Special integrals of motion in quantum integrable systems

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Abstract We investigate quantum integrals of motion in the sine-Gordon, Zhiber-Shabat and similar systems. When the coupling constants in these models take special values a new quantum symmetry appears. In those cases, correlation functions can be obtained, and they have a power law behavior.

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1. Introduction

Quantization of simple integrable systems is important as a test of quantization methods in more general cases. Any polynomial field operator must be well defined, and so must be regularized, respecting the demand of integrability. In ordinary integrable theory integrals of motion are polynomial field operators, and so have such infinities. Commutators of those integrals of motion also have their own infinities. Our problem is to determine those operators and a way to work with polynomial field operators which is free from all these infinities. We suppose that know the answer for quantum commutators such integrals of motion. If coupling constants don’t take special values integrals of motion must commute. So our problem to construct such operators. The sine-Gordon and Zhiber-Shabat models are very convenient for investigating
this problem.

In [1], two different methods of quantization are distinguished. The first is to construct the monodromy matrix \( T(\lambda) \) and impose

\[
[T(\lambda), T(\mu)] = 0.
\]

The second is to construct the quantum integrals of motion \( \hat{I}_i \) and demand

\[
[\hat{I}_i, \hat{I}_j] = 0.
\]

Classically, the methods are equivalent. At the quantum level, however, that is not the case. On the one hand, the generating function of quantum integrals of motion \( T(\lambda) \) is well defined. On the other hand \( \ln T(\lambda) \) is not well defined, because we have a problem connected with divergences [1]. Let us note that for factorization of the \( S \) matrix, we must have quantum integrals of motion (which commute with the Hamiltonian), i.e. the second method of quantization is preferred. This method have successfully tasted in quantum integrable sine-Gordon, Zhiber-Shabat system, quantum nonlinear Schrödinger equation, Korteweg-de Vries and modified Korteweg de Vries equations [1].

We have used this second method on the sine-Gordon and Zhiber-Shabat models and have obtained quantum integrals of motion for arbitrary coupling constants. If the coupling constants take special values, an interesting situation arises. Local conservation currents have been found and those currents have unusual commutation relations, i.e. new algebras. The generators of the algebras have two indices. This contrasts with ordinary conformal symmetry, where we have the Virasoro algebra with one-index generators [2]. Using this new symmetry, we can calculate correlation functions.

The power law behavior of the resulting correlation functions is connected with the violation of the standard equivalence [3] between the sine-Gordon and massive Thirring fermionic models. Let us recall basic features of this standard equivalence, and explain probably hidden difficulties of such approach.

We can consider work about equivalence of sine-Gordon/Thirring models [3] (work of Mandelstam and in perturbative approach works of Klaiber+Coleman) and explain our position. In our approach we use only Hamiltonian quantization. In the works of Mandelstam and Klaiber, another method has been followed. They solve the normal-ordered Lagrange equations and then check the equal-time anticommutators for fermionic fields. Reading those works carefully
reveals an equal-time anticommutator of the non-canonical form
\[
[\hat{\psi}(x,t),\hat{\psi}^+(y,t)]_+ = (x-y)^\sigma \delta(x-y) \neq \delta(x-y).
\]

We use the Hamiltonian approach and postulate the \emph{canonical} equal-time anticommutators (for initial operators) at the beginning of our calculations.

The Hamiltonian approach guarantees the conservation this equal-time anticommutator. It is easy to see this from the solution of the Hamiltonian equations
\[
\hat{\psi}(x,t) = \exp\left(-it\hat{H}\right) \hat{\psi}(x,0) \exp\left(it\hat{H}\right),
\]
where \(\hat{H}\) is the quantum Hamiltonian of the system. So we don’t have similar problems. Calculation of integrals of motion demands correct equal-time anticommutators because uses in fact basis property of \(\delta\) - function
\[
\int f(x)\delta(x-y)dx = f(y),
\]
and so we can not use above anticommutators as in the works of Mandelstam or Klaiber.

In our opinion, the equivalence of these theories must be investigated in the Hamiltonian approach and a connection between integrals of motion for sine-Gordon and massive Thirring models must be found.

