Photon echo quantum random access memory integration in a quantum computer

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Abstract

We have analysed an efficient integration of multi-qubit echo quantum memory (QM) into the quantum computer scheme based on squids, quantum dots or atomic resonant ensembles in a quantum electrodynamics cavity. Here, one atomic ensemble with controllable inhomogeneous broadening is used for the QM node and other nodes characterized by the homogeneously broadened resonant line are used for processing. We have found the optimal conditions for the efficient integration of the multi-qubit QM modified for the analysed scheme, and we have determined the self-temporal modes providing a perfect reversible transfer of the photon qubits between the QM node and arbitrary processing nodes. The obtained results open the way for realization of a full-scale solid state quantum computing based on the efficient multi-qubit QM.

(Some figures may appear in colour only in the online journal)

1. Introduction

Construction of a multi-qubit quantum computer (QC) is a genuine challenge for modern quantum physics and quantum engineering that imposes various critical requirements for controlled highly delicate quantum dynamics of qubits [1, 2]. A universal QC exploits a number of programming single- and two-qubit gates for the qubits incorporated in the QC where quantum processing demands arbitrary qubits to be addressed easily and requires convenient coupling of each pair of qubits. These and other physical demands to the QC have to be fulfilled with almost 100% fidelity and efficiency for all of the quantum gates [3].

The QC architecture must be scalable. Thus, solid state nanotechnologies seem to be promising for the construction of a QC. The first two-qubit solid state prototype of the QC was constructed in 2009 on the superconducting elements [4]. Later, three superconducting qubits were entangled [5, 6] and the prototype of the superconducting QC with quantum random access memory (QRAM) was demonstrated [7]. QRAM must have much longer decoherence times than that of processor qubits and provide a convenient way to address arbitrary qubits and their quantum superposition [8]. Therefore, the hybrid approach where superconducting qubits are used for processing and spin qubits are used for storage [9, 10] is perspective.

All of the requirements on a high figure of merit have to be satisfied not only for the processing nodes of QC, but also for multi-qubit quantum memory (QM) integrated in the QC circuit. Two of the four purified De’Vinzenco criteria on QC concern the QM [11]. QM must serve as the input and output terminals of the QC allowing, also, the temporal storage of qubit-preserving entanglement. Once the efficient multi-mode QM is created, the scalability will be achieved for the QC even with limited resources of processor units. Moreover, the availability of the QM in QC will allow parallel quantum computing.

The photon-echo-based QM (PEQM) with 69% efficiency has been demonstrated recently by using a solid state medium that had beaten the 50% threshold of quantum efficiency [12]. However, the QC requires the QM with almost 100% efficiency for the storage of the multi-qubit states, which will be a large long-term problem for future investigations. The first experimental step towards the realization of multi-qubit QM
in QC has been done recently [13]. Another related problem is the integration of the QM into QC for its effective coupling with processing gates and quantum communication with the external environment of the QC.

Recently, we have proposed an efficient multi-qubit PEQM in a single-mode QED resonator (cavity PEQM) [14–16]. Also, it was proposed in [17] without a spectral matching condition. The proposed PEQM provides desirable (almost 100%) quantum efficiency in a perfect retrieval of the stored quantum states for multi-qubit fields using moderate physical parameters of an atomic system in the optimal QED cavity and controlled inhomogeneously broadened resonant transition [18–22]. Considerable progress in the realization of efficient coupling with atomic ensembles in resonators has been recently demonstrated in [23–25].

Here, we elaborate on principles for the integration of the cavity PEQM in the QC architecture providing a perfect storage and reversible transfer of photon qubits between the QM and processing nodes. Finally, we summarize the obtained results and discuss other problems of the QM integration in the quantum processing.

2. Integrated multi-mode quantum memory

For the analysis of the QM, we use the quantum electrodynamics (QED) of the atomic ensembles in the single-mode QED cavity with the well-known Walls–Milburn formalism [26]. Here, we use two-level atomic ensembles for modelling the QM node and processing nodes in their resonant interaction with the cavity mode field (see figure 1). Also, we take the generalized Tavis–Cummings Hamiltonian [19, 27] for the atomic systems including the QM node with the processing gates and field modes where

\[ H_0 = \hbar \sum_{m=0}^{M} \sum_{j_m} \left( \frac{\hbar}{2} a^\dagger a + \sum_{n=1}^{N_m} b_m^\dagger \omega b_m(\omega) + \int b_m^\dagger(\omega) b_m(\omega) \, d\omega \right) \]  

is a basic Hamiltonian containing the main energies of atoms in the first term (\( S_m^z \) is a z-projection of the spin operator in the \( m \)th atomic node, \( m = 0 \) corresponds to the QM, \( M \) is a number of the atomic processing nodes, \( N_m \) is the number of atoms in the \( m \)th node); the second term is an energy of the cavity field mode (where \( a^\dagger \) and \( a \) are the creation and annihilation operators, respectively), energy of the waveguide field (\( n = 1 \)), and the third term is an energy bath field (\( n = 2 \)) (where \( b_m^\dagger \) and \( b_m \) are the creation and annihilation operators of the waveguide modes, respectively, with \( [b_m^\dagger(\omega'), b_m(\omega)] = \delta_{\omega',\omega} \delta(\omega' - \omega) \)).

