Classical-driving-enhanced parameter-estimation precision of a non-Markovian dissipative two-state system

Yan-Ling Li\textsuperscript{1}, Xing Xiao\textsuperscript{2,3,}\textsuperscript{*} and Yao Yao\textsuperscript{3†}

\textsuperscript{1} School of Information Engineering, Jiangxi University of Science and Technology, Ganzhou 341000, China
\textsuperscript{2} College of Physics and Electronic Information, Gannan Normal University, Ganzhou 341000, China
\textsuperscript{3} Beijing Computational Science Research Center, Beijing 100084, China

The dynamics of quantum Fisher information (QFI) of the phase parameter in a driven two-state system is studied within the framework of non-Markovian dissipative process. The influences of memory effects, classical driving and detunings on the parameter-estimation precision are demonstrated by exactly solving the Hamiltonian under rotating-wave approximation. In sharp contrast with the results obtained in the presence of Markovian dissipation, we find that classical driving can drastically enhance the QFI, namely, the precision of parameter estimation in the non-Markovian regime. Moreover, the parameter-estimation precision may even be preserved from the influence of surrounding non-Markovian dissipation with the assistance of classical driving. Remarkably, we reveal that the enhancement and preservation of QFI highly depend on the combination of classical driving and non-Markovian effects. Finally, a phenomenological explanation of the underlying mechanism is presented in detail via the quasimode theory.

PACS numbers: 42.50.Lc, 42.50.Ct, 03.65.Yz

I. INTRODUCTION

Quantum metrology is a fast developing field of current research in both theoretical and experimental physics[1, 2]. It is aimed to explore the capabilities of quantum systems that, when employed as probes sensing physical parameters, allow to attain resolutions that are beyond the ability of classical protocols [3, 4]. According to the quantum estimation theory, the ultimate achievable precision in parameter estimation scenarios is characterized by the quantum Cramér-Rao inequality [5]: \( \delta \phi \geq \frac{1}{\sqrt{NF}} \), where \( N \) denotes the number of measurement repetitions and \( F \) is the QFI. Namely, the ultimate precision is inversely proportional to the square root of QFI.

Phase estimation plays a central role in practical quantum metrology, such as optical interferometry [6, 7] and atomic spectroscopy [8, 9], since most of tasks can be attributed to the problem of estimating the relative phase. Unfortunately, any realistic quantum system inevitably couples to an uncontrollable environment which influences it in a non-negligible way [10]. Then the issue of robustness of quantum metrological protocols against various sources of decoherence has been raised [11–15], in particular, wondering whether such metrological schemes could be utilized not only to beat the standard quantum limit (SQL), but also to achieve the Heisenberg limit (HL) [1]. In this context, it is a pivotal task to preserve the precision scaling under the environmental noises.

The feasibility of preservation is greatly dependent on the intrinsic properties of the environment coupled to the system. According to the scale of correlation times, environments can be grouped into Markovian or non-Markovian types. The former case possessing a small decoherence time during which correlations disappear, has been proven to be harmful to quantum metrology [16–24]. Even a very low noise level can completely destroy the superiority of quantum metrological protocols and turn HL into SQL. However, non-Markovian environments which characterized by long correlation times or structured spectral features would be more general in many physical situations [25–29]. The research of non-Markovian effects (also known as memory effects) is attracting extensive attentions due to key developments in the analysis, understanding, and even simulation of non-trivial system-environment effects [30–37]. In a seminal work of Chin et al [38], they first pointed that the non-Markovian effects can guarantee the advantage of quantum metrological strategies in the presence of noise. This means non-Markovianity may serve as a new resource for enhancing estimation tasks in open systems. The aforementioned discussions are restricted to consider the influence of non-Markovian effects on the parameter estimation. However, to the best of our knowledge, few detailed investigations concerning the control of precision under environmental noises are available at present.

