The super-indeterminism in orthodox quantum mechanics does not implicate the reality of experimenter free will

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Abstract. The concept of ‘super-indeterminism’ captures the notion that the free choice assumption of orthodox quantum mechanics necessitates only the following requirement: an agent’s free-choice performance in the selection of measurement settings must not represent an exception to the rule of irreducible quantum indeterminism in the physical universe (i.e., “universal indeterminism”). Any additional metaphysical speculation, such as to whether quantum indeterminism, i.e., intrinsic randomness, implicates the reality of experimenter “freedom”, “free will”, or “free choice”, is redundant in relation to the predictive success of orthodox quantum mechanics. Accordingly, super-indeterminism views as redundant also, from a technical standpoint, whether an affirmative or a negative answer is claimed in reference to universal indeterminism as a necessary precondition for experimenter freedom. Super-indeterminism accounts, for example, for the circular reasoning which is implicit in the free will theorem by Conway and Kochen [1, 2]. The concept of super-indeterminism is of great assistance in clarifying the often misunderstood meaning of the concept of “free variables” as used by John Bell [3]. The present work argues that Bell sought an operational, effective free will theorem, one based upon the notion of “determinism without predetermination”, i.e., one wherein “free variables” represent universally uncomputable variables. In conclusion, the standard interpretation of quantum theory does not answer, and does not need to answer in order to ensure the predictive success of orthodox theory, the question of whether either incompatibilism or compatibilism is valid in relation to free-will metaphysics and to the free-will phenomenology of experimenter agents in quantum mechanics.

1. Introduction

The question concerning the reality of experimenter free will in science in general, and in quantum mechanics in particular, has a long and controversial history. As is well-known, the concept of free will, as such, has eluded universal definition, no less in the context of quantum theory and quantum indeterminism (e.g., Kane [4, 5]). The present analysis will limit itself to the introduction – into the larger discussion on “quantum free will” – of the concept of “super-indeterminism”. Its chief purpose is to draw attention to the fact that neither the metaphysical position of ‘determinism’ nor of ‘indeterminism’ is capable of offering any decisive answers towards the reality of experimenter free will in quantum mechanics; both, the compatibilist, and incompatibilist, interpretation of free will finds strong support in the community of free-will philosophers (Kane [4, 5]). For explanation, compatibilism is the position that determinism and free will are compatible, whereas incompatibilism states that they cannot be. Again, the critical
Supernormalism in the orthodox interpretation of quantum mechanics. Illustrated is the standard assumption that experimenter free choice depends on the existence of a process demonstrating intrinsic randomness and vice versa. Again, the reality of universal indeterminism depends equally—in turn—on experimenter free will being undetermined. For example, the concept of ‘super-indeterminism’ accounts for the much-cited, yet controversial, interpretation of experimenter free will in orthodox quantum mechanics by Conway and Kochen [1] who famously argued that “if indeed we humans have free will, then [so do] elementary particles.”

question of determinism versus indeterminism in quantum theory, such as, for example, in the evaluation of the possibility of de Broglie–Bohm theory (Bohm [6, 7]) need not be conflated with the additional question of whether determinism or indeterminism is compatible with, or even indicative of, the reality of experimenter freedom. Fig. 1 illustrates the metaphysical position of ‘super-indeterminism’.

1.1. Physical definitions of experimenter free will: indeterminism versus determinism

Regarding the free choice assumption, physical definitions in the literature on quantum foundations typically invoke concepts such as the complete independence from, or complete lack of correlation with, any past physical events in nature. A recent example of a concise definition of free choice in the context of orthodox quantum mechanics is the one by Colbeck and Renner [8]. They suggest that a choice called "... A is free if the only variables it is correlated with are those it could have caused." By that definition, it is apparent that a free choice action A is one that (i) must be “uncaused” itself, and (ii) sets in motion an entirely new causal chain, literally from "nothing", or at least starting from an assumption of “intrinsic randomness” (compare Fig. 1). The strict co-dependence of (non-causal) ‘intrinsic randomness’ and (uncaused) ‘experimenter freedom’ in the definition offered by Colbeck and Renner [8] is matched, for example, by the metaphysical assumptions also underlying the free will theorem by Conway and Kochen [1, 2]. These latter authors summed up the metaphysical import of their (free will) theorem by claiming that “if indeed we humans have free will, then [so do] elementary particles” (see Fig. 1). It is significant for the present work that Colbeck and Renner [8] conclude their own efforts at defining physically a free-will concept by referring to the authority of John Bell. They refer to Bell’s well-known, but often misinterpreted, statement concerning the meaning of “free variables” in the context of quantum-mechanical predictions (Bell [3]): “For me this means that the values of such variables have implications only in their future light cones.” In particular, Colbeck and Renner [8] have implicated a correspondence between Bell’s position on experimenter freedom and their own incompatibilist (e.g., super-indeterministic) position on free choice. In the following, the incongruence, however, of these separate views will be demonstrated. The subsequent analysis
will demonstrate that John Bell did not anticipate, and in fact denied, any conflict between experimenter freedom and an entirely deterministic world view (Bell [3, 9, 10]).

