Research Article

Performance of a New Time-Truncated Control Chart for Weibull Distribution Under Uncertainty

Ali Hussein AL-Marshadi¹, Ambreen Shafqat², Muhammad Aslam¹,*, Abdullah Alharbey¹

¹Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, 21551, Saudi Arabia
²Department of Statistics and Financial Mathematics, School of Science, Nanjing University of Science and Technology, Nanjing, Jiangsu, P.R. China

1. INTRODUCTION

A leading tool in the manufacturing process is the control chart that is applied to watch manufacturing shifts. A slight change in the quality target may cause big losses for a company. The variable control charts are designed to monitor the process using measurement data. Nonconforming items can be monitored using an attribute control chart. According to [1], "the traditional Shewhart np control charts are the statistical control scheme most commonly used for monitoring the number of non-conforming items." For more details, the reader may refer to [1–4].

Usually, control charts work when data is derived from a normal process. When data moves away from normality, the chart designed with normal distribution cannot monitor the process. Control charts designed with non-normal distributions present a good alternative. According to [5], "In most life testing, to reduce the test time of the experiment, a failure-censored (type-II) scheme, or time-censored (type-I) scheme is usually adopted." Therefore, the control chart designed with a time-truncated control can be used to save monitoring time. In the operational procedure of time-truncated charts, the experiment and watching times are fixed in advance, where an item is labeled as defective if its failure/service time is less than the specified time. The noted numbers of defects are plotted on the control chart to decide the state of the process. Previous studies [6] have proposed an attribute chart for Weibull distribution. Specifically, some studies [7] designed control charts using censored data. Others [8] have studied time-truncated charts for exponentiated half logistic distribution and Dagum distribution, respectively. Further, [9–12] show further control chart applications for various statistical distributions.

The Shewhart control charts given in the literature are unable to apply if some observations in the production data are uncertain or fuzzy. Fuzzy-based control charts are applied when uncertainty is presented in observations and parameters. According to [13], "fuzzy control charts are more sensitive than traditional ones; hence, they provide better quality products." Furthermore, Darestani et al. [14] proposed a u-chart using a fuzzy approach, whereas [15] proposed a chart using fuzzy logic and [16] proposed a c-chart using fuzzy logic [17]. More work can be seen in [17–21].

The major drawback of a fuzzy-based control chart is that it is less informative than other charts. As such, Smarandache [22] mentioned that the generalized form of fuzzy logic is known as neutrosophic logic. Moreover, Smarandache and Khalid [23] stated that neutrosophic logic is more efficient than fuzzy logic and interval-based analysis. Other researchers [24] have discussed neutrosophic logic applications. Neutrosophic statistics is a branch of statistics that analyzes neutrosophic data, and many studies [25] have presented methods to analyze the indeterminacy data. Other works [26] have introduced indeterminacy in an attribute chart and proposed attribute charts for neutrosophic statistics [27]. Moreover, Khan et al. [28] proposed an S-chart using neutrosophic statistics.
Lastly, Aslam et al. [29] proposed an efficient attribute chart. More information in neutrosophic statistics can be seen in [30,31].

The attribute control charts under neutrosophic statistics are presented in the literature. No previous work has designed a time-truncated attribute control chart for neutrosophic Weibull distribution using neutrosophic statistics. In this paper, we designed a time-truncated attribute control chart using neutrosophic statistics. The effect of the indeterminacy parameter was studied on the average run length (ARL). The advantages of the proposed chart are also discussed. In next section, we describe the design of the proposed chart. In Section 3, the comparison of the proposed chart with the counterpart chart and simulation study is described. The Illustrative example is described in Section 4. In the final section, the conclusion and future recommendations are displayed.

## 2. DESIGN OF THE PROPOSED CHART

First, we introduce the neutrosophic Weibull distribution. Then, we present the design of the current chart.

