Action for gravity: An emergent perspective

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Abstract. There is increasing recognition in recent years that the field equations of gravity may have the same conceptual status as the equations of fluid mechanics or elasticity, and hence, gravity could be thought of as an emergent phenomenon. We have shown that the action functional leading to the field equations of gravity too, can be given a thermodynamic interpretation, which supports the emergent paradigm of gravity. We have explored the action functional in three different ways, the key features of which we have summarized in this article.

1. Introduction
In recent years, it is recognized that the field equations of gravity may have the same conceptual status as the equations of fluid mechanics or elasticity, and hence, gravity could be thought of as an emergent phenomenon just like, say, fluid mechanics (for a review see [1]). This approach has a long history, originating from the work of Sakharov [2] and interpreted in many ways by different authors [3]. We have used the term ‘emergent’ in the specific and well-defined sense in terms of the equations of motion, rather than in more speculative vein, like considering the space and time themselves to be emergent, etc. The evidence for such a specific interpretation comes from different facts, like the possibility of interpreting the field equations in a wide class of theories as thermodynamic relations, the possibility of obtaining the field equations from a thermodynamic extremum principle, application of equi-partition ideas to obtain the density of microscopic degrees of freedom, etc. [1]. In such an approach, geometrical variables like the metric, etc. are derived concepts (e.g., pressure, density, etc. of a gas), and the dynamical equations governing them can be derived from the thermodynamic limit of an underlying micro-structure, say, by extremizing a suitably defined entropy functional. But on the other hand, we know from standard textbook description that one can obtain the field equations for gravity from an action functional in which the metric is varied. This raises the question:

If gravity is, indeed, an emergent phenomenon, should not the action functionals of gravity contain some signature of this fact?

We have shown that this is, indeed true, and the action functional does suggest that gravity is emergent. We have reviewed below three results based on earlier works in [4, 5, 6]

2. Holography in action
It is well-known that the Einstein-Hilbert action can be separated into a bulk term and a surface term. It is only the variations of the bulk $\Gamma$ term that contribute to the field equations. That is, the field equations (and their solutions) do not depend in any way on the surface term. It is, therefore, a mystery - in the conventional approach - that the surface term, which is ignored while

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obtaining the field equations can be used to determine the entropy of the horizons that arise in the theory! The solution to this mystery was first pointed out in [7], where it was emphasized that the bulk and surface terms in Einstein-Hilbert action are connected by a peculiar relation:

\[ L_{\text{sur}} = -\partial_i \left( g_{ab} \frac{\partial L_{\text{bulk}}}{\partial (\partial_i g_{ab})} \right), \]  

which allows the information about either one to be extracted from the other. The relationship between the bulk and boundary terms in the action was termed 'holographic', because the information about the bulk action functional (from varying of which, we can obtain the dynamical equations) is encoded in the boundary action functional. We continue to use this terminology 'holographic action' with this understanding. The holographic nature of the action fits very well with the thermodynamic approach to gravity, and can be thought of as the hidden signal in the action functionals indicating that the description of gravity is an emergent one. In fact, one can provide very general arguments to suggest that the action functional describing any theory of gravity that obeys the principle of equivalence and principle of general covariance will have a bulk and boundary term related holographically. If this is the case, one would like to explore the connection further and see what insights it can provide. We decompose the Einstein-Hilbert action into a particular surface and bulk term for any static spacetime using the time-time component of the Einstein tensor as:

\[ L_{\text{EH}} = -2G_0^0 + 2R_0^0, \]  

where \( 2R_0^0 \) can be shown to be total divergence in this case. In any static spacetime, one can show that \( L_{\text{bulk}} = -2G_0^0 \) is related to the ADM Hamiltonian in GR, while \( L_{\text{sur}} = -2R_0^0 \) is related to the Noether charge, associated with the diffeomorphism symmetry of the theory. One can show that the action functional then has direct thermodynamic interpretation in the Euclidean sector as:

\[ A_E \equiv -\beta E + S. \]  

The Euclidean-Einstein action has the interpretation as giving the (negative of) free energy of spacetime. One can then interpret extremizing the integral over \(-2G_0^0\) while keeping the surface term fixed at the boundary, as extremizing the bulk energy of the static spacetime while keeping the entropy fixed. From the previous work, we know that the resulting field equations can be interpreted as the thermodynamic identity \( dE = TdS - PdV \) on the horizon. In the usual thermodynamic systems, if we know the entropy functional \( S(E, V) \), we can obtain the other thermodynamic variables like \((T, P)\), etc. of the system. Alternatively, one can invert the form of \( S(E, V) \) to get energy \( E(S, V) \) in terms of entropy. Similarly, in the case of Einstein-Hilbert action, we can consider the extremization of \(-2G_0^0\), as equivalent to obtaining the thermodynamic relation \( dE = TdS - PdV \) (which is the same as field equations of the theory) from an energy functional. Further, if gravity is truly emergent, thermodynamic, phenomenon of an underlying microscopic theory, then one should be able to invert the energy functional \( E(S, V) \) of gravity to obtain the \( S(E, V) \) functional. This motivates us to ask: Can we obtain the surface term \( 2R_0^0 \) directly from the bulk term \(-2G_0^0\)? Holography answers this question. We can show by direct computation that \( L_{\text{sur}} = 2\sqrt{-g}R_0^0 \), and \( L_{\text{bulk}} = -2\sqrt{-g}G_0^0 \) are related by the holographic relation [4] as:

\[ L_{\text{sur}} = -\frac{1}{\sqrt{D} - 1} \partial_i \left( g_{ab} \frac{\delta L_{\text{bulk}}}{\delta (\partial_i g_{ab})} + \partial_j g_{ab} \frac{\delta L_{\text{bulk}}}{\delta (\partial_j g_{ab})} \right). \]  

