Gapped topological kink states and topological corner states in honeycomb lattice

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Based on the tight-binding calculations on honeycomb lattice and photonic experimental visualization on artificial graphene (AG), we report the domain-wall-induced gapped topological kink states and topological corner states. In honeycomb lattice, domain walls (DWs) with gapless topological kink states could be induced either by sublattice symmetry breaking or by lattice deformation. We find that the coexistence of these two mechanisms will induce DWs with gapless topological kink states. Significantly, the intersection of these two types of DWs gives rise to topological corner state localized at the crossing point. Through the manipulation of the DWs, we show AG with honeycomb lattice structure not only a versatile platform supporting multiple topological corner modes in a controlled manner, but also possessing promising applications such as fabricating topological quantum dots composed of gapless topological kink states and topological corner states.

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1. Introduction

The gapless topological kink states have been widely investigated in the domain walls (DWs) of graphene-type electronic and classical wave systems [1–9]. Generally, there are two different mechanisms for the formation of DWs. The first one is through sublattice symmetry breaking. For example, a DW separating two graphene sheets with oppositely staggered AB sublattice potential (AB-BA DW) hosts gapless topological kink states [1–4], which has been experimentally demonstrated in graphene [10–12], sonic and photonic graphene, etc. [13–19]. The second mechanism is lattice deformation, where gapless topological kink states localized along the interface between two sheets of graphene-type material with reversed $\sqrt{3} \times \sqrt{3}$ lattice deformation ($\sqrt{3} \times \sqrt{3}$ DW) [5]. Such a mechanism has also been experimentally visualized in various classical wave graphene-type systems [20–23]. Nevertheless, the topological states in the coexistence of these two mechanisms still remain unexplored.

Higher-order topological insulators (HOTIs) [24–40] have been attracting intensive attentions for its unique bulk-boundary corresponspendence. The non-trivial gapped edge states and corner states in two-dimensional (2D) HOTIs are protected by the bulk gap, which have been experimentally observed in various classical wave systems with square or kagome lattices [24,30–40]. It is natural to ask that whether these topological states can also be realized in graphene-type materials. Considering the manipulability of both the AB-BA and the $\sqrt{3} \times \sqrt{3}$ DWs and the rich topological phase diagram, graphene-type materials may open a new way integrating multiple topological states on a single chip.

In this paper, through theoretical analysis and photonic crystal (PC) experiments, we study the DW-induced topological states in artificial graphene (AG, an artificial honeycomb lattice structure) in the coexistence of sublattice symmetry breaking and lattice deformation. Introducing lattice deformation to the AB-BA DW (red lines in Fig. 1a, b) or breaking sublattice symmetry of a $\sqrt{3} \times \sqrt{3}$ DW (blue lines in Fig. 1a, b) will both induce gapped topological kink states. Significantly, the intersection of the AB-BA and the $\sqrt{3} \times \sqrt{3}$ DWs (Fig. 1a) gives rise to topological corner states (black dot in Fig. 1b) localized at the crossing point (Fig. 1c). Moreover, we show the manipulability of the DWs, thus demonstrating AG a versatile platform supporting multiple topological corner states with tunable interactions. Finally, we propose a scheme fabricating topological quantum dots (TQDs) possessing topological
corner states, gapless and gapped topological kink states, implying promising applications in mesoscopic physics.

2. Theoretical model

As shown in Fig. 2a, we begin with a modified honeycomb tight-binding model $H = -\sum_{i<j} t_{ij} c_i^\dagger c_j + \sum_i U_i c_i^\dagger c_i$, where $c_i^\dagger$ is the creation operator on site $i$. The $\sqrt{3} \times \sqrt{3}$ lattice deformation is considered by modulating the intra (inter)-unit-cell hopping strength $t_i$ as $t_1 = t + \delta t$ ($t_2 = t - \delta t$). Noticing that the enlarged new primitive unit cell contains six sites. The sublattice symmetry breaking is included by adding staggered potential $U_i = \Delta (-\Delta)$ on the A (B) sublattice. When $t_1$ and $t_2$ are exchanged (reversed $\delta t$) across a line, a $\sqrt{3} \times \sqrt{3}$ DW is formed. Similarly, an AB-BA DW is formed when $\Delta$ changes its sign [2].

