Abstract—Compressed sensing magnetic resonance imaging has shown great capability to accelerate data acquisition by exploiting sparsity of images under a certain transform or dictionary. Sparser representations usually lead to lower reconstruction errors, thus enduring efforts have been made to find dictionaries that provide sparser representation of magnetic resonance images. Previously, adaptive sparse representations are typically trained with K-SVD and the state-of-the-art image quality is achieved in image reconstruction. However, this reconstruction is time consuming because of the relatively slow training process. In this paper, we introduce a fast dictionary learning method, which is essentially an adaptive tight frame construction, into magnetic resonance image reconstruction. To enhance the sparsity, images are divided into classified patches according to the same geometrical directions and dictionary is trained within each class. We set up a sparse reconstruction model with the multi-class dictionaries and solve the problem with a fast alternative direction multiplier method. Experiments on real magnetic resonance imaging data demonstrate that the proposed approach achieves the lowest reconstruction error compared with several state-of-the-art methods and the computation is much faster than previous dictionary learning methods.

Index Terms—Compressed Sensing, Dictionary Learning, Magnetic Resonance Imaging

I. INTRODUCTION

THE compressed sensing (CS) theory proved that a sparse signal can be accurately reconstructed from a small number of random measurements [1, 2]. In magnetic resonance imaging (MRI), imaging speed is critical for MRI applications. CS is introduced into MRI and it has significantly reduced the data acquisition time [3]. This new imaging technology is short for CS-MRI. Besides, its combination with other fast MRI methods, e.g. parallel imaging [4-9], non-Cartesian sampling [10-12], CS-MRI. Overall, this new method makes use of random measurements [1, 2]. In magnetic resonance imaging (MRI), imaging speed is critical for MRI applications. CS is introduced into MRI and it has significantly reduced the data acquisition time [3].

Finding optimally sparse representations of images is an active research area since sparser representation usually leads to lower reconstruction error [20-22]. Within the field of MRI, MR images were sparsely represented by pre-specified dictionaries [3]. However, these dictionaries usually capture only one type of image features, and reconstruction qualities are not satisfactory. For example, contourlets [23], bandelet [24] are applicable to piecewise smooth images with smooth boundary. Combination of wavelets, contourlets and total variation [25] can suppress the artifacts produced by one transform but there is still loss of image structures when data are highly undersampled.

Recently, adaptive dictionaries have been explored by enforcing the sparsity on image patches [21, 22, 26-29]. Reconstructed images are significantly improved comparing with the previous pre-specified dictionaries in CS-MRI since these sparse representations are adaptive to the imaging data. K-SVD [20] is a typical dictionary learning method which has been applied in CS-MRI for a single image [21, 28, 30] or image series [31-34]. The main problem of using K-SVD in CS-MRI is its high computation cost [20]. As a result, these methods are time consuming in the iterative reconstruction of MRI [21] and may fail to sparsely represent some patches that are excluded in training to reduce the computation.

In this paper, we present a fast dictionary learning method on classified patches (FDLCP) to reconstruct MR image from highly undersampled data. The dictionaries training is implemented by a small singular value decomposition (SVD) and a thresholding operation, resulting in a simple computationally efficient algorithm. To improve the sparsity, multi-class dictionaries are trained on the classified image patches according to their geometrical directions. A sparse image reconstruction model is proposed on the multi-class dictionaries in CS-MRI. Overall, this new method makes use of the similarity of patches geometrical direction and provides a better sparse approximation for the target image.

To illustrate the benefits of our proposed method, we carried out experiments on both phantom data and realistic brain imaging MR imaging data. The experiments show that our proposed classified dictionaries provide a sparser representation than the similar adaptive but non-classified dictionary. Moreover, our method performs state-of-the-art MR reconstruction methods, dictionary learning MRI (DLMRI) [21], wavelets tree MRI (WaTMI) [35], and patch-based directional wavelets (PBDW) [22], in reducing artifacts, minimizing reconstruction error and saving computational time.

The rest of the paper is organized as follows. In Section II, we briefly review the CS-MRI and the fast dictionaries training
algorithm. Then, in Section III, we propose the multi-class dictionaries sparse reconstruction model for CS-MRI and derive an efficient iterative algorithm. Section IV demonstrates the performance of the proposed method. In Section V, we make the conclusion and discuss the future work.