2. John Bell’s own “free will theorem” in quantum mechanics is compatibilist

In contrast to both Conway and Kochen [1] and Colbeck and Renner [8], John Bell did not adopt an incompatibilist approach to the problem of free will in quantum mechanics, e.g., the super-indeterministic approach illustrated in Fig. 1. Instead, Bell was a compatibilist when it came to the free choice assumption and the notion of freedom in general. He argued consistently in the tradition of compatibilism starting in 1976 (Bell [3, 9, 10]). Although Bell never presented his position on free will as a theorem in name, he discussed in explicit terms the conceptual foundations, including key metaphysical assumptions, which lead him to propose a free choice concept on an operational basis, a position which might be called “effective indeterminism”, or “determinism without predetermination”. Specifically, Bell proposed that an experimenter agent serves in quantum correlation experiments as a source of effectively free variables, i.e., of variables that are, according to Bell, “… effectively free for the purposes at hand” (Bell [3]). “For me this means”, Bell explained, “that the values of such variables have implications only in their future light cones. They are in no sense a record of, and do not give information about, what has gone before. In particular they have no implications for the hidden variables v in the overlap of the backward light cones…” (Bell [3]). Contrary to the free choice concept proposed by Colbeck and Renner [8], Bell’s own approach to the free choice assumption is not founded upon intrinsic randomness or fundamental quantum indeterminism, i.e., his approach does not reflect super-indeterminism (see Fig. 1).

2.1. John Bell’s concept of “free variables” contradicts “super-indeterminism”

The fact that Bell was a compatibilist and that he thought of freedom as a concept that was not in opposition to ‘determinism’ is amply evident from his explanation in 1977 (Bell [3]). Then, Bell admitted making an error in his original presentation of 1976 when he had wrongly proposed that ‘free’ variables are “… not determined in the overlap of the backward light cones” (Bell [9]). “Here I must concede at once”, Bell [3] admitted “that the hypothesis becomes quite inadequate when weakened in this way. The theorem no longer follows. I was mistaken.” Following this admission, Bell affirmed that ‘free variables’ are variables that are neither ‘not determined’ nor ‘uncaused’ therefore, contradicting the orthodox concept of super-indeterminism described in Fig. 1. Instead, Bell’s free variables are defined operationally, as was mentioned already, as those that are “… effectively free for the purposes at hand” (Bell [3]). Bell’s crucial insight was this: a concept of effective experimenter free will must account for the fact that choices (actions) of the experimenter agent are not pre-determined by the past history of the universe. In particular, the variables associated with the choices by the epistemic agent must not be pre-determined, although they will be unpredictably determined, by “the hidden variables v in the overlap of the backward light cones…” (Bell [3]). For the technical meaning of the notion of “unpredictably determined” variables, in terms of ‘universally uncomputable’ variables, consult Sect. 4.

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That Bell was well aware of the crucial distinction between, on the one hand, “pre-determinism”, such as in the form of “super-determinism”, and, on the other hand, standard “determinism”, is clearly evident from his final statements on the free will problem (Bell [10]). “One can envisage theories”, Bell noted, “in which there just are no free variables... In such ‘super-deterministic’ theories the apparent free will of experimenters, and any other apparent randomness, would be illusory.” The fact that Bell was a compatibilist and that he argued for the possibility of effective experimenter free will in relation to a deterministic world view, is apparent from his
clear rejection of the super-deterministic approach. “I do not expect to see a serious theory of this kind”, Bell [10] explained further. Instead, he proposed that “I would expect a serious theory to permit ‘deterministic chaos’ or ‘pseudorandomness’, for complicated subsystems (e.g., computers) which would provide variables sufficiently free for the purpose at hand.”

