Interlayer hopping properties of electrons in layered metals

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A formalism is proposed to study the electron tunneling between extended states, based on the spin-boson Hamiltonian previously used in two-level systems. It is applied to analyze the out-of-plane tunneling in layered metals considering different models. By studying the effects of in-plane interactions on the interlayer tunneling of electrons near the Fermi level, we establish the relation between departure from Fermi liquid behavior driven by electron correlations inside the layer and the out of plane coherence. Response functions, directly comparable with experimental data are obtained.

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I. INTRODUCTION

Layered materials have been object of intensive study since they present important physics. A key feature of many layered materials is the anisotropy exhibited by its transport properties: while being metallic within the layers, the transport in the c-axis, perpendicular to the layers, may be coherent or incoherent and undergo a crossover with temperature from one regime to the other, thus changing the effective dimensionality of the system. Correlation effects increase as dimensionality decreases, therefore dimensionality is crucial for the electronic properties and to choose the appropriate model to study the system. Unusual properties are derived from the anisotropy and periodicity along the axis perpendicular to the planes i.e. the structure of collective excitations absent in two dimensional (2D) and three dimensional (3D) electron gases.

Among the most studied layered materials are the high-temperature cuprate superconductors. These compounds present a strong anisotropy and are treated as two-dimensional systems in many approaches. In the normal state the transport properties within the doped CuO$_2$ planes are very different from those along the c-axis: electron motion in the c-direction is coherent in contrast with the metallic behavior of the in-plane electrons as probed by the different $\rho_c$ and $\rho_{ab}$ resistivities and their different dependence with temperature. Optical conductivity measurements confirm the anomalous c-axis properties. These features have led to the assumption that the relevant physics of these substances lies on the CuO$_2$ planes, common to all the families. However, the importance of the c-axis structure is being intensively investigated since it offers a key to explain the differences of the critical temperature $T_c$ and optimal doping in the different compounds. The relevance of the nature of the conductance in the direction perpendicular to the CuO$_2$ planes to the nature of the superconducting phase has been remarked on both theoretical and experimental grounds. The anomalous behavior of the out of plane properties in the cuprates, and in analogy with the one dimensional Luttinger liquid, has led to the proposal of the failure of the conventional Fermi-liquid (FL) theory in these compounds.

Recent c-axis transport measurement focused on the resistivity anisotropy $\rho_c/\rho_{ab}$ ratio indicate that $\rho_c/\rho_{ab}$ quickly increases with decreasing temperature. It was remarkably found that, at moderate temperature (between 100 to 300K), $\rho_c/\rho_{ab}$ is almost completely independent of doping $x$ in the non superconducting regime ($0.01 \leq x \leq 0.05$), which suggest that the same charge confinement mechanism that renormalizes the c-axis hopping rate survives down to $x = 0.01$. On the contrary the $\rho_c/\rho_{ab}$ ratio quickly changes when $x$ enters into the superconducting regime, indicating that interlayer hopping quickly becomes easier as $x$ is increased above 0.05, i.e. the charges are increasingly less confined in this doping range. Measurements of the out-of-plane magnetoresistance(MR) of La$_{2-x}$Sr$_x$CuO$_4$ over a wide doping range and comparison with other systems suggest that a competition between the c-axis hopping rate and the in-plane scattering rate determines the behavior of the c-axis MR in layered perovskites with incoherent c-axis resistivity.

Many alternative models have been suggested to explain the puzzling properties, from Fermi liquid modified by strong electron-electron correlations to the non-Fermi Luttinger liquid. An alternative explanation of the emergence of incoherent behavior in the out of plane direction has been proposed in terms of the coupling of the interlayer electronic motion to charge excitations of the system. This approach implicitly assumes that electron-electron interactions modify the in-plane electron propagators in a non trivial way, at least at distances shorter than the elastic mean free path. The strong angular dependence of both the scattering rate and the interplane tunneling element can also lead to the observed anisotropies.

