The steady-state structure of accretion discs in central magnetic fields

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ABSTRACT
We develop a new analytic solution for the steady-state structure of a thin accretion disc under the influence of a magnetic field that is anchored to the central star. The solution takes a form similar to that of Shakura and Sunyaev and tends to their solution as the magnetic moment of the star tends to zero. As well as the Kramer’s law case, we obtain a solution for a general opacity. The effects of varying the mass transfer rate, spin period and magnetic field of the star and the opacity model applied to the disc are explored for a range of objects. The solution depends on the position of the magnetic truncation radius. We propose a new approach for the identification of the truncation radius and present an analytic expression for its position.

Key words: accretion, accretion discs – stars: dwarf novae – stars: magnetic fields – stars: neutron – stars: pre-main-sequence.

1 INTRODUCTION
There is a well-known one-dimensional solution for the structure of a thin accretion disc in the steady state. This solution, known as the Shakura–Sunyaev disc solution (Shakura & Sunyaev 1973), consists of a series of seven equations. The full solution contains expressions for disc surface density, scaleheight, density, temperature, opacity and viscosity. These expressions depend upon the Shakura–Sunyaev alpha parameter, a dimensionless parametrization of viscosity. In addition to the alpha parameter the solution is expressed as a function of radius, mass transfer rate, stellar mass and stellar radius.

We wish to reformulate the Shakura–Sunyaev disc solution to incorporate the effect of a torque from a stellar magnetic field. A solution for the structure of an accretion disc in a magnetic field is desirable because accretion discs are frequently found around magnetic stars. Such magnetic fields can allow the transfer of angular momentum between the disc and the star and can therefore significantly affect the structure of the disc and the spin evolution of the star (e.g. Armitage & Clarke 1996; Brandenburg & Campbell 1998). These effects can be important in young stars (e.g. Königl 1990; Armitage, Clarke & Tout 1999), cataclysmic variables (e.g. Schenker et al. 2002) and X-ray binaries (e.g. Pringle & Rees 1972; Romanova et al. 2003). Accretion discs can exhibit outbursts due to the thermal viscous disc instability, the interoutburst time, known as the recurrence time, can be increased by several orders of magnitude by truncating the inner disc (e.g. Matthews, Speith & Wynn 2004). Accretion disc truncation by various mechanisms has also been invoked to explain the ultraviolet (UV) lag in dwarf novae (e.g. Meyer & Meyer-Hofmeister 1994; King 1997). In this paper we treat magnetically enhanced accretion, but not the magnetic propeller effect.

A solution for the structure of a disc in a magnetic field is given by Brandenburg & Campbell (1998). By neglecting some mass transfer terms, they are able to find a detailed solution for the structure of the inner disc where a specific opacity model is assumed. In their solution magnetic pressure is considered in addition to magnetic torque, which improves the accuracy of the model for the inner disc regions. However, the azimuthal magnetic field is parametrized with respect to the shear between the disc and the stellar field using essentially the same form as used in this paper. As the effect of mass transfer into the disc is partially neglected, the solution is most valid in the very inner parts of the disc, where the magnetic field dominates the disc structure completely. A complementary solution is presented in this paper which, though less accurate in the centre of the disc, is continuous for the entire disc and can be applied to a range of opacity models. Furthermore, the solution derived below tends to the Shakura–Sunyaev solution in the case where the stellar magnetic field vanishes, and also at large radii.

The paper begins with a review of the necessary assumptions for the new solution. Then an approximate formulation for the magnetic torque is derived. This is applied, in conjunction with a new estimate of the magnetic truncation radius, to the derivation of a disc model. This model is generalized in order that it is compatible with any opacity that takes the form of a power law in density and temperature. We then explore how variables such as the mass transfer rate, magnetic field and stellar spin influence the disc structure.

2 THE SOLUTION
2.1 Assumptions
The derivation requires eight equations which follow in a form that is appropriate for our new magnetic case. The equations used here...
are similar to those presented by Frank, King & Raine (2002) in their derivation of the standard Shakura–Sunyaev solution. First, density in the disc can be approximated by

\[ \rho = \frac{\Sigma}{H}, \]  

(1)

where \( \Sigma \) represents the surface density and \( H \) is the scaleheight of the disc. For a thin disc the scaleheight can be approximated as

\[ H = \frac{c_s R^{1/2}}{(G M_* \mu)^{1/2}}, \]  

(2)

where \( c_s \) is the local speed of sound, \( R \) is the radial distance from the centre of the star in the plane of the disc and \( M_* \) is the mass of the primary star. The universal gravitational constant is represented by \( G \). This approximation is likely to break down close to the star, where a strong magnetic field may cause gas to stream along magnetic field lines. An ideal gas has an equation of state of the form

\[ P = \frac{\mu k T_c}{\mu m_u}, \]  

(4)

where \( k \) is the Boltzmann constant, \( T_c \) is the temperature in the mid-plane of the disc, the atomic mass unit is given by \( m_u \), and \( \mu \) is the mean molecular weight in the disc. This is frequently taken to be \( \mu \sim 0.6 \). Conservation of energy gives (Frank et al. 2002)

\[ \frac{4 \pi}{3} \frac{T_c^3}{\mu m_u} = \frac{9}{8} \rho \Sigma \frac{G M_*}{R^2}, \]  

(5)

If it is assumed that the disc is Keplerian until the boundary layer and that the disc has a large optical depth. Here \( \sigma \) represents the Stefan–Boltzmann constant and \( \tau \) is the opacity of the disc material. At this stage the form of the function \( v \Sigma \) is unknown, so that the equation must be kept in this general form. It is initially assumed that opacity may be approximated by Kramer’s law such that

\[ \tau = \kappa_0 \Sigma \rho \frac{T_c^{-7/2}}{t_c}, \]  

(6)

Kramer’s opacity is not a suitable model for all discs. Alternative opacity laws are discussed in Section 2.5. Following the same approach as for the normal Shakura–Sunyaev solution, we assume that the viscosity can be parametrized in the form

\[ v = \alpha c_s H. \]  

(7)

In addition, an expression for \( v \Sigma \) must be obtained. In the classical, non-magnetic Shakura–Sunyaev solution the relation

\[ v \Sigma = \frac{M}{3 \pi} \left[ 1 - \left( \frac{R}{R_*} \right)^{1/2} \right] \]  

holds, where \( R_* \) denotes the radius of the star. However, in the magnetic case the relation is more complex.

