Modified Variational Mode Decomposition for Power Line Interference Removal in ECG Signals

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ABSTRACT

Power line interferences (PLI) occurring at 50/60 Hz can corrupt the biomedical recordings like ECG signals and which leads to an improper diagnosis of disease conditions. Proper interference cancellation techniques are therefore required for the removal of these power line disturbances from biomedical recordings. The non-linear time varying characteristics of biomedical signals make the interference removal a difficult task without compromising the actual signal characteristics. In this paper, a modified variational mode decomposition based approach is proposed for PLI removal from the ECG signals. In this approach, the central frequency of an intrinsic mode function is fixed corresponding to the normalized power line disturbance frequency. The experimental results show that the PLI interference is exactly captured both in magnitude and phase and are removed. The proposed approach is experimented with ECG signal records from MIT-BIH Arrhythmia database and compared with traditional notch filtering.

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1. INTRODUCTION

Biomedical recordings are usually corrupted with power line disturbances and which results an erroneous data analysis. The main causes of power line disturbances in biomedical recordings are capacitive and magnetic coupling to building power lines and to non power line noise sources, nearby electrical appliances and mains wiring [1], [2]. The power line interference contains the fundamental frequency component at 50/60 Hz along with higher order harmonics. The removal of these interferences from the biomedical recordings is a complicated task since the behaviour of these disturbances is non stationary in nature. The PLI cancellation is important for proper interpretation of neural signals.

Several approaches have been proposed for the removal of power line interferences in biomedical recordings. The classical approach for removing power line disturbances is using a notch filter [3], [4]. But this filtering approach is not efficient due to the non stationary nature of the interferences and also due to frequency variations in the signal. Another common technique for interference cancellation is based on spectrum estimation but for real time data analysis this method is found inadequate [5]. Later several adaptive interference cancellation approaches have been proposed in various articles. In [6], proposes an algorithm based on Adaptive Notch Filter (ANF) approach for fundamental frequency estimation. Later the harmonics are estimated using discrete-time oscillators and then the amplitude and phase are measured using a simple recursive least square (RLS) algorithm. A discrete-time linear Kalman Notch filter based approach is used for PLI removal in [7]. Since the filter design is linear, this approach does not require any information.
about the phase and amplitude of the interferences. Lattice based second order infinite impulse response (IIR) notch filter is proposed in [8] for power line noise removal. Maniruzzaman et al, designed an adaptive filter based on least-mean-square concept for the removal of PLI from ECG recordings [9]. An Alpha-Beta filter based approach is used in [10]. An adaptive interference canceller with a second order PLL is proposed in [11]. The PLL can help to handle with the frequency deviations in the interference. This system is insensitive to baseline fluctuations and large amplitude variations. Weiner-Hopf equation can be used for finding the initial condition of the filter in [12] and based on that an adaptive system is designed for interference removal. Various signal processing algorithms are also employed for ECG noise removal [13-20]. In [13], Mateo et al, utilized the adaptability of Artificial Neural Network (ANN) algorithm to the time varying, non-linear features of ECG signals for interference removal. A sliding DFT based phase locking scheme is proposed in [14]. A least mean square based adaptive interference canceller is designed by replacing the squared-error at each sample by mean-square-error of an error vector in the LMS algorithm [15]. It is a modified version of the existing adaptive canceller included with error estimation in the neighbouring samples. A State Space Recursive Least Square (SSRLS) technique is employed for PLI removal in [16], [17]. The main advantage of this method is it does not require a separate reference power line for tracking the PLI. An FFT based algorithm for finding the central frequency of power line is used in [18]. Then subtracting the noise estimated from the corrupted signal for interference cancellation. Empirical mode decomposition (EMD) combined with filter approach is used for PLI removal in [19]. EMD is a data-driven adaptive signal decomposition algorithm and is used for capturing of power line noise in one of the IMF:

In this paper, modified variational mode decomposition (VMD), based approach for interference removal is discussed with experimental evaluation on noisy ECG data. Variational mode decomposition uses the concept of calculus of variation with Alternating Direction Method of Multipliers (ADMM) for determining the various modes present in the signal. The remaining section of the paper is organized as follows – section 2 describes the variational mode decomposition followed by the proposed approach for PLI removal. Section 3 describes the performance evaluation of the proposed approach on ECG signals and comparison with conventional notch filtering. Section 4 concludes the paper.

