α + $^{15}$O cluster structures in $^{19}$Ne and the α resonant scattering

R. Otani, M. Iwasaki, M. Tomita, and M. Ito

Department of Pure and Applied Physics, Kansai University, 3-3-35 Yamatecho, Suita, Osaka 564-8680, Japan.

E-mail: k592822@kansai-u.ac.jp

Abstract. The rotational bands of α + $^{15}$O(1/2−) in $^{19}$Ne are calculated by employing a simple potential model. The α−$^{15}$O interaction potential is constructed from the scattering calculation of its mirror system, α + $^{15}$N. The resonance states are identified by imposing the absorbing boundary condition. The present calculations predict the sequence of the spin-orbit splitting states in the unbound region of $^{19}$Ne, which are generated from the higher spin states in the $^{20}$Ne nucleus. The excitation function of the α + $^{15}$O elastic scattering is also predicted, and the appropriate condition to observe the resonances is also discussed.

1. Introduction

Cluster structures are well known to appear in the excited states of light nuclear systems. A typical example can be seen in $^{20}$Ne. The α + $^{16}$O cluster model is successful in reproducing the ground and excited rotational band structure in $^{20}$Ne [1]. In $^{19}$F, which is one proton deficient system of $^{20}$Ne, the formation of the α + $^{15}$N cluster structures are deeply analyzed [2, 3]. On the contrary, the α + $^{15}$O cluster structure, corresponding to the neutron deficient system of $^{20}$Ne, still remains unclear although the pioneering works can be seen in Refs. [2, 3].

In the present study, we investigate the α + $^{15}$O structure in the $^{19}$Ne systems by employing a simple potential model. A potential parameter set is determined from the calculation of the elastic scattering of the respective mirror system, α + $^{15}$N. From the nuclear potential of α + $^{15}$N, the energy levels of α + $^{15}$O are calculated by adding Coulomb force. In addition to the bound energy levels, the unbound resonant states are identified by the absorbing boundary condition [4, 5], and the rotational band structures are discussed in connection to the $^{20}$Ne = α + $^{16}$O system, which is a typical example of the α cluster system.

Moreover, the excitation function of the $^{15}$O resonant scattering by an α target are also investigated. This resonant scattering will be available in future experiment, and we predict the excitation function under the possible conditions reported in the recent experiment [6].

2. Theoretical Framework

The Woods Saxon (WS) potential is assumed for the nuclear potential in α + $^{15}$O, and its parameter set is fixed in the following two steps. First, the scattering problem of α + $^{15}$N is solved in the range of $E_\alpha = 6 \sim 55$ MeV. In the scattering calculation, the complex WS potential is employed, and we consider only the central part of the complex potential. The WS parameters of the real part is fixed over a whole energy range of the scattering calculation, while...
the parameters in the imaginary part are set to be energy dependent. All the parameters are optimized so as to reproduce the $\alpha + ^{15}$N elastic scattering as much as possible. The obtained parameters of the real part of WS are $V = -169.21$ MeV, $R = 2.52$ fm and $a = 0.796$ fm. The radial form of the potential calculated with this parameter set is similar to the folding type interaction used in the $\alpha + ^{16}$O OCM calculation [1]. The Coulomb potential is calculated by assuming the uniform charge distribution with the same radius $R$ as the nuclear potential.

Secondly, the spin-orbit interaction is introduced in the calculation of the energy levels of the $\alpha + ^{15}$O system. The strength of the spin-orbit potential is chosen to reproduce the energies of the bound levels of the $5/2^-$ and $3/2^-$ states, which are considered to be the pair levels in the spin-orbit splitting. The parameters of the radius $R_{LS}$ and the diffuseness $a_{LS}$ are taken to be the same values as those of the central potential, while the strength parameter, $V_{LS}$ is tuned. The resultant strength of the spin-orbit interaction is quite small, $V_{LS} = -0.3$ MeV, because the spin-orbit splitting of these two levels is fairly small ($\sim 300$ keV).

