The origin of the ankle
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(to appear in the CRIS 2006 proceedings)

The differential intensity of cosmic radiation shows a sequence of depressions referred to as knees in a large energy band above $10^{15}$ eV. The global depression entailed in the complete spectrum with respect to the extrapolated intensity based on low energy data amounts to a maximum factor of 8, occurring at $5 \times 10^{18}$ eV, where flux measurements exhibit a relative minimum, referred to as the ankle. It is demonstrated by a full simulation of cosmic ray trajectories in the Galaxy that the intensity minimum around the ankle energy is primarily due to the nuclear interactions of the cosmic ions with the interstellar matter and to the galactic magnetic field. Ankles signal the onset energies of the rectilinear propagation in the Milky Way at the Earth, being for example, $4 \times 10^{18}$ eV for helium and $6 \times 10^{19}$ eV for iron. The ankle, in spite of its notable importance at the Earth, is a local perturbation of the universal spectrum which, between the knee and the ankle, decreases by a round factor $10^9$, regaining its unperturbed status above $10^{19}$ eV.

1. INTRODUCTION

Energy measurements of giant air showers demonstrated the existence of a distinctive structure, the ankle, in the differential energy spectrum of the cosmic radiation above $10^{18}$ eV. Figure 1 shows the energy spectrum measured by four experiments [1,2,3,4] at these extreme energies. Cosmic ray intensity does not continue to decrease with a spectral index between 3 and 3.2, observed between $10^{16}$ and $10^{18}$ eV, but an enhancement of the intensity above $5 \times 10^{18}$ eV appears in all experiments. This enhancement is relative to the extrapolation at high energy with the index of 3 measured at lower energy, below $10^{18}$ eV. The ankle is the distinctive pattern in the spectrum consisting of a minimum of intensity followed by an enhancement observable approximately in the energy band $5 \times 10^{18}$-7 $\times 10^{19}$eV. The energy at which this enhancement occurs, and its magnitude, of this enhancement do not coincide in different experiments, though precise measurements of the minimum are available from the Hires Collaboration [4,5].

This contribution to the CRIS Conference 2006 follows recent studies [6,7] (hereafter Paper I and II, respectively) reporting a quantitative solution of the long-standing problem of the knee and ankle of the cosmic ray spectrum. A notable aspect of this solution is that the same mechanisms are responsible for the ankles and the knees of the individual ions. Even purposely it would be impossible to separate the explanation of the ankle from that of the knee.

The intrinsic mechanisms generating the ankles do not depend on the spectral indices of the cosmic rays at the sources nor on any extragalactic component of the cosmic radiation, but is an intrinsic property of the galactic cosmic rays (Sections 3 and 4). The extragalactic component, if any at the Earth around $10^{19}$ eV, might add to the galactic component, affecting the magnitude of the ankle, but its necessity and its experimental evidence remain unproved (Section 7). Note that the expected GZK depression or its absence [3] is beyond the energy band pertinent for the explanation of the ankles presented here.

2. COSMIC ION TRAJECTORIES

The method of calculation, used for over a decade, reconstructs ion trajectories by computer simulations and evaluates physical quantities of cosmic rays (intensities, energy spectra, ion ratios
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Figure 1. Cosmic ray spectra in the energy band around $10^{17}$ - $10^{20}$ eV from Haverah Park, Yakutsk, Agasa and Hires experiments. The two horizontal thin lines mark the discrepancy in intensity between the Agasa and Hires experiments.

etc) using the number of trajectories intercepting a given small volume in the Galaxy which in the present calculation can be the Earth (also local galactic zone or solar cavity), a sphere concentric to the Earth, or the entire disc volume (Section 9). The computational algorithms, described elsewhere [8,9,10] take into account the following astronomical, astrophysical and radioastronomy observations: (1) the spiral magnetic field; (2) the field strength (see figure 3 Paper I) of the spiral magnetic field; (3) a chaotic magnetic field with a field strength of about three times that of the regular field; (4) the form and the dimension of the Galaxy (see figure 1 in ref.[9]); (5) a uniform distribution of cosmic ray sources in the galactic disk (see eqn.(3) in ref.[9]); (6) the nuclear cross sections ion-hydrogen, $\sigma$; (7) the interstellar matter density in the disk, $d$, of 1.24 hydrogen atoms per cm$^3$; (8) the position of the solar cavity inside the disk, at 14 pc above the galactic midplane and 8.5 kpc from the galactic center; (9) the galactic wind (Section 2 in ref.[10]). Prerequisites of this study regarding the notion of galactic basin [11,10] have been previously discussed in detail.

