Modified majoron model for cosmological anomalies

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The vacuum expectation value \( v_s \) of a Higgs triplet field \( \Delta \) carrying two units of lepton number \( L \) induces neutrino masses \( \propto v_s \). The neutral component of \( \Delta \) gives rise to two Higgs particles, a pseudoscalar \( A \) and a scalar \( S \). The most general renormalizable Higgs potential \( V \) for \( \Delta \) and the Standard-Model Higgs doublet \( \Phi \) does not permit the possibility that the mass of either \( A \) or \( S \) is small, of order \( v_s \), while the other mass is heavy enough to forbid the decay \( Z \to AS \) to comply with LEP 1 data. We present a model with additional dimension-6 terms in \( V \), in which this feature is absent and either \( A \) or \( S \) can be chosen light. Subsequently we propose the model as a remedy to cosmological anomalies, namely the tension between observed and predicted tensor-to-scalar mode ratios in the cosmic microwave background and the different values of the Hubble constant measured at different cosmological scales. Furthermore, if \( \Delta \) dominantly couples to the third-generation doublet \( L_\tau = (\nu_\tau, \tau) \), the deficit of \( \nu_e \) events at IceCube can be explained. The singly and doubly charged triplet Higgs bosons are lighter than 280 GeV and 400 GeV, respectively, and could be found at the LHC.

I. INTRODUCTION

Although the Hot Big Bang Model and General Relativity are arguably very robust, they work only if an additional piece is added to the game: inflation. In order to explain the flatness, homogeneity and isotropy of the Universe and the absence of monopoles and other relics, a period of inflation is crucial.

But inflation is a framework comprising countless different inflationary models. Clearly all of them produce a flat, isotropic, homogeneous and relic-free Universe but each one leaves some specific imprints (as the inhomogeneities pattern of the model at hand in the CMB and structure formation) that can help us find out, which one of the plethora of models in the market is the correct one.

During inflation two types of perturbations are produced: scalar or matter perturbations and tensor (metric) perturbations (gravity waves). Each one can be characterized by its amplitude and the dependence on the scale of such amplitude. However, only a subset of two of these four quantities is independent and therefore all our insight of inflation is reduced to two parameters generally chosen to be the spectral index \( n_s \), i.e. the dependence on the scale of the matter perturbations, and the tensor to scalar (amplitude) ratio \( r \). This is the reason why all the inflationary models reduce to lines, points or regions in the \( n_s - r \) plane.

As a consequence to discriminate which region is favored by experiments is also to select which inflationary models remain in the game. The theoretical guidance

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at this stage is crucial. Specific particle physics models with their matter content and interactions should help shed some light on which are the inflationary potentials worth considering, while at the same time making predictions which can be tested elsewhere.

In recent years tensions of cosmological data with the predictions based on the SM and the ΛCDM model have emerged and a light scalar boson \( \phi \) interacting with neutrinos has been considered to alleviate these tensions. Specifically, favored regions for the ratio \( r \) of tensor (metric) to scalar (matter) perturbations inferred from the anisotropies of the cosmic microwave background (CMB) and the spectral index \( n_s \) can be significantly modified and therefore the selection rule for successful inflationary models is vastly affected [1–4]. Furthermore, the Hubble constant determined from local measurements disagrees with the value inferred from CMB data and new neutrino interactions might remedy this as well [5, 6].

Historically, the first interest into light scalars interacting with neutrinos was driven by the attempt to build majoron models breaking lepton number spontaneously, first realized through an SU(2) singlet [7, 8] or triplet [9] field, or with several fields in both representations [10]. The triplet models (and those employing doublet fields) did not comply with LEP 1 data on invisible \( Z \) decays [11] shifting the focus entirely to SU(2) singlet fields \( \phi \) [11–13]. But couplings of singlets to active neutrinos are tiny, because SU(2) forbids such couplings at dimension-4 level and the effective coupling is necessarily suppressed by small mixing angles (e.g. between active and sterile neutrinos). E.g. Ref. [11] finds coupling constants below \( 10^{-3} \). As we will see below, such small couplings do not permit solutions to the cosmological problems in the most interesting region with \( m_\phi > 30 \text{keV} \). Thus to date there is no viable model supporting the idea of Refs. [1–6]
in this mass range.

