Attracting divisors on projective algebraic varieties

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Abstract

We obtain sufficient and necessary conditions (in terms of positive singular metrics on an associated line bundle) for a positive divisor \( D \) on a projective algebraic variety \( X \) to be attracting for a holomorphic map \( f : X \rightarrow X \).

1 Preliminaries

Attracting sets play an important part in the study of dynamical systems. In recent years much attention has been devoted to attracting periodic points for maps in several complex variables, ([FS], [Ga], [Le] and other) and, to somewhat lesser extent, to attracting hypersurfaces ([BD], [BDM], [St1], [St2]). It would be interesting to find unifying framework for dealing with attracting periodic points on Riemann surfaces and hypersurfaces in higher dimensional complex manifolds as well as to undertake study of algebraic attracting sets of codimension greater than 1. We hope that initial steps towards both goals can be made by considering attracting divisors for holomorphic maps on projective algebraic varieties, which are the subject of the present paper. We characterize an attracting divisor \( D \) for a holomorphic map \( f : X \rightarrow X \) by behavior of a suitable positive singular hermitian metric on the line bundle \( [D] \) and of an associated class of \( \omega \)-plurisubharmonic functions. In sections 1 and 2 we gather theoretical tools and in Section 3 we prove our main result:

**Theorem.** The following are equivalent:

(i) \( D \) is attracting for \( f \);

(ii) there is a positive singular metric with weight \( \psi \) on the line bundle \( [D] \) and
a neighborhood $V$ of $A$ such that $\exists \beta < 1 : f_\ast \psi \geq \beta \psi$ in $V$;
(iii) there is a positive singular metric with weight $\phi$ on the line bundle $[D]$ such that $\exists \beta < 1 : f_\ast (PSH(X, \omega)) \subset \beta PSH(X, \omega)$, where $\omega = dd^c \phi$.

Let $X$ be an $n$-dimensional complex algebraic variety (we will make more assumptions about $X$ later). $\mathcal{O}_X$ will denote the sheaf of germs of holomorphic functions on $X$ and $\mathcal{M}$ the sheaf of germs of meromorphic functions on $X$. For detailed definitions and more information we refer the reader to [GR], [GH], [Fu] or [Ha].

**Definition 1.** ([Fu], B.4.1, B.4.2, B.4.5) A (Cartier) divisor $D$ on $X$ is defined by data $(U_\iota, f_\iota)$, where the $U_\iota$ form an open covering of $X$ and the $f_\iota$ are non-zero meromorphic functions, such that $f_\iota/f_\kappa$ is a unit (i.e., a holomorphic, nowhere vanishing function) on $U_\iota \cap U_\kappa$. The rational functions $f_\iota$ are called local equations for $D$ (they are determined up to multiplication by units on $U_\iota$). The support of $D$ is the set of all points $x \in X$ such that a local equation for $D$ is not in $\mathcal{O}^*_x$, (i.e., it is not a unit). A divisor $D$ is effective if local equations $f_\iota$ are sections of $\mathcal{O}$ on $U_\iota$.

Writing the local equation of $D$ on $U_\iota$ as $f_\iota = a_\iota/b_\iota$, with $a_\iota, b_\iota$ holomorphic on $U_\iota$, we see that the support of $D$ in $U_\iota$ consists of components of the set $a_\iota/b_\iota = 0$. Hence (see [L], II.5.3), the support of $D \neq 0$ in $X$ is an analytic subset of $X$ of pure codimension one.

We will use interchangeably the languages of divisors, line bundles and invertible sheaves, because of the following one-to-one correspondences (cf. [Fu], Appendix B.4.4 and [Ha], Ch. II):

A divisor $D$ on $X$ determines a line bundle on $X$, denoted by $[D]$. The sheaf of sections of $[D]$ may be identified with the $\mathcal{O}_X$-subsheaf of $\mathcal{M}$ (i.e., a $\mathcal{O}_X$-submodule of $\mathcal{M}$) generated on the open cover $U_\iota$ by $1/f_\iota$. Equivalently, transition functions for $[D]$ with respect to the covering $U_\iota$ are $g_{\iota\kappa} = f_\iota/f_\kappa$. And a section $\sigma = \{s_\iota\}, s_\iota = g_{\iota\kappa}s_\kappa$ of a line bundle $L$ on $X$ determines a divisor $D = \{(U_\iota, s_\iota)\}$.

