Vertical Specialization and Wage Inequality:
A Simple Model of Monopolistic Competition*

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Abstract

We analyze the relationship between vertical specialization and skilled-unskilled wage inequality by using a monopolistic competition model in which firms can disintegrate their production activities between countries. We develop a new diagrammatic exposition to show that vertical specialization arises, i.e., a country produces final goods by using domestic labor with imported inputs and exports final goods abroad. We show that a country with a larger share of vertical specialization has greater wage inequality.

JEL Classification: F11; F12; F23

Key words: monopolistic competition, offshoring, vertical specialization, wage inequality

1. Introduction

A disintegration of the production process is one of the important features in explaining the recent growth of international trade, i.e., firms have located various stages of production in many different countries, and as a consequence, international trade in intermediate goods and services has grown. 1) In particular, firms in developed countries specialize in research and development and offshore manufacturing of products to developing countries. 2) Such offshoring could result in growth of international trade in two aspects, exports of technologies and imports of products that embody such technologies. This pattern of international trade could

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1) See Hummels, Ishii, and Yi (2001) and Barba Navaretti and Venables (2004) for empirical evidence on the recent growth in trade resulting from the disintegration of the production process.

2) Corporations such as Mattel and Nike do the design and marketing of their products in the United States and outsource other activities including production of their products to low-wage countries in Asia (Feenstra, 1998). A more recent example is the Apple iPhone, the back of which says “Designed in California, Assembled in China.” It is composed of various parts produced in many different countries and assembled by Foxconn, a Taiwanese company, at its plant in Shenzhen, China (Irwin, 2015).
be called vertical specialization, since it is an exchange of final products for intermediate inputs. In this paper, we theoretically examine the relationship between vertical specialization and factor prices, in particular, wages of skilled and unskilled workers.

To examine the relationship, we develop a simple model based on a Heckscher–Ohlin framework with monopolistic competition. The model is a modified version of Krugman (1981). Krugman’s model consists of two industries in which firms use sector-specific labor and engage in monopolistic competition. With this model, he considers trade in final products between countries with different factor proportions and examines the income-distribution effects of trade within a country. While Krugman (1981) considers a factor intensity difference between industries, we focus on a factor intensity difference between production activities within an industry. In our model, production of firms consists of two activities, developing blueprints and manufacturing products, and each production activity requires activity-specific labor, skilled labor for developing blueprints and unskilled labor for manufacturing products. This modification enables us to examine the relationship between vertical specialization and skilled–unskilled wage inequality.

We develop a new diagrammatic exposition to show that vertical specialization arises, i.e., an unskilled-labor abundant country produces final goods with domestic unskilled-labor and imported blueprints, and exports final goods to a skilled-labor abundant country. Following the procedure developed by Hummels, Ishii, and Yi (2001), we construct a measure of vertical specialization. We show that the effects of trade on wage inequality crucially depend on the share of vertical specialization in total trade. Furthermore, we show that a country with a larger share of vertical specialization has greater wage inequality.

There is extensive literature on globalization of production and wage inequality. Empirical work such as Feenstra and Hanson (2003) reports that international trade in the form of foreign outsourcing contributes increases in the wage gap between skilled and unskilled labor in developed countries. Ohyama (1996) develops a Ricardian model of processing trade and examines mutual gains from vertical specialization. Jones (2000) and Arndt and Kierzkowski (2001) theoretically investigate the effects of international fragmentation of the production process on income distribution in various trade models. Grossman and Rossi-Hansberg (2008) develop a model based on a Heckscher–Ohlin framework to study the labor market effects of offshore outsourcing. More recently, Helpman (2018) provides a comprehensive survey on the theoretical and empirical analyses on the relationship between the inequality of earnings and globalization in the form of foreign trade and offshoring.

This paper is closely related to Helpman (1984) in that the disintegration of the production process arises to exploit differences in factor prices across countries. Our paper complements Helpman’s work in that we develop a new diagrammatic exposition based on Yomogida (2010) and analyze the relationship between vertical specialization and wage inequality.3)

3) Unlike the present paper, Yomogida (2010) considers the structure of production in which all of production factors are completely mobile across production activities, and examines issues that are totally different from those investigated in the present paper.
2. The Model

First, let us consider a closed economy of a monopolistic competition model with one industry. All individuals have the utility function,

$$U = \ln \left( \sum_{i=1}^{n} c_i^{\theta} \right)^{\frac{1}{\theta}}, \quad 0 < \theta < 1,$$

where $c_i$ is the consumption of the $i$th product.

