Control-Oriented Power Allocation for Integrated Satellite-UAV Networks

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Abstract—This letter presents a sensing-communication-computing-control (SC\textsuperscript{3}) integrated satellite unmanned aerial vehicle (UAV) network, where UAVs are equipped with sensors, mobile edge computing (MEC) servers, base stations and satellite communication modules. Like a nervous system, this integrated network is capable of organizing multiple field robots in remote areas, so as to perform mission-critical tasks which are dangerous for humans. Aiming at activating this nervous system with multiple SC\textsuperscript{3} loops, we present a control-oriented optimization problem. Different from traditional studies which mainly focused on communication metrics, we address the power allocation issue to minimize the sum linear quadratic regulator (LQR) control cost of all SC\textsuperscript{3} loops. Specifically, we show the convexity of the problem and reveal the relationship between the optimal transmit power and intrinsic entropy rates of different SC\textsuperscript{3} loops. For the assure-to-be-stable case, we derive a closed-form solution for ease of practical applications. After demonstrating the superiority of the control-oriented power allocation, we further highlight its difference from the classic capacity-oriented water-filling method.

Index Terms—Control parameter, linear quadratic regulator (LQR), power allocation, satellite-UAV network.

I. INTRODUCTION

FIELD robots could perform mission-critical tasks that are dangerous for humans. When being dispatched to remote areas without terrestrial cellular coverage, the operation of robots has to rely on non-terrestrial infrastructures, including satellites and unmanned aerial vehicles (UAVs) [1], [2]. For such scenarios, a UAV platform needs to have integrated functionalities to support various requirements of robots, e.g., sensing, communication, computing and controlling. For example, a UAV can be equipped with on-board sensors to collect scene information, with mobile edge computing (MEC) servers to analyze the situation and make quick decisions for robot control, with base stations to transmit control commands to robots, and with a satellite communication module to support real-time communication to the remote cloud center [3]. This leads to a sensing-communication-computing-control (SC\textsuperscript{3}) integrated satellite-UAV network, in which efficient resource orchestration for all the related functionalities is important.

In the SC\textsuperscript{3} integrated network, the whole control process, including sensing, computing, communication and control, is performed in a closed-loop manner, which is referred to as a SC\textsuperscript{3} loop. Intuitively, a SC\textsuperscript{3} loop can be regarded as a reflex arc, with the network as a nervous system [4], where multiple SC\textsuperscript{3} loops would share and compete for resources.

Existing studies on integrated satellite-UAV networks have mainly focused on communications. For example, [5] jointly optimized the channel allocation, power allocation, and hovering time of UAVs to maximize the data transmission efficiency. However, in a SC\textsuperscript{3} integrated network, control and communication are closely coupled, and one will be more concerned with the control performance, especially for mission-critical tasks where reliable control must be guaranteed.

Researchers in the control field have investigated the relationship between communication and control. It was shown that a noisy linear control system can be stabilized only if the communication throughput per cycle exceeds the intrinsic entropy rate of the control system [6]. Qiu et al. generalized this result to a multi-channel case [7]. Recently, a lower bound of the minimum data rate to achieve a certain linear quadratic regulator (LQR) cost was presented [8]. These achievements have indicated that jointly optimizing control and communication is promising and significant in the SC\textsuperscript{3} integrated satellite-UAV network. However, most of these works modeled communications as simple pipelines with simple parameters and left communication resource allocation undiscussed.

Inspired by the efforts in the control field, some recent works have considered the control part as constraints in communication design. Authors in [9] maximized the spectral efficiency of a wireless control system subject to the control convergence rate constraint. Chen et al. maximized the delay determinacy under the same constraint [10]. These studies have taken a great step toward control-oriented optimization. However, they still focused on communications as the objective. In mission-critical SC\textsuperscript{3} integrated networks, as has been discussed, the control performance is more important and should be treated as the objective rather than constraints. Compared with the convergence rate that was considered in [9], [10], the LQR cost, which has been widely used in the control theory [11], [12], is a more suitable objective for measuring the control performance, as it provides a simple form to balance the state deviation and control energy consumption.

