QUASI–SELF-SIMILAR EVOLUTION OF THE TWO-POINT CORRELATION FUNCTION: STRONGLY NONLINEAR REGIME IN $\Omega_0 < 1$ UNIVERSES

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ABSTRACT

The well-known self-similar solution for the two-point correlation function of the density field is valid only in an Einstein–de Sitter universe. We attempt to extend the solution for non–Einstein–de Sitter universes. For this purpose we introduce an idea of quasi–self-similar evolution; this approach is based on the assumption that the evolution of the two-point correlation is a succession of stages of evolution, each of which spans a short enough period to be considered approximately self-similar. In addition, we assume that clustering is stable on scales where a physically motivated “virialization condition” is satisfied. These assumptions lead to a definite prediction for the behavior of the two-point correlation function in the strongly nonlinear regime. We show that the prediction agrees well with $N$-body simulations in non–Einstein–de Sitter cases and discuss some remaining problems.

Subject headings: cosmology: theory — gravitation — large-scale structure of universe — methods: $n$-body simulations

1. INTRODUCTION

It is generally believed that the structure in our universe has grown out of tiny density fluctuations through gravitational instability. The growth of those density fluctuations is completely described by linear theory on scales much larger than the correlation length of the density field (e.g., Peebles 1980). The behavior of the nonlinear density field on smaller scales, however, needs to be modeled with additional assumptions, and the resulting predictions should be verified and calibrated through extensive comparison with $N$-body simulations.

The most popular prediction in the nonlinear regime is based on the combination of self-similar evolution and the stable clustering Ansatz (Peebles 1974, 1980; Davis & Peebles 1977); if the linear power spectrum of the density field follows a single power law $\propto k^n$, and the universe is described by the Einstein–de Sitter model, the evolution proceeds in a self-similar manner, since there is no characteristic scale in the system. This self-similarity, together with the Ansatz that the clustering is stable in the strongly nonlinear regime, predicts that the logarithmic slope of the two-point correlation function $\xi$ is equal to $-3(n + 3)/(n + 5)$.

This solution has been widely applied in modeling the nonlinear gravitational clustering (Hamilton et al. 1991; Peacock & Dodds 1994, 1996; Jain, Mo, & White 1995) and in understanding the pairwise velocity dispersions, and thus the redshift-space distortion (Suto & Jing 1997; Jing, Mo, & Börner 1998). In fact, the above prescription has been applied even in cases where the universe is not described by the Einstein–de Sitter model and/or the linear power spectrum is not of a power-law form. The fitting formula for the nonlinear power spectrum by Peacock & Dodds (1996), for instance, takes into account the non–power-law nature of the linear spectrum, but they have not examined the validity of the stable clustering solution in non–Einstein–de Sitter models because “if collapse occurs at high redshift, then $\Omega = 1$ may be assumed at that time” (Peacock & Dodds 1994).

We revisit this issue in detail for the non–Einstein–de Sitter case, i.e., when the matter density parameter $\Omega_0$ is smaller than unity. Using $N$-body simulations with $64^3$ particles, Suginohara et al. (1991) already found that while the stable solution is reproduced well in the Einstein–de Sitter model, the slope of $\xi$ becomes steeper in $\Omega_0 < 1$ models (see also Suto 1993). This is not at all surprising because the derivation of the solution relies heavily on the scale-free nature of the Einstein–de Sitter model; that is, (1) $a \propto t^{2/3}$, where $a$ is the cosmic scale factor, and (2) $D(t) \propto a$, where $D(t)$ is the linear growth rate of density fluctuations. This motivates us to find a suitable modification of the self-similar solution so that it may also be to the non–Einstein–de Sitter case.

In this paper we introduce an idea of quasi–self-similarity. This is based on the assumption that the evolution of the two-point correlation proceeds as a sequence of different quasi–self-similar stages, each of which is described by the locally self-similar solution determined by the cosmological parameters at that epoch. We also assume that the clustering becomes stable on scales where a “virialization condition” is satisfied. These two assumptions lead to a prediction of the behavior of the two-point correlation function in the strongly nonlinear regime. We show the extent to which the prediction agrees with the results of high-resolution $N$-body simulations.

