A de Broglie-Bohm Like Model for Dirac Equation
submitted to Nuovo Cimento

O. Chavoya-Aceves
Camelback H. S., Phoenix, Arizona, USA.
chavoyao@yahoo.com

September 11, 2018

Abstract

A de Broglie-Bohm like model of Dirac equation, that leads to the correct Pauli equations for electrons and positrons in the low-speed limit, is presented. Under this theoretical framework, that affords an interpretation of the quantum potential, the main assumption of the de Broglie-Bohm theory—that the local momentum of particles is given by the gradient of the phase of the wave function—wont be accurate. Also, the number of particles wont be locally conserved. Furthermore, the representation of physical systems through wave functions wont be complete.

PACS 03.53.-w Quantum Mechanics

1 Introduction

We'll show that Dirac equation can be interpreted as describing the motion of four currents of particles, under the only action of the electromagnetic field, with four-velocities given by the equations

\[ m v_{i,\mu} = -\partial_\mu \Phi_i - \partial_\mu S_i \pm \frac{q}{e} A_\mu, \]

where \( \pm S_i \) are the phases of the components of the wave function, in such way that there are not negative energies, and the functions \( \Phi_i \) are hidden variables. This leads to the correct Pauli equations for electrons and positrons in the low-speed limit, whenever the zitterbewegung terms can be neglected.

Within the theoretical framework of this work, the main assumption of the de Broglie-Bohm theory of quantum mechanics [1] is not valid and a picture of particles moving under the action of electromagnetic field alone, without any quantum potential, emerges. However, the number of particles is not locally conserved. Which is very well known, as a matter of fact. Also, given that hidden variables have been introduced—on the grounds of some general electrodynamic
considerations, given in the first section—the representation of physical systems through wave functions won’t be complete and, therefore, as foreseen by Einstein, Podolsky, and Rosen, quantum mechanics won’t be a complete theory of motion.

2 On the Motion of Particles Under the Action of Electromagnetic Field

Consider a non-stochastic ensemble of particles whose motion is described by means of a function

\[ \vec{r} = \vec{r}(\vec{x}, t), \]

such that \( \vec{r}(\vec{x}, t) \) represents the position, at time \( t \), of the particle that passes through the point \( \vec{x} \) at time zero—the \( \vec{x} \)-particle. This representation of motion coincides with the lagrangian representation used in hydrodynamics\textsuperscript{[2]}.

As to the function \( \vec{r}(\vec{x}, t) \), we suppose that it is invertible for any value of \( t \). In other words, that there is a function \( \vec{x} = \vec{x}(\vec{r}, t) \), that gives us the coordinates, at time zero, of the particle that passes through the point \( \vec{r} \) at time \( t \). Also, we suppose that \( \vec{r}(\vec{x}, t) \) is a continuous function, altogether with its derivatives of as higher order as needed to secure the validity of our conclusions.

According to the definition of \( \vec{r}(\vec{x}, t) \), the velocity of the \( \vec{x} \)-particle at time \( t \) is

\[ \vec{u}(\vec{x}, t) = \left( \frac{\partial \vec{r}}{\partial t} \right)_{\vec{x}}. \]

Writing \( \vec{x} \) as a function of \( \vec{r} \) and \( t \), we get the velocity field at time \( t \):

\[ \vec{u}(\vec{r}, t) = \vec{u}(\vec{x}(\vec{r}, t), t). \]

Let’s consider the corresponding four-velocity:

\[ v^\mu = \left( \begin{array}{c} c \\ \sqrt{1 - v^2/c^2} \end{array} \right) \]

Using four-dimensional tensorial notation, the derivative of \( v_\mu \) with respect to the proper time, along the world-line of the corresponding particle is:

\[ \frac{dv_\mu}{ds} = v^\nu \partial_\nu v_\mu \]

From the identity

\[ v^\nu v_\nu = c^2 \]

we prove that

\[ v^\nu \partial_\mu v_\nu = 0, \]

that allows us to write

\[ \frac{dv_\mu}{ds} = (\partial_\nu v_\mu - \partial_\mu v_\nu)v_\nu. \]
If the four-force acting on the ensemble of particles has an electromagnetic origin, it is given by the expression

\[ f_\mu = \frac{q}{c} F_{\mu\nu} v^\nu, \quad (3) \]

where

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (4) \]

is Faraday’s tensor, and \( A_\mu \) is the electrodynamic four-potential.

