HYPERNUCLEI AS CHIRAL SOLITONS.

V.B.Kopeliovich
Institute for Nuclear Research of the Russian Academy of Sciences,
Moscow 117312, Russia

The identification of flavored multiskyrmions with the ground states of known hypernuclei is successful for several of them, e.g. for isodoublet $^4\Lambda H - ^4\Lambda He$, isoscalars $^5\Lambda He$ and $^7\Lambda Li$. In other cases agreement is not so good, but the behaviour of the binding energy with increasing baryon number is in qualitative agreement with data. Charmed or beautiful hypernuclei within this approach are predicted to be bound stronger than strange hypernuclei. This conclusion is stable relative certain variation of poorly known heavy flavor decay constants.

1 Introduction

One of the actual questions of nuclear and elementary particle physics is the possibility of the existence of nuclear matter fragments with unusual properties, e.g. with flavor being different from that of $u$ and $d$ quarks. This issue can have interesting consequences in astrophysics and cosmology. The recently observed at Chandra X-ray Observatory stellar objects, RXJ1856 and 3C58, can be interpreted just as the strange quark matter stars. Experimental as well as theoretical studies of such nuclear fragments have been performed first for strangeness, see e.g. [1, 2] and references therein, and also, to some extent, for charm and beauty quantum numbers [3]-[6]. Theoretical approaches vary from standard nuclear potential models, to the topological soliton models (Skyrme model and its extensions). In the latter case, the extension of the original $SU(2)$ model into $SU(3)$ configuration space is necessary. It is known that in $SU(3)$ extensions of the model there are several different local minima in the configuration space [7]. The quantization of configurations near each of these minima is possible, leading to the prediction of spectrum of quantum states with different flavor quantum numbers. Here the quantization of $SU(2)$ bound skyrmions embedded into $SU(3)$ is considered, following papers [8, 9, 10]. The physical interpretation of such quantum states seems to be the simplest in comparison with others, because the lowest in energy states can be identified with usual nuclei. Previously we derived in this way some spectrum of ”flavored multiskyrmions” regardless of their interpretation [10]. Here we make an attempt to identify some of these states with known hypernuclei.

The chiral soliton models provide a picture of baryonic systems ($BS$) outside, from large enough distances, based on few fundamental principles and ingredients incorporated in the model lagrangian. The details of baryon-baryon interactions do not enter the calculations explicitly, although they make influence, of course, implicitly, via some integral characteristics of $BS$, such as their masses, moments of inertia ($\Theta_F$, $\Theta_T$ below), sigma-term ($\Gamma$), etc. The $SU(2)$ rational map (RM) ansatz [11] which approximates well the results of numerical calculations [12] was used as the starting point for the evaluation of static properties of bound states of skyrmions necessary for their quantization in the $SU(3)$ configuration space. The knowledge of the ”flavor” moment of inertia and the $\Sigma$-term allows us then to estimate the flavor excitation energies [8, 10]. The masses of the lowest states with strangeness, charm or beauty are calculated within the rigid oscillator version of the bound state approach, and the binding energies of baryonic systems with different flavors, $s$, $c$ or $b$, are estimated. Within the $RM$ approximation, at large enough $B$ the chiral field configuration has the form of the ”bubble” with universal properties of the shell, where the mass and baryon number of the $BS$ are concentrated. The width of the shell and its average mass density do not depend on the baryon number [13]. This picture can be
acceptable for not large $B$ ($B = A$ atomic number of the nucleus) say, up to $B \sim 16$, and by this reason we discuss here the hypernuclei not heavier than hyper-oxygen.

2 Lagrangian and the mass formula

The Lagrangian of the Skyrme model, which in its well known form depends on parameters $F_\pi$, $F_D$ - meson decay constants, the Skyrme constant $e$, etc., has been presented previously [9, 10], and we give here its density for completeness:

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \mathcal{L}^{SB}$$

with the term of the second order in chiral derivative

$$\mathcal{L}^{(2)} = -\frac{F_\pi^2}{16} Tr l_\mu l_\mu,$$

the antisymmetric $4-th$ order, or Skyrme term

$$\mathcal{L}^{(4)} = \frac{1}{32e^2} Tr [l_\mu l_\nu]^2,$$

the $6-th$ order term

$$\mathcal{L}^{(6)} = e_6 Tr ([l_\mu l_\nu][l_\gamma l_\delta])$$

and the symmetry (chiral and flavor) breaking terms

$$\mathcal{L}^{SB} = \frac{F_\pi^2 m_\pi^2}{16} Tr (U + U^\dagger - 2) +$$

$$\frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{24} Tr (1 - \sqrt{3}\lambda_8) (U + U^\dagger - 2) +$$

$$\frac{F_B^2 - F_\pi^2}{48} Tr (1 - \sqrt{3}\lambda_8) (U l_\mu l_\mu + l_\mu l_\mu U^\dagger)$$

