A Design Method for Labyrinth Sound Absorption Structure with Micro-perforated Plates

Dequn Yu¹, Xiaoming Wang¹ and Yulin Mei²

¹School of Mechanical Engineering, Dalian University of Technology, 116024 Dalian, China,
²School of Automotive Engineering, Dalian University of Technology, 116024 Dalian, China
meiyulin@dlut.edu.cn

Abstract. A kind of labyrinth sound absorption structure is constructed by embedding multi-layer micro-perforated plates into a pipeline, and a design method is proposed to optimize its parameters to ensure a good sound absorption performance. First, the effects of parameters of the labyrinth structure on the sound absorption performance are investigated. Second, a parameter optimization method is constructed based on the acoustic impedance transfer analysis and particle swarm optimization algorithm. Then, a simulation experiment is implemented by means of COMSOL to verify the optimization results. Finally, the labyrinth structures with straight pipeline, spiral pipeline and parallel pipeline are designed and their sound absorption performances are evaluated and compared. The research shows that, the effects of the perforation diameter, the plate thickness, and the perforation rate on the sound absorption performance are non-independent while the effect of the depth of cavity is independent; the weighting coefficient in the parameter optimization method could be adjusted to ensure a big absorption bandwidth in a low frequency range; in the frequency range of 30-4000 Hz, the labyrinth structure with straight pipeline and the labyrinth structure with spiral pipeline have the same sound absorption performance; for the labyrinth structure with parallel pipeline, the inner corner radius of pipeline has a great influence on the sound absorption performance, and the radius should be greater than 20mm.

1. Introduction
The micro-perforated plate absorber (MPA) was proposed by Maa for the application under severe conditions, such as high temperature, humidity and corrosion. Compared with other sound absorbing structures, the MPA has a wide range of applications, because its sound absorbing characteristics can be calculated accurately and it is relatively easy to design and manufacture.

In 1975, Maa [1] discussed the basic principle of micro-perforation plate in detail. In 1988, Maa [2] proposed an accurate method for calculating the sound absorption coefficient of micro-perforated plates. In 1997, Maa [3] further modified and improved the theoretical model through comparative analysis of experimental data and theoretical model. Zhao et al. [4] studied the effect of mechanical impedance on the absorption coefficient of the micro-perforation plates. Toyoda et al. [5] investigated the relationship between different boundary conditions and the sound absorption coefficient. These researches improved the average sound absorption coefficient of single-layer micro-perforated plate.

To further improve the sound absorption performance of the MPA in low-frequency field, scholars also studied the calculation and optimization methods for sound absorption coefficient of multi-layer
micro-perforated plates. Zhao et al. [6] used the genetic algorithm to optimize the sound absorption performance of multi-layer micro-perforated plates and realized the broadband absorption in the medium and high frequency range. Zhao et al. [7] compared different methods for calculating the sound absorption coefficient of multi-layer micro-perforated plates. Lee et al. [8] designed a multi-layer sound absorption structure composed of thin plates, micro-perforated plates and porous materials, and evaluated its sound absorption performance through theoretical analysis and experiment. Kim et al. [9] studied the influence of different boundary conditions on the sound absorption performance of multi-layer micro-perforated plates. Due to the good application prospects of the MPA, new multi-layer micro-perforated plate structures are constantly emerging [10].

In this paper, a kind of labyrinth sound absorption structure is constructed by embedding multi-layer micro-perforated plates into a pipeline, and the parameter design method for the labyrinth structure is proposed and verified. First, the effects of parameters of the labyrinth structure on its sound absorption performance are investigated. Second, based on the acoustic impedance transfer analysis and particle swarm optimization algorithm, an optimization method is proposed to design structure parameters, including the perforation diameter, the plate thickness, the perforation rate and the cavity depth. Then, a simulation experiment is implemented by means of COMSOL to verify the optimization results. Finally, the labyrinth structures with spiral pipeline and parallel pipeline are designed, and their sound absorption performances are compared with that of the labyrinth structure with straight pipeline.

