Nonsingular Cosmology from an Unstable Higgs Field

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The observed value of the Higgs mass indicates an instability of the Higgs scalar at large energy scales, and hence also at large field values. In the context of early universe cosmology, this is often considered to lead to problems. Here we point out that we can use the instability of the Higgs field to generate an Ekpyrotic phase of contraction. In the context of string theory it is possible that at very high energy densities extra states become massless, leading to an S-brane which leads to the transition between a contracting phase in the past and the current expanding phase. Thus, the Higgs field can be used to generate a non-singular bouncing cosmology in which the anisotropy problem of usual bouncing scenarios is mitigated.

I. INTRODUCTION

Based on the observed value of the Higgs mass \[1\], the Higgs potential is unstable at large energy scales where the quartic self coupling constant and hence the potential become negative \[2\], with important consequences for cosmology (see e.g. \[3,4\]). Since large field values lead to large energy densities, the instability will arise at large field values which are relevant to early universe cosmology. It is usually assumed that quantum gravity effects will cause the potential to turn positive again at very high values (conservatively speaking, at value where the Higgs energy density approaches the Planck density). There will be a global minimum of the potential which corresponds to Anti-de-Sitter space \[1\].

This instability is not a problem for the Standard Model of particle physics at the present time, since the usual Higgs vacua remain local vacua, and their life time is much larger than the age of the Universe. However, when applied to early universe cosmology some problems may arise. In the radiation phase of Standard Big Bang cosmology, finite temperature corrections to the potential are able to stabilize the Higgs field and localize it in the usual vacua. The Standard Big Bang Model phase of cosmology, on the other hand, must be preceded by an early phase which solves the horizon and flatness problems and allows a causal generation mechanism of fluctuations. The current paradigm for this early phase is inflation, an early phase of accelerated expansion \[5\]. In simple models of inflation, the energy scale at which inflation takes place is of the order of \(10^{16}\) GeV. Unless the Higgs field is directly coupled to the inflaton, the field which generates inflation, the Higgs will perform a random walk during the period of inflation with a typical amplitude of \(H\), and time scale of \(H^{-1}\), where \(H\) is the Hubble expansion rate during inflation. This random walk will lead to the Higgs field having a finite probability of landing in its true Anti-de-Sitter (AdS) vacuum. This is a serious problem for our universe \[6\] (see also \[7\] for earlier discussions of the implications of the Higgs instability for inflation, and \[8\] for related work).

In this paper we point out that, in the context to alternatives to inflation \[2\] the instability of the Higgs potential may be a virtue rather than a problem \[3\]. Specifically, we consider a contracting universe described by General Relativity coupled to Standard Model matter. Early in the contracting phase, the equation of state will be that of cold matter, followed by a radiation-dominated phase. Eventually, the instability of the Higgs potential will set in. As we show, this generates a phase of Ekpyrotic contraction, a period which smooths out any pre-existing anisotropies. At Planck densities we postulate that a new set of fields becomes effectively massless, an effect which leads to an “s-brane” in the low energy effective action. This s-brane leads to a non-singular transition from contraction to expansion, as studied in detail in a different context in \[11\]. The Higgs field itself bounces off a potential barrier and returns back to small field values as the universe expands.

Thus, we show that the instability of the Higgs field allows us to construct a “natural” non-singular cosmology which is free from the usual BKL instability \[12\] which faces bouncing cosmological models \[13\]. Since the universe starts in a matter phase of contraction, our model yields a simple realization of the “matter bounce” alter-

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\textsuperscript{1} An easy way to avoid this instability problem is to assume that there is new physics at scales lower than the inferred Higgs instability scale which uplifts the potential. In this paper we shall assume that this is not the case.