From the law of motion

\[ m \frac{dv_\mu}{ds} = f_\mu, \quad (2) \]

and equations (2) to (4) we prove that

\[ (\partial_\mu p_\nu - \partial_\nu p_\mu) v^\nu = 0, \quad (5) \]

where

\[ p_\mu = mv_\mu + \frac{q}{c} A_\mu. \quad (6) \]

There is a class of solutions of equations (5) where \( p_\mu \) is the four-gradient of a function of space-time coordinates

\[ p_\mu = -\partial_\mu \phi. \quad (7) \]

Therefore,

\[ \partial_\mu p_\nu - \partial_\nu p_\mu = 0, \]

and (5) is obviously satisfied.

Equation (6) can be written in the form

\[ mv_\mu = -\partial_\mu \phi - \frac{q}{c} A_\mu \quad (8) \]

If the four-potential meets the Lorentz condition,

\[ \partial^\mu A_\mu = 0, \quad (9) \]

equation (8) is analogous to the decomposition of a classical, three-dimensional field, into an irrotational and a solenoidal part. The electromagnetic field appears thus as determining the four-dimensional vorticity of the field of kinetic momentum \( mv_\mu \).

Equation (1) tells us that functions \( \phi \) are not arbitrary, but are subject to the condition:

\[ (\partial^\mu \phi + \frac{q}{c} A^\mu)(\partial_\mu \phi + \frac{q}{c} A_\mu) = m^2 c^2, \quad (10) \]

which is the relativistic Hamilton-Jacobi equation.\[ \text{[3, Ch. VIII]} \]

The components of the kinetic momentum are:

\[ \left( \frac{K}{c}, \vec{p} \right) = \left( -\frac{1}{c} \frac{\partial \phi}{\partial t} - \frac{q}{c} V, \nabla \phi - \frac{q}{c} \vec{A} \right), \quad (11) \]
where \( V \) and \( \vec{A} \) are the components of the electrodynamic four-potential.

In the low-speed limit, we have

\[
K \approx mc^2 + \frac{p^2}{2m}.
\]

From this and (11) we get:

\[
\frac{\partial \phi}{\partial t} + \frac{(\nabla \phi - \frac{q}{c} \vec{A})^2}{2m} + qV + mc^2 = 0, \tag{12}
\]

that, but for the constant term \( mc^2 \), is the non-relativistic Hamilton-Jacobi equation.

3 A de Broglie-Bohm Like Interpretation of Dirac Equation

Let us consider Dirac equation for particles of mass \( m \) and charge \( q \), under the action of an electromagnetic field that meets the Lorentz condition (9):

\[
\gamma^\mu (i\hbar \partial_\mu - \frac{q}{c}A_\mu) \psi = mc\psi \tag{13}
\]

or

\[
\bar{\psi} \gamma^\mu (-i\hbar \partial_\mu - \frac{q}{c}A_\mu) = mc\bar{\psi} \tag{14}
\]

where \( \bar{\psi} = \psi \gamma^0 \).

It is easy to show that:

\[
\partial_\mu \iota^\mu = 0 \tag{15}
\]

where

\[
\iota^\mu = c\bar{\psi} \gamma^\mu \psi. \tag{16}
\]

From (13) & (14)

\[
\bar{\psi} \gamma^\mu \gamma^\nu (i\hbar \partial_\nu - \frac{q}{c}A_\nu) \psi = m\bar{\psi} \gamma^\mu \psi, \]

\[
\bar{\psi} \gamma^\nu \gamma^\mu (-i\hbar \partial_\nu - \frac{q}{c}A_\nu) \psi = m\bar{\psi} \gamma^\mu \psi.
\]

Therefore:

\[
\iota^\mu = \frac{i\hbar (\bar{\psi} \gamma^\nu \gamma^\mu \partial_\nu \psi - \partial_\nu (\bar{\psi} \gamma^\nu \gamma^\mu \psi)) - \frac{q}{mc} \bar{\psi} \psi \gamma^\mu A_\mu}{2m} \tag{17}
\]

Considering that:

\[
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \mathbf{I}, \tag{18}
\]

we can prove now that

\[
\iota^\mu = \frac{i\hbar (\bar{\psi} \partial^\mu \psi - \partial^\mu (\bar{\psi} \psi)) + i\hbar \partial_\nu (\bar{\psi} [\gamma^\mu, \gamma^\nu] \psi) - \frac{q}{mc} \bar{\psi} \psi \gamma^\mu A_\mu}{2m} \tag{19}
\]
The divergence of the second term of the right side is zero. Therefore we guess that the current of particles could be given by:

\[ j_0^\mu = \frac{i\hbar(\bar{\psi}\partial^\mu \psi - \partial^\mu \bar{\psi}\psi)}{2m} - \frac{q}{mc} \bar{\psi}\psi A^\mu \]  

