Simultaneous impact of intensity and turbulence scale to a by-pass laminar-turbulent transition

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Abstract. The influence of turbulence intensity on transition is quite well learned, especially for the zero pressure gradient. To characterize the main features of turbulence it is necessary to know its intensity but also its scale (most often the dissipation length is used as a length scale), usually related to a velocity along a streamline. In so far the investigation of the influence of turbulence scale is far away of completeness. In this paper the results of experimental correlations for the transition inception are presented with the use of two new parameters $ITu$ and $ILu$ that are created as integrals taken from the leading edge of the plate till the transition begins, expressing this way the averaged values of intensity and turbulence scale. The turbulence scale is made non-dimensional by the momentum thickness of a boundary layer. A new experimental correlation for the point of transition inception $Re^{**}$ is calculated in form of a product of these two new parameters raised to power determined in the three-dimensional regression procedure.

Introduction

Turbulence of an external flow causes an earlier laminar-turbulent transition of flow in a boundary layer which is responsible for the increase in skin friction and heat transfer. The aim of the investigation reported in this paper is to develop a new correlation for the transition inception in the by-pass laminar-turbulent transition considering both intensity and scale of turbulence.

It is also supposed that the process of the transition depends on mutual interaction of the external flow and boundary layer from the leading edge to the point $x_t$ of turbulence inception and, of course, further to the end of transition. So, the history of the flow between these two points (leading edge and $x_t$), should be considered as important. In this sense the turbulence intensity and scale should be averaged between these two points.

Since earlier investigations, this combined influence of intensity and scale was suspected to be responsible for an early transition.

1. Influence of turbulence intensity or scale

There are some experimental correlations of the transition inception Reynolds number and turbulence intensity in the literature. The first was given by Abu-Ghanam & Shaw (1980):

\[ Re^{**}_t = 163 + exp(6.91 - Tu) \]  (1)
This correlation has two asymptotes: first the transition can not occur below \( Re^{**} = 163 \) and the transition can occur at the latest for \( Re^{**} = 1165 \) for \( Tu = 0 \). The simpler correlations, where the initial Reynolds number was not taken into account, which is according to Mayle (1991) more appropriate for the flow with enhanced turbulence level as in turbomachinery is given below:

\[
Re_t^{**} = kTu^{-m}
\]

where \( k = 460 \) and \( m = 0.65 \) (by Hourmouziadis (1989)), while Mayle (1991) has proposed: \( k = 400 \) and \( m = 5/8 \).

There are few investigations relating to the influences of the turbulence scale on laminar-turbulent transition. In his review, Mayle (1991) cited the results of Hall & Gibbings (1972) (used in Hall and Gibbings) and Blair (1983) where transition appears earlier when the mesh of the grid is greater. Unfortunately, these results are not dimensionless, therefore their usefulness is rather low.

Jonas & Mazur & Uruba (200) also showed the turbulence scale dependence on the inception and transition length. They made their investigations for the constant turbulence level \( Tu = 3\% \) at the leading edge of plate. They claim themselves that the use of their correlations are rather limited.

Dyban & Epik & Supron & Juszyna (1996) enumerated some lacks of these correlations: the influence of turbulence intensity is not taken into account, the length scale is not made dimensionless and also some discrepancies in comparison with the correlation of Abu-Ghanam and Show.

