Quantum heat engines with multiferroic working substance

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The work provides an overview on some recent advances in the area of quantum thermodynamics and quantum heat engines. A particular emphasis is put on the possibility of constructing finite time quantum cycles and adiabatic shortcuts. We discuss in details the particular quantum heat engines operating with a multiferroic working substance.

Keywords: Quantum thermodynamics, quantum heat engines, multiferroics, adiabatic shortcuts, quantum dissipative systems, Bochkov-Kuzovlev and Jarzynski equalities.

1. Introduction

For any thermal heat engines and thermodynamic cycles the three main criteria are central: The produced maximal work, the efficiency, and the output power of the engine. The high efficiency of the heat engine is important for performing operations with low energy consumption, while the amount of the produced work and the power of the heat machine are crucial for swift performance. Is it realistic to meet all those three criteria simultaneously? The present work addresses this question and provides a brief overview on the current status of knowledge.

To introduce the problem and notations we start by recalling the basics of the classical thermodynamics\textsuperscript{1}. For a thermally isolated system consisting of two parts which are not in thermal equilibrium, a certain work is performed on the surrounding bodies during the transition to the equilibrium state. We exclude the work associated with the general expansion, assuming that the total volume of the system is conserved. Then the produced work is a function of the internal energy of the system $|W| = U_0 - U(S)$ where $U_0$ is the initial energy. Since the transition to the equilibrium state

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might occur in a variety of ways the final energy and the entropy \( U(S) \) might be different. Considering the produced work as a functional of the system’s entropy we write \( \delta S[W] = - (\partial U(S) / \partial S)|_{V=\text{const}} = -T \). As \( \delta S[W] \) is always negative (we use the absolute temperature scale \( T > 0 \)), any increment in the entropy while producing the work lowers work. Hence, the maximal work \( |W|_{\text{max}} \) is produced during the process when the entropy of the system stays constant \( S = \text{const} \). More rigorous deliberation leads to the following formula for the maximal work \( |W|_{\text{max}} = -\delta(U - T_0S + P_0V) \).

Here \( T_0, P_0 \) stand for the temperature and the pressure of the environment, while \( U, S, V \) define the internal energy, the entropy, and the volume of the working body. If the volume and the temperature are constant during the process, the produced maximal work is equal to the change in the free energy \( |W|_{\text{max}} = -\delta F \). Note, the adiabaticity of the process excludes any direct energy exchange between the bodies with the different temperatures.

The ideal heat engine envisioned by Carnot has four strokes: The working substance at temperature \( T_h \) absorbs isothermally energy from the hot heat bath, and then is cooled down adiabatically to the temperature \( T_l \). Thereafter, the working substance releases isothermally energy to a cold heat bath at \( T_l \), and eventually returns adiabatically to the initial state. Two thermal baths with temperatures \( T_h > T_c \) are needed to perform a reversible cycle. The existence of two heat baths allows excluding a direct irreversible energy exchange between the systems with different temperatures. The efficiency and the produced work of the ideal heat engine read:

\[
\eta = \frac{T_h - T_l}{T_h}, \quad W_{\text{max}} = \frac{T_h - T_l}{T_h} Q_{\text{in}},
\]

where \( Q_{\text{in}} \) is the heat absorbed from the hot bath. The adiabaticity imposes certain restrictions on the cycle’s swiftness. An ideal cycle takes an infinite time \( \tau \rightarrow \infty \), and therefore the output power of the ideal Carnot cycle vanishes \( P = W_{\text{max}} / \tau = 0 \). So it is of relevance to find ways yielding a good cycle efficiency with a reasonable power. This issue is not only a technical but also it is a conceptual one, even for classical systems. For quantum heat engines additional aspects are important. When the size of the working medium is scaled down to the mesoscopic scale purely quantum effects such as quantum fluctuations and interlevel transitions become important. The problem of the thermally assisted interlevel transitions can be solved relatively easily by detaching the working body from the heat bath.

