MWP-BERT: A Strong Baseline for Math Word Problems

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Abstract

Math word problem (MWP) solving is the task of transforming a sequence of natural language problem descriptions to executable math equations. An MWP solver not only needs to understand complex scenarios described in the problem texts, but also identify the key mathematical variables and associate text descriptions with math equation logic. Although recent sequence modeling MWP solvers have gained credits on the math-text contextual understanding, pre-trained language models (PLM) have not been explored for solving MWP, considering that PLM trained over free-form texts is limited in representing text references to mathematical logic. In this work, we introduce MWP-BERT to obtain pre-trained token representations that capture the alignment between text description and mathematical logic. Additionally, we introduce a keyword-based prompt matching method to address the MWPs requiring common-sense knowledge. On a benchmark Math23K dataset and a new Ape210k dataset, we show that MWP-BERT outperforms the strongest baseline model by 5-10% improvement on accuracy.

1 Introduction

Math word problem (MWP) usually refers to a textual description of a mathematical problem, which contains several known variables and asks for the answer of an unknown variables when solving it. More exactly, as shown in Figure 1, MWP solving requires the generation of an equation showing how to get that unknown quantity. Not only accurate language understanding, but also strong mathematical reasoning ability is necessary in MWP solving.

Along the path of MWP solver development, the pioneer studies use traditional rule-based methods, machine learning methods and statistical methods (Yuhui et al., 2010; Kushman et al., 2014; Shi et al., 2015; Koncel-Kedziorski et al., 2015), which are limited on two aspects. Firstly, some of them rely on well-designed hand-crafted features, and are thus not efficient at all. Secondly, those solvers usually follow a template-based way to get answers and have poor generalization ability, since they can only solve problems with given answer templates. Afterwards, inspired by the development of sequence-to-sequence (Seq2Seq) models, MWP solving has been formulated as a pipeline of translating language descriptions to mathematical equations with neural network based encoder-decoder framework (Wang et al., 2018, 2019; Chiang and Chen, 2018; Liu et al., 2019; Li et al., 2019; Xie and Sun, 2019; Zhang et al., 2020b; Shen and Jin, 2020).

Although those neural based models have achieved better performance than traditional methods, the best model on Math23k benchmark dataset until now (Shen and Jin, 2020) still sets gated recurrent units (GRUs) (Cho et al., 2014) as the backbone to perform text understanding task and encode input tokens. As is shown on recent studies of natural language understanding, pre-trained language models such as BERT (Bidirectional Encoder Representations from Transformers) (Devlin et al., 2019) can perform perfect natural language understanding. It has been widely used in a large variety of NLP tasks and applications in other fields, like twitter text (Qudar and Mago, 2020), scientific text (Beltagy et al., 2019), biomedical data (Yoon et al., 2019; Lee et al., 2020), and financial communications (Yang et al., 2020). The major advantage of BERT is that it is pre-trained with enormous language data without complex annotations and can be easily plugged into downstream tasks. Therefore, in this work, we proposed an MWP solver called MWP-BERT, which is composed of a BERT-based encoder and a tree-based decoder.
In the yard there were 25 chickens and rabbits. Together they had 80 legs. How many rabbits were in the yard? (Chickens have 2 legs, rabbits have 4 legs)

To address the limits of BERT trained with free-form text on understanding mathematical text expression, we curate an effective subset called Ape-clean from Ape210k \(^1\), a new large-scale MWP solving benchmark. To the best of our knowledge, this is the first attempt in applying pre-trained language model as problem encoder to solve MWPs. Furthermore, although pre-trained language models have demonstrated strong capabilities of text understanding, solving MWPs often requires common-sense knowledge. For example, solving the second question in Figure 1 needs to know how many legs a rabbit has and a chicken has. In the prevailing benchmark Math23k (Wang et al., 2017), only \(\pi\) or 1 is involved as a common-sense constant, e.g., when answering the problem about the area or circumference of a circle given only the radius. However, in more realistic scenarios, there exist a large amount of MWPs that need a variety of common-sense constants to generate the correct solution. These common-sense constants are usually not explicitly mentioned in the problem description. We have verified that there exist many such cases in the larger benchmark Ape210k. To extract them and use them along with the seen quantities in the problem description under an easy to plug manner, we propose an efficient keyword-based prompt matching method to introduce the necessary constants. Also, as shown in the GPT series (Radford et al., 2018, 2019; Brown et al., 2020), prompt plays an important role in triggering strong understanding ability in pre-trained language model. Thus, we believe this prompt-based method is more consistent with our pre-trained style model and can take full advantage of the strong natural language understanding capability of MWP-BERT while injecting the required common-sense constants information.

