Characterisation of flare Soft X-ray distribution with solar magnetic activity

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Abstract. We analyse the 0.1 – 0.8 nm solar soft X-ray flux catalogue from Geostationary Operational Environmental Satellites (GOES), managed by NASA/NOAA, between September 1978 and September 2017, in order to investigate the possible role of solar activity and solar cycle epoch on the distribution of soft X-ray peak fluxes. We concentrate our attention on the last three solar cycles because solar activity proxies seem to indicate a decrease in the magnetic activity of our star. We know that flare soft X-ray peak fluxes are characterised by a power-law distribution with an index $\alpha \simeq 2$ that shows a minor dependence on solar cycle. More in detail, we study the dependence of the power-law parameters during each single solar cycle (cycles 21-24) and during different regimes of solar activity defined using three different proxies: i) Sunspot Number (SSN), ii) Mg II core-to-wing ratio (Mg II Index), and iii) solar radio flux at 10.7 cm or 2800 MHz (F10.7). The power-law estimation analysis is performed in maximum likelihood estimation (MLE) fitting method with goodness-of-fit based on Kolmogorov-Smirnov test. Preliminary results indicate that the power-law index shows a slight decrease as solar activity decreases. This except for the F10.7 proxy. More in-depth statistical analysis is necessary to confirm our findings.

1. Introduction
Solar flares are the most powerful and sudden phenomena into the Solar System; they can release up to $10^{25}$ joules in few hours, even in minutes, by way of radiation (from $\gamma$ rays to radio wavelength), thermal conduction, mass ejections (Coronal Mass Ejections, CMEs), wave propagation and production of high-energy particles (Solar Energetic Particles, SEPs). Flares appear as sudden increase in photon flux into the solar corona (usually observed in H$\alpha$ and EUV). They could trigger a chain of eruptive phenomena which produce disturbances in Earth’s space environment that can adversely affect some important technologies (e.g. satellites, electrical power grids, long range radio communication, etc.) and threaten the health and safety of astronauts. Their study is consequently of primary importance in the context of space weather. Soft X-ray (SXR) peak flux distribution shows a power-law relationship, $p(x) \sim x^{-\alpha}$, for various order of magnitude with an index $\alpha$ approximately equal to 2 (e.g. [1–3]). Such a behaviour, observed in a large variety of different physical complex systems, is related to the study of the physical process connected to magnetic instabilities responsible for the flare onset, the magnetic reconnection process. This is the fundamental process associated with magnetic topology change and energy release linked to solar flares. Usually, in order to reproduce the observed peak flux power-law distribution various approaches...
Table 1. Solar flares classification according to the peak flux emitted in soft X-ray.

| Class | Peak Flux (0.1 – 0.8nm) | W/m² |
|-------|------------------------|------|
| A     | 10⁻⁸ – 10⁷             |      |
| B     | 10⁻⁷ – 10⁶             |      |
| C     | 10⁻⁶ – 10⁵             |      |
| M     | 10⁻⁵ – 10⁴             |      |
| X     | > 10⁻⁴                 |      |

are suggested: turbulent MHD systems represented by shell models (e.g. [4]), n-body simulations considering photospheric organization scales (e.g. [5, 6]), pure MHD simulations (e.g. [7, 8]) or self-organized critical systems (e.g. [3, 9]). Although with different results, all these classes of models are able to reproduce power-law distributions of flare fluxes.

The statistical approach presented in this work tries to investigate the possible dependence of the index of the power-law with the solar activity level and with the solar cycle epoch.

The paper is organised as follows. Sec. 2 describes our datasets. Sec. 3 details our approach and the methodology. Finally sec. 4 presents and discusses the results.

2. Datasets

In this study, we consider flares occurred from September 1978 to September 2017 and reported in the GOES soft X-ray catalogue produced by National Oceanic and Atmospheric Administration (NOAA). NOAA classifies flares according to the peak flux emitted in soft X-ray in 5 classes (see Tab. 1). We considered only flares ≥ C class, producing a comprehensive dataset of 54,933 events (see Tab. 2). All data are available at NOAA-NCEI catalogue online.

Regarding the proxies of solar magnetic activity in this work we use the three listed in the following subsections because they provide continuous and long-period observations. Fig. 1 shows the evolution of selected proxies of solar activity and flare detected.

