Wave absorption of an orthotropic rectangular panel based on direct feedback

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Abstract. This paper presents a new methodology of realizing an active wave control for an orthotropic rectangular panel. In general, an ideal wave controller is expressed by non-causal function which is impossible to be realized practically. The static direct feedback is introduced to overcome this problem. Firstly, a transfer matrix method for an orthotropic rectangular panel is introduced to describe the wave dynamics of the structure. This is followed by the derivation of feedback control laws for absorbing reflected waves. Then, static direct feedback is presented to realize the perfect wave absorption at single frequency. Finally, from a viewpoint of numerical analyses, control effects of the proposed method are verified by evaluating the absolute displacement distribution. It is found that the reflected wave absorbing control enables the inactivation of all vibration modes in the controlled direction.

1. Introduction

Although many vibration control methods have been proposed regardless of feedforward or feedback control, its mainstream is modal-based approach which is based on modal analyses. However, this method encounters difficulties in controlling a distributed parameter structure since such a structure has an infinite number of vibration modes. To overcome this problem, active wave control which is based on the hypothesis of modal excitation has been studied in recent years. According to the hypothesis, the standing waves are the direct cause of modal excitation, so that if the reflected waves are eliminated with any control input, then no standing waves are produced, resulting in the inactivation of vibration modes.

Reviewing literature on active wave control for a rectangular panel, most of the control methods are proposed for a semi-infinite structure [1-3]. Pan and Hansen [1] presented power transmission control along a semi-infinite panel using in-phase or independent control forces, and investigated the theoretically achievable reduction for each type of control forces. Furthermore, Kessissoglou [2, 3] analytically and experimentally studied active control of the bending wave transmission through the reinforcing beam of a ribbed and semi-infinite panel. In the case of a finite structure, Sakano and Tanaka [4] proposed active wave control for generating a quiet zone in a finite and pinned-supported panel by decomposing eigenfunctions into two wave components; positive travelling wave and negative travelling wave. However, this method is not a natural extension of the one-dimensional active wave control, and hence its control properties are far less effective than the ideal case, which is expected for one-dimensional active wave control. To cope with this problem, the authors previously
presented feedforward-based active wave control of a rectangular panel based on a transfer matrix method, clarifying the best possible control effect of the proposed method [5]. Furthermore, the authors have extended the feedforward wave control of an isotropic rectangular panel to the one for an orthotropic rectangular panel [6].

The objective of this paper is to present feedback-based active wave control of an orthotropic rectangular panel. Firstly, a transfer matrix method for an orthotropic rectangular panel whose sides are parallel or normal to the principal axes of orthotropy is introduced to describe the wave dynamics of the structure. This is followed by the derivation of feedback control laws for absorbing reflected waves. Then, static direct feedback is presented to realize the perfect wave absorption at single frequency. Finally, from a viewpoint of numerical analyses, control effects of the proposed method are verified by evaluating the absolute displacement distribution. It is found that the reflected wave absorbing control enables the inactivation of all vibration modes in the controlled direction.

2. Description of wave dynamics of an orthotropic rectangular panel

Conventional modal expression is not suitable for treating the wave dynamics of an orthotropic rectangular panel. Therefore, this paper employs a transfer matrix method which explicitly treats the wave dynamics. Assuming that shear deformation and rotary inertia are negligible, a displacement of an orthotropic rectangular panel, \( w(x,y,t) \), satisfies the following equation:

\[
D_{xx} \frac{\partial^4 w(x,y,t)}{\partial x^4} + 2(D_{xy} + D_{yy}) \frac{\partial^4 (x,y,t)}{\partial x^2 \partial y^2} + D_{yy} \frac{\partial^4 w(x,y,t)}{\partial y^4} + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = f(x,y,t)
\]

