Calculation of the stiffness of reinforced concrete structures under the action of torsion and bending

V I Kolchunov, A I Dem'yanov, N V Naumov and M M Mikhaylov

South West State University, 94, 50 let Oktyabrya Street, Kursk, 305040, Russia

E-mail: vlik52@mail.ru, speccompany@gmail.com, kolua199200@yandex.ru

Abstract. The development of methods for calculating the stiffness of reinforced concrete structures with cracks at the limit states is an urgent task. The proposed method implies that the spatial cracks develop on special bilinear surfaces. It involves dividing a rectangular section into a series of squares, which are subsequently replaced by the circles inscribed in them. This approach requires the introduction of the concept of “equivalence” between the torsional stiffness characteristics of a rectangular section and the circumscribed circle. The equation for determining the tangential torsional stresses (taking into account the deplanation) at any point of the cross section is written in a cylindrical and Cartesian coordinate system. Two variants of modelling a real crack are proposed: in the form of stepwise located three-dimensional finite elements that are detached in common nodes, or using an imaginary crack along which pairs of finite elements are picked out. So, we use a special double cantilever model for the calculation. Having a picture of the applied force and deformation loads (crack opening) in the console nodes, it is possible to determine the values of the work in the state: “before detach” and “after detach” of the double-element model.

1. Introduction

Because of the modern acceleration of the terms for the reconstruction and commissioning of buildings and structures, the development of a method for calculating the stiffness of spatial reinforced concrete structures in the presence of cracks becomes an urgent need not only for calculating serviceability limit states, but also for determining the internal forces in these statically indeterminate systems, where they directly depend on stiffness [1–4].

Combined precast and cast-in-place reinforced concrete structures and structures that are strengthened during the reconstruction of buildings and structures are characterized by the general resistance of two or more concrete with different strength and deformability properties [5–8]. This determines a number of specific parameters for calculating and designing composite structures, leading to a redistribution of internal forces between old and new concrete.

From the foregoing follows that the development of a methodology for calculating the stiffness of reinforced concrete composite structures with spatial cracks is an urgent need, which is one of the most important problems of capital construction.

2. Key points of proposed method

The main provisions of the proposed methodology for determining the stiffness of reinforced concrete composite structures under the combined action of torsion and bending with spatial cracks in them are described below.
Spatial cracks develop along special bilinear surfaces. The values of the corner points (A, B, C, D) of the cross section of the reinforced concrete structure fit into the equation of the bilinear surface in parametric form [9], i.e. the equation of the bilinear surface is specified with reference to a given cross section:

\[ \begin{bmatrix} x_k ; y_k ; z_k \end{bmatrix} = \begin{bmatrix} x_A ; y_A ; z_A \end{bmatrix} \cdot (1- u_k) \cdot (1- w_k) + \begin{bmatrix} x_B ; y_B ; z_B \end{bmatrix} \cdot (1- u_k) \cdot w_k + \begin{bmatrix} x_C ; y_C ; z_C \end{bmatrix} \cdot u_k \cdot (1- w_k) + \begin{bmatrix} x_D ; y_D ; z_D \end{bmatrix} \cdot u_k \cdot w_k. \]  

(1)

A classification of discrete basic spatial cracks is introduced. Along with normal cracks, three types of spatial cracks are taken into account according to work [10]. There may be several levels of crack formation.

The proposed method involves dividing a rectangular section into a series of squares, which are subsequently replaced by the circles inscribed in them (figure 1, c).

Starting from the static geometry characteristics of the large circle, we determine tangential stresses of torsion \( \tau_{t,j,i} \) in the centre of the small i-circle with “dispersion” of these values over all points of the i-circle \( (\tau_{t,i,i,\text{spr}}) \), and then sum it with the tangential torsional stresses \( \tau_{t,i,A,\text{cond},f} \), determined by the static geometry characteristics of i-circle (treating them as local with the “condensation” of its actual static geometry characteristics).