Quantum adiabaticity is more subtle. To be precise we specify the concept of adiabaticity separately for quantum and classical systems. The stroke of the cycle, which is adiabatic in the sense of classical thermodynamics may be nonadiabatic for a quantum working substance. The reason lies in quan-
tum interlevel transitions that naturally occur in the case of fast driving and a finite time thermodynamic cycles. Thus, quantum adiabaticity implies not only a decoupling of the system from the thermal source, but also requires an elimination of interlevel transitions that are of a pure quantum nature. Shortcuts to quantum adiabaticity is a recent theoretical concept that allows eliminating the effect of those interlevel transitions. In what follows we provide a brief introduction to shortcuts to quantum adiabaticity. Before that we discuss the efficiency of the Carnot engine at maximum output power for a classical system.

2. Efficiency of the Carnot engine at a maximum output power

An ideal Carnot engine assumes that during isothermal strokes the working substance is in equilibrium with the thermal reservoirs, meaning that the isothermal strokes are performed infinitely slowly. Therefore, the power output of the engine is zero, since the finite work is produced in an infinite time. Ref. assumes that during the isothermal expansion the heat flux through the vessel enfolding the working medium is proportional to the temperature gradient across the vessel. Therefore, the expressions for the absorbed heat and the heat rejected to the heat sink read $Q_{in} = \alpha t_1 (T_h - T_{hw})$, $Q_{out} = \beta t_1 (T_{lw} - T_l)$. Here $\alpha, \beta$ are constants, $t_1, t_2$ are the durations of the isothermal strokes, and $T_{hw}, T_{lw}$ are the temperatures of the working medium during the isothermal strokes. The key issue in this assumption is that the duration of the isothermal strokes are finite. However, the cycle is reversible and the total entropy production is zero $Q_{in}/T_{hw} = Q_{out}/T_{lw}$. The output power of the cycle is $P = (Q_{in} - Q_{out})/(t_1 + t_2)$ and the total time spent for the two adiabatic strokes is $(\gamma - 1)(t_1 + t_2)$. With this expressions one can maximize the power output $\partial P/\partial x = 0, \partial P/\partial y = 0$, where $x = T_h - T_{hw}, y = T_{lw} - T_l$. After a little algebra for the cycle efficiency and the maximum output power one obtains: $\dot{\eta} = 1 - (T_l/T_h)^{1/2}$. Note a finite time cycle needed for the maximal power output comes at the cost of a lower efficiency $\dot{\eta} < \eta$. Unfortunately, the results obtained in Ref. are not directly applicable to quantum heat engines since in the quantum case pure quantum interlevel transitions may lift the adiabaticity. Adiabatic shortcuts are thus needed.
3. First law of thermodynamics for quantum systems

The first law of thermodynamics states that the change in internal energy of a system is equal to the heat added to the system minus the work done by the system. This is a very general formulation applicable to quantum systems as well. However, for quantum systems the definitions of a quantum heat and a quantum work need to be revisited.

At nonzero temperature the energy of a system can be evaluated as follows:

\[ U = \text{Tr}(\hat{\rho}\hat{H}) \]

Here \( \hat{H} \) is the Hamiltonian of the system and \( \hat{\rho} \) is the density matrix. For the change in the internal energy we deduce:

\[ dU = \sum_{n=1}^{N} \left( E_{n} d\rho_{nn} + \rho_{nn} dE_{n} \right) \]

The first term \( \delta Q = E_{n} d\rho_{nn} \) corresponds to the heat exchange and is related to the change of the level populations \( \rho_{nn} \) for \( E_{n} = \text{const} \).

The second term \( \delta W = \rho_{nn} dE_{n} \) corresponds to the produced work. The working substance produces work due to a change in the energy spectrum \( dE_{n} \) of the system. The relation of heat exchange and the quantum level populations is clear. The concept of a quantum work needs however further specification.

4. Bochkov-Kuzovlev and Jarzynski equalities

Let us consider a thermally isolated classical system \( H(p, q, \lambda(t)) \) driven by an external parameter \( \lambda \). A change in the parameter \( \lambda(0) = \lambda_0, \lambda(t_f) = \lambda_f \) produces work delivered to the system. This work is assumed to be small compared to the energy of the system. At \( t = 0 \) the system is thermalized to a temperature \( T = 1/\beta \). The work done on the system reads:

\[ W = \int_{0}^{t_f} \frac{\partial H}{\partial \lambda} d\lambda + \int_{0}^{t_f} \frac{\partial H}{\partial p} dp + \int_{0}^{t_f} \frac{\partial H}{\partial q} dq \]

