Multi-Throat Brane Inflation

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Abstract

We present a scenario where brane inflation arises more generically. We start with D3 and anti-D3-branes at the infrared ends of two different throats. This setup is a natural consequence of the assumption that in the beginning we have a multi-throat string compactification with many wandering anti-D3-branes. A long period of inflation is triggered when D3-branes slowly exit the highly warped infrared region, under a potential generically arising from the moduli stabilization. In this scenario, the usual slow-roll conditions are not required, and a large warping is allowed to incorporate the Randall-Sundrum model.
**I. INTRODUCTION**

Brane inflation [1,2] is an interesting idea to construct the inflationary models in string theory. Recent studies [3,4] have shown that such models can be realized in warped space. The setups typically have anti-D3-branes sitting at the infrared (IR) cut-off of the warped space (throat), while the position of the mobile D3-branes, which enter from the ultraviolet (UV) entrance, plays the role of the scalar field. Inflation can be achieved when the slow-roll conditions are satisfied, and ends by brane-anti-brane collision and annihilation. A common feature for these models is that various amount of fine-tuning of different parameters is required. The reason traces back to the usual slow-roll conditions which demand a very flat potential. Given the rich stringy phenomena that have been discovered recently, it would certainly be interesting to look for more generic situations where the inflation can happen. We will discuss such a model in this paper.

We first notice that the slow rolling of the transverse brane motion can occur in a warped space as well without a flat potential, essentially because the coordinate speed of light along the transverse direction decreases due to the warping. Silverstein, Tong [5] and Alishahiha [6] have recently made use of this fact and studied a scalar coupled inflationary cosmology with a non-trivial kinetic term, where the rolling scalar corresponds to the position of a D3-brane moving toward the IR end of a throat. However, because of the back-reaction of the relativistic D3-brane [5,7], the D3-brane quickly enters a non-comoving phase [7] and most of the IR portion of the warped space becomes irrelevant for our purpose.

Different from the previous setups, we start our mobile D3-branes from the IR end of another throat. We will explain how this situation can arise rather naturally from a multi-throat string compactification with many wandering anti-D3-branes. “Slow-roll” happens when the D3-branes exit the exponentially warped IR end under a generic quadratic potential with a right sign. The detailed form of the potential is not very important. We will also find that this scenario allows an exponentially large warping, thus the Randall-Sundrum model [8] can be incorporated. We will discuss some subtleties involved in the reheating process.

**II. THROAT CONSTRUCTION AND BRANE-FLUX ANNihilation**

In this section, we briefly introduce some background material on warped space in string compactification which will be useful to set up our scenario.

The simplest way to create a warped space in string compactification is to place a stack of $N$ D3-branes transverse to the compactified dimensions [9]. A probe brane moving in the near-horizon region of these D3-branes will feel an AdS background. If all the D3-branes
are coincident, the warp factor goes all the way to zero in the IR direction. There is no mechanism to stabilize a large but finite hierarchy in this setup.

As shown in [10,11], this can be achieved by replacing the D3-branes with three-form fluxes near a conifold singularity on a Calabi-Yau manifold. If there are \( M \) units of RR fluxes and \( K \) units of NSNS fluxes on two dual three-cycles respectively, we will have a warped space with a smooth end and minimum warp factor

\[
h_{\text{min}} \sim \exp(-2\pi K/3Mg_s),
\]

where \( g_s \) is the string coupling. Therefore a large hierarchy can be obtained from a moderate number of fluxes. Away from the IR tip region, the resulting geometry is approximately AdS and the characteristic length scale \( R \) is related to the flux number by \( R^4 = \frac{27}{4} \pi g_s N \alpha'^2 \) with \( N = MK \).

