The effect of extreme asset prices to the valuation of zero coupon bond with jump diffusion processes

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Abstract. In general, the logarithmic returns of asset prices are not normally distributed. Brownian motion and normal distribution have been widely used in the Black-Scholes-Merton bond framework to model the return of assets. Merton has provided a formula for the valuation of a zero coupon bond where the asset price process contains a continuous Poisson jump component, in addition to a continuous log-normally distributed component. This paper applies jump diffusion processes to derive some bond parameters, these are equity and default probability, when the asset prices have extreme values.

1. Introduction
Academic literature on credit risk has grown substantially over the last decades. One way of modeling credit risk is the structural approach which relies on a thorough description of the economics of default. A wide range of theoretical structural models have been proposed and they all contribute to the literature as they focus on the effects of various realistic economic considerations on credit risk valuation and prediction.

Credit risk is the risk resulting from credit events such as changes in credit ratings, restructuring, bankruptcy, etc. Formal definition of credit risk is the distribution of financial losses due to unexpected changes in the credit quality of counterparty in a financial agreement [1]. The point of view in credit risk is the default event, which happens when the firm cannot fulfill its legal obligations by bond contract.

Merton [2] was the seminal paper which builds a model based on the capital structure of the firm, which becomes the first of the structural model. He assumes that firm is financed by equity and a zero coupon bond with face value \( K \) and maturity date \( T \). In this approach, the company defaults at the bond maturity date \( T \) if its assets have smaller value than the face value of the bond at time \( T \).

The practical assumption in the Merton model is the return on company assets is normally distributed with constant volatility. However, it is often found that the return of assets that are not normally distributed, in this case have skewness and kurtosis values that do not meet the standard value. Several studies have found evidence that financial data is very sensitive to market conditions. It is due to the nature of financial data which tends to contain extreme data [3]. The extreme nature of a group of data is often identified based on the nature of heavy tail or fat tail from a data distribution.

Heavy tail or fat tail phenomenon from data distribution, namely the form of probability distribution that describes the nature of data that has a thicker shape than the normal distribution in the tail of the
distribution. This heavy tail phenomenon is often also followed by the occurrence of a large frequency of occurrence in the part of the mode of data distribution which is often referred to as the occurrence of lepökurtic or excess kurtosis. It also became a motivation for researchers to develop a non-Gaussian model [4, 5, 6].

Several theories have been put forward to solve this problem. Jarrow and Rudd [7] use the deviation from the Gaussian moment as an unknown distribution approach with Edgeworth series expansion. The Cornish-Fisher model is using α-th quantile development based on the transformation of the variable random Gaussian standard into a non-standard Gaussian random variable [8]. While Corrado and Su [9] has model stock price returns with Gram-Charlier expansion for option valuations.

Merton models are the main paper in reference to bond valuations. This theory has developed rapidly, including the existence of Merton Jump Diffusion Model which considers the existence of skewness and excess kurtosis which are not by the standard normal distribution. One paper that discusses this theory is Matsuda [10] with the Levy process approach.

It is of great importance for those in charge of managing risk to understand how financial asset returns are distributed. Practitioners often assume for convenience that the distribution is normal. It is now commonly accepted that financial asset returns are, in fact, heavy-tailed. However, empirical evidence has led many to reject this assumption in favor of various heavy-tailed alternatives. Heavy tailed performance show the existence of skewness and excess kurtosis performance.

In addition, the Gram-Charlier expansion as a generalization of the normal distribution to overcome the existence of skewness and kurtosis parameters is also quite widely used in this problem [11]. Another work has been done by Cheang & Chiarella [12]. The paper has extended the analysis of Merton to the case where the distribution of the jump-arrivals and the jump-sizes change under the change of measure and using a Radon-Nikodym derivative process. Burger [13] has studied a comparison theory between Merton jump diffusion model and Black Scholes model. In case of Indonesian data, Abdurakhman and Maruddani [14] provide an analysis of the effect of skewness and kurtosis on bond valuations on Indonesian corporate bond.

Based on the background of the discuss problems, this paper gives a mathematical model for bond valuation with a period of time with asset data having skewed and fat tails (kurtosis). The valuation includes calculating the estimated equity and the chance of bankruptcy of the company that issued the bond based on the company's assets that contained skewness and excess kurtosis. Modeling will be carried out based on Merton Jump Diffusion Model in the case of Indonesian corporate bond data.

