Correlation between the curvature and some properties of the neutron star

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Abstract. We calculate neutron star (NS) properties like mass $M$, radius $R$, compactness $C$ and surface curvature (SC) using relativistic mean-field (RMF) and Skyrme-Hartree-Fock (SHF) models. Predictive competence of the various RMF and SHF parameter sets are discussed and compared with the NICER and pulsar observational data. We try to correlate between some properties of the NS like $C$, $R$ and SC. To calculate the correlations coefficient, we employ the Pearson formula. We substantiate a correlation between the SC and $C$ for canonical NS, and it shows a strong correlation with a coefficient of 0.992. Similarly, we find another correlation between the radius and SC with a coefficient of 0.982 for the canonical star. The three-dimensional correlations are studied for the $C$, $R$ and SC with varying the masses of the NS. To visualise the correlation between these quantities as mentioned above, both for maximum and canonical mass, a $7 \times 7$ heat map is constructed, which gives the correlations between each pair of quantities. We find that there is a strong correlation between these pairs $SC_{1.4}-R_{1.4}$, $SC_{1.4}-C_{1.4}$. The correlations become weaker for the maximum mass NS. To find the correlation between the same quantities but for different masses of the NS, we calculated the correlations coefficients for 1.5-1.8 $M_{\odot}$ NS. There are also we get strong correlations between $C$, $R$ and SC.

Keywords: equation of state, neutron star, curvature

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1 Introduction

Understanding the nature of the fundamental interaction at supra-saturation density is currently considered as a prominent topic in fundamental physics. In the laboratory, we can create the dense matter up to few times of the saturation density [1, 2], which is not enough to understand the nuclear interaction at high degree of density and isospin asymmetry. This experimental inadequacy forces us to use neutron star (NS) as sole ingredient to study the high dense nature of the nuclear interaction [3]. NS is one of the most familiar members of the compact object family of the visible universe, having central density 5–6 times the density of the matter at nuclear centre [3, 4]. Most of the terrestrial experiments and analysis of the finite nuclear data give the information about the nature of the equation of state (EOS) around saturation density, while the higher density region still remains unexplored. Various theoretical models predict diverge behaviour for the EOSs in that region, leading to completely different values for the global properties of the NS. The global properties of the NS like maximum mass, radius, moment of inertia are used as an effective tool to put the strong constraints on the nature of the EOS [3].

In recent years, the tidal deformability from the first binary NS merger event, GW170817 [5, 6] can also spotlight the nature of matter enclosed inside the NS. The composition of the NS is not clear till now. In the conventional models, the NS is populated by the neutron and admixture of the proton and electron are inside the NS to maintain the $\beta$-equilibrium and charge neutrality conditions. The extreme conditions inside the NS favor various exotic phenomena to happen like hyperon production [7–9], kaon condensation [8, 10–14], deconfinement of quark [15–19], and presence of DM particle [20–27]. Recently, the Neutron Star Interior Composition Explorer (NICER) data [28–33] can put a strong constraints on the mass and radius of a NS. This simultaneous measurement of the mass and radius of a NS can constraint the nuclear EOS. The correlation between the tidal deformability and the radius of the canonical star is now a settled issue [2, 34–43]. But the value of the correlation coefficient is still a debatable issue, various theoretical formalism give different ranges of the correlation coefficients. Similarly, it is noticed that a correlation exists between the skin thickness of the heavy neutron-rich nuclei ($^{208}$Pb) and NS properties [44, 45]. It is the consequence of the fact that the slope of the symmetry energy determine the pressure of the neutron-rich skin and radius of the NS. Li et al. have showed that an unique relation between $K_{sym}$ and slope $L$ can constrain the high density behaviour of the $E_{sym}$ [46]. Along the same path Alam et al. have shown that there exist
a strong correlation between the NS radii and slope of the incompressibility of the nuclear matter (NM) [47]. Using Brueckner-Hartree-Fock formalism, Wei et al. found a strong correlation between NS radius, tidal deformability and pressure of the beta stable matter [48]. Therefore, it is a common practice to look for a correlation between the NM and NS properties. In this work, we try to find a new type of correlation between the NS properties, i.e correlation between the surface curvature (SC) and other bulk properties like $R$ and $C$.