Introducing anti-D3-branes at the IR end of the throat breaks the supersymmetry [12]. This has been used by KKLT [13] to lift the AdS vacuum and construct the dS space. If the warped space is generated by D3-branes, we expect some of them to be annihilated by the anti-D3-branes. But in the case of the three-form fluxes, there is no obvious way to annihilate the anti-D3-branes. A very interesting process of brane-flux annihilation is described in [12,14]. What happens is that, starting with \( p \) \( (p < M) \) number of anti-D3-branes in the IR tip, they tend to cluster and then puff up into a fuzzy NS5-brane in the presence of the fluxes. This NS5-brane will unwind an internal three-cycle, annihilating one unit of NSNS flux and creating \( M - p \) D3-branes to conserve the total D3-charge. The unwinding process can happen either through quantum tunneling if \( p \ll M \), or through classical rolling otherwise.

### III. BRANE DYNAMICS IN A THROAT

A detailed description of the brane dynamics in warped space will be very important to our scenario. In this paper, we are interested in the dynamics of a D3-brane exiting from the IR end of a throat in a background four-dimensional dS space. We approximate the warped space as a simple AdS space

\[
ds^2 = h^2(r) \left( -dt^2 + a^2(t)dx^2 \right) + h^{-2}(r)dr^2, \quad h(r) = r/R
\]

with a four-form potential \( C_{0123} = -h^4a^3 \). The \( a(t) \) is the scale factor of the dS space. The angular motion in the extra dimensions is neglected. We will start with a static D3-brane in the IR region. For D3-brane, the gravitational and four-form potentials cancel each other.
But in string compactification, a potential will arise from the volume stabilization [3]. We take such a potential to be

$$V(r) = -\frac{1}{2}H^2r^2$$  

in unit of brane tension $T_3$, where $H$ is the Hubble constant of the four-dimensional inflationary space. The absolute magnitude of this potential is of the same order as that arises from the conformal coupling [3]. In the brane inflation considered in [3,4], there are different sources of such mass terms. The slow-roll conditions require cancelations among them to a certain precision. Here we will just take the generic magnitude, but choose the sign to make sure that the D3-brane will exit the throat.

The homogeneous probe D3-brane obeys the following DBI action

$$S = T_3 \int d^4x \left[ -a^3(t)h^2 \sqrt{h^4 - \dot{r}^2} + a^3(t) \left( h^4 - V(r) \right) \right].$$  

The second term in (3.3) includes both the Chern-Simons potential from the RR five-form field and the potential (3.2). Under the acceleration of the potential (3.2), for $t \ll -H^{-1}$, the equation of motion gives the behavior of the D3-brane as

$$r = -\frac{R^2}{t} + \frac{9R^2}{2H^2t^3} + \cdots,$$  

where the leading order is that of light and the second order is determined by the potential. Note here that we have chosen the time $t$ to run from $-\infty$. In terms of $r$, the valid region for (3.4) becomes

$$r \ll R^2H.$$

The probe back-reaction can be characterized by the Lorentz contraction factor $\gamma = h^2/\sqrt{h^4 - \dot{r}^2}$ of the relativistic D3-brane [5,7]. It has to be much smaller than the strength of the background. This requires $\gamma \ll N$, because the background can be thought of as being created by $N$ D3-branes. Using the behavior (3.4), this requires

$$r \gg \frac{1}{3N}R^2H.$$  

We note that, while the lower bound (3.6) for $r$ is the consistency requirement to use the probe dynamics, the upper bound (3.5) is not a constraint – it gives a region where we can simply treat the brane as being highly relativistic, outside of which the D3-brane can only move slower than the speed of light.
IV. OUR SCENARIO

Now we are ready to describe our scenario. We assume that in the beginning we have a multi-throat configuration in the extra dimensions, resulting from a string compactification with three-form fluxes. This can be regarded as a generalization of the KS-GKP [10,11] model, but an explicit construction is beyond the scope of this paper. We start with many wandering anti-D3-branes in the bulk. These anti-D3-branes will be attracted to and settle down in the IR ends of different throats. Depending on the number of fluxes and anti-D3-branes associated with each throat, the anti-D3-branes will have different lifetime. For example, some of them may decay earlier as a result of brane-flux annihilation; the corresponding throat may become more shallow or even disappear; the resulting D3-branes may stay or come out of the throat and enter the other throats. During these processes, the usual slow-roll inflation may also happen under some special conditions.

