Magnetic field effects on the transport properties of high-Tc cuprates

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Abstract
Starting from a recently proposed comprehensive theory for the high-Tc superconductivity in cuprates, we derive a general analytic expression for the planar resistivity, in the presence of an applied external magnetic field \( H \) and explore its consequences in the different phases of these materials. As an initial probe of our result, we show it compares very well with experimental data for the resistivity of La\(_{2-x}\)Sr\(_x\)CuO\(_4\) (LSCO) at different values of the applied field. We also apply our result to Bi\(_{2201}\) and show that the magnetoresistivity (MR) in the strange metal (SM) phase of this material, exhibits the \( H^2 \) to \( H \) crossover, as we move from the weak to the strong field regime. Yet, despite of that, the MR does not present a quadrature scaling. Remarkably, the resistivity H-field derivative does scale as a function of \( \frac{H}{T} \), in complete agreement with recent magneto-transport measurements made in the SM phase of cuprates (Ayres et al 2020 arXiv:2012.01208). We, finally, address the issue of the \( T \)-power-law dependence of the resistivity of overdoped cuprates and compare our results with experimental data for Tl2201. We show that this provides a simple method to determine whether the quantum critical point associated to the pseudogap temperature \( T^* (x) \) belongs to the superconducting dome or not.

Keywords: high-Tc cuprates, transport properties, applied magnetic field

(Some figures may appear in colour only in the online journal)

1. Introduction

Any complete theory for superconductivity in the high-Tc cuprates must be capable to describe, besides the superconductivity mechanism itself, the properties of their normal phases. The comprehension of such phases of the cuprates, actually, seems to be as challenging as that of the superconducting (SC) phase itself.

An interesting issue, in connection to this, is the range of different functional dependences on the temperature, which are exhibited by the resistivity as we cross the \( T_c (x) \) SC dome. These are usually of the form \( \rho(T) \propto T^{\delta+\delta} \), where apparently \( \delta \in [0, 1] \). The precise value of \( \delta \), however, is strongly dependent on the specific region of the SC dome where we cross the SC transition and, consequently, the previously vanishing resistivity acquires a temperature dependence.

The situation becomes even richer, when we apply an external magnetic field and consider the resistivity dependence on it. Then, a wide range of effects can be observed, including the destruction of the SC phase.

A particularly interesting non-SC phase of the cuprates is the so-called strange metal (SM) phase [1–14], where the resistivity grows linearly with the temperature, with a slope that decreases with doping, proportionally to the pseudogap (PG) temperature \( T^* (x) \) [15]. Recent studies reveal, however, that specially in the case of overdoped (OD) cuprates [16], depending on the doping amount, we not always move directly from the SC phase to a linear dependent resistivity. In many cases, for some compounds, we rather observe...
a super-linear dependence on $T$ before we reach the linear regime [1].

Interesting experimental studies have also addressed the issue of the effect of an external magnetic field on the transport properties of OD cuprates [1]. Such studies reveal, for instance, the existence of a crossover in the magnetic field dependence of the magnetoresistivity (MR) in the SM phase, ranging from a quadratic behavior, at weak fields, to a linear one, in the strong field regime [1]. Such a behavior is analogous to the one observed in quantum critical phases of electron doped cuprates [17] and pnictide superconductors [18, 19].

In such systems, the crossover was ascribed to a quadrature scaling behavior, in which the planar MR behaves according to the empirical expression:

$$
\rho(T, H) - \rho(0, 0) = \sqrt{(\alpha k_B T)^2 + (\gamma \mu_B \mu_0 H)^2},
$$

where $\alpha$ and $\gamma$ are constant fitting parameters.

A benchmark of the quadratic behavior is that the quantity $\Delta \rho/T = (\rho(T, H) - \rho(0, 0))/T$ becomes a function of the ratio $H/T$, namely:

$$
(\rho(T, H) - \rho(0, 0))/T \propto \sqrt{1 + \left(\frac{\lambda \mu_{B_1} \mu_0 H^2}{k_B T}\right)^2},
$$

The study carried on in [1] on the cuprates Bi2201 and Tl2201 shows that in spite of exhibiting the $H^2$ to $H$ crossover in the MR field dependence, the MR data for cuprates in the SM phase do not scale as the quadrature would do, namely, as in (2).