\[ H_1 = \frac{\hbar}{2} \sum_{m=0}^{M} \sum_{j_m} \left( \Delta_{j_m}(t) + \delta \Delta_{j_m}(t) \right) S_m^z + \int \omega_i(\omega) b_m^\dagger(\omega) b_m(\omega) \, d\omega \]

+ \int \frac{\hbar}{2} \sum_{m=0}^{M} \sum_{j_m} \left( g_{jm} S_m^z a^\dagger - g_{jm}^* S_m^z a \right)

+ \int \frac{\hbar}{2} \sum_{n=1}^{2} \kappa_n(\omega) \left[ b_n(\omega) a^\dagger - b_n^\dagger(\omega) a \right] \]

is a perturbation Hamiltonian generalized by taking into account inhomogeneous broadening of the atomic frequencies and continuous spectral distribution of the field modes.

The first term in (2) comprises perturbation energies of atoms where \( \Delta_{j_m}(t) \) is a controlled frequency detuning of the \( j \)th atom \( (\Delta_{j_m}(t < \tau) = \Delta_{j_m} \) and \( \Delta_{j_m}(t > \tau) = -\Delta_{j_m} \) for the QM node in the analysed storage protocol); \( \delta \Delta_{j_m}(t) \) is a fluctuating frequency detuning of the \( j \)th atom determined by the local stochastic fields, and \( g_{jm} \) is a coupling photon–atom constant [26]. Ensemble distributions over the detunings \( \Delta_{j_m}(t) \) and \( \delta \Delta_{j_m}(t) \) determine the inhomogeneous and homogeneous broadenings of the resonant atomic lines in the atomic nodes. We assume that all atoms in the processing nodes \( (m = 1, \ldots, M) \) have equal frequency detunings \( \Delta_{j_m}(t) = \Delta_m(t) \). In the following, we use a Lorentzian shape for inhomogeneous broadening of the QM node and a typical anzatz for the ensemble average over the fluctuating detunings \( \delta \Delta_{j_m}(t) \):

\[ \sum_{j_m=1}^{N_m} \left| g_{jm} \right|^2 \exp[-i \Delta_{j_m}(t - t')] \Phi_{jm}(t,t') \]

\[ = N_m |\bar{g}_m|^2 \exp[-(\Delta_m + \gamma_{21})|t - t'|], \]

where \( \Phi_{jm}(t,t') = \exp[\text{i} \sum \Omega_j(t,t')] \); \( \Omega_j(t,t') = \int_{t_0}^{t_1} \text{d}t' \delta \Delta_{j_m}(t''), \gamma_{21} \) is a spectral width of the homogenous Lorentzian line, \( \Delta_m \) is an inhomogeneous broadening, and \( |\bar{g}_m|^2 \) is a quantity averaged over the atoms in the \( m \)th node. The second term in (2) contains the frequency detunings of the field \( n \)th modes. The third term is an interaction energy of atoms with the cavity mode in the rotating wave approximation (\( S_m^z \) and \( S_m^z \) are the transition spin operators). The fourth term is an interaction energy of the cavity mode with the waveguide (\( n = 1 \)) and bath modes (\( n = 2 \)) characterized by the coupling constants \( \kappa_n(\omega) \).

We note that \( [H_0, H_1] = 0 \), although some individual terms do not commute with each other where Hamiltonian \( H_0 \) characterizes the total number of excitations in the atomic system and photons which is conserved during the quantum evolution. \( H_0 \) gives a contribution only to the total evolution of the common phase of the wavefunction and can be singled out from the light–atom dynamics. \( H_1 \) determines a unitary operator causing a coherent evolution of the atomic and
field systems accompanied by the dynamical exchange and entanglement of the excitations between the systems.

In spite of a huge complexity of the compound light–atom system, we show here that their quantum dynamics governed by $H_1$ can be perfectly reversed in time on our demand in a simple robust way. Here, we assume that initially all atoms in the $m$th node ($j = 1, 2, \ldots, N_m$) stay in the ground state $|0\rangle_m = |0, 0, \ldots, 0\rangle_m$ ($m = 0, 1, 2, \ldots, M$), and we launch a signal containing multi-mode single-photon wave packets prepared in the quantum state $|\psi_m(t)\rangle_{ph} = \prod_{k=1}^{N_{ph}} |\psi_k^+\rangle (t - \tau_k) |0\rangle$, where $\psi_k^+ (t - \tau_k) = \int_{-\infty}^{\infty} df \int_{0}^{\pi} d\phi \exp (-i (t - \tau_k) b^+_k (\omega_k)) f_k (\omega_k)|1\rangle_{ph}$ is a number of temporal modes) to the optical cavity that is in a vacuum state $b_1(\omega) \cdot |0\rangle = 0$; $b_1(\omega)$ is the field containing $\psi_1\rangle$ of the QM node from the external waveguide. Additional bath field modes in the frequency detunings and in the coupling constants.

