Destroying Aliases from the Ground and Space: Super-Nyquist ZZ Cetis in K2 Long Cadence Data

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Abstract

With typical periods of the order of 10 minutes, the pulsation signatures of ZZ Ceti variables (pulsating hydrogen-atmosphere white dwarf stars) are severely undersampled by long-cadence (29.42 minutes per exposure) K2 observations. Nyquist aliasing renders the intrinsic frequencies ambiguous, stifling precision asteroseismology. We report the discovery of two new ZZ Cetis in long-cadence K2 data: EPIC 210377280 and EPIC 220274129. Guided by three to four nights of follow-up, high-speed (≤30 s) photometry from the McDonald Observatory, we recover accurate pulsation frequencies for K2 signals that reflected four to five times off the Nyquist with the full precision of over 70 days of monitoring (~0.01 μHz). In turn, the K2 observations enable us to select the correct peaks from the alias structure of the ground-based signals caused by gaps in the observations. We identify at least seven independent pulsation modes in the light curves of each of these stars. For EPIC 220274129, we detect three complete sets of rotationally split ℓ = 1 (dipole mode) triplets, which we use to astroseismically infer the stellar rotation period of 12.7 ± 1.3 hr. We also detect two sub-Nyquist K2 signals that are likely combination (difference) frequencies. We attribute our inability to match some of the K2 signals to the ground-based data to changes in pulsation amplitudes between epochs of observation. Model fits to SOAR spectroscopy place both EPIC 210377280 and EPIC 220274129 near the middle of the ZZ Ceti instability strip, with $T_{\text{eff}} = 11590 \pm 200$ K and $11810 \pm 210$ K, and masses $0.57 \pm 0.03 M_\odot$ and $0.62 \pm 0.03 M_\odot$, respectively.

Key words: asteroseismology – methods: data analysis – stars: individual (EPIC 210377280, EPIC 220274129) – stars: oscillations – white dwarfs

1. Introduction

Stellar pulsations are extremely sensitive to the detailed interior structures of stars, and Fourier analysis of photometric light curves can reveal the eigenfrequencies of these physical systems. This is the only method by which we directly constrain stellar interiors. However, the measurement of accurate pulsation frequencies for precision asteroseismology requires extended photometric monitoring with few gaps and short exposure times. For data that do not meet these criteria, frequency determination is hindered by the aliasing of pulsation signals.

The atmospheres of the majority of white dwarf stars are dominated by hydrogen (DA white dwarfs). As these stars cool, partial ionization of hydrogen eventually causes an outer convection zone to develop. The modulation of the flux by convection can drive global nonradial oscillations (Brickhill 1991) that manifest as photometric variability of the star. Near the mean mass of 0.64 $M_\odot$ (Tremblay et al., 2013), DA white dwarfs pulsate as ZZ Ceti variables (a.k.a., DAVs) in the range 12,500 ≥ $T_{\text{eff}}$ ≥ 10,800 K (Tremblay et al., 2013), exhibiting photometric variations with periods of 3–20 minutes.

White dwarf asteroseismology has flourished in the era of near-continuous time series photometry from Kepler. The spacecraft saves data for pre-selected targets of interest in two observing modes: long cadence, with continuous exposures of roughly 30 minutes; and short cadence, with one-minute exposures for a limited number of targets (Howell et al., 2014). Short-cadence observations are required to sufficiently oversample typical white dwarf pulsation periods for straightforward frequency measurements. Both in its original mission and while observing new fields along the ecliptic as K2, Kepler has targeted known and candidate pulsating white dwarf stars at short cadence, collecting the most extensive coverage of ZZ Ceti variability to date. This has enabled the precise determination of pulsation frequencies for asteroseismic analysis (Greiss et al., 2014; Hermes et al., 2014, 2015a, 2017a, 2017c; Bell et al., 2015), as well as the discovery of a pulsation-related outburst phenomenon that operates near the cool edge of the ZZ Ceti instability strip (Bell et al., 2015, 2016, 2017; Hermes et al., 2015b).

Long-cadence K2 observations can potentially reveal white dwarf pulsations, but the signals suffer dramatic aliasing against the Nyquist frequency, as well as amplitude reduction, since the 30-minute cadence severely undersamples the pulsations. While the sub-Nyquist pulsation frequency aliases measured from these data are extremely inaccurate, they are of exceptionally high precision owing to the long observational baseline of K2.

High-speed photometry from individual ground-based observatories causes a different kind of aliasing: gaps in the data from daylight and weather introduce cycle-count ambiguities into the pulsation record. Selecting the correct alias among the comb of peaks that make up the observational spectral window is nontrivial.

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2. Observations

2.1. Long-cadence K2 Photometry

EPIC 210377280 and EPIC 220274129 were observed at long cadence in K2 fields 4 and 8 for 70.82 and 78.66 days, respectively, as part of a search for white dwarf transits and variability with K2. Both were high-probability white dwarf candidates from the catalog of Gentile Fusillo et al. (2015), but they were not included in our short-cadence survey for pulsating white dwarfs since their \((u-g, g-r)\) colors were well outside our usual selection region for typical ZZ Cetics. Both stars exhibit multiple significant periods of variability in the long-cadence K2 photometry.

