USING A PHENOMENOLOGICAL MODEL TO TEST THE COINCIDENCE PROBLEM OF DARK ENERGY

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ABSTRACT

By assuming a phenomenological form for the ratio of the dark energy and matter densities \( \rho_X \propto \rho_m^{a_\xi} \), we discuss the cosmic coincidence problem in light of current observational data. Here, \( \xi \) is a key parameter to denote the severity of the coincidence problem. In this scenario, \( \xi = 3 \) and \( \xi = 0 \) correspond to \( \Lambda \)CDM and the self-similar solution without the coincidence problem, respectively. Hence, any solution with a scaling parameter \( 0 < \xi < 3 \) makes the coincidence problem less severe. In addition, the standard cosmology without interaction between dark energy and dark matter is characterized by \( \xi + 3\omega_X = 0 \), where \( \omega_X \) is the equation of state of the dark energy component, whereas the inequality \( \xi + 3\omega_X \neq 0 \) represents non-standard cosmology. We place observational constraints on the parameters (\( \Omega_X, \omega_X, \xi \)) of this model, where \( \Omega_X,0 \) is the present value of density parameter of dark energy \( \Omega_X \), by using the Constitution Set (397 supernovae of type Ia data, hereafter SNeIa), the cosmic microwave background shift parameter from the five-year Wilkinson Microwave Anisotropy Probe and the Sloan Digital Sky Survey baryon acoustic peak. Combining the three samples, we get \( \Omega_X,0 = 0.72 \pm 0.02 \), \( \omega_X = -0.98 \pm 0.07 \), and \( \xi = 3.06 \pm 0.35 \) at 68.3% confidence level. The result shows that the \( \Lambda \)CDM model still remains a good fit to the recent observational data, and the coincidence problem indeed exists and is quite severe, in the framework of this simple phenomenological model. We further constrain the model with the transition redshift (deceleration/acceleration). It shows that if the transition from deceleration to acceleration happens at the redshift \( z > 0.73 \), within the framework of this model, we can conclude that the interaction between dark energy and dark matter is necessary.

Key words: cosmology; miscellaneous – cosmology: theory – dark energy

Online-only material: color figures

1. INTRODUCTION

One of the most important issues of modern cosmology concerns the accelerating expansion of the universe, which has been discovered and verified by type Ia supernovae (SNeIa; Riess et al. 1998; Perlmutter et al. 1999; Hicken et al. 2009), cosmic microwave background (CMB; Spergel et al. 2003) and baryon acoustic oscillation (BAO; Eisenstein et al. 2005) observations (also see recent review: Frieman et al. 2008). After the discovery of this scenario, a great variety of attempts have been made to explain this acceleration. The nature of the accelerating expansion is one of the most outstanding problems of physics and astronomy today. Currently, the existing mechanisms for cosmic acceleration can be roughly divided into three kinds (see the reviews: Copeland et al. 2006; Caldwell & Kamionkowski 2009). (1) An exotic energy component with negative pressure, dubbed as dark energy, is introduced in the right-hand side of the Einstein equation. The nature of the dark energy is still unknown. Dark energy models include the \( \Lambda \)CDM model (Carroll et al. 1992; Riess et al. 1998; Peebles & Ratra 2003), the holographic dark energy model (Li 2004; Wu et al. 2008), the Chaplygin gas model (Kamenshchik et al. 2001; Benoãºm 2002; Zhu & Alcaniz 2004), and some scalar field models, such as quintessence (Caldwell et al. 1998; Zlatev et al. 1999). (2) The theory of gravity is modified at the Hubble scale, and the cosmic acceleration is due to gravity, without the help of an exotic negative pressure component. Examples of modified gravity theory include the braneworld model (Arkani-Hamed et al. 1998; Antoniadis et al. 1998; Randall & Sundrum 1999a; Randall & Sundrum 1999b; Dvali et al. 2000; Defayet 2001; Zhu & Alcaniz 2005), \( f(R) \) gravity (Carroll et al. 2004; Nojiri & Odintsov 2003; Song et al. 2007; Atazadeh et al. 2008; Wu & Zhu 2008), and the Cardassian model (Freese & Lewis 2002; Zhu & Fujimoto 2002, 2003, 2004; Sen & Sen 2003; Mosquera Cuesta et al. 2008). The above-mentioned two mechanisms are based on the cosmological principle, which claims that our universe is isotropic and homogeneous at large scales. (3) The local inhomogeneity of our universe is used to explain the acceleration (see George 2008 and corresponding references therein).

