Nucleon spin structure I: A dynamical
determination of gluon helicity distribution
in the nucleon

Wei Zhu and Jianhong Ruan

Department of Physics, East China Normal University, Shanghai 200062, P.R. China

Abstract

Gluon helicity distribution in the nucleon is dynamically predicted by using a nonlinear QCD evolution equation—the DGLAP equation with the parton recombination corrections—starting from a low scale, where the nucleon is almost only consisted of valence quarks. The comparisons of our predicted gluon helicity distribution with the available data are presented. We find that the contribution of the gluon helicity to the nucleon spin structure is much larger than the predictions of most other theories. This result suggests a significant orbital angular momentum of the gluons is required to balance the gluon helicity. A novel spin-orbital structure of the proton in the light-cone frame is described based on the quantitative calculations, and the nucleon spin crisis is discussed.

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1 Introduction

A precise determination of the gluon helicity distribution $\delta g(x, Q^2)$ is important in order to understand the structure of the nucleon including the origin of its spin. However, a direct measurement of the polarized gluon density in the nucleon is difficult. In the global analysis of the polarized lepton-nucleon deep inelastic scattering (DIS), the distribution $\delta g(x, Q^2)$ is extracted from the spin structure function $g_1(x, Q^2)$ through scaling violation as a higher-order effect of quantum chromodynamics (QCD). Unfortunately, such indirect determination of $\delta g$ is affected by large uncertainties because of the limited range in momentum transfer at fixed Bjorken-$x$ and almost entirely arbitrary input gluon distribution. In fact, the data suggests that such global fit with either positive or negative input gluon distributions provides equally good agreement. Recently, a high-precision measurement of the mid-rapidity in polarized pp collisions stringently constraint the above mentioned polarized parton distribution functions. The analysis of NNPDF collaboration [1] found an evidence for possibly large gluon helicity distribution, which is against the common belief that it is rather small.

In this work we use a QCD dynamic model of the parton distributions to predict the gluon helicity distribution in the nucleon without unknown input gluon distribution. Our model images that all gluons in the nucleon are radioactively generated from the intrinsic quarks beginning at a low resolution scale. Thus, we can predict the radioactive (un-polarized and polarized) gluon distributions provided the initial quark distributions are fixed by the well known spin structure functions. Such quark model for the nucleon was early proposed by [2] in 1977, but it is first realized for the unpolarized proton in our previous work [3], where the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [4] with the parton recombination corrections [5] is used to reproduce the unpolarized parton distributions of the nucleon at $Q^2 > 1 GeV^2$. Since the similar corrections of the parton recombination to the polarized DGLAP equation have been proposed in work [6], we can use these two modified evolution equations to dynamically predict the polarized gluon distribution in the nucleon.

Comparing with the global analysis via scaling violation, the gluon helicity distribution in this work is determined directly by the observed spin structure function $g_1(x, Q^2)$. We find that the contribution of the gluon helicity to the nucleon spin structure is surprisingly large, which in excess of previous estimations in theory. The reasons are as follows: (i) The shadowing effect of the gluon recombination in the evolution of the polarized parton distributions is weaker than that in the unpolarized case since $g\delta g << g^2$ at small $x$ in Eqs. (2.2) and (2.3). Therefore, much more strong polarized gluons are emitted by quarks inside a polarized proton through the long evolution length from $\mu^2$ to $Q^2 > 1GeV^2$; (ii) The positive contribution of the polarized gluon recombination, which is opposite to that of the unpolarized gluon recombination enhance the accumulation of the gluon helicity.
The QCD evolution of the parton distributions begins from a low bound state scale $\mu^2$ not only dynamically determine the polarized gluon distribution, but also exposes a novel spin-orbital structure of the nucleon in the light-cone frame, where the nucleon spin crisis has a possible solution.

The organization of this paper is as follows. We will present the DGLAP equation with the parton recombination corrections for the polarized parton distributions in Sec. 2 and Appendix. Then we predict the gluon helicity distribution in the nucleon in Sec.3. Where the comparisons of our predictions with the available experimental data are given. We find a large contribution of gluon helicity, therefore the significant orbital angular momentum of gluons is required to obtain a correct nucleon spin. In Sec. 4 a novel spin-orbital structure of the nucleon in the right-cone frame is described based on the quantitative calculations. Through a discussion of the helicity sum rules in the nucleon, we propose a solution about the nucleon spin crisis: the initial valence quarks inside a polarized nucleon in the light-cone frame have non-vanished orbital angular momentum, which changes the direction of the quark spin to deviate the polarized direction of the nucleon and gives $\Delta \Sigma < 1$. 
2 Nonlinear polarized QCD evolution equation

We use \( f_+(x, Q^2) \) and \( f_-(x, Q^2) \) to refer to parton \((f = q, \bar{q}, g)\) densities with positive and negative helicity which carry a fraction \(x\) of the nucleon momentum. The difference \(\delta f(x, Q^2) = f_+(x, Q^2) - f_-(x, Q^2)\) measures how much the parton of flavor \(f\) remembers its parent nucleon polarization. The ordinary spin averaged parton densities are given by \(f(x, Q^2) = f_+(x, Q^2) + f_-(x, Q^2)\).

