A QED Model for the Origin of Bursts from SGRs and AXPs

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ABSTRACT

We propose a model to account for the bursts from soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) in which quantum electrodynamics (QED) plays a vital role. In our theory, that we term “fast-mode breakdown,” magnetohydrodynamic (MHD) waves that are generated near the surface of a neutron star and propagate outward through the magnetosphere will be modified by the polarization of the vacuum. For neutron star magnetic fields $B_{\text{NS}} \gtrsim B_{\text{QED}} \approx 4.4 \times 10^{13}$ G, the interaction of the wave fields with the vacuum produces non-linearities in fast MHD waves that can steepen in a manner akin to the growth of hydrodynamic shocks. Under certain conditions, fast modes can develop field discontinuities on scales comparable to an electron Compton wavelength, at which point the wave energy will be dissipated through electron-positron pair production. We show that this process operates if the magnetic field of the neutron star is sufficiently strong and the ratio of the wavelength of the fast mode to its amplitude is sufficiently small, in which case the wave energy will be efficiently converted into an extended pair-plasma fireball. The radiative output from this fireball will consist of hard X-rays and soft $\gamma$-rays, with a spectrum similar to those seen in bursts from SGRs and AXPs. In addition, the mostly thermal radiation will be accompanied by a high-energy tail of synchrotron emission, whose existence can be used to test this theory. Our model also predicts that for disturbances with a given wavelength and amplitude, only stars with magnetic fields above a critical threshold will experience fast-mode breakdown in their magnetospheres. In principle, this distinction provides an explanation for why SGRs and AXPs exhibit burst activity while high-field radio pulsars apparently do not.

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1. Introduction

Soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are a subclass of neutron stars that presently includes about a dozen members. They are observed to emit pulsed X-ray radiation, with steadily increasing periods, as well as short bursts of hard X-rays and soft γ-rays. Compared to most radio pulsars, SGRs and AXPs have relatively long periods that are confined to the narrow range $P \approx 5 - 12$ s, and rapid spin-down rates, $\dot{P} \sim 10^{-11}$ s s$^{-1}$, but appear to be radio quiet. Compared with high-mass X-ray binaries, these objects have relatively low X-ray luminosities, $L_x \sim 10^{35} - 10^{36}$ erg/s, and soft spectra, but no detectable companions (for recent reviews see, e.g. Hurley 2000; Mereghetti et al. 2002).

The nature of SGRs and AXPs remains somewhat uncertain. From timing measurements, it is clear that they cannot be rotation-powered: their rate of loss of rotational energy, $|\dot{E}| \equiv 4\pi^2 I \dot{P}/P^3 \sim 10^{32.5}$, is orders of magnitude smaller than their persistent X-ray luminosities. Motivated by this fact, two types of models have been proposed in which the energy is supplied primarily either by magnetic fields or by accretion. In the “magnetar” picture (Duncan & Thompson 1992; Thompson & Duncan 1995 [hereafter, TD95]), SGRs and AXPs are ultramagnetized neutron stars, powered by magnetic field decay (Thompson & Duncan 1996; Heyl & Kulkarni 1998), possibly supplemented by residual thermal energy (e.g. Heyl & Hernquist 1997a,b). If magnetic fields and characteristic ages can be estimated from spin-down in the usual manner for radio pulsars (e.g. Manchester & Taylor 1977), then the observations imply $B_{NS} \sim 10^{19.5}(P\dot{P})^{1/2} \sim 10^{14} - 10^{15}$ G and $\tau_c \equiv P/2\dot{P} \sim 10^3 - 10^5$ years (e.g. Kouveliotou et al. 1998, 1999). Note that these estimates would not obtain if the field is not a dipole (e.g. Arons 1993).

In the second type of theories, the X-ray emission is powered by accretion from a low-mass or disrupted companion (e.g. Mereghetti & Stella 1995; van Paradijs et al. 1995; Ghosh et al. 1997), or a fossil disk (e.g. Corbet et al. 1995; Chatterjee et al. 2000; Alpar 2001; Marsden et al. 2001). According to these models, the magnetic field inferred from the observed luminosities and spin periods is $B_{NS} \sim 10^{11} - 10^{13}$ G, if the star is accreting near its equilibrium spin period, and the narrow observed range of spin periods and characteristic ages can be explained by the dependence of spin-down by the propeller effect on initial spin period and disk mass (e.g. Chatterjee & Hernquist 2000; Eksi & Alpar 2003).

Both classes of models have strengths and weaknesses in relation to existing observational data. The infrared and optical counterparts identified for some of these sources (e.g.

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1Throughout, we use the symbol $B_{NS}$ to denote the magnetic field of the neutron star at its surface.
Hulleman et al. 2000, 2001; Wang & Chakrabarty 2002; Israel et al. 2002, 2003a) can, in principle, be explained by viscous dissipation and radiative reprocessing by a disk (e.g. Perna et al. 2000; Perna & Hernquist 2000), provided that the disk is thin (e.g. Menou et al. 2001) and compact (see Israel et al. 2003a). The claims of pulsed optical emission from AXP 4U0142+61 (Kern & Martin 2002) would appear problematic for the accretion hypothesis. However, Ertan & Cheng (2003) have shown that optical pulsations can be produced in the context of disk-star dynamo gap models (Cheng & Ruderman 1991). It remains to be seen whether a fully consistent solution of this type can be found which satisfies all the requirements on e.g. the radial extent of the disk.

Tests of the magnetar model from observations in various spectral regimes have been somewhat inconclusive. At present, this theory does not make detailed predictions that can be confronted with the infrared and optical data. Partly for this reason, it is unclear why, in this picture, the infrared emission from AXPs appears not to be pulsed, unlike the optical emission, at least in the case of 4U0142 (for a discussion see, e.g. Israel et al. 2003b). However, the recent identification of a feature in the X-ray spectrum of SGR 1806-20 (Ibrahim et al. 2002, 2003) can be interpreted as a proton cyclotron resonance in a magnetic field $B_{NS} \sim 10^{15}$ G, in agreement with timing estimates.

