INFLUENCE OF ADDITIONAL FERMIONS AND GAUGE BOSONS ON $\text{Re}(\epsilon'/\epsilon)$

PAUL H. FRAMPTON

Institute of Field Physics, Department of Physics and Astronomy, University of North Carolina, Chapel Hill, NC 27599-3255, USA

The CP violation parameter $\text{Re}(\epsilon'/\epsilon)$ has previously been calculated in the standard model and the result depends on the top quark mass in an exciting way. We consider how the result is modified in two specific extensions of the standard model. The first adds only fermions, an extra vector-like quark doublet, and tends to reduce $\text{Re}(\epsilon'/\epsilon)$. The second contains three extended families with the third family treated differently and additional gauge bosons - the 331 model - and can accommodate larger values of $\text{Re}(\epsilon'/\epsilon)$ for a given $m_t$. When the experimental values of $m_t$ and $\text{Re}(\epsilon'/\epsilon)$ are known more accurately, it will allow discrimination between theories. If the lower experimental number $\text{Re}(\epsilon'/\epsilon) = (0.74 \pm 0.60) \times 10^{-3}$ (Fermilab, E731) is confirmed and $m_t$ is near the value 175GeV the standard model is acceptable. But, if the higher value $\text{Re}(\epsilon'/\epsilon) = (2.3 \pm 0.65) \times 10^{-3}$ (CERN, NA31) persists, a dynamics like the 331 model where the third family is treated differently from the first two and there exist additional gauge bosons seems to be favored.

1 Introduction

Symmetry under the combined operation of C (charge conjugation) and P (parity) holds only approximately for weak interactions. Violation of CP was a surprising discovery made in 1964 in the neutral kaon system. Since then the effect still has not been seen in any other system and is parameterised by the quantities $\epsilon$ and $\text{Re}(\epsilon'/\epsilon)$. The first parameter is known experimentally quite accurately and given as:

$$\epsilon = (2.26 \pm 0.09) \times 10^{-3} \exp(i\phi(\epsilon)) \quad (1)$$

where

$$\phi(\epsilon) = 43.68 \pm 0.14 \text{degrees}. \quad (2)$$

The experimental measurements of the second parameter $\text{Re}(\epsilon'/\epsilon)$ are less settled as stated in the abstract above.

In 1973 the KM mechanism was introduced. Following that, a long series of estimates have been made of the quantity $\text{Re}(\epsilon'/\epsilon)$, until the late 1980’s working with the assumption $m_t < M_W$ and subsequently with the correct inequality. Given $m_t > M_W$, it was found that the electroweak penguin diagrams are large and tend to cancel the gluon penguin diagrams. The cancellation becomes complete for $m_t$ about 220GeV.

Thus the high $m_t$ leads in the standard model to a small value of $\text{Re}(\epsilon'/\epsilon)$ close to the zero value predicted by the superweak theory.

The first extension of the standard model we consider is the rather ordinary one where we simply add a fourth family with vector-like quarks. This does not effect agreement with precision electroweak measurements (S,T parameters). We calculate for this case including QCD and electroweak corrections.

The other extension we consider is the 331 model which contains new fermions - the exotic quarks D, S and T - and new gauge bosons - the dileptons $(Y^-, Y^+), (Y^{++}, Y^+)$ and a $Z'$. In this case, the calculation of $\text{Re}(\epsilon'/\epsilon)$ is far richer in the number of Feynman diagrams. We find a generalized GIM mechanism for the dileptons. The result is calculated as a function of the dimensionless ratios $x_D = M_D^2/M_Y^2$ (we always assume $M_D = M_S$) and $x_T = (M_L^2/M_T^2)$. The results depend sensitively on these ratios and on $m_t$.

The calculation involves several steps and theoretical uncertainties. While the short-range physics is well controlled by perturbation theory, the hadronic matrix elements involve confinement physics of QCD and are naturally less accurately estimated. Nevertheless, such uncertainties should not effect the general conclusions for specific models.

2 Description of the calculation of $\text{Re}(\epsilon'/\epsilon)$

Here we outline the basic steps needed in the calculation of $\text{Re}(\epsilon'/\epsilon)$ in a general model which may extend the standard model. For the calculation of the effective Hamiltonian at low energy (1GeV) we integrate out the heavy quarks by using RG equations. One calculates the Feynman diagrams at $M_W$ scale to find the initial values of the Wilson coefficients which are produced by the strong and electroweak corrections to the original $(\Delta S = 1)$ operators. Thus, after integrating the RGEs between quark thresholds and dropping heavy quarks at each stage, one gets an effective Hamiltonian at $\mu = 1\text{GeV}$ scale as
where $\lambda_i = V_{ts}^* V_{td}$, and the superscripts refer to the quark flavors. Substituting the effective Hamiltonian into the definition of $Re(\epsilon'/\epsilon)$ one obtains an expression in terms of the Wilson coefficients $C_i$ and the hadronic matrix elements of operators $Q_i$.

