Simulation for Callable Convertible Discount Bonds with Monte Carlo Method

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Abstract. Pricing of Callable Convertible Discount Bonds is based on non-redemption constraints under a discrete framework. The Monte Carlo method is used to conduct a large number of random simulations on the change path of the underlying stock price on the future time $T$. And then average it and discount it at a risk-free rate. Finally, it can get the price of the callable convertible discount bonds. The value of convertible bonds under different stock prices is analyzed in the form of charts. At the same time, we also consider the effect of different parameter variables on the value of convertible bond. Therefore, we can have a comprehensive understanding of the pricing of the callable convertible discount bonds.

Introduction

The Monte Carlo method is widely applied in many fields. It is applicable to multi-dimensional derivative securities pricing characteristics and easy to deal with the realistic characteristics of discrete coupon dividends, path dependence and other convertible bonds. As a result, it has gradually become one of the most effective methods in the pricing of convertible bonds. In a risk-neutral world, stock prices are largely subject to geometric Brownian motion. Therefore, we can conduct a large number of repeated random simulations on the change path of stock prices on $T$ in the future, and then average the results of these simulations at a risk-free rate. In the end, we can get the value of the derivative.

Callable Convertible Discount Bonds referred to as CCDB. The restrictions are as follows:

When the price of the underlying stock continues to rise, the holder has the right to exercise the right of conversion at any time at a predetermined price within the remaining term of the convertible bond.

The publisher has the right to redeem at any time at a predetermined redemption price within the remaining term of the convertible bond.

This article assumes no redemption notice period because redemption notice period has little effect on the theoretical value of callable convertible discount bonds.

No resale terms, replacement terms and other non-standard terms.

At present, there are many methods to price convertible bonds. It mainly includes analytical solution method, partial differential equation based on solution analysis method and monte carlo simulation method. Callable convertible discount bond is a derivative asset. The different ending conditions of callable convertible discount bonds have different stock price paths. Different ending conditions of callable convertible discount bonds have different stock price paths. Different ending situations include the publisher exercises the right of redemption in advance, holder expires to execute the transformation policy and get the face value at maturity\cite{2}. Monte Carlo method is very suitable for the pricing of derivative assets with different path dependence\cite{3}. Financial statistics software Matlab can flexibly calculate and analyze the callable convertible discount bond pricing problem. Therefore, this article uses Monte Carlo method to analyze the pricing of callable convertible discount bonds.
Literature Review

Adoukonou Olivier, Andre Florence and Viviani Jean-Laurent research theory of Mayers to support that firms facing high overinvestment costs choose callable convertible bonds in order to implement an optimal sequential financing strategy. They studied the call provision embedded in the convertible bonds by comparing the investment and financing activities of the Western European firms around the exercise in detail. They use the Western European firms’ case to analyze the role of call provision in convertible bonds. The empirical results indicate that call provision does not play a significant role (in general) in sequential financing strategy. \[1\]

Exponential variance gamma process into the American-style convertible bond pricing framework can let price convertible bond become much more rigorous. \[6\]

Zhu Songping et al. uses integral equation method to price puttable convertible bonds. Two integral equation formulations are presented and then using numerical method to solve the integral equation. In the end, the bond price is then calculated from the integral representations in their respective domains. The key point is they obtain two free boundaries. The two free boundaries can be used to calculate the integral representations in their respective and then get the convertible price. Previous studies think it at most exist one free boundary. These two free boundaries can let price problem for callable convertible bonds become much more accurate. \[7\]

Natalya M. and Enrico P. research the relationship between convertible bonds and bank risk-taking. Bank convertible debt has risen rapidly to over 200 billion dollars in the last 5 years because of credit market disruption and spillovers to the real economy. Taking the example of American CoCo convertible bond to research how affect bank’s incentives. The model identifies explicitly the effect of contractual terms of contingent capital. The CoCo convertible bond can help to clarify the importance of going concern conversion which reduces leverage. \[4\]

Method Introduction

The basic idea of monte carlo simulation is to establish a statistical model or random process. The parameters are equivalent to the solution of the problem, and the estimated value and statistics of the parameters are calculated by repeated random sampling. So we can get an approximate solution to the problem. The more samples you have, the closer the approximation is to the real value. The first step is to generate the random path of the stock price. Then calculate the option yield according to the simulated random path. A large number of samples are generated and the benefits of each sample path are calculated by repeating these two steps over and over again. Then in the risk-neutral world, it can discount the risk-free interest rate to get the price of the callable convertible discount bond. It can get the price of the callable convertible discount bond by using simple arithmetic mean.

Assume that the Brownian motion of stock prices in a risk-neutral environment is as follows:

\[dS = \mu S dt + \sigma S dz\]  \hspace{1cm} (1)

\(dz\) is a standard Brownian motion. \(\mu\) is the yield in a risk-neutral world. \(S\) denotes the stock price. \(\sigma\) represents the standard deviation of expected instantaneous return on stock price and can also called volatility of the underlying stock price. \(dS\) is the derivative of the stock price. The above continuous model is discretized and a discrete equation of stock price movement is obtained.

\[S(t + \Delta t) - S(t) = \mu S(t) \Delta t + \sigma S(t) \epsilon \sqrt{\Delta t}\]  \hspace{1cm} (2)

In order to have a more accurate simulation in practical application. Not from stock \(S\), but from the price of logarithmic \(\ln(S)\) of the simulation. According to the law of Ito have \(\ln(S)\) to conform to the differential equations:

\[d\ln(S(t)) = (\mu - \sigma^2/2) dt + \sigma \epsilon \]  \hspace{1cm} (3)