Here we are most interested in the last step. Suppose there exists a throat with relatively large warping. After the anti-D3-branes annihilate with some number of NSNS fluxes either quantum mechanically or classically, the resulting D3-branes take the longest time to come out. This can happen either because the decay process is the slowest, or the D3-branes move most slowly due to the large warping as described in the last section. We will then concentrate on the dynamics of these D3-branes alone since all other processes have already happened. We will call this throat the brane-throat (B-throat). As mentioned, the gravitational and four-form potentials cancel each other for D3-branes. So for them to get out of the throat, a potential with a right sign such as (3.2) is needed.

The inflationary energy is provided by the anti-D3-branes in some other throats, where they have relatively longer lifetime – in fact, among them only that with the biggest warp factor provides the dominant inflationary energy, because the energy density of the others are red-shifted. The long period of inflation is triggered by the slow motion of the D3-branes in the deep B-throat.

After the D3-branes finally come out of the B-throat, they will then be attracted to different throats under similar potentials as (3.2) but with an opposite sign. They will annihilate some of the anti-D3-branes and end the inflation. Among all these throats, there are two of them which are especially interesting. One provides the dominant inflationary energy as mentioned above, which we will call the anti-brane-throat (A-throat); and another is where the standard model eventually lives, which we will call the standard-model-throat (S-throat). In the simplest case, the A-throat and S-throat can be the same throat.

All the processes are happening under the assumption that the shape and size moduli of the compactified dimensions are fixed, and that in the end of the inflation our universe
is in a meta-stable dS space with a small cosmological constant. This is in the sense of the KKLT [13] and KKLMMT [3] models. We will use the subscripts $B$, $A$ and $S$ to label the quantities related to $B$-throat, $A$-throat and $S$-throat, respectively.

**V. INFLATIONARY E-FOLDS AND DENSITY PERTURBATION**

We now calculate the resulting inflationary e-folds. The anti-D3-branes in the $A$-throat provide the inflationary energy density $V_A = 2T_A h_A^4$, where $T_A = n_A T_3$ is the total brane tension of $n_A$ anti-D3-branes and $h_A$ is the warp factor. A factor of two comes in due to the summation of the gravitational and four-form potentials. (A possible shift from the potential (3.2) may be included in the $n_A$ which is then no longer an integer.) Here we consider one D3-brane in the $B$-throat. We denote the warp factor at the position of the D3-brane as $h_B = r_B/R_B$. Starting from $r_B$, all of the inflationary e-folds $N_e$ are obtained within a range of order $N_e r_B$. One can check that the change of the potential (3.2) within this range is negligible. So the inflationary energy is nearly a constant during the inflation. We can then approximate $N_e$ as

$$N_e = \int H dt \approx HR_B h_B^{-1} \approx \sqrt{\frac{2}{3} \frac{\sqrt{T_A R_B}}{M_{pl}} h_A^2 h_B^{-1}}. \quad (5.1)$$

Using (5.1) we can rewrite the conditions (3.5) and (3.6) as (with $N$ specified as $N_B$)

$$\frac{N_e}{3N_B} r_B \ll r \ll N_e r_B. \quad (5.2)$$

The upper bound means that, in most of the inflationary period, this D3-brane is relativistic. This justifies our approximation in (5.1) where we estimate $\Delta t \approx R_B h_B^{-1}$ using (3.4). This can be readily understood as follows. Since our potential (3.2) does not satisfy the usual slow-roll conditions, the slow-roll behavior has to come from the causality constraint. This is the reason that the D3-brane travels nearly at the speed of light. The lower bound gives the maximum number of the e-foldings, up to $N_B$, that can be achieved in this model without triggering the back-reaction. So $N_e$ denotes the latest e-folds.\(^1\)

In terms of the fundamental parameters in the expression (5.1), a long period of inflation can be obtained easily. Using $T_3 = (2\pi)^{-3} g_s^{-1} \alpha'^{-2}$ and $R_B^4 = \frac{27\pi^2}{4} g_s N_B \alpha'^2$, Eq. (5.1) becomes

$$N_e \approx 0.1 \sqrt{n_A} g_s^{-1/4} N_B^{1/4} \frac{\alpha'^{-1/2}}{M_{pl}} h_A^2 h_B^{-1}. \quad (5.3)$$