Our contributions can be summarized as:

- We propose a novel MWP-BERT, and push the neural network’s understanding ability of MWP into a new stage.
- In order to solve problems that need common-sense knowledge, we propose a keyword-based prompt matching method to inject them into models. This can readily be used alongside any future improvements in pre-trained style MWP models.
- We curate a new dataset called Ape-clean from Ape210k, including around 125K samples. The dataset can support novel studies such as MWP model pre-training, MWP training dataset enlarging, and MWP solving with common-sense knowledge.
- Experiments on both Ape-clean and Math23k dataset show that our MWP-BERT achieves 5-10% higher accuracy than previous baselines.

2 Related Works

Math Word Problems Solving. MWP solving was presented as a task for artificial intelligence several decades ago (Fletcher, 1985), where earlier studies usually applied rule-based methods. Afterwards, researchers started to semantically parse MWP descriptions and generate answers by using pre-defined templates, and those methods are usually categorized into traditional statistical methods.

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\(^1\)https://github.com/Chenny0808/ape210k
(Kushman et al., 2014; Shi et al., 2015). Motivated by the recent success of neural network, deep neural solver (DNS) (Wang et al., 2017) was proposed as a pioneer to solve MWP via Seq2Seq models, which are still the mainstream solvers until now. Besides that, a tree-structured decoder (Liu et al., 2019; Xie and Sun, 2019) was presented to achieve goal-driven decoding. Li et al. (2019) borrowed the idea from Transformer and applied multi-head attention. Chiang and Chen (2018) used a stack to generate predicted equations in a suffix way. Moreover, many pre-processing methods and tricks were also very helpful to Seq2Seq solvers, such as number mapping (Wang et al., 2017), equation normalization (Wang et al., 2018), graph construction (Zhang et al., 2020b) and data augmentation (Liu et al., 2020). Recently, some new directions of MWP solving were also explored like knowledge distillation (Zhang et al., 2020a), weak supervision (Hong et al., 2021a), and situation model (Hong et al., 2021b). In this paper, we build a novel MWP solver based on pre-trained language models (e.g., BERT) to strengthen the understanding of the problem discription.

**Math Word Problems Solving Datasets.** There are several math word problem datasets. Alg514 (Kushman et al., 2014) contains 514 problems from a tutoring website. Dolphin18k (Huang et al., 2016) contains 18460 problems from Yahoo question answering website. AllArith (Roy and Roth, 2017) is a combination of several small MWP datasets (Hosseini et al., 2014; Roy and Roth, 2015; Koncel-Kedziorski et al., 2015) and contains 831 math word problems in total. MAWPS (Koncel-Kedziorski et al., 2016) is curated as a dataset with varying complexity from different sources. AS-Div (Miao et al., 2020) is a dataset with more diverse lexicon usage, for challenging the mathematical understanding of an MWP solver. MathQA (Amini et al., 2019) is an MWP dataset annotated by multiple-choice options and operation programs. Math23k (Wang et al., 2017) contains 23161 problems for elementary school students, which is the most commonly used dataset in MWP solving area because of its superiority in term of the size. The recently released Ape210k is the largest dataset so far, containing 210488 math word problems. A special feature (and challenge) of Ape210k is the inclusion of problems requiring common-sense knowledge for getting the solution. With the availability of Ape210k, we are motivated to build a pre-trained MWP-BERT model and design efficient methods for introducing the required common-sense knowledge.

