Effect of the soil pressure and that of filtration forces on a triangular retaining wall

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Abstract. The filtration forces considered as the forces determining the form and the performance characteristics of full-scale hydro-engineering gravity structures have a significance comparable with that of the main effects: the hydrostatic pressure and the own mass of the structure. The examples of determination of the stress state caused by filtration forces through the modelling method include the recommendations on the replacement of the aforesaid effects by the surface loadings applied to the gravity centre of their projections. This method reduces the accuracy of the results and requires (like with the construction of backpressure lines) the tests of many models. The authors carried out the modelling of the effect of the soil pressure and that of filtration forces in accordance with the modelling methodology based on the equivalence of effects. The article contains a solution for a retaining wall with a triangular cross-section under the effect of the soil pressure on the vertical side and that of volume filtration forces.

1. Introduction

The breakdown of the soil mass stability is often accompanied by considerable destructions of roads, bridges, civil and industrial buildings and other structures; sometimes there may be victims there. Therefore it is necessary to develop and carry out some active measures against the landslides and other soil mass instabilities.

One of the methods of improvement of the soil mass stability is the use of retaining walls.

This article considers the effect of the soil pressure on the vertical side and that of volume filtration forces on a retacting wall with a triangular cross-section.

2. Problem statement

A well-known solution for a retaining wall with a triangular cross-section under the effect of the hydrostatic water pressure on the vertical side and that of volume forces of the own weight of the structure has the following form (the solution was obtained with the use of the function of stresses in the form of a polynomial of degree [1, 2]):

\[
\sigma_x^{(w,y)} = \left( \gamma_w \cot \alpha - \gamma \right) x + \left( \gamma_w \cot \alpha - 2 \gamma_w \cot^3 \alpha \right) y,
\]

\[
\sigma_y^{(w,y)} = -\gamma_w x,
\]

(1)
where $\gamma_w$ - water density;
$\gamma_1$ - density of the retaining wall material.

It is known that the soil pressure on the vertical side of a retaining wall with the assumption of plane slip surfaces is determined through the following formula \([3, 4, 5]\):

$$p_x = \gamma x \tan^2 \left(45^\circ - \left(\varphi / 2\right)\right),$$  \(2\)

where $\gamma$ - soil density;
$\varphi$ - internal friction angle.

The stress values in the retaining wall caused by the soil mass pressure (i.e. those for the saturated soils with free continuous water) on the vertical side may be determined through (1) substituting there $\gamma_w$ for $\gamma' \tan^2 \left(45^\circ - \left(\varphi / 2\right)\right)$ ($\gamma'$ - soil density with consideration of the water hydrostatic weighing) and assuming $\gamma_1 = 0$.

It is also known that the volume filtration forces $F$ are connected with the head pressure losses at the filtration path through the following formula \([6, 7, 8]\):

$$F = -\gamma_w \nabla H,$$  \(3\)

where $\gamma_w$ - water density;
$H$ - head pressure function.

The directions and the absolute values of these forces may be determined with a sufficient accuracy through the calculation or the use of some experimental methods \([9, 10]\).

An intricate distribution of volume filtration forces complicates the theoretical solution of the problem. That is why the work \([9]\) suggests to substitute the effect of volume filtration forces (when evaluating the structure weighing) by the static equivalent surface loading distributed in accordance with the diagram of pressures in water in the plane of the structure base.

3. Theoretical principles

According to the conditions of the equivalence of effects \([11, 12, 13]\), the volume filtration forces (3) may be substituted by the surface loading $P$ and the forced deformations $\xi$ given in the following form:

$$P = -\gamma_w H,$$  \(4\)

$$\xi = -\frac{1-2\nu}{E} \gamma_w H$$  \(5\)

Then the stresses caused by the volume filtration forces may be determined through the formula:

$$\sigma_{ij}^{(F)} = \sigma_{ij}^{(p)} - \sigma_{ij}^{(2)} - \delta_{ij} P$$  \(6\)

It is known that the head pressure function $H$ for a constant filtration flow satisfies to the Laplace’s equation, i.e. $\nabla^2 H = 0$. Then the function of forced deformations (5) also is a harmonic one in the retaining wall area, hence $\sigma_{ij}^{(2)} = 0$ and the formula (6) takes the form

$$\sigma_{ij}^{(F)} = \sigma_{ij}^{(p)} - \delta_{ij} P$$  \(7\)

Thus the determination of stresses caused by volume filtration forces is reduced to the determination of the stresses caused by the surface forces (4) (Figure 1a).

Let us determine the surface forces. Consider the stress function as a sum of polynomials of degrees 2 and 3. Then the expression for the stresses have the form

$$\sigma_x^{(p)} = c_1 x + d_1 y + c_2,$$
$$\sigma_y^{(p)} = a_1 x + b_1 y + a_2,$$
$$\sigma_{xy}^{(p)} = -b_1 x - c_1 y - b_2$$  \(8\)
The constants in the aforesaid formulas may be determined from the boundary values on vertical and inclined sides.

On the vertical side we have $y = 0$, $\sigma_y = -\gamma_w H$, $\tau_{xy} = 0$.

Using the formulas (8), we obtain $a_1 = 0$, $a_2 = -\gamma_w H$, $b_1 = 0$, $b_2 = 0$.

