DARK MATTER AND DETECTOR CROSS SECTIONS

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We consider here the spin independent neutralino-proton cross section for a variety of SUGRA and D-brane models with R-parity invariance. The minimum cross section generally is \( \tilde{\sigma} \sim 1 \times 10^{-9} \text{pb} \) (and hence accessible to future detectors) except for special regions of parameter space where it may drop to \( \lesssim 10^{-12} \text{pb} \). In the latter case the gluino and squarks will be heavy (\( \gtrsim 1 \text{ TeV} \)).

Dark matter detectors have now achieved a sensitivity that they have begun to probe interesting parts of SUSY parameter space. It is thus of interest to see what sensitivity will be needed to explore the entire space. To examine this we consider here models based on gravity mediated supergravity (SUGRA), where the LSP is generally the lightest neutralino (\( \tilde{\chi}_1^0 \)). Neutralinos in the halo of the Milky Way might be directly detected by their scattering by terrestrial nuclear targets. Such scattering has a spin independent and a spin dependent part. For heavy nuclear targets the former dominates, and it is possible to extract then (to a good approximation) the neutralino-proton cross section, \( \sigma_{\tilde{\chi}_1^0-p} \). Current experiments (DAMA, CDMS, UKMDC) have sensitivity to halo \( \tilde{\chi}_1^0 \) for

\[
\sigma_{\tilde{\chi}_1^0-p} \gtrsim 1 \times 10^{-6} \text{ pb} \tag{1}
\]

and future detectors (GENIUS, Cryoarray) plan to achieve a sensitivity of

\[
\sigma_{\tilde{\chi}_1^0-p} \gtrsim (10^{-9} - 10^{-10}) \text{ pb} \tag{2}
\]

We consider here two questions: (1) what part of the parameter space is being tested by current detectors, and (2) what is the smallest value of \( \sigma_{\tilde{\chi}_1^0-p} \) the theory is predicting (i.e. how sensitive must detectors be to cover the full SUSY parameter space). The answer to these questions depends in part on the SUSY model one is considering and also on what range of theoretical and input parameter one assumes. In the following, we examine three models that have been considered in the literature based on grand unification of the gauge coupling constants at \( M_G \approx 2 \times 10^{16} \text{ GeV} \): (1) Minimal supergravity GUT (mSUGRA) with universal soft breaking at \( M_G \); (2) Nonuniversal soft breaking models for Higgs and third generation scalar masses at \( M_G \), and D-brane models (based on type IIB orientifolds) which allow for nonuniversal gaugino masses and nonuniversal scalar masses at \( M_G \).

While each of the above models contain a number of unknown parameters, theories of this type can still make relevant predictions for two reasons: (i) they allow for radiative breaking of \( SU(2) \times U(1) \) at the electroweak scale (giving a natural explanation of the Higgs mechanism), and (ii) along with calculating \( \sigma_{\tilde{\chi}_1^0-p} \), the theory can calculate the relic density of \( \tilde{\chi}_1^0 \), i.e \( \Omega_{\tilde{\chi}_1^0} = \rho_{\tilde{\chi}_1^0}/\rho_c \) where \( \rho_{\tilde{\chi}_1^0} \) is the relic density of \( \tilde{\chi}_1^0 \) and \( \rho_c = 3H_0^2/(8\pi G_N) \) (\( H_0 \) is the Hubble constant and \( G_N \) is the Newton constant). Both of these greatly restrict the parameter space. In general one has \( \Omega_{\tilde{\chi}_1^0}h^2 \sim \left( \int_0^{\infty} dx (\sigma_{\text{ann}}v) \right)^{-1} \) (where \( \sigma_{\text{ann}} \) is the neutralino annihilation cross section in the early universe, \( v \) is the relative velocity, \( x_f = kT_f/m_{\tilde{\chi}_1^0}, T_f \) is the freeze out temperature, \( \langle ... \rangle \) means thermal average and \( h = H_0/100 \text{ km s}^{-1}\text{Mpc}^{-1} \)). The fact that these conditions can be satisfied for reasonable parts of the SUSY parameter space represents a significant success of the SUGRA models.

In the following we will assume \( H_0 = (70 \pm 10) \text{ km s}^{-1}\text{Mpc}^{-1} \) and matter (m) and
baryonic (b) relic densities of $\Omega_m = 0.3 \pm 0.1$ and $\Omega_b = 0.05$. Thus $\Omega_{\chi^0 h^2} = 0.12 \pm 0.05$. The calculations given below allow for a 2σ spread, i.e. we take $0.02 \leq \Omega_{\chi^0 h^2} \leq 0.25$. It is clear that when the MAP and Planck satellites determine the cosmological parameters accurately, the SUGRA dark matter predictions will be greatly sharpened.

