Excitations in superfluid Neutron Star matter

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Abstract. We present a study of the excitation spectra and of the spectral functions for the elementary excitations in superfluid neutron star matter below the crust. We restrict the analysis to the proton and electron components within the Random Phase Approximation generalized to the superfluid case. The excitation spectrum shows the interplay of these two components and their relevance for the excitation modes. The results are relevant for neutrino physics and thermodynamical processes in neutron stars.

1. Introduction
Homogeneous matter below neutron star crust is expected to have neutron and proton superfluid components [1]. The elementary excitations of the matter affect the whole thermodynamics and long term evolution of the star. Collective modes in asymmetric nuclear matter have been studied previously, e.g. in Refs. [2, 3, 4]. In the astrophysical context, a study of the collective excitations in normal neutron star matter on the basis of the relativistic mean field method has been presented in Ref. [5]. The elementary excitations in superfluid neutron star matter have been studied by several authors, often with controversial results [6, 7, 8, 9, 10, 11]. As it is well known a neutral superfluid must present a Goldstone mode at low momentum, and this can have a strong influence on e.g. neutrino emission [6, 7, 8, 9, 10, 11, 12] or mean free path. For a single charged superfluid the Coulomb interaction suppresses the mode, which is replaced by the plasmon mode. In neutron star matter the physical situation is complicated by the multi-component structure, since the plasmon mode is in fact mainly an electron excitation, and furthermore the nuclear interaction couples neutron and proton excitation modes. We present the general theoretical scheme within the conserving approximations [13, 14], which guarantee current conservation and the fulfilment of the related Generalized Ward Identities (GWI) [15]. However, we will focus the study only on the proton component, leaving the complete treatment to future works. The reason of this choice is that the role of Coulomb interaction is quite crucial in this case, while the coupling with neutron is expected to be weak [16]. The aim of the paper is to have a qualitative picture of the overall excitation spectrum and of the corresponding strength functions. Therefore, for simplicity, in the numerical applications we will consider the effective mass equal to the bare one.
2. Formalism

According to the conserving approximation scheme [14] the density-density correlation function \( \Lambda \), defined by (\( \rho \) is the density)

\[
\Lambda(12; 3) = < T(\rho(3)\psi^\dagger(1)\psi(2)) >
\]

satisfies the integral equation

\[
\Lambda(12; 3) = \Lambda_0(12; 3) + \Lambda_0(12; 12')\mathcal{V}(12'; 3)\Lambda(45; 3)
\]

(2)

In these equations \( i \equiv (r_i, t_i, \sigma_i, \tau_i) \) stands for coordinate, time, spin and isospin variables for single particle states, and

\[
\Lambda_0(12; 3) = \frac{1}{i} G(13)G(32)
\]

(3)

The quantity \( G \) is the single particle Green’ function,

\[
G(12) = -i < T\{\psi(1)\psi^\dagger(2)\} >
\]

(4)

where \( \psi^\dagger, \psi \) are the creation and annihilation operators for the considered particles. Finally in Eq. (2) a bar over a symbol \( i \) indicates integration and summation over the corresponding set of variables. The effective particle-hole interaction \( \mathcal{V} \) is the key quantity and if the approximation has to be conserving it must be expressed by the functional derivative

\[
\mathcal{V}(12; 45) = i \frac{\delta \Sigma(12)}{\delta G(45)}
\]

where \( \Sigma \) is the single particle self-energy, defined according the Dyson’s Equation

\[
G^{-1}(12) = G_0^{-1}(12) - U(12) - \Sigma(12)
\]

(5)

with \( G_0 \) the Green’s function for non-interacting particles and \( U \) a possible external single particle potential, eventually local in space and time. In Eq. (5) the functional derivative is meant performed considering the Green’s function as a functional of the external potential. If the derivative is taken at \( U = 0 \), then \( \Sigma \) describes the linear response of the system. In this framework, if we express the self-energy in terms of the Green’s function according to some approximate scheme, the coupled Equations (2) and (5) for \( \Lambda \) and \( G \) ( or \( \Sigma \) ) define the corresponding conserving approximations. It has to be stressed that \( G \) and \( \Sigma \) must be calculated self-consistently. The simplest approximations for \( \Sigma \) are the Hartree or Hartree-Fock ones, where the self-energy is a linear functional of the Green’s function, with the bare or effective interaction. In particular the bare Coulomb interaction for the protons and electrons can be treated in this way. In this case Eq. (1) for \( \Lambda \) is the Random Phase Approximation (RPA). The excitations of the proton-electron system in Neutron Star conditions via Coulomb interaction only have been extensively studied in ref. [17]. It was shown that the electron screening suppresses the proton plasma excitation, which is then replaced by a sound-like mode, where the proton and electron components move in phase and are both strongly excited. On the contrary, the electron plasma excitation is almost unaffected, with a slightly reduced strength, and remains almost a pure electron mode. The inclusion of superfluidity can be formally achieved by including in the above scheme a further discrete variable \( \alpha \) labeling the destruction \( (\alpha = 1) \) and creation \( (\alpha = -1) \) operators, so that now the collective index \( i \equiv (r_i, t_i, \sigma_i, \tau_i, \alpha_i) \equiv (x_i, \alpha_i) \). Special care must be taken in the generalization of the basic equations, e.g. the one defining the inverse single particle Green’s function.
Figure 1. Proton excitation spectrum at saturation baryon density in Neutron Star matter, corresponding to proton Fermi momentum $k_F = 0.5596$ fm$^{-1}$, as the pairing gap $\Delta$ is varied. The four panels correspond to the gap values indicated by the corresponding labels. The electron component is included.