The recent detection of bursts from two AXPs (Gavriil et al. 2002; Kaspi et al. 2003) has shown that these objects behave similarly to SGRs in all respects. Various authors have suggested that the bursts are powered by accretion (e.g. Harwitt & Salpeter 1973; Colgate & Petschek 1981; Epstein 1985; Tremaine & Zytkow 1986; Blaes et al. 1990; Katz et al. 1994), but Thompson & Duncan (1996) have argued that these models have numerous difficulties satisfying all the observational constraints. In particular, there is growing evidence that the bursts are accompanied by an overall change in the properties of the star, such as its spin-down, pulse profile, energy spectrum, and flux (e.g. Woods et al. 2001, 2003; Kouveliotou 2003). In some cases, the bursts have been preceded by glitches, and it appears that the timing noise in AXPs and SGRs is correlated with spin-down rate (e.g. Heyl & Hernquist 1999a; Woods et al. 2000; Gavriil & Kaspi 2002). These observations can be explained by a rearrangement of the magnetic field, possibly driven by a deformation of the stellar crust which also affects the interior of the star and its magnetosphere. (See Ertan & Alpar 2003 for an accretion-based interpretation of some of these observations.)

In the magnetar model, bursts originate when a fracture of the crust sends Alfvén waves into the magnetosphere (TD95). These waves are proposed to ultimately damp, resulting in a cascade of power to high wavenumbers, producing a pair plasma fireball through magnetic reconnection and a burst. This general picture is appealing because of the observations which suggest a link between deformations of the crust and the bursts, but the detailed
physical mechanism for the conversion of wave energy into hard X-rays and soft $\gamma$-rays is not well-understood. Moreover, a number of radio pulsars have been discovered with inferred magnetic field strengths similar to those of magnetars and apparently exceeding the value $B_{\text{QED}} \approx 4.4 \times 10^{13}$ G (e.g. Camilo et al. 2000; Morris et al. 2002; McLaughlin et al. 2003a,b). It is not clear why these objects have magnetic fields comparable to e.g. AXPs but do not exhibit AXP-like emission.

In this paper, we propose a specific physical mechanism for the origin of bursts in SGRs and AXPs. Like in the TD95 theory for magnetars, we suppose that bursts are triggered by a disturbance in the star that sends MHD waves into the magnetosphere. However, rather than relying on magnetic reconnection to produce the bursts, we suggest that the subsequent evolution of these waves is driven by QED processes that, under certain conditions, causes dissipation of the wave energy through pair production. In our model, vacuum polarization in fields exceeding $B_{\text{QED}}$ leads to non-linear evolution of fast MHD waves resembling hydrodynamic shocks. When the fields associated with the waves develop large gradients on scales comparable to an electron Compton wavelength, rapid conversion of the wave energy into a pair plasma ensues, yielding a burst.

In §2, we summarize the basic physics required by our analysis and argue that QED non-linearities will affect fast-mode MHD modes, but not Alfvén waves, at least to lowest order. In §3, we derive an analytical expression for the opacity of a fast mode produced at the surface of a neutron star to develop a QED shock, and calculate the rate at which wave energy will be dissipated into pairs. We infer the conditions for “fast-mode breakdown” to occur in neutron star magnetospheres; i.e. when the optical depth to shock formation approaches and exceeds unity. We show that for a given wavelength and amplitude of strong fast modes generated at the stellar surface, there is a corresponding critical value of the surface magnetic field that must be exceeded for our mechanism to operate. Only those neutron stars with sufficiently strong magnetic fields and which have the capacity to produce large amplitude, short wavelength disturbances will be susceptible to fast-mode breakdown, the development of a pair-plasma fireball, and a subsequent burst of hard X-rays and soft $\gamma$-rays. In §4, we describe the properties of the fireball and the nature of the emission that would be seen by a distant observer. We outline a scenario in §5 that, in principle, can explain why SGRs and AXPs exhibit bursts, unlike radio pulsars with intense magnetic fields. In §6, we compare our model to that of TD95, and discuss possible connections to optical and infrared emission from AXPs. Finally, we summarize our theory in §7 and describe additional predictions that can be used to test its validity.
2. QED Processes

There has been considerable confusion in the recent astronomical literature about the applicability of various results from QED to the physics of the environments of magnetars. In view of this, we briefly summarize the formalism required by our analysis and discuss regimes of validity for the relevant expressions from QED.

2.1. The effective Lagrangian

For our purposes, vacuum polarization effects can be calculated using an effective Lagrangian for the electromagnetic field. Following the usual convention, we write

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \cdots. \]  

(1)

Here, \( \mathcal{L} \) is the full Lagrangian density, \( \mathcal{L}_0 \) is the classical term, and \( \mathcal{L}_1 \) includes vacuum corrections to one-loop order. Higher order radiative corrections would be described by additional terms. Dittrich & Reuter (1985) give an expression for the second-order correction to the Lagrangian and find that it is smaller than the one-loop term by a factor of the fine structure constant, regardless of field strength. The behavior of even higher order terms in the expansion for \( \mathcal{L} \) is an open question. In what follows, we assume that the relevant effects can be approximated using \( \mathcal{L}_0 \) and \( \mathcal{L}_1 \).

Both \( \mathcal{L}_0 \) and \( \mathcal{L}_1 \) can be expressed in terms of the Lorentz invariants

\[ I \equiv F_{\mu\nu}F^{\mu\nu} = 2 \left( |B|^2 - |E|^2 \right) \]  

(2)

and

\[ K \equiv \left[ \epsilon^{\lambda\rho\mu\nu} F_{\lambda\rho} F_{\mu\nu} \right]^2 = -4 \left( E \cdot B \right)^2, \]  

(3)

where \( \epsilon^{\lambda\rho\mu\nu} \) is the completely antisymmetric Levi-Civita tensor. The effective Lagrangian of the electromagnetic field was derived by Heisenberg & Euler (1936) and Weisskopf (1936) using electron-hole theory. In rationalized Gaussian units, we can write \( \mathcal{L}_0 \) and \( \mathcal{L}_1 \) as

\[ \mathcal{L}_0 = -\frac{1}{4} I, \]  

(4)

\[ \mathcal{L}_1 = \frac{\alpha}{2\pi} \int_0^{\infty} e^{-\zeta} \frac{d\zeta}{\zeta^3} \left[ i \zeta^2 \sqrt{-K} \frac{\cos(J_+ \zeta) + \cos(J_- \zeta)}{4 \cos(J_+ \zeta) - \cos(J_- \zeta)} + B_{\text{QED}}^2 + I \frac{\zeta^2}{6} \right], \]  

(5)

where

\[ J_\pm \equiv \frac{1}{2B_{\text{QED}}} \left[ -I \pm i \sqrt{-K} \right]^{1/2}, \]  

(6)
\( \alpha \equiv e^2/hc \) is the fine structure constant, \( B_{\text{QED}} \equiv m^2c^3/eh \approx 4.4 \times 10^{13} \) G, and a similar quantity can be defined for the electric field, \( E_{\text{QED}} \equiv m^2c^3/eh \approx 2.2 \times 10^{15} \) V/cm. The above expressions for \( L_0 \) and \( L_1 \) are identical to the corresponding terms in eq. (45a) of Heisenberg & Euler (1936).

