A Case for Applying an Abstracted Quantum Formalism to Cognition*

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This chapter outlines some of the highlights of efforts undertaken by our group to describe the role of contextuality in the conceptualization of conscious experience using generalized formalisms from quantum mechanics. Conscious experience is filtered not just through innate categories to give rise to stimulus-response reflexes, but also learned categories, including concepts such as ‘container’, ‘democracy’, ‘truth’ and ‘falsehood’. The meanings of these concepts are not rigid or static but shift fluidly depending on context, increasing dramatically our potential to both inform and be informed by the world. As Edelman and Tonini (2000, p. 101) put it: “Every act of perception is, to some extent, an act of creation, and every act of memory is, to some degree, an act of imagination.”

Elements of conscious experience, such as perceived stimuli, retrieved memories, and concepts are considered entities of the personal cognitive sphere, referred to collectively as ‘conceptual entities’. These can be modelled by considering them as configurations of properties, which are consistently testable through personal and interpersonal cognitive processes such as perception, reflection, and social interactions. Specifically, we are concerned with the interface between the conceptual entity and its extraneous surroundings: the context. The interrelation and concatenation of concepts consciously experienced as a stream of thought is particularly affected by context, being influenced not only by the ever-fluctuating associative structure of the conceptual network but also by drives and emotion, as well as environmental affordances including the social milieu.

One of the characteristic aspects of quantum mechanics is the measurement context provoking an indeterministic influence on the physical entity under consideration. The mathematical formalism of quantum mechanics describes precisely this influence and its corresponding probabilities. The situation of contextual influence in cognition is more complex than the one encountered in the micro-world, but generalizations of the mathematical formalisms of quantum mechanics are transferable to the modeling of the creative, contextual manner in which concepts are formed, evoked, and often merged together in cognition (Gabora 2001, Gabora and Aerts 2002a, Gabora and Aerts 2002b). Of course, the nature of the conceptual entity differs essentially from that of a physical entity in its transience and subjectivity, and in its ontological status. The operational formulation of ‘generalized’ quantum mechanics, however, allows a description of contextuality that does not specifically depend on the physical nature of the entities and contexts involved (Aerts 1982, Aerts 1983, Aerts 1986, Aerts 1998, Aerts 2002, Aerts and Aerts 1994, Aerts and Aerts 1997, Aerts et al. 1999a, Aerts et al. 1999b).

For clarity, we emphasize that it is the abstracted formalism which is ‘borrowed’ from quantum theory, not in any way its microphysical ontology of particles and fields. Our approach thus concerns the formal structure of models that are able to describe cognitive entities and processes with contextuality, not the substrate that implements them in the brain.

1 Formalization of Contextual Change

In this section we explain why the mathematical modeling of a certain kind of contextuality requires abstracted ‘quantum’ structures. Let us consider an entity—a conceptual entity in our case—and a

*Published as: Aerts, D., Broekaert, J. and Gabora, L. (2003), A case for applying an abstracted quantum formalism to cognition, in Mind in Interaction, ed. Campbell, R., John Benjamins, Amsterdam.
context—e.g., some situation or social framework. The entity, denoted $S$, can be in different states. We represent the set of relevant states of $S$ as the set $\Sigma$, and denote individual states by symbols $p, q, r, \ldots \in \Sigma$. The effect of the context, which we denote by $e$, on the entity $S$ is, in its most general form, that it changes the state $p$ of the entity to another state $q$ of the entity. In general this change of state under influence of the context is not deterministic, which means that a probability of change is involved. Let us denote by $\mu(p, q, e)$ the probability that the context $e$ changes state $p$ of $S$ to state $q$. Of course, for a modelling of the complete situation, not one but several different possible contexts must be considered. Let is denote the set of all relevant contexts $\mathcal{M}$, and specific contexts by the symbols $e, f, g, \ldots \in \mathcal{M}$. This means that the abstract form of the mathematical formalism we consider is determined by the triplet $(\Sigma, \mathcal{M}, \mu)$, where $\Sigma$ is the set of states of the entity $S$, $\mathcal{M}$ is the set of contexts for $S$, and $\mu: \Sigma \times \Sigma \times \mathcal{M} \to [0, 1]$, the probability function that describes the transition probability between different states of $S$ under the influence of a specific context.

1.1 Classical and Quantum Contextuality

The abstracted formalism gives us qualitative indications and quantitative measures of contextuality, tools with which we can evaluate and describe contextuality in various cognitive processes. As in quantum mechanics, there are two kinds of contextuality in cognition. In order to elaborate them, however, it is necessary to present a few more details—albeit a strict minimum—of the generalized quantum formalism.

Within the standard quantum mechanical formalism, two kinds of change under the influence of context are distinguished. First there is a nondeterministic change of state under influence of the measurement context. For this change, each state $p_u$ of the quantum entity $S$ under consideration is described by a unit vector $u$ of a complex Hilbert space $\mathcal{H}$. Let us denote this set of states by $\Sigma_{\mathcal{H}}$. Each (measurement) context $e_{\mathcal{H}}$ is described by a self-adjoint operator $H$ on this Hilbert space $\mathcal{H}$. Let us denote the set of contexts by $\mathcal{M}_{\mathcal{H}}$. The probability function $\mu$ is given as follows. Suppose the quantum entity is in state $p_u$ under the influence of context $e_{\mathcal{H}}$. The state $p_u$ is then changed to one of the eigenstates $p_v$ of the self-adjoint operator $H$, and the probability of change is given by:

$$\mu(p_v, p_u, e_{\mathcal{H}}) = |\langle u, v \rangle|^2$$

where $\langle u, v \rangle$ is the scalar product of vectors $u$ and $v$ in the Hilbert space $\mathcal{H}$.

