Topological inflation
induced by a non-minimally coupled massive scalar field

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Abstract

We reanalyze cosmology of a non-minimally coupled massive scalar field which was originally discussed by Futamase and Maeda in the context of chaotic inflation scenario. We find a new type of inflationary solution where inflation occurs inside a domain wall. This new solution relaxes constraints on the coupling constant for successful inflation.

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Among various inflationary models [1], Linde’s chaotic inflation [2] is a simple and natural scenario, which is induced by a minimally coupled scalar field with a polynomial potential. From a viewpoint of quantum field theory in curved spacetime, however, we expect that the scalar field may couple with spacetime curvature. Futamase and Maeda [3] investigated how the nonminimal coupling term \( (1/2)\xi R\phi^2 \) affects realization of chaotic inflation, and derived constraints on the coupling constant \( \xi \) from the condition for sufficient inflation. For the potential (i) \( V(\phi) = (1/2)m^2\phi^2 \), they obtained \( |\xi| \lesssim 10^{-3} \), while for the model (ii) \( V(\phi) = (1/4)\lambda\phi^4 \), \( \xi < \sim 10^{-3} \) [3]. Here, we have adopted the sign convention such that the conformal coupling corresponds to \( \xi = +1/6 \).

For the model (ii) or double-well potentials which includes a quartic term, density perturbations have been calculated by several authors independently [4], who have argued that the fine-tuning problem of the self-coupling constant \( \lambda \) (\( \lesssim 10^{-13} \)) is loosened by the non-minimal coupling term if \( |\xi| \) is large enough. Their results are confirmed by a more rigorous calculation by Makino and Sasaki [5]. Recently, constraints from tensor perturbations are also discussed [6].

The model (i), on the other hand, has not been highlighted because the result by Futamase and Maeda indicates that it needs a kind of fine-tuning, \( |\xi| \lesssim 10^{-3} \), just from the condition for a sufficient amount of inflation. In this paper, however, we shall show that another type of inflation may be possible if \( \xi < 0 \), which relaxes the constraint on \( \xi \).

The model with a non-minimally coupled massive scalar field is described by the action,

\[
S = \int d^4x \sqrt{-g} \left[ \frac{m_{\text{Pl}}^2}{16\pi} \mathcal{R} - \frac{1}{2}\xi \phi^2\mathcal{R} - \frac{1}{2}(\nabla \phi)^2 - V(\phi) \right] \quad \text{with} \quad V(\phi) = \frac{1}{2}m^2\phi^2, \tag{1}
\]

where \( m_{\text{Pl}} \) is the Planck mass, and we concentrate on the case \( \xi < 0 \). Following Futamase and Maeda [3], we apply the conformal transformation,

\[
\hat{g}_{\mu\nu} = (1 + \psi)g_{\mu\nu} \quad \text{with} \quad \psi \equiv -\xi \kappa^2\phi^2 \quad \text{and} \quad \kappa^2 \equiv \frac{8\pi}{m_{\text{Pl}}^2}, \tag{2}
\]

and introduce a new scalar field \( \Phi \) as

\[
\Phi \equiv \int d\phi \sqrt{1 + (1 - 6\xi)\psi} \quad \frac{1}{1 + \psi} \left[ \frac{1}{\kappa^2} \text{arcsinh}\sqrt{(1 - 6\xi)\psi} + \sqrt{\frac{3}{2}} \ln \left\{ \frac{\sqrt{1 + (1 - 6\xi)\psi} - \sqrt{-6\xi}\psi}{\sqrt{1 + (1 - 6\xi)\psi} + \sqrt{-6\xi}\psi} \right\} \right]. \tag{3}
\]

Then the model is described by the Einstein gravity with a canonical scalar field:

\[
\hat{S} = \int d^4\hat{x} \sqrt{-\hat{g}} \left[ \frac{\hat{\mathcal{R}}}{2\kappa^2} - \frac{1}{2}(\nabla \Phi)^2 - \hat{V}(\Phi) \right] \quad \text{with} \quad \hat{V}(\Phi) = \frac{V(\phi)}{(1 + \psi)^2}. \tag{4}
\]

