Study on the possible molecular state composed of $D_{s}^{*}\bar{D}_{s1}$ within the Bethe-Salpeter framework

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Abstract

Recently a vector charmonium-like state $Y(4626)$ was observed in the portal of $D_{s}^{+}\bar{D}_{s1}(2536)^{-}$. It intrigues an active discussion on the structure of the resonance because it has obvious significance for gaining a better understanding on its hadronic structure with suitable inner constituents. It indeed concerns the general theoretical framework about possible structures of exotic states. Since the mass of $Y(4626)$ is slightly above the production threshold of $D_{s}^{+}\bar{D}_{s1}(2536)^{-}$ whereas below that of $D_{s}^{*}\bar{D}_{s1}(2536)$ with the same quark contents as that of $D_{s}^{+}\bar{D}_{s1}(2536)^{-}$, it is natural to conjecture $Y(4626)$ to be a molecular state of $D_{s}^{*}\bar{D}_{s1}(2536)$, as suggested in literature. Confirming or negating this allegation would shed light on the goal we concern. We calculate the mass spectrum of a system composed of a vector meson and an axial vector i.e. $D_{s}^{*}\bar{D}_{s1}(2536)$ within the framework of the Bethe-Salpeter equations. Our numerical results show that the dimensionless parameter $\lambda$ in the form factor which is phenomenologically introduced to every vertex, is far beyond the reasonable range for inducing an even very small binding energy $\Delta E$. It implies that the $D_{s}^{*}\bar{D}_{s1}(2536)$ system cannot exist in the nature as a hadronic molecule in this model, so that we may not think the resonance $Y(4626)$ to be a bound state of $D_{s}^{*}\bar{D}_{s1}(2536)$, but something else, for example a tetraquark and etc.

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I. INTRODUCTION

In 2019 the Belle Collaboration observed a vector charmonium-like state \( Y(4626) \) in the portal of \( e^+e^- \rightarrow D_{s1}^+(2536)^- + c.c. \) and its mass and width are \( 4625.9^{+6.2}_{-5.9}(\text{stat.}) \pm 0.4(\text{syst.}) \) MeV and \( 49.8^{+13.9}_{-11.5}(\text{stat.}) \pm 4.0(\text{syst.}) \) MeV \[1\]. In 2008 the Belle Collaboration reported a near-threshold enhancement in the \( e^+e^- \rightarrow \Lambda_c^+\Lambda_c^- \) cross section and the peak corresponds to a hadronic resonance which is named as \( Y(4630) \)[2]. Recently a simultaneous fit was performed to the data analysis of \( e^+e^- \rightarrow \Lambda_c^+\Lambda_c^- \) and a peak with mass and width being \( 4636.1^{+9.8}_{-7.2}(\text{stat.}) \pm 8.0(\text{syst.}) \) MeV and \( 34.5^{+21.0}_{-16.2}(\text{stat.}) \pm 5.6(\text{syst.}) \) MeV\[3\] emerges. Due to their very close masses and widths, it is tempted to consider \( Y(4626) \) and \( Y(4630) \) as the same resonance. In Ref.\[4\] the authors explained \( Y(4626) \) and \( Y(4660) \) to be mixtures of two excited charmonia, whereas the authors\[5\] suggested \( Y(4626) \) as a molecular state \( D_s^*\bar{D}_{s1}(2536) \). In Ref.\[6, 7, 8\] \( Y(4626) \) was regarded as a tetraquark \( cs\bar{c}\bar{s} \).

Since 2003 many exotic resonances \( X, Y \) and \( Z \) bosons\[9–18\] have been experimentally observed, such as \( X(3872), X(3940), Y(3940), Z(4430), Y(4260), Z_c(4020), Z_c(3900), Z_b(10510) \) and \( Z_b(10650) \) (of course, not a complete list). The states have attracted attention of theorists because their structures obviously are beyond the simple \( q\bar{q} \) settings for mesons. If we can firmly determine their compositions, it would definitely enrich our knowledge on hadron structures and moreover shed light on the non-perturbative QCD effects at lower energy ranges. Studies with different explanations on the inner structures\[19\] have been tried, such as molecular state, tetraquark or dynamical effect\[20\]. Anyway, all the ansatz have a certain reasonability, but a unique picture or criterion for firmly determining the inner structures is still lacking. Nowadays, the majority of phenomenological researchers conjectures what the concerned exotic states are made of, just based on the available experimental data. Then by comparing the results with new data one can check the validity degree of the proposal. If the results obviously contradict to the new measurements of better accuracy, the ansatz should be abandoned. Following this principle we explore \( Y(4626) \) by assuming it to be a molecular state of \( D_s^*\bar{D}_{s1}(2536) \), and then using more reliable theoretical framework to check the scenario and see if the proposal from our intuition can be valid.

Concretely, in this work supposing \( Y(4626) \) as a \( D_s^*\bar{D}_{s1}(2536) \) molecular state, we employ the Bethe-Salpeter (B-S) equation which is a relativistic equation established on the basis of quantum field theory, to study the two-body bound state \[21\]. Initially, the B-S equation was used to study the bound state of two fermions\[22, 24\], later the method was generalized to the system of one-fermion-one-boson\[25\]. In Ref.\[26, 27\] the authors employed the Bethe-Salpeter equation to study some possible molecular states, such as \( K\bar{K} \) and \( B\bar{K} \) system. With the same approach the bound state of \( B\pi, D^{(*)}D^{(*)}, B^{(*)}B^{(*)} \) are studied\[28, 29\]. Recently the approach was applied to explore doubly charmed baryons\[30, 31\] and pentaquarks\[32, 33\]. In this work, we try to calculate the spectrum of \( Y(4626) \) composed of a vector meson and an axial vector meson.

If two constituents can form a bound state the interaction between them should be large enough to hold them into a bound state. The chiral perturbation theory tells us that two hadrons interact via exchanging a certain mediate meson(s) and the forms of the effective
vertices are determined by relevant symmetries, but the coupling constants generally are obtained by fitting data. For the molecular states, since two constituents are color-singlet hadrons the exchanged particles are some light mesons with proper quantum numbers. It is noted that even though there are many possible light mesons contributing to the effective interaction between the two constituents, generally one or several of them would provide the dominant contributions. Moreover, beyond it, most of time the scenario with some other mesons exchange should also be taken into account, because even though the extra contributions are small comparing to the dominant one(s), they sometimes are not negligible, namely it would make the secondary contribution to the effective interaction. Then the effective kernel for the B-S equation is set. For the $D_s^*\bar{D}_{s1}(2536)$ system, the contribution of $\eta$ \cite{34,36} dominates, whereas in Ref. \cite{34} the authors suggested $\sigma$ exchange makes the secondary contribution. In our case considering the concerned quark contents of $D_s^*$ and $D_{s1}(2536)$, the contribution of $f_0(980)$ should stand as the secondary one. The effective interactions induced by exchanging $\eta$ are deduced with the heavy quark symmetry\cite{34,36} and we present the formulas in the appendix A. With the effective interactions we can derive the kernel and establish the corresponding B-S equation.

