Supercritical Pomeron
and eikonal representation
of the diffraction scattering amplitude

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Abstract

The intercept of the supercritical Pomeron is examined with the use of different forms of the scattering amplitudes of the bare Pomeron. The one-to-one correspondence between the eikonal phase and the ratio of the elastic and total cross section is shown. Based on new experimental data of the CDF Collaboration, the intercept and power of the logarithmic growth of the bare and total Pomeron amplitude are analyzed. It is shown that as a result of the eikonalization procedure, the bare QCD Pomeron becomes compatible with experiment.

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Despite a long development of different QCD methods of the calculations of diffraction processes, the problem of the pomeron is still very topical. Now we recognize that the research of the pomeron exchange requires not only a pure elastic process but also for many physical processes involving electroweak boson exchanges. There are two approaches to the pomeron, the ”soft” pomeron built of multiperipheral hadron exchanges and more recent perturbative-QCD ”hard” pomeron built of the gluon-ladder. The ”soft” pomeron dominates in high energy hadron-hadron diffractive reactions while the ”hard” Pomeron dominates in high energy $\Upsilon - \Upsilon$ scattering [1] and determines the small $x$-behaviour of deep inelastic structure functions and spin-averaged gluon distributions.

The ”corner stone” for many models of the Pomeron is the power of the total cross sections growth. In the Regge dynamics, the total cross section in the high-energy asymptotic limit is described in terms of a Regge trajectory with intercept $\alpha_{pom}(0) = 1$. This leads to a constant total cross section as $s \to \infty$. The growth of the total cross section at ISR energies requires new or additional assumptions. The ”soft” pomeron of the standard form with $\alpha_{pom}(0) = 1 + \epsilon$ was introduced in [2]. New even and odd under crossing amplitudes were introduced in [3]: the ”Froissaron” and ”Odderon” which have a maximal power of growth $\ln^2 s/s_0$, where $s_0$ is a scale factor, and control the growth of cross sections in the asymptotic regime. The observed growth of inelastic cross sections and the multiplicity coincide with these idea.

Starting from the Low-Nussinov pomeron [4], many studies have been made for determining the pomeron intercept [5]. The perturbative QCD leading-log calculation of the gluon ladder diagrams gives the following result [6]

$$\epsilon = 12 \frac{\alpha_s}{\pi} \ln 2 \sim 0.5$$

This ”hard” pomeron is not yet observed experimentally. Really, the new global QCD
analysis of data for various hard scattering processes leads to the small $x$ behaviour of the gluon structure function determined by the ”hard” pomeron contribution [7].

$$g(x) \sim \frac{1}{x^{1+\epsilon}}$$

with $\epsilon = 0.3$.

So, two different pomeron contributions are observed in soft and hard processes. We think that this may be the manifestation of the same pomeron in different region of momenta transfer. In hard scattering processes, the interaction time is sufficiently small and the single ”hard” pomeron exchange with $\epsilon \sim 0.3$ contributes. In soft diffractive hadron reactions, the interaction time is large and the pomeron rescattering effects must be important. These contributions can decrease the power of the amplitude growth from $\epsilon \sim 0.3$ to $\epsilon_{soft} \sim 0.08$. The same point of view was expressed in [8].

A consistent calculation of the rescattering effects is still an open problem. In what follows, we shall use the most simple approximation of these effects, the eikonalization of the multiple pomeron contributions. It will be shown that the eikonal representation decreases $\epsilon \sim 0.14 \div 0.16$ of the bare pomeron contribution in the eikonal phase to $\epsilon_{soft} \sim 0.09$. To perform this analysis, we shall use the ratio of $\sigma_{el}/\sigma_{tot}$. It will be shown that this ratio leads to practically model independent results for the pomeron intercept.

There exist many discussions about the energy dependence of the elastic and total cross section in hadron - hadron scattering [3 4 11]. A recent analysis of the experimental data has been made in [11]. The conclusion has been drawn that this analysis gives strong evidence for a log($s/s_0$) dependence of $\sigma_{tot}$ at present energies rather than for log($s/s_0^2$) and indicates that the odderon is not necessary to explain experimental data. In the accordance with this work, in [12] the analysis of a phenomenological model has been made and it has been found that available experimental data at $t = 0$ do not indicate a growth of the total cross sections faster than the first power in logarithm of the energy.