Interestingly and remarkably, however, it was shown in [1] that the MR data for the resistivity field derivative, $(\partial \rho(T, H)/\partial H)$, do scale as in (2), namely,

$$
\frac{\partial \rho(T, H)}{\partial H} = f\left(\frac{H}{T}\right).
$$

In two recent publications [15, 20] we developed a comprehensive theory for the high-T$c$ cuprates, whose most distinguishable feature, perhaps, is to be testable. Indeed, our theory allows for the theoretical determination of several physical quantities, which can be directly compared with the experiments. Among these, we have obtained analytical expressions for the SC and PG transition temperatures $T_c$ and $T^*$ as a function of quantities such as the stoichiometric doping parameter, number of planes, pressure and external magnetic field [15, 20]. We have also obtained a general expression for the resistivity as a function of the temperature in the different non-SC phases of the high-T$c$ cuprates [15]. These results are in excellent agreement with the experiments for a wide range of cuprate systems with one, two and three planes per unit cell.

In the present work, we directly derive from the aforementioned theory, a general expression for the planar resistivity as a function of an applied external magnetic field $H$.

We firstly apply this result in order to describe the resistivity in LSCO and specially to determine how it is modified when the system is under the action of an external magnetic field.

We, then, consider our expression for the resistivity in the SM phase and show that, interestingly, our expression completely agrees with the experimental results found in [1] for Bi2201. In particular, it exhibits the $H^2$ to $H$ crossover, in spite of the fact that it does not present the quadrature scaling behavior. Yet, it satisfies the field derivative scaling (3).

Finally we address the issue of the super-linearity of the resistivity of OD cuprates, right above the SC transition and offer a simple explanation, which is illustrated by comparison with experimental data for Tl2201.

### 2. The resistivity

#### 2.1. General expression

The resistivity can be obtained as the inverse conductivity matrix, which is given by the Kubo formula:

$$
\sigma_{ij}^{\parallel} = \lim_{\omega \to 0} \frac{\Pi_{ij}^{\parallel}(\omega + ie, k)}{\omega},
$$

where $\Pi_{ij}^{\parallel}$ is the retarded, connected current-current correlation function:

$$
\Pi_{ij}^{\parallel} = \langle j^i f^j \rangle_c.
$$

This is given by the second functional derivative of the grand-canonical potential in the presence of an applied electromagnetic vector potential $A(\omega, k)$, namely,

$$
\langle j^i f^j \rangle_c(\omega, k) = \frac{\delta^2 \Omega[A]}{\delta A^i(\omega, k) \delta A^j(\omega, k)},
$$

$\Omega[A]$ relates to the grand-partition functional $Z[A]$ as:

$$
\Omega[A] = -\frac{1}{\beta} \ln Z[A],
$$

which is given by:

$$
Z[A] = \text{Tr}_{\text{Total}} e^{-\beta[H[A] - \mu N]}.
$$

In the expression above, $H[A]$, is our proposed Hamiltonian for the cuprates [15, 20], in the presence of an external field:

$$
A = \frac{1}{2} r \times B,
$$

which corresponds to a constant external magnetic field $B = \mu_0 H$.

The trace above can be evaluated with the help of the eigenvalues of $H[A] - \mu N$, which are given by [15, 20]:

$$
\mathcal{E}_l[A] = \sqrt{\Delta^2 + \left(\sqrt{v^2(hk + eA)^2 + M^2 + l^2} \right)^2},
$$

where $l = \pm 1$. The expression above is given in terms of the external field and the ground-state expectation values: of the
Cooper pair operator, $\Delta$, of the exciton operator, $M$ and of the chemical potential, $\mu$. The field dependence is conveniently expressed through the replacement:

$$
M^2 \rightarrow M^2 + 2ev \hbar k \cdot A + e^2 v^2 A^2, \\
M^2 \rightarrow M^2 - 2ev (L \cdot H) + \frac{1}{4} e^2 v^2 \langle r^2 \rangle H^2,
$$

