What does the CCFR measurement of the Gross–Llewelyn Smith sum rule tell us?

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Abstract

Recently the CCFR Collaboration reported the measurement of the Gross–Llewelyn Smith sum rule
\[ \int_0^1 dF_3^{\nu p}(x, Q^2 = 3 \text{ GeV}^2) = 2.50 \pm 0.018(\text{stat}) \]
Subsequently Kataev and Sidorov analyzed the \(Q^2\)–dependence of this sum rule and pointed out a discrepancy between the results obtained via integration of the NLO fits to \(xF_3(x, Q^2)\) and the purely perturbative prediction. We suggest an explanation of this disagreement and show that the result of the CCFR measurement of the GLS sum rule integral can be determined from the knowledge of the value of \(\Lambda_{\overline{\text{MS}}}\) only.

The Gross–Llewelyn Smith sum rule provides one of the important bridges between the QCD improved quark–parton model and the constituent quark model. Moreover, the quantity
\[ G(Q^2) \equiv \frac{1}{2} \int_0^1 dx \left( F_3^{\nu p}(x, Q^2) + F_3^{\nu p}(x, Q^2) \right) \]
is one of the few physical quantities calculated so far up to the NNLO of perturbative QCD [1]. In [2] the CCFR Collaboration reported the result
\[ G(Q^2 = 3 \text{ GeV}) = 2.50 \pm 0.018(\text{stat.}) \pm 0.078(\text{syst.}). \]

In [3] Kataev and Sidorov analyzed this result and addressed an important question of the \(Q^2\)–dependence of this quantity. They pointed out certain discrepancy between the predictions of perturbative QCD for this quantity and what they call experiment. In this note we explain the reason for their observation and argue that in a consistent treatment of the latter there is no place for such a discrepancy. We also comment on the procedure used in [2] to evaluate [1]. However, as the publication [2] contains only a very sketchy and incomplete description of this procedure, we frequently consulted the unpublished thesis [4] for details.

We start by recalling the familiar fact that in QCD the quantity \(G(Q^2)\) can be evaluated in two different ways. First, it can be written as a purely perturbative expansion in some
\[ \text{In the following } F_3(x, Q^2) \text{ stands for the average } \left( F_3^{\nu p}(x, Q^2) + F_3^{\nu p}(x, Q^2) \right) / 2. \]
renormalization scheme RS (we use \( a \equiv \alpha_s/\pi \) as an expansion parameter instead of \( \alpha_s \)) of the form
\[
G(Q^2) = 3 \left( 1 - a_{\text{RS}} \left( 1 + r_1(\text{RS})a_{\text{RS}} + r_2(\text{RS})a^2_{\text{RS}} + \cdots \right) \right)
\]
(3)
While \( r_1 \) is known already for some time from [3], \( r_2 \) has been calculated, for massless quarks and in \( \overline{\text{MS}} \) RS, only very recently in [1]. The factor 3 on the r.h.s. of the above relation is of crucial importance. In the quark–parton model it corresponds to the fact that there are 3 constituent (valence) quarks in the proton, but in QCD it has much deeper meaning and is based (see [6] for a recent review of this subject): on the validity of

- equal time commutation relations of currents made from the quark fields,
- the asymptotic freedom.

Both of these two properties are so fundamental to QCD that we cannot question the factor 3 in (3) without abandoning the perturbative QCD itself. On the other hand were it possible to really measure (1), the result would be, as emphasized in the original papers [7, 8], extremely important for the verification of the very foundations of QCD.

The second theoretical route to \( G(Q^2) \) exploits the nonsinglet QCD fit to the \( Q^2 \) dependence of \( xF_3(x, Q^2) \), which, beside theoretical ingredients, requires also the specification of some initial condition. In [4] this initial condition at some \( Q_0 \) was taken in the form
\[
xF_3(x, Q_0^2) = Ax^\alpha(1 - x)^\beta + Cx^\gamma
\]
(4)
while Kataev and Sidorov employed a simpler form, corresponding to \( C = 0 \) in the above expression. Fitting \( A, \alpha, \beta, C, \gamma, \) together with \( \Lambda_{\overline{\text{MS}}} \), to CCFR data one can compute \( F_3(x, Q^2) \) at any \( x \) and \( Q^2 \) and therefrom evaluate \( G(Q^2) \) at any \( Q^2 \)! The main observation in [3] concerns the disagreement between these two evaluations of \( G(Q^2) \). Similar observation has in fact been made already in [4], but there the reason for the appearance of such a discrepancy turns out to be different than in [3].