(2)

Here the left chiral derivative $l_\mu = \partial_\mu U U^\dagger$, the unitary matrix $U \in SU(3)$. In the original Skyrme model the $6-order$ term $\mathcal{L}^{(6)}$, which can be presented also as a baryon (topological) number density squared, was not included, and we shall omit it here as well. Recent calculations of flavor excitation energies, performed by A.M. Shunderyuk, provide the results which are close to the results obtained in [10] and in present paper. The Wess-Zumino term in the action, which can be written as a 5-dimensional differential form, plays a very important role in the quantization procedure, but it does not contribute to most of static properties of skyrmions, see e.g. [8, 10].

The physical values of these constants are: $F_\pi = 186$ Mev, $e$ is close to $e = 4$, we take here the value $e = 4.12$ [14]. The chiral symmetry breaking part of the Lagrangian depends on meson masses, the pion mass $m_\pi$, and the mass of $K, D$ or $B$ meson, we call it $m_D$. The flavor symmetry breaking ($FSB$) part of the Lagrangian is of the usual form, and was sufficient to describe the mass splittings of the octet and decuplet of baryons [14], within the collective coordinates quantization approach with configuration mixing. It is important that the flavor decay constant (pseudoscalar decay constant $F_K$, $F_D$ or $F_B$) is different from the pion decay constant $F_\pi$. Experimentally, $F_K/F_\pi \approx 1.22$ and $F_D/F_\pi \approx 2.28^{+1.1}_{-1.1}$ [15]. The $B$-meson decay constant is not measured yet. In view of this uncertainty, we take for our estimates two values of $r_c = F_D/F_\pi, r_c = 1.5$ and 2, and same for $r_b = F_B/F_\pi$, following also to theoretical estimates [16].

We begin our calculations with unitary matrix of chiral fields $U \in SU(2)$, as was mentioned above. The classical mass of $SU(2)$ solitons and other static characteristics
necessary for our purposes, in most general case depend on 3 profile functions: \( f, \alpha \) and \( \beta \). The general parametrization of \( U_0 \) for an \( SU(2) \) soliton we use here is given by \( U_0 = c_f + s_f \tilde{r} \tilde{n} \) with \( n_z = c_\alpha, n_x = s_\alpha c_\beta, n_y = s_\alpha s_\beta, s_f = \sin f, c_f = \cos f \), etc. For the RM ansatz \( f = \tilde{f}(r) \), i.e. the profile depends on one variable, only; the components of vector \( \tilde{n} \) are some rational functions of two angular variables which define the direction of radius-vector \( \tilde{r} \) [11].

The quantization of solitons in \( SU(3) \) configuration space was made in the spirit of the bound state approach to the description of strangeness, proposed in [17] and used in [18, 19]. We use here somewhat simplified and very transparent variant, the so called rigid oscillator version, proposed in [8]. The details of the quantization procedure can be found in [8]-[10], and we shall not reproduce them here. We note only, that the \((u,d,c)\) and \((u,d,b)\) \( SU(3) \) groups are quite analogous to the \((u,d,s)\) one, for the \((u,d,c)\) group a simple redefinition of hypercharge should be made.

The following mass formula has been obtained for the masses of states with definite quantum numbers: baryon (topological) number \( B \), flavor \( F \) (strangeness, charm or beauty), isospin \( I \) and angular momentum \( J \) [8, 10]:

\[
E(B,F,I,J) = M_{B,cl} + |F| \omega_{F,B} + \frac{1}{2\Theta_{T,B}} [c_{F,B} T_r (T_r + 1) + (1 - c_{F,B}) I (I + 1) +
\]

\[+(\bar{c}_{F,B} - c_{F,B}) I F (I F + 1)] + \frac{J(J + 1)}{2\Theta_{J,B}},
\]

(3)

\( \omega_{F,B} \) or \( \bar{\omega}_{F,B} \) are the frequencies of flavor (antiflavor) excitations:

\[
\omega_{F,B} = N_c B (\mu_{F,B} - 1) / (8\Theta_{F,B}), \quad \bar{\omega}_{F,B} = N_c B (\mu_{F,B} + 1) / (8\Theta_{F,B}).
\]