(11)
in the presence of an applied field, where we replaced $r^2$ and $L$ for their average values. Since the ground state, either $|p_i\rangle$ or $|p_i\rangle$ is a linear combination of $|l,m\rangle = |1,\pm 1\rangle$, it follows that $(L_z) = 0$ and the second term in (11) does not contribute, thus confirming the observation made in [1] that there is no contribution of the orbital coupling with the external field. This also leads to results that are independent of the specific direction of the applied external magnetic field, which is in agreement with the experimental observations reported in [1].

The grand-partition functional $Z[A]$ follows from equations (8) and (10), and after functional integration over the fermionic (holes), degrees of freedom, namely [15, 20]:

$$
Z[A] = \exp \left\{ -\beta \left( \frac{|\Delta|^2}{8\mu} + \frac{|M|^2}{8\rho} + N_\mu(x) - NTA \sum_{n=-\infty}^{\infty} \sum_{l=\pm 1} \int \frac{d^2k}{4\pi^2} \ln \left[ (\omega_n + i\omega_0)^2 + \xi_i^2[A] \right] \right) \right\}
$$

$$
= Z[0] \exp \left\{ -\beta T \sum_{n}\sum_{l=\pm 1} \int \frac{d^2k}{(2\pi)^2} \ln \left[ \frac{(\omega_n + i\omega_0)^2 + \xi_i^2[A]}{\omega_n^2 + \xi_i^2[0]} \right] \right\},
$$

(12)

where, we used (11), to define:

$$
\mathcal{M}^2 \equiv M^2 + e^2 v^2 A^2.
$$

(16)

Considering that,

$$
A^2 = \frac{1}{4} \langle r^2 \rangle (\mu_0 H)^2,
$$

(17)

where we have replaced the square of the position vector by its average value, related to the de Broglie wavelength:

$$
\langle r^2 \rangle \approx \left( \frac{\hbar}{mv} \right)^2 = \left( \frac{\hbar}{m_e v} \right)^2 \left( \frac{m}{m_e} \right)^2,
$$

(18)

where $m$ is the effective quasiparticle mass and $m_e$ the electron mass, we can express (16) as:

$$
\mathcal{M}^2 = M^2 + e^2 v^2 A^2 = M^2 + \left( \frac{e\hbar}{2m_e} \right)^2 \lambda_2^2 (\mu_0 H)^2,
$$

(19)

which implies,

$$
\frac{\mathcal{M}}{k_B T} = \sqrt{\frac{M^2 + (\lambda_2 \mu_0 H)^2}{k_B T}}
$$

$$
= \sqrt{\left( \frac{M}{k_B T} \right)^2 + \lambda_2^2 \left( \frac{\mu_0 H}{k_B T} \right)^2},
$$

(20)

where $\mu_0 = \frac{e^2}{2\pi\hbar}$ is the Bohr magneton, $\frac{m}{m_e} = 0.671 K/T$ and $\lambda_2 \approx \frac{\hbar}{m_e}.

The corresponding DC resistivity per CuO$_2$ plane, then, will be given by (we drop from now on, the $ij$-superscript):

$$
\rho_{dc} = \sum_{i,j} \frac{\hbar}{m_e v} \left( \frac{\Delta^2 + (M + l\mu)^2}{2k_B T} \right).
$$

(15)
\[
\rho = \left( \frac{\sigma_{\text{DC}}}{N} \right)^{-1} = \frac{\mathcal{M}}{h_\beta V^{-1} e^2 v^2 \sum_{l,l'=\pm 1} \frac{\mathcal{M} + i \mu}{\sqrt{\Delta^2 + \left( \mathcal{M} + i \mu \right)^2 + \hbar \omega_0}} \tanh\left( \frac{\sqrt{\Delta^2 + \left( \mathcal{M} + i \mu \right)^2 + \hbar \omega_0}}{2 k_B T} \right)},
\]

(21)

where \( V = d a^2 \) is the volume of the primitive unit cell, per \( \text{CuO}_2 \) plane, with \( d \) being the distance between planes, \( a \) the lattice parameter and \( v \), the characteristic velocity of the holes (such that for LSCO \( (\hbar v/a) \approx 2.86 \times 10^{-2} \, \text{eV} \) [20]).

