Impact of Single-Top Measurement to Littlest Higgs Model with T-Parity

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We show that a precise measurement of the single-top production cross section at the Tevatron and the LHC can strongly constrain the model parameters of the Littlest Higgs model with T-parity. A reduction in the single-top production rate from the Standard Model prediction implies new physics phenomena generated by the heavy T-parity partners of the top quark. We show that the degree of polarization of the top quark produced from the decay of its heavy T-odd partner (T−) can be utilized to determine the new physics energy scale, and the mass of T− can be measured from the missing transverse momentum distribution in the T− → b̄b event.

In spite of the great success of the Standard Model (SM) of particle physics, there is no good understanding why the mass of the SM Higgs boson is at the weak scale. One extension of the SM, as a low energy effective theory below the cutoff scale Λ, is the class of Little Higgs (LH) models [1] in which the electroweak symmetry is collectively broken and a weak scale Higgs boson mass is radiatively generated. At one loop order, the large Λ2 correction to the Higgs boson mass term induced by the top quark (t) is cancelled by its fermionic partner, and those induced by the electroweak gauge bosons are cancelled by their bosonic partners. Constraints from low energy precision data, especially the ρ-parameter measurement, require the symmetry breaking scale of the LH models to be so high that the predicted phenomenology has little relevance to the current high energy collider physics program [2]. To relax the constraints from low energy data, a discrete symmetry called T-parity was introduced in the Little Higgs models, to warrant the T-parity partners of the photon [3]. Consequently, the cutoff scale of the model, Λ = 4πf, can be as low as 10 TeV and the masses of new heavy resonances, at the scale of f, can be of sub-TeV [3]. This type of models is particularly interesting because it also provides a dark matter candidate which is the lightest T-odd particle (LTP) A_H, the heavy bosonic T-parity partner of photon [8]. Here, we shall focus on the “Littlest” Higgs model with T-parity (LHT), which is based on an SU(5)/SO(5) nonlinear sigma model whose low energy Lagrangian is described in detail in Ref. [3]. The new particle mass scale f of the model is bounded from below by low energy precision data to be about 500 GeV [4]. In this work, we will concentrate on the phenomenology associated with the T-even top partner (T+), T-odd top partner (T−) and T-odd partners of the electroweak gauge bosons [7].

After the electroweak symmetry breaking, the masses of the T-parity partners of the photon (A_H), Z-boson (Z_H) and W-boson (W_H) are generated as M_{A_H} = \frac{g'f}{\sqrt{2}}(1 - \frac{5v^2}{8f^2} + \cdots) and M_{Z_H} \simeq M_{W_H} = gf(1 - \frac{v^2}{8f^2} + \cdots). Here, v characterizes the weak scale (∼246 GeV), and at tree level the SM-like W and Z gauge boson masses can be expressed as M_W = \frac{g}{2}v and M_Z = \sqrt{g^2 + g'^2}v, respectively. Because of the smallness of the U(1)_Y gauge coupling constant g', the T-parity partner of the photon A_H tends to be the lightest T-odd particle in the LHT. We note that the mass of W_H is determined by f, for the SU(2)_Y gauge coupling constant g and the vacuum expectation value v have been fixed by the measured values of W and Z boson masses. Hence, if M_{W_H} can be directly measured from collider data, then the cutoff scale of the LHT can be determined. Another way to determine the scale f is to study the T-parity partners of the top quark which is to be shown below.

As shown in Ref. [3], the mass of top quark is generated from top-Yukawa interaction Lagrangian which depends on two parameters of the LHT. In this work they are chosen to be f and the mixing angle α which describes the amount of mixing among the fermionic degrees of freedom needed to cancel the quadratic divergence of Higgs boson mass term at the one loop level. The mass of top quark (m_t) has been measured to a good accuracy [6]. Given m_t and v, we can trade the two parameters f and α by the masses of the top quark T-parity partners T+ and T−. Up to the O(v^2/f^2) corrections, they can be expressed, respectively, as

M_{T+} = \frac{m_t f}{v s_α}, \quad M_{T−} = \frac{m_t f}{v c_α}, \quad (1)

where s_α denotes sin α and is bounded from above to be less than 0.96 by the unitarity requirement for considering the J = 1 partial wave amplitudes in the coupled system of (t̄t, T+T+, b̄b, WW, Zh) states [3]. Moreover, sin α cannot be exactly equal to zero because the “collective” symmetry breaking mechanism of the LHT only works for a non-zero s_α, and M_{T+} cannot be larger than the cutoff scale Λ. If we take the “naturalness” argument seriously for the Higgs mass corrections, then s_α has to be larger than about 0.1 for f to be around 1 TeV [2]. In Fig. 1 we show the contours of M_{T+} (left panel) and M− (right panel) in the plane of s_α and f.

