A Pedestrian Approach to the Measurement Problem in Quantum Mechanics

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The quantum theory of measurement has been a matter of debate for over eighty years. Most of the discussion has focused on theoretical issues with the consequence that other aspects (such as the operational prescriptions that are an integral part of experimental physics) have been largely ignored. This has undoubtedly exacerbated attempts to find a solution to the “measurement problem”. How the measurement problem is defined depends to some extent on how the theoretical concepts introduced by the theory are interpreted. In this paper, we fully embrace the minimalist statistical (ensemble) interpretation of quantum mechanics espoused by Einstein, Ballentine, and others. According to this interpretation, the quantum state description applies only to a statistical ensemble of similarly prepared systems rather than representing an individual system. Thus, the statistical interpretation obviates the need to entertain reduction of the state vector, one of the primary dilemmas of the measurement problem. The other major aspect of the measurement problem, the necessity of describing measurements in terms of classical concepts that lay outside of quantum theory, remains. A consistent formalism for interacting quantum and classical systems, like the one based on ensembles on configuration space that we refer to in this paper, might seem to eliminate this facet of the measurement problem; however, we argue that the ultimate interface with experiments is described by operational prescriptions and not in terms of the concepts of classical theory. There is no doubt that attempts to address the measurement problem have yielded important advances in fundamental physics; however, it is also very clear that the measure-
ment problem is still far from being resolved. The pedestrian approach presented here suggests that this state of affairs is in part the result of searching for a theoretical/mathematical solution to what is fundamentally an experimental/observational question. It suggests also that the measurement problem is, in some sense, ill-posed and might never be resolved. This point of view is tenable so long as one is willing to view physical theories as providing models of nature rather than complete descriptions of reality. Among other things, these considerations lead us to suggest that the Copenhagen interpretation’s insistence on the classicality of the measurement apparatus should be replaced by the requirement that a measurement, which is specified operationally, should simply be of sufficient precision.

I. INTRODUCTION

Since the beginning of quantum mechanics more than 80 years ago, physicists have argued about how to interpret the theoretical concepts introduced by the theory. Perhaps the most troublesome of all is the meaning of the wave function \( \Psi \) introduced by Schrödinger in 1926 [1]. Both the statistical rule, probability \( \sim |\Psi|^2 \), suggested by Born [2] and the concomitant phenomenon of quantum interference were anathemas to classical physics. The founders of quantum mechanics including Bohr, Heisenberg, Pauli, Schrödinger, and Einstein spent considerable effort worrying about how best to interpret the theory. In fact, there is more than one “interpretation” of quantum mechanics and the precise meanings of these remain the subject of much discussion. There has never been (nor, perhaps, ever will be) complete agreement on this issue.

Bohr’s point of view, commonly referred to as the Copenhagen interpretation (see below), was for many years considered to be the agreed upon viewpoint, at least as was declared in standard textbooks on quantum mechanics. Modern texts and the teachers that use them are often more circumspect and usually list a variety of interpretations such as the ensemble interpretation, the Copenhagen interpretation, decoherence theory, realist models such as Bohmian mechanics, the many worlds interpretation, etc. The truth of the matter is that few physicists actually know the details of any of these interpretations. Even the principal proponents of the Copenhagen interpretation, Bohr, Heisenberg, and Pauli disagreed on various aspects of it. Most physicists seem to arrive at some vague personal interpretation
of quantum mechanics and then stop worrying about it, following the David Mermin maxim embodied in his statement, “If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be ‘Shut up and calculate!’” [3]. That is, if one solves Schrödinger’s equation (or a relativistic equivalent) and uses the statistical Born rule to interpret the solution, then quantum theory seems to provide a complete description of all that can be observed. Further discussion as to the physical significance of $\Psi$ or as to whether or not it provides a complete description of reality is unnecessary and should be eschewed.

As a case in point, consider the so called Copenhagen interpretation. While physicists still argue about what is and what is not included in the Copenhagen interpretation, at the very least most would agree that, according to this interpretation, the wave function, or alternatively the density matrix, predicts the probability distribution of outcomes of particular measurements made on an ensemble of similarly prepared systems. However, as to how to perform these measurements, both theory and the accompanying interpretation are silent. Such predictions must be turned over to experimental physicists in whom both classical physics and the art of constructing apparatus are deeply ingrained. (While not usually acknowledged, the same is true in other domains of physics. That is, even after an interpretation is given, there is no implied prescription for how to perform an experiment.) Nevertheless, the Copenhagen interpretation often serves as a foil for discussions of interpretations of quantum mechanics and the measurement problem and it is useful to make explicit those aspects of it that are referred to in this paper, with the caveat that there is no well defined Copenhagen interpretation. In Heisenberg’s words: “...it may be a point in the Copenhagen interpretation that its language has a certain degree of vagueness, and I doubt whether it can become clearer by trying to avoid this vagueness.” [4] As mentioned above, the main proponents didn’t fully agree on what the Copenhagen interpretation entails. So we use the term simply to label the following interpretative statements that are often attributed to it and other interpretations, and to which many physicists ascribe.

According to Stapp [4], “The logical essence of the Copenhagen interpretation is summed up in the following two assertions: 1) the quantum theoretical formalism is to be interpreted \textit{pragmatically}; and 2) quantum theory provides for a \textit{complete} scientific account of atomic phenomena.” Several operational principles that are often associated with the Copenhagen interpretation are: i) the square of the magnitude of the wave function, $|\Psi|^2$, is associated with the probability of the occurrence of an event (the Born rule); ii) it is not possible to
determine, via measurement, all of the possible properties of a given system (the Heisenberg uncertainty principle); iii) the only possible values of a given property that can result from a measurement are the eigenvalues of the operator associated with that property; and iv) measurements must be (or invariably are) made with apparatus that are described in terms of classical physics. Item iii) invariably leads to wave function collapse (see below); however, this is not problematic for the Copenhagen or ensemble interpretations for which the wave function is considered to be a computation tool rather than an aspect of physical reality. This list is by no means exhaustive but will suffice for the purposes of this paper.

Perhaps the single most perplexing aspect of interpreting quantum mechanics is what is generally referred to as the measurement problem, i.e., the unresolved problem of how the outcome of a particular measurement arises from a quantum theory that, at most, renders a probabilistic distribution of all possible outcomes. We say “at most” because quantum theory itself says nothing at all about the measurement process. The probabilistic significance of the result of a quantum mechanical calculation arises from interpretive statements that accompany quantum mechanics but such statements are not, in themselves, intrinsic to the theory. For example, most interpretations maintain that the measurement of any observable of a system can only be an eigenvalue of the Hermitian operator associated with that variable and the probability of measuring a particular value is given by the absolute value squared of the corresponding eigenfunction, $|\lambda_i\rangle$, projected onto the wave function, $|\Psi\rangle$, of the system, i.e., $|\langle \lambda_i | \Psi \rangle |^2$, the Born rule. However, such an interpretation is not intrinsic to the mathematical structure of the theory. Furthermore, just what constitutes a measurement of an observable is not well defined and, in fact, constitutes one of the aspects of the measurement problem. We maintain that this is also the case for classical theory and contributes to a “classical measurement problem” (see Section VI). There are other interpretations that offer different explanations as to the significance of the wave function and they are all extrinsic to the mathematical formalism of quantum mechanics and are all vague about the details of what constitutes a measurement. Different people have emphasized different aspects of the measurement problem; however, the following four (related) issues are frequently raised:

1) Wave Function Collapse: Part of any standard interpretation of quantum mechanics is that the only possible outcomes of the measurement of a particular property of a system are eigenvalues of the quantum mechanical operator associated with that property. Even though it is not part of the Copenhagen interpretation, many physicists harbor the belief that
the wave function represents the real world and, furthermore, that it provides a complete description of an individual system. If this were so, then after a measurement has been made, it must be that the wave function of a system is transformed by the measurement from the initial wave function to the eigenfunction associated with the measured value. The problem is that such a transition is not part of the unitary evolution of the system as described by quantum theory. If it were, then presumably quantum theory would predict this transition and, hence, the exact outcome of the measurement, thereby contradicting the statistical interpretation that lies at the heart of the theory. This aspect of the measurement problem is confounded by the fact that wave function collapse, if assumed to be a physical process, would not be expected to occur instantaneously and should therefore be accessible to observation. Decoherence theory has been useful in understanding how the wave function of a system evolves as it interacts with a measuring device and the environment if these are also described quantum mechanically by a wave function. However, this evolution is necessarily distinct from wave function collapse [5].

2) The Consistency of Quantum and Classical Reality: In some sense, this is the flip side of the first issue. We perceive reality as a series of events involving the objects of our perceptions. These occur sequentially and definitively. On the other hand, quantum mechanics seems to say nothing what-so-ever about these events but only describes, exactly, the evolution of interacting wave functions. An interpretation of the wave functions is necessary to make the connection with events and, in the end, it is only a probabilistic statement about the real world. In short, the formalism of quantum mechanics precludes the occurrence of any specific event whereas, in our world, we know that specific outcomes always occur.

