CHARGE ASYMMETRIES OF LEPTON TRANSVERSE MOMENTA IN DRELL–YAN PROCESSES AT THE LHC

M.W. KRASNY
Laboratoire de Physique Nucléaire et des Hautes Énergies
Université Pierre et Marie Curie Paris 6
Université Paris Diderot Paris 7
CNRS–IN2P3
4 place Jussieu, 75252 Paris Cedex 05, France

W. PŁACZEK
The Marian Smoluchowski Institute of Physics, Jagiellonian University
Reymonta 4, 30-059 Kraków, Poland

(Received September 21, 2012)

Charged lepton transverse momenta in the Drell–Yan processes play an important role at the LHC in precision measurements of the Standard Model parameters, such as the $W$-boson mass and width, their charge asymmetries and $\sin^2\theta_W$. Therefore, their distributions should be described as accurately as possible by the Monte Carlo event generators. In this paper, we discuss the problem of matching the hard-process kinematics of the Monte Carlo generator WINHAC with the parton-shower kinematics of the PYTHIA 6.4 generator while interfacing these two programs. We show that improper assignment of the quark and antiquark effective momenta in the LO matrix element computations may affect considerably the predicted lepton transverse momenta and even completely reverse their charge asymmetries at the LHC. We propose two matching schemes in which the NLO QCD distributions of the leptonic kinematical variables can be well reproduced by the LO WINHAC generator.

DOI:10.5506/APhysPolB.43.1981
PACS numbers: 12.15.–y, 12.38.–t

* The work is partly supported by the program of the French–Polish co-operation between IN2P3 and COPIN no. 05-116, and by the Polish National Centre of Science grant no. DEC-2011/03/B/ST2/00220.
1. Introduction

Distributions of transverse momenta of charged leptons ($p_T^l$) produced in Drell–Yan (DY) processes are important observables in hadron collider experiments. Their sensitivity to the values of the Standard Model (SM) parameters and to the polarisation of $W$ and $Z$ bosons produced in the DY process can be used in precision tests of the Standard Model (SM) and in searches for new physics.

The measurement of the SM parameters: $W$-boson mass ($M_W$), its charge asymmetry ($M_{W^+} - M_{W^-}$) and width ($\Gamma_W$) at the LHC was investigated in Refs. [1–6]. It was shown that in order to reach a precision at the level of $\sim 10$ MeV, the $p_T^l$ distributions must be controlled experimentally to a comparable accuracy. Since the Monte Carlo (MC) event generators are indispensable tools to derive the values of the SM parameters from the measured distributions, they must predict these distributions to even higher precision.

MC event generators for Drell–Yan processes developed so far can be divided into two categories. The first one includes generators which are based on the precise calculations of matrix elements (ME) for hard processes: those including QCD effects to the NLO or even NNLO level, and those including QED and electroweak (EW) corrections. In these generators, called hereafter the ME generators, differential cross sections for the hard process are convoluted with the universal parton distribution functions (PDFs) of the hadron beams. These PDFs depend, apart from the factorisation scale, only on the longitudinal momenta of partons, $x$. In this type of the MC generators, partons entering the hard process are assumed to be parallel to hadron beam direction. The second category of MC generators includes the so-called parton-shower (PS) generators, such as PYTHIA [7], HERWIG [8], etc. They generate initial-state multi-parton radiation in form of the LO-type QCD/QED parton cascade and then perform hadronisation as well as some particle decays. In the PS generators, partons entering the hard process are no longer collinear with the hadron-beam directions, but acquire non-zero transverse momenta. The hard process itself is described by these generators usually at the LO level. Thus, as long as the ME precision is the key factor determining the overall accuracy of the measurement of a selected observable, they are inferior with respect to the ME generators. However, in the remaining cases they often provide better description of the hadronic energy flow associated with the DY process, in particular, for not to high transverse momenta of the $W$ and $Z$ bosons.

A very important, and at the same time difficult issue is how to combine these two types of generators, avoiding, on the one hand, a double counting of QCD corrections and, on the other, possible gaps in phase space present in some PS algorithms. As the state-of-the-art practical solutions to this
problem for the QCD effects are regarded the \textsc{MC@NLO} [9] and \textsc{POWHEG} [10] generators. They match the NLO QCD ME calculations with the parton shower generators, albeit in different ways. \textsc{MC@NLO} uses \textsc{HERWIG} or \textsc{HERWIG++} [11] for parton shower generation, while \textsc{POWHEG} is more universal, in principle, it can use an arbitrary parton-shower generator.