Motivated by the above discussion, we optimize the sum LQR control cost of all SC\textsuperscript{3} loops. Particularly, we establish...
Due to the limited computing capability of the MEC server on the UAV, some complex computing tasks will be offloaded to the remote cloud center through satellite. The queuing time is negligible as the transmission of the tasks is infrequent (only at the start of each cycle). Denoting the task data size as $D_k$, the transmission times from UAV to satellite and from satellite to control center can be calculated as $t_{k,T,U2S} = \frac{D_k}{R_{U2S}}$ and $t_{k,T,S2C} = \frac{D_k}{R_{S2C}}$, where $R_{U2S}$ and $R_{S2C}$ denote the corresponding transmission data rates. The processing time is $t_{k,P} = \frac{D_k}{\bar{f}_k \alpha_k}$, where $\alpha_k$ is the average CPU cycles to process the task per bit, and $\bar{f}_k$ is the CPU frequency. From the perspective of system robustness, we consider the maximum propagation delay based on the satellite orbit height $H_S$, the minimum elevation angle $\theta_{min}$, and the maximum central angle $\theta_{max}$, denoted as $\tau_{max} = \frac{(R_e+H_S) \sin \theta_{max}}{c \cos \theta_{min}}$ [15], where $R_e$ is the Earth radius. Therefore, the total delay, when a task is conducted on the cloud, can be calculated as

$$T_{k,1} = t_{k,T,U2S} + t_{k,T,S2C} + t_{k,P} + 4\tau_{max},$$

where the transmission time from the remote cloud center back to the UAV is omitted as the amount of the output data is small.

Denoting the control period of SC$^3$ loop $k$ as $T_k$, the available time duration for transmitting the control commands would be shortened to $T_{k,2} = T_k - T_{k,1}$, if a task is conducted remotely.

The communication data rate between the UAV and robots is limited, which may affect the control performance. According to [6], to stabilize control object $k$, the data throughput transmitted in each cycle needs to satisfy the condition

$$BT_{k,2} \log_2 \left( 1 + \frac{g_k \rho_k}{\sigma^2} \right) > h_k \triangleq \log_2 |\det A_k|,$$

where $B$ denotes the channel bandwidth, $\sigma^2$ is the noise variance and $h_k$ is the intrinsic entropy rate which denotes the stability of object $k$. A large $h_k$ indicates an unstable object, which requires a high transmission rate to stabilize.

In control theory, the control performance can be measured by the LQR cost function [11], [12]. In this letter, we consider the worst-case long-term average LQR cost, formulated as

$$l_k \triangleq \sup_{N \to \infty} \lim_{N \to \infty} \mathbb{E} \left[ \frac{1}{N} \sum_{t=1}^{N} (x_{k,t}^T Q_k x_{k,t} + u_{k,t}^T R_k u_{k,t}) \right],$$

where $Q_k$ and $R_k$ are semi-positive definite weight matrices. The terms $x_{k,t}^T Q_k x_{k,t}$ denotes the deviation of the system from the zero state, and the term $u_{k,t}^T R_k u_{k,t}$ denotes the input energy. These weight matrices balance the state and the energy, which can be set according to practical requirements. For example, one should set the entries of $Q_k$ to be large if he expects that the state of the system converges to zero quickly.

To achieve a certain LQR cost ($l_k$), an additional term has to be added to the condition previously described in (3), and the data throughput in each cycle now must satisfy

$$BT_{k,2} \log_2 \left( 1 + \frac{g_k \rho_k}{\sigma^2} \right) > h_k + \frac{n}{2} \log_2 \left( 1 + n N(v_k) \frac{|\det M_k|}{b_k - \text{tr}(\Sigma_k S_k)} \right),$$

where $N(v_k) \triangleq \frac{1}{2\pi} \exp \left( -\frac{1}{2} h(v_k) \right)$, $h(v_k)$ is the differential entropy of $v_k$, $\Sigma_k$ is its covariance matrix, and $M_k$ and $S_k$ are fixed.
are the solutions to control system Riccati equations shown in [8]. The right side of (5) is decreasing with \( l_k \), which means that a lower LQR cost requires a higher data rate in a single SC\(^3\) loop. When multiple SC\(^3\) loops share resources, e.g., the power, the control-oriented and communication-oriented resource allocation schemes will have essentially different results. This will be shown in the next section.

