Electromagnetic masses of the mesons and the generalization of Dashen’s theorem

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Abstract
In the framework of $U(3)_L \times U(3)_R$ chiral field theory, Dashen’s theorem is reexamined, and the well-known result of $m_{\pi^\pm}^2 - m_{\pi^0}^2$ obtained by Das, Guralnik, Mathur, Low, and Young is reproduced. We find that Dashen’s theorem, which automatically holds for pseudoscalar mesons in this theory, can be generalized to the sector of axial-vector mesons, however, fails for the sector of vector mesons.

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1 Introduction

In the recent years, studies of electromagnetic masses of mesons, Dashen's theorem \[1\] and its violation have attracted much attention \[2, 3, 4, 5, 6, 7, 8\]. Dashen's theorem, which states that the square electromagnetic mass differences between the charge pseudoscalar mesons and their corresponding neutral partners are equal in the chiral SU(3) limit, is expressed as follows,

\[
\begin{align*}
(m_{\pi}^2 & - m_{\pi^0}^2)_{EM} = (m_{K}^2 - m_{K^0}^2)_{EM}, \\
(m_{\pi^0}^2)_{EM} = 0, & \quad (m_{K^0}^2)_{EM} = 0. \quad (1)
\end{align*}
\]

The subscript EM denotes electromagnetic mass. There are three meson-octets $0^-(\text{pseudoscalar mesons})$, $1^-(\text{vector mesons})$, and $1^+ (\text{axial-vector mesons})$ which belong to the same representation of $SU(3) \subset U(3)$, but have different spin or parity. Thus, a natural question aroused here is whether Dashen’s theorem (which holds for pseudoscalar-octet mesons) could be generalized to the vector-octet and axial-vector-octet mesons or not. In this present paper, by employing $U(3)_L \times U(3)_R$ chiral theory \[9, 10\], it will be shown that the generalization to axial-vector mesons is valid, or, in the chiral SU(3) limit,

\[
\begin{align*}
(m_{\pi}^2 & - m_{\eta^0}^2)_{EM} = (m_{K}^2 - m_{K^0}^1)_{EM}, \\
(m_{\eta^0}^2)_{EM} = 0, & \quad (m_{K^0}^1)_{EM} = 0. \quad (2)
\end{align*}
\]

However, similar generalization fails for vector mesons. The latter is of the same conclusion as one given by Bijnens and Gosdzinsky \[14\].

$U(3)_L \times U(3)_R$ chiral theory of pseudoscalar, vector, and axial-vector mesons provides a unified description of meson physics in low energies. This theory has been extensively investigated in Refs.\[9, 10, 11, 7, 12, 13\], and its predictions are in good agreement with the experimental data. Vector meson dominance (VMD) \[15\] in the meson physics is a natural consequence of this theory instead of an input. This means that the dynamics of the electromagnetic interactions of mesons has been introduced and established naturally. Therefore, the present theory makes it possible to evaluate the electromagnetic self-energies of these low-lying mesons systematically. According to this pattern, for example, we can work out the well-known result of $(m_{\pi}^2 - m_{\pi^0}^2)_{EM}$ given by Das et al \[16\] (to see Sec. 2), which serves as leading order in our evaluations. This indicates that our pattern evaluating the electromagnetic self-energies of mesons is consistent with the known theories in the pion case under the lowest energies limit. Since the dynamics of mesons, including
pseudoscalar, vector and axial-vector, is described in the present theory, the method calculating EM-masses in this paper is legitimate not only for $\pi$ and $K$ mesons, but also for $a_1, K_1, \rho$ and $K^*$ mesons. Thus, both checking the Dashen’s argument of eq.(1) in the framework of the effective quantum fields theory and investigating its generalizations mentioned above become practical.

The basic lagrangian of this chiral field theory is (hereafter we use the notations in Refs.[9, 10])

$$\mathcal{L} = \bar{\psi}(x)(i\gamma \cdot \partial + \gamma \cdot v + e_0Q\gamma \cdot A + \gamma \cdot a\gamma_5 - mu(x))\psi(x)$$

$$+ \frac{1}{2}m_1^2(\rho^\mu \rho_\mu + \omega^\mu \omega_\mu + a^\mu a_\mu + f^\mu f_\mu)$$

$$+ \frac{1}{2}m_2^2(K_{\mu \nu}^{a a}K^{a \mu \nu} + K_1^{\mu \nu}K_1_{\mu \nu})$$

$$+ \frac{1}{2}m_3^2(\phi_\mu \phi^\mu + f_\mu f^\mu) + \mathcal{L}_{EM}$$

(3)

where $u(x) = exp[i\gamma_5(\pi_1, \pi_2, \pi_3 + \eta + \eta')](i=1,2,3$ and $a=4,5,6,7), a_\mu = \tau_i a^i_\mu + \lambda_a K_{1 \mu}^a + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)f_{sm}, v_\mu = \tau_i \rho^i_\mu + \lambda_a K_1^{a \mu} + (\frac{2}{3} + \frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3} - \frac{1}{\sqrt{3}}\lambda_8)\phi_\mu, A_\mu$ is the photon field, $Q$ is the electric charge operator of $u, d$ and $s$ quarks, $\psi$ is quark-fields, $m$ is a parameter related to the quark condensate, and $\mathcal{L}_{EM}$ is the kinetic lagrangian of photon fields. Here, the meson-fields are bound states of quarks, and they are not fundamental fields. The effective lagrangian for mesons is derived by performing path integration over quark fields. This treatment naturally leads to VMD. Following Refs.[9, 10], the interaction lagrangians of neutral vector meson fields and photon fields read

$$\mathcal{L}_{\rho \gamma} = -\frac{e}{f_\rho} \partial_\mu \rho^0_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu),$$

$$\mathcal{L}_{\omega \gamma} = -\frac{e}{f_\omega} \partial_\mu \omega_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu),$$

$$\mathcal{L}_{\phi \gamma} = -\frac{e}{f_\phi} \partial_\mu \phi_\mu (\partial^\mu A^\nu - \partial^\nu A^\mu).$$

