Efficient Quantum Signature and Its Application in On-line Quantum Payment System

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Abstract
Two arbitrated quantum signature schemes, one with message recovery and the other with appendix, are proposed. The two proposed schemes need not prepare quantum entanglement states, do not require comparing qubits, only require von Neumann measurement, and have a significant property that both the signatory and the receiver can share and use a long-term secret key with the arbitrator by utilizing the key together with a random number. In addition, the proposed scheme with message recovery can ensure the confidentiality of the message and achieve a higher transmission efficiency, while the proposed scheme with appendix can sign classical messages of any length by using the hash function to generate digests. Therefore, the efficiency of the two proposed schemes are greatly improved. Furthermore, we applies the quantum signature to quantum payment and propose an on-line quantum payment system.

Keywords: Quantum cryptography; Quantum signature; Quantum payment

1 Introduction
Quantum cryptography depends on fundamental quantum-mechanical law to provide unconditional security for communication. The idea of applying quantum mechanics to cryptography was first introduced in the 1970s by Wiesner (published in 1983) [1]. Bennett and Brassard developed the idea by proposing the famous quantum key distribution protocol in 1984 [2]. Since then, a range of quantum cryptographic protocols have been extensively studied, such as quantum key distribution [2, 3, 4], quantum secret sharing [5], quantum authentication [6], quantum bit commitment [7], quantum oblivious transfer [8] and quantum signature [9, 10, 11, 12, 13, 14, 15]. Especially, quantum key distribution has been proven to be unconditional secure both in theory and in practice [16, 17, 18].

Digital signature, as an electronic equivalent to hand-written signature, is an essential cryptographic primitive and particularly useful in electronic commerce. A valid digital signature can be used to authenticate the identity of the originator, ensure data integrity and provide non-repudiation service. Most classical signature schemes are designed based on certain unproven computational assumptions such as the infeasibility of factoring large integers and solving discrete logarithm. Unfortunately, quantum algorithms are capable of factoring large integers and solving discrete logarithm [19]. Fortunately, quantum signature (QS), whose security relies on quantum-mechanical law rather than on computational assumptions, promises to provide an alternative to classical signature.

Gottesman and Chuang proposed a QS scheme based on quantum one-way functions [20], which was absolutely secure even against quantum attack. However, their scheme is not an efficient scheme as signing an \( m \)-bit message uses up \( O(m) \) qubits of the public key. Zeng and Christoph presented an arbitrated QS scheme utilizing the correlation of the Greenberger-Horne-Zeilinger (GHZ) states and quantum one-time pad [10]. Lee et al. proposed two arbitrated QS schemes with message recovery based on GHZ states and the utilization of quantum one-time pad [11]. One scheme uses a public board and the other does not. Lü and Feng presented two arbitrated QS schemes which could sign unknown quantum states using quantum stabilizer codes [12, 13]. Wang et al. designed an arbitrated
QS with message recovery [14] and an arbitrated QS with appendix [15] without using entangle effect, thus the efficiency of the schemes is improved.

However, almost all previously presented arbitrated QS schemes [10, 11, 12, 13, 14, 15] did not consider how to reuse the shared key between the signatory and the arbitrator or between the receiver and the arbitrator. In fact, if each time when a signatory needs to sign, he has to obtain a new key shared with the arbitrator through quantum key distribution protocol such as [2, 3, 4], the efficiency of the QS protocols would be considerably affected. In this paper, we propose two arbitrated QS schemes, one with message recovery and the other with appendix, based on the work of the Hwang et al. about three-party authenticated quantum key distribution [20]. In the proposed schemes, no quantum entanglement states are used, comparing qubits is unnecessary, only von Neumann measurement is required, and both the signatory and the receiver can share and use a long-term secret key with the arbitrator. Besides, in the proposed scheme with message recovery, the confidentiality of the message can be guaranteed and a higher efficiency in transmission can be obtained, while in the proposed scheme with appendix, classical messages of arbitrary length can be signed by employing hash function to encode the message into quantum information of fixed length. Therefore, the efficiency of the proposed schemes are greatly improved. Furthermore, we construct an efficient on-line quantum payment system utilizing the proposed arbitrated QS.

