Electron spectroscopy of the shot noise reduction effect

O. M. Bulashenko and J. M. Rubí

Departament de Física Fonamental, Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain

V. A. Kochelap

Department of Theoretical Physics, Institute of Semiconductor Physics, Kiev 252028, Ukraine

(June 5, 1999)

A general formula for current noise in a two-terminal ballistic nondegenerate conductor under the action of long-range Coulomb correlations has been derived. The noise reduction factor (in respect to the uncorrelated value) is obtained for biases ranging from thermal to shot noise limits, and it is related to spatial variations in transport characteristics. The contributions of different energy groups of carriers to the noise are found, that leads us to suggest an electron energy spectroscopy experiment to probe the Coulomb correlations in ballistic conductors.

Shot-noise measurements become a fundamental tool to probe carrier interactions in mesoscopic systems. The term “shot noise”, appeared originally in the context of pure ballistic electron transmission in vacuum-tube devices, has acquired nowadays a much broader usage and refers to different mesoscopic structures, including diffusive conductors, resonant-tunneling devices, etc., where the carrier flow exhibits nonequilibrium noise proportional to the electric current.

In this Letter we address the case of shot noise in pure ballistic conductors under the action of long-range Coulomb correlations, thus going back to the genuine shot noise definition. The fact that shot noise may be affected by Coulomb interactions has been known since the times of vacuum tubes. Their importance in mesoscopic conductors has been emphasized by Landauer and Büttiker, and evidenced recently by Monte Carlo simulations. The purpose of our paper is twofold. First, we present for the first time a self-consistent theory of shot noise in ballistic nondegenerate conductors by solving analytically the kinetic equation. Second, basing upon this theory, we suggest an electron spectroscopy experiment to make the Coulomb correlations effect visible. The possibility of such an experiment is based on recent advances in nanoscale fabrication techniques and shot-noise measurements.

Consider a two-terminal semiconductor sample with plane parallel heavily-doped contacts at $x=0$ and $x=d$. We assume $\lambda_w \ll d \lesssim \lambda_p$, with $\lambda_w$ the electron wavelength and $\lambda_p$ the mean free path, so that electrons may be considered as classical particles moving ballistically between the contacts and interacting with each other electrostatically. The transport in such a system is described by the Vlasov system of equations, that is the collisionless kinetic equation for the electron distribution function $F(X,v_x,t)$ coupled self-consistently with the Poisson equation for the electrostatic potential $\varphi(X,t)$

$$\frac{\partial F}{\partial t} + v_x \frac{\partial F}{\partial X} + \frac{q}{m} \frac{\partial}{\partial v_x} \int \frac{d\varphi}{dX} F(X,v_x,t) = 0,$$

$$\frac{d^2 \varphi}{dx^2} = \frac{q}{\kappa} \int F(X,v_x,t)dv_x.$$  

Here, $v_x$ is the $X$-component of the electron velocity, $q$ the electron charge, $m$ the electron effective mass, and $\kappa$ the dielectric permittivity. The applied bias $U$ between the contacts is assumed to be fixed by a low-impedance external circuit. The distribution functions at the left (L) and right (R) contacts are supposed to consist of a stationary part and a small fluctuation

$$F(0,v_x,t)|_{v_x>0} = F_s(v_x) + \delta F_L(v_x,t),$$

$$F(d,v_x,t)|_{v_x<0} = F_s(v_x) + \delta F_R(v_x,t).$$  

Under nondegenerate and equilibrium conditions in the contacts, we assume for the stationary part of the injection function the half-Maxwellian distribution $F_s(v_x) = 2N_0/(\sqrt{\pi}v_0) e^{-v_x^2/v_0^2}$, where $N_0$ is the density of electrons injected from the contact, $v_0 = \sqrt{2k_BT/m}$ is the thermal velocity, $k_B$ the Boltzmann constant, and $T$ the temperature. The stochastic terms $\delta F_L$, $k = L,R$ in Eq. (3) are the only sources of noise under ballistic transport considered here, since the electron motion between the contacts is noiseless. Their correlation is given by $\langle \delta F^*_L(v_x,t)\delta F^*_R(v'_x,t') \rangle = F_s(v_x)\delta(v_x-v'_x)\delta(t-t')$. As a consequence of these fluctuations inside the contacts (whose origin is ultimately the carrier scattering processes), both the electron distribution function and electrostatic potential in the ballistic sample fluctuate, which leads to the current fluctuations. Introducing the Fourier transform for the fluctuations, the linearized Vlasov equations give