Graphite, another layered material, presents an inplane hopping much larger than the interplane hybridization. Many of the transport properties established in the past for this well known material are being questioned at present. Recent conductivity measurements re-
veal a suppression of the c-axis conductivity much larger than what would be predicted by the band calculations of the interlayer hopping. Band structure calculations are also challenged by very recent claims of observation of quantum Hall plateaus in pure graphite. The unconventional transport properties of graphite such as the linear increase with energy of the inverse lifetime (see also), suggest deviations from the conventional Fermi liquid behavior, which could be due to strong Coulomb interactions unscreened because of the lack of states at the Fermi level.

By assuming that electron correlations modify the in-plane electron propagator, we show that even in the clean limit, many-body effects can suppress the coherent contribution to the out of plane electron hopping. The clean limit is defined as that in which the elastic mean free path diverges. It is shown that for certain models the interplane hopping between extended states can be a relevant or an irrelevant variable, in the Renormalization Group (RG) sense, depending on the strength of the coupling constant. The scheme used here is based on the RG analysis as applied to models of interacting electrons. We will demonstrate the usefulness of the method, by rederiving known results for a non trivial system, the array of Luttinger liquids, and then we will apply the method to two well defined two-dimensional systems where ordinary perturbation theory fails, due to the existence of logarithmic divergences.

Recently there has been renewed interest in two-dimensional systems that support low energy excitations which can be described by Dirac fermions. Examples are the flux–phase of planar magnets, which can be described by Dirac fermions. Examples include Luttinger liquids, and quasiparticles of planar zero-gap semiconductors as graphite. Long range Coulomb interactions and disorder are modelled in these systems by means of gauge fields coupled to the Dirac fermions. Most of these systems show anomalous transport properties ranging from mild departures of Fermi liquid behavior as in pure graphite, to the total destruction of the quasiparticle pole. Disorder can be modelled as random gauge fields coupled to the Dirac quasiparticles. It affects the quasiparticle Green’s function in a computable way what in turn modifies the interlayer tunneling.

This paper is organized as follows. In section II we present the method of calculation and show the results it gives in one dimension. In section III we apply it to two-dimensional models which show deviations from FL behavior: in particular to systems with a vanishing density of states at the Fermi level as is the case of graphene sheets and to a system of planar electrons near a Van Hove singularity. The effect of disorder on these graphitic and related systems is discussed. The main physical consequences of our calculation are presented in section IV.

II. THE METHOD OF CALCULATION.

In the presence of electron-electron interactions, tunneling processes are modified by inelastic scattering events. The influence of inelastic scattering on electron tunneling has been studied, using equivalent methods, in mesoscopic devices which show Coulomb blockade, Luttinger liquids, and dirty metals. The simplest formulation of the method replaces the excitations of the system (such as electron-hole pairs) by a bath of harmonic oscillators with the same excitation spectrum. This approach can be justified rigorously in one dimension, and is always an accurate description of the response of the system when the coupling of the quasiparticles to each individual excitation is weak, although the net effect of the environment on the system under study can be large. The expression for the coupling between the electrons and the oscillators is obtained by assuming that the oscillators describe the charge oscillations of the system. Then, the coupling can be related, using perturbation theory, to the charge-charge correlations in the electron gas. This is consistent with the assumption that the modes in the environment are weakly perturbed by their interaction to the low energy electrons whose tunneling properties are been considered:

\[ H_{int} = c_i^\dagger c_i \sum_k V_i(\vec{k}) \hat{\rho}_k \]  

where \( c_i^\dagger (c_i) \) creates (destroys) an electron at site \( i \), and \( \hat{\rho}_k \) describes the charge fluctuations of the environment, which are to be described as a set of harmonic modes as stated above. The interaction in Eq. (1) is the simplest, and most common, coupling between the tunneling electrons and the excitations of the system, which is spin independent. Other spin-dependent or more complicated couplings can also be taken into account, provided that the appropriate response function is used.

