Evolution and Asteroseismology of Pulsating Low-Mass White Dwarfs

L. M. Calcaferro, A. H. Córsico, L. G. Althaus, A. D. Romero and S. O. Kepler

1 Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque s/n (1900) La Plata, Argentina
2 Instituto de Astrofísica La Plata, CONICET-UNLP, Paseo del Bosque s/n, 1900, La Plata, Argentina
3 Departamento de Astronomía, Universidade Federal do Rio Grande do Sul, Av. Bento Gonçalves 9500, Porto Alegre 91501-970, RS, Brazil
lcalcaferro.acorsico,althaus@fcaglp.unlp.edu.ar; alejandra.romero@ufrgs.br, kepler@if.ufrgs.br

Abstract

Many low-mass white dwarfs are being discovered in the field of our galaxy and some of them exhibit $g$-mode pulsations, comprising the extremely low-mass variable (ELMV) stars class. Although it is generally believed that these stars are characterized by thick H envelopes, the existence of low-mass WDs with thin H envelopes is also possible from stellar evolution considerations. We have performed detailed asteroseismological fits to all the known ELMVs to search for a representative model by employing a set of fully evolutionary models that are representative of low-mass He-core white dwarf stars with a range of stellar masses $[0.1554-0.4352] M_{\odot}$, effective temperatures $[6000-10000] K$, and also with a range of H envelope thicknesses $-5.8 \lesssim \log(M_{H}/M_\star) \lesssim -1.7$, hence expanding the space of parameters. We found that some of the stars under analysis are characterized by thick H envelopes, but others are better represented by models with a thin H envelope.

1 Introduction

Low-mass white dwarfs (LMWDs), which are characterized by $M_\star \lesssim 0.45 M_{\odot}$, are thought to be formed by strong mass-loss episodes at the red giant branch (RGB) of low-mass stars in binary systems before the occurrence of the He flash, so they are expected to harbor He cores (see Althaus et al., 2013; Istrate et al., 2016 for instance). Among them, there is a population of WDs with very low mass: ELMs with $M_\star \lesssim 0.18 - 0.20 M_{\odot}$, characterized by $5 \lesssim \log(g) \lesssim 7$ and $8000 \lesssim T_{\text{eff}} \lesssim 22000 K$. For stars with masses greater than $0.18 - 0.20 M_{\odot}$ the WD progenitors are expected to experience CNO flashes that reduce their hydrogen content, making them unable to sustain nuclear burning. Then, they are expected to have shorter evolutionary timescales in comparison with stars with masses lower than $0.18 - 0.20 M_{\odot}$, whose progenitors are not expected to experience H flashes, and hence, end up with thicker H envelopes. Consequently, they sustain residual nuclear burning with the resulting longer evolutionary timescales. The mentioned upper-mass limit for ELM WDs is not only motivated by these physical differences but also because they differ in their pulsational properties (Althaus et al., 2013; Córsico & Althaus, 2014a), however this value depends on the metallicity of the WD progenitors (see Istrate et al., 2016, for instance).

These stars are detected by the ELM, SPY, WASP and SDSS surveys, among others, and some of them show $g$-mode pulsation periods (Hermes et al., 2012, 2013a; Kilic et al., 2015; Bell et al., 2015, 2017; Pelisoli et al., 2018; Bell et al., 2018), allowing the study of their interiors by applying the tools of WD asteroseismology. In the theoretical plane, adiabatic pulsational analyses show that $g$ modes in ELMVs with $M_\star < 0.18 M_{\odot}$ are restricted mainly to the core regions (Steinfadt et al., 2010; Córsico et al., 2012b; Córsico & Althaus, 2014a). Then, we can constrain the core chemical structure of these stars. Nonadiabatic stability computations (Córsico et al., 2012b; Van Grootel et al., 2013; Córsico & Althaus, 2016) predict that there are unstable $g$ and $p$ modes excited by $\kappa - \gamma$ (Unno et al., 1989) mechanism acting at the H-ionization zone. Also, the $\varepsilon$ mechanism may destabilize some short period $g$ modes (Córsico & Althaus, 2014b).

