Maritime inventory routing problem with undedicated compartments: A case study from the cement industry

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Abstract. The cost of sea transportation in the cement industry located in an archipelago country has a large contribution of the overall cost because cement is a heavy product but the price is cheap and must be distributed to packing plants that are spread across several islands. The high level of competition among cement companies makes the issue of cost efficiency and inventory availability a major concern for companies. This paper proposed a solution to maritime inventory routing problems (MIRP) to minimize the cost of sea transportation faced by a cement company in the Indonesian archipelago in meeting the needs at the consumption port. Unlike other multi-product handling such as oil and other chemicals that require the use of special compartments on ships, undedicated ship compartments are generally used on multi-product cement carriers. To solve this problem, we developed a mixed-integer linear programming (MILP) model to minimize transportation costs with an inventory level that remains unfulfilled during the planning horizon period. The results of this optimization provide a global optimum with the total cost of transportation about IDR 3.69 billion for the second week period in October 2019 and maintained the availability of cement during this period.

1. Introduction
Past over a decade, the capacity of cement industry has increased by the entry of many new competitors in domestic cement industry. It is calculated that oversupply of cement will reach around 40 million tons per year in 2020. This oversupply then would be giving a tough competition to cement companies located in Indonesia and demand every single industry to increase its efficiency in every matter to get the price of goods as least as possible, so that a more reasonable sale price can possibly achieve a high market share. The expense of sea transportation is one of the major components of the cost that is need to be reviewed for an island area like in Indonesia. Fifteen percent of the total amount is contributed from the sea transportation expense.

The large percentage of the marine transportation cost component of the total cost is one of the focuses for companies to make efficiency in this sector. Besides maintaining the availability of cement in each packing plant is also a concern so as not to provide market opportunities for competitors. In fact, throughout 2019, each packing plant lacked cement stock for several days which resulted in providing opportunities for competitors to seize the existing market, especially in the second semester with a very high level of demand. The current problem in determining the assignment route for bulk cement distribution vessels is still determined manually without utilizing operational research methods. By developing an optimization of scheduling system [1], it provides an opportunity to reduce the cost of sea
transportation in the distribution of bulk cement and can avoid the shortage of cement stock at the packing plant.

This paper is a variation of a ship inventory routing and scheduling problem with undedicated compartments (sIRPSP-UC) [2], but with adjustments to problems encountered in a cement industry company. Adjustments to this problem such as routing rules where each ship's assignment can only visit one port only and must return to the production port. In contrast to assuming that partial unloading and visits to several ports are permitted [2].

This paper also takes references from previous papers for the model studied [3–7]. The problem with a single product model discussed [3–5]. Dedicated compartments problem for multi-product also discussed [4]. They supposed that compartments are assigned only to a specific product. This mean it is not permissible to assign a specific product to a compartment that has been used before by another product.

2. Problem description
The company has a bulk cement loading port called a production port. This port serves bulk cement needs for several units of packing plants that are scattered in various regions in Eastern Indonesia. This bulk cement is distributed using several ships that have heterogeneous characteristics in terms of cargo capacity, vessel dimensions, number of compartments, and transportation costs. The capacity and depth of the packing plant port dock vary. Considering the suitability between the dimensions of the ship and the depth of the dock at the port of the packing plant, only certain ships can visit a particular packing plant.

The cost of transportation with the basic freight system is predetermined in the contract agreement with the provider as the owner of the ship. Costs according to the contract vary for each ship and each destination of the packing plant. Other cost components such as port costs, demurrage fees, and others are not taken into account because both the origin and destination ports are the same company ownership assets.

The daily consumption capacity of each packing plant is different. Each consumption port has one or several silos for storing the cement. The level of inventory of bulk cement products at the production port is not required and is assumed to be unlimited, because the amount of production compared to the daily consumption of the packing plant is very large and can meet the needs of each packing plant in the planning horizon period. But for the consumption port at the packing plant, the product inventory level at the silo is required, both the maximum and minimum levels. At the beginning of the planning horizon, each silo will have a given initial inventory level. The minimum inventory level at the packing plant silo is determined by calculating travel time and daily consumption rating.

