Hierarchical contagions in the interdependent financial network

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Abstract

We model hierarchical cascades of failures among banks linked through an interdependent network. The interaction among banks include not only direct cross-holding, but also indirect dependency by holding mutual assets outside the banking system. Using data extracted from the European Banking Authority, we present the interdependency network composed of 48 banks and 21 asset classes. Since interbank exposures are not public, we first reconstruct the asset/liability cross-holding network using the aggregated claims. For the robustness, we employ 3 reconstruction methods, called Anan, Hala and Maxe. Then we combine the external portfolio holdings of each bank to compute the interdependency matrix. The interdependency network is much more dense than the direct cross-holding network, showing the complex latent interaction among banks. Finally, we perform macroprudential stress tests for the European banking system, using the adverse scenario in EBA stress test as the initial shock. For different reconstructed networks, we illustrate the hierarchical cascades and show that the failure hierarchies are roughly the same except for a few banks, reflecting the overlapping portfolio holding accounts for the majority of defaults. Understanding the interdependency network and the hierarchy of the cascades should help to improve policy intervention and implement rescue strategy.

Keywords: financial network, interdependent network, contagions, stress test, macroprudential

JEL: G01, G21, G32, G33, D85

1. Introduction

In recent years, network models, systemic stress testing and financial stability have attracted growing interest both among scholars and practitioners (Battiston and Martinez-Jaramillo, 2018). Regular stress tests conducted by authorities, such as the European Banking Authority, aim to evaluate the performance of individual banks in adverse scenarios, which are microprudential. Macroprudential outcomes are not simply the summation of microprudential changes. For example, when financial innovation reduces the cost of diversification, this may trigger a transition from stationary return dynamic to a nonstationary one (Corsi et al., 2016). Therefore, to be truly macroprudential, it is necessary to assess the role of network contagion in potentially amplifying systemic risk (Gai and Kapad, 2019).

There are different interactive channels among financial institutions. Figure 1 illustrates 3 types of financial networks: (a) interbank network, (b) bank-asset bipartite network and (c) interdependent network. The interbank network characterizes direct credit exposures to other banks and risk contagion can be caused by direct cross-holding. Take Dungey et al. (2020) for example, they empirically analyze the transmission of shocks between global banks, domestic banks and the non-financial sector for 11 Eurozone countries. Apart from direct connection, it’s apparent that banks are indirectly connected by holding overlapping portfolio outside the banking system as in Figure 1 (b). Barucca et al. (2021) empirically find significant overlapping equity and debt portfolios between different types of financial institution, providing evidence for the existence of a price-mediated channel of contagion between banks.

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The third type of network is much more complex, including not only direct cross-holding, but also indirect dependency by holding mutual assets as in Figure 1 (c). This interdependency has been shown as a realistic source of uncertainty in systemic risk (Roukny et al., 2018). Furthermore, Elliott et al. (2014) study cascading failures in an equilibrium model of interdependent financial network.

Since interdependent network model provides two contagion channels and is more realistic, it is worthy of further study. The main goal of this study is to identify cascade hierarchies in the interdependent financial network. Given the available literature our contribution is threefold. First, we slightly revised the model of Elliott et al. (2014) by separating the bank’s “value” delivered to final investors outside the banking system to external liabilities and equity value. Such division is in line with the balance sheet and can make clear the bank value in the general sense, although it does not change the derived form of interdependency matrix. This modification also facilitates empirical research for the European banking system, because the European Banking Authority (EBA) dataset does not provide liability items, but only asset items and some equity items such as Tier 1 capital. Second, we integrate microprudential stress test and macroprudential stress test together for the European banking system. Considering that the EBA’s stress test is microprudential for individual banks, we perform macroprudential stress test by using the adverse scenario in EBA’s stress test as the initial shock. Third, since granular data on interbank credit exposures is not public, we employ 3 reconstruction methods to form the cross-holding network and then study contagion hierarchies comparatively.

The remainder of the paper is organized as follows. Section 2 presents the literature review. Section 3 introduces the model and method of identifying cascade hierarchies. Section 4 shows the data and the empirical analyses. Section 5 concludes the paper.

2. Literature review

As in Figure 1, we review existing literature about network contagion according to the network structures adopted.

