Primordial Non-Gaussianity in Multi-Scalar Inflation

Shuichiro Yokoyama,† Teruaki Suyama,‡ and Takahiro Tanaka***

1Department of Physics, Kyoto University, Kyoto 606-8502, Japan
2Institute for Cosmic Ray Research, The University of Tokyo, Kashiwa 277-8582, Japan

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We give a concise formula for the non-Gaussianity of the primordial curvature perturbation generated on super-horizon scales in multi-scalar inflation model without assuming slow-roll conditions. This is an extension of our previous work. Using this formula, we study the generation of non-Gaussianity for the double inflation models in which the slow-roll conditions are temporarily violated after horizon exit, and we show that the non-linear parameter \( f_{NL} \) for such models is suppressed by the slow-roll parameters evaluated at the time of horizon exit.

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I. INTRODUCTION

Non-Gaussianity of the primordial curvature perturbation is a potentially useful discriminator of the many existing inflation models [1, 2]. Planck [3] is expected to detect the primordial non-Gaussianity if the so-called non-linear parameter, \( f_{NL} \), which parameterizes the magnitude of the bispectrum, is larger than \( 3 \) [4, 5]. Higher order correlation functions such as trispectrum would also be a useful probe of the primordial non-Gaussianity [5, 6, 7]. Hence it is important to theoretically understand the generation of non-Gaussianity.

Standard single slow-roll inflation model predicts rather small level of the non-linear parameter, \( f_{NL} \), suppressed by the slow-roll parameters [8]. In multi-scalar field inflation models with separable potential, explicit calculations in the slow-roll approximation show that \( f_{NL} \) is also of the order of the slow-roll parameters [9, 10, 11, 12, 13, 14, 15, 16]. This feature is likely to hold for more general potential that satisfies the slow-roll conditions [17, 18, 19]. On the other hand, the generation of non-Gaussianity due to the violation of the slow-roll conditions remains an open problem.

In this paper, we derive a formula which is an extension of that given in our previous paper [19] to the non-slow-roll case. We use the unit \( M_{pl}^2 = (8\pi G)^{-1} = 1 \).

II. FORMULATION

A. Background equations

In this section, we derive a formula which is an extension of that given in our previous paper [19] to the non-slow-roll case. We use the unit \( M_{pl}^2 = (8\pi G)^{-1} = 1 \).

We consider a \( N \)-component scalar field whose action is given by

\[
S = - \int d^4x \sqrt{-g} \left[ \frac{1}{2} h_{IJ} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J + V(\phi) \right],
\]

\[
(I, J = 1, 2, \ldots, N),
\]

where \( g_{\mu\nu} \) is the spacetime metric and \( h_{IJ} \) is the metric on the scalar field space. In the main text, we restrict our discussion to the flat field space metric \( h_{IJ} = \delta_{IJ} \) to avoid inessential complexities due to non-flat field space metric. Extension to the general field space metric is given in appendix B.

We define \( \varphi^I(i = 1, 2) \) as

\[
\varphi^I_1 \equiv \phi^I, \quad \varphi^I_2 \equiv \frac{d}{dN} \phi^I,
\]

where \( dN = H dt \) with \( H \) and \( t \) being the Hubble parameter and cosmological time, respectively. Namely, we take e-folding number, \( N \), as a time coordinate. For brevity, hereinafter, we use Latin indices at the beginning of Latin alphabet, \( a, b \) or \( c \), instead of the double indices, \( I, i \), i.e., \( X^a = X^I_I \).

Then, the background equation of motion for \( \varphi^a \) is

\[
\frac{d}{dN} \varphi^a = F^a(\varphi),
\]

where \( F^a(= F^I_I) \) is given by

\[
F^I_I = \varphi^I_2, \quad F^2_2 = - \frac{V}{H^2} \left( \varphi^I_2 + \frac{V^I}{V} \right),
\]
where \( V^I = \delta^{IJ}(\partial V/\partial \phi^J) \), and the Friedmann equation is
\[
H^2 = \frac{2V}{6 - \varphi_2^J \varphi_2^I},
\]
with \( \varphi_2^I = \delta_{1J} \varphi_2^J \).

**B. Perturbations**

In the \( \delta N \) formalism, the evolution of the difference between two adjacent background solutions determines that of the primordial curvature perturbation on super-horizon scales. In this paper, we use the word "perturbation" to denote the difference between two adjacent background solutions. In this subsection, we analyze the time evolution of the perturbation and relate the result to the curvature perturbation.

The solution of the background equation (3) is labelled by \( 2N \) integral constants \( \lambda^a \). Let us define \( \delta \varphi^a \) as the perturbation,
\[
\delta \varphi^a(N) \equiv \varphi^a(\lambda + \delta \lambda; N) - \varphi^a(\lambda; N),
\]
where \( \lambda \) is abbreviation of \( \lambda^a \) and \( \delta \lambda^a \) is a small quantity of \( O(\delta) \). \( \delta \varphi^a(N) \) defined by Eq. (6) represents a perturbation of the scalar field on the \( N = \) constant gauge.

For the purpose of calculating the leading bispectrum of the curvature perturbation, it is enough to know the evolution of \( \delta \varphi^a(N) \) up to second order in \( \delta \). For later convenience, we decompose \( \delta \varphi^a \) as
\[
\delta \varphi^a = \delta \varphi^{(1)} + \frac{1}{2} \delta \varphi^{(2)},
\]
where \( \delta \varphi^{(1)} \) and \( \delta \varphi^{(2)} \) are first and second order quantities in \( \delta \), respectively.

Evolution equation for \( \delta \varphi^{(1)} \) is given by
\[
\frac{d}{dN} \delta \varphi^{(1)}(N) = \frac{P^a}{b}(N) \delta \varphi^{(1)} b(N),
\]
where \( \frac{P^a}{b}(N) = \frac{\partial F^a}{\partial \varphi^b} \bigg|_{\varphi = \varphi(N)} \).

Here \( \varphi(N) \) represents the unperturbed trajectory. The explicit form of \( \frac{P^a}{b} \) is shown in appendix A. Formally, solution of this equation can be written as
\[
\delta \varphi^{(1)}(N) = \Lambda^a_b(N; N_e) \delta \varphi^{(1)} b(N_e),
\]
where \( \Lambda^a_b \) is a solution of
\[
\frac{d}{dN} \Lambda^a_b(N, N') = \frac{P^a}{a}(N) \Lambda^b_c(N, N') ,
\]
with the condition \( \Lambda^a_b(N, N) = \Lambda^a_b(N, N) = \delta^i_j \delta^j_i \).