3) Classicality of Experiments: According to Bohr’s statement of the Copenhagen interpretation, while a (microscopic) system under investigation is described quantum mechanically, the measurement apparatus that observes it must be (or, perhaps, always is) described classically. How does one decide which aspects of a system are to be described classically and which to be described quantum mechanically, i.e., what is the location of the quantum/classical divide? Many, including Heisenberg, have pointed out that this divide does not represent a discontinuity of physical systems but rather is simply a transition from one formalism to another. Nevertheless, it has historically been considered to be an important aspect of the measurement problem.
4) *Interference Effects*: One of the most amazing consequences of the quantum nature of matter is quantum interference, a phenomenon in which a particle’s wave function exhibits wave interference. For example, if a single particle wave function passes through a barrier with two slits, the parts of the wave function emerging from the two slits interfere and there will be periodic locations on a distant screen where there is (near) zero probability that the particle will strike. If one of the slits is covered, the interference fringes disappear. One wonders why, by closing off one possible path of the particle, the probability of it striking the screen at a previously inaccessible region becomes nonzero? This sort of interference is not the least bit surprising for inherently wave phenomena like sound or light; however, that “particles” should behave this way runs counter to our intuition. Furthermore, if one simply determines through which slit the particle passes without obstructing it, then quantum interference between the two parts of the wave function disappears. The only way this can be explained is that such a determination, in some sense, constitutes a measurement and the wave function of the particle collapses.

One rather curious aspect of the measurement problem should be kept in mind. It can certainly be argued that for the last 80 years, while many interesting resolutions of the problem have been suggested, the problem of quantum measurements remains largely unsolved. Yet, the advance of quantum mechanics and quantum field theory has been enormous and seems not to have been impeded in the least by the lack of a satisfactory resolution of the measurement problem nor even by the lack of an agreed upon interpretation of the theory. How can this be? On the other hand, we will show that this situation is not particularly remarkable in the context of the pedestrian approach presented here.

**II. DECOHERENCE THEORY**

Today, many physicists are of the mind that decoherence theory has largely resolved the measurement problem. While certainly relevant to the measurement problem, it is a far reach indeed to claim that decoherence theory has solved the problem. Decoherence theory is relevant to those aspects of the measurement problem that deal with the classicality of macroscopic measurement apparatus and it will be important to compare the perspective of decoherence theory with that of the pedestrian approach put forward in this paper. The details of decoherence theory would be much too large a diversion to undertake in this paper;
however, there are many accessible treatments in the literature including the books by Joos et al. [6] and Schlosshauer [7].

Decoherence theory is neither new physics nor a new interpretation of quantum mechanics; although, it is certainly relevant to questions of interpretation. In decoherence theory, the measuring apparatus and the environment with which it inevitably interacts are both treated as purely quantum mechanical systems. As a consequence of the interactions of the quantum system of interest with the measuring apparatus and it with its immediate environment, the three become entangled, i.e., strongly correlated with each other. All, or at least most, of the environmental quantum degrees of freedom are not observable (certainly, not observed) and, therefore, must be summed over to achieve a reduced state of the system plus apparatus. The net effect of the enormous number of environmental degrees of freedom is that off-diagonal terms of the reduced density matrix rapidly vanish, i.e., coherence between the different eigenstates of the system/apparatus is lost. Thus, decoherence theory demonstrates why it is that quantum coherence is seldom, if ever, observed at the classical (macroscopic) level, the fourth aspect of the measurement problem listed above [5].

Another aspect of the measurement problem that we have referred to is the Copenhagen interpretation’s requirement of a classical measurement, raising the immediate question as to what determines the divide between the quantum system and the classical measuring apparatus. It was Heisenberg’s view [8] that the dividing line between quantum and classical did not signify a discontinuity of the physical process but rather is defined by the nature of the measurement, which to a certain extent is at the discretion of the observer. In fact, its not that the measuring apparatus is “classical” but rather that classical physics formalism is used to analyze its behavior. The problem is how to merge the formalisms of quantum and classical physics so that the evolution of the systems can be followed through the measurement process. The Copenhagen and orthodox von Neumann interpretations lack a description of the interaction of systems across the quantum/classical divide and are, therefore, of no help in this respect. Decoherence theory has, in a sense, resolved the dilemma but only by treating the macroscopic measuring apparatus as a quantum mechanical system that interacts with the original quantum system via a quantum mechanical Hamiltonian. However, one is left with the problem of the interpretation of the measuring apparatus wave function in terms of ordinary experience. One still must apply the Born rule of the Copenhagen interpretation, which, in effect, assumes that only one of the outcomes actually
occurs. In this sense, decoherence theory does not address the problem that quantum mechanics alone is insufficient to explain why we do not experience mixed states in our classical world.

Finally, while decoherence theory is relevant to the measurement process in general and is a useful computational tool for characterizing many microscopic and mesoscopic systems of interest (e.g., in foundations of physics, quantum information, and quantum computing), it is far from useful in designing most experiments. Experimentalists have quite successfully created experiments with no consideration whatsoever of the form of the apparatus/environment interaction Hamiltonian. Indeed, in many cases the experiment is designed so that the coupling of the apparatus and measured system is strong enough to be able to neglect the effects of the environment. This is highly desirable; in an ideal experiment, the environment would play no role. Of course, one might say that experimentalists have simply learned by trial and error how to design experiments, after which these skills simply become part of one’s physical intuition, just as people (even professional cyclists) learn how to ride bicycles without any knowledge of rotational dynamics or conservation of angular momentum. These same comments also apply to the quantum/classical model of interactions discussed in the Appendix.

III. SYSTEM PREPARATION AND MEASUREMENT EXECUTION

Perhaps because of the emphasis of the Copenhagen interpretation on measurements, another aspect of the quantum/classical divide is frequently glossed over and that is system preparation. Preparation and measurement are fundamentally different. A measurement yields a numerical datum. The repeatable preparation process generates a statistical ensemble from which data are collected. But how is it that a complicated and entangled arrangement of macroscopic classical equipment manages to create a well defined quantum system that is to be the subject of a subsequent measurement? A quick check of the two decoherence review articles by Zurek and Schlosshauer reveals no discussion of system preparation whatsoever. To be fair, the authors do refer to measurements that leave a system in a specific eigenstate, which can be viewed as preparing such a quantum system for further observation. However, in general, system preparation need not involve a measurement and, indeed, experimentalists probably wouldn’t look at system preparation in
Consider, for example, the prescriptions for preparing an electron beam for its subsequent use in a double slit interference experiment. These might include boiling off electrons from a hot filament, which is heated by current from a power supply, accelerating these electrons through a known potential (generated from a high voltage generator), and then passing them through a small aperture so as to approximate a point source. At large distances from the aperture the electron beam is, to a good approximation, described as a plane wave momentum eigenstate. Nowhere in this description were we required to discuss the quantum nature of electrons. An apparently classical apparatus was used to generate a coherent quantum plane wave. One might consider the electrons initially to be in a pure or even mixed state that, by the correspondence principle, only appears to be in a classical state. Then system preparation is effected by a “measuring” apparatus (e.g., a magnetic field) that is used to select those electrons that are in the desired quantum state. However, this view seems to imply that an electron wave function possesses a reality prior to the experiment and independent of the experimenter. If so, then one is led to the view that wave functions are the real entities which inhabit the universe with all the concomitant problems (e.g., wave function collapse) that this entails.

The Copenhagen interpretation does not provide a straightforward account of system preparation. In fact, in neither the case of system preparation nor measurement, are there precise rules for associating the experimental specifications with the wave functions describing the systems. In Stapp’s “practical account of quantum theory” [4], he emphasizes the operational descriptions of both preparing and measuring devices. While certainly informed by classical and quantum theory, the effective “rules” of system preparation are arrived at by calibrating both the system preparation and measuring devices. The calibration procedure is facilitated by the leverage of many possible measurements of the different states of systems. This leverage was illustrated by [4] with the following example. Consider the matrix element between two different systems, $A$ and $B$, with $N_A$ possible eigenstates for the former and $N_B$ for the latter. Then there are $N_A + N_B$ unknown functions, $\Psi_A$ and $\Psi_B$ but $N_A \times N_B$ experimentally determinable quantities, $|\langle A|B \rangle|^2$.

"Using this leverage, together with plausible assumptions about smoothness, it is possible to build up a catalog of correspondences between what experimental physicists do and see, and the wave functions of the prepared and measured
systems. It is this body of accumulated empirical knowledge that bridges the
gap between the operational specifications $A$ and $B$ and their mathematical
images $\Psi_A$ and $\Psi_B$.

How scientists arrive at these operational prescriptions is an extremely interesting ques-
tion that involves theoretical models, physical intuition, historical precedent and, we sup-
pose, even sociology and psychology. Is such a topic ever the subject of study? One might
argue that every scientific book and paper ever written is, in part, an attempt to answer
this question.

After a system has been prepared, how does an experimentalist execute a measurement? In
general, the construction of a measurement apparatus and the subsequent measurement
are effected according to prescriptions that the experimenters have both created themselves
and acquired from others. While the designs of experiments might well rely on fundamental
quantum mechanical and classical calculations, they also rely on previous observations of
the behavior of systems that are then described phenomenologically, i.e., not derived from
fundamental theory, and in some cases on conventional wisdom even if that wisdom is not
completely understood. Even then, the ultimate realization of the experiment is generated
from a set of operational prescriptions and the skill of the experimentalist in realizing the
experiment. This is the art of experimentation. The behavior of all of the equipment
generated via these prescriptions can, in principle, be described by fundamental quantum
(or classical) theory; however, it is doubtful this has ever occurred when constructing, for
example, a soldering iron. Yet, even if a soldering iron were, somehow, the subject of
an experimental investigation to confirm a quantum mechanical prediction, the test would
presumably require yet another apparatus to perform the experiment. Our claim is that the
prescriptions that define an experiment are expressed neither in the language of quantum
theory nor in the language of classical theory but rather in the common (technical) language
that directs the actions of the experimenter.

