Magnetic focussing of electrons and holes in the presence of spin-orbit interactions

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In this work we theoretically investigate transverse magnetic focussing in two dimensional electron and hole gasses with strong spin orbit interactions. We present a general result for spin orbit interaction with singular winding numbers in the adiabatic limit. We then present results for systems with two spin orbit interactions of different winding number appear, using the concrete and experimentally relevant case of an applied in-plane magnetic field in hole systems with Rashba type interactions. We predict that the application of a large in-plane field is found to have a strong effect on the magnetic focussing spectrum.

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I. INTRODUCTION

The dynamics of charge carriers in spin-orbit coupled systems is a vital area of investigation for the extremely active field of spintronics. Controlling and manipulating the flow of electrons and holes serves as the foundation of an entire class of spintronic devices, most notably the Datta-Das spin transistor and its progeny$^1$. Experimental studies of spin-orbit systems, with an eye to these potential applications, are extensive. One particularly fruitful experimental technique is transverse magnetic focussing (TMF), which involves the mesoscopic transport of charge carriers from an injector quantum point contact (QPC) to a collector QPC, in a two dimension (2D) charge gas, through which the charge carrier can propagate ballistically on a scale of tens of microns. The charge carriers are “focused” by out of plane magnetic field $B = B_z$ as illustrated in Fig. 1. The two possible trajectories shown in Fig. 1 correspond to two possible spin polarisation. The figure illustrates that the problem under investigation is how the spin influences the semiclassical long-range orbital dynamics. As an experimental technique, TMF has provided important insights into the properties of heterostructures and Quantum point contacts. More recently, TMF has been applied in Graphene . Another recent development is the use of two dimensional systems with large spin orbit (SO) interactions in TMF, the goal to observe spin splitting in the magnetic focussing spectrum. Such a splitting has been observed in heavy hole systems using zinc blend heterostructures. Simultaneously there has been extensive theoretical investigation of TMF with Rashba SO interaction which is linear in the particle momentum using a variety of exact numerical, and strong and weak coupling semiclassical approximations. In addition, polarised photocurrents have been experimentally detected using TAF$^2$, and TMF has been used as an efficient method for detecting density differences in spin species in a QPC$^3$.4.

While extensive, theoretical investigation so far has been limited to the linear in the particle momentum SO interactions, like usual Rashba interaction. Though often dominant in two dimensional electron systems, for heavy hole based systems the spin-orbit Rashba-like interaction is cubic in momenta. Particular crystal lattice orientations with respect to the 2D heterstructure in zinc-blend semiconductors result in cubic in momenta Dresselhaus SO interaction. Application of an in-plane magnetic field to a heavy hole zinc-blend semiconductor heterostructure results in an effective SO interaction which is quadratic in the particle momentum. Furthermore, most 2D systems contain SO interactions of different orders in momentum simultaneously. Such a combination can suppress or enhance the spin splitting, depending on the growth orientation and confinement shape. Such partial, or complete compensation of spin orbit interactions forms the basis of many proposed spintronic devices.

In light of these considerations, in this work we pursue a theoretical investigation of TMF for a wide variety of SO interactions which scales as some power of the particle momentum. Moreover, we consider a case when SO interaction with different powers of momentum act simultaneously. We assume that the focusing magnetic field is weak and hence the evolution of the particle wave function is adiabatic.

The paper is structured as follows. In section II we present theory for SO interactions with a single winding number, and determining the peak position in a magnetic focussing experiment. In section III we extend this to the case of two SO interactions acting simultaneously, and determine the change to the magnetic focussing spectrum. Finally, in section IV we briefly examine the ques-
II. SPIN-ORBIT INTERACTIONS WITH A GIVEN WINDING NUMBER

Semiconductor heterostructures allow for a great diversity of SO interactions. To render our results as more general, we consider the following spin orbit interaction for a two dimension electron or hole gas (2DEG),

\[ H_{so} = \frac{i\gamma}{2} p^n \sigma_+ + h.c. \]

Here \( p \) is the particle momentum, \( p_\pm = p_x \pm ip_y \), and \( \sigma_\pm = \sigma_x \pm i\sigma_y \), are the Pauli matrices describing the effective spin 1/2. For electron systems, these matrices represent the electron spin, while for holes which have the internal angular momentum \( J = 3/2 \), the matrices describe the two level heavy hole subsystem with \( n \) other powers of \( J \), \( \gamma \) has \( n \) nonzero phase for \( n \geq 2 \). The only possible kinematic structure for the heavy holes Rashba-like interaction is \( H_{so} \propto (n \cdot [p \times S]) \), where \( S \) is electron spin and \( n \) is the unit vector orthogonal to the plane of 2DEG. This results in Eq. (1) with \( n = 1 \) and with a real coefficient \( \gamma = \gamma_1 \).