Since the excitations of the system (electron-hole pairs, plasmons) are modelled as bosonic modes, one can write an effective electron-boson hamiltonian of the type:

\[ H_{e-b} = H_{elec} + H_{env} + H_{int} \]

\[ = \sum t_{ij} c_i^\dagger c_j + \sum \omega_k b_k^\dagger b_k + \sum g_{k,i} c_i^\dagger (b_k^\dagger + b_k) \]

where \( H_{elec} \) describes the individual quasiparticles, \( H_{env} \) stands for the set of harmonic oscillators which describe the environment, and \( H_{int} \) defines the (linear) coupling between the two. The \( b_k^\dagger (b_k) \) are boson creation (destruction) operators, the \( t_{ij} \) describe the electronic hopping processes. The information about the interaction between the electron in state \( i \) and the environment is encoded in the spectral function:

\[ J_i(\omega) = \sum_k |g_{k,i}|^2 \delta(\omega - \omega_k) \]

The function \( J_i(\omega) \) describes the retarded interaction induced by the environment. In standard perturbative
treatments of the hamiltonian in Eq. (2), this function is simply the self energy of an electron at site i, as schematically shown in Fig. 1.

Using second order perturbation theory and Eq. (1), we can write

$$J_i(\omega) = \sum_{\mathbf{k}} V_i^2(\mathbf{k}) \Im \chi(\mathbf{k}, \omega) \quad (4)$$

where \( \chi(\mathbf{k}, \omega) \) is the Fourier transform of the density-density response of the system, \( \langle \hat{\rho}(\mathbf{k})\hat{\rho}_e(0) \rangle \). The electron-boson interaction leads to a Franck-Condon factor which reduces the effective tunneling rate. The Franck-Condon factor depends exponentially on the coupling between the particles and the oscillators, and when it diverges additional self consistency requirements between the hopping amplitudes and the oscillator frequencies included in the calculation have to be imposed. The electron propagators acquire an anomalous time, or energy, dependence, that can be calculated to all orders if the state is localized, or, which is equivalent, neglecting the hopping terms in Eq. (2). The present scheme can be considered a generalization to layered systems of the approach presented in reference \( ^{33,36} \) for a Luttinger liquid, where the scaling of tunneling is expressed in terms of an effective density of states. The tunneling will become irrelevant, as in \( ^{33,36} \), in one of the models considered here, that of electrons in the vicinity of a saddle point of the dispersion relation, which is consistent with the picture of the c-axis interlayer tunneling strongly suppressed by voltage fluctuations, proposed by Turlakov and Leggett \( ^{35} \).

Using the boson representation of the environment, we can write the electron propagator as:

$$\langle c_i^\dagger(t) c_i(t') \rangle \sim \langle c_i^\dagger(t) c_i(t') \rangle_0 \times \exp \left\{ - \int d\omega \left[ 1 - e^{i\omega(t-t')} \right] \frac{J_i(\omega)}{\omega^2} \right\} \quad (5)$$

where \( \langle c_i^\dagger(t) c_i(t') \rangle_0 \sim e^{-i\omega(t-t')} \) is the Green’s function in the absence of the interaction. This result can be viewed as the exponentiation of the second order self energy diagram shown in Fig. 1. The approximation of the excitations in the environment by bosonic modes allows us to sum an infinite set of diagrams like the one in Fig. 1.

In contrast to previous work, we analyze tunneling between coherent extended states. In order to do so, we need to generalize Eq. (5) to this case. We then need to know the Green’s function of coherent states in the individual layers, \( G(\mathbf{k}, \omega) \), including the correction due to the interaction to the environment. We firstly assume that Eq. (5) also holds in a system with extended states. For a standard metallic system, we must insert \( \langle c_i^\dagger(t) c_i(t') \rangle_0 \sim 1/(t-t') \) in Eq. (5). It can be shown that this approximation is exact at short times, \( W \ll (t-t')^{-1} \ll \Lambda \), where \( W \) is an energy scale related to the dynamics of the electrons, and \( \Lambda \) is the upper cutoff in the spectrum of the environment. This expression can be generalized, taking into account the spatial structure of the coupling to:

$$\langle \Psi^\dagger_i(t) \Psi_j(t') \rangle \sim \frac{1}{t-t'} \times \exp \left\{ \int \int d\mathbf{r} d\mathbf{r}' d^2\mathbf{k} e^{i\mathbf{k} \cdot (\mathbf{r}-\mathbf{r}')} \int d\omega \left[ 1 - e^{i\omega(t-t')} \right] \frac{V_{\text{eff}}(\mathbf{k}, \omega)}{\omega^2} \right\} \quad (6)$$

where \( \Omega \) is the region of overlap of the wavefunctions \( \Psi_i(\mathbf{r}) \) and \( \Psi_j(\mathbf{r}) \). This expression, which can be seen as the exponential of the leading frequency dependent self-energy correction to the electron propagator, has been extensively used in studies of tunneling in zero dimensional systems (single electron transistors) which show Coulomb blockade, \( ^{33} \), one dimensional conductors, \( ^{34} \), and disordered systems in arbitrary dimensions, \( ^{35} \).