The spin-dependent QCD evolution equation of parton distributions with parton recombination corrections was first derived by Zhu, Shen and Ruan in [6], it reads

\[
Q^2 \frac{dx \delta q_v(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} y \delta q_v(y, Q^2) \Delta P_{qq}(\frac{x}{y}),
\]

for flavor non-singlet quarks;

\[
Q^2 \frac{dx \delta q_i(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ y \delta q_i(y, Q^2) \Delta P_{qq}(\frac{x}{y}) + y \delta g(y, Q^2) \Delta P_{gg}(\frac{x}{y}) \right] - \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x \Delta P_{gg \rightarrow q}(x, y) [y g(y, Q^2) y \delta g(y, Q^2)] + \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x \Delta P_{gg \rightarrow q}(x, y) [y g(y, Q^2) y \delta g(y, Q^2)], \tag{2.1}
\]

for sea quarks;

\[
Q^2 \frac{dx \delta g(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[ y \sum_{i=1}^{2f} \delta q_i(y, Q^2) \Delta P_{qq}(\frac{x}{y}) + y \delta g(y, Q^2) \Delta P_{gg}(\frac{x}{y}) \right] - \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{dy}{y} x \Delta P_{gg \rightarrow g}(x, y) [y g(y, Q^2) y \delta g(y, Q^2)] + \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^x \frac{dy}{y} x \Delta P_{gg \rightarrow g}(x, y) [y g(y, Q^2) y \delta g(y, Q^2)], \tag{2.2}
\]

\(i f \ x \leq 1/2\), \(i f \ 1/2 \leq x \leq 1\).
\[
Q^2 \frac{d x \delta g(x, Q^2)}{dQ^2}
\]

\[
= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{d y}{y} \frac{x}{y} y \sum_{i=1}^{2f} \delta q_i(y, Q^2) \Delta P_{qq}(x, y) + y \delta g(y, Q^2) \Delta P_{gg}(x, y)
\]

\[
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{1/2} \frac{d y}{y} x \Delta P_{gg\rightarrow q}(x, y)[yg(y, Q^2) y \delta g(y, Q^2)], (i f \ 1/2 \leq x \leq 1)
\]

(2.3)

for gluon, where the factor \(1/(4\pi R^2)\) is from normalizing two-parton distribution, \(R\) is the correlation length of two initial partons, the linear terms are the standard spin-dependent DGLAP evolution and the recombination functions in the nonlinear terms are

\[
\Delta P_{gg\rightarrow g}(x, y) = \frac{27}{64} \frac{(2y - x)(-20y^3 + 12y^2x - x^3)}{y^5},
\]

(2.4)

\[
\Delta P_{gg\rightarrow q}(x, y) = \frac{1}{48} \frac{(2y - x)^2(-y + x)}{y^4}.
\]

(2.5)

The spin structure function \(g_1\) at leading order (LO) and \(Q^2 > 1 GeV^2\) is written as [5]

\[
g_1(x, Q^2) = \frac{1}{2} \sum_i e_i^2 [\delta q_i(x, Q^2) + \delta \overline{q_i}(x, Q^2)],
\]

(2.6)

where \(e_i\) are the electric charges of the (light) quark-flavors \(i = u, d, s\).

The solutions of Eqs (2.1-2.3) are coupled with the spin-averaged evolution equation, which is

\[
Q^2 \frac{d x q_v(x, Q^2)}{dQ^2}
\]

\[
= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{d y}{y} \frac{x}{y} y q_v(y, Q^2) P_{qq}(x, y),
\]

(2.7)

for valence quarks;

\[
Q^2 \frac{d x q_i(x, Q^2)}{dQ^2}
\]

\[
= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{d y}{y} \frac{x}{y} y q_i(y, Q^2) P_{qq}(x, y) + y g(y, Q^2) P_{gg}(x, y]
\]

\[
- \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_x^{1/2} \frac{d y}{y} x P_{gg\rightarrow q}(x, y)[yg(y, Q^2)]^2
\]

\[
+ \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{1/2} \frac{d y}{y} x P_{gg\rightarrow q}(x, y)[yg(y, Q^2)]^2, (i f \ x \leq 1/2),
\]

(2.8)
\[Q^2 \frac{dxq_i(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} [yq_i(y, Q^2)P_{qv}(\frac{x}{y}) + yg(y, Q^2)P_{vg}(\frac{x}{y})] + \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{1/2} \frac{dy}{y} x P_{gg\rightarrow q}(x, y)[yg(y, Q^2)]^2, \text{ if } 1/2 \leq x \leq 1, \quad (2.8)\]

for sea quarks;

\[Q^2 \frac{dxg(x, Q^2)}{dQ^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} [y \sum_{i=1}^{2f} q_i(y, Q^2)P_{qv}(\frac{x}{y}) + yg(y, Q^2)P_{vg}(\frac{x}{y})] - \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{1/2} \frac{dy}{y} x P_{gg\rightarrow g}(x, y)[yg(y, Q^2)]^2 + \frac{\alpha_s^2(Q^2)}{4\pi R^2 Q^2} \int_{x/2}^{1/2} \frac{dy}{y} x P_{gg\rightarrow g}(x, y)[yg(y, Q^2)]^2, \text{ if } x \leq 1/2, \quad (2.9)\]

for gluon, where the linear terms are the standard DGLAP evolution [4] and the recombination functions in the nonlinear terms are [5]

\[P_{gg\rightarrow g}(x, y) = \frac{9}{64} \frac{(2y - x)(72y^4 - 48xy^3 + 140x^2y^2 - 116x^3y + 29x^4)}{xy^5}, \quad (2.10)\]

\[P_{gg\rightarrow q}(x, y) = P_{gg\rightarrow \bar{q}}(x, y) = \frac{1}{96} \frac{(2y - x)^2(18y^2 - 21xy + 14x^2)}{y^5}. \quad (2.11)\]

