Fulde-Ferrell pairing instability of a Rashba spin-orbit coupled Fermi gas

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We theoretically analyze the pairing instability of a three-dimensional ultracold atomic Fermi gas towards a Fulde-Ferrell superfluid, in the presence of Rashba spin-orbit coupling and in-plane Zeeman field. We use the standard Thouless criterion for the onset of superfluidity, with which the effect of pair fluctuations is partially taken into account by approximately using a mean-field chemical potential at zero temperature. This gives rise to an improved prediction of the superfluid transition temperature beyond mean-field, particularly in the strong-coupling unitary limit. We also investigate the pairing instability with increasing Rashba spin-orbit coupling, along the crossover from a Bardeen-Cooper-Schrieffer superfluid to a Bose-Einstein condensate of Rashbons (i.e., the tightly bound state of two fermions formed by strong Rashba spin-orbit coupling).

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I. INTRODUCTION

The recent experiment achievement of an unequal two-component gas mixture of ultracold fermionic neutral atoms offers a unique opportunity to solve some longstanding problems in quantum many-body physics [1,2]. The key to ultracold atomic Fermi gases is the incredible purity and precise control afforded over both the interactions between particles and the confining environment. This gives theorists an idealized test-bed for developing new models that are not corrupted by complications due to unknown impurities or disorder, which are often encountered with solid-state materials. At present, the long-sought crossover from a Bardeen-Cooper-Schrieffer (BCS) superfluid to a Bose-Einstein condensate (BEC) of tightly bounded Cooper pairs has been realized experimentally [1], by using broad Feshbach resonances in a Fermi cloud of $^{40}\text{K}$ or $^{6}\text{Li}$ atoms [3,4]. The realization of an exotic inhomogeneous superfluid state, the so-called Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superfluid, in which the Cooper pairs carry nonzero center-of-mass momentum [5,6], has also been indirectly demonstrated [7,8], by tweaking the population imbalance of the two spin components [9,10].

Over the past year, a major advance in the field of ultracold atomic Fermi gases is the creation of synthetic spin-orbit coupling through the use of two counterpropagating Raman laser beams [11-19]. In solid-state systems, it is now widely known that spin-orbit coupling is responsible for certain novel classes of materials, where topological order plays a fundamental role [20,21]. Thus, ultracold atomic Fermi gases appear to be a new paradigm to explore these new types of topological materials. To date, a number of intriguing properties of spin-orbit coupled Fermi gases have been addressed, including the anisotropic bound state [22,23], anisotropic superfluidity [24,25,26], enhanced pseudogap [27], and particularly topological superfluid and Majorana fermions [28,29] - which may enable a form of quantum computing known as topological computation [30].

In this work, we theoretically investigate the possibility of inhomogeneous FFLO superfluidity in spin-orbit coupled atomic Fermi gases. Our research is motivated by the recent study by Zheng and co-workers [31] who showed that the change of topology of Fermi surfaces due to spin-orbit coupling and in-plane Zeeman field can provide a useful mechanics for inhomogeneous superfluidity, in addition to the population imbalance in two spin states. This enhanced inhomogeneous superfluidity was first discussed by Barzykin and Gor’kov in the context of surface superconductivity in solid-state materials such as WO$_3$:Na [32]. In contrast to solid-state superconductors, an important new ingredient of ultracold atomic Fermi gases is strong interaction, which is necessary in order to have an experimentally achievable superfluid transition temperature. As a first step, in the previous studies by Zheng and co-workers [31], as well as by many others [33,34], mean-field theory at zero temperature has been adopted, leading to a qualitative picture of inhomogeneous superfluidity at the BEC-BCS crossover.

