MIGRATION AND THE FORMATION OF SYSTEMS OF HOT SUPER-EARThS AND NEPTUnES

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ABSTRACT

The existence of extrasolar planets with short orbital periods suggests that planetary migration induced by tidal interaction with the protoplanetary disk is important. Cores and terrestrial planets may undergo migration as they form. In this paper we investigate the evolution of a population of cores with initial masses in the range 0.1–1 $M_\oplus$ embedded in a disk. Mutual interactions lead to orbit crossing and mergers, so that the cores grow during their evolution. Interaction with the disk leads to orbital migration, which results in the cores capturing each other in mean motion resonances. As the cores migrate inside the disk inner edge, scarring and mergers of planets on unstable orbits, together with orbital circularization, causes strict commensurability to be lost. Near commensurability however is usually maintained. All the simulations end with a population typically between two and five planets, with masses depending on the initial mass. These results indicate that if hot super-Earths or Neptunes form by mergers of inwardly migrating cores, then such planets are most likely not isolated. We would expect to always find at least one, more likely a few, companions on close and often near-commensurable orbits. To test this hypothesis, it would be of interest to look for planets of a few to about 10 $M_\oplus$ in systems where hot super-Earths or Neptunes have already been found.

Subject headings: planetary systems: formation — planetary systems: protoplanetary disks

1. INTRODUCTION

The recent announcements of the detection of extrasolar planets of a few earth masses ($M_\oplus$) (OGLE-05-390Lb, 5.4 $M_\oplus$, Beaulieu et al. 2006; Gliese 876d, 7.3 $M_\oplus$, Rivera et al. 2005) gives support to the initial solid core accumulation model for planet formation. Although it has been proposed that such “super-Earths” could have formed through gravitational instability in a gaseous protoplanetary disk leading to a giant planet, which subsequently lost its gaseous envelope through the action of external UV radiation (Boss 2006), a formation mechanism through the accumulation of planetesimals seems more natural. In addition to these super-Earths, nine planets with a mass comparable to that of Uranus or Neptune (in the range 10.5–18.5 $M_\oplus$) have been reported.

OGLE-05-390Lb is at a distance of 2.1 AU from the central star and was detected through microlensing. Gliese 876d, detected through radial velocity measurements, is at 0.02 AU from its parent star. Among the nine planets with masses similar to that of Neptune, four are within 0.1 AU of the central star.

Gliese 876 is an M-type star. The temperature at 0.02 AU from the star is therefore low enough for heavy elements to condense. Thus, it is possible to consider that Gliese 876 d formed in situ by accumulation of heavy material that spiraled in with the gas through the circumstellar disk. However, the existence of close orbiting giant planets, the so-called hot Jupiters and hot Neptunes, has been taken as an indication of the operation of large-scale migration induced by the interaction of recently formed protoplanets with protoplanetary disks (e.g., Papaloizou & Terquem 2006, and references therein). Such migration processes may have operated during the formation of lower mass planets as well. In particular, planets in the Earth mass range are expected to undergo type I migration (e.g., Ward 1997). Accordingly, here we envisage a scenario in which cores assemble further away from the central star and migrate inward due to tidal interaction with the disk. The disk is supposed to be truncated at some inner edge so that the planets do not fall onto the star. Mutual perturbations of the cores lead to orbit crossing, collisions, and possibly mergers during the migration phase, resulting in the formation of a smaller number of more massive planets on short-period orbits.

Such a scenario has been considered by Brunini & Cionco (2005), who calculated by means of N-body simulations the evolution of 100 protoplanets of 0.5 $M_\oplus$ together with 200 planetesimals of 0.1 $M_\oplus$ subject to their mutual interaction and the tidal interaction with the disk. Focusing on the mass and semimajor axis of the final largest solid core, they concluded that Neptune-like planets on short orbits should be common.

Here we focus on the final configuration of the multiplanet systems we obtain. Evolution of an ensemble of cores in a disk almost always leads to a system of a few planets, with masses that depend on the total initial mass, on short orbits with mean motions that frequently exhibit near commensurabilities and, for long enough tidal circularization times, apsidal lines that are locked together. Starting with a population of 10–25 planets of 0.1 or 1 $M_\oplus$, we typically end up with between two and five planets with masses of a few tenths of an Earth mass or a few Earth masses, depending on the total initial mass, inside the disk inner edge. Interaction with the central star leads to tidal circularization of the orbits, which, together with possible close scatterings and final mergers, tends to disrupt mean motion resonances that are established during the migration phase. The system, however, often remains in a configuration in which the orbital periods are close to commensurability.

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Apsidal locking of the orbits, if established during migration, is often maintained through the action of these processes.

The plan of the paper is as follows. In § 2 we review studies of the evolution of migrating resonant planets. We also show that tidal circularization by the central star results in the disruption of strict commensurability, although the mean motions may remain nearly commensurable and apsidal lines locked. In § 3 we describe the model we use to follow the evolution of a population of planets and describe our initial conditions. To calculate the mutual interactions between the cores, we use an N-body code. A dissipative force is included to model the tidal interaction with the disk, which leads to orbital decay and eccentricity and inclination damping. Relativistic effects and tidal interaction with the central star are also included, as they affect the eccentricity of the orbits close to the star. We also incorporate the possibility of corotation torques acting in the edge region. The potential importance of these in reversing type I migration has been indicated by Masset et al. (2006). We discuss the effects of such torques on eccentric orbits. In § 4 we describe our results. As mentioned above, all the runs end with a few planets on close orbits inside the inner cavity that frequently exhibit near commensurability and apsidal line locking. We study the effect of varying the circularization timescale and of a hypothetical reversal of the torque near the disk inner edge due to corotation or other effects. Finally, in § 5 we summarize and discuss our results.

