Analysis of Damage and Crack Propagation in Unidirectional Composite Laminates with a Peridynamic Model

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Abstract. A continuous function was applied to establish a peridynamics model to study the damage of unidirectional composite laminate, a kind of transverse isotropic material with three-dimensional (3D) structure. To express the interaction of nonadjacent layers and the influence of thickness on elastic deformation and damage, this paper employed a spherical material horizon. The spherical harmonics function was applied to express the relationship of the micro-elastic modulus and the bond-fiber angle, as well as critical bond stretch and the bond-fiber angle, respectively. Spatial stress or strain conditions were employed to calculate the strain energy density and solve the bond stiffness constant. The model was verified with unidirectional multilayer composite laminate with orifice. The damage mode and the crack propagation process of laminates with orifice were also consistent with that of experimental.

1. Introduction

Due to the heterogeneous nature and complex damage patterns, the numerical modeling of damage in composite laminates remains a challenge after decades of research efforts. Peridynamics, a nonlocal theory of continuum mechanics proposed by Silling [1], has unique advantages to solve the problem of modeling damage of composite laminates. As a nonlocal theory, it defines an interaction range of material points which called material horizon, each material point of the horizon interacts with its neighbors within a finite region. The interaction between two material points is called a “bond,” and the response of a given bond in a bond-based peridynamics model is assumed independent from other bonds. It is based on spatial integral equation instead of spatial derivatives in the governing equations, which is significant for describing discontinuities in continuum mechanics such as cracks. The spontaneous generation of cracks is controlled by the micromodulus of the governing equation, so the cracks can be spontaneously initiated and expanded in arbitrary direction when two originally interaction points separate more than the critical stretch.

In most of the peridynamics research about the unidirectional composite lamina, two independent function were adopted to describe the micromodulus, one for the fiber direction and another for all other directions [2, 3, 4, 5, 6, 7, 8]. However, the stiffness (off-axis modulus) changes continuously with respect to the fiber orientation in a unidirectional lamina, according to the macro-mechanics of
composite laminates. To describe the continue change of the stiffness, Ghajari et al. [9] proposed a spherical harmonic function to express the micro-modulus and critical bond stretch and applied it to 2D structure. The model have the advantage of avoiding the mesh dependence on either the fiber orientation or the discretization without special processing. Wenke Hu et al. [6] adopted two scaling factors to calibrate the micro-elastic modulus with the uniform grids which aligned with the fiber orientation. Zhou et al. [10] employed a continuous functions to model the relationship between the peridynamics parameters and the fiber direction when they dealt with the dynamic fracture problem of orthotropic materials. Ren et al. [11] decomposed the multidirectional layer composite laminates into isotropic matrix and transverse isotropic fiber sheet. For the fiber sheet, the spherical harmonic expansion theory was applied refers to Ghajari’s method. To combine the matrix and the fiber sheet, they defined a cylindrical horizon with only adjacent layers. However, Jiang et al. [12] pointed out that the construction of a cylinder model considering only adjacent lamination while ignored the influence of thickness, making the model insensitive to damage in the direction of lamination plate thickness, especially in lamination damage.

This paper extend the spherical harmonic function proposed by Ghajari et al. [9] to analyze the uniderectional multilayer fiber reinforced composite with 3D structure. A spherical material horizon was defined to describe the influence of thickness on elastic deformation and damage, and spatial stress and strain conditions were employed to calculate the strain energy density and solve the bond stiffness constant. The model was verified by the tensile test of the unidirectional composite laminates with orifice.

2. Peridynamics theory

2.1. Bond-based peridynamics theory

The Peridynamics theory is a kind of nonlocal theory, the key of which is that internal force between every pairs of material points is expressed in the form of integrals [1]. In the reference configuration $B$, there is an interaction between any material point $x$ and any other material point $x'$ in its horizon $\delta$ at any time $t$, as shown in Fig. 1.

![Figure 1. Schematic diagram of peridynamics theory.](image)

According to Newton's second law, the peridynamics equation of the material point $x$ at any time $t$ can be expressed by:

$$\rho(x)\ddot{u}(x, t) = \int_{H(x)} f(u' - u, x' - x, t) dH + b(x, t)$$  (1)

Where $H(x)$ is the horizon of material point $x$, which includes all the material points that the material point $x$ can interact with inside the body; $\rho$ is the density of the material point $x$; $u$ is the displacement vector of the point $x$; $\ddot{u}$ is the acceleration vector at the point $x$; $b$ is the external load vector per unit volume; $\delta$ is the radius of the horizon. Within this horizon, the action exerted on the material point at $x$ by another material point at $x'$ is characterized by the pairwise force vector $f$ called the constitutive force function.