2. Two Hamiltonian structures for the sine-Gordon equation

It is well known that the classical model obeying the sine-Gordon equation
\[
\partial^2_{tt}\varphi(x,t) - \partial^2_{xx}\varphi(x,t) + (m^2/\beta) \sin(\beta\varphi(x,t)) = 0 \tag{2.1}
\]
is an integrable Hamiltonian system [4]. Let us introduce equal-time canonical Poisson brackets for \(\varphi(x,t),\pi(x,t)\), where \(\pi(x,t) = \partial_t\varphi(x,t)\):
\[
\{\varphi(x,t),\pi(y,t)\} = \delta(x-y). \tag{2.2}
\]
Equation (2.1) can be written in Hamiltonian form with the Hamiltonian
\[
\mathcal{H}' = \int_{-\infty}^{+\infty} dx \left( \frac{1}{2}\pi^2 + \frac{1}{2}(\partial_x\varphi)^2 + (m^2/\beta^2)(1 - \cos(\beta\varphi)) \right).
\]
Integrals of motion are determined in terms of \(\varphi(x,t),\pi(x,t)\) by demanding that they commute with \(\mathcal{H}'\), using the brackets (2.2).
The equation of motion also has a second Hamiltonian structure. Let us introduce the coordinates \( \xi = x + t, \eta = x - t \), and define \( \chi(\xi, \eta) = \varphi((\xi + \eta)/2, (\xi - \eta)/2) \). In the new coordinates (2.1) becomes

\[
\partial_\xi \partial_\eta \chi(\xi, \eta) - (m^2/\beta) \sin(\beta \chi(\xi, \eta)) = 0. \tag{2.3}
\]

The canonical Poisson brackets for \( \chi(\xi, \eta) \) are

\[
\{\chi(\xi, \eta), \chi(\xi', \eta')\} = \varepsilon(\xi - \xi'). \tag{2.4}
\]

Equation (2.3) can therefore be written in Hamiltonian form with

\[
\mathcal{H} = (2m^2/\beta^2) \int_{-\infty}^{+\infty} d\xi (\cos(\beta \chi(\xi, \eta)) - 1). \tag{2.5}
\]

The Hamiltonian and integrals of motion can be determined at some initial time. The condition on the initial function is a holomorphic condition \( \partial_\eta \chi(\xi, \eta) = 0 \) (\( \eta \) is the time) if we have \( t = i\tau \). We should determine a quantum field of the theory as \( \hat{\chi}(\xi, \eta) = \exp(-i\eta \hat{\mathcal{H}}) \hat{\chi}(\xi) \exp(i\eta \hat{\mathcal{H}}) \). Integrals of motion of the Hamiltonian (2.5) and brackets (2.4) can be written like differential polynomials of \( \partial_\xi \chi(\xi) \). We will consider the following boundary condition \( \partial_\xi \chi(\xi) (\xi \to \pm \infty) \), and it has some singularity in a restricted region of the real axis \( \xi \). Let us consider \( \xi \) as a complex coordinate and expand \( \partial_\xi \chi(\xi) \) in some series. Indeed, any holomorphic function in the region \( r < |\xi - a| < R \) can be represented by a series in positive and negative powers of \( (\xi - a) \), the value of \( a \)-places where we have a singularity [5]. So we have

\[
\partial_\xi \chi(\xi) = \sum_{n \in \mathbb{Z}} a_n \xi^{-n-1},
\]

and

\[
\chi(\xi) = q + p \ln \xi - \sum_{n \neq 0} \frac{a_n \xi^{-n}}{n}. \tag{2.6}
\]

We have set \( a = 0 \) without loss of generality. Now let us consider quantization of our field. The commutation relations for the operators are

\[
[\hat{p}, \hat{q}] = 1, \quad [\hat{a}_n, \hat{a}_m] = n \delta_{n+m,0}.
\]

