No deconfinement in QCD?

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At a critical temperature QCD in the chiral limit undergoes a chiral restoration phase transition. Above the phase transition the quark condensate vanishes. The Banks-Casher relation connects the quark condensate to a density of the near-zero modes of the Dirac operator. In the Nambu-Goldstone mode the quasi-zero modes condense around zero, \( \lambda \rightarrow 0 \), and provide a nonvanishing quark condensate. The chiral restoration phase transition implies that above the critical temperature there is no any longer a condensation of the Dirac modes around zero. If a \( U(1)_A \) symmetry is also restored and a gap opens in the Dirac spectrum then the Euclidean correlation functions are \( SU(2N_f) \supset SU(N_f)_L \times SU(N_f)_R \times U(1)_A \)- symmetric. This symmetry implies that a free (deconfined) propagation of quarks in Minkowski space-time that perturbatively interact with unconfined gluons is impossible. This means that QCD above the critical temperature is not of a quark-gluon plasma origin and has a more complicated structure.

I. INTRODUCTION

Classically QCD with \( N_F \) degenerate flavors in a finite volume \( V \) and without exact zero modes of the Dirac operator (which are irrelevant in the \( V \rightarrow \infty \) limit) has a \( SU(2N_f) \supset SU(N_f)_L \times SU(N_f)_R \times U(1)_A \) symmetry [1]. A symmetry of hadrons in this case is \( SU(2N_F) \times SU(2N_F) \) for mesons and \( SU(2N_F) \times SU(2N_F) \times SU(2N_F) \) for baryons, which is a model-independent, pure analytical statement. The axial anomaly breaks the \( U(1)_A \) symmetry, and consequently also the \( SU(2N_F)_L \supset SU(N_f)_L \times SU(N_f)_R \) symmetry. In the thermodynamic limit \( V \rightarrow \infty \) the lowest lying eigenmodes of the Dirac operator condense around zero (the so-called near-zero modes) and provide according to the Banks-Casher relation a nonvanishing quark condensate. If effects of anomaly and of spontaneous breaking of chiral symmetry are encoded in the same near-zero modes, then a truncation of the near-zero modes should lead to a large symmetry of hadrons mentioned above. It explains naturally previous lattice observations of emergence of a symmetry of hadrons, that is larger than the chiral \( SU(N_f)_L \times SU(N_f)_R \times U(1)_A \) symmetry of the QCD Lagrangian, after an artificial subtraction of the near-zero modes of the Dirac operator [2, 3].

These results have a very nontrivial implication for QCD above the chiral restoration phase transition at a critical temperature \( T_c \). Here we discuss the chiral limit, where QCD undergoes a chiral restoration phase transition [8] (at finite quark masses the phase transition converts into a fast cross-over, as it follows from lattice measurements).

In this note we address QCD at zero chemical potential, because in this case there exists a rigorous connection between the quark condensate of the vacuum and a density of modes of the Dirac operator around zero - the Banks-Casher relation. In the Nambu-Goldstone mode of chiral symmetry, i.e. below the phase transition, the modes of the Dirac operator condense around zero, \( \lambda \rightarrow 0 \), and provide a nonvanishing quark condensate.

Above the phase transition the quark condensate of the vacuum vanishes. If in addition the \( U(1)_A \) symmetry is also restored and a gap opens in the Dirac spectrum around \( \lambda = 0 \) [10, 11], then the Euclidean QCD correlation functions become \( SU(2N_f) \) symmetric. Such a symmetry prohibits in Minkowski space-time a propagation of a free deconfined massless quark that perturbatively interact with gluons, because the magnetic interaction manifestly breaks the \( SU(2N_f) \) symmetry. The only possibility is that the chirally symmetric quarks are confined into \( SU(2N_F) \times SU(2N_F) \) and \( SU(2N_F) \times SU(2N_F) \times SU(2N_F) \) symmetric "hadrons".