In the case of ME generators that include QED/EW radiative corrections, the situation is simpler as combining them with PS generators usually does not lead to double counting\(^1\). One of such generators is the MC program \textsc{WINHAC} [12–14]. It includes higher-order QED effects for the final-state radiation (FSR) and initial-final state interferences in the Yennie–Frautschi–Suura (YFS) exclusive exponentiation scheme [15] together with the \(\mathcal{O}(\alpha)\) EW corrections for the full charged-current DY process [16]. For the QCD (and QED ISR) effects, it is interfaced with \textsc{PYTHIA} 6.4 [7]. However, we do not use the Les Houches Accord (LHA) scheme [17] but our own interface, in which the \textsc{PYTHIA} routines for parton-shower generation and hadronisation are called directly from the \textsc{WINHAC} program. The principal reason is that the LHA is not general enough to transmit the full information contained in the spin density matrix of the \(W\) and \(Z\) bosons between these two generators. Moreover, from the purely technical perspective, we avoid writing/reading events into/from disk files, which makes event generation easier and much more efficient. For example, in our studies presented in Refs. [1–6] and requiring generation of \(\mathcal{O}(10^{11})\) events, its efficiency was one of the principal optimisation targets.

In any interface which extends the LHA scheme to processes involving the spin-1 EW bosons as intermediate particles, a particular care must be taken of the proper matching of the ME-type kinematics with the PS-type kinematics. In the following, we show that this is particularly important to describe the charged lepton transverse momenta distributions in the Drell–Yan processes at the LHC. In particular, we find that using the original \textsc{PYTHIA} effective momenta of incoming quarks in the above matching results in strongly biased \(p_T\) distributions, which is particularly visible in their charge asymmetries. We propose solutions to this problem that seem to reproduce well the NLO QCD predictions for these asymmetries, as obtained \textit{e.g.} from \textsc{MC@NLO}. The goal of this exercise is to try to achieve the NLO QCD precision for the description of the leptonic observables in the DY process with the suitably matched LO QCD generator which incorporates the state-of-the-art EW corrections.

The paper is organised as follows. In the next section, we describe in detail the matching of the ME kinematics with PS kinematics as it is realised in the \textsc{WINHAC} MC event generator. In Sec. 3, we present numerical results

\(^1\) The only problem here may be the QED ISR, but since its numerical effects are rather small, it can be dealt sufficiently well by PS generators.
illustrating the above issues, discuss their meaning for the LHC physics and propose our solutions that match the NLO QCD predictions. Finally, in Sec. 4, we summarise the paper.

2. Matching of ME and PS kinematics

The WINHAC [12] MC event generator is dedicated to precision modelling of single $W$-boson production with leptonic decays, i.e. the charged-current Drell–Yan processes, in proton–proton, proton–antiproton and ion–ion collisions, with the main emphasis on the QED effects and electroweak corrections. It uses fully massive spin amplitudes to evaluate the hard process matrix elements. They can be computed in an arbitrary reference frame, in particular, they can be used to calculate polarised $W$-boson cross sections\(^2\). In terms of the perturbative QCD, the current version includes the LO hard-process matrix element. The QCD effects enter only through scaling-violating PDFs, taken from the LHAPDF library [18]. Therefore, in WINHAC incoming quarks producing a $W$-boson are collinear with hadron (ion) beams; their longitudinal momenta are given by the two $x$-variables which are generated according to PDFs and subsequently convoluted with the hard-process differential cross section. At this stage WINHAC is a ME-type MC generator. Its full event kinematics (i.e. all four-momenta of initial, intermediate and final state particles) is constructed for incoming partons collinear with the colliding beams. Let us call it the ME kinematics. At this level WINHAC has been cross-checked numerically to a high precision with independent calculations [13, 16, 19].

Events of the ME kinematics do not look very realistic from the experimental point of view for the following two reasons. Firstly, QCD radiation affect not only longitudinal momenta of partons but also their transverse momenta. Therefore, using purely collinear PDFs for the description of the QCD effects is not sufficient. Secondly, partons are not observed experimentally. What can be observed are the products of their hadronisation and decays. Therefore, in a realistic MC generator, to be used in an experimental data analysis, the above effects must be taken into account. In WINHAC this is done through the interface to the PYTHIA 6.4 generator which performs the initial-state LO-type QCD (and QED) parton shower, appropriate proton-remnant treatment, and necessary hadronisation/decays. In PYTHIA, partons entering the hard process are not collinear with the hadron beams. In the case of the charged-current DY processes, PYTHIA provides, in its event record, the momenta of the two effective on-mass-shell quarks producing $W$-bosons. A vectorial sum of these momenta gives a momentum

\(^2\) WINHAC provides several options for computing polarised $W$-boson processes in several reference frames.
of an appropriate $W$-boson. Such a $W$-boson, in contrast to the case of a ME-type MC generator, carries the transverse momentum, being a vectorial sum of the quarks transverse momenta. This has to be taken into account in constructing the hard-process event kinematics. Let us call the kinematics in which the incoming partons are not parallel to the beams (as a result of the aforementioned effects) the PS kinematics, and a corresponding MC generator — the PS generator.

