Intrinsic Symmetry of Spacetime

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In modern physics, a profound thought is "symmetry inducing interaction". In this paper, following this thought, we pointed out that a symmetry of the quantum state of the vacuum $|\text{vac}\rangle$ (the so-called "intrinsic symmetry") rather than the usual symmetry of the Lagrangian $\mathcal{L}$ or the action $S$ that plays the key role to construct a correct theory for quantum gravity. Based on a hypotheses about the intrinsic symmetry of the spacetime, we developed the theory for quantum gravity and found that the gravitational interaction naturally emerges.

Quantum electrodynamics (QED) and Quantum chromodynamics (QCD) are successful gauge theories of electromagnetic and strong interactions that agree with experiments very well. The key points of QED and QCD are the U(1) and SU(3) gauge symmetries, respectively. When the gauge symmetry of a given quantum system is acknowledged, the right formula of its Lagrangian $\mathcal{L}$ or action $S$ can be obtained straightforwardly. This leads to the belief of "symmetry induce interaction". What’s kind of symmetry that determines the gravitational interaction obeying general relativity? Following Utiyama\textsuperscript{[1]}, various attempts have been done to construct the Yang-Mills type gauge theories for gravity, where the basic fields are the gauge fields of an appropriate group and Higgs fields. Based on local supersymmetry (a "gauge" symmetry mixing fermions and bosons), supergravity is another way towards quantum gravity. However, all these theories based on various symmetries of the Lagrangian or the action have not solve the trouble from quantum gravity.

In this paper, we found that the development of a complete theory for quantum gravity also follows the belief of "symmetry induce interaction". However, we point out that it is misleading to derive the right theory solely based on the symmetry of the Lagrangian $\mathcal{L}$ or the action $S$ that characterizes the invariant properties of our spacetime. Instead, it is the symmetry of the quantum state of the vacuum that plays more important role to construct a correct theory for quantum gravity. We call the symmetry of the vacuum $|\text{vac}\rangle$ (or the ground state) for a system to be intrinsic symmetry. The intrinsic symmetry characterizes the detailed variant properties of our spacetime. A key question arises, what kind of intrinsic symmetry for our universe that becomes the necessary and sufficient condition to produce gravitational interaction? In this paper, we will answer the question based on a basic hypothesis of intrinsic symmetry. Based on the hypothesis of intrinsic symmetry, the gravitational interaction is induced as a way of general relativity.

Hypotheses of intrinsic symmetry for spacetime. Our starting point is massive Dirac model, that describes the dynamics of matter in our universe. The Hamiltonian is written as $\mathcal{H} = \int (\hat{\Psi}^\dagger \hat{H} \hat{\Psi}) d^3x$ where

$$\hat{H} = \mathbf{c}^2 \hat{p} + mc^2 \mathbf{T}. \quad (1)$$

The Gamma matrices $\Gamma^i (i = x, y, z)$ and $\Gamma^t$ obey Clifford algebra, i.e., $\{\Gamma^i, \Gamma^j\} \neq 0$, and $\{\Gamma^i, \Gamma^t\} \neq 0$. For example, We assume that $\Gamma^t = \tau^z \otimes \mathbb{I}$, $\Gamma^x = \tau^x \otimes \sigma^2$, $\Gamma^y = \tau^y \otimes \sigma^3$, $\Gamma^z = \tau^z \otimes \sigma^1$. $m$ is the particle mass. In the following parts, we set $c = 1$ and $\hbar = 1$. The corresponding Lagrangian is written as $\mathcal{L}_{4D} = \hat{\Psi} (i\gamma^\mu \partial_\mu - m) \hat{\Psi}$ where $\hat{\Psi} = \Psi^\dagger \gamma^0$, $\gamma^\mu$ are the Gamma matrices defined as $\gamma^1 = \gamma^0 \Gamma^z$, $\gamma^2 = \gamma^0 \Gamma^y$, $\gamma^3 = \gamma^0 \Gamma^x$, $\gamma^0 = \Gamma^t$.

