On Bose-Einstein Condensation in Any Dimension

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Abstract

Arbitrarily large ground state population is a general property of any ideal bose gas when conditions of degeneracy are satisfied; it occurs at any dimension D. For D = 1, the condensation is diffuse, at D = 2 it is a sort of quasi-condensate. The discussion is made by following a microscopic approach and for finite systems. Some astrophysical consequences are discussed, as well as the temperature-dependent mass case.

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1 What is Bose-Einstein condensation?

At present there is a renewed interest in Bose-Einstein condensation (BEC), particularly after the experimental realization of it [1]. Actually, BEC is one of the most interesting problems of quantum statistics. It occurs in a free particle Bose gas at a critical temperature $T_c$, and is a pure quantum phenomenon, in the sense that no interaction is needed to be assumed to exist among the particles. BEC is interesting for condensed matter (superfluidity, superconductivity) but has also increasing interest in high energy physics (electroweak phase transition, superfluidity in neutron stars). The consequences of its occurrence at dimensions different from $D = 3$ may have interest equally in these two fields of physics.

Bose-Einstein condensation is understood as the steady increase of particles in the state with zero energy [3], or as the macroscopically large number of particles accumulating in a single quantum state [2], and its connection with the theory of phase transitions is actually a property of BEC in dimensions $D > 2$, since it has a critical temperature at which the phenomenon of condensation starts. But as different from BEC, phase transitions theory assumes, in general, some interaction among the particles [2], and properties of non-analyticity of thermodynamic quantities appear in the thermodynamic limit $N = \mathcal{N}/V_{N,V \to \infty}$, where $\mathcal{N}$ and $V$ are respectively the number of particles and volume of the system. It has been also investigated the possible connection of BEC with spontaneous symmetry breaking (SSB) [4], [5]. Actually, there is a close analogy, but not a full correspondence among them. The SSB assumes also interaction among the fields, i.e., systems with infinite number of degrees of freedom. In systems of low dimensionality, it happens that no SSB of a continuous symmetry occurs in one or two spatial dimensions $D$ according the the Mermin-Wagner theorem [6]; (see also [8], for a proof that there are no Goldstone bosons in one dimension). Thre is, however, a close correspondence between phase transitions theory and SSB. Concerning BEC, it is usually stated [7] that in the thermodynamic limit BEC is not possible in $D = 2$ and that it neither occurs
for $D = 1$. In the present letter we want to consider again the occurrence of BEC in $D = 1, 2, 3$ and later in any dimension for the case of systems having a finite volume and number of particles. We will adopt the procedure of investigating the microscopic behavior of the density (in momentum space) which exhibit some interesting properties. We also discuss an example in the context of astroparticle physics in which there can be observable phenomena which would escape in the thermodynamic limit. We must mention finally that the general case of condensation in an arbitrary dimension was first studied by May [9] for $D \geq 2$ and later by Ziff, Uhlenbeck and Kac [7].

The Bose-Einstein distribution $(e^{(E-\mu)/T} - 1)^{-1}$ for $\mu < 0$ and $p \neq 0$ vanishes strictly at $T = 0$, which fact suggests that at $T = 0$ no excited states of a Bose gas can exist on the average and condensation in the ground state seems to be a general property, whenever the conditions of quantum degeneracy of the Bose-Einstein gas are satisfied. Quantum degeneracy is usually understood to be achieved when the De Broglie thermal wavelength $\lambda$ is greater that the mean interparticle separation $N^{-1/3}$. However, the remarkable discovery made by Einstein on the Bose distribution was that condensation may occur at temperatures different from zero, which is what is usually properly named BEC.

According to our previous considerations, there are two different ideas which usually are considered to be the same, concerning what is to be understood as BEC: 1) The existence of a critical temperature $T_c > 0$ such that $\mu(T_c) = E_0$, where $E_0$ is the single particle ground state energy. (This condition is usually taken as a necessary and sufficient condition for condensation; see i. e. [10]). Then for $T \leq T_c$, some significant amount of particles starts to condense in the ground state. 2) The existence a finite fraction of the total particle density in the ground state and in states in some neighborhood of it at some temperature $T > 0$. We shall name 1) the strong and 2) the weak criterion.