While interfacing the ME-type generator with the PS generator, one has to take care of the appropriate matching of the ME kinematics with the PS kinematics. This is particularly important for processes in which particles with non-zero spin, e.g. $W/Z$-bosons, are produced as intermediate states. In the following, we describe in detail how such a matching is performed in the WINHAC interface to PYTHIA 6.4. Then, we discuss possible pitfalls of the kinematical matching of these two types of MC generators.

In the WINHAC interface to PYTHIA the final hard-process event kinematics is constructed through the following steps:

1. The ME kinematics of a given MC event is generated in WINHAC — the four-momenta of: the incoming quarks, the intermediate $W$-boson, the final-state leptons, and the radiative photons are constructed in the frame in which quarks are collinear with the hadron beams, where the $+z$ axis is the direction of one of the beams at the collision point.

2. All the above four-momenta are then Lorentz-boosted along the $W$-boson direction to the $W$-boson rest frame, which is also the centre-of-mass frame of the incoming quarks. In this frame, quarks are still aligned along the $z$ axis.

3. The PS kinematics is generated by PYTHIA, in which the effective on-mass-shell quarks producing the $W$-boson are non-collinear with the beams as a results of the initial-state QCD/QED parton shower\(^3\). Their four-momenta are given in the LAB frame with the $+z$ axis along one of the hadron beams.

4. The above PYTHIA-quarks four-momenta are Lorentz-boosted along the sum of their momenta to their centre-of-mass frame, which is also the $W$-boson rest frame\(^4\). Contrary to the WINHAC, quarks in point 2,

\(^3\) Actually, for technical reasons PYTHIA performs the so-called backward QCD evolution. This aspect is not important for our discussion. Here, we are concerned mainly with the PS kinematics in which partons entering the hard process are not parallel to the hadron beams — this can be a result of any type of a parton-shower algorithm in which transverse degrees of freedom are not neglected (integrated out).

\(^4\) Instead of a single parallel boost, one might use a combination of two boost: along $p_{tW}$ and along $p_{zW}$. We have checked numerically that both methods are fully equivalent.
their momenta, although back-to-back, are not aligned with the \( z \) axis in this frame. Instead, their direction is rotated with respect to the \( z \) axis by the polar angle \( \theta_q \), and with respect to the \( x \) axis by the azimuthal angle \( \phi_q \).

5. From the \textsc{pythia}-quarks momenta, specified in the above frame, we calculate the angles \( (\theta_q, \phi_q) \) and then we perform rotations of all the \textsc{winhac} momenta specified in the \( W \)-rest frame (point 2) using these rotation angles. After such rotations the \textsc{winhac} quarks are aligned with the \textsc{pythia} quarks.

6. Finally, the whole \textsc{winhac} event is Lorentz-boosted from the above frame to the LAB frame along the sum of the original \textsc{pythia}-quarks momenta (the boost is opposite to the one in point 4).

In our opinion, all the above steps are needed for a proper matching of the \textit{ME} kinematics with the \textit{PS} kinematics in any interface between the ME-type MC event generator and the PS generator. This is particularly important for the production of the \( W \)-bosons which are spin-1 particles with \( V-A \) couplings to fermions. In such a case respecting all the spin correlations in the above matching is obligatory.

The above kinematical matching relies on the correct \textsc{pythia} generation of the incoming “effective” quarks momenta. Their spacial orientation is crucial for the spin correlations which, in turn, influence the angular distributions of the \( W \)-decay leptons and, as a consequence, their \( p_T \) distribution.

At this point, one may ask if such effective on-shell quarks make sense at all. It has been known for some time that a cross section corresponding to the real NLO QCD emission can be expressed as a linear combination of the LO matrix elements \cite{20,21}. The latter can be calculated using some effective incoming on-shell partons four-momenta, \textit{e.g.} with the help of spin amplitudes. However, care must be taken while constructing these effective four-momenta for each LO matrix element individually in order not to spoil spin correlations. Coefficients of these matrix elements can be expressed as functions of variables related to the radiated partons (\textit{e.g.} their momenta fractions and polar angles in an appropriate frame) and they are generally different for each LO matrix element. In a Monte Carlo approach, computations of the NLO cross section can be done with the use of the so-called branching algorithm, where in each branch a single LO matrix element is evaluated and a particular branch is picked up with a probability proportional to the coefficient of this matrix element. What is also interesting, flavours of these effective partons depend only on the respective LO process and can be different than flavours of partons initiating the NLO process. Recently, such a method has been adapted to implement the Drell–Yan processes in the \textsc{powheg} generator \cite{22}.
For the DY processes the differential cross section corresponding to the NLO real-parton radiation can be expressed as the following combination of the LO matrix elements

$$d\sigma_{\text{NLO}}^R = \sum_i C_i \left| \mathcal{M}_{q\bar{q}'} (\tilde{p}_q, \tilde{p}_{\bar{q}'}) \right|^2,$$

