Cooperative self-organization and sympathetic cooling of a multispecies gas in a cavity

Tobias Grießer, Wolfgang Niedenzu and Helmut Ritsch
Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria
E-mail: Helmut.Ritsch@uibk.ac.at

New Journal of Physics 14 (2012) 053031 (11pp)
Received 10 February 2012
Published 24 May 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/5/053031

Abstract. We study the dynamics of a multispecies mixture of laser-illuminated polarizable particles moving inside an optical resonator. Above a certain pump threshold the collective enhanced scattering of laser light into the cavity induces a phase transition from a homogeneous spatial distribution to a common crystalline order. We analytically show that adding particles of any mass and temperature always strictly lowers the minimum pump power required for self-ordering and trapping. This allows to capture and trap new species of atoms, molecules or even polarizable nanoparticles in combination with proven examples, for which a high phase-space density is readily available. Cooperative light scattering mediates effective energy exchange and thus sympathetic cooling between different species without the need for direct collisional interaction. The predicted ordering thresholds and cooling timescales are in the range of current technology for particles with a wide range of mass, polarizability and initial temperature.

1 Author to whom any correspondence should be addressed.
1. Introduction

Laser light-induced forces are routinely used to manipulate polarizable particles from atoms and molecules [1] to larger objects such as nanoparticles, micro beads or even protozoae [2]. Laser trapping and cooling, however, is limited to a finite class of atomic species, very few kinds of molecules [3] or isolated vibration modes of nanomechanical objects [4]. Cooling requires specific setups with specifically chosen laser frequencies and configurations for any species, so that their number only slowly increases with time [5].

In principle, self-organization and cooling by coherent light-scattering in cavities gives a general alternative to trap and cool any kind of polarizable particles within an optical resonator [6–8]. In practice, however, the required phase-space densities and laser intensities have so far only been achieved for atomic ensembles [9–11], where theoretical expectations of fast sub-Doppler cooling were even surpassed [12], but the required phase-space density to achieve self-ordering and trapping has not been reached for molecules or nanoparticles [7, 13].

As the solution we propose to put ensembles of different species simultaneously into the same optical resonator. We predict that under suitable conditions all species are simultaneously trapped and cooled using only a single laser frequency and optical resonator. Our central claim is that the simultaneous presence of any additional species always increases the collective light scattering and improves trapping and cooling. As a particularly interesting case we study the mixture of a dense atomic ensemble with a smaller ensemble of heavier molecules or nanoparticles. Even when it is impossible to reach the self-organization threshold for the latter alone, combined trapping and sympathetic cooling can be readily achieved in cooperation with the atoms. Due to the nonlocal interaction the different particles might even be located at different regions within the cavity. This setup opens up a novel way of simultaneous multispecies trapping and cooling without the need for a tailored laser configuration for each species. This can be improved further, using several cavity modes simultaneously [7].

2. The model

Consider a dilute classical gas consisting of $S$ kinds of $N_i$ polarizable point particles of mass $m_i$ within the overlap region of a high-$Q$ optical resonator and a standing-wave pump laser tuned close to resonance with a cavity mode (figure 1).
The particles scatter light into and out of the cavity mode and the resulting interference pattern creates dynamical optical potentials guiding the particle motion. For simplicity we approximate the pump and cavity field in the interaction region by plane standing waves and consider motion along the cavity axis only. This suffices for describing the essential physics of self-organization and cooling [9, 14, 15]. A practical experimental implementation can be envisaged by confining the particles by two crossed standing-wave pump lasers into a lattice of one-dimensional (1D) tubes along the cavity axis [8, 16]. Extension to 3D motion and field geometries is straightforward and is expected to induce only minor quantitative changes [15]. In terms of the effective pump amplitudes $\eta_s$, the light shifts per photon $U_{0,s}$, and the semi-classical cavity mode amplitude $a$, the optical potentials along the cavity axis are given by [14]

$$\Phi_s(x, a, a^*) = \hbar \eta_s (a + a^*) \sin(kx) + \hbar U_{0,s} |a|^2 \sin^2(kx),$$

