Modeling of Tunable DBRs Lasers Based on Transmission-Line Laser Model

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Abstract. The transmission-line laser model (TLLM) is extended to simulate the three-section distributed Bragg reflector (DBR) lasers, while a combined model of TLLM and digital filter approach is developed to study the sampled grating DBR (SGDBR) lasers. We integrate time-domain TLLM method and frequency-domain transfer matrix method (TMM) in the combined model. Characteristics of tunable DBR lasers and SGDBR lasers are all successfully simulated, such as L-I curve, and static tuning characteristics, which are in good agreement with the published experimental results. This new method is also used to study transient response and lasing spectrum of the devices, which are difficult to be obtained by the frequency domain models.

1. Introduction
Tunable lasers are important photonic integrated devices, and have potential wide applications in future high-capacity photonic networks based on dense wavelength division multiplexing and wavelength routing [1]. Among several structures, DBR based lasers seems more attractive due to their fast switching speed, such as distributed Bragg reflector (DBR) lasers [2], sampled grating DBR (SGDBR) lasers [3], superstructure grating DBR (SSGDBR) lasers [4], and digital supermode DBR (DSDBR) lasers [5] etc. In order to investigate the properties of these tunable DBRs lasers, powerful modeling tools are expected.

The TLLM [6]-[7] is a wide-bandwidth large signal dynamic laser model and developed by A J Lowery. It can take inhomogeneous effects, multiple longitudinal modes, carrier-induced refractive index change, and spontaneous emission noise into account. Moreover, it’s a distributed-element circuit model, electrical parasitic and matching circuits can also be included in as a part of the model. TLLM has been successfully used to model Fabry-Perot lasers [8], distributed feedback semiconductor lasers [9], external-cavity DBR mode-locked lasers [10], and semiconductor optical amplifier [11]. In this paper, the TLLM is introduced to analyze tunable three-section DBR lasers and widely tunable four-section SGDBR lasers.

The traditional TLLM is extended to model three-section DBR lasers first. For four-section SGDBR lasers, a combined theory of TLLM and digital filter approach is developed. The active regions and the phase regions are easy to be described in time domain. However, the two reflectors of SGDBR are difficult to be described in time domain, due to the complex structure of sampled grating.
Here, the TMM, a frequency domain model, is used to characterize the sampled grating regions, and then is transformed into time domain via the digital filter approach. It has been verified the validity that time domain and frequency domain can be transformed into each other by using the finite impulse response (FIR) filter as a bridge [12]-[13]. Based on the TLLM and digital filter approach, characteristics of tunable DBR lasers and SGDBR lasers, such as transient response, lasing spectrum, L-I curve, and static tuning characteristics are all successfully simulated.

The paper is organized as follows: In Section 2, a description of the TLLM used in the analysis is discussed. Different ways for the passive grating description of DBR laser and SGDBR laser, and the simulation results are given in Section 3. Finally, a brief conclusion is drawn in Section 4.

2. Model description

The basic principle of the TLLM is that the laser cavity length \( L \) is longitudinally divided into a number of equal length sections \( s \), with a length \( \Delta L \), as shown in Figure 1. In each section, a scattering matrix \( S \) is used to represent the optical process, including stimulation emission, spontaneous emission, and attenuation. The matrices of these sections are then connected by transmission-lines, which account for propagation delays of the optical wave. Each section includes a propagation delay of \( \Delta T \). This is related to the laser’s cavity length and the group velocity \( v_g \) inside the cavity, given by

\[ \Delta L = v_g \Delta T \]  \hspace{1cm} (1)

The connecting processes can be represented by a connecting matrix \( C \). From the iterations of scattering and connecting processes, the output optical field in the time domain can be obtained. Then, by applying fast Fourier transform (FFT), the laser output spectrum can be easily acquired. The independent carrier rate equation is used in each section to govern the carrier-photon interaction. In the following, we introduce the scattering and connecting matrices as well as the carrier density rate equation, separately.