In modern physics, the quantum state of vacuum $|\text{vac}\rangle$ plays very important role. However, people take it for granted that $|\text{vac}\rangle$ is trivial and has no more structure. Usually, it is assumed a trivial intrinsic symmetry at beginning, i.e., $\mathcal{T}(\Delta x^i) |\text{vac}\rangle = |\text{vac}\rangle$ and $\mathcal{T}(\Delta t) |\text{vac}\rangle = |\text{vac}\rangle$ where $\mathcal{T}(\Delta x^i)$ and $\mathcal{T}(\Delta t)$ are translation operators along spatial and tempo directions, respectively. In this paper, we will show that the triviality of vacuum may be an illusion of an effective theory in long wave limit under coarse graining. Thus, to develop a complete theory of quantum gravity, one must know the more information about the original system underlying the superficial, excited parts. To pursue this goal, we consider a phenomenological hypothesis about intrinsic symmetry (the symmetry of vacuum $|\text{vac}\rangle$) to learn the deep features of our spacetime in small size.

**Hypothesis:** The vacuum state $|\text{vac}(\vec{x}, t)\rangle$ of flat spacetime obeys the following spatiotemporal invariance,

$$\mathcal{T}(\Delta x^i) |\text{vac}(\vec{x}, t)\rangle = e^{i\Delta \phi^i} |\text{vac}(\vec{x}, t)\rangle, \quad \mathcal{T}(\Delta t) |\text{vac}(\vec{x}, t)\rangle = e^{i\Delta \phi^0} |\text{vac}(\vec{x}, t)\rangle \quad (2)$$

where $\Delta \phi^i = k_i \Delta x^i$, $k_i = k_0 = \frac{\pi}{l_P}$ and $\Delta \phi^0 = \frac{\Delta t}{l_P}$. Here, $l_P = \sqrt{\frac{\hbar c}{m}}$ is Planck length.

According to the Hypothesis, our spacetime looks like a non-Abelian version of space-time crystal\textsuperscript{[2]}. We call the portion with lattice distance $\Delta x^i = l_P$, $\Delta t = t_{\text{clock}}$ to be a unit in spacetime. After shifting the distance $\Delta x^i = l_P$ along an arbitrary spatial $x^i$-direction $(i = x, y, z)$, the phase angle of the ground state of the vacuum $|\text{vac}(\vec{x}, t)\rangle$ changes $\pi$, i.e., $\mathcal{T}(l_P) |\text{vac}(\vec{x}, t)\rangle = e^{i\pi} |\text{vac}(\vec{x}, t)\rangle$; After shifting the time interval $\Delta t = t_{\text{clock}} = m^{-1}$ along
a tempo direction, the phase angle of the ground state of the vacuum changes \( \pi \), i.e., \( \mathcal{T}(m^{-1}) |\text{vac}(\vec{x}, t)\rangle = e^{it\pi} |\text{vac}(\vec{x}, t)\rangle \). Therefore, the periodic motion of vacuum indicates the existence of an internal “clock” of our spacetime with a period of time \( t_{\text{clock}} = 1/m \).

In the following parts, we will show the physical consequences of the hypothesis and develop a systematical theory for quantum gravity, including its kinematics theory and dynamic theory.

**Kinematics theory for quantum gravity – the description for quantum states of curved spacetime.** Firstly, based on the Hypothesis, we discuss the description of the quantum states of flat spacetime. Now, we have an (Euclidian) 4D spacetime with coordinates \((\vec{x}, it)\) and uniform Gamma matrix \(\Gamma^I_{\text{flat}}\) along \( x^I \)-direction \((I = x, y, z, t)\). The anticommutation condition matrices \(\{\Gamma^I_{\text{flat}}, \Gamma^J_{\text{flat}}\} = 2\delta_{IJ} \) and \([\Gamma^I_{\text{flat}}, \Gamma^J_{\text{flat}}] \neq 0\) indicate that our spacetime show the properties of non-commutating geometry. The quantum state of flat spacetime is characterized by \(\Gamma^I\). According to above definition, we have \([\Gamma^I, \Gamma^J, \Gamma^K]_{\text{flat}} = (\tau^x \otimes \sigma^x, \tau^y \otimes \sigma^y, \tau^z \otimes \sigma^z, \tau^z \otimes 1)\).