(1)

which lead to the one-body Hamiltonian functions $H_s(x, p, a, a^*) = \frac{p^2}{2m_s} + \Phi_s(x, a, a^*)$, determining the dynamics of an individual particle belonging to the $s$th species through the canonical equations of motion [17]. $H_s$ depends parametrically on the cavity field amplitude $a$, which in turn is driven by the light scattered collectively by all the particles and by white noise $\xi$, modeling vacuum fluctuations. As detailed in [19, 21], for a statistical treatment of the dynamics it is convenient to redefine the state of the particles of the $s$th species $\{x_j(t), p_j(t)\}$ in terms of the Klimontovich distribution [18]

$$f_K^s(x, p, t) := \frac{1}{N^s} \sum_{j_s=1}^{N^s} \delta(x-x_j(t))\delta(p-p_j(t)).$$

(2)

Then the mode amplitude evolves according to

$$\dot{a} = (i\Delta_c - \kappa)a - \frac{i}{\hbar} \sum_{s=1}^S N_s \int \frac{\partial H_s}{\partial a^*} f_K^s(x, p, t) \, dx \, dp + \sqrt{\kappa} \xi,$$

(3)
where \( \kappa \) denotes the cavity decay rate and \( \Delta_c = \omega_p - \omega_c \) the detuning between pump and cavity frequency. We decompose the Klimontovich distribution according to \( f_K^s(x, p, t) = f_s(x, p, t) + \delta f_s(x, p, t) \), where \( f_s(x, p, t) := \{ f_K^s(x, p, t) \} \), averaged over an ensemble of suitable initial conditions and the realizations of the white noise, is called one-body distribution function. Note that \( f_s(x, p, t) \, dx \, dp \) is equal to the expected fraction of particles of the \( s \)th species in a phase space volume \( dx \, dp \) around the point \( (x, p) \) at time \( t \) and the average over its fluctuations vanishes, \( \langle \delta f_s(x, p, t) \rangle \equiv 0 \). Likewise, we decompose the mode amplitude into \( a = \alpha + \delta a \), where \( \alpha = \langle a \rangle \). The one-body distribution function exactly satisfies

\[
\frac{\partial f_s}{\partial t} + \frac{p}{m_s} \frac{\partial f_s}{\partial x} - \frac{\partial \langle \Phi_s \rangle}{\partial x} \frac{\partial f_s}{\partial p} = \left\{ \frac{\partial \delta \Phi_s}{\partial x} \frac{\partial f_s}{\partial p} \right\},
\]

in which predominantly the rhs describes statistical correlations. For \( N_s \to \infty \) these tend to zero and we recover the Vlasov (or mean-field) kinetic theory. There, \( \langle \Phi_s(x, a, a^*) \rangle \) is replaced by \( \Phi_s(x, a, a^*) \) and the rhs of (4) is set to zero, such that spatially homogeneous particle distributions scatter no light into the mode and constitute an equilibrium state at zero cavity field \([19]\).

3. Multispecies self-organization threshold

Following \([19]\), the multispecies self-organization threshold is obtained as the boundary of dynamical stability of spatially uniform distributions in the case of negative effective detuning \( \delta := \Delta_c - \frac{1}{2} \sum_s N_s U_{0,s} < 0 \), by an analysis of the linearized Vlasov equation \([20]\). For convenience we rescale the uniform equilibrium distributions as \( f_0_s(p) = (L m_s v_s)^{-1} G_s(\frac{p}{m_s v_s}) \) in terms of a typical velocity \( v_s \) and the cavity length \( L \). Assuming a strictly monotonic decrease in \( |p| \), as fulfilled by all relevant distributions (e.g. Gaussian, Bose–Einstein, \( q \)-Gaussian, etc), a given set of spatially homogeneous distributions is unstable if and only if