In the following we will present a model of neutrinos that can not only modify the allowed region in the $n_s - r$ plane changing this way the inflationary models that survive the experimental scrutiny but also provides a viable modification of the SU(2) triplet majoron idea [9]. Since triplet fields have renormalizable, dimension-4 couplings to leptons, even $O(1)$ couplings of the majoron to active neutrinos are possible a priori. Thus we reopen a large portion of the parameter space with consequences for other applications of a light scalar coupling to neutrinos. Furthermore, our model makes predictions beyond cosmology and neutrino physics and can be tested in forthcoming experiments.

This paper is organized as follows: In the following section we present a class of majoron models in which either $S$ or $A$ is light, while the other boson is heavy enough to forbid $Z \rightarrow AS$. Next we discuss phenomenological consequences and “smoking gun” features. Finally we conclude.

II. THE MODEL

An SU(2) triplet Higgs field

$$\Delta = \begin{pmatrix} \delta^+ \\ v_s + \frac{h_s + a_s}{\sqrt 2} \delta^+ \\ -\frac{\delta^+}{\sqrt 2} \end{pmatrix}$$  \hspace{2cm} (1)

developing a vacuum expectation value (vev) $v_s$ in the neutral component generates Majorana masses of light neutrinos in a natural way via its coupling to the lepton doublets. Electroweak precision data imply $v_s \ll v$, where $v = 174$ GeV is the vev of the doublet Higgs field $\Phi$ of the Standard Model (SM). The physical Higgs fields are mixtures of the components of $\Delta$ and $\Phi$; in particular there are two extra neutral Higgs bosons, a scalar $S$ and a pseudoscalar $A$. These approximately coincide with $h_s$ and $a_s$, respectively. By assigning two units of lepton number $L$ to $\Delta$ and choosing an $U(1)_L$-invariant Higgs potential one arrives at a model which breaks $U(1)_L$ spontaneously. Then $A$ is a massless Goldstone mode, the majoron [7–9].

Postulating an effective interaction of neutrinos with a light scalar $\phi$,\n
$$L_{\text{eff}} = g_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta \phi,$$  \hspace{2cm} (2)

where $\nu_\alpha$ generically denotes any of the light left-handed neutrino fields or its charge-conjugate, one can alleviate the tensions in cosmological data mentioned in the introduction while simultaneously modifying the $n_s - r$ region to include well motivated inflationary models which were previously ruled out [1–5]. In this paper we only consider couplings of $\phi$ to $\tau$-neutrinos $\nu_\tau$, for which terrestrial and astrophysical data do not imply strict bounds and it is not relevant for us whether $L$ is identified with the total lepton number or with $L_\tau$. We aim at the formulation of a minimal triplet majoron model addressing the cosmological anomalies and the majoron couplings to $\nu_e$ and $\nu_\mu$ are neither conceptually nor phenomenologically relevant for this.

$L_{\text{eff}}$ is not gauge invariant, a meaningful interaction must be formulated in terms of lepton doublets. Identifying $\phi$ with the majoron amounts to completing $L_{\text{eff}}$ to\n
$$L_{\text{eff}}^\Delta = \frac{y^\Delta}{2} \bar{L}_i \Delta L_3 + h.c. \hspace{2cm} \cap - \frac{m^\Delta}{2} (\bar{\nu}_\tau \nu^c_\tau + \bar{\nu}_\tau \nu_\tau)$$
$$- \frac{y^\Delta}{2\sqrt 2} \left[ (h_s + ia_s)\bar{\nu}_\tau \nu_\tau + (h_s - ia_s)\bar{\nu}_\tau \nu^c_\tau \right],$$  \hspace{2cm} (3)

where $L_3 = (\nu_\tau, \tau)^T$, $L_3^c = (\bar{\tau}, -\nu_\tau)^T$, and $m^\Delta = y^\Delta v_s$ is the contribution of $\Delta$ to the Majorana mass of $\nu_\tau$. However, all known Higgs potentials predict that $S$ and $A$ are either both light, with masses around $v_s$, or both heavy, with masses of order $v$ or larger. This finding holds for the most general renormalizable Higgs potential, irrespective of whether $L$ is broken spontaneously or explicitly. For this reason the original triplet majoron models were discarded with the advent of LEP 1 data on the invisible $Z$ width which left no room for the decay $Z \rightarrow AS$, whose decay rate is entirely fixed by the value of the SU(2) gauge coupling.\n
Finally, in models where $\phi$ in Eq. (2) is a singlet field mixing with a heavy $S$ or $A$ the coupling $g \equiv g_{\tau\tau}$ is suppressed by a tiny mixing angle and is far too small to solve the cosmological tensions in the interesting region with $m_\phi > 30$ keV. The same remark applies, if a singlet $\phi$ couples to heavy sterile neutrinos which mix with $\nu_\tau$.

