All Neural Networks are Created Equal

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Abstract

One of the unresolved questions in the context of deep learning is the triumph of GD based optimization, which is guaranteed to converge to one of many local minima. To shed light on the nature of the solutions that are thus being discovered, we investigate the ensemble of solutions reached by the same network architecture, with different random initialization of weights and random mini-batches. Surprisingly, we observe that these solutions are in fact very similar - more often than not, each train and test example is either classified correctly by all the networks, or by none at all. Moreover, all the networks seem to share the same learning dynamics, whereby initially the same train and test examples are incorporated into the learnt model, followed by other examples which are learnt in roughly the same order. When different neural network architectures are compared, the same learning dynamics is observed even when one architecture is significantly stronger than the other and achieves higher accuracy. Finally, when investigating other methods that involve the gradual refinement of a solution, such as boosting, once again we see the same learning pattern. In all cases, it appears as if all the classifiers start by learning to classify correctly the same train and test examples, while the more powerful classifiers continue to learn to classify correctly additional examples. These results are incredibly robust, observed for a large variety of architectures, hyperparameters and different datasets of images. Moreover, a point’s ranking is not affected by whether it is in the train or test set. Thus we observe that different classification solutions may be discovered by different means, but typically they evolve in roughly the same manner and demonstrate a similar success and failure behavior. For a given dataset, such behavior seems to be strongly correlated with effective generalization, while the induced ranking of examples may reflect inherent structure in the data.

1 Introduction

The recent success of deep networks in solving a variety of classification problems effectively, in some cases reaching human level precision, is not well understood. One baffling result is the incredible robustness of the learnt models: using variants of Stochastic Gradient Descent (SGD), with random weight initialization and random sampling of mini-batches, different solutions are obtained. Empirically, these solutions may correspond to different parameter values and possibly different local minima of the loss function, but they still achieve roughly the same overall accuracy. To shed light on this question, we need to be able to compare different network solutions. This is not straightforward, since there is hidden redundancy and ample symmetry in the underlying architecture. It is therefore meaningless to compare two architecture instances by measuring the similarity between their parameter values without taking into account those symmetries, at the very least. Instead, we represent each network instance by a binary vector in the size of the dataset, where each element in it is assigned 1 if the network classifies the corresponding data point correctly, and 0 otherwise.

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We measure the consistency of architecture. What happens when we compare different architectures? Now the overall accuracy is typically weaker than CNNs in image classification, and achieve lower accuracy. Yet upon inspection, we see that this pattern of results is very robust, independent of such choices as optimization method or hyperparameter values, the detailed architecture, or the particular dataset.

What happens when we compare different architectures? Now the overall accuracy is typically different when comparing two network instances. Still, in Section 4 we show that the same pattern of results occurs when analyzing different Convolutional Neural Network (CNN) architectures, including public domain architectures such as VGG, Alexnet, DenseNet and ResNet, and also deep linear networks. Different architectures may learn at a different speed and achieve different generalization accuracy, but they still learn in the same order. Specifically, they start by learning roughly the same examples, after which the more powerful networks continue to learn additional examples as learning proceeds. In Section 5 we show that this pattern of results also encompasses other standard families of classifiers, such as SVM and nearest neighbor classification. These classifiers are typically much weaker than CNNs in image classification, and achieve lower accuracy. Yet upon inspection, we see that the images these methods classify correctly correspond for the most part with the images that the more powerful CNNs classify correctly at the stage in which they reach similar accuracy.

In Section 6 we investigate cases where this robust pattern breaks down. We observe that when training a CNN on a random dataset - when the labels are randomly shuffled 1, or a dataset which is artificially constructed to be very difficult (by eliminating "easy" examples), the agreement between different classifiers is no longer observed. In all these cases, the model also exhibits poor generalization. This suggests that the pattern of results reported above is characteristic of the learnable domain, where the learnt neural network achieves effective generalization.

