Dynamics of a coupled spin vortex pair in dipolar spinor Bose-Einstein condensates

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The collisional and magnetic field quench dynamics of a coupled spin-vortex pair in dipolar spinor Bose-Einstein condensates in a double well potential are numerically investigated in the mean field theory. Upon a sudden release of the potential barrier the two layers of condensates collide with each other in the trap center with the chirality of the vortex pair exchanged after each collision, showing the typical signature of in-phase collision for the parallel spin vortex phase, and out-of-phase collision for the antiparallel phase. When quenching the transverse magnetic field, the vortex center in the single-layered condensate starts to make a helical motion with oval-shaped trajectories and the displacement of the center position is found to exhibit a damped simple harmonic oscillation with an intrinsic frequency and damping rate. The oscillation mode of the spin vortex pair may be tuned by the initial magnetic field and the height of the Gaussian barrier, e.g. the gyrotropic motions for parallel spin vortex pair are out of sync with each other in the two layers, while those for the antiparallel pair exhibit a double-helix-structure with the vortex centers moving opposite to each other with the same amplitude.

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I. INTRODUCTION

The multilayered structure with stacked vortices are intensively studied in the past few years, as their simple structure combined with highly non-trivial dynamical properties make them fascinating objects of research [1–12]. They also provide the possibility to store information by means of their chirality and polarity. As an ideal platform, a dipolar spinor Bose-Einstein condensate can form a coreless vortex spontaneously in a pancake-shaped trap as a result of the anisotropy of the magnetic dipolar-dipolar interaction (MDDI) [13–19]. At intermediate temperature, a time-dependent Hartree-Fock-Bogoliubov study [20, 21] has shown that an anomalous vortex pair may be spontaneously generated in the anomalous fraction with the condensed atoms filled the vortex core, when phases corresponding to the singly charged vortex are imposed in the condensed and the anomalous components of the gas. In particular, if an axial gaussian barrier is imposed upon the pancake-shaped trap, the condensate will be divided into two layers with parallel or antiparallel vortex in them for different barrier height [22], which resembles the trilayer films with a ferromagnetic-nonmagnetic-ferromagnetic (F-N-F) structure.

In fact, the collisional dynamics of two topological defects in the condensate has drawn great attention in recent years [23–29]. A quantum version of Newton’s cradle has been observed in the arrays of trapped one-dimensional (1D) bose gases by turning off the crossed dipole trap quickly, which is in the proximity of an integrable system [23]. Recently, an experiment on the collision of matter-wave solitons consisted of a degenerate gas of ⁷Li atoms has been explored by using a Gaussian laser beam to cut the condensate in half and then turning off the barrier quickly [24]. This would enable us to further explore the transition between integrable and non-integrable systems. Another experimental study [26, 27] on the collisional dynamics of two half-quantum vortices in a highly oblate ⁶⁷Na BEC is performed by means of a vortex-dipole generation technique based on a moving optical obstacle, which demonstrates the short range interaction between half-quantum vortices with different core magnetizations. To understand the collisional dynamics of a spin vortex pair [22] formed in the pancake-shaped dipolar spinor condensate needs a full three-dimensional numerical simulation, which may provide further insight into the face-to-face collisional dynamics of the spin vortex through the controlled formation of parallel or antiparallel vortex pair in the two layers of condensate.

A more practical motivation of this study lies in that magnetic vortex oscillators have been investigated for use as tunable microwave generators for future wireless communication [30–32]. Hence a better understanding of the fundamental physical properties of multilayered vortex structure is urgent in order to improve their functionality. Recently, investigations of dynamically coupled magnetic vortices have been reported in pairs [33–35], trios [36], chains [38, 40], and arrays [38, 41, 44] of microscopic ferromagnetic disks and squares [43, 42]. The spontaneous parallel and antiparallel spin vortex pairs provide an ideal platform for simulating multilayer magnetic vortices and a transverse magnetic field may be applied to displace the vortex centers [22], to achieve the similar spin structure.
in nanopillar devices. Moreover, the coupled motion of vortices in a trilayer structure has been addressed by time-resolved imaging technique with a scanning transmission x-ray microscope, which stimulates us to reveal the vortex dynamics in coupled layers of the condensate after quenching the transverse field to zero.

