Forced Granular Orifice Flow

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Abstract

The flow of granular material through an orifice is studied experimentally as a function of force $F$ pushing the flow. It is found that the flow rate increases linearly with $F$ — a new, unexpected result that is in contrast to the usual view that $F$, completely screened by an arch formed around the orifice, has no way of altering the rate. Employing energy balance, we show that this behavior results mainly from dissipation in the granular material.

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Although granular materials such as sand, rice etc. are familiar in daily life and industrial handling, their static and dynamic behavior is only beginning to be understood\cite{1}. In these systems, arch formation is an important effect, which may remarkably influence the mechanical properties of these materials\cite{2}. A classic example of a dynamic arch can be found in the simple system of a silo (fig.1a), in which mass is discharged from an orifice. Since ancient times, it has been supposed (and used for making ”hour glasses”) that the discharge rate $Q$ of the granular orifice flow (GOF) is independent of the height $H$ of the silo column — a phenomenon commonly believed to be due to the so called ”free fall arch” (FFA) formed around the orifice\cite{3}. Moreover, if the orifice diameter $D$ decreases, the FFA may jam into an static arch, at which point the flow stops. The jamming transition was studied in \cite{4} for the 2D case. Specifically the FFA is described as a semispherical surface spanning the orifice, the total energy of which is taken as minimal in the ”minimum energy theory” \cite{5}, and the normal stress taken as vanishing in the ”hour glass theory” \cite{6}.

These theories represent the most advanced continuous modeling of GOF, from which the empirical Beverloo formula for $Q$ can be derived\cite{3}. The FFA assumed here is rather unusual. It can resist all force $F$ given by the weight of the column $H$, and any load $L$ on its top, such that $Q$ is fully protected from being influenced. Clearly, this basic assumption can be directly checked, by measuring the rate $Q$ as a function of $F$. We performed this measurements and found a linear dependence (if $F$ is not too small),

$$Q = A (1 + \alpha F)$$

instead $\partial Q/\partial F = 0$. This demonstrates that the properties claimed for FFA in \cite{5,6} are, at the least, not rigorously valid. The difference between \cite{11} and the Torricelli’s law for ideal liquids: $Q \sim \sqrt{F}$ is clearly a result of granular arch and dissipation. Note that to measure the variation of $Q$ with $F$, we need to remove the Janssen effect\cite{7,8}, that the load $L$ and the weight of sand are typically redirected onto the side wall of silo, steering $F$ to a saturation value $F_{sat}$ determined by the frictional property between sand and wall, including the history of the preparation. The screening of $F$ can be removed, for instance, by allowing the side wall to move freely and vertically (see fig.1b), then $F$ is a sum of the load $L$, the weight of sand $M$, and the wall $M_W$ – a quantity that can be experimentally controlled and varied.

The variation of $Q$ with the orifice diameter $D$ has been studied for a long time \cite{8,9}.
The Beverloo formula,

\[ Q = C\rho \sqrt{g}(D - D_a)^{5/2} \]  

though well known, needs as a topic of current interests, validation and a clarification of its limitation, see e.g. [10]. Here, \( g \) is the gravitational acceleration, \( C \) a parameter, and \( D_a \) a correction to the orifice diameter of an empty annulus, necessary because the flow has a tendency to avoid the periphery of the orifice \([3, 5]\). More recently, Sheldon and Durian measured the variation of \( Q \) with the orifice tilt angle \( \theta \) \([11]\). Neither dependence (not to mention their combination) is well understood, posing a serious challenge to any granular theory. We shall consider only horizontal orifice planes \((\theta = 0)\), focusing our attention on the variation of \( Q \) with the pushing force \( F \). To our best knowledge, this property has not been addressed before. There is a simulation which shows an increasing of the GOF velocity with column height \([12]\), which seems similar to \([1]\), but its authors attributed this effect to the choice made in the simulation of the damping parameter, and did not take it as a general behavior of GOF. Moreover, strong vibration may break up the arches, dramatically altering the flow behavior, even rendering it to become the Torricelli one \([13, 14]\). Also, for dilute granular flows studied in \([15]\), a deviation from \([1]\) is expected, because no arches are formed.