For the phenomenological analysis of the relevant collider bounds and astrophysical constraints discussed below it does not matter whether $S$ or $A$ is the light scalar corresponding to $\phi$ in Eq. (2) and for definiteness we consider the case $\phi = S \simeq h_s$. Then Eq. (3) entails\n
$$g = -y^2 / (2\sqrt 2).$$

Global fits to cosmological data constrain the combination [4]

$$G_{\text{eff}} = \frac{g^2}{m_S^2} = \frac{y^\Delta}{8 m_S^2},$$  \hspace{2cm} (4)

while successful Big Bang Nucleosynthesis (BBN) is sensitive to the mass and coupling of the scalar particle in a different combination. More specifically, BBN is very sensitive to the amount of extra radiation. The observed primordial abundances of deuterium and helium set strong constraints on the coupling and the mass of our scalar particle. These bounds are reflected in figure 1.
FIG. 1. The contour of the extra radiation $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3 = 0.6$ at a temperature of 1 MeV in the $m_S$-$g$ plane. The (light-blue) region above the blue line is forbidden by primordial helium and deuterium abundances, thus $m_S$ is in the sub-keV range or $m_S \geq 30$ keV. The red region corresponds to $10^{-3}/\text{MeV}^2 < G_{\text{eff}} < 10^{-1}/\text{MeV}^2$, which is the 2$\sigma$ CMB favored region for $G_{\text{eff}} > 10^{-3}/\text{MeV}^2$ [4]. Unlike SU(2) singlet models our triplet model covers the whole allowed parameter region, up to $m_s \sim 1$ MeV. For the displayed region with $g \geq 10^{-7}$ the bound $m_{\nu_\tau} \leq 1$ eV implies $v_s \leq 3.5$ MeV.

where it can be seen that two regions are consistent with BBN and at the same time result in a $G_{\text{eff}}$ able to change the CMB temperature and polarization spectra.\#2 A heavy or MeV region with $0.03\text{MeV} \leq m_S \leq 1\text{MeV}$ and a light or keV region with $m_S \leq 0.1$ keV.

Bounds from meson decays are irrelevant in our case (and therefore not shown) as our new interaction concerns only tau neutrinos. We further stress that none of the bounds derived on $L_{\text{eff}}$ in Eq. (2) from $\tau$ or tauonic $Z$ decays (see e.g. [14] for a recent study) applies to a complete model like ours: The proper cancellation of infrared singularities (for $m_{\nu_\tau} \to 0$) requires the inclusion of virtual corrections, which in turn involve also heavy fields (like the charged components of $\Delta$) to cancel ultraviolet divergences.

The Higgs potential of $\Delta$ in Eq. (1), $\Phi = (\phi^+, v + \sqrt{\mu} \mu v^2)$, and $\Phi^c = (v + \frac{h - i a}{\sqrt{2}}, -\phi^-)$ reads:

$$V = -\mu^2 \Phi^\dagger \Phi - \mu_3^2 \text{Tr} (\Delta^\dagger \Delta) + \lambda (\Phi^\dagger \Phi)^2$$

$$+ \lambda_\Delta [\text{Tr} (\Delta^\dagger \Delta)]^2 + \alpha_1 \Phi^\dagger \Delta^\dagger \Phi + \alpha_2 \Phi^\dagger \Delta \Phi + \alpha_3 \Phi^\dagger \Delta^\dagger \Phi^\dagger$$

$$+ \delta_1 (\Phi^\dagger \Delta^\dagger \Phi + \Phi^\dagger \Delta \Phi^\dagger)^2$$

$$- \delta_2 (\Phi^\dagger \Delta^\dagger \Phi - \Phi^\dagger \Delta \Phi^\dagger)^2. \quad \text{(5)}$$