II. BACKGROUND AND RELATED WORK

A. CS-MRI

MR images can be reconstructed from undersampled data by employing the sparse reconstruction model. Let \( x \in \mathbb{C}^{N^2} \) be the reconstructed \( N \times N \) image in a vector form, \( F_u \in \mathbb{C}^{M \times N^2} (M < N^2) \) is the undersampled Fourier encoding matrix, and \( y \) is the undersampled k-space measurements. Define \( \Psi \in \mathbb{C}^{N^2 \times M} \) as the sparsifying transform, where images have sparse representations under this transform. A typical CS-MRI reconstruction is obtained by solving the following problem:

\[
\min_x \| \Psi x \| \quad s.t. \quad \| y - F_u x \| < \varepsilon ,
\]

(1)

where \( \varepsilon \) is a parameter controls the fidelity of the reconstruction to the measured data. Minimizing \( \| \Psi x \| \) promotes the image sparse representation and the constraint \( \| y - F_u x \| < \varepsilon \) enforces the data consistency. The Eq. (1) tries to find the sparsest representation among all possible solutions that are consistent with the acquired data.

B. Fast Dictionary Learning (FDL)

Most existing adaptive dictionaries are performed on image patches [20, 36]. The basic idea of these approaches is to find a set of atoms, columns in the dictionary, from image patches extracted so that image patches can be approximated by a sparse linear combination of these atoms.

Given an image \( x \in \mathbb{C}^{N^2} \), let \( x_i \in \mathbb{C}^{N^2} \) denotes the \( i \)-th image patch extracted from the image \( x \), \( D \in \mathbb{C}^{N^2 \times k} \) stands for the adaptive dictionary, and \( a_i \in \mathbb{C}^{k} \) is the sparse representation of \( x_i \) with respect to dictionary \( D \). The adaptive dictionary is obtained by solving the following minimization problem

\[
\min_{a_i \in \mathbb{C}^{k}} \sum_i \| x_i - Da_i \|^2 + \lambda^2 \| a_i \|^2.
\]

(2)

The popular K-SVD method [20] learns an over-complete dictionary, meaning \( k \) \( > \) \( n^2 \), from the input image according to Eq. (2). The K-SVD method adopts a greedy algorithm that is very computationally demanding. Even though signals are more likely to have a good sparse approximation under a redundant dictionary, finding an optimal over-complete dictionary is usually an ill-posed and under-constrained problem.

There exists other attempt on finding an adaptive dictionary \( D \) that satisfies orthogonality \( D^T D = I \) [36, 37]. When all possible patches are sampled, the orthogonal dictionary learning is equivalent to an adaptive wavelet tight frame construction [36]. It has shown that orthogonal dictionary learning achieves comparable performance to the K-SVD in image denoising but runs much faster [36]. It is expected that the dictionary will also lead to fast computation and adaptively sparse representation in CS-MRI.

Let \( X = [x_1, x_2, \ldots, x_J] \in \mathbb{C}^{M \times J} \) denote the set of trained images patches, \( A = [a_1, a_2, \ldots, a_J] \in \mathbb{C}^{M \times J} \) is the sparse approximation of images patches \( X \) under the orthogonal dictionary \( D \). The tight frame learning is achieved by [36]

\[
\min_{\hat{A}} \sum_{j=1}^{J} \| x_j - D^T \hat{A} x_j \|^2 + \lambda^2 \| \hat{A} \|_2 \quad s.t. \quad D^T D = I .
\]

(3)

The Eq. (3) is solved by alternately computing the sparse coding \( A \) with simple hard thresholding and updating the dictionary \( D \) with a small SVD decomposition in each iteration. Hence the dictionary learning algorithm is simple and the whole training process is much faster than the commonly used K-SVD [36]. We refer to [36] for more details.

III. PROPOSED METHOD

A. Fast Dictionary Learning on Classified Patches

Two properties of adaptive dictionary are expected. First, it is able to enforce patches sparsity of the target image. Second, the learning process is computationally efficient. While the latter one has been solved by SVD with hard thresholding, how to provide an optimal sparse representation is still challenging. Since image patches contain substantial and distinct features, adaptive dictionary learnt from all the images patches may not capture all the valid image features sufficiently.

