Neutral Higgs boson contributions to CP asymmetry of $B \to \Phi K_S$ in MSSM

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We have studied the neutral Higgs boson (NHB) contributions to the pure penguin process $B \to \Phi K_S$ in MSSM with middle and large tan$\beta$ (say, $> 8$). We show that the $\alpha_s$ order hadronic matrix elements of NHB induced operators can make sizable effects on both the branch ratio and time dependent CP asymmetry $S_{\phi K}$. Under the all relevant experimental constraints, the Higgs mediated contributions to $S_{\phi K}$ alone can provide a significant deviation from SM and, in particular, lead to a negative $S_{\phi K}$ which is reported by BaBar and Belle.

Among charmless hadronic B decays, the rare decay $B \to \Phi K_S$ is one of channels which provide a powerful testing ground for new physics. The reason is simply because it has no tree level contributions in the standard model (SM). The decay has been studied in SM and the predicted branching ratio (Br) in the perturbative QCD framework is $(3.5-4.3) \times 10^{-6}$ by using the BBNS approach \cite{2,3} or about $10 \times 10^{-6}$ by using the H.-n. Li et al’s approach \cite{4}, which is somewhat smaller than or consistent with the measured value $(8.4^{+2.5}_{-2.1}) \times 10^{-6}$, the current world average from BaBar \cite{9} and Belle \cite{7}.

The recently reported measurements of time dependent CP asymmetries in $B \to \Phi K_S$ decays by BaBar \cite{9} and Belle \cite{8} lead to the error weighted average

$$S_{\phi K} = \sin(2\beta(\Phi K_S))_{\text{ave}} = -0.39 \pm 0.41$$

with errors added in quadrature$^\ast$.

Here the time-dependent CP-asymmetry $S_{\phi K}$ is defined by

$$a_{\phi K}(t) = C_{\phi K} \cos(\Delta M_{B_d}^0 t) + S_{\phi K} \sin(\Delta M_{B_d}^0 t),$$

where

$$C_{\phi K} = \frac{1 - |\lambda_{\phi K}|^2}{1 + |\lambda_{\phi K}|^2}, \quad S_{\phi K} = \frac{2 \text{Im} \lambda_{\phi K}}{1 + |\lambda_{\phi K}|^2},$$

with $\lambda_{\phi K}$ being

$$\lambda_{\phi K} = \left( \frac{q}{p} \right) \frac{A(\bar{B}_d^0 \to \Phi K_S)}{B(\bar{B}_d^0 \to \Phi K_S)}.$$

In the SM the above asymmetry is related to that in $B \to J/\Psi K_S$ \cite{10} by

$$\sin(2\beta(\Phi K)) = \sin(2\beta(J/\Psi K)) + O(\lambda^2)$$

where $\lambda \simeq 0.2$ appears in Wolfenstein’s parameterization of the CKM matrix and

$$\sin(2\beta(J/\Psi K_{S,L}))_{\text{world-ave}} = 0.734 \pm 0.054.$$  \hspace{1cm} (6)

Therefore, \cite{11} violates the SM at the 2.7 $\sigma$ deviation.

Obviously, the impact of these experimental results on the validity of CKM and SM is currently statistics limited. However, they have attracted much interest in searching for new physics \cite{11,12,13}.

In the letter we inquire the possibility to explain the 2.7 $\sigma$ deviation in R-parity conservative supersymmetric models, e.g., the minimal supersymmetric standard model (MSSM). There are mainly two new contributions arising from the QCD and chromomagnetic penguins and neutral Higgs boson (NHB) penguins with the gluino and squark propagated in the loop. The QCD and chromomagnetic penguin contributions to $b \to s s \bar{s}$ have been analyzed in Refs.\cite{12,13}. The NHB penguin contributions have been discussed in Ref.\cite{13}. However, the conclusion on NHB contributions, $S_{\phi K}$ can

