Development of 3D particle method for calculating large deformation of soils

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ABSTRACT

In this study, a three-dimensional (3D) Particle method without using grid was developed for analyzing large deformation of soils instead of using ordinary finite element method (FEM) or finite difference method (FDM). In the 3D Particle method, the governing equations were discretized by various particle interaction models corresponding to differential operators such as gradient, divergence, and Laplacian. The Mohr-Coulomb failure criterion was incorporated into the 3D Particle method to determine soil failure. The yielding and hardening behavior of soil before failure was also considered by varying viscosity of soil. First of all, an unconfined compression test was carried out and the large deformation following soil yielding or failure was simulated by the developed 3D Particle method. The results were also compared with those of a commercial FEM software PLAXIS 3D. The developed 3D Particle method was able to simulate the 3D large deformation of soils due to soil yielding and calculate the variation of normal and shear stresses following clay deformation.

Keywords: particle method, large deformation, soil column, confined compressive stress

1 INTRODUCTION

Traditional numerical methods such as Finite Element Method (FEM) or Finite Difference Method (FDM) are routinely used to analyze an elasto-plastic behavior of soils. Such FEM and FDM use grids or meshes for calculating stress or deformation. Because of that, they have certain limitation to solve large deformation of soils observed in landslide, debris flow and heaving of soft soil. Due to such drawback of FEM or FDM, Cundall and Struck (1979) developed Discrete Element Method (DEM). This method is able to calculate the large deformation of soils by using particles instead of grids. However, DEM has some difficulties when it determines input parameters because its input parameters are not directly related to real soil properties such as cohesion and friction angle. Therefore, FEM, FDM and DEM are not suitable for analyzing large deformation of soils in geotechnical engineering.

A particle method calculates continuum’s movement as a particle movement. There are two kinds of representative particle methods. One is Smoothed Particle Method (SPH) developed by Lucy (1977), and the other is Moving Particle Method (MPS) developed by Koshizuka et al. (1995). In order to analyze fluid’s division and merge, a particle method was widely used in hydromechanics (Daly et al., 1965). The particle method was also used in analyzing ship crack caused by the collision between ship and waves.

We used a particle method to solve geotechnical deformation problems because it can simulate the characteristics behavior of both fluid and solid. Some Japanese researchers began to study how to analyze large deformations of soils in geotechnical engineering by using SPH and MPS (Bui et al., 2008; Maeda and Sakai, 2004; Naili et al., 2005). These researches are all based on SPH that was developed to solve fluid and solid dynamics. Bui et al. (2008) added elasticity-plasticity models such as Drucker-Prager model and Cam-Clay model into SPH to consider the interaction between water and soil in geotechnical engineering. The SPH developed by Bui et.al (2008) can be used in geotechnical engineering practices and has possibility to solve deformation problems such as landslide, and debris flow. Based on SPH, MPS is developed to analyze uncompressed fluid in shipping engineering problems and civil engineering problems.

This research developed 3D Particle method based on MPS method and incorporated Mohr-Coulomb failure criterion. Unconfined compression test was carried out and simulated by 3D Particle method. The results were also compared with those of a commercial FEM software PLAXIS 3D.

2 PARTICLE METHOD

2.1 Basic formulations and models

The governing equations for incompressible fluids are the continuity equation (1) and Navier-Stokes
equation (2). The Navier-Stokes equation is constituted by stress part, viscosity part and external force part to calculate the variation of velocity.

\[
\frac{D\rho}{Dt} = 0 \quad (1)
\]

\[
\frac{D\mu}{Dt} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \mu + \bar{F} \quad (2)
\]

where \( \rho \) is density, \( t \) is time, \( \mu \) is velocity’s vector, \( \nabla \) is gradient operator, \( P \) is force, \( \nu \) is coefficient of kinematic viscosity, \( \bar{F} \) is external force.

### 2.2 Plastic behavior

In this study, the Mohr-Coulomb failure criterion was incorporated to determine the soil particle failure. In other words, the failure of a particle can be determined depending on given cohesion and friction angle. The plastic deformation can be occurred before shear stress reaches to the Mohr-Coulomb failure criterion. So, this kind of plastic behavior can be called hardening behavior. Plastic behavior of soil as shown in Fig. 1 is determined with shear stress ratio. A plastic shear modulus as in (3) is used to determine the shear deformation of soil. The plastic shear modulus is a function of shear stress ratio. The maximum plastic shear modulus as in (4) is decided by undrained shear strength \( (c_u) \), plastic index \( (PI) \) and water content \( (w) \).

\[
G_p = G_{p}^{\text{max}} \left( 1 - 0.8 \cdot \left( \frac{\tau}{\sigma'} \right) \right)^2 \quad (3)
\]

\[
\frac{G_{p}^{\text{max}}}{\sqrt{c_u \cdot P_{a}}} = \frac{6.25}{3 + 44w^2} \sqrt{\frac{P_{I}^w}{0.44 \cdot P_{I} + 20}} \quad (4)
\]

where \( P_{a} \) is an atmospheric pressure. Equation (4) was modified from Massarsch (2004)’s equation. He suggested the correlation between the maximum elastic shear modulus and undrained shear strength. It is a function of water content and plasticity index of soils.

