Finite element calculation model of the piled rafts

A.Ya. Dzhankulaev*, N.A. Guenko, A.A. Dzhankulayev,
H.M. Dyshekov
Kabardino-Balkarian State University named after H.M. Berbekov,
173,Chernyshevsky St., Nalchik, Russian Federation
E-mail: dzhankulaevaj@yandex.ru

Abstract. Finite element model for calculating piled rafts in difficult geological conditions.

Introduction
In recent years, piled rafts have become a frequently used type of foundations in industrial and civil construction. The need for the construction of such foundations is becoming especially urgent due to the fact that the territories with favorable ground conditions for construction are developed, the number of buildings storeys and the load on the bases are increasing. It is also known that in some cases the pile foundations’ use on sufficiently strong soils and in cases where a layer of strong soil lies on the surface is more economically and technologically effective than the shallow foundations and the foundations of other structural solutions. In these cases, when the grillage is in contact with the ground, a part of the external load can be transmitted through the sole of the plate, and another part is perceived by the piles. For the structures on soils and fields of piles, this type of foundation construction is usually called the combined piled rafts.

Practice shows that the slab and foundation piles can transfer various parts of the load to the soil base depending on the actual characteristics and working conditions of the soil. Taking this fact into account is a very urgent task especially in pile-plate complex foundation structures. Here it is necessary to take into account the base work, both with the foundation slab and with each pile, depending on the characteristics of the soil at all base points.

Main part
Such foundation systems belong to the class of thick plates, where the shear components of the stress-strain state are commensurate with the bending parameters. Therefore, the basis of the proposal below is based on the results of a study by the scientific school of N.I. Karpenko on accounting for the volumetric stress-strain state and nonlinearity of the material. The most commonly used way of modeling these properties is the finite element method. When deriving the stiffness matrix of the foundation slab’s final element, an important step is the soil base model selection. Complex diverse phenomena in the foundations that occur during their loading are difficult to take into account in practical calculations. Therefore, various simplified models of the base are used. In the finite element calculations, the most common model is the Winkler elastic base [1]. In this case, the base resistance is taken directly proportional to the settlement of the foundation plate:

\[ P_f(x, y) = CW(x, y), \]  

(1)
where $C$ – is the base coefficient of subgrade resistance.

Let us assume that the deflection shape of a rectangular finite element on an elastic base is the same as in the absence of a base [2]. Then we obtain the following expression for determining the deflections in the field of the foundation slab:

$$ W(x, y) = \{S\}_1(q), $$

where $\{q\}^T = \{q_1, q_2, q_3, q_4\}$ - defines the vector of generalized nodal displacements for a rectangular four nodal finite element; $\{S\} = \{S_1, S_2, S_3, S_4\}$ - determines a vector which components are functions determining the finite element deflection shape. Here we use the classical compactly supported polynomial functions of two variables that are the same as for approximating the bending deflections of the rectangular finite element of the slab in the form of a twelve-membered polynomial [2]:

$$ W_m = a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 x y + a_6 y^2 + a_7 x^3 + $$
$$ + a_8 x^2 y + a_9 x y^2 + a_{10} y^3 + a_{11} x^3 y + a_{12} x y^3. $$

The potential energy of the foundation final element soil base in this case has the form of a double integral over the area $F$:

$$ U = \frac{1}{2} \iint_P W(x, y) dF. $$

Here the coefficient $\frac{1}{2}$ is before the integral, it is introduced on the basis of the assumption that the operation of the soil foundation is linear and the conditions are of static loading. In the case of taking into account the variability of the base coefficient of subgrade resistance during the foundation system’s operation for various reasons, the computational process, as a rule, is built by the step method, which allows the assumption of linearity at each stage of the iteration. In addition, in modern pile-plate systems, reinforced concrete is used as the main material. Calculations taking into account the volumetric stress-strain state of concrete for thick grill slabs and nonlinearity of materials have become common practice [3,4], and the organization of the computational step-iteration process has been studied quite well.

