Constraints on Parameters of the $R_F$ Parity Model from Quark and Neutrino Mass Matrices

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Abstract

The forms of quark and lepton mass matrices are severely restricted in the $R_F$ parity model. We determine the form of the quark mass matrix first and derive the form of the neutrino mass matrix. For this to be consistent with the present experiments, we conclude that the masses of the superpartners of the right–handed down-type squarks and sneutrino vacuum expectation values satisfy, $m_{\tilde{d}_R} \langle \tilde{\nu}_\mu \rangle \gtrsim 400 \text{ GeV}^2$ or $m_{\tilde{d}_R} \langle \tilde{\nu}_\tau \rangle \gtrsim 400 \text{ GeV}^2$. We also find that without a sterile neutrino it is difficult to obtain a large mixing angle solution of $\nu_\mu$ and $\nu_\tau$. With a sterile neutrino, we show a possibility for a large mixing angle solution of $\nu_\mu$ with $\nu_{\text{sterile}}$.

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I. INTRODUCTION

The strong CP problem has two most attractive possibilities: the spontaneously broken Peccei-Quinn (PQ) symmetry \[1\] and the massless quark solution \[2\]. For the case of axion, one can easily construct models for a very light axion \[3\], since the very light axion is just an addition to the standard model and its effect to the standard model is suppressed by power(s) of the axion decay constant \( F_a \sim 10^{12} \text{ GeV} \). For very light axions, there exists a good motivation from the superstring that a model-independent axion should exist \[4\]. Probably, this is the most attractive theoretical reason for the very light axion \[5\]. On the other hand, it has been very difficult to construct a consistent model for the massless up quark because any new symmetry to make the up quark massless affects the predictions of the standard model sector significantly. It has been conjectured that somehow a symmetry in superstring models would render up quark massless. But this is also disfavored by the problem of no global symmetry in superstring models except the one corresponding to the model-independent axion \[6\]. Even if one has a theory for massless up quark, it will satisfy many constraints related to the fermion mass matrix. Here lies the difficulty of building a massless up quark model. Therefore, it will be very interesting to construct phenomenologically allowable massless up quark models. Indeed, there exist an attempt to build a model toward \( m_u = 0 \) with a \( U(1) \) horizontal symmetry \[7\].

Recently, a kind of \( R \) parity named “\( R_F \) parity” was introduced to solve the strong CP problem with almost massless up quark \[8\]. The \( R_F \) parity model was constructed such that it satisfies the proton decay constraint and possibility for generating quark and lepton masses \[8\]. In this paper, we study the \( R_F \) parity model further to constrain its parameter space from the known quark and lepton mass matrix textures. We find that it is not impossible to satisfy the phenomenological constraints.

The \( R_F \) parity is defined as

\[
R_F = (-1)^{3B+L+2S}(-1)^{2I_F},
\]

where \( F = \delta_{f1} \) (\( f = 1, 2, 3 \)), and \( B, L, S, I \) and \( F \) are the baryon number, lepton number, spin, weak isospin, and the first family number, respectively. In this model the \( R_F \) parity distinguishes the first family from the second and the third ones. Thus, it is possible to make the first family massless. This is good for up quark but bad for electron and down quark. Therefore, it was necessary to break the \( R_F \) parity spontaneously by vacuum expectation values (VEV) of the sneutrino fields. The sneutrino fields have the same quantum number as the down type Higgs field \( H_1 \); thus their VEV’s render electron and down type quark massive\[1\]. Still up quark remains massless since the quantum numbers of sneutrinos are different from the up type Higgs field \( H_2 \).

Another kind of \( R \) parity can be defined such that \( U_1^c \) (the so-called righthanded up quark singlet superfield) has a different quantum number from the rest of the quark and lepton superfields. This possibility will make up quark massless, but does not explain why the other first family members are light. This possibility will be discussed in Sec. V after exploiting the phenomenological implications of the \( R_F \) parity model.

\[1\]With a different motivation, similar idea was considered in Ref. \[9\].
The $R$ parities responsible for the masslessness of up quark can have a root in superstring models. Presumably these $R$ parities can be a discrete subgroup of $U(1)_R$ global symmetry \[^{10}\]. Even though the string theory does not admit a global $U(1)_R$ symmetry, it can allow discrete groups. Therefore, the study of discrete groups toward massless up quark can have a far reaching extension toward superstring models.

In Sec. II, we discuss the textures of the quark mass matrix, and obtain bounds the sneutrino VEV’s. In Sec. III, we study the lepton mass matrix, and obtain bounds on the masses of zino and down type squarks. In Sec. IV, we introduce $U(1)$ horizontal gauge symmetry so that the desired mass matrices are obtained from the Froggatt-Nielsen idea \[^{11}\]. The possibility of the opposite $R$ parity charge for $U^c_i$ is discussed in Sec. V. Finally, we summarize our works in Sec. VI. In Appendix, we present superpotential terms obeying the $U(1)_X \times U(1)_Z$ symmetry, and suggest a mechanism of generating the $\mu$-term.

II. THE CKM MATRIX

The most general $d = 3$ superpotential consistent with the $R_F$ parity is

$$W_0 = \sum_{i,j} f^u_{ij} L_i E^c_j H_1(i \neq 1) + f^u_{ij} Q_i U^c_j H_2(i \neq 1) + f^d_{ij} Q_i D^c_j H_1(i \neq 1) + \lambda_{ijk} L_i L_j E^c_k(j \neq 1) + \lambda_{ijk} L_i Q_j D^c_k(j \neq 1).$$

This superpotential gives $m_e = m_d = m_u = 0$ naturally. To explain the nonzero $m_e$ and $m_d$, we introduce a soft $R_F$ parity violating term, which is hoped to mimic a general feature in other $R_F$ breaking models,

$$W_1 = M_S S^2 + \epsilon^2 S + f_{si} S L_i H_2(i \neq 1),$$

where $S$ is a singlet superfield with $Y = 0$ and $R_F = -1$ and we suppress the $d = 4$ baryon number violating terms. With this $R_F$ parity violation, the electron and the down quark obtain their masses through the vacuum expectation values (VEV’s) of the $S$ and sneutrino fields given by

$$\langle S \rangle \sim \frac{\epsilon^2}{M_S}, \quad \langle \tilde{\nu}_i \rangle \sim \frac{f_{si} \nu_2 \epsilon^2}{M_{\tilde{\nu}_i}^2},$$

where $\nu$’s are VEV’s of the neutral Higgs fields $\langle H^0_\alpha \rangle (\alpha = 1, 2)$ and $M_{\tilde{\nu}_i}^2$ denotes the mass of the $i$-th generation sneutrino field. Let us note that $\langle \tilde{\nu}_i \rangle$’s are not suppressed by $\frac{1}{M_S}$ due to the $M_S S^2$ term in $W_1$.