On the contrary, in [13] the overall analysis of experimental data has been made
on the basis of the soft and hard supercritical pomeron where the \( \log^2(s/s_0) \) behaviour has been obtained. Good agreement with the data on deep inelastic scattering and photoproduction is achieved when the non-perturbative component of the Pomeron is governed by the "maximal" behaviour, i.e. \( \ln^2 s \) [14].

One can make various comments on this and others analyses and on the used experimental material. Essential uncertainty in the values of \( \sigma_{tot} \) has been shown in [15]. Now we have a large discussion about the value of \( \sigma_{tot} \) at \( \sqrt{s} = 1.8 \) TeV. In [16] it has been found that at this energy \( \sigma_{tot} = 72.2 \text{mb} \). Recent results of the CDF Collaboration [17] are

\[
(1 + \rho^2)\sigma_{tot} = 62.64 \pm 0.95(\text{mb}) \quad \text{at} \quad \sqrt{s} = 546 \text{GeV},
\]

\[
(1 + \rho^2)\sigma_{tot} = 81.83 \pm 2.29(\text{mb}) \quad \text{at} \quad \sqrt{s} = 1.8 \text{ TeV},
\]

and

\[
\delta(s_1) = \frac{\sigma_{elast}}{\sigma_{tot}} = 0.210 \pm 0.002 \quad \text{at} \quad \sqrt{s} = 546 \text{ GeV},
\]

\[
\delta(s_2) = \frac{\sigma_{elast}}{\sigma_{tot}} = 0.246 \pm 0.004 \quad \text{at} \quad \sqrt{s} = 1.8 \text{ TeV}.
\]

The last two relations have small errors because of the cancellation of some errors.

As we will show further, the ratio of these two quantities is more interesting and allows us to obtain the intercept of the bare and eikonalized Pomeron. Let us denote

\[
\Delta(s_{12}) = \frac{\delta(s_1)}{\delta(s_2)} = \frac{\sigma_{elast}(s_1) \cdot \sigma_{tot}(s_2)}{\sigma_{elast}(s_2) \cdot \sigma_{tot}(s_1)}.
\]

We shall consider the properties of the scattering amplitude at \( \sqrt{s} \geq 540 \text{ GeV} \). At such high energies, we can neglect the contribution of the non-leading Regge terms, and in this analysis, for simplicity, we neglect the real part of the scattering amplitude. For the pomeron contribution we use the ordinary form

\[
T(s, t) = i\hbar s^{\alpha(t) - 1} e^{R_0^2 t/2}
\]

(1)

with the linear trajectories \( \alpha(t) = \alpha(0) + \alpha' t \) and \( \alpha(0) = 1 + \varepsilon \). The differential, elastic and total cross sections look as follows:

\[
\frac{d\sigma}{dt} = \pi |T(s, t)|^2; \quad \sigma_{el} = \int_{-\infty}^{0} \frac{d\sigma}{dt} dt; \quad \sigma_{tot} = 4\pi \text{Im} T(s, 0).
\]

4
The slope $B(s,t)$ at $t = 0$ will be defined as

$$B(s,0) = \frac{d}{dt} \ln \left( \frac{d\sigma}{dt} \right)$$

Using the scattering amplitude (II) we obtain

$$\sigma_{el} = \pi h^2 \int_{-\infty}^{0} s^{2(\alpha(t)-1)} e^{R^2_0 t} dt = 2\pi \frac{h^2 s^{2\varepsilon}}{R^2}$$

and

$$\sigma_{tot} = 4\pi h s^\varepsilon$$

where $R$ can be energy-dependent, $R^2 = R^2_0 (1 + \gamma \ln s)$.