Related work. How deep neural networks generalize is an open question 2. Neural networks’ expressiveness is very broad 3, and they can learn any arbitrary complex function 4. This extended capacity can indeed be reached, and neural networks have been shown to be able to memorize datasets with randomly assigned labels 1. Nevertheless, the dominant hypothesis today is that in natural datasets they "prefer" to learn an easier hypothesis that fits the data rather than memorize it all 15. Our work is consistent with a hypothesis which requires fewer assumptions, see Section 7.

In various machine learning methods such as curriculum learning 6, self-paced learning 7 and active learning 8, examples are presented to the learner in a specific order 9,10. Although conceptually similar, here we analyze the order in which examples are learnt, while the aforementioned methods seek ways to alter it. Likewise, the design of effective initialization methods is a striving research area 11,12,13. Here we do not seek to improve these methods, but rather analyze the properties of an ensemble of network instances generated by the same initialization methodology.

2 Notations

Given some neural network architecture \( f \), and a labeled dataset \( \mathcal{X} = \{(x_i, y_i)\}_{i=1}^M \) where \( x_i \in \mathbb{R}^d \) denotes a single data point and \( y_i \in [K] \) its corresponding label, we analyze the consistency of \( f \) when repeatedly trained on \( \mathcal{X} \) from scratch. Let \( \mathcal{S} \) denote the set of different extents (total epochs of \( \mathcal{X} \)) used to train \( f \), where \( |\mathcal{S}| = E \). We create \( N \cdot E \) instances of \( f \), denoted as \( f_{i,1}, \ldots, f_{i,N} \) for every \( e \in \mathcal{S} \). We initialize each \( f_{i,e} \) using Xavier initialization 12 independently for each instance, and train it for \( e \) epochs, supplying it with uniformly sampled mini-batches in each iteration.

We measure the consistency of architecture \( f \) by comparing the classification outcome of the different instances of \( f \) trained with the same set of examples, and analyze this consistency throughout the entire learning process. We define the consistency score of an example \((x, y)\) during epoch \( e \) as follows:
\[ c^e(x, y) = \frac{1}{N} \sum_{i=1}^{N} 1[f_i^e(x) = y] \]

The consistency score \( c^e(x, y) \) measures the classifiers’ agreement when \( x \) is correctly classified. However, it does not take into account the classifiers’ agreement when it is not. We therefore define in addition the consensus score, a complementary score that measures the consensus of the classifiers - the largest number of classifiers that classify the example by the same label:

\[ s^e(x, y) = \max_{k \in [K]} \frac{1}{N} \sum_{i=1}^{N} 1[f_i^e(x) = k] \]

We say that example \((x_i, y_i)\) is easier than example \((x_j, y_j)\) with respect to epoch \( e \), if \( c^e(x_i, y_i) \geq c^e(x_j, y_j) \). We say that example \((x_i, y_i)\) is learnt at least as fast as example \((x_j, y_j)\) if from some epoch \( e' \) and onward, example \((x_i, y_i)\) is easier than example \((x_j, y_j)\). This is formally given by: \( \exists e' \in S_E: \forall e > e' \; c^e(x_i, y_i) \geq c^e(x_j, y_j) \). We define an example as easiest if it is learnt at least as fast as all other examples. Examples for which \( \forall e : c^e(x, y) = 1 \) are trivially easiest. Similarly, we say that example \((x_i, y_i)\) is most difficult if all other examples are learnt at least as fast as \((x_i, y_i)\).

Examples for which \( \forall e : c^e(x, y) = 0 \) are trivially most difficult.

### 3 Diversity in a single architecture

Throughout this paper we empirically investigate the distribution of the two scores defined above, measuring consistency and consensus, in an ensemble of classifiers \( \mathcal{F} \). In this section, we start with an ensemble obtained from a single neural network architecture repeatedly trained on the same training set \( \mathcal{X} \), with different initial conditions and with independently sampled mini-batches. Specifically, using the above notations, we start by analyzing the ensemble \( \mathcal{F}^e = \{f_1^e, ..., f_N^e\}, \; e \in S_E \). We focus on convolutional neural networks trained on visual classification tasks, and analyze the distribution of consistency scores over the entire learning process.