In general case, the total polar moment of inertia of a rectangular cross-section is equal to the sum of the polar moments of inertia of the small squares-circles into which the rectangle is discretized (angular sections, because of their small influence on the values of tangential stresses, are not taken into account, figure 1,c):

\[ I_t = I_{t,1} + I_{t,2} + \ldots + I_{t,j} = \sum I_{t,j}. \]  

(2)

Analyzing the proposed approach, it becomes necessary to introduce the concept of “equivalence” between the characteristics of the torsional rigidity of a rectangular cross section and the circle described around this rectangular cross section. To do this, we can write the following equality:

\[ G_{\text{equ}} = \frac{G_{\text{rec}} \cdot I_{\text{rec}}}{I_{\text{bcir}}} = \frac{G_{\text{rec}} \cdot \sum_{i=1}^{n} (I_{t,i} + r_{j}^2 \cdot A_{i})}{I_{\text{bcir}}}. \]  

(3)

here \( I_{\text{bcir}} \) - polar moment of inertia of the large circle described around a rectangular cross-section; \( G_{\text{rec}} \) - shear modulus of a rectangular cross-section; \( I_{\text{rec}} \) - polar moment of inertia of a rectangular cross-section; \( I_{t,i,j} \) - polar moment of inertia of small squares-circles located inside the contour of rectangular cross-section; \( n \) - the number of small squares-circles located inside the contour of a rectangular cross-section; \( G_{\text{equ}} \) - “equivalent” shear modulus associated with “spreading” of an arbitrary cross section, which consists of small squares-circles into a large circle.; \( r_{j} \) - distance between centers of large circle \( O \) and small circle \( O_{i} \); \( A_{i} \) - area of the i-th small circle.

To bring the values of polar moments to a large circle, let us use the coefficient \( \alpha_{\text{equ}} \), which can be found by formula:

\[ \alpha_{\text{equ}} = \frac{G_{\text{rec}}}{G_{\text{equ}}}. \]  

(4)

The distribution of the torque acting in the cross-section is proportional to the ratio between the “equivalent” shear rigidity of the large circle described around a rectangular cross-section and the shear rigidity of each of the small circles (into which the cross section is discretized).
The distribution of the torque acting in rods with composite cross-sections is proportional to the ratio between the “equivalent” shear rigidity of the large circle described around a rectangular cross-section and the shear rigidity of each of the small circles (into which the cross-section is discretized) relative to the common geometric center of the cross-section:

\[ M_{t,i} = M_t \cdot \frac{A_i \cdot G_i}{A_{bcir} \cdot G_{eq}}. \]  

(5)

It is important to note that for the distributed torque to satisfy the following equation:

\[ \sum_{i=1}^{n} M_{t,i} = M_t. \]  

(6)

Formulas for determining tangential torsional stresses in the circle of the cross section, located at \( x \)-distance from the support, can be written in the cylindrical and Cartesian coordinate systems:

\[ \tau_{t,A} = \frac{M_{t,A,cond}}{r_{j,A}} + \frac{M_{t,i}}{r_{j,A}} \cdot \frac{\tau_{loc} + \tau_{conc,loc}}{I_{t,i} \cdot \alpha_{equ} - \frac{M_{t,i}}{I_{t,i} \cdot \alpha_{equ}} \cdot \frac{\tau_{loc} + \tau_{conc,loc}}{2} \leq \tau_{t,u}. \]  

(7)

