Mass generation with Gauss-Bonnet term: No van Dam-Veltman-Zakharov discontinuity in AdS space

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Abstract

We prove that in anti de Sitter space, there is no van Dam-Veltman-Zakharov discontinuity in the graviton propagator. Here we obtain the mass term of $M^2 \propto \Lambda^2$ from the Gauss-Bonnet term, which is a ghost-free one. The condition that the massless limit is smooth is automatically satisfied for this case.

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Recently there has been much interest in the massless limit of the massive graviton propagator \(1–3\). A key word of this approach is that van Dam-Veltman-Zakharov (vDVZ) discontinuity \(5\) is peculiar to Minkowski space and it does not arise in anti de Sitter space. They used the spin-2 Pauli-Fierz mass term \(6\) for this calculation. As is well known, there is no origin for this in the starting action. A criterion for introducing the Pauli-Fierz term is that it is a ghost-free term for a free, massive spin-2 propagation in the linearized level\(2\). We need another ghost-free term which can be derived from the starting action. A concrete example is the Gauss-Bonnet term \(7\), which is obviously a ghost-free combination. This can generate a mass term in anti de Sitter space.

In this letter, we investigate the the vDVZ discontinuity in anti de Sitter space using the Gauss-Bonnet term. For simplicity, we choose the harmonic gauge (transverse gauge: \(\nabla_\mu h^{\mu\nu} = \frac{1}{2} \nabla^\nu h\)). This choice is obvious for the massless propagation because this has the gauge symmetry. Although the gauge symmetry is broken in the massive case, we can use this as the transversal condition to study the massive propagation.

We start from the 4D gravity with a cosmological constant and the Gauss-Bonnet term \(7\)

\[
I = \int d^4x \sqrt{-g}\left\{ \frac{1}{16\pi G} R - \Lambda + \alpha (R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2) \right\}.
\] (1)

Here we set \(16\pi G = 2\) and \(\Lambda < 0\). Also we follow \(\eta_{\mu\nu} = \text{diag}(- + + +)\) convention. The equation of motion is given by

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (R - 2\Lambda) = g_{\mu\nu} \alpha (R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2) - 2\alpha (2RR_{\mu\nu} - 4R_{\mu\rho}R_{\nu}^\rho - 4R_{\mu\nu\rho\sigma}R^{\rho\sigma} + 2R_{\mu\rho\sigma\kappa}R^{\rho\sigma\kappa}).
\] (2)

The anti de Sitter space solution is expressed in terms of the metric (\(\bar{g}_{\mu\nu}\)) as

\[
\bar{R}_{\mu\nu\rho\sigma} = \frac{\Lambda}{3} (\bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu}\rho), \quad \bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}, \quad \bar{R} = 4\Lambda.
\] (3)

To study the propagation of the metric, let us introduce the perturbation around the background space

\[
g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}.
\] (4)

Hereafter we use the background values without ”overbar” (for example, \(\bar{g}_{\mu\nu} \to g_{\mu\nu}\)). Further we use the gauge condition for a simple calculation. After a lengthy calculation, its linearized equation to Eq.(2) with the external source \(T_{\mu\nu}\) takes the form

\[\text{(For a spin-3/2 supersymmetric particle, its correct massless limit is not } m \to 0 \text{ but } m \to \sqrt{-\Lambda/3} \text{. I thank Waldron for pointing out this.)}

\[\text{(However, if one considers the massive graviton from the Kaluza-Klein reduction, the Pauli-Fierz term could be expected to appear. I thank Papazoglou for pointing out this point.)}\]
\[ \Delta_L h_{\mu\nu} + \frac{1}{2} \nabla^2 h_{\mu\nu} - 2\Lambda (h_{\mu\nu} - \frac{1}{2} h g_{\mu\nu}) + \frac{64\Lambda^2\alpha}{9} (h_{\mu\nu} - \frac{1}{4} h g_{\mu\nu}) = 4T_{\mu\nu} \quad (5) \]

with the Lichnerowicz operator \( \Delta_L \) for spin-2 field. In deriving this, we use the relations

\[ 2\delta R_{\mu\nu\alpha\beta}(h) = -\nabla_\alpha \nabla_\mu h_{\nu\beta} - \nabla_\beta \nabla_\nu h_{\mu\alpha} + \nabla_\alpha \nabla_\nu h_{\mu\beta} + R_{\mu\nu\alpha\beta} h^{\gamma\rho} + R_{\nu\gamma\alpha\beta} h^{\gamma\rho}, \quad (6) \]

\[ 2\delta R_{\mu\nu}(h) = \Delta_L(h_{\mu\nu}) - \nabla^2 h_{\mu\nu} - 2R_{\rho\mu\alpha\nu} h^{\rho\alpha} + 2R_{\rho(\mu} h^{\rho\nu)}, \quad (7) \]

\[ \delta R(h) = g^\mu \delta R_{\mu\nu}(h) - h^\mu R_{\mu\nu}. \quad (8) \]

Here we observe that the Gauss-Bonnet term contributes to the linearized equation as a traceless combination. The trace of Eq.(5) takes the form

\[ \nabla^2 h + 2\Lambda h = 4T. \quad (9) \]