We first study the relationship between topological states in these two types of DWs. Fig. 2b plots the 1D energy spectrum for the $\sqrt{3} \times \sqrt{3}$ DW. Counter-propagating gapless states inside the bulk gap [41] are similar to the topological kink states in the AB-BA DW [2, 8, 42]. The low-energy effective Hamiltonian of $H$ following Refs. [41, 43] is derived as

$$H_{\text{eff}}(k) = \begin{pmatrix}
    t_1 - t_2 & -i\Delta & 0 \\
    -i\Delta_{+} & t_2 - t_1 & \Delta \\
    0 & -i\Delta_{+} & t_1 - t_2
\end{pmatrix},$$

where $k_i = k_i \pm ik_i$ and $v$ denotes the Fermi velocity (detailed derivation can be referred to the Supplementary materials). Eq. (1) with $\Delta = 0$ is analogous to the Hamiltonian of a sublattice-symmetry-breaking honeycomb lattice ($\Delta \neq 0, \delta t = 0$) in the basis of $\{k, k'\} \otimes \{A, B\}$. They are topologically equivalent to each other for $t_1 - t_2$ playing the role of $\Delta$. Notably, the $C_3$ crystalline symmetry is indispensable for the unitary transformation Eq. (S3) (online) relating the basis in Eq. (1) and the basis of $\{k, k'\} \otimes \{A, B\}$. Therefore, the gapless states in the $\sqrt{3} \times \sqrt{3}$ DW and the AB-BA DW have the same topological origin, where $t_1 - t_2$ and $\Delta$ play the role of the mass term, respectively. The sign change of the mass term across the DW leads to the bulk band inversion, which protects the gapless topological kink states. Formally, the gapless state in the AB-BA DW is related to the topological invariant as the valley Chern number difference across the AB-BA DW [44, 45]. For the $\sqrt{3} \times \sqrt{3}$ DW, in parallel, a “pseudospin” Chern number difference protecting the gapless kink state could be defined in another Hamiltonian basis obtained through a unitary transformation (see the Supplementary materials).

However, $t_1 - t_2$ and $\Delta$ possess different positions in Eq. (1), and the bulk gap $E_g = 2\sqrt{(t_1 - t_2)^2 + \Delta^2}$. These observations manifest that the mass terms $t_1 - t_2$ and $\Delta$ are orthogonal to each other. It indicates that even though one mass term changes its sign across the DW, the topological kink state is still gapped due to the presence of the other mass term. As plotted in Fig. 2c (2d), there is a $2|\Delta| (2|t_1 - t_2|)$ gap in the topological kink state of the $\sqrt{3} \times \sqrt{3}$ (AB-BA) DW by introducing lattice deformation (sublattice symmetry breaking).

We further study the intersection of these two types of DWs. For convenience, the AB-BA ($\sqrt{3} \times \sqrt{3}$) DW is assumed along the $y$ ($x$) direction (Fig. 1a). In both the left and the right sides of the AB-BA DW, the gapped topological kink state can be described as $h v k_0 \sigma_x + m_i \sigma_z$, where $m_i = t_1 - t_2$ is the mass term induced by lattice deformation. However, $m_i$ changes its sign due to the $\sqrt{3} \times \sqrt{3}$ DW, leading to a 1D band inversion. Consequently, a topological corner state emerges around the crossing point. Such topological corner state is mathematically related to the winding number of the 1D gapped topological kink state. Fig. 1b exhibits its eigen-energy spectrum, and a nearly-zero-energy state inside the gap of the topological kink states is observed only in the presence of the intersection of the AB-BA and $\sqrt{3} \times \sqrt{3}$ DWs, indicating the existence of the topological corner state.

