Unusual formations of the free electromagnetic field in vacuum

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Abstract

It is shown that there are exact solutions of the free Maxwell equations (FME) in vacuum allowing an existence of stable spherical formations of the free magnetic field and ring-like formations of the free electric field. It is detected that a form of these spheres and rings does not change with time in vacuum. It is shown that these convergent solutions are the result of an interference of some divergent solutions of FME. One can surmise that these electromagnetic formations correspond to Kapitsa’s hypothesis about interference origin and a structure of fireball.

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I. INTRODUCTION

The generally accepted opinion exists that solutions of the free Maxwell equations (FME) are well studied and do not boil down to any surprises. Nevertheless, we will show in the next Sections, such mathematically well-known solutions (see, e.g., [1], where general solutions of the Maxwell equations was obtained) lead, however, to the existence of rather unusual and unexpected electromagnetic formations in vacuum such as closed spherical magnetic surfaces (without electric field on this surface and where the magnetic field is tangential and its intensity depends on time) and ring-like formations of the electric field (without magnetic field in all points of the ring and where the electric field is tangential and depends on time). We will also show that these formations do not change their form with time in vacuum.

II. AN UNUSUAL SOLUTION OF THE FREE MAXWELL EQUATIONS

We found that a certain class of exact solutions of FME

\begin{align*}
\text{div } \mathbf{E} &= 0, \\
\text{rot } \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\
\text{div } \mathbf{B} &= 0, \\
\text{rot } \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t},
\end{align*}

exists which has some unexpected characteristics. The present work is devoted to the research of the such solutions.

We shall look for solutions of the system FME as follows:

\[ \mathbf{E}(\mathbf{r}, t) = \mathbf{e}(\mathbf{r}) \psi(t) \quad \text{and} \quad \mathbf{B}(\mathbf{r}, t) = \mathbf{b}(\mathbf{r}) \chi(t). \]  

where \( \psi(t) \) and \( \chi(t) \) are some functions of time, vectors \( \mathbf{e} \) is a polar vector and \( \mathbf{b} \) is an axial one.

And so substituting (5) in FME we obtain:

\begin{align*}
\text{div } \mathbf{e} &= 0, \\
\text{rot } \mathbf{e} &= -\frac{1}{c} \frac{\chi'}{\psi} \mathbf{b}
\end{align*}
\[
\text{div} \mathbf{b} = 0, \quad \text{(8)}
\]
\[
\text{rot} \mathbf{b} = \frac{1}{c} \frac{\psi'}{\chi} \mathbf{e}. \quad \text{(9)}
\]

It is obvious that these equations are consistent if and only if

\[
-\frac{\chi'}{\psi} = \Omega_1 \quad \text{and} \quad \frac{\psi'}{\chi} = \Omega_2, \quad \text{(10)}
\]

where index \(\{\prime\}\) means a derivative with respect to time, \(\Omega_1\) and \(\Omega_2\) are arbitrary constants.

In order to obtain solutions of this system with three constants only, and to obtain sinusoidal solutions, we propose that \(\Omega_1 = \Omega_2 = \Omega\). Thus, the general solution of the system (10) is

\[
\chi(t) = \mathcal{A} \cos(\Omega t - \eta) \quad \text{and} \quad \psi(t) = \mathcal{A} \sin(\Omega t - \eta),
\]

where \(\mathcal{A}\) and \(\eta\) are arbitrary constants, and equations for \(\mathbf{e}\) and \(\mathbf{b}\) become:

\[
\nabla \times \mathbf{e} = \frac{\Omega}{c} \mathbf{b} \quad \text{and} \quad \nabla \times \mathbf{b} = \frac{\Omega}{c} \mathbf{e}. \quad \text{(12)}
\]

In order to solve this system, let us at first note that formally summing two equations (12) we obtain

\[
\nabla \times (\mathbf{e} + \mathbf{b}) = \frac{\Omega}{c} (\mathbf{e} + \mathbf{b}) \quad \text{or} \quad \nabla \times \mathbf{a} = \frac{\Omega}{c} \mathbf{a}. \quad \text{(13)}
\]