Here, we approach the FFLO problem through the analysis of pair fluctuations and address the Fulde-Ferrell pairing instability of a normal Fermi gas at finite temperatures [35,36]. This leads to the so-called Thouless criterion for the onset of superfluidity, which, in the weakly interacting limit, gives exactly the same superfluid transition temperature as the mean-field approach. In the strongly interacting BEC-BCS crossover, however, it provides an improved prediction beyond mean-field, as the effect of pair fluctuations is partially taken into account by using a modified chemical potential. We note that the Thouless criterion is identical to finding out the two-particle bound state in the presence of Fermi surfaces. Our study is therefore a natural generalization of the previous analysis of pairing instability from the two-particle perspective [37,38].
The rest of the paper is organized as follows. In the next section (Sec. II), we present the model Hamiltonian for an ultracold atomic Fermi gas with Rashba spin-orbit coupling and in-plane Zeeman field. We describe the framework of pair-fluctuation analysis and the resulting Thouless criterion. The chemical potential of a normal Fermi gas is in general strongly affected by the pair fluctuations across the BEC-BCS crossover. A quantitative evaluation of such a pair-fluctuation effect, however, is extremely difficult, particularly in the presence of anisotropy in the momentum space due to the combined effect of spin-orbit coupling and in-plane Zeeman field. Therefore, in this work, we shall use an approximate chemical potential, the mean-field chemical potential. Therefore, in this work, we shall use an approximate chemical potential, the mean-field chemical potential at zero temperature. In Sec. III, we discuss in detail the Thouless pairing instability towards a Fulde-Ferrell superfluid, by analyzing the particle-particle vertex function of the Fermi cloud in the weakly interacting limit and unitary limit. In Sec. IV, we consider the pairing instability at the crossover from a BCS superfluid to a BEC of the so-called Rashbons, i.e., tightly bound pairs formed by strong Rashba spin-orbit coupling. Finally, Sec. IV is devoted to conclusions.

II. MODEL HAMILTONIAN AND THOULESS CRITERION

Let us consider a three-dimensional two-component Fermi gas of $^6$Li or $^{40}$K atoms with Rashba spin-orbit coupling $\lambda(\sigma_x \hat{k}_y - \sigma_y \hat{k}_x)$ and an in-plane Zeeman field along the $x$-direction $\hbar \sigma_x$. This configuration is yet to be experimentally realized [48]. Here $\hat{k}_x \equiv -i \partial_x$ and $\hat{k}_y \equiv -i \partial_y$ are the momentum operators, and $\sigma_x$ and $\sigma_y$ are the Pauli matrices. We have denoted the strength of Rashba spin-orbit coupling and of in-plane Zeeman field by $\lambda$ and $h$, respectively. Near a broad Feshbach resonance, the interacting Fermi system is well-described by a single-channel model Hamiltonian,

$$\mathcal{H} = \int dx \left\{ \hat{\psi} \sigma \left[ \hat{\xi}_k + \lambda(\hat{k}_y \sigma_x - \hat{k}_x \sigma_y) + h\sigma_x \right] \hat{\psi}^\dagger \right\} + U_0 \hat{\psi}^\dagger \left( x \right) \hat{\psi}^\dagger \left( x \right) \hat{\psi} \left( x \right) \hat{\psi} \left( x \right),$$

where $\hat{\xi}_k \equiv -\hbar^2 \nabla^2 / (2m) - \mu$ is the single-particle kinetic energy with atomic mass $m$, measured with respect to the chemical potential $\mu$, $\psi \left( x \right) \equiv \psi^\dagger \left( x \right) \psi \left( x \right)$ denotes collectively the annihilation field operators for atoms in the spin-state $\sigma = \uparrow, \downarrow$, $U_0$ is the interaction strength of the contact interaction between atoms with unlike spins. The use of the contact interatomic interaction leads to an ultraviolet divergence at large momentum or high energy. To overcome such a divergence, we express the interaction strength $U_0$ in terms of the $s$-wave scattering length $a_s$,

$$\frac{1}{U_0} = \frac{m}{4\pi \hbar^2 a_s} - \frac{1}{V} \sum_k \frac{m}{\hbar^2 k^2}$$

where $V$ is the volume of the system. Experimentally, by sweeping an external magnetic field across the broad Feshbach resonance, the scattering length $a_s$ can be tuned precisely to arbitrary values [2].