(4)

The direct couplings of photon to other mesons are constructed through the substitutions

$$\rho^0_\mu \rightarrow \frac{e}{f_\rho} A_\mu, \ \omega_\mu \rightarrow \frac{e}{f_\omega} A_\mu, \ \phi_\mu \rightarrow \frac{e}{f_\phi} A_\mu.$$  

(5)

where

$$\frac{1}{f_\rho} = \frac{1}{2}g, \ \frac{1}{f_\omega} = \frac{1}{6}g, \ \frac{1}{f_\phi} = -\frac{1}{3\sqrt{2}}g.$$  

(6)

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$g$ is a universal coupling constant in this theory. It can be determined by taking the experimental values of $f_\pi$, $m_\rho$ and $m_a$ as inputs \cite{g, g1, g2}. Thus, all interaction lagrangians between mesons and photon($\gamma$), $L_i(\Phi, \gamma, ...|_{\Phi=\pi,a,v, ...}$ are obtained.

Using $L_i(\Phi, \gamma, ...|_{\Phi=\pi,a,v, ...}$, we can evaluate the following S-matrix

$$S_\Phi = \langle \Phi | T \exp[i \int d^4x L_i(\Phi, \gamma, ...)] - 1 | \Phi \rangle |_{\Phi=\pi,a,v,...}. \tag{7}$$

On the other hand $S_\Phi$ can also be expressed in terms of the effective lagrangian of $\Phi$ as

$$S_\Phi = \langle \Phi | i \int d^4x L_{\text{eff}}(\Phi) | \Phi \rangle.$$  

Noting $L = \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} m_\Phi^2 \Phi^2$, then the electromagnetic interaction correction to the mass of $\Phi$ reads

$$\delta m_\Phi^2 = \frac{2i S_\Phi}{\langle \Phi | \Phi^2 | \Phi \rangle}, \tag{8}$$

where $\langle \Phi | \Phi^2 | \Phi \rangle = \langle \Phi | \int d^4x \Phi^2(x) | \Phi \rangle$. Thus, all of virtual photon contributions to mass of the mesons can be calculated systematically.

The paper is organized as follows. In Sec. 2, we shall reexamine Dashen’s theorem for pseudoscalar mesons in the framework of the present theory. In Secs. 3 and 4, the generalization of this theorem to axial-vector and vector mesons is studied respectively. Finally, we give the summary and conclusions.

2 To reexamine Dashen’s theorem in $U(3)_L \times U(3)_R$ chiral theory of mesons

Due to VMD(eq.(4)), the interaction lagrangians which contribute to electromagnetic mass of the mesons has to contain the neutral vector meson fields $\rho^0$, $\omega$ and $\phi$. When the calculations are of $O(\alpha_{em})$ and one-loop, there are two sorts of vertices contributing to electromagnetic self-energies of pseudoscalar mesons: four points vertices and three points vertices. The former must be the coupling of two pseudoscalar fields and two neutral vector mesons fields, and the latter must be the interaction of a pseudoscalar field to a neutral vector meson plus another field.

Thus, from Refs.\cite{g, g1}, the interaction lagrangians contributing to electromagnetic mass of pseudoscalar mesons($\pi$ and $K$) via VMD in the chiral limit read

$$L_{\rho\rho\pi\pi} = \frac{4F^2}{g^2 f_\pi^2} (\rho_\mu \rho^{0\mu} + \frac{1}{2\pi^2 F^2} \partial_\mu \rho_\nu \rho^{0\mu}) \pi^+ \pi^- \tag{9}$$

4
\[ L_{\rho a} = \frac{2i\gamma F^2}{f_\pi g^2} \rho_\mu \pi^+(a^{-\mu} - \frac{1}{2\pi^2 F^2} \partial^2 a^{-\mu}) + \text{h.c.}, \]  
\[ L_{K K^{-} v v} = \frac{F^2}{f_\pi^2 g^2} \{(\rho^0_\mu + v^8_\mu)^2 + \frac{1}{2\pi^2 F^2} (\partial^\nu \rho^0_\mu + \partial^\nu v^8_\mu)^2\} K^+ K^-, \]  
\[ L_{K^\pm K^0 v v} = \frac{i\gamma F^2}{g^2 f_\pi} (\rho^0_\mu + v^8_\mu) K^+ (K^{-\mu} - \frac{1}{2\pi^2 F^2} \partial^2 K^{-\mu}) + \text{h.c.}, \]  
\[ L_{K^0 K^0 v v} = \frac{F^2}{f_\pi^2 g^2} \{(\rho^0_\mu + v^8_\mu)^2 + \frac{1}{2\pi^2 F^2} (\partial^\nu \rho^0_\mu + \partial^\nu v^8_\mu)^2\} K^0 K^0, \]  
\[ L_{K^0 K_1^0 v v} = \frac{i\gamma F^2}{g^2 f_\pi} (\rho^0_\mu + v^8_\mu) K^0 (\bar{K}^0\mu) - \frac{1}{2\pi^2 F^2} \partial^2 \bar{K}^0\mu) + \text{h.c.} \]

where

\[
\pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \pm i\pi^2), \quad a^\pm = \frac{1}{\sqrt{2}}(a^1 \pm ia^2), \quad K^\pm = \frac{1}{\sqrt{2}}(K^4 \pm iK^5), \quad K^0(\bar{K}^0) = \frac{1}{\sqrt{2}}(K^6 \pm iK^7), \quad K^\pm_{1\mu}(\bar{K}^0_{1\mu}) = \frac{1}{\sqrt{2}}(K^6_{1\mu} \pm iK^7_{1\mu}).
\]

with

\[
F^2 = \frac{f^2_c}{1 - \frac{2c}{g}}, \quad c = \frac{f^2_\pi}{2gm^2_\rho}, \quad \gamma = (1 - \frac{1}{2\pi^2 g^2})^{-1/2}.
\]

where \( v \) denotes the neutral vector mesons \( \rho^0, \omega \) and \( \phi, v^8_\mu = \omega_\mu - \sqrt{2}\phi_\mu. \) \( f_\pi \) is pion’s decay constant, and \( f_\pi = 0.186\text{GeV}. \) Here, we neglect the contributions to electromagnetic mass of pions or kaons which are proportional to \( m^2_\pi \) or \( m^2_K \), because we are interested in the case of chiral limit.