## 3 The Proposed MWP-BERT Model

### 3.1 Problem Statement

The input of an MWP solver is a textual description, we denote it as $W$ with length $m$, thus $W = \{w_1, w_2, ..., w_m\}$. The output is an equation showing how to get the final answer. We denote it as $A$ with length $n$, where $A = \{a_1, a_2, ..., a_n\}$. The vocabulary of $A$ contains three parts, namely $V_{op}$, $V_{num}$ and $V_{cons}$. $V_{op}$ is the vocabulary for all operators, i.e. $+, -, \times, \div$ and $. V_{num}$ does not contain the actual value of quantities appeared in $W$, those quantities are denoted as $\{n_1, n_2, ..., n_k\}$, where $n_i$ means the $i$-th number from $W$ and $k$ is the maximum number of quantities in $V_{num}$ in order to fix the size of it. $V_{cons}$ contains necessary constant values e.g., $\pi$. Note that for simple problems like those in Math23k, only a few known constant values like $\pi$ need to be specified in $V_{cons}$. However, for more challenging problems in Ape210k, a variety of common-sense constants should be covered for presenting the common-sense knowledge, e.g., “rabbits have $4$ legs”, and “there are $24$ hours in a day”. We next introduce our solution on extracting and introducing the common-sense knowledge.

### 3.2 External Constant Introduction

In solving MWPs, the required common-sense knowledge is often regarding certain constants that are different from the seen quantities in the problem description. How to introduce these external constants is the key for including the required knowledge. It is infeasible to simply add potential external constants from as much sensible common-sense knowledge as possible. First, the size of $V_{cons}$ will be large, and the high-dimensional and sparse output will become difficult to predict. Second, only adding the external constants to $V_{cons}$ cannot tell the meaning of the constants and help the understanding of the knowledge to introduce.

From our observations, we find that most of the external constants are associated with keywords as explicit triggers. For example, the external constant $3$ is associated with the word “cone”, and $4$ is assoc-
associated with the words “rabbit” and “legs”. Therefore, we propose a keyword-based prompt matching method to introduce necessary constants into the problem description in the form of keyword-sentence pairs \((K, W_s)\). For a given problem \(W\), if \(K \in W\), we create a sentence \(W_s\) which contains associated constants with \(K\). The additional sentence \(W_s\) and the original problem description \(W\) will be jointly sent to the MWP encoder for embedding the external constants of introduced knowledge with the original problem description. Details of the created keyword-sentence pairs can be found in the supplementary material. The proposed method is easy to plug into any MWP solvers and can naturally work under the off-line setting because it can be applied during data pre-processing.

3.3 MWP-BERT Architecture

The overview of the proposed MWP-BERT model is shown in Figure 2. The encoder is a pre-trained BERT to extract features from problem descriptions, then the decoder based on goal-driven tree-structured solver (GTS) (Xie and Sun, 2019) generates the answer equation recursively.

3.3.1 Encoder

To understand jointly the additional sentence \(W_s\) and the original problem description \(W\), the encoder should have strong capability on natural language understanding. We employ BERT (Devlin et al., 2019) due to its success shown in a variety of NLP tasks. However, BERT pre-trained with free text may not understand well the mathematical expression and logic in MWPs. We thus perform masked language model (MLM) pre-training of our encoder on Ape-clean dataset, which has 125,675 samples from Ape210k by removing the low-quality data samples. We apply masks on 10% of tokens, randomly replace 10% of tokens by other tokens and keep 80% of tokens unchanged.

The pre-trained encoder model maps the original problem \(W\) with the auxiliary sentence \(W_s\) to a representation matrix \(Z \in \mathbb{R}^{(m+s) \times h}\), where \(s\) is the length of \(W_s\).

\[
Z = \text{encoder}([W_s; W]).
\]  
where \([-\cdot\] denotes concatenation. The representation vector corresponding to each word in \(Z\) will be used in the decoding process for generating the solution.

3.3.2 Decoder

By taking the problem embedding \(Z\), the decoder generates a binary tree in a top-down manner for the solution, as tree-based representations have been commonly used in MWP solving. It is easy to encode operation order into the top-down structure of a tree, avoiding brackets and simplifying the equation representation by tree traversal. Moreover, tree-based decoder (Xie and Sun, 2019) is a reflection of human-like goal-driven way of MWP solving, which can decompose unknown quantity into several sub-goals and solve them recursively.
In our approach, we transfer all equations into tree-based representation and its pre-order traversal result serves as the ground truth.

