Constraining $Z'$ From Supersymmetry Breaking

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Abstract

We suggest and analyze a class of supersymmetric $Z'$ models based on the gauge symmetry $U(1)_x = xY - (B - L)$, where $Y$ is the Standard Model hypercharge. For $1 < x < 2$, the $U(1)_x D$-term generates positive contributions to the slepton masses, which is shown to solve the tachyonic slepton problem of anomaly mediated supersymmetry breaking (AMSB). The resulting models are very predictive, both in the SUSY breaking sector and in the $Z'$ sector. We find $M_{Z'} = 2 - 4$ TeV and the $Z - Z'$ mixing angle $\xi \simeq 0.001$. Consistency with symmetry breaking and AMSB phenomenology renders the $Z'$ “leptophobic”, with $Br(Z' \rightarrow \ell^+\ell^-) \simeq (1 - 1.6)\%$ and $Br(Z' \rightarrow q\bar{q}) \simeq 44\%$. The lightest SUSY particle is either the neutral Wino or the sneutrino in these models.

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1 Introduction

One of the simplest extensions of the Standard Model (SM) is obtained by adding a $U(1)$ factor to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge structure. Such $U(1)$ factors arise quite naturally when the SM is embedded in a grand unified group such as $SO(10)$, $SU(6)$, $E_6$, etc \cite{1, 2}. While it is possible that such $U(1)$ symmetries are broken spontaneously near the grand unification scale, it is also possible that some of the $U(1)$ factors survive down to the TeV scale. In fact, if there is low energy supersymmetry, it is quite plausible that the $U(1)$ symmetry is broken along with supersymmetry at the TeV scale. The $Z'_\chi$ and $Z'_\psi$ models arising from $SO(10) \to SU(5) \times U(1)_\chi$ and $E_6 \to SO(10) \times U(1)_\psi$ are two popular extensions which have attracted much phenomenological attention \cite{1–8}. $Z'$ associated with the left–right symmetric extension of the Standard Model does not require a grand unified symmetry. Other types of $U(1)$ symmetries, which do not resemble the ones with a GUT origin, are known to arise in string theory, in the free–fermionic construction as well as in orbifold and $D$–brane models \cite{9–11}. Gauge kinetic mixing terms of the type $B^{\mu\nu}Z'_{\mu\nu}$ \cite{12} which will be generated through renormalization group flow below the unification scale can further disguise the couplings of the $Z'$.

The properties of the $Z'$ gauge boson – its mass, mixing and couplings to fermions – associated with the $U(1)$ gauge symmetry are in general quite arbitrary \cite{13}. This is especially so when the low energy theory contains new fermions for anomaly cancellation. In this paper we propose and analyze a special class of $U(1)$ models wherein the $Z'$ properties get essentially fixed from constraints of SUSY breaking. We have in mind the anomaly mediated supersymmetric (AMSB) framework \cite{14, 15}. In its minimal version, with the Standard Model gauge symmetry, it turns out that the sleptons of AMSB become tachyonic. We suggest the $U(1)$ symmetry, identified as $U(1)_x = xY - (B - L)$, where $Y$ is the Standard Model hypercharge, as a solution to the negative slepton mass problem of AMSB. This symmetry is automatically free of anomalies with the inclusion of right–handed neutrinos. It is shown that the $D$–term of this $U(1)_x$ provides positive contributions to the slepton masses, curing the tachyonic problem. The consistency of symmetry breaking and the SUSY spectrum points towards a specific set of parameters in the $Z'$ sector. For example, $1 < x < 2$ is needed for the positivity of the left–handed and the right–handed slepton masses. Furthermore, the $U(1)_x$ gauge coupling, $g_x$, is
fixed to be between 0.4–0.5. The resulting $Z'$ is found to be “leptophobic”\cite{16} with $Br(Z \rightarrow \ell^+\ell^-) \simeq (1 - 1.6)\%$ and $Br(Z \rightarrow q\bar{q}) \simeq 44\%$.

AMSB models are quite predictive as regards the SUSY spectrum. The masses of the scalar components of the chiral supermultiplets in AMSB scenario are given by \cite{14,15}

$$\left(m^2\right)_{\phi_i}^\phi_j = \frac{1}{2}M_{aux}^2 \left[\beta(Y)\frac{\partial}{\partial Y}\gamma_{\phi_i}^{\phi_j} + \beta(g)\frac{\partial}{\partial g}\gamma_{\phi_i}^{\phi_j}\right],$$

where summations over the gauge couplings $g$ and the Yukawa couplings $Y$ are assumed. $\gamma_{\phi_i}^{\phi_j}$ are the one–loop anomalous dimensions, $\beta(Y)$ is the beta function for the Yukawa coupling $Y$, and $\beta(g)$ is the beta function for the gauge coupling $g$. $M_{aux}$ is the vacuum expectation value of a “compensator superfield” \cite{14} which sets the scale of SUSY breaking. The gaugino mass $M_g$, the trilinear soft supersymmetry breaking term $A_Y$ and the bilinear SUSY breaking term $B$ are given by \cite{14,15}

$$M_g = \frac{\beta(g)}{g}M_{aux}, \quad A_Y = -\frac{\beta(Y)}{Y}M_{aux}, \quad B = -M_{aux}(\gamma_{H_u} + \gamma_{H_d}).$$

We see that the SUSY masses are completely fixed in the AMSB framework once the spectrum of the theory and $M_{aux}$ are specified.

The negative slepton mass problem arises in AMSB because in Eq. (1) the gauge beta functions for $SU(2)_L$ and $U(1)_Y$ are positive, $\gamma_{\phi_i}^{\phi_j}$ are negative, and the Yukawa couplings are small for the first two families of sleptons. In our $Z'$ models, there are additional positive contributions from the $U(1)_x D$–terms which render these masses positive.

In Ref. \cite{17} the negative slepton mass problem of AMSB has been solved with explicit Fayet–Iliopoulos terms added to the theory. In contrast, in our models, the $D$–term is calculable, which makes the $Z'$ sector more predictive. We find $M_{Z'} = 2 - 4$ TeV and the $Z - Z'$ mixing angle $\xi \simeq 0.001$. Constraints from the electroweak precision observables are satisfied, with the $Z'$ model giving a slightly better fit compared to the Standard Model.

Other attempts to solve the negative slepton mass problem of AMSB generally assume TeV–scale new physics \cite{18–20} or a universal scalar mass of non–AMSB origin \cite{21}. In Ref. \cite{20} we have shown how a non–Abelian horizontal symmetry which is asymptotically free solves the problem. Some of the techniques we use here for the symmetry breaking analysis are similar to Ref. \cite{20}.

The plan of the paper is as follows. In section 2 we introduce our model. In section 3 we analyze the Higgs potential of the model. In section 4 we present formulas for the SUSY
spectrum. Section 5 contains our numerical results for the SUSY spectrum as well as for the $Z'$ mass and mixing. In section 6 we analyze the partial decay modes of the $Z'$. In section 7 we analyze other experimental test of the model. Here we show the consistency of our models with the precision electroweak data. Section 8 has our conclusions. In an Appendix we give the relevant expressions for the beta functions, anomalous dimensions as well as for the soft masses.

2 $U(1)_x$ Model

We present our model in this section. We consider adding an extra $U(1)$ gauge group to the Standard Model gauge structure of MSSM. The model is then based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_x$, where the $U(1)_x$ charge is given by the following linear combination of hypercharge $Y$ and $B - L$:

$$U(1)_x = xY - (B - L).$$

The particle content of the model and the $U(1)_x$ charge assignment are shown in Table 1. Besides the MSSM particles, the model has new particles $\{\nu^c_i, \nu^c, \bar{\nu}^c, S_+ \text{ and } S_-\}$ which are all singlets of the Standard Model gauge group.

| Superfield | $Q_i$ | $u^c_i$ | $d^c_i$ | $L_i$ | $e^c_i$ | $H_u$ | $H_d$ | $\nu^c_i$ | $\nu^c$ | $\bar{\nu}^c$ | $S_+$ | $S_-$ |
|------------|------|--------|--------|------|--------|------|------|----------|--------|-----------|------|------|
| $U(1)_x$   | $\frac{x}{6} - \frac{1}{3}$ | $-\frac{2x}{3} + \frac{1}{3}$ | $\frac{x}{3} + \frac{1}{3}$ | $-\frac{x}{2} + 1$ | $x - 1$ | $\frac{4}{3}$ | $-\frac{4}{3}$ | $-1$ | $-1$ | $1$ | $2$ | $-2$ |

Table 1: Particle content and charge assignment of the $U(1)_x$ model. Here $i = 1 - 3$ is the family index.

In order for $\tilde{L}_i$ and $\tilde{e}^c_i$ sleptons to have positive mass–squared from the $U(1)_x D$–term, the charges of $L_i$ and $e^c_i$ must be of the same sign. This is possible only for $1 < x < 2$. We shall confine to this range of $x$, which is an important restriction on this class of models. The $\nu^c_i$ fields are needed for $U(1)_x$ anomaly cancellation. $S_+$ and $S_-$ are the Higgs superfields responsible for $U(1)_x$ symmetry breaking. The $\nu^c + \bar{\nu}^c$ pair facilitates symmetry breaking within the AMSB framework. The superpotential of the model consistent with the gauge symmetries is given by:

$$W = (Y_u)_{ij} Q_i H_u u^c_j + (Y_d)_{ij} Q_i H_d d^c_j + (Y_l)_{ij} L_i H_d e^c_j + \mu H_u H_d$$
Here $i, j = 1, 2, 3$ are the family indices. The mass parameters $\mu$ and $\mu'$ are of order TeV, which may have a natural origin in AMSB \cite{14}. In general, one can write additional mass terms of the form $M_i \nu_c^i \bar{\nu}^c$ in the superpotential. Such terms will have very little effect on the symmetry breaking analysis that follows. We forbid such mass terms by invoking a discrete symmetry (such as a $Z_2$) which differentiates $\nu_c$ from $\nu_c^i$.

Small neutrino masses are induced in the model through the seesaw mechanism. However, the $\nu_c^i$ fields, which remain light to the TeV scale, are not to be identified as the traditional right–handed neutrinos involved in the seesaw mechanism. The heavy fields which are integrated out have $U(1)_x$–invariant mass terms. Specifically, the following effective nonrenormalizable operators emerge after integrating out the heavy neutral lepton fields:

$$L_{\nu_{eff}}^{\nu} = \frac{Y_{\nu}^2}{M_N^2} L_i L_j H_u H_u S_L.$$  \hfill (5)

Here $M_N$ represents the masses of the heavy neutral leptons. For $M_N \sim 10^9$ GeV and $\langle S_- \rangle \sim$ TeV, sub–eV neutrino masses are obtained. Note that we have not allowed neutrino Dirac Yukawa couplings of the form $h_{\nu_j} L_i \nu_c^i H_u$, which would generate Majorana masses of order MeV for the light neutrinos. We forbid such terms by a global symmetry $G$, either discrete or continuous. In our numerical examples we shall assume this symmetry to be non–Abelian, with $\nu_c^i$ transforming as a triplet [for example, $G$ can be $O(3)$, $S_4$, $A_4$, etc.]. Such a symmetry would imply that $f_{\nu_c^i}$ in Eq. (4) are equal for $i = 1 \rightarrow 3$.

### 3 Symmetry Breaking

The scalar potential (involving $H_u$, $H_d$, $S_+$, $S_-$ fields) of the model is given by:

$$V = (M_{H_u}^2 + \mu^2)|H_u|^2 + (M_{H_d}^2 + \mu^2)|H_d|^2 + (M_{S_+}^2 + \mu'^2)|S_+|^2 + (M_{S_-}^2 + \mu'^2)|S_-|^2$$

$$+ B \mu (H_u H_d + h.c.) + B' \mu' (S_+ S_- + h.c.) + \frac{1}{8} (g_1^2 + g_2^2) (|H_u|^2 - |H_d|^2)^2$$

$$+ \frac{1}{2} g_x^2 |H_u H_d|^2 + \frac{1}{2} g_x^2 \frac{x}{2} |H_u|^2 - \frac{x}{2} |H_d|^2 + 2 |S_+|^2 - 2 |S_-|^2),$$  \hfill (6)
where the last term is the $U(1)_x D$ term. The $B$ and the $B'$ terms for the model are given by

$$B = - (\gamma_{H_u} + \gamma_{H_d}) M_{aux} \quad \text{and} \quad B' = - (\gamma_{S_+} + \gamma_{S_-}) M_{aux},$$

(7)

where the $\gamma$’s are the one–loop anomalous dimensions given in the Appendix, Eqs. (115)–(116), (120)–(121).

We parameterize the VEVs of $H_u, H_d, S_+$ and $S_-$ as

$$\langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle S_+ \rangle = z, \quad \langle S_- \rangle = y.$$  

(8)

In minimizing the potential, we have to keep in mind the fact that the VEVs of $\langle S_+ \rangle$ and $\langle S_- \rangle$ should be much larger than the VEVs of $\langle H_u \rangle$ and $\langle H_d \rangle$ for a consistent picture. In addition, the VEV of $\langle S_+ \rangle$ should be greater than the VEV of $\langle S_- \rangle$ in order for the $D$–term contribution to the slepton masses to be positive. We have checked explicitly that all the above–mentioned conditions are satisfied at the local minimum for a restricted choice of model parameters. The physical Higgs bosons as well as the sleptons acquire positive mass–squared, while generating a $Z'$ mass and $Z - Z'$ mixing angle consistent with experimental constraints.

