UPPER BOUND ON THE TENSOR-TO-SCALAR RATIO IN GUT-SCALE SUPERSYMMETRIC HYBRID INFLATION

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ABSTRACT: We explore the upper bound on the tensor-to-scalar ratio r in supersymmetric (F-term) hybrid inflation models with the gauge symmetry breaking scale set equal to the value 2.86 · 10^{16} \text{ GeV}, as dictated by the unification of the MSSM gauge couplings. We employ a unique renormalizable superpotential and a quasi-canonical Kähler potential, and the scalar spectral index ns is required to lie within the two-sigma interval from the central value found by the Planck satellite. In a sizable region of the parameter space the potential along the inflationary trajectory is a monotonically increasing function of the inflaton, and for this case, r \lesssim 2.9 \cdot 10^{-4}, while the spectral index running, |d\ln ns/d\ln k|, can be as large as 0.01. Ignoring higher order terms which ensure the boundedness of the potential for large values of the inflaton, the upper bound on r is significantly larger, of order 0.01, for subplanckian values of the inflaton, and |d\ln ns/d\ln k| \simeq 0.006.

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I. INTRODUCTION

Supersymmetric (SUSY) hybrid inflation based on F-terms, also referred to as F-term hybrid inflation (FHI), is one of the simplest and well-motivated inflationary models [1, 2]. It is tied to a renormalizable superpotential uniquely determined by a global U(1) R-symmetry, does not require fine tuned parameters and it can be naturally followed by the breaking of a Grand Unified Theory (GUT) gauge symmetry, such as

\begin{align*}
G_{B-L} &= G_\text{SM} \times U(1)_{B-L},
\end{align*}

[3], where G_\text{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y is the gauge group of the Standard Model (SM), G_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} [4, 5], and flipped SU(5) [6–9], with gauge symmetry G_{SM} = SU(5) \times U(1)_X. Let us clarify, in passing, that the term “GUT” is used in the sense of the gauge coupling unification within Minimal SUSY SM (MSSM), although the aforementioned gauge groups are not adopted. Such models can arise from string compactifications, see for e.g. Ref. [7, 10]. The embedding of the simplest model of FHI within a higher gauge group may suffer from the production of cosmic defects which can be evaded, though, in the cases of smooth [11] or shifted [12] FHI.

In the simplest implementation of FHI [1], we should note that the potential along the inflationary track is completely flat at tree level. The inclusion of radiative corrections (RCs) [1] produce a slope which is needed to drive inflaton towards the SUSY vacuum. In this approximation the predicted scalar spectral index ns \simeq 0.98, is in slight conflict with the latest WMAP [15] and PLANCK [16] data based on the standard power-law cosmological model with Cold Dark Matter and a cosmological constant (LCDM). Furthermore, the gauge symmetry breaking scale M turns out to be close to (but certainly lower than) its SUSY value, M_{\text{GUT}} \simeq 2.86 \cdot 10^{16} \text{ GeV}.

A more complete treatment which incorporates supergravity (SUGRA) corrections [26] with canonical (minimal) Kähler potential, as well as an important soft SUSY breaking term [14, 17], has been shown to yield values for ns that are fully compatible with the data [15, 16], with M in this case somewhat lower than the one obtained in Ref. [1]. A reduction of M is certainly welcome if FHI is followed by the breaking of an abelian gauge symmetry, since it helps to reconcile M with the bound [13] placed on it by the non-observation of cosmic strings [17–20].

The minimal FHI scenario described above, while perfectly consistent with the current observations, requires some modification if one desires to incorporate values of M that are comparable or equal to M_{\text{GUT}}. This is indispensable in cases where G_{\text{GUT}} includes non abelian factors besides G_\text{SM}, which are expected to disturb the successful gauge coupling unification within MSSM. In this letter, we would like to emphasize that the observationally favored values (close to 0.96) for ns with M equal to the SUSY GUT scale can be readily achieved within FHI by invoking a specific type of non-minimal Kähler potential, first proposed in Ref. [22]. In particular, a convenient choice of the next-to-minimal and the next-to-next-to-minimal term of the adopted Kähler potential generates [22–24] a positive mass (quadratic) term for the inflaton and a sizeable negative quartic term which assist us to establish FHI of hilltop type [25] in most of the allowed parameter space of the model. Our objectives can also be achieved in smaller regions of the allowed parameter space even with monotonic inflationary potential and therefore complications related to the initial conditions of FHI can be safely eluded. Acceptable ns values within this set-up are accompanied with an enhancement of the running of ns, \alpha_s, and the scalar-to-tensor ratio, r, which reach, thereby, their maximal possible values within FHI if we take into account that M’s larger than M_{\text{GUT}} are certainly less plausible. Note, in passing, that the reduction of ns by generating a negative mass (quadratic) term for the inflaton, as done in Ref. [21], is not suitable for our purposes since M remains well below M_{\text{GUT}}.

