Specker’s fundamental principle of quantum mechanics

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I draw attention to the fact that three recently proposed physical principles, namely “local orthogonality”, “global exclusive disjunction”, and “compatible orthogonality” are not new principles, but different versions of a principle that Ernst Specker noticed long ago. I include a video of Specker stating this principle in 2009 in the following terms: “Do you know what, according to me, is the fundamental theorem of quantum mechanics? (. . . ) That is, if you have several questions and you can answer any two of them, then you can also answer all of them”. I overview some results that suggest that Specker’s principle may be of fundamental importance for explaining quantum contextuality. Specker passed away in December 10, 2011, at the age of 91.

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INTRODUCTION

In a recent article entitled “Quantum contextuality from a simple principle?” [1], Henson pointed out that in my paper “A simple explanation of the quantum violation of a fundamental inequality” [2], in which I presented an explanation of the maximum quantum violation of the Klyachko-Can-Binicio˘ glu-Shumovsky (KCBS) inequality \( \mathbb{R} \) (Result 1 in [2]), and also pointed out that the proof of impossibility of Popescu-Rohrlich (PR) nonlocal boxes \( \mathbb{A} \) by Fritz et al. in [3] can be connected to the fact that the graph representing the exclusivity structure of the Clauser-Horne-Shimony-Holt (CHSH) inequality \( \mathbb{R} \) contains, induced, exactly the same exclusivity structure of the KCBS inequality (Observation 1 in [2]), and also introduced a family of quantum correlations exactly singled out by the principle (initially called “global exclusive disjunction”) that the sum of probabilities of pairwise exclusive events cannot be higher than 1 (Result 2 in [2]), I was implicitly assuming that “if all pairs in a set of events are pairwise exclusive, [then] the set can itself be considered exclusive”. Indeed, I was making that assumption. Henson called this assumption “consistent exclusivity”.

Later, Henson, in versions 2 and 3 of [1], added a note pointing out that “consistent exclusivity” is, “when the exclusive sets are defined with relevance to non-locality”, equivalent to the “local orthogonality” defined by Fritz et al. in [3]. Henson also pointed out that the word ‘local’ is inappropriate when applied to the KCBS scenario or general scenarios where different “parties” are not distinguished. Consequently, Henson proposed to call the principle “compatible orthogonality”.

SPECKER’S PRINCIPLE

The purpose of this note is to draw attention to the fact that this principle (in any of its formulations) is not new: Ernst Specker noticed it long ago and pointed out its fundamental importance. Moreover, this principle was already used to bound and single out quantum correlations. The story can be summarized as follows:

- The origin can be traced back to Specker’s 1960 paper “The logic of propositions which are not simultaneously decidable” [6] (cited as Ref. [9] in [2]), where Specker shows how this assumption exactly singles out classical (and quantum; see below) correlations for a specific three-box game (later called Specker’s parable of the overprotective seer [8]), while theories not satisfying this assumption can achieve higher values.

- It appears in the Kochen-Specker paper [9]. Simon Kochen has recently commented the following: “Ernst and I spent many hours discussing the principle (. . . ). The difficulty lays in trying to justify it on general physical grounds, without already assuming the Hilbert space formalism of quantum mechanics. We decided to incorporate the principle as an axiom in our definition of partial Boolean algebras. It appears on pp. 65–66 as follows: A partial Boolean algebra \( C \) is a union of a family \( F \) of Boolean algebras which is (i) closed under pairwise intersection of Boolean algebras, and such that (ii) if any two of a finite set \( S \) of elements of \( C \) lie in a common Boolean algebra in \( F \) then all the elements of \( S \) lie in a common Boolean algebra in \( F \). (. . . ) I have never found a general physical justification for (ii)” [10].

- Its importance as a fundamental principle was stressed by Specker in several occasions. For example, I heard it from Specker himself in June 17, 2009 in the following terms: “Do you know what, according to me, is the fundamental theorem of quantum mechanics? (. . . ) That is, if you have several questions and you can answer any two of them, then you can also answer all three of them. This seems to me very fundamental”. It is recorded in video [11]. Two different fragments of that evening’s
The importance of Specker’s three-box game was also noticed in Liang, Spekkens, and Wiseman’s paper significantly entitled “Specker’s parable of the overprotective seer: A road to contextuality, nonlocality and complementarity”, which was “inspired by Ernst Specker’s talk at the workshop ‘Information Primitives and Laws of Nature’, which took place at ETH, Zürich in May 2008” [8], p. 32.

