ABSTRACT
The environment near supermassive black holes (SMBHs) in galactic nuclei contains a large number of stars and compact objects. A fraction of these are likely to be members of binaries. Here we discuss the binary population of stellar black holes and neutron stars near SMBHs and focus on the secular evolution of such binaries, due to the perturbation by the SMBH. Binaries with highly inclined orbits with respect to their orbit around the SMBH are strongly affected by secular Kozai processes, which periodically change their eccentricities and inclinations (Kozai cycles). During periapsis approach, at the highest eccentricities during the Kozai cycles, gravitational wave (GW) emission becomes highly efficient. Some binaries in this environment can inspiral and coalesce at timescales much shorter than a Hubble time and much shorter than similar binaries that do not reside near an SMBH. The close environment of SMBHs could therefore serve as a catalyst for the inspiral and coalescence of binaries and strongly affect their orbital properties. Such compact binaries would be detectable as GW sources by the next generation of GW detectors (e.g., advanced-LIGO). Our analysis shows that ~0.5% of such nuclear merging binaries will enter the LIGO observational window while on orbits that are still very eccentric ($e \gtrsim 0.5$). The efficient GW analysis for such systems would therefore require the use of eccentric templates. We also find that binaries very close to the SMBH could evolve through a complex dynamical (non-secular) evolution, leading to emission of several GW pulses during only a few years (though these are likely to be rare). Finally, we note that the formation of close stellar binaries, X-ray binaries, and their merger products could be induced by similar secular processes, combined with tidal friction rather than GW emission as in the case of compact object binaries.

Key words: binaries: close – Galaxy: center – gravitational waves – stars: kinematics and dynamics

Online-only material: color figures

1. INTRODUCTION
The majority of stars reside in binary and higher multiplicity systems (Raghavan et al. 2010; Remage Evans 2011) and are observed to exist both in the field and in dense stellar environments such as globular clusters and galactic nuclei. In dense clusters, the dynamical evolution of binaries can be strongly affected by encounters with others stars. In the field, encounters with other stars are rare and do not affect the binary evolution. Such binaries may still dynamically evolve if they reside in triple (or higher multiplicity) systems. The inner binary in a triple can be affected by the perturbation of the third, outer companion and evolve through long-term secular processes. In particular, triples in which the relative inclination between the orbit of the inner binary and the outer binary is high can evolve through Kozai–Lidov oscillations (Kozai 1962; Lidov 1962) in which the eccentricity and inclination of the inner binary significantly change in a periodic/quasi-periodic manner.

In this study we explore the evolution of binaries in the environments of supermassive black holes (SMBHs), which are thought to reside at the center of most galaxies (e.g., Gebhardt et al. 2000, 2003); throughout this paper, we focus on galactic nuclei similar to the Galactic center (GC) of our Galaxy. Close to the center, where the gravitational potential is dominated by the mass of the SMBH, binaries are bound to the SMBH and effectively form a triple system, in which the binary orbit around the SMBH is the outer orbit of the triple. Binaries near an SMBH may therefore evolve both through gravitational scattering by other stars in the dense nuclear cluster and through secular evolution in the SMBH–stellar binary triple system (Antonini et al. 2010). Various types of binaries exist in such environments, including both main-sequence (MS) binaries, compact objects (COs) such as white dwarf (WD), neutron star (NS), or stellar black hole (BH) binaries, or mixed CO–MS binaries. In this paper we focus on CO binaries, their evolution into short periods, and the way in which they may become potentially observable gravitational wave (GW) sources for the advanced laser interferometer GW observatory (LIGO). Scattering with other stars is also accounted for, but only in the sense of the lifetime of binaries in the GC, i.e., we do not follow the evolution of binaries that are disrupted (evaporated) through encounters with other stars at timescales shorter than the relevant secular timescales (Kozai evolution).

In the following we begin with a discussion of binaries in the GC and their expected properties (Section 2) and then discuss the relevant timescales for the evolution of stars and binaries in galactic nuclei (Section 3). In Section 4 we discuss the secular evolution of binaries near an SMBH and their potential coalescence due to a combined Kozai-cycle–GW-loss mechanism; we then calculate the type and properties of the GW sources that form through this process. The secular evolution of other types of binaries is briefly discussed in Section 5. We summarize our results in Section 6.

2. BINARIES IN THE GALACTIC CENTER
At present, little is known about binaries in the GC. Massive binaries can be currently detected only by observations of stellar variability (Ott et al. 1999; DePoy et al. 2004; Martins et al. 2006; Rafelski et al. 2007), which can mainly detect
eclipsing or ellipsoidal variable binaries (Rafelski et al. 2007). The fraction of such binaries among O-stars outside the GC is 2%–11% (Garmany et al. 1980; Rafelski et al. 2007). Rafelski et al. (2007) conducted a variability survey of the central 5′ × 5′ of the GC and found one such binary (also observed by Ott et al. 1999; DePoy et al. 2004; Martins et al. 2006; Rafelski et al. 2007) among 15 massive stars (7%), a fraction consistent with the Galactic one. Other binary candidates were also found: a colliding wind binary (Rafelski et al. 2007) and another eclipsing binary (Peeples et al. 2007). Although these data are still insufficient for drawing strong conclusions about the total massive binary fraction in the GC, they do suggest that it is comparable to that observed outside the GC, among massive binaries. For reference, the massive binary fraction in the solar neighborhood is very high (>70%; Garmany et al. 1980; Mason et al. 1998; Kouwenhoven et al. 2007; Remage Evans 2011; Kobulnicky & Fryer 2007), and most massive binaries have semi-major axes of up to a few AU. Given our poor understanding of binary formation, it is unknown whether star formation in the unusual GC environment results in similar binary properties.

No observational data exist regarding low-mass binaries in the GC. Little is known about compact binaries in the GC; the observed overabundance of X-ray sources in the central pc (Muno et al. 2005) suggests that they are not rare, but their properties and origin are not known.

We focus on binaries very close to the SMBH, where secular Kozai evolution affects the binary dynamics. However, this population is continuously destroyed through various processes (evaporation, mergers, disruption). We therefore first discuss the binary population outside the central region near the SMBH (≥2 pc), which serves as a continuous source term repopulating the binaries in the central region. We then discuss the processes that can repopulate binaries in the central region, both from populations outside and from in situ formation of new binaries.

2.1. Binaries outside the Radius of Influence of the SMBH

In the following, we discuss the binary population outside the central region near the SMBH (≥2 pc). This binary population serves as a source term for binaries in the central region on which we focus, which are continuously destroyed through various processes.

The binary fraction and semi-major axis distribution of observed MS stars in the solar neighborhood have been analyzed in many binary surveys (e.g., Duquennoy & Mayor 1991; Lada 2006, and references therein). For COs the observational data are still lacking. However, many theoretical analyses have been done using stellar evolution and binary synthesis codes. Most of these studies focused on close binaries, usually in the context of cataclysmic variables, X-ray binaries, or GW sources (which are relatively short-lived and compose only a very small fraction of the binary population), and did not analyze the general distribution, which is needed for our analysis. Nevertheless, the general semi-major axis distribution of CO binaries has been found for isolated (i.e., not taking into account dynamical effects that could be important in dense stellar environments) WD–MS binaries (Willems & Kolb 2004) and BH binaries (Belczynski et al. 2004). For NSs, one can use the observed pulsar binary population as found in the ATNF pulsar catalog (available at http://www.atnf.csiro.au/research/pulsar/psrcat/; Manchester et al. 2005). The initial period distribution we consider for the different binary populations is shown in Figure 1. These distributions do not take into account dynamical evolution of binaries in the dense environment near the SMBH, mainly important in evaporating wide binaries (so-called soft binaries; Heggie 1975) due to distant stellar encounters. They also do not account for the long-term evolution of close CO binaries due to GW emission. We account for these evolutionary aspects using simplified methods.

The primordial binary fraction (not taking into account dynamical evolution such as evaporation of the binaries due to encounters), \( f_{\text{bin}} \), is different for the different binary types, MS, WD, NS, or BH binaries. It depends on both the progenitor binary fraction, \( f_{\text{bin}}^{\text{pro}} \) (i.e., BH and NS progenitors are high-mass stellar binaries, whereas WD binary progenitors have lower masses), and the fraction of surviving binaries, \( f_{\text{surv}} \), after going through their evolutionary stages.