2. Simultaneous influence of turbulence intensity and scale on transition inception

It is supposed that the process of by-pass transition in a boundary layer depends on mutual interaction of the external flow and boundary layer from the leading edge to the inception and, of course, further to the end of transition. So the history of the flow between these two points seems to be important. In this sense the turbulence intensity and scale should be averaged between these two points. This model neglects intentionally the phenomena leading to the occurrence of Tollmien-Schlichting waves which are also observed in a boundary layer with enhanced turbulence level of the external flow. The turbulence decay behind the grid is given by the formula:

\[
Tu = c \left( \frac{d}{x} \right)^m \left( \frac{M}{d} \right)^p
\]

where \( c, m \) and \( p \) are constants experimentally determined, \( d \) is diameter of the grid rod and \( M \) is a grid mesh size. Using the geometry shown in (1) (experimental rig) so integrating from the leading edge position \( Ls \) to \( Ls + xt \) the averaged turbulence intensity between leading edge and inception point is given by following formula:

\[
ITu_m = \frac{1}{xt} \int_{Ls}^{Ls + xt} Tu dx
\]

As a turbulence length scale \( Lu \), the dissipative length scale is considered according to the formula:

\[
Lu = -\frac{\left( \frac{u''}{U} \right)^{\frac{3}{2}}}{\delta_x
\]

Dividing \( Lu \) by the well known formula for the momentum thickness in the laminar boundary layer:

\[
\delta^{**} = 0.664xRe_x^{-0.5}
\]
we get the second dimensionless parameter:

\[
ILu_{\delta,m} = \frac{1}{x_t} \int_{L_S}^{L_s+x_t} \frac{Lu}{\delta^*} dx
\]  

(7)

Finally, the new experimental correlation for the transition inception is proposed in the following form:

\[
Re^* = k \cdot ITu_{m1} \cdot ILu_{m2}^\delta
\]  

(8)

3. Investigation rig

The investigation was carried out in the subsonic wind tunnel of low level of turbulence, \( Tu < 0.08\% \) and velocity up to 100 m/s. The sketch of the working section and the detail of the leading edge is shown in (fig. 1). The enhanced level of turbulence was generated by five grids of following dimensions: Grid 1 \((d=0.3 \text{ mm}, M=1 \text{ mm})\), Grid 2 \((d=0.6 \text{ mm}, M=3 \text{ mm})\), Grid 3 \((d=1.6 \text{ mm}, M=4 \text{ mm})\) and Grid 4 \((d=3.0 \text{ mm}, M=10 \text{ mm})\) and Grid 5 \((d=3.0 \text{ mm}, M=30 \text{ mm})\). Grids were placed at the different distances upstream of the leading edge of plate: \(L_s = 450, 410, 370\) and 330 mm. Also four different oncoming velocities were used: \(U = 6, 10, 15\) and 20 m/s.

Figure 1. Plate in wind tunnel: a) shape of leading edge, b) plate (1), grid (2) at distance \(L_s\) upstream of leading edge.

The coordinates systems for the turbulence intensity and scale measurements is fixed to the grid with \(x\) coordinate parallel to the mean velocity of flow. The coordinate system for boundary layer measurement is fixed to the leading edge of plate and the distance between the grid and leading edge of plate is equal to \(L_s\) as mentioned above.

To avoid separation at the leading edge the plate was set at the incidence angle \(i = -1.63^\circ\) therefore a rather small velocity gradient along the plate was measured and next the acceleration coefficient \(K\) calculated according to formula below

\[
K = \frac{\nu}{U^2} \frac{dU}{dx}
\]  

(9)

and its value is approximately equal to: \(K \approx 2.7 \cdot 10^{-7}\).

4. Investigation results

First the turbulence intensity behind the grid was measured by means of thermoanemometry, as a result the decay law for turbulence intensity was established. The turbulence intensity generated by grids and evolved into the flow was from about 0.4% to about 3.4% at the leading edge of the plate \((x_0)\). For this turbulence decay the correlations (3) was found, with the values for \(m = 0.562 \div 0.943\), \(p = 0.15\) and \(c = 0.193 \div 2.357\) depending on velocity and grid.
The dissipation scale of turbulence was found according to the formula (5). The $Lu$ range reached from 1.2mm to about 7.0mm at the leading edge.