To intuitively show the quantum states of spacetime, we introduce the concepts of virtual spacetime crystal and the corresponding matrix-network on it. As shown in Fig.1(a), a virtual 1+1D spacetime crystal becomes an “lattice”, of which the lattice distance along spatial/tempo direction is Planck length/time \((l_P/t_P)\). During an spatial/tempo shifting \(l_P/t_P\), the phase changing of the vacuum is \(\pi/(mt_P)\). The matrix-network are described by \(\Gamma^a\) and \(\Gamma^I\) on all links between two nearest-neighbor lattice sites; (b) is an illustration of units of 1+1D spacetime.

![Virtual 1+1D spacetime crystal](image)

**FIG. 1:** (Color online) (a) An illustration for virtual 1+1D spacetime crystal and the corresponding matrix-network. The lattice distance along spatial/tempo direction is Planck length/time \((l_P/t_P)\). During an spatial/tempo shifting \(l_P/t_P\), the phase changing of the vacuum is \(\pi/(mt_P)\). The matrix-network are described by \(\Gamma^a\) and \(\Gamma^I\) on all links between two nearest-neighbor lattice sites; (b) is an illustration of units of 1+1D spacetime.

See the illustration in Fig.2(b). Now, the distances between two nearest-neighbor lattice sites on virtual spacetime crystal deform, i.e., \(\vec{x}(\vec{x}, t) - \vec{x} = \delta \vec{x}(\vec{x}, t) = e(x, \vec{x}, t)\), \(t(\vec{x}, t) - t = \delta t(\vec{x}, t) = e_t(\vec{x}, t)\) where \(e(\vec{x}, t)\) are vierbein fields that are the difference between the geometric unit-vectors of the original frame and the deformed frame. With help of the vierbein fields \(e^a\), the space metric is defined by \(e^a e_a = g^a_a\), \(e^a_i e_a^i = \delta_a^\mu\), \(\eta_{ab} e^a e^b = g_{ab}\) where \(\eta_{ab}\) is the Minkowskian matrix \(\eta_{ab} = \text{diag}(-1,1,1,1)\). Furthermore, one needs to introduce spin connections \(\omega^{ab}(\vec{x}, t)\) and the Riemann curvature 2-form as \(R_a^b = d \omega^a_b + \omega^a_c \wedge \omega_c^b\), where \(R_a^b = e_{ab}(\vec{x}, t)\gamma_{ab}\) are the components of the usual Riemann tensor projection on the tangent space.

As a result, under the geometric description, the Lagrangian for particles on curved spacetime becomes

\[
S_{3D} = \int \sqrt{-g} \Psi (e^a_i \gamma^a (i \partial_\mu + i \omega_\mu) - m) \Psi \, dx^4
\]

where \(\omega_\mu = (\omega_{ij}^a \gamma^a / 2, \omega_{ij}^a \gamma^{ij} / 2) \) \((i, j = 1, 2, 3)\) and \(\gamma^{ab} = -\frac{1}{2} [\gamma^a, \gamma^b] \) \((a, b = 0, 1, 2, 3)\). This model described by \(S_{3D}\) is invariant under local Lorentz transformation \(\hat{U}_{\text{Loc}}(\vec{x}, t) = e^{\omega_{ab}(\vec{x}, t) \gamma_{ab}}\). \(\gamma^5\) is invariant under local Lorentz symmetry as \(\gamma^5 \rightarrow (\gamma^5)^{t} = \hat{U}_{\text{Loc}}(\vec{x}, t) \gamma^5 (\hat{U}_{\text{Loc}}(\vec{x}, t))^{-1} = \gamma^5\).