here \( \tau_{t,A,cond} \) and \( \tau_{t,i,A,cond} \) are the tangential stresses at an arbitrary point A of the large circle, described around an arbitrary cross section after “condensing” the static-geometric characteristics of the cross-section “dissolved” along this circle and the tangential stresses at an arbitrary point A of a small square-circle after “condensing”; \( \tau_{loc} \) - local shear stresses arising in each i-circle; \( r_{j,A}, y_{j,A}, z_{j,A} \) - the distance (coordinates in the general coordinate system YOZ) from the center of the large circle described around the cross section to the center of the small circle in which an arbitrary point A is located; \( M_t \) - torque, acting in the cross section of the rod; \( I_t \) - total polar moment of inertia of the cross-section approximated by small squares – circles; \( \tau_{conc}, \tau_{conc,loc} \) - shear stresses caused by force, geometrical and inter-medium concentration of deformations, and components caused by local concentration; \( r_{i,A}, y_{i,A}, z_{i,A} \) - the distance from the center of the small i-circle to an arbitrary point A located in the small i-circle, in which the values of tangential torsional stresses \( \tau_t \) are determined and its coordinates in the local coordinate system \( Y_i, O_i, Z_i \) respectively; \( M_{t,i} \) - torque, acting in a small i-circle into which the cross section of the rod is being discretized; \( I_{t,i} \) - polar moment of inertia, in the small i-circle (it consists of its own polar moment of inertia and added moment, equal to \( r^2 \cdot A_i \)); \( \tau_{t,u} \) - ultimate torsional shear stresses.

Let us consider \( \tau_{t,A} \) from formula (8) as the resultant of two components in the general coordinate system YOZ, which can be determined from the following dependencies:

\[ \tau_{t,A,xy} = \tau_{t,A} \cdot \sin \alpha_j \cdot \frac{M_t}{r_{j} \cdot \alpha_{equ}} \cdot \frac{2 \cdot y_{j,A} + z_{j,A}^2}{y_{j,A} + z_{j,A}^2 + \tau_{loc} + \tau_{conc,loc} \cdot \tau_{conc,loc}} \times \frac{y_j}{\sqrt{y_j^2 + z_j^2}} \leq \tau_{t,xy,ul}, \]  

(9)

\[ \tau_{t,A,xz} = \tau_{t,A} \cdot \cos \alpha_j \cdot \frac{M_t}{r_{j} \cdot \alpha_{equ}} \cdot \frac{2 \cdot y_{j,A} + z_{j,A}^2}{y_{j,A} + z_{j,A}^2 + \tau_{loc} + \tau_{conc,loc} \cdot \tau_{conc,loc}} \times \frac{z_j}{\sqrt{y_j^2 + z_j^2}} \leq \tau_{t,xz,ul}, \]  

(10)
here \( \tau_{t,xy,ul} \) and \( \tau_{t,xz,ul} \) - components of the ultimate values of tangential torsional stresses.

In addition to the resultant \( \tau_{t,A} \), found by the equation (8) for the corresponding circles, it is necessary to take into account the components associated with the deplanation of a rectangular section. The displacement due to deplanation of the cross section can be written in the form:

\[
W = \frac{M_t}{G_{rec} \cdot I_t} \cdot f(y, z) \cdot f_2(x) = \frac{M_t}{G_{rec} \cdot I_t} \cdot \frac{a_y^2 - b_z^2}{a_y^2 + b_z^2} \cdot y \cdot z \cdot I \cdot \left( 1 - \frac{x}{T} \right) + w_{loc}.
\]  

(11)

Thus, displacement \( W \) is a complex function, depending on the coordinates \( y, z, x \). When finding the relative angular (shear) deformations of the deplanation and using the Cauchy dependences, they take the form:

\[
\gamma_{d,yx} = \frac{\partial w}{\partial y} + \frac{\partial \omega}{\partial x} + \frac{\partial w}{\partial y} + 0 = \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} + 0 = \frac{\partial w}{\partial z},
\]

(12)

\( w, \nu, \omega \) are the displacements in the direction of the axes \( x, y, z \), respectively. As applied to the deplanation model described by equation (11), displacement \( \nu = \omega = 0 \). As a result, we have:

\[
\gamma_{dep,yx,locl} = \frac{M_t}{G_{rec} \cdot I_t} \cdot \frac{a_y^2 - b_z^2}{a_y^2 + b_z^2} \cdot I \cdot \left( 1 - \frac{x}{T} \right) \cdot \left( \frac{M_t}{I_t \cdot G_{equ}} \cdot \gamma_{j,A^*} \right) \leq \gamma_{dep,yx,ul},
\]