The DGLAP equation with the parton recombination corrections at leading order will be applied to the low \(Q^2\) region, where various higher order corrections become larger. In principle, we need to consider all these contributions, while it’s beyond our ability. Some of them (for example, the contributions of vector meson components in the virtual photon, see our next work [7]) are irrelevant to the evolution of parton distributions due to the factorization schema, and we neglect these corrections in this work. On the other
hand, the higher order (NLO, NNLO...) QCD corrections should modify the splitting and recombination functions in the evolution equation. For them we take a following methodology and technique: if the leading order contributions (or including necessary lowest order corrections) to a given process are compatible with the experimental data, one can conjecture that these neglected higher order corrections to this process may cancellable each other, or they are successfully absorbed by finite number of the free parameters.
3 Dynamically radioactive gluon helicity distribution

We focus on the gluon distribution, its evolution is dominated by the valence quark distributions. Therefore, the corrections of the asymmetry sea distributions in the nucleon are neglected in this work.

As we know, many effective QCD theories describe the nucleon as a bound state by three quarks at its rest frame. The distributions of these quarks in the light-cone configuration have a similar character as the valence quark distributions observed at high \( Q^2 \) in DIS. Besides, the QCD evolution equation shows that either the second moment (i.e., the average momentum fraction) of the unpolarized gluon distribution or the first moment (i.e., the total helicity) of the polarized gluon distribution increase as \( Q^2 \) increasing. A natural suggestion is that all partons (valence quarks, sea quarks and gluons) at high \( Q^2 \) scale are evolved from three initial valence quarks via QCD dynamics. Such idea was firstly proposed in 1977 for unpolarized parton distributions by [2]. They assumed that the nucleon consisted of valence quarks at a low starting point \( \mu^2 \sim 0.064 GeV^2 \) (but is still in the perturbative region \( \alpha_s(\mu^2)/2\pi < 1 \) and \( \mu > \Lambda_{QCD} \)), and the gluons and sea quarks are radioactively produced at \( Q^2 > \mu^2 \) using the DGLAP equation. However, such natural input is failed due to too steep behavior of the predicted parton distributions at small \( x \) since a long evolution range from \( \mu^2 \) to \( Q^2 > 1 GeV^2 \). Recently the above naive idea was realized in the unpolarized DGLAP equation with the parton recombination corrections at LO approximation in [3], where the input distributions at \( \mu^2 = 0.064 GeV^2 \) were extracted through fitting \( F_2^p(x, Q^2) \) and \( F_2^n(x, Q^2) \) and they have been fixed as

\[
x u_v(x, \mu^2) = 24.3x^{1.98}(1 - x)^{2.06},
\]

\[
x d_v(x, \mu^2) = 9.10x^{1.31}(1 - x)^{3.8},
\]

while

\[
g(x, \mu^2) = 0, \quad q_i(x, \mu^2) = \overline{q}_i(x, \mu^2) = 0,
\]

and the parameters in Eqs. (2.8) and (2.9) are \( \Lambda_{QCD} = 0.204 GeV \) and \( R = 4.24 GeV^{-1} \). We plot these input distributions in Fig.1.

Similarly, using the polarized DGLAP equation with the parton recombination corrections Eqs. (2.1)-(2.3) and combining Eqs. (2.7)-(2.9), we fit \( g_1^p(x, Q^2) \) and \( g_1^n(x, Q^2) \) using the data [8] in Fig.2 and extract the input polarized valence quark distributions in the proton as

\[
\delta u_v(x, \mu^2) = 40.3x^{2.85}(1 - x)^{2.15},
\]
Figure 1: Input valence quark distributions at $\mu^2 = 0.064 GeV^2$ for polarized and unpolarized densities.

Figure 2: Fitting data [8] for $g_1$ at $Q^2 = 5 GeV^2$ using the input distributions Eqs. (3.1)-(3.6).
\[ \delta d_v(x, \mu^2) = -18.22x^{1.41}(1-x)^{1.0}, \]  

(3.6)

and

\[ \delta g(x, \mu^2) = 0, \quad \delta q_i(x, \mu^2) = \delta \bar{q}_i(x, \mu^2) = 0, \]  

(3.7)

they are plotted in Fig. 1. Note that Eqs. (3.3) and (3.7) don’t exclude scalar and longitudinal component of gluon field in the nucleon, but the parton model takes a special gauge—the physical axial gauge, where all observed gluons are physical transverse gluons.

![Figure 3: Predicted polarized gluon distribution $x\delta g(x, Q^2)$ in the proton at the different $Q^2$-scales, which show the accumulation of radiative polarized gluons at small $x$ in the evolution.](image)

We predict the gluon helicity distribution at different $Q^2$ in Fig. 3. The results clearly show the accumulation of polarized gluons at small $x$. We call it as the large gluon helicity effect.

