FLAVOR-SPIN SYMMETRY AND THE TENSOR CHARGE*

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Exploiting an approximate phenomenological symmetry of the \(J^{PC} = 1^{+-}\) light axial vector mesons and using pole dominance, we calculate the flavor contributions to the nucleon tensor charge. The result depends on the decay constants of the axial vector mesons and their couplings to the nucleons.

1. Introduction

Investigations of the spin composition of the nucleon have led to surprising insights, beginning with the revelation that the majority of its spin is carried by quark and gluonic orbital angular momenta and gluon spin rather than by quark helicity. In addition, considerable effort has gone into understanding, predicting and measuring the transversity distribution, \(h_1(x)\), of the nucleon.\textsuperscript{\textdagger} Transversity, as combinations of helicity states, \(|\perp/\top >\sim (|+ >|\pm > -\rangle)\), for the moving nucleon is a variable introduced originally by Moravcsik and Goldstein\textsuperscript{\textdaggerdbl} to reveal an underlying simplicity in nucleon–nucleon spin dependent scattering amplitudes. In their analysis of the chiral odd distributions, Jaffe and Ji\textsuperscript{\textdaggerdbl} related the first moment of the transversity distribution to the flavor contributions of the nucleon tensor charge:

\[
\int_0^1 (\delta q^a(x) - \delta q^b(x)) \, dx = \delta q^a \text{ (for flavor index } a)\text{.}
\]

The leading twist transversity distribution function, \(\delta q^a(x)\), is as fundamental to understanding the spin structure of the nucleon as its helicity counterpart \(\Delta q^a(x)\). While the latter in principle can be measured in hard scattering processes, the transversity distribution (and thus the tensor charge) decouples at leading twist in deep inelastic scattering since it is chiral odd. Bounds placed on the leading twist quark distributions through positivity constraints suggest that they satisfy the inequality of Soffer\textsuperscript{\textdaggerdbl}, yet, the non-conservation of the tensor charge makes it difficult to predict. In contrast to

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the axial vector isovector charge, no sum rule has been written that enables a clear
relation between the tensor charge and a low energy measurable quantity. Among
the various approaches, from the QCD sum rule to lattice calculation models, there
appears to be a range of expectations and a disagreement concerning the sign of
the down quark contribution.

Recently, further insight into tranversity has come from its interpretation in
terms of Deeply Virtual Compton Scattering (DVCS) in the context of skewed
parton distributions (SPDs) wherein those distributions which flip the quark heli-
city are the skewed counterparts of the usual quark transversity distributions.
In the forward limit $H_T^q(x,\xi, t)$ reduces to the ordinary tranversity distribution,
$H_T^q(x,0,0) = \delta q^a(x)$. It’s first moment is nothing other than the $t \to 0$ limit of the
form factor associated with the quark helicity flip amplitude $A_{++,-,-}$ which survives
in the forward limit, namely the tensor charge.

Additionally, it has been pointed out by Diehl, that angular momentum con-
servation in these helicity flip amplitudes is accompanied by a transfer of orbital
angular momentum. This is indicated by nonzero intrinsic transverse momentum
transfer of the partons which is not observed in ordinary parton distribution func-
tions where the incoming and outgoing nucleon momentum are equal. We find this
essential correlation between helicity flip, orbital angular momentum and depen-
dence on transverse momentum transfer to persist in our model estimate of the
tensor charge.

2. Modeling The Tensor Charge

Here we present an approach to calculating the tensor charge that exploits the
approximate mass degeneracy of the light axial vector mesons ($a_1(1260)$, $b_1(1235)$
and $h_1(1170)$) and uses pole dominance to calculate the tensor charge. Our
motivation stems in part from the observation that the tensor charge does not mix
with gluons under QCD evolution and therefore behaves as a non-singlet matrix
element. In conjunction with the fact that the tensor current is charge conjugation
odd (it does not mix quark-antiquark excitations of the vacuum, since the latter is
charge conjugation even) suggests that the tensor charge is amenable to a valence
quark model analysis.

2.1. Pole Dominance and Spin-Flavor Symmetry

The flavor components of the nucleon tensor charge are defined from the local
operator nucleon matrix element of the tensor current,

$$\langle P, S_T | \bar{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = 2\delta q^a(\mu^2)(P^\mu S_T^\nu - P^\nu S_T^\mu).$$  (1)
We adopt the model that the nucleon matrix element of the tensor current is dominated by the lowest lying axial vector mesons

$$
\langle P, S_T | \bar{\psi} \sigma^{\mu \nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = \lim_{k^2 \to 0} \sum_{\mathcal{M}} \frac{\langle 0 | \bar{\psi} \sigma^{\mu \nu} \gamma_5 \frac{\lambda^a}{2} \psi | \mathcal{M} \rangle \langle \mathcal{M}, P, S_T | P, S_T \rangle}{M^2_\mathcal{M} - k^2}.
$$

(2)

