The influence of pairing on the nuclear matrix elements of the neutrinoless $\beta\beta$ decays

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The discovery of the massive character of the neutrinos in the recent measurement at Super-Kamiokande [1], SNO [2] and KamLAND [3], has opened a new era in the neutrino physics. However, these experiments are sensitive only to the mass differences between the three neutrino species. Their absolute mass scale and hierarchy are still unknown. In addition, we don’t know either if the neutrinos are Dirac or Majorana particles. The double beta decay is the rarest nuclear weak process. It takes place between two even-even isobars, when the decay to the intermediate nucleus is energetically forbidden due to the pairing interaction, that shifts the even-even and the odd-odd mass parabolas in a given isobaric chain. The two-neutrino decay is just a second order process in the weak interaction. It conserves the lepton number and has been already observed in several nuclei. A second mode, the neutrinoless decay $0\nu\beta\beta$ can only take place if the neutrino is a Majorana particle and demands an extension of the standard model of electroweak interactions, because it violates the lepton number conservation. Therefore, the observation of the double beta decay without emission of neutrinos will sign the Majorana character of the neutrino and will establish the absolute mass scale of the neutrinos, hence deciding their mass hierarchy.

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The expression for the neutrinoless beta decay half-life, in the $0^+ \rightarrow 0^+$ case, can be brought to the following form [4,5]:

$$[T_{1/2}^{(0\nu)}(0^+ \rightarrow 0^+)]^{-1} = G_{0\nu} \left( M^{(0\nu)} \left( \left\langle m_{\nu} \right\rangle / m_e \right) \right)^2$$ (1)

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left( \frac{g_{\nu}}{g_A} \right)^2 M_F^{(0\nu)} - M_T^{(0\nu)}$$ (2)

where $\left\langle m_{\nu} \right\rangle$ is the effective neutrino mass (a linear combination of the neutrino mass eigenvalues whose coefficients are the corresponding elements of the neutrino mixing matrix), and $G_{0\nu}$ is the kinematic phase space factor [6].

The important point at this stage is that, once the neutrinoless double beta decay be detected, to transform the measured half life in an accurate value of the effective neutrino mass would require a precise computation of the nuclear matrix elements (NME’s) of the decay operators. This, in turn, demands a detailed description of the structure of the nuclei involved in the process. A critical analysis of the available predictions for the NME’s of the potential $0\nu\beta\beta$ emitters (only about one dozen) was made recently by Bahcall et al. [7]. Their conclusion was rather pessimistic, owing to the large dispersion of the calculated values. In a subsequent paper, Rodin et al. [8] have shown that many of the quasi-particle RPA (QRPA) calculations taken into account in Bahcall’s survey were obsolete, and that, when these are not considered, the spread of the calculated values is much smaller. The aim of this work is to go one step further and to propose a much narrower band of values for the NME’s, based in the predictions of large scale applications of the Interacting Shell Model (ISM) and in the analysis of the QRPA results in terms of the pairing content of their solutions.

The matrix elements $M_{GT,F,T}^{(0\nu)}$ can be calculated in the closure approximation, that is good to better than 90% due to the large average energy of the virtual neutrino ($\sim 100$ MeV) [9]. For the Gamow-Teller channel it reads,

$$M_{GT}^{(0\nu)} = 0^+_1 \left| h(\vec{r}_1 - \vec{r}_2) \right| (\vec{\sigma}_1 \cdot \vec{\sigma}_2) (t_1 - t_2) 0^+_2$$ (3)

and similar expressions hold for the other matrix elements. $h(\vec{r}_1 - \vec{r}_2)$ is the neutrino potential ($\sim 1/r$) obtained from the neutrino propagator. Higher order contributions (hoc) to the nuclear current produce the tensor term and add extra contributions to the Gamow-Teller expression of Eq. [9,10].

Generically, the two body decay operators can be written in the Fock space representation as:

$$M_{GT}^{(0\nu)} = 0^+_1 \left| h(\vec{r}_1 - \vec{r}_2) \right| (\vec{\sigma}_1 \cdot \vec{\sigma}_2) (t_1 - t_2) 0^+_2$$ (3)
\[ \hat{M}^{(0\nu)} = \sum_{J} \left( \sum_{i,j,k,l} M_{i,j,k,l}^{(0\nu)} \left( a_{i}^{\dagger} a_{j}^{\dagger} a_{k} a_{l} \right)^{0} \right), \]

(4)

where the indices \( i, j, k, l \) run over the single particle orbits of the spherical nuclear mean field. Applying the techniques of ref. [11] we can factorize the operators as follows

\[ \hat{M}^{(0\nu)} = \sum_{\pi} \hat{P}_{J^\pi} \hat{P}_{J^\pi}^{\dagger} \]

(5)

The operators \( \hat{P}_{J^\pi} \) annihilate pairs of neutrons coupled to \( J^\pi \) in the father nucleus and the operators \( \hat{P}_{J^\pi}^{\dagger} \) substitute them by pairs of protons coupled to the same \( J^\pi \). The overlap of the resulting state with the ground state of the grand daughter nucleus gives the \( J^\pi \)-contribution to the NME. The –a priori complicated– internal structure of these exchanged pairs is dictated by the double 

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The ISM calculations reported in this letter are carried out in the spirit of the previous shell model works [12, 13, 14, 15]. For the A=76 and A=82 cases we make full calculations in the valence space (1p1/2, 0f5/2, 1p1/2, 0g9/2) using a newly built effective interaction that, starting with a G-matrix [16] has its matrix elements fitted to a large set of experimental data. For the A=124, A=128, A=130, and A=136 emitters, we make full calculations in the valence space (0g7/2, 1d5/2, 1d3/2, 2s1/2, 0h11/2) with another interaction obtained in a similar manner. These model spaces and interactions will be discussed in detail elsewhere [17].