2. QUASI–SELF-SIMILAR EVOLUTION

2.1. The Self-Similar Solution in the Einstein–de Sitter Model

For later convenience, we first briefly review the self-similar solution in the Einstein–de Sitter case (Peebles 1974, 1980; Davis & Peebles 1977). Suppose a system of particles with mass $m$. In the fluid limit, the system is described by the one-particle distribution function $f(x, p, t)$, where $x$ is the comoving coordinate, and $p = ma^2dx/dt$ is its canonical momentum. The distribution function $f(x, p, t)$ obeys the...
Vlasov equation (Inagaki 1976; Peebles 1980):
\[
\frac{\partial f}{\partial t} + \frac{p_i}{m a^2} \frac{\partial f}{\partial x_i} = m \frac{\partial \phi}{\partial x_i} \frac{\partial f}{\partial p_i} = 0 ,
\]
where the gravitational potential \( \phi \) satisfies
\[
\nabla^2 \phi = 4\pi Gma^{-1} \int f d^3 p .
\]

In the Einstein–de Sitter model \((a \propto t^{2/3})\), equation (1) admits a self-similar solution of the form
\[
f(x, p, t) = t^{-3\xi - 1} f\left(\frac{x}{t^2}, \frac{p}{t^{2+1/3}}\right) .
\]
The value of the parameter \( \alpha \) is determined by matching the solution in a linear regime as follows. First, note that equation (3) implies the following form for the two-point correlation function \( \xi(x, t) \):
\[
\xi(x, t) = \hat{\xi}(x/t^\alpha) .
\]
If the initial power spectrum of density fluctuations, \( P_{\text{initial}}(k) \), is proportional to \( k^n \), then \( \xi(x, t) \) at later epochs behaves as \( x^{-(n+3)\alpha + 1/3} \) on large scales where linear theory is valid. Thus, the value of \( \alpha \) is explicitly specified as
\[
\alpha = \frac{4}{3(n + 3)} .
\]

In the small-scale (nonlinear) limit, it is often assumed that the average proper separation of pairs remains constant (the stable clustering hypothesis):
\[
\langle v_{21}(x, t) \rangle = -a \dot{\xi} ,
\]
where \( v_{21} \) is the relative peculiar velocity for a pair, and the brackets denote the average over pairs at a given comoving separation \( x \). This assumption with equation (4) fixes the behavior of the correlation function in the nonlinear limit. Substituting equation (6) into the equation of the particle pair conservation,
\[
\frac{\partial \xi}{\partial t} + \frac{1}{ax^2} \frac{\partial}{\partial x} \left[ x^2 (1 + \xi) \langle v_{21}(x, t) \rangle \right] = 0 ,
\]
yields
\[
\frac{\partial \xi}{\partial t} = -a \frac{1}{x^2} \frac{\partial}{\partial x} (x^3 \xi) ,
\]
for \( \xi \gg 1 \). Thus, \( \xi \) in the nonlinear regime should be of the form
\[
\xi = a^n \xi(ax) ,
\]
where \( g \) is an arbitrary function at this point. Finally, the consistency with the above similarity solution requires that \( \xi \) should be given by the following power-law form:
\[
\xi(x, t) \propto x^{-3(n+3)/(n+5)} a^{4/(n+5)} \]
\[
\propto x^{-3(n+3)/(n+5)} a^{6/(n+5)} .
\]

2.2. Quasi–Self-Similar Clustering Solutions

Now we attempt to generalize the above self-similar solution in a hypothetical universe where \( a \propto t^p \) and \( D(t) \propto t^q \). In non–Einstein–de Sitter models, both \( p \) and \( q \) are not constant but change with time. As long as the timescale of the change of \( p \) and \( q \) is smaller than that of the cosmic expansion, however, they can be regarded as constants for a short period at each epoch.

We begin with the assumption of similarity for \( \xi(x, t) \):
\[
\xi(x, t) = \hat{\xi}(x/t^\alpha) .
\]
Unlike the Einstein–de Sitter case, any kind of self-similarity does not hold in a strict sense. However, the range of \( p \) and \( q \) considered here is close to that in the Einstein–de Sitter case, so it seems reasonable to assume that some form of self-similarity is realized in an approximate sense. The simplest possibility is that, as in the previous subsection, \( \xi(x, t) \) has the form given by equation (11). With this Ansatz we repeat the same procedure in the previous subsection, adopting \( P_{\text{initial}}(k) \propto k^n \) and the stable clustering hypothesis in the strongly nonlinear regime.