In Ref. [12], Cabello, Severini, and Winter derived some consequences from assuming this principle, showing that it provides an upper bound and sometimes singles out quantum correlations. Although its fundamental role is not explicitly emphasized in [12], this paper seems to be the inspiration of both Henson’s [1] and Fritz et al.’s [3].

**BOOLE, SPECKER, AND LOVÁSZ: AN OVERVIEW OF SOME RECENT RESULTS**

In the following, I overview some results that suggest that the principle that pairwise decidable propositions are jointly decidable, together with Boole’s condition that the sum of probabilities of jointly exclusive propositions cannot be higher than 1 [13], which I will collectively call Specker’s principle or simply $S$, may explain quantum contextuality. Specifically, by that here I mean explaining why the maximum quantum value for a convex combination of joint probabilities [14] of events whose exclusive disjunction structure is encoded in a graph $G$ in which events are represented by vertices and exclusive events are represented by adjacent vertices [16] is exactly given by the Lovász number of $G$, $\vartheta(G)$, as shown in [12,13]. In [12], it is shown that the maximum satisfying $S$ is exactly given by the fractional packing number of $G$, $\alpha^*(G, \Gamma)$ when $\Gamma$ is the set of all cliques of $G$. In this case, we will denote it as $\alpha^*(G)$, while for general probabilistic theories $\Gamma$ is a general hypergraph. $\alpha^*(G, \Gamma)$ is defined as $\max \sum_{i \in V(G)} w_i$, where the maximum is taken over all $0 \leq w_i \leq 1$ and for all cliques $C_i \subseteq \Gamma$, under the restriction $\sum_{i \in C_i} w_i \leq 1$. In order to single out structures with a quantum-classical separation, it is useful to recall that the maximum value for classical theories is given by the independence number of $G$, $\alpha(G)$ [12]. These tools allow us to summarize several old and recent results in a simple way.
S might not be enough to single out $\vartheta(G)$. — In [2], it is stated that “Navascués has a proof that the set of quantum correlations is strictly smaller than the set satisfying local orthogonality”. Without seeing this proof, I cannot judge whether this affects the program stated before [14].

$S$ singles out the quantum correlations for the KCBS inequality, which is the inequality associated to the simplest graph with quantum-classical separation. — This is Result 1 in [2]. In the language of [12], it follows from the fact that $\vartheta(C_5 \ast C_5) = \alpha^*(C_5 \ast C_5) = 25\frac{1}{2}$.

$S$ singles out quantum correlations in all those cases in which $\vartheta(G \ast G) = \alpha^*(G \ast G)$. — Result 2 in [2] identifies graphs with this property.

Every graph representing correlations with quantum-classical separation has, induced, (i) odd cycles of length 5 or more and/or (ii) their complements. — This result was introduced in [18]. It is an open problem whether the quantum correlations of these families are singled out by $S$. However, the accumulated knowledge in graph theory is compatible with/suggests that the quantum correlations of family (i)/(ii) are singled out by $S$. In any case, for family (ii), the difference between correlations satisfying $S$ and quantum correlations would be unappreciable in actual experiments. In the language of [12], this follows from the fact that $\lim_{m \to \infty} \vartheta(C_m^n) \approx \lim_{m \to \infty} \alpha^*(C_m^n)$, for $m$ odd $\geq 5$. If we could explain the maximum quantum contextuality of an arbitrary graph in terms of the maximum quantum contextuality of their induced elements of families (i) and (ii), then a proof that $S$ singles out quantum correlations for families (i) and (ii) would be a strong evidence that $S$ may be the fundamental principle of quantum contextuality.

Further investigations will solve this puzzle. The purpose of this note is to argue why, in my opinion, Specker deserves the credit for noticing this principle. It is also clear for me that the influence, direct or indirect, of his work is behind all these recent developments.

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