The binary fraction of massive stars (\( f_{\text{bin}}^{\text{pro}} = 0.7 \)) is more than two times that of lower mass stars (\( f_{\text{bin}}^{\text{pro}} = 0.3 \); Kobulnicky & Fryer 2007; Lada 2006). Stellar evolution further changes these fractions. Thus, for obtaining \( f_{\text{bin}} \) we either take the observed binary fractions in the solar neighborhood (in the cases of MS and BS binaries) or use the theoretical models for the survivability of the different binaries (Willems & Kolb 2002, 2004; Belczynski et al. 2004) to find \( f_{\text{surv}} \) and multiply it by the progenitor binary fractions \( f_{\text{bin}}^{\text{pro}} \) (i.e., \( f_{\text{bin}} = f_{\text{pro}}^{\text{bin}} \times f_{\text{surv}} \)). We finally get \( f_{\text{bin}} = 0.7, 0.3, 0.3, 0.07, \) and 0.1 (for high- and low-mass MS stars, WDs, NSs, and BHs, respectively) as the evolution of lower mass stars (and WDs) changes the semi-major axis distribution of the binaries but does not result in their disruption, whereas the evolution of BHs and NSs results in most cases in the binary disruption. We adopt these initial binary fractions throughout our calculations. The fractions of CO binaries used here already account for the initial high binary fraction and the stellar evolution of these binaries. However, these binary fractions are also expected to be reduced due to dynamical encounters with other stars and through coalescence due to GW emission (for COs) and tidal friction. In the following
calculations (specifically in Section 4.3), we assume that the initial CO binary fractions ($f_{bin}$) in the GC are similar to those obtained here, where the effects due to dynamical encounters and Kozai evolution of compact binaries are accounted for in a simplified manner, as we discuss later on. Evolution due to tidal friction, which is important for MS stars, will be discussed elsewhere and is only shortly discussed here.

2.2. The Binary Fraction near an SMBH

As discussed above, the binary fraction in the GC is unknown, and the current observational data are insufficient to prove any meaningful constraints. Very few theoretical studies explored the role and evolution of binaries near an SMBH. Bahcall & Wolf (1977) briefly discussed the effect of binaries on the dynamics of stars in a cusp around an SMBH. They showed that their effect on the cusp dynamics is small, since the vast majority of these binaries are soft, and the total energy that they can transfer to the cusp stars is small with respect to their orbital energy (in the potential of the SMBH). Perets et al. (2008) suggested that binaries in the observed stellar disk in the GC can have a non-negligible effect on the evolution of the cold disk (see also Cuadra et al. 2008). Binaries can also be important for a fast deposition of stars close to an SMBH, through binary disruption (Hills 1988; Gould & Quillen 2003; Perets et al. 2007; Madigan et al. 2009; Antonini et al. 2011b). These studies did not discuss the binary population very close to the SMBH; the only study to explore the general population of binaries near an SMBH was done by Hopman (2009), even though this pioneering study accounted only for regular relaxation processes. A full study of the binary population near an SMBH is highly desirable. However, this is beyond the scope of this study, which mainly focuses on the secular evolution of binaries due to the perturbation by an SMBH. Here we assume a simple model in which the binary population in galactic nuclei follows their population in the field, but we also account for their evaporation through taking their effective lifetimes to be the evaporation timescale at the distance from the SMBH where they reside. We discuss several possible replenishments of binaries in Section 4.4.

3. TIMESCALES

In the dense stellar environment of a galactic nucleus, otherwise rare dynamical processes can take place and affect the evolution of the stellar population near the center. Here we compute the timescale associated with such processes in the GC and compare it to the Kozai timescale associated with the secular evolution of binaries due to the perturbations by a central SMBH.

We consider stellar-mass binaries with components of masses $m_0$ and $m_1$ orbiting an SMBH of mass $M_*$. We denote the eccentricities of the inner and outer binaries, respectively, as $e_1$ and $e_2$, and semi-major axes $a_1$ and $a_2$. We also define $g_1$ as the argument of periastron of the inner binary relative to the line of the descending node, and $I$ as the mutual orbital inclination of the inner binary with respect to the outer one.

The mass density near the SMBH was modeled using the broken power-law density profile:

$$
\rho(r) = \rho_0 \left( \frac{r}{r_0} \right)^{-\gamma} \left[ 1 + \left( \frac{r}{r_0} \right)^2 \right]^{(\gamma-1.8)/2},
$$

with $\gamma$ the inner slope index and $r_0 = 0.5$ pc. Setting $\rho_0 = 5.2 \times 10^5 M_\odot$ pc$^{-3}$, this gives a good fit to the observed space density at the GC outside the core, normalized at 1 pc (Schödel et al. 2009). For the slope of the inner density profile we assumed two different values: $\gamma = 0.5$, representative of the observed distribution of stars at the GC (Buchholz et al. 2009; Do et al. 2009; Bartko et al. 2010), and $\gamma = 1.8$, corresponding approximately to a nearly relaxed configuration of stars near a dominating point-mass potential (Alexander 2005). We set $M_* = 4 \times 10^6 M_\odot$, similar to the mass of the SMBH at the GC (Ghez et al. 2008; Gillessen et al. 2009).

Kozai timescale. The Kozai timescale can be written in terms of the masses of the three bodies, the eccentricity of the outer binary, and their semi-major axes as

$$
T_K \approx \frac{2 P_1^2}{3 \pi P_1^2} (1 - e_2^2)^{3/2} = 1.1 \times 10^4 \text{yr} \left( \frac{a_2}{0.1 \text{pc}} \right)^3 
\times \left( \frac{M_*}{2 \times 10^6 M_\odot} \right)^{-1} \left( \frac{M_b}{2 M_\odot} \right)^{1/2} \left( \frac{a_1}{1 \text{AU}} \right)^{-3/2}
\times (1 - e_2^2)^{3/2},
$$

where $P_1$ is the period of the inner binary, $P_2$ is the period of the outer orbit, and $M_b = m_0 + m_1$.

Extra sources of periastron precession. Kozai cycles can be suppressed by additional sources of apsidal precession. The most relevant processes to consider are relativistic precession and, in the case of stellar binaries, precession due to the tidal bulge raised on each of the inner binary’s components. Writing the timescale of relativistic precession as

$$
T_{GR} = \frac{a_1^{5/2} c^2 (1 - e_2^2)}{3 G^{3/2} M_b^{5/2}},
$$

$$
= 1.9 \times 10^6 \text{yr} \left( \frac{M_b}{2 M_\odot} \right)^{-3/2} \left( \frac{a_1}{1 \text{AU}} \right)^{5/2} (1 - e_2^2),
$$

with $G$ the gravitational constant and $c$ the speed of light, it can be shown that general relativistic precession in the inner binary stops the Kozai oscillations when (Hollywood & Melia 1997; Blaes et al. 2002)

$$
\frac{a_2}{a_1} < 4200 \left( \frac{a_1}{1 \text{AU}} \right)^{1/3} \left( \frac{M_b}{2 M_\odot} \right)^{-2/3}
\times \left( \frac{M_*}{4 \times 10^6 M_\odot} \right)^{1/3} \left( \frac{1 - e_2^2}{1 - e_1^2} \right)^{1/2}.
$$

Tidal bulges always tend to promote periastron precession and therefore suppress Kozai cycles. The timescale of periastron precession due to non-dissipative tides is (Kiseleva et al. 1998; Eggleton & Kiseleva-Eggleton 2001)

$$
T_{Tide} = \frac{8 a_1^{13/2} (1 - e_2^2)^5}{15 (GM_b)^{1/2} (8 + 16 e_2^2 + e_2^4)}
\times \left[ \frac{m_1 k_0 R_0^5 + m_0 k_1 R_1^5}{m_0} \right]^{-1},
$$

with $R_0$ and $R_1$ the stellar radii and $k_0$ and $k_1$ the tidal Love numbers. Assuming equal objects in the binary and $k_0 = k_1 = 0.01$, we find

$$
T_{Tide} = 1.3 \times 10^{12} \text{yr} \left( \frac{a_1}{1 \text{AU}} \right)^{13/2} \left( \frac{M_b}{2 M_\odot} \right)^{-1/2} \left( \frac{R_0}{1 R_\odot} \right)^{-5}.
$$

(6)
\[ \frac{(1 - e_1^2)^5}{8 + 12e_1^2 + e_1^4}, \]  
where the stellar radii are obtained using the expression (Hansen et al. 2004)

\[ R = R_\odot \left( \frac{M}{M_\odot} \right)^{0.75}. \]

Comparing Equations (3) and (6), it can be seen that, in general, relativistic precession is initially the most rapid mechanism inducing apsidal motion, but, for very large eccentricities, when, for instance, the system approaches the maximum of a Kozai cycle, one expects tides to become important to the evolution. Consequently, for stellar binaries with relatively large semi-major axes, very large eccentricities (eventually resulting in mass transfer or even collisions) could be avoided due to apsidal precession induced by tidal bulges.