2.1. Interbank network contagions

This kind of model shows that contagion can be caused by direct credit exposures among banks. Rogers and Veraart (2013) model financial market as a directed graph of interbank obligations and study the occurrence of systemic risk. Gai and Kapadia (2010) develop an analytical network contagion model and suggest that financial systems exhibit a robust-yet-fragile tendency. That is, while the probability of contagion may be low, the influences can be extremely widespread when problems occur. Similarly, Acemoglu et al. (2015) argue that the extent of financial contagion exhibits a form of phase transition. In addition, many studies focus on how interbank network topology creates instability (Bardoscia et al., 2017; Eboli, 2019). Zhang et al. (2021) find that network connectedness of banks strengthens the relationship between liquidity creation and systemic risk. Brunetti et al. (2019) study the interbank
market around the 2008 financial crisis and find that the correlation network and the physical credit network behave different. During the crisis, the correlation network displays an increase in connection, while the physical credit network shows a marked decrease in connection.

2.2. Overlapping portfolio contagions

When a bank suffers a negative shock to its equity, a natural way to return to target leverage is to sell assets. Greenwood et al. (2015) present a model in which fire sales propagate shocks across banks. Huang et al. (2013) build a bipartite banking network model composed of banks and assets and present a cascading failure describing the risk propagation process during crises. Similarly, Caccioli et al. (2014) show the amplification of financial contagion due to the combination of overlapping portfolios and leverage, in terms of a generalized branching process. Furthermore, for quantifying the potential exposure to indirect contagion arising from deleveraging of assets in stress scenarios, Cont and Schaanning (2019) propose two indicators. Vodenska et al. (2021) build a bipartite network with weighted links between banks and assets based on sovereign debt holdings, and then model the systemic risk propagation.

2.3. Interdependent network contagions

This kind of model investigates how these two channels (the interbank channel and the overlapping channel) propagate individual defaults to systemic cascading failures. Caccioli et al. (2015) argue that neither channel of contagion results in large effects on its own. In contrast, when both channels are active, defaults are much more common and have large systemic effects. Aldasoro et al. (2017) likewise suggest that contagion occurs through deleveraging and interbank connection. The interdependent network models are also applied to characterize contagions in reinsurance and derivatives markets (Klages-Mundt and Minca, 2020; Paddrik et al., 2020).

Elliott et al. (2014) study cascading failures in an interdependent financial network. They show that discontinuous changes in asset values trigger further failures. Furthermore, when banks face potentially correlated risks from outside the financial system, the interbank connections can share these risks, but they also create the channels by which shocks can be propagated (Elliott et al., 2021). In addition, some studies find that the overlapping portfolio holding by banks accounts for the majority of defaults. Chen et al. (2016) confirm that the market liquidity effect has a great potential to cause systemic contagion. Dungey et al. (2020) show that deleveraging speed and concentration of illiquid assets play a critical role in cascades. Ma et al. (2021) further prove that illiquidity is a critical factor in triggering risk contagion and that higher interbank leverage can cause larger losses for both the banks and the external assets. Our results are consistent with these literature, in the sense that the general contagion hierarchies are mainly determined by the overlapping channel, while the structure of interbank network is also important for some specific banks.

3. Methodology

3.1. The model

The model follows Elliott et al. (2014), but separates the “value” in their paper, that any bank delivers to final investors outside the system of cross-holding, to external liabilities and equity value. Concretely, for every bank, its assets are divided into external assets and interbank assets, and its liabilities are divided into external liabilities and interbank liabilities. The equity value is the difference between its total assets and its total liabilities. Table 1 illustrates a balance sheet based on this.

| Assets          | Liabilities       |
|-----------------|-------------------|
| External assets | $\sum_k D_{ik} p_k$ |
| Interbank assets| $a_i \equiv \sum_j C_{ij} V_j$ |
| External liabilities | $l_{el} V_i$ |
| Interbank liabilities | $l_i \equiv \sum_j C_{ji} V_i$ |
| Net worth       | $v_i$             |
Assume that there are $N$ banks and $M$ external assets. The current value of asset $k$ is denoted $p_k$. Let $D_{ik} \geq 0$ be the fraction of the value of asset $k$ held directly by bank $i$ and let $D$ donate the matrix whose entry is equal to $D_{ik}$. A bank can also hold shares of other banks. Let $C_{ij} \geq 0$ is the fraction of bank $j$ owned by bank $i$, where $C_{ii} = 0$ for each $i$. The cross-holding matrix $C$ can be viewed as a network in which there is a directed link from $j$ to $i$ if cash flows in that direction, in other words, if $i$ owns a positive share of $j$.