From the observational point of view, it is well known that flat models with a very small cosmological term (\( \rho_\Lambda \lesssim 10^{-47} \) GeV\(^4\)) are in good agreement with almost all sets of cosmological observations, which makes them an excellent description of the observed universe. From the theoretical viewpoint, however, these scenarios are embarrassed by the so-called cosmological constant problems (Weinberg 1989; Weinberg 2000). One issue of the cosmological constant problems is to understand in a natural way why the observed value of the vacuum energy density \( \rho_\Lambda \) is so small. As it is known, the theoretical value of \( \rho_\Lambda \) is about 120 orders of magnitude larger than the observed value. This problem is called fine-tuning problem. Another issue of the cosmological constant problems is the so-called “why now?” or coincidence problem (Zlatev et al. 1999). Briefly put, it is to understand why \( \rho_\Lambda \) is not only small, but also the same order of magnitude as the present mass density of universe. The present epoch then is the very special time in the history of the universe, the only period when \( \Omega_\Lambda \approx \Omega_m \).

Although there is no convincing fundamental theory available to understand why the vacuum energy dominance happened only recently, several possible approaches have been adopted to explain or alleviate the coincidence problem. One approach involves some sort of anthropic principle to explain the coincidence problem (Weinberg 2000; Vilenkin 2001; Garriga et al.
Considering a flat FRW universe with \( \Omega_X + \Omega_m = 1 \) throughout, we obtain

\[
\Omega_X = \frac{\Omega_{X,0}a^\xi}{1 - \Omega_{X,0}(1 - a^\xi)},
\]

where \( \Omega_{X,0} \) is the present value of \( \Omega_X \). According to the energy conservation equation, one obtains

\[
\frac{d\rho_{tot}}{da} = \left(1 + \omega_X \Omega_X \right) \rho_{tot} = 0,
\]

where \( \rho_{tot} = \rho_m + \rho_X \) is the total density. Setting

\[
\rho_X = \kappa \rho_m a^\xi,
\]

based on Equation (3), we have

\[
\frac{d\rho_m}{da} + \frac{3}{a} \rho_m = - (\xi + 3\omega_X) \frac{\kappa a^{\xi-1}}{1 + \kappa a^\xi} \rho_m,
\]

where \( \kappa \) is a constant. Further, one can read

\[
\frac{d\rho_m}{da} + \frac{3}{a} (1 + \omega_X) \rho_m = - Q,
\]

where the interaction term \( Q = - (\xi + 3\omega_X) \frac{\kappa a^{\xi-1}}{1 + \kappa a^\xi} \rho_m \). The phenomenological interaction term \( Q \) is inspired from the interaction between the dilaton field \( \sigma \) and the matter field in the scalar-tensor theory of gravity (Kaloper & Olive 1998),

\[
\mathcal{L} = \sqrt{-g} \left[ -\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \left( \frac{\xi}{2} \right) - 2 \right] \mathcal{L}_{\sigma}.
\]

For a general coupling function \( \xi(\sigma) \), we get the interaction term \( Q = - 3 \rho_{tot} H (d(\ln \xi)/d(\ln a))/2 \) (Curbelo et al. 2006) which has the desired linear relationship between \( Q \) and \( \rho_m \).

Consequently, \( \xi > 3\omega_X \) or \( \xi + 3\omega_X < 0 \), corresponding to \( Q < 0 \), denotes the standard cosmology without interaction between dark energy and dark matter. In contrast, \( \xi + 3\omega_X > 0 \), corresponding to \( Q > 0 \), represents the non-standard cosmology with interaction between dark energy and dark matter. Furthermore, \( \xi + 3\omega_X = 0 \), corresponding to \( Q = 0 \), indicates that the energy is transferred from dark matter to dark energy. On the other hand, \( \xi + 3\omega_X < 0 \), corresponding to \( Q > 0 \), denotes that the energy transfer from dark energy to dark matter.

The solution to Equation (3) is

\[
\frac{\rho_{tot}}{\rho_0} = \exp \left( \int_a^1 \frac{da}{a} \left( 1 + \omega_X \Omega_X \right) \right).
\]

By assuming \( \omega_X = \text{const.} \), we find

\[
\rho_{tot} = \rho_0 a^{-3} \left( 1 - \Omega_{X,0}(1 - a^\xi) \right)^{-3\omega_X/\xi},
\]

so that

\[
E^2 = a^{-3} \left( 1 - \Omega_{X,0}(1 - a^\xi) \right)^{-3\omega_X/\xi},
\]

where \( E = H/H_0 \) is the dimensionless Hubble parameter.
This phenomenological model has three free parameters ($\Omega_{X,0}$, $\omega_X$, $\xi$), where $\Omega_{X,0}$ specifies the current density of dark energy, $\omega_X$ denotes its equation of state, and $\xi$ presents how strongly $\Omega_X/\Omega_m$ varies with redshift and quantifies the severity of the coincidence problem. In addition, the special cases $\xi = 3$ and $\xi = 0$ correspond to $\Lambda$CDM and the self-similar solution without coincidence problem, respectively. The parameters ($\Omega_{X,0}$, $\omega_X$, $\xi$) can be constrained from the cosmological data as we describe below.