From the point of view of finite-temperature quantum field theory, the strong criterion for BEC leads to the infrared $k^{-2}$ divergence of the Boson propagator,

$$\lim_{k_4 < k \to 0} G(k_4 - i\mu, \mathbf{k}, T) \simeq 1/ - \mu^2 + E_0^2 \simeq 1/k^2,$$
which is cancelled by the density of states $4\pi k^2$ when calculating the particle density. High temperature radiative corrections usually have mainly the effect of shifting the (longitudinal) boson mass in an amount $\delta M^2 \simeq T^2$ (Debye screening) and although many physical features in the investigation of Bose-Einstein condensation would appear in the non-relativistic limit (and in the one-loop approximation), we would discuss at the end briefly some features of the relativistic limit.

2 The critical temperature in the $D = 3$ case

Let us remind the origin of the critical quantities $\mu_c$, $T_c$ in the standard $3D$ theory of BEC. The chemical potential $\mu = f(N,T) < 0$ is a decreasing function of temperature at fixed density $N$, and for $\mu = 0$ one gets an equation defining $T_c = f_c(N)$. For temperatures $T < T_c$, as $\mu = 0$, the expression for the density gives values $N'(T) < N$, and the difference $N - N' = N_0$ is interpreted as the density of particles in the condensate. The mean interparticle separation is then $l = N^{-1/3}$.

In our considerations we will use integrals, as usually, understood as approximations of sums over discrete quantum states, without implying to take the thermodynamic limit. For usual macroscopic systems, as the separation between quantum states is $\Delta p = h/V^{1/3}$, the approximation of the sum by the integral quite justified.

Now, above the critical temperature for condensation

$$N = 4\pi \lambda^{-3} \int_0^\infty \frac{x^2 dx}{e^{x^2 + \bar{\mu}} - 1}$$

$$= \lambda^{-3} g_{3/2}(z)$$

where $\bar{\mu} = -\mu/T (> 0)$, $x = p/p_T$, is the relative momentum $p_T = \sqrt{2mT}$ being the characteristic thermal momentum and $\lambda = h/(2\pi m T)^{1/2}$ is the De Broglie thermal wavelength. The function $g_n(z)$ is (see i. e. [2])

$$g_n(z) = \frac{1}{\Gamma(z)} \int_0^\infty \frac{x^{n-1} dx}{z^{-1} e^x - 1}.$$
where \( z = e^{\mu/T} \) is the fugacity. At \( T = T_c \), we have \( g_n(0) = \zeta(n) \), and \( g_{3/2}(0) = \zeta(3/2) \), and the density is

\[
N_c = \frac{\zeta(3/2)}{\lambda^3}
\]

or in other words, \( l^{-3} \lambda^3 = \zeta(3/2) \approx 2.612 \). Let us have a microscopic look at BEC in the \( D = 3 \) case and to this end we investigate in detail the following quantity defined as the particle density in relative momentum space

\[
f_3(x, \tilde{\mu}) = \frac{x^2}{e^{x^2 + \tilde{\mu}} - 1}
\]

By calculating the first and second derivatives of this function, we find that for \( \tilde{\mu} \neq 0 \) it has a minimum for \( x = 0 \) and a maximum for \( x = x_\mu \) where \( x_\mu \) is the solution of

\[
e^{x^2 + \tilde{\mu}} = \frac{1}{(1 - x^2)}.
\]

In this sense, \( f(x, \tilde{\mu}) \) behaves in a very similar form to the Maxwell-Boltzmann distribution of classical statistics. But as \( \tilde{\mu} \to 0 \), \( x_\mu \to 0 \) also, and the maximum of the density, for strictly \( \tilde{\mu} = 0 \), is located at \( x = 0 \). The convergence to the limit \( x = 0 \) is not uniform. A finite fraction of the total density falls in the ground state.