2. Calculation model of sound absorption coefficient of micro-perforated plates

In this section, the calculation model is established to evaluate the acoustic absorption coefficient of a multi-layer micro-perforated plate structure [9]. The structure consists of n-layer micro-perforated plates and a tube. The micro-perforated plates are installed in the pipeline of the tube, as shown in Figure 1. The micro-perforated plate is rectangular with the edge lengths of $L_1 = L_2 = 50 \text{mm}$, and its thickness is $t_i$ (in mm). Circular perforations distribute evenly on the plate, and the diameter of perforation and perforation rate are $d_i$ (in mm) and $p_i$. $D_i$ (in mm) stands for the distance from the ith-layer plate to the (i+1)th-layer, describing the depth of cavity corresponding to the ith-layer plate. It is assumed that the micro-perforated plates and the tube are made of steel and a plane wave propagates in the structure. The inner wall of the tube and the bottom back plate are considered to be hard sound field boundaries, namely, all incident sound waves are reflected.

![Figure 1. Plane wave propagation in the n-layer micro-perforated plate structure.](image)

According to the multi-layer micro-perforated plate theory, the relative acoustic impedance $z_i$ of the ith-layer micro-perforated plate can be calculated through equations (1) to (5) [1-3]:

$$z_i = r_i + jw m_i$$

where, $j = \sqrt{-1}$; $w$ is the angular frequency of the incident plane wave; $r_i$ and $m_i$ are the relative acoustic resistance and the relative acoustic mass of the ith-layer micro-perforated plate. Under the
normal atmospheric pressure, it is assumed that the sound velocity is \( c = 340 \text{m/s} \) and the kinematic viscosity coefficient of air is \( \mu = 1.56 \times 10^{-5} \text{m}^2/\text{s} \). Then, \( r_i \) and \( m_i \) can be obtained by:

\[
 r_i = \frac{0.147 \cdot \frac{t_i}{d_i^2}}{p_i} \\
 m_i = 0.294 \times 10^{-3} \times \frac{t_i}{p_i} \cdot k_{m,i}
\]

where, \( k_{r,i} \) and \( k_{m,i} \) respectively represent the acoustic resistance constant and the sound mass constant of the \( i \)-th layer, satisfying

\[
 k_{r,i} = (1 + \frac{x_i^2}{32})^{-1/2} + \frac{2^{1/2} x_i d_i}{8 t_i}
\]

\[
 k_{m,i} = 1 + (9 + \frac{x_i^2}{2})^{-1/2} + 0.85 \frac{a}{t}
\]

where, \( x_i = d_i \times (0.1 \times f)^{1/2} \), and \( f \) represents the vibration frequency of incident plane wave.

According to the impedance transfer analysis method of acoustic propagation, the sound absorption coefficient of the \( n \)-layer micro-perforated plate structure [9] can be obtained by:

\[
a = 1 - \left[(Z_1 - 1)/(Z_1 + 1)\right]^2
\]

where, \( Z_i \) is the equivalent impedance of the structure consisting of the \( n \)-layer micro-perforated plates, and can be calculated iteratively from the \( n \)-th layer to the 1st layer plate.

For a single-layer micro-perforated plate structure, namely, when \( n = 1 \), we have:

\[
 Z_{n} = z_{n} - j \cot(k D_{n})
\]

where \( k = 2\pi f c^{-1} \).

If the number of layers continues growing, the equivalent impedance of the structure consisting of \( n \)-layer micro-perforated plates can be figured out from that of the structure consisting of \((n-1)\)-layer micro-perforated plates:

\[
 Z_{n-1} = z_{n-1} - j \cot(k D_{n-1}) + \frac{1 + \cot^2(k D_{n-1})}{Z_n - i \cot(k D_{n})} \\
 n \geq 3
\]

3. Parameter optimization

3.1. Analysis and optimization of the single-layer micro-perforated plate

The sound absorption performance of the MPA is mainly affected by the perforation diameter \( d_i \), the plate thickness \( t_i \), the perforation rate \( p_i \) and the cavity depth \( D_i \). In this section, the effects of these parameters on the sound absorption performance of single-layer micro-perforated plate are analyzed.

