Sound attenuation performance of periodic expansion chamber mufflers

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Abstract. Expansion chamber mufflers are often used for duct noise control of building ventilation systems. In this paper, the periodic arrangement of expansion chamber muffler is proposed to improve the sound attenuation performance. The sound attenuation of periodic expansion chamber mufflers is investigated theoretically with Bloch wave theory and a case is calculated for the distance between mufflers and the length of the expansion chamber are the same. The results reveal that the transmission loss of the expansion muffler is enhanced by the periodic arrangement.

1. Introduction
The single expansion chamber muffler is often used to reduce noise in piping systems or exhaust pipes. The expansion chamber results in acoustic impedance mismatch to prevent sound from propagating forward. The performance of a single expansion muffler is a periodic function of the chamber length and the sound wave number. Using different mufflers together is effective to enhance the attenuation performance [1, 2]. Periodically alignment of the muffler is used to get a different sound attenuation performance. Such a periodic structure exhibit stopbands and passbands in frequency domains.

Bradley [3, 4] first investigated sound propagation in periodic waveguides theoretically with Bloch waves functions and then the result was verified with experiments. His finding suggested that Bloch waves function can be applied to both the infinite and finite cases. Sugimoto and Horioka [5] later also used the Bloch waves to investigate the influences of a periodic array of Helmholtz resonators in a tunnel. Wang and Mak [6, 7] proved that a much broader transmission loss band can be produced with periodic Helmholtz resonators compared with the single Helmholtz resonator and later the effect of the disorder of periodic distances and geometries in periodic resonators was investigated [8].

This paper investigates the influence of the periodic arrangement on the sound attenuation performance of the periodic expansion chamber muffler. The transmission loss of the periodic expansion muffler is obtained with the transfer matrix method and the Bloch wave theory. The periodic alignment leads to a different transmission loss which may be applied in duct noise control design.
2. Theoretical analysis

Expansion chamber mufflers are arranged periodically along an infinite duct as shown in Fig. 1(a) and the diameters of the duct and the expansion chamber are $d_1$ and $d_2$ respectively. In Fig. 1(a), a uniform duct and an expansion chamber form a periodic cell. The length of the uniform duct is $d$; the chamber length is $L$; the overall length of a periodic cell is $h$.

Both in the duct and the mufflers only planar waves are assumed to be able to propagate. Bradley [3, 4] examined how the sound propagates in a periodic structure and proved that the periodic waveguide can be solved with the Bloch wave functions. As shown in Fig. 1(a), the waves between two adjacent periodic cells can be related with a transfer matrix $T$:

$$
\begin{bmatrix}
I_{n+1} \\
R_{n+1}
\end{bmatrix} =
T
\begin{bmatrix}
I_n \\
R_n
\end{bmatrix}
= \begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
$$

In each periodic cell, the sound wave is composed of the incident wave and the reflected wave. $I_n$ and $R_n$ are the incident wave and the reflected wave of the nth cell and $I_{n+1}$ and $R_{n+1}$ are the incident wave and the reflected wave in the n+1th periodic cell. According Bloch wave theory, the waves of the n+1th cell are related to that of the nth cell with $e^{-jqh}$ and $q$ is called the Bloch wave number. Therefore the determination of the transfer matrix $T$ and its eigenvalue $e^{-jqh}$ and eigenvector $v$ is the solution of the periodic waveguide:

$$
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
v_I \\
v_R
\end{bmatrix} = e^{-jqh}
\begin{bmatrix}
v_I \\
v_R
\end{bmatrix}
$$

The acoustic pressure in the nth cell can be expressed as:

$$
p_n(x) = I_n e^{-jk(x-x_n)} + R_n e^{jk(x-x_n)}
$$

where the center of the uniform duct of the nth cell is $x_n$. At the inlet of the nth expansion chamber, the acoustic pressures and the acoustic particle velocities are $p_{in}$, $u_{in}$ and at the outlet of the nth expansion chamber, the acoustic pressures and the acoustic particle velocities are $p_{out}$, $u_{out}$ respectively:
\[ p_{in} = I_n e^{-jkd/2} + R_n e^{jkd/2} \]
\[ \rho_0 c_0 u_{in} = I_n e^{-jkd/2} - R_n e^{jkd/2} \]
\[ p_{out} = I_{n+1} e^{jkd/2} + R_{n+1} e^{-jkd/2} \]
\[ \rho_0 c_0 u_{out} = I_{n+1} e^{jkd/2} - R_{n+1} e^{-jkd/2} \]

