Bose-Einstein condensation is known to occur in weakly and strongly interacting systems such as dilute gases and liquid $^4$He [1]. Since more than ten years it is also theoretically shown that for symmetric nuclear matter, below a critical density, $\alpha$-particle condensation is favored [2,4]. It is expected to occur at densities smaller than a fifth of the nuclear saturation density. At higher densities the $2\text{-nucleon deuteron condensation}$ prevails over the 4-nucleon condensation. This new possible phase of nuclear matter may have its counterpart in low-density states of self conjugate lighter nuclei, in the same way as superfluid nuclear and neutron matter. This means that under some circumstances, the alpha condensation, i.e. bosonic properties, might dominate over the nucleon properties even in finite nuclei. Thus, by showing that the Hoyle state (i.e. the first excited state $0^+$ of $^{12}\text{C}$) and the sixth $0^+$ state at 15.097 MeV of $^{16}\text{O}$ are described by $\alpha$-particle condensate type functions, Refs. [3,4] advance the idea that these states are candidates to observe $\alpha$ condensation. A common feature of these states is their diluteness. For instance, calculations [5] and recent experimental data [6] show that the rms radius of the Hoyle state exceeds by 45% the radius of $^{12}\text{C}$ in its ground-state. On the other hand the Hoyle state is known to play a decisive role in stellar nucleosynthesis of $^{12}\text{C}$ with the assumption that the only contribution to the alpha-particle width comes from the $^8\text{Be}_{g.s.} + \alpha$ reaction [7]. At present an experimental upper limit of 4% exists for the contribution from the direct $3\alpha$ channel to the alpha width [8]. A phase space calculation suggests that the probability ratio of the $3\alpha$ to the $\alpha + ^{9}\text{Be}$ decay process is $5 \times 10^{-4}$ [9]. However such a factor may be enhanced in relation with the peculiar properties of the Hoyle state previously discussed.

According to the present understanding, the compatibility of a nuclear state with $\alpha$-particle condensation may be judged upon its excitation energy close to the $N\alpha$ threshold, the emission simultaneity and both the low kinetic energy and kinetic energy dispersion. Then, it becomes clear that probably the most appropriate methodology should involve high velocity reaction products in the laboratory detected by a high granularity-high solid angle particle array, as needed in multifragmentation reaction studies [10].

The Letter reports on results obtained for the nuclear...
reaction $^{40}$Ca+$^{12}$C at 25 MeV per nucleon incident energy performed at INFN, Laboratori Nazionali del Sud in Catania, Italy. The beam impinging on a thin carbon target (320$\mu$g/cm$^2$) was delivered by the Superconducting Cyclotron and the charged reaction products were detected by the CHIMERA 4π multidetector \[13\]. The beam intensity was kept around $10^7$ ions/s to avoid pile-up events. CHIMERA consists of 1192 silicon-CsI(Tl) telescopes mounted on 35 rings covering 94% of the solid angle, with polar angle ranging from 1° to 176°. Among its most interesting characteristics, we mention the low detection and identification thresholds for light charged particles (LCP) and the very high granularity at forward angles. The mass and charge of the detected nuclei were determined by standard time of flight (TOF) for LCP stopped in silicon detectors and $\Delta E - E$ ($Z > 5$) and shape identification ($Z \leq 5$) techniques for charged products stopped in CsI(Tl). In particular $^8$Be nuclei (two equal-energy Os hitting the same crystal) were identified in CsI(Tl) \[14\]. The energy of detected nuclei was measured by the Si detectors calibrated using proton, carbon and oxygen beams at various energies ranging from 10 to 100 MeV. For $Z = 2$, dedicated energy calibrations of the fast component of CsI(Tl) light was realized using the time of flight (TOF). Though high-quality TOF charts allowing for a direct calibration of the CsI light exist only for 60% of the total number of modules, we have finally managed to calibrate more than 95% of modules from 1° to 62°. The modules for which TOF information was poor or missing were calibrated by comparing the fast component distribution with the benchmark distribution of the corresponding ring built out the telescopes with excellent TOF.

Invariant $v_{par} - v_{per}$ plots show the binary character of the collisions with the formation of two emitting sources, a quasi-target (QT) and a quasi-projectile (QP). These latter may be easily separated according to the smaller or larger velocity of reaction products with respect to $v_{proj}/2$. After a first event filtering according to the total $\alpha$ multiplicity ($m_{\alpha} \geq 3$) and the total detected charge ($Z_{tot} \leq Z_P + Z_T = 26$), we have focused exclusively on QP decay products with $m_{\alpha} \geq 3$.

The power of correlation functions was extensively discussed in the literature \[15\]–\[18\]. They can furnish space-time information taking advantage of proximity effects induced by Coulomb repulsion, reveal excited states (whose decay produces the detected correlated particles) and enlighten any production of events or subevents with specific partitions. In this Letter various correlation functions have been used: two-particle correlation functions to first judge the quality of energy calibrations and multi-particle correlation functions to identify possible $\alpha$-particle condensation states and define their de-excitation characteristics.

