D-Branes and Physical States

Sanjaye Ramgoolam and Lárus Thorlacius

Joseph Henry Laboratories
Princeton University
Princeton, NJ 08544, U.S.A.
ramgoola@puhep1.princeton.edu
larus@viper.princeton.edu

States obtained by projecting boundary states, associated with D-branes, to fixed mass-level and momentum generically define non-trivial cohomology classes. For on-shell states the cohomology is the standard one, but when the states are off-shell the relevant cohomology is defined using a BRST operator with ghost zero modes removed. The zero momentum cohomology falls naturally into multiplets of $SO(D-1,1)$. At the massless level, a simple set of D-brane configurations generates the full set of zero-momentum states of standard ghost number, including the discrete states. We give a general construction of off-shell cohomology classes, which exhibits a non-trivial interaction between left and right movers that is not seen in on-shell cohomology. This includes, at higher mass levels, states obtained from typical D-brane boundary states as well as states with more intricate ghost dependence.
1. Introduction

D-branes [1,2] have played a key role in the development of string duality, as they define solitonic states which are exchanged with perturbative string states under various duality transformations [2,3]. The study of D-brane boundary states has yielded considerable information about D-brane dynamics [4,5,6,7]. In the present paper, we will look for further insights by studying formal properties of boundary states, and their relationship to elementary string scattering states.

Boundary states can be usefully viewed as sources in closed string theory [8,9]. To each worldsheet boundary there corresponds a boundary state and the leading order effect of including worldsheets with boundaries is to add a right hand side to the usual closed string equation of motion:

\[(Q + \bar{Q})|\Psi\rangle = |B\rangle.\] (1.1)

The nilpotence of \((Q + \bar{Q})\) then implies that

\[(Q + \bar{Q})|B\rangle = 0.\] (1.2)

This equation has been proposed as a general criterion that selects admissible boundary states [8,10]. It suggests that boundary states are physical states of the closed string. Since \(Q + \bar{Q}\) commutes with \(L_0 + \bar{L}_0\), and with spacetime momenta, one can project boundary states to fixed momentum and mass level and still have BRST invariant states. In the following, we will focus on such projected boundary states, and ask if they represent non-trivial cohomology classes.

Consider first the Neumann boundary state of the bosonic string in \(R^{D-1,1}\) spacetime,

\[|N\rangle = \exp \left\{ -\sum_{n=1}^{\infty} \frac{1}{n} \alpha_{-n} \cdot \bar{\alpha}_{-n} + c_{-n} \bar{b}_{-n} + \bar{c}_{-n} b_{-n} \right\} \frac{1}{2} (c_0 + \bar{c}_0)|gh\rangle,\] (1.3)

where \(|gh\rangle\) is annihilated by all positive frequency ghost modes and all non-negative frequency anti-ghost modes. More general D-brane boundary states that can be found in the literature have the same simple ghost structure as \(|N\rangle\), but the BRST condition (1.2) does not rule out more complicated ghost dependence. The specification of the \(R^{D-1,1}\) spacetime means that \(p^\mu_L = p^\mu_R\), while the Neumann boundary conditions require \(p^\mu_L = -p^\mu_R\), so the momenta are forced to be zero. At higher mass levels, zero spacetime momentum means that the states are necessarily off-shell. The usual BRST operator does not have off-shell states in its cohomology. On the other hand there exist interesting amplitudes, for
example involving closed string exchange between D-branes \([2][11]\), that are off-shell from the closed string point of view. We are therefore led to look for a definition of ‘off-shell but physical’ states. There is a natural definition in the old covariant formalism, given in \([13]\). We will describe an adaptation of this to the BRST context, which involves a restricted BRST operator, \(Q'\), with ghost zero modes excised, acting on a certain subspace of the Fock space of matter and ghosts. When this restricted BRST operator is used to define off-shell cohomology, both on-shell and off-shell components of boundary states belong to non-trivial cohomology classes.

We describe the relation between components of a simple class of boundary states and the zero-momentum cohomology of the closed string. Boundary states associated with an appropriate set of D-brane configurations generate all the cohomology classes at the massless level. This includes ‘discrete states’ which only exist at zero momentum and do not appear to play any role in perturbative closed string theory. The extra discrete states at zero momentum combine with those obtained by continuing the ordinary perturbative states to form representations of \(SO(D - 1, 1)\).

We then give an algorithm for building closed string off-shell cohomology classes at arbitrary mass levels. By considering zero-momentum states at low-lying massive levels, (which are now cohomology classes of \(Q'\) but not of \(Q\)), we find that simple boundary states do not generate the full cohomology.

2. Basic properties of \(Q'\) cohomology

2.1. Definition of \(Q'\) cohomology

In the old covariant formalism there is a natural definition of ‘off-shell but physical’ \([13]\). Let us recall how that goes, first in the open string case. On-shell physical states \(|\chi\rangle\) are characterized by conditions expressed in terms of the matter Virasoro generators \(L_n^{(m)}\):

\[
(L_n^{(m)})|\chi\rangle = 0, \text{ for } n > 0,
\]

\[
(L_0^{(m)} - 1)|\chi\rangle = 0.
\]

(2.1)

Off-shell physical states are defined simply by dropping the condition on \(L_0^{(m)}\). For closed strings we similarly drop the condition on \(L_0^{(m)} + \bar{L}_0^{(m)}\).

1 For an early discussion of off-shell amplitudes see \([12]\) and references therein.
This definition has a natural extension to the modern covariant formalism with ghosts present. Let us make the zero mode dependence of $Q$ explicit, 

$$Q = c_0(L_0^{(m)} + L_0^{(gh)} - a) + b_0 M + Q', \quad (2.2)$$

where $M$ is constructed entirely from ghosts,

$$M = -2 \sum_{n=1}^{\infty} n c_{-n} c_n. \quad (2.3)$$

It follows from the nilpotence of $Q$ that

$$Q'^2 = -M(L_0 - a) = -(L_0 - a) M \quad (2.4)$$

and that

$$[Q', L_0] = 0, \quad (2.5)$$

$$[Q', M] = 0,$$

where $L_0 \equiv L_0^{(m)} + L_0^{(gh)}$. The cohomology of the $Q'$ operator can be studied, without loss of generality, on states annihilated by $b_0$. It is trivially repeated on states annihilated by $c_0$.

It follows from (2.4) that a linear space where $Q'^2$ is zero may be selected by imposing the condition $L_0 - a = 0$ or by imposing $M = 0$. If the first option is chosen, off-shell states are removed from the beginning. If the second is chosen, then the cohomology at $L_0 - a = 0$, contains the null states which are $Q'$ of states which do not satisfy $M = 0$. A third option, which we adopt below, is to select the appropriate cohomology according to whether or not $L_0 - a = 0$. For $L_0 - a \neq 0$, we take the cohomology of the $Q'$ operator computed on the space satisfying $M = 0$. For $L_0 - a = 0$, we take the standard $Q$ cohomology (which is closely related to $Q'$ on $L_0 - a = 0$, with no restriction on $M$ \[14\]).

