Novel and accurate solitary wave solutions of the conformable fractional nonlinear Schrödinger equation

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Abstract
In this paper, the Khater II analytical technique is used to examine novel soliton structures for the fractional nonlinear third-order Schrödinger (3-FNLS) problem. The 3-FNLS equation explains the dynamical behavior of a system’s quantum aspects and ultra-short optical fiber pulses. Additionally, it determines the wave function of a quantum mechanical system in which atomic particles behave similarly to waves. For example, electrons, like light waves, exhibit diffraction patterns when passing through a double slit. As a result, it was fair to suppose that a wave equation could adequately describe atomic particle behavior. The correctness of the solutions is determined by comparing the analytical answers obtained with the numerical solutions and determining the absolute error. The trigonometric Quintic B-spline numerical (TQBS) technique is used based on the computed required criteria. Analytical and numerical solutions are represented in a variety of graphs. The strength and efficacy of the approaches used are evaluated.

Keywords
Quantum mechanical system, Computational and numerical solutions, Soliton waves, Ultra-short optical fiber

AMS classification: 35J10; 35D30; 35R10; 65N15; 35Q51

Introduction
Numerous academics in diverse fields, including engineering and applied sciences, have recently concentrated their efforts on the physical interpretation of optical fibers.¹ Organic artificial materials used in the manufacturing of optical fibers include the well-known silica (drawing glass) and polymers with a thickness less than a human hair.² This procedure transforms the fiber into a translucent one with increased flexibility.³ This kind of fiber is often used to transfer light down a long rope of optical fiber, which is a critical step in fiber-optic communications.³ This fiber transfers data over a greater distance and at a higher bandwidth than electrical wires.³ Transmission procedures occur without a single signal being lost or with far less signal loss than metal lines do because fibers are resistant to electromagnetic interference.⁶ Illumination and imaging are regarded to be the second critical function of optical fiber.⁷ Where this occurs, the material is often wrapped in bundles to bring light into, or pictures out in small areas.⁸ Fiber scopes, fiber optic sensors, and fiber lasers are all examples of this approach.⁹

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Applications

Here, the analytical and approximate solutions of the conformable fractional NLS equation are investigated by employing the suggested schemes. This study’s goal is to construct novel solutions’ structures and check their accuracy by calculating the absolute value of error between analytical and approximate solutions (Figures 1–10).
Figure 1. Graphical representations of equation (8) for the real, imaginary, and absolute values of the $Q_{11}(x,t)$.

Figure 2. Graphical representations of equation (9) for the real, imaginary, and absolute values of the $Q_{12}(x,t)$. 
Figure 3. Graphical representations of equation (10) for the real, imaginary, and absolute values of the $Q_{II,1}(x, t)$

Figure 4. Graphical representations of equation (11) for the real, imaginary, and absolute values of the $Q_{II,2}(x, t)$
Figure 5. Graphical representations of equation (12) for the real, imaginary, and absolute values of the $Q_{III,1}(x,t)$

Figure 6. Graphical representations of equation (13) for the real, imaginary, and absolute values of the $Q_{III,2}(x,t)$
Applying the Khater II method to the considered model for investigating the soliton wave solutions, gets the following values of the above-shown parameters: Set I

\[
a_0 \rightarrow 0, a_1 \rightarrow \frac{\sqrt{6}q_1}{\sqrt{-l_1 - 2l_3}}, b_1 \rightarrow 0, p_1 \rightarrow i\frac{\sqrt{q_2 - 2\delta q_1}}{\sqrt{3}q_1}.
\]  

(5)
set II

\[ a_0 \rightarrow 0, \quad a_1 \rightarrow \frac{\sqrt{ab_1}}{\sqrt{\delta}}, \quad l_i \rightarrow \frac{-4ab_1^2l_3 - 3\delta q_1^2}{2ab_1^2}, \quad p_1 \rightarrow \frac{\sqrt{\delta q_1^2 - 2q_2^2}}{\sqrt{6\sqrt{q_1}}}. \]  

(6)

set III

\[ a_0 \rightarrow 0, \quad a_1 \rightarrow 0, \quad l_i \rightarrow -\frac{2(ab_1^2l_3 + 3\delta q_1^2)}{ab_1^2}, \quad p_1 \rightarrow \frac{\sqrt{-\delta q_1^2 - q_2^2}}{\sqrt{3\sqrt{q_1}}}. \]  