### 2.2 Magnetic torque

In order to obtain an expression for \( v \Sigma \) in the magnetic case, we integrate the continuity equation in the direction normal to the plane of the disc, which gives

\[ \frac{\partial \Sigma}{\partial t} + \frac{1}{R} \frac{\partial (R \Sigma v_R)}{\partial R} = 0. \]  

(9)

Similarly the radial component of the Navier–Stokes equation becomes

\[ \Sigma \left( \frac{\partial v_R}{\partial t} + v_R \frac{\partial v_R}{\partial R} - \frac{v_R^2}{R} \right) = - \frac{\partial P}{\partial R} - \Sigma \frac{GM}{R^2} \]  

\[ + \frac{4}{3 R^{7/2}} \frac{\partial}{\partial R} \left( R^{1/2} v_R \frac{\partial v_R}{\partial R} \right) - \frac{2}{3 R^3} \frac{\partial (R^2 v_R v_R)}{\partial R}, \]  

(10)

and the azimuthal component becomes

\[ \Sigma \left( \frac{\partial v_{\phi}}{\partial t} + v_R \frac{\partial v_{\phi}}{\partial R} \right) = \frac{1}{R^2} \frac{\partial}{\partial R} \left[ R^2 v_R \frac{\partial v_R}{\partial R} \left( \frac{l}{R^2} \right) \right] + \Sigma \Lambda, \]  

(11)

where we have added an external torque acting on the disc, with \( \Lambda \) representing the injection rate of specific angular momentum and \( I \) indicating the current specific angular momentum. This follows the same procedure as Lin & Papaloizou (1986), while a similar method is also employed by Pringle (1991).

Assuming that the disc is sufficiently cold and therefore that the dynamical time-scale is much smaller than the viscous time-scale, we can adopt a Keplerian approximation for the azimuthal motion such that \( v_{\phi} = (G M_*/R^{1/2}) \). Equation (12) can then be solved for the radial velocity in the disc,

\[ v_R = - \frac{3}{R^{7/2}} \frac{\partial (R^{1/2} v_{\phi})}{\partial R} + 2 \Lambda \frac{R^{1/2}}{\sqrt{G M_*}}, \]  

(13)

Inserting equation (13) into the continuity equation (9) yields an evolution equation for the surface density of the disc,

\[ \frac{\partial \Sigma}{\partial t} = \frac{3}{R^{1/2}} \frac{\partial}{\partial R} \left[ \frac{R^{1/2}}{\partial \Sigma} \frac{\partial (R^{1/2} v_{\phi})}{\partial R} \right] - \frac{1}{R^2} \frac{\partial}{\partial R} \left( 2 \Lambda \Sigma \frac{R^{1/2}}{\sqrt{G M_*}} \right), \]  

(14)

where the right-hand side is composed of a diffusion term, and an advection term that is due to the external torque.

To parametrize the specific torque \( \Lambda \), we can write

\[ \Lambda = \frac{I}{r_*} = \frac{\sqrt{G M_* R^{1/2}}}{t_\Lambda}, \]  

(15)

where \( t_\Lambda \) is the time-scale on which the local disc material gains angular momentum.

In the present case, the source of the torque is the magnetic interaction of a rotating, magnetic star with a partially ionized disc. The torque time-scale is therefore equivalent to the magnetic time-scale, \( t_\Lambda \sim t_{\text{mag}} \).

The inclusion of a magnetic field in such a hydrodynamic system introduces two additional terms to the Euler equation: a magnetic pressure term and a magnetic tension term, e.g. Dendy (1990). The magnetic pressure term is negligible where \( B \) is small. The magnetic tension term can be expressed as

\[ a_{\text{mag}} \sim \frac{1}{r_c} \left( \frac{B R B_\psi}{4 \pi} \right), \]  

(16)

where \( r_c \) represents the local radius of curvature of the field lines as a result of the torque, while \( B_\psi \) and \( B_R \) represent the vertical and azimuthal components of the magnetic field, respectively. We use the approximation \( r_c \sim H \) (Pearson, Wynn & King 1997). The ratio of vertical and azimuthal field strengths is related to the shear between the disc and the magnetic field. If it is assumed that the
field rotates with the star then this ratio can be expressed in the form (e.g. Livio & Pringle 1992)
\[
\frac{B_0}{B_i} \sim \frac{(\Omega_k - \Omega_*)}{\Omega_k},
\]
where \( \Omega_k \) represents the angular frequency of a body in a Keplerian orbit at a given radius and \( \Omega_* \) = \( \Omega_y \) denotes the angular frequency of the star and therefore of the magnetic field at all radii. The magnetic time-scale can now be defined in terms of magnetic acceleration and the Keplerian velocity by the relation
\[
\tau_{\text{mag}} \sim \frac{R \Omega_k}{a_{\text{mag}}} \sim -\frac{4\pi \gamma R^2 \rho H}{B_i^2} \left( \frac{\Omega_k^2}{(\Omega_k - \Omega_*)} \right).
\]
Finally, the volume density can be related to the surface density using equation (1) and for a dipole field, we have \( B_i \sim \mu B \) where \( \mu \) is the magnetic moment of the star according to its definition, in the steady state, the mass transfer rate \( \dot{M} \) may vary by a factor of the order of \( 10^3 \). This equation is not sensitive to the spin of the star. Indeed, by inspection we see that the Alfvén radius is greatly enhanced by the magnetic field, and the surface density so rapidly that the inner disc has a negligible surface density.