For quantum description of spacetime \(|\text{vac}(\vec{x}, t)\rangle\), a non-uniform Lorentz transformation \(\hat{U}_{\text{Loc}}(\vec{x}, t)\) acts on the ground state of vacuum \(|\text{vac}(\vec{x}, t)\rangle\), i.e.,

\[
|\text{vac}(\vec{x}, t)\rangle \rightarrow |\text{vac}(\vec{x}, t)\rangle^{t} = \hat{U}_{\text{Loc}}(\vec{x}, t) |\text{vac}(\vec{x}, t)\rangle
\]

where \(\hat{U}_{\text{Loc}}(\vec{x}, t) = e^{\omega_{ab}(\vec{x}, t) \gamma_{ab}}\). The quantum states are obtained following the non-uniform Lorentz
...gauge representation with the changing of the spatiotemporal coordinates \((\tilde{x}, \tilde{t})_\text{curved} = (\tilde{x}(\tilde{x}, t), \tilde{t}(\tilde{x}, t))\).

Now, we have an abnormal, very complex quantum gauge theory on flat spacetime – a gauge theory with high-gauge symmetry.

Under the gauge representation, for each 3D submanifold in 4D curved spacetime, there exists a corresponding gauge structure for different definition of \(\gamma^0\). For example, for the gauge description of \((x, y, z)\)-submanifold (this is just the 3D space in 4D spacetime, we set \(\gamma^0 = \Gamma^5\). Now, the total SO(4) Lorentz transformation \(\tilde{U}_{\text{Lor}}(\tilde{x}, t) = e^{\phi_{ab}(\tilde{x}, t)}\gamma^{ab}\) is a combination of spin rotation transformation \(\tilde{R}(\tilde{x}, t)\) and spatial transformation \(\tilde{U}_{xy/yz}(\tilde{x}, t) = e^{i\delta \tilde{b}(\tilde{x}, t)}\).

To characterize the inhomogeneity of local Lorentz transformation, we define an auxiliary gauge field \(A^{ab}_{\mu}(\tilde{x}, t)\) that is written into two parts: SO(3) parts

\[
A^{ij}(\tilde{x}, t) = \text{tr}(\gamma^{ij}(\tilde{U}_{\text{Lor}}(\tilde{x}, t))d((\tilde{U}_{\text{Lor}}(\tilde{x}, t))^{-1})
\]

and SO(4)/SO(3) parts

\[
A^{i0}(\tilde{x}, t) = \text{tr}(\gamma^{0i}\tilde{U}_{\text{Lor}}(\tilde{x}, t)d((\tilde{U}_{\text{Lor}}(\tilde{x}, t))^{-1}) = \gamma^{0i}d(\gamma^{i}(\tilde{x}, t)).
\]

The total field strength \(F^{ij}(\tilde{x}, t)\) of \(i, j = 1, 2, 3\) components can be divided into two parts \(F^{ij}(\tilde{x}, t) = F^{ij} + A^{i0} \wedge A^{0j}\). According to pure gauge condition, we have Maurer-Cartan equation,

\[
F^{ij}(\tilde{x}, t) = F^{ij} + A^{i0} \wedge A^{0j} \equiv 0
\]

or

\[
F^{ij} = dA^{ij} + A^{ik} \wedge A^{kj} \equiv -A^{0j} \wedge A^{0i}.
\]

In general, under the gauge description, for an arbitrary 3D sub-manifold in 4D spacetime, we set \(\gamma^0 = \alpha^1 + \beta^2 + \gamma^3 + \delta^4\) with \(\alpha^2 + \beta^3 + \gamma^4 + \delta^2 = 1\). Here, \(\alpha, \beta, \gamma, \delta\) are constant. Now, the SO(4) transformation are \(\tilde{U}_{\text{Lor}}(\tilde{x}, t) = e^{\phi_{ab}(\tilde{x}, t)}\gamma^{ab}\). The auxiliary gauge field \(\tilde{A}^{ab}(\tilde{x}, t)\) are defined by \(A^{ab}(\tilde{x}, t) = \text{tr}(\gamma^{ij}(\tilde{U}_{\text{Lor}}(\tilde{x}, t))d((\tilde{U}_{\text{Lor}}(\tilde{x}, t))^{-1})\) and the gauge field strength by \(F^{ij} = dA^{ij} + A^{ik} \wedge A^{kj} \equiv -A^{0j} \wedge A^{0i}\). As a result, there exist infinite classes of gauge fields. This quantum gauge field theory is really about a mathematical structure with high-gauge symmetry.