(4)

with \( \mu_{F,B} = [1 + 16\Theta_{F,B} (m_D^2 \Gamma_B + (F_B^2 - F_F^2) \Gamma_{B})] / (N_c B)^2]^{1/2}, \) \( N_c \) is the number of colors of the underlying QCD \((N_c = 3\) in all numerical estimates), \( m_D^2 = F_D^2 m_D^2 / F_F^2 = m_F^2 - m_F^2 \). The terms \( \pm N_c B / (8\Theta_{F,B}) \) in (4) arise from the Wess-Zumino term in the action which does not contribute to the masses and momenta of inertia of skyrmions [17, 8]. In terms of the quark models the difference \( \bar{\omega} - \omega = N_c B / (4\Theta_{F,B}) \) is the energy necessary for the production of additional \( q \bar{q} \) pair. The hyperfine structure constants \( c_{F,B} \) and \( \bar{c}_{F,B} \) are given by [8]

\[
c_{F,B} = 1 - \frac{\Theta_{T,B}(\mu_{F,B} - 1)}{2\Theta_{F,B} \mu_{F,B}}, \quad \bar{c}_{F,B} = 1 - \frac{\Theta_{T,B}(\mu_{F,B} + 1)}{\Theta_{F,B}(\mu_{F,B})},
\]

(5)

Evidently, when \( \mu \rightarrow \infty, \bar{c} \rightarrow 1 \). The contributions of the order of \( 1/\Theta \sim N_c^{-1} \) which depend originally on angular velocities of rotations in isospace and usual space are taken into account in (3). This expression was obtained by means of quantization of the oscillator-type Hamiltonian describing the motion of the \( SU(2) \) skyrmion in the \( SU(3) \) collective coordinates space. The classical mass \( M_{cl} \sim N_c \), and the energies \( \omega_F \sim N_c^0 = 1 \). The motion into ”flavor” direction, \( s, c \) or \( b \) is described by the amplitude \( D \) [8, 10] which is small for the lowest quantum states (lowest \( |F| \)): \( D \sim [16\Gamma_B \Theta_{F,B} m_D^2 + N_B^2 B^2]^{-1/2} / (2 |F| + 1) \). So, the amplitude \( D \) decreases with increasing mass \( m_D \) like \( 1 / \sqrt{m_D} \), as well as with increasing number of colors \( N_c \), and the method works for any value of mass \( m_D \), also for charm and beauty quantum numbers.

In (3) \( I \) is the isospin of the multiplet with flavor \( F \), \( T_r = p/2 \) is the so called ”right” isospin - the isospin of the not-flavored component of the \( SU(3) \) multiplet under consideration with \((p,q)\) - the numbers of upper and lower indices in the spinor which describes it. \( I_F \) is the isospin carried by flavored mesons which are bound by \( SU(2) \) skyrmion, \( \tilde{I} = T_r + I_F \). Evidently, \( I_F \leq |F|/2 \). In the rigid oscillator model the states predicted do not correspond to the definite \( SU(3) \) or \( SU(4) \) representations. How they can be ascribed to them was
shown in [8, 10]. For example, the state with $B = 1$, $|F| = 1$, $I = 0$ should belong to the octet of $(u, d, s)$, or $(u, d, c)$, $SU(3)$ group, if $N_c = 3$. Here we consider the quantized states of $BS$ which belong to the lowest possible $SU(3)$ irreps $(p, q)$, $p + 2q = 3B$: $p = 0$, $q = 3B/2$ for even $B$, and $p = 1$, $q = (3B - 1)/2$ for odd $B$. For $B = 3, 5$ and 7 they are 35, 80 and 143-plets, for $B = 4, 6$ and 8 - 28, 55 and 81-plets, etc. For even $B$, $T_r = 0$, for odd $B$, $T_r = 1/2$ for the lowest $SU(3)$ irreps (see Fig. 1).

The flavor moment of inertia which enters the above formulas [8, 10, 17] for arbitrary $SU(2)$ skyrmions is given by [10]:

$$
\Theta_F = \frac{1}{8} \int (1 - c_f) \left\{ F_D^2 + \frac{1}{c^2 f} \left[ (\bar{\partial} f)^2 + s_f^2 (\bar{\partial} n_i)^2 \right] \right\} d^3 \hat{r}.
$$