The rest of the paper is arranged as follows. In Section 2, we describe the preliminaries. In Section 3, two arbitrated QS schemes, one with message recovery and the other with appendix, are proposed and their security and efficiency are analyzed. In Section 4, an on-line quantum payment system is presented based on the proposed arbitrated QS. Finally, conclusions are drawn in Section 5.

2 Preliminaries

Before presenting our results we briefly depict basic knowledge about QS, and introduce some notations for understanding conveniently.

QS, as an analogy to manuscript signature and classical signature, should have the ability to authenticate the identity of the originator and make sure that the original content of the message has not been changed. Two secure requirements should be satisfied even if powerful quantum cheating strategies exist: one is that the attacker (including the malicious receiver) can not forge the signature and the other is the impossibility of disavowal by the signatory.

Generally, QS can be divided into two categories: QS with message recovery and QS with appendix. In the QS with message recovery, the signatory only sends the signature and later the receiver can obtain the original message by utilizing the secret information from the signature. While in the QS with appendix, the signatory sends both the signature and the message, so anyone can get the message.

The notations, which are necessary to better understand the subsequent results, are given as follows.

1. $U_i$: The $k$-bit identity string of one participant. $U_A$, $U_B$ and $U_a$ represent the identity of Alice, Bob and the arbitrator, respectively.
2. $h_1(\cdot)$, $h_3(\cdot)$, $h_3(\cdot)$: The one-way functions, where $h_3(\cdot)$ denotes the mapping $\{0, 1\}^* \rightarrow \{0, 1\}^{m_1}$, $h_2(\cdot)$ denotes the mapping $\{0, 1\}^* \rightarrow \{0, 1\}^{m_2}$ and $h_3(\cdot)$ denotes the mapping $\{0, 1\}^* \rightarrow \{0, 1\}^{m_3}$.
3. $r_i \in_R \{0, 1\}^l$: The $l$-bit string randomly chosen by the participant $U_i$.
4. $K_i$: The secret key string shared between the arbitrator and the participant $U_i$. The length of the secret key string is $n_1$ in the proposed QS with message recovery and $n_2$ in the proposed QS with appendix. Note that $n_1 = l + m_1$ and $n_2 = l + m_3$.
5. $P$: The n-bit message string. Notice that $n_1 = n + m_2 + k$ and $m_3 = l + k$.
6. $r_i || R_i$: The concatenation of the string $r_i$ and the string $R_i$.
7. $str_1 = str_2$: The equality between each bit of the string $str_1$ and that of the string $str_2$.
8. $str_1 \oplus str_2$: The bitwise XOR operation between the string $str_1$ and the string $str_2$. 


3 The proposed arbitrated QS schemes

In this section, we propose two arbitrated QS signature schemes, one with message recovery and the other with appendix, and analyze their security and efficiency. The basic idea of the arbitrated QS scheme with message recovery is similar to that of the scheme with appendix. The main difference is that the signed message is confidential in the first scheme, while public in the second scheme. Different kinds of QS influence the situations they are applied in. The presented schemes include three phases: Initializing phase, Signing phase and Verifying phase. Three partners, namely the signatory Alice, the receiver Bob and the arbitrator, are involved.

3.1 Arbitrated QS scheme with message recovery

A. Initializing phase

Alice shares her $n_1$-bit secret key $K_A$ with the arbitrator through quantum key distribution protocols [2, 3, 4] proved as unconditionally secure [16, 17, 18] and Bob obtains his $n_1$-bit secret key $K_B$ shared with the arbitrator in the same way. $K_A' = \{K_A[i]| i \in \{0,1\}\}$ represents the $i$th bit of the secret key $K_A$. Besides, Alice, Bob and the arbitrator share two hash functions: $h_1$ and $h_2$.