\hspace{1cm} * 2003 data is $-0.96 \pm 0.50^{+0.09}_{-0.11}$ by Belle \cite{8} and it is $+0.45 \pm 0.43 \pm 0.07$ by BaBar \cite{9}.
not smaller than 0.71, in Ref. [13] is valid only in some special cases, e.g., the extended minimal flavor violation (MFV) scenarios with the naive factorization of hadronic matrix elements of operators and the decoupling limit in MSSM, and it is not valid in (general) MSSM, as we shall show. We concentrate on the Higgs penguin contributions in MSSM in the letter. An interesting fact is that the Higgs penguins contribute to $B \to \phi K_S$ but not $B \to J/\psi K_S$ at the $\alpha_s$ order of hadronic matrix element calculations due to the non-match of color and flavor. At the tree level of hadronic matrix element calculations (i.e., the naive factorization) their contributions to the branch ratio of $B \to J/\psi K_S$ (the time dependent CP asymmetry $S_{J/\psi K}$) are very small (negligible). Therefore, we do not need to worry about the effects of the Higgs penguins on $S_{J/\psi K}$. It is shown that the Higgs mediated contributions in the case of middle and large $\tan\beta$ (say, $\geq 8$) are important under the constraints from the experimental bounds of $B_s \to \mu^+\mu^-$ and $\Delta M_s$ as well as $b \to s\gamma, s\gamma$, $B \to X_s\mu^+\mu^-$ and consequently can have significant effects on $B \to \Phi K_S$ due to the $\tan^2\beta$ enhancement of Wilson coefficients of the NHB induced operators in MSSM. Our results show that the Higgs mediated contributions to $S_{\phi K}$ alone can provide a significant deviation from the SM and a possible explanation of the $2.7 \sigma$ deviation, satisfying all the relevant experimental constraints.

The effective Hamiltonian induced by neutral Higgs bosons can be written as

$$\mathcal{H}_{\text{eff}}^{\text{neu}} = \frac{G_F}{\sqrt{2}} (-\lambda_i) \sum_{i=11,\ldots,16} (C_i Q_i + C_i' Q'_i) + \text{h.c.},$$

where $Q_{11}$ to $Q_{16}$, the neutral Higgs penguins operators, are given by

$$Q_{11(13)} = (\bar{s} b)_{S+P} \sum_q \frac{m_q}{m_b} (\bar{q} q)_{S-(+)P},$$

$$Q_{12(14)} = (\bar{s} t)_{S+P} \sum_q \frac{m_q}{m_b} (\bar{q} j)_{S-(+)P},$$

$$Q_{15} = \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b \sum_q \frac{m_q}{m_b} \bar{q} q_{S-(+)P},$$

$$Q_{16} = \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b \sum_q \frac{m_q}{m_b} \bar{q} q_{S-(+)P},$$

where $(\bar{q}_1 q_2)_{S_{\pm P}} = \bar{q}_1 (1 \pm \gamma_5) q_2$. The operators $Q'_i$s are obtained from the unprimed operators $Q_i$s by exchanging $L \leftrightarrow R$.

There are new sources of flavor violation in MSSM. Besides the CKM matrix, the $6 \times 6$ squark mass matrices are generally not diagonal in flavor (generation) indices in the super-CKM basis in which superfields are rotated in such a way that the mass matrices of the quark field components of the superfields are diagonal. This rotation non-alignment in the quark and squark sectors can induce large flavor off-diagonal couplings such as the coupling of gluino to the quark and squark which belong to different generations. These couplings can be complex and consequently can induce CP violation in flavor changing neutral currents (FCNC). It is well-known that the effects of the primed counterparts of quark and squark which belong to different generations. These couplings can be complex and consequently can induce CP violation in flavor changing neutral currents (FCNC). It is well-known that the effects of the primed counterparts of quark and squark which belong to different generations.