2.3 Truncation radius

The truncation radius \( R_t \) is the radius within which the accretion rate is greatly enhanced by the magnetic field, and the surface density falls rapidly to a value close to zero. The position of this radius is of particular interest as it can provide a boundary condition for equation (23). Mathematically the truncation radius can be identified with the largest real root of equation (23). In the standard Shakura–Sunyaev solution the boundary condition \( \nu \Sigma = 0 \) is applied at the boundary layer, near \( R = R_* \), as there is no viscous transport at that point. Physically, in the magnetically truncated case, the surface density at radii smaller than the truncation radius must always be zero. As equation (23) does not take this form, it is clear that the equation becomes unphysical where \( R < R_t \). It is therefore difficult to justify the use of the usual boundary condition in this case, as it will lie in an unphysical regime. If a truncation radius exists and can be identified, then it follows that \( \nu \Sigma = 0 \) at \( R = R_t \) may be used as a boundary condition.

There have been several attempts to locate the truncation radius due to the stellar magnetic field. The simplest of these estimates is the Alfvén radius. This is the radius at which magnetic pressure is equal to ram pressure. According to Frank et al. (2002) the Alfvén radius can be expressed by
\[
R_{\text{Alf}} = 5.1 \times 10^6 \frac{M_{16}^{-2/7} M_1^{-1/7} \mu_{30}^{4/7}}{c_s},
\]
where \( M_{16} \) represents the mass transfer rate in units of \( 10^{16} \) g s\(^{-1} \), \( M_1 \) is the stellar mass in solar masses and \( \mu_{30} \) is the stellar magnetic moment in units of \( 10^{30} \) G cm\(^3\). This equation is not sensitive to the spin of the star. Indeed, by inspection we see that the Alfvén radius can be related to \( \beta/M \) so that
\[
R_{\text{Alf}} = 1.25 (\beta/M)^{-7/2} \text{cm}.
\]
A more sophisticated approach than that of the Alfvén radius is adopted by Brandenburg & Campbell (1998), who include magnetic tension and magnetic pressure in their calculations. As some mass transfer terms are neglected, their solution is most valid very close to the star. Brandenburg & Campbell (1998) do not provide an analytic formulation for the truncation radius, but according to their data table the radii that they calculate are all close to the Alfvén radius.

The approach adopted here, which is consistent with Section 2.2 in that it treats the magnetic tension only, is to solve equation (23) for \( R_t \) by taking advantage of the fact that the inner accretion disc is empty in the case of magnetically enhanced accretion. If \( \Sigma (R_t) = 0 \), and if annuli at smaller radii also have a zero surface density then, in addition, \( \nu \Sigma = 0 \) at \( R = R_t \), and at immediately smaller radii. It is reasonable to conclude from this that \( \partial (\nu \Sigma) / \partial R = 0 \) at \( R = R_t \). Physically this can be interpreted as the effect of a rapid magnetic accretion. The inner edge of the disc loses angular momentum on a short time-scale and is dragged towards the stellar surface so rapidly that the inner disc has a negligible surface density. Some mass transfer terms are neglected, their solution is most valid very close to the star.
corotation radius from equation (23) so that
\[
\frac{M}{\beta} = 2\pi R_\gamma \gamma \left[ 1 - \left( \frac{R_t}{R_{co}} \right)^{3/2} \right]. \tag{26}
\]

This result can already tell us something concerning the behaviour of the truncation radius in this approach. It is clear that when \( M / \beta = 0 \), and the magnetic effect is at its most extreme, we have \( R_t = R_{co} \). It is not possible for \( R_t \) to exceed \( R_{co} \) providing that \( M / \beta > 0 \). This agrees with our previous assertion that the magnetic propeller does not have a steady-state solution. Of course when \( \beta / M = 0 \) there is no truncation and we find that \( R_t = 0 \). In reality, however, as the inner radius of the disc cannot be smaller than the stellar radius \( R_* \), the minimum value for the truncation radius is always \( R_t = R_* \). It is also clear that, according to this formulation, \( R_t \) is independent of the disc opacity, and of \( \alpha \) except in as much as \( M \) is related to \( \alpha \). There is no general analytic solution for \( R_t \) from equation (26), however, it is possible to obtain a solution for certain values of \( \gamma \). In order to do this, and to interpret the result, it is useful to make a parametrization of equation (26). We define
\[
Q = 2\pi R_{co} \gamma \left( \frac{\beta}{M} \right). \tag{27}
\]

Combining this with equation (26) yields
\[
Q = \frac{(R_t / R_{co})^{3/2}}{1 - (R_t / R_{co})^{3/2}}. \tag{28}
\]

The tension truncation radius given by equation (28), and with \( \gamma = 7/2 \), is plotted in Fig. 1. The Alfvén radius, which is also plotted, is smaller than the tension truncation radius for much of the regime of interest. Where the Alfvén radius is greater than the tension truncation radius the magnetic pressure may be said to dominate. However, when the Alfvén radius exceeds the corotation radius, it cannot represent the truncation radius of a steady-state accretion disc. For this reason the truncation radius given by equation (28) is preferable.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Plot to show how the Alfvén and tension truncation radii, as measured in units of corotation radii, vary as functions of the parameter \( Q \), defined in equation (26). The tension truncation radius is plotted for \( \gamma = 7/2 \). Note that the tension truncation radius never exceeds corotation.

