A Double-station Access Protocol for Optical Wireless Scattering Communication Networks

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Abstract

We propose a double-station access protocol based on CSMA/CA (DS-CSMA/CA) for optical wireless scattering communication networks (OWSCN), where two stations can transmit data to single destination simultaneously. The proposed protocol can avoid the frames colliding with each other. Furthermore, we propose the state transmission model to analyze the collision probability and throughput. We also propose to optimize the initial contention window and partner map for the protocol. Both numerical and simulation results imply that the proposed protocol can achieve remarkably higher throughput compared with traditional CSMA/CA.

Index Terms

DS-CSMA/CA, optical wireless scattering communication networks, collision probability, throughput.

I. INTRODUCTION

Optical wireless scattering communication can potentially offer high data rate transmission due to its large bandwidth [1]. Without emitting or being negatively affected by electromagnetic radiation, it can be applied to many scenarios where conventional radio-frequency (RF) communication is prohibited, for instance in the battlefield where radio silence is required [2]. Certain physical layer techniques can serve as the foundation of optical wireless scattering communication networks (OWSCN), including the signal detection of multi-user and multi-color communication [3], [4], information security [5], neighbor discovery [6] and error correction codes [7].

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On the other hand, IEEE 802.11 WLAN standard specifies the configurations of infrastructure and ad hoc network, where in the infrastructure network, mobile stations have to establish links via fixed access points, and in the ad hoc network, they can directly link with each other. For both types of network configurations, carrier sense multiple access with collision avoidance (CSMA/CA) mechanism is adopted to transmit asynchronous data frames from different mobile stations within a contention period. Furthermore, IEEE 802.11 employs Request to send/Clear to send (RTS/CTS) mechanism to reduce frame collisions introduced by the hidden node problem. The performance of exponential backoff mechanism and IEEE 802.11 standard is analyzed in and respectively. To further boost the performance of CSMA/CA, a multi-channel media access control (MAC) protocol is proposed based on frequency division multiple access (FDMA). Other techniques for WLAN’s behavior enhancement include the cooperative MAC protocol, multi-hop and beamforming antenna technology.

In this work, we consider an OWSCN based on ultra-violet communication. Due to the characteristics of infrastructure and ad-hoc network configuration, even if the transmitters try to send signals simultaneously, the received superimposed signal may suffer symbol boundary misalignment. Hence, we consider the physical-layer multi-user communication proposed in instead of other multiple access techniques. Since the signal superposition leads to higher achievable transmission rate compared with that without signal superposition, it is of interest to investigate the user access protocol that allows signal superposition. Consequently, we design a double-station protocol based on CSMA/CA (DS-CSMA/CA) for OWSCN, where two stations can transmit data frames to single destination simultaneously. Meanwhile, the proposed protocol can avoid data collision as much as possible via the exchanges of assisted frames. The pairs of stations than can transmit data simultaneously are characterized in a partner map which depends on the physical-layer achievable rate. To evaluate proposed protocol, we establish a state transition model and obtain the numerical solutions to the throughput and collision probability, which are validated by simulation results. Furthermore, we propose to approximately maximize the throughput with respect to the initial contention window and partner map. Numerical results demonstrate that the proposed protocol with optimized parameters can significantly outperform the conventional CSMA/CA.

The remainder of this paper is organized as follows. In Section II, we introduce the partner map
and specify the DS-CSMA/CA protocol. In Sections III and IV, we provide a state transition model to analyze the collision probability and throughput. In Section V, we propose to optimize the initial contention window and partner map to maximize throughput. Numerical and simulation results are shown in Section VI to evaluate the DS-CSMA/CA protocol compared with conventional CSMA/CA. Finally, Section VII concludes this work.

II. Media Access Control Protocol

A. Superimposed Frame Detection with symbol boundary misalignment for OWSCN

We consider an OWSCN consisting of \( N_s \) active stations, where each station has an immediate frame to transmit to a common destination. Since different stations’ transmissions cannot be perfectly synchronous, the frames can be superimposed with symbol boundary misalignment, where the relative delays are within one symbol duration, as shown in Figure 1. The achievable rate and frame detection for the superimposed transmission have been investigated in [23], [24], which can be adopted as the physical-layer technique to transmit data frames from two different stations in the OWSCN. In this work, we propose a double-station access protocol based on CSMA/CA that can utilize the joint detection of superposed signal from two different stations.

In the protocol, each station \( i \) deploys \( J_i \) independent time counters \( T_{i,1}, T_{i,2}, \ldots, T_{i,J_i} \) to control the packet transmission. For the convenience of specification, we define the following two important terms and other notations in Table 1:

- **Time counter pair** (TCPair): The two time counters in two stations that can transmit simultaneously with signal superposition at the receiver.
- **Partner time counter** (PTCounter): Within a TCPair, the two time counters are PTCounter of each other.
TABLE I
Specification of notations.

| Notation | Denotation |
|----------|------------|
| N<sub>s</sub> | Number of stations |
| Φ | Partner map |
| N | Number of TCPairs |
| \(T_{i,1}, T_{i,2}, \cdots, T_{i,J_i}\) | Time counters |
| \(W_{i,1}, W_{i,2}, \cdots, W_{i,J_i}\) | Current contention window |
| \(W_0\) | Minimum contention window |
| \(W_{\text{max}}\) | Maximum contention window |

B. Partner Map

A partner map \(Φ\) is adopted for the destination to store all indexes of TCPairs, where \(Φ = [\phi_{i,j}]_{1 \leq i \leq N_s, 1 \leq j \leq N_s}\) is a zero-one matrix depending on the link gains in the physical layer. Note that \(J_i = \sum_{k=1}^{N_s} \phi_{i,k}\) and \(\phi_{i,i'} = 1\) indicate that stations \(i\) and \(i'\) can have a superimposed transmission of data frames controlled by TCPair \((T_{i,j}, T_{i',j'})\), where \(j = \sum_{k=1}^{J_i} \phi_{j,k}\) and \(j' = \sum_{k=1}^{J_{i'}} \phi_{j',k}\); \(Φ\) is symmetrical and \(\phi_{i,i} = 0\) since TCPairs \((T_{i,j}, T_{i',j'})\) and \((T_{i',j'}, T_{i,j})\) must coexist for \(i \neq i'\). An example is that

\[
Φ = \begin{bmatrix}
0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1
\end{bmatrix},
\]

(1)

where \(\phi_{1,2} = \phi_{2,1} = 1\) indicates \(T_{1,1}\) of station 1 and \(T_{2,1}\) of station 2 can form TCPair \((T_{1,1}, T_{2,1})\); and \(T_{1,1}\) and \(T_{2,1}\) are PTCounters of each other.

C. Channel Contention

Before transmission, the initial values of all time counters \(T_{i,j}\) for \(1 \leq i \leq N_s, 1 \leq j \leq J_i\) are uniformly chosen from \([0, W_{i,j} - 1]\), where \(W_{i,j}\) denotes the contention window of \(T_{i,j}\). A common minimum contention window \(W_0\) and maximum contention window \(W_{\text{max}}\) are shared by all time counters.

For each time counter \(T_{i,j}\), \(W_{i,j}\) is selected as \(2^{m_{i,j}}W_0\), where \(m_{i,j}\) takes from 0, 1, 2, \cdots, \(M - 1\) with \(W_{\text{max}} = 2^{M-1}W_0\). Furthermore, the time counters are decremented as long as the channel is sensed idle, as the backoff process in the CSMA/CA protocol, and keep unchanged when the channel is sensed busy. More specifically, the proposed protocol is characterized into the following 3 parts.
1) Once a time counter $T_{i,j}$ of station $i$ decreased to 0, its backoff process is stopped, and station $i$ conducts the following active transmission:

1a) It sends the RTS frame (request to send frame) to the destination, which contains its 2-dimensional index $(i, j)$, as shown in Figure 2(a) and waits for the PTA frame (partner activate) from the destination.

1b) Once receiving the PTA frame, it waits for the DFTrigger frame (data frame trigger) from the destination. In case of missing the PTA frame, it doubles the contention window of $T_{i,j}$ by updating $W_{i,j} \leftarrow 2W_{i,j}$, and jump to step 1e).

1c) After receiving the DFTrigger, it transmits data frame to the destination, as shown in Figure 2(e) and waits for ACK (acknowledgement) from the destination.

1d) On receiving the ACK, it resets the contention window of $T_{i,j}$ to $W_0$, i.e., $W_{i,j} \leftarrow W_0$.

1e) Randomly choose an integer from $[0, W_{i,j} - 1]$ for $T_{i,j}$ as its initial backoff value, and restarts the backoff process in a DIFS. Other time counters also reactive their previous backoff process in a DIFS.

2) Once the destination receives the RTS frame from a certain station with the 2-dimensional index $(i, j)$ of $T_{i,j}$, it obtains the index $(i', j')$ of PTCounter $T_{i',j'}$ according to the partner map, and manages to receive data frames from stations $i$ and $i'$ into the following steps:

2a) It broadcasts the PTA frame containing $(i', j')$ to all stations, as shown in Figure 2(b) and waits for the SAK (superposition acknowledgement) from station $i'$.

2b) Once receiving the SAK, it broadcasts the DFTrigger frame to stations $i$ and $i'$, as shown in Figure 2(d) and waits for the superimposed data transmission.

2c) After successfully decoding the superimposed data frames, it broadcasts the ACK to all stations, as shown in Figure 2(f).

3) For station $\tilde{i} \neq i'$, upon receiving the PTA frame from the destination, its time counters stop backoff process such that the channel is available for stations $i$ and $i'$. Once receiving the ACK from the destination, its time counters reactive their previous backoff process in a DIFS. For station $i'$, upon receiving the PTA frame from the destination, its time counters stop the backoff process, and conducts the transmission via the following steps:

3a) As shown in Figure 2(c), it transmits the SAK frame to the destination, which contains whether it will have a superimposed transmission with station $i$. In case of refusing to
TABLE II
SPECIFICATION OF DIFFERENT TYPE OF FRAMES.

| Frame type | Function                                      |
|------------|-----------------------------------------------|
| RTS        | Ask for occupying the common channel          |
| PTA        | Activate the PTCounter and freeze other TCPairs |
| SAK        | Acknowledgement of a superimposed transmission |
| DFTrigger  | Flag of starting a superimposed transmission  |
| ACK        | Flag of a complete superimposed transmission  |

have superimposed transmission with station \(i\), it jump to 3d).

3b) After transmitting the SAK frame, it waits for the DFTrigger frame from the destination.

3c) Upon receiving the DFTrigger frame, it completes a superimposed transmission of data frame with station \(i\) to the destination, as shown in Figure 2(e), and waits for the ACK from the destination.

3d) Once receiving the ACK, \(T'_{i,j}\) resets the contention window to \(W_0\), randomly chooses a value from \([0, W_0 - 1]\) as its initial backoff value, and restarts the backoff process in a DIFS. Other time counters also reactivate their previous backoff process in a DIFS.

For clarification, we summarize all types of control frames in Table II. A successful superimposed transmission of data frames consists of the exchanges of RTS, PTA, SAK, DFTrigger, Data and ACK frames, as shown in Figure 3. A collision happens when more than one stations transmit RTSs to the destination simultaneously.

D. An Example of DS-CSMA/CA

We give an example on the proposed DS-CSMA/CA protocol in this subsection. We assume 5 stations with the partner map in Equation (1), where 8 TCPairs are deployed to control data transmission. Provides that \(W_{i,j} = W_0 = 32\) and \(W_{max} = 128\), and the states of 8 TCPairs at certain time point are shown in the first column of Table III.

