Reliability of n-Cascade Stress-Strength P(X<Y<Z) System for four different distributions

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Abstract. In this paper, the n-cascade with P(X<Y<Z) is used to find the reliability of stress-strength system, where X and Z are the strengths of a component subjected to stress Y. The n-cascade stress-strength system reliability expression is obtained for four different distributions which are [Exponential Pareto, Inverted Exponential, Exponentiated Invers Rayleigh, Frechet]. For some specific values of the parameters of the distribution, the numerical values of the four different reliabilities have also been computed, provided, and discussed.

1. Introduction

The reliability of the stress-strength system model component is defined as the probability that the stress working on it, is not greater than strength, where both x and y are random variables, so $R = P(X \geq Y)$. But here we consider the case that a component (or system) can work only when the stress Y on it is lies between two certain values of components (or systems) strengths (say X and Z), i.e. stress is within certain limits see [4],[6] and [7].

Under this system, the reliability can be defined as:

$R = P(X < Y < Z)$

Where X and Z are the component strengths (DMGO calls them lower and upper strengths), and Y is the stress on this component all X, Y, and Z are random variables.

In the source, Singh [7] has considered the reliability estimation under the assumption that the strength of a component y lies in an interval between two stress ($X_1$ and $X_2$), i.e. $R = P(X_1 < Y < X_2)$

The main aim of this paper is to obtain the reliability of the n-cascade stress-strength system $R_n$, where stress on the component is subjected to two strengths. The general model reliability is illustrated in section 2, the reliabilities obtained for four different distributions [Exponential Pareto, Inverted Exponential, Exponentiated Invers Rayleigh, Frechet] are given in section 3. Finally, the results and discussions of some numerical values of the reliabilities are given in section 4.

2. n-Cascade with P(X<Y<Z) System Reliability
After every failure in the cascade system, the stress is modified by a factor $(k)^{[5]}$. Let $y_i; i = 1, ..., n$ be the stress on $n$-components, then

$$y_2 = ky_1, y_3 = ky_2 = k^2y_1, ..., y_i = k^{i-1}y_1$$

The $i$th component works if the stress $k^{i-1}y_1$ lie in the interval $(x_i, z_i)$ the lower and upper strength of $i = 1, ..., n$ components, respectively. Whenever it falls outside these two limits, the component fails and another from standby takes the place of the failed component and the system continues to work until the $n$ component in cascade fails $[4]$. Assuming the components work independently then.

$$R_n = R(1) + R(2) + \cdots + R(n) \quad (1)$$

Where $R(r)$ is the marginal reliability of $r$th component.

Let $x_i, z_i$ and $y_i; i = 1, 2, ..., n$ are independent random variables with $F(x), H(z)$ cumulative distribution functions (cdf) and $g(y_i)$ probability density function (pdf), then

$$R(1) = P(x < y_1 < z) = P(y_1 > x) - P(y_1 > x, y_1 > z)$$

$$= \int_{y_1} F_x(ky_1)g(y_1)dy_1 - \int_{y_1} F_x(ky_1)H_x(y_1)g(y_1)dy_1 \quad (2)$$

And

$$R(2) = P(x < y_1 < z)P(x < y_2 < z)$$

$$= [1 - R(1)]P((y_2 = ky_1) > x) - P(ky_1 > x, ky_1 > z)$$

$$= [1 - R(1)] \left[ \int_{y_1} F_x(ky_1)g(y_1)dy_1 - \int_{y_1} F_x(ky_1)H_x(ky_1)g(y_1)dy_1 \right] \quad (3)$$

Similarly

$$R(3) = [1 - R(1)][1 - R(2)]$$

$$\left[ \int_{y_1} F_x(k^2y_1)g(y_1)dy_1 - \int_{y_1} F_x(k^2y_1)H_x(k^2y_1)g(y_1)dy_1 \right] \quad (4)$$

In general

$$R(n) = (1 - R(1))(1 - R(2)) \cdots (1 - R(n - 1))$$

$$\left[ \int_{y_1} F_x(k^{n-1}y_1)g(y_1)dy_1 - \int_{y_1} F_x(k^{n-1}y_1)H_x(k^{n-1}y_1)g(y_1)dy_1 \right] \quad (5)$$

3. Specific Distributions

In this section, we involve four different distributions for the random stress and strengths variables to obtain the mathematical expressions of reliabilities in equations (2-5) and then $n$-cascade system reliability in the equation (1).