The work averaged over the statistical ensemble is

\[ \langle W \rangle = \langle H(p_f, q_f, \lambda(t_f)) \rangle - \langle H(p_0, q_0, \lambda(0)) \rangle \]

According to the Bochkov-Kuzovlev equalities, not the work \( \langle W \rangle \) itself, but the exponential of the work \( \langle \exp(-\beta W) \rangle \) is the key point:

\[ \langle \exp(-\beta W) \rangle = \int \exp \left[ \beta(F_0 - H_0) \right] \exp \left[ -\beta(H_f - H_0) \right] d\Gamma_0. \]

Here \( \exp \left[ \beta(F_0 - H_0) \right] \) is the distribution function of the equilibrated system at \( t = 0 \), \( F_0 \) is the free energy, and \( \Gamma_0 = p_0q_0 \) is the phase volume of the system. Due to the Liouville theorem the phase volume of the system is...
invariant \( d\Gamma_0 = d\Gamma_f \). Therefore, one writes

\[
\langle \exp(-\beta W) \rangle = \exp(\beta F_0) \int \exp(-\beta H_f) d\Gamma_f.
\]

Finally we obtain the expression \( \langle \exp(-\beta W) \rangle = \exp(-\beta \Delta F) \), where \( \Delta F = F_f - F_0 \). This equality for cyclic process \( \lambda_f = \lambda \) was obtained by Bochkov and Kuzovlev in 1977 and by Jarzynski in 1997 for a more general setting \( \lambda_f \neq \lambda \).

According to statistical physics, for an adiabatic process when the parameter \( \lambda \) changes slowly compared to the system’s relaxation time, the produced work is equal to the change in free energy \( W = \Delta F \). For the non-adiabatic case \( W > \Delta F \) part of the work is wasted on the entropy production. The nonequilibrium entropy associated with the nonadiabatic process is defined as \( \Delta S_{ir} = \beta \langle W_{ir} \rangle \). Here \( \langle W_{ir} \rangle = \langle W \rangle - \Delta F \) is the difference between the delivered work in the nonadiabatic process and the change in free energy. We can rewrite Jarzynski equality in the following form (cf. Ref. 39):

\[
\Delta F = -T \log \langle \exp(-\beta W) \rangle.
\]

Noteably, the work for a quantum system is not an observable. This means that the average of the total work performed on the system doesn’t corresponds to the expectation values of an operator representing the work. In particular the work delivered to a quantum system is related to the time ordered correlation functions of the exponentiated Hamiltonian. The nonequilibrium entropy associated with a nonadiabatic process can be calculated straightforwardly 40. First we define the probability distribution of the quantum work

\[
p(W) = \sum_{n,m} \delta \left( W - \left( E_n^m - E_0^m \right) \right) p_{n,m}^0 p_{n}^0.
\]

Here

\[ p_n^0 = \exp \left( -\beta E_n^0 \right) / Z_0 \]

is the initial thermal Gibbs distribution, \( Z_0 \) is the partition function, \( E_n^0 \) are the initial energies, and \( p_{n,m}^0 \) are the transition probabilities. Using \( p(W) \) one can calculate the non-equilibrium quantum work:

\[
\langle W \rangle = \frac{1}{\beta} \sum_n p_n^0 \ln p_n^0 - \frac{1}{\beta} \sum_{n,m} p_{n,m}^0 \ln p_{n,m}^0 - \Delta F.
\]

\( p_n^0 \) is the final equilibrium distribution function. In the absence of purely quantum inter-level transitions \( p_{n,m}^0 = \delta_{nm} \) the first two terms disappear and the quantum work becomes equal to the change in free energy.
5. Adiabatic shortcuts and finite time quantum cycles

For a general discussion of shortcuts to adiabaticity and an overview of the interrelation between the various approaches as well as their historical developments we refer to the review article\textsuperscript{4} and references therein. Here we will basically follow Berry’s transitionless driving formulation\textsuperscript{5} which is equivalent to the counterdiabatic approach of Demirplak and Rice\textsuperscript{2,3}.

