Measurement of the Geometrical Decay of the Spin Hall Effect in Fe(CsAu)$_\nu$ Multi Layers

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December 23, 2021

Abstract

The anomalous Hall effect of Fe(CsAu)$_\nu$ is investigated ($\nu$ is an integer). Electrons with spin up and down experience a different degree of specular reflection at the FeCs interface. This yields a different mean free path for the two spin orientations. In the presence of an electric field parallel to the film plane one obtains a spin current in addition to the charge current. If one introduces impurities with a large spin-orbit scattering into the Cs host then the combination of spin current and spin-orbit scattering yields an anomalous Hall effect. By building in situ multi layers of CsAu (5nm of Cs and 0.04 atomic layers of Au) on top of an Fe film one can measure the relative magnitude of the spin current normal to the film. Within the accuracy of the experimental data the spin current in Fe(CsAu)$_\nu$ decays exponentially with a decay length of 20nm.

PACS: 73.50.-h, 72.25.Ba, 73.40.Jn, 73.21.Cd
1 Introduction

The investigation of spin currents and the anomalous Hall effect (AHE) have been attracting considerable interest in the past decades. The possible applications in spintronics [1], [2], [3], [4] will further accelerate the exploration of this interesting field.

An impurity with a strong spin-orbit interaction scatters spin up and down electrons with opposite left-right asymmetry. The asymmetric scattering of conduction electrons by spin-orbit scatterers was already investigated by Ballentine and Huberman [5], [6] almost 30 years ago. They explained the deviations of the Hall constant from the free electron values for heavy liquid metals such as Tl, Pb and Bi. They calculated the spin-orbit scattering using perturbation theory. One of the authors [7] has calculated exactly the AHE of a polarized electron gas in the presence of spin-orbit scattering (SOS) in terms of Friedel phase shifts.

The opposite left-right asymmetry of spin up and down scattering by spin-orbit interaction yields two related effect in zero magnetic field:

- In the presence of a spin current SOS yields an anomalous Hall effect. A spin current through a conductor with spin-orbit scattering yields a net transverse scattering and therefore a net transverse electric field. The spin up and down electrons in a spin current have opposite (drift-) velocities and therefore a spin-orbit scatterer deflects both spins to the same side. This generates an anomalous Hall effect, i.e., spin Hall effect with a transverse electric field.

- For a pure charge current the spin up and down electrons are scattered to opposite sides of the original trajectory. The SOS does not generate a charge imbalance. However, it creates opposite gradients in the chemical potential for spin up and down electrons. This effect has been predicted by Dyakonov and Perel [8] and Hirsch [9]. It has been recently observed in semiconducting samples [10], [11].

Both phenomena are two sides of the same effect. This effect became popular in recent years under the name of ”Spin Hall Effect”.

In this paper we investigate spin currents with the help of spin-orbit scattering impurities. Recently our group investigated the anomalous Hall effect (AHE) in double layers of FeCs which were covered with sub-mono layers of Pb and Au [12]. When the double layer FeCs with $d_{Fe} = 12.4nm$
and $d_{Cs} = 29.6\,nm$ was covered with 1/50 of a mono layers of Pb it showed a remarkable result. The AHE increased by about a factor of five when it was covered with 1/50 of a mono layers of Pb. (The geometry of the layers is shown in Fig.1a). In this spatial configuration the contribution of a Pb atom to the AHE was stronger by a factor of $2 \times 10^4$ than that of an Fe atom. If the FeCs double layer was covered with 1/50 atomic layer of Au then the AHE had the opposite sign and was about a factor four smaller.

