Gravitational Lenses, the Distance Ladder and the Hubble Constant: 
A New Dark Matter Problem

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ABSTRACT

In cold dark matter models, a galaxy’s dark matter halo is more spatially extended than its stars. However, even though the five well-constrained gravitational lenses with time delay measurements must have similar dark matter distributions, reconciling the Hubble constant estimated from their time delays with local estimates is possible only if that dark matter distribution is as compact as the luminous galaxy. The Hubble constant is $H_0 = 48^{+7}_{-4}$ km/s Mpc (95% confidence) if the lenses have flat rotation curves and $H_0 = 71^{+6}_{-7}$ km/s Mpc (95% confidence) if they have constant mass-to-light ratios, as compared to $H_0 = 72 \pm 8$ km/s Mpc (68% confidence) for local estimates by the HST Key Project. Either all five $H_0$ estimates based on the lenses are wrong, local estimates of $H_0$ are too high, or dark matter distributions are more concentrated than expected. The average value for $H_0$ including the uncertainties in the mass distribution, $H_0 = 62 \pm 7$ km/s Mpc, has uncertainties that are competitive with local estimates. However, by selecting a value for $H_0$ you also determine the dark matter distribution of galaxies.

Subject headings: cosmology: gravitational lensing; cosmology: Hubble constant; dark matter

1. Introduction

Gravitational lens time delay measurements can determine the Hubble constant $H_0$ given a model for the gravitational potential of the lens galaxy (Refsdal [1964]). The number of accurate delay measurements has begun to grow rapidly (see Schechter [2000]), leading to a sample of nine lenses. Five of these systems have time delay measurements, relatively simple environments, and a dominant lens galaxy with regular isophotes (RXJ0911+0551, Burud [2002b]; PG1115+080, Schechter et al. [1997]; Barkana [1997]; Impey et al. [1998]; SBS1520+530, Burud [2002b]; Faure et al. [2002]; B1600+434, Burud et al. [2000]; Koopmans et al. [2000]; and HE2149–2745, Burud et al. [2002]). The remaining four systems either have poorly determined properties (the lens positions in B0218+357 and PKS1830–211, see Lehar et al. [2000]; Winn et al. [2002]; Courbin et al. [2002]), or more complicated lenses (interacting galaxies in B1608+656, Koopmans & Fassnacht [1999]; a
brightest cluster galaxy in Q0957+561, see Keeton et al. (2000). With a sample of five relatively clean systems, we have reached the point where gravitational lenses become a serious alternative to the local distance ladder for the determination of \( H_0 \).

The determination of \( H_0 \) from a gravitational lens time delay requires a model for the mass distribution of the lens galaxy. In both models of particular time delay lenses (e.g. Keeton & Kochanek 1997, Impey et al. 1998, Koopmans & Fassnacht 1999, Lehar et al. 2000, Keeton et al. 2000, Winn et al. 2002) and from general analytic principles (Refsdal & Surdej 1994, Witt, Mao & Schechter 1995, Witt, Mao & Keeton 2000, Rusin 2000, Zhao & Pronk 2001, Wucknitz 2002, Oguri et al. 2002) we know there is usually a degeneracy between the radial mass distribution in the lens and the estimate of \( H_0 \), in the sense that more compact mass distributions lead to larger Hubble constants for a fixed time delay. This degeneracy can be broken by finding additional constraints (e.g. host galaxies, Kochanek, Keeton & McLeod 2001), or by requiring the mass distribution to be consistent with that of other lenses (e.g. Munoz, Kochanek & Keeton 2001), weak lensing studies (e.g. Guzik & Seljak 2002), local estimates from stellar dynamical observations (e.g. Rix et al. 1997, Romanowsky & Kochanek 1999, Gerhard et al. 2001, Treu & Koopmans 2002), X-ray observations (e.g. Fabbiano 1989, Lowenstein & White 1999) or theoretical models (e.g. Kochanek & White 2001, Keeton 2001). All these direct estimates support a dark matter dominated model in which the lenses have flat or slowly declining rotation curves near the Einstein ring.

We can also reverse the problem and simply determine the mass distribution the lenses must have in order to agree with local estimates of \( H_0 \) (e.g. Rusin 2000). We will use the measurement by the HST Key Project (Freedman et al. 2001) of \( H_0 = 72 \pm 8 \) km/s Mpc as our fiducial, independent estimate for \( H_0 \). There are two caveats about using this constraint. First, the Key Project estimate is significantly higher than the estimate of \( H_0 = 59 \pm 6 \) km/s Mpc by Saha et al. (2001) based on their Cepheid calibration for a sample of galaxies with Type Ia supernovae. Second, the uncertainties in both estimates are dominated by non-Gaussian systematic errors such as photometric calibrations, the distance to the LMC, Cepheid metallicity corrections and blending rather than statistical uncertainties (see the discussion in Freedman et al. 2001).