In consideration of the wide applications of magnetic vortex, in this paper we focus on the collisional and quench dynamics of the initial spin vortex ground states in Ref. by suddenly turning off either the potential barrier of the double well or the transverse magnetic field. The paper is organized as follows. In Sec. II we introduce our model and briefly review the ground state properties of this system, which is necessary to understand the following discussions. The numerical results on the collisional dynamics are presented in Sec. III. In Sec. IV we study the magnetic field quench dynamics of the spin vortex for different phases. Finally, we summarize our findings in Sec V.

II. FORMULATION

The dynamics of the spin vortex pair formed in a dipolar spinor condensate is governed by the following set of Gross-Pitaevskii equations in the mean-field treatment

\[ i\hbar \frac{\partial \psi_\alpha}{\partial t} = (T + U + c_0n) \psi_\alpha + g_F\mu_B B_{\text{eff}} \cdot \mathbf{F}_{\alpha\beta} \psi_\beta, \] (1)

where \( \psi_\alpha(\mathbf{r}) \) denotes the condensate wave function for the spin component \( \alpha = 0, \pm 1 \) respectively, \( n(\mathbf{r}) = \sum_\alpha |\psi_\alpha|^2 \) is the total density, \( T = -\hbar^2 \nabla^2/(2M) \) is the kinetic energy with \( M \) the atomic mass, and \( U(\mathbf{r}) \) is the trapping potential. The coefficient \( c_0 = 4\pi\hbar^2 (a_0 + 2a_2)/(3M) \) describes the spin-independent collisional interaction in the condensate with \( a_{0,2} \) being the s-wave scattering length for two-spin-1 atoms in the total spin channels 0 and 2, and \( g_F \) is the Landé g-factor of the atom, \( \mu_B \) is the Bohr magneton, and \( \mathbf{F} \) is the angular momentum operator. All spin related interactions are collected into the last term in a way of an effective magnetic field

\[ B_{\text{eff}}(\mathbf{r}) = B_{\text{ext}} + \frac{c_2}{g_F\mu_B} S(\mathbf{r}) + \frac{c_d}{g_F\mu_B} \int \frac{d^3r'}{|\mathbf{R}|^2} \left[ S(\mathbf{r}') - 3 \frac{|\mathbf{S}(\mathbf{r}') \cdot \mathbf{R}| \mathbf{R}}{|\mathbf{R}|^2} \right], \] (2)

with \( \mathbf{R} = \mathbf{r} - \mathbf{r}' \) and \( B_{\text{ext}} \) the external magnetic field. Here, \( S(\mathbf{r}) = \sum_\alpha \psi_\alpha^* \mathbf{F}_{\alpha\beta} \psi_\beta \) is the spin density, and \( c_2 \) and \( c_d \) characterize the spin exchange and dipole-dipole interaction, respectively.

The dipolar interaction in the condensate induces interesting spin vortex structure in the ground state for a pancake-shaped trap potential \( U(\mathbf{r}) \). To create a spin vortex pair, a Gaussian barrier of height \( A \) and width \( \sigma_0 \) is imposed upon a highly anisotropic harmonic potential with aspect ratio \( \lambda \), leading to a combined trapping potential \( U(\mathbf{r}) = \frac{1}{2}M\omega_\perp^2 (x^2 + y^2) + \frac{1}{2}M\omega_\parallel^2 z^2 + Ae^{-z^2/(2\sigma_0^2)} \) with \( \omega_\perp \) the radial trap frequency. In experiment, one may choose a Gaussian laser beam to cut the condensate into two halves and the system can be considered as quasi-two-dimensional clouds of atoms, nearly free to move in the planes. The vortex pair could be with either parallel or anti-parallel spin on the two sides of the barrier and the spin structures of the ground state denoted as the PSVP and ASVP are determined by the height of the Gaussian barrier \( A \) for a fixed barrier width \( \sigma_0 \) in the absence of an external magnetic field. The competition between the tunneling splitting of the two wells and the interlayer MDDI energy induces a phase transition between PSVP and ASVP. An increasing transverse magnetic field is found to remove the spin vortices sequentially before the condensate is fully polarized.