![Fig.1a: A single Au coverage of an FeCs double layer.](image)

![Fig.1b: The geometry of Fe(CsAu)$_\nu$ multi-layers.](image)

Since Pb and Au are non-magnetic they don’t yield a magnetic AHE. Instead the AHE has to be due to a spin current in the Cs. The origin of the spin current is due to the fact that spin up and down electrons experience a different exchange potential on the Fe side of the FeCs interface. This yields a different specular reflection of spin up and down electrons at the interface. As a consequence spin up and down electrons have different mean free paths (MFP) and, in the presence of an electric field, different drift velocities. (The large MFP in the Cs and the high degree of specular reflection at the upper surface (of the order of 80-90%) enhanced this effect). The spin current is the difference between the spin up and down currents. Since the host Cs is non magnetic the carrier concentration of spin up and down electrons...
is equal to half the electron density $n/2$ and the spin current is equal to $j_s = n (-e) (v_+ - v_-)/2$. Our observation of the AHE in FeCsPb layers was an early observation of the spin Hall effect. The mechanism will be discussed in more detail in the discussion and the appendix.

In this paper we investigate the development of the spin current as a function of film thickness. We prepare multi-layers consisting of an Fe film which is covered in several steps with sequences of 5nm of Cs and an impurity coverage of 0.04 atomic layer of a noble metal (see Fig.1b). For the noble metal we use (i) the strong spin-orbit scatterer Au and (ii) the weak spin-orbit scatterer Ag. The Au and Ag impurities differ strongly in their spin-orbit scattering but are otherwise very similar.

In this paper we use the following abbreviations: AHE=anomalous Hall effect, AHC=anomalous Hall conductance, AH ..=anomalous Hall .., MFP=mean free path, SOS=spin-orbit scattering.

2 Experiment

The preparation of the multi-layers and the measurements are performed in situ in an evaporation cryostat. During the whole experiment the evaporation cryostat is inserted into the liquid helium bath of a superconducting magnet. All the walls surrounding the sample are at liquid helium temperature, while the walls surrounding the evaporation sources are at liquid $N_2$ temperature. The vacuum in our system is better than $10^{-11}$torr.

A thin Fe film with a thickness of about 10$\text{nm}$ and a resistance of about 100$\Omega$ is quench condensed onto a 4.5$K$ cold quartz substrate. The Fe film is annealed to 40 $K$. Then the Fe film is covered with a Cs film of 5 $\text{nm}$ and annealed to 12 $K$. In the next step 0.04 atomic layer of Au is evaporated on top of the Cs film, acting as Au impurities. Again the sandwich is annealed to 12 $K$. This CsAu evaporation sequence is repeated several times. The Cs is evaporated from a SAES-Getters evaporation source. The evaporation rates of the Cs and Au sources are calibrated before and after the evaporation. After each Au evaporation, we have an Fe(CsAu)$_\nu$ sandwich (where $\nu$ can be 1, 2, 3, ...).

After each evaporation (and annealing) the magneto-resistance and the Hall resistance of the sample are measured. The measurements are conducted
in the external magnetic field range between -7 and +7 T and at the temperature of 8 K. An identical procedure is used to prepare and investigate Fe(CsAg)\nu multi-layers.

In Fig.2 the conductance \( L_{xx} \) of an Fe(CsAu)\nu multi-layer is plotted as a function of the Cs(Au) thickness \( d_{Cs} \). Each time the Cs surface is covered with 0.04 atomic layer of the noble metal the conductance is reduced.

\[ \sigma_{xx} = \frac{L_{xx} (d_{film}) - L_{xx} (d_{Fe})}{d_{film} - d_{Fe}} \]

Here \( d_{film} = d_{Fe} + d_{Cs} \) is the total film thickness. Using the free electron model \( \sigma_{xx} \) yields the effective MFP \( l_{eff} \) of the conduction electrons in the Cs with Au impurities. The same calculation is performed for the Fe(CsAg)\nu multi-layers. In Fig.3, the effective mean free paths \( l_{eff} \) of the conduction electrons in the Cs are plotted for (CsAu)\nu and CsAg\nu multi-layers as a function of the total Cs thickness \( d_{Cs} \). The similarity between the Fe(CsAu)\nu and the

![Graph showing conductance vs. thickness](image)
Fe(CsAg)$_{\nu}$ multi-layers is quite striking. The first 5nm thick Cs film has an $l_{eff}$ of about 12nm. Then the coverage with 0.04 atomic layer of Au or Ag reduces $l_{eff}$ by about 4nm. The data show that $l_{eff}$ increases with increasing total Cs thickness approaching a value of about 17nm.