In reference \[13\] it was observed that a simple extension of the DDF construction gives off-shell physical states satisfying (2.1). Here the observation takes the form:

**Proposition 2.1:** $Q'$ cohomology on $M = 0$, at $L_0 - a \neq 0$ contains off-shell continuations of the DDF states.

These states are annihilated by the positive matter Virasoro modes, and are built by acting with matter operators on the ghost vacuum $|gh\rangle$. For such states, the condition of annihilation by $Q'$ is precisely that they should be annihilated by the positive matter
Virasoro modes. The DDF states contain oscillators with polarization parallel to a light-like vector, and transverse oscillators. The analytic continuation of ref. \[13\] adjusts the momentum along the other light like direction, without changing the coefficients of any oscillators. It follows that the norm of a DDF state is unaffected by the continuation. Therefore such states cannot be in $\text{Im}(Q')$, for if they were, they would have to be null as the following argument shows. Let $|s\rangle = Q'|t\rangle$. Then the norm is

$$\langle t|(Q')^\dagger Q'|t\rangle$$

But $Q'$ is hermitian, using $(c_n)^\dagger = c_{-n}$, and $b_n^\dagger = b_{-n}$, so any $Q'$ closed and exact state is null.

We are interested in the non-chiral version $Q'$ cohomology for the closed string. We impose at the outset the conditions $L_0 - \bar{L}_0 = 0$, and $b_0 - \bar{b}_0 = 0$, which are certainly satisfied by D-brane boundary states. Defining $Q'_T = Q' + \bar{Q}'$, $Q_T = Q + \bar{Q}$, and $L_T = L_0 + \bar{L}_0$, we have

$$(Q'_T)^2 = -M(L_0 - a) - \bar{M}(\bar{L}_0 - a)$$

$$= -\frac{1}{2}(M + \bar{M})(L_0 + \bar{L}_0 - 2a)$$

(2.7)

In order to have a linear space where $(Q'_T)^2$ is zero we need to work in the space where $L_T - 2a = 0$ or the space where $(M + \bar{M}) = 0$. Our working definition of physical states, will be to consider, along the lines of the open string case, all states of fixed $L_T - 2a$: if $L_T - 2a = 0$, we use the standard $Q_T$ cohomology, whereas if $L_T - 2a \neq 0$, use $Q'_T$ cohomology at $(M + \bar{M}) = 0$.

By arguments similar to the above, this cohomology contains the analytic continuation of the DDF states. It is easy to see, however, that there are states which are $Q_T$ closed and exact (and hence null) when on-shell, but which give non-trivial $Q'_T$ cohomology when continued away from mass shell. For example two representatives of the cohomology class of the dilaton are

$$|s^{(1)}\rangle = \sum_{i=1}^{D-2} \alpha^i_{-1} \bar{\alpha}_{-i} |gh;p^0 = p, p^{D-1} = p\rangle$$

$$|s^{(2)}\rangle = \sum_{i=0}^{D-1} (\alpha^i_{-1} \bar{\alpha}_{-i} + c_{-1} \bar{b}_{-1} + \bar{c}_{-1} b_{-1}) |gh;p^0 = p, p^{D-1} = p\rangle,$$

(2.8)

where $|gh;p^0 = p, p^{D-1} = p\rangle$ is a state carrying equal non-zero momenta $p^0$ and $p^{D-1}$, and zero momentum in the remaining directions. The difference $|s^{(1)}\rangle - |s^{(2)}\rangle$ is $Q_T$ exact.
However when the momenta are continued to $|p^0| \neq |p^{D-1}|$ this state is still $Q'_T$ closed, and null, but not $Q'_T$ exact. Note that a state which is $Q'_T$ closed and exact is automatically null, a fact explained by equation (2.9), but a state which is $Q'_T$ closed and null is not necessarily exact. The fact that it is not exact follows simply from an $sl(2)$ structure in the problem (see section 2.6). Such null cohomology classes occur in standard $Q_T$ cohomology at zero momentum, but can occur at continuous momenta in off-shell cohomology. They appear in the space spanned by the projections of boundary states. Since they have zero norm, they do not contribute to closed string exchange amplitudes, but standard BRST decoupling arguments do not prevent them from contributing to higher order amplitudes.

2.2. Boundary states and cohomology

The projection of a boundary state onto a given spacetime momentum and mass-level will generically define an off-shell state, i.e. one for which $L_T - 2a = l \neq 0$. Such a state can be written

$$|s⟩ = A(c_0 + \bar{c}_0)|gh⟩,$$

(2.9)

where $A$ represents a string of matter and ghost creation operators, but containing no ghost zero modes. The following standard argument shows that a state of this form does not belong to a non-trivial $Q_T$ cohomology class.

We have $\{Q_T, b_0 + \bar{b}_0\} = L_T - 2a$. Consider the state

$$|t⟩ = \frac{1}{(L_T - 2a)}(b_0 + \bar{b}_0)A(c_0 + \bar{c}_0)|gh⟩$$

$$= \frac{2}{l} A|gh⟩.$$

(2.10)

It has the property that $Q_T|t⟩ = |s⟩$. This proves that $|s⟩$ is not only $Q_T$-closed, but also $Q_T$-exact. It is also easy to see that $(b_0 + \bar{b}_0)|s⟩$ is not in $Q_T$ cohomology. Using $\{Q_T, b_0 + \bar{b}_0\} = L_T - 2a$, we find :

$$Q_T(b_0 + \bar{b}_0)|s⟩ = l|s⟩ \neq 0.$$

(2.11)

Thus $(b_0 + \bar{b}_0)|s⟩$ is not in the cohomology of $Q_T$.

Off-shell components of boundary states do belong to non-trivial cohomology classes with respect to the restricted BRST operator $Q'_T$. Since off-shell $Q'_T$ cohomology is only found in $\text{Ker}(M + \bar{M})$, we first check that $M + \bar{M}$ annihilates known boundary states. For this we only need to consider the ghost part of any given boundary state, which is
identical to the ghost part of the Neumann state $|N\rangle$, for generic open string backgrounds (including D-branes). Applying $M + \bar{M}$ to the ghost part of $|N\rangle$ gives

$$(M + \bar{M}) \exp\left\{ \sum_{n=1}^{\infty} (c_{-n} \bar{b}_{-n} + \bar{c}_{-n} b_{-n}) \right\} \frac{1}{2} (c_0 + \bar{c}_0) |gh\rangle$$

$$= \sum_{m=1}^{\infty} m(c_{-m} c_m + \bar{c}_{-m} \bar{c}_m) \exp\left\{ \sum_{n=1}^{\infty} (c_{-n} \bar{b}_{-n} + \bar{c}_{-n} b_{-n}) \right\} (c_0 + \bar{c}_0) |gh\rangle$$

$$= \sum_{m=1}^{\infty} m(c_{-m} \bar{c}_{-m} + \bar{c}_{-m} c_{-m}) \exp\left\{ \sum_{n=1}^{\infty} (c_{-n} \bar{b}_{-n} + \bar{c}_{-n} b_{-n}) \right\} (c_0 + \bar{c}_0) |gh\rangle$$

$$= 0.$$  

Since $Q'_T$ commutes with the ghost zero modes, it is clear that $|s\rangle$ is also $Q'_T$ closed.