Note that there are no contributions to electromagnetic masses of \( \pi^0 \) in the chiral limit. This means

\[
(m^2_{\pi^0})_{\text{EM}} = 0, \quad \text{for massless quark.}
\]

It can be see from eqs.(9-14) that the kaon’s lagrangian is different from pion’s. All three neutral vector resonances \( \rho^0, \omega \) and \( \phi \) join kaon’s dynamics there, but only \( \rho^0 \) joins pion’s. In general, because of this difference the kaon’s electromagnetic mass due to VMD must be different from the pion’s. Taking a glance at this circumstance, it seems to be impossible to expect the
$K^0$'s electromagnetic mass vanishes like eq.(16) for $\pi^0$'s. Fortunately, however, after a careful consideration it will be shown below that $K^0$'s electromagnetic mass does vanish in the chiral SU(3) limit in our formalism. The point is that in the chiral SU(3) limit, $m_u = m_d = m_s = 0$ and $m_\rho = m_\omega = m_\phi$, the SU(3)-vector-meson symmetry and VMD make the total contribution to $(m_{K^0}^2)_{EM}$ vanished.

Let us show this point precisely now. The lagrangians contributing to electromagnetic masses of $K^0$ (eqs.(13)(14)) are different from $K^\pm$'s (eqs.(11)(12)). This difference is due to the structure constants of SU(3) group: $f_{345} = -f_{367} = \frac{1}{2}, f_{458} = f_{678} = \sqrt{3}/2$. In the $K^0$'s lagrangians, eqs.(13)(14), the neutral vector mesons emerge in combination forms: $(\partial_\nu \rho_0^\mu + v_8^\mu)$ or $(\partial_\nu \partial_\nu \rho_0^\mu + \partial_\nu v_8^\mu)$. Note that in the calculations of electromagnetic masses of pseudoscalar mesons, the vector meson fields ($\rho$, $\omega$ and $\phi$) and axial-vector meson fields ($a_1$ and $K_1(1400)$) in the above lagrangians must be abstracted into propagators in the corresponding S-matrices. Therefore, from eqs.(11)-(14) and SU(3) symmetry limit, $m_\rho = m_\omega = m_\phi$, it can be sure that $v_8^\mu$ is actually equivalent to $\rho_0^\mu$ in the calculations of electromagnetic masses of the mesons. Thus, the lagrangians contributing to electromagnetic masses of $K^0$ will vanish, then,

$$\left( m_{K^0}^2 \right)_{EM} = 0, \quad \text{in the chiral SU}(3) \text{ limit.} \quad (17)$$

Likewise, it can also be sure that the lagrangian (11) and (12) are exactly equivalent to lagrangians (9) and (10) respectively under the limit of $m_\rho = m_\omega = m_\phi$ and $m_a = m_{K_1}$. Then, according to eqs.(7) and (8) we have

$$\left( m_{K^\pm}^2 \right)_{EM} = \left( m_{\pi^\pm}^2 \right)_{EM}, \quad \text{in the chiral SU}(3) \text{ limit.} \quad (18)$$

Above arguments and conclusions can also be checked by manifest calculations which are standard in quantum fields theory. From eqs.(9)-(14) and VMD, the electromagnetic masses of $\pi^\pm$, $K^\pm$ and $K^0$ can be derived. To $\pi^\pm$, using the substitution eq.(5) and eqs.(9)(10), we get $\mathcal{L}_{\gamma\gamma\pi\pi}, \mathcal{L}_{\gamma\rho\pi\pi}$ and $\mathcal{L}_{\gamma\pi\alpha\pi}$. Then, the following S-matrices can be calculated

$$S_\pi(1) = \langle \pi | T \left[ \frac{e^2}{2i} \int d^4x_1 \mathcal{L}_{\gamma\gamma\pi\pi}(x_1) + \frac{e^2}{2!} 2 \int d^4x_1 d^4x_2 \mathcal{L}_{\gamma\rho\pi\pi}(x_1) \mathcal{L}_{\rho\gamma}(x_2) + \frac{e^2}{3!} 2 \int d^4x_1 d^4x_2 d^4x_3 \mathcal{L}_{\rho\rho\pi\pi}(x_1) \mathcal{L}_{\rho\gamma}(x_2) \mathcal{L}_{\rho\gamma}(x_3) \right] | \pi \rangle,$$

$$S_\pi(2) = \langle \pi | T \left[ \frac{e^2}{2} \int d^4x_1 d^4x_2 \mathcal{L}_{\pi\alpha\gamma}(x_1) \mathcal{L}_{\pi\alpha\gamma}(x_2) \right] | \pi \rangle.$$
\[ \frac{i^3}{3!} \int d^4x_1d^4x_2d^4x_3 \mathcal{L}_{\pi\rho\pi}(x_1) \mathcal{L}_{\pi\gamma}(x_2) \mathcal{L}_{\rho\gamma}(x_3) \\
+ \frac{i^4}{4!} \int d^4x_1d^4x_2d^4x_3d^4x_4 \mathcal{L}_{\pi\rho\pi}(x_1) \mathcal{L}_{\pi\rho\pi}(x_2) \mathcal{L}_{\rho\gamma}(x_3) \mathcal{L}_{\rho\gamma}(x_4) \| \pi \),
\]

where \( \mathcal{L}_{\rho\gamma} \) has been shown in eq.(4). Noting eq.(8) indicates

\[
(m^2_{\pi\pm})_{EM} = \frac{2i(S_\pi(1) + S_\pi(2))}{\langle \pi | \pi^2 | \pi \rangle},
\]

then we obtain

\[
(m^2_{\pi\pm})_{EM} = i \frac{e^2}{f^2_\pi} \int \frac{d^4k}{(2\pi)^4} (D - 1)m^4_\rho \frac{(F^2 + \frac{k^2}{2\pi^2})^2}{k^2(k^2 - m^2_\rho)} [1 + \frac{\gamma^2 F^2 + \frac{k^2}{2\pi^2}}{g^2 k^2 - m^2_a}],
\]