B. Signing phase

Alice randomly chooses a number $r_A \in_R \{0,1\}^l$ and computes $R_A = h_1(K_A, r_A) \oplus h_2(P, r_A)|U_A\rangle$. Alice generates her signature by encoding $P'|h_2(P, r_A)\rangle|U_A\rangle = h_1(K_A, r_A') \oplus R_A'$. Thus he can recover the message $P'$ and verify whether it has been changed by computing $h_2(P', r_A')$. The value of $U_A$ also can be verified. If $h_2(P, r_A) = h_2(P', r_A')$ and $U' = U_A$ he sets $\mu_a = 1$, else $\mu_a = 0$. Similarly, the arbitrator measures the received qubits $|S_A\rangle$ depending on the secret key $K_B$ shared with Bob. When the arbitrator gains the measuring results $r_B'|R_B\rangle$, he computes $F_B'R_B' = h_1(K_B, r_B) \oplus R_B'$. Then he checks whether $U_B$ equals $U_B$. If $U_B = U_B$ he keeps $\mu_b = 1$, else $\mu_b = 0$.

If $\mu_b = 0$, the arbitrator rejects Bob, confirms it to Alice and aborts the protocol. Otherwise the arbitrator does the operations as follows. He randomly chooses a number $r_a \in_R \{0,1\}^m$ and computes $R_a = h_1(K_B, r_a) \oplus \langle P||h_2(P, r_a, \mu_a, F_B')||U_a\rangle$. Then he encodes $r_a||R_a$ according to the secret key $K_B$ and gets the result $|y_\mu\rangle = M_{K_B}(r_a||R_a)$.

The arbitrator sends $|S_A\rangle$ and $|y_\mu\rangle$ to Bob with quantum channel.

Bob decodes the qubit string $|y_\mu\rangle$ relying on the secret key $K_B$ and obtains $r_a'||R_a'$. Then he computes $P'|h_2(P, r_a, \mu_a, F_B')\rangle|U_a' = h_1(K_B, r_a') \oplus R_a'$. Thus he can recover the message $P'$ and check either $h_2(P, r_a, \mu_a, F_B') = h_2(P', r_a', 1, F_B)$ or $h_2(P, r_a, \mu_a, F_B') = h_2(P', r_a', 0, F_B)$. The value of $U_a$ also can be checked. If $h_2(P, r_a, \mu_a, F_B') = h_2(P', r_a', 1, F_B)$ and $U_a' = U_a$. Bob would trust the arbitrator and accept Alice’s signature $|S_A\rangle$ of the message $P$. Otherwise he discards $|S_A\rangle$ and should restart the protocol.

3.2 Arbitrated QS scheme with appendix

A. Initializing phase

Alice shares her $n_2$-bit secret key $K_{A'}$ with the arbitrator through quantum key distribution protocols [2, 3, 4] proved as unconditionally secure [16, 17, 18] and Bob obtains his $n_2$-bit secret key $K_{B'}$ shared with the arbitrator in the same way. $K_{A'}' = \{K_{A'}[i]| i \in \{0,1\}\}$ represents the $i$th bit of the secret key $K_{A'}$. Besides, Alice, Bob and the arbitrator share two hash functions: $h_1$ and $h_2$.

B. Signing phase

Alice randomly chooses a number $r_{A'} \in_R \{0,1\}^l$ and computes $R_{A'} = h_1(K_{A'}, r_{A'}) \oplus h_2(P, r_{A'})|U_{A'}\rangle$. Alice generates her signature by encoding $P'|h_2(P, r_{A'})\rangle|U_{A'}\rangle = h_1(K_{A'}, r_{A'}) \oplus R_{A'}'$. Thus he can recover the message $P'$ and verify whether it has been changed by computing $h_2(P', r_{A'})$. The value of $U_{A'}$ also can be verified. If $h_2(P, r_{A'}) = h_2(P', r_{A'})$ and $U'_{A'} = U_{A'}$ he sets $\mu_a = 1$, else $\mu_a = 0$. Similarly, the arbitrator measures the received qubits $|S_{A'}\rangle$ depending on the secret key $K_{B'}$ shared with Bob. When the arbitrator gains the measuring results $r_{B'}'|R_{B'}\rangle$, he computes $F_{B'}R_{B'}' = h_1(K_{B'}, r_{B'}) \oplus R_{B'}'$. Then he checks whether $U_{B'}$ equals $U_{B'}$. If $U_{B'} = U_{B'}$ he keeps $\mu_b = 1$, else $\mu_b = 0$.