In addition, there exists an inevitable connection between gauge description and geometric description of the same dynamic matrix-network. For example, within the representation of \(\Gamma^5 = \gamma^0\), the relationship between \(e^{i}(\tilde{x}, t)\) and \(A^{0i}(\tilde{x}, t)\) is obtained as \(e^{i}(\tilde{x}, t) \equiv (2l_p)A^{0i}(\tilde{x}, t)\); within another representation of \(\Gamma^5 = \tilde{\gamma}^0\) we have \(e^{i}(\tilde{x}, t) = (2l_p)\tilde{A}^{0i}(\tilde{x}, t)\). After considering all these relationships, we have an accurate, complete description of the quantum states of curved spacetime through dynamic matrix-networks. See detailed calculations in supplementary materials.

**Dynamic theory for quantum gravity – the description for time evolution of quantum states of curved spacetime.** In this section, based on the Hypothesis, we discuss the description for time evolution of quantum states of our spacetime and develop the dynamic theory for quantum gravity.

An important fact is the existence of a topologically equivalence principle, that is a fermionic particle of arbitrary 3D subspace in 3+1D spacetime becomes topologically defect that traps monopole-like structure by topologically changing the Gamma matrices, i.e., \(\Gamma^\mu_{\text{flat}} \rightarrow \Gamma^\mu_{\text{curved}}(\tilde{x}, t)\). Along arbitrary direction \(\phi^0\), the local Gamma matrices around a fermionic particle at center are switched on the tangentia sub-spacetime: along given direction (for example \(x^I\)-direction), the local Gamma matrices on the tangential sub-space are switched by \(e^{i\phi^I, \Delta\phi^I} (\Delta\phi^I = \pi)\). As a result, due to the rotation symmetry in 4D spacetime, a fermionic particle in spacetime becomes monopole on arbitrary 3D sub-manifold in...
4D spacetime. Thus, the spacetime is not usual spacetime crystal. See detailed calculations in supplementary materials.

We then use Lagrangian approach to characterize the topologically equivalence principle.

Firstly, we consider he path-integral formulation to enforce the topologically equivalence principle on a (x, y, z)-sub-manifold. From the point view of gauge description, a fermionic quasi-particle becomes topological defect of space that traps a “magnetic monopole” of the auxiliary gauge field, i.e., \( \int \sqrt{-g} \Psi^I \Psi d^4x = q_m \) where \( \sqrt{-g} \Psi^I \) denotes density of fermionic quasi-particles and \( q_m = \frac{1}{4\pi} \int \epsilon_{ijk} \epsilon_{ljk} F_{ij} \cdot dS_l \) is the “magnetic” charge of an auxiliary gauge field \( A^I \). The local topological constraint for a topological unit can be re-written into

\[
\frac{i}{4} \text{tr} \sqrt{-g} \bar{\Psi} \gamma^0 \gamma^i (\gamma^0 / 2) \Psi = i \epsilon_{0ijk} \epsilon_{0ljk} \omega^0_0 \frac{1}{4\pi} \hat{D}_0 F_{jk} \tag{10}
\]

where \( \hat{D}_0 = i \partial_0 + \omega_0 \) is covariant derivative in 4D spacetime. \( \omega^0_0 \) is a field that plays the role of Lagrangian multiplier. The upper index \( i \) of \( \omega^0_0 \) denotes the local radial Gamma matrix around a fermion, along which the Gamma matrix doesn’t change. In the path-integral formulation, to enforce such a topological constraint, we may add a topological BF term in the action that is \( -\frac{1}{4\pi} \int \epsilon_{0ijk} R^{0i} \wedge F_{jk} \) where \( R^{0i} = d\omega^0_0 + \omega^0_0 \wedge \omega^0_0 \). From \( F_{jk} = -A^{j0} \wedge A^{k0} \) and \( e^i \wedge e^i = (2l_p)^2 A^00 \wedge A^{k0} \). As a result, we have

\[
\frac{1}{4\pi l_p^2} \int \epsilon_{0ijk} R^{0i} \wedge e^j \wedge e^k.
\]