Suppose that \( x_N \in [x_L, x_U] \) be a neutrosophic random variable having neutrosophic Weibull distribution, where \( x_L \) and \( x_U \) are the lower and upper values of neutrosophic random variable. The Weibull distribution neutrosophic probability density function (NPDF) and neutrosophic cumulative distribution function (NCDF) are defined as follows

\[
f(x_N) = \left\{ \left( \frac{x}{\alpha} \right)^{\beta-1} e^{-\left( \frac{x}{\alpha} \right)^\beta} \right\} I_N; \quad I_N \in [I_L, I_U], \quad x_N \in [x_L, x_U]
\]

\[
F(x_N) = 1 - \left\{ e^{-\left( \frac{x}{\alpha} \right)^\beta} \right\} (1 + I_N)
\]

where \( \beta \) is the shape parameter, \( \alpha \) is the scale parameter, and \( I_N \) shows the neutrosophic interval. The neutrosophic mean, say \( \mu_N \), of the neutrosophic Weibull distribution is given by

\[
\mu_N = a\Gamma(1 + 1/\beta) (1 + I_N)
\]

where \( \Gamma(.) \) denotes the gamma function. Let \( \mu_{N0} \) target the product’s lifetime and \( x_{N0} \) be a truncated time. The probability \( p_N \) that an item fails by time \( x_{N0} \) is given as

\[
p_N = 1 - \left\{ e^{-\left( \frac{x_{N0}}{\alpha} \right)^\beta} \right\} (1 + I_N)
\]

Let \( x_{N0} = a\mu_{N0} \) be the termination time, where \( a \) is constant and we express the unknown scale parameter \( \alpha \) in term of \( \mu_N \) using Eq. (3). Then, Eq. (4) can be rewritten as

\[
p_N = 1 - \left\{ e^{-\left( \frac{\mu_{N0}}{\mu_N} \right)^\beta} \left( (\Gamma(1+1/\beta)(1+I_N)) \right)^\beta \right\} (1 + I_N)
\]

When \( \mu_N = \mu_{N0} \), then the probability in Eq. (5) reduces to

\[
p_{N0} = 1 - \left\{ e^{-\left( \frac{\mu_{N0}}{\mu_{N0}} \right)^\beta} \right\} (1 + I_N)
\]

The \( np \) attribute control chart having lower control limit (LCL) and upper control limit (UCL) for the neutrosophic Weibull distribution is explained in the following steps:

**Step 1.** Count defective items of the products (D) from the selected sample.

**Step 2.** If \( D > UCL \) or \( D < LCL \), the process is considered out of control. The process is considered in control if \( LCL \leq D \leq UCL \).

Note that \( D \) follows a neutrosophic binomial distribution with parameters \( n \) and in-control probability \( p_{N0} \). Therefore, the control limits of the proposed \( np \) control chart are as follows:

\[
UCL = np_{N0} + k \sqrt{np_{N0}(1-p_{N0})}
\]

\[
LCL = \max \left[ 0, np_{N0} - k \sqrt{np_{N0}(1-p_{N0})} \right]
\]

where \( k \) is the coefficient or control constant of the control limits to be resolute. The \( p_{N0} \) values are normally unknown; therefore, the averages of failed items (D = defective items) such as \( \overline{D} \) are taken. Thus, the neutrosophic control limits for practical use are

\[
UCL_N = \overline{D} + k \sqrt{\overline{D} \left( 1 - \frac{\overline{D}}{n} \right)}
\]

\[
LCL_N = \max \left[ 0, \overline{D} - k \sqrt{\overline{D} \left( 1 - \frac{\overline{D}}{n} \right)} \right]
\]

The probability for the in-control process (i.e., \( p_{N0}^D \)) is given by

\[
p_{N0}^{(D)} = P \left( LCL \leq D \leq UCL | p_{N0} \right)
\]

\[= \sum_{d=LCL+1}^{UCL} \binom{n}{d} p_{N0}^d (1-p_{N0})^{n-d} \]

If \( LCL = 0, d \) should be 0 when applied. Let \( \mu_{N1} \) be the shifted average and the probability at the new mean is evaluated by

\[
p_{N1} = 1 - \left\{ e^{-\left( \frac{\mu_{N0}}{\mu_{N1}} \right)^\beta} \left( (\Gamma(1+1/\beta)(1+I_N)) \right)^\beta \right\} (1 + I_N)
\]

Let \( x_{N1} = a\mu_{N1} \) be the termination time.
If the shifted mean is $\mu_{Ni} = f\mu_{N0}$ for a constant $f$, then Eq. (10) can be rewritten as

$$p_{Ni} = 1 - \left\{ e^{-nf\beta -(\Gamma(1+1/n)\beta(1+I_0)^n)} (1 + I_N) \right\} + I_N\epsilon \left[ I_L, I_U \right] \epsilon X_{L1}, X_{U1}$$