(20)

which is made of four terms:

\[ mj_{0,1}^\mu = \rho_1(-\partial^\mu S_1 - \frac{q}{c} A^\mu) \]  

(21)

\[ mj_{0,2}^\mu = \rho_2(-\partial^\mu S_2 - \frac{q}{c} A^\mu) \]

\[ mj_{0,3}^\mu = \rho_3(-\partial^\mu S_3 + \frac{q}{c} A^\mu) \]

\[ mj_{0,4}^\mu = \rho_4(-\partial^\mu S_4 + \frac{q}{c} A^\mu) \]

where

\[ \psi_1 = \sqrt{\rho_1} e^{i \pi S_1} \]

\[ \psi_2 = \sqrt{\rho_2} e^{i \pi S_2} \]

\[ \psi_3 = \sqrt{\rho_3} e^{-i \pi S_3} \]

\[ \psi_4 = \sqrt{\rho_4} e^{-i \pi S_4} \]

are the components of the wave function.

The current (20) is thus seen as the sum of four fluxes. According to (6), the first two can be interpreted as fluxes of particles of charge \( q \), while the last would represent fluxes of particles of charge \(-q\). Equations (21) suggest that we could define the corresponding four-velocities from:

\[ mw_{1}^\mu = -\partial^\mu S_1 - \frac{q}{c} A^\mu \]  

(23)

\[ mw_{2}^\mu = -\partial^\mu S_2 - \frac{q}{c} A^\mu \]

\[ mw_{3}^\mu = -\partial^\mu S_3 + \frac{q}{c} A^\mu \]

\[ mw_{4}^\mu = -\partial^\mu S_4 + \frac{q}{c} A^\mu \]

but this is not sound, because in such case we should have:

\[ w_{i,\mu}^\mu w_{i,\mu} = c^2, \]

which is not true.

However, given that those four-vectors have already the required four-vorticity—for particles under the action of an electromagnetic field—, we suppose that there exist functions \( \Phi_1, \Phi_2, \Phi_3, \) and \( \Phi_4 \), such that the four-velocities are given by the equations:

\[ mw_{1}^\mu = -\partial^\mu (S_1 + \Phi_1) - \frac{q}{c} A^\mu, \]  

(24)
\[ mv^\mu_2 = -\partial^\mu (S_2 + \Phi_2) - \frac{q}{c} A^\mu, \]
\[ mv^\mu_3 = -\partial^\mu (S_3 + \Phi_3) + \frac{q}{c} A^\mu, \]
\[ mv^\mu_4 = -\partial^\mu (S_4 + \Phi_4) + \frac{q}{c} A^\mu, \]
in such way that
\[ v^\mu_i v^\mu_i = c^2 \] (25)

The divergence of the current of particles:
\[ j^\mu = \sum_{i=1}^{4} \rho_i v^\mu_i \] (26)

—that is not proportional to the electrical current—will then be:
\[ \partial^\mu j_\mu = \sum_{i=1}^{4} \partial^\mu (\rho_i \partial^\mu \Phi_i)/m, \]

which is not zero, and, consequently, the number of particles won’t be locally conserved.

From Dirac equation we can show that
\[ -\hbar^2 \partial^\mu \partial_\mu \psi - \frac{2i\hbar q}{c} A^\mu \partial_\mu \psi + \frac{q^2}{c^2} A^\mu A_\mu \psi = m^2 c^2 \psi + \frac{i\hbar q}{4c} [\gamma^\mu, \gamma^\nu] F^\mu\nu \psi, \] (27)

where \( F^\mu\nu \) is Faraday’s tensor or:
\[ -\hbar^2 \partial^\mu \partial_\mu \varphi - \frac{2i\hbar q}{c} A^\mu \partial_\mu \varphi + \frac{q^2}{c^2} A^\mu A_\mu \varphi = m^2 c^2 \varphi + \frac{\hbar q}{c} (\vec{\sigma} \cdot \vec{H} \varphi + i\vec{\sigma} \cdot \vec{E} \chi), \] (28)

\[ -\hbar^2 \partial^\mu \partial_\mu \chi - \frac{2i\hbar q}{c} A^\mu \partial_\mu \chi + \frac{q^2}{c^2} A^\mu A_\mu \chi = m^2 c^2 \chi + \frac{\hbar q}{c} (\vec{\sigma} \cdot \vec{H} \chi + \vec{\sigma} \cdot \vec{E} \varphi), \] (29)

where we have written the wave function in the form:
\[ \psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} \] (30)

