Design of robust $H_{\infty}$ technique using linear matrix inequalities for load frequency control

Ravneet Kaur¹, S K Jain²
Department of Electrical and Instrumentation Engineering
Thapar Institute of Engineering and Technology, Patiala
Punjab, India
rkaur1_me17@thapar.edu¹, skjain@thapar.edu²

Abstract. In this paper, a method for robust decentralised control is proposed as a solution to the problem of load frequency control (LFC). In order to achieve robustness against uncertainties, $H_{\infty}$ control design based on linear matrix inequalities (LMI) technique is presented. $H_{\infty}$ technique is used to tune the control parameters of the proportional-integral (PI) controller under LMI constraints. The robust performance of the proposed technique is tested on an interconnected two-area power system with non-reheat thermal turbine units for different load disturbances. The system is implemented under MATLAB/SIMULINK environment. The results obtained demonstrate the satisfactory dynamic response on introduction of robust control.

Keywords- Load frequency control, Linear matrix inequalities, $H_{\infty}$ technique, PID control, Area control error.

1. Introduction

Each individual control area in an interconnected system holds charge for meeting its own load demand along with maintaining the net power exchange which is scheduled with the neighbouring areas. The role of preserving the system frequency and power exchange at the tie-line at their nominal values is undertaken by Load Frequency Control (LFC). The primary input to LFC problem is given in the form of area control error (ACE) which holds the charge for keeping the error signal and tie-line power deviations under permissible limits so as to be in compliance with the North American Electric Reliability Council (NERC)’s standards of control performance. ACE is a function of product of deviation in frequency ($\Delta f$) and frequency bias factor ($\beta$) and the real power interchange deviations($\Delta P_{tie}$) from nominal values.

A number of design methods dedicated towards improving the robustness and performance of LFC take into account disturbance causing factors such as parameter variations, modelling uncertainties, nonlinearity, Generator Rate Constraints (GRC), effects of deregulation and load characteristics. These methods include control techniques namely, classical control, modern control and in recent years, intelligent control. The classical PID controllers are preferred in industrial automation system
due to the satisfactory result for a range of working conditions and processes. The general approach used for tuning of these controllers is online based trial-and-error. A number of optimization techniques have been projected which simulate the entire power system and not just the control area being studied. The underlying limitation of these proposals is the assumption that the power system under study is composed of identical subsystems[1][2]. The failure of these proposals therefore led to the introduction of a decentralized LFC[3][4][5][6][7]. However, this alternative did not meet the practical industry requirements because of the resulting complex state-feedback controllers. This paper offers a H∞ theory based robust controller for the purpose of LFC. For this, proportional-integral (PI) type controller is chosen as it presents itself as a practically ideal option for industrial applications. H∞ technique under linear matrix inequalities (LMI) constraints obtains gain constant values for the controller such that it exhibits robust characteristics whenever changes occur in the system owing to change in tie line power exchange. Modification of each area according to the H∞ based LMI technique[8][9][10] helps in achieving the same.

The designing and verification of the robust controller presented in this paper is achieved by using LMI control toolbox provided in MATLAB and simulations for testing are carried out in Simulink[11][12]. In control design, the controlled variables are frequency deviation, governor load set-point and area control error.

In the segments that follow, a technical background to H∞ technique based on LMI and modelling of control areas for LFC problem is presented in section 2 and 3 respectively.

2. Robust H∞ technique using linear matrix inequalities

Amongst all the feedback controllers, H∞ controller is the most robust. An overview of H∞ control based on LMI approach is presented here. For a stable linear time-invariant system, the H∞ norm is given by the highest input/output RMS gain of the transfer function G(s)[13].

As shown in figure 1, consider a linear fractional transformation model of the system, where,

- w is the disturbance input vector;
- u is the control input;
- z is the controlled output;
- y is the regulated output vector;
- state matrix P(s) is the transfer matrix of the plant;
- and state matrix K(s) is the transfer matrix of the controller[13].