There are several databases for the polarized parton distributions, which are extracted by the global fitting DIS data. We compare our results with the GRV distributions [9] in Fig. 4. The difference is obvious. It is not surprise that the gluon helicity distribution has large uncertainty, since the shape of the input gluon distribution is not constrained well enough by the DIS data alone.

In order to understand the large gluon helicity, we draw the evolution kernels $P_{gg}(z)$, $P_{gg}(z)$, $\Delta P_{gg}(z)$, $\Delta P_{gg}(z)$ and $yP_{gg\rightarrow g}(z)$, $yP_{gg\rightarrow q}(z)$, $y\Delta P_{gg\rightarrow g}(z)$, $y\Delta P_{gg\rightarrow q}(z)$ in Figs. 5 and 6. One can find that
Figure 4: Comparisons of our predicted polarized LO parton distributions at $Q^2 = 5 GeV^2$ with the GRV distributions [9].

Figure 5: The splitting functions.
(i) $P_{gg}(z) > 0$ and $\Delta P_{gg}(z) > 0$ imply that $\delta g(x, Q^2)$ is positive in our dynamic model;
(ii) Since $\Delta P_{gg}(z) < 0$ at small $z$, we have $d g_1(x, Q^2)/d \ln Q^2 \sim -\delta g(x, Q^2)$ at small $x$, i.e., a large positive $\delta g$ at small $x$ is expected to drive $g_1$ towards large negative values;
(iii) $\Delta P_{gg} \to g < 0$ and $\Delta P_{gg} \to q < 0$ lead the net positive corrections of the gluon fusion to the polarized parton distributions since a negative sign in the shadowing terms of Eq. (2.3). To illustrate this effect, in Fig.7 we present $x \delta g(x, Q^2)$ at $Q^2 = 1$ and $5 GeV^2$ with and without the corrections of gluon recombination corrections. One can find that the effects of the gluon recombination in the polarized gluon distribution is positive.

As we know that some approaches are planned to measure the gluon distributions. For example, the semi-inclusive deep inelastic scattering processes measure the $\delta g/g$ from helicity asymmetry in photon-gluon fusion. The COMPASS collaboration [10] have used this method and found a rather small value for $\delta g/g = 0.024 \pm 0.080 \pm 0.057$ at $x = 0.09$ and $Q^2 = 3 GeV^2$. However we think that although the value of $\delta g/g$ is small, the polarized gluon contribution to the spin of the nucleon may sizable since $g$ itself is large at small $x$. In order to compare with the data, one needs to assume a suitable form for the unpolarized gluon distribution $g(x, Q^2)$. Fortunately, both $\delta g(x, Q^2)$ and $g(x, Q^2)$ are calculated within a same dynamics in this work and we avoid a larger uncertainty in the determination of $g(x, Q^2)$. We compare our predicted $\delta g/g$ with the COMPASS data in Fig. 8.

The other direct probing of $\delta g$ is offered by jet and $\pi$ production in polarized proton-proton collisions available at BNL Relativistic Heavy Ion Collider (RHIC). A recent DSSV
Figure 7: Predicted polarized gluon distributions $x\delta g$ in the nucleon at $Q^2 = 1$ and $5 GeV^2$ with gluon recombination corrections (solid curves) and without gluon recombination corrections (dashed curves).

Figure 8: Comparison of dynamically predicted $\delta g/g$ with COMPASS data [10] at $Q^2 = 1$ and $5 GeV^2$. 
analysis [11] of high-statistics 2009 STAR [12] and PHENIX [13] data showed an evidence of non-zero gluon helicity in the proton. They found that the gluon helicity distribution of the proton is positive and away from zero in $0.05 < x < 0.2$, although the data presented has very large uncertainty in the small $x$ region. Figure 9 presents the comparisons of our predicted $g_1(x, Q^2)$ at $Q^2 = 10 GeV^2$ with the DSSV bounds with the 90% confidence level (C.L.) interval. Our results are beyond a up bound of the DSSV results, while they both suggest a sizable gluon polarization.

![Figure 9: Our predicted $x\delta g$ at $Q^2 = 10 GeV^2$. The dotted curves are the fits within the 90% confidence level (C.L.) interval by DSSV using RHIC measurements in [11].](image)

The NNPDF group has developed a new methodology [1] to unbiasedly extract polarized gluon distribution function. They use all essentially available data and get with an evidence for a positive gluon polarization in the medium and small $x$ region. This discovery is compatible with our results. Figure 10 shows a comparison of our predicted gluon helicity distribution with the NNPDF bounds. This example shows that a positive initial helicity distribution of the gluon at $\mu^2$ in the nucleon is impossible since it will obviously beyond the up-bound of the NNPDF analysis at $Q^2 \sim 10 GeV^2$. 
4 Discussions and Summary

The total helicities of partons are calculated by the first moments

\[ \Delta f(Q^2) = \int_0^1 dx \delta f(x, Q^2), \tag{4.1} \]

Note that our predicted \( \Delta q_s(Q^2) \) for the sea quarks is positive since the negative contributions from the asymmetric strange quarks are neglected in this work.