In this letter, we focus on control-oriented resource allocation, and aim to minimize the sum long-term average LQR cost of the SC\(^3\) loops by optimizing power allocation \( p = [p_1, p_2, \ldots, p_K] \). The problem is formulated as

\[
\begin{align*}
\min_{p} & \quad \sum_{k=1}^{K} l_k(p_k) \\
\text{s.t.} & \quad \sum_{k=1}^{K} p_k \leq P_{\text{max}},
\end{align*}
\]

(6a)

where \( I = [l_1, l_2, \ldots, l_K] \) and (6c) is the communication constraint imposed by the control performance requirements. In the following, we will transform this problem to a convex problem and analyze its optimal solution.

### III. Problem Transformation and Analysis

We first propose a lemma to show the convexity of problem (6), and further reveal the relationship between the optimal power allocation and other parameters.

**Lemma 1:** Problem (6) is equivalent to the convex problem

\[
\begin{align*}
\min_{p} & \quad \sum_{k=1}^{K} l_k(p_k) \\
\text{s.t.} & \quad \sum_{k=1}^{K} p_k \leq P_{\text{max}},
\end{align*}
\]

(7a)

where

\[
\begin{align*}
BT_{k,2} \log_2 \left( 1 + \frac{g_k p_k}{\sigma^2} \right) > h_k, & \quad k = 1, 2, \ldots, K, (7c)
\end{align*}
\]

(7c)

Proof: As the right side of (5) is decreasing with \( l_k \), the equality must hold to minimize the objective function. Therefore, we obtain the equation between \( l_k(p_k) \) as (8). Correspondingly, problem (6) is recast as problem (7), where constraint (7c) ensures a positive denominator in (8).

Next, we prove the convexity of (7). The second order derivative of \( l_k(p_k) \) is shown as (9) at the bottom of the page. From (9), we can find that \( \frac{\partial^2 l_k}{\partial p_k^2} > 0 \) as long as

\[
\begin{align*}
\frac{\partial^2 l_k}{\partial p_k^2} = 2BT_{k,2} \frac{2^\frac{3}{2} h_k N(v_k)}{\sigma^2} \det M_k \left( 1 + \frac{g_k p_k}{\sigma^2} \right) \left[ \frac{2BT_{k,2}}{n} 2^\frac{3}{2} h_k + \frac{2BT_{k,2}}{n} \left( 1 + \frac{g_k p_k}{\sigma^2} \right) \frac{2BT_{k,2}}{n} \frac{2^\frac{3}{2} h_k}{n} + \left( 1 + \frac{g_k p_k}{\sigma^2} \right) \frac{2BT_{k,2}}{n} - 2^\frac{3}{2} h_k \right]
\end{align*}
\]

(9)
We consider the assure-to-be-stable assumption, that the communication capability is significantly greater than the lowest capability requirement to keep the system stable in (3), i.e., $BT_k \log_2(1 + \frac{\eta k p_k}{g_k}) \gg h_k$. This assumption means that the control systems are far from the unstable point, and we can focus on the control performance instead of the stability.

**Proposition 2:** Under the assured-to-be-stable assumption, if all of the SCs loops have the same control period and computing delay, which means that $T_{1,2} = T_{2,2} = \cdots = T_{k,2} = T$, the optimal solution to problem (6) is obtained as (13), shown at the bottom of the page.

**Proof:** When $BT_k \log_2(1 + \frac{\eta k p_k}{g_k}) \gg h_k$, we have $(1 + \frac{9k p_k}{\sigma^2}) \gg 2^\frac{h_k}{\pi}$, which means that the term $2^\frac{h_k}{\pi}$ in the denominator of the left hand in (11) is negligible. Therefore, we can rewrite (11) as

$$p^*_k = \left(\frac{2BT |\det M_k| \frac{1}{\lambda} 2^\frac{h_k}{\pi} N(v_\lambda) g_k}{\lambda \sigma^2} \right) \frac{2BT + \pi}{\pi h_k} - 1 \left(\frac{\sigma^2}{g_k}\right).$$

(14)

On the other hand, due to the monotonicity of $l_k$ with respect to $p_k$, the equality in (7b) must hold, i.e.,

$$\sum_{k=1}^{K} p_k = P_{\max}.$$  

(15)

Based on (14) and (15), we obtain that the Lagrangian multiplier $\lambda$ satisfies equation (16), shown at the bottom of the page.