The following shows that $|t\rangle$ in (2.10) is $Q'_T$ closed:

$$Q'_T|t\rangle = (Q_T - 1/2(c_0 + \bar{c}_0)(L_T - 2a))|t\rangle$$

$$= |s\rangle - \frac{1}{2}(c_0 + \bar{c}_0)(b_0 + \bar{b}_0)|s\rangle$$

$$= 0.$$  

In the first line we used the fact that $b_0 M, \bar{b}_0 \bar{M}$, and $L_0 - \bar{L}_0$ all annihilate $|t\rangle$.

Now that we have seen that the components of a boundary state are annihilated by $Q'_T$, it remains to prove that they are not in $Im(Q'_T)$. Suppose that a state is $Q'_T$ closed and exact. Then it would have zero norm by (2.6). All we have to do, to prove that the projections of the boundary state are not $Q'_T$ exact, is to check that they don’t have zero norm. As an example consider a static D-brane in flat spacetime in the bosonic string theory. The norm of the state at level $l$ can be read off from the formula

$$\langle B | q^{L_0 + \bar{L}_0} | B \rangle = \prod_{n=0}^{\infty} \frac{1}{(1 - q^{2n})^{D-2}},$$

which exploits the factorized form of the D-brane boundary state as a product of exponentials. The relevant norm, according to the above formula, is manifestly positive. A similar expression applies for D-brane boundary states of the NS-NS sector of the superstring. This guarantees that at each level we have at least one cohomology class. In section 2.6 we will give an independent argument, which does not rely on norms, and shows that in the off-shell case any $Q'_T$ closed state of ghost number zero is in cohomology.
2.3. Examples at massless level

There are physically interesting boundary states which illustrate, already at the massless level, the fact that the components of boundary states do not always give $Q_T$ cohomology classes, but give $Q'_T$ cohomology. Consider a boundary state, which acts as a source of closed string states winding around a compact direction $X^i$. A non-vanishing value of the momentum $p^i_L = -p^i_R$ defines, in general, an off-shell state but is compatible with Neumann boundary condition. Such off-shell winding states are not in the cohomology of $Q_T$ but are in that of $Q'_T$.

2.4. Remark on decoupling of longitudinal states

For on-shell S-matrix calculations a simple formal argument can be given for tree level decoupling of longitudinal states \[15\]. We write the longitudinal state as $Q_T |t\rangle$ and then commute $Q_T$ through the remaining operators until it annihilates the vacuum. In fact the on-shell longitudinal states can be written as $Q'_T$-exact states, since

$$
Q_T |t\rangle = (Q'_T + \frac{1}{2} (b_0 + \bar{b}_0)(M + \bar{M}) + \frac{1}{2} (c_0 + \bar{c}_0)(L_T - 2a)) |t\rangle = Q'_T |t\rangle.
$$

(2.15)

In the first line we have used the level matching condition $(L_0 - \bar{L}_0)|t\rangle = 0$ and $(b_0 - \bar{b}_0)|t\rangle = 0$. In the second we have used the on-shell condition and $(b_0 + \bar{b}_0)|t\rangle = 0$. The decoupling argument can be used in the presence of boundary operators provided we use $Q'_T$ instead of $Q_T$. As was emphasized in \[16\], the state that enters into the calculation of D-brane amplitudes from the closed string point of view is $(b_0 + \bar{b}_0)|B\rangle$, rather than the $|B\rangle$ that appears in the equation of motion \[1.1\]. The off-shell components of $(b_0 + \bar{b}_0)|B\rangle$ are only annihilated by $Q'_T$, and not by $Q_T$. In applying decoupling arguments one has to be careful about boundaries of moduli space, but it is noteworthy that $Q'_T$, which gives a particularly simple definition of off-shell cohomology, also enters in a formal BRST discussion of decoupling in the presence of boundary states.

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2 An off-shell continuation of this type was found useful in \[8\] for regularization purposes.
2.5. Enhanced Lorentz symmetry

Standard Q cohomology provides representations of Lorentz symmetries. Consider the momentum of an on-shell massless particle, \( p^0 = p^{D-1} = p \). The cohomology at this momentum is computed by considering the action of \( Q \) (which is \( SO(D-1,1) \) invariant) on a space spanned by oscillators (which have well-defined \( SO(D-1,1) \) transformation properties) acting on the ground state \( |gh; p^0 = p, p^1 = p\rangle \). The cohomology is computed by considering the action of \( Q \) (which is \( SO(D-1,1) \) invariant) on a space spanned by oscillators (which have well-defined \( SO(D-1,1) \) transformation properties) acting on the ground state \( |gh; p^0 = p, p^1 = p\rangle \). The ground state is left invariant by an \( SO(D-2) \) subgroup of \( SO(D-1,1) \). Similarly, the momentum of a massive particle is left invariant by \( SO(D-1) \). At zero spacetime momentum, however, the cohomology provides representations of the full Lorentz group \( SO(D-1,1) \) as we illustrate in the following section. If we consider on-shell cohomology, \( p^\mu = 0 \) restricts us to the massless level. Given that the \( Q' \) (and \( Q'_T \)) operators are \( SO(D-1,1) \) invariant, the enhanced symmetry will occur at arbitrary mass level in the case of \( Q' \) (and \( Q'_T \)) off-shell cohomology.

2.6. Vanishing theorems for off-shell cohomology

Zero-momentum Q cohomology contains states at exotic ghost numbers, and one might expect this to be a generic feature of zero-momentum cohomology at higher levels. This is not the case, and off-shell \( Q' \) cohomology has a surprisingly simple structure.

