Performance analysis of hybrid MPAPM technique for deep-space optical communications

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Abstract
A hybrid multiple pulse amplitude and position modulation (H-MPAPM) scheme based on gradual multi-pulse pulse-position modulation (G-MPPM) and multi-level pulse amplitude modulation (M-PAM) schemes is proposed for deep-space optical communication. In this scheme, transmitted information is conveyed by different combinations of the positions and amplitudes of multiple optical pulses. The performance of the proposed scheme with multi-pulse pulse-position modulation (MPPM), G-MPPM and multiple pulse amplitude and position modulation (M-PAPM) schemes in terms of the number of transmitted symbols, data rate, symbol error rate (SER), bit error rate (BER), bandwidth utilization efficiency and power requirements are studied in this work. Our results reveal that, at the same frame size, the proposed scheme allows much more symbols per frame transmission, achieves higher transmission energy and further enhance the bandwidth utilization efficiency of MPPM, G-MPPM and M-PAPM schemes. Moreover, compared to the MPPM and G-MPPM, the proposed scheme achieves lower levels of SERs but at the expense of peak power levels. Furthermore, the proposed scheme outperforms G-MPPM and MPPM schemes by approximately 1 and 2 dB at a BER of 10^{-6}, respectively. In addition, the analytical expressions for the BER and SER of the proposed scheme are validated using Monte Carlo (MC) simulations.

1 | INTRODUCTION

Over the years, the recent advancements in free space optical (FSO) wireless communication technology has regained a great interest in laser-based deep-space optical communication (DSOC) applications [1–3]. As an alternative to the current radio frequency (RF) technologies, FSO communication also known as optical wireless communication (OWC) can provide high-speed data transmission for the next-generation communication systems that can be deployed relatively in different environments such as indoor, underwater, atmospheric and space [3, 4].

One of the most commonly modulation formats used in intensity modulation with direct detection (IM/DD) for optical direct-detection channels with photon-counting receiver in DSOC systems is the single-carrier pulse position modulation (PPM) scheme [5, 6]. PPM is an average energy-efficient modulation format and has the advantages of lower duty cycles, high peak-to-average-power ratio (PAPR) and better anti-noise performance over other pulse modulation schemes [7–10]. In the case of M-ary PPM (M-PPM), information is conveyed by positioning a single pulse in one of the M slots and blocks of m = \log_2(M) bits are mapped to one of the M slots.

However, bandwidth efficiency of M-PPM especially for higher values of M is low and its requirement of complex time-domain equalization can be problematic for most FSO links with severe channel conditions [11–13]. As such, multi-level modulation schemes such as multi-level pulse amplitude modulation (M-PAM) are preferred for bandwidth limited systems where the transmitted data can take multiple amplitude levels [14–19]. M-PAM achieves higher spectral efficiency when compared to M-PPM but comes at the cost of power efficiency [20].

To overcome these problems, combinatorial or multipulse PPM (MPPM) modulation scheme has been proposed to improve the band-utilization efficiency in single-pulse M-PPM scheme [21]. With the same transmission efficiency, MPPM reduces the required transmission bandwidth in M-PPM to about half i.e., improve the spectral efficiency of M-PPM and...
has higher optical power utilization [22]. Again, when the bandwidth efficiency is of great interest, MPPM is often not a good candidate [23]. Also, decoding in MPPM is very difficult as the information capacity is affected by the total number of pulse time slots for each symbol period; thus, well suited for tight bandwidth conditions. [24].

To further enhance the system performance, the advantages of $M$-PPM and $M$-PAM are combined in multiple pulse amplitude and position modulation (M-PAPM) [25]. Alternatively, various modulation formats with different hybrid approaches for OWC system have been proposed over the years [26–34]. Hybrid modulation techniques can simultaneously transmit higher data with both high spectral and power efficiencies over traditional modulation techniques [29].

Moreover, multi-carrier modulation (MCM) schemes are also proposed in order to address some of single-carrier modulation (SCM) drawbacks. In MCM, transmitted data stream is assigned to several sub-channels (subcarriers) in parallel using the orthogonal frequency division multiplexing (OFDM) [35]. Compared to SCM, MCM is less energy-efficient but more bandwidth-efficient [36]. However, the power and bandwidth efficiency of MCM is affected by the constraint of real-valued non-negative signalling in a negative way [37]. As alternative to the MCM, carrierless amplitude and phase modulation (CAP) modulation is becoming increasingly popular for OWC system [37]. Recently, authors in [38] have demonstrated a Gb/s full-duplex indoor OWC system using a low-bandwidth and low-cost vertical-cavity surface-emitting laser (VCSEL) based CAP format achieving a simultaneous operation of 10 and 2 Gb/s for downlink and uplink, respectively.