For each target, we utilize the light curves extracted and processed by the EVEREST 2.0 pipeline (Luger et al. 2016, 2017), discarding all points with “quality” flags set. We flatten the light curves further by dividing by a cubic spline fit, tuning the fitting weights manually to remove residual noise at the lowest frequencies without affecting the measured amplitudes of the signals of astrophysical interest in the Fourier transform (FT). We note that version 2.0 of the EVEREST pipeline accounts for contamination from nearby stars in the photometric aperture, which otherwise could dilute the amplitudes of variability measured in \textit{Kepler}’s 4′′ pixels.

The details of the K2 observations of both stars are summarized in Table 1, including the total number of long-cadence observations in the final light curves.

2.2. Time Series Photometry from the Ground

We confirmed the pulsational nature of the photometric variations of both targets with high-speed follow-up from the ground with the ProEM camera\(^8\) on the 2.1 m Otto Struve Telescope at McDonald Observatory. At Cassegrain focus, the ProEM camera has a 1′6 × 1′6 field of view, and with 4 × 4 binning, our effective plate scale is 0′038 pixel\(^{-1}\). Each object was observed on four different nights, as detailed in Table 2. The data were acquired through a broadband BG40 filter that transmits between 3300 and 6000 Å to reduce sky noise.

We use standard IRAF tasks to dark-subtract and flat-field the images with calibration frames that we acquired each night. We extract circular aperture photometry for the target and comparison stars in the field using CCD\textunderscore HSP, an IRAF script that uses tasks from PHOT (Kanaan et al. 2002). We divide the measured target counts by a weighted sum of comparison star counts, then by a low-order best-fit polynomial to correct for airmass and transparency variations using the WQED tools (Thompson & Mullally 2013). WQED also applies a barycentric correction and accounts for the latest leap seconds to enable the reliable combination of multiple nights of data.

2.3. Spectroscopy

We acquired spectroscopic observations for both stars with the Goodman spectrograph (Clemens et al. 2004) on the 4.1 m SOAR telescope on Cerro Pachón, Chile, using the setup described in Hermes et al. (2017a). The EPIC 210377280 and EPIC 220274129 spectra were obtained on 2016 September 02 and July 14, respectively.

We fit the data to 1D atmosphere models following the methodology of Tremblay et al. (2011), with the ML2/\(\alpha = 0.8\) mixing length theory parameterization. The uncertainties include the external errors from data reduction and flux calibration estimated by Gianninas et al. (2011). We also apply the 3D convection corrections of Tremblay et al. (2013). Both sets of atmospheric parameters are provided in Table 3. These parameters place both stars near the middle of the ZZ Ceti instability strip (Tremblay et al. 2013). A comparison of the best-fit models to the observed Balmer line profiles is displayed in Figure 1.

3. Comparability of Data Sets

There are a number of effects, both observational and astrophysical, that cause the signatures of pulsations in these stars

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\(^7\) Using tools from SciPy: http://www.scipy.org/.

\(^8\) https://www.princetoninstruments.com/products/ProEM-EMCCD
to differ between the K2 photometry and the follow-up ground-based light curves. We must factor in these considerations as we approach the task of matching signals between these data sets. In this section, we discuss the most significant of these effects and how we incorporate them into the analysis that follows.

3.1. Nyquist Aliasing

The Nyquist critical frequency for evenly sampled data is \( f_{\text{Ny}} = 1/(2\Delta t) \), where \( \Delta t \) is the constant spacing in time between observations. FTs of photometric light curves exhibit peaks at the accurate frequencies of intrinsic stellar brightness variations for signals that are bandwidth limited to below \( f_{\text{Ny}} \). However, frequencies of stellar photometric variability greater than \( f_{\text{Ny}} \) will be aliased into the range \( 0 < f < f_{\text{Ny}} \). Figure 2 demonstrates the relationship between the underlying signal frequency and the location of its \( 0 < f < f_{\text{Ny}} \) alias.

The long-cadence K2 light curves are acquired on board the spacecraft with a regular spacing of 29.43 minutes. These timestamps are then corrected for the changing distance between the spacecraft and the solar system barycenter, breaking the strict regularity of the time sampling. The Nyquist frequency is not so simply defined under these conditions. Eyer & Bartholdi (1999) show that the effective Nyquist frequency over which signals are exactly aliased is \( f_{\text{Ny}} = 1/(2\pi) \), where \( \pi \) is the greatest common factor of all time separations between pairs of observations, \( p \) is the longest period that the time series could be folded on to cause all samples to be coincident in phase. Koen (2006) points out that there is a practical lower limit on \( p \) (upper limit on \( f_{\text{Ny}} \)) set by the recorded timestamp precision.

FTs of the K2 data are significantly affected by Nyquist-like aliasing across lower frequencies due to the near-even spacing of K2 observations. This pseudo-Nyquist behavior occurs at \( f_{\text{Ny}} = 1/(2p^\star) \), where the time samples folded on \( p^\star \) are highly concentrated in phase. Murphy et al. (2013) exploited the difference in aliasing behavior between this and the true Nyquist frequency to identify the intrinsic frequencies for super-Nyquist pulsators in the original Kepler Mission data.

Following the framework of Eyer & Bartholdi (1999), we identify the pseudo-Nyquist frequency by direct calculation of the spectral window (the signature in the FT of a pure sinusoid sampled as the data). The frequency of the first peak beyond zero in the spectral window with amplitude near unity is twice \( f_{\text{Ny}}^\star \). The pseudo-Nyquist frequencies of EPIC 2103777280 and EPIC 22074129 are 283.2377 and 283.2388 \( \mu \text{Hz} \), respectively. The distinction is subtle, so we refer to \( f_{\text{Ny}}^\star \) as simply the Nyquist frequency for the remainder of this paper.

