Image Colorization Based on Locally Linear Embedding

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\section*{Abstract}

This paper deals with a digital image colorization problem and proposes a novel colorization algorithm. To recover the color image from a given grayscale image and several color pixels, we introduce locally linear embedding (LLE). Each patch of an image is considered as a point on a manifold, and the proposed algorithm colorizes the grayscale image by LLE using the formulation of a block Hankel matrix. Numerical experiments show that the proposed algorithm achieves high recovery performance.

\section{Introduction}

Digital image colorization is the recovery of a color image from a grayscale image and several color pixels. The existing methods can be classified into two types: semi-automatic algorithms \cite{1} - \cite{3} with user interaction and automatic algorithms \cite{4} - \cite{6} using deep neural networks such as convolutional neural networks. Because any color image can be given by a user, here we consider semi-automatic colorization.

Levin et al. proposed a colorization algorithm using optimization \cite{1}. Levin’s algorithm requires several color pixels and colorizes an image assuming that neighboring pixels have a similar color when their intensities have similar values. Based on the sparse optimization technique, Levin’s colorization algorithm has been improved \cite{2}, where to recover the color image from fewer given color pixels, the algorithm assumes that the change in the chrominance values of neighboring pixels is small. These algorithms can colorize a grayscale image appropriately if the number of given color pixels is sufficient. However, these algorithms cannot spread the given color information to a distant location; therefore, a user must provide a large amount of color information for each object and area.

To spread the given color information to a distant location, we propose a novel colorization algorithm based on locally linear embedding (LLE) \cite{7}. LLE assumes that each data point and its neighbors lie on or close to a locally linear patch of a manifold and characterizes the local geometry of these patches by linear coefficients that reconstruct each data point from its neighbors. That is, LLE is a method for reducing the dimension of nonlinear data manifolds, and we can obtain appropriate values of the blank points on the manifolds. Here, we assume that each patch of an image is expressed as a point on the manifold and is represented by a weighted average of some points, where the weights are calculated by LLE. To express a patch of an image as a point on a manifold, we convert an image to a block Hankel matrix. Then the colorization problem is formulated as a Frobenius norm minimization problem. Numerical experiments show that the proposed algorithm colorizes a grayscale image efficiently.

\section{Main Result}

We deal with the digital image colorization problem, where a color image is recovered from a grayscale image and several given color pixels.

First, each patch of an image is expressed as a point on a manifold by converting each patch to a column of matrix. That is, the matrix is considered as a manifold representing the characteristics of the image. Note that we can obtain three manifolds from a color image: a grayscale image and two chrominance images; however, the colorization problem gives only one manifold of the grayscale. Therefore, we consider the manifold obtained from the grayscale image. Here, we consider that the matrix obtained by the above approach is a block Hankel matrix as shown in Fig. 1, where matrix $A$ represents an image and $H(\cdot)$ is the conversion function to the block Hankel matrix, that is, $H(A)$ is the block Hankel matrix converted from $A$.

Next, using the given grayscale image, we calculate the relation of between each point by LLE. Let $Y \in \mathbb{R}^{m \times n}$ and $H(Y) \in \mathbb{R}^{p \times q}$ denote the grayscale image and the block Hankel matrix converted from the grayscale image, respectively, where $q$ is patch size and $p = (m - q + 1)(n - q + 1)$.
Figure 1: Block Hankel matrix (patch size is \(3 \times 3\))

\(K\) neighboring points on the manifold are calculated for each point using similarity \(J_{j,i}\) as follows:

\[
J_{j,i} = \|\mathcal{H}(Y)_i - \mathcal{H}(Y)_j\|_2^2 \sum_{j}(1-\exp(-\|e_j-e_l\|_2/2\sigma))
\]

(1)

where \(\sigma\) is a parameter, \(e_j \in \{1, \cdots, m-q+1\} \times \{1, \cdots, n-q+1\}\) denotes the upper left coordinates of the \(i\)th patch and \(\mathcal{H}(Y)_i\) denotes the \(i\)th column vector of \(\mathcal{H}(Y)\), where the vector comprises the elements of the \(i\)th patch. The \(i\)th patch is similar to the \(j\)th patch if the value of \(J_{j,i}\) is small. Then we calculate the weight matrix \(W \in \mathbb{R}^{p \times p}\) from \(\mathcal{H}(Y)\) by the following equation:

\[
W_i = \arg\min_{W_j} \|\mathcal{H}(Y)_i - \sum_{j \in K(i)} W_{j,i} \mathcal{H}(Y)_j\|_2^2
\]

(2)

where \(W_i\) denotes the \(i\)th column vector of \(W\) and \(K(i)\) is the set of \(K\) neighboring points of the point \(\mathcal{H}(Y)_i\). \(W_i\) satisfies \(\sum_j W_{j,i} = 1\). The weight \(W_{j,i}\) summarizes the contribution of the \(j\)th patch to the \(i\)th reconstruction.