3. CONSTRAINTS FROM THE RECENT OBSERVATIONS

3.1. Constraints from SNeIa

As it is known, the first direct evidence for cosmic acceleration came from SNeIa (Riess et al. 1998; Perlmutter et al. 1999), and they have provided the strongest constraints on the cosmic equation of state and other cosmological parameters (Riess et al. 2004, 2007; Astier et al. 2006; Wood-Vasey et al. 2007; Davis et al. 2007; Kowalski et al. 2008; Hicken et al. 2009). Since they are as bright as typical galaxies when they peak, SNeIa can be observed to large distances, recommending their utility as standardizable candles for cosmology. At present, they are the most effective and mature probe of cosmology. The present analysis uses the recently compiled “Constitution Set” of 397 SNeIa data covering a redshift range $0.015 \leq z \leq 1.551$ (Hicken et al. 2009).

Constrains from SNeIa can be obtained by fitting the distance modulus $\mu(z)$. The theoretical distance modulus is

$$\mu_0(z) = 5 \log_{10}(D_L(z)) + \mu_0,$$  

where $\mu_0 = 42.38 - 5 \log_{10} h$. The Hubble-free luminosity distance is given by

$$D_L(z; p, \mu_0) = H_0d_L = (1 + z) \int_0^z \frac{du}{E(u; p)},$$  

where $p \equiv (\Omega_{X,0}, \omega_X, \xi)$ is the complete set of parameters. The best-fit values of parameters can be determined by minimizing the function

$$\chi^2_{SNe}(p, \mu_0) = \sum_{i=1}^n \left[ \frac{\mu_{\text{obs},i}(z_i) - \mu_{\text{obs},i}(z_i)}{\sigma^2_{\mu_i}} \right],$$  

where $n = 397$ is the number of the SNeIa data and $\mu_{\text{obs},i}(z_i)$ is the distance modulus obtained from observations, $\sigma_{\mu_i}$ is the total uncertainty of the SNeIa data. Figure 1 shows the probability contours constrained from the 397 SNeIa data in the $(\omega_X, \xi)$ plane. The best-fit parameters in this case are found to be $\Omega_{X,0} = 0.71 \pm 0.03$, $\omega_X = -1.01 \pm 0.17$, and $\xi = 3.16 \pm 1.91$ with 68.3% confidence level.

3.2. Constraints from BAO

The recently observed baryon oscillations in the power spectrum of galaxy correlation function also is a powerful probe to explore dark energy and constrain cosmological model (Eisenstein et al. 2005). Before the universe had cooled sufficiently for neutral atoms to persist, it consisted of a hot plasma of photons, electrons, protons, baryons, and other light nuclei. The tight coupling between photons and electrons due to Thompson Scattering leads to oscillations in the hot plasma. As the universe expands and cools, electrons and protons combine into atoms making the universe neutral. The pattern of initial perturbations and expanding wavefronts is seen in the CMB, and is ultimately imprinted on the matter distribution and should be seen in the spectrum of galaxy correlations today. The primary representation of these BAOs is a feature at the “sound horizon” length $r_s$ which is the distance traveled by the acoustic waves by the time of plasma recombination (Copeland et al. 2006; Albrecht et al. 2006).

The size of BAO peak can be used as a “standard cosmological ruler” to constrain the cosmological parameters (Blake & Glazebrook 2003; Seo & Eisenstein 2003; Dolney et al. 2006), which was first successfully found by detecting of a peak in the correlation function of about 50,000 luminous red galaxies over 3800 deg$^2$ in the SDSS (Eisenstein et al. 2005). This peak can be denoted by a parameter $A$, which is independent of cosmological models and for a flat universe can be expressed as

$$A(z_{\text{BAO}}; p) = \sqrt{\Omega_{m,0} E(z_{\text{BAO}})}^{-1/3} \left[ \frac{1}{z_{\text{BAO}}} \int_{z_{\text{BAO}}}^{\infty} \frac{dz}{E(z; p)} \right]^{2/3},$$  

where $\Omega_{m,0} = 1 - \Omega_{X,0}$ and $z_{\text{BAO}} = 0.35$. The observational value is $A = 0.469 \pm 0.017$. The $\chi^2_{\text{BAO}}$ value is