(2.1)

where $C_i$ are the coefficients depending on the radiated parton variables, e.g. $C_i = C_i(x, \cos \theta)$, where $x$ and $\theta$ are the momentum fraction and the polar angle of this parton. The effective four-momenta $\tilde{p}_q$ and $\tilde{p}_{\bar{q}'}$ of the incoming on-shell quark $q$ and antiquark $\bar{q}'$ entering the LO matrix element are constructed in the NLO-process CM frame in such a way that the momentum of the “spectator” of the radiation is only rescaled without changing its direction, while the four-momentum of the “emitter” is calculated as a difference of the electroweak boson four-momentum and the “spectator” effective four-momentum, so its direction is different from that of the original “emitter” parton. The “emitter–spectator” assignment to the incoming partons is done on the Feynman-diagramatic basis, details can be found e.g. in [21, 22].

In order to compute the appropriate LO matrix elements and to match the LO kinematics with the NLO, one appropriate Lorentz transformations should be performed for the effective on-shell quarks four-momenta. Actually, they are analogous to the ones described above for the WINHAC–PYTHIA matching, see [22]. Based on this analogy we believe that our procedure for the kinematical matching between WINHAC and PYTHIA should be correct, at least up to the NLO. Of course, the parton shower provides only a leading-log (LL) type approximation of the NLO QCD corrections, but in PYTHIA the exact NLO matrix elements for real-parton emission can be taken into account through appropriate correcting weights. If this is done, the predictions of PYTHIA for the leptonic distributions in the DY processes should be exact at the NLO for the hard process, except for the normalisation. The latter will not be correct because PYTHIA does not include the NLO virtual corrections — this could be easily fixed by applying the NLO $K$-factor. For the lepton charge asymmetries, this $K$-factor is even not needed because it cancels out between numerators and denominators. In the discussed WINHAC to PYTHIA interface these NLO matrix element corrections are included by default. However, the PYTHIA predictions will not be exactly the same as from the fixed-order NLO calculations, because PYTHIA generates also the higher-order LL-type QCD corrections through parton showers. They will lead to additional distortions of the leptonic distributions, in particular that of $p_T^{l}$, but should not change them drastically.
3. Numerical results and discussion

An observable which is most sensitive to details of the kinematical matching between the ME-type MC generator and the PS generator in the charged-current DY process is the final-state charged lepton transverse momentum $p_T^l$. This is because its distribution is a strongly varying function with a sharp Jacobian peak. Its shape is considerably affected by the non-zero $W$-boson transverse momentum $p_T^W$, see e.g. [5]. Moreover, since $W$ is a vector boson and its coupling to fermions are of the $V–A$ type, the angular distributions of its decay leptons in the $W$-rest frame are highly asymmetric

$$\frac{d\sigma}{d \cos \hat{\theta}_{lq}} \propto \left(1 - Q_W \cos \hat{\theta}_{lq}\right)^2,$$

where $\hat{\theta}_{lq}$ is, in the limit of massless quarks, the angle between the outgoing charged lepton and incoming quark directions, and $Q_W = \pm 1$ is the $W$-boson electric charge in the units of the positron charge. Because of that, $p_T^l$ depends strongly not only on $p_T^W$ but also on the individual momenta of the quark and antiquark.

In our numerical comparisons we have used the following MC programs: WINHAC 1.35 [12], PYTHIA 6.401 [23], MC@NLO 4.03 [24] and MCFM 5.8 [25].

The lepton charge asymmetry observables are used to scrutinize the differences between the $W^+$ and $W^-$ mediated processes in these generators. For a given kinematical variable $a$, the charge asymmetry $\text{Asym}^{(+,−)}(a)$ is defined as

$$\text{Asym}^{(+,−)}(a) = \frac{d\sigma^+/da - d\sigma^-/da}{d\sigma^+/da + d\sigma^-/da},$$

where $+$ and $-$ refer to the electric charge of the $W$-boson (or the final-state charged lepton) and $d\sigma^\pm/da$ is the differential cross section of an observable $a$.

The asymmetry distributions have been obtained for the proton–proton collisions at $\sqrt{s} = 14$ TeV using the CTEQ 6.1 PDF set [26] and the particles properties from the PDG 2011 publication [27], for the following two cases: (1) without any kinematical restrictions for the outgoing lepton and (2) with the kinematical cuts

$$p_T^l > 20 \text{ GeV}, \quad |\eta| < 2.5, \quad E_T^{\text{miss}} > 25 \text{ GeV}.$$ (3.3)

In Fig. 1, we show the charge asymmetry distributions as a function of $p_T^l$ for electrons obtained from WINHAC interfaced with PYTHIA. The kinematical matching described the previous section was applied with the effective on-shell quarks four-momenta as provided by PYTHIA. These dis-
tributions are compared with the ones coming directly from PYTHIA 6.4 (left plots) and the ones obtained from MC@NLO (right plots). Lower plots show the differences of the distributions presented in the upper plots. A very good agreement between WINHAC and PYTHIA shows that all the technical aspects of the kinematical matching in their interface were done correctly. However, we see that the WINHAC and PYTHIA results differ considerably from the MC@NLO results, in particular we observe the opposite behaviour of the $p_T^l$ asymmetry above the Jacobian peak ($\gtrsim 40$ GeV). The region around the Jacobian peak is crucial for the $W$ mass measurements at the LHC, see e.g. Refs. [1, 3]. The source of this discrepancy must be understood to hope for any improvement of the present precision of the $W$-boson mass, width and their charge asymmetries at the LHC.