Below, we briefly review in Sec. II the basics of FHI when it is embedded in nonminimal SUGRA and briefly recall in Sec. III the observational and theoretical constraints imposed on our model. In Sec. IV we exhibit our updated results, and our conclusions are summarized in Sec. V.
II. FHI WITH NONMINIMAL KÄHLER POTENTIAL

1. Spontaneous Breaking of $G_{GUT}$. The standard FHI can be realized by adopting the superpotential

$$W = \kappa S (\Phi \Phi - M^2)$$

(1)

which is the most general renormalizable superpotential consistent with a continuous R-symmetry \([1]\) under which

$$S \rightarrow e^{i\alpha} S, \quad \bar{\Phi} \Phi \rightarrow \bar{\Phi} \Phi, \quad W \rightarrow e^{i\alpha} W.$$  

(2)

Here $S$ is a $G_{GUT}$-singlet left-handed superfield, and the parameters $\kappa$ and $M$ are made positive by field redefinitions. In our approach, $\Phi$, $\Phi$ are identified with a pair of left-handed superfields conjugate under $G_{GUT}$ which break $G_{GUT}$ down to $G_{SM}$. Indeed, along the D-flat direction $|\Phi| = |\bar{\Phi}|$ and the SUSY potential, $V_{\text{SUSY}}$, extracted (see e.g. ref. [23, 27]) from $W$ in Eq. (1), reads

$$V_{\text{SUSY}} = \kappa^2 \left((|\Phi|^2 - M^2)^2 + 2|S|^2|\Phi|^2\right).$$  

(3)

From $V_{\text{SUSY}}$ in Eq. (3) we find that the SUSY vacuum lies at

$$\langle S \rangle = 0 \quad \text{and} \quad |\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = M,$$  

(4)

where the vacuum expectation values of $\Phi$ and $\bar{\Phi}$ are developed along their SM singlet type components. As a consequence, $W_{\text{HI}}$ leads to the spontaneous breaking of $G_{GUT}$ to $G_{SM}$. We single out the following two cases:

- $G_{GUT} = G_{LR}$ where $\Phi$ and $\bar{\Phi}$ belong to the $(1, 1, 2, -1)$ and $(1, 1, 2, 1)$ representation of $G_{LR}$. The symmetry breaking in this case is

$$SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y.$$  

Therefore, 3 of the 4 generators of $SU(2)_R \times U(1)_{B-L}$ are broken, leading to 3 Goldstone bosons which are absorbed by the 3 gauge bosons which become massive. Among them, $W_{\text{HI}}^\pm$ with masses $m_{W_{\text{HI}}^\pm} = gM$ correspond to the charged $SU(2)_R$ gauge generators, and one, $A$, to a linear combination of the $SU(2)_R$ and $U(1)_{B-L}$ generator with mass $m_A = \sqrt{5/2}gM$, where $g$ is the SUSY gauge coupling constant at the GUT scale.

- $G_{GUT} = G_{5X}$, where $\Phi$ and $\bar{\Phi}$ belong to the $(10, 1)$ and $(\bar{10}, -1)$ representation of $G_{5X}$. In this case, 13 of the 25 generators of $G_{5X}$ are broken, giving rise via the Higgs mechanism to 13 massive gauge bosons. In particular, 12 gauge bosons which correspond to the generators of $SU(5)$ acquire masses $m_X^\pm = m_{\gamma^\pm} = gM$, and one gauge boson associated with a linear combination of the $SU(5)$ and $U(1)_X$ generators acquires a mass $m_A = \sqrt{32/34}gM$. In both cases no topological defects are generated during the breaking of $G_{GUT}$, in contrast to gauge groups such as $SU(4)_C \times SU(2)_L \times SU(2)_R \times SU(5)$ or $SO(10)$ which lead to the production of magnetic monopoles.

2. The Inflationary Stage. The superpotential $W_{\text{HI}}$ in Eq. (1) gives rise to FHI since, for large enough values of $|S|$, there exist a flat direction

$$\Phi = \bar{\Phi} = 0 \quad \text{where} \quad V_{\text{SUSY}} (\Phi = 0) = V_{\text{HI0}} = \kappa^2 M^4.$$  

(5)

