One-dimensional collision carts computer model and its design ideas for productive experiential learning

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Abstract

We develop an Easy Java Simulation (EJS) model for students to experience the physics of idealized one-dimensional collision carts. The physics model is described and simulated by both continuous dynamics and discrete transition during collision. In designing the simulations, we discuss briefly three pedagogical considerations namely (1) a consistent simulation world view with a pen and paper representation, (2) a data table, scientific graphs and symbolic mathematical representations for ease of data collection and multiple representational visualizations and (3) a game for simple concept testing that can further support learning. We also suggest using a physical world setup augmented by simulation by highlighting three advantages of real collision carts equipment such as a tacit 3D experience, random errors in measurement and the conceptual significance of conservation of momentum applied to just before and after collision. General feedback from the students has been relatively positive, and we hope teachers will find the simulation useful in their own classes.

Introduction

The conservation of linear momentum coupled with the conservation of kinetic energy in perfectly elastic versus inelastic collisions are difficult concepts [1] for many students. This is probably due to a combination of many factors. One of the main causes is the difficulty in ‘making sense’ of the phenomena, without learning by experiencing and contextualizing in ‘real-life referents’ [2], hence leading to what is commonly referred to as the abstract nature [3] of learning physics.

In traditional classrooms, students could be asked to imagine idealized frictionless surfaces where collision carts ‘move without a decrease in velocity’ [4] and collide in either perfectly elastic collisions without any loss of energy or perfectly inelastic collisions where the velocities of the collision carts become immediately the same. These conditions are almost impossible to achieve using real laboratory equipment [5], thus we argue that computer simulation could be an appropriate substitute for active learning referents, provided simulations are carefully developed [6].
used in an appropriate context [7], aided with challenging inquiry activities [8] and facilitated by teachers who believe [9] in the effectiveness of the tool.

Building on open source codes shared by the Open Source Physics (OSP) community, such as Esquembre’s example of ‘Collision in One Dimension’ [10], Duffy’s [11] ’One-Dimensional Collision Model’ for game design, and F-K Hwang’s many other examples in the NTNUJAVA Virtual Physics Laboratory, we customize an Easy Java Simulation (EJS) [12] computer simulation as a virtual laboratory, as shown in figure 1 [13], that we hope many teachers will find useful.

Physics model

In this simulation, the two-body collision carts model is simulated by both continuous dynamics and a discrete transition, where the system dynamics change discretely and the state values jump when the two carts collide. The continuous dynamics is simulated using the ‘Evolution page’ as in figure 2 in EJS by the following equations, assuming that the $x$ positions of the centres of carts 1 and 2 are $x_1$ and $x_2$ respectively and their instantaneous velocities $v_1$ and $v_2$ respectively:

$$\frac{dx_1}{dt} = v_1$$

$$\frac{dx_2}{dt} = v_2.$$  

Notice how easily these equations simulate carts that continue with uniform $x$ direction motion without any loss of energy.

The discrete transition before and after the collision is handled by an event handler in EJS by selecting ‘Type = State event’ that returns the carts to the position just when the two carts are in contact. The corresponding ‘Action’ to execute during this event is to assign the physical velocities to the carts as determined by assuming the concept
of coefficient of restitution $e$. The equations for the final velocities $v_1$ and $v_2$ of the carts of masses $m_1$ and $m_2$ as functions of initial velocities $u_1$ and $u_2$ are given by

$$v_1 = \frac{m_1 u_1 + m_2 u_2 - m_2 e(u_1 - u_2)}{(m_1 + m_2)} \quad (3)$$

$$v_2 = \frac{m_1 u_1 + m_2 u_2 + m_1 e(u_1 - u_2)}{(m_1 + m_2)}, \quad (4)$$

respectively.

Due to the assumption of a coefficient of restitution $e$

$$e = -\frac{v_1 - v_2}{u_1 - u_2}, \quad (5)$$

combined with equations (3) and (4), the simulation model allows students to discover that the total momentum of carts 1 and 2 just before and after collision is always the same, as shown commonly by

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2. \quad (6)$$

For the total kinetic energies of carts 1 and 2, kinetic energy loss $E_{\text{loss}} = 0$ for a perfectly elastic collision and $E_{\text{loss}} > 0$ for partially or perfectly inelastic collisions, as given by

$$\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + E_{\text{loss}}. \quad (7)$$

This physics model, when implemented in a simulation, allows experimentation by ‘messing about’ productively [7], serving as a powerful referent tool for learning.