3.1. Exponential Pareto Distribution (EPD)

Let the strengths random variables $X$ and $Z$ are independent identically distributed with EPD $(\rho, \lambda_1, \theta)$ and EPD $(\rho, \lambda_2, \theta)$, respectively, with cumulative distribution function (cdf) given as $[2]$ and $[3]:$

$$F(x) = 1 - e^{-\lambda_1(x/\rho)^\theta}, \quad x > 0$$

$$H(z) = 1 - e^{-\lambda_2(z/\rho)^\theta}, \quad z > 0$$

And let $y_1$ be EP $(\rho, \lambda, \theta)$ random stress variable with probability density function (pdf) given as:

$$g(y_1) = \frac{\lambda \rho^{\theta-1}}{\theta} e^{-\lambda(y_1/\rho)^\theta}, \quad y_1 > 0$$
Where $\int_{y_1} \left( \frac{y_1}{\rho} \right)^{\theta-1} e^{-\lambda \left( \frac{y_1}{\rho} \right)^{\theta}} \, dy_1 = \frac{\rho}{\lambda \theta}$

Since $g(y_1)$ is pd function.

Then from equation (2), we have.

$R(1) = A_1 - B_1$

Then

$$A_1 = \int_{y_1} \left( 1 - e^{-\lambda_1 \left( \frac{y_1}{\rho} \right)^{\theta}} \right) \frac{\lambda \theta}{\rho} \left( \frac{y_1}{\rho} \right)^{\theta-1} e^{-\lambda \left( \frac{y_1}{\rho} \right)^{\theta}} \, dy_1$$

$$= 1 - \frac{\lambda \theta}{\rho} \int_{y_1} \left( \frac{y_1}{\rho} \right)^{\theta-1} e^{-(\lambda + \lambda_1) \left( \frac{y_1}{\rho} \right)^{\theta}} \, dy_1$$

$$= 1 - \frac{\lambda \theta}{\rho (\lambda + \lambda_1) \theta} = 1 - \frac{\lambda}{(\lambda + \lambda_1)}$$

$$B_1 = \int_{y_1} F_X (y_1) H_2(y_1) g(y_1) \, dy_1$$

$$= \int_{y_1} \left( 1 - e^{-\lambda_1 \left( \frac{y_1}{\rho} \right)^{\theta}} \right) \left( 1 - e^{-\lambda_2 \left( \frac{y_1}{\rho} \right)^{\theta}} \right) g(y_1) \, dy_1$$

Then, we get

$$R(1) = 1 - \frac{\lambda}{(\lambda + \lambda_1)} - 1 + \frac{\lambda}{(\lambda + \lambda_2)} + \frac{\lambda}{(\lambda + \lambda_1, \lambda_2)} - \frac{\lambda}{(\lambda + \lambda_1 + \lambda_2)}$$

And now from equation (3), we have

$$R(2) = [1 - R(1)] [A_2 - B_2]$$

$$A_2 = \int_{y_1} F_X (ky_1) g(y_1) \, dy_1 = \int_{y_1} \left( 1 - e^{-\lambda (\frac{ky_1}{\rho})^{\theta}} \right) g(y_1) \, dy_1$$