In the area of visible light communication (VLC), a form of OWC technology, modulation schemes such as spatial modulation (SM), space shift keying (SSK) and colour shift keying (CSK) are also investigated. SM is a low-complexity and energy-efficient multiple-input and multiple-output (MIMO) transmission scheme [39]. In SM systems, multiple antennas are used with only one active transmit antenna at any instant, while other antennas remain silent [40]. The advantages of SM over MIMO have been shown by many researches [40]. Alternatively, SSK is a special form of SM proposed in order to trade-off receiver complexity for data rate [41, 42]. Both SM and SSK schemes are able to completely avoid inter-channel interference (ICI) in both time and space [42, 43]. CSK modulation scheme on the other hand, transmits a signal through the variation of colour intensities emitted by red, green and blue (RGB) light LEDs [44–46]. CSK is widely used in VLC systems to address issues such as limited dimming support, low data rate and non-negative optical intensity [47].

To further increase the data rate transmission and bandwidth utilization efficiency, we proposed a hybrid multiple pulse amplitude and position modulation (hybrid M-PAPM or H-MPAPM). Basically, hybrid M-PAPM is an improved multi-level version of gradual MPPM (G-MPPM) proposed in [48] where $M$-PAM scheme is employed to increase the generated symbol length. This allows the simultaneous representation of information by different combinations of the positions and amplitudes of multiple optical pulses in hybrid M-PAPM scheme. We analyse the data rate, transmitted symbols per frame, bandwidth utilization efficiency and power requirements for the proposed hybrid scheme. In addition, we derive the expressions for the average symbol error rate (SER) and upper bound bit error rate (BER) for the proposed hybrid scheme and compared them with that of MPPM and G-MPPM schemes. Furthermore, Monte Carlo (MC) simulations are provided to verify the accuracy of the derived theoretical expressions.

The rest of this paper is organised as follows. Section 2 describes the proposed hybrid M-PAPM system model and its characteristics. Section 3 gives an overview of $M$-PAM, MPPM and G-MPPM schemes and their corresponding SER and BER expressions. In Section 4, we present the average SER and BER expressions for the proposed hybrid M-PAPM scheme. Simulations results and analysis are presented in Section 5, and conclusion is given in Section 6.

## 2 | HYBRID M-PAPM SYSTEM MODEL

In this scheme, similar to the classical G-MPPM, one or more optical pulses (up to $n$ pulses) per frame are allowed to be transmitted with $M$-PAM used in increasing the generated symbol length by $k^n$ more combinations, i.e. the proposed hybrid modulation scheme simultaneously conveys information by the different combinations of both the amplitudes and positions of multiple pulses with each symbol duration positioned into $M$ different time slots. Using these combinations, the resultant number of transmitted symbols per frame for this scheme is expressed by summing up all possible symbols as

\[
L_{H\text{-MPPM}} = k^n \sum_{i=1}^{M} \binom{M}{i}
\]

where $k^n$ (with $k \in \{1, 2, \ldots\}$) denotes the set of possible amplitude levels with respect to the number of transmitted optical pulses ($n$) and \( \binom{M}{i} = \frac{M!}{(M-i)!i!} \). Alternatively, hybrid M-PAPM can be explicitly written as hybrid $k$-level $n$-pulse M-PAPM indicating clearly the level, number of optical pulses and the frame size time slots or hybrid $n$-pulse $M$-PAPM without the level(s). Thus, the overall number of transmitted bits mapped with one symbol is then

\[
B_{H\text{-MPPM}} = \left\lfloor \log_2 \left( k^n \sum_{i=1}^{M} \binom{M}{i} \right) \right\rfloor
\]

When $n > 1$, the resultant transmitted symbols per frame given by Equation (1) is rarely power of two and as such, the message bits are encoded in the hybrid M-PAPM symbol subset of size $2^{\left\lfloor \log_2 \left( k^n \sum_{i=1}^{M} \binom{M}{i} \right) \right\rfloor}$, where $\lfloor \cdot \rfloor$ is the integer floor operation.