So, at first we resolve Eq.(13) with respect to \(\mathbf{a}\), and then we obtain from the vector \(\mathbf{a}\) (which, obviously, has no polarity) the polar vector \(\mathbf{e}\) and the axial vector \(\mathbf{b}\). Actually, one can express polar and axial parts of any vector without polarity, in general, as follows:

\[
\mathbf{e}(\mathbf{r}) = \frac{1}{2} \left[ \mathbf{a}(\mathbf{r}) - \mathbf{a}(-\mathbf{r}) \right] \quad \text{(14)}
\]

and

\[
\mathbf{b}(\mathbf{r}) = \frac{1}{2} \left[ \mathbf{a}(\mathbf{r}) + \mathbf{a}(-\mathbf{r}) \right] \quad \text{(15)}
\]

Now, if we calculate a rotor of both parts of equations (14), (15) one can be satisfied that the system (12) is fulfilled:

\[
\nabla \times \mathbf{e}(\mathbf{r}) = \frac{1}{2} \left[ \nabla \times \mathbf{a}(\mathbf{r}) - \nabla \times \mathbf{a}(-\mathbf{r}) \right] = \frac{1}{2} \left[ \frac{\Omega}{c} \mathbf{a}(\mathbf{r}) + \frac{\Omega}{c} \mathbf{a}(-\mathbf{r}) \right] = \frac{\Omega}{c} \mathbf{b}(\mathbf{r}) \quad \text{(16)}
\]
\begin{align*}
\nabla \times \mathbf{b}(\mathbf{r}) &= \frac{1}{2} \left[ \nabla \times \mathbf{a}(\mathbf{r}) + \nabla \times \mathbf{a}(-\mathbf{r}) \right] = \frac{1}{2} \left[ \frac{\Omega}{c} \mathbf{a}(\mathbf{r}) - \frac{\Omega}{c} \mathbf{a}(-\mathbf{r}) \right] = \frac{\Omega}{c} \mathbf{e}(\mathbf{r}). \\
\n\end{align*}

Here we take into account that after inverting the coordinates, the equation \( \nabla \times \mathbf{a}(\mathbf{r}) = \frac{\Omega}{c} \mathbf{a}(\mathbf{r}) \) becomes \( -\nabla \times \mathbf{a}(-\mathbf{r}) = \frac{\Omega}{c} \mathbf{a}(-\mathbf{r}) \). Thus, one can see that if we find \( \mathbf{a} \) as a solution of Eq.(13) it means that we find \( \mathbf{e} \) and \( \mathbf{b} \) as solution of the system (12).

The equation (13) was already solved in the literature (see, e.g. [2,3]). And so, the solution of Eq.(13) in the spherical system of coordinates is\(^1\):

\[
\mathbf{a} = D \left\{ \frac{2\alpha}{r^3} \cos \theta \right\} \mathbf{e}_r + D \left\{ \frac{\gamma}{r^3} \right\} \mathbf{e}_\theta + D \left\{ \frac{\Omega \alpha}{cr^2} \sin \theta \right\} \mathbf{e}_\varphi. \tag{18}
\]

Finally, separating vectors \( \mathbf{e} \) and \( \mathbf{b} \) we obtain the solution of the system (12) expressed by components (cartesian and spherical ones):

\[
\mathbf{e} = D \left\{ \frac{-\alpha \Omega y}{cr^3}, \frac{\alpha \Omega x}{cr^3}, 0 \right\} = \frac{\Omega \alpha \sin \theta}{cr^2} D \mathbf{e}_\varphi \tag{19}
\]

and

\[
\mathbf{b} = D \left\{ \frac{\beta x z}{r^5}, \frac{\beta y z}{r^5}, \frac{2\alpha}{r^3} - \frac{\beta (x^2 + y^2)}{r^5} \right\} = \frac{2\alpha \cos \theta}{r^3} D \mathbf{e}_r + \frac{\gamma \sin \theta}{r^3} D \mathbf{e}_\theta, \tag{20}
\]

where

\[
\alpha = \cos \left( \frac{\Omega r}{c} - \delta \right) + \frac{\Omega r}{c} \sin \left( \frac{\Omega r}{c} - \delta \right) \nonumber \]

\[
\beta = 3\alpha - \frac{\Omega^2 r^2}{c^2} \cos \left( \frac{\Omega r}{c} - \delta \right) \text{ and } \gamma = \beta - 2\alpha. \nonumber \]

