Electroproduction of two light vector mesons in the next-to-leading approximation

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Abstract

We calculate the amplitude for the forward electroproduction of two light vector mesons in next-to-leading order BFKL. This amplitude is written as a convolution of two impact factors for the virtual photon to light vector meson transition with the BFKL Green’s function. It represents the first next-to-leading order amplitude ever calculated for a collision process between strongly interacting colorless particles.
1 Introduction

Collision processes between strongly interacting particles in the limit of large center-of-mass energy are the best candidates for the realization of the BFKL dynamics [1], provided that a “hard” scale exist which allows the use of perturbation theory in the strong coupling $\alpha_s$.

In the BFKL approach, both in the leading logarithmic approximation (LLA), which means resummation of leading energy logarithms, all terms $(\alpha_s \ln(s))^n$, and in the next-to-leading approximation (NLA), which means resummation of all terms $\alpha_s (\alpha_s \ln(s))^n$, the (imaginary part of the) amplitude for a large-$s$ hard collision process can be written as the convolution of the Green’s function of two interacting Reggeized gluons with the impact factors of the colliding particles (see, for example, Fig. 1).

The Green’s function is determined through the BFKL equation. The NLA singlet kernel of the BFKL equation has been achieved in the forward case [2], after the long program of calculation of the NLA corrections [3] (for a review, see Ref. [4]). For the non-forward case the ingredients to the NLA BFKL kernel are known since a few years for the color octet representation in the $t$-channel [5]. This color representation is very important for the check of consistency of the $s$-channel unitarity with the gluon Reggeization, i.e. for the “bootstrap” [6]. Recently the last missing piece has been determined for the determination of the non-forward NLA BFKL kernel in the singlet color representation, i.e. in the Pomeron channel, relevant for physical applications [7].

On the other side, NLA impact factors have been calculated for colliding partons [8] and for forward jet production [9]. Among the impact factors for transitions between colorless objects, the most important one from the phenomenological point of view is certainly the impact factor for the virtual photon to virtual photon transition, i.e. the $\gamma^* \rightarrow \gamma^*$ impact factor, since it would open the way to predictions of the $\gamma^* \gamma^*$ total cross section. Its calculation is rather complicated and only after year-long efforts it is approaching completion [10].

A considerable simplification can be gained if one considers instead the impact factor for the transition from a virtual photon $\gamma^*$ to a light neutral vector meson $V = \rho^0, \omega, \phi$. In this case, indeed, a close analytical expression can be achieved in the NLA, up to contributions suppressed as inverse powers of the photon virtuality [11]. In particular, it turns out that (a) the dominant helicity amplitude is that for the transition from longitudinally polarized virtual photon to longitudinally polarized vector meson; (b) the impact factor, both in the LLA and in the NLA, factorizes into the convolution of a hard scattering amplitude, calculable in perturbative QCD, and a meson twist-2 distribution amplitude [11].

The knowledge of the $\gamma^* \rightarrow V$ impact factor allows for the first time to determine completely within perturbative QCD and with NLA accuracy the amplitude of a physical process, the $\gamma^* \gamma^* \rightarrow VV$ reaction. This possibility is interesting first of all for theoretical reasons, since it could shed light on the role and the optimal choice of the energy scales entering the BFKL approach. Moreover, it could be used as a test-ground for comparisons
with approaches different from BFKL, such as DGLAP, and with possible next-to-leading order extensions of phenomenological models, such as color dipole and \( k_t \)-factorization. But it could be interesting also for the possible applications to the phenomenology. Indeed, the calculation of the \( \gamma^* \to V \) impact factor is the first step towards the application of BFKL approach to the description of processes such as the vector meson electroproduction \( \gamma^* p \to V p \), being carried out at the HERA collider, and the production of two mesons in the photon collision, \( \gamma^* \gamma^* \to VV \) or \( \gamma^* \gamma \to V J/\Psi \), which can be studied at high-energy \( e^+e^- \) and \( e\gamma \) colliders.

In this paper we concentrate on the NLA forward amplitude for the \( \gamma^* \gamma^* \to VV \) reaction. Such a process has been studied recently in Ref. [12] in the Born (2-gluon exchange) limit for arbitrary transverse momentum. First of all, we show how the available results for the \( \gamma^* \to V \) impact factor and the BFKL Green’s function can be put together to build up the NLA amplitude of the \( \gamma^* \gamma^* \to VV \) process in the \( \overline{\text{MS}} \) scheme. Then we restrict ourselves to the particular case of collision of virtual photons with equal virtualities and present some numerical estimates of our result, aimed at showing the extent of the contributions to the NLA amplitude from the impact factor and from the NLA kernel and the dependence on the energy scale introduced in the BFKL approach and on the renormalization scale which appears in the \( \overline{\text{MS}} \) scheme.

Moreover, we show that, despite being the NLA corrections large and of opposite sign with respect to the leading order, it is possible to achieve a well-behaved form of the amplitude, by a suitable choice of the energy and renormalization scale parameters. The procedure we adopted here to optimize the perturbative result will be compared with other approaches to optimization and also with methods based on the improvement of the NLA BFKL Green’s function [13] in forthcoming publications.

When the present paper was ultimated, a new paper appeared [14] in which forward amplitude of \( \gamma^* \gamma^* \to VV \) process is studied in the LLA and also an estimate for the NLA effects based on the specific assumptions is given. We found that at LLA level our results coincide with that ones in [14]. In order to clarify the physics behind the BFKL NLA corrections it is important to compare our exact NLA results with different approximate approaches to the NLA effects. However, such a study goes beyond the scope of the present paper. It will be given in a separate publication where, in particular, we plan to make a systematic comparison of exact NLA results with the estimates of [14].

