Measurement method for longitudinal displacement of wheel/rail contact point using strain gauges put on wheels

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Abstract
This paper describes a measurement method for longitudinal displacement of wheel/rail contact point in a railway vehicle using a wheelset to measure the wheel/rail contact forces, which is generally used for the sake of the running safety evaluation in Japan. The clarification of phenomenon related to the wheel/rail contact has very important role to discuss the curving performance and the running safety of railway vehicles. The longitudinal displacement of the contact point is related to the wheelset angle of attack, which is related to the steering performance of the wheelset. The authors propose algorithms to extract informations about the contact point from the cyclic signal of the strain gauges that are put on the wheel to measure the contact forces. Experiments on roller rigs were carried out to evaluate the reliability and the accuracy of the proposed algorithm. The running tests were also carried out to demonstrate the applicability of the proposed algorithm to the real condition used. Those test results show that the measurement result using proposed algorithm is reliable as a longitudinal displacement of the contact point.

Keywords : Running safety of railway vehicle, Wheelset angle of attack, Wheel/rail contact, Curving performance, Wheel/rail contact forces, Contact geometry

1. Introduction

As well-known, it is very important to clarify the phenomenon related to the wheel/rail contact to discuss a curving performance and a running safety of railway vehicles. In general, the contact point between the wheel and the rail displaces to the lateral and the longitudinal direction. The lateral displacement of the contact point is related to the contact angle which is a parameter of Nadal’s formula and has important role in the running safety evaluation. On the other hand, the longitudinal displacement is related to the wheelset angle of attack, which is related to the steering performance of the bogie. When the displacement of the contact points are measured during a running state, the measurement result could give a lot of information about the above-mentioned matter. This paper discusses the method for measuring the longitudinal displacement of the wheel/rail contact point.

Several researches have been carried out on the measurement of the lateral displacement of the contact point by using the strain gauges put on the wheel (Kanehara, et al., 2001; Ozawa, et al., 2018). Measurement methods of the lateral displacement using a thermos-graphic images are also investigated by several researchers (Burstow, et al., 2011; Yamamoto, 2018, 2019). In those method, however, the longitudinal displacement cannot be measured. Since the longitudinal displacement is related to the wheelset angle of attack as mentioned above, the measurement method of the wheelset angle of attack is also focused by various researchers (e.g. Miyamoto et al., 1992, 2002; Kataori et al., 2011). These approaches do not measure the contact point directly, although the wheelset angle of attack is compatible with the longitudinal displacement if the wheel shape, the rail shape and the lateral displacement of the contact point are known. In addition, those methods require the additional devices for measurement and the system would be complicated from the technical viewpoint.

The state estimation approaches are also widely investigated. In particular, the estimation methods of the wheel/rail contact state using a kind of observer (e.g. Kalman filter) have been proposed by many researchers (e.g. Yunshi, et
Configure the wheelset to measure the wheel/rail contact forces: 24 strain gauges are put on the wheel to measure the wheel load ($P$), the lateral force ($Q$) and the longitudinal tangential force ($T$). The bridge circuits $p_1$ and $p_2$, which consist of the strain gauges $1A$ to $8A$ and $1B$ to $8B$ are for measuring $P$ and $T$. The bridge circuits $q_1$ and $q_2$, which consist of the strain gauges $1a$ to $7a$ and $1a'$ to $7a'$ are for measuring $Q$.

al., 2014; Murata, et al., 2018a, 2018b; Heckman, et al., 2019). The observer approaches have common issues: how to design the plant model and how to decide the plant parameters. Since there are some uncertain parameters such as the friction coefficient and the creep coefficient, which are significantly related to the wheel/rail contact phenomenon, the direct measurement approaches have an advantage compared with the observer approaches.

In this paper, we propose a direct measurement method of the longitudinal displacement of the wheel/rail contact point using strain gauges put on the wheel. To measure the longitudinal displacement, the method uses the wheelset that is used to measure the contact forces. The wheelset is generally used in Japan to evaluate the running safety and the configuration of the strain gauges attached to the wheelset is illustrated in Fig. 1 (Ishida, et al., 1997; Suzuki, et al., 2019). Although the special wheelset is required to measure the longitudinal displacement, the wheelset is widely used in the running tests, and the configuration of the wheelset is assumed to be a de facto standard in Japan. The advantages of the proposing method are that the method does not require any additional hardware components such as additional strain gauges, additional cameras and so on, and is a direct measurement approach, which is robust against uncertainties on wheel/rail contact state.

This paper consists of 6 chapters including chapter 1. Chapter 1 describes the motivations of this study and reviews previous related works. Chapter 2 introduces the basic concept of the proposing algorithm briefly. Chapter 3 presents the concrete algorithms to measure the longitudinal displacement of the contact point. Chapter 4 and section 5 illustrate the experimental evaluations on the proposing algorithm. Chapter 4 illustrates the experiments on the roller rigs which confirms the accuracy of the measurement. Chapter 5 illustrates the running tests which demonstrate the performance of the proposed algorithm during the real running condition. Chapter 6 concludes the paper. Note that the lateral force and the wheel load are denoted as $Q$ and $P$ respectively in this paper.

2. Measurement principle of longitudinal displacement of contact point

In general, the axle of the wheel is not orthogonal to the rail when the railway vehicle is passing a curve as shown in Fig.2. The relative yaw angle between the wheel and the rail is called as the wheelset angle of attack. The wheelset angle of attack is related to the running safety of the railway vehicle since the lateral force becomes large when the wheelset angle of attack is large, which causes the increase of the derailment coefficient. When the axle of the wheel is not orthogonal to the rail, the contact point is not located under the axle: that is, the contact point is displaced longitudinally because of the conicity of the wheel. The aim of this paper is to discuss the measurement method for the displacement of the contact point.

