Recent results obtained, often in fruitful collaboration with Japanese colleagues, in the study of the interplay between single-particle and collective degrees of freedom in exotic nuclei are reviewed.

1 Introduction

The new renaissance in nuclear structure claimed by our colleague I. Tanihata in his talk, as produced by the use of radioactive beams and new generations of detectors, challenges the theorists with a number of issues. Among these, the interplay between single-particle and collective degrees of freedom still needs to be investigated systematically.

There is no need to emphasize, how strongly the coupling of the mean-field single-particle states calculated in the Hartree-Fock (HF) or in the Hartree-Fock-Bogoliubov (HFB) approach with the Random Phase Approximation (RPA or QRPA) collective surface vibrations renormalize the properties of the nuclear excitations, in terms of effective masses, charges, spreading widths etc. Decades of works in stable nuclei testify it.

Much less has been done in nuclei far from the stability line, with some pioneering works from the groups in Orsay and Milano. Recently, interesting results have been obtained, and they are shortly reported below.

It has been shown that the coupling with density vibrations of the single-particle HF states in $^{24}$O can lead to new shell structure, eventually accounting for the $N=16$ magic number observed experimentally.

Giant resonances (GR) are coupled as well to the density vibrations and are known to acquire a spreading width mainly through this mechanism in stable nuclei. In a very recent work, the low-lying dipole strength in neutron-rich oxygen isotopes $^{18}$O-$^{22}$O has been calculated and compared to the first experimental data obtained at GSI using the electromagnetic excitation process at beam energy around 600 MeV/u on a Pb target. It is
concluded that the spreading is even larger than in stable, heavy nuclei for which a large systematics exists.

It is well known, that *pairing*, the attraction correlating pairs between the least bound particles in a system, controls almost every aspect of nuclear structure close to the ground state. It determines, to a large extent, which nuclei are stable and which are not.

The very special role played by the pairing force in drip-line nuclei, is understood from the approximate relation between the Fermi level $\lambda$, pairing gap $\Delta$ and nucleon separation energy $S$

$$S \approx -\lambda - \Delta.$$  \hspace{1cm} (1)

Since for drip-line nuclei $S$ is very small, $\lambda + \Delta \approx 0$, meaning that the pairing component of the effective interaction can no longer be treated as a small perturbation important only near the Fermi surface. Thus, very recently the simple QRPA calculations performed using zero-range Skyrme interaction in the standard BCS approach\[13\] were compared for the first time with QRPA results obtained using consistently the finite-range Gogny force\[14\], not only for the ground state in the HFB approximation, but also in solving the QRPA equations to calculate the excited states.

The richness of the many-body effects in the particle-particle channel associated with the coupling to surface vibrations is also under rather active study\[15\].

## 2 Results

### 2.1 Giant Resonances

The calculation in particular for the GR have been done by extending a microscopic model developed in the last decade within the Milano-Orsay collaboration\[16\], in particular by including the pairing correlations in a simple way.

For this purpose, the effective, energy dependent, complex Hamiltonian\[10\]

$$\mathcal{H}(E) \equiv Q_1HQ_1 + W^\dagger(E)$$  \hspace{1cm} (2)

$$= Q_1HQ_1 + Q_1HQ_2 \frac{1}{E - Q_2HQ_2 + i\epsilon}Q_2HQ_1,$$  \hspace{1cm} (3)
used in closed-shell nuclei, has been generalized and the effects of the pairing interaction included, in a Skyrme HF+BCS+QRPA framework[13]. Thus, the eigenstates of $Q_1HQ_1$ are the QRPA eigenstates, including the low-lying quadrupole and octupole collective vibrations[13] to which the quasiparticle (qp) states couple in the $Q_2$ space via the $W^{\downarrow}(E)$ term. For each value of the excitation energy $E$, the QRPA equations for the full $\mathcal{H}$ are solved. The resulting sets of eigenstates $|\nu\rangle$ and complex eigenvalues $E_\nu - i\Gamma_\nu/2$ enable to calculate all relevant quantities, in particular the strength function for an operator $F$

$$S(E) = -\frac{1}{\pi} \sum_\nu \frac{|\langle \nu|F|g.s.\rangle|^2}{E - E_\nu + i\Gamma_\nu/2}. \quad (4)$$