The summation is over those mesons with quantum numbers, $J^{PC} = 1^{+-}$ that couple to the nucleon via the tensor current; namely the charge conjugation odd axial vector mesons – the isoscalar $h_1(1170)$ and the isovector $b_1(1235)$. To analyze this expression in the limit $k^2 \to 0$ we require the vertex functions for the nucleon coupling to the $h_1$ and $b_1$ meson

$$
\langle MP | P \rangle = \frac{ig_{MN \pi} \pi (P, S_T)}{2M_N} \sigma^{\mu \nu} \gamma_5 u (P, S_T) \varepsilon_\mu k_\nu,
$$

(3)

and the corresponding matrix elements of the meson decay amplitudes which are related to the meson to vacuum matrix element via the quark tensor current.

$$
\langle 0 | \bar{\psi} \sigma^{\mu \nu} \gamma_5 \frac{\lambda^a}{2} | \mathcal{M} \rangle = if_\mathcal{M} \varepsilon_\mu k_\nu - \varepsilon_\nu k_\mu.
$$

(4)

Here $P_\mu$ is the nucleon momentum, and $k_\mu$ and $\varepsilon_\nu$ are the meson momentum and polarization respectively. The former yield the nucleon coupling constants $g_{MN \pi}$ and the latter yield the meson decay constant $f_\mathcal{M}$. Taking a hint from the valence interpretation of the tensor charge, we exploit the phenomenological mass symmetry among the lowest lying axial vector mesons that couple to the tensor charge; we adopt the spin-flavor symmetry characterized by an $SU(6)_W \otimes O(3)$ multiplet structure.\[4\] Thus, the $1^{+-}$ $h_1$ and $b_1$ mesons fall into a $(35 \otimes 1) = 1$ multiplet that contains $J^{PC} = 1^{+-}, 0^{++}, 1^{++}, 2^{++}$ states, where these mesons couple “symmetrically” to baryons

$$
\text{Tr}(J \cdot \Phi) = g \left( \ldots + c_1 \frac{J_5^a}{4M_N} \cdot \frac{F_\mathcal{M}^{\mu \nu}}{4M_N} + c_2 J_\mu^a A_\mu^a + \ldots \right),
$$

where $J$ and $\Phi$ are the nucleon “super” current and meson “super” multiplet. Reducing this expression to 2-component form

$$
\mathcal{L}_{\mathcal{M}N}^{(SU(6) \otimes O(3))} = g \ N^\dagger \left( \ldots + \frac{\sigma}{3} \cdot \hat{k} \hat{P} \cdot \varepsilon_{b_1} + \frac{i}{\sqrt{2}} \left( \hat{P} \times \hat{k} \right) \cdot \varepsilon_{a_1} + \ldots \right) N,
$$

we identify the $SU(6)_W \otimes O(3)$ Yukawa couplings, the $c^{\mathcal{M}}$, and thus the $g_{MN \pi}$. Similarly, the meson decay constants are determined from the $SU(6)_W \otimes O(3)$ quark current couplings to the mesons,

$$
\mathcal{L}_{\mathcal{M}qq}^{(SU(6) \otimes O(3))} = f \ \chi^\dagger \left( \ldots + \sigma \cdot \hat{k} \hat{P} \cdot \varepsilon_{b_1} + \frac{i}{\sqrt{2}} \left( \hat{P} \times \hat{k} \right) \cdot \varepsilon_{a_1} + \ldots \right) \chi.
$$

This analysis enables us to relate the $a_1$ meson decay constant measured in $\tau$ decay, \[f_{a_1} = (0.19 \pm 0.03) \text{GeV}^2\], and the $a_1NN$ coupling constant $g_{a_1NN} = 7.49 \pm 1.0$ (as determined from $a_1$ axial vector dominance for longitudinal charge
as derived in Ref. 14 but using $g_A/g_V = 1.267$ to the meson decay constants and coupling constants. We find

$$f_{b_1} = \sqrt{2} M_{b_1} f_{a_1}, \quad g_{b_1 N N} = \frac{5}{3\sqrt{2}} g_{a_1 N N}, \quad (5)$$

where the $5/3$ appears from the $SU(6)$ factor $(1 + F/D)$ and the $\sqrt{2}$ arises from the $L = 1$ relation between the $1^{++}$ and $1^{+-}$ states. Our resulting value of $f_{b_1} \approx 0.21 \pm 0.03$ agrees well with a sum rule determination of $0.18 \pm 0.03$. The $h_1$ couplings are related to the $b_1$ couplings via $SU(3)$ and the $SU(6)$ $F/D$ value,

$$f_{h_1} = \sqrt{3} f_{b_1}, \quad g_{h_1 N N} = \frac{5}{3} g_{b_1 N N} \quad (6)$$

