A solution to the baryon and dark-matter coincidence puzzle in a $\tilde{N}$ dominated early universe

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Abstract

If a bosonic partner of a right-handed neutrino dominates the early universe sufficiently before its decay, important ingredients in the present universe are related to physics of the right-handed neutrino sector. In particular, we find that the ratio of the baryon to the dark-matter densities is given only by low-energy parameters such as a neutrino mass and a gravitino mass if the reheating temperature of inflation is much higher than $10^{12}$ GeV. Here, the gravitino is assumed to be the lightest supersymmetric particle and the dominant component of the dark matter. The observed ratio, $\Omega_B/\Omega_{DM} \simeq 0.21 \pm 0.04$, suggests the mass of the gravitino to be in the range of $\mathcal{O}(10)$ MeV provided the CP violating phase is of the order 1.
1 Introduction

The seesaw mechanism [1] is very attractive, since it explains naturally not only the observed small neutrino masses but also the baryon asymmetry in the present universe [2]. The important ingredient in the seesaw mechanism is the presence of right-handed neutrinos $N_i (i = 1 - 3)$ whose Majorana masses $M_i$ are very large such as $M_i \simeq 10^{9-15}$ GeV. In a supersymmetric (SUSY) extension of the seesaw mechanism the right-handed neutrinos $N_i$ are necessarily accompanied with SUSY-partner bosons $\tilde{N}_i$ (right-handed sneutrinos), and it is quite plausible [3] that the $\tilde{N}_i$ have very large classical values during inflation if the masses of the right-handed neutrinos are smaller than the Hubble constant of the inflation. If it is the case and coherent oscillations of the bosons $\tilde{N}_i$ dominate the early universe sufficiently before their decays, some of important parameters in the present universe are determined by the physics of the right-handed neutrino sector. In this letter, we point out that if a boson partner of a right-handed neutrino $N_1$ dominate once the early universe it may solve the coincidence puzzle of the baryon and dark-matter densities provided that the mass of gravitino is $O(10)$ MeV.

Before discussing the physics of $\tilde{N}_1$ we should note a generic problem in supergravity, that is the gravitino problem [4]. If the gravitino is unstable, it has a long lifetime and decays during or after the big-bang nucleosynthesis (BBN). The decay products destroy the light elements created by the BBN and hence the abundance of the relic gravitino is constrained from above. This leads to an upper bound of the reheating temperature $T_R$ of inflation. The recent detailed analysis [5] shows a stringent upper bound such as $T_R < 10^4$ GeV for the gravitino having hadronic decay modes. In the present scenario this reheating temperature means the temperature just after the decay of the coherent $\tilde{N}$ oscillation (i.e. the decay temperature $T_d$). Such a low decay temperature is nothing unnatural in the scenario, but the produced lepton (baryon) asymmetry is too small [3].

A solution to this gravitino problem is to assume that the gravitino is the lightest SUSY particle (LSP) and hence stable [6]. This solution is very interesting in the present scenario, since the ratio of $\Omega_B$ to $\Omega_{3/2}$ is independent of the unknown temperature $T_d$, but it is given by only low-energy parameters if the reheating temperature of inflation is sufficiently high. Here, $\Omega_B$ and $\Omega_{3/2}$ are mass density parameters of the baryon and the
gravitino, respectively. We find that the ratio is determined by masses of a neutrino, the gluino and the gravitino and an effective CP-violating phase (as shown in Eq. (20)). The observation \( \Omega_B/\Omega_{DM} \simeq 0.21 \pm 0.04 \) \(^7\) suggests \( m_{3/2} = \mathcal{O}(10) \) MeV. \( m_{3/2} \) is the mass of the gravitino.) Here, we have assumed that the gravitino is the dominant component of the cold dark matter, that is \( \Omega_{DM} \simeq \Omega_{3/2} \). The gravitino of mass in the range of \( \mathcal{O}(10) \) MeV will be testable in future experiments as discussed in Ref. \(^8\).

2 Matter from a coherent right-handed sneutrino

2.1 Baryon asymmetry from a coherent right-handed sneutrino

We consider a framework of the minimal supersymmetric standard model (MSSM) with three generations of heavy right-handed neutrinos \( N_i \) \( (i = 1 - 3) \). The \( N_i \) couple to the MSSM particles through a superpotential,

\[
W = \frac{1}{2} M_i N_i N_i + h_{i\alpha} L_{\alpha} H_u N_i,
\]

where \( M_i \) denote masses of the right-handed neutrinos and \( L_{\alpha} \) \( (\alpha = e, \mu, \tau) \) and \( H_u \) are the supermultiplets of lepton doublets and a Higgs doublet which couples to up-type quarks. The small left-handed neutrino masses are obtained via the seesaw mechanism \(^1\).