By using equation (5) we obtain an equation

$$\frac{d}{dt} b_{1m}(t) = -i (\omega - \omega_0) b_{1m}(t) - \kappa_1 (\omega) a(t),$$

$$\frac{d}{dt} \tilde{S}^m_\omega (t) = -\tilde{g}_{j_n} a - i [\Delta_{j_n} + \Delta_b] \tilde{S}^m_\omega (t),$$

$$\frac{d}{dt} a(t) = \sum_{m=0}^{M} \sum_{j_m} g_{j_m} \tilde{S}^m_\omega (t) - \frac{1}{2} (\gamma_1 + \gamma_2) a(t) + \left[ \sqrt{\gamma_1} b_{1m}(t) + \sqrt{\gamma_2} b_{2m}(t) \right],$$

where $\gamma_1 = 2\pi \kappa_j^2 (\omega_0)$, $\kappa = 1, 2$. The input signal field containing $N_{ph}$ temporally separated photon modes is given by $b_{1m}(t) = \sum_{k=1}^{N_{ph}} b_{1k} (t - \tau_k)$, where $b_{1k} (t - \tau_k) = (1/\sqrt{2\pi}) \int_{-\infty}^{\infty} df \exp [-i (\omega - \omega_0) \tau_k] b_{1\omega} |1\rangle_{ph}$ and $\tau_k$ is a moment of time when the $k$th field mode arrives at the QM node from the external waveguide.

By using a Fourier transformation for atomic coherences and field modes ($\tilde{S}^m_\omega (t), a(t), b_{1,2m}(t)) = \int_{-\infty}^{\infty} dv \int_{0}^{\pi} d\phi \exp [-i \omega (t - \tau_k)] \tilde{S}^m_\omega (v, a(v), b_{1,2m}(v))$ in equations (4–6), we find the excited atomic coherences in the processing nodes

$$\tilde{S}^m_\omega (v) = \frac{i g_{j_m} a(v)}{[\Delta_m - \omega + i \gamma_2]} ,$$

where we have replaced index $j_m$ by the common index ‘$m$’ in the frequency detunings and in the coupling constants.

By using equation (7) and a formal solution of equation (5) we obtain an equation

$$\left[ \frac{1}{2} (\gamma_1 + \gamma_2) + \frac{\Gamma_{tot}}{2[1 - i\omega/\Delta_m]} + \delta_m (v) \right] a(v) = \left[ \sqrt{\gamma_1} b_{1}(v) + \sqrt{\gamma_2} b_{2}(v) \right].$$

where $\delta_m (v) = \sum_{m=0}^{M} \Omega_m^2/|\Delta_m - \omega - i \gamma_2|$, and $\Gamma_{tot} = 2\Omega_m^2/\Delta_b$ is a rate of photon absorption by the QM node in the central unit spectral domain, $\Omega_m^2 = N_m |\tilde{g}_m|^2$ and $\Delta_b = \Delta_m + \gamma_2$.

Let us assume large enough spectral detuning in comparison with the spectral width of the signal fields $\Delta_m >> \delta_m$, and with the linewidths of the processing nodes $\Delta_m >> \gamma_2$ that simplifies to $\delta_m (v) = \delta_m (0) + i v B_m (v)$, where $\delta_m (0) = \sum_{m=0}^{M} N_m |\tilde{g}_m|^2/\Delta_m$ and

$$B_m (v) = \sum_{m=0}^{M} \Omega_m^2/\Delta_m (\Delta_m - v)$$

(9)
determines a spectrally (v) dependent frequency shift of the cavity mode due to the interaction with atoms in the processing nodes. The shift $\delta_m (0)$ can be controlled and reduced to zero by adiabatic changing of the spectral detunings $\Delta_m (t)$ in each nth atomic ensemble. So, we can put the shift $\delta_m (0) = 0$, thereby determining a resonant interaction of the cavity mode with the ensemble of QM atoms (for spectral component $v = 0$) which leads to the following formal solution for the cavity field mode:

$$a(v) = \frac{2[\sqrt{\gamma_1} b_{1m}(v) + \sqrt{\gamma_2} b_{2m}(v)]}{\{\gamma_1 + \gamma_2 + \frac{\Gamma_{tot}}{2[1 - i\omega/\Delta_m]} - 2iv \Pi_{tot}(v)\}}.$$ 

(10)