By preparing the system in the two distinct phases it is interesting to study the dynamics of coupled spin vortex pair by turning off either the barrier or the magnetic field, both of which are described by solving numerically the time dependent GP equation Eq. (1) with the help of the real time propagation method. To study the dynamics of the coupled vortex pair, the initial states for the quench dynamics of the condensate are prepared in the spin vortex state based on our previous work on the ground state of the system. For \(^{87}\text{Rb}\), we take \( c_2 = 0.01c_0 \) and deliberately increase the DDI parameter to \( c_d = |c_2| \) to accelerate the convergence of the numerical calculation as in Ref. In addition, we take the same parameters as in previous study, i.e. total atom numbers \( N = 5 \times 10^5 \), \( \omega_\perp = (2\pi)100 \text{ Hz} \), \( \sigma_0 = 0.65\xi_\perp \), and \( \lambda = 6 \) in our numerical calculation, and the dimensionless units are \( \hbar \omega_\perp \) for energy, \( \ell_\perp = \sqrt{\hbar/(M\omega_\perp)} \) for length, \( \omega_\perp^{-1} \) for time, and \( \ell_\perp^{-3/2} \) for the wave function, respectively. Enhancement of the DDI by increasing the number of particles only slightly changes the critical barrier height \( A^* \) between the PSVP and ASVP phases. For example, for \( N = 5 \times 10^5 \) adopted in this work, \( A^* = 246\hbar \omega_\perp \), while for \( N = 2 \times 10^5 \) we have \( A^* = 158\hbar \omega_\perp \). This, however, does not change the dynamics qualitatively. In fact, it was verified that the qualitative results can be reproduced by solely increasing \( N \) to \( 10^7 \) with \( c_d \) being that of the \(^{87}\text{Rb}\) atom.

III. COLLISION DYNAMICS

In order to provide an experimentally discernible signature of the PSVP and ASVP phases, we study the collision dynamics of the spin vortex pair by turning off the barrier quickly in a very short time once the pair is formed in the upper and lower planes, i.e. the system is prepared deliberately at the beginning in the ground state of the double well with \( A = 100\hbar \omega_\perp \) and 300\( \hbar \omega_\perp \), respectively. The atoms suddenly find themselves at the classical turning points of the harmonic trap in the \( z \) direction and begin to accelerate towards the centre. We carried out a full three dimensional calculation of the collision dynamics and the integrated density profiles are
The dynamics of magnetic vortices in nanodisks or multilayer structures manipulated by the magnetic field has attracted more and more attention [45–49]. The rotation of a vortex as a whole around its equilibrium position is found in a gyrotropic mode with a frequency far below the ferromagnetic resonance of the corresponding material [46]. Here we show similar oscillation mode of the spin vortex pair tuned by the magnetic field in the dipolar spinor condensate system. We focus on the motion of the vortex center in three typical structures, i.e. the single layer, PSVP and ASVP, when the transverse magnetic field is suddenly quenched to zero. In the following we show the time-resolved trajectories of the vortex centers in the $z = z_{\text{min}}$ plane with $z_{\text{min}}$ being the positions of the potential minima along the $z$ axis. In the three cases discussed here, we have $z_{\text{min}} = 0$ for single layer conden-

**FIG. 1:** (Color online) Collision dynamics of a coupled spin vortex pair in (a) PSVP ($A = 100\hbar \omega_\perp$) and (b) ASVP ($A = 300\hbar \omega_\perp$) phases of the condensate when the Gaussian barrier is turned off suddenly. Each image denotes the cloud density in $x - z$ plane after integrating along $y$ axis.

**FIG. 2:** (Color online) Spin vortex pair in the three collisional moments for the ASVP phase. Spin structures in the initial potential minima are shown in the upper and lower layers with the chiralities exchanged after each collision - the chiralities are inverted at $t = \tau/2$ and restored at $t = \tau$.

Presented in Fig. 1 for two oscillation cycles. As a result, the Newton’s cradle-like dynamics [23] is observed in our system, featured with distinct interference fringes and chirality exchange in the spin vortex pair.

The two sheets of cloud collide with each other in the center of the trap twice each full cycle, for instance at $t = \tau/4$ and $3\tau/4$ with $\tau = 2\pi/\omega_z = 1.05\omega_\perp^{-1}$, as illustrated in Fig. 1. When they first meet, the atoms in the PSVP phase produces global interference patterns in the whole condensate, while those in the ASVP phase are stirred only in the central area. At this moment, the collisional interaction among the three components is the strongest so that the exchange of the particle number reaches to most frequently. We find that the density of spin-0 component $n_0$, defined as the full space integration of $|\psi_0|^2$, decreases during the first collision time for the PSVP phase, indicating that this phase favors a particle exchange direction $2|0\rangle \rightarrow |1\rangle + |-1\rangle$, while it increases for the ASVP phase and an opposite particle exchange channel is preferred. It is worthwhile to mention that the magnetization of $z$ axis $M_z = \int dr (|\psi_1|^2 - |\psi_{-1}|^2)$ remains zero in the whole evolution process so that the particle number density of $\pm$ components always satisfies $n_1 = n_{-1}$.