With all necessary parameters being beforehand chosen and input, the B-S equation is solved numerically. In some cases the equation does not possess a solution if one or several parameters are set within a reasonable range, then a conclusion must be drawn that the proposed bound state should not exist in nature. On the contraries, a solution of the B-S equation with reasonable parameters implies that the corresponding bound state is formed. In that case, simultaneously the B-S wave function is obtained which can be used to calculate the rates of strong decays, which will help experimentalist to design new experiments for further measurements.

This paper is organized as follows: after this introduction we will derive the B-S equation related to possible bound state composed of $D_s^*$ and $\bar{D}_{s1}(2536)$ which are a vector and an axial vector meson respectively. In section III the formula for its strong decays are present. Then in section IV we will solve the B-S equation numerically. Since $Y(4626)$ is supposed to be a molecular bound state, the input parameters must be within a reasonable range, but our results say that this mandatory condition cannot be satisfied, thus we think that such a molecular state of $D_s^*\bar{D}_{s1}(2536)$ may not exist. However, as we deliberately set the parameters to a region which is not favored by all previous phenomenological works, we can obtain the required spectrum and corresponding wavefunctions. With the wavefunction we evaluate the strong decay rate of $Y(4626)$ and present our results by figures and tables. Section IV is devoted to a brief summary.

II. THE BOUND STATES OF $D_s^*\bar{D}_{s1}$

Since the newly observed resonance $Y(4626)$ contains hidden charms and its mass is close to the sum of the masses of $D_s^*$ and $\bar{D}_{s1}$ where $D_s^* - \bar{D}_{s1}$ corresponds to $D_s^{*+} - D_{s1}^-$ or $D_s^{*-} - D_{s1}^+$, a conjecture about its molecular structures composed of $D_s^*$ and $D_{s1}$ is favored. For a state with spin-parity being $1^-$, its spatial wave function is in $S$ wave. There are two...
possible states $Y_1 = \frac{1}{\sqrt{2}}(D_s^+D_{s1}^- + D_s^-D_{s1}^+)$ and $Y_2 = \frac{1}{\sqrt{2}}(D_s^+D_{s1}^- - D_s^-D_{s1}^+)$. We will focus on such an ansatz and try to find numerical results by solving the relevant B-S equation.

A. The Bethe-Salpeter (B-S) equation for $1^- D_s^s\bar{D}_{s1}$ molecular state

By the effective theory $D_s^s$ and $\bar{D}_{s1}$ interact mainly via exchanging $\eta$. The Feynman diagram at the leading order is depicted in Fig. 1. To take into account the secondary contribution induced by exchanging other mediate mesons, in Ref. [34] the authors consider a contribution of exchanging $\sigma$ to the effective interaction. Since there are neither $u$ nor $d$ constituents in $D_s^s$ and $\bar{D}_{s1}$, their coupling to $\sigma$ would be very weak, thus the secondary contribution to the interaction may come from exchanging $f_0(980)$ instead. The relevant Feynman diagrams are shown in Fig. 2. The relations between relative and total momenta of the bound state are defined as

$$p = \eta_2 p_1 - \eta_1 p_2, \quad q = \eta_2 q_1 - \eta_1 q_2, \quad P = p_1 + p_2 = q_1 + q_2,$$

where $p_1$ and $p_2$ ($q_1$ and $q_2$) are the momenta of the constituents, $p$ and $q$ are the relative momenta between the two constituents of the bound state at the both sides of the diagram, $P$ is the total momentum of the resonance, $\eta_i = m_i/(m_1 + m_2)$ and $m_i$ ($i = 1, 2$) is the mass of the $i$-th constituent meson. $k$ is the momentum of the exchanged mediator.
A detailed analysis on the Lorentz structure \cite{24,26,27} determines the form of the B-S wave function of the bound state comprising a vector and an axial vector mesons (\(D_s^*\) and \(\bar{D}_{s1}\)) in \(S\)-wave as

\[
\langle 0|T\phi_a(x_1)\phi_b(x_2)|V\rangle = \frac{\varepsilon_{abcd}}{\sqrt{6}M}\chi^d_p(x_1, x_2)P^c,
\]  

where \(a, b, c\) and \(d\) are Lorentz indices. The wave function in the momentum space can be obtained by carrying out a Fourier transformation

\[
\chi^a_p(p_1, p_2) = \int d^4x_1d^4x_2e^{ip_1x_1+ip_2x_2}\chi^a_p(x_1, x_2) = (2\pi)^4\delta(p_1 + p_2 + P)\chi^a_p(p).
\]

Using the so-called ladder approximation one can get the B-S equation deduced in earlier references\cite{21,23}

\[
\varepsilon_{abcd}\chi^d_p(p)P^c = \Delta_{1aa}\int \frac{d^4q}{(2\pi)^4} K^{\alpha\beta\mu\nu}(P, p, q)\varepsilon_{\mu\nu\sigma\omega}\chi^a_p(q)P^\omega \Delta_{2\beta}\,.
\]

where \(\Delta_{1aa}\) and \(\Delta_{2\beta}\) are the propagators of \(D_s^*\) and \(\bar{D}_{s1}\) respectively, \(K^{\alpha\beta\mu\nu}(P, p, q)\) is the kernel determined by the effective interaction between two constituents which can be calculated from the Feynman diagrams in Fig. 1 and 2. In order to solve the B-S equation, we decompose the relative momentum \(p\) into the longitudinal component \(p_l (= p \cdot v)\) and the transverse one \(p_t^{\mu} (= p^{\mu} - p_tv^\mu) = (0, \mathbf{p}_T)\) with respect to the momentum of the bound state \(P\) (\(P = Mv\)).

\[
\Delta^{\alpha\alpha}_1 = \frac{i[-g^{\alpha\alpha} + (1 - \xi)p_l^\alpha p_l^\alpha/(p_l^2 - \xi m_l^2)]}{(\eta_1M + p_l + \omega_t - i\epsilon)(\eta_1M + p_l - \omega_t + i\epsilon)},
\]

\[
\Delta^{\beta\beta}_2 = \frac{i[-g^{\alpha\alpha} + (1 - \xi)p_l^\beta p_l^\beta/(p_l^2 - \xi m_l^2)]}{(\eta_2M - p_l + \omega_t - i\epsilon)(\eta_2M - p_l - \omega_t + i\epsilon)},
\]

where \(M\) is the mass of the bound state \(Y(4626)\), \(\omega_t = \sqrt{p_t^2 + m_t^2}\) and \(\xi\) is the parameter in the gauge fixing. We set \(\xi\) to be 1 in our calculation.