2 The NLA amplitude

We consider the production of two light vector mesons \( (V = \rho^0, \omega, \phi) \) in the collision of two virtual photons, \( \gamma^* (p) \gamma^* (p') \to V (p_1) V (p_2) \).

\begin{equation}
\gamma^* (p) \gamma^* (p') \to V (p_1) V (p_2) .
\end{equation}
Here, $p_1$ and $p_2$ are taken as Sudakov vectors satisfying $p_1^2 = p_2^2 = 0$ and $2(p_1 p_2) = s$; the virtual photon momenta are instead

$$p = \alpha p_1 - \frac{Q_1^2}{\alpha s} p_2, \quad p' = \alpha' p_2 - \frac{Q_2^2}{\alpha' s} p_1,$$

so that the photon virtualities turn to be $p^2 = -Q_1^2$ and $(p')^2 = -Q_2^2$. We consider the kinematics when

$$s \gg Q_{1,2}^2 \gg \Lambda_{QCD}^2,$$

and

$$\alpha = 1 + \frac{Q_1^2}{s} + O(s^{-2}), \quad \alpha' = 1 + \frac{Q_2^2}{s} + O(s^{-2}).$$

In this case vector mesons are produced by longitudinally polarized photons in the longitudinally polarized state [11]. Other helicity amplitudes are power suppressed, with a suppression factor $\sim m_V/Q_{1,2}$. We will discuss here the amplitude of the forward scattering, i.e. when the transverse momenta of produced $V$ mesons are zero or when the variable $t = (p_1 - p)^2$ takes its maximal value $t_0 = -Q_1^2 Q_2^2 / s + O(s^{-2})$.

The forward amplitude in the BFKL approach may be presented as follows

$$\mathcal{I}_{ms}(A) = \frac{s}{(2\pi)^2} \int \frac{d^2 \vec{q}_1}{\delta_1^2} \Phi_1(\vec{q}_1, s_0) \int \frac{d^2 \vec{q}_2}{\delta_2^2} \Phi_2(-\vec{q}_2, s_0) \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left( \frac{s}{s_0} \right)^\omega G_\omega(\vec{q}_1, \vec{q}_2).$$  (5)

This representation for the amplitude is valid with NLA accuracy. Here $\Phi_1(\vec{q}_1, s_0)$ and $\Phi_2(-\vec{q}_2, s_0)$ are the impact factors describing the transitions $\gamma^*(p) \to V(p_1)$ and $\gamma^*(p') \to V(p_2)$, respectively. The Green's function in (5) obeys the BFKL equation

$$\delta^2(\vec{q}_1 - \vec{q}_2) = \omega G_\omega(\vec{q}_1, \vec{q}_2) - \int d^2 \vec{q} K(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_2),$$  (6)
where $K(\vec{q}_1, \vec{q}_2)$ is the BFKL kernel. The scale $s_0$ is artificial. It is introduced in the BFKL approach at the time to perform the Mellin transform from the $s$-space to the complex angular momentum plane and must disappear in the full expression for the amplitude at each fixed order of approximation. Using the result for the meson NLA impact factor such cancellation was demonstrated explicitly in Ref. [11] for the process in question.

It is convenient to work in the transverse momentum representation, where “transverse” is related to the plane orthogonal to the vector mesons momenta. In this representation, defined by

$$\hat{q} | \vec{q}_i \rangle = \vec{q}_i | \vec{q}_i \rangle$$  \hspace{1cm} (7)

$$\langle \vec{q}_1 | \vec{q}_2 \rangle = \delta^{(2)}(\vec{q}_1 - \vec{q}_2)$$ \hspace{1cm} (8)

the kernel of the operator $\hat{K}$ is

$$K(\vec{q}_2, \vec{q}_1) = \langle \vec{q}_2 | \hat{K} | \vec{q}_1 \rangle$$ \hspace{1cm} (9)

and the equation for the Green’s function reads

$$\hat{1} = (\omega - \hat{K}) \hat{G}_\omega$$ \hspace{1cm} (10)

its solution being

$$\hat{G}_\omega = (\omega - \hat{K})^{-1}.$$ \hspace{1cm} (11)

The kernel is given as an expansion in the strong coupling,

$$\hat{K} = \hat{\alpha}_s \hat{K}^0 + \hat{\alpha}_s^2 \hat{K}^1,$$ \hspace{1cm} (12)

where

$$\hat{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$ \hspace{1cm} (13)

and $N_c$ is the number of colors. In Eq. (12) $\hat{K}^0$ is the BFKL kernel in the LLA, $\hat{K}^1$ represents the NLA correction.

The impact factors are also presented as an expansion in $\alpha_s$

$$\Phi_{1,2}(\vec{q}) = \alpha_s D_{1,2} \left[ C_{1,2}^{(0)}(\vec{q}^2) + \hat{\alpha}_s C_{1,2}^{(1)}(\vec{q}^2) \right] , \quad D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}} \sqrt{N_c^2 - 1},$$ \hspace{1cm} (14)

where $f_V$ is the meson dimensional coupling constant ($f_\rho \approx 200$ MeV) and $e_q$ should be replaced by $e/\sqrt{2}$, $e/(3\sqrt{2})$ and $-e/3$ for the case of $\rho^0$, $\omega$ and $\phi$ meson production, respectively.