2. Theoretical Framework

Merton Jump Diffusion (MJD) model is one of Black-Scholes theory development to capture the skewness and excess kurtosis of the underline asset density by simple addition of a compound Poisson Process.

Merton uses Levy process by adding compound Poisson process (discontinuous jump process) into Brownian motion with drift (continuous diffusion process). The probability that an asset price jumps during a small time interval \( dt \) can be written using Poisson process \( dN \), as

\[
\begin{align*}
P\{\text{an asset price jumps once in } dt\} &= P\{ dN(t) = 1 \} \approx \lambda dt \\
P\{\text{an asset price jumps more than once in } dt\} &= P\{ dN(t) \geq 2 \} \approx 0 \\
P\{\text{an asset price does not jumps in } dt\} &= P\{ dN(t) = 0 \} \approx 1 - \lambda dt
\end{align*}
\]

Where the parameter \( \lambda \) is the intensity of the jump process which is independent of time \( t \).

MJD dynamics of asset prices which incorporate the above properties take the Stochastic Differential Equation with the form

\[
\frac{dA_t}{A(t)} = (\alpha - \lambda k)dt + \sigma dW(t) + (\gamma_t - 1)dN(t)
\]

\( y \) is nonnegative random variables drawn from lognormal distribution \( \ln Y_T \sim N(\mu, \delta^2) \)

Then the solution of Stochastic Differential Equation from the above equation is

\[
dA(t) = (\alpha - \lambda k)A(t)dt + \sigma A(t)dW(t) + (\gamma_t - 1)A(t)dN(t)
\]

Then the dynamics of asset prices is
\begin{equation}
A(T) = A(0) \exp \left\{ \left( r - \frac{\sigma^2}{2} - \lambda k \right) T + \sigma W(T) + \sum_{i=1}^{N_1} Y_i \right\}
\end{equation}

\[ W(T) \sim N(0,T) \text{ dan } \ln A(T) \sim N(\mu,\sigma^2), \text{ dengan } \mu = \ln A(0) + \left( r - \frac{\sigma^2}{2} \right) T \text{ dan variansi } \sigma^2 T. \]

In this theory, we add three parameters, \( \lambda, \mu, \text{ and } \delta \) to the original Black-Scholes model which give us to control skewness and excess kurtosis, which equation is given as follows: \[ 10 \]

\[ \text{Mean}[A(T)] = a - \frac{\sigma^2}{2} - \lambda \left( \exp \left( \mu + \frac{\delta^2}{2} \right) - 1 \right) + \lambda \mu \]

\[ \text{Variance}[A(T)] = \sigma^2 + \lambda \delta^2 + \lambda \mu^2 \]

\[ \text{Skewness}[A(T)] = \frac{\lambda(3\delta^2\mu + \mu^3)}{(\sigma^2 + \lambda \delta^2 + \lambda \mu^2)^{-3/2}} \]

\[ \text{Excess Kurtosis}[A(T)] = \frac{\lambda(3\delta^4 + 6\delta^2\mu + \mu^3)}{(\sigma^2 + \lambda \delta^2 + \lambda \mu^2)^{-3/2}} \]

Then we have equity expectation of the firm at maturity date is

\[ E_{T_0, MJD}^{\tau} = \sum_{i=1}^{N_1} \exp(-\lambda x)^i \Gamma (S_t, \tau = T - t, \sigma_t, r_t) \]

with:

\[ \tilde{\lambda} = \lambda(1 + k) = \lambda \exp \left( \mu + \frac{\delta^2}{2} \right) \]

\[ \sigma_t^2 = \sigma^2 + \frac{i \delta^2}{\tau} \]

\[ r_t = r - \lambda k + \frac{i \ln(1 + k)}{\tau} = r - \lambda \left( \exp \left( \mu + \frac{\delta^2}{2} \right) - 1 \right) + \frac{i \left( \mu + \frac{\delta^2}{2} \right)}{\tau} \]

\[ V_{BS} = \text{equity expectation of Black-Scholes model without jump} \]