II. \( SU(2N_F) \) HIDDEN CLASSICAL SYMMETRY OF QCD

In this section we review some results of ref. [1]. Non-perturbatively QCD is defined in Euclidean space-time in a finite box with a volume \( V \) with the lattice ultraviolet regularization. Consider the Lagrangian in Euclidean space-time for \( N_F \) degenerate quark flavors in a given gauge background,

\[
\mathcal{L} = \Psi^\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x) \tag{1}
\]

with

\[
D_\mu = \partial_\mu + ig\frac{t^a}{2}A^a_\mu, \tag{2}
\]

where \( A^a_\mu \) is the gluon field configuration and \( t^a \) are the \( SU(3) \)-color generators. Note that in Euclidean space-time only the Hermitian conjugation, \( \Psi^\dagger(x) \), can be used.

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See also the opposite statement [12] and its critique in ref. [11]. We will assume that conclusions of the JLQCD collaboration [10, 11] are correct [13].
The nonzero eigenvalues come in pairs \(\pm \lambda_n\), because

\[
i\gamma_\mu D_\mu \Psi_n(x) = -\lambda_n \Psi_n(x).
\]

The nonzero eigenvalues are defined through the following set of 15 generators:

\[
\{ (\tau^a \otimes \mathds{1}_D), (\mathds{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i) \},
\]

where \(\tau^a\) are isospin Pauli matrices. If the \((2N_F)^2 - 1\) dimensional rotation vector \(\epsilon\) is a constant for the whole 3+1-dim space, then the corresponding transformation is global, while with the space-dependent rotation \(\epsilon(x)\) it is local. Obviously, the Lagrangian (1) does not have these symmetries, because the Dirac operator does not commute with the \(SU(2)_{CS}\) generators.

They are chiral, \(L\) or \(R\). Given standard antiperiodic boundary conditions for fermions along the time direction the zero modes appear only in gauge configurations with a nonzero global topological charge. The difference of numbers of the left-handed and right-handed zero modes is fixed according to the Atiyah-Singer theorem by the topological charge of the gauge configuration. Consequently there is no one-to-one correspondence of the left- and right-handed zero modes: The zero modes induce an asymmetry between the left and the right at nonzero global topological charge. The \(SU(2)_{CS}\) chiralspin rotations (except for pure \(U(1)_A\) rotations) mix the left- and right-handed Dirac spinors. Such a mixing can be defined only if there is a one-to-one mapping of the left- and right-handed Dirac spinors in the Hilbert space. In other words the zero modes explicitly violate the \(SU(2)_{CS}\) invariance, see Appendix for details. The zero modes are the precise reason for absence of the \(SU(2)_{CS}\) invariance in the exponential (classical) part of the partition function (6).

It is well understood, however, that these exact zero modes are completely irrelevant since their contributions to the Green functions and observables vanish in the thermodynamic limit \(V \to \infty\) as \(1/V\), see e.g. refs. \[14–16\]. Consequently, in the finite-volume calculations we can ignore (or subtract) the exact zero-modes.

Now we will analyse symmetry properties of the partition function above assuming that there are no exact zero modes. The \(SU(2)_{CS} \supset U(1)_A\) (\(CS\)- chiralspin) transformations are defined as \[14\]

\[
\Psi \to \Psi' = e^{\frac{i}{2}\Sigma} \Psi,
\]

with the following generators

\[
\Sigma = \{ \gamma^4, i\gamma^5\gamma^4, -\gamma^5 \},
\]

that form an \(SU(2)\) algebra

\[
[\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk} \Sigma^k.
\]

The \(\gamma^4\) and \(\gamma^5\gamma^4\) matrices mix the right- and left-handed components of the fermion fields.

