A new single-dynamical-scalar-field model of dark energy

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A new single-dynamical-scalar-field model of dark energy is proposed, in which either higher derivative terms nor structures of extra dimension are needed. With the help of a fixed background vector field, the parameter for the effective equation of state of dark energy may cross $w = -1$ in the evolution of the universe. After suitable choice of the potential, the crossing $w = -1$ and transition from decelerating to accelerating occur at $z \approx 0.2$ and $z \approx 1.7$, respectively.

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I. INTRODUCTION

Recent observations\[1–3\] suggest that the universe consists of dark energy (73%), dark matter (23%) and baryon matter (4%). To understand the nature of the dark energy is one of essential tasks in cosmology. The most simple form of the dark energy is the cosmological constant, which fits well to the observation data \[4\] and which has the effective equation of state \(p = -\rho\), i.e. \(w = -1\). If the dark energy is totally contributed by the cosmological constant, the universe is asymptotically de Sitter one and we should study the de Sitter spacetime and asymptotically de Sitter spacetimes in all aspects, including in the framework of general relativity \[5\], in the framework of de Sitter invariant special relativity \[6\], and locally de Sitter invariant gauge theory of gravity \[7, 8\], etc.

However, there is an evidence \[9\] to show that the dark energy might evolve from \(w > -1\) in the past to \(w < -1\) today and cross \(w = -1\) in the intermediate redshift. Obviously, the simple dynamical dark energy models considered vastly in the literature as well as the cosmological constant cannot explain it. For example, \(w\) in the quintessence models \[10\] is always evolving in the range of \(-1 \leq w \leq 1\). In the phantom models \[11\], regardless its wrong-sign kinetic energy term, \(w\) always varies below \(-1\). The parameter \(w\) of the effective equation of state for the general k-essence models \[12\] also fails to cross \(w = -1\). Recently, many dark energy models with the property of crossing \(w = -1\) have been proposed \[13–17\]. In the dynamical-scalar-field models of dark energy \[13, 14\], either the malformed phantom fields or higher derivative term are needed. In \[15\], the structures in extra dimension are introduced to realize the parameter \(w\) crossing \(-1\). In \[16, 17\], the dark energy is supposed to be contributed from a vector field \[16\] or a gas of fermion particles interacting with a condensate represented by a vector field \(b_\mu\) \[17\]. The vector-field and fermion-field models of the dark energy do not need phantom fields, higher derivative terms and more complex structure in extra dimension and so on.

In the present paper, we shall propose a new single-dynamical-scalar-field model, as a toy, of dark energy, which may explain that the parameter \(w\) crosses \(-1\) at \(z \approx 0.2\) and the the universe experience a phase transition form decelerating to accelerating at \(z \approx 1.7\). The basic idea is that the dark energy is not direct effect of a fundamental field, while it is only effectively described by a dynamical scalar field. Taking it into account, we may consider the following model of a single dynamical scalar field coupled to an \textit{a priori} non-dynamical, background covariant vector field,

\[
S_{\phi}^{\text{eff}} = \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - g^{\mu\nu} \phi_{,\mu} A_{\nu} - V(\phi) \right) \tag{1.1}
\]

where the vector \(A_\mu\) describes some unknown effects of the universe and is supposed to have constant zeroth-component in a comoving system. In such a model, no phantom is needed to realize \(w < -1\) and \(w\) crossing \(-1\).

In the following, we shall introduce the model first. And then we shall make some numerical analysis with an ordinary potential and a potential inspired from supergravity \[18\]. Finally, we shall give some concluding remarks.

II. A NEW SINGLE-DYNAMICAL-SCALAR FIELD MODEL

Suppose that the universe is governed by the action

\[
S = S_{\text{EH}} + S_{\text{fluid}} + S_{\phi}^{\text{eff}}, \tag{2.1}
\]

where \(S_{\text{EH}}\) is the Einstein-Hilbert action of gravitation, \(S_{\text{fluid}}\) is the action of perfect fluid, describing the baryonic matter and cold dark matter, and \(S_{\phi}^{\text{eff}}\) is the effective action of dark energy, which has been given in Eq.(1.1). Since \(A_\mu\) is introduced as a part of the effective model of dark energy, we suppose that it only couples to the scalar field but does not couple to the matter. As the standard treatment in cosmology, we suppose that the universe is homogeneous and isotropic on large scale, and further suppose that the universe is flat for simplicity, so that the geometry of the universe is described by the Robertson-Walker metric

\[
ds^2 = dt^2 - a^2(t) \left( dr^2 + r^2 d\Omega_2^2 \right), \tag{2.2}
\]
where $d\Omega_2^2$ is the line-element on a unit sphere. Under the assumption of homogeneity and isotropy, all quantities, including $\rho$, $\phi$, etc., depend on $t$ only. In particular, the stress-energy tensor for the scalar field, defined by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \delta S_\phi^{\text{eff}}$$

reads

$$T_{\mu\nu} = \phi^2(t) \delta_{\mu}^{\alpha} \delta_{\nu}^{\beta} - 2 \phi(t) A_0 \delta_{\mu}^{\alpha} \delta_{\nu}^{\beta} - \frac{1}{2} g_{\mu\nu} \left( \phi^2(t) - 2 \phi(t) A_0 - 2 V[\phi(t)] \right).$$