Second there is a deterministic change, which is the ‘mechanic’ evolution. It is described by the Schrödinger equation, governing continuous change over time. For clarity, we mention that, within a classical mechanical formalism, only deterministic change under the influence of context is possible, and it is described by the equations of Newton.

To a first approximation we can say that the term classical contextuality refers only to situations where context works in a deterministic way on the state of the entity under consideration, while quantum contextuality includes nondeterministic change for measurement contexts and deterministic change for dynamical contexts.

1.2 The Statistical Situation

Of course in classical physics, models are built that include indeterminism, namely statistical mechanical models. This indeterminism, however, is of a specific and limited nature: it describes a lack of knowledge about the exact (pure) state of the physical entity under consideration. Thus, the notion of statistical state (or mixed state) is introduced. In our formalism it is similarly possible to describe entities (even conceptual entities) in statistical states. In this statistical case, the distinction between classical and quantum is subtle and depends on the structure of the probability model involved. If the probability model is Kolmogorovian—meaning that it satisfies Kolmogorov’s axioms, and hence the event space is a $\sigma$-algebra—only the classical statistical situation can be described.

It is known that the quantum probability model is not Kolmogorovian (Accardi 1982, Accardi and Fedullo 1982, Pitowsky 1989). It is possible to prove that the nonKolmogorovian nature of the quantum probability model is due to a lack of knowledge concerning how context interacts with the entity under consideration, i.e., by the presence of fluctuations in the interaction between context and entity. Even if we were to suppose that at the ontological level the interaction between context and entity engenders a change of state that is deterministic, a lack of knowledge about this interaction gives rise to a probability
model that does not satisfy Kolmogorov’s axioms. Hence a quantum-like probability model is needed to model this situation (Aerts 1986, Aerts 1987, Aerts 1995).

Because of the confusion often created by this subtlety, we will elaborate it a little further. If a model contains only one state of an entity and one context influencing this state, as is often the case in an isolated problem where no attempt is made to deliver a full model description of the entity, the distinction between classical and quantum described above vanishes. Consider the situation of ‘casting a die’. In probability theory, only one state and one context are given. Indeed, suppose we do not know exactly how the die is cast—as is the case in the standard probability model of this situation—then this lack of knowledge gives rise to one state, a statistical state that describes the situation “the die is cast”, without any further specification of how it is cast. There is just one context, for example the physical state of the table on which the die is cast. But we can inverse the description, and for example consider the state “the die is located within a cup that stands on the table”, which is a ‘pure’ state as compared to the former statistical state that we considered. Then the context becomes more intricate: “we, who take the cup, move it around, and cast the die on the table, plus the situation of the table”. Here the lack of knowledge concerns the context, not the state of the die. Obviously both situations give rise to the same mathematical model, and Kolmogorovian probability theory can be applied.

The die example shows that for a very simple situation that can be modeled using only one state and one context, the mathematical structure of the probability model cannot give us the information needed to distinguish between the two ontologically different situations: lack of knowledge about the state, and lack of knowledge about the context. Moreover, the situation can always be captured within a Kolmogorovian probability model. We note this explicitly because in cognition as well as in other fields where probability theory is used, the more complex situation is often cut down to a sample of fragmented simple situations, where only one state and one context are considered. Then each simple situation can be modeled by standard probability theory within a Kolmogorovian structure. It is only when the more complex situation of an entity changing and evolving over a range of different states is considered that the ontological difference between lack of knowledge of the state giving rise to a Kolmogorovian structure, and lack of knowledge of the context giving rise to a quantum structure, is revealed. This phenomenon is well known in axiomatics. Indeed, the Kolmogorovian structure of a probability model is an axiomatic structure, and often ontological differences are only revealed by means of the axiomatic structure if the situations considered are not too simple (not too small). Often in cognition and other fields of science, a complex situation is cut into fragmented simple situations without paying attention afterwards to how these fragmented situations are pasted back again into a complex model that still corresponds to reality. We believe that this at the origin of the fact that the quantum nature—or at least non-Kolmogorovian nature—of such complex situations was not identified much earlier. It has been proven that in the case of an entity with simple dual contexts (contexts that allow only two possible effects of change), at least three contexts are needed for the resulting quantum structure to be revealed mathematically (Accardi 1982, Accardi and Fedullo 1982).

2 Contextuality in Cognition

In cognition, we will always lack knowledge concerning the interaction between the context and the conceptual entity. Hence the situation of the presence of uncontrollable fluctuations on this interaction is the standard one that we have to consider. This is the strongest argument we put forward for the necessity of applying the mathematical structures developed for generalized quantum mechanics to the cognition. Again we remark that the quantum structure of the mathematical model will only be revealed if more than one state and one context are considered. A detailed analysis of this situation, taken into account the effect of the presence of fluctuations on the interaction between context and entity, giving rise to a generalized quantum structure and non-Kolmogorovian probability model, can be found in Gabora (2001), and Gabora and Aerts (2002b). Prior to this general approach, specific abstracted quantum models had been worked out for various cognitive situations by our group. Indeed, it was already obvious to us that quantum structures had to account for the phenomenon of contextualization.