Let $V_i$ be the total asset value of bank $i$. This is equal to the value of external assets holding by bank $i$ plus the value of its claims on other banks:

$$V_i = \sum_k D_{ik}p_k + \sum_j C_{ij}V_j.$$  

(1)

Equation (1) can be written in matrix notation as

$$V = Dp + CV$$  

(2)

and solved to yield

$$V = (I - C)^{-1}Dp.$$  

(3)

On the other hands, the total value of bank $i$ is also equal to its total liabilities plus its equity value $v_i$. Its total liabilities constitute of interbank liabilities $\sum_j C_{ji}V_i$ and external liabilities $l^{\text{e}}_iV_i$, where $l^{\text{e}}_i$ is the ratio of external liabilities to total assets. Hence, the equity value of bank $i$:

$$v_i = \sum_j C_{ij}V_j - \sum_j C_{ji}V_i + \sum_k D_{ik}p_k - l^{\text{e}}_iV_i.$$  

(4)

Now we denote the capital ratio (i.e. ratio of equity value to total value) of bank $i$ as $\tilde{C}_{ii}$, then

$$\tilde{C}_{ii} \equiv 1 - l^{\text{e}}_i - \sum_{j \in N} C_{ji}.$$  

(5)

Note that the off-diagonal entries of the matrix $\tilde{C}$ are defined to be 0. Hence, Equation (4) can be written in matrix notation as

$$v = CV - (I - \tilde{C})V + Dp = (C - (I - \tilde{C}))V + Dp.$$  

(6)

Substituting for the total asset value $V$ from (3), this becomes

$$v = (C - I + \tilde{C})(I - C)^{-1}Dp + Dp = (C - I + \tilde{C} + (I - C))(I - C)^{-1}Dp$$

$$= \tilde{C}(I - C)^{-1}Dp = A Dp.$$  

(7)

Here we refer to $A = \tilde{C}(I - C)^{-1}$ as the interdependency matrix.

As in Elliott et al. (2014), banks will lose some value in discontinuous ways if their values fall below certain critical thresholds. In fact, it’s these discontinuities that lead to cascading failures. If the equity value $v_i$ of a bank $i$ falls below some threshold level $v_i$, then the bank is said to fail and incurs failure costs $\beta_i v_i$. In many situations, a natural cap for $\beta_i$ is 1. That is, the maximum loss that can result from the failure of bank $i$ is its value at the time of failure.

The valuations in (3) and (7) are similar when we include the discontinuous failure costs, and so the total value of bank $i$ becomes

$$V_i = \sum_{j \neq i} C_{ij}V_j + \sum_k D_{ik}p_k - \beta_i I_{v_i < v_i},$$  

(8)

where $I_{v_i < v_i}$ is an indicator variable taking value 1 if $v_i < v_i$ and value 0 otherwise.
This leads to a new version of (3):

$$V = (I - C)^{-1}(Dp - b(v)), \quad (9)$$

where $b(v) = \beta_iv^iv_i$. Correspondingly, (7) is re-expressed as

$$v = C(I - C)^{-1}(Dp - b(v)) = A(Dp - b(v)). \quad (10)$$

An entry $A_{ij}$ of the interdependency matrix describes the proportion of $j$’s costs that $i$ pays when $j$ fails as well as $i$’s claims on the external assets that $j$ directly holds.

3.2. Identifying cascade hierarchies

Based on the interdependent model, we can trace the propagation path initiated by a specific shock. At step $t$, let $Z_t$ be the set of failed banks. Initialize $Z_0 = \emptyset$ and $\mathbf{y} = \theta v_0$. Assume an adverse scenario that causes prices of mutual assets to decline. Then the cascade hierarchies can be identified as following. At each step $t \geq 1$:

1. Let $\tilde{b}_{t-1}$ be a vector with element $\tilde{b}_i = \beta_i v_i$ if $i \in Z_{t-1}$ and 0 otherwise.
2. Let $Z_t$ be the set of all $k$ such that entry $k$ of the following vector is negative:

$$A(Dp - \tilde{b}_{t-1}) - \mathbf{y}. \quad (11)$$

3. Terminate if $Z_t = Z_{t-1}$. Otherwise return to step 1.

When this algorithm terminates at step $T$, the sets $Z_1, Z_2, ..., Z_T$ correspond to the failure hierarchies.

