Error Analysis Method and Sensitivity Analysis for Linkage Transmission Mechanism

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Abstract. Given the linkage transmission mechanism with mutual restraint, coupling, and multi constraint conditions, due to the existence of machining errors of parts, the error accumulation in the assembly process leads to the uncertainty of position and posture. At present, there is no quantitative analysis. In this study, the small displacement torsor (SDT) method was used to describe the error variation of geometric elements, and the error transmission attribute of the joint surface was proposed by using a homogeneous transformation matrix. The error accumulation value of each assembly joint surface for the linkage transmission mechanism was analyzed by using MATLAB numerical algorithm, to predict the actual position change value of the frame caused by machining error and the sensitivity of factors affecting the assembly accuracy. The sensitivity can guide for improving assembly accuracy and optimizing technology.

1. Introduction

In the process of assembly, the motion accuracy and sensitivity of the multi constraint linkage transmission mechanism with mutual restraint and coupling are the main indicators that affect the product performance, and the machining error of the part itself is one of the main factors affecting the motion accuracy. At present, the research mostly focuses on the influence of the assembly error formed in the assembly process on the motion accuracy.

Whitney [1] introduced the homogeneous transformation matrix method applied in the field of the robot into the statistical analysis of assembly tolerance and studied the assembly accuracy of assembly. Ferreira and Liu [2] described the error distribution of the machine tool by a quadratic linear model. In 2003, Camelo [3] proposed an evaluation method based on linear mechanics and state-space representation for dimension change transfer of multi-station flexible assembly system, aiming at the dimension quality deviation of parts meeting the requirements in the assembly process. Huston [4] put forward new concepts, which can expand the scope of application, including solving the motion equation of the multi-body mechanical system and the displacement between the bodies of the system. Zhou [5]
introduced the theory of robust design into the dimensional chain and put forward the concept of the sensitivity of the eligible probability of dimension. Wei [6] regarded the planar dimensional chain as interval variables and then calculated the influence of interval variables of each composing loop on interval variables of the close loop, namely sensitivity. In 2008, Han [7] made a simple comparative analysis of the local and global methods of sensitivity analysis. In 2015, Zhang [8] and others first established two kinematic models of the robot through the homogeneous transformation matrix, and further established the error analytic model of the robot's static pose through the small displacement composing method, and analyzed the robot's end pose error caused by the static error. In 2018, Zhao [9] and others studied the influence of joint clearance on the assembly accuracy of planar single-closed-loop mechanisms, introduced the rotatability law of the linkage, established the assembly error model considering the clearance, and gave the method of calculating the bound of assembly error. Qian [10] et al. described the error transfer model of the press based on the loop incremental theory to determine the key error factors affecting the precision of the servo press of the mechanical press. Angle and position errors were used to characterize the motion output accuracy, and the key error factors affecting the output accuracy were determined through sensitivity analysis. In 2019, Wang [11] et al. established an evaluation model and used the global sensitivity method to quantitatively analyze the error sensitivity of CNC machine tools.

It can be seen from the above literature that the kinematic accuracy and sensitivity are the key indicators for the multi constraint linkage transmission mechanism with mutual restraint and coupling, while the assembly accuracy is the key factor affecting the kinematic accuracy. At present, the research on assembly accuracy mainly focuses on the accumulation and transmission of errors in the assembly process, while the research on the manufacturing uncertainty factors affecting the assembly accuracy, such as machining error, is not enough. Especially, the lack of the quantitative analysis of its machining error, which bases on the motion accuracy of the assembled multi constraint linkage transmission mechanism with mutual restraint and coupling. At the same time, the error cumulative analysis model proposed in the previous literature is also more complex and not practical. Given the lack of quantitative analysis of the mapping relationship between the machining error of parts and the geometric error transfer in the assembly process, the small displacement torsor method described the error change of geometric elements, the error transfer relationship of the assembly joint surface of the multi-link mechanism is proposed by using the homogeneous transformation matrix, and the multi-link assembly error model with error is established. Hence, to improve the assembly accuracy and optimize the assembly process, the accumulated error values of each assembly joint surface of the connecting rod transmission mechanism were analyzed by the numerical algorithm, and the actual position and pose changes of the output position of the connecting rod mechanism caused by the machining geometric error of the hole making and axis machining of the connecting rod transmission mechanism and the sensitivity of various factors affecting the assembly accuracy were predicted.