The effective interaction can, in turn, be written in terms of the response function as:

$$V_{\text{eff}}(\mathbf{k}, \omega) = V^2(\mathbf{k}) \Im \chi(\mathbf{k}, \omega) \quad (7)$$

The time dependence in Eq. (6) is determined by the low energy limit of the response function. In a Fermi liquid, we have

$$\Im \chi(\mathbf{k}, \omega) \sim \alpha(\mathbf{k}) |\omega| \quad \omega \ll E_F \quad , (8)$$

where \( E_F \) is the Fermi energy. Eq. (6) then gives:

$$\lim_{(t-t') \to \infty} \langle \Psi^\dagger_i(t) \Psi_j(t') \rangle \sim \frac{1}{(t-t')^{1+\alpha}} \quad (9)$$

where

$$\alpha = \int d\mathbf{k} \int \int d^2\mathbf{r} d^2\mathbf{r}' d^2\mathbf{k} e^{i\mathbf{k} \cdot (\mathbf{r}-\mathbf{r}')} V^2(\mathbf{k}) \alpha(\mathbf{k})$$

(10)

The parameter \( \alpha \) gives the correction to the scaling properties of the Green’s functions. Integration in \( \mathbf{k} \) is restricted to \( |\mathbf{k}| \ll L^{-1} \) where \( L \) is the scale of the region where the tunneling process takes place. The value of \( L \) is limited by the length over which the phase of the electronic wavefunctions within the layers is well defined.

FIG. 1: Diagram corresponding to the lowest order contribution to the self energy of an electron at site i in the hamiltonian given in Eq. (4). This diagram can be used to describe the spectral function in Eq. (4).
The former approach is directly related to the theoretical model proposed in, where the limit of coherent tunneling is set by the value $\alpha = 1$. In sec. IIIB this procedure is applied to the study of interlayer tunneling in layered systems whose Fermi surface is close to a saddle point which has been proposed as a possible explanation of some properties of the cuprates.

We can also use Eq. (10) to analyze the interlayer tunneling by applying Renormalization Group methods. The simplest case where this procedure has been used is for the problem of an electron tunneling between two states, $i$ and $j$, which has been intensively studied. We integrate out the high energy bosons, with energies $\Lambda - d\Lambda \leq \omega_k \leq \Lambda$ and rescaled hopping terms are defined. As mentioned earlier, Eq. (10) is valid for this range of energies. The renormalization of the hoppings is such that the properties of the effective Hamiltonian at energies $\omega \ll \Lambda$ remain invariant. If the hoppings $t_{ij}$ are small, any physical quantity which depends on them can be expanded, using time dependent perturbation theory, in powers of:

$$t_{ij}^2 \langle c_i^\dagger(t)c_j(t)\rangle \approx t_{ij}^2 \langle c_i^\dagger(t')\rangle \langle c_j(t')\rangle$$

(11)

The integration of the high energy modes implies that the terms in Eq. (11) are restricted to $t \leq \Lambda^{-1}$, or, alternatively, the time unit have to be rescaled, $\tau' = e^{d\Lambda/\Lambda}$, where $\tau \sim \Lambda^{-1}$. Using Eq. (10), the condition of keeping the perturbation expansion in powers of the terms in Eq. (11) invariant implies that:

$$t_{ij}^2 \rightarrow t_{ij}^2 e^{2\alpha D/2}$$

(12)

which can also be used to define the scaling dimension of the hopping terms. The beta function of the hopping is then

$$\frac{\partial(t_{ij}/\Lambda)}{\partial l} \equiv \beta(t_{ij}) = -\alpha t_{ij}/\Lambda$$

(13)

where $l = \log(\Lambda_0/\Lambda)$, and $\Lambda_0$ is the initial value of the cutoff.