Bulk cement products that are distributed have two different types, namely OPC type, and PCC type. In this paper, we called by type K1 and K2. The two types of cement cannot be mixed if they are loaded on the same ship unless separated in different compartments. Each compartment is not dedicated to one particular type of product, so it can be loaded by all types of products but because it cannot be mixed, the compartment can only be loaded by one type of product at the same time. The purpose of this issue is finding a least cost solution that may be used for routing routes and the schedules of ship loading/unloading.

3. Formulation of model
In this part, we discuss a mathematical model dealt with the problem. This mathematical model is similar to previous papers but due to several modifications to account for ship routing and the suitability between ship and consumption port [2,4]. We have divided this section into 5 parts. They are objective function, routing constraints, loading/unloading constraints, and the last is inventory constraints, as shown below:
3.1. Objective function

The objective function for this problem is to minimize the cost of traveling. The traveling cost of ship \(v\) from port \(i\) to port \(j\) is denoted by \(CT_{ijv}\). \(q_{imvkc}\) is used for indicating a continuous variable which determines that at node \((i,m)\), the quantity of product \(k\) loaded/unloaded into/from the compartment \(c\) using ship \(v\). If there is no loading/unloading movement at the artificial nodes; \(q_{0(v)1vkc} = 0\). Given below is the objective function:

\[
\text{Min } Z = \sum_{\forall v \in V} \sum_{(i,m) \in E} \sum_{c \in C_v} CT_{ijv}q_{imvkc}, \quad i \neq j, i \neq 1
\]  

(1)

3.2. Routing constraints

Consider that there is a complete ship’s set indicated by \(V\), which has been scheduled as well as routed. An artificial node \(o(v)\) is present in the ship \(v \in V\). At this artificial port, the ship will start travelling. For multiple physical ports the ships are working and these ports are denoted by \(H\). The physical ports’ subsets that the ships have visited are denoted by \(H_v \subseteq H\). Therefore, each location that might be visited by ship is indicated by \(H_v \cup \{o(v)\}\). It can be suggested that the port \(i \in H\) have been visited multiple times since planning. The \(M_i > 0\) can be defined as the maximum number visited port \((i \in H)\) by ships. It can be observed that the artificial ports have been visited just once.

Assume that the directed graph is \(G = (N, A)\), where \(A\) indicates the set of arc & \(N\) indicates the set of nodes. For representation of ports’ pair as well as the amount of visits by ships at specific port, \(N = \{(i, m): i \in H, m = 1, ..., M_i\} \cup \{o(v), 1\}\). In this set, every expected destination of ship can be observed. After this, the \(N_v \subseteq N\), can be considered as the ship’s locations various subsets at different ports of production. Furthermore, \(A\) (set of arcs) indicates every association among 2 nodes. \(A_v \subseteq A\) can be used to indicate the subset of arcs for the ships.

Three binary variables are involved in routing formulation:

- \(x_{imjnv}\) is equivalent to 1, when the ships travel from the node \((i,m)\) towards node \((j,n)\) and 0 else;
- \(y_{im}\) is equivalent to 1 when the node \((i,m)\) is the unvisited node and 0 else;
- \(z_{imv}\) is equivalent to 1 when the route is completed by ship at node \((i,m)\) and 0 else;

The routing constraints are as follows:

\[
\sum_{(j,n) \in N} x_{0(v)1jnv} + z_{0(v)1v} = 1, \quad \forall v \in V
\]  

(2)

\[
\sum_{(j,n) \in N} x_{imjnv} - \sum_{(j,n) \in N} x_{imjnv} - z_{inv} = 0, \quad \forall (v,i,m) \in V x N, i \neq j
\]  

(3)

\[
z_{inv} = 1, \quad \forall v \in V
\]  