Based on the bond-based peridynamics theory, the constitutive force function in homogeneous isotropic elastic materials can be expressed as [13]:

```latex
\rho(x)\ddot{u}(x, t) = \int_{H(x)} f(u' - u, x' - x, t) dH + b(x, t)
```
\[
f(\eta, \xi) = \frac{\xi + \eta}{|\xi + \eta|} \mu(\xi, t) cs
\]  
(2)

Where \( \xi = x' - x \) and \( \eta = u' - u \) represent the relative position and displacement vector of the material point \( x \) and \( x' \), respectively. \( c \) is the micro-elastic modulus; \( s \) is the bond stretch, which is defined as:

\[
s = \frac{|\xi + \eta| - |\xi|}{|\xi|}
\]  
(3)

2.2. Failure criteria

The material point \( x \) interacts with the point \( x' \) in the horizon by a peridynamics bond. When the stretch between two points exceeds the critical bond stretch rate \( s_0 \), bonds break and these two points cease to interact. A scalar history-dependent function \( \mu(t, \xi) \) is defined as follows to represent the damage of material, which is assigned 0 and 1 depending on whether the bond is broken.

\[
\mu(\xi, t) = \begin{cases} 
1 & \text{if } s(t', \xi) \leq s_0 \text{ for all } 0 < t' < t, \\
0 & \text{otherwise} 
\end{cases}
\]  
(4)

Where \( s_0 \) is the critical stretch of the material point, as shown in figure 2.

\[\text{Figure 2. Relation between bond stretch } s \text{ and pairwise force function } f.\]

The damage of the material point \( x \) can be defined as:

\[
\varphi(x, t) = 1 - \frac{\int_R dv_n \mu(t, \xi)}{\int_R dv_{sr}}
\]  
(5)

2.3. Numerical implementation

In peridynamics, reference configurations are uniformly dispersed to a finite number of particles. The integral formula of the internal force function in the basic motion equation of peridynamics is expressed as a numerical form of finite summation [13]:

\[
\rho(x_i) \ddot{u}^n(x_i, t) = \sum_{j=1}^{N} f \left( u^n(x_j, t) - u^n(x_i, t), x_j - x_i \right) dV_j + b^n(x_i, t)
\]  
(6)
3. Peridynamics model based on 3D Transversely isotropic material

3.1. Bond stiffness constant

In transversely isotropic material, the elastic properties of materials in plane perpendicular and that of the principal material axes are equal. As shown in Figure 3, axis 1, axis 2 and axis 3 are mutually perpendicular, where axis 1 is the principal material axis, while the plane containing axes 2 and 3 is an anisotropic plane. The micro-elastic modulus of the bond was expressed with the angle between the bond and the principal material axis, which was applied to describe the anisotropy of transversely isotropic material [9]. The position vector of \( \mathbf{\xi} \) can be described with the polar coordinate \( (\xi, \theta, \phi) \), where \( \theta \) is the angle between \( \mathbf{\xi} \) and axis 1, while \( \phi \) is the angle between \( \mathbf{\xi} \) and axis 3.

\[
\mathbf{\xi}(\alpha) = A_{00} + A_{20}P_2^0(\cos \alpha) + A_{40}P_4^0(\cos \alpha) + A_{60}P_6^0(\cos \alpha) + A_{80}P_8^0(\cos \alpha)
\]  
\[
A_{00} = 0.0722c_1 + 0.9278c_2
\]  
\[
P_2^0(\cos \alpha) = \frac{1}{8}(35 \cos^4 \alpha - 30 \cos^2 \alpha + 3)
\]  
\[
P_4^0(\cos \alpha) = \frac{1}{16}(231 \cos^6 \alpha - 315 \cos^4 \alpha + 105 \cos^2 \alpha - 5)
\]  
\[
P_6^0(\cos \alpha) = \frac{1}{128}(6435 \cos^8 \alpha - 12012 \cos^6 \alpha + 6930 \cos^4 \alpha - 1260 \cos^2 \alpha + 35)
\]  

