Numerical simulation of the impact of high-speed metallic plates using two approaches

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Abstract. The paper is devoted to the mathematical modeling of the problem of two metal plates impact using two approaches. In the first approach the problem is solved using three-dimensional Euler equations and the stiffened-gas equation of state for the media. The parameters of the equation of state are calibrated using wide-range equations of state computations of the parameters of shock waves which form after the impact. The second approach is based on one-dimensional two-fluid seven-equation model. In simulations of metal plates impact we get two shocks after the initial impact that propagate to the free surfaces of the samples. The characteristics of shock waves are close (maximum relative error in characteristics of shocks is not greater than 7\%) to the data from the wide-range equations of states computations.

1. Introduction

The study of the mechanisms and the laws of hydrodynamic instability development and turbulent flows in different fundamental and practical problems is among the most complicated modern issues of the continuum media mechanics. The best practices in natural and numerical experiments show that the modeling of the hydrodynamic instability development should be based on three-dimensional (3D) models the program realization of which demands huge computational costs.

The authors of the paper have developed for several years expandable software for the numerical solution of 3D Euler equations using different numerical schemes including high-order methods [1, 2]. The software provides the solution of some problems of the hydrodynamic instability development theory. The obtained results however relates to the flows of ideal gas. At the same time the effects of hydrodynamic instability development are also take place in the media with much more complex equations of state (EOS). In natural experiments [3] on high-speed impact of metal plates, the 3D structures were found on the contact surface of the plates and one of the possible reasons for the effect is the development of Rayleigh–Taylor instability. In our previous works [4, 5] we got the principal effect of 3D structures formation on the contact surface between colliding plates in the statement qualitatively corresponded to the experiment in [3]. The 3D Euler equations simulations for the media with the ideal gas equation of state were used. The quantitative correspondence however was not good. So the goal of the current work is the development of the mathematical model and the computational
algorithm from [4] to obtain better agreement between the parameters of the process of high-speed metal plates impact.

The problem has a long history of investigations and for the theoretical studies different numerical approaches were used each with its own pros and cons. In the early studies of S.K. Godunov with co-authors, the problem was considered in two-dimensional Lagrangian statement [6, 7]. In [4, 8] as a result of two- and tree-dimensional gas-dynamics modeling the multidimensional structures at the contact surface were obtained and the assumptions about the mechanisms of the structures formation were done. In [9], the numerical simulations of waves formation under an oblique impact of metal plates during explosion welding on the basis of Maxwell relaxation model and molecular dynamics method were carried out. It was shown that the numerical simulation correctly reproduces the formation and evolution of waves on the contact boundary. The problem was also considered in [10] using powerful Euler–Lagrangian 3D computer code taking into account wide-range EOS of the metals.

The problem of high-speed impact of two plates is actually two-fluid problem at least at the initial stages of the process when the metals behave as non-mixing compressible fluids. At the same time, there is a lack of the works with consideration of the problem using the models of heterogeneous media mechanics. Another goal of the paper is the formulation of the one-dimensional (1D) two-fluid mathematical model, computational algorithm and fitting of the parameters of the numerical technology for both qualitative and quantitative description of the initial stage of two metal plates impact in the statement of [3].

2. Statement of the problem

Consider the interaction of the lead plate with the thickness 2 mm (parallelepiped 5 mm × 5 mm × 2 mm in 3D, a segment [3 mm; 5 mm] in 1D statement) and the initial density 7900 kg/m³ with the steel plate with the thickness 3 mm (parallelepiped 5 mm × 5 mm × 3 mm in 3D, a segment [0 mm; 3 mm] in 1D statement) and the initial density 7900 kg/m³ (see figure 1). The lead plate is thrown to the direction of steel one with the velocity 500 m/s. It is assumed that at the initial stage of the impact during first 10 μs the metals behave as pseudo-fluids [3] so the gas dynamics approach is valid. The initial pressure is 10⁵ Pa everywhere.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** To the statement of the problem: (a) 3D gas dynamics statement, predicted total density distribution in kg/m³ after 0.1 μs after an impact; (b) 1D two-fluid statement.
We set the inflow conditions at the boundaries $x = 0$ mm and $x = 5$ mm, periodic conditions at the others in 3D statement. The inflow values at $x = 0$ mm corresponds to the initial conditions for the steel plate and at $x = 5$ mm to the lead one. As a result of metal plates interaction, two shock waves (SW) are formed. The computation lasts up to the moment of SW arrival to the boundaries $x = 0$ mm or $x = 5$ mm. Computational grid is uniform with the cell size equal to $10^{-3}$ mm in 1D statement and contains 5 000 cells. Computational grid is uniform and contains $100 \times 100 \times 100 = 10^6$ cells in 3D statement.