3. Experimental observation

An AG can be experimentally realized through PCs, as hexagonal array of alumina $\text{Al}_2\text{O}_3$ cylinders with relative permittivity $\epsilon = 7.5$ (Fig. 2e). The hexagonal unit cell (honeycomb sublattice) is indicated in red (blue). The hopping strength $t_1 (t_2)$ in the tight-binding model is controlled by the intra (inter)-unit-cell separation between neighbouring cylinders $h_1 (h_2)$ [22]. The sublattice potential $\Delta$ is modulated by the diameters of the cylinder.
We firstly calculate the phase diagram with respect to parameters $h_1$, $h_2$, $d_0$ and $\Delta d$ (Fig. 2f). Here $d_0$ is set as 5.5 mm and lattice constant $a = 2h_1 + h_2 = 17\sqrt{3}$ mm. The dipole (quadruple)-like mode is a photonic analogue of the $p$ ($d$)-electron orbit. As shown in Fig. 2g, with the increase of the ratio $h_1/h_2$, the frequency of the dipole-like modes (blue triangles) exceeds the quadruple-like one (red circles), indicating the band inversion induced by the lattice deformation. Their frequency difference (in unit of GHz) is shown by the color in Fig. 2f. When the cylinder diameter changes (Fig. 2h), the eigen-frequency of the counter-clockwise magnetic field exceeds the clockwise counterpart, indicating 1D phase transition by staggered A/B sublattice [17]. Therefore the phase diagram can be separated into four quadrants in the coexistence of two different 1D phase transitions.

We choose one set of parameters from each quadrant (denoted by arrows in Fig. 2f) to construct the intersection of the two types of DWs. The photograph of the experimental setup is shown in Fig. 1a, where the blue line denotes the $\sqrt{3} \times \sqrt{3}$ DW with $h_1 = 19/\sqrt{3}$ mm ($h_1 = 13.5/\sqrt{3}$ mm) on its left (right) side. Cylinders with 5 mm (6 mm) in diameter are assembled at sites A (B) where they are swapped across the AB-BA DW (red line in Fig. 1a). The polarization of the electric field is parallel to the cylinders. In principle, there could be four different DWs while in order to facilitate the sample preparation, the parameters we choose give...
rise to three regions (Fig. 1a) and their energy spectra by tight-binding calculations are shown in Fig. 3a. Though the gapped topological kink state exists in all three regions I, II and III, their energy gaps are different. The local density of states (LDOS) distributions at typical Fermi energy E obtained by lattice Green’s function are plotted in Fig. 3b–d [46], where three patterns of gapped topological kink states are shown with the increase of E.

Excited by a monopole antenna (denoted by star in Fig. 1a), time-harmonic electric field distributions of photonic AG can be measured (Fig. 3e–h). In case of the frequency \( f = 7.1545 \) GHz slightly higher than the top of the bulk photonic bands (Fig. 3b), measured electric field is localized around all DWs, indicating topological kink states in all regions I, II, and III (Fig. 3e). At a higher \( f = 7.1895 \) GHz, only the states near region II and III (Fig. 3f) are excited. Such observations are consistent with the LDOS distributions in Fig. 3c where the Fermi energy only crosses the topological kink bands in region II and III, showing a strong evidence for the existence of gapped topological kink states in region I. When the frequency is increased to \( f = 7.2005 \) GHz, the gapped topological kink states in region I and II (Fig. 3d, g) are manifested. Significantly, at frequency \( f = 7.3590 \) GHz, only the states around \( x = y = 0 \) are excited (Fig. 1c). The spatial exponential decay of the corresponding electric field intensity (see the Supplementary materials for detailed analysis) clearly characterizes the topological corner modes. In addition, we note that the topological corner state is robust against disorder as for random dislocations of \( \text{Al}_2\text{O}_3 \) cylinders (Fig. 3h). Moreover, the gapped topological kink states and topological corner states are also quite robust against the variance of the crossing angle between the two types of DWs (detailed numerical results in tight-binding model, electromagnetic numerical simulation and microwave experimental setup are referred in the Supplementary materials). Though each of the four different domains in this honeycomb lattice possesses crystalline symmetry as \( C_3 \) or \( C_4 \), that specific crystalline symmetry is not required for the spatial arrangement of these two DWs. The robustness originates from the fact that the valley Chern number difference plays a similar role as the Chern number difference [47]. Finally, though the experiments here carried out are in the microwave regime, by fabricating honeycomb PC with much smaller sizes, we expect all these results can also be reproduced in the optical frequencies [19,48,49].