After 3 time slots, the TCPairs are illustrated in the second column of Table III where \(T_{1,2}\) and \(T_{4,3}\) are both reduced to 0. Accordingly, stations 1 and 4 both transmit the RTS frame to the destination, where a collision happens. Then, they conduct steps 1a), 1b) and 1e). The new initial backoff values are chosen from \([0, 64]\), e.g., \(T_{1,2} = 26, T_{4,3} = 58\).

After another 2 time slots, the TCPairs are self-decrement twice and turn to the states shown in the third column of Table III where only \(T_{3,2}\) is reduced to 0. Therefore, stations 3 and 4
Fig. 2. Illustration of a successful superimposed transmission.

Fig. 3. Frame sequence of a successful superimposed transmission.

conduct a successful superimposed transmission as shown in Figure 3 where station 3 conducts steps 1a)-1e); and station 4 conducts steps 3a)-3d).

III. STATE TRANSITION MODEL FOR BOOSTED CSMA/CA

Since the evolutions of different TCPairs are independent of each other, we can choose to investigate certain TCPair instead of the whole system. The subscript of time counters \((T_{i, j}, T_{i', j'})\) can be simplified into \((T_1, T_2)\) characterized by parameters \(m, n, i, j\), where \(m\) and \(n\) notate that the contention windows of the \(T_1\) and \(T_2\) equal \(W_m = 2^m W_0\) and \(W_n = 2^n W_0\), respectively; and \(i\) and \(j\) notate that \((T_1, T_2) = (i, j)\). Furthermore, let \(P(m, n, i, j)\) denote the probability of state \((m, n, i, j)\),
3 time slots later

5 time slots later

and $\mathbb{P}(m, n, i, j | m', n', i', j')$ denote the state transition probability from state $(m', n', i', j')$ to state $(m, n, i, j)$, where $0 \leq m, m', n, n' \leq M - 1, 0 \leq i, i' \leq W_m - 1, 0 \leq j, j' \leq W_n - 1$.

### A. State Transition Probabilities

First of all, for $0 \leq i \leq W_m - 1, 0 \leq j \leq W_n - 1$, both $T_1$ and $T_2$ must be reduced by one in the next slot. Hence the transition probability is given by

$$\mathbb{P}(m, n, i, j | m, n, i + 1, j + 1) = 1.$$ (2)

The remaining cases depend on parameter $p$, the probability that a collision happens in the channel due to other TCPairs’ contention. Probability $p$ is determined by parameters $N$ and $\eta$ as follows,

$$p = 1 - (1 - \eta)^{N - 1},$$ (3)

where $N = |\Phi| = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \phi_{i,j}$ denotes the number of TCPairs as shown in Table I, and

$$\eta = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \left[ \sum_{i=1}^{W_m-1} \mathbb{P}(m, n, i, 0) + \sum_{j=1}^{W_n-1} \mathbb{P}(m, n, 0, j) \right]$$ (4)

denotes the overall probability that one of the time counters is reduced to 0.

Secondly, for $i = 0$ or $j = 0$, it will lead to two cases of state-transmission. A new successful transmission, consisting of RTS, PTA, SAK, DFTrigger, Data frame superimposed transmission and ACK, may happen with probability $(1 - p)$. After a successful transmission, $T_1$ and $T_2$ are independently initialized uniformly in the range $[0, W_0 - 1]$. Accordingly, the transition

| Initial | 3 time slots later | 5 time slots later |
|---------|-------------------|-------------------|
| $(T_{1,1}, T_{2,1}) = (3, 28)$ | $(T_{1,1}, T_{2,1}) = (0, 25)$ | $(T_{1,1}, T_{2,1}) = (24, 23)$ |
| $(T_{1,2}, T_{4,1}) = (19, 13)$ | $(T_{1,2}, T_{4,1}) = (16, 10)$ | $(T_{1,2}, T_{4,1}) = (14, 8)$ |
| $(T_{1,3}, T_{5,1}) = (8, 30)$ | $(T_{1,3}, T_{5,1}) = (5, 27)$ | $(T_{1,3}, T_{5,1}) = (3, 25)$ |
| $(T_{2,2}, T_{3,1}) = (24, 6)$ | $(T_{2,2}, T_{3,1}) = (21, 3)$ | $(T_{2,2}, T_{3,1}) = (19, 1)$ |
| $(T_{2,3}, T_{4,2}) = (9, 31)$ | $(T_{2,3}, T_{4,2}) = (6, 28)$ | $(T_{2,3}, T_{4,2}) = (4, 26)$ |
| $(T_{2,4}, T_{5,2}) = (11, 17)$ | $(T_{2,4}, T_{5,2}) = (8, 14)$ | $(T_{2,4}, T_{5,2}) = (6, 12)$ |
| $(T_{3,2}, T_{4,3}) = (5, 33)$ | $(T_{3,2}, T_{4,3}) = (2, 0)$ | $(T_{3,2}, T_{4,3}) = (0, 56)$ |
| $(T_{3,3}, T_{5,3}) = (29, 7)$ | $(T_{3,3}, T_{5,3}) = (26, 4)$ | $(T_{3,3}, T_{5,3}) = (24, 2)$ |
probabilities are given by
\[ P(0, 0, i, j|m, n, i', 0) = \frac{1 - p}{W_0^2}, \quad P(0, 0, i, j|m, n, 0, j') = \frac{1 - p}{W_0^2}. \] (5)

Besides, the RTS may collide with that from other stations with probability \( p \). In this case, if \( m, n \neq M - 1 \), either \( T_1 \) or \( T_2 \) may be uniformly chosen in the range \([0, W_{m+1} - 1]\) or \([0, W_{n+1} - 1]\) with probability \( p \), and thus the transition probabilities are given by
\[ P(m + 1, n, i, j|m, n, 0, j + 1) = \frac{p}{W_{m+1}}, \quad P(m, n + 1, i, j|m, n, i + 1, 0) = \frac{p}{W_{n+1}}. \] (6)

Furthermore, if \( m = M - 1 \) or \( n = M - 1 \), the contention windows of either \( T_1 \) or \( T_2 \) will never become doubled in subsequent transmission, and the transition probabilities are given by
\[ P(M - 1, n, i, j|M - 1, n, 0, j + 1) = P(m, M - 1, i, j|m, M - 1, i + 1, 0) = \frac{p}{W_{M-1}}. \] (7)

In addition, if \( m = n = M - 1 \), neither \( T_1 \) or \( T_2 \) will be doubled the contention window in subsequent transmissions. Hence, the transition probabilities are given by
\[ P(M - 1, M - 1, i, j|M - 1, M - 1, 0, j + 1) = P(M - 1, M - 1, i, j|M - 1, M - 1, i + 1, 0) = \frac{p}{W_{M-1}}. \] (8)

Finally, considering \( i = j = 0 \), where \( T_1 \) and \( T_2 \) decrease to 0 simultaneously, where a collision must happen. For \( m, n \neq M - 1 \), \( T_1 \) and \( T_2 \) will uniformly take values in \([0, W_{m+1} - 1]\) and \([0, W_{n+1} - 1]\), respectively. The transition probability is given by
\[ P(m + 1, n + 1, i, j|m, n, 0, 0) = \frac{1}{W_{m+1}W_{n+1}}. \] (9)

For \( m = M - 1 \) or \( n = M - 1 \), either \( W_m \) or \( W_n \) is never doubled since it is up to the maximum contention window \( W_{\text{max}} \). Then, the transition probabilities are given by,
\[ P(M - 1, n + 1, i, j|M - 1, n, 0, 0) = \frac{1}{W_{M-1}W_{n+1}}, \]
\[ P(m + 1, M - 1, i, j|m, M - 1, 0, 0) = \frac{1}{W_{m+1}W_{M-1}}. \] (10)

For \( m = n = M - 1 \), neither \( W_m \) or \( W_n \) will be doubled, and the transition probabilities are given by
\[ P(M - 1, M - 1, i, j|M - 1, M - 1, 0, 0) = \frac{1}{W_{M-1}^2}. \] (11)
B. The State Probabilities

We obtain the state probabilities in this subsection based on the state transition probabilities. Since \( i \) and \( j \) take values in \([0, W_m - 1]\) and \([0, W_n - 1]\), respectively, it is convinced that \( \mathbb{P}(m, n, W_m, \bullet) = \mathbb{P}(m, n, \bullet, W_n) = \mathbb{P}(m, n, W_m, W_n) = 0 \) for \( 0 \leq m, n \leq M - 1 \). We summarize all possible state probabilities into the following 7 cases.

**Case 1:** For \( m = n = 0, 0 \leq i \leq W_0 - 1, 0 \leq j \leq W_0 - 1 \), as shown in Figure 4(a), the previous states of \((0, 0, i, j)\) are \((0, 0, i + 1, j + 1)\), \((0, 0, 0, 1)\), \((0, 0, 0, 2)\), \(\ldots\), \((M - 1, M - 1, 0, W_{M-1} - 1)\) as well as \((0, 0, 1, 0), (0, 0, 2, 0), \ldots, (M - 1, M - 1, W_{M-1} - 1, 0)\), and thus

\[
\mathbb{P}(0, 0, i, j) = \mathbb{P}(0, 0, i + 1, j + 1) + \frac{1 - p}{W_0^2},
\]

where \( \eta \) is given by Equation (4).

**Case 2:** For \( 0 < m < M - 1, n = 0, 0 \leq i \leq W_m - 1, 0 \leq j \leq W_0 - 1 \), as shown in Figure 4(b), the previous states of \((m, 0, i, j)\) are \((m, 0, i + 1, j + 1)\) as well as \((m - 1, 0, 0, j + 1)\), and thus we have

\[
\mathbb{P}(m, 0, i, j) = \mathbb{P}(m, 0, i + 1, j + 1) + \frac{\mathbb{P}(m - 1, 0, 0, j + 1)}{W_m} p.
\]

**Case 3:** For \( m = M - 1, n = 0, 0 \leq i \leq W_{M-1} - 1 \) and \( 1 \leq j \leq W_0 - 1 \), as shown in Figure 4(c), the previous states of \((M - 1, 0, i, j)\) are \((M - 1, 0, i + 1, j + 1)\), \((M - 2, 0, 0, j + 1)\) as well as \((M - 1, 0, 0, j + 1)\), and thus

\[
\mathbb{P}(M - 1, 0, i, j) = \mathbb{P}(M - 1, 0, i + 1, j + 1) + \frac{\mathbb{P}(M - 2, 0, 0, j + 1)}{W_{M-1}} p + \frac{\mathbb{P}(M - 1, 0, 0, j + 1)}{W_{M-1}} p.
\]

**Case 4:** For \( 0 < n \leq m < M - 1, 0 \leq i \leq W_m - 1, 0 \leq j \leq W_n - 1 \), as shown in Figure 4(d), the previous states of \((m, n, i, j)\) are \((m, n, i + 1, j + 1)\), \((m - 1, n - 1, 0, 0)\), \((m - 1, n, i, j + 1)\) as well as \((m, n - 1, i + 1, j)\), and thus

\[
\mathbb{P}(m, n, i, j) = \mathbb{P}(m, n, i + 1, j + 1) + \frac{\mathbb{P}(m - 1, n - 1, 0, 0)}{W_n W_m} + \frac{\mathbb{P}(m - 1, n, i, j + 1)}{W_m} p + \frac{\mathbb{P}(m, n - 1, i + 1, j)}{W_n} p.
\]