2. Error Model of Linkage Transmission Mechanism

2.1. Quantitative characterization of geometric errors of parts

The error change of the frame pose of the linkage transmission mechanism refers to the change between the actual coordinate pose and the ideal coordinate pose of the frame center. The small change of the frame pose is described by the Small displacement torsor method (six motion components are offset). Among the factors that lead to the frame accuracy error, this study only considers one of the important factors, which is the geometric machining error, just as the influence of the axis error. The rotation of small displacement can be expressed as \( D = (\delta_x, \delta_y, \delta_z, \delta_\theta, \delta_\phi, \delta_\psi) \), of which the first three represent displacement along the coordinate axis and the last three represent rotation offset around the coordinate axis. As shown in Figure 1, a plot of the location between the actual mating surface and the ideal mating surface after a small displacement torsor.

Because there are many uncertain factors in the assembly process, such as machining error, surface quality, assembly force error, and so on, the above manufacturing uncertainty is a lack of quantitative
analysis on the mechanism of geometric error transmission in the assembly process.

This study only takes the coaxiality error in the position error as an example, other position errors are similar to coaxiality error, and the geometric error of parts of the multi-link transmission mechanism is considered, which is defined as coaxiality. The range of micro displacement error caused by coaxiality error is $(-t/2, t/2)$, and the range of micro rotation error is $(-t/2L, t/2L)$. $L$ is the length of the cylindrical element generatrix, and the error components are expressed as $(\delta x, \delta y, \delta z, \delta \theta, \delta \varphi, \delta \psi) = (\delta x, \delta y, 0, \delta \theta, \delta \varphi, 0)$.

Figure 1 Coordinate system of ideal matching surface and actual matching surface

2.2. Geometric error modeling method of the linkage mechanism
Define the coordinate system before and after the shift of the central pose of the frame of the linkage transmission mechanism, as shown in Figure 2: Part $\theta$ as a fixed basic part, also known as the global coordinate system, expressed by $O_{\theta}X_{\theta}Y_{\theta}Z_{\theta}$; compared with the basic coordinate system, the joint surface of the base bearing hole $A$, which is with machining error, is called the local coordinate system, expressed by $O_{A}X_{A}^{'Y_{A}^{'Z_{A}^{'} }; and compared with the basic coordinate system, the ideal coordinate system of the joint surface of the base bearing hole $A$ is represented by $O_{i}X_{i}Y_{i}Z_{i}$, and the coordinate position relationship between $O_{i}X_{i}Y_{i}Z_{i}$ and $O_{A}X_{A}^{'Y_{A}^{'Z_{A}^{'} is shown in Figure 1, which will not be repeated here. The position relationship between $O_{\theta}X_{\theta}Y_{\theta}Z_{\theta}$ and $O_{A}X_{A}^{'Y_{A}^{'Z_{A}^{'} is shown in Figure 2.

Figure 2 definition of a coordinate system

The ideal local coordinate value of $O_{i}$ in the $O_{\theta}$ coordinate system is the first transfer of the linkage mechanism, expressed by $(x_{0i}, y_{0i}, z_{0i})$; the ideal local coordinate value of $O_{\theta}$ in the $O_{A}$ coordinate system is the second transfer of the linkage mechanism, expressed by $(x_{0\theta}, y_{0\theta}, z_{0\theta})$; and so on, $(x_{0i}, y_{0i}, z_{0i})$ represents the ideal local coordinate value of the $i$ local coordinate system in the $i-1$ local coordinate system.