The effective Hamiltonian results in the following matrix element for the decay $B \to \phi K_S$

$$\langle K_S \phi | \mathcal{H}_{\text{eff}}^{\text{neu}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle K_S \phi | T_p^{\text{neu}} | B \rangle,$$

where $T_p^{\text{neu}}$ is given by

$$T_p^{\text{neu}} = a_4^{\text{neu}} (\bar{s} b)_{V-A} \otimes (\bar{s} s)_{V-A}$$

\[\dagger\] Strictly speaking, the sum over $q$ in expressions of $Q_i$ should be separated into two parts: one is for $q=u, c$, i.e., upper type quarks, the other for $q=d, s, b$, i.e., down type quarks, because the couplings of upper type quarks to NHBs are different from those of down type quarks. In the case of large $\tan\beta$ the former is suppressed by $\tan^{-1}\beta$ with respect to the latter and consequently can be neglected. Hereafter we use, e.g., $C_{11}$ to denote the Wilson coefficient of the operator $Q_{11} = (\bar{s} b)_{S+P} \frac{m_c}{m_b} (\bar{c} c)_{S-}$. 
In Eq. (10)

\[ \frac{m_s}{m_b} \left[ -\frac{1}{2} a_{12}(\bar{s}b)_{V+A} \otimes (\bar{s}s)_{V-A} \\
- \frac{1}{2} a'_{12}(\bar{s}b)_{V-A} \otimes (\bar{s}s)_{V+A} \\
+ \frac{4m_s}{m_b} (a_{16} + a'_{16})(\bar{s}b)_{V-A} \otimes (\bar{s}s)_{V-A} \right]. \]  

In Eq. (11)

\[ a^\text{new}_4 = \frac{C_F \alpha_s}{4\pi} \frac{P^\text{new}_{\phi,2}}{N_c}, \]

\[ a_{12} = C_{12} + \frac{C_{11}}{N_c} \left[ 1 + \frac{C_F \alpha_s}{4\pi} \left(-V' - \frac{4\pi^2}{N_c} H_{\phi K}\right) \right], \]

\[ a_{16} = C_{16} + \frac{C_{15}}{N_c}, \]

with \( P^\text{new}_{\phi,2} \) being

\[ P^\text{new}_{\phi,2} = -\frac{1}{2} (C_{11} + C'_{11}) \]

\[ \times \left[ \frac{m_s}{m_b} \left( \frac{4}{3} \ln \frac{m_b}{\mu} - G_{\phi}(0) \right) + \left( \frac{4}{3} \ln \frac{m_b}{\mu} - G_{\phi}(1) \right) \right] \]

\[ + (C_{13} + C'_{13}) \left[ -2 \ln \frac{m_b}{\mu} G_{\phi}^0 - GF_{\phi}(1) \right] \]

\[ - 4(C_{15} + C'_{15}) \left[ - \frac{1}{2} - 2 \ln \frac{m_b}{\mu} C_{\phi}^0 - GF_{\phi}(1) \right] \]

\[ - 8(C_{16} + C'_{16}) \left[ \left( \frac{m_c}{m_b} \right)^2 \left( -2 \ln \frac{m_b}{\mu} C_{\phi}^0 - GF_{\phi}(s_c) \right) \right] \]

\[ - 8(C_{16} + C'_{16}) \left[ -2 \ln \frac{m_b}{\mu} C_{\phi}^0 - GF_{\phi}(1) \right] \]

where

\[ G_{\phi}^0 = \int_0^1 \frac{dx}{x} \Phi_{\phi}(x), \quad GF(s, x) = \int_0^1 dt \ln \left[ s - x t \bar{t} \right], \]

\[ GF_{\phi}(s) = \int_0^1 \frac{dx}{x} \frac{\Phi_{\phi}(x)}{x} GF(s - i \epsilon, \bar{x}) \]

with \( \bar{x} = 1 - x \) and \( \Phi_{\phi}(x) = 6x \bar{x} \) in the asymptotic limit of the leading-twist distribution amplitude. In calculations we have set \( m_{u,d} = 0 \) and neglected the terms which are proportional to \( m_b^2/m_\phi^2 \) in Eq. (12). We have included only the leading twist contributions in Eq. (11). In Eq. (11) \( a'_{i} \) is obtained from \( a_{i} \) by substituting the Wilson coefficients \( C_{ij}'s \) for \( C_{ij}s \). Here we have used the BBNS approach [2, 3] to calculate the hadronic matrix elements of operators. Notations not explicitly defined are the same as those in Ref. [3].