(11)

The in-process probability for the shifted mean can be written as

$$p_{N1}^1 = P\{LCL \leq D \leq UCL\} = p_{Ni}^1 (1 - p_{Ni})^{n-d}$$

(12)

The in-control ARL can be calculated as

$$ARL_0 = \frac{1}{1 - p_{N1}^1}$$

(13)

The ARL for the shifted process can be calculated as

$$ARL_1 = \frac{1}{1 - p_{N1}^1}$$

(14)

ARL values for various values (e.g., $I_N$, $f$, and $\beta$) are shown in Tables 1–3, where the following behavior can be noted.

1. When the indeterminacy parameter $I_N$ increases, the decreasing trend in $ARL_1$ is noted.
2. For the same values, the values of $ARL_1$ decreases as $\beta$ increases.

The values of $k$, $ARL_0$, and $ARL_1$ can be obtained through the following steps:

**Step 1:** Fix the values of $\beta$, $n$, and $I_N$, thus determining the values of $k$ via the grid search method.

**Step 2:** Several values of $k$ are noted during simulation, where $ARL_0 \geq r_0$ is a specified ARL value.

**Step 3:** Choose the $k$ value when $ARL_0$ is close to $r_0$.

**Step 4:** Determine $ARL_1$ values for various values of $f$.

### 3. COMPARATIVE STUDIES

Our proposed control chart becomes the same chart proposed in [6] when $I_N = 0$. Table 4 presents both control charts when $I_N = 0.1$ and $I_N = 0.5$. Table 4 shows that the proposed control chart provides smaller ARls values when compared to the chart proposed by [6]. For example, when $f = 0.9$, $\beta = 1.1$, $n = 30$, and $I_N = 0.1$, the proposed control chart detects the first out-of-control value at the 151st sample, whereas the existing chart proposed by [6] detects the first out-of-control value at the 177th sample. Similarly, when $f = 0.9$, $\beta = 1.1$, $n = 30$, and $I_N = 0.5$, the proposed control chart detects the first out-of-control value at the 131st sample, whereas and the existing chart proposed by [6] detects the first out-of-control value at the 177th sample. From this study, it can be seen that the proposed chart provides a quick indication about the shift in the process when compared to the control chart proposed by [6]. This study thus proves the efficiency of the proposed chart.

### 3.1. Simulation Study

The performance of the proposed control chart was compared with the existing chart proposed by [6] using simulated data. The data was generated when $a = 0.1548$, $n = 30$, $\beta = 1$, and $I_N = 0.1$. Among 50 observations, the first 20 values were generated when the process was in-control (IC) state when $\mu_0 = 1$. Further, the next 30 observations were generated from the shifted process when $f = 0.8$ by using the R software. The values of the number of defective items ($D$) under the proposed scheme are plotted in Figure 1 and the existing control chart displayed in Figure 2. Figure 1 indicates the shift at the 31st sample; the existing chart proposed by [6] indicates no shift. By comparing both figures, we can conclude that the proposed control chart indicates an issue in the existing control chart. Therefore, the application of the proposed control chart under uncertainty is helpful for minimizing the number of non-conforming items and getting conforming products in less time.

### 4. ILLUSTRATIVE EXAMPLE

The proposed control chart was applied in an automobile manufacturing company located in South Korea. The company was interested in monitoring the service time in months for specific sub-systems [6]. The same service data was applied by [6], who showed...
that the data followed the Weibull distribution with $\beta = 1$. For this study, let $a = 0.1883$, $k = 3.1368$, $n = 30$, $p_0 = 0.1722$, and $I_N = 0.1$, which leads to the value of $\beta = 1(1 + 0.1) = 1.1$ for the neutrosophic Weibull distribution. Let the value of $\mu_{\beta N} = 60$, which leads to time $x_{N0} = a\mu_{\beta N} = 0.1883 \times 60 = 11.29$. From 400 observations, the statistic $D$ is made by counting the observation as a failure if it is smaller than 11.29 months. The numbers of defects $D$ are shown in Figures 3 and 4, and also listed as follows:

\[3, 1, 4, 4, 3, 3, 6, 4, 3, 5, 2, 3, 3, 4, 4, 4, 6, 1, 4, 5, 5, 6, 7, 7, 2, 1, 7, 8, 3, 3, 9, 4, 5, 10, 5, 5, 7.\]