In the calculation of the energy spectra, we apply the basis expansion method. The shifted Gaussian, having a functional form of $re^{-\nu(r-S)^2}$, is employed as the basis function in the calculation of the reduced radial wave function of $rR(r)$. In the basis function, the width parameter $\nu$ is 3.8 fm$^{-2}$, while the distance parameter $S$ ranges from $S_{\text{min}} = 0.1$ fm to $S_{\text{max}} = 40.1$ fm with a constant mesh of $\Delta S = 0.5$ fm.

Above the $\alpha$ decay threshold, the absorbing boundary condition (ABC) [4, 5] is applied to identify the resonance parameters, such as the resonance energy $E_R$ and the decay width $\Gamma_R$. As for the absorbing potential, we employ the shifted polynomial function [4, 5], which starts at $r_a = 12$ fm. The power of the polynomial is taken to be $\beta = 2$ or 4, depending on the energy position of the resonances [4]. The strength of $\eta$ is determined to reduce the errors arising from the reflection by the absorber as much as possible [4].

The excitation function of the $\alpha + ^{15}$O resonant scattering is also calculated. In solving the scattering problem, we apply the Kohn-Hulthén-Kato (KHK) variational method [4, 7]. In the present calculation, the computational procedure is the same as the method with the trial function (II) in Ref. [7]. The width parameter is $\nu = 4.0$ fm$^{-2}$, while the matching radius $R_C$ is taken to be 12.0 fm. The distance parameters are $S_{\text{min}} = 0.1$ fm, $S_{\text{max}} = 14.1$ fm and $\Delta S = 0.35$ fm.

3. Results

3.1. Elastic scattering of the $\alpha + ^{15}$N system

Figure 1 shows the results of the angular distributions of the $\alpha + ^{15}$N system. The theoretical calculations, shown by the solid curve, nicely reproduce the global oscillating pattern of the experimental distribution, plotted by the solid circles. The volume integral of the imaginary potential per nucleon pair reveals the increase tendency above higher energy than $E_\alpha = 23.7$ MeV, while the value at $E_\alpha = 6.85$ MeV deviates from the increase tendency. The determination of the parameters at $E_\alpha = 6.85$ MeV a little difficult because the observed angular range is limited to the forward region of $\theta_{\text{c.m.}} = 50^\circ \sim 80^\circ$ at this energy.
3.2. Energy spectra of $^{19}\text{Ne}$

The energy spectra of $\alpha + ^{15}\text{O}$ is calculated from the nuclear potential of $\alpha + ^{15}\text{N}$, which is determined from the elastic scattering. The imaginary potentials are switched off in the calculation of the energy spectra. In the following calculations, we focus on the energy levels, which have the total quanta of $N \geq 8$ in the wave function of the $\alpha - ^{15}\text{O}$ relative motion. Here $N$ corresponds to the total oscillator quanta in the Harmonic Oscillator (HO) potential, and $N$ is given by $N = 2n + L$ with the radial node number $n$ and the relative spin $L$ in the $\alpha - ^{15}\text{O}$ relative wave function. This restriction of $N$ is based on the Pauli’s exclusion principle in the $^{20}\text{Ne} = \alpha + ^{16}\text{O}$ system [1].

In Fig. 2, the calculated spectra of the negative parity states in $^{19}\text{Ne}$ with $N = 8$ (middle levels) are compared with the spectra of $^{20}\text{Ne} = \alpha + ^{20}\text{Ne}$ (right levels). Here the spectra of $^{20}\text{Ne}$ are calculated from the WS potential, which simulates the folding type potential in Ref. [1]. The resonant levels above the $\alpha$ threshold are identified by the absorbing boundary condition [4].

Since the $^{15}\text{O}$ core contains one neutron $0p_{1/2}$ hole, the spin-orbit splitting in $^{19}\text{Ne}$ are generated from the coupling of the $\alpha - ^{16}\text{O}$ relative motion in $^{20}\text{Ne}$ and the neutron hole in $^{16}\text{O}$, as indicated by the dotted line attached to the energy levels. The present calculation seems to nicely reproduce the observed spectra (left levels) although the experimental information on the highly excited states is still insufficient. We have also performed the calculation of the positive parity states with $N = 9$ although the results are not shown here. The spin-orbit splittings form the $N = 9$ bands in $^{20}\text{Ne} = \alpha + ^{16}\text{O}$ are clearly confirmed.