Experimental setup is shown in fig.1b, where the side wall is a PVC cylinder of inner

![FIG. 1: Granular flow from a bottom orifice with (a): fixed wall, and (b): vertically movable wall. (See text).](image-url)
FIG. 2: $Q^{2/5}$ versus $D$. Full straight is the best linear fit. Inset shows a measured discharging $M(t)$.

diameter $\phi = 10.4$ cm, weight $M_W = 1.16$ kg. It is horizontally restricted by pulley wheels such that it can be moved freely in the vertical direction. The granular material consists of nearly mono-disperse glass beads of the diameter $d \sim 3$ mm. Bottom orifice is of $D = 14, 16, 18, 20, 22$ mm. Load $L$ ranges from 0 to about 40 kg. For each experiment, the flow starts with a granular column of the height $H_i = 51$ cm, prepared by direct pouring, and ends with $H_f = 37$ cm, the total flowed mass is about $M_0 = 1.8$ kg. The bulk density $\rho$ of the granular sample is about 1.5 g/cm$^3$. GOF rate $Q$ is measured by continuing recording (15 records/s) the flowed mass $M = \int_0^t Q dt'$ with an electronic balance of the precision $\pm 0.5$ g. It is observed that $M(t)$ is a fairly good straight line, indicating a constant rate $Q$ given by its slope (see e.g. fig.2 inset). For every measurement the flow rate is found reproducible, with a fluctuation less than 1%. First, we performed measurements with a fixed cylinder without load ($L = 0$), and obtained a rate dependence $Q_{fixed \; wall}^{L=0}(D)$ that follows the Beverloo law [2] excellently, as shown in fig.2. Best fitting gives $D_a \simeq 3.7$ mm and $C \simeq 0.51$, close to the values 4.5 and 0.58 given in the book [3].

Next, we release the cylinder, allowing it to move vertically, such that the silo screening effect is eliminated. In this situation, the bottom force $F$ pushing the flow is given by the total mass of the setup, which varies slightly with the time, $M = M(t)$. During the flow, which starts at $t = 0$ and terminates at $t_f$, the temporally averaged pushing force is

$$\langle F \rangle = \frac{1}{t_f} \int_0^{t_f} F dt = L + M_W + \frac{(H_i + H_f)}{2(H_i - H_f)} M_0.$$  

(3)
FIG. 3: Measured GOF rate $Q$ versus average pushing force $\langle F \rangle$ (symbols), and their best linear fit (straights), for various orifice diameters $D$.

Because no noticeable influences of this slight temporal dependence of $F$ on the rate $Q$ has been observed, (the measured $\int Q dt$ is always a good straight line as given in fig.2 inset), we shall assume $F \approx \langle F \rangle$ in what follows. Fig.3 shows the measured variation of $Q$ with $\langle F \rangle$, all straight lines, confirming Eq.(1) with $\alpha > 0$. Note that although the load, the cylinder and the upper part of the granular column go down slowly during the flow as a whole, no relative motion among them is observed.

To check the validity of the Beverloo law \(2\) for pushed GOF, we plot $Q^{2/5}$ versus $D$ for various forces $\langle F \rangle$ in fig.4a. Again linearity was obtained, implying a good agreement. By best linear fit, we found that both Beverloo parameters $C$, $D_a$ increase with pushing force (fig.4b and c), with the former showing a linear behavior,

$$C = C_0 (1 + \langle F \rangle / F_0)$$

(4)

(where $C_0 = 0.44$ and $F_0 = 35$ kg), while the latter is nonlinear, displaying a saturation behavior as $\langle F \rangle$ increases, approximately as

$$D_a = D_\infty - \frac{D_\infty - D_0}{1 + \langle F \rangle / F_0^*}$$

(5)

(where $D_\infty$ is its biggest value at the large force limit $\langle F \rangle \to \infty$, and $D_0$ the smallest one at the zero force limit). Best fit yields $F_0^* \simeq F_0$, $D_\infty \simeq 8.3$ mm, $D_0 \simeq 3.2$ mm, see fig.4c.

In the Beverloo law, $D_a$ accounts for clogging, which happens when the orifice is mesoscopic, of a size no more than a few grains. Our measurements show that clogging is slightly
FIG. 4: (a): $Q^{2/5}$ versus $D$ for various pushing force $\langle F \rangle$. Symbols are the data in fig.3, and full straights their best linear fits. Their Beverloo parameters are plotted in (b) and (c) (symbols) as functions of $\langle F \rangle$, where full curves are the model [4,5].

more likely with increased force. Note that as a finite size effect clogging cannot exist in the macroscopic limit, $D >> d$, where the Coulomb yield law holds. This means $D_o$ must saturate at a mesoscopic size.