If we go back and substitute the original integral over momentum by a sum over shells of quantum states of momentum (energy states), and write

\[
N_c = \frac{4\pi}{h^3} \sum_{i=0}^{\infty} \frac{p_i^2 \Delta p}{e^{p_i^2/2MT} - 1}.
\]

For \( i = 0 \), by taking \( \Delta p \approx h/V^{1/3} \), where \( V \) is the volume of the vessel containing the gas, the contribution of the ground state density is \( N_0 = \frac{4}{V^{1/3} \lambda^2} \). We have thus a fraction of

\[
N_0/N_c = 4\lambda\zeta(3/2)/V^{1/3} = 4\zeta(3/2)/N^{1/3}
\]

particles in the ground state, which is the most populated, as described by the statistical distribution, at \( T = T_c \). A numerical estimation for one liter of \( He \) gas leads to \( N_0/N \approx 10^{-6} \). In quantum states in a small neighborhood of the ground state, the momentum
density has slightly lower values. Thus, at the critical temperature for BEC, there is a
set of states close to the ground state, having relative large densities.

The reader may argue that in the thermodynamic limit (5) vanishes. It is true. But
(3) indicates an interesting relation: the larger the separation between quantum states,
the larger the population in the ground state at the critical temperature. In the thermo-
dynamic limit the quantum states form a continuum, and (3) has no meaning. However,
all systems of physical interest, in laboratory as well as in astrophysical and cosmological
contexts have finite $V$ and $N$.

We conclude that at the critical temperature for condensation the density of particles in
momentum space reaches its maximum at zero momentum, and by describing the density
as a sum over quantum states, a macroscopic fraction is obtained for the density in the
ground state and in neighbor states. Thus, at the critical temperature, the weak criterion
is satisfied. For values of $T < T_c$, the curve describing the density in momentum space
flattens on the $p$ (or $x$) axis, and its maximum decreases also. As we admit conservation
of particles, we get increased the ground state density by adding to $f(x)$ the quantity
$2N[1 - (T/T_c)^{3/2}][\theta(T_c - T)\lambda^3\delta(x)]$ as an additional density. The density in neighbor states
decreases as well as in larger momentum states. This leads to the usual Bose-Einstein
condensation. We must remark that in this case for values of $T$ smaller but close to $T_c$
both the weak and the strong criteria are satisfied.

3 The $D = 1$ and $D = 2$ cases

Let us see what happens for $D = 1$. In this case, the mean interparticle separation
$l = N^{-1}$. The density in momentum space coincides with the Bose- Einstein distribution,
$f_1(p, T, \mu) = (e^{p^2/2MT - \mu/T} - 1)$. This function has only one extremum, a maximum, at
$p = 0$. By using the previous change of variables, we have the expression for the density
of particles as
\[ N = 2\lambda^{-1} \int_0^\infty \frac{dx}{e^{x^2 + \bar{\mu}} - 1} = \frac{1}{\lambda} g_{1/2}(z). \] (6)

We have thus that \( N\lambda \simeq g_{1/2}(z) \). The fact that \( \lim g_{1/2}(z) \) diverges as \( z \to 1 \) indicates an enhancement of the quantum degeneracy regime. But this fact actually means that \( \mu \) is a decreasing function of \( T \) for \( N \) constant, and for very small \( \bar{\mu} \) one can write, approximately

\[ N \simeq 2\lambda^{-1} \int_0^{x_0} \frac{dx}{x^2 + \bar{\mu}} \simeq \frac{\pi \bar{\mu}^{-1/2}}{\lambda}, \] (7)

where \( x_0 = p_0/MT \), \( p_0 \) being some characterizing momentum \( p_0 \gg p_T \). Thus, \( \bar{\mu} \) does not vanishes at \( T \neq 0 \), and for small \( T \) it is approximately given by \( \bar{\mu} = \pi^2/4N^2\lambda^2 \). It is easy to obtain approximately the pressure \( P (= \text{force}/L^0) \) and energy density respectively as

\[ P \simeq \frac{\pi^2 T}{N\lambda^2} \quad \text{and} \quad U = \frac{1}{2} P \] (8)

which indicates that they vanish as \( T/N \to 0 \), the specific heat \( C_v = \partial U/\partial T \) decreasing as \( T^{1/2} \) as \( T \to 0 \). For high temperatures one can easily prove that \( C_v \) is constant; it indicates some correspondence with the behavior of \( C_v \) in the \( D = 3 \) case [2], but there is no discontinuity in its derivative with regard \( T \).