4. Empirical analyses for European banking system

4.1. Data

We use data collected by the European Banking Authority (EBA) for the 2018 EU-wide stress test. This public dataset covers a sample of 48 banks in 15 countries in the European Union and European Economic Area at the highest level of consolidation. Table 2 lists 48 banks and countries they belong to. This dataset not only provides the actual balance sheet figures and their International Financial Reporting Standard (IFRS) 9 restated figures, but also covers a three-year horizon baseline and adverse scenarios, which take the end-2017 data as the starting point.

The actual and restated figures give the exposure values in various asset classes. Table 3 lists 21 asset classes that we collect from the EBA dataset and provides corresponding EBA items and EBA exposure codes for each type of asset. Among them, type 2100 indicates the aggregated claims on other credit institutions that one bank holds. However, granular exposure data on banking networks is not public. The credit exposure networks can be reconstructed by some inference methods using only aggregated relational data (Anand et al., 2018). The other 20 classes are the external assets mutually holding by 48 banks.

The adverse scenario gives the corresponding exposure values of various asset classes under some assumed macroeconomic shocks, including a growth in gross domestic product (GDP) in the EU of -1.2%, -2.2% and 0.7% as of 2018, 2019 and 2020 respectively. This adverse scenario can be viewed as an ideal initial shock with which the proposed hierarchical contagion model will be tested.

4.2. Reconstruction of interbank network

In order to test the reliability of contagious hierarchies identified by the proposed model, we employ 3 network reconstruction methods to build the asset/liability cross-holding network. We call these 3 methods Anan (Anand et al., 2015), Hała (Halaj and Kok, 2013) and Maxe (Upper and Worms, 2004). Either of 3 methods can reconstruct interbank networks with aggregated assets and liabilities. However, the EBA dataset only provides asset exposures, no liability data. We refer to some empirical studies based on these data assuming that for bank $i$, the aggregated interbank assets $\sum_j C_{ij}V_j$ equal to the aggregated interbank liabilities $\sum_j C_{ji}V_i$ (Chen et al., 2016; Glasserman and Young, 2015). We now give a brief description for these 3 methods.