The great advantage of the \( Q \) parametrization is that once the value of \( Q \) has been established from equation (27) it is straightforward to read off the value of \( R_t / R_{co} \), which can only vary from zero to unity. This can be done either from Fig. 1 or from similar plots for different values of \( \gamma \). This means that only one plot is required for all the parameter space of interest. For the fully magnetized case, where \( \gamma = 7/2 \), an exact analytic solution cannot be found for \( R_t / R_{co} \) from equation (28). In the region where \( R_t / R_{co} \) is close to unity, the \( \gamma = 7/2 \) case can be closely approximated by setting \( \gamma = 3 \), which yields a quadratic equation in \( (R_t / R_{co})^{3/2} \). This, of course, has an exact analytic solution, and as this is very close to the solution for \( \gamma = 7/2 \) for all of the region of interest, for that case we can make the approximation
\[
\frac{R_t}{R_{co}} \approx \left[ \frac{Q}{2} \left( \sqrt{\frac{4}{Q} + 1} - 1 \right) \right]^{2/3}. \tag{29}
\]

While the fractional error in this approximation becomes larger when \( R_t / R_{co} \) approaches zero, the absolute error is always less than 0.05, and, in any case \( R_t / R_{co} \) can never become very small as physically it should be limited by \( R_* \).

### 2.4 Synthesis

The result in equation (23) was obtained in Matthews et al. (2004). However, in that work the standard non-magnetic Shakura–Sunyaev viscosity prescription was substituted into the equivalent of equation (23) in order to obtain a surface density profile. This is clearly not self-consistent. In this paper no assumption of the form of \( \nu \) will be made other than that in equation (7). Matthews et al. (2004) also used an arbitrary inner boundary condition such that \( \partial \Sigma / \partial R = 0 \) at \( R = R_* \). In the following derivation the inner boundary condition \( \Sigma(R_t) = 0 \), which is equivalent to the condition \( \nu \Sigma = 0 \) at \( R = R_t \), will be used, as discussed in Section 2.3. The value of \( R_t \) can be obtained using equation (29) or by another method. In any case, where \( R_t < R_* \), the boundary condition \( \Sigma(R_t) \) should be used instead, in agreement with the usual approach (e.g. Frank et al. 2002). The above boundary condition for \( R_t > R_* \) yields an expression for the constant \( C \), when applied to equation (23), so that
\[
\nu \Sigma = \frac{M}{3\pi} \left[ 1 - \left( \frac{R_t}{R} \right)^{1/2} \right] - \frac{\beta R^{-\gamma} h}{3(\gamma - 2)} = \frac{M}{3\pi} f^3 v, \tag{30}
\]

where
\[
h = \left( \frac{R}{R_{co}} \right)^{3/2} \left[ 1 - \left( \frac{R_t}{R} \right)^{1/2 - \gamma} \right] \left[ 1 - \left( \frac{R_t}{R} \right)^{1/2 - \gamma} \right] + \frac{(\gamma - 2)}{\gamma - 1/2} \left[ 1 - \left( \frac{R_t}{R} \right)^{1/2 - \gamma} \right], \tag{31}
\]

and
\[
f = \left[ 1 - \frac{R_t}{R} \right]^{1/2} \tag{32}
\]

and
\[
v = 1 - \frac{\beta}{M R^\gamma (\gamma - 2) f^3}. \tag{33}
\]

where \( R_t \) replaces \( R_* \) throughout if \( R_t < R_* \). In the limit \( \beta = 0 \), this therefore yields the non-magnetic result in equation (8) as expected. Substituting relation (30) into equation (5) gives
\[
\frac{4\pi T_3^4}{3\tau} = \frac{9GM_* M}{8 R^3 3\pi f^3 v}. \tag{34}
\]
Following a procedure similar to that of Frank et al. (2002), we can now obtain a full analytic solution for the thin accretion disc, to which a magnetic torque is applied. Solving the set of algebraic equations (1)–(4), (6), (7), (30) and (34) yields the following expression for surface density:

\[
\Sigma = \left( \frac{32\sigma}{27\kappa_0} \right)^{1/10} \left( \frac{\mu m_a}{k} \right)^{1/14} \times \alpha^{-4/5} \left( \frac{GM_*}{R^3} \right)^{1/4} \left( \frac{M f^4}{3\pi v} \right)^{7/10}.
\]  

(35)

If \( \mu = 0.6 \) then the result can be rearranged into a more familiar form so that

\[
\Sigma = 3.7\alpha^{-4/5} \mathcal{M}_{17/10} M_1^{1/4} R_1^{-3/4} f^{14/5} v^{7/10} \text{ g cm}^{-2}.
\]  

(36)

The radial distance from the star, in the plane of the disc, is represented by \( R \) in units of \( 10^{10} \text{ cm} \). The numerical coefficient is sensitive to the values adopted for both \( \mu \) and for opacity. Other than this the result in equation (36) differs from the usual form only by the presence of the correction term \( \nu \), and the substitution of \( R_i \) for \( R \) throughout. It is instructive that the correction term differs from unity by an amount that is proportional to the ratio of \( \beta \) to the mass transfer rate. This means that, as would be expected, a sufficiently high-mass transfer rate can overcome the magnetic field of a star and the disc will revert to a Shakura–Sunyaev form.

The remaining parts of the solution are also easily found and are collected below. In all cases they differ from the usual Shakura–Sunyaev form by the presence of the factor \( \nu \):  

\[
H = 1.7 \times 10^3 \alpha^{-1/10} \mathcal{M}_{17/10} M_1^{3/8} R_1^{-5/8} f^{3/5} v^{3/10} \text{ cm}
\]  

(37)

\[
\rho = 3.7 \times 10^{-8} \alpha^{-7/10} \mathcal{M}_{17/10} M_1^{5/8} R_1^{-15/8} f^{11/5} v^{11/20} \text{ g cm}^{-3}
\]  

(38)

\[
T_c = 2.5 \times 10^6 \alpha^{-1/5} \mathcal{M}_{17/10} M_1^{1/4} R_1^{-3/4} f^{6/5} v^{3/10} \text{ K}
\]  

(39)

\[
\tau = 180\alpha^{-4/5} \mathcal{M}_{17}^{1/5} f^{4/5} v^{1/5}
\]  

(40)

\[
\nu = 2.9 \times 10^{12} \alpha^{4/5} \mathcal{M}_{17/10} M_1^{-1/8} R_1^{-3/4} f^{6/5} v^{3/10} \text{ cm}^2 \text{ s}^{-1}.
\]  