Minimization of the potential leads to the following conditions:

$$\sin 2\beta = \frac{2B\mu}{2\mu^2 + M_{H_u}^2 + M_{H_d}^2},$$  

(9)

$$\frac{M_{Z'}^2}{2} = -\mu^2 + \frac{M_{H_d}^2 - M_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{x^2 g_s^2 v^2}{4} - \frac{x g_s^2 u^2 \cos 2\psi}{\cos 2\beta},$$  

(10)

$$\sin 2\psi = -\frac{2B'\mu'}{2\mu'^2 + M_{S_+}^2 + M_{S_-}^2},$$  

(11)

$$\frac{M_{Z'}^2}{2} = -\mu'^2 + \frac{M_{S_-}^2 - M_{S_+}^2 \tan^2 \psi}{\tan^2 \psi - 1} + \frac{x^2 g_s^2 v^2}{4} - \frac{x g_s^2 u^2 \cos 2\beta}{\cos 2\psi}.$$  

(12)

Here $M_{Z'}^2 = \frac{x^2 g_s^2 v^2}{2} + 8 g_s^2 u^2$, $\tan \beta = \frac{v_u}{v_d}$, $\tan \psi = \frac{z}{y}$, $\sqrt{v_u^2 + v_d^2} = v = 174$ GeV and $\sqrt{z^2 + y^2} = u$.

To see the consistency of symmetry breaking, we need to calculate the Higgs boson mass–squared and establish that they are all positive. We parameterize the Higgs fields (in the unitary gauge) as

$$H_u = \begin{pmatrix} H^+ \sin \beta \\ v_u + \frac{1}{\sqrt{2}}(\phi_2 + i \cos \beta \phi_3) \end{pmatrix}, \quad \langle H_d \rangle = \begin{pmatrix} v_d + \frac{1}{\sqrt{2}}(\phi_1 + i \sin \beta \phi_3) \\ H^- \cos \beta \end{pmatrix},$$  

$$S_+ = z + \frac{1}{\sqrt{2}}(\phi_4 + i \cos \psi \phi_5), \quad S_- = y + \frac{1}{\sqrt{2}}(\phi_6 + i \sin \psi \phi_5).$$  

(13)
The CP–odd Higgs bosons \( \{\phi_3, \phi_5\} \) have masses given by

\[
m^2_A = \frac{2B\mu}{\sin 2\beta}, \quad m^2_{A'} = -\frac{2B'\mu'}{\sin 2\psi},
\]

(14)

The mass matrix for the CP–even neutral Higgs bosons \( \{\phi_1, \phi_2, \phi_4, \phi_6\} \) is given by

\[
(M^2)_{cp-even} = \begin{pmatrix}
(M^2)_{11} & (M^2)_{12} & -2xg^2xv_dz & 2xg^2xv_dy \\
(M^2)_{12} & (M^2)_{22} & 2xg^2xv_u z & -2xg^2xv_u y \\
-2xg^2xv_dz & 2xg^2xv_u z & (M^2)_{33} & (M^2)_{34} \\
2xg^2xv_dy & -2xg^2xv_u y & (M^2)_{34} & (M^2)_{44}
\end{pmatrix},
\]

(15)

where

\[
(M^2)_{11} = m^2_A \sin^2 \beta + M_Z^2 \cos^2 \beta + \frac{1}{2}(x^2g^2xv^2 \cos^2 \beta),
\]

(16)

\[
(M^2)_{12} = -m^2_A \sin \beta \cos \beta - M_Z^2 \sin \beta \cos \beta - \frac{1}{2}x^2g^2xv^2 \sin \beta \cos \beta,
\]

(17)

\[
(M^2)_{22} = m^2_A \cos^2 \beta + M_Z^2 \sin^2 \beta + \frac{1}{2}(x^2g^2xv^2 \sin^2 \beta),
\]

(18)

\[
(M^2)_{33} = m^2_A' \cos^2 \psi + 8g^2x^2, 
\]

(19)

\[
(M^2)_{34} = -m^2_A' \sin \psi \cos \psi - 8g^2x^2yz,
\]

(20)

\[
(M^2)_{44} = m^2_A' \sin^2 \psi + 8g^2x^2y^2.
\]

(21)

It is instructive to analyze the effect of the \( U(1)_x \) \( D \)–term on the mass of the lightest MSSM Higgs boson \( h \). Consider the upper left \( 2 \times 2 \) subsector of the CP–even Higgs boson mass matrix. It has eigenvalues given by

\[
\lambda_{1,2} = \frac{1}{2} \left[ m^2_A + M_Z^2 + \frac{x^2g^2xv^2}{2} \right] \\
\pm \sqrt{\left( m^2_A + M_Z^2 + \frac{x^2g^2xv^2}{2} \right)^2 - 4m^2_A M_Z^2 \cos^2 2\beta - 4m^2_A \left( \frac{x^2g^2xv^2}{2} \right) \cos^2 2\beta}.
\]

(22)

From Eq. (22) we obtain an upper limit on \( m_h \):

\[
m_h \leq \sqrt{\frac{x^2g^2xv^2}{2} + M_Z^2 |\cos 2\beta|}.
\]

(23)

The mixing between the doublets and the singlets will reduce the upper limit further. In fact, we find this mixing effect to be significant.

The lower \( 2 \times 2 \) subsector of Eq. (15) has eigenvalues

\[
\lambda'_{1,2} = \frac{1}{2} \left[ 8g^2x^2u^2 + m^2_{A'} \pm \sqrt{(8g^2x^2u^2 + m^2_{A'})^2 - 4m^2_{A'} (8g^2x^2u^2) \cos^2 2\psi} \right].
\]

(24)
From Eq. (24) we obtain an upper bound of the lightest Higgs mass for the $SU(2)$ singlet sector:

$$m_{h'} \leq m_{A'} |\cos 2\psi|.$$  \hspace{1cm} (25)

The above upper limit on $m_{h'}$ is affected only minimally by the mixing between the doublet and the singlet Higgs fields.

As in the MSSM, the mass of the charged Higgs boson $H^\pm$ is given by

$$m_{H^\pm}^2 = m_{A}^2 + M_{W}^2.$$ \hspace{1cm} (26)

We now turn to the supersymmetric fermion masses. The (Majorana) mass matrix of the neutralinos $\{\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{B}', \tilde{S}_+^0, \tilde{S}_-^0\}$ is given by

$$\mathcal{M}^{(0)} = \begin{pmatrix}
M_1 & 0 & -\frac{\nu d}{\sqrt{2}} g_1 & \frac{\nu u}{\sqrt{2}} g_1 & 0 & 0 & 0 \\
0 & M_2 & \frac{\nu u}{\sqrt{2}} g_2 & -\frac{\nu d}{\sqrt{2}} g_2 & 0 & 0 & 0 \\
-\frac{\nu d}{\sqrt{2}} g_1 & \frac{\nu u}{\sqrt{2}} g_2 & 0 & -\mu & -\frac{\nu d}{\sqrt{2}} x g_x & 0 & 0 \\
\frac{\nu u}{\sqrt{2}} g_1 & -\frac{\nu d}{\sqrt{2}} g_2 & -\mu & 0 & \frac{\nu u}{\sqrt{2}} x g_x & 0 & 0 \\
0 & 0 & -\frac{\nu d}{\sqrt{2}} x g_x & \frac{\nu u}{\sqrt{2}} x g_x & M_1' & 2\sqrt{2} g_x z & -2\sqrt{2} g_x y \\
0 & 0 & 0 & 0 & 2\sqrt{2} g_x z & 0 & \mu' \\
0 & 0 & 0 & 0 & -2\sqrt{2} g_x y & \mu' & 0
\end{pmatrix},$$ \hspace{1cm} (27)

where $M_1$, $M_1'$ and $M_2$ are the gaugino masses for $U(1)_Y$, $U(1)_x$ and $SU(2)_L$. The physical neutralino masses $m_{\tilde{\chi}_i^0} \ (i = 1-7)$ are obtained as the eigenvalues of this mass matrix. We denote the diagonalizing matrix as $O$:

$$O\mathcal{M}^{(0)}O^T = \text{diag}\{m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0}, m_{\tilde{\chi}_6^0}, m_{\tilde{\chi}_7^0}\}. \hspace{1cm} (28)$$

In the basis $\{\tilde{W}^+, \tilde{H}_u^0\}$, $\{\tilde{W}^-, \tilde{H}_d^0\}$ the chargino (Dirac) mass matrix is

$$\mathcal{M}^{(c)} = \begin{pmatrix}
M_2 & g_2 \nu_d \\
g_2 \nu_u & \mu
\end{pmatrix}. \hspace{1cm} (29)$$

This matrix is diagonalized by a biunitary transformation $V^* \mathcal{M}^{(c)} U^{-1} = \text{diag}\{m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\}$.

The $Z - Z'$ mixing matrix is given by

$$\mathcal{M}_{Z-Z'}^2 = \begin{pmatrix}
M_Z^2 & \gamma M_Z^2 \\
\gamma M_Z^2 & M_{Z'}^2
\end{pmatrix}. \hspace{1cm} (30)$$
where
\[ \gamma = \frac{-x g_x}{\sqrt{g_1^2 + g_2^2}}, \]
\[ M_{Z}^2 = \frac{\nu^2}{2} \left( g_1^2 + g_2^2 \right), \quad M_{Z'}^2 = \frac{x^2 g_x^2 \nu^2}{2} + 8 g_x^2 u^2. \] (31)

The physical mass eigenstates \( Z_1 \) and \( Z_2 \) with masses \( M_{Z_1}, M_{Z_2} \) are
\[ Z_1 = Z \cos \xi + Z' \sin \xi, \] (32)
\[ Z_2 = -Z \sin \xi + Z' \cos \xi, \] (33)

where
\[ M_{Z_1, Z_2}^2 = \frac{1}{2} \left[ M_{Z}^2 + M_{Z'}^2 \pm \sqrt{(M_{Z}^2 - M_{Z'}^2)^2 + 4 \gamma^2 M_{Z}^4} \right]. \] (34)

The \( Z - Z' \) mixing angle \( \xi \) is given by
\[ \xi = \frac{1}{2} \arctan \left( \frac{2 \gamma M_{Z}^2}{M_{Z}^2 - M_{Z'}^2} \right) \approx -\gamma M_{Z}^2 / M_{Z'}. \] (35)

We have ignored kinetic mixing of the form \( B^\mu\nu Z'^\mu \) in the Lagrangian [12, 13].

The masses of the heavy right–handed neutrinos are given by
\[ m_{\nu^c_i} = f_{\nu^c_i} z, \] (36)
where \( i = 1 - 3 \) is the family index. The fourth right–handed neutrino \( \nu^c \) mixes with the \( \bar{\nu}^c \) field forming two Majorana fermions. The masses are the eigenvalues of the mass matrix
\[ M_{\nu^c \bar{\nu}^c} = \begin{pmatrix} f_{\nu^c z} & M_{\nu^c} \\ M_{\nu^c} & m_y \end{pmatrix}, \] (37)
where \( M_{\nu^c} \) is the mass parameter that appears in the superpotential of Eq. [4]. We denote the eigenstates of this matrix as \( \omega_1, \omega_2 \) and the mass eigenvalues as \( m_{\omega_1} \) and \( m_{\omega_2} \).

## 4 The SUSY Spectrum

### 4.1 Slepton masses

The slepton mass–squareds are given by the eigenvalues of the mass matrices
\[ M_i^2 = \begin{pmatrix} m_{\tilde{l}_i}^2 & m_{\tilde{e}_i} \left( A_{Y_i} - \mu \tan \beta \right) \\ m_{\tilde{e}_i} \left( A_{Y_i} - \mu \tan \beta \right) & m_{\tilde{e}_i}^2 \end{pmatrix}. \] (38)
where \(i = e, \mu, \tau\), and

\[
m^2_{l_i} = \frac{M^2_{\text{aux}}}{(16\pi^2)} \left[ Y_{l_i} \beta(Y_{l_i}) - \left( \frac{3}{2} g_2 \beta(g_2) + \frac{3}{10} g_1 \beta(g_1) + 2 \left(1 - \frac{x}{2}\right) g_x \beta(g_x) \right) \right] + m^2 \left( \frac{1}{2} + \sin^2 \theta_W \right) \cos 2\beta M^2_Z + 2g^2_2 \left(1 - \frac{x}{2}\right) (z^2 - y^2),
\]

\[
m^2_{\tilde{e}_i} = \frac{M^2_{\text{aux}}}{(16\pi^2)} \left[ 2Y_{\tilde{e}_i} \beta(Y_{\tilde{e}_i}) - \left( \frac{6}{5} g_1 \beta(g_1) + 2(x - 1)^2 g_x \beta(g_x) \right) \right] + m^2 \left( \frac{1}{2} + \sin^2 \theta_W \right) \cos 2\beta M^2_Z + 2g^2_2 (x - 1)(z^2 - y^2).
\]

The SUSY soft masses are calculated from the RGE given in the Appendix [Eqs. (124), (130)]. Note the positive contribution from the \(U(1)_x\) \(D\)-terms in Eqs. (39)–(40), given by the terms \(+2g^2_x(1 - \frac{x}{2})(z^2 - y^2)\) and \(+2g^2_x(x - 1)(z^2 - y^2)\). There are also negative contributions proportional to \(\beta(g_x)\), but in our numerical solutions, the positive \(D\)-term contributions are larger than the negative contributions. We seek solutions where \(z = \langle S_+ \rangle\) and \(y = \langle S_- \rangle\) are much larger than \(v_u, v_d\), of order TeV, with \(z > y\).