For transverse isotropic materials, it can be considered that the plane formed by axis 2 and axis 3 (plane of isotropy) is symmetric with respect to axis 1. Since the mesh dependence on either the fiber orientation or the discretization can be avoided by modeling the micromodulus with the 8th-order spherical harmonic, the micro-elastic modulus of bonds \( c(\alpha) \) are expressed as formula (7), where \( A \) and \( P \) are the basic constants of the mathematical model.

\[
\text{Figure 3. Three-dimensional model of a peridynamics bond.}
\]

In order to obtain a smooth transition of \( c(\alpha) \) between 0 and \( \pi/2 \), particularly for cases with \( c_1/c_2 \gg 1 \), it was assumed that \( c(0^\circ) = c_1 \) and \( c(\alpha) = c_2 \) for \( \alpha = 45^\circ, 55^\circ, 65^\circ \) and \( 90^\circ \). By solving the system of five equations, the following relations were obtained for the constant coefficients:

\[
A_{00} = 0.0722c_1 + 0.9278c_2
\]  
\[
A_{20} = 0.3021(c_1 - c_2)
\]  
\[
A_{40} = 0.3376(c_1 - c_2)
\]
\[ A_{60} = 0.2159(c_1 - c_2) \]  
\[ A_{80} = 0.0722(c_1 - c_2) \]  

(15)  

(16)

In the classical continuum theory, the strain energy density of the material point is:

\[ W^{CL} = \frac{1}{2} C_{ij} \varepsilon_i \varepsilon_j \]  

(17)

Where \( C_{ij} \) is the stiffness matrix, \( i \) and \( j = 1, 2, 3, 4, 5, 6 \). In the peridynamics theory, the strain energy density of a particle with a neighborhood fully contained within a body under spatial stress or strain conditions is:

\[ W^{PD} = \frac{1}{2} \int_0^{2\pi} \int_0^\pi \int_0^\delta \left[ \frac{c(\alpha)\xi^2}{2} \right] \xi^2 \sin \varphi \, d\xi \, d\varphi \, d\theta \]  

(18)

As presented in Table 1, the system of six equations \( W_n^{CL} = W_n^{PD} \) (\( n= 1, 2, 3, 4, 5, 6 \)) was solved with Matlab, which resulted in:

\[ c_1 = \frac{34.93C_{11} - 24.93C_{22}}{\pi \delta^4} \]  

(19)

\[ c_2 = \frac{10.07C_{22} - 0.07C_{11}}{\pi \delta^4} \]  

(20)

\[ C_{12} = C_{13} = C_{55} = C_{66} = 0.057C_{11} + 0.27C_{22} \]  

(21)

\[ C_{33} = C_{44} = \frac{1}{3} C_{22} \]  

(22)

\[ C_{33} = C_{22} \]  

(23)

For an isotropic material, \( C_{11} = C_{22} \), and Eqs. (19) to (23) can be simplified to:

\[ c_1 = c_2 = \frac{10C_{11}}{\pi \delta^4} \]  

(24)

\[ C_{12} = C_{13} = C_{23} = C_{33} = C_{44} = C_{55} = C_{66} = \frac{1}{3} C_{11} \]  

(25)

This is identical to the micro-elastic modulus obtained by Silling et al. [13] for isotropic materials.

| n  | Strain state          | Bond stretch                      |
|----|-----------------------|----------------------------------|
| 1  | \( \varepsilon_1 = \zeta \), and other components are zero | \( s = \zeta \cos^2 \theta \sin^2 \varphi \) |
| 2  | \( \varepsilon_2 = \zeta \), and other components are zero | \( s = \zeta \sin^2 \varphi \sin^2 \theta \) |
| 3  | \( \varepsilon_3 = \zeta \), and other components are zero | \( s = \zeta \cos^2 \varphi \) |
| 4  | \( \gamma_{23} = \zeta \), and other components are zero | \( s = \zeta \sin \theta \sin \varphi \cos \varphi \) |
| 5  | \( \gamma_{13} = \zeta \), and other components are zero | \( s = \zeta \sin \varphi \cos \varphi \cos \theta \) |
| 6  | \( \gamma_{12} = \zeta \), and other components are zero | \( s = \zeta \sin \varphi \cos \varphi \cos \theta \) |
3.2. Critical bond stretch

The spherical harmonic expansion was also used to define the dependency of the critical bond stretch on the position vector of $\xi$.

$$s^2(\alpha) = B_{00} + B_{20} P_2^0 (\cos \alpha) + B_{40} P_4^0 (\cos \alpha) + B_{60} P_6^0 (\cos \alpha) + B_{80} P_8^0 (\cos \alpha)$$ \hfill (26)