1 Calculational Details

In order to get reasonably accurate results, it is necessary to include a number of theoretical corrections in the analysis. We list here the main ones used in the calculations below: (i) In relating the theory at $M_G$ to phenomena at the electroweak scale, the two loop gauge and one loop Yukawa renormalization group equations (RGE) are used, iterating to get a consistent SUSY spectrum. (ii) QCD RGE corrections are further included below the SUSY breaking scale for contributions involving light quarks. (iii) A careful analysis of the light Higgs mass $m_h$ is necessary (including two loop and pole mass corrections) as the current LEP limits impact sensitively on the relic density analysis for $\tan\beta \lesssim 5$. (iv) L-R mixing terms are included in the squark (mass)$^2$ matrices since they produce important effects for large $\tan\beta$ in the third generation. (v) One loop corrections are included to $m_b$ and $m_\tau$ which are again important for large $\tan\beta$. (vi) The experimental bounds on the $b \to s\gamma$ decay put significant constraints on the SUSY parameter space and theoretical calculations here include the leading order (LO) and approximate NLO corrections. We have not in the following imposed $b \to \tau$ (or $t \to b - \tau$) Yukawa unification or proton decay constraints as these depend sensitively on unknown post-GUT physics. For example, such constraints do not naturally occur in the string models where $SU(5)$ (or $SO(10)$) gauge symmetry is broken by Wilson lines at $M_G$ (even though grand unification of the gauge coupling constants at $M_G$ for such string models is still required).

All the above corrections are under theoretical control except for the $b \to s\gamma$ analysis where a full NLO calculations has not been done. (We expect that while the full analysis might modify the regions of parameter space excluded by the $b \to s\gamma$ experimental constraint, the minimum and maximum values of $\sigma_{\chi^0 - p}$ would probably not be significantly changed.) The analysis of $\sigma_{\chi^0 - p}$, taking into account the above theoretical corrections has now been carried out by several groups obtaining results in general agreement. These results are presented below.

Accelerator bounds significantly limit the SUSY parameter space. In the following we assume the LEP bounds $m_h > 104(100)$ GeV for $\tan\beta=3(5)$ and $m_{\chi^\pm} > 104(100)$ GeV. (For $\tan\beta > 5$, the $m_h$ bounds do not produce a significant constraint.) For $b \to s\gamma$ we assume an allowed range of 2σ from the CLEO data. The Tevatron gives a bound of $m_\tilde{q} \gtrsim 270$ GeV (for $m_\tilde{q} \equiv m_3$).

Theory allows one to calculate the $\tilde{\chi}^0_1$ quark cross section and we follow the analysis of to convert this to $\chi^0_1 - p$ scattering. For this one needs the $\pi - N\sigma$ term, $\sigma_{\pi N}$ and $\sigma_0 = \sigma_{\pi N} - (m_u + m_d)(|q\tilde{s} s p|)$ and the quark mass ratio $r = m_s/(1/2)(m_u + m_d)$. We use here $\sigma_{\pi N} = 65$ MeV, from recent analyses based on new $\pi - N$ scattering data, $\sigma_0 = 30$ MeV and $r = 24.4 \pm 1.5$.

2 mSUGRA model

We consider first the mSUGRA model where the most complete analysis has been done. mSUGRA depends on four parameters and one sign: $m_0$ (universal scalar mass at $M_G$), $m_{1/2}$ (universal gaugino mass at $M_G$), $A_0$ (universal cubic soft breaking mass), $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$ (where $\langle H_2,1 \rangle$ gives rise to (up, down) quark masses) and $\mu/|\tilde{\mu}|$ (where $\mu$ is the Higgs mixing parameter in the superpotential, $W_\mu = \mu H_1 H_2$). One conventionally
restricts the range of these parameters by “naturalness” conditions and in the following we assume $m_0 \leq 1$ TeV, $m_{1/2} \leq 600$ GeV (corresponding to $m_{\tilde{g}} \leq 1.5$ TeV, $m_{\tilde{q}} \leq 240$ GeV), $|A_0/m_0| \leq 5$, and $2 \leq \tan \beta \leq 50$. Large $\tan \beta$ is of interest since SO(10) models imply $\tan \beta \geq 40$ and also $\sigma_{\tilde{g} \tilde{g}}$ increases with $\tan \beta$. $\sigma_{\tilde{g} \tilde{g}}$ decreases with $m_{1/2}$ for large $m_{1/2}$, and thus if one were to increase the bound on $m_{1/2}$ to 1 TeV ($m_{\tilde{g}} \leq 2.5$ TeV), the cross section would drop by a factor of 2-3.