We will first discuss the open string. It has been shown [17] that the ghost system contains an \( sl(2) \) algebra (and that only singlets under this \( sl(2) \) enter the construction of free string field actions). Indeed let \( N_{gh} \) be the ghost number operator (without the zero modes):

\[
N_{gh} = \sum_{n>0} (c_{-n}b_n - b_{-n}c_n), \tag{2.16}
\]

and \( N \) be an operator of ghost number \( -2 \) defined by

\[
N = -\sum_{n>0} \frac{1}{2n} b_{-n}b_n. \tag{2.17}
\]

Then we have the \( sl(2) \) relations

\[
[N_{gh}, M] = 2M, \\
[N_{gh}, N] = -2N, \tag{2.18}
\]

\[
[M, N] = N_{gh},
\]
where $M$ and $N$ are respectively raising and lowering operators of $sl(2)$. This allows a simple proof of the following vanishing theorem:

**Proposition 2.2:** $Q'$ cohomology groups, at $L_0 - a \neq 0$, are empty at non-zero ghost number.

An off-shell state can only be annihilated by $Q'$ if it is annihilated by $M$. There are no states in $Ker(M)$ at negative ghost number. This is simply because there are no finite dimensional representations of $sl(2)$ with a negative highest spin, and at fixed mass level (and momentum) there are only finitely many states. Any state $|s(n)\rangle$ of ghost number $n > 0$ annihilated by $M$ generates a finite dimensional representation when we act with the lowering operator $N$. Using the $sl(2)$ relations (2.18), we see that such a state is $Q'$ exact:

\[
|s(n)\rangle = \frac{N_{gh}}{n} |s(n)\rangle \\
= \frac{MN}{n} |s(n)\rangle \\
= Q'\left(\frac{-Q'}{L_0 - 1} \frac{N}{n} |s(n)\rangle\right).
\]

(2.19)

This completes the proof of the proposition.

Similarly for the bosonic closed string, we have

**Proposition 2.3:** $Q'_T$ cohomology groups, at $L_T - 2a \neq 0$, are empty at non-zero ghost number.

The proof proceeds as above, with $M$, $N$, and $N_{gh}$ replaced by $M + \bar{M}$, $N + \bar{N}$, and $N_{gh} + \bar{N}_{gh}$ respectively.

Another consequence of the above is that any off-shell ghost number zero state $|s\rangle$ annihilated by $Q'$ is in $Q'$ cohomology. Indeed, suppose

\[
Q'|s\rangle = 0 \quad \text{and} \quad |s\rangle = Q'|t\rangle,
\]

(2.20)

then $|t\rangle$ has ghost number $-1$ and $(Q')^2|t\rangle = 0$, which implies that $M$ annihilates $|t\rangle$. But we saw that there are no states in $KerM$ at negative ghost number. The same argument shows that any off-shell ghost number zero closed string state annihilated by $Q'_T$ is in cohomology.

Identical arguments apply for the NS and NS-NS sectors of open and closed superstrings respectively, the key feature being the $sl(2)$ symmetry of the ghost and superghost system (see Appendix A).
3. D-branes and cohomology in bosonic string theory

We have seen above that boundary states are not only BRST invariant but also gener-
ically give rise to non-trivial cohomology. In this section we will consider the massless level
and zero spacetime momentum. The relevant cohomology is then the standard one, and
is larger than at generic momentum. ‘Discrete states’ associated with such jumps in coho-
mology were first studied in detail in the context of the non-critical \( c \leq 1 \) string \([18,19,20]\)
where there are infinitely many of them. It was later pointed out \([21]\) that this phe-
nomenon also occurs in critical string theories. For a recent discussion of zero momentum
cohomology see \([22]\).

At the massless level the cohomology is spanned by \((D-2)^2\) states at generic mo-
momentum. They are obtained in a lightfront like frame by acting with \( D - 2 \) left moving
transverse DDF operators \( A_i \) and \( D - 2 \) right moving operators \( \bar{A}_i \), on a state carrying
momentum in the \( X^0 \) and \( X^{D-1} \) directions, for example. The DDF operators \( A_i \) and \( \bar{A}_i \)
carry spacetime indices running from 1 to \( D - 2 \).

At zero momentum however there are \( D^2 + 1 \) independent states in the zero momentum
cohomology. They are

\[
\alpha^-_{\mu} \bar{\alpha}^-_{\nu} |gh\rangle \quad \text{for} \quad 0 \leq \mu, \nu \leq (D-1),
\]

\[
|D_{gh}\rangle = (c_{-1} \bar{b}_{-1} + \bar{c}_{-1} b_{-1}) |gh\rangle.
\]  

(3.1)

These states include two scalars, a traceless symmetric tensor of \( SO(D-1,1) \) of dimension
\((D+2)(D-1)/2\), and an antisymmetric tensor of dimension \((D-1)(D)/2\). Note that the
ghost dilaton is \( Q_T \)-trivial, \( |D_{gh}\rangle = Q_T |\chi\rangle \), with

\[
|\chi\rangle = b_{-1} \bar{b}_{-1} (c_0 - \bar{c}_0) |gh\rangle.
\]

(3.2)

On the other hand \( |\chi\rangle \) is not annihilated by \( b_0 - \bar{b}_0 \), so \( |D_{gh}\rangle \) is not trivial in the semirelative
complex (\( i.e \) in the subspace of the matter-ghost Fock space annihilated by \( b_0 - \bar{b}_0 \)), which
defines closed string physical states \([23]\).

We will see that the projections of D-brane boundary states to the massless level span
the complete zero-momentum cohomology. The boundary state for a static p-brane with
Neumann boundary conditions along the directions \( i = 0 \) up to \( p \), and Dirichlet boundary
conditions along $i = p + 1$ up to $D - 1 (=25)$, has the following component at the massless level:

$$(- \sum_{\mu=0}^{p} \alpha_{-1}^{\mu}(\bar{\alpha}_{-1})_{\mu} + \sum_{\mu=p+1}^{D-1} \alpha_{-1}^{\mu}(\bar{\alpha}_{-1})_{\mu} + \bar{c}_{-1}b_{-1} + c_{-1}\bar{b}_{-1})|gh\rangle). \quad (3.3)$$

Along the Dirichlet directions we should sum over all momenta satisfying $p_L = p_R$. Non-zero Neumann momenta satisfying $p_L = -p_R$ are compatible with $Q'_T$ invariance and arise when some directions are compactified.

Using (3.3) we can write the following equations at zero momentum,

$$|D_M\rangle = 1/2(|B(-1)) - |B(D - 1))\rangle), \quad (3.4)$$

$$|D_{gh}\rangle = 1/2(|B(-1)) + |B(D - 1))\rangle),$$

where $|B(p))$ is the projection of the $p$-brane boundary state to the massless level, $|D_M\rangle$ is the zero-momentum matter dilaton, and $|D_{gh}\rangle$ is the zero-momentum ghost dilaton. In this sense the boundary states associated to the $(-1)$- and $(D - 1)$-brane give rise to the two scalars of $SO(D - 1, 1)$.