Before numerical calculations a remark is in place. From Eq. (12) we can see that the large contributions to matrix elements of the operators \( Q_i^{\mu}, i=11,...,16 \) arise from penguin contractions with b quark in the loop. It is the contributions which make the effects of Higgs mediated mechanism sizable at the \( \alpha_s \) order. At the \( \mu = m_w \) scale only non zero Wilson coefficients are \( C_{11,13}'s \), and \( C_{15,16}(\mu) \) are obtained from \( C_{13}'(m_w) \) due to the operator mixing under renormalization. Moreover, because of the mixing of \( Q_i, i=13,...,16, \) onto \( Q_{77,89}, C_{89}(\mu) (\mu \sim m_b) \) can be significantly enhanced [13], which can have large effects on both \( Br \) and \( S_{\phi K} \), as can be seen from the SM decay amplitude of \( B \rightarrow \phi K_s \).

In numerical calculations we use the SM decay amplitude of \( B \rightarrow \phi K_s \) given in Ref. [3]. We calculate the evaluation of Wilson coefficients to the next-to-leading for the SM operators and to the leading order for \( Q_{11,...,16} \) and their mixing with the SM operators. In order to see how large the effects of NHBs can be we switch off all other SUSY contributions except for those coming from NHB mediated penguin operators. We show model-independent in FIG. 1 \( S_{\phi K} \) versus the phase of \( C_{13}(m_w) \) with fixed values of \( [C_{13}(m_w)] \), setting \( C_{11}(m_w) \) and \( C_{11,13}'(m_w)=0 \) in order to see the essential feature of the \( \alpha_s \) order corrections due to the Higgs mediated mechanism. We find that \( S_{\phi K} \) can reach
FIG. 1: $S_{\phi K}$ versus the phase of $C_{13}(m_{w})$ for fixed $|C_{13}(m_{w})|$ for (a) $\mu = m_{b}$, (b) $\mu = m_{b}/2$. The solid, dotted and dashed curves correspond to $|C_{13}(m_{w})|= 0.05, 0.15$ and $0.25$ respectively.

FIG. 2: The correlation between $S_{\phi K}$ and $\text{Br}(B \rightarrow \phi K)$ for including only SM and NHB contributions. (a) is the tree level result for $\mu = m_{b}$, (b) is including the $\alpha_{s}$ corrections. In Fig. 2b green and red dots denote the results for $\mu = m_{b}$ and $\mu = m_{b}/2$ respectively. Current $1\sigma$ bounds are shown by the dashed lines.

a minus value if $|C_{13}(m_{w})|$ is equal to or larger than 0.10 for $\mu = m_{b}$ and 0.11 for $\mu = m_{b}/2$. $S_{\phi K}$ can be negative in quite a large range of the phase of $C_{13}$ for a larger value of $|C_{13}(m_{w})|$. $S_{\phi K}$ is dependent of but not sensitive to the characteristic scale $\mu$ of the process. For a specific model, e.g., MSSM which in the letter we are interested in, we calculate all relevant Wilson coefficients $C_{11,13}^{(i)}(m_{w})$ in reasonable regions of parameter space with middle and large $\tan\beta$ and present numerical results in FIG. 2 below.