The left–handed sneutrino masses are given by

\[
m^2_{\tilde{\nu}_L} = \frac{M^2_{\text{aux}}}{(16\pi^2)} \left[ \frac{3}{2} g_2 \beta(g_2) - \frac{3}{10} g_1 \beta(g_1) - 2 \left(1 - \frac{x}{2}\right) g_x \beta(g_x) \right] + \frac{1}{2} \cos 2\beta M^2_Z + 2g^2_2 \left(1 - \frac{x}{2}\right) (z^2 - y^2).
\]

### 4.2 Squark masses

The mixing matrix for the squark sector is similar to the slepton sector. The diagonal entries of the up and the down squark mass matrices are given by

\[
m^2_{\tilde{U}_i} = (m^2_{\text{soft}}\tilde{U}_i - m^2_{\tilde{U}_i} + \frac{1}{6} (4M^2_W - M^2_Z) \cos 2\beta + 2g^2_x \left(\frac{x}{6} - \frac{1}{3}\right) (z^2 - y^2),
\]

\[
m^2_{\tilde{D}_i} = (m^2_{\text{soft}}\tilde{D}_i + m^2_{\tilde{D}_i} - \frac{2}{3} (M^2_W - M^2_Z) \cos 2\beta + 2g^2_x \left(-\frac{2x}{3} + \frac{1}{3}\right) (z^2 - y^2),
\]

\[
m^2_{\tilde{D}_i} = (m^2_{\text{soft}}\tilde{D}_i - m^2_{\tilde{U}_i} - \frac{1}{6} (2M^2_W + M^2_Z) \cos 2\beta + 2g^2_x \left(\frac{x}{6} - \frac{1}{3}\right) (z^2 - y^2),
\]

\[
m^2_{\tilde{D}_i} = (m^2_{\text{soft}}\tilde{D}_i + m^2_{\tilde{D}_i} + \frac{1}{3} (M^2_W - M^2_Z) \cos 2\beta + 2g^2_x \left(\frac{x}{3} + \frac{1}{3}\right) (z^2 - y^2).
\]

Here \(m_{U_i}\) and \(m_{D_i}\) are quark masses of different generations, \(i = 1, 2, 3\). The squark soft masses are obtained from the RGE as

\[
(m^2_{\text{soft}}\tilde{Q}_i) = \frac{M^2_{\text{aux}}}{16\pi^2} \left[ Y_{u_i} \beta(Y_{u_i}) + Y_{d_i} \beta(Y_{d_i}) - \frac{1}{30} g_1 \beta(g_1) - \frac{3}{2} g_2 \beta(g_2)
\]

\[\right)
\]
\[- \frac{8}{3} g_3 \beta(g_3) - 2 \left( \frac{x}{6} - \frac{1}{3} \right)^2 g_x \beta(g_x) \] \quad (43)

\[ (m_{\text{soft}}^2)_{\tilde{\nu}_i^c} = \frac{M_{\text{aux}}^2}{16\pi^2} \left[ 2 Y_{u_i} \beta(Y_{u_i}) - \frac{8}{15} g_1 \beta(g_1) - \frac{8}{3} g_3 \beta(g_3) - 2 \left( -\frac{2x}{3} + \frac{1}{3} \right)^2 g_x \beta(g_x) \right] \quad (44) \]

\[ (m_{\text{soft}}^2)_{\tilde{\nu}_{\mu}^c} = \frac{M_{\text{aux}}^2}{16\pi^2} \left[ 2 Y_{d_i} \beta(Y_{d_i}) - \frac{2}{15} g_1 \beta(g_1) - \frac{8}{3} g_3 \beta(g_3) - 2 \left( \frac{x}{3} + \frac{1}{3} \right)^2 g_x \beta(g_x) \right] \quad (45) \]

### 4.3 Heavy sneutrino masses

The heavy right-handed sneutrinos ($\tilde{\nu}_i^c$) split into scalar ($\tilde{\nu}_{is}^c$) and pseudoscalar ($\tilde{\nu}_{ip}^c$) components with masses given by

\[ m_{\tilde{\nu}_{is}^c}^2 = \frac{M_{\text{aux}}^2}{(16\pi^2)} \left[ 4 f_{\nu i} \beta (f_{\nu i}) - 2 g_x \beta (g_x) \right] - 2 g_x^2 (z^2 - y^2) \]
\[ + \quad 2 \mu' f_{\nu i} y + 4 f_{\nu i}^2 z^2 + 2 f_{\nu i} A_{\nu i} z, \quad (46) \]

\[ m_{\tilde{\nu}_{ip}^c}^2 = \frac{M_{\text{aux}}^2}{(16\pi^2)} \left[ 4 f_{\nu i} \beta (f_{\nu i}) - 2 g_x \beta (g_x) \right] - 2 g_x^2 (z^2 - y^2) \]
\[ - \quad 2 \mu' f_{\nu i} y + 4 f_{\nu i}^2 z^2 - 2 f_{\nu i} A_{\nu i} z. \quad (47) \]

As for the fourth heavy sneutrino, there is mixing between the $\tilde{\nu}^c$ and the $\tilde{\nu}^c$ fields. This leads to two $2 \times 2$ mass matrices, one for the scalars, and one for the pseudoscalars. They are given by

\[ M_{\tilde{\nu}_{is}^c}^2 = \begin{pmatrix} m_{\tilde{\nu}_{is}^c}^2 & 2 M_{\nu s} \left( f_{\nu s} z + h y + \frac{B_{\nu s} h}{2} \right) \\ 2 M_{\nu s} \left( f_{\nu s} z + h y + \frac{B_{\nu s} h}{2} \right) & m_{\tilde{\nu}_{ip}^c}^2 + 2 M_{\nu s} \left( f_{\nu s} z + h y + \frac{B_{\nu s} h}{2} \right) \end{pmatrix}, \quad (48) \]

\[ M_{\tilde{\nu}_{ip}^c}^2 = \begin{pmatrix} m_{\tilde{\nu}_{ip}^c}^2 & 2 M_{\nu p} \left( f_{\nu p} z + h y + \frac{B_{\nu p} h}{2} \right) \\ 2 M_{\nu p} \left( f_{\nu p} z + h y + \frac{B_{\nu p} h}{2} \right) & m_{\tilde{\nu}_{ip}^c}^2 + 2 M_{\nu p} \left( f_{\nu p} z + h y + \frac{B_{\nu p} h}{2} \right) \end{pmatrix}, \quad (49) \]

where

\[ m_{\tilde{\nu}_{is}^c}^2 = \frac{M_{\text{aux}}^2}{(16\pi^2)} \left( 4 f_{\nu s} \beta (f_{\nu s}) - 2 g_x \beta (g_x) \right) - 2 g_x^2 (z^2 - y^2) \]
\[ + \quad 2 \mu' f_{\nu s} y + 4 f_{\nu s}^2 z^2 + 2 f_{\nu s} A_{\nu s} z + M_{\nu s}^2, \quad (50) \]

\[ m_{\tilde{\nu}_{ip}^c}^2 = \frac{M_{\text{aux}}^2}{(16\pi^2)} \left( 4 f_{\nu p} \beta (f_{\nu p}) - 2 g_x \beta (g_x) \right) - 2 g_x^2 (z^2 - y^2) \]
\[ - \quad 2 \mu' f_{\nu p} y + 4 f_{\nu p}^2 z^2 - 2 f_{\nu p} A_{\nu p} z + M_{\nu p}^2, \quad (51) \]

\[ m_{\tilde{\nu}_{is}^s}^2 = \frac{M_{\text{aux}}^2}{(16\pi^2)} \left( 4 h \beta (h) - 2 g_x \beta (g_x) \right) + 2 g_x^2 (z^2 - y^2) \]
\[ + \quad 2 \mu' h z + 4 h z^2 + 2 h A h y + M_{\nu s}^2, \quad (52) \]

\[ 10 \]
\[ m_{\tilde{\nu}_p}^2 = \frac{M_{\text{aux}}^2}{(16\pi^2)} (4h\beta(h) - 2g_x\beta(g_x)) + 2g_x^2(z^2 - y^2) - 2\mu'hz + 4h^2y^2 - 2hA_hy + M_{\nu_p}^2, \]

\[ B_{\tilde{\nu}_p\tilde{\nu}_p} = -M_{\text{aux}}(\gamma_{\tilde{\nu}_p} + \gamma_{\tilde{\nu}_p}). \]

Here \( s (p) \) stands for scalar (pseudoscalar). The beta functions, gamma functions and the \( A \) terms are given in the Appendix, Eqs. (125)–(131). We shall denote the mass eigenstates of the scalars as \( \tilde{\omega}_{1s}, \tilde{\omega}_{2s} \) with masses \( m_{\tilde{\omega}_{1s}}, m_{\tilde{\omega}_{2s}} \), and the pseudoscalars as \( \tilde{\omega}_{1p}, \tilde{\omega}_{2p} \) with masses \( m_{\tilde{\omega}_{1p}}, m_{\tilde{\omega}_{2p}} \).

### 5 Numerical Results for the Spectrum

As inputs at \( M_Z \) we choose the central values (in the \( \overline{MS} \) scheme) \[ \alpha_3(M_Z) = 0.119, \quad \sin^2\theta_W = 0.23113, \quad \alpha(M_Z) = \frac{1}{127.922}. \] (55)

We keep the top quark mass fixed at its central value, \( M_t = 174.3 \) GeV. We follow the procedure outlined in Ref. [20] to determine the parameter \( \tan \beta \) and the lightest MSSM Higgs boson mass \( m_h \). The gauge couplings and the top quark Yukawa coupling are evolved from the lower momentum scale to \( Q = 1 \) TeV, where the Higgs potential is minimized. We use the Standard Model beta functions for this evolution. In determining the top quark Yukawa coupling \( Y_t(m_t) \), we use 2–loop QCD corrections to convert the physical mass \( M_t \) into the running mass \( m_t(m_t) \).

For the lightest Higgs boson mass of MSSM we use the 2–loop radiatively corrected expression for \( m_h^2 = (m_h^2)_o + \Delta m_h^2 \), where \( \Delta m_h^2 \) is given in Ref. [23].

We present numerical results for two models: Model 1 with \( x = 1.3 \), and Model 2 with \( x = 1.6 \). In Model 1, the left–handed sleptons are heavier than the right–handed sleptons, while the reverse holds for Model 2.

The value of \( M_{\text{aux}} \) should be in the range \( M_{\text{aux}} = 40 - 100 \) TeV if the SUSY particles are to have masses in the range 100 GeV – 2 TeV. In Table 2 corresponding to Model 1, we choose \( M_{\text{aux}} = 56.398 \) TeV. In Table 7 (for Model 2) we choose \( M_{\text{aux}} = 59.987 \) TeV. We have included the leading radiative corrections [24] to \( M_1, M_2 \) and \( M_3 \) in our numerical study. In Model 1 we find \( M_1 : M_2 : M_3 = 3.0 : 1 : 7.1 \). The minimization conditions (Eqs. (9)–(10)) fix \( \tan \beta = 4.39 \) in this model. The choice of \( g_x = 0.41, f_{\nu_\alpha} = f_{\nu_\beta} = 0.28 \),
and \( h = 0.921 \) are motivated by the requirements of consistent symmetry breaking with 
\[ \langle S_+ \rangle \gtrsim \langle S_- \rangle \gg v_u, v_d, \] 
and the positivity of slepton masses. We find that the model parameters are highly constrained. Only small deviations from the choice in Table 2 are found to be consistent.

From Table 2 we see that the lightest Higgs boson of the MSSM sector has mass of 121 GeV. The lightest SUSY particle is the neutralino \( \tilde{\chi}_1^0 \), which is approximately a neutral Wino. This is a candidate for cold dark matter [25]. Note that \( \tilde{\chi}_1^0 \) is nearly mass degenerate with the lighter chargino \( \tilde{\chi}_1^\pm \) (which is approximately the charged Wino). The mass splitting \( m_{\tilde{\chi}_1^0} - m_{\tilde{\chi}_1^\pm} = 180 \text{ MeV} \), where the bulk (173 MeV) arises from finite electroweak radiative corrections [26], not shown in Table 2.

In the \( U(1)_x \) sector, there is a relatively light neutral Higgs boson \( h' \) with a mass of 60 GeV. This occurs since the parameter \( \tan \psi = \frac{z}{y} \) is close to 1 – a requirement for consistent symmetry breaking [see Eq. (25)]. \( h' \) is an admixture of \( S_+ \) and \( S_- \), and as such has no direct couplings to the Standard Model fields. Its mass being below 100 GeV is fully consistent with experimental constraints. The phenomenology of such a weakly coupled light neutral Higgs boson will be discussed in section 7.

The mass of the \( Z' \) gauge boson and the \( Z - Z' \) mixing angle are listed in Table 3 (for Model 1). In section 7 we show that these values are compatible with known experimental constraints.