When the coordinate axes are all ideal values $(x_{0i}, y_{0i}, z_{0i}) = \{0\}$, if the small displacement error $\Delta \bar{O}_{i}(\delta_{x}, \delta_{y}, \delta_{z})$ of $O_{i}$ occurs:
If there is no angle deviation in the global coordinate system of $O_0X_0Y_0Z_0$, the angle deviation of each part only affects the coordinate system position of the next level part but does not influence the coordinate system position of its part. If part $i$ has a small angular rotation deviation $\Delta \bar{O}_i(\delta_\theta, \delta_\phi, \delta_\psi)$ around three coordinate axes $X$, $Y$, and $Z$, the error change matrix of the interface between the ideal assembly surface and the actual assembly surface is as follows after the occurrence of small displacement and small angular deviation at the same time:

$$D_i(\delta_x, \delta_y, \delta_z, \delta_x, 0, 0, 0) = \begin{bmatrix} 1 & 0 & 0 & \delta_x \\ 0 & 1 & 0 & \delta_y \\ 0 & 0 & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

At the same time, the range of small-displacement error caused by the coaxiality error of the linkage is $(-t/2, t/2)$, the range of small-angle error is $(-t/2L, t/2L)$, and $L$ is the length of the cylindrical element generatrix, then the error component is expressed as

$$D_i(\delta_x, \delta_y, \delta_z, \delta_x, \delta_y, \delta_z) = \begin{bmatrix} 1 & -\delta_\psi & \delta_\phi & \delta_x \\ \delta_\psi & 1 & -\delta_\theta & \delta_y \\ -\delta_\phi & \delta_\theta & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2)

Where $i = 1, 2, 3$ n. $D_i$ is the error variation matrix expressed by small displacement torsor, $D_i^\prime$ is the matrix expression of the ideal coordinate value of the $i$ local coordinate system in the $i-1$ local coordinate system, and $P_i^\prime$ is the transpose of the actual coordinate value vector of the $i-1$ part.

2.3. Example verification

The error analysis of the transmission system is carried out by taking a linkage transmission mechanism with a simplified linkage structure of the coordinator as the research object.

2.3.1. Configuration analysis of linkage mechanism. As shown in Figure 1, because the multi-link mechanism has the properties of restraint and coupling, its kinematic accuracy and sensitivity are the key indicators, and the assembly accuracy is the key to affect the kinematic accuracy. For the manufacturing geometric error that affects the assembly accuracy, just as the axis error, the influence on the position accuracy of the frame of the multi-link mechanism can not be ignored. The multi-link transmission mechanism is a closed frame structure composed of a base, tension link, linkage arm, and frame. $AEFJ$ is the base, which plays a supporting and stabilizing role. $ACE$ and $JHF$ are tension links, and $BDGI$ is a linkage arm. $A$ represents the joint surface of the base bearing hole and linkage bearing assembly. The following is a unified description of joint surface $A$, Other joint surfaces are similar to this. As can be seen from Figure 3, each component of the connecting rod transmission mechanism, as shown in Figure 4, is the modeling algorithm.
2.3.2. Configuration simplification of the multi-link transmission mechanism. For the convenience of description, the multi-link transmission mechanism is simplified as follows: (1) only the coaxiality error of the transmission system is considered in this study, for example, the coaxiality error of the base hole relative to the ideal central axis at the joint surface A in Figure 3, \( t_{\text{coa}}=0.01 \), and other errors are not considered; (2) to mainly describe the calculation method of the model, the structural shape of the tension link and linkage arm is simplified.

2.3.3. Example verification of linkage mechanism. According to the location of local coordinates of each part in the measured 3D model, as shown in Figure 1, the value of local coordinates as shown in Table 1, and the coaxiality index of parts at the assembly position of shaft hole given by the drawing design are as follows, which are substituted into formula 2 and formula 3.