**Binary evaporation.** In a dense environment, binaries may evaporate due to dynamical interactions with field stars if

\[ |E|/(M_\odot \sigma^2) \lesssim 1, \]

with \( E \) the internal orbital energy of the binary and \( \sigma \) the one-dimensional (1D) velocity dispersion of the stellar background. The evaporation timescale is given by (Binney & Tremaine 1987)

\[ T_{EV} = 3.2 \times 10^7 \text{ yr} \left( \frac{M_\odot}{2 M_\odot} \right) \left( \frac{\sigma}{306 \text{ km s}^{-1}} \right)^{-1} \left( \frac{m}{M_\odot} \right)^{-1} \times \left( \frac{a_1}{1 \text{ AU}} \right)^{-1} \left( \frac{\ln \Lambda}{15} \right)^{-1} \left( \frac{\rho}{2.1 \times 10^6 M_\odot \text{ pc}^{-3}} \right)^{-1}, \]

where \( \ln \Lambda \) is the Coulomb logarithm, \( \sigma \) and \( \rho \) are the 1D velocity dispersion and mass density, respectively, and \( m \) is the mass of a typical object in the system, assumed to be solar-mass stars in what follows. The velocity dispersion, \( \sigma \), is calculated from the Jeans equation:

\[ \rho(r) \sigma(r)^2 = G \int r^{-2} d'r^{-2} (M_* + M_*(< r')) \rho(r'), \]

with \( M_*(< r) \) the total mass in stars within \( r \).

**Two-body relaxation.** Assuming equal-mass stars and an isotropic velocity distribution, the local two-body (non-resonant) relaxation time is defined as (Spitzer 1987)

\[ T_{NR} = 1.6 \times 10^{10} \text{ yr} \left( \frac{\sigma}{306 \text{ km s}^{-1}} \right)^3 \left( \frac{m}{M_\odot} \right)^{-1} \times \left( \frac{\ln \Lambda}{15} \right)^{-1} \left( \frac{\rho}{2.1 \times 10^6 M_\odot \text{ pc}^{-3}} \right)^{-1}. \]

Setting \( \gamma = 0.5 \) in the density model of Equation (1), this time is somewhat larger than the age of the Galaxy (\( \sim 10^{10} \text{ yr} \)) when computed at the radius of influence (\( \sim 3 \text{ pc} \)) of the SMBH (Merritt 2010).

**Resonant relaxation.** In the dense stellar environment near an SMBH, as long as the relativistic precession timescale is much longer than the orbital period, the mechanism that dominates the scattering of stars onto high-eccentricity orbits is resonant relaxation. Because in the potential of a point mass the orbits are fixed ellipses, perturbations on a test particle are not random but correlated. The residual torque \( |Q| \approx \sqrt{N} G m/r \), exerted by the \( N \) randomly oriented, orbit-averaged mass distributions of the surrounding stars, induces coherent changes in angular momentum \( \Delta L = Q \tau \) on timescales \( \tau \lesssim T_{coh} \), where the coherent time \( T_{coh} \) is fixed by the mechanism that most rapidly causes the orbits to precess (e.g., mass precession, relativistic precession). The angular momentum relaxation time associated with resonant relaxation is

\[ T_{RR} \approx \left( \frac{L_c}{\Delta L_{coh}} \right)^2 T_{coh}, \]

where \( L_c \equiv \sqrt{G M_* a} \) is the angular momentum of an orbit with radius \( r \approx a \) and \( |\Delta L_{coh}| \sim |Q T_{coh}| \) is the accumulated change over the coherence time. Assuming that the precession is determined by the mean field of stars (i.e., mass precession), the angular momentum relaxation time is (Rauch & Tremaine 1996)

\[ T_{RR} \approx 9.2 \times 10^8 \text{ yr} \left( \frac{M_*}{4 \times 10^6 M_\odot} \right)^{1/2} \left( \frac{a_2}{0.1 \text{ pc}} \right)^{3/2} \left( \frac{m}{M_\odot} \right)^{-1}. \]

At small radii relativistic precession becomes efficient at suppressing resonant relaxation and the gravitational perturbations are dominated by classical (two-body) non-resonant relaxation. The value of the angular momentum (the Schwarzschild barrier) at which this transition occurs is (Merritt et al. 2011)

\[ (1 - e_2^2)_{SB} \approx 6 \times 10^{-3} \left( \frac{C_{SB}}{0.7} \right)^2 \left( \frac{a_2}{0.1 \text{ pc}} \right)^{-2} \times \left( \frac{M_\odot}{4 \times 10^6 M_\odot} \right)^4 \left( \frac{m}{M_\odot} \right)^{-2} \left( \frac{N}{6000} \right)^{-1}, \]

where \( N \) is the number of stars within radius \( a_2 \) and \( C_{SB} \) is a constant of order unity.

**Vector resonant relaxation.** An additional fundamental process in changing the external orbit of the binary is vector resonant relaxation (VRR). The external orbital plane is randomized non-coherently on the VRR timescale (Hopman & Alexander 2006a):

\[ T_{RR,v} \approx 7.6 \times 10^6 \left( \frac{M_*}{4 \times 10^6 M_\odot} \right)^{1/2} \left( \frac{m}{M_\odot} \right)^{-1} \times \left( \frac{a_2}{0.1 \text{ pc}} \right)^{3/2} \left( \frac{N}{6000} \right)^{-1/2} \text{ yr}. \]

VRR changes the direction but not the magnitude of the external orbit angular momentum. The randomization process of the external orbital plane can possibly produce variations in the mutual inclination of the binary orbit with respect to its orbit around the SMBH.

In Figure 2, these different timescales are compared for the case of a circular orbit and for an eccentric orbit with \( e_2 = 0.95 \). In the latter case, the corresponding timescales are obtained by multiplying the quantities of Equations (12), (13), and (15) by a factor (1 - \( e_2 \)). To compensate for the fact that when the external eccentricity is large the binary spends most of the time at apoapsis, we divided the evaporation time given in Equation (10)
by a factor $\sqrt{1-e_i^2}$. At the radii of interest, for either the cusp or the core model, the Kozai timescale is much shorter than any other timescale associated with perturbations from dynamical interaction with field stars. VRR is the most rapid mechanism associated with gravitational interaction with field stars. For $a_2 \lesssim 0.1$ pc, since $T_{RR,V} \gg T_K$, VRR might lead to an increase of the number of binaries undergoing Kozai resonances with the SMBH: large inclined binaries will have already experienced multiple Kozai cycles and eventually merge before VRR could change appreciably their orbits, while low inclined binaries could move to high inclinations via VRR and enter the Kozai regime. The degree to which the binary dynamics depend on VRR and other dynamical processes is a complicated topic that would most likely require direct $N$-body simulations describing dense clusters of stars around SMBHs. In what follows, as a first approximation, we neglect VRR and use the time for binary evaporation ($T_{EV}$) as the timescale of reference to define the parameter space where our integrations are valid.

Figure 2: Timescales at the Galactic center (dashed lines) compared to the Kozai period ($T_K$; solid lines) for $(10+10) M_{\odot}$ (red lines) and $(1+1) M_{\odot}$ (black lines) binaries with $a_1 = 1$ AU. Rectangular regions show the age and radial extent of the S-stars and disk of WR/O-stars. Dotted lines give the radii at which Kozai cycles are fully suppressed by relativistic precession in the inner binary. Open circles indicate the radius where the timescale of periapsis precession due to tidal bulges with initially highly inclined orbits would have performed such that the closest approach between the stars is twice the sum of their radii, i.e., $e_2 \equiv (\sqrt{2} - 1) \times (R_0 + R_1)/a_2$. The lines corresponding to $T_{RR}$ are truncated where the condition (14) is satisfied. In the left and right panels we set $e_2 = 0.1$ to solve for $a_2$ in Equation (14). For shallow cusp models, one finds that the Schwarzschild barrier suppresses resonant relaxation processes at radii corresponding to the S-stars cluster.