The constant coefficients were obtained by using the same assumptions leading to Eq. (7), as follows:

$$B_{00} = 0.0722 s_{01}^2 + 0.9278 s_{02}^2$$ \hfill (27)

$$B_{20} = 0.3021 (s_{01}^2 - s_{02}^2)$$ \hfill (28)

$$B_{40} = 0.3376 (s_{01}^2 - s_{02}^2)$$ \hfill (29)

$$B_{60} = 0.2159 (s_{01}^2 - s_{02}^2)$$ \hfill (30)

$$B_{80} = 0.0722 (s_{01}^2 - s_{02}^2)$$ \hfill (31)

The energy required for a crack to split a body into two halves equals the sum of the rupture energy of the bonds that initially crossed the crack surface. The critical strain energy release rate is this energy divided by the area of the crack surface. This method can be applied to brittle materials, where other dissipative mechanisms are negligible compared to fracture. The critical strain energy release rates for mode I crack propagation in the planes normal to the axis 1 and axis 2, $G_{c1}$ and $G_{c2}$ respectively, can be determined from the following integrals:

$$G_{c1} = \int_0^\delta \int_0^{2\pi} \int_0^{\cos^{-1}(\xi)} \left[ c(\alpha) s_0^2(\alpha) \xi \right] \xi^2 \sin \alpha d\alpha d\xi d\varphi_1 dz$$ \hfill (32)

$$G_{c2} = \int_0^\delta \int_0^{2\pi} \int_0^{\cos^{-1}(\xi)} \left[ c(\alpha) s_0^2(\alpha) \xi \right] \xi^2 \frac{\cos \alpha}{\sin \varphi_2} d\alpha d\xi d\varphi_2 dz$$ \hfill (33)

Mathematical software was used to solve the above numerical value, and the following equation can be obtained:

$$s_{01}^2 = \frac{c_1 (345 G_{c1} - 377 G_{c2}) + c_2 (527 G_{c1} - 51 G_{c2})}{\delta^3 (\xi^2 + 28c_1 \xi^2 + 11c_2^2)}$$ \hfill (34)

$$s_{02}^2 = \frac{c_1 (490 G_{c2} - 395 G_{c1}) + c_2 (377 G_{c2} - 345 G_{c1})}{\delta^3 (\xi^2 + 28c_1 \xi^2 + 11c_2^2)}$$ \hfill (35)

For an isotropic material, Eqs. (34) and (35) were simplified to:

$$s_{01}^2 = s_{02}^2 = \frac{16 G_{ic}}{\pi \delta^3}$$ \hfill (36)

This is the same as the critical bond stretch obtained by Silling et al. [13] for isotropic materials.
4. Numerical calculation and model validation

4.1. Computational model
Fiber reinforced composite laminates were manufactured from unidirectional carbon fiber (T800) reinforced/epoxy prepreg. The mechanical properties of the materials are listed in Table 2. A universal testing machine of SUST was applied for tensile tests (in accordance with ASTM D5677/D7615). As it is shown in Figure 4, the dimensions of the specimen are: 0.2m × 0.036m × 0.002m (l × w × h) and the radius of the hole is 0.006 m. The laminates are comprised of ten laminate with the sequence: [0]_{10}, [45]_{10}, [90]_{10}. The tensile loading speed was 2mm/min, and the experiment was stopped when the sample was separated or the load dropped significantly. The simulation model of the laminate specimen was built with 0.2 mm grid spacing. The horizontal radius and width of the edge were specified as δ = 3Δx and d = 3Δx[14]. In order to ensure the convergence of simulation results, adaptive dynamic relaxation was adopted [15].

Table 2. Mechanical properties of carbon fiber/epoxy T800/X850 composite.

| Parameters                                      | Value         |
|------------------------------------------------|---------------|
| Logitudinal tensile modulus $E_1$ [Gpa]        | 195           |
| Transverse tensile modulus $E_2, E_3$ [Gpa]    | 8.58          |
| Longitudinal transverse shear modulus $G_{12}, G_{13}$ [Gpa] | 4.57          |
| Transverse shear modulus $G_{23}$ [Gpa]        | 2.90          |
| Longitudinal transverse Poisson’s ratio $\nu_{12}, \nu_{13}$ | 0.33          |
| Transverse Poisson’s ratio $\nu_{23}$          | 0.48          |
| Mode-I longitudinal energy release rate $G_{1c}$ [Mpa·m] | 2.37×10^{-3} |
| Mode-II transverse energy release rate $G_{2c}$ [Mpa·m] | 1.78×10^{-3} |
| Density $\rho$ [Kg/m$^3$]                      | 1570          |