#### Measuring inter-classifier agreement.
When all the classifiers in \( \mathcal{F}^e \) are identical, the consistency score of each example is either 0 or 1, and its consensus is 1. This results in a perfect bi-modal distribution of consistency scores; we quantify bi-modality using the following measure suggested by Pearson [14] for an RV \( X \): kurtosis \( X \) – skewness\(^2\) \( X \) – 1; the lower this measure is, the more bi-modal \( X \) is. On the other hand, when all classifiers independently err in their classification, the distribution of both scores is expected to resemble a Gaussian centered around the average accuracy of the classifiers for the consistency score, and some lower value for the consensus score. It follows that the higher the mean consensus is, and the more bi-modal-like the distribution of consistency scores is around 0 and 1, the more similar the set of classifiers is.

#### Learning dynamics: 4 phases.
Different architectures may learn at a different speed, while hyper-parameter values also affect the progress of learning. In spite of this, we observe that qualitatively, the learning dynamics is similar in all the datasets we have tested. A typical dynamics is composed of 4 phases: (i) Initialization: with random weight initialization, the distribution of consistency scores appears Gaussian around chance level, see Fig. [1a] (ii) Beginning of learning: a very short phase, 0-10 epochs into learning in all the datasets we have analyzed. In this phase, most examples achieve a consensus score of 0 (less than chance), with an exponentially decreasing tail, as shown in Fig. [1b]. It appears that all the network instances learn a very weak classifier at this point. (iii) Bi-modality: this is the longest phase in most problems, and often lasts until the end of the learning process. The distribution of scores is almost bi-modal around 0 and 1, see Fig. [1c] With the progress of time, more and more examples transfer from consistency 0 to 1 quite rapidly. An example of this dynamics can be seen when training ResNet-50 on the ImageNet dataset, as depicted in Fig. [2]. This phase shows that throughout the entire learning process the instances are very consistent, learning the same examples approximately at the same time. It suggests that the initial conditions and the random mini-batch sampling have little effect on the learning process. (iv) End of learning: a phase that only exists for very easy datasets and training sets. In this case, all the instances reach almost 100%

\(^1\)For large enough \( N \) this follows immediately from the central limit theorem.
accuracy, resulting in a uni-modal distribution around consistency 1 as shown in Fig. 1d. This case can be viewed as a degenerate case of the bi-modal phase.

Figure 1: The distribution of consistency scores in the 4 phases of learning, training st-VGG on the small-mammals dataset. Top: train data. Bottom: test data.

Figure 2: The distribution of consistency scores as it changes throughout the entire learning process, describing 27 instances of ResNet-50 [15] which classify the test set of ImageNet. Epochs shown, from left to right: 0, 1, 5, 30, 40, 100. Epoch 0: the initialization phase; epochs 1 and 5: beginning of learning phase; remaining epochs: the bi-modality phase.

Unlike the consistency score, the consensus score is also affected by the consistency of the chosen labels when the networks are erroneous. Thus a consensus score of 1 indicates that all networks have classified the image in the same way, regardless of whether it is correct or incorrect. Generally, the mean consensus score increases with training. In Fig. 3 we can see the distribution of consensus scores for the cases depicted in Fig. 1 showing that indeed all the network instances classify examples in almost the same way, even when they make mistakes.

Figure 3: The distribution of consensus scores during the 4 phases of learning, in corresponding epochs as in Fig. 1 for st-VGG trained on the small-mammals dataset.