Evolution equation for \( \delta \varphi^{(2)} \) is given by
\[
\frac{d}{dN} \delta \varphi^{(2)} a(N) = \frac{P^a}{b}(N) \delta \varphi^{(2)} a(N) + \frac{Q^a}{b}(N) \delta \varphi^{(1)} b(N) \delta \varphi^{(1)} c(N) ,
\]
where \( \frac{Q^a}{b}(N) \) is defined by
\[
\frac{Q^a}{b}(N) = \frac{\partial^2 F^a}{\partial \varphi^b \partial \varphi^c} \bigg|_{\varphi = \varphi(N)} = \frac{\partial P^a}{b}{(N) \bigg|_{\varphi = \varphi(N)}} .
\]

The explicit form of \( \frac{Q^a}{b}(N) = \frac{Q^a b}{b}(N) \) is shown in appendix A. Let us choose the integral constants \( \lambda^a \) as the initial values of \( \varphi^a \) at \( N = N_e \), namely, \( \lambda^a = \varphi^a(N_e) \). Then we have \( \delta \varphi^a(N_e) = \delta \lambda^a \). Hence \( \delta \varphi^{(0)}(N) \) vanishes at \( N_e \). Under this initial condition, the formal solution of Eq. (12) is given by
\[
\delta \varphi^{(2)} a(N) = \int_{N_e}^{N} dN' \Lambda^a b(N, N') Q^b c(N') \times \delta \varphi^{(1)} c(N') \delta \varphi^{(1)} d(N') .
\]

According to the \( \delta N \) formalism, the curvature perturbation on large scales evaluated at a final time, \( N = N_f \), is given by the perturbation of the e-folding number between an initial flat hypersurface at \( N = N_e \) and a final uniform energy density hypersurface at \( N = N_f \). Let us take \( N_e \) to be a certain time soon after the relevant length scale crossed the horizon scale, \( H^{-1} \), during the scalar dominant phase and \( N_f \) to be a certain time after the complete convergence of the background trajectories has occurred. At \( N > N_f \) the dynamics of the universe is characterized by a single parameter and only the adiabatic perturbations remain. Then, the e-folding number between \( N_e \) and \( N_f \) can be regarded as the function of the final time \( N_f \) and \( \varphi^a(N_f) \), which we denote \( N(N_e, \varphi(N_e)) \).

Based on \( \delta N \) formalism, the curvature perturbation on the uniform energy density hypersurface evaluated at \( N = N_f \) is given by
\[
\zeta(N_f) \simeq \delta N(N_e, \varphi(N_e)) = N_{eA} \delta \varphi^a + \frac{1}{2} N_{ab} \delta \varphi^a \delta \varphi^b + \cdots ,
\]
where \( \delta \varphi^a = \delta \varphi^a(N_e) \) represents the field perturbations and their time derivative on the initial flat hypersurface at \( N = N_e \). The left hand side in Eq. (15) is obviously independent of the initial time \( N_e \), and hence so is \( \delta N(N_e, \varphi(N_e)) \). Here we also defined \( N_{eA} = N_a(N_e) \).

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1 We consider the case in which isocurvature perturbations do not persist until later.
and $N_{ab} = N_{ab}(N_s)$ by

$$
N_a(N) = \frac{\partial N(N_c, \varphi)}{\partial \varphi^a} \bigg|_{\varphi = \varphi(0)} ,
$$
(16)

$$
N_{ab}(N) = \frac{\partial^2 N(N_c, \varphi)}{\partial \varphi^a \partial \varphi^b} \bigg|_{\varphi = \varphi(0)} ,
$$
(17)
evaluated at $N = N_s$.

It is well known that the curvature perturbations on an uniform density hypersurface, $\zeta$, remain constant in time for $N > N_c$. Hence, $\zeta(N_c)$ gives the final spectrum of the primordial perturbation.

Let us take $N_F$ to be a certain late time during the scalar dominant phase. Then we have

$$
\zeta(N_c) \simeq \delta N(N_c, \varphi(N_F))
= N_aF \delta \varphi_F^a + \frac{1}{2} N_{ab} \delta \varphi_F^a \delta \varphi_F^b + \cdots ,
$$
(18)

where $\delta \varphi_F^a = \delta \varphi(N_F)$, $N_aF = N_a(N_F)$ and $N_{ab} = N_{ab}(N_F)$. During the period with $N_s < N < N_F$, we can use the solutions for $\delta \varphi^a$ given by Eqs. (10) and (14). Using these solutions, we obtain the relations:

$$
N_{as} = N_{aF} A_{a}^b(N_F, N_s) ,
$$
(19)

$$
N_{as} = N_{cd} F \Lambda_{a}^b(N_F, N_s) \Lambda_{b}^d(N_F, N_s) + 2 \int_{N_s}^{N_F} d N_c \Lambda_{a}^c(N_c) Q_{dc}^e(N_c)
\times \Lambda_{b}^d(N_c, N_s) \Lambda_{c}^e(N_c, N_s) ,
$$
(20)

with

$$
N_a(N) \equiv N_{aF} A_{a}^b(N_F, N) .
$$
(21)

C. Non-linear parameter

In this subsection, we derive a formula for the non-linear parameter $f_{NL}$ by making use of the $\delta N$ formalism. We first give the definition of $f_{NL}$. It is defined as the magnitude of the bispectrum of the curvature perturbation $\zeta$,

$$
B_\zeta(k_1, k_2, k_3) = \frac{6}{5} \frac{I_{NL}}{(2\pi)^3/2} \left[ P_\zeta(k_1) P_\zeta(k_2) + P_\zeta(k_2) P_\zeta(k_3) + P_\zeta(k_3) P_\zeta(k_1) \right] ,
$$
(22)

where $P_\zeta$ is the power spectrum of $\zeta$. The definitions of $P_\zeta$ and $B_\zeta$ are, respectively,

$$
\langle \zeta(k_1) \zeta(k_2) \rangle \equiv \delta(k_1 + k_2) P_\zeta(k_1) ,
$$
(23)

$$
\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle \equiv \delta(k_1 + k_2 + k_3) B_\zeta(k_1, k_2, k_3) .
$$
(24)

Equation (22) restricts the form of the bispectrum. The bispectrum in general does not take that simple form.