A. Functional path-integral approach

To solve the model Hamiltonian, we employ the functional path integral approach and consider the partition function

$$Z = \int \mathcal{D} \left[ \psi, \bar{\psi} \right] \ e^{-\mathcal{S}[\psi(x,t),\bar{\psi}(x,t)]},$$

where $\beta = 1/(k_B T)$ is the inverse temperature, and $\mathcal{S}[\psi, \bar{\psi}] \equiv \int_0^\beta d\tau \left[ \int dx \left( \psi \sigma_\mu \partial_\mu \psi^\dagger + \mathcal{H}(\psi, \bar{\psi}) \right) \right]$ is the action obtained by replacing the field operators $\psi^\dagger$ and $\psi$ in $\mathcal{H}(\psi, \bar{\psi})$ with the Grassmann variables $\psi(x, \tau)$ and $\bar{\psi}(x, \tau)$, respectively. Following the standard procedure [49], we introduce the pairing field $\Delta(x, \tau)$ and decouple the quartic interaction term in $\mathcal{H}(\psi, \bar{\psi})$ into a quadratic form via Hubbard-Stratonovich transformation. By integrating out the original fermionic fields $(\psi, \bar{\psi})$ and expanding the pairing field around its saddle point solution $\Delta(x, \tau) = \Delta_0 + \delta \Delta(x, \tau)$, up to the level of gaussian pair fluctuations [49], we obtain

$$Z = \int \mathcal{D} \left[ \delta \Delta, \delta \bar{\Delta} \right] \ e^{-\mathcal{S}_{eff}[\delta \Delta(x, \tau), \delta \bar{\Delta}(x, \tau)]}.$$  

In accord with the expansion of the pairing field, the effective action $\mathcal{S}_{eff} = \mathcal{S}_{mf} + \delta \mathcal{S}$ consists of a mean-field saddle-point part

$$\mathcal{S}_{mf} = \int_0^\beta d\tau \int dx \left\{ \frac{|\Delta_0|^2}{U_0} - \frac{1}{2} \text{Tr} \left( -\mathcal{G}_0^{-1} \right) + \beta \sum_k \xi_k \right\},$$

and a gaussian-fluctuation part

$$\delta \mathcal{S} = \int_0^\beta d\tau \int dx \left\{ \frac{|\delta \Delta(x, \tau)|^2}{U_0} + \frac{1}{4} \text{Tr} \left( \mathcal{G}_0 \Sigma \right)^2 \right\}.$$ 

Here, the trace is over all the spin, spatial, and temporal degrees of freedom, $\xi_k = \hbar^2 k^2 / (2m) - \mu = \epsilon_k - \mu$, the single-particle Green function $\mathcal{G}_0$ is given by,

$$\mathcal{G}_0^{-1} = \left[ \begin{array}{cc}
-\partial_\tau - \hat{\xi}_k - \lambda(\hat{k}_y \sigma_x - \hat{k}_x \sigma_y) - h\sigma_x & i \Delta_0 \sigma_y \\
i \Delta_0 \sigma_y & -\partial_\tau + \hat{\xi}_k - \lambda(\hat{k}_y \sigma_x + \hat{k}_x \sigma_y) + h\sigma_x
\end{array} \right] \delta(x - x') \delta(\tau - \tau'),$$
and the self-energy $\Sigma$ takes the form,

$$
\Sigma = \begin{pmatrix}
0 & i\delta \Delta \sigma_y \\
-i\delta \Delta \sigma_y & 0
\end{pmatrix}.
$$

(8)

In our previous study [43], we have investigated the possibility of Fulde-Ferrell superfluidity, based on the mean-field saddle-point action $S_{mf}$ or its corresponding mean-field thermodynamic potential $\Omega_{mf} = k_B T S_{mf}$. The saddle-point solution of the pairing field $\Delta_{0}(x)$ has been shown to carry a non-zero center-of-mass momentum, whose magnitude is roughly proportional to the strength of the in-plane Zeeman field. Here, we aim to understand the instability of a normal Fermi gas with respect to the Fulde-Ferrell pairing, by analyzing the gaussian-fluctuation action $\delta S$. For this purpose, in the following we derive the particle-particle vertex function.

