Variability in Shell Models of GRBs

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Abstract. Many cosmological models of gamma-ray bursts (GRBs) assume that a single relativistic shell carries kinetic energy away from the source and later converts it into gamma rays, perhaps by interactions with the interstellar medium or by shocks within the shell. Although such models are able to reproduce general trends in GRB time histories, it is difficult to reproduce the high degree of variability often seen in GRBs. We investigate methods of achieving this variability using a simplified external shock model. Since our model emphasizes geometric and statistical considerations, rather than the detailed physics of the shell, it is applicable to any theory that relies on relativistic shells. We find that the variability in GRBs gives strong clues to the efficiency with which the shell converts its kinetic energy into gamma rays.

The “external shock” models of gamma-ray bursts (GRBs) assume that the gamma rays are emitted from a single shell of material traveling at highly relativistic speeds (for example, [1] and [2]). Fenimore, Madras, and Nayakshin [3] have found that the envelope of emission for a single relativistic shell fits the overall shape of a few GRBs, but it does not account for the wide variability found in GRBs. In this paper, we investigate methods of achieving the observed degree of variability in GRB time histories using randomly placed active regions on the shell.

DESCRIPTION OF THE MODEL

The “time” that is measured in GRB time histories is the time at which photons arrive at the detector (denoted by $T$), and is not the time of emission as would be measured in the rest frame of the detector (equivalent to the rest frame of the shell’s expansion, denoted by $t$). We set $t = 0$ to be the time at which the central explosion occurs, and $T = 0$ to be the arrival time of a photon emitted at $t = 0$; therefore

$$T = (1 - \beta \cos \theta)t = \Lambda \gamma^{-1}t$$

(1)

where $\Lambda$ is defined as $\Lambda = \gamma(1 - \beta \mu)$, and $\mu = \cos \theta$. By setting Equation 1 equal to a constant, one finds that the surface of constant arrival time is an ellipsoid
FIGURE 1.  

a. (left) The position of the spherical shell at four different times as it expands from \( t_o \) to \( t_{\text{max}} \). The two ellipses represent surfaces of constant arrival time. Photons emitted from any point on the inner ellipsoid will arrive at the detector simultaneously at time \( T \), and photons emitted from any point on the outer ellipsoid will arrive at the detector at time \( T + dT \). The cross-hatched region represents the volume that will be “seen” by the detector between these two times. Each small circle represents the volume swept out by an entity during its emission lifetime of \( \Delta T_p \). Only the white entities will be seen between \( T \) and \( T + \Delta T \).

b. (right) A three-dimensional view of the cross-hatched area shown at left. The volume of this annulus is given by \( \Upsilon(T)dT \).

[4]. All of the photons emitted from such an ellipsoid will arrive at the detector simultaneously.

Our model considers a thin shell (as required by the observations [3]) that expands outward from a central point so that \( R = \beta ct \). The shell expands to a radius of \( R_o = \beta ct_o \), and then emits gamma rays until it reaches a radius of \( R_{\text{max}} = \beta ct_{\text{max}} \). The gamma ray emission must occur from small, independent patches on the shell, which we call “entities” [3]. Each entity begins as a small perturbation, perhaps caused by the shell’s interactions with the interstellar medium or by instabilities within the shell. The entity grows at the sound speed \( c_s \sim c \) until it reaches a maximum size of \( \Delta R_\perp = \Gamma c \Delta T_p \) (see Table 2 in [3]). Thus, each entity represents a causally connected region, and many entities can fit on the surface of the shell. We assume that each entity emits isotropically with a power-law photon spectrum in its own reference frame, \( \Phi'(E') = E'^{-\alpha} \) photons (entity \( dt' dE' dO' \))\(^{-1} \) (primed quantities are measured in the rest frame of the entity).

Figure 1 gives a pictorial representation of our model. It is important to emphasize that we are considering a thin shell. Thus, the four dashed lines in Figure 1a represent the shell at four different times. The area between \( t_o \) and \( t_{\text{max}} \) represents the volume swept up by the shell over its emission lifetime. Likewise, the entities are small areas on the shell. The small spheres in Figure 1a represent the volumes...
swept out by the entities as the shell expands.

A more quantitative description can be made by calculating the volume enclosed in the cross-hatched region, which we denote as Υ(T)dT (Figure 1b):

\[ Υ(T)dT = 2\pi \int_{\min(R_o,R_{ell,in})}^{\max(R_{ell,out},R_{ell,in})} \int_{\mu(T)}^{\mu(T+dT)} r^2 d\mu \ dr \]  

(2)

where \( R_{ell,out} = \beta c(T+dT)/(1-\beta) \), and \( R_{ell,in} = \beta cT/(1+\beta) \). Equation 1 gives \( \mu(T) = [\beta^{-1}-Tc/r] \), and \( \mu(T+dT) = [\beta^{-1}-(T+\Delta T)c/r] \). Evaluating the integral yields the volume seen per dT:

\[ Υ(T) = \begin{cases} 0 & \text{if } T < T_o \\ \frac{\pi c (\beta c)^2}{(1-\beta)^2} \left( T^2 - T_o^2 \right) & \text{if } T_o < T < T_{max} \\ \frac{\pi c (\beta c)^2}{(1-\beta)^2} \left( T_{max}^2 - T_o^2 \right) & \text{if } T_{max} < T < \Gamma^2(1+\beta)^2 T_o \\ \frac{\pi c (\beta c)^2}{(1-\beta)^2} \left( T_{max}^2 - \frac{T^2}{\Gamma^4(1+\beta)^4} \right) & \text{if } \Gamma^2(1+\beta)^2 T_o < T < \Gamma^2(1+\beta)^2 T_{max} \\ 0 & \text{if } T > \Gamma^2(1+\beta)^2 T_{max} \end{cases} \]