In the tree structure, every parent node represents an operator (from $V_{op}$) and every leaf node contains a quantity (from $V_{num} \cup V_{con}$). During the top-down generation, the decoder has two tasks: predicting and decomposing. For each new node in the tree, firstly we need to predict whether it is an operator or a quantity. Then the predicted operator node is going to be decomposed with two children nodes, while the predicted quantity node will stay as a leaf node without further decomposition. At the beginning, we initialize the root node with the mean vector $\overline{Z} \in \mathbb{R}^h$ of the encoder output $Z$.

**Prediction Module.** This module predicts the token of tree node based on encoder output $Z$ and the node representation $q$, where $q_{root}$ is equal to $\overline{Z}$ as previously mentioned. Then the context vector $v$ is calculated by attention mechanism:

$$v = \sum_{i=0}^{m} s_i Z_i$$

(2)

where $Z_i \in \mathbb{R}^h$ is the hidden vector of $i$-th word in $W$, and attention score $s_i$ is:

$$s_i = \frac{\exp(W_1 \sigma(W_2[q : Z_i]))}{\sum_{i=0}^{m} \exp(W_1 \sigma(W_2[q : Z_i]))}$$

(3)

where $W_1$ and $W_2$ are both trainable matrices, $\exp$ denotes the exponential function, and $\sigma$ is the activate function and $[;]$ denotes concatenation.

After getting the context vector $v$, we define a probability function $\text{score}$, which predicts current token’s (node) probability distribution across $V_{num} \cup V_{con} \cup V_{op}$:

$$\text{score}(q, v, e_a) = W_3 \sigma(W_4[q : v : e_a])$$

(4)

where $W_3$ and $W_4$ are both trainable, $e_a$ is the embedding vector of token $a$ and $\text{score}(q, v, e_a)$ is the probability score of the token $a \in V_{num} \cup V_{con} \cup V_{op}$. When $a \in V_{con} \cup V_{op}$, we look up a randomly initialized and learnable embedding matrix $W_{\text{embed}}$ to get $e_a$, and $W_{\text{embed}}$ is shared by different MWPs because $V_{con} \cup V_{op}$ is the same for all MWPs. When $a \in V_{num}$, $e_a$ is defined to be equal to $Z_{idx(a)}$ where $idx(a)$ is the index of $a$ in problem description, because quantities in different problems have different meanings thus should be embedded differently. Eventually, we calculate the probability score for each token in target vocabulary as Equation 4 shows and select the largest one as our final prediction.

**Decomposition Module.** If the predicted token of a node is an operator, that node will be forwarded to this decomposition module. Firstly, the node representation of left child $q_{left}$ is calculated by a 2-layer gate module $\text{Gate}_1$:

$$q_{left} = \text{Gate}_1(q_p, v_p, e_p)$$

(5)

where $q_p$ is the hidden representation of parent node, $v_p$ and $e_p$ are context vector and embedding vector of predicted token, respectively.

We apply prediction module on $q_{left}$ and get another predicted token. We repeat this predict-decompose-predict process until we get a token $a \in V_{num} \cup V_{con}$. Then, we calculate the right child node representation by another 2-layer gate module $\text{Gate}_2$:

$$q_{right} = \text{Gate}_2(q_p, v_p, e_p, r_{left})$$

(6)

where $r_{left}$ is the hidden representation of left child node, which is different from $q_{left}$. Since we have got the predicted token $a_l$ of left child node using $q_{left}$, the representation of left node is updated by:

$$r_{left} = \begin{cases} e(a_l) & \text{if } a_l \in V_{con} \cup V_{num}, \\ \text{Gate}_3(r_{ll}, q_{lr}, e(a_l)) & \text{if } a_l \in V_{op}. \end{cases}$$

(7)

where $e(a_l)$ is the embedding of token $a_l$, $\text{Gate}_3$ is another gate module, $r_{ll}$ is the left child representation (updated) of left child, and $q_{lr}$ is the right child representation (not updated) of left child.