Let us suppose that the Hamiltonian of the system has the form \( \hat{H}(p, q, \lambda) \). Here \( p, q \) are canonical coordinates and \( \lambda \) is a parameter in the sense discussed above. For the solution of Schrödinger equation

\[
i\frac{\partial \Psi}{\partial t} = \hat{H}\Psi
\]

we implement the following ansatz:

\[
\Psi = \sum_n a_n(t) \varphi_n(p, q, \lambda) \exp \left\{ -i \int_{-\infty}^t E_n(\lambda) dt \right\},
\]

where \( E_n(\lambda) \) are the instantaneous quasi-energies that depend adiabatically on the parameter \( \lambda \). After standard derivations for the time dependent coefficients \( a_n(t) \) we obtain the iterative solution

\[
a_n^{(1)}(t) = -\int_{-\infty}^t d\tau \sum_{m \neq n} \frac{\langle \varphi_n| \frac{\partial}{\partial \lambda} | \varphi_m \rangle \lambda}{E_m - E_n} \times a_m(-\infty) \exp \left\{ -i \int_{-\infty}^\tau (E_m - E_n) d\tau' \right\}.
\]

The adiabatic approximation is valid when the following criteria hold

\[
a_n^{(2)} \sim \frac{\partial H}{\partial \lambda} \frac{1}{(E_m - E_n)^2}.
\]

Here \( a_n^{(2)} \) is a second order correction to \( a_n(t) \). If the characteristic time scale of the parameter \( \lambda \) is \( \dot{\lambda} \sim 1/\tau \) and \( \frac{1}{(E_m - E_n)^2} \gg 1 \) then the dynamic of the system is adiabatic and the effect of the non-adiabaticity is exponentially small.

In the case of an adiabatic evolution the general state \( |\Psi_n(t)\rangle \) driven by \( \hat{H}_0(t) \) is cast as

\[
|\Psi_n(t)\rangle = \exp \left[ -\frac{i}{\hbar} \int_0^t dt' E(t') \right.
\]

\[
- \int_0^t dt' \langle \Phi_n(t')| \partial_{t'} \Phi_n(t') \rangle \left| \Phi_n(t) \right\rangle.
\]

In essence the method of adiabatic shortcuts is an inverse engineering problem with the aim of finding of a new Hamiltonian for which the states \( (1) \)
behave as $i\partial_t \Psi_n(t) = \hat{H}(t) \Psi_n(t)$. Note that the time dependence of the new Hamiltonian can be arbitrary fast. With the aid of the unitary time-evolution operator

$$\hat{U}(t) = \sum_n \exp \left[ -\frac{i}{\hbar} \int_0^t dt' E(t') \right] \langle \Phi_n(t') | \Phi_n(t) \rangle,$$

we construct the auxiliary (counter-diabatic) Hamiltonian

$$\hat{H}_{CD}(t) = i\hbar (\partial_t \hat{U}(t)) \hat{U}^\dagger(t).$$

The reverse state engineering relies on the requirement that the states (1) solve for the Hamiltonian (3), meaning that

$$i\hbar \partial_t |\Psi_n(t)\rangle = \hat{H}_{CD}(t) |\Psi_n(t)\rangle.$$

In this way even for a fast driving the transitions between the eigenstates $|\Phi_n(t)\rangle$ are prevented. After a relatively simple algebra the counter-diabatic (CD) Hamiltonian $\hat{H}_{CD}(t)$ takes the form

$$\hat{H}_{CD}(t) = \hat{H}_0(t) + \hat{H}_1(t),$$

where

$$\hat{H}_1(t) = i\hbar \sum_{m \neq n} \frac{|\Phi_m\rangle \langle \Phi_m | \partial_t \hat{H}_0(t) |\Phi_n\rangle \langle \Phi_n |}{E_n - E_m}.$$ 

We adopt the initial conditions for the driving protocol as

$$\hat{H}_{CD}(0) = \hat{H}_0(0), \quad \hat{H}_{CD}(\tau) = \hat{H}_0(\tau).$$

Thus, on the time interval $t \in [0, \tau]$ we achieve a fast adiabatic dynamic by means of the counter-diabatic Hamiltonian $\hat{H}_{CD}(t)$.