2. PROPOSED APPROACH

The proposed approach uses modified variational mode decomposition for power line disturbance cancellation. The concept of VMD and how it is utilized for efficient power line noise cancellation is discussed in this section.

2.1. Variational Mode Decomposition

The concept of variational mode decomposition is proposed in [20]. VMD, decomposes the signal into various modes or intrinsic mode functions (IMF’s) using calculus of variation. Each mode of the signal is assumed to have compact frequency support around a central frequency. VMD tries to find out these central frequencies and IMF’s centered on those frequencies concurrently using an optimization methodology called Alternating Direction Method of Multipliers (ADMM). In VMD, a function that can measure the

Figure 1. Clean ECG signal and ECG corrupted with PLI

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bandwidth of IMF, \( u_k(t) \) is calculated. For that, first compute Hilbert transform of IMF, \( u_k^H(t) \) and formulate an analytic function \( \{ u_k(t) + j u_k^H(t) \} \). The frequency spectrum of this function is one sided and assumed to be centered on \( \omega_k \). By multiplying this analytical signal with \( e^{-j\omega_k t} \), the signal is frequency translated to be centered at origin. The integral of the square of the time derivative of this frequency translated signal is a measure of bandwidth of the IMF, \( u_k(t) \). Now the problem can be formulated as an optimization problem as follows,

\[
\min_{\omega_k, u_k} \sum_k \left\| \left\{ \left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right\} e^{-j\omega_k t} \right\|^2 \\
\text{s.t.} \sum_k u_k = f
\]

(1)

Where \( f \) is the original signal. That is the sum of the bandwidths of \( k \) modes is minimized subject to the condition that sum of the \( k \) modes is equal to the original signal. So the algorithm tries to find out \( k \) unknown central frequencies and \( k \) functions centered at those frequencies. Now this constrained optimization problem is converted into an unconstrained problem using the augmented Lagrangian multiplier method. The augmented Lagrangian multiplier corresponds to the above optimization is as follows;

\[
L(u_k, \omega_k, \lambda) = \alpha \sum_k \left[ \left\| \left( \delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right\| e^{-j\omega_k t} \right]^2 + \left\| f - \sum_k u_k \right\|^2 + \left\langle \lambda, f - \sum_k u_k \right\rangle
\]

(2)

Now this can be solved via the ADMM framework and the corresponding update equations are obtained. In ADMM, solve for one variable at a time assuming that all the other variables are known. The update for IMF, \( u_k(t) \) is,

\[
\hat{u}_k^{n+1} = \left( f - \sum_{i \neq k} u_i \right) \frac{1}{1 + 2(\omega - \omega_k)^2}, \quad \omega \geq 0
\]

(3)

The modes are updated in the frequency domain. The update equation for central frequency \( \omega_k \) is,

\[
\omega_k^{n+1} = \frac{\int_0^\pi \omega |\hat{u}_k(\omega)|^2 d\omega}{\int_0^\pi |\hat{u}_k(\omega)|^2 d\omega}
\]

(4)

And update for \( \lambda \) is,

\[
\hat{\lambda}^{n+1} \leftarrow \lambda^n + \tau \left( f - \hat{u}_k^{n+1}(t) \right)
\]

(5)

The Lagrangian multiplier \( \lambda \) is for exact reconstruction and is updated as dual ascent [20].