3.2. Phase Smearing

Finite exposure lengths of photometric observations have the effect of boxcar smoothing the underlying signal. The amplitudes measured from the data are therefore smaller than the intrinsic signal amplitudes, and a correction factor must be applied if accurate amplitudes are of interest. When exposures of duration \( t_{\text{exp}} \) are used to record a sinusoid of period \( P \), its amplitude will appear smaller by a factor \(^9\) of \( \eta \):

\[
\eta = \frac{A_{\text{measured}}}{A_{\text{intrinsic}}} = \sin(\pi t_{\text{exp}}/P).
\]

Figure 2 demonstrates this in terms of the ratio of the signal frequency to the Nyquist frequency, assuming continuous exposures with no overhead. This expression for phase smearing can be found in various works, including Hekker &

![Table 3](https://example.com/table3.png)

![Figure 1](https://example.com/figure1.png)

![Figure 2](https://example.com/figure2.png)

**Figure 1.** Balmer line profiles from the SOAR spectra of EPIC 2103777280 (left) and EPIC 22074129 (right). The best-fit 1D models (described in Tremblay et al. 2011) are overplotted in red, and their parameters are indicated at the bottom of each panel.

**Figure 2.** Effect of continuous time series sampling on the measured amplitude and frequency of a signal. Signals with intrinsic frequencies beyond the Nyquist frequency will be aliased into the sub-Nyquist regime (right axis) with significantly decreased amplitudes (left axis).
Christensen-Dalsgaard (2017), where it is referred to as “apodization.” A derivation is available in Murphy (2014, Section 1.2.2).

This effect has a major impact on the ZZ Ceti pulsation amplitudes measured from long-cadence K2 data, where \( t_{\text{exp}} > P \). The equation also expresses \( K2 \)’s lack of sensitivity to signals with periods that are near integer fractions of the exposure time.

### 3.3. Passband Differences

Variations of the emergent flux from a star due to pulsations are wavelength dependent. This causes the same pulsations to have different measured amplitudes in different photometric systems. *Kepler* has a broad response function spanning roughly 4300–8900 Å,\(^{10}\) while the McDonald observations were made through Earth’s atmosphere and a BG40 filter that spans 3300–6000 Å.

To account for this, we calculated the expected pulsation amplitude ratio as measured through these two passbands for a representative \( T_{\text{eff}} = 11,700 \, \text{K}, \log g = 8.0 \, \text{DA white dwarf} \) (within 1\( \sigma \) of the 3D-corrected spectroscopic parameters of both stars). We utilized the grid of spectroscopic model atmospheres described in Koester (2010)—with a ML2/\( \alpha = 0.8 \) treatment of convection—to model the emergent flux. We consider pulsations of spherical degree \( \ell = 1 \) and \( \ell = 2 \), since practically all available constraints on white dwarf pulsations are consistent with these two spherical degrees (e.g., Winget & Kepler 2008).

We expect the intrinsic amplitudes of \( \ell = 1 \) and \( \ell = 2 \) pulsation modes to be larger in the McDonald data than in the K2 observations by factors of roughly 1.126 and 1.133, respectively. We account for this effect by applying the average scaling factor of 1.130 when comparing measurements between data sets throughout this work.

### 3.4. Spectral Window

Gaps in data can cause confusion in the determination of pulsation frequencies by introducing an uncertainty in the number of cycles missed when observations were not being made. Our ground-based data suffer significant diurnal aliasing since they are distributed across multiple nights. As opposed to the intrinsic limitations on frequency precision set by the spectral resolution \( (\alpha 1/T, \text{where } T \text{ is the total baseline of observations; Montgomery & O’donoghue 1999}) \), aliasing introduces an additional extrinsic error to frequency determinations, since we might select an inaccurate alias peak.

The uncertainty introduced by these aliases is best understood by studying the spectral window: the signature of a pure sinusoid in the FT that arises solely from the time sampling. We plot the spectral windows for both stars as part of our analyses that follow. We characterize the magnitude of the potential frequency confusion by measuring the location of the highest noncentral alias in the spectral window.

Any pulsation modes included in our frequency solutions based on the ground-based data alone are flagged as potentially suffering from spectral window alias ambiguities. We emphasize that this extrinsic error is non-Gaussian, and that any attempt at asteroseismic inference should carefully consider alternate frequency solutions for these stars that employ other viable aliases.

### 3.5. Intrinsic Mode Variation

Despite our best efforts to account for how differences in instrumentation affect the comparability of pulsation signatures between these data sets, we face an astrophysical limitation that are more difficult to surmount: the intrinsic signatures of the stellar pulsations may have changed significantly between epochs of observation. The challenges of comparing multi-season pulsation measurements for ZZ Ceti variables that have cooled significantly beyond the hot edge of the instability strip is well established in the literature, particularly from Whole Earth Telescope (Nather et al. 1990) campaigns, GD 154 (Pfeiffer et al. 1996), G29–38 (Kleinman et al. 1998), HL Tau 76 (Dolez et al. 2006), and EC14012-1446 (Handler et al. 2008; Provencal et al. 2012), for example, show such dramatic amplitude changes that significant modes from one year can be completely absent the next. With nine- and seven-month gaps between space- and ground-based campaigns on EPIC 210377280 and EPIC 220274129, we have no expectation that the same eigenmodes will be excited to similar amplitudes in both data sets.