The above integral cannot be evaluated explicitly, in general. Heyl & Hernquist (1997c; hereafter HH97) have derived an analytic expression for \( L_1 \) as a power series in \( K \):

\[
L_1 = L_1(I, 0) + K \frac{\partial L_1}{\partial K} \bigg|_{K=0} + \cdots , \tag{7}
\]

where the first term in this series is

\[
L_1(I, 0) = \frac{\alpha}{4\pi} I X_0 \left( \frac{1}{\xi} \right) = \frac{\alpha}{4\pi} \int_0^\infty e^{-u/\xi} \frac{du}{u^3} \left( -u \coth u + 1 + \frac{u^2}{3} \right), \tag{8}
\]

and

\[
\xi = \frac{1}{B_{\text{QED}}} \sqrt{\frac{I}{2}}. \tag{9}
\]

The function \( X_0 \) can be evaluated analytically (as can the higher order terms in the expansion for \( L_1 \)) with the result (HH97)

\[
X_0(x) = 4 \int_0^{x/2-1} \ln(\Gamma(v+1)) \, dv - \frac{1}{3} \ln x + C - \left[ 1 + \ln \left( \frac{4\pi}{x} \right) \right] x + \left[ \frac{3}{4} + \frac{1}{2} \ln \left( \frac{2}{x} \right) \right] x^2, \tag{10}
\]

where

\[
C = 2 \ln 4\pi - 4 \ln A - \frac{5}{3} \ln 2 = 2.911785285, \tag{11}
\]

and the constant \( \ln A \) is related to the first derivative of the Riemann zeta function, \( \zeta^{(1)}(x) \), by

\[
\ln A = \frac{1}{12} - \zeta^{(1)}(-1) = 0.248754477. \tag{12}
\]

The integral of \( \ln \Gamma(x) \) can be expressed in terms of special functions (eqs. 18, 19 in HH97). (See also Dittrich et al. 1979; Ivanov 1992, but note the cautionary remark in HH97.)

The expression above for \( X_0(x) \) can be expanded in either a Taylor series in the weak field limit, \( \xi \ll 1 \), or an asymptotic series in the strong field limit \( \xi \gg 1 \), as can the higher order terms in equation (7), to give series expansions for \( L_1 \) as a function of either \( I \) and \( K \), or equivalently \( B \) and \( E \). In particular, to lowest order in the weak field limit

\[
L_1 = \frac{\alpha}{90\pi} \frac{1}{B_{\text{QED}}^2} \left[ (B^2 - E^2)^2 + 7 (E \cdot B)^2 \right] + \cdots \quad (\xi \ll 1). \tag{13}
\]
In the limit of an ultrastrong magnetic field, \( B \gg B_{\text{QED}} \), but for a weak electric field, \( E \ll E_{\text{QED}} \), \( \mathcal{L}_1 \) can be written

\[
\mathcal{L}_1 = \frac{\alpha}{6\pi} B^2 \left[ \ln \left( \frac{B}{B_{\text{QED}}} \right) - 12 \ln A + \ln 2 \right] + \cdots
\]  
(14)

(see e.g. eq. 29 in HH97 for the higher order terms). We note that equations (13) and (14) agree, respectively, with the corresponding terms in eqs. (43) and (44) of Heisenberg & Euler (1936). The analysis of HH97 generalizes the expressions of Heisenberg & Euler to arbitrary order.

Also, note that while our expression for \( \mathcal{L}_0 \) is identical to eq. (4-120) of Itzykson & Zuber (1980), equation (13) differs from their eq. (4-125) by a factor of \( 1/4\pi \), as a consequence of a difference in the system of units employed.\(^2\) Our expressions for \( \mathcal{L}_0 \) and \( \mathcal{L}_1 \) both differ from those in Berestetskii et al. (1982) by an overall factor of \( 1/4\pi \) (their eqs. 129.2 and 129.21); however, the dynamics of the fields are invariant with respect to rescalings of the Lagrangian.

We emphasize that the expression for the Lagrangian in the weak-field limit, equation (13), cannot be applied to magnetar fields which are thought to have \( B_{\text{NS}} \gg B_{\text{QED}} \). The use of the weak-field expressions to calculate, e.g. the index of refraction of the vacuum near the surface of a magnetar will result in estimates that are incorrect by more than an order of magnitude at the relevant field strengths. In this limit, the Lagrangian should instead be approximated by e.g. equation (14), which is an asymptotic series for \( \mathcal{L}_1 \) valid for \( B \gg B_{\text{QED}} \) and \( E \ll E_{\text{QED}} \).

### 2.2. The magnetized vacuum

The dependence of \( \mathcal{L}_1 \) on \( B \) and \( E \) means that the vacuum will behave in some respects like a material medium in the presence of strong magnetic and/or electric fields. This behavior will, in general, be more complex than for simple polarizable media because of the non-linear dependence of \( \mathcal{L}_1 \) on the fields (e.g. Erber 1966; Tsai & Erber 1975).

Many of these effects are subtle even for magnetar field strengths. For example, the vacuum will respond to an applied field like a non-linear paramagnetic substance (e.g. Mielnieczuk et al. 1988), so that \( B \) and \( H \) will differ (Klein & Nigam 1964a,b; Heyl & Hernquist

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\(^2\) Itzykson & Zuber (1980) use Heaviside’s units in defining the Coulomb force; \( E^2 \) and \( B^2 \) are smaller in this system than in ours by a factor of \( 4\pi \).
1997d). This will influence e.g. the spin-down rate of magnetars, but quantitatively the effect appears to be small (Heyl & Hernquist 1997e).

More promising is the impact that vacuum polarization can have on the propagation of waves through the magnetospheres of strongly magnetized neutron stars. Because a magnetized vacuum responds to applied fields in a non-linear manner, sinusoidal disturbances like electromagnetic waves can develop discontinuities that are analogous to hydrodynamical shocks. Lutzky & Toll (1959) and Zheleznyakov & Fabrikant (1982) studied this process in the weak-field limit, while Bialynicka-Birula (1981) considered stronger fields and Heyl & Hernquist (1998a; hereafter HH98) extended the analysis to fields characteristic of magnetars. In particular, HH98 used the effective Lagrangian described in §2.1 to derive an expression for the opacity of electromagnetic waves traveling through an arbitrarily strong magnetic field to experience shocks, used the method of characteristics to investigate the development on discontinuities in these waves and investigated the jump conditions across these discontinuities (see also Boillat 1972).

