Earthquake-induced displacement of cohesive-frictional slopes subject to cracks

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Abstract. The upper bound theorem of limit analysis together with Newmark’s method are employed to evaluate the displacement of soil slopes subject to cracks. The pseudo static approach has been routinely used in the literature to estimate the seismic displacement of soil slopes. However, the effect of cracks on the slope displacement has yet to be tackled. In this paper, a new technique is proposed to estimate the horizontal displacement at the slope toe due to a given earthquake postulating rough estimation of real time crack formation. Rotational failure mechanisms for intact slopes exhibiting the formation of cracks as part of the failure process and the case of cracks which are pre-existing in the slope were considered. On the basis of Newmark’s method, the seismic-induced displacement is calculated by incorporating a stepwise yield acceleration corresponding to the cracks occurring in the slope. Results of the proposed technique can reasonably bridge the gap between the conservatism of assuming the slopes subject to the most detrimental cracks, and the overestimation of slope stability resulted from the neglect of crack formation. An example illustrating the procedure for a given earthquake is presented. Also, charts providing the values needed to calculate the stepwise yield accelerations are proposed.

1. Introduction
Cracks can be found in soil slopes and embankments due to tensile stresses such as seismic action or external static loading, and/or due to desiccation and cycles of wetting and drying. Pre-existing cracks can cause significant reduction in the stability of soil slopes [1-3], especially if these slopes are subjected to seismic action [4]. The presence of a vertical crack can reduce the safety factor of the slope depending mainly on its location and depth, and it does not only form a potential part of the slip surface, but also an easy flow channel for rainfall water which reduces the soil strength and exerts a lateral stress, inducing the failure when the crack is filled with water.

Methods for assessing the seismic stability of slopes have been developed during the last century. The Mononobe-Okabe method is one of the first published works that address the stability of retaining walls and dams during earthquake incorporating dynamic earth pressure [5, 6]. Thereafter, several limit equilibrium methods were developed for this purpose (e.g. [7-9]) which remain the most commonly used by practitioners. More recently, numerical methods for continuum mechanics, such as the finite element method with strength reduction technique (e.g. [10, 11]) and finite element limit analysis [12] have provided the capability to reliably detect the onset of failure in slopes according to the approach of continuum mechanics. However, if cracks are present, a continuum approach no longer works since the onset of instability is ruled by the behaviour of single fractures. In this case, the Discrete Element Method can nowadays be employed for 3D analyses of slopes with cracks [13].
Recent algorithmic advances in terms of contact detection algorithms [14, 15] have substantially reduced the runtime of these analyses. However, when little information on the presence of cracks is available, extensive parametric analyses requiring large computational times are necessary. In this case, an analytical approach is very desirable so that numerical analyses would be run only for the case(s) identified by the analytical approach as the most critical.

Newmark’s analytical method [16] is popular among practitioners where a pseudo-static force is used instead of the dynamic excitation to calculate earthquake-induced displacements. The analytical solution for earthquake-induced displacements undergone by intact slopes subject to a rotational failure mechanism is presented by [17]. One of the main limitations of Newmark’s method is the neglect of the earthquake induced strength degradation of the soil, i.e. it assumes a constant yield acceleration throughout the analysis [18]. In this paper, however, earthquake induced crack occurrence and the consequent reduction of yield acceleration are accounted for. Seismic induced displacements are calculated based on a stepwise time varying yield acceleration.

2. Formulation of the Problem

The kinematic approach of limit analysis will be used to calculate the least upper bound on the yield (critical) coefficient of acceleration $K_y$ for a given uniform $c,\phi$ slope. The yield acceleration can be defined as the minimum level of horizontal acceleration (vertical acceleration being proportional to the horizontal acceleration) that brings the slope to failure (i.e. safety factor =1). According to Newmark’s method [16], slope displacements start to accumulate whenever the seismic induced acceleration exceeds the yield acceleration. Then, displacements occurring during the earthquake can be obtained by double integrating the differences between the applied accelerations and the yield one during the time intervals when the ground velocity is larger than zero.