All the plant elements are taken as real, rational and proper transfer functions. The relation of the input and output vectors to the plant transfer matrix P is:

$$\begin{bmatrix} Z(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} W(s) \\ U(s) \end{bmatrix}$$ (1)

For disturbance (w) to the controlled output (q), the closed loop transfer function is:
\[ T. F(P, K) = P_{11} + P_{12}K(I - P_{22})^{-1}P_{21} \]  

The closed-loop \( H_\infty \) control technique using parameter \( \gamma \) is used to design a controller \( K(s) \) such that:
- Internal stability of the control loop is guaranteed,
- The \( H_\infty \) norm of \( T. F(P, K) \) is strictly less than \( \gamma \), i.e. \( ||T_{zw}(s)||_{\infty} < \gamma \).

The minimal realization for the plant \( P(s) \) using state space approach under \( H_\infty \) norm is:
\[ P(s) = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} [sI - A]^{-1} (B_1 B_2) \]  

The corresponding state space equations are:
\[ \begin{align*}
\dot{x} &= Ax + B_1w + B_2u \\
y &= C_yx + D_{y1}w + D_{y2}u \\
z &= C_xx + D_{21}w + D_{22}u
\end{align*} \]  

\( \text{Lemma 1:} \) The assumptions applied to the plant parameters are that matrices \( A, C_y, B_2 \) are stabilizable and detectable while \( D_{y2} = 0 \). For existence of the matrix \( K \) in an \( H_\infty \) controller, existence of a symmetric matrix \( X \) is necessary such that:
\[ \begin{bmatrix} A_f^T X + X A_f & X B_f & C_f^T \\ B_f^T X & -\gamma I & D_f^T \\ C_f & D_f & -\gamma I \end{bmatrix} < 0 \]  

\[ X > 0 \]  

The rational controller \( K(s) \) is given by:
\[ K(s) = D_k + C_k(sI - A_k)^{-1}B_k; \]  

State space model of controller is
\[ \begin{align*}
\zeta &= A_k \xi + B_ky \\
u &= C_k \xi + D_ky
\end{align*} \]  

Closed loop transfer function realisation from \( w \) to \( z \) is:
\[ T. F(G, K)(s) = D_f + C_f(sI - A_f)^{-1}B_f \]  

The state space model which corresponds to this is:
\[ \begin{align*}
x_f &= A_f x_f + B_f w \\
z &= C_f x_f + D_f w
\end{align*} \]  

Where,
\[ x_f = \begin{bmatrix} \xi \\ \zeta \end{bmatrix} \]
\[ C_f = [C_x + D_{x2} D_k C_y \quad D_{x2} C_k] \]
\[ B_f = [B_1 + B_2 C_k D_{y1} \quad B_k D_{y1}] \]
\[ D_f = D_{21} + D_{x2} D_k D_{y1} \]
\[ A_f = \begin{bmatrix} A + B_2 C_k C_y & B_2 C_k \\ B_k C_y & A_k \end{bmatrix} \]  

\section*{3 Dynamic model}
Composition of a power system is multi-layered. It consists of numerous subsystems of interconnected generating units forming various sets of control areas. For a control area \( i \) containing \( n \) units of generation, the dynamic model is shown in Fig 2[8]. Coherency is assumed among the units of each area.
Figure 2 Dynamic-model for a control area $i$.

For the power system presented in figure 2, the $H_{\infty}$ design is presented using equation (4)

$$
\dot{x}_i = A_ix_i + B_{iu}u_i + B_{iw}w_i \\
z_i = C_i x_i + D_{iu}u_i \\
y_i = C_i x_i
$$