The production of each product consists of two activities, development of a blueprint and manufacturing of a product based on the blueprint. There are two factors of productions, each of which is specific to each activity, skilled labor is used only in the blueprint activity and unskilled labor is required only in the manufacturing activity. All firms have the same cost function,

$$m_i = w_s \alpha + w_u \beta x_i, \quad i = 1, \ldots, n,$$

where $w_s$ is a wage rate of skilled labor, $w_u$ is a wage rate of unskilled labor, and $x_i$ is output of firm $i$. That is, $w_s \alpha$ is a fixed cost of developing a blueprint and $w_u \beta x_i$ is a variable cost of manufacturing a product. Note that output of each product equals total consumption of the product.

The full employment condition of each type of labor is respectively,

$$\begin{align*}
L_s \alpha &= L_s = z, \\
\sum_{i=1}^{n} \beta x_i &= L_u = 2 - z
\end{align*}$$

where $L_s$ and $L_u$ are the endowment of skilled and unskilled labor, respectively. Also, by following Krugman (1981), we assume that the total labor force is set equal to 2 and $z$ is a measure of factor endowment ratios.

Now we can derive equilibrium in this model. First, profit maximization of a firm leads to a price that will be marginal cost plus a constant percentage markup,

$$p = \frac{w_u \beta}{\theta}.$$

Since all firms face the same constant elasticity of demand $\frac{1}{1-\theta}$, and have the same technology of production, they set the same price. Given the price, the profit of each firm is $\pi_i = px_i - m_i$. Due to the free entry condition, each firm’s profit will be zero, and this condition leads to output of each firm,

$$x = \left( \frac{w_s}{w_u} \right) \left( \frac{\alpha}{\beta} \right) \left( \frac{\theta}{1-\theta} \right).$$

The number of firms is determined by the full employment condition of skilled labor,

$$n = \frac{z}{\alpha}.$$

Given the number of firms, the full employment condition of unskilled labor is used to determine output of each firm,
Vertical Specialization and Wage Inequality: A Simple Model of Monopolistic Competition

\[ x = \frac{\alpha (2 - z)}{\beta z}. \]  

(7)

We can determine a relative wage by (5) and (7),

\[ \frac{w_s}{w_u} = \left( \frac{1 - \theta}{\theta} \right) \left( \frac{2 - z}{z} \right). \]  

(8)

The relative wage depends on two parameters, a degree of product differentiation \( \theta \) and the measure of factor endowment ratios \( z \).\(^4\) Given the measure of relative factor endowment ratios \( z \), a smaller \( \theta \) leads to a greater \( w_s / w_u \) i.e., skilled and unskilled wage inequality increases with the degree of product differentiation.\(^5\)

Finally, we can illustrate market equilibrium by a diagram. We begin by rewriting the utility function as

\[ u = \ln \left( \frac{1 - \theta}{\theta} \frac{X}{n} \right), \]  

(9)

where \( X = nx \).\(^6\) Given this form of the utility function, we can derive

\[ \frac{\partial u}{\partial n} / \frac{\partial u}{\partial X} = \left( \frac{1 - \theta}{\theta} \right) \frac{X}{n}, \]  

(10)

which shows a trade-off between the variety of products and aggregate output for the consumers, i.e., the absolute value of the slope of an indifference curve in the space of variety and output.

The zero profit condition (5) can be written as,

\[ \left( \frac{1 - \theta}{\theta} \right) \frac{X}{n} = \frac{w_s \alpha}{w_u \beta}, \]  

(11)

where the right hand side equals the absolute value of the slope of the GDP line, i.e., the aggregate budget line.\(^7\)

Now we are ready to illustrate equilibrium in the closed economy of this model. Since the slope of the indifference curve (10) equals the slope of the budget constraint (11), we can show equilibrium as point \( E \) in Figure 1. At the market equilibrium, the marginal rate of substitution (MRS) equals the ratio of a marginal cost of developing a blueprint to a marginal cost of manufacturing a product. This implies that monopolistic competition equilibrium generates the socially optimal combination of variety and quantity.

This result is somewhat surprising because market prices are distorted. Matsuyama (1995) explains why this result arises in a monopolistic competition model a la Dixit and Stiglitz (1977). He points out that there are two types of distortions: price distortions and incomplete markets, and he concludes that the market equilibrium generates the efficient resource allocation.

