Influence of rainfall dynamics uncertainty on sensitivity analyses results in hydrodynamic models of large urbanised catchments

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Abstract. This paper presents an analysis method of rainfall uncertainty influence on the variability of modelled runoff from a municipal catchment. The model of 41.75 km² basin prepared in SWMM 5.0 software was used in this study. The developed model consisted of 120 sub-catchments of area 5-100 ha and terrain slope 0.2 – 6%. The share of the impervious surface area of sub-catchments was in the range of 1 – 80%. The model was preliminary calibrated basing on measurement data for six rainfall-runoff events at four measuring cross-sections. To allow assessment of time-related rainfall intensity variability influence on maximal flow values registered at measuring cross-sections, the length of time step was changed by a division of registered data values on 30 minutes periods. The uncertainty parameterisation was performed for analysed rainfall events (resolution 30 minutes) by introduction the subsequent values of θ = {0.3; 0.4; 0.5; 0.6; 0.7}. Thus, it was assumed that for θ = 0.3, 30% of hourly rainfall dropped during the first 30 minutes, the remaining 70% during the next time step, etc. For the assumed θ = 0.3 – 0.7 values influence calculations of calibrated parameters variability (retention depth of impervious, impervious surfaces, the roughness of impervious surfaces, retention depth of pervious surfaces, and roughness of pervious surfaces) on modelled values of maximal flows at measuring cross-sections were performed. In order to supplement the analysis, the influence of impervious terrains spatial variability at measuring cross-sections on determined dynamics of maximal flows concerning SWMM 5.0 calibrated parameter values was assessed.

1. Introduction

Simulation of the variability of stormwater runoff from urban catchments is important due to the development of catchments, modernisations implemented in stormwater networks, and the quality of rainwater transported to the receiver [1-8]. Currently, several computer programs can be used for simulation tests (e.g. INFOWORKS, XPSWMM, MIKE URBAN). To capture the spatial-temporal rainfall dynamics and thus improve rainfall-runoff modelling telecommunication microwave links (MWL) can be used [9]. One of the tools commonly used for this purpose is the SWMM program (Storm Water Management Model), due to the free availability of the source code [10, 11]. Thus, its
modification and extension are possible [12, 13]. Nevertheless, to obtain the useful results of the simulation, there is a need to gather detailed information about the catchment area and, on the other hand, to measure rainfall and flow with adequate resolution. Due to the number of parameters included in the model, the pre-calibration stage is a sensitivity analysis to identify the parameters affecting the simulation results [14-17]. This is an important stage of model development, which may be confirmed by numerous papers in the field [18, 19]. Although sensitivity analysis has a significant impact on subsequent stages of model calibration, many aspects are still unexplained. However, the influence of catchments’ characteristics, resolution of rainfall events, and outflow hydrograph characteristics are omitted in developed procedures of model calibration and sensitivity analyses [20-22].

The article presents the results of a sensitivity analysis of a large urban catchment area for selected calibration sections with different catchment characteristics. The analyses were performed for two precipitation events with varying precipitation resolution and rainfall distribution.

2. Materials and methods

2.1. Object of study
The research was conducted for the Silnica River catchment area, located in Southern Poland, in the Świętokrzyskie Voivodeship (figure 1). The total area of the selected basin equals 47.75 km². The largest part of the catchment is located in the city of Kielce, and only a small part in the Kielce county. The strong impact of urbanisation on the catchment is visible both in the city and its suburban area. The upper part of the Silnica catchment up to the Dąbrowa cross-section (D) is mostly covered by forests. The number of impervious areas is only 4.16% of the total surface of the catchment area. Below the Dąbrowa cross-section, the number of impervious areas increases, while the number of previous areas decreases. Between the calibration sections of Dąbrowa (D) and Jesionowa (J), there is a storage reservoir with an area of 10.5 ha and a capacity of 170 000 m³ [23].

![Figure 1](image-url)  
**Figure 1.** Division of the catchment area due to the rainfall post assigned to the area, including land use [24].
For the Pakosz calibration cross-section (P) (located below the city centre) the number of impervious areas increases by up to 25.8% of the total catchment area.

Below, up to the Białołoga calibration section (B) estuary, the share of green areas increases, while impervious areas reduce their share in the total area to 23.95%. Figure 1 shows the division of the catchment area for the respective rainfall post and the land use of the Silnica catchment area.

The length of the analysed watercourse is 19.741 km, and the average slope is 7.3‰. In the upper reaches, the river flows through forest areas where natural processes of sand and gravel accumulation occur after rainfall. From the section of Dąbrowa indicated in figure 1, up to the retention reservoir, the river flows through the suburban area. This section is partially regulated by turf or crushed stone. The floodplain route is built-up in places, which increases the water levels in the riverbed. By entering the Kielce Reservoir, the watercourse is building a delta. The nature of the riverbed changes below the dam. The banks are regulated by concrete elements. High banks protect against the occurrence of water flooding above the banks. Below the city centre, the riverbed is reinforced with turf and crushed stone. There is intense intra-trough accumulation here, which is the effect of overloading the river with transported material from the city. At the mouth of the river, there are no artificial reinforcements of the riverbed, which causes intensive erosion processes.