As mentioned in the introduction, the special wheelset is generally used to measure the wheel/rail contact force and to evaluate the running safety of a railway vehicle. The strain gauges are put on the wheel and configured in the bridge circuit as shown in Fig.1 to measure the strain of the wheel. The strain is converted into the contact forces in a running
Fig. 2 The concept of the algorithm to measure the longitudinal displacement of the contact point from the strain gauge signals put on the wheel: If the contact phase is measured, the longitudinal displacement of the contact point can be estimated by subtracting the contact phase of the right wheel from that of the left wheel.

Fig. 3 Examples of the sensitivity functions: the contact phase is the contact position along the circumferential direction on the wheel surface, which is converted into the angle about the axis of the wheelset. The functions $f_1(\phi)$ and $f_2(\phi)$ define the relation between the contact phase and the wheel load sensitivity. The functions $g_1(\phi)$ and $g_2(\phi)$ define the relation between the contact phase and the longitudinal tangential force sensitivity. The functions $h_1(\phi)$ and $h_2(\phi)$ define the relation between the contact phase and the lateral force sensitivity. Those discrete sensitivity functions are obtained by calibration tests.

test for the sake of the running safety evaluation using the wheelset. The sensitivity coefficient used to the conversion depends on the position of the contact point along the circumferential direction on the wheel surface. In other words, the sensitivity is described as a function of the position, $\phi$. The strain outputs $\epsilon_{p1}$, $\epsilon_{p2}$, $\epsilon_{q1}$ and $\epsilon_{q2}$, with respect to each bridge circuit, are described as follows (Ishida, et al., 1997):

$$
\begin{align*}
\epsilon_{p1} &= Pf_1(\phi) + Tg_1(\phi) + Qe_1(\phi) \\
\epsilon_{p2} &= Pf_2(\phi) + Tg_2(\phi) + Qe_2(\phi) \\
\epsilon_{q1} &= Qh_1(\phi) \\
\epsilon_{q2} &= Qh_2(\phi),
\end{align*}
$$

where $P$ is the wheel load, $T$ is the longitudinal tangential force, $Q$ is the lateral force, $f_1$, $f_2$, $g_1$, $g_2$, $e_1$, $e_2$, $h_1$ and $h_2$ are the functions that describe relations between the sensitivity and the contact position. In this paper, we call those functions as the “sensitivity function.” The subscripts 1 and 2 are the identifier of the bridge circuit. That is, for example, 2 bridge circuits are used to measure the wheel load $P$. The bridge circuit 1 and 2 are configured so that the phase difference between the sensitivity functions $f_1$ and $f_2$ become $\pi/2$ rad. As a result, the phase difference between the sensitivity functions $g_1$ and $g_2$ also become $\pi/2$ rad. The same way of thinking is applied to the bridge circuits to measure the lateral force $Q$.

The cross sensitivity terms about the lateral force of Eq.(1) and Eq.(2), which are $Qe_1(\phi)$ and $Qe_2(\phi)$, are sufficiently small in general. Hence, this paper neglects the cross sensitivity terms to simplify the problem. Examples of the sensitivity functions are shown in Fig.3, which are obtained as a discrete relationship between the sensitivity and the position.
Fig. 4 Examples of the strain wave forms which are obtained in a running test: The wave forms are acquired during the circular curve negotiation. The strain signals $\epsilon_{p1}$ and $\epsilon_{p2}$ are the combined signal of the influence of the wheel load and the longitudinal tangential force. The distortion of the wave forms of $\epsilon_{p1}$ and $\epsilon_{p2}$ is caused by the action of the longitudinal tangential force. The decoupling of the wheel load and the longitudinal tangential force should be achieved as noted in the following section.

Fig. 5 Examples of the finite Fourier series approximation of the sensitivity functions: The plots indicate the approximation of the discrete sensitivity functions shown in Fig.3. The approximation degree, $D$ is equal to 11 in the examples.

$\phi$ through the stationary calibration test. The contact position, which is indicated as “contact phase” in the figure, is described as the angle about the axis of the wheelset. As shown in this figure, the sensitivity changes cyclically with respect to the contact phase.

The core concept of the algorithms which are proposed in this paper is to extract the contact phase information from the cyclic strain signals that are obtained during the vehicle running. When the contact phase is measured, the longitudinal displacement of the contact point can be estimated by subtracting the contact phase of the left wheel and that of the right wheel, as illustrated in Fig.2. The aim of the proposing algorithms is to extract the phase information from the strain data which is obtained in the running test of the real railway vehicle, as illustrated in Fig.4. In other words, the proposing algorithms provide a solution of the inverse problem of Eqs. (1), (2), (3) and (4) in terms of the contact phase $\phi$. The strain gauge signals, however, are influenced by the contact forces $P$, $Q$ and $T$ as well as the contact phase. The following section describes how to extract the contact phase information with eliminating the influence of the contact forces.

