Initial state fluctuations and higher harmonic flow in heavy-ion collisions

Björn Schenke
Physics Department, Brookhaven National Laboratory, Upton, NY

in collaboration with
C. Gale, S. Jeon (McGill), P. Tribedy (VECC), R. Venugopalan (BNL)

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Introduction

- Large elliptic flow has indicated fluid behavior of matter created at RHIC in early 2000’s. BNL announces “perfect liquid” in 2005 press release.

- The importance of fluctuations was realized later and analysis of odd flow harmonics began in 2010 since B. Alver, G. Roland, Phys.Rev. C81, 054905.

- Analysis of all flow harmonics can help determine initial state properties and transport properties of the QGP (and hadron gas).

- I will discuss systematics within event-by-event hydrodynamics, present a QCD based model for the initial state including geometric and color charge fluctuations, and make first comparisons to experimental data.

B. Schenke, S. Jeon, C. Gale, Phys. Rev. C85, 024901 (2012)
B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012)
B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1206.6805 (2012)
Initial state fluctuations: MC-Glauber model

To study systematics we use a simple geometric model.
Later, we improve significantly on this.

- Sample Woods-Saxon distributions to determine all nucleon positions (green and red circles).
- Sample impact parameter $b$ and overlap nuclei.

- Nucleon-nucleon collision occurs if distance is $< \sqrt{\sigma_{NN}/\pi}$
- At position of collision add 2D-Gaussian energy density distribution with width $\sigma_0$ (blue blobs).
  $\sigma_0$ (e.g. 0.4 fm) is a model parameter.

$\Psi_{PP2}$ and $\Psi_{PP3}$ are participant planes for ellipticity and triangularity.
Hydrodynamic evolution

Given the initial energy density distribution we solve

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \pi^{\mu\nu}$$

using only shear viscosity: $$\pi^{\mu\nu} = 0$$

initial \hspace{2cm} ideal \hspace{2cm} $$\eta/s = 0.16$$

3+1D event-by-event relativistic viscous hydrodynamic simulation

MUSIC  B. Schenke, S. Jeon, C. Gale, Phys. Rev. C82, 014903 (2010); Phys.Rev.Lett.106, 042301 (2011)
Flow analysis  B. Schenke, S. Jeon, C. Gale, Phys. Rev. C85, 024901 (2012)

After Cooper-Frye freeze-out and resonance decays in each event we compute

\[ v_n = \langle \cos[n(\phi - \psi_n)] \rangle \]

with the event-plane angle \( \psi_n = \frac{1}{n} \arctan \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle} \)

Sensitivity of event averaged \( v_n \) on

Sensitivity to viscosity and initial state structure increases with \( n \)
New model for the initial state

To make use of $v_n$ measurements we need a more rigorous understanding of the initial state and its fluctuations.

Gluon saturation $\rightarrow$ strong gluon fields, large occupation numbers at $k_T \leq Q_s$ $\rightarrow$ classical field approximation

Solve classical Yang-Mills equations event-by-event, including geometric and color charge fluctuations.
Sample nucleon positions from Woods-Saxon distributions.

Use IP-Sat model fit to HERA data to get $Q_s^2(x, b_\perp)$ for each nucleon. The color charge density squared $g^2 \mu^2$ is proportional to $Q_s^2$.

Add all $g^2 \mu^2(x_\perp)$ in each nucleus to obtain $g^2 \mu_1^2(x_\perp)$ and $g^2 \mu_2^2(x_\perp)$.