The asteroseismological techniques have been successfully applied to WD stars (Winget & Kepler, 2008; Fontaine & Brassard, 2008; Althaus et al., 2010). One of the main asteroseismological avenues, developed at La Plata Observatory, involves the calculation of fully evolutionary models characterized by chemical profiles resulting from all the processes experienced during the evolution of the WD progenitors, and this is the
approach we follow, that has already been employed in several cases; see, for instance, the cases for GW Virginis stars (Córscico et al., 2007a,b, 2008, 2009; Kepler et al., 2014; Calcaferro et al., 2016), DBV stars (Córscico et al., 2012a; Bognár et al., 2014) and ZZ Ceti stars (Kepler et al., 2012; Romero et al., 2012, 2013, 2017).

In Calcaferro et al. (2017) we have performed the first asteroseismological analysis of all the known ELMVs by computing period-to-period fits employing radial and non-radial g- and p-mode pulsation periods of low-mass He-core WD evolutionary models with stellar masses between 0.1554 and 0.4352M⊙, resulting from the computations of Althaus et al. (2013), that take into account the binary evolution of the progenitor stars. In that work, we were able to find solutions in most of the cases but in all of them, there were multiple possible solutions. In addition, for most of the stars the derived asteroseismological models are more massive in comparison with the spectroscopic determinations, something that could be related to the fact that the only low-mass He-core WD models considered are characterized by outer H envelopes coming from the stable mass loss scenario via Roche-lobe overflow. It cannot be discarded, however, that some of these stars can have thinner H envelopes that could result from common-envelope evolution of close binary systems (Nandez & Ivanova 2016; Ivanova & Nandez, 2016; Clayton et al., 2017), or from the loss of the envelope of a RGB star induced by an inspiralling giant planet (Nelemans & Tauris 1998; De Marco & Soker, 2002; Sabach & Soker, 2017), although this issue is currently under debate.

Then, by virtue of the mentioned considerations, we expand the space of parameters by introducing the H envelope thickness as an additional adjustable model parameter. Employing this new grid, we perform a thorough analysis by considering not only a range in stellar mass (0.1554 < M⋆ < 0.4352M⊙) and effective temperature (13000 ≲ T eff ≳ 6000 K) but also, in the H-envelope thickness (−5.8 ≲ log(MH/M⋆) ≲ −1.7, depending on the stellar mass). Thus, our fits are done over 17 000 WD configurations.

2 Methods

For the present analysis, we employed the realistic configurations for low-mass He-core WD stars computed by Althaus et al. (2013), that imitates the binary evolution of the progenitor stars assuming initial configurations consisting of a 1.0M⊙ Main Sequence (donor) star and a 1.4M⊙ neutron star companion as the other component. We refer the reader to that work for details of the physics and the numerical code (LP-PUL) employed. When the initial orbital period is varied (between 0.9 and 300d) different WD models are obtained (with M⋆ = 0.1554, 0.1612, 0.1650, 0.1706, 0.1762, 0.1805, 0.1863, 0.1921, 0.2025, 0.2390, 0.2707, 0.3205, 0.3624 and 0.4352M⊙). Adiabatic pulsation periods for non-radial g modes with ℓ = 1 and ℓ = 2 were taken from Córsico & Althaus (2014a) in the case of WD models with canonical H envelope thicknesses. For WD models having thinner H envelopes, we computed the periods for the present work. In both cases, the pulsation periods were computed employing the adiabatic version of the LP-PUL pulsation code Córsico & Althaus (2006).

To generate this new set of sequences with different H envelope thicknesses, for each sequence characterized by a given value of M⋆ and a thick (canonical) value of MΗ we have artificially replaced 1H by 4He from a given mesh point in order to obtain certain values of the H envelope thickness. This is done at very high Teff values at the final cooling track to ensure that any nonphysical transitory effects associated with this procedure have concluded long before the models get to the pulsating stage of ELMV WD stars. Time-dependent element diffusion was allowed to act after implementing the change in the thicknesses of the H envelope. Diffusion erodes considerably the chemical profiles at the transition regions. In Fig. 1, we show a graphical representation of the values that result for the different H envelope thicknesses, for every stellar mass under consideration, at Teff ~ 8000 K. A gray line connects the canonical values of the H-envelope mass as predicted by evolutionary models (Althaus et al., 2013). In the upper panel of Fig. 2, we display the internal chemical profiles for H corresponding to WD models at Teff ~ 8000 K with M⋆ = 0.2390M⊙, where the black line represents the profile corresponding to the canonical envelope and the lines of different colors correspond to the thin H envelopes. In all the envelopes of these models, the He/H transition region has a single-layered shape. The shape of the chemical profiles leaves significant signatures in the run of the squared critical frequencies, and in particular, in the Brunt–Väisälä frequency (N). In the lower panel of Fig. 2, we display the logarithm of the squared Brunt–Väisälä frequency, where we can see the clear connection between the chemical transition regions (upper panels) and the features in the run of the Brunt–Väisälä frequency for each model.