In practice (1) is very difficult to really measure. Because of experimental limitations the accessible range of \( x \) varies with \( Q^2 \) and prevents the measurement of \( G(Q^2) \) separately at each value of \( Q^2 \). At low \( Q^2 \) only very low \( x \) are accessible while for large \( Q^2 \) only data at large \( x \) are available. Any experimental measurement of \( G(Q^2) \) therefore inevitably involves extrapolations to unmeasurable regions and must consequently be considered as a combination of data and some kind of theoretical prejudice. In [2, 3] this extrapolation procedure actually exploits the result of a nonsinglet QCD fit to \( F_3(x, Q^2) \)! So the experimental measurement of \( G(Q^2) \) actually assumes the validity of perturbative QCD. There is nothing illegal on this assumption, but, as emphasized above, we have to be consistent and include in the fitting procedure the constraint leading to the factor 3 in (3) as well. While the original determination [2, 3] does include this constraint on the QCD fit, the analysis of [3] it does not.

Although the thesis [4] contains many essential details on the determination of \( G(Q^2) \), it fails to provide convincing and full description of the NLO QCD fits used in the evaluation of \( G(Q^2 = 3 \text{ GeV}^2) \) at so small \( Q^2 \). First, there is the question of the number \( n_f \) of massless quarks used in the evolution equations. As these are written for a fixed number of massless quarks, one must employ some kind of approximate procedure for crossing of quark mass thresholds. This is important in particular for the region of low \( Q^2 \), below the charmed quark
threshold, which gives nonnegligible contribution to \( G(Q^2) \) but which is certainly ineffective at \( Q^2 = 3 \text{ GeV}^2 \). On the other hand some formulations in \[4\] indicate that the fits used in determination of \( G(Q^2 = 3 \text{ GeV}^2) \) used data with \( Q^2 > 12 \text{ GeV}^2 \) only! In this case \( n_f = 4 \) would be a good approximation, but then the fits do not actually fit the data at \( Q^2 \) around 3 GeV²! Although our argument is independent of the quality of these fits, this point should certainly be clarified.

The problem of crossing the quark mass thresholds is only one of the subtle points in determining the GLS sum rule for so low \( Q^2 \). There are others as well:

- Target mass corrections. These can be easily included via the procedure of \[4\], but were ignored in extrapolation procedure of \[2, 4\].
- Higher twist terms. For \( Q^2 = 3 \text{ GeV}^2 \) these turn out to be very important \[10\]!
- Cabbibo mixing. Below the charmed quark threshold the \( d - c \) is ineffective, thereby reducing the quantity \( G(Q^2) \) by a factor \( \cos^2 \theta_c \).

In \[2\] the reason for evaluating the quantity \( G(Q^2) \) at so low \( Q^2 \) was motivated by the fact that low \( x \) region gives dominating contribution to it and 3 is just the average value of \( Q^2 \) for the lowest \( x \) value measured in the CCFR experiment. However, looking at the data in this bin (see Fig.4 below) it is clear that these data are obviously incompatible with the pure QCD evolution used in the extrapolation! Whether this is due to experimental problems, or physical effects not included in the pure perturbative QCD is another matter, but it is certainly much safer to follow the procedure employed in \[3\]. There only \( Q^2 \geq 12.6 \text{ GeV}^2 \) was used in NLO QCD fits to CCFR data on \( F_3(x, Q^2) \), thereby avoiding most of the problems mentioned above, and \( G(Q^2) \) obtained by integration of these fits was compared to the purely perturbative expression \[3\] taken to the LO, with the couplant \( a(Q^2) \) defined to the NLO (for reasons discussed below) and with \( n_f = 4 \). The main observation of \[3\] concerns the discrepancy, displayed in Fig.1, between these two results and in particular its \( Q^2 \)-dependence. In the rest of this note we identify the source of this discrepancy and show how to avoid it.