2. MIGRATION OF PLANETS AND ORBITAL RESONANCE

2.1. Resonant Capture during Migration

The existence of commensurabilities among the mean motions of pairs of satellites of Jupiter and Saturn is believed to be the result of capture into resonances following the differential expansion of their orbits induced by the dissipation of the tides raised in the central planet (Greenberg et al. 1972, 1973). Once established, such commensurabilities are stable due to the secular transfer between the satellites of the angular momentum fed into the satellite system by the tides (Goldreich 1965). Planets in a disk may also get locked into a resonance if their semimajor axes evolve at a different rate, causing their orbits to approach each other. Melita & Woolfson (1996; see also Haghighipour 1999) were the first to consider such a scenario to explain near commensurability between the periods of the major planets of the solar system. In their study, the evolution of the semimajor axes was assumed to be caused by accretion of gas by the planets and dynamical friction. Kley (2000) subsequently studied the evolution of two Jupiter-like planets embedded in a protoplanetary disk that underwent orbital migration due to tidal interaction with the disk. Formation and maintenance of commensurabilities in a system of migrating planets in this type of simulation was subsequently reported by Masset & Snellgrove (2001) and Snellgrove et al. (2001). Since then, different studies, motivated by the observation of extrasolar planetary systems exhibiting commensurabilities, have shown that capture of giant planets into resonances during migration is a natural expectation (Nelson & Papaloizou 2002; Lee & Peale 2002; Kley et al. 2004).

The planets subsequently migrate, maintaining the commensurability. The capture into resonance of migrating planets in the Earth mass range has also been studied (Papaloizou & Szuszkiewicz 2005; McNeil et al. 2005; Cresswell & Nelson 2006).

Once an embedded pair of planets is in resonance, the resonant angles (see below) and the angular difference of the apsidal lines are generally found to librate about fixed values. Note that the apsidal lines need not necessarily be aligned or antialigned. The relative orientation of the orbits may be phase locked at an angle that differs from 0° or 180°, depending on their eccentricities and the mass of the planets (Beaugé et al. 2003; Kley et al. 2004).

In this paper we study the commensurabilities that are established when a population of protoplanets with masses on the order of an Earth mass migrate together through a protoplanetary disk. As they migrate, the planets undergo collisions, which are assumed to result in mergers. This causes their masses to grow with time. We suppose that the protoplanetary disk has an inner edge inside of which is a cavity in which disk-planet interactions and induced orbital migration cease. Observations suggest the existence of magnetospheric cavities of this type with their extent controlled by the magnetic field of the central star (e.g., Bouvier et al. 2007). Inner disk boundaries in the range 0.05–0.1 AU might be expected. Once the planets enter such a cavity, both additional collisions and mergers, as well as tidal interaction with the star, cause strict commensurability to be lost, although near commensurability and locking of the apsidal lines of the orbits of pairs of planets may be maintained under the action of both these processes. Similar effects resulting from tidal interaction have been noted in the context of the solar system (e.g., Dermott et al. 1998) and in the context of extrasolar systems of giant planets (Novak et al. 2003). We now consider the dynamics of two planets in mean motion resonance with locked apsidal lines that are subject to tidal interaction with the central star that causes orbital circularization and shows how strict commensurability is lost, while the relative orientation of the apsidal lines can be maintained.

2.2. Loss of Commensurability through Tidal Dissipation

Once a planet has migrated to a small enough radius, tidal interaction with the central star becomes significant. If we assume that the rotation period of the central star is longer than the orbital period of the planet, which is expected for planets with orbital periods of ~4 days or less, tidal interaction between the star and the planet leads to eccentricity damping and orbital decay. The consequent reduction in the semimajor axis would tend to cause any previously formed commensurability to be lost. Here we neglect the tides raised on the star by the planet, as we only consider planets of a few Earth masses for which such tides are not expected to be significant (Goldreich & Soter 1966). To get some insight into the dynamics of the system, we consider a simple model in which there are only two planets orbiting a star of mass $M_*$. We denote by $m_i$, $a_i$, $e_i$, and $n_i$ the mass, semimajor axis, eccentricity, and mean motion of the inner planet ($i = 1$) and the outer planet ($i = 2$). The two planets are presumed to be in a mean motion resonance, so that $n_1/n_2 = (p + q)/p$, where $p$ and $q$ are integers. To simplify the discussion we only consider the $\epsilon^0$-eccentricity resonances with $q = 1$. The associated resonant angles are $\Phi_1 = p\lambda_1 - (p + 1)\lambda_2 + \dot{\omega}_1$ and $\Phi_2 = p\lambda_1 - (p + 1)\lambda_2 + \dot{\omega}_2$, where, for $i = 1$ and 2, $\lambda_i$ are the mean longitudes and $\dot{\omega}_i$ are the arguments of pericenter.

The rates of change of the semimajor axes and eccentricities for planets $i = 1$ and 2, induced by the resonant interaction, can be found from the following equations (e.g., Dermott et al. 1988):

\begin{equation}
\frac{da_i}{dt} = \frac{2}{m_i} \sqrt{\frac{a_i}{GM_*}} \frac{\partial U}{\partial a_i},
\end{equation}

\begin{equation}
\frac{de_i}{dt} = -\frac{1}{e_m n_i \sqrt{GM_* a_i}} \frac{\partial U}{\partial \dot{\omega}_i}.
\end{equation}
interaction potential, which, to first order in the eccentricities, is of the form

$$U = -\frac{m_1 m_2}{a_2^2} [e_1 S_1(\alpha) \cos \Phi_1 + e_2 S_2(\alpha) \cos \Phi_2],$$

(3)

where $\alpha = a_1/a_2$ and the $S_i$ are known functions (Goldreich 1965; Dermott et al. 1988), the detailed form of which does not affect our conclusions.