We perhaps further clarify this scheme by considering a numerical example in which $n = 3$ optical pulses are transmitted in $M = 6$ frame time slots with $k = 2$ amplitude levels. The number of possible symbols and the corresponding number of bits per frame that can be obtained using the hybrid M-PAPM is 328 symbols and 8.36 bits, respectively. Meanwhile, using the
TABLE 1 Number of symbols and their corresponding bits per frame for different modulation schemes

| Schemes  | M = 8 | M = 12 | M = 16 |
|----------|-------|--------|--------|
|          | # sym | # bit  | # sym | # bit  | # sym | # bit  |
| MPPM     | 56    | 5.8    | 220   | 7.8    | 560   | 9.1    |
| G-MPPM   | 92    | 6.5    | 298   | 8.2    | 696   | 9.4    |
| M-PAPM   | 448   | 8.8    | 1760  | 10.8   | 4480  | 12.1   |
| Hybrid M-PAPM | 736   | 9.5    | 2384  | 11.2   | 5568  | 12.4   |

TABLE 2 Bits per Symbol, Spectral Efficiency and Bandwidth Requirements for various MPPM based modulation schemes

| Scheme     | Bits per Symbol | Spectral Efficiency | Bandwidth requirement |
|------------|-----------------|---------------------|-----------------------|
| MPPM       | \[\log_2 \binom{M}{a}\] | \[\frac{1}{M} \log_2 \binom{M}{a}\] | \[\frac{MR_b}{M}\] |
| G-MPPM     | \[\log_2 \sum_{i=1}^{a} \binom{M}{i}\] | \[\frac{1}{M} \log_2 \sum_{i=1}^{a} \binom{M}{i}\] | \[\frac{MR_b}{M}\] |
| M-PAPM     | \[\log_2 k^e \sum_{i=1}^{a} \binom{M}{i}\] | \[\frac{1}{M} \log_2 k^e \sum_{i=1}^{a} \binom{M}{i}\] | \[\frac{MR_b}{M}\] |
| H-MPAPM    | \[\log_2 k^e \sum_{i=1}^{a} \binom{M}{i}\] | \[\frac{1}{M} \log_2 k^e \sum_{i=1}^{a} \binom{M}{i}\] | \[\frac{MR_b}{M}\] |

conventional MPPM, G-MPPM and M-PAPM, we obtain 20, 41 and 160 symbols corresponding to 4.32, 5.36 and 7.32 bits per frame, respectively. Clearly, a significant increase in both the number of transmitted symbols and bits is achieved using the proposed hybrid M-PAPM scheme. Table 1 provides further numerical examples with frame sizes M = 8, 12 and 16.

Moreover, the data rate \(R_b\) for the system adopting the hybrid scheme is given by

\[R_b = \frac{\log_2 k^e \sum_{i=1}^{a} \binom{M}{i}}{MT_s}\]  

where \(T_s\) represents the slot duration and is defined as the ratio of symbol duration \(T\) to frame size \(M\) slots, i.e. \(T_s = \frac{T}{M}\).

Thus, the bandwidth utilisation efficiency \(U\) is given by

\[U = \frac{\log_2 k^e \sum_{i=1}^{a} \binom{M}{i}}{M}\]  

Also, with the same number of optical pulses, the information rate ratio of H-MPAPM with respect to the conventional MPPM is given by

\[\text{Information rate ratio} = \frac{\log_2 k^e \sum_{i=1}^{a} \binom{M}{i}}{\log_2 \binom{M}{a}}\]  

Moreover, assuming that the transmitter sends signals at a fixed transmission rate of \(R_b\), then the bandwidth requirements of MPPM, G-MPPM, M-PAPM and H-MPAPM schemes can easily be obtained by taking the inverse of the slot duration. Table 2 summarised the transmitted bits per symbol, spectral efficiencies and bandwidth requirements of various MPPM based modulation schemes.

For DSOC systems, the IM/DD with photon-counting detectors are considered for the transceiver, i.e. IM and DD are assumed at the transmitter and receiver, respectively [49]. The receiver basically counts the number of photodetections by detecting the energy of the received signal during some observation interval and then makes a decision on the transmitted symbol using the photon counts [50]. In an effort to minimise the SER for the proposed hybrid scheme, we considered a DSOC channel model where channel fading effects such as atmospheric attenuation, atmospheric turbulence, pointing error, multipath and dispersion are not taken into account and only the background radiation is assumed. Moreover, to simplify the comparison analysis with the existing modulation schemes, a Poisson distribution with IM/DD discrete memoryless optical channel (DMC) model for both the signal and background optical radiation is considered. In addition, compared with high levels of background optical radiation, the detector thermal noise is also ignored. Most of the analysis here can, however, be extended to the aforementioned models in a straightforward manner.