With the help of another definition of reduced Gamma matrices \( \gamma^0_0 \), there exist different topological BF terms \( S_{\text{MBF},i} \) that enforces the topologically equivalence principle on other 3D sub-manifolds. After summarizing the contribution from all 3D sub-manifolds, the upper index of the topological BF term \( R^j \wedge e^k \wedge e^l \) must be symmetric, i.e., \( i, j, k, l \in \{1, 2, 3, 0\} \). In summary, we have

\[
S_{\text{MBF}} = \frac{1}{16\pi G} \int \epsilon_{ijkl} R^{ij} \wedge e^k \wedge e^l \] that is same to the Einstein-Hilbert action \( S_{\text{EH}} \).

Finally, the total action is obtained as

\[
S = S_{\text{4D}} + S_{\text{MBF}} = \int \sqrt{-g} (\Psi (e^\mu_a \gamma^a \hat{D}_\mu - m) \Psi) d^4x + \frac{1}{16\pi G} \int \sqrt{-g} R d^4x \tag{11}
\]

where \( \hat{D}_\mu = i \partial_\mu + i \omega_\mu \). \( G = l_p^2 \) is the Newton constant. The variation of the action \( S \) via the metric \( \delta g_{\mu\nu} \) gives the Einstein’s equations that describes time evolution of quantum states of our spacetime \( R_{\mu\nu} = \frac{1}{8\pi G} \delta g_{\mu\nu} = 8\pi G T_{\mu\nu} \) as this is just the result from general relativity! Under time evolution of quantum states, the spacetime smoothly changes. This process looks like a usual classical one. However, under gauge representation, the curved spacetime is really the deformation of matrix-network with the dynamic Gamma matrices \( \Gamma^I_{\text{curve}}(\vec{x}, t) = U_{\text{Lor}}(\vec{x}, t) \Gamma^I_{\text{flat}}(U_{\text{Lor}}(\vec{x}, t))^{-1} \). Therefore, the Einstein’s equations becomes a classical equation of motion describing the time evolution of quantum states of our spacetime.

Another shocking result here is the classical gravity is really a “topological” quantum theory! We give an explanation on the reason why a quantum theory turns into a dynamic classical theory. According to the intrinsic symmetry, “time” is highly nontrivial. A fermionic quasi-particle is not only phase switching along a spatial direction, but also becomes topological defect along tempo direction. Therefore, the addition fermionic particles change the Gamma matrix along tempo direction and consequently, change the internal “clock” of the vacuum. Effectively, other particles nearby start moving. This is the “topological” mechanism of gravitational interaction.

In addition, if we set the particle mass to be 1, \( m = 1 \), the Newton constant \( G \) becomes a dimensionless parameter as \( G = (m l_p)^2 \). It is obvious that a weak gravitational interaction comes from a tiny dimensionless parameter \( m l_p = m t_p \ll 1 \). A tiny \( m l_p \) indicates a very slow internal “clock” comparing with Planck time, i.e., \( t_{\text{clock}} \gg t_p \). As a result, the greater the mass \( m \), the stronger the gravitational interaction.

Collective excitations – spin-2 gravitational waves. An important issue for quantum gravity is the collective excitations of spacetime. In this section, we will show that the collective excitations of spacetime are just the spin-2 gravitational waves.