Motivated by the above considerations, this study is to discuss the role of classical driving in the precise estimation of relative phase. To this end, the QFI is examined
II. QUANTUM FISHER INFORMATION

The classical Fisher information originates from the statistical inference, in which we are given a set probability distributions \( p(x_i|\phi) \) with measurement outcomes \( \{x_i\} \). Here \( \phi \) is an unknown parameter that we wish to determine and \( \vec{X} = \{x_1, x_2, ..., x_N\} \) is an observable random variable. Without loss of generality, we assume \( \phi \) is a real parameter and the observable \( \vec{X} \) is discrete. The classical Fisher information is defined as

\[
F_\phi = \sum_i p(x_i|\phi) \left[ \frac{\partial \ln p(x_i|\phi)}{\partial \phi} \right]^2, \tag{1}
\]

which characterizes the inverse variance of the asymptotic normality of a maximum-likelihood estimator. Note that if the observable \( X \) is continuous, the summation should be replaced by an integral.

Quantum Fisher information is formally generalized from the classical one and is defined as

\[
F_\phi = \text{Tr}(\rho_\phi L_\phi^2) = \text{Tr}[\partial_\phi \rho_\phi L_\phi], \tag{2}
\]

where \( L_\phi \) is the so-called symmetric logarithmic derivative, which is defined by \( \partial_\phi \rho_\phi = (L_\phi \rho_\phi + \rho_\phi L_\phi)/2 \) with \( \partial_\phi = \partial/\partial \phi \). By diagonalizing the matrix as \( \rho_\phi = \sum_n \lambda_n |\psi_n\rangle \langle \psi_n|, \) one can rewritten the QFI as [18, 39]

\[
F_\phi = \sum_n \frac{(\partial_\phi \lambda_n)^2}{\lambda_n} + \sum_n \lambda_n F_{\phi,n} - \sum_{n \neq m} \frac{8 \lambda_n \lambda_m}{\lambda_n + \lambda_m} |\langle \psi_m| \partial_\phi \psi_n \rangle|^2, \tag{3}
\]

where \( F_{\phi,n} \) is the QFI for pure state \( |\psi_n\rangle \) with the form

\[
F_{\phi,n} = 4\left[|\langle \psi_n| \partial_\phi \psi_n \rangle|^2 - |\langle \psi_n| \partial_\phi \psi_n \rangle|^2\right]. \tag{4}
\]

Note that Eq. (3) suggests the QFI of a non-full rank state is only determined by the subset of \( \{|\psi_i\rangle\} \) with nonzero eigenvalues. Physically, the QFI can be divided into three parts [39, 40]. The first term is just the classical Fisher information determined by the probability distribution; The second term is a weighted average over the QFI for all the nonzero eigenstates; The last term stemming from the mixture of pure states reduces the QFI and hence the estimation precision below the pure-state case.

III. MODEL

We consider a two-level atom with Bohr frequency \( \omega_0 \) driven by a classical field of frequency \( \omega_L \) [41, 42]. The atom is embedded in a zero-temperature bosonic reservoir. Under the rotating-wave approximation, the Hamiltonian of the system can be written as (\( h = 1 \))

\[
H = \frac{\omega_0}{2} \sigma_z + \sum_k \omega_k a_k^\dagger a_k + \left( \sum_k g_k \sigma_+ e^{-i \omega_k t} \sigma_+ + h.c. \right), \tag{5}
\]

where \( \sigma_{x,y,z} \) are the Pauli operators, \( \sigma_+ \) and \( \sigma_- \) the atomic inversion operators, \( a_k^\dagger \) and \( a_k \) the creation and annihilation operators of the \( k \)th mode with frequency \( \omega_k \) of the reservoir and \( g_k \) the coupling constants between the atom and the reservoir. The Rabi frequency \( \Omega \), which has been assumed to be a real number, is taken to be small compared to the atomic and laser frequencies \( \Omega \ll \omega_0, \omega_L \).