(41)

The radial velocity of the disc material can be calculated directly from

\[
\nu_R = -\frac{\dot{M}}{2\pi} R^{-1} \Sigma^{-1}.
\]  

(42)

### 2.5 The general opacity case

Kramer’s opacity is valid for discs where \( T_c \gtrsim 1 \times 10^4 \text{ K} \). For discs around young stars, for example, this condition will not always be valid. The range of opacities in such discs is discussed in Semenov et al. (2003). It is therefore useful to examine some alternative opacity laws and apply them in a manner similar to that found above. The process is straightforward, providing the opacity can be approximated by the form

\[
\tau = \kappa \Sigma \alpha^{a} T_b^b,
\]  

(43)

where \( \kappa, a \), and \( b \) are constants. A collection of opacity prescriptions in this form can be found in the appendix of Bell & Lin (1994). The equations of the full solution are collected below, where \( d = 20 + 6a - 4b \).

\[
\Sigma = \left( \frac{32\sigma}{27\kappa_0} \right)^{4/d} \left( \frac{\mu m_a}{k} \right)^{(16-4b)/d} \left( \frac{GM_*}{R^3} \right)^{(4-a-2b)/d} \times \alpha^{-16-2a+4b/d} \left( \frac{M f^4}{3\pi v} \right)^{(12+2a-4b)/d}
\]  

(44)

\[
H = \left( \frac{32\sigma}{27\kappa_0} \right)^{-2/d} \left( \frac{\mu m_a}{k} \right)^{(-8+2b)/d} \left( \frac{GM_*}{R^3} \right)^{(-7-a+2b)/d} \times \alpha^{-2-2a/d} \left( \frac{M f^4}{3\pi v} \right)^{(4+2a)/d}
\]  

(45)

\[
\rho = \left( \frac{32\sigma}{27\kappa_0} \right)^{6/d} \left( \frac{\mu m_a}{k} \right)^{(24-4b)/d} \left( \frac{GM_*}{R^3} \right)^{(-11+4b)/d} \times \alpha^{-14+4b/d} \left( \frac{M f^4}{3\pi v} \right)^{(8-4b)/d}
\]  

(46)

\[
T_c = \left( \frac{32\sigma}{27\kappa_0} \right)^{-4/d} \left( \frac{\mu m_a}{k} \right)^{(4+6a)/d} \left( \frac{GM_*}{R^3} \right)^{(6+4a)/d} \times \alpha^{(-4-4a)/d} \left( \frac{M f^4}{3\pi v} \right)^{(8+4a)/d}
\]  

(47)

\[
\tau = \kappa_0 \left( \frac{32\sigma}{27\kappa_0} \right)^{(4+6a-4b)/d} \left( \frac{\mu m_a}{k} \right)^{(16+24a)/d} \times \left( \frac{GM_*}{R^3} \right)^{(4+10a+4b)/d} \alpha^{(-16-16a)/d} \times \left( \frac{M f^4}{3\pi v} \right)^{(12+10a+4b)/d}
\]  

(48)

\[
\nu = \alpha \left( \frac{32\sigma}{27\kappa_0} \right)^{-4/d} \left( \frac{\mu m_a}{k} \right)^{(-16+4b)/d} \left( \frac{GM_*}{R^3} \right)^{(-4+a+2b)/d} \times \alpha^{(16+2a-4b)/d} \left( \frac{M f^4}{3\pi v} \right)^{(8+4a)/d}.
\]  

(49)

### 3 Form of Functions

Some typical disc structures will now be illustrated, using the full disc solution quoted in equations (44)–(49). The effect upon the solution of altering some of the parameters will also be shown. In every case the truncation radius, which is required for the solution, is obtained from equation (29). It is therefore implicitly assumed that \( \gamma = 7/2 \). Fig. 2 shows the solution as applied to a disc surrounding a typical young stellar object (YSO). The disc is plotted from \( R = 0 \) to \( 3.5 \times 10^{12} \text{ cm} \). This is an arbitrary maximum radius, used to illustrate the behaviour of the inner disc as, in a real YSO, the disc would be much larger than this. In the case of a single YSO, which occupies a simple potential, the model can be extended indefinitely. It should be noted, however, that, in a model with a great radial extent, it is unlikely that the entire disc could be correctly represented with a single \( \alpha \) parameter (e.g. Gammie 1996), and a single opacity prescription. A further complication in these massive,
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Figure 2. Analytic model of a typical YSO disc showing the surface density, disc scaleheight, viscosity and central temperature as a function of radius. The ratio of radial to azimuthal velocity is also shown. The disc is plotted from $R = 0$ to $3.5 \times 10^{12}$ cm. The star has a radius $R_\star = 1 \times 10^{11}$ cm, mass $M_\star = 1 M_\odot$, spin period $P_{\text{spin}} = 3$ d and a surface magnetic field of $B = 500$ G. The mass transfer rate throughout the disc are from $\dot{M} = 5 \times 10^{-9}$ to $2 \times 10^{-8} M_\odot$ yr$^{-1}$.

An alpha parameter of $\alpha = 0.01$ is adopted, and Kramer’s opacity is assumed. The vertical lines illustrate the stellar radius and the corotation radius.

Relatively cool discs, is that the gravitational instability may also contribute towards the disc viscosity. In the present case, however, where only the inner disc is modelled, Kramer’s opacity is assumed throughout the simulated region and a global value of $\alpha = 0.01$ is adopted. This is undoubtedly a simplification, but is adequate for the purpose of illustration. This value of $\alpha$ in YSOs is consistent with FU Orionis recurrence times in the magnetic cases. (Matthews et al. 2004).