In the SC phase, we have \( \Delta \neq 0 \), which implies \( M = 0 \). In the absence of an applied magnetic field, we have \( M = M \) and, consequently, also \( M \to 0 \). Therefore, we can see, from (21), that in this case \( \rho \to 0 \) whenever \( \Delta \neq 0 \). In the presence of an applied magnetic field, conversely, having \( \Delta \neq 0 \) no longer implies \( M = 0 \) and, consequently, we may have a nonzero resistance, even for \( \Delta \neq 0 \). The critical temperature, however, is thereby reduced and eventually may vanish for a sufficiently strong magnetic field. We illustrate below this mechanism in full detail for the case of LSCO.

### 2.2. The scaling function

Outside the SC phases, we have \( \Delta = 0 \), which leads to the following expression for the resistivity:

\[
\rho = \frac{V k_B}{h e^2 v^2} \left[ \tanh\left( \frac{\Delta + \mu + \hbar \omega_0}{2 k_B T} \right) + \tanh\left( \frac{\Delta - \mu + \hbar \omega_0}{2 k_B T} \right) \right].
\]

(22)

Using the identity,

\[
\frac{2 \sinh(a)}{\cosh(a) + \cosh(b)} = \tanh\left( \frac{a + b}{2} \right) + \tanh\left( \frac{a - b}{2} \right),
\]

(23)

for the 1st + 4th and 2nd + 3rd terms above, this can be rewritten as:

\[
\rho = \frac{V k_B}{h e^2 v^2} \left[ \tanh\left( \frac{\mathcal{M}}{k_B T} \right) + \tanh\left( \frac{\mathcal{M} - \mu + \hbar \omega_0}{k_B T} \right) \right] \left[ \frac{\sinh\left( \frac{\mathcal{M}}{k_B T} \right)}{\cosh\left( \frac{\mathcal{M}}{k_B T} \right) + \cosh\left( \frac{\mu + \hbar \omega_0}{k_B T} \right)} + \frac{\sinh\left( \frac{\mathcal{M} - \mu + \hbar \omega_0}{k_B T} \right)}{\cosh\left( \frac{\mathcal{M} - \mu + \hbar \omega_0}{k_B T} \right) + \cosh\left( \frac{\mu - \hbar \omega_0}{k_B T} \right)} \right],
\]

(24)

or,

\[
\rho = \frac{V k_B}{h e^2 v^2} \frac{\mathcal{M} T}{2 \sinh\left( \frac{\mathcal{M}}{k_B T} \right)} \left\{ \frac{\left[ \cosh\left( \frac{\mathcal{M}}{k_B T} \right) + \cosh\left( \frac{\mu}{k_B T} \right) \cosh\left( \frac{\hbar \omega_0}{k_B T} \right) \right]^2 - \left[ \sinh\left( \frac{\mu}{k_B T} \right) \sinh\left( \frac{\hbar \omega_0}{k_B T} \right) \right]^2}{\cosh\left( \frac{\mathcal{M}}{k_B T} \right) + \cosh\left( \frac{\mu}{k_B T} \right) \cosh\left( \frac{\hbar \omega_0}{k_B T} \right)} \right\}.
\]