If $\mu_b = 0$, the arbitrator rejects Bob, confirms it to Alice and aborts the protocol. Otherwise the arbitrator does the operations as follows. He randomly chooses a number $r_{a'} \in_R \{0,1\}^m$ and computes $R_{a'} = h_1(K_{B'}, r_{a'}) \oplus \langle P||h_2(P, r_{a'}, \mu_{a'}, F_{B'}')||U_{a'}\rangle$. Then he encodes $r_{a'}||R_{a'}$ according to the secret key $K_{B'}$ and gets the result $|y_{a'}\rangle = M_{K_{B'}}(r_{a'}||R_{a'})$.

The arbitrator sends $|S_{A'}\rangle$ and $|y_{a'}\rangle$ to Bob with quantum channel.

Bob decodes the qubit string $|y_{a'}\rangle$ relying on the secret key $K_{B'}$ and obtains $r_{a'}'||R_{a'}'$. Then he computes $P'|h_2(P, r_{a'}, \mu_{a'}, F_{B'}')\rangle|U_{a'}' = h_1(K_{B'}, r_{a'}) \oplus R_{a'}'$. Thus he can recover the message $P'$ and check either $h_2(P, r_{a'}, \mu_{a'}, F_{B'}) = h_2(P', r_{a'}', 1, F_{B'})$ or $h_2(P, r_{a'}, \mu_{a'}, F_{B'}) = h_2(P', r_{a'}', 0, F_{B'})$. The value of $U_{a'}$ also can be checked. If $h_2(P, r_{a'}, \mu_{a'}, F_{B'}) = h_2(P', r_{a'}', 1, F_{B'})$ and $U_{a'}' = U_{a'}$. Bob would trust the arbitrator and accept Alice’s signature $|S_{A'}\rangle$ of the message $P$. Otherwise he discards $|S_{A'}\rangle$ and should restart the protocol.
Alice and the arbitrator share their $n_2$-bit secret key $K_A$ employing unconditionally secure quantum key distribution protocols. Bob shares his $n_2$-bit secret key $K_B$ with the arbitrator in the same way. In addition, Alice, Bob and the arbitrary share one hash functions $h_1$.

**B. Signing phase**

Alice randomly chooses a number $r_A \in \{0, 1\}^l$ and computes $R_A = h_3(K_A, r_A, P) \oplus (r_A||U_A)$.

Alice obtains her signature $|S_A\rangle = M_{K_A}(r_A||R_A)$ by encoding $r_A||R_A$ based on her secret key $K_A$. If $K^i_A = 0$, $|S^i_A\rangle = [0]$ (or $[1]$) when $(r_A||R_A)^i = 0$ (or 1). If $K^i_A = 1$, $|S^i_A\rangle$ is $|+\rangle$ (or $|-\rangle$) when $(r_A||R_A)^i = 0$ (or 1).

Alice sends the signature $|S_A\rangle$ followed by the message $P$ to Bob. That can be accomplished by transmitting $|S_A\rangle$ with quantum channel and $P$ with classical channel, or by transforming $P$ to qubits $|P\rangle$ according to the basis $R$ and then sending $|S_A\rangle$ and $|P\rangle$ using quantum channel.

**C. Verifying phase**

After Bob receives the signature $|S_A\rangle$ and $P$, Bob randomly chooses a number $r_B \in \{0, 1\}^l$. Then Bob computes $R_B = h_3(K_B, r_B) \oplus (r_B||U_B)$.

Bob obtains the qubit string $|y_B\rangle = M_{K_B}(r_B||R_B)$ by encoding $r_B||R_B$ relying on his secret key $K_B$ with the identical method as Alice does in the signing phase.

Bob transmits the signature $|S_A\rangle$, $|y_B\rangle$ and $P$ to the arbitrator.

The arbitrator measures the received qubits $|S_A\rangle$ depending on the secret key $K_A$ shared with Alice. If $K^i_A = 0$ the qubit is measured according to the basis $R$; otherwise the basis $D$. After the arbitrator gains the measuring outcomes $r_A'||R_A'$, he computes $r_A'||U_A' = h_3(K_A, r_A'||P) \oplus R_A'$. Thus he can verify whether $r_A'||U_A' = r_A'||U_A$. If they are equal, he sets $\mu_a = 1$, else $\mu_a = 0$. Similarly, the arbitrator measures the received qubits $|y_B\rangle$ based on the secret key $K_B$ shared with Bob. When the arbitrator gets the measuring results $r_B'||R_B'$, he computes $r_B'||U_B' = h_3(K_B, r_B'||P) \oplus R_B'$. Then he checks whether $r_B'||U_B' = r_B'||U_B$. If $r_B'||U_B' = r_B'||U_B$ he keeps $\mu_b = 1$, else $\mu_b = 0$.