We impose two important constraints from $B \rightarrow X_{s}\gamma$ and $B_{s} \rightarrow \mu^{+}\mu^{-}$. Considering the theoretical uncertainties, we take $2.0 \times 10^{-4} < \text{Br}(B \rightarrow X_{s}\gamma) < 4.5 \times 10^{-4}$ as generally analyzed in literatures. The current experimental upper bound of $\text{Br}(B_{s} \rightarrow \mu^{+}\mu^{-})$ is $2.6 \times 10^{-6}$ \cite{10}. Because the bound constrains $|C_{Q_{i}} - C_{Q_{i}}^{\prime}|$ (i=1, 2)\footnote{ $C_{Q_{1,2}}^{(i)}$ are the Wilson coefficients of the operators $Q_{1,2}^{(i)}$ which are Higgs penguin induced in leptonic and semileptonic $B$ decays and their definition can be found in Ref. \cite{23}. By substituting the quark-Higgs vertex for the lepton-Higgs vertex it is straightforward to obtain Wilson coefficients relevant to hadronic $B$ decays.} we can have values of $|C_{Q_{i}}|$ and $|C_{Q_{i}}^{\prime}|$ larger than those in constrained MSSM (CMSSM) with universal boundary conditions at the high scale and scenarios of the extended minimal flavor violation in MSSM in which $|C_{Q_{i}}^{\prime}|$ is much smaller than $|C_{Q_{i}}|$. We require that predicted $\text{Br}$ of $B \rightarrow X_{s}\mu^{+}\mu^{-}$ falls within 1 $\sigma$ experimental bounds. We also impose the current experimental lower bound $\Delta M_{s} > 14.4 ps^{-1}$ and experimental upper bound $\text{Br}(B \rightarrow X_{s}g) < 9\%$. In numerical analysis we fix $m_{b} = m_{t} = 400 GeV$ and $\tan\beta = 30$. We vary the NHB masses in the ranges of 91GeV $\leq m_{b} \leq 135$GeV, 91GeV $\leq m_{H} \leq 200$GeV with $m_{t} < m_{H}$ and 200GeV $\leq m_{A} \leq 250$GeV for the fixed mixing angle $\alpha = 0.6, \pi/2$ of the CP even NHBs and scan $\delta_{23}^{AA}$ in the range $|\delta_{23}^{AA}| \leq 0.1$ which arises from the constraints of $b \rightarrow s\gamma$ and $\Delta M_{s}$ \footnote{ $\delta_{23}^{AA}$ is the parameter in the usual mass insertion approximation \cite{21} and its definition can be found in Refs. \cite{13, 21}.}. We perform calculations of hadronic matrix elements of the Higgs penguin operators $Q_{1,2}^{(i)}$ at the $\alpha_{s}$ order (tree level), i.e., in the naive factorization formalism, and to the $\alpha_{s}$ order. The results for tree level (i.e., the naive factorization) and to the $\alpha_{s}$ order are shown in FIGs. 2a and 2b, respectively. Fig. 2a is given for $\mu = m_{b}$. In Fig. 2b green and red dots denote the results for $\mu = m_{b}$ and $\mu = m_{b}/2$ respectively. We find that although $S_{\phi K}$ can have a sizable deviation from SM but it can not be smaller than 0.6 if one use the naive factorization to calculate hadronic matrix elements. However, $S_{\phi K}$ for both $\mu = m_{b}$ and $\mu = m_{b}/2$ can have a significant deviation from SM when we include the $\alpha_{s}$ corrections. We find that there are regions of parameters where $S_{\phi K}$ falls in 1$\sigma$ experimental bounds and Br is smaller than $1.6 \times 10^{-5}$. The FIGs. 2a and 2b are plotted for $\alpha = \pi/2$. The similar figures are obtained for $\alpha = 0.6$. We stress that at tree level $S_{\phi K}$ can have a sizable deviation from SM because we can have values of $|C_{Q_{i}}|$ and $|C_{Q_{i}}^{\prime}|$ larger than those in CMSSM with universal boundary conditions at the high scale and the extended MFV scenarios in MSSM by a factor of 3 or so, as pointed out above. It is instructive to note that because the LR and RL mass insertions do not contribute to the Higgs penguin operators, the Higgs mediated
mechanism probes the LL and RR insertions, in contrast with the QCD and chromomagnetic penguins whose effects are too small to alter $S_{\phi K}$ significantly in the case of only LL and RR insertions\(^\dagger\). We have also carried through the calculations in the middle tan $\beta$ case, tan $\beta = 8$, within the range $|\delta_{23}^{AA}| \leq 1$ and the results are similar to those in the large tan $\beta$ case.