By substituting the last expression for \( \bar{\mu} \) back in (7), one has that

\[ N \simeq \frac{\lambda^{-1}}{\gamma} \int_{-\gamma}^{\gamma} \frac{\gamma dx}{x^2 + \gamma^2} \] (9)

where \( \gamma = \pi/2N\lambda \). Due to the properties of the Cauchy distribution, one can write

\[ \frac{1}{2} N = \frac{\lambda^{-1}}{\gamma} \int_{-\gamma}^{\gamma} \frac{\gamma dx}{x^2 + \gamma^2} \]

We can also write

\[ f_1(x, T, N)_{T/N \to 0} \simeq \frac{2N\lambda}{\pi} \delta(x) \]

Thus, for small \( T/N \) all the population tends to concentrate on the ground state, and although not an usual Bose- Einstein condensation, we claim that (as in the magnetic field case, [11]) we have a ”diffuse” condensation [12] when \( T/N \) is low enough. There is no critical point; there is no discontinuity in the derivative of the specific heat in
The condensation is the outcome of a continuous process in the sense that there is always some amount of particles in the ground state, and this quantity can be increased continuously to reach macroscopically significant values, by decreasing enough the ratio $T/N$, i.e., even for temperatures far from zero. The $D = 1$ case satisfies the weak criterion at any temperature $T$, but not the strong one. Closely connected with the $D = 1$ case is the problem of condensation of a gas of charged particles in presence of a strong magnetic field \[1\], where all the previous considerations apply, by substituting $\gamma = 2MeBT/\hbar c^2 N$, where $e$ is the electric charge and $B$ the magnetic field in Gauss. If $eB\hbar/McT \gg 1$, the system is confined to the Landau ground state $n = 0$, and the problem can be treated in close connection to $D = 1$ case. We will consider the model of a gas of kaons in a neutron star \[13\]. By taking a density $N \simeq 10^{44} \text{ cm}^{-3}$, $T \simeq 10^{8}\text{K}$ and local magnetic field $B \sim 10^{14} \text{e G}$, the condition of quantum degeneracy are exceedingly satisfied. In that case $\gamma = 10^{-30}$. By taking the dimensions of the star as $10^7 \text{ cm}$, the discrete quantum states would be spaced by an amount of $\delta p = 10^{-34} \text{ gcm/s}$. One half of the total density would be distributed in $\eta = 2\gamma/\Delta p = 10^4$ quantum states. The ground state density with zero momentum $p$ along the magnetic field, can be estimated then as $\Delta N = \eta^{-1}N = 10^{40}$, leading to observable effects: superfluidity and strong diamagnetic response to the applied field. (the magnetization $\mathcal{M} = eB\hbar/2Mc \simeq 10^{16} \text{ G}$ would exceed in two orders of magnitude the microscopic applied local field and would be the preponderating field). However, the quantity $\Delta N/N = \eta^{-1} = \pi\hbar Nc/MeBT^2$ tends to zero in the thermodynamic limit, and some of the physical effects or condensation would appear softened or even erased in that limit.