If $\mu_b = 0$, the arbitrator denies Bob, confirms it to Alice and aborts the protocol. Else the arbitrator performs the following operations. He chooses a random number $r_a \in \{0, 1\}^l$ and computes $R_a = h_3(K_B, r_a, P, \mu_a) \oplus (r_a||U_a)$. Then he encodes $r_a||R_a$ based on the secret key $K_B$ and obtains the result $|y_a\rangle = M_{K_B}(r_a||R_a)$.

The arbitrator sends $|S_A\rangle$, $|y_a\rangle$ and $P$ to Bob.

Bob measures the qubits $|y_a\rangle$ relying on the secret key $K_B$ and gains $r_a'||R_a'$. Then he checks whether $r_a'||U_a = h_3(K_B, r_a'||P, 1) \oplus R_a'$. If they are identical, Bob could believe in the arbitrator and accept $|S_A\rangle$ as Alice’s signature of the message $P$. Otherwise he discards $|S_A\rangle$ and should perform the protocol again.

### 3.3 Security Analysis of the Arbitrated QS Schemes

The security of the QS scheme generally involves two aspects: one is that the attacker (including the malicious receiver) can not forge the signature and the other is the impossibility of disavowal by the signatory and the receiver. Besides, the arbitrator should be trustworthy in an arbitrated QS scheme. Since the proposed schemes use one-way hash functions to map classical bit string of any length to that of fixed length and to authenticate partial information, we cannot evaluate the security precisely. However, all the transmitted data are encrypted and in the forms of random nonorthogonal quantum states, the attacker cannot obtain the useful information without disturbance. If substantial disturbance occurs, the protocol could be aborted. If such abnormal actions occur $\xi_{max}$ times ($\xi_{max}$ may be agreed before the protocol), the participants may consider key exposure; otherwise we assume the key will not be exposed in the proposed protocols and analyze the security of the proposed schemes as follows.

**Impossibility of Forgery** Assume that the attacker $A$ attempts to counterfeit Alice’s signature. Then he has to learn the secret key $K_A$ shared with the arbitrator. However, that is impossible due to unconditionally secure quantum key distribution. Hence, he can not get $r_A$ and $R_A$ which will be used in the verifying phase. If either of them is wrong, the arbitrator will discover the forgery. Furthermore, in the presented QS schemes, the preshared secret key is used together with a random number, the receiver Bob will not obtain the same polarization qubits even though the same message
is signed again. Therefore, even if the secret key $K_A$ is used several times, the adversary $A$ still cannot learn the secret key $K_A$ and forge Alice’s signature of the message $P'$ favorable to him.

**Impossibility of disavowal** Suppose Alice and Bob have disagreements or disputes. Then the arbitrator is needed to handle them. If Alice denies her signature, the arbitrator can confirm that Alice has signed the message since the information of Alice’s secret key $K_A$ is included in the signature $|S_A⟩$. Similarly, if Bob disavows the signature received, the arbitrator also can make sure that Bob has received the signature $S_B$ of the message $P$, because he needs the assistance of the arbitrator to verify whether the signature is valid. Likewise, since both $K_A$ and $K_B$ are used together with random numbers, which ensures that the polarization qubits they generate are different each time, they can be used repeatedly.

### 3.4 Efficiency analysis of the arbitrated QS schemes

The efficiency of the QS schemes is generally considered in two aspects [14]: (1) the total number of the transmitted quantum bits and classical bits when $n$-bit message is signed; (2) the complexity of performing a scheme, including the generation of initial information, quantum operations, comparison among quantum states, etc. In order to compare, we also adopt the formula $\eta = \frac{B_s}{Q_s + B_t}$ used to estimate the efficiency, where $B_s = n$, $Q_t$ and $B_t$ represent the number of signed message bits, transmitted quantum bits and exchanged classical bits, separately. Then we will consider these two aspects between our proposed schemes and other typical QS schemes.

