Stochastic Gravitational Wave Background from Global Cosmic Strings

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(Dated: October 14, 2019)

Global cosmic strings are generically predicted in particle physics beyond the Standard Model, e.g., a post-inflationary global $U(1)$ symmetry breaking which may associate with axion-like dark matter. We demonstrate that although subdominant to Goldstone emission, gravitational waves (GWs) radiated from global strings can be observable with current/future GW detectors. The frequency spectrum of such GWs is also shown to be a powerful tool to probe the Hubble expansion rate of the Universe at times prior to the Big Bang nucleosynthesis where the standard cosmology has yet to be tested.

\textbf{Introduction.} The recent discovery of gravitational waves (GW) by the LIGO/Virgo collaboration \cite{LIGOd15, LIGOd17, LIGOd18, LIGOd19} ushered in a new era of observational astronomy. In addition to facilitating a better understanding of large compact objects in our current Universe such as blackholes and neutron stars, GWs can also provide an unprecedented window to probe early Universe cosmology and new fundamental physics at the sub-atomic level.

Among potential cosmological sources for a stochastic GW background (SGWB) \cite{1994PhRvD..49.1800C}, a cosmic string network is one that can generate strong signals over a wide frequency range due to continuous emission of GWs throughout a long epoch of cosmic history \cite{Fischler1985, Fujita1995}. This source is among the primary targets for SGWB searches at GW experiments such as LIGO and LISA \cite{1999PhRvD..60f3003F, 2009PhRvD..80h3008F}. Cosmic strings are stable one-dimensional objects characterized by a tension $\mu$. They can arise from superstring theory \cite{Green1987, Green1987a} or from a vortex-like solution of field theory \cite{Kamenshchik1998} which typically descends from a spontaneously broken gauge or global $U(1)$ symmetry. Field theory string tension relates to the symmetry breaking scale $\eta$ by $\mu \propto \eta^2$ while for global strings there is an additional divergence factor $\sim \ln(\eta/H)$ ($H^{-1}$: horizon size) \cite{1983PhRvD..28..642G, 1983PhRvD..28..657G}. Once formed, the network consists of horizon-size long strings along with a collection of sub-horizon sized string loops due to long string intersections. The loops subsequently oscillate and radiate energy until they decay away.

Most of the existing studies on GW signals from cosmic strings are for Nambu-Goto strings or gauge strings where GW is the dominant radiation mode. In contrast, GWs from global strings have been largely neglected as it is in general subdominant to Goldstone boson or axion emission (only a few studies exist \cite{1993PhRvD..48.1337A, 1993PhRvD..48.1350A, 1994PhRvD..49.1800C}). However, since the detection prospect of the Goldstones are highly model-dependent, the GW signal, albeit rare, could be complementary or even a smoking-gun for discovery in the case when the Goldstones/axions have no non-gravitational interaction with the Standard Model (SM) (e.g. in the string axiverse scenario \cite{1999PhRvD..60f3003F}). It is particularly timely to re-examine this overlooked signal channel, in light of the growing interest in axion-like dark matter models where axion strings are inevitably present for post-inflationary $U(1)_{\text{PQ}}$ breaking.

In this Letter, we perform a state-of-the-art calculation for GW frequency spectrum originated from global strings and demonstrate that such GW signals can be observable with current/future GW detectors, such as LISA, DECIGO, BBO and SKA, while consistent with existing constraints. A comparison with gauge string generated GWs is also made. Furthermore, we investigate the effects of background cosmology on this spectrum, e.g. the well-motivated possibilities of early matter domination and kination epochs before the Big Bang nucleosynthesis (BBN), which could lead to signals within reach of LIGO. We thereby demonstrate that, analogous to (yet distinct from) the case with gauge strings \cite{2003PhRvD..68f3505K, 2014PhRvD..89b3506K}, global string induced GW spectrum can be a powerful tool to discern the energy composition of the Universe during the pre-BBN primordial dark age \cite{2006PhRvD..74d3507A, 2006PhRvD..74d3507Aa}.