Robustness. The results reported above are extremely robust, seen in all the datasets and architectures we have investigated. The analyzed datasets include: MNIST [16], CIFAR-10 [17], CIFAR-100 [17], tiny ImageNet [18], ImageNet [19], and some of subsets these datasets. The architectures include: AlexNet [20], DenseNet [21] and ResNet50 [15] for ImageNet, VGG16 [22] and a stripped version of VGG (denoted st-VGG) for CIFAR-10 and CIFAR-100, and several different handcrafted networks for other sets (see details in Appendix A). The results were replicated while changing the following hyper-parameters: learning rate, optimizer, batch size, dropout, L2-regularization, width, length and depth of layers, number of layers, number and size of kernels, and activation functions. In fact, as shown in Section 4.2 the results can be reproduced without the use of non-linearities as well.

2 All the code will be published upon acceptance.
The order in which data points are being learnt is also very robust. So robust, that the removal of examples from the training set has little to no effect on the general order of learning. To see this, we train $N = 100$ instances of st-VGG on the small-mammals super-class of CIFAR-100 (small-mammals dataset, see Appendix B). We calculate the consistency score of the training set, and use it to remove examples from the training set, followed by the training of the same architecture on the remaining examples from scratch. When moving the easiest examples (2.5% of the train set) from the train set to the test set, their classification accuracy still remains perfect after a single epoch. In contrast, when moving only the most difficult examples from the train set to the test set, their classification accuracy is low at the beginning of learning (0.08 accuracy at epoch 1), and approaches 0 rapidly. Finally, when moving random examples from the train to the test set, their classification accuracy remains close to the original test set accuracy throughout the learning process. This set of results seems to indicate that the learning order is determined by an inherent property of the data.

4 Cross architectures diversity - neural networks

We saw in section 3 that random instances of the same architecture are consistent with each other, learning the examples in the same order and pace as illustrated in Fig. 4a. We now report similar consistency between different architectures which are trained on the same dataset. To see that, we need to be able to match epochs between the different ensembles of networks; this is not straightforward as both the final accuracy and the learning pace are likely to be different for different architectures. Surprisingly, when matching epochs by accuracy across two such ensembles, and despite differences in the pace of learning, we find that the order of learning remains the same (Fig. 4b).

4.1 Comparing public domain convolutional neural networks

Let $F^e = \{f^e_1, ..., f^e_N\}$ and $G^e = \{g^e_1, ..., g^e_N\}$ denote two ensembles of convolutional neural network architectures $f$ and $g$ respectively, trained on the same dataset. For both ensembles we repeat the analysis described above, calculating the consistency score of each example in every training epoch. Since each architecture is characterized by its own learning pace, we compare the consistency scores in pairs of epochs $e_1, e_2$, in which the mean error of the two architectures is similar up to $\pm 1\%$ error. Each matched pair of epochs $e_1, e_2$ corresponds to two ensembles of networks $F^{e_1} = \{f^{e_1}_1, ..., f^{e_1}_{N_1}\}$ and $G^{e_2} = \{g^{e_2}_1, ..., g^{e_2}_{N_2}\}$. It defines a distribution of examples over the $2 - D$ plane, for pairs of consistency scores, as shown in Fig. 4c-4d. In this distribution, each point $(x, y)$ is assigned the fraction of examples with consistency score $x$ for $F^{e_1}$ and $y$ for $G^{e_2}$.

Figure 4: Comparing the order of learning in ResNet [15] and AlexNet [20] on the ImageNet dataset [15]. (a) Correlation between 13 instances of ResNet and another 13 instances of ResNet $r = 0.99$, $p \leq 10^{-10}$. (b) Correlation between 27 instances of ResNet and 22 instances of AlexNet $r = 0.87$, $p \leq 10^{-50}$, see text for details. (c-d) $2 - D$ distribution of the consistency scores of ResNet (x-axis) and AlexNet (y-axis), in epochs that reach similar performance. (c) Accuracy 0.39: ResNet - epoch 4, Alexnet - epoch 30. (d) Accuracy 0.71: ResNet - epoch 40, Alexnet - epoch 80. For visualization, all entries above 4% are saturated white.