By considering the perforation rate \( p_i \) and the cavity depth \( D_i \) to be constant, the variations of the sound absorption coefficient and absorption bandwidth with the perforation diameter \( d_i \) and the plate thickness \( t_i \) are investigated. Set \( p_i = 2\% \) and \( D_i = 50 \text{mm} \). The numerical analysis results are shown in Figure 2 and Table 1, where \( t_i \) is set as 0.2mm, 0.4mm, 0.6mm, 0.8mm and 1.0mm,
respectively. Figure 2 illustrates when the sound absorption coefficient is greater than 0.9, an optimal match is the plate thickness $t_i = 0.2 \text{mm}$ corresponding to the perforation diameter $d_i = 0.12 \text{mm}$, and the other is $t_i = 0.8 \text{mm}$ corresponding to $d_i = 0.24 \text{mm}$. Table 1 gives the plate thickness $t_i$, the perforation diameter $d_i$, the absorption frequency range, the absorption bandwidth and the absorption frequency at peak. Within the absorption frequency ranges shown in Table 1, the sound absorption coefficient of single-layer micro-perforated plate is greater than 0.9. Table 1 shows that, when the perforation rate $p_i$ and the cavity depth $D_i$ keep constant, as the plate thickness increases, the matching diameter increases, resulting in the decrease of the sound absorption frequency and absorption bandwidth.

Table 1. Sound absorption performance of the MPA with different plate thicknesses.

| Thickness of the plate (mm) | Diameter of the hole (mm) | Absorption frequency range (Hz) | Bandwidth (Octave) | Frequency at peak (Hz) |
|----------------------------|---------------------------|--------------------------------|--------------------|------------------------|
| 0.2                        | 0.12                      | 861-1813                       | 2.11               | 1263                   |
| 0.4                        | 0.17                      | 769-1486                       | 1.93               | 1069                   |
| 0.6                        | 0.21                      | 705-1277                       | 1.81               | 946                    |
| 0.8                        | 0.24                      | 656-1132                       | 1.73               | 858                    |
| 1.0                        | 0.27                      | 618-1025                       | 1.66               | 793                    |

By means of the numerical analysis results in Table 1, Figure 3 is given to further analyze the effect of the thickness $t_i$ and the diameter $d_i$ on the absorption bandwidth. It could be found that the perforation diameter $d_i$ is basically proportional to the plate thickness $t_i$, and is smaller than $t_i$; as the plate thickness $t_i$ decreases, the ratio of $d_i$ to $t_i$ increases and the corresponding sound absorption bandwidth increases.

Figure 3. Relationship between thickness, diameter and bandwidth.
In order to explore the variation of sound absorption coefficient with the perforation diameter $d_i$ and the perforation rate $p_i$, the numerical analysis is implemented by considering the plate thickness $t_i$ to be constant and respectively setting $p_i$ as 1%, 2%, 4%, 6% and 8%. The results are given in Figure 4 and Table 2. Figure 4 illustrates that, when the sound absorption coefficient is greater than 0.9, an optimal match is the perforation rate $p_i = 1\%$ corresponding to the perforation diameter $d_i = 0.38 \text{mm}$, and the other is $p_i = 4\%$ corresponding to $d_i = 0.16 \text{mm}$. Within the absorption frequency ranges shown in Table 2, the sound absorption coefficient is greater than 0.9. Table 2 shows that, when the plate thickness $t_i$ keeps constant, as the perforation rate $p_i$ increases, the matching diameter $d_i$ decreases, and the corresponding absorption frequency and absorption bandwidth increase.