Let us now turn our attention to the $D = 2$ case. Here $l = N^{-1/2}$ The distribution is

$$f_2(x, T, \bar{\mu}) = \frac{x}{e^{x^2+\bar{\mu}} - 1}$$

and has always an absolute minimum at $x = 0$, (the density vanish in the ground state) and a maximum at a value of $x > 0$ being a solution of $e^{x^2+\bar{\mu}} = \frac{1}{1-2x^2}$. We can write the
density as
\[ N = 2\lambda^{-2} \int_0^\infty \frac{e^{-(x^2+\mu)x}dx}{1 - e^{-(x^2+\mu)}} = 2\lambda^{-2} \ln(1 - e^{-\mu}). \] (10)
We have \( \bar{\mu} = -\ln(1 - e^{-N\lambda^2/2}) \); thus as \( T/N \to 0 \), \( \bar{\mu} \to e^{-Nh/4\pi MT} \). The solution \( \bar{\mu} = 0 \) makes the density \( N \) in (10) to diverge, as in the \( D = 1 \) case, except for \( T = 0 \). At nonzero \( \bar{\mu} \), the population in a closed small neighborhood of \( x = 0 \) is strictly zero, but the amplitude of this interval decreases as \( \bar{\mu} \to 0 \), that is, the maximum of the density in momentum space is reached at a value of the momentum \( x_{\text{max}} \neq 0 \) which decreases as \( T/N \to 0 \). The maximum of the density is in a neighborhood but not strictly in the ground state. None of the weak and strong criteria are satisfied at any temperature \( T \neq 0 \). Thus, there is no strict Bose-Einstein condensation in the sense that the value of the density in the ground state is zero for any \( T/N \neq 0 \). As one can write for \( x \) small
\[ \lim_{x\to 0} f_2(x, T, \bar{\mu}) = \lim \frac{1}{\pi} \frac{x}{x^2 + \bar{\mu}} = \delta(\sqrt{\bar{\mu}}), \]
i.e., the density at \( x = 0 \) is nonzero only for \( \bar{\mu} = 0 \). But as the maximum of the density increases continuously by decreasing \( T/N \), being located at \( x_m \to 0 \), (the convergence to \( x = 0 \) being non-uniform), we have a sort of quasi-condensate, in the spirit of the weak criterion, i.e. most of the density can be found concentrated in a small interval of values of momentum around the \( p = 0 \) state at arbitrary small temperatures.

The pressure \( P = \text{force/L} \) and energy desntity in such case are approximately given by
\[ P = 2\lambda^{-2}Te^{-N\lambda^2/2}(N\lambda/2 - 1) \quad U = P, \] (11)
and obviously vanish in the limit \( T \to 0 \). For high \( T \), these quantities vary linearly with \( T \), as can be easily verified. The behavior of \( C_v \) is roughly similar to the the \( D = 1 \) case.

4 The case \( D > 3 \)

In the case \( D > 3 \), the density in momentum space reads,
\[ f_D(x, T, \bar{\mu}) = \frac{x^{D-1}}{e^{x^2+\bar{\mu}} - 1} \]
This function has an absolute minimum at \( x = 0 \) and an absolute maximum at \( x = x_{\text{max}} \neq 0 \) given by the nonzero solution of the equation \( e^{x^2 + \mu} = 1/[1 - 2x^2/(D - 1)] \). The density is, thus, zero at the ground state, and in a small neighborhood of it, and in this sense differs from the \( D = 3 \) and the \( D = 1 \) cases. The total density is given by

\[
N = \frac{\lambda^{-D}}{\Gamma(D/2)} \int_{0}^{\infty} f_D dx = \lambda^{-D} g_{D/2}(z) \tag{12}
\]

In this case, the density \( \mu \) decreases as \( T \to 0 \) and \( N \) converges for exactly \( \mu = 0 \). Thus, there is a nonzero critical temperature \( T_c \) such that \( \mu(T_c) = 0 \). Then for \( T < T_c \), \( (12) \) is unable to account for the total density and if conservation of particles is imposed, one must admit that the lacking density \( N_0 = N - N' \) is exactly at the ground state. Thus, although the density of particles in an (open) neighborhood of the ground state is zero, exactly at the ground state it is given by \( 2N[1 - (T/T_c)^{D/2}]\lambda^D \delta(x) \).

