Brine rejection leads to salt-fingers in seasonally ice-covered lakes

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Key Points:

- Brine rejection-induced salt fingers occurs in most seasonally ice-covered lakes
- Salt fingers may not obviously perturb in the temperature stratification
- Rejected brine is often distributed evenly with depth

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Abstract
Brine rejection can lead to salt-finger formation under lake ice. We performed a series of experiments to visualize these fingers. We believe that we are the first to do so in a freshwater system. While we detected salt-fingers in our camera recordings, the signal of these fingers is nearly absent in the temperature record. Further, we estimate that the brine is often distributed evenly with depth. Comparing the salt fluxes in our experiments with estimated global averages, we argue that salt-fingers form in most seasonally ice-covered lakes.

Plain Language Summary
When ice forms on the surface of lakes, dissolved salts are pushed out of the ice into the liquid water below. If enough salt is rejected from the ice, the excess weight of the salt can lead to a long ‘fingers’ of salty fluid moving from the ice into the water below. We ran a series of experiments to investigate these ‘fingers’, and conclude that this process occurs in most freshwater lakes that freeze annually. This process is important for the evolution of lakes, and how it will be different as fewer lakes freeze.

1 Introduction
The world’s lakes are rapidly warming (O’Reilly et al., 2015; Woolway & Merchant, 2019). In turn, this warming is causing a decrease in ice cover for high latitude lakes (Magnuson, 2000; Sharma et al., 2019). The full effect of this change is not yet clear, and may result in harmful effects to wildlife (Hampton et al., 2015, 2017), lead to increased surface warming through a loss of albedo (Kim et al., 2018), and increased methane release (Greene et al., 2014). To predict the evolution of these systems, we need to characterize the lake processes that depend on ice cover. This paper is focused on one such under-ice process: brine rejection.

When a water surface freezes, dissolved constituents within the water are excluded from the ice. The concentration of those constituents, then, increases at the ice-water interface. The rejection of salts from the ice is of particular interest, as salts are pervasive and often form an important role in stratifying aquatic systems. For example, there has been significant interest in brine rejection from sea ice (Worster & Rees Jones, 2015). While there has been limited study of this phenomenon in freshwater systems, Pieters & Lawrence (2009) have demonstrated that brine rejection may play an essential role in suppressing turnover in lakes.

Brine rejection in freshwater is complicated by a nonlinear equation of state. Unlike seawater, freshwater systems have a temperature of maximum density above their freezing temperature (Chen & Millero, 1986). This nonlinear temperature dependence enables lakes to reverse stratify under ice, with colder-water (reaching the freezing temperature at the ice-water interface) above warmer water below. Despite this stable temperature stratification, Bluteau et al. (2017) and Olsthoorn et al. (2020) showed that once enough brine is rejected, double-diffusive plumes will form that transport salty water from the ice-water interface to the lake bottom. This type of double-diffusive salt-fingering has been shown to be complicated by the nonlinear equation of state (Özgökmen & Esenkov, 1998). The present paper builds upon this previous work by performing a set of physical experiments to visualize the brine rejection-induced plumes.

Our paper has three objectives. First, we will visualize the salt-plumes and demonstrate their structure. We believe that we are the first to have done so in freshwater laboratory experiments. Second, we will highlight that brine rejection can occur even at very-low salinity values, from which we suggest that these plumes will occur in most ice-covered lakes. At low salinity, these plumes do not obviously perturb the temperature stratifi-
cation, explaining why field measurements have often failed to measure their occurrence. Third, we highlight the role that these salt-plumes play in transporting water from the ice-water interface to the bottom of a lake. We hypothesize that this may provide an important oxygen transport pathway in light of the recent observation of high phytoplankton concentrations under the lake ice (Hampton et al., 2017; B. Yang et al., 2020).

2 The Buoyancy Flux Ratio

Brine rejection produces to a salt flux \( F_S \) at the ice-water interface, which is determined by the rate of ice growth as

\[
F_S = -r_S S_{z=H} \frac{dH}{dt}
\]

where \( S_{z=H} \) is the salinity at the ice-water interface, \( H \) is the depth of the liquid water, and \( r_S \) is the proportion of salt rejected by the ice. This brine rejection coefficient was \( \approx 0.95 \) in experiments with potassium chloride (Bluteau et al., 2017) and as high as 0.99 in field observations (Pieters & Lawrence, 2009). We did not measure the salinity of the ice in the present experiments, and so we approximate \( r_S = 1 \). Further, Ashton (1989) argues that thin ice grows linearly in time. We will show that this is a reasonable approximation for the experiments discussed in this paper. We further argue that the total amount of rejected brine is small compared to the initial mean water salinity, i.e. \( S_{z=H} \approx S_0 \). Combining these approximations, we model \( F_S \) as a case dependent constant.