![Figure 1. Schematic of the transmission line laser model](image)

2.1. TLLM description for active section

2.1.1. Scattering Matrix. The scattering processes represent the optical process, including stimulated emission, spontaneous emission, and attenuation, in each section, which can be expressed as [14]

\[ \kappa A(n) = S \cdot \kappa A(n) + \kappa A(n^s) \]  \hspace{1cm} (2)
where \( A(n) \) is the incident wave at the input of the \( n \) section at the \( k \) time step, \( A(n)^r \) is the reflected wave from that section, and \( A(n)^s \) is the wave due to the spontaneous emission. \( S \) represents the scattering matrix, given by

\[
S = \frac{1}{\gamma(1 + Z_s)} \begin{bmatrix}
\gamma(G + y)(Z_s - 1) & 2\gamma Y_C(Z_s - 1) & 2\gamma Y_L(Z_s - 1) & 2y \\
G(Z_s + 1) & (2Y_C - y)(Z_s + 1) & 2Y_L(Z_s + 1) & 0 \\
G(Z_s + 1) & 2Y_C(Z_s + 1) & (2Y_L - y)(Z_s + 1) & 0 \\
\gamma(G + y)2Z_s & 4\gamma Y_C Z_s & 4\gamma Y_L Z_s & y(1 - Z_s)
\end{bmatrix}
\]

where \( G \) can be expressed as

\[
G = \exp(\Gamma g \Delta L / 2) - 1
\]

\( \gamma \) is the attenuation in each section, given by

\[
\gamma = \exp(-\alpha_s \Delta L / 2)
\]

where \( \Gamma \) is the waveguide confinement factor, \( g \) is the material gain, \( \alpha_s \) is the absorption coefficient.

\( Z_s \) is the impedance of the phase shift stub, \( Y_C \) is the capacitive reactance, \( Y_L \) is the inductive reactance, and \( y = 1 + Y_C + Y_L \) is the total admittance of the RLC stub filter [7]. The phase shift stub is used to account the phase shift effect in the device. In fact, this phase shift is produced due to the impedance mismatching between the phase stub and the main transmission-line. The impedances of the phase shift stub are normalised to the cavity wave impedance and are given by

\[
Z_s = \left| \cot(\pi f n_{\text{eff}} l/c) \right|
\]

where \( f \) is the laser resonance frequency, \( n_{\text{eff}} \) is the effective refractive index, \( c \) is the velocity of light in a vacuum, \( l \) is the change in phase length across a section, given by

\[
l = \frac{\Gamma \Delta L (N(n) - N_p)}{n_g} \frac{dn}{dN}
\]

where \( N_p \) is an arbitrary carrier density for zero phase shift and is usually set to the threshold carrier density[15], \( n_g \) is the group effective refractive index, \( dn/dN \) is the refractive index carrier dependence coefficient. Therefore, the scattering process can be expressed as

\[
\begin{bmatrix}
A(n) \\
A_c(n) \\
A_L(n) \\
A_p(n)
\end{bmatrix}^k = S \cdot \begin{bmatrix}
A(n) \\
A_c(n) \\
A_L(n) \\
A_p(n)
\end{bmatrix}^k + \begin{bmatrix}
I_z Z_C y / 2 \\
0 \\
0 \\
0
\end{bmatrix}
\]
where \( I_s \) is the noise current representing spontaneous emission [14], \( A(n) \), \( A_c(n) \), \( A_l(n) \), and \( A_p(n) \) are the traveling waves in the main transmission-line, the capacitive stub, the inductive stub, and the phase shift stub, respectively. Spontaneous emission sources in each section are generated by a Gaussian distributed random number generator. The mean-square value of these sources \( I_s \) is shown to be

\[
I_s = mN(n)\sqrt{2\beta L hf B / Z_p}
\]  

(9)

where \( m \) is a unit constant with dimension of metres, \( N(n) \) is the carrier density in section \( n \), \( \beta \) is the spontaneous emission coupling coefficient, \( L \) is the laser cavity length, \( hf \) is the photon energy, \( B \) is the bimolecular recombination coefficients, and \( Z_p \) is the transverse-wave impedance of the waveguide [6].