(3)

where \( T_o = to/(1-\beta) \), and \( T_{max} = t_{max}/(1-\beta) \). Note the rather surprising result that the volume “seen” by the detector is constant for \( T_{max} < T < \Gamma^2(1+\beta)^2 T_o \) for the relativistically expanding shell. During these times, the number of entities in the cross-hatched region in Figure 1a is a constant with respect to \( T \). In comparison, in the non-relativistic case, the volume seen, and therefore the number of entities seen, would increase as the area of the shell increased.

The time history from such a shell is given by

\[ V(T)dT = 2\pi \int_{\min(R_o,R_{ell,in})}^{\max(R_{ell,out},R_{ell,in})} \int_{\mu(T)}^{\mu(T+dT)} \rho C' \Lambda^{-(\alpha+1)} r^2 d\mu \ dr \]  

(4)

where \( V(T) \) is the expected time history in photons \((dT\ dA_{det})^{-1}\), \( C' \) is the photon flux as observed in the rest frame of the entity, and \( \Lambda^{-(\alpha+1)} \) incorporates the relativistic effects. We define \( \rho \) as the “density” of entities so that \( \rho \ dV \) gives the number of entities within the cross-hatched region in Figure 1a. (The density \( \rho \) is proportional to the fraction of the shell’s surface that emits gamma rays during the shell’s evolution, and therefore gives some idea of how efficiently the shell converts its kinetic energy into gamma rays.) The envelope is [5]

\[ V(T) = \begin{cases} 0 & \text{if } T < T_o \\ \psi \frac{T^{\alpha+3} - T_o^{\alpha+3}}{T^{\alpha+1}} & \text{if } T_o < T < T_{max} \\ \psi \frac{T_{max}^{\alpha+3} - T_o^{\alpha+3}}{T^{\alpha+1}} & \text{if } T_{max} < T < \Gamma^2(1+\beta)^2 T_o \end{cases} \]

(5)
FIGURE 2. The volume seen by the detector per $\Delta T$ is shown as the solid line, and the resulting photon flux at the detector is shown as the dashed line. The volume, $\Upsilon(T)$, and the detector signal, $V(T)$, both increase as $T^2$ for $T_o < T < T_{\text{max}}$; this is identical to the results for a non-relativistic shell. However, in the case of a relativistic shell for $T > T_{\text{max}}$, the volume remains constant over a large range of $T$, and the signal shows a power-law decay. Since the detector sees a constant volume, and thus a constant number of entities, per $\Delta T$ for $T > T_{\text{max}}$, the decrease in the photon flux after $T_{\text{max}}$ is entirely determined by relativistic effects.

where $\psi$ is a constant (see Figure 2).

Equation 5 gives the average signal that would be expected from a collection of entities scattered over the surface of a relativistic shell. However, the number of entities seen in any volume $\Upsilon(T)dT$ is a random quantity; since the entities are independent and discrete, the randomness follows simple Poisson statistics. Therefore, this model predicts time histories with a mean given by Equation 5 and Poisson variations about that mean, which would look very similar to the “peaks” usually observed in GRB time histories. A low value for $\rho$ gives a low number of entities and correspondingly large Poisson fluctuations, leading to a “spiky” time history with many peaks. Conversely, a high efficiency leads to a smooth time history. To illustrate these points, we present two simulated time histories in Figures 3a and 4a with characteristics selected to match particular BATSE bursts. We estimate that the typical fraction of a single relativistic shell’s surface that emits gamma rays during its lifetime is $\sim 10^{-3}$ [6].

CONCLUSIONS

In order to achieve the observed variability in GRBs using a single-shell model, we have found that the gamma-ray emission must occur from small patches on the shell (entities). We have derived two significant characteristics of the time histories expected from such a model. First, the average number of entities contributing to the signal remains constant throughout much of the time history, although the overall photon flux decreases due to relativistic effects. Second, the “peaks” in a time history can be ascribed to Poisson variations in the actual number of entities contributing to the signal at any given time. Taken together, these properties imply
FIGURE 3.  

a. (left) Simulated time history using a high density of entities. Nearly 100% of the shell’s surface emitted gamma rays in this simulation. Note that the high efficiency gives a smooth profile.  
b. (right) BATSE trigger 1467.

FIGURE 4.  

a. (left) Simulated time history assuming a low density of entities. Only 1% of the shell’s surface emitted gamma rays in this simulation. Note that this low efficiency gives a spiky profile, with many “peaks”.  
b. (right) BATSE trigger 678.

that the relative variations in a GRB (i.e. the heights of the peaks relative to the average envelope of the signal) should remain constant throughout a time history. Qualitatively, this result is consistent with visual inspections of GRBs. In general, bursts are equally "spiky" during the first half of the time history as the second half.

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