And then, the two layers of condensate pass straightly through each other before reemerging on the other side. When the condensates separate into two parts again at $t = \tau/2$, we observed an interesting phenomenon that the chiralities of the spin vortex pair in the upper and lower layers, i.e. the circulations of the in-plane magnetization components colored by red and blue in Fig. 2 exchange with each other in the ASVP phase and are restored at $t = \tau$ except that the spin structures are a little distorted near the vortex center, while those in the PSVP phase are the same as they should be. The phase differences between the lower and upper parts $\phi_0$ (which is 0 in the PSVP and $\pi$ in the ASVP) leads to drastically different behavior in the wave functions: at the half period $t = \tau/2$ a density peak appears at the centre-of-mass of the condensate for the PSVP indicating an in-phase collision, which is not observed for the ASVP case implying an out-of-phase collision. These phenomena are in good agreement with the experimental observation of the collision of matter wave solitons [24]. Different with the 1D collision, the face-to-face oscillation here in two dimensional will be destructed in a few cycles, which can be understood as a result of the density-dependent inelastic collision.

**IV. QUENCH DYNAMICS OF THE TRANSVERSE MAGNETIC FIELD**
sate in a harmonic trap, and \( z_{\text{min}} = \pm \sigma_0 \sqrt{2 \ln (A/\lambda^2 \sigma_0^2)} \) for the double well structure.

**Dynamics of a spin vortex in a single-layered condensate:** We first consider the motion of the vortex center in a single-layered, pancake-shaped condensate starting from an initial state with a spin vortex for \( B_x = 25 \mu G \) which may be regarded as a reference for more complex structure. As demonstrated in Fig. 3(a), when the field is removed quickly, the vortex center in the \( z = 0 \) plane starts to make a helical motion with oval-shaped trajectory in \( xy \) plane. The projection of the center position on the \( y \)-axis, that is the displacement of the vortex center, is found to exhibit a well-defined periodic oscillation for quite a long time, as shown in Fig. 3(b). This gyrotropic motion of the vortex center has been fitted with a damped simple harmonic oscillation motion as \( \Delta y(t) = (y_0 \exp(-\beta t) + y_1) \cos(\omega_0 t) \), where \( y_0 = 1.7 \ell_\perp \), \( y_1 = 0.8 \ell_\perp \), \( \beta = 5.84 \times 10^{-7} \omega_\perp \), and \( \omega_0 = 0.46 \omega_\perp \). This can be explained by taking into account the fact that the magnetic vortex is driven to move by a precessional torque generated by turning off the transverse magnetic field suddenly. We also examine the time dependence of the magnetization along \( x \) axis \( M_x = \int dr \mathcal{S}_x(r) \) in Fig. 3(c), the oscillation there has been fitted with a formula \( M_x(t) = (M_0 \exp(-\beta t) + M_1) \cos(\omega_0 t) \) with \( M_0 = 0.17 \hbar \) and \( M_1 = 0.12 \hbar \). We find that the characteristic damping parameter \( \beta \) and the oscillation frequency \( \omega_0 \) match exactly with those in the evolution of displacement of the vortex centers \( \Delta y(t) \). This can be clearly seen in the Fourier spectral analysis of the two oscillations in Fig. 3(d). During the evolution, however, the magnetizations along \( y \) and \( z \) axis are found to be constantly zero.

Staring from a polarized state gives another scenario of the quench dynamics of the spin structure in a single-layered condensate, including the vortex formation and its helical motion. Fig. 4 shows the details: four vortices are formed after the quench of the magnetic field, and they merged sequentially into two, and then to a single vortex. Meanwhile the magnetization is found to undergo a rather slow damping oscillation. Once the vortex is stabilized, its center moves in just the same way as the helical motion with oval-shaped trajectory, the frequency of which tends to match the small transverse field case for \( B_x = 25 \mu G \) as shown in the bottom panel in Fig. 4. We therefore conclude that the oscillation frequency of the vortex center in the single-layered condensate is intrinsic and irrelevant to the quenching initial state.

