Restoring function of the orbiting electron wave packet

Vineet Kumar\textsuperscript{1,2}

\textsuperscript{1}Quanterro Technologies FZC LLC, Villa 21, Street 04, Khalifa City A, Abu Dhabi, UAE (68952)
\textsuperscript{2}AiFi Technologies LLC, Villa 13, Street 09, Khalifa City A, Abu Dhabi, UAE (68952)

vineet05k@gmail.com

Abstract. About the reference line as a closed curve along which electron moves at a discrete distance from the nucleus of atom, if asked, locally, is there any distribution alike then it is simple to answer yes. This is due to the group of wave of electron protrudal as wave packet. Revolving around the nucleus with certain kinetic energy determines the peculiar wave length, in a way that higher the energy lower the wavelength is as suggested by de Broglie, does this means that with respect to local points on closed curve the energy is distributed to every wave lines, yes it means. But, when asked in comparative to classical spring-mass oscillating system revolving, is there any restoring part in the orbiting electron wave packet then this is important to know. This paper based on the velocity operator cataloging along with its correspondence to energy-frequency relation discuss about the restoring function of orbiting electron wave packet and its importance in the orbital angular momentum.

1. Introduction

Based on the inherence in terms of energy content, is it possible to modify the energy-frequency relation of an orbiting electron in a stationary orbit \( m \) from \( E_m = h \nu_m \) to \( \sqrt{E_{om}^2 + \sigma_m^2} = h \nu_m \)? Where, \( h \nu_m \) hold equal to the terms given by equation (1) and \( h \) is the Planck’s constant. According to de Broglie a moving electron, whatever its nature, electron has a wave properties associated with it [1]. In other words, a particle is not only ‘protrudal’, but protrudal as the packet of lines \( l_r \) for \( r = 1, 2, \ldots, \infty \); i.e. \( l_1, l_2, l_3, \ldots, l_r, \ldots \), active such that each satisfy by equation and therefore determines the overall collective. Over space-time coordinate the solutions for equation of action of \( l_r \) can be some possible but depend upon the conditions, for instance like the string wave. The constant value solution determines the action of line \( l_r \) inactive and the absence of it as well.

\[
h \nu_m = \sqrt{(\int_0^{\nu_{om}} \! h d\nu)^2 + (m_e Q)^2} \tag{1}
\]
where, $Q \propto Q_{x_k} = 2^n x_k, v_{x_k}$

$$
\sqrt{\lambda_{x_k} \left[1 - \frac{1}{2^n x_k}\right]} \left( V_{\perp x_k} \psi \right)^* \left(V_{\perp x_k} \psi \right) dx_k \sqrt{\lambda_{x_k} \left[1 - \frac{1}{2^n x_k}\right]} \left(x_k \psi \right)^* \left(x_k \psi \right) dx_k
$$