\(^4\) Note that the elasticity of substitution between products is \( 1/(1 - \theta) \). A smaller \( \theta \) leads to a smaller elasticity of substitution, which implies a higher degree of product differentiation.

\(^5\) The pricing equation (4) implies that the parameter \( \theta \) is the ratio of the marginal cost to the average cost in equilibrium. This means that a lower value of \( \theta \) indicates the more importance of unexploited scale economies.

\(^6\) In autarky, the total consumption equals the total output.

\(^7\) \( GDP = pX = w_s \alpha n + w_u \beta X \)
allocation since these two types of distortions exactly offset each other.\footnote{The market incompleteness exists due to the following reason. In monopolistic competition models based on Dixit and Stiglitz (1977), the market for a new product is created when the product is introduced. This implies that consumers cannot signal how many varieties they want through the market, that is, the market for the variety does not exist.}

3. Pattern of Trade

Let us suppose that there are two countries, the home country and the foreign country. The home country is just described in Section 2. The foreign country is assumed to be identical to the home country except for factor endowments. Let* denote the variables associated with the foreign country. We have

\[
\begin{align*}
L_x &= z, \quad L_o = 2 - z, \\
L_x^* &= 2 - z, \quad L_o^* = z.
\end{align*}
\]

Clearly, the parameter $z$ can be regarded as an index of similarity in factor endowment ratios. If $z = 1$, then the countries are identical. Factor proportions will be dissimilar as $z$ decreases to zero. Without a loss of generality, we will consider the case in which $z \in (0,1)$, the home country is relatively more unskilled–labor abundant than the foreign country.

Suppose that the countries can trade at a transportation cost of zero. Also, firms can disintegrate the production activities geographically, i.e., they can develop blueprints in one country and manufacture products in the other. Then, offshoring of the manufacturing activities will arise to take advantage of a relative wage difference between the countries. In trade equilibrium, a difference in the costs of the production activities disappears and the relative wages are equalized between the countries,

\[
\frac{w_x}{w_o} = \frac{\overline{w}_x}{\overline{w}_o} = \frac{\overline{w}_x^*}{\overline{w}_o^*},
\]

where we use — on a variable to indicate the trade equilibrium. The zero profit condition implies that output of a firm depends on the relative wage of each country as in (5). The equalization of the relative wages leads to the same output for all firms.
Since preferences are the same, the consumers of each country have the same demand function for each product. Thus, the price of each product is given by the same form as in (4). Given the pricing policy, the zero profit condition of a firm can be written as

\[
\left(\frac{1-\theta}{\theta}\right)x = \frac{\alpha \bar{w}_s}{\beta \bar{w}_u},
\]

Note that the left hand side of (13) is equal to the slope of an indifference curve of the world representative consumer (see (10)).

Now we are ready to examine the pattern of trade. We begin with the pattern of production. Figure 2 shows a variant of the box diagram of the world. The home country’s origin is O and the foreign country’s counterpart is O*. E is the production point. The slope of the diagonal OO* equals the output \( \bar{x} \). The home firms integrate the production activities within the home country. The number of the home products is \( OH = n \), and the total output equals \( HP = n \bar{x} \).

There are two groups of the foreign firms. Firms in one group integrate the production activities within the foreign country, and \( \bar{n}_f^* \) is the number of firms (products) in such a group. Firms in the other group produce blueprints in the foreign country but manufacture the products in the home country, and \( \bar{n}_h^* \) is the number of firms (products) in such a group that disintegrates the production activities internationally. Obviously, the total number of the foreign products is \( O'F = \bar{n}_f^* + \bar{n}_h^* \). The total output of the firms that integrate production within the foreign country is \( FE = \bar{n}_f^* \bar{x} \). And the total output of the firms that offshore manufacturing to the home country is \( EP = \bar{n}_h^* \bar{x} \).

We turn to the pattern of consumption. Each country’s total consumption is proportional to its GDP, which is determined at point G at which the GDP line EG intersects with the diagonal OO*. The home country’s GDP relative to the world’s is \( OG / OO^* \). Since all of the variet-

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9) The home country’s aggregate output equals \( X = (2-z)/\beta \) and the foreign counterpart equals \( X^* = z/\beta \). The number of products (blueprints) developed in the home country equals \( n = z/\alpha \) and the foreign counterpart equals \( n^* = (2-z)/\alpha \).