Similar considerations apply to certain types of MHD waves. Heyl & Hernquist (1999b; hereafter HH99) employed a variational principle to investigate the propagation of MHD waves through strongly magnetized, relativistic plasmas, including QED effects (see Achterberg 1983 for the classical limit). HH99 showed that the fundamental modes of propagation in the ultrarelativistic limit are two oppositely directed Alfvén modes and the fast mode, in agreement with Thompson & Blaes (1998, hereafter TB98).

In the ultrarelativistic limit, the fast mode does not carry any current and behaves similar to a vacuum electromagnetic wave (TB98); consequently, it is not surprising that the fast MHD modes are subject to shocking in a manner identical to electromagnetic radiation traveling through an external magnetic field in the absence of a plasma (HH99). The ultrarelativistic limit results from neglecting the mass of the charge carriers; therefore, it corresponds to the long-wavelength limit and applies to waves whose frequency is well below the plasma frequency (TB98).

It is, thus, these long-wavelength fast modes that are susceptible to energy dissipation through pair production, at least to lowest order. On the other hand, HH99 found that because of the gauge and Lorentz invariance of QED, non-linearities do not develop in the propagation of single Alfvén modes.
3. Fast-Mode Breakdown

In a manner analogous to the general picture proposed by TD95, we suppose that bursts are triggered in SGRs and AXPs either by a fracture of the stellar crust or a pure rearrangement of the magnetic field. Shifts in the footprints of magnetic field lines will send MHD waves into the magnetosphere (e.g. fig. 1 in TD95). For the purposes of discussion, we assume that some fraction of the wave energy will be emitted in the fast mode. Whether the fast modes are produced directly or through the interaction of Alfvén waves is beyond the scope of this article. Unlike the scenario described in TD95, we do not consider the evolution of Alfvén waves since the analysis of HH99 indicates that they will not develop shocks from their interaction with the vacuum. (QED will introduce additional couplings between oppositely directed Alfvén waves, perhaps exciting other dissipative processes, but we do not pursue these interesting possibilities here.)

As we noted earlier, the evolution of the fast mode in a strongly magnetized, ultrarelativistic plasma is identical to the behavior of an electromagnetic wave in the absence of the plasma; therefore, we use the results of HH98 to describe the propagation of a fast wave through the magnetosphere of a neutron star.

To illustrate the effect, we consider waves in which the electric field is polarized perpendicular to the magnetic field of the star, and require the wave to travel perpendicular to the magnetic field. Under these assumptions, the opacity for a fast mode of wavenumber $k$ to form a shock is given by

$$\kappa = -k \frac{\alpha}{4\pi} \frac{b}{B_{\text{QED}}} \left[ X_0^{(1)} \left( \frac{1}{\xi} \right) \xi^{-2} - X_0^{(2)} \left( \frac{1}{\xi} \right) \xi^{-3} + X_0^{(3)} \left( \frac{1}{\xi} \right) \xi^{-4} \right],$$

where $\xi = B_{\text{NS}}/B_{\text{QED}}$, $B_{\text{NS}}$ is the strength of the stellar field, $b$ is the amplitude of the magnetic field of the fast mode, the function $X_0(x)$ is defined by equations (10-12), and $X_0^{(n)}(x) \equiv \frac{d^n}{dx^n}X_0(x)$. Using the definition of $X_0(x)$, we find

$$\kappa = k \frac{b}{B_{\text{QED}}} \frac{\alpha}{\pi} \left\{ \frac{1}{3\xi} + \frac{1}{\xi^2} \left[ \frac{1}{4} \ln (4\pi \xi) - \frac{1}{2} \ln \Gamma \left( \frac{1}{2\xi} \right) + \frac{1}{2} \right] + \frac{1}{4\xi^3} \Psi \left( \frac{1}{2\xi} \right) - \frac{1}{8\xi^4} \Psi^{(1)} \left( \frac{1}{2\xi} \right) \right\},$$

where $\Psi(x) \equiv d\ln \Gamma/dx$, and $\Psi^{(1)}(x) = d\Psi/dx$.

HH98 give expansions for $\kappa$ that are valid in the weak- and strong-field limits and which are useful for determining the asymptotic behavior of the opacity. In what follows, we are interested in the evolution of a wave emitted from the stellar surface, where the field would be stronger than $B_{\text{QED}}$ for a magnetar, as it propagates into regions in the magnetosphere where $B \ll B_{\text{QED}}$, requiring us to use the full expression for $\kappa$ above.
Note that according to equation (16), the opacity, and hence the integrated optical depth, depend on the properties of the wave only through the combination $kb/B_{\text{QED}}$; i.e. the wavenumber $(2\pi/\lambda)$ multiplied by the amplitude in units of $B_{\text{QED}}$.

The structure of the wave at a particular location is determined by integrating the opacity over the trajectory of the wave, giving the optical depth, $\tau = \int \kappa ds$. We assume that the wave is initially sinusoidal; therefore, the wave begins to lose energy to pair production when $\tau$ reaches unity. We use the Maxwell-equal-area prescription (Landau & Lifshitz 1987) to determine the structure of the wave after the shock forms and the ensuing rate of pair production.

In this way, we adopt a number of simplifying assumptions to describe the rate of energy dissipation. Strictly speaking, the effective Lagrangian we employ here is correct only if the magnetic field is locally uniform. This description is no longer valid when the field varies by of order unity on scales comparable to an electron Compton wavelength. Analyzing this situation would require a more general formulation, such as the proper-time method developed by Schwinger (1951). In practice, this means that we cannot describe the structure of the shocks. However, the shock jump conditions (HH98) make it possible to estimate the rate of dissipation through pair production without a detailed understanding of shock structure.

We also assume that the only dissipative mechanism is pair production. In particular, we ignore dispersive effects, as they are estimated to be small under the conditions appropriate for neutron star magnetospheres (e.g. Adler 1971). In addition, for simplicity, we do not include other processes, such as non-linear QED interactions between Alfvén waves.

To summarize, we assume that a sinusoidal fast mode is generated at the stellar surface and that it begins to steepen owing to vacuum polarization as it propagates outward (see fig. 4 in HH98). If the integrated optical depth exceeds unity along its trajectory, the field gradients in the wave will be large enough for pairs to be produced, leading to “fast-mode breakdown.”