(4)

The uniform duct and the expansion chamber can be expressed with transfer matrix method \[9\] and then the \( p_{out}, u_{out} \) at the outlet is:

\[
\begin{bmatrix}
    p_{out} \\
    \rho_0 c_0 u_{out}
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 \\
    0 & 1/m
\end{bmatrix}
\begin{bmatrix}
    \cos kL & -j \sin kL \\
    -j \sin kL & \cos kL
\end{bmatrix}
\begin{bmatrix}
    1 & 0 \\
    0 & m
\end{bmatrix}
\begin{bmatrix}
    p_{in} \\
    \rho_0 c_0 u_{in}
\end{bmatrix}
\]

(5)

where the cross-sectional areas of the duct and the expansion chamber are \( S_1 \) and \( S_2 \) respectively and \( m = S_2/S_1 \). Combining Eqs. (4) and (5), the relation of waves in \( n \)th cell and in the \( n+1 \)th cell is:

\[
\begin{bmatrix}
    I_{n+1} \\
    R_{n+1}
\end{bmatrix} =
\begin{bmatrix}
    0.5 e^{-jkd/2} & 0.5 e^{jkd/2} \\
    0.5 e^{jkd/2} & -0.5 e^{-jkd/2}
\end{bmatrix}
\begin{bmatrix}
    \cos kL & -jm \sin kL \\
    -jm \sin kL/m & \cos kL
\end{bmatrix}
\begin{bmatrix}
    e^{-jkd/2} & e^{jkd/2} \\
    e^{jkd/2} & -e^{-jkd/2}
\end{bmatrix}
\begin{bmatrix}
    I_n \\
    R_n
\end{bmatrix}
\]

(6)

Therefore, \( T \) is:

\[
T =
\begin{bmatrix}
    e^{-jkd} & 
    \frac{j}{2} \left( 1 - \frac{1}{m} \right) \sin kL \\
    \frac{j}{2} \left( \frac{1}{m} - 1 \right) \sin kL \\
    \frac{j}{2} \left( \frac{1}{m} - 1 \right) \sin kL \\
    \frac{e^{jkd}}{2} \left( 2 \cos kL + j \left( 1 + \frac{1}{m} \right) \sin kL \right)
\end{bmatrix}
\]

(7)

The \( q \) can be determined from the equation below:

\[
\cos (qh) = \frac{1}{2} (T_{11} + T_{22}) = \cos kL \cos kd - \frac{1}{2} \left( 1 + \frac{1}{m} \right) \sin kL \sin kd
\]

(8)

The solution of \( q \) can be obtained from Eqs. (8) and then the eigenvalue \( e^{jqh} \) will be determined. The \( q \) influences the relation of waves \( e^{jqh} \) between adjacent periodic cells which resulting in sound attenuation or not. \( q \) is multivalued as a result of the inverse cosine function. \( q \) is real when the absolute value of the right side of Eq. (8) is less than or equal to one. The relation between two adjacent cell is \( e^{jkh} \) and the absolute value of \( e^{jkh} \) is 1 and therefore the waves passing through each cell are only associated with a phase change. The amplitudes of the waves are the same and the frequency band are referred as passbands, where there is no amplitude attenuation when waves propagate from one periodic cell to another cell.

\( q \) is complex when the absolute value of the right side of Eq. (8) is more than one, and \( q \) can be divided into the real part \( q_r \) and the imaginary part \( q_i \). The coefficient \( e^{q_r} \) will be \( e^{n_b} e^{-n_b} \). It is obviously the \( e^{q_i} \) is the attenuation factor when waves pass through each periodic cell. As the waves propagate the infinite periodic structure, the waves are eliminated and vanish. The frequency bands when \( q \) is complex is catalogued into stopbands where the sound waves prevented by the periodic structure. The stopbands mean stopping sound propagation and reveal the attenuation performance of the periodic expansion chamber mufflers.