The contribution of quark helicity to the nucleon spin is \( \Delta \Sigma(Q^2 = 5 GeV^2) \approx 0.30 \) in our simple LO estimations. This value is compatible with the world average values \( \Delta \Sigma(Q^2 = 10 GeV^2) = 0.31 \pm 0.07 \) [14], and \( \Delta \Sigma(Q^2 = 5 GeV^2) = 0.333 \pm 0.011 \pm 0.025 \pm 0.028 \) [15], which include the higher order QCD corrections. This result supports our approximation: using a simple LO approximation with suitable input parameters to replace the complicated higher order corrections.

In Fig. 11 we plot the evolutions of \( \Delta \Sigma(Q^2) \) and \( \Delta g(Q^2) \) with increasing \( Q^2 \). We find that \( \Delta q_v(Q^2) \equiv 0.296 \), but \( \Delta q_s(Q^2) \) is slowly increasing from 0 at \( \mu^2 \) to 0.016 at \( Q^2 = 10^3 GeV^2 \) due to the parton recombination corrections. On the other hand, the contribution of gluon helicity \( \Delta g(Q^2) \) increases with \( \ln Q^2 \) beginning from zero at \( \mu^2 \), and it becomes large at \( Q^2 > 0.3 GeV^2 \).

The above mentioned \( \Delta \Sigma \) and \( \Delta g \) should be balanced by the orbital angular momenta of the quarks and gluons. For this sake, we write the nucleon helicity sum rule
Figure 11: Contributions of spin and orbital motion of the partons to the proton helicity and their evolutions with $Q^2$.

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma(Q^2) + \Delta g(Q^2) + \sum_q L_q^z(Q^2) + L_g^z(Q^2),
\]

(4.2)

where $L_{q,g}^z$ denote the contribution from orbital angular momentum. In our model the sum rule at $Q^2 = \mu^2$ is

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma(\mu^2) + \sum_q L_q^z(\mu^2),
\]

(4.3)

where

\[
\Delta \Sigma(\mu^2) = \Delta u_v(\mu^2) + \Delta d_v(\mu^2),
\]

(4.4)

\[
\sum_q L_q^z(\mu^2) = L_u^z(\mu^2) + L_d^z(\mu^2).
\]

(4.5)

On the other hand, the helicity sum rule of the polarized proton in its rest frame according to the constituent quark model is

\[
\frac{1}{2} = \frac{1}{2} \sum_q \Delta q^c,
\]

(4.6)

where $\Delta q^c$ denotes the helicity of the constituent quark $q$ with its "spin" $\vec s_q$. Comparing Eq. (4.6) with Eq. (4.3), we assume
\[
\frac{1}{2} \sum_q \Delta q^c = \frac{1}{2} \Delta \Sigma(\mu^2) + \sum_q L_q^z(\mu^2). \tag{4.7}
\]

One can find that we have \(\Delta \Sigma(\mu^2) < \Delta u^c + \Delta d^c\) if \(\sum_q L_q^z(\mu^2) > 0\).

Taking the SU(6) symmetry in the proton rest frame, we have

\[
\Delta u^c = \frac{4}{3}, \quad \Delta d^c = -\frac{1}{3}. \tag{4.8}
\]

From Eqs. (3.5) and (3.6) we know

\[
\Delta u_v(\mu^2) = 0.644, \quad \Delta d_v(\mu^2) = -0.348. \tag{4.9}
\]

Using Eq. (4.7) we obtain

\[
L_u^z(\mu^2) = 0.345, \quad L_d^z(\mu^2) = 0.007, \tag{4.10}
\]

it implies that a polarized proton at scale \(\mu^2\) has two rotating u-quarks, while the d-quark is located at the center of the proton since it has almost zero-orbital momentum.

We discuss the physics behind Eq. (4.7): Where are \(\sum_q L_q^z(\mu^2)\) from ? What reasons fix this coupling form of the spin-orbital angular momentum? We noticed that Ma [16] first suggested that the orbital angular momentum \(\sum_q L_q^z(\mu^2)\) in Eq.(4.7) may origin from the transversity distribution of the constituent quarks in the rest frame of the nucleon by the Melosh-Wigher rotation [17]. The Melosh-Wigher rotation is a pure kinematic effect in the frame transformation, we consider that this effect should keep the angular momentum conservation, for say,

\[
\frac{2}{3} \tilde{s}_u = \frac{2}{3} \hat{s}_u + \frac{1}{2} \tilde{L}_u, \tag{4.11}
\]

where \(\tilde{s}_u\) and \(\tilde{L}_u\) are the spin and orbital angular momentum of a u-valence quark at \(\mu^2\) in the light-cone frame of the polarized proton; 2/3 is from the \(SU(6)\)-distribution. Because the spin as an elemental physical quantity always has \(|\hat{s}_u| = |\tilde{s}_u| \equiv 1/2\), therefore, the orbital angular momentum \(\tilde{L}_u\) changes only the direction of the spin from \(\hat{s}_u\) (it also is the polarized direction of the proton) to \(\tilde{s}_u\). Under these constraint conditions, the spin and orbital motion of a quark are not independent. In fact, as shown in Fig. 12, once the value of \(L_q^z(\mu^2)\) is determined, the coupling angle between \(\tilde{s}_u\) and \(\tilde{L}_u\) in Eq. (4.11) is completely fixed and it leads the helicity \(\Delta u_v(\mu^2) < \Delta u^c\) in Eq. (4.7), i.e., the nucleon spin crisis.