**B. Particle-particle vertex function**

In the normal phase where the pairing field vanishes, i.e., $\Delta_0 = 0$, the inverse single-particle Green function $G_0^{-1}$ is diagonal. In the momentum space, it can be easily inverted to give

$$
G_0(k, i\omega_m) = \begin{pmatrix}
g_+(k, i\omega_m) & 0 \\
0 & g_-(k, i\omega_m)
\end{pmatrix},
$$

(9)

where $\omega_m \equiv (2m + 1)\pi k_B T$ ($\nu_n \equiv 2n\pi k_B T$) is the fermionic (bosonic) Matsubara frequency; $g_+$ and $g_-$ are given by

$$
g_+ = \frac{(i\omega_m - \xi_k) + (\lambda k_y + h) \sigma_x - \lambda k_x \sigma_y}{(i\omega_m - \xi_k)^2 - [(\lambda k_y + h)^2 + \lambda^2 k_x^2]}
$$

(10)

and

$$
g_- = \frac{(i\omega_m + \xi_k) + (\lambda k_y - h) \sigma_x + \lambda k_x \sigma_y}{(i\omega_m + \xi_k)^2 - [(\lambda k_y - h)^2 + \lambda^2 k_x^2]}
$$

(11)

respectively. After some algebra, we obtain the gaussian-fluctuation part of the action as

$$
\delta S = k_B T \sum_{q \mu n} \left[ -\Gamma^{-1}(q, i\nu_n) \right] \delta \Delta(q, i\nu_n) \delta \bar{\Delta}(q, i\nu_n),
$$

(12)

where the inverse vertex function is given by

$$
\Gamma^{-1} = \frac{1}{U_0} + \frac{k_B T}{V} \sum_{k, i\omega_m} \left[ \frac{1}{(i\omega_m - E_{k, +})(i\nu_n - i\omega_m - E_{q-k, +})} + \frac{1}{(i\omega_m - E_{k, -})(i\nu_n - i\omega_m - E_{q-k, -})} - A_{res} \right],
$$

(13)

with the single-particle energy

$$
E_{k, \pm} = \xi_k \pm \sqrt{\lambda^2 k_x^2 + (\lambda k_y + h)^2},
$$

(14)

$$
E_{q-k, \pm} = \xi_{q-k} \pm \sqrt{\lambda^2 (q_x - k_x)^2 + (\lambda q_y - \lambda k_y + h)^2},
$$

(15)

and

$$
A_{res} = \frac{\sqrt{\lambda^2 k_x^2 + (\lambda k_y + h)^2} \sqrt{\lambda^2 (q_x - k_x)^2 + (\lambda q_y - \lambda k_y + h)^2 + \lambda^2 k_x (q_x - k_x) + (\lambda k_y + h) (\lambda q_y - \lambda k_y + h)}}{(i\omega_m - E_{k, +})(i\omega_m - E_{k, -})(i\nu_n - i\omega_m - E_{q-k, +})(i\nu_n - i\omega_m - E_{q-k, -})}.
$$

(16)

By performing explicitly the summation over $i\omega_m$, replacing $k$ by $q/2 + k$ and re-arranging the terms, we find that

$$
\Gamma^{-1} = \frac{m}{4\pi \hbar^2 a_s} + \frac{1}{2V} \sum_{k} \left[ \frac{f(E_{q/2+k, +}) + f(E_{q/2-k, +}) - 1}{i\nu_n - E_{q/2+k, +} - E_{q/2-k, +}} + \frac{f(E_{q/2+k, -}) + f(E_{q/2-k, -}) - 1}{i\nu_n - E_{q/2+k, -} - E_{q/2-k, -}} - \frac{1}{\xi_k} \right]