The normal ordering of non-commuting operators obeys

\[
\hat{p}\hat{q} = :\hat{p}\hat{q}: + 1, \quad \hat{q}\hat{p} = :\hat{p}\hat{q}:
\]
\[ \hat{a}_n \hat{a}_{-n} =: \hat{a}_n \hat{a}_{-n} : + n, \quad \hat{a}_{-n} \hat{a}_n =: \hat{a}_{-n} \hat{a}_n :, \quad n > 0. \]

The relation for field is the following

\[ \check{\chi}(\xi) \check{\chi}(\xi') =: \check{\chi}(\xi) \check{\chi}(\xi') : + \ln(\xi - \xi'). \]

Quantum integrals of motion are differential polynomials of \( \check{\chi} \) with some coefficients. These coefficients should be defined by

\[ [\hat{\mathcal{H}}, \hat{I}_k] = 0, \quad (2.7) \]

where the quantum Hamiltonian \( \hat{\mathcal{H}} \) of the theory is

\[ \hat{\mathcal{H}} = \int_{-\infty}^{+\infty} : \exp(\alpha \check{\chi}(\xi)) : d\xi + \int_{-\infty}^{+\infty} : \exp(-\alpha \check{\chi}(\xi)) : d\xi. \]

When we integrate over \( \xi \), we obtain a singularity. Indeed, when we expand the product of ordered operators, we have, for example

\[ : \exp(\alpha \check{\chi}(\xi)) : (\check{\chi}(\xi'))^k := \sum_{n=0}^{\infty} \frac{1}{(\xi - \xi')^n} : \exp(\alpha \check{\chi}(\xi)) \hat{P}_n^k(\xi') : . \]

We should determine the function at the singularity. Let us consider the commutator:

\[ [\hat{\mathcal{H}}, \hat{i}_k(\xi')] = \hat{\mathcal{H}} \hat{i}_k(\xi') - \hat{i}_k(\xi') \hat{\mathcal{H}}, \]

where \( \hat{i}_k(\xi) \) is the density of the integrals of motion. In the first piece we take the contour of integration which is above \( \xi' \), and in the second piece we take the contour of integration which is below \( \xi' \). Then we add integration over \( C_{R \to \infty} \) which is equal to zero. The closed contour can be deformed and we obtain

\[ [\hat{\mathcal{H}}, \hat{i}_k(\xi')] = \oint_{\xi'} d\xi \hat{h}(\xi) \hat{i}_k(\xi'). \]

### 3. Quantum symmetry of integrable theory

Using the definition of the commutator, we can calculate the integrals of motion. If the coupling constants take generic values, we have

\[ \hat{i}_4 = \hat{(\partial \check{\chi})}^4 + \frac{(\alpha^4 - 6\alpha^2 + 4)}{\alpha^2} : (\partial^2 \check{\chi})^2 :, \]
\[ i_6 = \frac{1}{2} (\partial \hat{\chi})^6 + \frac{5}{3} \frac{(-\alpha^4 + 8\alpha^2 - 4)}{\alpha^2} (\partial \hat{\chi})^3 \partial^3 \hat{\chi} + \frac{3 \alpha^8 - 40\alpha^6 + 155\alpha^4 - 160\alpha^2 + 48}{6\alpha^4} (\partial^3 \hat{\chi})^2, \]

and they satisfy

\[ [\hat{T}_4, \hat{T}_6] = 0. \]

But if the coupling constant takes special values, some interesting results appear. We have obtained

\[ [\hat{H}, \hat{J}^k(\xi)] = 0, \]

if the coupling constant \( \alpha = 1 \) in the sine-Gordon theory. Here we found

\[ \hat{J}^k(\xi) = \frac{1}{(k + 1)!} : (\partial^{1+k} \exp \hat{\chi}(\xi)) \partial^k \exp - \hat{\chi}(\xi) : + \frac{1}{(k + 2)(k + 1)} : (\partial^{2+k} \exp \hat{\chi}(\xi)) \exp - \hat{\chi}(\xi) : , \quad k \in \mathbb{N}. \]

Similar cases are listed in the Appendix.

