GEOMETRICAL SCALING IN HIGH ENERGY COLLISIONS AND ITS BREAKING*

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We analyze geometrical scaling (GS) in Deep Inelstic Scattering at HERA and in pp collisions at the LHC energies and in NA61/SHINE experiment. We argue that GS is working up to relatively large Bjorken $x \sim 0.1$. This allows to study GS in negative pion multiplicity $p_T$ distributions at NA61/SHINE energies where clear sign of scaling violations is seen with growing rapidity when one of the colliding partons has Bjorken $x \geq 0.1$.

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1. Introduction

In this short note, following Refs. [1–5] where also an extensive list of references can be found, we will focus on the scaling law, called geometrical scaling (GS), which has been introduced in the context of DIS [6]. Recently, it has been shown that GS is also exhibited by the $p_T$ spectra at the LHC [1–3]. An onset of GS in heavy ion collisions at RHIC energies has been reported in Ref. [3]. At low Bjorken $x < x_{\text{max}}$, proton is characterized by an intermediate energy scale $Q_s(x) =$ called saturation scale [7, 8] — defined as the border line between dense and dilute gluonic systems within a proton (for review, see e.g. Refs. [9, 10]). For the present study, however, the details of saturation are not of primary interest, it is the very existence of $Q_s(x)$ which is of importance.

Here, we present analysis of three different pieces of data which exhibit both emergence and violation of geometrical scaling. In Sect. 2 we briefly describe the method used to assess the existence of GS. Secondly, in Sect. 3 we describe our recent analysis [4] of combined HERA data [11] where it has

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been shown that GS in DIS works very well up to relatively large $x_{\text{max}} \sim 0.1$ (see also [12]). Next, in Sect. 4, on the example of the CMS $p_T$ spectra in central rapidity [13], we show that GS can be extended to hadronic collisions. For particles produced at non-zero rapidities, one (larger) Bjorken $x = x_1$ may leave the domain of GS, i.e. $x_1 > x_{\text{max}}$, and violation of GS should appear. In Sect. 5 we present analysis of very recent $pp$ data from NA61/SHINE experiment at CERN [14] and show that GS is indeed violated once rapidity is increased. We conclude in Sect. 6.

2. Method of ratios

Geometrical scaling hypothesis means that some observable $\sigma$ that, in principle, depends on two independent kinematical variables, say $x$ and $Q^2$, in fact, depends only on a specific combination of them denoted as $\tau$

$$\sigma(x, Q^2) = F(\tau)/Q_0^2. \quad (1)$$

Here, function $F$ in Eq. (1) is a dimensionless function of scaling variable

$$\tau = Q^2/Q_s^2(x) \quad (2)$$

and

$$Q_s^2(x) = Q_0^2 (x/x_0)^{-\lambda} \quad (3)$$

is the saturation scale. Here, $Q_0$ and $x_0$ are free parameters which can be extracted from the data within some specific model for $\sigma$, and exponent $\lambda$ is a dynamical quantity of the order of $\lambda \sim 0.3$. Throughout this paper, we shall test the hypothesis whether different pieces of data can be described by formula (1) with constant $\lambda$, and what is the kinematical range where GS is working satisfactorily.

In view of Eq. (1), observables $\sigma(x_i, Q^2)$ for different $x_i$’s should fall on one universal curve, if evaluated not in terms of $Q^2$ but in terms of $\tau$. This means, in turn, that ratios

$$R_{x_i, x_{\text{ref}}}(\lambda; \tau_k) = \frac{\sigma(x_i, \tau(x_i, Q_k^2; \lambda))}{\sigma(x_{\text{ref}}, \tau(x_{\text{ref}}, Q_{k, \text{ref}}^2; \lambda))} \quad (4)$$

should be equal to unity independently of $\tau$. Here, for some $x_{\text{ref}}$, we pick up all $x_i < x_{\text{ref}}$ which have at least two overlapping points in $Q^2$.

For $\lambda \neq 0$, points of the same $Q^2$ but different $x$’s correspond, in general, to different $\tau$’s. Therefore, one has to interpolate $\sigma(x_{\text{ref}}, \tau(x_{\text{ref}}, Q^2; \lambda))$ to $Q_{k, \text{ref}}^2$ such that $\tau(x_{\text{ref}}, Q_{k, \text{ref}}^2; \lambda) = \tau_k$. This procedure is described in detail in Refs. [4].
By tuning $\lambda$, one can make $R_{x_i,x_{\text{ref}}}(\lambda; \tau_k) \to 1$ for all $\tau_k$. In order to find optimal value $\lambda_{\text{min}}$ that minimizes deviations of ratios (4) from unity, we form the chi-square measure

$$
\chi^2_{x_i,x_{\text{ref}}} (\lambda) = \frac{1}{N_{x_i,x_{\text{ref}}}} \sum_{k \in x_i} \left( \frac{R_{x_i,x_{\text{ref}}}(\lambda; \tau_k) - 1}{\Delta R_{x_i,x_{\text{ref}}}(\lambda; \tau_k)} \right)^2 ,
$$

(5)

where the sum over $k$ extends over all points of given $x_i$ that have overlap with $x_{\text{ref}}$, and $N_{x_i,x_{\text{ref}}}$ is a number of such points.