In this case, one has \( \xi \propto x^{-(n+3)\alpha + 2\alpha} \) in linear regime, and equation (5) is replaced by
\[
\alpha = \frac{2q}{n + 3} .
\]
Combining equation (9) for \( \xi \) in the strongly nonlinear regime yields, instead of equation (10),
\[
\xi(x, t) \propto x^{-3(n+3)/(n+3+2f)/a^{4/(n+3+2f)}}
\]
\[
= (ax)^{-3(n+3)/(n+3+2f)} a^3 ,
\]
for \( \xi \gg 1 \). The quantity \( f \equiv q/p \) in the above expression is in fact the familiar logarithmic derivative of \( D \) with respect to \( a \). An excellent approximation to \( f \) is (Lahav et al. 1991)
\[
f = \frac{d \ln D}{d \ln a} \sim \Omega(a)^{0.6} + \frac{\lambda(a)}{70} \left[ 1 + \frac{\Omega(a)}{2} \right] ,
\]
where
\[
\Omega(a) = \frac{\Omega_0}{\Omega_0 + (1 - \Omega_0 - \lambda_0)(a/a_0) + \lambda_0(a/a_0)^3}
\]
and
\[
\lambda(a) = \frac{\lambda_0(a/a_0)^3}{\Omega_0 + (1 - \Omega_0 - \lambda_0)(a/a_0) + \lambda_0(a/a_0)^3} ,
\]
with \( \lambda_0 \) being the dimensionless cosmological constant at the present epoch \( a_0 \).

2.3. Comparison With N-Body Simulations

Let us compare in detail the self-similar solution (eq. [10]) and the quasi–self-similar solution (eq. [13]) with high-resolution N-body simulations (Jing 1998, 2002, in preparation). The simulations consist of two Einstein–de Sitter models, two open models \((\Omega_0 = 0.1, \lambda_0 = 0)\), and two spatially flat models \((\Omega_0 = 0.1, \lambda_0 = 0.9)\). All the models employ scale-free initial power spectra \( P_{\text{initial}}(k) \propto k^n \) with \( n = -1 \) and \( n = -2 \). Gravitational force calculation is based on the particle-particle-particle-mesh (P³M) algorithm. Simulation parameters are listed in Table 1, and further details of the simulations are described in Jing (1998, 2002 in preparation).

Figure 1a plots the two-point correlation functions that are appropriately scaled with respect to \( x \) according
TABLE 1

| Simulation Parameters | \( n \) | \( N^a \) | \( \epsilon_{grav}/L_{box}^b \) |
|-----------------------|--------|--------|-----------------|
| \((\Omega_0, \lambda_0)\) |        |        |                 |
| \((1.0, 0.0)\).........| \(-1\) | \(-2\) | \(256^4\) \(5.9 \times 10^{-4}\) |
| \((0.1, 0.0)\).........| \(-1\) | \(-2\) | \(200^4\) \(7.5 \times 10^{-4}\) |
| \((0.1, 0.9)\).........| \(-1\) | \(-2\) | \(200^4\) \(7.5 \times 10^{-4}\) |

* Number of particles.

b Gravitational softening length in units of the box size \(L_{box}\).

This comparison points to the following two suggestions: (1) in \(\Omega_0 < 1\) models the conventional self-similar solution fails to describe the behavior of \(\xi\) for \(\xi \geq 100\), and (2) taking into account the dependence of the slope on the cosmological parameters, as in equation (13), does not yet yield an acceptable prediction. With these points in mind, we attempt to improve the quasi–self-similar solution in the next section.

3. AN IMPROVED MODEL

3.1. Virialization Condition and Scale Dependence of the Slope of \(\xi\)

In the previous section we have applied the quasi–self-similarity only at the present epoch. In reality, however, the logarithmic slope of the correlation function,

\[
\frac{d \ln \xi}{d \ln x} = -\frac{3(n + 3)}{n + 3 + 2f(\alpha_{vir}(x))},
\]

should be fixed locally at the epoch of the virialization of the corresponding scale, \(\alpha_{vir}(x)\). This naturally generates additional scale-dependence on the resulting solution through the time-dependence of \(f\). More specifically, we attempt to incorporate this effect and to improve the model in the previous subsection as follows.