2 See \[9\] for a review of various alternatives to cosmological inflation.

3 See also \[10\] where the instability of the Higgs was used to construct the contracting phase preceding a bounce.
II. MODEL

We will be considering the Higgs sector of the Standard Model of particle physics. The Lagrangian is

\[ \mathcal{L}_0 = X - V(h), \]

where \( h \) denotes the Higgs field, \( X \) is the standard kinetic term \((X = \frac{1}{2}(\partial_{\mu} h)^2)\) and \( V(h) \) is the Higgs potential energy. The bare potential energy is given by

\[ V(h) = \frac{1}{4} \lambda_0 (h^2 - v^2)^2, \]

where \( \lambda_0 \) is the tree level renormalized coupling constant and \( v \) is the vacuum expectation value of the field.

The bare potential is subject to quantum corrections. At one loop level, the corrections have been studied in detail in \[16\] (see also \[17\]). There are logarithmic corrections to the potential which enter with a positive sign for bosons and with a negative sign for fermions. For the observed value of the top quark the fermion contribution dominates and the one loop effective potential can be written as

\[ V^{(1)}(h) = \frac{1}{4} \lambda(h)(h^2 - v^2)^2, \]

Following \[11\], we assume that at high energy densities a new sector of states becomes effectively massless. This follows if we consider a superstring model which reaches an enhanced symmetry point at some critical density \[18, 19\]. In this case these states must be included in the low energy effective action as a term arising only at a particular density, or equivalently at a particular time \( t_c \) when this critical density is achieved. The action \( S \) hence contains a term of the form

\[ \delta S = \mu \delta(t - t_c). \]

Such a term is called an \( S \)-brane, and \( \mu \) is its tension which is set by the mass scale of the new physics.

An \( S \)-brane can be viewed as a relativistic topological defect which is space-like. It is well known that for such defects the pressure in direction of the defect is negative (i.e. positive tension), and the pressure perpendicular to the defect vanishes. For an \( S \)-brane, this implies that the induced energy density vanished and the pressure is negative. Hence, for the model we are considering the energy density \( \rho \) and pressure \( p \) are given by

\[ \rho = X + V \]
\[ p = X - V - \mu \delta(t - t_c). \]

with

\[ \lambda(h) = \lambda_0 - b \ln \left( \frac{h^2}{\Lambda^2} \right). \]

where \( b \) is a positive number set by the top quark Yukawa coupling constant, and \( \Lambda \) gives the scale at which the instability sets in, which according to the current best Higgs and top quark mass measurements is about 10^{10} \text{GeV} \[2\]. In the following we shall work with the one-loop effective potential but we will omit the superscript. In our numerical study we will work with a larger value of \( \Lambda \) in order to reduce the magnitude of the hierarchy in energy and time scales.

At field values which correspond to Planck-scale or string scale energy densities we assume that there will be corrections which uplift the potential. We consider (following \[14\]) the leading order non-renormalizable term

\[ \delta V = \frac{g h^6}{M^2}, \]

where the coupling constant \( g \) can be absorbed into the mass scale \( M \) and we can set \( g = 1 \). In Figure 1 we present a sketch of the one loop effective potential including the above term.

From the above it follows that an \( S \)-brane leads to the violation of the weak energy condition which allows for a transition between a contracting phase and an expanding phase.

The cosmological scenario which we have in mind is now the following. We begin in a contracting universe with the Higgs field close to today’s minimum \( h = v \). Initially, \( h \) is oscillating about \( v \) with an amplitude which is small compared to \( v \). The corresponding equation of state is that of (when averaged over time) cold matter, i.e. \( p = 0 \). The amplitude of field oscillations will grow as the universe contracts, and eventually it reaches of order \( v \). After that point, \( h \) is free also to explore negative field values. In fact, after some time of contraction the local Higgs potential barrier at \( h = 0 \) becomes negligible and \( h \) will oscillate in a potential which looks quartic. At this point the equation of state will change to that of (again averaged over time) radiation, i.e. \( p = \frac{4}{3} \rho \). The early dynamics of \( h \) in our model is shown in Fig. 2, as follows from numerically solving the equations of motion discussed in the following section.
FIG. 1: Sketch of the one loop effective potential of the Higgs field including the extra term coming from postulated quantum gravity effects. The instability of the Higgs sets in at field values above $\Lambda$. Note that the local minimum of $V(h)$ at $h = 0$ is not visible on this plot due to the large hierarchy of scales between the electroweak symmetry breaking scale and the Planck scale. We have used the following values of the constants appearing in the potential: $v = 246\text{GeV}$, $\lambda_0 = 0.129$, $b = 0.0187$, $\Lambda = 2.4 \times 10^{15}\text{GeV}$, $g = 1$ and $M = 2.4 \times 10^{18}\text{GeV}$. The first three of these parameters reflect the measured masses of the Higgs and the top quark, the final term reflects the assumption of a quantum gravity-induced wall in the potential at values of $h$ corresponding to the reduced Planck mass.