The right-handed sneutrinos may have large classical values during inflation if their effective masses are smaller than the Hubble parameter \( H_{\text{inf}} \) \(^3\). Hereafter, we restrict our discussion to the lightest right-handed sneutrino \( \tilde{N}_1 \), for simplicity, and treat the amplitude \( \tilde{N}_1^{\text{init}} \) during the inflation as a free parameter.

After the end of the inflation, the Hubble parameter \( H \) decreases and the \( \tilde{N}_1 \) starts to oscillate when \( H \) becomes smaller than its mass \( M_1 \).\(^1\) The coherent oscillation of the \( \tilde{N}_1 \) decays into \( L\tilde{H}_u \) or \( \tilde{L}H_u \) and their CP-conjugates when \( H \simeq \Gamma_{N_1} \), where \( \Gamma_{N_1} \simeq (1/4\pi) \sum_{\alpha} |h_{1\alpha}|^2 M_1 \) is the decay rate of the \( \tilde{N}_1 \). The decay produces the lepton number density as, \( n_L = \epsilon \times n_{\tilde{N}_1} \), where \( n_{\tilde{N}_1} \) is the number density of the \( \tilde{N}_1 \) at the decay time, and \( \epsilon \) is the lepton asymmetry produced in the \( \tilde{N}_1 \) decay. Assuming \( M_1 \ll M_2, M_3 \), the

\(^1\)We assume the potential for the \( \tilde{N}_1 \) is given by a mass term, \( V = M_1^2 |\tilde{N}_1|^2 \). We discuss the validity of this simplification of the potential for analyzing the dynamics of the \( \tilde{N}_1 \) in the next section.
explicit form of $\epsilon$ is given by \cite{9,10}

$$
\epsilon \simeq (1 - 2) \times 10^{-10} \left( \frac{M_1}{10^6 \text{GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{eV}} \right) \sin \delta_{\text{eff}},
$$

\begin{equation}
(2)
\end{equation}

where $\delta_{\text{eff}}$ is an effective CP violating phase and $m_{\nu_3}$ corresponds to the heaviest neutrino mass, we have used $\langle H_u \rangle = 174 \text{ GeV} \times \sin \beta$, assuming $\sin \beta \simeq 1/\sqrt{2} - 1$.

When the $\tilde{N}_1$ dominates the universe, we can write the energy density ($\rho$) and the entropy density ($s$) of the universe at the decay time as

$$
\rho \simeq M_1^2 |\tilde{N}_1^{\text{decay}}|^2 \simeq \frac{\pi^2}{30} g_\ast(T_d) T_d^4 \simeq 3 M_{\text{pl}}^2 \Gamma_{\tilde{N}_1}^2, \quad (3)
$$

$$
s \simeq \frac{2\pi^2}{45} g_\ast(T_d) T_d^3.
$$

Here, $T_d$ is the temperature of radiation right after the $\tilde{N}_1$ decay, $g_\ast$ the number of effective degrees of freedom which is 230 for the temperature $T \gg 1 \text{ TeV}$ in the MSSM and $M_{\text{pl}} \simeq 2.4 \times 10^{18} \text{ GeV}$ the reduced Planck scale. In the above equation, we have assumed instantaneous decay of the $\tilde{N}_1$ and used the energy conservation. Hereafter, we only focus on the scenario in which the $\tilde{N}_1$ domination is the case.

The resultant lepton number is converted to the baryon-number asymmetry \cite{2}, which is given by \cite{3,11}

$$
\frac{n_B}{n_\gamma} = -\frac{8}{23} \left( \frac{n_L}{n_\gamma} \right) = -\frac{8}{23} \left( \frac{n_L}{\rho} \right) \left( \frac{\rho}{s} \right) \left( \frac{s}{n_\gamma} \right) = -\frac{8}{23} \left( \frac{\epsilon}{M_1} \right) \left( \frac{3T_d}{4} \right) \left( \frac{s}{n_\gamma} \right),
$$

\begin{equation}
(4)
\end{equation}

where we have used $s/n_\gamma \simeq 7.04$ at the present and Eqs. \cite{3} in the last equation. We take $m_{\nu_3} \simeq 0.05 \text{ eV}$ as suggested from the atmospheric neutrino oscillation and assume $\sin \delta_{\text{eff}} \simeq 1$. Then, the observed baryon asymmetry $n_B/n_\gamma = 6.5^{+0.4}_{-0.3} \times 10^{-10}$ \cite{7} implies