2.1.2. Connecting Matrix. The connection matrix describes the cross coupling between forward and backward traveling waves. It provides the incident wave of the scattering section from the reflected wave of the previous section, which may be described by

\[
k_{\pm 1} A(n) = C \cdot k A(n) \tag{10}
\]

where \( C \) is the connecting matrix. For the active section and phase section of the tunable DBRs lasers, the connecting matrix of the main transmission-line can be written as:

\[
\begin{bmatrix}
A(n+1) \\
B(n)
\end{bmatrix}_{k+1} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}_k
\begin{bmatrix}
A(n) \\
B(n+1)
\end{bmatrix}_k
\]  

(11)

For the inductive stubs in each section, the connection matrix is

\[
\begin{bmatrix}
A_l(n) \\
B_l(n)
\end{bmatrix}_{k+1} = \begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}_k
\begin{bmatrix}
A_l(n) \\
B_l(n)
\end{bmatrix}_k
\]  

(12)

For the capacitive stubs in each section, the connection matrix is

\[
\begin{bmatrix}
A_c(n) \\
B_c(n)
\end{bmatrix}_{k+1} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}_k
\begin{bmatrix}
A_c(n) \\
B_c(n)
\end{bmatrix}_k
\]  

(13)

For the phase-adjusting stubs, which may be inductive or capacitive when the cotangent in eqn. 6 is negative, we have

\[
\begin{bmatrix}
A_p(n) \\
B_p(n)
\end{bmatrix}_{k+1} = \begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}_k
\begin{bmatrix}
A_p(n) \\
B_p(n)
\end{bmatrix}_k
\]  

(14)

and when it is positive, we have
2.1.3. Carrier Density Rate Equation. The time-dependent carrier density in the active region \( N \) satisfies

\[
\frac{dN}{dt} = \eta \frac{I}{eV} - AN - BN^2 - CN^3 - 2g_v S_p
\]

(16)

where \( \eta \) is the injection efficiency, \( I \) is the injection current, \( e \) is the free electron charge, \( V \) is the volume of each section, \( A, B, \) and \( C \) are nonradiative linear, bimolecular and Auger recombination coefficients, respectively. \( S_p \) stands for the photon density, which is related to the incident waves from either side by\[14\]

\[
S_p(n) = \frac{[(A'(n))^2 + (B'(n))^2]n_{eff}}{m^2\hbar cZ_p}
\]

(17)

The gain for multiple quantum well (MQWs) can be modeled by\[16\]

\[
g = \frac{g_0 \ln(N/N_T)}{2(1+\varepsilon S_p)}
\]

(18)

where \( g_0 \) is the gain coefficient, \( N_T \) is the transparent carrier density, and \( \varepsilon \) is the gain compression factor.

2.2. TLLM Description for Phase Section

The phase section has scattering and connecting processes, so the TLLM description for it is almost as same as that for the active section. There is no stimulated emission in the phase section, only attenuation is needed to be considered.

In the next section, different ways for the passive grating description of the DBR laser and SGDBR laser will be given.

3. Simulation results and disussion

3.1. Three-section tunable DBR lasers

In the DBR section, the refractive index \( n \) and the internal absorption \( \alpha \) can be changed by injection currents of the DBR section, according to the free-carrier plasma effect \[17\]. The change of the carrier-induced index and internal absorption \( \Delta n \) and \( \Delta \alpha \) can be described by

\[
\Delta n = \Gamma \frac{dn}{dN} N
\]

(19)

\[
\Delta \alpha = \Gamma \frac{d\alpha}{dN} N
\]

(20)

where \( dn/dN \) and \( d\alpha/dN \) are material coefficients determined by the free-carrier plasma effect.

The carrier density related to the injection current of the passive section is given by
\[
\frac{dN}{dt} = \eta I e^{-V_p} - AN - BN^2 - C_p N^3
\]  

(21)

where \(V_p\) and \(C_p\) stand for waveguide volume, and Auger recombination coefficient in the passive waveguide, respectively.

For the Bragg grating regions of three-section DBR lasers, the coupling exists between forward and backward waves due to the modulation of the refractive index. As shown in figure 2, this effect can be described by the impedance transition of the transmission-line in the TLLM [10].

\[
\begin{bmatrix}
1 + kΔL & -kΔL \\
 kΔL & 1 - kΔL
\end{bmatrix}
\begin{bmatrix}
A(n+1) \\
B(n)
\end{bmatrix}
= 
\begin{bmatrix}
A(n) \\
B(n+1)
\end{bmatrix}
\]

(22)

and for a high-low impedance boundary, we have

\[
\begin{bmatrix}
1 - kΔL & kΔL \\
-kΔL & 1 + kΔL
\end{bmatrix}
\begin{bmatrix}
A(n+1) \\
B(n)
\end{bmatrix}
= 
\begin{bmatrix}
A(n) \\
B(n+1)
\end{bmatrix}
\]

(23)

\(k\) is the grating coupling per unit length.