4. GRAVITATIONAL WAVE SOURCES

In the following, we address the possibility that the merger time of compact binaries residing in galactic nuclei can be significantly accelerated due to their interaction with a central SMBH. Some systems that otherwise would have no chance of merging before being disrupted by gravitational encounters with background stars (or stellar remnants) will actually merge due to the combined effect of Kozai cycles and GW loss. For an isolated binary, the GW merger timescale can be approximated as (Peters 1964)

$$ T_0 \approx \frac{3}{85} \frac{a_1}{c} \left( \frac{a_1^3 c^6}{G^3 \mu^2 M_b} \right) \left( 1 - e_i^2 \right)^{7/2} \times \left( \frac{2 \times 10^{10} M_\odot}{m_0 m_1 M_b} \right) \frac{a_1}{0.1 \text{ AU}} \left( 1 - e_i^2 \right)^{7/2}. $$

Given the strong dependence of $T_0$ on the eccentricity, the induced oscillations in the binary orbital eccentricity via secular Kozai processes can clearly decrease dramatically the merger timescale (e.g., Thompson 2011). The GW signal emitted by such compact binaries will be completely dominated by

In this section, we have assumed that the perturbers are solar-mass objects, but we note here that gravitational scattering can
repeated pulses emitted during periapsis passages and at high eccentricities where the GW frequency is maximized (Gould 2011).

4.1. N-body Integrations

In this section, we describe high-accuracy N-body simulations that were used to study the evolution of a limited selection of systems representing binaries of BHs orbiting an SMBH. These simulations were carried out using the ARCHAIN integrator (Mikkola & Merritt 2008), which includes PN corrections to that were used to study the evolution of a limited selection circular and equal-mass binaries with components of masses \(10 M_\odot\). The periapsis distance of the external binary was \(3.3 \times r_h\), and we fixed the initial inclination of the inner orbits with respect to the external orbit around the SMBH at \(I = 88^\circ\). We then varied \(e_1\) and \(a_1\) in order to study the dependence of the merger timescale on the latter quantities. The choice of a large inclination was also motivated by the short merger timescale of such systems allowing for an efficient computation of their dynamical evolution.

Figure 3 gives the time evolution of the semi-major axis, eccentricity, and GW frequency (\(f_{GW}\)) of the inner binaries. The peak GW frequency was approximated by (Wen 2003)

\[
f_{GW} = \frac{\sqrt{GM_b}}{\pi} \frac{1}{\left[ a_1 \left( 1 - \frac{e_1^2}{1+e_1} \right)^{1/2} \right]^{1.1954}}, \tag{18}
\]

with semi-major axis and eccentricity obtained from the relative radial expressions given in Equations (3.6a) and (3.6b) of Damour & Deruelle (1985).

The oscillations in the binary inner eccentricity and GW frequency induced by the SMBH perturbations speed up the merger time by many orders of magnitude. For example, the nominal GW merger timescale for the cases shown in the middle panels of the figure without SMBH is \(\sim 10^{10}\) yr, while it is just a few years in our simulations. The oscillations in eccentricity and inclination observed in the computed orbits are reminiscent of the “Kozai cycles” generally discussed in the context of hierarchical triple systems (e.g., Harrington 1968; Wu et al. 2007; Naoz et al. 2011b; Prodan & Murray 2012). However, the distance of closest approach to the SMBH for the orbits of Figure 3 is too short (about three times the binary tidal disruption radius) for the system to be treated exactly as hierarchical. In other words, the strong interaction at periapsis causes high-order terms (beyond the octupole order) in the equations of motion to become important to the evolution. The amplitude of the eccentricity oscillations changes essentially at each cycle, eventually making the inner binary periapsis distance small enough that efficient GW loss takes place. Interestingly, we found that the changes in eccentricity, for initially high eccentric orbits (\(e_2 \gtrsim 0.5\)), always occur very rapidly during the closest approach of the binary to the SMBH rather than in a smooth-like way along the entire orbit, as would be the case if we had used the secular perturbation equations instead of direct N-body integrations. This behavior is evident in Figure 3, but see also Figure 7 in Antonini et al. (2010).

We note also that in some cases (e.g., upper left panel of Figure 3) it is possible that the binary first enters a regime in which GW loss starts to dominate the evolution, suddenly causing a drop of the binary semi-major axis; during this phase, the binary is then brought back to a small-eccentricity configuration where GW loss becomes again negligible with the subsequent evolution occurring at approximately constant semi-major axis.

Figure 4 displays the evolution of a stellar BH binary with \(a_1 = 0.1\) AU, inclination \(I = 88^\circ\), and \(g_1 = 0^\circ\). The binary is on a circular orbit of radius 20 AU from the central BH. During the oscillations in eccentricities, at the maximum of a Kozai cycle, the binary GW frequency is such that the system could be a detectable source for LISA-like GW detectors. The signal from such sources in a LISA-like detector would result in repeated pulses emitted during periapsis approach. The system merges in only \(\sim 20\) yr, after many eccentricity oscillations have occurred, and enters the LIGO observational window (\(\gtrsim 10\) Hz) with a very large eccentricity (\(e_1 \approx 0.99995\) before the fast orbital circularization due to GW loss begins. Wen (2003) showed that, in the context of stellar BH triples that might form in the core of globular clusters, for some particular configurations it is possible that the inner binary GW frequency enters the LIGO observational band before the maximum eccentricity expected for the Kozai oscillations is reached. In these cases, the eccentricities can remain significantly large (\(\sim 0.9\)) at \(f_{GW} \sim 10\) Hz. We stress that the system shown in Figure 4 is qualitatively different from those discussed in this previous work: only after multiple periodic oscillations in the orbital elements have already occurred does the binary reach very high eccentricities and enter the LIGO frequency band. Such a situation occurs before gravitational radiation becomes important, allowing the eccentricity to be so large at such high frequencies.

4.2. Approximate Method

Given the significant computational resources per run made necessary by the high accuracy of the N-body integrations presented in the previous section, no attempt was made to systematically explore the large parameter space of the three-body interactions. In order to perform such a broad exploration, we use an approximate method that is detailed below.

If the external orbit of the inner binary is wide enough, i.e., the gravitational perturbations from the SMBH on the binary are weak, the system can be regarded as a hierarchical triple and it is possible to treat the dynamics of the entire system as if it consists of an inner binary of point masses \(m_0\) and \(m_1\) and an external binary of masses \(M_b\) and \(m_1 + M_\star\). We define the angular momenta \(G_1\) and \(G_2\) of the inner and outer binaries and total angular momentum \(H = G_1 + G_2\):

\[
G_1 = m_0 m_1 \left[ \frac{G a_1 (1 - e_1^2)}{M_b} \right]^{1/2} \tag{19}
\]

and

\[
G_2 = M_b M_\star \left[ \frac{G a_2 (1 - e_2^2)}{M_b + M_\star} \right]^{1/2}. \tag{20}
\]

The quadrupole-level secular perturbation equations describing the evolution of the inner binary orbital elements are (Ford et al. 2000)

\[
\frac{d a_1}{d t} = \frac{-64 G^3 m_0 m_1 M_b}{5 c^5 a_1^7 (1 - e_1^2)^{7/2}} \left( 1 + \frac{73}{24} e_1^2 + \frac{37}{96} e_1^4 \right), \tag{21}
\]
Figure 3. Time evolution of the gravitational wave frequency ($f_{\text{GW}}$), eccentricity ($e_1$), and semi-major axis of binary black holes orbiting an SMBH. In these integrations, the distance of closest approach of the binary to the SMBH is $r_{\text{per}} = 3.3 \times r_{\text{bt}}$, with $r_{\text{bt}}$ the tidal disruption radius of the binary. We set the initial parameters to $e_1 = 0$, $m_0 = m_1 = 10 \, M_\odot$, $I = 88^\circ$, and $g_1 = 0^\circ$ and adopt different values for the orbital eccentricity $e_2$ and inner semi-major axis $a_1$. The nominal merger timescales of the binaries in isolation are $\sim 10^{14}$ yr ($a_1 = 1\,\text{AU}$; bottom right panel), $\sim 10^{13}$ yr ($a_1 = 0.5\,\text{AU}$; two bottom leftmost panels), $\sim 10^{10}$ yr ($a_1 = 0.1\,\text{AU}$; middle panels), and $\sim 10^9$ yr ($a_1 = 0.05\,\text{AU}$; top panels). The presence of an external perturber, the SMBH in this case, causes periodic variation of the inner binary eccentricity, greatly reducing the merger timescales of these systems.