Let $X, Y$ be non-singular compact complex varieties, $f : X \to Y$ be a proper holomorphic map. Recall that the pushforward operator on $(p, q)$-currents on $X$, $f_\ast : \mathcal{D}^{(p,q)}(X) \to \mathcal{D}^{(p,q)}(Y)$ is defined by $f_\ast T, \phi = \langle T, f^\ast \phi \rangle$ for all $T \in \mathcal{D}^{(p,q)}(X)$ and all $\phi \in \mathcal{D}^{(p,q)}(Y)$. This operator commutes with the differential operators $d$ and $d^c$, hence $dd^c f_\ast = f_\ast d d^c$. If $T = dd^c u$, where $u$ is a plurisubharmonic function on $X$ and $f$ has finite fibers, then $f_\ast u(y) = \sum_{x \in f^{-1}(y)} u(x)$, $y \in Y$.

For $X, Y$ and $f$ as above it is possible to define (up to an isomorphism) the pushforward $f_\ast L$ of a line bundle $L$ over $X$ corresponding to a sheaf $\mathcal{F}$ as the line bundle corresponding to the sheaf $f_\ast \mathcal{F}$. The pushforward of an invertible sheaf is constructed as follows ([GR], theorem 2.3.4 + Appendix A): Let $y \in Y$ and
let \(x_1, \ldots, x_t\) be different points in the fiber \(f^{-1}(y)\), \(U'_1, \ldots, U'_t\) pairwise disjoint open neighborhoods of \(x_1, \ldots, x_t\) and \(V'\) an open neighborhood of \(y\). Then there exists \(V \subset V'\) an open neighborhood of \(x_1, \ldots, x_t\) and \(V'\) pairwise disjoint neighborhoods of the points \(x_j\), \(j = 1, \ldots, t\) (in particular, \(f(U_j) \subset V\), \(j = 1, \ldots, t\)). For a sheaf \(F\) on \(X\) these give canonical bijections 

\[
(f_{U_j,V})_\ast (F(U_j)) = F(U_j) \exp(-\psi_f) = \bigotimes_{j=1}^t F(U_j),
\]

where \(f_{U_j,V} = f|_{U_j\cap V} : U_j \to V\). These bijections allow us to define \(f_\ast (F(V)) = \bigotimes_{j=1}^t F(U_j)\).

For pushforward of divisors and associated line bundles, the following proposition holds:

**Proposition 1.** Let \(X, Y\) be non-singular compact complex varieties, \(f : X \to Y\) be a proper holomorphic surjection and let \(D\) be a divisor in \(X\). Then:

(a) The current \(f_\ast D\) is a divisor in \(Y\);

(b) \(f_\ast [D] = [f_\ast D]\), where \([D]\) is the line bundle associated with the divisor \(D\).

**Proof.** Part (a) is Lemma 3.1 from [Ji]. From its proof it can be deduced (cf. also [Fu], section 1.4) that if \(g_{i\kappa}\) are transition functions in \([D]\), then \(f_\ast (g_{i\kappa})\) are transition functions in \([f_\ast D]\), which implies part (b). \(\square\)

### 2 Singular hermitian metrics on line bundles and \(\omega\)-plurisubharmonic functions

In the treatment of singular hermitian metrics on line bundles we will follow the approach according to Demailly (cf. [De2], [De3]).

**Definition 2.** ([De3], Definition 3.12) Let \(L\) be a complex line bundle over a complex manifold \(X\). A singular (hermitian) metric on \(L\) is a metric which is given in any trivialization \(\theta : L|_U \to U \times \mathbb{C}\) by

\[
\|\xi\| = |\theta(\xi)| \exp(-\psi(x)), \quad x \in U, \xi \in L_x,
\]

where \(\psi \in L^1_{\text{loc}}(U)\) is a function called the weight of the metric with respect to the trivialization \(\theta\).