We can combine the \(SU(2)_{CS}\) rotations with the flavor \(SU(N_F)\) transformations into one larger group, \(SU(2N_F)\). In the case of two flavors the \(SU(4)\) transformations

\[
\Psi \to \Psi' = e^{ie T/2} \Psi,
\]

are defined through the following set of 15 generators:

\[
\{ (\tau^a \otimes \mathds{1}_D), (\mathds{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i) \},
\]

are the following form:

\[
\bar{\Psi} \Psi = \text{constant}.
\]

Given standard antiperiodic boundary conditions for fermions along the time direction the zero modes appear only in gauge configurations with a nonzero global topological charge. The difference of numbers of the left-handed and right-handed zero modes is fixed according to the Atiyah-Singer theorem by the global topological charge of the gauge configuration. Consequently there is no one-to-one correspondence of the left- and right-handed zero modes: The zero modes induce an asymmetry between the left and the right at nonzero global topological charge. The \(SU(2)_{CS}\) chiralspin rotations (except for pure \(U(1)_A\) rotations) mix the left- and right-handed Dirac spinors. Such a mixing can be defined only if there is a one-to-one mapping of the left- and right-handed Dirac spinors in the Hilbert space. In other words the zero modes explicitly violate the \(SU(2)_{CS}\) invariance, see Appendix for details. The zero modes are the precise reason for absence of the \(SU(2)_{CS}\) invariance in the exponential (classical) part of the partition function (6).

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Now we will directly read off the symmetry properties of the partition function \[14\] on the subspace that does not include the zero modes. For any \(SU(2)_{CS}\) and \(SU(2N_F)\) transformation the \(\bar{\Psi}(n)\) and \(\bar{\Psi}_m\) Dirac bispinors transform as

\[
\Psi_n \to U \Psi_n, \quad \Psi^\dagger_m \to (U \Psi_m)^\dagger,
\]

where \(U\) is any unitary transformation from the groups \(SU(2)_{CS}\) and \(SU(2N_F)\), \(U^\dagger = U^{-1}\). It is then obvious that the exponential part of the partition function, which is a functional with fixed \(\lambda_n, c_n, \bar{c}_k\), is invariant.

\[2\] Very often instead of \(\Psi^\dagger\) the \(\Psi\) notation is used in Euclidean space. Then it should be kept in mind that under Euclidean Lorentz transformations (\(SO(4)\)) \(\Psi\) transforms as \(\Psi^\dagger\).
with respect to global and local \( SU(2)_{CS} \) and \( SU(2N_F) \) transformations, because

\[
(U\Psi_k(x))^\dagger U\Psi_n(x) = \Psi_k(x)\Psi_n(x).
\] (14)

This equation is well defined on a subspace of Dirac modes that does not include the zero modes, see Appendix for a proof. The exact zero modes contributions

\[
\Psi_{0}^\dagger(x)\Psi_n(x), \Psi_{k}^\dagger(x)\Psi_{0}(x), \Psi_{0}^\dagger(x)\Psi_0(x),
\]

for which the equation (14) is not defined, are irrelevant in the thermodynamic limit and can be ignored\(^3\). In other words, QCD classically without the irrelevant exact zero modes has in a finite volume \( V \) both global and local \( SU(2N_F) \) symmetries. These \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries are hidden\(^4\) classical symmetries of QCD. We emphasize, to avoid any confusion, that the symmetry properties of the Dirac operator in (1) and the symmetry properties of the classical part of the partition function on the subspace that does not include the exact zero modes need not be the same. Only with the full Hilbert space of the Dirac eigenmodes, that necessarily includes zero modes, these \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries must be absent in the integrand. This has been proven in the Appendix. However, once we consider a subspace of the eigenmodes there is no a general constraint on a symmetry. And indeed, we have demonstrated that a symmetry of the integrand in (6), once the zero modes are ignored, is higher - it is \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetric, contrary to the Lagrangian (1).

The axial anomaly, that stems from the noninvariance of the measure \( \prod_{k,n} d\bar{e}_kdc_n \) under a local \( U(1)_A \) transformation \( [17] \), breaks the classical \( U(1)_A \) symmetry. Since the \( U(1)_A \) is a subgroup of \( SU(2)_{CS} \), the axial anomaly breaks either the \( SU(2)_{CS} \) and \( SU(2N_F) \) \( \rightarrow \) \( SU(N_F)_L \times SU(N_F)_R \). In other words, the fermion determinant is not invariant under \( SU(2)_{CS} \) and \( SU(2N_F) \) because it does contain effect of the anomaly.