The sample star has typical properties for a low-mass YSO so that $R_\star = 1 \times 10^{11}$ cm, $M_\star = 1 M_\odot$, $P_{\text{spin}} = 3$ d. The vertical magnetic field at the surface is $B_z = 500$ G, which is quite low when compared with recent measurements which give fields of the order of kG (e.g.
However, there is an uncertainty in how $\beta$ relates to field strength, which makes comparison with measured field strengths imprecise. The disc is illustrated at mass transfer rates of between $M = 5 \times 10^{-6}$ and $2 \times 10^{-8}$ M$_\odot$ yr$^{-1}$. This is a typical $M$ as measured at the star (Gullbring et al. 1998) and must, in steady state, be the $M$ into every annulus of the disc.

Fig. 2(a) shows the surface density as a function of radius. The form is similar to that of the usual non-magnetic Shakura–Sunyaev solution. It differs, however, in that the inner disc is truncated further out than the stellar surface. In this plot only results at radii greater than $R_*$ are shown as within $R_*$ the solution becomes unphysical. In reality this region may contain more complex accretion streams on to the magnetic poles of the star, which is difficult to reproduce in a one-dimensional treatment. Truncation should occur in the region where the magnetic time-scale and viscous time-scale become comparable. In this region the magnetic field is also sufficiently influential to produce effects such as magnetic warping (O’Sullivan et al. 2004). As the accretion rate increases, so does the surface density. As expected, an increased accretion rate also pushes the truncation radii in to smaller radii.

The disc scaleheight (Fig. 2b) follows an approximately linear relation with radius, as with the usual Shakura–Sunyaev solution. However, truncation occurs some distance from the star in the same manner and at the same radii as with the surface density. The disc becomes thicker at higher accretion rates, but for realistic parameters remains of the order of $H \sim 0.1 R$ so that the thin-disc approximation appears to be justified with these parameters. The temperature of the disc mid-plane, as illustrated by Fig. 2(d), follows a series of curves similar in form to those of the surface density. This is to be expected as $T_\phi \propto \Sigma^2$ for Kramer’s opacity. Fig. 2(e) plots $|v_R/v_\phi|$ as a function of radius where $v_\phi$ is assumed to be Keplerian and $v_R$ is calculated using equation (42). It can be seen that $v_R \ll v_\phi$ outside the truncation radius, so the assumption that the disc is cool is valid for the region modelled.

The advantage of the solution derived in Section 2.5 is that it can be applied to a general opacity case, providing that the opacity law follows the relation given by equation (43). The discs around young stars are likely to be dusty and cooler than those in most cataclysmic variables or X-ray binaries. The surface density and temperature of such protoplanetary discs are plotted in Fig. 3 for three different opacity laws. The opacity prescriptions for those plots were obtained from Bell & Lin (1994). The shape of the curves are affected only slightly by the change in opacity, but the magnitude of the densities and of the temperatures are altered by several orders of magnitude. It is notable, although unsurprising given the form of equation (26), that the truncation radius does not vary with opacity. Plotting the temperature of the disc mid-plane provides a useful check of consistency for opacity assumptions. It is clear for example that here the inner disc should not be represented by an opacity law which models the behaviour of ice crystals, as $T_\phi > 1 \times 10^3$ K in much of the inner disc if such an assumption is made.

In addition to the above-mentioned uncertainties regarding the disc viscosity, the young stellar objects have limitations as test cases for the new disc solution. The study of outburst behaviour is particularly difficult with YSOs. As only the very inner part of the disc is modelled, the mass transfer rate into the simulated region would in reality be related to the viscous state of the outer disc so that $\alpha$ and $\dot M$ are not independent. This complication is avoided for dwarf novae, where the whole disc can be simulated. In this case the mass transfer rate into the accretion disc is unrelated to the viscous state of the disc. Moreover, the behaviour of dwarf nova discs is in general better understood than that of those around YSOs (Warner 1995). In the hot viscous state dwarf nova discs are very close to the steady state.

In the case of binaries, however, the disc occupies a more complicated potential and an axisymmetric model such as that described in Section 2 must, unavoidably, be an incomplete one. Effects such as tidal resonances cannot easily be modelled in one dimension. A one-dimensional model should, however, be a reasonable approximation, whilst magnetic and viscous forces dominate over tidal ones as they do in the inner disc.

Fig. 4 shows the steady-state density profiles and viscosity profiles of a typical dwarf nova disc in a hot, high alpha and in a cold, low alpha viscous state. The disc is plotted from $R = 0$ to $2 \times 10^{10}$ cm, while the central white dwarf has a radius of $R_* = 1 \times 10^8$ cm. The white dwarf is spinning with a period of $P_{\text{spin}} = 100$ s and the

**Figure 3.** Analytic model of a typical YSO disc showing the surface density and the temperature for three different opacity prescriptions. The disc is plotted from $R = 0$ to $3.5 \times 10^{12}$ cm. The star has a radius $R_* = 1 \times 10^{11}$ cm, mass $M_* = 1$ M$_\odot$, spin period $P_{\text{spin}} = 3$ d and a surface magnetic field of $B = 500$ G. The disc has a mass transfer rate of $\dot M = 1 \times 10^{-8}$ M$_\odot$ yr$^{-1}$, an alpha parameter of $\alpha = 0.01$. In addition to Kramer’s opacity ($\kappa_0 = 5 \times 10^{25}$, $a = 1$, $b = -3.5$), two opacity prescriptions from Bell & Lin (1994) are illustrated: metal grains ($\kappa_0 = 0.1$, $a = 0$, $b = 0.5$) and ice grains ($\kappa_0 = 2 \times 10^{-2}$, $a = 0$, $b = 2$). The vertical lines illustrate the stellar radius and the corotation radius.
surface magnetic field is $B = 2 \times 10^3$ G, which would normally be insufficient for the binary to be considered magnetic, as intermediate polar binaries have fields of $B \sim MG$ (Warner 1995). It should be noted, however, that magnetic field strengths for white dwarfs with weaker fields are not generally well known. The disc opacity is modelled using Kramer’s law throughout this and subsequent disc models.