Let’s consider a component of \( \psi \):
\[ \omega = \sqrt{m} e^{\pm \frac{i}{c} S}, \]

The terms in the left side of equation (28) are
\[ -\frac{2i\hbar q}{c} A^\mu \partial_\mu \omega = \left( \pm \frac{2q\sqrt{\rho}}{c} A^\mu \partial_\mu S - \frac{2i\hbar q}{c} A^\mu \partial_\mu \sqrt{\rho} \right) e^{\pm \frac{i}{c} S}, \]
and

\[-h^2 \partial^\mu \partial_\mu \omega = \left[-h^2 \partial^\mu \partial_\mu \sqrt{\rho} + \sqrt{\rho} \partial^\mu \partial_\mu \sqrt{\rho} \right] \mp i h (2 \partial^\mu \sqrt{\rho} \partial_\mu \rho + \sqrt{\rho} \partial^\mu \partial_\mu \rho) + e^{\pm \frac{i \hbar}{\hbar} \rho} \]

From this we can separate the real and imaginary parts of the equation that results from left multiplying (28) by \( \Phi^\dagger \):

\[
\sum_{i=1,2} \rho_i \left(-\partial^\mu S_i - \frac{q}{c} A^\mu \right)(-\partial_\mu S_i - \frac{q}{c} A_\mu) = \frac{q}{c} m^2 c^2
\]

(31)

and

\[
\sum_{i=1,2} h^2 \left(\sqrt{\rho_i} \partial^\mu \partial_\mu \sqrt{\rho_i} \right) + \frac{h q}{c} \varphi^\dagger \sigma \cdot H \varphi + \frac{h q}{c} \text{Re}(i \varphi^\dagger \sigma \cdot \vec{E} \chi)
\]

In a similar fashion:

\[
\sum_{i=3,4} \rho_i \left(-\partial^\mu S_i + \frac{q}{c} A^\mu \right)(-\partial_\mu S_i + \frac{q}{c} A_\mu) = \frac{q}{c} m^2 c^2
\]

(33)

and

\[
\sum_{i=1,2} h^2 \left(\sqrt{\rho_i} \partial^\mu \partial_\mu \sqrt{\rho_i} \right) + \frac{h q}{c} \chi^\dagger \sigma \cdot H \chi + \frac{h q}{c} \text{Re}(i \chi^\dagger \sigma \cdot \vec{E} \phi)
\]

and

\[
\sum_{i=3,4} \partial^\mu (\rho_i (-\partial_\mu S_i + \frac{q}{c} A_\mu)) = -\frac{q}{c} \text{Im}(i \varphi^\dagger \sigma \cdot \vec{E} \chi)
\]

(34)

Adding equations (32 & 34), considering that \( \sigma \cdot \vec{E} \) is an hermitian operator, we obtain the equation:

\[
\partial^\mu j_{0,\mu} = 0,
\]

which was already known.

From equations (24):

\[
(-\partial^\mu S_i \mp \frac{q}{c} A^\mu)(-\partial_\mu S_i \pm \frac{q}{c} A_\mu) = (mv_i^\mu + \partial^\mu \Phi_i)(mv_i,\mu + \partial_\mu \Phi_i) = m^2 c^2 + 2mv_i^\mu \partial_\mu \Phi_i + \partial^\mu \Phi_i \partial_\mu \Phi_i
\]

(35)

This allows us rewrite equations (31 & 33) in the form:

\[
\sum_{i=1,2} \rho_i \left(v^\mu_i \partial_\mu \Phi_i + \frac{\partial^\mu \Phi_i \partial_\mu \Phi_i}{2m} \right) = \sum_{i=1,2} \frac{h^2}{2m} \sqrt{\rho_i} \partial^\mu \partial_\mu \sqrt{\rho_i} + \frac{h q}{2mc} \varphi^\dagger \sigma \cdot \varphi + \frac{h q}{2mc} \text{Re}(i \varphi^\dagger \sigma \cdot \vec{E} \chi),
\]