The rates of change of $e_i$ and $a_i$ also have additional contributions $(de_i/dt)_i$ and $(da_i/dt)_i$ arising from tidal dissipation (hereafter, the subscript $t$ denotes variations due to tidal effects). Thus, by use of equations (1) and (3), we obtain

$$\frac{da_1}{dt} = 2p \frac{m_1}{m_2} \sqrt{\frac{a_1}{GM_*}} F + \left( \frac{da_1}{dt} \right)_t,$$

(4)

$$\frac{da_2}{dt} = -\frac{2(p+1)}{m_2} \sqrt{\frac{a_2}{GM_*}} F + \left( \frac{da_2}{dt} \right)_t,$$

(5)

where

$$F = \frac{m_1 m_2}{a_2^2} [e_1 S_1(\alpha) \sin \Phi_1 + e_2 S_2(\alpha) \sin \Phi_2].$$

(6)

In addition, we find, from equations (2) and (3), for the rate of change of the eccentricities,

$$\frac{de_1}{dt} = -\frac{1}{e_1 m_1 \sqrt{GM_* a_1}} F_1 + \left( \frac{de_1}{dt} \right)_t,$$

(7)

$$\frac{de_2}{dt} = -\frac{1}{e_2 m_2 \sqrt{GM_* a_2}} F_2 + \left( \frac{de_2}{dt} \right)_t,$$

(8)

where

$$F_i = \frac{m_1 m_2}{a_2^2} e_i S_i(\alpha) \sin \Phi_i, \quad i = 1, 2,$$

(9)

and we have $F = F_1 + F_2$.

We are now going to show from the above equations that, if the resonance is maintained on average, the eccentricities must decrease with time. This means that the existence of the resonance acting through the two resonant angles cannot prevent the decay of the eccentricities due to the action of the tides. Indeed, for the resonance to be maintained on average, we must have $d \ln (a_1/a_2)/dt = 0$. Equations (4) and (5) then indicate that, on average

$$\frac{da_1}{dt} = \frac{2p}{m_1 a_1} \sqrt{\frac{a_1}{GM_*}} + \frac{2(p+1)}{m_2 a_2} \sqrt{\frac{a_2}{GM_*}} F = -\left[ \frac{d \ln (a_1/a_2)}{dt} \right]_t,$$

(10)

We can now use equations (7) and (8) to obtain

$$m_1 \sqrt{GM_* a_1} \frac{d e_1^2}{dt} + m_2 \sqrt{GM_* a_2} \frac{d e_2^2}{dt} = -2F + m_1 \sqrt{GM_* a_1} \left( \frac{d e_1^2}{dt} \right)_t + m_2 \sqrt{GM_* a_2} \left( \frac{d e_2^2}{dt} \right)_t.$$

(11)

Now, the effect of the tides causing circularization is to decrease $e_i$, i.e., $(de_i/dt)_i < 0$, while conserving the angular momentum of $m_1$. Thus, $a_i$ also decreases. But tides are stronger in $m_1$, which is closer to the central star, which means that $|d \ln (a_1/a_2)/dt| < 0$. Equation (10) then implies that $F$ is positive. It then follows that the right-hand side of equation (11) is negative, from which we can deduce that the eccentricities decrease with time as long as any one of them is nonzero. This implies that if the resonance is maintained, the $e_i$ must ultimately decrease.

However, for the resonance to be maintained, one requires that the change in $a_1$ produced by tides in one libration period be much smaller than the amplitude of the oscillation in $a_1$ in resonance, $\Delta a_1$ (adiabatic criterion; see, e.g., Dermott et al. 1988). We now show that this is not expected to be compatible with a decrease of the eccentricities. Indeed, when the eccentricity decreases, the libration period increases and $\Delta a_1$ decreases, so that at some point the resonance is broken. To see how this happens, as long as the eccentricities are not too small, one can use a perturbation scheme with the small parameter $e = (m_1/M_*)^1/2$, where $m_1$ is the largest of the planet masses. For resonance libration we expect $d/\Delta t = O(e_i/n_1)$. As $d\dot{\omega}/\dot{\Omega} = O(e_i^2 n_1)$, to lowest order this is commonly neglected (Dermott et al. 1988). Under this scheme (which in addition requires $e_i \gg e_i^2/n_1$), the angles $\Phi_1$ and $\Phi_2$ obey the same equation (Dermott et al. 1988)

$$\frac{d\Phi_1}{dt} = \frac{d\Phi_2}{dt} = p n_1 - (p+1)n_2,$$

(12)

and $\Phi_1 - \Phi_2 = \tilde{\omega}_1 - \tilde{\omega}_2$ is constant. Thus, we find

$$\frac{d^2 \Phi_1}{dt^2} = \frac{d^2 \Phi_2}{dt^2} = -\frac{3F}{\sqrt{GM_*}} \left[ \frac{p^2 n_1}{m_1 a_1} + \frac{(p+1)^2 n_2}{m_2 a_2} \right]$$

$$+ \frac{p}{m_1} \left( \frac{dn_1}{dt} \right) - (p+1) \left( \frac{dn_2}{dt} \right).$$

(13)

This can be further reduced to a forced pendulum equation (e.g., Goldreich 1965):

$$\frac{d^2 \xi}{dt^2} = -\omega_\xi^2 \sin \xi + p \left( \frac{dn_1}{dt} \right) - (p+1) \left( \frac{dn_2}{dt} \right),$$

(14)

where $\xi = \Phi_1 - \delta$, with

$$\tan \delta = \frac{e_2 S_2 \sin (\tilde{\omega}_1 - \tilde{\omega}_2)}{e_1 S_1 + e_2 S_2 \cos (\tilde{\omega}_1 - \tilde{\omega}_2)}.$$
Note that to the order we have worked above, $\Delta \omega = \omega_1 - \omega_2$ is constant, corresponding to a secular resonance for which apsidal alignment is maintained. As this type of resonance does not require a mean motion commensurability, it is possible for it to be maintained during the circularization process, as well as some scattering and merger events (see below).

3. MODEL AND INITIAL CONDITIONS

We consider a system consisting of a primary star and $N$ cores or protoplanets embedded in a gaseous disk surrounding it. The cores undergo gravitational interaction with each other and the star and are acted on by tidal torques from the disk.

Work by several authors (e.g., Kley et al. 2004; Papaloizou & Szuszkiewicz 2005; Cresswell & Nelson 2006) has demonstrated that the essential aspects of their motion can be captured by $N$-body integration. The effect of the disk torques and dissipative forces are included in the integration. Such a procedure has been shown to give results very similar to those obtained when the disk response induced by a planetary perturber and the torques acting back on the protoplanet are calculated using hydrodynamic simulations.