A cosmic ion emanated from a source travels, on average, a mean length, $L$, in the disk volume, primarily determined by $\sigma$ and $d$. Typically, this global length $L$ is subdivided into thousands and thousands of segments, depending on the particular region of propagation and the energy of the cosmic ion. The ion propagation normal to the regular spiral field is generated by the chaotic field leading to a transverse-to-longitudinal displacement ratio compatible with the quasi-linear theory of ion propagation in an astrophysical environment. The chaotic or turbulent magnetic field is materialized by magnetic cloudlets (see, for instance, ref.[12,13]) with variable dimensions and a field strength three times greater than that of the regular field.

Trajectories at low energy are quite different from those at very high energy. Figures 2 and 3 display a typical low energy trajectory.

3. WHAT ARE THE KNEES OF THE INDIVIDUAL IONS

The differential energy spectrum of the cosmic rays, $dn/dE$, depends on the number of cosmic ray trajectories intercepting the local galactic zone, $n_g$. Figure 4 shows $n_g$ versus energy for helium, taken as an example. Three energy regions of $n_g$ are clearly distinguishable: (1) a high plateau; (2) a rapid descent; (3) a low plateau. The intensity gap between the high and the low plateau is of capital importance to comprehend the origin of the ankle. This gap depends on the matter thickness encountered by the cosmic helium in the Galaxy and on the nuclear cross section helium-hydrogen, $\sigma$(He).

The form of the spectrum shown in figure 4 is quite general, and simple mechanisms concur in its creation as clarified below.

In the energy band $10^{10}$-$10^{15}$ $n_g$ is approximately constant, though slightly decreasing. Forcing a power law interpolation on $n_g$ with a
constant spectral index, a value of 0.13 for helium results. This decreasing feature is due to rising cross sections as explained in detail in Paper I, Sections 3 and 4. In the high plateau region, the indices of \( n_g \) for different nuclides tend to decrease with the atomic number. An elegant derivation of this result is feasible by the notion of galactic basin [10,11].

The second portion of the spectrum, referred to as the knee, is due primarily to the break in the efficiency of the ion bending inherent the galactic magnetic field. This break has been analyzed in detail elsewhere (Paper I Section 4 and Paper II Section 5). The break energy is defined as that particular energy characteristic of a nuclear species, where the bend of \( n_g \) occurs and a sharp decrease dominates. Though the effect of the magnetic field on \( n_g \) is dominant, the exact form of \( n_g \) emerges only after including the position of the solar cavity, the nuclear cross sections and the disc size. The decrease of the cosmic ray intensity, or equivalently the descent in the bending efficiency and concomittant effects on \( n_g \) of the other parameters, cannot continue indefinitely as the energy increases above the break energies. In fact, the average value of the magnetic field strength is finite (say, 3 \( \mu G \)), and because of this finiteness, at some energy, for a specified ion, trajectories remain completely unbent, and consequently, the bending efficiency vanishes. This condition corresponds to the rectilinear propagation which is at the origin of the ankles as discussed in Section 8.

Figure 2. Example of an iron trajectory in the disc and its time evolution (vertical axis) during \( 3.5 \times 10^6 \) years. Note that the zig-zag, caused by the turbulent field, disappears for trajectories around and above \( 10^{18} \) eV, the rectilinear propagation region.

Figure 3. Side view of the same iron trajectory shown in figure 2, with a length of 334 098 pc and a straight-line distance between source and Earth of 671 pc. The star denotes the Fe source at \( x=5.404, y=2.826 \) and \( z=-0.148 \) kpc while the cross shows the Earth’s position.
4. POSTULATING CONSTANT SPECTRAL INDICES IN THE COSMIC RAY SPECTRUM