Fig.3: The effective mean-free path of the (CsAg)$_{\nu}$ and (CsAu)$_{\nu}$ multi-layers as a function of the total Cs film thickness, including the Ag or Au impurities.

The measurement of the Hall resistance yields the AH conductance. In Fig.4 the anomalous Hall conductance is plotted as a function of the applied magnetic field for the multi-layers Fe(CsAu)$_{\nu}$ where $1 \leq \nu \leq 7$. The normal Hall conductance is subtracted and the anti-symmetric part $[L_{xy}(+B) - L_{xy}(-B)]/2$ is plotted. The AHC is constant at large fields and can be extrapolated to $B = 0$ (as shown in the upper curve). This yields the AHC $L_{xy,0}$ in zero magnetic field.
Fig. 4: The anomalous Hall conductance of an Fe(CsAu)\(\nu\) multi layer as a function of the applied magnetic field for different numbers of layers \(\nu\).

In Fig. 5 the zero field AHC \(L_{xy}^0\) is plotted versus the thickness of the multilayers of (CsAu)\(\nu\) and (CsAg)\(\nu\) (the Fe thickness subtracted). Obviously the two systems behave very differently.
3 Discussion

3.1 The Fe(CsAg)$_{\nu}$ multi-layers

The pure but very disordered Fe film has an AH conductance of about $1.5 \times 10^{-5} \Omega^{-1}$. When the first Cs film is superimposed the AH conductance increases to a value of about $5.5 \times 10^{-5} \Omega^{-1}$. This induced AHE is a well-known phenomena which has been analyzed by one of the authors [13], [14] in a simple model. Its origin is briefly recalled in the appendix.

3.2 The Fe(CsAu)$_{\nu}$ multi-layers

Since the mean free path in Fe(CsAu)$_{\nu}$ is essentially identical to that in the Fe(CsAg)$_{\nu}$ multi-layers it experiences the same induced AH conductance as the Fe(CsAg)$_{\nu}$ multi-layers. But there is obviously an additional effect. In
this paper we will focus on the additional effect, i.e., the difference between the two multi-layers.

The AHC drops clearly when the first 0.04 atomic layer of Au are condensed onto the Cs. The difference between the CsAu and the CsAg has to be due to the spin-orbit scattering of the Au. It indicates that there is a spin current in the Cs layer.

To simplify the discussion we have plotted in Fig.6 the difference in the AHC between the Fe(CsAg)$_\nu$ and the Fe(CsAu)$_\nu$ multi-layers. It is important to remember that the additional contribution of the Fe(CsAu)$_\nu$ is negative and Fig.6 shows the absolute value of the difference. This AHC $\Delta L_{xy}$ is caused by the spin-orbit scattering of the Au. Again this proves that we have a spin current in the Cs film. The Au atoms measure the local spin current density, and $\Delta L_{xy}$ is proportional to the integrated spin current density.

\[
\Delta L_{xy} = 1.4 \times 10^{-4} \ast \left\{ 1 - \exp \left[ -\frac{(d_{CsAu} - 3nm)}{20nm} \right] \right\} \Omega^{-1}
\]

First we discuss the effect of the combined CsAu evaporations. The magnitude of $\Delta L_{xy}$ increases with $\nu$ and levels off for larger thicknesses of the (CsAu)$_\nu$. This means that the spin current extends quite deep into the (CsAu)$_\nu$ multi-layer. We expect that the electrons in the (CsAu)$_\nu$ are essentially unpolarized except very close to the FeCs interface. Therefore the
spin current is solely caused by the different drift velocities of the spin up and down electrons. As a consequence a scattering process that destroys the drift velocity will also destroy the spin current. Therefore we expect that the spin current decays exponentially with the distance from the interface as \( j_s^0 \exp \left( -\frac{z}{l_{chr}} \right) \) and the decay length \( l_{chr} \) is of the order of the MFP. If the Au atoms would be homogeneously distributed in the Cs then the AH conductance would integrate the spin Hall effect over the film thickness. This yields for \( \Delta L_{xy} \)

\[
\Delta L_{xy} \propto \int_0^d \exp \left( -\frac{z}{l_{chr}} \right) dz \propto \left[ 1 - \exp \left( -\frac{d}{l_{chr}} \right) \right]
\]