\[
\sum_{s=1}^{S} \frac{N_s \eta_s^2}{k_B T_s} \left( \int_{-\infty}^{\infty} -\frac{1}{2u} \, du \right) > \frac{\kappa^2 + \delta^2}{h|\delta|},
\]

with \( k_B T_s = m_s v_s^2 / 2 \) and \( P \) the Cauchy principal value. For thermal (i.e. Gaussian) momentum distributions the integral in (5) is unity and the threshold condition assumes the simple form

\[
\sum_{s=1}^{S} \frac{N_s \eta_s^2}{k_B T_s} \geq \frac{\kappa^2 + \delta^2}{h|\delta|}.
\]

This threshold formula is one of the central results of this work. Above threshold, density perturbations and the electric field amplitude grow exponentially and evolve if the light shift is not too large, i.e. \( N_s |U_{0,s}| \left( 1 + \sum_{s' \neq s} \frac{N_{s'} \eta_{s'}}{N_s \eta_s} \right) \lesssim |\delta| \) for all species, towards an ordered quasi-stationary state (figure 2) with growth exponent \( \gamma > 0 \) fulfilling

\[
(\gamma + \kappa)^2 + \delta^2 = \sum_{s=1}^{S} \frac{N_s \eta_s^2 h \delta}{2k_B T_s} \int_{-\infty}^{\infty} \frac{u \, dG_s/du}{(\gamma/k v_s)^2 + u^2} \, du.
\]

Note that, while the rhs of equations (5) and (6) only depends on cavity parameters, all terms in the sum on the lhs are manifestly positive and proportional to the pump intensity. This has the important consequence that inserting any extra particle species into the cavity
will lower the power needed to start the self-organization process, regardless of temperature or polarizability of the added particles. Note that we neglect absorption of the pump beam, consistent with our assumption of a dilute and optically thin gas. At higher temperatures, where \((kv_s)^2 \gg \kappa^2 + \delta^2\), the field amplitude’s growth rate is, from (7), given by

\[
\gamma = -\kappa + \left( \sum_{j=1}^{S} \frac{h|\delta|}{k_B T_s} N_j \eta_s^2 - \delta^2 \right)^{1/2}.
\]  

We thus find strong sympathetic enhancement, i.e. the field grows faster the more species contribute such that the required power and time needed for self-organization is lowered by combining several species.

4. Long-term dynamics and equilibrium

For a large but finite number of particles, the Vlasov kinetic theory, which neglects all dynamical correlations, provides an accurate description on a time scale essentially fixed by the solution to equation (7). The long-term evolution of the system and in particular its statistical equilibrium
state are, on the other hand, governed by precisely these correlations [21]. In this section, we shall deal with this stage of the time evolution in the limit of weak coupling, i.e. \( \sum N_i |U_{ij}| \ll |\Delta_c| \), where we can neglect the terms \( \hbar U_{ij} |a|^2 \sin^2(kx) \) in the optical potentials (1), rendering them linear functions of the mode amplitude. We perform, for each species separately, a canonical transformation of variables \((x, \phi) \rightarrow (I_i, \theta_i)\). Here, \( I_i \) denotes the one-body action based on the ensemble-averaged Hamiltonian function \( \langle H_i(x, p, a, a^*) \rangle \equiv H_i(x, p, \alpha, \alpha^*) \),

\[
I_i = \pm \frac{1}{2\pi} \int \sqrt{2m_i} \left[ \langle H_i \rangle - \langle \Phi_i(x') \rangle \right] \, \mathrm{d}x',
\]

and \( \theta_i \) its canonically conjugate angle variable

\[
\theta_i = \frac{\partial S_i}{\partial I_i},
\]

obtained from the generating function \( S_i = \pm \int^x \sqrt{2m_i} \left[ \langle H_i \rangle - \langle \Phi_i(x') \rangle \right] \, \mathrm{d}x' \). The reason for doing this is that at the end of the initial, mean-field governed dynamics, the one-particle distributions depend on \((x, p)\) solely through the ensemble-averaged one-body Hamiltonian functions and thus on the actions alone. From that point onwards, they are slowly modified by the dynamical correlations in such a way that the system evolves towards statistical equilibrium in a sequence of mean-field steady states [23]

\[
f_s(x, p, t) \simeq f_s(I_s, t).
\]