From the above $R_F$ parity conserving and violating terms, the mass matrices of up- and down-type quarks and charged leptons are given by

$$(M_u)_{ij} = f^u_{ij} \frac{\nu_2}{\sqrt{2}}(j \neq 1),$$

$$(M_d)_{ij} = f^d_{ij} \frac{\nu_1}{\sqrt{2}}(j \neq 1) + \lambda_{ij} \langle \tilde{\nu}_1 \rangle(j \neq 1) + \sum_{k=2,3} \lambda_{kji} \langle \tilde{\nu}_k \rangle \delta_{j1},$$

$$(M_l)_{ij} = f^l_{ij} \frac{\nu_1}{\sqrt{2}}(j \neq 1) + \lambda_{ij} \langle \tilde{\nu}_1 \rangle(j \neq 1) + \sum_{k=2,3} \lambda_{kji} \langle \tilde{\nu}_k \rangle \delta_{j1}.$$
The vanishing elements of the first column of mass matrix of the up-type quark guarantees
the massless up quark at the tree level. And it is obvious that the elements of the first
columns of mass matrices of the down-type quarks and the charged leptons are also zero
without the soft $R_F$ parity violation, i.e. without sneutrino VEV’s, $\langle \tilde{\nu}_2 \rangle$ and
$\langle \tilde{\nu}_3 \rangle$.

Here, we try to satisfy the phenomenologically known quark mass hierarchy. This hier-
archy is expressed in terms of a small expansion parameter, $\lambda \simeq 0.22$ which is sine of the
Cabibbo angle. Using renormalization group equations,\footnote{We find that the phases and $O(\lambda^4)$ term of $U_L$ do not change our results.} we have

$$\begin{align*}
\frac{m_d(m_t)}{m_b(m_t)} &\sim \lambda^4, \quad \frac{m_s(m_t)}{m_b(m_t)} \sim \lambda^2, \quad \frac{m_c(m_t)}{m_t(m_t)} \sim \lambda^4, \\
\frac{m_b(m_t)}{m_t(m_t)} &\sim \lambda^3.
\end{align*}$$
\hfill (6)

These ratios of masses indicate that the unitary matrix defining mass eigenstate up type
quarks in terms of the weak eigenstate quarks has the form $U_L = 1 + O(\lambda^4)$ up to phases.\footnote{The matrices $U_L, D_L, u_L$ and $d_L$ should not be confused with the particle names with the same notations.}

Therefore, we consider only the down-type quark mixing from now on, or $V_{\text{CKM}} = D_L + O(\lambda^4)$.

In case that only down-type quarks mix, $M_d$ and $V_{\text{CKM}}$ are constrained by the following
relation,

$$V_{\text{CKM}}^\dagger M_d^\dagger M_d V_{\text{CKM}} = \text{diag}(m_d^2, m_s^2, m_b^2)$$
\hfill (7)

which allows us to extract constraints on the parameters of $M_d$ from the measured values of
the $V_{\text{CKM}}$ elements.

Another experimental data we take into account in the $R_F$ model are the decay modes
of $K^+$ \cite{12},

$$\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) \leq 5.2 \times 10^{-9}, \quad \mathcal{B}(K^+ \to \pi^0 \nu e^+) \leq 0.0482,$$
\hfill (8)

from which one can derive a bound on $\lambda'_{(i\neq 1)1j}$ \cite{12},

$$\left|\lambda'_{(i\neq 1)1j}\right| \lesssim 0.012 \left(\frac{m_{\tilde{d}_{Rj}}}{100 \text{ GeV}}\right),$$
\hfill (9)

where $m_{\tilde{d}_{Rj}}$ denotes the right–handed down–type squark mass. Combining this upper bound
with constraints from $V_{\text{CKM}}$ will lead us to lower bounds on the sneutrino VEV’s.

If $R_F$ parity is exact, then the mixing matrices of up- and down-type left-handed quarks,
which diagonalize $M_u^\dagger M_u$ and $M_d^\dagger M_d$, are given by \footnote{Only the strong interaction coupling is dominant at energy scales lower than the top quark mass.}

$$U_L = \begin{bmatrix} 1 & 0 \\ 0 & u_L \end{bmatrix}, \quad D_L = \begin{bmatrix} 1 & 0 \\ 0 & d_L \end{bmatrix},$$
\hfill (10)
where $u_L$ and $d_L$ are $2 \times 2$ matrices. From these, the CKM matrix is given by

$$V_{CKM}^{RF} = \begin{bmatrix} 1 & 0 \\ 0 & u_L^d d_L \end{bmatrix}.$$  \hspace{1cm} (11)

Therefore, mixing of the first family with the other families and a CP phase do not appear in $V_{CKM}^{RF}$. Thus the $R_F$ parity must be broken spontaneously to render phenomenologically acceptable angles. The spontaneous symmetry breaking of the $R_F$ parity is achieved by the VEV’s of sneutrino fields, $\langle \tilde{\nu}_i \rangle$ ($i = 2, 3$).

In the presence of sneutrino VEV’s, let us parameterize the down-type quark mass matrix as

$$M_d = \begin{bmatrix} \epsilon_1 (m_2)_1 (m_3)_1 \\ \epsilon_2 (m_2)_2 (m_3)_2 \\ \epsilon_3 (m_2)_3 (m_3)_3 \end{bmatrix} \equiv (\vec{\epsilon}, \vec{m}_2, \vec{m}_3).$$  \hspace{1cm} (12)

Then $M_d^d M_d$ is given by

$$M_d^d M_d = \begin{bmatrix} |\vec{\epsilon}|^2 & \vec{\epsilon}^* \cdot \vec{m}_2 & \vec{\epsilon}^* \cdot \vec{m}_3 \\ \vec{\epsilon} \cdot \vec{m}_2 & |\vec{m}_2|^2 & \vec{m}_2^* \cdot \vec{m}_3 \\ \vec{\epsilon} \cdot \vec{m}_3^* & \vec{m}_2 \cdot \vec{m}_3^* & |\vec{m}_3|^2 \end{bmatrix}.$$  \hspace{1cm} (13)