Using the 0-brane and the D-instanton we can similarly write:

$$\alpha^0_{-1}(\bar{\alpha}_{-1})_0|gh\rangle = 1/2(-|B(0)) + |B(-1))\rangle). \quad (3.5)$$

Let $|B(1; i))$ be the 1-brane, with the direction $i$ as the spatial Neumann direction. Then

$$\alpha^i_{-1}\bar{\alpha}_{-1i}|gh\rangle = \frac{1}{2}(|B(0)) - |B(1; i))\rangle). \quad (3.6)$$

Now rotate this 1-brane in the $(ij)$ plane by an angle $\theta$ and call the corresponding boundary state $|B(1; ij, \theta))$. Then

$$\begin{aligned}
\left(\alpha^i_{-1}\bar{\alpha}_{-1j} + \alpha^j_{-1}\bar{\alpha}_{-1i}\right)|gh\rangle &= \left. \frac{\partial}{\partial \theta}|B(1; ij, \theta)\right|_{\theta=0} \\
\end{aligned} \quad (3.7)$$

States with the corresponding antisymmetric combination can be obtained by considering 1-branes with a background magnetic field $F_{ij}$.

States of the form $\alpha^i_{-1}(\bar{\alpha}_{-1})_0 + \alpha^0_{-1}\bar{\alpha}_{-1i}$ can be obtained from boundary states of 0-branes boosted along the $i$ direction [5] and the corresponding antisymmetric combination can be obtained using a D-string aligned along the $i$ direction with a background worldvolume electric field.

\footnote{In this section we find it convenient to work with the boundary states without the $(c_0 + \bar{c}_0)$ factor.}
The above states exhaust the set of zero-momentum cohomology classes. The states obtained by projecting the various boundary states to the massless level are not all linearly independent. We have described one generating set made of the D-instanton, the zero brane (stationary and boosted), D-strings (with and without constant background electromagnetic fields), and the \((D - 1)\)-brane. Note that both the D-instanton and the \((D - 1)\)-brane are present in this generating set. Although there is a lot of freedom in the choice of generating set, we always need these two, as they are the only \(p\)-brane configurations that give rise to the two scalars of \(SO(D - 1, 1)\). Given the special properties of the D-instanton in D-brane scattering \([24, 25]\), it is interesting that it is required in order to give the complete set of zero-momentum states.

We close this section with some further comments on the relation between D-branes and discrete states. Consider \(R^{D-1,1}\) spacetime where boundary states will carry zero momentum in each Neumann direction. For Dirichlet directions there will be a sum over all momenta, but since the BRST operator commutes with momentum, we can restrict to one momentum at a time, and still have BRST invariant states. Precisely at zero momentum for all directions, we can form linear combinations from all types of D-branes and extra cohomology classes may be expected.

We can understand why the discrete states do not admit continuation to \(p_L = -p_R\) by considering toroidal compactifications. For instance the boosted D-brane with a boost along the \(i\) direction will only exist when this direction is non-compact. This explains why states \(\alpha_{-1}^0 \bar{\alpha}_{-1i} + \alpha_{-1}^i (\bar{\alpha}_{-1})_0\) do not admit continuations to \(p_L^i = -p_R^i \neq 0\). Similarly the boundary states associated with a \(D\)-string with worldvolume electric field, which generate the antisymmetric combinations of timelike and space-like oscillators, do not exist when the spatial worldvolume direction of the D-string is compact and the electric fields are quantized \([26]\).

4. Boundary states and cohomology in Type II superstrings

The definitions of off-shell cohomology developed for the bosonic string in section 2 admit a simple generalization to type II superstrings as outlined in appendix A. Here we will describe generating sets of D-branes for zero-momentum cohomology at the massless level. In doing this, we will be comparing with cohomology computed in a fixed picture as in \([27, 28]\) and references given there. In terms of operators which act on the \(sl(2)\) invariant vacuum this amounts to considering \(Q\)-invariant operators up to \(Q\)-commutators, in the
space of operators consisting of $e^{q\phi}$ times polynomials in $\beta, \gamma$ and their derivatives (and the obvious generalization in the non-chiral case) together with matter and ghost operators. Moreover a simple comparison is possible, which closely resembles the bosonic discussion, when we compare zero-momentum cohomology classes with the combined set of boundary states of D-branes of both type IIA and type IIB superstrings.

Boundary states contain a sum over an infinite number of pictures $(q_L, q_R)$ with $q_L + q_R$ fixed (at $-2$ in the NS-NS sector). The sum ensures that the D-brane boundary state is annihilated by half the supersymmetries, but as far as BRST invariance is concerned there is no need to consider the whole sum. In the NS-NS sector for example we can work in the $(-1, -1)$ picture and the other terms are related by picture changing. Recall that the boundary state which appears in (1.1) is proportional to $(c_0 + \bar{c}_0)|gh\rangle$, but in using the boundary states to compute amplitudes one works with $(b_0 + \bar{b}_0)|B\rangle$. So, in general, we will be interested in two types of cohomology classes, which have representatives of the form

\[ X|gh; q_L, q_R\rangle, \tag{4.1} \]

or

\[ X(c_0 + \bar{c}_0)|gh; q_L, q_R\rangle, \tag{4.2} \]

where $X$ is a ghost number zero operator, and $|gh; q_L, q_R\rangle$ is a vacuum state satisfying:

\[ b_n, \bar{b}_n|gh; q_L, q_R\rangle = 0 \text{ for } n \geq 0, \]

\[ c_n, \bar{c}_n|gh; q_L, q_R\rangle = 0 \text{ for } n > 0, \]

\[ \beta_n|gh; q_L, q_R\rangle = 0 \text{ for } n > -q_L - 3/2, \]

\[ \gamma_n|gh; q_L, q_R\rangle = 0 \text{ for } n \geq q_L + 3/2, \]

\[ \bar{\beta}_n|gh; q_L, q_R\rangle = 0 \text{ for } n > -q_R - 3/2, \]

\[ \bar{\gamma}_n|gh; q_L, q_R\rangle = 0 \text{ for } n \geq q_R + 3/2. \tag{4.3} \]

We will refer to the states in (4.1) and (4.2) as states of “standard ghost structure”.