Table 4 lists the eigenvectors of the neutralino mass matrix. These will become relevant in discussing the decays of the \( Z' \) gauge boson. Tables 5 and 6 give the eigenvectors of the chargino and the CP–even Higgs bosons, which will also be used in the study of \( Z' \) decays.

Tables 7–11 are analogous to Tables 2–6, except that they now apply to Model 2 (with \( x = 1.6 \)). In this case, \( \tan \beta = 5.83 \) and \( m_h = 126 \text{ GeV} \). Here the right–handed sleptons are heavier than the left–handed sleptons. In fact, in this Model, the LSP is the left–handed sneutrino. This can also be a candidate for cold dark matter in the AMSB framework, as the decay of the moduli fields and the gravitino will produce \( \tilde{\nu}_{Li} \) with an abundance of the right order [18, 27].
| Particles                          | Symbol                  | Mass (TeV)          |
|-----------------------------------|-------------------------|---------------------|
| Neutralinos                       | \{m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}\} | \{0.175165, 0.517, 0.980, 0.980\} |
| Neutralinos                       | \{m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}\} | \{0.206, 1.644, 3.278\} |
| Charginos                         | \{m_{\tilde{\chi}^\pm}, m_{\tilde{\chi}^\pm}\} | \{0.175171, 0.983\} |
| Gluino                            | \text{M}_3              | 1.239               |
| Neutral Higgs bosons              | \{m_{h}, m_{H}, m_{A}\} | \{0.121, 0.793, 0.792\} |
| Neutral Higgs bosons              | \{m_{h'}, m_{H'}, m_{A'}\} | \{0.060, 2.394, 0.241\} |
| Charged Higgs bosons              | \text{m}_{H^\pm}        | 0.796               |
| R.H sleptons                      | \{m_{\tilde{\nu}_R}, m_{\tilde{\nu}_R}, m_{\tilde{\nu}_1}\} | \{0.215, 0.215, 0.205\} |
| L.H sleptons                      | \{m_{\tilde{\nu}_L}, m_{\tilde{\nu}_L}, m_{\tilde{\nu}_2}\} | \{0.249, 0.249, 0.257\} |
| Sneutrinos                        | \{m_{\tilde{c}}, m_{\tilde{c}}, m_{\tilde{c}}\} | \{0.220, 0.220, 0.220\} |
| R.H down squarks                  | \{m_{\tilde{d}_R}, m_{\tilde{d}_R}, m_{\tilde{b}_1}\} | \{1.284, 1.284, 1.284\} |
| L.H down squarks                  | \{m_{\tilde{d}_L}, m_{\tilde{d}_L}, m_{\tilde{b}_2}\} | \{1.186, 1.186, 1.028\} |
| R.H up squarks                    | \{m_{\tilde{u}_R}, m_{\tilde{u}_R}, m_{\tilde{t}_1}\} | \{1.098, 1.098, 0.644\} |
| L.H up squarks                    | \{m_{\tilde{u}_L}, m_{\tilde{u}_L}, m_{\tilde{t}_2}\} | \{1.184, 1.184, 1.099\} |
| R.H scalar neutrinos              | \{m_{\tilde{\nu}_i}\} | \{0.605\} |
| R.H pseudoscalar neutrinos        | \{m_{\tilde{\nu}_i}\} | \{0.413\} |
| Heavy scalar neutrino (\tilde{\nu}_c, \tilde{\nu}_c) | \{m_{\tilde{\nu}_1}, m_{\tilde{\nu}_2}\} | \{1.142, 3.644\} |
| Heavy pseudoscalar neutrino (\tilde{\nu}_c, \tilde{\nu}_c) | \{m_{\tilde{\nu}_3}, m_{\tilde{\nu}_2}\} | \{0.595, 1.439\} |
| R.H neutrinos                     | \{m_{\nu}\} | \{0.455\} |
| Heavy neutrinos (\nu^c, \nu^c)   | \{m_{\omega_1}, m_{\omega_2}\} | \{0.933, 1.635\} |

Table 2: Sparticle masses in Model 1 \((x = 1.3)\) for the choice \(M_{aux} = 56.398\) TeV, \(\tan^\psi = -1.295\), \(u = 2.054\) TeV, \(f_{\nu^c} = 0.28\), \(f_{\nu^c} = 0.28\), \(h = 0.921\), \(g_x = 0.41\), \(M_{\nu^c} = 1\) TeV and \(M_l = 174.3\) GeV. This corresponds to \(\tan^\beta = 4.39\), \(\mu = -0.977\) TeV, \(\mu' = 0.214\) TeV, \(y_b = 0.03\).

| \(Z'\) boson mass | \(M_{Z'}\) | 2.383 TeV |
|-------------------|------------|-----------|
| \(Z - Z'\) mixing angle | \(\xi\) | 0.001 |

Table 3: \(Z'\) mass and \(Z - Z'\) mixing angle in Model 1 for the same set of input parameters as in Table 2
| Fields | $\tilde{\chi}^0_1$ | $\tilde{\chi}^0_2$ | $\tilde{\chi}^0_3$ | $\tilde{\chi}^0_4$ | $\tilde{\chi}^0_5$ | $\tilde{\chi}^0_6$ | $\tilde{\chi}^0_7$ |
|--------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $\tilde{B}$ | -0.003 | 0.998 | 0.051 | 0.025 | 0.000 | -0.001 | 0.000 |
| $\tilde{W}^0_3$ | -0.997 | 0.001 | -0.052 | -0.058 | 0.000 | 0.002 | 0.000 |
| $\tilde{H}^0_d$ | 0.078 | 0.054 | -0.703 | -0.704 | -0.002 | 0.030 | 0.001 |
| $\tilde{H}^0_u$ | -0.004 | 0.019 | -0.707 | 0.706 | 0.001 | -0.042 | 0.016 |
| $\tilde{B}'$ | 0.000 | 0.000 | -0.004 | -0.023 | -0.026 | -0.612 | -0.790 |
| $\tilde{S}_+$ | 0.000 | 0.000 | -0.011 | 0.039 | -0.597 | 0.642 | -0.479 |
| $\tilde{S}_-$ | 0.000 | 0.000 | -0.009 | 0.026 | 0.802 | 0.458 | -0.382 |

Table 4: Eigenvectors of the neutralino mass matrix in Model 1. The unitary matrix $O$ in Eq. (86) is the transpose of this array.

| $U_{11}$ | $U_{12}$ | $U_{21}$ | $U_{22}$ | $V_{11}$ | $V_{12}$ | $V_{21}$ | $V_{22}$ |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.994 | 0.110 | -0.110 | 0.994 | 1.000 | 0.006 | -0.006 | 1.000 |

Table 5: Eigenvectors of the chargino mass matrix in Model 1, where $U$, $V$ are the unitary matrices that diagonalize the chargino mass matrix ($V^*M^{(c)}U^{-1} = M^{(c)}_{\text{diag}}$).

| Fields | $h$ | $h'$ | $H$ | $H'$ |
|--------|-----|-----|-----|-----|
| $H^0_d$ | 0.226 | -0.025 | 0.974 | -0.007 |
| $H^0_u$ | 0.967 | -0.110 | -0.227 | 0.027 |
| $S_+$ | -0.050 | -0.612 | -0.010 | -0.790 |
| $S_-$ | 0.104 | 0.783 | -0.008 | -0.613 |

Table 6: Eigenvectors of the CP–even Higgs boson mass matrix in Model 1. This array corresponds to $X$ used in Eqs. (82) – (84) and Eq. (109) of the text.
| Particles                   | Symbol                      | Mass (TeV)                          |
|----------------------------|-----------------------------|-------------------------------------|
| Neutralinos                | \{m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}\} | \{0.185.851, 0.550, 1.049, 1.050\} |
| Neutralinos                | \{m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}, m_{\tilde{\chi}^0}\} | \{0.498, 2.840, 4.539\}           |
| Charginos                  | \{m_{\tilde{\chi}^\pm}, m_{\tilde{\chi}^\pm}\}            | \{0.185855, 1.051\}               |
| Gluino                     | \text{M}_{3}                | 1.298                               |
| Neutral Higgs bosons       | \{m_{h}, m_{H}, m_{A}\}     | \{0.126, 0.625, 0.625\}            |
| Neutral Higgs bosons       | \{m_{h'}, m_{H'}, m_{A'}\}  | \{0.023, 3.436, 0.125\}            |
| Charged Higgs bosons       | \text{m}_{H^\pm}            | 0.630                               |
| R.H sleptons               | \{m_{\tilde{e}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\} | \{0.383, 0.383, 0.385\}           |
| L.H sleptons               | \{m_{\tilde{e}_L}, m_{\tilde{\mu}_L}, m_{\tilde{\tau}_2}\} | \{0.213, 0.213, 0.210\}           |
| Sneutrinos                 | \{m_{\tilde{\nu}_L}, m_{\tilde{\nu}_R}, m_{\tilde{\nu}_\tau}\} | \{0.174, 0.174, 0.174\}           |
| R.H down squarks           | \{m_{\tilde{d}_R}, m_{\tilde{d}_R}, m_{\tilde{b}_1}\}       | \{1.370, 1.370, 1.369\}           |
| L.H down squarks           | \{m_{\tilde{u}_L}, m_{\tilde{u}_L}, m_{\tilde{b}_2}\}       | \{1.267, 1.267, 1.087\}           |
| R.H up squarks             | \{m_{\tilde{u}_R}, m_{\tilde{\mu}_R}, m_{\tilde{\tau}_1}\} | \{1.031, 1.031, 0.406\}           |
| L.H up squarks             | \{m_{\tilde{d}_L}, m_{\tilde{d}_L}, m_{\tilde{b}_2}\}       | \{1.264, 1.264, 1.1141\}          |
| R.H scalar neutrinos       | \{m_{\tilde{\nu}_i}\} (i = 1 - 3) | 1.583                               |
| R.H pseudoscalar neutrinos | \{m_{\tilde{\nu}_i}\} (i = 1 - 3) | 1.129                               |
| Heavy scalar neutrino \(\tilde{\nu}^{\pm}, \bar{\tilde{\nu}}^{\pm}\) | \{m_{\tilde{\nu}_{1s}}, m_{\tilde{\nu}_{2s}}\} | \{1.852, 4.700\}                  |
| Heavy pseudoscalar neutrino \(\tilde{\nu}^{\pm}, \bar{\tilde{\nu}}^{\pm}\) | \{m_{\tilde{\nu}_{ps}}, m_{\tilde{\nu}_{2ps}}\} | \{1.398, 2.586\}                  |
| R.H neutrinos              | \{m_{\nu_i}\}              | 0.829                               |
| Heavy neutrinos \(\nu^c, \bar{\nu}^c\) | \{m_{\nu_1}, m_{\nu_2}\} | \{1.174, 2.070\}                  |

Table 7: Sparticle masses in Model 2 \((x = 1.6)\) for the choice \(M_{aux} = 59.987\ \text{TeV}, \tan \psi = -1.202, u = 2.697\ \text{TeV}, f_{\tilde{\psi}} = 0.4, f_{\tilde{\psi}} = 0.4, h = 1.0, g_x = 0.45, M' = 2.197\ \text{TeV}, M_{\tilde{\psi}} = 1\ \text{TeV}\) and \(M_t = 174.3\ \text{GeV}\). This corresponds to \(\tan \beta = 5.83, \mu = -1.046\ \text{TeV}, \mu' = -0.505\ \text{TeV}, y_h = 0.06\).

| \(Z'\) boson mass | \(M_{Z'}\) | 3.433 \(\text{TeV}\) |
|-------------------|------------|----------------------|
| \(Z - Z'\) mixing angle | \(\xi\) | 0.00068 |

Table 8: \(Z'\) mass and \(Z - Z'\) mixing angle in Model 2 for the same set of input parameters as in Table 7.
Table 9: Eigenvectors of the neutralino mass matrix in Model 2. The unitary matrix $O$ in Eq. (86) is the transpose of this array.

| Fields | $\tilde{\chi}_0^1$ | $\tilde{\chi}_0^2$ | $\tilde{\chi}_0^3$ | $\tilde{\chi}_0^4$ | $\tilde{\chi}_0^5$ | $\tilde{\chi}_0^6$ | $\tilde{\chi}_0^7$ |
|--------|------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $\tilde{B}$ | -0.001 | 0.998 | -0.052 | 0.023 | 0.000 | 0.000 | 0.000 |
| $\tilde{W}_3^0$ | -0.997 | 0.002 | 0.053 | -0.052 | 0.000 | -0.001 | 0.000 |
| $\tilde{H}_d^0$ | -0.074 | -0.052 | -0.703 | 0.705 | -0.002 | 0.011 | 0.001 |
| $\tilde{H}_u^0$ | 0.000 | -0.020 | -0.707 | -0.707 | -0.001 | -0.021 | 0.016 |
| $\tilde{B}'$ | 0.000 | 0.000 | 0.006 | -0.004 | 0.023 | 0.0563 | 0.826 |
| $\tilde{S}_+$ | 0.000 | 0.000 | 0.011 | 0.018 | -0.648 | -0.620 | 0.441 |
| $\tilde{S}_-$ | 0.000 | 0.000 | 0.007 | 0.017 | 0.761 | -0.546 | 0.350 |

Table 10: Eigenvectors of the chargino mass matrix in Model 2, where $U, V$ are the unitary matrices that diagonalize the chargino mass matrix ($V^*M^{(c)}U^{-1} = M^{(c)}_{\text{diag}}$).