The proposed hybrid M-PAPM transmitter and receiver block diagrams (not detailed in this work) are shown in Figures 1 and 2, respectively. As illustrated in the figures, the proposed system model consists of a transmitter, an optical channel and a receiver. The most commonly optical or light sources used in FSO systems are either coherent sources (laser diodes (LDs)) or non-coherent sources (light emitting diodes (LEDs)). Generally, LEDs are mostly employed for indoor applications, i.e. for short link and moderate data rates whereas, LDs are preferred for outdoor applications with high data rate requirements due to their highly directional beam profile and high optical power outputs [34]. Here, we proposed using an incoherent detection for better receiver sensitivity. As shown in Figure 1, data bits are first fed to the transmitter’s digital signal processing (DSP) unit, which divides them into several blocks. Each block contains \(\log_2 k^e \sum_{i=1}^{a} \binom{M}{i}\) bits. The first \(\log_2 \sum_{i=1}^{a} \binom{M}{i}\) bits are encoded using the G-MPPM scheme which determine the positions of the transmitted \(n\) pulses within the frame. Each of the G-MPPM optical pulse is then
further modulated using the $M$-PAM modulator by $k^n$ more combinations, i.e. the G-MPPM optical pulses simply take multiple amplitude levels using the $M$-PAM modulator. The transmitter then sends photons through a DMC. The photons are detected by a photodiode (PD) at the receiver side as shown in Figure 2. The receiver DSP unit then decodes the transmitted bits by counting the number of received photons according to the Poisson channel distribution [51].

3 | REVIEW OF M-PAM, MPPM AND G-MPPM SCHEMES

3.1 | Review of M-PAM

The optical signal of an $M$-PAM can be expressed as

$$s_i(t) = a_i \cdot g_T(t) \cdot \cos(2\pi f t), 0 \leq t < T_i$$

where $T_i$, $a_i$, $g_T(t)$, $t$ represent the slot width of symbol, the $i$th pulse amplitude, the rectangular pulse and the optical frequency, respectively [52]. In an $M$-PAM scheme, information is encoded in the amplitude of the transmitted signal pulse carrier and each pulse conveys $\log_2 M$ bits per symbol with $M = 2^k$ possible $k$ bit blocks of symbols. The closed-form expression for the BER of $M$-PAM under additive white Gaussian noise (AWGN) channels is given by [53] as

$$P_b = \frac{1}{\log_2 M} \sum_{k=1}^{\log_2 M} P_b(k),$$

with

$$P_b(k) = \frac{1}{M} \sum_{i=1}^{M-1} A_i^k \cdot \text{erfc}(\sqrt{B_i} \gamma)$$

where

$$A_i^k = (-1)^{i-1} k^i \cdot M \cdot 2^{k-1} \cdot \left( \frac{1}{2} + \frac{1}{2} \right)$$

$$B_i = \frac{(2i + 1)^2 \cdot 3 \log_2 M}{M^2 - 1}$$

where $\gamma = \frac{E_i}{N_0}$ denotes the signal-to-noise ratio (SNR) per bit, $[x]$ denotes the largest integer to $x$ and erfc($\cdot$) is the complementary error function given by

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt.$$ (11)

The SER of an $M$-PAM scheme in AWGN channel is given in [54] as

$$P_i = \frac{2(M - 1)}{M} Q(\sqrt{2\gamma})$$

where $\alpha = 6 \log_2 M/(M^2 - 1)$ and $Q(\cdot)$ is Marcum’s Q-function, given by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} dy$$

3.2 | Review of MPPM

In MPPM scheme [21], the symbol duration $T$ is partitioned into $M$ time slots with each duration $T = T/M$ and $n$ optical pulses are sent every symbol duration. The MPPM symbol for any $n \in \{1, 2, ..., M\}$ and time slots $M > 1$ is defined by an $M$-dimensional vector $B$ selected from the set [36]

$$B_{\text{MPPM}} \overset{\text{def}}{=} \left\{ B \in \{0, 1\}^M : \sum_{i=1}^{M} B_i = n \right\}$$