The amplitude of the (anharmonic) oscillations of $h$ will continue to grow until it reaches the local maximum of the potential at $h \sim \Lambda$. At that point the instability of $h$ sets in and $h$ will start rolling down the potential towards negative values of $V$. Once the potential becomes negative, the equation of state of the Higgs field becomes of Ekpyrotic type \cite{20}, i.e.

$$w \equiv \frac{p}{\rho} > 1.$$  \hspace{1cm} (9)

As was realized in \cite{21} in the context of the matter bounce scenario, this phase of Ekpyrotic contraction has the virtue of diluting anisotropies \cite{22}. Thus, our scenario in a completely natural way solves the main problem of a bouncing scenario, namely the anisotropy problem.

Once the energy density in the contracting phase approaches the string scale (or Planck scale) density, two effects take place. Firstly, the Higgs field hits the “potential wall” where $V(h)$ sharply rises due to the extra contribution $\delta V$ to the potential from \cite{5}. This causes $h$ to slow down and $V(h)$ to turn positive. Secondly, at the string density the S-brane \cite{6} is encountered. We will take this to happen at the time when $h$ comes to rest, i.e. $X = 0$ (we comment later on this assumption). At this point, a transition between contraction and expansion takes place. Because for a symmetric bounce the kinetic energy is negligible immediately before and after the bounce the equation of state parameter $w$ will approach $w = -1$ from both sides. At the bounce point itself, the S-brane leads to a value of $w$ which is formally $w = -\infty$ (this shows the consistency of our analysis with the general theorems of \cite{23} that the single fluid matter with a crossing of $w = -1$ is required to obtain a bounce \cite{24}).

After the bounce, the Higgs field $h$ will roll back down the potential (towards the origin) picking up kinetic energy, and then using this kinetic energy to roll back to $h \sim \Lambda$ and back into the local minimum at $h = v$. The dynamics of $h$ in the phase of Higgs instability and around the S-brane bounce is shown in Figs. 3 and 4. Once again, these plots are obtained by numerically solving the equations of motion discussed in the following section.

In the above we have focused on the Higgs sector of the Standard Model matter. We should keep in mind that the
FIG. 2: The evolution of the Higgs field early in the contracting phase. The initial conditions were taken to be \( h = v + \delta h \) with a small offset \( \delta h = 0.1v \) and \( \dot{h} = 0 \). Initially, \( h \) oscillates about \( h = v \), but because of the Hubble antidamping the amplitude of oscillations grows and eventually \( h \) will be able to cross over the local potential maximum at \( h = 0 \). Subsequently, \( h \) executes oscillations about \( h = 0 \). Before the transition the time-averaged equation of state is \( w = 0 \), afterwards \( w = 1/3 \). The figure shows the evolution of \( h \) (vertical axis) as a function of time (horizontal axis). The field values are in units of \( v \), the time values in units of \( v^{-1} \). In this numerical simulation, the \( h \) dependence of the quartic coupling constant and the extra \( h^6 \) terms are neglected since they are not important since they are not important.

In order to see both the increase in the amplitude and the oscillation period easily, we have set \( v = 10^{-3} \) in Planck units.

other particles of the Standard Model also contribute to the dynamics. If we have in mind starting the contracting phase in a state which looks like the time reverse of our present universe, the rest of the Standard Model matter will initially be in a cold matter-dominated state as well. The radiative degrees of freedom of the Standard Model matter will lead to a transition between the matter-dominated phase and the radiation phase earlier than if we only consider the Higgs field dynamics. This does not change our scenario. It simply causes the Higgs field to climb its potential towards the instability point \( h \sim \Lambda \) at a different rate. Once the Higgs field starts its descent towards negative values, the Ekpyrotic equation of state of the Higgs field will cause the Higgs to rapidly come to dominate matter, and from then on the analysis is exactly how it was described above.