As noted in the introduction, the salinity in most limnological contexts is low enough that the freezing temperature of the water is lower than the temperature of maximum density \( \tilde{T}_{md} \). Thus, the liquid water beneath the ice is be stably stratified in temperature \( T \). As brine is rejected from the ice, the salt stratification is unstably stratified. Thus, there is a competition between the stabilizing temperature field and the destabilizing effect of salinity.

The buoyancy frequency \( (N^2) \) is a measure of the strength of the buoyancy stratification and is computed

\[
N^2 = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} = g\alpha \frac{\partial T}{\partial z} - g\beta \frac{\partial S}{\partial z} = N^2_T + N^2_S,
\]

where \( \alpha \equiv \alpha(T, T_{MD}, S_0) \) is the thermal expansion coefficient (ignoring variations in pressure), and \( \beta \) is the constant haline contraction coefficient. Before ice forms, the density stratification is dominated by temperature \( (N^2_S = 0) \). After ice forms, the rejected salt decreases \( N^2 \). We write the relative buoyancy frequency ratio as

\[
\frac{N^2}{N^2_T} = 1 + \frac{N^2_S}{N^2_T} = 1 - \tau R_0,
\]

such that if \( \tau R_0 > 1 \), then \( N^2 < 0 \) and the system is hydrostatically unstable. Here, \( \tau \) is the ratio of the thermal to saline diffusivities, which is nearly constant \( (\tau \approx 127 \text{ near } 0 ^\circ C) \). Note that the product \( 1/(\tau R_0) = \frac{\partial T}{\partial z}/\frac{\partial S}{\partial z} = R_\rho \) is the density ratio typically defined in double diffusive systems (Radko, 2013). At the ice-water interface, we compute

\[
R = \frac{\beta F_S}{\alpha F_T},
\]

where \( F_T \) is temperature flux into the ice. This buoyancy flux ratio \( R \) computes the relative strength of the salinity to temperature stratification at the ice-water interface, where both the temperature and salinity stratification are the largest. We will show that \( R \) is the controlling parameter in this system.
Figure 1. (a) A schematic of the laboratory setup. Heat loss through the side-walls produced domain-scale recirculation cells below the growing ice. Brine rejection produced long finger-like features that propagate from the ice-water interface to the base of the tank. Note that the third acrylic side-wall was not included in the diagram. (b) Water depth as a function of time for Case 4. The water depth decreased approximately linearly with time. Time \( t = 0 \) is the time of ice formation.

3 Laboratory Setup

We performed a series of laboratory experiments designed to visualize the brine-rejection-induced convection in low-salt environments. These experiments were performed by placing a 50x50x50 cm\(^3\) acrylic tank (Figure 1a) in a commercial walk-in freezer. The tank was partially filled with a sodium-chloride solution to a depth of approximately 20 cm. The walls of the tank were triple-pane to thermally insulate the water from side-wall heat loss. The surface of the water was exposed to freezing atmospheric temperatures \( T_{\text{Air}} \) leading to surface ice growth. While the dominant heat loss was through the surface, weak side-wall cooling did produce tank-scale recirculation cells between the centre and the sides of the tank.

To visualize the flow within the tank, we seeded the saline water with 20\(\mu\)m polyamid particles. We illuminated these particles with a vertical laser sheet (660nm) through middle of the tank, which were then recorded at one frame per ten seconds. From these images, we computed with flow velocity with DigiFlow (Olsthoorn & Dalziel, 2017). The speed of the slowest salt-fingers were comparable with the settling velocity of the particles \(3 \approx 10^{-6} \text{ m/s}\). As we will discuss below, we compute the RMS velocity values to remove the error from both the settling velocity and the recirculation cells.

