Proof of the method of paired zeroing of numbers in a residue system

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Abstract. The paper studies the problem of determining the lower boundary of number correction speed in computing systems operating in the basis of non-positional arithmetic in residual classes. The topicality of the problem is necessitated by the search of methods allowing reducing the time for digital processing of signals in non-positional neuroprocessors. The work considers the variants for error correction with single and multiple control bases of the residue system. The proof is provided that allows justifying the adequacy of the modified method that implements paired zeroing of numbers in the system of residual classes. It was shown that the suggested solutions allow considerably reducing the time expenses for digital processing of signals in neuroprocessors specialized in summation and multiplication operations. The proof is elaborated by the method of mathematical induction.

1. Introduction

Large amount of calculations necessary for processing telemetry of oil and gas wells, modeling of bottom-hole zone treatment under complex geological conditions requires new solutions for reliable and fast implementation of basic arithmetic operations of summation and multiplication. The development of a non-positional neuroprocessor considerably accelerates the calculations. However, the provision of reliable calculations is still a challenge. For instance, the modified method of paired zeroing of numbers in a system of residual classes requires proof.

In [1], the authors consider the introduction of the term zeroing of numbers. Outstanding works allow developing the theory of error correction in a non-positional system of residual classes and suggesting a method of paired zeroing of numbers in a residue system. The peculiarity of the method is its focus on cutting time consumption for correction of single and multiple errors. The problem proving the convergence of the method was solved.

2. Materials and Methods

Zeroing is one of the methods for determining the correctness of a number. It involves conversion of initial number \( A = (\alpha_1, \alpha_2, ... \alpha_n, \alpha_{n+1}) \) into number \( A^{(n)} = (0,0, ..., 0,\gamma_{n+1}) \) using the transformation
sequence that does not allow a single escape from working range \( M = (m_1 \times m_2 \ldots \times m_n) \), where \( m_i \) are the bases of the moduli of residual class system [1–3].

The process of number zeroing envisages consecutive subtraction of constants from this number.

\[
(m_{11}, m_{22}, \ldots, m_{n+1n}), m_{11} = 1, 2, \ldots, m_1 - 1;
\]

\[
(0, m_{22}, \ldots, m_{n+12}), m_{22} = 1, 2, \ldots, m_2 - 1;
\]

\[
(0, 0, \ldots, m_{nn}, m_{n+1n}), m_{nn} = 1, 2, \ldots, m_n - 1.
\]

In this case, number \( A = (\alpha_1, \alpha_2, \ldots, \alpha_{n+1}) \) is consecutively transformed into \( \hat{A} = (0, \alpha'_2, \ldots, \alpha'_{n+1}) \), then into \( A'' = (0, 0, \alpha''_2, \ldots, \alpha''_{n+1}) \), and so on.

After \( n \) iterations, we get \( A^{(H)} = (0, 0, \ldots, 0, \gamma_{n+1}) \).

If \( \gamma_{n+1} = 0 \), then the initial number is correct (lies within interval \([0, M)\)); if \( \gamma_{n+1} \neq 0 \), then the number is incorrect and lies within interval \([jM, (j+1)M)\) for \( j = 1, 2, \ldots, m_{n+1} - 1 \). The value of interval number \((j+1)\), where gets operand \( \hat{A} \) is determined from

\[
\hat{A} = (\beta_1, \beta_2, \ldots, \beta_{n+1}) + (0, 0, \ldots, \Delta \alpha_i, \ldots, 0).
\]

Then, evidently, \( \Delta A \) lies not in the first interval \([0, M)\), because number \( A_0 = (0, 0, \ldots, 0) \) lies in the first interval.

Let \( \Delta A \) lie in the \( k \)-th interval:

\[
(k - 1)M \leq \Delta A < kM.
\]

In the system of inequalities, let us

\[
\begin{align*}
0 & \leq A < M \\
(k - 1)M & \leq \Delta A < kM
\end{align*}
\]

sum up

\[
(k - 1)M \leq A + \Delta A < (k + 1)M.
\]

Let us assume \( j = k - 1 \), then we can write \( jM \leq \hat{A} < (j + 2)M \), i.e. an error can turn correct operand into incorrect one lying only in one of the two intervals, \([jM, (j + 1)M)\) or \([(j + 1)M, (j + 2)M)\).