In reality the Au impurities are located at discrete distances of \( \nu d_{Cs} \) (\( \nu = 1, 2, \ldots \)) from the FeCs interface. This modifies the result for \( \Delta L_{xy} \) slightly. We find a good fit of the experimental data with the function \( \Delta L_{xy} = 1.4 \times 10^{-4} \times \left\{ 1 - \exp \left[ -\left( d_{CsAu} - 3nm \right) / 20nm \right] \right\} \Omega^{-1} \). This function is plotted in Fig. 6 as a dashed curve. The experimental characteristic length is \( l_{chr} = 20nm \). In the appendix we derive the spatial dependence of the spin current.

### 3.2.1 The Au-surface effect

Next we discuss the effect of the Cs and Au evaporation in more detail. Only for \( \nu = 1 \) does the Au condensation yield an increase of the AHC. For \( \nu = 2 \) the effect of the Au condensation is very small and for larger \( \nu \) the AHC actually decreases. From Fig.2 one recognizes that the condensation of Au onto the Cs reduces the conductance of the multi-layer. This reduction is equivalent to the conductance of about 2nm to 2.5nm of Cs.

This agrees well with a rather surprising observation that we made in a previous investigation. There we condensed Pb and Au onto an FeCs double layer. The Pb and Au introduced an additional AHC, but the dependence of the \( \Delta L_{xy} \) on the coverage of the surface impurities was quite unexpected. For a coverage of \( d_{Pb} \) or \( d_{Au} \) of about 0.03 atomic layer \( \Delta L_{xy} \) showed a large extremum, and at a coverage of 0.2 atomic layer \( \Delta L_{xy} \) had almost completely disappeared. Our interpretation of this behavior was that impurities at the surface introduced so much disorder close to the surface that (i) most of the spin current decayed before it reached the Pb (Au) impurities at the surface and (ii) the spin-orbit scattered electrons could not generate much current in y-direction because their MFP was so short. Our new experimental data in Fig.5 or Fig.6 demonstrate that this reduction of the MFP is so dominant
that after each Au evaporation $\Delta L_{xy}$ reduces. The new Au impurities do not generate any significant additional AHC, and the AHC of the lower Au layer is reduced because of the MFP reduction.

This reduction of the MFP in the last Cs layer can be (i) a reduction in the whole last Cs layer by a factor of two or (ii) a reduction to zero in the upper 2nm of Cs. The latter case corresponds to electron localization at the upper surface due to the impurities. In the appendix we will give some arguments for the latter case.

4 Conclusion

The anomalous Hall effect in Fe(CsAu)$_\nu$ multi-layers has been investigated. It is compared with the AHE of Fe(CsAg)$_\nu$ multi-layers. Although the two kinds of impurities cause almost identical normal scattering and the MFPs in the resulting multi-layers are essentially the same, the AHE is quite different. The difference in the AH conductance $\Delta L_{xy}$ is due to a spin current which experiences the spin-orbit scattering of the Au impurities. This effect has been baptized “Spin Hall Effect”. The spin current is due to the different specular reflection of spin up and down electrons at the FeCs interface. By condensing a series of CsAu layers the size of the spin current can be analyzed geometrically. It decays as $[1 - \exp(-z/l_{chr})]$ where $z$ is the distance from the interface and $l_{chr}$ is a length which has a value of $l_{chr} = 20nm$.