At present, we are far from a generally applicable quantum-like approach to describing what happens in the mind. However, the effectiveness of applying appropriate adaptations of the quantum formalism to specific problems in the field is considerable. In the next few sections, we illustrate how some of perplexing problems in the formal description of human conscious experience can be handled in this way. In each case, we explain how non-classical features arise with respect to the relevant cognitive entities
and processes and how their contextual aspects are described through the introduction of quantum-like formalisms.

### 2.1 Violation of Bell Inequalities by Entangled Concepts

The presence of entanglement—i.e. genuine quantum structure—can be tested for by determining whether correlation experiments on a joint entity violate Bell inequalities (Bell 1964). Pitowsky (1989) proved that if Bell inequalities are satisfied in an experiment, it follows a classical Kolmogorovian probability scheme. The probability can then be explained as being due to a lack of knowledge about the precise state of the system in a classical manner. If, however, Bell inequalities are violated, Pitowsky proved that no such classical Kolmogorovian probability model exists. In that case the interwoven nature of the compound entity is exposed.

Bell inequalities have been successfully applied to an elementary cognitive setting (Aerts et al. 2000), with the inequalities functioning as a quantitative indicator of entanglement. The application in cognition requires a brief presentation of basic elements of the formalism. We need to introduce four experiments $e_i$ each having only two possible outcomes, with $i$ running from 1 to 4. Furthermore, we need the expectation values $E_{ij}$, with $i, j$ each running from 1 to 4, for coincidence experiments, i.e. joint experiments of the type $e_i e_j$. The expectation values are defined as:

$$E_{ij} = P(o_i(u), o_j(u)) + P(o_i(d), o_j(d)) - P(o_i(u), o_j(d)) - P(o_i(d), o_j(u))$$

(2)

where originally $u$ stands for ‘up’ and $d$ for ‘down’, but actually they can stand for any outcomes out of the two possibilities. So ‘yes’ and ‘no’ are valid as well. We assume that the associated outcome values $o$ are either +1 (for $o(u)$) or -1 (for $o(d)$), the correlation outcome function $P$ are then +1 or -1 as well. Finally, from the assumption that the correlation $E_{ij}$ is local, Bell derived his inequalities:

$$|E_{13} - E_{14}| + |E_{23} + E_{24}| \leq 2$$

(3)

The essential point here is that when this inequality holds it indicates that the entity is not entangled; that is, the aspects of the entity exposed by the different experiments are not interdependent.

We illustrate its cognitive application as follows. Let there be a person—the respondent in our experiment—who is very acquainted with two cats Glimmer and Inkling which sometimes sport tinkling bells on their necklaces. Suppose now that we submit our respondent to some basic experiments, which are nothing more than asking for a reply to one of the questions,

- **D**: “Think of one of the cats, which one?” (Directive question)
- **S**: “Do you hear its bell ring?” (Sensory question)

while at the same time *one of the cats—which first were out of sight—can jump on the scene*. In the coincidence experiments we consider combinations of the basic ones just mentioned, with of course the possibility here that both cats appear in front of the respondent.

We can summarize the basic experiments and outcomes as follows:

| Experiments | Outcome up | Outcome down |
|-------------|------------|--------------|
| $e_1$       | “Glimmer appears” | “Glimmer!” | “Inkling!” |
| $e_2$       | “Inkling appears” | “ring” | “silence” |
| $e_3$       | “Glimmer appears” | “Inkling!” | “Glimmer!” |
| $e_4$       | “Inkling appears” | “ring” | “silence” |

The necessary conditions such that Bell inequalities are violated in this experiment are as follows. As a fact we must have *i*) both cats are wearing bells around their necks that day. We require the respondent *ii*) to abide the question experiments, *iii*) to have an activated categorical concept ‘cat’. Most crucially is however the feature of yielding to the coercive nature of internalized context *iv*) when Glimmer is seen, there is a change of mind state and the instance ‘Glimmer’ is reported, and when Inkling is seen ‘Inkling’ is reported. Then the outcome table for joint experiments in this configuration can be immediately obtained:
This leads to the correlation values: $E_{13} = -1$, $E_{14} = +1$, $E_{23} = +1$, $E_{24} = +1$. As a consequence we have the violation of the Bell inequality:

$$ |E_{13} - E_{14}| + |E_{23} + E_{24}| = +4$$

The reason that Bell inequalities are violated is that the respondent’s state of mind changes from activation of the abstract categorical concept ‘cat’, to activation of either ‘Glimmer’ or ‘Inkling’. We can thus view the state ‘cat’ as an entangled state of these two instances of it. The relationship between a concept and specific instances of it with a coercive context lead to the violation of the Bell inequality (Aerts et al. 2000). Thus we have evidence that this formalism reflects the underlying structure of concepts. In Aerts et al. (2000) we show that this result is obtained because of the presence of EPR-type correlations amongst the features or properties of concepts. The EPR nature of these correlations arises because of how concepts exist in states of potentiality, with the presence or absence of particular properties being determined in the process of the evoking or actualizing of the concept. In such situations, the mind handles concept combination in a quantum manner.

### 2.2 The Opinion Poll

We extend the foregoing analysis of entangled states of concepts to a cognitive setting where an individuals opinions are probed. Opinions may appear to be more stable than other sorts of conceptual entities that manifest as fleeting contents of conscious experience, such as impressions and ideas. However, even opinions can exhibit quantum-like contextuality. The formalism employed here for the identification and description of nonclassical contextuality in cognition can be applied experimentally, for instance, to the analysis of opinion poles (Aerts and Aerts 1994, Aerts and Aerts 1997). To explain this we first have to introduce the sphere-elastic model, which is a quantum model for the spin of a spin 1/2 quantum entity (Aerts 1986, Aerts 1987, Aerts 1993, Aerts 1995).