$V$ is complete up to terms of dimension 4\#3 and the dimension-6 terms involving $\delta_{1,2}$ are instrumental to lift the mass of either $S$ or $A$ above $M_Z$. All parameters are chosen real, so that $V$ is invariant under charge conjugation $C$, and we only consider solutions with real vevs. $L$ is a good symmetry of $V$ for $\beta = \delta_1 - \delta_2 = 0$. Any other dim-6 operator can be expressed as a linear combination of the terms in Eq. (5) and a $\Delta L = 0$ operator, which does not contribute to the $A$-$S$ mass splitting. The minimization conditions $\partial V/\partial \nu = 0 = \partial V/\partial a_\nu$ read:

$$\mu^2 = 2\lambda v^2 + \mathcal{O}\left(\frac{v^4}{v^2}\right),$$

$$\beta = v_s \left(\frac{m^2}{v^2} + 4\delta v^2 + 2\lambda \Delta v_s^2\right)$$

with

$$m^2 = -\mu^2 + \alpha_2 v^2 + \alpha_3 v^2 \left(6\right)$$

$$\mu_\Delta, \alpha_2 v^2, \alpha_3 v^2 \leq \mathcal{O}(m^2), \quad \text{(7)}$$

The parameter $m^2$ will govern the mass of the desired light state. Avoiding fine-tuning between different terms in Eq. (7) we must have

$$\mu_\Delta, \alpha_2 v^2, \alpha_3 v^2 \leq \mathcal{O}(m^2),$$

$$\text{(8)}$$

Using the minimization conditions in Eq. (6) we trade $\mu$ and $\beta$ for $v$ and $v_s$ in the formulae below for fields and masses. Neglecting $\mathcal{O}\left(\frac{v^5}{v^2}\right)$ terms the physical states are

$$G = a + 2\frac{v_s}{v} a_s, \quad A = a_s - 2\frac{v_s}{v} a,$$  

$$H = h - \frac{2\mu_\Delta^2 + 8\delta_1 v^4}{m^2 - 4\lambda v^2 + 4\delta_1 v^4} \frac{v_s}{v} h_s,$$  

$$S = h_s + \frac{2\mu_\Delta^2 + 8\delta_1 v^4}{m^2 - 4\lambda v^2 + 4\delta_1 v^4} \frac{v_s}{v} h,$$  

$$G^+ = \phi^+ + \sqrt{2} \frac{v_s}{v} \delta^+, \quad H^+ = \phi^+ - \sqrt{2} \frac{v_s}{v} \phi^+,$$  

$$H^{++} = \delta^{++}. \quad \text{(11)}$$

Here $G$ and $G^+$ are the massless Goldstone bosons eaten by $Z$ and $W^+$, respectively. Neglecting subdominant terms the desired (squared) Higgs masses read

$$m_A^2 = 4\delta_2 v^4 + m^2 + 2\lambda \Delta v_s^2 \left(13\right)$$

$$m_S^2 = 4\delta_1 v^4 + m^2 + 6\lambda \Delta v_s^2 \left(14\right)$$

$$m_H^2 = 4\lambda v^2 = (125 \text{GeV})^2, \quad \text{(15)}$$

$$m_{H^{++}}^2 = 2m_{H^{++}}^2 = \alpha_1 v^2. \quad \text{(16)}$$

We start our discussion with the role of $U(1)_L$ symmetry in $V$. For this it is helpful to use Eq. (6) to trade $m^2 + 2\lambda \Delta v_s^2$ for $\beta$ in $m_A^2$ in Eq. (13) to find

$$m_A^2 = 4(\delta_2 - \delta_1) v^4 + \frac{\beta v^2}{v_s}. \quad \text{(17)}$$

For exact $U(1)_L$ symmetry with $\beta = 0$ and $\delta_1 = \delta_2$ we verify that $A$ is the massless Goldstone boson of this broken symmetry. This solution corresponds to the vanishing of the bracket in Eq. (6). (For the second solution with $v_s = 0$ the symmetry is unbroken and Eq. (17) does not hold.) An interesting case is $\beta = \delta_1 = 0$, with $\delta_2 \neq 0$: \#2 As was shown in [4] $G_{\text{eff}}$ in the ballpark of $3.5 \cdot 10^{-2}$ MeV$^{-2}$ opens up the allowed window in the $n_S - r$ plane, significantly affecting the selection criteria for acceptable models of inflation. \#3 The term $\lambda_\Delta \text{Tr} (\Delta^\dagger \Delta) \text{Tr} (\Delta \Delta)$ is phenomenologically irrelevant.
U(1)$_L$ is explicitly broken by a higher-dimensional term only: The minimization equation (6) is not sensitive to this term and features spontaneous U(1)$_L$ as in the original, renormalizable majoron models. The phenomenological effect of $\delta_2 \neq 0$ is to render $m_A$ massive, with the possibility of $m_A > M_Z$, and we may view this case as spontaneous symmetry breaking without Goldstone.