In the above generalization of the Beverloo law which includes the pushing force, variations of $C$, $D_o$ as modeled by [4,5] is considered. Unfortunately, this way of generalization obscures the linear relation between $Q$ and $F$. Alternatively, we may consider a different generalization that keeps the linearity: Letting $A$ and $\alpha$ in [1] vary with $D$ as

$$A = \rho \sqrt{5} C_0 (D - D_0)^{5/2},$$

$$\alpha = \left(1 - \frac{D_1}{D}\right) \frac{1}{F_0}.$$  \hspace{2cm} (6)

When the model constants $C_0$, $D_0$, $F_0$ take the same values as in [4,5], the two models become identical in the limit $\langle F \rangle \rightarrow 0$ or $D >> D_{0,\infty,1}$. As the difference between them is very small (see fig.5a), it can not be decided with the present data which version of generalization is more appropriate. Using the values for $A$, $\alpha$ given by the straight lines of fig.3, we obtain a good agreement with [6,7], if $D_1 \simeq 13$ mm (fig.5b,c).

It is worth noting that the above empirical formulas for $Q(F,D)$ are invalid in the limits of small pushing forces $F \rightarrow 0$ and clogging orifice diameter $D \rightarrow D_{\text{clog}} > D_{\infty}$, where $Q$ is vanishing but the formulas do not show it. The diameter $D_{\text{clog}}$, at which GOF is clogged,
FIG. 5: (a): Comparison between the model (14) (circles) and the model (67) (curve). Inset is an amplified figure for small \( D \). (b): variation of \( \alpha \), (c): of \( A \), with \( D \). Symbols are measurements, and curves are the model (67).

is \( F \) dependent, and not a parameter of these models. For the experiments of this work, we have \( D_1 > D_{c\log} > D_\infty > D_0 \). That \( D_{c\log} \) could not be obtained with the models is probably due to the fact that the transition from flow to clogging is not a sharp one, but occurs via an intermittent flow. The above values for \( D_{0,\infty} \) is well below the transition at which no flow is possible, while \( D_1 \) belongs to the intermittent regime, which starts from \( D \sim 12 \text{ mm} \) for \( L = 0 \), and \( \sim 14 \text{ mm} \) for \( L = 40 \text{ kg} \). Moreover, in addition to causing the flow, \( F \) also increases clogging, as it jams the grains around the orifice and FFA. At \( D \sim D_1 \), these two effects compensate each other, and we have \( \alpha \sim 0 \), implying \( Q \)'s independence of \( F \).

The dependence of the GOF rate on the pushing force \( F \) can be qualitatively understood employing the energy balance: The power \( W_{in} \) injected into the granular system is equal to the sum of the power \( W_{out} \) carried by the flow and the heat production \( W_D \) inside it:

\[
W_{in} = W_D + W_{out}. \tag{8}
\]

If the height of the granular column is not too small, we have,

\[
W_{in} = \frac{4gFQ}{\pi \rho \phi^2}, \quad W_{out} = \frac{8Q^3}{\pi^2 D^4 \rho_{out}^2} + W_{fluct}^{out}. \tag{9}
\]

The contribution \( W_{fluct}^{out} \) accounts for the kinetic energy of the fluctuating motion of the grains, which we assume is much smaller than the first contribution and may hence be
ignored; $\rho_{\text{out}}(< \rho)$ is the density of the orifice flow. For ideal liquid, $W_D = 0$, and the balance is given between $W_{\text{in}}$ and $W_{\text{out}}$, resulting in the square root dependence $Q \sim \sqrt{F}$ of Torricelli. But for GOF, dissipation is so strong that the balance is mainly between $W_{\text{in}}$ and $W_D$. With the present experiment, (assuming $\rho_{\text{out}} = \rho/2$,) we estimate $W_{\text{out}}/W_{\text{in}} \lesssim 10^{-2}$, indicating that $W_{\text{out}}$ is negligible. In this situation, the rate $Q(F)$ is determined by the dependence of the heat production power on the pushing force, $W_D(F)$. Tailor expanding and comparing the result with (11), we have

$$W_{\text{in}} = W_D = W_D^{(1)} F + W_D^{(2)} F^2 + ...$$

and

$$A = \pi \phi^2 \rho W_D^{(1)}/4g,$$

$$\alpha = W_D^{(2)}/W_D^{(1)}.$$  

Further study needs to consider the heat produced in the system. Employing Granular Solid Hydrodynamics (GSH) as given in [16], we show a order-of-magnitude estimate for $W_D^{(1)}$ below.