A singular metric can be thus given by a collection of functions \(\psi = \{\psi_i\}\), \(\psi_i \in L^1_{\text{loc}}(U_i)\) satisfying \(\psi_i = \psi_\kappa + \log |g_{i\kappa}|\) in \(U_{i\kappa}\), where \(U_i\) is a trivializing cover for \(L\) and \(g_{i\kappa}\) are the transition functions. The metric is called positive if the functions \(\psi_i\) are plurisubharmonic.

The following proposition is simple but useful:
Proposition 2. If the collection \( \{ \psi_i \} \) defines a positive singular metric in a line bundle \( L \) on \( X \) and \( f : X \to Y \) is a proper holomorphic surjective map, then \( \{ f_*(\psi_i) \} \) defines a positive singular metric in the line bundle \( f_*L \) on \( Y \).

Proof. If \( f \) is holomorphic and proper, the functions \( f_*(\psi_i) \) are plurisubharmonic if \( \psi_i \) are (see e.g. [De 3], proposition 1.13). Further, \( \psi_i = \psi_i + \log |g_{i,\kappa}| \) gives \( f_*(\psi_i)(z) = f_*(\psi_i)(z) + \log \prod_{x \in f^{-1}(z)}|g_{i,\kappa}(x)| \) in \( U_{\kappa} \). The latter term is the logarithm of the modulus of a transition function in the bundle \( f_*L \) (cf. Proposition [1]), so the compatibility conditions also hold. \( \square \)

Let now \( D \) be an effective divisor on \( X \) and \( [D] \) its associated line bundle. One can give a positive singular metric on \([D]\) as in Example (3.14) in [De3]. Assume that \( \sigma_0, \sigma_1, \ldots, \sigma_N \) are nonzero holomorphic sections of \([D]\). They define a singular hermitian metric on \([D]\) by

\[
\|\xi\|^2 = \frac{|\theta(\xi)|^2}{|\theta(\sigma_1(x))|^2 + \ldots + |\theta(\sigma_N(x))|^2}
\]

with respect to a trivialization \( \theta \). The weight function for this metric is \( \psi(x) = \log(\sum_{j=1}^N|\theta(\sigma_j(x))|^2)^{1/2} \), which is a plurisubharmonic function.

This metric can be viewed as introduced by a metric on \( H^0(X, [D]) \) as follows: Let \( \sigma_0, \sigma_1, \ldots, \sigma_N \) be a basis for the linear system \([D]\) of all divisors linearly equivalent to \( D \) and \( B_{[D]} = \bigcap \sigma_j^{-1}(0) \) its base locus. There is a meromorphic map \( \Phi_{[D]} : X \setminus B_{[D]} \to \mathbb{P}^N, \quad \Phi_{[D]}(x) = (\sigma_0(x) : \ldots : \sigma_N(x)) \). Then the positive closed \((1,1)\)-current \( \frac{1}{2\pi} \Theta(F) = dd^c \psi \) is equal to the pullback over \( X \setminus B_{[D]} \) of the Fubini-Study metric \( \omega_{FS} = \frac{1}{2\pi} dd^c \log(|z_0|^2 + \ldots + |z_N|^2) \) of \( \mathbb{P}^N \) by \( \Phi_{[D]} \).

In what follows we will assume that \( D \) is very ample, i.e., that the map \( \Phi_{[D]} : X \to \mathbb{P}^N \) associated with the linear system \([D] = \mathbb{P}(H^0(X, [D])) \) is a regular embedding. Then in particular \( B_{[D]} = \emptyset \). According to Theorem II.7.1 in [Ha], \( \Phi^*(\mathcal{O}(1)) \) is an invertible sheaf on \( X \), which is generated by the global sections \( \sigma_0, \ldots, \sigma_N, \sigma_j = \Phi^*(\tau_j), \quad j = 0, \ldots, N \). Moreover, as \( \Phi \) is an embedding, each open set \( X_j := X \setminus \sigma_j^{-1}(0), \quad j = 0, \ldots, N \) is affine ([Ha], Proposition II.7.2).