Sample $\rho^a$ from local Gaussian distribution for each nucleus

$$\langle \rho^a(x_\perp)\rho^b(y_\perp) \rangle = \delta^{ab}\delta^2(x_\perp - y_\perp)g^2 \mu^2(x_\perp)$$
Gauge fields before the collision

Color currents:
\[ J_1^\nu = \delta^{\mu^+} \rho_1(x^-, x_\perp) \]
\[ [D_\mu, F^{\mu\nu}] = J_1^\nu \]
\[ J_2^\nu = \delta^{\mu^-} \rho_2(x^+, x_\perp) \]
\[ [D_\mu, F^{\mu\nu}] = J_2^\nu \]

Correlations and fluctuations in the gluon fields:

Shown is the correlator of the Wilson lines
\[ C_{(1,2)}(x_\perp) = \frac{1}{N_c} \text{Re}[\text{tr}(V(1, 2)^\dagger(0, 0)V(1, 2)(x, y))] \]

The length scale of fluctuations is \(1/Q_s\) - not the nucleon size
Energy density

B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012)

Solve for gauge fields after the collision in the forward lightcone
Compute energy density in the fields at $\tau = 0$ and later times with CYM evolution
Lattice: Krasnitz, Venugopalan, Nucl. Phys. B557 (1999) 237

Very different initial energy density distributions in the models
MC-KLN: Drescher, Nara, nucl-th/0611017
mckln-3.52 from http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html with defaults, energy density scaling
Solve for gauge fields after the collision in the forward lightcone
Compute energy density in the fields at $\tau = 0$ and later times with CYM evolution

Lattice: Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

Very different initial energy density distributions in the models

MC-KLN: Drescher, Nara, nucl-th/0611017

mckln-3.52 from http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html with defaults, energy density scaling
$dN_g/dy$ at finite time $\tau = 0.4$ fm in transverse Coulomb gauge $\partial_i A^i = 0$

$N_{\text{part}}$ from MC-Glauber with $\sigma_{NN} = 42$ mb and 64 mb respectively

Experimental data: PHENIX, Phys.Rev.C71 034908 (2004) and ALICE, Phys.Rev.Lett. 106, 032301 (2011)

Scaled by $2/3$ to compare to charged particles.

Some freedom in normalization - will need to account for entropy production.
$dN_g/dy$ at finite time $\tau = 0.4 \text{ fm}$ in transverse Coulomb gauge $\partial_i A^i = 0$ $N_{\text{part}}$ from MC-Glauber with $\sigma_{NN} = 42 \text{ mb}$ and $64 \text{ mb}$ respectively

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Scaled by $2/3$ to compare to charged particles.
Some freedom in normalization - will need to account for entropy production.
\[ P(dN_g/dy) \text{ at time } \tau = 0.4 \text{ fm with } P(b) \text{ from a Glauber model} \]

Experimental data: STAR, Phys. Rev. C79, 034909 (2009)

Glasma model gives a convolution of negative binomial distributions
No need to put them in by hand
Eccentricities

\[ \varepsilon_n = \sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2} \]

Averages are weighted by the energy density

\[ \langle r^n \rangle \]

- \( \varepsilon_n \) larger in Glasma model for odd \( n \)
- \( \varepsilon_n \) smaller in Glasma model for \( n = 2 \) (for \( b > 3 \) fm)
  - about equal for \( n = 4 \), larger for \( n = 6 \)
Compute all components of $T^{\mu\nu}$

Determine energy density and $(u^x, u^y)$ at $\tau > 0$ fm from $u_\mu T^{\mu\nu} = \varepsilon u^\nu$
as input for hydrodynamic simulations

No instabilities (need full 3+1D Yang-Mills for that):

system is far from equilibrium - cannot yet match $\Pi^{\mu\nu}$

Energy density and $(u_x, u_y)$ at $\tau = 0.4$ fm/c
Centrality selection and flow
Centrality selection and flow

![Glasma centrality selection graph](image)

![Distribution of b in 20-30% central bin](image)
Centrality selection and flow

Glasma centrality selection

0-5%
5-10%
10-20%
20-30%
30-40%
40-50%
50-60%

Distribution of $b$ in 20-30% central bin

$P(b)$

$\langle v_n^2 \rangle^{1/2}$ vs. $p_T$ [GeV]

ATLAS 20-30%, EP

$\tau_{\text{switch}} = 0.2$ fm/c

$\eta/s = 0.2$
Centrality selection and flow

Glasma centrality selection

P(dN_g/dy)