Table 2. Sound absorption performance of the MPA with different perforation rates.

| Perforation rate (%) | Diameter of the hole (mm) | Absorption frequency range (Hz) | Bandwidth (Octave) | Diameter-depth ratio of the hole |
|----------------------|----------------------------|--------------------------------|-------------------|--------------------------------|
| 1                    | 0.38                       | 513-765                        | 1.49              | 0.745                          |
| 2                    | 0.24                       | 656-1132                       | 1.73              | 0.3                             |
| 4                    | 0.16                       | 789-1552                       | 1.97              | 0.2                             |
| 6                    | 0.13                       | 853-1783                       | 2.05              | 0.1625                         |
| 8                    | 0.11                       | 893-1917                       | 2.15              | 0.1325                         |

By means of the numerical analysis results in Table 2, Figure 5 gives the relationship between the perforation rate $p_i$, the diameter $d_i$ and the absorption bandwidth. It could be found that when the thickness $t_i$ is constant, $p_i$ is negatively correlated with $d_i$, and the big perforation rate and the small perforation diameter can result in a big absorption bandwidth.

Figure 4. Variation of sound absorption coefficient with $d_i$ and $p_i$.

Figure 5. Relationship between perforation rate, diameter and bandwidth.
The effect of the cavity depth on the sound absorption performance of the MPA is relatively independent. The numerical analysis is implemented by considering the plate thickness $t_i$ to be constant and respectively setting the cavity depth $D_i$ as 50mm, 100mm and 150mm, and the results are shown in Table 3. Within the absorption frequency ranges shown in Table 3, the sound absorption coefficient is greater than 0.9. It could be found that when the cavity depth $D_i$ increases, the matching diameter changes little, resulting in a big bandwidth in the low frequency range.

Table 3. Sound absorption performance of the MPA with different cavity depths.

| Cavity depth (mm) | Diameter of the hole (mm) | Absorption frequency range (Hz) | Bandwidth (Octave) |
|------------------|--------------------------|--------------------------------|-------------------|
| 50               | 0.24                     | 656-1132                       | 1.73              |
| 100              | 0.24                     | 390-763                        | 1.96              |
| 150              | 0.23                     | 281-582                        | 2.07              |

In summary, the effects of the perforation diameter, the plate thickness, and the perforation rate on sound absorption performance are non-independent, and these parameters could be optimized and matched to maximize the absorption bandwidth; the effect of the cavity depth on the sound absorption performance is independent, and a deep cavity can result in a big bandwidth in a low absorption frequency range.

3.2. Analysis and optimization of the multi-layer micro-perforated plates.

In this section, a sound absorption structure consisting of multi-layer micro-perforated plates is designed, and the absorption frequency range is set as 30Hz-4000Hz. In combination with actual needs and the numerical analysis results of the single-layer micro-perforated plate, the design parameters of the structure are optimized within the following ranges.

$$0.1 \leq d_i \leq 1, \quad 0.1 \leq t_i \leq 2, \quad 0.1 \leq p_i \leq 10, \quad 10 \leq D_i \leq 300 \quad i = 1, 2, \ldots, n$$ (9)

where, the unit of perforation diameter, plate thickness and cavity depth is mm, and the perforation rate is $p_i \%$. Because the sound absorption coefficient does not increase significantly when the number of layers of micro-perforated plates is bigger than 10, the number $n$ is chosen as 10. A labyrinth sound absorption structure is constructed by embedding 10-layer micro-perforated plates into a straight pipeline, as shown in Figure 1. By means of particle swarm optimization algorithm, the objective function can be written as:

$$f(x) = -\left(b_1 \cdot \sum_{i=1}^{n} a_i \cdot a_{\text{min}} - b_2 \cdot \sum_{i=1}^{n} l \right)$$ (10)

where, $a_i$ is the sound absorption coefficient; $a_{\text{min}}$ denotes the minimum sound absorption coefficient within the frequency range of 30-4000 Hz; $b_1$ and $b_2$ are weighting coefficients. The frequency step is 2.