3. Algorithms to extract contact phase

3.1. Contact phase extraction algorithm using bridge circuit for measuring lateral force

In this section, an algorithm to extract the contact phase using the bridge outputs, which are $\epsilon_{q1}$ and $\epsilon_{q2}$, is developed. The Eq. (3) and the Eq. (4) are configuring the closed 2 variables simultaneous equations. The lateral force $Q$ and the contact phase $\phi$ are calculated by solving the simultaneous equations when the bridge outputs $\epsilon_{q1}$ and $\epsilon_{q2}$ are given. As shown in Fig.3, the sensitivity functions $h_1$ and $h_2$ can be approximated by trigonometric functions. From the fact that the phase difference between the function $h_1$ and $h_2$ is approximated as $\pi/2$, the bridge output $\epsilon_{q1}$ and $\epsilon_{q2}$ are described as follows:

$$\epsilon_{q1} \approx Q \cos \phi$$

$$\epsilon_{q2} \approx Q \sin \phi.$$
The validity of the approximation is discussed in the following subsection. Under the approximation, the contact phase, $\phi$ is calculated as follows:

$$\phi = \text{atan2}(\epsilon_{q2}, \epsilon_{q1}),$$

(7)

where atan2 is the function that calculates the arc tangent with considering the singularity at $\pm\pi/2$ rad. The atan2 function is widely implemented in common programming languages. The advantage of this method is the simpleness of the calculation. When the lateral force is small, however, this algorithm does not work well since the strain signals do not have enough strength to calculate the accurate contact phase. Examples of such situations are running on a straight track, a wet condition of rail and the running with a steering bogie. On the other hand, it is unusual for the wheel load to be significantly small compared with the lateral force. Therefore, it can be said that the bridge circuits for measuring the wheel load are better than that for measuring the lateral force from the viewpoint of the reliability of the contact phase measurement.

### 3.2. Contact phase extraction algorithm using bridge circuit for measuring wheel load

In this subsection, we discuss the method to extract the contact phase from the output signal of the bridge circuits for measuring wheel load. The following two problems should be solved to extract the contact phase:

(1) The sensitivity functions $f_1$ and $f_2$ cannot be assumed as simple trigonometric functions. How to express the functions as analytical functions is one of the two problems.

(2) The bridge outputs $\epsilon_{q1}$ and $\epsilon_{q2}$ are influenced not only by the wheel load and the contact phase but also the longitudinal tangential force. In other words, the Eq. (1) and the Eq. (2) are not configuring the closed simultaneous equations since the number of unknown variables is 3 while the number of equations is 2.

#### 3.2.1. Analytical expression of sensitivity functions

In order to solve the above-mentioned problem (1), we propose an expression method of the sensitivity functions using the finite Fourier series as follows:

$$f_i(\phi) = \sum_{n=1}^{D} (S_{fui} \sin n\phi + C_{fui} \cos n\phi)$$

(8)

$$g_i(\phi) = \sum_{n=1}^{D} (S_{gui} \sin n\phi + C_{gui} \cos n\phi)$$

(9)

$$h_i(\phi) = \sum_{n=1}^{D} (S_{hui} \sin n\phi + C_{hui} \cos n\phi),$$

(10)

where $i = 1$ or 2 is the index to identify the bridge circuit, $D$ is the approximation degree of the finite Fourier series and $S_{fui}$, $C_{fui}$, $S_{gui}$, $C_{gui}$, $S_{hui}$ and $C_{hui}$ are the Fourier coefficients. The Fourier coefficients are calculated by solving the liner least square problem of which the basis function is the Fourier basis. The calibration test, which is a kind of the stationary test applied to the single wheelset, is carried out to calibrate the sensitivity that is used to convert the strain signal to the force. As a result of the calibration test, the discrete sensitivity functions are obtained as shown in Fig.3. The liner least square method is applied to the discrete sensitivity functions and the Fourier coefficients are $S_{fui}$, $C_{fui}$, $S_{gui}$, $C_{gui}$, $S_{hui}$ and $C_{hui}$ calculated as characteristic values of the wheelset.

Examples of the finite Fourier series approximation of the sensitivity functions are shown in Fig.5. Those plots indicate continuous expressions of the discrete sensitivity functions shown in Fig.3. The discrete sensitivity functions are well approximated by using the finite Fourier series. The components of the finite Fourier series of the functions $h_1$ and $h_2$, which are related to the measurement circuit for the lateral force, are shown in Fig.6. The amount of the 1st degree Fourier coefficient is quite larger than that of the other components for both of $h_1$ and $h_2$. In addition, the coefficient related to the cosine function is dominant in the function $h_1$ while coefficient related to the sine function is dominant in the function $h_2$. Those results show that the sensitivity functions $h_1$ and $h_2$ are well approximated by the single trigonometric functions as discussed in the section 3.1. The characteristics of the Fourier coefficients of the functions $f_1$, $f_2$, $g_1$ and $g_2$ are discussed in the following subclause.

#### 3.2.2. Elimination method of influence of longitudinal tangential force

The simultaneous equations, composed of Eq. (1) and Eq. (2) are solved if the longitudinal tangential force can be neglected. In a traveling situation of a real railway vehicle, however, the influence of the longitudinal tangential force is not
negligible. In this section, we propose an elimination method of the influence of the longitudinal tangential force focusing on the high frequency components of the sensitivity functions. To this end, we discuss the qualitative characteristics of the Fourier coefficients of the sensitivity functions $f_1, f_2, g_1$ and $g_2$ at first.

The components of the finite Fourier series of the functions $f_1, f_2, g_1$ and $g_2$, which are related to the measurement circuit for the wheel load and the longitudinal tangential force, are shown in Fig.7. The amplitude $A_n$ in those plots means the square root of sum of squares of the $n$-th degree coefficient, and is calculated as follows:

$$A_n = \sqrt{S_{m}^{2} + C_{m}^{2}}.$$ (11)

The important qualitative characteristics of the Fourier coefficients are follows:

(1) Only the 1st degree component is dominant in the functions $g_1$ and $g_2$, which are related to the influence of the longitudinal tangential force, while the functions $f_1$ and $f_2$ contain the higher degree component at a certain level. In particular, the amplitude of the components which have an odd number of degree are relatively large.