Obviously, $V_{\text{HI0}}$ provides us with a constant potential energy density which can be used to drive inflation. The realization of FHI in the context of SUGRA requires a specific Kähler potential. We consider here a fairly generic form of the Kähler potential, which does not deviate much from the canonical one \([17, 26]\); further it respects the R symmetry of Eq. (2). Namely we take

$$K = |S|^2 + |\Phi|^2 + |\bar{\Phi}|^2 + \frac{1}{4} k_{4S} |S|^4 + \frac{1}{6} k_{6S} |S|^6$$

$$+ \frac{1}{8} k_{8S} |S|^8 + \frac{1}{10} k_{10S} |S|^{10} + \frac{1}{12} k_{12S} |S|^{12} + \cdots$$  

(6)

where $k_{4S}, k_{8S}, k_{12S}$ and $k_{10S}$ are positive or negative constants of order unity and the ellipsis represents higher order terms involving the waterfall fields ($\Phi$ and $\bar{\Phi}$) and $S$. We can neglect these terms since they are irrelevant along the inflationary path. Finally, we include the RCs. These originate from a mass splitting in the $\Phi - \bar{\Phi}$ supermultiplets, caused by SUSY breaking along the inflationary valley \([1]\). We end up with the following inflationary potential – see e.g. Ref. [23, 24]:

$$V_{\text{HI}} \simeq V_{\text{HI0}} \left(1 + c_{2K} \sum_{\nu=1}^{5} (-1)^\nu c_{2\nu K} \left(\frac{\sigma}{\sqrt{2}M}\right)^{2\nu}\right),$$  

(7)

where $\sigma = \sqrt{2}|S|$ is the canonically (up to the order $|S|^2$) normalized inflaton field. The contribution of RCs read

$$c_{\text{HI}} = \frac{\kappa^2 N}{12\pi^2} \left(2 \ln \frac{\kappa^2 M^2}{Q^2} + f_{\text{rc}}(x)\right),$$  

(8a)

where $N$, for our cases, is the dimensionality of the representations to which $\Phi$ and $\bar{\Phi}$ belong. We have $N = 2 \ [N = 10]$ when $G_{GUT} = G_{LR} [G_{GUT} = G_{5X}]$. Also $Q$ is a renormalization scale, $x = \sigma^2/2M^2$, and

$$f_{\text{rc}}(x) = (x+1)^2 \ln (1 + 1/x) + (x-1)^2 \ln (1 - 1/x).$$  

(8b)

The remaining coefficients, $c_{2\nu K}$, in Eq. (7) can be expressed as functions of the $k$‘s in Eq. (6) \([23, 24]\). From them only the first two play a crucial role during the inflationary dynamics; they are

$$c_{2K} = k_{4S} \quad \text{and} \quad c_{4K} = \frac{1}{2} - \frac{7k_{4S}}{4} + k_{4S}^2 - \frac{3k_{6S}}{2}$$  

(9)

The residual higher order terms in the expansion of Eq. (7) prevent a possible runaway behavior of the resulting $V_{\text{HI}}$ – see point 8 of Sec. III. For completeness, we include also them:

$$c_{6K} = -\frac{2}{3} + \frac{3k_{4S}}{2} - \frac{7k_{4S}^2}{4} + k_{4S}^3 + \frac{10k_{6S}}{3}$$

$$-3k_{4S}k_{6S} + 2k_{8S},$$  

(10a)
III. Constraining the Model Parameters

Under the assumptions that (i) the observed curvature perturbation is generated wholly by \( \sigma \) and (ii) FHI is followed in turn by matter and radiation era, our inflationary set-up can be qualified by imposing a number of observational (1-3) and theoretical (4-8) requirements specified below:

1. The number of e-foldings that the scale \( k_* \) = 0.05/Mpc undergoes during FHI is at least enough to resolve the horizon and flatness problems of standard Big Bang cosmology. Employing standard methods [16, 23], we can derive the relevant condition:

\[
N_{HI*} = \int_{\sigma_t}^{\sigma} \frac{d\sigma}{m_{\text{Pl}} V_{HI}} \simeq 19.4 + \frac{2}{3} \ln \left( \frac{V_{HI}^{1/4}}{1 \text{ GeV}} \right) + \frac{1}{3} \ln \left( \frac{T_{\text{th}}}{1 \text{ GeV}} \right),
\]

where the prime denotes derivation w.r.t. \( \sigma \), \( \sigma_* \) is the value of \( \sigma \) when \( k_* \) crosses outside the horizon of FHI, and \( \sigma_t \) is the value of \( \sigma \) at the end of FHI. This coincides with either the critical point \( \sigma_c = \sqrt{2} M \) appearing in the particle spectrum of the \( \Phi - \bar{\Phi} \) system during FHI – see Eq. (8b) –, or the value for which one of the slow-roll parameters [28]

\[
\epsilon \simeq m_{\text{Pl}}^2 \left( V_{HI}' / \sqrt{2} V_{HI} \right)^2 \quad \text{and} \quad \eta \simeq m_{\text{Pl}}^2 V_{HI}'' / V_{HI} \quad (12)
\]

exceeds unity. Since the resulting \( \kappa \) values are sizably larger than \( (M/m_{\text{Pl}})^2 \) – see next section – we do not expect the production of extra e-foldings during the waterfall regime, which in our case turns out to be nearly instantaneous – cf. Ref. [29].