In the thermodynamic limit \( V \rightarrow \infty \) the otherwise finite lowest eigenvalues \( \lambda \) condense around zero and provide according to the Banks-Casher relation at \( m \rightarrow 0 \) a nonvanishing quark condensate in Minkowski space \( [9] \)

\[
\lim_{m \to 0} < 0\bar{\Psi}(x)\Psi(x)|0 > = -\pi \rho(0).
\] (15)

Here a sequence of limits is important: first an infinite volume limit is taken and only then - a chiral limit. The quark condensate in Minkowski space-time,

\[
< 0\bar{\Psi}(x)\Psi(x)|0 >,
\]

breaks all \( U(1)_A, SU(N_F)_L \times SU(N_F)_R, SU(2)_{CS} \) and \( SU(2N_F) \) symmetries to \( SU(N_F)_V \). Consequently, the hidden classical \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries are broken both by the anomaly and spontaneously.

III. RESTORATION OF \( SU(2)_{CS} \) AND \( SU(2N_F) \) AT HIGH TEMPERATURE

The hidden classical \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries are broken by the anomaly and by the quark condensate. Above the chiral restoration phase transition the quark condensate vanishes. If in addition the \( U(1)_A \) symmetry is also restored \( [10, 11] \) and a gap opens in the Dirac spectrum, then it follows that above the critical temperature the \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries are manifest. The precise meaning of this statement is that the correlation (Green) functions and observables are \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetric.

IV. NO FREE DECONFINED QUARKS ABOVE THE CHIRAL RESTORATION PHASE TRANSITION

What do these \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries of Euclidean correlation functions imply for Minkowski space-time, where we live? They imply that there cannot be deconfined free quarks and gluons at any finite temperature.

Assume that above \( T_c \) QCD is in a deconfined phase. Then, according to the definition of deconfinement and of the quark-gluon plasma phase, there must be free propagating quarks. Free propagating quarks, interacting with gluons, are solutions of the Dirac equation and have the following Lagrangian

\[
\bar{\Psi}i\gamma^\mu D_\mu \Psi = \bar{\Psi}i\gamma^0 D_0 \Psi + \bar{\Psi}i\gamma^k D_k \Psi.
\] (16)

The first term describes an interaction of the quark charge density with the chromo-electric part of the gluonic field and the second term contains a kinetic term for a free quark as well as an interaction of the spatial current density with the chromo-magnetic field.

While the chromo-electric part of the Dirac Lagrangian is invariant under global and space-local \( SU(2)_{CS} \) and \( SU(2N_F) \) transformations, the kinetic term and the quark - chromo-magnetic field interaction - are not. Consequently the Green functions and observables calculated in terms of unconfined quarks and gluons in Minkowski space (i.e. within the perturbation theory) cannot be \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetric at any finite temperature, because the magnetic interaction necessarily breaks both symmetries.

Also above \( T_c \) QCD is in a confining regime.

In contrast, color-singlet \( SU(2N_F) \)-symmetric "hadrons" (with not yet known properties) are not prohibited

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\(^3\) The exact zero modes break the \( SU(2)_{CS} \) and \( SU(2N_F) \) symmetries, see Appendix. Consequently these symmetries are absent at the QCD Lagrangian level.

\(^4\) Because they are not seen at the Lagrangian level and become visible only when the irrelevant exact zero modes are subtracted.
by the restored hidden $SU(2N_F)$ symmetry of QCD and can freely propagate. "Hadr... in Minkowski space can be constructed [18].