(7)

thus, the traveling wave solutions of the conformable fractional nonlinear NLS equation are structured as follows

For \( \delta \neq 0 \), we obtain

\[ Q_{i,1}(x, t) = \frac{-\sqrt{6\sqrt{\delta q_1}e^{(p_1^2 + p_1^2 + p_1)l_3}}}{\sqrt{-l_1 - 2l_3}} \tan \left( \sqrt{\delta} \left( q_1x - \frac{q_2^2}{\gamma} \right) \right), \]  

(8)
\[ \mathcal{Q}_{I,1}(x,t) = \frac{\sqrt{6} \sqrt{\delta q_1 e^{ip_2 t + p_3 x + p_4}}}{\sqrt{-l_1 - 2l_2}} \cot \left( \sqrt{\delta} \left( q_1 x - \frac{q_2 t}{\gamma} \right) \right), \]

(9)

\[ \mathcal{Q}_{II,1}(x,t) = b_1 e^{ip_2 t + p_3 x + p_4} \left( \sec \left( \sqrt{\delta} \left( q_1 x - \frac{q_2 t}{\gamma} \right) \right) - \sqrt{\alpha} \tan \left( \sqrt{\delta} \left( q_1 x - \frac{q_2 t}{\gamma} \right) \right) \right), \]

(10)

\[ \mathcal{Q}_{II,2}(x,t) = b_1 e^{ip_2 t + p_3 x + p_4} \left( \sqrt{\alpha} \cot \left( \sqrt{\delta} \left( q_1 x - \frac{q_2 t}{\gamma} \right) \right) + \csc \left( \sqrt{\delta} \left( q_1 x - \frac{q_2 t}{\gamma} \right) \right) \right), \]

(11)

\[ \mathcal{Q}_{III,1}(x,t) = b_1 e^{ip_2 t + p_3 x + p_4} \sec \left( \sqrt{\delta} \left( q_1 x - \frac{q_2 t}{\gamma} \right) \right), \]

(12)

\[ \mathcal{Q}_{III,2}(x,t) = b_1 e^{ip_2 t + p_3 x + p_4} \csc \left( \sqrt{\delta} \left( q_1 x - \frac{q_2 t}{\gamma} \right) \right). \]

(13)
Approximate solutions

Investigating the above-obtained solutions’ accuracy by calculating the absolute error between analytical and numerical solutions. Checking the following solutions equations (8), (10), (12) when \( \delta = -1, l_1 = -7, l_3 = 2, p_1 = 2, p_3 = 4, p_2 = 6, q_1 = -1, q_2 = 3 & b_1 = 0.5, \delta = -4, q_1 = 0.1, q_2 = -0.2 & b_1 = 2, \delta = -9, q_1 = 6, q_2 = 4 \) to evaluate the initial and boundary conditions of the studied model, gets

\[
Q(x,t) = \begin{cases} 
\sqrt{2}\tanh(3t + x), \\
n(\tanh(2(0.2t + 0.1x)) + \sech(2(0.2t + 0.1x))), \\
3\sech(4t - 2x)
\end{cases}
\]  

(14)

using these conditions in the suggested numerical scheme’s framework demonstrates the matching between analytical and approximate solutions through the following Tables 1–3.