By the Feynman diagrams shown in Fig. 1 and 2 the kernel \(K^{\alpha\beta\mu\nu}(P, p, q)\) is written as

\[
K^{\alpha\beta\mu\nu}(P, p, q) = C_1g_{D_{s1}D_{s1}^*}g_{D_{s1}D_{s1}^*}^\gamma(\sqrt{6}k^{\mu}k^{\alpha} - \sqrt{3}k^2g^{\mu\alpha} + \sqrt{3}k \cdot p_l k \cdot q_1 g^{\mu\alpha}/m_1/m_1')
\]

\[
(\sqrt{6}k^{\nu}k^{\nu} - \sqrt{3}k^2g^{\nu\nu} + \sqrt{3}k \cdot p_l k \cdot q_1 g^{\nu\nu}/m_2/m_2')\Delta(k, m_n)F^2(k, m_n)
\]

\[
-\frac{2}{3}C_2g_{D_{s1}D_{s1}^*}g_{D_{s1}D_{s1}^*}g^{\sigma\mu\alpha\omega}k_0 p_{1\omega} + q_{1\omega} g^{\sigma\nu\beta\rho}k_0 p_{2\rho} + q_{2\rho}\Delta(k, m_n)F^2(k, m_n)
\]

\[
+\frac{2}{3}C_3g_{D_{s1}D_{s1}^*}g_{D_{s1}D_{s1}^*}g^{\mu\alpha\nu\delta}k_0 \Delta(k, m_n)F^2(k, m_n)
\]
where $m_s$ is the mass of the exchanged meson, $C_1=1$ for $Y_1$ and -1 for $Y_2$, $C_2=1$, $C_3=1$, $g_{D_{s1}^*D_{s1}^*}$, $g_{D_{s1}^*D_{s1}^*}$, $g_{D_{s1}^*D_{s1}^*}$ and $g_{D_{s1}^*D_{s1}^*}$ are the concerned coupling constants and $\Delta(k, m_s) = i/(k^2 - m_s^2)$.

Since the two constituents of the molecular state are not on shell, at each interaction vertex a form factor should be introduced to compensate the off-shell effect. The form factor is employed in many references [37–40], even though it has different forms. Here we set it as:

$$F(k, m_s) = \frac{\Lambda^2 - m_s^2}{\Lambda^2 + k^2}, \quad (8)$$

where $k$ is the three-momentum of the exchanged meson and $\Lambda$ is a cutoff parameter. Indeed, the form factor is introduced phenomenologically and there lacks any reliable knowledge on the value of the cutoff parameter $\Lambda$. $\Lambda$ is often parameterized to be $\Lambda_{QCD} + m_s$ with $\Lambda_{QCD} = 220$ MeV which is adopted in some references [37–40]. As suggested, the order of magnitude of the dimensionless parameter $\lambda$ should be close to 1. In our later numerical computations, we set it to be within a wider range of $0 \sim 4$.

The wave function can be written as

$$\chi^d_P(p) = f(p)e^d,$$ \quad (9)

where $\epsilon$ is the polarization vector of the bound state and $f(p)$ is the radial wave function. The three-dimension spatial wave function is obtained after integrating over $p_l$

$$f(|Pr|) = \int \frac{dp_l}{2\pi} f(p). \quad (10)$$

Substituting Eqs. (8) and (9) into Eq. (11) and multiplying $\varepsilon_{abfg}x^a_f(x_1, x_2)P^f$ on both sides one can sum over the polarizations of both sides. Employing the so-called covariant instantaneous approximation $q_l = p_l$ i.e. using $p_l$ to replace $q_l$ in $K(P, p)$, the kernel $K(P, p, q)$ does not depend on $q_l$ any longer. Then we take a typical procedure: integrating over $q_l$ on the right side of Eq. (11), multiplying $\int \frac{dp_l}{(2\pi)}$ on both sides of Eq. (11), and integrating over $p_l$ on the left side, to reduce the expression into a compact form. Finally we obtain

$$6M^2 f(|pr|) = \int \frac{dp_l}{(2\pi)} \int \frac{d^3q_T}{(2\pi)^3} \left[ \frac{f(|q_T|)}{((\eta_1 M - p_l)^2 - \omega^2 + i\epsilon)[((\eta_1 M - p_l)^2 - \omega^2 + i\epsilon)]} \right]$$

$$[C_1 g_{D_{s1}^*D_{s1}^*} F^2(k, m_s) + C_1 \frac{p_T \cdot q_T + C_2(p_T \cdot q_T)^2 + C_3(p_T \cdot q_T)^3 + C_4(p_T \cdot q_T)^4}{(p_T - q_T)^2 - m_s^2}$$

$$-C_2 g_{D_{s1}^*D_{s1}^*} F^2(k, m_s) - C_0 \frac{p_T \cdot q_T + C_2(p_T \cdot q_T)^2 + F^2(k, m_{f_0})}{(p_T - q_T)^2 - m_s^2} - 6M^2 \frac{C_0 g_{D_{s1}^*D_{s1}^*} g_{D_{s1}^*D_{s1}^*}}{(p_T - q_T)^2 - m_s^2}], \quad (11)$$

with

$$C_0 = \frac{4M^2}{m_s^2 m_{f_0}^2 + m_s^2(m_s^2 + (p_T^2 + q_T^2)^2)}.$$
\[ C_1 = \frac{8M^2 (p_T^2 + q_T^2) (-2m_1^2m_2^2 + p_T^2q_T^2)}{m_1^2m_2^2}, \]
\[ C_2 = \frac{4M^2 (4m_1^2m_2^2 - p_T^4 - 4p_T^2q_T^2 - q_T^4)}{m_1^2m_2^2}, \]
\[ C_3 = \frac{8M^2 (p_T^2 + q_T^2)}{m_1^2m_2^2}, \]
\[ C_4 = \frac{-4M^2}{m_1^2m_2^2}, \]
\[ C_0 = \frac{-16M^2 (\eta_2 M - p_i) (\eta_1 M + p_i) (p_T^2 + q_T^2)}{3m_1m_2}, \]
\[ C_1 = \frac{32M^2 (\eta_2 M - p_i) (\eta_1 M + p_i)}{3m_1m_2}. \]

While we integrate over \( p_i \) on the right side of Eq. (11) there exist four poles which are located at \(-\eta_1 M - \omega_1 + i\epsilon, -\eta_1 M + \omega_1 - i\epsilon, \eta_2 M + \omega_2 - i\epsilon \) and \(\eta_2 M - \omega_2 + i\epsilon\). By choosing an appropriate contour we only need to evaluate the residuals at \( p_i = -\eta_1 M - \omega_1 + i\epsilon \) and \( p_i = \eta_2 M - \omega_2 + i\epsilon \).