The equations of motion are

$$\frac{d^2 r_i}{dt^2} = -\frac{GM_i r_i}{|r_i|^3} - \frac{\sum_{j=1, j \neq i}^N Gm_j (r_i - r_j)}{|r_i - r_j|^3} - \Gamma_i + \Gamma_r,$$

(17)

where $M_i$, $M_d$, and $r_i$ denote the mass of the central star, that of planet $i$, and the position vector of planet $i$, respectively. The acceleration of the coordinate system based on the central star (indirect term) is

$$\Gamma = \sum_{j=1}^N \frac{Gm_j r_j}{|r_j|^3},$$

(18)

and that due to tidal interaction with the disk and/or the star is dealt with through the addition of extra forces as in Papaloizou & Larwood (2000):

$$\Gamma_i = -\frac{1}{t_{m,i}} \frac{dr_i}{dt} - \frac{2}{|r_i|^3 t_{f,i}} (dr_i \cdot r_i) r_i - \frac{2}{t_{e,i}} \frac{dr_i}{dt} \cdot e_z,$$

(19)

where $t_{m,i}$, $t_{f,i}$, and $t_{e,i}$ are the timescales over which, respectively, the angular momentum, the eccentricity, and the inclination with respect to the unit normal $e_z$ to the gas disk midplane change. Evolution of the angular momentum and inclination is due to tidal interaction with the disk, whereas evolution of the eccentricity occurs due to both tidal interaction with the disk and the star. We have

$$\frac{1}{t_{e,i}} = \frac{1}{t_{e,i}^d} + \frac{1}{t_{e,i}^t},$$

(20)

where $t_{e,i}^d$ and $t_{e,i}^t$ are the contributions from the disk and tides raised by the star, respectively. Relativistic effects are included through $\Gamma_r$ (see Papaloizou & Terquem 2001).

3.1. Orbital Circularization due to Tides from the Central Star

The circularization timescale due to tidal interaction with the star is given by Goldreich & Soter (1966) as

$$t_{e,i}^t = 4.065 \times 10^4 \left( \frac{M_\odot}{M_i} \right)^{2/3} \left( \frac{20a_i}{1 \text{ AU}} \right)^{6.5} Q' \text{ yr},$$

(21)

where $a_i$ is the semimajor axis of planet $i$. Here and below we have adopted cgs units in which the planet has a density of 1. The parameter $Q' = 3Q/(2k_2)$, where $Q$ is the tidal dissipation function and $k_2$ is the Love number. For solar system planets in the terrestrial mass range, Goldreich & Soter (1966) give estimates for $Q'$ in the range $10^{-5}$ and $k_2 \sim 0.3$. The values of these quantities are clearly very uncertain under the very different physical conditions likely to be appropriate to extrasolar planets. We have performed simulations with $Q' = 10^0$, $10^3$, and $10^6$. In the first case, tidal circularization is very effective, while in the last it only produces very small effects over practical simulation times.

For a 1 $M_\odot$ planet at 0.05 AU and $Q' = 10^0$, we get $t_{e,i}^t = 4 \times 10^6$ yr, whereas at 0.1 AU and for $Q' = 10^3$, we get $t_{e,i}^t = 4 \times 10^9$ yr. Thus, a range of circularization timescales may apply, going from short compared to the formation time to comparable to the age of the system.

3.2. Type I Migration

In the local treatment of type I migration (e.g., Tanaka et al. 2002), if the planet is not in contact with the disk, there is no interaction between them, so that $t_{m,i}$, $t_{e,i}^d$, and $t_{e,i}^t$ are taken to be infinite. When the planet is in contact with the disk, disk-planet interactions occur, leading to orbital migration, as well as eccentricity and inclination damping (e.g., Ward 1997). In that case, away from the disk edge, we adopt

$$t_{m,i} = 146.0 \left[ 1 + \frac{e_i}{1.3H/r} \right]^5 \left[ 1 - \left( \frac{e_i}{1.1H/r} \right)^4 \right]^{-1} \times \left( \frac{H/r}{0.05} \right)^2 \left( \frac{M_d}{M_i} \right)^{1/2} \left( \frac{a_i}{1 \text{ AU}} \right)^{3/2} \text{ yr},$$

(22)

$$t_{e,i}^d = 0.362 \left[ 1 + 0.25 \left( \frac{e_i}{H/r} \right)^4 \right] \left( \frac{H/r}{0.05} \right)^{3/2} \left( \frac{a_i}{M_d/M_i} \right)^{1/2} \left( \frac{1 \text{ AU}}{1 \text{ AU}} \right)^{3/2} \text{ yr},$$

(23)

and $t_{e,i}^t = t_{e,i}^d$ (eqs. [31] and [32] of Papaloizou & Larwood 2000 with $f_x = 0.6$). Here $e_i$ is the eccentricity of planet $i$, $H/r$ is the disk aspect ratio, and $M_d$ is the disk mass contained within 5 AU. We have assumed here that the disk surface mass density varies as $r^{-3/2}$. For a 1 $M_\odot$ planet on a quasi-circular orbit at 1 AU, we get $t_{e,i}^t \sim 10^5$ yr and $t_{e,i}^t \sim 500$ yr for $M_d = 10^{-3} M_\odot$, and $H/r = 0.05$. Note that the timescales given by equations (22) and (23) can be used not only for small values of $e_i$, but also for eccentricities larger than $H/r$. The eccentricity dependence of these timescales is supported by the simulations of Cresswell & Nelson (2006). Their absolute normalization can be varied by scaling the disk surface density. We have checked that the general simulation outcomes are robust to varying the ratio $t_{e,i}^d/t_{m,i}$ by a factor of 3.

3.3. Corotation Torques

Type I migration, as discussed above, is caused through the excitation of density waves at Lindblad resonances. However, torques due to corotation resonances may also act. These depend on the gradient of specific vorticity or vortensity (Goldreich & Tremaine 1979). They are generally small, actually vanishing in a Keplerian disk with a surface density profile $\propto r^{-3/2}$, except where the surface density varies fairly rapidly. Based on a three-dimensional linear response calculation, for a planet on a circular orbit and disk surface density $\Sigma \propto r^{-3}$, Tanaka et al. (2002) find that migration stops for $\alpha = -27/11$ and is outward for surface density profiles that decrease more rapidly inward.
In our simulations, for simplicity we have either adopted equations (22) and (23) with no interaction inside of the disk inner edge, which from the above discussion we expect to correspond to a moderate taper with $\alpha = -27/11$, or allowed for a very sharp edge, as described below.