We also recorded the temperature and salinity of the water. The water temperature was measured with an in-house built temperature chain, constructed from 10 BR14 NTC-type thermistors (Thermometrics Corp., CA), calibrated with an error of \(\sim 0.01^\circ\text{C}\). These 10 thermistors were attached to a vertical rod at 2 cm spacing. One thermistor failed over the course of the experiments and was removed from the analysis. Further, these thermistors saturate at sufficiently low temperatures, at which point they were also omitted. The temperature chain was positioned in the center of the water bath.

A HOBO conductivity logger (Hoskin Scientific), positioned against the far wall of the tank, continuously measured salinity in the water. Further, we recorded reference salinity values of the water before and after each experiment with a handheld salinity
The net decrease in water depth over each experiment is denoted $\Delta H$.

Table 1. Table of the basic parameters associated with each experiment. The initial water depth and initial mean water salinity was $H_0$ and $S_0$, respectively. The net decrease in water depth over each experiment is denoted $\Delta H$.

| Case | SetPoint [°C] | $H_0$ [mm] | $S_0$ [g/kg] | $\Delta H$ [mm] | $F_S$ [$10^{-9}$ g/kg m/s] | $R$ |
|------|--------------|------------|--------------|----------------|---------------------------|-----|
| 0 0  | -15 | 223  | $1.0 \times 10^{-03}$ | 7.0 | 0.2 | – |
| 0 1  | -15 | 215  | $1.3 \times 10^{-02}$ | 7.8 | 3.3 | 0.03-0.04 |
| 0 2  | -15 | 212  | $2.2 \times 10^{-02}$ | 8.0 | 6.0 | 0.04-0.08 |
| 0 3  | -15 | 187  | $3.8 \times 10^{-02}$ | 7.4 | 10.3 | 0.05-0.12 |
| 0 4  | -15 | 205  | $8.5 \times 10^{-02}$ | 6.6 | 22.0 | 0.08-0.18 |
| 0 5  | -15 | 213  | $3.5 \times 10^{-01}$ | 7.2 | 99.4 | 0.24-1.20 |
| 0 6  | -15 | 220  | $1.2 \times 10^{+00}$ | 6.7 | 270.3 | 0.45-3.07 |
| 0 7  | -15 | 226  | $3.5 \times 10^{+00}$ | 4.6 | 659.8 | 0.70-6.91 |
| 0 8  | -10 | 193  | $3.7 \times 10^{-02}$ | 4.9 | 6.1 | 0.02-0.04 |
| 0 9  | -10 | 215  | $8.2 \times 10^{-02}$ | 5.4 | 12.9 | 0.06-0.13 |
| 0 10 | -10 | 216  | $3.5 \times 10^{-01}$ | 4.0 | 51.2 | 0.14-0.44 |
| 0 11 | -10 | 210  | $1.2 \times 10^{+00}$ | 2.7 | 151.3 | 0.35-0.73 |
| 0 12 | -10 | 224  | $3.6 \times 10^{+00}$ | 2.9 | 460.0 | 1.06-3.00 |

meter (Cole-Palmer Cond 3110). We converted between raw conductivity and salinity using a constructed calibration curve following Pawlowicz (2008). A case-specific linear correction was also included to the Hobo record to correct for temperature-driven salinity drift prior to ice formation. The Hobo probe measured the salinity increase due to the brine rejected from the ice.

In total, we ran 13 laboratory cases with initial salinity values of $S_0 = 0 - 4$ g/kg. Complementary experiments were performed with a freezer setpoint temperatures of $-10^\circ$C and $-15^\circ$C. Table 1 includes the individual case parameters. In each case, the camera images detected the ice formed on the water surface. We selected the time of ice formation as $t = 0$. For freezer setpoint $-15^\circ$C, the ice formed nearly instantaneously across the water surface. For freezer setpoint $-10^\circ$C, often one side would freeze before the other (a time difference of up to 1hr) leading to an asymmetry in the brine rejection rate. To account for this early asymmetric freezing, the data was omitted until the initial salt-plumes hit the bottom and the ice formation was more even.

4 Results

4.1 Ice Growth

We measured the mean water depth $H$ over time from the camera images (Figure 1b). Before ice formation ($t < 0$), $H$ is constant; As ice grows ($t > 0$), $H$ slowly decreased. We defined the ice formation time $t = 0$. Due to the change in opacity of the ice as it forms, the detection algorithm often struggled to detect the initial growth of ice (see the blip around $t = 0.5$hr in Figure 1b). This was worse at high salinity values. As discussed above, the ice growth was approximately linear in all cases and was primarily a function of the freezer setpoint temperature. Typical final ice thicknesses were around 0.5cm.