Recall that as a consequence of the factorization theorem in QCD the non–singlet (NS) structure function \( F_3(x, Q^2) \) can be written as a convolution

\[
F_3(x, Q^2) = \int_0^1 \frac{dy}{y} q_{\text{NS}}(y, M) C_{\text{NS}} \left( \frac{Q}{M}, \frac{x}{y}, a(\mu) \right) \tag{5}
\]

between the NS distribution function \( q_{\text{NS}}(x, M) \), satisfying the evolution equation

\[
\frac{dq_{\text{NS}}(x, M)}{d \ln M} = \int_0^1 \frac{dy}{y} q_{\text{NS}}(y) P_{\text{NS}} \left( \frac{x}{y}, a(M) \right) \tag{6}
\]

and the “hard scattering cross–section” \( C(Q/M, z, a(\mu)) \). The scales \( M, \mu \) appearing in \[3\] are the so called factorization and renormalization scales and the couplant \( a(M) \) satisfies the familiar equation

\[
\frac{da(M)}{d \ln M} = -ba^2 (1 + ca(M) + \cdots) , \tag{7}
\]
where the first two coefficients $b, c$ are unique functions of $n_f$. The branching function $P_{\text{NS}}$ as well as the hard scattering cross-section $C_{\text{NS}}$ admit the following perturbative expansions

$$P_{\text{NS}}(z, a(M)) = a(M)P^{(0)}_{\text{NS}}(z) + a^2(M)P^{(1)}_{\text{NS}}(z) + \cdots$$  \hspace{1cm} (8)

$$C_{\text{NS}}(Q/M, z, a(\mu)) = \delta(z) + a(\mu)C^{(1)}_{\text{NS}}(Q/M, z) + \cdots$$  \hspace{1cm} (9)

In the NLO approximation only the first two terms in each of the expansions (7), (8), (9) are taken into account. Turning the above expressions into the corresponding ones for the NLO approximation, we get an explicit expression for $F_{\delta}(N, Q^2)$ [11]

$$F_{\delta}(N, Q^2) = q_{\text{NS}}(N, M)C_{\text{NS}}(Q/M, N, a(\mu))$$  \hspace{1cm} (11)

where

$$q_{\text{NS}}(N, M) = A_N \left[ \frac{a(M)}{1 + ca(M)} \right]^{-d^{(0)}_N/b} (1 + ca(M))^{-d^{(1)}_N/bc}$$  \hspace{1cm} (12)

$$C_{\text{NS}}\left(\frac{Q}{M}, N, a(\mu)\right) = \left[ 1 + a(\mu) \left( d^{(0)}_N \ln \frac{Q}{M} + \frac{d^{(1)}_N}{b} + \kappa_N \right) \right]$$  \hspace{1cm} (13)

and $d^{(i)}_N$, $i = 0, 1$ are the moments of the LO and NLO branching functions $P^{(0)}_{\text{NS}}(z)$ and $P^{(1)}_{\text{NS}}(z)$ respectively, and the hard scattering scale $\mu$ is in general different from the factorization scale $M$. Recall also that while $d^{(0)}_N$ is unique, $d^{(1)}_N$ is essentially arbitrary, defining the so called factorization convention (FC) [11, 12]. Two invariants appear in (11): the quantity $\kappa_N$, which is independent of both $M$ and $d^{(1)}_N$, but depends for general $N$ on the choice of the renormalization scheme of the couplant [12]; and the constants $A_N$, which are independent of anything. As argued by Politzer [11], these constants must be kept fixed when the factorization scale $M$ or factorization convention, defined at the NLO by $d^{(1)}_N$, are varied! Moreover, these constant provide alternative way of specifying the necessary boundary condition on the solution of the evolution equation. Note, however, that fixing $A_N$ does not correspond to specifying the initial condition on $F_{\delta}(x, Q^2)$ at any finite $Q^2_0$. Rather the constants $A_N$ determine the asymptotic behaviour of $F_{\delta}(N, M^2)$ as $M \rightarrow \infty$.