This result can be straightforwardly generalized to systems with a degenerated spectrum. In this case we have

$$\hat{H}_1(t) = i\hbar \sum_{m \neq n} \sum_{q=1}^{\lambda_m} \sum_{k=1}^{\lambda_m} \frac{|\Phi_m^k\rangle \langle \Phi_m^k | \partial_t \hat{H}_0(t) |\Phi_n^q\rangle \langle \Phi_n^q |}{E_n - E_m}.$$ 

Here we assumed that the eigenvalue $E_m$ is $\lambda_m$ times degenerated and $|\Phi_m^k\rangle$ are the corresponding degenerated eigenfunctions $k = 1, \ldots \lambda_m$. 
6. Superadiabatic quantum heat engine with a multiferroic working medium

A central point for any quantum heat engine is the choice of the appropriate working substance. We identified multiferroics (MF) and in particular magnetoelectrics nanostructures as promising candidates\(^{10,31}\). MFs are materials of nanostructures with coupled order parameters such as elastic, magnetic, and ferroelectric ordering\(^{42-50,52-54}\) and can be well integrated in solid-state electronic circuits (in particular in oxide-based electronics). Hence, an engine based on a MF substance performs magnetic, electric and possibly (via piezoelectricity) mechanical works, at the same time. Particularly relevant are quantum spiral magnetoelectric substances\(^55\). A prototypical one dimensional chiral MF system can be modeled by a frustrated spin = 1/2 chain of \(N\) sites aligned along the \(x\) axis. Spin frustrations is due to competing ferromagnetic nearest neighbor \(J^1 > 0\) and antiferromagnetic next-nearest neighbor \(J_2 < 0\) interactions. We apply a time dependent electric field \(\mathcal{E}(t)\) which is linearly polarized along the \(y\) axis, and an external magnetic field \(B\) is applied along the \(z\) axis. The corresponding Hamiltonian reads

\[
\hat{H}_0(t) = \hat{H}_S + \hat{H}_{SF}(t),
\]

\[
\hat{H}_S = -J_1 \sum_i \vec{\sigma}_i \cdot \vec{\sigma}_{i+1} - J_2 \sum_i \vec{\sigma}_i \cdot \vec{\sigma}_{i+2} - \gamma eB \sum_i \sigma^z_i,
\]  

Here \(\hat{H}_S\) is time independent, while \(\hat{H}_{SF}\) is time dependent and contains the coupling of the external electric field to the electric polarization of the chain. The electric polarization \(\vec{P}_i\) tagged to spin non-collinearity reads

\[
\vec{P}_i = g_{ME} \vec{e}_{i,i+1} \times (\vec{\sigma}_i \times \vec{\sigma}_{i+1}),
\]

where \(\vec{e}_{i,i+1}\) is the unit vector connecting the sites \(i\) and \(i+1\), \(g_{ME}\) is a magneto-electric coupling constant. The spatially homogeneous, time dependent electric field \(\varphi(t)\) couples to the chain electric polarization \(\vec{P}\) such that

\[
\varphi(t) \cdot \vec{P} = d(t) \sum_i (\vec{\sigma}_i \times \vec{\sigma}_{i+1})^z,
\]

with \(d(t) = \varphi(t)g_{ME}\). The quantity \((\vec{\sigma}_i \times \vec{\sigma}_{i+1})^z\) is known as the \(z\) component of the vector chirality. With this notation \(\hat{H}_{SF}(t)\) reads

\[
\hat{H}_{SF}(t) = -\varphi(t) \cdot \vec{P} = d(t) \sum_i (\sigma^x_i \sigma^y_{i+1} - \sigma^y_i \sigma^x_{i+1}).
\]  

(9)
The quantum Otto cycle consists of two quantum isochoric and two adiabatic strokes. The quantum isochoric strokes correspond to a heat exchange between the working substance and the cold and the hot heat baths. During the quantum isochoric strokes the level populations are altered, see Fig. 1.