Hence, the relation $\sigma_{el}/\sigma_{tot}$ is

$$\frac{\sigma_{el}}{\sigma_{tot}} = \delta(s) = \frac{h}{2R^2} s^\varepsilon.$$  \hfill (4)

Using $\Delta(s_{12})$, we can find the intercept of the pomeron

$$\varepsilon = \frac{\ln(\Delta(s_{12}))}{\ln(s_1/s_2)} + \ln[\frac{1 + \gamma \ln(s_1)}{1 + \gamma \ln(s_2)}]/\ln(s_1/s_2)$$

$$= \varepsilon_0 + \varepsilon_1.$$ \hfill (5)

If we take the amplitude in the logarithmic form of the energy dependence

$$T(s,t) = i h \cdot \ln^n(s) \cdot e^{R^2_0 t/2},$$ \hfill (6)

we can calculate $n$

$$n = \frac{\ln(\Delta(s_{12}))}{\ln(\ln(s_1)/\ln(s_2))} + \ln[\frac{1 + \gamma \ln(s_1)}{1 + \gamma \ln(s_2)}]/\ln(\ln(s_1)/\ln(s_2)).$$ \hfill (7)

$$= n_0 + n_1$$

Now let us consider the eikonal representation of the scattering amplitude which is a simple form of summation of pomeron rescattering effects:

$$T(s,t) = i \int \rho d\rho J_0(\rho \Delta)(1 - e^{i\chi(s,\rho)}),$$ \hfill (8)

where

$$i\chi(s,\rho) = i \int \Delta J_0(\rho \Delta)T^B(s,-\Delta^2) d\Delta.$$ \hfill (9)
and $T^B$ is the amplitude of the bare pomeron. Let us use for $T^B$ the form (1) and then for the eikonal phase we have

$$i\chi(s, \rho) = \frac{\hbar s e^{iB}}{R^2} e^{-\rho^2/(2R^2)} = -X \cdot e^{-\rho^2/(2R^2)}.$$  \hfill (10)

It can be shown that the total and elastic cross section can be represented as

$$\sigma_{tot} = -R^2 \sum_{k=1}^{\infty} \frac{(-X)^k}{k!},$$

$$\sigma_{el} = -R^2 \left\{ 2 \sum_{k=1}^{\infty} \frac{(-X)^k}{k!} + \sum_{k=1}^{\infty} \frac{(-2X)^k}{k!} \right\}.$$  \hfill (12)

Hence, their ratio depends only on $X$.

The numerical calculation shows that for the experimental values we have the one-to-one correspondence with $X_i$:

$$\delta(546) = 0.210 \leftrightarrow X(546) = 1.38 \quad \delta(1800) = 0.246 \leftrightarrow X(1800) = 1.862$$  \hfill (13)

Based on these values of $X_i$, we can obtain the intercept $\varepsilon^B$ or a power of the logarithmic growth $n^B$. They have nearly the same form as (5), (7)

$$\varepsilon^B = \frac{\ln[X(s_1)/X(s_2)]}{\ln(s_1/s_2)} + \ln\left[\frac{1 + \gamma\ln(s_1)}{1 + \gamma\ln(s_2)}\right]/\ln(s_1/s_2)$$

$$= \varepsilon_0^B + \varepsilon_1;$$

$$n^B = \frac{\ln[X(s_1)/X(s_2)]}{\ln(\ln(s_1)/\ln(s_2))} + \ln\left[\frac{1 + \gamma\ln(s_1)}{1 + \gamma\ln(s_2)}\right]/\ln(\ln(s_1)/\ln(s_2))$$

$$= n_0^B + n_1.$$  \hfill (15)

Using the recent experimental data we can calculate

$$\varepsilon_0 = 0.066 \quad \varepsilon_0^B = 0.126,$$

$$n_0 = 0.913, \quad n_0^B = 1.73.$$
The values of $\varepsilon_1$ and $n_1$ weakly depend on $\gamma$. The values of the total cross sections heavily depend on these values of $\gamma$.