Glasma centrality selection

0-5%
5-10%
10-20%
20-30%
30-40%
40-50%
50-60%

Hydro evolution

MUSIC

Hydro evolution

MUSIC

ALICE data v_n{2}, p_T > 0.2 GeV

η/s = 0.2

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)
ALICE collaboration, Phys. Rev. Lett. 107, 032301 (2011)
Event-by-event distributions of $v_n$

comparing to all new ATLAS data:
https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CONFNOTES/ATLAS-CONF-2012-114/

see talk by Jiangyong Jia in Session 4A, today, 11:20 am

Preliminary results: Statistics to be improved.
Event-by-event distributions of $v_n$ comparing to all new ATLAS data:
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Björn Schenke (BNL)
QM2012
Effect of initial flow

Weak effect of initial flow on hadron $v_n(p_T)$

Expect stronger effect for photon $v_n$:
Photons are mostly produced early at high temperatures

Effect of different switching time $0.4\,\text{fm}/c$ is very weak

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)
Temperature dependent $\eta/s$

Use $\eta/s(T)$ as in Niemi et al., Phys.Rev.Lett. 106 (2011) 212302 and arXiv:1203.2452

$v_n(p_T)$ for given $\eta/s(T)$ indistinguishable from constant $\eta/s = 0.2$

More detailed study needed

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)
Experimental data:
extracted in Retinskaya et al., Phys.Rev.Lett. 108 (2012) 252302
from ALICE data in K. Aamodt et al., Phys. Lett. B 708, 249 (2012)
Summary and conclusions

- Higher flow harmonics are sensitive to viscosity and fluctuating initial states.

- **IP-Glasma model**
  - includes geometric and color charge fluctuations
  - produces negative binomial fluctuations
  - has different eccentricities than previous CGC based models
  - provides initial flow profile from the non-equilibrium stage
  - describes flow coefficients up to at least $v_5$ with $\eta/s = 0.2$

- Initial flow has weak effect on hadronic $v_n$
  - Photon study underway

- Temperature dependent $\eta/s$ not distinguishable from average $\eta/s$

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B. Schenke, S. Jeon, C. Gale, Phys. Rev. C85, 024901 (2012)
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Gauge fields before the collision

Color currents:

\[ J_1^\nu = \delta^{\mu+} \rho_1(x^-,x_\perp) \]
\[ [D_\mu, F^{\mu\nu}] = J_1^\nu \]
\[ J_2^\nu = \delta^{\mu-} \rho_2(x^+,x_\perp) \]
\[ [D_\mu, F^{\mu\nu}] = J_2^\nu \]

Solution in covariant gauge:

\[ A_{\text{cov}(1,2)}^+(x^-,x_\perp) = \frac{-g \rho_{(1,2)}(x^-,x_\perp)}{\nabla_\perp^2 + m^2} \]

with infrared cutoff \( m \) of order \( \Lambda_{\text{QCD}} \).

Solution in light cone gauge:

\[ A_{(1,2)}^+(x_\perp) = A_{(1,2)}^-(x_\perp) = 0 \]
\[ A_{(1,2)}^i(x_\perp) = \frac{i}{g} V_{(1,2)}(x_\perp) \partial_i V_{(1,2)}^\dagger(x_\perp) \]

\( V \) is the path-ordered exponential of \( A_{\text{cov}(1,2)}^+ \).
Gauge fields before the collision

The correlator of the Wilson lines

\[ C_{(1,2)}(x_\perp) = \frac{1}{N_c} \text{Re}[\text{tr}(V(1, 2)\dagger(0, 0)V(1, 2)(x, y))]] \]

with

\[ V_{(1,2)}(x_\perp) = P \exp \left( -ig \int dx^- \frac{\rho_{(1,2)}(x^-, x_\perp)}{\nabla^2_\perp + m^2} \right) \]

shows the degree of correlations and fluctuations in the gluon fields.