| $U_{11}$ | $U_{12}$ | $U_{21}$ | $U_{22}$ | $V_{11}$ | $V_{12}$ | $V_{21}$ | $V_{22}$ |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.994 | 0.105 | -0.105 | 0.994 | 1.000 | 0.000 | -0.000 | 1.000 |

Table 11: Eigenvectors of the CP–even Higgs boson mass matrix in Model 2. This array corresponds to $X$ used in Eqs. (82) – (84) and Eq. (109) of the text.

| Fields | $h$ | $h'$ | $H$ | $H'$ |
|--------|-----|-----|-----|-----|
| $H_d^0$ | 0.176 | 0.002 | 0.984 | 0.005 |
| $H_u^0$ | 0.984 | 0.010 | -0.176 | -0.025 |
| $S_+$ | -0.012 | -0.640 | 0.007 | -0.768 |
| $S_-$ | -0.023 | 0.768 | 0.006 | -0.640 |
6 \ Z' \text{ Decay Modes and Branching Ratios}

The \ Z' \text{ gauge boson of our model has substantial coupling to the quarks. With its mass in the range 2-4 TeV, it will be produced copiously at the LHC via the process } pp \to Z'. \text{ The reach of LHC is about 5 TeV for a } Z' \text{ with generic quark and lepton couplings}^28. \text{ Our model will then be directly tested at the LHC. Once produced, the } Z' \text{ will decay into various channels. It is important to identify the dominant decay modes of the } Z' \text{ and calculate the corresponding branching ratios. This is what we do in this section. We will see that our } Z' \text{ is almost leptophobic, with } Br(Z' \to e^+e^-) = (1 - 1.6)\%. \text{ Direct limits on such a } Z' \text{ are rather weak, however, the } Z - Z' \text{ mixing which occurs in our models at the level of 0.001 does provide useful constraints.}

We now turn to the dominant 2–body decays of \ Z'. \text{ In this analysis we can safely ignore the small } Z - Z' \text{ mixing for the most part.}

The Lagrangian for \ Z' \text{ coupling to the Standard Model fermions can be written as}

\[ \mathcal{L} = g_x \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f Z_{\mu}' . \] (56)

The \ Z' \text{ decay rate into a fermion–antifermion pair is then}

\[ \Gamma(Z' \to \bar{f} f) = C_f \frac{g_x^2}{12\pi} M_{Z'} \left[ v_f^2 \left( 1 + 2 \frac{m_f^2}{M_{Z'}^2} \right) + a_f^2 \left( 1 - 4 \frac{m_f^2}{M_{Z'}^2} \right) \right] \sqrt{1 - 4 \frac{m_f^2}{M_{Z'}^2}} . \] (57)

Here \( C_f = 3 \ (1) \) for quarks (leptons), \( M_{Z'} \) is the \( Z' \) mass and \( g_x \) is the \( U(1)_x \) gauge coupling. The vector and the axial–vector couplings \( (v_f, a_f) \) are related to the \( U(1)_x \) charges of the fermions as

\[ v_f = \frac{1}{2} (Q(f_L) + Q(f_R)) , \] (58)

\[ a_f = \frac{1}{2} (Q(f_L) - Q(f_R)) . \] (59)

Here \( Q \) is the \( U(1)_x \) charge of \( f_L \) (listed in Table 1) and \( Q(f_R) = -Q(f_L') \).

The decay width for \( Z' \to \nu_{L_i} \nu_{L_i} \) and \( Z' \to \nu_{i}^c \nu_{i}^c \) are:

\[ \Gamma(Z' \to \nu_{L_i} \nu_{L_i}) = \frac{g_x^2}{24\pi} Q_{\nu_{L_i}}^2 M_{Z'} , \] (60)

\[ \Gamma(Z' \to \nu_{i}^c \nu_{i}^c) = \frac{g_x^2}{24\pi} Q_{\nu_i}^2 M_{Z'} \left( 1 - 4 \frac{m_{\nu_i}^2}{M_{Z'}^2} \right)^{\frac{3}{2}} . \] (61)
There is mixing between the heavy vector–like νc and the νc [Cf. Eq. (37)], with the mass eigenstates (ω1, ω2) given by

\[
\begin{pmatrix}
ν^c \\
\bar{ν}^c
\end{pmatrix} = \begin{pmatrix}
\cos θ_{ν^c} & \sin θ_{ν^c} \\
-\sin θ_{ν^c} & \cos θ_{ν^c}
\end{pmatrix}
\begin{pmatrix}
ω_1 \\
ω_2
\end{pmatrix}
\tag{62}
\]

Since \(Q_{ν^c} = -Q_{ν^c}\), the Lagrangian for the \(Z'\) coupling to these neutrino is given by

\[
\mathcal{L} = \frac{g_x}{2} Q_{ν^c} \left( \cos 2θ_{ν^c} ω_1 \gamma^\mu γ_5 ω_2 - \sin 2θ_{ν^c} ω_2 \gamma^\mu γ_5 ω_2 \right)
- \sin 2θ_{ν^c} ω_2 \gamma^\mu γ_5 ω_1 Z'_μ. \tag{63}
\]

This leads to the decay rates

\[
\Gamma(Z' → ω_1ω_1) = \frac{g_x^2}{24π} M_{Z'} Q^2_{ν^c} \cos^2 2θ_{ν^c} \left( 1 - \frac{4m^2_{ω_1}}{M_{Z'}^2} \right)^{\frac{3}{2}}, \tag{64}
\]

\[
\Gamma(Z' → ω_2ω_2) = \frac{g_x^2}{24π} M_{Z'} Q^2_{ν^c} \cos^2 2θ_{ν^c} \left( 1 - \frac{4m^2_{ω_2}}{M_{Z'}^2} \right)^{\frac{3}{2}}, \tag{65}
\]

\[
\Gamma(Z' → ω_1ω_2) = \frac{g_x^2}{12π} M_{Z'} Q^2_{ν^c} \sin^2 2θ_{ν^c} \left[ 1 - \frac{(m^2_{ω_1} + m^2_{ω_2})}{2M^2_{Z'}} - \frac{(m^2_{ω_1} - m^2_{ω_2})^2}{2M^2_{Z'}} - 3m^2_{ω_1}m^2_{ω_2}{M^2_{Z'}} \right]
\times \sqrt{\left( 1 - \frac{(m^2_{ω_1} + m^2_{ω_2})}{M^2_{Z'}} \right) \left( 1 - \frac{(m^2_{ω_1} - m^2_{ω_2})}{M^2_{Z'}} \right)}. \tag{66}
\]

Here \(m_{ω_1}(m_{ω_2})\) are the masses of the physical Majorana fermions.

The \(Z'\) interaction with the sfermions is described by the Lagrangian

\[
\mathcal{L} = ig_s (v_f ± a_f) \tilde{f}^* L,R \partial_μ \tilde{f}_{L,R} Z'_μ. \tag{67}
\]

The rate for the decay \(Z'\) to sfermions is given by

\[
\Gamma(Z' → \tilde{f}^*_L,R \tilde{f}_{L,R}) = C_f \frac{g_x^2}{48π} M_{Z'} (v_f ± a_f)^2 \left( 1 - \frac{4m^2_{f_{L,R}}}{M^2_{Z'}} \right)^{\frac{3}{2}}, \tag{68}
\]

where the \(+(-)\) sign is for the left (right)–handed sfermions and \(m_{f_{L,R}}\) is the left (right)–handed sfermion mass. \(v_f\) and \(a_f\) are as given in Eqs. (58)–(59).

In the top squark sector, there is non–negligible mixing between the left and the right–handed sfermions. This leads to the following modification of the Lagrangian:

\[
\mathcal{L} = ig_s \left( (v_f ± a_f \cos 2θ_j) \tilde{f}_{1,2}^* \partial_μ \tilde{f}_{1,2} - a_f \sin 2θ_j (\tilde{f}_1^* \partial_μ \tilde{f}_2 + \tilde{f}_2^* \partial_μ \tilde{f}_1) \right) Z'_μ, \tag{69}
\]
where $\theta_f$ is the left–right sfermion mixing angle. The decay rate is given by

$$\Gamma(Z' \to f_{1,2}^* f_{1,2}) = C_f \frac{g_x^2}{48\pi} M_{Z'} (v_f \pm a_f \cos 2\theta_f)^2 \left(1 - \frac{m_{f_{1,2}}^2}{M_{Z'}^2}\right)^\frac{3}{2}, \quad (70)$$

$$\Gamma(Z' \to \bar{f}_{1,2} f_{1,2}) = C_f \frac{g_x^2}{48\pi} M_{Z'} (a_f \sin 2\theta_f)^2 \left[1 + 2 \frac{(m_1^2 + m_2^2)}{M_{Z'}^2} + \frac{(m_1^2 - m_2^2)^2}{M_{Z'}^4}\right]^{\frac{3}{2}}. \quad (71)$$

The $\tilde{\nu}^c$ and $\tilde{\nu}^c$ splits into two scalar and two pseudoscalar which mix (see Eqs. (48)–(49)). The mass eigenstate $\tilde{\omega}_{is}$ and $\tilde{\omega}_{ip}$ are given as

$$\left(\begin{array}{c}
\tilde{\nu}^c_s \\
\tilde{\nu}^c_p
\end{array}\right) = \left(\begin{array}{cc}
\cos \theta_{ws} & \sin \theta_{ws} \\
-\sin \theta_{ws} & \cos \theta_{ws}
\end{array}\right) \left(\begin{array}{c}
\tilde{\omega}_{1s} \\
\tilde{\omega}_{2s}
\end{array}\right). \quad (72)$$

$$\left(\begin{array}{c}
\tilde{\nu}^c_s \\
\tilde{\nu}^c_p
\end{array}\right) = \left(\begin{array}{cc}
\cos \theta_{wp} & \sin \theta_{wp} \\
-\sin \theta_{wp} & \cos \theta_{wp}
\end{array}\right) \left(\begin{array}{c}
\tilde{\omega}_{1p} \\
\tilde{\omega}_{2p}
\end{array}\right). \quad (73)$$

The Lagrangian for the $Z'$ coupling to the scalar–pseudoscalar pair is given by:

$$\mathcal{L} = g_x \left[(Q_{\nu c} \cos \theta_{ws} \cos \theta_{wp} + Q_{\nu c} \sin \theta_{ws} \sin \theta_{wp}) \tilde{\omega}_{1s} \overset{\leftrightarrow}{\partial}_\mu \tilde{\omega}_{1p}
+ (Q_{\nu c} \sin \theta_{ws} \sin \theta_{wp} + Q_{\nu c} \cos \theta_{ws} \cos \theta_{wp}) \tilde{\omega}_{2s} \overset{\leftrightarrow}{\partial}_\mu \tilde{\omega}_{2p}
+ (Q_{\nu c} \cos \theta_{ws} \sin \theta_{wp} - Q_{\nu c} \sin \theta_{ws} \cos \theta_{wp}) \tilde{\omega}_{1s} \overset{\leftrightarrow}{\partial}_\mu \tilde{\omega}_{2p}
+ (Q_{\nu c} \sin \theta_{ws} \cos \theta_{wp} - Q_{\nu c} \cos \theta_{ws} \sin \theta_{wp}) \tilde{\omega}_{2s} \overset{\leftrightarrow}{\partial}_\mu \tilde{\omega}_{1p}\right] Z'^\mu. \quad (74)$$

This leads to the decay rate

$$\Gamma(Z' \to \tilde{\omega}_{is} \tilde{\omega}_{ip}) = \frac{g_x^2}{48\pi} Q_{\nu c}^2 \left[1 - 2 \frac{(m_{\omega_{is}}^2 + m_{\omega_{ip}}^2)}{M_{Z'}^2} + \frac{(m_{\omega_{is}}^2 - m_{\omega_{ip}}^2)^2}{M_{Z'}^4}\right]^{\frac{3}{2}}, \quad (75)$$

where $Q_{ij}$ is identified with the appropriate coupling to $\tilde{\omega}_{is} \tilde{\omega}_{ip}$ term in the Lagrangian of Eq. (74).

The supersymmetric partners of $\nu^c_i$ split into a scalar ($\tilde{\nu}^c_{is}$) and a pseudoscalar ($\tilde{\nu}^c_{ip}$). The decay of $Z'$ to these fields is similar to those analyzed in Eq. (75):

$$\Gamma(Z' \to \tilde{\nu}^c_{is} \tilde{\nu}^c_{ip}) = \frac{g_x^2}{48\pi} Q_{\nu c}^2 \left[1 - 2 \frac{(m_{\tilde{\nu}^c_{is}}^2 + m_{\tilde{\nu}^c_{ip}}^2)}{M_{Z'}^2} + \frac{(m_{\tilde{\nu}^c_{is}}^2 - m_{\tilde{\nu}^c_{ip}}^2)^2}{M_{Z'}^4}\right]^{\frac{3}{2}}, \quad (76)$$

where $m_{\tilde{\nu}^c_{is}}$ and $m_{\tilde{\nu}^c_{ip}}$ are the masses of the scalar and the pseudoscalar.