\[
\frac{de_1}{dt} = 30K e_1^2 (1 - e_1^2) (1 - \cos^2 I) \sin 2g_1 - \frac{304G^3 m_0 m_1 M_{\text{bh}} e_1}{15c^3 a_1^4 (1 - e_1^2)^{5/2}} \left(1 + \frac{121}{304} e_1^2\right) , \tag{22}
\]

\[
\frac{dg_1}{dt} = 6K \left(\frac{1}{G_1} \left[4 \cos^2 I + (5 \cos 2g_1 - 1)(1 - e_1^2 - \cos^2 I)\right] + \frac{\cos I}{G_2} \left[2 + e_1^2(3 - 5 \cos 2g_1)\right]\right) + \frac{3}{c^2 a_1 (1 - e_1^2)} \left[\frac{GM_{\text{bh}}}{a_1}\right]^{3/2} , \tag{23}
\]
Equations (22)–(25) can be considered as describing the interaction between two weighted ellipses instead of point masses in orbits and are accurate as long as the energy of the system and its angular momentum are approximately conserved quantities within each Kozai cycle. These equations include the effects of relativistic precession and energy loss due to GW emission, and they can be integrated numerically much more rapidly than the full equations of motion. They are used here to investigate the dynamical evolution of compact binaries near massive BHs.

We set the mass of the binary components similar to that typical of stellar BHs: $m_0 = m_1 = 10 M_{\odot}$ (Woosley et al. 2002). In this case, Equations (22)–(24) are equivalent to the octupole-level secular perturbation equations given in Blaes et al. (2002). In fact, from Equation (19) of Blaes et al. (2002) we see that when $m_0 = m_1$ the octupole terms vanish. We also run integrations for unequal-mass binaries and including octupole terms to see the effect that higher order terms have on the binary evolution. These integrations gave results very similar to those obtained using the quadrupole approximation. The fact that the octupole terms have only a small effect on the binary dynamics is easily explained if we parameterize the relative contribution of the octupole-level approximation using the parameter (Naoz et al. 2011a)

$$\epsilon_M = \left( \frac{m_0 - m_1}{m_0 + m_1} \right) \left( \frac{a_1}{a_2} \right) \frac{e_2}{1 - e_2^2}. \quad (27)$$

In our systems $a_2 \gg a_1$ and even for very large eccentricities ($e_2 \sim 0.9$) the contribution of the octupole terms is negligible.

In Figure 5 we give the merger time, $T_M$, as a function of the semi-major axis of the outer orbit; this last quantity is given in units of the initial binary semi-major axis ($a_1$). We set $e_1 = e_2 = 0.1$, $a_1 = (0.1, 0.05, 0.01)$ AU, and we consider several values of the inclination angle. The merger time is compared to the evaporation time ($T_{\text{EV}}$) of the binary systems that we computed using Equation (10). The time for evaporation in a shallow density profile is greatly increased with respect to the cusp model and is independent of radius since, close to the SMBH, both $\rho$ and $\sigma$ in Equation (10) go like $\sim r^{-0.5}$.

Highly inclined binaries ($I \geq 70^\circ$) that lie close to the central BH inspiral and coalesce over timescales much shorter than a Hubble time and typically shorter than their evaporation timescale on such environments. At large distances from the SMBH due to GW loss. Here we used $m_0 = m_1 = 10 M_{\odot}$, $e_1 = e_2 = 0.1$, and $g_1 = 0^\circ$. Black filled circles were obtained by direct three-body integrations including PN corrections up to order 2.5.

(A color version of this figure is available in the online journal.)
SMBH, relativistic precession becomes efficient at suppressing Kozai cycles and the merger time approaches its value when the binary evolves in isolation. Note that, since the perturber in our simulations is much more massive than the inner binary, there is essentially no difference between retrograde and prograde simulations is much more massive than the inner binary, there is essentially no difference between retrograde and prograde.

Filled black circles in the figure were obtained by direct integrations of the three-body system carried out using the ARCHAIN integrator. The agreement between direct integrations and the results obtained by using the secular perturbation equations is indeed very good.

Dashed lines in Figure 5 give the time required for the stellar BHs to spiral into the central BH. This timescale was approximated by considering the compact binary as a single point mass and by using Peters (1964) formula

\[ T_0, \ast \approx 1.8 \times 10^{12} \text{yr} \left( \frac{20 M_\odot}{M_b} \right) \left( \frac{4 \times 10^6 M_\odot}{M_\ast} \right)^2 \times \left( \frac{a_2}{\text{mpc}} \right)^4 (1 - e_2^2)^{3/2} \] (28)

For small inclinations ($\lesssim 70^\circ$), within $a_2/a_1 \lesssim 700$, the merger time of the binary into the SMBH can be shorter than the time required for the stellar BHs to merge due to GW loss. However, the compact binary will be broken apart by tidal forces if its center of mass approaches the SMBH within the tidal breakup radius, $r_{\text{bt}}$, which is much larger than the Schwarzschild radius of the central BH: $r_{\text{sc}} \approx 0.04 \text{ AU}$. If the compact binary does not merge due to the perturbations by the SMBH, it will be disrupted before the two stellar BHs separately inspiral and merge with the central object. In this case, each of the two inspiral events cannot be treated as an isolated two-body problem and the mutual perturbations between the stellar BHs could produce a GW signal in a LISA-like detector that would differ significantly from the idealized waveform obtained in the case of an isolated extreme-mass ratio inspiral (Amaro-Seoane et al. 2012).

It is interesting to explore the effect of changing outer and inner orbital eccentricities on the binary merger time. In the upper panel of Figure 6, each curve corresponds to a fixed periapsis distance and inclination $I = 85^\circ$. As expected from Equation (2), larger external eccentricities lead initially to faster mergers, but, as the orbital period of the external orbit becomes comparable to the timescale for relativistic precession in the inner binary, the merger time starts to increase rapidly with the orbital radius, or apoapsis, of the external orbit. For $a_1 = 0.1 \text{ AU}$, setting $r_{\text{per}} \approx 20 \text{ AU}$, the binary merger time is $10^{-3} \times T_0$ when $r_{\text{apo}} \approx 0.01 \text{ pc}$. This is also an estimate of the limiting galactocentric radius within which we expect Kozai oscillations to significantly affect the dynamical evolution of the BH binaries. The lower panel of the same figure gives $T_M$ as a function of $e_1$ for $I = 85^\circ$, $a_1 = 0.05 \text{ AU}$, and $a_2 = 130, 100$, and $70 \text{ AU}$. Note that $T_0$ is also a function of $e_1$; for eccentricity larger than about $e_1 \gtrsim 0.7$ the effect of the Kozai resonance on the merger time becomes progressively less important since $e_1$ approaches (or exceeds) the value of the maximum eccentricity that would be attained by the binary via Kozai processes, which is mainly fixed by the initial inclination ($e_{\text{max}} = \sqrt{1 - \sin^2 I}$, neglecting additional sources of apsidal precession).