For fixed $G_{\text{eff}}$ the perturbativity limit $y_A^2 \lesssim 2$ entails an upper limit on $m_S$ through Eq. (4). In the scenario with spontaneous U(1)$_L$ breaking (where $\beta = 0$) one necessarily has $m \lesssim v_s$. As a consequence, $m_{\nu_\tau} = y_\tau v_s \lesssim 1\text{ eV}$ pushes $y_\tau^2$ and $m_S$ far below their otherwise theoretically allowed upper bounds. Specifically, $m_S \lesssim 10\text{ keV}$ for $G_{\text{eff}} \gtrsim 10^{-4}\text{ MeV}^{-2}$ and the BBN constraint of figure 1 tightens this to $m_S \lesssim 0.3\text{ keV}$. In the scenario, with explicit U(1)$_L$ breaking, however, one easily infers from Eq. (6) that one can choose $m$ (and thereby $m_S$) and $v_s$ independently thanks to the free parameter $\beta$.

It is easy to find a UV completion generating the dim-6 terms in Eq. (5): Consider heavy real scalar singlets $\chi_{1,2}$ coupling as

$$V_s = (\rho_1 + i\sigma_1)\chi_1 \Phi^c \Delta^\dagger \Phi + (\rho_2 + i\sigma_2)\chi_2 \Phi^c \Delta^\dagger \Phi + \text{H.c.}$$

(18)

with real $\rho_{1,2, \sigma_{1,2}}$. Under charge conjugation we have $\Delta \leftrightarrow \Delta^\dagger, \Phi \leftrightarrow \Phi^*$. Choosing further $\chi_1 \leftrightarrow \chi_2, \chi_2 \leftrightarrow -\chi_1$ under $C$ and demanding $C$ invariance of $V_s$ implies $\sigma_1 = \rho_2 = 0$. Integrating out the heavy singlet fields as shown in Fig. 2, gives $\delta_1 = \rho_1^2/(2m_{\chi_1}^2)$ and $\delta_2 = \sigma_2^2/(2m_{\chi_2}^2)$, and e.g. $m_{\chi_1} \gg m_{\chi_2}$ produces the scenario with heavy $A$ and light $S$. Thus the UV completion in Eq. (18) generates the desired dim-6 terms in Eq. (18) without producing any other dim-6 operators.

Instead of invoking $C$ symmetry one can also work with $L$, by assigning $L = 2$ to $\chi = \chi_1 + i\chi_2$ enforcing $\rho_2 = i\sigma_2 = i(\rho_1 + i\sigma_1)$. Compared to the minimal model described above the UV sector must be richer and break $L(\tau)$ in a way to produce the desired mass hierarchy. Loop effects and/or a small vev of $\chi_1$ may render $\beta \neq 0$, which is a welcome feature as discussed in the paragraph above Eq. (18). Note that $\chi$ does not need to acquire a vev. If one chooses to consider scenarios with a non-zero $\chi$ vev, this vev must be small to avoid large corrections to $\beta$ spoiling $|\beta| \lesssim O(v_\nu)$ implied by Eq. (6). Note that all other dim $\lesssim 4$ terms in V conserve $L$ and therefore do not receive contributions linear in the $L$-breaking vev of $\chi$. We do not aim at an exhaustive discussion of all attractive UV completions here. Instead, we consider it as an advantage that $V$ in Eq. (5) allows us to fully study the low-energy phenomenology (including loop effects where needed) without specifying the underlying fundamental theory.