Finally, we consider how to recover color images. To recover the color image, we solve the following problem:

\[
X = \arg\min_{X_{k,l}} \|\mathcal{H}(X)_i - \sum_{j \in K(i)} W_{j,i} \mathcal{H}(X)_j\|_2^2
\]

s.t. \(X_{k,l} = X_{k,l}^{\star}\) for \((k,l) \in \mathcal{I}\)

(3)

where \(X\) is the design variable, \(X_{k,l}\) denotes the \((k,l)\)-element of \(X\), \(X_{k,l}^{\star}\) is the given chrominance value of the \((k,l)\)-element, and \(\mathcal{I}\) is the index set of the given color pixels. Then the problem (3) can be rewritten as the following formula:

\[
\min \|\mathcal{H}(X)(I - W)\|_F^2
\]

s.t. \(X_{k,l} = X_{k,l}^{\star}\) for \((k,l) \in \mathcal{I}\)

(4)

where \(\|\cdot\|_F\) denotes the Frobenius norm and \(I\) denotes the identity matrix. To solve problem (4), we use the gradient method. Algorithm 1 shows the proposed method, where \(X_0\) is the initial value of the proposed algorithm and is given as follows:

\[
X_0 = \begin{cases} 
X_{k,l}^{\star} & (k,l) \in \mathcal{I} \\
0 & \text{otherwise}
\end{cases}
\]

(5)

### 3. Numerical Experiments

To show the efficiency of the proposed algorithm, we compare the proposed algorithm with Levin’s method using test images. The image size is \(50 \times 50\) and the given color pixels are randomly selected and have true values. In all examples, we use \(q = 3, \sigma = 0.1, \epsilon = 10^{-5}\) and the number of neighbors of each data point is five points, that is, \(K = 5\).

Figure 2 shows the colorized images of Peppers, Lena, and Mandrill using 27, 21, and 28 color points, respectively. The given color pixels are shown at the center of each red circle in Fig. 2. The proposed algorithm achieves a high recovery performance, especially for the resulting image of Mandrill. In the resulting image obtained using Levin’s algorithm, the blue color of the mandrill’s face has unnaturally spread to its hair. However, the resulting image obtained with the proposed algorithm is natural.

Table 1 shows the peak signal-to-noise ratio (PSNR) of the resulting colorized images, which is defined as follows:

\[
\text{PSNR} = 10 \log_{10} \frac{255^2}{\text{MSE}}
\]

(6)

\[
\text{MSE} = \frac{1}{3mn} \sum_{C \in \{R,G,B\}} \sum_{i=1}^{n} \sum_{j=1}^{m} (C_{i,j} - C'_{i,j})^2
\]

(7)

where \(R, G, B\) are the RGB components of the true image, and \(R', G', B'\) are the RGB components of the image recov-
Table 1: Results of PSNR[dB]

| image name | Peppers | Lena  | Mandrill |
|------------|---------|-------|----------|
| number of given colors | 27 points | 21 points | 28 points |
| Levin’s method | 26.43 | 30.77 | 25.02 |
| proposed method | 30.6 | 34.17 | 28.23 |

Table 2: Results of SSIM

| image name | Peppers | Lena  | Mandrill |
|------------|---------|-------|----------|
| number of given colors | 27 points | 21 points | 28 points |
| Levin’s method | 0.979 | 0.990 | 0.925 |
| proposed method | 0.987 | 0.993 | 0.948 |

tered by the colorization algorithm. Table 2 shows the structural similarity (SSIM) [8] of the resulting colorized images. As can be seen, the proposed method is more effective than Levin’s method. We can see that the PSNR and SSIM have higher values for the proposed method is higher values.

4. Conclusions

We have proposed a new colorization algorithm based on LLE. The proposed method projects each patch of an image to a manifold and recovers a color image assuming that chrominance images have the same weight values as those of a grayscale image. Numerical experiments show that the proposed algorithm achieves a high recovery performance.

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Figure 2: Left column: Pepper, middle column: Lena, right column: Mandrill. First row: original image, second row: given color positions, third row: results of Levin’s method, and fourth row: results of proposed method.