We should emphasize that these operational prescriptions have little to do with the styl-
ized experiments and simplified procedures that one might find in articles on quantum
measurement theory. The latter are those that can, and invariably are, characterized by the
mathematical formalism of quantum mechanics. (There are good accounts in the literature
which emphasize these more formal aspects of system preparation [11, 12].) By operational
procedures, we are referring instead to the much more complicated prescriptions actually
used by experimental physicists, engineers, and technicians when they carry out their work. While the need for such prescriptions is sometimes addressed in the literature [4, 9, 13], the prescriptions themselves have received surprisingly little attention and the important role that they play is usually not acknowledged. The expression of operational prescriptions is not theoretical in the sense that there is no well defined set of consistent mathematical relations that define them. The fact that nearly all analyzes of the measurement problem avoid discussions of this type of operational prescription has both been responsible for the problem being posed in purely “theoretical” terms and has exacerbated attempts to find a solution. This is not a criticism leveled at theoretical physicists. Experimentalists often use similar simplifications in their research papers. They frequently give detailed recipes to coworkers or share crucial prescriptive aspects of their work at meetings but, more often than not, such details are not discussed in the literature.

The Copenhagen interpretation seems to tacitly acknowledge this issue by insisting on the classicality of measurement apparatus, albeit with the “certain degree of vagueness” to which Heisenberg referred [4]. One of the essential elements of the Copenhagen and von Neumann interpretations is the existence of the quantum/classical divide, the Heisenberg cut if you will. This postulate asserts that the quantum system is separated from the classical measuring apparatus by the Heisenberg cut and somewhere on the far side of this cut there is a classical apparatus and a classical description. In the Copenhagen/von Neumann interpretation, the measurement problem is sidestepped by postulating a correspondence between the quantum world and the classical description. However, this correspondence is not easy to characterize. The Copenhagen interpretation lacks a theory that describes the interaction across the cut and, in fact, presumes that what happens at the cut is not mathematically describable. Any attempt to describe that interaction in a mathematically consistent way, inevitably leads to a corresponding measurement theory for classical mechanics which picks up some of the features of the quantum world. Again, the Heisenberg cut does not refer to a discontinuity of the physical process but rather to a discontinuity in the formalism used to treat the system and the measuring apparatus and the Copenhagen interpretation provides no theory of how to bridge this divide.

In the Appendix, we describe an approach that provides a way to merge the formalisms of quantum and classical theory; however, we maintain that this resolution does not solve the measurement problem. There is, in fact, another divide and that is between quan-
tum/classical formalism and the operational prescriptions of experiments. In some ways, this divide is more insidious in that it is difficult to imagine a theory that would connect the mathematical formalism of theoretical physics (classical or quantum) with the non-mathematical prescriptions that define experiments. While the pedestrian approach of this paper falls short of resolving the measurement problem (more on this later), it would not make the theory any less testable nor any less useful than orthodox quantum theory, with its Heisenberg cut.

An interesting question is why Bohr insisted that measurements always be described via classical physics. Perhaps it is not because the measurement process is well described by classical physical theory, but rather because of the happenstance that much of the same language is used both for the descriptions of measurements and the formulation of classical theory. This might lead one to the conclusion that all aspects of measurements are well understood in terms of fundamental classical theory whereas, in fact, much of our understanding of measuring apparatus is phenomenological and is supplemented by precisely the sort of operational prescriptions referred to above.

Before discussing further the role of operational prescriptions and their impact on the measurement problem, we turn to some aspects of quantum and classical mechanics that are particularly relevant to our paper.

IV. PROBABILITY AND QUANTUM MECHANICS

Max Born, in the 1926 paper in which he introduced the concept of the probability of a state [14], gives the following description of the new physics: “The motion of particles follows probability laws, the probability itself however propagates according to the law of causality.” In this sentence, as Abraham Pais remarked, Born “expressed beautifully the essence of wave mechanics” [15]. The probabilistic interpretation developed rapidly; the early history of probability in quantum mechanics may be reconstructed from a footnote in Heisenberg’s 1927 paper on the uncertainty relation [16] where he lists the main contributions up to that date, starting with Einstein’s statistical interpretation of de Broglie waves in his 1925 paper on the quantum gas and continuing with papers of Born, Heisenberg, Jordan, Pauli and Dirac. The literature on this topic is enormous and we will limit ourselves to some historical considerations that are particularly relevant to this paper.
The Born rule sets $P = |\Psi|^2$. It is straightforward to reformulate quantum mechanics so that the probability $P$ plays a central role. As is well known, the polar decomposition $\Psi = \sqrt{P} \ e^{iS/\hbar}$ maps the complex Schrödinger equation for $\Psi$ to a pair of real, nonlinear equations for two real variables $P$ and $S$ which now become the fundamental variables of the theory. This transformation was first carried out by Madelung [17]. However, as is clear from the title of his paper, his aim was to reformulate quantum mechanics as a hydrodynamic theory and, in accordance with this, he interpreted $P$ as a mass density function, following the first interpretation proposed by Schrödinger rather than the new statistical interpretation of Born (Madelung’s paper was published in 1927 but submitted in October 1926, a few months after the papers in which Born introduced his new rule).

In this formulation, the function $S$ plays the role of a potential for the velocity field $v$ of the fluid, which is defined according to $v = \nabla S/m$. Making use of this interpretation of $S$, Madelung shows that the real and imaginary parts of the Schrödinger equation correspond to a hydrodynamical equation of continuity for the mass density function and to an equation for irrotational fluid flow but with a (nonclassical) term which Madelung describes somewhat vaguely as due to the action of “internal” forces of the continuum.

Madelung’s formulation was revived in the 1950’s by Takabayasi [18, 19] and, in modified form, by Bohm and Vigier [20] and Schönberg [21]. Takabayasi in particular developed and expanded the theory considerably, presenting it as a new “hydrodynamical” quantization procedure. He recognized that full equivalence with wave mechanics required a topological condition which takes the place of single-valuedness of the wave function (i.e., that $\oint_C dS/\hbar$ must be an integer for all loops $C$ in configuration space), introduced the concept of “quantum stress,” and extended the theory to spin degrees of freedom, relativistic equations, and fields. Bohm and Vigier further developed the hydrodynamical approach by adding fluctuations to the Madelung fluid and modeling particles as highly localized inhomogeneities that moved with the local fluid velocity. Their physical model, which was presented as an extension of the formulation of quantum mechanics developed by Bohm [22], was in part motivated by criticisms (by Pauli [23] and Keller [24]) regarding the assumption that the probability distribution of an ensemble of particles coincides with $|\Psi|^2$. The work of Schönberg, while related to the physical picture of the hydrodynamic model, goes far beyond it by introducing the second quantization of the Madelung fluid.

The authors who revisited Madelung’s approach in the 1950s considered the physical pic-
ture of a fluid to be a useful model. At the same time, they were aware that it was necessary to introduce substantial modifications or reformulations of the original Madelung interpretation to bring it in line with Born’s statistical interpretation. That this was Schönberg’s motivation for introducing second quantization becomes clear from the following sentence from the introduction of his paper: “It is well known that the Madelung model does not lead to a satisfactory interpretation of the Schrödinger equation. By considering the Madelung formalism as the classical theory of the motion of a fluid medium and applying to it the second quantization we can get a satisfactory interpretation.”

It should also be mentioned that stochastic mechanics, which was introduced by Fényes [25] in the 1950s and later formulated in a different manner by Nelson [26], may also be seen as a later extension of Madelung’s approach, in the sense that the stochastic process that provides the basis of the theory leads to the Madelung equations. This has been emphasized by Guerra, who writes in his review article [27] that “a natural and straightforward particle interpretation of the Madelung fluid is indeed possible, but only by allowing a random character to the underlying trajectories. In the semiclassical limit $h \to 0$ the randomness disappears and the trajectories become those of the classical theory, while the Madelung fluid, through the vanishing of the quantum potential, reduces to the Hamilton-Jacobi fluid.”

The most familiar formulation of quantum mechanics which makes use of $P$ and $S$ variables is the de Broglie-Bohm theory [22, 28–30]. It is, however, conceptually very different from Madelung’s formulation and therefore it should not be considered an extension of it. The approach is based on the co-existence of particles and a wavefunction that is assumed to evolve according to the Schrödinger equation: instead of replacing the wavefunction by variables $P$ and $S$, the standard quantum mechanical description is completed by adding point particles which follow definite trajectories which are determined by the wave function. The ontology therefore includes not only particles but also the wavefunction. The field $v = \nabla S/m$ acquires a new interpretation, in that it describes the motion of individual particles rather than the average motion of particles, as in the hydrodynamical approach.

One may adopt a “minimalist” approach and drop Madelung’s hydrodynamical picture altogether (i.e., the assumption that $P$ is associated with the mass density function of a fluid) and interpret $P$ instead in Bornian fashion as the probability density of particles. We will follow this route here. This shift in interpretation leads to a formulation in which $P$ and $S$ are still fundamental variables of the theory, but now quantum mechanics no longer
appears in the guise of a hydrodynamic theory— it becomes a statistical theory of ensembles on configuration space. We now give an overview of some of the general features of such theories.

The description of physical systems by ensembles on configuration space (ECS) may be introduced at a very fundamental level, without making reference to quantum mechanics. The starting point is simply a probability density $P(x)$ on the configuration space with coordinates $x$, with $P(x) \geq 0$ and $\int dx P(x) = 1$.