The factor $\Pi_{tot}(v) = \{1 + B_m (v)\}$ in equation (10) characterizes solely all the difference in the interaction between the cavity field and atoms of the QM node caused by the presence of the processing nodes. Therefore, one can perform further all the calculations for the quantum storage of the input signal field similar to [15]. By using equations (10) and (5), we obtain the atomic coherences $S_j^{\pm,0}$ in the QM and processing nodes and find after algebraic calculations the storage quantum efficiency as $Q_{ST}^M = \bar{P}_{rec,1}/\bar{n}_1$, where $P_{rec,1} = \sum_{j=0}^{N} \langle S_j^{\pm,0}\rangle_0$ is an excited level population of atoms after the interaction with the last nth signal field mode (i.e. for $t > t_n + \delta$, index ‘$M$’ in $Q_{ST}^M$ means the presence of $M$ processing nodes). The total number of photons in the input signal field is $\bar{n}_1 = \int_{-\infty}^{\infty} dv \int_{0}^{\pi} d\phi \exp [-i \omega (t - \tau_k)] b_{1\omega} |1\rangle_{ph}$, where $\bar{n}_1 = \int_{-\infty}^{\infty} dv \int_{0}^{\pi} d\phi \exp [-i \omega (t - \tau_k)] b_{1\omega} |1\rangle_{ph}$ is the number of photons in the 4th temporal mode, (i.e. 4th) is determined by the quantum averaging over the 4th mode state $\psi_4(t - \tau_k) |0\rangle$. The total storage efficiency of the signal field will be given by $Q_{ST}^M = (1/\bar{n}_1) \sum_{j=0}^{N} Q_{ST,j}^M b_{1j} |0\rangle_1$, with the particular quantum storage efficiency of the 4th mode

$$Q_{ST,j}^M = \int_{-\infty}^{\infty} dv Z^M(v, \Delta_m, \gamma_1, \gamma_2, \Gamma_{tot}) \left| \tilde{b}_j^+(v) b_1(v) \right|^2/\bar{n}_1,$$

(11)

where the modified spectral function $(\Delta_m >> \gamma_2$ is assumed)

$$Z^M(v, \Delta_m, \gamma_1, \gamma_2, \Gamma_{tot}) = \frac{\Delta_m^2}{(\Delta_m + v^2) \gamma_1 + \gamma_2 + \Gamma_{tot}/(1 - i\omega/\Delta_m) - 2iv \Pi_{tot}(v)}.$$ 

(12)

characterizes spectral properties of the quantum storage (we have taken into account the expectation values $\langle \tilde{b}_j^+(v) b_1(v) \rangle = \langle S_{j,m}^+(0) S_{j,m}^-(0) \rangle = 0$ and $m = 0, 1, \ldots, M$, for the used initial state).
For a relatively narrow spectral width $\delta \omega$, of the signal field and with weak atomic decoherence rate $\gamma_2$ in comparison with the inhomogeneously broadened width $(\delta \omega \ll \Delta_0 < \Delta_0 + \gamma_2)$, we obtain from equations (16) and (17):

$$Q_{ST, k}^M (\delta \omega, \gamma_1) = \frac{\gamma_1}{(\gamma_1 + \gamma_2) \frac{4 \Gamma_{tot}}{\Gamma_{tot} + (\gamma_1 + \gamma_2) \gamma_1}}.$$  

(13)

Quantum efficiency $Q_{ST, k}^M$ reaches unity at $\Gamma_{tot}/\gamma_1 = 1$ and $\gamma_2/\gamma_1 \ll 1$ similar to the matching condition of the QM in a resonator without processing nodes [14, 15] which shows the possibility of perfect storage for a multi-mode signal field in the presence of $M$ processing modes (where the number of the temporal signal modes is limited by $M_{max} \approx \Delta_0/\gamma_2$).

Note that $\gamma_1 = \Gamma_{tot}$ is a condition of perfect optimal matching between the waveguide modes and the QM atomic system in the single-mode cavity. In this case, the signal field containing many temporal modes enters into the cavity and transfers to the QM atomic system in one step without any escape from the cavity similar to the simple absorption in a resonator [28, 29]. The single-step storage of the multi-mode field is possible for inhomogeneously broadened atomic optical (electron spin) transition providing a perfect absorption (storage) for the arbitrary temporal profile of the light fields with finite spectral width.

Retrieval of the stored light fields is realized here via the CRIB procedure [15, 18–22] based on symmetry properties of the light–atom dynamics for the signal absorption and echo field emission stages. After the perfect storage due to the coupling matching condition, we change the sign of the frequency detunings $\Delta_j \rightarrow -\Delta_j$ at the time moment $t = \tau$ by changing an electric or magnetic field polarity similar to the experimental techniques [12, 19, 30]. For the case of negligibly weak interaction with the bath modes and slow atomic decoherence, i.e. assuming $\gamma_2 \approx 0$, $\gamma_2/\gamma_1 \ll 1$, we find that the initial quantum state of the multi-mode signal field will be reproduced at $t = 2\tau$ due to complete unitary reversibility in the echo emission getting the field spectrum inverted relative to the central frequency $\omega_0$ in comparison with the original one. By taking into account the atomic decoherence and effective coupling with waveguide field modes similar to the absorption storage, we find the echo field to be as follows:

$$a_{echo} = -\exp (-\gamma_2 t) \left( -\gamma_1 \right) \sum_{k=1}^{M} \frac{d\int_{-\infty}^{\infty} dz' Z_M (v, \Delta_0, \gamma_1, \gamma_2, \Gamma_{tot}, v) b_{1,k} (v) \exp \left\{ i v (t + \tau - 2\tau) \right\}}{\sqrt{2\pi} \gamma_1}.$$  