\textbf{GW spectrum from global cosmic strings.} Results from the simulations for Abelian-Higgs string or Nambu-Goto string network have demonstrated that after formation the network quickly reaches a scaling regime where it tracks the cosmic background energy density with a constant fraction $\sim G\mu$, and GW radiation is the leading energy loss mechanism \cite{1983PhRvD..28..642G, 1983PhRvD..28..657G, 2008PhRvD..78d3520D, 2008PhRvD..78f3523D, 2011PhRvD..83d3521D, 2012PhRvD..86d3521D, 2013PhRvD..88d3522D, 2014PhRvD..90b3521D}. In contrast, simulation for global strings is much more challenging due to the need to cover a large hierarchy in the relevant physical scales: the string core size $\sim \eta^{-1}$ and the inter-string separation scale $H^{-1}$. Recent years have seen rapid development in global string simulations \cite{2015PhRvD..92f4035C, 2016PhRvD..93d3504D, 2017PhRvD..95d3502T, 2018PhRvL.120x1107T, 2018PhRvL.121e1102C} partly driven by its connection to axion physics, while uncertainties remain to be resolved with future higher resolution simulations, in particular whether there is a logarithmic deviation from scaling \cite{2018PhRvD..98h3516D, 2019PhRvD..99h3506D, 2020PhRvD..91h3522D}. Note that the discrepancy among different simulation results could be due to different numerical algorithms and different diagnostics for counting the strings \cite{2018PhRvL.121e1102C, 2019PhRvD..99h3506D}, in addition to the different ranges of excess tension ($\mathcal{N}$, see Eq. 1) they explored. As in \cite{2019PhRvD..99h3506D}, we take the available simulation results at face value for our studies. To integrate the string network evolution into our studies on GW signal, we adopt the approach in \cite{2018PhRvL.121e1102C, 2019PhRvD..99h3506D, 2020PhRvD..91h3522D} which is based on the analytical
velocity-dependent one-scale (VOS) model, with key parameters calibrated with recent numerical simulation results. We focus on oscillating string loops, expected to be the leading sources of both GW and Goldstone emissions [7, 35]. Given the present uncertainties on global string simulation, we will first consider a simple monochromatic loop size distribution at formation time \( t_i = a t_i \). Inspired by the recent gauge string simulations [36, 37], we consider a benchmark scenario that 10% of the network energy releases to \( \alpha \approx 0.1 \) large loops while the remaining goes to kinetic energy of smaller loops that dissipates by redshifts. Later we will also present results for a log uniform distribution up to \( \alpha \approx 1 \) as suggested in [28] based on simulating the first a few e-folds of Hubble expansion after the formation of a global string network. More comprehensive discussion on the sensitivity to loop distribution and the effect of possible deviation from scaling behavior will be presented in [38].

Global strings are characterized by a time-dependent string tension [7, 13, 14]

\[
\mu(t) = 2\pi \rho_0 t^2 \ln \left( L/\delta \right) \equiv 2\pi \eta^2 N, \quad (1)
\]

where \( L \approx H^{-1} \xi^{-1} \) is the string correlation length, \( \xi \) is the number of long strings per horizon volume, \( \delta \approx 1/\eta \) is the string thickness, and \( N \equiv \ln (L/\delta) \approx \ln (\eta \xi^{-1} t) \) is time-dependent. Once reaching the scaling regime, the long string energy density evolves as

\[
\rho_\infty = \xi(t) \frac{\mu(t)}{t} \sim \xi, \quad (2)
\]

where \( \xi(t) \) quickly approaches a constant for gauge or Nambu-Goto strings, yet needs to be determined for global strings. Based on the evolution equation of string correlation length \( L \), the analytical VOS model predicts relations between time-dependent variables \( \xi, N, \bar{v} \) (average velocity) and parameters such as the loop chopping rate \( \bar{c} \), momentum parameter \( k \) and Goldstone radiation parameter \( \sigma \) (the leading dissipation mode for global strings). Therefore, with a set of \( \xi, N, \bar{v} \) given by the simulation results that cover a finite range of the network evolution history, we will be able to fit for model parameters and infer \( \xi(t), \bar{v}(t) \) for other times. For instance, using recent simulation data [27–32] we find \( \xi, \bar{v} \) approaching constant values at times deep into radiation domination (RD) era (\( N \geq 20 \)): \( \xi \approx 4.0, \bar{v} \approx 0.57 \) [82]. Our results based on the VOS model suggests that \( \xi(t) \) evolves as \( \xi \propto 1 - 1/N \) (\( N > 1 \)), which can be approximated as linear growth for very small \( N \), but approaches a constant at large \( N \). A persisting linear growth in \( N \), as suggested by extrapolating simulation results based on very early evolution [28, 31], cannot be reproduced in VOS model, in agreement with [32].