2. The amplitude \( A_s \) of the power spectrum of the curvature perturbation, which is generated during FHI and can be calculated at \( k_* \) as a function of \( \sigma_* \), must be consistent with the data [15, 16], i.e.

\[
\sqrt{A_s} = \frac{1}{2 \sqrt{3 \pi m_{\text{Pl}}^3}} \left( \frac{V_{HI}^{3/2} (\sigma_*)}{|V_{HI,\sigma} (\sigma_*)|} \right) \simeq 4.686 \cdot 10^{-5} \quad (13)
\]

3. The (scalar) spectral index \( n_s \), its running, \( \alpha_s = d n_s / d \ln k \), and the scalar-to-tensor ratio, \( r \), given by

\[
n_s = 1 - 6 \epsilon_* + 2 \eta_* \quad (14a)
\]

\[
\alpha_s = 2 \left( 4 \eta_*^2 - (n_s - 1)^2 \right) / 3 - 2 \xi_* \quad \text{and} \quad r = 16 \epsilon_* \quad (14b)
\]

where \( \xi \simeq m_{\text{Pl}}^4 V_{HI}'' / V_{HI}^2 \) and all the variables with the subscript * are evaluated at \( \sigma = \sigma_* \), must be in agreement with the observational data [15, 16] derived in the framework of the \( \Lambda \)CDM model:

\[
n_s = 0.9603 \pm 0.014 \Rightarrow 0.945 \lesssim n_s \lesssim 0.975, \quad (15a)
\]

\[
\alpha_s = -0.0134 \pm 0.018 \quad \text{and} \quad r < 0.11, \quad (15b)
\]

at 95% confidence level (c.l.). Limiting ourselves to \( \alpha_s \)’s consistent with the assumptions of the power-law \( \Lambda \)CDM model, we further impose the following upper bound:

\[
|\alpha_s| \ll 0.01, \quad (16)
\]

since, within the cosmological models with running \( \alpha_s \), |\( \alpha_s \)|’s of order 0.01 are encountered [15, 16].

4. The \( G_{\text{GUT}} \) breaking scale in Eq. (4) has to be determined by the unification of the MSSM gauge coupling constants, i.e.,

\[
g M \simeq 2 \cdot 10^{16} \text{ GeV}, \quad (17)
\]

with \( g \approx 0.7 \) being the value of the unified gauge coupling constant. Here \( g M \) is the mass at the SUSY vacuum, Eq. (4), of the non singlet under \( G_{\text{SM}} \) gauge bosons \( W_R^\pm \) if \( G_{\text{GUT}} = G_{\text{LR}} \) or \( X^\pm \) and \( Y^\pm \) if \( G_{\text{GUT}} = G_{5X} \) – see Sec. II.

5. The expression of \( V_{HI} \) in Eq. (7) is expected to converge at least for \( \sigma \sim \sigma_* \). This fact can be ensured if, for \( \sigma \sim \sigma_* \), each successive term \( c_{nR} \) in the expansion of \( V_{HI} \) (and \( K \) Eq. (7)) and (Eq. (6)) is smaller than the previous one. In practice, this objective can be easily accomplished if the \( k \)’s in Eq. (6) – or Eq. (7) – are sufficiently low.

6. It is reasonable to ask \( V_{HI} \) to be bounded from below as \( \sigma \to \infty \). Given our ignorance, however, for the pre-inflationary (i.e. for \( \sigma \gg \sigma_* \)) cosmological evolution we do not impose this requirement as an absolute constraint.

7. Depending on the values of the coefficients in Eq. (7), \( V_{HI} \) is either a monotonic function of \( \sigma \) or develops a local minimum and maximum. The latter case may jeopardize the implementation of FHI if \( \sigma \) gets trapped near the minimum of \( V_{HI} \). It is, therefore, crucial to indicate the regions where \( V_{HI} \) is a monotonically increasing function of \( \sigma \).