In the collinear factorization approach the meson transition impact factor is given as a convolution of the hard scattering amplitude for the production of a collinear quark–antiquark pair with the meson distribution amplitude (DA). The integration variable in this convolution is the fraction $z$ of the meson momentum carried by the quark ($\bar{z} \equiv 1 - z$ is the momentum fraction carried by the antiquark):

$$C_{1,2}^{(0)}(\vec{q}^2) = \int_0^1 dz \frac{\vec{q}^2}{\vec{q}^2 + z\bar{z}Q_{1,2}^2} \phi_{\parallel}(z).$$ \hspace{1cm} (15)
The NLA correction to the hard scattering amplitude, for a photon with virtuality equal to \( Q^2 \), is defined as follows

\[
C^{(1)}(\vec{q}^2) = \frac{1}{4N_c} \int_0^1 dz \frac{\vec{q}^2}{\vec{q}^2 + zQ^2} [\tau(z) + \tau(1-z)] \phi_{\parallel}(z),
\]

with \( \tau(z) \) given in the Eq. (75) of Ref. [11]. \( C^{(1)}_{1,2}(\vec{q}^2) \) are given by the previous expression with \( Q^2 \) replaced everywhere in the integrand by \( Q^2_1 \) and \( Q^2_2 \), respectively.

The distribution amplitude may be presented as an expansion in Gegenbauer polynomials

\[
\phi_{\parallel}(z, \mu_F) = 6z(1-z) \left[ 1 + a_2(\mu_F)C^3/2_2(2z-1) + a_4(\mu_F)C^3/2_4(2z-1) + \ldots \right].
\]

The scale dependence of \( a_n(\mu_F) \) is well known [15]:

\[
a_n(\mu_F) = L^{\gamma_n/\beta_0} a_n(\mu),
\]

where \( L = \alpha_s(\mu_F)/\alpha_s(\mu) \) and

\[
\beta_0 = \frac{11N_c}{3} - \frac{2n_f}{3}
\]

is the leading coefficient of the QCD \( \beta \)-function, with \( n_f \) the number of active quark flavors. The anomalous dimensions \( \gamma_n \) are positive and grow with \( n \). Therefore any DA approaches the asymptotic form \( \phi_{\parallel}^{as}(z) = 6z(1-z) \) at large \( \mu_F \).\(^1\)

Below we will use the DA in the asymptotic form. Besides the simplicity of the following presentation, the reason is twofold. Presumably, the form of DA chosen at low \( \mu_F \) will affect mainly only the overall normalization of the amplitude but not the sum of BFKL energy logarithms and the resulting dependence of the amplitude on the energy in which we are primarily interested in this study. Another point is that, according to QCD sum rules estimates [16], \( a_2(1 \text{ GeV}) \) is \( 0.18 \pm 0.10 \) for \( \rho \) and \( 0 \pm 0.1 \) for \( \phi \). Therefore \( \phi_{\parallel}^{as} \) may be indeed a good approximation for the DA of light vector mesons. Integrating over \( z \) in (15) with \( \phi_{\parallel}(z, \mu_F^2) = \phi_{\parallel}^{as}(z) \), we obtain, for photon virtuality \( Q^2 \),

\[
C^{(0)}(\alpha = \frac{\vec{q}^2}{Q^2}) = 6 \alpha \left[ 1 - \frac{\alpha}{c} \ln \frac{2c + 1}{2c - 1} \right],
\]

where \( c = \sqrt{\alpha + 1/4} \). \( C^{(0)}_{1,2} \) are given by the previous expression with \( Q^2 \) replaced by \( Q^2_1 \) and \( Q^2_2 \), respectively. For the NLA term \( C^{(1)}_{1,2}(\vec{q}^2) \) the integration over \( z \) can be performed by a numerical calculation.

To determine the amplitude with NLA accuracy we need an approximate solution of Eq. (11). With the required accuracy this solution is

\[
\hat{G}_\omega = (\omega - \hat{\alpha}_s \hat{K}^0)^{-1} + (\omega - \hat{\alpha}_s \hat{K}^0)^{-1} \left( \hat{\alpha}_s^2 \hat{K}^1 \right) (\omega - \hat{\alpha}_s \hat{K}^0)^{-1} + \mathcal{O} \left( \left( \hat{\alpha}_s^2 \hat{K}^1 \right)^2 \right).
\] \(^1\)The dependence of the resulting amplitude on \( \mu_F \) is subleading. Due to the collinear counterterm, see Eq. (72) of [11], the NLA correction to the meson impact factor contains a term proportional to \( \ln(\mu_F) \), see Eq. (75) of [11], which compensates in the amplitude with NLA accuracy the effect of the meson DA variation with \( \mu_F \).
The basis of eigenfunctions of the LLA kernel,

\[ \tilde{K}^0|\nu\rangle = \chi(\nu)|\nu\rangle, \quad \chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right), \quad (22) \]

is given by the following set of functions:

\[ \langle \bar{q} | \nu \rangle = \frac{1}{\pi \sqrt{2}} \left(\bar{q}^2\right)^{i\nu - \frac{1}{2}}, \quad (23) \]

for which the orthonormality condition takes the form

\[ \langle \nu' | \nu \rangle = \int \frac{d^2 \bar{q}}{2\pi^2} \left(\bar{q}^2\right)^{i\nu - i\nu' - 1} = \delta(\nu - \nu'). \quad (24) \]