2.2. Modified VMD for PLI Removal

In this approach, a modified VMD algorithm is proposed for removing the 50/60Hz power line interferences in ECG records. \( \omega \) represents the central frequency corresponding to the IMF’s. In the modified algorithm, the \( \omega \) is fixed to the normalized frequency corresponding to 50/60 Hz. The \( \omega \) varies in the range from 0 to \( \pi \). In the updating procedure, the \( \omega \) that we have fixed remains the same and the remaining \( \omega \) will be updated each time. The modes corresponding to each \( \omega \) will get updated until the algorithm converges. As the frequency of the power line interference is 50/60Hz, the mode fixed with the corresponding normalized frequency will capture this power line disturbance. All other frequency components presents in the signal will be captured by other IMF’s. Similar to the original theory [20], combining all the IMF’s gives original signal. The error between the original signal and the sum of all modes is negligible. Now, by removing the mode with PLI disturbance, the reconstructed signal results in a power line interference free signal. One of the main issues with the original VMD algorithm is that, if the component of interest is of less
power compared to other components present in the signal, VMD could not remove it. This problem is avoided in the modified VMD algorithm. In this algorithm, by fixing the central frequency, we are able to capture the low power components also. By this same way, we can extract higher order power harmonics present in the signal. During this operation, it must be observed that the nearby frequencies are not all affected. So the proposed methodology, acts as a very sharp notch filter to remove the specified frequency.

**Modified VMD algorithm**

1. Initialize \( \hat{u}_{k1}, \hat{o}_{k1}, \hat{\lambda}_{k1}, n \leftarrow 0 \)
2. Initialize \( \hat{o}_{k} \) as the normalized frequency corresponding to 50/60 Hz

**Repeat**

3. \( n \leftarrow n + 1 \)
4. for \( k = 1 : K \) do
5. update \( \hat{u}_{k} \) for all \( \omega \geq 0 \)
6. if \( k = 2 \), then no update of \( \omega \)
   else
7. update \( \omega_{k} \) for all \( \omega \geq 0 \)
8. Dual ascent for all \( \omega \geq 0 \)
9. until convergence: \( \sum_{k} \| \hat{u}_{k}^{n+1} - \hat{u}_{k}^{n} \| / \| \hat{u}_{k}^{n} \| < \epsilon \)

**3. RESULTS AND DISCUSSION**

The performance of the proposed approach is evaluated with MIT-BIH Arrhythmia database [21]. The ECG signal records containing power line disturbances at 60 Hz is considered for the experimenting purpose. It does not contain any harmonics. In figure 2, it shows a frame of ECG record 228 with 60 Hz power disturbance along with its power spectral density (PSD) plot. From the PSD plot, the presence of 60 Hz power disturbances can be seen as a small lobe at 60 Hz.

This ECG signal is then given to modified VMD algorithm, where the second mode is fixed with normalized frequency of 60 Hz. VMD creates IMF's based on the parameters such as total number of modes, bandwidth constraint, time step of the dual ascent etc. The modes thus obtained are shown in figure 3. The second mode is made to operate for 60 Hz. The power disturbances at 60 Hz will get captured in the same mode. This can be clearly visible from figure 3. It is observed that the reconstructed signal has some loss of height (or magnitude) at the location where there are R-peaks. The heights of R-peaks are really crucial for analysis purpose. The lost magnitude of the peak can be seen in the mode corresponding to interference. So a thresholding step is introduced in the proposed methodology to avoid the loss of information. The importance of thresholding can be understood by observing the small peak like signal samples in the second mode at several locations. These locations are marked with red rings in figure 3. Hence, by finding a threshold as an average on energy, those small peaks can be extracted, and is added back to the reconstructed signal. Now the reconstructed signal will be the clear ECG signal without PLI. Figure 4 shows PSD of PLI removed reconstructed ECG signal. From the PSD it can be observed that the lobe at 60 Hz is not present. This exhibits the power of modified VMD algorithm to exactly take out the specified frequency and the nearby frequencies are not all affected.
Consider another scenario, where the ECG signal contains 50Hz power line noises along with harmonics in it. In this case, the second mode of VMD algorithm is fixed for capturing frequencies of 50 Hz and the third mode is fixed for capturing the odd harmonics of 150 Hz present in the signal. The signal and its PSD plot are given in figure 5. The corresponding modes obtained are given in figure 6. From the modes obtained, it is observed that the second and third modes captured the power line fundamental frequency and the odd harmonics present in the noisy ECG signal exactly. Hence, for reconstruction, removing those modes will result in a PLI free signal. Figure 7 shows the noisy and noise removed ECG signals along with the PSD plot.