\( (\bar{\partial} n_i)^2 = (\bar{\partial} \alpha)^2 + s_\alpha^2 (\bar{\partial} \beta)^2 \). It is simply connected with $\Theta_F^{(0)}$ for the flavor symmetric case:

$$
\Theta_F = \Theta_F^{(0)} + (F_D^2/F_s^2 - 1) \Gamma/4, \Gamma \text{ is defined in (7) below. The flavor } \text{inertia increases with } B \text{ almost proportionally to } B. \text{ The isotopic moments of inertia are the diagonal components of the corresponding tensor of inertia, in our case this tensor of inertia is close to unit matrix multiplied by } \Theta_T.
$$

The quantities $\Gamma$ (or $\Sigma$-term), which defines the contribution of the mass term to the classical mass of solitons, and $\bar{\Gamma}$ in $\mu F, B$ are given by:

$$
\Gamma = \frac{F^2}{2} \int (1 - c_f) d^3 \hat{r}, \quad \bar{\Gamma} = \frac{1}{4} \int c_f [(\bar{\partial} f)^2 + s_f^2 (\bar{\partial} n_i)^2] d^3 \hat{r}.
$$

For the $RM$ ansatz the formulas (6), (7) can be slightly modified [10], but in such general form they look simple enough already. The masses of solitons have been calculated in [12] and [10], moments of inertia, $\Gamma$ and $\bar{\Gamma}$ were calculated in [10] for several values of $B$, the
missing quantities are calculated here. The contribution to the $\mu_{F,B}$ proportional to $\tilde{\Gamma}_B$ is suppressed in comparison with the term $\sim \Gamma$ by the small factor $\sim F_D^2/m_D^2$, and is more important for strangeness.

### 3 Strange and beautiful hypernuclei

It is convenient to calculate the energy difference between the state with flavor $F$ belonging to the $(p,q)$ irrep, and the ground state with $F = 0$ and the same $B$, $J$ and $(p,q)$ \cite{10}:

$$\Delta E_{B,F} = |F|\omega_{F,B} + \frac{\mu_{F,B} - 1}{4\mu_{F,B}\Theta_{F,B}}[I(I + 1) - T_r(T_r + 1)] + \frac{(\mu_{F,B} - 1)(\mu_{F,B} - 2)}{4\mu_{F,B}^2\Theta_{F,B}} I_F(I_F + 1),$$

(8)

It was used in deriving (3) and (8) that the so called ”interference” moment of inertia which makes a contribution to the lagrangian proportional to the product of angular velocities of rotation in the isotopic and ordinary 3D space, is negligible compared to the isotopic and orbital tensors of inertia \cite{20} for all multiskyrmions except those with $B = 1, 2$. Note also, that (8) does not depend on $\Theta_T$, only on $\Theta_F$, when the formulas for hyperfine splitting constants are used.

For the state with isospin $I = 0$ and unit flavor number, $|F| = 1$, the binding energy difference in comparison with the ground state of the nucleus with the same $B$, $(p,q)$ and $|F| = 0$, is

$$\Delta \epsilon_{B,F} = \omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1} - 1)}{8\mu_{F,1}^2\Theta_{F,1}} + \frac{3(\mu_{F,B} - 1)}{8\mu_{F,B}^2\Theta_{F,B}}$$

(9)

Such states can exist for odd $B$, with $I_F = T_r = 1/2$, see Fig.1a. In the case of antifavor excitations we have similar formulas, with the substitution $\mu \rightarrow -\mu$.

| $\Lambda A$ | $\omega_s$ | $\Delta \epsilon_s$ | $\epsilon_{tot}^{exp}$ | $\omega_{tot}=1,3$ | $\Delta \epsilon_b$ | $\epsilon_{tot}^{exp}$ | $\omega_{tot}=2$ | $\Delta \epsilon_b$ |
|------------|------------|----------------------|------------------------|-----------------|--------------------|------------------------|-----------------|--------------------|
| $1$        | 306        | —                    | —                      | 4501            | —                  | 4805                   | —                | —                  |
| $\frac{3}{2}^H$ | 289       | -3                   | 5.23                  | 4424            | 75                 | 4751                   | 53              | 61                 |
| $\frac{3}{2}^H e$ | 287      | -6                   | 33.14                 | 4422            | 76                 | 4749                   | 54              | 81                 |
| $\frac{3}{2}^L i$ | 282       | -3                   | 37.67                 | 4422            | 81                 | 4744                   | 59              | 97                 |
| $\frac{3}{2}^B e$ | 291       | -13                  | 63.2                  | 4459            | 97                 | 4773                   | 31              | 88                 |
| $\frac{3}{2}^B$ | 294       | -16                  | 59                    | 4478            | 96                 | 4786                   | 18              | 93                 |
| $\frac{3}{2}^C$ | 295       | -18                  | 78                    | 4488            | 106                | 4793                   | 11              | 107                |
| $\frac{3}{2}^A$ | 300       | -23                  | 91                    | 4515            | 118                | 4810                   | -7              | 108                |