Next we consider the case of large $m_A$. In the limit $M_A^2 \gg M_{\tilde{g}}^2$ (for $M_A \geq 300$ GeV), the charged, heavy CP-even and CP-odd neutral Higgs bosons are nearly mass degenerate, $M_{\tilde{g}} \approx M_{\tilde{A}} \approx M_A$, $\sin(\beta - \alpha)$ approaches 1, and the properties of the lightest CP-even neutral Higgs boson $h$ are almost identical to those of the SM Higgs boson (so called the decoupling limit). In this case, $C_{1,2}^{\mu}$ approaches zero if $m_A m_h \ll m_h^2$. So $S_{\phi K}$ for $\mu \sim m_\mu$ cannot be smaller than 0.7 under the constraint from $B_s \to \mu^+ \mu^-$. However, as shown in Ref.\(^\ddagger\), the decoupling limit can be relaxed in some regions with $M_A^2 \gg M_{\tilde{g}}^2$ of the parameter space which are allowed by experiments of $(g - 2)_{\mu}$, $b \to s \gamma$ and lower bounds of Higgs and sparticle masses, due to the large off-diagonal scalar top and scalar bottom mass matrix elements contributing to the Higgs sector by radiative corrections. In the regions, the charged, heavy CP-even and CP-odd Higgs bosons are not mass degenerate, and $\sin^2(\beta - \alpha)$ can be damped from 1. We take two typical set of parameters in the regions given in the TABLE 1 Case B of Ref.\(^\ddagger\) to calculate Wilson coefficients $C_{1,2}^{(i)}(m_{\tilde{g}})$, $i = 11, 13, 17$. Corresponding numerical results are that $S_{\phi K} = -0.53$ and $-0.19$, $B_s = [0.77$ and $0.98] \times 10^{-5}$, respectively.

We now consider the case of only a LL insertion. Then $C_{11,15,16} = 0$ and consequently the experimental bound of $B_s \to \mu^+ \mu^-$ constrains $|C_Q|$ ($i = 1, 2$). We scan $\delta_{23}^{LL}$ in the range $|\delta_{23}^{LL}| \leq 0.1$ with other parameters same as above under all relevant constraints. The result is shown in Fig. 3a. We find that $S_{\phi K}$ can significantly deviate the SM value but it can not be smaller than 0.4. As pointed out above, if using the naive factorization of hadronic matrix elements, the minimal value of $S_{\phi K}$ is 0.7, in contrast with that of including the $\alpha_s$ corrections of hadronic matrix elements. The similar result is obtained for the case of only a RR insertion.

Finally we switch on other SUSY contributions of which the contributions of the chromomagnetic penguin with gluino-down type squark in the loop are dominant. We scan $|\delta_{23}^{LL,RR}| \leq 0.1$, with values of other parameters the same as those above under the constraints from $b \to s \gamma$ and $\Delta M_s$ as well as other relevant experiments mentioned above. The result including both the chromomagnetic and QCD penguin contributions and NHB penguin contributions is shown in FIG. 3b. One can see from the figure that there are regions of parameters where $S_{\phi K}$ falls in $1\sigma$ experimental bounds and Br is smaller than $1.6 \times 10^{-5}$. Comparing FIG. 3b with FIG. 2b, one can see that the regions of parameter space in the case including both contributions are larger than those when including only the NHB contributions.