The proposed modified VMD algorithm is compared with the conventional notch filtering approach. In notch filtering, when the interference fundamental frequency is slightly deviated from the 50/60 Hz, the notch filters fail to remove the interference present in the signal. From the experiments, it can also observed that the notch filters completely fails to pick up the deviations occurring in higher order harmonics of the interference. However the proposed algorithm successfully removes the interferences under both situations. Figure 8.a and 8.b represents the PSD plot of the proposed modified VMD algorithm and figure 8.c and 8.d represents the traditional notch filtering PSD plot. From the plots it can be seen that the notch filtering completely fails to pick up the interference when the fundamental frequency is slightly deviated from 60 Hz. Where as the proposed approach successfully removes the interference present in the signal.
Figure 4. (a), (b) Noisy record 228 and its PSD; (c), (d) denoised record 228 and PSD of denoised portion

Figure 5. ECG signal with PLI at 50 Hz and its harmonics and corresponding PSD of the signal

Figure 6. Modes obtained through modified VMD
3.1. Signal to Noise Ratio

The performance of the proposed approach is also evaluated in terms of input and output signal to noise ratio (SNR). The input SNR (SNR$_{in}$) is calculated by finding the ratio of the power of the clean ECG signal to the power of the interference signal. When the power line disturbance increases, the value of input SNR will be low. The output SNR (SNR$_{out}$) is calculated by finding the ratio of the power of the estimated signal to the power of the error in the estimation. The correctness of the proposed approach is checked by varying the SNR$_{in}$ from -10 dB to 30 dB and the corresponding SNR$_{out}$ values are tabulated in Table 1. To do the experiments, ECG signal record 101 of MIT-BIH Arrhythmia database is chosen. While it was observed that, for low SNR$_{in}$ signal, the last mode of the VMD doesn’t contain any signal information. Hence, during reconstruction, avoiding this mode gives a signal with improved SNR$_{out}$.

Table 1. Results of evaluation on ECG record 101

| Signal Record | SNR$_{in}$ | SNR$_{out}$ |
|---------------|------------|-------------|
| 101           | 28.6982    | 49.8585     |
| 101           | 22.6776    | 47.6108     |
| 101           | 16.6570    | 44.0847     |
| 101           | 11.7963    | 30.5947     |
| 101           | 2.6776     | 22.6983     |
| 101           | -0.8442    | 20.5207     |
| 101           | -6.8648    | 14.6240     |
| 101           | -9.3636    | 12.1169     |
4. CONCLUSION

The proposed approach using modified variational mode decomposition clearly removes the 50/60 Hz interference and its harmonics from the ECG recordings. By correctly fixing the boundaries for the signal in VMD mode calculation, it accurately extracts the specified frequency, without altering the nearby frequencies. During the reconstruction of signal by excluding the mode that captured the noise and harmonics, it is shown that the reconstructed signal have a high $\text{SNR}_{\text{out}}$ compared to $\text{SNR}_{\text{in}}$. From the observations made it can be concluded that, the proposed modified VMD based approach is appropriate for power line interference cancellation from ECG signals. It can also conclude that the proposed approach acts as a very sharp notch filter by removing the specified frequency.

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