Let us now write the solution (5) in the explicit form, taking into account Eqs.(11), (19) and (20):

\[
\mathbf{E} = \left[ \frac{\Omega \alpha \sin \theta}{cr^2} D \mathbf{e}_\varphi \right] \sin(\Omega t - \eta) \tag{21}
\]

and

\[
\mathbf{B} = \left[ \frac{2\alpha \cos \theta}{r^3} D \mathbf{e}_r + \frac{\gamma \sin \theta}{r^3} D \mathbf{e}_\theta \right] \cos(\Omega t - \eta), \tag{22}
\]

\(^1\)\( D \) is a dimension constant \( [D] = M^{1/2} L^{5/2} T^{-1} \)
where $\delta$ and $\eta$ are arbitrary constants.

It follows from the solutions (21), (22) that the necessary (not sufficient!) condition in order for these solutions to not diverge in $r = 0$ is:

$$\alpha(0) = \left\{ \cos\left(\frac{\Omega r}{c} - \delta\right) + \frac{\Omega r}{c} \sin\left(\frac{\Omega r}{c} - \delta\right) \right\}_{r=0} = 0 \quad \Rightarrow \\
\cos \delta = 0 \quad \Rightarrow \quad \delta = (n + \frac{1}{2})\pi, \text{ where } n = 0, \pm 1, \pm 2, \ldots$$

In order to satisfy oneself that the solutions (21), (22) converge, one can calculate the following limits for $\delta = \frac{\pi}{2}$:

$$\lim_{r \to 0} \frac{\alpha}{r^2} = 0; \quad \lim_{r \to 0} \frac{\alpha}{r^3} = \frac{\Omega^3}{3c^3}; \quad \lim_{r \to 0} \frac{\gamma}{r^3} = -\frac{2\Omega^3}{3c^3},$$

and corresponding limits for $E$, $B$ and the energy density $w = \frac{E^2 + B^2}{8\pi}$ are:

$$\lim_{r \to 0} E = 0; \quad \lim_{r \to 0} B = \frac{2D\Omega^3 \cos(\Omega t)}{3c^3}k; \quad \lim_{r \to 0} w = \frac{D^2\Omega^6}{18\pi c^6} \cos^2(\Omega t),$$

where $k$ is $Z$-ort of the cartesian system.

The constant $\eta$ just defines an initial wave phase of the fields $E$ and $B$. So without loss of generality we can just write one non-divergent solutions for $\delta = \frac{\pi}{2}, \eta = 0$:

$$E = D \left[ \frac{\alpha \Omega \sin \theta}{cr^2}e_\varphi \right] \sin(\Omega t); \quad B = D \left[ \frac{2\alpha \cos \theta}{r^3}e_r + \frac{\gamma \sin \theta}{r^3}e_\theta \right] \cos(\Omega t),$$

where

$$\alpha = -\frac{\Omega r}{c} \cos \left(\frac{\Omega r}{c}\right) + \sin \left(\frac{\Omega r}{c}\right) \quad \text{and} \quad \gamma = \alpha - \frac{\Omega^2 r^2}{c^2} \sin \left(\frac{\Omega r}{c}\right).$$

Note that the solution (25) can be found directly from the general solution of the Maxwell equations obtained by G. Mie [1]

Generally speaking, taking into account (19) and (20) one can see that an infinity divergent solutions for $E$ and $B$ exists as well as convergent ones. Curiously enough, but the convergency of these solutions is defined by the constant $\delta$. Solutions converge in $r = 0$ if and only if

$$\delta = (n + \frac{1}{2})\pi, \text{ where } n = 0, \pm 1, \pm 2, \ldots$$

We show that the convergent solution (25)\textsuperscript{3} is represented as an interference of diver-

\textsuperscript{2}we calculate these limits expanding $\alpha$ and $\gamma$ in series of powers of $r$.