The proposed QS scheme with message recovery involves several security parameters, which could influence the efficiency of the scheme. The larger of the number of the signed message bits $n$ and the smaller of the number of the security parameters $l$ and $m_1$, the higher is the efficiency of the scheme. For instance, if $l = \frac{n}{2}$ and $m_1 = \frac{3n}{2}$, in terms of the formula, the efficiency of our QS scheme with message recovery is 10%; if $l = \frac{n}{4}$ and $m_1 = \frac{3n}{4}$, the efficiency of the scheme is 12%. According to the formula, the efficiency of Zeng’s scheme [10]. Lee’s scheme with a public board [11]. Lee’s scheme without a public board [11]. Lü’s scheme [12] and Wang’s scheme [14] is 9%, 12%, 11%, 11% and 11%, respectively. While the scheme proposed by Gottesman and Chuang is not an efficient scheme [9], since signing an $m$-bit message uses up $O(m)$ qubits of the public key. The QS scheme with appendix presented by Wang et al. could encode the classical message of any length into the quantum information of fixed length by using hash function to generate message digest [15]. While the proposed QS scheme with appendix also can sign the classical message of any length by utilizing hash function to produce the digest of the string including the message and random bits.

The second aspect about efficiency of QS schemes concerns the complexity of carrying out a scheme. The scheme proposed by Gottesman and Chuang [9] does not need the arbitrator, but it needs a trusted key distributions center used to distribute keys to other participants. Zeng’s scheme [10] requires preparing and distributing GHZ states, and involves some complicated quantum operations such as performing a joint measurement on each message qubit and GHZ particles, carrying out Bell measurement on Bell states and comparing quantum states. Lee’s schemes [11] also require using GHZ states, measuring GHZ particles and comparing qubit strings. Lü’s scheme [12] is rather complicated due to the use of quantum stabilizer codes and syndromes. Wang’s schemes [14] need not using GHZ states and only need performing von Neumann measurement, but the additional random secret bit string as message authentication code besides the key is needed in the initializing phase and the comparison of the qubit strings is still required. In addition, almost all the previously presented arbitrated QS schemes [10] [11] [12] [13] [14] [15] did not consider how to use the key shared by the signatory and the arbitrator or by the receiver and the arbitrator repeatedly. Actually, if each time when a signatory needs to sign, the signatory and the receiver have to obtain a new key shared with the arbitrator via quantum key distribution protocol, the efficiency of the protocols would be affected greatly. While in our proposed QS schemes, GHZ states are not necessary, comparing qubit strings is not required, only von Neumann measurement is needed, and the signatory and the receiver can share and use a long-term key with the arbitrator by utilizing the key together with a random number. Therefore, the efficiency of the proposed QS schemes is largely improved.
4 On-line quantum payment system using the arbitrated QS

Like classical payment system, quantum payment system, mainly involving quantum cash-like payment system [1] and quantum check-like payment system [22], should be done on-line or off-line. Generally, the large-value payment transactions requiring higher security should rather be settled on-line whereas low-value payment transactions requiring less security could be handled off-line. In this paper, an on-line quantum check-like payment system taking the advantage of the arbitrated QS with appendix is proposed, which can provide high security. As a matter of fact, if the transaction information needs to be confidential, the arbitrated QS with message recovery can be utilized to construct a similar payment system.

In the presented quantum check payment system, the check information consists of two parts somewhat similar to that in [22], the first part which must be generated and signed by the payer is $P_1 = (p_1 || p_2 || p_3 || p_4 || p_5 || p_6 || p_7)$, where $p_1$ is the name of the payee, $p_2$ is the name of the payer, $p_3$ is the account number of the payer, $p_4$ is the amount of money, $p_5$ is the use of the check, $p_6$ is the written date of the check and $p_7$ is the expiration date of the check. While the other part which should be generated and endorsed by the payee is $P_2 = (p_8 || p_9 || p_{10})$, where $p_8$ is the payee’s name, $p_9$ is the payee’s account number and $p_{10}$ is the amount of money.