Similarly, to \( K^\pm \) and \( K^0 \), from eqs.(11)-(14), we have

\[
(m^2_{K\pm})_{EM} - (m^2_{K^0})_{EM} = i \frac{e^2}{f^2_\pi} \int \frac{d^4k}{(2\pi)^4} (D - 1)(F^2 + \frac{k^2}{2\pi^2})(1 + \frac{\gamma^2 F^2 + \frac{k^2}{2\pi^2}}{g^2 k^2 - m^2_{K_1}})
\times \left[ \frac{m^2_\rho m^2_\omega}{3 k^2(k^2 - m^2_\rho)(k^2 - m^2_\omega)} + \frac{2 m^2_\omega m^2_\phi}{3 k^2(k^2 - m^2_\rho)(k^2 - m^2_\phi)} \right]
\]

\[
(m^2_{K^0})_{EM} = i \frac{e^2}{4 f^2_\pi} \int \frac{d^4k}{(2\pi)^4} (D - 1)(F^2 + \frac{k^2}{2\pi^2})(1 + \frac{\gamma^2 F^2 + \frac{k^2}{2\pi^2}}{g^2 k^2 - m^2_{K_1}}) k^2
\times \left[ \frac{1}{k^2 - m^2_\rho} - \frac{1}{3 k^2 - m^2_\rho} - \frac{2}{3 k^2 - m^2_\phi} \right]^2
\]

where \( D = 4 - \varepsilon \). Obviously, taking \( m_\rho = m_\omega = m_\phi \), and \( m_a = m_{K_1} \), the contribution of eq.(21) is zero, and eq.(20) is exactly eq.(19). Thus, eq.(18) holds, and Dashen’s theorem(eq.(1)) is automatically obeyed in this theory.

It should be emphasized here that unlike \( (m^2_{\pi\alpha})_{EM} = 0 \), \( (m^2_{K^0})_{EM} = 0 \) is not only due to the chiral limit, but also due to the SU(3) symmetry limit for vector mesons. Similar conclusion has also been obtained by Donoghue and Perez in Ref.[6] in the framework of chiral perturbation theory. Therefore, when one investigates the violation of Dashen’s theorem, the contributions due to \( m_\rho \neq m_\omega \neq m_\phi \) should be taken into account.[6]

In the above, the calculations on EM-masses are up to the fourth order covariant derivatives in effective lagrangians[6, 11]. In the remainder of this Section, we find that the result of \( (m^2_{\pi\pm} -
$m^2_{\pi a})_{EM}$ given by Das et al \cite{16} can be reproduced if the electromagnetic self-energy of pions receives the contributions only from the second order derivative terms. In this case, the interaction lagrangians $L_{\rho\rho\pi\pi}$ and $L_{\rho\pi a}$ (eqs.(9)(10)) will be simplified as follows

$$L_{\rho\rho\pi\pi} = \frac{4F^2}{g^2 f^2} \rho^0_\rho \partial_\mu \pi^+ \pi^- , \quad L_{\rho\pi a} = \frac{2iF^2}{f^2 g^2} \rho^0_\rho \pi^+ a^{-\mu} + h.c..$$

Thus, in the chiral limit, the electromagnetic self-energy of pions is

$$(m^2_{\pi^\pm} - m^2_{\pi^0})_{EM} = (m^2_{\pi^\pm})_{EM} = i \frac{3e^2}{f^2} \int \frac{d^4k}{(2\pi)^4} m^4_\rho \frac{F^2}{k^2 - m^2_\rho} (1 + \frac{F^2}{g^2 (k^2 - m^2_\rho)})$$

(22)

The Feynman integration in eq.(22) is finite. So it is straightforward to get the result of $(m^2_{\pi^\pm} - m^2_{\pi^0})_{EM}$ after performing this integration, which is

$$(m^2_{\pi^\pm} - m^2_{\pi^0})_{EM} = \frac{3\alpha_{em}}{8\pi f^2} \left\{ \frac{2F^2}{m^2_\rho} - \frac{2F^4}{g^2 (m^2_\rho - m^2_a)} (\frac{1}{m^2_\rho} + \frac{1}{m^2_a - m^2_\rho} \log \frac{m^2_\rho}{m^2_a}) \right\}$$

(23)

where $\alpha_{em} = \frac{e^2}{4\pi}$. Because we only consider the second order derivative terms in the lagrangian, the relation between $m_a$ and $m_\rho$ is $m^2_a = \frac{F^2}{g^2} + m^2_\rho$ instead of eq.(27) in Ref.\cite{9}. Thus, using eq.(15) we can get

$$(m^2_{\pi^\pm} - m^2_{\pi^0})_{EM} = \frac{3\alpha_{em}}{4\pi} \frac{m^2_a m^2_\rho}{m^2_a - m^2_\rho} \log \frac{m^2_a}{m^2_\rho}$$

(24)

When substituting the relation $m^2_a = 2m^2_\rho$, which can be derived from the Weinberg sum rules \cite{17}, into eq.(24), we have

$$(m^2_{\pi^\pm} - m^2_{\pi^0})_{EM} = \frac{3\log 2}{2\pi} \alpha_{em} m^2_\rho$$

(25)

which is exactly the result obtained by Das et al \cite{16}, and serves as the leading term of eq.(19).

