Peculiarities of Steam Treatment of Food Plant Products and Simulation Thereof

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Abstract. Methods of water steam treatment of disperse food raw materials at various pressures have become widely used in the processing industry. Humidity and temperature conditioning, drying and blanching are typical non-stationary processes progressing at a uniform or non-uniform (decreasing) rate. This article describes an analytical solution to the system of differential equations of joint heat and mass transfer outlining distribution of heat and moisture fields at the material heating stage. The following peculiarities of the food raw materials processing have been considered to set the appropriate boundary value problem: some of the processed food quality horticultural products (harvested potatoes to be dried, washed fruits or pretreated sunflower seeds prior to vegetable oil expression) were included with the group of products characterized by a tight-fitting subspherical fruit (seed) coat, which is why we consider them a single, homogenous, and isotropic particle with averaged thermophysical properties. The difference in temperatures of the heated air and the treated material results in condensation and formation of a liquid film on the surface of the objects, which is why heat is spontaneously transferred from the condensate film to the body primarily on account of heat conductivity and moisture distribution from the periphery to the center. The issues of storing agricultural plant products closely related to the discussed ones are no less important. Rational natural resource management globally acknowledged as the dominant economic development trend presumes the fullest possible use of processed human-consumed plant products. For instance, vegetable proteins prevail over meat ones in food patterns in most countries. In Japan, this ratio is 78.3/21.7, in Ukraine - 72.3/27.7, in the USA - 65.3/32.9, in the United Kingdom - 61.7/32.4, in Germany - 65.3/34.7, in France - 60/40, in Canada - 68.4/31.6, in China - 87.3/12.7, in Italy - 74.6/25.4. Adequate storage of the harvested plant products prepared for processing significantly contributes to the solution of this problem. Given the considerable amount of root vegetables, grain and seeds in food plant products, it is clear why foodstuff manufacturers show interest to conditions and modes of storage thereof. Modern warehouses used for these purposes are equipped with various means to ensure required hygrothermal modes and even to control composition of the environment within the units intended to store preserved bulk root vegetables, grain and seeds. That is why the discussed problems of analytical evaluation of storage conditions for various steam-treated bulk agricultural raw materials are of genuine interest.

1. Introduction
If food plant products are steam-treated as a bulk of separate objects, the problem is reduced to solving the system of differential equations of associated heat and mass transfer. Therefore, the physical model of solid body / steam interaction proposed by I.A. Mikhailov may be used to provide an analytical solution to the system of differential equations of joint heat and mass transfer at the stage of heating of...
separate subspherical particles. The setting of problems of heat and mass transfer in stored agricultural plant products depends not only on storage conditions, but also on the physiological activity of bulk components. The methods of storing such products treated to achieve required humidity and temperature are airtight warehouses [1, 2]. Although the method of airtight storage of humid products has been used since Ancient times, it has become significantly more popular in recent years due to wide use of new harvesting machinery. Regardless of the ambient humidity, the airtight storage method is based on the principle of reducing oxygen concentration in the warehouse to such a level that insects and yeasts die or become inactivated. Airtightly stored raw materials preserve nutritional, feeding, and technological properties.

2. Problem setting and discussion of obtained solutions

Such a solution will help to calculate the change of temperature and moisture content fields in a separate object, heating rate and power consumption.

\[
\frac{\partial [r_t(r, \tau)]}{\partial \tau} = a_q \frac{\partial^2 [r_t(r, \tau)]}{\partial r^2} \quad (R_1 < r < R_2) \quad (1)
\]

\[
\frac{\partial [r_U(r, \tau)]}{\partial \tau} = a_m \frac{\partial^2 [r_U(r, \tau)]}{\partial r^2} + \varepsilon \rho \frac{\partial [r_U(r, \tau)]}{\partial \tau} \quad (0 < r < R_1) \quad (2)
\]

at the following boundary conditions:

\[
t_0(r, 0) = t_{0c} = \text{const} ;
\]

\[
t(r, 0) = t_0 = \text{const} ;
\]

\[
U(r, 0) = U_0 = \text{const} ;
\]