In fact, sub-horizon perturbations of fields give different $k$-dependent form of the bispectrum [17]. However, the sub-horizon contribution to the bispectrum is suppressed by the slow-roll parameters evaluated at the time of horizon exit. In contrast, the super-horizon evolution always gives the bispectrum in the form of Eq. (22) independent of the number of fields (see below). If $f_{NL} \gtrsim 1$, which is an interesting case from the observational point of view, then the contribution due to super-horizon evolution dominates the total bispectrum.

We assume that the slow-roll conditions are satisfied at $N = N_s$. Then, to a good approximation, $\delta \varphi^I_1$ becomes a Gaussian variable [8, 17] with its variance given by

$$
\langle \delta \varphi^I_1 \delta \varphi^I_1 \rangle \equiv \delta^{IJ} \left( \frac{H_s}{2\pi} \right)^2 ,
$$
(25)

and $\varphi^I_2$ becomes function of $\varphi^I_1$. Differentiating $\varphi^I_2 \simeq -\varphi^I_1$, we have

$$
\langle \delta \varphi^I_2 \delta \varphi^I_2 \rangle \equiv \frac{V^{IJ}}{V} - \frac{V_I}{V} \delta \varphi^I_1 + \cdots .
$$
(26)

The higher order terms are also suppressed by the slow-roll parameters. Hence, $\delta \varphi^I_2$ is Gaussian as is $\delta \varphi^I_1$, to a good approximation. Then, we can write down the variance of $\delta \varphi^I_2$ as

$$
\langle \delta \varphi^I_2 \delta \varphi^I_2 \rangle \simeq A^{IJ} \left( \frac{H_s}{2\pi} \right)^2 .
$$
(27)

At the first order both in the field perturbation and slow-roll limit, the matrix $A^{IJ}$ can be written as

$$
A^{IJ} = \epsilon ^{IJ} ,
$$
(28)

where

$$
\epsilon ^{IJ} \equiv \left[ \frac{V^{IJ}(\phi)}{V(\phi)^2} \right]_{\phi = \varphi(N_s)} .
$$
(29)

Since $\epsilon ^{IJ} = O(\epsilon, \eta)$, we find that $\langle \delta \varphi^I_2 \delta \varphi^I_2 \rangle$ and $\langle \delta \varphi^I_2 \delta \varphi^I_3 \rangle$ are suppressed by the slow-roll parameters. At the same order, $O(\epsilon, \eta)$, it is known that we need to add the slow-roll correction terms to $A^{IJ}$ in Eq. (28). Such corrections to $A^{IJ}$ are given in [28].

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2 Here, we consider the case that inflation is induced by the canonical scalar fields. In such a case, current observations of the spectrum of curvature perturbation restrict the models so that the slow-roll conditions are satisfied until the cosmologically relevant scales exit of the horizon scale. Otherwise, for example, in Dirac-Born-Infeld (DBI) inflation model, the sub-horizon contribution to the bispectrum not only has different $k$-dependent form, but also can be large enough to be detectable in the future experiment without inconsistency with current observations [23].
Using these equations, to the leading order, the non-linear parameter is written as

\[
\frac{6}{5} f_{NL} \simeq \frac{N_{aa} N_{bb} N_{cd} A_{ae} A_{bd}}{(N_{aa} N_{af} A_{ef})^2}
\]

\[
= \frac{1}{(N_{aa} \Theta_1^a)^2} \left[ N_{abF} \Theta^a(N_F) \Theta^b(N_F) + \int_{N_*}^{N_F} dN' N_{c} N(N') Q_{ab}^c(N') \times \Theta^a(N') \Theta^b(N') \right],
\]

(30)

where

\[
\Theta^a(N) \equiv A^a_c(N,N_*) A^{cb} N_{bs},
\]

and \(\Theta^a = \Theta^a(N_*)\). As we mentioned before, we have neglected the non-Gaussianity from the sub-horizon contributions in deriving Eq. (30). Eq. (30) shows that, aside from the scope of the present paper.

First, we solve Eq. (32) backward till \(N = N_*\) under the initial conditions \(N_a(N_F) = N_{af}\). Then we solve Eq. (33) forward till \(N = N_F\) under the initial conditions \(\Theta^a(N_*) = A^{ab} N_{bs}\). Substituting these solutions into Eq. (30), we obtain \(f_{NL}\).

The evolution equation of \(\Theta^a(N)\), Eq. (33), is identical to that of \(\delta \varphi^b\), Eq. (9), at the linear level. Since Eqs. (32) and (33) are mutually dual, a variable composed of \(N_a(N_*)\) and \(\Theta^a(N_*)\):

\[
W(N_*) = N_a(N_*) \Theta^a(N_*)
\]

becomes constant irrespective of \(N_*\). This constancy of \(W(N_*)\) corresponds to the constancy of \(\delta N(N_*, \varphi(N_*)\) which was mentioned below Eq. (16).

D. Non-linearity generated in scalar dominant phase

In order to evaluate Eq. (30), we need to know \(N_{af}\) and \(N_{abF}\). If one takes into account the evolution of the curvature perturbations after the scalar dominant phase, isocurvature perturbations may remain during preheating/reheating era after inflation. In such cases, in order to calculate \(N_{af}\), \(N_{abF}\), we need to investigate the evolution of the background e-folding number with the effect of short wavelength/radiation component, which is beyond the scope of the present paper.