Now let us consider the evolution of the sum rule (4.2) with \(Q^2\). The evolution equation for the quark and gluon orbital angular momenta at the leading order approximation was derived by Ji, Tang and Hoodbhoy in [18], it reads
Figure 12: A schematic diagram of the proton spin crisis: Orbital angular momentum $\vec{L}_u$ of a valence $u$-quark at a bound state scale $\mu^2$ impels the direction of the $u$-quark spin ($\vec{s}_u$) to deviate the polarized direction of the proton ($\vec{s}_c$) and gives $\Delta \Sigma < 1$. Note that in this example, the spin-orbital coupling form of a $u$-quark is completely fixed by the value of $\Delta u_v(\mu^2) = 0.644$.

\[
\frac{d\sum_q L^z_q(t)}{dt} = \frac{\alpha_s(t)}{2\pi} \left[ -\frac{4}{3} C_F \sum_q L^z_q(t) + \frac{n_f}{3} L^z_g(t) \right] + \frac{\alpha_s(t)}{2\pi} \left[ -\frac{2}{3} C_F \Sigma(t) + \frac{n_f}{3} \Delta g(t) \right],
\]

\[
\frac{dL^z_g(t)}{dt} = \frac{\alpha_s(t)}{2\pi} \left[ 4 C_F \sum_q L^z_q(t) - \frac{n_f}{3} L^z_g(t) \right] + \frac{\alpha_s(t)}{2\pi} \left[ -\frac{5}{6} C_F \Sigma(t) - \frac{11}{2} \Delta g(t) \right],
\]

where $C_F = 4/3$, $n_f$ is the number of active quark flavors, $t = \ln(Q^2/\Lambda^2)$ and $t_0 = \ln(\mu^2/\Lambda^2)$. The solutions are

\[
\sum_q L^z_q(Q^2) = -\frac{1}{2} \Delta \Sigma(Q^2) + \frac{1}{2} \frac{n_f}{16 + 3n_f} + \left( \frac{t}{t_0} \right)^{-2(16+3n_f)/9\beta_0} \left[ \sum_q L^z_q(\mu^2) + \frac{1}{2} \Delta \Sigma(\mu^2) - \frac{1}{2} \frac{3n_f}{16 + 3n_f} \right],
\]

and

\[
L^z_g(Q^2) = -\Delta g(Q^2) + \frac{1}{2} \frac{16}{16 + 3n_f} + \left( \frac{t}{t_0} \right)^{-2(16+3n_f)/9\beta_0} \left[ L^z_g(\mu^2) + \Delta g(\mu^2) - \frac{1}{2} \frac{16}{16 + 3n_f} \right],
\]

where $\beta_0 = 11 - 2n/3$. Because of $\Delta \Sigma(\mu^2) = 0.296$, $L^z_q(\mu^2) = 0$, $\Delta g(\mu^2) = 0$, we can fix

\[
\sum_q L^z_q(\mu^2) = 1/2 - \Delta \Sigma(\mu^2)/2 = 0.352.
\]

Table. The contributions of various components to the proton helicity at different $Q^2$. vskip 1.0cm
In Fig. 11 we add the curves of $\sum_q L_q^z(Q^2)$ and $L_g^z(Q^2)$ taking $n_f = 3$. These results describe a following novel spin-orbital picture of the proton in the light-cone frame: The proton is mainly constructed by one d-valence quark and two u-valence quarks, the d-quark is located at the center of the proton, and two u-quarks rotating with $\sum_q L_q^z(\mu^2) \simeq 0.35$ around the d-quark at a bound state scale, with the $Q^2$ increasing the valence quarks radiate gluons and then following with sea quarks, the former build fast rotating glue cloud (see Table), but their rotating direction is opposite to the u-quarks. Remind that the above mentioned possible orbital angular momentum of the partons in the polarized proton can be checked in the special experiments [19].

According to the above mentioned discussions, we suggest a possible picture of the nucleon spin. The nucleon at rest frame is constructed by three valence quarks with the scalar and longitudinal gluons on the non-perturbative QCD vacuum, which are confined inside a small range and have intrinsic transverse momentum. The nucleon spin in this case is completely composed of the quark helicity. While in DIS process the probe with higher $Q^2$-scale will see other picture of the nucleon: the transverse gluons are omitted by quarks at $Q^2 > \mu^2$, while the scalar and longitudinal gluons are hidden in the axial gauge. The nucleon spin is composed of the helicity and orbital angular momenta of quarks and gluons via QCD evolution, where the primary orbital angular momentum of the quarks are produced by the intrinsic transverse momentum of the valence quarks through the Melosh-Wigner rotation.