Extended, continuous records from *Kepler* and K2 have provided new insights into mode variations (e.g., Hermes et al. 2014; Bell et al. 2015, 2016). By inspecting the FTs of 27 ZZ Cetis observed by *Kepler* and K2, Hermes et al. (2017a) discovered a dichotomy of mode behavior: while some modes are notably coherent, most with periods longer than 800 s undergo significant modulation and appear as multiple closely spaced peaks in the FT. These bands of power are well fit by Lorentzians with half widths at half maximum of the order of 1 \( \mu \text{Hz} \). The amplitudes of the many individual peaks that make up these power bands are typically smaller than the instantaneous, intrinsic amplitudes of the corresponding pulsation modes. Unlike for phase smearing and passband effects, we are unable to apply a simple corrective factor that accounts for this mode incoherence.

### 4. Comparing Data Sets

The ZZ Ceti pulsation signals in long-cadence K2 data may have suffered any integer number of Nyquist reflections. We assess these candidate intrinsic frequencies through comparison with ground-based observations from the McDonald Observatory. Pulsation frequencies that we are able to positively match between these data sets do not suffer measurement ambiguities from either the Nyquist aliasing of the K2 data or the window function aliasing of the multi-night ground-based photometry. We conservatively accept only unique, unambiguous matches between data sets.

To determine the signals of statistical significance in the K2 data, we use a bootstrapping approach to calculate a threshold in the FT that corresponds to a 0.1% false alarm probability (FAP). This is similar to the calculations made in, e.g., Greiss et al. (2014) and Bell et al. (2015, 2016).\(^{11}\) Signals are

\(^{10}\) https://keplerscience.arc.nasa.gov/CalibrationResponse.shtml

\(^{11}\) We note that the application in these previous works was not strictly bootstrapping, as the resampled flux values were drawn without replacement (i.e., shuffled). The 0.1% FAP thresholds calculated through resampling with replacement agree to within 1% of the values from resampling without replacement.
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significant to better than 99.9% confidence\textsuperscript{12} if they exceed the 99.9 percentile of maximum peak amplitudes detected in the full FTs of 10,000 random bootstrap samplings of the K2 light curves of each star.

The rule-of-thumb significance criterion for ground-based light curves is to reject the null hypothesis (that peaks can be ascribed to pure noise) for peaks in excess of four times the mean amplitude evaluated in a local region of the FT (4\(A\); Breger et al. 1993). These peaks would be accepted as signal with an FAP of \(\approx 0.1\%\) (Kuschnig et al. 1997). For these data sets, a high density of pulsation signatures convolved with a broad spectral window makes this evaluation of the local noise level a challenge, and we instead interpolate between 4\(A\) values calculated in regions on each side of the frequency range of pulsational power: 0–800 \(\mu\)Hz and 6000–8000 \(\mu\)Hz.

All FTs examined in this work were computed with the PERIOD04 software package (Lenz & Breger 2005). We oversample the spectral resolution by a factor of 20 to obtain representative peak amplitudes. We average the ground-based light curves into 60 s bins (matching the least common multiple of exposure times used on different nights) to avoid giving runs with shorter exposure times undue weight in the FT.

While our primary analysis utilizes the K2 light curves extracted by the EVEREST pipeline, we test the robustness of our signal detections by confirming their presence in the PDC_SAP light curves\textsuperscript{13} produced by the K2 Pipeline (Van Cleve et al. 2016) and the extractions described by Vanderburg & Johnson (2014).

4.1. EPIC 210377280

The FT of the long-cadence K2 light curve of EPIC 210377280 is displayed in the top panel of Figure 3. Our bootstrap calculation yields a 0.1% FAP significance threshold at 0.148% amplitude, which is indicated by the dashed red line. The peak marked with a \(\times\) symbol is within 0.3 \(\mu\)Hz of a typical instrumental artifact at \(\approx 50 \mu\)Hz caused by K2’s thruster firings (Van Cleve et al. 2016), so we exclude it from our analysis. We mark four significant signals of astrophysical interest with colored arrows. Their properties, computed from least-squares fits to the entire K2 light curve, are listed in Table 4.

The spectral window from four nights of ground-based observations of EPIC 210377280 from the McDonald Observatory is displayed in the bottom panel of Figure 3. Any signals in the ground-based data are convolved with this complex aliasing structure in the FT, making it difficult to select the correct pulsation frequencies. The x-axes of both panels of Figure 3 have the same scale, emphasizing the relative imprecision of ground-based signal detections. We aim to use the precise K2 data to guide our selection of the correct alias peaks; in doing so, we determine the number of Nyquist bounces of the K2 signals and recover accurate pulsation frequencies at K2 precision. When the K2 data do not assist our peak selection, we risk choosing the wrong aliases and adopting frequencies that are off by the order of 11.6 \(\mu\)Hz, as discussed in Section 3.4.