Fig. 1. The comparisons of the $p_T^l$ charge asymmetries from WINHAC and PYTHIA 6.4 (left plots), and from WINHAC and MC@NLO (right plots); lower plots show the differences between the programs.

In Fig. 2, we present the comparisons of the $p_T^l$ charge asymmetry distributions for muons without kinematical cuts and with the cuts of Eq. (3.3) between WINHAC (with the same kinematical matching as in Fig. 1) and MC@NLO. Similar discrepancies as for the electrons are observed for the fully inclusive distributions. In the presence of cuts, the asymmetry distribution changes its shape and the differences between the two programs are smaller but still unacceptable.
We have also compared the charge asymmetry distributions as a function of: $p_T^W$, $y_W$ and $\eta_l$, where:

$$p_T^W = \sqrt{(p_x^W)^2 + (p_y^W)^2},$$  \hspace{1cm} (3.4)  

$$y_W = \frac{1}{2} \ln \left( \frac{E^W + p_z^W}{E^W - p_z^W} \right),$$  \hspace{1cm} (3.5)  

$$\eta_l = -\ln (\tan (\theta_l/2)),$$  \hspace{1cm} (3.6)  

and found a good agreement between WINHAC and MC@NLO. In Fig. 3, we present charge asymmetries for $\eta_l$. Except for the large values of $\eta_l$, i.e. except for the region which is beyond the measurement domain of the ATLAS and CMS experiments, the agreement between the two programs is good. The large discrepancies between the two programs in the restricted phase-space, as specified by Eq. (3.3), are thus important only for the charge asymmetries of the transverse lepton momentum distributions.

One may argue that these discrepancies result from differences in the shape of the $p_T^W$ distributions between WINHAC and MC@NLO. Indeed, we have found that the $p_T^W$ distributions differ for WINHAC and MC@NLO, but mainly at low $p_T^W (< 6 \text{ GeV})$, while for higher values their ratio is flat. In the PYTHIA PS algorithm used by WINHAC, the $p_T^W$ distribution at low values
Fig. 3. The $\eta_l$ charge asymmetries from WINHAC and MC@NLO without cuts (left plots) and with the typical ATLAS and CMS cuts (right plots); lower plots show the differences between the programs.

is affected mainly but the so-called intrinsic partonic $k_T$ which is generated from a Gaussian distribution with an adjustable width. We have used this dependence and generated the samples of events with amplified differences of the $p^W_T$ distributions between the above two generators in the range which is well beyond the present measurement uncertainties. We have observed that the corresponding charge asymmetries of the $p^l_T$ distributions remained hardly changed. We have also compared these asymmetries for $p^W_T > 6$ GeV, where the ratio of the $p^W_T$ distributions from WINHAC and MC@NLO is flat, and found similar results. Finally, have checked that, in spite of differences in the absolute $p^W_T$ distributions, their charge asymmetries agree very well between the two programs. This proofs that the differences in $p^W_T$ do not explain the large discrepancies in the $p^l_T$ asymmetries between WINHAC and MC@NLO. Thus, the latter must be attributed to the differences in the effective polarisation of the WINHAC and MC@NLO W-bosons.

Can we find simple physical arguments to explain the shape of the $p^l_T$ charge asymmetry distribution? Can we say which program is right and which is wrong? In order to try to answer the above two questions, we first produced the WINHAC distributions for the case of $p^W_T = 0$, i.e. without the PYTHIA parton shower in WINHAC (using purely beam-collinear quarks from the standard PDFs). These distributions are shown in Fig. 4 and compared
with the previous MC@NLO result. As one can expect, for the WINHAC \( p_T^W = 0 \) case the charge asymmetry distribution is flat below the Jacobian peak position (\( \approx 40 \) GeV), and then rises very slowly with increasing \( p_T^l \). Its average value below the peak position reflects the difference between the total cross sections for positively and negatively charged DY processes which is driven by the effective excess of the \( u \) quarks with respect to the \( d \) quarks producing the \( W \)-bosons at the LHC. For the \( p_T^l \) values above the Jacobian peak \( W \)-bosons must be off-shell if \( p_T^W = 0 \). Since higher invariant mass prefers harder quarks and since \( u \) is, on average, harder than \( d \), the relative number of produced \( W^+ \) rises with respect to \( W^- \). In the following, this effect will be called the isospin effect.

\[
W^\pm \rightarrow \mu^\pm \nu_\mu : \text{no cuts} \quad W^\pm \rightarrow \mu^\pm \nu_\mu : \text{with cuts}
\]

Fig. 4. The \( p_T^l \) charge asymmetries from WINHAC with \( p_T^W = 0 \) and MC@NLO, without cuts (left plots) and with the typical ATLAS and CMS cuts (right plots); lower plots show the differences between the programs.