### Table 2 | ARLs of the proposed chart when $r_0 = 370$.  
\[\beta = 1.1, n = 30\]

| $f$ | $k = 3.13684, I_N = 0.1, a = 0.1883$ | $k = 3.19848, I_N = 0.2, a = 0.1584$ | $k = 3.1863, I_N = 0.4, a = 0.1332$ | $k = 3.21308, I_N = 0.5, a = 0.1310$ |
|-----|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1.0 | 371.62                           | 371.68                           | 371.37                           | 374.92                           |
| 0.9 | 151.50                           | 150.41                           | 141.13                           | 131.00                           |
| 0.8 | 59.51                            | 58.61                            | 51.77                            | 46.61                            |
| 0.7 | 22.75                            | 22.22                            | 18.61                            | 16.04                            |
| 0.6 | 8.67                             | 8.40                             | 6.78                             | 5.68                             |
| 0.5 | 3.47                             | 3.35                             | 2.69                             | 2.27                             |
| 0.4 | 1.64                             | 1.50                             | 1.34                             | 1.21                             |
| 0.3 | 1.07                             | 1.00                             | 1.01                             | 1.00                             |
| 0.2 | 1.00                             | 1.00                             | 1.00                             | 1.00                             |
| 0.1 | 1.00                             | 1.00                             | 1.00                             | 1.00                             |

### Table 3 | ARLs of the proposed chart when $r_0 = 370$.  
\[\beta = 2, n = 30\]

| $f$ | $k = 3.58595, I_N = 0.1, a = 0.2167$ | $k = 3.58132, I_N = 0.2, a = 0.2224$ | $k = 2.89913, I_N = 0.4, a = 0.2472$ | $k = 3.3222, I_N = 0.5, a = 0.2428$ |
|-----|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 1.0 | 372.83                           | 370.03                           | 371.83                           | 370.58                           |
| 0.9 | 136.70                           | 120.64                           | 88.27                            | 79.58                            |
| 0.8 | 47.76                            | 37.90                            | 21.24                            | 17.70                            |
| 0.7 | 16.20                            | 11.85                            | 5.64                             | 4.53                             |
| 0.6 | 5.58                             | 3.96                             | 1.94                             | 1.62                             |
| 0.5 | 2.17                             | 1.64                             | 1.09                             | 1.03                             |
| 0.4 | 1.17                             | 1.05                             | 1.00                             | 1.00                             |
| 0.3 | 1.00                             | 1.00                             | 1.00                             | 1.00                             |
| 0.2 | 1.00                             | 1.00                             | 1.00                             | 1.00                             |
| 0.1 | 1.00                             | 1.00                             | 1.00                             | 1.00                             |