**Case 5:** For \( m = M - 1, 0 < n < M - 1, 0 \leq i \leq W_{M-1} - 1, 0 \leq j \leq W_n - 1 \), as shown in Figure 4(e), the previous states of \((M - 1, n, i, j)\) are \((M - 2, n - 1, 0, 0)\), \((M - 1, n - 1, 0, 0)\), \((M - 2, n, 0, j + 1)\), \(\ldots\), and thus

\[
\mathbb{P}(M - 1, n, i, j) = \mathbb{P}(M - 1, n, i + 1, j + 1) + \frac{\mathbb{P}(M - 2, n - 1, 0, 0)}{W_n W_m} + \frac{\mathbb{P}(M - 1, n - 1, 0, 0)}{W_m} p + \frac{\mathbb{P}(M - 2, n, 0, j + 1)}{W_n} p.
\]
1), \((M - 1, n - 1, i + 1, 0)\) as well as \((M - 1, n, j + 1)\), and thus

\[
\mathbb{P}(M - 1, n, i, j) = \mathbb{P}(M - 1, n, i + 1, j + 1) + \frac{\mathbb{P}(M - 2, n - 1, 0, 0)}{W_{M-1}W_n} + \frac{\mathbb{P}(M - 1, n - 1, 0, 0)}{W_{M-1}W_n}p + \frac{\mathbb{P}(M - 1, n, i + 1, 0)}{W_{M-1}}p + \frac{\mathbb{P}(M - 1, n, j + 1)}{W_{M-1}}p
\]

\[(16)\]

**Case 6:** For \(m = n = M - 1, 0 \leq i, j \leq W_{M-1} - 1\), as shown in Figure 4 (f), the previous states of \((M - 1, M - 1, i, j)\) are \((M - 1, M - 1, i + 1, j + 1), (M - 2, M - 1, 0, j + 1), (M - 1, M - 2, i + 1, 0), (M - 1, M - 1, 0, j + 1), (M - 1, M - 1, i + 1, 0), (M - 1, M - 1, 0, 0), (M - 2, M - 2, 0, 0), (M - 2, M - 1, 0, 0), (M - 1, M - 2, 0, 0)\) as well as \((M - 1, M - 1, 0)\). Therefore, we have

\[
\mathbb{P}(M - 1, M - 1, i, j) = \mathbb{P}(M - 1, M - 1, i + 1, j + 1) + \frac{\mathbb{P}(M - 2, M - 1, 0, 0)}{W_{M-1}W_n} + \frac{\mathbb{P}(M - 1, M - 2, i + 1, 0)}{W_{M-1}}p + \frac{\mathbb{P}(M - 1, M - 1, 0, 0)}{W_{M-1}W_n}p + \frac{\mathbb{P}(M - 1, M - 1, 0, 0)}{W_{M-1}}p
\]

\[(17)\]

**Case 7:** For \(0 \leq m < n \leq M - 1, 0 \leq i \leq W_m - 1\) and \(0 \leq j \leq W_n - 1\), we have

\[
\mathbb{P}(m, n, i, j) = \mathbb{P}(n, m, j, i).
\]

\[(18)\]

Finally, we summarize the state probabilities and related state transition of the 7 cases in the second and third columns of Table IV and the proof is detailed in Appendix A.

**IV. Collision Probability and Throughput**

**A. Numerical Solution of Collision Probability**

For the collision probability in Equation (5), we adopt Newton method to obtain its numerical solution of \(\hat{p}\), where the iteration is given by

\[
\hat{p}^{(v+1)} = \hat{p}^{(v)} - \left[\left(1 - \eta_{p=\hat{p}^{(v)}}\right)^{N-1} + \hat{p}^{(v)} - 1\right] \left[(N - 1)\left(1 - \eta_{p=\hat{p}^{(v)}}\right)^{N-2} \frac{\partial \eta}{\partial p}_{p=\hat{p}^{(v)}}\right]^{-1}
\]

\[(19)\]

where \(\hat{p}^{(v)}, \eta_{p=\hat{p}^{(v)}}\) and \(\frac{\partial \eta}{\partial p}_{p=\hat{p}^{(v)}}\) are the numerical values of \(p\), \(\eta\) and \(\frac{\partial \eta}{\partial p}\) in the \(v\)-th iteration, respectively; and the initial \(\hat{p}^{(0)}\) should take value in range \((0, 1)\). For Equation (19), we give the method to calculate \(\eta\) and \(\frac{\partial \eta}{\partial p}\) based on the state transition model.
(a) State transition of cases 1, \( m = n = 0 \).
(b) State transition of case 2, \( 0 < m < M - 1, n = 0 \).
(c) State transition of case 3, \( m = M - 1, n = 0 \).
(d) State transition of case 4, \( 0 \leq m, n \leq M - 1 \).
(e) State transition of case 5, \( 0 < m < M - 1, n = M - 1 \).
(f) State transition of case 6, \( m = n = M - 1 \).

Fig. 4. State transition of the cases 1-6.

According to Equation (4), \( \eta \) depends on the state probabilities with either \( i = 0 \) or \( j = 0 \). Let \( \epsilon_{m,n} = \mathbb{P}(m,n,0,0), r_{m,n} = [\mathbb{P}(m,n,0,1),\mathbb{P}(m,n,0,2), \cdots ,\mathbb{P}(m,n,0,W_n - 1)]^T \) and \( d_{m,n} = [\mathbb{P}(m,n,1,0),\mathbb{P}(m,n,2,0), \cdots ,\mathbb{P}(m,n,W_m - 1,0)]^T \) for \( 0 \leq m, n \leq M - 1 \). The state probabilities in Equations (12)-(18) can be expressed in vector form in Theorem 1 based on the transition matrices \( A_{r,r,m,n}, A_{d,r,m,n}, A_{r,d,m,n} \) and \( A_{d,d,m,n} \) given in Figures 5(a)-5(d) respectively.

**Theorem 1.** The vector form of state probabilities corresponding to the 7 cases in Equations (12)-(18) can be summarized as follows,
Case 1: For \( m = 0, n = 0 \) and \( 0 \leq i, j \leq W_0 - 1 \), we have

\[
    r_{0,0,i} = d_{0,0,i} = (W_0 - i)W_0^{-1} \epsilon_{0,0}.
\]  (20)

Case 2: For \( 0 < m < M - 1 \) and \( n = 0 \), we have

\[
    \begin{align*}
    r_{m,0} & = W_m^{-1} pA_{r,r,m,n} r_{m-1,0}, \\
    d_{m,0} & = W_m^{-1} pA_{d,d,m,n} r_{m-1,0}, \\
    \epsilon_{m,0} & = W_m^{-1} p1^T_{W_{m-1}} r_{m-1,0}.
    \end{align*}
\]  (21)

Case 3: For \( n = 0 \) and \( m = M - 1 \), we have

\[
    \begin{align*}
    r_{M-1,0} & = (I - W_{M-1}^{-1} pA_{r,r,m,n})^{-1} (W_{M-1}^{-1} pA_{r,r,m,n} r_{M-2,0}), \\
    d_{M-1,0} & = W_{M-1}^{-1} pA_{d,d,m,n} (r_{M-2,0} + r_{M-1,0}), \\
    \epsilon_{M-1,0} & = W_{M-1}^{-1} p1^T_{W_{m-1}} (r_{M-2,0} + r_{M-1,0}).
    \end{align*}
\]  (22)

Case 4: For \( 0 < n \leq m < M - 1 \), we have

\[
    \begin{align*}
    r_{m,n} & = W_m^{-1} pA_{r,r,m,n} r_{m-1,n} + W_n^{-1} pA_{d,d,m,n} d_{m,n-1} + W_m^{-1} W_n^{-1} \epsilon_{m-1,n-1} u_{n,n}, \\
    d_{m,n} & = W_m^{-1} pA_{r,d,m,n} r_{m-1,0} + W_n^{-1} pA_{d,d,m,n} d_{m,n-1} + W_m^{-1} W_n^{-1} \epsilon_{m-1,n-1} u_{m,n}, \\
    \epsilon_{m,n} & = W_m^{-1} p1^T_{W_m} \epsilon_{m-1,n-1} + W_n^{-1} p1^T_{W_n} d_{m,n-1,[1,W_m]} + W_m^{-1} \epsilon_{m-1,n-1}.
    \end{align*}
\]  (23)

where \( u_{m,n} = [W_{m}, W_{n}, \ldots, W_{n}, W_{n-1}, \ldots, 1]. \)
Case 5: For \( m = M - 1 \) and \( 0 < n < M - 1 \), we have
\[
\begin{align*}
\mathbf{r}_{M-1,n} &= (I - W_{M-1}^{-1}p\mathbf{A}_{r,r,m,n})^{-1}\left[ W_{M-1}^{-1}p\mathbf{A}_{r,r,m,n} \mathbf{r}_{M-2,n} + W_n^{-1}p\mathbf{A}_{d,d,m,n} \mathbf{d}_{M-1,n-1} + W_{M-1}^{-1}W_n^{-1}(\varepsilon_{M-2,n-1} + \varepsilon_{M-1,n}) \mathbf{u}_{n,n} \right], \\
\mathbf{d}_{M-1,n} &= W_{M-1}^{-1}p\mathbf{A}_{d,d,m,n}(\mathbf{r}_{M-2,n} + \mathbf{r}_{M-1,n}) + W_n^{-1}p\mathbf{A}_{d,d,m,n} \mathbf{d}_{M-1,n-1} + W_{M-1}^{-1}W_n^{-1}(\varepsilon_{M-2,n-1} + \varepsilon_{M-1,n-1}) \mathbf{u}_{M-1,n}, \\
\varepsilon_{M-1,n} &= W_{M-1}^{-1}p\mathbf{1}_W^T \mathbf{W}_{M-1}^{-1}(\mathbf{r}_{M-2,n} + \mathbf{r}_{M-1,n}) + W_n^{-1}p\mathbf{1}_W^T \mathbf{d}_{M-1,n-1,1,W_n} + W_{M-1}^{-1}(\varepsilon_{M-2,n-1} + \varepsilon_{M-1,n-1}).
\end{align*}
\] (24)

Case 6: For \( m = n = M - 1 \), we have
\[
\begin{align*}
\varepsilon_{M-1,M-1} &= \left[ 1 - 2W_{M-1}^{-3}p\mathbf{1}_W^T (I - W_{M-1}^{-1}p\mathbf{A}^\sigma)^{-1} \mathbf{u}_{M-1,M-1} - W_{M-1}^{-1} \right]^{-1} \left[ 2W_{M-1}^{-1}p\mathbf{1}_W^T \mathbf{d}_{M-1,M-2} \\
&\quad + W_{M-1}^{-1}\varepsilon^\sigma + 2W_{M-1}^{-1}p\mathbf{1}_W^T (I - W_{M-1}^{-1}p\mathbf{A}^\sigma)^{-1}(W_{M-1}^{-1}p\mathbf{A}^\sigma \mathbf{d}_{M-1,M-2} + W_{M-1}^{-1}\varepsilon^\sigma \mathbf{u}_{M-1,M-1}) \right], \\
\mathbf{r}_{M-1,M-1} &= \mathbf{d}_{M-1,M-1} \\
&\quad = (I - W_{M-1}^{-1}p\mathbf{A}^\sigma)^{-1}\left[ W_{M-1}^{-1}p\mathbf{A}^\sigma \mathbf{d}_{M-1,M-2} + W_{M-1}^{-2}(\varepsilon^\sigma + \varepsilon_{M-1,M-1}) \mathbf{u}_{M-1,M-1} \right],
\end{align*}
\] (25)
where \( \mathbf{A}^\sigma = \mathbf{A}_{r,r,M-1,M-1} + \mathbf{A}_{r,d,M-1,M-1} = \mathbf{A}_{d,d,M-1,M-1} + \mathbf{A}_{d,r,M-1,M-1} \) and \( \varepsilon^\sigma = \varepsilon_{M-2,M-2} + \varepsilon_{M-1,M-1} + \varepsilon_{M-1,M-2} \).