1https://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2018
| Country code | Country         | Bank                                                        | Bank abbr. |
|-------------|----------------|-------------------------------------------------------------|------------|
| AT          | Austria        | Raiffeisen Bank International AG                            | RBI        |
| AT          | Austria        | Erste Group Bank AG                                         | EBS        |
| BE          | Belgium        | KBC Group NV                                                | KBC        |
| BE          | Belgium        | Bellius Banque SA                                           | Belfius    |
| DE          | Germany        | DZ BANK AG Deutsche Zentral-Genossenschaftsbank              | DZ Bank    |
| DE          | Germany        | Landesbank Baden-Wurttemberg                                | LBBW       |
| DE          | Germany        | Deutsche Bank AG                                            | DBK        |
| DE          | Germany        | Commerzbank AG                                              | CBK        |
| DE          | Germany        | Norddeutsche Landesbank - Girozentrale -                    | NORD/LB    |
| DE          | Germany        | Bayerische Landesbank                                       | BayernLB   |
| DE          | Germany        | Landesbank Hessen-Thuringen Girozentrale AdoR               | Helaba     |
| DE          | Germany        | NRW.BANK                                                   | NRW        |
| DK          | Denmark        | Danske Bank                                                 | Danske     |
| DK          | Denmark        | Jyske Bank                                                  | JYSK       |
| DK          | Denmark        | Nykredit Realkredit                                         | Nykredit   |
| ES          | Spain          | Banco Santander S.A.                                        | SAN        |
| ES          | Spain          | Banco Bilbao Vizcaya Argentaria S.A.                        | BBVA       |
| ES          | Spain          | CaixaBank, S.A.                                             | CABK       |
| ES          | Spain          | Banco de Sabadell S.A.                                      | SAB        |
| FI          | Finland        | OP Financial Group                                          | OP         |
| FR          | France         | BNP Paribas                                                 | BNP        |
| FR          | France         | Groupe Credit Agricole                                      | ACA        |
| FR          | France         | Societe Generale S.A.                                       | GLE        |
| FR          | France         | Groupe Credit Mutuel                                        | GCM        |
| FR          | France         | Groupe BPCE                                                 | BPCE       |
| FR          | France         | La Banque Postale                                           | LABP       |
| GB          | United Kingdom | Barclays Plc                                                | BARC       |
| GB          | United Kingdom | Lloyds Banking Group Plc                                     | LLOY       |
| GB          | United Kingdom | HSBC Holdings Plc                                           | HSBC       |
| GB          | United Kingdom | The Royal Bank of Scotland Group Plc                        | RBS        |
| HU          | Hungary        | OTP Bank Nyrt.                                              | OTP        |
| IE          | Ireland        | Bank of Ireland Group plc                                   | BIR        |
| IE          | Ireland        | Allied Irish Banks Group plc                                | AIB        |
| IT          | Italy          | UniCredit S.p.A.                                            | UNCRY      |
| IT          | Italy          | Intesa Sanpaolo S.p.A.                                      | ISP        |
| IT          | Italy          | Banco BPM S.p.A.                                            | BPM        |
| IT          | Italy          | Unione di Banche Italiane Societa Per Azioni                | UB1        |
| NL          | Netherlands    | N.V. Bank Nederlandse Gemeenten                             | BNG        |
| NL          | Netherlands    | ABN AMRO Group N.V.                                         | ABN        |
| NL          | Netherlands    | ING Groep N.V.                                              | ING        |
| NL          | Netherlands    | Cooperatieve Rabobank U.A.                                  | Rabobank   |
| NO          | Norway         | DNB Bank Group                                              | DNB        |
| PL          | Poland         | Powszechna Kasa Oszczednosci Bank Polski SA                  | PKO        |
| PL          | Poland         | Bank Polska Kasa Opieki SA                                  | PEO        |
| SE          | Sweden         | Skandinaviska Enskilda Banken - group                       | SEB        |
| SE          | Sweden         | Nordea Bank - group                                         | Nordea     |
| SE          | Sweden         | Swedbank - group                                            | SWDB       |
| SE          | Sweden         | Svenska Handelsbanken - group                               | SHB        |
Table 3: Asset classes and their EBA data reference codes.

| EBA Item | EBA Exposure | Asset classes |
|----------|--------------|---------------|
| 183203, 183303 | **1100** | Central banks and central governments |
| 183203, 183303 | **1200** | Regional governments or local authorities |
| 183904, 183905 | **1300** | Public sector entities |
| 183904, 183905 | **1400** | Multilateral Development Banks |
| 183904, 183905 | **1500** | International Organisations |
| 183203, 183303 | **1600** | General governments |
| 183904, 183905 | **1700** | Credit institutions |
| 2100 | **2200** | Other financial corporations |
| 3000 | | Corporates (Credit Risk) / Non Financial corporations (NPE- Forbearance) |
| 3000 | | Secured by mortgages on immovable property |
| 4110 | | Retail - Secured by real estate property - SME |
| 4120 | | Retail - Secured by real estate property - Non SME |
| 4200, 4310, 4320 | | Retail - Other - SME |
| 4200, 4310, 4320 | | Retail - Other - Non SME |
| 4500 | | Retail - SME |
| 4700 | | Households |
| 5000 | | Secured by mortgages on immovable property |
| 6400 | | Items associated with particularly high risk |
| 6500 | | Covered bonds |
| 6600 | | Claims on institutions and corporates with a ST credit assessment |
| 6700 | | Collective investments undertakings (CIU) |

4.2.1. Anan

Anand et al. (2015) propose a method combining information-theoretic arguments with economic incentives to keep the realistic features of interbank network. The authors argue that the Minimum Density (MD) method is suitable for sparse networks such as financial markets, and is able to minimize the cost of additional linkages to reconstruct the network.

