Cluster radioactivity of Th isotopes in the mean-field HFB theory.

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Cluster radioactivity is described as a very mass asymmetric fission process. The reflection symmetry breaking octupole moment has been used in a mean field HFB theory as leading coordinate instead of the quadrupole moment usually used in standard fission calculations. The procedure has been applied to the study of the “very mass asymmetric fission barrier” of several even-even Thorium isotopes. The masses of the emitted clusters as well as the corresponding half-lives have been evaluated on those cases where experimental data exist.

1. Introduction

Cluster radioactivity, first predicted theoretically by Sandulescu et al.\textsuperscript{11} in 1980, was discovered in 1984 by Rose and Jones\textsuperscript{2} in the spontaneous reaction $^{223}\text{Ra} \rightarrow ^{14}\text{C} + ^{209}\text{Pb}$. Although cluster emission is a very exotic process, with a relative branching ratio to $\alpha$-decay of the order of $10^{-10} - 10^{-17}$, it has been observed in many actinide nuclei from $^{221}\text{Fr}$ to $^{242}\text{Cm}$. Clusters emitted in these reactions are light nuclei from $^{14}\text{C}$ to $^{34}\text{Si}$, whereas the heavy mass residue is a nucleus that differs from the doubly magic $^{208}\text{Pb}$ by no more than four nucleons.

Cluster emission fills up the gap in the fragment’s mass spectrum of the nuclear decay between $\alpha$ radioactivity and spontaneous fission, where the masses of the fragments are typically greater than 60. From a theoretical point of view, cluster radioactivity may be treated as the emission of a pre-formed cluster inside the nucleus in close analogy to $\alpha$-decay. The alternative approach is to consider these reactions as a particular case of very mass asymmetric fission.

In this paper the potential energy surfaces (PES) of several even mass Th isotopes obtained with the help of the Hartree-Fock-Bogoliubov (HFB) theory and the D1S Gogny force are analyzed. As it is well established, this methodology and force have been successfully applied in the calculation of the spontaneous fission prop-
properties of heavy nuclei. Therefore, it seems natural to think that this method could also be applied to investigate very asymmetric fission leading to the emission of clusters. Preliminary explorations in this direction have shown that this is indeed the case and with the present calculation we want to extend the description to other nearby region of the Nuclide Chart.

2. Theoretical Model and Results

Fission barriers are obtained in the mean-field models by calculating potential energy curves as a function of a convenient quantity by considering a constraint applied on the system. Usually a single constraint on some elongation parameter e.g. quadrupole moment is used and separate fission paths are obtained. This procedure sometimes may lead to rather incomplete or even misleading conclusions about the topology of the fission barrier and therefore calculations with simultaneous constraints on quadrupole and octupole moments have been performed in our case. As a consequence a bidimensional PES has been created as a function of both the elongation and reflection-asymmetry parameters of the nuclear system. Afterwards, fission paths have been found in the bottom of the valleys of the surface. Such procedure ensures that all fission paths and the passes connecting them are properly described.

In Fig. 1 we have plotted, as an example, the PES of $^{230}$Th as a function of quadrupole and octupole moments. The part of the curve below the dotted line represents compact solutions, whereas the part above this line corresponds to the system made of two separated fragments. Two valleys leading from the ground state to scission can be easily found on the surface. The fission paths, which are the bottoms of these valleys are marked with thin solid lines. One of them goes initially along $Q_3 = 0$ axis and relatively small reflection asymmetry can be found from around $Q_2 = 50$ b. This valley leads to normal spontaneous fission. On the other valley, the octupole moment is all the time different from zero and increases almost linearly with quadrupole moment. The very big asymmetry of masses of the fragments suggests that fission along this path leads to cluster radioactivity.

The very mass asymmetric fission path could, in principle, be characterized and obtained by the constraint on the quadrupole moment but it turns out in this and other examples that using a single constraint in the octupole moment $Q_3$ suffices to follow that path and therefore it is the octupole and not the quadrupole moment the “natural” coordinate to be used in this kind of studies. The similar conclusions can be deduced from Fig. 2, where the sequence of the density distribution plots of $^{230}$Th shows the evolution of the shape of the nucleus at the cluster emission path with increasing octupole moment. One can clearly see there that increasing $Q_3$ leads straightforwardly to very mass asymmetric fission.