In the following we will take a closer look at the dynamics of the Higgs system. Since in the Ekpyrotic phase spatial gradient terms also get washed out we will focus on the homogeneous dynamics.

III. EQUATIONS OF MOTION

As usual, it is convenient to write the dynamics in terms of a rescaled field

\[ u \equiv ah, \]

and in terms of conformal time \( \eta \) which is related to the physical time \( t \) via \(^4\)

\[ dt \equiv a(t) d\eta. \]

The Higgs field equation of motion then becomes

\[ u'' - (\mathcal{H}^2 + \mathcal{H}')u = -a^3 V_h, \]

where \( \mathcal{H} \) is the Hubble expansion rate in conformal time, namely \( \mathcal{H} = a'/a \), and a prime denotes the derivative with respect to \( \eta \).

During the radiation phase of contraction the second term on the left hand side of (12) vanishes and, in the limit \( |h| \gg v \) the equation reduces to

\[ u'' = -\lambda u^3. \]

For field values \( |h| \ll \Lambda \), i.e. far from the Higgs instability point, we can take \( \lambda \) to be a positive constant, and hence the solutions yield anharmonic oscillations.

Beyond the instability point the sign of \( \lambda \) changes. As mentioned before, the runaway of \( h \) to values with negative potential leads to an Ekpyrotic equation of state.

\(^4\) Note that in the contracting phase \( \eta \) is negative and approaches \( \eta = 0 \).
FIG. 3: The evolution of the Higgs field in the contracting phase after the onset of the instability. The initial conditions were taken to be $h$ just to the left of the local maximum of the potential with a small initial velocity sufficient to push $h$ over the barrier (specifically $h = 2 \times 10^{-2}$ and $\dot{h} = 2 \times 10^{-5}$ in reduced Planck units). The left panel shows the evolution of $h$ (vertical axis) as a function of time (horizontal axis). The field and time values in Planck units. To guide the eye we have drawn horizontal dashed lines at the value of $h$ corresponding to the onset of the Higgs instability. The numerical result shows that at late times the Higgs field becomes localized in the metastable Higgs region near $h = 0$. The right panel shows the equation of state parameter $w$ (solid curve) and the Hubble parameter (dashed curve) (each on the vertical axis) as a function of time. The field is seen to rapidly accelerate down the potential, reach negative values of the potential with a resulting Ekpyrotic equation of state $w \gg 1$. After crossing the minimum of the potential, $h$ climbs up the steep potential and comes to rest. At the point when $h$ comes to rest we have inserted an S-brane chosen to yield a symmetric bounce, i.e. $H$ simply changes sign, and the values of $h$ and $\dot{h}$ do not change. As can be seen, after the bounce $h$ falls back down to the minimum value of its potential, climbs back up towards $h = 0$, and ends up oscillating about $h = 0$. At the large field values considered here, the double well structure of the potential near $h = 0$ has a negligible effect. Hence, we have here taken $v = 0$. $w \gg 1$. This leads to slow contraction

$$a(t) \sim t^p,$$

with $p \ll 1$. In this case, the terms of (12) involving $\mathcal{H}$ and its derivatives are negligible. During a short time interval during which the change in the logarithm in (4) can be neglected the equation of motion takes the form

$$u'' = \lambda u^3,$$

where $\lambda > 0$, where has rapidly growing solutions of the form

$$u(\eta) = \frac{f}{\eta},$$

with

$$f^2 = \frac{2}{\lambda}.$$

The above rapid growth of $h$ leads to the potential energy which scales as

$$V \sim -\frac{2 f^2}{\eta^4} a^{-4}.$$
which show that the energy density in the Higgs field rapidly comes to dominate over all other forms of energy density (in particular that of regular radiation which scales as $a^{-4}$).