Based on this method, $c$ is defined as the fixed cost of establishing a link, $N$ represents the number of banks. $C$ notes the matrix of aggregated exposure values. The aggregated interbank assets of bank $i$ are $\sum_{j=1}^{N} C_{ij}$, and its aggregated liabilities are $\sum_{j=1}^{N} C_{ji}$. Then, the MD method is formulated as:

$$
\min c \sum_{i=1}^{N} \sum_{j=1}^{N} 1[C_{ij} \geq 0], \quad s.t.
$$

$$
\sum_{j=1}^{N} C_{ij} = a_i \quad \forall i = 1, 2, ..., N
$$

$$
\sum_{i=1}^{N} C_{ij} = a_j \quad \forall j = 1, 2, ..., N
$$

where the integer function $1$ is equal to one, only if bank $i$ lends to bank $j$, and zero otherwise. Here, the authors design a heuristic to solve this computationally expensive problem.

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4.2.2. Hala

Halaj and Kok (2013) propose an iterative algorithm to randomly generate a series of interbank networks. At the initial network, assume that the possibility of all links is the same that all entries in the matrix $C_0$ are equal to zero, and the unmatched interbank assets and liabilities are initiated as $a_0 = a$ and $l_0 = l$. When iterating to the $k + 1$ step, a pair of banks $(i, j)$ are randomly selected. Next, extract the random number $f$ from the unit interval to re-scale the matrix to update the weight $C_{ij}^{k+1}$ as follows:

$$C_{ij}^{k+1} = C_{ij}^k + f^{k+1} \min\{a_i^k, l_j^k\}$$

(13)

and the unmatched assets and liabilities are:

$$a_{i}^{k+1} = a_i^k - \sum_{j=1}^{N} C_{ij}^{k+1} \quad \text{and} \quad l_{j}^{k+1} = l_j^k - \sum_{i=1}^{N} C_{ij}^{k+1}$$

(14)

The iteration is repeated until no more interbank assets are left to be assigned.

4.2.3. Maxe

Maxe is the maximum entropy method, the basis of iterative methods (Upper and Worms, 2004). In the initial guess network, the exposure of bank $i$ to bank $j$ is equal to the aggregated exposure of bank $i$ multiplied by the aggregated exposure of bank $j$, namely, $Q_{ij} = a_i a_j$. Next, the network is re-scaled until the constraints are satisfied. This entails maximizing the entropy function:

$$- \sum_{i,j} C_{ij} \log C_{ij} / Q_{ij}$$

(15)

Entropy optimization can achieve network reconstruction through an effective iterative algorithm. Paltalidis et al. (2015) employ this method to reconstruct interbank network to study transmission channels of systemic risk.

4.2.4. Reconstructed European interbank networks

Table 4 reports the network statistics we compute for reconstructed networks using above 3 approaches. It’s shown that the reconstructed networks are very different. Network generated by Maxe has the largest number of links, the highest density and degree, so as to clustering and core size. This is because Maxe network is fully connected. Compared Anan with Hala, we find that the Anan network is more sparse, having lower density and clustering, smaller average degree and core size. The lender/borrower dependency is defined as the average of the market share of the largest borrower or lender, respectively. The HHI (Herfindahl-Hirschman Index) describes the concentration of both assets and liabilities. Due to the sparsity of Anan network, it’s reasonable that this network has higher dependency and concentration. The assortativity characterizes the preference for a network’s nodes to attach to others that are similar. Both Anan and Hala have negative assortativities, which is consistent with the statistic of the genuine interbank networks computed in Anand et al. (2018).

Figure 2 displays the European interbank network (direct cross-holding matrix $C$) reconstructed by Anan and Hala respectively. The widths of the arrows are proportional to the sizes of the cross-holdings. The area of bank node is proportional to its equity value. The banks with the same color are belong to the same country. The arrow direction means that the origin bank has claims on the destination bank. Consistent with Table 4, the Anan network is more sparse than the Hala network. We can also find that in both reconstructed networks, banks from the UK (the pink node), Germany (the brown node) and France (the blue node) are located in more central positions, showing that these banks are connected densely.

