Study on support time of tunnel lining based on non-equilibrium evolution

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Abstract. According to non-equilibrium evolution stability and control theory, over force is the internal effective driving force of unrecoverable time-dependent deformation. The distribution of over force reflects the failure position and mode of tunnel. The opposite force of over force is the optimal reinforcement force to prevent unrecoverable time-dependent deformation and provides a quantitative and accurate reinforcement design method for tunnel. The plastic complementary energy (PCE) is the norm of over force. The curve of PCE versus time provides a unified quantitative criterion of real time dynamic stability evaluation of tunnel in time-dependent deformation process under various external disturbing factors. Based on the curve of PCE versus time, the stresses of lining structure in different supporting time after excavation are studied comparatively. The results show that the stress of lining structure will be very high if supporting time is too early. The surrounding rock of tunnel will be unstable due to damage evolution if supporting time is too late. There is an optimal supporting time when the stress of lining structure will not be too high and the surrounding rock will not lose stability. The curve of PCE versus time can be used to guide reinforcement design and evaluate reinforcement effect.

1. Introduction

Time-dependent deformation will inevitably occur after excavation disturbance of tunnel surrounding rock. Under the condition of low situ-stress, the deformation can gradually converge and stabilize, but
under the condition of high situ-stress, the deformation will continue to develop, forming plastic zone or even damage zone. This process can be regarded as a non-equilibrium evolution process. On the one hand, the structure will give full play to its own bearing capacity, gradually tend to a stable state, showing a trend of self-equilibrium evolution; on the other hand, the occurrence of plastic zone and damage zone leads to the deterioration of surrounding rock parameters, the structure self-supporting capacity becomes worse, tends to an unstable state, deviates from the equilibrium state. Therefore, it is necessary to find an optimal support time, which can make the surrounding rock give full play to its bearing capacity, reduce the thickness of lining structure, and avoid tunnel instability.

2. Viscoplastic and viscodamage constitution model

2.1. Viscoplastic model

As unrecoverable time-dependent deformation, viscoplastic deformation is defined by overstress in Duvaut-Lions model [1]. The viscoplastic strain rate is calculated by:

$$\dot{\varepsilon}^{vp} = \Gamma^{vp} D^{-1} : (\sigma - \hat{\sigma})$$

(1)

$\Gamma^{vp}$ is positive viscoplastic viscosity coefficient, $D$ is fourth-order elastic tensor, $\hat{\sigma}$ is second-order inviscid stress tensor, $\sigma$ is second-order stress tensor. According to closest point projection method (CPPM), $\hat{\sigma}$ is the closest point projection of $\sigma$ onto the yield surface $f(\sigma)=0$ in the sense of the energy norm $\|\sigma - \hat{\sigma}\|_{D^{-1}} = \sqrt{(\sigma - \hat{\sigma}) : D^{-1} : (\sigma - \hat{\sigma})}$. $\hat{\sigma}$ is unique for any $\sigma$ provided that the yield and plastic potential function surfaces are convex. For the Drucker-Prager (D-P) yield function and associated flow rules [2], the viscoplastic strain rate is calculated by:

$$\dot{\varepsilon}^{vp} = \Gamma^{vp} \left( \frac{f(\sigma)}{9\alpha^2 K + G} \left[ \alpha I + \frac{1}{2\sqrt{J_2}} s \right] \cdot \left( \alpha I - 9\alpha^2 K / G \sqrt{J_2} - \frac{k}{\kappa} \right) \right)$$

(2)

$$\dot{\varepsilon}^{vp} = \Gamma^{vp} \left[ \frac{1}{9K} \left( I - \frac{\kappa}{\alpha} \right) I + \frac{1}{2G} s \right] \cdot \left( \alpha I - 9\alpha^2 K / G \sqrt{J_2} - \kappa > 0 \right)$$

(3)

$I$, is first stress tensor invariant, $J_2$ is second deviotoric stress tensor invariant, $\alpha$ is pressure-sensitive parameter associated with the internal friction angle, $\kappa$ is initial yield stress, $K$ is bulk modulus, $G$ is shear modulus.