Since a unitary transformation does not change the eigenvalues of the system, in the rotating reference frame through a unitary transformation \( U_R = e^{-i \omega_0 t/2} \), the Hamiltonian in equation (5) is equivalently transferred to an effective Hamiltonian

\[
H_e = \frac{\Delta}{2} \sigma_z + \Omega \sigma_x + \sum_k \omega_k a_k^\dagger a_k + \left( \sum_k g_k \sigma_+ e^{i \omega_k t} \sigma_+ + h.c. \right), \tag{6}
\]

where \( \Delta = |\omega_0 - \omega_L| \). Note that the first two terms on the right side can be diagonalized in the new dressed bases

\[
|E\rangle = \cos \frac{\eta}{2} |e\rangle + \sin \frac{\eta}{2} |g\rangle,
\]

\[
|G\rangle = -\sin \frac{\eta}{2} |e\rangle + \cos \frac{\eta}{2} |g\rangle,
\]

with \( \eta = \tan^{-1}(2\Omega/\Delta) \). Then in the dressed-state bases, the effective Hamiltonian can be rewritten as

\[
H'_e = \frac{\omega_D}{2} \rho + \sum_k \omega_k a_k^\dagger a_k + \cos^2 \frac{\eta}{2} \sum_k (g_k e^{i \omega_k t} a_k \rho_+ + h.c.), \tag{8}
\]

where \( \omega_D = \sqrt{\Delta^2 + 4\Omega^2} \) is the dressed frequency. The new inversion operator \( \rho_+ \) is given by \( \rho_+ = |E\rangle\langle E| - |G\rangle\langle G| \).
\(|G⟩⟨G|\) and the new raising operator \(ρ_+\) is defined as 
\(ρ_+ = \langle E|G\) Here, the terms \(akρ_+e^{iωLt}, akρ_−e^{−iωLt}\) and 
their complex conjugates have been neglected by using the usual rotating-wave approximation.

We consider the situation of no more than one excitation in the whole system, so the subspace spanned in the dressed bases is given by: 
\(|ψ⟩ = |G⟩S \otimes |0⟩_R, |ψ⟩ = |E⟩S \otimes |0⟩_R, |ψ_k⟩ = |G⟩S \otimes |k⟩_R,\) where \(|k⟩_R\) indicates that there is a photon in the \(k\)th mode of the reservoir. 
Assuming the environment is initially prepared in the vacuum state, then it follows that any initial state of the form

\(|Ψ(0)⟩ = c_0|ψ₀⟩ + c_1(0)|ψ₁⟩,\) (9)
evolves after time \(t\) into the state

\(|Ψ(t)⟩ = c_0|ψ₀⟩ + c_1(t)|ψ₁⟩ + \sum_k c_k(t)|ψ_k⟩.\) (10)

Note that the amplitudes \(c_1(t)\) and \(c_k(t)\) depend on time, while the amplitude \(c_0\) is constant in time because \(H_p|ψ₀⟩ = 0\). By solving the Schrödinger equation, we could obtain a closed integro-differential equation for \(c_1(t)\)

\(\dot{c}_1(t) = −\cos t \frac{η}{2} \int_0^t dt_1 f(t − t_1)c_1(t_1).\) (11)

The kernel \(f(t − t_1)\) is given by a certain two-point correlation function of the reservoir \([10]\).

\(f(t − t_1) = \int dω J(ω) \exp[\imath(ωD + ωL − ω)(t − t_1)].\) (12)

In above, we have used the limitation of a continuum of reservoir modes \(\sum_k |g_k|^2 \rightarrow \int J(ω)dω,\) where \(J(ω)\) is the spectral density function, characterizing the reservoir spectrum. Until now, our result is valid for an environment with a generic spectral density since no restrictive hypothesis is made on the environment.

In order to study in more detail the QFI in a non-Markovian environment, we need to specify the spectral density of the reservoir. We focus on the Lorentzian spectral density of the form

\(J(ω) = \frac{1}{2\pi} \frac{γ_0 \lambda^2}{(ω_0 − ω − δ)^2 + λ^2},\) (13)

where \(δ = ω_0 − ω_c\) is the detuning between atomic frequency \(ω_0\) and the center frequency of the structured environment \(ω_c\). The parameter \(λ\) defines the spectral width and \(γ_0\) is related to the decay of the excited state of the atom in the Markovian limit \([43]\). Usually, there are two regimes \([44]\): weak-coupling regime \((γ_0 < λ/2),\) where the behavior of the system is Markovian, and strong-coupling regime \((γ_0 > λ/2),\) where non-Markovian dynamics occurs. Note that Eq.\,(13) is one of the most studied spectrums of bosonic environments since it leads to an electromagnetic field inside an imperfect cavity supporting the mode \(ω_0\), which is typical of dissipative systems in several physical contexts. In this model, the reservoir correlation function is given by an exponential form

