On the design of deep-sea optical elements made of PMMA

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Abstract. This paper considers some issues related to the design of the optical assembly of a deep-sea porthole made of polymethyl methacrylate (PMMA). The considered element is planned to be used as part of a lighting system operable at a depth of more than 5000 meters. The calculation is made without taking into account the influence of temperatures.

1. Introduction
The study of the sea bottom and the bosom of the sea may not be possible without the use of lighting systems. An integral part of both the surveillance system itself and the backlight system is the porthole. The requirements for the material of the porthole of deep-sea observation systems should take into account its use under extreme operating conditions: a large pressure, temperature gradient, and resistance to chemical influences. The main materials for practical use in this case are fused quartz K8 (Schott N-BK7) and PMMA polymer. Studies on the subject are presented in [1-4].

2. Research objective
When designing a deep-sea porthole, it is necessary to maintain a balance between reliability and preservation of the optical transparency of the element during operation.

In terms of the geometric shape, the most popular are: a flat cylinder (disk, round thick plate), the frustum of a cone and a frustum sector of a sphere (“dome”). The main optical materials are fused quartz, for example, K8 (Schott N-BK7) and polymers, for example polymethyl methacrylate (PMMA). Glass offers several advantages in comparison with polymer: simplicity of technological surface treatment, relative simplicity of analytical prediction of the porthole working capacity. However, glass compared to polymer has a significant drawback - this is the weight of the product and impact strength. Note that the strength analysis of the use of K8 (Schott N-BK7) as an optical element of a deep-sea study is presented in [5–7]. The use of PMMA as an optical material is widely reported in J.D. Stachiw’s works. However, this work is more of a fundamental experimental character.

Below, by the example of designing an optical element made of PMMA, the main difficulties in predicting the working capacity of this element are considered.

3. Material features
PMMA is an amorphous polymer, the physical and mechanical properties of which depend on temperature, strain rate, molar mass and the characteristics of the external load, manufacturing process (in this case, the injection manufacturing is considered). The relationship between stresses both in the glassy state of the polymer and in the elastic state at the initial stage can be almost linear, but this does not mean that the deformation is elastic within this section. The deformation in the polymer even in a
linear section is not always reversible, because determined by the movement of the links, and not by the change in interatomic distances.

The temperature limits that characterize the mechanical properties of polymers (brittle, elastic state) are not constants of the material, but depend on the speed or deformation time. In addition, a special case is a comprehensive compression or extension, here the specificity of the polymer is almost completely lost and should be considered as an ordinary solid [8]. Moreover, in theoretical calculations, the ideal material usually appears without defects, the small size of which reduces the strength by several times [9].

4. Strength and durability criterion
Considered load can be short-term and long-term. For a short-term load, within the framework of the task, as for a brittle material, it is important to use the Pisarenko – Lebedev strength criterion [10]:
\[
\sigma_{eq} = \chi \sigma_i + (1 - \chi) \cdot \sigma_i \leq \sigma_{max},
\]
where \(\sigma_i, \sigma_2\) – components of the main stress vector; \(\sigma_i\) – stress intensity, defined as \(\sigma_i = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \cdot \sigma_2}; \chi\) – plasticity coefficient, is experimentally, numerically equal to:
\[
\chi = \frac{\sigma_i}{\sigma_s}; \sigma_s, \sigma_c \quad – \text{ultimate tensile and compressive strength, respectively}; \sigma_{max} \quad – \text{maximum allowable voltage value, numerically coincides with} \ \sigma_i.
\]

In the case of a long-term load, Zhurkov’s criterion of durability is used [5,11]:
\[
\tau = \tau_0 \exp \left( \frac{U_0 - \gamma \sigma}{RT} \right),
\]
where \(U_0, \gamma, \tau_0\) – factors defining the strength characteristics of the material: value \(U_0\) characterizes the activation energy of the destruction process; \(\tau_0 = 10^{-13}\) seconds coincides with the period of thermal vibrations of atoms; \(\gamma\) – coefficient proportional to overvoltage at interatomic bonds; \(R\) – universal gas constant; \(T\) – absolute temperature °K.

5. The source data
The numerical definition of the stress-strain state of the optical element was carried out in conjunction with the porthole body. The following are all the main characteristics used in modeling the stress-strain state and in analytical estimates of the element thickness.