Applying local energy conservation, the loop formation rate per unit volume at the formation time \( t_i \) is given by

\[
\frac{dn_{loop}}{dt} = F_\alpha \frac{C_{\text{eff}}(t_i)}{\alpha} t_i^{-4}. \quad (3)
\]

As said, we will first consider the simple case of \( F_i \approx 10\%, \alpha \approx 0.1 \). \( C_{\text{eff}}(t_i) \) can be predicted based on VOS model solutions and depends on the redshift scaling of the background energy density \( \rho \) of universe. For a single dominating source, \( \rho \) scales as

\[
\rho \propto a^{-\alpha}, \quad (4)
\]

where \( a \) is cosmic scale factor. For \( n = 3 \) (matter domination), \( 4 \) (radiation domination), and \( 6 \) (kination, to be discussed later), we find \( C_{\text{eff}} \approx 1.32, 2.26, 2.70 \) at large \( N \), respectively [38].

Once formed, a loop oscillates and loses energy by the rate [7, 15, 39, 40]

\[
dE/dt = -\Gamma G\mu^2 - \Gamma_a \eta^2, \quad (5)
\]

where the right hand side represents GW and Goldstone radiation in order. Studies show that the dimensionless constants \( \Gamma \approx 50 \) [36, 37, 41, 42], \( \Gamma_a \approx 65 \) [7, 38]. Note that considering the \( 2 \ln (L/\delta) \) factor in \( \mu \) (Eq. 1), GW and Goldstone radiation rates can be comparable at late times for sufficiently large \( \eta > 10^{15} \) GeV). For simplicity we ignore the radiation of heavy radial modes which is suppressed relative to Goldstone emission [28], and would not noticeably affect the result for GW signal.

Consequently the length of a loop after its formation time \( t_i \) would evolve as

\[
\ell(t) \approx \alpha t_i - \Gamma G \mu (t - t_i) - \kappa (t - t_i), \quad (6)
\]

where \( \kappa \equiv \Gamma_a/(2\pi N) \).

Now we are ready to move on to calculate the GW radiation spectrum originated from the network. The string loops emit GWs from normal mode oscillations at frequencies \( f_{\text{emit}} \approx 2k/\ell \) with \( k \in \mathbb{Z}^+ \). The emitted GW frequencies then redshift as

\[
f \approx \frac{a(t_i)}{a(t_0)} \frac{2k}{\ell(t)}, \quad (7)
\]

where \( t \) is GW emission time, \( t_0 \) is the current time. Summing up contributions from all harmonic modes and using Eqs. 3, 6, the GWs relic density spectrum as observed today is

\[
\Omega_{\text{GW}}(f) = \frac{f}{\rho_c} \frac{d\rho_{\text{GW}}}{df} = \sum_k \Omega_{\text{GW}}^{(k)}(f). \quad (8)
\]

with

\[
\Omega_{\text{GW}}^{(k)}(f) = \frac{1}{\rho_c} \frac{2k \mathcal{F}_a}{f} \alpha \int_{t_F}^{t_0} d\ell \frac{\Theta(t_i, \ell)}{\alpha + \Gamma G \mu + \kappa} C_{\text{eff}}(\lbrack t(k) \lbrack_i) \left( \frac{a(t_i)}{a(t_0)} \right)^5 \left( \frac{a(t_i)}{a(t)} \right)^3, \quad (9)
\]

and the decomposed radiation constant as

\[
\Gamma^{(k)} = \frac{\Gamma k^{-4}}{\sum_{m=1}^{\infty} m^{-4}}, \quad (10)
\]
with the causality and energy conversation conditions

$$\Theta(t_i, \tilde{t}) \equiv \theta(\tilde{t} - t_i).$$ (11)

Numerically, \(\sum_{m=1}^{\infty} m^{-\frac{4}{3}} \simeq 3.60\) and \(\sum_k \Gamma^{(k)} = \Gamma, t_F\)
is the formation time of the string network, and \(\rho_c = 3H_0^2/8\pi G\) is the critical density. Note that to consider radiation of Goldstones, we can define \(\Omega_a(f)\) in analogy to Eq. 9, and simply replace \(\Gamma \rightarrow \Gamma_a, \Gamma G\mu^2 \rightarrow \Gamma_a \eta^2\).