According to the general theory of relativity, the energy and momentum of whatsoever matter or radiation present in the universe is strongly correlated with the curvature of space-time. The strength of the curvature depends on the distortion originated by a massive object in space-time, which mechanically is similar to the functioning of a trampoline. To unfold this unified theory of gravitation in terms of simple geometric algebra, certain mathematical quantities like compactness ($C \equiv M/R$), Riemann tensor, Ricci scalar, Kretschmann scalar have been defined. The more detailed interpretation and derivation of these mathematical quantities which holds a lot of information about the curvature can be found in the Ref. [49]. Among these quantities, the quantities $K$ and $W$ are more prominent to measure the space-time curvature both inside and outside the star. Our previous analysis have calculated all these defined curvatures with and without dark matter (DM) inside the NS. We found that DM has very significant effects on the curvatures and compactness of the NS. Hence, in this present work, we want to explore more on this curvature and correlate with some NS properties because the correlations between curvatures and some bulk properties of the NS gives a comprehensive picture of their nature.

This paper is organised as follow: in Sec. 2, we discuss the theoretical formalism of the relativistic mean-field formalism used to calculate the bulk properties of the NS. The last subsection of the Sec. 2 is dedicated to discuss the theoretical formalism various curvature of the NS. In Sec. 3, we discuss our numerical results. Finally, Sec. 4 is devoted to a concluding remarks of our results and discussion.

2 Framework

This section outlines some quantities to calculate the curvature both inside and outside of the NS for different EOSs. To study the NS properties, we take relativistic mean-field (RMF), Skyrme-Hartree-Fock (SHF) and density-dependent RMF (DDRMF) EOSs. For the last few decades, the assumed models provided a better platform to predict the finite and infinite NM properties. Unlike the non-relativistic model, the RMF model obeys the casual limit up to very high-density [50]. The casual nature of the RMF model provides a natural way to go from finite nuclei to NS, which has a very high density. Moreover, the considered SHF models do not become acausal for masses below $2M_\odot$.

2.1 Mathematical form for different curvature

In this Sub-Sec., we adopt the curvature quantities from the Refs. [49]. The quantities are Ricci scalar, full contraction of Ricci tensor, Kretschmann scalar (full contraction of the Riemann tensor) and the full contraction of the Weyl tensor to measure the curvature of the space-time.

The Ricci scalar

$$\mathcal{R}(r) = 8\pi \left[ \mathcal{E}_{NS}(r) - 3P_{NS}(r) \right], \quad (2.1)$$

the full contraction of the Ricci tensor

$$\mathcal{J}(r) \equiv \sqrt{\mathcal{R}_{\mu\nu}} \mathcal{R}^{\mu\nu} = 8\pi \left[ \mathcal{E}_{NS}^2(r) + 3P_{NS}^2(r) \right]^{1/2}, \quad (2.2)$$

$$- 2 -$$
the Kretschmann scalar

\[ K(r) \equiv \sqrt{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}} \]

\[ = 8\pi \left[ \left\{ 3\mathcal{E}_{NS}^2(r) + 3P_{NS}^2(r) + 2P_{NS}(r)\mathcal{E}_{NS}(r) \right\} - \frac{128\mathcal{E}_{NS}(r)m(r)}{r^3} + \frac{48m^2(r)}{r^6} \right]^{1/2} \]  

(2.3)

and the full contraction of the Weyl tensor

\[ \mathcal{W}(r) \equiv \sqrt{\mathcal{C}_{\mu\nu\rho\sigma}\mathcal{C}^{\mu\nu\rho\sigma}} = \left[ \frac{4}{3} \left( \frac{6m(r)}{r^3} - 8\pi\mathcal{E}_{NS}(r) \right) \right]^{1/2} . \]  

(2.4)