After a lengthy calculation in these new variables, the system’s long-term evolution can be cast into a set of coupled nonlinear Fokker–Planck equations

\[
\frac{\partial f_s}{\partial t} = \frac{\partial}{\partial I_s} \left( A_s f_s + B_s \frac{\partial f_s}{\partial I_s} + \sum_{r=1}^S C[f_s, f_r] \right)
\]

for the distributions. The quasi-stationary ensemble-averaged mode amplitude is determined by the implicit equation

\[
\alpha = \frac{2\pi}{\Delta_c + \imath \kappa} \sum_{s=1}^S N_s \eta_s \int f_s(I_s) g_{0,s}(I_s, \alpha) \, \mathrm{d}I_s,
\]

wherein

\[
g_{n,s}(I_s, \alpha) := \frac{1}{2\pi} \int_0^{2\pi} \sin(kx) e^{-\imath n \theta_i} \, \mathrm{d}\theta_i.
\]

The rhs of equation (12), describing the redistribution of particles among the orbits, consists of two contributions originating from fluctuations and decay of the mode amplitude

\[
A_s[f_s] = -4\hbar \Delta_c \kappa \omega_0 \sum_{n=-\infty}^{\infty} \frac{n^2 \eta^2_n |g_{n,s}|^2}{|D(in\omega_0)|^2},
\]

\[
B_s[f_s] = \hbar^2 \kappa \sum_{n=-\infty}^{\infty} \frac{n^2 \eta^2_n |g_{n,s}|^2}{|D(in\omega_0)|^2} \left( \kappa^2 + \Delta_c^2 + n^2 \omega_0^2 \right),
\]

and a generalized Balescu–Lenard operator [24–26]

\[
C[f_s, f_r] = \sum_{n,m=-\infty}^{\infty} \int w_{nm}(I_s, I_r) \left( n \frac{\partial f_s}{\partial I_s} f_r' - m \frac{\partial f_r}{\partial I_r} f_s \right) \, \mathrm{d}I_s, \quad \mathrm{d}I_r,
\]

New Journal of Physics 14 (2012) 053031 (http://www.njp.org/)
 where

\[ w_{nm}(I_s, I'_s) := 8\pi^2\hbar^2\Delta_c^2 n_r \frac{\eta_n^2|g_{n,s}|^2 \eta_{n'}^2|g_{n',s'}|^2}{|D(i\omega_n)||D(i\omega_{n'})|} n\delta (n\omega_n - m\omega_{n'}). \] (18)

Here, \( \omega_n(I_s) = \partial (H_s) / \partial I_s \) is the nonlinear orbital frequency and the prime denotes the function at \( I_s = I'_s \). For spatially uniform ensembles \( I_s \to p/k \) and the expressions for \( A_s \) and \( B_s \) given in [21] are recovered. \( D(i\omega) \), here called the dielectric function, is given by

\[ D(i\omega) = (i\omega + \kappa)^2 + \Delta_c^2 - 4\pi\hbar\Delta_c \sum s \sum n \infty N_s \eta_s^2 \int \frac{\partial f_s}{\partial I_s} \frac{n|g_{n,s}|^2}{\omega + n\omega_s - i0} dI_s \] (19)