The states of the sphere-elastic model correspond to the points $P$ of the surface of a sphere, denoted $surf$, with center $O$ and radius 1. This means that we can denote a state $p_v$ by indicating the unit-vector $v$ corresponding to a point of $surf$ (see Fig. 1,a), and the collection of all possible states is $\Sigma = \{p_v \mid v \in surf\}$. For each point $u \in surf$, we introduce the context $e_u$ in the following way. We consider the diametrically opposite point $-u$, and install an elastic band of length 2, such that it is fixed with one of its end-points in $u$ and the other end-point in $-u$. The influence of the context $e_u$ is the following. Once the elastic is installed, the particle $P$ falls from its original place $v$ orthogonally onto the elastic, and sticks to it (Fig. 1,b). The elastic then breaks and the particle $P$, attached to one of the two pieces of the elastic (Fig. 1,c), moves to one of the two end-points $u$ or $-u$ (Fig. 1,d).

![Fig. 1 : A representation the sphere model. In (a) $P$ indicates a state $p_v$ in the point $v$, and the elastic corresponding to the context $e_u$ is installed between the two diametrically opposed points $u$ and $-u$. In (b) $P$ falls orthogonally onto the elastic and sticks to it. In (c) the elastic breaks and $P$ is pulled towards the point $u$, such that (d) it arrives at the point $u$.](image-url)
The state $p_v$ is changed by the context $e_u$ into one of the two states $p_u$ or $p_{-u}$. We make the hypothesis that the elastic band breaks uniformly, which means that the probabilities of state transition under influence of the contexts are:

$$
\mu(p_u, p_v, e_u) = \frac{1 + \cos \theta}{2} = \cos^2 \frac{\theta}{2} \quad (5)
$$

$$
\mu(p_{-u}, p_v, e_u) = \frac{1 - \cos \theta}{2} = \sin^2 \frac{\theta}{2} \quad (6)
$$

We can easily show that the sphere-elastic model is an entity of which the description is isomorphic to the quantum description of the spin of a spin 1/2 particle. Hence, speaking in the quantum jargon, the sphere-elastic model is a model for the spin of a spin 1/2 quantum particle. This means that we can describe it using the ordinary quantum formalism with a two-dimensional complex Hilbert space as the carrier for the set of states of the entity. It is easy to see on the sphere-elastic model the effect of the lack of knowledge on the contexts: this corresponds to the lack of knowledge of where the elastic will break during the interaction of the context with the state of the entity.

While in quantum mechanics the effect of context on the outcome is complete, or coercive, in cognition it is intermediate, or tempered. The tempered quantum-like contextuality can be described using the $\epsilon$-model (Aerts et al. 1993, Aerts and Durt 1994a, Aerts and Durt 1994b, Aerts et al. 1999d), which is an obvious generalization of the sphere elastic model. For the $\epsilon$-model we introduce for each context $e_u$ a parameter $\epsilon \in [0, 1]$, and make the hypothesis that the elastic corresponding to the context $e_u$ can now only break in an interval of length $2\epsilon$ around the middle point of the elastic. This means that for $\epsilon = 1$, the $\epsilon$-model reduces to the original sphere-elastic model, hence a pure quantum model, while for $\epsilon = 0$, each elastic can only break in its middle point, which means that we have a classical situation, of contexts that have a deterministic effect on the state of the entity under consideration (with exception of what happens in the middle point, but this is a classical type of indeterminism, as in the case of a classical unstable equilibrium position). For intermediate values of $\epsilon$, smaller than 1 and greater than 0, the $\epsilon$-model describes the tempered situation of quantum-like contextuality.

Let us describe shortly how the $\epsilon$-model can be used to model the quantum-like contextuality that appears in an opinion poll situation. We consider three different questions for the opinion poll.

- $u_1$: “Are you in favor of the use of nuclear energy?”
- $u_2$: “Do you think it would be a good idea to legalize soft-drugs?”
- $u_3$: “Do you think capitalism is better than social-democracy?”

We have chosen typical questions on which many respondents will not have predetermined opinions. Since the respondent has to respond with ‘yes’ or ‘no’, she or he, not having an opinion before the questioning will ‘form’ an opinion during the process of questioning itself. The $\epsilon$-model can be applied to fingerprint this effect of creation in the opinion+context interaction.

To simplify the situation, but without touching the essence, we make the following assumptions about the probabilities that are involved. We suppose that in all cases 50% of the persons have answered the question $u_1$ with ‘yes’, but only 15% of the persons had a predetermined opinion. This means that 70% of the persons formed their answer during the process of questioning. For simplicity we make the same assumptions for $u_2$ and $u_3$. We can represent this situation in the $\epsilon$-model as shown in Figure 2. We also make some assumptions of the way in which the different opinions related to the three questions influence each other. One can see how a person can be a strong proponent for the use of nuclear energy,
while having no predetermined opinion about the legalization of soft drugs (area 1 in Figure 3). Area (4) corresponds to a sample of persons that have predetermined opinion in favor of legalization of soft drugs and in favor of capitalism. For area (10) we have persons that have predetermined opinion against the legalization of soft drugs and against capitalism. All the 13 areas of Figure 3 can be described in such a simple way.

