Scrutinizing the Green’s functions of QCD: Lattice meets Schwinger-Dyson

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Outline of the talk

- Schwinger-Dyson equations in non-Abelian gauge theories: difficulties with the conventional formulation
- Gauge-invariant truncation scheme: Pinch Technique
- Lattice results for gluon propagator
- Dynamical mass generation (Schwinger mechanism)
- Comparing SD results with lattice simulations
- IR finite effective charge
- Kugo-Ojima revisited
- Conclusions
The (quarkless) QCD Lagrangian

\[ \mathcal{L} = - \frac{1}{4} G_{\mu \nu}^a G^{a \mu \nu} + \frac{1}{2 \xi} (\partial^\mu A^a_\mu)^2 + \partial^\mu \bar{c}^a \partial_\mu c^a + g f^{abc} (\partial^\mu \bar{c}^a) A^b_\mu c^c \]

where gluonic field strength tensor

\[ G_{\mu \nu}^a = \partial^\mu A^a_\nu - \partial^\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu \]

\[ < 0 | T[A^a(x)_\mu A^b(y)_\nu] | 0 > = \Delta^{ab}_{\mu \nu}(x - y) \]

\[ < 0 | T[\bar{c}^a(x)c^b(y)] | 0 > = D^{ab}(x - y) \]
Off-shell Green’s functions

- **Gauge dependent**

- **Renormalization point** $\langle \mu \rangle$ and **scheme dependent**

  $\implies$ **Not really physical**

**However...**

- They **capture** characteristic ingredients of the **underlying dynamics** (perturbative and non-perturbative)

- When appropriately combined **give rise** to physical observables

  $\implies$ **crucial pieces** for completing the **QCD puzzle**.
Beyond perturbation theory...

**Non-perturbative effects**
Lattice, Schwinger-Dyson equations

**Asymptotic Freedom**
Equations of motion for off-shell Green’s functions.
Derived formally from the generating functional.

Infinite system of coupled non-linear integral equations.
Inherently non-perturbative.
Self-consistent truncation scheme must be used.
Difficulties with conventional SD series

\( q^\mu \Pi_{\mu \nu}(q) = 0 \)

The most fundamental statement at the level of Green’s functions that one can obtain from the BRST symmetry. It affirms the transversality of the gluon self-energy and is valid both perturbatively (to all orders) as well as non-perturbatively.

Any good truncation scheme ought to respect this property.

Naive truncation violates it.
\[ \Delta_{\mu\nu}(q) = -1_{\mu} = -1_{\nu} + \frac{1}{2} \]

\[ + + + \]

\[ q^\mu \Pi_{\mu\nu}(q) \big|_{(a)+(b)} \neq 0 \]

\[ q^\mu \Pi_{\mu\nu}(q) \big|_{(a)+(b)+(c)} \neq 0 \]

**Main reason**: Full vertices satisfy complicated Slavnov-Taylor identities.
The **pinch technique** defines a good truncation scheme.

**Diagrammatic rearrangement** of perturbative expansion (to all orders) gives rise to effective Green’s functions **with special properties**.

J. M. Cornwall, Phys. Rev. D 26, 1453 (1982)
J. M. Cornwall and J.P., Phys. Rev. D 40, 3474 (1989)
D. Binosi and J.P., Phys. Rev. D 66, 111901 (2002)
M. Binger and S.J.Brodsky, Phys. Rev. D 74:054016 (2006)

**Longitudinal momenta** trigger **Slavnov-Taylor identities** inside diagrams:

\[
\begin{align*}
k \nu \gamma' &= (k + \not p - m) - (\not p - m) \\
&= S_0^{-1}(k + p) - S_0^{-1}(p),
\end{align*}
\]
Pinch technique rearrangement:
Simple, QED-like Ward Identities, instead of Slavnov-Taylor Identities, to all orders

\[ q_1^\mu \tilde{\Gamma}_{\mu\alpha\beta}^{abc}(q_1, q_2, q_3) = g f^{abc} \left[ \Delta^{-1}_{\alpha\beta}(q_2) - \Delta^{-1}_{\alpha\beta}(q_3) \right] \]