During the preparation of the multi-layers a thin 0.04 atomic layer thick Au layer is condensed onto a fresh 5nm thick Cs layer. Each time the longitudinal conductance $L_{xx}$ is reduced by $\Delta L_{xx}$. This $\Delta L_{xx}$ corresponds to the conductance of about 2nm of Cs. It is suggested that this is a (precursor of) surface localization of the conduction electrons in the Cs. Such an effect has recently been observed in the superconducting proximity effect of PbK double layers [15].
5 Appendix

5.1 Origin of the induced anomalous Hall effect

The pure but very disordered Fe film in the Fe(CsAg)$_\nu$ multi-layer has an AH conductance of about $1.5 \times 10^{-5} \Omega^{-1}$. When Fe is covered with the CsAg film the AH conductance increases with the coverage (as is shown in Fig.5). This is due to the following facts [13], [14]:

(i) If the electric field in the Fe points in the x-direction then the current has a finite y-component (which represents the AH conductance of the Fe film $L_{yx}(\text{Fe}) = d_{Fe} \star j_y / E_x$). A fraction of the conduction electrons cross the interface to the Cs and carry with them their y-component of the current. This current in y-direction yields an additional contribution the Hall conductance $L_{xy}$. This induced AH conductance increases with the Cs thickness. It saturates when the Cs thickness reaches about half the mean free path in the Cs.

(ii) The second contribution is caused by the electrons which are accelerated in the Cs. When those electrons cross into the Fe they have a much larger drift velocity than the native Fe electrons. They therefore yield a strongly enhanced AH conductance within a thin layer of the Fe. This layer has a thickness of about half the mean free path of the Fe electrons.

Both contributions are of the same order of magnitude and contribute to the increase in the AH conductance of the Fe(CsAg)$_\nu$ multi-layer. The role of the Ag impurities lies in the limitation of the mean free path to about 17nm. Therefore the increase of the AH conductance levels out at about 10nm. This mechanism of the induced AHE depends on AHE in the Fe layer and the mean free paths in the Fe and the Cs.

5.2 Origin of the spin current

We can imagine two different origins of the spin current in the Cs film, (i) transfer of the spin current in the ferro magnetic Fe film through the FeCs interface and (ii) different specular reflection of the spin up and down electrons in the Cs at the FeCs interface. The experimental data favor the second mechanism as the following discussion shows.

(i) The spin up and down electrons in the Fe have different densities and MFP's. Therefore the spin up and down current densities in the Fe are different. This represents a spin current which could cross the FeCs interface
and enter the Cs film. The resulting spin current density $j_s$ at the upper Cs surface should be at best independent of the Cs thickness. Any scattering within the Cs should reduce $j_s$ at the free Cs surface. Experimentally we observe, however, a larger AH conductance for the first Au coverage when the Cs thickness is larger. In a previous experiment we covered a FeCs double layer ($d_{cs} = 27.5\text{nm}$) with 0.02 atomic layer of Au and observed $\Delta L_{xy} = -8.8 \times 10^{-5} \Omega^{-1}$. In the present experiment we covered the FeCs double layer ($d_{cs} = 5\text{nm}$) with 0.04 atomic layer of Au and observed $\Delta L_{xy} = 1.2 \times 10^{-5} \Omega^{-1}$. Although the Au coverages are not identical it is obvious that the larger Cs thickness yields a large spin current density at the free surface.

(ii) The spin up and down electrons in the Cs experience a different degree of specular reflection at the FeCs interface. These electrons have accumulated a finite drift velocity before they are reflected. This drift velocity increases with larger Cs thickness because the effective MFP increases. The resulting spin current after the reflection is proportional to the current before the reflection. Therefore the spin current density at the free surface increases with increasing Cs thickness. This is the experimental observation.