$$= 1 - \frac{\lambda \theta}{\rho} \int_{y_1} \left( \frac{y_1}{\rho} \right)^{\theta-1} e^{-(\lambda + k \theta_1) \left( \frac{y_1}{\rho} \right)^{\theta}} \, dy_1$$
\begin{align*}
B_2 &= \int_{y_1} f_{x}(k y_1) h_{x}(k y_1) g(y_1) \, dy_1 \\
&= \int_{y_1} \left(1 - e^{-\lambda_1 \left(\frac{y_1}{\rho} \right)^\theta} \right) \left(1 - e^{-\lambda_2 \left(\frac{y_1}{\rho} \right)^\theta} \right) g(y_1) \, dy_1 \\
&= \int_{y_1} \left(1 - e^{-k^\theta \lambda_1 \left(\frac{y_1}{\rho} \right)^\theta} \right) \left(1 - e^{-k^\theta \lambda_2 \left(\frac{y_1}{\rho} \right)^\theta} + e^{-k^\theta (\lambda_1 + \lambda_2) \left(\frac{y_1}{\rho} \right)^\theta} \right) g(y_1) \, dy_1 \\
&= 1 - \frac{\lambda \theta}{\rho} \frac{\rho}{(\lambda + k^\theta \lambda_1)} - \frac{\lambda \theta}{\rho} \frac{\rho}{(\lambda + k^\theta \lambda_2)} + \frac{\lambda \theta}{\rho} \frac{\rho}{(\lambda + k^\theta (\lambda_1 + \lambda_2))} \\
&- \frac{\lambda}{(\lambda + k^\theta \lambda_1)} - \frac{\lambda}{(\lambda + k^\theta \lambda_2)} + \frac{\lambda}{(\lambda + k^\theta (\lambda_1 + \lambda_2))}
\end{align*}

So, \( R(2) = \left(1 - R(1)\right) \left(1 - \frac{\lambda}{(\lambda + k^\theta \lambda_1)} - 1 + \frac{\lambda}{(\lambda + k^\theta \lambda_1)} + \frac{\lambda}{(\lambda + k^\theta \lambda_2)} - \frac{\lambda}{(\lambda + k^\theta (\lambda_1 + \lambda_2))}\right) \)

Then, \( R(3) = \left[1 - R(1)\right]\left[1 - R(2)\right] \)

\( R(3) = \left(1 - R(1)\right)\left(1 - R(2)\right) \left(\frac{\lambda}{(\lambda + k^2 \theta \lambda_2)} - \frac{\lambda}{(\lambda + k^2 \theta (\lambda_1 + \lambda_2))}\right) \)

So,

\( R(n) = \left(1 - R(1)\right)\left(1 - R(2)\right) \ldots \left(1 - R(n - 1)\right) \left(\frac{\lambda}{(\lambda + k^{(n-1)} \theta \lambda_2)} - \frac{\lambda}{(\lambda + k^{(n-1)} \theta (\lambda_1 + \lambda_2))}\right) \)

Substituting the values of \( R(r) \), \( r = 1, 2, \ldots, n \); we can obtain \( R_n \), the reliability of the system.

\( R(4) = \left(1 - R(1)\right)\left(1 - R(2)\right)\left(1 - R(3)\right) \left(\frac{\lambda}{(\lambda + k \theta \lambda_2)} - \frac{\lambda}{(\lambda + k \theta (\lambda_1 + \lambda_2))}\right) \)

3.2. Inverted Exponential Distribution (IED)

The random variables \( X \) and \( Z \) are strengths independent identically distributed with \( x \sim \text{IE}(\alpha_1) \) and \( z \sim \text{IE}(\alpha_2) \), respectively, with cdf given as[8]:

\[
F(x) = e^{-\left(\frac{x}{a_1}\right)^2}, \quad x > 0
\]

\[
H(z) = e^{-\left(\frac{z}{a_2}\right)^2}, \quad z > 0
\]

And let \( y_1 \sim \text{IE}(\alpha) \) be a random stress variable with pdf given as:
\begin{equation}
g(y_1) = \frac{\alpha}{y_1^2} e^{-\left(\frac{\alpha}{y_1}\right)}, \quad y_1 > 0
\end{equation}

Where \( \int_{y_1}^{1} \frac{1}{y_1^2} e^{-\left(\frac{\alpha}{y_1}\right)} dy_1 = \frac{1}{\alpha} \)

Since \( g(y_1) \) is pd function, then from equation 2 we have.

\[ R(1) = A_1 - B_1 \]