Due to the mixing between t and T+, the couplings of
$W^+_T b$ and $W^+ \bar{T} + b$ are expressed, respectively, as

$$V_{tb} \left( \frac{g}{\sqrt{2}} c_L \gamma_\mu P_L \right) \quad \text{and} \quad V_{tb} \left( \frac{g}{\sqrt{2}} s_L \gamma_\mu P_L \right), \quad (2)$$

where $V_{tb}$ is the value of the $(t, b)$ element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $c_L = \sqrt{1-s^2_L}$ with $s_L = \sin \theta_W + \cdots$, and $P_L = \frac{1-s_L}{2}$ and $P_R = \frac{1+s_L}{2}$ are the left-handed and right-handed projection operators, respectively. In the above expression, the product of $V_{tb}c_L$, which is denoted as $V_{tb}^{eff}$, should be identified with the CKM matrix element determined from the low energy processes up to the one-loop order. In the SM, the value of $V_{tb}$ element is constrained by the unitarity of CKM matrix, which requires its value to be very close to 1 (about 0.999\cite{10}). For simplicity, we will take $V_{tb} = 1$ in our numerical analysis. When the parameter $s_\alpha$ varies, the effective coupling strength of $W^+tb$, hence the single-top production rate at the Fermilab Tevatron and the CERN Large Hadron Collider (LHC), also varies. As $s_\alpha \to 0$, it is approaching to the SM $W^+tb$ coupling strength. Furthermore, since $|s_\alpha|$ is bounded by 1, $c_\alpha$ has to be larger than $\sqrt{1-(v/f)^2}$ in the LHT; with $f = 500$ GeV, $c_\alpha > 0.88$. Hence, $V_{tb}^{eff}$ is consistent with the Tevatron measurement on the decay branching ratio of $t \to bW^+$ in the $t\bar{t}$ events\cite{10}. It is also consistent with the most recent measurement of single-top event rate at the Tevatron: $|V_{tb}| = 1.0^{+0.1}_{-0.0}$\cite{11}.

Since the strength of the $W^+_T b$ coupling in the LHT is always smaller than that in the SM, the single-top production rate at the Tevatron and the LHC will also be smaller than that predicted by the SM. Hence, the measurement of the single-top production cross section provides a crucial test to the LHT. The deviations of the cross sections of the single-top production from the SM predictions ($\delta \equiv \Delta \sigma / \sigma_{SM}$) can be expressed in terms of $s_\alpha$ and $f$ as

$$\delta = \frac{\sigma_{SM} - \sigma_{LHT}}{\sigma_{SM}} = f^2 \frac{v^2}{f^2} + O \left( \left( \frac{v}{f} \right)^4 \right). \quad (3)$$

For illustration, we show in Fig.\textbf{1} the constraints on the parameter $s_\alpha$ and $f$ for $\delta = 2\%$ (yellow dashed-line), 5\% (red dashed-line) and 8\% (blue dashed-line), respectively. The shadowed region respects the electroweak precision test (EWPT), with $V_{tb}$ set to be 1, at the 95\% confidence level\cite{2}. In the same figure we also show the contours of $M_{T_+}$ (left panel) and $M_{T_-}$ (right panel). The pattern of the contour lines can be easily understood from Eq.\textbf{1}. We note that the allowed parameter space is strongly constrained when $\delta$ is small. For example, when $\delta \lesssim 2\%$, $f \geq 780$ GeV and $0.67 < s_\alpha < 0.78$; when $\delta \lesssim 5\%$, $f \geq 600$ GeV and $0.74 < s_\alpha < 0.85$; and when $\delta \lesssim 8\%$, $f \geq 550$ GeV. The above constraints can be translated into non-trivial limits of $M_{T_+}$ and $M_{T_-}$, which are summarized in Table\textbf{1}.

At hadron colliders, single-top events can be produced via three processes: $s$-channel ($u\bar{d} \to t\bar{b}$), $t$-channel ($u\bar{b} \to dt$) and $tW$ associated channel ($gb \to tW^-$); each process generates distinct event distributions and can be measured separately. In the LHT, the deviations of the single-top production rates of these three processes from the SM predictions have to be identical at the tree level, i.e. $\delta(s) = \delta(t) = \delta(tW)$. This is an important test of the LHT. In contrast, the above relation does not hold for LH models without $T$-parity.