Here, let us evaluate \(\zeta(N_F)\) on the uniform energy density hypersurface at \(N = N_F\), neglecting the later evolution of the curvature perturbations during the period with \(N_c > N > N_F\). In this case we can obtain explicit forms of \(N_{af}\) and \(N_{abF}\), which appeared in Eqs. (21) and (30), written in terms of the background quantities at \(N = N_F\). On the super-horizon scales, the uniform energy density hypersurface is equivalent to the constant Hubble hypersurface. Then, \(\zeta(N_F)\) is evaluated by the time shift \(\delta N\) measured from the \(H = \text{constant}\) hypersurface. Therefore at \(N = N_F\) we have the equation:

\[
H(\varphi(N_F + \zeta(N_F))) = H(\varphi(0)(N_F))).
\]

(35)

The Hubble parameter \(H\) is given by Eq. (6). Solving Eq. (35) with respect to \(\zeta(N_F)\), we obtain

\[
\zeta(N_F) \approx N_{af} \delta \varphi_F + \frac{1}{2} N_{abF} \delta \varphi_F \delta \varphi_F,
\]

(36)

with

\[
N_{af} = - \frac{H_a(\varphi)}{H_F(\varphi) F(\varphi)} |_{\varphi(0)(N_F)},
\]

(37)

\[
N_{abF} = - \frac{U_{abF}(\varphi)}{H_c(\varphi) F(\varphi)} |_{\varphi(0)(N_F)},
\]

(38)

where

\[
U_{ab} = H_{ab} - \frac{2H_a}{H_d} (H_c P_{bc} + F_c H_{eb}) \frac{2H_a H_b}{(H_c H_F)^2} (F_c H_{ad} F_{eb} + F_c P_{eb} H_d).
\]

(39)

and \(H_a = \partial H/\partial \varphi^a\), \(H_{ab} = \partial^2 H/\partial \varphi^a \partial \varphi^b\). The explicit forms of \(H_a\) and \(H_{ab}\) are shown in appendix A. The right-hand sides of Eqs. (37) and (38) are written in terms of local quantities, i.e., those depending only on \(\varphi^i\) and \(d\varphi^i/dN\) at \(N = N_F\). Hence, once we specify a central background trajectory \(\varphi^0(\varphi)\), we can readily determine \(N_{af}\), \(N_{abF}\). \(\delta N\) evaluated using the determined \(N_{af}\) and \(N_{abF}\) depends on \(N_F\) unless \(N_F > N_c\), where \(N_c\) is a time when the complete convergence of the background trajectories occurs.

III. DOUBLE INFLATION MODEL WITH LARGE MASS RATIO

Substituting Eqs. (37) and (38) to Eq. (30), the non-linear parameter, \(f_{NL}\), can be also defined as a function of the final time, \(N_F\). As mentioned above, the curvature perturbations on uniform energy density hypersurface, \(\zeta\), evolve when \(N_F < N_c\), and hence \(f_{NL}\) also evolves.

In this section, using the formulation given in the previous section [11] we calculate the non-linear parameter \(f_{NL}(N_F)\) for the double inflation models, which violate
the slow-roll conditions for a certain period when the cosmologically relevant scales are well outside the horizon. The potential is given by \[ V(\phi, \chi) = \frac{1}{2}m_\phi^2 \phi^2 + \frac{1}{2}m_\chi^2 \chi^2 . \] (40)

We assume a large mass ratio, i.e., \( m_\chi/m_\phi \gg 1 \). Because of \( m_\chi/m_\phi \gg 1 \), the energy density of \( \chi \)-field, \( \rho_\chi \), decays faster than that of \( \phi \)-field, \( \rho_\phi \). Hence if \( \rho_\phi \gg \rho_\chi \) at initial time, \( \rho_\chi \) never dominates the energy density during the later evolution of the universe. In such a model, only a single chaotic inflation induced by \( \phi \)-field occurs. It is known that in a single scalar slow-roll inflation, \( f_{NL} \) is suppressed by the slow-roll parameters. Here we assume the opposite, \( \rho_\chi \gg \rho_\phi \), at the initial time. In this case the slow-roll conditions are badly violated when \( \phi - \chi \) equality \( (\rho_\phi \simeq \rho_\chi) \) occurs. We assume that both fields are in the slow-roll phase at the initial time and the inflation induced by \( \phi \)-field also occurs after \( \phi - \chi \) equality. Denoting the initial values of fields at \( N = N_* \) as \( \phi_* \) and \( \chi_* \), this assumption implies \( \phi_*, \chi_* \gg 1 \).

There are several works in which the primordial non-Gaussianity in the two-scalar chaotic inflation model whose potential is given by Eq. (40) was investigated. In Ref. [10], the authors analyzed the primordial non-Gaussianity generated during \( \mathcal{N} \)-flation model and gave a simple analytic formula using the slow-roll approximation even on super-horizon scales based on \( \delta N \) formalism. In Ref. [10], using their slow-roll formula the authors also calculated the non-linear parameter \( f_{NL} \) for the two-scalar chaotic inflation model whose potential is given by Eq. (40) with the mass ratio \( m_\phi/m_\chi = 9 \). In such case, the slow-roll conditions are not violated during scalar dominant phase, so they could use the slow-roll formula. They compared the result obtained by using their analytic formula with that obtained numerically, and they found that the non-linear parameter does not become large in such case. In Ref. [10], the authors also provided another formulation for the non-linearity generated on super-horizon scales without slow-roll conditions. They also calculated the non-linear parameter using their formulation for the potential given by Eq. (40) with the mass ratio \( m_\phi/m_\chi = 12 \). They also found that the non-linear parameter evaluated at the end of inflation does not become large in such model.

Here, using the formulation given in the previous section, we numerically calculated \( f_{NL} \) for the mass parameters \( m_\phi = 0.05, m_\chi = 1.0 \) and compared the result with the previous work [10].

A. Numerical calculation for \( m_\chi/m_\phi = 20 \)

Using the formulation given in the previous section, we numerically calculated \( f_{NL} \) for the mass parameters \( m_\phi = 0.05, m_\chi = 1.0 \). Hence the mass ratio is \( m_\chi/m_\phi = 20 \). Initial value of fields are set to \( \phi_* = \chi_* = 10 \). We choose the \( e \)-folding number as a time coordinate and set the initial time \( N_* \) to 0. The central background trajectory in the field space is shown in Fig. 1 and the evolution of these two fields as a function of \( N \) is shown in Fig. 2. We define slow-roll parameters as

\[ \epsilon \equiv -\frac{1}{2} \frac{dH}{dN} , \]

\[ \eta_{\phi\phi} \equiv \frac{m_\phi^2}{V} , \eta_{\chi\chi} \equiv \frac{m_\chi^2}{V} . \]

The evolution of the slow-roll parameters is shown in Figs. 3, 4 and 5.