### 4. Applications

In addition to the simple HOTI possessing single topological corner mode, one can create a number of topological corner modes and manipulate their interactions by arranging the spatial positions of these DWs. A device with three topological corner modes is illustrated in Fig. 4a, which can be described by Hamiltonian \( \mathcal{H}_1 = -t_1a_1^\dagger a_2 - t_2a_2^\dagger a_3 + h.c. \). Its electric field distributions at frequencies \( f = 7.246 \) and \( 7.260 \) GHz are measured. In Fig. 4c, three peaks of the electric field are observed, corresponding to the eigen-vector \( \frac{1}{\sqrt{2}} \left[ -1, 0, 1 \right]^T \) for the lowest eigen-energy of \( \mathcal{H}_1 \). Conversely, the eigen-vector at medium eigen-energy is \( \frac{1}{\sqrt{2}} \left[ 1, 0, -1 \right]^T \), therefore the electric field vanishes in the central region as confirmed in Fig. 4d. Due to the leakage from excitation antenna (dashed circle in Fig. 4d), the field near the leftmost corner mode is excessively strong. One can also periodically arrange the DWs to construct a Su–Schrieffer–Heeger (SSH) chain [50] by topological corner modes (Fig. 4b). There are two topological corner modes with bond length \( l_1 \) and \( l_2 \) in a unit cell. Fig. 4e, f show simulated energy spectra. Increasing \( l_1 \) and \( l_2 \) simultaneously will decrease both the inter- and intra-unit-cell coupling strength, therefore the band width also decreases (Fig. 4e). When \( l_1 > l_2 \), the inter-unit-cell coupling strength is stronger than the intra-unit-cell one. A band gap emerges and increases with the difference between \( l_1 \) and \( l_2 \) (Fig. 4f). Since topological corner modes can be used to construct different tight-binding models with tunable parameters, such platform bears similarities to the optical lattice in cold atom [51] and is much more convenient in sample fabrication and measurement.

The coexistence of multiple kinds of topological states in AGs, especially the PC-based AG, provides the possibility of fabricating unique topological devices. We propose two TQDs as illustrated in Fig. 5a, b by combining different topological states. For both two devices, an AB-BA DW is formed across the dashed line, while

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**Fig. 3.** (Color online) (a) Energy spectrum for the device with DWs I, II, and III. The parameters of the four quadrants are \( (\delta, \gamma) = (0.0675\gamma, 0.09\gamma), (0.06\gamma, 0.09\gamma), (-0.06\gamma, -0.09\gamma), \) and \( (0.0675\gamma, -0.09\gamma) \), respectively. (b)–(d) LDOS distributions at energies (b) \( E = -0.14 \), (c) \( E = -0.12 \), (d) \( E = -0.10 \). (e)–(h) Experimentally measured electric field distributions at frequencies \( f = 7.1545, 7.1895, 7.2005, \) and \( 7.4115 \) GHz, respectively. (h) Topological corner mode survives in the presence of disorders.
the $\sqrt{3} \times \sqrt{3}$ lattice deformation is presented only in the filled region (Fig. 5a, b). The gapped topological kink states exist in such regions, acting as two barriers separating the central region from the two leads with gapless topological kink states. The finite-size confinement introduce the bound states in the central region [52]. Fig. 5c, d show the conductance $G$ versus Fermi energy $E$ for these two TQDs. The key feature is that $G$ shows resonant tunneling behavior when $E$ approaches the energy of the bound state $e$ [53]. For a very small $L$ (Fig. 5c), the device is similar to the system in Fig. 1a. A sharp peak of $G$ emerges around $E = 0$ only for the device in Fig. 5b. These results agree with Fig. 1b, where zero-energy topological corner state is localized at the crossing of the

Fig. 4. (Color online) (a) Schematic diagram of the alignment of DWs with three topological corner modes. (b) Illustration of an SSH chain. For each unit, there are two topological corner modes with bond length $l_1$, $l_2$. (c), (d) Experimentally measured field distributions with topological corner modes coupling at $f = 7.246$, and 7.260 GHz, respectively. (e), (f) Energy spectra for (b) in the presence of different $l_1$, $l_2$'s ($\Delta = 0.18t$, $\delta t = 0.06t$).