- \frac{1}{4V} \sum_{k} \left[ 1 + \frac{(\lambda q_x/2)^2 + (\lambda q_y/2 + h)^2 - \lambda^2 (k_x^2 + k_y^2)}{\sqrt{\lambda^2 (q_x/2 + k_x)^2 + (\lambda q_y/2 + \lambda k_y + h)^2} \sqrt{\lambda^2 (q_x/2 - k_x)^2 + (\lambda q_y/2 - \lambda k_y + h)^2}} \right] C_{res},
$$

(17)
where $f(E) \equiv 1/(e^{\beta E} + 1)$ is the Fermi distribution function and

$$
C_{\text{res}} = \frac{1}{2} \left[ \left( \frac{f(E_{q/2,k,+}) + f(E_{q/2,k,-}) - 1}{\nu_n - E_{q/2,k,+} - E_{q/2,k,-}} \right) + \left( \frac{f(E_{q/2,k,+}) + f(E_{q/2,k,-}) - 1}{\nu_n - E_{q/2,k,+} - E_{q/2,k,-}} \right) \right]
$$

By integrating out the quadratic pairing-fluctuation term of $\delta S$, we obtain the contribution of the guassian pair fluctuations to the thermodynamic potential as

$$
\delta \Omega = k_B T \sum_{q} \ln \left[ -\Gamma^{-1}(\mathbf{q}, i\nu_n) \right].
$$

### C. Thouless criterion

Within the approximation of keeping gaussian pair fluctuations only [48], the particle-particle vertex function $\Gamma(q, i\nu_n)$ can be physically interpreted as the Green function of “Cooper pairs”. This is already evident in Eq. (19), as the thermodynamic potential $\Omega_B$ of a free bosonic Green function $G_B$ is formally given by $\Omega_B = k_B T \sum_{\mathbf{q}, i\nu_n} \ln [-G^{-1}(\mathbf{q}, i\nu_n)]$. Therefore, by neglecting the interactions between Cooper pairs, which is consistent with the approximation of gaussian pair fluctuations, the superfluid phase transition occurs when the particle-particle vertex function develops a pole at zero frequency $i\nu_n = 0$. This is the so-called Thouless criterion [46-49].

$$
\max \Gamma^{-1}(\mathbf{q}, i\nu_n = 0) |_{T = T_c} = 0.
$$

Here, at the superfluid transition temperature $T_c$, the maximum of the inverse vertex function may not occur at zero momentum $\mathbf{q} = 0$. If happens, the phase coherence arises firstly among Cooper pairs that carry a non-zero center-of-mass momentum. This is precisely the pairing instability towards a Fulde-Ferrell superfluid.

### D. Approximate chemical potential

To use the Thouless criterion, we need to know the chemical potential $\mu$ at the superfluid transition temperature $T_c$, which is to be determined by the number of particles in the Fermi cloud, consisting of both fermions $n_F = -\partial \Omega_B/\partial \mu$ and Cooper pairs $n_C = -\partial \Omega_B/\partial \mu$. In the strongly interacting regime, the number of Cooper pairs $n_C$ is significant, leading to a sizable suppression of the chemical potential. Within the guassian pair fluctuation theory, however, such a suppression is very difficult to determine, as the calculations of the vertex function $\Gamma(q, i\nu_n)$ and consequently the thermodynamic potential $\delta \Omega$ are now greatly complicated by the anisotropy in the momentum space arising from spin-orbit coupling and in-plane Zeeman field. Therefore, we consider an approximate scheme for the chemical potential, based on the following two observations: (1) In the superfluid phase, the temperature dependence of the chemical potential becomes weak, even in the strongly interacting regime [53]. Thus, we may set $\mu(T_c) \approx \mu(T = 0)$; (2) At zero temperature, the mean-field theory provides a reasonable qualitative description of the BEC-BCS crossover [1]. Thus, we may approximate $\mu(T = 0) \approx \mu_{\text{mf}}(T = 0)$. Using these two observations, in the end we shall approximate