V. DISCUSSION

Early arguments about deconfinement at high temperature and transition to the quark-gluon plasma are all based on perturbative derivation of a Debye screening of the color charge. QCD is however never perturbative and what these perturbative calculations mean is not clear. Such calculations are self-contradictory: They rely on unconfined quarks and gluons and at the same time try to address confining properties without any clear definition what deconfinement would mean. While something drastic might indeed happen with gluodynamics at high T, whether it means deconfinement or not is by far not clear.

The Wilson and Polyakov loop criteria of confinement-deconfinement are applicable only for a pure glue theory. While lattice measurements of the Wilson and Polyakov loops (and of related Z3 symmetry) do show that indeed some properties of a pure glue theory rapidly change at the critical temperature, it is by far not clear whether it means deconfinement or not. To conclude about confinement-deconfinement one invokes as an intermediate step an interpretation of the Wilson loop and of a correlator of the Polyakov loops as a "potential" between the static color charges. What this "potential" would mean for quarks that move and whether they are confined or not is not clear.

Here in contrast we rely on the truly nonperturbative and rigorous Banks-Casher relation and on a symmetry of QCD above the chiral restoration phase transition. Namely, we have shown that given manifest $SU(N_F)_L \times SU(N_F)_R$ and $U(1)_A$ chiral symmetries the actual symmetry of QCD with $N_F$ degenerate flavors is $SU(2N_F)$ that prohibits in Minkowski space-time an on-shell propagation of free deconfined quark that interact with perturbative gluons.

VI. PREDICTIONS

Appearance of the $SU(2)_{CS}$ and of $SU(2N_F)$ symmetries at $T > T_c$ can be directly tested on the lattice. By definition QCD is said to be symmetric under some symmetry group $U$ if the diagonal correlation functions calculated with a set of operators $O_1, O_2, \ldots$ that form an irreducible representation of the group $U$ are identical, and if the off-diagonal cross-correlators vanish.

Transformation properties of meson and baryon operators under $SU(2)_{CS}$ and $SU(2N_F)$ groups are given in refs. [3], [4]. In particular, three isovector $J = 1$ mesonic operators $\bar{\Psi} \gamma^4 \gamma^i \Psi, (1^{-+}); \bar{\Psi} \gamma^0 \gamma^i \Psi, (1^{-+}); \bar{\Psi} \gamma^0 \gamma^i \gamma^4 \gamma^i \Psi, (1^{++})$ form an irreducible representation of $SU(2)_{CS}$. One expects that below $T_c$ all three diagonal correlators will be different and the off-diagonal cross-correlator of two $(1^-)$ operators will be not zero. Above $T_c$ a $SU(2)_{CS}$ restoration requires that all three diagonal correlators should become identical after a common normalization at some point and the off-diagonal correlator of two $(1^-)$ currents should vanish. A restoration of $SU(2)_{CS}$ and of $SU(N)_L \times SU(N)_R$ (the latter can be tested e.g. through a coincidence of the diagonal correlators with the vector- and axial-vector currents) implies a restoration of $SU(2N_F)$.

A similar prediction can be made with the baryon operators.

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VII. APPENDIX

In this appendix we prove the following statements:
(i) The $SU(2)_{CS}$ and $SU(2N_F)$ transformations are well defined on the subspace of the nonzero eigenmodes of the Dirac operator and this subspace is invariant under these transformations.
(ii) These transformations are not defined on the full Hilbert space of Dirac eigenmodes that includes the zero modes.

For convenience we will use the chiral representation and generators of $SU(2)_{CS}$ are the following matrices:

\begin{align}
\gamma_4 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
\gamma_5 \gamma_4 &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \\
\gamma_5 &= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.
\end{align}

The left- and right-handed projections are defined as

\begin{align}
\Psi_L &= \frac{1 - \gamma_5}{2} \Psi; \\
\Psi_R &= \frac{1 + \gamma_5}{2} \Psi
\end{align}

with

\begin{align}
\Psi &= \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}.
\end{align}