3 Generalization of Dashen’s theorem to axial-vector meson sector

It is straightforward to extend the studies in the previous section to the axial-vector octet mesons. In the $U(3)_L \times U(3)_R$ chiral fields theory of mesons, the lagrangians contributing to electromagnetic
masses of axial-vector mesons ($a_1$ and $K_1$) read

$$\mathcal{L}_{a_{pp}} = -\frac{4}{g^2}[\rho^0_\mu \rho^0_\mu a^+ a^\nu - \frac{\gamma^2}{2} \rho^0_\mu \rho^0_\mu (a^+a^\nu + a^-a^{+\nu})],$$

$$\mathcal{L}_{a_{ap}} = \frac{2i}{g}(1 - \frac{\gamma^2}{\pi^2 g^2})\partial^\nu \rho^0_\mu a^+ a^\nu - \frac{2i}{g} \rho^0_\mu a^+ \mu (\partial^\nu a_\mu - \gamma^2 \partial_\mu a^-) + h.c.,$$

$$\mathcal{L}_{a_{ap\rho}} = \frac{2i}{g} (\beta_1 \rho^0_\mu \pi^+ a^- - \beta_2 \rho^0_\mu \pi^- a^- + \beta_3 \rho^0_\mu a^+ \mu (\partial^2 \pi^- - \beta_4 \rho^0_\mu \pi^+ \partial^2 a^-)
- \beta_5 \rho^0_\mu \partial_\nu a^+ \mu (\partial^\nu \pi^-) + h.c.)$$

$$\mathcal{L}_{K_1^+, K_1^- \ pi v} = -\frac{1}{g^2}[(\rho^0_\mu + \nu^8_\mu)^2 K_1^+, K_1^-\nu
- \frac{\gamma^2}{2} (\rho^0_\mu + \nu^8_\mu) (\rho^0_\mu + \nu^8_\mu) (K_1^{+\mu} K_1^-\nu + K_1^{-\mu} K_1^+\nu)],$$

$$\mathcal{L}_{K_1^+, K_1^- v} = \frac{i}{g}(1 - \frac{\gamma^2}{\pi^2 g^2})(\partial^\nu \rho^0_\mu + \partial^\nu \nu^8_\mu) K_1^{+\mu} K_1^-\nu
- \frac{i}{g} (\rho^0_\mu + \nu^8_\mu) [K_1^{+\mu} (\partial^\nu K_1^-\mu - \gamma^2 \partial_\mu K_1^-\nu) + h.c.],$$

$$\mathcal{L}_{K_1^+, K_1^- v} = \frac{i}{g} (\beta_1 \rho^0_\mu + \nu^8_\mu) K_1^{+\mu} K_1^-\mu + \frac{i}{g} \beta_2 (\rho^0_\mu + \nu^8_\mu) \partial^\nu K_1^{+\mu} K_1^-\mu
+ \frac{i}{g} \beta_3 (\rho^0_\mu + \nu^8_\mu) \partial^2 K_1^-\mu - \frac{i}{g} \beta_4 (\rho^0_\mu + \nu^8_\mu) \partial^2 K_1^{+\mu}
- \frac{i}{g} \beta_5 (\rho^0_\mu + \nu^8_\mu) \partial_\nu K_1^{+\mu} \partial^\nu K^- + h.c.)$$

$$\mathcal{L}_{K_1^0, K_1^- \ pi v} = \mathcal{L}_{K_1^+, K_1^- \ pi v} \{\rho^0 \leftrightarrow -\rho^0, K_1^+ \leftrightarrow K_1^0 (K_1^-)\},$$

$$\mathcal{L}_{K_1^0, K_1^- \ pi} = \mathcal{L}_{K_1^+, K_1^- \ pi} \{\rho^0 \leftrightarrow -\rho^0, K_1^+ \leftrightarrow K_1^0 (K_1^-)\},$$

$$\mathcal{L}_{K_1^0, K_1^- v} = \mathcal{L}_{K_1^+, K_1^- v} \{\rho^0 \leftrightarrow -\rho^0, K_1^+ \leftrightarrow K_1^0 (K_1^-), K^\pm \leftrightarrow K^0 (K^0)\}.$$
From the above, it is found that due to the SU(3) symmetry the structure of these axial-vector meson’s lagrangians, eqs.(26)-(34), are similar to the pseudoscalar meson’s (eqs.(9)-(14)). So it is possible to find out the EM-masses relations between $a_1$’s and $K_1$’s in the chiral SU(3) limit through similar analyses done in the previous section. Firstly, like the case of $\pi^0$, there are no couplings to $a_0^0$ field in lagrangian eqs.(26)-(28), therefore
\[
(m_{a_0^0})_{EM} = 0. \tag{35}
\]
Secondly, according to the previous section, we can take $v_\mu^8 = \rho_\mu^0$ in the calculations of EM-masses of $K_1$ mesons in the chiral SU(3) limit. Then, from lagrangians of eqs.(32)-(34), we have
\[
(m_{K_1^0})_{EM} = 0. \tag{36}
\]
Finally, using $v_\mu^8 = \rho_\mu^0$ again, noting $m_\pi = m_K = 0$ in chiral limit and comparing $K^\pm$’s lagrangians (eqs.(29)-(31)) with $a_1$’s (eqs.(26)-(28)), we have
\[
(m_{a_\pm}^2)_{EM} = (m_{K_1^\pm})_{EM}. \tag{37}
\]
Eqs.(35), (36) and (37) are just eq.(2). Then we conclude that the generalization of Dashen’s theorem to axial-vector mesons is legitimate.

Above conclusion can also be checked by manifest calculations like the case of pseudoscalar mesons in the previous section. For completeness, we provide these in follows. The calculations from lagrangians of eqs.(26)-(34) (together with VMD) to the desired EM-masses are somewhat lengthy, but the procedures here are clear without any obstructions, which is like the case of $\pi$’s(eq.(19)).