Pre-existing cracks, i.e. cracks exiting in the slope before any seismic excitation occurs, can significantly reduce the yield acceleration for a given slope, depending on their locations and depths [4]. Here, we shall consider an initially intact slope subject to the formation of tension cracks as a result of the earthquake. In this case, the cracks are formed as part of the failure process at the first time the slope yield acceleration is exceeded. Then, in order to calculate the slope displacements generated by the earthquake, a new yield acceleration, accounting for the presence of the cracks formed the first time the yield acceleration of the intact slope was exceeded, needs to be calculated for all the subsequent steps. Four cases are considered in this paper:

I. Slopes made of rocks / cohesive soils of unlimited tensile strength, hence not subject to tension cracks.
II. Slopes made of rocks / cohesive soils of limited tensile strength.
III. Slopes made of rocks / cohesive soils of zero tensile strength.
IV. Slopes subject to the most unfavourable crack from a stability point of view pre-existing the onset of the earthquake.

The procedure for calculating the stepwise time varying yield acceleration is outlined as follows:

1- Determine the yield acceleration for an initially intact slope subject to the formation of tension cracks $K_{y(1)}^{int}$. Vertical tension cracks are formed as part of the occurring failure mechanism since energy is needed to form any crack [3]. Therefore the yield acceleration of a slope subject to the formation of tension cracks is lower than the yield acceleration of a slope of unlimited tensile strength $K_{y(1)}^{int}$, i.e. $K_{y(1)}^{1} \leq K_{y(1)}^{int}$. This acceleration is used to calculate the displacements at the first time that the seismic acceleration exceeds the yield acceleration $K_{y(1)}^{1}$.

2- Determine the yield acceleration for the same slope but accounting for the presence of the crack generated in step 1, treated now as a pre-existing crack (i.e. the crack is already present
so that no energy is dissipated for crack formation). This new value of yield acceleration, \( K^2_{y(c)} \) is used to calculate the displacements in all the subsequent steps.

3- Determine the accumulated wedge displacement \( D_i \) with respect to the ground surface, at each time step \((i)\) when the seismic acceleration exceeds \( K^2_{y(c)} \).

4- Calculate the dimensionless coefficient \( C \) that relates the displacement of the slope toe to the integral of the earthquake acceleration record above the level of yield acceleration.

5- Determine the accumulated horizontal displacement at the slope toe \( D_{iL} \), where \( D_{iL} = C \times D_i \), and then the total horizontal displacement \( D \) is to be found.

It should be noted that, although several tension cracks at different locations in the slope may form during an earthquake, only the crack which has the worst detrimental effect on slope stability needs to be considered in the calculation, since according to the kinematic approach of limit analysis, the failure mechanism taking place is the most critical mechanism for the stability of the slope among all the kinematically feasible mechanisms.

3. Calculation of Yield Acceleration

The upper bound theorem of limit analysis is employed here to calculate the yield acceleration for both intact and cracked slopes. The analytical expressions for the calculation of the external work done by soil masses sliding along composite log-spiral failure surfaces, which requires the use of fictitious wedges bordered by a log-spiral, were first presented in [19, 20] for the case of slopes with horizontal upper part subject to a sequence of landslides, and for more general case of slopes with an inclined upper part, see [21]. Note that these calculations apply to slopes made of bonded granulates [22, 23] as well. In [24], the calculation of the work done by a wedge enclosed by two log-spirals was first presented. The analytical solution is derived here for the case of a horizontal upper slope surface and vertical pre-existing cracks from the upper slope (see figure 1). However, the solution can be straightforwardly extended to the case of a non-horizontal upper slope and that of cracks departing from the slope face, such an extension is reported in [2].