It is clear that the $\gamma_4$ and $\gamma_5 \gamma_4$ generators of $SU(2)_{CS}$ mix the left- and right-handed spinors:
\[ \gamma_5 \Psi_L = -\Psi_R; \quad \gamma_5 \Psi_R = \Psi_L; \]  \hspace{1cm} (22)

\[ \gamma_4 \Psi_L = \Psi_R; \quad \gamma_4 \Psi_R = \Psi_L; \]  \hspace{1cm} (23)

\[ \gamma_5 \gamma_4 \Psi_L = \Psi_R; \quad \gamma_5 \gamma_4 \Psi_R = -\Psi_L. \]  \hspace{1cm} (24)

Then, given (3) and (4) for all \( n \neq 0 \) we have:

\[ \Psi_{nL} = \frac{1}{2} \Psi_n - \frac{1}{2} \overline{\Psi}_n, \]  \hspace{1cm} (25)

\[ \Psi_{nR} = \frac{1}{2} \Psi_n + \frac{1}{2} \overline{\Psi}_n, \]  \hspace{1cm} (26)

where \( \Psi_{-n} = \gamma_5 \Psi_n \). Consequently, on the subspace \( n \neq 0 \) of the full Hilbert space there is a one-to-one mapping of the left- and right-handed spinors and in this case the \( SU(2)_{CS} \) transformations are well defined. It is also obvious that this subspace is invariant under \( SU(2)_{CS} \) and \( SU(2N_F) \) because any element from these groups transforms a vector from this subspace to one and only one vector of the same subspace. This concludes the proof of the statement (i).

For a noninteracting fermionic field a general form of \( \Psi_n \) at \( n \neq 0 \) is

\[ \Psi_n = \begin{pmatrix} \chi \\ \overline{\chi} \end{pmatrix}, \]  \hspace{1cm} (27)

where \( \chi \) is a two-component spinor.

Now we will prove the statement (ii). We will consider for simplicity only the \( Q = 1 \) sector (\( Q \) is the global topological charge of the gauge configuration), that contains one left-handed zero mode and does not contain any right-handed zero mode. A generalization to any \( Q \neq 0 \) is obvious.

The zero mode solution in this case is

\[ \Psi_0 \equiv \Psi_{0L}. \]  \hspace{1cm} (28)

If we act with the \( SU(2)_{CS} \) generators on this spinor we obtain:

\[ \gamma_5 \Psi_{0L} = -\Psi_{0L}, \quad \gamma_4 \Psi_{0L} = \phi, \quad \gamma_5 \gamma_4 \Psi_{0L} = \phi, \]  \hspace{1cm} (29)

where \( \phi \) is some right-handed spinor that does not satisfy the Dirac equation, \( \gamma_\mu D_\mu \phi \neq 0 \), because there is no right-handed zero mode in the \( Q = 1 \) sector.

Now we will prove that \( \phi \) does not belong to the Hilbert space. Assume that it does. Then it can be expanded over the right-handed non-zero modes:

\[ \phi = \sum_{n=\pm 1, \pm 2, \ldots} \alpha_n \Psi_{nR}. \]  \hspace{1cm} (30)

We multiply this equation with \( \gamma_4 \):

\[ \gamma_4 \phi = \sum_{n=\pm 1, \pm 2, \ldots} \alpha_n \gamma_4 \Psi_{nR} \]  \hspace{1cm} (31)

or

\[ \Psi_{0L} = \sum_{n=\pm 1, \pm 2, \ldots} \alpha_n \Psi_{nL}. \]  \hspace{1cm} (32)

The latter relation means that \( \Psi_{0L} \) should be linear independent with a set \( \Psi_{nL}, \quad n \neq 0 \). The latter is however not true since the zero and nonzero modes form a linear independent basis of the Hilbert space. Consequently, our assumption is not true. In other words, \( \phi \) is a spinor that is outside the Hilbert space. This means that the \( SU(2)_{CS} \) and \( SU(2N_F) \) transformations are not defined on the full Hilbert space that includes the zero modes.

Q.E.D.

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