Taking the background as the spatially flat Friedmann-Robertson-Walker spacetime,

\[
d\hat{s}^2 = -d\hat{t}^2 + a^2(\hat{t})d\mathbf{x}^2 = (1 + \psi)[-d\hat{t}^2 + a^2(\hat{t})d\mathbf{x}^2], \tag{5}
\]

the field equations for the homogeneous parts read
\[ \dot{H}^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{3} \left( \frac{1}{2} \Phi_i \dot{\phi} + \dot{V} \right), \quad \Phi_{,ii} + 3H\Phi_{,i} + \dot{V}_\phi = 0, \]  

(6)

where \( \dot{i} \equiv d/d\dot{t} \) and \( \dot{V}_\phi \equiv d\dot{V}/d\Phi \).

Thanks to the standard form of the field equations (6), we can discuss the qualitative behavior of \( \Phi \) (or \( \phi \)) in terms of the potential shape. We depict \( \dot{V}(\Phi) \) for \( \xi < 0 \) in Fig. 1. A distinguished feature of this potential is that it has a maximum at \( \Phi = \Phi_{\text{max}} \) corresponding to \( \psi = 1 \) or \( \phi = 1/\kappa \sqrt{-\xi} \equiv \phi_{\text{max}} \). Hence, if the initial value of the scalar field, \( \Phi_i \), is larger than \( \Phi_{\text{max}} \) and if the energy density of the scalar field \( E_\phi = \Phi_i^2/2 + \dot{V}(\Phi) \) is below \( \dot{V}(\Phi_{\text{max}}) \), \( \Phi \) cannot reach the origin \( \Phi = \phi = 0 \), but it will run away to infinity as long as the universe is expanding in the conformal frame with \( \dot{H} > 0 \). In this case, because the potential has an asymptotic form,

\[ \dot{V} = \frac{m^2}{2\xi^2\kappa^2} e^{-\sqrt{\frac{-2\xi}{\kappa^2}}} \text{ with } \hat{\rho} \equiv \frac{1 - 6\xi}{-2\xi} > 3 \text{ for } \Phi \gg \Phi_{\text{max}}, \]  

(7)

we find power-law inflation [7] in the conformal frame,

\[ \dot{a} \propto \hat{\rho}, \quad \kappa \Phi = \frac{\sqrt{\hat{\rho}} \ln \left( \frac{2m^2\hat{\rho}^2}{(1 - 6\xi)(3 - 16\xi)} \right)}{2}. \]  

(8)

In the original frame, on the other hand, the corresponding asymptotic solution is exponential inflation,

\[ a \propto e^{H_{as}t}, \quad \phi \propto e^{\alpha mt} \text{ with } H_{as} \equiv \frac{(1 - 4\xi)m}{\sqrt{-2\xi(1 - 6\xi)(3 - 16\xi)}}, \quad \alpha \equiv \frac{-2\xi}{\sqrt{(1 - 6\xi)(3 - 16\xi)}}. \]  

(9)

This inflating region does not enter a reheating phase, and hence it cannot lead to the present universe. Nevertheless, this solution arouses our interest because the spacetime approaches exact de Sitter while the scalar field also increases exponentially. To check this result, one may also examine the field equations in the original frame [5],

\[ H^2(1 + \psi) = \frac{\kappa^2}{3} \left( \frac{1}{2} \dot{\phi}_\tau^2 + V + 6\xi H\dot{\phi}_\tau \right), \quad \phi_{,tt} + 3H\phi_{,t} + 6\xi(H_{,t} + 2H^2) + V_{,\phi} = 0. \]  

(10)

One can easily confirm that the solution (9) satisfies the above equations in the asymptotic regime where \( 1 + \psi \) in the right-hand-side (RHS) of the first equation can be approximated by \( \psi \).