From Eq. (7), we obtain the following relations,

$$|\vec{\epsilon}| = \left(\sqrt{1 + A^2 |z|^2} + \mathcal{O}(\lambda)\right) \frac{m_d}{\lambda} \approx \sqrt{1 + A^2 |z|^2} \lambda m_s,$$

$$|\vec{m}_2| = \left(\sqrt{1 + A^2} + \mathcal{O}(\lambda)\right) m_s,$$

$$|\vec{m}_3| = \left(1 + \mathcal{O}(\lambda^3)\right) m_b,$$

$$\vec{\epsilon}^* \cdot \vec{m}_2 = |\vec{\epsilon}| |\vec{m}_2| \left(\frac{1 + zA^2}{\sqrt{(1 + A^2)(1 + A^2 |z|^2)}} + \mathcal{O}(\lambda)\right),$$

$$\vec{\epsilon}^* \cdot \vec{m}_3 = |\vec{\epsilon}| |\vec{m}_2| \left(\frac{zA}{\sqrt{1 + A^2 |z|^2}} + \mathcal{O}(\lambda)\right),$$

$$\vec{m}_2 \cdot \vec{m}_3 = |\vec{m}_2||\vec{m}_3| \left(\frac{A}{\sqrt{1 + A^2}} + \mathcal{O}(\lambda)\right),$$  \hspace{1cm} (14)

where $z = \rho - i \eta$ and $\lambda$, $A$, $\rho$, and $\eta$ are the conventional Wolfenstein parameters \[13\]. Therefore, for example, one can obtain the following down-type quark mass matrix which has eigenvalues of $m_d$, $m_s$ and $m_b$ and gives a right form for the CKM mixing matrix

$$M_d = \begin{bmatrix} \mathcal{O}(\lambda^4) & \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^4) \\ \lambda^3 & \lambda^2 & \mathcal{O}(\lambda^3) \\ z^* A \lambda^3 & A \lambda^2 & 1 \end{bmatrix} m_b.$$  \hspace{1cm} (15)

From Eqs. (7) and (13), one can derive the following relations
\[ \lambda'_{211} \langle \bar{\nu}_\mu \rangle + \lambda'_{311} \langle \bar{\nu}_\tau \rangle = \mathcal{O}(\lambda^4) m_b, \]
\[ \lambda'_{212} \langle \bar{\nu}_\mu \rangle + \lambda'_{312} \langle \bar{\nu}_\tau \rangle = \lambda^3 m_b, \]
\[ \lambda'_{213} \langle \bar{\nu}_\mu \rangle + \lambda'_{313} \langle \bar{\nu}_\tau \rangle = z^* A \lambda^2 m_b. \]  

(16)

If one of \( \langle \bar{\nu}_\mu \rangle \) or \( \langle \bar{\nu}_\tau \rangle \) dominates, the relative sizes of \( \lambda' \)'s can be fixed independently of the sizes of the sneutrino VEV's. The numerical values of the CKM matrix elements are given for \( \lambda = 0.22 \) and \( |z^* A| = 0.34 \) \[14\]. Therefore, a typical size of the products \( \lambda'_{i\mu} \langle \bar{\nu}_i \rangle \) with \( i = 2, 3 \) and \( j = 1, 2, 3 \) is \((0.01 - 0.05) \text{ GeV}\). On the other hand, it is not easy to derive some meaningful information on the other \( \lambda'_{1\mu} \)'s because these couplings contribute to \( M_d \) together with the conventional Yukawa terms proportional to unknown parameters \( f^d_{ij} \) and \( v_1 \). From Eqs. (9) and (16), it is obvious that at least one of \( \langle \bar{\nu}_\mu \rangle \) and \( \langle \bar{\nu}_\tau \rangle \) should satisfy the following inequality:

\[ \langle \bar{\nu}_{\mu,\tau} \rangle \gtrsim 4 \text{ GeV} \left( \frac{100 \text{ GeV}}{m_{\tilde{d}_R}} \right). \]  

(17)

Note that the lower bound becomes smaller for a larger squark mass.

Though the up–quark mass is zero at tree level, it can be generated radiatively when \( S \) field has a vacuum expectation value as shown in Fig. 1 \[8\]. The one-loop up–quark mass is given by

\[ \delta m_u \sim \sum_{i,k=2,3} \frac{f^u_{ik} \lambda'_{ik}}{16\pi^2} m^d_k f_{Si} \langle S \rangle \sim \sum_{i=2,3} \frac{f_{Si} \lambda'_{1i} \epsilon^2}{16\pi^2} \frac{m_b f^u_{3i}}{M_{\text{SUSY}} M_S}, \]  

(18)

where \( m^d_2 = m_s \) and \( m^d_3 = m_b \) and we used \( \langle S \rangle = \epsilon^2 / M_S \). The combination \( f_{Si} \lambda'_{1i} \epsilon^2 \) is constrained as in Eq. (11) through the relation \( \langle \bar{\nu}_i \rangle \sim f_{Si} v_2 \epsilon^2 / M_{\tilde{\nu}_i}^2 \). Taking \( m_b = 5 \text{ GeV} \), \( v_2 = 100 \text{ GeV} \) and \( M_{\tilde{\nu}_i}^2 = 1 \text{ TeV} \) gives \( f_{Si} \lambda'_{1i} \epsilon^2 \sim 180 \text{ GeV}^2 \). This one–loop up–quark mass should be small to solve the strong CP problem: \( \delta m_u < 10^{-13} \text{ GeV} \) \[5\]. This leads to

\[ \frac{f^u_{31}}{(M_S/\text{GeV})} < 2 \times 10^{-11}. \]  

(19)

This bound is stronger than the rough estimation given in Ref. \[8\] by one order of magnitude.

### III. NEUTRINO MIXING AND STERILE NEUTRINO

Due to the VEV's of the sneutrino and \( S \) fields in our case, three neutrinos mix with four neutralinos and one \( S \) field in the \( 8 \times 8 \) mass matrix. In the \( (\nu_i; \tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0, S) \) basis, the mass matrix is given by

\[ M_0 = \begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}, \]  

(20)

where we neglected the contributions coming from the one-loop diagrams. Here, \( 0 \) is the \( 3 \times 3 \) matrix with 0 entries and \( m_D \) is a \( 3 \times 5 \) matrix.
\[
\begin{pmatrix}
-\frac{1}{2}g'(\bar{\nu}_e) & \frac{1}{2}g(\bar{\nu}_e) & 0 & -\mu_1 & 0 \\
-\frac{1}{2}g'(\bar{\nu}_\mu) & \frac{1}{2}g(\bar{\nu}_\mu) & 0 & -\mu_2 & f_{S_2} \frac{v}{\sqrt{2}} \\
-\frac{1}{2}g'(\bar{\nu}_\tau) & \frac{1}{2}g(\bar{\nu}_\tau) & 0 & -\mu_3 & f_{S_3} \frac{v}{\sqrt{2}} \\
\end{pmatrix},
\]

where \(\mu_{2,3} = f_{S(2,3)}\langle S\rangle\). \(M\) in Eq. \((21)\) is a 5 \times 5 mass matrix of the neutralinos and \(S\) field

\[
M = \begin{pmatrix}
cM_2 & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 & 0 \\
0 & M_2 & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 & 0 \\
-\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu & 0 \\
\frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu & 0 & (f_{S_2} + f_{S_3})\langle S\rangle \\
0 & 0 & 0 & (f_{S_2} + f_{S_3})\langle S\rangle & M_S \\
\end{pmatrix},
\]

where \(c \equiv M_1/M_2 = (5/3)\tan^2\theta_W \simeq 0.5\), assuming the unification relation.