Deliberately we have chosen the different fractions of people in such a way that the conditional probabilities fit into the $\epsilon$-model for a value of $\epsilon = \sqrt{2}$. It can be proven mathematically that these values of conditional probabilities corresponding to these questions $u_1$, $u_2$ and $u_3$ can neither be fitted into a Kolmogorovian probability model nor into a pure quantum probability model (Aerts and Aerts 1994, Aerts and Aerts 1997, Aerts 1995). This indicates that only part of the properties are created during the process of testing, and that the contextual influence is therefore not fully coercive. If the respondents already had a fixed opinion about the question asked, the relevant probability model is Kolmogorovian and contextual influence is nihil.

This purely technical development of ‘interactive’ statics does imply, with the prerequisite of feasibility of standardized experimental set up as described here, the classical analysis of the opinion pole could be modified in order to identify opinion-context created effects.

### 2.3 The Generalized Liar Paradox

The abstracted quantum formalism is applied next to the modeling of a compound conceptual entity: in casu the generalized Liar Paradox. When one reasons logically over a Liar Paradox — e.g. “this sentence is false”—, with an initial conceptualization of it as ‘true’, then one experiences a build-up of cognitive dissonance as one recognizes by inference the inconsistency, which is followed by its conceptualization as ‘false’. After which the logical oscillatory process can be reiterated incessantly. We treat the conceptual entity of the Liar Paradox as a consistently testable configuration of truth properties expressed by sentences, and subject to our capacity of logical inference, which figures here as a coercive internal context.

These conceptual entities have the advantage of simplicity through their autonomy relative to extraneous input other than the personal dispositions of logical inference, e.g. contingent mind frame, emotional state, environment. And still the reflection on these entities is non-trivial with respect to the dynamics of self-reference.

The complete quantum formal description of the logical properties of the Liar Paradox conceptual entity contrasts with the description of the more generally encountered non-classical contextuality in the previous subsections. Next to the non-deterministic contextual collapse evolution, this model of the Liar Paradox allows the introduction of the continuous deterministic evolution by reasoning at any subsequent instance of time as well. We detail the coincidence of the effect of ‘inner’ logical context and reasoning in our concluding remarks.

Not only the simpler forms of the double Liar Paradox (Aerts et al. 1999a, Aerts et al. 1999b) but also $m$-sentence generalizations (Aerts et al. 2002a) allow a ‘complete’ quantum description. These particular entities—the generalized $m$-sentence Liar Paradoxes—are e.g. with $m = 5$:

\[
\begin{align*}
1 & \text{ sentence 3 is false} \\
2 & \text{ sentence 5 is false} \\
3 & \text{ sentence 2 is true}
\end{align*}
\]
The first number is the sentence pointer, then follows the corresponding proposition. The sentence-pointer numbers onto proposition-content numbers, and vice versa, link together the sentences in a closed ‘daisy chain’ configuration.

In the present context we merely expose the basic components of the formal model, while deleting most of the technical details (Aerts et al. 2002a). The model represents the state of the Liar Paradox entity by tensor products of state vectors of the sentence sub-spaces. Initially when the conceptual entity is in its potential state no truth value of the entity should be explicited. The truth and falsehood measurements on each sentence correspond to appropriately chosen projectors, while each next step of the dynamics is achieved by endorsing the inferred truth value resulting in the ensuing projection.

We have shown the model needs to be constructed in a $(2m)^m$ dimensional Hilbert space. However, the evolution of the given paradox constrains the dynamics to a mere $2m$-dimensional subspace. Given the choice of the truth and falsehood by hypothesis operators, for sentence $i$:

$$T_i = 1_1 \otimes ... 1_{i-1} \otimes T \otimes 1_{i+1}... \otimes 1_m$$

$$F_i = 1_1 \otimes ... 1_{i-1} \otimes F \otimes 1_{i+1}... \otimes 1_m$$

with

$$T = \begin{pmatrix} 0 & ... & 0 & 0 \\ ... & ... & ... & ... \\ 0 & ... & 1 & 0 \\ 0 & ... & 0 & 0 \end{pmatrix}_{2m \times 2m} \quad \text{and} \quad F = \begin{pmatrix} 0 & ... & 0 & 0 \\ ... & ... & ... & ... \\ 0 & ... & 0 & 0 \\ 0 & ... & 0 & 1 \end{pmatrix}_{2m \times 2m}$$

and given a particular $m$-sentence configuration, the corresponding reasoning sequence of $2m$ vectors can be constructed. All residual entries of the state vector are unequivocally fixed by a ‘filling’ procedure. With respect to this procedure, we obtained consistently interpretable state functions in terms of “truth/falsehood by reference” and “truth/falsehood by hypothesis”.

The first step of the reasoning on the Liar Paradox is represented by letting a projector corresponding to the chosen logical hypothesis, act on the initial superposition state $\Psi_0$, an equiponderate expression of all possible truth and falsehood states of component sentences. All subsequent steps of inferences-hypothesis when reasoning through the generalized Liar Paradox are discrete in their effect and can therefore be conceived as the operation of a step matrix. In the 5-Liar Paradox example, this discrete evolution submatrix is given by;

$$UD|_{\text{sub}} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}_{2m \times 2m}$$

The discreteness in the temporal process features explicitly in the logical reasoning. In the present model, the formalism of quantum mechanics on the other hand allows a continuous time parameter of evolution, allowing ‘intermediate’ states of reasoning on the Liar Paradox. This is done by extracting the phenomenological Hamiltonian—the infinitesimal time propagator of the system—through the application of Stone’s Theorem (see Figure 4).

In the full quantum model an initial hypothesis about one sentence engenders a time evolution of build up and collapse of logical states without end. Evidently any real world reasoning on the Liar paradox does not expose this compulsory machine-like continuation of the process.