On the other hand, a coefficient of kinematic viscosity in the calculation of particle method is defined by plastic shear modulus as in (5).

\[
\nu = \frac{1112 \cdot G_{p}^{\text{p}}}{P_{a}} \cdot \nu_{w} \quad (5)
\]

where \( \nu_{w} \) is a coefficient of kinematic viscosity of water. In other words, the hardening behavior of soil is simulated as viscous fluid in this particle method of this study. Before failure, viscosity of soil is determined by shear plastic modulus which depends on stress ratio, but the viscosity can be regarded as water’s viscosity when soil’s condition meets the soil failure criterion. So, Equation (5) is used to calculate soil’s behavior before failure.
3 SIMULATION OF UNCONFINED COMPRESSION TEST

3.1 Unconfined compression test

Marine clay was retrieved from Jinhae, Korea. Its materials properties are shown in Table 1. A clay specimen with 5 cm in diameter and 10 cm in height was prepared in this test and sheared at 10 mm/min. Fig. 3 showed the deformation of a clay column at 30, 90, 180, 360 sec during unconfined compression testing. The test result was compared with FEM and 3D Particle method in Fig. 5.

![Unconfined compression test on clay](image)

Fig. 3. Unconfined compression test on clay

| Table 1. Specimen data of unconfined compression test. |
| --- | --- | --- |
| Symbol | Quantity | Value |
| $\phi$ | Friction angle | 32.6 deg |
| $c$ | Cohesion | 25.6 kPa |
| $\rho$ | Density | 1750 kg/m$^3$ |
| $Gs$ | Specific gravity | 2.65 |

3.2 Comparison of Particle method, unconfined compression test and FEM

PLAXIS 3D was used for analyzing unconfined compression test of clay. Fig. 4 showed the simulation result of particle method. The loading was given at 10 mm/min as the unconfined compression test. The primary input parameters for both 3D Particle method and FEM are summarized in Tables 2 and 3. The stress-strain curves calculated from PLAXIS 3D, 3D Particle method and unconfined compression test were compared in Fig. 5. The results of FEM, 3D Particle method and unconfined compression test are similar before the clay yielded. The peak strength is 86 kPa by 3D Particle method, 82 kPa by PLAXIS 3D (FEM), and 83 kPa by unconfined compression test. After the clay failed, the stress-strain behavior was quite different between PLAXIS 3D and unconfined compression test. On the other hand, the result of 3D Particle method was similar to that of unconfined compression test. The program of PLAXIS 3D was developed by FEM method and this study used soft soil creep model in this commercial program. However, Mohr-Coulomb model was used in the program which we developed by using 3D Particle method.

![3D Particle method simulation of unconfined compression test](image)

Fig. 4. 3D Particle method simulation of unconfined compression test

| Table 2. Input parameters for particle method. |
| --- | --- | --- |
| Symbol | Quantity | Value |
| $\phi$ | Friction angle | 32.6 deg |
| $c$ | Cohesion | 25.6 kPa |
| $\rho$ | Density | 1750 kg/m$^3$ |
| $\nu$ | Kinetic viscosity | 0.14 |

| Table 3. Input parameters for FEM. |
| --- | --- | --- |
| Symbol | Quantity | Value |
| $\phi$ | Friction angle | 32.6 deg |
| $c$ | Cohesion | 25.6 kPa |
| $\rho$ | Density | 1750 kg/m$^3$ |

![Results of 3D Particle method, unconfined compression test and FEM](image)

Fig. 5. Results of 3D Particle method, unconfined compression test and FEM
4 CONCLUSIONS

This study simulated and analyzed unconfined compression test by using particle method. Mohr-Coulomb failure criterion was incorporated into the developed 3D Particle method. The results of 3D Particle method were compared with unconfined compression test and a commercial FEM software PLAXIS 3D.

1) The normal stress in this particle method was calculated through the dead-weight and the change of velocity between particles.

2) Mohr-Coulomb failure criterion was used in 3D Particle method and it determined that particle fails or not. On the other hand, before meeting Mohr-Coulomb failure criterion, the viscosity is used to simulate the plastic behavior of soil.

3) The comparison of FEM, particle method and unconfined compression test showed that the normal stress of clay was similar before the clay yielded, however the behavior was different after the clay yielded.

4) In this study, the stress can be calculated while loading was increasing and the deformation phenomenon can be simulated. Then, the prediction and analysis of deformation problems in geotechnical like landslide is prospected by using particle method.

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