$$\left(v_x \frac{\partial}{\partial X} + \frac{q}{m} \frac{d\varphi}{dv_x} \frac{\partial}{\partial v_x} \right) \delta F(X,v_x) = -q \frac{\partial F}{\partial v_x} \frac{\partial}{\partial X},$$

$$\frac{d^2 \varphi}{dx^2} = q \kappa \int \delta F(X,v_x)dv_x,$$  

where in the low-frequency regime of interest here ($\omega \ll \tau_T^{-1}$, with $\tau_T$ being an average transit time between the contacts) we have omitted the term $\propto i\omega$. It is seen, that the calculation of fluctuations requires the knowledge of the stationary distributions $F$ and $\varphi$, which, in turn, can be determined by solving the self-consistent steady-state problem. It is advantageous to introduce the dimensionless potential $\psi(x) = q\varphi(x)/(k_BT)$, and to scale $n = N/(2N_0)$, $x = x/L_D$, with $L_D = \sqrt{\kappa k_BT/(2q^2N_0)}$ being the Debye screening length. In such units our problem contains only two dimensionless parameters: (i) the length of the sample $\lambda = d/L_D$, and (ii) the applied voltage bias $V = qU/(k_BT)$. Let the space charge be such,
that a potential minimum $\psi_m \equiv -V_m$ occurs at $x = x_m$, which acts as a barrier for the electrons by reflecting a part of them back to the contacts. Since Eq. (1) is equivalent to \(\langle dF/dt \rangle_{\text{trajectory}} = 0\), the distribution function at any plane $X$ may be expressed through the injection distribution functions given at the contacts. By making use of the Maxwellian stationary injection and the contribution of different groups of carriers (transmitted and reflected), we obtain the electron density $n = \int F(x,v_x) dv_x$ as a functional of the potential

$$n(\eta) = n_m e^{\eta[1 + \beta \text{erf}(\sqrt{\eta})]}, \quad (6)$$

where $\eta(x) = \psi(x) - \psi_m$ is the shifted potential measured from the minimum, $erf$ stands for the error function, $n_m = \frac{1}{\sqrt{2\pi}} e^{-V_m}(1 + e^{-V})$ is the electron density at the potential minimum, and $\beta = \tanh(V/2)$. Here, and throughout the paper, we shall use the upper sign for the left side of the potential minimum $0 < x < x_m$, and the lower sign for the right side $x_m < x < \lambda$. Note that in equilibrium, $V=0$, $\beta=0$, the Boltzmann distribution $n(x) = e^{\psi(x)}$ is recovered throughout the sample.

The obtained Eq. (6) is then used to solve the Poisson equation $\frac{d^2\eta}{dx^2} = n$, subject to the boundary conditions at the contacts $\eta(0) = V_m$, $\eta(\lambda) = V_m + V$, and the condition at the potential minimum $\eta(x_m) = 0$. Integration leads to the electric-field distribution

$$E = -\int_0^x \frac{dn}{d\eta} \frac{d\eta}{\sqrt{h_V(\eta)}}, \quad 0 < x < x_m, \quad \sqrt{2n_m h_V(\eta)}, \quad x_m < x < \lambda, \quad -\sqrt{2n_m h_V(\eta)}, \quad (7)$$

$$h_V(\eta) = e^{\eta} - 1 + \beta \left[e^{\eta \text{erf}(\sqrt{\eta})} - \frac{2}{\sqrt{\pi}} \right], \quad (8)$$

where $\beta$ and $n_m$ are functions of $V$ as specified above. Integrating Eq. (8), one obtains the distribution of the potential in an implicit form where, for the given $V$, $\lambda$, the only unknown parameter is the potential minimum $V_m$. The latter is found from the matching at $x = x_m$

$$\lambda \sqrt{2n_m} = \int_0^{V_m} \frac{dn}{\sqrt{h_V(\eta)}} + \int_{V_m + V}^{\lambda} \frac{dn}{\sqrt{h_V(\eta)}}. \quad (9)$$

This brief description of the steady-state is then completed by the expression for the stationary current $I = qA \int v_x F dv_x$, for which we find

$$I = I_c e^{-V_m}[1 - e^{-V}] = 2I_c n_m \beta, \quad (10)$$

where $I_c = \frac{1}{\sqrt{\pi}} q N_0 v_0 A$ is the emission current from each contact. The above relations solve completely the steady-state problem for a ballistic conductor under a space-charge-limited transport regime, for which Eqs. (1) and (4) determine the current-voltage characteristics.

