Detecting chiral singularities in lattice QCD at strong coupling

Costas G. Strouthos

Division of Science and Engineering, Frederick Institute of Technology, Nicosia 1303, Cyprus

We study the difficulties associated with detecting chiral singularities in strongly coupled lattice QCD at finite nonzero temperature. The model undergoes a second order transition which belongs to the $O(2)$ universality class [1]. At fixed finite temperature the model is described by a 3d $O(2)$ continuum scalar field theory in its broken phase. At this temperature there is a range of quark masses where the light pions are describable by a 3d continuum chiral perturbation theory (ChPT). The effective action can be written in terms of $\vec{S}$, an $O(2)$ vector field with the constraint $\vec{S} \cdot \vec{S} = 1$. The partition function of lattice QCD with one staggered fermion interacting with $U(3)$ gauge fields can be mapped into a monomer-dimer system in the strong coupling limit [8]. In this study we use a very efficient algorithm discovered recently to solve the model in the chiral limit [4]. In this work we choose $L_x = L_y = L$. One can study the thermodynamics of the model by working on asymmetric and anisotropic lattices with $L_t \ll L$ and allowing the temporal staggered fermion phase factor to vary continuously [8].

In order to study chiral physics we focus on the chiral condensate $\Sigma_m$, the chiral susceptibility $\chi$, the helicity modulus $Y$ and the pion mass $M_\pi$. As discussed in [4], one can define the pion decay constant at a quark mass $m$ to be $F_m^2 = \lim_{L \to \infty} F_m^2$. For $m = 0$ we then obtain $F = \lim_{m \to 0} F_m$, the pion decay constant introduced in Eq.[1]. We can also define $\Sigma = \lim_{m \to 0} \Sigma_m$, where $\Sigma_m$ is the infinite volume chiral condensate.

1. Introduction

Given the difficulties associated with understanding chiral singularities in a realistic QCD calculation, we explore this subject in strong coupling lattice QCD with one staggered fermion at finite temperature. The model undergoes a second order transition which belongs to the $O(2)$ universality class [1]. At fixed $T < T_c$, within the $O(2)$ scaling window, the long distance physics of our model is described by a 3d $O(2)$ continuum scalar field theory in its broken phase. At this temperature there is a range of quark masses such that $1 \ll \xi \ll 1/M_\pi$, where $M_\pi$ is the pion mass. In this region we expect that $\Sigma$, $M_\pi$ and $F^2$ satisfy the expansion

$$\langle O \rangle = z_0 + z_1 \sqrt{m} + z_2 m + z_3 m^2 + \ldots,$$

where the $\sqrt{m}$ behavior is the power-like singularity arising due to the infrared pion physics [2].

2. Results

For massless quarks we fit $F^2$ vs. $T$ to $F^2 = C(T_c - T)^\nu$ for $T < T_c$. In the fit we fix $T_c = 7.47739(3)$ [11] and $\nu = 0.67155(27)$ [12]. For $7.05 \leq T \leq 7.42$ we get $C = 0.2217(1)$ with $\chi^2/DOF = 1.03$. Including $T = 7.0$ in the fit makes $\chi^2/DOF$ jump to 2.0. At $T$ close to $T_c$, $\xi$ is very large and it is difficult to satisfy $\frac{F}{M_\pi} \gg \xi$. On the other hand when $\xi \sim 1$ lattice artifacts become important. We estimate that $T = 7.0$ is at the edge of the $O(2)$ scaling window. All the results discussed...
below were obtained at \( T = 7.0 \). We fix \( L_t = 4 \) and vary spatial volume from \( L = 8 \) to \( L = 96 \) for \( 0 \leq m \leq 0.25 \).

The first four terms in the chiral expansion of \( F_m \) in 3d \( O(N) \) ChPT are given by \( 6 \):

\[
F_m = F[1 + a_1\sqrt{m} + a_2 m + a_3 m \sqrt{m}],
\]

(3)

where \( a_1 \propto (N - 2) \). Since in our case \( N = 2 \) we expect \( a_1 = 0 \). We fit data in the range \( 0 \leq m \leq 0.00625 \) (see Fig.1). We find \( F = 0.3674(1) \), \( a_2 = 34.0(7) \) and \( a_3 = -182(6) \) with \( \chi^2/\text{DOF} = 1.2 \). The prediction of ChPT that \( a_1 = 0 \) is in excellent agreement with our results.