Even if the individual time delay lenses provide no direct constraints on their dark matter distributions, our general understanding of galaxies and the CDM paradigm set some broad limits. In our analysis we consider two limiting possibilities for the mass distributions of the gravitational lenses. The first model is a dark matter model with a flat rotation curve. These models set a lower bound on \( H_0 \) unless typical galaxies have rising rotation curves on the spatial scales of Einstein rings (typically 1.0–1.5\( R_e \)). The second model is a constant mass-to-light ratio (constant \( M/L \)) model based on the photometry obtained by the CfA/Arizona Space Telescope Lens Survey (CASTLES, Falco et al. 2001). No galaxy models in a cold dark matter (CDM) dominated universe should have mass distributions which are more centrally concentrated than the luminosity distribution (e.g. Mo, Mao & White 1998). Hence, the constant \( M/L \) models set an upper bound on \( H_0 \) (Impey et al. 1998). In §2 we discuss the data and our analysis methods.
In §3 we present the estimates of $H_0$ for the two limiting mass distributions and compare them to the local estimates. We are forced to conclude that there is a problem in either the lens estimates of $H_0$, the local estimates of $H_0$ or our understanding of dark matter distributions.

2. The Available Data

There are nine lenses with reasonably accurate time delay measurements (see Schechter 2000, Burud 2002b) of which we consider only five: RXJ0911+0551, PG1115+080, SBS1520+530, B1600+434 and HE2149–2745. The four lenses we do not include in our analysis are B0218+357, Q0957+561, B1608+656 and PKS1830–211. We first discuss why we used only five of the nine lenses. Then we provide a brief synopsis of the data and previous models of the five lenses we include in our analysis.

In B0218+357 and PKS1830–211 the problem lies in making an unambiguous measurement of the lens galaxy position relative to the images because there is a strong degeneracy between the lens position and the value of $H_0$ even for a fixed mass distribution (see Lehar et al. 2000). In B0218+357 the small image separation (0′′.35) made it impossible to reliably determine the lens position from existing HST images (Lehar et al. 2000), with the best estimates favoring very low values of $H_0$. Using the centroid of the radio ring leads to a higher value of $H_0$ (Biggs et al. 1999), but the center of an Einstein ring is more closely related to the position of the source than of the lens (see Kochanek et al. 2001). Wucknitz (2001) estimated the lens position by modeling the radio ring. At this position the delay implies very high values of $H_0$, but the models are inconsistent with the VLBI data for the radio cores. Given these problems, we do not include B0218+357 in our analysis. A new attempt to measure the position directly is planned using the Advanced Camera for Surveys (ACS, Cycle 11, PI N. Jackson).

While we are confident of our estimate of the lens position in PKS1830–211 (Winn et al. 2002), an independent analysis of the same data by Courbin et al. 2002 suggests an alternate interpretation involving multiple lens components. Winn et al. (2002) find that the lens is a single spiral galaxy with a well-determined position leading to estimates of $H_0$ very similar to those for the five lenses we will analyze here ($H_0 = 44 \pm 9$ km/s Mpc for an SIE model of PKS1830–211). The quality of the images makes it impossible to derive an accurate photometric model for the galaxy, so we cannot produce an estimate of $H_0$ under the assumption of a constant mass-to-light ratio. If, on the other hand, the multiple lens interpretation of Courbin et al. (2002) is correct, then the PKS1830–211 system is useless for estimates of $H_0$ given the available data. Deeper HST images of the system could resolve the problem, but none are currently scheduled.

We neglect Q0957+561 and B1608+656 because they are compound lenses with many more parameters than constraints. The Q0957+561 lens is dominated by a brightest cluster galaxy sitting near the center of its cluster (see Chartas et al. 2002). Keeton et al. (2000, also Bernstein et al. 1997) were able to use the properties of the lensed images of the quasar host galaxy to show
that all earlier models of the system were inconsistent with the observed structure of the host galaxy, but were unable to use the host galaxy to obtain a robust estimate of $H_0$. The structure of the host requires a cluster near the lens galaxy, as confirmed by the Chartas et al. (2002) X-ray observations, and it also requires that the lens galaxy dominate the image splitting. Essentially all earlier estimates of $H_0$ using Q0957+561 violated these two requirements (see Keeton et al. 2000). Very deep infrared images planned for HST Cycle 11 should provide the constraints needed to make reliable estimates of $H_0$ from this system.