4.2 Time Evolution

We discuss the time evolution of these experiments and the generation of salt fingers. As a representative case, we will initially focus on Case 4, before comparing this case with the remainder.
Figure 2. (a) Temperature timeseries of the bottom water temperature ($T_B$) as a function of time. (b)-(e) Snapshots of the vertical velocity ($v$) at the times indicated by the red dots in panel (a). Data is presented for Case 4
After ice formation, the water reverse stratified and continued to cool in all cases. Figure 2a is a plot of the bottom water temperature ($T_B$) over time for Case 4. Initially ($t \approx 0$) the water preferentially cooled near the surface such that the bottom water temperature $T_B$ was approximately constant. As the system continued to cool, $T_B$ decreased.

Figure 2b-e contains snapshots of the vertical velocity $v$ at the times indicated by the red dots in Figure 2a. Prior to the formation of salt-fingers, side-wall heat losses generated tank-scale recirculation cells (see Figure 2b). In particular, the heat loss at the side-walls induced a negatively buoyant jet (red), which then induced a counter-flow of the bulk interior fluid moving down (blue). Note that the thermistor chain in the domain centre ($x \approx 0.25$cm) perturbs the PIV and the affected region is removed in our subsequent analysis.

By $t = 2.5$ hr, we observe the formation of finger-like plumes (Figure 2c), which grow at the surface and propagate down towards the tank bottom (Figure 2d-e). These plumes induce their own counter-flow between the fingers. The recirculation cells accelerated the downward propagation of the plumes. Importantly, these plumes transport salt from the ice-water interface to the tank bottom. We suggest that these plumes will also transport oxygen and other dissolved constituents, which has not been highlighted in the freshwater literature.

### 4.3 Heat Transport

The evolution of the observed depend upon the brine rejection rate. Figure 3 contains contour plots of (a,c,e) the vertical velocity and (b,d,f) the temperature stratification for Cases 0, 4, and 6. These cases covered a range of water salinities from 0.0 – 1.3 g/kg, and highlight the difference between distilled, low-salinity, and medium-salinity environments. The horizontal dashed lines in the temperature plots (Figure 3b,d,f) denote the position of the top and bottom thermistor, with temperature above the top thermistor interpolated with an error-function. Before ice formation ($t < 0$), the water temperature was well mixed due to surface driven convection. After ice formation, the water reverse stratified and continued to cooled over the experiment.
We ran an experiment with distilled water (Case 0, \( S_0 \approx 0 \) g/kg) to underscore that the observed plumes result from brine rejection. As we expect, no plumes were observed in the absence of salt (Figure 3a). In the case of distilled water, the temperature stratification cooled smoothly, enhanced by the side-wall heat losses.

For low-salinity environments (Case 4, \( S_0 = 0.09 \) g/kg, as with Figure 2), salt fingers form but they are sufficiently weak that no identifiable signal was measured in the thermistor chain data (Figure 3d). Brine rejection produced salt fingers in very low salinity environments; salt fingers are not always identifiable in the temperature record.

As the salinity increased (Case 6, \( S_0 = 1.2 \) g/kg), brine rejection resulted in more energetic plumes (Figure 3e). These more energetic plumes generated convective motions that perturbed the temperature stratification (see Figure 3f around \( t = 1 \) hr). As the initial bulk salinity \( S_0 \) increased, the speed of the plumes increased and induced larger perturbations to the temperature field. The plumes also form earlier (See Appendix A).

The saline plumes stirred the water, which enhanced the vertical transport of heat. We estimated the magnitude of this effect through an average turbulent thermal diffusivity \( \kappa^* \) (See Appendix B for more details). Figure 3g is a plot of \( \kappa^* \) for each of the three plotted cases. The sidewall convection increased \( \kappa^* \) by a factor of 10 from its molecular value \( \kappa_0 = 1.33 \) m\(^2\)/s. Note that Case 0 and 4 have approximately the same thermal diffusivity dominated by the recirculation cells. As the brine-rejection-induced convection increased, \( \kappa^* \) further increased by a factor of 2. Thus, the saline plumes do increase the heat transport in the system but this is likely to be subdominant to other physical processes occurring in a natural system. This does not mean that the salt transport is negligible as we will see below.