In the presence of the kinematical cuts we see a good agreement between the two programs below the Jacobian peak, and the discrepancy begins above this peak but is smaller than without cuts and also than when PYTHIA is used. The sharp cut at \( p_T^l = 25 \) GeV for the WINHAC results comes from the cut on \( E_T^{\text{miss}} \) (in the case of \( p_T^W = 0 \) they are equivalent). It is rather striking that below the Jacobian peak the charge asymmetry of the \( p_T^l \) distribution is generated mainly by the cut on the lepton pseudorapidity, \( \eta_l \), and is hardly sensitive to the \( W \)-bosons transverse momentum spectrum.
What is the reason for the observed shape of the distribution for $p_{T}^{W} = 0$? In the $W$-rest frame the events with high $p_{T}^{l}$ correspond to $\eta_{l} \approx 0$, while the ones with low $p_{T}^{l}$ to large positive and large negative $\eta_{l}$. If we take the $+z$ axis along the quark momentum, then the $W$-bosons will have preferably positive rapidity in such a frame, since the quarks are, on average, harder than the antiquarks. Thus, when we perform a boost to the LAB frame in the presence of symmetric cuts on $\eta_{l}$, the events with negative $\eta_{l}$ will migrate in, while the ones with positive $\eta_{l}$ will migrate out of the selected kinematical region (the $W$-boson rapidity just adds to the lepton pseudorapidity). Since for $W^{-}$ charged leptons are emitted preferably along its direction, then more events with low $p_{T}^{l}$ will move out than move in, while for $W^{+}$ it will be opposite. This is why we observe the decrease of the asymmetry with increasing $p_{T}^{l}$ up to the value close to the Jacobian peak position. Close to the peak position majority of leptons must have $\eta_{l} \approx 0$ in the $W$-rest frame and the migration mechanism discussed above can be neglected. For such $p_{T}^{l}$ values and above the discussed earlier mechanism related to the relative hardness of the distributions of the $u$ and $d$ quarks takes over and the asymmetry rises.

Having understood the influence of the migration and the quark–isospin effects on the charge asymmetry distribution, let us try to answer our main question: do we understand the $p_{T}^{l}$ charge asymmetry when $p_{T}^{W} > 0$?

As discussed before, the shape of the distribution below the Jacobian peak position is determined by the migration mechanism and is hardly dependent on the underlying distribution of $p_{T}^{W}$. Therefore, in this region our previous analysis holds. We thus concentrate on the region of the large $p_{T}^{l}$ (above the position of the Jacobian peak). The main difference here with respect to the $p_{T}^{W} = 0$ case is that in addition to the isospin effect another effect comes into play and becomes dominant: the effect of hard QCD radiation which influences the effective polarisation of the $W$-bosons. In the discussion presented below, the $W$-polarisation is specified in the reference frame in which the spin quantisation axis is parallel to the direction of the $W$-boson.

It has been recently shown that for the processes of $W + \text{jets}$ production at the LHC left-handedly polarized $W$s dominate over the right-handedly polarized $W$s [28]. For the left-handed $W$s the charged leptons are emitted preferably in the $W^{-}$ direction and opposite to the $W^{+}$ direction (and vice versa for the right-handed $W$s). Therefore, the non-zero $p_{T}^{W}$ increases, on the average, the transverse momentum of the negatively charged lepton and decreases it for the positively charged one. This is what we observe in the left plot of Fig. 4, where for MC@NLO the asymmetry decreases for high $p_{T}^{l}$. There is, of course, some contribution from longitudinally polarized $W$s, but it never dominates [28] and, what is more important, charged lepton angular
distributions are in this case identical for $W^+$ and $W^-$. Moreover, the isospin effect, which could potentially counterbalance such a decrease, is sizeably smaller in magnitude due to a steeply falling Breit–Wigner distribution. Therefore, the MC@NLO results do have a rather convincing explanation of the $p_T^l$ charge asymmetry behaviour while the PYTHIA results do not.

In order to check the validity of the above reasoning, we have generated events using the Monte Carlo program MCFM [25] which calculates the fixed-order QCD corrections to the hard process convoluted with the collinear PDFs. The charge asymmetry distributions for $W + 1 \text{ jet}$ and $W + 2 \text{ jets}$ are presented in Fig. 5 and compared with the ones from WINHAC with the standard PYTHIA parton-shower matching. The $p_T^l$ asymmetries predicted by MCFM are close to those from MC@NLO, which supports the conclusion that the MC@NLO predictions on the $p_T^l$ asymmetries are more likely to be correct than those of PYTHIA.