From Fig. 2 we see that initially \( \chi \)-field decays rapidly while \( \phi \)-field remains almost at its initial value due to large Hubble friction. At around \( N = 26 \), \( \chi \)-field reaches zero and starts damped oscillation. As shown in Fig. 4, around this time, the slow-roll parameter \( \epsilon \) exceeds 1 for a moment and the inflation ends once. From Figs. 4 and 5 also \( \eta_{\chi\chi} \) exceeds 1 after \( \chi \)-field settles to the minimum, while, \( \phi \)-field is slow-rolling during this phase.

In calculating the non-linear parameter \( f_{NL} \), we regard \( f_{NL} \) as a function of \( N_F \) in the same sense as \( \zeta(N_F) \) in Eq. (39). We show the evolution of the non-linear parameter \( f_{NL}(N_F) \) in Fig. 6. From this figure, we find that the non-linear parameter \( f_{NL}(N_F) \) temporarily increases with the maximum amplitude reaching \( \sim 8 \). As soon as \( \chi \)-field settles to its minimum, the universe becomes dominated by \( \phi \)-field due to the decay of \( \rho_\chi \), and the universe starts the second inflation driven by \( \phi \)-field. After the second inflation starts, iso-curvature perturbation decays rapidly due to the decay of \( \chi \). Hence the time \( N_e \) is before
FIG. 2: (color online) This figure shows evolution of $\phi$ (dashed red line) and $\chi$ (solid blue line). We choose the $e$-folding number, $N$, as a time coordinate and set the initial time, $N_*$, to 0.

FIG. 3: This figure shows the evolution of the slow-roll parameter $\epsilon$ defined by Eq. (41).

FIG. 4: This figure shows the evolution of the slow-roll parameter $\eta_{\phi\phi}$ defined by Eq. (42).

FIG. 5: This figure shows the evolution of the slow-roll parameter $\eta_{\chi\chi}$ defined by Eq. (42).

FIG. 6: This figure shows the evolution of non-linear parameter, $f_{NL}(N_F)$. After the second inflation, $N_F > 30$, the background trajectories in phase space have completely converged. Thus, the curvature perturbation on a constant Hubble hypersurface, $\zeta(N_F)$, remains constant on large scales at $N_F > 30$ and the $f_{NL}(N_F)$ also remains constant in this era.
the end of the second inflation in this model. Then, the
sum of \( f_{NL}(N_F > 30) \) and the sub-horizon contributions
gives the primordial non-Gaussianity, independent of the
reheating process. The final value of \( f_{NL}(N_F > 30) \) is
0.01004. Hence the generation of large non-Gaussianity
due to the violation of the slow-roll conditions in this
model does not occur.

In Ref. [16], the authors numerically calculated the
non-linear parameter for the model whose potential is
given by Eq. (46), with mass ratio \( m_\phi/m_\chi = 12 \). They
used their own formalism, which is different from \( \delta N \)
formalism used in this paper. The behavior of the non-
linear parameter \( f_{NL} \) obtained by their calculation seems
similar to our result shown in Fig. 6.

B. Approximate analytical expression for \( f_{NL} \)

In the previous subsection (III A), by numerical calculation,
we found that the final value of \( f_{NL} \) is much
less than 1, although the curvature perturbation becomes
highly non-Gaussian (\( f_{NL} \approx 8 \)) for a moment. We can
derive the analytical expression for the final value of \( f_{NL} \)
in the limit of large mass ratio \( m_\chi/m_\phi \gg 1 \), which is
the subject of this subsection.

The background equations are given by
\[
\begin{align*}
H \frac{d}{dN} \left( H \frac{d\phi}{dN} \right) + 3H^2 \frac{d\phi}{dN} + m_\phi^2 \phi &= 0, \quad (43) \\
H \frac{d}{dN} \left( H \frac{d\chi}{dN} \right) + 3H^2 \frac{d\chi}{dN} + m_\chi^2 \chi &= 0, \quad (44) \\
H^2 &= \frac{m_\phi^2 \phi^2 + m_\chi^2 \chi^2}{6 + \left( \frac{d\phi}{dN} \right)^2 + \left( \frac{d\chi}{dN} \right)^2}. \quad (45)
\end{align*}
\]

Within the slow-roll approximation, these equations reduce to
\[
\begin{align*}
\frac{d}{dN} \phi &\simeq -\frac{m_\phi^2 \phi}{3H^2}, \quad \frac{d}{dN} \chi \simeq -\frac{m_\chi^2 \chi}{3H^2}, \\
H^2 &\simeq \frac{1}{6} \left( m_\phi^2 \phi^2 + m_\chi^2 \chi^2 \right). \quad (46)
\end{align*}
\]

At the initial time, \( N = N_*, \) both fields are in the slow-
roll phase and \( \phi_* \simeq \chi_* \gg 1. \) Then the ratio of the time
derivatives of the scalar fields is
\[
\frac{d\phi}{dN}/d\chi/dN \simeq \frac{m_\phi^2 \phi}{m_\chi^2 \chi} \ll 1, \quad (47)
\]

and also \( \rho_\chi \gg \rho_\phi \) in the limit of large mass ratio. Hence,
to a good approximation, \( \phi(N) \simeq \phi_* \) until the energy
density of \( \phi \)-field dominates the total energy density of
the universe, i.e., \( \rho_\phi \simeq \rho_\chi. \) On the other hand, solving
Eq. (48), the evolution of \( \chi \)-field during the slow-roll
phase can be obtained as
\[
\chi(N) \simeq \sqrt{\chi_*^2 - 4N}. \quad (48)
\]

Here we recall that we have set \( N_* = 0. \) After the time
when \( \chi \sim 1, \) \( \chi \)-field is no longer in the slow-roll phase and \( \rho_\chi \) evolves as;
\[
\rho_\chi(N) \simeq \rho_\chi(\chi(0)) \exp \left[ -3(N - \chi(0)) \right], \quad (49)
\]

where \( \chi(0) \) represents the time when \( \chi \)-field starts oscil-
lation. From Eq. (48), we have
\[
\chi(N) \approx \frac{\chi_*^2}{4} - \frac{\chi(0)^2}{4}. \quad (50)
\]