To determine the viable range of \(\eta\) to be considered, we first take into account the self-consistency requirement that the \(U(1)\) symmetry breaking occurs or is restored after inflation. In the context of axion cosmology, such a condition is given as \([43]\) \(\eta < \text{Max}\{T_{GH}, T_{\text{max}}\}\), the \(T_{GH} = H_I/2\pi\) is the Gibbons-Hawking temperature of de sitter space during inflation \([44, 45]\), \(H_I\) is the Hubble rate during inflation, and the \(T_{\text{max}} \lesssim \sqrt{M_{\text{Pl}} H_I}\) is the maximum thermalization temperature after inflation \([43, 46]\), \(M_{\text{Pl}}\) is reduced Planck mass. By considering the recent CMB bound on the standard single-field slow-roll inflation model \([47–49]\), we find the constraint \(\eta \lesssim 1.2 \times 10^{16}\text{GeV}\). With these in mind, we will consider \(\eta \lesssim O(10^{15})\text{GeV}\) in our benchmark examples of GW spectra.

In Fig. 1, assuming standard cosmological history, we show GW frequency spectrum originated from global cosmic strings with varying \(\eta\). The gauge string GW spectra \([19, 20]\) are also shown for comparison. One can see that the global string GW amplitudes are more sensitive to \(\eta\): \(\Omega_{\text{gauge}} \propto \eta^4\) vs. \(\Omega_{\text{global}} \propto \eta\) \([38]\). Another observation is that, relative to gauge string spectrum, the global string GW spectrum overall shifts to lower frequency, and the magnitude of the shift depends on \(\eta\). This can be explained by the key relationship between the loop lifetime \(\tau\) and \(\eta\) by solving Eq. (6):

$$\tau \simeq \frac{\alpha + \Gamma G\mu + \kappa}{\Gamma G\mu + \kappa} t_i,$$ (12)

where gauge string scenario is restored with \(\kappa = 0\). We can see that due to the strong Goldstone emission rate, global string loops typically decay within one Hubble time after loop formation, i.e., have much shorter lifetime than their gauge string counterpart, and more so for smaller \(\eta\). Because of this, for loops formed at the same time, radiation (both GWs and Goldstones) from global strings on average experiences a longer period of redshift, rendering a spectrum shifted towards lower frequency. Another notable difference from gauge string spectrum is that, instead of a long nearly flat plateau corresponding to radiation-dominated era, global string spectrum sees a gradually declining plateau towards high \(f\) due to the logarithmic time-dependence of \(\mu\).

In Fig. 2, a benchmark example of GW frequency spectrum is given \((\eta = 10^{15}\text{GeV}, \alpha = 0.1)\) with standard cosmological history. For comparison, we also show the results based on an alternative scenario where sub-horizon loops are formed with logarithmic uniform distribution in the region of \((\eta^{-1}, H^{-1})\), as suggested by \([28]\). We can see that albeit visible differences, the GW spectrum is mostly robust against such an uncertainty/variation in loop distribution. The spectrum based on log uniform distribution extends to slightly lower \(f\) due to the formation of larger loops closer to horizon size. We also show the current sensitivity bands of LIGO \([1, 2, 50, 51]\) and the projected sensitivities for LISA \([52]\), DECIGO/BBO \([53]\), Einstein Telescope (ET) \([54, 55]\) and Cosmic Explorer (CE) \([56]\). In the lower \(f\) region, European Pulsar Timing Array (EPTA) \([57]\) imposes a strong constraint of \(\eta \lesssim 3.2 \times 10^{15}\text{GeV}\), with the expected sensitivity of SKA shown below \([58]\). The presence of cosmic strings potentially distort CMB power spectrum and leads to additional constraint of \(\eta \lesssim 10^{16}\text{GeV}\) \([59–61]\) \([83]\). Furthermore, the total energy densities of GWs and
radiation-like Goldstones are bounded by CMB and BBN as $\int d(\ln f) f \Omega_{GW,a}(f) h^2 \lesssim 3.8 \times 10^{-6}$ [62–64], leading to the constraint $\eta \lesssim 10^{15}$ GeV. In general CMB polarization data potentially yields stronger bound on $\Omega_{GW}$ in the range $f \sim 10^{-17} - 10^{-14}$ Hz [65–67]. However, GWs from global strings safely evade this bound since the GW signal in this very low frequency range is not populated until after photon decoupling time (more on time-frequency correspondence later), thus is not present at the CMB epoch. In summary, Fig. 2 demonstrates an example that GWs from global strings can lead to signals within reach of foreseeable GW detectors while satisfying existing constraints [84].