The three-section tunable DBR laser for the simulation includes a 450-\(\mu\)m-long active section, a 150-\(\mu\)m-long phase section, and a DBR section with \(\kappa L = 2\), Bragg wavelength at 1550nm, and the cavity length equal to 150\(\mu\)m. The other simulation parameters are given in Table 1.

| Table 1. Simulation Parameters |
|-------------------------------|
| Parameters | Symbol | Value |
| Waveguide width | \(w\) | 2 \(\mu\)m |
| Waveguide thickness | \(d\) | 0.05 \(\mu\)m |
| Waveguide loss | Active region | \(\alpha_a\) | 3000 m\(^{-1}\) |
| | Passive region | \(\alpha_p\) | 200 m\(^{-1}\) |
| Waveguide confinement factor | Active region | \(\Gamma_a\) | 0.35 |
| | Passive region | \(\Gamma_p\) | 0.5 |
| Effective refractive index | \(n_{\text{eff}}\) | 3.23 |
| Group refractive index | \(n_g\) | 3.7 |
The turn-on transient response and the output spectrum are simulated and shown in figure 3 and figure 4, respectively. The active section is initially biased at 0 mA, and then modulated with a step current of 100 mA. The phase current and DBR current are stable at 0 mA. The optical spectrum is obtained by performing FFT of output optical field after the model running for some iteration. It should be noted that optical spectrum during the oscillation transient of the laser is somehow different from the one obtained when the laser is settled. Figure 4 gives a spectrum when the output field of the laser is stable.

![Figure 3](image1.png) **Figure 3.** Turn-on transient response of DBR laser.

![Figure 4](image2.png) **Figure 4.** Output spectrum of DBR laser.

Figure 5 shows the output power versus bias current of the active section without application of the phase section and DBR section current. The simulations are performed with sufficient iterations for the turn-on transient to settle. As shown in Figure 5, the threshold current is about 18 mA, and the differential efficiency is about 0.104 mW/mA.
The DBR section can be seen as a frequency selective mirror. The refractive index changes with the injection current allows the peak wavelength to be tuned, thus selects one of the longitudinal mode of the active region as the dominate mode. The lasing wavelength versus injection current of DBR section is simulated and shown in Figure 6. The tuning curve is like a ladder, each step stands for a longitudinal mode. We can see that each step slants to short wavelength, since the longitudinal mode will shift toward the blue side with the increase of the DBR current.

3.2. Tunable SGDBR lasers
As mentioned above, the impedance transition of the transmission-line is used in the TLLM to model the DBR section of three-section DBR lasers. However, the two reflectors of SGDBR are difficult to be described in time domain, due to the complex structure of sampled grating. Here, the front sampled-grating (FSG) and rear sampled-grating (RSG) sections are first characterized by TMM in the frequency domain, and then they are transformed into time domain via the digital filter approach. As shown in figure 7, reflection and transmission filter are used to describe the reflectance and transmission characteristics of FSG and RSG sections, respectively.

![Figure 7. Schematic of the digital filter approach for the TLLM](image)

FSR/RSG sections are first modeled by TMM [18]-[19] to get the reflectivity and transmission coefficient, then the FIR filter coefficients in the time domain $x(t)$ can be obtained easily by a reverse

**Figure 5.** The output power versus bias current of the active section.

**Figure 6.** The lasing wavelength versus injection current of DBR section.
fast Fourier transform (FFT) transformation of the reflectivity and transmission coefficient function in
the frequency domain, given by [12]

\[ x(t) = \frac{1}{M} \sum_{k=0}^{M-1} X(f) e^{2\pi j f \Delta t} \]  

where \( X(f) \) is reflectivity and transmission coefficient of FSR/RSG sections in the frequency
domain, \( M \) is the FIR coefficient number.

The output optical field or the reflected field can be expressed by the \( M \) input field \( y^k \) and
reflectivity and transmission FIR filter coefficients at the specific time \( k \) and earlier as [12]

\[ F_{out}^n = \sum_{k=0}^{M} x^k(t) y^{n-k} \]  

where \( x^k(t) \) is the digital filter coefficients.

The SGDBR laser for the simulation includes a 450-μm-long active section, a 150-μm-long phase
section, a 10-period front sampled-grating mirror with 6-μm-wide bursts using a 58.5-μm period and a
12-period rear sampled-grating mirror with 6-μm-wide bursts and 64.5-μm period. The other
simulation parameters are given in Table 1.