To solve the fluctuation problem (1)–(4), we first find the fluctuation of the distribution function $\delta F$ in a given electrostatic potential $\varphi(x) + \delta \varphi(x)$ by solving the perturbed kinetic equation (4). The fluctuation $\delta F$ consists of the contributions corresponding to the transmitted and reflected groups of carriers, the expressions for them are quite cumbersome and will be presented elsewhere [13]. Then, for the current fluctuation we obtain

$$\delta I = \int_{V_m}^{\lambda} \delta I_L(\varepsilon) d\varepsilon - \int_{V_m + V}^{\lambda} \delta I_R(\varepsilon) d\varepsilon - \delta V_m, \quad (11)$$

where $\delta V_m \equiv -\delta \psi(x_m)$ is the potential minimum fluctuation and $\delta I_k(\varepsilon)$ is the fluctuation of the contact injection current per energy interval $[x \delta F_k(\varepsilon)]$, with $\varepsilon$ being the kinetic energy normalized by $k_B T$. The latter is given by the self-consistent potential fluctuation (long-range Coulomb correlations), that compensates the current fluctuation and may result in the noise reduction.

The potential barrier fluctuation $\delta V_m$, which is of prime interest, we find from the linearized Poisson equation (3). Let us introduce the perturbation of the potential $\delta \eta_x = \delta \psi(x) - \delta \psi(x_m)$, referenced to the fluctuating potential minimum, so that at the minimum $\delta \eta_{x_m} = 0$. Thus, Eq. (3) gives the stochastic differential equation

$$\frac{d^2 \delta \eta_x}{dx^2} = n(x)[\delta \eta_x - \delta \eta_0] \pm \frac{J \delta \eta_x}{4\pi \eta(x)} + \delta n_{x}^{\text{inj}}, \quad (12)$$

subject to the boundary conditions $\delta \eta_0 = \delta \eta_\lambda = \delta V_m$, where $J \equiv I/I_e$ is the normalized current and

$$\delta n_{x}^{\text{inj}} = \frac{1}{2\pi I_e} \sum_{k=L,R} \int_{\psi_k - \psi_m}^{\infty} \frac{\delta I_k(\varepsilon) d\varepsilon}{\varepsilon + \psi(x) - \psi_k}$$

$$+ \frac{1}{2\pi I_e} \left\{ \begin{array}{cl} \int_{\psi_{R} - \psi(x)}^{\psi_{R} - \psi_{m}} \frac{\delta I_{L}(\varepsilon) d\varepsilon}{\varepsilon + \psi(x) - \psi_{L}} & 0 < x < x_m, \\ \int_{\psi_{L} - \psi(x)}^{\psi_{L} - \psi_{m}} \frac{\delta I_{R}(\varepsilon) d\varepsilon}{\varepsilon + \psi(x) - \psi_{R}} & x_m < x < \lambda. \end{array} \right. \quad (13)$$

is the electron-density fluctuation due to the injection from the contacts, which is obtained by considering the contributions from both the transmitted and reflected groups of carriers. The terms $\sim \delta \eta_{x}$ in the rhs of Eq. (13) are related to the fluctuations of the potential profile induced by injected electrons. Note that Eq. (12) is a second-order nonhomogeneous differential equation with spatially dependent coefficients. To find its solution in a general form is a complicated problem. An additional difficulty is due to the term $1/\sqrt{4\pi \eta(x)}$ which is singular at $x = x_m$. Nevertheless, we solve it analytically without any approximation by making use of the method recently applied for a stochastic drift-diffusion equation [4], and obtain

$$\delta V_m = \frac{1}{\Delta} \int_0^x u(x) \delta n_{x}^{\text{inj}} dx, \quad (14)$$