(1)

where

\[
D_{xx} = \frac{E_x h^3}{12(1-\nu_x\nu_y)} , D_{yy} = \frac{E_y h^3}{12(1-\nu_x\nu_y)} , D_{xy} = \nu_y D_{xx} = \nu_x D_{yy} , D_{ss} = \frac{G_{xy} h^3}{6}
\]

(2)

and \( E_x \) and \( E_y \) are the Young's modulus for the \( x \) and \( y \) direction, \( \nu_x \) and \( \nu_y \) are the Poisson's ratio for the \( x \) and \( y \) direction, \( G_{xy} \) is the shear modulus, \( \rho \) is the density, \( h \) is the thickness, and \( f(x, y, t) \) is the external pressure applied to the panel. Assuming that the structure is subjected to the harmonic excitation, the time-dependent term of Eq.(1) is \( e^{j\omega t} \). If this is the case, a general solution to Eq.(1) is given by

\[
w(x,y,t) = \sum_{l=1}^{\infty} W_l(x,y)e^{j\omega t}
\]

(3)

In the conventional modal analysis, \( W_l(x,y) \) in the above equation is expressed as the product of a modal coefficient and a normalized modal function. This paper, however, aims to express the structural vibration by wave motion, employing the separation of the spatial function \( W_l(x,y) \) for each direction, that is,

\[
W_l(x,y) = w_{l,m}(x)w_{j,n}(y)
\]

(4)

where \( m \) and \( n \) denote the modal index in the \( x \) and \( y \) direction, respectively. Note that Eq. (4) is valid if \( w_{l,m}(x) \) or \( w_{j,n}(y) \) is a harmonic function. In this paper, the latter is assumed to be a harmonic function. If this is the case, \( w_{j,n}(y) \) satisfies the following equation:

\[
\frac{d^2 w_j(y)}{dy^2} = -\beta_n^2 w_n(y)
\]

(5)

where \( \beta_n \) is a wavenumber in the \( y \) direction. It should be noted that the classical boundary condition which satisfies this assumption are a pinned support and a sliding support. However, in the region where the near-field effect is relatively small, the assumption in Eq. (5) approximately holds.
Then, substituting Eqs. (3), (4) and (5) into a homogeneous form of Eq. (1), an ordinary differential equation for the $x$ direction is derived as

$$\frac{d^4w_m}{dx^4} - 2\beta_n^2 D_1 \frac{d^2w_m}{dx^2} + \left(\beta_n^4 D_2 - k^4\right)w_m = 0$$

(6)

where

$$D_1 = \frac{D_{xx} + D_{xx}}{D_{xx}}, D_2 = \frac{D_{yy}}{D_{xx}}, k^4 = \frac{\rho h\omega^2}{D_{xx}}$$

(7)

Solving Eq. (6), the progressive wave solution in the $x$ direction of the panel is derived as

$$w_m(x) = c_1 e^{a_n x} + c_2 e^{b_n x} + c_3 e^{-a_n x} + c_4 e^{-b_n x}$$

(8)

where $c_1$, $c_2$, $c_3$ and $c_4$ are the coefficients defined by the boundary conditions and external inputs. Furthermore, $a_n$ and $b_n$ are described as

$$a_n = \sqrt{\beta_n^2 D_1 + \gamma_n}, \quad b_n = \sqrt{\beta_n^2 D_1 - \gamma_n}$$

(9)

$$\gamma_n = \sqrt{\beta_n^4 D_1^2 - \beta_n^4 D_2^2 + k^4}$$

(10)

As shown in Eq. (8), $w_m(x)$ depends on only the modal index for the $y$ direction, $n$. Next, substituting Eqs. (4) and (8) into Eq. (3), a general solution to Eq. (1) is given by

$$w(x, y) = \sum_{n=1}^{\infty} \left( c_{1n} e^{a_n x} + c_{2n} e^{b_n x} + c_{3n} e^{-a_n x} + c_{4n} e^{-b_n x} \right) w_n(y)$$

(11)