Then what does the complete quantum description reflect? The autonomy of dynamics does not necessarily intend an ontological reality of the isolated conceptual entity. The latter could be inferred from too literal interpretation of the physical analogy with the obtained complete quantum description. On the other hand, the construction procedure of the step evolution reflects the cognitive person’s motivation by reasoning. The obtained dynamical model extends the ‘entity + context’ configuration which is static.
It is to be expected that more general conceptual entity with variable internal context and environmental context will not fit a complete quantum model, *a fortiori* one with a permanent propagator. We can interpret the descriptive coincidence of the ‘autonomous evolution of an ontological state’ and ‘coercion by inner logical context’ as the mechanism that lends the conceptual entity its *intentionality*. The conceptual entity ‘liar paradox’ refers over time and consistently to the inner context of logical inference, which is indeed, in this particular case, sufficient ground for recognizing its intentionality.

Finally, does the quantum model ‘solve’ the liar paradox? The feature of the non-classical contextuality reflects a distinct ontological status of the liar Paradox as a highly contextual entity when compared to conceptual entities with fixed classical truth or falsehood. The model therefore suggests that entities exist which are not subject to classical logical categories. When we try to understand the paradox through these classical categories, we are caught in a contradiction which, in this model, can be avoided by separation of incompatible values over time.

### 2.4 Thought as a Cognitive Form of Context-driven Actualization of Potential

As we mentioned in section 1.1 in quantum mechanics there are two fundamental forms of change: (1) continuous dynamical evolution in the absence of a measurement as described by the Schrödinger equation, and (2) quantum collapse when a measurement takes place. We propose that conscious experience similarly consists of two phases: (1) a continuous, predictable unfolding of states when the situation does not require a decision to be made and presents no outstanding ambiguity, and (2) context-driven collapse of the conceptual network when a decision must be made, or an ambiguous stimulus or situation resolved (Gabora 2001). In this second situation, we say that prior to the decision or resolution of ambiguity, the mind is in a state of potentiality. This is demonstrated schematically in Figure 5, which shows the potentiality state of the concept ‘diamond’ and the various different contexts that actualize to differing degrees the various different properties of ‘diamond’. Thus, for example, in the context of wanting to cut metal, the property ‘strength’ would be actualized, whereas in the context of wanting to marry someone, the property of being a symbol of love would be actualized. In the context of wanting to make money, the property ‘valuable’ is actualized. This context might pull in properties that are not generally associated with diamond, such as that a fake diamond can be constructed from plastic.
We can view situations as ‘testing’ or measuring the state of the conceptual network, revealing something of its hidden structure by showing how it collapses in response to a certain kind of context.

The collapse is demonstrated schematically in Figure 6. In this case, the context is that of wanting to make money. This context causes the concept ‘diamond’ to collapse from a superposition type state where many elements are potentially relevant to an ‘eigenstate’ (end state) where specific elements are actually relevant.

More broadly, both the continuous and discontinuous aspects of both conscious experience and quantum mechanics can be viewed as different means of instantiating Context Driven Actualization of Potential or CAP (Gabora 2001). Both of the continuous forms of change can be seen as the degenerate case of the discontinuous; with respect to the present context there is only one possible outcome, thus the outcome is determined. Since the potentiality is limited to one outcome there is no need for collapse; this one alternative just plays itself out.

It is interesting to note that two of the greatest stumbling blocks we now face in the formalization of conscious processes are extremely similar to the two main problems that have been identified in quantum mechanics. These problems can be referred to as the measurement problem and the entanglement problem.

- **The measurement problem**: Classical mechanics cannot describe situations where the measurement affects the state of an entity. For quantum mechanics the effect of change provoked by the measurement context on the state of the quantum entity is an essential ingredient of the theory. Similarly, when a concept (e.g. ‘diamond’) is evoked, it is evoked in some context, which affects its state. Therefore we need a formalism that incorporates context into description of entity.

- **The entanglement problem**: If in classical mechanics the composed entity consisting of two entities is described, this composed entity does not contain ‘new’ states, that cannot be reduced to product states. In quantum mechanics this is essentially the case, the composed entity consisting of two quantum entities, always contains ‘new’ states, the so called entangled states. Similarly, in creative process, concepts merge to generate something new with properties not reducible to the given constituents. Again, we need a formalism for describing how two entities combine into one and how new states are formed during this combination.

The similarity of the problems faced in the two domains should make it clear that the application of formalisms from one to the other was not pulled out of a hat. In fact, it is part of an effort to gather different forms of change—physical, chemical, geological, biological, cognitive, cultural, social, economic, and so forth—under one umbrella that identifies and formalizes their similarities and differences with respect to: (1) Degree of contextuality, (2) Degree of indeterminism due to context, (3) Degree of context dependence, and (4) Degree to which context-driven change is retained in future lineage(s). For present purposes it would not be appropriate to go into this general framework in detail. Our point is merely to show that the application of quantum formalisms to cognition is part of an interdisciplinary effort to develop a general framework for comparing and contrasting processes of change.
3 The Ontology of Cognition and Consciousness

We now outline the relationship of the developments of abstracted quantum formalism in cognition described here to the general question of the ontology of cognition and consciousness. This approach provides possibly a sufficient framework for a process of consciousness in systems with the capacity of conceptualization. We will juxtapose this with an amplification process model of consciousness (Gabora 2002), which is also consistent with the presented intrinsic contextuality formalism, but extends it with the additional assumption the consciousness is a fundamental primitive.