The proposed method incorporate this information to benefit dictionary learning, inspired by the critical role of estimating geometrical directions in sparse representation of images [22, 24]. We choose the geometrical direction estimation proposed in [24] because the computation is fast and this method can preserve the image directions very well for CS-MRI [22].

Let \( R_j \in \mathbb{R}^{N^2 \times J} \) stands for an operator extracting the \( j \)-th image patch and the total of patches is \( J \), the optimal direction \( \omega_{i,j} \) of the \( j \)-th patch is estimated according to

\[
\min_{\omega_{i,j} \in \{1, 2, \ldots, n^2\}} \| \hat{e}_{i,j} - W^T G_{\omega} R_j x_i \|_2^2
\]

(4)

where \( G_{\omega} \) is an operator that re-arrange pixels along a candidate geometrical directions \( \{\theta_1, \ldots, \theta_{n^2}\} \), \( W^T \) is the forward 1D Haar wavelets and \( \hat{e}_{i,j,\omega} \) is the preserved 25% largest wavelet coefficients. The geometrical directions provide optimal sparsity among candidate directions. Details of the direction estimation are skipped here and can be found in [24]. As shown in Fig. 1(a) and (b), the red lines imply that the direction of image patches are estimated very well.

Dictionary training is performed on patches of the same class which are classified according to their geometrical directions. For example, one class of patches with anti-diagonal
geometrical directions is formed in the Fig. 1(c). Then the
dictionary training is performed within each class according to
\[ \min_{D_o} H_{st} \omega \omega \omega \omega \lambda - + = D_A \omega X A \omega D D I , \quad (5) \]
where \( D_o \) is the dictionary for the patches \( \omega \) that shares the
same geometrical directions \( \omega \).

The Eq. (5) is solved by alternately computing the sparse
coding \( A \) and updating the dictionary \( D_o \) with SVD in each
iteration.

The sparse coding sub-problem is
\[ \min_{A} \| X_o - D_o A \|_F^2 + \lambda^2 \| A \|_1 \quad \text{s.t.} \quad D_o^H D_o = I , \quad (6) \]
where \( X_o \) is the dictionary for the patches \( X_o \) that shares the
same geometrical directions \( \omega \).

The Eq. (5) is solved by alternatingly computing the sparse
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same geometrical directions \( \omega \).

The Eq. (5) is solved by alternatingly computing the sparse
coding \( A \) and updating the dictionary \( D_o \) with SVD in each
iteration.

The dictionary updating sub-problem is
\[ \min_{D} \| X_o - D_o A \|_F^2 \quad \text{s.t.} \quad D_o^H D_o = I , \quad (9) \]
that is solved by
\[ D_o^{(k+1)} = P V^H \quad (10) \]
where \( P \) and \( V \) are the orthogonal matrices of the following SVD
\[ X_o A^H = P A V^H . \quad (11) \]

The process of FDLCP is summarized in Algorithm 1.

![Algorithm 1 Fast dictionary learning on classified patches](image)

**Initial:** Set the initial dictionary \( D_o^{(0)} (\omega = \omega_1, \cdots, \omega_Q) \)

1. Estimate geometric directions of patches according to
   Eq. (4);
2. Form classes of patches sharing the same directions;
3. For each geometric direction \( \omega \in \{ \omega_1, \cdots, \omega_Q \} \)
   
   - For iterations \( k = 1, 2, \ldots, K \)
   - Do the sparse coding following Eq. (7);
   - Run the SVD following Eq. (11);
   - Update the dictionary according to Eq. (10);
   - If converge, save one class dictionary \( D_o \); else, go to step 4;
4. Output multi-class dictionaries \( D_o (\omega = \omega_1, \cdots, \omega_Q) \).