In the second part of experiment the characteristics of the boundary layer and especially the local shear stress coefficient $C_f$ were obtained. The values of $C_f$ were used to observe the region of transition and determine the intermittency factor $\gamma$ by means of formula (10)

$$\gamma = \frac{C_f - C_{fl}}{C_{ft} - C_{fl}}$$

where $C_{fl}$ and $C_{ft}$ are local skin friction coefficient in laminar and turbulent regions of the flow.

Next another formula for intermittency factor $\gamma$ was used to set the characteristics of laminar-turbulent transition on a plate. To describe the intermittency coefficient $\gamma$ in the transition region the cumulative distribution function of three-parametric Weibull probability distribution was used, Wierciski (1997).

$$\gamma(x) = 1 - \exp\left(-\frac{Re^{**} - Re_{\Theta}^{**}}{Re_{\Theta}^{**} - Re_{fl}^{**}}\right)^{\alpha}$$

In the above formula $\alpha$ is a shape parameter, $Re_{fl}^{**}$ - the point where transition begins, $Re_{\Theta}^{**}$ - the point where intermittency factor $\gamma$ is equal to $(e - 1)/e = 0.6321$, where $e$ is the base of natural logarithm. Finally, the correlation (8) for the transition beginning was proposed. The searched constants were found by means of the three-parametric regression procedure as follows: $k = 307.5$, $m_1 = -0.22$, $m_2 = -0.05$. Correlation coefficients for the formula (8) are: $r_{12} = -0.91$, $r_{13} = -0.81$, $r_{23} = 0.85$ (axes 1, 2, 3 corresponds to $Re_{fl}^{**}$, $ITu_m$, $ILu_{m,\delta}$ accordingly).

Figure 2. Three parametric correlation (8)

Comparing our results with the known formulas (1 and 2) and taking turbulence intensity at the leading edge, we can see (fig. 3) that coefficients $k$ and $m$ seem to differ for different grids. The lower are the values of $d$ and $M$, the lower is the value of coefficient $k$. Only the results for grid 4 are similar to the theoretical correlations. This might be because the Mayle investigations were made for $Tu \geq 3$ at the leading edge, and only grid 4 meets such conditions.
Figure 3. Momentum thickness Reynolds number at the onset of transition as a function of turbulence level.

So, it seems the three-dimensional correlation is necessarily to characterize the relation between $IT_u m$, $IL_u \delta,m$ and $Re_t^{**}$, because it can be used for all grids and velocities. Relation of turbulence scale at the leading edge ($x_0$) and momentum thickness Reynolds number at the onset of transition was made by Jonas & Mazur & Uruba (200). In his opinion the function:

$$Re_t^{**} = k Lu^m$$

(12)

decreases as $Lu$ increases. This is true if we make correlation for all grids together. But when we make this for every grid separately, the result seems to become disturbing. As it is shown on figure 4, for grids 1-4 apart, $Re_t^{**}$ increases as $Lu$ increases. Besides, the higher are the values of $d$ and $M$, the higher is the value of coefficient $k$. A different situation is, when turbulence scale increases thus the momentum Reynolds number becomes lower than the critical value $Re_t^{**} = 201$. For Grid 5 coefficient $m$ becomes negative, as Jonas suggested. These results need further verification.

5. Conclusions

The results of correlation for simultaneous influence of the turbulence intensity and scale on transition parameters are presented, especially on the begin of laminar-turbulent transition $Re_t^{**}$ in a boundary layer. For the first time, the two non-dimensional mean parameters for the turbulence intensity $IT_u m$ and scale $IL_u m$ as an averaged integral values from the leading edge to the point of transition inception are applied to get the correlation. The three-dimensional regression procedure was applied to this analysis. Taking into account the influence of both the $IT_u m$ and $IL_u \delta,m$ makes the results dispersion much lower. The experimental correlation for the transition inception as well the theoretical values of $IT_u$ and $IL_u$ can probably be rather easy implemented in the CFD calculations.

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Figure 4. Momentum thickness Reynolds number at the onset of transition as a function of turbulence scale

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