Applying the formulae provided by material mechanics to Eq. (11), slope $\theta(x, y)$, internal bending moment $m_i(x, y)$ and internal shear force $q_i(x, y)$ in the $x$ direction are obtained. The state vector $z(x, y)$ at arbitrary position on the rectangular panel is then defined as

$$z(x, y) = \left[ w(x, y) \quad \theta(x, y) \quad m_i(x, y) \quad q_i(x, y) \right]^T = \sum_{n=1}^{\infty} z_n(x) w_n(y)$$

(12)

where the superscript T denotes the transpose of the expression, and $z_n(x)$ is the state vector in the $x$ direction at modal index, $n$, which is defined as

$$z_n(x) = \left[ w_n(x) \quad \theta_{x,n}(x) \quad m_{x,n}(x)/D \quad q_{x,n}(x)/D \right]^T = K_n \cdot w_n(x)$$

(13)

where $K_n$ is the wavenumber matrix which is defined as

$$K_n = \begin{bmatrix}
1 & 1 & 1 & 1 \\
-b_n & -a_n & -a_n & -b_n \\
\nu_y \beta_n^2 - a_n^2 & \nu_y \beta_n^2 - b_n^2 & \nu_y \beta_n^2 - b_n^2 & \nu_y \beta_n^2 - a_n^2 \\
\left\{ \beta_n^2 D_3 \left( a_n^2 - \nu_y \beta_n^2 \right) \right\} & \left\{ \beta_n^2 D_3 \left( b_n^2 - \nu_y \beta_n^2 \right) \right\} & \left\{ \beta_n^2 D_3 \left( b_n^2 - \nu_y \beta_n^2 \right) \right\} & \left\{ \beta_n^2 D_3 \left( a_n^2 - \nu_y \beta_n^2 \right) \right\} \\
1 & 1 & 1 & 1 \\
-a_n & b_n & b_n & a_n \\
\nu_y \beta_n^2 - a_n^2 & \nu_y \beta_n^2 - b_n^2 & \nu_y \beta_n^2 - b_n^2 & \nu_y \beta_n^2 - a_n^2 \\
\left\{ \beta_n^2 D_3 \left( a_n^2 - \nu_y \beta_n^2 \right) \right\} & \left\{ \beta_n^2 D_3 \left( b_n^2 - \nu_y \beta_n^2 \right) \right\} & \left\{ \beta_n^2 D_3 \left( b_n^2 - \nu_y \beta_n^2 \right) \right\} & \left\{ \beta_n^2 D_3 \left( a_n^2 - \nu_y \beta_n^2 \right) \right\}
\end{bmatrix}$$

(14)
$w_n(x) = \frac{D_3}{D_{xx}}$ 

and $w_n(x)$ is the wave vector which is defined as

$w_n(x) = \begin{bmatrix} c_{1,n}e^{i\alpha x} & c_{2,n}e^{i\beta x} & c_{3,n}e^{-i\alpha x} & c_{4,n}e^{-i\beta x} \end{bmatrix}^T$ 

Furthermore, the wave vector expands to

$w_n(x) = D_n(x)e_n$ 

where

$D_n(x) = \begin{bmatrix} e^{i\alpha x} & 0 & 0 & 0 \\ 0 & e^{i\beta x} & 0 & 0 \\ 0 & 0 & e^{-i\alpha x} & 0 \\ 0 & 0 & 0 & e^{-i\beta x} \end{bmatrix}$

Next, consider a panel element along the $x$ axis with node $i-1$ and $i$ at both ends of the element. Let $z_n(x_{i-1})$ and $z_n(x_i)$ be $i$-$i$th $z_n$ and $z_n$ for brevity, and apply a local coordinate to these state vectors. Hence, by setting $i$-$i$th $z_n$ to be a local origin, $z_n$ is written as a function of the distance between the two nodes, $l$. Then, from Eqs. (13) and (17), the state vectors at each node are described as