Profound connection with Background Field Method

Special transversality properties
Transversality enforced loop-wise and field-wise

The gluonic contribution

\[ q^\mu \Pi_{\mu\nu}(q)|_{(a1)+(a2)} = 0 \]

The ghost contribution

\[ q^\mu \Pi_{\mu\nu}(q)|_{(b1)+(b2)} = 0 \]
New Schwinger-Dyson series

Transversality is enforced separately for gluon- and ghost-loops, and order-by-order in the “dressed-loop” expansion!

A. C. Aguilar and J. P., JHEP 0612, 012 (2006)
D. Binosi and J. P., Phys. Rev. D 77, 061702 (2008); JHEP 0811:063,2008.
A. Cucchieri and T. Mendes, PoS LAT2007, 297 (2007).
I. L. Bogolubsky, et al, PoS LAT2007, 290 (2007)

... looks like a massive propagator.
\[ \Delta(q^2) = \frac{1}{q^2[1 + \Pi(q^2)]} \]

- If \( \Pi(q^2) \) has a pole at \( q^2 = 0 \) the vector meson is massive, even though it is massless in the absence of interactions.

  - **J. S. Schwinger**, Phys. Rev. 125, 397 (1962); Phys. Rev. 128, 2425 (1962).

- Requires massless, *longitudinally coupled*, Goldstone-like poles \( \sim 1/q^2 \)

- Such poles can occur dynamically, even in the absence of canonical scalar fields. Composite excitations in a strongly-coupled gauge theory.

  - **R. Jackiw and K. Johnson**, Phys. Rev. D 8, 2386 (1973)
  - **J. M. Cornwall and R. E. Norton**, Phys. Rev. D 8 (1973) 3338
  - **E. Eichten and F. Feinberg**, Phys. Rev. D 10, 3254 (1974)
Dynamics enters through the three-gluon vertex:

\[ \begin{align*}
\text{Satisfies the correct Ward identity} & \quad q_1^{\mu} \tilde{T}_{\mu \alpha \beta}^{abc}(q_1, q_2, q_3) = g f^{abc} \left[ \Delta_{\alpha \beta}^{-1}(q_2) - \Delta_{\alpha \beta}^{-1}(q_3) \right] \\
\text{longitudinally coupled massless bound-state poles} & \quad \sim 1/q^2, \text{ instrumental for } \Delta^{-1}(0) > 0.
\end{align*} \]
Numerical results and comparison with lattice

**Infrared Finite**

![Graph showing numerical results and comparison with lattice.](image)

- A. C. Aguilar, D. Binosi and J. P., Phys. Rev. D 78, 025010 (2008).
- I. L. Bogolubsky, *et al*., PoS LAT2007, 290 (2007)
- A. Cucchieri and T. Mendes, PoS LAT2007, 297 (2007); Phys. Rev. Lett. 100, 241601 (2008)
The gluon “mass” is not “hard” but momentum-dependent

\[ m^2(q^2) \sim \frac{\langle G_{\mu\nu}^2 \rangle}{q^2} \]

\[ \langle G_{\mu\nu}^2 \rangle: \text{dimension four gauge-invariant gluon condensate} \]

J. M. Cornwall, Phys. Rev. D 26, 1453 (1982).
M. Lavelle, Phys. Rev. D 44, 26 (1991).
A. C. Aguilar and JP, Eur. Phys. J. A 35, 189 (2008).
The ghost sector: SD equation

\[(\frac{\phantom{k}}{q})^{-1} = (\phantom{k}q)^{-1} + \frac{k}{q} (k + q)\]