Recently, Masset et al. (2006) have proposed that inward protoplanet migration can be halted near sharp disk inner edges, which act as traps. Here we study the migration of protoplanets into an inner evacuated cavity, so we thus consider the possibility of corotation torques produced in a narrow region near the disk inner cavity boundary. We comment that the calculation of corotation torques is very uncertain, being dependent not only on the details of the edge profile but also on the degree of resonance saturation, which depends on the amount of turbulence and viscosity present (Masset et al. 2006). We here point out two possible effects that may act to reduce the effectiveness of such edge torques when there is a system of interacting planets. The first is planet–planet scattering, which could move the semimajor axis of a planet across the edge. The second is that protoplanets on eccentric orbits will only sample the edge for a fraction of the orbit and accordingly suffer a reduced torque. Note that orbital eccentricity is more likely to be sustained when the protoplanet orbit is only partly contained within the disk, as then the effectiveness of disk damping is reduced.

Corotation torques can act to produce outward torques in the inner edge domain $R_{in} - \Delta_r/2 < r < R_{in} + \Delta_r/2$, where the surface density changes rapidly. Here the edge is centered on $r = R_{in}$, and the total width of the domain is $\Delta_r$. As in the case of Lindblad torques, the effect of orbital eccentricity is to reduce such corotation torques, and this effect has been incorporated in our modeling.

Masset et al. (2006) indicate that when $a_i = R_{in}$, outward corotation torques may exceed the normal inward type I torques by a factor of 5 when $e = 0$. To investigate the possible role of such torques, in some of our simulations we adopted the following approximate procedure. In the edge domain we replaced $t_{m,i}$ by $-0.2(t_{m,i,0}(1 + e_i r/H))$, where $t_{m,i,0}$ denotes $t_{m,i}$ evaluated for $e_i = 0$. The factor in parentheses accounts for the fact that for large $e_i$, the effective values of the azimuthal number $m$ contributing to the corotation torque are reduced by a factor $H/(r e_i)$.

In practice, this detail is not important for the simulations we carried out, because $e_i$ never significantly exceeds $H/r$ in these cases. Thus, a planet in the center of the domain with $e_i = 0$ experiences an outward torque of the required magnitude, while for larger $e_i$, the time-averaged torque will decline through the factor $H/(r e_i)(\Delta_r/(2r e_i))$. The second factor here estimates the fractional reduction of the time spent in the edge domain once $e_i > \Delta_r/(2R_{in})$.

Although we have focused on the reversed-edge torque as being due to a corotation effect, it is possible that similar features could be produced by, e.g., the presence of a toroidal magnetic field near the inner edge (Terquem 2003).

3.4. Numerical Integration and Initial Conditions

The equations of motion are integrated using the Bulirsch-Stoer method (e.g., Press et al. 1992). All the planets are supposed to have an identical mass density $\rho = 1$ g cm$^{-3}$. If the distance between planets $i$ and $j$ becomes less than $\left[3M_i/(4\pi \rho)^{1/3} + 3M_j/(4\pi \rho)^{1/3}\right]^{1/3}$, a collision occurs and as is commonly assumed in studies of this kind, the planets merge. They are subsequently replaced by a single planet of mass $M_i + M_j$, which is given the position and the velocity of the center of mass of planets $i$ and $j$.

The simulations begin by placing $N$ planets on coplanar orbits in an annulus with outer and inner radii $r_{out}$ and $r_{in} = \pi r_{out}$, respectively. In some cases, the planets $i = 1, 2, \ldots, N$ were given the radial coordinate $r = r_{in}1 + (x^3 - 1)(3i - 2)^{1/3}$ together with the polar angle $\varphi = 2\pi i/3$. In other cases, $r$ and $\varphi$ were chosen randomly. The planets were then given the local circular velocity in the azimuthal direction. We have fixed $r_{out} = 1$ or 2 AU and $x = 0.1$. Note that the range of radii over which the cores are initially spread does not affect the outcome of the simulations. Indeed, if $r_{out}$ were larger, the cores would just take longer to migrate in. The disk is supposed to be truncated at some inner edge radius $R_{in}$ in the neighborhood of which a corotation torque may apply.

Initially, all the planets have the same mass $M_p$. We have fixed $M_p = 0.1 M_\odot$ or $M_p = 1 M_\odot$. This range of masses has been chosen because they have expected migration times from 5 AU to the central regions of the disk that are comparable to disk lifetimes. Cores more massive than 1 $M_\odot$ can subsequently form through mergers. We investigate what final systems of planets may be produced inside of the disk inner boundary.

Time $t = 0$ marks the beginning of the simulations. It corresponds to the time when the cores considered begin to migrate from the initial positions allocated to them. As it takes at least close to a million years to form these cores, $t = 0$ for the scenario envisaged here should correspond to a time when the disk has already evolved significantly. Note that some cores may have begun to form further away from the central star than the initial positions allocated to them. Here we assume that we can take $t = 0$ to be the time at which all cores formed in the disk that are able to migrate down to the inner cavity within the disk lifetime are contained within the radius $r_{out}$ of the initial distribution. The total mass contained in these cores is varied between 1 and 25 $M_\odot$. We comment that either increasing $r_{out}$ or decreasing the initial core mass has the effect of extending the evolution time, which is inversely proportional to the core mass. Simulations performed with either larger $r_{out}$, such as A2, A9, B3, or initial core masses reduced by a factor of 10, such as A10, indicated below, produce qualitatively similar end results but with correspondingly reduced final total masses in the latter case. This is indicative that drawing out the evolution time does not alter the qualitative behavior.