The discussion above is all based on the infinite assumption of the periodic structure which have the passbands and stopbands. While only finite structure exists in practice as shown in Fig. 1(b). Bradley[4] also prove that Bloch wave functions are suitable to the finite case. Bloch wave functions can be catalogued into forward-traveling \( [v_{1f}, v_{1R}]^T \) and backward-traveling \( [v_{2f}, v_{2R}]^T \). The Bloch number of \( [v_{1f}, v_{1R}]^T \) is \( q_1 \) and the Bloch number of the backward-traveling Bloch wave \( [v_{2f}, v_{2R}]^T \) is
respectively. Any arbitrary termination impedance can be got with the composition of two types
Bloch wave functions.

Fig. 1(b) shows n finite periodic expansion chamber mufflers located along a duct with an identical
distance. The left end of the duct is a loudspeaker and the right end is set to be anechoic. The acoustic
waves in the first periodic cell can be expressed with the eigenvectors:

\[
\begin{bmatrix}
I_1 \\
R_1
\end{bmatrix} = a \begin{bmatrix}
v_{1l} \\
v_{1R}
\end{bmatrix} + b \begin{bmatrix}
v_{2l} \\
v_{2R}
\end{bmatrix}
\]  

(9)

As mentioned before, the duct end is set as an anechoic termination which means no reflection occurs
in the last cell. Therefore, the reflected wave in n+1th cell \(R_{n+1}\) is zero and then \(b/a\) can be calculated.
The transmission loss of a structure is acoustic power ratio of the incident wave and the transmitted
wave. For the n periodic expansion chamber mufflers, the incident wave and the transmitted wave of
are \(I_1\) and \(I_{n+1}\) respectively and the transmission loss is:

\[
TL = 20 \log_{10} \left| \frac{I_1}{I_{n+1}} \right| = 20 \log_{10} \left| \frac{v_{1l} + b/a v_{2l}}{e^{-j\omega L} v_{1l} + b/a e^{-j\omega L} v_{2l}} \right|
\]  

(10)

3. Results and discussion

As shown in Fig. 1(a), the infinite structure results in no reflected waves exist. Inside of the infinite
periodic waveguide, the incident wave from n expansion chambers is \(a v_{1l}\) and the transmitted wave
\(e^{-j\omega L} a v_{1l}\). The transmission loss can be calculated as \(TL_{\text{inf}} = 20 \log_{10} \left| a v_{1l} / \left( e^{-j\omega L} a v_{1l} \right) \right|\). Eqs. (10)
gives the transmission loss of finite structure. Fig. 2 show the comparison of the finite structure and
the infinite structure. The dimensions are \(d_1 = 0.05\) m, \(d_2 = 0.1\) m, and \(d = L = 0.4\) m.

According to Fig. 2, the transmission loss of finite structure is approaching to that of the infinite
structure as the number n increases and when n is larger than two, the stopbands and passbands are
close to that of the infinite waveguide. The periodic structure produces higher transmission loss with a
narrower frequency band compared to a single expansion chamber which is shown in the top left of
Fig. 2(n=1).

![Figure 2](image_url)

Figure 2. The transmission loss of n finite periodic expansion chambers (solid lines are for
finite structure and the dash lines are for infinite condition).
4. Conclusions
The single expansion chamber is a very common duct noise control device. This paper focuses the performance of the periodic expansion chamber and utilise the Bloch wave theory to investigate the transmission loss of the periodic structure. The transmission loss of the infinite and finite structure is discussed with the Bloch waves. A case is simulated when the periodic distance and the expansion chamber length are the same. The results show that the transmission loss of finite expansion mufflers is close to that of the infinite condition when the chamber number increases. The transmission loss of the periodic expansion chamber mufflers can be greatly enhanced as chamber number increases.

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