The existence of the ankle is incubated in the quantity \( n_g \) versus energy shown in figure 4. Subsequent analysis indicates how the low plateau in figure 4 transforms into a physical ankle accessible to experimentation. Figure 5 shows an idealization of the universal spectrum of the cosmic radiation between \( 10^{10} \) and \( 10^{21} \) eV. A notable feature of this spectrum is its linearity in logarithmic scales. The ankle is a small perturbation of the differential intensity. In fact the maximum difference between extrapolated and measured intensity is a factor 8 occurring around \( 5 \times 10^{18} \) eV (see figure 20 in Paper I and the data in ref.[4]), while in the knee-ankle energy range the differential intensity falls by a round factor \( 10^9 \), hence the term small perturbation. A similar conclusion is reached in ref.[15] where other empirical arguments are used. The remarkable linearity shown in figure 5 suggests that individual ions may also have approximate linearities. For logical coherence in the explanation of the ankle this linearity is converted here in a postulate. Notice that this postulate constitutes a useful ordo rerum in the experimental data below \( 10^{15} \) eV and above \( 10^{17} \) eV for a number of reasons [16]. Also, the quantitative account of the knee and the ankle given here would demand a much softer hypoth-
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Figure 6. Energy spectra of six nuclear species obtained by ideally assuming equal source powers of any galactic ions called here unphysical ion blend. This ideal spectra transform into real, physical spectra, shown in figure 7, once source abundances and spectral indices are considered.

The powers of the galactic sources are equal

Intensity (arbitrary units)

Energy (eV)

proton
He
N
Si
Ca
Fe

Figure 7. Computed energy spectra of individual ions according to the elemental abundances and spectral indices of the blend 3 of table 1. These curves are just one of the many examples of how the energy spectra given in figure 6 may be transformed by a particular ion blend.

The alteration of n_g caused by the spectral indices does not spoil the existence of the ankle already present in n_g versus energy as a low plateau (see figure 6). Nor could the particular values of the spectral indices of all ions influence the existence the ankle since the sum of the individual ion spectra, on average, should compensate, in order to recover imperatively, the linearity shown in figure 5. How the function n_g is altered by the particular spectral indices is illustrated in figure 7 for helium with an index of 2.72, and also for other ions.

5. COMPUTED AND MEASURED INDIVIDUAL KNEES

The indices and elemental abundances used in the present calculation are shown in Table 1, inspired from the results of some experiments (Kas-
Figure 8. Comparison between computed and measured proton spectrum. The computed He and Fe spectra with indices of 2.6 (He) and 2.5 (Fe) are also shown as a relevant grid of ion fluxes. The spectra normalization assumes flux equality for proton and iron at $10^{15.3} \text{ eV}$ and a $\text{He}/\text{Fe}$ flux ratio different from that of table 1.

The computed proton spectrum agrees fairly well with the measurements; taking a softer index (for example, 2.72 instead of 2.6) the agreement with the experimental data becomes excellent. The energy spectrum obtained by the QJSjet algorithms in Kascade gives a similar accord, since the resulting spectrum rigidly shifts in intensity. Note that this shift in intensity is absorbed in the normalization and the accord persists. The computed proton spectrum seems in disagreement with that measured by the TibetAS\(\gamma\) Collaboration [21] which has a lower energy bend and a softer descent compared to these calculations and to the Kascade data.

The computed helium and iron spectra also exhibit a good agreement with the Kascade and Eas-top data as discussed in detail in Paper I and II.

Let us remark here that by altering the values of the ion blends, within plausible empirical limits, the accord between computed and measured spectra persists.

### 6. THE COMPLETE SPECTRUM BETWEEN THE KNEE AND THE ANKLE

The complete spectrum of the cosmic radiation is the sum of the partial spectra of the in-

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**Table 1**

| Ion      | Blend 1 | Blend 2 |
|----------|---------|---------|
| Ion      | Comp. p. cent | Index | Comp. p. cent | Index |
| H        | 32.8    | 2.72    | 37.9    | 2.74    |
| He       | 29.7    | 2.72    | 27.4    | 2.72    |
| CNO      | 11.7    | 2.65    | 10.8    | 2.65    |
| Ne-S     | 10.0    | 2.65    | 9.3     | 2.65    |
| Fe (17-26) | 15.0 | 2.60    | 13.8    | 2.50    |
| Ca       | 0.8     | 2.60    | 0.8     | 2.60    |
| \(\gamma\) | 3.05    |         | 3.06    |         |
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dividual ions. Of course, it is assumed that electrons, positrons, antiprotons and other elementary particles have negligible fluxes compared to the global nuclide flux.