\(^1\)Since a signal takes a time of $N_e H^{-1}$ to reach $r_B$ from the bulk, it is also necessary to start the inflation earlier in order to assume that the Hubble constant is independent of the extra dimensions.
For example, if \(0.1\sqrt{n_A} \ g_s^{-1/4} N_B^{1/4} \alpha'^{-1/2}/M_{Pl} \sim 1\), then we need \(h_B \sim h_A^2/N_e\). In terms of the flux number in (2.1), this is easy to satisfy.\(^2\)

In a dS space, the amplitude of the scalar quantum fluctuations being stretched out of the horizon is \(H/2\pi\). Translated to the transverse brane coordinate \(r_B\), this gives the perturbation \(\delta r_B = H/(2\pi \sqrt{T_3}) \approx r_B N_e/\tilde{N}\), where \(\tilde{N} \equiv (27N_B/8)^{1/2}\). (Note that the Lorentz contraction \(\gamma_B^{-1}\) in the transverse direction is cancelled by a dilation factor \(\gamma_B\) which gives a Hubble constant \(H\gamma_B\) for a moving observer on the D3-brane.) These ripples on the D3-brane cause a spatially dependent delay \(\delta t\) for the ending of the inflation to an unperturbed observer. Using (3.4) and (5.1), this gives the density perturbation \([15]\)

\[
\delta = 2 H r_B \delta t \approx 2 H r_B R^2 \left( \frac{1}{r_B} - \frac{1}{r_B + \delta r_B} \right) \approx \frac{2H r_B}{5H} \frac{N_e^2}{\tilde{N} + N_e},
\]

where \(H_r\) is the Hubble constant during the reheating and is proportional to the squared warp factor of the S-throat \(h_S^2\).

In this section we assume that matter created during the reheating comes from the energy density released from the brane-anti-brane annihilation, and the conversion is efficient. In the simplest case where the A-throat and S-throat are identical, this implies \(H_r \approx H\). For a normal value of \(\tilde{N}\), this gives a too big density perturbation. Therefore we consider the case where they are different. In our scenario the reheating on the standard-model anti-D3-branes is not a problem even if the S-throat is different from the A-throat, since the inflation is not caused by D3-branes moving near the UV entrance of the A-throat as in KKLMMT. In our case, there can be many D3-branes coming out of the B-throat. They will then scatter into different throats, including the S-throat, colliding and annihilating anti-D3-branes there as a reheating mechanism. (But to ensure that the subsequent cosmological evolution is dominated by the S-throat, the anti-D3-branes in the A-throat have to be all annihilated, and the possible extra D3-branes will have to come out and enter other throats, including the S-throat, which have \(h < \sim h_S\).)

For example, the experiments indicate that the density perturbation in (5.4) is of order \(10^{-5}\). If we take \(\tilde{N} \sim 100\) and \(N_e \sim 60\), this gives \(H_r/H \sim 10^{-6}\). That is, ignoring the factors \(n_S\) and \(n_A\), the S-throat has a warp factor roughly seven e-folds smaller than the A-throat. Since only the relative ratio \(h_S^2/h_A^2\) appears in (5.4), extremely small numbers are allowed for \(h_S\) and \(h_A\). Therefore our model can incorporate the Randall-Sundrum model \([8]\) to solve the hierarchy problem.