$$u(x) = \frac{1}{n(x)} + E(x) \times \left\{ \begin{array}{cl} \int_0^x \frac{J_{y}(y) + n(y)}{n^2(y)} dy - \frac{1}{n_{L} E_{L}}, & 0 < x < x_m, \\ \int_x^\lambda \frac{J_{y}(y) - n(y)}{n^2(y)} dy - \frac{1}{n_{R} E_{R}}, & x_m < x < \lambda, \end{array} \right. \quad (15)$$
where \( \Delta \equiv (\lambda/2) + E_L^{-1} - E_R^{-1} \), \( \nu(x) \equiv 1/\sqrt{4\pi n(x)} \), and \( n(x) \), \( E(x) \) are the steady-state spatial profiles of the electron density and electric field, which take the values at the left and right contacts \( n_L, E_L \) and \( n_R, E_R \), respectively. The obtained analytical expressions yield the fluctuation of the barrier height in terms of the spatially distributed "noise source" \( \delta \nu_{x, \lambda} \) which, in turn, is given by the fluctuations \( \delta I_L, \delta I_R \) at the contacts. The function \( u(x) \) shows the relative contributions of the "noise sources" to the potential barrier fluctuations. Substituting the obtained formula for \( \delta V_m \) into Eq. (11), we obtain the current fluctuation as

\[
\delta I = \int_0^\infty \gamma_L(\epsilon) \delta I_L(\epsilon) \, d\epsilon + \int_0^\infty \gamma_R(\epsilon) \delta I_R(\epsilon) \, d\epsilon,
\]

(16)

\[
\gamma_L(\epsilon) = \begin{cases} 
-2J \int_0^\lambda K(x, \epsilon) \, dx, & \epsilon < V_m, \\
1 - J \int_0^\lambda K(x, \epsilon) \, dx, & \epsilon > V_m,
\end{cases}
\]

(17)

\[
\gamma_R(\epsilon) = \begin{cases} 
-2 \epsilon \int_0^\lambda K(x, \epsilon - V) \, dx, & \epsilon < V_m + V, \\
1 - J \int_0^\lambda K(x, \epsilon - V) \, dx, & \epsilon > V_m + V,
\end{cases}
\]

(18)

where \( K(x, \epsilon) = u(x)/[2\sqrt{\pi} \Delta \sqrt{\epsilon + \psi(x)} \), and \( x_L, x_R \) are found from \( \epsilon = -\psi(x_L) = V - \psi(x_R) \). The functions \( \gamma_L(\epsilon) \) introduced here for each contact have a meaning of the current fluctuation transfer functions, since they represent the ratio of the transmitted current fluctuation to the injected current fluctuation for a particular injection energy \( \epsilon \). The terms proportional to the current \( J \) are originated from the potential minimum fluctuations, whereas the constant contributions \( \pm 1 \) represent the direct transmission of fluctuations to the opposite contact. Eq. (16) leads to the spectral density of current fluctuations

\[
S_I = 2qI_0 \int_0^\infty \left[ \gamma_L^2(\epsilon) + \gamma_R^2(\epsilon) \right] e^{-\epsilon} \, d\epsilon.
\]

(19)

This equation with \( \gamma_k(\epsilon) \) given by formulas (17) and (18) is the final result of our derivations. It allows us to obtain the current-noise spectral density, for the given length of the conductor \( \lambda \) and applied voltage \( V \), from the steady-state distributions of the potential \( \psi(x) \), electric field \( E(x) \), and electron density \( n(x) \) by direct integration. Thus, the current-noise level is directly related to the transport inhomogeneity in the system. Note that the obtained formulas are exact for biases ranging from thermal to shot noise limits under a space-charge-limited transport conditions.