This approach has been successfully used to describe inelastic tunneling in different situations. The analysis which leads to Eq. (12) can be generalized to study hopping between extended states, provided that we can estimate the long time behavior of the Green’s function, $G(k_l, t - t')$. We assume that, in a translationally invariant system, there is no dependence on the position of the local orbital, $i$. This result implies that the frequency dependence of the Green’s function, in a continuum description, can be written as:

$$\lim_{|\vec{r} - \vec{r}'| \to 0} G(\vec{r} - \vec{r}', \omega) \propto |\omega|^\alpha$$

(15)

Equation (14) is related to Eq. (15), by:

$$\lim_{|\vec{r} - \vec{r}'| \to 0} G(\vec{r} - \vec{r}', \omega) = \int d^D \vec{k} G(\vec{k}, \omega)$$

(16)

where $D$ is the spatial dimension. In the cases discussed below, the interaction is instantaneous in time, and the non interacting Green’s function can be written as:

$$G_0(\vec{k}, \omega) \propto \frac{1}{\omega^{\frac{D}{2}}} F\left(\frac{k^2}{\omega}\right)$$

(17)

and $F(\omega)$ is finite. Thus, from the knowledge of the real space Green’s function, using Eq. (9), we obtain $\alpha$, which, in turn, determines the exponent $\alpha - D/z$ which characterizes $G(\vec{k}, \omega)$. Generically, we can write:

$$G_{loc,ext}(\omega) \sim |\omega|^{\delta_{loc,ext}}$$

(19)

where the subindices $loc, ext$ stand for localized and extended wavefunctions. In terms of these exponents, we can generalize Eq. (13) to tunneling between general states to:

$$\frac{\partial(t_{loc,ext}/\Lambda)}{\partial l} = -\delta_{loc,ext} t_{ij}/\Lambda$$

(20)

Before proceeding to calculations of $\delta_{loc}$ and $\delta_{ext}$ for various models, it is interesting to note that, in general, the response function of an electron gas in dimension $D > 1$ behaves as

$$\lim_{\omega \to 0, |k| \to 0} \chi(\vec{k}, \omega) \sim |\omega|/|\vec{k}|$$

so that, from Eq. (18),

$$\lim_{L \to \infty} \alpha \sim L^{(1-D)}$$

Thus, for $D > 1$, the contribution of the inelastic processes to the renormalization of the tunneling vanishes for delocalized states, $L \to \infty$. This result is consistent with the existence of Fermi liquid behavior above one dimension. Note that anisotropic Fermi surfaces, with inflection points, can lead to marginal behavior above one dimension.11, 12.
It is easy to show that, in an isotropic Fermi liquid in D dimensions, \( \lim_{L \to \infty} \alpha \propto L^{1-D} \), where \( L \) is the linear dimension of the (localized) electronic wavefunctions \( \Psi(\vec{r}) \). This result is due to the dependence on \( \vec{k} \) of the response functions which goes as

\[
\text{Im} \chi(\vec{k}, \omega) \sim \frac{1}{|k_L^{D-1}| |\vec{k}|}.
\]

Thus, for \( D > 1 \), we recover coherent tunneling in the limit of delocalized wavefunctions.

In one dimension, one can use the non-interacting expression for \( \text{Im} \chi_0(\vec{k}, \omega) \), to obtain:

\[
\int d^2 \vec{r} d^2 \vec{r}' d^2 \vec{k} e^{i(\vec{r}-\vec{r}') \vec{k}} \int d\omega \left[ 1 - e^{i\omega(t-t')} \right] \frac{V_{eff}(\vec{k}, \omega)}{\omega^2} \propto \left( \frac{U}{E_F} \right)^2 \times \begin{cases} 0 & t-t' \ll L/v_F \\ \log[v_F(t-t')/L] & t-t' \gg L/v_F \end{cases}
\]

where we have assumed a smooth short range interaction, parametrized by \( U \). Hence, the Green’s functions have a non trivial power dependence on time, even in the \( L \to \infty \) limit, in agreement with well known results for Luttinger liquids. In order to obtain the energy dependence of the effective tunneling between \( \vec{k} \) states near the Fermi surface, one needs to perform an additional integration over \( d\vec{r} \). In general, near a scale invariant fixed point, \( \omega \propto |\vec{k}|^2 \), and for a 1D conductor one knows that \( z = 1 \).