There are four kinds of participants in the on-line quantum check payment system: Payer, Payee, Issuer (or Issuing bank) and Acquirer (or Acquiring Bank). The presented payment system includes four phases: Registration phase, Payment phase, Capture phase and Settlement phase. The payer owns the account at the issuing bank and the payee obtains the account at the acquiring bank through the registration phase. The quantum check payment system begins with the payment phase in which the payer sends the signed check to the payee. In the capture phase, the payee transmits the endorsed check to the acquiring bank who will receive the money from the issuing bank in the settlement phase. Figure 1 shows the basic model and detailed descriptions of the four phases are given in the following.

![Figure 1: Basic model of quantum check payment system](image)

4.1 Registration phase

If the payer wants to finish the payment by check, he has to register at the issuing bank and open a check account which can make him have the right to use checks. The payer can come to the issuing bank to complete the registration phase. This also can be accomplished using the following operations.

1. The payer and the Issuer share more than $n_2$-bit secret key $K$ through quantum key distribution protocols [2, 3, 4] proved as unconditionally secure [16, 17, 18].

2. The payer and the Issuer keep the first $n_2$-bit secret key $K_1$ as a long-term shared key. The Issuer encrypts the payer’s identity information $U_1$ and check account information $CAI_1$ with the other bits of the secret key $K$ using quantum one-time pad, and sends the encrypted information to the payer.
3. The payer decodes the encrypted information and gets $U_1 \| CAI_1$.

Thus, after the registration, the payer and the Issuer share the secret key $K_1$, the payer gets the check account information $CAI_1$ and the Issuer stores $U_1 \| CAI_1$ in the database. Then the payer can deposit some money to his account at any convenient time and pay by check.

The payee who may receive a check can register in the similar way at the acquiring bank. After the registration, the payee and the Acquirer share the secret key $K_2$, the payee obtains the normal account information $NAI_2$ and the Acquirer stores $U_2 \| NAI_2$ in the database.

4.2 Payment phase

When the payer and the payee carry out the transaction, the payer chooses to pay the goods or other services by check. Hence, in the payment phase, the payee generates and signs the first part information of check, and then sends the signed check to the payee. The detailed performing process is as follows.

1. The payer creates the first part information of the check: $P_1 = (p_1 || p_2 || p_3 || p_4 || p_5 || p_6 || p_7)$.

2. The payer chooses a random number $r_1 \in_R \{0, 1\}^l$ and computes $R_1 = h_3(K_1, r_1, P_1) \oplus (r_1 || U_1)$.

3. The payer obtains the signature $|S_1\rangle = M_{K_1}(r_1 || R_1)$ of $P_1$ by encoding $r_1 || R_1$ relying on his secret key $K_1$. If $K_1 = 0$, $|S_1\rangle$ is $|0\rangle$ (or $|1\rangle$) when $r_1 || R_1$ is $0$ (or $1$). If $K_1 = 1$, $|S_1\rangle$ is $|+\rangle$ (or $|-\rangle$) when $(r_1 || R_1)$ is $0$ (or $1$).

4. The payer sends $|S_1\rangle$ and $P_1$ to the payee. This can be finished by transmitting $|S_1\rangle$ with quantum channel and $P_1$ with classical channel, or by transforming $P_1$ to qubits $|P_1\rangle$ based on the basis $R$ and then sending $|S_1\rangle$ and $|P_1\rangle$ utilizing quantum channel.

4.3 Capture phase

In the capture phase, the payee generates and endorses the second part information of the check, and latter transmits the endorsed check to the acquiring bank. This can be completed by performing the following operations.

1. After receiving $|S_1\rangle$ and $P_1$, the payee produces the second part information of the check: $P_2 = (p_8 || p_9 || p_{10})$.

2. The payee randomly chooses a number $r_2 \in_R \{0, 1\}^l$ and computes $R_2 = h_3(K_2, r_2, P_2) \oplus (r_2 || U_2)$.

3. The payee endorses $P_2$ by encoding $r_2 || R_2$ according to his secret key $K_2$ and obtains the results $|S_2\rangle = M_{K_2}(r_2 || R_2)$. The encoding way is similar to that the payer does in the payment phase.

