The parameters influencing sphere cooling in a cold liquid

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Abstract The processes of heat and mass transfer during cooling of heated sphere in the subcooled liquid are studied. When a heated solid surface is submerged in a liquid with a considerably lower saturation temperature, a vapor film can be formed around the heater. Using of non-equilibrium boundary conditions gives the possibility to solve this problem with taking into account peculiarities of the transport processes at the vapor–liquid interface. The Fourier thermal conductivity law is used for description of heat transport process in vapor film. The heat from external boundary of vapor film is removed through natural convection. Pressure balance is provided by hydrostatic pressure and non-equilibrium boundary condition. The obtained calculation results are compared with the previous calculation data and known experimental results.

1. Introduction
Progressive trends towards enhancement of heat and mass transfer processes at the micro level, the search for highly efficient means of cooling of electronic devices causes an increased interest in heat transfer at extremely high levels of thermal loads. As is known, under these conditions the most effective way of heat removal is boiling process. At the same time the film boiling process can be realized under such conditions. Understanding the film boiling dynamics is crucial for predicting heat transfer performance. Analysis of film boiling is quite challenging due to the process of film boiling which involves two-phase heat and mass transport across the separating interface with widely varying thermo-physical properties. Also, the multifactorial nature of the processes occurring both in technical systems and in natural conditions. The models that can be used to describe processes occurring in vapor film should be choosed taking the following considerations. The continuum mechanic approaches may not be justified in the cases where characteristic process time is several nanosecond, or geometric dimensions of the object are comparable with several mean free path of the vapor molecules. In these cases the methods of molecular kinetic theory or molecular-dynamic simulation approaches should be used. The dynamic models describing the transport processes on the interfacial surface can also be used in the study (consideration) of phenomena such as the eruption of underwater volcanoes [1], modeling emergency situations at nuclear power plants associated with the ingress of molten metal into cooling water [2], etc. Experimental data of papers [3, 4] show that film boiling of subcooled liquids is sensible for different parameters. For example, some surface defects can promote more intense local heat transfer. In this paper we discuss about influence of different parameters on steady film characteristics.
2. Problem formulation

The following physical model of subcooled water film boiling on the sphere of radius $R_w$ immersed into the depth $h$ is proposed. The temperature of this sphere is higher than critical temperature of water. Thus stable vapor film of radius $R_1$ is formed due to the high temperature of the hot sphere $T_w$.

Pressure over the free surface of liquid $P_b$ as well as temperature of cold water $T_b$ are known (Fig. 1).

As can be seen from the schematic diagram of the problem, the liquid is separated from the heater by the vapor film. The heater is cooled by liquid and heater temperature $T_w$ decreases in time. Heat is supplied to the interface from the vapor side. Vapor film is closed from the outside space. Interface is nearly of spherical shape. Possible fluctuations of vapor film are not considered.

![Figure 1. Schematic diagram of the problem.](image)

The first approach is quasi steady state problem.

3. Steady-state problem

The following assumptions are accepted for the mathematical description formulation. Interphase temperature is constant on cross section of vapor film. As the heater is cooled down, the size of vapor film $R_1$ and temperature of vapor-liquid interface $T_1$ are changed in time. Heat and mass transfer processes in two-phase system are considered as being quasisteady. At that, mass flux on the vapor-liquid interface is equal to zero. In this formulation the mathematical problem consists in determination of time dependence of heater temperature considering heat mass transfer peculiarities through vapor-liquid interface. The system of equations is simple enough [5] and includes the heat and pressure balance correlations on the vapor-liquid interface, as well as correlation describing heater temperature change.

Heat balance on the vapor-liquid interface is determined by thermal conductivity and radiation from vapor side and natural convection in liquid (Fig. 2):

$$
\frac{\overline{\kappa}^v(T_w - T_1)R_w}{(R_1 - R_w)R_1} + \varepsilon \sigma_\odot (T_w^4 - T_1^4) = \alpha(T_1 - T_b)
$$

(1)

where $(R_1 - R_w) = \delta$ is the vapor film thickness, $\overline{\kappa}^v$ is the coefficient of average vapor thermal conductivity, $\varepsilon$ is the emissivity factor; $\sigma_\odot$ is Stefan-Boltzmann constant; $\alpha$ is the coefficient of heat transfer. Due to high level of temperature difference between heater and water the radiation can reach 30% of interface heat flux.

Heat flux in liquid is transferred by natural convection:

$$
\alpha = Nu \frac{\lambda^l}{2R_1},
$$

(2)

where $\lambda^l$ is the coefficient of average liquid thermal conductivity.
Nusselt number (Nu) is determined from regime parameters [6]:

\[ Nu = C \cdot Ra^n, \]  

(3)

where \( Ra = Gr \cdot Pr = \frac{g\beta(T_i - T_w)(2R)}{\mu' C_p} \), \( \beta \) is the coefficient of liquid thermal expansion, \( C_p \) is the specific heat, \( \mu' \) is the dynamic viscosity, \( \nu' \) is the kinematic viscosity, \( \rho' \) is the liquid density, \( g \) is the acceleration of gravity.

The values of \( C \) and \( n \) are determined from the following correlations

\[ C = 0.13, \quad n = 1/3 \quad \text{at} \quad Ra > 10^8, \]  

(4)

\[ C = 0.5, \quad n = 1/4 \quad \text{at} \quad 10^5 > Ra > 1, \]  

(5)

\[ Nu = 2 \quad \text{at} \quad Ra < 1 \]  

(6)

As can be seen from correlations (2) – (6) the heat transfer regime depends on heater diameter and temperature difference in liquid.