8. Hilltop FHI proceeds such that \( \sigma \) rolls from \( \sigma_{\text{max}} \), which is the point where the maximum of \( V_{HI} \) lies, down to smaller values. Therefore a mild tuning of the initial conditions is required [21] in order to obtain acceptable \( n_s \) values, since for lower \( n_s \) values we must set \( \sigma_* \) closer to \( \sigma_{\text{max}} \). We quantify the amount of tuning in the initial conditions via the quantity [21]:

\[
\Delta_{m*} = (\sigma_{\text{max}} - \sigma_*) / \sigma_{\text{max}}. \quad (18)
\]

Large \( \Delta_{m*} \) values correspond to a more natural FHI scenario.
IV. RESULTS

Our inflationary model depends on the parameters:

\[ \kappa, k_{4S}, k_{6S}, k_{8S}, k_{10S}, k_{12S}, N, T_{rh}, \text{ and } \sigma_* , \]

with \( M \) fixed from Eq. (17). In our computation, we use as input parameters \( k_{8S} \), \( k_{10S} \) and \( k_{12S} \). We also fix \( T_{rh} \approx 10^{10} \text{ GeV} \), which saturates the conservative gravitino constraint and results in \( N_{\text{HI}} \approx 50 \). Variation of \( T_{rh} \) over \(-1-2\) orders of magnitude is not expected to significantly alter our findings – see Eq. (9). We restrict \( \kappa \) and \( \sigma_* \) such that Eqs. (11) and (13) are fulfilled. The restrictions on \( n_s \) from Eq. (15a) can be met by adjusting \( k_{4S} \) and \( k_{6S} \), whereas the last three parameters of \( K \) control mainly the boundedness and the monotonicity of \( V_{\text{HI}} \); we thus take them into account only if we impose restriction 6 of Sec. III. In these cases we set \( k_{10S} = -1 \) and \( k_{12S} = 0.5 \) throughout and we verify that these values do not play a crucial role in the inflationary dynamics. We briefly comment on the impact of the variation of \( k_{8S} \) and \( N \) on our results. Using Eq. (14b) we can extract \( \alpha_n \) and \( r \).

Following the strategy of Ref. [22] we choose the sign of \( c_{2K} = k_{4S} \) to be negative – cf. Ref. [21]. As a consequence, fulfilling of Eq. (15a) requires a negative \( c_{4K} \) or positive \( k_{6S} \) – see Eq. (9). More explicitly, \( V_{\text{HI}} \) given by Eq. (7) can be approximated as

\[ V_{\text{HI}} \approx V_{\text{HI}0} \left( 1 + c_{\text{HI}} + |c_{2K}| \frac{\sigma^2}{2m_p^2} - |c_{4K}| \frac{\sigma^4}{4m_p^4} - |c_{6K}| \frac{\sigma^6}{8m_p^6} + |c_{8K}| \frac{\sigma^8}{16m_p^8} \right), \]

and it may develop a non-monotonic behavior in a sizable portion of the allowed parameter space. Employing Eq. (19), we can show that \( V_{\text{HI}} \) reaches a local maximum at the inflaton-field value:

\[ \sigma_{\text{max}} \approx m_p \sqrt{\frac{\pi|c_{2K}| + \sqrt{\pi^2 |c_{2K}| + N\kappa^2 |c_{4K}|}}{2|c_{4K}|}}, \]

and a local minimum at the inflaton-field value:

\[ \sigma_{\text{min}} \approx m_p \sqrt{\frac{3|c_{6K}| + \sqrt{9|c_{6K}| + 32|c_{4K}|c_{6K}}}{2|c_{8K}|}}. \]

In deriving Eq. (20a) we keep terms up to the fourth power of \( \sigma \) in Eq. (19), whereas for Eq. (20b) we focus on the last three terms of the expansion in the right-hand side of Eq. (19). For this reason, the latter result is independent of \( c_{\text{HI}} \) and \( c_{2K} \).

The structure of \( V_{\text{HI}} \) is displayed in Fig. 1 where we show the variation of \( V_{\text{HI}} \) as a function of \( \sigma \) for \( \kappa = 0.018 \) and \( k_{4S} = -0.0443, k_{6S} = 0.736, k_{8S} = -1 \) (gray line) or \( k_{4S} = -0.0415, k_{6S} = 0.656, k_{8S} = -0.5 \) (light gray line). These parameters yield \( n_s = 0.96, r \approx 0.00019 \) and \( \alpha_n \approx 0.0054 \) \( \alpha_n \approx 0.0037 \) (gray [light gray] line). The values of \( \sigma_* / M \approx 19.03 \) \( \sigma_* / M \approx 18.4 \) (gray [light gray] line) and \( \sigma_f / M \approx 1.42 \) are also depicted. In the first case (gray line) \( V_{\text{HI}} \) remains monotonic due to the larger \( |k_{8S}| \) value employed. Contrarily, \( V_{\text{HI}} \) develops the minimum-maximum structure, in the second case (light gray line) with the maximum located at \( \sigma_{\text{max}} / M = 26.6 \) and the minimum at \( \sigma_{\text{min}} / M = 53.8 \). The values obtained via Eqs. (20a) and (20b) are indicated in curly brackets. We find that \( \Delta_{\text{ms}} \approx 0.31 \).