Here \( d^3q_T = q_T^2 \sin(\theta) d|q_T| d\theta d\phi \) and \( p_T \cdot q_T = |p_T||q_T|\cos(\theta) \), one can integrate out the azimuthal part and then Eq. (11) is reduced into a one-dimensional integral equation

\[
 f(|p_T|) = \frac{|q_T|^2 f(|q_T|)d|q_T|}{12M^2(2\pi)^2} \left\{ \frac{C_1 g_{D_{31}D_{31}^{*}} g_{D_{31}D_{31}^{*}} (\omega_1 + \omega_2)}{\omega_1\omega_2[M^2 - (\omega_1 + \omega_2)^2]} (C_0 J_0 + C_1 |p_T||q_T|J_1) 
+ C_2 |p_T|^2 |q_T|^2 J_2 + C_3 |p_T|^3 |q_T|^3 J_3 + C_4 |p_T|^4 |q_T|^4 J_4 \right\} 
- \frac{C_2 g_{D_{31}D_{31}^{*}} g_{D_{31}D_{31}^{*}}}{\omega_1[(M + \omega_1)^2 - \omega_2^2]} (C_0' J_0 + C_1' |p_T||q_T|J_1)|_{p_i = -\eta_1 M - \omega_1} 
- \frac{g_{D_{31}D_{31}^{*}} g_{D_{31}D_{31}^{*}}}{\omega_2[(M - \omega_2)^2 - \omega_1^2]} (C_0' J_0 + C_1' |p_T||q_T|J_1)|_{p_i = \eta_2 M - \omega_2} 
\frac{C_3 g_{D_{31}D_{31}^{*}} g_{D_{31}D_{31}^{*}}}{\omega_1\omega_2[M^2 - (\omega_1 + \omega_2)^2]} 6M^2 J_0',
\]

with

\[
 J_0 = \int_0^\pi \frac{\sin \theta d\theta}{-(p_T - q_T)^2 - m_\eta^2} F^2(k, m_\eta), \quad J_1 = \int_0^\pi \frac{\sin \theta \cos \theta d\theta}{-(p_T - q_T)^2 - m_\eta^2} F^2(k, m_\eta),
\]
\[
 J_2 = \int_0^\pi \frac{\sin \theta \cos^2 \theta d\theta}{-(p_T - q_T)^2 - m_\eta^2} F^2(k, m_\eta), \quad J_3 = \int_0^\pi \frac{\sin \theta \cos^3 \theta d\theta}{-(p_T - q_T)^2 - m_\eta^2} F^2(k, m_\eta),
\]
\[
 J_4 = \int_0^\pi \frac{\sin \theta \cos^4 \theta d\theta}{-(p_T - q_T)^2 - m_\eta^2} F^2(k, m_\eta), \quad J_0' = \int_0^\pi \frac{\sin \theta d\theta}{-(p_T - q_T)^2 - m_{f_0}^2} F^2(k, m_{f_0}).
\]
B. Normalization condition for the B-S wave function

In analogy to the cases in Refs. [26, 27], the normalization condition for the B-S wave function of a bound state should be

$$\frac{i}{6} \int \frac{d^4 p d^4 q}{(2\pi)^8} \varepsilon_{abcd} \varepsilon_p \frac{P_c}{M} \frac{\partial}{\partial P_0} \left[ I^{\alpha \beta}(P, p, q) + K^{\alpha \beta}(P, p, q) \right] \varepsilon_{\alpha \beta \mu \nu} \varepsilon_p = \frac{P_0}{M} = 1,$$

where $P_0$ is the energy of the bound state which is equal to its mass $M$ in the center of mass frame. $I(P, p, q)$ is a product of reciprocals of two free propagators with a proper weight.

$$I^{\alpha \beta}(P, p, q) = \frac{\delta^4(p - q)}{(2\pi)^4} \left( \Delta^{\alpha}_{1} \right)^{-1} \left( \Delta^{\beta}_{2} \right)^{-1}.$$

(14)

In our earlier work [29], we found that the term $K^{\alpha \beta}(P, p, q)$ in brackets is negligible, so that now we ignore it as done in Ref. [41].

With $\left( \Delta^{\alpha}_{1} \right)^{-1} = -i g^{a\alpha} (p_1^2 + m_1^2)$, $\left( \Delta^{\beta}_{2} \right)^{-1} = -i g^{b\beta} (p_2^2 + m_2^2)$ and $\varepsilon^d \varepsilon^\nu = -g^{d\nu} + P^d P^\nu / M^2$ the normalization condition is

$$i \int \frac{d^4 p d^4 q}{(2\pi)^8} f^*(p) \frac{\partial}{\partial P_0} \frac{\delta^4(p - q)}{(2\pi)^4} \left( p_1^2 + m_1^2 \right) \left( p_2^2 + m_2^2 \right) f(q) = 1.$$

(15)

After performing some manipulations we obtain the normalization of the radial wave function

$$\frac{1}{\pi^2} \int \frac{d^3 P_T}{(2\pi)^3} f^2(|P_T|) \frac{M \omega_1 \omega_2}{\omega_1 + \omega_2} = 1.$$

(16)

III. THE STRONG DECAYS OF THE MOLECULAR STATE $Y(4626)$

Now we investigate the strong decays of $Y(4626)$ using the effective interactions.

A. Decay to $D_s^*(1^-) + \bar{D}_{s0}(2317)(0^+)$

The relevant Feynman diagram is depicted in Fig. 3 (a) where $\bar{D}_{s0}$ represents $\bar{D}_{s0}(2317)$. The amplitude is,

$$\mathcal{A}_a = g_{D_s D_s^*} g_{D_{s0} D_{s0}} \int \frac{d^4 p}{(2\pi)^4} \frac{2}{3} k_0 \varepsilon_{1\mu} \varepsilon^{\nu \alpha \beta} \left( \frac{P_{1\beta}}{m_1} + \frac{q_{1\beta}}{m_1'} \right) \chi^{d}_{\nu} \frac{P_c}{M} k_0 \Delta(k, m_s) P^2(k, m_s),$$

(17)

where $k = p - (\eta_2 q_1 - \eta_1 q_2)$, $\varepsilon_1$ is the polarization vector of $D_{s}^*$. We still take the approximation $k_0 = 0$ to carry out the calculation.

The amplitude can be parameterized as [42]

$$\mathcal{A}_a = g_0 M \varepsilon_1 \cdot \varepsilon^* + \frac{g_2}{M} \left( q \cdot \varepsilon_1 q \cdot \varepsilon^* - \frac{1}{3} q^2 \varepsilon_1 \cdot \varepsilon^* \right).$$

(18)

The factors $g_0$ and $g_2$ are extracted from the expressions of $\mathcal{A}_a$.