The dimensions of the shell model bases reach in some cases \( O(10^3) \). The present calculations adopt the closure approximation, and model the short range and finite size corrections as in [12]. We use \( r_0 = 1.2 \text{ fm} \) to make the matrix elements dimensionless and \( g_A = 1.25 \). The choice of \( g_A = 1.25 \) instead of the quenched value \( g_A = 1.0 \) needed for the pure Gamow-Teller processes in nuclei is consistent with the use of the closure approximation, in which the multipole decomposition of the decay plays no role at all. In addition, even without closure, the need of a quenched \( g_A \) in the \( J^\pi = 1^+ \) channel of the 0\( \nu \) decay, that has no reason to resemble the pure Gamow-Teller operator of the 2\( \nu \) decay, is not guaranteed. Higher order contributions to the nuclear current (hoc) [10] are explicitly included for the first time in the ISM context, leading to reductions of the NME’s in the 15\% range. Our final predictions for \( M^{(0\nu)} \) are gathered in Table I.

Except for doubly magic \( ^{48}\text{Ca} \), whose NME is severely quenched, our predictions cluster around a value \( M^{(0\nu)} \approx 2 \). The upper bounds on the neutrino mass for a half life of \( 10^{25} \text{ y} \), that incorporate the phase space factors, show a mild preference for the potential emitters with A=82, 124, 130 and 136. The matrix elements are dominated by the Gamow-Teller contribution. The influences of the restrictions in the valence space and of the choice of the effective interaction in the ISM NME’s have been studied in [13]. In all, these should result in a 20\% uncertainty of our predictions. Treating the short range correlations with a prescription softer than Jastrow might produce an increase of NME’s, that we have not evaluated yet in the ISM context, but we do not expect it to go beyond 10-15\%.

In order to explore the structure of the 0\( \nu \beta\beta \) two body transition operators, we have plotted in Figure 1 the contributions to the 0\( \nu \) GT matrix element as a function of the \( J^\pi \) of the decaying pair. The results are very sug-
the cases that we have studied, as can be seen in table II. Notice that the cancellations are substantial. These features are also present in the QRPA calculations, in whose context they had been discussed in refs. [8, 19].

TABLE II: J=0 vs J>0 pair contributions to the Gamow Teller matrix element (no hoc).

|         | M_{GT}^{(0)} | M_{GT}^{(0)} (J=0) | M_{GT}^{(0)} (J>0) |
|---------|--------------|-------------------|-------------------|
| 76Ge → 76Se | 2.35         | 5.59              | -3.24             |
| 82Se → 82Kr | 2.25         | 5.32              | -3.07             |
| 130Te → 130Xe | 2.12        | 6.58              | -4.46             |
| 136Xe → 136Ba | 1.77        | 5.72              | -3.95             |

To grasp better this mechanism, we have expressed the matrix elements in a basis of generalized seniority s (s counts the number of unpaired nucleons in the nucleus): |0_i\rangle = \sum_j \alpha_j |s_i\rangle; |0_f\rangle = \sum_j \beta_j |s_f\rangle. The J=0 terms provide essentially all the contribution to M^{(0)} that is diagonal in s. The canceling parts, J>0, produce almost exclusively cross terms with Δs=+4. The matrix elements f(s|M^{(0v)}|s_i) are roughly proportional to (b_{max}−s), averaged in parent and grand daughter, while the cross terms f(s + 4|M^{(0v)}|s_i) are constant—in both cases scaled by the larger oscillator quantum number in each valence space—. The two body matrix elements of the operator M^{(0v)}(J = 0) are almost identical to those of the isovector pairing of the nuclear effective interaction, that is why it acts as a “pair counter”. At present we cannot offer a similarly simple explanation for the behavior of the J>0 terms. Obviously, when the initial and final states have seniority zero, the s=0 contribution is maximized and the canceling terms are null, hence, M^{(0v)} becomes maximal.