$$
\mu(T_c) \approx \mu_{\text{mf}}(T = 0).
$$

This approximate scheme for the chemical potential may be examined for an ordinary BEC-BCS crossover Fermi gas without Rashba spin-orbit coupling. In the unitary limit, where the $s$-wave scattering length $a_s$ diverges, the recent accurate measurement at MIT [53] reported that $\mu(T_c) \approx 0.42 E_F$, in units of the Fermi energy $E_F$. Although the mean-field prediction of zero temperature chemical potential [1], $\mu_{\text{mf}}(T = 0) \approx 0.59 E_F$, has about 40% overestimation of $\mu(T_c)$, it is much better than the value commonly used in the weakly interacting limit, i.e., $\mu(T_c) = E_F$. In Fig. [1] we show the zero-temperature mean-field chemical potential of a Rashba
spin-orbit coupled Fermi gas at two dimensionless interaction parameters: $1/k_F a_s = -1$ and $1/k_F a_s = 0$, to be used later in the numerical calculations. The results are obtained by minimizing the mean-field thermodynamic potential or action Eq. (10) with respect to a Fulde-Ferrell order parameter $\Delta_0(x) = \Delta e^{iqy}$, by treating $\Delta$ and $q$ as the independent variational parameters. For more details, see Ref. [43].

III. FULDE-FERRELL PAIRING INSTABILITY AT BEC-BCS CROSSOVER

We now determine the superfluid transition temperature of a Rashba spin-orbit coupled Fermi gas with in-plane Zeeman field, by using the Thouless criterion Eq. (20). As discussed in the previous section, we take the zero-temperature mean-field chemical potential as the approximate chemical potential at $T_c$.

In Fig. 2 we report the temperature dependence and momentum dependence of the inverse vertex function for a weakly interacting spin-orbit coupled Fermi gas with an in-plane Zeeman field $h = 0.3E_F$. The maximum of the inverse vertex function reaches zero when the temperature decreases down to $0.053T_F$, indicating the onset of superfluid transition. Remarkably, at this superfluid transition temperature, the inverse vertex function is an anisotropic function of momentum and its maximum occurs at a nonzero momentum $q = q_{FF} \hat{c}_y$, where $q_{FF} \simeq 0.35k_F$ and $\hat{c}_y$ is the unit vector along the $q_y$ direction. This strongly indicates that the resulting state is an inhomogeneous Fulde-Ferrell superfluid which breaks the spatial translation invariance. We note that the Fulde-Ferrell momentum obtained from the Thouless criterion is consistent with the mean-field prediction obtained in the superfluid phase at zero temperature [43], which gives nearly the same number. This consistency is easy to understand, as the properties of the Fermi condensate remains roughly the same in the superfluid phase. The preference of the Fulde-Ferrell momentum along the $q_y$ direction is uniquely determined by the change of topology of the two Fermi surfaces [43].

By calculating the superfluid transition temperature at different in-plane Zeeman fields, we construct the finite temperature phase diagram at $1/k_F a_s = -1$, as shown in Fig. 3. For comparison, we also show the mean-field critical temperature at some typical Zeeman fields by using solid circles. In this weakly interacting regime, the chemical potential is not significantly modified by the presence of emerging Cooper pairs. As a result, the Thouless criterion and the mean-field calculation predict roughly the same superfluid transition temperature, as we may an-
We now consider the strongly interacting unitary limit and we anticipate. These two different approaches also give nearly the same results for the Fulde-Ferrell momentum at the superfluid phase transition, as illustrated by the inset of Fig. 3.

IV. FULDE-FERRELL PAIRING INSTABILITY OF A RASHBON CONDENSATE

We so far focus on a particular spin-orbit coupling strength $\lambda k_F/E_F = 1$. With increasing the Rashba spin-orbit coupling, it is known that a tightly-bound pair of two fermions can form, even with a weak attractive interatomic interaction [22, 24]. This new type of bound pairs, referred to as Rashbons [22, 54], underlies an exotic anisotropic fermionic superfluid in the absence of...
ent spin-orbit coupling strengths, $\lambda_k$ inverse vertex function along the different spin-orbit coupling strengths, line), weak interaction strength $1$, transition temperatures are indicated by arrows. Here we take an in-plane Zeeman field $h = 0.3E_F$.