Since a typical scale for \(M\) is much larger than that of \(m_D\), it is enough to use the see-saw formula to find the following reduced neutrino mass matrix \([15]\) :

\[
m_{\nu}^{\text{eff}} = -m_D M^{-1} m_D^T = \frac{cg^2 + g'^2}{D} \left( \begin{array}{cc}
\Lambda_e^2 & \Lambda_e \Lambda_\mu \\
\Lambda_e \Lambda_\mu & \Lambda_\mu^2
\end{array} \right) - \frac{v^2}{2M_S} \left( \begin{array}{ccc}
0 & 0 & 0 \\
0 & f_{S_2} & f_{S_2} f_{S_3} \\
f_{S_2} f_{S_3} & f_{S_3}^2 & \end{array} \right) + \mathcal{O}\left(\frac{1}{M_S^2}\right),
\]

where \(\Lambda_i\) and \(D\) are given by

\[
\Lambda_i = \mu\langle \bar{\nu}_i \rangle - v_\mu i, \\
D = 2\mu \left[2cM_2 \mu + v_1 v_2 \left( cg^2 + g'^2 \right) \right].
\]

Neglecting the \(\mathcal{O}(1/M_S^2)\) contributions, the neutrino mass matrix, Eq. \((23)\), has one zero eigenvalue and two nonzero eigenvalues

\[
m_{\nu_1} = 0, \\
m_{\nu_2} = \frac{[(f_{S_2} \Lambda_\tau - f_{S_3} \Lambda_\mu)^2 + (f_{S_2}^2 + f_{S_3}^2) \Lambda_\mu^2] v_2^2}{2\Lambda^2 M_S}, \\
m_{\nu_3} = \frac{(cg^2 + g'^2)\Lambda^2}{D} - \frac{(f_{S_2} \Lambda_\mu + f_{S_3} \Lambda_\tau)^2 v_2^2}{2\Lambda^2 M_S},
\]

where \(\Lambda^2 = \Lambda_e^2 + \Lambda_\mu^2 + \Lambda_\tau^2\).

Naively, one would expect that \(\Lambda_{\mu,\tau}\) are smaller than \(\Lambda_e\) because \(\Lambda_e\) comes from the \(R_F\)-parity conserving parts. But, this expectation is not true because \(\langle \bar{\nu}_{\mu,\tau} \rangle\) given in Eq. \((3)\) is not suppressed by \(M_S\). Namely, the sneutrino \(\bar{\nu}_e\), preserving the \(R_F\) parity, and sneutrinos \(\bar{\nu}_{\mu,\tau}\), violating the \(R_F\) parity, can have comparable VEV’s, or even \(\langle \bar{\nu}_e \rangle\) can be smaller than \(\langle \bar{\nu}_{\mu,\tau} \rangle\) depending on the sizes of the corresponding soft SUSY breaking terms. This is because below the \(R_F\) breaking scale their VEV’s are determined by the respective potentials. Then, the mixing matrix \(V_\nu\) which diagonalizes \(m_{\nu}^{\text{eff}}\) satisfies \(V_\nu^T m_{\nu}^{\text{eff}} V_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\), and has the following form

\[
V_\nu = \begin{pmatrix}
\Lambda_\tau/\Lambda & -(\Lambda_e \Lambda_\mu)/(\Lambda \Lambda') & -\Lambda_e/\Lambda \\
0 & \Lambda'/\Lambda & \Lambda_\mu/\Lambda \\
-\Lambda_e/\Lambda & -(\Lambda_\mu \Lambda_\tau)/(\Lambda \Lambda') & \Lambda_\tau/\Lambda
\end{pmatrix} + \mathcal{O}\left(\frac{1}{M_S}\right),
\]

where \(\Lambda = \sqrt{\Lambda_e^2 + \Lambda_\mu^2 + \Lambda_\tau^2}\).
where $\Lambda' \equiv \Lambda^2 + \Lambda^2$. On the other hand, the mixing matrix of the left–handed charged leptons $\rho (L_L)$ which satisfies $L_L^\dagger M_L L_L = \text{diag}(m_e, m_\mu, m_\tau)$ does not mix the first family with the other family members without $R_F$–parity violation as the left–handed quark mixing matrices in Eq. (10) do not mix the first family members. Therefore, in the presence of $R_F$–parity violation we obtain

$$\nu_e = \left( L_L^\dagger V_\nu \right)_{1k} \nu_k \approx \frac{(\Lambda_\tau/\Lambda')}{\nu_1} - \frac{(\Lambda_\mu/\Lambda)}{\nu_2} + \frac{(\Lambda_\tau/\Lambda)}{\nu_3} \nu_3$$

where $\nu_i \ (i = 1, 2, 3)$ are the mass eigenstates.

From the lower bounds on $\langle \tilde{\nu}_{\mu, \tau} \rangle$ given in Eq. (17), one can obtain the following lower bound on $m_{\nu_3}$:

$$m_{\nu_3} \geq \frac{c g^2 + g'^2}{D} \langle \tilde{\nu}_{\mu, \tau} \rangle^2 \sim 80 \text{ KeV} \left( \frac{300 \text{ GeV}}{M_2} \right)^2 \left( \frac{1 \text{ TeV}}{m_{\tilde{d}_Rj}} \right)^2,$$

where we take $c = 0.5$ as given by the unification condition. This means that $\Lambda_\mu$ and/or $\Lambda_\tau$ are significantly greater than $\Lambda_e$ to avoid the present mass bound on the electron neutrino: $m_{\nu_e} < O(1) \text{ eV}$.