(2) The amplitude of the each component of finite Fourier series is almost the same regardless of the difference in the sensitivity function $f_1$ and $f_2$. Specifically, since the amplitude of $n$-th degree component in the sensitivity function $f_1$ and $f_2$ are almost the same value, they are assumed to have the common value, $A_n$.

(3) The difference in the phase between function $f_1$ and $f_2$ is approximated as $\pi/2$ for each odd number component of the Fourier series as shown in Fig.8.

The characteristic (1) is the most important factor to discuss the elimination of the influence of the longitudinal tangential force. It means that if we use a high frequency component of the sensitivity function for the extraction of the contact phase, the influence of the longitudinal tangential force can be eliminated. The amplitude ratios of the $n$-th component of the longitudinal tangential force to the wheel load $r_{mi}$, which is defined as follows:

$$r_{mi} = \frac{\sqrt{S_{mi}^{2} + C_{mi}^{2}}}{\sqrt{S_{jmi}^{2} + C_{jmi}^{2}}},$$ (12)

are shown in Table 1 for the components of odd number of degree. The amplitude ratios of the 3rd and the 5th components are small, while that of the 1st component is large.

We introduce an algorithm to extract the contact phase from a data that is obtained by the real running test. We suppose that the contact phase of arbitrary timing in a real running situation is $\phi_0$, and the time series strain data set during a rotation of wheel is available to extract the contact phase. The output of bridge circuit $\epsilon_{p1}$ is approximated as follows:

$$\epsilon_{p1} = \hat{P} \left\{ \sum_{n=1}^{D} (S_{fn} \sin(n(\phi - \phi_0)) + C_{fn} \cos(n(\phi - \phi_0))) \right\} + \hat{T} \left\{ \sum_{n=1}^{D} (S_{gn} \sin(n(\phi - \phi_0)) + C_{gn} \cos(n(\phi - \phi_0))) \right\},$$ (13)

where $\hat{P}$ and $\hat{T}$ are, respectively, the average wheel load and the longitudinal tangential force during a rotation of the wheel. Note that $\hat{P}$ and $\hat{T}$ are the unknown variables in this problem. The similar approximation is applied to the output of the bridge circuit $\epsilon_{p2}$. Eq. (13) is an approximated formula by using the continuous sensitivity functions, which are obtained by the calibration test. On the other hand, the Fourier series approximation can also be applied to the data during the real running directly, as follows:

$$\epsilon_{p1} = \sum_{n=1}^{D} (S_{n} \sin(n\phi) + C_{n} \cos(n\phi)),$$ (14)

where $S_{n}$ and $C_{n}$ are the Fourier coefficients related to the one rotation data during the real running. We assume that the Eq. (13) and the Eq. (14) are the equivalent. The following expression is obtained by applying the compound angle formulas and the R-alpha method to the Eq. (13).

$$S_{nl} = \sqrt{\hat{P}S_{f1n} + \hat{T}S_{g1n}} + \sqrt{\hat{P}C_{f1n} + \hat{T}C_{g1n}} \sin(n\phi_0 + \phi_{S1n})$$ (15)

$$C_{nl} = \sqrt{\hat{P}S_{f1n} + \hat{T}S_{g1n}} + \sqrt{\hat{P}C_{f1n} + \hat{T}C_{g1n}} \sin(n\phi_0 + \phi_{C1n}).$$ (16)
Fig. 6  The components of the finite Fourier series of the functions $h_1$ and $h_2$: The plots indicate the amount of the Fourier coefficients with respect to each term of the finite Fourier series. The blue bars indicate the amount of the Fourier coefficients that are related to the sine function, and the red bars indicate the amount of the Fourier coefficients that are related to the cosine function. Those results show that the sensitivity functions $h_1$ and $h_2$ are well approximated by the single trigonometric functions.

Fig. 7  The components of the finite Fourier series of the functions $f_1$, $f_2$, $g_1$ and $g_2$: The plots indicate the Fourier coefficients with respect to each term of the finite Fourier series. The blue bars indicate the amount of the Fourier coefficients that is related to the sine function, the red bars indicate the amount of the Fourier coefficients that is related to the cosine function and the green bars indicate the amplitude with respect to each term of the finite Fourier series. Only the 1st degree term is dominant in the functions $g_1$ and $g_2$, which are related to the influence of the longitudinal tangential force.

Fig. 8  The phases and phase differences between the sensitivity functions $f_1$ and $f_2$ of each odd number of degree: the difference in the phase between function $f_1$ and $f_2$ is approximated as $\pi/2$ for each odd number component of the Fourier series.
where $\phi_{Cl_n}$ and $\phi_{Cl_n}$ are angles that are satisfying the following equations:

$$
\cos \phi_{Sl_n} = \frac{\hat{P}C_{fl_n} + \hat{T}C_{gl_n}}{\sqrt{(\hat{P}s_{fl_n} + \hat{T}s_{gl_n})^2 + (\hat{P}C_{fl_n} + \hat{T}C_{gl_n})^2}}
$$

$$
\sin \phi_{Sl_n} = \frac{\hat{P}S_{fl_n} + \hat{T}s_{gl_n}}{\sqrt{(\hat{P}s_{fl_n} + \hat{T}s_{gl_n})^2 + (\hat{P}C_{fl_n} + \hat{T}C_{gl_n})^2}}
$$

$$
\cos \phi_{Sl_n} = \frac{\hat{P}C_{fl_n} + \hat{T}C_{gl_n}}{\sqrt{(\hat{P}s_{fl_n} + \hat{T}s_{gl_n})^2 + (\hat{P}C_{fl_n} + \hat{T}C_{gl_n})^2}}
$$