In any case, the above runaway solution is irrelevant to our Universe, and Futamase and Maeda [3] obtained a constraint \(|\xi| < 10^{-3}\) so that \( \phi_{\text{max}} > 5m_{\text{Pl}} \) and the initial value of \( \phi \) required for sufficient chaotic inflation \( \phi_i \sim 5m_{\text{Pl}} \) lies on the left of the potential peak.

We can argue, however, that sufficient inflation may be possible even if \( \phi_{\text{max}} < 5m_{\text{Pl}} \) because the plateau around \( \phi = \phi_{\text{max}} \) may cause another channel of inflation. Seen in the conformal frame, some domains of the universe relax to \( \Phi = 0 \) and others run away to \( \Phi \to \infty \) as the universe expands. In between these two classes of regions exist domain walls where large potential energy density \( \dot{V}(\Phi_{\text{max}}) \) is stored. If the curvature of the potential is sufficiently small there, such domain walls will inflate. This is nothing but topological
inflation proposed by Linde and Vilenkin [8]. A similar phenomenon has been found in the model with higher-curvature gravity [9]: a potential maximum appears in the conformal frame which permits topological inflation.

Let us discuss whether inflation can take place at $\phi = \phi_{\text{max}}$, following the arguments of Linde and Vilenkin [8], which have been verified by numerical analysis [10]. The conditions for slow-roll inflation ($|\dot{\Phi}, \ddot{\Phi}| \ll |3\dot{H}\Phi|$ and $\Phi^2/2 \ll \dot{V}$) are equivalent to

$$\left| \frac{\dot{V}}{\kappa V} \right| \ll \sqrt{6}, \quad \left| \frac{\dot{\Phi}\dot{\Phi}}{\kappa^2 V} \right| \ll 3. \quad (11)$$

For the potential in (11), inequalities (11) lead to

$$\frac{|1 - \psi|}{\sqrt{1 + (1 - 6\xi)\psi\kappa\phi}} \ll \frac{3}{2}, \quad \frac{|1 - 6\psi - (5 - 35\xi)\psi^2 \pm 2(1 - 6\xi)\psi^3|}{[1 + (1 - 6\xi)\psi^2]^{\kappa^2\phi^2}} \ll \frac{3}{2}, \quad (12)$$

where we have used (3). At the potential maximum, the first condition is trivially satisfied; hence we have only to check the second condition. Substituting $\phi = \phi_{\text{max}}$ ($\psi = 1$) into the second one in (12), we find

$$\frac{|\xi|}{1 + 3|\xi|} \ll \frac{3}{4}. \quad (13)$$

The similar relation is derived from the condition that the thickness of the wall characterized by the curvature scale of the potential at the maximum, $\hat{R}_w$, is greater than the horizon, $\hat{H}^{-1}$ [8,9]:

$$\hat{R}_w \hat{H} = \sqrt{\frac{\kappa^2 \dot{V}(\Phi_{\text{max}})}{3\dot{V},\Phi(\Phi_{\text{max}})}} = \sqrt{\frac{1 + 3|\xi|}{12|\xi|}} \gtrsim 1. \quad (14)$$

Inequalities (13) and (14) suggest that inflation actually takes place at the top of the potential if $|\xi| \ll 1$. Once inflation sets in, it continues forever inside a domain wall. This solution indicates exponential inflation both in the conformal frame and in the original frame, because the conformal factor changes only slowly during slow roll-over topological inflation.

In order to obtain more precise conditions for sufficient inflation, we solve field equations (6) numerically. We assume initial values as $\Phi_i = \delta\Phi_Q \equiv \dot{H}/2\pi$ and $\dot{\Phi}_i = 0$, and observe the e-fold number of inflation after the classical dynamics dominates over quantum fluctuations, i.e., $|\dot{\Phi}|/\dot{H} > \delta\Phi_Q$. Sufficient expansion requires typically

$$\frac{a_f}{a_c} = \sqrt{\frac{1 + \psi_c \dot{a}_f}{1 + \psi_f \dot{a}_c}} > e^{65}, \quad (15)$$