- **Landau gauge** \(\Rightarrow \Gamma_\mu = \text{tree level}\)

\[
D^{-1}(p^2) = p^2 - g^2 C_A \int_k \left[ p^2 - \frac{(p \cdot k)^2}{k^2} \right] \Delta(k) D(p + k)
\]
In the deep IR: $p^2 D(p^2) \rightarrow \text{constant} \quad \Rightarrow \quad \text{No \textit{“power-law enhancement”!}}$

$\Rightarrow \text{At odds with the \textit{“ghost-dominance”} picture.}$

C. S. Fischer, J. Phys. G 32, R253 (2006)
Abelian Ward identities \( \widehat{Z}_1 = \widehat{Z}_2, \ Z_g = \widehat{Z}_A^{-1/2} \)

\[ \equiv \text{Renormalization-group invariant combination} \]

\[ \widehat{a}_0(q^2) \equiv g_0^2 \widehat{\Delta}_0(q^2) = g^2 \widehat{\Delta}(q^2) \equiv \widehat{a}(q^2) \]

\( \Delta(q^2) \) and \( \widehat{\Delta}(q^2) \) are connected by the formal relation:

\[ \Delta(q^2) = \left[ 1 + G(q^2) \right]^2 \widehat{\Delta}(q^2) \]

D. Binosi and J. P., Phys. Rev. D 66, 025024 (2002) .
where

\[ G(q) = \Delta \]

\[ D \]

- Important Green’s function!

- Its SDE is (Landau gauge)

\[
G(q^2) = -\frac{C_A g^2}{3} \int_k \left[ 2 + \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k + q).
\]
Checking the perturbative behavior of $\hat{\Delta}(q^2)$

- Enforces $\beta$ function coefficient in front of UV logarithm ($b = 11 C_A/48 \pi^2$).

\[ g^2 \hat{\Delta}(q^2) = \frac{g^2 \Delta(q^2)}{[1 + G(q^2)]^2} \]

\[ 1 + G(q^2) = 1 + \frac{9}{4} \frac{C_A g^2}{48 \pi^2} \ln(q^2/\mu^2) \]

\[ \Delta^{-1}(q^2) = q^2 \left[ 1 + \frac{13}{2} \frac{C_A g^2}{48 \pi^2} \ln(q^2/\mu^2) \right] \]

\[ \downarrow \]

\[ \hat{\Delta}^{-1}(q^2) = q^2 \left[ 1 + b g^2 \ln(q^2/\mu^2) \right] \]
The $\mu$-dependent ingredients from the SDE

**Gluon Propagator**

- $\alpha(\mu^2) = 0.21$ and $\mu = 4.3$ GeV
- $\alpha(\mu^2) = 0.16$ and $\mu = 10$ GeV
- $\alpha(\mu^2) = 0.13$ and $\mu = 22$ GeV

**Dependence on $\mu$**
Forming the $\mu$-independent $\hat{d}(q^2)$

\[ \chi \frac{g^2(\mu)}{[1+G(q^2)]^2} \]

No $\mu$-dependence
Defining the effective charge

Cast the dimensionful \( \tilde{d}(q^2) = g^2 \Delta(q^2) \) in the form:

\[
\tilde{d}(q^2) = \frac{4\pi \bar{\alpha}(q^2)}{q^2 + m^2(q^2)},
\]

where the dimensionless effective charge is

\[
\bar{\alpha}(q^2) = \frac{1}{4\pi b \ln \left( \frac{q^2 + \rho m^2(q^2)}{\Lambda^2} \right)}
\]

- It displays asymptotic freedom in the UV.
- **Freezes** at a finite value in the low energy regime

\[
\bar{\alpha}^{-1}(0) = 4\pi b \ln \left( \frac{\rho m^2(0)}{\Lambda^2} \right) \implies \text{Infrared Fixed Point for QCD}
\]
Infrared finite effective charge