(14)

where we have assumed a slower atomic decoherence in comparison with the temporal duration of the input light pulses $(\delta t \gamma_2 \ll 1)$. Using equation (14) we find the total number of photons in the echo signal:

$$n_{echo} = \int d\omega b_{echo}^\dagger (\omega, t \gg 2\tau) b_{echo} (\omega, t \gg 2\tau) = M \sum_{k=1}^{M} n_{echo,k},$$  

(15)
parameters of equation (20): \( M \) with dashed), 8 (dotted), 16 (solid line) where we have used spectral units the comparison of (a), (b) and (c), the influence of the processing range \( /\Omega_1//\Delta_1 \). We stress a principal advantage of the proposed multi-mode QM in the QED cavity for QRAM of the fault tolerant QC working with an operation error smaller than 10\(^{-4} \) [31] quantum storage within the broadened spectral range \( /\Delta_1 \) as depicted in figure 2(a).

We stress a principal advantage of the proposed multi-mode QM in the QED cavity for QRAM of the fault tolerant QC working with an operation error smaller than 10\(^{-4} \) [31] with respect to the QMs based on the PEQMs for travelling light modes [22, 30, 32–34] since there 100% efficiency occurs only for a theoretically infinite optical depth of the coherent resonant atomic system \( (\alpha L \gg 1) \). Thus, the analysed highly efficient multi-mode integrated QM opens a door for practical application in quantum storage and processing with many-photon qubits.

3. Quantum transfer between QM and processing nodes

Here, let us consider a simplified principal scheme of the QC containing only four atomic nodes. The four nodes contain the multi-qubit QM (0th node) and three processing nodes. The QM node is placed in the external gradient electric or magnetic field providing a controlled inhomogeneous broadening of atomic frequencies \( \Delta m \gg \delta \omega_j \), whose central atomic frequency coincides with the frequency of the QED cavity mode \( \omega_{QM} = \omega_0 \). We describe three processing nodes by using \( N \) two-level resonant atoms in each node with equal frequencies \( \omega_{j,2,3} \) within each node which can be tuned far away from the frequency \( \omega_0 \). (Similarly instead of these atomic ensembles, we can use just three two-level squids or quantum dots characterized by the enhanced dipole moment of resonant transition.)

Below we demonstrate a perfect transfer of an arbitrary photon qubit between the QM node and one of the processing nodes (first node) in the QED cavity. We take into account the multi-qubit initial state encoded in the \( n \) temporally separated photon modes when each \( k \)th temporal mode is stored in the QM node at the moment of time \( t \equiv t_k \) (where \( t_1 < t_2 < \cdots < t_k < t_{k+1} \cdots < t_n \) as presented in the previous section. When the storage procedure is completed (\( t = \tau \)), we switch off the inhomogeneous broadening and detune the QM atomic frequency from resonance with the cavity frequency \( \omega_{QM} \neq \omega_0 \).

In order to transfer one arbitrary \( k \)th qubit from the QM node to the first node we switch off the coupling with the external waveguide (that leads to \( \gamma_1 \approx 0 \)) and then we launch rephasing of the atomic coherences in the QM node by reversion of the atomic detunings for time \( \tau \geq \tau \): \( \Delta_{j,1} (t \geq \tau) \rightarrow -\Delta_{j,1} \). It is obvious that the atomic coherence of the \( k \)th qubit state will be rephased at \( t_k = t_k + 2(\tau - t_k) = 2\tau - t_k \)—the moment of time of the \( 4 \)th echo emission. We note that it is possible to suppress the light–atom interaction for all rephasing qubits by using an appropriate additional control of the inhomogeneous broadening preserving these qubits from complete rephasing. Before rephasing of the \( k \)th mode we equalize the frequencies of the QM-node and of the first node with the cavity frequency \( \omega_{QM} = \omega_1 = \omega_{0} \) at the moment of time \( t > 2\tau - t_{k+1} + \delta t \), where the temporal duration of each temporal mode is \( \delta t \ll t_{k+1} - t_k \). The quantum dynamics of the \( 4 \)th atomic coherence in the QM node (\( m = 0 \)) of atomic coherences in three processing nodes and of the cavity mode will be determined by the following system of equations:

\[
\frac{d}{dt} S_{-;m}^j = - (g_m^j)^a i\Delta_{j,m} S_{-;m}^j, \quad (21)
\]
\[
\frac{d}{dt}a = \sum_{m=0}^{N_n} \sum_{j=1}^{N_a} g_j^0 \Omega_j^m S_{m,n}^j,
\]
where we have ignored a weak atomic decoherence for the used timescale \((\gamma_2 t < 1)\).