In B1608+656 the lens consists of two interacting galaxies lying inside the Einstein ring defined by the images (see Koopmans & Fassnacht 1999). The two galaxies have irregular isophotes, due to the interactions and the presence of dust, and the position estimates for the two galaxies depend on the wavelength of the observation (see Koopmans & Fassnacht 1999, Surpi & Blandford 2001). Dark matter models by Koopmans & Fassnacht (1999) find higher values of $H_0$ than for the lenses we analyze, with $H_0 = 66 \pm 7$ km/s Mpc for their standard model, although they obtain a broader range of values under different assumptions for the position of the two lens galaxies and the degree to which the model should be constrained by the observed orientations and ellipticities. If the luminosity distributions are complex, it seems reasonable to expect complex mass distributions which may not be well described by standard mass models. This system may be better modeled using the non-parametric approach of Williams & Saha (2000), which can include the effects of transient mass distributions produced by the merger. For the more regular lenses, the non-parametric solutions must be employed with care because the positive surface density constraint used to limit parameter space allows too much freedom in the models (permitting negative density distributions, negative distribution functions or dynamically unstable solutions). This is in contrast to standard parametric models, which are guaranteed to correspond to physical dynamical models at the price of restricting the freedom in the mass distribution. While we regard our reasons for neglecting B1608+656 as legitimate, it is the one system we drop whose properties may run counter to our general discussion.

We now discuss the five systems (RXJ0911+0551, PG1115+080, SBS1520+534, B1600+434 and HE2149−2745) we include in our analysis.

We fit RXJ0911+0551 based on the CASTLES photometric data, the Burud (2002) delay of 150 ± 6 days (68% confidence) between the A-C cusp images and image D, and the Morgan et al. (2001) centroid for the nearby X-ray cluster $12^\circ 0 \pm 3^\prime 0$ East and $-39^\circ 8 \pm 3^\prime 0$ South of image B. The lens galaxies were modeled by two de Vaucouleurs components. The primary lens has a major axis effective radius of $0^\prime 77 \pm 0^\prime 07$, a major axis position angle of $-39^\circ \pm 15^\circ$ and an axis ratio of 0.79 ± 0.06. The satellite galaxy (only 8% of the H-band flux) has a major axis effective radius of $0^\prime 36 \pm 0^\prime 14$, and is indistinguishable from round in the available data. To simplify the fits we used a round satellite in both the constant $M/L$ and dark matter lens models. Test runs showed this assumption had no significant effects on the estimates of $H_0$. The isophotes of the two components are well separated and regular (unlike B1608+656).
The gravitational potential of the cluster near RXJ0911+0551 has significant effects on the estimates of $H_0$. We modeled the cluster as a singular isothermal sphere (SIS), finding reasonable fits for cluster velocity dispersions of 1100 km/s for the dark matter models and 1250 km/s for the constant $M/L$ models. The SIS models are in reasonable agreement with the 840 ± 200 km/s velocity dispersion of the galaxies (Kneib, Cohen & Hjorth 2000, see Burud 2002b). An SIS model has equal shear and convergence, $\kappa_c = \gamma_c = b_c/2r \simeq 0.25$ where $b_c$ is the cluster critical radius and $r$ is the distance to the cluster center. We could obtain Hubble constants $1/(1 - \kappa_c) \simeq 0.38$ higher than those for the SIS models by assuming a very compact cluster with $\kappa_c = 0$ at the location of the lens because of the mass sheet degeneracy in the models (Falco, Gorenstein & Shapiro 1985). Thus, our RXJ0911+0551 models really supply the likelihood for $H_0(1 - \kappa_c)/(1 - \kappa_{SIS})$ where $\kappa_{SIS}$ is the convergence for the SIS model and $\kappa_c$ is the convergence for the true cluster density profile, both measured at the lens. Like the lens galaxies, the cluster rotation curve should also be flat or falling on these scales and must have positive density, so $0 \leq \kappa_c \leq \kappa_{SIS}$. This means that we can use RXJ0911+0551 only to obtain lower bounds on $H_0$ or to estimate the true cluster surface density $\kappa_c$.

We fit PG1115+080 based on the photometric data from Impey et al. (1998) and the time delay estimates by Barkana (1997) based on Schechter et al. (1997). The longest delay is 25 ± 4 days (95% confidence). The model consists of a de Vaucouleurs primary lens galaxy and a perturbing isothermal group. Kochanek et al. (2001) found that the structure of the Einstein ring image of the quasar host galaxy weakly (2σ) ruled out constant $M/L$ models in favor of models with a flat rotation curve and lower values of $H_0$. Historically, the low estimates of $H_0$ from PG1115+080 have been regarded as anomalous (e.g. see Koopmans & Fassnacht 1999). Our models reproduce the earlier estimates by Impey et al. (1998).