\[
t_0(R_2, \tau) = t_{0l} = \text{const} ;
\]

\[
U(R_2, \tau) = U_{n} = \text{const} ;
\]

\[
\lambda_q \frac{\partial t(R_1, \tau)}{\partial r} = \lambda_q \frac{\partial t(R_1, \tau)}{\partial r} \quad (8)
\]

\[
t_0(R_1, \tau) = u (R_1, \tau)
\]

\[
\frac{\partial U(0, \tau)}{\partial \tau} = \frac{\partial U(0, \tau)}{\partial \tau} \quad (9)
\]

\[
t_0(\tau) = U(0, \tau) \quad (10)
\]

where

\[
t_0(r, \tau), t(r, \tau) \text{- temperatures of the coat (liquid film) and the core, respectively;}
\]

\[
t_{0c}, t_0 \text{- baseline temperatures;}
\]

\[
U(r, \tau) \text{- core moisture content;}
\]

\[
U_0 \text{- baseline moisture content;}
\]

\[
R_1 \text{- core radius;}
\]

\[
R_2 \text{- coat radius;}
\]

\[
a_q, a_m \text{- thermal diffusivity factors of the coat and the core, respectively;}
\]

\[
t_{0l} \text{- temperature of condensing steam;}
\]

\[
U_n \text{- moisture content of saturated steam;}
\]

\[
\lambda_q, \hat{\lambda}_q \text{- thermal conductivity factors of steam and the body (core), respectively.}
\]
Equations (3)-(5) demonstrate establishment of uniform baseline temperature and moisture distributions; 
equations (6)-(7) - first-order boundary conditions defining temperatures on the coat surface and 
moisture content in the boundary layer between the coat and the core; 
equations (8)-(9) - fourth-order boundary conditions setting the balance of temperatures and heat 
flows at the coat/core boundary; 
(10) - symmetry conditions; 
iequation (11) - conditions of physical limitations of temperature and moisture content.

The set (1)-(11) boundary value problem was solved by means of the Laplace integral transform. 
The obtained distributions of temperature and moisture fields are as follows:

\[
T(X, Fo) = \sum_{n=1}^{\infty} \sum_{i=1}^{2} \frac{2}{\varepsilon Ko X} A_n \left(1 - v^2_i\right) \sin(v_i \mu_n X) \exp\left(-\mu_n^2 Fo\right);
\]

\[
\Theta(X, Fo) = 1 + \frac{T_c}{\varepsilon Ko X} \sum_{n=1}^{\infty} \sum_{i=1}^{2} \frac{2}{\varepsilon Ko X} A_n \left(1 - v^2_i\right) \sin(v_i \mu_n X) \exp\left(-\mu_n^2 Fo\right);
\]

\[
T(X, Fo) = \frac{t(r \tau) - t_0}{t_0 - t_{0c}}; \quad -\text{dimensionless temperature};
\]

\[
T_c = \frac{t_c(r \tau) - t_{0c}}{t_0 - t_{0c}};
\]

\[
T_{0c} = \frac{t_{0c}}{t_0}; \quad T_{0hl} = \frac{t_{hl}}{t_0} - t_{0c};
\]

\[
\Theta(X, Fo) = \frac{U(r \tau) - U_0}{U_{hl} - U_0}; \quad -\text{dimensionless moisture content};
\]

\[
X = \frac{r}{R^2}; \quad -\text{dimensionless coordinate};
\]

\[
Fo = \frac{a_n a\tau}{R^2}; \quad -\text{Fourier number};
\]

\[
Lu = \frac{a_n}{a_q}; \quad -\text{Lykov number};
\]

\[
Ko = \frac{\rho(U_{hl} - U_0)}{c_q t_0}; \quad -\text{Kossovich number};
\]

\[
Pn = \frac{\delta t_0}{U_{hl} - U_0}; \quad -\text{Posnov number};
\]

\[
\mu_n - \text{sequential positive characteristic roots}
\]