The action of the full NLA BFKL kernel on these functions may be expressed as follows:

\[ \tilde{K}|\nu\rangle = \tilde{\alpha}_s(\mu_R)\chi(\nu)|\nu\rangle + \tilde{\alpha}_s^2(\mu_R) \left( \chi^{(1)}(\nu) + \frac{\beta_0}{4N_c} \chi(\nu) \ln(\mu_R^2) \right)|\nu\rangle + \tilde{\alpha}_s^2(\mu_R) \frac{\beta_0}{4N_c} \chi(\nu) \left( i \frac{\partial}{\partial\nu} \right)|\nu\rangle, \quad (25) \]

where the first term represents the action of LLA kernel, while the second and the third ones stand for the diagonal and the non-diagonal parts of the NLA kernel. The function \( \chi^{(1)}(\nu) \), calculated in [2], is conveniently represented in the form

\[ \chi^{(1)}(\nu) = -\frac{\beta_0}{8N_c} \left( \chi^2(\nu) - \frac{10}{3} \chi(\nu) - i\chi'(\nu) \right) + \bar{\chi}(\nu), \quad (26) \]

where

\[ \bar{\chi}(\nu) = -\frac{1}{4} \left[ \frac{\pi^2 - 4}{3} \chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} \right] + \frac{\pi^2 \sinh(\pi\nu)}{2\nu \cosh^2(\pi\nu)} \left[ 3 + \left( 1 + \frac{n_f}{N_c} \right) \frac{11 + 12\nu^2}{16(1 + \nu^2)} + 4\phi(\nu) \right], \quad (27) \]

\[ \phi(\nu) = 2 \int_0^1 dx \frac{\cos(\nu \ln(x))}{(1 + x)\sqrt{x}} \left[ \frac{\pi^2}{6} - \text{Li}_2(x) \right], \quad \text{Li}_2(x) = -\int_0^x dt \frac{\ln(1 - t)}{t}. \quad (28) \]

Here and below \( \chi'(\nu) = d(\chi(\nu))/d\nu \) and \( \chi''(\nu) = d^2(\chi(\nu))/d\nu^2 \).

We will need also the \(|\nu\rangle\) representation for the impact factors, which is defined by the following expressions

\[ \frac{C_1^{(0)}(\bar{q}^2)}{\bar{q}^2} = \int_{-\infty}^{+\infty} d\nu' c_1(\nu') \langle \nu'|\bar{q}\rangle, \quad \frac{C_2^{(0)}(\bar{q}^2)}{\bar{q}^2} = \int_{-\infty}^{+\infty} d\nu c_2(\nu) \langle \bar{q}|\nu\rangle, \quad (29) \]

\[ c_1(\nu) = \int d^2 \bar{q} C_1^{(0)}(\bar{q}^2) \frac{(\bar{q}^2)^{i\nu - \frac{1}{2}}}{\pi \sqrt{2}}, \quad c_2(\nu) = \int d^2 \bar{q} C_2^{(0)}(\bar{q}^2) \frac{(\bar{q}^2)^{i\nu - \frac{1}{2}}}{\pi \sqrt{2}}, \quad (30) \]
and by similar equations for $c_{1}^{(1)}(\nu)$ and $c_{2}^{(1)}(\nu)$ from the NLA corrections to the impact factors, $C_{1}^{(1)}(q^{2})$ and $C_{2}^{(1)}(q^{2})$.

Using (21) and (25) one can derive, after some algebra, the following representation for the amplitude

$$\frac{\Im m_{s}(A)}{D_{1}D_{2}} = \frac{s}{(2\pi)^{2}} \int_{-\infty}^{+\infty} d\nu \left( \frac{s}{s_{0}} \right)^{\bar{\alpha}_{s}(\mu_{R})} \alpha_{s}^{2}(\mu_{R}) c_{1}(\nu) c_{2}(\nu) \left[ 1 + \bar{\alpha}_{s}(\mu_{R}) \left( \frac{c_{1}^{(1)}(\nu)}{c_{1}(\nu)} + \frac{c_{2}^{(1)}(\nu)}{c_{2}(\nu)} \right) \right] + \bar{\alpha}_{s}^{2}(\mu_{R}) \ln \left( \frac{s}{s_{0}} \right) \left( \chi(\nu) + \frac{\beta_{0}}{8N_{c}} \chi(\nu) \right) \left[ -\chi(\nu) + \frac{10}{3} + i \frac{d\ln(c_{1}(\nu))}{d\nu} + 2 \ln(\mu_{R}^{2}) \right]. \tag{31}$$

We find that

$$c_{1,2}(\nu) = \left( \frac{Q_{1,2}^{2}}{\sqrt{2}} \right)^{\frac{3}{2} + \frac{\nu}{2} \pm i\nu} \frac{\Gamma(3 \pm 2i\nu)}{\Gamma[3 \pm 2i\nu] \cosh(\pi \nu)} 6\pi,$$  

$$c_{1}(\nu) c_{2}(\nu) = \frac{1}{Q_{1}Q_{2}} \left( \frac{Q_{1}^{2}}{Q_{2}^{2}} \right)^{\nu} \frac{9 \pi^{3}(1 + 4\nu^{2}) \sinh(\pi \nu)}{32\nu(1 + \nu^{2}) \cosh^{3}(\pi \nu)},$$

$$i \frac{d\ln(c_{1}(\nu))}{d\nu} = 2 \left[ \psi(3 + 2i\nu) + \psi(3 - 2i\nu) - \psi\left( \frac{3}{2} + i\nu \right) - \psi\left( \frac{3}{2} - i\nu \right) - \ln(Q_{1}Q_{2}) \right]. \tag{34}$$