and characterizes the system’s collective response. Let us remark that the coupled kinetic equations (12) constitute another central result of the present work. In their derivation we assumed that the particle distribution functions \( f_s(I_s, t) \) are always strongly Vlasov stable. This assumption breaks down close to the self-organization threshold and thus (12) is valid only away from the transition point. The interaction contained in the Balescu–Lenard collision operator (17) quantifies the energy and momentum exchange between particles of like and different species and involves orbits \( I_r, I_s \) with \( n\omega_r = m\omega_s \). In mechanics, such orbits with commensurable frequencies are called resonant. The origin of the energy exchange term (17) lies in the scattering of laser light into the cavity by a first particle and subsequent backscattering into the laser mode by a second, resonant particle. This is in effect a pair collision albeit entirely nonlocal. The appearance of the dielectric function also reveals that the remaining particles participate collectively as a medium in that process. These quasi-collisions can be used for efficient sympathetic cooling as demonstrated below. The source of the remaining terms in (12) involves only single scattering events and subsequent loss through the cavity mirrors. It is worth remarking that the quasi-collision operator (17) vanishes for thermal distributions with equal temperatures, and thus the quasi-collisions tend to establish global thermal equilibrium.

Stable equilibria of (12) exist only for \( \Delta_c < 0 \). Below threshold, they are homogeneous with vanishing field and, independent of the number of species, \( q \)-Gaussian momentum distributions:

\[ f_{s, eq}(p) \propto \exp_{q_s} \left( \frac{-p^2}{2m_s k_B T} \right), \] (20)

where

\[ k_B T := \hbar \frac{\kappa^2 + \Delta_c^2}{4|\Delta_c|} \] (21)

and

\[ \exp_q(u) = \left[ 1 + (1 - q)u \right]^{1/\gamma} \] (22)

is the \( q \)-exponential with parameter \( q_s = 1 + \omega_{R,s}/|\Delta_c| \). For \( q_s \to 1 \) the distribution becomes an ordinary Gaussian. The recoil frequencies are given by \( \omega_{R,s} := \hbar \kappa^2 / 2m_s \) and \( k_B T \) denotes a ‘thermal’ energy with a minimum of \( \hbar \kappa^2 / 2 \) for \( \Delta_c = -\kappa \). The reason why the equilibrium state of a given species is unaffected by the presence of others is because of the vanishing of interspecies scattering \( C[f_{s, eq}, f_{r, eq}] \equiv 0 \).
Sufficiently far above threshold, the ordered equilibria are well approximated by the Maxwell–Boltzmann distributions

$$f_{s, \text{eq}}(x, p) \propto \exp \left( \frac{-H_s}{k_B T_{\text{kin}}^s} \right),$$

with kinetic temperatures

$$k_B T_{\text{kin}}^s := \frac{\langle p^2 \rangle}{m_s} = k_B T + \frac{\hbar \omega_{0,s}^2}{|\Delta_c|}$$

and trap frequencies $$\omega_{0,s}^2 = 4\eta_1 \omega_{R,s} |Re \alpha|$$ proportional to the cavity field generated commonly by all species. Therefore, unlike below threshold, the equilibrium of a given species is affected by the presence of the others. Figure 3 shows the formation and properties of a two-species self-organized steady state. The initial increase of the kinetic energy originates from the fast growth of the cavity intensity due to instability and is followed by cooling in the trapped state.

5. Sympathetic cooling

Most interestingly, the energy exchange between different sorts of particles reduces the cooling time for any species in the presence of another via collisionless sympathetic cooling. At this point it is necessary to clarify the notion of cooling. We shall associate with cooling a reduction of the extension of the sth species in one-body phase space. As the quantity $$\langle J_s \rangle := \int J_s f_s(I_s) dI_s$$, with $$J_s = I_s$$ for transient and $$J_s = I_s / 2$$ for trapped orbits, provides a measure of this extension, cooling therefore corresponds to a decrease of $$\langle J_s \rangle$$.