Figure 3 displays the interdependent matrix $A$ in European banking system reconstructed by Anan and Hala. The widths of the arrows are proportional to the degrees of inter-dependency. Note that the interdependent matrix $A$ not only describes the direct cross-holding among banks, but also the indirect claims on the external assets that other banks hold. Therefore, the interdependent network $A$ are more dense than the direct interbank network $C$. This is exactly explain what is interdependency and the difference between interdependency model and simple cross-holding model.
Table 4: Network statistics for reconstructed interbank networks.

|                          | Anan | Hala | Maxe |
|--------------------------|------|------|------|
| Number of Links          | 99   | 344  | 2256 |
| Density                  | 4.388| 15.248|100.000|
| Avg Degree               | 2.063| 7.167| 47.000|
| Med Degree               | 1    | 7    | 47   |
| Assortativity            | -0.308| -0.321| NaN  |
| Clustering               | 0.678| 21.794|100.000|
| Lender Dependency        | 83.718| 57.910| 10.708|
| Borrower Dependency      | 86.334| 71.905| 10.708|
| Mean HHI Assets          | 0.785| 0.463| 0.045|
| Median HHI Assets        | 1.000| 0.439| 0.045|
| Mean HHI Liabilities     | 0.824| 0.639| 0.045|
| Median HHI Liabilities   | 1.000| 0.620| 0.045|
| Core Size (% banks)      | 10.417| 18.750| 97.917|

Figure 2: Direct cross-holding matrix $C$ in European banking system reconstructed by Anan and Hała. The widths of the arrows are proportional to the sizes of the cross-holdings. The area of bank node is proportional to its equity value. The banks with the same color are belong to the same country.

4.3. Cascades

To illustrate the hierarchical cascades, we consider the adverse scenario in EBA 2018 EU-wide stress test. The initial shock to the values of 20 types of external assets is extracted from the adverse scenario as of 2020. The failure thresholds $\theta$ are set to $\theta$ times the IFRS 9 restated figures at the end-2017 (which is the actual balance sheet data).
Various levels of $\theta$ are chosen to test the cascade process. If a bank fails, then the loss in value is $\beta v_i$, where $\beta$ is set to 0.3 for lower failure cost and 0.8 for higher failure cost.

We examine the results for Anan network, Hała network and Maxe network respectively. In Table 5, Panel A and B display the hierarchies of cascades for Anan reconstructed network. In case of $\theta = 0.971$, there are 5 banks hit its failure point under the initial shock. For both levels of failure costs, cascades do not occur. We then raise $\theta$ to 0.973 and see how cascades occur. In this case, there are 17 banks failed under the initial shock. Then DZ Bank, BayernLB and ING are triggered by a contagion when $\beta = 0.3$. When failure cost is raised to 0.8, three more banks (EBS, GLE and UNCRY) are failed in this hierarchy. In the next cascading round, when $\beta = 0.3$, LBBW and BBVA are triggered to fail due to their exposures to the former two rounds of failed banks. For example, both LBBW and BBVA have claims on DZ Bank (see Figure 2(a)). Pushing $\beta$ up to 0.8, there are two more banks (Belfius and OP) failed due to taking higher failure cost.

Panel C and D in Table 5 display the hierarchies of cascades for Hała reconstructed network. The initial failed banks are the same as the Anan cases. However, cascades are triggered in case of $\theta = 0.971$, that is, causing UBI to fail. This is due to the fact that cross-holding network reconstructed by Hała has higher connection and density compared to the Anan network. In the next round, UBI’s failure further causes RBI to fail because RBI has claims on UBI (see Figure 2(b)). When failure cost is raised to 0.8, similar with the Anan case, there are more banks failed in each cascading round and finally up to four failure hierarchies. Pushing $\theta$ up to 0.973 leads to more banks failed and would cause failures at earlier levels, but would not change the ordering. Take $\beta = 0.8$ for example, in case of $\theta = 0.971$, the DZ Bank failed at the third hierarchy, while in case of $\theta = 0.973$, the DZ Bank failed at second hierarchy.