Case 7: For \( 0 \leq m < n \leq M - 1 \), we have \( \mathbf{r}_{m,n} = \mathbf{d}_{n,m}, \mathbf{d}_{m,n} = \mathbf{r}_{n,m}, \varepsilon_{m,n} = \varepsilon_{n,m} \).

Proof: Please refer to Appendix B.

Based on Theorem 1, we can use \( \varepsilon_{0,0} \) to obtain \( \mathbf{r}_{m,n,i}, \mathbf{d}_{m,n,j} \) and \( \varepsilon_{m,n} \) for \( 0 \leq m, n \leq M - 1 \), where \( \mathbf{r}_{m,n,i} \) and \( \mathbf{d}_{m,n,j} \) are the \( i \)-th and \( j \)-th element of \( \mathbf{r}_{m,n} \) and \( \mathbf{d}_{m,n} \), respectively, for \( 0 \leq i \leq W_m - 1, 0 \leq j \leq W_n - 1 \). Since \( \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \sum_{i=0}^{W_m-1} \sum_{j=0}^{W_n-1} \mathbb{P}(m, n, i, j) = 1 \), letting \( \mathbb{P}(m, n) = \sum_{i=0}^{W_m-1} \sum_{j=0}^{W_n-1} \mathbb{P}(m, n, i, j) \) denote the probability that the contention windows of \( T_1 \) and \( T_2 \) equal \( W_m \) and \( W_n \), respectively, we have the following Theorem 2 to calculate \( \mathbb{P}(m, n) \) for \( 0 \leq m, n \leq M - 1 \).

Theorem 2. Corresponding to the 7 cases in Theorem 1, probability \( \mathbb{P}(m, n) \) can be calculated as follows,

Case 1: For \( m = n = 0 \), we have
\[
\mathbb{P}(0,0) = \frac{1}{6}(2W_0 + 1)(W_0 + 1)\varepsilon_{0,0}.
\] (26)

Case 2: For \( 0 \leq m < M - 1 \) and \( n = 0 \), we have
\[
\mathbb{P}(m,0) = W_m^{-1}p \sum_{i=1}^{W_m-1} \left[ \frac{1}{2}i^2 + \left( W_m + \frac{1}{2} \right)i \right].
\] (27)
Case 3: For $m = M - 1$ and $n = 0$, we have
\[
\mathbb{P}(M - 1, 0) = W^{-1}_{M-1}p \sum_{i=1}^{W_{0-1}} (r_{M-2,0,i} + r_{M-1,0,i}) \left[ -\frac{1}{2}i^2 + \left(W_{M-1} + \frac{1}{2}\right)i \right].
\] (28)

Case 4: For $0 < n \leq m < M - 1$, we have
\[
\mathbb{P}(m, n) = W^{-1}_m \sum_{i=1}^{W_{m-1}} r_{m-n,i} \left[ -\frac{1}{2}i^2 + \left(W_m + \frac{1}{2}\right)i \right] + W^{-1}_n \sum_{i=1}^{W_{n-1}} d_{m,n-1,i} \left[ -\frac{1}{2}i^2 + \left(W_n + \frac{1}{2}\right)i \right]
\]
\[
+ W^{-1}_n \sum_{i=W_m}^{W_{n-1}} d_{m,n-1,i} \left(-\frac{1}{2}W^2_n + \frac{1}{2}W_n\right) + W^{-1}_m W^{-1}_n \epsilon_{m-1,n-1} \left(-\frac{1}{6}W^2_n + \frac{1}{2}W_nW_n + \frac{1}{2}W_mW_n + \frac{1}{6}W_n\right).
\] (29)

Case 5: For $m = M - 1$ and $0 < n < M - 1$, we have
\[
\mathbb{P}(M - 1, n) = W^{-1}_{M-1}p \sum_{i=1}^{W_{m-1}} (r_{M-2,n,i} + r_{M-1,n,i}) \left[ -\frac{1}{2}i^2 + \left(W_{M-1} + \frac{1}{2}\right)i \right] + W^{-1}_n \sum_{i=1}^{W_{n-1}} d_{M-1,n-1,i} \left[ -\frac{1}{2}i^2 + \left(W_n + \frac{1}{2}\right)i \right]
\]
\[
+ W^{-1}_n \sum_{i=W_m}^{W_{n-1}} d_{M-1,n-1,i} \left(-\frac{1}{2}W^2_n + \frac{1}{2}W_n\right) + W^{-1}_m W^{-1}_n \epsilon^\sigma \left(-\frac{1}{6}W^2_n + \frac{1}{2}W_nW_n + \frac{1}{2}W_{M-1}W_n + \frac{1}{6}W_n\right).
\] (30)

where $\epsilon^\sigma = \epsilon_{M-2,n-1} + \epsilon_{M-1,n-1}$.

Case 6: For $m = M - 1$ and $n = M - 1$, we have
\[
\mathbb{P}(M - 1, M - 1) = W^{-1}_{M-1}p \sum_{i=1}^{W_{m-1}} d^\sigma_i \left[ -\frac{1}{2}i^2 + \left(W_{M-1} + \frac{1}{2}\right)i \right] + W^{-2}_{M-1} \epsilon^\sigma \left(-\frac{1}{3}W^3_{M-1} + \frac{1}{2}W^2_{M-1} + \frac{1}{6}W_{M-1}\right).
\] (31)

where $d^\sigma_i = r_{m-1,n,i} + r_{m,n,i} + d_{m,n-1,i} + d_{m,n,i}$ and $\epsilon^\sigma = \epsilon_{M-2,M-2} + \epsilon_{M-2,M-1} + \epsilon_{M-1,M-2} + \epsilon_{M-1,M-1}$.

Case 7: For $0 \leq m < n \leq M - 1$, we have that $\mathbb{P}(m,n) = \mathbb{P}(n,m)$.

Proof: Please refer to Appendix C.

Based on Theorems 1 and 2, we can utilize $\epsilon_{0,0}$ to obtain $\mathbb{P}(m,n)$ for $0 \leq m, n \leq M - 1$, and adopt the condition that $\sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \mathbb{P}(m,n) = 1$ to calculate $\epsilon_{0,0}$. Furtherly, we adopt Theorem 1 to obtain $r_{m,n,i}$, $d_{n,0,i}$, and $\epsilon_{m,n}$ for $0 \leq i \leq W_m - 1, 0 \leq j \leq W_n - 1, 0 \leq m, n \leq M - 1$ based on $\epsilon_{0,0}$, and calculate $\eta_{|p=\tilde{p}(v)}$ according to Equation (4) or its equivalent vector as follows,
\[
\eta = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \mathbf{1}_{W_{m-1}}^T r_{m,n} + \mathbf{1}_{W_{m-1}}^T d_{m,n}.
\] (32)
Moreover, in order to calculate $\frac{\partial \eta}{\partial p}|_{p=p^0}$ in Equation (19), we have that

$$\frac{\partial \eta}{\partial p} = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} T_{W_n-1}^T \nabla_p \mathbf{r}_{m,n} + T_{W_n-1}^T \nabla_p \mathbf{d}_{m,n},$$

(33)

where $\nabla_p \mathbf{r}_{m,n} = \left[ \frac{\partial \mathbf{r}(m,n,0,1)}{\partial p}, \frac{\partial \mathbf{r}(m,n,0,2)}{\partial p}, \ldots, \frac{\partial \mathbf{r}(m,n,W_n-1)}{\partial p} \right]^T$ and $\nabla_p \mathbf{d}_{m,n} = \left[ \frac{\partial \mathbf{d}(m,n,0,1)}{\partial p}, \frac{\partial \mathbf{d}(m,n,0,2)}{\partial p}, \ldots, \frac{\partial \mathbf{d}(m,n,W_n-1)}{\partial p} \right]^T$. According to Theorem 2, we have Lemma 1 to calculate $\nabla_p \mathbf{r}_{m,n}, \nabla_p \mathbf{d}_{m,n}$ and $\frac{\partial \epsilon_{m,n}}{\partial p}$ as follows, where the proof is omitted since it is based on standard calculus.

**Lemma 1.** Corresponding to the 7 cases in Theorem 1, $\nabla_p \mathbf{r}_{m,n}, \nabla_p \mathbf{d}_{m,n}$ and $\frac{\partial \epsilon_{m,n}}{\partial p}$ can be characterized as follows,

**Case 1:** For $m = n = 0$, and $0 \leq i, j \leq W_0 - 1$, we have

$$\frac{\partial r_{0,0,i}}{\partial p} = \frac{\partial d_{0,0,i}}{\partial p} = (W_0 - i)W_0^{-1} \frac{\partial \epsilon_{0,0}}{\partial p}.$$  
(34)

**Case 2:** For $0 < m < M - 1$ and $n = 0$, we have

$$\nabla_p \mathbf{r}_{m,0} = W_m^{-1} \mathbf{A}_{r,m,0} (\nabla_p \mathbf{r}_{m-1,0} + p \nabla_p \mathbf{r}_{m-1,0}),$$

$$\nabla_p \mathbf{d}_{m,0} = W_m^{-1} \mathbf{A}_{d,m,0} (\nabla_p \mathbf{r}_{m-1,0} + p \nabla_p \mathbf{r}_{m-1,0}),$$

$$\frac{\partial \epsilon_{m,0}}{\partial p} = W_m^{-1} T_{W_n-1}^T (\nabla_p \mathbf{r}_{m-1,0} + p \nabla_p \mathbf{r}_{m-1,0}).$$  
(35)

**Case 3:** For $m = M - 1$ and $n = 0$, we have

$$\nabla_p \mathbf{r}_{M-1,0} = (I - W_{M-1}^{-1} p \mathbf{A}_{r,m,0})^{-1} W_{M-1}^{-1} p \mathbf{A}_{r,m,0} (\nabla_p \mathbf{r}_{M-1,0} + p \nabla_p \mathbf{r}_{M-1,0}),$$

$$\nabla_p \mathbf{d}_{M-1,0} = W_{M-1}^{-1} \mathbf{A}_{d,m,0} (\nabla_p \mathbf{r}_{M-1,0} + p \nabla_p \mathbf{r}_{M-1,0}),$$

$$\frac{\partial \epsilon_{M-1,0}}{\partial p} = W_{M-1}^{-1} T_{W_n-1}^T (\nabla_p \mathbf{r}_{M-1,0} + p \nabla_p \mathbf{r}_{M-1,0}).$$  
(36)