At the surface \( m \to M \) due to \( r \to R \). The Ricci tensor and Ricci scalar vanish outside the star because they depend on the \( \mathcal{E}_{NS}(r) \), \( P_{NS}(r) \), which are zero outside the star. But, there is a non-vanishing component of the Riemann tensor which does not vanish; \( R^1_{010} = -\frac{2M}{R^3} = -\xi \), even in the outside of the star \([49, 51]\). So the Riemann tensor is the more relevant quantity to measure the curvature of the stars than others. Kretschmann scalar is the square root of the full contraction of the Riemann tensor. The vacuum value for both \( K \) and \( \mathcal{W} \) is \( \frac{4\sqrt{3}M}{R^3} \), one can easily see from Eqs. (2.3) and (2.4). Therefore, one can take \( K \) and \( \mathcal{W} \) as two reasonable measures for the curvature within the star. The SC is defined as the ratio of curvature at the surface of the NS \( K(R) \) to the curvature of the Sun \( K_{\odot} \). \( SC = \frac{K(R)}{K_{\odot}} \). This ratio \( \frac{K(R)}{K_{\odot}} \approx 10^{14} \) i.e the NS curvature is \( 10^{14} \) times more than the Sun. Hence, we calculate the correlations between SC and some bulk properties such as \( M \), \( R \) and \( C \) in the following section.

3 Results and Discussions

3.1 Equation of state of the NS

The structure of an NS is divided into the four regions outer crust, inner crust, outer core and inner core. The outer crust consists of nuclei distributed in a solid body-centred-cubic (bcc) lattice filled by a free electron gas. As we go from outer crust to inner crust, the density increases and eventually the nuclei in the crust become so neutron-rich that neutrons start to drip from them. In this situation, the inner crust contains the free electron and neutron gases and formed different types of pasta structure. With the help of nuclear models, the neutron drip density determined the boundary between the outer and inner crust. But, the transition density from the crust to the core is much more uncertain and strongly model-dependent. When the average density reaches a value about half of the NM saturation density, the lattice structure escapes due to energetic reasons and the system changes to a liquid phase.

In Fig. 1, we plot some selected EOSs from RMF and SHF as representative case. The lowest density part of the EOS represents the outer crust of the NS. The inner crust ranges from a density \( \sim 3 \times 10^{-4} \) to \( \sim 8 \times 10^{-2} \) fm\textsuperscript{-3}. The core part starts from a density \( \sim 2 \times 10^{-2} \) fm\textsuperscript{-3} and it extends up to the centre of the NS, which is 5–6 times the density of the NM. At the lower density part all the EOSs shows almost same in nature but at the higher density part, all shows different behaviour. This is due to that we take same crust EOS ( both for outer and inner crust ) which is BCPM EOSs [52] for all assumed parameter sets. In the present context, we take 30 well-known parameter sets of the RMF models HS [53], LA [54], H1 [55], LZ [56], NL3 [57], NL3* [58], NL-SH [59], NL1 [60], GM1 [9], GL85 [8], GL97 [61], NL3-II [57], NLD [54], NL-RA1 [62], TM1 [63], TM2 [63], PK1 [64], FSU [65], FSU2 [66], IUFSU [67], IUFSU* [68], SINPA [69], SINPB [69], G1 [70], G2 [70], G3 [71], IOPB-I [72], FSUGarnet [73], FSU2R [74], and FSU2H [74], 15 SHF models such
Figure 1. (color online) Some selected EOSs from RMF, SHF and DD-RMF sets are shown to know its behaviour mainly at the core part.

as BSk20, BSk21 [75], BSk22, BSk23, BSk24, BSk25 [76], KDE0v1 [77], SkI2, SkI3, SkI5 [78], SkI6 [79], SLY2 [80], SLY4 [81], SL230a [82], Rs [83] and 6 DDRMF sets DD2 [84], DD2Y [85], DDME1 [86], DDME2 [87], DDME2Y [85], DDLZ1 [48] to study various properties of the NS and investigate the existing correlation between various quantities. Moreover, one can take unified EOSs as given in Refs. [85]. The main idea to take a large sets of the parameter sets is to cover the wide range of the NM parameter value and establish a correlation with more accuracy.