During the optimization procedure, the 10-layer micro-perforated plates are divided into three groups, and each group has the same design parameters. By setting $b_1 = 1$, $b_2 = 1/20$, acceleration constants $c_1 = c_2 = 1.6$, the inertia factor $w = 0.7$, an iterative procedure is implemented to optimize the structure parameters. The number of population particles is 2048, and the number of iteration is 1000. Table 4 gives the optimal parameters, including the perforation diameter, the plate thickness, the perforation rate and the cavity depth; Figure 6 shows the iteration procedure; Figure 7 illustrates the sound absorption coefficient of the labyrinth structure with these optimal parameters. The iteration converges around the 500th step, and the sound absorption coefficient of the optimized labyrinth structure is bigger than 0.9 in 30Hz-4000Hz.
Table 4. Optimal parameters of the 10-layer micro-perforated plates.

| Layers | Diameter of the hole (mm) | Thickness of plate (mm) | Perforation rate (%) | Cavity depth (mm) |
|--------|---------------------------|-------------------------|----------------------|------------------|
| 1-4    | 0.1                       | 0.1                     | 10                   | 35.6             |
| 5-8    | 0.1                       | 0.1                     | 5.94                 | 300              |
| 9-10   | 0.1                       | 0.1                     | 1.71                 | 268.9            |

Figure 6. Iterative procedure.

Figure 7. Sound absorption coefficient.

In order to investigate the influence of the weighting coefficient on the optimization, the iteration is implemented by respectively setting $b_2 = 1/10$ and $1/30$, as shown in Figure 8 and Table 5. Table 5 shows that when the weighting coefficient $b_2$ is smaller than $1/20$, the optimization result is better, resulting in a big absorption bandwidth and a short length of the structure.
Table 5. Influence of the weighting coefficient on the optimization.

| $b_2$ | Bandwidth (Octave) | Length of the structure (mm) | $\beta_{\text{min}}$ |
|-------|--------------------|-----------------------------|---------------------|
| 1/10  | 9                  | 1808                        | 0.89                |
| 1/20  | 132                | 1880                        | 0.9                 |
| 1/30  | 132                | 1870                        | 0.9                 |

4. Simulation experiment and labyrinth structures

A simulation experiment is designed to measure the sound absorption coefficient of the labyrinth structure constructed by embedding the multi-layer micro-perforated plates into a pipeline. The data measured by the simulation experiment will be compared with the numerical analysis results calculated in section 3.2 to verify the parameter optimization method proposed in section 3.

COMSOL is used to simulate an impedance tube shown in Figure 9 to measure the sound absorption coefficient by the transfer function method. $P_i$ is the incident wave from the left side; $P_r$ is the reflected wave; sensor 1 is located at $x_1 = 30\text{mm}$; sensor 2 is located at $x_2 = 20\text{mm}$; the blue part is the test sample.

![Figure 9. Schematic diagram of an impedance tube.](image)

The sound absorption coefficient of the test sample can be obtained by:

$$\alpha = 1 - |r|^2$$  \hspace{1cm} (11)

where, $r$ is the reflection coefficient, and can be calculated by:

$$r = \frac{H_{12} - H_{1}e^{j k_0 x_1}}{H_{2} - H_{12}}$$  \hspace{1cm} (12)

where, $H_1$, $H_2$, and $H_{12}$ are the transfer functions of incident wave, reflected wave and total sound field, respectively; $k_0 = 2\pi f/c$ is the wave number. Using $p_1$ and $p_2$ to stand for the sound pressure at sensor 1 and sensor 2, we have

$$H_1 = e^{-jk_0 x_1}, \quad H_2 = e^{jk_0 x_2}, \quad H_{12} = \frac{p_2}{p_1}$$  \hspace{1cm} (13)

The test sample is the labyrinth structure constructed by embedding 10-layer micro-perforated into a straight pipeline, as shown in Figure 1, and the structure parameters are the same as those in Table 4. The sound absorption performance of the labyrinth structure with straight pipeline is measured by the simulation experiment. The average pressure of the measuring plane is calculated, and the sound pressure map is illustrated in Figure 10. Figure 11 shows the data measured by the simulation experiment and the numerical analysis results calculated in section 3.2. It could be found that these results are consistent, verifying the parameter optimization method in section 3.
In order to explore the influence of the shape of pipeline on the sound absorption performance of the labyrinth structure, labyrinth structures with spiral pipeline and parallel pipeline are constructed.