2.2. Viscodamage model

Under different loading conditions, damage evolution depends on stress level and equivalent viscoplastic strain [3]. The damage rate is calculated by:

$$\dot{\phi} = \Gamma^{vd} \left( \frac{Y}{Y_0} \right) \exp(kp)$$

(4)

$\Gamma^{vd}$ is viscosity coefficient of viscodamage, $q$ is stress dependency parameter, $k$ is material parameter, $Y$ is damage driving force calculated by $Y = \alpha I_1 + \sqrt{J_2}$, $Y_0$ is reference damage driving force, $P$ is accumulative equivalent viscoplastic strain, $\phi$ is scalar damage variable ranging between $\phi = 0$ (intact) and $\phi = 1$ (failure).

2.3. Coupling of viscoplastic and viscodamage models

For small deformation assumption, the total strain is divided into elastic and viscoplastic strains:
\[ \varepsilon = \varepsilon^e + \varepsilon^p \]  

(5)

\( \varepsilon \) is nominal total strain tensor, \( \varepsilon^e \) is nominal elastic strain tensor, \( \varepsilon^p \) is nominal viscoplastic strain tensor.

According to the continuum damage mechanics, the relationship between effective stress and nominal stress is [4]:

\[ \bar{\sigma} = \sigma / (1 - \phi) \]  

(6)

\( \bar{\sigma} \) is effective strain tensor, \( \sigma \) is nominal strain tensor.

The strain equivalence hypothesis is used:

\[ \bar{\varepsilon} = \varepsilon, \bar{\varepsilon}^e = \varepsilon^e, \bar{\varepsilon}^p = \varepsilon^p \]  

(7)

\( \bar{\varepsilon} \) is effective total strain tensors, \( \bar{\varepsilon}^e \) is effective elastic strain tensor, \( \bar{\varepsilon}^p \) is effective viscoplastic strain tensor.

3. Non-equilibrium evolution stability and control theory

3.1. Over force and plastic complementary energy in viscoplasticity

In the theory of viscoplasticity, the stress state beyond the yield surface determines the rate of viscoplastic deformation [1, 5]. The stress \( \sigma \) at any time satisfies the equilibrium condition but may not satisfy the yield condition. If \( f(\sigma) \leq 0 \), \( \dot{\varepsilon}^p = 0 \), there is no time-dependent deformation. If \( f(\sigma) > 0 \), \( \dot{\varepsilon}^p > 0 \), time-dependent deformation occurs. The overstress in Duvaut Lions model which determines the viscoplastic strain rate is [1]:

\[ \Delta \sigma = \sigma - \dot{\sigma} \]  

(9)

The equivalent nodal force of overstress \( \Delta \sigma \) is defined as over force, i.e.

\[ \Delta \mathbf{Q} = \sum_{V} \int_{V} B^T \Delta \sigma dV \]  

(10)

The norm of over stress \( \Delta \sigma \) is defined as plastic complementary energy (PCE):

\[ \text{PCE} = \frac{1}{2} \int_{V} (\sigma - \dot{\sigma} : C : (\sigma - \dot{\sigma}) dV \]  

(11)

\( C \) is elastic compliance tensor.

Over force is the equivalent nodal force of overstress, which indicates that the over force is the internal effective driving force of unrecoverable time-dependent deformation during the non-equilibrium evolution process. The existence of the over force indicates the structure has no sufficient resistance and will deform continuously until failure occurs. The distribution of the over force shows the dominant patterns of local failure. To secure the self-unsupported structure, a reinforcement force that is equal and opposite to the over force, i.e., \( -\Delta \mathbf{Q} \), may be exerted. Therefore, the opposite force of the over force is the reinforcement force to prevent unrecoverable time-dependent deformation or failure. This reinforcement force has been shown to be optimal by the principle of minimum PCE. PCE is an energy norm of both the viscoplastic strain rate field and the over force. Thus, viscoplastic structures under given conditions deform towards the limit steady state at which the viscoplastic deformation rate or the over force is minimised in the sense of PCE. The minimum steady value is greater than zero for the perfect viscoplastic material and is zero for strain hardening material [6]. If a structure with a minimum PCE greater than zero is not reinforced, the structure suffers steady
viscoplastic flow at the minimum rate until failure occurs. The non-equilibrium evolution law of rock structure follows the principle of minimum PCE. The relation between time-dependent deformation, damage evolution and reinforcement force during excavation unloading process can be well demonstrated by the ground reaction or response curve (GRC) in New Austria Tunneling Method (NATM) [7, 8]. The typical relation between the required support force and the convergence of the tunnel wall after excavation unloading is indicated by the GRC in figure 1.

