Multi-aircraft cooperative detection strategy based on formation

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Abstract. A filter algorithm combining Interactive Multiple Model (IMM) algorithm with adaptive transition probability and Unscented Kalman Filter (UKF) is proposed for the issue of state estimation of highly maneuvering targets with unknown motions. The centralized fusion structure is adopted to process measurements of multi-aircraft to improve the accuracy of target state estimation. The cooperative detection strategy based on formation is studied from two aspects of the aircrafts number and configuration radius, and conclusions are drawn through simulation: In general, the more the number of aircrafts, the better the detection effect; when the configuration radius is 250km and 100km, the better cooperative detection effect can be obtained; considering the cost issue, 4 aircrafts can be selected for cooperative detection in a rectangular formation with a half diagonal of 100km.

1. Introduction
The detection and tracking technologies for flying targets have always been key research in the aerospace field. However, in recent years, with the improvement of aircrafts manoeuvrability, the increasingly complex electromagnetic environment, and the emergency of group combat, it has become more difficult to detect and track flying targets. Multi-sensor cooperative detection is proposed in this situation. Multi-sensor systems can not only enhance the survivability and reliability of the system, but also increase the credibility of measurement information, while expanding the space and time coverage and improving detection performance.

During cooperative detection of multiple aircrafts, there are many factors that affect the accuracy of cooperative detection, such as the number of aircrafts, the relative position between aircrafts, the distance to the targets, the precision of the sensors, and the field of view. In this paper, two formation parameters, the number of aircrafts and configuration radius of aircrafts, are studied, and conclusions are obtained to guide the setting of formation parameters.

This paper focuses on the following aspects: Firstly, the target state estimation algorithm is studied for the detection of manoeuvring targets. Secondly, the multi-sensor data fusion algorithm is studied. Finally, the relationship between formation parameters and cooperative detection accuracy is analysed.

2. Target state estimation
Kalman filter is generally used for target state estimation, and the state equation of the target is written as equation (1). The measurement equation, which is generally nonlinear, is expressed as equation (2):

\[ x(k) = \Phi(k, k-1)x(k-1) + \Gamma(k, k-1)w(k-1) \]  
\[ z(k) = h[x(k), k] + v(k) \]  

\[ \text{(1)} \]

\[ \text{(2)} \]
2.1. Unscented Kalman Filter

It is necessary to select a filtering method suitable for the nonlinear system. Commonly used are Extended Kalman filter (EKF), Unscented Kalman filter (UKF), Particle filter (PF) and so on. Among these filtering methods, Unscented Kalman Filter (UKF) uses UT transformation to approximate the probability density distribution of the nonlinear function. It has the same amount of calculation as EKF but doesn’t need to ignore the second-order component or calculate the Jacobian matrix, which makes its precision higher and application range wider [1]. The basic steps of UKF are as follows:

① Set initial value.

② When \( k > 1 \), calculate \( 2n + 1 \) sigma points.

\[
[\hat{x}(k-1), \kappa(n + \kappa)^{-1}],
\]

\[
\hat{x}(k-1) \pm [\sqrt{(n + \kappa)P(k-1)}]_{i} , [2(n + \kappa)]^{-1}, \quad i = 1 \ldots n \quad (3)
\]

\[
\hat{x}(k-1) \pm [\sqrt{(n + \kappa)P(k-1)}]_{i} , [2(n + \kappa)]^{-1}, \quad i = n + 1 \ldots 2n
\]

Where, \( n \) is the dimension of the target state vector, and \( \kappa \) is the scaling factor, which is generally taken as \( \kappa + n = 3 \).