5.3 Theoretical description of the spin current

5.3.1 Homogeneous spin current

Let us first consider a spin current in a non-magnetic metal film with a constant spin current density $j_s$. The metal contains normal and spin-orbit scattering impurities. The total relaxation time $\tau_0$ is the same for spin up and down electrons. In addition the SOS impurities scatter spin up and down electrons to the opposite film edges. One of the authors calculated the AH conductance for a spin current in the presence of SOS [7]. It can be expressed by an anomalous Hall cross section $a_{xy}$. (We use the symbol $a$ for the cross section instead of $\sigma$ to avoid confusion with the conductivity $\sigma$). If we restrict ourselves to (s,p)-scattering then the dominant term of the cross section $a_{xy}$ is

$$a_{xy} \approx -\frac{4\pi}{3k_F^2} \sin(\delta_{l,+} - \delta_{l,-}) \sin\delta_0 \cos(\delta_{l,+} + \delta_{l,-} - \delta_0)$$

(1)

(The full, lengthy expression for $a_{xy}$ is given in ref. [7]). Here $k_F$ is the Fermi wave vector and $\delta_{l,\pm} = \delta_{l\pm1/2,l}$ are the Friedel phase shifts for spin up and down electrons with the orbital and total angular momenta $l$ and $j = l \pm 1/2$. 

13
The physical meaning of this cross section is the following: A current density \( j_{x,\uparrow} \) of spin up electrons (\( \uparrow \parallel \hat{z} \)) encounters a SOS impurity. Due to the asymmetric SOS a fraction of the current is deflected in y-direction. This fraction is a current \( I_{y,\uparrow} \) which is equal to the current density times the cross section \( \alpha_{xy} \): \( I_{y,\uparrow} = a_{xy}j_{x,\uparrow} \). This current decays after the time \( \tau_0 \) i.e., after the distance of the MFP \( l_0 \). For a SOS impurity concentration \( n_i \) one obtains an AH current density in y-direction of

\[
j_{y,\uparrow} = l_0n_i a_{xy} \dot{j}_{x,\uparrow}
\]

For the spin down electrons one obtains an AH current density with the opposite sign

\[
j_{y,\downarrow} = -l_0n_i a_{xy} \dot{j}_{x,\downarrow}
\]

So the total current density in the y-direction is

\[
j_y = l_0n_i a_{xy} (j_{x,\uparrow} - j_{x,\downarrow}) = pl_0n_i a_{xy} j_c
\]

where \( p = (j_{x,\uparrow} - j_{x,\downarrow}) / (j_{x,\uparrow} + j_{x,\downarrow}) \) is the polarization of the current density. The resulting (spin Hall) conductivity is

\[
\sigma_{xy} = pm_i a_{xy} \frac{ne^2}{\hbar k_F l_0^2} \tag{2}
\]

The resulting AH angle is \( \tan \alpha = j_y/j_x = pl_0n_i a_{xy} \). The AH angle, the MFP and the impurity density are experimentally known. The AH cross section \( a_{xy} \) is known in terms of the Friedel phase shifts. If someone would calculate the Friedel phase shifts \( \delta_{i\pm1/2,1} \) one could directly measure the polarization of the current by means of the AH conductance.

### 5.3.2 Inhomogeneous spin current

In our experiment the FeCs interface acts the source of spin current. The appropriate treatment would be using the Boltzmann equation. If one includes the size effect in the thin films then the problem becomes rather involved. We treat the problem here in a simplified fashion. We divide the charge current density \( j_c(z) \) in the Cs film into \( [j^+(z) + j^-(z)] \). The superscript gives the sign of the \( k_z \)-component of the involved electrons. Here \( j^-(z) = \)
\[ j^+_\uparrow(z) + j^+_\downarrow(z) \] is the part of the charge current with \( k_z < 0 \) which moves towards the interface. Similarly \( j^+_s(z) = [j^+_\uparrow(z) - j^+_\downarrow(z)] \) is the (part of the) spin current with \( k_z > 0 \) which moves away from the surface. At the FeCs interface the spin up and down electrons experience a different degree of specular reflection, \( \lambda^\uparrow \) and \( \lambda^\downarrow \). The specular part of the reflected electrons maintains the accumulated drift velocity and contributes a current density \( j^+(0) = \lambda^\uparrow j^-\uparrow(0) + \lambda^\downarrow j^-\downarrow(0) \) at \( z = 0 \). When \( \lambda^\uparrow \neq \lambda^\downarrow \) then the interface generates a spin current \( j^+_s \) which moves away from the interface into the Cs and is given by \( j^+_s(0) = \lambda j^-\uparrow(0) \) where \( \lambda = \lambda^\uparrow - \lambda^\downarrow \). From now on the electric field has no effect on the size of the spin current.