**Dynamics of a spin vortex pair in the PSVP and ASVP phases:** For the PSVP or ASVP ground state, the number of spin vortices decreases from 2 to 0 with a growing transverse field in the three stages of the magnetization process, i.e. double-vortex, single-vortex, and polarized states. We first examine a small transverse field with \( B_x = 25 \mu G \), in which case the spin vortex pair is formed in two sheets of condensate with the same chiralities and the centers displaced along the same direction for the PSVP state, and with opposite chiralities and the two centers separated away along opposite directions for the ASVP state. Quenching this transverse field abruptly to zero, we find that the dynamics of the vortex centers is much different from that of a single-layered condensate due to the existence of a significant interplay between the interwell MDDI and tunneling splitting of the double-well potential. For the PSVP phase, the vortex centers of the
two layers make a gyrotropic motion but out of sync with each other, as shown in Fig. 5(a) and their projection on y axis in Fig. 5(b). Clearly the average position of the two vortex centers is not zero due to the frequency difference of the oscillations in the upper and lower layers (the green line in Fig. 5b). We also examine the time dependence of the magnetization $M_z$ in Fig. 5c, where the magnetization in the two parts oscillate in the same way as the displacement of the vortex centers, just as what happens in the single-layered condensate. In particular, we make a Fourier transform of the magnetization dynamics over a period of $100\omega_\perp^{-1}$ and find that for each layer a beat phenomenon appears in the dynamics with two slightly different characteristic frequency peaks as shown in Fig. 5d. Further numerical simulations show that the weight of two frequencies in the dynamics changes along with the width of the Gaussian barrier. As the distance between the layers increases, the principle peak moves to higher frequency which represents the gyroscopic vortex motion, while the other peak corresponds to its higher harmonics. This can be understood as following: Note that both the interwell dipolar interaction (proportional to $1/\sigma_0^3$) and tunneling splitting (roughly proportional to $e^{-\tilde{A}\sigma_0}$) decrease when the two layers are separated wider at a fixed height of the interwell barrier $\tilde{A}$ which is measured from the minima of the double-well potential. In the PSVP phase, the dipolar interaction inside each layer thus plays a major role so that the dynamics is dominated by a relatively strong nonlinear interaction as in the single layer case with the principle characteristic frequency more prominently located at $0.5\omega_\perp$ for $\sigma_0 = 1.2\ell_\perp$ as can be seen in the inset of Figure 5d.

As forming an ASVP state needs a higher barrier $A$, the tunneling interaction between the two wells will be weakened so that the interwell dipolar interaction plays a key role in the process of the evolution. The trajectories of the vortex centers in ASVP phase are shown in Fig. 5(e) and their projections in Fig. 5(f). We find that starting from two opposite positions in the $xy$ plane, the two vortices with opposite chiralities exhibit a double-helix-structured gyrotropic motion and exhibit damped oscillation behaviors. It is worthwhile to notice that the motion of the vortex centers in the two layers are opposite to each other, but with the same amplitude, leading to a nearly zero average displacement of the vortex centers (the green line in Fig. 5(f)). The magnetization $M_z$ in each layer shows perfect match during the damped evolution (Fig. 5g). Obviously the beat phenomenon exists neither in the motion of vortex cores nor the evolution of the magnetization $M_z$ (Fig. 5f and 5g) just as in the single layer case. The Fourier transform of the oscillation is a single peaked spectrum with the characteristic frequency located at $0.56\omega_\perp$ for $\sigma_0 = 1.2\ell_\perp$ as shown in the inset of Fig. 5h). The damping rate $\beta$ of the oscillation can be adjusted with a varying barrier separation $\sigma_0$, for instance it is three times faster for a closer space $\sigma_0 = 0.5\ell_\perp$ for which we have $\beta = 0.03\omega_\perp$ than a wider separation $\sigma_0 = 1.2\ell_\perp$ for which we have $\beta = 0.01\omega_\perp$ with the fitting frequency $\omega = 0.52\omega_\perp$.