Then, Eq. (49) can be rewritten as
\[
\rho_\chi(N) \approx \rho_\chi \exp \left( -3N + \frac{3}{4} \chi_*^2 \right), \quad (51)
\]

where \( \rho_\chi \) is independent of \( N, \chi_* \) and \( \phi_* \). Since \( \rho_\chi \) de-
cays in time for \( N > \chi(0), \) \( \rho_\phi \) dominates the total energy
density of the universe at some time and \( \phi \)-field starts slow-rolling. Let us denote the \( \epsilon \)-folding number at the
time when \( \rho_\phi \simeq \rho_\chi \) as \( N_{eq}. \) For \( N > N_{eq}, \) the evolution of \( \phi \)-field is given by
\[
\phi(N) \simeq \sqrt{\phi_*^2 - 4(N - N_{eq})}, \quad (52)
\]
during the slow-roll phase. \( N_{eq} \) is obtained from the equation,
\[
\rho_\phi(N_{eq}) \simeq \frac{1}{2} m_\phi^2 \phi(N_{eq})^2 \simeq \rho_\chi(N_{eq}). \quad (53)
\]

Combining Eq. (51) with Eq. (53), we have
\[
N_{eq} \simeq \chi_*^2/4 - \frac{1}{3} \log \frac{m_\phi^2 \phi_*^2}{2 \rho_\chi}. \quad (54)
\]

The total energy density after \( N = N_{eq} \) is approximately
\( \rho_\phi. \) Hence, using Eqs. (52) and (54), we have
\[
\rho(N) \simeq \rho_\phi(N) \simeq \frac{1}{2} m_\phi^2 \phi^2(N) \simeq \frac{1}{2} m_\phi^2 \phi_*^2 \left[ \phi_*^2 + \chi_*^2 - 4 \left( N + \frac{1}{3} \log \frac{m_\phi^2 \phi_*^2}{2 \rho_\chi} \right) \right]. \quad (55)
\]

After \( N = N_{eq}, \) the universe is dominated by the single
slow-roll component \( \phi \)-field, and the curvature pertur-
bation on the uniform density hypersurface becomes
constant in time on super-horizon scales. From Eq. (55),
we obtain an expression for the \( \epsilon \)-folding number at the
final time in terms of \( \phi_* \) and \( \chi_* \) as
\[
N(N_F, \phi_*, \chi_*) \simeq \frac{1}{4} \left( \phi_*^2 + \chi_*^2 \right) - \frac{2}{3} \log \phi_* \\
- \frac{\rho_F}{2 m_\phi^2} - \frac{1}{3} \log \frac{m_\phi^2}{2 \rho_\chi}, \quad (56)
\]
where \( \rho_F = \rho(N_F) \). This gives
\[
- \frac{6}{5} f_{NL}(N_F) \simeq \frac{N_I N_J N_{IJ}}{(N_K N_K)^2}
\]
\[
= \frac{2}{\phi^2 + \chi^2} \left( 1 + \mathcal{O}(\phi_*^{-2}, \chi_*^{-2}) \right), \tag{57}
\]
where \( N_I = \partial N/\partial \phi_*^I \), \( N_{IJ} = \partial^2 N/\partial \phi_*^I \partial \phi_*^J \). Hence \( f_{NL} \) is \( \mathcal{O}(\phi_*^{-2}, \chi_*^{-2}) \), and it is much less than 1. \( f_{NL} \) estimated using this expression is \( \approx 0.01 \) for \( m_* = 0.05 \), \( m_\chi = 1.0 \) and \( \phi_* = \chi_* = 10 \), which agrees well with the numerical result in the previous subsection (see IIIA).

The reason why we get small \( f_{NL} \) in the double inflation model can be understood as follows. In the case of a large mass ratio, evolution of the universe can be clearly divided into three stages. First one is the inflation induced almost by a single field, i.e., \( \chi \)-field. The second one is the non-inflationary phase where the energy density of the oscillating \( \chi \)-field is still larger than that of the \( \phi \)-field. The third one is the inflation induced by a single field, i.e., \( \phi \)-field. Then the total \( e \)-folding number during all these stages mainly comes from the two inflationary stages, i.e., the first and the third stages. The perturbation of the \( e \)-folding number generated during the second stage is negligible. Hence as a crude approximation, the dynamics can be regarded as just the sum of two single field inflationary phases. This picture gives the first term of Eq. (56). The correction to this picture is represented by the second term, which is indeed minor for \( N \gtrsim 60 \).

The derivation of Eq. (57) can be straightforwardly extended to more general double inflation models with
\[
V(\phi, \chi) = c_1 \phi^{2p} + c_2 \chi^{2p}, \tag{58}
\]
where \( p \) is an arbitrary positive integer. In this case, the final value of \( f_{NL} \) is more suppressed than Eq. (57) by a factor \( 1/p \).

### C. Comment on \( f_{NL} \) in \( N \)-flation model

Discussion of the previous subsection about the small \( f_{NL} \) in the double inflation model gives us some insight into the generation of the non-Gaussianity in the so-called \( N \)-flation model \[29\], in which the potential is given by
\[
V(\phi) = \sum_{I=1}^N \frac{m_I^2}{2} \phi_I^2. \tag{59}
\]
This is a generalization of the double inflation model to an arbitrary number of fields. We consider the case in which all fields have large initial amplitude, i.e., \( \phi_I \gg 1 \). We also assume \( m_\phi < m_\chi < \cdots < m_N \). In this case, \( \phi_N \) field decays first. If \( \rho_N \) dominates the total energy density of the universe until \( \phi_N \sim 1 \), then the inflation is almost like the single field inflation by \( \phi_N \)-field. After \( \phi_N \)-field decays, then \( \phi_{N-1} \)-field decays and so on. Hence the leading \( e \)-folding number would be given by
\[
N \approx \frac{1}{3} \sum_{I=1}^N \phi_I^2. \tag{60}
\]
Then the corresponding \( f_{NL} \) is
\[
- \frac{6}{5} f_{NL} \sim 2 \left( \sum_{I=1}^N \phi_I^2 \right)^{-1} \sim \frac{1}{2N}. \tag{61}
\]
Roughly speaking, \( f_{NL} \) is suppressed by the inverse of the \( e \)-folding number. This result is quite similar to that in Ref. \[9\], where the authors studied under the slow-roll approximation.