| Value of \( x \) | Computational | Numerical | Error |
|------------------|---------------|-----------|-------|
| 0                | 0.5           | 0.5       | 0     |
| 0.03125          | 0.503115193844475 | 0.501937305889842 | 0.00117788795463303 |
| 0.0625           | 0.50621661454248 | 0.503257774697122 | 0.00295283984535843 |
| 0.09375          | 0.509286023769424 | 0.504758545778733 | 0.00524747799969053 |
| 0.125            | 0.512341187163975 | 0.506233282790194 | 0.00610790437383153 |
| 0.15625          | 0.515375874399654 | 0.507744877966136 | 0.00763099643351772 |
| 0.1875           | 0.518389859254452 | 0.509284219977527 | 0.00910563927692487 |
| 0.21875          | 0.521382919678445 | 0.510861728140095 | 0.0105211915383552 |
| 0.25             | 0.524354837859385 | 0.512482845660283 | 0.0118719281991012 |
| 0.28125          | 0.527305400286126 | 0.514154650904803 | 0.0131507493812224 |
| 0.3125           | 0.530334397810024 | 0.515884127303917 | 0.0143502705051067 |
| 0.34375          | 0.533141625704094 | 0.517687673029703 | 0.0154629526743865 |
| 0.375            | 0.536026883719795 | 0.519546013571158 | 0.0164808701481863 |
| 0.40625          | 0.538889976142713 | 0.521494278349788 | 0.0173956797922497 |
| 0.4375           | 0.5417307118432    | 0.523532037888795 | 0.0181986740540151 |
| 0.46875          | 0.544548904327377 | 0.525668356009301 | 0.0188805483190754 |
| 0.5              | 0.547344371789091 | 0.527912845019664 | 0.0194315267694269 |
| 0.53125          | 0.550116937456142 | 0.530275724958807 | 0.01984121286053 |
| 0.5625           | 0.552866428090777 | 0.532767889999535 | 0.0200985380932416 |
| 0.59375          | 0.555592667713748 | 0.53540981167391 | 0.0201919658526351 |
| 0.625            | 0.558295521623361 | 0.538187466332522 | 0.0201080552998389 |
| 0.65625          | 0.560974803814055 | 0.541140730888795 | 0.0198340730331669 |
| 0.6875           | 0.56330370869359 | 0.544275170785648 | 0.019355200837102 |
| 0.71875          | 0.566262074907929 | 0.547606325171496 | 0.0186557497364332 |
| 0.75             | 0.568869773029139 | 0.551150920528514 | 0.0177188525006257 |
| 0.78125          | 0.571453327339189 | 0.554927303141453 | 0.016526024197736 |
| 0.8125           | 0.574012604974758 | 0.558954502662785 | 0.0150581021311973 |
| 0.84375          | 0.576547487124174 | 0.563256483612032 | 0.0133905945121422 |
| 0.875            | 0.579057824046117 | 0.567847150349095 | 0.0112106736070223 |
| 0.90625          | 0.581543525085849 | 0.572787237420961 | 0.0087562876648867 |
| 0.9375           | 0.584004468688968 | 0.57797327044526 | 0.0060311914444221 |
| 0.96875          | 0.586440547412716 | 0.583526785320522 | 0.00251476209219426 |
| 1               | 0.588851658934815 | 0.588851658934815 | 0.0 |
Results and discussion

This part explores the paper’s findings and innovation by exhibiting the benefits of the analytical and numerical techniques utilized, the acquired results, and their comparison to previously published solutions, ultimately proving the correctness of the found solutions. Finally, we will analyze the physical meaning of the numbers presented. Mostafa M. A. Khater discovered the Khater II technique for the first time. He created this extended technique in order to get innovative structures for explicit wave solutions to a class of nonlinear evolution equations. The efficacy and potency of this technique have been established. The TQBS scheme has been employed to find the numerical solutions of the investigated model based on the obtained solutions (8), (10), (12). The analytical solutions have been explained through some different graphs (1, 2, 3, 4, 5, 6). While the matching between both solutions is explained through some distinct plots (7, 8, 9). Comparing our solutions to show the solutions’ accuracy leads to verify our obtained solutions and superiority of equation (8) over other constructed solutions.

Conclusion

The 3-FNLS model was effectively studied in this research work using the Khater II approach. Numerous separate precise solutions for moving and isolated waves have been discovered. These solutions have been illustrated using a variety of drawings that demonstrate additional and unexpected aspects of the fractional models under consideration. Our acquired answers have been discussed in terms of their correctness and uniqueness. Additionally, the potency and usefulness of the approaches utilized are discussed and validated.

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Authors’ contribution

Dexu Zhao and Mostafa Khater have revised the conceptualization, data curation, and methodology. Dianchen Lu and Mostafa Khater have revised Data curation, Investigation, and Software. Samir Salama and Piayphong Yongphe have revised the physical meaning of the obtained solutions and raised the given graphs resolutions. All authors have read and agreed to the published version of the manuscript.

Declaration of conflicting interests

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Availability of data and material

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Code availability

The used code of this study is available from the corresponding author upon reasonable request.

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