It can be useful to separate from the NLA correction to the impact factor the terms containing the dependence on $s_{0}$ and on $\beta_{0}$,

$$C^{(1)}(q^{2}) = \int_{0}^{1} dz \frac{q^{2}}{q^{2} + z\bar{z}Q^{2}} \phi_{\parallel}(z) \tag{35}$$

$$\times \left[ \frac{1}{4} \ln \left( \frac{s_{0}}{Q^{2}} \right) \ln \left( \frac{\alpha + z\bar{z}}{\alpha^{2}z\bar{z}} \right) + \frac{\beta_{0}}{4N_{c}} \ln \left( \frac{\mu_{R}^{2}}{Q^{2}} \right) + \frac{5}{3} - \ln(\alpha) + \ldots \right].$$

Accordingly, one can write

$$c_{1,2}^{(1)}(\nu) = \tilde{c}_{1,2}^{(1)}(\nu) + \bar{c}_{1,2}^{(1)}(\nu),$$

where $\tilde{c}_{1,2}^{(1)}(\nu)$ are the contributions from the terms isolated in the previous equation and $\bar{c}_{1,2}^{(1)}(\nu)$ represent the rest. After straightforward calculations we found that

$$\frac{\tilde{c}_{1}^{(1)}(\nu)}{c_{1}(\nu)} + \frac{\bar{c}_{2}^{(1)}(\nu)}{c_{2}(\nu)} = \ln \left( \frac{s_{0}}{Q_{1}Q_{2}} \right) \chi(\nu) + \frac{\beta_{0}}{2N_{c}} \left[ \ln \left( \frac{\mu_{R}^{2}}{Q_{1}Q_{2}} \right) + \frac{5}{3} \right]$$

$$+ \psi(3 + 2i\nu) + \psi(3 - 2i\nu) - \psi\left( \frac{3}{2} + i\nu \right) - \psi\left( \frac{3}{2} - i\nu \right). \tag{37}$$

Using Eq. (31) we construct the following representation for the amplitude

$$\frac{Q_{1}Q_{2} \Im m_{s}A}{D_{1}D_{2} s} = \frac{1}{(2\pi)^{2}} \alpha_{s}(\mu_{R})^{2} \tag{38}$$

$$\times \left[ b_{0} + \sum_{n=1}^{\infty} \bar{\alpha}_{s}(\mu_{R})^{n} b_{n} \left( \ln \left( \frac{s}{s_{0}} \right)^{n} + d_{n}(s_{0}, \mu_{R}) \ln \left( \frac{s}{s_{0}} \right)^{n-1} \right) \right].$$
where the coefficients
\[ \frac{b_n}{Q_1 Q_2} = \int_{-\infty}^{+\infty} d\nu c_1(\nu) c_2(\nu) \frac{\chi^n(\nu)}{n!}, \] (39)
are determined by the kernel and the impact factors in LLA. Note that
\[ b_0 = \frac{9\pi}{4} (7\zeta(3) - 6), \] (40)
therefore in the Born (the 2-gluon exchange) limit our result coincides with that of Ref. [12].

The coefficients\(^2\)
\[ d_n = n \ln \left( \frac{s_0}{Q_1 Q_2} \right) + \frac{\beta_0}{4N_c} \left( n + 1 \right) \frac{b_{n-1}}{b_n} \ln \left( \frac{\mu_R^2}{Q_1 Q_2} \right) - \frac{n(n-1)}{2} \]
\[ + \frac{Q_1 Q_2}{b_n} \int_{-\infty}^{+\infty} d\nu (n + 1) f(\nu) c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n - 1)!} \]
\[ + \frac{Q_1 Q_2}{b_n} \int_{-\infty}^{+\infty} d\nu c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n - 1)!} \left[ \frac{c_1^{(1)}(\nu)}{c_1(\nu)} + \frac{c_2^{(1)}(\nu)}{c_2(\nu)} + (n - 1) \frac{\chi(\nu)}{\chi(\nu)} \right] \] (41)
are determined by the NLA corrections to the kernel and to the impact factors. Here we use the notation
\[ f(\nu) = \frac{5}{3} + \psi(3 + 2i\nu) + \psi(3 - 2i\nu) - \psi \left( \frac{3}{2} + i\nu \right) - \psi \left( \frac{3}{2} - i\nu \right). \] (42)

One should stress that both representations of the amplitude (38) and (31) are equivalent with NLA accuracy, since they differ only by next-to-NLA (NNLA) terms. Actually there exist infinitely many possibilities to write a NLA amplitude. For instance, another possibility could be to exponentiate the bulk of the kernel NLA corrections
\[ \frac{\mathcal{I} m_s (A)}{D_1 D_2} = \frac{s}{(2\pi)^2} \int_{-\infty}^{+\infty} d\nu \left( \frac{s}{s_0} \right)^{\alpha_s(\mu_R)\chi(\nu) + \bar{\alpha}_s^2(\mu_R)\chi(\nu) + \frac{\beta_0}{8N_c} \chi(\nu) \left[-\chi(\nu) + \frac{1}{\pi} \right]} \] \[ \times \left[ 1 + \bar{\alpha}_s(\mu_R) \left( \frac{c_1^{(1)}(\nu)}{c_1(\nu)} + \frac{c_2^{(1)}(\nu)}{c_2(\nu)} \right) + \bar{\alpha}_s^2(\mu_R) \ln \left( \frac{s}{s_0} \right) \frac{\beta_0}{8N_c} \chi(\nu) \left( i \frac{d\ln(\chi(\nu))}{d\nu} + 2 \ln(\mu_R^2) \right) \right]. \] (43)
This form of the NLA amplitude was used in [17] (see also [18]), without account of the last two terms in the second line of (43), for the analysis of the total $\gamma^* \gamma^*$ cross section.