Or

$$
z_i = \begin{bmatrix} \beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22} \end{bmatrix} \begin{bmatrix} A C E_i \\
\beta_{31} \Delta P_{Cl} \end{bmatrix}^T
$$

$$
w_i = \begin{bmatrix} \eta_i & \Delta P_{Di} \end{bmatrix}^T
$$

Where

$$
x_i^T = [x_{ia}^T x_{1i}^T x_{2i}^T ... x_{ni}^T], \ u_i = \Delta P_{Cl}
$$

$$
y_i^T = \begin{bmatrix} A C E_i \end{bmatrix}^T \int A C E_i
$$

$$
\eta_i = \sum_{j=1}^{N} T_{ij} \Delta f_j
$$

$$
x_{ia}^T = \begin{bmatrix} \Delta f_{i} \\
\Delta P_{Cl_{i}} \end{bmatrix}, \ x_{1i}^T = \begin{bmatrix} \Delta P_{T1} \\
\Delta P_{V1} \end{bmatrix}
$$

$$
x_{2i}^T = \begin{bmatrix} \Delta P_{Tn} \\
\Delta P_{Vn} \end{bmatrix}
$$

The state space model of the dynamic system for designing the controller is obtained by using equation (9), (10) and (11) and is given below:

$$
A_{i} = \begin{bmatrix} A R E_{i} & M P_{i} \\
D R O O P_{i} & T G_{i} \end{bmatrix}
$$

$$
C_i = \begin{bmatrix} C_{i}^{**} \\
C_{i_{ei}}^{**} \end{bmatrix}
$$

$$
C_{ie} = \begin{bmatrix} C_{i_{ei}}^{*} \\
C_{i_{ei}}^{**} \end{bmatrix}
$$

$$
B_{iw} = \begin{bmatrix} B_{iw}^{**} \\
0 \end{bmatrix}
$$

$$
B_{iu} = \begin{bmatrix} 0 \\
B_{iu}^{**} \end{bmatrix}
$$

$$
A R E_{i} = \begin{bmatrix} -D/T_{pi} & -1/T_{pi} & 0 \\
2\pi \sum_{j=1}^{N} T_{ij} & 0 & 0 \\
B_{i} & 1 & 0 \end{bmatrix}
$$
\[
MP_i = \begin{bmatrix}
\frac{1}{T_{P_1}} & 0 & \cdots & \frac{1}{T_{P_i}} & 0 \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix}
\]

\[
T_{G_i} = \begin{bmatrix}
\frac{-1}{T_{T_1}} & 0 & \cdots & 0 & \frac{-1}{T_{T_1}} \\
0 & \frac{-1}{T_{T_1}} & \cdots & 0 & \frac{-1}{T_{T_1}} \\
\frac{-1}{R \, T_{H_1}} & 0 & \cdots & 0 & 0
\end{bmatrix}, \quad B_{iw}^* = \begin{bmatrix}
\alpha \left( \frac{T_{H_n}}{\eta} \right) \\
\cdots \\
\alpha \left( \frac{T_{H_n}}{\eta} \right)
\end{bmatrix}
\]

\[
DROOP_i = \begin{bmatrix}
\frac{-1}{R \, T_{H_1}} & 0 & \cdots & 0 & 0 \\
0 & \frac{-1}{R \, T_{H_1}} & \cdots & 0 & 0 \\
\frac{-1}{R \, T_{H_1}} & 0 & \cdots & 0 & 0
\end{bmatrix}, \quad B_{iw}^* = \begin{bmatrix}
0 & -2\pi \frac{1}{T_{P_i}} \\
0 & 0 & \cdots & 0 & 0
\end{bmatrix}
\]

\[
C_{ia} = \begin{bmatrix}
\beta_{1i} & 0 & 0 \\
0 & \beta_{2i} & 0 \\
0 & 0 & \beta_{3i}
\end{bmatrix}, \quad D_{ia} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

Where,

- \( P_T, P_C, P_D, P_V, P_{tie} \) are the turbine power, governor load set-point, power demand, governor valve and net tie-line flow respectively.

- \( f, \eta, \alpha \) are area frequency, area interface and ramp rate factor respectively.

- \( T_{P_i}, T_{T_i}, T_{H_i} \) are area aggregate inertia, tie-line synchronizing coefficient between area \( i \) and \( j \), turbine time constant and governor time constant respectively.

