers the neighborhood of the zero point \( p_i \) once, the function \( \bar{\phi} \) covers the corresponding region in \( \bar{\phi} \)-space \( \beta_i \) times, and \( \eta_i = \text{sign} J(\frac{\phi}{v})_{p_i} = \pm 1 \) is the Brouwer degree of \( \phi \)-mapping. Substituting this expression of \( c_i \) and (15) into (12), we gain the total expansion of the rank-\( k \) topological tensor current

\[
j_{\mu_1 \cdots \mu_k} = \frac{1}{\sqrt{g_x}} \sum_{i=1}^{l} \frac{\beta_i \eta_i \sqrt{g_v}}{J(\frac{\phi}{x})_{p_i}} \delta(N_i) J_{\mu_1 \cdots \mu_k}(\phi). \tag{19}
\]

From the above equation, we conclude that the inner structure of \( j_{\mu_1 \cdots \mu_k} \) is labelled by the total expansion of \( \delta(\bar{\phi}) \), which includes the topological information \( \beta_i \) and \( \eta_i \).

It is obvious that, in (6), when \( u^I \) and \( u^I (I = 2, \ldots, k) \) are taken to be time-like evolution parameter and space-like parameters, respectively, the inner structure of \( j_{\mu_1 \cdots \mu_k} \) just represents \( l(k - 1) \)–dimensional topological defects moving in the \( n \)–dimensional Riemann manifold \( G \). The \( k \)-dimensional singular submanifolds \( N_i \) \( (i = 1, \cdots, l) \) are their world sheets. Here we see that the defect objects are generated from where \( \bar{\phi} = 0 \) and, the Hopf indices \( \beta_i \) and Brouwer degree \( \eta_i \) classify these defects. In detail, the Hopf indices \( \beta_i \) characterize the absolute values of the topological quantization and the Brouwer degrees \( \eta_i = +1 \) correspond to defects while \( \eta_i = -1 \) to antidefects. It must be pointed that the relationship between the zero points of the \( m \)–dimensional order parameter field \( \bar{\phi} \) and the space location of these topological defects is distinct and clear and it is obtained rigorously without tie on any concrete model or hypothesis.
4 Evolution equation of the defect objects

At the beginning of this section, we firstly give some useful relations to study many defects theory. On the $i$-th singular submanifold $N_i$ we have

$$\phi^a(x)|_{N_i} = \phi^a(z_1^i(u), \ldots, z^n_i(u)) \equiv 0,$$

which leads to

$$\partial_\mu \phi^a \frac{\partial x^\mu}{\partial u^I}|_{N_i} = 0, \quad I = 1, \ldots, k$$

Using this relation and the expression of the Jacobian matrix $J(\hat{\phi}_v)$, we can obtain

$$J_{\mu_1 \cdots \mu_k} \left( \frac{\phi}{x} \right) |_{\phi=0} = \frac{1}{m!} \epsilon^{\mu_1 \cdots \mu_k \mu_{k+1} \cdots \mu_n} \epsilon_{a_1 \cdots a_m} \frac{\partial \phi^{a_1}}{\partial x^{\mu_{k+1}}} \cdots \frac{\partial \phi^{a_m}}{\partial x^{\mu_n}}$$

$$= \frac{1}{m!} \epsilon^{\mu_1 \cdots \mu_k \mu_{k+1} \cdots \mu_n} \epsilon_{a_1 \cdots a_m} \frac{\partial \phi^{a_1}}{\partial v^{A_1}} \cdots \frac{\partial \phi^{a_m}}{\partial v^{A_m}} \partial x^{\mu_{k+1}} \partial x^{\mu_n}$$

$$= \frac{1}{m!} \epsilon^{\mu_1 \cdots \mu_k \mu_{k+1} \cdots \mu_n} \epsilon_{A_1 \cdots A_m} \partial v^{A_1} \cdots \partial v^{A_m} \frac{\partial v^{A_1}}{\partial x^{\mu_{k+1}}} \cdots \frac{\partial v^{A_m}}{\partial x^{\mu_n}}, \quad (20)$$

then from this expression, the rank–$k$ tensor current can be expressed by

$$j^{\mu_1 \cdots \mu_k} = \frac{1}{m! \sqrt{g_v}} \sum_{i=1}^l \beta_i \eta_i \sqrt{g_v} \delta(N) \epsilon^{\mu_1 \cdots \mu_k \mu_{k+1} \cdots \mu_n} \epsilon_{A_1 \cdots A_m} \frac{\partial v^{A_1}}{\partial x^{\mu_{k+1}}} \cdots \frac{\partial v^{A_m}}{\partial x^{\mu_n}}, \quad (21)$$

Thus, using the above formulas and (11), on $M_i$, the integral of the dual tensor $\tilde{j}_{\nu_1 \cdots \nu_m}$ gives the following result

$$\int_{M_i} \tilde{j}_{\nu_1 \cdots \nu_m} dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_m} = \beta_i \eta_i.$$ 

This shows that, for the first time, we gain the topological charges of these defects which are determined by the Hopf indices and Brouwer degrees of the $\phi$–mapping.

Corresponding to the rank–$k$ topological tensor currents $j^{\mu_1 \cdots \mu_k}$, it is easy to see that the Lagrangian of many defects is just

$$L = \sqrt{\frac{1}{m!} g_{\mu_1 \nu_1} \cdots g_{\mu_k \nu_k} j^{\mu_1 \cdots \mu_k} j^{\nu_1 \cdots \nu_k}} = \delta(\hat{\phi})$$

which includes the total information of arbitrary dimensional topological defects in $G$ and is the generalization of Nielsen’s Lagrangian[21]. The action in $G$ is expressed by

$$S = \int_G L \sqrt{g_v} d^m x = \sum_{i=1}^l \beta_i \eta_i \int_{N_i} \sqrt{g_u} d^k u = \sum_{i=1}^l \beta_i \eta_i S_i.$$
where $S_i$ is the area of the singular manifold $N_i$. It must be pointed out here that the Nambu–Goto action\cite{22}, which is the basis of many works on defect theory, is derived naturally from our theory. From the principle of least action, we obtain the evolution equations of many defect objects

$$\sum_{IJ} g^{IJ} \frac{\partial x^\lambda}{\partial u^I} \frac{\partial x^\nu}{\partial u^J} \frac{\partial x^\mu}{\partial x^\nu} \frac{\partial \phi}{\partial x^\mu} = 0, \quad I, J = 1, ..., k.$$ \hspace{1cm} (22)

As a matter of fact, this is just the equation of harmonic map\cite{23}.

5 Conclusion

In summary, we have studied the topological property of the topological defects in general case by making use of the $\phi$–mapping topological current theory. As a result, the topological defects are generated from the zero point of $\phi$–mapping and the topological quantum number of these defects are the Winding number which are determined by the Hopf indices and the Brouwer degrees of $\phi$–mapping. The action and the evolution equations of these defects in Riemannian manifold $G$ are also obtained from our theory. We would like to point out that the theory of topological defects in this paper is an unified theory of describing the topological properties of the arbitrary dimensional topological defects and all the results are gained from the viewpoint of topology without any particular models or hypothesis. It is much more important in understanding the origin and the formation of these topological defects in early universe. Moreover, as we see in the present work, $\phi$–mapping is a useful method which can provide an important window into the topological structure of physical systems.

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