3. Mathematical models and numerical algorithms

For the 3D statement the mathematical model is based on 3D non-stationary two-component Euler equations [1, 2] supplemented by stiffened gas EOS [7] with the parameters $\gamma$ and $P_0$. Numerical algorithm is based on the spatial directions splitting technique and hybrid grid-characteristics method [1, 2]. The Roe averaging procedure is used for all matrixes (Jacobian, the matrixes of left and right eigenvectors for the determinative system of equations) coefficients computations. The procedure takes into account the stiffened gas EOS [11]. The numerical approach in use is valid for the general EOS if the partial derivatives $\left(\frac{\partial p}{\partial \varepsilon}\right)_\rho$ and $\left(\frac{\partial p}{\partial \rho}\right)_\varepsilon$ are known (here and further the notations are standard).

The second approach is based on the system of equations from [12] which describes two-fluid compressible flows. Governing system of equation comprises the mass, momentum and energy conservation laws for both fluids. The specific features of the model are compressibility of fluids, velocities and pressures non-equilibrium, the absence of conservative form of notation. Conservation laws are supplemented by transfer equation for the volume fraction for one of the fluids. The stiffened gas equations of state are used for each fluid with different values of $\gamma$ and $P_0$ parameters. Computational algorithm for one time step integration consists of three stages. At first, the hyperbolic step is carried out using HLL numerical scheme and then the velocities relaxation procedure and the pressures relaxation procedure [13].

4. Results of the numerical experiments

4.1. 3D gas dynamics simulations

To reproduce quantitatively reasonable characteristics of metal plates impact, the parameters of stiffened gas EOS should be fitted using either experimental data or the results of simulations taking into account the wide-range EOS for metals. We use the results of simulations [14] obtained using wide-range EOS [15] and the numerical approach [10]. The results from [14] will be referred as reference values further, see table 1. Here and elsewhere SW in steel plate will be called the “left” one and in lead plate – the “right” one in accordance with the statement in figure 1.

| SW     | Pressure, GPa | Density, kg/m$^3$ | Gas velocity, m/s | SW speed, km/s |
|--------|---------------|--------------------|-------------------|----------------|
| In steel | 7.99          | 8 246              | 211               | 4.72           |
| In lead  | 7.99          | 12 830             | 211               | 1.97           |

| SW     | Pressure, GPa | Density, kg/m$^3$ | Gas velocity, m/s | SW speed, km/s |
|--------|---------------|--------------------|-------------------|----------------|
| In steel | 7.24 (–7%)    | 8 595 (+4%)        | 272 (+29%)        | 3.40 (–28%)    |
| In lead  | 7.24 (–7%)    | 12 294 (–5%)       | 272 (+29%)        | 2.38 (+28%)    |

Figure 2 illustrates the predicted fields of density and pressure at the time moment 0.5 μs after the impact. The density distribution in figure 1a at the earlier time moment shows both shock waves and contact surface which moves in the direction of the steel plate. The figures 1a and 2 correspond to the
parameters of stiffened gas EOS $\gamma = 3.0$ and $P_0 = 25$ GPa. The parameters were found as a result of parametric calculations in which we compared the calculated characteristics of both SWs with the reference values. Table 2 demonstrates the finally achieved agreement with the relative errors.

![Image](predicted_distributions.png)

**Figure 2.** Predicted distributions of (a) velocity in m/s and (b) pressure in GPa after 0.5 μs after an impact in 3D statement.

### 4.2. 1D two-fluid model simulations

To satisfy the hyperbolicity conditions for the determinative system of equations, all computational cells should contain both fluids so on the interval [0 mm; 3 mm] we set the volume fraction of lead equal to $10^{-6}$ and on the interval [3 mm; 5 mm] we set the volume fraction of steel equal to $10^{-6}$. A series of numerical experiments was carried out in order to fit the results of the calculations with stiffened gas EOS to the results of modeling using wide-range EOS. The parameters $\gamma$ and $P_0$ for lead and steel plates were varied and the shock speeds $D_L$ and $D_R$, contact velocity $v_{cont}$ and pressure $P_{cont}$, post-shock densities (see table 3) were compared with the reference values (see table 1). For each parameter, the relative error in comparison with the reference value was measured.