According to the upper bound theorem of limit analysis, the failing wedge E-D-C-B rotates rigidly and log-spirally around the centre of rotation P, as yet undefined, with the ground lying on the right of the log-spiral piece D-C and of the vertical crack C-B remaining at rest. The equation of log-spiral D-C is:

\[
r = r_\chi \exp[\tan(\phi(u - \chi))]
\]

with \( r \) being the distance of a generic point of the spiral to its centre, \( \phi \) being the angle of internal friction, \( r_\chi \) identifying the distance of point F of the spiral to its centre, and \( \chi, \nu \) being the angles made by segment P-F and segment P-D with the horizontal, respectively (see figure 1).

The upper bound on the yield acceleration \( K_y \) will be derived imposing energy balance for the failing wedge E-D-C-B:

\[
\dot{D} = \dot{W}
\]

where \( \dot{D} \) and \( \dot{W} \) are the rate of dissipated energy and of external work, respectively. The dissipated energy \( \dot{D} \) has two terms as follow:

\[
\dot{D} = \dot{D}_{B-C} + \dot{D}_{C-D}
\]

where \( \dot{D}_{B-C} \) and \( \dot{D}_{C-D} \) are the rates of dissipated energy along the crack (B-C) and the log-spiral segment (D-C), respectively. The analytical expression for the energy dissipated to form a new crack is given in [3]:

\[
\dot{D}_{B-C} = \dot{D}_{C-D} = \pi \rho \sigma_0 \frac{1}{2K_y^2} 
\]

\[ \pi \frac{1}{2K_y^2} 
\]
\[ \dot{D}_{B-C} = \hat{\theta} r_x \left( \frac{\sin \chi}{\tan \theta} \right)^2 \left[ \frac{f_c}{2} \int \frac{1 - \sin \theta}{\cos^2 \theta} d\theta + \frac{f_t}{1 - \sin \phi} \int \frac{\sin \theta - \sin \phi}{\cos^2 \theta} d\theta \right] \]

with \( \theta_i \) : is the angle made by the segment P-B with the horizontal (see Fig. 1), \( f_c \) and \( f_t \) are the unconfined compressive and tensile strength of the geo-material, respectively. According to the Mohr-Coulomb failure criteria (see figure 2), they can be expressed as:

\[ f_c = 2c \frac{\cos \phi}{1 - \sin \phi} \]

\[ f_t = 2\alpha c \frac{\cos \phi}{1 + \sin \phi} \]

where \( \alpha \) is a dimensionless coefficient introduced here to express the tensile strength of the soil (i.e. \( 0 \leq \alpha \leq 1 \)). In this Paper, values of \( \alpha \) of 0, 0.5 and 1 will be considered. Substituting equation (5) and equation (6) into equation (4), the following expression is obtained:

\[ \dot{D}_{B-C} = c \hat{\theta} r_x f_{B-C}(\chi, \nu, \zeta, \phi, \alpha) = c \hat{\theta} r_x \left( \frac{\sin \chi}{\tan \theta} \right)^2 \left[ \frac{\cos \phi}{1 - \sin \phi} \int \frac{1 - \sin \theta}{\cos^3 \theta} d\theta + \frac{2\alpha \cos \phi}{1 - \sin^2 \phi} \int \frac{\sin \theta - \sin \phi}{\cos^3 \theta} d\theta \right] \]

**Figure 1.** Failure mechanism. Note that \( \phi \neq \phi \). The wedge of soil enclosed by black lines D-C (logarithmic spiral failure line), B-C (pre-existing crack), B-E (upper surface of the slope) and E-D (slope face) rotates around point P.
Figure 2. Yield stress criteria for (a) soil of limited tensile strength. (b) soil of zero tensile strength (tension cut-off), after [3].

Also energy is dissipated along the log-spiral segment (D-C). Its analytical expression, reported in, is:

$$D_{C-D} = c \dot{\theta}_x^2 f_{C-D} (x, \nu, \zeta, \phi) = c \dot{\theta}_x^2 \exp \left[ 2 \tan \phi (\zeta - \chi) \right] \exp \left[ 2 \tan \phi (\nu - \zeta) \right] - 1 \over 2 \tan \phi$$ (6).