\textsuperscript{3}We designate it by $E_c$ and $B_c$ in this section.
After simple algebraic transformation one can represent the solution (25) as a superposition of two waves spreading in opposite directions in every point:

\[ E_c = E_{(\rightarrow)} + E_{(\leftarrow)} \quad \text{and} \quad B_c = B_{(\rightarrow)} + B_{(\leftarrow)}, \]  

(28)

where

\[ E_{(\rightarrow)} = \frac{\Omega \sin \theta}{2cr^2} \left[ \cos \left( \frac{\Omega r}{c} - \Omega t \right) + \frac{\Omega r}{c} \sin \left( \frac{\Omega r}{c} - \Omega t \right) \right] \partial e_\varphi, \]  

(29)

\[ E_{(\leftarrow)} = -\frac{\Omega \sin \theta}{2cr^2} \left[ \cos \left( \frac{\Omega r}{c} + \Omega t \right) + \frac{\Omega r}{c} \sin \left( \frac{\Omega r}{c} + \Omega t \right) \right] \partial e_\varphi, \]  

(30)

\[ B_{(\rightarrow)} = \frac{\cos \theta}{r^3} \left[ \sin \left( \frac{\Omega r}{c} - \Omega t \right) - \frac{\Omega r}{c} \cos \left( \frac{\Omega r}{c} - \Omega t \right) \right] \partial e_r + \frac{\sin \theta}{2r^3} \left[ -\frac{\Omega r}{c} \cos \left( \frac{\Omega r}{c} - \Omega t \right) + \left( 1 - \frac{\Omega^2 r^2}{c^2} \right) \sin \left( \frac{\Omega r}{c} - \Omega t \right) \right] \partial e_\theta, \]  

(31)

\[ B_{(\leftarrow)} = \frac{\cos \theta}{r^3} \left[ \sin \left( \frac{\Omega r}{c} + \Omega t \right) - \frac{\Omega r}{c} \cos \left( \frac{\Omega r}{c} + \Omega t \right) \right] \partial e_r + \frac{\sin \theta}{2r^3} \left[ -\frac{\Omega r}{c} \cos \left( \frac{\Omega r}{c} + \Omega t \right) + \left( 1 - \frac{\Omega^2 r^2}{c^2} \right) \sin \left( \frac{\Omega r}{c} + \Omega t \right) \right] \partial e_\theta, \]  

(32)

Labourless calculation shows also that

\[ E_{(\rightarrow)} = \frac{1}{2} (-E_d + E_c); \quad E_{(\leftarrow)} = \frac{1}{2} (E_d + E_c) \]  

(33)

and

\[ B_{(\rightarrow)} = \frac{1}{2} (-B_d + B_c); \quad B_{(\leftarrow)} = \frac{1}{2} (B_d + B_c), \]  

(34)

where \( E_d, B_d \) are divergent solutions of the system (1)-(4):

\[ E_d = \mathcal{D} \left[ \frac{\alpha_d \Omega \sin \theta}{cr^2} e_\varphi \right] \cos(\Omega t); \quad B_d = \mathcal{D} \left[ \frac{2\alpha_d \cos \theta}{r^3} e_r + \frac{\gamma_d \sin \theta}{r^3} e_\theta \right] \sin(\Omega t), \]  

(35)

here

\[ \alpha_d = \cos \left( \frac{\Omega r}{c} \right) + \frac{\Omega r}{c} \sin \left( \frac{\Omega r}{c} \right) \quad \text{and} \quad \gamma_d = \alpha - \frac{\Omega^2 r^2}{c^2} \sin \left( \frac{\Omega r}{c} \right). \]

It is obvious that the functions \( E_{(\rightarrow)}, E_{(\leftarrow)}, B_{(\rightarrow)}, B_{(\leftarrow)} \) diverge in \( r = 0 \) and they are also solutions of FME.
III. STABLE ELECTROMAGNETIC SPHERES AND RINGS IN VACUUM AS A CONSEQUENCE OF THE SOLUTION (25)

As we will show below the solution (25) of FME leads to an existence of unusual spherical formations of the free electromagnetic field.