$$
T_d \simeq 10^6 \text{ GeV} - 10^7 \text{ GeV}.
$$

Before closing this subsection, we should mention washout effects of the lepton asymmetry. When the decay temperature of the $\tilde{N}_1$ is close to its mass, $T_d \simeq M_1$, the produced lepton-number asymmetry is washed out by lepton-number violating interactions mediated by $N_1$. Thus, in order to avoid the washout effect, we require $T_d < M_1$, and this
condition is rewritten by using Yukawa coupling constants in Eq. (1) as (11),

\[ \left( \sum_{\alpha} |h_{1\alpha}|^2 \right)^{\frac{1}{2}} \simeq 5 \times 10^{-6} \left( \frac{T_d}{10^6 \text{ GeV}} \right)^{\frac{1}{2}} \left( \frac{T_d}{M_1} \right)^{\frac{1}{2}} < 5 \times 10^{-6} \left( \frac{T_d}{10^6 \text{ GeV}} \right)^{\frac{1}{2}}. \] (6)

Here, we have used Eq. (3) to relate \( M_1 \) and \( T_d \). We require the Yukawa couplings \( h_{1\alpha} \) to be as small as the Higgs coupling to the electron. We may explain naturally such small Yukawa coupling constants by a spontaneously broken discrete \( Z_6 \) flavor symmetry (11, 12).

### 2.2 Conditions for \( \tilde{N} \) domination

In the previous subsection, we consider the \( \tilde{N}_1 \) to dominate the energy density of the early universe. We discuss, here, conditions for the \( \tilde{N} \) domination.

We classify the history of the energy density of the early universe by the reheating temperature \( T_R \) of inflation, the initial amplitude \( |\tilde{N}_{1\text{init}}| \) and the decay temperature \( T_d \) of the right-handed sneutrino.\(^2\) If the Hubble parameter at the end of the reheating process of inflation is smaller than \( M_1 \), the \( \tilde{N}_1 \) starts to oscillate around its minimum before the end of the reheating. The domination of the \( \tilde{N}_1 \) starts at a temperature \( T_{\text{dom}} \) which is estimated as

\[ T_{\text{dom}} \simeq T_R \times \left( \frac{|\tilde{N}_{1\text{init}}|^2}{3M_{\text{pl}}^2} \right). \] (7)

Thus, a condition for the domination of the \( \tilde{N}_1 \) is

\[ T_{\text{dom}} \simeq T_R \times \left( \frac{|\tilde{N}_{1\text{init}}|^2}{3M_{\text{pl}}^2} \right) > T_d. \] (8)

On the other hand, if the Hubble parameter at the end of the reheating of inflation is larger than \( M_1 \), the \( \tilde{N}_1 \) starts to oscillate after the end of the reheating process. The temperature of the background radiation when the \( \tilde{N}_1 \) oscillation starts is given by

\[ T_{\text{osci}} \simeq \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{M_{\text{pl}}M_1}. \] (9)

In this case, the \( \tilde{N}_1 \) dominates the universe soon after it starts the oscillation. As in the previous case the temperature \( T_{\text{dom}} \) at which the domination of the \( \tilde{N}_1 \) begins is estimated

\(^2\)\( T_R \) is defined as a temperature of the radiation right after the end of the reheating process of inflation.
as

\[ T_{\text{dom}} \simeq T_{\text{osci}} \times \left( \frac{|\tilde{N}_1^{\text{init}}|^2}{3M_{\text{pl}}^2} \right). \]  

(10)

Thus, the condition for the \( \tilde{N}_1 \) to dominate the universe is,

\[ T_{\text{dom}} \simeq T_{\text{osci}} \times \left( \frac{|\tilde{N}_1^{\text{init}}|^2}{3M_{\text{pl}}^2} \right) > T_d. \]  

(11)

As we have seen in the previous subsection, we consider \( T_d \approx 10^6 \text{ GeV} - 10^7 \text{ GeV} \), and hence the above conditions Eqs. (8) or (11) can be satisfied for a wide range of the initial amplitude of the \( \tilde{N}_1 \), \( T_R \) and \( M_1 \).

2.3 Dark-matter genesis

As discussed in the introduction, we assume the gravitino to be the LSP and the dominant component of the cold dark-matter (CDM). As we see below, the relic gravitino density is proportional to the decay temperature of the \( \tilde{N}_1 \) if the gravitinos are produced mainly by the \( \tilde{N}_1 \) decay. Thus, the ratio between \( \Omega_B h^2 \) and \( \Omega_{3/2} h^2 \) becomes independent of the decay temperature \( T_d \) (see Eq. (11)) and is determined only by low-energy parameters.