![Figure 4. Geometry model of unidirectional laminates with a hole (unit: mm).](image)

4.2. Results and discussion
The comparison of the force-strain curve obtained from the proposed model and test results of universal material testing machine for the three different laminates ([0]_{10}, [45]_{10}, [90]_{10}) is shown in Figure 5. For the laminates with the sequence [0]_{10}, the force-strain curve remains essentially linear (Fig. 5). When the load was exceeded to the ultimate load, a slight crisp sound was heard, which indicated a small amount of fiber breakage. The results were very satisfied with that of the experiment, and the relative error was only 8.7%. For the laminates with the sequence [45]_{10}, the relative error was 13.6% (Fig. 5). For the laminates with the sequence [90]_{10}, the force-strain curve also remains essentially linear (Fig. 5) and the relative error was only 5.4%. When the load was exceeded to the ultimate load, the ultimate damage was instantaneous brittle fracture. It can be found that the calculation results of the model are basically consistent with the experimental results for unidirectional laminates with different fiber direction.
Figure 5. Force-strain curve of unidirectional laminates with 0°(left), 45°(center), 90°(right) fiber orientation under uniaxial tension.

In view of the advantages of peridynamics in solving discontinuity problems, the damage model of section 3.2 was applied to simulate the progressive damages of the unidirectional laminates with center-orifice under uniaxial tensile. The damage degree ranges from 0 to 1 was expressed by Eq. (5).

The longitudinal displacement fields on both sides of the orifice should be symmetrically distributed along the fiber direction, according to the experiments. The simulation results are the same as that of the experiments. Fig. 6 show the longitudinal deformation field of laminates with the sequence: \([0]_{10}\) (left), \([45]_{10}\) (center), \([90]_{10}\) (right), when the displacement of the loading head is 0.15 mm. At this point, the longitudinal displacement is small so the laminate is not damaged. When the displacement increased to a certain extent, the damages occurs. Fig. 7 - 9 show the progressive damage of laminates with the sequence \([0]_{10}, [45]_{10}, [90]_{10}\), respectively. Fiber damage has not been observed in the experimental process and the simulation. The simulation show that the matrix damages emerge on the orifice edge and then propagated to surrounding areas. The ultimate damage was in accordance with the experiment (Fig. 10). Hence, the failure morphology of the laminate with different fiber direction shows that the damage mode was mainly affected by the matrix damage during the process of damage expansion.

Figure 6. Longitudinal (up row) and transverse (bottom row) deformation field of laminates with the sequence: \([0]_{10}\) (left), \([45]_{10}\)(center), \([90]_{10}\)(right)
Figure 7. Progressive damage of matrix of the laminates with $[0]_{10}$ (left: $t=10000$; center: $t=13000$; right: $t=20000$)

Figure 8. Progressive damage of matrix of the laminates with $[45]_{10}$ (left: $t=18500$; center: $t=19500$; right: $t=20000$)

Figure 9. Progressive damage of matrix of the laminates with $[90]_{10}$ (left: $t=17500$; center: $t=18000$; right: $t=18300$)

Figure 10. Damages of the laminates with different sequence (experiments), left: $[0]_{10}$; center: $[45]_{10}$; right: $[90]_{10}$

5. Conclusions
A peridynamics model has been proposed based on the 3D transverse isotropic composite laminates. In 3D structure transverse isotropic composite laminates, a material point in a lamina interacts not only with points inside its lamina, but also with its adjacent and nonadjacent laminate. To describe and model this situation, this paper defined a spherical material horizon to express the interaction of adjacent and nonadjacent layers, and model the influence of thickness on elastic deformation and damage. The 8th spherical harmonics function was employed to establish the models of bond micromodulus and critical bond stretch, which can avoid the mesh dependence on either the fiber orientation or the discretization and simulate progressive damages of composite laminates with arbitrary fiber direction. Spatial stress or strain conditions were applied to calculate the strain energy density and solve the bond stiffness constant. The model was verified with unidirectional multilayer composite laminates. By comparing the orifice laminate tensile results of simulation with that of the experiment, the simulation results of the initial damage position, extension path and final failure mode were also in accordance with that of the experiment.
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