Then the partial width is expressed as

$$d\Gamma_a = \frac{1}{32 \pi^3} |\mathcal{A}_a|^2 \frac{|q_2|^2}{M^2} d\Omega.$$

(19)
B. Decay to $D_s(0^-) + D_s(2460)(1^+)$

The corresponding Feynman diagram is depicted in Fig. 3(b) where $D_{s1}'$ denotes $D_s(2460)$ through the whole paper. The amplitudes is

$$\mathcal{A}_b = g_{D^*_s D_s} g_{D_{s1} D_s(2460)} \int \frac{d^4 p}{(2\pi)^4} \frac{2}{3} k^a \bar{\chi}^d(p) \varepsilon_{abcd} \frac{P^c}{M} \epsilon_{2\mu} \epsilon^{\nu\mu\lambda\omega} (\frac{p_{1\omega}}{m_1} + \frac{q_{1\omega}}{m_1^*}) k_{\nu} \Delta(k, m_s) F^2(k, m_s).$$

The amplitude can also be parameterized as

$$\mathcal{A}_b = g'_0 M \epsilon_2 \cdot \epsilon^* + \frac{g'_2}{M} (q \cdot \epsilon_2 q \cdot \epsilon^* - \frac{1}{3} q^2 \epsilon_2 \cdot \epsilon^*),$$

where $\epsilon_2$ is the polarizations of $D_{s1}'(2460)$. The factors $g'_0$ and $g'_2$ can be extracted from the expressions of $\mathcal{A}_b$.  

FIG. 3: The decays of $Y(4626)$ by exchanging $\eta$.  

9
C. Decay to $D_s(2460)(1^+)$ + $\bar{D}_s^*(1^-)$

The Feynman diagram for the process of $Y(4626) \to D_s(2460)(1^+) + \bar{D}_s^*(1^-)$ is depicted in Fig. 3(c). The amplitudes is

$$\mathcal{A}_c = g_{D_s^*D_s(2460)}^2g_{D_sD_s(2460)}\int \frac{d^4p}{(2\pi)^4} \frac{2}{3i\epsilon_1} k_\omega \left( \frac{p_1^\gamma}{m_1} + \frac{q_1^\gamma}{m_1'} \right) \epsilon_1^d(p) \epsilon_{abcd}^c \frac{P^c}{M} \left( -3k^b \frac{k}{v} + k^2 g^{b\nu} - k \cdot p_2 k \cdot q_2 g^{b\nu}/m_2/m_2' \right) \epsilon_2 \Delta(k, m_s) F^2(k, m_s),$$

where $\epsilon_1$ and $\epsilon_2$ are the polarization vectors of $D_s(2460)$ and $\bar{D}_s^*$ respectively. The total amplitude can be parameterized as

$$\mathcal{A}_c = g_{10} \epsilon^{\mu\nu\alpha\beta} P_\mu \epsilon_{1\nu} \epsilon_{2\alpha} \epsilon_{2\beta}^* + \frac{g_{11}}{M^2} \epsilon^{\mu\nu\alpha\beta} P_\mu q_\nu \epsilon_{1\alpha} \epsilon_{2\beta} \epsilon_2.$$

The factors $g_{10}, g_{11}$ and $g_{12}$ are extracted from the expressions of $\mathcal{A}_c$.

D. Decay to $D_s^*(1^-)$ + $D_s(2460)(1^+)$

The Feynman diagram for $Y(4626) \to D_s^*(1^-) + D_s(2460)(1^+)$ is depicted in Fig. 3(d). The amplitude is

$$\mathcal{A}_d = g_{D_s^*D_s(2460)}^2g_{D_sD_s(2460)}\int \frac{d^4p}{(2\pi)^4} \frac{2}{3i\epsilon_1} k_\omega \left( \frac{p_1^\gamma}{m_1} + \frac{q_1^\gamma}{m_1'} \right) \epsilon_1^d(p) \epsilon_{abcd}^c \frac{P^c}{M} \left( -3k^b \frac{k}{v} + k^2 g^{b\nu} - k \cdot p_2 k \cdot q_2 g^{b\nu}/m_2/m_2' \right) \epsilon_2 \Delta(k, m_s) F^2(k, m_s),$$

where $\epsilon_1$ and $\epsilon_2$ are the polarization vectors of $D_s^*$ and $D_s(2460)$ respectively.

The total amplitude for the strong decay of $Y(4626) \to D_s^*(1^-) + \bar{D}_s(2460)(1^+)$ can also be expressed as

$$\mathcal{A}_d = g_{11}^\prime \epsilon^{\mu\nu\alpha\beta} P_\mu \epsilon_{1\nu} \epsilon_{2\alpha} \epsilon_{2\beta}^* + \frac{g_{12}^\prime}{M^2} \epsilon^{\mu\nu\alpha\beta} P_\mu q_\nu \epsilon_{1\alpha} \epsilon_{2\beta} \epsilon_2.$$

The factors $g_{11}', g_{12}'$ and $g_{12}''$ are extracted from the expressions of $\mathcal{A}_d$.

E. Decay to $D_s(0^-)$ + $\bar{D}_s(2572)(2^+)$

The Feynman diagram is depicted in Fig. 3(e) where $\bar{D}_s$ represents $\bar{D}_s(2572)$. The amplitudes is,

$$\mathcal{A}_e = g_{D_s^*D_s\eta}^2g_{D_sD_s\eta}\int \frac{d^4p}{(2\pi)^4} \frac{2}{3i\epsilon_1} k_\omega \left( \frac{p_1^\gamma}{m_1} + \frac{q_1^\gamma}{m_1'} \right) \epsilon_1^d(p) \epsilon_{abcd}^c \frac{P^c}{M} \left( -3k^b \frac{k}{v} + k^2 g^{b\nu} - k \cdot p_2 k \cdot q_2 g^{b\nu}/m_2/m_2' \right) \epsilon_2 \Delta(k, m_s) F^2(k, m_s),$$
where $\epsilon_2$ is the polarization tensor of $D_s(2572)(2^+)$. The total amplitude is written as

$$A_e = \frac{g_{20}}{M^2} \epsilon^{\mu\nu\alpha\beta} P_\mu \epsilon_{2\nu\sigma} q_\alpha \epsilon^*_\beta q_\sigma. \quad (27)$$

The factors $g_{20}$ can be extracted from the expressions of $A_e$.

F. Decay to $D_s(0^-) + D_s(2536)(1^+)$

The Feynman diagram is depicted in Fig. 3 (f) where $\bar{D}_{s1}$ represents $D_s(2536)$. The amplitudes is

$$A_f = g_{3D_s \eta} g_{D_s \eta} \int \frac{d^4p}{(2\pi)^4} \frac{2}{3} k^a \chi^d(p) \epsilon_{abcd} P^c \epsilon_{2\mu\lambda\omega} (\frac{p_{2\omega}}{m_2} + \frac{q_{2\omega}}{m_2}) k_\nu \Delta(k, m_2) F^2(k, m_2), \quad (28)$$

where $\epsilon_2$ is the polarization vector of $D_s(2536)$.