In performing the comparison between cohomology and projections of boundary states we may use, in the NS-NS case, either $|B\rangle$ or $(b_0 + \bar{b}_0)|B\rangle$, since the cohomology over $(c_0 + \bar{c}_0)|gh\rangle$ is identical to that over the $|gh\rangle$ vacuum. In the R-R and R-NS cases this is not always true.
In the NS-NS sector the zero momentum cohomology at standard ghost number closely parallels that of the bosonic string. Working in the $(-1, -1)$ picture, it is spanned by

$$
\psi_{-1/2}^{\mu} \bar{\psi}_{-1/2}^{\nu} |gh; -1, -1), \quad 0 \leq \mu, \nu \leq 9,
$$

$$
|D_{gh}⟩ = (\beta_{-1/2} \bar{\gamma}_{-1/2} - \bar{\beta}_{-1/2} \gamma_{-1/2}) |gh, -1, -1⟩. \quad (4.4)
$$

Note that in the $(-1, -1)$ picture all the positive $\beta$ and $\gamma$ zero modes annihilate the vacuum. As in the bosonic case, the dilaton is $Q_T$ of something which is not annihilated by $b_0 - \bar{b}_0$, and is physical in the semirelative complex,

$$
|D_{gh}⟩ = (Q + \bar{Q}) \beta_{-1/2} \bar{\beta}_{-1/2} (c_0 - \bar{c}_0) |gh, -1, -1⟩. \quad (4.5)
$$

Again the states naturally fall into representations of $SO(D - 1, 1)$ ($= SO(9, 1)$ in this case).

The boundary states of $[4]$ are linear combinations of states in the NS-NS sector and the R-R sector. They are separately BRST invariant. D-branes and anti D-branes differ in the relative sign between the two sectors. Thus we can isolate states purely in the NS-NS or R-R sector by taking linear combinations. In the NS-NS sector all zero-momentum physical states are generated by D-branes, at rest or with constant boosts, or constant electromagnetic field backgrounds. The argument is identical to the one we gave for the bosonic string.

In the R-R sector the zero-momentum limit is more subtle. The boundary states in $[4]$, for example, contain states in the $(-1/2, -1/2)$ picture which have projections that vanish in the zero-momentum limit. They can, however, be related via picture changing to states in the $(-3/2, -1/2)$ picture $[29]$, and after this a finite zero-momentum limit may be obtained. We hope to give a more detailed account of the relations between boundary states and cohomology in the R-R and R-NS sectors in future work.

5. Construction of off-shell closed string cohomology classes

5.1. General construction

The condition for $Q'_T$ to vanish, in the off-shell case where $L_T \neq 0$, is $M + \bar{M} = 0$. There are many states which satisfy this condition without satisfying $M = \bar{M} = 0$. This allows for a more intricate mixing between left- and right-moving degrees of freedom in
the states that are in the cohomology of $Q'_T$ than is possible in the on-shell cohomology of $Q + \bar{Q}$ where $Q$ and $\bar{Q}$ separately square to zero.

We now describe a construction of $Q'_T$ cohomology classes, which illustrates this non-trivial left-right mixing. As given here, the discussion will apply directly to the bosonic string case, and the NS-NS sector of the superstring. We expect that small modifications will work for the R-R and R-NS sectors. The input will be a pair of states, $A_{(-n)}$ of left moving ghost number $-n$ and $\bar{B}_{(-n)}$ of right moving ghost number $-n$, which satisfy a set of conditions:

\[
\begin{align*}
(Q'_T)^{2n+1}A_{(-n)} &= 0, \\
(\bar{Q}'_T)^{2n+1}\bar{B}_{(-n)} &= 0, \\
\langle (A_{(-n)})^\dagger (Q'_T)^{2n}A_{(-n)} \rangle &\neq 0, \\
\langle (\bar{B}_{(-n)})^\dagger (\bar{Q}'_T)^{2n}\bar{B}_{(-n)} \rangle &\neq 0.
\end{align*}
\]  

In the actual cases of the bosonic string or the NS-NS sector, where we will apply this construction, the last two conditions in (5.1) are not really necessary since any $Q'_T$ closed state of ghost number zero is in cohomology, but we present the discussion in a way that does not use the $sl(2)$ structure of section 2.6 and may thus be more general.

Let us define

\[
\begin{align*}
A_{(-n+k)} &\equiv Q'^k A_{(-n)}, \\
\bar{B}_{(-n+k)} &\equiv (\bar{Q}'_T)^k \bar{B}_{(-n)}.
\end{align*}
\]  

The subscripts denote ghost numbers. In some of our examples $A$ and $\bar{B}$ have spacetime indices which we have suppressed here.

We will prove that the following state represents a non-trivial cohomology class:

\[
|s\rangle = \sum_{i=-n}^{-n} (-1)^{\sigma(i)} A_{(i)} \bar{B}_{(-i)},
\]  

where $(-1)^{\sigma(i)} = \pm 1$ as determined by the following rule. If $A_{(i)}$ is fermionic, then $\sigma(i+1) = \sigma(i) + 1$. If $A_{(i)}$ is bosonic, then $\sigma(i+1) = \sigma(i)$. If we fix the sign of the first term to be positive say, this rule determines all the remaining signs. For $|s\rangle$ to be $Q'_T$-closed we need, for all $i$,

\[
Q'((-)^{\sigma(i)} A_{(i)} \bar{B}_{(-i)}) = -\bar{Q}'((-)^{\sigma(i+1)} A_{(i+1)} \bar{B}_{(-i+1)}).
\]
It is easily checked that the definition of $\sigma(i)$ given above guarantees this equality. Together with the first two equations in (5.1), this ensures that $|s\rangle$ is $Q'_T$-closed. The norm of the state is given by

$$
\langle s|s\rangle = (-1)^{\sigma(A(-n))}\langle (A_0\bar{B}_0)^\dagger(A_0\bar{B}_0)\rangle
$$

$$
+ \sum_{i=1}^n (-1)^{\sigma(i)+\sigma(-i)}\langle ((A_{(i)}\bar{B}_{(-i)})^\dagger(A_{(-i)}\bar{B}_{(i)})) + \langle (A_{(-i)}\bar{B}_{(i)})^\dagger(A_{(i)}\bar{B}_{(-i)})\rangle
$$

$$
= \langle A_{0}^\dagger A_{0}\rangle \langle \bar{B}_{0}^\dagger \bar{B}_{0}\rangle
$$

$$
+ \sum_{i=1}^n (-1)^{\sigma(i)+\sigma(-i)}\langle (A_{(i)}^\dagger A_{(-i)})\langle \bar{B}_{(-i)}^\dagger \bar{B}_{(i)}\rangle + \langle A_{(-i)}^\dagger A_{(i)}\rangle \langle \bar{B}_{(i)}^\dagger \bar{B}_{(-i)}\rangle
$$

(5.6)

All other terms give zero because of ghost number conservation. Using the equation

$$
\langle A_{(-i)}^\dagger A_{(i)}\rangle
$$

$$
= \langle A_{(i)}^\dagger A_{(-i)}\rangle
$$

$$
= \langle A_{(-n)}^\dagger (Q')^{(n-i)+(n+i)} A_{(-n)}\rangle
$$

$$
= \langle A_{(-n)}^\dagger (Q')^{2n} A_{(-n)}\rangle,
$$

(5.7)

and its antichiral analog, it is clear that $\langle s|s\rangle$ is proportional to

$$
\langle A_{(-n)}^\dagger (Q')^{2n} A_{(-n)}\rangle \langle \bar{B}_{(-n)}^\dagger (Q')^{2n} \bar{B}_{(-n)}\rangle.
$$

The description of the signs given after (5.3) can be used to check the coefficient in (5.3).