The system of equations is completed with the Friedmann-Robertson-Walker equations which take on a simpler form in terms of physical time. During the stages when the Higgs field dominates the energy-momentum tensor of matter these equations are

$$H^2 = \frac{8\pi G}{3}(X + V)$$  \hspace{1cm} (19)

where $H \equiv \dot{a}/a$ is the Hubble expansion rate in terms of physical time, and $G$ is Newton’s gravitational constant, and

$$\frac{\dot{a}}{a} = -\frac{4\pi G}{3}(4X - 2V - 3\mu \delta(t - t_c)).$$  \hspace{1cm} (20)

More specifically, inserting the expressions for the equation for the change in $H$ becomes

$$\dot{H} = -8\pi GX + 4\pi G\mu \delta(t - t_c),$$  \hspace{1cm} (21)

where $t_c$ is the time when the energy density has increased to the point that the S-brane appears.

From (21) it follows that once the S-brane is hit, the value of the Hubble constant jumps by an amount

$$\Delta H = 4\pi G\mu.$$  \hspace{1cm} (22)

In order to obtain a cosmological bounce this jump in $H$ has to be large enough to change the sign of $H$. Let us estimate the numbers. For concreteness let us assume that the S-brane is due to string states becoming massless, as in the scenario of $[11]$. In this case the value of $\mu$ is given by dimensional analysis by

$$\mu t_s^{-1} = m_s^4,$$  \hspace{1cm} (23)

where $m_s$ is the string scale and $t_s$ is the associated time, namely

$$t_s = G^{-1/2}m_s^{-2}.$$  \hspace{1cm} (24)

Hence

$$\mu = G^{-1/2}m_s^2.$$  \hspace{1cm} (25)

This value has to be compared with the value of $H$ at the time when the Higgs energy density reaches the string scale density $m_s^4$. By the first Friedmann equation this is given by

$$H \sim G^{1/2}m_s^2.$$  \hspace{1cm} (26)

Thus, we see that the expected jump in $H$ due to the S-brane is large enough to change contraction into expansion.

Whether the cosmological bounce is symmetric or not appears to depend on details of the construction. If the time $t_c$ is the time when the Higgs field comes to rest when running up the potential wall as stringy values, then the bounce will be symmetric. If the time $t_c$ arises earlier, then after the bounce the Higgs field will continue to roll up the hill for a while before turning around. Since the universe keeps contracting until the S-brane is hit, even if $h$ has already turned around, it is also possible for the bounce to occur after $h$ has already reached its maximal value.

Let us take a closer look at the matching conditions across the S-brane. Given the change of $H$ across the
brane by the amount \( \Lambda \), the values of \( h \) and \( \dot{h} \) across the brane are also determined. We will assume that \( h \) does not jump. In this case, the equations
\[
H^2 = \frac{8\pi G}{3} [V_+ + X_+],
\]
\[
H^2 = \frac{8\pi G}{3} [V_+ + X_+] = (H_+ + \Delta H)^2,
\]
where the subscripts \( - \) and \( + \) indicate the quantities right before and right after the bounce. These equations determine the value of \( \dot{h} \) after the bounce. For a symmetric bounce with \( X_- = 0 \) and \( H_+ = -H_- \) we obtain \( X_+ = 0 \), i.e. \( h \) starts at rest after the bounce.

In the numerical simulation whose results are presented in Figs. 3 and 4 we have solved the equations of motion without the brane for times before the bounce is reached. We assume a symmetric bounce, i.e. that the bounce occurs when \( h \) has climbed up the potential at large field values to positive values and comes to rest. At this point, we reverse the sign of the value of the Hubble parameter, as discussed above. After the bounce we again solve the equations of motion without the brane. Note that at the bounce point \( w = -1 \) since \( X = 0 \).