Using a nonlinear QCD evolution equation-the DGLAP equation with the parton recombination corrections, we find that the contribution of the polarized gluons to the proton spin is much larger than the predictions of most other theories. This result is compatible with the recent NNPDF analysis and suggests a significant orbital angular momentum of gluons to balance the gluon helicity. In concretely, the proton helicity at a bound state scale $\mu^2$ is composed by $\sim 30\%$ quark helicity and $\sim 70\%$ orbital angular momentum of the quarks, where two u-valence quarks are rotating around a d-valence quark in the polarized proton. With increasing $Q^2$, the omitted gluons accumulate a larger positive helicity, which is mainly balanced by their orbital momentum. Therefore, there are two rotating groups in a polarized proton at $Q^2$: a slower quark group and a faster gluon group with opposite rotating directions. In particularly, the initial valence quarks inside a polarized proton in the light-cone frame have non-vanished orbital angular momentum, which cause the direction of the quark spin to deviate from the polarized

| $Q^2$ | $0.064 GeV^2$ | $1 GeV^2$ | $10 GeV^2$ | $100 GeV^2$ |
|-------|---------------|-----------|-------------|-------------|
| $\frac{1}{2} \Delta \Sigma$ | 0.148 | 0.149 | 0.151 | 0.153 |
| $\sum_q L_q^z$ | 0.352 | 0.124 | 0.096 | 0.080 |
| $\Delta g$ | 0 | 1.056 | 1.993 | 2.889 |
| $L_g^z$ | 0 | -0.829 | -1.74 | -2.622 |
| Total | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
direction of the nucleon and gives $\Delta \Sigma < 1$, which is the so called nucleon spin crisis.
From Ref.[6], we have LO polarized gluon recombination functions

\[ P_{g^+g^+\rightarrow g^+} = \frac{9}{4} \frac{(x_1 + x_2 - x)^3}{x_2^2(x_1 + x_2)^3 x_1^2} (x_1^4 - 2 x_1^3 x + x_1^2 x^2 + x_2^4 - 2 x_2^3 x + x_2^2 x^2) \]

\[ + x_1^2 x_2^2 - x_1^2 x_2 x - x_1 x_2^2 x + x_1 x_2^2 x^2) \]  
(A.1)

\[ P_{g^+g^-\rightarrow g^+} = \frac{9}{4} \frac{(x_1 + x_2 - x)}{x_2^2(x_1 + x_2)^3 x_1^2} (6 x_1^4 x_2 x + 6 x_1^3 x_2^2 x - 3 x_1^3 x_2 x^2 - 7 x_1^2 x_2 x^3 \]

\[ + 11 x_1^2 x_2^2 x^2 + 6 x_2^4 x_1 x + 6 x_2^3 x_1^2 x - 3 x_2^3 x_1 x^2 - 7 x_2^2 x_1 x^3 + 2 x_1 x_2 x^4 \]

\[ + x_1^6 + x_2^6 + 2 x_1^5 x_2 + 2 x_1^4 x_2^2 - x_1^4 x^2 - 2 x_1^3 x^3 + 2 x_1^2 x^4 + 2 x_2 x_1^5 \]

\[ + 2 x_2^4 x_1^2 - x_2^4 x^2 - 2 x_2^3 x^3 + 2 x_2^2 x^4 + 2 x_1^3 x_2 x^3) \]  
(A.2)

\[ P_{g^+g^-\rightarrow g^-} = \frac{9}{4} \frac{(x_1 + x_2 - x)}{x_2^2(x_1 + x_2)^3 x_1^2} (141 x_1^7 x_2 x + 42 x_1^4 x_2^2 - 100 x_1^8 x_2 x \]

\[ + 19 x_1^5 x_2^2 - 5 x_2^3 x_1 x + 39 x_1^2 x_2^4 + 137 x_1^3 x_2^2 x - 86 x_1^6 x_2 + 26 x_1^9 x_2 + 2 x_1^5 x_2^5 - 5 x_1^4 x_2^4 \]

\[ + 155 x_1^4 x_2^2 x - 40 x_1 x_2 x^2 x^2 - 212 x_1^6 x_2^3 \]

\[ - 124 x_1^3 x_2^3 x - 196 x_1^1 x_2^3 x + 79 x_1^2 x_2^2 x^2 + 6 x_1^4 x_2^4 x + 44 x_2 x_1^2 x \]

\[ - 177 x_1^5 x_2^3 x - 209 x_1^7 x_2 x^2 - 40 x_1^3 x_2^4 x - 33 x_1^3 x_2^3 x^2 \]

\[ + 2 x_2^6 x_2^2 x^2 + 319 x_1^5 x_2^3 x^2 + 291 x_1^6 x_2^2 x^2 - 35 x_1^7 x_2 x^2 + 13 x_2 x_1^5 x_2 \]

\[ + 64 x_2 x_1^3 + 6 x_2^4 x_1^2 + 26 x_1^9 x_2 + 57 x_1^8 x_2 + 64 x_1^7 x_2^2 + 34 x_1^6 x_2^3 + 4 x_2 x_1^4 x_2 \]

\[ + 12 x_1^5 x_2^5 + 34 x_1^4 x_2^6 - 4 x_2^6 x_4 + 2 x_2^5 x_3 + 30 x_1^8 x_2^2 \]

\[ - 20 x_1^7 x_2 + 5 x_1^6 x_4 - 20 x_1^9 x_2 \]  
(A.3)

\[ P_{g^+g^-\rightarrow g^-} = \frac{9}{4} \frac{(x_1 + x_2 - x)}{x_2^2(x_1 + x_2)^3 x_1^2} (-31 x_1^7 x_2 x + 27 x_1^4 x_2^2 - 7 x_1^8 x_2 + 9 x_1^5 x_2^2 \]

\[ - 104 x_1^2 x_2 x + 54 x_1^2 x_2 x^2 - 206 x_1^3 x_2^6 x + 3 x_1^6 x_2 x + 26 x_2^9 x_1 + 5 x_2^{10} \]

