ENHANCED DIAGRAMS IN HIGH ENERGY HADRONIC AND NUCLEAR SCATTERING

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High energy hadronic and nuclear interactions are described within Gribov’s Reggeon scheme. An approach to re-summation of enhanced Pomeron graphs is proposed. The latter is applied to develop a new Monte Carlo model which treats non-linear interaction effects explicitly in individual hadronic collisions. On the other hand, we discuss a possible generalization of the scheme, which allows to account also for pQCD effects. In particular, this offers a possibility to fix model parameters on the basis of data on hadron-hadron cross sections, hadronic multiparticle production, and on total and diffractive proton structure functions.

1 Introduction

Investigations of high energy hadronic and nuclear interactions remains an interesting and grateful field of research. An important ingredient of such studies is the development of corresponding Monte Carlo (MC) models-generators, the latter being extensively used for projecting new experiments, analyzing and interpreting measured data.

Still, despite a significant progress in QCD over the past few decades, only a phenomenological treatment is generally possible for minimum-bias hadron-hadron (hadron-nucleus, nucleus-nucleus) collisions, typically being based (explicitly or implicitly) on the Gribov’s Reggeon approach. In the latter scheme a high energy hadron-hadron collision is described as a multiple scattering process, where elementary re-scatterings, corresponding to microscopic parton cascades, are treated phenomenologically as Pomeron exchanges – Fig. 1.

The Pomeron exchange amplitude is typically chosen as

\[ f_{\text{P}}^{\text{ad}}(s, b) = \frac{i \gamma_a \gamma_d (s/s_0)^{\alpha_p(0)-1}}{\lambda_{\text{ad}}^{\text{P}}(s)} \exp \left( -\frac{b^2}{4 \lambda_{\text{ad}}^{\text{P}}(s)} \right) \]  

(1)

\[ \lambda_{\text{ad}}^{\text{P}}(s) = R_{a}^2 + R_{d}^2 + \alpha'_p(0) \ln(s/s_0), \]  

(2)
where $s_0 \simeq 1 \text{ GeV}^2$ is the hadronic mass scale, $\alpha_P(0)$ and $\alpha'_P(0)$ are the intercept and the slope of the Pomeron Regge trajectory, and $\gamma_a$, $R_a^2$ are the coupling and the slope of Pomeron-hadron $a$ interaction vertex. Assuming an over-critical Pomeron trajectory ($\alpha_P(0) > 1$) it is characterized by a power-like energy increase and a Gaussian impact parameter shape, with the corresponding width (slope) rising with energy.

Using the optical theorem and calculating various unitarity cuts of elastic scattering diagrams of Fig. 1 according to the so-called Abramovskii-Gribov-Kancheli (AGK) cutting rules, one can obtain expressions for total and inelastic cross sections as well as for relative probabilities of particular interaction configurations, e.g., for a given number of elementary inelastic processes ("cut" Pomerons), all being expressed via the Pomeron amplitude $f^P_{ad}(s, b)$. Furthermore, identifying such "cut" Pomeron processes with string formation and break-up allowed to propose powerful model approaches, like Quark-Gluon String or Dual Parton models, which in turn opened the way to develop corresponding MC generators of hadronic and nuclear collisions.

Such a scheme is characterized by a great simplicity and flexibility, a general high energy collision being just a superposition of a number of elementary processes – Pomeron exchanges, by small number of adjustable parameters and by a parameter-free generalization to hadron-nucleus and nucleus-nucleus case. Nevertheless, its validity is subject to condition that such elementary processes proceed independently of each other.

2 Enhanced Pomeron Diagrams

The above-mentioned condition is not expected to be valid in the "dense" regime, i.e. in the limit of high energies $s$ and small impact parameters $b$ of the interaction. There, a large number of elementary scattering processes occurs and corresponding underlying parton cascades largely overlap and interact with each other. Such effects are traditionally described by so-called enhanced diagrams, which involve Pomeron-Pomeron interactions, however, taking into consideration only triple-Pomeron coupling. Here we shall rather stay close to the $\pi$-meson dominance approach, where all multi-Pomeron vertexes have been expressed via the triple-Pomeron coupling constant $r_{3P}$ and where an asymptotic re-summation has been proposed.

Recently, a re-summation procedure for higher orders of such diagrams has been proposed, however, taking into consideration only triple-Pomeron coupling. Here we shall rather stay close to the $\pi$-meson dominance approach, where all multi-Pomeron vertexes have been expressed via the triple-Pomeron coupling constant $r_{3P}$ and where an asymptotic re-summation has been proposed.

At sufficiently small energies one can restrict himself with just lowest in $r_{3P}$ contribution (one multi-Pomeron vertex) – Fig. 2 left, which involves any transitions of $m \geq 1$ into $n \geq 1$ Pomerons minus the Pomeron self-coupling ($m = n = 1$). Going to higher energies it is the subtracted self-coupling graph which gives the largest contribution. Adding also higher order diagrams, which can be obtained from the one in Fig. 2 left) iterating the multi-Pomeron vertex in $t$-channel, one obtains the "dense" limit (large $s$, small $b$) result as a sum over corresponding self-coupling graphs – Fig. 2 right. The latter corresponds to the usual (quasi-)eikonal Pomeron scheme, described in the Introduction, however, with a re-normalized Pomeron intercept:

$$\tilde{\alpha}_P(0) = \alpha_P(0) - 4\pi r_{3P}/\gamma_{\pi}$$  (3)
In the general case one has to account for all essential enhanced diagrams in order to obtain a smooth transition between the mentioned “dilute” (small \( s \), large \( b \)) and “dense” limits. To this end we obtain the so-called “fan” contribution, defined via a recursive equation of Fig. 3(left), and introduce also a “generalized fan” – using a similar equation of Fig. 3(right); the difference between the two being due to vertexes with both “fans” connected to the projectile and ones connected to the target:

\[
\sum_{n=1}^{\infty} (-1)^n x_n, b_n + \sum_{n=1}^{\infty} x_n, b_n
\]

Figure 3: Recursive equations for the “fan” (left) and “generalized fan” (right) contributions.