For transverse polarized Dirac particles, $S^\mu = (0, S_T)$ these values, in turn, enable us to determine the isovector and isoscalar parts of the tensor charge,

$$\delta q^v = \frac{f_{b_1} g_{h_1 N N} \langle k_2^2 \rangle}{\sqrt{2} M_N M_{b_1}^2}, \quad \delta q^s = \frac{f_{h_1} g_{h_1 N N} \langle k_2^2 \rangle}{\sqrt{2} M_N M_{h_1}^2}, \quad (7)$$

respectively (where, $\delta q^v = (\delta u - \delta d)$, and $\delta q^s = (\delta u + \delta d)$). Transverse momentum appears in these expressions because the tensor couplings involve helicity flips that carry kinematic factors of 3-momentum transfer, as required by rotational invariance. The squared 4-momentum transfer of the external hadrons goes to zero in Eq. (2), but the quark fields carry intrinsic transverse momentum. This intrinsic $k_2$ of the quarks in the nucleon is determined from Drell-Yan processes and from heavy vector boson production. We use a Gaussian momentum distribution, and let $\langle k_2^2 \rangle$ range from $(0.58$ to $1.0 \text{ GeV}^2)$.  [17]

3. Mixing and Results

In relating the $b_1(1235)$ and $h_1(1170)$ couplings in Eq. (5) we assumed that the latter isoscalar was a pure octet element, $h_1(8)$. Experimentally, the higher mass $h_1(1380)$ was seen in the $K + \bar{K} + \pi^+ \pi^- \pi^0$ decay channel while the $h_1(1170)$ was detected in the multi-pion channel. This decay pattern indicates that the higher mass state is strangeonium and decouples from the lighter quarks – the well known mixing pattern of the vector meson nonet elements $\omega$ and $\phi$. If the $h_1$ states are mixed states of the $SU(3)$ octet $h_1(8)$ and singlet $h_1(1)$ analogously, then it follows that

$$f_{h_1(1170)} = f_{b_1}, \quad g_{h_1(1170) N N} = \frac{3}{\sqrt{2}} g_{b_1 N N}, \quad (8)$$

with the $h_1(1380)$ not coupling to the nucleon (for $g_{h_1(1) N N} = \sqrt{2} g_{h_1(8) N N}$). These symmetry relations yield the results

$$\delta u(\mu^2) = (0.58 \text{ to } 1.01) \pm 0.20, \quad \delta d(\mu^2) = -(0.11 \text{ to } 0.20) \pm 0.20. \quad (9)$$

These values are similar to several other model calculations: from the lattice; to QCD sum rules; the bag model; and quark soliton models. The calculation has
been carried out at the scale $\mu \approx 1$ GeV, which is set by the nucleon mass as well as being the mean mass of the axial vector meson multiplet. The appropriate evolution to higher scales (wherein the Drell-Yan processes are studied) is determined by the anomalous dimensions of the tensor charge which is straightforward but a slowly varying.

It is interesting to observe that the symmetry relations that connect the $b_1$ couplings to the $a_1$ couplings in Eq. (5) can be used to relate directly the isovector tensor charge to the axial vector coupling $g_A$. This is accomplished through the $a_1$ dominance expression for the isovector longitudinal charges derived in

$$\Delta u - \Delta d = g_A g_{VNN} \frac{\sqrt{2} f_{a_1} g_{a_1 NN}}{M_{a_1}^2}.$$  \hspace{1cm} (10)

Hence for $\delta q^v$ we have

$$\delta u - \delta d = \frac{5}{6} g_{VNN} \frac{g_{a_1} M_{a_1}^2}{M_{b_1}^2 M_N M_{b_1}} \langle k^2 \rangle.$$  \hspace{1cm} (11)

It is important to realize that this relation can hold at the scale wherein the couplings were specified, the meson masses, but will be altered at higher scales (logarithmically) by the different evolution equations for the $\Delta q$ and $\delta q$ charges. To write an analogous expression for the isoscalar charges ($\Delta u + \Delta d$) would involve the singlet mixing terms and gluon contributions, as Ref. 14 considers. However, given that the tensor charge does not involve gluon contributions (and anomalies), it is expected that the relation between the $h_1$ and $b_1$ couplings in the same $SU(3)$ multiplet will lead to a more direct result

$$\delta u + \delta d = \frac{3}{5} M_{NN}^2 \delta q^v,$$  \hspace{1cm} (12)

for the ideally mixed singlet-octet $h_1(1170)$. These relations are quite distinct from other predictions.

4. Conclusions

Our axial vector dominance model with $SU(6)_W \otimes O(3)$ coupling relations provide simple formulae for the tensor charges. This simplicity obscures the considerable subtlety of the (non-perturbative) hadronic physics that is summarized in those formulae. These results support the view that the underlying hadronic physics, while quite difficult to formulate from first principles, is essentially a $1^{+-}$ meson exchange process. Further, through the kinematics of DVCS and those indicated by low momentum transfer we find the essential correlation between helicity flip, orbital angular momentum and dependence on transverse momentum transfer to persist in our model estimate of the tensor charge.
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