Confronting FHI with the constraints of Sec. III, we can identify the allowed regions in the \( \kappa - (-k_{4S}), \kappa - k_{6S}, \kappa - |\alpha_n| \) and \( \kappa - r \) planes – see Fig. 2. The conventions adopted for the various lines are also shown. In particular, the thick and thin gray dashed [dot-dashed] lines correspond to \( n_s = 0.975 [n_s = 0.946] \), whereas the thick and thin gray solid lines are obtained by fixing \( n_s = 0.96 \) – see Eq. (15a). The thick lines are obtained setting \( k_{8S} = -1.5 \) which – together with the universally selected \( k_{10S} \) and \( k_{12S} \) above – ensures the fulfillment of restriction 6 of Sec. III; the faint lines correspond to the choice \( c_{6K} = c_{6K} = c_{10K} = 0 \), which does not ensure the boundedness of \( V_{\text{HI}} \). From the panels (a), (b) and (c) we see that the thin lines almost coincide with the thick ones for \( \kappa \leq 0.01 \), and then deviate and smoothly approach some plateau. The regions allowed by imposing the constraints 1-6 of Sec. III are denoted by light gray shading. In the hatched subregions, requirement 7 is also met. On the other hand, the regions surrounded by the thin lines are actually the allowed ones, when only the restrictions 1-5 of Sec. III are satisfied. The various allowed regions are cut at low \( \kappa \) values since the required \( k_{6S} \) reaches rather high values (of order 10), which starts looking unnatural. At the other end, Eq. (16) and \( \sigma_* \approx m_p \) bounds the allowed areas in the case of bounded or unbounded \( V_{\text{HI}} \) respectively. For both cases, we remark that \( |k_{4S}| \) increases with \( \kappa \) whereas \( k_{6S} \) drops as \( \kappa \) increases. For fixed \( \kappa \), increasing \( |k_{4S}| \) means decreasing \( k_{8S} \). Moreover, \( |k_{4S}| \) is restricted to somewhat small values in order to avoid the well-known [27, 28] \eta \) problem of FHI. On the other hand, no tuning for \( k_{6S} \) is needed since it is of order unity for most \( \kappa \) values.

\[ \text{FIG. 1: The variation of } V_{\text{HI}} \text{ in Eq. (19) as a function of } \sigma \text{ for } n_s = 0.96 \text{ taking } N = 10, \kappa = 0.018, k_{10S} = -1, k_{12S} = 0.5 \text{ and } k_{4S} = -0.0443, k_{6S} = 0.736, k_{8S} = -1.5 \text{ [gray [light gray] line]. The values of } \sigma_*, \sigma_f, \sigma_{\text{max}} \text{ and } \sigma_{\text{min}} \text{ are also depicted.} \]
From Fig. 2-(c) we observe that for increasing $\kappa$ beyond $0.01$, $|\alpha_s|$ corresponding to the bold lines precipitously drops at $\kappa \simeq 0.02$, changes sign and rapidly saturates the bound of Eq. (16) along the thick black solid line. In other words, for every $\kappa$ in the vicinity of $\kappa \simeq 0.2$ we have two acceptable $k_{6S}$ values, as shown in Fig. 2-(b) with two different $\alpha_s$ values of either sign. Furthermore, from Fig. 2-(d) we remark that $r$ is largely independent of the $n_s$ value, and so the various types of lines coincide for both bound and unbound $V_{HI}$. We also see that $r$ increases almost linearly with $\kappa$ and reaches its maximal value which turns out to be: (i) $r \simeq 2.9 \cdot 10^{-5}$ as $\alpha_s$ approaches the bound of Eq. (16), for bounded $V$; (ii) $r \simeq 0.01$ as the inequality $\sigma_s \leq m_P$ is saturated for $n_s \simeq 0.975$ and unbounded $V_{HI}$. Therefore, lifting restriction 6 of Sec. III allows larger $r$. However, non vanishing $c_{6K}$’s perhaps corresponds to a more natural scenario.