The coaxiality error of the front, rear, left and right inner holes of the base are \( t_{\text{coa}}=0.01 \), the coaxiality error of the front, rear, left and right inner holes of the linkage arm is \( t_{\text{coa}}=0.01 \), and the coaxiality error of the shaft of the upper part of the connecting rod and the inner hole of the frame is \( t_{\text{coa}}=0.02 \).

Table 1 local coordinate figure of assembly joint surface of the multi-link mechanism

| Coordinate position | system | Local coordinate figure | Coordinate position | system | Local coordinate figure |
|---------------------|--------|-------------------------|---------------------|--------|-------------------------|
| O                   |        | (0, 0, 0)               | F                   |        | (0, 160, 0)             |
| A                   |        | (0, -80, 0)             | G                   |        | (26, 0, 46)             |
According to the above data, the movement and rotation error vectors of the end position frame of the multi-link transmission mechanism are shown in Table 2. The angle between the global coordinate system of the end position (frame) center and the ideal coordinate system is 0.0435°, which is the rotation angle error of the linkage transmission mechanism. Through the rotation angle error and the data in Table 2, the parallelism between the frame and the ground is 0.046 mm. It meets the design accuracy requirements.

| direction                              | offset error of end position/mm |
|----------------------------------------|---------------------------------|
| X-direction displacement               | 0.0349                          |
| Y-direction displacement               | 0.0024                          |
| Z-direction displacement               | 0.0496                          |
| Rotation in the X direction            | 9.1789E-05                     |
| Rotation in the Y direction            | 3.6288E-04                     |
| Rotation in the Z direction            | 3.7981E-04                     |

Table 2 pose change of end position (frame) of the linkage transmission mechanism

3. Sensitivity Analytical Method of Assembly Error Accuracy

3.1. Sensitivity coefficient analytical method
Based on the error accumulation formula in the first chapter, the sensitivity analysis of the geometric error of each assembly part of the multi-link transmission mechanism is carried out, to quantitatively determine the influence of the geometric coaxiality error of each part on the end position accuracy of the system.

If $Y$ is defined as the vector of the end (frame) position of the multi-link transmission mechanism after assembly, the small error variation matrix $T_Y$ of the multi-link transmission mechanism can be obtained by equation 3:

\[
T_Y = \begin{bmatrix}
1 & -\delta_{y\phi} & \delta_{y\theta} & \delta_{yz} \\
-\delta_{y\phi} & 1 & -\delta_{y\theta} & \delta_{yz} \\
\delta_{y\phi} & \delta_{y\theta} & 1 & \delta_{yz} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

At the same time, $Y$ is defined as the total error vector of displacement error and rotation error of the end (frame) position of the assembled multi-link transmission mechanism.

\[
Y = F\left(U_{F_1}, \ldots, U_{F_m}\right)
\]

Since $Y$ is a continuously differentiable function, the partial derivative of $Y$ to each small error component of $U$ is defined as $\partial F/\partial U$, then the function $Y$ is expanded according to the first-order Taylor formula as follows:

\[
F(U + \Delta U) = F(U) + \frac{\partial F}{\partial U} \Delta U + O(\Delta U) \approx Y + \Delta Y
\]

Then the sensitivity analysis model of error vector $U$ is obtained as follows:

\[
\Delta Y = \frac{\partial F}{\partial U} \Delta U = SU
\]

$S = \partial F/\partial U$ is defined as the sensitivity of variable $U$.

The geometric errors of the parts in the assembly parts of the multi-link transmission mechanism
cause the small variation components of the end (frame) position of the assembly system after assembly, which position has a great influence on them can be reflected by the sensitivity value \[12\].

\[
S_{p_e} = \left| \frac{\partial Y_p}{\partial U_g} \right| = \begin{bmatrix}
\frac{\partial Y_p}{\partial \delta_{g\theta}} & \frac{\partial Y_p}{\partial \delta_{gy}} & \frac{\partial Y_p}{\partial \delta_{gy}} & \frac{\partial Y_p}{\partial \delta_{gy}} & \frac{\partial Y_p}{\partial \delta_{gy}} & \frac{\partial Y_p}{\partial \delta_{gy}}
\end{bmatrix}
\] (8)

Where \(p=[x, y, z, \theta, \phi, \psi]\), the influence of coaxiality error of the joint surface of the \(g\)-th part on the end position accuracy of the system can be judged by the obtained \(S_{pg}\) value.