(4)

\[
\sum_{v \in V} \sum_{(j,n) \in N \cup \{o(v)\}} x_{jimnv} + y_{im} = 1, \quad \forall (i,m) \in N, i \neq j
\]  

(5)

\[
y_{im} y_{i(m-1)} \geq 0, \quad \forall (i,m) \in N, m \neq 1
\]  

(6)

\[
x_{imnv} = 0, \quad \forall v \in V, \forall (i,m,j,n) \in Av, \quad i \neq j, i > 1, j > 1
\]  

(7)

\[
x_{imjn1} = 0, \quad \forall v \in V, \forall (i,m,j,n) \in Av, \quad j \neq 1, j \neq 4
\]  

(8)

\[
x_{imjn2} = 0, \quad \forall v \in V, \forall (i,m,j,n) \in Av, \quad j \neq 1, j \neq 3
\]  

(9)

\[
x_{imjn} \in \{0,1\}, \quad \forall v \in V, \forall (i,m,j,n) \in Av
\]  

(10)

\[
z_{inv} \in \{0,1\}, \quad \forall v \in V, \forall (i,m) \in N \cup \{o(v)\}
\]  

(11)
Constraint (2) confirms every single ship leaves from or is present at its initial position. Constraints (3) and (4) confirms that at the harbor, the \( mth \) arrival should leave or finish the route here. Constraint (5) make sure that only one ship can occupy node \((lm)\). Constraint (6) enforces that the precedence constraint should not be spoiled. Constraint (7) ensures that no route form consumption port to consumption port. Constraints (8) and (9) declares compatibility ship and port. Then, constraints (10) to (12) state the binary variables for those variables that have been found indulged in the routing constraints.

3.3. Loading and unloading constraints

There are two sets involved in these constraints. The first one, \( K \), it is the set of all possible items or products. An item \( k \subseteq K \) has its own storage. Suppose that storage can only hold 1 product, the index and notation of products also denote their loadings. Jik is the abbreviation that is used to indicate the consumption port or it is a port of demand, that is similar to +1 if \( i \) port is creator of product \((k)\), and is equal to -1 if is a consumer of product. Furthermore, \( K_i \subseteq K \) is the subset of products created or consumed in port \( i \) and \( P_k \subseteq K \) is subset of items which ship can transport. Later one i.e. \( C \), is the set of every potential sections and \( C_v \subseteq C \) is the subset of the ship compartments. The \( CM_{vc} \) is used to indicate the maximum capacity of \( v \) ship’s section \( c \). During beginning of this, the product \((k)\)' initiating capacity loaded in the ship’s compartment can be represented by \( QQ_{vkc} \).

Basically 3 different variables have been used in this the constraints:

- \( o_{lmvkc} \) is taken as a binary variable which equivalent to 1 only when the ship \( v \) travels to node \((i, m)\) and then loads or unloads its product \( k \) over there from or into compartment \( c \), and 0 if else;
- \( l_{lmvkc} \) is considered as a continuous variable that basically indicates the product’s quantity \( k \) that has been loaded in ship’s compartment \( c \) after moving from node \((i, m)\);
- \( q_{lmvkc} \) is also considered as a continuous variable that is used to specify the product’s quantity \( k \) that has been loaded or unloaded from node \((i, m)\) of ship’s compartment \( c \). This suggests that no loading or even unloading has been done at artificial nodes, \( q_{o(v)1kc} = 0 \)

The loading and unloading constraints are as follows:

\[
\begin{align*}
\forall v \in V, (i, m) \in N, (j, n) \in N_p, & \quad \forall (k, c) \in P_v \times C_v, i \neq j
\end{align*}
\]

\[
x_{injv} (l_{lmvkc} + l_{jk} q_{lmvkc} - l_{lmvkc}) = 0, \quad \forall v \in V, (i, m) \in N, (j, n) \in N_p, \\
\forall (k, c) \in P_v \times C_v, i \neq j
\]