We fit SBS1520+530 based on the CASTLES photometric data (see also Faure et al. 2002) and the Burud (2002b) delay of 130 ± 3 days (68% confidence). The lens was constrained to be a de Vaucouleurs model with a major axis effective radius of $R_e = 0''.60 \pm 0''.02$, a major axis position angle of $-26^\circ \pm 2^\circ$ and an axis ratio of $0.46 \pm 0.04$. The models included an external shear restricted by a Gaussian prior to the range $\gamma = 0.05 \pm 0.05$ (see the discussion of HE2149–2745).

We fit B1600+434 based on photometric fits to the CASTLES data and the optical time delay of 51 ± 4 days (95% confidence) measured by Burud et al. (2000). The optical delay agrees with the radio delay of 57 ± 6 days (68% confidence) measured by Koopmans et al. (2000) but it has smaller uncertainties. In the constant $M/L$ models we did not constrain the mass ratio of the disk and the bulge, which allowed the models more freedom than strictly necessary. The constant $M/L$ model consisted of a de Vaucouleurs bulge and an exponential disk forced to have a common centroid and major axis position angle ($44^\circ \pm 6^\circ$). The bulge (disk) has a major axis scale length of $0''.45 \pm 0''.06$ ($0''.84 \pm 0''.06$) and an axis ratio of $0.69 \pm 0.05$ ($0.16 \pm 0.06$). These values are broadly consistent with the estimates by Maller et al. (2000), differing mainly in our requirement that the two components share a common orientation. The dust lane in the lens galaxy makes its position relatively uncertain, and we adopt the position of $(\Delta RA = -0''.13 \pm 0''.05, \Delta Dec = 0''.21 \pm 0''.05)$
from image B used by Koopmans, de Bruyn & Jackson (1998). The estimates for $H_0$ in Burud et al. (2000) and Koopmans et al. (2000) used hybrid (disk + bulge + halo) mass models developed by Koopmans et al. (1998) and Maller et al. (2000). These models give values of $H_0$ between our dark matter and constant $M/L$ limiting models for the same lens position.

We fit HE2149–2745 based on the CASTLES photometric data and the time delay of $103 \pm 12$ days (68% confidence) reported by Burud et al. (2002). The model consists of a de Vaucouleurs primary lens galaxy in a perturbing external (tidal) shear. We must always allow for a local shear in realistic models (see Keeton, Kochanek & Seljak 1997, Kochanek 2002), although it has little effect on the estimate of $H_0$ ($\delta H_0/H_0 \sim \gamma$, see Witt et al. 2000). The external shear is constrained by a weak prior ($\gamma = 0.05 \pm 0.05$) to the typical range observed in four-image lenses (see Keeton et al. 1997, Kochanek 2002). In RXJ0911+0551, PG1115+080 and B1600+434, the shear perturbations were supplied by the nearby group or galaxy.

Burud et al. (2002) modeled the system using pseudo-Jaffe models (Keeton & Kochanek 1997, Munoz et al. 2001) to explore the dependence of $H_0$ on the radial mass distribution. They adopted a break radius for the density distribution such that the ellipticities of the mass and the light agreed leading to an estimate of $H_0 = 66 \pm 8$ km/s Mpc. This criterion is arbitrary because we only have evidence for the alignment of the mass and the light in lens galaxies rather than for a similarity in shape (see Kochanek 2002). Based on theoretical arguments and other observations that halos tend to be rounder than the luminous galaxies (see Sackett 1999), we can plausibly use the observed ellipticity only as an upper bound on the models. Models of HE2149–2745 with more dark matter (a larger break radius) are rounder, just as we might expect, and allow significantly lower values of $H_0$. The addition of the external shear further loosens any constraint associated with the ellipticity of the lens.