We can calculate the commutation relations between the \( \hat{J}^k_n \)

\[ [\hat{J}^k_n, \hat{J}^p_m] = (k + 1)! \sum_{l=0}^{k} \frac{\hat{J}^{p+l'}_{n+m}}{l!(k + 1 - l)!} \prod_{j=0}^{k-l'} (m + p + 1 - j') - (p + 1)! \sum_{l=0}^{p} \frac{\hat{J}^{k+l}_{n+m}}{l!(p + 1 - l)!} \prod_{j=0}^{p-l} (n + k + 1 - j) + \delta_{n+m,0} \frac{(p + 1)!(k + 1)!}{2(k + p + 3)!} \times \]

\[ \times \left( (-1)^{k+1} \prod_{j=0}^{2+k+p} (n + k + 1 - j) - (-1)^{p+1} \prod_{j=0}^{2+k+p} (m + p + 1 - j) \right), \]

where \( \hat{J}_n^k = \frac{1}{2\pi i} \oint_0 \hat{J}^k(\xi) \xi^{k+1+n} d\xi \). This is a new type of infinite-dimensional algebra, with central extension. Using this quantum symmetry we can calculate
the correlation functions in a system of this sort.

4. Evolution of operators in the Heisenberg picture

In the Heisenberg picture, the operator $\hat{A}(\xi, \eta)$ can be obtained from the operator $\hat{A}(\xi)$:

$$\hat{A}(\xi, \eta) = \exp(-i\eta\hat{H}) \hat{A}(\xi) \exp(i\eta\hat{H}).$$

The correlation function of the operators has the form

$$\langle 0| \hat{A}_1(\xi_1, \eta_1) \cdots \hat{A}_n(\xi_n, \eta_n)|0\rangle =$$

$$\langle 0| \exp(-i\eta_1\hat{H}) \hat{A}_1(\xi_1) \exp(i\eta_1\hat{H}) \cdots \exp(-i\eta_n\hat{H}) \hat{A}_n(\xi_n) \exp(i\eta_n\hat{H})|0\rangle,$$

where $|0\rangle$ is the vacuum of the theory. If $\hat{A}(\xi, \eta)$ is a complicated function of operator $\hat{\chi}$, we should fix a certain ordering. We choose, for example,

$$\exp(\alpha \hat{\chi}(\xi, \eta)) = \exp(-i\eta\hat{H}) : \exp(\alpha \hat{\chi}(\xi)) : \exp(i\eta\hat{H}),$$

$\hat{\chi}(\xi)$ is the field of theory and it has the decomposition (2.6). The normal ordering $:\ :$ was discussed in section 2.

5. One-point correlation function and properties of the vacuum

Let us consider the one-point correlation function

$$\langle \hat{J}(\xi, \eta) \rangle = \langle 0| \exp(-i\eta\hat{H}) \hat{J}(\xi) \exp(i\eta\hat{H}) |0\rangle = \langle 0|\hat{J}(\xi)|0\rangle.$$

where $\hat{J}(\xi, \eta)$ is conservation current in the theory. In a space-invariant theory, any one-point correlation function is equal to a constant $\langle \hat{J}(\xi, \eta) \rangle = C$. This property means that

$$\langle 0|\hat{J}_n|0\rangle = \delta_{n,0}C, \quad n = 0, \pm 1, \pm 2 \ldots .$$

And we have the property that $\langle 0| \hat{J}_n|0\rangle$ are orthogonal to each other for all $n \neq 0$. We can obtain this property if we assume that

$$\hat{J}_n|0\rangle = 0, \quad n \in \mathbb{N}.$$
In our case we have
\[
\hat{J}^k_n|0\rangle = 0, \quad n \in \mathbb{N}; \quad k \in \mathbb{N}.
\] (5.1)

We should have the properties of invariance of vacuum as follows:
\[
\hat{P}|0\rangle = 0,
\] (5.2)

where \(\hat{P}\)-is the operator of translation (\(\hat{P} = \frac{1}{2} \int_{-\infty}^{+\infty} : (\partial_\xi \hat{\chi}(\xi))^2 : d\xi\)). We know that
\[
[\hat{P}, \hat{J}^k_n] = (k + n + 1) \hat{J}^k_{n-1}.
\] (5.3)