3.2 Mass-radius of the NS

In Fig. 2, we show the mass-radius relation for RMF EOSs. We also put the masses of pulsar like PSR J1614-2230 [88], PSR J0348+0432 [89] and PSR J0740+6620 [90]. Recently, the secondary components of the GW190814 events left us to speculate whether it is the lowest massive black hole (BH) or a super heavy NS. The observed mass was $2.50 \pm 0.07 M_\odot$. Many authors have claimed that it was a binary BH merger [91, 92], heaviest NS [93, 94], super-fast pulsar [95] etc. The NICER results are also put to constraint both mass-radius from the x-ray study of the millisecond pulsars PSR J0030+0451 [29, 32]. The constraint on mass-radius are found to be $M = 1.44^{+0.15}_{-0.14} M_\odot$ ($M = 1.34^{+0.15}_{-0.16} M_\odot$) and $R = 13.02^{+1.24}_{-1.06}$ km ($R = 12.71^{+1.14}_{-1.15}$ km) by Miller et al. [29] (Riley et al. [32]) which are indicated in two boxes.

Few parameter sets such as GL97, GL85, TM2, FSU2, IFSU*, SINPA, SINPB, G1, G2, G3, IOPB-I, FSUGarnet and FSU2R lie in the range of the observed maximum masses. Most of the RMF parameter sets satisfy the mass-radius constraint from the NICER observation. We can see from Fig. 2 that the NICER data can not discard many EOSs based on the mass-radius measurement. The NICER data is related to the radius measurement of the NS of mass around $1.4 M_\odot$, which has not very high central density. The central density of the canonical NS lies $2-3 \rho_0$. As all the EOSs are fitted to the saturation properties, they show a convergent behaviour up to a few times the saturation density. This is also a fact that the properties of the small NS have a strong correlation with the NM properties due to their low central density [96]. Few parameter sets like HS, LA, LZ, NL-SH, NL1 and IUFSU parameter sets do not obey the NICER mass-radius constraint. However, only eight
parameter sets like GM1, GL85, FSU2, IUFSU*, G1, IOPB-I, FSUGarnet and FSU2R obey both the NICER and recent maximum NS mass constraints.

In the case of SHF and DD-RMF, the mass and radius of the NS are also depicted in Fig. 2. Except for DDLZ1, DDME1, DDME2, DD2, all satisfies the maximum mass constraints given by different PSR measurements. The NICER limit is also satisfied by all parameter sets except the KDE0v1 and SLY family. The maximum mass predicted by DDLZ1, which lies in the limit given by GW190814.

3.3 Radius of the canonical NS

Along with the maximum mass, in recent years, the radius of the canonical star is also used as an important constraint to control the nature of the EOS in the high-density region [67]. It is now a well-known fact that the radius of the canonical NS and the canonical tidal deformability are strongly correlated [34, 97, 98]. Various models give a different form of correlation. Different analysis of tidal deformability data of GW170817 suggest different range of canonical radius [34–43, 98–104]. We plot the ranges of the canonical radius in Fig. 3. In some cases, the analysis gives a very narrow range of the canonical radius, for example, Most et al. [37]. However, in many cases, the acceptable range is too wide, which is not much suitable to constraint the EOS. If we consider the maximum and minimum radius predicted by the RMF parameter sets, we can find the radius range of the canonical star is $\sim 11.4 - 14.3$ km, which entirely lies in the range reported by the NICER results [29] except HS, LA, LZ, NL-SH, NL1 and IUFSU. This shows that RMF formalism is good enough to reproduce the NICER data. If we do not consider the maximum mass limit and stick to the NICER constraint only, then almost all RMF parameter obey the mass-radius constraint except a few. From Fig. 2, it is clear that too soft EOS (low maximum mass) and too stiff EOS (high maximum mass) are discarded by both NICER and maximum mass constraint. Both SHF and DDRMF sets also lie in the limit given by the NICER.
Figure 3. (color online) We compile the ranges of the canonical radius from some refs. [34–43, 98–104]. The left and right arrow represents no fixed lower and upper limit for canonical NS as compare to others. The NICER radius range given by Miller and Riley et al. [29, 32] are depicted with solid green and dark red lines.

In Fig. 3, we compile the canonical radius of the NS predicted from different approaches. We present the NICER results on top of it with solid green and dark red lines. The upper limit of the canonical radius from various approaches lies in the range of NICER results. For Miller et al. limit is fully satisfied by Most et al. and more than 50% limit is satisfied by Malik, Fattoyev, Annala, Radice and Coughlin et al. as shown in Fig. 3. But in case of Riley et al. limit, only Malik and Most et al. approaches are within the range but other approaches also satisfy ≥ 50% limit.