A. Some details of the energy distribution in the field (25)

Let us write, after some transformations, the expression for the energy density for the solution (25). One can show that the energy density contains both the time-independent part and time-dependent one:

\[
    w = \frac{E^2 + B^2}{8\pi} = \frac{D^2}{16\pi} \left\{ \frac{\Omega^2 \alpha^2 \sin^2 \theta}{c^2 r^4} + \left[ \frac{4\alpha^2}{r^6} \cos^2 \theta + \frac{\gamma^2}{r^6} \sin^2 \theta \right] \right\} + \\
    + \frac{D^2}{16\pi} \left\{ \left[ \frac{4\alpha^2}{r^6} \cos^2 \theta + \frac{\gamma^2}{r^6} \sin^2 \theta \right] - \frac{\Omega^2 \alpha^2}{c^2 r^4} \sin^2 \theta \right\} \cos(2\Omega t). \tag{36}
\]

Let us find from (36) the locus where \( w \) does not depend on \( t \). It is obvious that the loci are

1) along the axis \( Z \) in the points where \( \tan \left( \frac{\Omega z}{c} \right) = \frac{\Omega z}{c} (\theta = 0, \pi; \alpha = 0) \);

2) at surfaces where \( r \) satisfies the equation \( \gamma^2 = \alpha^2 \left( \frac{\Omega^2 r^2}{c^4} - 4 \cot^2 \theta \right) \). One can see the cross-section of these surfaces in Fig.3 (discontinuous curves)\(^4\)

Now we calculate the electromagnetic energy \( E \) within a sphere of the radius \( R \) with the centre in the coordinate origin:

\[
    E_{\oplus} = \int_0^R \int_0^{2\pi} \int_0^\pi r^2 \sin \theta \ w(r, \theta, \varphi, t) = E(R) + E(R, t), \tag{37}
\]

where

\[
    E(R) = \frac{D^2}{6R^3} \left[ \frac{\Omega^4 R^4}{c^4} - \frac{\Omega^2 R^2}{c^2} \sin^2 \left( \frac{\Omega R}{c} \right) - \alpha^2 \right], \tag{38}
\]

\[
    E(R, t) = -\frac{D^2}{6R^3} \alpha \gamma \cos(2\Omega t). \tag{39}
\]

\(^4\) All figures in this work were performed in the program “Mathematica-4.0”.

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Here $\alpha = -\frac{\Omega R}{c} \cos \left(\frac{\Omega R}{c}\right) + \sin \left(\frac{\Omega R}{c}\right)$ and $\gamma = \alpha - \frac{\Omega^2 R^2}{c^2} \cos \left(\frac{\Omega R}{c}\right)$.

One can show from Eq.(39) that electromagnetic energy within spheres of the radiiuses $R$ which are solutions of the equations

$$\tan \left(\frac{\Omega R}{c}\right) = \frac{\Omega R}{c},$$  

(40)

or

$$\tan \left(\frac{\Omega R}{c}\right) = \frac{\Omega R}{1 - \frac{\Omega^2 R^2}{c^2}},$$

(41)

does not change with time.

Note that every root of Eq. (40) is placed at the number line between two neighbour roots of Eq. (41) and vice versa. One can show that a distance between theseighboring spherical surfaces tends to $\frac{cT}{2\pi}$ when $R \to \infty$. Let us also direct attention to an interesting fact that at the surfaces of the spheres of the radius (40) only the magnetic field is present, and the electric field at these surfaces does not exist. It follows directly from Eq.(25) for $\alpha = 0$.

B. Analysis of the Poynting vector’s field corresponding to the wave field (25)

The Poynting’s vector corresponding to the wave field (25) is

$$\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{B} = \frac{D^2}{8\pi} \left[ \frac{\Omega \alpha^2 \sin(2\theta)}{r^5} \mathbf{e}_\theta - \frac{\Omega \alpha \gamma \sin^2 \theta}{r^5} \mathbf{e}_r \right] \sin(2\Omega t).$$

(42)

Let us calculate the total momentum and the angular momentum of the electromagnetic field (25) within a sphere of the arbitrary radius $r$ with the center in the coordinate origin. Because the Poynting’s vector is proportional to the vector of the density of momentum in the same point we can just calculate the integral of the Poynting’s vector over volume of the sphere.