Figure 1 depicts the conditions for a fast mode to break down in this manner in the magnetosphere of a neutron star with a given surface field. Here, we assume, that the fast mode is polarized with its electric field perpendicular to the stellar field, that the wave travels radially in the equatorial plane of the star, whose radius is ten kilometers, and that the neutron star field is a pure dipole. For simplicity, we have not included the gravitational redshift of the wave which would decrease the critical wavelength by at most 30%. Although the shape of the fast modes with wavelengths larger and amplitudes smaller than traced by the lines will change as they propagate through the magnetosphere, they do not form shocks,
Fig. 1.— The conditions for a fast mode to break down in a neutron star magnetosphere, expressed as a ratio of the wavelength to the amplitude of the wave. Only waves with sufficiently small wavelength and/or large amplitude will break down. The calculation for the upper curve assumes that the wave is plane parallel (i.e. its strength does not diminish with radius from the star). The lower curve is for a spherical wave.
and so no energy is dissipated.

If the ratio of the wavelength to amplitude of strong fast modes generated at the surface of the neutron star is limited from below, then Figure 1 demonstrates that only stars with surface fields greater than a particular value will support fast-mode breakdown in their magnetospheres. For example, if the minimum wavelength of a fast mode is one kilometer, one finds that only neutron stars with $B_{\text{NS}} > 10B_{\text{QED}}$ will exhibit fast-mode breakdown for disturbances with amplitude $b = B_{\text{NS}}$. Our model thus naturally leads to a cut-off behavior in the production of a pair plasma through this mechanism, depending on the manner in which waves are produced at the stellar surface.

In Figure 2, we focus on the evolution of a particular wave in greater detail. Here, we examine a fast mode of six meter wavelength (to be precise, $2\pi R/10^4$) whose initial amplitude is equal to one percent of the surface magnetic field. We consider surface fields of $4.4 \times 10^{14}$ G and $1.2 \times 10^{15}$ G. The left panel shows both the cumulative fraction of wave energy that is converted into pairs, along with the differential fraction per radial bin. For the more strongly magnetized neutron star, the bulk of the energy of the wave dissipates within one stellar radius from the surface. Only about 30% of the energy in the wave traveling through the magnetosphere of the more weakly magnetized neutron star is dissipated. Again, the bulk of the pair production is within a few stellar radii. The right panel depicts the energy production rate for the two situations.

4. The Fireball

In our model, most of the pairs are produced near the surface of the star, within a few stellar radii. To place the energy generation rates in perspective, the active volume of pair production is $\sim 10^{19}$ cm$^3$, yielding a total energy generation rate of $\sim 10^{47}$ erg/s for the more strongly magnetized star in Figure 2. The typical energy of an SGR burst is $\sim 10^{40}$ ergs, so the pair-production will last $\sim 100$ ns or a few oscillations of the fast mode.

However, the total volume of the fireball is much larger than this because each electron has a large cross section to absorb X-rays. A mere $10^{13}$ ergs of pairs per cubic centimeter is sufficient to make a distance of several kilometers opaque to X-rays. Figure 3 depicts the Thomson optical depth for X-rays to escape to infinity radially from various distances from the center of the neutron star. The fireball extends for several tens or hundreds of stellar radii from the surface of the neutron star for the models depicted in Figures 2 and 3. Additionally, as long as the fast mode is not discontinuous initially, no pairs are produced at the surface of the star. This gap between the fireball and the stellar surface prevents a
Fig. 2.— Rate of power dissipation of a fast-mode traveling through a magnetar magnetosphere. The initial amplitude of the fast mode is 1% of the surface magnetic field, and its wavelength is 6.3 meters. Similar results obtain as the amplitude and wavelength are increased proportionately. The absolute power dissipated increases as the square of the wave amplitude. In the left panel, we show the cumulative (monotonic curves) and differential (peaked curves) energy dissipated for stellar fields $B_{NS} = 30B_{QED} \approx 1.2 \times 10^{15}$ G (upper curves) and $B_{NS} = 10B_{QED} \approx 4.4 \times 10^{14}$ G (lower curves). The right panel shows the energy production rate for the two cases, with the upper and lower curves for $B_{NS} = 30B_{QED}$ and $B_{NS} = 10B_{QED}$, respectively.
large baryon contamination of the pair plasma.

The size of the fireball depends on the properties of the wave, specifically on $kb$, the product of the wavenumber and the amplitude of the wave. Using the asymptotic behavior of the shock opacity for weak fields (HH98) and the evolution of the wave for large values of $\tau$, we find that the outer radius of the fireball goes as $(kb)^{-2/5}$. The distance between the surface of the star and the edge of the fireball is proportional to $(kb)^{-1}$. To make things a bit more concrete, for a neutron star with surface dipole field of $B_{NS} = 30B_{QED}$, the initial fireball extends from 4 km above the surface of the 10 km star to 800 km from the center of the star for $kb = 0.3B_{QED}m^{-1}$. If we look at a wave with $k = 1m^{-1}$ and $b = B_{NS}$ ($kb = 30B_{QED}m^{-1}$), the fireball starts a mere 40 m above the surface and extends to 120 km from the star. Because the pair production begins only after the wave has traveled one optical depth, there is always a gap between the stellar surface and the fireball.

The formation of the fireball and the fireball itself will produce two types of emission. The vast majority of this emission will be in the form of blackbody radiation from the expanding fireball (as in TD95). The model presented here differs in the mechanism that produces the pair-plasma fireball but the fireball itself is qualitatively similar, so the specific radiative processes outlined by TD95 operate here as well. We can obtain an estimate for the effective temperature of the blackbody emission by assuming that the fireball emits at the Eddington limit in the strong magnetic field surrounding the neutron star. TD95 give the following result (their eq. 44):

$$T_{\text{max}}(\parallel) = 7.6(Y_{e^-} + Y_{e^+})^{-1/6} \left[ \frac{B(R)}{B_{QED}} \right]^{1/3} \left( \frac{R}{R_{NS}} \right)^{-1/3} \left( \frac{g_{NS,14}}{2} \right)^{1/6} \text{keV.}$$

Fast-mode breakdown produces a fireball many tens of stellar radii in diameter, so the value of $R$ at the outside of the fireball is much greater than the radius of the star. However, the bulk of the energy of the fireball is dumped within a few radii of the star and the fireball is contained within the closed field lines; therefore, emission can escape from near the surface of the star, and the bulk of the flux will be characterized by a blackbody temperature $\sim 10$ keV.

As the schematic illustrations in Figure 4 show, some radiation will escape before the fireball forms and from outside the fireball. The fraction of the emission that escapes before the formation of the fireball is approximately the reciprocal of the peak optical depth of the fireball, $\sim 10^{-9}$ for the models considered here.