Comparing the stress-energy tensor (2.4) with the one for perfect fluid, one may define the effective energy density and pressure of scalar field by

$$\rho_\phi(t) = T_{00} - \frac{1}{2} \phi^2(t) - \dot{\phi}(t) A_0 + V[\phi(t)],$$

$$p_\phi(t) = -g^{11} T_{11} = -\frac{1}{2} \phi^2(t) - \dot{\phi}(t) A_0 - V[\phi(t)].$$

As the effective description of dark energy, $\rho_\phi = \frac{1}{2} \dot{\phi}(\phi - 2 A_0) + V$ should always be positive. The parameter $w$ in the effective equation of state $p_\phi = w \rho_\phi$ is then

$$w = \frac{\dot{\phi}^2 - 2 \phi A_0 - 2 V(\phi)}{\dot{\phi}^2 - 2 \phi A_0 + 2 V(\phi)}.$$

It is obvious that for $A_0 > 0$,

$$w \begin{cases} 
\geq -1, & \phi \geq 2 A_0 \text{ or } \phi \leq 0, \\
< -1, & 2 A_0 > \phi > 0 \text{ and } \dot{\phi}^2 - 2 A_0 \dot{\phi} + 2 V(\phi) > 0,
\end{cases}$$

irrelevant,

$$\begin{cases} 
\geq -1, & 0 > \phi > 2 A_0 \text{ and } \dot{\phi}^2 - 2 A_0 \dot{\phi} + 2 V(\phi) \leq 0,
\end{cases}$$

and for $A_0 < 0$,

$$w \begin{cases} 
\geq -1, & \phi \geq 0 \text{ or } \phi \leq 2 A_0, \\
< -1, & 0 > \phi > 2 A_0 \text{ and } \dot{\phi}^2 - 2 A_0 \dot{\phi} + 2 V(\phi) > 0,
\end{cases}$$

irrelevant,

$$\begin{cases} 
0 > \phi > 2 A_0 \text{ and } \dot{\phi}^2 - 2 A_0 \dot{\phi} + 2 V(\phi) \leq 0.
\end{cases}$$

Namely, the parameter $w$ may cross $-1$ in the evolution of the universe.

As mentioned in the previous section, we suppose that $A_\mu$ has a constant norm and constant zeroth-component in Robertson-Walker metric. Under the assumption, the equation of motion for $\phi(t)$ takes the form

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 3 H A_0,$$

where $H = \dot{a}/a$ is the Hubble parameter. Comparing with the standard form of the equation of motion for a scalar field, there exists an additional current due to the existence of $A_0$. Now, the Friedmann equation reads

$$H^2 = \frac{8 \pi G}{3} \left( \frac{1}{2} \dot{\phi}^2 - \phi A_0 + V(\phi) + \rho_{\text{fluid}} \right),$$

where the last term on the right-hand-side is the energy density of matter as a perfect fluid. In the present paper, we are only interested in the late stage of the evolution of the universe, in which both the baryon matter and cold dark matter can be treated as pressureless perfect fluid. Thus, we may set

$$\rho = \rho_0 a^3/\alpha^3,$$

where $\rho_0$ is a constant, denoting the present value of the energy density of all matter. The subscript 0, except in $A_0$ and the following $b_0$, represents to take the value today.
III. NUMERICAL ANALYSIS

In order to solve Eqs.(2.10,2.11) numerically, we recast them as the first-order differential equations with respect to the redshift \( z = (a_0/a) - 1 \),

\[
\frac{d\varphi}{dz} = -\frac{\chi}{h(1+z)} \tag{3.1}
\]

\[
\frac{d\chi}{dz} = \frac{3\chi - b_0}{1+z} + \frac{v'(\varphi)}{h(1+z)} \tag{3.2}
\]

\[
h^2 = \frac{1}{2}\chi^2 - b_0\chi + v(\varphi) + \Omega_{\Lambda 0}(1+z)^3, \tag{3.3}
\]

where

\[
\varphi = \sqrt{\frac{8\pi G}{3}} \phi, \quad \chi = \sqrt{\frac{8\pi G}{3}} \frac{\dot{\phi}}{H_0}, \quad b_0 = \sqrt{\frac{8\pi G A_0}{3}} \frac{H_0}{H_0}, \tag{3.4}
\]

\[
h = \frac{H}{H_0}, \quad v(\varphi) = \frac{8\pi GV(\phi)}{3H_0^2}, \quad \Omega_{\Lambda 0} = \frac{8\pi G \rho_0}{3H_0^2}. \tag{3.5}
\]