Note that in the $n = 0$ case this construction is just the tensor product of a pair of states which are in cohomology of $Q'$ and $\bar{Q}'$ respectively. In this case, the last two conditions in (5.1) reduce to the statement that the states are not null:

$$
\langle A_{0}^\dagger A_{0}\rangle \neq 0,
$$

$$
\langle \bar{B}_{0}^\dagger \bar{B}_{0}\rangle \neq 0,
$$

(5.8)

which implies that they are not $Q'$ exact. In the following we will consider some examples of the above construction. It would be interesting to determine whether this construction gives a complete basis in the $Q'_T$ off-shell cohomology.
5.2. Examples at low-lying levels

We now consider some examples at zero momentum, first in the case of the bosonic string. At \( L_0 - 1 = 2 \), the state \( A_{\mu \nu} \alpha_{-1} \alpha_{-1}^\nu |gh \rangle \) is \( Q' \) closed if \( A_{\mu \nu} \) is traceless. Taking this together with an analogous \( \bar{Q}' \) closed state, we can apply the above construction with \( n = 0 \).

Another example is given by solutions to \( Q'^3 |s \rangle = 0 \), such as

\[
b_{-2} |gh \rangle, \quad b_{-1} \alpha_{-1}^\mu |gh \rangle.
\]

On both states \( Q'^3 \) is clearly zero because there is no ghost number 2 state at this mass level. Using the fact that \( Q'^2 \) is proportional to \( M \), it also follows immediately that \((5.1)\) is satisfied. If we take \( A_{\mu-1}^\mu = b_{-1} \alpha_{-1}^\mu |gh \rangle \) on the left and on the right \( \bar{B}_{-1}^\nu = \bar{b}_{-1} \alpha_{-1}^\nu |gh \rangle \) we get a set of states:

\[
|s\rangle^{\mu \nu} = (A_{\mu-1}^\mu \bar{B}_{(1)}^\nu - A_{\mu-1}^\mu \bar{B}_{(0)}^\nu - A_{\mu-1}^\mu \bar{B}_{(-1)}^\nu)
= (2b_{-1} \alpha_{-1}^\mu \bar{c}_{-1} \bar{\alpha}_{-1}^\nu - \alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu - 2c_{-1} \alpha_{-1}^\mu \bar{b}_{-1} \bar{\alpha}_{-1}^\nu) |gh \rangle
= (-\alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu - 2(c_{-1} \bar{b}_{-1} + \bar{c}_{-1} b_{-1}) \alpha_{-1}^\mu \alpha_{-1}^\nu) |gh \rangle.
\]

Applying the same construction, using on the left the state \( A_{(-1)}^\mu = b_{(-1)} \alpha_{-1}^\mu |gh \rangle \) and on the right \( B_{(-1)} = b_{-2} |gh \rangle \) yields the off-shell physical state:

\[
[\alpha_{-1}^\mu (\bar{\alpha}_{-1}^\nu + 6 \bar{c}_{-1} \bar{b}_{-1}) + 4 \alpha_{-1}^\nu (c_{-1} \bar{b}_{-2} + 2 \bar{c}_{-2} b_{-1})] |gh \rangle.
\]

Let us also consider examples of this construction in the NS-NS sector of the superstring at \( L_0 - 1/2 = \bar{L}_0 - 1/2 = 1/2 \). The first example is shown for its simplicity rather than its physical interest since it is projected out by GSO. The conditions \((5.1)\) are satisfied by the state \( \beta_{-1/2} |\bar{\psi}_{-1/2}^\mu gh; -1 \rangle \). Taking \( A_{(-1)}^\mu = \beta_{-1/2} \psi_{-1/2}^\mu gh; -1 \rangle \) and \( \bar{B}_{(-1)} = \bar{\beta}_{-1/2} \bar{\psi}_{-1/2}^\nu gh; -1 \rangle \) we obtain the following state:

\[
|s\rangle^{\mu \nu} = A_{(-1)}^\mu \bar{B}_{(1)}^\nu + A_{(0)}^\mu \bar{B}_{(0)}^\nu - A_{(1)}^\mu \bar{B}_{(-1)}^\nu
= [\alpha_{-1}^\mu \bar{\alpha}_{-1}^\nu + 2(\beta_{-1/2} \gamma_{-1/2} - \bar{\beta}_{-1/2} \bar{\gamma}_{-1/2}) \psi_{-1/2}^\mu \bar{\psi}_{-1/2}^\nu] |gh; -1, -1 \rangle.
\]

One can also construct examples which survive the GSO projection. At \( L_0 - 1/2 = \bar{L}_0 - 1/2 = 1 \), take \( A_{(-1)}^\mu = (b_{-1} \psi_{-1/2}^\mu + \beta_{-1/2} \alpha_{-1}^\mu) |gh; -1 \rangle \) and \( \bar{B}_{(-1)} = \bar{\beta}_{-3/2} |gh; -1 \rangle \), to obtain

\[
|s\rangle^\mu = A_{(-1)}^\mu \bar{B}_{(1)} + A_{(0)}^\mu \bar{B}_{(0)} - A_{(1)}^\mu \bar{B}_{(-1)}
= [(b_{-1} \psi_{-1/2}^\mu + \beta_{-1/2} \alpha_{-1}^\mu) \gamma_{-3/2}
+ \psi_{-3/2}(\bar{\alpha}_{-1} \bar{\psi}_{-1/2}^\nu - 2(\bar{b}_{-1} \bar{\gamma}_{-1/2} - \bar{c}_{-1/2} \bar{\beta}_{-1/2}))
- (c_{-1} \psi_{-1/2}^\nu + \gamma_{-1/2} \alpha_{-1}^\mu) \bar{\beta}_{-3/2}] |gh; -1, -1 \rangle.
\]
5.3. Examples at arbitrary $n$

Finally we give an example to show that solutions to (5.1) exist for arbitrary $n$. Take the bosonic string, at zero momentum and $L_0 = n(n+1)/2$. The state $A(-n) = b_{-1}b_{-2} \cdots b_{-n}|gh\rangle$ satisfies

$$(Q')^{2n}A(-n) = C_1 (M)^n A(-n) = C_2 c_{-1}c_{-2} \cdots c_{-n}|gh\rangle,$$

where $C_1$ and $C_2$ are non-zero constants. Now $Q'$ acting on this should have ghost number $n + 1$. Such a state has at least $L_0 = (n+1)(n+2)/2$, which implies $(Q')^{2n+1}A(-n) = 0$. Using the form of $(Q')^{2n}$ (5.1) is easily seen to be satisfied. Similar arguments show that in the NS sector an example is given by $(\beta_{-1/2})^n|gh;-1\rangle$.