**Case 4:** For $0 < n \leq m < M - 1$, we have

$$\nabla_p \mathbf{r}_{m,n} = W_m^{-1} \mathbf{A}_{r,m,n} (\nabla_p \mathbf{r}_{m-1,n} + p \nabla_p \mathbf{r}_{m-1,n}) + W_n^{-1} \mathbf{A}_{d,m,n} (\nabla_p \mathbf{d}_{m,n-1} + p \nabla_p \mathbf{d}_{m,n-1}) + W_m^{-1} W_n^{-1} \frac{\partial \epsilon_{m-1,n-1}}{\partial p} \mathbf{u}_{m,n},$$

$$\nabla_p \mathbf{d}_{m,n} = W_m^{-1} \mathbf{A}_{d,m,n} (\nabla_p \mathbf{r}_{m-1,n} + p \nabla_p \mathbf{r}_{m-1,n}) + W_n^{-1} \mathbf{A}_{d,d,m,n} (\nabla_p \mathbf{d}_{m,n-1} + p \nabla_p \mathbf{d}_{m,n-1}) + W_m^{-1} W_n^{-1} \frac{\partial \epsilon_{m-1,n-1}}{\partial p} \mathbf{u}_{m,n},$$

$$\frac{\partial \epsilon_{m,n}}{\partial p} = W_m^{-1} W_n^{-1} T_{W_n-1}^T (\nabla_p \mathbf{r}_{m-1,n} + p \nabla_p \mathbf{r}_{m-1,n}) + W_n^{-1} T_{W_n-1}^T (\nabla_p \mathbf{d}_{m,n-1,1} + p \nabla_p \mathbf{d}_{m,n-1,1}) + W_m^{-1} \frac{\partial \epsilon_{m-1,n-1}}{\partial p}.$$  
(37)
Case 5: For \( m = M - 1 \) and \( 0 < n < M - 1 \), we have

\[
\nabla_p r_{M-1,n} = (I - W_{M-1}^{-1}pA_{r,r,m,n})^{-1}\left[W_{M-1}^{-1}A_{r,r,m,n}(\nabla_p r_{M-2,n} + \nabla_p r_{M-1,n} + p\nabla_p r_{M-2,n}) + W_n^{-1}A_{d,d,m,n}\right]
\]

\[
\nabla_p d_{M-1,n} = W_{M-1}^{-1}A_{r,d,m,n}(\nabla_p r_{M-2,n} + \nabla_p r_{M-1,n} + p\nabla_p r_{M-2,n} + p\nabla_p r_{M-1,n}) + W_n^{-1}pA_{d,d,m,n}(\nabla_p d_{M-1,n-1} + p\nabla_p d_{M-1,n-1}) + W_{M-1}^{-1}W_n^{-1}\left(\frac{\partial \epsilon_{M-2,n-1}}{\partial p} + \frac{\partial \epsilon_{M-1,n-1}}{\partial p}\right)\mathbf{u}_{M-1,n}.
\]

\[
\frac{\partial \epsilon_{M-1,n}}{\partial p} = W_{M-1}^{-1}1^T_{W_n^{-1}}(\nabla_p r_{M-2,n} + \nabla_p r_{M-1,n} + p\nabla_p r_{M-2,n} + p\nabla_p r_{M-1,n}) + W_n^{-1}1^T_{W_n^{-1}}(\nabla_p d_{M-1,n-1}1_{W_n} + p\nabla_p d_{M-1,n-1}1_{W_n}) + W_{M-1}^{-1}\left(\frac{\partial \epsilon_{M-2,n-1}}{\partial p} + \frac{\partial \epsilon_{M-1,n-1}}{\partial p}\right).
\]

(38)

Case 6: For \( m = n = M - 1 \), we have

\[
\frac{\partial \epsilon_{M-1,M-1}}{\partial p} = \left[1 - 2W_{M-1}^{-1}p1^T_{W_{M-1}}(I - W_{M-1}^{-1}pA^\sigma) - 1\right]\left\{2W_{M-1}^{-1}1^T_{W_{M-1}}(\nabla_p d_{M-1,M-2} + p\nabla_p d_{M-1,M-2}) + W_{M-1}^{-1}\frac{\partial \epsilon^\sigma}{\partial p} + 2W_{M-1}^{-1}p1^T_{W_{M-1}}(I - W_{M-1}^{-1}pA^\sigma) - 1\right\}[W_{M-1}^{-1}A^\sigma
\]

\[
(\nabla_p d_{M-1,M-2} + \nabla_p d_{M-1,M-1} + p\nabla_p d_{M-1,M-2}) + W_{M-1}^{-2}\frac{\partial \epsilon^\sigma}{\partial p}u_{M-1,M-1} + W_{M-1}^{-1}\left(\frac{\partial \epsilon_{M-2,M-2}}{\partial p} + \frac{\partial \epsilon_{M-2,M-1}}{\partial p} + \frac{\partial \epsilon_{M-1,M-2}}{\partial p}\right)\mathbf{u}_{M-1,M-1}.
\]

\[
\nabla_p r_{M-1,M-1} = \nabla_p d_{M-1,M-1}
\]

\[
= (I - W_{M-1}^{-1}pA^\sigma)^{-1}\left[W_{M-1}^{-1}pA^\sigma(\nabla_p d_{M-1,M-2} + \nabla_p d_{M-1,M-1} + p\nabla_p d_{M-1,M-2})
\]

\[
+ W_{M-1}^{-2}\left(\frac{\partial \epsilon^\sigma}{\partial p} + \frac{\partial \epsilon_{M-2,M-1}}{\partial p}\right)\mathbf{u}_{M-1,M-1} + W_{M-1}^{-1}\left(\frac{\partial \epsilon_{M-2,M-2}}{\partial p} + \frac{\partial \epsilon_{M-2,M-1}}{\partial p} + \frac{\partial \epsilon_{M-1,M-2}}{\partial p}\right)\mathbf{u}_{M-1,M-1}.
\)

(39)

where \( A^\sigma = A_{r,r,M-1,M-1} + A_{r,d,M-1,M-1} = A_{d,d,M-1,M-1} + A_{d,r,M-1,M-1} \); and

\[
\frac{\partial \epsilon^\sigma}{\partial p} = \frac{\partial \epsilon_{M-2,M-2}}{\partial p} + \frac{\partial \epsilon_{M-2,M-1}}{\partial p} + \frac{\partial \epsilon_{M-1,M-2}}{\partial p}.
\]

(40)

Case 7: For \( 0 \leq m < n \leq M - 1 \), we have that \( \nabla_p r_{m,n} = \nabla_p d_{n,m} \), \( \nabla_p d_{m,n} = \nabla_p r_{n,m} \), \( \frac{\partial \epsilon_{m,n}}{\partial p} = \frac{\partial \epsilon_{n,m}}{\partial p} \) for \( 0 \leq m, n \leq M - 1 \). Furthermore, the normalization condition of \( \nabla_p r_{m,n}, \nabla_p d_{m,n} \) and \( \frac{\partial \epsilon_{m,n}}{\partial p} \) is given by the following equation,

\[
\sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \frac{\partial \mathbb{P}(m,n)}{\partial p} = \frac{\partial}{\partial p} \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \mathbb{P}(m,n) = 0.
\]

(41)
Therefore, we propose the following Lemma 2 to calculate \( \frac{\partial \tilde{P}(m,n)}{\partial p} \) using \( \nabla_p r_{m,n}, \nabla_p d_{m,n} \) and \( \frac{\partial \epsilon_{m,n}}{\partial p} \) for \( 0 \leq m, n \leq M - 1 \).

**Lemma 2.** Based on the 7 cases in Theorem 2, the partial derivative \( \frac{\partial \tilde{P}(m,n)}{\partial p} \) for \( 0 \leq m, n \leq M - 1 \) can be computed as follows,

**Case 1:** For \( m = n = 0 \), we have

\[
\frac{\partial \tilde{P}(0,0)}{\partial p} = \frac{1}{6}(2W_0 + 1)(W_0 + 1) \frac{\partial \epsilon_{0,0}}{\partial p}.
\] (42)

**Case 2:** For \( 0 < m < M - 1, \) and \( n = 0 \), we have

\[
\frac{\partial \tilde{P}(m,0)}{\partial p} = W_m^{-1} \sum_{i=1}^{W_0-1} \left( k \frac{\partial}{\partial p} r_{m-1,0,i} + p \frac{\partial r_{m-1,0,i}}{\partial p} \right) \left[ - \frac{1}{2} i^2 + \left( W_m + \frac{1}{2} \right) i \right].
\] (43)

**Case 3:** For \( m = M - 1 \) and \( n = 0 \), we have

\[
\frac{\partial \tilde{P}(M-1,0)}{\partial p} = W_{M-1}^{-1} \sum_{i=1}^{W_0-1} \left( k \frac{\partial}{\partial p} r_{M-2,0,i} + k \frac{\partial}{\partial p} r_{M-1,0,i} + p \frac{\partial r_{M-2,0,i}}{\partial p} + p \frac{\partial r_{M-1,0,i}}{\partial p} \right) \left[ - \frac{1}{2} i^2 + \left( W_{M-1} + \frac{1}{2} \right) i \right].
\] (44)

**Case 4:** For \( 0 < n \leq m < M - 1 \), we have

\[
\frac{\partial \tilde{P}(m,n)}{\partial p} = W_m^{-1} \sum_{i=1}^{W_n-1} \left( k \frac{\partial}{\partial p} d_{m-1,n,i} + p \frac{\partial d_{m-1,n,i}}{\partial p} \right) \left[ - \frac{1}{2} i^2 + \left( W_m + \frac{1}{2} \right) i \right] + W_n^{-1} \sum_{i=1}^{W_m-1} \left( \frac{\partial}{\partial p} d_{m,n-1,i} + p \frac{\partial d_{m,n-1,i}}{\partial p} \right) + W_m^{-1} W_n^{-1} \frac{\partial \epsilon_{m-1,n-1}}{\partial p}
\]
\[
\left( - \frac{1}{6} W_n^3 + \frac{1}{2} W_m W_n^2 + \frac{1}{2} W_m W_n + \frac{1}{6} W_n \right).
\] (45)

**Case 5:** For \( m = M - 1 \) we have \( 0 < n < M - 1 \), we have

\[
\frac{\partial \tilde{P}(M-1,n)}{\partial p} = W_{M-1}^{-1} \sum_{i=1}^{W_n-1} \left( k \frac{\partial}{\partial p} d_{M-2,n,i} + k \frac{\partial}{\partial p} d_{M-1,n,i} + p \frac{\partial d_{M-2,n,i}}{\partial p} + p \frac{\partial d_{M-1,n,i}}{\partial p} \right) \left[ - \frac{1}{2} i^2 + \left( W_{M-1} + \frac{1}{2} \right) i \right]
\]
\[
+ W_n^{-1} \sum_{i=1}^{W_m-1} \left( \frac{\partial}{\partial p} d_{M-1,n-1,i} + p \frac{\partial d_{M-1,n-1,i}}{\partial p} \right) \left[ - \frac{1}{2} i^2 + \left( W_m + \frac{1}{2} \right) i \right] + \left( - \frac{1}{2} W_n + \frac{1}{2} \right) \sum_{i=W_m}^{W_{M-1}-1} \left( \frac{\partial}{\partial p} d_{M-1,n-1,i} + p \frac{\partial d_{M-1,n-1,i}}{\partial p} \right)
\]
\[
+ \left( - \frac{1}{6} W_n^3 + \frac{1}{2} W_{M-1} W_n^2 + \frac{1}{2} W_{M-1} W_n + \frac{1}{6} W_n \right).
\] (46)
TABLE IV
THE EXPRESSION OF VECTOR FORM OF STATE PROBABILITIES.

| State transition | State probability | \( r_{m,n}, d_{m,n}, e_{m,n} \) | \( P(m,n) \) | \( \nabla_p r_{m,n}, \nabla_p d_{m,n}, \frac{\partial \epsilon_{m,n}}{\partial p} \) | \( \frac{\partial \epsilon_{m,n}}{\partial p} \) |
|------------------|-------------------|-----------------|------------|---------------------------------|-----------------|
| \( m = n = 0 \)  | Fig. 4(a)         | (12)            | (20)       | (26)                            | (34)            |
| \( m = M - 1, n = 0 \) | Fig. 4(b)      | (13)            | (21)       | (27)                            | (35)            |
| \( 0 < m < M - 1, n = 0 \) | Fig. 4(c)    | (14)            | (22)       | (28)                            | (36)            |
| \( 0 < n \leq m < M - 1 \) | Fig. 4(d)    | (15)            | (23)       | (29)                            | (37)            |
| \( m = M - 1, 0 < n < M - 1 \) | Fig. 4(e)    | (16)            | (24)       | (30)                            | (38)            |
| \( m = n = M - 1 \)  | Fig. 4(f)       | (17)            | (25)       | (31)                            | (39)            |
| \( 0 < m < n < M - 1 \) | --              | (18)            | \( r_{m,n}, d_{m,n}, e_{m,n} \) | \( P(m,n) \) | \( \nabla_p r_{m,n}, \nabla_p d_{m,n}, \frac{\partial \epsilon_{m,n}}{\partial p} \) | \( \frac{\partial \epsilon_{m,n}}{\partial p} \) |