The advantage of classifying patches is illustrated in Fig. 1. It
shows that the sparsest representation is obtained using the
proposed FDLCP. The trained dictionaries using FDL and
FDLCP are adapted to images thus provide sparser
representation than the non-adaptive Haar wavelets (Fig. 1(d)).
The trained dictionary using FDL (Fig. 1(e)) represents main
directions of all patches but may not sufficiently capture one
direction contained in single class of patches due the
orthogonality of dictionary. On the contrary, the trained
dictionary (Fig. 1(f)) using FDLCP tends to fit patches with a
specific geometrical direction. Therefore, the proposed FDLCP
achieve the sparsest representation in Fig. 1(g), where it leads
to the fastest decay of approximation error, the relative \( \ell_2 \)
norm error (RLNE) \[22, 29\], when certain portions of largest
coefficients are preserved.

**B. Sparse Reconstruction Model with Multi-class Dictionary**

Equipped with the trained dictionaries, we are ready for the
undersampled MR image reconstruction.

Let \( \Psi = D^H_o \) is a dictionary with the geometrical direction
\( \omega \) for the \( j \)th image patch, an MR image is reconstructed by
solving the following minimization problem:
\[ \min_{\lambda} \frac{1}{2} \| y - F C \|_2^2 + \sum_{j=1}^{Q} \| \Psi_{\omega_j} R X \|_1 \quad \text{s.t.} \quad \Psi^H_{\omega_j} \Psi_{\omega_j} = I . \quad (12) \]

The \( l_1 \) norm term in the Eq. (12) enforces the fidelity of the
reconstruction to the undersampled k-space data. The \( l_1 \) norm
term promotes the patches sparse representation with respect to
corresponding trained dictionaries. The regularization parameter \( \lambda \) trades off between sparsity and data fidelity.

As the patches are extracted from the image with the same
where $c$ is the overlap factor.

Define a transform $\Psi = [\Psi_{a_1} R_1, \Psi_{a_2} R_2, \ldots, \Psi_{a_I} R_I]'$ that satisfies

$$\Psi^H \Psi = \left[ \begin{array}{cccc} R_1^H \Psi_{a_1}^H & R_1^H \Psi_{a_2}^H & \cdots & R_1^H \Psi_{a_I}^H \\ R_2^H \Psi_{a_1}^H & R_2^H \Psi_{a_2}^H & \cdots & R_2^H \Psi_{a_I}^H \\ \vdots & \vdots & \cdots & \vdots \\ R_I^H \Psi_{a_1}^H & R_I^H \Psi_{a_2}^H & \cdots & R_I^H \Psi_{a_I}^H \end{array} \right]$$

and let $\Phi = \frac{1}{\sqrt{c}} \Psi$, one has

$$\Phi^H \Phi = I \quad (14)$$

This shows that the rows of $\Phi$ form a tight frame in image space. Therefore, the proposed FDLCP is actually an adaptive tight frame construction method. With Eq. (14), MR image reconstruction in Eq. (12) can be rewritten as

$$\min_{x,z} \frac{1}{2} \| y - F x \|_2^2 + \| \Phi z \|_2^2 \quad \text{s.t.} \quad \Phi^H \Phi = I \quad (15)$$

This model means that the target MR image is reconstructed by enforcing its sparsity under a transform embedded with the geometrical directions and trained dictionaries.

How to solve Eq. (15) numerically is presented below.

C. Numerical Algorithm

To solve Eq. (15), we follow the split Bregman for tight frame image restoration [38]. First, an auxiliary variable $\alpha = \Phi x$ is introduced to split the $\ell_1$ norm and $\ell_2$ norm terms. Eq. (15) is equivalent to

$$\min_{x,a} \frac{1}{2} \| y - F x \|_2^2 + \| a \|_2^2 \quad \text{s.t.} \quad \alpha = \Phi x \quad (16)$$

Then we utilize alternative direction multiplier method (ADMM) [39] to solve Eq. (16).

$$\min_{x,a} \frac{1}{2} \| y - F x - h \|_2^2 + \| \Phi x - a - d \|_2^2 + \frac{\beta}{2} \| a \|_2^2 \quad (17)$$

where $d$ and $h$ are two Lagrangian multipliers. The ADMM technique turns Eq. (15) into iteratively solving following sub-problems:

$$\chi^{(k+1)} = \arg \min_{x,a} \frac{1}{2} \| y - F x \|_2^2 + \frac{\beta}{2} \| \Phi x - a^{(k)} - d^{(k)} \|_2^2$$

$$a^{(k+1)} = \arg \min_{a} \| a \|_2 + \frac{\beta}{2} \| \Phi x^{(k+1)} - a - d^{(k+1)} \|_2^2$$

$$h^{(k+1)} = h^{(k)} - \delta_h (F x^{(k+1)} - y)$$

$$d^{(k+1)} = d^{(k)} - \delta_d (\Phi x^{(k+1)} - a^{(k+1)})$$

where $\delta_h$ and $\delta_d$ are two constant step size.