4. NUMERICAL RESULTS

For the runs presented here, we fixed $M_* = 1 M_\odot$, $H/r = 0.05$, and $M_d = 10^{-3} M_\odot$. The prescriptions for disk planet torques, eccentricity, and inclination damping rates were, apart from possible modifications listed in Table 1, as described in § 3. With these parameters specified, the quantities characterizing a run were the initial number of planetary cores $N$, their initial mass $M_p$, the tidal dissipation parameter $Q'$, the radius of the inner disk edge $R_{in}$, and the bounding radii of the initial planet distribution $r_{in}$ and $r_{out}$. Table 1 lists the parameters corresponding to the different runs. The initial number of planets $N$ is either 10, 12, or 25. The outer radius of the initial distribution, $r_{out}$, is either 1 or 2 AU.

4.1. General Outcome

All the runs start with the same qualitative evolution. A few collisions and mergers take place very close to the beginning of the simulation, but significant migration occurs. These result from the initial unstable distribution of orbits. Then as the planets migrate inward, further collisions and mergers occur on a timescale somewhat shorter than the complete migration timescale. Finally, the runs end with a stable configuration with a few planets, typically between 2 and 5, which will be inside the inner cavity when no edge corotation torques are applied. In that case, the most massive planets tend to be on the tightest orbits, as they migrate faster. Mean motion resonances are always established.
during the migrating phase. Some rearrangement may take place as the planets approach the inner cavity and further collisions occur, but by the time all the planets left over finally enter the cavity, mean motion resonances between almost all pairs of planets have been established. At that stage, residual scattering/mergers, should they occur, together with tidal interaction with the central star, leading to the circularization of the orbits on a timescale which is shorter for closer in planets, results in the disruption of strict commensurabilities (see § 2.2). The semimajor axes usually do not evolve very significantly, however, so the mean motions can stay near commensurate.

We now describe in more detail run A3, which is a typical run. It begins with \( N = 12 \) planets each having a mass \( M_p = 1 M_{\oplus} \) and for which \( r_{\text{out}} = 1 \, \text{AU} \) and \( r_{\text{in}} = 0.05 \, \text{AU} \). The time evolution of the semimajor axes and eccentricities of the planets up to \( 3 \times 10^4 \) yr is shown in Figure 1. Within the first 100 yr after the start of the calculation, before any migration has occurred, five pairs of planets merge. Another merger occurs after \( \sim 10^4 \) yr. After \( \sim 2 \times 10^4 \) yr, the four innermost planets enter the inner cavity, where their semimajor axes do not evolve any more. When one of the planets still in the disk finally approaches the inner cavity, after \( \sim 3 \times 10^4 \) yr, it pumps up the eccentricity of the innermost planets, which results in two pairs of planets undergoing collisions and mergers. At that point, three planets are left in the cavity, where they are joined after \( \sim 7 \times 10^4 \) yr by the outermost planet. Pairs of orbits are then in mean motion resonances. Subsequent tidal circularization, which acts on a timescale of a few million years and is seen on the bottom panel of Figure 1, will disrupt the resonances, but the orbits may stay nearly commensurate.

To study the action of tidal circularization on a commensurability formed by disk-planet interaction, for purely illustrative purposes we consider a simple example with two planets. In the

\[
\frac{P_1}{P_2} = \frac{M_2}{M_1} \approx \frac{Q_1}{Q_2} = \frac{m_2}{m_1} = \frac{1}{3}
\]
We also point out that the commensurabilities given in Table 1 are late, so that the systems are not formally exactly commensurable.

Note that this is not always the case. In some of the runs we have performed, these angles circulate, so that the systems are not formally exactly commensurable. We also point out that the commensurabilities given in Table 1 are accurate to within at least 1% and often to within 0.1%.

4.2. Tidal Circularization

To investigate the effect of changing the orbital circularization rate, we performed simulations A8 with $Q' = 10$ and B2 with $Q' = 1000$. Both these runs have inner cavity radius $R_{in} = 0.1$ AU. In the former case, orbital circularization is manifest in the simulation, while in the latter case, it is too long to be manifest.

The simulation A8 ended with three planets in the inner cavity in near but not exact commensurability. The evolution of the semimajor axes is shown in Figure 4, as is the evolution of the angular differences of the apsidal lines $\Delta \omega$ for the two innermost planets and the innermost and outermost of the three planets. These librate about alignment in the former case and antialignment in the latter case. In Figure 5 the evolution of the semimajor axes is shown for run B2. The right panel of this figure shows the evolution of the angular difference of the apsidal lines for the two planets that remain in the inner cavity. This oscillates around the antialigned position. This case has a very long circularization time. It is possible that for some of these cases with larger cavity

We have run a case with a lower total mass (A10, $N = 10$ and $M_p = 0.1 M_\odot$) and a case with a higher total mass (A11, $N = 25$ and $M_p = 1 M_\odot$). The results of these runs are similar to those described above, only the mass of the planets left in the inner cavity changes, roughly scaling with the initial total mass.

In the run A3 described above, we found that $\Delta \omega$ and $\Phi$ were librating about some fixed values. Note that this is not always the case. In some of the runs we have performed, these angles circulate, so that the systems are not formally exactly commensurable. We also point out that the commensurabilities given in Table 1 are accurate to within at least 1% and often to within 0.1%.

Fig. 2.—Evolution of the resonant angle $\Phi \equiv -\Phi_1 = 5 \dot{\psi}_2 - 4 \dot{\psi}_1 - \omega_1$ expressed in radians for two interacting planets that disk interaction caused to enter a 5:4 resonance under the action of orbital circularization. The left panel is for a case with a circularization rate 10 times faster than that illustrated in the middle panel. Both panels show the early stages of the evolution during which the libration amplitude increases as the planets begin to move out of the 5:4 resonance. Note that the evolution illustrated in the middle panel is 10 times slower than that illustrated in the left panel, showing that the evolution is driven by the orbital circularization. In the case with faster circularization, the angle $\Phi$ begins to show circulation after a time $\sim 7 \times 10^5$ yr, as the system moves toward the 4:3 resonance. The right panel shows this circulation in a high time resolution plot taken after $\sim 7.4 \times 10^5$ yr. The circulation period is $\sim 12$ days, while the orbital period of the inner planet is $\sim 3$ days.