Three pedagogical design considerations

To add to the body of knowledge surrounding why simulations could be more effective tools compared to real laboratory equipment [6, 7], we share three pedagogical design insights which, although not exhaustive, have emerged as being able to push the effectiveness of the tool to a greater usefulness to students. Teachers interested in other useful features in simulation design can refer to Physics Education Technology (PhET) at the University of Colorado at Boulder [14].

Consistent simulation world view with pen and paper representation

We realized that the side view of the real equipment setup is easy for students to associate with reality, especially with clear colour association (green for cart 1, orange for cart 2, for example) and rotating wheels to represent motion.

Customization of the simulation with vector representations of velocities $u_1, u_2, v_1$ and $v_2$ in a traditional whiteboard representation, with velocity vectors pointing to the right as being positive, is implemented with appropriate emphasis via an increase in font size and colour change, allowing students to relate to the classroom discussion of problems as in figure 3. Several other simulations either do not represent the velocities [15] or show them pointing to the left [16], while showing a negative magnitude [17] or failing to simulate negative velocities [18]. Thus, a customized simulation consistent with our pen and paper representations by pointing...
to the right even with negative values minimizes confusion, especially when substituting values in equations (6) and (7).

**Table of data, scientific graphs and symbolic mathematical representations**

We found the table of data arrangement with columns ‘Cart 1’, ‘Cart 2’ and ‘Total’ and rows of before and after momentum and kinetic energy, as in figure 4, to be useful in our attempts to use the simulation in a guided inquiry approach to physics learning. The before and after quantities to be studied in the inquiry laboratory can be easily referred to. This helps to promote productive [7] activities, as most other simulations [15–18] need to record at different times the before and after quantities. A graphical representation of momentum and kinetic energy are plotted versus time to give students a time-dependent visualization of the physics quantities to be studied, as in figure 5. This graph shows, for the case of perfectly inelastic collision ($e = 0$), how the total momentum is the same before and after collision (red line) while the total kinetic energies register an energy loss $E_{loss} > 0$ after collision (purple line). The ability to select cart 1, cart 2, or both, allows for clearer visualization and understanding. A symbolic mathematical representation in the form of hints has been added to elicit predictive thinking from students and get them to deduce the curriculum learning outcomes of conservation of momentum and associated kinetic energies, as in figure 6.

In other words, these multiple forms of representation aim to give the students a means to make coherent sense by representing the same phenomena through a table of data, scientific graphs and symbolic mathematical representations.
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Figure 7. In the game mode, students can test their understanding and receive feedback on their answers from the simulation, the level of customized feedback is dependent on the teachers' customized input.

representations, supporting the world view given earlier.

Game for concept testing
To further enhance the engagement and interactivity through the simulation, we suspected that a simple game or puzzle [19] could be fun for the students to do by means of input fields and feedback text. Here students decide which input field to key in first. There could be hints like ‘no! hint $v_1 = v_2$’ for perfectly inelastic collision or ‘no! out by 2.0’ and, if the answers are correct, the feedback could be ‘yes! $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$’ to reaffirm the students’ attempt at the problem game, as shown in figure 7.

Strengths of physical world learning to augment simulation virtual learning
Although simulations have their strengths for extending and promoting learning, we would like to present a more balanced view on learning by connecting and suggesting when a real physical world setup should provide the grounding augmented by simulation.

The tacit 3D experience to learning, through a physical world collision carts setup, provides a very valuable experience to students, especially for performance in the real world [20].

Random errors in the measurement of quantities such as velocity; which is part of any scientific measurement, and absent in our simulation, is another example of this complementary augmentation to the strengths of physical world learning. For example, as in figure 8, using Tracker [21] to analyse an elastic collision of a moving cart with a stationary cart of equal mass from Bryan’s website [22], which shows the random errors in measurement of momentum $p_x$ for both masses and the validity of the conservation of momentum using velocities just before and just after collision.

Figure 8. Using Tracker [21] to analyse an elastic collision of a moving cart with a stationary cart of equal mass from Bryan’s website [22], which shows the random errors in measurement of momentum $p_x$ for both masses and the validity of the conservation of momentum using velocities just before and just after collision.

physical world video, such as the ones taken by students themselves and others like Bryan’s [22] video, notice the scientific graphs on the right clearly show the momentum of the two masses as having statistical fluctuations as a random error in measurement. In addition, the ease of conceptual realization of the significance of the velocities just before and just after, in the verification of conservation of momentum, are valuable experiences when using real equipment, currently not designed in our simulation.