The component $B_i$ is ‘0’ or ‘1’ if the slot is non-signal and signal, respectively. The number of bits per symbol in an MPPM is $\left\lfloor \log_2(M) \right\rfloor$ with a maximum for $n = \lfloor M/2 \rfloor$. The SER of an $n$-pulse $M$-PAM is given by [55] as

$$\text{SER}_{\text{MPPM}} = \sum_{k_{\text{min}}=0}^{\infty} \sum_{i=1}^{M-n} \sum_{j=1}^{\left\lfloor \frac{M}{2} \right\rfloor} \frac{n}{m} \frac{(M-n)}{j} p_i(k_{\text{min}})^m$$

$$\times (1 - P_1(k_{\text{min}})^m + P_1(k_{\text{min}})^j')$$

$$\times (1 - P_1(k_{\text{min}})^j + P_1(k_{\text{min}})')$$

$$\times R_i(k_{\text{min}} - 1)^{M-n} \left[ 1 - \frac{1}{\left( \gamma_{+n}^{+m} \right)} \right]$$

where $M$, $n$, and $k_{\text{min}}$ represents the number of time slots per symbol, the number of signal slots per symbol and the minimum photon count over all signal slots, respectively. $p_i(\cdot)$ and $p_i(\cdot)$ denote the conditional probabilities that a received slot has a value $i$ given a ‘0’ (non-signal or no pulse) and a ‘1’ (signal or pulse) is transmitted in a slot, respectively. $R_i(\cdot)$ and $P_1(\cdot)$ also represent their cumulative distributions, respectively. For any $k \in \{0, 1, 2, ..., \}$, the expressions for $p_i(\cdot)$, $p_i(\cdot)$, $P_0(\cdot)$ and $P_1(\cdot)$ are given by [55]

$$p_0(k) = \left( K_i \right)^k \frac{k!}{k!} e^{-K_i}$$

$$p_1(k) = \left( K_i + K_i \right)^k \frac{k!}{k!} e^{-K_i - K_i}$$

$$R_0(k) = \sum_{j=0}^{k} \left( K_i \right)^j \frac{j!}{j!} e^{-K_i}$$

$$P_1(k) = \sum_{j=0}^{k} \left( K_i + K_i \right)^j \frac{j!}{j!} e^{-K_i - K_i}$$
where $K_b$ is the mean value of a non-signal slot and $K_s + K_b$ is the mean value of a signal slot (signal plus background noise). In general, a closed form expression to calculate the BER of MPPM is very difficult and as a result, an upper bounded relation between the BER and SER of an $n$-pulse $M$-PAM is given by [56]

$$\text{BER}_{MPPM} \leq \frac{2^{\left\lfloor \log_2 (\frac{n}{M}) \right\rfloor} - 1}{2^{\left\lfloor \log_2 (\frac{n}{M}) \right\rfloor} - 1} \text{SER}_{MPPM} \quad (20)$$

### 3.3 Review of gradual MPPM (G-MPPM)

Gradual MPPM is a modified version of the conventional MPPM proposed to improve the performance and bandwidth utilization efficiency. G-MPPM increases the number of transmitted symbols per frame while maintaining a reasonable number of slots per frame [48]. Unlike MPPM that allow a fixed number of optical pulses per frame, G-MPPM allow the transmission of one or more pulses per frame. Thus, the number of bits per symbol in G-MPPM is $\left\lfloor \log_2 \sum_{i=1}^{M} \frac{M}{i} \right\rfloor$. The SER for the gradual $n$-pulse $M$-PAM scheme on a DMC is given by [48] as

$$\text{SER} = 1 - \frac{1}{\sum_{j=1}^{n} \binom{M}{1}} \cdot \left[ \left( \frac{M}{1} \right) \cdot P(c)_A + \sum_{j=2}^{n-1} \binom{M}{j} \cdot P(c)_B + \binom{M}{n} \cdot P(c)_C \right] \quad (21)$$

with

$$P(c)_A = \sum_{k_{\max}=1}^{M-1} \sum_{l=0}^{M-1} I(l, m) \sum_{i=1}^{M-1} p_i(k_{\max})^i$$

$$P(c)_B = \left[ 1 - P_{Th}(Th - 1) \right] \times \left[ 1 - P_{Th}(Th - 1)^{M-i} \right] \quad (22)$$

$$P(c)_C = \sum_{k_{\min}=1}^{M-n} \sum_{l=0}^{M-1} I(l, m) \sum_{i=1}^{M-1} p_i(k_{\min})^i$$

$$\times P_{Th}(k_{\min} - 1)^{M-n-1} P_{Th}(k_{\min})^{n-M} \times (1 - P_{Th}(k_{\min}))^{n-M} \quad (23)$$

where $Th = k_i / \ln(1 + k_i)$ denotes the threshold level, $I(l, m)$ is the probability of making the correct decision, $k_{\max} = \max(k_{\max}, 1, \ldots, k_{\max})$ is the maximum photon count over all signal slots, $P(c)_A$ represents the probability of correct transmission for symbols that contain only one signal slot, $P(c)_B$ represents the probability of correct transmission for symbols that contain a number of signal slots $i$ where $n(1 < i < n)$ and $P(c)_C$ is the probability of correct transmission for the symbols that contain $n$ signal slots.