6. Boundary states and cohomology at higher mass levels

In this section, we compare $Q'_T$ cohomology with states that can be obtained by projecting known boundary states, to zero momentum and higher mass levels.

A large class of off-shell physical states can be written in terms of projections of boundary states associated with simple D-brane configurations. For example consider for $I \neq J \neq K \neq L$, the state

$$(\alpha^I_{-1} \bar{\alpha}_{-1K} - \alpha^K_{-1} \bar{\alpha}_{-1I})(\alpha^J_{-1} \bar{\alpha}_{-1L} - \alpha^L_{-1} \bar{\alpha}_{-1J})|gh\rangle$$

which can be obtained by applying the construction of the previous section at $n = 0$. It can also be expressed in terms of $\frac{\partial}{\partial F^K_I} \frac{\partial}{\partial F^L_J}|B(F^K_I, F^L_J)\rangle$, where $|B(F^K_I, F^L_J)\rangle$ is the boundary state of a D-brane with constant electromagnetic fields $F^K_I$ and $F^L_J$.

Another example is given by

$$[(\alpha^I_{-2} \bar{\alpha}_{-2J} - \alpha^J_{-2} \bar{\alpha}_{-2I}) + 2(\alpha^I_{-1} \bar{\alpha}_{-1J} - \alpha^J_{-1} \bar{\alpha}_{-1I})(\bar{c}_{-1}b_{-1} + \bar{c}_{-1}b_{-1})]|gh\rangle.$$  

States of this form can be obtained by construction of the previous section and can also be expressed in terms of projections of $\frac{\partial}{\partial F^J_I}|B(F^J_I)\rangle$ with the derivative evaluated at zero field strength.

There are also states, however, in $Q'_T$ cohomology which cannot be obtained from known boundary states in this way. An example is provided by (5.11). Analogously, in the NS-NS sector of the superstring there is the example (5.13). The non-trivial ghost
structure of these states cannot be obtained from projections of standard boundary states, which satisfy the usual linear ghost boundary conditions:

\[ c_{-n} = -\bar{c}_n, \]
\[ b_{-n} = \bar{b}_n. \] (6.3)

The existence of these exotic off-shell physical states has important implications for the physical interpretation of \( Q'_{T} \) cohomology. It remains to be seen whether they correspond to a new class of boundary states. Another possibility is that these states have to be projected out of the off-shell cohomology in some way.

7. Summary

We have extended to the BRST context a definition of off-shell physical states, and described some of its basic properties and related vanishing theorems. We have discussed in some detail open and closed bosonic strings, and type II strings in the NS-NS sector. Off-shell closed string physical states exhibit a mixing of left and right moving degrees of freedom that is not possible in the case of on-shell cohomology.

At the massless level D-branes generate the entire zero-momentum cohomology of standard ghost number. This includes states which are continuations of the standard states of closed string perturbation theory around a flat background, as well as discrete states. At zero momentum, states fall naturally in multiplets of \( SO(9,1) \), whereas at non-zero momentum they fall in multiplets of \( SO(8) \) in the massless case and \( SO(9) \) in the massive case. This association of perturbative and discrete states into multiplets of a larger Lorentz group is reminiscent of the considerations of \[30\] and it would be interesting to see if the \( SO(9,1) \) structure can be simply interpreted in the framework of the higher dimensional formulations of string theory \[31,32,33,34,35\].

We have shown that a large class of non-trivial cohomology classes in this off-shell cohomology come from boundary states associated with D-branes. At higher mass levels we also found cohomology classes which do not come from standard boundary states. If \( Q + \bar{Q} \) closure and \( Q'_{T} \) cohomology, are the only criteria selecting boundary states, this means that more general boundary states exist with a non-trivial mixing between matter and ghosts. An alternative possibility is that further conditions have to be imposed on boundary states. If the first possibility turns out to be correct it would be interesting to look for a group theoretic structure relating the new boundary states to previously known ones, generalizing the way \( SO(9,1) \) relates known boundary states. If, on the other hand, the second possibility proves true, it remains to find the appropriate algebraic characterization of acceptable boundary states beyond \( Q_T \) closure and \( Q'_{T} \) cohomology.


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**Appendix A. Definition of off-shell cohomology for superstrings**

In the NS sector there are no superghost zero modes so the discussion is very similar to the bosonic case. We will first discuss the open superstring. We separate out the ghost zero modes:

\[
Q = c_0 (L_0 - 1/2) + b_0 M + Q' \tag{A.1}
\]

Using \(Q^2 = 0\), we find \((Q')^2 = -M(L_0 - 1/2)\). For off-shell states we can define \(Q'\) cohomology on the subspace where \(M = 0\). It contains analytic continuations of the DDF states.

Vanishing theorems for the NS sector can be proved by following the steps of section 2.6. The relevant \(sl(2)\) algebra (see for example [36] and references there) is given by:

\[
\begin{align*}
M &= -2 \sum_{n>0} n c_{-n} c_n + \gamma_{-n} \gamma_n, \\
N &= \sum_{n>0} -\frac{1}{2n} b_{-n} b_n + \frac{1}{2} \beta_{-n} \beta_n, \\
N_{gh} &= \sum_{n \neq 0} : c_{-n} b_n : - : \beta_{-n} \gamma_n : .
\end{align*}
\tag{A.2}
\]

As in section 2.6, we can show that any state of ghost number zero annihilated by \(Q'\) is in cohomology. The extension of these statements to the NS-NS sector of closed superstrings proceeds along the same lines.

In the R sector of the open superstring, a simple definition of off-shell cohomology is again possible. As before we make the ghost and superghost zero mode dependence explicit:

\[
Q = c_0 L_0 + b_0 M - \gamma_0^2 b_0 + \beta_0 G_0 + \gamma_0 K + Q' \tag{A.3}
\]

The nilpotence of \(Q\) then implies that \(Q'^2 = -M L_0 + G_0 K\) (see [28] and references therein). Away from the mass-shell condition we can consider \(Q'\) cohomology on the subspace where it squares to zero. This \(Q'\) operator commutes with \(SO(9,1)\) so multiplets of this group can be expected at zero spacetime momentum.

The BRST operator for the off-shell closed string case is defined by the sum of appropriate \(Q'\) and \(\bar{Q}'\) operators, and the subspace where the sum is zero is simply characterized in terms of \(M, \bar{M}, K\) and \(\bar{K}\). Vanishing theorems for off-shell cohomology in the R sector of open strings, and the R-R and R-NS sectors of closed strings remain to be investigated.
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