The amplitude is still written as

$$A_0 = g_0'' M \epsilon_2 \epsilon^* + \frac{g_2''}{M} (q \cdot \epsilon_2 q \cdot \epsilon^* - \frac{1}{3} q^2 \epsilon_2 \cdot \epsilon^*). \quad (29)$$

The factors $g_0''$ and $g_2''$ are extracted from the expressions of $A_f$.

IV. NUMERICAL RESULTS

A. the numerical results

Before we numerically solve the B-S equation all necessary parameters should be priori determined as accurate as possible. The masses $m_{D_s^*, m_{D_s\eta}, m_{D_s\eta'}, m_{D_s\pi}, m_{D_s \eta}}$ and $m_\rho$ come from the databook [13]. The coupling constants in the effective interactions $g_{D_s \eta}, g_{2D_s \eta}, g_{D_s \eta'}, g_{2D_s \eta'}, g_{D_s \eta'}, g_{D_s \eta}, g_{D_s \eta}, g_{D_s \eta}$ and $g_{D_s \eta}$ are taken from the relevant literatures and their values and related references are collected in the Appendix.

With these input parameters the B-S equation Eq. (12) can be solved numerically. Since it is an integral equation, an efficient way for solving it is discretizing it and then turns solving the integral equation to an algebraic equation group. Concretely, we let the variables $|P_T|$ and $|Q_T|$ be discretized into $n$ values $Q_1, Q_2, ... Q_n$ ($n=129$ in our calculation) and the equal gap between two adjacent values as $Q_{n-1} = \frac{Q_n - Q_1}{n-1}$. Here we set $Q_1 = 0.001$ GeV and $Q_2 = 2$ GeV. The $n$ values of $f(|P_T|)$ constitute a column matrix on the left side of the equation and the $n$ elements $f(|Q_T|)$ construct another column matrix on the right side of the equation as shown below. In this case, the functions in the curl bracket of Eq. (12) multiplying $\frac{|q_T|^2}{12M^2(2\pi)^2}$ would be an effective operator acting on $f(|Q_T|)$. It is specially noted that because discretizing the equation, even $\frac{|q_T|^2}{12M^2(2\pi)^2}$ turns from continuous integration variable into $n$ discrete values which are involved in the $n \times n$ coefficient matrix. Substituting the $n$ pre-set
$Q_i$ values into those functions, the operator turns into an $n \times n$ matrix which associates the two column matrices. It is noted that $Q_1$, $Q_2$,...$Q_n$ should take sequential values.

$$
\begin{pmatrix}
  f(Q_1) \\
  \vdots \\
  f(Q_{129})
\end{pmatrix} = A(\Delta E, \lambda)
\begin{pmatrix}
  f(Q_1) \\
  \vdots \\
  f(Q_{129})
\end{pmatrix}.
$$

As is well known, if a homogeneous equation possesses non-trivial solutions, the necessary and sufficient condition is $\det[A(\Delta E, \lambda) - I] = 0$ ($I$ is the unit matrix) where $A(\Delta E, \lambda)$ is just the aforementioned coefficient matrix. Thus solving the integral equation just turns to a sort of eigenvalue searching problem which is a familiar issue in quantum mechanics, in particular, the eigenvalue is required to be unit in this problem. Here $A(\Delta E, \lambda)$ is a function of the binding energy $\Delta E = m_1 + m_2 - M$ and parameter $\lambda$. The following procedure is a bit tricky. Inputting a supposed $\Delta E$, we vary $\lambda$ to make $\det[A(\Delta E, \lambda) - I] = 0$ hold. One can note that the matrix equation $(A(\Delta E, \lambda)_{ij}) (f(j)) = \beta (f(i))$ is exactly an eigenequation. Using the values of $\Delta E$ and $\lambda$, we seek out all possible “eigenvalues” $\beta$. Among them only $\beta = 1$ is the solution we expect. In the process of solving the equation group, the value of $\lambda$ is determined, and actually it is the solution of the equation group with $\beta = 1$. Meanwhile using the obtained $\lambda$, one achieve the corresponding wavefunction $f(Q_1), f(Q_2)...f(Q_{129})$ which just is the solution of the B-S equation.

Generally $\lambda$ should be within the range around the order of unit. In Ref.\[37\] the authors fixed $\lambda$ to be 3. In our earlier paper\[40\] the value of $\lambda$ varied from 1 to 3. In Ref.\[33\] we set the value of $\lambda$ within a range of 0 ~ 4 by which as believed, a bound state of two hadrons can be formed. When the obtained $\lambda$ is much beyond this range, one would conclude that the molecular bound state may not exist or at least is not a stable state. But it is really noted that the form factor is phenomenologically introduced and the parameter $\lambda$ is usually fixed via fitting data, i.e. neither the form factor nor the value of $\lambda$ are derived from an underlying theory, but based on our intuition (or say, a theoretical guess). Since the concerned processes are dominated by the non-perturbative QCD effects whose energy scale is about 200 MeV, we have reason to believe that the cutoff should fall within a range around a few hundreds of MeV to 1 GeV, and by this allegation one can guess that the value of $\lambda$ should be close to unit. However, from other aspect, this guess does not have a solid support, further phenomenological studies and a better understanding on low energy field theory are needed to get more knowledge on the form factor and value of $\lambda$. So far, even though we believe this range for $\lambda$ which sets a criterion to draw our conclusion at present, we cannot absolutely rule out the possibility that some other values of $\lambda$ beyond the designated region may hold. That is why we proceed to compute the decay rates of $Y(4626)$ based on the molecule postulate. (see below the numerical results for clarity of this point).

By our strategy, for the state $Y_2$ we let $\Delta E = 0.021$ GeV which is the binding energy of the molecular state as $M_{D^*_s} + M_{D_{s1}(2536)} - M_{Y(4626)}$. Then we try to solve the equation $|A(\Delta E, \Lambda) - I| = 0$ by varying $\lambda$ within a reasonable range. In other words, we are trying to determine a $\lambda$ whose value falls in the range of 0 to 4 as suggested in literature, to satisfy the equation.
As our result, we have searched a solution of \( \lambda \) within a rather large region, but unfortunately find that there is no solution which can satisfies the equation.

However, for the \( Y_1 \) state if one still keeps \( \Delta E = 0.021 \) GeV but set \( \lambda = 10.18 \), the equation \( |A(\Delta E, \lambda) - I| = 0 \) holds while the contributions induced by exchanging both \( \eta \) and \( f_0(980) \) are included. Instead, if the contribution of exchanging \( f_0(980) \) (Fig. 2) is ignored, with the same \( \Delta E \) one could get a value of \( \lambda \) which is very close to that without the contribution of \( f_0(980) \). It means that the contribution from exchanging \( f_0(980) \) is very small and can be ignored safely. Meanwhile by solving the eigenequation we obtain the wavefunction \( f(Q_1), f(Q_2)...f(Q_{129}) \). The normalized wavefunction is depicted in Fig. 4 with different \( \Delta E \).