\[
\frac{\mu v^2_1(1 - v^2_2)}{v^2_2 - v^2_1} (N \text{ctg}(v_1 \mu) - \text{ctg}(v_2 \mu)) - k_2 \left(\frac{\mu}{\sqrt{K_n}} \text{ctg} + 1\right) + 1 = 0;
\]

\[
A_n = -\frac{1}{(v^2_2 - v^2_1)\mu_n \phi_n} \{k_2 \sin(v_2 \mu_n) \left[\varepsilon Ko - T_{0c} \left(1 - v^2_2\right)\right]\}.
\]
\[
\mathbf{A}_2 = \left( \sin \gamma_n + \frac{\mu_n}{\sqrt{K\alpha}} \sin \gamma_n \right) - \frac{k_R}{\sqrt{K\alpha}} T_{0,II} \left( 1 - \nu_2^2 \right) \mu_n - 
\]
\[
- \varepsilon K_\alpha \sin \gamma_n (\sin (v_2 \mu_n) - v_2 \mu_n \cos (v_2 \mu_n)) \; ;
\]
\[
A_2 K = - \left( \frac{1}{\nu_2^2 - \nu_1^2} \right) \left( k_\alpha \sin (v_1 \mu_n) \left( \varepsilon K_\alpha - T_{0e} \left( 1 - \nu_1^2 \right) \right) \times \right.
\]
\[
\left. \times \left( \sin \gamma_n + \frac{\mu_n}{\sqrt{K\alpha}} \cos \gamma_n \right) - \frac{k_R}{\sqrt{K\alpha}} T_{0,II} \left( 1 - \nu_1^2 \right) \mu_n - \right)
\]
\[
- \varepsilon K_\alpha \sin \gamma_n (\sin (v_1 \mu_n) - v_1 \mu_n \cos (v_1 \mu_n)) \; ;
\]
\[
\varphi_n = \cos \gamma_n \left[ \frac{k_\alpha}{\sqrt{K\alpha}} \gamma_n \sin (v_1 \mu_n) \sin (v_2 \mu_n) \cos \gamma_n - \frac{k_\alpha}{\sqrt{K\alpha}} \times \right.
\]
\[
\left. \times \left( \sin (v_1 \mu_n) \sin (v_2 \mu_n) + \mu_n \left( v_1 \cos (v_1 \mu_n) \sin (v_2 \mu_n) + v_2 \cos (v_2 \mu_n) \sin (v_1 \mu_n) \right) \right) \right. 
\]
\[
+ \sin \gamma_n \left[ \frac{k_\alpha}{\sqrt{K\alpha}} \gamma_n \sin (v_1 \mu_n) \sin (v_2 \mu_n) - (k_\alpha - 1) \times \right.
\]
\[
\left. \times (v_1 \cos (v_1 \mu_n) \sin (v_2 \mu_n) + v_2 \sin (v_1 \mu_n) \cos (v_2 \mu_n)) + \frac{v_2 (1 - \nu_2^2)}{v_2^2 - \nu_1^2} \right) \times 
\]
\[
\left. \times (N \cos (v_1 \mu_n) \sin (v_2 \mu_n) - \cos (v_2 \mu_n) \sin (v_1 \mu_n) \right) + 
\]
\[
+ \mu_n (\sin (v_1 \mu_n) \sin (v_2 \mu_n) - v_1 \nu_2 \cos (v_1 \mu_n) \cos (v_2 \mu_n)) \right];
\]
\[
K_\varphi = \frac{a_{qc}}{a_\varphi} ; \; K_\alpha = \frac{\lambda_{qc}}{\lambda_\alpha} ; \; k_R = \frac{R_k}{R_1} ; \; \gamma_n = \frac{k_R - 1}{\sqrt{K_\varphi}} \mu_n ; 
\]
\[
\nu_i^2 = \left[ \alpha + (1 - 1) \sqrt{\alpha^2 - \frac{4}{L_n}} \right], (i=1,2) ;
\]
\[
\alpha = 1 + \frac{1}{L_n} + \varepsilon K_\alpha \mu_n ;
\]