The electrons (of the spin current) with \( k_z > 0 \) move with an average velocity \( \alpha v_F \) away from the interface into the Cs. Here \( v_F \) is the Fermi velocity and \( \alpha \) is of the order of 1/2. Within the Cs film both electron spins experience the same relaxation time \( \tau_0 \). Therefore \( \tau_0 \) is the decay time for both the charge current and the spin current, for example \( \frac{dj^+_s}{dt} = -j^+_s/\tau_0 \).

In the stationary situation this yields a spatial decay of the spin current.

\[ \frac{\partial j^+_s}{\partial z} = \frac{dj^+_s}{dt} / dz = -\frac{1}{\tau_0} \frac{1}{\alpha v_F} j^+_s \]

which yields

\[ j^+_s(z) = j^+_s(0) \exp \left( -\frac{z}{\alpha v_F \tau_0} \right) = j^+_s(0) \exp \left( -\frac{z}{l_{chr}} \right) \quad (3) \]

\[ = \lambda j^-\uparrow(0) \exp \left( -\frac{z}{l_{chr}} \right) = \frac{\lambda}{2} j^-\uparrow(0) \exp \left( -\frac{z}{l_{chr}} \right) \]

If there is a finite specular reflection \( r \) at the upper surface then part of the spin current \( j^+_s \) is reflected at the upper surface. This results in a spin current which propagates in the negative \( z \)-direction and has the form

\[ j^-_s(z) = j^-_s(d_{Cs}) \exp \left( -\frac{d_{Cs} - z}{l_{chr}} \right) = \frac{r\lambda}{2} j^-\uparrow(0) \exp \left( -\frac{2d_{Cs} - z}{l_{chr}} \right) \]

In the following we neglect the specular reflection at the upper surface and we treat the charge current density as constant in the film. Then the polarization \( p(z) \) of the current density is position dependent.

\[ p(z) = \frac{j^+_\uparrow(z) - j^+_\downarrow(z)}{j^+_\uparrow(z) + j^+_\downarrow(z)} = \frac{\lambda}{2} \exp \left( -\frac{z}{l_{chr}} \right) \]
To obtain the total AH conductance $L_{xy}$ one has to integrate equation (2) over $dz$.

$$L_{xy} = n_i a_{xy} \frac{ne^2}{\hbar k_F} \left[ \int_0^d p(z) \, dz \right]$$

$$= \frac{1}{2} \lambda n_i a_{xy} \frac{ne^2}{\hbar k_F} l_{chr}^2 \left[ 1 - \exp \left( -\frac{d_{Cs}}{l_{chr}} \right) \right]$$

(4)

Our experimental results agree within the accuracy of the measurement with the contribution of equation (4).

5.4 Interface localization

Our group recently investigated the effect of surface impurities on the conductance of alkali films using the superconducting proximity effect [15]. In these experiments it was demonstrated that the conductance of a 5nm thick K film on top of a Pb film lost its conductance when covered with a sub-mono layer of Pb. But it maintained its full ability to reduce the superconducting transition temperature of the Pb film. As discussed in [15] this is the behavior of localized electrons in the K whose localization length is larger than the film thickness. Then electrons in the layer can move perpendicular but not parallel to the layer.

Such a surface localization would explain why the AHE of Pb or Au impurities on top of an FeCs double layer disappears when the impurity coverage exceeds 0.03 atomic layer. The conductance and AHC plot in Fig.2 and Fig.5,6 suggest that the Au coverage repeatedly introduces surface localization or its precursor for the Cs conduction electrons. The condensation of the next Cs layer essentially removes this localization so that the Au impurities contribute to the AHC. It is also likely that the MFP of the conduction electrons in the (CsAu)$_\nu$ layers is anisotropic. The perpendicular MFP will be larger than the parallel one.