Material stress-strain properties [11] are presented in Table 1:

| Material          | Young’s modulus E, GPa | Poisson’s ratio ν | ratio D | Density ρ, kg/m³ | \(\sigma_s\), MPa | \(\sigma_c\), MPa | \(U_0\), kJ/mol | \(\gamma\), 10^{-28} m³ |
|-------------------|------------------------|-------------------|---------|------------------|------------------|------------------|-----------------|-------------------|
| Plexiglass PMMA   | 3                      | 0.3               | 1.190   | 1190             | 82               | 125              | 150             | 13.2              |
| Titanium BT1-0(Ti6Al4V) | 110                  | 0.32              | 4.450   | 4450             | 860              | –                | –               | –                 |

Calculation was made on the condition of material nonlinearity by the finite element method: at a short-term load, the deformation diagram [12, 13] corresponded to 20°C; it can be analytically approximated in the form of an exponential dependence.

\[
\sigma(\varepsilon) = E_0 \cdot \varepsilon - \left( \frac{\sigma_p}{1 + \exp \left( \frac{k \varepsilon}{\sigma_p} \right)} \right),
\]
where $E_0 = 3354.498045$ MPa – value proportional to Young’s modulus of the material in question, 
$\sigma_p = 5640.757009$ MPa, $k = 1$ – proportionality coefficient, $m = 0.471646$ – numerically derived constants.

Loads and boundary conditions. Static case:
- Internal working pressure $P_1 = 1$ atm. (0.1 MPa); when modeling, it must be taken into account that when the volume is immersed, water exerts pressure on the entire structure and induces an increase in pressure inside the volume.
- External hydrostatic pressure $P_2 = 630$ atm. (63 MPa);
- Rigid fixing of the frame in accordance with the drawing, Figure 1;
- At the interface between the PMMA media - index 1 and Titanium - index 2, the conjugation condition:
  \[ \sigma_{n1} + \sigma_{n2} = 0; \bar{u}_1 = \bar{u}_2 \]

In all calculation schemes, the diameter of the optical component is constant and equal to $D = 80$ mm. Product thickness $- h = 25...35$ mm. The considered form of the product: a cone, a “dome”. Taper angle $\varphi$ is equal to $60..100^\circ$. Outer surface radius is $R_b = 250$ mm. The inside is flat ($R_a = \infty$).

![Figure 1](image)

**Figure 1.** Load diagram: for a cone-shaped porthole (a); dome-shaped porthole (b).

6. **Calculation of a deep-sea porthole in the shape of a frustum cone and a frustum spherical sector ("dome")**
In the case of a cone, under the influence of hydrostatic pressure, the porthole bends and crimps along the conical supporting surface. Moreover, on the surface of small diameter, the area near its center will be under equal biaxial compression. Near the edges of this surface, in the area of contact of dissimilar materials, there will be a zone of maximum compressive stresses, the so-called stress concentrator.

Note that the dome-shaped porthole has a large surface area and the loads are distributed more evenly. In this case, the object in question is a complex model in the form of a spherical sector, characterized by: the diameter of the porthole, the inner and outer radii, and, accordingly, and the opening angle of the cone.

In this regard, an analysis of equivalent stresses was carried out according to formula 1, depending on the change in the shape of the windows, namely the angle of taper and the thickness of the glass. The results obtained in the course of numerical calculations are presented in Figure 2.
Figure 2. Ratio of equivalent stresses to ultimate stresses at various taper angles.

Figure 2 shows that, to a large extent, the strength of both porthole glass structures is determined by the correct choice of thickness and shape. In this case, an acceptable solution would be the choice of parameters: \( h = 30..35\text{ mm}, \varphi = 70^\circ..85^\circ \).

In the contact region of the optical element with the metal surface, there is a stress concentration point. In the selected range of parameters of the element thickness and taper, there is a slight advantage of the alternative form of the porthole compared to the conical.

Similar calculations were made taking into account the nonlinearity of the material based on formula (3). Comparative results are shown in Figure 3. It can be seen from the graphs that the curve on the basis of the analytical expression in the considered period is 5-6% higher in absolute value than the calculation with the stress-strain curve obtained on the basis of experimental data, which, ceteris paribus, does not impair the strength forecast.

Figure 3. Strength calculation using various stress-strain curves.
In addition to calculating equivalent stresses, an assessment of the glass material durability was made. According to formula (2), based on the kinetic concept of the breaking of interatomic bonds in a substance, under the conditions of the assigned task, the product durability without taking into account cycle of operation was about 20 hours.

7. Conclusion
Work carried out considers the models of acrylic portholes in the form of a frustum cone and a “dome”. In the Comsol Multiphysics finite element package, numerical results are obtained that take into account the physical nonlinearity of PMMA. The stress-strain diagram at $20^\circ$C and strain rate 0.001 $s^{-1}$ can be approximated by formula (3), which gives an overestimated result on equivalent stresses by 5-6% with thicknesses of $h = 30..35$ mm, which does not worsen the strength forecast.

Based on the data obtained, recommendations similar to those previously presented in [1] can be formulated. However, it should be noted that when designing polymer elements, one should take into account the significant dependence of PMMA on the nature of external loads and operating conditions. More accurate estimates can be obtained if PMMA is considered as a viscoelastic material, but additional parameters, for example, relaxation time, which are not a constant or a reference value, are needed.

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