So,

\begin{align*}
A_1 &= \int_{y_1} e^{-\left(\frac{\alpha \gamma_1}{y_1^2} + \frac{\alpha}{y_1}\right)} dy_1 = \alpha \int_{y_1} \frac{1}{y_1^2} e^{-\left(\frac{\alpha \gamma_1 + \alpha}{y_1}\right)} dy_1 = \frac{\alpha}{\alpha + \alpha_1 + \alpha_2} \\
B_1 &= \int_{y_1} e^{-\left(\frac{\alpha \gamma_1}{y_1^2} + \frac{\alpha}{y_1}\right)} dy_1 = \alpha \int_{y_1} \frac{1}{y_1^2} e^{-\left(\frac{\alpha \gamma_1 + \alpha}{y_1}\right)} dy_1 = \frac{\alpha}{\alpha + \alpha_1 + \alpha_2} \\
R(1) &= \frac{\alpha}{\alpha + \alpha_1} - \frac{\alpha}{\alpha + \alpha_1 + \alpha_2} = \frac{\alpha_2}{(\alpha + \alpha_1)(\alpha + \alpha_1 + \alpha_2)}
\end{align*}

And now from equation (3), we have

\[ R(2) = (1 - R(1))[A_2 - B_2] \]

\begin{align*}
A_2 &= \int_{y_1} F_x(ky_1) g(y_1) dy_1 = \int_{y_1} e^{-\left(\frac{\alpha_1 \gamma_1}{k y_1^2} + \frac{\alpha}{k y_1}\right)} dy_1 = \alpha \int_{y_1} \frac{1}{y_1^2} e^{-\left(\frac{\alpha \gamma_1 + \alpha}{y_1}\right)} dy_1 = \frac{\alpha}{\alpha + \alpha_1 + \alpha_2} \\
B_2 &= \int_{y_1} F_x(ky_1) H_x(ky_1) g(y_1) dy_1
\end{align*}

\[ B_2 = \int_{y_1} e^{-\left(\frac{\alpha_1 \gamma_1}{k y_1^2} + \frac{\alpha_2}{k y_1}\right) + \frac{\alpha_1}{k y_1^2} + \frac{\alpha_2}{k y_1}} dy_1 = \alpha \int_{y_1} \frac{1}{y_1^2} e^{-\left(\frac{\alpha \gamma_1 + \alpha}{y_1}\right)} dy_1 = \frac{\alpha}{\alpha + \alpha_1 + \alpha_2} \]

So

\[ R(2) = \left(1 - \frac{\alpha_2}{(\alpha + \alpha_1)(\alpha + \alpha_1 + \alpha_2)}\right) \left(\frac{\alpha}{\alpha + \alpha_1 k^2} - \frac{\alpha}{\alpha + \alpha_1 k^2 + \alpha_2 k^2}\right) \]

And

\[ R(3) = (1 - R(1))(1 - R(2)) \left(\frac{\alpha}{\alpha + \alpha_1 k^2} - \frac{\alpha}{\alpha + \alpha_1 k^2 + \alpha_2 k^2}\right) \]

So

\[ R(n) = (1 - R(1))(1 - R(2)) \ldots (1 - R(n - 1)) \left(\frac{\alpha}{\alpha + \frac{\alpha_1}{k(n-1)}} - \frac{\alpha}{\alpha + \frac{\alpha_1 + \alpha_2}{k(n-1)}}\right) \]

3.3. Exponentiated Inverse Rayleigh Distribution (EIRD)

Let the random variables X and Z are strengths independent identically distributed with \( x \sim EIR (\varphi_1, \phi_1) \) and \( z \sim EIR (\varphi_2, \phi_2) \), respectively, with cdf given as [1]:

\[ F(x) = \left( e^{-\left(\frac{\varphi_1}{x^2}\right)} \right) = e^{-\left(\frac{\varphi}{x^2}\right)}, \quad x > 0 \]
\[ H(z) = e^{-\left(\frac{z}{2}\right)^2}, \quad z > 0 \]

And, let \( y_1 \) be EIR (\( \rho, \phi \)) random stress variable with pdf given as:
\[
g(y_1) = \frac{2\phi}{y_1^3} e^{-\left(\frac{\phi}{y_1^2}\right)} \quad \text{for} \quad y_1 > 0
\]

Where \( \int_{y_1 \geq 0} \frac{1}{y_1^3} e^{-\left(\frac{\phi}{y_1^2}\right)} dy_1 = \frac{1}{2\phi} \)

Since \( g(y_1) \) is pd function, then from equation (2), we have.