Since as we will shortly see the NLA corrections are very large, the choice of the representation for the NLA amplitude becomes practically important. In the present situation, when an approach to the calculation of the NNLA corrections is not developed

\(^2\text{The following expression holds actually for } n > 1. \text{ The coefficient } d_1 \text{ coincides with } d_1^{\text{imp}}, \text{ see Eq. (46).}\)
yet, the series representation (38) is, in our opinion, a natural choice. It includes in some sense the minimal amount of NNLA contributions; moreover, its form is the closest one to the initial goal of the BFKL approach, i.e. to sum selected contributions in the perturbative series.

It is easily seen from Eqs. (38)-(42) that the amplitude is independent in the NLA from the choice of energy and strong coupling scales. Indeed, with the required accuracy,

\[
\bar{\alpha}_s(\mu_R) = \bar{\alpha}_s(\mu_0) \left( 1 - \frac{\bar{\alpha}_s(\mu_0) \beta_0}{4N_c} \ln \left( \frac{\mu_R^2}{\mu_0^2} \right) \right)
\]

and therefore terms \(\bar{\alpha}_s^n \ln^{n-1} s \ln s_0\) and \(\bar{\alpha}_s^n \ln^{n-1} s \ln \mu_R\) cancel in (38).

One can trace the contributions to each \(d_n\) coefficient coming from the NLA corrections to the BFKL kernel and from the NLA impact factors

\[
d_n = d_n^{\text{ker}} + d_n^{\text{imp}},
\]

\[
d_n^{\text{imp}} = n \ln \left( \frac{s_0}{Q_1 Q_2} \right) + \frac{\beta_0}{4N_c} \left( b_{n-1} \ln \left( \frac{\mu_R^2}{Q_1 Q_2} \right) + \frac{Q_1 Q_2}{b_n} \int_{-\infty}^{+\infty} d\nu f(\nu) c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!} \right) \]

\[
+ \frac{Q_1 Q_2}{b_n} \left( \int_{-\infty}^{+\infty} d\nu c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!} \left[ c_1^{(1)}(\nu) + c_2^{(1)}(\nu) \right] \right).
\]

The first coefficient, \(d_1\), is entirely due to the NLA corrections to the impact factors,

\[
d_1 = d_1^{\text{imp}}, \quad d_1^{\text{ker}} = 0.
\]

Let us note that in the BFKL formalism the NLA contribution to the impact factors guarantees not only independence of the amplitude from the energy scale, \(s_0\), but it contains also a term proportional to \(\ln \mu_R\) which is important for the renorm-invariance of the predicted results, i.e. the dependence of the amplitude on \(\mu_R\) and \(s_0\) is subleading to the NLA accuracy.

### 3 Numerical results

In this Section we present some numerical results for the amplitude given in Eq. (38) for the \(Q_1 = Q_2 \equiv Q\) kinematics, i.e. in the “pure” BFKL regime. The other interesting regime, \(Q_1 \gg Q_2\) or vice-versa, where collinear effects could come heavily into the game, will not be considered here. We will emphasize in particular the dependence on the renormalization scale \(\mu_R\) and \(s_0\) in the NLA result.

In all the forthcoming figures the quantity on the vertical axis is the L.H.S. of Eq. (38), \(\mathcal{I}m_s(A)Q^2/(s D_1 D_2)\). In the numerical analysis presented below we truncate the series in the R.H.S. of Eq. (38) to \(n = 20\), after having verified that this procedure gives a very good approximation of the infinite sum for the \(Y\) values \(Y \leq 10\). We use the two-loop running coupling corresponding to the value \(\alpha_s(M_Z) = 0.12\).
We have calculated numerically the $b_n$ and $d_n$ coefficients for $n_f = 5$ and $s_0 = Q^2 = \mu_R^2$, getting

$$
\begin{align*}
\begin{array}{cccccc}
b_0 &= 17.0664 & b_1 &= 34.5920 & b_2 &= 40.7609 & b_3 &= 33.0618 & b_4 &= 20.7467 \\
b_5 &= 10.5698 & b_6 &= 4.54792 & b_7 &= 1.69128 & b_8 &= 0.554475 \\
d_1 &= -3.71087 & d_2 &= -11.3057 & d_3 &= -23.3879 & d_4 &= -39.1123 \\
d_5 &= -59.207 & d_6 &= -83.0365 & d_7 &= -111.151 & d_8 &= -143.06 \\
\end{array}
\end{align*}
$$

(48)

In this case contributions to the $d_n$ coefficients originating from the NLA corrections to the impact factors are

$$
\begin{align*}
d_{\text{imp}}^1 &= -3.71087 & d_{\text{imp}}^2 &= -8.4361 & d_{\text{imp}}^3 &= -13.1984 & d_{\text{imp}}^4 &= -18.0971 \\
d_{\text{imp}}^5 &= -23.0235 & d_{\text{imp}}^6 &= -27.9877 & d_{\text{imp}}^7 &= -32.9676 & d_{\text{imp}}^8 &= -37.9618 \\
\end{align*}
$$

(49)

Thus, comparing (48) and (49), we see that the contribution from the kernel starts to be larger than the impact factor one only for $n \geq 4$.

These numbers make visible the effect of the NLA corrections: the $d_n$ coefficients are negative and increasingly large in absolute values as the perturbative order increases. The NLA corrections turn to be very large. In this situation the optimization of perturbative expansion, in our case the choice of the renormalization scale $\mu_R$ and of the energy scale $s_0$, becomes an important issue. Below we will adopt the principle of minimal sensitivity (PMS) [19]. Usually PMS is used to fix the value of the renormalization scale for the strong coupling. We suggest to use this principle in a broader sense, requiring in our case the minimal sensitivity of the predictions to the change of both the renormalization and the energy scales, $\mu_R$ and $s_0$.