The maximum $\sigma_{\tilde{g} \tilde{g}}$ arise then for large $\tan \beta$ and small $m_{1/2}$. This can be seen in Fig.1 where $(\sigma_{\tilde{g} \tilde{g}})_{\text{max}}$ is plotted vs. $m_{\tilde{g}}$ for $\tan \beta = 20, 30, 40$ and 50. Current detectors obeying Eq (1) are then sampling the parameter space for large $\tan \beta$, small $m_{\tilde{g}}$ (and also small $\Omega_{\tilde{g} h^2}$) i.e

$$\tan \beta \approx 25; \ m_{\tilde{g}} \approx 90 \text{ GeV}; \ \Omega_{\tilde{g} h^2} \approx 0.1.$$ \hfill (3)

To discuss the minimum cross section, it is convenient to consider first $m_{\tilde{g}} \approx 150$ GeV ($m_{1/2} \leq 350$) where no coannihilation occurs. The minimum cross section occurs for small $\tan \beta$. One finds

$$\sigma_{\tilde{g} \tilde{g}} \approx 4 \times 10^{-9} \text{ pb}; \ m_{\tilde{g}} \approx 140 \text{ GeV} \hfill (4)$$

which would be accessible to detectors that are currently being planned (e.g. GENIUS).

For larger $m_{\tilde{g}}$, i.e. $m_{1/2} \approx 150$ the phenomena of coannihilation can occur in the relic density analysis since the light stau, $\tilde{\tau}_1$, (and also $\tilde{e}_L$, $\tilde{\mu}_R$) can become degenerate with the $\tilde{\chi}_1^0$. The relic density constraint can then be satisfied in narrow corridor of $m_0$ of width $\Delta m_0 \approx 25$ GeV, the value of $m_0$ increasing as $m_{1/2}$ increases. Since $m_0$ and $m_{1/2}$ increase as one progresses up the corridor, $\sigma_{\tilde{g} \tilde{g}}$ will generally decrease.

We consider first the case $\mu > 0$. One finds in general that $\sigma_{\tilde{g} \tilde{g}}$ also decreases as $A_0$ increases. Fig.2 shows $\sigma_{\tilde{g} \tilde{g}}$ in the domain of large $A_0$ and for two values of $\tan \beta$. One sees that the smaller $\tan \beta$ still gives the lower cross section, though the difference is mostly neutralized at larger $m_{1/2}$. (For large $\tan \beta$, $m_0$ also becomes large to satisfy the relic density constraint i.e $m_0 \approx 700$ GeV for $\tan \beta = 40, m_{1/2} = 600$ GeV.) We have in general for this regime

$$\sigma_{\tilde{g} \tilde{g}} \approx 1 \times 10^{-9} \text{ pb}; \ m_{1/2} \leq 600 \text{ GeV}, \ \mu > 0, A_0 \leq 4 m_{1/2}. \hfill (5)$$

This is still within the sensitivity range of proposed detectors.

When $\mu$ is negative an “accidental” cancellation can occur in part of the parameter space in the coannihilation region which can greatly reduce $\sigma_{\tilde{g} \tilde{g}}$. This can be seen in Fig.3, where starting with small $\tan \beta$ the cross section decreases, leading to a minimum at about $\tan \beta = 10$, and then increases again for larger $\tan \beta$. At the minimum one has $\sigma_{\tilde{g} \tilde{g}} \approx 1 \times 10^{-12}$ when $\tan \beta = 10$ and $m_{1/2} = 600$ GeV. More generally one has

$$\sigma_{\tilde{g} \tilde{g}} \approx 1 \times 10^{-10} \text{ pb} \hfill (6)$$

for the parameter domain when $4 \approx \tan \beta \lesssim 20, m_{1/2} \approx 450 \text{ GeV}(m_{\tilde{g}} \approx 1.1 \text{ TeV}), \ \mu < 0$. In this domain, $\sigma_{\tilde{g} \tilde{g}}$ would not be accessible to any of the currently planned detectors. However, mSUGRA also then predicts that this could happen only when the gluino and squarks have masses greater than 1 TeV (and for only a restricted region of $\tan \beta$) a result that could be verified at the LHC.

3 Nonuniversal SUGRA Models

In the discussion of SUGRA models with nonuniversal soft breaking, universality for the first two generations of squark and slepton masses at $M_G$ is usually maintained to suppress flavor changing neutral currents. One allows, however, the Higgs and third generation squark and slepton masses to become nonuniversal. We maintain gauge and gaugino mass unification at $M_G$.