To $a_1$ mesons, the results are as follows,
\[
(m_{a_0^0})_{EM} = 0 \tag{38}
\]
\[
(m_{a_\pm}^2)_{EM} = (m_{a_\pm}^2)_{EM}(1) + (m_{a_\pm}^2)_{EM}(2) + (m_{a_\pm}^2)_{EM}(3) \tag{39}
\]
with
\[
(m_{a_\pm}^2)_{EM}(1) = ie^2 \gamma^2 \frac{\langle a | \int d^4x a_{\mu} a_{\nu} | a \rangle - \langle a | \int d^4x a_{\lambda} a_{\lambda} | a \rangle g^{\mu\nu}}{\langle a | \int d^4x a_{\mu} a_{\nu} | a \rangle} \frac{\langle a | \int d^4x a_{\mu} a_{\nu} | a \rangle g^{\mu\nu}}{\langle a | \int d^4x a_{\mu} a_{\nu} | a \rangle} \times \int \frac{d^4k}{(2\pi)^4} \frac{m_{\rho}^4}{k^2(k^2 - m_{\rho}^2)^2}(g_{\mu\nu} - k_{\mu}k_{\nu}) (k^2 - m_{\rho}^2), \tag{40}
\]
\[ (m_{a^\pm})_{EM} (2) = \frac{ie^2}{\langle a | \int d^4 x a^\mu a^\mu_p | a \rangle} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - 2p \cdot k - (k^2 - m_p^2)^2} \]

\[ \times \{ \langle a | \int d^4 x a^\mu a^\mu_p | a \rangle [4m_{a^\pm}^2 + (b^2 + 2\gamma^2p \cdot k)^2 + 2\gamma^4p \cdot k - \frac{4(p \cdot k)}{k^2} ] \]

\[ - \frac{1}{m_a^2} (b k^2 - (b - \gamma^2p \cdot k)^2) + \langle a | \int d^4 x a^\mu a^\mu_p | a \rangle k^\mu k^{\nu} [(b k^2 - 2(1 - \gamma^2p \cdot k)^2)] \}, \] (41)

\[ (m_{a^\pm})_{EM} (3) = \frac{-ie^2}{\langle a | \int d^4 x a^\mu a^\mu_p | a \rangle} \int \frac{d^4k}{(2\pi)^4} \frac{1}{m_k^2} \frac{m_p^4}{k^2} \]

\[ \times \{ \langle a | \int d^4 x a^\mu a^\mu_p | a \rangle (\beta_1 - 3\beta_2p \cdot k + \beta_3k^2)^2 + \langle a | \int d^4 x a^\mu a^\mu_p | a \rangle k^\mu k^{\nu} [\beta_2m_{a^\pm}^2 - \frac{(\beta_1 - 2\beta_2p \cdot k + \beta_3k^2)^2}{k^2}] \}. \] (42)

where \( \beta_1 = 1 - \frac{\gamma^2}{\pi^2g^2} \),

\[ \beta' = \beta_1 + (\beta_3 + \beta_4 - \beta_5)m_{K_1}^2. \]

To \( K_1 \) mesons, we have

\[ (m_{K_1^{\pm}}^2)_{EM} - (m_{K_1^{\mp}}^2)_{EM} = [(m_{K_1^{\pm}}^2)_{EM} (1) - (m_{K_1^{\mp}}^2)_{EM} (1)] + [(m_{K_1^{\pm}}^2)_{EM} (2) - (m_{K_1^{\mp}}^2)_{EM} (2)] + [(m_{K_1^{\pm}}^2)_{EM} (3) - (m_{K_1^{\mp}}^2)_{EM} (3)] \]

with

\[ (m_{K_1^{\pm}}^2)_{EM} (1) - (m_{K_1^{\mp}}^2)_{EM} (1) = \frac{ie^2}{\langle K_1 | \int d^4 x K_1^{\mu +} K_1^{\nu -} | K_1 \rangle - \langle K_1 | \int d^4 x K_1^{\mu +} K_1^{\nu -} | K_1 \rangle} \]

\[ \times \{ \langle K_1 | \int d^4 x K_1^{\mu +} K_1^{\nu -} | K_1 \rangle \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - 2p \cdot k} \}

\[ \times \{ \langle K_1 | \int d^4 x K_1^{\mu +} K_1^{\nu -} | K_1 \rangle [4m_{K_1}^2 + (b^2 + 2\gamma^2p \cdot k)^2 + 2\gamma^4p \cdot k - \frac{4(p \cdot k)^2}{k^2} ] \} \] (43)
\begin{align*}
&\frac{-1}{m_{K_1}^2}(bk^2 - (b - \gamma^2)p \cdot k)^2] + \langle K_1| \int d^4x K^+_{1\mu} K^-_{1\nu}|K_1\rangle k^\mu k^\nu[-(3b^2 - 4b + 4) \\
&+ D(b + \gamma^2)^2 + 4\gamma^2 - 6b\gamma^2 - 2\gamma^4 - \frac{2\gamma^4 p \cdot k}{k^2} + \frac{1}{m_{K_1}^2 k^2}(bk^2 - 2(1 - \gamma^2)p \cdot k)] \\
&\times \left[ \frac{1}{3} \frac{m_{\rho}^2 m_{\omega}^2}{k^2 - m_{\rho}^2}(k^2 - m_{\omega}^2) + \frac{2}{3} \frac{m_{\rho}^2 m_{\phi}^2}{k^2 - m_{\rho}^2}(k^2 - m_{\phi}^2) \right],
\end{align*}

\begin{align*}
(m_{K_1^\pm})_{EM}(3) - (m_{K_1^0})_{EM}(3) &= \frac{-ie^2}{\langle K_1| \int d^4x K^+_{1\mu} K^-_{1\nu}|K_1\rangle} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p - k)^2 - m_K^2} \\
&\times \{ \langle K_1| \int d^4x K^+_{1\mu} K^-_{1\nu}|K_1\rangle(\beta'_1 - 3\beta_2 p \cdot k + \beta_3 k^2)^2 + \\
&\langle K_1| \int d^4x K^+_{1\mu} K^-_{1\nu}|K_1\rangle k^\mu k^\nu[\beta_2 m_{K_1}^2 - \frac{\beta_1^2 - 2\beta_2 p \cdot k + \beta_3 k^2}{k^2}] \} \\
&\times \left[ \frac{1}{3} \frac{m_{\rho}^2 m_{\omega}^2}{k^2 - m_{\rho}^2}(k^2 - m_{\omega}^2) + \frac{2}{3} \frac{m_{\rho}^2 m_{\phi}^2}{k^2 - m_{\rho}^2}(k^2 - m_{\phi}^2) \right],
\end{align*}