2.2. SWMM model

Sub-catchments were determined using the SAGA GIS software. For this purpose, DTM (Digital Terrain Model) with a spatial resolution of 1 m was used, made as part of the ISOK (IT System of the Country Protection) [25] project, and information on the course of the natural river network, obtained from BDOT (Topographic Objects Database) [26]. The surface area of each sub-catchment was determined in the QuantumGIS program. In the QGIS program, employing zone analyses, the average inclination of the catchment surface was determined, which ranges from about 2% to 24%. The catchments with small areas not exceeding 10a were characterised by extreme values of determined slopes.

The value of Manning’s roughness coefficient of terrain was used depending on the type of surface. The final value of the parameter for the sub-catchment was determined using a weighted average method considering the surface area and its type. The maximum roughness coefficient obtained for pervious areas was 0.09 s · m$^{-1/3}$ while for impervious areas 0.021 s · m$^{-1/3}$. For two types of surfaces, the internal flow option was chosen as “Outlet”.

The percentage share of impervious surface area to the area of the whole basin was determined analogically to calculations of surface area for determination of terrain roughness. The maximum value of the parameter is $CN = 98$, for a small area that is located in the centre of the Kielce city. Rainwater infiltration was determined by the SCS method [27] (developed by Soil Conservation Service) using the dimensionless parameter – $CN$, which takes values from the range 0-100. Where in $CN = 1$ means the perfectly pervious surface and $CN = 100$, completely impervious. The value of CN determined by the SCS method was 51. Figure 1 presents the model of the analysed area with designated sub-catchments, main sections of the stormwater drainage network, discharging rainwater to the Silnica, and the river network.

Based on the main cross-sections and designated intermediate sections, the course of the watercourse was mapped and the basic information about the analysed river was introduced into the model, i.e. the height and ordinates of the bottom of the riverbed. The lengths between successive nodes were determined from the vector layer of the natural river network obtained from the topographic object database (BDOT) [26].

Information on the course of the pipes, i.e. their actual location in the area, length of sections, and diameters were obtained from the geodetic records of terrain utilities from 2015 (GESUT) [28]. The roughness coefficient ($n$) for all pipes was assumed as 0.016 s · m$^{-1/3}$. According to reality, the cross-sectional shape of the pipes was adopted as circular.

One of the most important output parameters in hydrodynamic modelling of stormwater drainage is rainfall data. Measurements were carried out at four points in the studied area (Dąbrowa, Jesionowa,
Pakosz, and Białogon). The amount of precipitation [mm] and the instantaneous maximum flow rate were measured [dm$^3$ \cdot s$^{-1}$]. The applied measurements were performed from May 1999 to August 2003, with a time step of 1 hour. Three precipitation events were selected for the calibration of the model, which resulted in the formation of a hydrograph with the maximum flow registered in the first measuring stand ranging from 500-1300 dm$^3$ \cdot s$^{-1}$. Simulation tests were carried out for a time step of 1h, and the impact of precipitation uncertainty overtime for a 30-minute step. To determine the amount of precipitation in the area without hydrometeorological control the equal drop polygon method, also called the Thiessen method was used.[29].

2.3. Calculations of surface runoff and the level of sewage in the river and pipes
The outflow was calculated for each separated sub-catchment. Calculations in the SWMM simulation program were carried out by transforming effective precipitation into an outflow from a catchment. For this purpose, the so-called flat non-linear tank algorithm was used. The aforementioned division of the catchment area into two main types: pervious and impervious is important. The outflow is calculated separately for each of these areas and then added up. The time-varying effective drainage $Q_m$ from the catchment area, using a non-linear reservoir model, is calculated from a system of equations (1)[11]:

$$
\begin{cases}
Q_m = W \cdot \left( h - h_p \right)^{\frac{5}{3}} \cdot \frac{n_p}{i_p} \cdot i_p^{0.5} \\
\frac{dV}{dt} = F \frac{dh}{dt} = FI_e - Q_m
\end{cases}
$$

where: $W$ – hydraulic width of catchment [m], $h$ – effective height of precipitation [m], $h_p$ – height of surface retention [m], $n_p$ – Manning’s roughness coefficient [s \cdot m$^{1/3}$], $i_p$ – catchment slope [-], $V$ – volume of rainwater [m$^3$], $t$ – time [s], $F$ – catchment area [m$^2$], $I_e$ – time-related intensity of effective rainfall [mm \cdot s$^{-1}$].