We will also assume that \( X \) is normal, i.e., the linear system \([D]\) on \( X \) giving the embedding \( \Phi : X \to \mathbb{P}^N \) is complete. This means (cf. [GH], p. 177) the restriction map \( H^0(\mathbb{P}^N, \mathcal{O}(H)) \to H^0(X, \mathcal{O}(H)) \) is surjective, where \( H \) denotes the hyperplane bundle.

It will be convenient to consider another positive singular metric on the line bundle \([D]\) associated with a divisor \( D \), which can be introduced using an isomorphism between \( H^0(X, [D]) \) and \( \{ u \in M(X)^* : \text{div } u \geq -D \} \). Let \( \sigma_0, \sigma_1, \ldots, \sigma_N \) be a basis for \( H^0(X, [D]) \). We can assume that \( D = \text{div } \sigma_0 \), so \( \text{supp } D = \{ x \in X : \sigma_0(x) = 0 \} \). Then, for \( u = \tau/\sigma_0 \), one just takes \( \|u\| = |u| = |\tau| \exp(-\log |\sigma_0|) \) ([De 3], Example 3.13). Hence the weight for this metric coincides with the logarithm of the local equation of \( D \) in the trivialization \( \theta \) for \([D]\). The following relation holds:
Proposition 3. The metrics $\| \cdot \|_{\sim}$ and $\| \cdot \|$ are equivalent, i.e., $\exists c > 0 : \forall \xi : c^{-1} \| \xi \| \leq \| \xi \|_{\sim} \leq c \| \xi \|$.

Proof. One uses Rudin-Sadullaev estimates on $X$ in exactly the same way as in Lemma 2 in [St2].

The metric $\| \cdot \|$ is particularly useful in measuring the distance from a point in $X$ to the support of $D$. Consider the subspace $Y = \text{span}(\sigma_1, \ldots, \sigma_N)$ of $H^0(X, [D])$. Then $\mathbb{C} \sigma_0 \times \{(0, \ldots, 0)\}$ is the linear complement of $\{0\} \times Y$ in $H^0(X, [D])$. Taking the norm $\| (\tau \sigma_0, z) \| = |\tau| + \| (0, z) \|$ in $\mathbb{C} \sigma_0 \times Y$, where $\| \cdot \|$ is the metric in $H^0(X, [D])$ introduced above, one can construct the following neighborhood base for the set $Y_\infty = \mathbb{P}(0 \times Y)$, (which equals the support of $D$):

$$\Omega_K = \{ \lambda \in \mathbb{P}(\mathbb{C} \sigma_0 \times Y) : \lambda \subset \{(\tau, y) : |\tau| < (1/K) \| y \| \}, K > 0 \},$$

which is the same as $\{ z \in \mathbb{C}^N : |z| \geq K \} \cup Y_\infty$ (II, VII.3.4).

Let $H$ be a hyperplane in the linear space $Y$ such that $0 \notin H$. There is a unique linear form $\lambda_H \in Y^*$ such that $H = \{ z : \lambda_H = 1 \}$. Let $H_\ast = \ker \lambda_H$. On the space $\mathbb{P} \mathbb{C} \times Y$ we have a chart $\beta_H : \mathbb{P}(\mathbb{C} \times Y) \setminus \mathbb{P}(\mathbb{C} \times H_\ast) \ni z \mapsto z/\lambda_H(z) \in \mathbb{C} \times H$. The domains of the charts $\beta_H$ cover the set $Y_\infty = \mathbb{P}(0 \times Y)$. Having chosen a norm in $Y$ and the norm $\| (t, z) \| = |t| + \| z \|$ in $\mathbb{C} \times H_\ast$, we can take $\rho_{\beta_H}(z, Y_\infty \setminus \mathbb{P}(\mathbb{C} \times H_\ast) = |\lambda_H(z)|^{-1}, z \in Y \setminus H_\ast$, as the distance between $Y_\infty$ and $z \in Y$. In the set $H_i = X_i \setminus (X_i \cap \{ \sigma_0 = 0 \})$, $\lambda_{H_i} = (\sigma_i/\sigma_0)$ is the reciprocal of the local equation of $D$, $i = 1, \ldots, N$.