$i$-$i$th $z_n = K_{i,n}D_n(0)e_n = K_{i,n}e_n$ 

$i$-$i$th $z_n = K_{i,n}D_n(l)e_n$ 

Multiplying Eq. (20) by $K_{i,n}^{-1}$ and substituting the resultant equation into Eq. (21) leads to the state equation of a panel expressed in the form,

$i$-$i$th $z_n = K_{i,n}D_n(l)K_{i,n}^{-1}z_n = i$-$i$th $T_n z_n$ 

where denotes the transfer matrix of the state vectors in the $x$ direction at modal index, $n$, between the node $i$-$1$ and $i$. Next, using Eq. (14), Eq. (22) is transformed as

$i$-$i$th $w_n = D_n(l)i$-$i$th $w_n$ 

As seen from Eq. (23), the matrix $D_n$ is found to be a transfer matrix of the wave vectors, that is, propagation matrix.

Next, consider the case where a disturbance force $f_d$ acts at node 2 and $N_c$ control forces ($f_1, f_2, \ldots, f_{N_c}$) placed parallel to the $y$ axis act to node 1, as shown in Fig. 1. The state equation of the rectangular panel in the $x$ direction at modal index, $n$, is then written as

$r$-$r$th $z_n = r$-$r$th $T_n f_{r,c,n} + r$-$r$th $T_n f_{r,d,n}$ 

where

$f_{c,n} = \begin{bmatrix} 0 & 0 & \tilde{f}_{c,n}/D_{xx} \end{bmatrix}^T, \tilde{f}_{c,n} = c_f \sum_{j=1}^{N_c} f_j w_n(y_j)$

$f_{d,n} = \begin{bmatrix} 0 & 0 & \tilde{f}_{d,n}/D_{xx} \end{bmatrix}^T, \tilde{f}_{d,n} = c_f \tilde{f}_{d,n} w_n(y_d)$
where $c_f$ is a constant determined by the boundary conditions at $y=0$ and $y=L_y$. Here, the boundary conditions at $x=0$ and $x=L_x$ are given by setting two out of four state variables in the state vector to be zero. Supposing that the $i$th and $j$th state variables at the right end are zero, and the $p$th and $q$th state variables at the left end, $L_z p, n$ and $L_z q, n$, are non-zero, Eq. (24) expands to the following equations:

$$\begin{align*}
R_i L_{1} t_{p,n} L_z &+ R_i L_{1} t_{q,n} L_z + R_i t_{14,n} f_{c,n} + R_i t_{14,n} f_{d,n} = 0 \\
R_i L_{1} t_{p,n} L_z &+ R_i L_{1} t_{q,n} L_z + R_i t_{14,n} f_{c,n} + R_i t_{14,n} f_{d,n} = 0
\end{align*}$$

(27)

(28)

where $\beta_{ij}$ denotes the $i$th row and $j$th column variable in the transfer matrix $T_{ij}$. Then, from Eqs. (27) and (28), the non-zero state variables at the left end (node $L$) are given in the matrix form:

$$\begin{bmatrix}
L_z p, n \\
L_z q, n
\end{bmatrix} = -A_n^{-1} B_n \begin{bmatrix}
f_{c,n} \\
f_{d,n}
\end{bmatrix}$$

(29)

where

$$A_n = \begin{bmatrix}
R_i L_{1} t_{p,n} L & R_i L_{1} t_{q,n} L \\
R_i L_{1} t_{p,n} L & R_i L_{1} t_{q,n} L
\end{bmatrix}$$

(30)

$$B_n = \begin{bmatrix}
R_i t_{14,n} & R_i t_{14,n} \\
R_i t_{14,n} & R_i t_{14,n}
\end{bmatrix}$$

(31)