Pressure balance (Fig. 3) is determined by hydrostatics \( (P_b + \rho'gh) \), non-equilibrium boundary condition [7], and Laplace equation:

\[ P_s(T_1) + 0.44 \frac{\kappa(T_w - T_1) R}{R_i(R_i - R_g) \sqrt{2RT_1}} - (P_b + \rho'gh) = \frac{2\sigma}{R_i}, \]  

(7)

where \( R \) is the individual gas constant, \( T_1 \) is the interface surface temperature, \( P_s(T_1) \) is the saturation pressure at the temperature \( T_1 \), \( \sigma \) is the surface tension, and \( \rho' \) is the liquid density.
The system of equations (1)-(7) can be solved for prescribed temperature of heater $T_w$. After this the balance correlation for heater temperature can be used to find time dependence $T_w(\tau)$:

$$\frac{dT_w}{d\tau} = -\frac{3q_w}{R_w \rho_w C_w},$$

(8)

where $q_w = \frac{\lambda''(T_w - T_1')R_1}{(R_1 - R_w)R_w} + \varepsilon \sigma_o (T_w^4 - T_1'^4)$.

The system of equations was solved numerically by standard MathCad procedures.

4. Results and discussion

The dependences of Rayleigh number and heat transfer coefficient on sphere radius for cases of different water temperature are presented in Figure 4. As is obvious from these results, the heat transfer coefficient decreases with increasing heater radius and also depends on temperature difference between vapor-liquid interface and cold water.

It should be noted that vapor-liquid interface temperature changing should be taken into account when calculating heat transfer coefficient. For example, if we proposed that vapor film thickness is much less than heater radius, the Rayleigh number can be rewritten in the following manner:

$$Ra = \frac{g \beta(T_1 - T_b)[2(R_w + \delta)]^3 \mu' C_p}{(\mu' \rho')^2 \lambda'} = \frac{g \beta(T_1 - T_b)[2R_w \left(1 + \frac{\delta}{R_w}\right)]^3 \mu' C_p}{(\mu' \rho')^2 \lambda'} \approx \frac{g \beta(T_1 - T_b)[2R_w]^3 \mu' C_p}{(\mu' \rho')^2 \lambda'}.\quad (9)$$

The correlation (2) can be rewritten by following manner using correlations (3)-(5) and assuming that the difference $T_1 - T_b$ is constant:

$$\alpha = C \left( \frac{g \beta(T_1 - T_b)(2R_w)^3 \mu' C_p}{(\mu' \rho')^2 \lambda'} \right) \frac{\lambda'}{2R_w} = Const \cdot R_w^{3n-1}.\quad (10)$$

In this case the following correlation for $\alpha$ can be obtained:

$$\alpha = Const, Ra > 10^8$$

$$\alpha = Const \cdot R_w^{-0.25}, 1 < Ra < 10^8$$

One can see that the heat transfer in liquid mainly depends on heater diameter and temperature difference. At that, the corresponding dependences are presented in figure 4. The space between two lines is not quite wide and values of Ra difference are proportional to temperature difference. For accounting the effect of natural convection on hot sphere cooling, the known correlation for heat transfer during natural convection is used. At this the correlation for spheres with the diameter up 10 $\mu$m is formulated for thermal conductivity without taking into account liquid motion.
Figure 4. Dependences of Ra number and heat transfer coefficient on sphere radius:

1 – $T_b=80^\circ$C; 2 – $T_b=20^\circ$C.

Further, the results obtained using correlation (1), (7) and (8) are presented. The dependences of relative thickness of vapor film on sphere radius for two different cold water temperatures are presented on figure 5. As is obvious from these results, the water temperature influences significantly vapor film values. For example, for cases of small heat droplet radius $10\mu m \leq R_w \leq 550\mu m$ for case of hotter water the vapor film is three times thicker than for the case of less heated water. It should be noted that for case of $T_b=20^\circ$C the relative thickness of vapor film decreases linearly with increasing radius. On the other hand, the minimum thickness value is observed in case of $T_b=80^\circ$C. In this situation the relative thickness of vapor film decreases linearly when droplet size changed from $10\mu m$ to $550\mu m$. When droplet radius becomes larger than $550\mu m$ the relative vapor thickness begins to increase. This effect is observed at this case because the liquid cannot remove all heat flux arriving from the heater.

Figure 5. Dependences of relative thickness of vapor film on sphere radius:

1 – $T_b=80^\circ$C; 2 – $T_b=20^\circ$C.
The cooling rate of the heated body for cases of different radius are presented in figure 6. In these cases the solid body heated to temperature 600°C is submerged into a liquid with a temperature of 20°C. As can be seen from this figure, the cooling rate depends strongly on sphere radius. On the one hand, this result seems quite obvious, but as can be seen from figure 5, the vapor film thickness for different heater radius is not the same. Consequently, the quantitative estimates concerning droplet cooling rate can be made based on the system (1), (7), (8) solution. It should be noted that the cooling rate for the sphere of 32 mm in diameter can be estimated as 5-10 K/s in the experiment [3] for different sub-cooling temperature of water.

![Figure 6](image)

**Figure 6.** Dependences of heater temperature on time:

A – $R_w=10^{-3}$ m; B – $R_w=10^{-4}$ m.

5. Conclusion

The influence of heated body size on its cooling rate is presented. The cooling rates for the spheres of different sizes were obtained based on solution results of vapor thickness and interface temperature values. The heat balance at the interface surface is considered. In this case the heat flux arriving to the interface surface from heater by conductivity should be equal the heat flux in a liquid transferred by a natural convection. On the one hand, the flow regime of natural convection depends on heater size, and on the other hand, the actual vapor pressure is determined by heat flux value. Consequently the mechanical equilibrium is considered. The results have shown that when heater size decreases, the intensity of heat transfer in a liquid increases. At the same time the relative thickness of the vapor film increases.

Acknowledgments

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