2.3. John Bell’s concept of “free variables” implies emergent unpredictability
The chief characteristic of “deterministic chaos”, i.e., of the strictly nonlinear, emergent phenomenon that is accounted for by self-organization or complexity theory, is the uncertainty, or unpredictability, of observable outcomes. This is despite the fact that the chaotic behavior is entirely governed by deterministic relations. In principle, this essential feature of deterministic chaos, holds true for physical, chemical, biological, and even psycho-physical systems, including in neurophysiological brain states potentially associated with the free-choice performance by an experimenter agent (for an overview, e.g., Walacezek [11]). In short, the image used by Bell of “deterministic chaos” represents well the notion of “effective indeterminism” or “determinism without predeterminism”. At the same time, Bell’s reference to deterministic chaos and pseudorandomness is, of course, wholly incompatible with the intrinsic randomness assumption of orthodox quantum mechanics shown in Fig. 1. Consequently, it is misleading to characterize John Bell’s own, well-documented views on free will (Bell [3]) as being in any way consistent with the incompatibilist free-choice concept described by Colbeck and Renner [8].

Finally, while the “super-indeterminism” in orthodox quantum mechanics has been recognized before, although not as a general metaphysical position (e.g., Hosenfelder [12]), it was nevertheless argued by many others that a compatibilist free will concept, i.e., determinism, must be in violation of the non-signalling constraint (e.g., Barrett et al. [13]; Kofler et al. [14]). The non-signalling constraint mediates the apparent conflict between quantum theory and the theory of special relativity (e.g., Eberhard [15]), and its role in deciding the viability of compatibilist free-will concepts in quantum mechanics is reviewed next.

3. The non-signalling theorem in the decision whether only an incompatibilist free will concept is viable for quantum mechanics
A standard view holds that only quantum theories invoking universal or fundamental indeterminism could successfully reproduce the predictions that are yielded by orthodox quantum mechanics. Specifically when negating the involvement of “cosmic conspiracy” theories, such as super-determinism (e.g., Bell [10]), in the interpretation of measurement outcomes, this view is widely thought to hold true. One major reason is the suggested violation of the standard, i.e., axiomatic, non-signalling constraint in any possible ontological quantum theory (e.g., Colbeck and Renner [16, 17]; Gallego et al. [18]). Axiomatic non-signalling, which was presumed in Bell’s original interpretation of his theorem (Bell [19]), is often considered mandatory, therefore, from the mainstream perspective (see Fig. 2). However, as was argued recently, and is illustrated in Fig. 2, the case for, or against, an axiomatic non-signalling assumption is undecidable on logical grounds alone (Walacezkek and Grössing [20, 21]). The reason is the non-reducible, relational interdependence of the metaphysical assumptions that underlie such considerations (see legend to Fig. 2).

3.1. The two interpretations by John Bell of the non-signalling theorem
As was reviewed recently, John Bell shifted his interest – after 1976 – from an “axiomatic non-signalling” assumption (Figs. 2 and 3) towards an assumption of “effective non-signalling” (Fig. 3), which is the non-signalling assumption that is compatible with de Broglie–Bohm theory (Walacezkek and Grössing [21]). In the context of the present analysis, an effective non-signalling theorem only, but not an axiomatic one, is compatible also with Bell’s notion of “free variables” as described in Sect. 2. That is, an effective non-signalling theorem allows for nonlocal
An axiomatic non-signalling assumption does not represent a conclusive argument for or against determinism and nonlocal hidden-variables theories in quantum mechanics (from Walacezek and Grössing [21]). As illustrated in Fig. 1 also, there exists – in orthodox quantum mechanics – an interdependence of free-choice (A) and intrinsic randomness (B) assumptions. The validity of an axiomatic non-signalling assumption (C), a standard assumption in orthodox quantum mechanics, depends in turn on the independent validity of assumptions A and B. Importantly, the concept of super-indeterminism (Fig. 1) demonstrates that an independent validation is out of reach however. This implies that the validity of the axiomatic interpretation of the non-signalling theorem cannot be proven based on present knowledge (for more details see Walacezek and Grössing [21]).