$$
\sin \phi_{Sl_n} = \frac{\hat{P}S_{fl_n} + \hat{T}s_{gl_n}}{\sqrt{(\hat{P}s_{fl_n} + \hat{T}s_{gl_n})^2 + (\hat{P}C_{fl_n} + \hat{T}C_{gl_n})^2}}
$$

which are phase biases determined by $\hat{P}$, $\hat{T}$ and the Fourier coefficients as the known characteristic values of the wheelset. The contact phase $\phi_0$ can be calculated using the Eqs.(15) and Eqs.(16) if $\hat{P}$ and $\hat{T}$ are known. In this problem, however, $\hat{P}$ and $\hat{T}$ are unknown and simultaneous equations, Eqs.(15) and Eqs.(16) can not be solved. At least, the number of unknown variables should be reduced to 2. To this end, the algorithm uses the fact that the influence of the longitudinal tangential force $\hat{T}$ can be neglected in the high frequency components of the Fourier coefficients of the sensitivity functions. The Eqs. (15) to the Eqs. (20) are rewritten as follows for the high frequency components:

$$
\hat{S}_{nl} = \hat{P} \sqrt{(S_{fl_n} + C_{fl_n})^2 \sin(n\phi_0 + \phi_{Sl_n})}
$$

$$
\hat{C}_{nl} = \hat{P} \sqrt{(S_{fl_n} + C_{fl_n})^2 \sin(n\phi_0 + \phi_{Cl_n})}
$$

$$
\cos \phi_{Sl_n} = \frac{S_{fl_n}}{\sqrt{(S_{fl_n} + C_{fl_n})^2}}
$$

$$
\sin \phi_{Sl_n} = \frac{C_{fl_n}}{\sqrt{(S_{fl_n} + C_{fl_n})^2}}
$$

$$
\cos \phi_{Cl_n} = \frac{-S_{fl_n}}{\sqrt{(S_{fl_n} + C_{fl_n})^2}}
$$

$$
\sin \phi_{Cl_n} = \frac{C_{fl_n}}{\sqrt{(S_{fl_n} + C_{fl_n})^2}}
$$

Note that the $\phi_{Sl_n}$ and $\phi_{Cl_n}$ are defined by using the result of the calibration test. Then, the unknown variables in the above equations are the average wheel load $\hat{P}$ and the contact phase $\phi_0$.

The same discussion in the former paragraph is also valid about the strain output $\epsilon_2$. The bridge output $\epsilon_2$ can also be utilized for the extraction of the contact phase. The Fourier coefficients $\hat{S}_{fl_n}$, $\hat{C}_{fl_n}$, $\hat{S}_{fl_n}$ and $\hat{S}_{fl_n}$, which are related to the $n$-th component of strain outputs $\epsilon_{p1}$ and $\epsilon_{p2}$, are approximated as follows by using the above-mentioned assumptions (2) and (3):

$$
\hat{S}_{nl} \approx \hat{P}A_n \sin(n\phi_0 + \phi_{Sl_b})
$$

$$
\hat{C}_{nl} \approx \hat{P}A_n \sin(n\phi_0 + \phi_{Cl_b})
$$

$$
\hat{S}_{n2} \approx \pm \hat{P}A_n \cos(n\phi_0 + \phi_{Sl_b})
$$

$$
\hat{C}_{n2} \approx \pm \hat{P}A_n \cos(n\phi_0 + \phi_{Cl_b}),
$$

where $\phi_{Sl_b}$ and $\phi_{Cl_b}$ are the phase offset. The signs of the Eq. (29) and the Eq. (30) depend on the number of degree $n$. Using these result, the contact phase $\phi_0$ is calculated as follows:

$$
n\phi_0 = \pm \text{atan}2(\hat{S}_{nl}, \hat{S}_{n2}) - \phi_{Sl_b}.
$$
Fig. 9 The block diagram of the proposed algorithm: The whole algorithm consists of 5 blocks. The “bias elimination” block eliminates the offset of the cyclic wave form of the strain outputs. The “wheel angular velocity estimation” block estimates the rotation speed from the strain outputs to extract the 1 cycle wave form of the strains in the “1 cycle wave extraction” block. The “calculation of Fourier coef.” block calculates the Fourier coefficients of the 1 cycle strain wave form and the contact phase is calculated by the “phase calculation” block.

The contact phase is also calculated by using the Fourier coefficients related to the cosine functions.

3.2.3. Extraction algorithm of contact phase difference

The strain signals to measure the wheel loads are obtained from the left wheel and the right wheel, which are \( \epsilon_{p1}^L, \epsilon_{p2}^L, \epsilon_{p1}^R \) and \( \epsilon_{p2}^R \). The above discussion is applied to left and right wheels, then the contact phases \( \phi_0^L \) and \( \phi_0^R \) are calculated for the left wheel and the right wheel respectively. Using the results, the contact phase difference \( \Delta \phi \) is calculated as follows:

\[
\Delta \phi = \phi_0^L - \phi_0^R.
\]

