An explicit model of temperature-dependent thermal conductivity for nanofluids

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Abstract. Nanofluids is a material that has good prospects to increase the efficiency of the heat transfer process. With the addition of small fraction nanoparticles, the thermal conductivity of the nanofluids can increase significantly compared to the fluid base. Although it has excellent application prospects, the study of thermal conductivity models is still a problem. The uniqueness of the characteristics and rapid development of nanofluids experiments make the proposed model continue to undergo validation testing. In this paper, a model of temperature-dependent the effective thermal conductivity of nanofluids is proposed. This model is a development of a previously proposed model that only involves aspects of the fraction of the volume and diameter of nanoparticles. The model construction comes from the modification of the model proposed by Mintsa-Roy-Nguyen-Doucet, which is equipped with aspects of the nano-convection mechanism as proposed by Jang-Choi.

1. Introduction
As a tropical country, Indonesia has an abundance of solar energy [1]. Solar energy can be utilized through its light (electromagnetic radiation) and through its heat energy. Devices that can convert sunlight energy into electricity are known as photovoltaics [2]. While devices that utilize the solar’s heat in the work process are often referred to as solar collectors [3].

Solar collectors have wide applications such as water heating systems, alcohol distillation, and etc. The ability to absorb solar thermal energy is the main variable that can determine the performance of solar collectors. This is because the thermal energy is used or transferred through the fluid (called the working fluid) which is located in the pipes of the solar collector [4].

To improve the performance of solar collectors, researchers currently use working fluids in the form of nanofluids. Nanofluids is a fluid with dispersed nanoparticles or suspensions containing nanoparticles [5, 6]. Nanofluids can increase the thermal conductivity of the fluid. Therefore, in addition to solar collectors, nanofluids are also applied to devices that require heat transfer mechanisms such as nuclear reactors, industrial refrigeration, micro-electromechanical, and nano-electromechanical systems. Several studies have shown that the thermal conductivity enhancement of water-based nanofluids can reach 50% compared to the water fluid base [7, 8].

Based on these criteria, the number of studies related to nanofluids, both experimentally and modeling continues to increase. Although the application of nanofluids in various fields is promising,
the theoretical research of nanofluids is still evolving today. Nanofluids have unique properties and heat transfer mechanisms that can be viewed from multiple sides. Some models of thermal conductivity of nanofluids that are widely developed include the ones proposed by Maxwell and Jang and Choi [9].

Several factors that influence the heat transfer performance of nanofluids are the size, shape, and volume fraction of nanoparticles [10]. In a previous study, one of us has proposed a model of the thermal conductivity of nanofluids based on these parameters through a nano-convection mechanism as proposed by Jang and Choi [8]. However, the model has not explained the relation between the thermal conductivity of nanofluids and temperature explicitly. The parameters that determine the thermal conductivity of nanofluids depend on temperature. The experimental data obtained by Mintsa et al. have confirmed the thermal conductivity of nanofluids increases with increasing temperature [11]. However, on the other hand, Yang and Han showed that water-based nanofluids with dispersed nanoparticles in the shape of nanorods had the opposite characteristic [12]. Therefore, based on the complexity of the relationship between parameters in the conductivity of nanofluids and temperature, in this paper, we will construct a model of temperature-dependent thermal conductivity of nanofluids.

2. Model of Temperature Dependent Thermal Conductivity of Nanofluids

The model described in this paper is the development of a model we proposed previously [8]. The model was built based on the heat transfer mechanism proposed by Jang and Choi [9]. The heat transfer mechanism of nanofluids is assumed to occur based on four modes: (i) due to interaction of base fluid molecules, (ii) nanoparticles, (iii) collisions between nanoparticles, and (iv) nano-convection. However, between these modes, the third mode is a mode with very little energy contribution compared to the other and we can ignore it.

In the first heat transfer mechanism caused by interactions between base fluid molecules, net flux is given by,

$$J_1 = -k_{bf} (1-f) \frac{dT}{dz}$$  \hspace{1cm} (1)

where $k_{bf}$ is the thermal conductivity of the base fluids, $f$ is the volume fraction of nanoparticles in nanofluids, $T$ is the temperature, and $z$ is the plane related to net energy flux. In the second heat transfer mechanism caused by nanoparticles, the net flux is given by,

$$J_2 = -k_p f \frac{dT}{dz}$$  \hspace{1cm} (2)

where $k_p$ is the thermal conductivity of nanoparticles dispersed in the nanofluids, it appears that if there are no volume fraction of nanoparticles dispersed in nanofluids then the second heat transfer mechanism will disappear and the first heat transfer mechanism will reduce the heat transfer mechanism for the base fluid. The last mechanism, namely the mechanism involving nano-convection, the net energy flux is given by,