**Effects of non-standard cosmologies.** As can be seen by Eq. 9, relic GW spectrum from cosmic strings is influenced by an extended period of cosmic history and consequently populates signals over a wide range of $f$. As a result, such a spectrum can be used to test standard cosmology and probe potential deviations prior to the BBN epoch. This idea has been investigated in the context of gauge strings [19, 20, 68]. Here we demonstrate the results for global strings, which are analogous to yet distinct from the gauge string case.

![Gravitational wave frequency spectrum from a global cosmic string network with $\alpha = 0.1$, $F_a = 0.1$ with $\eta = 10^{15}$ GeV (solid lines) and $\eta = 10^{14}$ GeV (dotted lines). The black lines show the GW spectrum with the standard cosmological evolution, while the colored lines correspond to cases with transition to an early period of $n = 6$ kination or $n = 3$ matter domination around the given temperature $T_\Delta = 10$ GeV or 100 GeV. The related experimental sensitivities are also shown.](image)

We consider two types of well-motivated non-standard histories. The first is a period of early matter domination ($n = 3$ in Eq. 4) before the onset of the standard RD era. This can be due to a temporary energy dominance by a long-lived massive particle or an oscillating scalar moduli field in a quadratic potential [69]. Such epochs end with reheating when the relevant particles decay away. The second modification to standard cosmology is a period of kination, with $n > 4$, which can arise from the oscillation of scalar field in non-renormalizable potential in quintessence models for dark energy or inflation [70, 71]. In particular, these scenarios have been considered in recent work on axion cosmology [72, 73]. We define the $t_\Delta(T_\Delta)$ as the time (radiation temperature) when the Universe transits to standard RD. The evolution of the energy density of the Universe before and after such a transition can then be parametrized as

$$
\rho(t) = \begin{cases} 
\rho_{st}(t) \left( \frac{a(t_\Delta)}{a(t)} \right)^n & : t < t_\Delta \\
\rho_{st}(t) & : t \geq t_\Delta
\end{cases}
$$

(13)

where $\rho_{st}$ is the energy density assuming standard cosmology, and $n = 3$ ($n = 6$) for early matter (kination) domination. In addition, to be consistent with BBN [74], in both scenarios the transition temperature should satisfy $T_\Delta \gtrsim 5$ MeV. In Fig. 3 we demonstrate the effects of non-standard cosmologies on GW spectrum from global strings, along with experimental sensitivities. Benchmark parameters are: $\eta = 10^{15}, 10^{14}$ GeV, and $T_\Delta = 10, 100$ GeV. One can see dramatic deviations from the standard prediction: a distinct falling (rising) at high $f$ spectrum due to the transition to an early matter-dominated (kination) phase. LIGO data already excluded the kination case with $T_\Delta \lesssim 20$ GeV, $\eta \gtrsim 10^{15}$ GeV. Potential overproduction of GWs and radiation-like Goldstones imposes additional constraint on the kination case. We have checked that other cases shown in Fig. 3 are consistent with these limits. Furthermore, these bounds can be alleviated depending on the onset/duration of kination domination [38].

As shown in Fig. 3, the transition to a non-standard cosmology at $T_\Delta$ results in a deviation in the GW spectrum at certain frequency $f_\Delta$. This correspondence between characteristic GW frequency and radiation temperature in RD era can be derived rigorously, analogous to the method for the gauge string case [38], or approximately with Eq. 7 (considering $k \simeq 1$ mode dominance [7, 75]) based on the observation that the global string loops are short-lived. The analytical estimate agrees with the numerical result:

$$
f_\Delta \simeq 3.02 \times 10^{-6} \text{Hz} \left( \frac{T_\Delta}{1 \text{ GeV}} \right) \left( \frac{\alpha}{0.1} \right)^{-1} \left[ \frac{g_*(T_\Delta)}{g_*(T_{eq})} \right]^{1/4}.
$$

(14)

In Fig. 4 we illustrate the above relation and the comparison with its counterpart for gauge strings [19, 20]. We can see that, unlike in the gauge string case, $\eta$-dependence is nearly absent in the $f_\Delta - T_\Delta$ relation (also as shown in Eq. 14), and with the same $f_\Delta$ band, GWs from global strings generally probe higher $T_\Delta$ or earlier times. These differences are due to the aforementioned fact that with the same formation time, global string loops on average have much shorter lifetimes and thus the radiated...
GWs experience more redshift in frequency. As a result, with detectors such as ET/CE GWs from global strings can probe cosmology up to a very early epoch of \( T_\Delta \sim 10^8 \text{GeV} \), even well before the reach of gauge strings of \( T_\Delta \sim 10^4 \text{GeV} \) [19] (for gauge strings, \( \eta \gtrsim 10^{13} \text{GeV} \) is excluded by EPTA assuming standard cosmology). Meanwhile, as shown in Fig. 4 for global strings the latest possible transition at \( T_\Delta \sim 5 \text{ MeV} \) is within the coverage of SKA/EPTA. (Note that for the signal amplitude to be within sensitivity of a certain detector, \( \eta \) needs to be sufficiently large.)