Fig. 4c shows the distribution over consistency scores when comparing ResNet50 $(N_1 = 27)$ and AlexNet $(N_2 = 22)$, both trained on the ImageNet dataset. We can see that this distribution is bi-modal, peaking around $(0, 0)$ and $(1, 1)$, where the values are an order of magnitude higher than the rest of the distribution. Moreover, the main diagonal of the $2 - D$ distribution is also quite prominent. This suggests that at the point in time when the architectures reach the same error, both have learnt the same examples, classifying most of the data in a similar way. These results have been reproduced for many pairs of epochs throughout the learning curve and corresponding to different
error rates, and for all datasets and architectures we have tested (see Appendix C for a complete list of experiments).

Next, let us measure the speed of learning of an example by averaging its consistency score across all training epochs. High score in this metric indicates that the example has been learnt sooner, and by more instances. In accordance, we compute the speed of learning of ImageNet examples using $N_1 = 27$ instances of ResNet on the one hand, and $N_2 = 22$ instances of AlexNet on the other. The correlation between these two functions in plotted in Fig. 4b. Even though the AlexNet error is significantly larger than ResNet (AlexNet Top-1 error: 0.45, ResNet Top-1 error: 0.24), the correlation between the order of learning is very high ($r = 0.87$, $p \leq 10^{-50}$), another indication that the two architectures indeed learn in a similar order.

Discussion: The two type of results reported here suggest that while different architectures may learn at a different speed, the data is learnt in the same order. It follows that all architectures which reach a certain error rate, classify the same examples in the same manner. We also see that stronger architectures, which reach a lower generalization error, seem to start off by first learning the examples that weaker architectures classify correctly, followed by the learning of more difficult examples.

4.2 Linear networks

Convolutional neural networks where the internal operations are limited to linear operators [23] define an important class of CNNs, as they are often used in the theoretical investigation of deep learning. It is natural to wonder, therefore, whether the bi-modal behavior observed in general convolutional neural networks also occurs in this special class of CNNs. The answer is in the affirmative.

We train $N = 100$ st-VGG networks on the small-mammals dataset, replacing all the activation layers by the identity operator, and changing the max-pooling into average-pooling, resulting in a linear CNN architecture. The performance of the linear networks is weaker (0.43 accuracy) than the original non-linear networks (0.56 accuracy). Still, the distribution of the consistency scores throughout the entire learning procedure is bi-modal (Pearson: 0.055), and this bi-modality is even more pronounced than the bi-modality in the non-linear (Pearson: 0.22) case previously discussed, see Fig. 5c.

![Figure 5](image)

Figure 5: Linear networks. (a-b) Comparing linear to non-linear networks. In blue, the distribution of consistency score for st-VGG trained on the small-mammals dataset. In red, the distribution of a subset of the examples which are classified correctly by a linear version of st-VGG at the given epoch: (a) epoch 1, (b) epoch 140. (c) Distribution of consistency scores for linear st-VGG trained on the small-mammals dataset.

In accordance with the previous results, the order by which examples are learnt by the linear networks is also similar to the order observed in the learning of non-linear networks. Moreover, the examples learnt by the linear networks are mostly the examples learnt by the non-linear networks early on. It is, however, difficult to visualize the dynamics of this learning as is done in Fig. 4, for example, since linear networks learn very fast. Instead, in Figs. 5a, 5b we plot (in blue) the consistency distribution induced by the non-linear networks. Superimposed in red is the fraction, among these points, of all the images correctly classified by linear networks, at the beginning of learning of both classifiers (epoch 1) and at the end of learning (epoch 140) respectively. We can see that the examples which are classified correctly by the linear network are almost always classified correctly by the non-linear networks, and this similarity is most pronounced at the ‘beginning of learning’ phase, after which the linear networks gradually fall behind the non-linear networks.
5 Cross architectures diversity - other learning paradigms