The FT of the ground-based observations of EPIC 210377280 in the full range of pulsational power is presented in the top panel of Figure 4. The pink shaded region corresponds to the 0.1% FAP level for the K2 data (dashed line in Figure 3), corrected for the effects of phase smearing (Section 3.2) and passband differences (Section 3.3). This essentially represents the sensitivity of the long-cadence K2 observations to the pulsation signatures measured from the ground; if these light curves were obtained simultaneously, we would expect the signals above the pink shaded region in Figure 4 to rise above the 0.1% FAP threshold in Figure 3. Integer multiples of the Nyquist frequency are marked in the figure with vertical red lines, and the 4\(A\) significance threshold for the ground-based data is displayed as the dashed red line. Notably, the K2 observations would not be sensitive to the majority of significant pulsations that we detect from the ground.

\textsuperscript{12} This is a slightly conservative criterion, since this approach treats astrophysical signals as additional sources of noise.

\textsuperscript{13} https://keplerscience.arc.nasa.gov/k2-data-release-notes.html

**Table 4. Significant K2 Aliases for EPIC 210377280**

| Frequency (\(\mu\)Hz) | Period (s) | Amplitude (%) |
|------------------------|------------|---------------|
| F\(_1\) | 68.569(13) | 14584(3) | 0.17(3) |
| F\(_2\) | 118.297(11) | 8453.3(8) | 0.20(3) |
| F\(_3\) | 122.630(9) | 8154.6(6) | 0.24(3) |
| F\(_4\) | 236.362(13) | 4230.8(2) | 0.17(3) |

**Figure 3.** Top: Fourier transform of the K2 observations of EPIC 210377280 out to the Nyquist frequency with a 0.1% false alarm probability (FAP) significance threshold (see the text). Significant signals are marked with colored arrows. Bottom: the spectral window from four nights of McDonald Observatory observations of EPIC 210377280 over a five-night span in 2016 February. The highest alias is located 11.6 \(\mu\)Hz away from the central peak. The x-axes of both panels have the same scale.
Figure 4. Top: Fourier transform of the McDonald observations of EPIC 210377280 with the K2 sensitivity function overlaid (K2 observations are not sensitive to the shaded region to an FAP of 0.1%). Solid vertical lines mark integer multiples of the K2 Nyquist frequency. The possible intrinsic frequencies and expected amplitudes (corrected for phase smearing and bandpass differences) corresponding to the measured aliases in the K2 FT are indicated with diamond markers (color-coded to match the top panel of Figure 3). Bottom: prewhitening sequence for EPIC 210377280 (progresses left to right, then down). Black triangles mark alias frequency selections supported by K2 observations; unfilled triangles point to peaks selected from ground-based data alone. The red dashed line in all panels shows the 4(A) significance threshold for the ground-based data. The last panel highlights the residual power in our fully prewhitened light curve.
Figure 5. Frequency solution from Table 5 overlaid on the ground-based observations of EPIC 210377280 from McDonald Observatory.

| Mode   | Frequency (μHz) | Period (s) | Amplitude\(^{a}\) |
|--------|-----------------|------------|------------------|
| \(f_1\) | 1255.581(9)     | 796.444(6) | 3.33(14)         |
| \(2f_1\) | 2511.162(19)   | 398.222(3) | 0.64(14)         |
| \(f_2\) | 1513.14(3)     | 660.88(6)  | 1.66(14)         |
| \(f_3\) | 1004.17(13)    | 995.85(13) | 1.62(14)         |
| \(f_4\) | 1059.07(13)    | 944.22(12) | 1.57(14)         |
| \(f_5\) | 1591.54(19)    | 628.32(7)  | 1.11(14)         |
| \(f_6\) | 1872.49(19)    | 534.05(5)  | 1.09(14)         |
| \(f_7\) | 2259.8(2)      | 442.51(4)  | 1.05(14)         |

Notes.

\(^{a}\) Amplitude based on the fit to ground-based data only.

\(^{b}\) Alias matched to the \(K_2\) signal.

\(^{c}\) Harmonic of a previously found signal.

\(^{d}\) Detected in ground-based data alone and may be an incorrect alias of the spectral window (see the text).

Figure 4 also indicates the possible intrinsic frequencies and expected amplitudes corresponding to the significant signals measured in the \(K_2\) data with diamonds that are color-coded to match Figure 3 and Table 4.

After adopting each new frequency from the ground-based data, we use a least-squares optimization in PERIOD04 to refine the overall solution. We then subtract (prewhiten) the best fit and search for additional significant signals in the FT of the residuals. The sequence of FTs in the smaller panels of Figure 4 demonstrates our process of frequency selection. Those signals marked with solid black triangles were informed by the precision \(K_2\) data, while signals marked with white triangles were selected as the highest-amplitude peaks in the ground-based data and may be the incorrect aliases.

The modes are characterized in Table 5 in order of adoption, and this best-fit model is plotted over the McDonald light curve in Figure 5. We refer to frequencies detected in the \(K_2\) data with a capital \(F_n\) (as in Table 4), and our final frequencies as \(f_n\) (Table 5). For frequencies matched to a \(K_2\) signal, we refine the frequency value and uncertainty by doing a final least-squares fit of the correct alias to the \(K_2\) data; otherwise, the values come from least-squares fits to the McDonald light curve. Formal intrinsic uncertainties are quoted following Montgomery & O’Donoghue (1999).