In figure 9, the computed energy spectra for the ion blend 2, in the interval $10^{10}$-$10^{21}$ eV, are given. The Tibet [22] and Haverah Park data [1] are reported in figure 9 while those of the Akeno and Agasa experiments [3] were shown previously (figure 12 Paper II). The computed spectrum is required to agree with the experimental data only at arbitrary energy point of $1.0 \times 10^{16}$ eV where the intensity ratio between Tibet and Kascade experiments is taken 1.4. Since no other constraint is imposed the overall accord is fairly good. The spectral index of 3.06 for this ion blend 2 in the energy interval $6.0 \times 10^{15}$-$10^{17}$ eV is in excellent agreement with the data of the quoted experiments and others not shown, to avoid confusion in data superposition. The ion blends 1 and 3 give 3.05 and 3.0, respectively, for the global spectral index $\gamma$.

7. GALACTIC AND EXTRAGALACTIC COMPONENTS AROUND AND ABOVE THE ANKLE ENERGY

The extragalactic component of cosmic rays at the Earth, if it exists, should cluster between $3.0 \times 10^{18}$ (proton ankle) and $5.0 \times 10^{19}$ eV (iron ankle). This fundamental result, derived elsewhere (Paper II, Section 7), is almost independent of the method of calculation. Taking advantage of this outcome two components of the cosmic ray flux at the Earth may be disentangled: galactic and extragalactic. The distinction in two components rests on the initial sites (Milky Way or outside) of the acceleration regions once secondaries (secondary protons, $^{3}$He, Be, B, etc.) generated in the wakes of the primaries may be neglected. The term extragalactic would denote a number of possibilities: (a) debris of normal galaxies in the cosmic vicinity; (b) reentrant particles overflowing from the Milky Way; (c) debris of peculiar powerful galaxies; (d) extragalactic particles of very high energy accelerated in space between galaxies. For the case (b) the more appropriate term extradisc cosmic rays is used.

The ratio between the galactic and extragalactic intensities at Earth, $I_g/I_e$, in the band $3.0 \times 10^{18}$ to $5.0 \times 10^{19}$ eV depends on the ion blend adopted for the galactic component. Detailed calculations with the blend 2 (Paper II Section 6) give the energy spectrum of the extradisc component, peaking around $10^{19}$ eV, as shown in figure 9. At this particular energy, the difference between the measured intensity and the computed galactic intensity is regarded, by definition, as an extradisc component. For the data shown in figure 9 the ratio $I_g/I_e$ is 2.7 while the Agasa data with the same blend 2 give $I_g/I_e=1.6$.

![Figure 9. Comparison between computed and measured spectra for the blend 2 given in table 1 and the data from the Tibet and Haverah Park experiments covering the energy range $10^{14}$-$10^{20}$ eV.](image-url)
8. THE RECTILINEAR PROPAGATION AND THE ION ANKLES

The galactic magnetic field is the basic element (though insufficient) for the comprehension of the processes generating the ankle. The form and strength of the galactic magnetic field with its intrinsic turbulence are known with relatively high precision, adequate for this study. The adequacy is justified elsewhere (Paper II Appendix B). The average magnetic field strength fixes two milestones along the energy axis related to the alteration of the ion trajectories in the Milky Way: the ankles occur at the energies where the rectilinear propagation of cosmic ions commences, while the knees are placed at energies where the ion bending efficiency departs from an approximately uniform distribution, typical at lower energies. By a uniform bending is meant an approximately isotropic distribution in the ion directions in the vicinity of the source. The average distances travelled by cosmic ions in the Milky Way (source vicinity) have been previously calculated [10].

One should notice that the quantity \( n_g \) versus energy changes by more than two orders of magnitude from the low to the high plateau (see figure 4). This computed gap is close to physical reality because of the accord with the data displayed in figures 8 and 9. The upper limits of the measured anisotropies in the arrival direction of cosmic rays at the Earth around the knee energy are about \( 10^{-3} \), and recent measurements [23,24] indicate the persistence of this figure even above \( 10^{18} \) eV [5]. Consequently, the intrinsic mechanisms producing the knee dominate by several orders of magnitude those producing anisotropies. It follows that the accuracy required for knee and ankle calculations is, by far, less demanding that that required for anisotropy calculations.