And the default probabilities of the firm at maturity date \( (T) \) is: \[ 15 \]

\[ P(A_T < K) = \sum_{i=0}^{\infty} \exp(-\lambda T)(\lambda T)^i \left( \frac{\ln(k) - \ln(A_T) - \left( r - \frac{\sigma^2}{2} - \lambda k \right)T - i \mu}{\sqrt{\sigma^2 T + i \sigma^2}} \right) \]

\[ 3. \text{Data and methods} \]

Indonesia corporate bond data is derived from publicly available databases obtained from Indonesian Bond Pricing Agency (IBPA) in 2017 on website [www.ibpa.co.id](http://www.ibpa.co.id). We use bond data issued by CIMB Niaga Bank in 2017 with code CIMB named “Obligasi Berkelanjutan II Bank CIMB Niaga Tahap III 2017 Seri C” which is issued on November 2, 2107. The profile structure of this bond is given at table 1 [16].

| Outstanding | Listing Date | Maturity Date | Issue Term | Coupon Structure |
|-------------|--------------|---------------|------------|------------------|
| 843.000.000.000.00 | November, 2017 | November, 2022 | 5 years | Fixed 7.75 % |

Total asset data of the firm is published by Indonesian Bank consists of monthly prices October 2012 until September 2017 on website [www.bi.go.id](http://www.bi.go.id). Investors’ main concern will be on the return on
investment which refers to the percentage growth in the value of an asset. If $X$ is asset value on the day, then the return from day $i$ to day $t+1$ is given by

$$R_t = \ln \frac{x_t}{x_{t-1}}$$ (6)

If $n$ is the number of returns in the sample, then the drift $\mu$ can be presented by the mean of the returns distribution and volatility $\sigma$ can be represented by the sample standard deviation. Procedure to valuate bond performance is as follow:
1. Calculate the value of ln return of assets for each $t$
2. Calculate all the parameters
3. Test the normality distribution of ln return assets
4. Calculating default probability of bond using Merton Jump Diffusion Model

4. Results and discussions
For deriving the probability of default, equity, and liability of the bond, we have to do some steps for fulfilling the assumptions. First, we have to estimate some parameters. In Table 2, we give summarize for those parameters.

| Parameter                        | Value       |
|----------------------------------|-------------|
| Asset Value at October 2012 ($A$)| 247,724,200.000.000 |
| Interest risk-free rate ($r$)    | 4.65 %      |
| Mean ($\mu$)                     | 0.004571849 |
| Variance ($\sigma^2$)            | 0.0003864503 |
| Volatility ($\sigma$)            | 0.06809848  |
| Minimum                          | -0.06831622 |
| Maximum                          | 0.05537749  |
| Skewness                         | -0.4763716  |
| Kurtosis                         | 5.254458    |

Then, we have to check whether the natural logarithm of total assets data is normally distribution or not. We can see the normality from the histogram. The result is given in Figure 1. From the figure, we can see that the ln return data is not normally distributed. It also can be saw from kurtosis value at Table 1. The kurtosis value is more than 3, so the excess kurtosis is shown from this value. It can be concluded that the data on the assets of CIMB is heavy tail. Alternatively, we can do hypothesis test of normality distribution with Jarque Bera Test formally. The result of the Jarque-Bera test is given in Table 3. Because of the p-value (0.0005598) is less than $\alpha$ (0.05), then we can conclude that this series is not normally distributed.
Based on the Jarque-Bera test, it is concluded that the data is not normally distributed. It is an indication that there is extreme data on asset data. So the Merton Jump Diffusion Model can be applied to this case.

Then, we can compute equity and probability of default using equation (4), (5), and (6). We summarize the result on table 4.

Using R programming and based on the calculation results, Table 4 shows that the probability of bankruptcy at maturity payments is very small on those two models. The expectation of equity also shows good performance. It can be seen from the value on those two models shows exceeds from debt value. It indicates that the opportunity of PT. CIMB Niaga Tbk cannot pay its debts when the maturity is very small. The expectation value of equity in the model also far exceeds the debt of the company, so it can be said that PT. CIMB Niaga Bank has good enough performance to be able to pay bond debt until maturity.

![Histogram of ln return CIMB Niaga data assets](image)
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