As noted above, the value of $s_\alpha$ cannot be zero in order for the LH mechanism to take place. Therefore, a heavy $T_+$ can be produced singly in hadron collisions via weak charged current ($W^+T_+b$) interaction, similar to the SM single-$t$ production, and is referred as single-$T_+$ production in this work. In Fig.\textbf{2} we present the inclusive cross sections of single $T_+$ production at the LHC in the plane of $s_\alpha$ and $f$. (Its production rate is too small to be observed at the Tevatron for $f$ greater than 500 GeV.) In the same figure, we also show the constraints from the single-$t$ production rate measurement on the parameters $s_\alpha$ and $f$ for $\delta = 2\%$ (yellow dashed-line), 5\% (red dashed-line) and 8\% (blue dashed-line), respectively. Again, the gray region is excluded by EWPT. We note that the large single-$T_+$ cross sections ($\gtrsim 50 \text{ fb})$ occur in the regime of $f < 750$ GeV and $s_\alpha \sim 0.75$, where the single-$t$ production rates are reduced as compared to the SM rates by about $3\% \sim 8\%$. Should no deviation be found in the single-$t$ production, e.g. $\delta \lesssim 2\%$ (below the yellow dashed curve), it will be very difficult to directly observe the single-$T_+$ signal at the LHC due to its small cross section ($\lesssim 13 \text{ fb}$). Therefore, the correlation of the single top process to the single-$T_+$ production can be used to test the LHT.

| $\delta (%)$ | $[5,8]$ | $[2,5]$ | $< 2$ |
|-------------|--------|--------|--------|
| $M_{T_+}$   | $[800,1000]$ | $[870,1600]$ | $> 1100$ |
| $M_{T_-}$   | $[500,620]$  | $[580,950]$  | $> 830$  |

\textbf{TABLE 1: Mass limits (in unit of GeV) of $T_+$ and $T_-$ quarks for various $\delta$ values. Here the brackets denote the range of $\delta$, $M_{T_+}$ and $M_{T_-}$, respectively.}
While \( T_+ \) is mainly produced via single-\( T_+ \) process, the T-odd heavy quark \( T_- \) is predominantly produced in pairs via strong QCD interaction because of the requirement of T-parity symmetry. Since the coupling of gluon to \( T_- T_- \) pair is fixed by the QCD gauge interaction, the \( T_- T_- \) pair production rate is determined by the mass of \( T_- \), and its dependence on the parameters \( s_\alpha \) and \( f \) is shown in Fig. 2. Again, we see that a precise measurement of the single-\( t \) production cross section can provide a stringent test on the LHT. A reduction in the single-top production rate by more than 5% would imply the \( T_- T_- \) pair production rate at the LHC to be larger than about 200 fb and \( f \) to be less than about 700 GeV. We also note that the \( T_- T_- \) cross section is more sensitive to \( f \) than \( s_\alpha \), as compared to the single-\( T_+ \) cross section.

The \( T_- \) quark preferentially decays into \( t \) plus \( A_H \), so the collider signature of the \( T_- T_- \) pair event is

\[
q\bar{q}(gg) \rightarrow T_- \bar{T}_- \rightarrow A_H A_H t \bar{t},
\]

where the two \( A_H \)'s produce the missing transverse energy signature. For the \( t \bar{t} \) plus missing transverse energy signature, the intrinsic SM background is generated from the process \( q\bar{q}(gg) \rightarrow t\bar{t}Z \), where \( Z \) decays into a pair of neutrinos, whose cross section is about 60 fb at the LHC. To observe the \( T_- T_- \) signal, one has to suppress this large background. Usually, this is done by making kinematic selections to enhance the signal-to-background ratio, such as the study done in Ref. 2 which concluded that a large background rate remained even after imposing a set of kinematic cuts. Here, we propose a new method to largely suppress the SM background rate by measuring the degree of polarization of the top quark (or top anti-quark) in the final state.

An interesting feature of the \( T_- \) decay is that the top quark from \( T_- \) decay is predominately right-handedly polarized because the left-handed component of the \( A_H \) coupling is suppressed by a factor of \( s_\alpha \), for

\[
g_{A_H T_- t} = \frac{2}{3} \frac{g'}{f} s_\alpha \gamma_\mu \left( s_\alpha \frac{v}{f} P_L + P_R \right),
\]

where the \( W \) boson decays leptonically. For the single-\( T_+ \) production, the SM backgrounds mainly come from