③ Time update.

\[
[\hat{x}(k, k-1), \hat{z}(k, k-1)] = \left\{ \sum_{i=0}^{2n} W_{i} [\Phi(k, k-1)\chi_{i}], \sum_{i=0}^{2n} W_{i} h(\chi_{i}, k) \right\}
\]

\[
P(k, k-1) = \sum_{i=0}^{2n} W_{i} [\Phi(k, k-1)\chi_{i} - \hat{x}(k, k-1)][\Phi(k, k-1)\chi_{i} - \hat{x}(k, k-1)]^{T} + FQ_{k-1}F^{T}
\]

\[
P_{z}(k) = \sum_{i=0}^{2n} W_{i} [h(\chi_{i}, k) - \hat{z}(k, k-1)][h(\chi_{i}, k) - \hat{z}(k, k-1)]^{T} + R_{k-1}
\]

④ Measurement update.

\[
K(k) = \left\{ \sum_{i=0}^{2n} W_{i} [\Phi(k, k-1)\chi_{i} - \hat{x}(k, k-1)][h(\chi_{i}, k) - \hat{z}(k, k-1)]^{T} \right\} P_{z}^{-1}(k)
\]

\[
\hat{x}(k) = \hat{x}(k, k-1) + K(k)[z(k) - \hat{z}(k, k-1)]
\]

\[
P(k) = P(k, k-1) - K(k)P_{z}(k)K^{T}(k)
\]

2.2. IMM-UKF with adaptive transition probability algorithm

Because the target motion form is generally unknown and the target may maneuver, it is considered to use multiple target models to filter in parallel. IMM algorithm is to treat the real motion state of the target as a combination of states generated by multiple target models, and each model has a certain probability to match the real motion [2]. Assuming that there are \( r \) target models, the state equations and measurement equations for each model can be written by equations (1) and (2). The transition probability matrix that controls model set switching is introduced as

\[
T_{p} = \begin{bmatrix}
p_{11} & \cdots & p_{1r} \\
\vdots & \ddots & \vdots \\
p_{r1} & \cdots & p_{rr}
\end{bmatrix}
\]

\( p_{ij} \) means the probability that the model \( i \) will be transferred to the model \( j \) in the next step. At time \( k \), filter steps are as follows:

① Reinitialization.

② When \( k > 1 \), calculate \( 2n + 1 \) sigma points.
\[ c^j(k) = \sum_{i=1}^{r} T^{i,j}_p u_{\text{IMM}}^j(k) \quad j = 1, \ldots, r \quad (10) \]

Where, \( c^j \) means the probability of model \( j \) after the transition; \( T^{i,j}_p \) means the probability of transferring model \( i \) to model \( j \); \( u_{\text{IMM}}^j \) means the probability of model \( j \) at the end of previous step.

Next perform input interaction, and get the reinitialized state and covariance:

\[ \hat{x}_0^j (k) = \sum_{i=1}^{r} \hat{x}^i (k) T^{i,j}_p u_{\text{IMM}}^j (k) c^j (k)^{-1} \quad (11) \]

\[ p_0^j (k) = \sum_{i=1}^{r} \left[ T^{i,j}_p u_{\text{IMM}}^j (k) c^j (k)^{-1} \left[ p^i (k) + \left( \hat{x}^i (k) - \hat{x}_0^i (k) \right) \left( \hat{x}^i (k) - \hat{x}_0^i (k) \right)^T \right] \right] \quad (12) \]

Where, \( \hat{x}^i (k) \) means the target state estimation of the model \( i \) at the last moment (k moment);

② Filtering.
③ Model probability updating.

Assuming the filtering residual of model \( j \) obeys Gaussian distribution, its likelihood function is

\[ A^j (k+1) = \left| 2 \pi S^j (k+1) \right|^{-1/2} \exp \left\{ -\frac{1}{2} \left[ v^j (k+1) \right]^T \left[ S^j (k+1) \right]^{-1} v^j (k+1) \right\} \quad (13) \]

Where, \( S^j (k+1) \) means the residual covariance matrix of model \( j \), and \( v^j (k+1) \) means the residual of measurements \( z(k+1) \) and predicted measurements \( z^p_{k+1} \) of model \( j \).