For holes in an in-plane magnetic field \( B \) the only possible kinematic structure is \( H_{so} \propto (J \cdot p)^2 (J \cdot B) \). To project this Hamiltonian to the \( J_z = \pm 3/2 \) heavy hole subspace one has to replace \( J^2_{\pm} \to \sigma_+, J^3_{\pm} \to \sigma_- \). All other powers of \( J \) are projected to zero. Hence Eq. (1) has \( n = 2 \) and a complex coefficient \( \gamma = \gamma_2 e^{-i\varphi} \), where \( \gamma_2 \) is real and proportional to the magnetic field. The phase \( \varphi \) is the angle of the in-plane magnetic field with respect to the y-axis, \( B_x = -B_y \sin \varphi, B_y = B_y \cos \varphi \), see Fig. 1.

The only possible kinematic structure for the heavy holes Rashba-like interaction is \( H_{so} \propto (J \cdot p)^2 (n \cdot [p \times J]) \). Again, projecting it to the \( J_z = \pm 3/2 \) subspace with the replacements \( J^2_{\pm} \to \sigma_+, J^3_{\pm} \to \sigma_- \) we come to Eq. (1) with \( n = 3 \) and with a real coefficient \( \gamma = \gamma_3 \).

An odd \( n \) in Eq. (1) implies the change of sign under inversion, therefore these interactions are due the lack of inversion symmetry in the semiconductor heterostructure. In Rashba-like interactions the inversion asymmetry is described by the vector \( n \). Alternatively, the \( n=3 \) case can originate from the Dresselhaus SO interaction which is due to the lack of inversion symmetry in the bulk of a zinc blend semiconductor11. In this case the magnitude and the phase of \( \gamma \) depends on the orientation of the heterostructure with respect to crystal axes.

The coefficient \( \gamma \) in Eq. (1) can be presented as

\[ \gamma = \gamma n e^{-i\varphi} \]

\[ \varphi = 0 \quad if \quad n = 1, 3 \]

\[ \varphi \neq 0 \quad if \quad n = 2 \]

\[ \gamma_n = \gamma n \frac{\epsilon_F}{k_F^2} , \quad k_F = \sqrt{2me_F} \]  

where \( \epsilon_F \) is the Fermi energy (chemical potential). The nonzero phase for \( n = 2 \) is determined by the orientation of the in-plane magnetic field as it is explained above. The dimensionless coefficient \( \gamma_n \) represents the value of the SO interaction at \( p = k_F \) in units of the Fermi energy. For the Rashba-like interaction (lack of inversion asymmetry in the heterostructure) at \( n = 1, 3 \) the coefficient can as large as \( |\gamma_n| \sim 0.1 - 0.2 \), dependent on the heterostructure quantum well. For the \( n = 2 \), the interaction induced by an in-plane magnetic field, in GaAs the value of the coefficient can be a large as \( |\gamma_n| \sim 0.02/\text{Tesla}^2 \), depending on the particulars of the heterostructure. The Dresselhaus interaction results is a relatively small value of \( |\gamma_3| \), in GaAs \( |\gamma_3| \sim 0.01 \). It is worth noting that in this case the phase can be nonzero.

We can consider the SO interaction as a momentum dependent effective Zeeman magnetic field, \( \mathcal{B}(k) \). From (1),

\[ H_{so} = -\mathcal{B} \cdot \mathbf{\sigma} \]

\[ \mathcal{B} = \gamma_n k^n (-\sin(n\theta + \varphi), \cos(n\theta + \varphi), 0) \]

\[ k = k (\cos \theta, \sin \theta, 0) \]

where \( \theta \) is the axial angle in the \( k \)-space. In the absence of a focussing out-of-plane magnetic field electrons (holes) move in straight lines, and the spin is parallel (or antiparallel) to the SO effective magnetic field,

\[ \langle \chi_s | \mathbf{\sigma} | \chi_s \rangle = s (-\sin(n\theta + \varphi), \cos(n\theta + \varphi), 0) \]