The Lagrangian for the $Z'$ coupling to the charged Higgs bosons is given by

$$\mathcal{L} = i g_x (Q_{H_d} \sin^2 \beta - Q_{H_u} \cos^2 \beta) \bar{H}^+ \overset{\leftrightarrow}{\partial}_\mu H^- Z'^\mu
+ g_x (Q_{H_d} + Q_{H_u}) \sin \beta \cos \beta M_W (W^+_{\mu} H^- + W^-_{\mu} H^+) Z'^\mu, \quad (77)$$

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where \( Q_{H_d} (Q_{H_u}) \) is the \( U(1)_x \) charge of \( H_d (H_u) \) field. The decay rates of \( Z' \) to \( H^+ H^- \) and \( W^\pm H^\mp \) are given by

\[
\Gamma(Z' \to H^+ H^-) = \frac{g_x^2}{48\pi} M_{Z'} (Q_{H_d} \sin^2 \beta - Q_{H_u} \cos^2 \beta)^2 \left( 1 - 4 \frac{m_{H^\pm}^2}{M_{Z'}^2} \right)^{\frac{3}{2}}, \tag{78}
\]

\[
\Gamma(Z' \to W^\pm H^\mp) = \frac{g_x^2}{192\pi} M_{Z'} (Q_{H_d} + Q_{H_u})^2 \left[ 1 + 2 \left( \frac{5M_W^2 - m_{H^\pm}^2}{M_{Z'}^2} \right) + \frac{(M_{W}^2 - m_{H^\pm}^2)^2}{M_{Z'}^4} \right] \times \sqrt{1 - 2 \left( \frac{M_W^2 + m_{H^\pm}^2}{M_{Z'}^2} \right) + \frac{(M_{W}^2 - m_{H^\pm}^2)^2}{M_{Z'}^4}}. \tag{79}
\]

Here \( m_{H^\pm} \) is the mass of the \( H^\pm \) Higgs boson and \( M_W \) is the mass of the \( W \)-boson.

The \( Z W^+ W^- \) coupling of the Standard Model will induce, through \( Z - Z' \) mixing, a \( Z' W^+ W^- \) coupling. The decay of \( Z' \) to a pair of \( W^+ W^- \) is found to be \[29\]

\[
\Gamma(Z' \to W^+ W^-) = \frac{g_x^2}{192\pi} \cos^2 \theta_W \sin^2 \xi M_{Z'} \left[ \frac{M_{Z'}^4}{M_W^4} \left( 1 + 20 \frac{M_W^2 - m_{H^\pm}^2}{M_{Z'}^2} + 12 \frac{M_{W}^4}{M_{Z'}^4} \right) \left( 1 - 4 \frac{M_{W}^2}{M_{Z'}^2} \right)^{\frac{3}{2}} \right]. \tag{80}
\]

We now discuss the decays of \( Z' \to Zh, ZH, Z'h, Z'H' \) as well as \( Z' \to hA, h'A' \) etc.. The relevant Lagrangian is

\[
\mathcal{L} = 2g_x M_{Z'} \sum_{i=1}^{4} (Q_{H_d} \cos \beta X_{1i} - Q_{H_u} \sin \beta X_{2i}) Z'^\mu Z_\mu H_i \\
- g_x \sum_{i=1}^{4} (Q_{H_d} \sin \beta X_{1i} + Q_{H_u} \cos \beta X_{2i}) Z'^\mu H_i^0 \leftrightarrow A \\
- g_x \sum_{i=1}^{4} (Q_{S_+} \cos \psi X_{3i} + Q_{S_-} \sin \psi X_{4i}) Z'^\mu H_i^0 \leftrightarrow A', \tag{81}
\]

where \( H_i^0 (= h, h', H, H') \) are the neutral CP–even Higgs bosons, \( m_{H_i} \) are the masses of the corresponding Higgs boson, \( Q_{S_+} (Q_{S_-}) \) is the \( U(1)_x \) charge of the \( S_+ (S_-) \) field and \( X_{ij} \) are the matrix elements of the unitary matrix that diagonalizes the CP–even mass matrix of Eq. \[15\]. The decay rates are then

\[
\Gamma(Z' \to ZH_i^0) = \frac{g_x^2}{48\pi} M_{Z'} (Q_{H_d} \cos \beta X_{1i} - Q_{H_u} \sin \beta X_{2i})^2 \times \left[ 1 + 2 \left( \frac{5M_{Z}^2 - m_{H_i}^2}{M_{Z'}^2} \right) + \frac{(M_{Z}^2 - m_{H_i}^2)^2}{M_{Z'}^2} \right] \sqrt{1 - 2 \left( \frac{M_{Z}^2 + m_{H_i}^2}{M_{Z'}^2} \right) + \frac{(M_{Z}^2 - m_{H_i}^2)^2}{M_{Z'}^2}}, \tag{82}
\]

\[
\Gamma(Z' \to H_i A) = \frac{g_x^2}{48\pi} M_{Z'} (Q_{H_d} \sin \beta X_{1i} + Q_{H_u} \cos \beta X_{2i})^2 \times \left[ 1 - 2 \left( \frac{m_A^2 + m_{H_i}^2}{M_{Z'}^2} \right) + \frac{(m_A^2 - m_{H_i}^2)^2}{M_{Z'}^2} \right]^{\frac{3}{2}}. \tag{83}
\]
\[
\Gamma(Z' \to H_i A') = \frac{g_x^2}{48\pi} M_{Z'} (Q_{S+} \cos \psi X_{3i} + Q_{S-} \sin \psi X_{4i})^2
\times \left[ 1 - 2 \left( \frac{m_{A'}^2 + m_{H_i}^2}{M_{Z'}^2} \right) + \frac{(m_{A'}^2 - m_{H_i}^2)^2}{M_{Z'}^4} \right]^{\frac{3}{2}},
\]

(84)

where \( m_A \) and \( m_{A'} \) are the pseudoscalar Higgs boson masses.

We parameterize the interactions between the neutralinos \((\tilde{\chi}_i^0, \tilde{\chi}_2^0, ... \tilde{\chi}_7^0)\) and the \( Z' \) boson as

\[
\mathcal{L} = \sum_{i,j} g_{ij} \tilde{\chi}_i^0 \gamma^\mu \gamma^5 \tilde{\chi}_j^0 Z'_\mu. \tag{85}
\]

Here the coupling \( g_{ij} \) is obtained from the eigenvectors of the neutralino mass matrix of Eq. (27) as

\[
\hat{g} = \frac{g_x}{2} O \left( \begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{x}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{array} \right) O^T, \tag{86}
\]

with \( g_{ij} = (\hat{g})_{ij} \). Here \( O \) is the orthogonal matrix that diagonalizes the neutralino mass matrix. The \( Z' \) partial decay rates into neutralinos is found to be

\[
\Gamma(Z' \to \tilde{\chi}_i^0 \tilde{\chi}_i^0) = \frac{g_x^2}{6\pi} M_{Z'} \left( 1 - 4 \frac{m_i^2}{M_{Z'}^2} \right)^{\frac{3}{2}},
\]

(87)

\[
\Gamma(Z' \to \tilde{\chi}_i^0 \tilde{\chi}_j^0) = \frac{(g_{ij} + g_{ji})^2}{12\pi} M_{Z'} \left[ 1 - \frac{(m_i^2 + m_j^2)}{2M_{Z'}^2} - \frac{(m_i^2 - m_j^2)^2}{2M_{Z'}^4} - 3 \frac{m_i m_j}{M_{Z'}^2} \right]
\times \sqrt{\left( 1 - \frac{(m_i + m_j)^2}{M_{Z'}^2} \right) \left( 1 - \frac{(m_i - m_j)^2}{M_{Z'}^2} \right)} \quad (i \neq j) \tag{88}
\]

where \( m_i \) are the neutralino masses. (Here our result disagrees with Eq. (48) of Ref. [3] by a factor of 2.)

The Lagrangian for the couplings of \( Z' \) to the charginos is given by [3]

\[
\mathcal{L} = \frac{1}{2} g_x \sum_{i,j=1}^2 \tilde{\chi}_i^{\pm} \gamma^\mu (v_{ij} + a_{ij} \gamma_5) \tilde{\chi}_j^{\pm} Z'_\mu. \tag{89}
\]
The $Z'$ decay rate into the chargino pair is then

$$\Gamma(Z' \to \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp) = \frac{g^2_{Z'}}{48\pi M_{Z'}} \left[ (v_{ij}^2 + a_{ij}^2)(1 - \frac{(m_i^2 + m_j^2)}{2M_{Z'}^2} - \frac{(m_i^2 - m_j^2)^2}{2M_{Z'}^4}) + 3(v_{ij}^2 - a_{ij}^2) \frac{m_im_j}{M_{Z'}^2} \right] \times \sqrt{\left(1 - \frac{(m_i + m_j)^2}{M_{Z'}^2}\right) \left(1 - \frac{(m_i - m_j)^2}{M_{Z'}^2}\right)}.$$ \hfill (90)

Here $m_i$ is the chargino mass, $v_{ij}$ and $a_{ij}$ are given in terms of the charges $Q_{H_u}$, $Q_{H_d}$ and the matrices $U$ and $V$ which diagonalize the chargino mass matrix Eq. \hfill \text{(29)}, can be explicitly written as \hfill [3]

$$v_{11} = Q_{H_d} V_{12}^2 - Q_{H_u} U_{12}^2, \quad (91)$$

$$a_{11} = Q_{H_d} V_{21}^2 + Q_{H_u} U_{21}^2, \quad (92)$$

$$v_{12} = v_{21} = Q_{H_d} V_{12} V_{11} - \delta Q_{H_u} U_{12} U_{11}, \quad (93)$$

$$a_{12} = a_{21} = Q_{H_d} V_{12} V_{11} + \delta Q_{H_u} U_{12} U_{11}, \quad (94)$$

$$v_{22} = Q_{H_d} V_{11}^2 - Q_{H_u} U_{11}^2, \quad (95)$$

$$a_{22} = Q_{H_d} V_{22}^2 + Q_{H_u} U_{22}^2, \quad (96)$$

where $\delta = sgn(m_{\tilde{\chi}_i^\pm}) \times sgn(m_{\tilde{\chi}_j^\mp}).$

In Table \hfill \text{12} we present the partial decay rates of $Z'$ to two fermions and to two scalars in Model 1. The total width of $Z'$ is 106 GeV (this ignores three body decays, which are more suppressed). One sees from Table 7 that the $Z'$ decays dominantly to $q\bar{q}$ with $Br(Z' \to q\bar{q}) \simeq 43.93\%$. On the other hand, $Br(Z' \to e^+e^-) \simeq 1.16\%$ in this case. Thus this $Z'$ is leptophobic. We also see that $Z' \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and $Z' \to \tilde{\chi}_i^\pm\tilde{\chi}_j^\mp$ are significant. There are also non–negligible decays into two Higgs particles, with $Z' \to h'\ell'$ being the dominant mode in this class. The decay of $Z'$ into sfermions is a new production channel for supersymmetric particles. Decays into sneutrino pairs is the dominant mode in this category, with $Br(Z' \to \tilde{\nu}_L\tilde{\nu}_L) \sim 7.74\%$. The signature will be $pp \to Z' \to \tilde{\nu}_L\tilde{\nu}_L \to \ell_i^- \ell_i^- \tilde{\chi}_1^+\tilde{\chi}_1^+$, where the sneutrino decays into $\ell_i^- \tilde{\chi}_1^+$, with the subsequent decay $\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 + \pi^\pm$, etc.

In Table \hfill \text{13} we list the $Z'$ partial decay rates in Model 2. $Br(Z' \to e^+e^-) \simeq 1.60\%$ in this case. Other features are very similar to the case of Model 1 (Table 7).
| Decays of $Z'$                                      | Width (GeV)               |
|----------------------------------------------------|---------------------------|
| $Z' \rightarrow \{\bar{u}u, \bar{c}c, \bar{t}t\}$ | $\{4.75, 4.75, 4.64\}$  |
| $Z' \rightarrow \bar{d}d$ ($ss, bb$)               | 9.59                      |
| $Z' \rightarrow \bar{e}e$ ($\bar{\mu}\mu, \bar{\tau}\tau$) | 1.13                      |
| $Z' \rightarrow \nu_{eL}\nu_{eL}$ ($\nu_{\mu L}\nu_{\mu L}, \nu_{\tau L}\nu_{\tau L}$) | 0.65                      |
| $Z' \rightarrow \nu_{eR}\nu_{eR}$ ($\nu_{\mu R}\nu_{\mu R}, \nu_{\tau R}\nu_{\tau R}$) | 4.19                      |
| $Z' \rightarrow \bar{\omega}_1\omega_1$          | 0.50                      |
| $Z' \rightarrow \{\bar{\chi}_1\bar{\chi}_3, \bar{\chi}_1\bar{\chi}_4, \bar{\chi}_2\bar{\chi}_4, \bar{\chi}_3\bar{\chi}_5, \bar{\chi}_4\bar{\chi}_5, \bar{\chi}_5\bar{\chi}_5, \bar{\chi}_5\bar{\chi}_6\}$ | $\{0.01, 0.01, 0.01, 3.38, 0.01, 0.05, 3.34, 5.65\}$ |
| $Z' \rightarrow \{\bar{\chi}_2\bar{\chi}_2, \bar{\chi}_1\bar{\chi}_2, \bar{\chi}_1\bar{\chi}_2^+\}$ | $\{3.36, 0.02, 0.02\}$  |
| $Z' \rightarrow \bar{\tilde{u}}_R\tilde{u}_R$ ($\bar{\tilde{c}}_R\tilde{c}_R$) | 0.13                      |
| $Z' \rightarrow \{\tilde{t}_R^*\tilde{t}_R, \tilde{t}_L^*\tilde{t}_L, \tilde{\tau}_R^*\tilde{\tau}_L\}$ | $\{0.88, 0.13, 0.13\}$  |
| $Z' \rightarrow \bar{\tilde{e}}_L^*\tilde{e}_L$ ($\bar{\tilde{\mu}}_L^*\tilde{\mu}_L, \bar{\tilde{\tau}}_L^*\tilde{\tau}_L$) | 0.30                      |
| $Z' \rightarrow \bar{\tilde{e}}_R^*\tilde{e}_R$ ($\bar{\tilde{\mu}}_R^*\tilde{\mu}_R, \bar{\tilde{\tau}}_R^*\tilde{\tau}_R$) | 0.23                      |
| $Z' \rightarrow \tilde{\nu}_{eL}\tilde{\nu}_{eL}$ ($\tilde{\nu}_{\mu L}\tilde{\nu}_{\mu L}, \tilde{\nu}_{\tau L}\tilde{\nu}_{\tau L}$) | 2.52                      |
| $Z' \rightarrow \tilde{\nu}_{1s}\tilde{\nu}_{1p}$ ($\tilde{\nu}_{2s}\tilde{\nu}_{2p}, \tilde{\nu}_{3s}\tilde{\nu}_{3p}$) | 1.94                      |
| $Z' \rightarrow \tilde{\omega}_{1s}\tilde{\omega}_{1p}$ | 0.36                      |
| $Z' \rightarrow Zh$                                | 1.11                      |
| $Z' \rightarrow \{hA', HA, h'A'\}$                | $\{0.03, 0.47, 0.62\}$  |
| $Z' \rightarrow H^+H^-$                            | 0.46                      |
| $Z' \rightarrow W^+W^-$                            | 1.08                      |
| $Z' \rightarrow W^\pm H^\mp$                      | 0                         |