### 4.3. Merger Fraction and Eccentricity Distribution

The number of GW sources from binary mergers in galactic nuclei, $N_{\text{GW}}$, is the number of binaries residing in some region close to the SMBH where they can efficiently merge ($r < R_{\text{merge}}$):

\[ N_{\text{GW}} = N_e (r < R_{\text{merge}}) \times f_{\text{bin}} \times f_{\text{bin}}(r < a_{\text{merge}}) f_{\text{merge}}(i > |90 - i_{\text{merge}}|), \] (29)

where $N_e (r < R_{\text{merge}})$ is the number of stars in the enclosed region, $f_{\text{bin}}$ is the binary fraction for the given stellar type discussed earlier, $f_{\text{bin}}(r < a_{\text{merge}})$ is the fraction of close enough binaries ($a < a_{\text{merge}}$) that survive in this region, and $f_{\text{merge}}$ is the fraction of binaries for which Kozai evolution leads to merger. In the following we estimate the rate of binary mergers and the properties of the coalescing binaries, which will determine their GW signal characteristics. In order to do so, we use a Monte Carlo approach that accounts for all the relevant parameters in Equation (29), through random sampling of binaries with the appropriate properties. Binary properties are sampled from the large phase space of the various binary properties, which we describe below. Through the random sampling, we obtain a large ensemble of binaries and use the approximate method described in Section 4.2, to evolve each of the binaries and check whether it could coalesce before it evaporates due to

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**Figure 6.** Upper panel: merger time (in units of the merger time of the binary evolved in isolation) vs. $r_{\text{apo}}/r_{\text{per}} = (1 + e_2)/(1 - e_2)$. At large semi-major axis, relativistic precession “de-tunes” the secular Kozai effects and decreases the maximum eccentricity achieved during a cycle. This results in longer merger times. Bottom panel: merger time as a function of the internal eccentricity of the compact binary. For $e_1 \gtrsim 0.7$, the effect of the Kozai resonance on the merger time becomes progressively less important as $e_1$ increases. In these computations we used $m_0 = m_1 = 10 M_\odot$, $e_1 = 0.1$, $I = 85^\circ$, and $g_1 = 0^\circ$. (A color version of this figure is available in the online journal.)
encounters with other stars. For those binaries that coalesce we save the binary eccentricity as it enters the LIGO band; these are then used to determine the overall eccentricity distribution of the GW sources. We first determine the relative fractions of merging binaries arising from different separations around the SMBH (see Table 1) and then normalize the overall rates by the expected number of binaries residing in that region (which depends on the total number of stars multiplied by the binary fraction). In the following, we describe the method in detail.

We considered the idealized case of binaries with equal components of masses 10 $M_{\odot}$ or 2 $M_{\odot}$ corresponding to BH–BH and NS–NS binaries, respectively. Given the uncertainty in the spatial distribution and properties of compact binaries in galactic nuclei, the results presented in this section constitute a set of baselines for making predictions about the expected GW signal produced by such systems.

We sampled the binaries’ external orbits from the distribution of orbital elements:

$$N(a, e^2) da de^2 = N_0 a^{-2-\beta} da de^2,$$

(30)

which generates steady-state phase-space distributions for an isotropic density cusp near a dominating point-mass potential. We first used the density model of Equation (1) with $\gamma = 0.5$ and 1.8 to determine the evaporation timescale of the binary systems, and we assumed that they follow the same density distribution of background perturbers, solar-mass stars in this case. In a further set of integrations, the evaporation timescale was computed from the combined density profile of stars and BHs corresponding to a mass-segregated cusp near Sgr A* (Hopman & Alexander 2006a):

$$\rho(r) = \rho_{\star}(r) + \rho_{\text{BH}} \left(\frac{r}{1 \text{ pc}}\right)^{-2},$$

(31)

with $\rho_{\text{BH}} = 1 \times 10^4 M_{\odot} \text{ pc}^{-3}$. The functional form of the stellar density profile, $\rho_{\star}(r)$, is that of Equation (1) with $\gamma = 1.4$. The power-law index in Equation (30) was set to $\beta = 2$ and $\beta = 1.5$ for BHs and NSs, respectively. We imposed a lower limit for the periastron of the external orbit of $a_2(1-e_2) > 4r_{\text{BH}}$ based on the fact that at shorter distances the binary is unstable due to the strong perturbations from the central BH and Equations (3)–(6) become a poor description of the dynamical evolution of the triple system. The distribution of inner semi-major axes, $a_1$, follows the binary distribution given in Figure 1.

Given a certain distribution of orbital elements, we then estimated the number of binaries that would merge in the galactic nucleus due to Kozai resonances induced by the central BH. Table 1 gives the total fraction of merging binaries for which the external orbits are sampled inside some fiducial radius $r_0$. The overall merger rate was obtained with the technique described in Wen (2003) and summarized below. Including the contribution of relativistic precession, but in the absence of gravitational radiation, the maximum eccentricity in a Kozai cycle can be estimated by considering that the Hamiltonian of the system, in its quadrupole form, is a conserved quantity. The total Hamiltonian is $H = k W$, where $k = 3Gm_{\odot}m_2a_1^2/8M_1a_2(1-e_2)^{1/2}$ and (Miller & Hamilton 2002)

$$W(g_1, e_1) = -2(1 - e_1) + (1 - e_1) \cos^2 I + 5e_1 \sin^2 g_1 (\cos^2 I - 1) + \frac{4}{\sqrt{1 - e_1}} \frac{M_\odot}{M_\bullet}$$

$$\times \left( \frac{a_2 \sqrt{1 - e_2^2}}{a_1} \right)^3 \left( \frac{2GM_\odot}{a_1 e_1^2} \right),$$

(32)

with the last term accounting for relativistic precession. The system starts from initial eccentricity $e_1$, argument of periapsis $g_1$, and mutual inclination $I$ and evolves to a maximum eccentricity $e_{\text{max}}$ and critical $g_{\text{crit}}$ and $I_{\text{crit}}$. The latter quantities can be related to each other setting $de_1/dt = 0$ in Equation (22); this gives

$$\sin^2 g_{\text{crit}} = \frac{152G^3 m_{\odot}m_1 M_\bullet}{e^5 a_1^4 (1 - e_{\text{max}}^2)^{7/2}} \left( 1 + \frac{121}{304} e_{\text{max}}^2 \right)$$

$$\times \frac{G_1}{K(1 - \cos^2 I_{\text{crit}})}.$$  

(33)

Using Equation (26) to relate $I_{\text{crit}}$ to $e_{\text{max}}$, and assuming that $a_1$, $a_2$, and $e_2$ are constant quantities, one obtains $e_{\text{max}}$ solving the implicit equation

$$W(g_1, e_1) - W(g_{\text{crit}}, e_{\text{max}}) = 0.$$  

(34)

An approximation to the merger time is

$$T_{\text{merge}} = T_0(a_1, e_{\text{max}})/\sqrt{1 - e_{\text{max}}^2}.$$  

(35)

Table 1 gives the total fraction of merging binaries for which the external orbits are sampled inside some fiducial galactocentric distances. We find that for BH–BH binaries, the SMBH–Kozai-induced mergers represent $\geq 40\%$ of the total number of mergers occurring at $r \lesssim 0.01$ pc. The overall fraction of mergers in models characterized by a cusp in the distribution of background perturbers (i.e., mass-segregated and cusp models) is slightly lower with respect to that of models with a core due to the shorter evaporation timescale at small radii in these former systems. When we sampled the external orbits within larger radii (>0.1 pc), the merger probability increases since the evaporation time becomes large with respect to the merger time of the binaries. However, at larger radii, both evaporation processes and relativistic precession become more important in quenching Kozai cycles that contribute only $\sim 10\%$ to the overall merger rate. In the table, we also give the total

| BH–BH | NS–NS |
|-------|-------|
| $f(\%)$ | $f(\%)$ | $f(\%)$ | $f(\%)$ |
| $r < 0.5 \text{ pc}$ | $r < 0.1 \text{ pc}$ | $r < 0.01 \text{ pc}$ |
| Core | 8.9 | 8.1 | 2.2 | 1.7 x 10^{-4} | 9.46 | 8.15 | 14.0 | 8.18 |
| Cusp | 5.9 | 5.2 | 3.6 | 3.6 x 10^{-3} | 4.01 | 3.06 | 2.81 | 1.49 |
| Segregated | 5.5 | 4.8 | 49.5 | 4.8 x 10^{-2} | 3.01 | 2.61 | 2.03 | 1.16 |

Note. All the GW rate estimates are as per Milky-Way-like galaxy.
fraction \( f' \) of binary systems with merger time in isolation shorter than their evaporation timescale. These systems would merge even with no help from the Kozai mechanism. Comparing these numbers with those corresponding to mergers induced by the gravitational perturbations from the SMBH, we see that the process described in this paper could contribute only a small fraction of LIGO sources in galactic nuclei; nevertheless, these are qualitatively different from regular mergers, as they contribute high-eccentricity GW sources as shown below.