For the dipole strength in the oxygen isotopes $^{18}\text{O}$, $^{20}\text{O}$ and $^{22}\text{O}$, the coupling to the doorway $Q_2$ states (of 4 qp character), increases the low-lying strength below 15 MeV up to 35% compared to the QRPA value in the first two isotopes, bringing the cross section in more than qualitative agreement with the new data[12], as clearly shown in Table 2 and 3 and Fig. 2 and 3 of the published work[11]. The strong experimental decrease in $^{22}\text{O}$ is also reproduced. The same trend is found in a shell model calculation[17]. Fig. 4 of the same paper[11] shows the strong effect of the mixing, eventually stronger than in stable, heavier nuclei, because of the reduced collectivity of the low-lying dipole strength with few component wave functions, reported in Table 3 and 4 of Ref.[11] and the asymmetry of the particle and hole phase-space. All this prevents the strong cancellation[2] among the different contributions to the mixing, reported for the first time in Fig.1 of Ref. [11] for the quasiparticle case.

It is clear, that a systematic appearance and understanding of low-lying dipole strength in neutron-rich nuclei will have deep implications in the astrophysical context, e.g. for the $r$-process, being the statistical $(n,\gamma)$ rate related to the dipole strength function[18].

### 2.2 Magic Numbers

The coupling of vibrations to nucleons moving in levels lying close to the Fermi energy ($E_F$) in atomic nuclei is expected to lead to a number of effects in the s.p. self-energy $\Sigma(E)_j$ added to the mean-field: (i) shifts of the single-particle levels towards the Fermi energy and thus an increase of the level
density, because of the opposite effects\cite{3} on the occupied and unoccupied states of the "polarization" and ‘correlation contributions\cite{3} (the diagrams in Fig.1 of paper\cite{3} (ii) single-particle depopulation, and thus average spectroscopic factors $Z_\omega$ different from unity.

A new frontier of these $\Sigma(E)_j$ calculations is found in the physics of the exotic nuclei, facing the experimental evidence for the appearance of new magic numbers, which starts to be collected. Theoretically, novel features of $\Sigma_j$ connected to the low-energy strength of the collective vibrations and to the coupling with the continuum were discussed in works of the Milano group\cite{8}. The difference with the results in stable nuclei may be qualitative, with level inversion and decreasing of the level density at $E_F$, again because of different cancellation effects among the quoted polarization and correlation contributions.

This is indeed the case, for example, of the nucleus $^{24}$O recently discussed\cite{9}. The s.p. energy of the $1d_{5/2}$ state is lowered by the coupling to the collective $2^+$ states instead of moving up. Moreover, the energy of the $2s_{1/2}$ state is also lowered by the coupling to the $3^-$ vibration. These lowering of occupied s.p. states are very specific because of the blocking of the available phase-space for the other contributions of Fig.1 in Ref.\cite{9}. This results in a small energy gap between $1d_{5/2}$ and $2s_{1/2}$ states and a very large (larger than 6 MeV) energy gap of a new magic number $N=16$, as shown in Fig. 3 of Ref.\cite{9}. This will be consistent with the recent observation of separation energies and interaction cross sections\cite{10}.

\section{Pairing}

In the revival of nuclear structure studies, produced by the availability of exotic nuclei, the problem of nuclear pairing has again become one the forefront of the theoretical interest. Indeed, the existence of neutron halo is due to pairing force\cite{19} and in heavier proton rich $N \approx Z$ nuclei, the proton-neutron pairing may play an important role.

A key problem in the pairing treatment, is the choice of the force. Since the gap equation is already a kind of in-medium two-body Schrödinger equation, one can not use a $G$ matrix which in itself is a solution of the in-medium two-body problem\cite{20}. Apart from the use of a simple interaction with constant matrix elements in a reduced space around the Fermi surface\cite{11}, still done e.g. in the above quoted works\cite{13,11}, it has become popular in the
study of exotic nuclei the use of a density-dependent zero range forces with a cutoff, as first introduced in Ref.[19]. It reads

\[ V(\vec{r}_1, \vec{r}_2) = V_0(1 - x[\rho((\vec{r}_1 + \vec{r}_2)/2)/\rho_0]^\alpha)\delta(\vec{r}_1 - \vec{r}_2), \]  

(5)
depending on 3 parameters ($\rho_0$ is the saturation density) and an energy-space cutoff value, otherwise the gap equation would diverge. Eventually, the cutoff value and $V_0$ may be chosen to reproduce at zero density the scattering length.

On the other side, the finite-range Gogny force exists[21, 22], which allows to perform pairing calculations without introducing new parameters and consistently with the force used in the mean-field. Although it should be considered as a $G$ matrix, it is found[20] that in the $1S_0$ channel it acts very much like a realistic bare force, especially in the D1S version[22] and at least up to the Fermi energy.