By taking the spiral curve \( r(\theta) = 30 \times \pi^{-1} \times \theta \) as a reference line, the labyrinth structure with spiral pipeline is designed, as shown in Figure 12. The pipe width is 50mm; the helix angle ranges from 1.58\( \pi \) to 6.5\( \pi \); the first micro-perforated plate is located at 6.5\( \pi \); the incident plane wave and two sensors are located at 50mm, 30mm, and 20mm from the left side of the first micro-perforated plate.

Figure 13 shows that the sound absorption coefficient of the spiral-pipeline labyrinth structure is basically the same as that of the straight-pipeline labyrinth structure within the frequency range of 30-4000 Hz, but when the frequency is above 4000Hz, the sound absorption coefficient deviates greatly from that of the straight pipe. On the one hand, for the spiral-pipeline labyrinth structure shown in Figure 12, the different lengths between the inner and outer walls and the variable depth of cavity with the radius cause the deviation. On the other hand, as the frequency of the incident wave increases, the wavelength will decrease and the decreasing wavelength will cause an increasing deviation of the sound absorption coefficient.

The labyrinth structures with parallel pipeline are designed, as shown in Figures 14-16. In order to investigate the effect of inside corner radius of pipeline on the sound absorption coefficient, the radius is set as 50mm, 20mm or 10mm, and the other design parameters are same. Simulation results are given in Figures 14-17 and Table 6. Figure 17 shows, when the inside corner radius is 10mm, the...
sound absorption performance of the structure degrades at 550Hz and 3220Hz. So, the inside corner radius of pipeline should be greater than 20mm to ensure a sound absorption coefficient larger than 0.9.

The parallel labyrinth structure with straight corners is also studied, as shown in Figure 18. The labyrinth pipe is installed in a rectangular plate of 370 * 420mm², and the distance between the adjacent pipes is 20mm. Figure 19 shows that the straight corner degrades the sound absorption performance greatly, especially at the frequencies of 600Hz and above 2000Hz. The reason is that the plane wave is reflected by the straight corner, as shown in Figure 18.

Figure 14. Labyrinth structure with a radius of 50mm and sound pressure map (30-4000Hz).

Figure 15. Labyrinth structure with a radius of 20mm and sound pressure map (100-4500Hz).

Figure 16. Labyrinth structure with a radius of 10mm and sound pressure map (550-3200Hz).

Figure 17. Sound absorption coefficient.
Table 6. Effect of inside corners on sound absorption performance.

| Inside corner radius (mm) | Outer corner radius (mm) | Average error | Maximum error | Structure size (mm²) |
|---------------------------|--------------------------|---------------|---------------|---------------------|
| 10                        | 60                       | 0.015         | 0.101         | 370 * 420           |
| 20                        | 70                       | 0.014         | 0.097         | 410 * 430           |
| 50                        | 100                      | 0.010         | 0.090         | 500 * 520           |

Figure 18. Labyrinth structure with straight corner and sound pressure map (600-2500Hz).

Figure 19. Sound absorption coefficient.

5. Summary
A kind of labyrinth sound absorption structure is constructed by embedding multi-layer micro-perforated plates into a pipeline. Based on the acoustic impedance transfer analysis and particle swarm optimization algorithm, the optimization method is proposed to design the structure parameters, and the optimization results are verified through the finite element analysis. The following conclusions are drawn:

(1) The effects of the perforation diameter, the plate thickness and the perforation rate on the sound absorption performance are non-independent, and these parameters could be optimized and matched to maximize the absorption bandwidth;

(2) The effect of the depth of cavity on the sound absorption performance is independent, and a deep cavity can result in a big bandwidth in a low frequency range;

(3) The weighting coefficient in the parameter optimization method could be adjusted to optimize the sound absorption performance, including a big absorption bandwidth and a small length of the structure;

(4) In the frequency range of 30-4000 Hz, the labyrinth structure with straight pipeline and the labyrinth structure with spiral pipeline have the same sound absorption performance;

(5) For the labyrinth structure with parallel pipeline, the inner corner radius of pipeline should be greater than 20mm, and straight corners could greatly degrade the sound absorption performance.

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