**Dynamics of a single vortex in the PSVP and ASVP:** Next, we choose an intermediate magnetic field such that the initial state has a single vortex in, say, the lower layer, while the upper layer is polarized with $M_z(z < 0) < M_z(z > 0)$. Depending on the magnetic phases of the condensate, the system can support different combinations of vortex states. We compare the results for PSVP and ASVP in Fig. 6 which shows the time dependence of the magnetization $M_z$ of the condensate for $A = 100$ and $300\hbar\omega_\perp^{-1}, \sigma_0 = 0.65\ell_\perp$ after quenching the initial magnetic field $B_z = 50\mu G$ for PSVP, and $B_z = 60\mu G$ for ASVP, to zero. As we can see, the evolution here is similar to the single layer case and can be divided into two stages: (i) vortex formation stage - compared to the stabilization of the PSVP spin vortex pair with the same chirality, it takes a little longer time for the two layers in the ASVP phase to form a spin vortex pair with opposite chiralities, and (ii) vortex oscillation stage - the centers of the stabilized vortices start to make the typical helical motion after $t = 25\omega_\perp^{-1}$ for PSVP and $t = 50\omega_\perp^{-1}$ for ASVP. We note that both the magnetization oscillation and the dis-
placement of the vortex center are rather small for PSVP phase, while those in the ASVP phase show quite a larger amplitude and lower frequency. Also, the two vortex centers in the ASVP phase follow the same trajectory as in Fig. 6(d), while the magnetizations in each layer are opposite to each other leading to a nearly vanishing total magnetization as indicated by the black dash-dotted line in Fig. 6(b).

**Dynamics of a polarized state in the PSVP and ASVP:**
At last, we consider a polarized initial state in which all spins are polarized along $x$ direction at a large magnetic field $B_x = 70 \mu G$. The fate of this state in a long time is a spin vortex pair with opposite chiralities, irrelevant with the initial PSVP or ASVP phases. In this regime the magnetization in the two layers show uniform behavior in both PSVP and ASVP phases, and in Fig. 6(e-h) we still observe that the magnetization and the vortex centers of each layer oscillate with time for PSVP, in the ASVP phase, however, the oscillation is greatly suppressed, i.e. the vortex center hardly move with their centers pinned at the origin of the $xy$ plane once the vortices are stabilized. These features may be exploited to identify the individual configurations of PSVP and ASVP experimentally.

V. CONCLUSION

Before concluding, we would like to point out the connections of this study to the existing experimental works in some related fields. The appearance and dynamics of vortex and half-vortex (HV) in polariton condensate and spin-1 anti-ferromagnetic condensate are connected with spontaneous symmetry breaking and phase transitions. The cores of the primary half-vortex in the polariton condensate are seen moving along orbital-like trajectories around the density maximum of the counter-polarized Gaussian state, keeping itself orbiting during a few tenths of picoseconds, while the trajectories of the twin singularities upon full-vortex injection undergo a spiraling at large polariton densities. Such dynamical configuration resembles the gyrotropic motions of the vortex center by manipulating the transverse field in double-layered dipolar condensate (see Fig. 5) and ferromagnetic nanodisks. By means of in situ magnetization-sensitive imaging, pairs of half-quantum vortices with opposite core magnetization are generated when singly charged quantum vortices are injected into the easy-plane polar phase of an antiferromagnetic spinor Bose-Einstein condensate. The separation distance of the pair and the magnitude of the core magnetization gradually saturate that is consistent with the short-range repulsive interactions between half-quantum vortices with opposite core magnetization. In our system, both the displacement of the vortex center and the magnetization along $x$ axis exhibit well-defined periodic oscillation, which is attributed to the long-range dipole-dipole interaction in the condensate.

In conclusion, we have investigated the collisional and magnetic field quench dynamics of the coupled spin vortex pair formed in a weakly interacting dipolar spinor BEC in a double well potential. We show that, in the absence of a magnetic field, the two sheets of the cloud collide with each other with their chiralities exchanged after each collision, when the potential barrier is turned off suddenly. Especially, the evolutions of the vortex pair in PSVP and ASVP phases can be easily distinguished from the density distribution images at half of the oscillation period $\tau/2$ where the in-phase and out-of-phase collisions differ markedly depending on the relative phase between the upper and lower layers. Motivated by the experimental studies of the dynamics of magnetic vortices in nanodisks or multilayer structures, we subsequently study the quench dynamics of the vortex in the ground states of a single- and double-layered condensate when the initial transverse field is removed quickly. The motion of the vortex center is found in a gyrotropic mode on account of the spin torque generated by quenching the transverse field. The displacement of the vortex center and the corresponding transverse magnetization are fitted with a damped simple harmonic oscillation motion with an intrinsic frequency and damping rate. These interesting dynamics provide an experimentally discernible signature of the PSVP and ASVP phases in the reach of current techniques of cold atom experiments. In future works we intend to develop the non-zero temperature theory to understand the effect of spin fluctuations.
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