### 4.4 Salinity Increase

The salt-plumes transport salt from the ice-water interface into the open water. Eventually, the salt-fingers reach the bottom of the tank, where the salinity \( \left( S_B \right) \) was continuously measured. Figure 4a is a plot of the salinity perturbation \( S' = (S_B - S_0) / S_0 \) for Cases 1-7 as a function of time. The data have been offset so that the individual cases can be observed. Notice that after some time, the bottom water salinity starts to increase. We identify this time \( t_M \), using a piecewise linear curve, which is indicated by the black squares in Figure 4a. This measurement time \( t_M \) identifies when the rejected brine first reaches the tank bottom.

It takes time for the salt-fingers to reach to the tank bottom. Therefore, the time of measured salt increase

\[
t_M = t_f + \Delta t_p
\]

is determined by the finger formation time \( (t_f) \) plus the propagation time \( (\Delta t_p) \). We estimate the formation time \( t_f \) through back-propagation of the first fingers, and denote the \( t_f \) estimate with circles in Figure 4a. We find that the plumes form earlier at higher salinity values, consistent with the prediction that the time required for the salt-plumes to form depends on the brine rejection rate (see Appendix A). At the highest salinity values, \( t_f \) was not included as the fingers formed nearly instantaneously with ice formation; \( t_f \approx 0 \). The propagation time \( (\Delta t_p = t_M - t_f) \) for the plumes to reach the tank bottom similarly decreased with salinity. That is, the plumes travelled more quickly as at higher salinities (see §4.5).

It is worth highlighting that for \( S_0 = 0.01 \) g/kg, no plumes were detected before the experiment ended. Similarly, the salinity did not increase above its initial value. If we had continued this experiment, we believe that we would have observed a salinity increase, even at this low salinity value. Nevertheless, we observed that salt-fingers at salinity values as low as 0.02 g/kg, at the low end of the limnological range of salinity val-
Figure 4. (a) Salinity perturbation as a function of time for Cases 1-7. The black squares indicate the time when the salinity perturbation starts to increase ($t_M$). Circles are denoted as the estimated time of finger formation ($t_f$). (b) Vertical velocity RMS as a function of $R$. For low $R \lesssim 1$, the $w_{rms}$ scales as $R^{\frac{2}{3}}$. For $R \gtrsim 1$, $w_{rms} \approx 2w_{rms,0}$. (c) The flux ratio $F$ plotted as a function of the buoyancy ratio $R$. The horizontal dashed line is given at $F = 1$ and the vertical dashed line denotes the estimated transition from (i) pooling to (ii) uniform distribution at $R = 0.5$. (d) Two simulated salinity profiles from Olsthoorn et al. (2020) (Case 5, $S_0 = 1$ g/L, $H = 0.1$m). (e) Two field salinity measurements from BML ($S_0 = 1.75$ g/L, $H = ?$m).
ues (see §5 for more details). Based upon this observation, we suggest that brine rejection is a significantly more ubiquitous under-ice process than has been previously recognized.

4.5 RMS Velocity

The saline plumes propagate faster at higher $S_0$. As noted in §2, the stable temperature stratification also controls the descent of these plumes. While the temperature stratification does vary with depth (see Figure 3b,d,f), the rate of formation of the salt fingers is controlled by the temperature stratification at the ice water interface. The controlling parameters of the system is $\mathcal{R} = \frac{\beta F_s}{\alpha F_T}$.

We compute average plume speed as the root-mean-squared vertical velocity $w_{\text{rms}}$ over the domain. We applied a high-pass spatial filter to remove the effect of the recirculation cells. In addition, we removed the velocity data within 5cm of the side-wall due to the boundary layers, along with the data affected by the thermistor chain. More sophisticated algorithms, such as Dynamic Mode Decomposition, produced similar results; we decided to use a simple filter. Further, we filter out data for $t < 0.5$ hr to avoid any remnant pre-ice convection. Figure 4b is a plot of $w_{\text{rms}}$ as a function of $\mathcal{R}$. The median values for each case are plotted as black squares, and the error bars are the standard deviation along each axis. In this plot, we scaled $w_{\text{rms}}$ by the appropriate reference velocity $w_{\text{rms},0} = \frac{v_f}{d}$, where length scale $d$ is the finger width ($d \approx 7.5$ mm in all cases). This is a common scaling in the literature (Radko, 2013). We estimate $w_{\text{rms},0} \approx 1.8 \times 10^{-5}$ m/s.