The total number of GW sources from compact binary mergers in Milky-Way-like nuclei can now be inferred by multiplying the fractions we obtained with the total number of binaries in such regions, i.e., the fraction of CO binaries in the stellar population. For the cusp and core models, we assume that the fraction of COs resembles their population in the field. Following Hopman & Alexander (2006b), we take the fraction of BHs and NSs to be 0.001 and 0.01 of the total stellar population, respectively (in the absence of mass segregation). For the mass-segregated models, we follow the distributions obtained by Hopman & Alexander (2006b) (this time after mass segregation occurred); i.e., we take \( n_{BH}(r) \propto r^{-2} \) and \( n_{NS}(r) \propto r^{-1.5} \) radial distributions, with the total numbers normalized such that 1800 BHs and 374 NSs reside inside 0.1 pc. We assume that the binary fractions follow our discussion in Section 2 (0.1 and 0.07 for BHs and NSs, respectively). Taking these numbers, we calculate the total number of merging compact binaries in a given Milky-Way-like galactic nucleus (see Table 1).

Figure 7 shows the number distribution of eccentricities at \( f_{GW} = 10 \text{ Hz} \) (i.e., the beginning of the LIGO band) for merging BH–BH binaries. This plot was obtained by integrating the set of Equations (21)–(24) and sampling the initial conditions as detailed above but considering only systems with inclinations \( I \geq 80^{\circ} \). The permitted parameter space was defined by imposing the evaporation time of the binary to be longer than its merger time. As an additional constraint we imposed the merger time of the binary in isolation to be shorter than the local evaporation time (i.e., \( T_0(a_1, e_1) > T_{EV}/\sqrt{1-e_1^2} \)) since these systems would rapidly merge even without undergoing Kozai resonance.

In all cases, at \( f_{GW} = 10 \text{ Hz} \) about 90% of the merging systems have \( e_1 < 0.1 \). However, in the cusp and mass-segregated distributions a small fraction (about \( \sim 0.1\% \)) of merging binaries have very large eccentricities \( e_1 \sim 1 \) at the moment they enter the LIGO band. These are the cases in which the gravitational radiation effect dominates the evolution within one Kozai cycle (see, e.g., Figure 8 of Wen 2003). About 0.5% of the mergers have eccentricities larger than 0.5 at 10 Hz. Systems that reach such high eccentricities at the first Kozai maximum will be very short-lived, which decreases the probability that they can be observed. On the other hand, eccentric mergers might be detectable to larger distances (e.g., O’Leary et al. 2009, \( \sim 1 \) Gpc for advanced LIGO and for 10 \( M_\odot \) BHs) and greater BH masses than circular mergers. Finally, we note that the fraction of eccentric mergers found here is a conservative estimate due to the lower limit that we imposed for the periapsis of the external orbit (i.e., \( a_2(1 - e_2) > 4r_{bh} \)). In fact, we expect that for smaller periapsis, the binary could evolve through a non-secular evolution that could lead to very large eccentricities in only a few years (e.g., Figure 4).

4.4. GW Rates from Coalescing Binaries in Galactic Nuclei

In the previous section we calculated the total number of merging compact binaries, for a given galactic nucleus, the gravitational perturbations from the SMBH, we see that the process described in this paper could contribute only a small fraction of LIGO sources in galactic nuclei; nevertheless, these are qualitatively different from regular mergers, as they contribute high-eccentricity GW sources as shown below.

The total number of GW sources from compact binary mergers in Milky-Way-like nuclei can now be inferred by multiplying the fractions we obtained with the total number of binaries in such regions, i.e., the fraction of CO binaries in the stellar population. For the cusp and core models, we assume that the fraction of COs resembles their population in the field. Following Hopman & Alexander (2006b), we take the fraction of BHs and NSs to be 0.001 and 0.01 of the total stellar population, respectively (in the absence of mass segregation). For the mass-segregated models, we follow the distributions obtained by Hopman & Alexander (2006b) (this time after mass segregation occurred); i.e., we take \( n_{BH}(r) \propto r^{-2} \) and \( n_{NS}(r) \propto r^{-1.5} \) radial distributions, with the total numbers normalized such that 1800 BHs and 374 NSs reside inside 0.1 pc. We assume that the binary fractions follow our discussion in Section 2 (0.1 and 0.07 for BHs and NSs, respectively). Taking these numbers, we calculate the total number of merging compact binaries in a given Milky-Way-like galactic nucleus (see Table 1).

Figure 7 shows the number distribution of eccentricities at \( f_{GW} = 10 \text{ Hz} \) (i.e., the beginning of the LIGO band) for merging BH–BH binaries. This plot was obtained by integrating the set of Equations (21)–(24) and sampling the initial conditions as detailed above but considering only systems with inclinations \( I \geq 80^{\circ} \). The permitted parameter space was defined by imposing the evaporation time of the binary to be longer than its merger time. As an additional constraint we imposed the merger time of the binary in isolation to be shorter than the local evaporation time (i.e., \( T_0(a_1, e_1) > T_{EV}/\sqrt{1-e_1^2} \)) since these systems would rapidly merge even without undergoing Kozai resonance.

In all cases, at \( f_{GW} = 10 \text{ Hz} \) about 90% of the merging systems have \( e_1 < 0.1 \). However, in the cusp and mass-segregated distributions a small fraction (about \( \sim 0.1\% \)) of merging binaries have very large eccentricities \( e_1 \sim 1 \) at the moment they enter the LIGO band. These are the cases in which the gravitational radiation effect dominates the evolution within one Kozai cycle (see, e.g., Figure 8 of Wen 2003). About 0.5% of the mergers have eccentricities larger than 0.5 at 10 Hz. Systems that reach such high eccentricities at the first Kozai maximum will be very short-lived, which decreases the probability that they can be observed. On the other hand, eccentric mergers might be detectable to larger distances (e.g., O’Leary et al. 2009, \( \sim 1 \) Gpc for advanced LIGO and for 10 \( M_\odot \) BHs) and greater BH masses than circular mergers. Finally, we note that the fraction of eccentric mergers found here is a conservative estimate due to the lower limit that we imposed for the periapsis of the external orbit (i.e., \( a_2(1 - e_2) > 4r_{bh} \)). In fact, we expect that for smaller periapsis, the binary could evolve through a non-secular evolution that could lead to very large eccentricities in only a few years (e.g., Figure 4).

4.4. GW Rates from Coalescing Binaries in Galactic Nuclei

In the previous section we calculated the total number of merging compact binaries, for a given galactic nucleus.
the number of CO binaries, given a nucleus model, divided by the relaxation time, and we get $\Gamma_{GW} = N_{GW}/\min(\tau_{rep}, \tau_H)$. The calculated GW rates, given these values, are shown in Table 1. The relaxation timescales are taken from Figure 2; these are approximately constant for the cusp model throughout the central pc ($\tau_{rep}(r) \sim \text{const} = 10^9 \text{ yr}$) and are longer than a Hubble time for the core models (and we therefore take $\Gamma_{GW} = N_{GW}/\tau_H$).

**Comparison with other GW sources and rate estimates.** When compared with other GW sources outside galactic nuclei, the GW rates from the processes discussed here are typically lower than most other estimates discussed in Abadie et al. (2010), reflecting the small number of COs residing in a galactic nucleus compared with the total number of such objects in a galaxy. The maximum BH–BH coalescence rate we find (for the segregated cusp model) provides a coalescence rate that is five times larger than the low estimates in Abadie et al. (2010), 10% of the realistic rate, and 0.1% of the high/optimistic rate estimate. The contribution to NS–NS coalescence rate is small in all cases; the highest rate we find (for the cusp model) is 10%, 1%, and 0.1% of the low, realistic, and high estimates in Abadie et al. (2010), respectively. The overall contribution of GW sources from galactic nuclei is therefore likely to be small, however, the eccentric signature of a fraction of these sources is qualitatively different from that of regular GW sources.

**Other replenishment processes.** Various other processes may affect the replenishment rate of compact binaries and/or their progenitors in the GC. In the following we qualitatively discuss such processes, but we do not account for them in our rate estimates (beside star formation, effectively integrated into the assumptions used in the observation-based estimates), as their complete analysis is beyond the scope of this paper.