Table 1. The collective motion contributions to the binding energies of the isoscalar hypernuclei with unit flavor, strangeness or beauty, $S = -1$ or $b = -1$, in Mev. $\omega_s$ and $\omega_b$ are the strangeness and beauty excitation energies, $\Delta \epsilon_{s,b}$, in Mev, are the changes of binding energies of lowest BS with flavor $s$ or $b$, $|F| = 1$, in comparison with usual $(u, d)$ nuclei with the same $B$-number. $\epsilon_{tot}$ is the total binding energy of the hypernucleus. Experimental values $\epsilon_{tot}^{exp}$ are taken from \cite{1, 2}. The energies $\omega$ for $B = 1$ are given for comparison. For beauty the first 3 columns correspond to $r_b = F_B/F_\pi = 1.5$, and the last 3 - to $r_b = 2$.

| $\Lambda A$ | $\omega_s$ | $\Delta \epsilon_s$ | $\epsilon_{tot}^{exp}$ | $\omega_{tot}=1,3$ | $\Delta \epsilon_b$ | $\epsilon_{tot}^{exp}$ | $\omega_{tot}=2$ | $\Delta \epsilon_b$ |
|------------|------------|----------------------|------------------------|-----------------|--------------------|------------------------|-----------------|--------------------|
| $\frac{3}{2}^H - \frac{1}{2}^H e$ | 283       | -23                  | 5.3                   | 10.52; 10.11    | 4402               | 71                     | 99              | 4735               | 52                 | 80                 |
| $\frac{5}{2}^H e - \frac{3}{2}^L i$ | 287       | -22                  | 10.31                 | 31.7; 30.8      | 4430               | 52                     | 84              | 4752               | 40                 | 72                 |
| $\frac{3}{2}^L i - \frac{1}{2}^B e$ | 288       | -20                  | 36.5                  | 46.05; 44.4     | 4443               | 43                     | 99              | 4765               | 33                 | 89                 |
| $\frac{5}{2}^B e - \frac{3}{2}^A B$ | 292       | -23                  | 42                    | 67.3; 65.4      | 4465               | 24                     | 89              | 4778               | 20                 | 85                 |
| $\frac{5}{2}^A B - \frac{1}{2}^C$ | 294       | -24                  | 67                    | 87.6; 84.2      | 4481               | 10                     | 102             | 4788               | 11                 | 103                |
| $\frac{3}{2}^C - \frac{1}{2}^A N$ | 299       | -28                  | 77                    | 169.3; 106.3    | 4506               | -14                    | 91              | 4805               | -5                 | 100                |
| $\frac{1}{2}^A N - \frac{3}{2}^A O$ | 301       | -30                  | 97                    | —               | 4521               | -28                    | 100             | 4815               | -14                | 114                |
Table 2. The binding energies of the isodoublets of hypernuclei with unit flavor, $S = -1$ or $b = -1$ in $Mev$. Other notations and peculiarities as in Table 1.

For the states with maximal isospin $I = T_r + |F|/2$ the energy difference can be simplified to [10]:

$$
\Delta E_{B,F} = |F| \left[ \omega_{F,B} + T_r \frac{\mu_{F,B} - 1}{4\mu_{F,B}\Theta_{F,B}} + \frac{([F] + 2)(\mu_{F,B} - 1)^2}{8\Theta_{F,B}\mu_{F,B}^2} \right].
$$

(10)

The case of isodoublets, even $B$, is described by (8) with $T_r = 0$, see Table 2 and Fig.1b. It follows from (10) that when a nucleon is replaced by a flavored hyperon in $BS$ the binding energy of the system with $|F| = 1, T_r = 0$ changes by

$$
\Delta \epsilon_{B,F} = \omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1} - 1)}{8\mu_{F,1}\Theta_{F,1}} - \frac{3(\mu_{F,B} - 1)^2}{8\mu_{F,B}^2\Theta_{F,B}}.
$$

(11)

For strangeness Eq. (11) is negative indicating that stranglets should have binding energies smaller than those of nuclei with the same $B$-number.