Let it be remarked that also the Bohr–Sommerfeld correspondence principle between integer multiples of $$\hbar$$ of the action variable and quantum mechanical energy eigenstates suggests this definition.
Figure 4. Time evolution of the kinetic energy (lower red pair of curves) and phase space volume $\langle J \rangle$ (upper blue pair) of a heavy species alone (dashed lines) and in the presence of a lighter species (solid lines). The rising solid curve depicts the intra-cavity intensity (a.u.) for sympathetic cooling. The threshold and enhanced cooling due to the second species is clearly visible. The inset depicts the distribution of the heavy particles at final time. The vertical lines are the action values separating trapped from untrapped particle orbits. In the case of the presence of the second species, almost all particles are finally trapped, whereas in the other case, almost no particle is trapped. Parameters: $N_1 = 1500$, $N_2 = 100$, $m_2 = 80m_1$, $\sqrt{N_1} \eta_1 = 400\omega_{R,1}$, $\sqrt{N_2} \eta_2 = 245\omega_{R,1}$, $\kappa = 100\omega_{R,1}$ and $\delta = -\kappa$.

For carefully chosen parameters, such that the trap frequencies of the species roughly coincide and thus allow for quasi-collisions even in the self-organized regime, the sympathetic cooling effect persists in principle also above threshold.

Finally, the energy flow per particle from species two to species one, $\dot{Q}_{21}$, for two spatially homogeneous ensembles can, from equation (17), be estimated as

$$\dot{Q}_{21} \approx \frac{2N_1 \eta_1^2 \eta_2^2 \hbar \Delta^2_\eta}{(\kappa^2 + \Delta^2_\eta)^2} \sqrt{\frac{m_1}{m_2}} \sqrt{\frac{\pi \hbar \omega_{R,2}}{k_B T_1}} \left[ 1 - \frac{T_2}{T_1} \right] \left[ 1 + \frac{m_1 T_2}{m_2 T_1} \right]^{-\frac{1}{2}}.$$

(25)
Here we assume that the first species is already cold, i.e. $2k_B T_1 / \hbar \kappa \ll \kappa / \omega_{R,1}$, and far from instability. It is maximal if $\Delta_c = -\kappa$.

6. Conclusions

In summary, we have shown that if the self-organization threshold can be reached with a certain species, any species can be added and will be trapped and cooled as well. The final temperatures are only limited by the resonator linewidth and, importantly, the cooling time of a given species can be reduced by means of energy exchange with a second, already colder and lighter species. Because the general effect has successfully been demonstrated in single-species experiments [12, 27, 28], we are confident that the multispecies generalization proposed here is well within the reach of current technology. New phases can also be expected in the case of a crystallization of a multispecies quantum gas close to absolute zero [29]. We expect that simultaneous additional cooling of one species will help us to cool all others.

Acknowledgments

We thank Peter Asenbaum, Nikolai Kiesel and Matthias Sonnleitner for discussions and Andreas Grießer for graphical support on the sketch. We acknowledge support from the Austrian Science Fund FWF through the projects SFB FoQuS P13 and P20391.