In accordance with the previous section, we can choose the parameters of the second and third nodes which provide a resonant interaction of the QM node and first processing node with the cavity field mode. Here, we take the frequencies of the second and third processing nodes tuned symmetrically far away from the cavity frequency \(\omega_3 - \omega_0 = -(\omega_2 - \omega_0) = -\Delta_2\) (assuming also an equal number of atoms and photon–atom coupling constants in the nodes: \(N_3 = N_1, g_3 = g_1\)). Finally, we take into account large enough spectral detuning \(\Delta_2\), where we can ignore the weak dispersion effects caused by the second and third nodes to the interaction between the QM node, first processing node and cavity mode. By using a formal solution of equation (21) in equation (22) and taking into account equal atomic frequencies in the processing node, \(\Delta_1 = 0\), we obtain the field equation

\[
\frac{d}{dt}a(t) = N_0 \tilde{g}_0 S_{0,0}(t) + N_1 \tilde{g}_1 S_{1,0}(t) - \Omega_0^2 \int_t^{t'} dt' \exp(-\Delta_0(t - t'))a(t') - \Omega_1^2 \int_t^{t'} dt' \exp(-\Delta_1(t - t'))a(t'),
\]
(23)

where the initial atomic coherences of QM node \(S_{0,0}(t)\) and of the first processing node \(S_{1,0}(t)\) are

\[
N_0 \tilde{g}_0 S_{0,0}(t) = \sum_{j=1}^{N_j} g_0^j \xi S_{0,0}^j(t) \exp[i\Delta_j^0(t - \tilde{t}_j)],
\]
(24)

\[
N_1 \tilde{g}_1 S_{1,0}(t) = \sum_{j=1}^{N_j} g_1^j \xi S_{1,0}^j(t).
\]
(25)

Below we evaluate the expectation values of the field and atomic coherences \(\langle a(0) \cdots | \Psi(t) \rangle_a\) for the initial state of atoms \(\langle \Psi(t) \rangle_a\) in the QM node that corresponds to the \(k\)th stored temporal mode. By taking into account the initial ground state of atoms in the first node, we obtain \(\langle S_{0,0}^j(t) \rangle = 0\) and \(\langle S_{1,0}^j(t) \rangle = 0\) in equations (23) and (25). The initial atomic coherence in the QM node will be determined by rephasing the process of the \(k\)th stored field mode state

\[
\hat{\xi}_0 S_{0,0}^j(t - \tilde{t}_j) = N_0^{-1} \sum_{j=1}^{N_i} g_0^j \xi S_{0,0}^j(t) \exp[i\Delta_j^0(t - \tilde{t}_j)],
\]
(26)

\[
\hat{\xi}_0 S_{1,0}^j(t - \tilde{t}_j) = N_0^{-1} \sum_{j=1}^{N_i} g_0^j \xi S_{1,0}^j(t) \exp[i\Delta_j^0(t - \tilde{t}_j)],
\]
(27)

where we have fixed rephasing of the \(k\)th atomic coherence \(\langle S_{0,0}^j(t) \rangle\) in the QM node.

By using Fourier transformations for the field mode \(a(t) = \int_{-\infty}^{\infty} a(v) e^{-i\nu t} dv\) and atomic coherences, we obtain a general solution of the field equation (23) as follows:

\[
\langle a(t) \rangle = N_0 \tilde{g}_0 \int_{-\infty}^{\infty} dt' \Gamma(t - t') \langle S_{0,0}^j(t') - \tilde{t}_j \rangle^0 \hat{\xi}_0, \]
(28)

\[
\Gamma(t) = i \int_{-\infty}^{\infty} \frac{dv}{2\pi} \frac{v e^{-i\nu t}}{\prod_{m=1}^{3} (v - v_m)},
\]
(29)

where \(v_1 = -i(\Delta_{in} - 2n), v_2 = -(S + i), v_3 = S - in, n = \Delta_{in}/3 - b/3, p = \Omega_0^2 + \Omega_1^2 - \Delta_0^2/3, D = (p/3)^3 + (q/2)^2\) and \(q = \Delta_{in} (\Omega_0^2 + 2\Delta_{in}/\Omega_1^2/3 - \Omega_1^2)/3\), we have here for \(p > 0\): \(b = 3(u - v)/2, S = \sqrt{3(u + v)/2, u = \sqrt{D - q/2}, v = \sqrt{D + q/2}\) and for \(p < 0\): \(b = 3R \cdot C \phi/3, S = \sqrt{3R} \cdot S \phi/3, R = \sqrt{-p/3}, \phi = q/2R^3\).