4. Study on supporting time of tunnel lining

The developed viscoplastic and viscodamage constitution model is numerically implemented based on user material subroutine UMAT in Abaqus software and used to study and calculate support time of tunnel lining. The finite element model of tunnel is shown in figure 2 and figure 3. The model width is 300m. The model length is 66m. The buried depth is 1100m. The rock mass and concrete are simulated by 8-node 6-facet solid element, and the bolt is simulated by rod element. The lining thickness is 20cm. The excavation diameter is 13.1m. The range of bolt arrangement is 240 degrees of the top arch of the tunnel, and the bolt arrangement is alternate and staggered. The length of bolt is 4.5m(Φ25mm) and 6m(Φ28mm) respectively. The physical and mechanical parameters of materials are shown in table 1. The viscoplastic and viscodamage model parameters of rock are shown in table 2. Except for rock, other materials are calculated according to the elastic model.

### Table 1. The physical and mechanical parameters of materials.

| Material     | \( \rho (\text{kg m}^{-3}) \) | \( E (\text{GPa}) \) | \( v \)  | \( f \) | \( c (\text{MPa}) \) |
|--------------|-------------------------------|----------------------|---------|--------|-----------------|
| Rock         | 2650                          | 6                    | 0.25    | 0.7    | 0.6             |
| Lining       | 2500                          | 28                   | 0.167   | 1      | 1.78            |
| Shotcrete    | 2500                          | 22.4                 | 0.167   | 1      | 1.78            |
| Bolt         | 7800                          | 200                  | —       | —      | —               |

*Figure 1. Relation between support force and tunnel convergence in NATM.*

*Figure 2. The finite element model of tunnel.*

*Figure 3. The finite element model of lining.*
Table 2. Viscoplastic and viscodamage model parameters of rock.

| Material | $\Gamma^p(h^{-1})$ | $\Gamma^q(h^{-1})$ | $Y_0$ (MPa) | $q$ |
|----------|---------------------|---------------------|--------------|-----|
| Rock     | 1                   | $5 \times 10^{-3}$  | 10           | 1   |

The PCE versus time of different support measures without damage are shown in figure 4 and figure 5. The enlarged view of PCE versus time of different elastic modulus of bolt without damage are shown in figure 6. The enlarged view of PCE versus time under different support time of lining without damage are shown in figure 7. The damage evolution is not considered in these cases. The meaning of the symbols in the picture is as follows: B(2GPa)-S-L(t=50h) means the support measure of bolt with elastic modulus of 2GPa, shotcrete and lining 50 hours after excavation. When the tunnel is not supported after excavation, the PCE is the largest and its attenuation is the slowest in the whole process. When bolt and shotcrete support are conducted, The PCE changes obviously and decays faster. The stiffness of the bolt has a significant impact on the support effect. The larger the elastic modulus of bolt, the better the support effect, as shown clearly in figure 6. When bolt, shotcrete and secondary lining are used, the change of PCE is more obvious and its attenuation is further accelerated. The lining support time also has a significant impact on the support effect. The earlier the support time, the better the support effect, as shown clearly in figure 7. However, the earlier the support time is, the greater the lining stress is. When the lining is applied immediately after excavation, the maximum compressive stress of the lining is 87.2MPa. When the lining is applied 50 hours after excavation, the maximum compressive stress of the lining is 30.5MPa, as shown in table 3, figure 8 and figure 9. At the initial stage of excavation, the deformation rate of surrounding rock is very large, and with the increase of time, the deformation rate will continue to decrease and tend to be stable. Therefore, if the lining is applied too early, the stress of the lining will be great, and the lining may be crushed. Therefore, it is necessary to find a suitable optimal lining time to ensure the support effect and not to make the lining stress too large. The optimal lining time is usually at the stage when the PCE begins to decrease to approximately constant.