\(f(t − t_1) = \frac{γ_0 λ}{2} \exp[−M(t − t_1)],\) (14)

where \(M = λ + iδ − iω_D\). Then the probability amplitude can be easily calculated \(c_1(t) = c_1(0)\xi(t),\) where \(\xi(t)\) is expressed as

\(\xi(t) = e^{−Mt/2}[\cosh(\frac{Kt}{2}) + \frac{2M}{K}\sinh(\frac{Kt}{4})],\) (15)

with \(K = \sqrt{4M^2 − 2γ_0λ(1 + \cos \eta)^2}\). Briefly, the total evolution could be described by the following state map

\(|G⟩S \otimes |0⟩_R \rightarrow |G⟩S \otimes |0⟩_R,\) (16)
\(|E⟩S \otimes |0⟩_R \rightarrow \xi(t)|E⟩S \otimes |0⟩_R + \sqrt{1 − \xi^2(t)}|G⟩S \otimes |k⟩_R\).

IV. CLASSICAL-DRIVING-ENHANCED PARAMETER-ESTIMATION PRECISION

With above state map in mind, we assume that the qubit is initially in the state

\(|ψ⟩_a = \cos \frac{θ}{2}|0⟩ + \cos \frac{φ}{2}|1⟩,\) (17)

and the reservoir is in the vacuum state, then the state of the total system is

\(|Ψ(t)⟩ = [c_0 \cos \frac{η}{2} + c_1(1) |\sin \frac{η}{2}|0⟩_a |0⟩_R + \[−c_0 \sin \frac{η}{2} + c_1(1) |\sin \frac{η}{2}|0⟩_a |0⟩_R + [d(t) \sin \frac{η}{2}|0⟩_a + d(t) |\sin \frac{η}{2}|1⟩_a |k⟩_R,\) (18)

where \(c_0 = \cos \frac{θ}{2} \cos \frac{φ}{2} − \cos \frac{θ}{2} \cos \frac{φ}{2} c_1(1) + \cos \frac{θ}{2} \sin \frac{φ}{2} c_1(1), d(t) = \sqrt{1 − |c_1(1)|^2 − |\cos \frac{η}{2}|^2} \cdot \frac{1}{2} \sum_k |g_k|^2 = \sum_k |g_k|^2 \cdot \frac{1}{2} \sum_k |g_k|^2.\) The reduced density matrix of the atomic qubit can be given by using the reservoir’s degrees of freedom

\(ρ(t) = ρ_{00}|0⟩⟨0| + ρ_{01}|0⟩⟨1| + ρ_{10}|1⟩⟨0| + ρ_{11}|1⟩⟨1|,\) (19)

where

\(ρ_{00} = \cos^2 \frac{η}{2} + c_1(1)^2 |\cos \frac{η}{2}|^2 + \frac{1}{2}[c_0 c_1^* + c_1^* c_1] |\sin \η|,\)
\(ρ_{01} = \left[−\frac{1}{2} + |c_1(1)|^2 |\cos \frac{η}{2} |^2 + \frac{1}{2}[c_0 c_1^* + c_1^* c_1] |\sin \η|,\)
\(ρ_{10} = \left[−\frac{1}{2} + |c_1(1)|^2 |\sin \frac{η}{2}|^2 + \frac{1}{2}[c_0 c_1^* + c_1^* c_1] |\cos \frac{η}{2}|^2 \right],\)
\(ρ_{11} = \sin^2 \frac{η}{2} + c_1(1)^2 |\cos \frac{η}{2}|^2 − \frac{1}{2}[c_0 c_1^* + c_1^* c_1] |\sin \η|,\) (20)

For the single qubit state, according to Eq.\,(3), an explicitly expression of QFI could be obtained. In the Bloch sphere representation, any qubit state can be written as