Thus, the formulas (8) for stresses will take the form

$$
\sigma_x^{(p)} = c_3 x + d_3 y + c_2, \\
\sigma_y^{(p)} = -\gamma_w H, \\
\tau_{xy}^{(p)} = -c_3 y
$$

The boundary conditions on the inclined side have the form

$$
-\sigma_x^{(p)} \sin \alpha + \tau_{xy}^{(p)} \cos \alpha = \gamma_w (H-x) \sin \alpha, \\
-\tau_{xy}^{(p)} \sin \alpha + \sigma_y^{(p)} \cos \alpha = -\gamma_w (H-x) \cos \alpha
$$

Substituting the expressions (9) into the aforesaid equations and considering the equation of the inclined side $y = x \tan \alpha$, we obtain

$$
- (c_3 x + d_3 x \tan \alpha + c_2) \sin \alpha - c_3 x \tan \alpha \cos \alpha = \gamma_w (H-x) \sin \alpha, \\
c_3 x \tan \alpha \sin \alpha - \gamma_w H \cos \alpha = -\gamma_w (H-x) \cos \alpha
$$

Solving these equations, we obtain $c_2 = -\gamma_w H$; $c_3 = \gamma_w \cot \alpha$; $d_3 = -2\gamma_w \cot^3 \alpha + \gamma_w \cot \alpha$

Let us write down the final expressions for the stresses caused by the substituting surface forces:

$$
\sigma_x^{(p)} = \gamma_w x \cot^2 \alpha + \gamma_w \cot \alpha \left(1 - 2 \cot^2 \alpha\right) y - \gamma_w H, \\
\sigma_y^{(p)} = -\gamma_w H, \\
\tau_{xy}^{(p)} = -\gamma_w y \cot^2 \alpha
$$

This solution may be obtained in a different way. Using the superposition principle for the forces, we can divide the surface loading $P$ into two parts (Figure 1b, c)

$$
P = p - \gamma_w (H-x),
$$

where $p$ – hydrostatic water pressure.

The solution corresponding to the hydrostatic water pressure has the form

$$
\sigma_x = \gamma_w x \cot^2 \alpha - 2\gamma_w y \cot^3 \alpha.
$$
\[
\begin{align*}
\sigma_y &= -\gamma_w \Delta x, \\
\tau_{xy} &= -\gamma_w \Delta y \tan^2 \alpha .
\end{align*}
\]

The solution corresponding to the second part of the surface loading obtained by analogy with the solution (10) with the use of the stress function in the form of the sum of polynomials of degrees 2 and 3 has the form

\[
\begin{align*}
\sigma_x &= \gamma_w \Delta x \tan \alpha - \gamma_w \Delta H , \\
\sigma_y &= -\gamma_w (H - \Delta x) , \\
\tau_{xy} &= 0
\end{align*}
\]

It is easy to prove that the sum of expressions (11) and (12) also gives the solution (10).

Then, according to (4) and (7), we obtain for the stresses caused by volume filtration forces

\[
\begin{align*}
\sigma_x^{(F)} &= \gamma_w \Delta x \tan \alpha + \gamma_w \Delta y \tan \left(1 - 2 \tan^2 \alpha \right) y , \\
\sigma_y^{(F)} &= 0 , \\
\tau_{xy}^{(F)} &= -\gamma_w \Delta y \tan^2 \alpha .
\end{align*}
\]

It is interesting to compare the stress diagrams for volume filtration forces with those for other effects (hydrostatic water pressure and soil pressure on the vertical side, own weight volume forces).

4. Research result

Fig. 2a, b presents the stress diagrams for volume filtration forces and those for hydrostatic water pressure and soil pressure on the vertical side with consideration of the own weight of the structure for the following initial data: \( \alpha = 30^\circ \), \( \phi = 30^\circ \), \( \gamma' = 10 kN / m^3 \), \( \gamma_w = 10 kN / m^3 \), \( \gamma_t = 20 kN / m^3 \). These stress diagrams correspond to a horizontal section situated at a given level \( x = \text{const} \) (\( 0 \leq x \leq H \)).

Figure 2. a - stress diagrams for volume filtration forces; b - stress diagrams of hydrostatic water pressure and soil pressure; c - diagrams the total stresses.
Comparing the corresponding stress diagrams, we come to the following conclusions.

The diagrams for all three constituent stresses are similar.

The stresses $\sigma_x$ on the vertical side are positive (stretching ones); the stresses caused by volume filtration forces are greater than those caused by the combined actions of hydrostatic water pressure, soil pressure and own weight volume forces. The stresses on the inclined side are otherwise (they are compressive ones and they are less).

The stresses $\sigma_y$ from volume filtration forces equal zero, while those from other considered actions are negative ones.

Tangential stresses from volume filtration forces are less than those from a combined action of hydrostatic water pressure, soil pressure and own weight volume forces. In addition, the tangential stresses on the vertical side equal zero and those on the inclined side have their maximum absolute values.

Figure 2c presents the diagrams for the total stresses from a combined action of hydrostatic water pressure, soil pressure, own weight volume forces and volume filtration forces.

5. Conclusions

The presented results also show that among the effects concerned the filtration volume forces have the significance comparable with that of other effects, so they should be considered in the process of solution of such problems.

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