- \( R, \Delta, D \) are the droop characteristics, deviation from nominal values and area load governing characteristic respectively.

- \( B, N \) are the frequency bias and number of control areas respectively.

The objectives of design of the controller are:

1. Regulation of the ACE and deviation in frequency \((\Delta f)\);

2. Reduction of excess unit deterioration and manipulation caused by sudden changes to the system parameters. The \( \beta_{1i}, \beta_{2i}, \beta_{3i} \) in (14) are weighting coefficients and their values used in this paper are 0.5, 1, and 500, respectively [8]. For the purpose of reducing the control exertion caused as a result of reversal of the governor load set-point and overshoot, a large value of coefficient is selected.

4. Case study

The system under study is shown in figure 3 and is an interconnected two-area power system with non-reheat thermal turbine units. The parameters of the system are tabulated in table 3. A robust decentralized load frequency controller (PID type) designed in accordance with algorithm given in section 3 is implemented on each area. The robust performance of both areas is compared with conventional industrial PID controller for various scenarios (table 1) of load disturbance. In each of these scenarios, an increment in the form of a step increase in load demand is applied to each area.
The only purpose of testing the system for three scenarios is to check whether the proposed controller depicts robust characteristics against frequently occurring small as well as some rare large disturbances. The load demand increment is very rarely large as it may lead to penalization of the party responsible for such a huge mismatch between power demand and supply. The only purpose of introduction large disturbances is to check the system robustness under extreme conditions.

**Table 1** Scenarios of load disturbances

| Scenario No. | % Step load increment in area1 ($\Delta P_{D1}$) | % Step load increment in area 2 ($\Delta P_{D2}$) |
|--------------|-----------------------------------------------|-----------------------------------------------|
| 1            | 5%                                            | 5%                                            |
| 2            | 3%                                            | 4%                                            |
| 3            | 10%                                           | 10%                                           |

**Figure 3** Two-area interconnected LFC model for thermal power system hosting non-reheat turbine units

For the two control areas shown above, the robust performance index of controller based on $H_\infty$ - LMI is given in table 2.

**Table 2** Robust performance index

| Control Design | $Y_{area1}$ | $Y_{area2}$ |
|----------------|-------------|-------------|
| $H_\infty$     | 500.0091    | 500.3490    |

The performance comparison of a conventional PID controller and $H_\infty$ technique-based controller for three scenarios of load disturbances is given in figure 4-6. Performance evaluation of both the controllers is carried on the basis of three parameters i.e. ACE, deviation in frequency ($\Delta f$), and governor-load-set-point ($\Delta P_g$).

**4.1 Simulation results**

The proposed controller gives a much better performance in terms of:

1. successfully driving the frequency error and ACE to zero in a lesser time than that taken by PID controller;
2. reducing the undershoot and overshoot;
3. damping out the oscillations for large disturbances;
4. and providing a better governor-load-set point response for the proposed controller as the oscillations during transient period are very small

**Table 3** Generating unit parameters

| MVA_{base} (1000 MW) | Unit | Area 1 | Area 2 |
|-----------------------|------|--------|--------|
| Rate | MW  | 1000 | 800  |
|------|-----|------|------|
| D    | pu/Hz | 0.0150 | 0.0140 |
| $T_p$ | pu-sec | 0.1667 | 0.1200 |
| $T_f$ | Sec | 0.4 | 0.36 |
| $T_H$ | Sec | 0.08 | 0.06 |
| $R$ | Hz/pu | 3.00 | 3.00 |
| $B$ | pu/Hz | 0.3483 | 0.3473 |
| $\alpha$ | - | 0.4 | 0.4 |