\(ρ = \frac{1}{2}(1 + \vec{W} ⋅ \vec{δ}),\) (21)
where \( \hat{W} = (W_x, W_y, W_z)^T \) is the real Bloch vector and \( \hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z) \) denotes the Pauli matrices. Therefore, for the single qubit state, \( F_\phi \) can be represented as follows [45]

\[
F_\phi = \begin{cases} 
|\partial_\phi \hat{W}|^2 + \frac{\hat{W} \cdot \partial_\phi \hat{\sigma}}{1 - |\hat{W}|^2}, & \text{if } |\hat{W}| < 1, \\
|\partial_\phi \hat{W}|^2, & \text{if } |\hat{W}| = 1.
\end{cases}
\tag{22}
\]

Substituting the Bloch vector components \( W_x = \rho_{01} + \rho_{10}, W_y = i(\rho_{01} - \rho_{10}), W_z = 2\rho_{00} - 1 \) into Eq. (3), then the dynamics of the QFI of parameter \( \phi \) can be obtained exactly by Eq.(22). However, the explicit expression is too complicated to present in the text. Nevertheless, numerical results indicate that the dynamics of QFI in non-Markovian reservoir by classical driving show interesting properties.

In order to observe the effect of classical driving on the estimation precision clearly, in Fig. 1, we show the dynamics of the QFI (\( F_\phi \)) with respect to different strength of classical driving. In order to make results comparable, we have plotted the results in both Markovian and non-Markovian regimes. It is clearly shown that the classical driving may slightly retard the QFI loss during the time evolution in the Markovian regime, but the \( F_\phi \) still decays rapidly to zero. The decay of \( F_\phi \) reflects that the estimation of parameter \( \phi \) becomes more inaccurate in this situation. In contrast, the behaviors of QFI are more complicated and interesting in the non-Markovian regime. As shown in Fig. 1(b), the QFI experiences damped oscillations in the absence of classical driving. The reason is that the QFI flows back and forth between the system and its non-Markovian reservoir due to the memory effect (a long correlation time of the reservoir) [32, 37]. However, it is remarkable that in the non-Markovian regime the classical driving plays an important role in the dynamics of QFI. We find the classical driving can dramatically protect the QFI from the influence of non-Markovian noises and enhance the precision of parameter estimation. The stronger the classical driving is, the higher is the estimation precision. These results can be understood as follows. On the one hand, we know that the oscillations of the atomic inversion represent the exchanging of energy between the atom and the field. When the Rabi frequency becomes larger, the energy exchange will become more rapid [46]. On the other hand, when the Rabi frequency increases, the effective coupling between the qubit and the reservoir decreases that suppresses the information exchange. Therefore the outflow of the information from the qubit is suppressed, i.e., the decay of QFI slows down. Due to the memory effects, this phenomenon is more evidently in the non-Markovian regime than Markovian regime. However, what’s the relation between the QFI preservation, non-Markovian effects and the classical driving?

To get a better understanding of the effects of classical driving and non-Markovian characteristics on the QFI preservation, we plot Fig. 2 to show the asymptotic behaviors of QFI as a function of the dimensionless quantity \( \Omega/\gamma_0 \) with \( \theta = \pi/2, \Delta = 0, \delta = 0 \) and \( \gamma_0 t = 50 \). Here, \( \lambda = 5\gamma_0 \) corresponds to the Markovian reservoir, while \( \lambda = 0.05\gamma_0, \lambda = 0.1\gamma_0, \lambda = 0.5\gamma_0 \) correspond to different non-Markovian reservoirs, respectively.

In order to observe the effect of classical driving under the classical driving. The other parameters are \( \theta = \pi/2, \Delta = 0 \) and \( \delta = 0 \). (a) In the Markovian regime \( \lambda = 10\gamma_0 \). (b) In the non-Markovian regime \( \lambda = 0.05\gamma_0 \).

FIG. 1. (color online) Quantum Fisher information \( F_\phi \) as a function of \( \gamma_0 t \) under the classical driving. The other parameters are \( \theta = \pi/2, \Delta = 0 \) and \( \delta = 0 \). (a) In the Markovian regime \( \lambda = 10\gamma_0 \). (b) In the non-Markovian regime \( \lambda = 0.05\gamma_0 \).