We observe that $w_{\text{rms}}$ increases with $\mathcal{R}$. For large values of $\mathcal{R} \gtrsim 1$, $w_{\text{rms}}$ appears to saturate with a value of $w_{\text{rms}}/w_{\text{rms},0} \approx 2$. We associate this saturation with the expected regime change around $\mathcal{R} \approx 1$ from double-diffusive fingers to salt-driven convection (Olsthoorn et al., 2020). Olsthoorn et al. (2020) similarly observed a saturation in the relative heat flux and associated it with the same regime transition.

For $\mathcal{R} < 1$, we find that the $w_{\text{rms}}/w_{\text{rms},0}$ scales with $\sim \mathcal{R}^{\frac{3}{2}}$. We leave theoretical explanation for this scaling for future work. However, this scaling indicates that $\mathcal{R}$ is indeed the correct controlling parameter and that the plume velocities increase sublinearly with increasing salt flux.

4.6 Salt distribution

Salt-plumes transport salt through the background thermal stratification. As the plumes descend, the transported salt may

(i) pool at the bottom creating a stable bottom saline layer,
(ii) be uniformly distributed with depth, or
(iii) be continuously entrained into a salt stratification.

As we have discuss in §7, the stable temperature stratification can support an unstable salinity stratification while remaining hydrostically stable ($N_S^2 < 0$, $N^2 > 0$). However, option (iii) is unlikely to occur as the salt stratification would rapidly overcome the stabilizing effect of the temperature stratification. We can determine how salt is being transported by comparing the amount of brine rejected with the amount of salt that reaches the tank bottom.

How does the brine rejection rate compare with the bottom salinity measurement $S_B$? Assuming that the salt is completely rejected from the ice, salt is conserved in the
water such that
\[
\frac{d}{dt} \int_0^H S \, dz = 0 \implies F_S = \frac{H}{F_B} \frac{d}{dt} S_B + \int \frac{\partial}{\partial t} (S - S_B) \, dz.
\]

Here, \( F_B \) is the salt flux estimate assuming the water were uniformly mixed to \( S_B \). That is, if the plumes deposit salt uniformly with depth, the salinity flux ratio
\[
F = \frac{F_B}{F_S} = \frac{H}{F_B} \frac{d}{dt} S_B = 1.
\]

The final term in (6) represents the salinity deficit or excess associated with either (i) salt pooling at the bottom of the tank \((F > 1)\), or (iii) entraining into the stratification \((F < 1)\).

We want to highlight that \( F = 1 \) does not imply that the salt stratification is uniform. Rather, a uniform salt deposition will keep the shape of the salinity profile steady, while increasing in time. As we will discuss in §5, there are field observations of salinity profiles that maintain their shape over time, while becoming increasingly saline.

Figure 4c is a plot of \( F \) as a function of \( R \). The error bars are the standard deviation of the scaled salinity error from linear growth, and the standard deviation of \( R \) over the experiment. We note that both axes contain the brine rejection rate \( F_S \), and, in that sense, are not orthogonal quantities. We include the data from the two-dimensional numerical simulations from Olsthoorn et al. (2020). For the numerical data, the mean salinity in the bottom 5% of the domain was used as a numerical proxy for \( S_B \). We observe that to a reasonable approximation, \( F \approx 1 \) for most cases, with some scatter. For low \( R \), the data are biased towards the (i) pooling regime where \( F \gg 1 \). Due to the large spread in the data, we do not attempt to come up with a precise scaling law for \( F \) vs \( R \). Instead, we simply estimate that the transition between (i) pooling \((F \gg 1)\) and (ii) uniform deposition \((F \approx 1)\) occurs around \( R \approx 0.5 \).