To set the probabilities in motion, assume that the dynamics of $P$ are generated by an action principle. This is a rather mild assumption which is valid for a very large class of systems; in particular, it holds for the types of systems that we consider in this paper. We develop the theory using a Hamiltonian formalism for fields (which, for our purposes, is more convenient than using a Lagrangian formalism). Then, to get equations of motion for $P$, introduce an auxiliary field $S$ which is canonically conjugate to $P$ and a corresponding Poisson bracket for any two functionals $F[P, S]$ and $G[P, S]$,

$$\{F, G\} = \int dx \left\{ \frac{\delta F}{\delta P} \frac{\delta G}{\delta S} - \frac{\delta F}{\delta S} \frac{\delta G}{\delta P} \right\}.$$

(1)

The equations of motion for $P$ and $S$ take the familiar form

$$\dot{P} = \left\{ P, \tilde{H} \right\} = \frac{\delta \tilde{H}}{\delta S}, \quad \dot{S} = \left\{ S, \tilde{H} \right\} = -\frac{\delta \tilde{H}}{\delta P},$$

(2)

where $\tilde{H}[P, S]$ is the ensemble Hamiltonian that generates time translations.

The fundamental variables of our phase space are the probabilities $P$ and the auxiliary function $S$. One may introduce the notion of an observable on this phase space, as any functional $A[P, S]$ that satisfies certain requirements \[31, 32\]. For example, the infinitesimal canonical transformation generated by any observable $A$ must preserve the normalization of $P$. This implies the condition $A[P, S + c] = A[P, S]$; i.e., gauge invariance under $S \rightarrow S + c$.

Up to now, the discussion has been very general: since the ensemble Hamiltonian has not been specified, the formalism may be applied to a large class of statistical theories. The following ensemble Hamiltonians are of interest in that they lead to equations that describe the evolution of quantum and classical non-relativistic systems \[31\]:
For example, the equations of motion derived from $\tilde{H}_Q[P,S]$ are given by

$$\frac{\partial P}{\partial t} + \nabla \cdot \left( P \frac{\nabla S}{m} \right) = 0, \quad \frac{\partial S}{\partial t} + \frac{|\nabla S|^2}{2m} + V + \frac{\hbar^2}{2m} \nabla^2 P^{1/2} = 0$$

while the equations of motion derived from $\tilde{H}_C[P,S]$ are the same as Eq. (5) but with $\hbar = 0$. The first equation in Eq. (5) is a continuity equation, the second equation is the classical Hamilton-Jacobi equation when $\hbar = 0$ and a modified Hamilton-Jacobi equation when $\hbar \neq 0$. Defining $\Psi := \sqrt{P} \, e^{iS/\hbar}$, Eq. (5) takes the form

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi,$$

which is the usual form of the Schrödinger equation. Thus, in the ECS approach, classical physics is not given a probabilistic description in a phase space with coordinates $x$ and momenta $p$ using a phase space probability $\rho(x,p)$. The two probabilistic descriptions are not equivalent in that in general a $\rho(x,p)$ has to be described by a mixture of configuration space states $P(x)$ and $S(x)$ [33, 34].

Notice that quantum and classical particles are treated on an equal footing in this more general framework, with differences being primarily due to the different forms of the respective ensemble Hamiltonians. One may ask whether the functions $P$ and $S$ can be interpreted in a similar way regardless of whether we are discussing a classical or a quantum system. We will show that this is indeed possible provided we do not try to assign properties to $P$ and $S$ that go beyond what is required of a statistical theory; i.e., these are quantities that should be used to describe the state of ensembles, to enable us to make predictions that can be compared to experiments.

Before looking at the role of probability, we consider the interpretation of $S$, which was introduced above as an auxiliary variable conjugate to $P$. One may define local energy and momentum densities in terms of $S$. If $\tilde{H}[\lambda P, S] = \lambda \tilde{H}[P, S]$, which holds true for the ensemble Hamiltonians of Eq. (3), one can show that $\partial S/\partial t$ is a local energy density. Furthermore, $\int dx \, P \nabla S$ is the canonical infinitesimal generator of translations and therefore $P \nabla S$ can be considered a local momentum density. These results are generally valid [33]; i.e., they hold true for both classical and quantum systems and thus provide a common physical interpretation of $S$ that is appropriate for a statistical theory.

To maintain full generality, $S$ should not be regarded as a “momentum potential” for individual particles. In particular, for an ensemble of classical particles it is not necessary
to assume that the momentum of a member of the ensemble is a well-defined quantity proportional to the gradient of $S$, as it is done in the usual deterministic interpretation of the Hamilton–Jacobi equation. Such an assumption would go beyond the requirements of a statistical theory and it is unnecessary. (This avoids forcing a similar deterministic interpretation in the quantum case which would correspond to the de Broglie-Bohm formalism. A deterministic picture can be recovered for classical ensembles precisely in those cases in which trajectories are operationally defined.)

We have assumed that $P$ is a probability density; i.e., that it is possible to measure the state of the system and that $P(x')dV$ describes the probability of finding the system in the particular configuration $x'$ within the configuration space volume $dV$.

It is important to point out that such an interpretation of $P$ is generally valid (i.e., regardless of whether we are considering a classical or quantum system), despite the fact that it has been claimed that probability theory does not apply to quantum mechanics. Indeed, as B. O. Koopman pointed out in a seminal paper written in 1957, “Ever since the advent of modern quantum mechanics in the late 1920’s, the idea has been prevalent that the classical laws of probability cease, in some sense, to be valid in the new theory.”

In his paper, Koopman goes on to refute this claim. In the introduction, he writes:

“\text{The primary object of this presentation is to show that the thesis in question is entirely without validity and is the product of a confused view of the laws of probability. The situation can be straightened out at a very elementary level: all that is needed is to make quite clear that and explicit the concept of event. It will not be necessary to either to adopt any particular position regarding the controversial matters at the foundations of probability or to commit oneself at all deeply on the level of physical law.}”

Koopman’s main point is that the claim that probability theory ceases to be valid in quantum mechanics is to a large extent the result of not distinguishing between \textit{compatible} and \textit{incompatible} events, but once this distinction is made, it can be seen that the axioms of probability theory are not violated in quantum mechanics. Events are interpreted in an operational sense, as is clear from the following quote:

“A thorough examination of all the concrete applications of the theory of probability shows that the concept of event can always be interpreted as a statement
concerning the state of a material system on a specified occasion. It is essential, furthermore, that the statement be meaningful according to the simplest interpretation of Bridgman’s operational standards: in principle, capable of verification (true or false) by an observation.”

Koopman’s article, which focuses on the double-slit experiment, seems to have been motivated by a publication of Feynman [36]. The observations of Koopman do not seem to be very well known; most textbooks of quantum mechanics do not incorporate them (two notable exceptions are the textbooks of Ballentine [12] and Peres [13]).

An explicit proof that quantum mechanics satisfies the axioms of probability theory was given later by Ballentine [37], who used the standard representation of observables in terms of operators and verified, for both pure states and density matrices, that the axioms are satisfied. More recently, a similar result was obtained by Goyal and Knuth, this time using a different approach which allowed them to prove that Feynman’s rules are compatible with probability theory by explicitly deriving Feynman’s rules on the assumption that probability theory is generally valid [38].

The misconception that quantum mechanics and probability theory are incompatible is unfortunately widespread. Ballentine reviews some erroneous applications of probability to quantum mechanics which have resulted in claims of inconsistency between probability theory and quantum theory. These typically involve mistakes where conditional probabilities are handled incorrectly. Probabilistic formulas that involve joint probability distributions also require some care because, as is well known, quantum mechanics lacks an expression for the joint probability distribution of variables whose operators do not commute. A formula that involves a joint probability distribution is no longer applicable if the joint probability distribution is not defined.

It is important to stress that the arguments discussed here are independent of the choice of interpretation of probability, as Koopman already pointed out in his paper, because they are based on the axioms of probability theory and these are common to all interpretations. Therefore, the $P$ that is used in the description of statistical systems by means of ensembles on configuration space plays the same operational role independent of whether the ensemble describes a classical or a quantum system.
V. A STATISTICAL DESCRIPTION OF CLASSICAL PHYSICS

One of the assertions of this paper is that classical physics is, in a very real sense, a statistical model of nature. It is, therefore, not the statistical character of quantum mechanics that distinguishes it from classical physics but rather must be something else. The ECS formalism presented in the previous section can be used to illustrate this and provides as well a model for treating interacting quantum and classical systems (see the Appendix). This might tempt one to offer the ECS theory as a resolution to the “classicality of experiments” aspect of the measurement problem as well as providing a mathematical model for the Heisenberg cut. However, we have already argued in Section III that experiments are described neither by classical nor quantum physics but rather by operational prescriptions that lie outside both the formalisms of classical and quantum theory. In fact, we will argue in Section VI that, rather than resolving any aspect of the quantum measurement problem, the statistical description of classical physics brings to light a concomitant measurement problem in classical physics.