It is easy to calculate this integral if we express spherical system orts by cartesian system orts:

$$\mathbf{e}_r = i \sin \theta \cos \varphi + j \sin \theta \sin \varphi + k \cos \theta \quad \text{and} \quad \mathbf{e}_\theta = i \cos \theta \cos \varphi + j \cos \theta \sin \varphi - k \sin \theta.$$  

\footnote{it follows from $\alpha = 0$ and $\gamma = 0$ correspondingly.}
Thus, integrating (42) over volume of the sphere we obtain:

\[
\iiint S r^2 \sin \theta \, dr \, d\theta \, d\phi = -\frac{D^2 4\pi \Omega \sin(2\Omega t)}{32\pi} k \left[ \frac{\alpha^2}{r^3} \sin^4 \theta \right]_0^\pi \, dr = 0. \tag{43}
\]

It means that the total momentum of the electromagnetic field (25) in volume bounded by an arbitrary sphere with a center in the coordinate origin is zero at any time. Analogically one can show that the total angular momentum of this field configuration is zero.

Let us now find the loci where Poynting’s vector is zero at any instant of time. It follows from Eq.(42) that conditions when Poynting’s vector is zero are:

\[
\alpha^2 \sin(2\theta) = 0 \quad \text{and} \quad \alpha \gamma \sin^2 \theta = 0. \tag{44}
\]

From the first equation of the conditions (44) we have the following possibilities:

(i) \( \alpha = 0 \). This automatically satisfies both conditions (44). From \( \alpha = 0 \) we obtain

\[
\tan \left( \frac{\Omega r}{c} \right) = \frac{\Omega r}{c}. \tag{45}
\]

Hence, the loci for the case (i) are spheres whose radii satisfy Eq.(45).

(ii) \( \sin(2\theta) = 0 \). It means that \( \theta \) can be 0, \( \frac{\pi}{2} \) or \( \pi \).

(ii-1) If \( \theta \) is 0 or \( \pi \), in this case both equations fulfill the conditions (44). So the locus is axis \( Z \).

(ii-2) If \( \theta = \frac{\pi}{2} \), this gives us two possibilities in order to satisfy the conditions (44): either \( \alpha = 0 \) (it is the case (i), see above) or \( \gamma = 0 \). From the last we have

\[
\tan \left( \frac{\Omega r}{c} \right) = \frac{\frac{\Omega r}{c}}{1 - \frac{\Omega^2 r^2}{c^2}}. \tag{46}
\]

So the loci corresponding the case \( \theta = \frac{\pi}{2} \), and \( \gamma = 0 \) are rings at the plane \( XY \) with radiuses satisfying Eq.(46). Note that in all points of these rings the magnetic field is zero.

Now we consider spheres whose equators are mentioned rings. These spheres are defined by the condition \( \gamma = 0 \). One can see from Eq.(42) that at these surfaces the Poynting’s vector in all points has tangential components only. Due to this fact the conservation of the energy within spheres of the radiuses (41) becomes more clear.
Thus, adjusted total in looking-for the loci where the Poynting’s vector for the field (25) is zero in any instant of time is:

*Locus 1:* Axis $Z$. We call this axis *magnetic axis* because an electric field does not exist there.

*Locus 2:* Rings at the plane $z = 0$ with radiiues satisfying Eq.(46). We call these rings *electric rings* because a magnetic field does not exist there.

*Locus 3:* Spheres with centers in the origin with radiiues satisfying Eq.(45). We call these spheres *magnetic spheres* because the electric field does not exist on them.

In order to elucidate better the results of the last analysis, let us adduce the graphic (FIG.1) where the distribution of the Poynting’s vector field is shown.
FIGURES

FIG. 1. Poynting’s vector field distribution for given instant of time in the plane $x = 0$, $Y$-axis is the abscissa and $Z$-axis is the ordinate. Here $c = 1, \omega = 1$.

We consider this distribution, for example, in the plane $x = 0$ (because of the axial symmetry of the energy density and energy-flux density distribution it is sufficient to consider this cross-section only).

We call spheres whose equator is the electric ring E-sphere. We call the magnetic spheres M-spheres. In the Fig. 1 one can see the vertical magnetic axis (coinciding with the $Z$-axis), the first E-sphere, the first M-sphere and the second E-sphere in the given instant of time. Within E-spheres the total electromagnetic energy conserves because the energy-flux vector at the surface of this sphere has tangential component only. The energy transfers along this surface from pole to equator (electric ring) and after a certain period of time does reverse movement. Within the first E-sphere the energy transfers from the magnetic axis to the electric ring and after a certain time returns.

The Poynting vector is zero in every point of the first M-sphere so the energy within this sphere conserves too. One can see that the energy transfers from the surface of the first magnetic sphere to the electric rings of the first and the second E-spheres. Analogical exchange of the energy takes place between next E- and M-spheres.