To obtain the values of total binding energy of hypernuclei shown in Tables, we add the calculated difference of binding energies given by (9) or (11) to the known value of binding energy of usual $(u,d)$ nucleus. E.g., for $B = 3$ it is the average of binding energies of $^3H$ and $^3He$, for $B = 4$ it is the binding energy of $^4He$ $(5.3\ Mev = (28.3 - 23)\ Mev)$, etc., see Fig.1. A special care should be taken about spin of the nucleus. For $^3\Lambda H$ and $^3\Lambda He$ and $^4\Lambda e$, $^5\Lambda Li$ and $^9\Lambda Li$, $^{12}\Lambda C$ and $^{13}\Lambda C$, and in few other cases the spins of the ground states of hypernucleus and nucleus coincide. For $^3\Lambda He$ ($J = 1/2$) and $^5\Lambda He$ ($J = 3/2$), $^3\Lambda Be$ ($J = 1/2$) and $^9Be$ ($J = 3/2$), $^{12}\Lambda C$ ($J = 1$) and $^{12}\Lambda C$ ($J = 0$) and in some other cases the difference in the rotation energies $E_j = J(J + 1)/(2\Theta_j)$ should be taken into account. E.g., for $^7\Lambda Li$ this difference decreases the theoretical value of binding energy by about $7\ Mev$, we have $29\ Mev$ instead of $36\ Mev$. In those cases when the spin of hypernucleus is not known, this correction was not included in Tables 1,2. Beginning with $B \sim 10$, the correction to the energy of quantized states due to nonzero angular momentum is small and decreases with increasing $B$ since the corresponding moment of inertia increases proportionally to $\sim B^2$.

Since $\Theta_{F,B}$ increases with increasing $B$ and $F_D (m_D)$ this leads to the increase of binding with increasing $B$ and mass of the "flavor", in agreement with [9, 10]. For beauty (and charm, see below) Eq. (11) is positive for $3 \leq B \leq 12$. As it follows from Tables 1,2, our method underestimates the binding energy of strangeness in nuclei, beginning with $B = A \sim 9$. It means that the other sources of binding should be taken into account, besides the collective motion of $BS$ in the $SU(3)$ configuration space.

4 Charmed hypernuclei

In this section the binding energies of charmed hypernuclei are presented for two values of the charm decay constant which correspond to the ratio $r_c = F_\pi/F_\pi = 1.5$ and $r_c = 2$. Although the measurement of this constant has been performed in [15], in view of its big uncertainty variation of this constant in some interval seems to be reasonable. As it follows from Table 3, the predicted binding energies of charmed hypernuclei differ not essentially for the values $r_c = 1.5$ and $r_c = 2$, this difference being increasing with increasing atomic number. For light hypernuclei this difference is considerably smaller than for beauty quantum number (see Section 3).

For the charm, the repulsive Coulomb interaction is greater than for ordinary nuclei with the same atomic number. Moreover, since the charmed nucleus has somewhat smaller
dimensions than the ordinary nuclei - the effect which has not been taken into account by present consideration - this repulsion can decrease the binding energies for charm by several Mev. This, however, does not change our qualitative conclusions. For \( B = A = 5 \) and 13 our results shown in Table 3 agree, within 15 – 20 Mev with early result by Dover and Kahana [4] where binding of the charm by several nuclei has been studied within potential approach. In general, we can speak about qualitative agreement with results of such approach for \( B \approx 5 - 10 \) [5, 6] (the results of the potential approach have been reviewed in [6]).

\[
\begin{array}{cccccc}
\lambda A & \omega_{c}^{r=1.5} & \Delta \epsilon_{c} & \epsilon_{c}^{\text{tot}} & \omega_{c}^{r=2} & \Delta \epsilon_{c} & \epsilon_{c}^{\text{tot}} \\
\hline
\Lambda \text{He} & 1504 & 27 & 35 & 1647 & 24 & 32 \\
\Lambda \text{Li} & 1505 & 25 & 52 & 1646 & 25 & 52 \\
\Lambda \text{Be} & 1497 & 32 & 70 & 1641 & 30 & 68 \\
\Lambda B & 1518 & 11 & 68 & 1654 & 17 & 74 \\
\Lambda C & 1525 & 4 & 79 & 1658 & 13 & 87 \\
\Lambda N & 1529 & 0 & 96 & 1660 & 10 & 106 \\
\Lambda O & 1540 & -11 & 103 & 1668 & 3 & 117 \\
\end{array}
\]

Table 3a.