The first small panel of Figure 4 depicts the first significant frequency that we adopt. This is a clear demonstration of the combined strength of these two data sets: one of the highest aliases (but not the highest; see the discussion in Section 5) of the largest signal from the ground-based photometry precisely matches the four-Nyquist-bounce candidate intrinsic signal underlying \(F_3\) in both amplitude and frequency. We fit and prewhiten this signal from the light curve to search for additional signals in the residuals. We also identify the second harmonic of this mode, which is displayed in the second small panel of Figure 4; the exact 2:1 ratio of harmonics informs the selection of the correct peak corresponding to 2\(f_1\), which is the sixth-highest-amplitude alias of that signal. Since \(F_3\) has been positively matched to a ground-based peak, we exclude other possible underlying solutions for this mode in later panels. Our final frequency constraint on \(f_1\) in Table 5 has a total uncertainty of only 0.009 μHz.

The other frequencies that we adopt do not unambiguously resolve \(K_2\) aliases. For instance, \(F_2\) has candidate frequencies close to aliases of both \(f_1\) and \(f_2\). We simply adopt the highest alias peaks associated with each ground-based signal in Table 5. At the amplitudes observed in the McDonald data, many of these would not be detectable at long cadence by \(K_2\) due to amplitude suppression from phase smearing (Section 3.2). A notable example is \(f_4\), which appears very close to \(8 \times F_{Ny}\). An exception to this is \(f_2\), which was most likely not excited to such high amplitude throughout the \(K_2\) observations, as discussed in Sections 3.5 and 5.

The bottom-right panel of Figure 4 displays part of the FT of the final residuals relative to our significance threshold. While a few remaining peaks exceed our adopted significance criterion, alias selection is nontrivial and not attempted.

4.2. EPIC 220274129

EPIC 220274129 is 1.7 mag brighter than EPIC 210377280 and exhibits a greater number of signals that exceed the 0.1% FAP threshold (0.059% amplitude), as displayed in the top panel of Figure 6. These are characterized in Table 6. We are able to exploit the physics of stellar oscillations to recognize relationships between many of the \(K_2\) signals, greatly simplifying the task of matching them to the ground-based data.

Rotation can break the azimuthal degeneracy of spherical harmonic pulsation patterns, splitting a mode of spherical degree \(l\) into 2\(l + 1\) multiplet components of integer azimuthal
order number \(-\ell \leq m \leq \ell\). For the nonradial gravity mode pulsations of white dwarfs, modes of different \(m\) show frequency spacings of

\[
\delta \nu_{\ell m} (\mu \text{Hz}) = (m/P_{\text{rot}}) \times (1 - C_{\ell \ell}),
\]

where \(P_{\text{rot}}\) is the rotation rate (assumed solid body) and \(C_{\ell \ell}\) is the Ledoux constant that describes the effect of the Coriolis force on a particular mode (Ledoux 1951). We use the observational convention that \(m = +1\) is the higher frequency component. In the asymptotic limit of high radial order, \(k\), \(C_{\ell \ell} \approx 1/\ell (\ell + 1)\), but generally \(C_{\ell \ell} \approx 1/\ell (\ell + 1)\).

We identify among the \(K2\) signals three sets of likely rotationally split \(\ell = 1\) triplets that share a similar frequency spacing with a mean of 11.56 \(\mu\)Hz. These are marked with connected arrows in Figure 6. The highest detected frequency is close enough to the Nyquist that it may have reflected one more, or fewer, time than the other components of the \(F_5\) triplet (dashed arrow in Figure 6), so we exclude it from our calculation of the mean frequency splitting. Following the approach of Hermes et al. (2017a), we assume a typical \(C_{\ell \ell} = 0.47\) and a 10\% systematic uncertainty to asteroseismically determine the rotation period of \(EPIC\ 220274129\) to be \(12.73 \pm 1.3\) hr. This period is typical of white dwarfs with measured rotation rates (Kawaler 2015; Hermes et al. 2017a).

The two signals with the lowest frequencies, \(F_1\) and \(2F_1\), are consistent with having a 1:2 ratio. We interpret that these are not super-Nyquist signals and have accurate periods of 24.658 \(\pm 0.017\) and 12.324 \(\pm 0.003\) hr. We discuss the possible nature of these signals at the end of this section, but otherwise exclude them from our search for pulsations in the ground-based data.

The spectral window of our McDonald observations is displayed in the bottom panel of Figure 6. The near coincidence between the location of the highest alias at 11.5 \(\mu\)Hz and the splitting of \(\ell = 1\) multiplets underscores the insensitivity of single-site ground-based observations for asteroseismic measurements of typical white dwarf rotation rates.

Because we have measured the splittings of the \(\ell = 1\) triplets, we can determine the intrinsic frequencies of each set to \(K2\) precision by resolving the super-Nyquist ambiguity of any one component. For triplets that we match to the ground-based data, we are also able to identify which signals correspond to the \(m = +1\) versus \(m = -1\) components.

We exclude from Table 6 a few peaks that reach significant amplitudes that are within 1 \(\mu\)Hz of the higher-amplitude signals associated with \(F_2\) (both \(m = \pm 1\) components) and \(F_6\) (lowest frequency component). These are likely caused by amplitude modulation during the \(K2\) observations distributing the power into a set of closely spaced peaks (Section 3.5). Though it is just below our conservative significance threshold, we include the highest frequency component of \(F_6\) because it is found at the expected location given the rotational frequency splitting observed in other modes.

The top panel of Figure 7 depicts our comparison of possible frequencies underlying the \(K2\) pulsation signals to the FT of the McDonald data in the full range of significant pulsational power. Like Figure 4 for the previous object, the pink shaded region highlights the insensitivity of the \(K2\) data, and the diamonds are color-coded to match Figure 6 and Table 6.