![Fig. 5. The $p_T^l$ charge asymmetries from WINHAC and MCFM for $W + 1 \text{ jet}$ (left plots), and MCFM for $W + 2 \text{ jets}$ (right plots); lower plots show the differences between the programs.](image)

But can we find the simple reason why the PYTHIA predictions are so grossly wrong? From our numerical tests and discussion presented above it becomes obvious that the problem must be related to the modelling of the effective polarisation of $W$-bosons. In the LO approximation and for on-shell partons, the $W$-polarisation is uniquely driven by the asymmetry in the
distributions of the momenta of the effective quark and antiquark entering the DY processes (see Eq. (2.1)), rather than by their sum which determines $y_W$ and $p_W^T$. Inspecting the PYTHIA 6.4 manual [7], we have found that the construction of these effective on-shell partons momenta should agree at NLO with that of Ref. [22]. PYTHIA, of course, generates through the parton-shower more than a single NLO emission, however, they should not change considerably (or even revert) the NLO effective partons momenta as such additional emissions are mainly soft and collinear. Therefore, the PYTHIA predictions for the $p_T^l$ asymmetries should not differ much from the NLO ones.

At this point, we started searching not only for possible conceptual but also for the technical errors affecting the spatial orientation of the quark and antiquark momenta. We have made several technical checks of the PYTHIA generator along this line. One of the checks done was to swap the transverse momenta of the effective on-shell quark and antiquark. To our great surprise, once this was done on the event-by-event basis, we have obtained a very good agreement with the MC@NLO charge asymmetry distribution, both in the full phase-space and in the restricted kinematical region. The comparisons are shown in Fig. 6 for the $p_T^l$ and in Fig. 7 for the $\eta_l$ dependence of the lepton

![Graphs showing charge asymmetries](image)

Fig. 6. The $p_T^l$ charge asymmetries from WINHAC with the transverse momenta of the effective quarks swapped and MC@NLO, without cuts (left plots) and with the typical ATLAS and CMS cuts (right plots); lower plots show the differences between the programs.
charge asymmetry. This agreement may be accidental but it may also sugge

s that the transverse momenta are, perhaps, not correctly assigned to the
effective quark and antiquark in PYTHIA. Whether or not such a hypothesis
is true, can, however, be verified only by the authors of the PYTHIA generator.

\[ W^\pm \rightarrow \mu^\pm \nu \mu : \text{no cuts} \]

\[ W^\pm \rightarrow \mu^\pm \nu \mu : \text{with cuts} \]

\[ \delta = W - M \]

Fig. 7. The \( \eta_l \) charge asymmetries from WINHAC with the transverse momenta of the effective quarks swapped and MC@NLO, without cuts (left plots) and with the typical ATLAS and CMS cuts (right plots); lower plots show the differences between the programs.

On the conceptual side, we have investigated the mechanism which drives the effective LO polarisation of the W bosons in the DY process involving on-shell quarks. We have found that skipping the rotations of leptons momenta in the W-boson rest frame, described in point 5 of the previous section, gives better agreement of the PYTHIA \( p_T \) charge asymmetry with the MC@NLO one. The results are shown in Fig. 8. The agreement with MC@NLO is slightly worse than in Fig. 6, however much better than in Fig. 2. Note that skipping these rotations is equivalent to retaining the PS-initial (parton-shower unaffected) effective on-shell quark helicities rather than those corresponding the PS-final ones (following the parton-shower).

We have implemented the above two options in the new version of WINHAC [12]. These versions cannot replace the future state-of-the-art NLO programs with the NLO PS and the full set of EW radiative corrections.
However, as long as such programs are not available, they may be of use in the initial phase of the measurement of the lepton charge asymmetries at the LHC. First of all, they can be of help in the unfolding of the measured charge lepton asymmetries in the experimental procedures where the precision of the EW and the real photon radiative correction matters. More importantly, the above versions, providing the simplified LO picture of the effective polarisation of $W$-bosons at the LHC, may help in designing the new polarisation-dependent observables for the studies of the electroweak symmetry breaking (EWSB) mechanism.