\[
\begin{array}{cccccc}
\lambda A & \omega_{c}^{r=1.5} & \Delta \epsilon_{c} & \epsilon_{c}^{\text{tot}} & \omega_{c}^{r=2} & \Delta \epsilon_{c} & \epsilon_{c}^{\text{tot}} \\
\hline
\Lambda \text{He} - \Lambda \text{Li} & 1493 & 12 & 40 & 1639 & 16 & 44 \\
\Lambda \text{Li} - \Lambda \text{Be} & 1504 & 9 & 41 & 1646 & 14 & 46 \\
\Lambda \text{Be} - \Lambda B & 1510 & 7 & 63 & 1648 & 15 & 71 \\
\Lambda B - \Lambda C & 1520 & 0 & 65 & 1655 & 10 & 75 \\
\Lambda C - \Lambda N & 1526 & -4 & 88 & 1659 & 7 & 99 \\
\Lambda N - \Lambda O & 1536 & -14 & 91 & 1666 & 1 & 106 \\
\Lambda O - \Lambda F & 1543 & -19 & 109 & 1670 & -2 & 126 \\
\end{array}
\]

Table 3b.

Table 3. The binding energies of the charmed hypernuclei, isoscalars in Table 3a and isodoublets in Table 3b, with unit charm, \( c = 1 \) in Mev. \( \Delta \epsilon_{c} \) in Mev, and \( \epsilon_{c}^{\text{tot}} \) is the same as in Tables 1, 2, for the charm quantum number. The results are shown for two values of charm decay constant, corresponding to \( r_{c} = 1.5 \) and \( r_{c} = 2 \). The chemical symbol is ascribed to the nucleus according to its total electric charge.

As in the \( B = 1 \) case, the absolute values of masses of multiskyrmions are controlled by the poorly known loop corrections to the classic masses, or the Casimir energy [21]. And as was done for the \( B = 2 \) states, the renormalization procedure is necessary to obtain physically reasonable values of the masses of multibaryons. This generates an uncertainty of about few tens of Mev, as the binding energy of the deuteron is 30 Mev instead of the measured value 2.225 Mev, so \( \sim 30 \) Mev characterises the uncertainty of our approach [10]. This uncertainty is cancelled mainly in the differences of binding energies \( \Delta \epsilon \) shown in Tables 1-3.

5 Comments and conclusions

The version of the bound state soliton model proposed in [8] and modified in [9, 10] for the flavor symmetry breaking case \( (F_{D} > F_{\pi}) \), allows to calculate the binding energy differences of ground states of flavored and unflavored nuclei, and combined with few phenomenological arguments it is very successful in some cases of light hypernuclei, e.g. isoscalars \( \Lambda \text{He} \) and \( \Lambda \text{Li} \). In other cases the accuracy of binding energies description is at the level of 10 – 30 Mev, expected for the whole method, which takes into account the collective motion of the baryonic systems, only. There is also general qualitative agreement with data.
in the behaviour of binding energy with increasing atomic number. It should be stressed that it is, probably, one of interesting examples when field theoretical model provides the results which can be directly compared with observation data, and can be considered as an additional argument in favor of applicability of the chiral soliton approach to the description of realistic properties of nuclei. For the charm and beauty quantum numbers the results only slightly depend on the poorly known values of the decay constants $F_D$ or $F_B$.

The tendency of decrease of binding energies with increasing $B$-number, beginning with $B \sim 10$, is connected with the fact that the rational map approximation, leading to the one-shell bubble structure of the classical configuration [11, 12, 13], is not good for such values of $B$. At large values of the $FSB$ mass we have approximately $\omega_F \approx m_D \sqrt{F/\Theta_F} F_D/(2F_\pi)$. For $RM$ configurations at large $B$ the sigma-term $\Gamma$ grows faster than the inertia $\Theta_F$, because the contribution of the volume occupied by the chiral field configuration, is more important for $\Gamma$ [13]. For larger $B = A$, beginning with several tens, the configurations of the type of skyrmion crystals seem to be more realistic than $RM$ type configurations.

The variation of the only model parameter, Skyrme constant $\epsilon$, makes small influence on the results presented here, negligible for charm or beauty quantum numbers. The quantities $\Gamma$ and inertia $\Theta_F$ both scale like $1/(F_\pi e^3)$, if the pion mass term in the minimized classical mass is omitted, therefore, the flavor excitation energies given by (4), depending on their ratio at large values of $m_D$, are scale invariant. The inclusion of the pion mass term slightly changes this conclusion, more for strangeness.

The hypernuclei with $|F| \geq 2$ can be studied using similar methods [10], and it will be done elsewhere. Consideration of the hypernuclei with ”mixed” flavors is possible in principle, but is technically more involved. For example, the isodoublet $\frac{3}{2}^+ s,cH\frac{3}{2}^+ s,cHe$ consisting of $(n, \Lambda, \Lambda_c)$ and $(p, \Lambda, \Lambda_c)$ is expected.