3. Results
The calculation of the proton pairing gap is a difficult task, because of the several competing many-body effects that must be taken into account. We will consider the value of the proton pairing gap as a parameter to be varied in a reasonable interval that can be considered typical of nuclear matter. The pairing interaction will be therefore adjusted accordingly within the BCS scheme. It is interesting to follow the evolution of the spectrum as the pairing gap $\Delta$ is changed. In Fig. 1 are reported the branches of the spectrum corresponding to excitations where the proton component is large. The left upper panel corresponds to the normal system. One identifies in the upper branch the sound mode, while the lower branch is strongly over-damped, and it does not correspond to a real excitation. The spectrum terminates at a certain momentum and the excitations are strongly damped beyond this momentum. As the gap increases the spectrum develop in a well defined manner. Below $2\Delta$ only one branch is present. It corresponds to a pseudo-Goldstone mode. This mode is present for superfluid systems where only the pairing interaction is present, and it corresponds to the Goldstone mode associated with the breaking of gauge invariance. When other interactions are active, the mode is mixed with the sound mode, but the branch is still present below $2\Delta$ with a different velocity. In any case, the energy of the mode is linear in the momentum at low enough momentum. Besides the pseudo-Goldstone, a mode above $2\Delta$ appears, that can be described as a pair-breaking mode. The two branches meet at an energy close to $2\Delta$, where they undergo a quasi-crossing phenomenon. The pseudo-Goldstone mode merges then into the ordinary sound mode, while the pair-breaking mode merges into the over-damped branch of the normal system. As the pairing gap increases, the part of the spectrum above $2\Delta$ shrinks and finally disappears. It has to be stressed that besides these branches, other branches are present, not shown in the figure, where the electron component is dominant. Again a lower branch is over-damped, while the upper branch is mainly the electron plasmon mode. However, as the pairing gap increases, this plasmon mode starts to be damped because of the coupling with the proton excitations, and it disappears even at moderately low...
values of the gap, typically 100-200 KeV. In Fig. 2 are reported the spectral functions of protons and electrons at density $0.16 \text{fm}^{-3}$ of the neutron star matter as the momentum of the excitation is varied. The pairing gap is $\Delta = 0.5 \text{ MeV}$. At low momenta the peak of the pseudo-Goldstone below $2\Delta$ is apparent. Above $2\Delta$ some strength is present, corresponding to a pair-breaking mode. As the momentum increases, the pseudo-Goldstone merges into the sound mode of the normal system. All that is consistent with the structure of the spectrum discussed above.

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References

[1] S.L. Shapiro and S.A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (Jhon Wiley and Sons, New York, 1983).

[2] P. Haensel, Nucl. Phys. A627, 53 (1978).

[3] F. Matera and V. Yu. Denisov, Phys. Rev. C 49, 2816 (1994).

[4] V. Greco, M. Colonna, M. Di Toro, and F. Matera, Phys. Rev. C 67, 015203 (2003).

[5] C. Providência, L. Brito, A. M. S. Santos, D. P. Menezes and S.S. Avancini, Phys. Rev. C 74, 045802 (2006).

[6] S. Reddy, M. Prakash, J. Lattimer and J. Pons, Phys. Rev. C 59, 2888 (1999).

[7] J. Kundu and S. Reddy, Phys. Rev. C 70, 055803 (2004).

[8] L.B. Leinson and A. Perez, Phys. Lett. B 638, 114 (2006).

[9] A. Sedrakian, H. Müther and P. Schuck, Phys. Rev. C 76, 055805 (2007).

[10] L.B. Leinson, Phys. Rev. C 78, 015502 (2008).

[11] E. E. Kolomeitsev and D. N. Voskresensky, Phys. Rev. C 77, 065808 (2008) ; arXiv:1003.2741 [nucl-th].

[12] D. G. Yakovlev, A. D. Kaminker and K. P. Levenfish, A&A 343, 650 (1999).

[13] G. Baym and L.P. Kadanoff, Phys. Rev. 124, 287 (1961).

[14] G. Baym, Phys. Rev. 127, 1391 (1962).

[15] J.R. Schrieffer, *Theory of Superconductivity*, W.A. Benjamin, Inc., N.Y., 1964.

[16] M. Baldo and C. Ducoin, Phys. Rev. C79, 035801 (2009).

[17] M. Baldo and C. Ducoin, Phys. of Atomic Nuclei 72, 1188 (2009).