Taking into account the expressions (1) and (2) from (3), we obtain the integral for calculating the stiffness matrix of the soil base:

$$ \left[ K \right]_f = \iint_F \{S\}^T C \{S\} dF. $$

The elements of the base matrix stiffness $\left[ K \right]_f$ are determined through the double integral over the FE area:

$$ K_{fij} = C \int_a^b \int_o^o \{S_i S_j\} dx dy. $$

The general stiffness matrix of the foundation plate’s finite element [4] is obtained as the sum of two matrices:

$$ \left[ K \right]_o = \left[ K \right] + \left[ K \right]_f, $$
\[ [K] \] - is the FE stiffness matrix of the foundation plate [4] without taking into account the resistance of the soil base and piles.

The values of the coefficient of subgrade resistance under the foundation is often not a fixed value, but varies depending on the base properties, depending on many physical, climatic, operational and other factors. Moreover, in the plate field, the values of the base coefficient of subgrade resistance are changed stepwise, which leads to a complication of calculation schemes and an increase in computational operations.

Let us consider the method of setting the distribution functions of the base coefficient of subgrade resistance and the base finite element stiffness matrix derivation without these calculated disadvantages.

In the expression (1), it was assumed that the base coefficient of subgrade resistance \( C = \text{const} \) over the field of the finite element. Using a functional dependence instead of a constant does not affect the calculation of the integral (4).

We choose the nodal points of the finite element as the base and denote the values of the base coefficients of subgrade resistance coefficients at these points are: \( C_{nm}^1, C_{nm}^2, C_{nm}^3, C_{nm}^4 \), as shown in Figure 1.

The approximation of the coefficients of subgrade resistance base at an arbitrary point inside the finite element with coordinates \((x, y)\) is given in the following expression form:

\[
C_{nm}^* (x, y) = \sum_{i=1}^4 C_{nm}^i N_i (x, y).
\]  

(7)

Here \( N_i \) - defines the form functions and they are given by the linear formulas:

\[
N_1 = (\xi, \eta) = 1 - \xi - \eta + \xi \eta, \\
N_2 = (\xi, \eta) = \eta - \xi \eta, \\
N_3 = (\xi, \eta) = \xi \eta, \\
N_4 = (\xi, \eta) = \xi - \xi \eta.
\]

where \( \xi = x / a, \eta = y / b \) - are the relative coordinates. In this case, the distribution law of the base coefficient of subgrade resistance over the field of the finite element has the character of a ruled surface, and it does not have steps and gaps over the slab field at different bed coefficients,
which improves the calculation results’ accuracy and gives better convergence of the iterative step process. In this case, the piles work is recorded through the corresponding values of the base coefficients of subgrade resistance $C_{nm}^{i}$ in the places of their location along the foundation field. Moreover, through the values of these coefficients, it is possible to specify various deformation characteristics of the piles without introducing them into the calculation as separate elements of the foundation structure while maintaining the two-dimensional design scheme.

Taking the approximation (7) into consideration, we rewrite the stiffness matrix of the soil base (4) in the form of an integral:

$$K_{fhn} = \int_{a}^{b} \int_{o}^{l} \sum_{m=1}^{4} S_{m}^{T} \sum_{k=1}^{4} C_{nm}^{k} N_{k} \sum_{n=1}^{4} S_{n}^{T} \, dx \, dy.$$  

Substituting the expression (8) into (6) gives the stiffness matrix CE of the pile-plate structure, which allows calculating the foundation systems taking into account the smoothed change in the actual coefficients of subgrade resistance along the field of the soil base.

However, the values of the base coefficient of subgrade resistance of the foundation slab part and the stiffness of the piles can vary under the influence of various factors, both upward and downward. The practice of modern high-rise civil construction, complex industrial and special structures does not imply a great opportunity to find a suitable territory for this with the necessary ground conditions. Often it is necessary to design unfavorable in the geological sense with different soil conditions on the sites along a horizontal surface and along a vertical section. In addition, the characteristics of such soils can greatly change under the influence of natural and climatic conditions, technological disasters, technological impacts and many other factors. This is especially true for our country due to the huge geographical distances and, as a consequence, the great climatic diversity. It is necessary to conduct survey, calculation, design and construction work on the sites with a huge spread not only of the physical and mechanical soil base characteristics, but also a wide range of temperature, humidity and other factors affecting the structure performance, especially its foundation and soil parts. To take this into account in the calculations of the “base-foundation” system, we rewrite the formula (1) as a multiplication:

$$P_{f}(x, y) = \varphi(z)CW(x, y)$$  

Here $\varphi(z)$ – is a function that corrects the values of the base coefficients of subgrade resistance depending on the soil base work nature. Under $Z$ various factors that affect the soil base performance are meant: foundation sediment, operating time, humidity change, temperature changes, etc. If several similar factors are simultaneously taken into account, this function becomes dependent on many variables and the number of integrals with their boundaries increases.