The flow in the pipes is calculated using the full version of the de Saint-Venant equation system, for transient motion, using a dynamic wave model. The dynamic wave model is described by the formulas of the continuity equation (2) and the dynamic equation (3)[11]:

$$
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0
$$

$$
\frac{\partial Q}{\partial t} + \frac{\partial \left( \frac{Q^2}{A} \right)}{\partial x} + gA \frac{\partial H}{\partial x} + gAS_f - gAh_L = 0
$$

where: $A$ – cross-sectional area of the pipe [m$^2$], $Q$ – flow [m$^3$ \cdot s$^{-1}$], $t$ – [s], $x$ – distance from the beginning along the watercourse axis [m], $h_l$ – energy loss on length [-], $S_f$ – hydraulic loss calculated using the Chezy-Manning equation [-], $g$ – gravitational acceleration [m \cdot s$^{-2}$], $H$ – hydraulic height [m].

2.4. Parameterisation of rainfall uncertainty in a rainfall event
In order to assess the impact of varying rainfall intensity overtime on the maximum flow values recorded in measurement sections, the time step was changed by dividing the recorded data into 30-minute periods. In the analysed rainfall events (30 minutes resolution), parameterisation of rainfall uncertainty was performed by entering further values $\theta$ ($\theta = \{0.3; 0.4; 0.5; 0.6; 0.7\}$) (where: $\theta$ – parameter describing the uncertainty of rainfall identification in a time step). This means that for $\theta = 0.3$, 30% of the hourly rainfall fell in the first thirty minutes of rain, while in the next step fell 70%,
etc. The figure 2 shows the division diagram for i = \{1,2,3,4,...n\}. The total amount of rainfall (P) (figure 2) in a precipitation event is determined as follows (4):

\[ \sum P(t) = P \]  \hspace{1cm} (4)

Figure 2. The proposed scheme of rainfall parameterisation.

As a part of this work, an assessment of the impact of selected parameters describing the model on the obtained values of instantaneous maximum flow was analysed. The following factors were tested: N-imperv – Manning’s roughness (n) for impervious surface (N_{imp}), N-perv – Manning’s roughness (n) for pervious surface (N_{perv}), Dstore-imperv – Retention depth for impervious surface (D_{imp}), Dstore-perv – Retention depth for pervious surface (D_{perv}).

2.5. Assessment of the impact of rainfall uncertainty on the results of local sensitivity analysis calculations

In the work examined the impact of rainfall uncertainty on the results of simulation of a drain hydrograph from the catchment area by determining the model sensitivity based on the equation (5) [30]:

\[ \delta = \frac{Q_m(x_1,x_2,...x_j + \Delta x_j) - Q_m(x_1,x_2,...x_j)}{Q_m(x_1,x_2,...x_j)} \]  \hspace{1cm} (5)

where: \( Q_m \) – values of the maximum outflow from the catchment area in a calculated cross-section, \( x_j \) – initial value of the calibrated parameter, \( \Delta x_j \) – change of the parameter value covered by the calibration.
3. Results

Prepared rainfall-outflow model, to reflect the real conditions in the catchment must be calibrated. The calibration process consists of determining optimal parameter values, which ensure that the calculation results match the previous measurements [23]. The parameter values were determined for the selected criterion, which in this paper was the instantaneous maximum flow. Rainfall events of 2, 6, and 7 were used for the analysis. The first two were formed during heavy rain, which was characterised by high rainfall intensity, but short time duration. And the last overflow comes from the occurrence of lasting rainfall, characterised by steady high rainfall. Table 1 shows the flows recorded during the occurrence of accepted overflows and the values of model calculations.

| Overflows | Dąbrowa | Jesionowa | Pakosz | Białogon |
|-----------|---------|-----------|--------|----------|
| No. 2     | P 657   | O 702     | P 1482 | O 2284   |
|           |         |           |        |          |
|           |         |           |        |          |
| No. 6     | P 873   | O 1289    | P 1395 | O 1764   |
|           |         |           |        |          |
|           |         |           |        |          |
| No. 7     | P 532   | O 477     | P 757  | O 969    |
|           |         |           |        |          |
|           |         |           |        |          |

Each of the cross-sections is characterised by a different fit of the measured and computational results. The first cross-section, Dąbrowa, shows the best fit. This is due to the aforementioned land use, which is characterised by an exceptionally low degree of surface sealing. For further analysis, the data forming the overflows number 2. and 7. were used.