\[
x_{injv} (l_{lmvkc}) = 0, \quad \forall v \in V, (i, m) \in N \cup \{o(v), 1\}, \quad \forall (j, n) \in N_p, \\
\forall (k, c) \in P_v \times C_v, i \neq j
\]

\[
QQ_{vkc} = l_{o(v)1vc}, \quad \forall v \in V, (i, m) \in N \cup \{o(v), 1\}, \forall (k, c) \in P_v \times C_v
\]

\[
q_{lmvkc} \leq \sum_{(j, n) \in N \cup \{o(v), 1\}} CM_{vc} x_{lnmv}, \quad \forall v \in V, (i, m) \in N, (j, n) \in N_p, \\
\forall (k, c) \in P_v \times C_v, i \neq j
\]

\[
l_{lmvkc} \leq \sum_{(j, n) \in N \cup \{o(v), 1\}} CM_{vc} x_{lnmv}, \quad \forall v \in V, (i, m) \in N, (j, n) \in N_p, \\
\forall (k, c) \in P_v \times C_v, i \neq j
\]

\[
q_{lmvkc} \leq CM_{vc} o_{lmvkc}, \quad \forall v \in V, (i, m) \in N \cup \{o(v), 1\}, \forall (k, c) \in P_v \times C_v
\]

\[
l_{lmvkc} \leq CM_{vc} (1 - o_{lmvkc}), \quad \forall v \in V, (i, m) \in N_p, \\
\forall (k, m') \in P_v, \forall c \in C_v, k' \neq k
\]
\[ \sum_{k \in P_v} o_{mvkc} \leq 1, \forall v \in V, \forall (i,m) \in N \cup \{o(v),1\}, \forall c \in C_v \] (20)
\[ o_{mvkc} \in \{0,1\}, \forall v \in V, \forall (i,m) \in N \cup \{o(v),1\}, \forall (k,c) \in P_v \times C_v \] (21)
\[ l_{mvkc} \cdot q_{mvkc} \geq 0, \forall v \in V, \forall (i,m) \in N \cup \{o(v),1\}, \forall (k,c) \in P_v \times C_v \] (22)

Constraint (13) keeps track of the quantity on board formerly and afterward a visit at node \((i,m)\) and the constraint (14) enforces that sections over-board should be unfilled when the ship departs for ports of production. Constraint (13) and constraint (14) are intermittent and expressed as similar linear restraints as explained by [3,4]. Constraint (15) guarantees that the product’s quantity in ship at beginning of travel is equal to the product’s initial \(k\) amount that was loaded in the \(c\) compartment. Constraints (16) to (18) limit the quantity unloaded (or loaded) and the quantity on board within their limits. Constraint (19) declares that only the same product \(k\) can be loaded in the compartment \(c\) on ship \(v\). Constraints (20) have applied the limitation that just an individual ship is capable of loading/unloading. Then, constraints (21) and (22) constraint state the binary variables and continuous variables for the variables included in loading or unloading constrains.

3.4. Time and scheduling constraints

Assume that the planning horizon denoted by \(TH\) and \(TT_{ijv}\) is used to denote the total travelling time of ship from one port \(i\) towards the port \(j\). \(TQ_{ik}\) is a parameter used to associate the actions performed at port that indicated the time spent in loading or unloading of product \(k\) at \(j\) port.

A continuous variable that is involved with time constraints: \(t_{im}\) which is demonstrated as time of ship’s arrival at node \((i,m)\) and the time constraints and scheduling are given below:

\[ t_{im} - t_{i(m-1)} \geq 0, \forall (i,m) \in N, m \neq 1 \] (23)
\[ t_{im} - t_{i(m-1)} - TQ_{ik}q_{mvkc} \geq 0, \forall (i,m) \in N, m \neq 1, i \neq 1 \] (24)
\[ t_{im} \leq TH, \forall (i,m) \in N \cup \{o(v),1\} \] (25)
\[ x_{imijn} \left[ t_{im} + \sum_{k \in P_v} \sum_{c \in C_v} TQ_{ik}q_{mvkc} \right] + TT_{ijv} - t_{jn} \leq 0, \forall v \in V, \forall (i,m,j,n) \in A_v, i \neq j \] (26)
\[ t_{im} \geq 0, \forall (i,m) \in N \cup \{o(v),1\} \] (27)