Let’s assume $\Lambda_\tau \gg \Lambda_e$ and the off-diagonal elements of charged lepton mass matrix $M_l$ are zero, then we obtain

$$\nu_e \sim \nu_1,$$

$$\nu_\mu \sim (\Lambda_\tau \nu_2 + \Lambda_\mu \nu_3)/\Lambda,$$

$$\nu_\tau \sim (-\Lambda_\mu \nu_2 + \Lambda_\tau \nu_3)/\Lambda.$$

If one of $\Lambda_\mu$ and $\Lambda_\tau$ dominates, then $\nu_\mu$ or $\nu_\tau$ will be the mass eigenstate $\nu_3$ whose mass is greater than $\sim 100 \text{ KeV}$. This implies that it is hard to explain the observation of the large mixing and $\sqrt{\Delta^2_{\text{atm}}} \sim 5 \times 10^{-2} \text{ eV}$ given by the Super-Kamiokande in terms of $\nu_\mu - \nu_\tau$ oscillation [14]. To explain the Super-Kamiokande observation, let’s introduce the sterile neutrino $N$ which is a singlet superfield with $Y = 0$ and $R_F = -1$. The relevant additional superpotential is given by

$$W_2 = M_N N^2 + M_{NS} NS + f_{N_i} N L_i H_2 (i \neq 1) + \frac{\lambda_{Nijk}^\mu}{M_\mu} N U_i^c D_j^c D_k^c,$$

where $M_\mu$ is the Planck mass. The VEV of $N$ field $\langle N \rangle$ induced by $\langle S \rangle$ is

$$\langle N \rangle \sim \frac{\epsilon M_{N S}}{M_N^2} + \frac{\epsilon M_N M_{N S}}{M_S M_N^2} + \frac{(f_{S_i} + f_{N_i}) M_N \langle \tilde{\nu}_i \rangle v_2}{M_N^2}.$$

If $M_N$ and $M_{NS}$ are negligible, then $\langle N \rangle \simeq 0$ and hence we can avoid the fast proton decay, arising from the last term of Eq. (28). The explicit mixing between $S$ and $N$ gives two mass eigenstates $S'$ and $N'$ with

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5The matrix $L_L$ should not be confused with the left-handed lepton superfield.
\[ M_{S'} \approx M_S \quad \text{and} \quad M_{N'} \approx M_N - M_{NS}^2/M_S. \quad (32) \]

With appropriate \( M_N \) and \( M_{NS} \) values, one could obtain almost vanishing sterile neutrino mass \( M_{N'} \).

Assuming \( |m_{\nu_\mu}^2 - m_{\nu_N}^2| \approx \Delta_{\text{atm}}^2 \) and \( \Lambda_\tau \gg \Lambda_\mu \), from Eqs. (4) and (25), we obtain
\[ m_{\nu_\mu} \sim m_{\nu_2} \sim \frac{f_{N_2}^2 v_2^2}{2M_S} \sim \frac{M_{N_2}^4 \langle \tilde{\nu}_\mu \rangle^2}{2\epsilon^4 M_S}. \quad (33) \]

Taking \( \sqrt{\Delta_{\text{atm}}} \approx m_{\nu_\mu} \) assuming almost vanishing sterile neutrino mass, we obtain
\[ M_S \sim 10^{10} \text{ GeV} \left( \frac{M_{N_2}}{\epsilon} \right)^4 \left( \frac{\langle \tilde{\nu}_\mu \rangle^2}{1 \text{ GeV}^2} \right). \quad (34) \]

For \( M_{N_2} \approx 10 \text{ TeV} \), \( \epsilon \approx 100 \text{ GeV} \), and \( \langle \tilde{\nu}_\mu \rangle \approx 10^{-3} \text{ GeV} \), the mass parameter \( M_S \) should be around \( 10^{12} \text{ GeV} \) which is the intermediate scale needed in supergravity and invisible axion \[17\]. Also, the large mixing between \( N \) and \( \nu_\mu (\sim \nu_2) \) could be obtained taking \( f_{N_2} v_2 \approx m_{\nu_2} \) with \( f_{N_2} \gg f_{N_3} \).

Note that the assumed hierarchy between the sneutrino VEV’s \( \langle \tilde{\nu}_\tau \rangle \gg \langle \tilde{\nu}_\mu \rangle, \langle \tilde{\nu}_e \rangle \) is consistent with the neutrino data since we introduced a sterile neutrino. In this case, \( \nu_\tau \) can decay to a lighter neutrino plus photon, which may affect the evolution of the universe. The neutrino interaction with electromagnetic fields due to nonzero transition magnetic moment is described by
\[ f' \mu_B (\bar{\nu}_i \sigma_{\mu\nu} \nu_\tau) F^{\mu\nu} + \text{H.c.}, \quad (35) \]
where \( \mu_B \) denotes the Bohr magneton. Then the partial lifetime of the tau neutrino is \[18\]
\[ \tau_{\nu_\tau \to \nu_i} \approx \frac{2\pi}{m_{\nu_\tau} f'_i \mu_B^2 \epsilon} \approx 4.5 \times 10^{-3} \left( \frac{100 \text{ KeV}}{m_{\nu_\tau}} \right)^3 \left( \frac{10^{-7}}{f'_i} \right)^2 \text{ sec}. \quad (36) \]

The decay, \( \nu_\tau \to \nu_i + \nu_j + \bar{\nu}_k \), is negligible compared to the above photon mode.

Note also that our assumed hierarchy on the sneutrino masses that the third generation VEV is much smaller than those of the first and second ones, \( M_{\tilde{\nu}_3} \ll M_{\tilde{\nu}_1,2} \), is possible in the so-called effective SUSY framework \[19\].

**IV. HORIZONTAL SYMMETRY**

The mass hierarchy of quarks and charged leptons can be explained by introducing an *abelian horizontal symmetry* \( U(1)_X \times U(1)_Z \) in our framework. This kind of horizontal symmetry at high energy scale, presumably at the Planck scale, might be necessary to introduce an expansion parameter in the mass matrix \[14\]. In string models, there appear

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\[6\] This effective SUSY was proposed to suppress the SUSY contributions to low-energy flavor changing neutral current (FCNC) or CP violating processes.
many gauge U(1) symmetries which act as horizontal symmetries in models distinguishing all fixed points [20]. In this case, the expansion parameter(s) is the ratio(s) of VEV(s) of SM singlet field(s) and the string scale. In this spirit, we try to search for a possibility of introducing the mass matrices discussed in the previous sections.