The shape of the cluster emission barriers of the Th nuclei as a function of $Q_3$ is plotted in the lower panels of Fig. 3. The barriers are around 25 MeV high and these are huge values in comparison with spontaneous fission. The barriers con-
Fig. 1. The PES of $^{230}$Th as a function of quadrupole and octupole moments. Thick solid lines show possible fission paths. Scission points is shown by a dotted line.

The potential energy in the two-fragment branch of the fission path decreases with increasing octupole moment mainly due to the decrease of the Coulomb repulsion between the two outgoing fragments. This should be manifested for large $Q_3$ values (large separation between fragments), where the nuclear interaction between fragments is no longer relevant. To check this assumption we will approximate the energy by the expression

$$V_{\text{two fragments}}(Q_3) = E_0 - Q + V_{\text{Coul}}(Q_3),$$

where $E_0$ is the ground state energy calculated in the HFB theory and the $Q$ value is obtained from experimental binding energies. The Coulomb energy can
be expressed as a function of $Q_3$

$$V_{\text{Coul}}(Q_3) = e^2 \frac{Z_1 Z_2}{R(Q_3)},$$

(2)

by means of a relation between the octupole moment and the distance between the centers of mass of the fragments. By assuming that the fragments are spherical and with a constant density we obtain

$$Q_3 = f_3 R^3,$$

(3)

with

$$f_3 = \frac{A_1 A_2}{A} \frac{(A_1 - A_2)}{A}.$$  

(4)

The potential energy from Eq. (1) is plotted in Fig. 3 with a thin line in the PES panel. We observe quite large energy differences between this formula and the HFB results specially at large values of $Q_3$. The differences can be attributed to slight deviations of the fragment’s density distribution from sphericity but first of all to a not big enough size of the harmonic oscillator basis used. For small $Q_3$ values,
Cluster radioactivity of Th isotopes in the mean-field HFB theory.

Fig. 3. Fission paths of $^{226}$Th, $^{228}$Th, $^{230}$Th and $^{232}$Th isotopes. Potential energy (bottom panels), inertia parameter $B(Q_3)$ (middle panels) and the number of nucleons in light fragment after scission (top panels) are plotted as a function of $Q_3$. Approximate values of the energy Eq. (1) and collective inertia Eq. (7) are marked with thin lines.
around the scission point, the HFB energy looks closer to the values of Eq. (1) as can be seen in Fig. 3. This is a consequence of a smaller spatial size of the system that requires a smaller basis size for its description and also to the fact that the nuclear interaction between fragments probably is not very relevant in that case.

In order to calculate half-lives of cluster emission the WKB approximation has been used.\(^6\) The half life is given by

\[
\begin{align*}
  t_{1/2} &= 2.86 \times 10^{-21} (1 + \exp(2S)), \\
  S &= \int_a^b dQ_3 \sqrt{2B(Q_3)(V(Q_3) - E_0)}.
\end{align*}
\]

Here \(a\) and \(b\) are turning points at the ground state energy \(E_0\) and \(B(Q_3)\) is collective quadrupole inertia (computed with the standard approximation of neglecting the residual interaction in its evaluation) with \(Q_3\) as the collective variable. The values of \(B(Q_3)\) are plotted in the middle panels of Fig. 3. Again, for the branch of the barrier after scission we have used the approximate formula

\[
  B(Q_3) = \frac{\mu}{9Q_3^{4/3} f_3^{2/3}},
\]

with effective mass:

\[
  \mu = m_n \frac{A_1 A_2}{A_1 + A_2}.
\]

obtained by assuming that the mass is the reduced mass of two spherical fragments and expressed in terms of the octupole moment. The oscillations of \(B(Q_3)\) observed in the part of the diagram before scission are due to the shell effects in the deforming nuclei. The collective mass after scission is smooth as the fragments almost do not change their deformation. As can be checked in Fig. 3 the approximate expression of Eq. (7) gives here results similar to the HFB calculations.