It is interesting, and possibly counterintuitive, that a change in $\alpha$ does not, in itself, cause $R_c$ to migrate. That this must be the case is very clear, however, from equation (26). In fact, for a given steady-state solution, the only parameters that set the position of the truncation radius are $M$, $\beta$, $R_c$, and $P_{\text{spin}}$. However, the truncation radius is not truly independent of $\alpha$ as, during the change of the viscous state, $\dot{M}$ also changes and $R_c$ is a function of $M$.

Although in reality the disc of a dwarf nova does not occupy a true steady state during either outburst or quiescence, it is a reasonable approximation to represent these two phases by two steady states.

The outburst cycle can then be interpreted as a cycle between these two steady states. The first plot in Fig. 4 illustrates the density profile of the disc in the hot and cold states where $\alpha = 0.1$ and 0.01, respectively, while $M = 1 \times 10^{-11}$ and $1 \times 10^{-10}$. The second plot shows disc viscosity for the same two steady states. If the viscous time-scale is given by

$$t_{\text{visc}} = \frac{R^2}{\nu}$$

then in the hot state the viscous time-scale is reduced from $t_{\text{visc}} \sim 1000$ to $\sim 100$ d at $R = 2 \times 10^{10}$ cm. This allows viscous processes to remove the inner disc material more rapidly and reduce the build up of mass.

It was noted in Section 2.4 that the new magnetic solution modifies the form of the standard Shakura–Sunyaev model only by the function $\nu$, which in turn varies from unity according to the ratio $\beta/M$, and by replacing $R_c$ by $R_\star$. This means that when $\beta$ tends to zero, and $R_c$ tends to $R_\star$, we recover the Shakura–Sunyaev solution and that for high $M$ the magnetic effect becomes less important. However, as $M$ also appears elsewhere in the solution, the effects of varying $M$ and $\beta$ are best examined separately. Figs 5 and 6 show a cold dwarf nova disc, where the white dwarf has the same properties as in Fig. 4. In Fig. 5 the effect on surface density of a change in $B$ is illustrated. In Figs 5(a) and (b), $B$ is varied from $2 \times 10^3$ to $1.15 \times 10^3$ G. The function $\nu$ is plotted for the same parameters in Fig. 5(b). As has been mentioned, there is no steady state for a magnetic propeller. However, the regime illustrated here is that of a ‘near-propeller’. The truncation radius is very close to its maximum radius, that of the corotation radius, and moves very little with increased field strength. Therefore, the great majority of the disc mass lies outside corotation and is propelled outwards. The stronger the field is, the greater this tendency will be. If the magnetic advection term is to be balanced by a viscous diffusion term then, for an increasing magnetic field, an increased surface density is required. In this case the function $\nu$ acts to enhance the surface density over most of the disc. Mathematically, there is no limit to the surface density enhancements that may be obtained in this manner. Physically, however, there are two reasons why this is not so. Either the disc will, at some point, reach so high a surface density that a thermal–viscous outburst will occur, or the disc will become very thick and the thin-disc treatment will cease to be valid.

For lower field strengths the disc behaves differently. This ‘strong-accretor’ regime is illustrated in Figs 5(c) and (d). Where $\beta = 0$ we have the Shakura–Sunyaev solution and $\nu$ retains a value of unity at all radii. As $\beta$ is increased the truncation radius migrates outwards, and in this case the surface density decreases throughout the disc. The effect is, however, more pronounced in the inner disc and the solution tends to the non-magnetic case with increasing $R_c$, as can clearly be seen from the form of $\nu$, which never exceeds unity, and so always reduces $\Sigma$.

Fig. 6 shows how the steady-state solution for a dwarf novae system, which is otherwise identical to that shown in Fig. 5, varies when $M$ is changed and $\beta$ is held constant. In both plots $M$ is varied from $1 \times 10^{-11}$ to $3 \times 10^{-11}$ $M_\odot$ yr$^{-1}$. Care is taken that the ratios $\beta/M$ are, in the first plot, the same as in Fig. 5(a) and, in the second plot, are the same as in Fig. 5(c), although the $B = 0$ plot has no analogue here. It can be seen that, while the truncation radii are identical to those in the case where $B$ is decreased, the surface densities are higher in the case where $M$ is increased. In fact, even in the ‘strong-accretor’ case, the surface density increases with increasing $M$. This is as expected from the form of equation (44). It
Figure 5. Analytic model of a typical dwarf nova disc showing the surface density $\Sigma$ as functions of radius for a range of magnetic field strengths. The disc is plotted from $R = 0$ to $2 \times 10^{10}$ cm. The star has a radius $R_\star = 1 \times 10^9$ cm, mass $M_\star = 1 M_\odot$ and spin period $P_{\text{spin}} = 100$ s. The disc has a mass transfer rate of $M = 1 \times 10^{-11} M_\odot$ yr$^{-1}$ and Kramer's opacity is assumed. The alpha parameter takes the value of $\alpha = 0.01$. The stellar magnetic field ranges from $B = 2 \times 10^4$ to $1.15 \times 10^4$ G in the upper two plots and from $B = 2 \times 10^3$ to $0$ G in the lower plots. These values are selected so that the ratio $\beta/M$ is the same as in the three graphs plotted in Fig. 6, in order that the effects of $M$ and $\beta$ may be seen independently.

Figure 6. Analytic models of a typical dwarf nova disc with a range of mass transfer rates. The disc is plotted from $R = 0$ to $2 \times 10^{10}$ cm. The star has a radius $R_\star = 1 \times 10^9$ cm, mass $M_\star = 1 M_\odot$, spin period $P_{\text{spin}} = 100$ s and a surface magnetic field of $B = 2 \times 10^4$ G in the first plot and $B = 2 \times 10^3$ G in the second. The disc has a range of mass transfer rates from $M = 1 \times 10^{-11}$ to $3 \times 10^{-11} M_\odot$ yr$^{-1}$ and Kramer's opacity is assumed. The alpha parameter takes the value of $\alpha = 0.01$. Again, the vertical lines illustrate the stellar radius and the corotation radius.
is significant that for $M > 2 \times 10^{-11} M_\odot \text{yr}^{-1}$ the truncation radii obtained by magnetic truncation are comparable to those of the order of $R_t \sim 10^8 \text{cm}$ predicted by King (1997) as a result of irradiation. However, at lower mass transfer rates the magnetic $R_t$ will become larger than that due to irradiation.