In broader terms, the rate of respiration of fruits and vegetables increases along with temperature and follows the Arrhenius law within the conventional storage temperature range of 0-40 °C, i.e. it is characterized by the following function:

\[
F = \exp \left( - \frac{E}{RT} \right)
\]

where \( E \) - activation energy, that is presumed to be rather high, \( T \) - absolute temperature, \( R \) - universal gas constant. According to article [3], the dependence of specific respiration heat of flax seeds of varying humidity stored for up to 10-15 days at +25 °C may be formulated as follows:

\[
\varphi_1 = \varphi_{101} \exp (\kappa_1 t) 
\]

The same pattern is observed when wheat grain is stored isothermally or adiabatically [4]. Factors \( \varphi_0 \) and \( \kappa_1 \) in equation (1) are functions of the baseline seed humidity, \( \varphi_0 \) - specific seed respiration heat at 0 °C.

The temperature of lower layers of raw materials is usually higher than the temperature of outside (upper) layers due to carbon dioxide release in bulk seeds caused by biological processes, humidity, and excessive heat, as well as to low thermal conductivity [5]. Studies of similar situations demonstrate that the
following exponential law of seed respiration rate reduction with layer height may be assumed sufficiently accurately: 

\[ q_2 = q_{20} \exp(-k_2 z) \]

where \( z \) is a coordinate, \( q_{m} \) - specific seed respiration heat at 0 °C at \( z = 0 \).

Therefore, the total dependence of the specific seed respiration heat on time \( \tau \) and coordinate \( z \) is as follows:

\[ q = q_2 \exp(k_1 \tau - k_2 z) \]

For more complete setting of the problem of storing bulk products considered a homogenous and isotropic medium in cylindrical containers the lateral surface whereof is heat-insulated, and temperatures of the base and the upper surface change spontaneously with time, the heat transfer problem may be formulated as follows [6]: it is necessary to solve a non-homogenous thermal conductivity equation for an infinite plate (the height of bulk products is smaller than the container's diameter) or a finite rod (the height of bulk products is considerably larger than the container's diameter)

\[ (0 < z < h, \ \tau > 0) \]

at initial condition

\[ t(z,0) = t_0 = \text{const} \]  

(14)

and at boundary conditions:

\[ t(0,\tau) = f_1(\tau); \]  

(15)

\[ t(h,\tau) = f_2(\tau) \]  

(16)

where \( a \) - thermal diffusivity factor;

\( h \) - bulk height;

\( t_0 \) - product temperature at the beginning of storage;

\( c \) - specific heat capacity of bulk products;

\( f_1(\tau) AND f_2(\tau) \) - SET CONFINED AND CONTINUOUS FUNCTIONS.

The following substitution

\[ \varphi(z,\tau) = \psi(z,\tau) + \frac{q_{20}}{c(k_1 - ak_2^2)} \exp(k_1 \tau - k_2 z) \]

where \( \psi(z,\tau) \) is a new required function, let us reduce the aforegiven differential equation to a homogenous equation, i.e. without a heat source

\[ \frac{\partial \psi}{\partial \tau} = a \frac{\partial^2 \psi}{\partial z^2} \]

but with new initial and boundary conditions:

\[ \psi(z,0) = t_0 - \frac{q_{20}}{c(k_1 - ak_2^2)} \exp(-k_2 z) = \varphi(z) \]

\[ \psi(0,\tau) = f_1(\tau) - \frac{q_{20}}{c(k_1 - ak_2^2)} \exp(k_1 \tau) = \varphi_1(\tau) \]

\[ \psi(h,\tau) = f_2(\tau) - \frac{q_{20}}{c(k_1 - ak_2^2)} \exp(k_1 \tau - k_2 h) = \varphi_2(\tau) \]

We used methods of mathematical physics [7] (the integral finite sine transform method) to obtain an analytical solution to the given boundary value problem of distribution of the field of temperatures in bulk products.