We modeled these five lenses using the lensmodel package (Keeton 2001). Each lens was fit using either a dark matter model or a constant $M/L$ model. The dark matter model used a singular isothermal ellipsoid (SIE) to represent the overall mass distribution of the system. The constant $M/L$ models used ellipsoidal de Vaucouleurs models and exponential disks constrained by the photometric model for the galaxies. We use the photometry only as a constraint on the mass distribution – the masses of the components are determined by the lens models with no constraints based on estimates of stellar mass-to-light ratios. Perturbing galaxies and clusters were modeled as singular isothermal spheres (SIS). All models included either an SIS perturber (RXJ0991+0551, PG1115+080, B1600+434) or a constrained tidal shear (SBS1520+530, HE2149–2745). We assumed 20% errors in the image fluxes, where the uncertainties are dominated by systematic problems such as substructure (see Dalal & Kochanek 2002) rather than measurement uncertainties. We also adopt a minimum time delay error of 5% to encompass systematic errors such as convergence fluctuations from large scale structure (e.g. Seljak 1994, Barkana 1996). The goodness of fit was determined by a $\chi^2$ statistic measured on the lens plane for the images combined with the fit to the observed time delays. Both the dark matter and the constant $M/L$ models produce statistically acceptable fits to the image configurations all lenses except
RXJ0911+0551. For this system the constant \( M/L \) model is formally ruled out (\( \Delta \chi^2 = 20 \)), but this is in part an artifact of using a round satellite galaxy to speed the calculations. We use maximum likelihood methods to analyze the results by examining the likelihood ratio \( 2 \ln(L/L_{\text{max}}) = \chi^2(H_0) - \chi^2_{\text{min}}(H_0) \) for each model sequence. We adopt an \( \Omega_0 = 0.3 \) flat cosmological model for our analysis and discussion.

3. Results and Conclusions

As discussed in §1, we focused on the dark matter and constant \( M/L \) models because they represent the limiting cases for physically possible mass distributions given our current understanding of dark matter. Unless galaxies have rising rotation curves near the Einstein ring of the lens (typically 1.0–1.5 effective radii from the lens galaxy), the SIE models provide lower bounds on \( H_0 \) for a fixed time delay. Constant \( M/L \) models are the most compact mass distributions allowed for a gravitational lens, so they lead to firm upper bounds on the \( H_0 \) (see Impey et al. [1998]). Dark matter cannot be more centrally concentrated than the stars unless their are fundamental flaws in our understanding of cold dark matter. These are two extreme limits, and all our theoretical expectations and estimates of mass distributions in other lenses or by other means suggest that reality must be closer to the dark matter model than to the constant \( M/L \) model. Our results, in the form of model likelihoods as a function of the Hubble constant, are shown in Figures 1 and 2.

Under either assumption about the mass distribution, the five lenses produce consistent estimates for \( H_0 \) (see Figure 1). If we exclude RXJ0911+0551, because of the added uncertainty created by the dark matter distribution of the nearby cluster (see below), we find that \( H_0 = 48^{+7}_{-4} \) km/s Mpc (95% confidence) for the dark matter model and \( H_0 = 71 \pm 6 \) km/s Mpc (95% confidence) for the constant \( M/L \) model. The jackknife and bootstrap error estimates are comparable to those from the combined likelihoods. Because the \( H_0 \) estimates for the lenses are very similar when we assume they have the same radial mass profiles, the radial mass distributions must be similar. For example, for potentials \( \phi \sim R^\beta \), where \( \beta = 1 \) is the isothermal profile we use for our dark matter model and the limit \( \beta \to 0 \) is a point mass, estimates of the Hubble constant roughly scale as \( H_0 \propto (2 - \beta) \) (Witt et al. 2000). If we assume that three of the four lenses have the dark matter (\( \beta = 1 \)) profile, then we find that the slope for the remaining lens must be \( \beta = 0.83^{+0.18}_{-0.16}, 1.09^{+0.09}_{-0.21}, 1.19^{+0.11}_{-0.15} \) and \( 0.98^{+0.22}_{-0.28} \) (68% confidence) for PG1115+080, SBS1520+530, B1600+434 and HE2149–2745 respectively. The jackknife estimate for the variance in the slopes

\(^{1}\)All cited values of \( H_0 \) from the lenses are scaled to this cosmological model. While the \( H_0 \) estimates from lenses do depend on the cosmological model (Refsdal 1966), the magnitude of the variations is too small to be relevant to our discussion. A demonstration that the cosmological model estimated by lens time delay measurements agrees with other estimates will be important as part of any claim that the errors in the \( H_0 \) measurements are smaller than about 5%. As an experiment, we estimated \( \Omega_m \) for flat cosmological models based on the estimates of \( H_0 \) for the dark matter models, finding that \( \Omega_m = 1.0 \) is ruled out compared to our standard model at 0.1\( \sigma \)!
Fig. 1.— Maximum likelihood estimates for $H_0$.

(Bottom) The likelihood functions for the constant $M/L$ models of the individual lenses. Obtaining higher values for $H_0$ than the constant $M/L$ models requires dark matter distributions which are more compact than the luminosity distribution. The dashed curve is RXJ0911+0551.

(Middle) The likelihood functions for the dark matter (DM) models of the individual lenses. Obtaining lower values for $H_0$ than the DM models requires a mass distribution with a rising rotation curve at the Einstein ring of the lens (at $1-2R_e$). The dashed curve is RXJ0911+0551.