### 4 BER AND SER FOR HYBRID M-PAPM

In this section, we derive the expressions for the average SER and an upper bound BER for the proposed H-MPAPM scheme.

#### 4.1 BER for hybrid M-PAPM

As shown in Figure 1, the hybrid M-PAPM scheme depends on both G-MPPM and $M$-PAM schemes. Therefore, an upper bound expression of the BER for the proposed hybrid scheme can be calculated by taking the average BERs of both G-MPPM and $M$-PAM as follows:

$$\text{BER}_{H} \leq \frac{1}{\log_2 k^s} \sum_{i=1}^{n} \binom{M}{i} \times \text{BER}_{G,MPPM} + \log_2 k^s \sum_{i=1}^{n} \binom{M}{i}$$

$$\times \left( (1 - \text{SER}_{G,MPPM}) \text{BER}_{M,PAM} + \frac{\text{SER}_{G,MPPM}}{2} \right) \quad (25)$$

where $\text{BER}_{M,PAM}$ is the bit error rate of $M$-PAM, $\text{BER}_{G,MPPM}$ and $\text{SER}_{G,MPPM}$ are the bit error and symbol error rates for G-MPPM scheme, respectively. The BER expression given by Equation (25) is divided into two parts. The first term $\log_2 k^s \sum_{i=1}^{n} \binom{M}{i} \times \text{BER}_{G,MPPM}$ accounts for the transmitted errors caused by the incorrectly decoding of the G-MPPM bits. The second term accounts for the BER of the remaining $\log_2 k^s \sum_{i=1}^{n} \binom{M}{i}$ bits and it also consists of two parts. The first part $(1 - \text{SER}_{G,MPPM}) \times \text{BER}_{M,PAM}$ considers the bit errors caused by incorrectly decoding of the $M$-PAM symbols when the G-MPPM signal-slots are correctly decoded, while the other part considers the case when the G-MPPM signal-slots are incorrectly decoded. Finally, the BER of the $M$-PAM is then calculated by taking the average of the two parts.

#### 4.2 SER for hybrid M-PAPM

The average SER expression for the proposed hybrid M-PAPM scheme can be obtained as the result of independent events of the SER$_{G,MPPM}$ and BER$_{M,PAM}$ as

$$\text{SER}_H = 1 - (1 - \text{SER}_{G,MPPM})(1 - \text{BER}_{M,PAM})^n \quad (26)$$

where $n_H$ represents the number of optical pulses in the proposed hybrid frame. Substituting the expressions for the SER$_{G,MPPM}$ and BER$_{M,PAM}$, Equation (26) can be finally written as shown in Equation (27).

The first parentheses in Equation (26) accounts for the event that the G-MPPM symbol is correctly decoded and its pulses’ positions are correctly determined. The second parentheses of the equation accounts for the event that all $M$-PAM symbols within the hybrid frame are correctly decoded. Clearly, the
5 1 SIMULATION RESULTS AND ANALYSIS

In this section, the performance evaluation of H-MPAPM, G-MPPM, M-PAPM and MPPM schemes is presented. The performance is compared in terms of the information rate ratio, bandwidth utilisation efficiency, average received power, BER and SER. Furthermore, Monte Carlo (MC) simulation results are used to validate the theoretical expressions.

Figure 3 shows the information rate ratios for H-MPAPM, G-MPPM and M-PAPM schemes normalized to the conventional MPPM scheme for the case $M = 16$ and optical pulses $(n)$ ranging from 2 to 10. It can be seen from the figure that the information rate ratio curves for all the schemes increases as the number of optical pulses per frame increases. As expected, the information rate ratio of H-MPAPM scheme increases allowing more symbols to be available for transmission.

Figure 4 compares the bandwidth utilization efficiencies for the various schemes at a fixed frame size of $M = 16$ and $n$ values ranging from 1 to 16. It can be seen from Figure 4 that the maximum achievable efficiencies for MPPM, G-MPPM and M-PAPM schemes are about 85% at 8 pulses, 100% at about 10 pulses and 145% at 11 pulses, respectively whereas for the H-MPAPM scheme, the utilization efficiency is about 160% at 12 pulses. We observed that the hybrid M-PAPM scheme achieves higher bandwidth utilization efficiency compared to the other modulation schemes. Thus, the H-MPAPM can provides better utilization of limited bandwidth during communication.