Update model probability with a normalized likelihood function:

\[ u_{\text{IMM}}^j (k+1) = A^j (k+1) \left[ \sum_{i=1}^{r} A^i (k+1) \right]^{-1} c^j (k) \left[ \sum_{j=1}^{M} A^j (k+1) \left[ \sum_{i=1}^{r} A^i (k+1) \right]^{-1} c^j (k) \right]^{-1} \quad (14) \]

④ Fusion.

\[ \hat{x}(k+1) = \sum_{j=1}^{M} u_{\text{IMM}}^j (k+1) \cdot \hat{x}^j (k+1) \quad (15) \]

\[ p(k+1) = \sum_{j=1}^{M} u_{\text{IMM}}^j (k+1) \left[ p^j (k+1) + \left[ \hat{x}^j (k+1) - \hat{x}(k+1) \right] \left[ \hat{x}^j (k+1) - \hat{x}(k+1) \right]^T \right] \quad (16) \]

⑤ Adaptive transition probability.

The transition probability matrix \( T_p \) is generally set to a fixed value, but due to the maneuverability of the target, it should be adaptive to better reflect the actual transformation of target motion modes. Given that the probability of model \( j \) at time \( k \) is \( u_{\text{IMM}}^j (k) \) , at time \( k+1 \) is \( u_{\text{IMM}}^j (k+1) \), the difference between \( u_{\text{IMM}}^j (k) \) and \( u_{\text{IMM}}^j (k+1) \) reflects the change of the matching degree between model \( j \) and the actual motion pattern. This information can be used to modify the transition probability \( T_p \). Considering the non-negativity of probability, the change rate of model probability can be taken in logarithmic form, so the adaptive transition probability can be calculated as

\[ \hat{T}_p^{i,j} (k+1) = \exp \left[ u_{\text{IMM}}^j (k+1) - u_{\text{IMM}}^j (k) \right] T_p^{i,j} (k) \quad (17) \]

Considering that the sum of the transition probabilities of a model transferring to all other models (including itself) at time \( k \) should be 1, equation (17) is normalized:
\[ T_p^{i,j}(k+1) = \frac{\tilde{T}_p^{i,j}(k+1)}{\sum_{j=1}^{M} \tilde{T}_p^{i,j}(k+1)} \]  

(18)

The above is the IMM-UKF with adaptive transition probability algorithm. It can be simply represented as shown in Figure 1.

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3. Multi-sensor data fusion

Data fusion is a combination of characteristic information of the same target to improve the accuracy of target state estimation. Fusion systems can be divided into three types: centralized, distributed and multi-level. Centralized fusion means that sensors transmit their measurements to the fusion center. It has optimal fusion results because of no information loss, but the cost would be high when the information transmitted is too much. In contrast, distributed fusion has lower cost but lower accuracy, because local filtering results were transmitted. Multi-level fusion can achieve high-precision estimation at a relatively low cost but with a complex structure \[4\]. Since the number of sensors considered in this paper is not very large, the communication channel will not be very demanding. So, we choose the centralized fusion system, to obtain a higher-precision results.

The centralized fusion system generally adopts measurement expansion algorithm and sequential filtering algorithm, both of which have the same estimation accuracy, but the calculation form of measurement expansion is simpler and easier to implement, so it is selected in this paper\[5\]. Assuming that the seeker \(S_i\) has measurements \(z_i\) and measurement error covariance matrix \(R_i\), and its measurement equation is \(h_i(x_i, S_i)\). The expressions of measurement expansion are as follows:

\[ z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_l \end{bmatrix} \]  

(19)

\[ R = \text{diag}(R_1, R_2, \ldots, R_l) \]  

(20)

\[ h = \begin{bmatrix} h_1(x_i, S_1) \\ h_2(x_i, S_2) \\ \vdots \\ h_l(x_i, S_l) \end{bmatrix}^T \]  

(21)

Equations (19) to (21) should be substituted into the filtering equations to estimate the target state.

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4. Multi-aircraft cooperative detection strategy based on formation

The formation of aircrafts will affect the effect of cooperative detection. This paper studies the influence of the two formation parameters of the number and configuration radius of aircrafts on the effect of cooperative detection through simulation, so that the formation parameters can be designed to optimize the cooperative detection effect.