### Table 4 | Comparison of the proposed chart with the existing chart in ARLs.  
\[\text{[6] chart (} I_N = 0\text{)}\]

| $I_N = 0.1$ | $I_N = 0.5$ |
|-------------|-------------|
| $\beta = 1$ | $\beta = 1.1$ | $\beta = 1$ | $\beta = 1.1$ |
| $k = 3.40009, a = 0.1344$ | $k = 3.33227, a = 0.1394$ | $k = 3.26994, I_N = 0.1, a = 0.1545$ | $k = 3.32217, \beta = 1, a = 0.09385$ | $k = 3.21308, I_N = 0.5, a = 0.1310$ |
| ARL         | ARL         | ARL         | ARL         | ARL         |
| 1.0         | 370.32      | 370.80      | 374.93      | 370.43      | 374.92      |
| 0.9         | 180.12      | 177.71      | 165.38      | 151.50      | 152.48      | 131.00      |
| 0.8         | 83.88       | 81.21       | 70.06       | 59.51       | 60.33       | 46.61       |
| 0.7         | 37.40       | 35.50       | 28.65       | 22.75       | 23.13       | 16.04       |
| 0.6         | 16.05       | 14.94       | 11.47       | 8.67        | 8.79        | 5.68        |
| 0.5         | 6.74        | 6.18        | 4.65        | 3.47        | 3.48        | 2.27        |
| 0.4         | 2.91        | 2.66        | 2.07        | 1.64        | 1.61        | 1.21        |
| 0.3         | 1.45        | 1.35        | 1.18        | 1.07        | 1.05        | 1.00        |
| 0.2         | 1.03        | 1.01        | 1.00        | 1.00        | 1.00        | 1.00        |
| 0.1         | 1.00        | 1.00        | 1.00        | 1.00        | 1.00        | 1.00        |
The control limits for the proposed chart are $UCL = 8$ and $LCL = 0$ when the values of parameters values are used from Table 2. The proposed chart is displayed in Figure 3 and the existing chart is shown in Figures 4, respectively. Figure 3 shows the shift in service time means the shift values detect out of control at 31st sample. The existing chart indicates that no action is needed for service time because all values are in control. By comparing both charts, it can be seen that the proposed control chart indicates an issue in monthly service time for the specific subsystem of the car while the existing chart does not show any issue in monthly service time for the specific subsystem of the car.

5. CONCLUSIONS

The time-truncated attribute control chart was herein presented using the neutrosophic Weibull distribution. The measurement's indeterminacy effect was studied on the performance of the proposed control chart with the counterpart control chart. The proposed chart was the generalized version of the attribute control chart under a neutrosophic environment. From the results presented in Tables 1–4, we observed that the indeterminacy parameter significantly affected ARL values. The out-of-control ARL values reduced when indeterminacy increased. The comparative study proved the current chart's efficiency. The proposed control chart was proven to monitor service time in the automobile industry. The proposed control chart has limitations, i.e., it can only be applied when the life/service time follows the neutrosophic Weibull distribution. Secondly, the proposed control chart cannot be applied for variable data. The proposed control chart can be applied in the automobile industry, aircraft industry, and mobile industry for monitoring the defective items. In the future, the proposed control chart should be used for other neutrosophic statistical dis-
The authors declare no conflict of interest.

AUTHORS’ CONTRIBUTIONS

A.H.A.M, A.S, M.A and A.A wrote the paper.

ACKNOWLEDGMENTS

The authors are deeply thankful to the editors and reviewers who offered invaluable suggestions for improving this manuscript. This article was supported by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah under project No. (G-22-130-1441). The authors, therefore, gratefully acknowledge the DSR technical and financial support.

REFERENCES

[1] A.A. De Araujo Rodrigues, E.K. Epprecht, M.S. De Magalhaes, Double-sampling control charts for attributes, J. Appl. Stat. 38 (2011), 87–112.
[2] E.K. Epprecht, A.F. Costa, F.C. Mendes, Adaptive control charts for attributes, Iie Trans. 35 (2003), 567–582.
[3] A.F. Costa, M. Rahim, Joint–X and R charts with two-stage samplings, Qual. Reliab. Eng. Int. 20 (2004), 699–708.
[4] Z. Wu, Q. Wang, An np control chart using double inspections, J. Appl. Stat. 34 (2007), 843–855.
[5] C.-H. Jun, S. Balamurali, S.-H. Lee, Variables sampling plans for Weibull distributed lifetimes under sudden death testing, IEEE Trans. Reliab. 55 (2006), 53–58.
[6] M. Aslam, C.-H. Jun, Attribute control charts for the Weibull distribution under truncated life tests, Qual. Eng. 27 (2015), 283–288.
[7] L. Zhang, G. Chen, EWMA charts for monitoring the mean of censored Weibull lifetimes, J. Qual. Technol. 36 (2004), 321–328.
[8] G.S. Rao, A control chart for time truncated life tests using exponentiated half logistic distribution, Appl. Math. Inf. Sci. 12 (2017), 125–131.
[9] M. Aslam, O.H. Arif, C.-H. Jun, An attribute control chart for a Weibull distribution under accelerated hybrid censoring, PloS One. 12 (2017), e0173406.
[10] S.R. Gadde, A. Fulment, P. Josephtat, Attribute control charts for the Dagum distribution under truncated life tests, Life Cycle Reliab. Saf. Eng. 8 (2019), 329–335.
[11] J.A. Adewara, K.S. Adekeye, O.L. Aako, On performance of two-parameter gompertz-based control charts, J. Probab. Stat. 2020 (2020), 1–9.
[12] S. Ali, T. Zafar, I. Shah, L. Wang, Cumulative conforming control chart assuming discrete Weibull distribution, IEEE Access. 8 (2020), 10123–10133.
[13] H. Razali, L. Abdullah, T.A. Ghani, N. Aimran, Application of fuzzy control charts: a review of its analysis and findings, in: M. Awang, S. Emamian, F. Yusof (Eds.), Advances in Material Sciences and Engineering, Lecture Notes in Mechanical Engineering, Springer, Singapore, 2020, pp. 483–490.