**Triple disruption.** Perets (2009; see also Ginsburg & Perets 2011) suggested that the disruption of triple stars could leave behind a binary in a close orbit around the SMBH; this could serve as a continuous source of replenished binaries close to the SMBH. This study estimated a triple disruption rate ($f_{\text{triple}}$) that is $\sim$3%–5% of the binary disruption rate of massive stars (OB stars; we do not consider the disruption of triple COs as these are likely to be rare). In half of these cases a binary is captured around the SMBH, where its companion is ejected. If the S-stars observed close to the SMBH in the GC arise from binary disruptions and/or similarly the observed population of hypervelocity stars arises from such processes, then these could be used to calibrate the binary and hence triple disruption rates. Capture rate calculations by Perets et al. (2007; see also Madigan et al. 2009; Perets et al. 2009) suggest that the binary disruption scenario (induced by massive perturbers scattering) could be consistent with the observed number of the young B-stars in the GC (and similar processes are likely to be at work in other galaxies: Perets & Alexander 2008). If this is the case, then binaries are continuously replenished into close regions around the SMBH and into eccentric orbits around the SMBH, and our use of the observed number of B-stars could be justified, with some important differences. Only $\sim$1.5%–3% of the observed B-stars would be the result of triple disruption and would actually be binaries. On the other hand, the captured binaries are likely to have smaller separations than typical binaries in the field, which are more likely to form compact binary progenitors that would efficiently merge. Taken together, it is likely that such a replenishment mechanism would produce a GW rate that is 10–20 times lower than the high estimate for the NS stars. This scenario, however, is not likely to produce progenitors massive enough to form BHs, such as the observed young O/WR stars in the GC (which reside in a disk-like configuration and have low eccentricities). The GW rate from BH coalescence is therefore not likely to gain much from this process.

**In situ star formation.** As mentioned above, our GC contains a large population of young massive O-stars, many of which reside in a stellar disk. These stars were most likely to form in situ following the fragmentation of a gaseous disk formed from an infalling gaseous clump (Genzel et al. 2010, and references therein). Such star formation bursts may occur continuously throughout the evolution of the stellar cusp around the SMBH. As mentioned earlier, eclipsing and close binaries are observed among the GC young stars, suggesting star formation as an additional process that can repopulate the binary population of the GC. Continuous formation of binaries in the GC is part of the assumptions made in our high GW rate estimate.

The other assumptions in the high estimate are related to the binary population properties and their distribution around the SMBH. Although some pioneering work has been done on the properties of binaries formed in the GC (Alexander et al. 2008), these are generally unknown. Another important issue is whether the orbits of binaries formed in a disk could later dynamically evolve to achieve close approach to the SMBH, where efficient secular evolution as studied here can occur. Although the O-stars observed in the disks are not observed closer than 0.05 pc to the SMBH, various mechanisms were suggested for their inward migration closer to the SMBH. Baruteau et al. (2011) discussed the disk migration of binaries through a similar migration scenario of planets in gaseous disks; they show that the binaries not only migrate but also shrink their inner orbit and possibly merge before they finish their migration. Madigan et al. (2009, 2011) proposed that binaries (and single stars) could be excited to high eccentricities through an eccentric disk instability scenario, and their periapsis approach could become small enough for the binaries to be disrupted. Both these mechanisms suggest the existence of binaries at close approach to the SMBH. In the disk migration scenario the binaries’ orbits around the SMBH are likely to be relatively circular, and the binaries lose their orbital energy and migrate on a short timescale of the order $T_{\text{mag}} \sim 10^4 \text{ yr}$; it is not clear, however, whether the gaseous disk enabling the migration would extend close to the SMBH and/or allow for migration in the innermost regions.4 In the eccentric disk instability binaries remain at large semi-major axis orbits around the SMBH but gradually increase their eccentricity and therefore come closer to the SMBH only at periapsis. The timescale for the close approach is relatively short, $T_{\text{sec}} \sim \text{few} \times 10^5$, and binaries may be left at eccentric orbits at the end of this process.

**Cluster infall.** Nuclear stellar clusters may result from the continuous infall of globular clusters that disperse close to or in the close vicinity of the SMBH (see Antonini et al. 2011a; Antonini 2012, and references therein). In particular, such clusters may harbor an inner core cluster of BHs that formed during the cluster evolution. If these BHs are retained in the cluster, this mechanism may contribute to the BH population in the GC. Further work needs to be done to explore the implications of this scenario to the replenishment of binaries in the GC.

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4 It is interesting to note, however, that compact binary mergers in a gaseous disk could provide additional and different channels for the formation of GW sources, if compact binaries exist in the gaseous disk, as could be the case for long-lived active galactic nucleus disks. We leave further study of these issues to future work.
Resonant relaxation. In our discussion on replenishment processes above, we considered the replenishment of binaries through non-resonant relaxation processes in which stars diffuse through random encounters with other stars. Resonant relaxation could change the eccentricities and inclination of stars much faster than regular relaxation. One may therefore consider the timescale faster than regular relaxation. One may therefore consider the timescale $\tau_{\text{rel}}$ for diffusional replenishment to be the scalar resonant relaxation timescale $\tau_{\text{RR}}$, which could be much shorter than the non-resonant relaxation timescale assumed in the low estimate (see Figure 2); this could raise the low estimates by 1–2 orders of magnitude. For the high estimate, resonant relaxation may support the assumption that the distribution of stars rapidly achieves a steady-state thermal distribution of eccentricities, as assumed in our models.

The role of the much faster VRR, which changes the inclination of an orbit around the SMBH, is less clear. It depends on the dynamical effect on the inner binary inclination with respect to its orbit around the SMBH. We may consider two extreme possibilities. If the binary orbit conserves its mutual inclination, i.e., acts as a stiff object, then VRR would not affect the secular evolution of the inner binary. However, if the binary orbit plays the role of a gyroscope, an initial co-planer configuration (e.g., zero mutual inclination), in which Kozai evolution is ineffective, could be transformed into a high mutual inclination configuration at which Kozai evolution plays an important role. To further complicate the problem, the timescales for VRR could be short enough as to be comparable to the Kozai timescale, in which case the coupling of both processes should be considered. All of the above-mentioned issues for VRR of binaries require a dedicated study, which is beyond the scope of this paper.

5. COMBINED KOZAI CYCLES AND TIDAL FRICTION EVOLUTION OF STELLAR BINARIES NEAR SMBHs

In the previous sections, we discussed the secular evolution of CO binaries near an SMBH. A similar secular evolution could be important for stellar binaries and/or MS-star–CO binaries. In these cases, GW emission that served to dissipate energy and induce the inspiral of the CO binaries is not likely to play an important role. However, binary stars can dissipate energy through tidal friction. The secular evolution of the triple system, SMBH–stellar binary, can therefore lead to similar outcomes to those studied in the context of stellar triples (see also Perets 2009).

Mazeh & Shaham (1979) first suggested that Kozai cycles and tidal friction in triples can induce their inspiral and the formation of compact binaries. Later studies by Eggleton & Kisseleva-Eggleton (2006) suggested that short-period binaries typically form through this process (which could be even more effective than previously thought when accounting for octupole approximation; Naoz et al. 2011a). Perets & Fabrycky (2009) suggested that such a process can induce mergers of binary stars and the formation of blue stragglers. Such mergers could be therefore a possible source of young stars at the GC (Perets & Fabrycky 2009; Antonini et al. 2011a). Similarly, other types of close binaries and their products could be formed in a similar way. We may therefore expect that the perturbative effect of SMBHs on binaries provides an efficient channel for the formation of close binaries and their products, such as X-ray binaries, supernovae, and gamma-ray bursts (following the merger of two WDs or NSs, or through the formation of a WD accreting binary; see also Thompson 2011). We note that the binary fractions of MS stars are generally higher than those of COs, as discussed in Section 2, and tidal fraction is more efficient in inducing binary shrinkage/coalescence than is GW emission. Taken together, we therefore expect the rate of shrinkage/merger of MS stars to be higher than our estimates of CO binary mergers. Since this study focuses on secular evolution of CO binaries, we postpone further quantitative discussion of the evolution of binaries with MS star components to another study.

6. CONCLUSION

In this study we explored the secular evolution of compact binaries orbiting a massive BH and considered the effects of the stellar environment of the binaries. We have shown that the SMBH can drive the binary (inner) orbit into high eccentricities, at which point the BHs can efficiently coalesce through GW emission, at times shorter than the evaporation times of the binaries in the hostile stellar environment of the SMBH. Such coalescing binaries can have non-negligible eccentricity when they enter the LIGO band and therefore have a unique observable signature. In addition, we found that binaries at very close orbits to the SMBH can interact with it on a dynamical timescale and present peculiar evolution leading to several GW pulses over a short timescale. The latter sources are likely to be rare and not be detected with current or planned instruments. Finally, we note that similar processes can be important for the formation and merger of close stellar binaries, in which cases the GW dissipation leading to a binary inspiral is replaced with tidal friction.

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