However, the gravitino is forced into thermal equilibrium by the scattering process if the decay temperature \( T_d \) is sufficiently high. If it is the case, the density of the gravitino is not proportional to \( T_d \), making the above argument invalid. The freeze-out temperature of the gravitino from the thermal bath is given by [13]

\[ T_f \approx 10^9 \text{ GeV} \left( \frac{g_*(T_f)}{230} \right)^{1/2} \left( \frac{m_{3/2}}{10 \text{ MeV}} \right)^2 \left( \frac{1 \text{ TeV}}{m_{\text{gluino}}} \right)^2, \]  

(12)

where \( m_{\text{gluino}} \) denotes the mass of the gluino. We should note here that our conclusion does not change as long as \( T_d < T_f \). We check in the next subsection that this condition is satisfied.

On the other hand, when the reheating temperature \( T_R \) of inflation is higher than \( T_f \), the gravitino is kept in the thermal equilibrium and its resultant density is estimated as

\[ \Omega_{3/2} h^2 \approx 5.0 \times 10^3 \left( \frac{m_{3/2}}{10 \text{ MeV}} \right) \left( \frac{230}{g_*(T_f)} \right). \]  

(13)
If $T_R$ is lower than $T_f$, the gravitino cannot be in the thermal equilibrium and its resultant density is given by

$$\Omega_{3/2}h^2 \simeq 2.1 \times 10^3 \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left( \frac{10 \text{ MeV}}{m_{3/2}} \right) \left( \frac{m_{\text{gluino}}}{1 \text{TeV}} \right)^2. \quad (14)$$

However, the gravitino density from the reheating process of the inflation is diluted by entropy production from the $\tilde{N}_1$ decay. By assuming the instantaneous decays of the $\tilde{N}_1$, which is accurate enough for the present purpose, we obtain the dilution factor (from the energy conservation) as

$$\Delta \equiv \left( \frac{s_{\text{after}}}{s_{\text{before}}} \right) \simeq \frac{T_{\text{dom}}}{T_d} \simeq \begin{cases} \frac{T_R}{T_d} \left( \frac{|\tilde{N}_1^{\text{init}}|^2}{3M_{\text{pl}}^2} \right) & (T_R < T_{\text{osci}}), \\ \frac{T_{\text{osci}}}{T_d} \left( \frac{|\tilde{N}_1^{\text{init}}|^2}{3M_{\text{pl}}^2} \right) & (T_R > T_{\text{osci}}), \end{cases} \quad (15)$$

where we have used Eq. (8) and (11). As a result, the present gravitino density is written as

$$\Omega_{3/2}h^2 = \Omega_{3/2}(T_d)h^2 + \frac{1}{\Delta} \Omega_{3/2}(T_R)h^2, \quad (16)$$

where $\Omega_{3/2}(T)h^2$ denote the gravitino density in Eqs. (13) or (14) at each temperatures $T = T_d$ or $T_R$. The first term in Eq. (16) represents the density of the gravitino produced in the $\tilde{N}_1$ decay, while the second term is the resultant density of the gravitino produced in the reheating process of inflation.

### 2.4 A solution to the coincidence puzzle

As we have seen, the baryon asymmetry in the present universe comes from the $\tilde{N}_1$ decay, and the resultant baryon density $\Omega_Bh^2$ is given by

$$\Omega_Bh^2 \simeq (6.3 - 13) \times 10^{-3} \left( \frac{T_d}{10^6 \text{ GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right) \sin \delta_{\text{eff}}, \quad (17)$$

where we have used the proton mass $m_p \simeq 0.938 \text{ GeV}$. (See Eq. (14).) On the other hand, from Eq. (16) the dark matter (the gravitino LSP) density is written as

$$\Omega_{\text{DM}}h^2 \simeq 0.21 \times \left( \frac{T_d}{10^6 \text{ GeV}} \right) \left( \frac{10 \text{ MeV}}{m_{3/2}} \right) \left( \frac{m_{\text{gluino}}}{1 \text{TeV}} \right)^2 \times k_N, \quad (18)$$
where $k_N$ is defined as

\[
k_N = \begin{cases} 
  1 & + 0.2 \times \left( \frac{10^{12} \text{ GeV}}{T_R} \right) \left( \frac{m_{3/2}}{30 \text{ MeV}} \right)^2 \left( \frac{1 \text{ TeV}}{m_{\text{gluino}}} \right)^2 \left( \frac{3 M_{\text{pl}}^2}{|N_1^{\text{init}}|^2} \right) \quad (T_R > T_f, \ T_R < T_{osc}), \\
  1 & + 0.2 \times \left( \frac{10^{12} \text{ GeV}}{T_{osc}} \right) \left( \frac{m_{3/2}}{30 \text{ MeV}} \right)^2 \left( \frac{1 \text{ TeV}}{m_{\text{gluino}}} \right)^2 \left( \frac{3 M_{\text{pl}}^2}{|N_1^{\text{init}}|^2} \right) \quad (T_R > T_f, \ T_R > T_{osc}), \\
  1 & + \left( \frac{3 M_{\text{pl}}^2}{|N_1^{\text{init}}|^2} \right) \quad (T_R < T_f, \ T_R < T_{osc}), \\
  1 & + \left( \frac{T_R}{T_{osc}} \right) \left( \frac{3 M_{\text{pl}}^2}{|N_1^{\text{init}}|^2} \right) \quad (T_R < T_f, \ T_R > T_{osc}),
\end{cases}
\]

for each values of $T_R$, $T_f$ and $T_{osc}$.