FIG. 2. (color online) Asymptotic behaviors of QFI as a function of the dimensionless quantity \( \Omega/\gamma_0 \) with \( \theta = \pi/2, \Delta = 0, \delta = 0 \) and \( \gamma_0 t = 50 \). Here, \( \lambda = 5\gamma_0 \) corresponds to the Markovian reservoir, while \( \lambda = 0.05\gamma_0, \lambda = 0.1\gamma_0, \lambda = 0.5\gamma_0 \) correspond to different non-Markovian reservoirs, respectively.
the enhancement of QFI greatly benefits from the combination of classical driving and non-Markovian effects.

In above, only the resonantly driving case is discussed, but the detuning (i.e. $\Delta \neq 0$) case would be more reasonable considering the frequency shift induced by the interaction with the environment. $F_\phi$ as a function of dimensionless quantity $\Delta/\gamma_0$ and $\Omega/\gamma_0$ with $\gamma_0 t = 50$ is plotted in Fig. 3. It follows from the numerical analysis that the detuning $\Delta$ has an adverse influence upon the precision of parameter estimation. This result is to be expected since the large detuning make the coupling between the qubit and the driving field weaker. Fortunately, this negative influence can be suppressed by increasing the the strength of classical driving.

Another factor on the precision of parameter estimation is the detuning $\delta = \omega_0 - \omega_c$ between the qubit frequency $\omega_0$ and the center frequency of the structured reservoir $\omega_c$. In Ref. [37], the author has shown that the enhancement of the QFI may occur by adjusting the reservoir-qubit detuning. However, we should point out that the result is a little different in our model. It is noted that the enhancement of QFI depends on the absolute values of $\delta$ in Ref. [37], namely, both positive and negative detunings would enhance the QFI equally. While in our model, considering the presence of classical driving, the symmetry of $F_\phi$ (with respect to $\delta = 0$) is broken and positive $\delta$ outperforms the negative one (with the same absolute value) for the enhancement of precision, as shown in Fig. 4. In addition, we note that negative $\delta$ first fails to improve the precision in a small region $[-(\sqrt{\Delta^2 + 4\Omega^2} - \Delta), 0]$, but regains the ability when $\delta < -(\sqrt{\Delta^2 + 4\Omega^2} - \Delta)$. This phenomenon could be understood clearer in Sec. V.

In a nutshell, from Figs. 1-4, we note some features as follows: (1) The increasing of Rabi frequency $\Omega$, non-Markovian effects $\gamma_0/\lambda$ and the positive detuning $\delta$ can effectively enhance the parameter-estimation precision. (2) This drastic enhancement occurs only when the conditions of the non-Markovian effects and the classical driving are satisfied simultaneously. These results provide a good method to enhance the precision of phase estimation in open quantum systems and would be benefit for quantum metrology.

V. PHYSICAL INTERPRETATION

In order to construct a more intuitive physical insight into the phenomenon of classical-driving-enhanced precision in non-Markovian environment, we utilize the quasi-mode Hamiltonian [44, 47–49] in the following analysis. For the Hamiltonian described by Eq. (8), we can observe it in another rotating reference frame $U = \exp[\hat{i}\omega_{st} t]/2$, then an effective Hamiltonian can be obtained as

$$H'_e = \frac{\omega_0'}{2} \rho_z + \sum_k \omega_k a_k^\dagger a_k + \sum g_k d_k \rho_+ + h.c.,$$

where $\omega_0' = \omega_0 + \sqrt{\Delta^2 + 4\Omega^2} - \Delta$, $g_k = \cos^2(\eta/2)g_k$. The effective Hamiltonian $H'_e$ is an exact unbiased spin-boson model. A noteworthy feature is that the basis states have been changed to \{|$E\rangle, |G\rangle$\} when the atom coupled with the structured reservoir with the assistance of the classical driving. Assuming the spectral function of the reservoir is still Lorentzian, the corresponding quasi-mode Hamiltonian can be given by

$$H_{\text{quasi}} = H'_0 + H_{\text{memory}} + H_{\text{dis}},$$

with

$$H'_0 = \frac{1}{2}\omega_0' \rho_z + \omega_c D^\dagger D + \int \nu C^\dagger(\nu) C(\nu) d\nu,$$