Since the dependence of results on $s_0$ is a feature typical of the BFKL approach and is somewhat new for the application of PMS, we will first illustrate the success of PMS in this respect on the following QED result known since a long time. In 1937 Racah calculated the total cross section for the production of $e^+e^-$ pairs in the collisions of two heavy ions at high energies [20],

$$
\sigma = \frac{28\alpha_{EM}Z_1^2Z_2^2}{27\pi m_e^2} \left( l^3 + Al^2 + Bl + C \right) + O \left( \frac{1}{(p_1p_2)} \right) ; 
$$

(50)

here $Z_{1,2}$ are the ions' charges, $m_e$ is electron mass, the ions' four-momenta are $p_{1,2}$,

$$
l = \ln \frac{2(p_1p_2)}{m_1m_2},
$$

(51)

is the energy logarithm and $m_{1,2}$ are the masses of the ions. The contributions suppressed by the power of energy are denoted as $O(1/(p_1p_2))$.

The coefficients in front of the subleading logarithms are large and have alternating signs

$$
A = -178/28 = -6.35714 \\
B = \frac{1}{28}(7\pi^2 + 370) = 15.6817 
$$

(52)
Figure 2: $\sigma/\sigma_0$ as a function of the energy logarithm $l$ for the cases of exact result of Racah, approximated result with $l_0 = -A/3$ (PMS optimal choice) and $l_0 = 0$ (kinematical scale for energy logarithms).

$$C = -\frac{1}{28} \left( 348 + \frac{13}{2} \pi^2 - 21 \zeta(3) \right) = -13.8182.$$ To illustrate the application of PMS, imagine that we know only the coefficient $A$ in front of the first subleading logarithm. Then using this knowledge we can construct the following approximation

$$\sigma^{app} = \sigma_0 \left( (l - l_0)^3 + (A + 3l_0)(l - l_0)^2 \right), \quad \sigma_0 = \frac{28\alpha^4_{EM} Z_1^2 Z_2^2}{27 \pi m_e^2},$$

(53) (an analog of NLA in the BFKL approach) where we shift the energy scale introducing the parameter $l_0$. Note that the dependence of the cross section on $l_0$ is subleading in the approximation used in Eq. (53). We fix $l_0$ by requiring the minimal sensitivity of (53) to the change of this parameter. It is not difficult to find that this procedure gives $l_0 = -A/3 = 2.11905$.\footnote{Note that in this example PMS gives the value of the parameter $l_0$ for which the correction to the lowest approximation, $(l - l_0)^3$, vanishes. Therefore in this case PMS gives a result which coincides with the one given by another alternative approach to optimize the approximation, the fast apparent convergence prescription [21].} In Fig. 2 we present three curves for $\sigma/\sigma_0$ as a function of the energy logarithm $l$; the first one was calculated using the exact result of Racah (with all subleading logarithms), the other two curves were calculated using (53) with $l_0 = 0$ and with the PMS value $l_0 = 2.11905$.

We see that the PMS approach gives a very good approximation to the Racah result\footnote{The negative cross section at $l < 2$ is due to the fact that terms subleading in energy, $O(1/(p_1 p_2))$ in (50), are not taken into account.}. On the other hand the procedure with $l_0 = 0$, which means that a kinematical scale for...
energy logarithms is used in the approximate formula, makes an awfully bad job for the whole $l$ range presented in the figure.

Returning to our problem, we apply PMS to our case requiring the minimal sensitivity of the amplitude (38) to the variation of $\mu_R$ and $s_0$. More precisely, we replace in (38) $\ln(s/s_0)$ with $Y - Y_0$, where $Y = \ln(s/Q^2)$ and $Y_0 = \ln(s_0/Q^2)$, and study the dependence of the amplitude on $Y_0$.

The next two figures illustrate the dependence on these parameters for $Q^2=24$ GeV$^2$ and $n_f = 5$. In Fig. 3 we show the dependence of amplitude on $Y_0$ for $\mu_R = 10Q$, when $Y$ takes the values 10, 8, 6, 4, 3.

We see that for each $Y$ the amplitude has an extremum in $Y_0$ near which it is not sensitive to the variation of $Y_0$, or $s_0$. Our choice of $\mu_R$ for this figure is motivated by the study of $\mu_R$ dependence. In Fig. 4 we present the $\mu_R$ dependence for $Y = 6$; the curves from above to below are for $Y_0=3, 2, 1, 0$.

Varying $\mu_R$ and $Y_0$ we found for each $Y$ quite large regions in $\mu_R$ and $Y_0$ where the amplitude is practically independent on $\mu_R$ and $Y_0$. We use this value as the NLA result for the amplitude at given $Y$. In Fig. 5 we present the amplitude found in this way as a function of $Y$. The resulting curve is compared with the curve obtained from the LLA prediction when the scales are chosen as $\mu_R = 10Q$ and $Y_0 = 2.2$, in order to make the LLA curve the closest possible (of course it is not an exact statement) to the NLA one in the given interval of $Y$. The two horizontal lines in Fig. 5 are the Born (2-gluon exchange) predictions calculated for $\mu_R = Q$ and $\mu_R = 10Q$.
Figure 4: \( I_{\mu}(\mathcal{A})Q^2/(s D_1 D_2) \) as a function of \( \mu_R \) at \( Y=6 \). The different curves are, from above to below, for \( Y_0 \) values of 3, 2, 1 and 0. The photon virtuality \( Q^2 \) has been fixed to 24 GeV\(^2\) \((n_f = 5)\).