2.1. Sunspot Number (SSN)

The SunSpot Number [10] dataset is taken from SIDC - Royal Observatory of Belgium and derives from multiple observations from all over the world. Sunspots are the oldest evidence of the solar activity cycle. They are created by a concentration of a kGauss magnetic field that partially block the energy transport by convection from the solar nucleus. They appear darker than the surrounding quiet photosphere because they are 1,500 – 3,000K cooler than it. Magnetic active regions (ARs), hosting sunspots, are the most likely regions for the occurrence of solar flares. In particular when they appear as bipolar pair, or groups, of sunspots forming polarity inversion lines (PIL) [11]. According to Shibata et al. [12] the total energy stored in a typical active region of size $L$ is: $E_{mag} \approx \frac{B^2}{8\pi} L^3 \approx 10^{33} \left( \frac{B}{10^3 G} \right)^2 \left( \frac{L}{5 \times 10^8 m} \right)^3 \text{erg}$.

2.2. Radio Flux (F10.7 cm)

Solar radio emissions at F10.7 cm [13] are provided by ground-based radio telescopes. The Sun presents a continuous radio background emitted by many different processes (e.g. free-free bremsstrahlung, plasma emissions). In this work we use the F10.7 historical dataset. This proxy

1 www.ngdc.noaa.gov
2 www.sidc.be/silso
Figure 1. Solar activity: September 1978 - September 2017. Panel a) SSN, F10.7 and Mg II (normalized between 0 and 1) trend, while panel b) the monthly amount of flares.

correlates very well with SSN and solar UV emission. It is reported in solar flux units (1sfu = 1 W·s/m²), and it has been measured continuously in Canada since 1947³.

2.3. Mg II Index
The Mg II index [14] is defined as a core-to-wing ratio from the Mg II doublet emission centered at 279.9nm. It is one of the most variable proxies with the solar cycle, indeed it could change by a factor from 2 to 10 between minimum and maximum phase of the Sun. It has been considered as a reliable proxy for geomagnetic storms/substorms, since the EUV radiation is the principal energy input for the ionosphere [13,14]. The solar Mg II feature is measured operationally on a daily basis by various NOAA satellites, then all data are combined to form a single time-series starting from 1978. Data are available at UVSAT - Bremen University⁴.

3. Statistical parameter estimation
We know that soft X-ray peak flux distribution shows a power-law relationship. Upper and lower tails of power-law distributions can be affected by instrumental cut-off (e.g. sampling windows, sampling cadence, noise, etc.). This experimental condition requires special care in determining the distribution’s parameters. In this work, in order to estimate the distribution parameters we use Maximum-Likelihood Estimation (MLE) method coupled with goodness-of-fit test based on Kolmogorov-Smirnoff statistics [16].

In more detail, we create a procedure as follows:

(i) **Smoothing Data:** smoothing solar proxies time-series with a moving average of 27-day (solar rotation period). This operation is necessary for creating more balanced activity bands removing large or intermittent fluctuations;

(ii) **Defining Activity Bands:** dividing the solar activity into three different bands, minimum, medium and maximum. First we minmax-normalise the range of each proxy between 0 and 1, then define three bands with the same amplitude.

³ www.spaceweather.gc.ca
⁴ www.iup.uni-bremen.de/UVSAT
Table 2. NOOA - NCEI flares data. In relation to Fig. 1 it is evident that the number of flares within a cycle depends on the activity level of the cycle itself. Cycle 24 is only partial.

| solar cycle | 21   | 22   | 23   | 24   |
|-------------|------|------|------|------|
| C flares    | 14710| 12439| 13141| 7759 |
| M flares    | 2186 | 2020 | 1442 | 741  |
| X flares    | 168  | 152  | 126  | 49   |
| Total Nr.   | 17064| 14611| 14709| 8549 |

(iv) Sorting Flares: According to the solar cycle they belong and the value of the proxies, solar flares are divided into three subsets (flares_at_min, flares_at_med and flares_at_max) for each proxy, hence we are dealing with nine different subsets to analyse.

Since the MLE method provides an analytical expression for estimating the power-law index ($\alpha$), thus we apply, for each subset related to an activity regime and for each proxy, a statistical study that combines MLE fitting with Kolmogorov-Smirnov one-sample test. This procedure is applied to the different flare datasets and produces distributions shown in Fig.2. The K-S distance method highlights the fluctuations on the tails of the power-law distribution, indeed in the initial and final part of the distribution the distance is the highest, meaning that the estimate distribution is the farthest from the empirical one.