3 Results

With the aim of finding an asteroseismological model whose periods best match the observed periods of every ELMV, we assess the quality function given by:

\[
\chi^2(M_\star, T_{\text{eff}}, M_\text{H}) = \frac{1}{n} \sum_{i=1}^{n} \min[(H_i^0 - H_i^T)^2],
\]
with \( n \) being the number of observed periods. The ELM model with the lowest value of \( \chi^2 \), if it exists, is adopted as the “best-fit model.” We compute this merit function \( \chi^2 = \chi^2(M_*, T_{\text{eff}}, M_H) \) for our set of stellar masses, covering a wide range in effective temperature \( 13000 \lesssim T_{\text{eff}} \lesssim 6000 \) K and also considering the thickness of the H envelope in the interval \(-5.8 \lesssim \log(M_H/M_*) \lesssim -1.7\). Firstly, we consider that all of the observed periods (\( \Pi_i^O \)) for each ELMV are associated with \( \ell = 1 \) g modes and we assess the quality function given by Eq. (1). Next, we consider a mix of g modes associated with both \( \ell = 1 \) and \( \ell = 2 \). Given that the solutions we obtain are more appropriate for the latter, in this work we only show those cases. In Figures 3 and 4 we show the projection on the effective temperature versus the stellar mass plane of the inverse of the quality function, \( (\chi^2)^{-1} \), for the ELMV under consideration, taking the corresponding set of observed periods into account, in analogy with Calcaferro et al. (2017). In these figures we include \( T_{\text{eff}} \) and \( M_* \) of the target star, along with their uncertainties for the 1D (orange box) and 3D (Tremblay et al., 2015, green box) model atmosphere determinations. For all stellar masses, we considered an uncertainty of 15% of the total mass, which is the characteristic difference in the value of the mass as derived from independent sets of evolutionary tracks (Calcaferro et al., 2017). Each point in the maps corresponds to a H envelope mass value \( (M_H/M_*) \) that maximizes the value of \( (\chi^2)^{-1} \) for that stellar mass and effective temperature. If there is a single maximum for a given star, we adopt the corresponding model as the asteroseismological solution. However, as in the cases under analysis where multiple possible solutions exist, we need to apply an external constraint, i.e. the uncertainty in the effective temperature given by the spectroscopy and, at variance with Calcaferro et al. (2017), we additionally employ the constraint of the stellar mass as given by the spectroscopic determinations.

We applied the mentioned procedure to all the known (and suspected) ELMVs, but here, we only show the results for two of them as an example. In Fig. 3 we show the case for SDSS J151826.68+065813.2 (J1518, for short). According to the 1D model atmosphere, this star is characterized by \( T_{\text{eff}} = 9990 \pm 140 \) K, \( \log(g) = 6.80 \pm 0.05 \) [cgs] and \( M_* = 0.220 \) \( M_\odot \) (Hermes et al., 2013b), and for the 3D model, \( T_{\text{eff}} = 9650 \pm 140 \) K, \( \log(g) = 6.68 \pm 0.05 \) [cgs] and \( M_* = 0.197 \) \( M_\odot \) (Tremblay et al., 2015). The seven periods observed for this star are \( \Pi_i^O = 1335.318 \pm 0.003, 1956.361 \pm 0.003, 2134.027 \pm 0.004, 2268.203 \pm 0.004, 2714.306 \pm 0.003, 2799.087 \pm 0.005 \) and \( 3848.201 \pm 0.009 \) s, according to Hermes et al. (2013b). As we can see in the figure, there are not any solutions within the spectroscopic boxes, however there is a possible solution at \( \sim 9487 \) K, characterized by \( 0.2390 \) \( M_\odot \), \( \log(M_H/M_*) = -3.67 \) and \( (\chi^2)^{-1} = 0.07 \). This is the best period fit in the considered ranges, and it lies closely to the spectroscopic parameters, so we adopt it as a solution for J1518. Once a model is adopted, it is worth determining the difference between the observed and theoretical periods. In this way, we assess the absolute period differences defined as \( \delta \Pi_i = |\Pi_i^O - \Pi_i^T| \) and we show in Table 1 the results for this case. In column 6 we also indicate the value of the linear non-adiabatic
growth rate, $\eta$ ($\eta \equiv -3\Im(\sigma)/\Re(\sigma)$, with $\Re(\sigma)$ and $\Im(\sigma)$ being the real and the imaginary part, respectively, of the complex eigenfrequency $\sigma$ computed with the nonadiabatic version of the LP-PUL pulsation code (Cór Disco et al., 2006; Cór Disco & Althaus, 2016)). If $\eta$ is positive (negative), the mode is unstable (stable). From this table we can see that most of the periods corresponding to the adopted asteroseismological model are associated with pulsationally unstable modes.