The block diagram of the whole algorithm for the contact phase extraction is shown in Fig.9. The “bias elimination” block eliminates the offset of the cyclic wave form of the strain outputs. This block is a kind of high-path filter and the similar function is implemented in the continuous measurement device of wheel/rail contact forces (Ishida, et al., 1997). The “wheel angular velocity estimation” block estimates the rotation speed from the strain outputs to extract the 1 cycle wave form of the strains in the “1 cycle wave extraction” block. A discrete Fourier transform algorithm, such as Fast Fourier Transform (FFT), can be used to estimate the rotation speed. When the rotation speed of the wheel \( \omega \) is known, the number of samples \( s \) included in the 1 cycle wave form is calculated as follows:

\[
s = \left\lfloor \frac{T}{\Delta t} \right\rfloor,
\]

where \( T = 2\pi/\omega \) and \( \Delta t \) is the sampling interval of the strain data. If the rotation speed of the wheelset is constant and known, we can also extract the 1-cycle signals by designating the constant number of samples. In the roller rig experiment, which is mentioned in the following section, the designation of the constant number of samples is used to extract 1-cycle signals. The “calculation of Fourier coef.” block calculates the Fourier coefficients of the 1-cycle strain wave form. In this block, a discrete Fourier transform algorithm can also be used. In this paper, we implemented the block by using FFT. The “phase calculation” block calculates the contact phase and the contact phase difference by using the calculated Fourier coefficients.

4. Roller rig experiment

The roller rig experiments are carried out to evaluate the quantitative validity of the proposed algorithms by using the creep force test rig, which can roll a wheelset on the roller rigs.
| Item                                          | Value          | Note                              |
|-----------------------------------------------|----------------|-----------------------------------|
| Wheel diameter                                | 836mm          |                                   |
| Wheel profile                                 | Modified arc profile |                |
| Roller diameter                               | 1600mm         |                                   |
| Roller cross-sectional shape                  | JIS 60kg rail  | Decided by the calibration procedure |
| Lateral position of the left side roller from the track center | -568.62mm | Decided by the calibration procedure |
| Lateral position of the right side roller from the track center | 569.8mm     |                                   |

4.1. Test overview and evaluation method

The relative yaw angle between the roller rigs and the wheelset is controlled by using the hydraulic actuators as shown in Fig.10. The vertical loads between the frames and the axle boxes are also controlled by using the hydraulic actuators, which are also shown in Fig.10. The 3D position and orientation are changed by adjusting the yaw angle and the vertical loads. The principal specifications of the experimental setup are shown in Table 2.

The wheelset to measure the contact forces is installed on the roller rigs and its yaw angle and the lateral displacement are measured by the two laser displacement sensors which are located nearby the wheel R as shown in Fig.10. The contact phase difference in the experiment is evaluated as a reference data by aiding the measured yaw angle, the measured lateral displacement and the 3D contact geometry. The 3D contact geometry calculates the contact point between the wheels and the roller rigs, and the contact phase difference is calculated by using those calculation results. The measured yaw angle and the lateral displacement are used as inputs of the 3D contact geometry. The relative displacement can be measured precisely by using the laser displacement sensors. However, it is difficult to determine the absolute reference point of the yaw angle and the lateral displacement. It is important to determine the absolute reference point as precisely as possible when the reference data is calculated by using the 3D contact geometry. To this end, the following procedure is performed:

- The two rolling tests are carried out to determine the absolute reference point roughly. The first test leads to the flange root contact between the wheel R and the right side roller. The second test leads to the flange root contact between the wheel L and the left side roller. In those two test, the absolute values of the target yaw angle of the wheelset are the same, and as a result, the absolute values of the actual yaw angles become almost the same. The average value of the measurement results of the lateral displacement in the two tests is assumed to be the temporarily absolute reference point, \( \hat{y}_0 \). The average value of the measurement result of the yaw angle in the two tests is assumed to be the absolute reference yaw angle, \( \psi_0 \). The measurement values of the lateral displacement and the yaw angle are shifted by \( \hat{y}_0 \) and \( \psi_0 \) respectively.
- The lateral positions of the contact point between the wheels and the rollers are also measured by using the coating material, as shown in Fig.11. On the other hand, the lateral positions of the contact points are calculated by using the shifted lateral displacement of the wheelset, the shifted yaw angle and the 3D contact geometry. The lateral positions of the roller rigs in the contact geometry are calibrated so that the calculated lateral contact positions become close values to the measurement lateral position of the contact point. The reference values of the contact phase difference are calculated by using the calibrated contact geometry model.

Fig.13 shows the comparison between the measured lateral contact positions and the calculated lateral contact positions, which are calculated by using the calibrated contact geometry model.

4.2. Evaluation of the algorithm using bridge circuits for measuring wheel load

An example of the time series data of the measured contact phase difference is shown in Fig.14. The calculation result of the contact phase difference is not stable at the beginning of the data processing since the bias elimination block cannot eliminate the bias adequately. Hence, the time average value of the calculated contact phase differences from 80 seconds to 180 seconds is used in the following evaluations.

The results of the roller rig experiment are shown in Fig.15. In those figures, the plots “flange root contact” indicate the test conditions such that the lateral contact position is located on the flange root as shown in Fig.11-(a) (ID VR:61A). The plots “flange contact” indicate the test conditions such that the lateral contact position is located on the flange as shown in Fig.11-(b) (ID VR:64A). The lateral contact area on the wheel profile in the 2 cases are shown in Fig.12. In the cases of flange contact, the contact area is located on the flange area, of which gradient of the profile is large. On the other hand, in the cases of flange root contact, the contact area is located on the flange root area, which has smaller gradient than flange area. Fig.15-(a) shows the relation between the wheelset angle of attack measured by the displacement sensors and the measured contact phase difference. Note that the constant of the proportionality of the relation between...
Fig. 10 Overview of the roller rig test machine and the experimental setup: The 2 laser displacement sensors are used to measure the lateral displacement and the yaw angle of the wheelset. The yaw angle of the wheelset is controlled by using the 2 hydraulic actuators.