Panel E and F in Table 5 display the hierarchies of cascades for Maxe reconstructed network. It is found that the
Table 5: Hierarchies of cascades in macroprudential stress test for the European banking system. Three restructure algorithms (i.e. Anan, Hala and Maxe) for the interbank cross-holding network are considered. This table reports the test results with different failure thresholds $\theta$ and different failure cost coefficients $\beta$.

| Panel | Algorithm | $\beta=0.3$ | $\beta=0.8$ |
|-------|-----------|-------------|-------------|
| A     | Anan, $\theta=0.971$ | JYSK, GCM, Rabobank, DNB, SEB, SHB | JYSK, GCM, Rabobank, DNB, SEB, SHB |
| First Failure | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB |
| Second Failure | DZ Bank, BayernLB, ING | EBS, DZ Bank, BayernLB, GLE, UNCRY, ING |
| Third Failure | LBBW, BBVA | Belfius, LBBW, BBVA, OP |
| B     | Anan, $\theta=0.973$ | JYSK, GCM, Rabobank, DNB, SEB, SHB | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB |
| First Failure | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB |
| Second Failure | DZ Bank, BayernLB, ING | EBS, DZ Bank, BayernLB, GLE, UNCRY, ING |
| Third Failure | LBBW, BBVA | Belfius, LBBW, BBVA, OP |
| C     | Hala, $\theta=0.971$ | JYSK, GCM, Rabobank, DNB, SEB, SHB | JYSK, GCM, Rabobank, DNB, SEB, SHB |
| First Failure | RBI | RBI, DZ Bank, ACA, AIB |
| Second Failure | UBI | LBBW |
| Panel D: Hala, $\theta=0.973$ | JYSK, GCM, Rabobank, DNB, SEB, SHB | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB |
| First Failure | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB |
| Second Failure | DZ Bank, UNCERY | Belfius, DZ Bank, LBBW, BBVA, UNCERY |
| Third Failure | LBBW | Helaba, OP |
| Panel E: Maxe, $\theta=0.971$ | JYSK, GCM, Rabobank, DNB, SEB, SHB | JYSK, GCM, Rabobank, DNB, SEB, SHB |
| First Failure | RBI | RBI, DZ Bank, ACA, AIB |
| Second Failure | UBI | LBBW |
| Panel F: Maxe, $\theta=0.973$ | JYSK, GCM, Rabobank, DNB, SEB, SHB | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB |
| First Failure | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB | RBI, CBK, Danske, JYSK, BNP, ACA, GCM, HSBC, AIB, UBI, Rabobank, DNB, PEO, SEB, Nordea, SWDB, SHB |
| Second Failure | DZ Bank, UNCERY | Belfius, DZ Bank, LBBW, BBVA, UNCERY |
| Third Failure | LBBW | Helaba, OP |

Cascading hierarchies are exactly the same as the Hala case. Even compared with the Anan case, the failure banks and the cascading hierarchies are roughly the same. It’s reasonable since banks’ external assets holdings weight more and play a key role in cascading dynamics. However, the structure of cross-holding network is also important for some specific banks. For example, Helaba failed in the cases of Hala and Maxe, while not in the Anan case. Our results are consistent with Chen et al. (2016), who find that the market liquidity effect has a greater potential than the network effect to cause systemic contagion.
5. Concluding remarks

Based on a simple model of interdependent financial networks, we have examined cascades in the European banking system. The interdependency means that the connections between banks include not only direct cross-holding (interbank network) but also indirect dependency by holding mutual assets outside the banking system (bipartite network). Through analyzing bank’s balance sheet, an equilibrium matrix is derived to characterize this interdependency.

We use data extracted from the European Banking Authority to illustrate the interdependency. First, we collect 20 classes of external assets mutually holding by 48 banks. For the cross-holding, interbank exposures are not available but the aggregated claims are public. Then we employ 3 network reconstruction methods to build the asset/liability cross-holding network. Finally, we compute the interdependency matrix. The interdependency network is much more dense than the direct cross-holding network, showing the complex latent interaction among banks.

Next we perform macroprudential stress tests for the European banking system, using the adverse scenario in EBA 2018 EU-wide stress test as the initial shock. For different reconstructed networks, we illustrate the hierarchical cascades and show that the failure hierarchies are roughly the same except for a few banks, reflecting the overlapping portfolio holding accounts for the majority of defaults.

Clearly the above tests are based on moderate scenario taken by EBA (recalling that they assume GDP in the EU only decreases -1.2%, -2.2% and even increases 0.7% as of 2018, 2019 and 2020 respectively), so that the default threshold must be set to a very high value (i.e. 0.97) to successfully trigger the initial failures. Nonetheless, we emphasize that understanding the interdependency network and the hierarchy of the cascades can help to improve policy intervention and implement rescue strategy.

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