The rate of external work $\dot{W}$ for the sliding wedge E-B-C-D can be calculated as the work of block E-D-F minus the work of block B-C-F. The work of block E-D-F is calculated by algebraic summation of the work of blocks P-D-F, P-E-F and P-D-E that are here called $\dot{W}_1$, $\dot{W}_2$ and $\dot{W}_3$, respectively. The work of block B-C-F is calculated by algebraic summation of the work of blocks P-C-F, P-B-F and P-C-B that are here called $\dot{W}_4$, $\dot{W}_5$ and $\dot{W}_6$, respectively. So $\dot{W}$ can be calculated from the following summation:

$$\dot{W} = \dot{W}_1 - \dot{W}_2 - \dot{W}_3 - (\dot{W}_4 - \dot{W}_5 - \dot{W}_6) = \dot{W}_1 - \dot{W}_2 - \dot{W}_3 + \dot{W}_4 + \dot{W}_5 + \dot{W}_6$$ (7).

The expressions for $\dot{W}_i$ are derived for each block by calculation of the vectorial product of the displacement rate $\dot{u}$ of the block (see figure 1) times their weight force [2]. Here instead, in addition to the weight force, a horizontal pseudo-static force, $F_h = mK_g \gamma = \gamma K_g A$ with $\gamma$ being the gravitational acceleration and $m$ the mass of the wedge, and a vertical one, $F_v = mK_v \gamma = \gamma K_v A$, are added to account for seismic action. For the sake of space, only the final expressions of the yield acceleration are reported here:

$$\dot{W}_i = \dot{\theta} \gamma r_x^3 \left[ \left( 1 + K_v \right) f_v (x, \nu, \chi, \phi) + k_h f_{h_x} (x, \nu, \chi, \phi) \right]$$ (8).

$$= \dot{\theta} \gamma r_x^3 \left[ \begin{array}{c} \left( 1 + K_v \right) \exp \left[ 3 \tan \phi (\nu - \chi) \right] \left( 3 \tan \phi \cos \nu + \sin \nu \right) - 3 \tan \phi \cos \chi - \sin \chi \right] \\ \left[ K_h \exp \left[ 3 \tan \phi (\nu - \chi) \right] \left( 3 \tan \phi \sin \nu - \cos \nu \right) - 3 \tan \phi \sin \chi + \cos \chi \right] \right]$$

with $\dot{\theta}$ being the rate of angular displacement of the failing wedge E-B-C-D. For block P-E-F instead:

$$\dot{W}_2 = \dot{\theta} \gamma r_x^3 \left[ \left( 1 + K_v \right) f_{2v} (x, \nu, \chi, \phi) + K_h f_{2h} (x, \nu, \chi, \phi) \right]$$ (9).
\[
\begin{align*}
\dot{\gamma} r_\gamma^3 &\left[(1 + K_v) \frac{L}{6r_\gamma} \sin \chi \left(2 \cos \chi - \frac{L}{r_\gamma} \right) + K_h \frac{L}{3r_\gamma} \sin^2 \chi \right] \\
\text{for block P-D-E:} & \\
\dot{W}_5 = \dot{\gamma} r_\gamma^3 \left[(1 + K_v) f_3, (\chi, \nu, \phi) + K_h f_3b (\chi, \nu, \phi) \right] \\
&= \dot{\gamma} r_\gamma^3 \left[(1 + K_v) \exp[\tan \phi (\nu - \chi)] \left(\sin (\nu - \chi) - \frac{L}{r_\gamma} \sin \nu\right) \right] \\
&\quad \left[\cos \chi - \frac{L}{r_\gamma} + \exp[\tan \phi (\nu - \chi)] \cos \nu\right] \\
&= \dot{\gamma} r_\gamma^3 \left[\frac{K_h}{6} \exp[\tan \phi (\nu - \chi)] \left(\sin (\nu - \chi) - \frac{L}{r_\gamma} \right) \right] \\
&\quad \left[\sin \chi + \exp[\tan \phi (\nu - \chi)] \sin \nu\right] \\
\text{for block P-C-F:} & \\
\dot{W}_4 = \dot{\gamma} r_\gamma^3 \left[(1 + K_v) f_4, (\chi, \nu, \phi) + K_h f_4b (\chi, \nu, \phi) \right] \\
&= \dot{\gamma} r_\gamma^3 \left[(1 + K_v) \exp[3 \tan \phi (\zeta - \chi)] \left(3 \tan \phi \cos \zeta + \sin \zeta\right) - 3 \tan \phi \cos \chi - \sin \chi \right] \\
&= \dot{\gamma} r_\gamma^3 \left[\frac{K_h}{3} \exp[3 \tan \phi (\zeta - \chi)] \left(3 \tan \phi \sin \zeta - \cos \zeta\right) - 3 \tan \phi \sin \chi + \cos \chi \right] \\
\text{for block P-B-F:} & \\
\dot{W}_5 = \dot{\gamma} r_\gamma^3 \left[(1 + K_v) f_5, (\chi, \nu, \phi) + K_h f_5b (\chi, \nu, \phi) \right] \\
&= \dot{\gamma} r_\gamma^3 \left[(1 + K_v) \frac{L}{6r_\gamma} \sin \chi \left(2 \cos \chi - \frac{L}{r_\gamma} \right) + K_h \frac{L}{3r_\gamma} \sin^2 \chi \right] \\
\text{for block P-C-B:} & \\
\dot{W}_6 = \dot{\gamma} r_\gamma^3 \left[(1 + K_v) f_6, (\chi, \nu, \phi) + K_h f_6b (\chi, \nu, \phi) \right] \\
&= \dot{\gamma} r_\gamma^3 \left[\frac{1}{3} \exp[2 \tan \phi (\zeta - \chi)] \cos^2 \zeta \left(\exp[\tan \phi (\zeta - \chi)] \sin \zeta - \sin \chi\right) + \right] \\
&\quad \frac{K_h}{6} \exp[\tan \phi (\zeta - \chi)] \cos \zeta \left(\exp[2 \tan \phi (\zeta - \chi)] \sin^2 \zeta - \sin^2 \chi\right) \\
\text{Substituting equation (3) and equation (7) into equation (2), the following expression is obtained:} \\
c \dot{\gamma} r_\gamma^3 (f_{b-c} + f_{c-d}) = \dot{\gamma} r_\gamma^3 \left[(1 + K_v) \left(f_{3v} - f_{2v} - f_{3v} - f_{4v} + f_{5v} + f_{6v}\right)\right. \\
&\quad + K_h \left(f_{1b} - f_{2b} - f_{3b} - f_{4b} + f_{5b} + f_{6b}\right) \\
&= \dot{\gamma} r_\gamma^3 \left[(1 + K_v) \left(f_{3v} - f_{2v} - f_{3v} - f_{4v} + f_{5v} + f_{6v}\right)\right. \\
&\quad + K_h \left(f_{1b} - f_{2b} - f_{3b} - f_{4b} + f_{5b} + f_{6b}\right) \right]
\end{align*}
\]