\[ \psi_{kS}(\mathbf{x}) \propto e^{ik \cdot \mathbf{r}} \chi_s , \quad \chi_s = \begin{pmatrix} i e^{i(n\theta + \varphi)} & 0 \\ 0 & -8 \end{pmatrix} \]

where \( s = \pm 1 \) is the spin polarization, \( \psi_{kS} \) is the particle wave function, and \( \chi_s \) is the spin wave function. Note that the effective spin rotates \( n \)-times when \( k \) rotates only once in the momentum space. For \( n = 1 \) this is illustrated in Fig. 2a. This is why we say that the Hamiltonian (1) has a given winding number. A combination of Hamiltonians with different values of \( n \) obviously violates this property. It is worth noting that for \( n = 1 \) the winding number represents the rotation of the real spin. However, this is not true for higher values of \( n \). The point is that for \( n = 2, 3 \) the effective spin \( \mathbf{\sigma} \) describes phases of the heavy hole components \( J_z = \pm 3/2 \). The expectation value of \( \langle \mathbf{\sigma} \rangle \) is not equal to the expectation value of \( J \). For example for \( n = 2 \) \( \langle J \rangle \) does not change its direction, it is directed along \( \mathbf{B}_y \), so it does not wind. Nevertheless, for our purposes the effective spin matters, and therefore the classification by the winding numbers makes sense and is important.
It is convenient to perform transformation to the SO field is no longer constant, but rotates in time, see Fig. 2(b) where different points in the k-space correspond to different particles. Hereafter we include the Zeeman coupling in addition to the spin through some angles in the k-space corresponding to different particles.

A particle in the focusing magnetic field. Both the particle momentum and the particle spin evolves in time. Importantly, there is a nonzero z-component of spin.

The full Hamiltonian, in a transverse magnetic field is

\[ \mathcal{H} = \frac{\pi^2}{2m} + \left( \frac{i\gamma}{2} \sigma^z \right) \frac{\pi^2}{2m} + \left( \frac{i\gamma}{2} \sigma^z \right) \]  

where \( \pi = p - eA \) with \( A \) being the vector potential corresponding to the focusing field \( B = B_z \), \( B = \nabla \times A \). Besides the vector potential we include Zeeman coupling to the focusing field, \( g \) is the g-factor for the electron (or hole). Hereafter we include the Zeeman coupling in the effective SO magnetic field,

\[ B \to \gamma_n k^\parallel (-\sin(n\theta+\varphi), \cos(n\theta+\varphi), 0) + \frac{1}{2} g\mu_B B_z (0, 0, 1), \]  

yielding a finite z component of \( \mathcal{B} \).

When a weak transverse magnetic field is applied, the trajectory of the charge carriers is curved, rotating the states through some angles \( \theta(t) = \omega t \), after some time \( t \), \( k \to k(\cos\omega t, \sin\omega t, 0) \). For such a curved trajectory, the SO field is no longer constant, but rotates in time, see Eq. (4). It is convenient to perform transformation to the reference frame corotating with the SO field

\[ \chi_s = U \chi_{s0} \]

where

\[ U(t) = e^{-i\pi^z(n\theta+\varphi)}, \quad \theta = \omega t. \]

Substitution of \( \chi_s \) in Schrödinger equation shows that the spin wave function in the corotating frame obeys the following equation

\[ i\frac{\partial}{\partial t} U(t) \chi_{s0} = \mathcal{H} U(t) \chi_{s0} \]  

Solution of Eq. (9) is straightforward since \( B_0 \) is time independent. There are two eigenstates with energies \( \pm |B_0| \) corresponding to \( s = \pm 1 \). In a superposition of these states the spin oscillates around direction of \( B_0 \). In the eigenstates the effective spin has the following values

\[ \langle \sigma_x \rangle = 0 \]
\[ \langle \sigma_y \rangle = \frac{\gamma_n k^\parallel}{|B_0|} \]
\[ \langle \sigma_z \rangle = s \frac{(g\mu_B B_z + n\omega)/2}{|B_0|}. \]  

It is worth noting that the term proportional to \( n\omega \) in \( \langle \sigma_z \rangle \) is a direct consequence of the Coriolis force. Eq. (10) gives the effective spin in the corotating frame. In the laboratory frame this gives

\[ \langle \sigma_+ \rangle = \frac{s i e^{i\gamma \pi^z n\Omega}}{|B_0|} \]
\[ \langle \sigma_- \rangle = \frac{s (g\mu_B B_z + n\omega)/2}{|B_0|}. \]  

The effective spin evolution in the laboratory frame is illustrated in Fig. 2(b).