3.4 Correlation between compactness and curvature

The compactness of the NS is defined as the ratio of the mass and the corresponding radius. In Fig. 4, we plot the $C$ with SC for assumed parameter sets for the canonical star. The graph shows that there is a strong correlation between them for the canonical star. The correlation looks cubic and the value of the correlation coefficient 0.992 i.e these are strongly correlated. The three-dimensional correlation between the SC and $C$ at different masses of the NS are also studies. We use the Pearson formula to calculate the three-dimensional correlation between some properties of the NS although it is valid for linear correlation. Therefore, a linear correlation between any two quantities $x$ and $y$ given by formula as [105]

$$
\xi = \frac{\sum_{xy}}{\sqrt{\sum_{xx}\sum_{yy}}},
$$

(3.1)
Figure 4. (color online) Left: We plot the SC with compactness for canonical star for various parameter sets of the RMF model. We find correlation with co-efficient $\zeta = 0.992$. Right: SC with radius and the correlation co-efficient $\xi = 0.982$.

where

$$\Sigma_{xy} = \frac{1}{N} \sum_{i=0}^{N} x_i y_i - \frac{1}{N^2} \left( \sum_{i=0}^{N} x_i \right) \left( \sum_{i=0}^{N} y_i \right),$$  \hspace{1cm} (3.2)

$$\Sigma_{xx} = \frac{1}{N} \sum_{i=0}^{N} x_i^2 - \frac{1}{N^2} \left( \sum_{i=0}^{N} x_i \right)^2,$$  \hspace{1cm} (3.3)

and

$$\Sigma_{yy} = \frac{1}{N} \sum_{i=0}^{N} y_i^2 - \frac{1}{N^2} \left( \sum_{i=0}^{N} y_i \right)^2.$$  \hspace{1cm} (3.4)

and $N$ is total number of models used and $i$ runs over $N$. The SC and radius of the different parameter set of the RMF model shown in Fig. 4. Here, also exist a strong correlation between them, and it looks to be inverse cubic. The correlation coefficient found to be 0.982. This correlation is comparatively weaker than the SC and compactness.

To show more specifically the mass dependence, we plot the SC, $C$, and $R$ of the NS for the different parameter sets for different masses of the NS in Fig. 5. One can see that there is correlation at each mass of the NS. We calculated the correlation between a pair of quantities using Eqs. (3.1) and (3.5) and present in Fig. 6. The $7 \times 7$ matrix representing the correlations between a pair of quantities, and the correlations coefficient is written inside the box. We find a noticeable correlation between SC with $R$ and $C$. There is a strong correlation between these pairs $SC_{1.4}-R_{1.4}$, $SC_{1.4}-C_{1.4}$. But the correlation between $R_{\text{max}}$ and $SC_{\text{max}}$ is around 85% but for $C_{\text{max}}$ and $SC_{\text{max}}$ is very poor.

In a similar fashion, we also calculate the value of $\xi$ between each pair of quantities such as SC, $C$ and $R$ for 1.4-1.8 $M_\odot$ and depicted in Fig. 7. The value of $\xi$ is $> 85\%$ between each pair for 1.5-1.8 $M_\odot$ NS. Hence we also find a strong correlation between SC, $C$ and $R$ for these masses of
Figure 5. (color online) Left: The correlation between SC and compactness for considered parameter sets is shown for different masses of the NS. Right: Correlation between SC with radius.

Figure 6. (color online) The heat map plot represents the correlation between different quantities like $M_{\text{max}}$, $R_{1.4}$, $R_{\text{max}}$, $SC_{1.4}$, $SC_{\text{max}}$, $C_{1.4}$ and $C_{\text{max}}$. Each number inside the box represents the correlation coefficients calculated using the Pearson formula between that corresponding pair.
The heat map represents the correlation between SC and compactness of the NS with masses 1.4, 1.5, 1.6, 1.7 and 1.8 $M_\odot$. Right: Heat map for SC and radii of different masses of the NS.