Table 3. Minimum principal stress of lining at $t = 150h$ for different support time of lining without damage.

| Support time (h) | Min principal stress (MPa) | Support time (h) | Min principal stress (MPa) |
|------------------|-----------------------------|------------------|-----------------------------|
| 0                | 87.2                        | 30               | 39.2                        |
| 5                | 63.3                        | 40               | 34.5                        |
| 10               | 54.8                        | 50               | 30.5                        |
| 20               | 45.4                        |                  |                             |
The PCE versus time of total, self-balance and damage evolution is shown in figure 10. When the damage evolution is considered, the tunnel without support will enter a complete evolutionary process usually including three evolution phases: primary evolution phase, steady evolution phase and accelerated evolution phase. The total evolution curve indicates the actual stability evolution process and is a comprehensive result of the self-balance evolution and damage evolution processes. The self-balance evolution curve indicates the deformation self-adjustment process without damage and is a
process from the non-equilibrium state to an equilibrium steady state. The damage evolution curve indicates the damage process due to parameter degradation and is a process tending to an unsteady state. Therefore, the damage evolution has a great influence on the spatio-temporal evolution of the tunnel, and also on the lining support time. If the lining time is too late, the surrounding rock will be irreversibly damaged and unstable.

The PCE versus time under different support time of lining including damage are shown in figure 11 and figure 12. In the case of no support after excavation, the tunnel will soon enter the third stage of accelerated evolution, and then instability occurs. However, the damage evolution process of tunnel can be controlled by bolt and shotcrete support immediately after excavation. So that it does not enter the accelerated evolution stage. Therefore, the bolt and shotcrete support immediately after excavation have good support effect on the tunnel. Then through lining support, the effect of tunnel support is further enhanced. When the lining is applied 10 hours after excavation, the maximum compressive stress of the lining is 70MPa. When the lining is applied 40 hours after excavation, the maximum compressive stress of the lining is 48.2MPa, as shown in table 4, figure 13 and figure 14.

### Table 4. Minimum principal stress of lining at t = 150h for different support time of lining.

| Support time (h) | Min principal stress (MPa) | Support time (h) | Min principal stress (MPa) |
|-----------------|---------------------------|-----------------|---------------------------|
| 10              | 70.0                      | 30              | 52.7                      |
| 20              | 59.1                      | 40              | 48.2                      |

![Figure 10. PCE versus time of total, self-balance and damage evolution.](image)

![Figure 11. PCE versus time under different](image)

![Figure 12. Enlarged view of PCE versus](image)
5. Conclusions

(1) Based on the viscoplasticity and damage mechanics theory, the non-equilibrium evolution stability and control theory is used to simulate the spatial-temporal evolution process of tunnel. The competition between two evolutionary trends of self-balance evolution and damage evolution determines the total evolution law.

(2) Over force is the internal effective driving force of unrecoverable time-dependent deformation during the non-equilibrium spatial-temporal evolution process. The distribution of over force reflects the failure position and mode of tunnel. The opposite force of over force is the optimal reinforcement force to prevent unrecoverable time-dependent deformation and provides a quantitative and accurate reinforcement design method for tunnel.

(3) The plastic complementary energy (PCE) is the norm of over force. The curve of PCE versus time provides a unified quantitative criterion of real time dynamic stability evaluation of tunnel in time-dependent deformation process under various external disturbing factors.

(4) Based on the curve of PCE versus time, the stresses of lining structure in different supporting time after excavation are studied comparatively. The results show that the stress of lining structure will be very high if supporting time is too early. The surrounding rock of tunnel will be unstable due to damage evolution if supporting time is too late. There is an optimal supporting time when the stress of lining structure will not be too high and the surrounding rock will not lose stability. The curve of PCE versus time can be used to guide reinforcement design and evaluate reinforcement effect.

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