Thus, quite recently[14] in our group, we coupled to the HFB code[25] a QRPA code, which allows to calculate for the first time the excited states of a superfluid nucleus with the Gogny D1S[22] interaction. The continuum is discretized by using a box of appropriate dimensions, (see the works[23, 24] for a discussion of the role of the continuum in the pairing problem) and a s.p. Wood-Saxon (WS) basis is used in the expansions. All the details will be found in a large article in preparation. Below, we will report shortly the results obtained for the low-lying $2^+$ state and for the dipole strength distribution in some O and Sn isotopes.

In the isotopes $^{18,20,22}$O, the experimental gap values $\Delta(N, Z)$ obtained with the formula

\[ \Delta(N, Z) = 1/4(S_n(N + 1, Z) + S(N - 1, Z) - 2S_n(N, Z)) \]  

(6)
where $S_n$ is the neutron separation energy, are nicely reproduced, being of the order of 2 MeV. However, the results for excitation energies and $B(E2)$ of the first $2^+$, are in strong disagreement with the experimental data, very much like in the simplified approach HF+Skyrme+BCS of Ref.[13]. The excitation energy are overestimated, being of the order of 4 MeV, while the $B(E2)$ strongly underestimated. A deeper discussion will appear in the paper in preparation. Also the dipole strength distribution is very similar to the one obtained in the work[11], in particular requiring the coupling to 4qp states to
reproduce the experimental strength below 15 MeV, see the discussion above in the subsection Giant Resonances.

In the Sn isotopes, from A=102 to A=130, the calculated energy of the first $2^+$ is of the order of 2 MeV, the experimental one being 1 MeV, and again the $B(E2)$ much lower than the known experimental ones. In the 120 and 124 isotopes the centroid energy of the dipole strength over 18 MeV, is about 3 MeV higher than the experimental ones, as already noted in Ref. [26] for doubly-magic heavy nuclei, while strength below 15 MeV is clearly found in $^{132}$Sn, a result of large astrophysical importance [18].

Having understood the continuum effects [24], much attention is paid to the many-body effects beyond the mean field (BCS or HFB) approaches, both in nuclear matter [27] and in finite nuclei [15]. In the last, in particular, are connected, once again, to the coupling of the s.p. motion to the low-lying surface vibrations (quantal size effects). While the self-energy effects are expected to reduce the pairing contributions, because the nucleons spend part of their time in more complicated configurations ($Z_\omega$ smaller than unity), and to change level density (nucleon effective mass $m^*$) [3], the exchange of collective surface vibrations between nucleons moving in time reversed states near the Fermi surface (induced, core polarization contribution to the effective interaction), is found [28] to lead to a conspicuous contribution to the nuclear pairing gap. This is very much like the attraction among electrons generated by the exchange of lattice phonons in the low-temperature superconductivity [29].

More quantitatively, we may note that, in the extreme weak coupling limit, the effective mass $m^*$ and the $Z_\omega$ factor appear in the expression for $\Delta$ as

$$\Delta \propto E_F \exp\left(-\frac{1}{m^* Z_\omega^2 K^*}\right)$$

A consistent calculation of all many-body effects, treating self-energy and induced pairing interaction on equal footing (not to forget the vertex corrections), is achieved by solving the Dyson equation (also called in this context Nambu-Gor’kov equation [29]) written as

$$G_j^{-1}(E) = G_j^{0-1}(E) - \Sigma_j(E).$$

Each term is a 2x2 matrix, and the diagonal and off-diagonal elements of $G^{-1}$ are the single-particle and pairing (connected to $\Delta$) Green function.
respectively. $G^{0-1}$ is the unperturbed one. In a recent work\cite{15}, this Dyson equation has been solved in the Sn isotopes with phenomenological inputs, a WS with $m^* / m = 0.75$, vibrational states reproducing the experimental ones and standard monopole pairing force with constant matrix element of strength $G$. It is found\cite{15} that the experimental value of the pairing gap of the order of 1.4 MeV is reproduced solving the full Dyson equation with a value of $G$ of the order of 0.17 MeV against a value of 0.23 MeV in the case of no particle-vibration coupling.

This approach, applied to the $^{11}$Li case\cite{30}, allowed to reproduce the extreme halo properties of this nucleus. In a different context, starting from the results of a shell-model calculation, the vibrational properties of this nucleus were discussed also in Ref.\cite{31}.

Any progress done in the description of the pairing correlations in extreme conditions (also with respect to the effective interaction to be used) will have deep consequences in our understanding of the physics of neutron stars, as vortices, cooling etc.\cite{32}.

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