The length scale of fluctuations is \(1/Q_s\). Not the nucleon size.
Gauge fields after the collision (Glasma)

Initial condition on the lightcone: require that fields match smoothly on the lightcone.

Solution:

\[ A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)} \]
\[ A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2} [A^i_{(1)}, A^i_{(2)}] \]

On the lattice the Wilson lines in the future lightcone are obtained from the condition:

\[ \text{tr} \left\{ t^a \left[ \left( U^i_{(1)} + U^i_{(2)} \right) \left( 1 + U^i_{(3)} \right) - \left( 1 + U^i_{(3)} \right) \left( U^i_{(1)} + U^i_{(2)} \right) \right] \right\} = 0 \]

where \( t^a \) are the generators of \( SU(N_c) \) in the fundamental representation. Solve iteratively.

Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

\[ U^i_{(1,2),j} = V^i_{(1,2),j} V^\dagger_{(1,2),j} + e_i \]

(gauge transform of 1: pure gauge)
Negative binomial fluctuations

Fluctuations in the total energy per unit rapidity produce negative binomial distribution (NBD).

B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1202.6646, Phys. Rev. Lett. 108, 252301 (2012)

\[
P(n) = \frac{\Gamma(k + n)}{\Gamma(k)\Gamma(n + 1)} \frac{\bar{n}^n k^k}{(\bar{n} + k)^{n+k}}
\]

Good, since multiplicity in pp collisions can be described well with NBD.

In AA, convolution of NBDs at all impact parameters describes data well too.

P. Tribedy and R. Venugopalan
Nucl. Phys. A850 (2011) 136-156

MC-KLN does not do that - these fluctuations need to be put in by hand.

see Dumitru and Nara arXiv:1201.6382
Negative binomial fluctuations

Extract $k$ and $\bar{n}$ using a fit with

$$P(n) = \frac{\Gamma(k + n)}{\Gamma(k) \Gamma(n + 1)} \frac{\bar{n}^n k^k}{(\bar{n} + k)^{n+k}}$$

at fixed impact parameters.

Ratio of $k/\bar{n}$ is $> 1$ for small $b$ and becomes small $\sim 0.14$ for large $b$. That is close to the value extracted for $p + p$ collisions: Dumitru and Nara arXiv:1201.6382
NBDs and Glasma flux tubes

Glasma flux tube picture:

\[ k = \zeta \frac{N_c^2 - 1}{2\pi} Q_s^2 S_\perp \]

Gelis, Lappi, McLerran, arXiv:0905.3234.

Width of NBD is inversely proportional to the number of flux tubes \( Q_s^2 S_\perp \).

\( S_\perp = \) interaction area.

\( \zeta \) should be close to constant in the flux tube picture.

B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1206.6805

\( \zeta \) should be close to constant in the flux tube picture.
NBDs and Glasma flux tubes

$\zeta$ is not constant because geometric fluctuations are very important. Were not considered in the derivation of

$$k = \zeta \frac{N^2}{2\pi} - \frac{1}{Q_s^2 S_\perp}$$

Eliminate by using smooth nucleon distributions:

![Graph showing the relationship between $Q_s^2 S_\perp$ and $\zeta$. The graph includes data points representing average nucleon positions with fixed $b=0$ fm.]

B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1206.6805
More centrality classes: IP-Glasma + MUSIC

\[ \left\langle v_n \right\rangle^{1/2} \]

\[ p_T \text{ [GeV]} \]

\[ \eta/s = 0.2 \]

\[ \tau_{\text{switch}} = 0.2 \text{ fm/c} \]

\[ \text{ATLAS 0-5\%, EP} \]

\[ \text{ATLAS 10-20\%, EP} \]

\[ \text{ATLAS 30-40\%, EP} \]

\[ \text{ATLAS 40-50\%, EP} \]

Björn Schenke (BNL)
Using $\eta/s = 0.16$ overestimates all $v_n$

Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)