For the current analysis of the free will concept, the critical reason for introducing an effective, instead of an axiomatic, non-signalling condition is the following: an effective non-signalling concept eliminates the danger of contradiction between the free choice assumption and the non-signalling principle, for example, in the context of de Broglie–Bohm theory (Walacezek and Grössing [21]). Next, given the different options for interpreting the notion of experimenter freedom – compatibilism versus incompatibilism, Sect. 4 will offer an outlook towards the possible future of the free will concept in quantum mechanics and beyond.

4. The future of free will in quantum mechanics and beyond: How is freedom possible?
What is meant by the reality of experimenter free will? This work defined the reality of freedom – until now – simply as the negation of the statement that free will is an illusion. This preliminary (circular) definition drew on the earlier statement by John Bell [10] who had explained that in “... ‘super-deterministic theories’ the apparent free will of experimenters, and any other apparent randomness, would be illusory.” Thus, the concept of “super-determinism” implicates the complete absence of experimenter freedom. However, for those theories that do not implicate super-determinism, how could experimenter free will be possible?

An informal, intuitive understanding of free will assumes that the free-willed agent is “free to choose” among different options or alternatives. Also known as the principle of alternative possibilities (e.g., Kane [5]), the intuitive understanding instantly raises the problem of “control” in free-willed choices: Who or what is in charge when is performed the “intentional” selection
Figure 3. The two different interpretations by John Bell of the non-signalling theorem (from Walleczeck and Grössing [21]). Bell’s first interpretation (Bell [19]), here called “axiomatic non-signalling”, rejects any form of deterministic quantum mechanics, upon assuming experimenter freedom in the incompatibilist, super-indeterministic sense (Fig. 1). The second, subsequent interpretation (Bell [9]), here called “effective non-signalling”, is compatible with ontological, deterministic quantum approaches of the nonlocal type. Importantly, unlike axiomatic non-signalling, an effective non-signalling theorem does not prohibit the nonlocal transfer of uncontrollable, i.e., non-Shannon, signals. For definitions of Shannon and non-Shannon signals in the context of Shannon’s mathematical theory of communication see Walleczeck and Grössing [21]. Effective non-signalling ensures the possibility of de Broglie–Bohm theory, under an effective free choice assumption, in agreement with Bell’s compatibilist notion of “free variables” (Bell [3]).

of one option from a set of many alternative options? The concept of ‘experimenter freedom’ cannot, therefore, be reasonably discussed without considering also the concept of ‘experimenter control’. Indeed, generations of free-will philosophers have pointed out the close connection between freedom and control (for reviews see Kane [5]). For example, free will would be an illusion if an experimenter agent lacks (operational) control over his or her performance, such as in setting up a certain experimental configuration for the purpose of “asking specific questions of nature”. An agent who can serve as a “source of operational control” was previously defined as a type-1 agent, whereas a “source of random variables” – without the element of control – was characterized as a type-2 source. For explanation, a ‘random event generator’ represents a type-2 source, but it does not possess type-1 agency, according to these definitions (Walleczek and Grössing [21]).

4.1. Quo Vadis quantum mechanics? Dualism or monism?
Again, since “randomness” is usually viewed as the opposite of “control” it is hard to comprehend how ‘universal indeterminism’ – by itself – could provide a realistic basis for the free-willed actions of (type-1) agents. Similarly, “non-randomness”, in the form of (pre)determinism, is equally unable to provide for the capacity of controlling physical events by an agent’s own free will. Consensus appears to be building towards the view that neither determinism nor indeterminism provides an obvious pathway towards freedom and control (e.g., Kane [4,5]). Regarding the free-willed control by an agent of physical events in nature, only an “extra-physical factor” might advance the possibility of genuine freedom in the strong sense of manifesting agent control. It certainly appears to be impossible to conceive of a scenario in which this extra-physical “causal power” (e.g., a concept of “free mental causation” by an agent) would not be in violation of the central tenet of modern science: the rejection of dualism in the form of the Cartesian separation of agent “mind” from “matter” (e.g., compare Cartesian ‘Res Cogitans’ versus ‘Res Extensae’). Quo Vadis quantum mechanics? Un-physical dualism or physical monism? The final Fig. 4 illustrates a comparison of the two competing metaphysical frameworks that underlie the
available options – universal indeterminism and determinism. What do these options mean for the future of quantum mechanics, e.g., regarding the pressing issue of operationalism versus realism in quantum theory?