The half-lives of cluster emission of Th isotopes are presented in Fig. 4 and Table 1. Good agreement is found between the theoretical results and the experimental data. The differences do not exceed two orders of magnitude. This could be considered as a rather poor agreement but it is to be noted that this kind of errors is typical for other theoretical predictions in the field of fission. It is also worth to stress here that in our model there are no free parameters to be fitted, making the degree of agreement between our results and experiment quite outstanding.

In \(^{230}\)Th and \(^{232}\)Th the predicted masses of the fragments in the HFB model differ slightly, by two protons, from experimental data. In both nuclei the HFB fission path corresponds to a \(^{22}\)O fragment. Nevertheless, the half-lives for cluster emission of Na isotopes are shorter than those found at the bottom of the HFB fission valley. What is possibly happening in this case is that the fission path leading at scission to a Na cluster has a lower action integral than the fission path located at the fission valley (as a consequence of a lower collective mass). As a consequence
Cluster radioactivity of Th isotopes in the mean-field HFB theory.

Fig. 4. Half-lives of cluster emission of Th isotopes calculated in the HFB theory are compared with experimental data.9

Table 1. Half-lives of cluster emission of Th isotopes calculated in the HFB theory are compared with experimental data.9

| Emitter  | Cluster | log(t₁/₂[s]) | log(t₁/₂[s]) |
|----------|---------|--------------|--------------|
| ²²⁶Th    | ¹⁸Ο     | 17.80        | >16.76       |
| ²²⁸Th    | ²⁰Ο     | 20.53        | 20.73        |
| ²³⁰Th    | ²²Ο     | 28.83        | —            |
| ²³⁰Th    | ²⁴Νε    | 26.22        | 24.63        |
| ²³²Th    | ²²Ο     | 33.39        | —            |
| ²³²Th    | ²⁴Νε    | 31.74        | >29.20       |
| ²³²Th    | ²⁶Νε    | 31.36        | >29.20       |

the Na cluster path yields a lower half-life making it the preferred decay mode. To explore this possibility a study in terms of the minimum of the action integral instead of the minimum energy path would be necessary and work in this direction is in progress. It is worth to stress here again that in the HFB framework the masses of the fragments are determined at the scission point and not in the fission path far from this point.

3. Conclusions
The analysis of the potential energy surface in a mean-field model such as the Hatree-Fock-Bogoliubov theory with the D1S Gogny force can be successfully applied to explain many features of cluster radioactivity. The very mass asymmetric fission valley can be easily found in the potential energy surface of the HFB theory.
To determine the fission path at the bottom of this valley the octupole moment as the collective coordinate leading to fission is required. The results for Th isotopes described in this paper are in reasonable good agreement with experimental fragment’s masses and half-lives.

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References
1. A. Sandulescu, D.N. Poenaru and W. Greiner, Sov. J. Part Nucl. 11, 528 (1980).
2. H.J. Rose and G.A. Jones, Nature 307, 245 (1984).
3. M. Warda, J. L. Egido, L. M. Robledo and K. Pomorski, Phys. Rev. C 66, 014310 (2002).
4. M. Warda, K. Pomorski, J. L. Egido and L. M. Robledo, Int. J. Mod. Phys. E 14, 403 (2005).
5. M. Warda, K. Pomorski, J. L. Egido and L. M. Robledo, J. Phys. G: Nucl. Part. Phys. 31, S1555 (2005).
6. J.L. Egido and L.M. Robledo, Nucl. Phys. A 738, 31 (2004).
7. L.M. Robledo and M. Warda, in preparation.
8. G. Audi, A.H. Wapstra and C. Thibault, Nucl. Phys. A 729, 337 (2003).
9. G. Audi, O. Bersillon, J. Blachot and A.H. Wapstra, Nucl. Phys. A 729, 3 (2003).