$$J_3 = h(T_p - T_f) f \approx -hf \delta_f \frac{dT}{dz}$$  \hspace{1cm} (3)

where $h$ and $\delta_f$ is the convection heat transfer coefficient and the thickness of the boundary layer, respectively. Based on these definitions, the thermal conductivity of nanofluids ($k_{nf}$) can be expressed as,

$$k_{nf} = k_{bf} (1-f) + \alpha k_p f + hf \delta_f$$  \hspace{1cm} (4)

where $\alpha$ is the empirical constant related to the conductivity of nanoparticles. On a nanoscale, boundary layer can be expressed based on the relationship between the Prandtl number and the equivalent diameter of the base fluid, $\delta_f \sim \frac{d_{eq}}{Pr}$. The convection heat transfer coefficient $h$ can be
determined based on the Nusselt number (Nu), \( h = \frac{k_w Nu}{d_p} \). Nusselt numbers in nanofluid have many expressions that vary based on the model proposed. However, Nusselt numbers are functions of Reynolds numbers, Prandtl numbers, and nanoparticle volume fractions. In the model that we proposed previously, the heat transfer coefficient of convection can be expressed as,

\[ h = \frac{k_{nf}}{d_p} \left( \gamma \text{Re}^m \text{Pr}^n f \right) \]  

(5)

where \( \gamma, m, \) and \( n \) are empirical constants related to experimental data. According to the relationships, \( k_r \) can be expressed as [8],

\[ k_r = \frac{k_{nf}}{k_{bf}} = (1 - f) + \frac{k_{nf}}{k_{bf}} f + \frac{\varepsilon d_{nf}}{d_p \text{Pr}} \left( \text{Re}^m \text{Pr}^n f \right) \]  

(6)

In the model proposed by Jang and Choi [9], the Reynolds number of a nanofluids can be defined as \( \text{Re} = \frac{\lambda C_m d_p}{v} \), where \( C_m, v \), and \( \lambda \) are random velocities of nanoparticles, kinematic viscosity, and correction factors. The definition of \( C_m \) in nanofluids can be obtained from the relationship between the Einstein diffusion coefficient (\( D \)) and the mean free path of base fluids (\( l_{bf} \)), \( C_m = \frac{D}{l_{bf}} \). By considering the nanoparticle (\( K \)) shape factor, \( C_m \) can be written as [12],

\[ C_m = \frac{k_{bf} T}{6 \pi \mu_{nf} a K} \]  

(7)

where \( \mu_{nf} \) is the dynamic viscosity of nanofluids, \( k_B \) is the Boltzmann constant and \( a \) is the equatorial semi-axis of the ellipsoid. In the case of spherical particles, \( a \) is related to its radius. The value of the form factor (\( K \)) can be determined based on the relationship [12],

\[ K = \frac{k_B T}{6 \pi \mu_{nf} a K} \left( \frac{4}{3} b^2 - 1 \right) \]  

(8)

where \( b \) is the aspect ratio between the major axis and the minor axis nanoparticles. For spherical particles, \( b = 1 \) and the value of \( K \) in eq. (8) reduce to unity thus eq. (7) reduce to the random velocities of nanoparticles as proposed by Jang and Choi. Expression of shape factor values in the eq. (8) will be useful when we calculate the thermal conductivity of non-spherical nanoparticles such as nanorods and it can answer the problems related to decreasing the thermal conductivity of nanofluids due to increased temperature.

If we assume the value of the Prandtl number is one and through the relationship between dynamic viscosity and kinematic viscosity, eq. (6) can be written as,

\[ k_r = (1 - f) + \frac{k_{nf}}{k_{bf}} f + \frac{\varepsilon d_{nf}}{d_p} \left( \eta C_m d_p \rho_{nf} \right)^n \]  

(9)

The relationship of parameters in the mechanism of heat transfer in nanofluids with temperature can be determined based on the model proposed by Mintsa-Roy-Nguyen-Doucet (MRND) [11],

\[ k_r = \frac{k_{nf}(T)}{k_{bf}(T)} \]  

(10)
In the MRND model, it is stated that the thermal conductivity of the nanofluids and base fluid is a function of temperature. In the case of water-based nanofluids, the value of thermal conductivity of water can be expressed as a linear relationship,

\[ k_w = k_{w1} + k_{w2}T \]  

(11)

where \(k_{w1}\) and \(k_{w2}\) are empirical constants. Based on experimental data on thermal conductivity of water in ref. [13] the value of thermal conductivity of water in the range 283 - 383 K is given by \(k_w = 0.2593 + 0.00116 \times T\). Comparison between the linear relationship with the experimental data is given by Fig. 1 (\(R^2 = 0.963\)).