It turns out that a much larger fraction of the energy is emitted from outside the fireball. Figure 5 makes this more quantitative. In both cases about $10^{-5}$ to $10^{-6}$ (or $\sim 10^{34}$ erg) of the burst energy emerges as synchrotron emission extending to $E = 2m_ec^2$ (above this energy the photons would produce pairs in the strong magnetic field of the neutron star).
Fig. 3.— The Thomson optical depth for X-rays to escape from a given radius to infinity for the models depicted in Fig. 2. To calculate the density of pairs, we have assumed that the fireball subtends one steradian, the total energy of the burst is $10^{40}$ ergs and that this energy is initially in the form of the rest-mass energy of the pairs. The stronger field case has a larger peak optical depth $\sim 10^9$ because the pair production in the strong field is typically closer to the star.
The schematic depicts that this emission has a power-law distribution, but this is merely for convenience. The calculation of the energy distribution of this high-energy radiation is beyond the scope of this paper.

The asymptotic scalings show that the emission from outside a particular radius is proportional to \((kb)^{-2}r^{-3}\). Combining this with the result for the location of the photosphere yields that the fraction of the emission that is produced outside the fireball is proportional to \((kb)^{-4/5}\).

Our estimates for the structure of the fireball are sensitive to the global properties of the neutron star magnetic field. Here, we have assumed a pure dipole; higher order multipoles would alter some of our estimates. For example, if the field included a large quadrupole contribution, the fireball would be more compact than the estimates given above.

5. SGRs and AXPs vs. High-Field Radio Pulsars

One of the most puzzling aspects of SGRs and AXPs is why they appear to behave so differently from those radio pulsars with similar timing characteristics. Originally, it was believed that SGRs and AXPs might be distinguished from radio pulsars purely on the basis of the strengths of their magnetic fields. However, a number of radio pulsars are now known that have \(B_{\text{NS}} \gtrsim B_{\text{QED}}\) and, in some cases, fields even larger than those of AXPs. (Assuming that SGRs and AXPs are isolated objects and are not accreting.)

Presently, there are five radio pulsars with magnetic fields \(B_{\text{NS}} \gtrsim B_{\text{QED}}\) (Camilo et al. 2000; Morris et al. 2002; McLaughlin et al. 2003a,b); in particular, the magnetic field of PSR J1847-0130 is \(B_{\text{NS}} = 9.4 \times 10^{13}\) G. It is interesting to compare PSR J1847-0130 with 1E2259+586, which is perhaps the most well-studied AXP. In many respects, these stars appear to be very similar. The period and spin-down rate are 6.71 secs and \(1.27 \times 10^{-12}\) s s\(^{-1}\) for PSR J1847-0130 (McLaughlin et al. 2003a) versus 6.98 secs and \(4.84 \times 10^{-13}\) s s\(^{-1}\) for AXP 1E2259+586 (e.g. Gavriil & Kaspi 2002). The implied field strengths and characteristic ages are \(B_{\text{NS}} = 9.4 \times 10^{13}\) G and \(\tau_{c} = 83,000\) years for PSR J1847-0130 and \(B_{\text{NS}} = 5.9 \times 10^{13}\) G and \(\tau_{c} = 230,000\) years for AXP 1E2259+586.

But, in other ways, these two objects are completely different. PSR J1847-0130 is a radio pulsar, whereas none of the AXPs have been detected as radio sources. Bursts of the type analyzed here were discovered from AXP 1E2259+586 (Kaspi et al. 2003), but no radio pulsar has exhibited similar behavior. These differences cannot be explained on the basis of magnetic field intensity since, in this case, the more “ordinary” radio pulsar has the larger inferred value of \(B_{\text{NS}}\). Similarly, the differences cannot be ascribed to the spin-down rate,
Fig. 4.— A schematic illustration of the formation of the pair fireball (Not to scale). The left panel shows the configuration before sufficient plasma has accumulated to make the vicinity of the neutron star optically thick. The right panel depicts the fireball and the continuing fast-mode breakdown outside the fireball. The fireball emits as a blackbody but before the formation of the fireball and outside of the fireball itself the pairs generate a distribution of high-energy synchrotron radiation up to $E = 2m_e c^2$ (insets).

Fig. 5.— The two panels depict the fraction of the pair-production outside of a given radius or below a particular optical depth. The energy dissipated below an optical depth of unity will escape to infinity as synchrotron emission. The upper curve in both panels is for $B_{NS} = 10B_{QED}$ and the lower curve is for $B_{NS} = 30B_{QED}$. 
because the more “peculiar” AXP has the smaller measured $\dot{P}$. Clearly, the true explanation is more involved.

In what follows, we speculate on reasons that may account for the contrasting properties of PSR J1847-0130 and AXP 1E2259+586. Our scenario is motivated by two other differences between these objects. First, AXP 1E2259+586 is associated with a supernova remnant (CTB 109) with an estimated age of $\tau_{\text{SNR}} = 10^4$ years, while PSR J1847-0130 appears to be isolated. Second, the persistent X-ray luminosity of the AXP in the 2 - 10 keV band is much higher: $L_x \sim 10^{34.5} - 10^{35}$ erg/s compared to an upper limit of $L_x < (2 - 8) \times 10^{33}$ erg/s for PSR J1847-0130 (e.g. McLaughlin et al. 2003a).

One interpretation of these differences is that AXP 1E2259+586 is considerably younger than PSR J1847-0130, in spite of its much larger timing age (230,000 vs. 83,000 years). This would account at least partly for its much higher persistent X-ray luminosity (e.g. Heyl & Hernquist 1997a,b), as well as its association with a $10^4$ year old supernova remnant.

In the magnetar model, it is usually stated that the neutron star is “magnetically powered,” in the sense that the magnetic field represents the dominant available supply of energy. However, it is not clear that this is always the case. The magnetic energy is

$$\mathcal{M} = \frac{1}{8\pi} \int B^2 dV. \tag{18}$$

If the magnetic field throughout the star is uniform, then for AXP 1E2259,

$$\mathcal{M} \sim 6 \times 10^{44} R_6^3 \left( \frac{B_{\text{NS}}}{5.9 \times 10^{13} G} \right)^2 \text{ergs}, \tag{19}$$

where $R_6$ is the radius of the star in units of $10^6$ cm. This can be compared with the available thermal energy of the star, $dU/dT = C_V$, where $C_V$, the heat capacity (e.g. Shapiro & Teukolsky 1983), is proportional to the temperature. For a uniform density sphere of non-relativistic neutrons, this can be integrated to give

$$U \sim 10^{47} R_6^2 M_{1.4}^{1/3} T_{8.5}^2 \text{ ergs}, \tag{20}$$

where $M_{1.4}$ and $T_{8.5}$ are the mass and core temperature of the star in units of $1.4M_\odot$ and $10^{8.5}$ K, respectively.