Similar procedure was applied to a lower value of the photon virtuality, \( Q^2=5 \) GeV\(^2\) and \( n_f = 4 \), where we again found that the choice of scales \( \mu_R = 10Q \) and \( Y_0 = 2.2 \) makes the LLA amplitude almost the closest to the NLA curve. The results are presented in Fig. 6.

Note that in calculating the NLA amplitude for Figs. 5, 6 we use optimal values of \( \mu_R \) and \( Y_0 \) found to be a bit different for different \( Y \) values from \( \mu_R = 10Q \) and \( Y_0 = 2.2 \).

We stress that one should take with care BFKL predictions for small values of \( Y \), since in this region the contributions suppressed by powers of the energy should be taken into account. At the lowest order in \( \alpha_s \) such contributions are given by diagrams with quark exchange in the \( t \)-channel and are proportional in our case to \( \alpha_{EM}\alpha_s^2 f_V^2/Q^2 \). At higher orders power suppressed contributions contain double logarithms, terms \( \sim \alpha_s^n \ln^{2n} s \), which can lead to a significant enhancement. Such contributions were recently studied for the total cross section of \( \gamma^*\gamma^* \) interactions [22].

If the NLA (and LLA) curves in Figs. 5, 6 are compared with the Born (2-gluon exchange) results, one can conclude that the summation of BFKL series gives negative contribution to the Born result for \( Y < 6 \) if one chooses for the scale of the strong coupling in the Born amplitude the value given by the kinematics, \( \mu_R = Q \). We believe that our calculations show that one should at least accept with some caution the results obtained in the Born approximation, since they do not give necessarily an estimate of the observable from below.

Another important lesson from our calculation is the very large scale for \( \alpha_s \) (and therefore the small \( \alpha_s \) itself) we obtain using PMS. It appears to be much bigger than the
Figure 5: $\Im m_s(A)Q^2/(s D_1 D_2)$ as a function of $Y$ for optimal choice of the energy parameters $Y_0$ and $\mu_R$ (curve labeled by “NLA”). The other curves represent the LLA result for $Y_0 = 2.2$ and $\mu_R = 10Q$ and the Born (2-gluon exchange) limit for $\mu_R = Q$ and $\mu_R = 10Q$. The photon virtuality $Q^2$ has been fixed to 24 GeV$^2$ ($n_f = 5$).

kinematical scale and looks unnatural since there is no other scale for transverse momenta in the problem at question except $Q$. Moreover one can guess that at higher orders the typical transverse momenta are even smaller than $Q$ since they “are shared” in the many-loop integrals and the strong coupling grows in the infrared. In our opinion the large values of $\mu_R$ we found is not an indication of the appearance of a new scale, but is rather a manifestation of the nature of the BFKL series. The fact is that NLA corrections are large and then, necessarily, since the exact amplitude should be renorm- and energy scale invariant, the NNLA terms should be large and of the opposite sign with respect to the NLA. We guess that if the NNLA corrections were known and we would apply PMS to the amplitude constructed as LLA + NLA-corrections + NNLA-corrections, we would obtain in such calculation more natural values of $\mu_R$.

We conclude this Section with a comment on the possible implications of our results for mesons electroproduction to the phenomenologically more important case of the $\gamma^*\gamma^*$ total cross section. By numerical inspection we have found that the ratios $b_n/b_0$ we got for the meson case agree for $n = 1 \div 10$ at $1 \div 2%$ accuracy level with the analogous ratios for the longitudinal photon case and at $3.5 \div 30%$ accuracy level with those for the transverse photon case. Should this similar behavior persist also in the NLA, our predictions could be easily translated to estimates of the $\gamma^*\gamma^*$ total cross section.
Figure 6: The same as in Fig. 5 for photon virtuality $Q^2$ fixed to $5 \text{ GeV}^2$ ($n_f = 4$).

4 Conclusions

We have determined the amplitude for the forward transition from two virtual photons to two light vector mesons in the Regge limit of QCD with next-to-leading order accuracy. This amplitude is the first one ever written in the next-to-leading approximation for a collision process between strongly interacting colorless particles. It is given as an integral over the $\nu$ parameter, which labels the eigenvalues of the leading order forward BFKL kernel in the singlet color representation. This form is suitable for numerical evaluations. The result obtained is independent on the energy scale $s_0$, and on the renormalization scale $\mu_R$ within the next-to-leading approximation.

Using a series representation of the amplitude which includes the dependence on the energy scale and on the renormalization scale at subleading level, we performed a numerical analysis in the kinematics when the two colliding photons have the same virtuality, i.e. in the “pure” BFKL regime. We have found that the next-to-leading order corrections coming from the kernel and from the virtual photon to light vector meson impact factors are both large and of opposite sign with respect to the leading order contribution.

An optimization procedure, based on the principle of minimal sensitivity method, has proved to work nicely and has lead to stable results in the considered energy interval, which allows us to predict the energy behavior of the forward amplitude. The procedure consists in evaluating the amplitude at values of the energy parameters for which it is the least sensitive to variations of them. We have found that there are wide regions of values of $s_0$ and $\mu_R$ where the amplitude remains almost flat.

The optimal choices of $s_0$ and $\mu_R$ are much larger that the kinematical scales of the problem. More than being the indication of appearance of another scale in the problem,
this could be related to the nature of the BFKL series. The renorm- and energy scale invariance, together with the large next-to-leading approximation corrections, call for large next-to-next-to-leading order corrections, which are most probably mimicked by unnatural optimal values for $s_0$ and $\mu_R$.

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