Fig. 5. (Color online) (a), (b) Illustration of the TQD devices, which includes QD (region with length $L$), barriers (region with length $l$) and semi-infinite leads (here $l = 1$, $L = 2$, and $N = 4$). The $\sqrt{3} \times \sqrt{3}$ DW is absent (presented) in (a) ((b)). (c), (d) Conductance $G$ vs. Fermi energy $E$. The blue (red) lines represent the device in (a) ((b)). In both two leads and the QD, $(\delta t, \Delta) = (0, 0.09t)$. In two barriers, $(\delta t, \Delta) = (0.06t, 0.09t)$. The other parameters are (c) $L = 2$, and (d) $L = 40$. 

The gapped topological kink states exist in such regions, acting as two barriers separating the central region from the two leads with gapless topological kink states. The finite-size confinement introduce the bound states in the central region [52].
AB-BA and the $\sqrt{3} \times \sqrt{3}$ DWs. In case of large $L$, a half-period shift of $G$ peaks between two devices is observed, because the $\sqrt{3} \times \sqrt{3}$ DW induces a special boundary condition and changes the bound

equation from $e^{i\theta} = \frac{e^{i\sqrt{3}m_{1}^2-2}}{e^{i\sqrt{3}m_{1}^2-2}}$ (Fig. 5a) to $e^{i(\theta+\pi)} = \frac{e^{i\sqrt{3}m_{1}^2-2}}{e^{i\sqrt{3}m_{1}^2-2}}$ (Fig. 5b). Since $\varepsilon = 0$ is always the solution of the latter equation, such topological corner states are examples of the celebrated Jackiw-Rebbi zero-mode [54]. The detailed derivations of $\varepsilon$ for the two TQDs and the comparison to the normal QDs are summarized in the Supplementary materials. Considering the key role that QD plays in mesoscopic device, this proposal may open up a new path simulating mesoscopic physics in PCs.

5. Discussion and conclusion

Though our experimental demonstrations are performed in PCs, all these non-trivial topological states are independent of the specific physical platform. In other words, both the gapped topological kink states and the topological corner states can be realized in a variety of condensed matter systems. For example, antidot lattice in 2D electron gas, in which graphene-type band structure has been experimentally realized [55,56] and both the AB-BA and the $\sqrt{3} \times \sqrt{3}$ DWs can be presented [57]. Molecular graphene on metallic surface could be an additional example [58], if lattice deformation and sublattice symmetry breaking are introduced [59]. Such an advantage may imply possible applications in a number of research fields. Moreover, we also notice some very recent theoretical proposals for the realization of topological corner states in graphene-like systems [60-62].

In summary, DW-induced gapped topological kink states and topological corner states are theoretically predicted and experimentally observed in AG. The gapped topological kink states in DWs are protected by the orthogonality between the two mass terms induced by sublattice symmetry breaking and lattice deformation, while the topological corner states emerge in the presence of the intersection between these two types of DWs. Furthermore, through manipulating the DWs, we also illuminate two applications by integrating multiple topological states in a single device. A versatile platform is proposed where the number of topological corner modes and their interactions are tunable. The advantages of applying mesoscopic physics concepts in PC-based AG are also demonstrated by constructing TQDs where gapped topological kink states and topological corner states coexist.

Conflict of interest

The authors declare that they have no conflict of interest.

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Author contributions

Hua Jiang and Zhi Hong Hang conceived the idea and supervised the project. Hua Jiang did the theoretical analysis with the input from YiJia Wu. Yuting Yang, Ziyuan Jia and Zhi Hong Hang did the experiments. Rui-Chun Xiao did DFT simulations. Yuting Yang, Yijia Wu, Zhi Hong Hang, Hua Jiang and X. C. Xie analyzed the data and wrote the manuscript. All authors contributed to scientific discussion of the manuscript.

Appendix A. Supplementary materials

Supplementary materials to this article can be found online at https://doi.org/10.1016/j.scib.2020.01.024.

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