Parity is clearly broken in this coupling. In order to quantify the parity violation effects, we define an asymmetry quantity \( A_{LR} \) as

\[
A_{LR} \equiv \frac{\sigma(t_L) - \sigma(t_R)}{\sigma(t_L) + \sigma(t_R)},
\]

where \( t_L \) and \( t_R \) denote the left-handedly and right-handedly polarized top quark, respectively. The \( Z \) boson preferentially couples to a left-handedly polarized top quark, respectively. The \( Z \) boson decays leptonically. For the single-\( T_- \) production, the SM intrinsic background rate can be largely suppressed by demanding a negative value of \( A_{LR} \). In Fig. 3 we present the contour of \( A_{LR}^{LHT} \) in the plane of \( s_\alpha \) and \( f \). We note that \( A_{LR} \) mainly depends on \( f \) and is not sensitive to \( s_\alpha \). This result leads to an important observation that the new particle mass scale parameter \( f \) can be determined by measuring the asymmetry \( A_{LR} \) in the production rates of left-handed and right-handed top quarks in the events with the \( t \bar{t} \) plus missing transverse energy signature. For example, if \( A_{LR} \) takes the value around \(-0.69 \), then \( f \) is about 550 GeV. For a larger value of \( f \), the asymmetry \( A_{LR} \) approaches to \(-1 \). After measuring \( f \) via \( A_{LR} \), one can uniquely determine \( s_\alpha \) from the measurement of \( M_{T_-} \) or \( M_{T_+} \). Should all three observables \( (A_{LR}, M_{T_-}, M_{T_+}) \) be measured, together with the single-top precision measurements, one can test the consistency of the LHT with data.

In the rest of the paper, we discuss how to directly measure \( M_{T_+} \) and \( M_{T_-} \) to test the LHT. \( T_+ \) quark has four tree level decay channels which produce separately the \( W^+ b \), \( Ht \), \( Zt \) and \( A_H T_- \) final states. Their decay branching ratios generally depend on the model parameters \( f \) and \( s_\alpha \). In this study we focus on the \( T_+ \rightarrow W^+ b \) decay mode. The single-\( T_+ \) event could be detected via

\[
q b \rightarrow q' T_+ \rightarrow q' b W^+ ,
\]
the single-top processes and the top quark pair production. The discovery potential of the LHC for the single-
T_+ production has been studied in the literature [12].
If the single-t rate is found to be much smaller than the SM prediction, then the LHT would predict a siz-
able single-T_+ signal which is characterized by a much larger transverse mass (or scalar sum of transverse energy) as compare to that predicted for the SM single-
t signal. The mass reconstruction of the T_+ quark is straightforward. It is similar to the mass reconstruction of the single-top event. One can first determine the lon-
gitudinal momentum of the neutrino from the W-boson mass constraint and then reconstruct the T_+ invariant mass
\[ M_{T_+} = \sqrt{(p_b + p_\nu + p_{f\nu})^2} \] [13].

One of the experimental signatures of the T_−T̅_− pair production at the LHC is
\[ q\bar{q}(gg) \rightarrow T_−\bar{T}_− \rightarrow 2A_H + t(\rightarrow bW^+) + \bar{t}(\rightarrow \bar{b}W^-), \]
where W^+ decays leptonically and W^- decays hadronically.
We have performed a Monte Carlo study and found a strong correlation between the location of the (broad) peak of the missing transverse energy (caused by the two LTP A_H bosons) distribution in the T_−T̅_− event and the mass of T_−. As shown in Fig. 4 the distribution \( E_T \) peaks about half of \( M_{T_−} \) for a wide range of \( f \) values. This special feature originates from the spin correlation in the T_−T̅_− production.

In summary, we have shown that the measurement of the single-top production is important for testing the LHT. Depending on the amount of its deviation from the SM prediction, the masses of the heavy T-even (T_+) and T-odd (T_−) partners of the top quark would be highly constrained. Furthermore, the single-T_+ and the T_−T̅_− pair production rates strongly depend on the result of the single-t production cross section measurement. We also proposed a new method to suppress the SM background for detecting the T_−T̅_− events by noting that the signal process tends to produce right-handed top quark from T_− decay while its SM background process (ttZ) tends to produce left-handed top quark. The asymmetry in the production rates of right-handed versus left-handed top quarks in the events with \( \ell \bar{\nu} \) plus \( \not{E}_T \) signature can be utilized not only to largely suppress the SM intrinsic background, but also to provide a measurement of the \( f \) parameter itself. We also point out that because of the spin correlations in the \( T_−T̅_− \) production and decay processes, the \( \not{E}_T \) distribution peaks around half of the mass of \( T_− \), which in turn provides a new method for measuring \( M_{T_−} \). From the measured values of \( f \) and \( M_{T_−} \), one can determine the remaining parameter \( s_α \).

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