First, note that in the present setup, dissipation occurs only in a small volume $V_D$ close to the orifice at the bottom. Assuming that this region is a dense granular gas with a granular temperature $T_g$, we have, according to GSH, the following relations for the pressure and heat production (up to quadratic order in $T_g$)

$$P = \frac{a\rho}{2\rho_c} \frac{\rho b_0 T_g^2}{(1 - \rho/\rho_c)^{1-a}}$$

$$W_D = V_D \gamma_0 T_g^2$$

where $\rho_c \sim 1.0667\rho$ is the random closest packing density, the exponent $a \sim 0.1$, and $b_0, \gamma_0$ material parameters, of granular matter. Eliminating $T_g$ with Eqs. (13,14) and noting that $P \sim 4gF/\pi \phi^2$ we get

$$W_D = V_D \gamma_0 \frac{8(\rho_c - \rho) g F}{a \pi \phi^2 \rho}$$

where

$$\gamma_0 = \frac{\gamma_0}{\rho b_0 (1 - \rho/\rho_c)^a}$$

can be considered as a constant of about 0.5 Hz [16]. Inserting $W_D^{(1)}$ as given by (15) into (11) we get

$$A = 2\gamma_0 V_D \frac{(\rho_c - \rho) / a}{a}.$$
This $A$ is comparable to the measurements if we take $V_D$ as 10 times the orifice volume $D^3$. To estimate $\alpha$, higher order expansion terms are needed. The detailed analysis will be presented separately.

Orifice flows are generally pushed by the force difference between the in- and outside of the orifice. The increase of the flow rate with the force obeys the Torricelli’s law if an arch is absent, such as in the case of Newtonian liquids. Granular materials form FFA, the cause of the linear increase of flow rates measured in this work. The increase has not been observed before, perhaps because it is obscured by the Janssen effect. Our result shows that neither the minimum energy theory nor the hour glass theory is fully satisfactory, and the common view that the FFA is able to resist all loads on it is at most an approximation. As dissipative mechanisms are probably responsible for the behavior of dynamic arches, analyzing heat production should help to solve the long-term mystery of GOF. Moreover, we observed that pushing forces favorite a jamming transition, an effect also operative in the slight increase of the empty annulus size $D_a$ with the force.

[1] P G de Gennes, Rev. Mod. Phys., 71, S374(1999); H M Jaeger, S R Nagel, and R P Behringer, Rev. Mod. Phys., 68, 1259(1996); GDR Midi, Eur. Phys. J. E 14, 341(2004)
[2] J Duran, Sands, Powders, and Grains, Springer-Verlag, 2000, New York
[3] R M Nedderman, Statics and Kinematics of Granular Material, Cambridge University press, 1992, New York. (chapter 10)
[4] Kiwing To, Phys. Rev. E 71, 060301(R)(2005); Kiwing To, Pik-Yin Lai & H K Pak, Phys. Rev. Lett., 86, 71(2001)
[5] R C Brown and J C Richards, Principle of Powder Mechanics, Pergamon, 1970, New York.
[6] S B Savage, Br. J. Mech. Sci. 16, 1885(1967); J F Davison & R M Nedderman, Trans. Inst. Chem. Eng. 51, 29(1973)
[7] M Sperl, Granular Matter, 8, 59(2006)
[8] B P Tighe and M Sperl, Granular Matter, 9, 141(2007)
[9] W A Beverloo, H A Liniger, J Van de Velde, Chem. Engng. Sci. 15, 260(1961).
[10] C Mankoc, A Janda, R Avévalo, J M Pastor, I Zuriguél, A Garcimartín, D Maza, Granular Matter, 9, 407 (2007)
[11] G H Sheldon & D J Durian, arXiv:0810.0495v3 [cond-mat.soft] 2009
[12] D Hirshfeld & D C Rapaport, Eur. Phys. J., E 4, 193(2001)
[13] M L Hunt, R C Weathers, A T Lee, C E Brennen, C R Wassgren, Phys. Fluids, 11, 68(1999);
    K Chen et al, Phys. Rev. E 74, 011306(2006); C Mankoc et al, Phys. Rev. E 80, 011309(2009)
[14] H P Martinez, H J van Gerner, J C R Suárez, Phys. Rev. E 77, 021303(2008)
[15] M Hou, W Chen, T Zhang, K Lu and C K Chan, Phys. Rev. Lett., 91, 204301(2003)
[16] Yimin Jiang & Mario Liu, Granular Matter, 11, 139 (2009); Phys. Rev. Lett., 99, 105501(2007)