(25)
We can express the resistivity in the presence of an applied magnetic field in terms of a three-variable scaling function \( G(K_1, K_2, K_3) \), where

\[
K_1 = \frac{M}{k_B T} ; \quad K_2 = \frac{\mu}{k_B T} ; \quad K_3 = \frac{\mu_B J_0}{k_B T},
\]

namely,

\[
\rho(x, T) = BT^2 G \left( \frac{M}{k_B T}, \frac{\mu}{k_B T}, \frac{\mu_B J_0 H}{k_B T} \right), \tag{27}
\]

where,

\[
G(K_1, K_2, K_3) = \frac{\sqrt{K_1^2 + (\lambda_2 K_3)^2}}{2 \sinh \left( \sqrt{K_1^2 + (\lambda_2 K_3)^2} \right)} \left[ \frac{\left( \cosh \sqrt{K_1^2 + (\lambda_2 K_3)^2} + \cosh K_2 \cosh K_3 \right)^2 - (\sinh K_2 \sinh K_3)^2}{\cosh \sqrt{K_1^2 + (\lambda_2 K_3)^2} + \cosh K_2 \cosh K_3} \right], \tag{28}
\]

and \( B \) is given by:

\[
B = \frac{\hbar}{e^2} \frac{d}{2\pi} \left( \frac{a}{\hbar v} \right) \frac{2}{k_B}. \tag{29}
\]

For LSCO, we have \( B_{LSCO} = 2.4457 \) nT/cm K\(^{-2} \) and, in general, we write \( B = \lambda B_{LSCO} \).

Different studies have approached the MR in cuprates by using a scaling analysis \([7, 25]\). In our treatment such scaling function appears naturally.

Notice that in the zero field limit, \( K_3 \to 0 \) and our expression for the resistivity reduces to the one in \([15]\).

\[
\rho = \frac{\lambda_1 \lambda_2 B T^* T K_3}{\sinh (\lambda_2 K_3)} \left[ \frac{1}{\cosh (\lambda_2 K_3) + \cosh (K_3 + D/k_B)} + \frac{1}{\cosh (\lambda_2 K_3) + \cosh (K_3 - D/k_B)} \right], \tag{30}
\]

where \( T^* \) is the PG temperature.

2.4. The zero magnetic field regime

In the \( H \to 0 \) limit, we have \( K_3 \to 0 \). In this case, (28) reduces to:

\[
G(K_1, K_2, K_3) \to G(K_1, K_2)
\]

\[
G(K_1, K_2) = \frac{K_1}{2 \sinh K_1} [\cosh K_1 + \cosh K_2]. \tag{31}
\]

In the SM phase, where we also have \( K_1 = 0 \), accordingly, the scaling function becomes:

\[
G(K_1, K_2) = C T^* T, \tag{32}
\]

and the resistivity, according to (27), becomes:

\[
\rho(T) = CT^* T. \tag{33}
\]

3. The resistivity of LSCO

Let us consider here a sample of LSCO, with a doping parameter \( x = 0.19 \), which has a \( T_c = 38.5 \) K, that has been studied in \([19]\).

In figure 1 we plot our expression (24), for the zero field resistivity (solid blue line), together with the experimental data from \([19]\). In figures 2 and 3, we represent the curves corresponding to our expression (24), respectively for an applied magnetic field of 50 T and 80 T, along with the experimental data from \([19]\). In figure 4, we depict the three curves together, along with the one for 30 T. In all cases used the fact that \( M \) vanishes at the SC transition, \( T_c = 38.5 \) K, by assuming a linear behavior \( M \propto (T - 38.5) \) around the transition. Notice that for magnetic fields up to about 50 T, the SC phase shrinks but persists, while for stronger fields it is completely destroyed.

We see that our expression for \( \rho(T, H) \) is in excellent agreement with the experimental data for LSCO.
4. The magnetoresistivity of Bi2201

Let us now consider our general expression for the resistivity, in the SM phase, equation (30), taken as a function of the applied magnetic field $H$. Let us apply it for the sample of Bi2201, having $T_c \simeq 1$ K at a fixed temperature $T = 4.2$ K studied in [1].

According to our expression for the SC transition temperature of cuprates [15, 20], for Bi2201 a critical SC temperature of $T_c \simeq 1$ K corresponds to a stoichiometric doping parameter $x = 0.377$.