For the third and the fourth cases in Eq. (19), the gravitino densities depend on the initial amplitudes of $\tilde{N}_1$. On the other hand, the second terms are negligible for the first and the second cases in Eq. (19) if the reheating temperature $T_R$ or the oscillation temperature $T_{osc}$ are much higher than $10^{12}$ GeV. (The model discussed in the next section gives most likely $|\tilde{N}_1^{\text{init}}| \simeq M_{\text{pl}}$.)

In Fig. 1 we plot the ratio $\Omega_B/\Omega_{DM}$ as a function of $T_{R,osc}$ for the first and the second cases in Eq. (19). We find that the ratio becomes independent of the $|\tilde{N}_1^{\text{init}}|$ and $T_{R,osc}$ for sufficiently high temperatures $T_{R,osc}$ and it is determined only by the low-energy parameters. In those regions, the ratios $\Omega_B/\Omega_{DM}$ are given by

\[
\frac{\Omega_B h^2}{\Omega_{DM} h^2} \simeq (0.1 - 0.2) \left( \frac{m_{3/2}}{30 \text{ MeV}} \right) \left( \frac{1 \text{ TeV}}{m_{\text{gluino}}} \right)^2 \left( \frac{m_{\nu 3}}{0.05 \text{ eV}} \right) \sin \delta_{\text{eff}}. \quad (20)
\]

Comparing Eq. (20) with the WMAP result $\Omega_B/\Omega_{DM} \simeq 0.21 \pm 0.04$ [7], we obtain the mass of the gravitino as

\[
m_{3/2} \simeq 30 \text{ MeV} - 60 \text{ MeV}, \quad \text{for } \sin \delta_{\text{eff}} \simeq 1, \ m_{\text{gluino}} \simeq 1 \text{ TeV}, \quad (21)
\]

which suggests a gauge mediation SUSY breaking (GMSB) [15]. Therefore, the coincidence puzzle between the baryon and the dark-matter densities can be naturally solved in the GMSB model when the both of the densities come dominantly from the $\tilde{N}_1$ decay.

Notice that we obtain $T_f \simeq 10^{10}$ GeV from Eq. (12) in the parameter region Eq. (21) and hence the condition $T_d < T_f$ discussed in the previous subsection is satisfied since $T_d \simeq 10^{6-7}$ GeV.
Finally, we comment on constraints from the Big Bang Nucleosynthesis (BBN). For the gravitino LSP scenario, the next to the lightest SUSY particle (NLSP) has a long lifetime and it may spoil the success of the BBN, in general. However, in our scenario of $m_{3/2} = O(10)$ MeV, the lifetime of the NLSP is sufficiently short as

$$\tau_{\text{NLSP}} \simeq 2 \times 10^{-2} \text{sec.} \left( \frac{m_{3/2}}{10 \text{ MeV}} \right)^2 \left( \frac{300 \text{ GeV}}{m_{\text{NLSP}}} \right)^5.$$ (22)

Thus, the NLSP can escape from the BBN constraints [16].

### 3 Some discussion

#### 3.1 A model for the right-handed neutrino sector

In the previous section, we have used a potential for the $\tilde{N}_1$, $V \simeq M_1^2 |\tilde{N}_1|^2$. However, if we assume the broken $U(1)_{B-L}$ gauge symmetry to generate the Majorana masses of $N_i$, the other MSSM fields are destabilized through a D-term potential of $U(1)_{B-L}$ during the $\tilde{N}_1$ oscillation. This problem is not present, if the gauge coupling constant of $U(1)_{B-L}$ is extremely small.
trace and hence it becomes difficult to predict the cosmic baryon asymmetry.\(^4\)

To avoid the above problem, we consider a model with \(U(1)_R \times Z_4^{B-L}\) symmetry whose charge assignments are given in Table 1\(^5\). Here, \(U(1)_R\) is the \(R\) symmetry. A simple superpotential allowed by the symmetry is

\[
W = yX(S^2 - v^2) - \frac{1}{2} f_i S N_i^2 + h_{i\alpha} L_\alpha H_u N_i + W_{\text{MSSM}},
\]

where we have added two MSSM singlets \(X\) and \(S\), \(y\) and \(f_i\) denote the Yukawa coupling constants, the parameter \(v\) the breaking scale of the \(Z_4^{B-L}\) symmetry, and \(W_{\text{MSSM}}\) the superpotential consists of the MSSM fields. As we see below, the evolution of the \(\tilde{N}_1\) can be analyzed by using the potential \(M_1^2|\tilde{N}_1|^2\) as long as the Hubble parameter during the inflation is much smaller than \(v \simeq 10^{15}\) GeV.