Up to now we investigated a variety of neural network architectures, revealing a common learning pattern in many classification problems defined by some data $X$. The next question now presents itself - can we see the same pattern in other classification methods and paradigms? We first investigate the boosting methodology based on linear classifiers as weak learners. Neural networks and AdaBoost have built-in mechanisms for measuring examples’ difficulty; in neural networks, we can measure the accuracy over time, and in AdaBoost the accuracy over the number of weak learners. Subsequently, we investigate other machine learning paradigms which do not necessarily contain a trivial way to measure difficulty, including SVM, KNN classifier, perceptron, decision tree, random forest and Gaussian naïve Bayes. Interestingly, we still find a strong correlation with the order of learning as defined above, in that these weaker classifiers tend to fit the easier examples which the neural networks first learn.

Boosting linear classifiers. We use AdaBoost [24] with $k$ weak linear classifiers, trained on the small-mammals dataset. As commonly observed, adding more classifiers (increasing $k$) improves the final performance. In Fig. 6a we plot accuracy for 3 groups of examples - easy, intermediate and difficult, where grouping is based on the consistency score of the CNN described above. The accuracy of boosting over easy examples is significantly higher than the general accuracy, and does not improve as we increase $k$. This result suggests that most of the easy examples can be classified linearly, and are learnt first by both boosting and our CNNs. On the other hand, accuracy when classifying difficult examples is worse than the general accuracy, and slowly decreases with $k$. For intermediate difficulty, the accuracy significantly improves with $k$. This suggests that the addition of classifiers to the AdaBoost enables the learning of some examples of intermediate difficulty. Overall, the order in which neural networks learn the data is positively correlated with the order of AdaBoost, see Fig. 6b.

Other classifiers. We train several machine learning paradigms on the small-mammals dataset: SVM (accuracy: 0.48), KNN classifier (accuracy: 0.36), perceptron (accuracy: 0.35), decision tree (accuracy: 0.31), random forest (accuracy: 0.48) and Gaussian naïve Bayes classifier (accuracy: 0.38). All classifiers under-perform the CNN architecture described above (accuracy: 0.56). As in the case of boosting, for each method, most of the examples which are classified correctly are among the first to be learned by the CNN architecture. In other words, for all these methods, which achieve low accuracy, it is the easier examples which are being learnt for the most part. This is illustrated in Fig. 7, demonstrating the phenomenon that during the learning process, most of the examples that are being learnt by other paradigms are the ones already learnt by neural networks.

6 The four phases of learning: are they always there?

In Section 3 we have introduced the 4 phases of learning, illustrated in Fig. 1 which appear very robustly in all the experiments presented until now, in both the train and test sets. Importantly, phase 3 - where the consistency score shows a bi-modal distribution, is the phase where most of the learning
takes place: in almost all the cases we have studied, most of the epochs induce a bi-modal distribution. In this section, we put this empirical observation to the test, and investigate the circumstances under which the bi-modal distribution of consistency scores disappears.

**Random labels.** We take the small-mammals dataset, and rearrange the labels such that every image is assigned a random label \([1]\). In this case, the training accuracy can reach 100\% when disabling drop-out (that acts as a regularizer), which indicates that the networks are able to memorize the data. Interestingly, the distribution of consistency scores in phase 3 is no longer bi-modal (minimum Pearson score of 1.07 on training data and 1.35 on test data during the entire learning process) but rather resembles a Gaussian centered around 0.8 during training and 0 during the test, see Fig. 8b. This may indicate that the bi-modality is related to the generalization property of learning rather than mere memorization, and might even indicate the level of overfit present in the current classifier.