References

[1] Metcalf H J and van der Straten P 1999 Laser Cooling and Trapping (New York: Springer)
[2] Thalhammer G, Steiger R, Bernet S and Ritsch-Marte M 2011 Optical macro-tweezers: trapping of highly motile micro-organisms J. Opt. 13 044024
[3] Doyle J, Friedrich B, Krems R V and Masnou-Seeuws F 2004 Quo vadis, cold molecules? Eur. Phys. J. D 31 149–64
[4] Kippenberg T J and Vahala K J 2008 Cavity optomechanics: back-action at the mesoscale Science 321 1172
[5] Shuman E S, Barry J F and DeMille D 2010 Laser cooling of a diatomic molecule Nature 467 820–3
[6] Vuletić V and Chu S 2000 Laser cooling of atoms, ions, or molecules by coherent scattering Phys. Rev. Lett. 84 3787–90
[7] Lev B L, Vukics A, Hudson E R, Sawyer B C, Domokos P, Ritsch H and Ye J 2008 Prospects for the cavity-assisted laser cooling of molecules Phys. Rev. A 77 023402
[8] Nimmrichter S, Hammerer K, Asenbaum P, Ritsch H and Arndt M 2010 Master equation for the motion of a polarizable particle in a multimode cavity New J. Phys. 12 083003
[9] Vuletić V, Chan H W and Black A T 2001 Three-dimensional cavity doppler cooling and cavity sideband cooling by coherent scattering Phys. Rev. A 64 033405
[10] Slama S, Bux S, Krenz G, Zimmermann C Ph. and Courteille W 2007 Superradiant Rayleigh scattering and collective atomic recoil lasing in a ring cavity Phys. Rev. Lett. 98 053603
[11] Baumann K, Guerlin C, Brennecke F and Esslinger T 2010 Dicke quantum phase transition with a superfluid gas in an optical cavity Nature 464 1301–6
[12] Black A T, Chan H W and Vuletić V 2003 Observation of collective friction forces due to spatial self-organization of atoms: from Rayleigh to Bragg scattering Phys. Rev. Lett. 91 203001
[13] Deachapunya S, Fagan P J, Major A G, Reiger E, Ritsch H, Stefanov A, Ulbricht H and Arndt M 2008 Slow beams of massive molecules Eur. Phys. J. D 46 307–13

New Journal of Physics 14 (2012) 053031 (http://www.njp.org/)
[14] Asbóth J K, Domokos P, Ritsch H and Vukics A 2005 Self-organization of atoms in a cavity field: threshold, bistability and scaling laws Phys. Rev. A 72 053417
[15] Salzburger T and Ritsch H 2009 Collective transverse cavity cooling of a dense molecular beam New J. Phys. 11 055025
[16] Haller E, Gustavsson M, Mark M J, Danzl J G, Hart R, Pupillo G and Nägerl H-C 2009 Realization of an excited, strongly correlated quantum gas phase Science 325 1224
[17] Domokos P, Horak P and Ritsch H 2001 Semiclassical theory of cavity-assisted atom cooling J. Phys. B: At. Mol. Opt. Phys. 34 187
[18] Klimontovich Y L 1995 Statistical Theory of Open Systems 1st edn (Dordrecht: Kluwer)
[19] Grießer T, Ritsch H, Hemmerling M and Robb G R M 2010 A Vlasov approach to bunching and selfordering of particles in optical resonators Eur. Phys. J. D 58 349–68
[20] Landau L D 1946 On the vibrations of the electronic plasma J. Phys. USSR 10 574
[21] Niedenzu W, Grießer T and Ritsch H 2011 Kinetic theory of cavity cooling and selforganisation of a cold gas Europhys. Lett. 96 43001
[22] Krapchev V B and Ram A K 1980 Adiabatic theory for a single nonlinear wave in a Vlasov plasma Phys. Rev. A 22 1229
[23] Chavanis P-H 2007 Kinetic theory with angle-action variables Physica A 377 469–86
[24] Balescu R 1960 Irreversible processes in ionized gases Phys. Fluids 3 52
[25] Lenard A 1960 On Bogoliubov’s kinetic equation for a spatially homogeneous plasma Ann. Phys. 10 390–400
[26] Luciani J F and Pellat R 1987 Kinetic equation of finite Hamiltonian systems with integrable mean field J. Physique 48 591–9
[27] Kruse D, Ruder M, Benhelm J, von Cube C, Zimmermann C, Courteille W Ph., Elsässer Th., Nagorny B and Hemmerich A 2003 Cold atoms in a high-q ring cavity Phys. Rev. A 67 051802
[28] Ritter S, Brennecke F, Baumann K, Donner T, Guerlin C and Esslinger T 2009 Dynamical coupling between a Bose–Einstein condensate and a cavity optical lattice Appl. Phys. B 95 213–8
[29] Gopalakrishnan S, Lev B L and Goldbart P M 2009 Emergent crystallinity and frustration with Bose–Einstein condensates in multimode cavities Nature Phys. 5 845–50