Long-range interactions and roton minimum softening in a spin-orbit coupled Bose-Einstein condensate

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We experimentally probe the collective excitations of a spin-orbit coupled Bose-Einstein condensate and show the existence of roton structures. These are due to the long-range interactions mediated by the spin-orbit coupling, which is generated by Raman dressing of atomic hyperfine states. When the Raman detuning is decreased, the roton mode softens and in our experiment we directly observe this effect by performing Bragg spectroscopy. We also show that for the parameters of our system, this softening stops at a finite excitation gap, which precludes a transition to a supersolid-like phase. A time-reversal like symmetry inherent in the system’s Hamiltonian is confirmed by showing that the roton softening is symmetric under a sign change of the Raman detuning. Finally, using a moving barrier that is swept through the BEC, we also show that the presence of a roton can lead to interesting consequences for the fluid dynamics.

PACS numbers: 03.75.Kk, 03.75.Mn, 32.80.Qk, 71.70.Ej

The flow properties and excitation spectra of superfluids have been fascinating research subjects since the first discovery of frictionless flow in liquid $^4$He. To explain the thermodynamic properties and the critical velocity below which superfluidity occurs, Landau semi-empirically introduced the existence of a particular quasiparticle called roton [1], which manifests itself through a minimum in the excitation spectrum $E(k)$ at finite $k$. Later, Feynman, based on Bijl’s wave-function [2], linked the excitation spectrum to the static structure factor $S(k)$ by a relation now known as the Bijl-Feynman formula, $E(k) = \hbar^2 k^2 / 2 m S(k)$. Here $\hbar k$ denotes the momentum of the excitations and $m$ is the atomic mass. The roton minimum therefore corresponds to a maximum in $S(k)$. Since the static structure factor is a measurement of density-density correlations, non-local density-density correlations can be thought of as being responsible for the existence of rotons.

While rotons were first predicted and confirmed experimentally in superfluid helium [3, 4], later on many other physical systems were also shown to support these excitations. Examples include systems exhibiting the fractional quantum Hall effect [5], antiferromagnets [6] and superfluids with long-range or finite-range interactions [7, 10]. Among the latter are atomic Bose-Einstein condensates (BECs) with dipolar interactions, however the experimental observation of rotons in dipolar BECs is still elusive [11, 14].

The fundamental nature of the roton as a collective excitation gives it a pivotal role when describing superfluidity and its properties are therefore of large interest. One of the most fundamental effects is roton minimum softening, which, when reaching a critical value or on closing the roton gap, can trigger a first-order liquid-solid like phase transition [10, 12]. The roton minimum softening therefore provides a potential route to the formation of supersolids which possess crystalline and superfluid properties at the same time. Very recently, the roton minimum softening and a possible supersolid phase were observed experimentally for the first time in a BEC with cavity-mediated long-range interactions [16].

Here we report on the experimental observation of roton mode softening in a spin-orbit coupled (SOC), dilute gas BEC. The excitation spectrum of such a system can exhibit a phonon-maxon-roton feature [13, 19], which we identify as stemming from the presence of long-range interactions mediated by the spin-orbit coupling. In our experiment we measure the mode excitations of the ground state by performing Bragg spectroscopy [20, 22]. Spin-orbit coupling is implemented by Raman dressing of two or more atomic hyperfine states, which play the role of different (pseudo-)spins [23, 32]. Previous theoretical studies have described roton excitations of SOC BECs for the case of vanishing Raman detuning [18, 19]. In our experiment, we demonstrate the softening of the roton minimum when, starting from a finite value, the Raman detuning is decreased. This softening can result in the closure of the roton gap, and a novel phase given by a one dimensional supersolid is predicted to appear [33]. However, in the case of $^{87}$Rb used in our experiment, the specific set of nonlinear couplings would require an exceedingly low Raman coupling strength to reach this closure point, which currently makes it unfeasible to collect measurable signals. We find a symmetry in the data for the roton softening, which can be explained by a time-reversal like symmetry in the Gross-Pitaevskii Hamiltonian governing the system. Finally, we report on an experiment directed at probing the influence of the roton mode on the hydrodynamical properties of the BEC.

The generation of spin-orbit coupling in our BECs is based on Raman coupling of two selected bare energy hyperfine states of $^{87}$Rb [23, 28]. The Hamiltonian resulting from the Raman coupling is

$$H_{soc} = -\frac{\hbar^2}{2} \frac{\partial^2}{\partial x^2} - i \gamma \frac{\partial}{\partial x} \sigma_z + \frac{\delta}{2} \sigma_z + \frac{\Omega}{2} \sigma_x,$$

(1)

where we have used $2 E_{Raman} = (\hbar k_{Raman})^2 / m$ as the unit of energy, with $2\hbar k_{Raman}$ being the momentum imparted...
on the atoms during a Raman transition. The spin–orbit coupling occurs along the \( x \) direction and is characterized by a coupling strength \( \gamma \). The detuning of the Raman beams from the atomic transition between the hyperfine states is denoted by \( \delta \) and the Rabi frequency by \( \Omega \).

