In-Situ Measurement of the Density Ratio of the K-Rb-\(^{21}\)Ne Comagnetometer Based on Electron Spin Relaxation Rate

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Abstract. Accurate measurement of alkali metal density ratio in hybrid vapor cells is essential for high sensitivity of comagnetometers. We introduce and investigate a new method of measuring the density ratio based on electron spin-relaxation rate. The Bloch equations including the parameter of density ratio are studied, and the density measurement results show that measurement uncertainty is less than 12%. The advantage of this method is measuring the in-situ density ratio, which is more precise and useful for optimizing sensitivity of comagnetometers by adjusting the density ratio in real time.

Keywords. Atomic comagnetometer, measurement of the density ratio, electron spin relaxation rate.

1. Introduction
Spin-exchange relaxation free (SERF) comagnetometers are highly sensitive to non-magnetic interaction, such as rotation and other anomalous fields [1, 2]. So comagnetometers have extensive applications in inertial navigation[3] and fundamental physics, including violations of Lorentz invariance[4, 5], fifth force[6, 7], and dark matter[8]. The K-Rb-\(^{21}\)Ne comagnetometer, which contains two alkali metals, is designed to achieve higher sensitivity [9, 10]. By spin exchange collisions, the ensemble has more uniform and efficient spin polarization. Therefore, the density ratio of alkali metal is crucial to electron spins polarization and high sensitivity of comagnetometers.

laser- absorption-spectroscopy (LAS) is widely used in measuring atomic density in vapor cell[11, 12]. This method derives the density from absorption line shapes, so it is not suitable for high density or high temperature, which absorbs most of laser and leads flat area at the peak of fitting line. Faraday-rotation method can measure optically thick alkali metal, but it introduces large magnetic field which destroys SERF regime[13]. Recently some method based on character of magnetometer and comagnetometer have been proposed. The schemes, which apply DC magnetic field to measure magnetic resonance linewidth[14] or use phase-frequency analysis of magnetometer[15], are only applicable to one species alkali metal. The method with measuring mix light shifts of K and Rb can directly measure density ratio of a K-Rb cell[16]. However, it is complex to find zero point of light shifts and limited to measure small density ratio.

Here, a novel method is proposed to measure density ratio of hybrid alkali metal atoms in K-Rb-\(^{21}\)Ne comagnetometers based on electron spin relaxation rate. We analysis the Bloch equations including the parameter of density ratio, and derive the density ratio from relationship between
electron spin-relaxation rate and pump light intensity. In-situ measuring the density ratio is advantageous when the comagnetometer works in SERF regime.

2. Method

Hybrid optical pumping is utilized to achieve uniform polarization of atomic ensemble. The atomic density of K is normally smaller than that of Rb in the K-Rb-\(^{21}\)Ne comagnetometer. K are directly pumped using circularly polarized light of D1 line. Then, the Rb atoms are polarized through spin exchange collisions with K. Finally, the electron-spin polarization is transferred to nuclear spins of \(^{21}\)Ne. In SERF regime, the K atoms and Rb atoms have almost the same polarization by the rapid spin-exchange collisions. Bloch equations are used to depict the behavior of the comagnetometer [17, 18].

\[
\frac{\partial P^e}{\partial t} = \frac{\gamma}{Q} (B + \lambda M P^e + L) \times P^e + \frac{(D, R_s s_p + R_s s_m) - (D, R_s + R_s s_m) P^e}{Q} + \frac{R^e (P^e - P^e)}{Q} + \frac{R^e e P^e}{Q}
\]

(1)

Here \(P^e\) and \(P^m\) denote the coupled electron polarization and \(^{21}\)Ne nuclear polarization, respectively. \(D_r\) stands for the density ratio of K to Rb atoms. B is the external magnetic field, and \(\Omega\) is the rotation vector, while \(L\) is the light shift. \(R_p\) is the pump rate of K, and the effective pump rate for Rb is \(D_r R_p\).

In the SERF regime, the Bloch equations could be linearized, and the steady state solution of equation (1) is obtained. The sensitivity of the comagnetometer signal to angular velocity is shown as:

\[
S = K_m P^e / \left( R^e e / \gamma^e \right) = \left( L + \delta B_y \right) \left( \frac{\delta B_x}{B_y} - \frac{\Omega_z}{\gamma^e} \right)
\]

(2)

where \(K_m\) denotes the scale factor which transforms the electron polarization at X-axis \(P^e_X\) into voltage signal S. \(B_y\) and \(\Omega_y\) stand for the magnetic field and input velocity at Y-axis, respectively. \(\delta B_y\) is the residual magnetic field at Z-axis around the compensation point. \(R^e\) is the total electron spin relaxation rate and is described as:

\[
R^e = D_r R_p + R^e_{sd}
\]

(3)

The spin-destruction relaxation rate \(R^e_{sd}\) consists of spin-destruction collisions with alkali metal and gases[19].

\[
R^e_{sd} = \sigma^e_{K} n_K + \sigma^e_{Rb} n_{Rb} + \sigma^e_{Ne} n_{Ne} + \sigma^e_{N_2} n_{N_2}
\]

(4)

where \(\sigma^e_{X} n_X\) stand for spin-destruction cross-section, the average thermal velocity and the number density of K, Rb, Ne, \(\text{N}_2\), respectively.