As we pointed out in the previous section, a statistical description of classical mechanics may be formulated using ensembles on configuration space. For example, in the ECS approach, the motion of a classical ensemble of particles under the influence of a potential \( V(x) \) is described in terms of the ensemble Hamiltonian \( \bar{H}_C \) of Eq. (3) and the resulting equations of motions are Eqs. (5) with \( \hbar = 0 \),

\[
\frac{\partial P}{\partial t} = -\nabla_x \left( P \frac{\nabla S}{m} \right), \quad \frac{\partial S}{\partial t} = -\frac{\left| \nabla S \right|^2}{2m} - V. \tag{6}
\]

In the limit of an initial \( \delta \) function probability distribution, the equations reduce to the exact classical equation of motion of a single particle subject to no uncertainty. Therefore, this formalism includes both the equation of motion of an ensemble of particles and the exact equation of motion of a single particle. In this sense, the above formulation might well be considered to be more fundamental than the classical Newton’s equations of motion. The obvious interpretation is that \( P(x, t) \) and \( [P(x, t) \nabla_x S(x, t)] \) describe the statistical distribution of the results of measurements performed on an ensemble of similarly prepared individual particles, which is analogous to the ensemble interpretation of quantum mechanics.

At this point, it is important to stress that this statistical description of classical mechanics, while unfamiliar to many, is not particularly new or revolutionary. It is well known that the Hamilton-Jacobi equation provides a formulation of classical mechanics that may
be considered as fundamental as the Hamiltonian and Lagrangian formulations. It is also well known that given a solution $S(x, t; c)$ of the Hamilton-Jacobi equation (where the $c$ are constants that specify the particular solution $S$), it is always possible to associate with this solution a whole family of conserved densities which satisfy the continuity equation. Hamilton-Jacobi theory is fundamentally a theory of ensembles \[34, 39\]. Any normalizable density that satisfies the corresponding continuity equation may be used to describe a physically allowed classical state.

How is it that the probability density $P$ can be interpreted as a fundamental description of a classical system? To be sure, all measurements of systems, whether classical or quantum, are subject to uncertainty but, for classical systems, these uncertainties are usually attributed to noise in the experiment. While, in principle, such uncertainties can be made arbitrarily small, in practice this is not the case as is well known to experimentalists. Rarely do the experimental uncertainties associated with a measurement apparatus even approach the fundamental limitations imposed by quantum mechanics, e.g., $\delta p \delta x \geq \hbar$. In reality, the exact physical states in classical physics are just as inaccessible as they are in quantum mechanics. Nevertheless, classical experimental uncertainties are routinely expressed as “errors” associated with the result of a measurement and rarely, if ever, considered to be attributes of the system under investigation and/or the measuring apparatus. But does this have to be the case? If one considers the statistical formulation to be a legitimate description of classical physics, then it appears that the uncertainties associated with classical systems should be as much a part of their theoretical descriptions as are the fundamental uncertainties associated with quantum mechanics.

One might ask why the uncertainties in classical physics are largely attributed to the preparation and measurement processes rather than to the theory. Perhaps, this happenstance is an historical accident. The uncertainties might just as well been attributed to classical theory as discussed above. However, in most cases the relative uncertainties in classical systems are so small that ignoring them or considering them to be uncertainties related to making measurements is entirely understandable. If classical physics had been generally concerned with very small (mesoscopic) systems or if our present environment had been one that included a great deal of randomly fluctuating forces, then this might not have been the case. Even so, there are cases in which uncertainties are considered to be part of the theory. One of the earliest such classical theories is that of Brownian motion, but there
are certainly other situations where this arises. For example, predictions of the large-scale structure in the universe are invariably statistical in nature due to an inherent randomness in the state of the early universe. The usual assertion is that the source of the randomness is quantum fluctuations; however, this claim is more a matter of conjecture than deduction.

There are aspects of the exact state formalism of classical mechanics that are as troublesome as those of quantum mechanics. Among the primary theoretical constructs of classical physics are point particles, particle trajectories, continuous media, and rigid bodies. While these may be useful approximations to what we observe in the physical world, their primary function in classical mechanics is as part of the formalism that is used to predict the statistical outcomes of experiments/observations. Because the constructs of classical physics were created from our everyday observations of the physical world, it is understandable that we often identify them with reality whereas in actuality they are simply part of the theoretical formalism that is necessary for a self-consistent classical mechanics.

As soon as one moves from formalism to the physical world, it becomes abundantly clear the theoretical constructs are fundamentally different from objects in the real world. For example, the finite size, density, stiffness, and viscosity of real bodies are all determined experimentally with no fundamental classical understanding of how they arise. These properties of matter are generally agreed to be quantum mechanical in origin even though most are much too complex to be computed within quantum mechanics and so are still determined experimentally [40]. In fact, many aspects of the consistency of classical physics collapse under close scrutiny because of the underlying quantum nature of the phenomena. In any case, there seems to be no compelling reason to cling to the precise, deterministic formulation of classical mechanics in lieu of the statistical formulation that, in any case, formally includes exact determinism by employing δ function probability densities as pointed out above.

Perhaps a more problematic aspect of the statistical formulation is the lack of a precise method of specifying the initial configuration space probabilities of a system. On the other hand, recall that neither is there a precise method for determining the quantum mechanical state of a system without employing either the notion of preparation or measurement of a system. As discussed above in Section III, quantum mechanical state preparation usually proceeds via a set of operational prescriptions that have been complied through the process of calibration. It seems reasonable that the same language could be used to assign a statistical distribution to a classical state. Certainly, experimentalists spend a great deal,
probably most, of their effort in understanding and estimating uncertainties associated with the experimental setup. These efforts might just as well be described as determining the probability density of various components of the system.

VI. THE MEASUREMENT PROBLEM IN CLASSICAL PHYSICS

A statistical formulation of classical physics brings with it a classical measurement problem. The probability function $P(x, t)$ only specifies the statistical outcome of an ensemble of similarly prepared states. There is nothing in the formalism to indicate that only one of the possibilities actually occurs. Consider a position measurement: If one insists that the probability function provides a description of an individual system, then the original probability density must be updated (i.e., must “collapse”) to one that has a lower uncertainty by some process associated with the measurement (in general, the canonically conjugate function $S(x, t)$ will also require updating). The description of this process must occur outside the theoretical formalism of classical mechanics. Otherwise, as in the analogous quantum case, the theory would predict the collapse and an exact prediction could be made, negating the statistical nature of the theory. So it seems that a statistical classical theory shares this feature of quantum theory. Because the contention is that physical theories, classical and quantum, are statistical in nature, any implied “collapse” will necessitate a process that is not included in the theory. It should be noted that, in the classical case, there is no inherent linearity in the theory and so the violation of linearity by the collapse is not the culprit.

It should be pointed out that this argument is not predicated on the ECS formalism introduced in Section IV which is only an example, albeit a compelling one, of how a statistical classical theory might be formulated. The “collapse” problem would arise in any statistical theory of classical physics.

Another aspect of the measurement problem that the statistical classical theory shares with quantum mechanics is the Copenhagen interpretation’s requirement of the classicality of experiments. At first glance, this may seem absurd; after all, how could classicality be a problem for classical physics? However, what is meant by the term classicality is not entirely clear. For example, the following is one of Bohr’s explanations of what he meant by “classical concepts” (although it should be noted that Bohr addressed this topic in many different ways, not all of which were completely consistent) [42]:
“The decisive point is to recognize that the description of the experimental arrangement and the recordings of observations must be given in plain language, suitably refined by the usual terminology. This is a simple logical demand, since by the word ‘experiment’ we can only mean a procedure regarding which we are able to communicate to others what we have done and what we have learnt.”

Stapp chooses to emphasize this pragmatic view of classicality by using the word specifications, i.e., 4.

“Specifications are what architects and builders, and mechanics and machinists, use to communicate to one another conditions on the concrete social realities or actualities that bind their lives together. It is hard to think of a theoretical concept that could have a more objective meaning. Specifications are described in technical jargon that is an extension of everyday language. This language may incorporate concepts from classical physics. But this fact in no way implies that these concepts are valid beyond the realm in which they are used by technicians.”

The point is that descriptions of experiments are invariably given in terms of operational prescriptions or specifications that can be communicated to the technicians, engineers, and the physics community at large. In some sense, even the words we use to write journal articles to present the results of an experiment might be considered to be part of the measurement apparatus. Are these operational prescriptions part and parcel of classical theory? Are they couched in terms of point particles, rigid solid bodies, and Newton’s laws? Of course not. They are part of Bohr’s “procedure regarding which we are able to communicate to others what we have done and what we have learnt.”

So why is it that the word “classical” can be taken in so many ways by physicists? There seems to be a tendency among physicists to think of every aspect of physics before quantum mechanics to be part of the classical picture or, perhaps, every subject of any scientific discipline that doesn’t employ a quantum mechanical analysis should be viewed as classical. While in a certain sense this is true, such a point of view tends to conflate the theoretical language of classical mechanics with ordinary (albeit technical) language, thereby removing the division between the theory of classical physics and the description of physical measurements. We have already pointed out that classical theory proper is quite formal and contains theoretical constructs that are fundamentally different from objects in the real world. On the
other hand, most of “classical physics” is phenomenological and makes no pretense to being fundamental. Classical physics does not represent an all encompassing, although incorrect, theory of the world. It is more patchwork of theoretical classical mechanics, electromagnetism, and phenomenological truths combined with the conventional wisdom and standard prescriptions of experimental physics.

So how do we treat measurements in classical physics? The same way as we do in quantum mechanics, with operational prescriptions in plain language so that the results can be communicated to the scientific community. These prescriptions are not contained within classical theory. In this sense, the description of a measurement in classical physics must be in terms of language that falls outside the theory. Of course, the measuring apparatus itself can be described (statistically) in terms of the (sometimes phenomenological) concepts of classical physics in the same way that it can be described quantum mechanically (although it rarely is). But to the extent that the apparatus is treated as part of the classical (quantum mechanical) system, it becomes part of the system under investigation and can no longer be considered part of the measurement. Heisenberg expressed this in the extreme (quoted in Ref. [8]): “One may treat the whole world as one mechanical system, but then only a mathematical problem remains while access to observation is closed off.”