A great advantage of the derived formulas is that one may estimate a relative contribution to the noise from different energy groups of carriers. Indeed, Fig. 1 shows the functions \( \gamma_k(\epsilon) \) for a fixed \( \lambda \) and various biases \( V \). In the low-voltage limit, \( \gamma_k \) tend to the step functions with a step at the barrier height. This means that only electrons able to pass over the barrier contribute to the equilibrium (thermal) noise. For this case, one can easily obtain the Nyquist noise formula \( S_{eq}^\text{eq} = 4qI_0 e^{-V_m} = 4k_B T G_0 \), where \( G_0 = dI/dU \mid_{U=0} \) is the zero-bias conductance. With increasing the bias \( V \), all electrons contribute to the noise: those transmitted over the barrier, and those reflected back to the contacts. The electrons for which \( \gamma_k(\epsilon) < 0 \) reduce the current fluctuations. The most efficient in such a compensation carriers are those with the energies in the vicinity of the potential barrier energy, where \( \gamma_k \rightarrow -\infty \). They provide an overcompensation of the injected from the contacts fluctuation. There also exist the specific energy \( \epsilon^\ast \), for which the compensation fluctuation is exactly equal to the injected fluctuation, giving no noise at all \( \gamma_L(\epsilon^\ast)=0 \).

The obtained current-noise spectral density \( S_I \), which accounts for the long-range Coulomb correlations, may be compared with the uncorrelated value through the so-called noise reduction factor \( \Gamma = S_I/[2qI \coth(V/2)] \). By this definition, both the thermal noise and shot noise limits are included. The results (both the noise and the steady-state spatial profiles) are in excellent agreement with the Monte Carlo simulations. Fig. 2 shows \( \Gamma \) vs applied voltage \( V \). At low values of \( \lambda, \Gamma \approx 1 \). As \( \lambda \) increases, the noise level becomes substantially reduced at \( k_B T \lesssim qU < qU_{cr} \), where \( U_{cr} \) is a critical voltage for which the potential minimum vanishes. At \( U > U_{cr} \) the full shot noise level is abruptly recovered. This sharp increase in the noise intensity when observed in an experiment would indicate on the disappearance of the potential barrier controlling the current.

The obtained exact solutions allows us to investigate in great detail the correlations between different groups of carriers. While the injected carriers are uncorrelated, those in the volume of the conductor become strongly correlated. Those correlations may be observed experimentally by making use of a combination of two already realized techniques: a hot-electron spectrometer and shot-noise measurements. The electron spectrometer, placed behind the receiving semitransparent contact, acts as an analyzer of electron distribution over the energy. In this way a spectroscopic information, that is the average partial currents \( I(\tilde{\epsilon}) \) and their fluctuations \( \delta I(\tilde{\epsilon}) \) may be measured for different energies \( \tilde{\epsilon} \) of electrons collected at the contact. The partial current of transmitted electrons at the receiving (right) contact is given by \( I(\tilde{\epsilon}) = I_e e^{-\tilde{\epsilon} - V_m} \). By integrating over the energies \( \tilde{\epsilon} \), the full resolution of the potential barrier is obtained as

\[
\delta I(\tilde{\epsilon}) = \delta I_L(\tilde{\epsilon} + V_m)\theta(\tilde{\epsilon}) - I_e e^{-V_m} \delta V_m \delta(\tilde{\epsilon}).
\]

(20)

Thus, the correlation function \( \langle \delta I(\tilde{\epsilon}) \delta I(\tilde{\epsilon}') \rangle \) for outgoing electrons may be expressed through that for injected uncorrelated electrons. A simple analysis shows that for \( \tilde{\epsilon}, \tilde{\epsilon}' > 0 \) the outgoing carriers remain uncorrelated since \( \langle \delta I_L(\tilde{\epsilon}) \delta I_L(\tilde{\epsilon}') \rangle \propto \delta(\tilde{\epsilon} - \tilde{\epsilon}') \) due to the imposed injection conditions leading to the full shot noise. In such a case, an interesting question arises: what is the reason for the noise reduction obtained for the total (integrated over the energies) current fluctuations? The answer is found looking at the electrons with energies close to the threshold energy \( \tilde{\epsilon}=0 \) ("tangent" electrons). All other electrons are anticorrelated with that group. This means that if there is a positive fluctuation of over-barrier electrons, there should be a negative one for the "tangent"
electrons and vice versa. This anticorrelation explains the overall noise reduction. The “tangent” electrons can be thought as over-correlated. The dispersion \(\langle \delta I^2(\varepsilon) \rangle\) has a sharp peak at \(\varepsilon=0\) and then decreases with energy at \(\varepsilon > 0\). This peak is divergent (\(\delta\)-shaped) in our collisionless theory. A small probability of scattering will lead to its broadening and finite magnitude. Therefore, by measuring the dispersion of the partial current fluctuations and/or their cross-correlations, one may observe a sharp peak and anticorrelation of electrons, thus making the Coulomb correlations effect visible.