where $\dot{a}_c$ denotes the scale factor in the conformal frame when $|\dot{\Phi}|/\dot{H} = \delta\Phi_Q$, and $\dot{a}_f$ that when the slow-roll conditions break down. The allowed region is plotted in Fig. 2. This shows that inflation continues sufficiently even if $|\xi| \cong 0.1$, contrary to the previous result [3].
Next, we investigate density perturbations generated during inflation, and constrain the model from the 4yr COBE-DMR data \cite{11}. Because Makino and Sasaki \cite{5} showed that the density perturbation in the original frame exactly coincide with that in the conformal frame, we can easily calculate the amplitude and the spectral index with the well-known formulas. The amplitude of perturbation on comoving scale \( l = 2\pi/k \) is given in terms of Bardeen’s variable \( \Phi_A \) \cite{12} as

\[
\Phi_A \left( \frac{2\pi}{k} \right) = \frac{\sqrt{3k^3\dot{V}_\phi}}{10\pi V,\Phi} = \frac{\sqrt{3k^3m\phi^2}\sqrt{1 + (1 - 6\xi)\psi}}{20\sqrt{2\pi}(1 - \psi^2)},
\]

where all quantities in the RHS are calculated at the time \( t_k \) when \( k \)-mode leaves the Hubble horizon during inflation, i.e., when \( k = aH \). The large-scale anisotropy of cosmic microwave background measured by the COBE-DMR leads \cite{11}

\[
\frac{\delta T}{T} = \frac{1}{3} \Phi_A(k \cong a_0H_0) \cong 10^{-5},
\]

where a subscript 0 denotes the present epoch. The concordant values of \( \xi \) and \( m \) are also plotted in Fig. 2. The spectral index \( n \) is given by \cite{13}

\[
n - 1 \equiv \frac{d\ln \Phi_A^2}{d\ln k} = \frac{3\dot{V}_\phi^2}{\kappa^2V^2} + \frac{2\dot{V}_{\phi\Phi}}{\kappa^2V} = -\frac{4}{\kappa^2\phi^2} \left[ \frac{3(1 - \psi)^2}{1 + (1 - 6\xi)\psi} + \frac{1 - 6\psi - (5 - 35\xi)(1 - 6\xi)^2}{1 + (1 - 6\xi)^{3/2}} \right].
\]

The values of \( n \) which satisfy the COBE-DMR normalization are plotted in Fig. 3. With the observational data, \( n = 1.2 \pm 0.3 \) \cite{11}, we have a constraint \( |\xi| \lesssim 10^{-2} \). To summarize, we have reexamined cosmology of a non-minimally coupled massive scalar field. We have shown that inflation can occur at the local maximum of the potential in the conformal frame, that is, an inflationary universe is created inside a domain wall just as in the topological inflation scenario of Linde and Vilenkin \cite{8}. In this case, sufficient inflation is possible even if \( |\xi| \cong 0.1 \), contrary to the previous constraint \( |\xi| \lesssim 10^{-3} \). We have also calculated the spectrum of density perturbations. The COBE-DMR data requires \( |\xi| \lesssim 10^{-2} \), which is again much less stringent than that claimed by Futamase and Maeda \cite{3}. In this model our Universe exists in a domain which experienced sufficient topological inflation and relaxed to \( \phi = 0 \). It is surrounded by a domain wall which is eternally inflating with the Hubble parameter \( H_w = m/(2\sqrt{3|\xi|}) \). Beyond the domain wall is an asymptotically de Sitter spacetime with a larger expansion rate \( H_{as} \).

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FIG. 1. The potential $\hat{V}(\Phi)$ in a fictitious world for $\xi < 0$ [3]. A plateau at $\Phi = \Phi_{\text{max}}$ leads topological inflation.
FIG. 2. Cosmological constraints on $\xi$ and $m/m_{Pl}$. The dashed curve is a constraint from sufficient inflation. The solid curve represents the concordant values with the amplitude of the COBE-DMR data.

FIG. 3. The spectral index of density perturbations, which satisfies the COBE normalization.