For a given magnetic field configuration in the Galaxy, the rectilinear propagation of an ion occurs at a distinctive, particular energy. Ion bending expressed by the curvature radius \( R \) in a small volume permeated by a uniform constant field obeys the equation \( R = p/QB \), where \( p \) is the ion momentum, \( Q \) the charge and the \( B \) the average field strength in the region where the ion propa-

![Figure 10. Helium grammage versus energy in the galactic disc. The arrow indicates the particular minimum energy of \( 4 \times 10^{18} \) eV at which the grammage attains its asymptotic value. The shape of the grammage agrees with that inferred (via residence time) from measurements of cosmic ray anisotropy versus energy [25] and it disagrees with the unphysical, erroneous grammage extrapolated from \( B/C \) flux ratio and other secondary-to-primary ratios measured at very low energy.](image-url)
the exact determination of the knee energies with the above equation. To each ion corresponds a galactic basin around the Earth with an intrinsic dimension [10,11] dependent on the ion, and the correct estimate of the energy threshold of the rectilinear propagation requires the position of the solar cavity, the disc size and the nuclear cross sections.

The mechanisms generating the ankles are the same as for the knees, but the physical conditions at the ankle energy region are less intricate.

9. THE NUCLEAR CROSS SECTIONS, ION ANKLES AND THE ANKLE

Dissecting the phenomena which concur in the formation of the ankle and individual ankles we are finally left with the role of the nuclear cross sections. The results reported in figures 4, 6 and 7, along with those shown in the figure 11, suffice to highlight this role. It is obvious from the spectra shown in figure 7 how the function $n_g$, appearing in figure 4, contributes to the formation of the ankle (the spectral indices are incorporated in figure 7). Imagine an ideal, unreal situation where the gap between the low and high plateau is close to zero. Neither a change of slope in the spectrum above $6.0 \times 10^{15}$ eV nor the ankle would have been observed. In this circumstance, the universal spectrum would have been entirely bound and regulated by the spectral indices, unaffected by this unreal compression of $n_g$, with no ankles at all. On the contrary, with a finite gap between the low and high plateau, the appearance of the ankle is a necessity.

The position and magnitude of the ankle and its necessity may be further appreciated by taking into account: (1) the difference between the high and low plateau for the same ion; (2) the difference between the low and high plateau in different ions; (3) the difference between low and high plateau for extragalactic cosmic rays.

The high plateau of any ion is fixed by the average matter thickness (grammage) encountered by galactic cosmic rays and by $\sigma$. In the energy region of the low plateau the physical conditions present in the high plateau region are unchanged, except for lower grammages (see figure 10) which ultimately imply low $n_g$ values. To elucidate the point (2) let us consider two different ions: He and Fe, for example. Since $\sigma$(Fe) is higher than $\sigma$(He), galactic iron suffers severe losses in the disc with respect to He losses. Therefore $n_g$ for iron becomes smaller than that for helium, a result displayed in figure 6; similarly for other ion pairs with different $\sigma$. The low plateau of iron is similar to that of helium, and all low plateaux are almost equal for all ions (see figure 6). This is simply due to the exponential function governing nuclear collisions via the attenuation length $\lambda =$
\( A/\sigma dN_A \). Since \( d \) is extremely small, less than 0.008 g/cm\(^2\), the difference in the product \( \sigma d \) for different ions is quite modest and the quantities \( n_g \) for different ions cluster around a minimum value (see figure 6). By this argument it follows that the proton gap must be higher than that of helium and the helium gap higher than that of iron etc.

A quite different result pertains to the extragalactic component for which the grammage is higher than that of the galactic component and the quantities \( n_g \) in the energy band of the low plateau largely differ. Above the energy of the rectilinear propagation the relative intensities of all extradisc ions have to be in the same ratios as do nuclear cross sections: \( \sigma(p)/\sigma(\text{He}) \), \( \sigma(\text{Fe})/\sigma(\text{He}) \) etc. This aspect is beautifully displayed in figure 11 above \( 10^{18} \text{ eV} \) for He, Fe and protons using the number of nuclear collisions in the disc instead of the companion variable \( n_g \).

The origin of the ankle described here may be further appreciated in comparison with other recent studies [26] where the ankle is a mere effect caused by the extragalactic component, introduced ad hoc.

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