For $N = 0$, corresponding to the sum rule (3), the situation is particularly simple, as $d^{(0)}_0 = 0$ and in the FC employed in both [2] and [3] (called universal) also $d^{(1)}_0 = 0$. Moreover the value of $\kappa_0$ is unique and equal to $-1$. Consequently, $q_{\text{NS}}(N = 0, Q^2) = A_0$ is $Q^2$–independent and (11) reduces to

$$F_{\delta}(0, Q^2) = G(Q^2) = A_0(1 - a(\mu))$$  \hspace{1cm} (14)

which coincides with the purely perturbative expression (3) in the LO provided $A_0 = 3$ and the same couplant $a(\mu)$ is used in both formulae. In the general case, when $\mu \neq M$, the couplant $a(\mu)$ used in (3) can be taken only to the LO approximation, but if we identify, as is the usual practice, $\mu = M$ and use the same couplant in both (8), (3), $a(M)$ in (14) is in the NLO approximation.
The crucial point of this note is the observation that for the quantity \( G(Q^2) \) the basic principles of QCD imply \( A_0 = 3 \)!. Taking this into account in the NLO fits to the data on \( F_3(x, Q^2) \), the integrals over these fits must coincide with the results of purely perturbative predictions [3]!. There is simply no place for the kind of discrepancy observed in [3]!. This conclusion holds independently of the quality of the NLO QCD fit to the data. Note also that while in the universal FC the constraint \( A_0 = 3 \) is equivalent to the condition

\[
\int_0^1 dx q_{NS}(x, Q^2) = 3
\]  

for all \( Q^2 \), in general FC this is not the case and \([13]\) may depend on \( Q^2 \).

The discrepancy observed in [3] is thus a direct consequence of the fact that in their NLO fits Kataev and Sidorov did not impose the constraint \( A_0 = 3 \). The technique employed in [3] to write down the QCD predictions for the \( Q^2 \) evolution of \( F_3(x, Q^2) \) is based on the idea [13, 14, 15] of expanding the convolution (3) in the set of Jacobi polynomials \( \Theta_{\alpha\beta}^k(x) \), orthogonal on the interval \((0,1)\) with the weight \( x^\alpha (1-x)^\beta \)

\[
F_3(x, Q^2) = x^\alpha (1-x)^\beta \sum_{k=0}^{\infty} \Theta_{\alpha\beta}^k(x) a_{\alpha\beta}^k(Q^2) 
\]  

where the “Jacobi moments” \( a_{\alpha\beta}^k(Q^2) \) are given as linear combinations of conventional moments \( F_3(j, Q^2) \) [14]:

\[
a_{\alpha\beta}^k(Q^2) \equiv \sum_{j=0}^{k} c_{kj} a_{\alpha\beta}^j(Q^2) 
\]  

In [3] the boundary condition, necessary to specify such a convolution, is taken in the form of the initial condition (4) with \( C = 0 \). From the fit of \( F_3(x, Q^2) \) to the CCFR data at all \( Q^2 > 12 \text{ GeV}^2 \) its authors then determine the values of the parameters \( A, \alpha, \beta, \Lambda_{\overline{MS}} \), describing \( F_3(x, Q^2) \), and evaluate \( G(Q^2) \) at \( Q_0^2 \). Repeating this procedure with \( Q_0^2 \) corresponding to each of the 14 values of \( Q^2 \) at which the data of [2] exist, yields the points with error bars in Fig. 1. Their comparison with the purely perturbative LO results (continuous curves) reveals the discrepancy mentioned above. In principle even a single fit is sufficient to evaluate \( G(Q^2) \) at all \( Q^2 \), but to integrate the expansion (16) multiplied by \( 1/x \) down to \( x = 0 \) is potentially dangerous because of the oscillatory character of the Jacobi polynomials. To integrate the initial condition (4) is, on the other hand, a straightforward and safe matter. It must, however, be kept in mind that although all these 14 fits to \( F_3(x, Q^2) \) are equally legal NLO approximations, they lead in general to numerically different results at all \( Q^2 \).