We adopt and prewhiten pulsation signals from the ground-based data in each of the smaller panels. First, we recognize that the largest signal from the ground \((f_1)\) matches a candidate frequency for the highest-amplitude component of the \(F_5\)
Figure 7. Same as Figure 4, except for EPIC 220274129. Mode-by-mode frequency adoption and prewhitening progresses in the sequence of smaller panels from left to right, then top to bottom. The vertical dashed lines and diamonds indicate candidate frequencies and amplitudes underlying the K2 signals, color-coded to match the triangles in Figure 6. Signals marked with black triangles were informed by K2 data, while unfilled triangles mark signals selected solely from the ground-based observations. The last panel highlights residual power in the fully prewhitened light curve.
three components by matches the $\ell$ because there is a ground-based aliasing degeneracy that light curve. These precise frequencies are provided in Table 7.

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Figure 7. Frequency solution overlaid on the ground-based observations of EPIC 220274129 from the McDonald Observatory.

Table 7: Frequency Solution for EPIC 220274129

| Mode       | Frequency ($\mu$Hz) | Period (s) | Amplitude (%) |
|------------|--------------------|------------|---------------|
| $f_1$ ($m = -1)^a$ | 1460.731(6)        | 684.589(3) | n/a           |
| $f_1$ ($m = 0)^b$  | 1472.007(5)        | 679.345(2) | n/a           |
| $f_1$ ($m = +1)^b$ | 1483.276(4)        | 674.1832(17) | 3.62(7) |
| $2f_1$ ($m = -1)^b$ | 2966.553(8)        | 337.992(3)  | 0.64(7)       |
| $f_2$ ($m = -1)^b$ | 1392.117(4)        | 718.331(2)  | 1.92(7)       |
| $f_2$ ($m = 0)^b$  | 1404.099(10)       | 712.200(5)  | n/a           |
| $f_2$ ($m = +1)^b$ | 1415.085(11)       | 706.671(6)  | n/a           |
| $f_2$ ($m = 0)^b$  | 1417.303(11)       | 705.565(5)  | n/a           |
| $f_2$ ($m = +1)^b$ | 2192.95(5)         | 456.006(11) | 1.85(7)       |
| $f_2$ ($m = 0)^b$  | 2875.83(11)        | 347.725(13) | 0.90(7)       |
| $f_2$ $y$        | 2320.67(12)        | 430.91(2)   | 0.81(7)       |
| $f_6$ $y$        | 1457.98(10)        | 685.88(5)   | 0.97(7)       |
| $f_7$ $y$        | 3688.13(16)        | 271.140(11) | 0.63(7)       |
| $f_8$ $y$        | 1315.75(16)        | 760.02(9)   | 0.61(7)       |
| $f_9$ $y$        | 720.89(19)         | 1387.2(4)   | 0.51(7)       |

Notes:

- $^a$ Amplitude based on fit to ground-based data only, where applicable.
- $^b$ Alias refined by the K2 signal.
- $^c$ Harmonic of the previously found signal.
- $^d$ Proximity of the K2 signal to $f_{\text{pul}}$ admits two candidate solutions to this triplet component.
- $^e$ Detected in ground-based data alone and may be incorrect alias of spectral window (see the text).
- $^f$ This peak is suspected to be redundant with the $f_1$ triplet that could not be completely fit out of the ground-based data.

The signals $F_4$ and $2F_1$ were likely detected at their intrinsic frequencies and do not represent independent pulsation frequencies. These were detectable thanks to the extent and high signal-to-noise of the K2 data, and while generally beyond the goals of this paper, are deserving of some discussion. We consider two plausible physical interpretations for these signals: combination frequencies and rotational modulation signatures.

Combination frequencies appear at the sums and differences of ZZ Ceti pulsation frequencies, presumably due to the nonlinear response of the convection zone to pulsations (Brickhill 1992; Wu 2001). The frequencies of the sub-Nyquist signals are similar to the differences between components of $\ell = 1$ rotational triplets.
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The top panel of Figure 9 displays these sub-Nyquist signals in greater detail. The signal near 22.54 μHz actually consists of two closely spaced, significant peaks (with a third compelling but formally insignificant peak nearby). Individual ZZ Ceti pulsation modes that vary in amplitude during observations appear as clusters of closely spaced peaks in Kepler/K2 data (Section 3.5). Hermes et al. (2017a) find that long-period pulsations (periods longer than 600 s) often exhibit these amplitude modulation signatures. Empirically, modes in this period regime begin to be driven to observable amplitude near the effective temperature of EPIC 220274129 (T_{eff,3D} = 11,810 K), which is also approximately where nonlinear combination frequencies reach their largest observed amplitudes in ZZ Cetis (Hermes et al. 2017a). The closely spaced peaks of the lowest frequency component of F3 (intrinsic period 674.183 s) displayed in the bottom panel of Figure 9 are indicative of significant amplitude modulation. Any combination frequencies involving pulsations that are undergoing intrinsic variations will also vary. For example, the differences between the two peaks of the m = +1 component and the m = −1 component of F3 mimic the substructure of the 2F1 signal. While these are compelling candidates for the principle modes of this difference frequency, all ℓ = 1 triplets share similar rotational splittings, including any that were suppressed below our detection threshold by phase smearing. The specific sub-Nyquist structure observed may be the combined signature of multiple difference frequencies in this star.