From the superpotential Eq. (23), the scalar potential which is relevant to the dynamics of the \(\tilde{N}_1\) is given by

\[
V = |y(\tilde{S}^2 - v^2)|^2 + |2y\tilde{S}\tilde{X} - f\frac{1}{2} \tilde{N}_1^2|^2 + |h\phi^2 - f\tilde{S}\tilde{N}_1|^2 + |h\tilde{N}_1\phi|^2, \quad (24)
\]

where \(\phi\) denotes the flat direction in the MSSM defined by \(H_u = 1/\sqrt{2}(0, \phi)^T\), \(\tilde{L} = 1/\sqrt{2}(\phi, 0)^T\), and we have omitted the flavor index from the Yukawa coupling constants for abbreviation.\(^6\) In the following discussion, we focus on the evolution of the \(\tilde{N}_1\), \(\tilde{X}\) and \(\tilde{S}\), assuming \(M_1 \ll H_{\text{inf}} \ll M_2, M_3\) and \(\phi = 0\). The dynamics of \(\phi\) is discussed in the next subsection, where we see that the thermal mass term sets \(\phi\) to the origin.

If \(H_{\text{inf}} \ll v\), \(\tilde{S}\) and \(\tilde{X}\) are fixed to their minima during inflation (we have required \(y\) be not too small). We also require \(f|\tilde{N}_1^{\text{init}}| \ll v\) not to destabilize the minimum of \(\tilde{S}\).\(^7\) Thus, the scalar fields are fixed in the end of inflation at

\[
\tilde{N}_1 = \tilde{N}_1^{\text{init}}, \quad \tilde{X} = \left(\frac{f}{4yv}\right)(\tilde{N}_1^{\text{init}})^2, \quad \tilde{S} = v.
\]

\(^4\)If \(B\) or \(L\) violating non-renormalizable terms exist in the MSSM superpotential, the Affleck-Dine baryogenesis \([17]\) may work, which changes our result in the previous section. Even if there is no such \(B\) or \(L\) violating terms, decay processes of the multi-field oscillations are not so simple and the fate of the \(\tilde{N}_1\) oscillation is difficult to be predicted.

\(^5\)The three right-handed neutrinos are required to cancel \(Z_4^{B-L}\) gauge anomalies.

\(^6\)We can easily extend our discussion to the case where the Hubble mass terms are induced by the supergravity effects.

\(^7\)Even for \(|\tilde{N}_1^{\text{init}}| \simeq M_{\text{pl}}\) this condition can be easily realized by a spontaneously broken discrete \(Z_6\) symmetry \([11, 12]\), where \(f\) may be as small as \(10^{-5}\).
After the end of inflation, the Hubble parameter $H$ becomes smaller than $M_1$ and the $\tilde{N}_1$ starts to oscillate around its origin. Since the time scale of the motion of $\tilde{X}$ and $\tilde{S}$ ($\sim 1/(yv)$) is much smaller than the one of the $\tilde{N}_1$ oscillation ($\sim 1/M_1$), $\tilde{X}$ and $\tilde{S}$ trace their minima along with the $\tilde{N}_1$ oscillation;

$$\tilde{X}(t) \simeq \left( \frac{f}{4gyv} \right) \tilde{N}_1(t)^2, \quad \tilde{S}(t) \simeq v.$$  

(26)

Therefore, we find that our assumption in the previous sections to take the scalar potential of the $\tilde{N}_1$ as $V \simeq M_1^2 |\tilde{N}_1|^2$ is valid. Thus, we expect the initial amplitude of the $\tilde{N}_1^{\text{init}}$ to be of the order of $M_{\text{pl}}$.

| Fields       | $Q_L$, $U_R$, $E_R$ | $L_L$, $D_R$ | $H_u$, $H_d$ | $\tilde{N}_1$ | $\tilde{X}$ | $S$ |
|--------------|-------------------|--------------|--------------|---------------|-----------|-----|
| $R$ charges  | 1                 | 1            | 0            | 1             | 2         | 0   |
| $Z_4$ charges| 1                 | -3           | 2            | 1             | 0         | 2   |

Table 1: Here, $Q_L$ and $L_L$ denote the SU(2)$_L$ doublet quarks and leptons, $U_R$, $D_R$ and $E_R$ are the SU(2)$_L$ singlet up- and down-quarks and leptons, and $H_{u,d}$ the up-type and down-type Higgs.