Fig. 11 Measurement of the lateral position of the contact point by using a coating material.

Fig. 12 The lateral contact area on the wheel profile in test ID VR:61A and test ID VR:64A on the wheel profile: the center of tread is located on which \( Y = 0 \)mm.
Fig. 13  Comparison between the measured and the calculated lateral contact positions using the 3D contact geometry.

Fig. 14  An example of the set of time series data of the strain signals and the measured contact phase difference, $\Delta \phi$. The calculation result of the contact phase difference is not stable at the beginning of the data processing since the bias elimination block cannot eliminate the bias adequately at the beginning of the processing.
Fig. 15 Results of the roller rig experiment: The plots “flange root contact” indicate the test conditions such that the lateral contact position is located on the flange root. The plots “flange contact” indicate the test conditions such that the lateral contact position is located on the flange. (a) indicates the relation between the wheelset angle of attack measured by the displacement sensors and the measured contact phase difference. If the lateral position of the contact point is almost same, they have a large correlation. (b) indicates the relation between the measured and calculated contact phase difference. They have a large correlation regardless of the lateral contact position.

(a) Comparison between wheelset angle of attack and the measured contact phase difference

(b) Comparison between the measured and the calculated contact phase difference

Fig. 16 Comparison between the measurement result of the contact phase difference using the bridge circuits for measuring the lateral force and the wheel load: (a) shows the comparison of time average values of each test. (b) shows the comparison between the relative error of the 2 measurement results and the minimum peak-to-peak value of the strain signal, which is an indicator of the strength of the strain signal for measuring the lateral force. The relative error becomes large as the minimum peak-to-peak value becomes small.

(a) Comparison of the time average values of each test

(b) Comparison between the relative error and the minimum peak to peak value

The wheelset angle of attack and the contact phase difference depends on the contact angle. In other words, the contact phase difference depends not only on the wheelset angle of attack but also the lateral position of the contact point. In the 6 cases of the flange root contact condition, the lateral position of the contact points are almost the same as shown in Fig.13. The measured contact phase difference and the wheelset angle of attack have large correlation in regards to the 6 cases. Fig.15-(b) shows the relation between the measured and the calculated contact phase difference. From Fig.15-(b), it is found that they have a large correlation regardless of the lateral contact position. In summary, it is found that the measured contact phase difference by using the bridge circuits for measuring wheel load is reliable.

4.3. Evaluation of the algorithm using bridge circuits for measuring lateral force

We evaluated the reliability of the measurement result of the contact phase difference using the bridge circuits for measuring the lateral force by comparing with the measurement result using the bridge circuits for measuring the wheel load. The evaluation results are shown in Fig.16. Fig.16-(a) shows the comparison of time average values of each test. The measurement results of each method in the test IDs 5, 6, 7, 8 and 9 are almost the same value, while large errors appeared in the test IDs 1, 2, 3 and 4.
As mentioned in section 3.1, it is assumed that the reliability of the measurement result using the bridge circuits for the lateral force becomes low when the applied lateral force is small. We evaluated the relation between the error and the strength of the strain signals for measuring the lateral force to investigate the cause of the error. Fig.16-(b) shows the comparison between the relative error of the 2 measurement results and the minimum peak-to-peak value of the strain signal for measuring the lateral force. The minimum peak-to-peak value is defined as follows:

- 4 peak-to-peak values are calculated for each strain signals, $\epsilon_{p1}^L, \epsilon_{p2}^L, \epsilon_{p1}^R$ and $\epsilon_{p2}^R$.
- The minimum one is defined as the minimum peak-to-peak value.

The minimum peak-to-peak value is an indicator of the strength of the strain signal for measuring the lateral force. The relative error, $e$ is defined as follows:

$$e = \left| \frac{\Delta\phi_p - \Delta\phi_q}{\Delta\phi_p} \right|, \quad (34)$$

where $\Delta\phi_p$ is a measurement result of the contact phase difference using the bridge circuits for measuring the wheel load and $\Delta\phi_q$ is a measurement result of that using the bridge circuits for measuring the lateral force. The plot shows that the relative error becomes large when the amplitude of the strain signals are small. In other words, this test results certify that the reliability of the measurement result using the bridge circuits for measuring lateral force becomes high when the applied lateral force is large.

5. Application to running test data

The running test was carried out to evaluate the applicability of the proposed method during a real running condition in MIHARA test center, which is located on premises of Mitsubishi Heavy Industries Engineering, LTD.

5.1. Test overview

The configuration of the test train is shown in Fig.17-(a). The test train consists of the motor cars and the trailer car. The measurement is carried out in the trailer car. The wheelset to measure not only the contact forces between the wheel and rail, but also measure the contact phase difference is installed in front of the trailer car. The structure of the trailer car is similar to the vehicles for the commuter trains, which are generally used in Japan. The strain signals from the wheels are transferred through the slip-ring device installed on the axle box. The wheel profile of the measurement wheelset is the modified arc wheel profile, which is widely used in the Japanese meter-gauge railway vehicles. The configuration of the test track is shown in Fig.17-(b). The test track consists of the straight section and the compound curve section which has 2 radiuses of curve, R120 and R160. The rail profile of the test track is EN54E1 and the gradient of the inclined base-plate is 20:1. The contact phase difference is calculated off-line by using the strain data acquired in the running test. In this section, we tested the measurement algorithm using the bridge circuits for wheel load since the running test data includes the straight and transition curve sections, where the lateral force is small.