In conclusion, by solving analytically the kinetic equation coupled self-consistently with a Poisson equation, we have derived a general formula for the current noise in a ballistic nondegenerate conductor which accounts for the Coulomb correlations. We propose an evident experiment to discover these correlations by monitoring the Coulomb interaction and noise in small-size ballistic devices, like ballistic transistors, point contacts, etc.

This work has been supported by the Dirección General de Enseñanza Superior, Spain and the NATO linkage grant HTECH.LG 974610.

[1] R. Landauer, Nature 392, 658 (1998).
[2] M. J. M. de Jong and C. W. J. Beenakker, in Mesoscopic Electron Transport, edited by L. P. Kowenhoven, G. Schön, and L. L. Sohn (Kluwer, Dordrecht, 1997), p. 225.
[3] D. O. North, RCA Rev. 4, 441 (1940); A. van der Ziel, Noise (Prentice-Hall, Englewood Cliffs, NJ, 1954).
[4] R. Landauer, Phys. Rev. B 47, 16 427 (1993); Physica B 227, 156 (1996).
[5] M. Büttiker, J. Math. Phys. 37, 4793 (1996).
[6] T. González, O. M. Bulashenko, J. Mateos, D. Pardo, and L. Reggiani, Phys. Rev. B 56, 6424 (1997).
[7] O. M. Bulashenko, J. Mateos, D. Pardo, T. González, L. Reggiani, and J. M. Rubí, Phys. Rev. B 57, 1366 (1998).
[8] Nondegeneracy of an electron gas in our theory doesn’t mean that we address only the case of moderately doped semiconductors. The theory is also applicable to quantum heterostructures with over-barrier transport, where current is determined by a tail in the distribution function (ballistic-injection, real-space-transfer devices, etc.).
[9] V. V. Mitin, V. A. Kochelap, and M. A. Stroscio, Quantum Heterostructures (Cambridge Univ., NY, 1999).
[10] M. Reznikov et al., Phys. Rev. Lett. 75, 3340 (1995).
[11] A. Kumar et al., Phys. Rev. Lett. 76, 2778 (1996).
[12] R. J. Schoelkopf et al., Phys. Rev. Lett. 78, 3370 (1997).
[13] Both the current and its fluctuation are conserved along the sample due to the conservation of an electron energy under the ballistic motion.
[14] O. M. Bulashenko, G. Gomila, J. M. Rubí, and V. A. Kochelap, Appl. Phys. Lett. 70, 3248 (1997).
[15] O. M. Bulashenko, J. M. Rubí, and V. A. Kochelap (unpublished).
[16] Since the potential \(\delta \eta\) is referenced to the fluctuating minimum, its values on the contacts are not zero, while in a stationary frame \(\delta \varphi_L=\delta \varphi_R=0\) due to a fixed-applied-voltage conditions.
[17] The singularity in \(\gamma_k(\varepsilon)\) at \(\varepsilon \to V_m\) is of the logarithmic type, so it gives the well defined net noise after the integration over \(\varepsilon\).
[18] J. R. Hayes, A. F. J. Levi, and W. Wiegmann Phys. Rev. Lett. 54, 1570 (1985).
[19] M. Heiblum, M. I. Nathan, D. C. Thomas, and C. M. Knoedler, Phys. Rev. Lett. 55, 2200 (1985).

FIG. 1. Current fluctuation transfer functions for each injecting contact \(\gamma_L(\varepsilon)\) (solid), \(\gamma_R(\varepsilon + V)\) (dots) vs injecting energy \(\varepsilon\) for \(\lambda=30\) and various biases \(V\). The argument of \(\gamma_R\) is shifted by \(V\), so that both \(\gamma_k\) are singular \((\gamma_k \to -\infty)\) at the same energy corresponding to the barrier height \(\varepsilon = V_m\).

FIG. 2. Current-noise reduction factor \(\Gamma\) vs bias \(U\) for different lengths of the sample \(\lambda=d/L_D\). For the case of \(\lambda=50\), different contributions to \(\Gamma\) are shown: from over-barrier electrons transmitted from the left (L) and right (R) contacts, and those reflected by the potential barrier (refl).