Now let us look at the spatial dynamics of the particle in the laboratory frame. We are interested in a semiclassical wave packet propagating along a trajectory. To approach the problem we use the Heisenberg representation, where the wavefunctions are time independent, while the operators evolve in time. The Heisenberg equations of motion with the Hamiltonian \( \mathcal{H} \) read

\[ \dot{\chi}_+ = \frac{i}{\hbar} \mathcal{H}, \chi_+ = \frac{\pi^+}{m} + ni\gamma\pi^- \]  

where

\[ \dot{\chi}_+ \]  

The second of these equations represents the usual Lorentz force, interestingly it is not influenced by the SO interaction. On the other hand the first equations which gives relation between momentum and velocity depends on the SO interaction. In the semiclassical limit we have to replace the operators \( \pi_+ \), \( v_+ \), and \( \sigma_+ \) in Eqs. (12) by their expectation values. The expectation value of spin is given by Eq. (11). Hence, in the semiclassical limit...
Eqs. (12) are transformed to
\[ v_{+s} = v_{+s} e^{iω_s t}, \]
\[ \pi_{+s} = k_{Fs} e^{iω_s t}, \]
\[ v_{Fs} = \frac{k_{Fs}}{m} \left( 1 - \frac{1}{2} sn \gamma_n \right), \]
\[ iω_s k_{Fs} = imω_c v_{Fs}, \]
\[ ω_c = \frac{eB_z}{m} > 0. \]

Note that while in Eq. (12) velocity and momentum are operators, in Eq. (13) these are usual numbers. Note also that the Fermi momentum and the Fermi velocity depend on spin. When transforming from (12) to (13) we assume that the SO interaction is small compared to the Fermi energy, \( γ_n \ll 1 \), we expand in powers \( (s\gamma_n) \) keeping only the leading term. Note that \( (s\gamma_n)^2 = 1 \) is independent of spin. Therefore Eq. (13) is robust, the neglected spin dependent term is of the third order, \( \sim s\gamma_n^3 \). On the other hand we assume that the SO interaction is sufficiently strong to justify adiabatic approximation for spin dynamics, \( \gamma_n \gg \omega_c/\epsilon_F \). From Eq. (13) we immediately find that the frequency of cyclotron motion depends on spin
\[ ω_s = ω_c \left( 1 - \frac{1}{2} sn \gamma_n \right). \]

In a focussing experiment particles are injected at energy equal to the chemical potential \( \epsilon_F \) independent of spin. However momentum of the particle depends on spin. Substitution of the spin expectation value \( \langle \pi_z \rangle \) in the Hamiltonian \( \mathcal{H} \) gives the following momentum and velocity
\[ k_{Fs} = k_F \left( 1 - \frac{1}{2} s\gamma_n \right), \]
\[ v_{Fs} = \frac{k_F}{m} \left( 1 + \frac{1}{2} s\gamma_n [1 - n] \right). \]

Note that both \( k_{Fs} \) and \( v_{Fs} \) are real, this implies that the momentum and the velocity are always parallel. Finally the focusing distance, see Fig.1 is
\[ L_s = v_{Fs} \frac{2}{ω_s} = 2r_c \left( 1 + \frac{1}{2} s\gamma_n \right), \]

where
\[ r_c = \frac{k_F}{mω_c} \]
is the cyclotron radius. Remarkably, when expressed in terms of \( \gamma_n \) the focussing length is independent of the winding number \( n \). It does depend on the spin polarisation, and this leads to the double focusing peak. For the case of \( n = 1 \), the solution, Eq. (10), was attained by Zülicköök.

The solution \( n = 1 \) has been obtained in terms of momentum and velocity, see Eq. (12). This solutions does not use explicitly the out-of-plane component of spin \( \langle \sigma_z \rangle \) presented in Eq. (11), the Coriolis force remains “under the carpet”. In Appendix A we present an alternative solution for \( n = 1 \). The alternative solution is presented in terms of the velocity operator and its time derivative. The alternative solution is essentially based on \( \langle \sigma_z \rangle \) and the role of the Coriolis force is explicit, with the final answer identical to Eqs. (13) and (15).