It was found, based on the calculations made, that in the rainfall event 1 $N_{imp}$ and $D_{perv}$ had a significant impact on the variability of the modelled maximum outflow in calculation sections. In turn, the values of $D_{imp}$ and $N_{perv}$ had a negligible impact on the variability of the modelled maximum outflow. However, in the rainfall event 2, the key impact of $N_{imp}$ and $D_{perv}$ on the modelled maximum outflow in calculation sections was demonstrated. The obtained results confirmed the significant impact of rainfall intensity distribution on the model’s sensitivity [30]. Besides, based on calculations, it was found that the development of the area had a significant impact on the sensitivity of the model. This confirms the varied course of the curves $\delta = f (x_j)$ for calculation sections. In the case of $N_{imp}$, a greater impact of changes in the value of the calibrated parameter for the Jesionów cross-section was demonstrated than in the Dąbrowa case. In the case of the Jesionów section, for $N_{imp} = 0.015$ the value of $\delta = 10.28\%$, while for the Dąbrowa section this value drops to 1.59%. In turn, approximate values of $\delta = f (N_{imp})$ were obtained for the Pakosz and Białogon cross-sections. The large variation in the curves obtained for events 1. and 2. confirm that, apart from the location of the calculation sections, the distribution of rainfall intensity and its dynamics in a precipitation event are of key importance.

The key aspect in model calibration and model sensitivity analysis is the uncertainty of rainfall, which is confirmed by the determined curves (figures 3 and 4). This is also confirmed by the analysis performed by Mrowiec (2008) [30]. Analysing the obtained curves (precipitation event 1), it was found that for the cross-section Jesionowa for $\theta = 0.2$ and $\theta = 0.8$ the highest values of $\delta$ were obtained. While for $\theta = 0.6$ the least sensitivity was obtained (figure 3a). In the Dąbrowa cross-section (figure 3a) the highest sensitivity was obtained for $\theta = 0.2$ value $\delta = 3.86\%$, and the lowest for $\theta = 0.8$ value $\delta = 1.59\%$. For Pakosz and Białogon cross-sections, the greatest sensitivity of the model was demonstrated for $\theta = 0.2$. For example, for $N_{imp} = 0.015$, the value of $\delta$ is 13.36% and 11.66%, respectively. Considering $D_{per}$ (figure 2b), the highest sensitivity of the model was found for the Pakosz and Białogon cross-sections for $\theta = 0.8$ and the lowest for $\theta = 0.20$. For example, for $D_{per} = 2.5$, the largest values of $\delta$ for Pakosz and Białogon are 13.16% and 7.86%, respectively. Analysing
the variation $\delta = f(N_{perv})$ (figure 4a), the greatest sensitivity of the model was demonstrated for the cross-section of Jesionów and Pakosz, which is confirmed by the calculated values of $\delta$. In the case of the Dąbrowa and Pakosz cross-sections, the greatest sensitivity of the model was demonstrated for $\theta = 0.6$. The smallest sensitivity was obtained for the Jesionowa section for $\theta = 0.2$ equal to 6.77%, and for the Pakosz section for $\theta = 0.8$, where for $x = 0.05$ the value of $\delta = 5.52\%$. Analysing the variability of $\delta = f(D_{perv})$, it was found that for $D_{perv} > 4.0$ mm, the $\theta$ value does not affect the sensitivity of the model.

**Figure 3.** The impact of $N_{imp} = f(\theta)$ on $\delta$ for the cross-section (a) Dąbrowa, Jesionowa and (b) Pakosz, Białogon and the impact of $D_{perv} = f(\theta)$ on $\delta$ for the cross-section (c) Dąbrowa, Jesionowa and (d) Pakosz, Białogon for rainfall event 1.

**Figure 4.** The impact of $N_{perv} = f(\theta)$ on $\delta$ for cross-section (a) Pakosz, Jesionowa and the impact of $D_{perv} = f(\theta)$ on $\delta$ for cross-section (b) Dąbrowa, Pakosz for rainfall event 2.
4. Conclusions
The results of calculations of instantaneous maximum flow confirm, on the one hand, the complex impact and interactions between the calibrated model parameters and the catchment area characteristics. On the other hand, they indicate the important role of identifying rainfall distribution variability on simulation results and sensitivity analysis results. Based on the results, it was observed that both, the distribution of rainfall intensity and land use have a significant impact on the sensitivity of the model.

This is important from the point of model calibration, collection of measurement data, application of appropriate methods of measurement, and location of calibration cross-sections in large urban catchments.

Based on the calculations, it was found that land development has a significant impact on the sensitivity of the model. According to the value of the roughness coefficient for impervious surfaces, the greater influence of the calibrated parameter changes was observed for the Jasionów cross-section than Dąbrowa. For the Jasionów cross-section for $N_{imp} = 0.015$, the obtained value was $\delta = 10.28\%$, and for the Dąbrowa cross-section, this value drops to $1.59\%$. In the Dąbrowa cross-section, the highest sensitivity was obtained for $\theta = 0.2$, the value of $\delta = 3.86\%$, and the lowest for $\theta = 0.8$, the value of $\delta = 1.59\%$.

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