NUCLEAR DEFORMATION EFFECTS TO THE FORMATION CROSS SECTION OF $\eta'$-MESIC NUCLEUS*

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We study the nuclear deformation effects to the formation spectrum of the $\eta'$-mesic nucleus theoretically. We find that the deformation effects could significantly change the spectrum shape and the effects should be considered appropriately.

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1. Introduction

The $\eta'$ meson is known to have the exceptionally large mass within light pseudoscalar mesons. Since the $U_A(1)$ anomaly has been considered to be the dominant origin of the large mass, we expect to obtain new information on the aspects of the strong interaction symmetry related to the $U_A(1)$ anomaly from the in-medium properties of the $\eta'$ meson, especially from its mass shift [1]. However, we have only poor knowledge of $\eta'$ in nucleus at present. In this context, we are very much interested in the structure and formation of the $\eta'$-nucleus bound states, which have not been observed yet.

Search for the $\eta'$ bound states was firstly proposed in Ref. [2]. Then, the experimental search by the $^{12}\text{C}(p,d)$ reaction was performed at GSI [3] based on the studies in Refs. [4, 5]. In the observed spectrum of the emitted deuteron, the peak structure corresponding to the bound-state formation was not observed. The upper limit of the formation cross section was deduced from the data and reported in [3], which was used to determine the range of the potential strength consistent to the upper limit assuming the energy-independent simple potential form.

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In this report, we consider the possible deformation effects of the nucleus in the \( \eta' \) bound states to the reaction spectrum to get deeper insight of the observed spectrum.

2. Formulation

To simulate the nuclear deformation in the \( \eta' \)-mesic nucleus, we construct the nuclear densities from the harmonic oscillator (H.O.) wave function and change the density of \( ^{11}\text{C} \) in the final state of the \( ^{12}\text{C}(p,d)^{11}\text{C} \otimes \eta' \) reaction by varying the harmonic oscillator parameter \( \omega \), which is the only parameter in the present model to determine the nuclear deformation. The density of \( ^{11}\text{C} \) is defined by the H.O. radial wave function \( R_{nl} \) as

\[
\rho(r) = \sum_{s_{1/2},p_{3/2}} \frac{2l + 1}{4\pi} |R_{nl}|^2
\]

by taking the sum of the single-particle densities of \( s_{1/2} \) and \( p_{3/2} \) levels and by normalizing it as to be \( \int \rho(r) d^3r = 1. \) We vary the parameter \( \omega \) as \( \omega/\omega_0 = 0.75, \ 1.00, \ 1.25, \ 1.50 \) with the fixed \( \omega_0 = 40 A^{-1/3} \approx 17.47 \text{ MeV} \) with \( A = 12 \). The bound \( \eta' \) states are obtained by solving the Klein–Gordon equation with the complex potential

\[
V(r) = (V_0 + iW_0) \rho(r) / \rho_0,
\]

where \( \rho(r) \) is defined in Eq. (1) and \( \rho_0 = 0.17 \text{ fm}^{-3} \). The strength of the \( \eta' \) potential has been studied in Refs. [6–11].

The formation spectra of the \( \eta' \)-mesic nucleus are calculated by the effective number approach [12] as

\[
\frac{d^2\sigma}{d\Omega dE} = \sum_{n \otimes \eta'} \frac{\Gamma}{2\pi (\Delta E)^2 + \Gamma^2/4} \left( \frac{d\sigma}{d\Omega} \right)^{\text{ele}} N_{\text{eff}},
\]

and the effective number \( N_{\text{eff}} \) is defined by the wave functions participating in the reaction as

\[
N_{\text{eff}} = \sum_{JM} \left| \int \left[ \phi_{\eta'}^* (r) \otimes \psi_n (r) \right]_{JM} e^{iq \cdot r} D(z, b) d^3r \right|^2,
\]

where \( n \) indicates the neutron state in the target, the elementary cross section is assumed to be

\[
\left( \frac{d\sigma}{d\Omega} \right)^{\text{ele}} = 30 \ [\mu b/\text{sr}],
\]

as in Ref. [5] and \( D(z, b) \) is the distortion factor. The detailed explanation of the effective number approach is given in Ref. [12] and related articles.
3. Numerical results

We show the calculated density distribution of $^{11}$C in Fig. 1 for four values of the H.O. parameter. These densities are used to evaluate the $\eta'$-nuclear optical potential in Eq. (2). The properties of the $\eta'$ bound states are obtained by solving the Klein–Gordon equation, and the binding energies and widths are compiled in Table I for $V_0 = -150$ MeV cases as indicated in Refs. [6, 7]. We found that the bound states become monotonically deeper for larger $\omega$ values with larger widths.

![Fig. 1. Density distributions of the $^{11}$C nucleus based on the H.O. wave functions with different values of the H.O. parameter $\omega$ as shown in the figure with $\omega_0 = 17.47$ MeV.](image)

The calculated binding energies and the widths of $\eta'$ bound states in unit of MeV for different values of the H.O. parameter $\omega$ as shown in the table with $\omega_0 = 17.47$ MeV. The potential strengths in Eq. (2) are assumed to be $(V_0, W_0) = -(150, 5)$ MeV.

| $\omega/\omega_0$ | 0.75 | 1.00 | 1.25 | 1.50 |
|-------------------|------|------|------|------|
| nl                |      |      |      |      |
| 0s                | -69.05 | 6.20 | -113.37 | 10.19 | -168.04 | 15.38 | -234.52 | 22.25 |
| 1s                | -13.88 | 3.32 | -29.60 | 5.84 | -50.77 | 8.96 | -77.38 | 12.75 |
| 0p                | -41.35 | 5.15 | -72.19 | 8.51 | -110.88 | 12.74 | -157.88 | 18.05 |
| 1p                | —     | —     | -2.52 | 2.81 | -10.58 | 5.34 | -22.72 | 8.25 |
| 0d                | -14.10 | 3.84 | -31.43 | 6.56 | -54.64 | 9.93 | -83.76 | 14.01 |
| 0f                | —     | —     | —     | —     | -4.27 | 6.71 | -17.37 | 9.88 |

We show in Fig. 2 the calculated spectra for the formation of $\eta'$-mesic nucleus in the $^{12}$C($p, d$) reaction for $V_0 = -150$ MeV cases. The shape of the spectra was changed significantly according to the change of the density...
distribution of $^{11}$C. In Fig. 3, we also show the spectra for $V_0 = -80$ MeV cases [8] to see the dependence on the strength of the attractive potential. We see that the spectra shape is simpler for the shallower potential case. However, the deformation effects to the formation spectra are important again. The spectrum will be even simpler for the shallower potential like $V_0 = -40$ MeV as reported in Refs. [9–11] due to the smaller number of the bound states. Nevertheless, the importance of the nuclear deformation effect is considered to be very common to the formation spectra.

![Fig. 2. The calculated forward spectra of $^{12}$C($p, d$) reaction for the formation of the $\eta'$-mesic nucleus at the incident proton energy of 2.5 GeV for different values of the H.O. parameter $\omega$ as shown in the figures with $\omega_0 = 17.47$ MeV and $V_0 = -150$ MeV.](image-url)
4. Summary

We have investigated the deformation effects of the $^{11}$C nucleus to the formation spectra in the $^{12}$C($p,d$)$^{11}$C$\otimes\eta'$ reaction. We found that the deformation of the $^{11}$C nucleus could change the formation spectra significantly, and thus, the effects should be considered appropriately. To evaluate the deformation effects in more realistic manner, we will make use of the theoretical results in Ref. [13] as a next step, where the self-consistent relativistic mean field framework is adopted.

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