### 5 Implications for Lakes

Lerman et al. (1995) estimated the global salt concentration for lakes (excluding saline lakes with salinity > 3 g/kg) to be 0.2 g/kg. It is not clear how one would define a global estimate for a lake freezing rate required to compute an average brine-rejection rate. In lieu of this, we estimated the freezing rate from the St. Lawrence field data of Ashton (1989) to be \( \approx 0.2 \) mm/hr over 10 days. Further, we estimated a freezing rate of \( \approx 0.3-0.4 \) mm/hr over two months from the data of Y. Yang et al. (2013). Taking 0.2 mm/hr as a global estimate for thin ice growth over a day, we predict a salt flux of \( 2 \times 10^{-8} \) g/kg m/s. Clearly, a global estimate for a brine rejection rate is highly variable but this daily estimate gives us a starting point. Nonetheless, we have observed brine-rejection-induced salt fingers for salt fluxes at a third this value within several hours. It is further challenging to compare values of \( R \). However, our thermal stratification is stronger than typically found in lakes under ice (B. Yang et al., 2021), suggesting that lake values of \( R \) would be even larger than the experiments with similar salt flux rates \( F_S \). In summary, we argue that brine-rejection-induced salt fingers are present in most seasonally ice-covered lakes.

While our experiments do demonstrate the prevalence of brine-rejection-fingers, we do omit several physical processes found in real lake systems. Namely, we do not include the effect of wind, or precipitation that play a role in the freezing rate. Additionally, the

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\(^1\) Boreal lakes have a mean ice cover of 0.5m over the winter (Kirillin et al.; 2012), which would predict \( F_s = 6 \times 10^{-9} \) g/kg m/s, assuming a linear ice growth over 6 months.
freezer temperature was fixed (modulo periodic oscillations from the freezer), while daily atmospheric temperatures can vary significantly. Nevertheless, these additional parameters primarily affect the surface salt-flux $F_s$, and hence $R$, modifying the evolution of the salt fingers, but not eliminating them.

Measuring $F \approx 1$ is an indication that the surface salt flux is distributed evenly with depth. This is supported by the 2D numerical results of Olsthoorn et al. (2020), which showed that the structure of the salt stratification remained nearly fixed, while its value increased over time (Figure 4d). The observation that the salinity profiles 'shift' to higher salinity agrees with the measurement $F \approx 1$. Where $R \lesssim 0.5$, the salinity partially pools, such that there is a preferential increase in salinity at the tank bottom.

A shift in the salinity stratification is also observed in field measurements. Figure 4e is a plot salinity profiles taken from Base Mine Lake ($S_0 \approx 1 \text{ g/kg}$), a lake in central Alberta, Canada (Tedford et al., 2019). We find that the shape of the salinity profiles are consistent after a month, while the average value has increased under the ice.

Certainly, brine rejection is expected to play a larger role for lakes with higher salinity, such as those in industrially impacted water bodies (e.g. tailing ponds Pieters & Lawrence, 2009). However, even in low-salinity lakes, brine rejection is able to penetrate the temperature stratification and transport water from the ice-water interface down to the lake bottom. Other under-ice mixing processes, such as radiatively driven convection (Kirillin et al., 2012; Ulloa et al., 2018), are localized near the ice-water interface and necessarily rely on mixing of the temperature stratification. As highlighted by B. Yang et al. (2020), weak salinity gradients at depth can balance the buoyancy effects of the temperature stratification and thus, even 'small' salt fluxes can effect the stratification at lake bottoms. This work highlights that brine rejection may play a role transporting salt, and other dissolved constituents, to otherwise inaccessible regions.

6 Conclusion

We have shown that brine rejection enhances salt transport between the ice-water interface and lake bottoms. While limited supporting field measurements exist, we argue that salt-fingering will occur in most seasonally ice-covered lakes. Given that many freshwater lakes are becoming increasingly saline due to anthropogenic forcing (Dugan et al., 2017), we expect brine rejection to play an increasingly important role in the future.

Appendix A Time to Instability

When will salt fingers form? As above, we implicitly assume that the flux of salt under the ice is constant with time. If this were to continue indefinitely, eventually the gradient of salt at the ice-water interface will increase the fluid density to the point that the system will become unstable. If you keep adding salt, eventually the salt will be heavy enough to sink. In this section, we will discuss the earliest time at which the system will go unstable to brine-rejection-induced instabilities.