Table 12: Decays modes for $Z'$ in Model 1 for the parameters used in Table 2. The total decay width is $\Gamma(Z' \rightarrow all) = 97.68$ GeV.
| Decay Modes of $Z'$ | Width (GeV) |
|---------------------|-------------|
| $Z' \to \{\tilde{u}u, cc, tt\}$ | {15.00, 15.00, 14.86} |
| $Z' \to \tilde{d}d$ ($\tilde{s}s, \tilde{b}b$) | 20.90 |
| $Z' \to \tilde{e}e(\tilde{\mu}\mu, \tilde{\tau}\tau)$ | 3.69 |
| $Z' \to \nu_{eL}\nu_{eL}$ ($\nu_{\mu L}\nu_{\mu L}, \nu_{\tau L}\nu_{\tau L}$) | 0.37 |
| $Z' \to \nu_{eR}\nu_{eR}$ ($\nu_{\mu R}\nu_{\mu R}, \nu_{\tau R}\nu_{\tau R}$) | 6.19 |
| $Z' \to \{\tilde{\omega}_1\omega_1, \tilde{\omega}_1\omega_2\}$ | {1.41, 0.06} |
| $Z' \to \{\tilde{\chi}_1\tilde{\chi}_3, \tilde{\chi}_1\tilde{\chi}_4, \tilde{\chi}_2\tilde{\chi}_4, \tilde{\chi}_3\tilde{\chi}_5, \tilde{\chi}_4\tilde{\chi}_5, \tilde{\chi}_5\tilde{\chi}_5, \tilde{\chi}_5\tilde{\chi}_6\}$ | {0.03 0.03, 0.03, 10.99, 0.01, 0.04, 1.63, 6.64} |
| $Z' \to \{\tilde{\chi}_2^+ \tilde{\chi}_2^-\}$ | {10.96} |
| $Z' \to \tilde{u}_L^* \tilde{u}_L$ ($\tilde{c}_L^* \tilde{c}_L$) | 0.02 |
| $Z' \to \tilde{u}_R^* \tilde{u}_R$ ($\tilde{c}_R^* \tilde{c}_R$) | 3.80 |
| $Z' \to \{\tilde{t}_R^* \tilde{t}_R, \tilde{t}_L^* \tilde{t}_L, \tilde{t}_R^* \tilde{t}_L\}$ | {5.93, 0.45, 0.45} |
| $Z' \to \tilde{d}_L^* \tilde{d}_L$ ($\tilde{s}_L^* \tilde{s}_L, \tilde{b}_L^* \tilde{b}_L$) | 0.02 |
| $Z' \to \tilde{d}_R^* \tilde{d}_R$ ($\tilde{s}_R^* \tilde{s}_R, \tilde{b}_R^* \tilde{b}_R$) | 3.77 |
| $Z' \to \tilde{e}_L^* \tilde{e}_L$ ($\tilde{\mu}_L^* \tilde{\mu}_L, \tilde{\tau}_L^* \tilde{\tau}_L$) | 0.18 |
| $Z' \to \tilde{\nu}_R^* \tilde{\nu}_R$ ($\tilde{\mu}_R^* \tilde{\mu}_R, \tilde{\tau}_R^* \tilde{\tau}_R$) | 1.54 |
| $Z' \to \tilde{\nu}_{eL}^* \tilde{\nu}_{eL}$ ($\tilde{\nu}_{\mu L}^* \tilde{\nu}_{\mu L}, \tilde{\nu}_{\tau L}^* \tilde{\nu}_{\tau L}$) | 4.54 |
| $Z' \to \tilde{\nu}_L^* \tilde{\nu}_L$ | 1.04 |
| $Z' \to \tilde{\omega}_1 \omega_1$ | 0.91 |
| $Z' \to Zh$ | 2.96 |
| $Z' \to \{hA', HA, h'A'\}$ | {0.01, 2.38, 0.60} |
| $Z' \to H^+ H^-$ | 2.38 |
| $Z' \to W^+ W^-$ | 2.81 |
| $Z' \to W^\pm H^\mp$ | 0 |

Table 13: Decay modes for $Z'$ in Model 2 for the parameters used in Table 7. The total decay width is $\Gamma(Z' \to all) = 229.93$ GeV.
7 Other Experimental Signatures

In this section we discuss experimental signatures of the model other than $Z'$ decays.

7.1 $Z$ Decay and Precision Electroweak Data

The $Z - Z'$ mixing angle and the direct coupling of $Z'$ to the Standard Model fermions leads to modification of $Z$ decays. Precision electroweak data from LEP and SLC can be used to constrain such a $Z'$ in the mass range of a few TeV. Typically one finds the $Z - Z'$ mixing angle $\xi$ bounded to be less than a few $\times 10^{-3}$ [4], which is satisfied in our models.

The mixing of $Z$ with $Z'$ shifts the mass of the $Z$ boson from its SM value, while leaving the $W$ mass unaffected. This leads to a positive shift in the $\rho$ parameter:

$$\rho = \rho_{SM} \left( 1 + \xi^2 \frac{M^2_{Z'}}{M^2_Z} \right).$$  \hspace{1cm} (97)

The partial decay width $\Gamma(Z \to f\bar{f})$ is modified to

$$\Gamma(Z \to f\bar{f}) = \frac{\alpha M_Z}{12 \sin^2 \theta_W \cos^2 \theta_W} \left[ (g_V \cos \xi + \kappa v_f \sin \xi)^2 + (g_A \cos \xi + \kappa a_f \sin \xi)^2 \right].$$  \hspace{1cm} (98)

where

$$g_V = (T_3 - 2q \sin^2 \theta_W), \quad g_A = T_3, \quad \kappa = \frac{2g_V \sin \theta_W \cos \theta_W}{\kappa},$$  \hspace{1cm} (99)

with $q$ being the electric charge of the fermion. $v_f$ and $v_a$ are given in Eqs. (58) and (59).

Partial widths of the $Z$ will deviate from the Standard Model values owing to the shift in the coupling of $Z$ to fermions as well as due to a change in the derived value of $\sin^2 \theta_W$.

We define

$$\Delta_f = \frac{\Gamma(Z \to f\bar{f})}{\Gamma(Z \to f\bar{f})_{SM}} - 1.$$  \hspace{1cm} (100)

We use $\sin^2 \theta^SM_W = 0.23113$ (the best fit in the Standard Model) for evaluating $\Gamma(Z \to f\bar{f})_{SM}$. We do not perform a global fit to the available data, but we present a specific fit which is at least as good as the Standard Model and perhaps slightly better. We choose to set $\Delta_\ell = 0$, which yields $\sin^2 \theta_W = 0.230717$ in Model 1. With this value of $\sin^2 \theta_W$ we find

$$\{\Delta_u, \Delta_d, \Delta_\nu\} = \{0.00100, 0.00171, 0.00206\} \quad \text{(Model 1)}. \hspace{1cm} (101)$$
This leads to the following modifications of decay widths:

\[
\Gamma_{\text{had}} = \Gamma_{\text{had}}^{SM} + \Delta_d (2\Gamma_d^{SM} + \Gamma_b^{SM}) + 2\Delta_u \Gamma_u^{SM} = 1.74545 \text{ GeV}, \tag{102}
\]

\[
\Gamma_{\text{inv}} = (1 + \Delta_{\nu}) \Gamma_{\text{inv}}^{SM} = 502.793 \text{ MeV}, \tag{103}
\]

\[
R_\ell = \frac{\Gamma_{\text{had}}}{\Gamma(Z \to \ell^+\ell^-)} = 20.7744. \tag{104}
\]

We see that \(\Gamma_{\text{had}}\) is closer to the experimental value of 1.7444 GeV compared to the Standard Model value of 1.7429 GeV. Similarly \(R_\ell\) is closer to the experimental value (20.767 ± 0.025) than the Standard Model value (20.744). On the other hand, \(\Gamma_{\text{inv}}\) is somewhat worse than the Standard Model fit (501.76 MeV) to be compared with the experimental value of (499.0 ± 1.5 MeV). This deviation is still within acceptable range.

Here for our numerical fits we used the central values \(\Gamma_d^{SM} = 0.383185 \text{ GeV}, \Gamma_b^{SM} = 0.375926 \text{ GeV}\) and \(\Gamma_u^{SM} = 0.300302 \text{ GeV}\), and \(\Gamma_{\text{had}}^{SM} = 1.7429 \text{ GeV}\) [22].

The predicted value of \(M_W\) is modified as

\[
M_W = \sqrt{\left[1 + \frac{\xi^2 M_{Z'}^2}{M_Z^2} \left(1 - \sin^2 \theta_W \right) \sin^2 \theta_W \right]} M_W^{SM} = 80.4427 \text{ GeV}, \tag{105}
\]

where \(M_W^{SM} = 80.391 \text{ GeV}\) is used. This value is closer to the direct measurement \(M_W = 80.446\) than the Standard Model value.

In Model 2 we find, following the same procedure, \(\sin^2 \theta_W = 0.230783, \Delta_d = 0.00131, \Delta_u = 0.00089, \Delta_{\nu} = 0.00138\) and \(\Gamma_{\text{had}} = 1.74493 \text{ GeV}, \Gamma_{\text{inv}} = 502.453 \text{ MeV}, R_\ell = 20.7682, M_W = 80.4356 \text{ GeV}\).

The radiative correction parameter in \(\mu\) decay, \(\Delta r\), is slightly different in our model compared to the Standard Model. In the on–shell scheme we have

\[
\frac{M_W^2 \sin^2 \theta_W}{(M_W^2 \sin^2 \theta_W)^{SM}} = \frac{(1 - \Delta r)_{SM}}{(1 - \Delta r)}. \tag{106}
\]

We obtain \(\Delta r = 0.03501\) (in Model 1) using the Standard Model value of \(\Delta r = 0.0355 \pm 0.0019\). Clearly, such a shift is consistent with experimental constraints \(((\Delta r)_{\text{exp}} = 0.0347 \pm 0.0011)\).

### 7.2 \(Z'\) Mass Limit

The direct limit on the mass of \(Z'\) with generic couplings to quarks and leptons is \(M_{Z'} > 600 \text{ GeV}\). There is also a constraint on \(M_{Z'}\) from the process \(e^+e^- \to \mu^+\mu^-\). LEP II has
set severe constraints on lepton compositeness \[30, 22\] from this process. We focus on one such amplitude, involving all left–handed lepton fields. In our model, the effective Lagrangian for this process is

\[ L_{\text{eff}} = -g_x^2 \left(1 - \frac{x}{2}\right)^2 \frac{1}{M_{Z'}^2}(e_L \gamma_\mu e_L)(\bar{\mu}_L \gamma^\mu \mu_L). \]  

(107)

Comparing with \( \Lambda_{\bar{L}L}(ee\mu\mu) > 6.3 \) TeV \[22\], we obtain \( \frac{M_{Z'}}{g_x} \geq (1 - \frac{x}{2})^{2} 51 \) TeV. For \( g_x = 0.41 \) (0.45) and \( x = 1.3 \) (1.6) this implies \( M_{Z'} \geq 361 \) (226) GeV. For the choice of parameters in Tables 2 and 7, the above constraint is easily satisfied.