The obtained result is applied for determining an alternative combination of numbers in the system of residual classes by the method of zeroing.

The time of zeroing is determined as \( T = 2n\tau_\Sigma \), where \( \tau_\Sigma \) is time of zeroed number summation with zeroing constant. The number of zeroing constants equals

\[
k = \sum_{i=1}^{n} m_i - n,
\]

while the number of digits of the storage of the constants in the non-positional neuroprocessor equals

\[
c = \left( \sum_{i=1}^{n} m_i - n \right) (n - i).
\]

Obviously, the most important of the strong points of the system of residual classes—high operation speed of modular operations—is lost during zeroing. The time of operation implementation is comparatively high, which lowers the efficiency of residue system application.

However, there is the method of paired zeroing of numbers in the system of residual classes [4–5]. According to the method, at each step, the zeroing is performed simultaneously in two bases. Zeroing time decreases twofold and equals \( T = n\tau_{\Sigma L} \).

The total number of zeroing constants equals

\[
k = \sum_{i=1}^{n/2} m_i m_{n-i+1} - n/2,
\]

while the number of digits of zeroing constants equals

\[
c = \left( \sum_{i=1}^{n/2} m_i m_{n-i+1} - n/2 \right) (n - 2i).
3. The study
In the method of paired zeroing of numbers with preliminary sampling of digits, the operations of
summing and sampling of the next zeroing constant are combined in certain time cycles. In addition, the
sampling of the next constant is performed with preparation of values of numbers that will be used at
the next zeroing stage to select a new zeroing constant for the number.

\[(0, ..., 0, \alpha_1, \alpha_{i+1}, ..., \alpha_{n-i}, \alpha_{n-i+1}, 0, ..., \beta_{n+1})\]

Values \(\alpha_i\) and \(\alpha_{n-i+1}\) in an elementary member working in bases \(m_{i+1}\) and \(m_{n-i}\) can be used to
prepare values \(\alpha'_{i+1}, \alpha''_{n-i}\) that will be exploited at the next zeroing stage to sample the constant. Indeed, values \(\Delta\alpha_{i+1}, \Delta\alpha_{n-i}\) that will be subtracted from \(\alpha_{i+1}\) and \(\alpha_{n-i}\), correspondingly, are determined only
by values \(\alpha_i\) and \(\alpha_{n-i+1}\). During constant sampling using values \(\alpha_i\) and \(\alpha_{n-i+1}\), from corresponding
tables values \(\alpha'_{i+1}\) and \(\alpha''_{n-i}\) can be sampled in a single cycle. In this case, it is unnecessary to have
digits in bases \(m_{i+1}\) and \(m_{n-i}\), which will allow decreasing the capacity of zeroing constants

\[C = \left(\sum_{i=1}^{\lfloor n/2 \rfloor} m_i m_{n-i+1} - n/2\right) (n - 2i - 2).\]

The number of summations in suggested zeroing variant equals \(\left\lceil \frac{n+1}{2} \right\rceil\), since the zeroing is
simultaneously performed in all informational bases of the system of residual classes in pairs. After
every two summations, one additional cycle is required for the formation of the next address and
reference to the storage of the zeroing constants. In this connection every two summation cycles
\((\tau_0 = T_0)\) correspond to one cycle without summation. If conventional paired zeroing at \(n = 5\)
(summation of two zeroing constants) requires four conditional time cycles \(4T_0\), then the considered
method requires three cycles \(3T_0\).

Generally, the analytical dependence of the zeroing time on the number of information bases of the
system of residual classes can be represented as

\[T = \left\lceil \frac{n+1}{2} \right\rceil \tau_\Sigma + \left\lceil \frac{n+1}{2} + 1 \right\rceil \tau_{mem}.\]

where \([x]\) is the integral part of \(x\), that does not exceed \(x\), \(\tau_{mem}\) is the time of reference to the storage
(table memory) of zeroing constants.