### 3.2 Stability of the $LH_u$ flat direction

We give a comment on stability of the $LH_u$ flat direction $\phi$ during the $\tilde{N}_1$ oscillation. Instability of the $LH_u$ direction comes from a cross term in the scalar potential between the $LH_u$ direction and the $\tilde{N}_1$ in Eq. (24).\footnote{We thank K. Hamaguchi for pointing out this problem.} However, we find that thermal effects stabilize the $LH_u$ direction $\phi$.

The $LH_u$ flat direction $\phi$ is at the origin when the $\tilde{N}_1$ has a large amplitude, since it has a large positive mass term $|h\tilde{N}_1|^2|\phi|^2$. After the $\tilde{N}_1$ starts to oscillate, the positive mass term $|h\tilde{N}_1|^2|\phi|^2$ decreases and the cross term between $\phi$ and the $\tilde{N}_1$ becomes more significant than the positive mass term. When the $\tilde{N}_1$ becomes smaller than $M_1/h$, (see the last two terms in Eq. (24)), $\phi$ seems to depart from the origin for the $\tilde{N}_1 \lesssim M_1/h$. However, we should note here that there is a thermal mass term for $\phi$ from the thermal mass.

\footnote{Our approximation of the potential Eq. (24) is no longer valid for $\tilde{N}_1 \gg M_{\text{pl}}$ in the supergravity theory.}
background, and hence the effective potential for $\phi$ is given by

$$
V \simeq |h\phi^2 - M_1 \tilde{N}_1|^2 + |h\tilde{N}_1 \phi|^2 + \alpha^2 T^2 |\phi|^2,
$$

(27)

where $T$ denotes the temperature of the thermal background. Here, the coefficient $\alpha$ is estimated as $\alpha^2 \simeq 3g^2_2/8 + g^2_1/8 \simeq 1/4$ for $\phi \ll T$, and we have omitted the thermal effects for the $\tilde{N}_1$.\(^{10}\) As we see below, the flat direction $\phi$ is still stabilized at the origin by the thermal mass term in the course of the $\tilde{N}_1$ oscillation.

If $T_R > T_{osci}$ (see Eq. (10)), the $\tilde{N}_1$ starts to oscillate during the radiation dominated era, and hence $|\tilde{N}_1|$ and $T$ decrease with $a(t)^{-3/2}$ and $a(t)^{-1}$, respectively. Here, $a(t)$ denotes the scale factor of the universe. To discuss the stability of $\phi$, it is convenient to define the temperature $T_{\text{back}} \propto a(t)^{-1}$ during the $\tilde{N}_1$ domination, which corresponds to the temperature without the $\tilde{N}_1$ decay.\(^{11}\) Since $(T_{\text{back}})^2$ decreases faster than $M_1|\tilde{N}_1|$, $\phi = 0$ is a stable point until the decay time of the $\tilde{N}_1$, if the condition,

$$
2hM_1|\tilde{N}_{1\text{decay}}|^2 \ll \alpha^2 (T_{\text{back}}^1)^2,
$$

(28)
is satisfied at the $\tilde{N}_1$ decay time. Here, $T_{\text{back}}^1$ is a background temperature at the $\tilde{N}_1$ decay time, which is given by

$$
T_{\text{back}}^1 = T_{\text{dom}} \left( \frac{a(t_{\text{dom}})}{a(t_{\text{decay}})} \right)^{2/3} = T_{\text{dom}} \left( \frac{H_d}{H_{\text{dom}}} \right)^{2/3} = T_{\text{dom}} \left( \frac{T_d}{T_{\text{dom}}} \right)^{4/3} = T_{\text{dom}} \left( \frac{T_d}{T_{\text{dom}}^1} \right)^{1/3},
$$

(29)

where $t_{\text{decay}}$ denotes the decay time of the $\tilde{N}_1$, and $H_{d,\text{dom}} \propto T_{d,\text{dom}}^2/M_{\text{pl}}$ the Hubble parameters at the decay time of the $\tilde{N}_1$ and at the beginning of the $\tilde{N}_1$ domination, respectively. From the energy conservation at the decay time of the $\tilde{N}_1$, we find that the amplitude $|\tilde{N}_{1\text{decay}}|$ satisfies

$$
M_1^2 |\tilde{N}_{1\text{decay}}|^2 = \frac{\pi^2}{30} g_*(T_d) T_d^4.
$$

(30)

Thus, the condition Eq. (28) can be written as

$$
\frac{\alpha^2}{2} \left( \frac{30}{\pi^2 g_*} \right)^{1/2} \left( \frac{T_d}{T_{\text{dom}}} \right)^{2/3} \gg h.
$$

(31)

\(^{10}\)Possible thermal effects to the motion of the $\tilde{N}_1$ are discussed in Ref. [11] which shows that those effects are irrelevant as long as $M_1 \gtrsim T_d$.