The constant $\lambda_{x_k}$ of eqn (1) are the de Broglie’s wavelength of electron along orthogonal $xyz$ axes and $1/\nu/2^n$ is the elapsed time for circumferential length of $\lambda/2^n$ related to $\vec{V}_|| = \vec{V}_\parallel (V_{x_k} x_k, \partial_{x_k}; t)$. The $\langle \sigma^n \rangle$ part of $E_m$ corresponds the double integral term of eqn (1), as alternating one, both over spatial and temporal variables, whereas $E_{om}$ mark the component $hv_{om} \cdot m_e$ is the mass of electron and $n_{x_k}$ are positive integers & equal. For $n$ equal to let say 4 bit, the total $2^4 = 16$ bins possible within the wavelength range $\lambda_{x_k}$ and given by the difference between upper and lower limit of integration of eqn (1). From eqn (1), it is not the energy difference $\int_0^{\nu_{||}} hdv - \int_0^{\nu_{om}} hdv = \int_0^{\nu_{om}} hdv$ between the stationary orbits $l$ and $m$ for photon exchange but it is the difference $\int_0^{\nu_{im}} hdv = E_{im}$ only [2], however the lines $\nu$ of $\psi$ in case of process $\left[ \frac{1}{2} \left( \int_0^{\nu_{mi}} hdv - \int_0^{\nu_{im}} hdv \right) + \int_0^{\nu_{im}} hdv \right]$ restricted by $\int_0^{\nu_{im}} hdv$ get modified. The integral part of eqn (1), so realize, is based on the mechanics analogous to develop eqn (4). Of a considered integral, the term $V_{\perp x_k} \psi = V_{\perp x_k} (V_{x_k} x_k, \partial_{x_k}; t) \psi$ determines the transverse action in per unit time $1/\nu/2^n$ and the term $\langle x_k \psi \rangle$ of position coordinate with in the spatial limiting range of $\lambda_{x_k}/2^n$. * is the complex conjugate. The velocity vector $\vec{V}_{||}$ over differentials $V_{x_k} \partial_{x_k}$ at any time $t$ is different from the $\vec{V}_{\parallel}$ such that the resultant of two is given by eqn (2) as the linear operator. $(\partial_x \partial_y \partial_z; t) = (V_{x_k} x_k; \partial_{x_k}; t)$ are the differentials over spatial coordinate $(xyz) = (V_{x_k} x_k)$, and $\partial_x = \partial/\partial x$. $i$ is imaginary symbol and $\hat{x}, \hat{y}$ & $\hat{z}$ are unit vectors along $x, y$ & $z$ axes respectively. $V_{\perp x_k}$ for all $x_k = xyz$ are the different components of $\vec{V}_{\perp}$, similarly of $\vec{V}_{||}$ as well. $\hbar = \frac{\hbar}{2\pi}$

$$
\vec{V}_{\perp} \left( \bigvee_k x_k, \partial_{x_k}; t \right) + \vec{V}_{\parallel} \left( \bigvee_k x_k, \partial_{x_k}; t \right) = - \frac{i\hbar}{m_e} \sum_{x_k=xyz} \hat{x}_k \partial_{x_k}
$$

The zero value of $Q$ possible when the subjection of $\vec{V}_{\perp}$ in consideration with electron wave function $\psi$ gives 0, which is not possible. This $\vec{V}_{\perp x_k} \psi = 0$ remarks the line of motion of electron around the nucleus with no transverse effect. However, the subjection of $V_{\parallel x_k}$ in consideration of $\psi$ with a get rid of circular effect condition due to the extraneous in a way modified in case of rectilinear motion. Keeping the question about the eqn (1) of subsistence and their related outcomes, the structure of this paper is as follows: it begins with introduction in Sect 1, then followed by explanation in Sect 2. And, finally the paper ends with conclusion in Sect 3. Here, the consideration of bit $n$ is to bind the integral limit with in $\lambda/2^n$ and also for the related time term $T/2^n$ to approximate the relation. For the possible $n$ bit, the energy contain in each of the different bins are equal. Now the question is how does $2^n$ with both $\lambda$ and $T$ involve? To answer this; consider the $Q$ term of $h\nu_m$ which is equivalent to $\langle \sigma_m^2 \rangle$; when observe it’s a same value over the entire $\lambda$ or $\lambda/2$. Even same, when half of $\lambda/2$, i.e. $\lambda/4 = \lambda/2^2$ is there, so for the sake of considered integral that shows correspondence between $\lambda_{x_k}$ and $v_{x_k}$ the $2^n$ bins assumed. With the first cut step of $\lambda$, the $\lambda/2$ as crest part and $\lambda/2$ as trough part obtained in first bit; in second bit each of $\lambda/2$ of crest and $\lambda/2$ of trough divided to give $\lambda/4$, $\lambda/4$ of $\lambda/2$ of crest part and $\lambda/4$, $\lambda/4$ of $\lambda/2$ of trough part respectively. In 3 bit each $\lambda/4$ determine crest and $\lambda/4$ & $\lambda/4$ of trough parts divided to give $\lambda/8$, $\lambda/8$ of $\lambda/4$ & $\lambda/8$, $\lambda/8$ of $\lambda/4$
The product between \( d_t K(t) \) and \( d_t P(t) \) at any time be let, \( p(t) = m_e V(t) d_t V(t) k_s w(t) d_t w(t) = - (k_s w(t) d_t w(t))^2 \). Also \( p(t) \) equals to \( k_s w(t) d_t w(t) m_e V(t) d_t V(t) = - (m_e V(t) d_t V(t))^2 \). This indicates that there are no any chance that \( p(t) \) ever be positive for such restoring system. Similarly, the expressions \( q(t) = \sqrt{-k_s w(t) d_t w(t)m_e V(t) d_t V(t)} = |m_e V(t) d_t V(t)| \) always be positive real. Also \( q(t) = |k_s w(t) d_t w(t)| \). If \( |k_s w(t) d_t w(t)| = k_s w(t) d_t w(t) \) then for harmonic motion \( |m_e V(t) d_t V(t)| = - m_e V(t) d_t V(t) \). Using \( q(t) \) in \( U \), it represent eq_n (3).