\[
R(1) = A_1 - B_1
\]

\[
A_1 = \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_2}{y_1^2}\right)} e^{-\left(\frac{\phi}{y_1^2}\right)} dy_1 = \frac{2\phi}{y_1^3} \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_1}{y_1^2}\right)} dy_1 = \frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2}
\]

\[
B_1 = \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_2}{y_1^2}\right)} \frac{2\phi}{y_1^3} e^{-\left(\frac{\phi}{y_1^2}\right)} dy_1 = \frac{2\phi}{y_1^3} \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_1}{y_1^2}\right)} dy_1 = \frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2}
\]

So

\[
R(1) = \frac{\phi \rho \phi + \phi_1 + \rho_2 \phi_2}{(\phi + \rho \phi_1)(\phi + \phi_1 + \rho_2 \phi_2)}
\]

And now from equation (3), we have

\[
R(2) = [1 - R(1)] [A_2 - B_2]
\]

\[
A_2 = \int_{y_1 > 0} F_x(ky_1) g(y_1) dy_1 = \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_2}{ky_1^2}\right)} e^{-\left(\frac{\phi}{y_1^2}\right)} dy_1
\]

\[
= \frac{2\phi}{y_1^3} \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_1}{ky_1^2}\right)} dy_1 = \frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2}
\]

\[
B_2 = \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_2}{ky_1^2}\right)} \frac{2\phi}{y_1^3} e^{-\left(\frac{\phi}{y_1^2}\right)} dy_1 = \frac{2\phi}{y_1^3} \int_{y_1 > 0} e^{-\left(\frac{\phi + \rho \phi_1}{ky_1^2}\right)} dy_1 = \frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2}
\]

So

\[
R(2) = \left(1 - \frac{\phi \rho \phi + \phi_1 + \rho_2 \phi_2}{(\phi + \rho \phi_1)(\phi + \phi_1 + \rho_2 \phi_2)}\right) \left(\frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2} - \frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2}\right)
\]

Then, \( R(3) = (1 - R(1))(1 - R(2)) \left(\frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2} - \frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2}\right) \)

And, \( R(4) = (1 - R(1))(1 - R(2))(1 - R(3)) \left(\frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2} - \frac{\phi}{\rho \phi + \phi_1 + \rho_2 \phi_2}\right) \)

So
\[ R(n) = (1 - R(1))(1 - R(2)) \cdots (1 - R(n-1)) \left( \frac{\phi}{\phi + \frac{\phi_1}{\kappa(n-1)}} - \frac{\phi}{\phi + \frac{\phi_1 + \phi_2}{\kappa(n-1)^2}} \right) \]

3.4. Frechet Distribution (FD)

Let the random variables \( X \) and \( Z \) are strengths independent identically distributed with \( X \sim F(\delta_1, \beta) \) and \( Z \sim F(\delta_2, \beta) \), respectively, with cdf given as [6]:

\[ F(x) = e^{-\delta x^{-\beta}}, \quad x > 0 \]
\[ H(z) = e^{-\delta z^{-\beta}}, \quad z > 0 \]

And, let \( Y_1 \) be \( F(\delta, \beta) \) random stress variable with pdf given as:
\[ g(y_1) = \delta \beta y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}}, \quad y_1 > 0 \]

Where \( \int y_1 y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 = \frac{1}{\delta \beta} \)

Since \( g(y_1) \) is pdf function, then from equation (2), we have:

\[ R(1) = A_1 - B_1 \]
\[ A_1 = \int y_1 e^{-\delta_1 y_1^{-\beta}} \delta \beta y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 = \delta \beta \int y_1 y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 \]
\[ = \frac{\delta}{\delta + 1} \]
\[ B_1 = \int y_1 e^{-\delta_1 y_1^{-\beta}} e^{-\delta_2 y_2^{-\beta}} \delta \beta y_2^{-(\beta+1)} e^{-\delta y_2^{-\beta}} \, dy_1 \]
\[ = \delta \beta \int y_1 y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 = \frac{\delta}{\delta + \delta_1 + \delta_2} \]

And
\[ R(1) = \frac{\delta \delta_2}{(\delta + \delta_1)(\delta + \delta_1 + \delta_2)} \]