3. Feedback control law for wave absorption and its realization using static output feedback

In this section, the control law for absorbing reflected waves is derived. As discussed in the previous section, the wave dynamics is individually described each in the ($\ast$, $n$) mode group in the transfer matrix method. Thus, a feedback loop should be formed each for the mode group. Employing the $n$th
x-dependent displacement component at \(x=x_m\) (node \(m\)) in the region between the nodes \(L\) and 1 as a feedback signal, the corresponding control force is described as

\[
\tilde{f}_{c,n} = D_{nx} g_n w_n(x_m)
\]  

(32)

where \(g_n\) is a feedback wave control law for the \((*, n)\) mode group. Furthermore, the \(n\)th displacement component is given by

\[
w_n(x_m) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \chi_n(x_m) = m_L t_{1,p,n} L z_{p,n} + m_L t_{1,q,n} L z_{q,n}
\]  

(33)

Substituting Eqs. (32) and (33) into Eqs. (27) and (28), the non-zero initial state vector with a feedback loop is described as

\[
\begin{bmatrix}
L z_{p,n} \\
L z_{q,n}
\end{bmatrix} = C_n^{-1} b_n \tilde{f}_{d,n}(s) / D_{xx}
\]  

(34)

where

\[
C_n = \begin{bmatrix}
\Delta_n + g_n(s) \alpha_{11,n,m,L} t_{1,p,n} & g_n(s) \alpha_{12,n,m,L} t_{1,q,n} \\
g_n(s) \alpha_{21,n,m,L} t_{1,p,n} & \Delta_n + g_n(s) \alpha_{22,n,m,L} t_{1,q,n}
\end{bmatrix}
\]  

(35)

\[
b_n = \begin{bmatrix}
\alpha_{12,n} \\
\alpha_{22,n}
\end{bmatrix}
\]  

(36)

In the above equations, \(\alpha_{q,p,n}\) is a numerator of the \(i\)th row and \(j\)th column element of the matrix \(A_n\) and \(\Delta_n\) is the determinant of \(A_n\).

A reflected wave is defined as the wave propagating from a control point (node 1) to a disturbance point (node 2), that is, a positive travelling wave in Element 2 which is the region between nodes 1 and 2. From Eqs. (17) and (21), the target wave element is given by

\[
w_{r,n}(x_n) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} K_{n}^{-1} a_{L} T_n \left( 1_{L} T_{n} L z_n + f_{c,n} \right)
\]  

(37)

Moreover, using Eq. (34), we have the following equation:

\[
1_{L} T_n L z_n + f_{c,n} = \begin{bmatrix}
\varepsilon_{1,n} g_n - \varepsilon_{2,n} \\
\varepsilon_{3,n} g_n - \varepsilon_{4,n} \\
\varepsilon_{5,n} g_n - \varepsilon_{6,n} \\
\varepsilon_{6,n} g_n - \varepsilon_{8,n}
\end{bmatrix} \tilde{f}_{d,n}(s) / D_{xx} \det C_n
\]  

(38)

where

\[
\varepsilon_{1,n} = 1_{L} t_{1,p,n} \gamma_{1,n} + 1_{L} t_{1,q,n} \gamma_{2,n}
\]  

(39)

\[
\varepsilon_{2,n} = \Delta_n \left( 1_{L} t_{1,p,n} \alpha_{12,n} + 1_{L} t_{1,q,n} \alpha_{22,n} \right)
\]  

(40)

\[
\varepsilon_{3,n} = 1_{L} t_{2,p,n} \gamma_{1,n} + 1_{L} t_{2,q,n} \gamma_{2,n}
\]  

(41)

\[
\varepsilon_{4,n} = \Delta_n \left( 1_{L} t_{2,p,n} \alpha_{12,n} + 1_{L} t_{2,q,n} \alpha_{22,n} \right)
\]  

(42)

\[
\varepsilon_{5,n} = 1_{L} t_{3,p,n} \gamma_{1,n} + 1_{L} t_{3,q,n} \gamma_{2,n}
\]  