Now it is easy to obtain
\[
\hat{J}^0_n|0\rangle = 0, \quad \hat{J}^1_{n-1}|0\rangle = 0, \quad \hat{J}^2_{n-2}|0\rangle = 0, \ldots \quad \hat{J}^k_{n-k}|0\rangle = 0,
\]

and for the left vacuum:
\[
\langle 0 | \hat{J}^k_n = 0, \quad \langle 0 | \hat{J}^1_k = 0, \quad \langle 0 | \hat{J}^2_2 = 0, \ldots \quad \langle 0 | \hat{J}^k_{k+1} = 0.
\]

It is easy to understand that
\[
\langle 0 | \hat{A}_1(\xi_1, \eta_1) \ldots \hat{A}_n(\xi_n, \eta_n) \hat{J}^1_p|0\rangle = 0, \quad p = 0, \pm 1, \pm 2.
\]

We should know the commutator \([\hat{J}^1_n, \hat{A}_k(\xi_k, \eta_k)]\). We can calculate this commutator if \(\hat{A}(\xi) = : \exp(\alpha \hat{\chi}(\xi)):\)
\[
[\hat{J}^1_n, : \exp(\alpha \hat{\chi}(\xi)) :] =
\]
\[
= \xi^{n+2}(\hat{J}^1_{n+1} : \exp \alpha \hat{\chi}(\xi)) _\xi^{(\alpha^3 - \alpha)} (n+2)(n+1)\xi^n : \exp \alpha \hat{\chi}(\xi) :.
\]

Now we write the differential equations for the correlation function
\[
\sum_{j=1}^{n} \xi_j^{p+2} \langle 0 | \hat{A}_1(\xi_1, \eta_1) \ldots \exp(-i\eta_j \hat{H}) (\hat{J}^1_{j-2} : \exp \alpha_j \hat{\chi}(\xi_j) :) \exp(i\eta_j \hat{H}) \ldots \hat{A}_n(\xi_n, \eta_n)|0\rangle =
\]
\[
= D^1_p f, \quad p = 0, \pm 1, \pm 2,
\]

where \(D^1_p\) are known differential operators. For the two-point correlation function, we have three equations
\[
\alpha(\xi_2 - \xi_1) \partial_{\xi_1} f + \beta(\xi_1 - \xi_2) \partial_{\xi_2} f - \frac{(\alpha^3 - \alpha + \beta^3 - \beta)}{3} f = 0,
\]
\[\alpha(\xi_2 - \xi_1)(2\xi_1 + \xi_2)\partial_{\xi_1}f + \beta(\xi_1 - \xi_2)(\xi_1 + 2\xi_2)\partial_{\xi_2}f\]
\[-((\alpha^3 - \alpha)\xi_1 + (\beta^3 - \beta)\xi_2)f = 0,\]

\[\alpha(\xi_2 - \xi_1)(3\xi_1^2 + 2\xi_1\xi_2 + \xi_2^2)\partial_{\xi_1}f + \beta(\xi_1 - \xi_2)(\xi_1^2 + 3\xi_2^2 + 2\xi_1\xi_2)\partial_{\xi_2}f\]
\[-(2(\alpha^3 - \alpha)\xi_1^2 + 2(\beta^3 - \beta)\xi_2^2)f = 0.\]

They have a solution (if we set \(\beta = -\alpha\))

\[f(\xi_1, \xi_2, \eta_1, \eta_2) = C(\alpha)G(\eta_1, \eta_2)(\xi_1 - \xi_2)^{(1 - \alpha^2)},\]

where \(G(\eta_1, \eta_2), C(\alpha)\) are arbitrary functions. The equation (2.3) in light-cone coordinates has the symmetry \((\xi \leftrightarrow \eta)\), and so we have \(f(r) \sim r^{2(1 - \alpha^2)}\) where \(r\) is the distance between the points. We can investigate other Ward identities from another current \(\hat{J}_2(\xi)\) and obtain the same formula for \(f(r)\).