To get the relative importance of each error more intuitively, the sensitivity figures of six error components of the small error variation matrix \(TY\) of the multi-link transmission mechanism are normalized, in which \(SC_{pg}\) is the sensitivity coefficient corresponding to the error \(S_{pg}\) of the part[12].

\[
SC_{p_e} = \frac{S_{p_e}}{\sum S_{p_e}}
\] (9)

The sensitivity of the key parts which affect the end (frame) position error of the multi pull linkage mechanism after assembly is calculated. The coordinate system is established on the corresponding fitting surface. The relationship between the total error and the geometric error of each part is calculated by using the geometric error transfer model. The derivatives of the six motion components of the end (frame) position error to the error components of each joint surface are obtained from the sensitivity calculation formula (8) - (9), that is, the sensitivity figure and the normalized sensitivity coefficient.

3.2. Example analysis of error sensitivity of linkage mechanism

By solving the influence of small component changes caused by the coaxiality error of each part on the position and pose of the end position of the multi-link transmission mechanism, the sensitivity and sensitivity coefficient of the end position can be obtained, where \(\delta_{x}, \delta_{y}, \delta_{z}, \delta_{\theta}, \delta_{\phi}, \delta_{\psi}\) are the small displacement error components in the end (frame) position error matrix of the linkage system after assembly, \(\delta_{x}, \delta_{y}, \delta_{z}, \delta_{\theta}, \delta_{\phi}, \delta_{\psi}\) are the error components of the single assembly joint surface that affect the end position of the multi-link transmission mechanism, as shown in Fig. 5 and table 3. It is summarized as follows:

![Sensitivity coefficient of \(\delta Y_x\)](a)

![Sensitivity coefficient of \(\delta Y_y\)](b)
Fig. 5 sensitivity of each error component affecting the total error vector of the system

Table 3 sensitivity of each error component affecting the total error vector of the system

| Displacement error component | $\delta Y_x$ | $\delta Y_y$ | $\delta Y_z$ |
|------------------------------|--------------|--------------|--------------|
| $\delta a_{20}$             | 0.055325     | 0.072792     | 0.099278     |
| $\delta a_{19}$             | 0.055325     | 0.072792     | 0.099278     |
| $\delta a_{18}$             | 0.055325     | 0.057279     | 0.17148      |
| $\delta a_{17}$             | 0.055325     | 0.057279     | 0.17148      |
| $\delta a_{16}$             | 0.065698     | 0.072792     | 0.086643     |
| $\delta a_{15}$             | 0.0656985    | 0.072792     | 0.086643     |
| $\delta a_{14}$             | 0.055325     | 0.057279     | 0.110108     |
| $\delta a_{13}$             | 0.055325     | 0.057279     | 0.110108     |
| $\delta a_{12}$             | 0.055325     | 0.065632     |              |
| $\delta a_{11}$             | 0.055325     | 0.065632     |              |
| $\delta a_{10}$             | 0.038036     | 0.065632     |              |
| $\delta a_{9}$              | 0.038036     | 0.065632     |              |
| Sum of sensitivity coefficients | 0.65         | 0.7828       | 0.935018     |

It is summarized as follows:

(1) It can be seen from the results in Fig. 5 and table 3 that the rotation error component around the $Z$-axis coordinate system, which is one of the six motion quantities in the coaxiality error component of the $F$, $G$, $H$, $I$ and $J$ assembly joint surface in Fig. 3, has a relatively great influence on the error component $\delta Y_x$ of the end position of the multi-link transmission mechanism.