(36)

\[
\sum_{i=3,4} \rho_i \left(v^\mu_i \partial_\mu \Phi_i + \frac{\partial^\mu \Phi_i \partial_\mu \Phi_i}{2m} \right) = \sum_{i=3,4} \frac{h^2}{2m} \sqrt{\rho_i} \partial^\mu \partial_\mu \sqrt{\rho_i} + \frac{h q}{2mc} \chi^\dagger \sigma \cdot \chi + \frac{h q}{2mc} \text{Re}(i \chi^\dagger \sigma \cdot \vec{E} \phi),
\]

(37)
\[
\frac{\hbar}{2mc} \chi^\dagger \cdot \chi + \frac{\hbar}{2mc} Re(i\chi^\dagger \cdot \vec{E} \phi),
\]

In the low-speed limit:

\[
v^0 \approx c \ (v^1, v^2, v^3) \rightarrow \vec{v}
\]

where \( \vec{v} \) is the classical velocity. Therefore:

\[
v^\mu \partial_\mu \Phi + \frac{\partial^\mu \Phi \partial_\mu \Phi}{2m} \approx 0
\]

\[
\frac{\partial \Phi}{\partial t} + \vec{v} \cdot \nabla \Phi - \frac{(\nabla \Phi)^2}{2m} = 0
\]

\[
\frac{\partial \Phi}{\partial t} + (\vec{v} - \frac{\nabla \Phi}{m}) \cdot \nabla \Phi + \frac{(\nabla \Phi)^2}{2m} = 0
\]

In this limit also:

\[
\vec{v} = \nabla S + \nabla \Phi \pm \frac{q}{c} \vec{A}
\]

—where \( \vec{A} \) is the vector potential of the electromagnetic field—from equations \( 36, 37 \) & \( 39 \), we get an approximation for the kinetic energy:

\[
K_\phi = \sum_{i=1,2} \rho_i (-\frac{\partial \Phi_i}{\partial t} + mc^2 + \frac{(\nabla S_i - \frac{q}{c} \vec{A})^2}{2m} - \frac{\hbar^2 \Delta \sqrt{\rho_i}}{2m \sqrt{\rho_i}} + \frac{\hbar}{2mc} \frac{\phi^\dagger \sigma \cdot \vec{H} \phi}{\phi^\dagger \phi} + U_\phi)
\]

\[
K_\chi = \sum_{i=3,4} \rho_i (-\frac{\partial \Phi_i}{\partial t} + mc^2 + \frac{(\nabla S_i + \frac{q}{c} \vec{A})^2}{2m} - \frac{\hbar^2 \Delta \sqrt{\rho_i}}{2m \sqrt{\rho_i}} + \frac{\hbar}{2mc} \frac{\chi^\dagger \sigma \cdot \vec{H} \chi}{\chi^\dagger \chi} + U_\chi),
\]

where

\[
U_\phi = \frac{\hbar}{2mc} Re \left( \frac{\phi^\dagger \sigma \vec{E} \chi}{\phi^\dagger \phi} \right)
\]

\[
U_\chi = \frac{\hbar}{2mc} Re \left( \frac{\phi^\dagger \sigma \vec{E} \chi}{\chi^\dagger \chi} \right)
\]

From the definition of the four-velocity:

\[
K_\phi = \sum_{i=1,2} \rho_i (-\frac{\partial \Phi_i}{\partial t} - \frac{\partial S_i}{\partial t} - qV)
\]

\[
K_\chi = \sum_{i=3,4} \rho_i (-\frac{\partial \Phi_i}{\partial t} - \frac{\partial S_i}{\partial t} + qV)
\]

where \( V \) is the scalar potential of the electromagnetic field.

From these equations and \( 40 \& 41 \), we find that the components of \( \phi \) and \( \chi \) can be obtained in the low-speed limit as solutions of the system:
\[
\sum_{i=1,2} \rho_i \left( \frac{\partial S_i}{\partial t} + mc^2 + \left( \nabla S_i - \frac{\mathbf{A}}{c} \right)^2 - \frac{\hbar^2}{2m} \frac{\Delta \sqrt{\rho_i}}{\sqrt{\rho_i}} \right) + qV + \frac{\hbar q}{2mc} \frac{\varphi^\dagger \mathbf{\sigma} \cdot \mathbf{H} \varphi}{\varphi^\dagger \varphi} + U_\varphi = 0
\]

and
\[
\sum_{i=3,4} \rho_i \left( \frac{\partial S_i}{\partial t} + mc^2 + \left( \nabla S_i + \frac{\mathbf{A}}{c} \right)^2 - \frac{\hbar^2}{2m} \frac{\Delta \sqrt{\rho_i}}{\sqrt{\rho_i}} \right) - qV + \frac{\hbar q}{2mc} \frac{\chi^\dagger \mathbf{\sigma} \cdot \mathbf{H} \chi}{\chi^\dagger \chi} + U_\chi = 0,
\]

including (32 & 34).