**Infrared fixed point**

**Running charge** $m_0 = 600$ MeV

- $\alpha(\mu^2) = 0.21$ and $\mu = 4.3$ GeV
- $\alpha(\mu^2) = 0.16$ and $\mu = 10$ GeV
- $\alpha(\mu^2) = 0.13$ and $\mu = 22$ GeV

**Asymptotic freedom**
The Kugo-Ojima story

Consider the Green’s function

\[
\int d^4 x \ e^{-i \cdot (x-y)} \langle T \left[ (D_\mu c)_m^n \ (f_{nrs} A_{\nu}^n c^s)_y \right] \rangle = P_{\mu \nu}(q) \delta^{mn} u(q^2)
\]

A heavy-duty formal study concludes that if

\[
u(0) = -1
\]

⇒ **Color Confinement** and **infrared-enhanced** ghost

T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. 66, 1 (1979).

However, the ghost IS NOT enhanced. And, in addition ...
No evidence of $u(0) \rightarrow -1$

A. Sternbeck, arXiv:hep-lat/0609016
A special relation allows us to write

$$u(q^2) = -\frac{1}{3} C_A g^2 \int_k \left[ 2 + \frac{(k \cdot q)^2}{k^2 q^2} \right] \Delta(k) D(k + q).$$

Plug in the lattice results for $\Delta$ and $D$, and see what happens.
Our results

\[ u(0) \sim -\frac{2}{3} \]
A prediction and a challenge

Relates a “ghostly” Green’s function to two gluon propagators, defined at two completely different quantization schemes.

For the Kugo-Ojima criterion \[ u(0) = -1 \] to be valid, we must have: \[ \Delta(0) \rightarrow \infty \quad ... \]

No way!

Instead ...
Our prediction:

Infrared Finite

The challenge: Compute $\hat{\Delta}$ on the lattice!

Lattice formulation of Background Field Method exists.

R. F. Dashen and D. J. Gross, Phys. Rev. D 23, 2340 (1981)
Conclusions

- **Self-consistent description** of the non-perturbative QCD dynamics in terms of an **IR finite gluon propagator** appears to be within our reach.

- **Gauge invariant truncation** of SD equations furnishes reliable non-perturbative information and strengthens the synergy with the lattice community.

- **Meaningful contact with phenomenological studies.**
Effective low-energy field theory describing the gluon mass: massive gauge-invariant Yang-Mills

\[ \mathcal{L}_{MYM} = \frac{1}{2} G_{\mu\nu}^2 - m^2 \text{Tr} \left[ A_\mu - g^{-1} U(\theta) \partial_\mu U^{-1}(\theta) \right]^2 \]

\[ U(\theta) = \exp \left[ i \frac{1}{2} \lambda_a \theta^a \right], \theta^a: \text{scalar (Goldstone-like) fields} \]

Locally gauge-invariant under combined

\[ A'_\mu = VA_\mu V^{-1} - g^{-1} [\partial_\mu V] V^{-1}, \quad U' = U(\theta') = VU(\theta) \]

Gauged non-linear sigma model:
⇒ non-renormalizable (because \( m = \text{const} \)).
But, from the SD analysis, \( m = m(q^2) \), vanishes in the UV
⇒ renormalizability restored.
What about confinement?

If gluons are “massive”, where does the long range force associated with confinement come from?

- $\mathcal{L}_{MYM}$ admits vortex solutions, with a long-range pure gauge term in their potentials (like Nielsen-Olesen).

- Vortices have topological quantum number corresponding to the center of the gauge group, $Z_N$ for $SU(N)$.

- Center vortices of thickness $\sim m^{-1}$ form a condensate: their entropy is larger than their action $\Rightarrow \langle G_{\mu\nu}^2 \rangle$. 

The topological linking (Gauss linking) between the (fundamental representation) Wilson loop and center vortices with a finite density in the vacuum

⇒ area law
⇒ quark confinement

(not fully demonstrated!)