5.2. Measurement results of contact phase difference

The measurement results of the contact phase difference in the running test are shown in Fig.18. The 2 variations of the degree of the Fourier coefficient, $n = 1$ and 3, and the 3 variations of the running speed, $V = 20km/h, 30km/h$ and
Fig. 18 Measurement results of the contact phase difference in the running test: The 2 variations of the degree of the Fourier coefficient and the 3 variations of the running speed are tested. The measurement results using the 3rd degree of the Fourier coefficients \((n = 3)\) are valid both qualitatively and quantitatively, although the measurement result using the 1st degree \((n = 1)\) is not valid since the 1st degree coefficient is influenced by the longitudinal tangential force.

Table 3 Average values of the calculated contact phase difference \((n = 3)\) in R160 and R120 circular curves.

|        | \(V = 20\) km/h | \(V = 30\) km/h | \(V = 40\) km/h |
|--------|-----------------|-----------------|-----------------|
| R160   | 0.0179 rad (7.697 mm) | 0.0158 rad (6.794 mm) | 0.0141 rad (6.063 mm) |
| R120   | 0.0272 rad (11.70 mm) | 0.0256 rad (11.01 mm) | 0.0195 rad (8.385 mm) |

As seen in the measurement result of using the 1st degree coefficient, the calculated contact phase differences in the R120 section are smaller than that in the R160 section. The amount of the calculated contact phase differences is about 0.1 to 0.3 rad. Those results are invalid from the viewpoint of the following matters:

- In general, the wheelset angle of attack becomes large as the radius of curve becomes small (e.g. Takai et al., 2002). As mentioned in the introduction, the longitudinal displacement of the contact point, which is equivalent to the contact phase difference, is related to the wheelset angle of attack. More specifically, the contact phase difference is proportional to the wheelset angle of attack when the flange contact is assumed. The calculation results using the 1st degree coefficient is invalid qualitatively from the viewpoint of the matter.

- The relation between the wheelset angle of attack \(\psi\) and the contact phase difference \(\Delta \phi\) is approximated by the following formula when the contact between flange and rail is assumed.

\[
\Delta \phi \approx \psi \tan \alpha,
\]

(35)

where \(\alpha\) is the angle of flange. According to the previously carried out test in MIHARA test center, the wheelset angle of attack which is obtained by the fixed point observation at R160 circular curve was about 0.5 to 1 degree in the front wheelset (Tanaka, et al., 2016). Substituting those values and the angle of flange \((\approx 65\) deg for the modified arc profile) into Eq. (35), the realistic range of the contact phase difference is about 0.019 to 0.037 rad. The order of the calculation result by using the 1st order coefficient is quite large compared with the estimated realistic range. Then, the calculation results using the 1st degree coefficient is invalid quantitatively.

From those results, it is found that it is important to eliminate the influence of the longitudinal tangential force in the real running state of a railway vehicle.

On the other hand, the calculation results using the 3rd degree coefficient seem to be valid both qualitatively and quantitatively. The average values of the calculated contact phase difference in the R120 circular curve are larger than 40km/h are tested. The regions of time which correspond to the circular curve sections R160 and R120 are indicated in the plot.
that in the R160 circular curve as shown in Table 3. The table also shows the values that is converted to the dimension of distance when the wheel radius is assumed to be 430mm. If the flange contact is assumed, the estimated wheelset angle of attacks are 0.48 deg for $V = 20$ km/h, 0.42 deg for $V = 30$ km/h and 0.38 deg for $V = 40$ km/h in the R160 circular curve. The average values of the calculated contact phase difference become small as the running speed increases. A similar relation between the wheelset angle of attack and the running speed is obtained by the previously carried out experiments and simulations (Miyamoto et al., 1992).

The proposed method assumes implicitly that the one point contact occurs between the wheel and rail. On the running test condition in MIHARA test center, however, two point contact on the flange and the tread would occur because of the large base-plate angle of EN standard compared with Japanese standard base-plate (Hondo et al., 2016). It is presumed that the load center of the distributed forces was close to the contact point on the flange. Then, the estimated wheelset angle of attack was close to the previously measured one. More detail investigations about the relation between the contact phase difference calculated by the proposed algorithm and the contact states such as two point contact are required to improve the algorithm.

The reliability of the calculated contact phase difference from the perspective of the dynamic behavior should also be investigated. A low frequency vibrations are observed in the R120 circular curve section when the running speeds are 30 km/h and 40 km/h, as shown in Fig.18. However, it is difficult to assess whether the calculated result is expressing an actual phenomenon or not at the moment, although the order of the mean values of the calculation result seem to be valid according to the comparison with the realistic range of the contact phase difference which is calculated by using the previously measured wheelset angle of attack (Tanaka, et al., 2016).

Although the above-mentioned matters are left to the future works of this research, the basic validity of the proposed method is presented by the roller rig experiment and the running test of the real railway vehicle.

6. Conclusions

This paper proposed a measurement method of the longitudinal displacement of the wheel/rail contact point by using the strain gauges put on the railway wheels. The longitudinal displacement is measured as the difference in the contact phase of the left and right wheels. We evaluated the reliability and the accuracy of the proposed measurement algorithm through the roller rig experiments. The measured contact phase difference was close to the calculated contact phase difference using the measured lateral displacement of the wheelset, the measured wheelset angle of attack and the 3D contact geometry. We also carried out the running test to evaluate the feasibility of the proposed algorithm during a real running condition of a railway vehicle. The measurement result was valid qualitatively and quantitatively.

Future works are as follows:
- The evaluation of the dynamic performance of the proposed algorithm.
- A detailed investigation of the proposed algorithm under a 2-point contact condition, especially focusing on the contact condition (e.g. dry or wet) and running condition (e.g. running speed and track conditions).
- The construction of a real-time measurement device.

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