3.1 The ‘Reflected’ Internal Context Model of Consciousness

The abstracted quantum formalism we apply to cognition sustains a view of consciousness as an emergent, integrated process involving reflection of the entity via the context. We now reiterate the underlying arguments for and expand upon this view.

First we mention that the presence of quantum-like probability structure is not a sufficient indication for emergent processes in the conceptual entity-context interaction, i.e. with genuine new states resulting. The quantitative tool of statistics discerns classical and quantum-like probability structure, given the constraints of multiple states and multiple contexts, and does so depending on contextual uncertainty or interaction uncertainty. This leaves open a number of ontologically very different decisive factors for the quantum-like probability structure: ‘fuzzy’ context, fluctuating interaction, and indeed entity-integration. An interaction process giving rise to a non-Kolmogorovian probability structure can still have an underlying deterministic mechanics. Potentially this type of dynamics provides emergence but fails to procure the new state properties in terms of the created entity, and shunts the typical features of the quantum approach in which it is embedded (cf below). On the other hand an emergent integration process of conceptual entity+context will provide the necessary supplementary variability—it has new features—in the interaction to certainly break the Kolmogorov axioms, and thus lead to the quantum-like probability structures. But this effect is not unequivocally expressed since the quantum-like structure could already have been accounted for by mere contextual uncertainty. So we find in quantum-like probability structure no decisive indication of emergence.

More potent arguments come from the various examples of quantum-like features in cognitive processes using different quantum approaches (Bell, Schrödinger, $\epsilon$-model, CAP evolution); these do strongly suggest an emergent process. In quantum physics the non-deterministic change, i.e. the measurement collapse, has an extra-ordinary function: it relates the single individual entity to the coercive superstructure of the measuring context. The measurement collapse thus integrates hybrid components, formally it subjects the individual entities state function to the projection operator of the context. Physically although we understand that a connection between both is possible; the superstructure of the context is in fact a highly structured and organized compound system of elementary entities. The architecture and organization of the superstructure supplies it with supplementary functionality, the acquired ‘new’ properties. These emergent features can be understood in the manner of complex dynamics (a process widely accounted for in the literature, e.g. Scott (1995)). The von Neumann measurement collapse is therefore a generic procedure and formalism for the interaction of an entity and its relative superstructure. In cognition exactly the same relation is present between the conceptual entity and the cognitive context. The effective context in the cognitive scheme refers to a vast network of related concepts, memories, drives, stimuli and supplemented by the induced effects of the external environment, which constitute finally the present mind frame. The cognitive context shows to some extent the same hybrid relation with conceptual entities and has to some extent the same hybrid coercive effect. Only on these grounds we can identify the quantum-like features in our model with emergent processing.

Finally we try to relate parsimoniously the ‘intrinsic contextuality approach to cognition’ to a model of consciousness. From a conservative point of view, consciousness indicates a capacity ‘to perceive the being in the world’, as such reducing it to a cognitive state be it a centric one. To analyze this process in terms of the intrinsic contextuality approach. As a prerequisite, we let conceptualization be an acquired functionality of a context stimulated, feedback regulated, consistently referent, activity of the neural network. Attention is directed by a number of factors which are expressed in the internal context and the shift of this context, and provides the present cognitive experience. Their relatively hybrid nature integrates and procures the cognitive experience. To extend cognition to consciousness is then to direct the mental context to this proper context itself as subject. In common cognitive tasks, the
mental context is in an emergent relation to the conceptual entities, the refocusing in consciousness does not fit that common configuration. The practical ‘impossibility’ to subject the mental context to itself in an act of thought, indicates to some extent the evanescent nature of consciousness. In effect such an attempt probably converts to experiencing these components of the internal context that relate to proper existence.

The present elementary and speculative account for the process of consciousness is based directly on the intrinsic contextuality model in cognition and its translation of ‘proper reflection’. Its merit lies in the consistency with the ontology of precedent concept-context analysis and its parsimonious plainness. This account puts the essential dynamics of cognition and consciousness in the context directing capacity of the mind, a process not accounted for here.

3.2 The Amplification Model of Consciousness

Fundamental approaches bypass the problem of getting consciousness from non-conscious components by positing that consciousness is a universal primitive (Chalmers 1996, Dyson 1979, Feigl 1958/1979, Edelman and Tonini 2000, Foster 1989, Ghose and Aurobindo 1998, Griffin 1998, Hartshorne 1968, Lockwood 1989, Montero 2001, Nagel 1979, Russel 1926, Scott 1995, Stoljars 2001, Strawson 2000, Whitehead 1929). The double aspect theory of information, for example, holds that information has a phenomenal aspect (Chalmers 1996). How then do you get from phenomenal information to human consciousness? Our emphasis here on the interface between cognitive state and context might appear to suggest we believe that consciousness is uniquely associated with a particular kind of cognitive, or perhaps organic, structure. Our position, however, is consistent with the view that subjectivity of some very primitive form is ubiquitous, and that these structures merely locally amplify this subjectivity to give rise to what we generally view as full-fledged consciousness (Gabora 2000, Gabora 2002). It has been proposed that an entity is conscious to the extent it amplifies information, first by trapping and integrating it through closure, and second by maintaining dynamics at the edge of chaos through simultaneous processes of divergence and convergence. The origin of life through autocatalytic closure induced phase transitions in the degree to which information, and thus consciousness, is locally amplified. Another such phase transition may have taken place with the origin of an interconnected worldview through conceptual closure (Gabora 1998, Gabora 1999, Gabora 2002, Gabora and Deses 2002).