Using this, Eq.(2) leads to

\[ \Delta_L h_{\mu\nu} - 2\Lambda h_{\mu\nu} + M^2_{GB} (h_{\mu\nu} - \frac{1}{4} h g_{\mu\nu}) = 4T_{\mu\nu} - 2T g_{\mu\nu}, \quad (10) \]

where the Gauss-Bonnet mass of the graviton is determined as \( M^2_{GB} = \frac{64\Lambda^2\alpha}{9} \). Now we are interested in the transverse traceless metric propagation (\( h^{tt}_{\mu\nu} \)) in anti de Sitter space. This corresponds to a true graviton propagation. Let us compare Eq.(10) with Eq.(3) in ref. [2]

\[ \Delta_L h^{tt}_{\mu\nu} - 2\Lambda h^{tt}_{\mu\nu} + M^2_{GB} h^{tt}_{\mu\nu} = 4T^{tt}_{\mu\nu} - 2T g_{\mu\nu}, \quad (11) \]

Under the harmonic gauge, two equations are the same forms except the trace term (\( = \cdots h g_{\mu\nu} \)). However this term is irrelevant to our purpose because we consider the transverse traceless sector. In order to study this propagation, we take

\[ \Delta_L h^{tt}_{\mu\nu} - 2\Lambda h^{tt}_{\mu\nu} + M^2_{GB} h^{tt}_{\mu\nu} = 4T^{tt}_{\mu\nu}. \quad (12) \]

Here the transverse traceless source (\( \nabla^\rho T^{tt}_{\rho\nu} = 0, T^{tt} = 0 \)) is given by [2]

\[ T^{tt}_{\mu\nu} = T_{\mu\nu} - \frac{1}{3} T g_{\mu\nu} + \frac{1}{3} (\nabla_\mu \nabla_\nu + g_{\mu\nu} \Lambda/3)(\nabla^2 + 4\Lambda/3)^{-1} T. \quad (13) \]

According to Porrati in [2], the result is clear. The propagator with both \( \Lambda \) and \( M^2_{GB} \) has only a physical pole at \( \nabla^2 = M^2_{GB} - 2\Lambda \). Its residue is given by

\[ 2T_{\mu\nu} T^{\mu\nu} + T \left[ \frac{2\Lambda - 2M^2_{GB}}{3M^2_{GB} - 2\Lambda} \right] T. \quad (14) \]

For \( \Lambda \to 0 \) at the fixed \( M^2_{GB} (> \Lambda, \alpha \to \text{large}) \), we have one-particle amplitude for the massive spin-2 field as

\[ A_{\text{massive}} = \frac{2}{\nabla^2 + M^2_{GB}} \left( T^{\mu\nu} T_{\mu\nu} - \frac{1}{3} T^2 \right). \quad (15) \]
This is positive (the ghost is free) \[8\]. On the other hand, for \( M_{GB}^2(< \Lambda, \alpha \to \text{small}) \to 0 \) at finite \( \Lambda \), one finds the massless spin-2 amplitude

\[
A_{\text{massless}} = \frac{2}{-\nabla^2} \left( T^{\mu\nu} T_{\mu\nu} - \frac{1}{2} T^2 \right)
\]

which leads also to a positive one. Concerning the smooth massless limit of \( M^2/\Lambda \to 0 \) \[4\], our case provides a good limit. This is so because \( M_{GB}^2 = \frac{64\Lambda^2\alpha}{9} \) vanishes faster than the cosmological constant \( \Lambda \), as \( \Lambda \to 0 \). For this case we assume that the Gauss-Bonnet parameter \( \alpha \) is small and finite. If we include the Pauli-Fierz mass term instead of the Gauss-Bonnet term of \( M_{GB}^2(h_{\mu\nu} - h g_{\mu\nu}/4) \), this contributes \( M_{PF}^2(h_{\mu\nu} - h g_{\mu\nu}) \) to Eq.(4). For comparison, we note the contribution of the cosmological term as \(-2\Lambda(h_{\mu\nu} - h g_{\mu\nu}/2)\) with \( \Lambda < 0 \). All of these play the same role of the mass-like parameter in anti de Sitter space.

At this stage we wish to comment on the origin of our mass. Miemiec \[10\] showed that the lowest mass of a localized graviton in 4D anti de Sitter space gets a quadratic one of \( \Lambda \). That is, \( M^2 \propto |\Lambda|^2 \). Also Schwartz \[11\] argued that in addition to a bare mass (\( M^2 \propto |\Lambda| \)) of graviton, there also exists a CFT correction to the graviton propagator via the AdS/CFT correspondence. This takes a form of \( \delta M^2 \propto |\Lambda|^2 \). We note that the Gauss-Bonnet term may arise from the quantum correction of the matters in the curved space. Hence our mass term which is proportional to \( \Lambda^2 \) is not an unphysical one but it may give us a new evidence for the AdS/CFT correspondence.

In conclusion, we obtain the mass term from the Gauss-Bonnet term. This is a ghost-free combination. Importantly we generate the mass term through the starting action. Although the trace part of its linearized equation differs slightly from the Pauli-Fierz mass term, the transverse traceless part is exactly the same as Eq.(3) in ref. \[2\]. Hence we find that there is no the vDVZ discontinuity in the graviton propagator.

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