In the end, all nodes in the binary tree cannot be further decomposed and have a predicted token. We then can easily transfer that tree to a pre-order traversal sequence and compare it with the ground truth. The implementation details of $\text{Gate}_1$, $\text{Gate}_2$ and $\text{Gate}_3$ can be found in (Xie and Sun, 2019).

**3.3.3 Training Criteria**

Following the Seq2Seq tasks aiming to transfer a sequence into another sequence, our overall training target is to maximize the probability of answer prediction, i.e., minimizing the negative log probability:

$$L = - \log P(A|W)$$

(8)

where $W$ is the MWP description and $A$ is the corresponding answer equation. Both encoder and decoder are trainable, the only difference is that the encoder is a pre-trained model while the decoder is updated in the training process.
4 Experiments

To show the superiority of our proposed MWP-BERT, we make a comprehensive comparison study among our model and the most representative baselines. We use the value accuracy as our testing metric. Because the answer value can be calculated from different equations, it is more reasonable to check if the final answer value is correct rather than checking the equations.

4.1 Datasets

Math23k. Math23k (Wang et al., 2017) is a widely used dataset to evaluate MWP solvers. It contains 21,162 Chinese math word problems in the training set and 1000 problems in the test set. Evaluation results can be reported on test set, while researchers also bind its training and test set together and perform 5-cross validation on this dataset. We report the results in both settings.

Ape-clean. Ape210k is a recently released large dataset of Chinese MWPs, including 210,488 problems. The problems in Ape210k are more diverse and difficult than those in Math23k, not only because of the requirement of common-sense knowledge for getting solutions, also because problems may not have their ground-truth solution equations nor answers. The problems without answers are not usable. The problems without the equations but only answer values are challenging to use. We leave this open issue for our future study. Therefore, we follow the rules below to select the usable problems from Ape210k to construct an Ape-clean dataset.

- We remove all MWPs that have no answer values nor equations.
- We remove all MWPs that only have answer values without equations.
- We remove all MWPs with a problem length $m > 100$ or an answer equation length $n > 20$, as they will bring obstacles for training. We cannot perform truncation on over-length problems or answers because incomplete problems are unsolvable and incomplete answers are meaningless to work on.

After data cleaning, the Ape-clean dataset contains 125,675 MWPs, including 122,588 training problems and 3,087 testing problems. Although around half of problems in Ape210k are filtered out, the over 125K high-quality problems are sufficient to be used for pre-training MWP-BERT.

4.2 Implementation Details

We train our model on an NVIDIA TESLA V100 graphic card with 32G memory, all implementation of training and testing is coded in Python with Pytorch framework. The model was trained for 50 epochs with a batch size of 32. Adam (Kingma and Ba, 2014) optimizer is applied with an initial learning rate of 5e-5, which would be halved every 30 epochs. Dropout rate of 0.5 is set during training to prevent over-fitting. During testing, we use 5-beam search to get reasonable solutions. The hyper-parameters setting of our BERT encoder is 12 layers of depth, 12 heads of attention and 768 dimensions of hidden features.

4.3 Experimental Results

The evaluation metric is answer accuracy. We firstly transfer the generated equation tree into a value, then compare it with the ground truth value. The reported accuracy is the percentage of the correctly answered problems in the testing set.

4.3.1 Evaluation on Math23k Dataset

We first compare our approach with the most recent representative baselines on the benchmark Math23k dataset. The first baseline is DNS which is the pioneer work using Seq2Seq model to solve MWPs. Math-EN (Wang et al., 2018) proposes an equation-normalization method and uses vanilla Seq2Seq model to get solutions. S-Aligned (Chiang and Chen, 2018) applies a stack to generate target equations. T-RNN (Wang et al., 2019) develops a template-based method with recurrent neural network. GROUP-ATT (Li et al., 2019) is inspired by Transformer and implements a multi-head attention mechanism. GTS (Xie and Sun, 2019) proposes a goal-driven tree-based decoder and achieves great results. KA-S2T (Wu et al., 2020) proposes a novel knowledge-aware model that can incorporate background knowledge. However, their background knowledge still cannot bring external constants to equation generation, which means this model still struggles to solve difficult MWPs with external constants. Graph2Tree (Zhang et al., 2020b) constructs two graphs during data pre-processing to extract extra relationships from text descriptions. Multi-ED (Shen and Jin, 2020) is the latest approach which...
proposes a multi-encoder multi-decoder structure and achieves the best results on Math23K before our approach. To avoid the implementation error that may cause unreproducible results of baseline models, we reported the results of these baselines from the papers where they were published, as many previous papers (Zhang et al., 2020b; Shen and Jin, 2020) did.