III. PHENOMENOLOGY

Studies of perturbativity for the SM [15] and 2HDM [16, 17] have shown that self-couplings should be smaller than $\approx 5$. Applying this bound to $\alpha_1$ in Eq. (16) implies that $m_{H^{++}} \lesssim 400\text{ GeV}$ and $m_{H^+} \lesssim 280\text{ GeV}$. Current collider bounds are much weaker, because the production of these heavy charged Higgs bosons is an electroweak gauge process (e.g. vector boson fusion at the LHC). Favorable decays are $H^{++} \rightarrow \tau^+\tau^+$ and $H^+ \rightarrow \tau^+\nu_\tau$, unless $y_\tau^2$ is too small. In the latter case one must resort to gauge-coupling driven decays like $H^+ \rightarrow W^+ S$, $W^+ A$. For a cut-off scale of $\Lambda \sim 0.5\text{ TeV}$ (and $O(1)$ couplings in the UV completion) we have $\delta_2 y_\tau^2 \sim 0.1$ and Eq. (14) gives $m_A \sim 120\text{ GeV}$. $A$ is produced through gauge interactions, and now a small $y_\tau^2$ is welcome to suppress the decay into neutrinos. Detection through $A \rightarrow ZS$ will fail if $M_A - M_Z$ is smaller than the trigger threshold for missing transverse momentum. It is therefore advisable to focus on the searches for the charged bosons.

The model can be tested by its astrophysical signatures as well. Depending whether our scalar field is in the MeV or keV range different signals can be expected. As mentioned before, CMB experiments are sensitive only to $G_{\text{eff}}$ and therefore both ranges give exactly the same phenomenology cosmology-wise. This is not the case regarding astrophysical experiments. For scalars in the MeV range, the interaction introduced will make high energy ($\sim\text{TeV}$) tau neutrinos from astrophysical sources scatter resonantly with the CMB tau neutrinos and therefore a deficit of tau neutrinos can be expected. More precisely a dip in the tau neutrino spectrum corresponding to the resonant energy [4]

$$E_{\text{resonant}} \simeq \frac{m_\nu^2}{2m_{\nu_\tau}}$$

(19)

is to be expected in experiments like IceCube and KM3Net. Remarkably, IceCube seems to be seeing

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#4 Note that when the tau neutrino mass drops below the neutrino...
a deficit in tau neutrinos although the effect is not significant yet.

For scalars in the keV range the resonant energies involved make it ideal to detect such interactions in experiments sensitive to the diffuse supernova neutrino background, like T2HK. A detailed analysis of both signals will be given elsewhere.

As mentioned before, another very attractive property of this model is that it provides a higher value of the Hubble constant $H_0$, i.e. at 95% C.L. $H_0 = 70.06^{+2.21}_{-2.31}$ km/s/Mpc as compared with the ΛCDM value of $H_0 = 68.35^{+1.94}_{-1.84}$ km/s/Mpc.

This is specially welcome, as the CMB provides a 2-3σ [18] lower value of the Hubble constant than local measurements do [19]. In a simple 1-parameter extension, by means of $G_{eff}$ this tension is only weakened but not resolved. However, including also $N_{eff}$ and $\sum m_\nu$ as free parameters to the analysis completely relaxes the tension in the Hubble parameter. Such an extension is however beyond the scope of the present work.

IV. CONCLUSIONS

The possibility of a light scalar particle interacting with neutrinos receives a lot of attention to alleviate several tensions in cosmological data. The particle physics community is interested in this kind of interaction as a means to break lepton number $L$ spontaneously, providing a natural framework for neutrino Majorana masses. So far all cosmological analyses have employed the effective interaction of Eq. (2), which violates electroweak SU(2) symmetry. In this paper we have presented a viable realization of a model of a scalar interacting with neutrinos, by complementing the original SU(2) triplet models (featuring spontaneous or explicit $L$ violation) with higher-dimensional terms and devising possible UV completions generating these terms. With our lagrangian one can now consistently calculate constraints from $Z$, charged lepton, and meson decays, which was not possible with Eq. (2). To bypass these constraints we have discussed the cosmological and astrophysical implications for the case that the light scalar couples dominantly to tau neutrinos. Depending on the mass range of the scalar particle, characteristic signals are possible at the IceCube or T2HK experiments. The LHC can search for the charged members of the SU(2) triplet, whose masses must be below 400 GeV. For appropriate choices of the parameters our Higgs potential conserves $L$ at the level of the renormalizable terms and shares essential features of the orginal triplet majoron model [9].

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