\[
U = m_e \omega \left[ \int (\mp \sqrt{-w(t)d_t w(t)V(t)d_t V(t)} dt) + \int \left( \pm \sqrt{V(t)d_t V(t)w(t)d_t w(t)} dt \right) \right] (3)
\]

This \( U \) of eq_n (3), let \( U_1 \), is for a single system. In case of multiple systems in different directions with common end connected, the total restoring energy of such classical system is \( \sum U_j \). Along with \( \sum U_j \) there is rotational kinetic energy \( I_{cm} \omega_c^2 / 2 \) and potential energy \( P_c \) given by eq_n (4), if the system is revolving about an axis. \( \omega_j \) is the angular frequency of \( j^{th} \) system having same mass \( m_e \) and \( I_{cm} \) is the moment of inertia of system. This \( E \) term of eq_n (4) is like the \( Q \) part of the wave packet which comprises of a group of wave each with certain velocity, wavelength, phase and amplitudes such that the centre of mass is the local constructive point. Similarly, the part \( I_{cm} \omega_c^2 / 2 \) of term \( E_1 \) can also be compare with the operator \( \hat{V} |_1 \), but need to apply the condition for such. That is, if there is somehow like crest and trough portions in the path of revolving system about an axis then the velocity which corresponds the \( \omega_c \) only earlier, in this case responsible for a wave like motion also not only subject to the local connected point but to all the points of system. In other words, each mass \( m_e \) besides \( \omega_j \) in time period \( 2\pi / \omega_c \) of rotation also posses other angular velocity as well due to crest and trough portion of the rotating orbit.

\[
E = \left( \frac{1}{2} I_{cm} \omega_c^2 + P_c \right) + \frac{m_e \sum_j \omega_j \left[ \int (\mp \sqrt{-w_j(t)d_t w_j(t)V_j(t)d_t V_j(t)} dt) + \int \left( \pm \sqrt{V_j(t)d_t V_j(t)w_j(t)d_t w_j(t)} dt \right) \right]}{E_1} (4)
\]
concern of $V_{||x_k}$ able to make differentiated out from eq_n (2). The classification of such respective part and even of $V_{||x_k}$ is very complex and cannot be possible therefore, but this does not means that the $Q$ part of eq_n (1) is not there, it is there in terms of restoration over space-time variables, which can be seen in orbital angular momentum. The orbital angular momentum operator $\hat{L}(V_k x_k, \partial x_k; t)$ of electron and its component is given by eq_n (5) and eq_ns (5.1), (5.2) & (5.3) respectively.

$$\hat{L}
\left(\bigvee_{k} x_k, \partial x_k; t\right)
= \hat{r}
\left(\bigvee_{k} x_k, t\right)
\times m_e
\left[\hat{V}_{||} \left(\bigvee_{k} x_k, \partial x_k; t\right) + \hat{V}_{\perp} \left(\bigvee_{k} x_k, \partial x_k; t\right)\right]$$

or equivalently,

$$L_x \left(\bigvee_{k} x_k, \partial x_k; t\right)
= m_e \cdot det
\left[
\begin{array}{cc}
y & z \\
V_{||y} + V_{\perp y} & V_{||z} + V_{\perp z}
\end{array}
\right]$$

$$L_y \left(\bigvee_{k} x_k, \partial x_k; t\right)
= m_e \cdot det
\left[
\begin{array}{cc}
z & x \\
V_{||z} + V_{\perp z} & V_{||x} + V_{\perp x}
\end{array}
\right]$$

$$L_z \left(\bigvee_{k} x_k, \partial x_k; t\right)
= m_e \cdot det
\left[
\begin{array}{cc}
x & y \\
V_{||x} + V_{\perp x} & V_{||y} + V_{\perp y}
\end{array}
\right]$$