The vertical coefficient of acceleration is introduced by $\lambda$ as a ratio of the horizontal acceleration, where $\lambda = K_v / K_h$. Consistently with figure 1, the + sign indicates vertical downward acceleration, whereas the – sign indicates vertical upward acceleration. An upper bound on the coefficient of yield acceleration $K_y$ is obtained by solving equation (14) with respect to $K_h$ as follow:
The global minimum of $f_i(\chi, \nu, \zeta, \phi, c/\gamma H, \beta, \lambda, \alpha)$ over the three geometrical variables $\chi, \nu, \zeta$ provides the least upper bound on the coefficient of yield acceleration, assuming that the most unfavourable crack for the slope is present. However, mathematical constraints could be applied to equation (15) in order model a slope with a prescribed crack depth or crack location. Using equation (15), charts providing the yield acceleration for soil slopes with either zero tensile strength (i.e. $\alpha = 0$) or half the Mohr-Coulomb tensile strength (i.e. $\alpha = 0.5$) are presented in figure 3. It should be noted that the solid lines in figure 3 refer to soil slopes that are initially intact but they undergo crack formation as part of the developing failure mechanism. The dashed lines, however, are for soil slopes with an earthquake induced crack, since in this case the crack is already formed and treated here as an open crack. The stepwise yield acceleration, proposed in this paper, can be found using these two lines (i.e. solid and dashed), where for a given soil slope properties, two values of yield acceleration are obtained. The one obtained from the solid line represents the starting value of the yield acceleration which steps down to the value obtained from the dashed line as soon as exceeded for the first time by the applied acceleration, given by the earthquake record. The definition of the stepwise yield acceleration is detailed by an illustrative example later in this paper. It should also be noted that failure passes below the slope toe were not permitted during the calculations for these charts. This type of failure might occur for gentle slope with low angle of internal friction [25].

4. Calculation of Seismic Displacements

The maximum horizontal displacement of the slope face occurs at the slope toe [17]. This displacement is denoted here as $\delta u_x$ (see figure 4). Based on Newmark’s method, the rate of $\delta u_x$ can be calculated as [25]:

$$\delta u_x = r_c \sin \nu \delta \theta = r_c \sin \nu \int \frac{\delta \theta}{dt} dt = C \int (K_i - K_f) g dt$$  \hspace{1cm} (16)$$

where $\delta \theta$ is the angular displacement $\bar{\theta}$ is the angular acceleration $K_i$ is the applied horizontal coefficient of acceleration at step $i$, and $C$ is a dimensionless coefficient that relates the displacement of the slope toe to the integral of the earthquake acceleration record above the level of yield acceleration. Performing the calculations, this coefficient can be expressed as:

$$C = \gamma r_c^3 \exp[\tan \phi (\nu - \chi)] \sin \nu \left[ \frac{\lambda}{G} \left( f_{1v} - f_{2v} - f_{3v} + f_{4v} + f_{5v} + f_{6v} \right) + \left( f_{1h} - f_{2h} - f_{3h} + f_{4h} + f_{5h} + f_{6h} \right) \right]$$  \hspace{1cm} (17)$$

with $G$ being the weight of the potential sliding mass and $l$ is the distance from point P to the centre of gravity of that mass. The calculations for $G$ and $l$ are listed in Appendix A. To this end, the seismic induced displacements can be calculated using equation (16) by assigning an earthquake record and calculating the yield acceleration for the slope of interest.
Figure 3. Yield horizontal acceleration with zero vertical acceleration. Left hand side charts (a), (b) and (c) are for soil slopes with zero tensile strength while (d), (e) and (f) are for soil slopes with limited tensile strength (i.e. half of Mohr-Coulomb’s tensile strength).
5. Illustrative Example

Here we shall consider a slope with $\beta = 70^\circ$, $\phi = 20^\circ$, $\lambda = 0$ and $c/\gamma H = 0.15$ subjected to the Northridge earthquake (1994), whose main characteristics are listed in table 1. Four cases are considered: case (I) soil slope with full tensile strength, (i.e. $\alpha = 1$), therefore not subject to tension cracks, case (II) soil slope of limited tensile strength, in this case $\alpha = 0.5$, case (III) soil slope of zero tensile strength, (i.e. $\alpha = 0$ ), and case (IV) soil slope subjected to the most adverse pre-existing crack.

Now, according to the procedure mentioned earlier in this paper, the stepwise yield acceleration for soil with limited or zero tensile strength is illustrated in figure 5(a). It can be noticed that the yield acceleration for a soil slope with limited tensile strength is reduced significantly when it is exceeded for the first time by the applied acceleration. This is because the crack formed as part of the failure at that instance, is then treated as a pre-existing one. Consequently, this could increase the estimated displacement as shown in figure 5(b).