Considering that \(\tau_\Sigma = \tau_{mem}\), we get relation

\[T = \left(\frac{n+1}{2} + \left\lceil \frac{n+1}{2} + 1 \right\rceil \right) \tau_\Sigma. \quad (1)\]

At even \(n\):

\[T' = \left(\frac{n}{2} + \left\lceil \frac{n+1}{2} + 1 \right\rceil \right) \tau_\Sigma. \quad (2)\]

If \(\frac{n}{2}\) is even, then

\[T'_{\text{even}} = \frac{3}{4} n \tau_\Sigma. \quad (3)\]

If \(\frac{n}{2}\) is odd, then

\[T'_{\text{odd}} = \left(\frac{3n+2}{4}\right) \tau_\Sigma. \quad (4)\]

At odd \(n\),

\[T'' = \left(\frac{n+1}{2} + \left\lceil \frac{n+1}{2} + 1 \right\rceil \right) \tau_\Sigma. \quad (5)\]

If \(\frac{n+1}{2}\) is even, then

\[T''_{\text{even}} = \frac{3}{4} (n + 1) \tau_\Sigma. \quad (6)\]

If \(\frac{n+1}{2}\) is odd, then

\[T''_{\text{odd}} = \left(\frac{3n+5}{4}\right) \tau_\Sigma. \quad (7)\]

Let us prove relation (1) by the method of mathematical induction on \(n\).
Proof.
Step one. According to equation (1), for $n = 3, T = 3\tau_\Sigma$.

Step two. Let us assume that equation (1) is also valid for $n = k$, i.e.
$$T_k = \left(\left(\frac{k+1}{2}\right) + \left[\frac{k+1}{2}\right]\right)\tau_\Sigma.$$  

Step three. Let us prove that equation (1) is also valid for $n = k + 1$, i.e.
$$T_{k+1} = \left(\left(\frac{k+2}{2}\right) + \left[\frac{k+2}{2}\right]\right)\tau_\Sigma.$$  

At even $k$, $(k + 1$ is odd) 
$$T'_{k+1} = \left(\frac{k}{2} + 1 + \left[\frac{k+2}{2}\right]\right)\tau_\Sigma.$$  

if $\frac{k}{2}$ is even, then 
$$T_{even(k+1)}' = \left(\frac{2k+2}{4}\right)\tau_\Sigma.$$  

if $\frac{k}{2}$ is odd, then 
$$T_{odd(k+1)}' = \left(\frac{2k+4}{4}\right)\tau_\Sigma.$$  

At odd $k$, $(k + 1$ is even) 
$$T''_{k+1} = \left(\left(\frac{k+1}{2}\right) + \left[\frac{k+1}{2}\right]\right)\tau_\Sigma.$$  

if $\frac{k+1}{2}$ is even, then 
$$T_{even(k+1)}'' = \left(\frac{2k+2}{4}\right)\tau_\Sigma.$$  

if $\frac{k+1}{2}$ is odd, then 
$$T_{odd(k+1)}'' = \left(\frac{2k+4}{4}\right)\tau_\Sigma.$$  

Then:
$$\frac{3k+3}{4}\tau_\Sigma = \frac{3}{4}(k+1)\tau_\Sigma;$$  
$$\frac{3k+5}{4}\tau_\Sigma = \frac{3}{4}(k+1)\tau_\Sigma;$$  
$$\frac{3k+7}{4}\tau_\Sigma = \frac{3}{4}(k+1)\tau_\Sigma;$$  
$$\frac{3k+9}{4}\tau_\Sigma = \frac{3}{4}(k+1)\tau_\Sigma.$$  

Thus, equation (1) is also valid for $n = k + 1$, what was to be demonstrated.

The received relations (1–2) and (5) refine the lower limit of error correction speed for specified values $n$ and $\tau_{next}$ (see Fig. 1) In practical calculations, the error correction time should be determined by equations (3–4) and (6–7).
Figure 1. Error correction time for different zeroing methods: standard and paired (modified).

4. Conclusion
Above, we have proven the method of paired zeroing of numbers in the system of residual classes with preliminary sampling of digits. The method allows determining the lower limit of error correction speed in a residue system. The decrease of the time for convergence of an alternative combination to the error basis by 50% allows sharply increasing the efficiency of application of the system of residual classes through the possibility of correction of multiple errors.

The proven method can serve as the basis for synthesizing the error correction unit similar to that described in [6]. The unit should include operation and storage registers, deciphers, selectors, unit of zeroing of constants, memory unit for errors of constants, summation unit, valves, keys and commutators. The practical application of the studies is aimed at the solution of problems described in [7–10].

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