\(^{11}\)The actual background temperature is much higher than the temperature $T_{\text{back}}$, since the decay of the $\tilde{N}_1$ reheats up the radiation. Thus, the condition in Eq. (28) is a sufficient one to stabilize the $LH_u$ flat direction by the thermal effects.
By using the definition of $T_d$ in Eq. (33) and $T_{\text{dom}} < (90/\pi^2 g_*)^{1/4} \sqrt{M_1 M_{\text{pl}}}$, we obtain the sufficient condition for the $\phi$ stabilization as

$$\frac{1}{4\pi} \left( \frac{\alpha^2}{2} \right)^3 \left( \frac{30}{\pi^2 g_*) \right)^{3/2} \gg h. \tag{32}$$

This is satisfied when $h$ satisfies the condition Eq. (33). Therefore, the flat direction $\phi$ remains at its origin if $T_R > T_{\text{osci}}$.

On the other hand, if $T_R < T_{\text{osci}}$, the $\tilde{N}_1$ starts to oscillate before the completion of the reheating of inflation and the situation becomes rather complex. Since $T$ decreases with $a(t)^{-3/8}$ during the inflaton dominated era [18], we should also require

$$2hM_1|\tilde{N}_1^{\text{init}}| \ll \alpha^2 T^2, \tag{33}$$

at the beginning of the $\tilde{N}_1$ oscillation for the stability of $\phi$. The temperature of the background radiation at the beginning of the $\tilde{N}_1$ oscillation is estimated as

$$T = T_R \left( \frac{a(t_R)}{a(t_{\text{osci}})} \right)^{3/8} = T_R \left( \frac{H_{\text{osci}}}{H_R} \right)^{1/4} \simeq (M_1 M_{\text{pl}})^{1/4} T_R^{1/2}, \tag{34}$$

where $t_R$ and $t_{\text{osci}}$ denote the cosmic times of the end of the reheating and the beginning of the $\tilde{N}_1$ oscillation, respectively, and $H_R$ and $H_{\text{osci}}$ the Hubble parameters at those times. Here, we have used $H \propto a^{-3/2}$ in the inflaton dominated era, $H_{\text{osci}} \simeq M_1$, and $H_R = (\pi^2 g_*/90)^{1/4} T_R^2 / M_{\text{pl}}$. Thus, for $T_R < T_{\text{osci}}$, we should also require in addition to Eq. (32)

$$T_R \gg \frac{2}{\alpha^2} \sqrt{M_1 M_{\text{pl}} \left( |\tilde{N}_1^{\text{init}}| / M_{\text{pl}} \right)} \simeq \frac{2 \sqrt{4\pi}}{\alpha^2} T_d \left( |\tilde{N}_1^{\text{init}}| / M_{\text{pl}} \right), \tag{35}$$

which is naturally satisfied in the $\tilde{N}_1$ dominated scenario. Thus, we find that the flat direction $\phi$ remains also at the origin for $T_R < T_{\text{osci}}$.\(^{12}\)

Finally, we give a summary of the conditions which we should require to the right-handed neutrino sector.

\(^{12}\)If the background temperature $T$ in Eq. (31) is larger than the inflaton mass $M_{\text{inf}}$, the actual background temperature at the beginning of the $\tilde{N}_1$ oscillation is $T \simeq M_{\text{inf}}$ [19]. Then, the condition in Eq. (33) is modified to

$$\alpha^2 M_{\text{inf}}^2 \gg hM_1 M_{\text{pl}}, \tag{36}$$

where $M_{\text{inf}}$ is the inflaton mass.
• $M_1 \ll H_{\text{inf}}$; for a large initial amplitude of the $\tilde{N}_1$.
• $T_R \gg T_d \left(3M_{\text{pl}}^2/|N_1^{\text{init}}|^2\right)$, for the $\tilde{N}_1$ domination.
• $T_d < M_1$, to avoid washout effect of the lepton asymmetry.
• $v \gg H_{\text{inf}}$, to fix $\tilde{X}$ and $\tilde{S}$ as in Eq. (25) during the inflation.
• $f|\tilde{N}_1^{\text{init}}| \ll v$, not to destabilize the $\tilde{S} \simeq v$.

These conditions are easily satisfied, for example,

$$M_1 \simeq 10^{9-10} \text{ GeV}, \; v \simeq 10^{15} \text{ GeV}, \; f \simeq 10^{-5}, \; h \simeq 10^{-6}.$$  \hfill (37)

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