We take a collective excitation moving along z-direction as an example. The perturbation of the spacetime comes from dynamical fluctuating Gamma matrices, i.e., \( \Gamma^I_{\text{curved}} = U_{\text{Lor}}(\vec{x}, t) \Gamma^I_{\text{flat}}(U_{\text{Lor}}(\vec{x}, t))^{-1} \) where \( U_{\text{Lor}}(\vec{x}, t) = e^{i3\pi \phi_0 \sin(\omega t - k z)} \). We define the vierbein fields \( e^a(\vec{x}, t) \) that are supposed to transform homogeneously under the local symmetry, and to behave as ordinary vectors under a wave-like Lorentz transformation along z-direction,

\[
\delta \phi^x = \frac{\pi}{a} \delta x(t) = \frac{\pi \phi_0}{2l_p} y_0 \sin(\omega t - k z), \tag{12}
\]

\[
\delta \phi^y = \frac{\pi}{l_p} \delta y(t) = \frac{\pi \phi_0}{2l_p} x_0 \sin(\omega t - k z).
\]

Because a space rotation on xy-plane corresponds to a rotation on spin-xy-plane, the spin of this transverse collective wave is 2. This is a gravitational wave with × polarizations.

In addition, there exists a new quantum effect for gravitational waves: quantum transverse spin resonance effect. Because the gravitational waves is obtained by a non-uniform Lorentz transformation \( U_{\text{Lor}}(\vec{x}, t) = e^{i3\pi \phi_0 \sin(\omega t - k z)} \), when there exists a gravitational wave, the corresponding spin eigenstates \( |\psi\rangle \) for fermionic particles turns into \( U_{\text{Lor}}(\vec{x}, t) |\psi\rangle = e^{i3\pi \phi_0 \sin(\omega t - k z)} |\psi\rangle \). This a new quantum effect for gravitational waves which is to be observed in experiments. If the dimensionless amplitude “h” is introduced to describe the maximum displace-
ment per unit length from the polarization of gravitational waves, the quantum transverse spin resonance effect is estimated to be order of $h$. Nature sets a natural amplitude of the spin perturbation $\phi_0 \sim h \sim 10^{-21}$. Another new quantum effect for gravitational waves is quantum non-Abelian interference effect. For a gravitational wave with $\times$ polarizations on $yz$-plane move along $x$-direction $\hat{U}_{1,\text{Lor}}(\vec{x}, t) = e^{i\Gamma x \phi_0 \sin(\omega t - kz)}$, when another gravitational wave with $\times$ polarizations on $xz$-plane move along $y$-direction $\hat{U}_{2,\text{Lor}}(\vec{x}, t) = e^{i\Gamma y \phi_0 \sin(\omega t - ky)}$, there exists a weak gravitational wave with polarizations on $xy$-plane move along $(x+y)$-direction and twice wave length of original waves, i.e., $e^{i\Gamma z \delta \phi_y(y, t) \delta \phi_x(x, t)/2}$. In the classical limit $h \to 0$, the quantum non-Abelian interference effect disappears.

Because the Einstein-Hilbert action comes from topological BF terms, there is no renormalization effect on the topological BF terms from self-interaction between gravitational waves. The situation is similar to that in 2+1D nonAbelian Chern-Simons theory.

Discussion and conclusion. In this paper, instead of focusing on the usual symmetry of Lagrangian $\mathcal{L}$ or action $\mathcal{S}$, we concentrate on the intrinsic symmetry – the symmetry of the quantum state of vacuum $|\text{vac}(\vec{x}, t)\rangle$ in a physical system. We found that under a hypothesis of intrinsic symmetry, the gravitational interaction naturally emerges. This Hypothesis is proposed about the symmetry characteristics of the spacetime that helps us develop the theory for quantum gravity. Based on the picture of dynamic matrix-networks, gauge representation becomes an accurate, complete description of the quantum states of curved spacetime. In addition, the topology characteristics of particles on spacetime is explored that helps us develop the dynamic theory for quantum gravity. The Einstein-Hilbert action is really a topological BF term that exactly reproduces the low energy physics of the general relativity. Now, the spacetime could be globally curved by adding additional particles and thus particles become interacting each other by disturbing same spacetime. This work would definitely help researchers to understand the all mysteries in quantum gravity and unify different theories.

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