Similar conservation currents exist in other models (see the Appendix). Now let us consider integrals of motion in systems with Hamiltonian

\[\mathcal{H}_k = \int_{-\infty}^{+\infty} : \exp(\alpha \hat{\chi}(\xi)) : d\xi + \int_{-\infty}^{+\infty} : \exp(k\alpha \hat{\chi}(\xi)) : d\xi.\]

We have obtained nontrivial integrals of motion if \(k = -1, -2, \frac{1}{\alpha^2}, \frac{2}{\alpha^2}, \frac{4}{\alpha^2}\) and local conservation currents (see Appendix). When \(k = -1\) we have the sine-Gordon theory and if \(k = -2\), the Zhiber-Shabat theory. The theories with other \(k\) have no classical limit \((\alpha \to 0)\).

6. Conclusion

Computer calculations confirm an absolutely general behavior of quantum integrable systems with exponential interactions. We have obtained some examples of local conserved currents (densities) in all known models. Correlation functions in these cases can probably be obtained by a similar procedure. We have obtained a new infinite-dimensional symmetry in quantum integrable theories, and we calculated two-point correlation function using this symmetry.

There is an interesting connection between this calculation and a similar calculation in fermionic models [6], where we used the Hamiltonian method to solve the massless Thirring model and obtained an anomaly for conservation of the energy-momentum tensor. We in fact reveal the hidden problem in fermionic anticommutators by demonstrating on obvious problem– the
anomaly. So the massless Thirring model is not a conformal field theory (solvable theory) and we cannot use Coleman’s way of proving the equivalence of the massive Thirring model and sine-Gordon theories using perturbative methods.

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Appendix A.

Here we report the arrangement and quantities $m_n$ of nontrivial currents at levels $n$. A current which is a differential of another current is trivial. The results of computer calculations are listed in the Table below. We investigate the space of quantum integrals of motion for the Hamiltonian $\hat{H}_k$. This problem can move to investigation of system $S_n$ ordinary linear equations with parameters $\alpha, k$, degeneration of this system can correspond to appear the current at level $n$. Determinant of the system for $n = 4, 6$ have a form

$$\det S_4 = (\alpha^2 - 1)(1 - k)^2(\alpha^2 k + 2)^2(k + 1)(-\alpha^2 k + 2),$$

$$\det S_6 = (\alpha - 1)^2(\alpha + 1)^2(3\alpha^2 - 2)(\alpha^2 - 3)(1 - k)^4(k\alpha^2 + 2)^4(k + 1)(k + 2) \times$$

$$\times (2k + 1)(-k\alpha^2 + 2)(-k\alpha^2 + 4)(-k\alpha^2 + 1)(-2\alpha^2 k^2 + k\alpha^2 - 2k - 3)$$

After we find solutions for current when $\det S_n = \det S_4 = 0$, which demand some conditions for parameters $\alpha, k$.

Table
| $\alpha$ | k  | $m_2$ | $m_3$ | $m_4$ | $m_5$ | $m_6$ | $m_7$ | $m_8$ | $m_9$ | $m_{10}$ | $m_n$-conjecture |
|--------|----|-------|-------|-------|-------|-------|-------|-------|-------|-----------|------------------|
| 1      | -1 | 0     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1         | $m_n = 1$, for $n > 2$ |
| $\sqrt{2}$ | -1 | 1     | 0     | 1     | 0     | 1     | 0     | 1     | 0     | 1         | $m_{2p} = 1$, $p \in \mathbb{N}$ |
| $\sqrt{3}$ | -2 | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 0     | 1         | $m_{6+4(p-1)}$, $p \in \mathbb{N}$ |
| $\sqrt{3}$ | -2 | 0     | 0     | 0     | 1     | 0     | 1     | 0     | 0     | 1         | $m_{6+2(p-1)}$, $p \in \mathbb{N}$ |
| $\sqrt{3}$ | $\frac{1}{\alpha}$ | 0     | 0     | 0     | 0     | 0     | 1     | 0     | 0     | 1         | $m_{8+2(p-1)}$, $p \in \mathbb{N}$ |
| $\sqrt{3}$ | $\frac{1}{\alpha}$ | 0     | 0     | 0     | 0     | 1     | 0     | 1     | 0     | 1         | $m_{6+4(p-1)}$, $p \in \mathbb{N}$ |

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