$\det[ ]$ is the determinant of matrix $[ ]$. The matrix $[ ]^x_{||y+\perp}$ is given by eq_n (5.1) can be separate by the $[ ]_{||y+\perp}$ and given by eq_n (6.1). Similarly, the matrices $[ ]_{||z+\perp}$ and $[ ]_{||x+\perp}$ are given by eq_ns (6.2) and (6.3) respectively.

$$\begin{bmatrix}
y \\
V_{||y} + V_{\perp y} \\
V_{||z} + V_{\perp z}
\end{bmatrix}
= \begin{bmatrix}
y & z \\
V_{||y} & V_{||z}
\end{bmatrix}
\cdot \begin{bmatrix}
y \\
V_{\perp y}
\end{bmatrix}
+ \begin{bmatrix}
y & z \\
V_{\perp y} & V_{\perp z}
\end{bmatrix}
\cdot \begin{bmatrix}
z \\
V_{||z}
\end{bmatrix}$$

$$\begin{bmatrix}
z \\
V_{||z} + V_{\perp z} \\
V_{||x} + V_{\perp x}
\end{bmatrix}
= \begin{bmatrix}
z & x \\
V_{||z} & V_{||x}
\end{bmatrix}
\cdot \begin{bmatrix}
z \\
V_{||x}
\end{bmatrix}
+ \begin{bmatrix}
z & x \\
V_{||x} & V_{||z}
\end{bmatrix}
\cdot \begin{bmatrix}
z \\
V_{||z}
\end{bmatrix}$$

$$\begin{bmatrix}
x \\
V_{||x} + V_{\perp x} \\
V_{||y} + V_{\perp y}
\end{bmatrix}
= \begin{bmatrix}
x & z \\
V_{||x} & V_{||y}
\end{bmatrix}
\cdot \begin{bmatrix}
x \\
V_{||y}
\end{bmatrix}
+ \begin{bmatrix}
x & z \\
V_{||y} & V_{||x}
\end{bmatrix}
\cdot \begin{bmatrix}
x \\
V_{||x}
\end{bmatrix}$$
$$\begin{align*}
\left[ V_{||x} + V_{\perp x} \ V_{||y} + V_{\perp y} \right] = & \begin{bmatrix} \frac{x}{2} & \frac{y}{2} \\ \frac{x}{2} & \frac{y}{2} \end{bmatrix} + \begin{bmatrix} \frac{x}{2} & \frac{y}{2} \\ \frac{x}{2} & \frac{y}{2} \end{bmatrix}
\end{align*}$$

(6.3)

From eq_ns (6.1) to (6.3) this can be viewed that, for instance, if the rotation is along the z axis then the parts (6.1.1) of (6.1) and (6.2.1) of (6.2) over $\psi$ are applicable but not (6.1.2) and (6.2.2), however the eq_n (6.3) has nothing to do with it. The part $[.]_{(y, z)}$ of eq_n (6.1.1) and $[.]_{(z, x)}$ of eq_n (6.2.1) is due to the $Q$ part of the wave packet. Again, it is this non-zero value of $V_{\perp x_k}(\mathcal{V}_k \mathcal{X}_k; \partial \mathcal{X}_k; t)\psi$ by which the commutation relation of all for the electron orbiting around the nucleus satisfy.

3. Conclusions

Based upon the considered integral given by eq_n (1) due to the eq_n (2), an important aspect about the orbital angular momentum of electron in atom is covered and given by eq_ns from (6.1) to (6.3), such that if the orbit is orthogonal to any of the axis then even in that case the components along the rest of the other axes is not zero. It is due to the energy part describe by eq_n (1) the commutation relation of all, particularly depend on orbital and their components, hold. Eq_ns (1) and (2) can be better realize by the spring-mass classical system describe by eq_n (4).

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References

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