To prove our claim we have repeated the analysis of [3], evaluating for each initial \( Q_0^2 \) the value of the constant \( A_0 \). We have used the same method of Jacobi polynomials, described above, as in [3]. Technical aspects of our calculations (number of terms taken into account in the expansion [13], accuracy achieved etc.) can be found in [14]. For the class of conditions (4) \( A_0 \) can easily be expressed as a function of \( A, C, \alpha, \beta, \gamma \)

\[
A_0 = \left[ A \frac{\Gamma(\alpha)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+1)} + \frac{C}{\gamma} \right] \frac{1}{1 - a(Q_0)} 
\]  

setting \( \mu = Q_0 \) in (14). Turned around the condition \( A_0 = 3 \) implies a constraint between the mentioned parameters, for instance

\[
A = \left[ A_0(1 - a(Q_0)) - \frac{C}{\gamma} \right] \frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha)\Gamma(\beta+1)} 
\]
In Fig. 2 we plot \( r(Q_0^2) \equiv A_0(Q_0)/3 \), as given in (18), with \( A, \alpha, \beta \) determined from fits of (16) to CCFR data, using the parametrization (3) with \( C = 0 \), (i.e. the one used in (3)), as a function of \( Q_0 \). We have repeated this exercise also for three indicated cuts on \( Q^2 \) used in the fits. For all of them the same pattern of the dependence \( A_0(Q_0) \) emerges: for low \( Q_0^2 \) \( A_0 \) is below 1, while for large \( Q_0^2 \) it rises above 1. Note that \( r(Q_0^2) \) is equal to the ratio of the two results in Fig. 1. For all chosen samples there is thus a value of \( Q_0^2 \) for which the unconstrained fit is consistent with the condition \( A_0 = 3 \). This point depends somewhat on the \( Q_2^2 \) cut, but lies roughly around 100 GeV\(^2\). The corresponding values of \( \Lambda_{\overline{\text{MS}}} \) and \( \chi^2/d.f. \) are plotted as functions of \( Q_0^2 \) in Fig. 3.

In the next step we imposed (for the simplest case \( C = 0 \)) on our fits the constraint (19), guaranteeing thus \( A_0 = 3 \). The dashed curves in Fig. 3 describe the results of these constrained fits. From Figs. 2, 3 we conclude that

1. The value of extracted \( \Lambda_{\overline{\text{MS}}} \) is only weakly dependent on the initial \( Q_0^2 \), but
2. depends sensitively on the \( Q^2_{\text{cut}} \)
3. for unconstrained fits \( \chi^2/d.f. \) is an increasing function of \( Q_0^2 \), which
4. decreases with increasing \( Q^2_{\text{cut}} \).
5. As expected, the constrained fits yield essentially the same values of \( \Lambda_{\overline{\text{MS}}} \) and \( \chi^2/d.f. \) for those \( Q_0^2 \) for which \( r(Q_0^2) = 1 \). For lower, as well as higher, values of \( Q_0^2 \) the fits have substantially higher \( \chi^2/d.f. \) and yield dramatically different \( \Lambda_{\overline{\text{MS}}} \).

The last point demonstrates the importance of a proper choice of the initial \( Q_0^2 \) if an unconstrained fit is performed and its correlation with the assumed form of the initial condition (4) and the value of \( Q^2_{\text{cut}} \) parameter. To further illustrate the difference between the constrained and unconstrained fits and indicate the importance of the choice of \( Q_0^2 \), we compare in Fig. 4 their results to CCFR data for the sample with \( Q^2_{\text{cut}} = 12 \text{ GeV}^2 \). The values of extracted the values of the extracted \( \Lambda_{\overline{\text{MS}}} \) (see also Fig. 3) between 260 – 270 MeV are in reasonable agreement with the result \( \Lambda_{\overline{\text{MS}}} = 237 \pm 36 \text{ MeV} \), obtained in (4).

In principle we could use more general form of the initial condition than in (4), adding further parameters and looking for such forms which would exhibit flatter dependence \( r(Q_0^2) \) than in Fig. 2. In our own investigations we have taken slightly different form of the additional term (to the conventional \( Ax^\alpha(1 - x)^\beta \)) in the initial condition (4) and namely

\[
F_3(x, Q_0^2) = Ax^\alpha(1 - x)^\beta(1 + \gamma x)
\]

The corresponding results are not significantly different and we therefore show only some typical examples in Fig. 3a. The basic feature of these results is, as expected, lower \( \chi^2/d.f. \) of the fits. More detailed investigation of this question certainly makes sense but would take us beyond the immediate subject of this note.