Constraints (23) and (24) permits that several ships can be present on the production port at a time and restrict that only one ship for consumption port at a time. Constraint (25) limits the arrival time in the planning horizon. Constraint (26) keeps a check on the total time of routing between two nodes. Just like in constraint (13) and constraint (14), this constraint has been found non-linear and suggested to be redesigned as similar yet linear constraint. Then, constraint (27) This constraint declares the continuous variables of arrival time.

3.5. Inventory constraints

There are 4 parameters dealt with inventory constraints. \(IS_{ik}\) denote item’s initial inventory at port \(i\) for product \(k\). \(SM_{ik}\) and \(SX_{ik}\) are used to indicate the maximum and the minimum product’s \(k\) levels at port \(i\). \(R_{ik}\) is then used to indicate the product’s rate of production and consumer at port \(i\). \(s_{imk}\) is associated with this variable and is basically an inventory constraint that is used to represent the product’s \(k\) level when the ship reaches at node \((i,m)\). Given below are the inventory constraints:

\[ q_{mvkc} \leq s_{mk} + J_k R_{ik} \left[ \sum_{k \in P_v} \sum_{c \in C_v} TQ_{ik}q_{mvkc} \right], \forall v \in V, \forall (i,m) \in N_p, \forall k \in P_v, \forall c \in C_v \] (28)
\[ s_{1ik} = IS_{ik} + J_k R_{ik} l_{i1}, \forall (i,k) \in H \times K_i \] (29)
\[
\sum_{v \in V} \sum_{c \in C_v} J_{ik} q_{ik} + J_{ik} R_{ik} (t_{im} - t_{i(m-1)}) \cdot s_{imk} = 0, \quad \forall (i,m,k) \in N \times K_i, m \neq 1
\]

\[
SM_{ik} \leq s_{imk} \leq SX_{ik}, \quad \forall (i,m,k) \in N \times K_i
\]

\[
SM_{ik} \leq s_{imk} - \sum_{v \in V} \sum_{c \in C_v} J_{ik} q_{imvkc} + J_{ik} R_{ik} (TQ_{ik} q_{imvkc}) \leq SX_{ik},
\]

\[
\forall (i,m,k) \in N \times K_i
\]

Constraint (28) limits the loading quantity product \( k \) must not exceed the available product \( k \) in the silo. Constraint (29) keeps a check on the storage level of product \( k \) at the time of first arrival. Constraint (30) tracks the level of storage among existing and earlier visits. Constraints (31) and (32) ensure that the level of storage must be in the limit when the ships arrive as well as when it leaves the port. The storage levels have been restricted under the constraint (33) regarding the completion of the planning horizon. Then, constraint (34) declares the continuous variables of inventory level.

4. Computational study

Here we represent the actual case description the computational result.

4.1. Case description

In this model, there is P1 as the production port with P2, P3 as well as P4 as consumers. Among these 3 customers the first customer has 2 products (K1 and K2) while the latter two customers have only 1 product (K1). The detailed information about the ports and their storage is depicted in Table 1.

There are 4 ships (V1, V2, V3, and V4) for serving these ports. Ship V1 and V2 only have 1 compartment, but V3 and V4 have 2 undedicated compartments (C1 and C2). The transportation cost for every assignment from P1 to P2, P3 and P4 are stated IDR 240 Million, IDR 183 Million and IDR 105 Million respectively. The information about the ships is depicted in Table 2. Table 3 shows information about travelling and un/loading time.