And now from equation (3), we have

\[ R(2) = [1 - R(1)][A_2 - B_2] \]
\[ A_2 = \int y_1 F_x(ky_1)g(y_1) \, dy_1 = \int y_1 e^{-\delta_1 (ky_1)^{-\beta}} \delta \beta y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 \]
\[ = \delta \beta \int y_1 y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 = \frac{\delta}{\delta + \delta_1 k^{-\beta}} \]
\[ B_2 = \int y_1 F_x(ky_1)H_z(ky_1)g(y_1) \, dy_1 = \int y_1 e^{-\delta_1 (ky_1)^{-\beta}} e^{-\delta_2 (ky_1)^{-\beta}} \delta \beta y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 \]
\[ = \delta \beta \int y_1 y_1^{-(\beta+1)} e^{-\delta y_1^{-\beta}} \, dy_1 = \frac{\delta}{\delta + \delta_1 k^{-\beta} + \delta_2 k^{-\beta}} \]

So
\[ R(2) = (1 - R(1)) \frac{\delta \delta_2 k^{-\beta}}{(\delta + \delta_1 k^{-\beta})(\delta + \delta_1 k^{-\beta} + \delta_2 k^{-\beta})} \]

Then, \( R(3) = (1 - R(1))(1 - R(2)) \frac{\delta \delta_2 (k^{-\beta})}{(\delta + \delta_1 (k^{-\beta}))(\delta + \delta_1 (k^{-\beta}) + \delta_2 (k^{-\beta}))} \]

So
\[ R(n) = (1 - R(1))(1 - R(2)) \cdots (1 - R(n - 1)) \left( \frac{\delta \delta_{2}(k^{(n-1)})^{-\beta}}{(\delta + \delta_{1}(k^{(n-1)})^{-\beta})} \left( \frac{\delta_{1}(k^{(n-1)})^{-\beta} + \delta_{2}(k^{(n-1)})^{-\beta}}{\delta + \delta_{1}(k^{(n-1)})^{-\beta}} \right) \right) \]

Then, \( R_{n} = R(1) + R(2) + \cdots + R(n) \)

4. Numerical Evaluation and Results

For the four different distribution and by specific values of the parameters involved in mathematical expressions of \( R(r), r = 1, 2, 3, 4 \) the system reliability \( R_{n} \) and the marginal reliabilities \( R(1), R(2), R(3) and R(4) \) (i.e. n=4) are evaluated and recorded in the following cases.

4.1. For Exponential Pareto Distribution (EPD)

- \( \lambda_{1} = 0.9, \lambda_{2} = 2.4 and k = 3 \)
  
  When drawing the parameter \( \lambda \) against reliability and for the different values of parameter \( \theta \), we notice that the curves start from one point with good reliability with a small value, where the curve escalates to the largest value, and then it begins to decrease and stability, so that the decrease with the increase in the value of parameter \( \lambda \), but when the value of Parameter \( \theta \) to greater values, reliability begins to decrease gradually until steady state is reached and then decreasing.

- \( \lambda_{1} = 0.9, \lambda_{2} = 2.4 and k = 3 \)

  And at the same default values for both parameters \( \lambda_{1} and \lambda_{2} \) and the coefficient \( k \), the reliability graph against parameter \( \theta \) at the different values of parameter \( \lambda \), gave a direct decreasing form at the small value of parameter \( \lambda \) and when increasing it led to a very slight increase in reliability (An indication of the increased achievement required in the system) and from then the direct drop to the state of decreasing, while at the greater value of the parameter \( \lambda \), it gave a significant increase in reliability until it reached the inflection point with a considerable distance and then begins to decrease.

- \( \lambda = 0.9, k = 3 \)

  For indicators that plot reliability against parameter \( \theta \) with different values for each of the parameters \( \lambda_{1} and \lambda_{2} \), when the values of the parameter \( \lambda_{1} \) exceed the values of the parameter \( \lambda_{2} \), the reliability is decreasing with slight stability in the second case.

  The reliability behavior is as shown in the following figures with different values of shape parameters for the Exponential Pareto Distribution.

![Figure 1. The Reliability curve against parameter \( \lambda \)]
4.2. For Inverted Exponential Distribution (IED)

- **k=2**
  When drawing the relationship between reliability $R$ with parameter $\alpha$ for different values of two parameters $\alpha_1$ and $\alpha_2$, it gave a form that initially increased the reliability $R$ sharply from the point of origin and then inflection to decrease after $R$ exceeded the value (0.5) that is $k=2$.