While these models contain a large number of new parameters, their effects on $\sigma_{\tilde{g} \tilde{g}}$ can be charcterized approximately by the
signs of the deviations from universality. One choice can greatly increase $\sigma_{\tilde{\chi}^0_1-p}$, by a factor of 10-100 compared to the universal case, and the reverse choice can reduce $\sigma_{\tilde{\chi}^0_1-p}$ (though by a much lesser amount). Thus it is possible for detectors to probe regions of smaller $\tan\beta$ with nonuniversal breaking, and detectors obeying Eq. (1) can probe part of the parameter space for $\tan\beta$ as low as $\tan\beta \approx 4$.

The minimum cross section occurs (as in mSUGRA) at the lowest $\tan\beta$ and at the largest $m_{1/2}$ i.e. in the coannihilation region. We limit ourselves here to the case where only the Higgs masses are nonuniversal. One finds then results similar to mSUGRA i.e. $\sigma_{\tilde{\chi}^0_1-p} \approx 10^{-9}$ pb for $\mu > 0$, $m_{1/2} \leq 600$ GeV. For $\mu < 0$, there can again be a cancellation of matrix elements reducing the cross section to $10^{-12}$ pb when $m_{1/2} = 600$ GeV in a restricted part of the parameter space when $\tan\beta \approx 10$.

4 Summary

We have examined here the neutralino-proton cross section for a number of SUGRA type models. In all the models considered, there are regions of parameter space with $\chi^0_1 - p$ cross sections of the size that could be observed with current detectors. Thus with the sensitivity of Eq. (1), detectors would be sampling regions of the parameter space for mSUGRA where $\tan\beta \approx 25$, $m_{\chi^0_1} \approx 90$GeV and $\Omega_{\chi^0_1}h^2 \approx 0.1$. Nonuniversal models can have larger cross sections and so detectors could sample down to $\tan\beta \approx 4$, while for the D-brane models considered, detectors could sample down to $\tan\beta \approx 15$.

The minimum cross sections these models predict are considerably below current sensitivity. Thus for mSUGRA one finds for $\mu > 0$ that $\sigma_{\chi^0_1-p} \approx 1 \times 10^{-9}$ pb for $m_{1/2} \leq 600$ GeV, $\mu > 0$, where $m_{1/2} = 600$ GeV corresponds to $m_{\tilde{g}} \approx 1.5$ TeV, $m_{\chi^0_1} \approx 240$ TeV. This is still in the range that would be accessible to detectors being planned (such as GENIUS or Cryoarray). For $\mu < 0$, a cancellation can occur in certain regions of parameter space allowing the cross sections to fall below this. Thus

$$\sigma_{\chi^0_1-p} < 1 \times 10^{-10} \text{pb for } 4 \approx \tan\beta \approx 20,$$

$$\mu < 0, m_{1/2} \approx 450 \text{ GeV}$$

and reaching a minimum of $\sigma_{\chi^0_1-p} \approx 1 \times 10^{-12}$ pb for $\tan\beta = 10$, $m_{1/2} = 600$ GeV, $\mu < 0$. This domain would appear not to be accessible to future planned detectors. Since $m_{1/2} = 450$ GeV corresponds to $m_{\tilde{g}} \approx 1.1$ TeV, this region of parameter space would imply a gluino squark spectrum at the LHC above 1 TeV.

The above results holds for the mSUGRA model. While a full analysis of coannihilation has not been carried out for the nonuniversal and D-brane models, results similar to the above hold for these over large regions of parameter space. Thus for nonuniversal Higgs masses and for the D-brane model one finds $\sigma_{\chi^0_1-p} \approx 10^{-9}$ pb for $\mu > 0$, while a cancellation allows $\sigma_{\chi^0_1-p}$ to fall to $10^{-12}$ pb for $\mu < 0$ at $\tan\beta \approx 10$ (with again a gluino/squark mass spectrum in the TeV domain).

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![Figure 1](image1.png)

Figure 1. ($\sigma_{\tilde{\chi}_1^0-p}^{\mu>0}$)$_{\text{max}}$ for mSUGRA obtained by varying $A_0$ over the parameter space for $\tan\beta = 20, 30, 40, \text{and } 50$[9]. The relic density constraint has been imposed.

![Figure 2](image2.png)

Figure 2. ($\sigma_{\tilde{\chi}_1^0-p}^{\mu<0}$) for mSUGRA in the coannihilation region for $\tan\beta = 40$ (upper curve) and $\tan\beta = 3$ (lower curve), $A_0 = 4m_{1/2}$, $\mu > 0$.

![Figure 3](image3.png)

Figure 3. ($\sigma_{\tilde{\chi}_1^0-p}^{\mu<0}$) for mSUGRA and $\mu < 0$ for (from top to bottom on right) $\tan\beta = 20, 5$ and 10. Note that for $\tan\beta \geq 10$, the curves terminate at the left due to the $b \to s\gamma$ constraint.