In the absence of any fluid motion, the temperature and salinity fields will simply diffuse. That is,

$$\frac{\partial T}{\partial t} = \kappa_T \nabla^2 T, \quad \frac{\partial S}{\partial t} = \kappa_S \nabla^2 S,$$

We can solve for $T, S$ analytically given the appropriate boundary conditions. As the instability is expected to occur significantly prior to the diffusive timescale, we will make the simplifying approximation that the bottom is infinitely far away to simplify the an-
alytical solution. These semi-infinite solutions are

\[ T = -T_f - (T_f + T_{B_0}) \text{erf} \left( \frac{z}{\sqrt{4\kappa_T t}} \right), \]  
\[ S = \frac{F_S}{\kappa_S} \left( \frac{4\kappa_S t}{\pi} \exp \left( -\frac{z^2}{4\kappa_S t} \right) + \text{erf} \left( \frac{z}{\sqrt{4\kappa_S t}} \right) + S_0 \right). \]  

(A2)  
(A3)

As the domain is semi-infinite, we set \( z = 0 \) as the ice-water interface. As \( \kappa_S \gg \kappa_T \), for low \( F_S \), the temperature stratification will be much more diffuse than the salinity stratification when the system goes unstable. Near the ice-water interface, then, the temperature field will be nearly uniform and will not have a dominant effect on the stability of the saline boundary layer. Note, that we are assuming that the ice-water interface is approximately fixed in time, and does evolve much slower than the diffusive timescale for temperature.

As discussed in detail in Drazin & Reid (2004), this system will go unstable once the Rayleigh number of this system surpasses some critical value. The instability will be primarily dependent on the salt stratification and, in particular, the salinity difference over the salinity boundary layer height. The depth of the boundary layer is given as \( \delta \sim \sqrt{\kappa_S t} \). The salinity difference over this boundary layer is given \( \Delta S = S(\tilde{z} = H) - S_0 \sim F_S \sqrt{\frac{t}{\kappa_S}} \). In this case, the Rayleigh number is

\[ \text{Ra}_S = \frac{g\beta \Delta S \delta^3}{\nu} \sim \frac{g\beta F_S \nu}{\kappa_S} t^2. \]  

(A4)

Thus, the critical time of instability will occur at

\[ t_c = C \left( \frac{g\beta F_S}{\nu} \right)^{-\frac{1}{2}}, \]  

(A5)

with some constant \( C \).

From the laboratory experiments, we estimated the time of finger formation through a linear regression of the position of initial salt fingers as a function of time. We then compared this finger formation time to the predicted finger formation time (A5). Figure A1 is a comparison plot to these two quantities, normalized by the temperature diffusion timescale \( \tau_T = H^2/\kappa_T \). We find that the scaling (A5) does approximate the laboratory observations. As this estimate does not include the effect of the temperature stratification, this would partially explain some of the noise in the fit.

**Appendix B Optimal Diffusivity**

We use an pseudospectral solver, built in-house, to estimate the the optimal diffusivity \( \kappa_* \) between the laboratory data and the diffusion equation. That is, we found \( \kappa_* \) by minimizing the functional

\[ J = \int \int (T_{Lab} - T)^2 \, dz \, dt + \int \int \theta^* \left( \partial_t T - \partial_z (\kappa_0 \partial_z T + F) \right) \, dz \, dt. \]  

(B1)

By construction, the solved temperature \( T \) satisfies the forced diffusion equation. Here, \( T_{Lab} \) is the laboratory temperature record interpolated onto a 64-point grid. The variables \( \theta \) and \( \kappa_0 \) are a Lagrangian multiplier and a constant reference diffusivity. The forcing \( F \) is then used to solve for \( \kappa_* \) as

\[ F = \kappa_* \partial_z T. \]  

(B2)

The optimal \( \kappa_* \) is then averaged over a suitable time range and re-interpolated onto the sparse grid of the temperature sensors and the position of the ice-water interface.

Figure B1 is a comparison plot between the laboratory data and the optimized diffusion solution. Note that the diffusion equation does a reasonable job at approximating the laboratory data.
Figure A1. Time of finger formation as a function of $\frac{g\beta F_2}{V}\tau_T^2$. Both axes have been normalized by the temperature diffusion timescale $\tau_T = \frac{H^2}{\kappa_T}$.

Figure B1. Comparison between the interpolated laboratory data (top) and the optimized diffusion solution (bottom).
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