![Scheme of the quantum Otto cycle based on a chiral multiferroic chain. Adapted from 10](image)

The MF working substance produces work during the adiabatic process. Changing the amplitude of the applied external electric field modifies the energy spectrum of the system. This is the mechanism behind producing work. The quantum Otto cycle and the MF-based engine is detailed in recent works. A particular type of the time dependence for the external electric field is given by

\[
d(t) = \epsilon \left( \frac{t^3}{3\tau} - \frac{t^2}{2} \right) + d_0,
\]

which ensures that the requirement for the shortcuts of adiabaticity

\[
\hat{H}_{CD}(0) = \hat{H}_0(0), \quad \hat{H}_{CD}(\tau) = \hat{H}_0(\tau),
\]

is fulfilled. The power output of the quantum Otto cycle is given by

\[
\mathcal{R} = \frac{-\langle W_2 \rangle_{\text{ad}} - \langle W_4 \rangle_{\text{ad}}}{\tau_1(T_H) + \tau_2 + \tau_3(T_L) + \tau_4}.
\]

Here \(\tau_1(T_H), \tau_3(T_L)\) are the relaxation times of the MF working substance in contact with the hot and the cold thermal baths (strokes 1 \(\rightarrow\) 2, and...
3 \rightarrow 4), \tau_2 \text{ and } \tau_4 \text{ correspond to the duration of the adiabatic strokes, } \\
\langle W_2 \rangle_{\text{ad}} \text{ and } \langle W_4 \rangle_{\text{ad}} \text{ correspond to the work produced during the quantum adiabatic strokes. The condition} \\
\langle W_2 \rangle_{\text{ad}} + \langle W_4 \rangle_{\text{ad}} + Q_{\text{in}} + Q_{\text{out}} = 0, \\
during the whole cycle should be satisfied. The corresponding absorbed heat \(Q_{\text{in}}\) \text{ and the released heat } \(Q_{\text{out}}\) \text{ by the working substance are defined as follows} \\
\begin{align*}
Q_{\text{in}} &= \sum_n E_n(0) \left( \frac{e^{-\beta_H E_n(0)}}{\sum_n e^{-\beta_H E_n(0)}} - \frac{e^{-\beta_L E_n(\tau)}}{\sum_n e^{-\beta_L E_n(\tau)}} \right), \\
Q_{\text{out}} &= \sum_n E_n(\tau) \left( \frac{e^{-\beta_L E_n(\tau)}}{\sum_n e^{-\beta_L E_n(\tau)}} - \frac{e^{-\beta_H E_n(0)}}{\sum_n e^{-\beta_H E_n(0)}} \right). 
\end{align*}
\tag{12}

We adopt the dimensionless parameters \\
\(J_1 = 1, J_2 = -1, B = 0.1, d_0 = 2.5, \epsilon = 1.\)

In explicite units these parameters correspond to the one phase MF material\(^{56}\) \(\text{LiCu}_2\text{O}_2\), \(J_1 = -J_2 = 44\,[[\text{K}]].\) The external driving fields strengths \(B = 3\,[[\text{T}]], \varphi = 5 \times 10^3\,[[\text{kV/cm}]].\) We assume that the duration of the adiabatic strokes of the cycle are equal to \(\tau_2 = \tau_4 = \tau.\) The time unit in our calculations corresponds to the \(\hbar/J_1 \approx 0.1\,[[\text{ps}]].\) CD driving allows reducing \(\) the driving time. Implementing a short driving protocol is supposed to maximize the output power of the cycle. Duration of the isochoric strokes 

We supplement the CD Hamiltonian \(\hat{H}_{\text{CD}}(t)\) by the Hamiltonian of the heat bath \(\hat{H}_{\text{bath}}\) and the system-bath interaction \(\hat{H}_{\text{int}}.\) In addition, we assume that the phononic heat bath is coupled to the \(z\) component of the vector chirality \(K_n^z = (\sigma_n^x\sigma_{n+1}^y - \sigma_n^y\sigma_{n+1}^x).\) The argument behind this doing is that the vector chirality is a characteristic measure for the non-collinearity in the spin order and is directly influenced by the lattice distortion and the phononic modes

\begin{align*}
\hat{H} &= \hat{H}_{\text{CD}}(t) + \hat{H}_{\text{int}} + \hat{H}_{\text{bath}}, \\
\hat{H}_{\text{bath}} &= \int dk \omega_k \hat{b}_k^{\dagger} \hat{b}_k, \\
\hat{H}_{\text{int}} &= \sum_{n=1}^4 K_n^z \int dk g_k (\hat{b}_k^{\dagger} + \hat{b}_k). 
\end{align*}
\tag{13}
Here $\hat{b}^{\dagger}_{k}, \hat{b}_{k}$ are the phonon creation and annihilation operators, and $g_{k}$ is the coupling constant between the system and the bath. After a straightforward derivations we obtain