$$H_{\text{memory}} = \sqrt{\gamma_0} \lambda/2 (\rho_+ D + \rho_- D^\dagger),$$

$$H_{\text{dis}} = (\lambda/\pi) \frac{1}{2} \int d\nu [D^\dagger C(\nu) + DC^\dagger(\nu)].$$
and the continuum modes. Therefore, in the Markovian regime \((\lambda \gg \gamma_0)\), the dynamics of QFI is dominated by the dissipative interaction \(H_{\text{dis}}\), and the QFI decays exponentially and vanishes only asymptotically, as depicted in Fig. 1(a). Whereas in the non-Markovian regime \(\lambda \ll \gamma_0\), the dissipative interaction is suppressed and the information could exchange back and forth between the system and discrete quasimode before it is completely dissipated into the continuum quasimodes. So the QFI vanishes with a damping of its revival amplitude in the non-Markovian regime, as shown in Fig. 1(b).

Next, we demonstrate how the classical driving affect the dynamics of QFI. In the quasimode Hamiltonian, the memory part is also a Jaynes-Cummings model, and the effective detuning between the qubit and discrete quasimode is

\[
\Delta_{\text{eff}}(\Omega, \delta, \delta) = |\sqrt{\Delta^2 + 4\Omega^2} - \Delta + \delta|, \tag{28}
\]

where \(\Delta = |\omega_0 - \omega_L|\), \(\delta = \omega_0 - \omega_c\), and Rabi frequency \(\Omega\). It is well known that for the Jaynes-Cummings model, the larger the absolute value of detuning, the weaker coupling between the qubit and the mode of the field \([46]\). Then the information transfer between qubit and field slows down. With this conclusion in mind, the results in Sec. IV could be explained clearly via the effective detuning \(\Delta_{\text{eff}}\). As shown in Fig. 5(b), \(\Delta_{\text{eff}}\) is a monotonic increasing function of \(\Omega\) and positive \(\delta\), while it is a decreasing function of \(\Delta\). Therefore, increasing \(\Omega\) and positive \(\delta\) can effectively enhance the precision of parameter estimation, whereas increasing \(\Delta\) reduces the precision. That is why increasing \(\Omega\) and positive \(\delta\) are beneficial to the enhancement of precision, while increasing \(\Delta\) is harmful. However, due to the presence of classical driving, negative \(\delta\) first reduces the effective detuning \(\Delta_{\text{eff}}\) in a small region and then increases it when \(\delta < -(\sqrt{\Delta^2 + 4\Omega^2} - \Delta)\).

We shall first exploit the physical mechanism of the QFI dynamics without classical driving. From the quasimode Hamiltonian, we find that the system only couples to a discrete quasimode, which interacts with a set of continuum quasimodes, as shown in Fig. 5(a). Note that the coupling strength between the discrete quasimode and continuum quasimodes just depends on the width of the spectral density \(\lambda\). The discrete quasimode functions as a memory between the system and the dissipative environment. The behavior of the new system composed by the qubit and the discrete quasimode is completely Markovian \([44]\). Since the qubit interacts only with the discrete quasimode, while the dissipative process happens only in the interaction between the discrete mode and the continuum modes.

VI. CONCLUSIONS

In summary, we have investigated the parameter-estimation precision of a driven two-state system coupled to a bosonic environment at zero temperature. The pre-
cision is just slightly improved, and still decays asymptotically to zero in the Markovian environment under the classical driving. However, it could be greatly enhanced and preserves a quite long time in non-Markovian environment with the assistance of classical driving. We also find that increasing the Rabi frequency or the degree of the non-Markovian effects can make further improvement on the precision, while non-resonant driving will reduce the precision. Moreover, we should emphasize that the drastic enhancement is based on the combination of classical driving and non-Markovian effects. The above results provide an active method to combat the influence of decoherence on quantum metrology. Finally, according to the quasimode theory, an intuitive physical interpretation has been provided about the precision enhancement under the classical driving.

ACKNOWLEDGMENTS

This work is supported by the Funds of the National Natural Science Foundation of China under Grant Nos. 11247006, 11365011, 11447118 and the China Postdoctoral Science Foundation under Grant No. 2014M550598.
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