### III. SPIN-ORBIT INTERACTIONS WITH MIXED WINDING NUMBERS

In this section we consider the general SO interaction. The Hamiltonian is
\[ \mathcal{H} = \frac{\pi^2}{2m} - \mathcal{B} \cdot \sigma. \]

Again we assume that the spin dynamics are adiabatic, the SO field is everywhere much larger than the cyclotron frequency, \( |\mathcal{B}| \gg \omega_c \). Adiabaticity is the essential physical approximation. Besides that we assume that \( |\mathcal{B}| \ll \epsilon_F \). This assumption is not essential, but it allows us to perform calculations analytically. Practically the inequality is always valid. Because of the spin adiabaticity \( \sigma \rightarrow s\mathcal{B}/|\mathcal{B}| \) and hence (18) is reduced to the classical Hamiltonian
\[ H_{cl} = \frac{\pi^2}{2m} - s|\mathcal{B}|, \]

where \( \mathcal{B} \) is a function of \( \pi \). Hence Hamilton Eqs. of motion are
\[ v_+ = \frac{\partial H_{cl}}{\partial \pi_-} = \frac{\pi_+}{m} - s \frac{\partial \mathcal{B}^2}{\partial \pi_-}, \]
\[ \pi_+ = iv \omega_c m v_+, \]

where the z-component in \( \mathcal{B}^2 = \mathcal{B}_z^2 + \mathcal{B}_y^2 + \mathcal{B}_z^2 \) can be safely neglected. Let us look for solution in the form
\[ \pi_+ = k e^{iθ}, \]
\[ v_+ = v_0 e^{iθ}, \]

where \( k \) is real. Expanding in powers of \( |\mathcal{B}|/\epsilon_F \) Eqs. (19), (20) are transformed to
\[ k = k_F \left( 1 + s \frac{|\mathcal{B}|}{2\epsilon_F} \right), \]
\[ v = \frac{k_F}{m} \left[ 1 + s \frac{|\mathcal{B}|}{2\epsilon_F} - s \frac{k_F}{2|\mathcal{B}|} e^{-iθ} \frac{\partial \mathcal{B}^2}{\partial \pi_-} \right], \]
\[ \dot{θ} = \omega_c \left[ 1 - s \frac{k_F}{4|\mathcal{B}|} \left( e^{-iθ} \frac{\partial \mathcal{B}^2}{\partial \pi_-} + e^{iθ} \frac{\partial \mathcal{B}^2}{\partial \pi_+} \right) \right]. \]

Note that \( v \) is generally complex while \( θ \) is of course real. Complex \( v \) with real \( k \) indicates that the momentum and the velocity are generally not parallel.

Integration of Eqs. (22) is straightforward. To be specific we apply (22) to the physically interesting case of
heavy holes in asymmetric quantum well and in external in-plane magnetic field of few to several Tesla. This experimental setup is frequently employed for the polarisation of the QPC. The presence of the in-plane magnetic field leads to the simultaneous action of \( n = 3 \) and \( n = 2 \) SO interactions.

\[
H_{so} = \frac{1}{2} \left( \gamma_3 \pi_3^3 + \gamma_2 e^{-i\phi} \pi_2^2 \right) \sigma_+ + h.c.
\]  

(23)

In this case

\[
B^2 = \gamma_3^2 \pi_3^3 + \gamma_2 \gamma_3 \left( e^{i\phi} \pi_3^3 + e^{-i\phi} \pi_2^2 \right) + \gamma_2^2 \pi_2^2 \pi_1^2.
\]

(24)

The condition of the spin adiabaticity implies that \( b(\theta) \) does not vanish, \( |\gamma_3| b(\theta) \gg \omega_e/\epsilon_F \). Substitution of \( |\gamma_3| b(\theta) \gg \omega_e/\epsilon_F \) in (24) gives the following

\[
v_+ = \frac{k_F}{m} e^{i\phi} \left[ 1 + \frac{s}{2} \frac{\gamma_3^3 b(\theta)}{b(\theta)} - \frac{3}{2} \frac{\gamma_3^3 a(\theta) - i c(\theta)}{b(\theta)} \right],
\]

\[
\dot{\theta} = \omega_c \left[ 1 - \frac{3}{2} \frac{\gamma_3^3 a(\theta)}{b(\theta)} \right],
\]

\[
a(\theta) = 1 + \left( \frac{5}{3} / \gamma_3 \right) \cos(\theta - \varphi) + \left( \frac{2}{3} / \gamma_3 \right)^2 \gamma_2^2 sin(\theta - \varphi).
\]