### 7.3 \( h \to h'h' \) Decay

Since the neutral Higgs boson \( h' \) is lighter than the Standard Model Higgs \( h \), the decay \( h \to h'h' \) can proceed for part of the parameter space. The decay rate is given by

\[ \Gamma(h \to h'h') = \frac{G_{hh'}^2}{8\pi m_h} \sqrt{1 - \frac{4m_{h'}^2}{m_h^2}}, \]  

(108)

where

\[ G_{hh'}^2 = \frac{\left(g_x^2 + g_y^2\right)}{4\sqrt{2}} \left[(v_d X_{11} - v_u X_{21})(X_{12}^2 - X_{22}^2) + 2(v_d X_{12} - v_u X_{22})(X_{11} X_{12} - X_{21} X_{22})\right] \]

\[ + \frac{g_x^2}{4\sqrt{2}} \left[2(4X_{31} X_{32} - 4X_{41} X_{42} - x X_{11} X_{12} + x X_{21} X_{22})\right] \]

\[ \times (-xv_d X_{12} + xv_u X_{22} - 4y X_{42} + 4z X_{32}) \]

\[ - (4X_{32}^2 - 4X_{42}^2 - x X_{12}^2 + x X_{22}^2)(xv_d X_{11} - xv_u X_{21} + 4y X_{41} - 4z X_{31})\right]. \]  

(109)

Here \( X \) is the unitary matrix that diagonalizes the CP–even Higgs mass matrix of Eq. (15). In principle this can compete with the dominant decay \( h \to b\bar{b} \). However we find that in Model 1 of Table 2 the decay is kinematically suppressed, while in Model 2 of Table 7 due to the small admixture of \( h \) in \( S_+ \), \( S_- \), this decay is suppressed: \( \Gamma(h \to h'h') = 1.48 \times 10^{-7} \) GeV (see Table 11). It is worth noting that if the mixings are as large as in Table 6 and if the decay is kinematically allowed, then \( \Gamma(h \to h'h') \sim 0.1 \) MeV is possible. Once produced, the dominant decays of \( h' \) will be \( h' \to b\bar{b} \) and \( h' \to c\bar{c} \) with comparable partial widths, as can be seen from \( H^0_u \) and \( H^0_d \) components in \( h' \) (see Table 6).
7.4 Signatures of SUSY Particles

The supersymmetric particles, once produced in $pp$ ($p\bar{p}$) collisions, will decay into the LSP. The LSP is $\tilde{\chi}_1^0$ (the neutral Wino) in Model 1 while it is the scalar neutrino $\tilde{\nu}_L$ in Model 2. In Model 1, $\tilde{\chi}_1^0$ is nearly mass degenerate with the lightest chargino $\tilde{\chi}_1^\pm$, with a mass splitting of about 180 MeV. The decay $\tilde{\chi}_1^0 \rightarrow \pi^\pm \chi_1^\pm$ will then occur within the detector. At the Tevatron Run 2 as well as at the LHC, the process $p\bar{p}$ (or $pp$) $\rightarrow \tilde{\chi}_1^0 + \tilde{\chi}_1^\pm$ will produce these SUSY particles. Naturalness suggest that $m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_1^\pm} \lesssim 300$ GeV (corresponding to $m_{\text{gluino}} \lesssim 2$ TeV). Strategies for detecting such a quasi-degenerate pair has been carried out in Ref. [31, 32]. In the case where the LSP is the left-handed sneutrino, the decay $\tilde{\chi}_1^\pm \rightarrow \ell^\pm \tilde{\nu}_L$ will be allowed. In this case $\tilde{\chi}_1^0$ will decay dominantly to $\tilde{\chi}_1^0 \rightarrow \tilde{\nu}_L\nu_L$.

8 Conclusions

We have suggested in this paper a new class of supersymmetric $Z'$ models motivated by the anomaly mediated supersymmetry breaking framework. The associated $U(1)$ symmetry is $U(1)_x = xY - (B - L)$, where $Y$ is the Standard Model hypercharge. For $1 < x < 2$, the charges of the lepton doublets and the lepton singlets have the same sign. This implies that the $U(1)_x$ $D$-term can induce positive masses for both the doublet and the singlet sleptons and can cure the tachyonic problem of AMSB. We have shown explicitly that this is indeed possible in this class of models. In achieving this, the parameters of the model get essentially fixed. We have found that $M_{Z'} = 2 - 4$ TeV and the $Z - Z'$ mixing angle $\xi \simeq 0.001$. The phenomenologically viable $Z'$ turns out to be leptophobic – with $Br(Z' \rightarrow \ell^+\ell^-) \simeq (1 - 1.6)\%$. The dominant decay of $Z'$ is to $q\bar{q}$ pair with $Br(Z' \rightarrow q\bar{q}) \simeq 44\%$. Decays into supersymmetric particles and Higgs bosons are also significant.

In Tables 2 and 11 we present our spectrum for two models, Model 1 (with $x = 1.3$) and Model 2 (with $x = 1.6$). The lightest SUSY particle is the neutral Wino (Model 1) or the sneutrino (Model 2). The partial decay widths of $Z'$ are listed in Tables 12 and 13. These models are compatible with precision electroweak data, with the $Z'$ models giving slightly better fits to the data than the Standard Model. This $Z'$ should be within reach of LHC. The correlations between the $Z'$ decays and the supersymmetric spectrum.
should make this class of models distinguishable from other $Z'$ models.

A Appendix

In this Appendix we give the one-loop anomalous dimension, beta-function and the soft SUSY breaking masses for the various fields in our model.

A.1 Anomalous Dimensions

The one–loop anomalous dimensions for the fields in our model are:

\begin{align}
16\pi^2 \gamma_{L_{ij}} &= (Y_l Y_l^\dagger)_{ji} - \delta_i^j \left( \frac{3}{10} g_1^2 + \frac{3}{2} g_2^2 + 2 \left(1 - \frac{x}{2}\right)^2 g_x^2 \right), \\
16\pi^2 \gamma_{e_{ij}} &= 2(Y_l^\dagger Y_l)_{ij} - \delta_i^j \left( \frac{6}{5} g_1^2 + 2(-1 + x)^2 g_x^2 \right), \\
16\pi^2 \gamma_{Q_{ij}} &= (Y_d Y_d^\dagger)_{ji} + (Y_u Y_u^\dagger)_{ji} - \delta_i^j \left( \frac{1}{30} g_1^2 + \frac{3}{2} g_2^2 + \frac{8}{3} g_3^2 + 2 \left(\frac{x}{6} - \frac{1}{3}\right)^2 g_x^2 \right), \\
16\pi^2 \gamma_{U_{ij}} &= 2(Y_u^\dagger Y_u)_{ij} - \delta_i^j \left( \frac{8}{15} g_1^2 + \frac{8}{3} g_3^2 + 2 \left(\frac{2}{3} x + \frac{1}{3}\right)^2 g_x^2 \right), \\
16\pi^2 \gamma_{D_{ij}} &= 2(Y_d^\dagger Y_d)_{ij} - \delta_i^j \left( \frac{2}{15} g_1^2 + \frac{8}{3} g_3^2 + 2 \left(\frac{x}{3} + \frac{1}{3}\right)^2 g_x^2 \right), \\
16\pi^2 \gamma_{H_d} &= 3Y_{d_3}^2 + Y_{l_3}^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 - 2 \left(-\frac{x}{2}\right)^2 g_x^2, \\
16\pi^2 \gamma_{H_u} &= 3Y_{u_3}^2 - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 - 2 \left(-\frac{x}{2}\right)^2 g_x^2, \\
16\pi^2 \gamma_{\nu_c} &= 4f_{\nu_c}^2 - 2g_x^2, \\
16\pi^2 \gamma_{\bar{\nu}_c} &= 4f_{\bar{\nu}_c}^2 - 2g_x^2, \\
16\pi^2 \gamma_{S^+} &= 4h^2 - 2g_x^2, \\
16\pi^2 \gamma_{S^-} &= 2 \sum_{i=1}^3 f_{\nu_i}^2 + 2f_{\bar{\nu}_c}^2 - 8g_x^2, \\
16\pi^2 \gamma_{S^0} &= 2h^2 - 8g_x^2.
\end{align}

A.2 Beta Function

The beta functions for the Yukawa couplings appearing in the superpotential, Eq. (4), are:

\begin{equation}
\beta(Y_{d_3}) = \frac{Y_{d_3}}{16\pi^2} \left( 6Y_{d_3}^2 + Y_{u_3}^2 + Y_{l_3}^2 - \frac{7}{15} g_1^2 - 3g_2^2 - \frac{16}{3} g_3^2 - \frac{4 + 2x + 7x^2}{9} g_x^2 \right).
\end{equation}
\[ \beta(Y_{u_3}) = \frac{Y_{u_3}}{16\pi^2} \left( 6Y_{u_3}^2 + Y_{d_3}^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 - \frac{4 - 10x + 13x^2}{9}g_x^2 \right), \] (123)

\[ \beta(Y_{t_3}) = \frac{Y_{t_3}}{16\pi^2} \left( 4Y_{t_3}^2 + 3Y_{d_3}^2 - \frac{9g_1}{5}g_t^2 - 3g_2^2 - (4 - 6x + 3x^2)g_x^2 \right), \] (124)

\[ \beta(f_{\nu_e}) = \frac{f_{\nu_e}}{16\pi^2} \left( 10f_{\nu_e}^2 + 2f_{\nu_\mu}^2 + 2f_{\nu_\tau}^2 - 12g_x^2 \right), \] (125)

\[ \beta(f_{\nu_\mu}) = \frac{f_{\nu_\mu}}{16\pi^2} \left( 10f_{\nu_\mu}^2 + 2f_{\nu_e}^2 + 2f_{\nu_\tau}^2 - 12g_x^2 \right), \] (126)

\[ \beta(f_{\nu_\tau}) = \frac{f_{\nu_\tau}}{16\pi^2} \left( 10f_{\nu_\tau}^2 + 2f_{\nu_e}^2 + 2f_{\nu_\mu}^2 - 12g_x^2 \right), \] (127)

\[ \beta(f_{\nu_c}) = \frac{f_{\nu_c}}{16\pi^2} \left( 10f_{\nu_c}^2 + 2f_{\nu_\mu}^2 + 2f_{\nu_\tau}^2 - 12g_x^2 \right), \] (128)

\[ \beta(h) = \frac{h}{16\pi^2} (10h - 12g_x^2). \] (129)

The gauge beta function of our model are

\[ \beta(g_i) = b_i \frac{g_i^3}{16\pi^2}, \] (130)

where \( b_i = (\frac{33}{5}, 1, -3, (11x^2 - 16x + 26)) \) for \( i = 1, 2, 3, x \).

### A.3 A terms

The trilinear soft SUSY breaking terms are given by

\[ A_Y = -\frac{\beta(Y)}{Y} M_{\text{aux}}, \] (131)

where \( Y = (Y_{u_1}, Y_{d_1}, Y_{t_1}, f_{\nu_e}, f_{\nu_\mu}, h) \).

### A.4 Gaugino Masses

The soft masses of the gauginos are given by:

\[ M_i = \frac{\beta(g_i)}{g_i} M_{\text{aux}}, \] (132)

where \( i = 1, 2, 3, x \), corresponding to the gauge groups \( U(1)_Y, SU(2)_W, SU(3)_C, U(1)_x \) with \( \beta(g_i) \) given as in Eq. (130) with \( M_x = M'_x \).

### A.5 Soft SUSY Masses

The soft masses of the squarks and the sleptons are given in the text. For the \( H_u, H_d, \nu^c, S_+, S_- \) fields they are:

\[ (\tilde{m}_{s_{\text{soft}}}^2)_{H_u} = \frac{M_{\text{aux}}^2}{16\pi^2} \left( 3Y_{u_3}^2 \beta(Y_{u_3}) - \frac{3}{10}g_1^2 \beta(g_1) - \frac{3}{2}g_2^2 \beta(g_2) - 2 \left( \frac{x}{2} \right)^2 g_x^2 \beta(g_x) \right), \] (133)
\begin{align}
(\tilde{m}_{soft}^{2})_{H_d}^{H_d} &= \frac{M_{aux}^{2}}{16\pi^2} \left( 3Y_{d_1}^3(Y_{d_3}) + Y_{t_3}^3(Y_{t_3}) - \frac{3}{10}g_1\beta(g_1) - \frac{3}{2}g_2\beta(g_2) \right) \\
&- 2 \left( -\frac{x}{2} \right)^2 g_x\beta(g_x) ,
(134) \\
(\tilde{m}_{soft}^{2})_{S_+}^{S_+} &= \frac{M_{aux}^{2}}{16\pi^2} \left( 2 \sum_{i=1}^{3} f_{\nu c}^i(1-f_{\nu c}^i) + 2f_{\nu e}^i(1-f_{\nu e}^i) - 8g_x\beta(g_x) \right) ,
(135) \\
(\tilde{m}_{soft}^{2})_{\bar{S}_-}^{\bar{S}_-} &= \frac{M_{aux}^{2}}{16\pi^2} \left( 2h\beta(h) - 8g_x\beta(g_x) \right) ,
(136) \\
(\tilde{m}_{soft}^{2})_{\nu c}^{\nu c} &= \frac{M_{aux}^{2}}{16\pi^2} \left( 4f_{\nu c}^i(1-f_{\nu c}^i) - 2g_x\beta(g_x) \right) ,
(137) \\
(\tilde{m}_{soft}^{2})_{\bar{\nu} c}^{\bar{\nu} c} &= \frac{M_{aux}^{2}}{16\pi^2} \left( 4h\beta(h) - 2g_x\beta(g_x) \right) .
(138)
\end{align}

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