Because it is very difficult to study the general mass matrix, we are guided first by the phenomenologically plausible relation $M_l \approx M_d$ (but with a modification a la Georgi and Jarlskog [21]), and a phenomenological hierarchy $m_2^2(m_t)/m_4^2(m_t) \sim \lambda^4$. As explained before, then $|U_L| = 1 + \mathcal{O}(\lambda^4)$; and we can take the following forms for the mass matrices using $\lambda$ as an expansion parameter

$$M_u \sim \begin{bmatrix} 0 & \lambda^6 & \lambda^2 \\ 0 & \lambda^5 & \lambda \\ 0 & \lambda^4 & 1 \end{bmatrix} m_t, \quad M_d \sim \begin{bmatrix} \lambda^7 & \lambda^6 & \lambda^7 \\ \lambda^6 & \lambda^5 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^3 \end{bmatrix} m_t, \quad M_l \sim \begin{bmatrix} \lambda^7 & \lambda^6 & \lambda^7 \\ \lambda^6 & \lambda^5 & \lambda^6 \\ \lambda^6 & \lambda^5 & \lambda^3 \end{bmatrix} m_t. \quad (37)$$

Some elements arise as higher order corrections. For example, the first and the second rows of $M_u$ and $(1,i)$ and $(2,3)$ elements of $M_d$ arise as higher order corrections. Note that the lower right $2 \times 2$ submatrix of $M_u$ gives a determinant of order $\lambda^5 m_t^2$. However, this does not imply $m_c \sim \lambda^5 m_t$ since $M_u$ must be diagonalized by a biunitary transformation, implying this determinant is not invariant. To get eigenvalues, we diagonalize $M_u M_u^\dagger$ in which case we obtain $m_c \sim \lambda^4 m_t$.

Let us introduce three SM singlet fields, $\theta_1$, $\theta_2$ and $\theta_3$, to explain the above mass hierarchies, which seems to be our minimal choice. We further assume a universal VEV to simplify the analysis,

$$\langle \theta_1 \rangle^3 \approx \langle \theta_2 \rangle^3 \approx \langle \theta_3 \rangle^3 \approx \lambda M^3, \quad (38)$$

where $M$ is the energy scale where nonrenormalizable interactions are introduced but still the $U(1)_X \times U(1)_Z$ symmetry is preserved below $M$. In string models, one can identify $M = M_{\text{string}}$.

We find that the following $U(1)_{X,Z}$ charge assignments are enough to explain the hierarchies of the mass matrices given in Eq. (37).

| Table 1. $U(1)_X \times U(1)_Z$ charges. |
| Q_X | Q_2 | Q_3 | D^c_1 | D^c_2 | D^c_3 | U^c_1 | U^c_2 | U^c_3 |
|---|---|---|---|---|---|---|---|---|
| Q_X | 9/2 | 0 | -5 | -2 | -2 | 4 | 1/3 | 1/3 | 1/3 |
| Q_Z | 6 | -1 | 1 | -9 | -6 | -3 | -1 | 2 | 5 |

| L_1 | L_2 | L_3 | E^c_1 | E^c_2 | E^c_3 | H_1 | H_2 | S |
|---|---|---|---|---|---|---|---|---|
| Q_X | 4 | -1/2 | -11/2 | -3/2 | -3/2 | 9/2 | -1 | 14/3 | x |
| Q_Z | 2 | -5 | -3 | -5 | -2 | 1 | -5 | -6 | z |

where $Q_X$ and $Q_Z$ denote the $U(1)_X$ and $U(1)_Z$ charges, respectively. The above charge assignment is valid for $\tan \beta \sim \mathcal{O}(1)$. For large $\tan \beta$, a somewhat different charge assignment will be obtained. Here, we have not fixed the charges of the $S$ field. Its charges can be fixed once we know the size of $M_S/M$. One has to cancel anomalies. Since we are considering a low energy effective theory, we will ignore the $U(1)_{X,Z}$ related anomalies except
$U(1)_{X,Z} - SU(3)_c - SU(3)_c$ anomaly. This is because an introduction of colored scalars at intermediate scale is dangerous for proton stability. If the symmetry breaking scale of $U(1)_{X,Z}$ is near the grand unification scale, then the proton stability problem reduces to that of supersymmetric models \[22\]. Anyway, we assume that there is no colored scalars below the scale $M$. The other anomalies can be canceled by introducing $Q_X$ and $Q_Z$ carrying color singlet superfields which we do not try to specify due to our ignorance of the super high energy physics. Note that as in any hierarchical model of this type, the ratio of charges of horizontal symmetry $U(1)_{X,Z}$ can include a large number.

The Yukawa couplings rendering quark masses are of the form $Q_i(D_c$ or $U_c)H_\alpha (\alpha = 1$ or $2)$. Among these, only $Q_3U_c^3H_2$ is allowed by the $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_Z$ symmetry. To introduce other Yukawa couplings through nonrenormalizable terms, we need at least three singlet superfields $\theta_1$, $\theta_2$ and $\theta_3$ which are assigned with the charges:

Table 2. $U(1)_X \times U(1)_Z$ charges of superheavy singlets.

| $\theta_1$ | $\theta_2$ | $\theta_3$ |
|------------|------------|------------|
| $Q_X$      | 0          | $-1$       | 1          |
| $Q_Z$      | 1          | $-1$       | 0          |

Then the needed SM invariants become neutral through (non)renormalizable interaction with the $\theta_i$’s. The above charge assignments do not allow the interactions which can give the nonzero elements in the first column of $M_u$. All the other elements of $M_u$ and $M_{d,l}$ anticipated in Eq. (37) can be obtained through interactions which are neutral under the $U(1)_X \times U(1)_Z$ horizontal symmetry. These interactions are shown in the Appendix.

Note that the above charge assignment suggests the relation $\langle \theta_1 \rangle = \langle \theta_2 \rangle = \langle \theta_3 \rangle$ since only the combination of $\theta_1\theta_2\theta_3$ is neutral. The superpotential of $\theta$ fields is given by

$$W_\theta = \lambda_1 (\theta_1\theta_2\theta_3) + \lambda_2 (\theta_1\theta_2\theta_3)^2 + \cdots.$$  \hspace{1cm} (39)