We once more emphasize that the Poynting’s vector field takes opposite direction with time, due to the existence of the function $\sin(2\Omega t)$ in Eq.(42).

For more demonstrativeness we adduce here the graphic (FIG. 2) of cross-section of the Poynting’s vector field in the plane $z = 0$.

FIG. 2. Poynting’s vector field distribution for given instant of time in the plane $z = 0$, $X$-axis is the abscissa and $Y$-axis is the ordinate.

At last we adduce here the common graphic (FIG. 3) of cross-section ($x = 0$) of the surfaces where the energy density is constant and first M-sphere, first and second E-spheres.

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6This period is defined by the function $\sin(2\Omega t)$ from Eq.(42).
FIG. 3. Cross-section of the surfaces of the constant energy density and first M-sphere and first E-spheres in the plane \( x = 0 \), Y-axis is the abscissa and Z-axis is the ordinate. Here \( c = 1, \omega = 1 \).

We emphasize that these surfaces do not deform, do not displace and do not rotate with time in vacuum.

IV. DISCUSSION

Thus, we obtained a stationary free electromagnetic field which can be consequence of some interference processes. Why one can speak here about interference? Actually, we see that in this electromagnetic formation, surfaces (discontinuous curves in Fig.3) and points (in Z-axis) where the energy density is constant exist. From this one can surmise that these surfaces and points are nodes of wave. It is well-known also that standing electromagnetic waves are a result of interference processes.

Of course, the solution of the free Maxwell equations corresponding to these ball-like electromagnetic formations was obtained for vacuum. But if we call to mind that in air, the values \( \varepsilon = 1, \mu = 1 \) we can be practically sure that the solution (25) is valid for air, taking into account that air does not have free charges and currents. So it is easy to draw an analogy between our solution and Kapitsa’s hypothesis about the interference nature of ball lightning [4]. Actually, the electric field of electromagnetic waves which “voyage” within M-spheres and especially the electric field of the aforementioned electric rings have to ionize the air converting it to plasma. A size of the critical region of ionization is defined by the radius of the magnetic sphere, in which a density energy is still adequate for ionizing air. This ultimate magnetic sphere in turn plays the role of a magnetic trap for plasma confinement. One can indeed see from Eq.(36) that the energy density within the magnetic spheres decreases as \( \frac{1}{r^2} \). It means that at a certain distance the energy density is less than the critical value which is necessary to ionize the air. This condition has to define the radius of the ultimate magnetic sphere within which conditions of the ionization still exist. Taking into account this limited value of the radius of this ultimate magnetic sphere one can speak about the fireballs.

It goes without saying that it is just our hypothesis, but the analogy between Kapitsa’s idea and our ball-like solutions doubtless takes place. It should also be stated that other
ball-like stable formations in the radiation field were obtained in the paper “Is there yet an explanation of ball lightning?” by G.H. Arnhoff [5] and in the paper “Ball lightning as a force-free magnetic knot” by A. F. Rañada et al [6] (see also [7]). It follows from these works that the electromagnetic energy contained in a spherical volume, cannot escape (the energy corresponding to our solutions behaves in the same way). According to [5] outside of this volume there is only an quasi-electrostatic field, rotating with constant angular velocity about the axis (in this point our and Arnhoff’s solutions are different). In turn A.F. Rañada et al [6,7] proposed ball-like electromagnetic formations as a solution based on the idea of the “electromagnetic knot”, an electromagnetic field in which any pair of magnetic lines or any pair of electric lines form a link - a pair of linked curves.

Thus the famous hypothesis of the Nobel prizewinner P.L.Kapitsa that fireballs (or balls lightning) are standing electromagnetic waves of unusual configuration as a result of some interference process from the day of its formulation (in 1955) never (to the present day) got a theoretical (mathematical) support. One can see that our work first gives a theoretical support to this hypothesis.

In a subsequent work we are going to research the process of the genesis of these unusual electromagnetic formations.

And in conclusion we just note that A.O. Barut was right when claimed that “Electrodynamics and the classical theory of fields remain very much alive and continue to be the source of inspiration for much of the modern research work in new physical theories” [8].

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