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Fig. 1.—Two-point correlation functions in the simulations scaled according to (a) conventional self-similar solution and (b) quasi–self-similar solution with scale-independent slope. The solid lines are the results for \(\Omega_0 = 1\) models, and the symbols are those for \(\Omega_0 = 0.1\) models with either \(\lambda_0 = 0\) or \(\lambda_0 = 0.9\). Error bars are estimated from three realizations for each model; only the error bars for \(n = -2\) and \((\Omega_0, \lambda_0) = (0.1, 0.9)\) are shown. Other models have smaller error bars, which are not shown for clarity. The arrows correspond to \(\xi(x) = 100\) for each model.
In order to determine \( a_{\text{vir}}(x) \) at each scale \( x \), we need to make an assumption about the scale on which the system has been virialized at a given epoch \( a \). Here, we mainly adopt the following assumption: the system has just been virialized at a scale \( x_{\text{vir}}(a) \), where the volume average of the two-point correlation function reaches the critical overdensity of a virialized halo, \( \Delta_{\text{vir}}(a) \), predicted by the spherical collapse model,

\[
\xi(x_{\text{vir}}(a)) = \Delta_{\text{vir}}(a),
\]

where

\[
\xi(x) = \frac{3}{\pi^2} \int_0^x y^2 \xi(y) \, dy.
\]

It seems natural to use \( \xi \) rather than \( \xi \) for the present purpose because the right-hand side of equation (18) is the average density of a halo that has just been virialized. In \( \lambda_0 = 0 \) models, the critical density \( \Delta_{\text{vir}}(a) \) is explicitly written as (Lacey & Cole 1993)

\[
\Delta_{\text{vir}}(a) = 4\pi^2 \left( \frac{\cosh \eta - 1}{\sinh \eta - \eta} \right)^2,
\]

\[
\cosh \eta = \frac{2}{\Omega(a)} - 1,
\]

and its accurate fitting formula in \( \lambda_0 = 1 - \Omega_0 \) models (Nakamura & Suto 1997) is given by

\[
\Delta_{\text{vir}}(a) \approx 18\pi^2 \left\{ 1 + 0.4093 \left[ \frac{1}{\Omega(a)} - 1 \right]^{0.9052} \right\}.
\]

The “virialization condition” (eq. [18]) may be somewhat arbitrary, but it is perhaps the most natural and physically motivated among other choices.

Furthermore, we assume that, for \( x \leq x_{\text{vir}}(a) \), the stable clustering condition is satisfied, i.e., \( \xi(x, a) \) is of the form of equation (9). This is equivalent to saying that, at a fixed physical scale \( r = ax \), the slope of \( \xi \) is kept constant while its amplitude grows in proportion to \( a^2 \) (cf. eq. [13]).

The above assumptions are not yet sufficient in predicting \( \xi(x) \) for a given model. The amplitude of \( \xi \) at an arbitrary point needs to be specified by hand because our model does not predict the overall amplitude of \( \xi(x) \). Once the value of \( \xi \) is fixed at a scale \( x = \epsilon \) [for example, in the next subsection, we take \( \epsilon_{\text{grav}} \), the gravitational softening length, as \( \epsilon \) and use the value of \( \xi(\epsilon_{\text{grav}}) \) in the simulations], we can compute \( \xi(x) \) at \( a = a_0 \) up to \( x_{\text{vir},0} = x_{\text{vir}}(a_0) \). The procedure is as follows:

1. Set boundary condition. Below an extremely small-scale \( x_i \), where \( \xi \) is sufficiently large, our model prediction should reduce to the conventional self-similar solution in the Einstein–de Sitter universe, i.e.,

\[
\xi \propto x^{-3(n + 3)/(n + 5)} \quad \text{for} \quad x \leq x_i.
\]

This is because the epoch of the virialization of the corresponding scale is sufficiently early; at that epoch the universe is indistinguishable from the Einstein–de Sitter model (Peacock & Dodds 1994). Equation (23) implies that

\[
\xi(x_i) = \frac{(n + 5)}{2} \xi(x_i).
\]

We first choose \( x_i \) arbitrarily and set the value of \( \xi(x_i) \) to be sufficiently large; say, \( 10^9 \). This is the starting point of our computation.