and

\begin{align*}
(m_{K_1^0})_{EM} &= (m_{K_1^0})_{EM}(1) + (m_{K_1^0})_{EM}(2) + (m_{K_1^0})_{EM}(3)
\end{align*}

with

\begin{align*}
(m_{K_1^0})_{EM}(1) &= ie^2 (\gamma^2 \langle K_1| \int d^4x K^0_{1\mu} \bar{K}^0_{1\nu}|K_1\rangle - \langle K_1| \int d^4x K^0_{1\mu} \bar{K}^0_{1\nu}|K_1\rangle g^{\mu\nu} \\
&\times \int \frac{d^4k}{(2\pi)^4} \left(k^2 g_{\mu\nu} - k_{\mu} k_{\nu}\right) \left[ \frac{1}{k^2 - m_{\rho}^2} - \frac{1}{3} \frac{1}{k^2 - m_{\omega}^2} - \frac{2}{3} \frac{1}{k^2 - m_{\phi}^2} \right]^2,
\end{align*}

\begin{align*}
(m_{K_1^0})_{EM}(2) &= \frac{ie^2}{4\langle K_1| \int d^4x K^0_{1\mu} \bar{K}^0_{1\nu}|K_1\rangle} \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{k^2 - 2p \cdot k} \right] \\
&\times \left\{ \langle K_1| \int d^4x K^0_{1\mu} \bar{K}^0_{1\nu}|K_1\rangle \left[ 4m_{K_1}^2 + (b^2 + 2b\gamma^2)k^2 + 2\gamma^4 p \cdot k - \frac{4(p \cdot k)^2}{k^2} \right] \\
&- \frac{1}{m_{K_1}^2}(bk^2 - (b - \gamma^2)p \cdot k))^2 \right\} + \langle K_1| \int d^4x K^0_{1\mu} \bar{K}^0_{1\nu}|K_1\rangle k^\mu k^\nu[-(3b^2 - 4b + 4) \\
&+ D(b + \gamma^2)^2 + 4\gamma^2 - 6b\gamma^2 - 2\gamma^4 - \frac{2\gamma^4 p \cdot k}{k^2} + \frac{1}{m_{K_1}^2 k^2}(bk^2 - 2(1 - \gamma^2)p \cdot k)] \} \\
&\times \left[ \frac{1}{3} \frac{1}{k^2 - m_{\rho}^2} - \frac{1}{3} \frac{1}{k^2 - m_{\omega}^2} - \frac{2}{3} \frac{1}{k^2 - m_{\phi}^2} \right]^2,
\end{align*}

\begin{align*}
(m_{K_1^0})_{EM}(3) &= \frac{-ie^2}{4\langle K_1| \int d^4x K^0_{1\mu} \bar{K}^0_{1\nu}|K_1\rangle} \int \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{(p - k)^2 - m_K^2} \right]
\end{align*}
\[
\times \langle K_1 | \int d^4x K_1^0 \bar{K}_1^0 | K_1 \rangle \left( \beta'_1 - 3\beta_2 p \cdot k + \beta_3 k^2 \right)^2 + \\
\langle K_1 | \int d^4x K_1^0 \bar{K}_1^0 | K_1 \rangle k^\mu k'^\nu \left[ \beta_2 m_K^2 - \frac{\beta'_1 - 2\beta_2 p \cdot k + \beta_3 k^2}{k^2} \right] \}
\times k^2 \left[ \frac{1}{k^2 - m_\rho^2} - \frac{1}{3} \frac{1}{k^2 - m_\omega^2} - \frac{2}{3} \frac{1}{k^2 - m_\phi^2} \right].
\]

(48)

where \( p^2 = m_K^2 \).

Taking \( m_\rho = m_\omega = m_\phi \), and \( m_a = m_K \) (chiral SU(3) limit), we immediately obtain

\[
(m_{a^\pm})_{EM}(i) = \left( m_{K_\pm} \right)_{EM}(i), \quad (m_{K_1})_{EM}(i) = 0, \quad i = 1, 2, 3.
\]

Then the conclusion of eqs.(35)-(37) (eq.(2)) has been confirmed.

4 Generalization of Dashen’s theorem to vector meson sector

Now let us consider the vector meson octet sector. According to Refs.[9, 10], the lagrangians which contribute to the electromagnetic masses of \( \rho^\pm \) and \( K^{*\pm} \) read

\[
\mathcal{L}_{\rho \rho \rho} = -\frac{4}{g^2} \rho^0 \rho^\mu \rho^\nu \rho^\nu + \frac{2}{g^2} \rho^0 \rho^\mu (\rho^{\mu \nu} \rho^{-\nu} + \rho^{-\mu} \rho^{\nu}),
\]

(49)

\[
\mathcal{L}_{\rho \rho} = \frac{2i}{g} \partial_\nu \rho^0 \rho^{+\mu} \rho^{-\nu} - \frac{2i}{g} \rho^0 \rho^\nu (\partial^\mu \rho^{-\nu} - \partial^{\mu} \rho^{-\nu}) + h.c.,
\]

(50)

\[
\mathcal{L}_{\rho \omega \pi} = -\frac{3}{\pi^2 g^2 f_\pi} \varepsilon^{\mu \nu \alpha} \partial_\mu \omega_\nu \rho^\alpha + h.c.
\]

(51)

\[
\mathcal{L}_{K^{*+} K^{*-} \omega} = \frac{1}{g^2} \left( \rho^0 + v_\mu^8 \right)^2 K^{*+} K^{-}\nu
\]

\[
+ \frac{1}{2g^2} (\rho^0 + v_\mu^8)(\rho^0 + v_\nu^8)(K^{+\mu} K^{-\nu} + K^{-\mu} K^{+\nu}),
\]