ChPT predicts that the first four terms in the chiral expansion of \( \Sigma_m \) are given by

\[
\Sigma_m = \Sigma[1 + b_1\sqrt{m} + b_2 m + b_3 m \sqrt{m}].
\]

(4)

As shown in \( 6 \) \( b_1 \propto (N - 1) \), implying that \( b_1 \neq 0 \) in our case. At \( m = 0 \) we compute \( \Sigma \) by using the finite size scaling formula for \( \chi \) given by \( 6 \):

\[
\chi = \frac{1}{N} \sqrt{\frac{\Sigma}{L}} \left[ 1 + 0.226(N - 1) + \frac{1}{F^2 L} + \frac{\alpha}{L^2} + \ldots \right],
\]

(5)

where \( \alpha \) depends on the higher order low energy constants of the chiral Lagrangian. When \( F \) is fixed to 0.3675 obtained earlier, our results for \( 16 \leq L \leq 144 \) fit very well to this formula. We find \( \Sigma = 2.2648(10) \), \( \alpha = 4.6(3) \) with \( \chi^2/\text{DOF} = 0.87 \). Fitting our data to Eq. (6) in the region \( 0 \leq m \leq 0.00625 \) gives \( \Sigma = 2.2642(10) \), \( b_1 = 1.36(4) \), \( b_2 = 23(1) \), \( b_3 = -135(10) \) with a \( \chi^2/\text{DOF} = 1.1 \). We note that we cannot find a mass range within our results in which we can find a good fit when we fix \( b_2 = b_3 = 0 \). However, in the range \( 0 \leq m \leq 0.00175 \) we can set \( b_3 = 0 \) to obtain \( \Sigma = 2.2643(10) \), \( b_1 = 1.44(3) \) and \( b_2 = 15.2(7) \) with a \( \chi^2/\text{DOF} = 1 \). Note that \( b_2 \) changes by about 30\% when this different fitting procedure is used, while \( b_1 \) is more stable. The data and the fits are shown in Fig.2.

The first four terms in the chiral behavior of the pion mass are predicted to be of the form

\[
M_\pi = \sqrt{\frac{\Sigma}{F^2 \sqrt{m}} \left[ 1 + c_1 \sqrt{m} + c_2 m + c_3 m \sqrt{m} \right]},
\]

(6)

Further, the chiral Ward identities imply that \( c_1 = b_1/2 - a_1 \). Fitting our data to Eq. (6) after fixing \( \Sigma = 2.2648 \) and \( F = 0.3675 \) obtained above, we find \( c_1 = 0.55(16) \), \( c_2 = -21(6) \) and \( c_3 = 110(50) \) with \( \chi^2/\text{DOF} = 0.5 \). We see that our data is consistent with the relation \( c_1 = b_1/2 - a_1 \) although the error in \( c_1 \) is large. Fixing \( c_1 = b_1/2 = 0.68 \) obtained from fitting the chiral condensate, yields \( c_2 = -25.8(8) \) and \( c_3 = 149(14) \)
without changing the quality of the fit. This fit along with our data for $M_\pi$ are shown in Fig. 3.

3. Discussion

It is expected that the natural expansion parameter for ChPT in three dimensions is $x \equiv M_\pi/(4\pi F^2)$. Let us find the values of $x$ where the chiral expansion up to a certain power of $x$ is sufficient to describe the data reasonably. The solid lines in Fig. 4 are the best fits to the form $\Sigma_m + z_1 x + z_2 x^2 + z_3 x^3$, while the dot-dashed lines are the best fits to a smaller range in $x$ with $z_3$ set to 0. As the graph indicates the relative error in $\Sigma_m$ due to the absence of the $z_3$ term is smaller from the error in $F^2_m$ at a given value of $x$. In order to determine $F^2_m$ within say 5% we need the $z_3$ term even at $x \sim 0.15$. Thus, we conclude that in our model the chiral expansion converges rather poorly, especially for the pion decay constant. It is striking that almost all previous reliable unquenched lattice QCD simulations with staggered fermions have used $m \geq 0.005$ (in lattice units), irrespective of whether the study was of QCD thermodynamics or the QCD vacuum, whether the studies were done at strong couplings or weaker couplings. The value $m \sim 0.005$ corresponds to $x \sim 0.18$ in our model. We can ask how the chiral extrapolations will look if we use the data for $x \geq 0.18$ to fit to the expected chiral form. In Fig. 4 we show the best fits for large masses using the dashed lines. As expected in order to use chiral extrapolations reliably it is important to know the range of their validity.

Acknowledgements

This work was done in collaboration with Shailesh Chandrasekharan.

REFERENCES

1. S. Chandrasekharan and F-J. Jiang, Phys. Rev. D68, 091501 (2003).
2. D.J. Wallace and R.K.P. Zia, Phys. Rev. B12, 5340 (1975).
3. P. Rossi and U. Wolff, Nucl. Phys. B248, 105 (1984).
4. D.H. Adams and S. Chandrasekharan, Nucl. Phys. B662, 220 (2003).
5. S. Chandrasekharan and C.G. Strouthos, Phys. Rev. D69, 091502 (2004).
6. P. Hasenfratz and H. Leutwyler, Nucl. Phys. B343, 241 (1990).
7. M. Campostrini et al, Phys. Rev. B83, 214503 (2001).