All three roots of the cubic equation related to equations (21) and (22), \(v_{1,2,3}\), have negative imaginary parts revealing a causality in the field evolution in equation (23) that leads to \(\Gamma(t) < 0\). By taking into account (28) and temporal properties of the response \(\Gamma(t)\)-function, we introduce a \(k\)th atomic coherence with a temporal mode related to the response \(\Gamma(t)\)-function in the following way:

\[
\langle S_{0,0}^j(t - \tilde{t}_j) \rangle = \xi \int \frac{dt''}{\Gamma(\tilde{t}_j - t'')} = \xi \Phi(\tilde{t}_j - t''),
\]
(30)

where the mode \(\Phi(\tau < 0) = 0\), and we call this mode a temporal self-mode of QC as depicted in figure 4, \(F_m = (S^2 + b(\Delta_{in} - n)^2 + S^2(\Delta_{in} - n - b)^2), \theta(\alpha) = \Theta(\Delta_{in} - n - b)^2\) and \(\xi\) is a normalizing constant. We find the constant \(\xi\) by assuming that each \(k\)th temporal mode contains a single-photon wave packet. Here, the field mode is given by the following equation:

\[
\xi \Phi(\tau) = \int_{-\infty}^{\infty} \frac{dv}{\pi} \frac{\Delta_{in}(\nu)^2}{\nu^2 + \Delta_{in}} f(\nu) e^{-iv\tau}.
\]
(31)

By using equations (29), (30) and (31), we find

\[
f(v) = \frac{\xi}{2\Delta_{in}} \frac{\nu (\nu + i\Delta_{in})}{\nu^2 + \Delta_{in}^2} \exp \left[ -i\nu \right],
\]
(32)

where \(|f(v)|^2\) determines a probability of finding \(j\)th atom of the QM node in the excited state, i.e.

\[
\sum_{j=1}^{N_i} |f(v_j)|^2 = N_0 \int_{-\infty}^{\infty} \frac{dv}{\pi} \frac{\Delta_{in}(\nu)^2}{\nu^2 + \Delta_{in}^2} |f(\nu)|^2 = 1.
\]
(33)

This leads to the following normalizing constant: \(\xi = \sqrt{2\Delta_{in}/(N_0 K)}\), where

\[
K = -i \sum_{j=1}^{N_i} \prod_{m=1}^{3} (v_m - v_n)^2
\]
(35)

By using equations (30) and (31), we find a complete depopulation of the \(k\)th field mode at \(t \leq t_k\):

\[
\langle a(t - \tilde{t}_j) \rangle = \xi N_0 \tilde{g}_0 [\Phi(0) - \Phi^2(\tilde{t}_j)] = \xi \tilde{g}_0 N_0 \tilde{g}_0 \Phi(0) = 0.
\]
(36)

where we have assumed a large depopulation of the initial \(k\)th atomic coherence in the QM node \(\langle S_{0,0}^j(t - \tilde{t}_j) \rangle = 0\). We notice that the excited field mode \(\langle a(t) \rangle\) is a continuous function of time in contrast to the \(\Gamma(t - t')\)-function at \(t - t'\).

By using equations (28) and (30), we obtain the atomic coherence of the first processing node as follows:

\[
\langle S_{0,0}^j(t) \rangle = \xi \tilde{g}_0 N_0 \tilde{g}_0 \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} \frac{dv}{\pi} \frac{\Delta_{in}(v + i\nu)^2}{\nu^2 + \Delta_{in}^2} \Phi(\nu) \langle \tilde{t}_j \rangle = \xi \tilde{g}_0 \frac{2\Delta_{in}^2}{\Omega_0^2 \Omega_1^2} P_1(\tilde{t}_j),
\]
(37)

where \(P_1(\tilde{t}_j) = \int_{0}^{\infty} dt \Phi(t^2)\).
Quantum efficiency of the $k$th mode transfer to the first processing node at $t = \tilde{t}_k$ will be given by

$$Q_1(\tilde{t}_k) = N_1|\langle S_1(\tilde{t}_k) \rangle_0|^2 = \frac{2 \Delta_{in}}{K} \Omega_0^2 \Omega_1^2 \rho^2(\tilde{t}_k).$$  \hspace{1cm} (38)

In order to obtain a perfect quantum transfer of the $k$th temporal field mode, we have to find parameters of the atomic nodes providing an ideal mode transfer to the processing node at some fixed moment of time. Numerical evaluation of the quantum efficiency $Q_1(\tilde{t}_k)$ for various atomic parameters of QM and processing nodes ($\Delta_{in}$, $\Omega_0$, and $\Omega_1$) is presented in figure 4 for $\Omega_0 = 1$, $\Omega_1 = 0.3$. Here, we have made the calculations in the units of $\Omega_0$-Rabi frequency and inhomogeneous broadening in the spectral region: $0.2 < \Delta_{in} < 10$. As seen in figure 3, the temporal self-mode has a smooth temporal shape only for the intermediate spectral range $0.5 < \Delta_{in} \leq 5.5$ that can be explained in the following way. For small inhomogeneous broadening $\Delta_{in} < 0.5$, we observe an obvious long-term quantum oscillation between the QM and processing nodes at the state transfer. However, if we increase $\Delta_{in} > 5$ at the same magnitude of $\Omega_0 = 1$, we will reduce the resonant coupling strength between the QM node and QED cavity that will increase the transfer time to the processing node. Thus, there are optimal atomic parameters for fast and robust transfer between the QM and processing node. Also, we note an interesting property of the state transfer revealing a continuous increase of the quantum efficiency $Q_1(\tilde{t}_k)$ with inhomogeneous broadening $\Delta_{in}$ (figure 5). We must obtain high enough quantum efficiency $Q_1(\tilde{t}_k) > 0.9999$ for the realization of the fault tolerant QC (see [21]). As seen in figure 6, we can reach the necessary quantum efficiency for $\Delta_{in} > 5.5$ where the temporal field mode has a still smooth shape and short temporal duration which seems useful for practice.