### IV. Summary

Based on the \( \delta N \) formalism, we have derived a useful formula for calculating the primordial non-Gaussianity due to the super-horizon evolution of the curvature perturbation in multi-scalar inflation without imposing slow-roll conditions. This formula can apply for the inflation models with general field space metric, \( h_{IJ} \), as far as super-horizon contributions are concerned. Generally, when one calculates the non-Gaussianity of the curvature perturbations, one has to solve the second order perturbation equations. In doing so for a multi-scalar inflation, there appear tensorial quantities with respect to the indices of the field components. Our formula reduces the problem of calculating the non-linear parameter \( f_{NL} \) to solving only first order perturbation equations for two vector quantities. This reduces \( \mathcal{O}(N^2) \) calculations to \( \mathcal{O}(N) \) ones where \( N \) is the number of the scalar field components. Hence our formalism has a great advantage for the numerical evaluation of \( f_{NL} \) in the inflation model composed of a large number of fields.

We have also studied the primordial non-Gaussianity in double inflation model as an example that violates slow-roll conditions by using our formalism. We found that, although \( f_{NL} \) defined for the curvature perturbation on a constant Hubble hypersurface exceeds 1 for a moment around the time when the slow-roll conditions are violated, the final value of \( f_{NL} \) is suppressed by the slow-roll parameters evaluated at the time of horizon exit. We have shown that this can be understood even analytically in the \( \delta N \) formalism. This result is straightfor-
wardly extended to more general double inflation model and $N$-flation model.

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**APPENDIX A: SPECIFIC EXPRESSION**

In this appendix, we show the explicit forms of $P_{ij}^a (= P_{ij}^{aj})$ in Eq. (9), $Q_{ab,ijk} (= Q_{ijk})$ in Eq. (13), $H_a (= H_1^i)$ in Eq. (36) and $H_{ab} (= H_{ijk})$ in Eq. (39). $P_{ij}^a$ is given by

\[
P_{ij}^a = 0 , \quad P_{ij}^{aj} = \delta^a_j ,
\]

\[
P_{ij}^{ai} = -\frac{V}{H^2} \left( \frac{V_j}{V} - \frac{V^i V_j}{V^2} \right) ,
\]

\[
P_{ij}^{ai} = \varphi_2 V^{ai} \varphi_2 - \frac{V}{H^2} \delta^a_j.
\]

$Q_{ijk}^{ab}$ by

\[
Q_{ijk}^{123} = Q_{ijk}^{12} = Q_{ijk}^{13} = Q_{ijk}^{23} = 0 ,
\]

\[
Q_{iJK}^{11} = -\frac{V}{H^2} \left( \frac{V_J}{V} - \frac{V^i V_J}{V^2} - \frac{V^j V_J}{V^2} + 2 \frac{V^i V_J V_K}{V^3} \right) ,
\]

\[
Q_{iJK}^{12} = \left( \frac{V^i}{V} - \frac{V^i V_J}{V^2} \right) \varphi_2 K ,
\]

\[
Q_{iJK}^{21} = \left( \frac{V^j}{V} - \frac{V^j V_K}{V^2} \right) \varphi_2 K ,
\]

\[
Q_{iJK}^{22} = \delta^i_K \varphi_2 J + \delta^i_J \varphi_2 K + \delta_{JK} \left( \varphi_2^2 + \frac{V^i}{V} \right) ,
\]

$H_1^a$ is given by

\[
H_1^a = H \frac{V_i}{V} , \quad H_1^2 = \frac{H^3}{2V} \varphi_2 J ,
\]

and $H_{ij}^{a}$ by

\[
H_{ij}^1 = \frac{H}{2} \left( \frac{V_i J}{V} + \frac{V J_i}{V^2} \right) ,
\]

\[
H_{ij}^2 = \frac{H^3}{4V^2} V J_2 J , \quad H_{ij}^1 = \frac{H^3}{4V^2} V J_2 J ,
\]

\[
H_{ij}^2 = \frac{H^3}{2V^2} \left( \delta_{ij} + \frac{3H^2}{2V} \varphi_2 J \varphi_2 J \right) .
\]

**APPENDIX B: EXTENSION TO THE GENERAL FIELD SPACE METRIC**

In this appendix, we will extend the formulation given in sec. II for the flat field space metric $h_{iJ} = \delta_{iJ}$ to the general case. For general case, the background equations corresponding to Eq. (3) for flat field space are given by

\[
\frac{d \varphi_1^a}{dN} = F_1^a , \quad \frac{d \varphi_2^a}{dN} = F_2^a ,
\]

where $DA^i \equiv dA^i + \Gamma^i_{JK} A^J \varphi^K$, $\Gamma^i_{JK}$ is Christoffel symbol with respect to the field space metric $h_{iJ}$, and

\[
F_1^a = \varphi_2^a , \quad F_2^a = -\frac{V}{H^2} \left( \varphi_2^2 + h^{ij} V J_i V J^i V J^j \right) .
\]

In the same way as Eq. (6), we can define the perturbation as

\[
\delta \varphi^a (N) = \varphi^a (\lambda + \delta \lambda; N) - \varphi^a (\lambda; N) ,
\]

and we decompose $\delta \varphi^a$ as

\[
\delta \varphi^a = \delta \varphi^{(1)} + \frac{1}{2} \delta \varphi^{(2)} .
\]

The curvature perturbation can be expanded in terms of $\delta \varphi^{(1)}$ as

\[
\zeta \simeq N_a \delta \varphi^a + \frac{1}{2} N_a b \delta \varphi^a \delta \varphi^b + \cdots .
\]