The error component $\delta Y_x$ of the end position of the multi-link transmission mechanism has been relatively greatly influenced by the rotation error component around the $Z$-axis coordinate system of one of the six motion quantities of the coaxiality error component at the $D$, $E$, $I$, and $J$ assembly joint surface in Fig. 3, and the rotation error component around the $Y$-axis coordinate of one of the six motion quantities of the coaxiality error component at the $C$ and $H$ assembly joint surface.

The error component $\delta Y_x$ of the end position of the multi-link transmission mechanism has been
relatively greatly influenced by the rotation error of one of the six motion quantities of the coaxiality error component at the assembly joint surface of $C$ and $H$ around the coordinate axis $X$ in Fig. 3 and the rotation error of one of the six motion quantities of the coaxiality error component at the assembly joint surface of $D$ and $E$ around the coordinate axis $Y$ in Fig. 3.

(2) The factors influencing the error component $\delta_{Y\theta}$ of the end position of the multi-link transmission mechanism are $\delta_{\phi3}$, $\delta_{\phi6}$, $\delta_{\phi15}$, $\delta_{\phi16}$ with sensitivity coefficients of 0.25; the factors that have a relatively great influence on the error component $\delta_{Y\psi}$ of the end position of the multi-link transmission mechanism are the rotation error components around the $Y$-axis of other joint surfaces except for the four rotation error components around the $Y$-axis of $\delta_{\phi5}$, $\delta_{\phi6}$, $\delta_{\phi15}$, and $\delta_{\phi16}$, and the sensitivity coefficients are all 0.0625; the error component around the $Z$-axis of each joint surface, of which the sensitivity coefficients are 0.05, affects the error component $\delta_{Y\psi}$ of the end position of the linkage.

From the above analysis, it can be seen that the influence of coaxiality error on the sensitivity of assembly accuracy of the multi-link transmission mechanism in geometric machining error. The displacement components $\delta_{Y\psi}$, $\delta_{Y\theta}$, and $\delta_{Y}$ of the error vector at the end position of the linkage transmission mechanism are mainly affected by the rotation error. The rotation error is amplified with the structure size of the parts, that is, the influence of the rotation error on the displacement component has a great correlation with the structure size of the parts. The rotation components $\delta_{Y\psi}$, $\delta_{Y\theta}$ and $\delta_{Y\theta}$ of the error vector of the end position of the linkage transmission mechanism are only affected by the rotation errors of their respective direction axes, and the sensitivities of the relevant influencing factors are equal.

According to the actual assembly, the empirical value is consistent with the trend of theoretical calculation and analysis. Therefore, by controlling the geometric error of parts with a large sensitivity coefficient and the accuracy of the assembly position, it can be used as the basis for the design of parts and systems.

4. Conclusion
Because the multi-link mechanism has the properties of restraint and coupling, its kinematic accuracy and sensitivity are the key indicators, and the assembly accuracy is the key to affect the kinematic accuracy. The manufacturing geometric error that affects the assembly accuracy can’t be ignored, just as the influence of the axial error on the position accuracy of the multi-link mechanism frame.

(1) Aiming at the lack of quantitative analysis of the mapping relationship between the machining error and the geometric error transfer in the assembly process which affect the assembly accuracy of the multi-link mechanism, this study introduces the small displacement torsor method to describe the error change of the geometric elements, and puts forward the error transfer relationship of the assembly joint surface of the multi-link mechanism by using the homogeneous transformation matrix. The error model of multi-link assembly with errors is established, and the component of error change in the actual position and pose of the end position frame center of the link due to the accumulation of coaxiality errors of multiple parts is calculated. Quantitative expression of manufacturing geometric error, as the relationship between axis error and assembly accuracy.

(2) The small error variation at the end position frame of multi-link transmission mechanism is caused by accumulation of coaxiality error. The sensitivity coefficients are obtained, by solving the partial differential equation of the former relationship of the small error variation and coaxiality error, to predict the influence of coaxiality error at each position on the end position of the system. The calculation method is efficient and practical, providing a theoretical basis for the precision design of the multi-link transmission mechanism.

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