The calculation can be conveniently divided into four steps:

1. Finding the Wilson coefficients at the $M_W$ scale.
2. Bringing down the energy scale to $\mu < m_c$ from $\mu = M_W$ by using the renormalization group equations.
3. Estimating the hadronic matrix elements.
4. Calculating the phase parameter in the CKM matrix, $\delta$.

2.1 Initial value of Wilson coefficients

The original four-quark operator corresponding to $\Delta S = 1$ generates a new set of four-quark operators through the box diagrams, $B(x)$; the Z-penguin diagrams, $C(x)$; the photon penguin diagrams, $D(x)$; and the gluon penguins, $E(x)$. At one-loop order, one finds a complete set of four-quark operators generated by these diagrams and by additional diagrams, if any.

The expressions for the functions $B(x)$, etc. in the standard model as well as the resulting values for the Wilson coefficients $C_i(M_W)$ exist in the literature. We calculate them for the extensions of the standard model.

2.2 Wilson coefficients at $\mu = 1 \text{GeV}$

The evolution of the Wilson coefficients from the energy scale $\mu = M_W$ to energy scale $\mu = m_c$ are governed by the RGEs which are soluble in terms of an evolution operator which requires the evaluations of all the relevant anomalous dimensions.

2.3 Hadronic matrix elements.

The effective Hamiltonian after the decoupling of heavy quark states contains seven operators whose matrix elements need to be evaluated. This can be done using any one of several methods such as QCD sum rules, lattice gauge theories, factorization, or the $1/N$ expansion.

We follow the $1/N$ expansion approach and the matrix elements using this are given as:

\[ < Q_1 >_0 = (1/3)(2/N - 1)X B_1, \text{etc.} \]  
\[ < Q_1 >_2 = < Q_2 >_2 = \sqrt{2}/3(1 + 1/N)X B_{27}, \text{etc.} \]

where $B_{1,27}$ are bag parameters.

2.4 Calculation of $\delta$ in CKM matrix

The phase parameter in the CKM matrix is obtained using the experimental value of $\epsilon$. By definition

\[ \epsilon = e^{(i\pi/4)}(Im M_{12} + 2 Re M_{12})/ (\sqrt{2} \Delta M) \]

where

\[ \xi = Im A_0/Re A_0 \]

$M_{12}$ is the off-diagonal matrix element in the neutral K-meson mass matrix and $\Delta M$ is the $K_1 - K_2$ mass difference. The theoretical expression for $M_{12}$ is evaluated from the usual box diagram in terms of the "bag" parameter $B_K$.

3 Model with Additional Vector-like Quarks

We consider the extension of the standard model which contains a "fourth family" of quarks. Thus we simply add one $SU(2)$ doublet of vector-like quarks:

\[ \left( \begin{array}{c} U^\alpha \\ D^\alpha \end{array} \right)_L \right) \left( \begin{array}{c} U^\alpha \\ D^\alpha \end{array} \right)_R \]

Only the left-handed doublet participates and makes contributions to all the Feynman diagrams of the standard model i.e. box, gluon-penguin, Z-penguin and photon-penguin. The new particles do not lead to diagrams of new topologies.

This is similar to a fourth generation except that the anomalies have been cancelled directly by the quarks and we do not need to consider an additional charged lepton or a massive neutrino.

The functions $B(x_1, x_Q)$ are, of course, functions of the mass $M_Q$ of the extra quarks.

The result may be summarized by saying that the value of $Re(\epsilon'/\epsilon)$ is always made more negative. Thus, if $Re(\epsilon'/\epsilon)$ turns out to be zero or even negative, the inclusion of extra fermions is favored. On the other hand, if the answer is as large as the central CERN result, then additional fermions alone will not accommodate the empirical value.

4 Model with Additional Gauge Bosons

In the present section, we shall study a richer and more interesting extension which contains both additional fermions and gauge bosons.

In particular, we use the 331 model which is motivated by accommodation of three families. The model has gauge group $SU(3)_c \times SU(3)_L \times U(1)_X$ where $SU(3)_L \times U(1)_X$ contains the standard electroweak gauge group. The larger gauge group is broken at a scale $U$ by the vacuum expectation of a Higgs triplet.
The fermions transform as follows. The quarks of the first and second families are in left-handed triplets and right-handed singlets of $SU(3)_L$,

\[
\begin{pmatrix}
w^\alpha \\
d^\alpha \\
D^\alpha
\end{pmatrix}_L
\]

and similarly for c, s, S. The third family of quarks is in a left-handed anti-triplet $3^*_L = (b, t, T)$ and right-handed singlets.