These results highlight the role of the seniority structure of the nuclear wave functions in the build-up of the 0ν NME’s, and we shall examine this issue for the competing theoretical approaches. In the first place, we have plotted the results of the ISM calculations of the NME’s as a function of the seniority in Fig. 2. The values with maximum seniority provide the exact ISM results in the corresponding valence spaces. Two aspects are worth to underline; a) the strong reduction of the NME as the maximum allowed seniority increases (up to a factor five); and b) the fact that, at s≤4, the NME’s of the A=76, 82, 128, and 130 decays miss convergence by factors 2-3. On the contrary, in the A=48, A=124, and 136 cases the convergence at s≤4 is much better. The reason why these decays behave differently is very illuminating; 124Sn has only neutrons in the valence space, hence, its wave function is dominated by low seniority components and its NME at s≤4 is quite close to the exact result; in the A=136 decay, the s≤4 calculation for 136Xe is exact, therefore, at s≤4, the NME is also close to the exact one; finally, in the A=48 decay the s>4 components are negligible both in doubly magic 48Ca and in 48Ti.

We can now proceed to compare in detail the “state of the art” ISM and QRPA [21, 22] NME’s in Fig. 3. The QRPA results for 124Sn are not yet available. The range of QRPA values shown in the figure is that given by the authors, and derives from the different choices of g_{pp} and g_A, as well as from their use or not of a renormalized version of the QRPA. The larger values correspond to g_A=1.25 and should therefore be preferred in the comparison with our predictions. Both the QRPA and the ISM calculations include the higher order corrections from ref. [10]. For a proper comparison, the TU07 NME’s have been increased by 10% due to their different choice of r_0. In all the calculations the short range

FIG. 2: (color online) The neutrinoless double beta decay NME’s, defined in equation 4 as a function of the maximum seniority of the wave functions.

FIG. 3: (color online) The neutrinoless double beta decay NME’s; comparison of ISM and QRPA calculations. Tu07; QRPA results from ref. [21]. Jy07; QRPA results from ref. [22]. ISM s≤4 and ISM; present work. 

FIG. 3: (color online) The neutrinoless double beta decay NME’s; comparison of ISM and QRPA calculations. Tu07; QRPA results from ref. [21]. Jy07; QRPA results from ref. [22]. ISM s≤4 and ISM; present work.
correlations are modeled by the same Jastrow factor.

Several interesting conclusions stem from this figure. First, the fact that the different QRPA calculations are now compatible. In addition, they produce NME’s that are strikingly close to the ISM ones calculated at the truncation level $s \leq 4$. In the $A=136$ decay, in which the $s \leq 4$ truncation is a good approximation to the full result, the QRPA values and the ISM ones do agree (this seems to be also the case for the $A=124$ decay \cite{24}). This suggests that, somehow, $s \leq 4$ is the implicit truncation level of the QRPA. In the QRPA calculations in a spherical basis that we are discussing, the ground states of parent and grand daughter, calculated in the BCS approximation, have generalized seniority zero. The RPA ground state correlations of multipole character (quadrupole, octupole, etc.), bring components with $s \geq 4$ into these wave functions. But, for the RPA approximation to remain valid, their amplitudes should decrease with $s$. Indeed, in our ISM $s \leq 4$ results, the percentage of $s=0$ components is always larger than 70\%, a figure compatible with a QRPA description. However, in the full calculation for the $A=76$, $A=82$, $A=128$, and $A=130$ decays, this percentage can be as low as 25\% (actually, in $^{76}$Se, the $s=4$ components almost double the percentage of the $s=0$ ones). In these cases, the QRPA is bound to overestimate the amount of $s=0$ components and, consequently, the value of the NME’s. In a sense, the QRPA can be said to be a “low seniority approximation”, roughly equivalent to the $s \leq 4$ ISM truncations, that overestimate the NME’s when the nuclei that participate in the decay are strongly correlated by the multipole part of the effective nuclear interaction. The extent of the overestimation depends on the degree of validity of the low seniority approximation in each decaying pair.

The values of $M^{(0\nu)}$ predicted by the present ISM calculation for the $A=76$, $A=82$, $A=128$, and $A=130$ decays, are smaller that the QRPA (central) ones by factors 1.5-2. Therefore, for a given value of the effective neutrino mass, the predicted ISM half-lives of the $0\nu\beta\beta$ decays are 2-4 times longer than the QRPA ones. Equivalently, for a given lower bound on the half-life, the ISM NME’s produce upper bounds on the effective neutrino mass that are larger than those of the QRPA by factors 1.5-2. For instance, a bound on $T_{1/2}(^{76}$Ge $\rightarrow ^{76}$Se) of $10^{25}$ y. results in an effective neutrino mass of 1.05 eV with the ISM NME, and 500 meV with the QRPA one. The same bound for the half-life of the $^{130}$Te $\rightarrow ^{130}$Xe decay would lead to bounds on the neutrino mass of 410 meV and 270 meV respectively.

In summary, we have analyzed the $0\nu\beta\beta$ NME’s in terms of the $J^o$ of the decaying neutron pair. We have found that in the seniority zero limit the decays are strongly favored. When the non zero seniority components of the wave functions, originated by the multipole terms of the nuclear effective interaction, are properly taken into account, the matrix elements are drastically reduced. In particular, when the multipole correlations are large, the low seniority truncations, $s \leq 4$, similar to those implicitly present in the spherical QRPA approaches based in a BCS treatment of the pairing interaction, are shown to overestimate the NME’s. Hence, we surmise that, when the QRPA and ISM results do not agree, the true NME’s should be much closer to the ISM predictions than to the QRPA ones.

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