Heyl & Hernquist (1997b) studied the thermal evolution of ultramagnetized neutron stars and found that if these objects follow conventional cooling tracks then the core temperature is given by

$$T_9 \sim t_y^{1/6}, \tag{21}$$
where $t_{\text{yr}}$ is the age in years, and this equation is valid for $t_{\text{yr}} \lesssim 10^5$ yrs and $B_{\text{NS}} \lesssim 10^{15}$ G. If the true age of AXP 1E2259 is $\sim 10^4$ years, then $T_{8.5} \approx 0.7$ and from equations (19) and (20), $U \sim 100 M$, and so the thermal component represents the largest energy supply in this star. This conclusion would not obtain if unconventional sources of cooling are included. In this case, however, the expected thermal emission would not contribute significantly to the X-ray spectrum of AXP 1E2259 (e.g. Heyl & Hernquist 1997a). Moreover, Heyl & Kulkarni (1998) have shown that magnetic field decay for $B \lesssim 10^{15}$ G contributes negligibly to the reservoir of thermal energy of such a young neutron star. Consequently, the thermal radiation from AXP 1E2259 is probably not strongly affected by field decay.

Thus, there is the possibility that at least in the case of AXP 1E2259 the evolution is driven primarily by thermal, rather than magnetic effects. In principle, this could distinguish that object from PSR J1847-0130 because the radio pulsar may be much older and, hence, its supply of thermal energy would be depleted relative to the AXP (Heyl & Hernquist 1997b).

Blandford, Applegate & Hernquist (1983) proposed that a vertical heat flow through the outer layers of a neutron star can amplify a horizontal seed field by thermoelectric effects. Whether or not this mechanism can actually generate the magnetic fields associated with AXPs is unclear, but the analysis of Blandford et al. suggests that the structure of a pre-existing magnetic field can be altered by the thermal flux. In particular, the flow of heat may yield a disordered, evolving pattern of magnetic flux on the surface of the star, whether or not the crust is seismically active, and regardless of how the field originated. The thermal evolution of the star could in principle lead directly to magnetic activity in the magnetosphere, producing stresses in the crust and ultimately starquakes. We note also that the thermoelectric effect studied by Blandford et al. is most efficient if the outer layers of the star consist of light elements, as is required for the cooling calculations to match the observed X-ray luminosity (Heyl & Hernquist 1997a).

In the examples presented earlier, we assumed a simple dipole geometry in estimating the opacity for MHD waves to develop shocks. However, our model depends not on the dipole component of the field but the total field near the neutron star, $B_{\text{NS}}$, which could be larger and more complicated than a dipole. A key difference between AXP 1E2259 and PSR J1847 is that 1E2259 was identified as an AXP; this means that its X-ray emission exceeds the spin-down luminosity by many orders of magnitude. Regardless of whether this emission is due to the cooling of the neutron star or magnetic field decay, this heat emerges through the crust of the neutron star (this does not apply for accretion models of AXPs), and could drive evolution in the surface field.

Thus, we propose that the thermal flux in AXP 1E2259 may be responsible for maintaining magnetic activity in this star, possibly supplemented by seismic effects, owing to
its young age. This magnetic activity seeds the mechanism for bursts described in this paper by generating MHD waves in a continuous manner, some of which exceed the threshold for shocking owing to their short wavelengths and/or large amplitudes. As such a star cools, the surface field begins to stabilize when the thermal energy is radiated. The star becomes more quiescent as the magnetic field becomes mainly dipolar and the source of MHD waves to trigger bursts is no longer available. Eventually, it may evolve into an object like PSR J1847, which resembles an ordinary radio pulsar, albeit slowly rotating and unusually strongly magnetized.

In the context of our model, AXP 1E2259 would differ from PSR J1847 not on the basis of its magnetic field strength or spin-down rate, but because it is much more magnetically active. One piece of evidence that indirectly supports this interpretation is the discrepancy between the timing age of AXP 1E2259 and the age of its associated supernova remnant or the cooling age inferred from its X-ray luminosity. As we indicated earlier, if the magnetic field structure of AXP 1E2259 is complex and dynamic, timing estimates for a simple dipole will not lead to accurate values for the true age of the star. This is consistent with the notion that the magnetic field of AXP 1E2259 is disordered, owing to the impact of thermal processes on the structure of the field. (This explanation is not unique, however, as the discrepancy between the timing age and the age of the supernova remnant can also be accounted for even if the field is dipolar, provided that the star was born rotating slowly.)

To the extent that AXPs and SGRs are the same type of object, as suggested by observations, the arguments presented here may be relevant to this entire subclass of neutron stars. From equations (19) and (20), we can obtain a condition for the approximate equality of magnetic and thermal energies:

\[ B_{15} \sim \tilde{\rho}_{15}^{1/6} T_{8.5}^{0.5} \]  

(22)

where \( \tilde{\rho}_{15} = \tilde{\rho}/10^{15} \) is the mean density of the star in gm/cm\(^3\) and \( B_{15} = B_{\text{NS}}/10^{15} \)G. The largest estimated fields for AXPs and SGRs are \( B_{\text{NS}} \sim 10^{15} \) G, and these stars have ages \( \sim 10^4 \) years, implying core temperatures \( T_{8.5} \sim 0.7 \), according to equation (21). Thus, it is possible that heat and not magnetism represents the dominant global energy supply. Detailed calculations of the thermal structure of ultramagnetized neutron stars (e.g. Heyl & Hernquist 1998b, 2001), combined with modeling of the interplay between magnetic fields and the heat flux in the outer layers of the stars will be required to understand the implications of the energetics more fully. But, we note that the analysis of Heyl & Kulkarni (1998) supports our argument that thermal effects dominate under the relevant circumstances.
6. Discussion

Our theory for the bursts from SGRs and AXPs requires that the magnetic field in the vicinity of the neutron star exceed the quantum critical field $B_{\text{QED}}$. If the wavelength to amplitude ratio of fast modes is bounded from below, then there is a magnetic field below which no bursts can occur. This cutoff behavior is in contrast to the model proposed by TD95, in which neutron stars with weaker fields could, in principle, exhibit weaker bursts.