At the critical temperature \( N_c \lambda^D = \zeta(D/2) \), which tends to unity with increasing \( D \). We may conclude that the \( D \)-dimensional gas becomes less degenerate with increasing \( D \). For any dimension, the relation between energy and pressure \( P = 2U/D \) holds. Also, below the critical temperature, \( C_v \sim T^{D/2} \). We see that the \( D > 3 \) case satisfies the strong but not the weak criterion for \( T < T_c \). Our results for \( D \geq 3 \) are in agreement with those obtained in ref. \[10\].

Expression \( (12) \) is valid for continuous \( D \). It can be easily checked that for \( 1 > D > 0 \) the quantity \( f_D(x) \) diverges for \( x = 0 \), and \( N \) remains finite for \( \mu \neq 0 \). Thus the ”diffuse” condensation takes place in the interval \( 1 \geq D > 0 \). The \( D = 2 \)-like behavior occurs for \( 2 \geq D > 0 \), whereas, as demonstrated by May \[9\], usual condensation occurs for \( D > 2 \). However, for \( 2 < D < 3 \), both criteria, weak and strong are satisfied, \( f_D \) being divergent at \( x = 0 \), whereas \( N \) remains finite.

5 The relativistic case

We shall revisit the relativistic case. Here the conservation of particles must reflect some invariance property of the Lagrangian. We are keeping in mind the simplest case of a
charged massive scalar field.

In that case, the conserved quantity, derived from the Noether theorem is the charge (in \(D\) spatial dimensions) and as different from \([\[4]\]), we must include the contribution of antiparticles: this is a natural consequence of a relativistic finite-temperature treatment of the problem and we cite only some authors \([14]\), \([4]\), \([3]\). At high \(T\) one must consider the natural excitation of pairs particle-antiparticle, and the conserved quantities depend usually from the difference of their average densities.

\[
Q = i \int_0^\infty j_0(x) d^D x
\]

(13)

where

\[
j_\nu = \psi^* \partial_\nu \psi - (\partial_\nu \psi^*) \psi
\]

After building the density matrix for the Grand Canonical ensemble, one can write the thermodynamic potential, and from it the conserved charge as an expression which contains the difference of average number of particles minus antiparticles:

\[
< Q > = \frac{2\pi^{D/2} T^D}{\Gamma(D/2) \hbar^D} \int_0^\infty x^{D-1} dx (n_p - n_a),
\]

(14)

where \(n_p = [(e^{E-\bar{\mu}} - 1)^{-1}, n_a = [e^{E+\bar{\mu}} - 1)^{-1}]\) are the particle and antiparticle densities, \(E = (x^2 + \bar{M}^2)\), and \(x = p/T, \bar{M} = Mc/T\). Condensation in \(D \geq 3\) occurs for \(\bar{\mu} \rightarrow \bar{M}\). For \(D < 3\) the condensation is very well reproduced by the infrared (non-relativistic) limit already seen, by taking the chemical potential as \(\mu' = \mu - M\).

A very interesting case occurs when \(M = 0\). In that case (14) demands \(\mu = 0\) for not to have negative population densities of particles or antiparticles. All the charge must be concentrated in the condensate and the critical temperature for condensation, as suggested in \([3]\) is \(T = \infty\).
6  The case $M = M(T)$

In some systems the interactions at high temperature behave in such a way that can be described effectively as free particle systems with variable mass; i.e., the temperature-dependent interaction leads to the arising of a mass $M = M(T)$. If $M(T) \to 0$ we have BEC with increasing temperatures, as discussed in [13]. We have in that case two regions for condensation: the low temperature and the extremely high one. It is interesting to consider also the case in which $M(T)$ decreases enough to have conditions for condensation in some interval $T_1 \leq T \leq T_2$, where $T_1 \neq 0$, $T_2 \neq \infty$ are the two critical points. For $D=3$ we would have condensation in some ”hot” interval of temperatures; i.e. superfluid or superconductive effects may appear in some intervals of temperature even far from $T = 0$.

All our previous considerations for condensation in $D \neq 3$, would also be valid in such case.

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