Finally, we have tabulated the calculated properties such as its mass, radius, central density, SC and compactness for both canonical and maximum mass NS as given in Table 1.

4 Conclusions

In summary, we calculate some quantities such as mass, radius, curvatures and compactness of NS obtained from RMF and SHF sets. For the last four decades, the RMF and SHF formalism has been well suited for studying NM and NS properties. To study the correlations, we have taken 30 different RMF, 15 SHF and 6 DDRMF EOSs, which covers a broader range of nuclear saturation properties. We have analysed the range of the canonical radius from different approaches. We have also compared the RMF results with NICER data. We find that the lower limit of the canonical radius from different approaches lies completely, such as Most and Malik et al. data, and more than 50% is inside by other approaches. Also, we see that only a few parameter sets can both the maximum mass and NICER constraints. Fig. 2 shows the none of the parameter sets can satisfy both the constraint from GW170817 and GW190814. Only a few parameters sets satisfy the constraint from NICER, GW190814 and the observed maximum masses of NSs. We can choose the most suitable parameter sets, which obey the constraints from NICER, GW170817 and recent maximum mass limit simultaneously. These parameter sets are SINPB, GL97, FSUGarnet, FSU2R, IOPB-I, G1, GL85. This analysis shows that some parameter sets can reproduce the maximum mass of the supermassive neutron star (not confirmed yet super-massive NS or smallest black hole) but simultaneously unable to reproduce other constraints. So it gives a hint that we can modify these equations of state in the lower density region to satisfy the constraints from the GW170817 and NICER data. We calculate the curvature of the neutron star using RMF and SHF EOSs. The interesting conclusion is that we find a correlation between surface curvature and compactness at different masses of the NS. They follow a cubic correlation between them. Using the Pearson formula, we have calculated the correlation coefficient. We found the correlation coefficient between the curvature and compactness is $\xi = 0.992$ for the canonical star which represents that the correlation is strong enough. Similarly, we find another correlation between the curvature and the neutron star’s radius, having the correlation coefficient,
Table 1. The properties of NS maximum mass $M_{\text{max}}$, radius $R$, central density $\varepsilon_c$, surface curvature SC and compactness $C$. 

| Parameter sets | $M_{\text{max}}$ ($M_\odot$) | $R$ (km) | $\varepsilon_c$ (MeV fm$^{-3}$) | SC | $C$ |
|----------------|-----------------------------|----------|----------------------------------|-----|-----|
|                | $R_{1.4}$ | $R_{\text{max}}$ | $\varepsilon_{1.4}$ | $\varepsilon_{\text{max}}$ | $C_{1.4}$ | $C_{\text{max}}$ |
| HS             | 2.980  | 14.31 | 14.03 | 226 | 750 | 1.414 | 3.183 | 0.144 | 0.313 |
| LA             | 2.981  | 14.32 | 14.04 | 228 | 755 | 1.416 | 3.185 | 0.144 | 0.313 |
| H1             | 2.987  | 14.23 | 14.03 | 226 | 750 | 1.431 | 3.188 | 0.145 | 0.314 |
| L1             | 2.981  | 14.76 | 14.16 | 226 | 740 | 1.285 | 3.095 | 0.140 | 0.311 |
| NL3            | 2.774  | 14.08 | 13.16 | 270 | 870 | 1.477 | 3.584 | 0.147 | 0.311 |
| NL3*           | 2.760  | 14.56 | 13.51 | 254 | 830 | 1.492 | 3.610 | 0.147 | 0.311 |
| NL-SH          | 2.795  | 14.35 | 13.42 | 249 | 830 | 1.394 | 3.141 | 0.144 | 0.307 |
| G1             | 2.370  | 13.56 | 11.96 | 314 | 1100 | 1.654 | 4.085 | 0.152 | 0.292 |
| GL85           | 2.168  | 14.17 | 11.93 | 311 | 1150 | 1.449 | 3.769 | 0.152 | 0.292 |
| GS7           | 2.003  | 12.70 | 10.67 | 440 | 1440 | 2.012 | 4.865 | 0.162 | 0.277 |
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$\xi = 0.982$ for the canonical star. The correlations between surface curvature, compactness and radius are more stronger for 1.5-1.8 $M_\odot$. 

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