(25)

The velocity is not necessarily aligned with the momentum, due to the spin orbit interaction. Integration of these equations results in the following particle trajectory

\[
\dot{\theta} = \omega_c t + \theta_0 - s \frac{3}{2} \frac{\gamma_3^3}{b(\theta')} \left[ \frac{a(\theta')}{b(\theta')} \right] d\theta',
\]

\[
x + iy = \frac{k_F}{\omega_c} \left( i e^{i\phi_0} - e^{i\phi} \right)
\]

\[
+ s \frac{3}{2} \int_{\theta_0}^{\theta} d\theta' \left[ \frac{a(\theta')}{b(\theta')} + 3i \frac{c(\theta')}{b(\theta')} \right].
\]

(26)

Hete \( \theta_0 \) is the angle which momentum of the particle makes with the x-axis at the point of injection, \( t = x = y = 0 \), \( k_x = k \cos \theta_0 \), \( k_y = k \sin \theta_0 \). We reiterate again that generally the group velocity of the particle is not parallel to its momentum. In particular, assume that the particle is injected along the x-axis, \( \theta_0 = 0 \), \( k_y = 0 \). This does not imply that \( v_y = 0 \), one can easily find from Eq. (25) that \( v_y = -s \gamma_3^3 \). 

In principle Eq. (24) solves the problem. However, we still need to specify what is the initial value of the injection angle \( \theta_0 \). Intuitively it is zero, \( \theta_0 = 0 \). In the next section we put this statement on a solid ground by considering a model of the QCP injector. Below in this Section we set \( \theta_0 = 0 \).

Numerical evaluation of Eq. (26) is straightforward. Trajectories for \( \gamma_3 = 0.1 \), \( s = \pm 1 \) and zero in-plane magnetic field, \( \gamma_2 = 0 \), are shown in panel a of Fig. 3 by dashed blue and red lines. These are simple semicircles as we discussed in Section II. By solid blue and red lines in the same panel a of Fig. 3 we show \( s = \pm 1 \) trajectories for \( \gamma_3 = 0.1 \) and \( \gamma_2 = 0.05 \). A strong dependence of the focusing distance on the in-plane magnetic field is evident. The trajectories depend on direction of the in-plane magnetic field, Fig. 3 corresponds to \( \varphi = -\pi/2 \), the field is parallel to the direction of injection. In the panel b of Fig. 3 we show trajectories for magnetic field perpendicular to the direction of injection, \( \varphi = \pi/2 \). The dashed lines are identical to that in Fig. 3, \( \gamma_3 = 0.1 \) and \( \gamma_2 = 0 \) (zero magnetic field). The solid lines account for the magnetic field corresponding to \( \gamma_2 = 0.05 \). We see that for this orientation of the in-plane magnetic field the focusing distance is independent of the field.

Let us take the focusing shift due to the Rashba interaction, Eq. (16),

\[
\Delta L_R = s r e^{\gamma_3} (27)
\]

as a reference value. Then the total focusing shift is

\[
\Delta L = \Delta L_R (1 + \delta_B),
\]

(28)
function inside the 1D channel is
\[ \psi \propto e^{ik_x \cos \theta_y}, \quad -w/2 < y < w/2 \]
\[ \varphi = \frac{\pi}{w} \]

We assume that \( k_x \ll \varphi \), hence, since the total energy is equal to Fermi energy, we conclude that \( \varphi \approx k_F \). The Hamiltonian inside the channel reads
\[ H \approx \frac{k_x^2}{2m} + \frac{\varphi^2}{2m} - \epsilon_F \left( 3\gamma_3 k_x/\kappa + \gamma_2 \cos \varphi \right) \sigma_y + \gamma_2 \sin \varphi \sigma_x \] (30)

The spin orbit interaction given by the second line in this equation determines the spin polarisation inside the channel.