5.5 Local mean free path

In Fig.3 the average MFP of the conduction electrons in Cs with Au or Ag impurities is plotted. In this evaluation the conductance of the whole Cs film divided by the total Cs is evaluated. The MFP is calculated from the resulting conductivity. This evaluation is appropriate for pure films. The structure
of their initial layer recrystallizes somewhat when the initial layer is covered with the same material. When, however, the individual Cs layers are separated by a thin Au or Ag sub-mono layer then the structure of the previous layers can be frozen. In this case a local MFP might be more appropriate. It is calculated from the local conductivity $\Delta L_{xx}/5\text{nm}$. $\Delta L_{xx}$ is the conductance of the multi-layer minus the conductance of the last Fe(CsAu)$_{\nu}$. For the upper points in Fig.6 one has $\Delta L_{xx} = L_{xx}(\text{Fe(CsAu)$_{\nu}$Cs}) - L_{xx}(\text{Fe(CsAu)$_{\nu}$})$ and for the lower points $\Delta L_{xx} = L_{xx}(\text{Fe(CsAu}_{\nu+1}) - L_{xx}(\text{Fe(CsAu)$_{\nu}$})$.

![Fig.6: The local mean free path in the Fe(CsAu)$_{\nu}$ multi-layer.](image)

### 5.6 The normal Hall effect in Fe(CsAu)$_{\nu}$ multi-layers

The measurement of the Hall effect of the multi-layers also yields the Hall constant. Fig.7 shows the local Hall constant as a function of the Cs thickness. The upper points represent the last Cs mini layer and the lower points the last Cs layer with Au coverage. While the Hall constant of the individual Cs layers lies in the range of $-(63 \pm 3) \times 10^{-11} m^3/As$ the condensation of the noble metal increases the value to $-(77 \pm 2) \times 10^{-11} m^3/As$. The condensation of the Au or Ag impurities acts as if 25% of the Cs layer do no
longer contribute to the normal Hall effect.

![Graph](image)

Fig. 7: The local Hall constant for the Fe(CsAu)$_ν$ multi-layers as function of the total Cs thickness.

### 5.7 Asymmetric Hall curve

In Fig. 8 the Hall curves for Fe(CsAu)$_4$ and Fe(CsAg)$_4$ are plotted as a function of the magnetic field. The normal Hall conductance is subtracted, but the curves are not symmetrized. It is very obvious that the Hall curve for Fe(CsAu)$_4$ is not anti-symmetric. Such a behavior can be easily observed in the magnetoresistance of a ferromagnetic film or multi-layer. It occurs when the magnetic field is aligned perfectly perpendicular to the film. If the magnetic field is lowered and approaches $B = 0$ then the magnetic domains align parallel to the film plane. The orientation within the film plane is arbitrary. Therefore the angle between the current and the magnetization of the single domain will be arbitrary too. Generally the resistance is anisotropic and depends on the angle between current and magnetization. Then the resistance at zero and small fields depends on the accidental orientation of the magnetic domains within the film plane. One observes hysteresis. This can be avoided by slightly tilting the film substrate so that the magnetic field is no longer perfectly perpendicular to the film.

On the other hand such hysteresis is generally not observed for the AHE. Only the $z$-component of the magnetization $M$ contributes to the AHE. The
The z-component of the magnetization, $M_z$, is equal to the applied magnetic flux $B$ (for $B < M$). This is demonstrated by the perfectly anti symmetric shape of the AH curve of Fe(CsAg)$_4$. Therefore we believe that the asymmetric shape of the AH curve for Fe(CsAu)$_4$ contains interesting information about the system. It might suggest that the orientation of the electron spins in this field range is not parallel to the magnetic field. So far we have not investigated further this observation. It does not effect any of our conclusions because we only used the AHC for the large fields and extrapolated the linear part back to zero.

Fig.8: The anomalous Hall curves for Fe(CsAu)$_4$ and Fe(CsAg)$_4$ as a function of the magnetic field. The normal Hall effect is subtracted but the curves are not symmetrized.

Acknowledgement: The research was supported by the National Science Foundation NIRT program, DMR-0439810.
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