We observe that the optimistic restriction 7 in Sec. III can be met in very limited slices of the allowed (lightly gray shaded) areas, only when the boundedness of $V_{HI}$ has been ensured. In these regions $\sigma_s$ also turns out to be rather large (10 $M$), and we therefore observe a mild dependence of our results on $c_{6K}$ (or $k_{6S}$). This point is further clarified in Table I where we list the model parameters and predictions for $n_s \simeq 0.96$, $N = 10$, $\kappa = 0.005, 0.01, 0.02$ and various $k_{6S}$ values. We remark that for $\kappa = 0.005$ the results are practically unchanged for varying $k_{6S}$. The dependence on $k_{6S}$ starts to become relevant for $\kappa \simeq 0.01$ and crucially affects the results for $\kappa = 0.02$: here, for $k_{6S} = -2$ the solution obtained belongs to

![Figure 2: Allowed (lightly gray shaded) region, as determined by the restrictions 1-6 of Sec. III, in the $\kappa - (\kappa_{4S})$ (a), $\kappa - k_{6S}$ (b), $\alpha - |\alpha_s|$ (c) and $\kappa - r$ (d) plane for $N = 10$, $k_{6S} = -1.5$, $k_{10S} = -1$ and $k_{12S} = 0.5$. In the hatched regions $V_{HI}$ remains monotonic. The conventions adopted for the various lines are also shown. The thin lines are obtained by setting $c_{6K} = c_{8K} = c_{10K} = 0$ in Eq. (7).](image)

| $k_{6S}$ | $\kappa$ | $\sigma_s/\Delta m$ | $k_{4S}$ | $k_{6S}$ | $\Delta_{m^*}$ | $\alpha_s$ | $r$ |
|---------|---------|-----------------|--------|--------|------------|---------|----|
|         | (10^{-2}) | (10^{-2}) | (% ) | (10^{-3}) | (10^{-5}) |
| 0.5     | 0.5     | 6.7            | 3.46   | 2.29   | 28         | 3.7     | 1.5|
| 0.5     | 1       | 11.7           | 3.94   | 1.04   | 29         | 5       | 6.6|
| 0.5     | 2       | 20.4           | 4.2    | 0.61   | 3          | 5.3     | 23 |
| 1.5     | 0.5     | 6.7            | 3.46   | 2.29   | 28         | 3.7     | 1.5|
| 1.5     | 1       | 11.5           | 3.98   | 1.1    | 30         | 4.8     | 5.9|
| 1.5     | 2       | 23.64          | 4.68   | 0.715  | 21.6      | 23.6    | 1.5|
| 2       | 0.5     | 6.7            | 3.46   | 2.29   | 28         | 3.7     | 1.5|
| 2       | 1       | 11.54          | 3.98   | 1.11   | 30         | 4.7     | 6.3|
| 2       | 2       | 23.4           | 5.2    | 0.785  | -8.3      | 23      | 23|
the branch with $\alpha_s < 0$ and not in the branch with $\alpha_s > 0$, as is the case with $\kappa = 0.005$ and 0.01. Listed is also the quantity $\Delta_{m^*}$ which takes rather natural values for the selected $\kappa$ – the entries without a value assigned indicate that $V_{HI}$ is a monotonic function of $\sigma$. 

As shown in Fig. 2-(a), $|k_{4S}|$ ranges between about 0.015 and 0.05 for the case with bounded $V_{HI}$ or 0.042 for unbounded $V_{HI}$. For each of these $k_{4S}$ values and every $\kappa$ in the allowed range found in Fig. 2, we vary $k_{GS}$ in order to obtain $n_s$ in the observationally favored region of Eq. (15a) and we extract the resulting $r$. Our results are presented in Fig. 3, where we display the allowed region in the $n_s - r$ plane for bounded (upper plot) or unbounded (lower plot) $V_{HI}$. Along the dashed lines of both plots $k_{GS}$ ranges between 9 and 26 whereas along the solid line of the upper [lower] plot $k_{GS}$ varies between 0.69 and 0.75 [0.39 and 1.15]. From the upper plot we see that the maximal for $r$ is about $2.9 \cdot 10^{-4}$ and turns out to be nearly independent of $n_s$. Interestingly, this value is included in the region with monotonic $V_{HI}$ depicted by the hatched region. From the lower plot we see that there is a mild dependence of the largest $r$ from $n_s$; thus, the maximal $r = 0.006$ is achieved for $n_s = 0.975$. No region with monotonic $V_{HI}$ is located in this case, however.

Summarizing our findings from Figs. 2 and 3 for $n_s$ in the range given by Eq. (15a) and imposing the restrictions 1-7 of Sec. III, the various quantities are bounded as follows:

$$
\begin{align}
\{4.9 \cdot 10^{-2}\} & < \frac{\kappa}{10^{-2}} \lesssim 2.3, \\
\{1.4\} & \lesssim \frac{-k_{4S}}{10^{-2}} \lesssim 7.95, \\
0.68 & \lesssim k_{GS} \lesssim 0.77 (10), \\
\{5.7 \cdot 10^{-2}\} & \lesssim \frac{\alpha_s}{10^{-2}} \lesssim 1, \\
\{1.7 \cdot 10^{-3}\} & \lesssim \frac{r}{10^{-4}} \lesssim 2.9.
\end{align}
$$