(Top) The upper panel shows the combined likelihood functions for the dark matter (DM) models (left solid/dash), the constant $M/L$ models (right solid/dash) and a Gaussian model for the local estimate by the HST Key project (dashed-dot). The solid (dashed) curves for the dark matter and constant $M/L$ models exclude (include) RXJ0911+0551.

The horizontal dashed lines show the 68% ($1\sigma$), 95% ($2\sigma$) and 99% ($3\sigma$) confidence limits based on the likelihood ratio.
Fig. 2.— Final likelihood estimates for $H_0$. The light solid line shows the estimate from the lens time delays assuming the mass distribution is bounded by the dark matter and constant $M/L$ models but we are unable to discriminate between any intermediate model. The lower bounds are due to the dark matter models and the upper bounds are due to the the constant $M/L$ models. RXJ0911+0551 is included assuming the cluster surface density is bounded by $\kappa_{SIS} \geq \kappa_c \geq 0$. The dashed line is the HST Key Project estimate and the heavy solid line is the joint likelihood. Compared to the mean and variance of $H_0 = 72 \pm 8$ km/s Mpc of the Key Project, the lenses have $H_0 = 62 \pm 7$ km/s Mpc and the joint likelihood has $H_0 = 67 \pm 5$ km/s Mpc. The variance in the $H_0$ estimate differs from the 68% confidence region of 53 km/s Mpc $< H_0 < 73$ km/s Mpc because the likelihood distribution is not Gaussian. The problem with the region permitted by the joint likelihood is that it corresponds to models with little dark matter compared to the expectations for CDM.
is $\sigma_\beta \simeq 0.23$, roughly half of which is due to the measurement uncertainties. The intrinsic scatter must be relatively small ($\sigma_\beta \simeq 0.15$). As a result, our estimates of $H_0$ vary little if we use any three of the four lenses (which is unfortunate, see the Appendix).

Despite the presence of the cluster, RXJ0911+0551 agrees with this general picture. If we assume that the lens galaxy in RXJ0911+0551 has a similar structure to the other lens galaxies, then we can estimate the dark matter surface density in the cluster from the requirement that all five systems must agree on the value of $H_0$. We find that the cluster convergence is $\kappa_c = 0.24 \pm 0.08$ for the dark matter models and $\kappa_c = 0.25 \pm 0.06$ for the constant $M/L$ models. These values are very similar to the SIS convergences of $\kappa_{SIS} = 0.25$ and 0.31, as we might expect when the lens lies $200 h^{-1}$ kpc from the cluster center where the cluster density profile is close to $\rho \sim r^{-2}$ for typical models (e.g. Bullock et al. 2001).

Unfortunately, while the time delay lenses must have very similar mass profiles, we cannot determine which profile is appropriate given the available data. We can try to break the degeneracy by using an independent estimate of $H_0$ (e.g. Rusin 2000), in particular the local estimate by the HST Key Project that $H_0 = 72 \pm 8$ km/s Mpc (Freedman et al. 2001). The agreement between the lenses and the Key Project is very good for the constant $M/L$ models and terrible for the dark matter models (see Figure 1). Even with the freedom to adjust the cluster it is difficult to reconcile the dark matter model of RXJ0911+0551 with the Key Project estimate because it requires a negative cluster surface density ($\kappa_c = -0.13 \pm 0.20$). The local estimate breaks the degeneracy, but it also means that the lens galaxies cannot have massive, extended dark halos. Such a conclusion disagrees with the CDM picture of galaxies and the abundant evidence for halos in galaxies and gravitational lenses similar to the time delay systems.

There are three possible solutions.