The figures show that the scenario argued for in the above approximate analytical considerations is indeed obtained. In particular, there is an Ekpyrotic phase of contraction followed by a non-singular bounce, and at late times in the expanding phase the Higgs field ends up in its “regular” vacuum state with \( |h| = v \). A blowup of Figure 4 shows that for the value of \( \Lambda \) chosen, \( \Lambda = 10^{-3} \) in Planck units, the Ekpyrotic phase of contraction lasts for about 300 Planck times. We have verified numerically that the length of the period of Ekpyrotic contraction scales roughly as \( \Lambda^{-1} \). Thus, for the value \( \Lambda \sim 10^{-7} \) in Planck units indicated by the current measurements of the Higgs and top quark mass, we obtain about \( 3 \times 10^6 \) Planck times of Ekpyrotic contraction. Whether this is sufficient to completely solve the anisotropy problem depends on the initial anisotropy at the beginning of the evolution.

On the other hand, the tuning on the parameter \( \mu \) required to obtain a sufficiently symmetric bounce to allow the Higgs field to relax to \( |h| = v \) at late times becomes more severe the smaller \( \Lambda \) is.

IV. COSMOLOGICAL SCENARIO

The cosmological scenario we have developed is the following. We start in a matter-dominated phase of contraction with the Higgs field close to its current vacuum value \( v \). After some amount of contraction there will be a smooth transition to a radiation phase of contraction. During both periods \( h \) will be oscillating (initially about \( v \) and later about \( 0 \)). Once the amplitude of oscillation reached a value of order \( \Lambda \), the Higgs instability will set in. The Higgs field rolls off to negative values of the potential, leading to an Ekpyrotic phase of contraction during which the energy the Higgs energy density comes to dwarf the energy density in all other forms of matter, and also smooths out anisotropies. Eventually the energy density increases to the point when the dynamics hits an S-brane, at which point a non-singular transition from contraction to expansion sets in. In the expanding phase the Higgs field re-traces its evolution in the contracting phase, eventually landing it back in its vacuum state \( h = v \).

This scenario provides a realization of the \textit{matter bounce} alternative to cosmological inflation as a theory for the origin of structure in the universe, and as a solution of the horizon problem. According to this scenario, we start early in the contracting phase with fluctuations in their quantum vacuum state. The growth of the curvature fluctuations on super-Hubble scales in the contracting phase then transforms the spectrum from a vacuum spectrum to a scale-invariant one on length scales which exit the Hubble radius during the matter phase of contraction \cite{14, 15}.

The key question for late time cosmology is whether the scale-invariance of the spectrum of curvature fluctuations survives the bounce. This question has been studied in many toy models on non-singular bouncing cosmology, e.g. in the quintom bounce \cite{24, 25}, in the Horava-Lifshitz gravity bounce \cite{26}, the bounce \cite{27} obtained in Horava-Lifshitz gravity \cite{28} and in the ghost condensate and Galileon bounces \cite{29} (see \cite{30, 31} for the background models), and it was found that on length scales larger than the time scale of the bounce phase the spectral shape is unchanged. The issue is more subtle, however, in the case of an S-brane bounce, as discussed in detail in \cite{32}. In this case, the final spectrum depends sensitively on the coordinate system in which the brane is defined. In the case of purely adiabatic fluctuations the scale-invariance of the spectrum is preserved.

V. DISCUSSION

We have shown that the instability of the Higgs potential can have positive consequences for early universe cosmology when considered not in the context of the inflationary scenario, but in the context of a bouncing scenario. More specifically, we have shown that the instability of the Higgs field generates an Ekpyrotic phase of contraction which smooths out anisotropies and hence solves the key problem facing bouncing cosmologies.

We have argued that an S-brane arising from stringy effects can lead to a non-singular bounce. When the S-brane is considered to have infinitesimal thickness in temporal direction there will be a jump in the value of the Hubble constant, but if the S-brane is smeared out in the same way that topological defects are (they have finite thickness) then the cosmological evolution is completely non-singular.

Our scenario hence gives a realization of the matter bounce alternative to the inflationary scenario of struc-
ture formation.

Note that to obtain a symmetric bounce a certain amount of tuning of $\mu$ is required. For asymmetric bounces the danger is that $h$ will not be able to relax to $h = \pm v$ at late times in the expanding phase, but comes to rest in a AdS minimum.

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