Fig. 3.—Evolution of the angular difference of the apsidal lines $\Delta \omega$ and of the resonant angle $\Phi$ (in deg) for planets A and B vs. time (in yr) starting at $8 \times 10^6$ yr after the beginning of the simulation for the same run as in Fig. 1. The angles librate about some fixed values with an amplitude of a few degrees, which indicates mean motion resonance (3:2 here) and apsidal locking.

notation of § 2.2, these had masses $m_1 = 2 M_\odot$ and $m_2 = 8 M_\odot$. We performed two simulations with the same disk parameters as A3, but, for practical reasons, the circularization rates were taken to be 10 and 100 times faster. A comparison of these cases indicates that the form of the evolution is the same, but with the timescale appropriately stretched. These planets initially migrated into the inner cavity and formed a 5:4 commensurability for which the resonant angle $\Phi \equiv -\Phi_1 = 5 \dot{\psi}_2 - 4 \dot{\psi}_1 - \omega_1$ has a small libration about zero. The subsequent evolution under the action of orbital circularization is shown in Figure 2. As expected, the libration amplitude increases as the planets begin to move out of the 5:4 resonance. In the case with faster circularization, the angle $\Phi$ eventually begins to show circulation, approaching the 4:3 resonance. In this particular example, the eccentricity attains very small values $\leq 0.001$ while away from the center of resonances. The indication is that the system separates as it moves away from resonant configurations associated with high eccentricities.

For run A3, the four planets left at the end of the run, which we label A, B, C, and D, have a mass of 2, 6, 3, and 1 $M_\odot$, respectively. The two innermost planets, A and B, are in a 3:2 mean motion resonance, with $(\dot{n}_A / n_B - 3/2) \sim 10^{-7}$, where $n_A$ and $n_B$ are the mean motions of planets A and B, respectively. In Figure 3 we plot the angular difference of the apsidal lines $\Delta \omega$ and the resonant angle $\Phi = 3 \dot{n}_C - 2 \dot{n}_A - \dot{\omega}_A$, where $\dot{n}_A$ and $\dot{n}_B$ are the mean longitudes of planets A and B, respectively, and $\dot{\omega}_A$ is the argument of pericenter of planet A. After $\sim 10^2$ yr, both $\Delta \omega$ and $\Phi$ librate about some fixed values ($344^\circ$ and $207^\circ$, respectively) with an amplitude of a few degrees, which indicates apsidal locking and mean motion resonance. Note that $\Delta \omega$ and $\Phi$ do not necessarily librate about $0^\circ$ or $180^\circ$ when there is a mean motion resonance. As shown by Beaugé et al. (2003), the equilibrium value of these angles tends to depart from either $0^\circ$ or $180^\circ$ when the eccentricity of the planets is not small (typically higher than $\sim 0.1$).

We have run a case with a lower total mass (A10, $N = 10$ and $M_p = 0.1 M_\odot$) and a case with a higher total mass (A11, $N = 25$ and $M_p = 1 M_\odot$). The results of these runs are similar to those described above, only the mass of the planets left in the inner cavity changes, roughly scaling with the initial total mass.

In the run A3 described above, we found that $\Delta \omega$ and $\Phi$ were librating about some fixed values. Note that this is not always the case. In some of the runs we have performed, these angles circulate, so that the systems are not formally exactly commensurable. We also point out that the commensurabilities given in Table 1 are accurate to within at least 1% and often to within 0.1%.
Fig. 4.—Left: Evolution of the semimajor axes for run A8, as in Fig. 1. Right: Evolution of the angular differences of the apsidal lines $\Delta \omega$ (in radians) for the two innermost, according to semimajor axis, planets (points clustered about 0) and the innermost and outermost of the three planets (points clustered about $\pi$ and $-\pi$). Note that because of periodicity, any multiple of $2\pi$ can be added.

Fig. 5.—Left: Evolution of the semimajor axes for run B2, as in Fig. 4. Right: Evolution of the angular difference of the apsidal lines $\Delta \omega$ (in radians) for the remaining two planets. This oscillates around the antialigned position.
radii, some orbital eccentricity remains on $10^9$ yr timescales, in which case the alignment/antialignment of the apsidal lines could be observed. The evolution of the eccentricities in the simulations B2 and A8 is shown in Figure 6. In the latter case, a small amount of decay for the innermost planets can be seen, while in the former, the circularization rate due to tides induced by the central star is too small to have any effect.

We also performed simulation B1, which has parameters identical to A8 except the inner cavity radius $R_{in} = 0.05$ AU, and simulation B3, which is identical to B2 apart from the size of the initial domain in which the planets were started. In both simulations, the number of remaining planets and final period ratios were similar.

4.3. Migration Halted at the Disk Inner Edge by Corotation Torques

In the runs presented above, the interaction between the planets and the disk leads to inward migration of the planets. After a planet enters the inner cavity, it is no longer pushed in by the disk, but it can still be pushed in by planets further away, which enter the cavity at a subsequent time. Note that when a planet approaches the inner cavity, it may gently push in the planets that are already inside, but it may also perturb them in such a way that a merger occurs. Both processes are seen in run A3, displayed in Figure 1.

It has been argued recently (Masset et al. 2006) that when an embedded planet reaches a region of the disk where the mass density decreases sharply, the tidal torque from the disk is reversed, so that migration is halted and the planet is trapped at this location. We have performed runs in which the torque at the inner edge of the disk is reversed according to the prescription described in § 3.3 to test whether planets penetrate inside the cavity. In simulations C1, C2, and C3, such torques were applied. The boundary torques that we applied were strong enough to prevent entry into the inner cavity while the disk was present. We emphasize that this is the important feature of the torque prescription that we adopted and that otherwise results should be independent of details.

We considered two phases in these cases for which the disk edge torque prevented entry into the inner cavity. While embedded in the disk, the strong orbital circularization allowed close commensurabilities to form among six to seven planets in a similar manner to that described for simulation A3. These are indicated in Table 1. A state was reached for which the semimajor axes became almost constant, such that angular momentum transferred from inner to outer planets prevented their inward migration. After this state was reached, the disk was removed (by setting the induced migration and circularization rates to zero) in order to study the further evolution and in particular the effect of removing the stabilizing influence of disk eccentricity damping.