In this balanced view, we suggest there are possible situations where the strengths of physical world learning should be relied upon more while any potential weaknesses are compensated for by the strengths afforded by virtual simulations, thus maintaining a connected experience to learning in the physical world augmented with simulations.
Table 1. Survey results for a 1 h 40 min hands-on lesson with guided inquiry worksheets (N = 64).

| Question                                                                 | 1 very much (%) | 2 | 3 | 4 | 5 very little (%) |
|--------------------------------------------------------------------------|-----------------|---|---|---|-------------------|
| (1) How much do you know about the physics taught today before lesson?  | 0               | 5 | 23| 44| 28                |
| (2) How much do you know about the physics taught today after the lesson?| 6               | 50| 41| 0 | 3                 |
| (3) I enjoyed learning physics through this activity                     | 8               | 45| 34| 11| 2                 |
| (4) Rate the lesson as a valuable learning experience                    | 14              | 58| 25| 2 | 2                 |

Feedback from students

Feedback about the lesson using an earlier version of the simulation is summarized in table 1. Survey questions 1 and 2 indicate an increase from 5% to 56% in the students perceiving themselves as knowing very much (scale 1) and much (scale 2) more than before the lesson and a reduction from 72% to 3% from self-reporting that they still know little (scale 4) or very little (scale 5) after the lesson.

Survey questions 3 and 4 suggest the benefits, with more than 50% and more than 70% rating the learning experiences as enjoyable and valuable respectively. We speculate that factors such as improving the teacher’s belief in the effectiveness of technology supported lessons, their level of experience in facilitating lessons using technology [23] and a more challenging guided inquiry task should be able to bring the percentages higher for future lessons.

We include excerpts from the qualitative survey results and informal interviews with the students to give some themes and insights into the conditions and processes during the laboratory lessons. Words in brackets [] are added to improve the readability of the qualitative interviews.

(1) Active learning can be fun

‘...It [is] an eye opener...[we] do not usually get to learn with virtual learning environment...and it makes learning fun and interesting’.

‘The lesson was fun and makes us think instead of just listen[ing] to teacher and remember[ing] whatever the teacher said’.

‘It makes learning much more interesting and fun. It makes us want to learn and find out more about the topic’.

(2) Need experience to understand

‘...It [this lab] lets me figure out the concepts rather than just listen[ing] and believing what is taught without understanding’.

‘Normally people would have to experience any physics concepts themselves hands [-] on to really remember concepts. Lectures on the other hand may not be effective since maybe what the lecturer is bringing through us is unclear, and thus practical lessons to learn concepts is a great learning deal’.

(3) Simulation can support inquiry learning and thinking like real scientists

‘These kinds of lesson force us to think critically. It makes us look at the results, analyse and then find the trend within, which is a really good way to learn independently. It also gives us confidence and a sense of accomplishment when the conclusions we arrive at are correct.’

‘Such vlab [virtual lab] lesson effectively utilizes the IT resources to enhance lessons, making physics lessons less dry. Besides, by identifying trends in values first hand, I can remember it easier rather than via lecture notes and slides’.

(4) Need for strong inquiry learning activities

‘The activity worksheet did not generate much thinking and concept understanding, just simply presents a set of values to copy to get the answers’.

‘It [virtual lab] helps hasten the process of learning but the exchange of data [in the worksheet activities] is troublesome’.
(5) Need for testing and well-designed simulation [7]

Some students suggest visual and audio enhancements such as ‘better quality so that the simulations could be more interesting and appealing’ and ‘add sound effects’.

A good suggestion that surfaced is to make the ‘program [simulation] designed as a game, thereby making it more interactive. At the end a table can be provided and it would provide us [students] with the values. From there, we do analysis.’

This suggestion inspired us to design ‘Game for concept testing’ described earlier.

(6) Appreciative learners

‘I [student] really thank you for spending time coming up with this program. You are really an educator who cares and dares to try new things. Thanks! Hope you can come up with even better programs so that they can empower students in physics subject.’

‘Thank you teachers for spending time to develop this app.’

Conclusion

The theoretical physics model of a two-body collision system in one dimension is discussed and implemented in EJS and equations (1)–(7) should be applicable to any modelling tool, such as VPython [24] or Modellus [25].

Three pedagogical considerations in simulation design, (1) a consistent simulation world view with pen and paper representations, (2) a data table, scientific graphs and symbolic mathematical representations for ease of data collection and multiple representational visualizations [26] and (3) a game for simple concept testing, are implemented in our simulation, which we believe can further support learning.

We also suggest using a physical world setup to augment simulation by highlighting three advantages of real collision carts equipment, such as a tacit 3D experience, random errors in measurement and conceptual significance of conservation of momentum applied to just before and after collision.

General feedback from the students has been relatively positive, as derived from the survey questions, interviews with students and discussions with teachers, and we hope teachers will find the simulation useful in their own classes.

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