The leptons for all three families are in anti-triplets:

\[
\begin{pmatrix}
e^- \\
\nu_e \\
e^+
\end{pmatrix}_L
\]

All anomalies cancel with three families, although not for each family separately.

The additional five gauge bosons are a $Z'$ and two doublets of dileptons $(Y^{--}, Y^{-+}, Y^{++}, Y^-)$ with lepton numbers $L = 1, -2$ respectively. The breaking by $\Phi_3$ gives masses to $Z'$ and $Y$ by the Higgs mechanism. The quarks $Q = D, S, T$ also acquire mass from $\Phi >$ through their Yukawa couplings. We shall take $M_D = M_S$ for simplicity but allow $M_T$ to vary independently. The dilepton mass $M_Y$ must be above 300GeV but can be up to 1100GeV. We denote $x_D = M_D^2/M_Y^2$ and $x_T = M_T^2/M_Y^2$ and allow them to vary in the range $0.1 \leq x_{D,T} \leq 10$.

The new particles introduce additional Feynman diagrams beyond the usual standard model for $K \to \pi\pi$ decay at one-loop level. The ones of the same topology modify the functions $B, C, D, E$ which now depend on $x_D$ and $x_T$. A new topology enters through the mixings of $Z'$ with $\gamma, Z$ but this is used only to cancel certain divergences in the electroweak penguin diagrams, the finite part being very small due to the large $Z'$ mass.

After evaluation of the modified $B, C, D, E$ functions, it is interesting to note the following points:

1. For box and gluon-penguin diagrams, the infinities are cancelled by a generalized GIM mechanism and the functional dependence of these diagrams on the respective $x_Q$ turns out to be the same as results in the standard model.

2. The coupling between the exotic quarks and the $Z$ boson is vector-like. As a consequence of this and of gauge invariance, the finite part of the $Z$-penguin vanishes at order $(q^2/M_Z^2)$, where $q_\mu$ is the four-momentum transfer.

3. Though the generalized GIM mechanism does not remove completely the divergences of the photon-penguin, they do nevertheless cancel as a result of gauge invariance after including the new-topology diagrams mentioned above. Another interesting point in the photon-penguin is that all three exotic quarks contribute with the same sign: the charge in the closed loop flows in the opposite sense for the $T$ relative to $D$ and $S$ and negates the sign change normally arising from the unitarity of the CKM matrix. This is a result of the asymmetric treatment of the third family, and is crucial in obtaining a more positive result for $Re(\epsilon'/\epsilon)$.

The result for $Re(\epsilon'/\epsilon)$ is that the minimum value is essentially identical, within one per cent, to the standard model. The maximum value can accommodate $2.3 \times 10^{-3}$ if we allow acceptable choices of $\Lambda_{\overline{MS}}$ and $m_s$.

5 Discussion and Summary

The values of the direct CP violation parameter is generally non-zero if there is direct $\Delta S = 1$ CP violation as in the standard model; it is very small $\ll 10^{-4}$ only from delicate cancellations.

In the standard model, the prediction for $Re(\epsilon'/\epsilon)$ was thought to be about $+3 \times 10^{-3}$ when $m_t \ll M_W$ was assumed, and the $Z$-penguins were neglected. With the heavy top quark, and inclusion of $Z$-penguins, the prediction is now in the range from $0.8 \times 10^{-3}$ down to zero if we take conservative values for all the other parameters.

We have considered how $Re(\epsilon'/\epsilon)$ can be effected by additional fermions and gauge bosons.

What we find is that the simplest additional fermions tend to decrease the prediction compared to the standard model. On the other hand, the extension which treats the third family differently can increase the prediction to a value $2 \times 10^{-3}$.

The future should bring us accurate experimental values, both for $Re(\epsilon'/\epsilon)$ as well as for the CKM parameters, $\Lambda_{\overline{MS}}, m_s$, and $B_K$. As further information on $Re(\epsilon'/\epsilon)$ emerges resolving the discrepancy between the current Fermilab (0.74 $\times 10^{-3}$) and CERN (2.3 $\times 10^{-3}$) central values, it will indicate not only whether new physics is involved but what form is required.

If the smaller Fermilab value for $Re(\epsilon'/\epsilon)$ is verified, the standard model can accommodate it. If a vanishingly small value emerges, superweak theory will be favored. If a negative value appears, extra fermions are indicated.

If the larger CERN central value is confirmed then it hints that there is something beyond the standard model, and of the scenarios we have considered the most favored is that with additional gauge bosons and an asymmetrical treatment of the third generation.

The full calculation is published in: J. Agrawal and P. H. Frampton, Nucl. Phys. B419, 254-278 (1994). The U.S. Department of Energy is thanked for support. An extensive bibliography, for which inadequate space remains, is provided in the Nucl. Phys. B article.