Then, according to our expression for the PG temperature $T^*$ [15, 20] of cuprates, such doping parameter corresponds to $T^* = 3.15$ K. The sample of Bi2201, studied in [16] at a temperature of $T = 4.2$ K, therefore must be in the SM phase, where $M = 0$ and $\mu = DT$ [15, 20].
Figure 5. Magnetoresistivity of Bi2201. Our theoretical expression, derived from first principles (green line), accurately describes the experimental result obtained in [1], for a sample of Bi2201 with $T_c \approx 1$ K, which corresponds to a PG temperature $T^* = 3.15$ K [20]. The measurement is made at a temperature $T = 4.2$ K, which is larger than $T^*$, implying that the material is in the SM phase. The (linear) dashed line is added just to emphasize the crossover of the dependency of $\rho$ with $H$.

Figure 6. Magnetoresistivity of Bi2201, for the same sample of figure 5. The different curves represent our analytical result corresponding to temperatures of 4.2 K (green), 5 K (red), 10 K (blue), 20 K (gold) and 30 K (cyan). Experimental data are for the 4.2 K sample (same as in figure 5).

Using our expression (30) at a fixed temperature of $T = 4.2$ K, for $M = 0$ and choosing $\lambda_1 = 25.32$, $\lambda_2 = 3$ and a residual resistivity $\rho_0 = 100$ $\mu\Omega \text{cm}$, we obtain the curve depicted in green in figure 5. The experimental data are from [1].

In figure 6 we show the MR curves for the same sample of B12201 at different temperatures.

5. The resistivity of Ti2201 and the location of the QCP

Let us take the case of Ti2201, in order to address the issue of the power-law dependence of the zero field resistivity near the SC dome in OD cuprates. As it turns out, knowledge of such power-law will enable to clarify the issue concerning the location of the quantum critical point (QCP) associated to the SM phase. For this purpose, we are going to use the results obtained in [15], according to which, we have the following power-law regimes for the resistivity just outside the SC dome: SM, fermi liquid (FL), crossover (C):

$$
\begin{align*}
\text{SM} & : \rho \propto T \\
\text{FL} & : \rho \propto T^2 \\
\text{C} & : \rho \propto T^{1+\delta} \quad \delta \in [0, 1].
\end{align*}
$$

(34)

We also recall that the resistivity behavior in the upper PG phase shares the $T$-linear behavior with the SM phase [15].

The scaling function $G(K_1, K_2)$ has the following types of behavior in each of the regions above [15]:

$$
\begin{align*}
\text{SM} & : G \propto T \\
\text{FL} & : G \propto C \\
\text{C} & : G \propto \left(\frac{T^*}{T}\right)^{1-\delta} \quad \delta \in [0, 1].
\end{align*}
$$

(35)

Let us consider now, the two following scenarios for the phase diagram of cuprates, which we illustrate for the case of Ti2201.

In the first scenario (I), depicted in figure 7, the QCP where the PG line $T^*$ (x) ends, is located precisely at the edge of the SC dome, while in the second scenario, (II) which is depicted in figure 8, the QCP is located inside the SC dome.

Attentive inspection of these phase diagrams allows for the following conclusion. In the first scenario, the transition from the SC dome, in the OD region, always leads to a linear $\rho \propto T$ behavior of the resistivity. In the second scenario, conversely, according to figure 8, the resistivity behavior depends on where we cross the SC dome: if we do it below the green line on the right-hand-side, we shall have a $\rho \propto T^2$ behavior. When we cross the SC dome between the dashed line and the green line on the right-hand-side, we shall have, conversely, a $\rho \propto T^{1+\delta}$, super-linear behavior. Finally, when we cross the SC dome in between the dashed line on the left-hand-side and the green line, we shall have a linear behavior, $\rho \propto T$. In any of the three cases, however, as we raise the temperature, we will eventually reach a $T$-linear behavior of the resistivity.

From the behavior of the resistivity of a given cuprate material in the OD region one may infer about what type of scenario we will observe in its phase diagram, concerning
especially the PG temperature line $T^*(x)$ and the position of the QCP.

For the case of LSCO, for instance, the behavior exhibited in figure 1, strongly suggests that scenario I applies to this material.