Hence,

\[
\text{Im} G(\omega, k_F) \propto \omega^{z+\alpha} \sim \omega^{-1+\alpha}.
\]

The flow of the hopping terms under a Renormalization Group scaling of the cutoff is:

\[
\frac{\partial(t/\Lambda)}{\partial t} = \begin{cases} -\alpha & \text{localized hopping} \\ 1 - \alpha & \text{extended hopping} \end{cases}
\]

where \( t \) denotes a hopping term, between localized or extended states. In the latter case, the hopping becomes an irrelevant variable for \( \alpha > 1 \).

III. LAYERED MODELS

A. Vanishing density of states at the Fermi level.

The simplest two dimensional model for interacting electrons where it can be rigorously shown that the couplings acquire logarithmic corrections in perturbation theory is a system of Dirac fermions \( (\epsilon_k = v_F |\vec{k}|) \), with Coulomb, \( 1/|\vec{r} - \vec{r}'| \), interaction. This model can be used to describe isolated graphene planes, and can help to understand the anomalous behavior of graphite observed in recent experiments. Nodal quasiparticles in d-wave superconductors are also describable in these terms.

In order to apply the procedure outlined in the previous section, one needs the Fourier transform of the interaction,

\[
V_{eff}(\vec{k}) = e^2/(\epsilon_0 |\vec{k}|),
\]

where \( e \) is the electronic charge, and \( \epsilon_0 \) is the dielectric constant, and the susceptibility of the electron gas. For a single graphene plane, this quantity has been computed in and is:

\[
\chi_0(\vec{k}, \omega) = \frac{1}{8} \frac{|\vec{k}|^2}{\sqrt{v_F^2 |\vec{k}|^2 - \omega^2}}
\]

These expressions need to be inserted in equations (7) and (8).

For simplicity, we consider the expression in Eq. (20), as it allows us to obtain analytical results. The imaginary part, \( \text{Im} \chi_0(\vec{k}, \omega) \), is different from zero \( \omega > v_L |\vec{k}| \).

We cut off the spatial integrals at a scale, \( L \), of the order of the electronic wavefunctions involved in the tunneling. Performing the same computation as in the case of \( D=1 \) we obtain an expression similar to that in Eq. (21) except that the prefactor \( (U/E_F)^2 \) is replaced by the squared of effective coupling constant of the model, \( e^2/(\epsilon_0 v_F)^2 \). Thus, also in the graphene model, the propagators acquire an anomalous dimension what was advocated in as pointing out to a departure of the model from Fermi liquid behavior. As in 1D, the value of the exponent \( z \) which relates length and time scales is \( z = 1 \). The scaling of the hoppings now are:

\[
\frac{\partial(t/\Lambda)}{\partial t} = \begin{cases} -1 - \alpha & \text{localized hopping} \\ 1 - \alpha & \text{extended hopping} \end{cases}
\]

The extra constant in the first equation with respect to Eq. (22) reflects the vanishing of the density of states at the Fermi level for two dimensional electrons with a Dirac dispersion relation.

In graphite, the dimensionless coupling constant, \( e^2/v_F \), is of order unity. Under renormalization, it flows towards zero. Thus, despite the departure from Fermi liquid behavior, interplane tunneling is a relevant variable and a coherent out of plane transport in clean graphene samples should be observed. The former picture can change in the presence of disorder that is known to change the anomalous dimension of the fields.

B. Saddle point in the density of states.

The Fermi surface of most hole-doped cuprates is close to a saddle point of the dispersion relation. The possible relevance of this fact to the superconducting transition as well as to the anomalous behavior of the normal state was put forward in the early times of the cuprates and gave rise to the so-called Van Hove scenario.
where \( \alpha \) the wavefunction of the tunneling electron, and \( \Omega \) the anomalous dimension of the propagator in this case.