Our results show that both the Br and $S_{\phi K}$ of $B \to \phi K_S$ are dependent of the scale $\mu$, which can be understood as follows. Let us look at, e.g., the contributions coming from $Q_{13,14,15}$ in $a_{4u}^{\text{new}}$ (see, Eq. (12)), we have:

\[
\frac{da_{4u}^{\text{new}}}{d \ln \mu} = d \left[ \frac{d}{d \ln \mu} \frac{C_F (C_{13} 2 \ln \mu - 4 C_{15} 2 \ln \mu - 8 C_{16} 2 \ln \mu) G_\phi^0}{4 \pi 4 N_C} \right] + \ldots
\]

\[
= \frac{\alpha_s C_F}{4 \pi 4 N_C} \left[ (C_{13} 2 - 4 C_{15} 2 - 8 C_{16} 2) \right] G_\phi^0 + \alpha_s^2 \ln \mu \text{ terms} + \ldots,
\]

\(^\dagger\) It is possible that effects of the QCD and chromomagnetic penguins are significant to alert $S_{\phi K}$ greatly for only a LL or RR insertion because there can be an induced LR or RL insertion if $\mu \tan \beta$ is large enough.\(^\ddagger\)
where "..." denotes the terms coming from $Q_{11}$ and $Q_i'$, $i=11,13,...,16$. The first term in Eq. (14) is cancelled by

$$\frac{d(-2C_8gG_0)}{d\ln \mu} = \frac{\alpha_s}{4\pi} \frac{C_F}{4N_c} \left[ -2(C_{13} - 4C_{15} - 8C_{16}) \right] G_0^0 + \ldots,$$

where $-2C_8gG_0$ is the contribution coming from the chromomagnetic dipole operator and "..." denotes the terms independent of $C_{13,...,16}$. However, the $\alpha_s^2\ln \mu$ terms in Eq. (14) is left. Although their appearance is of $\alpha_s^2$ order, their magnitude indeed are of $\alpha_s$ order. Similar analysis can be done for the other terms in $\alpha_s^2\ln \mu$.

It is necessary to make a theoretical prediction in SM as precision as we can in order to give a firm ground for finding new physics. For the purpose, we calculate the twist-3 and weak annihilation contributions in SM using the method in Ref. [23] by which there is no any phenomenological parameter introduced. The numerical results show that the annihilation contributions to $\text{Br}(B \to \phi K)$ are of order $10^{-8}$ which are negligible, the twist-3 contributions to Br are also very small, smaller than one percent, and both the annihilation and twist-3 contributions to $S_{\phi K}$ are negligible.

In summary, we have shown that the NHB contributions in MSSM can have a significant effect on both the Br and $S_{\phi K}$ of $B \to \phi K_S$. We can have large Wilson coefficients (comparable with those in SM) of the NHB penguin induced operators $Q^i_p$, in some regions of the parameter space in MSSM under all relevant experimental constraints, which leads to that even at the tree level of hadronic matrix element calculations the time dependent CP asymmetry $S_{\phi K}$ can deviate from SM sizably. Due to the large contributions to the hadronic elements of the operators at the $\alpha_s$ order arising from penguin contractions with b quark in the loop, both the Br and $S_{\phi K}$ are sizably different from those in SM. The mixing $Q_{13,...,16}$ with $Q_8g$ makes the Wilson coefficient $C_8g(\mu)$ ($\mu \sim m_b$) enhanced a lot, which has large effects on both $S_{\phi K}$ and the Br. In some regions of the parameter space in MSSM, the NHB contributions alone can lead to negative $S_{\phi K}$ which is reported by BaBar and Belle in 2002 and even make $S_{\phi K}$ agreed with 2003 Belle data in a 1σ experimental bounds. Our results show that both the Br and $S_{\phi K}$ of $B \to \phi K_S$ are dependent of the scale $\mu$. The sizable scale dependence, which implies hadronic uncertainties up to the $\alpha_s$ order, comes mainly from the $O(\alpha_s)$ corrections of hadronic matrix elements and also from leading order Wilson coefficients $C_{i}^{(1)}$, $i=11,...,16$. However, despite there are hadronic uncertainties, the conclusion that the NHB contributions in MSSM can have a significant effect on both the Br and $S_{\phi K}$ of $B \to \phi K_S$ can still be drawn definitely.

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