First, we may fundamentally misunderstand the distribution of dark matter in galaxies. Our problem is the opposite of the more familiar “dark matter crisis” where models of dwarf and low surface brightness galaxy rotation curves seem to require dark matter distributions which are less centrally concentrated than theoretical predictions (see the review by Moore 2001). The issue for fitting rotation curves is the structure of central density cusps, with gravitational lenses favoring steeper cusps than those expected from CDM models rather than the shallower cusps suggested by the rotation curves (see Rusin & Ma 2001, Keeton 2001, Munoz et al. 2001). Whatever the structure of the cusps, all these models still have dark matter distributions which are less concentrated than the stars (baryons), while we need to make the dark matter at least as concentrated as the stars in order to solve the $H_0$ problem illustrated by Fig. 1. At present, the lensing constraints on the 5 systems cannot determine the radial mass distributions robustly. However, the lenses where we can make such estimates favor models close to the dark matter limit (see Munoz et al. 2001 and references therein), as do weak lensing studies (e.g. Guzik & Seljak 2002), local stellar dynamical observations (e.g. Rix et al. 1997, Romanowsky & Kochanek 1994, Gerhard et al. 2001, Treu & Koopmans 2002), and X-ray observations (e.g.
Second, the Key Project estimate that $H_0 = 72 \pm 8 \text{ km/s Mpc}$ could be too high or have significantly underestimated uncertainties. For example, the lensing results are more compatible with the estimate of $H_0 = 59 \pm 6 \text{ km/s Mpc}$ from Cepheid calibrations of the distances to Type Ia supernovae (Saha et al. 2001). Freedman et al. (2001) argue strongly against the Saha et al. (2001) analysis for a range of legitimate technical issues, which is why we adopted the Key Project estimate as our standard for local estimates. There are also arguments for significantly larger corrections for metallicity (e.g. Sasselov et al. 1997, Kochanek 1997) and blending (e.g. Mochejska et al. 2000) in the Cepheid distances. Finally, the standard error in the Key Project estimate of $H_0$ is dominated by a quadrature sum of systematic errors. This makes a very specific assumption about the probability distributions of the systematic errors, in the sense that conspiracies between the errors to systematically shift the estimate of $H_0$ in one direction are assumed to be very unlikely. A more conservative assumption about the behavior of systematic errors, such as a direct sum of the systematic errors rather than a quadrature sum, would lead to significantly larger uncertainties and less of a conflict with the estimates from the lenses. On the other hand, the check of the Cepheid distance scale against the maser distance to NGC4258 suggests that the Key Project distances may moderately underestimate $H_0$ (Newman et al. 2001).

The third possibility is that the estimates of $H_0$ from gravitational lenses are systematically low. Our result is a significant change from the previous analysis of the available lens sample by Koopmans & Fassnacht (1999), who found $H_0 = 68 \pm 7 \text{ km/s Mpc}$ for the dark matter models using B0218+357, Q0957+561, PG1115+080, B1608+656 and PKS1830–211. In our analysis we used only one of these 5 systems and added the four new systems RXJ0911+0551, SBS1520+534, B1600+434 and HE2149–2745. The high value found by Koopmans & Fassnacht (1999) was due to including the lenses we rejected as being unusable because the lens galaxy position was unknown or the system is an under-constrained compound lens (see §2 for a full discussion). For example, Koopmans & Fassnacht (1999) assign an estimate of $H_0 \approx 85 \text{ km/s Mpc}$ to PKS1830–211, while the best current estimate is $H_0 = 44 \pm 9 \text{ km/s Mpc}$ due to an improved, but disputed, direct estimate of the lens position (Winn et al. 2002, Courbin et al. 2002). The system with interacting lens galaxies, B1608+656, is the one system which may disagree with our present analysis.

Given that the five lenses we use are relatively clean, well constrained, and produce consistent results, it is unlikely that errors in the time delay measurements or the model constraints are responsible for the results. Nonetheless, the systems should be monitored further to both confirm the delay estimates and reduce the measurement errors to the point (under 5%) where they make a negligible contribution to the overall uncertainties. Deep, high resolution imaging to measure the structure of the lensed host galaxies will provide additional constraints on the lens models and can be used to determine the radial density profile (Kochanek et al. 2001). Lensed images of the host galaxy are seen in four of the current lenses with time delays (Q0957+561, Keeton et al. 2001, PG1115+080, Impey et al. 1998, B1600+434, Kochanek et al. 1999, and B1608+656, Koopmans & Fassnacht 1999). Additional HST observations are the key to including the rejected systems in
later analyses. The extra constraints from deep images of the host galaxies in Q0957+561 and B1608+656 may allow us to include them despite being compound lenses, and improved HST imaging of B0218+357 and PKS1830−211 can resolve the problems associated with the lens galaxy positions. The most important check on the results from the lenses is to measure precise time delays in more systems, particularly since the statistics of time delays provides another probe of the mass distribution (see Oguri et al. 2002). In order to avoid the current situation, in which 4 of 9 lenses with well-measured time delays are difficult to include in quantitative estimates of \( H_0 \), monitoring campaigns should focus on systems with one dominant lens galaxy having regular isophotes whose properties can be easily measured with HST observations.