\(^2\)From here we also see that the energy density of the moving D3-branes, which is \(T_B h_B^4 (\gamma_B - 1) \approx \frac{1}{3} N_e T_B h_B^4\), is negligible comparing to the inflationary energy density \(V_A\), as assumed in (3.3).
As we can see, certain details discussed above, including whether the S-throat can be identified with the A-throat, depends on the magnitude of the density perturbation in (5.4). As we will discuss in the next section, there are some subtleties regarding the reheating process in the deep throat. So further investigation is needed.\(^3\)

We now turn to some quantities which are less dependent of the overall magnitude of the density perturbation – the spectral index \(n_s\) and the running of this index. Since the number of the latest e-folds \(N_e\) is related to the wave-number \(k\) by \(d \ln k = -dN_e\), we get

\[
\begin{align*}
    n_s - 1 &\equiv 2 \frac{d \ln \delta}{d \ln k} \approx -\frac{4}{N_e} + \frac{2}{N + N_e}, \\
    \frac{dn_s}{d \ln k} &\approx -\frac{4}{N_e^2} + \frac{2}{(N + N_e)^2}.
\end{align*}
\]

These give a red-tilted scale invariant spectrum with negative running. Modifications of (5.4), (5.5) and (5.6) may come from the modification of the approximate geometry in (3.1), especially in the tip region of the throat [17]. But perhaps the most interesting possibility comes from the following consideration.

Consider the case \(\gamma_B H \delta t \gtrsim 1\), which happens for \(N_e \gtrsim \sqrt{3} \tilde{N}^{1/4}\). In this case, for a moving observer on D3-branes the transverse brane fluctuations \((\gamma_B \delta r_B \approx \gamma_B \delta t h_B^2)\), generated quantum mechanically from the vacuum in a Hubble time \((\gamma_B^{-1} H^{-1})\), travels faster than the speed of light \((h_B^2)\) in the transverse direction. This is impossible. The reason is that the field theory description on the D3-branes has broken down. It is unusual because the Hubble energy scale is now approximately greater than the red-shifted string scale, i.e. \(\gamma_B H \gtrsim \sqrt{2\pi T_B^{1/4} h_B}\), due to the large warping in the B-throat. The amplitudes of the stringy quantum fluctuations should be bound by the causality, but details are not well studied so far. This is an interesting case where the stringy effects are directly responsible for seeding the structure formation.

VI. DISCUSSION

Reheating for brane inflation in warped space is usually caused by brane-anti-brane annihilation in inflationary energy scale, as we followed in the last section. (In some sense, Eq. (5.4) is already different since \(H_r\) and \(H\) can be very different.) In this section, we discuss some subtle effects [7] which are different from the conventional field theory expectation due

\(^3\)A more detailed study [16] shows that such subtleties will modify the dependence of Eq. (5.4) on warp factors.
to fast-rolling D3-branes in the S-throat. They affect the overall magnitude of the density perturbation, and the relation between $N_e$ and $k$.

There are two different processes – collision and annihilation – that create open strings on the residue anti-D3-branes. For annihilation [18] this is only possible from loop diagrams connecting the decaying branes and anti-D3-branes [19]. The resulted energy density cannot exceed $2n_SH_S^2$, where $n_S$ is the number of anti-D3-branes annihilated in the S-throat and $h_S$ is the corresponding warp factor. In addition, they have to compete with the disk [20] and loop diagrams which create closed strings to the bulk.

We next look at the collision. As studied in [5,7], when the D3-branes enter from the UV entrance and move toward the IR end, their kinetic energy density may approach $N_S$ times the brane tension and enter a non-comoving phase [7]. Then their final energy density cannot grow too much greater. This is relatively independent of the total warping or the initial velocity, and is determined by the strength of background geometry characterized by $N_S$. The collision will transfer an energy density of order $N_ST_3h_S^4$, which is dominant over that from the annihilation since $N_S$ is a big number. This may be a good news – the collision can be a much more efficient process to create open strings especially if the corresponding oscillating scalar has bigger coupling to matter than gravity.

Furthermore, at the beginning of the reheating, the effective warp factor at the IR end is much greater than its original value, due to the back-reaction. After things cool down, it restores to $h_S$. This has the rescaling effect [7] (to a Poincare observer) which stretches the wavelength of the density perturbation by a ratio that can be as big as several e-folds below the total warping. On the branes, if we stay in a frame in which the time and length do not rescale, the effective Planck mass increases from an exponentially low value to the present value, as the IR warp factor restores. It is certainly interesting to see if any dramatic effects can be caused to the density perturbation.

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