[14] S.A. Darestani, A.M. Tadi, S. Taheri, M. Raeeszadeh, Development of fuzzy U control chart for monitoring defects, Int. J. Qual. Reliab. Manag. 31 (2014), 811–821.

[15] N.P. Alakoc, A. Apaydin, A fuzzy control chart approach for attributes and variables, Eng. Technol. Appl. Sci. Res. 8 (2018), 3360–3365.

[16] H. Ercan-Teksen, A.S. Anagün, Intuitionistic Fuzzy C-control charts using fuzzy comparison methods, in International Conference on Intelligent and Fuzzy Systems, Istanbul, Turkey, 2019, pp. 1161–1169.

[17] M.-H. Shu, H.-C. Wu, Fuzzy X and R control charts: fuzzy dominance approach, Comput. Ind. Eng. 61 (2011), 676–685.

[18] M.H. Zavvar Sabegh, A. Mirzazadeh, S. Salehian, G. Wilhelm Weber, A literature review on the fuzzy control chart; classifications analysis, Int. J. Supply Oper. Manag. 1 (2014), 167–189.

[19] S. Şentürk, N. Erginel, İ. Kaya, C. Kahraman, Fuzzy exponentially weighted moving average control chart for univariate data with a real case application, Appl. Soft Comput. 22 (2014), 1–10.

[20] M.Z. Khan, M.F. Khan, M. Aslam, S.T.A. Niaki, A.R. Mughal, A fuzzy EWMA attribute control chart to monitor process mean, Information. 9 (2018), 312.

[21] A. Özdemir, Development of fuzzy X–S control charts with unbalanced fuzzy data, Soft Comput. 25 (2021), 4015–4025.

[22] F. Smarandache, Neutrosophy. Neutrosophic probability, set, and logic, proquest information & learning, Ann Arbor, Michigan, USA. 105 (1998), 118–123.

[23] F. Smarandache, H.E. Khalid, Neutrosophic Precalculus and Neutrosophic Calculus, Infinite Study, 2015. https://arxiv.org/ftp/arxiv/papers/1509/1509.07723.pdf

[24] Y. Guo, A. Şengür, J.-W. Tian, A novel breast ultrasound image segmentation algorithm based on neutrosophic similarity score and level set, Comput. Methods Prog. Biomed. 123 (2016), 43–53.

[25] F. Smarandache, Introduction to Neutrosophic Statistics, Infinite Study, 2014.

[26] M. Aslam, A.H. Al-Marshadi, Design of a control chart based on COM-poisson distribution for the uncertainty environment, Complexity. 2019 (2019), 1–9.

[27] M. Aslam, R.A. Bantan, N. Khan, Design of a new attribute control chart under neutrosophic statistics, Int. J. Fuzzy Syst. 21 (2019), 433–440.

[28] Z. Khan, M. Gulistan, R. Hashim, N. Yaqoob, W. Chammam, Design of S-control chart for neutrosophic data: an application to manufacturing industry, J. Intell. Fuzzy Syst. 38 (2020), 4743–4751.

[29] M. Aslam, R.A. Bantan, N. Khan, Design of NEWMA np control chart for monitoring neutrosophic nonconforming items, Soft Comput. 24 (2020), 16617–16626.

[30] M. Aslam, N. Khan, A new variable control chart using neutrosophic interval method-an application to automobile industry, J. Intell. Fuzzy Syst. 36 (2019), 2615–2623.

[31] M. Aslam, A new goodness of fit test in the presence of uncertain parameters, Complex Intell. Syst. 7 (2021), 359–365.