**Case 6:** For \( m = n = M - 1 \), we have

\[
\frac{\partial P(M - 1, M - 1)}{\partial p} = W_{M-1}^{-1} \sum_{i=1}^{W_{M-1}} \left( k \frac{\partial}{\partial p} d_{i}^{\sigma} + p \frac{\partial d_{i}^{\sigma}}{\partial p} \right) \left( -1 + \frac{1}{2} W_{M-1} + \frac{1}{2} \right) + \frac{\partial \epsilon^{\sigma}}{\partial p} \left( \frac{1}{3} W_{M-1} + \frac{1}{2} + \frac{1}{6} W_{M-1} \right),
\]

where

\[
\frac{\partial d_{i}^{\sigma}}{\partial p} = \frac{\partial r_{m-1,n,i}}{\partial p} + \frac{\partial d_{m-1,n-1,i}}{\partial p} + \frac{\partial d_{m,n-1,i}}{\partial p} + \frac{\partial \epsilon_{m-1,n-1,i}}{\partial p},
\]

\[
\frac{\partial \epsilon^{\sigma}}{\partial p} = \frac{\partial \epsilon_{M-2,n-2}}{\partial p} + \frac{\partial \epsilon_{M-2,n-1}}{\partial p} + \frac{\partial \epsilon_{M-1,n-1}}{\partial p} + \frac{\partial \epsilon_{M-1,n}}{\partial p}.
\]

**Case 7:** For \( 0 \leq n < m \leq M - 1 \), we have that \( \frac{\partial P(m,n)}{\partial p} = \frac{\partial \epsilon_{m,n}}{\partial p} \).

Based on Lemmas 1 and 2, we can use \( \epsilon_{0,0} \) and \( \frac{\partial \epsilon_{0,0}}{\partial p} \) to characterize \( \frac{\partial P(m,n)}{\partial p} \), and utilize the normalization condition given by Equation (41) to obtain \( \frac{\partial \epsilon_{0,0}}{\partial p} \). Furthermore, we can calculate \( \nabla_p r_{m,n}, \nabla_p d_{m,n} \) and \( \frac{\partial \epsilon_{m,n}}{\partial p} \) for \( 0 \leq m, n \leq M - 1 \) by \( \frac{\partial \epsilon_{m,n}}{\partial p} \) based on Lemma 1, and achieve \( \frac{\partial m}{\partial p} \) according to Equation (33). Therefore, we can further calculate \( p \) according to Equation (19), where the key variables are shown in Table IV.

**B. Numerical Solution of Throughput**

According to Section IV in [11], the expectation of successful transmissions is given by \( L_p N \eta(1 - \eta)^{N-1} \); and that of total transmission equals \( T_s N \eta(1 - \eta)^{N-1} + T_c [1 - N \eta(1 - \eta)^{N-1} - (1 - \eta)^N] + \tau(1 - \eta)^N \), where \( L_p \) denotes the number of transmitted symbols within one data frame transmission; \( T_s \) and \( T_c \) denote the average channel busy due to a successful transmission and a
collision, respectively, given by
\[ T_s = \text{RTS} + \text{SIFS} + \text{PTA} + \text{SIFS} + \text{SAK} + \text{SIFS} + \text{DFTrigger} + \text{SIFS} + \text{PHY-H} + \text{MAC-H} + L_p + \text{SIFS} + \text{ACK} + \text{DIFS}; \] (48)
\[ T_c = \text{RTS} + \text{DIFS}; \]

and PHY-H as well as MAC-H denote the header of physical and MAC layers, respectively.

Generally, let \( C \) denote the throughput given by the following equation,
\[ C = \frac{L_p N \eta (1 - \eta)^{N-1}}{T_c N \eta (1 - \eta)^{N-1} + T_c [1 - N \eta (1 - \eta)^{N-1} - (1 - \eta)^N]} + \frac{\tau (1 - \eta^N)}{T_s + \tau L_o - T_c}, \] (49)
where \( \tau \) denotes the duration of once backoff; \( L_p, T_s, T_c \) as well as \( \tau \) have to be characterized by same unit; and \( L_o \) is given by
\[ L_o = \frac{N \eta (1 - \eta)^{N-1}}{T_c \tau^{-1} - (1 - \eta)^N (T_c \tau^{-1} - 1)}. \] (50)

V. OPTIMIZATION FOR INITIAL CONTENTION WINDOW AND PARTNER MAP

A. Optimization for Initial Contention Window

We propose to optimize initial contention window \( W_0^* \) to maximize the system throughput \( C \) given the number of TCPairs \( N \) in this subsection. According to Equations (49) and (50), \( C \) depends on \( \eta \) and thus depends on \( W_0 \), where \( W_0 \in 2^{N^*}, \) and \( 2^{N^*} \triangleq \{2^1, 2^2, \cdots \}. \) In order to optimize \( W_0 \) in a tractable manner, we slack \( W_0 \) from \( 2^{N^*} \) to \( \mathbb{R}, \) and obtain the optimal \( \bar{W}_0^* = \arg \max_{W_0 \in \mathbb{R}} C. \) Afterwards, based on \( W_{0,l}, W_{0,r} \in 2^{N^*}, \) the most two adjacent values to the optimal \( \bar{W}_0^* \), we have that \( W_0^* = \arg \max_{W_0 \in [W_{0,l}, W_{0,r}]} C. \) Generally, \( \bar{W}_0^* \) can be approximatively estimated from Theorem 3.

Theorem 3. The approximated slacked optimal initial contention window \( \bar{W}_0^* \) is given by
\[ \bar{W}_0^* \approx \frac{3}{2} \sqrt{2} N \gamma - \frac{3}{4} + \sqrt{\frac{9}{8} N^2 \gamma^2 - \frac{21 \sqrt{2}}{8} N \gamma + \frac{1}{16}}, \] (51)
where \( \gamma = \sqrt{T_c \tau^{-1}}, \) and \( T_c \) as well as \( \tau \) are denoted in Section IV.B.

Proof: Please refer to Appendix E.
B. Optimization for TCPair Number

For the optimization of partner map, we propose to optimize the TCPair number to maximize system throughput $C$ given initial contention window $W_0$. Via slacking $N$ from $\mathbb{N}^+$ to $\mathbb{R}$, we can obtain the optimal $\tilde{N}^*$ that satisfies $\tilde{N}^* = \arg \max_{N \in \mathbb{R}} C$, and its two adjacent integral neighbours $N_l$ and $N_r$. Then, we select the optimal $N^*$ by $N^* = \arg \max_{N \in \{N_l, N_r\}} C$. Generally, the optimal $\tilde{N}^*$ can be approximatively calculated according to Theorem 4.

**Theorem 4.** The approximated slacked optimal number of TCPairs $\tilde{N}^*$ is given by

$$\tilde{N}^* \approx \left(\frac{T_c}{\tau} - 1\right)^{-1} \left[\sqrt{\frac{\eta^2}{4} + \eta + \frac{\eta T_c}{\eta^2}} - 1 - \left(1 + \frac{\eta}{2}\right)\right] \eta^{-1},$$

(52)

where $\eta = (W_0 - 1)[\frac{1}{3}W_0^2 + \frac{1}{2}W_0 + \frac{1}{6}]^{-1}$.

**Proof:** Please refer to Appendix F.

C. Optimization for the Partner Map

Assuming that the system’s physical-layer-connectivity is characterized in an adjacent matrix $S \in \{0, 1\}^{N_s \times N_s}$ ($S = S^T$), we optimize the partner map $\Phi$ based on $S$ and the optimal number of TCPs $N^*$. In order to guarantee reliable transmission for each station, we minimize the variance of all stations’ PTCounters numbers, i. e.,

$$\Phi = \arg \min \mathcal{D}(J_1, J_2, \cdots, J_{N_s}),$$

s. t. $\sum_{i=1}^{N_s} \sum_{j=1}^{N_s} \phi_{i,j} = N^*$; $\Phi = \Phi^T$, $\Phi \circ S = \Phi,$

(53)

where $J_i$ denotes the number of PTCounters of station $i$; $\mathcal{D}[\bullet]$ denotes the variance function; $\Phi \circ S$ denotes the element-wise product of matrices $\Phi$ and $S$. Letting $\mathcal{R}(S) = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} s_{i,j}$ and $Q(S) = \sum_{i=1}^{N_s} [\mathcal{R}(S)]^2$, the objective function can be transformed into $\Phi = \arg \min_{B \in \{0, 1\}^{N_s \times N_s}} Q(B)$.

Let $S_0 = \{S\}$ and $\mathcal{B}^v(S_0)$ denote the set given by

$$\mathcal{B}^v(S_0) = \left\{ B \mid B \in \{0, 1\}^{N_s \times N_s}, B = B^T, B \circ S = B, \mathcal{D}(B, S) = 2v, S \in S_0 \right\},$$

(54)

where $\mathcal{D}(B, S) = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} (b_{i,j} - s_{i,j})^2$ denotes the Hamming distance of matrices $B$ and $S$. Therefore, we can convert the constrained optimization into $\Phi = \arg \min_{B \in \mathcal{B}^v(S_0)} Q(B)$, and resort a
the following equation to minimize $Q(B)$,

$$\min_{B \in B'(S_0)} Q(B) \approx \min_{B \in B^{-1}(S_0)} \left\{ Q(B) - \max_{1 \leq i, j \leq N_s} s_{i,j} [R_i(B) + R_j(B)] \right\} + 2.$$  \hspace{1cm} (55)

Furthermore, to reduce the solution space, letting

$$S_v = \{ B \mid Q(B) = \min_{B \in B'(S_0)} Q(B) \},$$  \hspace{1cm} (56)

we can transform Equation (55) into the following recursion equation,

$$\min_{B \in B'(S_0)} Q(B) \approx \min_{B \in S_{v-1}} Q(B) - \max_{B \in S_{v-1}} \max_{1 \leq i, j \leq N_s} s_{i,j} [R_i(B) + R_j(B)] + 2.$$  \hspace{1cm} (57)

According to Equation (56), we have $S_{v+1}$ given by

$$S_{v+1} = \left\{ (\hat{B}, \hat{i}, \hat{j}) \mid \hat{B} \in S_v, B \in B(S_v), s_{\hat{i}, \hat{j}} = s_{\hat{j}, \hat{i}} = 0, \hat{s}_{\hat{i}, \hat{j}} = \hat{s}_{\hat{j}, \hat{i}} = 1 \right\},$$  \hspace{1cm} (58)

where $g_v^* = \max_{B \in S_v} \max_{1 \leq i, j \leq N_s} s_{i,j} [R_i(B) + R_j(B)]$; and $B^{-1}[B(S_v)]$ is given as follows,

$$B^{-1}[B(S_v)] = \left\{ (\hat{B}, \hat{i}, \hat{j}) \mid \hat{B} \in S_v, B \in B(S_v), s_{\hat{i}, \hat{j}} = s_{\hat{j}, \hat{i}} = 0, \hat{s}_{\hat{i}, \hat{j}} = \hat{s}_{\hat{j}, \hat{i}} = 1 \right\}. $$  \hspace{1cm} (59)

The iterative processure terminates as $2v = N - N^*$, where $S_v$ is exactly the set of the optimal $\Phi$ that satisfies Equation (53).