Thus, the quantum efficiency $Q_1$ can be very close to 100% for relatively large inhomogeneous broadening $\Delta_{in}$ in comparison with the quantum Rabi frequencies of the QM and processing nodes ($\Delta_{in} > \Omega_0$ and $\Delta_{in} > \Omega_1$) as was shortly noted in [16] for QC on multi-atomic ensembles. The discussed properties of the optimal temporal self-modes reveal experimentally achievable coupling strength of QM and processing nodes in the QC scheme that somehow reminds us of the properties of optimal quantum storage processes studied recently in [35, 36]. It is worth noting again that the proposed integration of the QM is also applicable for the QC with superconducting or quantum dot processing nodes characterized by a two-level system with sufficiently high Rabi frequency $\Omega_1$. In order to fix the transferred state in the processing node, we have to switch off abruptly at $t = \tilde{t}_k$ the resonant coupling of the cavity mode with the QM and

**Figure 4.** Temporal shape of self-mode as a function of inhomogeneous broadening width $\Delta_{in}$ presented for two spectral ranges: (a) $0 < \Delta_{in} < 1.8$ and (b) $1.8 < \Delta_{in} < 10$, where we use the units of QM node Rabi frequency $\Omega_0 = N\delta_0^2 = 1$, $\Omega_1 = N\delta_1^2 = 0.3$.

**Figure 5.** Quantum efficiency of quantum transfer from the QM node to the processing node as a function of inhomogeneous broadening width $\Delta_{in}$, where we use the units $\Omega_0 = N\delta_0^2 = 1$, $\Omega_1 = N\delta_1^2 = 0.3$.

**Figure 6.** High quantum efficiency of quantum transfer from the QM node to the processing node in the optimal spectral range of inhomogeneous broadening width $\Delta_{in}$, where we use the units $\Omega_0 = N\delta_0^2 = 1$, $\Omega_1 = N\delta_1^2 = 0.3$. 
processing nodes providing a freezing of the transferred photon qubit in the first processing node.

In order to realize a backward transfer of the photon qubit to the QM node, we can use an interesting symmetry property of equations (21) and (22) under the following symmetry transformation: \( t \to -t', \Delta_{j,m} \to -\Delta_{j,m} \) (i.e. inversion of time and of atomic frequency detunings) plus a sign change of the field mode operator \( a \to -b \). This symmetry transformation has been proposed for general studies of the time reversibility of the PEQM based on the travelling field modes [20], also used for finding general quantum states of the light fields retrieval [21] and has been generalized recently for new reversible schemes in [37]. The symmetry transformation leads now to the system of equations for new operators \( b(t'), S_{jn}(t') \) which coincides completely with equations (21) and (22). This fact automatically leads to the following important obvious conclusion. The dynamics started equations (21) and (22). This fact automatically leads to the transformation leads now to the system of equations for the PEQM based on the travelling field modes (2012) 124017 S A Moiseev and S N Andrianov

4. Conclusion

In this work, we have elaborated in detail on the multi-mode QM integration into the QC scheme. Here we should be able to control the carrier frequencies of the QM and processing nodes in order to provide resonant interaction between the nodes with the cavity mode and external input light field. We have obtained modified optimal conditions for the integration of efficient multi-qubit QM in the QC and revealed a perfect temporal shape of the self-temporal QC modes providing an almost ideal reversible transfer of the photon qubits between the QM node and the arbitrary processing nodes. It should be stressed that we have theoretically demonstrated QC with QM in Von Neumann architecture using QM on the inhomogeneously broadened line where we have no macroscopic dipole moment during the storage stage in contrast to QM on the homogeneously broadened line. Moreover, the dephasing and rephasing of atomic coherence are performed automatically in the QM node without using any additional external fields. These properties of the described QM eliminate the relevant sources of decay and decoherence, making the creation of practical QRAM more feasible.

We note that the elaborated QM and quantum transport can be realized with almost 100% efficiency for the optimal moderate parameters of the atomic ensembles. The described integration schemes open promising possibilities for the practical realization of quantum protocols in the QC with a limited number of processing nodes but with multi-qubit QM. In particular, the outlined framework for the efficient coupling between the QM and processing node can be applied for superconducting and quantum dots QC or QC on multi-atomic ensembles. The detailed analysis of addressing the arbitrary qubits stored in QM and its quantum superposition will be a subject of further work.

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