Rewrite the above random process into discrete form:

\[\ln(S(t + \Delta t)) - \ln(S(t)) = (\mu - \sigma^2/2) \Delta t + \sigma \epsilon \sqrt{\Delta t}\]  \hspace{1cm} (4)
Equivalent form is as follows:

\[ S(t + \Delta t) = S(t) e^{(\bar{\mu} - \sigma^2/2)\Delta t + \sigma \epsilon \sqrt{\Delta t}} \]  

(5)

One of the advantages of doing a price log simulation is that \( \ln S(t) \) follows the broad Wiener process. This means that listing can be rewritten as follows:

\[ S(t + \Delta t) = S(t) e^{(\bar{\mu} - \sigma^2/2)\Delta t + \sigma \epsilon \sqrt{\Delta t}} \]  

(6)

Parameter introduction:

- \( B_F \) represents the par value of a convertible bond,
- \( B_C \) represents the redemption price of a convertible bond,
- \( P_1 \) expresses the conversion price of a convertible bond,
- \( S_0 \) indicates the current price of the stock,
- \( S_T \) represents the stock price in the future \( \tau \). \( S_T \) denotes the price of the stock when the convertible bond expires, \( S_c \) denotes the price of the underlying stock that makes the convertible value of the convertible bond equal to its redemption price \( B_c \).

If the price of the underlying stock reaches \( S_c = (B_c/B_F)P_1 \) before expiration, the publisher will announce the execution of redemption based on the best execution strategy. If the underlying stock price is never reached \( S_c = (B_c/B_F)P_1 \) before expiration, the issuer will not execute the redemption. It is an optimal option for the holder not to perform the conversion before maturity. According to the price of the underlying stock, if \( S_T > P_1 \) at expiration, the holder will perform conversion rights. So it can get the benefit of \( (B_F/P_1)S_T \). If \( S_T < P_1 \), the holder will get the benefit of \( B_F \).

The theoretical value of callable convertible discount bonds is shown in table 1:

| Convertible bond | Path of discount value | Exercise time | Exercise of rights |
|------------------|------------------------|--------------|--------------------|
| CCDB(\( S_0, T \)) | \( e^{-rT}(B_F/P_1)S_T \) | \( \tau_c<T \) | The publisher exercises the right of redemption |
|                  | \( e^{-rT}(B_F/P_1)S_T \) | \( \tau_c=T, S_T > P_1 \) | The holder exercises the right of conversion |
|                  | \( e^{-rT}B_F \) | \( \tau_c=T, S_T \leq P_1 \) | No right |

The path trend diagram of the underlying stock is shown in figure 1:

![Figure 1. The trigger of convertible bonds under different stock price paths.](image)

**Model Simulation**

According to the general market situation of China's convertible bond market, the numerical example of a callable convertible discount bond is as follows:

- Nominal value \( B_F = 100 \), the conversion price \( P_1 = 20 \), the redemption price \( B_C = 130 \), risk-free rate \( r = 0.03 \), share price volatility \( \sigma = 0.4 \), then \( S_c = (B_C/B_F)P_1 = 26 \). The value range of the current target stock price is divided into three ranges: 20~24, 24~28 and 28~32. In this way, the callable convertible discount bonds can be respectively in the price state, the parity state and the off-price state. The step length is assumed as 0.1 to simulate. 10,000 simulations and 300-time intervals make pricing more accurate and reliable. MATLAB simulation diagram is shown in figure 2,3,4 as follows:
Nominal value $B_F = 100$, the conversion price $P_1 = 20$, the redemption price $B_C = 130$, risk-free rate $r = 0.03$, share price volatility $\sigma = 0.4$. The initial target stock price is set as 22, 26 and 30. Considering the effect of the change in the remaining time $T$ from 1 to 3 years, 3 to 5 years and 5 to 7 years on the value of convertible bonds. As shown in figure 5, 6, 7 below:

Nominal value $B_F = 100$, the conversion price $P_1 = 20$, the redemption price $B_C = 130$, risk-free rate $r = 0.03$, time remaining $T = 5$. The initial target stock price is set as 22, 26 and 30. Considering the target price volatility is from 0.1-0.3, 0.3-0.5. the effect of stock price volatility on the value of convertible bonds. As shown in figure 8, 9 below:

Nominal value $B_F = 100$, the conversion price $P_1 = 20$, the redemption price $B_C = 130$, time remaining $T = 5$. Share price volatility $\sigma = 0.4$. The initial target stock price is set as 22, 26 and 30. Considering the risk-free rate $r$ from 0.02 to 0.04, 0.04 to 0.06. The effect of the change of riskless interest rate on the value of convertible bonds. As shown in figure 10, 11 below:

Conclusions

The key problem with applying monte Carlo simulation to the pricing of convertible bonds is that the present value calculation of convertible bonds under different stock price paths. In this article, Matlab statistical analysis software is used to price the callable convertible discount bonds under 10000 times
of simulation using monte Carlo method. The path simulation is carried out according to the constraint conditions of callable convertible discount bond. The theoretical value is obtained by a large number of experiments. With the increase of convertible bond constraints, the path will become much more complex. The main purpose of this article is to provide some ideas for the pricing of China's convertible bonds.

In the process of simulation, the effect of the different price on the value of convertible bonds is considered. The rest of the time T, the price volatility of the shares $\sigma$ and the changes in the riskless interest rate $r$ affect the price of the callable convertible discount bond. It enables readers to have a deeper understanding of the impact of changes in different variables on the price of convertible bonds. So that we can understand the full range of callable convertible discount bonds.

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