Conceptual closure is a process whereby memories become increasingly able to evoke one another mediated by the formation of abstract concepts, and eventually form an interconnected internal model of the world. It has been proposed that the capacity for conceptual closure came about approximately two million years ago when the human memory became large enough that episodes could afford to be more widely distributed, leading to the formation of a hierarchical network of abstract concepts. It has also been proposed that the capacity to alternate between focused and defocused modes of thought led to the capacity for conceptual closure at multiple hierarchical levels spanning different domains, and that this is what gave rise to the creative revolution of the Upper Paleolithic. Such a conceptual structure is able to creatively fine-tune plans, predictions, ideas and fantasies by evaluating situations in terms of past experiences, stories, and schemas, update opinions by considering them from different perspectives, and merge concepts to generate new concepts (which, liked quantum entities, often have properties not present in the constituents). All of these involve contextuality, revising one thing by viewing it in the context of another. Thus we move toward a picture in which closure, quantum structure, and contextuality are three intimately connected aspects of the process through which consciousness has become amplified.

At the deep level of formal theories, there is in fact a tight connection between quantum structure, and closure, which we discuss here briefly to give the reader a flavor of what this could imply for cognition. We frame this explanation using the lattice theoretic generalized quantum formalism, because it is here that this connection is revealed most clearly, though since the generalized quantum formalisms are translatable one into the other, the connection between closure and quantum structure could be analyzed using any one of them. In the lattice approach, the basic mathematical concepts for describing a physical entity, whether it be purely classical, purely quantum, or of the more general sort of structure referred to as quantum-like, are (1) a state space and (2) a set of properties. Axioms are defined on the state space and the set of properties, thus turning the whole into a state-property system (Aerts 1998, Aerts 2002). The entity under consideration is a classical entity if the lattice is Boolean, and pure quantum if the lattice is irreducible. It can be shown that if all the axioms are satisfied, the state-property system can be represented as the direct union over a set Ω of lattices formed out of the closed subspaces of a generalized
Hilbert space (Aerts 1983, Aerts et al. 2001, Aerts and Deses 2002).

The pure classical situation corresponds to the case where all these generalized Hilbert spaces have dimension one, hence this direct union reduces to the set $\Omega$ itself, which represents the phase space of the classical entity. So the general theory reduces to a pure classical mechanical theory over phase space. The pure quantum situation corresponds to the case where the set $\Omega$ is a singleton, hence the direct union reduces to one generalized Hilbert space, which for the case wherein the division ring is the complex number field, reduces to the Hilbert space of pure quantum mechanics. So the general theory reduces to a pure quantum mechanical theory over Hilbert space. The general quantum entity is described by a direct union over a phase space $\Omega$ of closed subspaces of generalized Hilbert spaces. It is $\Omega$ that enables one to describe the classical aspects of the entity, and the closed subspaces of generalized Hilbert spaces that allow for description of the quantum aspects.

It was recently proven that a state property system in the general quantum situation is categorically isomorphic with a closure structure on the state space (Aerts et al. 1999c, Van der Voorde 2000, Van Steirteghem 2000, Aerts et al. 2002b). The connected components of this closure structure are the pure quantum components of the generalized Hilbert spaces in the direct union, while the disconnected components are distributions over the phase space $\Omega$ that represents the classical aspects of the considered entity (Aerts at al. 2001, Aerts and Deses 2002). Thus it appears that, at a deep formal level, the connected components of a closure system are the quantum components of that system, and the disconnected components are the classical aspects. With respect to cognition, this tentatively suggests that closure of the conceptual network which causes it to become interconnected by hierarchical levels of increasingly abstract concepts, causes the mind to acquire more quantum-like structure.

4 Conclusions

The contextual nature of conscious experience suggests that in order to formally model it we should look to the domain of science where contextuality has been most seriously addressed: quantum mechanics. We know that conceptual entities always get evoked in some context. However the incorporation of context into the formal description of a conceptual entity is one of the biggest unsolved problems in cognitive science. Our formalism lends itself precisely to this problem.

An abstracted quantum mechanical representation of the entity-context interaction, with its hidden creation of new states, was adapted to the description of the conceptualization process for various different kinds of cognitive situations. We showed that Bell inequalities—the definitive test for quantum structure—are violated in the relationship between an abstract concept and instances or exemplars of that concept.

A formal $\epsilon$-model derived from quantum mechanics was adapted to the setting of an opinion pole, and identifies possible effects of intrinsic integration of opinion and questioning context, i.e. contextually-elicited opinions.

The full quantum model of the specific conceptual entity the ‘liar paradox’ provided a case for complete coercive contextuality and time evolution of a contextually subjected entity.

As in quantum mechanics, conscious experience consists of segments of dynamical evolution, which are not contextual and do not involve resolution of ambiguity or decision, and collapse events, which are context-dependent and involve a decision or the resolution of ambiguity. In both the quantum and the cognitive situation, both dynamical evolution and collapse are instances of context-driven actualization of potential. We believe that this actualizing of potential through contact with a context is the fundamental basis of evolution and change.

Our application of the general formalism still needs to be worked out in greater detail, and the predictions of the resulting ‘contextualized’ theory of cognition need to be empirically tested against other theories (such as prototype and exemplar theories of concepts), both through experiments with human subjects, and through simulations. We are also working on a description of the process by which pre-existing concepts or ideas merge to form new ones using the mathematics of entanglement and collapse.

In conclusion, we believe consciousness to be an emergent, integrated process involving reflection of the entity via the context, which can be fully described using an abstracted quantum formalism, and that amplification through closure may also be involved.
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