Taking into account (13) we can calculate $\sigma_{\text{tot}}$ at $\sqrt{s} = 546$ and 1800 GeV for different $R^2_0$. So, for $R^2 = R^2_0(1 + \gamma \ln(s))$ we have, for example:

1. for $\gamma = 0$

   \[
   R^2_0 = \begin{array}{cccccc}
   6.1 & 6.2 & 6.3 & 6.4 & 6.5 \\
   \sigma_{\text{tot}}(546) = & 60.78 & 61.77 & 62.77 & 63.77 & 64.77 \\
   \sigma_{\text{tot}}(1800) = & 75.03 & 76.26 & 77.49 & 78.72 & 79.95 \\
   \end{array}
   \]

   and

2. for $\gamma = 0.04$

   \[
   R^2_0 = \begin{array}{cccc}
   7.8 & 8.0 & 8.2 \\
   \sigma_{\text{tot}}(546) = & 58.45 & 59.9 & 61.45 \\
   \sigma_{\text{tot}}(1800) = & 76.74 & 78.7 & 80.7 \\
   \end{array}
   \]

Let us recall that the experimental values of $\sigma_{\text{tot}}$ are

\[
(1 + \rho^2)\sigma_{\text{tot}}(546) = 62.64 \text{ mb CDF} \quad (1 + \rho^2)\sigma_{\text{tot}}(546) = 63.5 \text{ mb UA4} \\
(1 + \rho^2)\sigma_{\text{tot}}(1800) = 81.83 \text{ mb CDF} \quad \text{and } \sigma_{\text{tot}}(546) = 63. \text{ mb UA4/2}
\]

the last number being a new result of UA4/2 Collaboration [18].

It is clear from the above Tables that the value of $\gamma$ is unlikely to be more than 0.4 and it is rather preferential to be near zero otherwise we will obtain a great divergence from the experimental data on $\sigma_{\text{tot}}$. Moreover, we can conclude that the experimental value of $\sigma_{\text{tot}}$ at $\sqrt{s} = 1800$ GeV cannot be less than $78 \div 80$ mb; otherwise it will contradict either the value of the ratio $\sigma_{\text{el}}/\sigma_{\text{tot}}$ or the value of $\sigma_{\text{tot}}$ at $\sqrt{s} = 546$ GeV.

The calculation for $\gamma = 0.04$ gives

\[
\varepsilon_1 = 0.0257, \quad n_1 = 0.35.
\]
Hence, as a result, we have for $\gamma = 0.04$,

\[
\varepsilon = 0.066 + 0.026 = 0.092, \quad \varepsilon^B = 0.126 + 0.026 = 0.152;
\]

\[
n = 0.926 + 0.35 = 1.28, \quad n^B = 1.73 + 0.35 = 2.08.
\]

Now, let us examine a more general case of the representation of the eikonal phase [19]:

\[
i\chi(s, \rho) = -h(s) \cdot \exp(-\mu(s) \cdot \sqrt{\rho^2 + b^2(s)})
\]

(16)

From this representation, one can obtain, in the limit $b \to 0$ or $b \to \infty$ the exponential or the gaussian form of the eikonal phase with various energy dependences of $h(s)$ and $b(s), \mu(s)$. This form of the eikonal phase corresponds to the scattering amplitude

\[
T(s, 0) = -\frac{i}{\mu^2} \sum_{n=1}^{\infty} \frac{(-X)^n}{n^2 n!} (1 + n \mu b);
\]

(17)

here and below $X = h(s) \cdot \exp(-\mu \cdot b)$ is the eikonal phase at $\rho = 0$. After calculations one can obtain $\sigma_{el}$ and $\sigma_{tot}$:

\[
\frac{\sigma_{el}}{\sigma_{tot}} = \frac{1}{2} \frac{[2T_1(X) - T_1(2X)] + b \cdot \mu [2T_2(X) - T_2(2X)]}{T_1(X) + b \cdot \mu T_2(X)}
\]

(18)

Here

\[
T_1(X) = -\sum_{n=1}^{\infty} \frac{(-X)^n}{n^2 n!}; \quad T_2(X) = -\sum_{n=1}^{\infty} \frac{(-X)^n}{nn!}.
\]

(19)

In the case when either $b$ and $\mu$ do not depend on energy or

\[
b(s) = b_0 \cdot \kappa(s); \quad \mu(s) = \mu_0 \cdot /\kappa(s)
\]

(20)

the relation (18) depends on energy only through $X$. The calculations show that the value of (18) very weakly depends on the value of $b(s) \cdot \mu(s)$. Again from the value of ratio $\sigma_{el}(s_i)/\sigma_{tot}(s_i)$ we can obtain the value of $X$ and then calculate $\sigma_{tot}(s_i)$ and the slope $B(s_i, 0)$.