(13)

\[
\gamma_{dep,zx,locl} = \frac{M_t}{G_{rec} \cdot I_t} \cdot \frac{a_y^2 - b_z^2}{a_y^2 + b_z^2} \cdot I \cdot \left( 1 - \frac{x}{T} \right) \cdot \left( \frac{M_t}{I_t \cdot G_{equ}} \cdot \lambda_{j,A^*} \right) \leq \gamma_{dep,zx,ul}.
\]

(14)

The components of the ultimate values of tangential stresses due to the deplanation are determined from the dependencies:

\[
\tau_{dep,yx} = \gamma_{dep,yx,locl} \cdot G_{equ} \leq \tau_{dep,yx,ul}, \quad \tau_{dep,zx} = \gamma_{dep,zx,locl} \cdot G_{equ} \leq \tau_{dep,zx,ul}.
\]

(15)

If the torque changes along the longitudinal axis of the reinforced concrete structure, then an additional dependence is introduced, based on the proportionality of the ratio between the torques in section \( k \) and section \( I - I \):

\[
\frac{K_T \cdot K_{pr,T} \cdot M_{T,k}}{M_{T,I}} = \frac{a}{c \cdot 0.5b \cdot \sin \alpha}, \quad M_{T,I} = \frac{a \cdot M_{T,k}}{K_T \cdot K_{pr,T} \cdot (c \cdot 0.5b \cdot \sin \alpha)}.
\]

(16)

Here \( K_T \) - a numerical coefficient that takes into account the static loading scheme from the position of additional torques along the length of the rod; \( K_{pr,T} \) - coefficient of ratio (it is known) between \( R_{sup} \) and \( T \); \( a \) - the horizontal distance from the support to section \( I - I \).

Summarizing the components of the tangential stresses from torsion \( \tau_{t,xy}, \tau_{t,xz}, \tau_{dep,yx} \) and \( \tau_{dep,zx} \) we obtain the result of stress \( \tau_{sum} \):

\[
\tau_{sum,A} = \sqrt{(\tau_{t,A,xy} + \tau_{dep,sum,xy})^2 + (\tau_{t,A,xz} + \tau_{dep,sum,xz})^2},
\]

(17)
here $\tau_{\text{dep,sum,xy}}$ and $\tau_{\text{dep,sum,zx}}$ - summed components of the shear stresses of the general and local deplanation, averaged in the i-th circle:

Their signs are determined automatically, using the second multiplier in equations (13) and (14), which controls quadrants of rectangular section. If within the i-circle the signs of the tangential deplanation stresses are different and it is difficult to average them, then the radii of the approximating circles must be reduced.

For the components of local deplanation, we select the “+” sign if we take into account the geometric concentration, “-” – if we are making “return” due to a sharp difference in stiffness in small circles adjacent to circles with zero stiffness.

Knowing $\tau_{\text{sum,A}}$ from the equation (17), we can find the torque per i-th circle of the compressed zone in section I – I:

$$T_c = M_{t,c} = \frac{\tau_{\text{sum,A}} \cdot I_{t,i}}{\sqrt{y^2 + z^2}}.$$  

(18)

Summarizing all $M_{t,i}$ for all i-th circles m located in the compressed zone of section I – I, we have

$$M_{t,c} = \sum_{i=1}^{m} M_{t,i}.$$  

(19)

The torque perceived by the concrete of the stretched zone equal:

$$T_R = M_{t,R} = \frac{\tau_{\text{sum,A}} \cdot \psi_{R,T} \cdot I_{t,i}}{\sqrt{y^2 + z^2}}.$$  

(20)

here $\psi_{R,T}$ is a parameter that takes into account the presence of adjacent spatial cracks in the stress-strain state due to torsion of the stretched zone of the middle section I – I.

