Summary: We study skew-amenable topological groups, i.e., those admitting a left-invariant mean on the space of bounded real-valued functions left-uniformly continuous in the sense of Bourbaki. We prove characterizations of skew-amenability for topological groups of isometries and automorphisms, clarify the connection with extensive amenability of group actions, establish a Følner-type characterization, and discuss closure properties of the class of skew-amenable topological groups. Moreover, we isolate a dynamical sufficient condition for skew-amenability and provide several concrete variations of this criterion in the context of transformation groups. These results are then used to decide skew-amenability for a number of examples of topological groups built from or related to Thompson’s group $F$ and Monod’s group of piecewise projective homeomorphisms of the real line.

MSC:

22A10 Analysis on general topological groups
43A07 Means on groups, semigroups, etc.; amenable groups
54H20 Topological dynamics (MSC2010)

Keywords:
topological group; group action; amenability; extensive amenability; isometry group

References:
[1] Proof. This is a consequence of Proposition 8.2, Remark 8.1, and Corollary 7.6. Alternatively, Corollary 8.3 above may be proved using Corollary 3.13 and Remark 8.1, via a more concrete rendering of the argument proving Proposition 7.5. Similarly, one may deduce Corollary 9.2 below from Corollary 3.13 and amenability of $Z$, by suitably reproducing the argument from the proof of Proposition 7.5. Furthermore, let us note the following immediate consequences of our observations above: Corollary 8.4. Let $A$ be a unital subring of $R$. The following hold:

[2] H.A/ u is skew-amenable.

[3] An interesting discussion of properties of a topological space $X$ ensuring that the associated compact-open topology and the topology of pointwise convergence agree on Homeo.X/ is to be found in [12, Remark 3, footnote 2, pp. 3-4].

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