Two- or multiple-particle correlation functions, with only one variable $E$ are defined as:

$$1 + R(E) = \frac{Y_{corr}(E)}{Y_{uncorr}(E)}, \quad (1)$$

where the role of the generic variable $E$ is equivalently played by the total kinetic energy of the particles of interest in their center-of-mass frame $E_{tot}$ or by the excitation energy of their emitting source/state, $E_{ex} = E_{tot} - Q$. In Eq. (1) $Y_{corr}$, the correlated yield spectrum, is constructed with the considered particles in the same event and $Y_{uncorr}$ stands for the uncorrelated yield spectrum constructed by taking particles in different events. In addition to these, by comparing the width of the peaks of the correlation function with the natural width of the excited states, one can estimate the distortions caused by the finite granularity. Fig. (1) presents the two-particle correlation functions for $\alpha - \alpha$ (a) and $\alpha - d$ (b) in terms of total kinetic energy and, respectively, excitation energy. The error bars have been calculated taking into account the statistical errors on both the correlated and uncorre-
related spectra. For $\alpha - \alpha$ we considered the QP event sample described in the paragraph above. For $\alpha$-d, we have additionally set the deuteron multiplicity to at least 1. One may see that the $\alpha - \alpha$ correlation function shows a narrow peak centered at 78 keV ($\Gamma=88$ keV) which corresponds to the ground state of $^8$Be ($Q=-92$ keV) and a much broader peak centered at 3.17 MeV ($\Gamma=4.80$ MeV), corresponding to the first excited state at 3.03 MeV. For $\alpha$-d we get a well formed peak centered at the excitation energy 2.21 MeV ($\Gamma=0.36$ MeV) which definitely corresponds to the first excited state of $^6$Li at 2.186 MeV. The broadening of correlation peaks is the genuine consequence of detector finite granularity. The finite granularity is responsible also for a certain imprecision in the determination of the energy. To illustrate this, Fig. 1 presents the correlation functions obtained under different hypotheses on the angle under which the detected particles were emitted. In the first case (solid symbols) we considered that all the particles which hit a certain module have been emitted under the angle corresponding to the geometrical center of that module. An alternative solution is to attribute to each particle a random angle in the domains allowed by the geometrical extension of the detector (open symbols). As we have noticed that the first hypothesis leads to smoother distributions and does not introduce extra broadening, the fits are done on the corresponding curves. This first hypothesis will be used for what follows. Given the complexity of the apparatus, one may conclude that the energy calibration is excellent and proceed to further spectroscopic analyses.

To do this, we exploit multi-particle correlation functions as shown in Fig. 2 for $3\alpha$ (a) and $4\alpha$ (b) emissions. In the first case we consider the QP events with $m_\alpha = 3$ and in the second case (poorer statistics) we consider QP events with $m_\alpha \geq 4$. The error bars are calculated considering only the statistical errors of the correlated spectra. Indeed errors on the uncorrelated spectra have been reduced to negligible values by increasing the number of uncorrelated events as compared to correlated events. The $3\alpha$ correlation function with standard event mixing for building the uncorrelated yield spectrum (i.e. each particle belongs to a different event, solid symbols) shows two peaks, at $E_{ex}^\alpha=7.61$ MeV ($\Gamma=0.18$ MeV) and 9.62 MeV ($\Gamma=1.33$ MeV). They correspond respectively to the Hoyle state ($E_{exp}^\alpha=7.654$ MeV, $\Gamma_{exp}=8.5$ eV) and to the complex excited region of $^{12}$C, characterized by the strong $E_{exp}^\alpha=9.64$ MeV, $3^-$ state and by the broad $E_{exp}^\alpha=10.3$ MeV, $0^+$ state submerging a possible $2^+$ state at 9.7 MeV. To have a first indication on the extent to which these states may be considered candidates for $\alpha$-particle condensation, it is mandatory to check whether the decay is simultaneous or if it proceeds partially via $^8$Be. To do this, we adopt the procedure proposed in Ref. [21] and plot with open symbols the correlation function obtained with partial event mixing (i.e. the uncorrelated yield spectrum is built by selecting two $\alpha$ particles from the same event and the third one from a different event). As one may see, the two peaks survive, but while the peak at higher excitation energy is almost unchanged, the magnitude of the Hoyle state is diminished. This is a qualitative indication that decays proceeding via $^8$Be, $^{12}$C, $2 \times ^8$Be intermediate states. The arrows correspond to centroids of Breit-Wigner distributions (solid lines).

![Figure 2](image-url)
In conclusion, the nuclear reaction \(^{40}\text{Ca}+^{12}\text{C}\) at 25 MeV/nucleon bombarding energy was used to produce states theoretically predicted as \(\alpha\)-particle condensate states. Supposing that equal values of kinetic energy of the emitted \(\alpha\)-particles represent a sufficient criterion for deciding in favor of \(\alpha\)-particle condensation, we found 39 events corresponding to the direct de-excitation of the Hoyle state of \(^{12}\text{C}\) which satisfy the criterion. It also clearly appears that the major part of Hoyle states deexcite via the \(^8\text{Be}+\alpha\) channel. Though the poor statistics prevents drawing a definite conclusion in the case of \(^{16}\text{O}\), we mention the existence of a small number of events originating from the decay of the 15.1 MeV state which

\[ 1 + R(\sigma_{E_{\alpha}}, < E_{\alpha}>) = \frac{Y_{\text{corr}}(\sigma_{E_{\alpha}}, < E_{\alpha}>) }{Y_{\text{uncorr}}(\sigma_{E_{\alpha}}, < E_{\alpha}>)} \quad (2) \]

Here, for a given alpha multiplicity, the numerator is the yield of events with given average kinetic energy of alpha particles, \(< E_{\alpha} >\), and given root mean square, \(\sigma_{E_{\alpha}}\).
fulfill the present criteria. An experiment with higher statistics is planned to better study the $^{16}$O case.

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