Due to existence of an error tolerance on measurements of the mass spectrum, we are allowed to vary \( \Delta E \) within a reasonable range to fix the values of \( \lambda \) again, for the \( D_{s1}\bar{D}_s \) system, the results are presented in Tab. 1. Apparently for a reasonable \( \Delta E \) any \( \lambda \) value which is obtained by solving the discrete B-S equation is far beyond 4. Does the result imply that \( D_{s1}\bar{D}_s \) fails to form a bound state? We will further discuss its physical significance in next section.

A new resonance \( Y(4626) \) has been experimentally observed[1], and it is the fact everybody acknowledges, but what composition it has, demands a theoretical interpretation. The molecular state explanation is favored by an intuitive observation. However our theoretical study does not support the allegation that \( Y(4626) \) is the molecule of \( D_s\bar{D}_{s1} \).

On other respect, the above conclusion is based on a requirement: \( \lambda \) must fall in a range of 0~4, which is determined by phenomenological studies done by many researchers. However, \( \lambda \) being in 0~4 is by no means a mandatory condition because it is not deduced form an underlying principle and lacks real foundation. Therefore even though our result does not favor the molecular structure for \( Y(4626) \), we still proceed to study the transitions \( Y(4626) \rightarrow D_s\bar{D}_s(2317) \), \( Y(4626) \rightarrow D_s\bar{D}_s(2460) \), \( Y(4626) \rightarrow D_s(2460)\bar{D}_s \), \( Y(4626) \rightarrow D_s\bar{D}_s(2573) \) and \( Y \rightarrow D_s\bar{D}_{s1}(2536) \) under the assumption of the molecular composition of \( D_s\bar{D}_{s1} \).

Using the wave function we calculate the form factors \( g_0, g_2, g_0', g_2', g_10, g_11, g_{12}, g_{10}', g_{11}', g_{12}', g_{20}, g_0'', g_2'' \) defined in Eqs. (18, 21, 23, 25, 27 and 29). With these form factors we get the decay widths of \( Y(4626) \rightarrow D_s\bar{D}_s(2317) \), \( Y(4626) \rightarrow D_s\bar{D}_s(2460) \), \( Y(4626) \rightarrow D_s(2460)\bar{D}_s \), \( Y(4626) \rightarrow D_s\bar{D}_s(2573) \) and \( Y(4626) \rightarrow D_s\bar{D}_{s1}(2536) \) which are denoted as \( \Gamma_a, \Gamma_b, \Gamma_c, \Gamma_d, \Gamma_e \) and \( \Gamma_f \) presented in Table 1. Theoretical uncertainties originate from the experimental errors, namely the theoretically predicted curve expands to a band.

Of course, exchanging two \( \eta \) mesons can also induce a potential as the next-to-leading order (NLO) contribution, but it undergoes a loop suppression therefore, we do not consider that contribution i.e. one-boson-exchange model is employed in our whole scenario.

V. CONCLUSION AND DISCUSSION

In this work we explore the bound state composed of a vector and an axial vector within the B-S equation framework. Concretely we study the resonance \( Y(4626) \) which is assumed
TABLE I: the cutoff parameter $\lambda$ and the corresponding binding energy $\Delta E$ for the bound state $D_s^*\bar{D}_{s1}$

| $\Delta E$ (MeV) | 3  | 9  | 15 | 21 | 27 |
|-----------------|----|----|----|----|----|
| $\lambda$      | 9.45 | 9.76 | 9.99 | 10.19 | 10.37 |

FIG. 4: The normalized wave function $f(|p_T|)$ for $D_s^*\bar{D}_{s1}$

to be a molecular state made of $D_s^*$ and $\bar{D}_{s1}(2536)$. According to the Lorentz structure we construct the B-S wave function of a vector meson and an axial one. Using the effective interactions induced by exchanging one light meson, the interaction kernel is obtained and the B-S equation for the $D_s^*\bar{D}_{s1}(2536)$ system is established. In our calculation exchanging $\eta$-meson provides the dominant contribution and that induced by exchanging $f_0(980)$ can be neglected.

Under the covariant instantaneous approximation the four-dimension B-S equation can reduce into a three-dimension B-S equation. Integrating out the azimuthal component of the momentum we obtain a one-dimension B-S equation which is an integral equation. With all input parameters such as the coupling constants and the corresponding masses of mesons we solve the equation for the molecular state of $D_s^*\bar{D}_{s1}(2536)$. When we input the binding energy $\Delta E = M_{Y(4626)} - M_{D_s} - M_{\bar{D}_{s1}(2536)}$, we search for $\lambda$ which satisfies the one-dimension B-S equation. Our criterion is that if there is no solution for $\lambda$ or the value of $\lambda$ is not reasonable, the bound state should not exist in the nature. On contrary, if a "suitable" $\lambda$ is

TABLE II: the decay widths (in units of MeV) for the transitions

| $\Gamma_a$ | $\Gamma_b$ | $\Gamma_c$ | $\Gamma_d$ | $\Gamma_e$ | $\Gamma_f$ |
|------------|------------|------------|------------|------------|------------|
| 0.111~0.550 | 0.290~0.862 | 0.0623~0.0679 | 0.0224~0.0226 | 0.00914~0.00930 | 0.0777~0.0998 |
found as a solution of the B-S equation, we would claim that resonance could be a molecular state. From the results shown in table I one can find that even for a small binding energy (we deliberately vary the value of the binding energy), the $\lambda$ which makes the equation to hold, must be larger than 9 which is far beyond the favorable one in literature so that we tend to think the molecular state of $D_s^* \bar{D}_{s1}(2536)$ does not exist unless the coupling constants get larger than those given in Appendix.

As aforementioned discussion, the $\lambda$ in the form factor at each vertex is phenomenologically introduced and does not receive a solid support from any underlying principle, therefore, we may suspect its application regime which might be the pitfall of the phenomenology. Thus we try to overcome this barrier to extend the value to a region which obviously deviates from the region favored by the previous works. As a $\lambda$ value beyond 9, the solution of the B-S equation exists, and the B-S wavefunction is constructed. Just using the wavefunctions, we calculate the decay rates of $Y(4626) \rightarrow D_s^* \bar{D}_{s1}(2517)$, $Y(4626) \rightarrow D_s \bar{D}_s(2460)$, $Y(4626) \rightarrow D_s(2460) \bar{D}_s^*$, $Y(4626) \rightarrow D_s^* \bar{D}_s(2460)$, $Y(4626) \rightarrow D_s \bar{D}_{s2}(2573)$ and $Y(4626) \rightarrow D_s \bar{D}_{s2}(2536)$ under the assumption that $Y(4626)$ is a bound state of $D_s^* \bar{D}_{s1}(2536)$. Our results indicate the decay widths are small comparing with the total width of $Y(4626)$.