6.1. Comparison with TD95

Many of the details of the TD95 model depend on the strength of the stellar magnetic field, so processes similar to theirs could produce bursts from more weakly magnetized neutron stars. For example, TD95 consider a pair-plasma fireball in a weaker magnetic field in order to account for Type-II X-ray bursts. Bursts such as these have not been detected from any radio pulsar even though the radio pulsars (including the most strongly magnetized ones) are typically ten to one hundred times closer to Earth than the SGRs, and instruments such as HETE-2 routinely detect such events from accreting sources. If the Alfvénic cascade outlined by TD95 does not operate, the cutoff behavior of fast-mode breakdown would naturally explain why these radio pulsars do not burst.

There are other significant differences between the Alfvénic cascade scenario of TD95 and the fast-mode breakdown model presented here. HH99 argued from the principles of gauge and Lorentz invariance that a single Alfvén wave does not suffer non-linearities in relativistic MHD (even when coupled to QED); only oppositely directed waves interact. Consequently, an Alfvén wave produced at the surface must propagate across the magnetosphere of the neutron star and reflect off the surface before it can interact with itself and instigate a cascade. Unless the magnetic field is tangled, the wave would have to travel several stellar radii before the cascade could begin, and furthermore it would have to reflect efficiently rather than be absorbed. Because the cascade requires both waves, the energy of the fireball would be limited to the energy of the reflected wave. At each reflection, the interaction time-scale is limited by the duration of the wave. If the stellar magnetic field is sufficiently strong, the fast modes will break down through the mechanism proposed here within a stellar radius and dump a large fraction of their energy into pairs, creating a fireball well before an Alfvénic cascade can begin.

Simulating an Alfvénic cascade would be amendable to the techniques used here and in HH98. The non-linearities in the evolution of Alfvén waves comprise two effects, one magnetohydrodynamic and one quantum-electrodynamic. The QED process is essentially the
same as that which operates for the fast modes. A detailed simulation of an Alfvénic cascade may reveal that it cannot operate within the confines of a neutron-star magnetosphere unless the QED contribution is sufficiently strong. Whether or not such a magnetic cutoff exists for the Alfvén modes is left for future work.

6.2. Non-thermal emission

Fast-mode breakdown predicts that high energy emission extending up to 1 MeV should accompany the thermal emission from the pair plasma. Although the fluence of this high-energy emission is much weaker than that of the fireball ($\sim 10^{-5}$), its flux may actually be comparable because it is produced promptly by fast-mode breakdown itself, rather than by the cooling fireball. Fast-mode breakdown also dumps a large number of pairs into the magnetosphere. If the lifetime of the pairs is sufficiently long or the smaller-scale and more common fast modes break down without forming a large fireball (the estimates of the preceding section assumed that the initial disturbance subtends a steradian of the stellar surface), fast-mode breakdown can populate a neutron-star magnetosphere with high-energy electron-positron pairs. This could fuel a pulsar-like emission process, possibly accounting for the optical/IR flux from AXPs (Hulleman et al. 2000, 2001; Wang & Chakrabarty 2002; Israel et al. 2002, 2003a). We have not discussed in detail the properties of the non-thermal component of fast-mode breakdown emission, but depending on the relative locations of the optical and infrared emission, one might also explain the relative variability in the two wave-bands (Kern & Martin 2002).

7. Conclusions

We have presented a model for the origin of bursts from SGRs and AXPs which we call “fast-mode breakdown.” In this theory, certain MHD waves propagating through the magnetospheres of neutron stars with sufficiently strong magnetic fields will develop discontinuities in response to the QED process of vacuum polarization and dissipate as a pair plasma fireball. To function, our mechanism requires both a strong stellar magnetic field, $B_{\text{NS}} \gtrsim B_{\text{QED}} \approx 4.4 \times 10^{13} \text{ G}$, as well as an active source of short wavelength, large amplitude MHD waves. Owing to the dependence of fast-mode breakdown on $B_{\text{NS}}$ and the properties of the wave, this phenomenon is subject to a cutoff, and will not operate below a limiting value of $B_{\text{NS}}$, for a given spectrum of MHD waves.

We have argued that a primary difference between AXPs/SGRs and high-field radio
pulsars may be in their ability to sustain the production of MHD waves of the appropriate wavelength and amplitude. On the basis of our model, we propose that only those neutron stars born with $B_{\text{NS}} \gtrsim B_{\text{QED}}$ will exhibit AXP- or SGR-like behavior when they are young. If the source of the waves involves an interplay between thermal and magnetic processes, then magnetars may evolve into radio pulsars as they age and lose their thermal energy. This transition would occur on roughly the cooling time-scale, which depends on the strength of the magnetic field (Heyl & Hernquist 1997b), but is typically $\lesssim 10^5$ years. This would explain the absence of old AXPs and SGRs. Conceivably, this picture could also account for the lack of AXPs or SGRs with $B_{\text{NS}} \gg 10^{15}$ G, if complete magnetic domination ultimately suppresses the effects that excite the wave activity required by our model.

Our work suggests interesting directions for future research on related topics. As we indicated earlier, the effective Lagrangian we have employed cannot describe the structure of electromagnetic and MHD shocks because it assumes that the magnetic field is uniform across an electron Compton wavelength. Analyzing this situation will require a more general approach, such as the proper-time method of Schwinger (1951).

We have ignored dispersive effects in describing the propagation of the waves, on the basis of estimates like those given by Adler (1971). If this approximation is not always appropriate, it raises the possibility that a competition between wave-steepening from QED and dispersion from a background plasma could lead to a situation in which solitary waves (photon torpedoes?) may exist. The consequences of such solutions are not currently predicted by our theory.

Here, we have focused on the evolution and break-down of individual fast-mode waves. However, as noted by HH98, QED effects will enable non-linear interactions between Alfvén waves in neutron star magnetospheres that could drive other paths for wave energy to be dissipated. The various shocks we have described could also interact with background plasma, producing accelerated particles with characteristics similar to ultra high-energy cosmic rays.

Finally, it is of interest to examine the evolution of magnetic fields in neutron stars, particularly in interaction with the heat flow through the outer layers of these objects. Direct simulation of e.g. thermoelectric coupling between magnetic fields and thermal flux should be possible, leading to greater understanding of the consequences of these processes.

We have suggested how our theory could be tested by, for example, searching for the high energy tail of emission that should accompany the mostly thermal spectra of the bursts. If our model is supported by such observations, it will clarify the energy source of bursts from SGRs and AXPs, add to the growing evidence that these objects have magnetic fields $B_{\text{NS}} \gtrsim B_{\text{QED}}$, and demonstrate the importance of QED effects in astrophysical sources.
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