3.3. Excitation function of the resonant $\alpha + ^{15}\text{O}$ scattering

In recent experiments, an $\alpha$ resonant scattering by the $\beta$-unstable nucleus has been developed by employing the technique of the inverse kinematics [6]. The $\alpha$ resonant scattering is one of useful tools in identifying the resonance energy and the $\alpha$ decay width, which are essential in astrophysical calculations. The scattering of the $^{15}\text{O}$ beam will be available in future experiments. Thus, the evaluation of its excitation function is very important before the realistic experiments. We calculate the excitation function under the same condition as in the recent experiment of the $\alpha + ^{7}\text{Li}$ scattering [6] and confirm whether the experimental setup, such as the energy resolution and the detection angle, is valid or not for the identification of the resonant structures.

The calculated excitation function is shown by the solid curve in Fig. 3. In this calculation, the scattering angle is fixed to $\theta_{e.m.} = 170^\circ$, which is almost the same condition to the experimental setting [6]. The energy step in the calculation is taken to be 0.01 MeV, which is much smaller than the experimental resolution, about 0.1 MeV [6]. In this figure, the resonance energies are plotted by the solid circles, and the decay width of the individual resonances are shown by the error bars. In the higher energy region of $E_{e.m.} \geq 2 \text{ MeV}$, the three peaks of the positive parity resonances can be seen: $E_{e.m.} \sim 2.8 \text{ MeV} (3/2^+ \text{ and } 1/2^+)$, $E_{e.m.} \sim 4.0 \text{ MeV} (7/2^+ \text{ and } 5/2^+)$,
and $E_{c.m.} \sim 8.0 \text{ MeV}$ ($11/2^+$ and $9/2^+$). The width of these resonances ranges from 0.2 MeV to 1 MeV, and the resonance enhancements appear with the smooth energy variation. On the contrary, the resonance width of the negative parity states, $13/2^-$ and $11/2^-$, are extremely sharp, and two sharp peaks are superposed on the broad bump appearing around $E_{c.m.} \sim 4 \text{ MeV}$. There are two resonances of $9/2^-$ and $7/2^-$ in the lower energy region of $E_{c.m.} \leq 2 \text{ MeV}$ but these resonances are completely masked by the enhanced back ground, arising from the kinematic factor of $E_{c.m.}^{-1}$ in the cross section.

We investigate the sensitivity of the excitation function to the energy resolution and the detection angle. If we smear the excitation function by the finite width of the experimental energy resolution, typically $\Delta E = 0.1 \text{ MeV}$ [6], the broad peaks for the positive parity resonances are almost unchanged, but the sharp peak structures of the negative parities are completely disappeared. We also vary the detection angle in the range of $\theta_{c.m.} = 40^\circ$ to $\theta_{c.m.} = 180^\circ$. We have confirmed that the peak to valley structure becomes the most prominent in the angular region of $\theta_{c.m.} \approx 170^\circ$. Therefore, the experimental setup [6], such as the energy resolution ($\Delta E = 0.1 \text{ MeV}$) and the detection angle ($\theta_{c.m.} \sim 170^\circ$), is considered to be optimal for the observations of the positive parity resonances.

4. Summary
In summary, we have calculated the energy levels of $\alpha + ^{15}\text{O}$ in $^{19}\text{Ne}$, and the excitation function in the resonant scattering of $^{15}\text{O}$ by an $\alpha$ target. The interaction potential of $\alpha - ^{15}\text{O}$ is determined from the elastic scattering of the respective mirror system, $\alpha + ^{16}\text{N}$. The present calculation predicts the rotational band structure, generated from the spin-orbit splitting of the one neutron-hole state in the $\alpha + ^{16}\text{O}$ cluster structure. In the $\alpha + ^{15}\text{O}$ resonant scattering, the present calculation strongly support the possible observation of the positive parity states because their decay width matches the recent experimental setting. The observation of the excitation function and the identification of the higher resonances, predicted in the present calculations, are strongly desired in future experiments.

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