10) Obviously, \( n = \bar{n} \) holds.
ies are consumed in each country, the consumption point of the home country is determined at point \( C \). The relative GDP of the foreign country is given by \( O^* G / O^* O \), and the foreign consumption pattern is determined at point \( C^* \).

Trade is composed of the two types of exchanges, vertical and horizontal exchanges. On the one hand, vertical trade, the exchange of blueprints for products, arises since the home country exports products and the foreign country exports blueprints. On the other hand, horizontal trade, the exchange of differentiated products, occurs as two-way trade in products.¹¹)

According to Hummels, Ishii, and Yi (2001), we can define vertical specialization for the home country as

\[
VS = \frac{\bar{\bar{w}}_s \bar{n}_s^* \alpha}{p(n + \bar{n}_h^*)^x} \times \frac{p^c}{c^*} \left( n + \bar{n}_h^* \right)
\]

(14)

\( VS \) is the imported input content of exports. On the right hand side of (14), the first term is the share of imported inputs (blueprints) in gross production for each foreign offshoring firm and the second term is the value of final good exports by foreign offshoring firms. Multiplying the first term by the second term is a value for imported blueprint content of final good exports by foreign offshoring firms.¹²)

Hummels, Ishii, and Yi (2001) use the share of \( VS \) in total exports as a measure of vertical specialization. In our setting, the share of \( VS \) in total exports equals the imported blueprint share of gross output and it can be written as¹³)

\[
\text{The share of } VS = \frac{\bar{\bar{w}}_s \alpha}{p^x} \times \frac{\bar{n}_h^*}{n + \bar{n}_h^*}
\]

(15)

On the right hand side of (15), the first term, the share of blueprints in production, decreases with \( \theta \), which implies that the cost share of blueprints is positively related with the degree of product differentiation. The second term, the share of exports by foreign offshoring firms in total exports, decreases with \( z \), which implies that the share of offshoring firm exports is negatively related with the index of similarity in labor endowment ratios.¹⁴)

¹¹) In the home country, the total expenditure equals the total income, \( p(n + \bar{n}_h^* + \bar{n}_s^*) \) = \( n \bar{w}_s \alpha + (n + \bar{n}_h^*) \bar{w}_s \beta \), which can be written as \( p(n + \bar{n}_h^*) (x - \bar{c}) - \bar{p} \bar{m}_s \bar{c} = n \bar{w}_s \alpha + \bar{w}_s \beta x \). This implies that the value of net exports of differentiated products equals the value of imports of blueprints in the home country.

¹²) In our setting, \( VS^* \) for the foreign country would be zero since it does not import any inputs from the home country. However, this does not necessarily mean that the foreign country does not engage in vertical specialization. The foreign country exports blueprints and imports final products that embody technologies developed by foreign offshoring firms. From the view point of the foreign country, \( VS \) can be interpreted as a value of exported blueprint content of final good imports. This implies that \( VS \) could be regarded as a measure of vertical specialization for the foreign country as well as the home country.

¹³) See Appendix for the proof.

¹⁴) Note that, for the foreign country, the share of \( VS \) is the share of the exported blueprint content of imports in total imports. In this sense, it could be regarded as a measure of vertical specialization for the foreign country as well.
4. Gains and Losses from Trade

Suppose that an individual with type $k$ labor receives a wage $w_k$ and has the utility function (1). The utility depends on the number of products and the wage relative to the price,

$$U_k = \frac{1-\theta}{\theta} \ln n + \ln \frac{w_k}{p}.$$ 

Let $\bar{U}_k$ denote the utility in the trade equilibrium. The change in the utility due to the movement from autarky to trade is

$$\bar{U}_k - U_k = \frac{1-\theta}{\theta} \left[ \ln \left( n + \frac{n_h}{\theta} + \frac{n_f}{\theta} \right) - \ln n \right] + \ln \frac{\widehat{w}_k}{p} - \ln \frac{w_k}{p}.$$ 

The effects on the utility consist of two components, a change in the variety of products and a change in a real wage in terms of the product. Since trade increases the number of products available for all consumers, the variety effect is positive for all types of individuals in every country. The real wage effect depends on location and type. First, in both countries, trade does not affect the real wages of unskilled labor since (4) implies that the markup is constant. Second, trade reduces the real wages of skilled labor in the home country but the change is reversed in the foreign country. The reason is straightforward. The home country is relatively skilled-labor scarce as compared to the foreign country.