In Figures 5 and 6, we plot the SERs versus the average number of received photons per frame $(K_{av})$ at $M = 8$ and 16. To guarantee that all the modulation schemes have the same transmission rate, we maintained the same slot duration throughout the analysis. Also noticed that the comparisons are performed under the average power constraint where usually the power resources of the transmitter is limited by the number of transmitting photons emitted per frame. At the same average energy level, the modulation scheme which achieves higher performance is highly desired and the average power can be replaced.
with the average $K_{av}$. Moreover, for equal average power constraint, we also maintained the same $K_{av}$ for all the schemes.

For the case of Figure 5 at $M = 8$, we considered the background radiation by the mean of the average number of received background photons ($K_b$) set to $K_b = 1$ and 5. The compared schemes here are chosen to be hybrid 2-pulse 8-PAPM, gradual 3-pulse 8-PPM and 4-pulse 8-PPM where the number of pulses per symbol are $n_H = 2$, $n_G = 3$ and $n_M = 4$, respectively. The selection is done such that the number of transmission symbols are almost or nearly the same for better comparison analysis. The number of transmission symbols for the hybrid 2-pulse 8-PAPM, gradual 3-pulse 8-PPM and 4-pulse 8-PPM are 144, 92 and 70 symbols, respectively. At $K_{av} = 40$ and $K_b = 1$, hybrid 2-pulse 8-PAPM achieves a reduction of 10 and 15 dB in SER compare to gradual 3-pulse 8-PPM and 4-pulse 8-PPM, respectively. Moreover, at $K_b = 5$, hybrid 2-pulse 8-PAPM scheme also achieves a reduction of 5 and 10 dB in SER compare to gradual 3-pulse 8-PPM and 4-pulse 8-PPM, respectively. These reductions in the SERs clearly shows the robustness of the hybrid $n$-pulse $M$-PAPM to noise.

For the case $M = 16$ as shown in Figure 6, the comparison is carried out between hybrid 4-pulse 16-PAPM, gradual 6-pulse 16-PPM and 8-pulse 16-PPM. Again, this selection is done to achieve almost or nearly the same number of transmission symbols for all the schemes. The transmission symbols for the above mentioned modulation schemes are 40256, 14892 and 12870, respectively. At $K_{av} = 120$ photons, hybrid 4-pulse 16-PAPM achieves a reduction of 16 and 21 dB in SER at $K_b = 1$ and a reduction of 10 and 16 dB at $K_b = 5$ over the gradual 6-pulse 16-PPM and 8-pulse 16-PPM schemes, respectively. These figures further shows the superior performance of the hybrid MPAPM scheme over the two compared schemes. Additionally, these results also shows that at the same average number of received photons per frame, the pulse power is much higher in the hybrid scheme than that in both G-MPPM and MPPM schemes which in turn causes the hybrid scheme to perform better in a noisy channel situation.

The SER performance is further explained by Figures 7 and 8. The figures show the effect of $K_{av}$ with respect to the $K_b$. Here, $K_{av}$ and $K_b$ represents the metric for the average and peak power, respectively. Clearly, with the same frame sizes, the average number of signal slots per frame in the H-MPAPM scheme is much less than that in both G-MPPM and MPPM schemes. Under the same number of received photons per frame transmission, there is a noticeable increase in the $K_b$ values in the H-MPAPM scheme than that of the G-MPPM and MPPM schemes. However, this noticeable increase in the number of received photons in the H-MPAPM scheme comes with the price of increased peak power levels. Moreover, on the numerical aspect of Figures 7 and 8, we can see that the peak power increases linearly...
as the average number of received photons per frame increases. However, when the same maximum peak power level is considered, the performance of the H-MPAPM scheme may be limited especially at lower values.

Figures 9 and 10 compare the BERs performances for hybrid $k$-level $n$-pulse $M$-PAPM ($k = 2$), gradual $n$-pulse $M$-PPM and conventional $n$-pulse $M$-PPM schemes with respect to the average received optical signal-to-noise ratio ($\text{OSNR}_{av}$) ratio. The comparison analysis is performed under average power constraint. Specifically in Figure 9 at $M = 8$ and $K_b = 1$, hybrid $M$-PAPM shows an improvement of about 0.6 and 1.7 dB at BER $= 10^{-6}$ when compared to G-MPPM and MPPM, respectively. More improvements of 1 dB and 2 dB is seen in Figure 9 when $K_b = 5$ at BER $= 10^{-6}$. Figure 10 further shows the improvement of the proposed hybrid scheme at BER $= 10^{-6}$ when $M = 16$ and $K_b = \{1, 5\}$. These results show the performance superior-