In terms of new variable, Eq.(2.7) becomes

\[
w = \frac{\chi^2 - 2\chi b_0 - 2v(\varphi)}{\chi^2 - 2\chi b_0 + 2v(\varphi)}. \tag{3.6}
\]

The initial values of integration are assigned to fit the present observation data [3, 9]:

\[
h_0 = 1, \quad \Omega_{\Lambda 0} := \left(\frac{1}{2}\chi^2 - b_0\chi + v\right)|_0 = 0.73, \quad \Omega_{\Lambda 0} = 0.27, \tag{3.7}
\]

\[
w_0 = -1.33, \quad \chi_0(x_0 - 2b_0) = -0.24, \quad v_0 = 0.85. \tag{3.8}
\]

Now, we consider several examples. The first example is the universe with a quadratic potential

\[
V(\phi) = \frac{1}{2}m^2\phi^2. \tag{3.9}
\]

We may further take \( b_0 = -0.6 \) and

\[
\frac{dw}{dz}|_0 = 1. \tag{3.10}
\]

then \( \chi_0 \approx -0.255, d\varphi/dz|_0 = -\chi_0 \),

\[
v'(\varphi)|_0 = \frac{-\frac{1}{4}(\chi_0(\chi_0 - 2b_0) + 2v_0)^2 - 6v_0(\chi_0 - b_0)^2}{\chi_0(\chi_0 - 2b_0) + 2v_0(\chi_0 - b_0)} = -0.117, \tag{3.11}
\]

\( \varphi_0 = -14.6 \) and \( \frac{v_0^2}{H_0^2} = 8.0 \times 10^{-3} \). FIG. 1 gives the evolution of \( w \) for a flat universe with a quadratic potential. It is obvious that the equation of state across \( w = -1 \) at \( z \approx 0.2 \), which is consistent with the SNe Ia observation [19] and that the transition redshift between the accelerating and decelerating phases occurs at \( z \approx 0.5 \), which is lower than the observational constraint on the transition redshift interval \( 0.6 < z < 1.7 \) [1, 2, 20]. The limit of \( w \) at large \( z \) is 1. For \( b_0 = 0.6 \) the behavior of \( w \) is similar to the case \( b_0 = -0.6 \). It can be shown that \( w \) has the similar behavior for the quartic potential

\[
V(\phi) = \frac{1}{4}4\phi^4. \tag{3.12}
\]
and the inverse power-law potential

\[ V(\phi) = V_0 \phi^{-\alpha}. \tag{3.13} \]

The next example is the universe with the potential inspired from supergravity [18],

\[ V(\phi) = V_0 \phi^{-\alpha} e^{4\pi G \phi^2}, \tag{3.14} \]

where \( \alpha \geq 11 \) is a constant. For the potential, we may take \( \alpha = 12, b_0 = -0.62, w_1 = 1.47 \), thus \( v_0'(\phi) = -0.129, \varphi_0 = -2.0, (\frac{1}{3\pi G})^5 \frac{V_0}{V_0'} = 8.6 \). Fig. 3 gives the evolution of \( w \) for the model. The equation of state across \( w = -1 \) at \( z \approx 0.2 \) and the transition redshift between the accelerating and decelerating phases occurs at \( z \approx 1.6 \), which is consistent with the SNe Ia observation [1, 2, 19, 20]. However, the dependence of \( w \) on \( z \) is not monotonic. \( w \) becomes less than \(-1\) between \( z \approx 0.7 \) and \( z \approx 1.3 \) again. The universe experienced another accelerated expansion around \( z \approx 3.8 \).
IV. CONCLUDING REMARKS

The single-dynamical-scalar-field model of dark energy may explain the parameter $w$ in the effective equation of state crossing $-1$ during the evolution of the universe, as long as a fixed background vector field is introduced. For some potentials, the parameter $w$ crossing $-1$ and transition from decelerating to accelerating occur at $z \approx 0.2$ and $z \approx 1.7$, respectively, which is consistent with the observations. The dependence of the parameter $w$ on the redshift $z$ is far from the linear relation. Thus, it is needed to reanalyze the observation data based on more general form of $w(z)$. By the way, without the introduction of oscillation potential, the universe may also exhibit some oscillation behaviors.

The present model is in the almost standard framework of physics, e.g. in general relativity in 4-dimension. There does not exist phantom in the model, which will lead to theoretical problems in field theory. Instead, an a priori vector field, $A_{\mu}$, is introduced, which is selected in an unspecified way by the cosmological background. It will result in the violation of the Lorentz invariance and might be contributed by the collective effects of the vacuum polarization of some fundamental fields. It is needed to further explore the physical meaning of such a vector field.

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