Thus, in the foreign country, both types of individuals necessarily benefit from trade. In the home country, individuals with unskilled labor gain but those with skilled labor may lose from trade. Whether individuals having skilled labor gain or not depends on the two parameters, $\theta$ and $z$. We can show that home skilled-labor would gain if

$$\theta < f(z) = \frac{\ln z - \ln 2}{2 \ln z - \ln 2 - \ln (2 - z)},$$

where $z \in (0, 1)$ and $\theta \in (0, 1)$. Otherwise, trade would hurt home skilled labor. Figure 3 shows how the welfare effects of trade for home skilled labor depend on the two parameters, $\theta$ and $z$. If labor endowment ratios are sufficiently dissimilar between the countries ($z$ is close to zero), the degree of product differentiation must be large enough ($\theta$ must be small enough) for home skilled labor to gain from trade.

Recall that the measure of vertical specialization is the share of $VS$, which is positively related with the degree of product differentiation but negatively related with the index of similarity in factor proportions. Given labor endowment ratios, home skilled labor would be more likely to benefit from trade as the share of vertical specialization is larger ($\theta$ becomes smaller). Trade is more likely to cause the conflicts of interests between skilled and unskilled labor in the home country as the share of vertical specialization is larger ($z$ becomes smaller), given that the degree of product differentiation is small enough, $\theta > 1/2$. No conflict of interests would arise due to trade if the degree of product differentiation is sufficiently large, $\theta < 1/2$.

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15) We can show that $f'(z) > 0$, $f(1) = 1$, and $\lim_{z \to 0} f(z) = 1/2$ (see Appendix for the proof). Krugman (1981) derived a similar diagram to examine gains and losses from horizontal trade.
5. Wage Inequality

In this section, we will examine the effects of trade on wage inequality. In autarky, the relative wage of each country is respectively given by

\[
\frac{w_s}{w_u} = \left( \frac{1-\theta}{\theta} \right) \frac{2-z}{z},
\]

(17)

\[
\frac{w_s^*}{w_u^*} = \left( \frac{1-\theta}{\theta} \right) \frac{z}{2-z}.
\]

(18)

In the trade equilibrium, wages of both types of labor are equalized between the countries and wage inequality between skilled and unskilled labor in either country can be derived as follows, 16)

\[
\frac{\bar{w}_s}{\bar{w}_u} = \frac{1-\theta}{\theta}.
\]

(19)

Unlike in the autarky equilibrium, the gap in wages between skilled and unskilled labor is independent of the index of similarity in factor proportions, \( z \), and it depends only on the degree of product differentiation, \( \theta \), in the trade equilibrium. If \( \theta < 1/2 \), then skilled labor earns more than unskilled labor. The smaller is \( \theta \), the wage gap between skilled and unskilled labor is larger.

Now we are ready to examine the effects of trade on wage inequality. First, let us look at how the effects depend on the index of similarity in factor proportions. Figure 4 shows that, in the autarky equilibrium, \( w_s / w_u \) decreases with \( z \) in the home country but the opposite is true in the foreign country, i.e., \( w_s^* / w_u^* \) increases with \( z \). This implies that trade-induced changes in wage inequality are larger as the countries have more dissimilar labor endowment ratios.

Next, let us examine how the income-distribution effects of trade depend on the degree of product differentiation as well as the index of similarity in labor endowment ratios. Figure 5 shows that there are four different cases (a)-(d).

(a) If \( 1 - z / 2 < \theta < 1 \), then unskilled labor earns higher wages than skilled labor in both coun-

16) See Appendix for the proof.
Vertical Specialization and Wage Inequality: A Simple Model of Monopolistic Competition

tries and this ranking order of wages holds under both autarky and trade.

(b) If $1/2 < \theta < 1 - z/2$, then trade affects the ranking order of wages in the home country, i.e., skilled labor earns higher wages than unskilled labor in autarky, but unskilled labor’s wages become higher than skilled labor’s due to trade. In the foreign country, the ranking order does not change, i.e., unskilled labor’s wages are higher than skilled labor’s under both autarky and trade.

(c) If $z/2 < \theta < 1/2$, then a trade-induced change in the ranking order of wages arises in the foreign country, i.e., unskilled labor’s wages are higher than skilled labor’s in autarky, but skilled labor earns higher wages than unskilled labor under trade. In the home country, the ranking order does not change due to trade, i.e., skilled labor’s wages are higher than unskilled labor’s under both autarky and trade.

(d) If $0 < \theta < z/2$, then the wages of skilled labor are higher than those of unskilled labor in both countries and this ranking order holds both before and after trade. Trade raises wage inequality in the foreign country, but reduces it in the home country.