4. Summary

In this paper, we have discussed the generic problem of kinematical matching of a parton shower generator with a matrix element generator for the Drell-Yan processes involving spin 1 intermediate particles. We have argued that the Les Houches Accord must be extended to take into account the spin correlations at all the stages of the event generation. We have described in detail our kinematical matching procedure which is used
in the interface of our WINHAC generator with the PYTHIA 6.4 generator. We have demonstrated that the momentum vectors of the on-shell quark and antiquark, carrying in the LO approximation the full information on the $W$-boson polarisation, must be well defined. Any error in directions of these vectors has dramatic consequences for the $p_T^l$ dependence of the charge asymmetries at the LHC. In particular, using the transverse momenta of the effective quarks provided in PYTHIA 6.4 leads to completely different behaviour of the above asymmetry than predicted by the NLO (and beyond) calculations, e.g. MC@NLO and MCFM. We have found that simple swapping of the effective quark and antiquark transverse momenta in PYTHIA, or skipping the rotation of the outgoing lepton momenta, results in the $p_T^l$ charged asymmetries that match the NLO predictions of MC@NLO. We have implemented the corresponding matching schemes in the new version of the WINHAC generator.

The issue of the proper matching between the matrix element calculations and the parton-shower generators respecting the spin correlations is important not only for the charged-current Drell–Yan processes but also for any process of production and decay of non-zero spin particles at the LHC. It needs to be readdressed in the more general context of matching the NLO matrix elements with the NLO parton shower such that a handle is given to the experimentalists to control the relative contributions of all the spin density matrix elements of the decaying particles, thus allowing for an experimental verification of the implemented Monte Carlo mechanism which drives the polarisation of non-zero spin particles at the LHC.

We would like to thank S. Jadach for useful discussions.

REFERENCES

[1] M.W. Krasny et al., Eur. Phys. J. C69, 379 (2010) [arXiv:1004.2597 [hep-ex]].
[2] M.W. Krasny et al., PoS ICHEP2010, 091 (2010).
[3] F. Fayette, M.W. Krasny, W. Placzek, A. Siodmok, Eur. Phys. J. C63, 33 (2009) [arXiv:0812.2571 [hep-ph]].
[4] F. Fayette, M.W. Krasny, W. Placzek, A. Siodmok, PoS EPS-HEP2009, 363 (2009) [arXiv:0909.1443 [hep-ex]].
[5] F. Fayette, arXiv:0906.4260 [hep-ex], Ph.D. Thesis.
[6] K. Rejzner et al., PoS HCP2009, 095 (2009).
[7] T. Sjostrand, S. Mrenna, P. Skands, J. High Energy Phys. 05, 026 (2006) [arXiv:hep-ph/0603175].
[8] G. Corcella et al., J. High Energy Phys. 01, 010 (2001) [arXiv:hep-ph/0011363].
[9] S. Frixione, B.R. Webber, J. High Energy Phys. 06, 029 (2002) [hep-ph/0204244].
[10] P. Nason, J. High Energy Phys. 11, 040 (2004) [arXiv:hep-ph/0409146].
[11] M. Bahr et al., Eur. Phys. J. C58, 639 (2008) [arXiv:0803.0883 [hep-ph]].
[12] W. Płaczek, S. Jadach, WINHAC version 1.35: The Monte Carlo event generator for single W-boson production with leptonic decays in hadron collisions, available from http://cern.ch/placzek/winhac
[13] W. Płaczek, S. Jadach, Eur. Phys. J. C29, 325 (2003) [arXiv:hep-ph/0302065].
[14] W. Placzek, PoS EPS-HEP2009, 340 (2009) [arXiv:0911.0572 [hep-ph]].
[15] D.R. Yennie, S. Frautschi, H. Suura, Ann. Phys. (NY) 13, 379 (1961).
[16] D. Bardin et al., Acta Phys. Pol. B 40, 75 (2009) [arXiv:0806.3822 [hep-ph]].
[17] J. Alwall et al., Comput. Phys. Commun. 176, 300 (2007) [arXiv:hep-ph/0609017].
[18] M.R. Whalley, D. Bourilkov, R.C. Group, arXiv:hep-ph/0508110.
[19] C.M. Carloni Calame et al., Acta Phys. Pol. B 35, 1643 (2004) arXiv:hep-ph/0402235.
[20] R. Kleiss, Phys. Lett. B180, 400 (1986).
[21] M.H. Seymour, Nucl. Phys. B436, 443 (1995) [arXiv:hep-ph/9410244].
[22] K. Hamilton, P. Richardson, J. Tully, J. High Energy Phys. 0810, 015 (2008) [arXiv:0806.0290 [hep-ph]].
[23] T. Sjostrand, S. Mrenna, P. Skands, PYTHIA 6.401, http://home.thep.lu.se/~torbjorn/pythiaaux/recent.html
[24] F. Frixione et al., MC@NLO 4.03, http://www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/
[25] J.M. Campell, R.K. Ellis, MCFM 5.8, http://mcfm.fnal.gov
[26] D. Stump et al., J. High Energy Phys. 10, 046 (2003) [arXiv:hep-ph/0303013].
[27] Particle Data Group, The review of particle physics, 2011, http://pdg.lbl.gov
[28] Z. Bern et al., Phys. Rev. D84, 034008 (2011) [arXiv:1103.5445 [hep-ph]].