The results in Table 1 show that our approaches outperform previous methods by a large margin (6% improvement on accuracy) and firstly bring the accuracy on Math23K over 80%. Moreover, the BERT encoder with pre-training on Ape-clean dataset can further improve the accuracy, although this evaluation is on Math23k rather than Ape-clean.

### 4.3.2 Evaluation on Ape-clean and Math23k When Trained by a Joint MWP Set

Since Ape-clean is five times larger than Math23k, we are interested to evaluate whether MWP solvers trained on Ape-clean dataset can perform well on the simple problems in Math23k testing set. We combine the training set of Math23k and Ape-clean to train the MWP-BERT model, and then measure the accuracy on the testing set of Math23k and Ape-clean separately.

Results shown in Table 2 convey interesting evaluation observations. Surprisingly, the accuracy of our models on Math23k reaches above 96%, which is marvelously high, because previous state-of-the-art methods can hardly reach 80% (Shen and Jin, 2020). Comparing to the results in Table 1, even GTS has a higher accuracy when trained with the big joint MWP set of Ape-clean and Math23k. We can thus confirm that our curated Ape-clean dataset is able to serve as a high-quality MWP training set to improve solvers’ generalization ability to solve problems in Math23k.

Moreover, the accuracy on Ape-clean of our approach is also much higher than other baselines (9% higher than the strongest baseline GTS). Note that not all the baseline methods in Table 1 are presented in the comparison in Table 2 for several reasons. First, baselines like Graph2Tree and Multi-ED require specific data pre-processing like semantic parsing and POS tagging on new datasets. It is thus hard for us to run them on the new Ape-clean dataset. Inappropriate pre-processing may lead to unfair and unreliable comparison. Second, the experimental results in Table 1 have already shown the superiority of our approach over all other baselines. Last, GTS is widely acknowledged as a strong baseline for MWP solving and all following studies like Graph2Tree, KA-S2T and Multi-ED are based on GTS. The performance of GTS in Table 2 implies that Graph2Tree and Multi-ED will benefit as well from the Ape-clean dataset. We will leave the evaluation confirmation to the community by releasing Ape-clean in public soon.

### Table 1: Comparison of answer accuracy (%) between our proposed models and different baselines on Math23k dataset, separated by the middle split line. Math23k column shows the results when evaluating on the public test set of Math23k. Math23k* column shows the result of 5-fold cross validation on Math23k dataset. The best results are highlighted with bold font. “w/o MLM” denotes “without masked language model pre-training on Ape-clean” and “w MLM” means “with the pre-training on Ape-clean”.

| Model               | Math23k | Math23k* |
|---------------------|---------|----------|
| DNS                 | -       | 58.1     |
| Math-EN             | 66.7    | -        |
| S-Aligned           | -       | 65.8     |
| T-RNN               | 66.9    | -        |
| Group-ATT           | 69.5    | 66.9     |
| GTS                 | 75.6    | 74.3     |
| KA-S2T              | 76.3    | -        |
| Graph2Tree          | 77.4    | 75.5     |
| Multi-E/D           | 78.4    | 76.9     |
| MWP-BERT w/o MLM    | 83.8    | 82.0     |
| MWP-BERT w MLM      | 84.4    | 82.3     |

### Table 2: Comparison of answer accuracy (%) between our proposed models and baselines when they are all trained by the combination of the training set from Ape-clean and Math23k dataset. Math23k column shows the results when evaluating on the public test set of Math23k. Ape-clean column shows the result on Ape-clean test set. The best results are in bold.