On the other hand, proceeding to general resolving equations for section I – I, we can use the equation of equilibrium of the moments of internal and external forces acting on an x axis perpendicular to this section and passing through the point of application of the resultant forces in the compressed zone ($T_{b,pl}=0$):

$$M_{t,R} = M_{t} - M_{t,c}.$$  

(21)

From equation (21), can be determined a parameter $\psi_{R,T}$, that takes into account the presence of adjacent spatial cracks in the stress-strain state due to the torsion of the stretched zone of the middle section I – I:

$$\psi_{R,T} = \frac{(M_{t} - M_{t,c}) \sqrt{y^2 + z^2}}{\tau_{\text{sum,A}} \cdot I_{t,i}}.$$  

(22)

It should be noted that the stresses $\tau_{t,xy,ul}$, $\tau_{t,zx,ul}$, $\tau_{\text{dep,xy,ul}}$, $\tau_{\text{dep,zx,ul}}$ are known (they are located on the horizontal sections of the “strain – stress” diagrams), since the plastic state occurs simultaneously for tangential and normal stresses.

From the equilibrium equation of the projections of the internal transverse (force Q) and external forces acting in section I – I on the Y axis ($\sum Y = 0$; the dowel forces in the working reinforcement in the middle section I – I are equal to zero):

$$-\tau_{\text{pl,x}} \cdot x \cdot b - \gamma_{Q,I} \cdot \tau_{\text{pl,x}} \cdot \psi_{R,Q} \cdot (h_0 - x) \cdot b + K_M \cdot R_{\text{sup}} = 0,$$  

(23)
here \( \tau_{pl,x} \) is the shear stress determined in the second stage of stress-strain state.

From the equation (23):

\[
\gamma_{Q,t} = \frac{K_M \cdot R_{sup} - \tau_{pl,x} \cdot x \cdot b}{\tau_{pl,x} \cdot \psi_{R,Q} \cdot (h_0 - x) \cdot b},
\]

(24)

here \( K_M \) is a numerical coefficient, that takes into account the loading scheme with additional bending moments along the length of the bar (if necessary, the field of local stresses \( \Delta_M \) is taken into account according to the proposals of S.P. Timoshenko, so we consider \( \Delta_M \) as known value).

In this case, the transverse force perceived by the concrete of the compressed zone \( (Q_{I,b}) \) equals to:

\[
Q_{I,b} = \tau_{pl,x} \cdot x \cdot b.
\]

(25)

The transverse force perceived by the concrete of the stretched zone \( (Q_{II,t}) \) equals to:

\[
Q_{II,t} = \gamma_{Q,t} \cdot \tau_{pl,x} \cdot \psi_{R,Q} \cdot (h_0 - x) \cdot b.
\]

(26)

On the other hand:

\[
Q_{II,t} = Q - Q_{I,b}.
\]

(27)

The last equation can be used to determine a parameter \( \psi_{R,Q} \) that takes into account the presence of adjacent spatial cracks in the stress-strain state of the extended zone of the middle section \( I-I \):

\[
\psi_{R,Q} = \frac{Q - Q_{I,b}}{\gamma_{Q,t} \cdot \tau_{pl,x} \cdot (h_0 - x) \cdot b}.
\]

(28)

To date, the development of the finite-element models for the calculation of reinforced concrete structures (RSC) taking into account their nonlinear deformation has reached a rather high level. According to it, the methodology for calculating stiffness and modeling discrete cracks in RCS is advisable to develop with the use of the most advanced software systems.

During this, it is necessary to take into account the nature of the development and opening of cracks in RCS \([11]\). For making calculation models of buildings with RCS it is reasonable to use three-dimensional finite elements (figure 2) \([12, 13]\).

When modeling complexly loaded reinforced concrete structures using three-dimensional finite elements, spatial cracks are approximated by the parallelepipeds inscribed in them.

The bottom line of the proposed model of cracks \([14, 15]\) consists in the fact that after replacing the actual crack (described by equation (1)) with a model in the form of stepwise located three-dimensional finite elements that are detached in common nodes simulating a crack, and the crack opening is set in the form of a deformation effect. The effect of discontinuity of reinforced concrete is taken into account by introducing into the model a variable crack opening width depending on the distance from crack to the axis of the longitudinal or transverse working reinforcement.

When solving the inverse problem – determining the width of crack opening, the deformation effect is not set, and only the seam of the smallest possible width is modeled with the help of detach. The width of crack opening is defined as the displacement of the edges of this seam.