Figure 8: (color online) The momentum dependence of the inverse vertex function along the $q_y$ direction, at three different spin-orbit coupling strengths, $\lambda_k/E_F = 1.5$ (solid line), 2.0 (dashed line) and 3.0 (dot-dashed line), and at a weak interaction strength $1/k_Fa_s = -1$. We use an in-plane Zeeman field $h = 0.3E_F$.

Zeeman field $h = 0.3E_F$. Hereafter, we restrict ourselves in the weakly interacting regime ($1/k_Fa_s = -1$) and take an in-plane Zeeman field $h = 0.3E_F$.

In Fig. 7 we present the maximum of the inverse particle-particle vertex function at three different spin-orbit coupling strengths. The corresponding momentum distributions along the $q_y$ direction at the superfluid transition temperature are shown in Fig. 8. With increasing the spin-orbit coupling, the superfluid transition temperature increases significantly, due to the formation of Rashbons. However, the Fulde-Ferrell momentum at the transition becomes smaller, indicating that Fulde-Ferrell superfluidity is inherently akin to the many-body environment.

Figure 9: (color online) Phase diagram of the crossover from a BCS superfluid to a Rashbon BEC, at the in-plane Zeeman-field $h = 0.3E_F$ and the interatomic interaction strength $1/k_Fa_s = -1$. The critical temperatures given by Thouless criterion and mean-field theory are shown by solid and dashed lines, respectively. The inset shows the Fulde-Ferrell momentum at the phase transition, predicted by the Thouless criterion (line) and the mean-field theory (solid circles).

Fulde-Ferrell superfluidity is inherently akin to the many-body environment.

In Fig. 9 we present a finite-temperature phase diagram of the crossover from a BCS superfluid to a BEC of Rashbons. As the superfluid transition temperature increases rapidly with spin-orbit coupling, it is already experimentally accessible at a modest coupling strength $\lambda_k/E_F = 2$, even with a small interaction parameter $1/k_Fa_s = -1$. The Fulde-Ferrell momentum $q_{FF}$ at this coupling strength is about $0.2k_F$, as shown in the inset, which might already be large enough to be detected experimentally by, for example, the momentum-resolved radio-frequency spectroscopy [39]. We note that, due to the use of the approximate chemical potential, the superfluid transition temperature in the Rashbon limit is overestimated by the Thouless criterion. It should saturate to about $0.193T_F$ in the limit of $\lambda \to \infty$.

V. CONCLUSIONS

In summary, we have investigated theoretically the Fulde-Ferrell pairing instability in a normal, spin-orbit coupled Fermi gas with an in-plane Zeeman field near a broad Feshbach resonance, by using the standard Thouless criterion. In addition to complementing the existing mean-field theoretical studies, we have predicted an improved superfluid phase transition temperature, based on an approximate scheme for chemical potential. We have shown that at the typical parameters, for example, with Rashba spin-orbit coupling strength $\lambda_k/E_F = 1$, in-plane Zeeman field $h = 0.5E_F$ and in the unitary
limit, the Fermi cloud will become a Fulde-Ferrell superfluid at about the experimentally attainable temperature $0.2T_F$. We have also presented a finite-temperature phase diagram along the crossover from a Bardeen-Cooper-Schrieffer superfluid to a Bose-Einstein condensate of Rashbions.

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[54] More precisely, according to the two-body picture, a Rashbon corresponds the two-particle bound state at the limit $\hbar^2/(m\lambda a_s) = 0$. Therefore, we may reach the Rashbon limit by increasing the spin-orbit coupling strength (at a fixed interaction parameter $1/k_F |a_s| \sim 1$).