The Hamiltonian can easily be diagonalized leading to the eigenspectrum \( E_{\pm}(k) = \frac{k^2}{2} \pm \sqrt{(\gamma k + \frac{\Omega}{2})^2 + \frac{\delta^2}{4}} \). The corresponding dressed eigenstates, \( \Phi_d \), are connected to the bare ones, \( \phi_b \), by a momentum-dependent rotation through \( \Phi_d = \mathcal{M}(k)\phi_b \) with \( \mathcal{M}(k) = \exp(\imath k_0 \sigma_y/2) \) and \( \tan k_0 = \Omega/(2\gamma k + \delta) \).

In the experiment we initially prepare a BEC in the lowest-energy dressed state \( |1\rangle \). Although the interactions between atoms in the bare states are s-wave and thus short-range, the Raman dressing leads to momentum-dependent interactions stemming from the coupling of momentum-related wave-functions. By performing a Fourier transform, it then becomes clear that such momentum-dependent interactions lead to long-range interactions in the spatial domain, which in turn can produce roton excitations. We note that in Ref. \[34\] the Raman dressed momentum-dependent interactions were used to produce higher-order partial waves in atomic collisions.

The mean-field Gross-Pitaevskii (GP) description of our many-particle system is given by

\[
H_{\text{soc}} \Psi + g \left( |\Psi_1|^2 + |\Psi_2|^2 \right) \Psi = \mu \Psi, \tag{2}
\]

where \( \Psi = (\Psi_1, \Psi_2)^T \) is the spinor describing the two-components, \( \mu \) is the chemical potential, and \( g \) denotes the nonlinear coefficient of the bare energy states. While the theoretical analysis neglects the trapping potential present in the experiment, we will show below how this can be compensated for. We also assume that all scattering lengths are equal, which is a very good approximation for our experimental system \([33]\). It is worth noting that the GP equations and the Hamiltonian \( H_{\text{soc}} \) possess a time-reversal-like symmetry described by \( \mathcal{R}_3 K \sigma_x \), where \( \mathcal{R}_3 \) flips the sign of the detuning, \( \mathcal{R}_3 \delta \mathcal{R}_3^\dagger = -\delta \), and \( K \) is the operator for complex conjugation. This guarantees the symmetry of ground states of the GP equations when the sign of the detuning is changed.

Collective excitations of a spin–orbit coupled BEC can be measured by probing its linear response to a sudden perturbation through a Bragg pulse. The general wavefunction spinor including the ground state, \( \Psi_{1,2} \), and the perturbations, \( \delta \Psi_{1,2} \), can be written as

\[
\Psi_{1,2}(x,t) = e^{-i\mu t + i k x} [\Psi_{1,2}(x) + \delta \Psi_{1,2}(x,t)],
\]

where \( \mu \) and \( k \) are the chemical potential and quasi-momentum of the ground states, respectively. The perturbations can be parameterized as \( \delta \Psi_{1,2}(x,t) = U_{1,2}(x) \exp(\imath q x - \imath \omega t) + U_{1,2}^\ast(x) \exp(-\imath q x + \imath \omega t) \), where \( U \) and \( V \), \( q \), and \( \omega \) are the two amplitudes, the quasi-momentum and the frequency of the perturbations, respectively. After substituting the general wavefunctions into the time-dependent GP equations and retaining only terms linear in the perturbations, we arrive at the Bogoliubov-de Gennes (BdG) equations

\[
\begin{pmatrix}
H_1(k) + \gamma(q + k) & \Omega_2 + g \Phi^*_1 \Phi_2 \\
\frac{\Omega_2}{2} + g \Phi^*_1 \Phi_2 & H_2(k) - \gamma(q + k) \\
-g \Phi^*_1 \Phi^*_2 & -g \Phi^*_1 \Phi^*_2 \\
-g \Phi^*_1 \Phi^*_2 & -g \Phi^*_1 \Phi^*_2
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2 \\
V_1 \\
V_2
\end{pmatrix}
= \omega
\begin{pmatrix}
U_1 \\
U_2 \\
V_1 \\
V_2
\end{pmatrix}, \tag{3}
\]

where

\[
H_1(k) = -\mu + \frac{(q+k)^2}{2} + 2g|\Phi^*_1|^2 + g|\Phi^*_2|^2 + \frac{\delta}{2},
\]

\[
H_2(k) = -\mu + \frac{(q+k)^2}{2} + 2g|\Phi^*_1|^2 + 2g|\Phi^*_2|^2 - \frac{\delta}{2}.
\]

The resulting BdG spectrum for realistic parameters is shown in Fig. \textbf{1}(a). One can clearly see that in the direction of the Raman momentum vector (i.e. on the right hand side of the plot), the BdG spectrum is characterized by a local minimum near 1.62 \( \hbar k_{\text{Raman}} \), which is the roton minimum. Reducing the Raman detuning allows to soften the roton mode, i.e. to decrease the excitation energy at the position of the minimum, without significantly affecting the long-wavelength phonon modes (Fig. \textbf{1}(b)).