| Model               | Math23k | Ape-clean |
|---------------------|---------|-----------|
| DNS                 | 53.6    | 67.1      |
| GTS                 | 91.4    | 74.9      |
| MWP-BERT w/o MLM    | 96.0    | 83.9      |
| MWP-BERT w MLM      | 96.2    | 84.3      |
| Training/Test Data | $P_{All}$ | $P_{simple}$ | $P_{KNWL}$ |
|-------------------|----------|-------------|-----------|
| Ape210k + w/o ECP | 64.4     | 67.5        | 53.0      |
| Ape-clean + w/o ECP | 79.8     | 83.8        | 65.1      |
| Ape-clean       | 83.9     | 82.7        | 88.3      |

Table 3: The answer accuracy (%) of MWP-BERT on solving the various testing problems in Ape-clean, when trained on different datasets. “w/o ECP” stands for “without external constants prompt”. $P_{simple}$ and $P_{KNWL}$ are the simple testing problems and common-sense knowledge required testing problems, respectively. $P_{All}$ means all testing problems.

test at 0.01 level.

### 4.4 Ablation Study on Ape-clean

To verify the effectiveness of our dataset cleansing and external constants introduction, we conduct the experiments as Table 3 shows. First in the column of $P_{All}$, we train an MWP-BERT model with the original Ape210k dataset without external constants introduction. The accuracy is only 64.4% on the test set of Ape-clean (for fair comparison). When trained with Ape-clean, the accuracy increased to 79.8%, which is still lower than the accuracy (83.9%) when external constants introduction is implemented. Second in the column of $P_{simple}$ when testing on only simple problems (78.5% of all samples), again, we can see that Ape-clean provides high-quality training data and promotes the accuracy to 83.8%. It is worth mentioning that the introduction of external constants doesn’t help on solving these simple problems, and thus lower the accuracy slightly to 82.7%. Third in the last column of $P_{KNWL}$ when testing on only problems (21.5% of all samples) that require common-sense knowledge, the accuracy difference between models with and without external constants introduction is more than 23%. This verifies the effectiveness of our keyword-based prompt matching method when solving problems requiring common-sense knowledge.

### 5 Case Study

We perform case study as shown in Table 4. The first problem is from Math23k dataset. Both Graph2Tree and Multi-E/D fail to generate the right solution, while our proposed MWP-BERT solves it correctly. The second problem is from Ape-clean and requires common-sense knowledge.

| Math23k Problem | In salt water weighing 200 grams, the weight of salt accounts for 20% of the water. After adding more water, the weight of salt is 12.5% of the water. How many grams of water were added? |
|----------------|----------------------------------------------------------------------------------------------------------------------------------|
| Graph2Tree:    | $x = 200 - (20\% \times 12.5\%)/200$ (✗)                                                                                       |
| Multi-E/D:     | $x = 200/200/(20\% \times 12.5\%)$ (✗)                                                                                          |
| MWP-BERT:      | $x = 200 \times 20\% / 12.5\% - 200$ (✓)                                                                                       |

| Ape-clean Problem | The area of a trapezoid is 4.5 square dm, the upper base is 15 cm, and the height is 2 dm. How many decimeters is the lower base? |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------|
| w/o ECP:          | $x = 4.5 \times 2/2 - 15$ (✗)                                                                                                  |
| MWP-BERT:         | $x = 4.5 \times 2/2 - (15/10)$ (✓)                                                                                             |

Table 4: Our case study. “w/o ECP” denotes without applying external constants prompt.

The solver without external constants prompt does not transform “15 cm” into “1.5 dm”, leading to a wrong answer. The two above cases again show that our approach not only has a stronger capability for MWP solving, our external constant prompt also empowers the solver to deal with common-sense demanding problems.

### 6 Conclusion

Math word problem solving has been attracting researchers’ attention for a long time. In this paper, we propose MWP-BERT, which to the best of our knowledge is the first usage of pre-trained language model on MWPs. Moreover, an external constant prompt is leveraged to introduce common-sense knowledge into MWP descriptions. We curate a new dataset Ape-clean by filtering out unsolvable problems from Ape210k, which can be used for both MWP-BERT pre-training and evaluation. Experimental evaluation results show that our proposed approach achieves the highest accuracy, 5-10% higher than the baselines. We believe that our study can serve as a pioneer work of both employing pre-trained language models and introducing commonsense-related constants in MWP solving area, hinting future studies.
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