Modern science rejects the Cartesian split, i.e., metaphysical dualism, between agents and world. Instead, modern science posits that experimenter agents are integral elements (of reality) of the physical universe, i.e., agents and universe – together – constitute a relational physical continuum. This non-dual continuum principle is indicated by the open line in Fig. 4 which encloses the (operational) presence of the experimenter agent as an intrinsic element of the universe. Importantly, natural science posits that the behavior of both experimenter agent and the rest of the physical universe is constrained by the same Laws of Nature, e.g., quantum theory and relativity theory. This reflects the (monistic) position of scientific physicalism. Must the physicalist position be radically revised in light of orthodox quantum mechanics and of the concept of super-indeterminism in Fig. 1?

4.2. Sources of unpredictability in universal indeterminism and determinism

The above analysis suggests that the relationship between experimenter freedom and experimenter control is neither addressed by universal indeterminism (Fig. 4a) nor by universal determinism (Fig. 4b). To repeat, the notion of ‘control’ is usually identified with determination, whereas the notion of ‘freedom’ with indetermination. In any case, essential to any minimal conception of experimenter freedom, as defined in the beginning of Sect. 4, appears to be the in-principle unpredictability of individual events in the universe. Naturally, this must include also an agent’s free-choice performance in the selection of measurement settings as an entirely unpredictable event itself (compare Fig. 4). On the one hand, orthodox quantum mechanics offers the traditional metaphysical position in regards to the source of in-principle unpredictability: “intrinsic randomness” (Fig. 4a). On the other hand, concepts such as Bell’s notion of “free variables”, for example, offer an alternative metaphysical position: “intrinsic complexity”. This alternative position is consistent with a universally deterministic universe (see

![Figure 4](https://example.com/figure4.png)

**Figure 4.** Illustrating (A) universal indeterminism and (B) universal determinism. (A) In ‘universal indeterminism’, agents and the physical universe are subject to the same fundamental indeterminism. Here, the in-principle unpredictability of physical observables is a function of “intrinsic randomness”. Note that – by itself – indeterminism neither rejects nor affirms the reality of “free will” or “free choice” as a result of super-indeterminism (see Fig. 1). (B) In ‘universal determinism’, agents and the physical universe are subject to the same fundamental determinism. Here, the in-principle unpredictability of physical observables is a function of “intrinsic complexity”. Note that determinism need not be identified with pre-determinism. This unnecessary, even false, identification of determinism with pre-determinism would render any notions of “free will” or “free choice” obsolete as part of scenario (B).
4.3. John Bell’s “free variables” as universally uncomputable variables

Free variables are those that are “... effectively free for the purposes at hand” (Bell [3]). As was reviewed in Sect. 2, in reference to “free variables”, John Bell raised the similarity to phenomena from nonlinear dynamics such as ‘deterministic chaos’, e.g., ‘pseudorandomness’. The present work suggests that Bell’s concept of free variables can be justifiably defined as variables that emerge as a function of the intrinsic complexity of universally deterministic processes (compare Fig. 4b). These complex and self-referential processes may generate physical observables whose values are universally uncomputable, i.e., their computation would require an infinite amount of computational resources (e.g., compare the halting problem in computational theory). In this scenario, even Laplace’s demon, and “the universe itself”, would be incapable of predicting with absolute certainty the future outcome of individual microscopic events in nature. The concept of universal uncomputability, which is inherent in self-referential systems dynamics, plausibly offers the kind of effective unpredictability which John Bell sought. For example, see also the notions of “effective indeterminism” and “determination without predetermination” in Sect. 2.