Summarizing the preceding considerations we claim that provided

- the NS fits are performed with the constraint \( A_0 = 3 \), i.e. for instance (19)
- and the LO purely perturbative approximation to (2) uses the same NLO couplant \( a(M) \) as the NLO fits to \( F_3(x, Q^2) \)

\[
\text{Summarizing the preceding considerations we claim that provided}
\]
these two ways of evaluating the GLS sum rule \( G(Q^2) \) must give exactly the same result. The discrepancy noted in [3] is entirely due to the fact that in their fits Kataev and Sidorov do not impose the constraint \( A_0 = 3 \).

Finally, let us briefly return to the original determination of the GLS sum rule in [4], which contains basically the same observation as in [3]. There is, however, a subtle difference between the procedures employed in these two papers. In [4] the value \( G(Q^2 = 3 \text{ GeV}^2) = 2.50 \pm 0.018 \) is obtained by extrapolating the NLO fits to \( Q^2 = 3 \text{ GeV}^2 \), fitting this extrapolation to the form (4) with \( C = 0 \) and integrating the resulting analytical formula. Using the LO term of (3), but with LO couplant \( a(M) \) only, they get, on the other hand, \( 2.65 \pm 0.03 \). They speculate that higher order terms and/or higher twist effects are, perhaps, responsible for this marked difference. In fact the reason is much simpler and again related to the constraint \( A_0 = 3 \). As already emphasized, in the FC used in [4] this constraint is equivalent to the condition

\[
\int_0^1 q_{NS}(x, Q^2) dx = 3
\]

which is explicitly imposed on their fits. Had they integrated the extrapolated form of their fits at \( Q^2 = 3 \text{ GeV}^2 \), they would get exactly the result (4) with \( a^{\text{NLO}}(Q) \), the NLO couplant. Had they furthermore used the purely perturbative expression (3) to LO, but with \( a^{\text{NLO}}(Q) \), their two results would coincide. However, by fitting the extrapolated form of \( F_3(x, Q^2) \) to (4) (with \( C = 0 \)) there is no more any guarantee that the resulting fit will satisfy the constraint \( A_0 = 3 \). And indeed, it does not! Substituting the values of \( A, \alpha, \beta \), published in [4, 2] to (18) we find \( A_0 = 2.73 \). This, together with the fact that the purely perturbative result with the NLO couplant equals (for the fitted value of \( \Lambda_{\text{MS}} = 237 \pm 36 \text{ MeV} \)) \( 2.71 \pm 0.03 \), explains quantitatively most of the effect \( ((2.73/3) \times 2.71 = 2.46) \). Note that the value \( \Lambda_{\text{MS}} = 210 \pm 28 \text{ MeV} \) mentioned in [4, 16] comes from fit to a combined \( F_2 \) (at large \( x \)) and \( F_3 \) data.
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Figure captions

**Fig. 1:** The quantity $G(Q^2)$ as obtained by purely perturbative expansion (solid and dashed lines) compared with its evaluation via integration of non-singlet fits to $F_3(x, Q^2)$ (points with error bars). Dashed lines correspond to $1\sigma$ error of $\Lambda_{\overline{\text{MS}}}$. Taken from the ref. [3].

**Fig. 2:** $Q_0^2$ dependence of the quantity $r(Q_0^2) = A_0/3$ extracted from nonsinglet fits to $F_3(x, Q^2)$ for three different cuts on $Q^2$ and two options on $\gamma$ in (20). Typical errors (not shown) of $r(Q_0^2)$ are 0.03.

**Fig. 3:** $\Lambda_{\overline{\text{MS}}}$ and $\chi^2/d.f.$ corresponding to nonsinglet fits with variable initial $Q_0^2$ for the same three classes of fits as in Fig. 2 and $\gamma = 0$. In a) also the results corresponding to variable $\gamma$ are displayed. Errors (not displayed) of $\Lambda_{\overline{\text{MS}}}$ are typically 40 MeV.

**Fig. 4:** The results of our fits to CCFR data on $F_3(x, Q^2)$ for two values of initial $Q_0^2 = 10, 100$ GeV$^2$, with or without the constraint $A_0 = 3$. Only data at $Q^2 > 12$ GeV$^2$ are included in the fits.
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