The dynamical exponent (17) in this case is \( z = 2 \). The effective relation

\[
\varepsilon_{A,B}(\mathbf{k}) = \mp (t \mp 2t') k_x^2 \pm (t \pm 2t')k_y^2 , \tag{26}
\]

The dynamical exponent in this case is \( z = 2 \). The dispersion relation (20) allows to formulate a renormalizable effective model based on the hamiltonian:

\[
\mathcal{H} = \sum_{i=A,B,k,s} \varepsilon_{i,k,i,s} c_i^\dagger c_i + \sum_{i,i',i''} u_{i,i';i''} c_{i'}^\dagger c_{i''} c_i, \tag{27}
\]

where \( \Lambda_0 \) is a high energy cutoff which sets the limits of the effective description. The particle–hole susceptibility has been computed in \( \mathcal{O} \):

\[
\text{Im } \chi(\mathbf{k},\omega) = \frac{1}{4\pi \varepsilon(\mathbf{k})} \left( |\omega + \varepsilon(\mathbf{k})| - |\omega - \varepsilon(\mathbf{k})| \right) , \tag{28}
\]

where \( \varepsilon(\mathbf{k}) \) is the dispersion relation (20).

The long time dependence of the Green’s function is determined by the low energy behavior of \( \chi \):

\[
\lim_{\omega \to 0} \text{Im } \chi(\mathbf{k},\omega) \sim \frac{\omega}{\varepsilon(\mathbf{k})} .
\]

Inserting this expression in eqs. (6) and (7), we can see that, irrespective of the details of the interaction, in the presence of a Van Hove singularity the exponent \( \alpha \) in the time dependence of the Green’s function goes as:

\[
\lim_{\Omega \to \infty} \alpha \sim \log(L) ,
\]

where \( L \), as before, is the length scale which characterizes the wavefunction of the tunneling electron, and \( \Omega \propto L^2 \) is the size of the integration region in Eq. (6). The details of the anomalous dimension of the propagator in this case depend on the nature of the interactions which determine

\[
V_{\text{eff}}(\mathbf{k},\omega) = V^2(\mathbf{k}) \text{Im } \chi(\mathbf{k},\omega) .
\]

To make contact with the work of Turlakov and Leggett about the \( c \)-axis interlayer tunneling in the cuprates, we have computed the parameter \( \alpha \) for different possible interactions. In \( \mathcal{O} \), several models, describing the voltage noise, are examined to estimate the parameter \( \alpha \) in order to describe the \( c \)-axis transport behavior.

Here we consider three examples of potential: a long-range Coulomb interaction, a short-range Coulomb interaction and the intermediate situation between them, the Thomas–Fermi screened potential. The parameter \( \alpha \) is calculated in the three cases.

a) Unscreened Coulomb potential, \( V(\mathbf{k}) = \frac{2\pi e^2}{e_0 |\mathbf{k}|} \). Due to the highly singular interaction, the \( \omega \) dependence of the effective potential is not linear and \( \alpha \) is not well defined in this case. We can still analyze the tunneling by computing the effective potential Eq. (6). It is computed to be:

\[
V_{\text{eff}}(\mathbf{k},\omega) = \frac{e^4}{e_0^2} \left[ 1 + \log \left( \frac{\omega}{\omega_0} \right) \right] ,
\]

where \( \omega_0 = |E_F - E_{VH}| \) is a low-energy cutoff, \( E_F \) is the Fermi energy and the position of the saddle point is at \( E_{VH} \). \( \omega_0 \) keeps the chemical potential away from the singularity and is required to avoid infrared divergences in the integrals. From Eq. (6) it can be seen that the tunneling in this case is exponentially suppressed.

b) Consider now a screened interaction of the Hubbard type, \( V(\mathbf{k}) = U a^2 \), where \( a \) is the lattice unit. In this case the effective potential has a linear term in \( \omega \) and the formalism described proceeds straightforward to give a value of the exponent \( \alpha \) which is

\[
\alpha = \frac{4U^2}{\pi e^2} \frac{1}{(2\pi)^2} K_M \left[ 1 + \frac{1}{2} \log \left( \frac{\Lambda}{\omega_0} \right) \right] ,
\]

where \( \Lambda \) is a high-energy cutoff, and

\[
K_M = \frac{1}{2} \log \left( \frac{k_x + k_y}{k_x - k_y} \right) ,
\]

where \( k_x, k_y \sim \sqrt{\omega_0} \). The RG analysis of \( \mathcal{O} \) shows that the coupling constant \( U \) of the model renormalizes to large values making the interlayer tunneling irrelevant.