4. The payee transmits $|S_1\rangle$, $|S_2\rangle$, $P_1$ and $P_2$ to the acquiring bank. This can be done by sending $|S_1\rangle$ and $|S_2\rangle$ with quantum channel and $P_1$ and $P_2$ with classical channel, or by transforming $P_1$ and $P_2$ to qubits $|P_1\rangle$ and $|P_2\rangle$ according to the basis $R$ and then sending $|S_1\rangle$, $|S_2\rangle$, $|P_1\rangle$ and $|P_2\rangle$ via quantum channel.

4.4 Settlement phase

Once the Acquirer receives $|S_1\rangle$, $|S_2\rangle$, $P_1$ and $P_2$, the settlement phase can begin. The Acquirer cooperates with the Issuer to check whether the received check is valid. If the check is valid, they finish the virement between the payer’s account and the payee’s account. This can be done through the inner bank transaction. For simplicity, here we assume that the Acquirer and the Issuer are the same bank. The following is the detailed performing process.

1. The bank measures the received qubits $|S_1\rangle$ depending on the secret key $K_1$ shared with the payer. If $K_1 = 0$, then the qubit is measured based on the basis $R$; otherwise the basis $D$. When the bank obtains the measuring outcomes $r_1' \| R_1'$, he computes $r_2' \| U_1' = h_3(K_1, r_1', P_1) \oplus (r_1' || U_1')$. Hence he can verify whether $r_2' \| U_1'$ is the same as $r_1' \| U_1$. If $r_2' \| U_1' = r_1' \| U_1$, the bank would trust the payer and accept the payer’s signature $|S_1\rangle$ of $P_1$. Otherwise he confirms to the payee that there is something wrong with $|S_1\rangle$ and $P_1$ and the transaction should be terminated.

2. The bank measures the received qubits $|S_2\rangle$ relying on the secret key $K_2$ shared with the payee in the similar way. When the bank gains the measuring results $r_2' \| R_2'$, he computes $r_2' \| U_2' = ...
Thus he can check whether $r''_2 || U'_2$ is identical to $r'_2 || U_2$. If $r''_2 || U'_2 = r'_2 || U_2$, the bank would believe in the payee and accept $|S_2\rangle$ as the payee’s signature of $P_2$. Else he confirms to the payer that the payee is not honest and the payment should be canceled.

3. If $|S_1\rangle$, $|S_2\rangle$, $P_1$ and $P_2$ are all valid, the bank continues to check. If $p_1 = p_8$ and $p_4 = p_{10}$, the bank debits the payer’s account with the amount of money $p_4$ and credits the payee’s account with the amount of money $p_{10}$; otherwise the bank rejects the virement between the payer’s account and the payee’s account and sends notifications to both of them.

5 Conclusions

In this paper, we propose two arbitrated quantum signature schemes, one with message recovery and the other with appendix, based on the three-party authenticated quantum key distribution protocol presented by Hwang et al. [20]. In the proposed schemes, we need not prepare quantum entanglement states, do not require comparing qubits, only need implement von Neumann measurement, and provide a significant feature that both the signatory and the receiver can share and use a long-term secret key with the arbitrator. Thus our schemes can be performed with high efficiency.

In addition, we construct an on-line payment system based on the proposed arbitrated QS with appendix. An on-line payment system is generally used in the large-value payment transactions and requires higher security. Compared with the classical payment system, the presented quantum payment system does not depend on unproven computational assumptions such as the intractability of factoring large integer and solving discrete logarithm, which might be broken with a quantum computer [19], but depend on basic principles of quantum mechanics. Thus the on-line quantum payment system can provide higher security and is more applicable for the large-value payment transactions. Compared with the quantum payment system presented by Al-Daoud [22], the quantum payment system proposed in this paper utilizes the arbitrated QS, needs not generate and distribute GHZ states, does not require performing complicated quantum operations such as CNot operation and Bell measurement and just needs to carry out von Neumann measurement. Therefore, the efficiency of the proposed system is greatly improved.

In summary, we have proposed two efficient arbitrated QS schemes and have demonstrated the possibility of applying the arbitrated QS to the on-line quantum payment system. The work of this paper may promote the research of designing different QS schemes with special property to adapt to various types of quantum payment systems.

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