As the displacement corresponding to a slope with the most detrimental pre-existing crack seems over conservative, at the same time, assuming an intact slope that remains intact during the earthquake may underestimate the displacement. However, assuming a limited tensile strength for the soil slope seems to reasonably bridge the gap between the conservatism, corresponding to a slope with the most detrimental pre-existing crack, and the underestimation of the displacement when ignoring the crack formation (i.e. intact slope). Figure 5(c) provides an insight as to the way the limited tensile strength could change the crack properties and the orientation of the failure mechanism.

Figure 4. Illustration of the horizontal displacement at the slope toe $\delta u$, and the angular displacement $\delta \theta$. 

![Illustration of the horizontal displacement at the slope toe $\delta u$, and the angular displacement $\delta \theta$.](image)
Figure 5. Slope with $\beta=70$, $\phi=20$, $c/\gamma H=0.15$. (a) Illustration of the calculated yield for the four cases considered employing the Northridge earthquake (1994) as seismic input. (b) Accumulated horizontal displacement at the slope toe, for the four cases considered. (c) Failure mechanisms associated with the calculated yield accelerations.
Table 1. Main characteristics of the earthquakes considered in the example case.

| Date        | 17/1/1994 |
|-------------|-----------|
| Station     | 24283 Moorpark - Fire Sta. |
| Magnitude   | 6.7       |
| Direction   | 180°      |
| Peak acceleration (g) | 0.292     |
| Epicentre distance (km) | 23        |

6. Conclusions
The upper bound theorem of limit analysis together with the pseudo static approach were employed to evaluate the displacements of cohesive frictional slopes subject to the formation of tension cracks. The formation of earthquake-induced tension cracks and their effect on the displacements were considered. The assumption of the stepwise yield acceleration can be used to reasonably bridge the gap between the conservatism corresponding to a slope with the most detrimental pre-existing crack, and the underestimation of the displacement when ignoring the crack formation throughout the analysis. Four cases were considered here: intact slopes of unlimited tensile strength, intact slopes of limited tensile strength, intact slopes with no tensile strength, and slopes subject to cracks pre-existing the seismic event. Charts providing the values needed to calculate the stepwise yield acceleration are presented.

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Appendix A
The calculations of the weight of the sliding mass, called $G$ in the manuscript, are detailed as follow:

$$G = \gamma A$$

(A.1)

with $A = A_1 - A_2 - A_3 - A_4 + A_5 + A_6$

$$A_1 = \frac{1}{2} r_{\gamma}^2 \left[ \exp \left( \frac{2 \tan \phi (\nu - \chi)}{2 \tan \phi} \right) - 1 \right]$$

(A.2)

$$A_2 = \frac{1}{2} r_{\gamma} L_{\gamma} \sin \chi$$

(A.3)
The arm length of the weight, $l$, is given by:

$$l = \sqrt{\left(\gamma r^2 (f_1 - f_2 - f_3 - f_4 + f_5 + f_6)\right)^2 + \left(\gamma r^3 (f_{h1} - f_{h2} - f_{h3} - f_{h4} + f_{h5} + f_{h6})\right)^2 \over G}$$

(A.8)

$$A_3 = \frac{1}{2} r^2 H \exp \left[ \tan \phi (\nu - \chi) \right] \sin (\beta + \nu) \sin \beta$$

(A.4)

$$A_4 = \frac{1}{2} r^2 \left[ \exp \left[ 2 \tan \phi (\zeta - \chi) \right] - 1 \right]$$

(A.5)

$$A_5 = \frac{1}{2} r L_2 \sin \chi$$

(A.6)

$$A_6 = \frac{1}{2} \delta r \cos \zeta = \frac{1}{2} r \delta \left[ \exp \left[ \tan \phi (\zeta - \chi) \right] \cos \zeta \right]$$

(A.7)

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