Following the Huygens principle the \( k_y \)-momentum distribution after emission from the injector is determined by the Fourier component \(^{16}\) of \( \omega \).
\[ \frac{dW}{dk_y} \propto \left| \int_{-w/2}^{w/2} dy e^{ik_y \cos \theta_y} \right|^2. \] (31)

Converting this to angular distribution we find
\[ \frac{dW}{d\theta_y} \propto \cos \theta_y \left| \cos \left( \frac{\pi}{2} \sin \theta_0 \right) \right|^2. \] (32)

The most important conclusion is that the distribution \(^{22}\) plotted in the right panel of Fig. 5 is peaked at \( \theta_0 = 0 \). This is what we used in the previous section. One can say that this conclusion trivial, but it is not quite. According to Eq. (25) this implies that the velocity distributions for injected particles with \( s = \pm 1 \) are peaked at nonzero angles
\[ \theta_{\text{max}} = -\frac{3}{2} \gamma_2 \sin \varphi. \] (33)

The derivation thus far takes no account of adiabatic opening of the QPC, reflecting the geometry chosen for this analysis. A more realistic geometry includes the adiabatic opening of the QPC, from the minimum width of the constriction, \( W_{\text{min}} \), to the point at which adiabatic transport breaks down, \( W_{\text{max}} \). At the minimum width of the channel, \( k_y = \pi/W_{\text{min}} \). As the transport through the QPC is adiabatic, at the exit, the transverse momentum is \( k_{y,x=0} = \pi/W_{\text{max}} \), reducing the angular spread of the injected electron/holes.\(^{15}\) The magnitude of this collimation effect depends on the ratio between \( W_{\text{min}} \) and \( W_{\text{max}} \), and hence the details of the experimental device. In the above geometry, \( W_{\text{min}} = W_{\text{max}} \) and hence \( k_y W_{\text{max}} \sim 1 \), leading to a very large angular spread, with no diffraction fringes. We note that while flaring can add significant collimation to the distribution, it does not effect peak, which will still be at \( k_y = 0 \), with non-zero velocity, as presented in \(^{33}\).
We present the alternative solution for usual, linear in momentum, Rashba interaction. Of course final answers of both methods are identical.

We start with usual Rashba spin orbit interaction,
\[ \mathcal{H} = \frac{\pi^2}{2m} + \gamma \pi (\sigma \times \hat{z}) - \frac{1}{2} g \mu_B B_z \sigma_z \] (A1)

which corresponds to the \( n = 1 \) for Eq. (1). The velocity is
\[ \mathbf{v} = \frac{\mathbf{p}}{m} + \gamma_1 \sigma \times \hat{z} \] , (A2)

and hence up to terms quadratic in the spin orbit interaction the energy is
\[ \mathcal{E} = \frac{m \mathbf{v}^2}{2} - \frac{1}{2} g \mu_B B_z \sigma_z . \] (A3)

Heisenberg equation of motion for velocity reads
\[ \dot{\mathbf{v}} = \omega_c \mathbf{v} \times \hat{z} + \{\mathbf{v} \times \hat{z}, \mathbf{\sigma} \cdot \mathbf{\dot{z}}\} + g \mu_B \mathbf{B} \gamma_1 \mathbf{\sigma} \] (A4)

We note the presence of a spin dependent term in the right-hand-side of the second equation, which has been considered as a ‘spin force’, in analogy with the usual Lorentz, force (the first term). The third term is the result of the coupling between the spin-orbit component of velocity, and the Zeeman interaction with the out of plane field. We stress that the ‘spin force’ appears only within this technique, it does not appear within the momentum-velocity technique considered in the main text, Eqs. (12), (20). In semiclassical approximation the ‘spin force’ term can be decoupled as \( \{\mathbf{v} \times \hat{z}, \mathbf{\sigma} \cdot \mathbf{\dot{z}}\} \rightarrow 2[\mathbf{v} \times \hat{z}](\sigma_z) \). Due to the ‘spin force’ the out-of-plane spin projection \( \sigma_z \), which was “under the carpet” in the main text solution, here plays an explicit role. Substitution of \( \langle \sigma_z \rangle \) from Eq. (11) gives
\[ \dot{\mathbf{v}} = \omega_c \left( 1 - \frac{s}{2} \right) \mathbf{v} \times \hat{z} \] (A5)

Hence we get a corrected cyclotron frequency identical to Eq. (14) with \( n = 1 \). We note that the additional spin dependent term due to the Zeeman effect does not result in any finite contribution to \( \dot{\mathbf{v}} \) in the absence of the rotation induced correction to \( \langle \sigma_z \rangle \). Due to Eq. (A3) the particle speed is spin independent, \( |\mathbf{v}| = k_F/m \) and we arrive to the same answer as that in Section II with \( n = 1 \).