The proposed method provides an iterative process based on a change of the crack opening width, which is determined by the formula (31) below.

Another variant of modeling discrete cracks is also possible. It is used in the case when the renumbering of the nodes of the finite element scheme of the reinforced concrete structure (building or construction), associated with the need for detach, considered in the first embodiment, is undesirable. In this case, at the first stage of modeling discrete cracks only imaginary discrete cracks are used, the development of which is predicted according to the introduced classification of cracks. At the second stage of modeling cracks along the trajectory of an imaginary crack, pairs of finite elements, adjacent
to the crack from two opposite sides, are identified. These pairs are considered in two states: before detach and after it.

A special double-element cantilever model of the reinforced concrete element (figure 2) is used for the calculation, and iterative analysis of the stress-strain state of the structure. At the same time, not only detach is important, but also a deformation load, which takes into account the effect of concrete discontinuity. The distributed reinforcement in such an element is replaced by two (for a two-dimensional model) and four (for a three-dimensional model) beam finite elements in each mutually perpendicular direction, respectively.

\[ \tau_{Q}/, \tau_{p.l,x} \]

\[ y_Q, \tau_{p.l,x} \]

\[ h_{p.d.x} \]

\[ x \]

\[ m.O \]

\[ m.O_1 \]

\[ T \]

\[ T_1 \]

\[ m.b \]

\[ h_{p.d.x} \]

\[ x_1 \]

\[ x_1 \]

Figure 1. Diagram of shear stresses \( \tau_{Q} \) from shear force in the middle section I – I

(a, b): approximation of a rectangular section I-I using squares and circles inscribed in them and the distribution of torques in compressed and extended zone in the I – I

(c)

The displacements of the nodes are determined from the calculation of the double-element cantilever calculation model with the loads attached to its nodes (nodal forces). In this case, the support mounts of two nodes in a two-dimensional model and four nodes in a three-dimensional model (alternating pivotally motionless and pivotally movable supports) it is necessary to set, taking into account the variations: left - right, front - rear, bottom - up. It is also important that, along with the nodal forces in the double-element model, are set the deformation effects associated with the crack opening width and the effect of discontinuity.

The deformation load is set in each node (except the support ones) in three directions in accordance with figure 2, where \( l, m \) and \( n \) are the direction cosines of the main crack opening vectors to the \( x, y \), and \( z \) axes, respectively.

Having a scheme of the applied forces and displacements in the cantilever nodes, it is possible to determine the values of the work in two states: “before detach” and “after detach” of the double-element model. From the condition of the equality of these works, the thickness of the finite elements in the state “before detach” decreases. This procedure is performed for all pairs of finite elements adjacent to the crack from different sides along horizontal, vertical or their lateral surfaces. As a result, along the virtual crack, the thickness of the finite elements decreases. This provokes the formation and development of cracks according to the criterion of regular dispersed cracks, without detach and renumbering of finite elements along the entire surface of a discrete crack.

The average efforts in nodes in different directions for a double-element cantilever model are determined from a physically nonlinear calculation of the entire structure. For this we use nodal forces
in the corresponding finite elements of concrete and reinforcement. In the places of transitions from horizontal sections of simulated cracks to vertical and lateral faces, the work in the corner finite elements is determined by averaging their values.

As a result, the new thickness of the finite elements adjacent to the crack can be found from the equation:

$$b = \frac{W_1}{W_2} \cdot b_1,$$

(29)

here $W_1$ and $W_2$ are the work of the double-element model “before detach” and “after detach”, respectively.

Figure 2. The proposed model of cracks in a complexly stressed element, modeled with the help of “detach” of spatial finite elements (a) and deformation effects $\Delta_1 = a^l_{cr}, \Delta_2 = a^m_{crc}, \Delta_3 = a^n_{crc},$ (b); the scheme of double-element spatial model: “before detach” (c) and “after detach” (d): 1 - 255 FE before detach; 2 - 201 (210) FE; 3 - 255 FE after detach; 4 - 233 (236) FE

The proposed model provides an iterative process that is controlled by the achieved accuracy of the thickness of the finite elements that are adjacent to the virtual (imaginary) cracks.