**Diagram:**

- **Δf** vs. Time (sec)
- **ACE** vs. Time (sec)
Figure 4 Response of Area 1 for scenario 1; Solid graph is for $H_{\infty}$, dash-dotted is for PID
Figure 5. Response of area 2 for scenario 1; Solid graph is for $H_\infty$, dash-dotted is for PID
Figure 6. Response of area 1 for scenario 2; Solid graph is for $H_{\infty}$, dash-dotted is for PID
Figure 7. Response of area 2 for scenario 2; Solid graph is for $H_\infty$, dash-dotted is for PID.
Figure 8. Response of area 1 for scenario 3; Solid graph is for $H_\infty$, dash-dotted is for PID
5. Conclusion

The purpose of this study is to find a solution to the problem of load frequency control by designing a robust controller. For this, an $H_\infty$ controller, designed using linear matrix inequalities (LMI) technique is presented. The state space analysis based on $H_\infty$ design under LMI technique constraints is carried out for a dynamic model of two-area interconnected power system with non-reheat thermal turbine units. The testing of the designed controller is carried out for three cases of load disturbances introduced in the system and the controller is verified for its robust performance. The performance evaluation is carried out for normal and rare large load changes occurring on a system. It can be concluded from the simulation results, that the response of $H_\infty$-LMI based robust controller to load fluctuations is better than PID controller. Therefore, LMI based controllers provide better robustness against possible disturbances and give an effective control performance.

6. References

[1] P. Kundur, N. J. Balu, and M. G. Lauby, *Power system stability and control*, 17th ed. McGraw-Hill, 2009.
[2] M. L. Kothari, N. Sinha, and M. Rafi, “Automatic generation control of an interconnected power system under deregulated environment,” in *Power Quality ’98*, 1998, pp. 95–102.
[3] Y. Zhang, L. Dong, and Z. Gao, “Load Frequency Control for Multiple-area Power Systems,” in *Proceedings of the 2009 Conference on American Control Conference*, 2009, pp. 2773–2778.
[4] A. Dev and M. K. Sarkar, “Robust Higher Order Observer Based Non-linear Super Twisting Load Frequency Control for Multi Area Power Systems via Sliding Mode,” *Int. J. Control. Autom. Syst.*, May 2019.
[5] M. H. Rahi and A. Feliachi, “$H_{\infty}$ robust decentralized controller for nonlinear power systems,” in *Proceedings of Thirtieth Southeastern Symposium on System Theory*, 1998, pp. 268–270.
[6] H. Trinh and M. Aldeen, “Decentralized load-frequency control of interconnected power systems,” in *1991 International Conference on Advances in Power System Control, Operation and Management, APSCOM-91.*, 1991, pp. 815–820 vol.2.
[7] A. Saikia and R. K. Mehta, “Extended state observer based decentralized load frequency control of three area interconnected system,” in *2016 International Conference on Advanced Communication Control and Computing Technologies (ICACCCT)*, 2016, pp. 783–788.
[8] D. Rerkpreedapong, A. Hasanovic, and A. Feliachi, “Robust load frequency control using genetic algorithms and linear matrix inequalities,” IEEE Trans. Power Syst., vol. 18, no. 2, pp. 855–861, May 2003.

[9] C. Koisap and S. Kaitwanidvilai, “A novel robust load frequency controller for a two area interconnected power system using LMI and Compact Genetic Algorithms,” in TENCON 2009 - 2009 IEEE Region 10 Conference, 2009, pp. 1–6.

[10] S. K. Pandey, S. R. Mohanty, N. Kishor, and R. P. Payasi, “Iterative linear matrix inequality algorithm based decentralized controller for load frequency control of two-area thermal power systems,” in 2013 International Conference on Power, Energy and Control (ICPEC), 2013, pp. 431–436.

[11] G. J. Balas, A. K. Packard, M. G. Safonov, and R. Y. Chiang, “Next generation of tools for robust control,” in Proceedings of the 2004 American Control Conference, 2004, vol. 6, pp. 5612–5615 vol.6.

[12] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, LMI control toolbox user’s guide. 1995.

[13] P. Gahinet and P. Apkarian, “A Linear Matrix Inequality Approach to H ∞ control,” Int. J. Robust Nonlinear Control, vol. 4(4), pp. 421–448, 1994.