Let us consider now the case of Tl2201. We evaluated the zero field resistivity, just above the SC transition for four samples, having, respectively, transition temperatures $T_c = 7$ K, 22 K, 35 K, 57 K. We did the calculation using (27), with the different scaling functions in (35). The blue curves were obtained by using the scaling function of the FL phase. The red curves, conversely, were obtained with the scaling function of the Crossover.

The result, compared with experimental data of [16], is shown in figures 9–12. It shows unequivocally that the blue
Figure 11. Resistivity of Tl2201 at zero magnetic field, for a sample with $T_c = 35$ K. The blue line is our theoretical expression, calculated with the scaling function appropriate for the FL phase, while the red line would be the result, should we did the calculation with the one corresponding to the Crossover. Experimental data from [16].

Figure 12. Resistivity of Tl2201 at zero magnetic field, for a sample with $T_c = 57$ K. The red line is our theoretical expression, calculated with the scaling function appropriate for the Crossover, while the blue line would be the result, should we did the calculation with the one corresponding to the FL phase. Experimental data from [16].

Curves are the ones that correctly describe the resistivity of Tl2201, for the 7 K, 22 K and 35 K samples of Tl2201 while the red curve correctly describe the resistivity of the 57 K sample. We conclude that this sample undergoes the SC transition into the Crossover region while the other three samples do it from SC to FL phases. Remarkably, we can confirm the previous conclusions by visual inspection of the phase diagram in figure 8, where the four red dashed lines represent the above samples of Tl2201.

The results above, consequently, strongly suggest that scenario II applies for Tl2201.

6. Quadrature and scaling

It was pointed out in [1] the existence of a crossover in the MR in cuprates, from a quadratic behavior at low fields to a linear behavior in the high-fields regime. Our theoretical expression reproduces the experimentally observed crossover (see figures 5 and 6).

This type of behavior, in materials such as electron doped cuprates and iron pnictides is usually ascribed to a quadrature scaling of the MR supposed to be associated to quantum critical phases.

In the case of cuprates, however, the same study of the MR in the SM phase indicates that the quadrature scaling is violated, in spite of the $H$-field crossover. Moreover, the MR field derivative is shown to scale as function of $H/T$.

Our results indicate that despite exhibiting the quadratic to linear crossover, which is observed experimentally, the resistivity of cuprates in the SM phase does not show a quadrature scaling dependence. Rather it depends on $H$ and $T$, through the function in (30), which was derived from our general theory for the cuprates [15, 20].

In figure 13, we display the field derivative of our expression (30), plotted as a function of the ratio $H/T$, namely, for $y = \frac{H}{T}$:

$$\frac{\partial \rho_{SM}(H, T)}{\partial H} = \frac{\partial \rho_{SM}(y, T)}{\partial y} \frac{\partial y}{\partial H} = \frac{T}{T} \frac{df(y)}{dy} = f'(y), \quad (36)$$

where we used that $\rho_{SM}(H, T) = Tf(y)$. 
The resulting expression has precisely the form of the collapsed experimental data exhibited in [1], which indicates the correctness of the MR derived from our theory.

7. Conclusion

We have derived, from our recently proposed theory for the high-Tc cuprates, an analytic expression for the resistivity in the presence of an external magnetic field. This shows an excellent agreement with the experimental data for the resistivity of LSCO at different values of the applied magnetic field.

The associated MR presents the crossover from parabolic to linear dependence despite the fact that it does not satisfy a quadrature scaling. Yet, the magnetic field derivative of the MR presents a $H/T$ scaling, in complete agreement with the experimental data near quantum critical points [1].

We introduced a method to determine whether the QCP associated to the PG temperature $T^*$ and the SM phase is located inside or outside (at the edge) of the SC dome. This is based on the observation of the power-law behavior of the resistivity, as a function of $T$, just above $T^*$, in the OD region. Our results indicate that the QCP is inside the dome for Tl2201 and on its very edge, for LSCO.

We see that our theory for the high-Tc cuprates has a considerable predictive power, providing accurate theoretical description for a wide variety of phenomena related to these compounds. Nevertheless, the model has not yet been employed in the description of the charge ordering phenomena associated to the PG temperature $T^*$.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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