$$\chi^2_{\text{BAO}}(p) = \frac{(A(z_{\text{BAO}}; p) - 0.469)^2}{0.017^2}.$$

In this subsection, our analysis consider the SNeIa data combining with BAO. The best-fit values for parameters $p \equiv (\Omega_{X,0}, \omega_X, \xi)$ can be determined by minimizing

$$\chi^2_{\text{total}} = \chi^2_{\text{SNe}} + \chi^2_{\text{BAO}}.$$

Figure 2 shows the contours constrained from the BAO data in addition to the SNeIa data in $(\omega_X, \xi)$ plane. The results are $\Omega_{X,0} = 0.72 \pm 0.02$, $\omega_X = -0.99 \pm 0.18$, and $\xi = 3.17 \pm 1.83$ at 68.3% confidence level. Compared to Figure 1, the allowed regions of parameters are not considerably reduced.
3.3. Constraints from CMB

The CMB observations are playing a key role in this era of precision cosmology (Barreiro 2009). In 1965 nearly isotropic background of microwave radiation was discovered, which has provided a wealth of new cosmological data. Subsequently, a series of experiments, such as COBE (Salopek 1992), BOOMERANG (Maukof et al. 2000; de Bernardis et al. 2000), MAXIMA (Hanany et al. 2000), Archeops (Benoit et al. 2003), VSA (Rubino-Martin et al. 2003; Rebolo et al. 2004), DASI (Kovac et al. 2002; Leitch et al. 2005), and WMAP (Spergel et al. 2003, 2007; Komatsu et al. 2009), are designed to detect the CMB. Most notably, the WMAP satellite has imposed strong constraints on cosmological parameters.

The structure of the anisotropy of the CMB radiation depends on two eras in cosmology, namely, the last scattering era ($z_s$) and today ($z = 0$), that can be applied to limit the model parameters by using the shift parameter $R$. For a flat universe, $R$ can be expressed as

$$ R(z_s; p) = \sqrt{\Omega_m \Omega} \int_0^{z_s} \frac{dz}{E(z; p)}, $$

where the last scattering redshift $z_s = 1089$. From the five-year WMAP data results (Komatsu et al. 2009), one can get the observational value $R = 1.710 \pm 0.019$. The $\chi^2_{\text{CMB}}$ value is

$$ \chi^2_{\text{CMB}} = \frac{(R - 1.710)^2}{0.019^2}. $$

We combine the above three data sets to minimize the total $\chi^2_{\text{total}}$:

$$ \chi^2_{\text{total}} = \chi^2_{\text{SNe}} + \chi^2_{\text{BAO}} + \chi^2_{\text{CMB}}. $$

Figure 3 shows the contours constrained from the joint analysis of SNeIa, BAO, and CMB data. The values of the parameters are $\Omega_{X,0} = 0.72 \pm 0.02$, $\omega_X = -0.98 \pm 0.07$, and $\xi = 3.06 \pm 0.35$ (68.3% confidence level). Compared to Figures 1 and 2, the allowed region of $\xi$ is remarkably reduced.

In addition, Figure 4 presents other results constrained from the joint analysis of SNeIa, BAO, and CMB data. The left panel of Figure 4 displays the evolutions of $\Omega_p(z)$ and $\Omega_X(z)$. The right panel of Figure 4 presents the evolution of their ratio $r(z) = \Omega_p(z)/\Omega_X(z)$. It shows that the values of $\Omega_p(z)$ and $\Omega_X(z)$ are the same order of magnitude in the redshift range $0.0 \leq z \leq 1.6$.

4. CONSTRAINTS FROM THE TRANSITION REDSHIFT $z_T$

The transition redshift (deceleration/acceleration) has been proved to provide an efficient way for constraining the models (Zhu & Fujimoto 2004; Zhu & Alcaniz 2005). According to the definitions of the decelerating parameter $q \equiv -(\ddot{a}/a)/\dot{a}^2$ and the Hubble parameter $H \equiv \dot{a}/a$, one obtains

$$ q = \left(\frac{-\ddot{a}}{a}\right) / H^2 = \frac{dH^{-1}}{dt} - 1. $$

By using the relations $a_0/a = 1 + z$ and $E(z) = H/H_0$, Equation (21) can be written as

$$ q(z) = \frac{1}{2E^2(z)} \frac{dE^2(z)}{dz}(1 + z) - 1, $$

where $E^2(z)$ is given by Equation (11). The transition redshift $z_T$, at which the expansion underwent the transition from deceleration to acceleration, is obtained by solving the equation

$$ q(z = z_T) = 0. $$

From Equation (11), Equation (22), and Equation (23), we find

$$ (1 + z_T)^{-\xi} = \frac{\Omega_{X,0} - 1}{\Omega_{X,0}(1 + 3\omega_X)}. $$
that the acceleration could have started at high redshift, even up to \( z \approx 3 \), if dark energy interacts strongly with dark matter. In contrast, the standard noninteracting models hardly even reach \( z_T \simeq 1 \). This is consistent with our results. If the transition redshift can be estimated by a model-independent measurement, it could be used as an important test to distinguish between coupled and uncoupled quintessence classes of models.