![Figure 8: Distribution of consistency scores during the learning process using: (a)-(c) st-VGG trained on a randomized small-mammals dataset; top - train data, bottom - test data. (d) st-VGG trained on an artificially generated hard dataset; top - beginning of training, bottom - end of training.](image)

**Hard classification problem.** In the absence of easy examples, the bi-modality disappears as well. To see this, once again we train \(N = 100\) instances of st-VGG on the small-mammals dataset. We remove examples from the dataset iteratively: in each iteration, we remove the 2\% easiest examples as determined by the consistency score. When maintaining only 20\% of the most difficult examples, we no longer see phase 3 and its characteristic bi-modal distribution, see Fig. 8d. Moreover, the generalization accuracy drops dramatically from 0.56 to 0.3. In contrast, when maintaining 20\% of the training set arbitrarily chosen, the generalization accuracy only drops to 0.47. This seems to indicate that the availability of easy examples in the training set is necessary for learning to achieve the desired generalization. Once again, it seems like bi-modality is correlated with successful generalization.

7 Conclusions and discussion

Our empirical study suggests that the order in which data is learnt is an internal property of the data, and its discovery is strongly correlated with effective generalization. Importantly, the same order is observed regardless of whether the particular data point is in the train or test set. This result is very robust - insensitive to the details of the architecture, dataset, or optimization choices. Moreover, not only do different neural network architectures learn data in the same order, but many other machine learning paradigms learn the data in a correlative manner.

What does it tell us about the generalization of neural networks, a question which is considered by many to be poorly understood? Neural networks can memorize an almost limitless number of examples, it would seem. In order to achieve generalization, most training protocols employ some regularization mechanism which does not allow for unlimited data memorization. As a result, the network fits only the train and test examples it would normally learn first, which are, based on our analysis, also the "easier" (or more typical) examples. We hypothesize that this may explain why a regularized network discovers robust solutions, with little variability among its likely instances.
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Appendix

A Architectures

In addition to the public domain architectures we used, as described in section 3, we also experimented with some handcrafted networks. Such networks are simpler and faster to train, mainly for less commonly used datasets, such as the small-mammals dataset and tiny ImageNet. Here, we list all such architectures.

st-VGG  The stripped version of VGG which we used for most experiments, is a convolutional neural network, containing 8 convolutional layers with 32, 32, 64, 64, 128, 128, 256, 256 filters respectively. The first 6 layers have filters of size $3 \times 3$, and the last 2 layers have filters of size $2 \times 2$. Every second layer there is a $2 \times 2$ max-pooling layer and a 0.25 dropout layer. After the convolutional layers, the units are flattened, and there is a fully-connected layer with 512 units followed by 0.5 dropout layer. When training with random labels, we removed both dropout layers to enable proper training, as suggested in [25]. The batch size we used was 100. The output layer is a fully connected layer with output units matching the number of classes in the dataset, followed by a softmax layer. We trained the network using the SGD optimizer, with cross-entropy loss. When training st-VGG, we used a learning rate of 0.05 which we decayed by a factor of 1.8 every 20 epochs. Parameters were chosen after coarse grid-search, to yield the highest performance, although that in the context of this work they don’t affect the results.

small st-VGG  To compare st-VGG with another architecture, we created a smaller version of it: we used another convolutional neural network, containing 4 convolutional layers with 32, 32, 64, 64 filters respectively, with filters of size $3 \times 3$. Every second layer there is a $2 \times 2$ max-pooling layer and a 0.25 dropout layer. After the convolutional layers, the units are flattened, and there is a fully-connected layer with 128 units followed by 0.5 dropout layer. The output layer is a fully connected layer with output units matching the number of classes in the dataset, followed by a softmax layer. We trained the network using the SGD optimizer, with cross-entropy loss. We trained this network with the same learning rate and batch size as st-VGG.

MNIST architecture  when experimenting with the MNIST dataset, we used some arbitrary small architecture for simplicity, as all architectures we tried reached over 0.99 accuracy. The architecture we used had 2 convolutional layers, with 32 and 64 filters respectively of size $3 \times 3$. After the convolutions, we used $2 \times 2$ max-pooling, followed by 0.25 dropout. Finally, we used a fully connected layer of size 128 followed by 0.5 dropout and Softmax. We used a learning rate of 1 for 12 epochs, using AdaDelta optimizer and a batch size of 100.