\begin{table}[h]
\centering
\caption{Data for ports and storages}
\begin{tabular}{|c|c|c|c|}
\hline
Ports & Product Type & Level max. (ton) & Level min. (ton) & Consumption rate (ton) \\
\hline
P2 & K1 & 12.000 & 2624 & 719 \\
    & K2 & 12.000 & 1018 & 279 \\
P3 & K1 & 12.000 & 1549 & 811 \\
    & K1 & 6.000  & 632  & 527 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Data for ships and their compartments}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Ship & Compartment capacity (ton) & Initial level (ton) & Initial position \\
& C1 & C2 & C1 & C2 & Destination port & Remaining time (days) \\
\hline
V1 & 1500 & 0 & 0 & 0 & P1 & 0 \\
\hline
\end{tabular}
\end{table}
Table 3. Data for travelling time and loading/unloading time

| Port | Loading/unloading time (days/ton) | Travelling time (days) |
|------|----------------------------------|------------------------|
|      |                                  | P1 to PP | PP to P1 |
| P1   | 0.0002                           | -        | -        |
| P2   | 0.0002                           | 3.45     | 2.8      |
| P3   | 0.0001                           | 1.71     | 1.4      |
| P4   | 0.0003                           | 1.1      | 0.87     |

4.2. Computational result

The solution of routing by MILP solved with using software LINGO version 18 unlimited version using an Intel Core 2 Core i7 with memory (RAM) 8 GB 1600 MHz DDR3 machine to operating the software. The solver LINGO is presented in Fig.1a for routing of ship V1, Fig.1b for routing ship V2, Fig.2a for routing ship V3, and Fig.2b for routing ship V4. Only 2 minutes and 20 seconds are required by the software in order to find out the appropriate solution for the planning horizon of 15 days and it involves the expense of IDR 3.689.538.000. This figure indicates the node \((i,m)\) of port \(i\) as well as the total number of visits at port \((m)\).

The inventory level of port P2 is presented in Table 4 and is described in Figure 3. There are 2 visiting on port P2, on the first loading on the 5th day was 5417 tons and the second loading on the 13th day amounted to 1438 tons. The inventory level of port P3 is presented in Table 5 and is described in Figure 4. There are 2 visiting on port P3, on the first loading was 5200 tons and the second loading on the 6th day amounted to 3041 tons. The inventory level of port P4 is presented in Table 8 and is described in Figure 5. There are 2 visiting at port P4, on the first loading was 367 tons the second loading on the 6th day amounted to 4500 tons.

Figure 1. Routing solutions for ship V1 and V2. (a) Routing for ship V1, (b) Routing for ship V2
Figure 2. Routing solutions for ship V3 and V4. (a) Routing for ship V3, (b) Routing for ship V4

Table 4. Storage level of products for P2

| Days | Storage level of product K1 (ton) | Storage level of product K2 (ton) |
|------|----------------------------------|----------------------------------|
|      | Initial level | Loading | End level | Stock | Loading | End level |
| 0    | 6554       | 0       | 6554      | 5098  | 0       | 5098      |
| 5    | 2959       | 5417    | 8376      | 3703  | 0       | 3703      |
| 13   | 2624       | 1438    | 4062      | 1471  | 105     | 1576      |
| 15   | 2624       | 0       | 2624      | 1018  | 0       | 1018      |

Figure 3. Storage level of products for P2. (a) Storage level of product K1, (b) Storage level of product K2

Table 5. Storage level of products for P3

| Days | Stock (ton) | Loading (ton) | End level (ton) |
|------|-------------|---------------|-----------------|
| 0    | 5473        | 0             | 5473            |
| 1    | 4662        | 5200          | 9862            |
| 6    | 5807        | 3041          | 8848            |
| 15   | 1549        | 0             | 1549            |
5. Conclusion

In this paper, a mathematical model for a ship inventory routing and scheduling problem with undedicated compartments is developed and then solved by LINGO optimization software. The objective of the problem is to find to a minimize transportation costs with an inventory level that remains unfulfilled during the planning horizon period. The results of this optimization provide a global optimum with the total cost of transportation about IDR 3.69 billion for 15 days planning horizon period in October 2019 and maintained the availability of cement during this period.

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