There is rough agreement of our results with the results of [19, 20] where the flavor excitation frequencies had been calculated within another version of the the bound state approach, and using the collective coordinates quantization method, for strangeness. Some difference in details takes place, however, and it would be of interest to reproduce our results within other variants of chiral soliton model. The model we used overestimates the strangeness excitation energies, but is more reliable for differences of energies which enter (9), (11), and for charm and beauty quantum numbers. Further theoretical studies and experimental searches for the baryonic systems with flavor different from $u$ and $d$ could shed more light on the dynamics of heavy flavors in baryonic systems. Results of this work have been presented at the workshops on Physics on Japan Hadron Facility NP01 (December 2001), NP02 (September 2002) and Particle and Nuclear Physics International Conference, PANIC02 (October 2002). I’m indebted to A.M.Shunderyuk for checking numerical calculations and to V.Andrianov, A.Gal, T.Nagae for discussions and remarks. The work has been supported by the Russian Foundation for Basic Research, grant 01-02-16615.

References

1. H. Bando, T. Motoba, J. Zofka, Internat. J. Mod. Phys. 21, 4021 (1990)
2. Proc. of the International Conference on Hypernuclear and Strange Particle Physics, Nucl.Phys. A639 (1998); A691 (2001)
3. A.A. Tyapkin, Yad.Fiz. 22, 181 (1975)
4. C.B. Dover, S.H. Kahana, Phys.Rev.Lett. 39, 1506 (1977)
5. H. Bando, M. Bando, Phys.Lett. B109, 164 (1982); H. Bando, S.Nagata, Prog.Theor.Phys. 69, 557 (1983); B.F. Gibson et al, Phys. Rev. C27, 2085 (1983)
6. N.I. Starkov, V.A. Tsarev, Nucl. Phys. A450, 507c (1990); S.A. Bunyatov, V.V. Lyukov, N.I. Starkov, V.A. Tsarev, Sov. J. Part. Nucl. 23, 253 (1992)
7. A.P. Balachandran et al, Phys. Rev. Lett. 52, 887 (1984); V.B. Kopeliovich, B. Schwesinger, B.E. Stern, Pis’ma v ZhETF, 62, 177 (1995); T.A. Ioannidou, B.M.A.G. Piette, W.J. Zakrzewski, J. Math. Phys. 40, 6353 (1999), hep-th/9811071; V.B. Kopeliovich, W.J. Zakrzewski, B.E. Stern, Phys. Lett. B492, 39 (2000)
8. K.M. Westerberg, I.R. Klebanov, Phys. Rev. D50, 5834 (1994); I.R. Klebanov, K.M. Westerberg, Phys. Rev. D53, 2804 (1996)
9. V.B. Kopeliovich, Pis’ma v ZhETF. 67, 854 (1998); hep-ph/9805296
10. V.B. Kopeliovich, W.J. Zakrzewski, Pis’ma v ZhETF, 69, 675 (1999); Eur. Phys. J C18, 389 (2000); V.B. Kopeliovich, ZhETF 120, 499 (2001)
11. C. Houghton, N. Manton, P. Sutcliffe, Nucl. Phys. B510, 507 (1998)
12. R.A. Battye, P.M. Sutcliffe, Phys. Rev. Lett. 79, 363 (1997); Rev. Math. Phys. 14, 29 (2002)
13. V.B. Kopeliovich, Pis’ma v ZhETF, 73, 667 (2001); J. Phys. G 28, 103 (2002)
14. B. Schwesinger, H. Weigel, Phys. Lett. B267, 438 (1991)
15. J.Z. Bai et al (BES Collaboration), Phys. Lett. B429, 188 (1998)
16. A.A. Penin and M. Steinhauser, Phys. Rev. D65, 054006 (2001); D. Ebert, R.N. Faustov and V.O. Galkin, hep-ph/0204167
17. C.G. Callan, I.R. Klebanov, Nucl. Phys. B262, 365 (1985)
18. D.O. Riska, N.N. Scoccola, Phys. Lett. B265, 188 (1991)
19. M. Schvellinger, N.N. Scoccola, Phys. Lett. B430, 32 (1998)
20. C.L. Schat, N.N. Scoccola, Phys. Rev. D61, 034008 (2000); Phys. Rev. D62, 074010 (2000)
21. B. Moussalam, Ann. of Phys. (N.Y.) 225, 264 (1993); F. Meier, H. Walliser, Phys. Rept. 289, 383 (1997); H. Walliser, Phys. Lett. B432, 15 (1998)