Our experiments start with a BEC confined in a crossed dipole trap with harmonic trap frequencies given by \( (\omega_x, \omega_y, \omega_z) = 2\pi \times (39, 153, 189) \) Hz. Spin–orbit coupling in the \( x \)-direction is induced by two Raman laser beams with \( \lambda_{\text{Raman}} \approx 789 \) nm, which intersect at the position of the BEC and are arranged with an angle of 90° between each other. The Raman lasers couple the \( |1, 0\rangle = |1\rangle \) and \( |1, -1\rangle = |2\rangle \) hyperfine states in the \( F = 1 \) manifold, which are split by a 10 G magnetic bias field. The accompanying quadratic Zeeman shift places the \( |1, +1\rangle \) state 7.4 \( E_{\text{Raman}} \) away from resonance and the BEC therefore resembles an effective spin-1/2 system. For the experiments described below, the Raman dressing is adiabatically increased starting at large detuning and ending at a final value close to resonance. In this way the BEC is loaded near the minimum of the lowest spin–orbit band.

We probe the collective excitation spectrum by performing Bragg spectroscopy. For this, two Bragg laser beams with a wavelength of \( \lambda \approx 1540 \) nm and small frequency difference, \( \Delta \nu_{\text{Bragg}} \), are pulsed on for 1 ms. The Bragg beams are collinear with the Raman beams (see Fig. \textbf{2}(a)) so that \( k_{\text{Bragg}} = 2\pi/\sqrt{2} \lambda_{\text{Bragg}} \). The Bragg
FIG. 1. (a) BdG spectrum of a spin-orbit coupled BEC for a nonlinear coefficient of $g = 0.186$, a Raman detuning of 500 Hz and a Raman coupling strength of $2.5E_{\text{Raman}}$. (b) Mode softening with decreasing Raman detuning (1000 Hz, 500 Hz and 0 Hz from top to bottom).

FIG. 2. Bragg spectra for BECs without spin-orbit coupling. (a) Schematic of the Bragg beam geometry. (b) Experimental image belonging to the black data set (squared symbols) of (c), taken near the peak of the resonance. The fainter cloud on the left shows the Bragg scattered atoms. (c) Red dots: Bragg spectrum for a trapped BEC. Black squares: Bragg pulse applied to a BEC after a 3 ms expansion time. The black data is an average over four measurement, a typical error bar is shown near the peak. The red data is from a single measurement.

The BdG analysis presented above describes a homogeneous BEC, whereas the experimental system is confined in a harmonic trap. To remedy this discrepancy and account for the spatial variation of the density in the experiment, one can introduce an effective scattering length, $a_{\text{eff}}$. To determine the value of $a_{\text{eff}}$ for our experiment, we first measure the Bragg spectrum for a BEC with $10^5$ atoms without spin-orbit coupling (red dots in Fig. 2(c)) and find a peak located at $\Delta \nu_{\text{Bragg}} = 2.7$ kHz. This peak position can be reproduced by the formula for the BdG spectrum of a homogeneous BEC if the density is taken to be equal to the central density of the trapped BEC and all scattering lengths are set to $a_{\text{eff}} = 53.7a_0$ where $a_0$ is the Bohr radius [35]. In our trapping geometry it is a good approximation that the same value of $a_{\text{eff}}$ is valid for a BEC with spin-orbit coupling. We will show below that this leads to excellent agreement between theory and experiment. To demonstrate the dependence of the BdG spectrum on the interatomic interactions, the black squares in Fig. 2(c) show the spectrum for a BEC that has been released from the trap and allowed to expand for 3 ms before the application of the Bragg pulse. In this case, the interaction energy has predominantly been transformed into kinetic energy and the peak position is simply given by $E/\hbar = (2\hbar k_{\text{Bragg}}^2)/2m\hbar = 1934$ Hz. This corresponds to the kinetic energy of a non-interacting atom after receiving a momentum kick of $2\hbar k_{\text{Bragg}}$ during the Bragg pulse.