The most plausible case would be (d). Such a situation could arise when the degree of product differentiation is sufficiently high and the countries are significantly dissimilar in fac-

Figure 4: Wage inequality and similarity in factor proportions

Figure 5: Four different cases in the effects of trade on wage inequality
tor proportions. Combining Figure 3 with Figure 4, we can confirm that no one loses from trade in case (d).

In case (d), we can also derive the relationship between vertical specialization and wage inequality. As we have shown in (15), the share of VS depends on the degree of product differentiation, \( \theta \), and the index of similarity in factor proportions, \( z \). Given the degree of product differentiation, a smaller \( z \) leads to a larger share of VS. Figure 4 shows that the impacts of trade on wage inequality become greater as \( z \) becomes smaller. These observations imply that trade would have greater effects on wage inequality as the share of vertical specialization is larger. This result originates from a feature of the Heckscher–Ohlin framework, i.e., a difference in factor proportions leads to vertical specialization which equalizes factor prices between the countries.

Furthermore, the product variety, a feature of monopolistic competition model, results in a positive relationship between vertical specialization and wage inequality. From (19), a smaller \( \theta \) leads to greater wage inequality under trade. Given factor endowment ratios, the share of VS is negatively related with \( \theta \). These results imply that a country with a larger share of vertical specialization has greater wage inequality. This outcome is obtained because a greater degree of product differentiation generates the larger relative demand for skilled labor, and as a result, a relative wage of skilled labor rises and blueprint contents of offshore production expand.

6. Concluding Remarks

We develop a simple trade model in which monopolistically competitive firms could disintegrate their production activities between the countries and analyze the relationship between vertical specialization and skilled–unskilled wage inequality. As the countries become more dissimilar in factor proportions, a share of vertical specialization is larger and trade has more substantial effects on wage inequality. Furthermore, as products are more highly differentiated, a share of vertical specialization is larger and wage inequality between skilled and unskilled labor becomes greater.

There are possible extensions with the use of our framework. The present paper examines the income–distribution effects of vertical specialization by comparing two extreme situations, autarky and trade with the disintegration of production. There is another situation in terms of a trade regime, i.e., countries are allowed to trade differentiated final products only. We could consider this situation as a benchmark and examine the effects of vertical specialization on wage inequality. We could also consider international labor migration and compare its income–distribution effects with those of the disintegration of production. These tasks are beyond the scope of the present paper and left for our future work.

Appendix

First, we will derive (15). Using the price in the trade equilibrium, \( \tilde{p} = \tilde{w} \beta / \theta \), with (13), we can obtain the share of blueprints in production as \( \tilde{w} \alpha / \tilde{p} x = 1 - \theta \). Trade with vertical specialization can achieve the resource allocation in the integrated equilibrium, a hypothetical construct, at which production factors as well as goods are perfectly mobile across the coun-
tries. Thus, in the trade equilibrium, the output of each firm is derived under the labor constraints for the world in which the demand for each type of labor equals its endowment of the world, i.e., \((n + n')\beta x = 2\) and \((n + n')_\alpha \alpha = 2\). With these constraints, we can derive the output of each firm, \(x = \alpha / \beta\). In the trade equilibrium, the constraint for unskilled labor at the home country is \(\beta(n + \bar{n}_h)x = 2 - z\). The number of blueprints developed in the home country is \(n = z / \alpha\). Using these equations with \(x = \alpha / \beta\), we can derive the share of final good exports by foreign offshoring firms as \(\bar{n}_h / \left(n + \bar{n}_h\right) = 2(1 - z) / (2 - z)\).

Second, we will derive (16). The effects of trade on the utility of home skilled labor can be obtained as

\[\bar{U}_s - U_s = \frac{1 - \theta}{\theta} \ln \frac{z}{2} + \ln \frac{z}{2 - z}.\]

Using the above equation, we can show \(\bar{U}_s > U_s\) if and only if (16) holds. In addition, we can show that

\[f' (z) = \frac{(2 - z)\left[\ln 2 - \ln (2 - z)\right] + z(\ln 2 - \ln z)}{z(2 - z)\left[2 \ln z - \ln 2 - \ln (2 - z)\right]} > 0.\]

Finally, we will derive (19). As we have shown above, the equilibrium output of each firm under trade is \(x = \alpha / \beta\). Using this with (13), we can obtain (19).

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M. Yomogida

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