For ℓ = 1 modes, C_{11} ≤ 0.5, and the separation of rotationally split sectoral (m = ±1) components is >P_{rot} (Equation (2)). While the difference frequency nature of the sub-Nyquist signals is preferred by Occam’s razor, we cannot rule out the interpretation that one of the peaks at 2F1 corresponds to 1/P_{rot}, manifest by some surface inhomogeneity like starspot modulation. This does not fully explain the observed substructure of the 2F1 signal, as we do not expect photometric rotation to show amplitude modulation on month timescales (the spot modulation of the DA white dwarf BOKS 53856 remained stable throughout 89 days of Kepler observations, for instance; Holberg & Howell 2011); therefore, difference frequencies are likely also required to explain the presence of multiple significant peaks near this frequency, as described in the previous paragraph. If the highest-amplitude component of 2F1 could be confirmed as the rotation rate, this would yield a more precise surface rotation period of 12.322 ± 0.003 hr, allowing us to resolve the remaining ambiguity of the m = +1 component of the f3 signal (the higher frequency is required for C_{11} ≤ 0.5). This would also enable asteroseismic tests of radial differential rotation models, as well as empirical constraints on values of C_{11} (as proposed for the spot-modulated, pulsating helium-atmosphere (DB) white dwarf PG 0112+104 by Hermes et al. 2017b). Furthermore, the precise 1:2 ratio of F1 to 2F1 suggests the possible nature of F1 as a subharmonic to the rotation rate. Though such a signal is not expected, EPIC 220274129 could be considered a candidate white-dwarf analog to KIC 10195926, a mAp star with a rotational subharmonic that Kurtz et al. (2011) suggest represents a torsional (r-mode) oscillation.

5. Discussion and Conclusions

Using a few nights of ground-based photometry, we have successfully resolved the frequency ambiguities underlying a few of the super-Nyquist signals in the long-cadence K2 observations of two new ZZ Ceti pulsating white dwarfs, EPIC 210377280 and EPIC 220274129. While these data sets are individually nonideal for asteroseismic measurements, they complement each other such that we can measure pulsation frequencies to a precision of ~0.01 μHz. We used a similar approach in Hermes et al. (2017c) to confirm the super-Nyquist nature of a rotationally split ℓ = 1 triplet centered on 109.15103s in the short-cadence K2 light curve of the ZZ Ceti star EPIC 211914185, but this work is the first example of recovering frequencies beyond four to five times the Nyquist.

By using the K2 data to resolve the ground-based aliasing, we have demonstrated the difficulty in selecting the correct peaks from single-site observations with diurnal gaps. The K2 data selected the second most probable alias of f1 from the FT of McDonald data on EPIC 210377280, rather than the marginally higher-amplitude peak. For the K2-informed second harmonics of f1 in both stars, the correct peaks do not have the highest amplitudes locally. Frequency selection from <1 week of single-site ground-based data alone can be off by many times the daily aliasing of ~11.6 μHz, as many of the frequencies in our solutions certainly are. This error can be much larger than the offset from assuming the wrong azimuthal order, m, which Metcalfe (2003) found to affect but not negate the usefulness of asteroseismic investigations. Prewhitenin by incorrect aliases can also adversely impact the properties of pulsation modes later inferred from the residuals. Any comparison of asteroseismic models to data with gaps should ideally consider the many viable combinations of aliases that can describe the light curves.

Our identification of rotationally split ℓ = 1 triplets in EPIC 220274129 enabled us to asteroseismically infer the stellar rotation period of 12.7 ± 1.3 hr. We also observe sub-Nyquist signals in this star that likely correspond to nonlinear
difference frequencies between different components of the same $\ell = 1$ multiplets.

Besides those pulsation signals that we were able to refine to $K2$ precision by positively matching between data sets, we identify $\geq 6$ additional pulsation signatures from the ground-based light curves of each star. These are measured to lower precision and may be inaccurate aliases from the spectral window, but they will still be useful for asteroseismically constraining the interior structures of these rich ZZ Ceti pulsators. The $K2$ data would not have been sensitive to most of these at their observed amplitudes.

There are also significant signals in the $K2$ data that we were unable to confidently match to the McDonald observations. As discussed in Section 3.5, pulsation amplitudes of ZZ Ceti stars are unable to constrain the interior structures of these rich ZZ Ceti pulsators. The $K2$ data would not have been sensitive to most of these at their observed amplitudes.

It is possible that the difference between the pseudo-Nyquist frequency and true Nyquist behavior (described in Section 3.1) could be exploited to place additional constraints on the intrinsic frequencies of the $K2$ signals as in Murphy et al. (2013, applied to Kepler data). This would provide an important confirmation to our frequency solutions and enable the precise identification of the $K2$ aliases that we did not match in our ground-based data. The addition of these precise frequencies to the solutions for these stars would increase our leverage on asteroseismically constraining their interiors. The sensitivity of this method to $K2$ data, where light-curve durations are much shorter than a spacecraft orbit, has not yet been demonstrated (S. Murphy 2017, private communication).

The spectra and light curves of EPIC 210377280 and EPIC 220274129 are available online at http://www.k2wd.org, alongside the published data for short-cadence ZZ Cetis observed with Kepler and $K2$ (Herms et al. 2017a).

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