V. SUMMARY

We consider magnetic focusing of electrons/holes in presence of a strong spin orbit interaction. The spin orbit interaction is considered in adiabatic approximation, spin follows the effective magnetic field. We classify spin orbit interactions by winding number \( n \), the number of rotations of the effective spin along a close path particle trajectory. First we consider the case of a singular winding number and extend previously known result for \( n = 1 \) (usual Rashba interaction) to larger values of \( n \), especially to experimentally relevant cases \( n = 2, 3 \). We show that in the case of a singular winding number the particle trajectory is circular with radius dependent of the spin orbit interaction. Second we consider the most interesting case of a combination of spin orbit interactions with different winding numbers. Particle dynamics in this case are significantly more complex. We derive general semiclassical equations of motion linearized in spin orbit interaction. Using the developed technique we consider in details the case of a combination of \( n = 3 \) (cubic Rashba) and \( n = 2 \) (in-plane magnetic field). This analysis can be relevant to dynamics of holes in semiconductors and provides a general approach to the problem of spin orbit dynamics while ever the adiabatic approximation is valid. We predict that the application of a large in-plane magnetic field will have a significant effect on the magnetic focussing spectrum, and will be very sensitive to the field orientation.

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Appendix A: Alternative solution of \( n = 1 \) case in terms of velocity operator

Our solution in Section II is obtained in terms of momentum and velocity, this method is technically the simplest one. An alternative method is to work with velocity without referring to momentum. Intuitively this method is more natural in the semiclassical limit. Here we present the alternative solution for usual, linear in momentum, Rashba interaction. Of course final answers of both methods are identical.

We start with usual Rashba spin orbit interaction,
\[ \mathcal{H} = \frac{\pi^2}{2m} + \gamma \pi (\sigma \times \hat{z}) - \frac{1}{2} g \mu_B B_z \sigma_z \] (A1)

which corresponds to the \( n = 1 \) for Eq. (1). The velocity is
\[ \mathbf{v} = \frac{\mathbf{p}}{m} + \gamma_1 \mathbf{\sigma} \times \hat{z} \] , (A2)

and hence up to terms quadratic in the spin orbit interaction the energy is
\[ \mathcal{E} = \frac{m \mathbf{v}^2}{2} - \frac{1}{2} g \mu_B B_z \sigma_z . \] (A3)

Heisenberg equation of motion for velocity reads
\[ \dot{\mathbf{v}} = \omega_c \mathbf{v} \times \hat{z} + \{\mathbf{v} \times \hat{z}, \mathbf{\sigma} \cdot \mathbf{\dot{z}}\} + g \mu_B \mathbf{B} \gamma_1 \mathbf{\sigma} \] (A4)

We note the presence of a spin dependent term in the right-hand-side of the second equation, which has been considered as a ‘spin force’, in analogy with the usual Lorentz, force (the first term). The third term is the result of the coupling between the spin-orbit component of velocity, and the Zeeman interaction with the out of plane field. We stress that the ‘spin force’ appears only within this technique, it does not appear within the momentum-velocity technique considered in the main text, Eqs. (12), (20). In semiclassical approximation the ‘spin force’ term can be decoupled as \( \{\mathbf{v} \times \hat{z}, \mathbf{\sigma} \cdot \mathbf{\dot{z}}\} \rightarrow 2[\mathbf{v} \times \hat{z}](\sigma_z) \). Due to the ‘spin force’ the out-of-plane spin projection \( \sigma_z \), which was “under the carpet” in the main text solution, here plays an explicit role. Substitution of \( \langle \sigma_z \rangle \) from Eq. (11) gives
\[ \dot{\mathbf{v}} = \omega_c \left( 1 - \frac{s}{2} \right) \mathbf{v} \times \hat{z} \] (A5)

Hence we get a corrected cyclotron frequency identical to Eq. (14) with \( n = 1 \). We note that the additional spin dependent term due to the Zeeman effect does not result in any finite contribution to \( \dot{\mathbf{v}} \) in the absence of the rotation induced correction to \( \langle \sigma_z \rangle \). Due to Eq. (A3) the particle speed is spin independent, \( |\mathbf{v}| = k_F/m \) and we arrive to the same answer as that in Section II with \( n = 1 \).
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