There is on one more parameters than in the early simplest form of the eikonal phase [3] so it is needed also to extract experimental information on the slope $B(s, 0)$. We can calculate the total cross sections, the slope $B$ at $\sqrt{s} = 546$ and 1800 GeV.
as functions of $\mu_0$ and choose the value of $b_0$ so that $\sigma_{\text{tot}}(546) = 62 \text{ mb}$. After that we calculate $\sigma_{\text{tot}}$ at $\sqrt{s} = 1800 \text{ GeV}$, the slopes for both energies and the intercept. These calculations are shown in Fig.1. In this figure, the solid lines are the calculated values of $\sigma_{\text{tot}}(1800)$, slopes $B$ at $\sqrt{s} = 546 \text{ GeV}$ (lower curve) and at $\sqrt{s} = 1800 \text{ GeV}$ (upper curve) and intercept of the bare pomeron; the dot-dashed lines show the experimental values and the dotted line their errors; the vertical lines show the bounds on $\mu_0$ put by two upper parts of the figure and hence the values of intercept of the bare pomeron.

It is clear that the experimental magnitude of $\sigma_{\text{tot}}(1800) = 80 \text{ mb}$ restricts weakly the value of $\mu_0$, whereas the magnitudes of slopes at different energies provide strong bounds.

We can see that if $\sigma_{\text{tot}}$ at $\sqrt{s} = 546 \text{ GeV}$ is equal to or larger then $62 \text{ mb}$ and we take into account the values of the relation $\sigma_{\text{el}}(s_i)/\sigma_{\text{tot}}(s_i)$, we cannot obtain $\sigma_{\text{tot}}$ at $\sqrt{s} = 1800 \text{ GeV}$ less than $77 \text{ mb}$.

It is seen from Fig.1 that the intercept $\epsilon_{\text{bare}} = 0.14 \pm 0.01$. The examination of variants with the energy dependence of $\mu$ and $b$ leads to the same result.

So we can see that in the examined energy range the power of the logarithmic growth of $\sigma_{\text{tot}}$ is larger than 1 but smaller than 2. It is clear that we have the $\ln^2$ term in the total cross section \cite{20} but with a small coefficient, and now we are very far from the asymptotic range. The ratio $\sigma_{\text{el}}/\sigma_{\text{tot}} = 0.246$ tells us the same. It is necessary to note that the analysis performed above clearly shows that the data obtained for the relations $\sigma_{\text{el}}/\sigma_{\text{tot}}$ gives a very strong evidence that $\sigma_{\text{tot}} \simeq 80 \text{ mb}$ at $\sqrt{s} = 1800 \text{ GeV}$.

This conclusion is in agreement with one given already in \cite{20}

In our opinion, this calculation shows that the value $\epsilon^{QCD}_{\text{QCD}} = 0.15 \div 0.17$ calculated in the framework of the QCD \cite{21} does not contradict the phenomenological value $\epsilon = 0.08$. The $\epsilon^{QCD}$ is to be compared with the intercept of the bare pomeron $\epsilon^B$, that is $\sim 0.15$, as it is evident from our analysis of experimental data. This is a consequence of the the interaction time being large in soft diffractive hadron reactions and we must take into account the pomeron rescattering effects. Hence the intercept which enters into the structure function can be sufficiently large in agreement with the value of the
bare pomeron intercept.

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Fig. 1 The dependence of $\sigma_{tot}(s_i)$, slope-$B(0, s_i)$ and intercept of bare Pomeron on $\mu_0$. 
Figure Captions

**Fig.1** The dependence of $\sigma_{tot}(s_i)$, slope-$B(0, s_i)$ and intercept of bare Pomeron on $\mu_0$. 