Substituting (16) into (14), we obtain (13) immediately, which completes the proof.

**Remark 2:** From (13), we see that $p^*_k$ is increasing with $h_k$ and $N(v_\lambda)$, which verifies the conclusion of Proposition 1. In addition, it is shown that the term $\frac{2BT + \pi}{\pi h_k}$, as the power of the control parameters, evaluates the influence of the control parameters on the allocation results. In the special case that $\frac{2BT + \pi}{h_k} \rightarrow 0$, we have $p^*_k = P_{\max} - \frac{1}{g_k} \log_{\eta_k}^{-1}(1/g_k)$, which is irrelevant to the control part of the system.

IV. Simulation Results

We assume 5 objects, which are randomly and evenly distributed in a circular area with a radius of 5000 m. The UAV is at the center of the circle. The UAV height, denoted as $H$, is set as 100m unless otherwise specified. Other parameters are set as $B = 5$ kHz, $\beta_0 = -60$ dB and $\sigma^2 = -110$ dBm [17]. We assume Low Earth Orbit (LEO) satellite, with $R^S = 1100$ km.

$$p^*_k = \frac{P_{\max} + \sum_{i=1}^{K} \sigma^2_i}{\sum_{i=1}^{K} |\det M_i| \frac{1}{\pi} 2^\frac{h_k}{\pi} N(v_i) \frac{2BT + \pi}{\pi h_k} - 1} \left(\frac{\sigma^2_i}{g_k}\right)$$

(13)

$$\frac{1}{\lambda} = \frac{P_{\max} + \sum_{k=1}^{K} \sigma^2_k}{\sum_{k=1}^{K} 2BT |\det M_k| \frac{1}{\pi} 2^\frac{h_k}{\pi} N(v_K) \frac{2BT + \pi}{\pi h_k} - 1} \left(\frac{\sigma^2_k}{g_k}\right)$$

(16)
the system stability under constraint (7c).

cation method still allocates power to every channel to ensure
channels with poor conditions, while the control-oriented allo-
filling method in this case will not allocate any power to some

choice for $SC$.

deterioration of the control performance. For this considera-
increases. This is because the time for command commu-

Fig. 4. LQR cost under different satellite latency.

Fig. 3a compares the power allocated to each channel obtained with different methods, with $P_{\text{max}} = 5\, \text{dBW}$. The intrinsic entropy rates of each system are set as 5. The channels are sorted such that $g_1 \geq g_2 \geq \cdots \geq g_8$. In Fig. 3a, the difference between these two methods is clearly shown.

The control-oriented allocation method tends to allocate more power to channels with bad conditions, while the water-filling method behaves oppositely. To show the difference more clearly, we compare the power allocation results with lower maximum power and more robots in Fig. 3b, where $P_{\text{max}} = -10\, \text{dBW}$ and $K = 10$. It is seen that the water-filling method in this case will not allocate any power to some channels with poor conditions, while the control-oriented allocation method still allocates power to every channel to ensure the system stability under constraint (7c).

In Fig. 4, we show the impact of satellite latency. It is seen that the LQR cost increases as the satellite latency increases. This is because the time for command communication decreases with larger satellite latency, resulting in deterioration of the control performance. For this consideration, Geostationary Earth Orbit satellite might not be a suitable choice for $SC^3$ networks, since its latency could be longer than the control period, resulting in an unstable system.

V. CONCLUSION

In this letter, we investigated a $SC^3$ integrated satellite-UAV network. A control-oriented power allocation problem was formulated. We transformed it into a convex problem and proved that more power should be allocated to the loops with higher intrinsic entropy rates to improve the control performance. We further derived the closed-form expression of the optimal power in the assure-to-be-stable case. Simulation results showed the difference between the control-oriented method and the capacity-oriented water-filling method. Specifically, the former will allocate more power to the channel with worse conditions while the latter behaves oppositely.

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