In this case, the FE soil base potential energy expression (3) takes the form:

$$U = \frac{1}{2} \int_{F} \int_{h} P_{f}(x, y)W(x, y)dzdF.$$  

Taking into account the dependencies (9) and (2), from the integral expression (10) we obtain the stiffness matrix of the base, taking into account various factors of the soil base non-linearity:

$$[K]_{f} = \int_{F} \int_{h} [S]^{T} \varphi(z)C[S]dzdF.$$  

(11)
Matrix Elements $[K]_f$ are determined taking into account (7) from the expression (11) according to the following formula:

$$K_{mn} = \iiint_{a \leq x \leq b} \sum_{m=1}^{4} \sum_{k=1}^{4} \sum_{n=1}^{4} s_m^T \phi(z) c_{nm}^k n_k \sum_{n=1}^{4} s_n^T dz \, dx \, dy$$

(12)

The general stiffness matrix of the pile-slab system FE $a$ is obtained as the sum of two matrices:

$$[K]_a = [K]_o + [K]_f,$$

(13)

$[K]_o$ - is the stiffness matrix of the FE plate without regard to the base. It is defined as the integral of three matrices multiplication [5]:

$$[K] = \iiint_{V} [B]_T [D] [B] dV.$$

(14)

$[B]$ – is the connection matrix of finite element strains with generalized nodal displacements, $[D]$ – is the plate material elasticity matrix.

If the calculation is carried out without taking into account the physical nonlinearity of the material, then the elasticity matrix $[D]$ is a constant value and accordingly the stiffness matrix $[K]$ is also fixed.

In the case of taking into account the slab material’s nonlinearity and the features of the soil base during operation, the total stiffness matrix of the entire system’s final element “foundation slab - pile - soil base” is obtained from (13) in the form of iterative dependence:

$$[K]_o^k = [K]_o^{k-1} + [K]_f^{k-1}$$

(15)

Using the formula (15), a single stepwise iterative calculation process is constructed. The vector magnitude of the generalized nodal displacements $\{q\}^i$ is determined from the main equation solution of the finite element method, which is a system of linear equations:

$$[K]_o^k \{q\}^i = \{F\}$$

(16)

Here $\{F\}$ – is a nodal load vector.

**Summary**

This finite element makes it possible to simulate the piled rafts’ operation in accordance with the real strength and deformation characteristics of the soil base on the two-dimensional design schemes’ basis. At the same time, it is possible to use the dependences of the grill slabs volumetric stress-strain state taking into account the nonlinear physical and mechanical characteristics of the material. In this case, the deformation and strength calculation can be combined into a single step-by-step iterative computational process. At the same time, many important features of the interaction of a piled rafts structure and a soil foundation are taken into account. These are the calculated parameters that increase the results’ reliability of modeling the work of similar foundation structures: the flexibility or rigidity of each pile, the unevenness of the pile field, the simultaneous interaction of the grillage slab with soil and piles, transferring the load to the plate and pile parts of the foundation without postulating the redistribution law, taking into account various factors of soil performance at all operation stages.
References

[1] Palatnikov E A 1961 Calculation of reinforced concrete slabs covering airports (Oborongiz, M.).

[2] Segerlind L 1979 Application of the finite element method (Mir, Moscow).

[3] Dzhankulaev A Ya, Karpenko N I 1992 Methodology for taking into account volumetric stress-strain state in the calculations of reinforced concrete slabs. Materials of the 29th international conference in the field of concrete and reinforced concrete (Stroyizdat, Moscow).

[4] Dzhankulaev A Ya 2000 A method for determining shear strains Bulletin of KBSU. Series “Engineering” 4.

[5] Dzhankulayev A Y, Likhov Z R, Shogenov O M 2018 The Finite Element of the Plate with the Account of the Transformation of the Cross Section and the Nonlinear Foundation Conference: 2018 IEEE International Conference “Quality Management, Transport and Information Security, Information Technologies” (IT&QM&IS) 8525048.