B Datasets

The small-mammals dataset used in the paper is the relevant super-class of the CIFAR-100 dataset. It contains 2500 train images divided into 5 classes equally, and 500 test images. Each image is of size $32 \times 32 \times 3$. This dataset was chosen due to its small size, which allowed conducting experiments efficiently. All the results observed on this dataset were reproduced on large, public domain datasets, such as CIFAR-100, CIFAR-10, and ImageNet.

C Additional results

Induced class hierarchy.  The ranking of training examples induced by the consistency scores typically induces a hierarchical structure over the different classes as well. To see this, we train $N = 100$ networks on the small-mammals dataset, and calculate for each image the most frequent class label assigned to it by the ensemble of networks. In Fig. 9, we plot the histogram of the consistency score (as in Fig. 1), but this time each image is assigned a color, which identifies its most frequent class label (1 of 5 colors). It can be readily seen that at the beginning of learning, only images from 2 classes reach a consistency score of 1. As learning proceeds, more class labels slowly emerge. This result suggests that classes are learnt in a specific order, across all networks. Moreover, we can see a pattern in the erroneous label assignments, which suggests that the classifiers initially use fewer class labels, and only become more specific later on in the learning process.
Dynamics of individual image consistency. We now focus on consistency scores of individual images, as they evolve throughout the entire learning process. For most examples, the score may climb up from random (0.2) to 1 in 1 epoch, it may dip down to 0 and then go up to 1 after a few epochs, or it may go rapidly down to 0. Either, the score remains 1 or 0. These patterns are shown in Fig. 11 and support the bi-modality results we report above. The duration in which a certain example maintains a consistency score 0 correlates with the order of learning: the longer it has 0 consistency, the more difficult it is. A minority of the training examples exhibit different patterns of learning. For example, a few images (the green curve in Fig. 11) begin with a very high consistency score (near 1), but after a few epochs their score drops rapidly to 0 and remains there.

Diversity in single architecture. The bi-modal phase we report in section 3 was seen in all unmodified datasets we tested, across all architectures. Specifically, we’ve tested ImageNet on AlexNet (N = 22), ResNet50 (N = 27), DenseNet (N = 7). Mnist on the Mnist architecture (see Appendix A) with N = 100, CIFAR-10 and CIFAR-100 with VGG-16 (N = 20) and st-VGG (N = 100), tiny ImageNet with st-VGG (N = 100), small-mammals dataset with st-VGG (N = 100) and small st-VGG (N = 100), and finally randomly picked super-classes of CIFAR-100, specifically "aquatic-mammals", "insects" and "household furniture" with st-VGG (N = 100). The number of instances N is chosen according to our computational capabilities. However, in all cases, picking much smaller N suffice to yield the same qualitative results.

In addition to hyper-parameters which may differ between various architectures, we also experimented with changing the hyper-parameters of st-VGG trained on the small-mammals dataset, always observing the same qualitative result. All experiments used N = 100 instances. Specifically, we tried a large range of learning rates, learning rate decay, SGD and Adam optimizers, large range of batch sizes, dropout and L2-regularization.

Cross architectures diversity. In addition to the results in section 4, the same qualitative results were obtained for all 2 architectures we trained on the same unmodified dataset. We conducted the following experiments: ImageNet dataset: ResNet-50 vs DenseNet, AlexNet vs DenseNet. Aquatic-mammals and small-mammals super-classes of CIFAR-100: st-VGG vs small st-VGG, Tiny ImageNet: st-VGG vs small st-VGG, CIFAR-10 and CIFAR-100: VGG16 vs st-VGG. All of which yielding similar results to the ones analyzed in section 4.
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