When we apply Bragg spectroscopy to a BEC with spin-orbit coupling, we find that each Bragg spectrum contains several distinct peaks. A typical spectrum for a BEC with $4 \times 10^4$ atoms, a Raman coupling strength of $3.5E_{\text{Raman}}$ and a Raman detuning of 500 Hz is shown in Fig. 3(a). Each peak corresponds to a different Bragg resonance within the BdG band as indicated in Fig. 3(b). The peak $\beta$ can be identified as probing the region of the roton. The reduced amplitude of the peaks with positive $\Delta \nu_{\text{Bragg}}$ is due to the difference in spin composition of the initial and final states of the Bragg transition: unlike the Raman beams, the Bragg beams do not change the spin state due to their large detuning from the Rb D1 and D2 lines.

To demonstrate the mode softening, we measure Bragg spectra for a range of different Raman detunings and record the positions of the Bragg peaks, see Fig. 1. The three data sets shown correspond to the peaks $\alpha$ (lowest lying curve), $\beta$ (middle curve) and $\gamma$ (highest lying curve) of Fig. 3. One can see that the peak positions significantly shift as a function of the Raman detuning, with the roton mode ($\beta$ peak, middle curve) clearly soft-
The theoretical ob-
sound (≈ sweep velocity at 2 mm/s). Note that a barrier sweep with these parameters leads to significant heating of the BEC. After the sweep we measure the spin composition of the BEC now has a quasimomentum opposite to the initial state. The lines represent the result of theoretical calculations.

en for a decreasing positive value. However, its energy does not reach zero in our system and a finite gap remains, which precludes a supersolid-like phase transition. For the specific parameters of Fig. 4, this zero detuning gap is 421 Hz. When the Raman detuning becomes negative, the shape of the dispersion relation of the SOC BEC changes such that the ground state of the BEC now has a quasimomentum opposite to the value at positive detuning. The symmetry between the data points for positive and negative detuning provides direct evidence for the existence of the time-reversal-like symmetry $R_{K\sigma}$. To demonstrate the excellent agreement between the data and the model using the homogeneous BdG equations with an effective interatomic scattering length of $a_{at} = 53.7\,a_0$, we overlay in Fig. 4 the theoretically obtained curves. The effective scattering length leads to a nonlinear coupling parameter in the GP Eq. of $g = 0.186$. The calculated spectrum is in very good agreement with the experimental data.

The existence of rotons can have direct consequences for the hydrodynamical behaviour of the BEC. As a particular example we probe the response of an SOC BEC to a repulsive light sheet that is swept through the condensate along the x-direction, see Fig. 5. The light sheet is formed by a laser with a wavelength of 660 nm and Gaussian waist sizes of $w_x = 12\,\mu m$ and $w_y = 70\,\mu m$. The central barrier height is approximately three times larger than the chemical potential of the BEC and the sweep velocity at 2.5 mm/s exceeds the central speed of sound ($\approx 1\,mm/s$). Note that a barrier sweep with these parameters leads to significant heating of the BEC. After the sweep we measure the spin composition $N_2 = N_1$ of the cloud, where $N_2$ ($N_1$) is the number of atoms in the $|2\rangle$ ($|1\rangle$) state in the low-momentum components after time-of-flight imaging. Two different scenarios are shown in Fig. 5(b): a barrier moving towards the roton direction (red filled circles), and a barrier moving in the opposite direction (black squares). When the Raman detuning is chosen such that the BdG spectrum supports a roton minimum (i.e. to the left of the shaded region in Fig. 5(a)), a significant difference in spin composition for the two cases is clearly visible. For spin-orbit parameters that do not support a roton minimum (i.e. to the right of the shaded region), the difference in spin composition is much reduced. The fact that this change occurs around the parameters where the roton minimum disappears possibly indicates the excitation of roton quasiparticles by the moving light sheet. A more detailed investigation of these effects is the topic of future work.

We hope that the presented experimental results and their theoretical interpretation will stimulate further studies of the fluid dynamics in SOC BECs. In superfluid helium, the roton minimum limits the flow speed in the superfluid phase according to the Landau criterion. In our system, the Raman dressing, which is employed to generate the spin-orbit coupling, breaks Galilean invariance so that the Landau criterion might not be applicable here. It would be interesting to extend our study along this direction. Furthermore, our experiment can be potentially extended to the case of a spin-orbit coupled lattice where rotons also exist but their manifestation is complicated.

Note added: During the completion of this manuscript, two further manuscripts appeared reporting the observation of roton excitations in BECs through Grant No. PHY-1306662.

ACKNOWLEDGMENTS

YZ and TB acknowledge support from OIST Graduate University. PE, MAK and CH acknowledge financial support from the National Science Foundation (NSF) through Grant No. PHY-1306662.
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