| No | Lead $\gamma$ | $P_{\text{in}}, \text{GPa}$ | Steel $\gamma$ | $P_{\text{in}}, \text{GPa}$ | $P_{\text{cont}}, \text{GPa}$ | $v_{\text{cont}}, \text{m/s}$ | $D_L, \text{km/s}$ | $D_R, \text{km/s}$ |
|----|---------------|-----------------|---------------|-----------------|-----------------|----------------|----------------|----------------|
| 1  | 3.9           | 65.5            | 3.0           | 30.0            | 10.1 (+26%)     | 215 (+2%)     | 6.0 (+27%)     | 2.7 (+38%)     |
| 2  | 3.9           | 65.5            | 3.0           | 60.0            | 11.9 (+49%)     | 251 (+19%)    | 6.0 (+27%)     | 3.8 (+94%)     |
| 3  | 3.9           | 65.5            | 4.0           | 30.0            | 11.0 (+38%)     | 232 (+10%)    | 6.1 (+29%)     | 3.2 (+64%)     |
| 4  | 3.9           | 65.5            | 3.0           | 15.0            | 8.5 (+6%)       | 182 (~14%)    | 6.0 (~27%)     | 1.9 (~4%)      |
| 5  | 3.0           | 65.5            | 3.0           | 15.0            | 8.0 (~0%)       | 196 (~7%)     | 5.2 (~11%)     | 1.8 (~7%)      |
| 6  | 2.9           | 65.5            | 3.0           | 15.0            | 8.0 (~0%)       | 198 (~6%)     | 5.1 (~8%)      | 1.8 (~7%)      |
| 7  | 2.7           | 65.5            | 3.0           | 15.0            | 7.8 (~2%)       | 202 (~4%)     | 5.05 (~7%)     | 1.9 (~4%)      |

We started with the EOS parameters for the steel plate from [7] and some default parameters for the lead one. The largest error was obtained for the contact surface pressure and SW speeds. In
calculations No. 2 and 3 the values of $\gamma$ and $P_0$ for steel plate were increased. The errors for mentioned parameters became larger. The post-shock densities remained almost the same and it is the general tendency for the following numerical experiments (the maximum relative error is 7% for all numerical experiments). The positive dynamics for the contact pressure and right SW speed was obtained with the decrease of $P_0$ for the steel plate in calculation No. 4. At the same time, the left SW speed was insensitive to the variation of the EOS parameters in calculations No. 1 – 4. In calculation No. 5 – 7 we successively decreased the value of $\gamma$ for the lead plate and obtained the error 7% for the left SW speed. Figure 3 illustrates the predicted distributions of steel volume fraction and density, lead density and contact pressure for the EOS parameters No. 7 in table 3. Note that the numerical scheme is quite diffusive and should be improved (see for example [16]) but at the same time provides the robust calculation for the huge pressure gradients up to the 5 orders of magnitude.

Figure 3. Predicted distributions at the successive time moments in 1D two-fluid simulation. Red lines correspond to 0.3 mcs time moment, blue lines – 0.4 mcs, green lines – 0.5 mcs. Density scales are in kg/m$^3$, pressure scale is in GPa.

5. Conclusions
The 3D mathematical model based on the multicomponent Euler equations supplemented with the stiffened gas equation of state for the numerical investigation of the metal plates impact is described.
The parameters of the stiffened gas equation of state are calibrated on the basis of the computations using wide-range equation of state for the metals. For the characteristics of the shock waves which are formed after the impact of two metal plates, the maximum error is in the range of 30%. The proposed numerical technique is suitable for the 3D numerical investigations of intensive directed energies flows on the substances taking into account the instabilities development on the contact surface.

The parametric numerical study of two metal plates impact using two-fluid approach in comparison with the simulations taking into account wide-range EOS for the metals is carried out. In calculations, the formation of two SW is obtained with the characteristics with maximum relative error 7% in comparison with the reference values. The determinative system of equations [12] is used. Both metals are considered to be weakly compressible media with stiffened gas equations of state. The computational algorithm is based on the operator splitting approach and HLL method for the approximation of the numerical flux and the non-conservative right-hand side terms.

The work of S.V. Fortova, P.S. Utkin and V.V. Shepelev (sections 1 – 3 and 4.2) is supported by the Russian Science Foundation under grant 17-11-01293 and performed at the Institute for Computer Aided Design of RAS.

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