\(R_p\) is optical pumping rate and can be described as[20]:

\[
R_p = \Phi(v) \sigma(v) = \frac{P}{A h v} \pi r_e c f \left( \frac{\Gamma}{2} \right) \left( \frac{\Gamma}{2} \right)^2
\]

(5)

where \(\Phi(v)\) is pump laser power density at pump laser frequency \(v\), \(r_e\), \(c\), \(f\) are the classical electron radius, light velocity, and oscillator strength, respectively. \(P\) is pump-light power and \(A\) is spot area. \(\sigma(v)\) is the absorption Lorentzian shape with the linewidth due to the pressure broadening \(\Gamma\).

Substituting \(R_p\) with equation (5), equation (3) can be expressed as:
\[
R'_{se} = D_r \frac{1}{Ah \nu} \pi r_c f \frac{\Gamma / 2}{(v - v_0)^2 + (\Gamma / 2)^2} P + R^c_{sd}
\]  (6)

According to equation (4), \( R^c_{sd} \) is a constant when the operating temperature of comagnetometer is stable. \( R_{tot}^c \) is the linear function of pump light intensity, and the density ratio \( D_r \) can be obtained by fitted slope based on equation (6).

3. Experiments

The experimental setup of the comagnetometer is shown in figure 1. A 12-mm-diameter spherical aluminosilicate glass vapor cell is installed in the center of a ferrite barrel and a four-layer cylindrical \( \mu \)-metal shield, which are used to reduce remanence. The cell is filled with K-Rb mixture, 2516 Torr 21Ne (70% isotope enriched), and 52 Torr N\(_2\) for quenching. The mole fraction of K atoms is 0.198, which was measured with laser-absorption-spectroscopy. The vapor cell is heated to 200 °C by AC currents. \( D_r \) is 0.0407 at 200 °C using Raoult’s law [21]. A three-axis coil compensates and provides a magnetic calibration field.

![Figure 1](image)

Figure 1. Schematic of the comagnetometer setup. The inset shows a sketch of hybrid optical pumping in a vapor cell. The following components are used: PBS (polarization beam splitter), BE (beam expander), M (reflective mirrors), GT (Glan-Taylor polarizer), PEM (photo elastic modulator), PD (photodiode).

The pump light is tuned to 770.108 nm and expanded to cover the cell at the Z-axis. While the probe light is tuned to 795.497nm and propagates at the X-axis. The probe beam is modulated by a PEM with a modulation frequency of 50 kHz. The light intensity coming from the Glan-Taylor polarizer is detected by a photodetector and demodulated with a lock-in amplifier (Zürich Instruments HF2LI). The magnetic field and input velocity can be derived from the demodulated signal.

4. Results and Discussion

When the comagnetometer operates in SERF regime, a small \( B_y \) square wave modulation and a series of \( \delta B_z \) along the Z-axis around the zero magnetic value are applied to the system. The difference of the signal response \( \Delta S \) is measured with different pump laser power. The relationship between \( \Delta S \) and \( \delta B_z \) is shown in figure 2 and the measured datas are fitted with equation (2).
Figure 2. The signal $\Delta S$ to By-square-wave modulation at different pump laser power density. The solid lines are fitted lines using equation (2) and $R_{\text{tot}}^e$ can be obtained from fitted lines.

Figure 3. Total relaxation rate of the electron $R_{\text{tot}}^e$ at different pump laser power. Measured data is fitted with a linear function when the pump light power density is lower than 200 mW/cm$^2$. The intercepts with the Y-axis represent $R_{\text{sd}}^e$.

Figure 3 shows the relationship between $R_{\text{tot}}^e$, which was measured using the By-modulation response, and pump-light power density. According to equation (6), $R_{\text{tot}}^e$ is the linear function of light intensity and the intercept with Y-axis represents $R_{\text{sd}}^e$. The intercept is $593 \pm 143$ s$^{-1}$, which is consistent with the theoretically calculated $R_{\text{sd}}^e$ values $455$ s$^{-1}$. The density ratio $D_e$ calculated by fitted slope is $0.043 \pm 0.005$. $R_{\text{tot}}^e$ increased nonlinearly when the light intensity exceeded 200 mW/cm$^2$. The difference between measured data and fitted line may be led by the nonzero spin-exchange relaxation of large electron magnetic field.

5. Conclusion
In this paper, we investigate a new method of measuring the density ratio of hybrid vapor cells based on electron spin relaxation rate. The density ratio can be calculated from slope of the electron spin-relaxation rate and the atomic number density can be further obtained by Raoult’s law. The experimental results show that the measurement uncertainty is less than 12%. This method can be applied during normal operation of comagnetometer and suitable to monitor the density ratio in real time, which provides a reference for optimization sensitivity of comagnetometers.

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