The bottom line is that classical physics is faced with the same two major aspects of the measurement problem as quantum mechanics: 1) The theory is fundamentally statistical in nature and any attempt to interpret it for single systems requires a “collapse” that necessarily lies outside the theory; and 2) The descriptions (specifications) of experiments must be operational prescriptions that are outside of theoretical physics, and the results must be communicated in plain language rather than with theoretical concepts. Our pedestrian approach to the former is simply that both quantum and classical physics are theories, not of individual systems, but rather of the statistical behavior of ensembles of systems. The quantum part of this statement is consistent with the minimalist interpretation of quantum mechanics espoused by Einstein [43], Ballentine [44], and others. As for the latter aspect of the measurement problem, our pedestrian approach is similar to Bohr’s and Heisenberg’s as espoused in the Copenhagen interpretation: “the description of the experimental arrangement and the recordings of observations must be given in plain language” and not in terms of theoretical constructs. (However, it should be noted that Bohr used the terms ‘in plain language’ and ‘classical physical concepts’ interchangeably.) Without these operational de-
scriptions, both quantum mechanics and classical theory are mathematical formalisms that necessarily remain detached from the real world.

The description of the classical measurement problem presented here leads necessarily to a reevaluation of some of the issues raised in Section [I] regarding the quantum measurement problem. Part of the classicality of experiments aspect of the quantum measurement problem is the issue of where to place the quantum/classical divide. We pointed out above, locating the divide goes only part way. One must also introduce a method to bridge the two formalisms, quantum and classical, in order to describe the occurrence of a measurement. Decoherence theory avoids this aspect of the measurement problem by treating the measuring apparatus as another quantum mechanical system. However, in so doing, the actual measurement is simply pushed out further until it can be described by operational prescriptions that define it. The divide between the theoretical predictions and the measurement remains unaccounted for. The ECS formalism (see the Appendix) is capable of dealing with interactions between one system described by quantum formalism and another by classical formalism; however, here again, the description of the operationally defined measurement falls outside both quantum and classical theory. This topic is addressed in Section [VII].

Another aspect of the measurement problem listed in the introduction, “the consistency of quantum and classical reality” must be rephrased as “the consistency of quantum and classical physics with reality.” That is, how do we reconcile the statistical nature of quantum and classical physics with our observed perceptions of the world. Here again the minimalist (ensemble) interpretation of our pedestrian approach obviates this problem by restricting the predictions of both classical and quantum physics to the statistical outcomes of measurements performed on ensembles of similarly prepared systems. The modesty of this restricted interpretation of the domain of physical theory will certainly be distasteful to many physicists. We will comment on this in Section [VIII].

VII. OPERATIONAL PRESCRIPTIONS, CLASSICALITY, AND SUFFICIENT PRECISION

If the above claim of the statistical nature of both quantum and classical physics is taken seriously, then perhaps we are in need of a more general Copenhagen-type interpretation. A possible version of such was alluded to in the previous section; however, questions remain.
For example, what is it that defines the transition from the theoretical evolution of the system to the measurement, which is defined operationally? For Bohr, Heisenberg, and Pauli, the demarcation is simply when one stops talking about quantum mechanics and starts talking about classical physics, with the acknowledgement that this transition is, to a certain extent, at the discretion of the experimenter.

However, now quantum and classical interacting systems, taken together, are considered to be part of the theoretical (statistical) evolution of the combined system. Then what constitutes a measurement? A crucial aspect of a measurement is that it be the declaration of a precise result [45], which can only be compared to the statistical predictions of the theory. The ensemble interpretation characterizes these predictions as corresponding to the distribution of measurements made on an ensemble of similarly prepared systems provided multiple measurements are possible; when this cannot be realized, it is still possible to interpret the prediction in a Bayesian sense, i.e., where probability is defined using the notion of degree of belief. How do we normally interpret experimental uncertainty? In general, a one standard deviation (1σ) value, for example, reflects the belief that were one to repeat the experiment many times, the results would be scattered about a mean value with ∼ 68% of the results falling within 1σ of the mean. This is precisely the prediction made by the statistical ensemble interpretation of a classical experiment. In some cases, experiments are actually performed many times and the quoted uncertainties represent the statistical distributions of the results. In this case, it is usually the standard deviation of the mean that is quoted as the measurement uncertainty, which is interpreted as the spread in the probability distribution of the mean of the results of multiple measurements if these were to be repeated in an ensemble of multiple measurements. In the case that an experiment cannot be repeated, e.g., the determination of the large-scale structure of the universe, then one must interpret predictions as degree of belief.

In cases where uncertainty is established by multiple measurements, one might wonder how it is that the statistical properties of the results of multiple measurements can be somehow attributed to the statistical state of the system instead of interpreted as uncertainties in the measurement. However, this situation can simply be regarded as a calibration that enables the specification of the statistical state of the system. The same circumstances can occur in a quantum mechanical system that is determined, after the fact, by multiple experiments on similarly prepared systems. In other cases, when the predominant uncertainty can
be predicted or measured beforehand, then one can simply assign a probability distribution to the system. Of course, there may be many sources of uncertainty that require distinct probability functions to be assigned to different parts of the system, which are then allowed to interact with one another. Recall that components of the “measuring” apparatus are to be considered as components of the whole system under investigation.

If the “measuring apparatus” is to be considered part of the combined quantum/classical system, what is the actual measurement that is assumed to lie outside of the theoretical description of the system, i.e., what marks the division between the theoretical description of the system and the operational description of the measurement? The answer to this question brings us to the notion of precision [45]. The contention is that the measurement occurs at the point in the evolution of the system at which there is no further uncertainty that may affect repeatability. Then the result is of sufficient precision that a measurement has been made. This is an imprecisely defined transition in that the specification of sufficient precision is left up to the experimenter. If the scientific question being addressed requires more precision then a part of the sequence that was formally considered an operational description of the measurement might, instead, have to be specified probabilistically and be included in a theoretical treatment of the system. In some ways, this is analogous to the imprecisely defined quantum/classical divide of the Copenhagen interpretation.

Once a precise measurement has been made, it has a permanence that can then be communicated to others visually, in writing, verbally, in technical language, in English, in Spanish, etc. Because measurements are considered to be exact and permanent, they become a matter of record. However, even at this point one might wish to introduce uncertainty into the process. What if one wishes to account for transcription errors, errors in graphical illustration, errors in language translation, linguistic errors, interpretational errors, erratic human behavior, etc.? In principle, these aspects of communication might also be treated theoretically using information theory, with specific probability distributions determined by calibration and then such communications could be considered to be part of the theoretical evolution of the system. However, in this case, it is doubtful that one would characterize this part of the system as belonging to classical theory. At some point one simply draws the line after which errors are dismissed with the proverbial “mistakes were made.”

So far, we have discussed the evolution of a quantum measurement as the following sequence: 1) follow an operational prescription to prepare the quantum mechanical state of
the system and the classical state of the measuring apparatus; 2) determine the quantum mechanical evolution of the system; 3) determine the joint quantum/classical evolution of the system and measurement apparatus (if an ECS type formalism is available) or the joint quantum evolution of the system and measuring apparatus (if a decoherence analysis is performed); 4) determine the classical evolution of the apparatus; 5) follow operational prescriptions to determine the precise result of the measurement; and 6) communicate the result to others. Of course, measurements needn’t proceed in such a linear fashion. There can be many quantum and classical components of the system and these can interact at different times resulting in complicated mixed quantum/classical states. Or, it might be possible to prepare a quantum state, let the state evolve, and then perform the “measurement” following a set of operational prescriptions without any involvement of a truly classical system (as decoherence theory would maintain); however, no specific examples of such experiments come immediately to mind. With regard to 3) above, even in the absence of a consistent model of quantum/classical interactions, it is undoubtedly the case that approximate (semi-classical) models are available with sufficient accuracy to represent the evolution of the systems.

One topic we have not touched upon is the relation of quantum mechanics and information. As far back as Bohr and Heisenberg, there have been interpretations of quantum mechanics that emphasized the relation of the quantum wave function to an experimenter’s knowledge of or information about the real world as opposed to postulating that the wave function is a representation of reality. The operational prescriptions discussed above should then be described as prescriptions for increasing an observer’s knowledge about a given system. The issue of how the operational prescriptions are related to an increase in our knowledge is important, but it is outside of the scope of this paper. At a minimum, it would require bringing in concepts from information theory (to quantify the amount of information provided by an experiment which is carried out according to an operational prescription) and probability theory (to quantify how the uncertainty is diminished by the data). These fields provide tools that are extremely important for the description of the experimental issues that play such a fundamental role in both classical and quantum measurement theory. While some important aspects of the operational prescriptions will very likely remain recipes that cannot be described in a mathematical way, efforts to provide a clearer description of those aspects that can be formulated in the more formal language of information theory and
probability theory are needed. However, we believe that such attempts at rigorous formulations should not be carried out in such a way that the complex experimental issues encoded in operational prescriptions are ignored.