The important and detectable issue is the decay patterns deduced above. This would compose a crucial challenge to the phenomenological scenario. If the decay patterns deduced in terms of the molecule assumption are confirmed (within an error tolerance), it would imply that the constraint on the phenomenological application of form factor which is originating from the chiral perturbation can be extrapolated to a wider region. By contrary, if the future measurements negate the predicted decay patterns, one should claim that the assumption that $Y(4626)$ is a molecular state of $D_s^* \bar{D}_{s1}(2536)$ fails. The resonance would be in different structure, such as tetraquark or hybrid etc.

We lay our hope on the future experimental measurements on those decay portals, which can help us to clarify the structure of $Y(4626)$.

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15
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Appendix A: The effective interactions

The effective interactions can be found in \cite{34,36}

\begin{align}
\mathcal{L}_{D^*D_1P} &= D^* \frac{D^*}{D_1} \rho[3D^*_{\mu b}(\partial_\mu \partial_\nu \mathcal{M})_{ba}D^*_{a\mu} - D_{\mu b}(\partial_\nu \partial_\rho \mathcal{M})_{ba}D^*_{a\mu}] \\
&\quad + \frac{1}{m_{D^*}m_{D_1}} \partial_\nu D_{\mu b}(\partial_\nu \partial_\rho \mathcal{M})_{ba} + c.c., \\
\mathcal{L}_{D_0D_1P} &= g_{D_0D_1P} D^*_{\mu b}(\partial_\mu \mathcal{M})_{ba}D^*_{a\mu} + c.c., \\
\mathcal{L}_{D^*D^*P} &= g_{D^*D^*P}(\partial_\mu \mathcal{M})_{ba}D^*_{a\mu} + c.c. 
\end{align}

\[ (A1) \]
\[ \mathcal{L}_{D_1D_1} = g_{D_1D_1} \frac{D_{1b}^{\mu} \partial_{\nu} M_{D_1}}{M_{D_1}} D_{1a}^{\alpha\dagger} (\partial^{\mu} \mathcal{M})_{ba} \varepsilon_{\mu\nu\alpha\beta} + c.c., \] (A4)

\[ \mathcal{L}_{D^*_1D_1} = g_{D^*_1D_1} \frac{D_{1b}^{\mu} \partial_{\nu} M_{D_1}}{M_{D_1}} D_{1a}^{\alpha\dagger} (\partial^{\mu} \mathcal{M})_{ba} \varepsilon_{\mu\nu\alpha\beta} + c.c., \] (A5)

\[ \mathcal{L}_{D^*_1D_1} = i g_{D^*_1D_1} \left[ \partial_{\nu} D_{1b}^{\mu} (\partial^{\mu} \mathcal{M})_{ba} D_{1a}^{\alpha\dagger} \right] - \frac{D_{1b}^{\mu} (\partial^{\mu} \mathcal{M})_{ba}}{M_{D_1}} \partial_{\nu} D_{1a}^{\alpha\dagger} + c.c., \] (A6)

\[ \mathcal{L}_{D^*_1D_1} = g_{D^*_1D_1} \left( \frac{\partial^{\mu} D_{1b}^{\mu} D_{1a}^{\alpha\dagger}}{m_{D_1}} - \frac{D_{1b}^{\mu} \partial^{\mu} D_{1a}^{\alpha\dagger}}{m_{D_1}^*} \right) (\partial^{\mu} \mathcal{M})_{ba} \varepsilon_{\mu\nu\alpha\beta} + c.c., \] (A7)

\[ \mathcal{L}_{D_1D_2} = g_{D_1D_2} (D_{1a\mu}) (\partial_{\nu} \mathcal{M})_{ba} D_{2a}^{\mu\nu} + c.c., \] (A8)

\[ \mathcal{L}_{D_1D_1f_0} = g_{D_1D_1f_0} (D_{1a}^{\mu}) D_{1a}^{\mu\dagger} f_0 + c.c., \] (A9)

\[ \mathcal{L}_{D^*_1D^*_1f_0} = g_{D^*_1D^*_1f_0} (D_{1a}^{\mu}) D_{1a}^{\mu\dagger} f_0 + c.c., \] (A10)

where \( c.c. \) is the complex conjugate term, \( a \) and \( b \) represent the flavors of light quarks, \( f_0 \) denotes \( f_0(980) \), \( \mathcal{M} \) is a 3 \times 3 hermitian and traceless matrix \( \mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\eta^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & K^0 & -\frac{\sqrt{2}}{3} \eta \end{pmatrix} \).

In the chiral and heavy quark limit, the above coupling constants are

\[ g_{D_1D_1} = -\frac{\sqrt{6}}{3} h_1 + h_2 \frac{\Lambda_f}{f_\pi} \sqrt{M_{D_1} M_{D_{1\gamma}}}, \]

\[ g_{D_1D_2} = \frac{2 \sqrt{6}}{3} \frac{\tilde{h}}{f_\pi} \sqrt{M_{D_1} M_{D_{1\gamma}}}, \]

\[ g_{D_1D_1} = \frac{g}{f_\pi}, \]

\[ g_{D_1D_1} = \frac{5 \kappa}{6 f_\pi}, \]

\[ g_{D_1D_2} = -\frac{2 g}{f_\pi} \sqrt{M_{D_1} M_{D_{1\gamma}}}, \]

\[ g_{D_1D_2} = \frac{h}{f_\pi} \sqrt{M_{D_1} M_{D_{1\gamma}}}, \]

\[ g_{D_1D_2} = \frac{\sqrt{6} \tilde{h}}{6 f_\pi} \sqrt{M_{D_1} M_{D_{1\gamma}}}, \]

\[ g_{D_1D_2} = -\frac{\sqrt{6} \kappa}{3 f_\pi} \sqrt{M_{D_1} M_{D_{2\gamma}}}, \]

and we suppose

\[ g_{D_1D_2} = -g_{D_1D_2} = -2 g_{\sigma} M_{D_{1\gamma}}, \]

\[ g_{D_1D_1} = -g_{D_1D_1} = -2 g_{\sigma} M_{D_{1\gamma}}. \]

with \( \Lambda_f = 1 \text{GeV}, f_\pi = 132 \text{MeV}[33], h_1 = 0.56, h_2 = 0.43, g = 0.64[36], \kappa = g, \)

\( \tilde{h} = 0.87[44], g_{\sigma} = 0.76[45], g'' = g_{\sigma}[46]. \)