Another parameter in the formula for thermal conductivity of nanofluids which is influenced by temperature is the density of nanofluids. In general, it can be defined as [14],

\[ \rho_n = (1 - f) \rho_f + f \rho_p \]  

(12)

where \(\rho_n\) is the base fluid density while \(\rho_p\) is the nanoparticle density. Assuming the density of the base fluid is easier to change compared to the density of nanoparticles due to changes in temperature, we can say that changes in the density of nanofluids due to temperature are dominated by changes in the density of the base fluids. For water-based nanofluids, the relationship between density and temperature is given by the polynomial relationship [15],

\[ \rho_n = \frac{999.839 + 16.945t - 7.9870 \times 10^{-3}t^2 - 46.170 \times 10^{-4}t^3 + 105.563 \times 10^{-3}t^4 - 280.542 \times 10^{-12}t^5}{1 + 16.87985 \times 10^{-3}t} \]  

(13)

where \(t\) is the temperature (°C). Dynamic viscosity is a parameter that changes due to changes in temperature. The viscosity of a nanofluids can be expressed as the relationship between the base fluid viscosity and the volume fraction of the dispersed nanoparticles [16],

\[ \mu_n = \mu_f \left( \frac{1}{(1 - f)^{12}} \right) \]  

(14)

For water-based nanofluids the base fluid viscosity can be expressed as,

\[ \mu_w = A \times 10^{B/(T-C)} \]  

(15)

where \(A = 2.414 \times 10^{-4}\), \(B = 247.8\), and \(C = 140\) (SI unit).

Through the explicit expressions of the parameters above, we confirm the model that has been proposed with experimental data of Al₂O₃ water-based nanofluids [11], as shown in Figure 2. It appears that the model we proposed has good accuracy. For Al₂O₃-nanofluids, \(f = 4\%\), \(R^2 = 0.999\) while for Al₂O₃-nanofluids, \(f = 5\%\), \(R^2 = 0.998\).
3. Conclusions
Through the development of thermal conductivity models involving aspects of nanoconvection as proposed in [9] and [8] we can obtain models of time-dependent thermal conductivity of nanofluids with high accuracy. Model development is carried out based on the assumption of the thermal conductivity of nanofluids and the thermal conductivity of the base fluid as a function of temperature as stated in [11]. The parameters that depend on temperature in this paper are base fluid conductivity, density, and dynamic viscosity.

![Graph](a)

![Graph](b)

**Figure 2.** The relationship between temperature and $k_r$ in Al$_2$O$_3$ water-based nanofluids: (a) $f = 4\%$, $d_p = 47$ nm; (b) $f = 6\%$, $d_p = 36$ nm. Lines are the proposed model while the dots are experimental data [11].

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References
[1] Tiandho Y, Dinata I, Sunanda W, Gusa R F and Novitasari D 2019 *IOP Conf. Series: Earth and Environmental Science* 257 012022
[2] Tiandho Y, Sunanda W, Afriani F, Indriawati A and Handayani T 2018 *Latvian Journal of Physics and Technical Sciences* 55 15
[3] Verma S and Tiwari A 2015 *Energy Conversion and Management* 100 324
[4] Hussein A 2016 *Renewable and Sustainable Energy Reviews* 62 767
[5] Warrier P, Yuan Y, Beck M and Teja A 2010 *AIChE Journal* 56 3243
[6] Bellos E, Tzivanidis C, Antonopoulus K and Gkinis G 2016 *Renewable Energy* 94 213
[7] Khanafer K and Vafai K 2018 *Renewable Energy* 123 398
[8] Tiandho Y, Gusa R F, Dinata I and Sunanda W 2018 *E3S Web of Conferences* 73 01015
[9] Jang S and Choi S 2007 *Journal of Heat Transfer* 129 617
[10] Taylor R, Phelan P, Otanicar T, Walker C and M. Nguyen 2011 *J. Renew. Sustain. Energy* 3 023104
[11] Mintsa H, Roy G, Nguyen C and Doucet D 2009 *International Journal of Thermal Sciences* 48 363
[12] Yang B and Han Z 2006 *Applied Physics Letters* **89** 083111
[13] Kell G 1972 *Water: A Comprehensive Treatise* (New York: Plenum Press) pp. 363-412
[14] Vajjha R, Das D and Mahagonkar B 2009 *Petroleum Science and Technology* **27** 612
[15] Kell G 1975 *Journal of Chemical and Engineering Data* **20** 97
[16] Mahbubul I, Saidur R and Amalina M 2013 *Procedia Engineering* **56** 310