(43)
\begin{align}
\varepsilon_{6,n} &= \Delta_n \left( 1, L t_{3,p,n} \alpha_{12,n} + 1, L t_{3,q,n} \alpha_{22,n} \right) \\
\varepsilon_{7,n} &= 1, L t_{4,p,n} \gamma_{1,n} + 1, L t_{4,q,n} \gamma_{2,n} - \Delta_n \left( m, L t_{1,p,n} \alpha_{12,n} + m, L t_{1,q,n} \alpha_{22,n} \right) \\
\varepsilon_{8,n} &= \Delta_n \left( 1, L t_{4,p,n} \alpha_{12,n} + 1, L t_{4,q,n} \alpha_{22,n} \right) \\
\gamma_{1,n} &= \alpha_{11,n} \alpha_{22,n} m, L t_{1,q,n} - \alpha_{12,n} \alpha_{21,n} m, L t_{1,q,n} \\
\gamma_{2,n} &= \alpha_{12,n} \alpha_{21,n} m, L t_{1,p,n} - \alpha_{11,n} \alpha_{22,n} m, L t_{1,p,n}
\end{align}

Therefore, the control law for absorbing the reflected wave component which is the third wave component in Element 2 is derived as

\begin{align}
g_n &= \frac{\kappa_{41,n} \varepsilon_{2,n} + \kappa_{42,n} \varepsilon_{4,n} + \kappa_{43,n} \varepsilon_{6,n} + \kappa_{44,n} \varepsilon_{8,n}}{\kappa_{41,n} \varepsilon_{1,n} + \kappa_{42,n} \varepsilon_{3,n} + \kappa_{43,n} \varepsilon_{5,n} + \kappa_{44,n} \varepsilon_{7,n}}
\end{align}

where \( \kappa_{kl,n} \) indicates the \( k \)th row and \( l \)th column variable in the matrix \( K_n^{-1} \).

The control law derived above is expressed as a non-causal function, and hence it must be approximately realized. This study employs the static output feedback that realizes the perfect wave absorption at single frequency. Since the control law is described as a complex function, it is rewritten as

\begin{align}
g_n &= \text{Re} \left[ g_n(\omega) \right] + j \omega \text{Im} \left[ \frac{g_n(\omega)}{\omega} \right] = g_{d,n}(\omega) + j \omega g_{r,n}(\omega)
\end{align}

As \( j \omega \) is a differential operator in the frequency domain, \( g_{r,n} \) indicates velocity feedback gain while \( g_{d,n} \) is displacement feedback gain. Therefore, by tuning the angular frequency in each feedback gain as a target value, perfect wave absorption is realized.

As seen in the theoretical development described above, a feedback wave control is constructed for each mode group. There are two options for realizing this control system. One is the use of smart sensors and actuators [7]. In this method, ideal modal control system is achieved, and hence spillover effect does not occur. The other is the use of a finite number of point sensors and actuators. If this is the case, a cluster control method which is based on collocation is effective, since this method enables to provide a required control input to a target mode group and guarantees sufficient stability margins [8].

4. Numerical example

This section presents the control effects of the proposed method from a viewpoint of absolute displacement distribution using physical and geometric parameters of the panel listed in Table 1.

| Specification of the target orthotropic rectangular panel |
|----------------------------------------------------------|
| **Length in the x direction** | **Length in the y direction** | **Width** | **Young's modulus in the x direction** | **Young's modulus in the y direction** |
|-------------------------------|-------------------------------|-----------|--------------------------------------|--------------------------------------|
| 0.33 [m]                     | 0.175 [m]                     | 0.001 [m] | 21 [Gpa]                             | 0.93 [Gpa]                           |

| Poisson's ratio in the x direction | Poisson's ratio in the y direction | Density [kg/m³] |
|----------------------------------|----------------------------------|----------------|
| 0.09                             | 0.04                             | 1770           |
Unlike the analytical development discussed in the previous sections, the numerical analysis deals with a small amount of loss factor, \( \eta = 0.005 \), which is necessary to avoid numerical overflow at modal frequencies of the panel. The minute loss factor, however, will not have any significant influence on the simulation reliability. Furthermore, as previously described, \( w_n(y) \) must be a harmonic function, so that it is assumed that all boundary conditions are the pinned support in the simulation. If this is the case, the following equations are determined:

\[
\begin{align*}
    w_n(y) &= \sin \beta_n x, \\
    \beta_n &= \frac{n\pi}{L_y}, \\
    c_f &= \frac{2}{L_y}
\end{align*}
\]