VIII. DISCUSSION

As alluded to earlier, it is curious that the measurement problem has persisted over eight decades even while advances in fundamental physics, quantum field theory in particular, have been remarkable to say the least. Perhaps one reason for its persistence is because physicists have sought theoretical solutions to what is essentially an experimental problem. As a consequence, in debates about the measurement problem, the experimental side of physics is largely ignored except in sweepingly general statements about experiments that, if not inaccurate, are certainly of limited validity. Many quantum interpreters seem to believe that fundamental physical theories should provide an accurate (or as an accurate as is possible) representation of reality and that the theories themselves should provide the basis for interpreting experiments. Many experimentalists would not necessarily subscribe to this principle. In this respect, the following quote by Gutzwiller [46] seems particularly relevant:

“The discussion of the so-called ‘thought experiments’ in most cases is singularly crude, i.e., removed from any awareness of the practical considerations in a real experiment. Time and effort is spent on purely mathematical relations. With few exceptions, the hard work of writing down, and then solving the relevant equations for a specific laboratory set-up has not even begun; in particular, the inevitable presence of noise is ignored most of the time.”

In the early days of quantum mechanics, its domain was essentially limited to the microscopic world, i.e., the world of atoms. The Copenhagen interpretation was often expressed in statements like, “quantum mechanics is a theory that makes statistical predictions about measurements made on microscopic systems with classical, macroscopic measuring apparatus.” It seems that Bohr, Heisenberg, and Pauli were comfortable with this situation and were willing to limit the applicability of quantum mechanics to microscopic (atomic) systems. Nevertheless, there are significant reasons to consider quantum mechanics to be much
more fundamental than classical physics. Today, most physicists view the quantum theory of matter and radiation as a truly fundamental theory of nature. About this, there can be little doubt. Because quantum coherence is critical to the measurement problem, physicists often point to macroscopic coherent quantum behavior, e.g., superconductivity and superfluidity, to demonstrate that quantum mechanics applies even to large scale structures. One needn’t, however, appeal to such exotic phenomena. In a very real sense, the phase information in electromagnetic waves is due to quantum coherence in a Bose condensation of photons. Moreover, virtually every property of ordinary macroscopic matter can be understood only in the context of quantum theory. On the other hand, in classical physics nearly every property of matter is understood only phenomenologically. Indeed, the behavior of an ordinary metal spring that obeys the “classical” Hooke’s law can also be taken as observational evidence in support of quantum mechanics because without quantum mechanics the very quality of the stiffness of the spring is an unexplained phenomenon. There are certainly less fundamental quantum theories (e.g., the non-relativistic Schrödinger equation) that follow from more fundamental theories (the relativistic Dirac equation); however, for example, the classical Hooke’s law doesn’t follow from any classical theory. To be sure, both classical electrodynamics and general relativity were considered to be classical fundamental theories; however, the former of these is known to be only the classical limit of the more fundamental quantum electrodynamics. The same is usually assumed to be true for general relativity even though as yet there is no accepted quantum theory of gravity.

These arguments together with the great predictive successes of quantum mechanics and quantum field theory (as embodied by the twelve decimal place agreement of the quantum electrodynamical prediction with the measured g-2 value of the electron) have probably led most physicists to accept that everything in nature is quantum mechanical in origin and to infer that classical physics corresponds to an approximation to quantum theory that is quite accurate in the limit of macroscopic systems with large energies. In fact, nothing that has been discussed in the present paper would contradict this view. This isn’t to say that classical theory isn’t enormously useful in our description of the world. In fact, the majority of calculations in physics are surely classical computations. Furthermore, there is little doubt that quantum mechanics would be absolutely useless in effecting solutions to these problems; the complexity of most phenomena assures us of this. So how do we answer the Einstein-type question about the nature of the real world? Does the fundamentalness of
quantum mechanics trump the usefulness of classical physics or should we be content with Bohr’s contention that both quantum (microscopic) and classical (macroscopic) accounts must be employed?

At this point, it may be prudent to remind ourselves of what we often tell our students and the public for that matter. That is, our theories should be considered to be simply “models” that are useful devices in describing the real world. If one were to ask most scientists whether a particular model coincided with the “real world,” the answer would probably be “no, they are just models” [47]. It is the fact that physical theories (models) are amenable to change in the face of new evidence that, in large part, is the strength of the discipline. Granted, some models are extremely good. But as good as they are, they are only models which we use to make sense out of the natural world and often have only a limited domain. If this is the case, then why should we be upset by the incongruous union of quantum theory and classical experimentation? Here again, the 12 decimal place agreement between quantum electrodynamical theory and experiment leads many to take the position that quantum theory, or some future version of it, is the embodiment of reality. However, one should be wary of deeming a single (or even several) high precision measurement(s) as strong evidence in support of a theory. [49] On the other hand, if one takes seriously the view that quantum theory is simply a model of reality with a necessarily limited domain of applicability, then perhaps the Copenhagen interpretation isn’t so distasteful.

Some might object to referring to quantum mechanics as “simply a model of reality” with a “limited domain of applicability.” Because quantum mechanics deals with the fundamental building block of nature, is it not surely the fundamental theory from which all others follow? The problem is that other theories don’t necessarily follow from quantum mechanics nor is quantum mechanics at all useful in dealing with most of the phenomena in nature. Consider, for example, the problem of predicting the motion of an irregularly shaped solid object tumbling in free space. A solution to this problem is relatively straightforward in terms of classical mechanics; although, the addition of the elastic properties of the solid would complicate the problem significantly. Of course, the elastic properties of the object are phenomenological at the classical level and can only be derived from first principles from the quantum mechanical behavior of the elementary particles that make up the solid. On the other hand, solving for the motion of the solid from a field theoretic account of the behavior of these particles is essentially impossible. In fact, its not even clear that
quantum field theory is capable of formulating the problem. Instead, we invent a model for the behavior of macroscopic solid objects, classical mechanics, which provides extremely accurate predictions in such instances. If one moves from such a simple system, an irregular solid body, to systems of extreme complication, e.g., human behavior, the connection to quantum mechanics is even more remote. Finally, there is the fact that the predictions made by quantum mechanics (or classical mechanics for that matter) require interpretative statements and observational prescriptions that lie outside the theory.

One might argue, from a reductionist point of view, that surely the theory of the “smallest” components of matter and energy, out of which everything in the physical world is composed, occupies a privileged place in natural philosophy. The fundamental principles inherent in physics in general and quantum mechanics in particular are surely to be thought of as fundamental principles of nature. In a sense, this is true; however, in reality these principles are the rules governing the models we construct and not of nature herself. For example, Newton’s universal law of gravitation, embodied by \( F = G \frac{m_1 m_2}{r^2} \) and \( F = ma \), are rules that govern the behavior of the constructs of the theory, massive point particles and the trajectories of those particles. There are no point particles and exact trajectories in nature. These are theoretical constructs of a model that has proved to provide an extremely useful description of some aspects of nature. The same is true for quantum mechanics, the principles of which govern the deterministic behavior of wave functions (or Hilbert state vectors), the theoretical constructs of the theory. This model has proved incredibly useful in describing the behavior of matter and radiation (when combined with interpretive statements and observational prescriptions), but the principles of quantum mechanics are not, in themselves, laws of nature but rather the rules that govern the theoretical constructs of a particular model of nature.

In fact, it is the mistaken identification of the theoretical constructs of a theory with entities in the natural world that sets the stage for the measurement problem. Although we have argued that classical mechanics has its own measurement problem, it is understandable why this is usually overlooked. The constructs of classical mechanics, e.g., point particles, solid bodies, and continuous media, all bear a strong resemblance with objects we see in nature and so we tend to identify these constructs with the reality that they are intended only to model. In quantum mechanics, following this practice of identifying theoretical constructs with reality immediately leads to serious conceptual difficulties. To what objective reality
does a wave function or Hilbert space vector correspond? If we identify wave functions with objective reality we are inexorably led to the measurement problem. As to whether or not quantum mechanics is the embodiment of reality, the above arguments suggest that quantum mechanics is neither complete nor does it even provide an accurate description of reality. It is, rather, an extremely powerful and useful model which helps us understand the physical world around us.

One of the most pernicious aspects of the measurement problem is the imprecisely defined quantum/classical divide. It is largely the inability of the Copenhagen and von Neumann interpretations of quantum mechanics to model the quantum/classical interaction that renders them incapable of resolving the measurement problem. In our pedestrian approach, we argue that it is not the quantum/classical divide that is problematic but rather the divide between the quantum/classical analysis and the operational prescriptions that characterize the measurement process. The fact that this latter divide is not mathematically describable is not, we believe, a problem because the operational prescriptions themselves are not theoretical constructs of physics but rather are given in Bohr’s “...plain language...to communicate to others what we have done and what we have learnt.” We do not claim that this distinction resolves the measurement problem but rather that it reveals that the measurement problem is not a theoretical problem to be resolved. The realization is that all physical theories, quantum and classical, derive their meaning by appealing to concepts that must lie outside the theory. On the other hand, our pedestrian approach must still deal with the interactions between the quantum system and the classical measuring apparatus that take place prior to the measurement. The formalism discussed in the Appendix provides an alternative, self-consistent way to treat the interaction of a quantum system with a classical measuring apparatus. However, like decoherence theory, this formalism is much too difficult to apply in all but the simplest cases. Therefore, one cannot claim that either of these two formalisms could have provided much help for physicists in analyzing the results of a given experiment. It seems reasonable that the liberal utilization of pragmatic, semi-classical, and heuristic arguments in the context of understanding complex systems has served to finesse such situations.