Consider a formation of aircrafts in a circular configuration, as shown in Figure 2.
We consider a multi-to-one simulation scenario, in which the number of targets is 1, and the number of aircrafts varies from 2 to 6. Choose two aircrafts $S_1$ and $S_4$, which are equivalent to a straight formation, and apply their position information to the 2 to 1 simulation situation. Three aircrafts $S_1$, $S_3$, $S_5$ in an equilateral triangle formation are chosen for a 3 to 1 situation. Similarly, four aircrafts $S_1$, $S_3$, $S_4$, $S_6$ in a rectangular formation, five aircrafts $S_1$, $S_2$, $S_3$, $S_4$, $S_5$ in a pentagonal formation, and six aircrafts $S_1$, $S_2$, $S_3$, $S_4$, $S_5$, $S_6$ in a regular hexagon formation, are chosen for corresponding simulations. The configuration radius $R$ also changes.

The simulation step is 0.01s. For the convenience of comparison, the initial filter value is set to the true value. Three target models are selected in the filter algorithm, which are constant acceleration model (CA), constant speed coordinate turn model (CSCT), and current statistical model (CSM) [6].

The aircrafts configuration radius and the number of aircrafts change at the same time. As shown in Figure 3, the horizontal axis represents the number of aircrafts participating in cooperative detection, and the vertical axis represents the mean error of target displacement estimation. Each line corresponds to a configuration radius, and each point is calculated after ten Monte Carlo simulations.
Firstly, observe the influence of the number of aircrafts on the cooperative detection effect, and the effect is represented by the estimation accuracy of target displacement. The relationship between the detection effect and the number of aircrafts can be expressed as follows:

\[
\begin{align*}
    J_6 & > J_4 > J_5 > J_3 > J_2 \\
    R &= 50 \cdot 100 \cdot 150 \cdot 200 \text{km} \\
    J_6 & > J_4 > J_5 > J_3 > J_2 \\
    R &= 250 \text{km}
\end{align*}
\]

(22)

Where, \( J_i \) means the cooperative detection effect of \( i \) aircrafts, and \( R \) means the configuration radius.

Secondly, observe the influence of configuration radius on the cooperative detection effect. The relationship between detection effect and configuration radius can be expressed as follows:

\[
\begin{align*}
    Q_{250} & > Q_{100} > Q_{50} > Q_{200} > Q_{150} \\
    N &= 3 \cdot 5 \cdot 6 \\
    Q_{100} & > Q_{250} > Q_{50} > Q_{200} > Q_{150} \\
    N &= 2 \cdot 4
\end{align*}
\]

(23)

Where, \( Q_R \) means the cooperative detection effect when the configuration radius of aircrafts is \( R \) km, and \( N \) means the number of aircrafts participating in the cooperative detection.

Finally, combining the two parameters, the following conclusions can be drawn:
① For the same configuration radius, generally, the larger the number of aircrafts, the better the detection effect. But when the number is 5, the detection effect under some certain radii is worse than that of 4 aircrafts;
② For the same number of aircrafts, generally, the detection effect is best when the configuration radius is 250km. However, the detection effect of the configuration radius of 100km may be better than that of 250km under some certain number of aircrafts;
③ Considering the cost issue, four aircrafts can be selected for cooperative detection in a rectangular formation with a half diagonal of 100km.

5. Conclusion

In this paper, the target state estimation algorithm was firstly studied. Combining the IMM algorithm with adaptive transition probability and the UKF algorithm, a high-precision estimation of motion state of highly maneuvering targets had been achieved. Secondly, during multi-aircraft cooperative detection, a centralized fusion structure which adopted measurement expansion algorithm was selected for high-precision data fusion. Finally, the cooperative detection strategy based on formation is studied, and the conclusions are as follows:
① In general, the more the number of aircrafts, the better the detection effect;
② When the configuration radius is 250km and 100km, better cooperative detection effect can be achieved;
③ Considering the cost issue, four aircrafts can be selected for cooperative detection in a rectangular formation with a half diagonal of 100km.

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