The disturbance point is fixed at \((x_d, y_d) = (0.3 \text{m}, 0.03 \text{m})\) through the simulation, and the control and measurement lines are placed parallel to the \( y \) axis at \( x = 0.01 \text{m} \).

Figures 2 and 3 show absolute displacement distribution of the orthotropic rectangular panel with and without control at the (2,1) and (2,2) modal frequencies. In the case of non-control, standing waves are formed by the disturbance force, and nodal lines appear according to the modal indices. This indicates that both positive and negative travelling waves exist in the structural vibration and their amplitudes are identical. In this case, resonant phenomena occur since the standing waves are tuned to the modal shapes, and the maximum amplitudes are 0.5 mm at the (2,1) modal frequency and 0.21 mm at the (2,2) modal frequency, respectively. In contrast, when the proposed method is applied to the panel, the nodal lines existing parallel to the \( y \) axis before control vanish. This means that reflected waves that are the control target are perfectly absorbed by the control inputs. In this case, the standing wave which is the direct cause of the modal excitation does not exist, so that the vibration

![Figure 2. Absolute displacement distribution at the (2,1) modal frequency. (a) Without control. (b) With control](image1)

![Figure 3. Absolute displacement distribution at the (2,2) modal frequency. (a) Without control. (b) With control](image2)
mode is made inactive. The maximum amplitudes are reduced to 10.2 μm at the (2,1) modal frequency and 11.3 μm at the (2,2) modal frequency, and those are, respectively, 2 % and 5.7 % of the uncontrolled amplitudes.

Next, control effort of the proposed method is considered. The magnitude ratio of control input and disturbance input (MRCD) is employed as a measure for evaluating the control effort, which is written as

\[
M_n = \left| \frac{f_{c,n}}{f_{d,n}} \right|
\]  

(53)

Table 2 shows the MRCD of the first two mode group at the (2,1) and (2,2) modal frequencies. As shown in the table, at the (2,2) modal frequency, the MRCD has similar value at the first two modal group. The reason their values are larger than 1 is that the control line in the x axis is close to the end boundary compared to the disturbance line. On the other hand, in the case of the (2,1) modal frequency, the MRCD for the (*,2) mode group is almost zero. This result implies the amplitude of the target wave component is minute at the control line. In fact, this frequency is lower than the cut-on frequency of the (*,2) mode group, and hence the amplitude of the target component decays exponentially from the disturbance line.

| Table 2. MRCD of the first two mode group at the (2,1) and (2,2) modal frequencies |
|-----------------------------------------------|-----------------|------------------------------|
|                                             | \( n=1 \)       | \( n=2 \)                     |
| the (2,1) modal frequency                    | 2.81            | 0.001                        |
| the (2,2) modal frequency                    | 2.56            | 2.74                         |

5. Conclusions
This paper has proposed static output feedback control for absorbing the reflected waves propagating in an orthotropic rectangular panel. Firstly, a transfer matrix method for a rectangular panel was introduced to describe the wave dynamics of the structure. Next, the feedback control law for absorbing reflected waves is derived. Then, the methodology of realizing the perfect wave absorption at single frequency is presented. Finally, from a viewpoint of numerical analyses, control effects of the proposed method were verified by evaluating the absolute displacement distribution. It was found that the reflected wave absorbing control enables the inactivation of all vibration modes in the controlled direction.

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