**VI. Numerical and Simulation Results**

We evaluate proposed DS-CSMA/CA protocol based on simulation and numerical analysis, where the parameters are $M = 4$, $L_p = 8184$, MAC-H = 272, PHY-H = 128, RTS = 160, PTA = 72, SAK = DFTrigger = 36, SIFS = 28, DIFS = 128 and $\tau = 50$, all in the unit of symbol duration.

Figures 6 and 7 plot the collision probability and overall throughput versus initial contention window length $W_0$, respectively, where the number of TPCAnes $N = 30$, and the throughput of conventional CSMA/CA is shown for comparison. It is demonstrated that the proposed analytical model for DS-CSMA/CA protocol is accurate enough, and the numerical results approach the simulation. Furthermore, the overall throughput of DS-CSMA protocol is remarkably higher than that of the conventional CSMA/CA.

Figures 8 and 9 illustrate the overall throughput versus the number of TPCAnes $N$ and initial
Fig. 6. The numerical and simulation result of collision probability.

Fig. 7. The numerical and simulation result of overall throughput.

Fig. 8. The throughput versus different initial contention window.

Fig. 9. The throughput versus different number of TCPairs.

contention window length $W_0$, respectively. It is observed that for fixed contention window, the optimal number of TCPairs are varied, and vice versa. Furthermore, we can obtain the optimal $W_0^*$ and $N^*$ given in Table V corresponding to different $N$ and $W_0$, respectively. It is also shown in Figures 8 and 9 that the maximum overall throughput can always be obtained based on the optimal $W_0^*$ and $N^*$.

VII. CONCLUSION

In this work, based on the physical-layer multi-user communication with symbol boundary misalignment, we have proposed a DS-CSMA/CA protocol for OWSCN, which can avoid
### Table V

| Given N | 20  | 50  | 100 | 200 | 500 |
|---------|-----|-----|-----|-----|-----|
| Optimal $W_0^{*}$ | 128 | 256 | 512 | 1024 | 4096 |
| Optimal $N^{*}$ | 4   | 9   | 17  | 35  | 138 |

collision and enhance the overall throughput. Furthermore, we have proposed a state transition model for the collision probability and throughput analysis, and for optimizing the initial contention window and partner map. Both numerical and simulation results show that the proposed DS-CSMA/CA protocol with the optimal initial contention window and partner map can significantly achieve higher throughput compared with traditional CSMA/CA.

### VIII. Appendix

#### A. Derivation of State Probabilities in Section III

For case 1, $m = n = 0$, $1 \leq i \leq j \leq W_0 - 1$,

$$
P(0,0,i,j) = P(0,0,i+1,j+1) + \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \sum_{i'=0}^{W_m-1} P(0,0,i,j|m,n,i',0)P(m,n,i',0)
$$

$$
+ \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \sum_{j'=0}^{W_n-1} P(0,0,i,j|m,n,0,j')P(m,n,0,j')
$$

$$
= P(0,0,i+1,j+1) + \frac{1-p}{W_0^2} \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} \sum_{i'=0}^{W_m-1} P(m,n,i',0) + \sum_{j'=0}^{W_n-1} P(m,n,0,j')
$$

$$
= P(0,0,i+1,j+1) + \eta \frac{1-p}{W_0^2}.
$$

For case 2, $0 < m < M - 1$, $n = 0$, $1 \leq i < W_m - 1$, $0 \leq j \leq W_0 - 1$,

$$
P(m,0,i,j) = P(m,0,i+1,j+1) + P(m,0,i,j|m-1,0,0,j+1)P(m-1,0,0,j+1)
$$

$$
= P(m,0,i+1,j+1) + \frac{P(m-1,0,0,j+1)}{W_m} p.
$$

For cases 3–7, the proof of Equations (14)–(18) is similar to that of case 2, and it is omitted.
B. Proof of Theorem 1

For case 1, \( m = n = 0 \), we have the following equation for \( 1 \leq i \leq j \leq W_0 - 1 \),

\[
\begin{align*}
P(0, 0, i, j) &= P(0, 0, i + 1, j + 1) + \eta \frac{1 - p}{W^2_0} = P(0, 0, i + 2, j + 2) + 2\eta \frac{1 - p}{W^2_0} \\
&= (W_0 - j)\eta \frac{1 - p}{W^2_0}.
\end{align*}
\]

Similarly, we have \( P(0, 0, i, j) = P(0, 0, j, i) \) for \( 1 \leq i, j \leq W_0 - 1 \). With \( i = j = 0 \), we have \( \epsilon_{0,0} = \eta(1 - p)W_0^{-1} \). Furthermore, letting \( i = 0, 1 < j \leq W_0 - 1 \), we have \( r_{0,0,j} = d_{0,0,j} = (W_0 - j)\eta \frac{1 - p}{W^2_0} = (W_0 - j)W_0^{-1}\epsilon_{0,0} \).

For case 2, \( 0 < m < M - 1, n = 0 \), we have that

\[
\begin{align*}
P(m, 0, i, j) &= P(m, 0, i + 1, j + 1) + W_m^{-1}pP(m - 1, 0, 0, j + 1) \\
&= P(m, 0, i + 2, j + 2) + W_m^{-1}p[P(m - 1, 0, 0, j + 2) + P(m - 1, 0, 0, j + 1)] \\
&= W_m^{-1}p \sum_{k=1}^{W_0 - 1 - j} P(m - 1, 0, 0, j + k),
\end{align*}
\]

when \( 0 \leq i \leq j + W_m - W_0 + 1 \leq W_m - 1 \); and

\[
P(m, 0, i, j) = W_m^{-1}p \sum_{k=1}^{W_0 - 1 - j} P(m - 1, 0, 0, j + k),
\]

when \( 0 \leq j + W_m - W_0 + 1 < i \leq W_m - 1 \).

Letting \( i = j = 0 \), we have \( \epsilon_{m,0} \) given by

\[
\epsilon_{m,0} = P(m, 0, 0, 0) = W_m^{-1}p \sum_{k=1}^{W_0 - 1} P(m - 1, 0, 0, k);
\]

when \( i = 0 \) and \( 1 \leq j \leq W_m - W_0 - 1 \), \( r_{m,0,j} \) is given by

\[
r_{m,0,j} = P(m, 0, 0, j) = W_m^{-1}p \sum_{k=1}^{W_0 - 1 - j} P(m - 1, 0, 0, j + k);
\]

when \( 1 \leq i \leq W_m - W_0 + 1 \) and \( j = 0 \), \( d_{m,0,i} \) is given by

\[
d_{m,0,i} = P(m, 0, i, 0) = W_m^{-1}p \sum_{k=1}^{W_0 - 1} P(m - 1, 0, 0, k);
\]
when \( W_m - W_0 + 1 < i \leq W_m - 1 \) and \( j = 0 \), \( d_{m,0,i} \) is given by

\[
d_{m,0,i} = \mathbb{P}(m, 0, i, 0) = W_m^{-1} p \sum_{k=1}^{W_m-1-i} \mathbb{P}(m-1, 0, 0, k).
\] (68)

Note that \( A_{r,m,n} \) and \( A_{d,m,n} \) are given in Figures 5(a) and 5(c), respectively, we can obtain the transition equations as shown in Equation (21).

For case 3, as similar to case 2, we have that

\[
\begin{align*}
\text{For case 3, as similar to case 2, we have that} \\
&\quad \text{For cases 4 – 6, the proof of Equations (23)–(25) is similar to that of cases 2 and 3, and we omit it here. For case 7, we can directly reach the conclusion from Equation (18).}
\end{align*}
\]

**C. Proof of Theorem 2**

For case 1, \( m = n = 0 \), according to Equation (62), we have that \( \mathbb{P}(0, 0, i, 0) = \mathbb{P}(0, 0, i, 1) = \cdots = \mathbb{P}(0, 0, i, i) = \mathbb{P}(0, 0, i-1, i) = \cdots = \mathbb{P}(0, 0, 0, i) = (W_0 - i)W_0^{-1} \) for \( 0 \leq i \leq W_0 - 1 \), and we can hereby separate the items of \( i = W_0 - 1 \) and \( j = W_0 - 1 \) from \( \mathbb{P}(0, 0) = \sum_{j=0}^{W_0-1} \sum_{i=0}^{W_0-1} \mathbb{P}(0, 0, i, j) \) to obtain the following result,

\[
\begin{align*}
\mathbb{P}(0, 0) &= \mathbb{P}(0, 0, W_0 - 1, W_0 - 1) + \sum_{i=0}^{W_0-2} \mathbb{P}(0, 0, i, W_0 - 1) + \sum_{j=0}^{W_0-2} \mathbb{P}(0, 0, W_0 - 1, j) + \sum_{j=0}^{W_0-2} \sum_{i=0}^{W_0-2} \mathbb{P}(0, 0, i, j) \\
&= (2W_0 - 1)W_0^{-1} \epsilon_{0,0} + \sum_{j=0}^{W_0-2} \sum_{i=0}^{W_0-2} \mathbb{P}(0, 0, i, j).
\end{align*}
\] (71)
Similarly, we can also separate the items of \( i = W_0 - 2 \) as well as \( j = W_0 - 2 \), and \( \mathbb{P}(0, 0) \) is simplified as follows,

\[
\mathbb{P}(0, 0) = (2W_0 - 1)W_0^{-1}e_{0,0} + \mathbb{P}(0, 0, W_0 - 2, W_0 - 2) + \sum_{i=0}^{W_0-3} \mathbb{P}(0, i, W_0 - 2) + \sum_{j=0}^{W_0-3} \mathbb{P}(0, 0, W_0 - 2, j) + \sum_{j=0}^{W_0-3} \sum_{i=0}^{W_0-3} \mathbb{P}(0, 0, i, j)
\]

\[
= (2W_0 - 1)W_0^{-1}e_{0,0} + (2W_0 - 3)2W_0^{-1}e_{0,0} + \sum_{j=0}^{W_0-3} \sum_{i=0}^{W_0-3} \mathbb{P}(0, 0, i, j)
\]

\[
= W_0^{-1}e_{0,0} \sum_{i=1}^{2} (2W_0 - 2i + 1)i + \sum_{j=0}^{W_0-3} \sum_{i=0}^{W_0-3} \mathbb{P}(0, 0, i, j).
\]  

(72)

After separating the items of \( i, j = 0, 1, \ldots, W_0 - 1 \), we have that

\[
\mathbb{P}(0, 0) = W_0^{-1}e_{0,0} \sum_{i=1}^{W_0} (2W_0 - 2i + 1)i = \frac{1}{6}(2W_0 + 1)(W_0 + 1)e_{0,0}.
\]  

(73)

For case 2, \( 0 < m < M - 1, n = 0 \), we have that \( \mathbb{P}(m, 0) = \sum_{j=0}^{W_0-1} \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, j) \). According to Equation (13), we have \( \mathbb{P}(m, 0, i, 0) = \mathbb{P}(m, 0, i + 1, 1) + W_m^{-1}pr_{m-1,0,1} \) for \( 0 \leq i \leq W_m - 2 \), and \( \mathbb{P}(m, 0, W_m - 1, 0) = W_m^{-1}pr_{m-1,0,1} \). We separate the items of \( j = 0 \) and combine them with those of \( j = 1 \), which is given as follows,

\[
\mathbb{P}(m, 0) = \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, 0) + \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, 1) + \sum_{j=2}^{W_0-1} \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, j)
\]

\[
= \sum_{k=1}^{2} \sum_{i=0}^{W_m-k} \mathbb{P}(m, 0, i, 1) + W_m^{-1}pW_m r_{m-1,0,1} + \sum_{j=2}^{W_0-1} \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, j).
\]  