The neighborhoods $\Omega_K$ have an alternative interpretation:

Proposition 4. $\Omega_K = \{ (x, \xi) : |\beta_{H_i}(\xi)| \exp(-\psi(x)) < 1/K \}$ is a strongly pseudoconvex $1/K$-tube around $A$.

Proof. Note that the existence of a strongly pseudoconvex neighborhood of the zero section is equivalent to the negativity of a holomorphic line bundle, see [EG], Proposition VI.6.1 and VI.6.2. We will use $\mathcal{O}(-1)$, the tautological line bundle over the projective space $\mathbb{P}(\mathbb{C} \times \mathbb{C}^N) = \mathbb{P}(H^0(X, [D]))$. A point $\lambda \in \mathbb{P}(\mathbb{C} \times \mathbb{C}^N)$ can be identified with the fiber of $\mathcal{O}(-1)$. Under this identification, the set $Y_\infty = A$ corresponds to the zero section of $\mathcal{O}(-1)$ and $\Omega_K$ is a strongly pseudoconvex $1/K$-tube around $A$, for all $K > 0$.

We will also need a notion of $\omega$-plurisubharmonic functions on a compact connected Kähler manifold $X$ with respect to a closed real current $\omega$ of bidegree $(1,1)$ on $X$. (On complex projective spaces such functions were first considered in [BT1]). Under the assumptions we made, $X$ is a projective algebraic manifold. The class of $\omega$-plurisubharmonic functions is defined as follows (see [GZ]):

$$PSH(\omega, X) = \{ \phi \in L^1(X, \mathbb{R} \cup \{-\infty\}) : dd^c \phi \geq -\omega, \phi \text{ is upper semicontinuous} \}$$

Let $L$ be a holomorphic line bundle on $X$ and $h = \{ h_z \}$ a smooth metric $h = \{ h_z \}$ in $L$. The curvature $\omega = dd^c h_z$ in $U_z$ defines a real closed current globally on
$X$, because of compatibility conditions for $h$. It was observed in [GZ] (at the beginning of §5) that the class $PSH(X,\omega)$ is in 1-to-1 correspondence with the set of positive singular metric in $L$ on $X$ (that is, with the set of their weights). Namely, for a metric $\psi$ the function $\phi = \psi - h$ satisfies $dd^c\phi \geq -\omega$, and conversely, for a $\phi \in PSH(X,\omega)$, the collection $\psi = \{\psi_i = \phi + h_i\}$ defines a positive singular metric in $L$ on $X$.

3 Attracting divisors

We will use the following definition of an attracting set, which is of topological character:

**Definition 3.** (see [SY] or [SE] and the references there) Let $(X,d)$ be a metric space and let $f : X \to X$ be a continuous map. A closed set $A \subset X$ is attracting for $f$ if and only if $f(A) = A$ and there exists a neighborhood $U$ of $A$ of the form $U = \{x \in X : d(x,A) \leq \varepsilon\}$, such that $f(U) \subset \text{int } U$ and $\bigcap_{n \geq 0} f^n(U) = A$.

Recall our working setting: $X$ is a normal projective algebraic manifold, $D$ is a very ample divisor on $X$ given by a holomorphic section $\{\sigma_0 = 0\}$ (by Kodaira Embedding Theorem, $D$ is positive), $A$ is the support of $D$, $f : X \to X$ is a proper holomorphic map with finite fibers, satisfying $f(A) = A$. Then a neighborhood base for $A$ is given by $\{x : \psi(x) \leq \varepsilon\}$, where $\psi$ is the weight of a positive singular metric on the line bundle $[D]$. We will say that $D$ is attracting for $f$ if its support $A$ is an attracting set for $f$ in the sense of the above definition.