5. Conclusions

The here introduced concept of super-indeterminism for orthodox quantum mechanics, which was shown to underlie the free will theorem by Conway and Kochen [1, 2], is consistent only with the metaphysical position shown in Fig. 4a: intrinsic randomness. By contrast, John Bell’s search for an effective free will concept was shown to be consistent only with the position shown in Fig. 4b: intrinsic complexity. It is likely that significant advances towards a realist quantum mechanics will depend on new insights into what might be called also “quantum complexity”, i.e., complexity at the smallest dimensions of reality. Summarizing, this work has identified two distinct routes towards avoiding the “cosmic pre-determination” of the free-choice performance by an experimenter agent: intrinsic randomness and intrinsic complexity (Fig. 4). Importantly, neither route was found to address the problem of the “free-willed control” – by an agent – of any physical events in nature. Importantly, both routes were argued to satisfy the minimal requirement of unpredictability, i.e., lack of pre-determination, of an agent’s choices based upon past information about the universe. Finally, a better understanding of the limits and possibilities of the alternative position of quantum complexity – as a source of unpredictability – will require the development of a deeper understanding of the relationship between quantum theory and the theory of complexity, self-organization, and emergence. The prospects for the possibility of an emergent quantum mechanics critically depends on fashioning a deeper understanding also – in agreement with Bell’s effective free will concept – of the relational nature of an emergent universe, including of emergent space and time.

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References

[1] Conway J and Kochen S 2006 The free will theorem Found. Phys. 36(10) 1441–1473 (Preprint arXiv:quant-ph/0604079)
[2] Conway J and Kochen S 2009 The strong free will theorem Notices of the AMS 56(2) 226–232
[3] Bell J S 1977 Free variables and local causality *Epistemol. Lett.* **15** (2) (February) repr. in *Dialectica* **39** (1985) 103-106.

[4] Kane R 2005 *A Contemporary Introduction to Free Will* (New York, NY: Oxford University Press) ISBN 978-0-19-514970-8

[5] Kane R (ed) 2011 *The Oxford Handbook of Free Will* 2nd ed (UK: Oxford University Press) ISBN 978-0-19-539969-1

[6] Bohm D 1952 A suggested interpretation of the quantum theory in terms of "hidden" variables. I *Phys. Rev.* **85**(2) 166–179

[7] Bohm D 1952 A suggested interpretation of the quantum theory in terms of "hidden" variables. II *Phys. Rev.* **85**(2) 180–193

[8] Colbeck R and Renner R 2013 A short note on the concept of free choice (*Preprint arXiv:1302.4446 [quant-ph]*)

[9] Bell J S 1976 The theory of local beables *Epistemol. Lett.* **9** (3) (March) repr. in *Dialectica* **39** (1985) 85-96.

[10] Bell J S 2004 La nouvelle cuisine, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge, UK: Cambridge University Press) pp 232–248 revised ed ISBN 978-0-511-81567-6

[11] Wallaczek J (ed) 2000 *Self-Organized Biological Dynamics and Nonlinear Control* (Cambridge, UK: Cambridge University Press) ISBN 0-521-62436-3

[12] Hossenfelder S 2012 The free will function (*Preprint arXiv:1202.0720 [physics.hist-ph]*)

[13] Barrett J, Hardy L and Kent A 2005 No signaling and quantum key distribution *Phys. Rev. Lett.* **95**(1) 010503

[14] Kofler J, Paterek T and Brukner C 2006 Experimenter’s freedom in Bell’s theorem and quantum cryptography *Phys. Rev. A* **73**(2) 022104 (*Preprint arXiv:quant-ph/0510167*)

[15] Eberhard P H 1978 Bell’s theorem and the different concepts of locality *Nuov. Cim.* B **46**(2) 392–419

[16] Colbeck R and Renner R 2011 No extension of quantum theory can have improved predictive power *Nature Commun.* **2** 411–415

[17] Colbeck R and Renner R 2012 Free randomness can be amplified *Nature Phys.* **8**(6) 450–454 (*Preprint arXiv:1105.3195 [quant-ph]*)

[18] Gallego R, Masanes L, De La Torre G, Dhara C, Acílita L and Acín A 2013 Full randomness from arbitrarily deterministic events *Nature Commun.* **4** 2654 (*Preprint arXiv:1210.6514 [quant-ph]*)

[19] Bell J S 1964 On the Einstein Podolsky Rosen paradox *Physics* **1**(3) 195–200

[20] Wallaczek J and Grössing G 2014 The non-signalling theorem in generalizations of Bell’s theorem *J. Phys.: Conf. Ser.* **504** 012001 (*Preprint arXiv:1403.3588 [quant-ph]*)

[21] Wallaczek J and Grössing G 2016 Nonlocal quantum information transfer without superluminal signalling and communication *Found. Phys.* In press. (*Preprint arXiv:1501.07177 [quant-ph]*)