In relation to composite reinforced concrete structures, a special double-element calculation model, (similar to described above) is introduced for simulating the joint between concretes. Program complex LIRA-SAPR is used to determine the physically nonlinear stress-strain state of this double-element computational model, with the reduction of size of finite elements adjacent to the seam (at
least two rows on each side of the seam). For making the “load - shear” \( Q - \Delta Q \) dependence, which is necessary to specify in series of small-sized finite elements the parameters \( G(\lambda), E(\lambda), \mu(\lambda) \) (to model the area around the seam), is used the experimental data of composite prisms, tested during shear in the joints between different concretes [16, 17, 18].

In accordance with this hypothesis, the task of determining the width of the crack opening \( a_{crc} \) reduces to finding the relative mutual displacements \( \varepsilon_g(y) \) of the reinforcement and concrete in the areas between the cracks:

\[
a_{crc} = 2 \int_0^1 \varepsilon_g(y) \, dy + \frac{\eta l_{crc}}{1 - \varepsilon_g(y) \, dy}.
\]

After integration and some simplifications, we get:

\[
a_{crc} = -\frac{2\Delta T}{G} - \frac{2B_{a,2}}{B} - \frac{2B_2}{B} \ln \left(1 + \frac{B_{a,2} \cdot A_{sw} \cdot E_{sw}}{q_{sw} S + B_{a,1} \cdot A_{sw} \cdot E_{sw}}\right).
\]

Here \( G \) – the conditional strain modulus of adhesion of reinforcement and concrete; \( S \) – the perimeter of the reinforcement cross section; \( \varepsilon_s \) - deformation of the reinforcement in the crack; \( A_{sw} \) - cross-sectional area of transverse reinforcement.

Given the shear stresses from the transverse force \( \tau_Q \), the equation (9) takes the form:

\[
\tau_{t,A,xy,*} = \tau_{t,A} \sin \alpha_j \pm \tau_Q = \left(\frac{M_l}{l_{t,equ}} \cdot \sqrt{2\varepsilon_{j,A}^2 + \varepsilon_{j,A}^2 + \tau_{loc} + \tau_{conc,loc} \pm \tau_{sw,loc}}\right) \cdot \sqrt{\frac{\gamma_j}{\sqrt{\varepsilon_{j,A}^2 + \gamma_j^2}}} \pm \tau_Q \leq \tau_{t,xy,ul}.
\]

Equation (17) for determining the resulting shear stress, taking into account equation (32) takes the following form:

\[
\tau_{sum,A,*} = \sqrt{\left(\tau_{t,A,xy,*} + \tau_{dep,sum,xy}\right)^2 + \left(\tau_{t,A,xyz} + \tau_{dep,sum,xyz}\right)^2}.
\]

Knowing the resulting shear stress from the equation (17), we can determine the relative angular deformation \( \gamma_{sum,A} \):

\[
\gamma_{sum,A} = \frac{\tau_{sum,A}}{G_{equ}}.
\]

Knowing \( \gamma_{sum,A} \) from the equation (34) we can determine the relative twist angle \( \theta_A \):

\[
\theta_A = \frac{\gamma_{sum,A}}{\tau_{j,A}}.
\]

Taking into account the obtained value of \( \theta_A \), we find the twist angle \( \varphi_A \) over the length from fixed support \( l(x) \):

\[
\varphi_A = \theta_A \cdot l(x).
\]

In works of N. I. Karpenko [19,20], the deformation model is described by a matrix:

\[
D = \begin{pmatrix}
\frac{1}{r} &= B_{11,*} \cdot M + B_{12,*} \cdot M_t + B_{13,*} \cdot N \\
\varphi &= B_{21,*} \cdot M + B_{22,*} \cdot M_t + B_{23,*} \cdot N \\
\varepsilon_{ox} &= B_{31,*} \cdot M + B_{32,*} \cdot M_t + B_{33,*} \cdot N
\end{pmatrix}.
\]
Matrix (37) has the same shape, but differs in the physical meaning of the elements $B_{ij,*}$, determined from the equation of the bilinear surface (1).