Note that the limiting values obtained without imposing the monotonicity of $V_{HI}$ – requirement 7 in Sec. III – are indicated in curly brackets. In the corresponding region, $\Delta_{m^*}$ ranges between 16 and 32%. As can be deduced from the data of Fig. 2, $\Delta_{m^*}$ increases with $\kappa$’s. Small $\Delta_{m^*}$ values indicate a second mild tuning (besides the one needed to avoid the $\eta$ problem), which is however a common feature in the models of hilltop inflation. If we ignore requirement 6 of Sec. III, then confining $n_s$ in the range of Eq. (15a) we obtain the following ranges:

$$
\begin{align}
4.9 \cdot 10^{-3} & \lesssim \frac{\kappa}{10} \lesssim 1, \\
1.4 & \lesssim \frac{-k_{4S}}{10^{-2}} \lesssim 4.7, \\
0.4 & \lesssim k_{GS} \lesssim 10, \\
5.7 \cdot 10^{-1} & \lesssim \frac{\alpha_s}{10^{-3}} \lesssim 6, \\
1.4 \cdot 10^{-5} & \lesssim \frac{r}{10^{-2}} \lesssim 1.
\end{align}
$$

Obviously, no solutions with monotonic $V_{HI}$ are achieved in this case whereas $\Delta_{m^*}$ varies between 16 and 29%. The maximal $r$ is reached for the maximal $n_s$ in Eq. (15a) and as $\alpha_s \sim m_{Pl}$.

So far we focused on $G_{5S}$, employing $N = 10$ in our investigation. However, our results are not drastically affected even in the case of $G_{LR}$ for most values of $\kappa$, as can be inferred by comparing the results (for $k_{GS} = -1.5$) listed in Tables I and II where we use $N = 10$ and $N = 2$ respectively. This signals the fact that the SUGRA corrections to $V_{HI}$ originating from the last term in the sum of Eq. (7) dominate over the radiative corrections.
corrections which are represented by $c_{HI}$. The discrepancy between the two results ranges from 6 to 20%, increasing with $\kappa$, and it is essentially invisible in the plots of Fig. 2. On the other hand, we observe that in the $N = 10$ case the enhanced $c_{HI}$ creates a relatively wider space with monotonic $V_{HI}$; this space is certainly smaller for $N = 2$, as shown from our outputs for $\kappa = 0.02$.

V. CONCLUSIONS

Inspired by the recently released results by the PLANCK collaboration on the inflationary observables, we have reviewed and updated the nonminimal version of SUSY hybrid inflation arising from F-terms, also referred to as FHI. In our formulation, FHI is based on a unique renormalizable superpotential, employs an quasi-canonical Kähler potential and is followed by the spontaneous breaking at $M_{GUT}$ of a GUT symmetry which is taken to be $G_{LR}$ or $G_{5x}$. As suggested first in Ref. [22] and further exemplified in Ref. [23, 24], $n_s$ values close to 0.96 in conjunction with the fulfillment of Eq. (17) can be accommodated by considering an expansion of the Kähler potential – see Eq. (6) – up to twelfth order in powers of the various fields with suitable choice of signs for the coefficients $k_{4S}$ and $k_{6S}$.

Fixing $n_s$ at its central value, we obtain \( \{7.8 \cdot 10^{-2}\} 1.57 \leq \kappa/10^{-2} \leq 2.2 \) with \( \{2.4 \leq -k_{4S}/10^{-2} \leq 7.2 \) and \( 0.72 \leq k_{6S} \leq 0.79 \) \{10\}, while \( |\alpha_s| \) and \( r \) assume the values \( \{0.1 \) 0.45\} \( \cdot 10^{-2} \) and \( \{3.5 \cdot 10^{-3}\} 1.4 \) \( \cdot 1.9 \) \( \cdot 10^{-4} \) respectively – recall that the limiting values in the curly brackets are achieved without imposing the monotonicity of $V_{HI}$. With a non-monotonic $V_{HI}$, $\Delta m$ ranges between 16 and 30%. It is gratifying that there is a sizable portion of the allowed parameter space where $V_{HI}$ remains a monotonically increasing function of $\sigma$; thus, unnatural restrictions on the initial conditions for inflation due to the appearance of a maximum and a minimum of $V_{HI}$ can be avoided. On the other hand, if we do not insist on the boundedness of $V_{HI}$, $\kappa$ reaches 0.1 with $k_{4S} = -0.046$ and $k_{6S} = 0.4$ with the resulting $\alpha_s$ and $r$ being both 0.006, that is close to 0.01. Finally FHI can be followed by a successful scenario of non-thermal leptogenesis [30] for both $G_{GUT}$’s considered here – cf. Ref. [9, 24].

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