Simulation C3 had $Q' = 1000$ and the larger inner cavity radius $R_{in} = 0.1$ AU. The evolution of the semimajor axes and the eccentricity of the outermost planet are shown in Figure 7. As indicated in Table 1, this run produced a system of six planets stably locked in a series of commensurabilities (9:8, 4:3, 7:6, 7:6, 5:4). In this state, the negative torques acting on the outer planets were effectively balanced by the corotation torque acting on the innermost planet, so that evolution of the semimajor axes ceased. The disk was removed at time $1.32 \times 10^7$ yr. Shortly after that time, two of the planets merged, leaving a system of five somewhat more widely spaced planets that survived with almost constant semimajor axes (see Fig. 7). The period ratios moving outward were then 1.125, 1.44, 1.25, and 1.26. The stability of systems of low-mass planets has been considered by Chambers et al. (1996). They considered systems of three objects, which in our case would correspond to $3 M_\oplus$ planets for up to $\sim 10^7$ inner
orbits, and we have evolved our systems for similar or longer times. They found that the systems must be more widely separated to ensure stability for longer times. This is of course a statistical statement; there are no guarantees in specific cases. We also note the additional potential stabilization provided by orbital circularization in our case. Nonetheless, if one makes the very arbitrary assumption that their results can be simply extrapolated to inner orbits, which would correspond to Gyr timescales, period ratios of \(1.25\) would be required. This corresponds to a spacing between planets of 10.75 Hill radii. This is similar to what our systems show except that, in the case of run C3, the innermost pair are very close to a 9:8 commensurability. Although these two planets maintained locked apsides, the appropriate resonant angle \(\Phi\) showed long-term variations but did not librate. It is possible that some of the planets in such systems could later merge, forming a more widely separated system with fewer planets, but the general character is likely to be preserved. Simulations C1 and C2, which had smaller inner cavity radii, led to similar configurations while the disk was present. However, in these cases, the systems remained stable when the disk was removed. This may be because of the increased importance of orbital circulation, especially in the case of C2, which had seven remaining planets. In this case, with \(Q' = 10\), orbital circularization cannot be neglected in the simulation run time and might be expected to assist system stability by preventing the slow buildup of orbital eccentricities that could result in orbit crossing.

5. SUMMARY AND DISCUSSION

We have calculated the evolution of a population of cores/planets with masses in the range 0.1–1 \(M_\oplus\) embedded in a disk. They evolve due to gravitational interaction with the central star, mutual gravitational interactions, and tidal interaction with the disk and the star. Mutual interactions lead to orbit crossing and mergers, so that the cores grow during their evolution. Interaction with the disk leads to orbital migration. As cores with different masses and at different locations migrate with different rates, they capture each other in mean motion resonances. Such captures enable planets to migrate inside the cavity inside of the disk inner edge. As they approach closer to the central star, for small enough cavities their orbits are circularized through tidal dissipation and strict commensurabilities are lost. That process may be also aided by scatterings and mergers of planets on unstable orbits that occur inside of the disk inner edge. Near commensurability, however, may be maintained. Note too that if apsidal locking is established during migration, it can be preserved through the operation of these processes. All the simulations typically end with a population between two and five planets, with masses depending on the initial mass. Note that the disk tidal torque may be reversed near the disk inner edge. When this is the case, although it is possible that some planets can still penetrate inside the cavity due to scatterings and/or weakening of the torques because of a finite eccentricity, some planets may be left in the disk, just beyond the inner edge, until that disperses.

The qualitative results do not depend on the details of the initial conditions. As long as a population of cores is able to migrate inward at different rates, the system evolves toward a family of a few planets, which are almost always on near-commensurate orbits.

The orbital migration and eccentricity damping timescales we have adopted in this paper have been derived for type I migration in inviscid disks. Note that type I migration has been shown to follow a random walk in a turbulent disk (Nelson & Papaloizou 2004). The studies done in the present paper assume that the cores can migrate down to the disk inner edge in regions where the gas is ionized and magnetic turbulence can develop (Fromang et al. 2002). These studies would of course not apply if there

Fig. 7.—Evolution of the semimajor axes (left) and the later evolution of the eccentricity of the outermost planet (right) for run C3. In this case, the disk was removed at time 1.32 \(\times 10^9\) yr. Shortly after that time, two of the planets merged, leaving a system of five planets.
were no systematic inward migration of the cores/protoplanets of the type we consider here. As of today, there is no indication that type I migration in a turbulent disk has a systematic trend (Nelson 2005), but this cannot be ruled out either. It is likely that type I migration does depend at least to some extent on the torque exerted by the material that corotates with the planet, which so far has not been taken into account in the simulations, which lack the required resolution. If there is a systematic trend, as long as the time-averaged migration rate is inward, by the averaging principle and as confirmed in test studies, short-term fluctuations do not qualitatively change the results we have presented in this paper. It has very recently been suggested (Paardekooper & Mellema 2006) that, because of effects arising from radiation trapping, type I migration in an optically thick laminar disk could be outward. This result, should it be confirmed for both laminar and turbulent disks, suggests that cores would not migrate inward in the disk’s inner parts as long as the dust opacity is high enough there. However, after the dust settles and agglomerates to form cores, the opacity decreases, and migration could then resume.

It has indeed an observational fact that the disk’s inner parts (within a few AU) become optically thin before the rest of the disk is depleted.

The calculations done in this paper show that if hot super-Earths or Neptunes form by mergers of inwardly migrating cores, then such planets are most likely not isolated. We would expect to always find at least one, more likely a few, companions on close and often near-resonant orbits. To test this hypothesis, it would be of interest to look for planets of a few to $\sim 10 M_\oplus$ in systems where hot super-Earths or Neptunes have already been found and there is no destabilizing influence of a giant planet close by.

It has been speculated that the cores of giant planets could form in a way similar to that investigated here, by accumulation of cores in the disk’s inner parts (e.g., Papaloizou & Terquem 1999). The calculations presented in this paper suggest that to assemble a massive core in the inner disk, significantly more mass in smaller cores may be needed to begin with. Indeed, most of the simulations end with at least three planets and not with a single planet containing all the initial mass.