Since this superpotential is symmetric under the exchanges of $(1,2,3)$, it may hints the same VEV’s of $\theta_1$, $\theta_2$, and $\theta_3$. But at this stage only the hyperplane $\theta_1\theta_2\theta_3$ is guaranteed. The D-flat conditions are

$$\sum_{a=1}^{3} Q_X^a |\theta_a|^2 = 0, \quad \sum_{a=1}^{3} Q_Z^a |\theta_a|^2 = 0,$$  \hspace{1cm} (40)

where $Q_X^a$ and $Q_Z^a$ are the $U(1)_X$ and $U(1)_Z$ charges of $\theta_a$, respectively. Therefore, $\theta_1 = \theta_2 = \theta_3$ is the D-flat direction. In addition, the introduction of a universal soft term in the potential $m_{3/2}(\theta_1^2 + \theta_2^2 + \theta_3^2)$ would not destroy the relation $\langle \theta_1 \rangle = \langle \theta_2 \rangle = \langle \theta_3 \rangle$. From this consideration, we conclude that $\theta$ fields have the same magnitude of the vacuum expectation values

$$\langle \theta_1 \rangle = \langle \theta_2 \rangle = \langle \theta_3 \rangle \equiv \lambda^{\frac{1}{3}} M$$  \hspace{1cm} (41)

where $\lambda$ is the expansion parameter. Let us consider the $(2,2)$ element of $M_u$ of Eq. (37) as an illustration. The relevant term which is neutral under the horizontal symmetry is

$$\Theta_1^{10} \Theta_5^5 Q_2 U_2^c H_2,$$  \hspace{1cm} (42)
where the dimensionless parameter $\Theta_i$ are defined as $\Theta_i \equiv \theta_i / M$ and we assume an $O(1)$ coupling. The VEV of $\theta_i$ of order of $\lambda^{1/3}$ and $\langle H_2 \rangle = v \sin \beta$ give

$$\lambda^5 v \sin \beta U_2 U_2' \approx \lambda^5 m_t U_2 U_2'^c$$

which is the $O(\lambda^5)$ given in Eq. (37). Similarly, other elements of $M_u$ can be obtained through interactions which are neutral under the $U(1)_X \times U(1)_Z$ horizontal symmetry.

Now, let us consider the down type quark mass matrix $M_d$. For example, the terms which give the $(1,1)$ element of $M_d$ are

$$\Theta_i^6 \Theta_i^3 L_3 Q_1 D_1'^c \quad \text{and} \quad \Theta_i^{10} \Theta_i^2 L_2 Q_1 D_1'^c$$

which are responsible for a nonvanishing down quark mass. After $\Theta_i$'s develop VEV's of order of $\lambda^{1/3}$ and assuming $\langle \tilde{\nu}_2 \rangle \approx 0$ and $\langle \tilde{\nu}_3 \rangle \approx \lambda^4 m_t$, we obtain

$$\lambda^3 \langle \tilde{\nu}_3 \rangle D_1 D_1'^c \approx \lambda^7 m_t D_1 D_1'^c.$$  

(45)

Similarly, other elements of $M_u$ could be obtained through interactions which are neutral under the $U(1)_X \times U(1)_Z$ horizontal symmetry. Of course, the above dimensionless coupling constants are given at very high energy scale, e.g. at $M$.

Note that our $U(1)_X \times U(1)_Z$ symmetry includes the result of the $R_F$ parity model [8], namely the $U(1)_X$ among $U(1)_X \times U(1)_Z$ does not allow $R_F$ parity violating terms. But the $U(1)_X \times U(1)_Z$ is more restrictive than the $R_F$ parity: it dictates the fermion mass hierarchy through the mass matrix Eq. (37).

V. THE OPPOSITE $R$ PARITY CHARGE FOR $U_1'^c$

Another choice of making the up quark massless is to distinguish $U_1'^c$ from the others. Thus, an $R$ parity called $R_U$ can be defined such that only $U_1'^c$, $H_1$ and $H_2$ have different quantum numbers from the rest of the quark and lepton superfields. For this purpose, let us assign the $R_U$ quantum numbers as

$$R_U(U_1'^c) = R_U(H_1) = R_U(H_2) = +1 \quad \text{and} \quad R_U(\text{other fields}) = -1. \quad (46)$$

Then, the $R_U$-parity conserving superpotential is given by

$$W_{R_U} = f_{i,j}^c L_i E_j^c H_1 + f_{i,j}^u Q_i U_j'^c H_2 (j \neq 1) + f_{i,j}^c Q_i D_j^c H_1 + \chi_{ij}^c U_1'^c D_j^c D_k^c + \mu H_1 H_2,$$  

(47)

where $\chi_{ij}^c = -\chi_{ij}^u$. This superpotential gives the following $Q_{em} = 2/3$ and $Q_{em} = -1/3$ quark mass matrices,

$$M^{(2/3)} = \begin{pmatrix} 0 & 0 & 0 \\ H_2 & H_2 & H_2 \\ H_2 & H_2 & H_2 \end{pmatrix}, \quad M^{(-1/3)} = \begin{pmatrix} H_0^0 & H_0^0 & H_1^0 \\ H_0^0 & H_1^0 & H_1^0 \\ H_1^0 & H_1^0 & H_1^0 \end{pmatrix},$$

(48)

where we have suppressed the Yukawa couplings. The rows count the singlet anti-quarks, and the columns count the doublet quarks. The charged lepton matrices have the same form as the $Q_{em} = -1/3$ quark mass matrices. It is obvious that $\text{Det} M^{(2/3)} = 0$, implying the massless u-quark. But, it does not explain why the other first family members are light. Note that the $R_U$-parity prevents the proton from decaying into ordinary particles such as to $e^+ + \pi^0$. However, the proton can decay if gravitino or axino is lighter than proton \[23\]. In this case, $\chi_{ij}^c$ should be severely constrained by some reasons.
VI. CONCLUSION

We have explored the phenomenological constraints of the $R_F$ parity model which gives $m_u \simeq 0$ consistent with the strong CP solution. In addition, we put an ansatz for the hierarchical fermion mass matrices. Within this framework, we first required that the $R_F$ parity model describes the charged lepton and quark masses and mixing angles properly. In particular, from the observed down type quark mass matrix we derived a plausible form for the sneutrino VEV’s

$$m_{\tilde{d}_{Rj}} \langle \tilde{\nu}_{\mu,\tau} \rangle \gtrsim 400 \text{ (GeV)}^2.$$  \hspace{1cm} (49)

Then we applied this constraint to the recent neutrino oscillation data. We find that without a sterile neutrino a large mixing angle solution of $\nu_\mu$ and $\nu_\tau$ is not possible in the $R_F$ parity model. However, introducing a sterile neutrino, it is possible to generate a large mixing angle solution, if the parameter $M_S$ is of order intermediate scale and $\langle \nu_\mu \rangle$ is of order GeV.

Since the hierarchical fermion mass matrix in the $R_F$ parity model is given in Eq. (37), we introduced a horizontal $U(1)_{X} \times U(1)_{Z}$ symmetry such that the fermion mass matrix arises naturally. This kind of horizontal symmetry can appear in string models frequently. We also pointed out another kind of $R$ parity, $R_U$ parity, rendering up quark massless, but its phenomenological consequences are not explored.