**Theorem.** The following are equivalent:
1. $D$ is attracting for $f$;
2. there is a positive singular metric with weight $\psi$ on the line bundle $[D]$ and a neighborhood $V$ of $A$ such that $\exists \beta < 1 : f_*\psi \geq \beta \psi$ in $V$;
3. there is a positive singular metric with weight $\phi$ on the line bundle $[D]$ such that $\exists \beta < 1 : f_*\psi \leq \beta \psi$ in $X$.

**Proof.** (i) $\Rightarrow$ (ii): Observe that $-\psi(x) = -\log |\sigma_0(x)| = -\log \text{dist}(x,A)$ is a plurisubharmonic exhaustion on $X \setminus A$. The equivalence is the same as Lemma 1 in [SY].

(ii) $\Rightarrow$ (iii): Let $\psi$ be a metric satisfying the condition (ii) and let $\omega = dd^c\psi$. As in Example 1.2 in [GZ], one can establish the following one-to-one correspondence between $PSH(X,\omega)$ and the class $\{u \in PSH(X \setminus A) : u - \psi \leq c_u\}$: if $u \in PSH(X \setminus A)$ satisfies $u \leq \psi + c_u$ with some constant $c_u$ depending only on $u$, then $\nu(u) = \begin{cases} u(x) - \psi(x), x \notin A \\ \limsup_{(x,A) \ni y \to x} (u(y) - \psi(y)) \end{cases}$ is in $PSH(X,\omega)$. A similar correspondence can be established for $PSH(X,\omega + f_*\omega)$, involving the weight $f_*\psi$. Let $v \in PSH(X,\omega)$. By (ii), $f_*\psi - \psi$ has a well-defined $\limsup_{(x,A) \ni y \to x} (f_*\psi(y) - \psi(y)) \leq \limsup_{(x,A) \ni y \to x} (f_*\psi(y) - f_*\psi(y))$.
for \( x \in A \), hence \( f_\ast v \in PSH(X, \omega) \).

(iii) \( \Rightarrow \) (ii): By (iii), \( f_\ast (-\phi) \in PSH(X, \beta \omega) \), so \( f_\ast (-\phi) + \beta \phi \) defines a metric in \([D]\). On the set \( \text{supp } D \) this metric is equal to 0, hence \( f_\ast \phi - \beta \phi \to \infty \) as \( x \to A \), which implies that close enough to \( A \), \( f_\ast \phi - \beta \phi > 0 \). Thus \( \psi = \phi \) is a metric as in condition (ii).

\[ \square \]

Remarks. (1) Even though trivializations were used to formulate the definition of a singular metrics, the Theorem is independent of trivializations defining the line bundle \( L = [D] \). Indeed, two sets of trivializations \( \theta_i, \theta'_i \) defining \( L \) in the open cover \( U_i \), are related by \( \theta'_i = \eta_i \cdot \theta_i \), with \( \eta_i \in \mathcal{O}^*(U_i) \), so the form \( \omega \) in the Theorem will be replaced by a cohomologous form \( \omega' = \omega + dd^c \chi \), where \( \chi \) is a function on \( X \) which is integrable with respect to any smooth volume form. Accordingly, the metric \( \psi \) will be replaced by \( \psi - \chi \) and the class \( PSH(X, \omega) \) by \( PSH(X, \omega') = PSH(X, \omega) + \chi \) (the last equality is Proposition 1.3.3 in [GZ]).

(2) It was observed in [DS], section 6, that the classes of \( \omega \)-plurisubharmonic functions are in general not stable under the operator \( f_\ast \). Condition (iii) of our Theorem gives an example of situation in which stability occurs, since \( \beta PSH(X, \omega) \subset PSH(X, \omega) \) for \( 0 < \beta < 1 \).

(3) If \( A \) is a regular hypersurface in \( X \), then the first adjunction formula (see e.g. [FG], IV.5.10, [GH], p. 145) says that on \( A \) the line bundle \([A]\) coincides with the normal bundle \( N_X(A) \) of \( A \) in \( X \). The neighborhoods in the definition of the attracting set and in the Theorem can then be chosen as tubular neighborhoods of \( A \) (cf. [4]).

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