2. Compute \( a_{\text{vir}}(x) \). From the above assumptions it follows that

\[
\bar{\xi}(x) = \left[ \frac{a_{\text{vir}}(x)}{a_0} \right]^{-3} \Delta_{\text{vir}}(a_{\text{vir}}(x)).
\]

At the current scale \( x \), we solve equation (25) for \( a_{\text{vir}}(x) \).

3. Advance \( x \). Substituting the value of \( a_{\text{vir}}(x) \) into equation (17) gives the local slope \( d\ln \xi/d\ln x \) at \( x \). Then we can advance \( x \) by a small interval \( \Delta x \) and compute \( \xi(x + \Delta x) \), and then \( \xi(x + \Delta x) \).

4. Repeat procedures 2 and 3 until \( \xi(x) \) becomes equal to \( \Delta_{\text{vir}}(a_0) \), which corresponds to the present virialization scale \( x_{\text{vir,0}} \).

5. Normalization.—Finally, we shift the resulting solution so that it can match the given amplitude at \( x = \epsilon \), using the fact that, if \( \xi^{(0)}(x) \) is a solution to equation (17), so is \( \xi^{(0)}(x\epsilon) \), with an arbitrary constant \( \epsilon \).

3.2. Comparison With N-Body Simulations

Figure 2 compares our improved model predictions with the N-body results. Using the scaling relation described above, we match the amplitude of our solution to that of the simulations at \( x = \epsilon_{\text{grav}} \) for each model (the innermost symbols). The length scale \( x \) is normalized by \( x_{\text{vir},0} = x_{\text{vir}}(a_0) \), and we show the results only in the virialized regime, \( x < x_{\text{vir},0} \). Clearly, our predictions are in good agreement with simulations for \( \xi \gtrsim 200 \) in all models. In particular, the predicted dependence on the spectral index \( n \) is excellently reproduced in the simulation results. Given the simplicity of our procedure, this can be regarded as a considerable success. For \( \xi < 200 \), however, the slopes of our predicted correlation functions are shallower than those of simulations (Figs. 2a and 2b).

Let us compare our model predictions with the conventional self-similar solution (eq. [10]) in more detail. The power-law slope of the conventional self-similar solution is shown by the dot-dashed lines in Figure 2. In \( \Omega_0 = 0.1 \) and \( \lambda_0 = 0.9 \) cases, both our model and the simulation have roughly the same slope as the conventional one near \( x = \epsilon_{\text{grav}} \), but in both of them the slope becomes steeper as the scale approaches \( x_{\text{vir},0} \). More impressively, in \( \Omega_0 = 0.1 \) and \( \lambda_0 = 0.0 \) cases, both our model and the simulation have a steeper slope on all scales shown in the figure than the conventional one.

Also shown in Figure 2 are the correlation functions obtained by Fourier transforming the fitting formulae for the nonlinear power spectra by Peacock & Dodds (1996). The Peacock-Dodds formula is extremely useful because it gives not only the shape but also the amplitude of the two-point correlation function from linear to strongly nonlinear regimes. The agreement between the Peacock-Dodds formula and the simulations is very good except for \( \Omega_0 = 0.1 \) and \( \lambda_0 = 0 \) models. In these cases, the Peacock-Dodds formula systematically underpredicts the simulation results. Nevertheless, this could be adjusted somehow by shifting the amplitude of the Peacock-Dodds formula so as to match the simulation at \( x = \epsilon_{\text{grav}} \) (see Fig. 2, dotted lines). Rather an important advantage of our model over the Peacock-Dodds prescription is that it does successfully predict the slope of \( \xi \) up to a scale where deviation from the
conventional one, $-3(n + 3)/(n + 5)$, is significant. Note that the slope of $\xi$ computed from the Peacock-Dodds formula is, on scales where it deviates from $-3(n + 3)/(n + 5)$, simply an interpolation from the numerical simulations. In this sense, our result implies that there is room for further improvement in the original Peacock-Dodds formula, and our present model may be useful for that purpose.

One may wonder if it is possible to improve our model predictions by somehow varying the condition given by equation (18). Figures 3 and 4 present those results. The top panels in those figures adopt $\Delta_{m,0}^2$ instead of $\Delta_{m,0}$ (Equation 18). While the agreement between the quasi-self-similar prediction and the simulations is indeed improved for models, this is not the case for models.

