Developmental Trajectories in the Understanding of Everyday Uncertainty Terms
– Supplementary Material –

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1 Developmental Trajectories in the Understanding of Everyday Uncertainty Terms: Supplementary material

Data and R code for reproducing the analyses and plots is available at https://osf.io/g2c6x/.

1.1 Correlation of Mean Estimates Across Age Groups

We first computed the mean estimate for each of the 14 terms separately for each age group, and then computed the Pearson correlation across age groups’ mean estimates. Figure 1 shows the full correlation matrix across age groups.

Figure 1: Correlation of mean quantitative estimates across age groups.

|                | 4-5 years | 6 years | 7 years | 8 years | 9 years | 10 years | 11-14 years |
|----------------|-----------|---------|---------|---------|---------|----------|-------------|
| 6 years        | .847      | .86     | .936    | .744    | .924    | .882     | .991        |
| 7 years        | .748      | .936    | .894    |         |         |          |             |
| 8 years        | .744      | .924    | .882    | .991    |         |          |             |
| 9 years        | .728      | .885    | .861    | .965    | .982    |          |             |
| 10 years       | .725      | .859    | .819    | .949    | .974    |          |             |
| 11-14 years    | .647      | .785    | .755    | .917    | .938    | .947     | .986        |

a) All terms

|                | 4-5 years | 6 years | 7 years | 8 years | 9 years | 10 years | 11-14 years |
|----------------|-----------|---------|---------|---------|---------|----------|-------------|
| 6 years        | .745      | .869    | .948    | .575    | .909    | .804     | .992        |
| 7 years        | .748      | .903    | .801    | .997    | .996    |          |             |
| 8 years        | .648      | .899    | .782    | .981    | .981    | .972     |             |
| 9 years        | .525      | .809    | .703    | .968    | .959    | .953     | .986        |
| 10 years       | .793      | .743    | .853    | .835    | .902    | .945     | .983        |
| 11-14 years    | .704      | .822    | .913    | .898    | .953    | .985     |             |

b) Frequency terms

|                | 4-5 years | 6 years | 7 years | 8 years | 9 years | 10 years | 11-14 years |
|----------------|-----------|---------|---------|---------|---------|----------|-------------|
| 6 years        | .94       | .949    | .963    | .948    | .976    | .997     | .987        |
| 7 years        | .935      | .852    | .919    | .91     | .96     |          |             |
| 8 years        | .881      | .822    | .913    | .898    | .953    | .985     |             |
| 9 years        | .793      | .743    | .853    | .835    | .902    | .945     | .983        |
| 10 years       | .704      | .822    | .913    | .898    | .953    | .985     |             |
| 11-14 years    | .647      | .785    | .755    | .917    | .938    | .947     | .986        |

c) Probability terms
1.2 Re-Analysis of Biehl & Halpern-Felsher (2001)

Biehl and Halpern-Felsher (2001) elicited the numerical estimates of 30 linguistic uncertainty terms by asking participants to assign a percentage between 0 and 100 to each of them. Their sample consisted of four age groups: 5th-graders ($M_{age} = 10.05$, $N=20$), 7th-graders ($M_{age} = 11.91$, $N=45$), 9th-graders ($M_{age} = 14.07$, $N=45$), and young adults ($M_{age} = 26.24$, $N=34$). They reported mean estimates and standard deviations for each term and age group (Table 1 in Biehl & Halpern-Felsher, 2001).

Figure 2 shows the mean estimates ($\pm$ 95% CI) across all terms (ordered by adults’ mean estimates). While Biehl and Halpern-Felsher (2001) reported some significant differences among the numerical estimates between age groups, the overall agreement is substantial, as indicated by the high correlations of children’s estimates with those of adults and the small mean absolute deviation (MAD). Both metrics indicate that in this age range children’s intuitions about the meaning of the verbal terms further approximate those obtained from the adult participants.

Figure 2: Mean numerical estimates assigned to different linguistic terms in Biehl & Halpern-Felsher (2001, Table 1). $r$ denotes the Pearson correlation of children’s mean estimates with adults’ mean judgments, MAD denotes the mean absolute deviation from adults’ estimates.
1.3 Probabilistic Modeling of Everyday Uncertainty Terms

We represent the vagueness of verbal terms using probability distributions, where a density function on the interval \([0, 1]\) is used to denote the likelihood of different numerical values belonging to the concept. In this view, each numerical value has a certain likelihood of belonging to a particular term, and the dispersion of the distribution encodes the phrases’ inherent vagueness. The appendix of the paper provides a detailed description of this approach.

Figure 3 shows the densities for the different frequency terms across age groups; Figure 4 shows the densities for the probability terms (see the online supplemental materials at https://osf.io/g2c6x/ for numerical values of all shape parameters).

Figure 3: Beta distributions and empirical densities (dashed line) for the investigated frequency terms. Each distribution has the same mean and variance as the empirical estimates.

Note. We used the \(\hat{f}_1\) beta-kernel estimator recommended by Chen (1999) to estimate the empirical densities. Because the Beta distribution is not defined on the bounds of the unit interval, we shifted data points located exactly on the bounds (i.e., judgments of 0 and 1, respectively) by 0.001.
Figure 4: Beta distributions and empirical densities (dashed line) for the investigated probability terms. Each distribution has the same mean and variance as the empirical estimates.

Probability terms: Beta distributions and empirical densities

Note. We used the $f_1$ beta-kernel estimator recommended by Chen (1999) to estimate the empirical densities. Because the Beta distribution is not defined on the bounds of the unit interval, we shifted data points located exactly on the bounds (i.e., judgments of 0 and 1, respectively) by 0.001.

2 References

Biehl, M., & Halpern-Felsher, B. L. (2001). Adolescents' and adults' understanding of probability expressions. *Journal of Adolescent Health, 28*(1), 30-35.

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