If we neglect the zitterbewegung term:
\[
\frac{q}{c} \chi^\dagger \mathbf{\sigma} \cdot \mathbf{E} \varphi \approx 0,
\]
in equations (32 & 34), considering equations (22), we can see that the components of \( \varphi \) can be chosen as solutions of a Schrödinger non-linear equation (Chapter 3.):
\[
i \hbar \frac{\partial \psi}{\partial t} = \left( -i \hbar \nabla - \frac{\mathbf{A}}{c} \right)^2 \psi + qV \psi + \frac{\hbar q}{2mc} \frac{\varphi^\dagger \mathbf{\sigma} \cdot \mathbf{H} \varphi}{\varphi^\dagger \varphi} \psi + U_\varphi \psi,
\]
whilst the components of \( \chi \) can be chosen as solutions of:
\[
-i \hbar \frac{\partial \psi}{\partial t} = \left( i \hbar \nabla + \frac{\mathbf{A}}{c} \right)^2 \psi - qV \psi + \frac{\hbar q}{2mc} \frac{\chi^\dagger \mathbf{\sigma} \cdot \mathbf{H} \chi}{\chi^\dagger \chi} \psi + U_\chi \psi
\]
(50)

Therefore, at least in the limit where the spinorial part of the Hamiltonian can be considered small, the components of \( \varphi \) and \( \chi \) can be chosen as solutions of the non-linear Pauli equations:
\[
i \hbar \frac{\partial \varphi}{\partial t} = \left( -i \hbar \nabla - \frac{\mathbf{A}}{c} \right)^2 \varphi + qV \varphi + \frac{\hbar q}{2mc} \varphi^\dagger \mathbf{\sigma} \cdot \mathbf{H} \varphi + U_\varphi \varphi,
\]
(51)
and
\[
-i \hbar \frac{\partial \chi}{\partial t} = \left( i \hbar \nabla + \frac{\mathbf{A}}{c} \right)^2 \chi - qV \chi + \frac{\hbar q}{2mc} \chi^\dagger \mathbf{\sigma} \cdot \mathbf{H} \chi + U_\chi \chi,
\]
(52)
that become linear when \( U_\varphi \) and \( U_\chi \) are neglected.

Actually, through the Madelung substitutions (22), it can be shown that any solution of the system (51-52) is a solution of (47-48-32-34), whenever the zitterbewegung term can be neglected.

In the general, relativistic, case, we recover a picture of particles moving under the action of the electromagnetic field alone, although the number of particles is not conserved. Which is well known to happen, as a matter of fact.

As to the nature of the functions \( \Phi_i \), we see that they are restricted by the condition:
\[
\sum_{i=1,2} \partial^\mu (\rho_i \partial_\mu \Phi_i) - \sum_{i=3,4} \partial^\mu (\rho_i \partial_\mu \Phi_i) = 0,
\]
required by the principle of conservation of electrical charge. Also, they are solutions of equations (36 & 37).
4 Conclusions and Remarks

If the ideas exposed in this paper are proven to be valid:

1. The main assumption of the de Broglie-Bohm theory—that the local impulse of quantum particles is given by the gradient of the phase of the wave function—won’t be accurate.

2. However, there will be still room for a classical interpretation of quantum phenomena, in terms of particles moving along well defined trajectories, under the action of the electromagnetic field.

3. The number of particles won’t be locally conserved.

4. Given that four hidden variables have been introduced—after some considerations on electrodynamics—the representation of physical systems through wave functions won’t be complete and, therefore, as foreseen by Einstein, Podolsky, and Rosen, quantum mechanics won’t be a complete theory of motion.

References

[1] D. Bohm. A Suggested Interpretation of Quantum Theory in Terms of “Hidden” Variables I and II; Physical Review, 85, 155-193(1952).

[2] R. Aris; Vectors, Tensors, and the Basic Equations of Fluid Mechanics; Dover (1989) p. 76.

[3] C. Lanczos; The Variational Principles of Mechanics; Dover 4th Ed. (1986).

[4] P. R. Holland; The Quantum Theory of Motion; Cambridge University Press (1993).