### 3. Deep Inelastic Scattering at HERA

In the case of DIS, the relevant scaling observable is $\gamma^* p$ cross section and variable $x$ is simply Bjorken $x$. In Fig. 1 we present 3d plot of $\lambda_{\text{min}}(x,x_{\text{ref}})$ which has been found by minimizing (5).

![Fig. 1. Three dimensional plot of $\lambda_{\text{min}}(x,x_{\text{ref}})$ obtained by minimization of Eq. (5).](image)

Qualitatively, GS is given by the independence of $\lambda_{\text{min}}$ on Bjorken $x$ and by the requirement that the pertinent value of $\chi^2_{x,x_{\text{ref}}} (\lambda_{\text{min}})$ should be small (for the discussion of the latter, see Refs. [4]). We see from Fig. 1 that the stability corner of $\lambda_{\text{min}}$ extends up to $x_{\text{ref}} \lesssim 0.1$, which is well above the original expectations. In Refs. [4] we have shown that

$$
\lambda = 0.32 - 0.34 \quad \text{for} \quad x \leq 0.08 .
$$

(6)
4. Central rapidity $p_T$ spectra at the LHC

In hadronic collisions at c.m. energy $W = \sqrt{s}$ particles are produced in the scattering process of two patrons carrying Bjorken $x$’s

$$x_{1,2} = e^{\pm y} \frac{p_T}{W}.$$  

For central rapidities, $x = x_1 \sim x_2$. It has been shown that in this case charged particle multiplicity spectra exhibit GS [1]

$$\frac{dN}{dy d^2 p_T} \bigg|_{y \geq 0} = \frac{1}{Q_0^2} F(\tau),$$

where $F$ is a universal dimensionless function of the scaling variable

$$\tau = \frac{p_T^2}{Q_s^2(x)} = \frac{p_T^2}{Q_0^2} \left( \frac{p_T}{(x_0 \sqrt{s})} \right)^\lambda.$$  

Therefore, the scaling observable is $\sigma(W, p_T^2) = dN/dy d^2 p_T$ and the method of ratios is applied to the multiplicity distributions at different energies ($W_i$ taking over the role of $x_i$ in Eq. (4)). For $W_{\text{ref}}$, we take the highest LHC energy of 7 TeV. Therefore, one can form two ratios $R_{W_i, W_{\text{ref}}}$ with $W_1 = 2.36$ and $W_2 = 0.9$ TeV. These ratios are plotted in Fig. 2 for the CMS single non-diffractive spectra for $\lambda = 0$ and for $\lambda = 0.27$, which minimizes (5) in this case. We see that original ratios plotted in terms of $p_T$ range from 1.5 to 7, whereas plotted in terms of $\sqrt{\tau}$ they are well concentrated around unity. The optimal exponent $\lambda$ is, however, smaller than in the case of DIS. Why this is so, remains to be understood.

![Fig. 2. Ratios of CMS $p_T$ spectra [13] at 7 TeV to 0.9 (blue circles) and 2.36 TeV (red triangles) plotted as functions of $p_T$ (left) and scaling variable $\sqrt{\tau}$ (right) for $\lambda = 0.27$.](image-url)
5. Violation of geometrical scaling in forward rapidity region

For $y > 0$, two Bjorken $x$’s can be quite different: $x_1 > x_2$. Therefore, looking at the spectra with increasing $y$ one can eventually reach $x_1 > x_{\text{max}}$ and GS violation should be seen. To this end, we shall use $pp$ data from NA61/SHINE experiment at CERN [14] at different rapidities $y = 0.1–3.5$ and at five different energies $W_1,...,5 = 17.28, 12.36, 8.77, 7.75, \text{and} \, 6.28 \text{ GeV}$.

In Fig. 3 we plot ratios $R_{1i} = R_{W_1,W_i} (4)$ for $\pi^-$ spectra in central rapidity for $\lambda = 0$ and 0.27. For $y = 0.1$, the GS region extends towards the smallest energy because $x_{\text{max}}$ is as large as 0.08. However, the quality of GS is the worst for the lowest energy $W_5$. By increasing $y$, some points fall outside the GS window because $x_1 \geq x_{\text{max}}$, and finally for $y \geq 1.7$ no GS should be present in NA61/SHINE data. This is illustrated nicely in Fig. 4.

![Fig. 3. Ratios $R_{1k}$ as functions of $\sqrt{\tau}$ for the lowest rapidity $y = 0.1$: (a) for $\lambda = 0$ when $\sqrt{\tau} = p_T$ and (b) for $\lambda = 0.27$ which corresponds to GS.](image)

![Fig. 4. Ratios $R_{1k}$ as functions of $\sqrt{\tau}$ for $\lambda = 0.27$ and for different rapidities (a) $y = 0.7$ and (b) $y = 1.3$. With an increase of rapidity, gradual closure of the GS window can be seen.](image)
6. Conclusions

We have shown that GS in DIS works well up to rather large Bjorken $x$’s with exponent $\lambda = 0.32 - 0.34$. In $pp$ collisions at the LHC energies in central rapidity GS is seen in the charged particle multiplicity spectra, however, $\lambda = 0.27$ in this case. By changing rapidity, one can force one of the Bjorken $x$’s of colliding patrons to exceed $x_{\text{max}}$ and GS violation is expected. Such behavior is indeed observed in the NA61/SHINE $pp$ data.

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