### Table 3. Performance analysis of H-MPAPM, MPPM and G-MPPM modulation schemes

| Performance analysis | MPPM | G-MPPM | H-MAPPM |
|----------------------|------|--------|---------|
| # Sym / # Bit        | Low  | Medium | High    |
| Bandwidth efficiency | 85%  | 100%   | 160%    |
|                      | at 8 pulses | at 10 pulses | at 12 pulses |
| Transmission rate    | Low  | Medium | High    |
| Complexity order     | $\mathcal{O}(nM)$ | $\mathcal{O}(n(M + 1))$ | $\mathcal{O}(n(k + M))$ |
| SER @ $10^{-6}$      | compare reduced 15 dB | reduced 10 dB | - |
|                      | to H-MPAPM. | $M = 8, K_b = 1$ | |
|                      | compare reduced 10 dB | reduced 5 dB | - |
|                      | to H-MPAPM. | $M = 8, K_b = 5$ | |
| BER @ $10^{-6}$      | compare reduced 1.7 dB | reduced 0.6 dB | - |
|                      | to H-MPAPM. | $M = 8, K_b = 1$ | |
|                      | compare reduced 2 dB | reduced 1 dB | - |
|                      | to H-MPAPM. | $M = 8, K_b = 5$ | |

The reason for this improvement is that the influence of noise is reduced by increasing the average received OSNR$_{av}$ ratio to a certain point. The performance analysis carried out in this section are summarised in Table 3.

Figures 11 and 12 shows the MC simulation results for the BER and SER derived in Equations (25) and (26), respectively. For the MC simulation, a total of $10^5$ random numbers were run in order to reduce the statistical uncertainties of the BER and SER. In addition, we also assumed perfect synchronization by the receiver. It is observed from the figures that the MC
simulation results are most closely matched with the obtained results as shown by Figures 5, 6, 9 and 10. The plotted MC simulation results shows the validity of the derived theoretical expressions.

In terms of the complexity of the proposed hybrid M-PAPM, it can be deduced that the performance improvement achieved by the H-MPAPM comes with additional complexities in the transmitter and receiver structure compared with both G-MPPM and MPPM schemes. These additional complexities arise from the need for periodical estimation of multiple threshold levels at the receiver side along with the need to implement complex synchronization schemes to compensate for synchronization errors caused by false detected slots. However, the bit encoding and decoding along with the detail transmitter and receiver designs are not in the scope of this paper and they represent important design issues that could be considered in other research works.

Moreover, in terms of the big $O$ notation complexity comparison, the complexity of traditional $n$-pulse $M$-PPM scheme could be obtained as: after computing the number of received photos (integrating the received electrical current) in each slot of the $M$ slot, $n^*M$ comparison have to be carried to determine the $n$ signal slots and therefore the complexity is of order $O(n * M)$ [30]. For Gradual $n$-pulse $M$-PPM scheme, $n^*M$ comparisons are carried first to select the $n$ slots with largest received signals. Then $(n-1)$ comparisons with predefined threshold are carried to determine the signal and non-signal slots. The complexity is of order $O(n(M + 1))$ [48]. For the proposed hybrid $k$-level $n$-pulse M-PAPM scheme, in addition to $n^*M + (n-1)$ comparisons to determine the signal slots there will be $(k-1) * n$ comparisons with $(k-1)$ thresholds to decode signal slots. Therefore, the complexity is of order $O(n(k + M))$ which is clearly larger than the other schemes.

## 6 | CONCLUSION

A hybrid multiple pulse amplitude and position modulation scheme based on gradual MPPM and $M$-PAM has been proposed in this work. The proposed hybrid scheme simultaneously encode information in the position and amplitude of multiple optical pulses. The performance of the proposed hybrid M-PAPM scheme in terms of the transmitted bits/symbols per frame, data rate, average BER, SER, bandwidth utilization efficiency and power requirements has been investigated. We have shown through simulations that the proposed hybrid scheme further improve the spectral efficiency of the system by transmitting more information per symbol than conventional MPPM, M-PAPM and G-MPPM. In addition, reduction in the SER of the MPPM and G-MPPM is shown when the proposed hybrid scheme is employed. It is also shown that the proposed hybrid scheme outperforms both MPPM and G-MPPM in terms of average BER. We further validate the derived theoretical expressions for the BER and SER of the proposed scheme under average power constraint is investigated as well.

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