The middle and bottom panels show the results for $n = -1$ and $n = -2$ (Equation 18). In each case we obtain acceptable agreement for both $n = -1$ and $n = -2$ models simultaneously, although the modified conditions lose the physical basis and should be regarded as empirical at best.

### 3.3. Validity of the Stable Clustering Hypothesis

The stable clustering hypothesis is an essential ingredient in our model, but it has been somewhat in doubt in recent literature (Yano & Gouda 2000; Ma & Fry 2000; Caldwell et al. 2001). We argue here, nevertheless, that the hypothesis still remains a reasonable assumption.

Yano & Gouda (2000) relate the inner density profile of virialized halos with the velocity parameter $h = -(v_{2,1}^2)/a$. They claim that $h$ should approach zero in the nonlinear limit if the logarithmic slope of the inner density profile is larger than $-3/2$. As pointed out by Ma & Fry (2000), their argument is valid only when all halos have an equal mass,
and should be completely altered for a realistic mass function.

On the basis of simulations by Jain (1997), Caldwell et al. (2001) find for a variety of cosmological models a universal relation between $f^2$ and $h$ and propose a fitting formula describing the relation. Although extrapolating this formula implies that $h \to 0$ in the nonlinear limit, they claim that their formula is valid for $f^2 \lesssim 10^3$. Thus, their results are not inconsistent with the idea that $h \sim 1$ in the strongly nonlinear regime. In fact, more recent numerical work supports the stable clustering assumption (Jing 2001; Fukushige & Suto 2001).

4. DISCUSSION

We have shown that the conventional self-similar solution in the Einstein–de Sitter universe does not describe the behavior of two-point correlation functions in $\Omega_0 \neq 1$ models for strongly nonlinear regimes of cosmological interest, $\xi < 10^4$. Instead, we have proposed a simple model to describe the two-point correlation functions in strongly nonlinear regimes by introducing the quasi–self-similar Ansatz. In fact, we have shown that the resulting model predictions in non–Einstein–de Sitter universes agree better with the high-resolution $N$-body simulations.

On the other hand, our current model is not fully successful yet, in the sense that the predicted behavior for $\xi < 200$ systematically differs from that observed in simulations. Empirically, this situation can be improved by an appropriate choice of the virialization threshold $\xi_{\text{vir}}$. While the physical meaning of this procedure is not clear, this may be related to some other physics that we omit in the present simple prescription. After all, the regime for $\xi < 200$ may not be completely dominated by the stable clustering evolution, and the effect of linear and quasi-linear evolution is likely to be important as well. In fact, it may be the case that the stable condition (eq. [6]) is not realized instantaneously when the virial condition (eq. [18]) is satisfied, and that the slope at the corresponding scale approaches the value
implied by equation (17) only gradually as the scale becomes more strongly nonlinear (for instance, $\xi > 200$) and completely decoupled from the linear evolution of the environment. We plan to check this interpretation using the time evolution of the simulation results in due course.

One of the most important applications of the conventional self-similar solution is the fitting formula for the nonlinear power spectrum by Peacock & Dodds (1996), which is based on an idea originally proposed by Hamilton et al. (1991). In the strongly nonlinear regime, on which we focus in this paper, the Peacock-Dodds formula agrees well with and also for and $\lambda_0 > 0$. We have found, however, that in $\lambda_0 < 0$ cases the formula fails to fit the simulations. This can be understood as follows. In such cases, the slope of $\xi$ asymptotically approaches that in conventional self-similar solution only on scales where $\xi$ is extremely large. This means that the gap between these asymptotic scales and linear scales is rather large, and it cannot be simply interpolated. In contrast, our model predicts the shape of $\xi$ up to the scale corresponding to $\xi \sim a$ few hundred, so interpolation between this scale and the linear scale may be much easier. Thus, our model may be useful in improving the Peacock-Dodds formula, especially in $\Omega_0 < 1$ and $\lambda_0 = 0$ cases.

In summary, although our proposed model still needs to be improved, the degree of success for $\xi \gtrsim 200$ is encouraging and useful, at least empirically. We hope that the idea of quasi-self-similar evolution may give a useful insight toward a better understanding of nonlinear dynamics of the mass density field in the universe. In particular, we plan to improve the existing fitting formula for the nonlinear power spectrum (Peacock & Dodds 1994, 1996) by applying the quasi-self-similar Ansatz.

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