In Fig. 4(b) we show the case for SDSS J184037.78+642312.3 (J1840, for short), with spectroscopic parameters $T_{\text{eff}} = 9390 \pm 140$ K, $\log(g) = 6.49 \pm 0.06$ [cgs] and $M_* = 0.183\,M_\odot$ for the 1D model atmosphere (Hermes et al., 2012), and $T_{\text{eff}} = 9120 \pm 140$ K, $\log(g) = 6.34 \pm 0.05$ [cgs] and $M_* = 0.177\,M_\odot$, for the 3D model atmosphere (Tremblay et al., 2015). We consider the set of the five observed periods ($P_i^D = 1164.15 \pm 0.38, 1578.7 \pm 0.65, 2376.07 \pm 0.74, 3930.0 \pm 300$ and $4445.3 \pm 2.4$ s) according to Hermes et al., 2012). The figure shows the existence of multiple possible solutions, however there is a very good solution that lies within the 3D model atmosphere at $\approx 9007$ K, for 0.1805 $M_\odot$ and $\log(M_H/M_*) = -2.44$ (i.e., a model with a canonical envelope), with $(\chi^2)^{-1} = 0.21$. The fact that one of the observed periods has a meaningful uncertainty ($P_0^D = 3930.0$ s, $\sigma = 300$ s) led us to repeat the analysis but considering a set with $-\sigma$ and another one with $+\sigma$ and we show the results in Figs 4(a) and 4(c). We found that solutions change considerably when varying that period and, as a result, we were not able to find a unique solution, but a range of possible solutions with parameters between $M_* = 0.1554 - 0.1869\,M_\odot$, $T_{\text{eff}} \approx 8997 - 9244$ K, and $M_H/M_* = 6.53 \times 10^{-6} - 2.04 \times 10^{-2}$.

4 Conclusions

In this work we have presented an asteroseismological analysis carried out on pulsating ELM WD stars on the basis of our complete set of fully evolutionary models representative of low-mass He-core WDs with a range of H envelope thicknesses. We generated a new grid of models for every stellar mass in our set and performed an asteroseismological analysis to all the known (and suspected) ELMV stars, as in Calcaferro et al. (2017), but employing a larger set of evolutionary sequences that expands the parameter space by incorporating the thickness of the H envelope as a free parameter. We found multiple solutions in all the cases, that may be due to the few periods detected in these stars. Only with the inclusion of external constraints (i.e., spectroscopic parameters) were we able to adopt a model, however in three cases, we could only indicate an interval of possible solutions. These results are summarized in Table 2. Also, some datasets exhibit one or more periods with significant uncertainty ($\sigma$), then we carried the same analysis out but considering $\pm \sigma$, for the most uncertain period. We found a meaningful variation in the results, one of these cases being J1840. Furthermore, some of the solutions are characterized by thick (canonical) H envelopes and some by thin H envelopes. This reinforces the findings of Calcaferro et al. (2018) about the possible existence of ELM WDs with thin H envelopes, hence leading to the possibility that they could have been formed through unstable mass loss, maybe via common-envelope episodes or from the lost of the envelope of a RGB star induced by an inspiralling giant planet.