We considered only the extreme limits for the radial mass distribution of the lenses, and we could obtain intermediate values of \( H_0 \) by slowly adding a dark matter halo to the constant \( M/L \) models. It is easy to mimic such models, and the results are shown in Figure 2.\footnote{For each lens, we set the likelihood ratio to be that of the dark matter (constant \( M/L \)) model for \( H_0 \) below (above) the Hubble constant corresponding to the model’s maximum likelihood. Since we are assuming no ability to distinguish the best fitting constant \( M/L \) and dark matter models, the likelihood ratios for all intermediate values of \( H_0 \) are set to unity. Thus, the likelihood distribution for each lens is a top hat with edges set by the physically limiting mass models softened by the uncertainties in the model fits.} Despite including the full, physically plausible range of uncertainties in the dark matter distributions, the uncertainties in the \( H_0 \) estimates from the lenses are comparable to those of the local distance scale. The mean and variance of the \( H_0 \) estimate from the lenses is \( H_0 = 62 \pm 7 \) km/s Mpc, compared to 72 ± 8 for the Key Project. The shapes of the distributions differ in detail because the systematic errors for the lenses were modeled as a top hat while those of the Key Project were modeled as Gaussians. Combining the two likelihood distributions gives \( H_0 = 67 \pm 5 \) km/s Mpc, which has reduced uncertainties but implies lens galaxies with little dark matter. Whatever the final resolution to the apparent conflicts, gravitational lenses are rapidly approaching the precision of the local distance scale methods of determining the Hubble constant despite the uncertainties and degeneracies in the present generation of models.

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A. Determining \( H_0 \) From Lens Samples With Inhomogeneous Mass Distributions

When we started this analysis, the motivation was to explore the consequences for the determination of \( H_0 \) from using time delay lenses having a range of radial mass distributions. This seemed a requirement since available summaries (e.g. Koopmans & Fassnacht 1999) favored...
giving PG11115+080 a tidally truncated halo so as to bring it into agreement with the remainder of the sample. However, once we defined the requirements for our clean lens sample, the need for dissimilar mass distributions evaporated. Far from being useful, this convergence of the mass distributions makes the problem harder because a sample of lenses with time delays and very different dark matter distributions can be used to estimate $H_0$ even though every individual lens suffers from a model degeneracy between the estimate of $H_0$ and the dark matter distribution.

The basic argument is easily understood using the simple analytic scalings of Witt et al. (2000) where there is a degeneracy between $H_0$ and the exponent of the density distribution, $\Delta t \propto (2 - \beta)/H_0$ for $\phi \propto R^\beta$ and $0 \leq \beta \leq 1$ covers the range from point masses to flat rotation curves. Suppose we have a sample of lenses whose true density exponents are $\beta_{\text{true},i}$. Given the true Hubble constant $H_{\text{true}}$, we measure time delays $\Delta t_i \propto (2 - \beta_{\text{true},i})/H_{\text{true}}$. We then model the systems assuming a total degeneracy in the lens models, finding an estimates for the Hubble constant of $H_i = H_{\text{true}}(2 - \beta)/(2 - \beta_{\text{true},i})$ for a model exponent of $\beta$. When all lenses have similar mass distributions, $\beta_{\text{true},i} \approx \beta_{\text{true},j}$, then the Hubble constant estimates suffer from the global degeneracy we observe in the current lens data – the requirement that all the lenses agree on the same value for the Hubble constant provides little leverage for separating the effects of $\beta$ and $H_0$ on the time delay.

If, however, the true mass distributions are widely scattered, then the simple requirement that all lenses must agree on the same value of $H_0$ allows us to determine $H_0$ even though every individual lens suffers from a complete degeneracy between $H_0$ and $\beta$ restricted only by the minimal requirement that the distribution is bounded by the limits of a flat rotation curve ($\beta = 1$) and a point mass ($\beta \to 0$). Each lens permits values of $H_0$ bounded by $H_{\text{true}}/(2 - \beta_{\text{true},i}) \leq H_0 \leq 2H_{\text{true}}/(2 - \beta_{\text{true},i})$ where the lower (upper) limit corresponds to a model with $\beta = 1$ ($\beta = 0$). If $0 \leq \beta_{\text{min}} \leq \beta_{\text{true},i} < \beta_{\text{max}} \leq 1$, then the ensemble of lenses restricts the Hubble constant to the range $H_{\text{true}}/(2 - \beta_{\text{max}}) \leq H_0 \leq 2H_{\text{true}}/(2 - \beta_{\text{min}})$. The fractional range of the estimates,

$$\frac{\Delta H}{H_{\text{true}}} = \frac{1}{2 - \beta_{\text{min}}} \left[ 1 - \frac{\beta_{\text{max}} - \beta_{\text{min}}}{2 - \beta_{\text{max}}} \right],$$

is always reduced by having a spread of mass distributions, becoming zero when the range of the true distributions matches that imposed on the models. Thus, it is better to have a mixture of lenses with and without dark matter than to have the homogeneous mass distributions the data seem to require.

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