4. Results and Discussion

Considering all values of the index obtained with different proxies, and for different magnetic activity regimes, it is possible to obtain an average value for the power-law index, $\overline{\alpha} = 2.18 \pm 0.04$. Comparing this with results from Feldman et al. (about 1,000 flares between 1993 and 1995, $\alpha = 1.88 \pm 0.21$) [17], and Aschwanden & Freeland (more than 300,000 flares from 1974 to 2012, $\alpha = 1.98 \pm 0.11$) [3], it appears significantly steeper, while it is compatible with studies from Yashiro et al. (about 6,000 events, $\alpha = 2.16 \pm 0.03$) [2] and Veronig et al. (50,000 flares $\geq$ C-

![Figure 2](image)

**Figure 2.** Left panel: it shows the trend of K-S distance (blue circles), the standard error (green dashed) and the error normalised over the value of $\alpha$ (orange dashed). Right panel: the variation of the power-law index, $\alpha$, with respect to the initial cut-off. Taking $x_{min}$ at the minimum value of the K-S distance (left panel) it gives the best estimate of $\alpha$ (right panel). $x_{min}$ is expressed in $\log(X_{Class}/10^{-6})$ (C-class flare).
class, $\alpha = 2.11 \pm 0.13$ [1]. Such behaviour probably relies upon differences between data with and without background subtraction. Indeed, according to Aschwanden & Freeland [3], data without background subtraction tend to overestimate minor events, especially during phases of maximum activity, resulting hence in a steeper distribution. In the future, using data from GOES XRS lightcurve it would be possible to remove background from NOAA/GOES events in order to confirm such hypothesis.

First we analysed the power-law index for the four solar cycles ($\alpha_{21} = 2.19 \pm 0.04$, $\alpha_{22} = 2.15 \pm 0.02$, $\alpha_{23} = 2.18 \pm 0.02$ and $\alpha_{24} = 2.22 \pm 0.03$). Despite a decrease in the global magnetic flux during cycles (see Fig. 1), the index of the power-law does not show a significant trend, and differences in values are always close to the standard error.

Results from the power-law fitting in relation to the solar magnetic activity are summarised in Tab. 3. We observe that, except for F10.7, the power-law index appears slightly steeper during phases of maximum activity. But the variations observed in power-law index result often below the standard deviation (see Fig. 3), and moreover uncertainties over data remain still fairly relevant. These uncertainties could be explained mainly by two arguments:

i) As previously stated, data without background subtraction tend to overestimate minor flares, especially during periods of high background noise (maximum regime); this effect introduces a bias in the estimation of the power-law distribution parameters. In particular it artificially enhances the power-law index during periods of maximum solar activity.

ii) Other sources of uncertainty could be introduced by arbitrary thresholds of the detection algorithm [18]. The detection of flares represents still nowadays a complex task, thus the events definition can significantly affects statistical results, as observed by Ryan et al. [18]. NOAA/GOES SXR dataset is the most widely accepted and used (e.g [1, 17]), but it results that different catalogues, compiled using detection algorithm based on arbitrary thresholds, lead to different statistical results.

In order to confirm such results we believe it is important to evaluate the impact of the background noise during different activity phases, the comparison between various flare datasets, as well as the analysis of flares per single active region.

![Figure 3](image)

Figure 3. Panels describe complementary cumulative distribution functions (CCDFs) of GOES XRS flares between 1978 and 2017 for the three different proxies, respectively SSN, F10.7cm, MgII. As the solar magnetic activity start to increase (from min to med) $\alpha$ becomes slightly steeper, with the exception of the $F10.7 \text{ cm}$ (central panel). That suggests a generally rise of the number of small flares, not observed with the SSN (left panel). The global behaviour of the distributions, however, seems to remain unchanged.
Table 3. Values of $x_{\text{min}}$ are expressed in $\log(X\text{Class}/E_0)$, where $E_0 = 10^{-6}$ W/m$^2$ (C-class).

| act. phase: | SSN | F$_{10.7}$ | Mg II |
|------------|-----|-----------|-------|
| index ($\alpha$) | min. | med. | max. | min. | med. | max. | min. | med. | max. |
| 2.17 | 2.18 | 2.20 | 2.26 | 2.13 | 2.24 | 2.15 | 2.15 | 2.24 |
| 0.03 | 0.02 | 0.02 | 0.03 | 0.01 | 0.01 | 0.03 | 0.01 | 0.02 |
| 6.0 | 4.8 | 6.3 | 6.5 | 5.6 | 6.0 | 5.8 | 4.9 | 5.9 |

Acknowledgments
We thank the fundamental data service and space weather support from NOAA - NASA/GSFC Solar Data Analysis Center (SDAC) for X-ray flare lists, from SIDC/SILSO - World Data Center at Royal Observatory of Belgium for daily sunspot number, from Space Weather - Natural Resources Canada for radio flux (10.7cm) catalogues, and UVSAT at Bremen University for composite MgII index dataset.

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