(74)

Similarly, \( \mathbb{P}(m, 0, i, 1) = \mathbb{P}(m, 0, i + 1, 2) + W_m^{-1}pr_{m-1,0,2} \) for \( 0 \leq i \leq W_m - 2 \) and \( \mathbb{P}(m, 0, W_m - 1, 0) = W_m^{-1}pr_{m-1,0,2} \). Separating the items of \( j = 1 \) and combining them with those of \( j = 2 \), we have

\[
\mathbb{P}(m, 0) = \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, 2) + \sum_{k=1}^{2} \sum_{i=0}^{W_m-k} \mathbb{P}(m, 0, i, 1) + W_m^{-1}pW_m r_{m-1,0,1} + \sum_{j=3}^{W_0-1} \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, j)
\]

\[
= \sum_{k=1}^{3} \sum_{i=0}^{W_m-k} \mathbb{P}(m, 0, i, 2) + W_m^{-1}p \sum_{j=1}^{2} r_{m-1,0,j} \sum_{k=1}^{j} \sum_{i=k}^{W_m} 1 + \sum_{j=3}^{W_0-1} \sum_{i=0}^{W_m-1} \mathbb{P}(m, 0, i, j).
\]  

(75)
Then, we separate the items of \( j = 2, 3, \cdots, W_0 - 1 \), and have the following equation,

\[
P(m, 0) = W_m^{-1} p \sum_{j=1}^{W_0} r_{m-1,0,j} \sum_{k=1}^{j} \sum_{i=k}^{W_m} 1 = W_m^{-1} p \sum_{j=1}^{W_0} r_{m-1,0,j} \left[ -\frac{1}{2} j^2 + \left(W_m + \frac{1}{2}\right) j \right].
\] (76)

For cases 3 – 6, the demonstration for Equations (28)–(31) can be achieved similar to case 2, and is omitted here; for case 7, we can directly reach the conclusion from Equation (18).

D. Proof of Theorem 3

We obtain \( \bar{W}_0^* \) by solve the equation \( \bar{W}_0^* = \arg(\eta = \eta^*) \), where \( \eta^* = \arg(\frac{\partial C}{\partial \eta} = 0) \). From Equations (49) and (50), we have that

\[
\frac{\partial C}{\partial \eta} = \frac{L_p \tau}{L_o^2 [T_s + \tau L_o^{-1} - T_c]^{-1}} \frac{\partial L_o}{\partial \eta} \triangleq C_0 C'_{\eta},
\] (77)

where \( C_0 = L_p \tau N^{-1} \eta^{-2} (1 - \eta)^{-N} [T_s + \tau L_o^{-1} - T_c]^{-2} > 0 \), and

\[
C'_{\eta} = (1 - \eta)^N - T_c \tau^{-1} [N \eta - (1 - (1 - \eta)^N)].
\] (78)

Hence, \( \eta^* = \arg(\frac{\partial C}{\partial \eta} = 0) = \arg(C'_{\eta} = 0) \). Based on the assumption that \( \eta << 1 \), we adopt the conclusion of [11] to calculate \( \eta^* \) by

\[
\eta^* = \arg(C'_{\eta} = 0) \approx \sqrt{2}(N\gamma)^{-1},
\] (79)

where \( \tau = \sqrt{T_c \tau^{-1}} \).

We furtherly consider to approximate the proposed optimization to uniform contention window with \( W_{max} = W_0 \). Accordingly, based on Equation (27), we have that \( \epsilon_{0,0} \approx \left[ \frac{1}{3} W_0^2 + \frac{1}{2} W_0 + \frac{1}{6} \right]^{-1} \), and \( \eta \) can be calculated by

\[
\eta \approx \sum_{i=1}^{W_0} \mathbb{P}(0, 0, i, 0) + \sum_{j=1}^{W_0} \mathbb{P}(0, 0, j, 0) = (W_0 - 1) \left[ \frac{1}{3} W_0^2 + \frac{1}{2} W_0 + \frac{1}{6} \right]^{-1}.
\] (80)

Then, solving \( (W_0 - 1) \left[ \frac{1}{3} W_0^2 + \frac{1}{2} W_0 + \frac{1}{6} \right]^{-1} = \sqrt{2}(N\gamma)^{-1} \), we have \( \bar{W}_0^* \) as follows,

\[
\bar{W}_0^* \approx \frac{3}{2\eta^*} - \frac{3}{4} + \sqrt{\frac{9}{4\eta^*^2} - \frac{21}{8\eta^*} + \frac{1}{16}}.
\] (81)

Substituting \( \eta^* \) in Equation (79) into Equation (81), we can readily obtain \( \bar{W}_0^* \) as Equation (51).
E. Proof of Theorem 4

Based on Equation (50) and the chain rule of derivation, we have that

$$\frac{\partial L_o}{\partial N} = \frac{\partial L_o(\eta, N)}{\partial \eta} \frac{\partial \eta}{\partial N} + \frac{\partial L_o(\eta, N)}{\partial N}. \quad (82)$$

We also consider to approximate the solution of proposed optimization by where $W_{\text{max}} = W_0$. In this case, from Equation (80), we have $\frac{\partial \eta}{\partial N} = 0$. Hence, $\tilde{N}^*$ satisfies

$$\tilde{N}^* = \arg \left\{ \frac{\partial L_o(\eta, N)}{\partial N} = 0 \right\} = \arg \left\{ (1 - \eta)^N + T_c \tau^{-1} \{ N \log(1 - \eta) + [1 - (1 - \eta)^N] \} = 0 \right\}. \quad (83)$$

For $\eta << 1$, we deploy the following approximation for $(1 - \eta)^N$ and $\log(1 - \eta)$,

$$(1 - \eta)^N \approx 1 - N\eta + \frac{N(N - 1)}{2} \eta^2, \quad \log(1 - \eta) \approx -\eta - \frac{\eta^2}{2}. \quad (84)$$

Substituting Equation (84) into Equation (83), we have that $\tilde{N}^*$ approximately satisfies

$$1 - \left( \eta + \frac{\eta^2}{2} \right) N - \left( \frac{T_c}{\tau} - 1 \right) \frac{\eta^2}{2} N^2 = 0. \quad (85)$$

where $\eta$ is given by Equation (80), and the solution is given by Equation (52).

References

[1] R. M. Gagliardi and S. Karp, “Optical communications,” New York, Wiley-Interscience, 1976. 445 p., 1976.
[2] Z. Xu and B. M. Sadler, “Ultraviolet communications: potential and state-of-the-art,” IEEE Commun. Mag., vol. 46, no. 5, pp. 67–73, 2008.
[3] G. Wang, C. Gong, and Z. Xu, “Signal characterization for multiple access non-line of sight scattering communication,” IEEE Trans. Commun., vol. 66, no. 9, pp. 4138–4154, 2018.
[4] T. Xiao, C. Gong, Q. Gao, and Z. Xu, “Channel characterization for multi-color vlc for feedback and beamforming design,” in IEEE ICC Workshop on Optical Wireless Communications, May 2018.
[5] D. Zou, C. Gong, and Z. Xu, “Secrecy rate of miso optical wireless scattering communications,” IEEE Trans. Commun., vol. 66, no. 1, pp. 225–238, 2018.
[6] Y. Li, L. Wang, Z. Xu, and S. V. Krishnamurthy, “Neighbor discovery for ultraviolet ad hoc networks,” IEEE J. Sel. Area Comm., vol. 29, no. 10, pp. 2002–2011, 2011.
[7] Y. Wang, N. Wu, and Z. Xu, “Study of raptor codes for indoor mobile vlc channels,” in IEEE GlobeCom Workshop on Optical Wireless Communications, pp. 9–13, Dec. 2018.
[8] IEEE802.11 Working Group, “Part 11: Wireless lan medium access control (mac) and physical layer (phy) specifications,” in ANSI/IEEE Std 802.11, Sept. 2019.
[9] C. L. Fullmer and J. Garcia-Luna-Aceves, “Solutions to hidden terminal problems in wireless networks,” in SIGCOMM, vol. 27, pp. 39–49, ACM, 1997.
[10] K. Xu, M. Gerla, S. Bae, et al., “How effective is the ieee 802.11 rts/cts handshake in ad hoc networks?,” in GlobeCom, vol. 2, pp. 72–76, 2002.

[11] G. Bianchi, “Performance analysis of the ieee 802.11 distributed coordination function,” IEEE J. Sel. Area Comm., vol. 18, no. 3, pp. 535–547, 2000.

[12] B.-J. Kwak, N.-O. Song, and L. E. Miller, “Performance analysis of exponential backoff,” IEEE/ACM Trans. Network., vol. 13, no. 2, pp. 343–355, 2005.

[13] A. Nasipuri, J. Zhuang, and S. R. Das, “A multichannel csma mac protocol for multihop wireless networks,” in WCNC, vol. 3, pp. 1402–1406, IEEE, 1999.

[14] S.-L. Wu, C.-Y. Lin, Y.-C. Tseng, and J.-L. Sheu, “A new multi-channel mac protocol with on-demand channel assignment for multi-hop mobile ad hoc networks,” in J-SPAN, pp. 232–237, IEEE, 2000.

[15] J. Chen, S.-T. Sheu, and C.-A. Yang, “A new multichannel access protocol for ieee 802.11 ad hoc wireless lans,” in PIMRC, vol. 3, pp. 2291–2296, IEEE, 2003.

[16] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity–part i: system description,” IEEE Trans. Commun., vol. 51, no. 11, pp. 1927–1938, 2003.

[17] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity. part ii. implementation aspects and performance analysis,” IEEE Trans. Commun., vol. 51, no. 11, pp. 1939–1948, 2003.

[18] R. Lin and A. P. Petropulu, “A new wireless network medium access protocol based on cooperation,” IEEE Trans. Signal Proces., vol. 53, no. 12, pp. 4675–4684, 2005.

[19] Y.-B. Ko, V. Shankarkumar, and N. H. Vaidya, “Medium access control protocols using directional antennas in ad hoc networks,” in InfoCom, vol. 1, pp. 13–21, IEEE, 2000.

[20] R. R. Choudhury, X. Yang, R. Ramanathan, and N. H. Vaidya, “Using directional antennas for medium access control in ad hoc networks,” in MobisCom, pp. 59–70, ACM, 2002.

[21] R. Ramanathan, J. Redi, C. Santivanez, D. Wiggins, and S. Polit, “Ad hoc networking with directional antennas: A complete system solution,” IEEE J. Sel. Area Comm., vol. 23, pp. 496–506, Mar. 2005.

[22] O. Bazan and M. Jaseemuddin, “A survey on mac protocols for wireless adhoc networks with beamforming antennas,” IEEE Commu. Surv. Tut., vol. 14, no. 2, pp. 216–239, 2012.

[23] G. Wang, C. Gong, Z. Jiang, and Z. Xu, “Characterization on asynchronous multiple access in non-line of sight scattering communication,” in 2018 IEEE ICC Workshops on Optical Wireless Communication, pp. 1–6, Mar. 2018.

[24] G. Wang, C. Gong, Z. Jiang, and Z. Xu, “Multi-layer superimposed transmission with symbol boundary offset for optical wireless scattering communication,” in eprint ArXiv:1805.02199, 2019.