Algorithm 2 MR image reconstruction with FDLCP.

**Initialize:** Input the undersampled k-space data $y$ and the adaptive dictionaries; set $x^{(0)} = F^H y$.

**Main:**

1. Update $x$ by solving normal equation Eq. (19);
2. Compute the sparse coefficients $a$ in Eq. (20);
3. Update the multiplier $z$ and $h$ in Eq. (18);
4. Until converged

**Output:** The reconstructed image $\mathbf{x}$.

For fixed $\alpha^{(0)}$, $z^{(0)}$ and $h^{(0)}$, $\alpha^{(n+1)}$ has a close-form solution:

$$\alpha^{(n+1)} = S_{\lambda \beta} (\Phi x^{(n+1)} - z^{(n)})$$

$$= \max \left( \Phi x^{(n+1)} - z^{(n)}, -\frac{1}{\beta} \mathbf{0} \right) \Phi x^{(n+1)} - z^{(n)}$$

The numerical algorithm is summarized in Algorithm 2.

D. The Whole Process of the Proposed Method

The complete process of our proposed method is shown in Fig. 2. It consists of four stages, which are reference image forming, patch classification, dictionaries learning and sparse MR image reconstruction. First, a reference image is reconstructed from undersampled k-space data using shift invariant discrete wavelet (SIDWT) [22, 29]. Second, geometrical directions are estimated on the patches of the reference image and patches sharing the same direction belong to the same class. Third, one dictionary is trained within single class and multi-class dictionaries are constructed for all classes. Last, image is reconstructed using the multi-class dictionaries. As the initial reference image usually contains obvious artifacts which may reduce the accuracy of patch classification, reference will be updated once again to improve the reconstruction. Using the SIDWT to obtain the initial reference images is not new and has been used in CS-MRI before [22, 29], thus it is not the focus of this paper. In this paper, we mainly
discuss how to train multi-class dictionaries fast and reconstruct sparse MR image accordingly.

IV. RESULTS

In this section, image reconstructions on phantom and in vivo MR data are carried out to evaluate the performance of the proposed method. Cartesian sampling with random phase encoding [3], 2D random sampling [3, 22, 29] and pseudo radial sampling [16] are adopted here. The proposed FDLCP method is compared with three state-of-the-art CS-MRI methods: WaTMRI [35] which utilizes the wavelet tree sparsity in MR images, DLMRI [21] which is a typical dictionary learning method in CS-MRI, and PBDW [22] which enforces the sparsity using patch-based directional wavelets.

In all FDLCP experiments, we use 3-level Daubechies wavelets in SIDWT [29] to obtain the reference image, and set each image patch size as 8×8 with maximum patch overlap. We pre-define 71 different geometrical directions for 8×8 patches classification that is typically set in PBDW [22].

Reconstruction performance is quantified by relative $l_2$ norm error (RLNE) [22] and structure similarity index (SSIM) [40]. Lower RLNE means the reconstructed image is more consistent to the fully sampled image, and higher SSIM indicates the structures are better preserved in the reconstruction.