In order to illustrate the effect of a change in $P_{\text{spin}}$ upon the disc, a more rapidly rotating star is advantageous. For this reason we use a neutron star in a low-mass X-ray binary. In Fig. 7 the disc is plotted from $R = 0$ to $8 \times 10^9 \text{cm}$. This is only a model of the centre of the disc, as this is where the solution diverges most from the non-magnetic case. The central star has typical properties for a neutron star in a low-mass X-ray binary, with radius $R_\star = 1 \times 10^6 \text{cm}$, mass $M_\star = 1.4 M_\odot$ and a range of spin periods from $P_{\text{spin}} = 10$ to 0.1 s. The stellar magnetic field is set to $B = 2 \times 10^6 \text{G}$ for all plots. The mass transfer rate has a constant value of $\dot{M} = 1 \times 10^{-9} M_\odot \text{yr}^{-1}$. The disc is assumed to be in a cool state and to obey Kramer’s opacity throughout. As we only model the centre of the disc it is reasonable to ignore tidal effects. The corotation radii here vary according to $P_{\text{spin}}$ and are marked as vertical lines with the same line style as their associated plots.

The surface density plot shows that an increased spin rate causes the magnitude to increase and brings $R_t$ closer to the star. To understand what causes this it is useful to examine the form of $h$, defined in equation (31), which represents the potential for departure from the non-magnetic model as a function of $R$ and $P_{\text{spin}}$. This function is independent of the $\beta/M$ ratio, which is in any case held constant throughout Fig. 7. The function is made up of two terms, the first of which depends upon the stellar spin and the second of which does not. The two terms have opposing signs for $\gamma > 2$. The expression can be interpreted so that the second term represents the residual accretion effect of a notional non-rotating magnetic star. For long spin periods, and hence for large values of $R_{\text{ crit}}$, it is clear that $h$ tends to this non-rotational case. The first term in $h$ is the propeller-like term which becomes more important when $R > R_{\text{ crit}}$. The transition between accretor and propeller is not, however, so abrupt or straightforward as might be imagined because of the different powers of $R$ in the two terms. The first case in Fig. 7 represents a ‘near-propeller’ solution, which has the signature of a negative $h$, which increases the surface density. In this case a ‘near-propeller’ solution requires a stronger field than a ‘strong-accretor’ solution to truncate the disc at the same radius. This may seem counterintuitive, but it agrees with the results of Livio & Pringle (1992) and can be understood by comparing the mechanisms of the magnetic propeller and the accretor. With a given spin period, and disc mass, the magnetic propeller can potentially clear a larger hole in the disc, because it is not limited by the corotation radius. However, because a magnetic propeller has to act in opposition to viscous processes, and because it causes a build up of mass at the edge of the hole, a stronger field is required to maintain the same hole size in the propeller case. For faster spinning stars, the corotation radius is smaller and so the propeller mechanism operates from closer to the star. This causes the truncation radius to become smaller.

4 DISCUSSION

A new self-consistent analytic solution has been developed for a thin accretion disc, under the influence of a central magnetic field. As this model represents a steady-state disc it is not applicable to all accretion discs, and cannot be used for the magnetic propeller. The model was obtained in a similar manner to the non-magnetic Shakura–Sunyaev model, and indeed the result tends to that non-magnetic case when the magnetic parameter $\beta$ tends to zero. The two models also converge at large radii. The analytic model has also been expanded to include the effects of a range of opacities in addition to Kramer’s law. The solution has been applied in the cases of young stellar objects, cataclysmic variables and X-ray binaries.

The new model produces a solution in which the inner disc is frequently truncated further out than the stellar boundary layer. This is caused by magnetically induced accretion. The results depend upon how the position of the truncation radius $R_t$ varies as a function of the disc and star parameters. It has been shown that the truncation radius is independent of the opacity prescription used. The magnetic interaction model is uncertain, so that neither the exact values of the magnetic parameters $\beta$ and $\gamma$ nor their dependence on the field strength of the star are well known so far. Further work is therefore required to better constrain the relationship between the surface field and the truncation radius. In addition, a comparison of the position of the truncation radius between the one-dimensional
solution and three-dimensional techniques, such as smoothed particle hydrodynamics, could be used to ensure that the one-dimensional inner boundary condition acts in the same manner as the three-dimensional analogue.

The critical radius at which thermal viscous outbursts begin varies almost linearly with radius (Cannizzo, Shafter & Wheeler 1988). Therefore, a truncated disc can store more mass before it goes into outburst. As a result magnetic fields can exert a strong influence on outburst cycles in accretion discs by forcing outbursts to begin further out in the disc. This is discussed in more detail by Matthews et al. (2004). It would be instructive to perform full outburst simulations in which disc annuli are permitted to switch between the hot and cold state according to local triggers, and on a thermal time-scale. Similar work has been carried out to model dwarf nova outbursts in smoothed particle hydrodynamics (e.g. Truss, Wynn & Wheatley 2004). The steady-state viscosity prescription should not be applied to a time evolution model because it becomes inaccurate at low densities. In addition a negative value of $\nu$, which can occur for $R < R_t$, will cause equation (14) to have sinusoidal solutions. More realistic simulations could be performed by relating the viscosity to the surface density or by using a more physically motivated viscosity such as that generated by the magneto-rotational instability (e.g. Balbus & Hawley 1991).

Additional future work will involve the generalization of the new solution to encompass torques from other sources such as planets embedded in the disc. This would have important implications for the study of planetary formation and migration in discs. The effect of the magnetic torque on the spin of the central star could also be investigated in detail. The long-term effect of such spin evolution may be found to have a significant effect on the structure and behaviour of the disc.

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