- **k=1.5**
  As for the lower value of the coefficient $k$, the figures remained the same but with increasing the value of the inflection to decreasing and approaching (0.6) for the case $\alpha_1 = 0.8$ and $\alpha_2 = 0.4$, also for the other two.

- **k=0.7**
  With a smaller value than the previous state of the coefficient $k$, the shapes were more deploy and better than their predecessors. Therefore, we note that as the value of $k$ decreases the chance of reliability increases and it takes a longer period of time before the process of decreasing begins.

The reliability behavior is as shown in the following figures with different values of coefficients for the Inverted Exponential Distribution.
Figures 3. The Reliability curve against parameter $\alpha$

4.3. For Exponentiated Inverse Rayleigh Distribution (EIRD)

- $\theta_1 = 0.78, \theta_2 = 0.23, k = 1.2$
  Here we notice the reliability value starting from the origin point it increases rapidly with the small values of the parameter $\theta$ takes a sharp inflection point at the value of ($\theta = 0.8$), and then starts decreasing rapidly when ($k = 1.2$).

- $\theta_1 = 0.78, \theta_2 = 0.23, k = 0.8$
  When the value decreases to ($k = 0.8$), the curve of the inflection from increasing to decreasing becomes wider and continues to the middle of the parameter value $\theta$, before it begins to decrease.

- $\phi_1 = 10.3, \phi_2 = 7.2, k = 1.5$
  For the parameter $\phi$ at different values of the parameters $\theta$, $\theta_1$, $\theta_2$ and constant values of the two parameters $\phi_1$, $\phi_2$ and $k = 1.5$, the reliability curve increases with increasing the values of this parameter, and it begins with a constant distance to a significant distance and then decreases and is similar in that the increasing and overlapping values of the parameters $\theta$, $\theta_1$, $\theta_2$. As for the case of a
decrease, which is the second case referred to by the continuous line it gives a decrease faster than the previous two.

- **$\phi_1 = 10.3, \phi_2 = 7.2, k = 2$**
  As the value $k$ increases to 2, the reliability is in the form of increasing curves symmetrical with the possibility of decreasing at values greater than (10) for the parameter $\phi$.

- **$\phi_1 = 0.4, \phi_2 = 1.3, k = 1.2$ and $\phi_1 = 0.4, \phi_2 = 1.3, k = 0.8$**
  In Figures (5) and (6) respectively, lower values were given for the two parameters $\phi_1, \phi_2$, so that draws the reliability against a parameter $\phi$, we notice that it gave a fast decrease in Fig.5, while in Fig.6 it was a lower decrease.

The reliability behavior is as shown in the following figures with different values of shape parameters for the Exponentiated Inverse Rayleigh Distribution.

**Figures 4.** The Reliability curve against parameter $\varphi$
4.4. For Fréchet Distribution (FD)

- $\delta_1 = 0.39, \delta_2 = 0.87, k = 1.5$
  When drawing the delta parameter $\delta$ against different values of the beta parameter $\beta$, we note that the reliability value does not start from the origin point and increases converging until a sharp inflection point of decreasing reliability values at ($k = 1.5$).

- $\delta_1 = 0.39, \delta_2 = 0.87, k = 0.63$
  When the value of the coefficient decreases to ($k = 0.63$), we observe the difference and divergence of the curves, so the best reversal condition is when plotting reliability against parameter $\beta$ at certain values of parameter $\delta$.

- $\delta_1 = 0.39, \delta_2 = 0.87, k = 1.5$ and $\delta_1 = 0.39, \delta_2 = 0.87, k = 0.63$
  In figures (3) and (4), we observe a constant and continuous decrease in reliability when the coefficient value is $k = 1.5$, while we have inflection from increasing to decreasing non-convex at $k = 0.63$, respectively.

The reliability behavior is as shown in the following figures with different values of shape parameters for the Fréchet Distribution.
5. Conclusion

Formulas for system reliability were found for four different distributions, and it found that there are various effects of the special distributions forms for each of the stress and strength variables.

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