$$\frac{d\rho_{S}(t)}{dt} = \sum_{\omega, \omega'} \sum_{\alpha, \gamma} e^{i(\omega - \omega')t} \Gamma(\omega) \langle K_{\alpha}^{\dagger}(\omega) \rho_{S}(t) K_{\alpha}^{\dagger}(\omega') \rangle$$

$$- K_{\alpha}^{\dagger}(\omega') \langle K_{\beta}(\omega) \rho_{S}(t) \rangle + h.c.,$$

$$\Gamma(\omega) = \int_{0}^{\infty} d\omega' s e^{i\omega' s} \langle B^{\dagger}(t) B(t - s) \rangle.$$

Here $B(t) = \int dkg_{k}(\hat{b}^{\dagger}_{k} e^{i\omega k t} + \hat{b}_{k} e^{-i\omega k t})$, $K_{\alpha}^{\dagger}(\omega) = \sum_{E_{m} \rightarrow E_{n}} \pi(E_{n}) K_{\alpha}^{\dagger}(\omega)$

and $\pi(E_{n}) = |\Psi_{n}\rangle\langle \Psi_{n}|$ is the projection operator onto the eigenstates $|\Psi_{n}\rangle$ of the CD Hamiltonian. For the bath correlation functions $\Gamma(\omega)$ we deduce

$$\gamma(\omega) = \Gamma(\omega) + \Gamma^{*}(\omega),$$

$$\gamma(\omega) = \pi J(\omega) \begin{cases} \frac{1}{\exp[\beta|\omega|] - 1}, & \omega < 0 \\ \frac{1}{\exp[\beta|\omega|] - 1} + 1, & \omega > 0 \end{cases}.$$ (15)

Here $J(\omega) = \frac{1}{2} \sum_{j} g_{j}^{2} \delta(\omega - \omega_{j}) = \pi \gamma$ is the spectral density of the thermal bath\(^{57}\). The efficiency of the cycle reads $\eta = \frac{\delta Q_{H} + \delta Q_{c}}{\delta Q_{H}}$.

7. Summary

Shortcuts to adiabaticity allows realizing transitionless fast quantum adiabatic dynamics to achieve a finite power output from a quantum engine. We derived analytical results for the mean square fluctuation for the work, the irreversible work and output power of the cycle. We observed that the work mean square fluctuations are increasing with the duration of the adiabatic strokes $\tau$ (cf. Fig. 2). The irreversible work exhibits a non-monotonic behavior (see Fig. 3) and has a maximum for $\tau = 0.26(\text{ps})$. At the end of the adiabatic stroke the irreversible work becomes zero confirming so that the cycle is reversible. We found that the quantum heat engine with a MF working medium has an optimal duration corresponding to the largest power output (see Fig. 4)

By implementing a Lindblad master equation we studied the thermal relaxation of the system. We evaluated the transferred heat $\delta Q_{H}$ to the
Fig. 2. Standard deviation of the work $\Delta W_{ad}$ in scaled units for two different hot and cold bath temperatures. The other parameters are: $\varepsilon = 1$, $J_1 = 1$, $J_2 = -1$, $B = 0.1$, $d_0 = 2.5$. Unscaled unit of $\Delta W_{ad}$ amounts to $6 \times 10^{-22} [J]$. Adapted from\textsuperscript{10}.

Fig. 3. $\langle W_{ir} \rangle$ for the values of the parameters $J_1 = 1$, $J_2 = -1$, $B = 0.1$, $d_0 = 2.5$. Adapted from\textsuperscript{10}.
working substance and the heat released by system to the cold bath $\delta Q_c$. We inferred a cycle efficiency of $\eta = 1 + \delta Q_c/\delta Q_H \approx 47\%$. If the system thermalizes to the Gibbs ensemble the efficiency is lower at $\eta \approx 23\%$ (see Fig. 5).

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Fig. 5. A complete quantum Otto cycle (a) using the level population corresponding to Gibbs distribution and (b) the level population obtained from the Lindblad master equation for the parameters $\gamma = 0.1, T_H = 40, T_L = 10, d_0 = 2.5$ and $d_1$ as in the figures. Adapted from $^{10}$.

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