We have also found that a similar interpretation continues to hold even for higher curvature theories such as Lanczos Lovelock.
3. The partition function for Lanczos-Lovelock gravity

Consider further, that the static spacetime is also spherically symmetric in $D$ dimensions described by the metric of the form:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2,$$

such that $f(r)$ vanishes at some surface $r = a$, say, with $f'(a) = B$, say) remaining finite. The class of metrics in Eq. (5) with the behaviour $f(a) = 0, f'(a) = B$ constitute a canonical ensemble at constant temperature, since they all have the same temperature $T = |B|/4\pi$. Let us consider the partition function for a general Lanczos-Lovelock theory, of order $m$ for this ensemble, which is given by the path integral sum:

$$Z(\beta) = \sum_{g \in S} \exp(-A^{(m)}_{LL}(g)). \quad (6)$$

The sum in Eq. (6) is restricted to the set $S$ of all metrics of the form in Eq. (5) with the behaviour $f(a) = 0, f'(a) = B$, and the Euclidean-Lagrangian is a functional of $f(r)$. The spatial integration will be restricted to a region bounded by $(D-2)$-spheres at $r = a$ and at the asymptotic boundary $r = b$. For the metric of the form in Eq.(5), the bulk term $2\sqrt{-g}G_0^0$ too, becomes a total derivative and the contribution to the Euclidean action then comes only from the boundary values. However, we have ignored the contribution from the asymptotic boundary, since it will only lead to a factor $Z_Q$, in deciding about the dependence of $Z(\beta)$ on the form of the metric near $r = a$. The final result can be written in a very suggestive form [5]:

$$Z(\beta) = Z_Q \exp \left[ \frac{(D-2)!}{(D-2m)!} \frac{\Omega a^{D-2m}}{4} - \beta \left( \frac{(D-2)!}{(D-2m-1)!} \frac{\Omega}{16\pi} a^{D-2m-1} \right) \right],$$

where $\Omega$ is the area of a unit $(D-2)$-sphere. Here, $S^{(c)}(a)$ and $E(a)$ turn out exactly the same as the expression for Wald entropy, and energy of the horizon of the corresponding Lanczos-Lovelock theory obtained through other methods. Another feature is that the partition function also highlights the connection between the ADM Hamiltonian, and the quasi-local energy of a horizon in a spherically symmetric static spacetime.

4. Action principle for the fluid-gravity correspondence

It is known that the black hole horizon can be interpreted as a dissipative membrane with Einstein’s equations projected on to it, taking a form very similar (but not identical) to the Navier-Stokes equation in fluid mechanics. This work formed the basis for the development of membrane paradigm [8]. Recently [1], it was shown that, when Einstein’s equations are projected on to any null surface and the resulting equations are viewed in the freely falling frame, they become identical to Navier-Stokes equation. Figuratively speaking, this result shows that a spacetime filled with null surfaces can be equivalently thought of as hosting a fluid (with the fluid variables related to the structure of the null surface at any given event), which satisfies the Navier-Stokes equation in the local inertial frame. In the approaches taken in all the previous work in this subject, one usually obtains the field equation by extremizing the Einstein-Hilbert action with respect to the variations of the metric and then project them on to a black hole horizon or on to a generic null surface.

This is, however, conceptually not very satisfactory in the emergent paradigm for two reasons. First, it would be nice if equations of macroscopic dynamics could be obtained from a thermodynamic extremum principle rather than a field theoretic action principle. Second, given
the fact that final equations are expressed in terms of variables defined using a null surface, it would be appropriate if the same variables are used in the extremum principle rather than the metric. Here, we have provided a thermodynamic extremum principle from which one directly obtains the Navier-Stokes equation rather than first obtaining or assuming Einstein's field equations and then deriving the Navier-Stokes equation by a projection to a null surface. The degrees of freedom varied in the action principle are the null vectors in the spacetime and not the metric tensor. The form of the action in terms of the fluid variables, which are in turn related to the null vectors defining the null surface is given in [6]. Here, we have pointed out an interesting feature related to this action functional.

Consider the spacetime to be completely foliated by null surfaces. Further, let the (non-affine) parameter along the null geodesics generating the null surface $S$ be denoted by $\lambda$. Consider an infinitesimal cross sectional area $\delta A$ of the 2-surface $S_t$, which is the intersection of the null surface $S$ with the constant time spacelike hypersurface $\Sigma_t$. Then, we have shown that the action can be written in a form:

$$\frac{dS}{dt} \propto -dE + TdS_H + Pd\delta A,$$

(8)

where (i) $dE$ denotes the loss in energy, because of viscous dissipation during evolution of the small area element $\delta A$ of the null surface from $\lambda_1$ to $\lambda_2$ along the null congruence, (ii) the second term corresponds to rise in the gravitational entropy proportional to the increase of the area of the horizon, which is due to the familiar information loss processes, and (iii) the third term is the (virtual) work done by the horizon against the pressure $P$ during its area expansion $d\delta A$. This suggests that we can interpret the rate of change of local action as an on-shell local entropy production rate of the given spacetime.

Thus, we have highlighted three features of the action leading to the field equations of gravity, which allow us to give a thermodynamic interpretation to the action, thus, supporting the paradigm that gravity is emergent.

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