**Conclusion/Discussion.** Stochastic GW background (SGWB) from a global cosmic string network is a universal signature and potential discovery mode for the underlying theory typically involving a global \( U(1) \) symmetry breaking which may associate with axion-like dark matter physics. In this work we compute the frequency spectrum originated from such a GW source based on recent analytical modeling and numerical simulation for global/axion strings. We demonstrate that, despite the apparent dominance of Goldstone radiation over GWs, global string generated SGWB can be within reach of an array of current/planned GW detectors, while compatible with existing constraints. We have also shown how such a GW spectrum depends on the equation of state of the background cosmology, and thereby can be utilized to probe the early Universe far before the BBN epoch which would be unaccessible with other observational means.

The GW spectrum from global strings has dependence on loop distribution at formation time and whether the global string network exhibits logarithmic deviation from scaling behavior at late times as suggested by some of the recent simulations. In this work we have taken into account some of these uncertainties, while more expansive study will be conducted in [38].

Although this work focused on global strings associated with massless, radiation-like Goldstones (i.e. no explicit breaking of the \( U(1) \) symmetry), it may also lead to a new avenue for probing massive axion-like particle (ALP) dark matter models. For axion models with post-inflationary \( U(1)_{PQ} \) breaking, axion topological defects including strings and domain walls are indispensable companions to axion dark matter particles. Recently there have been a substantially increased interest in studying axion topological defects [27, 28, 30, 31, 76–78]. Given that detection strategy for axion particles is highly model-dependent, the universal GW signals from axion topological defects can be highly complementary or even the smoking-gun, in particular for hidden axions without non-gravitational interaction with the SM. Due to the non-zero mass of axions and the related late-time formation of domain walls, the calculation of GW spectrum in ALP models is more complex than the case for pure global strings as we studied here. We will explore this direction in future work [38], which can be an important complement to the existing literature on ALP models.

**Acknowledgments.** We thank Edward Hardy, Hitoshi Murayama, Thomas Schwetz-Mangold and Wei Xue for helpful discussion. We also thank Marek Lewicki and David Morrissey for commenting on the manuscript. The authors are supported in part by the US Department of Energy grant DE-SC0008541. CC thanks Yin Chin Foundation of U.S.A. for its support. YC thanks the Kavli Institute for Theoretical Physics, Erwin Schrödinger International Institute, and Galileo Galilei Institute for hospitality while the work was being completed.

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[81] Most studies including the very recent [26] have reached consensus of GW domination for Abelian-Higgs strings, with the exception of e.g. [25]. As suggested in [26] the discrepancy in [25] could be due to insufficient resolution used in simulation on cosmological scales.

[82] We applied most of the data points used in [32] except for those from [29] which only simulates very low $N$ region $(N \lesssim 5)$ when the string network just begins to enter scaling regime and thus is sensitive to initial condition. This choice moderately improves the quality of the numerical fit. Meanwhile, relative to [32] we included a new data point at $N = 6$ from [30]. The data from another recent simulation [31] is left out since we found the fitting is very much off from VOS model prediction. This may be due to the same reason that leads to the discrepancy between their result and that in [28], i.e., the specific way that string length is measured in their simulation which may introduce a different overall rescaling factor [31]. We will further investigate the effects of these uncertainties in [38].

[83] The exact bound on $G_{M}$ due to CMB distortion depends on the type/specifies of the cosmic string network, and is uncertain for global strings related to simulation uncertainties.

[84] [79] drew a pessimistic conclusion for observing GW signals from global strings based on numerical simulation and by considering only the minimum contribution from long strings and horizon-size loops that is sourced at super-horizon scale as in [17, 80]. By considering the (stronger) contribution from sub-horizon loops and based on the semi-analytic VOS model which allows evolving the string network in the entire time range of interest, our study demonstrates a more optimistic prospect for GW signal from global strings.