However, $\delta \varphi^{(1)}$ defined in this way does not transform as a vector under the "coordinate transformation", $\delta^i \rightarrow \tilde{\delta}^i (\phi)$ because Eq. (13) takes the difference of variables at different points in field space. Hence the evolution equation for $\delta \varphi^{(1)}$ loses the manifest covariance. To avoid this drawback (though not essential), we introduce new perturbation variables $\delta \tilde{\varphi}^a = \delta \varphi^{(1)} + \frac{1}{2} \delta \varphi^{(2)}$ defined by

\[
\delta \tilde{\varphi}^{(1)} = \frac{d \varphi^{(1)}_1}{d \lambda} \delta \lambda , \quad \delta \tilde{\varphi}^{(2)} = \frac{D \varphi^{(1)}_1}{d \lambda} \delta \lambda ,
\]

\[
\delta \tilde{\varphi}^{(1)} = \frac{d \varphi^{(2)}_2}{d \lambda} \delta \lambda , \quad \delta \tilde{\varphi}^{(2)} = \frac{D^2 \varphi^{(2)}_2}{d \lambda^2} \delta \lambda ,
\]

which transform as vectors. Differentiating the background equations \([13]\) with respect to $\lambda$, we obtain the evolution equations for the perturbations of the scalar
fields in curved field space metric. To the first order, we have
\[
\frac{D}{dN} \delta \varphi^a(N) = (P^a_b(N) + \Delta P^a_b(N)) \delta \varphi^b(N), \tag{B8}
\]
where the coefficient is given by
\[
P^a_b = \left. \frac{D^2 F^a}{\partial \varphi^b \partial \varphi^c} \right|_{\varphi = \varphi(N)}, \tag{B9}
\]
and \(\Delta P^a_b\), which vanishes in the flat field space metric representing terms due to the curvature of the field space, is given by
\[
\Delta P^I_{1J} = \Delta P^{I2}_{1J} = \Delta P^{I2}_{2J} = 0, \\
\Delta P^{I2}_{1J} = -R^I_{LJK} \varphi^L_{2F} \varphi^K. \tag{B10}
\]
Here, \(R^I_{LJK}\) represents Riemann tensor associated with the field space metric \(h_{1J}\). Evolution equation for \(\delta \varphi^a\) is also given by
\[
\frac{D}{dN} \delta \varphi^a(N) = (P^a_b(N) + \Delta P^a_b(N)) \delta \varphi^b(N) + (Q^a_{bc}(N) + \Delta Q^a_{bc}(N)) \delta \varphi^b(N) \delta \varphi^c(N), \tag{B11}
\]
where
\[
Q^a_{bc} = \left. \frac{D^2 F^a}{\partial \varphi^b \partial \varphi^c} \right|_{\varphi = \varphi(N)}, \tag{B12}
\]
and the explicit form of \(\Delta Q^a_{bc}\) is given by
\[
\Delta Q^{I1}_{1J} = -R^I_{JTL} \varphi^L_{2F}, \\
\Delta Q^{I2}_{1J} = \Delta Q^{I2}_{11} = \Delta Q^{I2}_{12} = 0, \\
\Delta Q^{I2}_{22} = -\nabla_K R^I_{LJK} \varphi^L_{2F} \varphi^K, \\
\Delta Q^{I2}_{22} = -2R^I_{KLJ} \varphi^L_{2F} \varphi^K, \\
\Delta Q^{I2}_{2J} = 0. \tag{B13}
\]
By introducing the matrix \(\Lambda\) as the solution of
\[
\frac{D}{dN} \Lambda^a_b(N, N_s) = (P^a_b(N) + \Delta P^a_b(N)) \Lambda^c_b(N, N_s), \tag{B14}
\]
with the condition \(\Lambda^I_{1J}(N) = \delta^I_J \delta^1_1\), formal solutions of the first and second order equations are given by
\[
\begin{align*}
\delta \varphi^a(N) &= \Lambda^a_b(N, N_s) \delta \varphi^b(N), \\
\delta \varphi^2(N) &= \int_N \frac{dN'}{N_s} \Lambda^a_b(N, N') \\
&\times [Q^b_{cd}(N') + \Delta Q^b_{cd}(N')] \delta \varphi^c(N') \delta \varphi^d(N'). \tag{B15}
\end{align*}
\]
The expression for the curvature perturbation in terms of \(\delta \varphi^a\), Eq. (B5), can be also rewritten as that in terms of \(\delta \varphi^a\). To make the expression more concise, we introduce new variables \(\tilde{N}_a\) and \(\tilde{N}_{ab}\) as expansion coefficients of \(\tilde{\zeta}\) in terms of \(\delta \varphi^a\) so that we have
\[
\tilde{\zeta} = \tilde{N}_{aF} \delta \varphi^a_F + \frac{1}{2} \tilde{N}_{abF} \delta \varphi^a_F \delta \varphi^b_F + \cdots. \tag{B16}
\]
Comparing the two expressions (B5) and (B10), we find
\[
\begin{align*}
\tilde{N}_1 &= N_1^1 - N_2^2 \Gamma^K_{1J} \varphi^J_F, \\
\tilde{N}_2^1 &= N_2^1, \\
\tilde{N}_2^2 &= N_1^2 + N_2^2 \Gamma^K_{2J} \varphi^J_F, \\
&+ (N_1^1 + N_1^2) \Gamma^K_{11} \varphi^J_F, \\
- N_2^1 \nabla_F \Gamma^K_{1J} \varphi^J_F, \\
&N_1^2 \nabla_F \Gamma^K_{2J} \varphi^J_F, \\
\tilde{N}_{IJ} &= N_1^I - N_2^I \Gamma^K_{IJ} \varphi^J_F, \\
&+ N_2^I \nabla_F \Gamma^K_{IJ} \varphi^J_F, \\
\tilde{N}_{IJ} &= N_1^J - N_2^J \Gamma^K_{IJ} \varphi^K_F. \tag{B17}
\end{align*}
\]
From the constancy of \(\tilde{N}_{aF}^{(1)}\), we find
\[
\tilde{N}_a(N) = \tilde{N}_{aF} \Lambda^a_{aF}(N_F, N). \tag{B18}
\]
We also redefine \(\Theta^I\) in Eq. (31) as
\[
\Theta^a(N) = \Lambda^a_b(N, N_s) \Lambda^b_{cs}, \tag{B19}
\]
where \(\Lambda^a_{bc}\) is defined by
\[
\langle \delta \varphi^a_N \delta \varphi^b_N \rangle = \Lambda^{ab} \left( \frac{H_N}{2\pi} \right)^2. \tag{B20}
\]
\(\Lambda^{ab}\) for general case also depends on the field space metric, \(h_{1J}\).

Then, the expression for the non-linear parameter in general cases is also given by Eq. (30) with \(N_c(N')\) and \(Q^c_{ab}(N')\) replaced by \(\tilde{N}_c(N')\) and \(\tilde{Q}^c_{ab}(N')\).
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