c) Finally we will analyze the case of a Thomas–Fermi screened potential which is an intermediate situation with respect to the previous cases.

\[
V(\mathbf{k})^2 = \frac{(2\pi e^2)^2}{e_0^2 (k_F^2 + k_F^2)} .
\]

In this case \( \alpha \) can also be defined and it is computed to be

\[
\alpha \sim \frac{2e^4 k_F^2}{e_0 \pi e^2} \log^2 (k_T F/k) ,
\]

where \( k_T F \) is the Thomas–Fermi wavevector, and, \( k \) is a momentum cutoff \( k \sim \sqrt{\omega_0} \). In this case the longer range of the interaction makes the divergence softer as the dependence of the cutoff is as a squared log instead of a linear log.
In all the three cases studied α diverges as $E_F \rightarrow E_{VH}$. Thus, interlayer hopping is an irrelevant variable, and scales towards zero as the temperature or frequency is decreased. The additional logarithmic dependence found can be seen as a manifestation of the $\log^2$ divergences which arise in the treatment of this model\cite{1}. Note that, as in the graphene case, the coupling constants are also energy dependent, but in the Van Hove case they have an unstable flow and grow at low energies, suppressing even further the interlayer tunneling. This behavior of the interlayer hopping is in agreement with that obtained in Ref.\cite{2}, where it is found that c-axis interlayer tunneling is suppressed by voltage fluctuations.

In the case of a layered system, we can use the Random Phase Approximation (RPA), and include the effects of interplane screening\cite{3}:

$$\chi_{RPA}(\mathbf{k},\omega) = \frac{\sinh(|\mathbf{k}|d)}{\sqrt{[\cosh(|\mathbf{k}|d) + \frac{\pi e}{|\mathbf{k}|d} \sinh(|\mathbf{k}|d)\chi_0(\mathbf{k},\omega)]^2 - 1}}$$

(29)

where $d$ is the interplane spacing.

A numerical computation of\cite{20} provides the effective potential Eq. (7). The imaginary part $\chi_{RPA}$ gives the quantity usually known as the loss function that can be experimentally determined. This calculation is shown in Fig. 2.

The calculated effective potential compares well with the experimental plots of the energy loss function of Bi$_2$Sr$_2$CaCu$_2$O$_8$, measured by transmission energy loss spectroscopy in the low energy region\cite{20}, what reveals that the Van Hove model is also compatible with transport experiments. The results for the parameter $\alpha$ extracted from the numerical computation are in qualitative agreement with the analytical expressions given in cases of b) screened Hubbard interaction and c) Thomas-Fermi screened potential, studied above.

In a recent publication\cite{21} it is argued that Umklapp processes are crucial to determine the spectrum of density fluctuations, especially in two dimensional systems. It is worth to notice that the dispersion relation of the Van Hove model\cite{20} is a case where Umklapp processes enhance significantly the response function at low frequencies and wavevectors.

**IV. CONCLUSIONS.**

We have discussed the supression of interlayer tunneling by inelastic processes in two dimensional systems in the clean limit. Our results suggest that, when perturbation theory for the in–plane interactions leads to logarithmic divergences, the out of plane tunneling acquires a non trivial energy dependence. This anomalous scaling of the interlayer hopping can make it irrelevant, at low energies, if the in–plane interactions are sufficiently strong. A well known problem where this non trivial scaling takes place is the tunneling between one dimensional Luttinger liquids\cite{22,23,24}.

In two dimensions, the scaling towards zero of the out of plane hopping is always the case if the Fermi level of the interacting electrons lies at a van Hove singularity (note that the Fermi level can, in certain circumstances, be pinned to the singularity\cite{25}). In this situation, insulating behavior in the out of plane direction is not incompatible with gapless or even superconducting in–plane properties. If the Fermi level is not tuned to the singularity, the scaling presented here is only valid for energies larger than the distance of the Fermi level to the Van Hove singularity. Within this range of energies or temperatures, insulating behavior can be expected, and Fermi liquid (coherent) behavior will set in at lower energies. The appearance of instabilities can render this low energy regime unobservable.

Clean graphene planes show the opposite behavior, as electron-electron interactions become irrelevant at low energies.

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