Because some physicists assert that decoherence theory has largely resolved the measurement problem, it is important to point out the limitations of this claim. As discussed above, decoherence theory does demonstrate why it is that macroscopic measuring appa-
ratus, even if treated quantum mechanically, behave, statistically, according to notions of classical physics. Whether one chooses to perceive this as verifying how classical physics emerges from quantum mechanics or only that classical behavior is consistent with quantum mechanics, the primary dilemma of the measurement problem remains. That is, how is it that a continuous, evolving wave function leads to our immediate perception that specific events occur in space and time. Finally, while decoherence theory can explain how the reduced (averaged over environmental degrees of freedom) density matrix implies the same behavior as a proper mixed state, it is silent on the meaning and origin of proper mixed states. Of course, this is a problem of quantum mechanics in general, that is, mixed states are never the result of quantum evolution but rather must be posited initially using a combination of quantum and classical arguments. In short, in the context of quantum theory, where does ordinary experimental uncertainty arise? Finally, as we argued above, even if these issues are dismissed, in the end we encounter operational prescriptions expressed in plain language about which decoherence theory has nothing to say.

A primary message of the present paper is that classical physics is subject to the same measurement dilemma as quantum mechanics. The resolution is, in short, that both quantum mechanics and classical physics are probabilistic in nature and can only be interpreted as providing statistical interpretations of the outcomes of measurements made on ensembles of similarly prepared systems. In addition, the experiments themselves are ultimately described operationally in plain language and the outcome of any given experiment is a sufficiently precise result that can only be understood in the context of a predicted statistical distribution. In this sense, our pedestrian approach is very similar to that provided by the Copenhagen interpretation of quantum mechanics. As to whether the evolution of the systems is purely quantum mechanical, purely classical or a mixture of the two is, to a degree, at the discretion of the observer. Where does this leave us with respect to the criticism that a theory which only describes the statistical behavior of an ensemble of systems cannot be considered a complete description of reality? One is left with two options: Either the pedestrian approach has resolved or perhaps clarified the measurement problem or the measurement problem is even more pervasive than before, encompassing classical as well as quantum physics. We choose to embrace the former.

Suppose one were to accept the above pedestrian approach as a solution to the measurement problem. This would certainly imply that quantum mechanics cannot provide
a complete description of reality nor can a combination of quantum and classical physics. Wouldn’t this constitute a crisis, the result of which might be a serious impediment for theoretical physics? This is doubtful. As pointed out above, the quantum measurement problem has been with us for the last 80 years and yet it seems not, in the least, to have impeded the advance of quantum physics. Physical theories are models we construct to comprehend nature. The fact that they are limited in their validity should not necessarily be considered problematic. That some such theories rely on “classical physics” or the art of experimentation should not be taken as detracting from their validity. On the other hand, neither do we suggest that searching for new “resolutions” to the measurement problems is necessarily a futile endeavor. Certainly past efforts have contributed to a deeper understanding of fundamental physics and we undoubtedly have not learned all there is to know about quantum mechanics. Also, it’s conceivable that some future theory, some extension of quantum mechanics, will be capable of explaining everything in terms of the fundamental concepts of the theory; however, we doubt that this will ever happen.

Quantum electrodynamics (and its 12 decimal place accurate prediction of the g-2 factor of the electron) is a fantastic confirmation of the success of a fundamental physical theory regardless of whether this theory relies on other products of human imagination for its confirmation. We admit that one of the incentives for our current views on the meaning of quantum mechanics has been a desire to present gravity as a completely classical phenomenon with no quantum aspects. However, even if superstring theory turns out to provide a unified quantum view of all the four forces of nature, including gravity, it will not affect the arguments given above. Such a unified theory of everything would, undoubtedly, still be framed in terms of probabilities and would still depend on the operational description of experiments to derive its meaning. Does this qualification make such a “theory of everything” less prized? No. If such a theory is ever constructed it would still rank among the greatest achievements of mankind.

We pointed out in the abstract that past attempts to resolve the measurement problem have led to significant advances in our understanding of quantum mechanics with important consequences in the mesoscopic domain. If the measurement problem were to be categorized as a faux problem as this paper suggests, would that not have the effect of stifling investigations of the interactions of microscopic, mesoscopic, and macroscopic systems? This is highly doubtful. The current advances in the understanding of such interactions and their
applications to quantum computing, molecular biology, nanophysics, etc., have been extraordinary. These successes have spawned a vigorous, ongoing enterprise whose proponents rarely, if ever, frame questions in terms of the measurement problem. In fact, reframing the measurement problem as we have in this paper might well motivate new and fruitful investigations of the foundations of quantum mechanics.

Physics is often said to be an experimental (and observational) science. The real world confronts us every day with a plethora of phenomena and it is the job of science, physics in particular, to construct models to help us to understand what we observe as well as to predict the outcome of various situations. Because of the vast extent of nature, this understanding quite naturally takes the form of a patchwork of complementary models [4]. The quest for a single, all encompassing, consistent theory that provides a complete and accurate description of the entirety of nature strikes us as the extreme of hubris. It’s not necessarily useful or even desirable to view nature as evolving according to a single fundamental law. What we can do is to construct improved models to make sense of the universe. That we have been so successful in doing so is already nothing short of amazing, as is expressed in the well-known declaration (often attributed to Einstein), “The most incomprehensible thing about the world is that it is comprehensible.” This view of nature isn’t held by all scientists and certainly not by all physicists. (We refer you to M. Tegmark’s notion that the real world is precisely its mathematical structure [53].) The current patchwork of models leaves open the possibility that future theories (models), quantum or classical, may be capable of describing individual systems in the way that our current models do not. However, at this time it seems that such theories are not required and that, at least until future experiments raise new possibilities, the current (patchwork) structure still has a great deal to offer and that many new and exciting phenomena and theories will be discovered within this context.

Appendix A: Coupling of classical and quantum systems

We have seen in Section IV that both quantum and classical mechanics may be described using the same theory of ensembles on configuration space. The formalism allows for the coupling of classical and quantum systems in a natural and self-consistent way [31–33]. For example, consider the interaction of a quantum particle with mass \( m_q \) and configuration space coordinates \( q \) and a classical particle of mass \( m_x \) and configuration space coordinates \( x \).
Further suppose that the interaction between the two particles is represented by a potential $V(q, x, t)$ that depends on both classical and quantum coordinates. Then the total ensemble Hamiltonian for the system becomes

$$\tilde{H}_{QC}[P, S] = \int dq \, dx \, P \left[ \frac{|\nabla_x S|^2}{2m_x} + \frac{|\nabla_q S|^2}{2m_q} + \frac{\hbar^2}{4} \frac{|\nabla_q \log P|^2}{2m_q} + V \right]. \quad (A1)$$

There have been many suggestions on how to couple quantum and classical systems as well as many arguments as to why the coexistence of quantum and classical systems is inconsistent, which are offered as “proofs” that the real world must be entirely quantum mechanical in origin. Most of the latter arguments have been shown to be fallacious or to lack generality while specific models of quantum/classical interactions, for the most part, have serious shortcomings (see the discussions in [31] and [32] and references therein). The ECS formalism overcomes these problems, e.g., it allows for back reaction on the classical system, positivity of probability, conservation of both probability and energy, the correct equations of motion in the classical limit, the correct equations of motion for both classical and quantum systems in the limit of no interaction, automatic decoherence of quantum ensembles, the uncertainty relations for conjugate quantum variables, and seems capable of providing descriptions of physically interesting interactions [31–33]. In short, it provides a consistent model for classical-quantum interactions.

In this paper we are interested in the ECS formalism to the extent that it provides a model of measurement that may be used to describe in a consistent way the interaction between a classical apparatus and a quantum system that is being measured. The equations of motion that follow from the ensemble Hamiltonian $\tilde{H}_{QC}$ of Eq. (A1) are non-linear. However, if the coupling between the classical apparatus and the quantum system is weak, which would be the case for a measurement that does not disturb the quantum system too much, then the departure from linearity will be minimal for the quantum system. Of course, non-linearity would be expected for any formalism that deals with the measurement problem. After all, wave function collapse is non-linear as, in some sense, is the Copenhagen interpretation which imposes a statistical meaning for the wave function through the non-linear Born rule, $P \sim |\Psi|^2$. Because of its inherent non-linearity, the ECS formalism can be extremely difficult to deal with computationally. Therefore, it seems highly unlikely that it will ever serve as a general computational tool with which to solve general quantum mechanical measurement problems. Nevertheless, it does provide an extremely useful analysis for foundational
problems in physics. In addition, because it treats quantum and classical phenomena on equal footings, it could provide a vehicle for classical models of fundamental physics forces, for example gravity [54].

As a successful model of classical-quantum interactions, the ECS formalism does reduce some of the vagueness of the Copenhagen interpretation and, therefore, offers some answers to the measurement problem. There would no longer be any question as to the location of the quantum-classical divide. It is located precisely when and where the classical-quantum Hamiltonian specifies it to be [32]. Even so, Heisenberg’s view that the divide is, in some sense, at the discretion of the experimenter is still tenable because of the inherent freedom in constructing a sensible Hamiltonian relevant to a particular experimental setup. In other words, different choices of classical-quantum coupling terms are possible. The Schrödinger cat paradox doesn’t arise because the cat belongs to the classical side of the system and is not, therefore, represented by a wave function. Interference and the superposition principle are not part of the classical world. The classicality of experiments is no longer a problem because there is now a completely classical configuration space in which the measuring apparatus resides. In fact, it would seem that the ECS model provides a complete solution to the measurement problem; however, the very probabilistic description that allows classical and quantum mechanical systems to be treated side by side brings with it a classical measurement problem, on which we elaborated in Section VI.

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