For a particular case of constant temperatures in the bulk base, i.e. at
\[ t(0, r) = t_1 = \text{const}, \quad t(h, \tau) = t_2 = \text{const}, \] and given the known ratios \([8]\), we obtained the following solution \([9]\):

\[
t(z, \tau) = \frac{q_0}{c(k_1 - ak_2^2)} \exp(k_1 - k_2^2) + t_1 + (t_2 - t_1) \frac{z}{h} + \\
+ \left(\frac{2}{\pi}\right) \sum_{n=1}^{\infty} \frac{t_0 - t_1 - (-1)^n(t_0 - t_2)}{n\pi} \sin \frac{n\pi}{h} \exp\left(\frac{an^2\pi^2\tau}{h^2}\right) - \frac{2q_0}{c(k_1 - ak_2^2)} \sum_{n=1}^{\infty} \left[ (-1)^n \exp(-k_2h) \right] \sin \frac{n\pi}{h} \times \\
\left[ \frac{\exp\left(-\frac{an^2\pi^2\tau}{h^2}\right)}{(k_2h)^2 + (n\pi)^2} + \frac{k_2h^2}{a} + (n\pi)^2 \right].
\]

The obtained correspondence was used to simulate the bulk storage process. The data on warehouse storage of sunflower seeds (lot No. 1) provided in article \([10]\) were employed as experimental. As the difference in humidities of layers when the lot was piled \((x = 0)\) varied insignificantly \((\pm 0.2\%)\), difference \(k_2\) was set to zero. Humidity of seeds in a layer at baseline \(U(Z,0) = 8.4\%\), temperature \(t_0 = 16 ^\circ\text{C}\), \(k_t = 0.8 \times 10^7 \text{c}^{-1}\), \(q_0 = 0.013 \text{W/kg}\). This corresponds with estimates of heat of seeds of equivalent humidity and temperature calculated by other authors \([11-13]\).

Calculation results for a 3 meter-high bulk are provided in Table 1.

### Table 1. Experimental and calculated temperature values for the storage of sunflower seeds \((t_1 = 24 ^\circ\text{C}; t_2 = 22 ^\circ\text{C})\).

| \(\text{Fo}, 10^3\) | \(X = 0.17\) | \(X = 0.5\) | \(X = 0.83\) |
|-----------------|-----------|-----------|-----------|
|                 | \(t_{\text{exp}}\) | \(t_{\text{calc}}\) | \(t_{\text{exp}}\) | \(t_{\text{calc}}\) | \(t_{\text{exp}}\) | \(t_{\text{calc}}\) |
| 1               | -         | 20.6      | -         | 20.1      | -         | 19.2      |
| 3               | 22        | 21.4      | 20        | 21.2      | 13        | 20.8      |
| 6               | -         | 21.9      | -         | 21.5      | -         | 21.0      |
| 9               | -         | 22.3      | -         | 21.8      | -         | 21.3      |
| 13              | 23        | 22.6      | 21        | 22.0      | 20        | 21.5      |
| 17              | -         | 23.0      | -         | 22.2      | -         | 21.7      |
| 20              | 23        | 23.2      | 31        | 22.3      | 29        | 21.9      |
| 25              | -         | 23.4      | -         | 22.4      | -         | 21.9      |
| 30              | -         | 23.5      | -         | 22.5      | -         | 21.9      |

\[
F_o = \frac{aX}{h^2}, \quad X = \frac{z}{h}
\]

where \(-\) Fourier number, \(-\) dimensionless coordinate

### 3. Conclusion

Analysis of obtained solutions indicates that the specific pattern of isotherm distribution over the volume of the object subjected to steam treatment is determined for each type of food raw materials on the computer-assisted basis using observed thermophysical properties.

Adaptability of the proposed model to the real process if the height of bulk seeds is low and the Fourier number \(Fo < 0.02\) is clear. In other cases, calculated and experimental values do not match due to a range of reasons. On the one hand, the heat source function does not consider the change in humidity over the course of storage \((0.6-1.0\%)\). Furthermore, the model does not consider the accelerating effect of temperature on the heat emission intensity. The difference in temperatures of the middle layer at the beginning and at the end of storage reached 15 °C. We may assume that it is necessary to shift to the models considering joint heat and moisture transfer on the one hand and
nonlinearity in temperature on the other hand. The proposed mathematical models allow predicting and managing temperature fields in bulk products and thus affect their quality and storage term.

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