Our results show that with the current amount of observed periods of all the ELMVs, it is not possible to find a unique solution compatible with the spectroscopic determinations. It becomes an even more difficult task when one (or more) periods have a large uncertainty. Considering that we are employing a complete set of fully evolutionary models representative of He-core ELM WDs, with different H-envelope thicknesses, we have now reached a limit regarding the possibility of the asteroseismology of adopting a model through the period fit on the basis of this grid, in order to determine the internal structure of these stars. This indicates the

Figure 3: Projection on the $T_{\text{eff}}$ vs $M_*$/\(M_\odot\) plane of the inverse of the quality function for \(\ell = 1, 2, g\) modes, considering the set of observed periods for SDSS J151826.68+065813.2. The value of the thickness of the H envelope for each stellar mass corresponds to the sequence with the largest value of the inverse of the quality function for that stellar mass. The boxes depict the spectroscopic parameters for this star, along with their uncertainties, for the 1D and 3D model atmosphere. These spectroscopic boxes are defined considering $\pm \sigma$. The ranges in the three axes are focused on values of interest.
necessity of having richer observations of the pulsations of these stars, to be able to find more robust asteroseismological solutions. Finally, the discovery of new ELMVs is also a pressing need in order to have further knowledge of their internal structure, the nature of their progenitors, and the evolutionary channels that originates them (see Calcaferro et al., 2018).

Table 1: Observed and theoretical periods ($\ell = 1, 2$) for the asteroseismological model for J1518 with $M_\star = 0.2390\, M_\odot$, $T_{\text{eff}} \sim 9487\, K$ and $\log(M_H/M_\star) = -3.67$. The harmonic degree $\ell$, the radial order $k$, the absolute period difference, and the non-adiabatic growth rate for each theoretical period are also displayed.

| $\Pi^{a}_{0}$ [s] | $\Pi^{a}_{1}$ [s] | $\ell$ | $k$ | $|\delta\Pi^{a}|$ [s] | $\eta [10^{-5}]$ | Remark |
|-------------------|-------------------|-------|-----|-----------------|----------------|--------|
| 1335.318          | 1329.599          | 2     | 28  | 5.719           | 0.463          | unstable |
| 1956.361          | 1959.913          | 1     | 24  | 3.552           | 0.653          | unstable |
| 2134.027          | 2131.306          | 2     | 46  | 2.721           | 0.504          | unstable |
| 2268.203          | 2266.188          | 1     | 28  | 2.015           | 0.766          | unstable |
| 2714.306          | 2717.686          | 2     | 59  | 3.380           | $-0.373$       | stable  |
| 2799.087          | 2802.873          | 1     | 35  | 3.786           | 1.14           | unstable |
| 3848.201          | 3851.967          | 2     | 84  | 3.766           | $-4.96$        | stable  |

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Table 2: Main features of the adopted asteroseismological models for the known (and suspected) ELMVs.

| Star    | $T_{\text{eff}}$ [K] | $\log(g)$ [cgs] | $M_\star [M_\odot]$ | $\log(M_{\text{H}}/M_\star)$ | $\log(R_\star/R_\odot)$ | $\log(L_\star/L_\odot)$ |
|---------|----------------------|-----------------|----------------------|-----------------------------|--------------------------|--------------------------|
| J1840   | [8979,9244]          | [5.8276,6.7524] | (0.1554,0.1869)      | [-5.19,-1.69]               | [-1.5216,-1.0992]        | [-2.2166,-1.4249]        |
| J1112*  | 9301                 | 5.9695          | 0.1612               | -1.76                       | -1.1623                  | -1.4932                  |
| J1518   | 9487                 | 6.9994          | 0.2390               | -3.67                       | -1.5916                  | -2.3200                  |
| J1614   | 8989                 | 6.4468          | 0.1612               | -4.35                       | -1.4009                  | -2.0232                  |
| J2228*  | 7710                 | 6.1738          | 0.1554               | -1.69                       | -1.2725                  | -2.0409                  |
| J1738   | [8839,9273]          | [6.0506,6.9323] | [0.1612,0.1921]      | [-5.43,-1.76]               | [-1.5057,-1.2029]        | [-2.2548,-1.6560]        |
| J1618   | [8919,9231]          | [6.2661,6.7568] | [0.1650,0.1921]      | [-5.06,-1.89]               | [-1.5178,-1.2982]        | [-2.2447,-1.8401]        |
| J1735*  | 8075                 | 6.2241          | 0.1612               | -1.76                       | -1.2899                  | -1.9957                  |
| J2139   | 8173                 | 6.3355          | 0.1612               | 2.49                        | -1.3453                  | -2.0820                  |

Note: * Solution with canonical H envelope.

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