(52)

\[
\mathcal{L}_{K^{*+} K^{*-} \nu} = \frac{i}{g} (\partial_\nu \rho^0 + \partial_\nu v_\mu^8) K^{+\mu} K^{-}\nu
\]

\[-\frac{i}{g} (\rho^0 + v_\nu^8) K^{*+} (\partial_\mu K^{-\mu} - \partial^{\mu} K^{-\nu}) + h.c.,
\]

(53)

\[
\mathcal{L}_{K^{*\pm} K^{\mp} \nu} = -\frac{3}{\pi^2 g^2 f_\pi} \varepsilon^{\mu \nu \alpha} \partial_\mu K^{*+} \nu \partial_\alpha K^{-\nu} \left( \frac{1}{2} \partial_\alpha \rho_\mu^0 + \frac{1}{2} \partial_\alpha \omega^\alpha + \frac{\sqrt{2}}{2} \partial_\nu \phi_\alpha \right) + h.c.
\]

(54)
Eq. (51) and eq. (54) come from the abnormal part of the effective lagrangian \( \mathcal{L}_{IM} \) (to see Refs. [9, 10]). Thus, similar to the above, we conclude without any doubt that \( \rho^\pm \) and \( K^{*\pm} \) receive the same electromagnetic self-energies in the chiral SU(3) limit,

\[
(m_{\rho^\pm})_{EM} = (m_{K^{*\pm}})_{EM}
\] (55)

However, \( \rho^0 \) and \( K^{*0} \) can also obtain electromagnetic masses even in the chiral SU(3) limit, which is different from the case of neutral pseudoscalar and axial-vector mesons. The lagrangian contributing to the electromagnetic masses of \( K^{*0} \) is

\[
\mathcal{L}_{K^{*0}\bar{K}^{*0}v} = \mathcal{L}_{K^{*+}\bar{K}^{*+}v} \{ \rho^0 \leftrightarrow -\rho^0, K^{*\pm} \leftrightarrow K^{*0} (\bar{K}^{*0}) \},
\]

\[
\mathcal{L}_{K^{*0}\bar{K}^{*0}v} = \mathcal{L}_{K^{*+}\bar{K}^{*+}v} \{ \rho^0 \leftrightarrow -\rho^0, K^{*\pm} \leftrightarrow K^{*0} (\bar{K}^{*0}) \},
\]

\[
\mathcal{L}_{K^{*0}\bar{K}^{*0}v} = \mathcal{L}_{K^{*\pm}\bar{K}^{*\pm}v} \{ \rho^0 \leftrightarrow -\rho^0, K^{*\pm} \leftrightarrow K^{*0} (\bar{K}^{*0}), K^{\pm} \leftrightarrow K^{0} (\bar{K}^{0}) \}.
\]

Note that in eq. (58), the combination of the neutral vector mesons is \(-\rho_\mu + \omega_\mu + \sqrt{2}\phi_\mu \) instead of \(-\rho_\mu + \omega_\mu + \sqrt{2}\phi_\mu \) emerging in eqs. (56)(57) and the lagrangians contributing to the electromagnetic masses of \( K^0 \) and \( K^0_1 \). Therefore, even in the chiral SU(3) limit, the electromagnetic masses of \( K^{*0} \) is nonzero due to the contribution coming from eq. (58)(the contributions of eqs. (56) and (57) vanish in the limit of \( m_\rho = m_\omega = m_\phi \)).

To electromagnetic masses of \( \rho^0 \)-mesons, the circumstances are more complicated. The contributions to \( (m_{\rho^0})_{EM} \) from \( \mathcal{L}_{IM} \) is

\[
\mathcal{L}_{\rho\omega\pi} = -\frac{3}{\pi^2 g^2 f_\pi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \omega_\nu \rho_\alpha^0 \partial_\beta \pi^0
\]

Distinguishing from the case of \( K^{*0} \), the direct \( \rho^0 \)-photon coupling which comes from VMD(eq. (4)) can bring both the tree and one-loop diagrams contributing to the electromagnetic masses of \( \rho^0 \) in the chiral limit. Thus, \( (m_{\rho^0})_{EM} \) is also nonzero. Furthermore, from the point of view of large-\( N_c \) expansion, the tree diagrams are \( O(N_C) \) while the one-loop diagrams are \( O(1) \) [1, 18]. In general, we can not expect that \( (m_{\rho^0})_{EM} = (m_{K^{*0}})_{EM} \) in the chiral SU(3) limit(only loop diagrams can contribute to \( (m_{K^{*0}})_{EM} \)). So the generalization of Dashen’s theorem fails for vector meson octet. Similar result has also been found by Bijnens and Gosdzinsky [14].
5 Summary and Discussions

Employing $U(3)_L \times U(3)_R$ chiral fields theory of mesons and in the chiral SU(3) limit, we have obtained

$$(m^2_{\pi^\pm})_{EM} = (m^2_{K^\pm})_{EM}, \quad (m^2_{a^\pm})_{EM} = (m^2_{K_1^\pm})_{EM}, \quad (m^2_{\rho^\pm})_{EM} = (m^2_{K^*_\pm})_{EM},$$

and the electromagnetic masses of $\pi^0$, $K^0$, $a_1^0$ and $K_1^0$ vanish. Therefore, Dashen’s theorem (eqs. (1) and (2)) holds for both pseudoscalar and axial-vector mesons in this effective fields theory. However, the effective lagrangian makes non-zero contributions to electromagnetic masses of $\rho^0$ and $K^{*0}$ even in the chiral SU(3) limit, and VMD yields the direct coupling of $\rho^0$ and photon (eq. (4)), which provides other contributions to the electromagnetic masses of $\rho^0$. Generally, $(m^2_{\rho^0})_{EM} \neq (m^2_{K^{*0}})_{EM}$.

Therefore, the generalization of Dashen’s theorem fails for vector-mesons.

Dashen’s theorem is valid only in the chiral SU(3) limit. The violation of this theorem beyond the lowest order has been investigated extensively [2, 3, 4, 5, 7, 8], and a large violation has been revealed in Refs. [2, 3, 5, 8]. In particular, a rather large violation of eq. (2) has been obtained in Ref. [7].

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