Then, neglecting so-called “loop” and “chess-board” graphs, whose contributions always remain sub-dominant, and taking into consideration all other enhanced diagrams of ”net” type, with arbitrary topologies, one can express the full hadron-hadron scattering amplitude via these elementary ”building blocks” in a rather simple form:

\[
= + \sum_{n=1}^{\infty} (x_n, b_n) + \sum_{n=1}^{\infty} x_n, b_n
\]

Figure 2: Left: lowest order enhanced graph. Right: dominant contributions in the ”dense” regime.

To describe particle production one has to consider all relevant unitarity cuts of this amplitude. Proceeding in the usual way, i.e. applying AGK cutting rules and summing together contributions of cuts of certain topologies, one can obtain positively defined probabilities for various configurations of the interaction, which allows to develop corresponding MC generation procedure and to treat non-linear interaction effects explicitly in individual hadronic and nuclear collisions.

3 Matching with QCD?

An essential drawback of the described scheme is that the high energy behavior of hadron-hadron scattering amplitude is governed by a phenomenological Pomeron intercept, not being constraint.
by perturbative QCD results. On the other hand, it lacks a microscopic description of high \(p_t\) parton processes and thus does not allow to calculate corresponding observables.

A possible solution could be to employ the phenomenological Pomeron description only for ”soft” cascades of partons of small virtualities \(|q^2| < Q_0^2\), while treating perturbative parton evolution at \(|q^2| > Q_0^2\) within pQCD framework, \(Q_0^2\) being some cutoff for pQCD being applicable. Without Pomeron-Pomeron interactions one thus obtains the usual linear scheme described in the Introduction, however, based on the “general Pomeron”. The latter consists of two contributions: phenomenological “soft” Pomeron for a pure non-perturbative process (all \(|q^2| < Q_0^2\)) and a so-called “semi-hard Pomeron”, being a \(t\)-channel iteration of the “soft” Pomeron and the QCD ladder, for a cascade which at least partly develops in the high virtuality region (some \(|q^2| > Q_0^2\)) – Fig. 4(left).

To account for corresponding non-linear effects we assume that Pomeron-Pomeron interactions are dominated by partonic processes at comparatively low virtualities, \(|q^2| < Q_0^2\). Then we obtain precisely the scheme described in the preceding Section, now based on the “general Pomeron” and with multi-Pomeron vertexes involving only interactions between ”soft” Pomerons or between “soft ends” of “semi-hard Pomerons” – Fig. 4(right). The parameters of such a scheme can be fixed based on data on hadron-hadron cross sections, hadronic multi-particle production, and on the measured total and diffractive proton structure functions.

**References**

1. V. N. Gribov, *Sov. Phys. JETP* **26**, 414 (1968); **29**, 483 (1969).
2. A. B. Kaidalov and K. A. Ter-Martirosian, *Phys. Lett. B* **117**, 247 (1982); A. Capella et al., *Phys. Rep.* **236**, 225 (1994).
3. V. A. Abramovskii, V. N. Gribov, and O. Kancheli, *Sov. J. Nucl. Phys.* **18**, 308 (1974).
4. N. N. Kalmykov and S. S. Ostapchenko, *Phys. Atom. Nucl.* **56**, 346 (1993).
5. L. Gribov, E. Levin, and M. Ryskin, *Phys. Rep.* **100**, 1 (1983).
6. O. Kancheli, *JETP Lett.* **18**, 465 (1973).
7. J. L. Cardy, *Nucl. Phys. B* **75**, 413 (1974).
8. S. Bondarenko et al., *Nucl. Phys. A* **683**, 649 (2001).
9. A. B. Kaidalov, L. A. Ponomarev, and K. A. Ter-Martirosyan, *Sov. J. Nucl. Phys.* **44**, 468 (1986).
10. S. Ostapchenko, [hep-ph/0412332](https://arxiv.org/abs/hep-ph/0412332), [hep-ph/0501093](https://arxiv.org/abs/hep-ph/0501093).
11. N. N. Kalmykov, S. S. Ostapchenko, and A. I. Pavlov, *Bull. Russ. Acad. Sci. Phys.* **58**, 1966 (1994); *Nucl. Phys. Proc. Suppl.* **52B**, 17 (1997).
12. H. J. Drescher et al., *J. Phys. G: Nucl. Part. Phys.* **25**, L91 (1999); S. Ostapchenko et al., *J. Phys. G: Nucl. Part. Phys.* **28**, 2597 (2002).
13. E. Levin and C. I. Tan, [hep-ph/9302308](https://arxiv.org/abs/hep-ph/9302308).

*A similar approach is the ”heterotic Pomeron” scheme.*

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Figure 4: Left: a “general Pomeron” (l.h.s.) consists of the “soft” and the “semi-hard” Pomerons – correspondingly the 1st and the 2nd contributions on the r.h.s. Right: contributions to the triple-Pomeron vertex from interactions between ”soft” and ”semi-hard” Pomerons.