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APPENDIX

In this appendix, we present the interactions which are neutral under $U(1)_X \times U(1)_Z$ horizontal symmetry. These are given by:

- $QD^cH_1$ type
  $$\Theta_1^4\Theta_3^2 Q_3 D_3^c H_1, \quad \Theta_1^{12}\Theta_2^3 Q_2 D_3^c H_1, \quad \Theta_1^{10}\Theta_3^8 Q_3 D_2^c H_1,$$
  $$\Theta_1^{12}\Theta_3^3 Q_2 D_2^c H_1, \quad \Theta_1^{13}\Theta_3^8 Q_3 D_1^c H_1, \quad \Theta_1^{15}\Theta_3^3 Q_2 D_1^c H_1. \hspace{1cm} (50)$$

- $LE^cH_1$ type
  $$\Theta_1^7\Theta_3^3 L_3 E_3^c H_1, \quad \Theta_1^{12}\Theta_2^3 L_2 E_3^c H_1, \quad \Theta_1^{10}\Theta_3^8 L_3 E_2^c H_1,$$
  $$\Theta_1^{12}\Theta_3^3 L_2 E_2^c H_1, \quad \Theta_1^{13}\Theta_3^8 L_3 E_1^c H_1, \quad \Theta_1^{15}\Theta_3^3 L_2 E_1^c H_1. \hspace{1cm} (51)$$

- $QU^cH_2$ type
  $$Q_3 U_3^c H_2, \quad \Theta_1^7\Theta_2^5 Q_2 U_3^c H_2, \quad \Theta_1^3 Q_3 U_3^c H_2,$$
  $$\Theta_1^{10}\Theta_2^5 Q_2 U_2^c H_2, \quad \Theta_1^5 Q_3 U_1^c H_2, \quad \Theta_1^{13}\Theta_2^5 Q_2 U_1^c H_2. \hspace{1cm} (52)$$
\begin{itemize}
  \item \textbf{LQD}^c type
    \begin{align*}
    &\Theta_1^3 \Theta_3^3 L_3 Q_1 D_1^c, \quad \Theta_1^3 \Theta_3^3 L_3 Q_1 D_2^c, \quad \Theta_1^3 \Theta_3^3 L_3 Q_1 D_3^c, \\
    &\Theta_1^1 \Theta_2^2 L_2 Q_1 D_1^c, \quad \Theta_1^1 \Theta_2^2 L_2 Q_1 D_2^c, \quad \Theta_1^1 \Theta_2^2 L_2 Q_1 D_3^c, \\
    &\Theta_1^1 \Theta_2^2 L_1 Q_2 D_1^c, \quad \Theta_1^1 \Theta_2^2 L_1 Q_2 D_2^c, \quad \Theta_1^1 \Theta_2^2 L_1 Q_2 D_3^c, \\
    &\Theta_1^1 \Theta_2^3 L_1 Q_3 D_1^c, \quad \Theta_1^1 \Theta_2^3 L_1 Q_3 D_2^c, \quad \Theta_1^1 \Theta_2^3 L_1 Q_3 D_3^c.
    \end{align*}
\end{itemize}

\begin{itemize}
  \item \textbf{LLE}^c type
    \begin{align*}
    &\Theta_1^6 \Theta_3^3 L_1 L_3 E_1^c, \quad \Theta_1^3 \Theta_3^3 L_1 L_3 E_2^c, \quad \Theta_1^3 \Theta_3^3 L_1 L_3 E_3^c, \\
    &\Theta_1^1 \Theta_2^2 L_1 L_2 E_1^c, \quad \Theta_1^1 \Theta_2^2 L_1 L_2 E_2^c, \quad \Theta_1^1 \Theta_2^2 L_1 L_2 E_3^c.
    \end{align*}
\end{itemize}

Note that the \( U^cD^cD^c \) type terms are not allowed, which ensures that the proton stability.

Note that the so-called \( \mu \) term, \( \mu H_1 H_2 \), is not allowed in the model presented above. However, we need an electroweak scale \( \mu \) term toward a successful \( SU(2) \times U(1) \) symmetry breaking [24]. One example for the \( \mu \) term is through \( \Phi_1 H_1 H_2 \) where \( \Phi_1 \) represents the color singlet superfields introduced to cancel anomalies in Sec. IV. For example, one can introduce the appropriate \( \mu \) term by taking \( Q_X(\Phi_1) = -11/3, Q_Z(\Phi_1) = 11 \) and \( \langle \Phi_1 \rangle = \mu \). This kind of charge assignment does not generate the dangerous term \( U^cD^cD^c \), but can generate the \( L_1 H_2 \) term

\[ \Theta_2^7 \Theta_3^2 \Phi_1 L_1 H_2 = \lambda^3 \mu L_1 H_2 \approx \frac{\mu}{100} L_1 H_2. \]

Note that the relative size of the induced \( \mu_1 \) to \( \mu \) is independent of the charges of \( \Phi_1 \) since the factor \( \lambda^3 \) is given by the differences between charges of \( H_1 \) and \( L_1 \). This size of the induced \( \mu_1 \) could break the relation \( \Lambda_f \gg \Lambda_e \), which we needed to avoid the present mass bound on \( m_{\nu_e} \). Therefore, to suppress \( \Lambda_e \), we require some cancellation between \( \langle \nu_e \rangle \) and \( \lambda^3 v_1 \) or \( \mu_1 \) and the induced \( \mu_1 \).

To give VEV to \( \Phi_1 \) field, one can introduce a field \( \Phi_2 \) with opposite \( U(1)_X \times U(1)_Z \) charge, \( Q_X(\Phi_2) = +11/3, Q_Z(\Phi_2) = -11 \) and then, the relevant superpotential and scalar potential are given by

\[ W = f_\Phi \Phi_1 H_1 H_2 + M_\Phi \Phi_1 \Phi_2, \]
\[ V = M_1^2 |\Phi_1|^2 + M_2^2 |\Phi_2|^2 + (A_\Phi \Phi_1 H_1 H_2 + B_\Phi M_\Phi \Phi_1 \Phi_2 + H.c.), \]

where \( M_1^2 = M_{\Phi_1}^2 + M_{\Phi_2}^2, M_2^2 = M_{\Phi_1}^2 + M_{\Phi_2}^2 \), and \( M_{\Phi_1}, M_{\Phi_2}, A_\Phi \) and \( B_\Phi \) are the soft SUSY breaking parameters. Running \( M_1^2 \) can be driven negative by the large Yukawa coupling \( f_\Phi \). This radiative breaking which is the same mechanism as the radiative electroweak symmetry breaking, generates the VEV’s for \( \Phi \) fields as \( \langle \Phi_1 \rangle \sim \langle \Phi_2 \rangle \sim M_\Phi [25] \).
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FIG. 1. The one–loop $u$ quark mass.