\[ + 5 x_2^{10} + 115 x_1^4 x_2^4 - 211 x_1^2 x_2^5 x^3 + 125 x_1^6 x_2 x^3 - 192 x_1^3 x_2^3 x^2 \]

\[ - 92 x_1^4 x_2 x^3 + 319 x_1^2 x_2^6 x^2 - 25 x_1^4 x_2^5 x - 217 x_2^7 x_2^6 x + 136 x_1^5 x_2^4 x \]

\[ - 24 x_1^5 x_2^3 x^2 + 34 x_1^7 x_2 x^2 + 36 x_1^3 x_2^3 x^2 + 307 x_1^3 x_2^5 x^2 - 106 x_2^6 x_1^3 \]

\[ - 41 x_1^5 x_2^6 x^3 - 67 x_1^6 x_2^2 x^2 + 157 x_2^7 x_1^2 x + 27 x_2^5 x_1 x + 64 x_2^7 x_1^3 \]

\[ + 57 x_2^8 x_1^2 + 26 x_1^9 x_2 + 57 x_1^8 x_2 + 64 x_1^7 x_2^2 + 34 x_1^6 x_2^3 + 12 x_1^5 x_2^5 + 34 x_1^4 x_2^6 - 20 x_2^9 x + 5 x_1^6 x_4 - 20 x_2^7 x_3^3 + 30 x_2^8 x_2^2 - 5 x_1^8 x_2^2 + 2 x_1^7 x_3^3 \]

\[ + 2 x_1^6 x^4 - 4 x_1^9 x \]  
(A.4)

Setting \( x_1 = x_2 = y \), one can find

\[ \Delta P_{g\rightarrow g} = [P_{g^+g^+\rightarrow g^+} - P_{g^+g^-\rightarrow g^-} - P_{g^+g^-\rightarrow g^+} + P_{g^+g^-\rightarrow g^-}] \]
\[
= [P_{g+g\rightarrow g+} - P_{g+g\rightarrow g-} + P_{g+g\rightarrow g+} - P_{g+g\rightarrow g-}]
= \frac{27}{64} \left(2y - x\right) (-20y^3 + 12y^2x - x^3)
\]

Similar,

\[
P_{g+g\rightarrow q+} = \frac{1}{12} \left(\frac{1}{x_1 + x_2 - x}\right)^2 \left(4x_1^4 + 7x_1^3x_2 - 8x_1^3x + 2x_1^2x_2^2 - 6x_1^2xx2
+ 4x_1^2x_1^2 + 4x_1^2x_1^2 - x_1x_1^3 + 2x_1x_1^2 - x_1x_1^2\right)
\]

\[
P_{g+g\rightarrow q-} = \frac{1}{12} \left(\frac{1}{x_1 + x_2 - x}\right)^2 \left(4x_1^2x_1^2 + 4x_1^2x_1^2 + 8x_1x_1^3 - 8x_1x_1^2 + 4x_1^2
- 8x_1x_1^2 + 4x_1^2x_1^2 - x_1x_1^2\right)
\]

\[
P_{g+g\rightarrow q+} = \frac{1}{12} \left(\frac{1}{x_1 + x_2 - x}\right)^2 \left(4x_1^6x_1^2 + 4x_1^4x_1^4 - 8x_1^3x_2 + 24x_1^6x_2^2 + 16x_1^5x_2^3
+ 16x_1^7x_2 + 4x_1^6 + 24x_1^2x_1^3 + 4x_1^2x_2^6 - 10x_1^3x_1^3 + 33x_1^2x_1^2 - x_1x_1^2
- 8x_1^4x_1^2 + 24x_1^5x_2^2 + 33x_1^4x_1^2x_2 + 14x_1^3x_2x_2^2 - 40x_1^6x_2
- 53x_1^5x_1^2 - 8x_1^4x_2x_1^2 - 9x_1x_1^2\right)
\]

\[
P_{g+g\rightarrow q-} = \frac{1}{12} \left(\frac{1}{x_1 + x_2 - x}\right)^2 \left(24x_1^6x_1^2 + 16x_2^7x_1 - 8x_2^7x + 16x_2^5x_1^3 + 4x_2^8
+ 4x_1^6x_2^2 + 4x_1^4x_2^4 + 6x_1^2x_1^2 + 4x_1^2x_2^6 + 8x_1^3x_1^2x_2 + 15x_1^2x_1^2x_2
- 13x_1^4x_1^2 + 24x_1^5x_2^2 + 51x_1^4x_2^2x_2 - 35x_1^3x_2x_2^2 + x_1^5x_1^2
- 35x_2x_1^2 - 22x_1x_2^2\right)
\]

Setting \(x_1 = x_2 = y\), we have

\[
\Delta P_{gg\rightarrow q} = [P_{g+g\rightarrow q+} - P_{g+g\rightarrow q-} - P_{g+g\rightarrow q+} + P_{g+g\rightarrow q-}]
= [P_{g+g\rightarrow q+} - P_{g+g\rightarrow q-} + P_{g+g\rightarrow q+} - P_{g+g\rightarrow q-}]
= \frac{1}{48} \left(\frac{(2y - x)^2(-y + x)}{y^4}\right)
\]

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