5. DISCUSSIONS AND CONCLUSIONS

A phenomenological scaling solution \( \rho_X \propto \rho_a a^4 \) with minimal underlying theoretical assumptions appears to be a quite effective tool for analyzing the relationship between the two dark components of our universe. In this phenomenological model, the standard cosmology without interaction between dark energy and dark matter is characterized by \( \xi + 3\omega_X = 0 \), whereas \( \xi + 3\omega_X \neq 0 \) denotes non-standard cosmologies. The value of \( \xi \) quantifies the severity of the coincidence problem while the special cases \( \xi = 3 \) and \( \xi = 0 \) correspond, respectively, to \( \Lambda \)CDM and the self-similar solution without coincidence problem. Hence, any solution with a scaling parameter \( 0 < \xi < 3 \) makes the coincidence problem less severe. We have investigated the constrains imposed by the recent observations. Using the Constitution Set (397 SNeIa data) solely, we obtain \( \Omega_{X,0} = 0.71 \pm 0.03 \), \( \omega_X = -1.01 \pm 0.17 \), and \( \xi = 3.16 \pm 1.91 \) (68.3% c.l.). When the BAO data are taken into account in addition to the SNeIa data, the results are \( \Omega_{X,0} = 0.72 \pm 0.02 \), \( \omega_X = -0.99 \pm 0.18 \), and \( \xi = 3.17 \pm 1.83 \) (68.3% c.l.). Finally, combining SNeIa (397 data), BAO, and CMB data, the result is that \( \Omega_{X,0} = 0.72 \pm 0.02 \), \( \omega_X = -0.98 \pm 0.07 \), and \( \xi = 3.06 \pm 0.35 \) (68.3% c.l.).

Figures 1–3 show the observational contours from SNeIa data, SNeIa+BAO data and SNeIa+BAO+CMB data in the \( (\omega_X, \xi) \) plane, respectively. From these figures, it is rather obvious that the SNeIa and BAO data do not provide stringent constraints on \( \xi \), but inclusion of the CMB data significantly reduces the allowed region of this parameter. This implies that the high redshift may be able to give tighter constraint on the parameter \( \xi \). In the three cases, they all display obviously that the self-similar solution \( \xi = 0 \) without coincidence problem is excluded from the data. As we see from the three contours, the \( \Lambda \)CDM model, which corresponds to the point \( (\omega_X, \xi) = (-1, 3) \), is within the 1\( \sigma \) contour bound. It shows that the \( \Lambda \)CDM model still remains a good fit to the recent observational data,
as well as, the coincidence problem indeed exists and is quite severe. In addition, there is a tendency in the contours that $\xi$ decreases as $\omega_X$ increases.

The theoretical constrains from the transition redshift $z_T$ show that if the transition from deceleration to acceleration happens at the redshift $z_T > 0.73$, in the framework of this model, the interaction between dark energy and dark matter should be taken into account. On the other hand, if it happens at the redshift $z_T \leq 0.73$, we just cannot confirm whether the interaction is necessary by using the transition redshift only.

Two problems deserve to be pointed out here. Firstly, the line $\xi + 3\omega_X = 0$, corresponding to the standard cosmology without interaction between the two dark components, runs through the $\sigma_1, \sigma_2$, and $\sigma_3$ regions in the $(\omega_X, \xi)$ plane. It denotes that the recent observational data are insufficient to discriminate between the standard cosmology and the non-standard cosmology. This problem may be resolvable by using the transition redshift test, if the transition redshift can be obtain by a model-independent measurement. Secondly, the data also cannot discriminate between $\omega_X > -1$ and $\omega_X < -1$. In order to break the degeneracy, we pin our hope on the future observational data of high redshift SNeIa data from SNAP, etc. (Albrecht et al. 2006) and more precise CMB data from the ESA Planck satellite (Balbi 2007), as well as other complementary data, such as Gamma Ray Bursts data (Schaefer 2007; Liang & Zhang 2008), and gravitational lensing data (see Albrecht et al. 2006 and corresponding references therein).

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