Submatrix $F(\alpha, \beta, \theta)$, can be used to determine the angles of a bilinear surface.

3. Conclusions

1. The proposed method involves dividing a rectangular section into a series of squares, which are subsequently replaced by the circles inscribed in them. This approach requires the introduction of the concept of “equivalence” between the torsional stiffness characteristics of a rectangular section and the circle described around it.

2. Two variants of modeling a real crack are possible: in the form of stepwise located three-dimensional finite elements inscribed in it, which are detached in common nodes, or with the use of an imaginary crack along which pairs of finite elements are picked out. Thus, a special double-element cantilever model of a reinforced concrete element is used for the calculation. Having a picture of the applied force and deformation loads (crack opening) in the console nodes, it is possible to determine the values of the work in the state: “before detach” and “after detach” of the double-element model.

3. The proposed algorithm for modelling discrete cracks provides an iterative process, based on a change of the crack opening width (applied as a deformation load $\Delta = a_{crc,j}$ to the elements adjacent to the crack from two opposite sides).

4. When solving the inverse problem – determining the width of crack opening, the deformation effect is not set, and only the seam of the smallest possible width is modeled with the help of detach. The width of crack opening is defined as the displacement of the edges of this seam from the LIRA-SAPR finite element model.

References

[1] Ogawa Y, Kawasaki Y, Okamoto T 2014 Construction and Building Materials 67 165–69
[2] Dem’yanov A, Yakovenko I and Kolchunov V 2017 News Higher Educ. Inst. Tech Textile Ind. 4 370 246–51
[3] Dem’yanov A, Kolchunov V and Pokusaev A 2017 Struct. Mech. Eng. Constr. and Build 6 37–44
[4] Salnikov A, Kolchunov VL and Yakovenko I 2015 Appl. Mech. Mater. 725–26 784–89
[5] Lukina A, Kholopova I, Alpatova V and Solovieva A 2016 Procedia Engineering 153 414–18
[6] Chandrasekar E, Dhanaraj R and Santhakumar R 2007 Electronic Journal of Structural Engineering 7 1–7
[7] Jariwala V, Patel P and Purohit S. 2013 Procedia Engineering 51 282–89
[8] Rahal K and Collins M 2006 ACI Structural Journal. 103 3 328–38
[9] Demyanov A, Kolchunov V and Yakovenko I 2017 Industrial and Civil Engineering 9 18-24
[10] Demyanov A, Naumov N and Kolchunov V 2018 Building and reconstruction 4 78 3–19
[11] Bondarenko V 2004 The Computational Model of a Power Resistance of Reinforced Concrete (Moskow: ASV Press.) 472 p
[12] Kirsanov M 2014 Magazine of Civil Engineering 5 49 37–43
[13] Pavlenko A, Rybakov V, Pikht A and Mikhailov E 2016 Magazine of Civil Engineering 7 67 55–69
[14] Golishev A 2009 The Resistance of Reinforced Concrete (Kiev: Osnova Publ.) 432 p
[15] Kolchunov VI, Yakovenko I and Lyman I 2015 Building and reconstruction 5 61 17–24
[16] Merkulov S and Starodubtsev S 2012 Building and reconstruction 2 40 20–24
[17] Bashirov H, Kolchunov V, Fedorov V and Yakovenko I 2017 Reinforced concrete composite constructions of buildings and structures (Moscow: ASV Publ.) p 248
[18] Alkadi S, Demyanov A and Osovskih E 2017 Struct. Mech. Eng. Constr. and Buildings 5 72–80
[19] Karpenko N, Eryshev V and Latysheva E 2014 Vestnik MGSU. 3 168–78
[20] Karpenko N 1996 General Mechanics Model of Reinforced Concrete (Moscow: Stroyizdat Publ.) p 416