Figure 8 is the turn-on transient response of SGDBR with a current step of 100 mA in the active
section. The active section is initially biased at 0 mA. The FSG current, RSG current and phase current
are stable at 3mA, 8mA, and 0mA, respectively. Figure 9 is the output spectrum obtained by FFT of
the output optical field.

Figure 8. Turn-on transient response of the
SGDBR laser.

Figure 9. Output spectrum of the SGDBR
laser.

Figure 10 shows the output power versus bias current in the active section for three different
wavelength. It can be seen that the threshold currents are different for different lasing wavelength.
These simulation results are agreed with the experimental report [20]. Figure 11 and figure 12 show
the tuning curve of lasing wavelength versus current of FSG section and RSG section, respectively.
We can see that these curves are also like the ladder. Unlike the figure 6, each step stands for a super-
mode, which represents a significant alignment of the comb-like reflection peaks of FSG and RSG,
and the small steps are used to denote the change of different longitudinal modes. In figure 13, we
show the output spectrum of different lasing wavelengths in the same graph. The tuning range is about
40 nm covered from 1533 to 1573 nm, which is about the same with the published experimental
results [21].
4. Conclusion

The TLLM is used to model the tunable DBRs lasers, but different approaches are applied to simulate the passive grating section. The traditional TLLM developed by A J Lowery is used to simulate the three-section DBR lasers, while a combined theory of TLLM and digital filter approach is developed to model the SGDBR lasers. Characteristics of tunable DBR lasers and SGDBR lasers are all successfully simulated, including L-I curve, static tuning characteristics, lasing spectrum and transient response of the devices. Simulated results are in good agreement with the published experimental results. This method could be useful for investigating the complex optoelectronic devices.

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References

[1] Buus J and Murphy E J 2006 J. Lightwave Technol. 24 5-11
[2] Zhang L and Cartledge J C 1995 IEEE J. Quantum Electron. 31 75-81
[3] Jayaraman V, Chuang Z-M and Coldren L A 1993 *IEEE J. Quantum Electron.* 29 1824-34
[4] Tomori Y, Yoshikuni Y, Ishii H, Kano F, Tamamura T, Kondo Y and Yamamoto M 1993 *IEEE J. Quantum Electron.* 29 1817-22
[5] Ponnampalam L, Whitbread N D, Barlow R, Giacinto, Ward A J, Duck J P and Robbins D J 2006 *IEEE J. Quantum. Electron.* 42 223-30
[6] Lowery A J 1987 *IEEE Proceedings* 134 281-89
[7] Lowery A J 1989 *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields* 2 249-65
[8] Nguyen L V T, Lowery A J, Gurney P C R and Novak D 1995 *Optical and Quantum Electron.* 27 663-78
[9] Lowery A J 1989 *Electron. Lett.* 25, 1307-08
[10] Lowery A J 1991 *IEE Proceedings* 138 39-46
[11] Lowery A J 1988 *IEE Proceedings* 135 242-50
[12] Li Wei, Huang Wei-Ping and Li Xun 2004 *IEEE J. Quantum Electron.* 40 473-80
[13] Dong Lei, Zhang Ruikang, Wang Dingli, Zhao Shengzhi, Jiang Shan, Yu Yonglin and Liu Shuihua 2009 *J. Lightwave Technol.* 27 3181-88
[14] Lowery A J 1990 *IEE Proceedings* 137 293-300
[15] Lowery A J 1988 *IEE Proceedings* 135 126-32
[16] Yu S F and Ngo N Q 2002 *IEEE J. Quantum Electron.* 38 1062-74
[17] Buus J, Amann M C and Blumenthal D J 2005 *Tunable laser diodes and related optical sources*, Second Edition (New Jersey: John Wiley & Sons, Inc.)
[18] Shi Kai, Yu Yonglin, Zhang Ruikang, Liu Wen and Barry L P 2009 *Optics Communications* 282 81-7
[19] Björk G and Nilsson O 1987 *J. Lightwave Technol.* LT-5 140-6
[20] Lee San-Liang, Heimbuch M E, Cohen D A, Coldren L A and DenBaars S P 1997 *IEEE J. Sel. Top. Quantum Electron.* 3 615-27
[21] Mason B, Fish G A, DenBaars S P and Coldren L A 1999 *IEEE Photon. Technol. Lett.* 11, 638-40