A. Experiments on Phantom Data

Fig. 3 shows reconstrced images on a phantom data. The fully sampled phantom, as shown in Fig. 3(a), is acquired from 3T Simens MRI scanner using a turbo spin echo sequence (matrix size = 384×384, TR/TE=2000/9.7ms, field of view = 230×187mm$^2$, slice thickness = 5.0mm). This phantom is full of geometrical directivity features.
WaTMRI introduces obvious artifacts while DLMRI causes ringing around the edges. PBDW reconstructs images much better but produces artifacts in the smooth region in Fig. 3(d) and loses some edges in Fig. 3(i). The proposed FDLCP reconstructs the image best in Fig. 3(e) and leads to minimal loss of image features in Fig. 3(j). The quality metrics listed in Table I implies that FDLCP achieves the lowest RLNE and highest SSIM among all methods.

| Images   | WaTMRI | DLMRI | PBDW | FDLCP |
|----------|--------|-------|------|-------|
| Phantom  | 0.1542 | 0.0835| 0.0461| 0.0315|
|          | 0.7164 | 0.8693| 0.9681| 0.9846|
| Brain    | 0.1607 | 0.1414| 0.1145| 0.0959|
|          | 0.8243 | 0.8650| 0.9468| 0.9630|

B. Experiments on Brain Imaging Data

The T2-weighted and T1-weighted brain imaging data are obtained from different scanners. Fig. 4(a) and Fig. 7(d), T2-weighted brain MR images, are two slices measured from a healthy volunteer at a 3T Simens Trio Tim MRI scanners using the T2-weighted turo spin echo sequence (matrix size = 256×256, TR/TE = 6100/99ms, field of view = 220×220mm², slice thickness = 3.0mm). Fig. 6(a) is another T2-weighted image measured from a healthy volunteer at another 3T Simens scanner using a turbo spin echo sequence (matrix size = 384×324, TR/TE = 5000/97ms, field of view = 230×187mm², slice thickness = 5.0mm). T1-weighted brain images, Fig. 7 (e) and (f), are two slices obtained from a healthy volunteer at 1.5T Philips MRI scanner with fast-field-echo sequences (matrix size = 256×256, TR/TE = 1700/390ms, field of view = 230×230mm², slice thickness = 5mm).

The reconstruction errors in Fig. 4 show that FDLCP has lowest errors near edges and the fewest aliasing artifacts in the smooth region. Visual inspection is consistent to the two reconstruction metrics. The RLNEs and SSIMs in Table I points out that FDLCP leads to the lowest reconstruction error and highest reconstruction structure similarity among four reconstruction methods.

Using the same Cartesian sampling schemes with different sampling rates, consistent reduction on the reconstruction relative error RLNE and improvement on the structure similarity index SSIM is observed in Fig. 5. Therefore, the proposed reconstruction method outperforms the three state-of-the-art methods in this case.

![Fig. 5. Brain image reconstruction qualities versus different sampling rates. (a) RLNE versus different sampling rates; (b) SSIM versus different sampling rate.](image)

We also test the performance of FDLCP with different sampling patterns. Pseudo radial sampling is employed in Fig. 6. The error image of FDLCP is less structured, which indicates that FDLCP preserves the image features better than other methods. Besides, the superior RLNE and SSIM metrics, shown in Fig. 6(g) and (h), also implies the advantage of FDLCP. Another two different sampling patterns on more brain images were tested in Fig. 7. The RLNE and SSIM metrics are listed in Table II. The result implies that FDLCP always performs better than the compared methods.

![Fig. 6. Reconstruction error using radial sampling pattern. (a) A fully sampled brain image; (b) Pseudo radial sampling pattern with 0.18 full data; (b-c-e-f) Reconstruction errors for WaTMRI, DLMRI, PBDW and FDLCP, respectively; (g) RLNEs versus different sampling rates; (h) SSIM versus different sampling rates.](image)

![Fig. 7. Three sampling patterns and more brain images. (a) The Cartesian sampling pattern of sampling rate 0.20; (b) The 2D random sampling pattern of sampling rate 0.15; (c) The pseudo radial sampling pattern of sampling rate 0.18; (d) A T2-weighted brain image; (e-f) Two different slices T1-weighted brain images.](image)
A new MR image reconstruction method based on fast multi-class dictionary learning is proposed. Image patches are classified according to their geometrical directions and dictionaries are trained fast within each class. The alternative direction multiplier method is adopted to reconstruct the image efficiently. Results on phantom and brain imaging data demonstrate the superior performance of the proposed method in suppressing artifacts and preserving image edges. The proposed method outperforms the state-of-the-art MR image reconstruction methods and its computation is much faster than typical dictionary learning methods. In the future, how to optimally classify image patches and provides sparser image representation in CS-MRI will be further developed.

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