Identifying the source of super-high energetic electrons in the presence of pre-plasma in laser–matter interaction at relativistic intensities

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Abstract
The generation of super-high energetic electrons influenced by pre-plasma in relativistic intensity laser–matter interaction is studied in a one-dimensional slab approximation with particle-in-cell simulations. Different pre-plasma scale lengths and laser intensities are considered, showing an increase in both particle number and cut-off kinetic energy of electrons with the increase of pre-plasma scale length and laser intensity, the cut-off kinetic energy greatly exceeding the corresponding laser ponderomotive energy. A two-stage electron acceleration model is proposed to explain the underlying physics. The first stage is attributed to the synergetic acceleration by longitudinal electric field and counter-propagating laser pulses, and a scaling law is obtained with efficiency depending on the pre-plasma scale length and laser intensity. These electrons pre-accelerated in the first stage could build up an intense electrostatic potential barrier with maximal value several times as large as the initial electron kinetic energy. Some of the energetic electrons could be further accelerated by reflection off the electrostatic potential barrier, with their final kinetic energies significantly higher than the values pre-accelerated in the first stage.

Keywords: plasma physics, intense laser, plasma heating

(Some figures may appear in colour only in the online journal)
numerically modelled ponderomotive scaling [15] and other following works [16–18]. However these scalings do not address the dependence of hot electrons on finite-scale length preformed plasma in front of the target. The first systematic study of the sources of super energetic electrons with energy exceeding ponderomotive energy is the stochastic heating and acceleration model [19, 20]. It is found that when electrons experience the interaction of two counter-propagating laser pulses, efficient acceleration of the electrons in plasma or a vacuum can be realized provided that some threshold amplitudes of the lasers are exceeded. In a real situation, the counter-propagating laser can be produced automatically during the propagation of an intense laser pulse in the plasma through processes such as stimulated Raman backscattering or reflected laser fields from high density regions. However, recently fast electron generation due to relativistic intensity laser–matter interaction influenced by preformed plasma has been addressed in a number of experimental and theoretical studies [21–28], suggesting that the presence of pre-plasma can significantly affect fast electron distributions. The new findings in experimental and theoretical studies, addressing the influences of pre-plasmas, seem beyond the interpretations of stochastic heating and acceleration models. Both experiments and numerical simulations have reported an increase of fast electron generation efficiency with the increase of pre-plasma scale length. Recent particle-in-cell simulations [25] have observed super-high energetic electrons with cut-off kinetic energy as high as 100 MeV at laser intensity $10^{20}$ W cm$^{-2}$ and pre-plasma scale length 10 $\mu$m. Paradkar et al extend the stochastic heating and acceleration model, suggesting that the electron acceleration in under-dense plasma is a stochastic process in the presence of two counter-propagating laser pulses and an electrostatic potential [25]. Kemp and his cooperators also realize that the electrostatic potential caused by low-density preformed plasma plays an important role in electron heating [28]. However the underlying physics, (i) the increase in the generation efficiency of energetic electrons with the increase of pre-plasma scale length and laser intensity, and (ii) the acceleration mechanism of super-high energetic electrons with kinetic energy greatly exceeding the ponderomotive energy, is still unclear. To thoroughly understand the underlying physics, a theoretical scaling law for the cut-off electron energy enhancement are figured out analytically and confirmed through electrostatic PIC simulations.

This paper is arranged as follows: the details of numerical modelling and simulation results are demonstrated in section 2. The two-stage acceleration model suggested by analysing the simulation results is proposed in section 3 to explain the impact of pre-plasma and identify the sources of energetic electrons. Conclusions and discussions are given in section 4.

2. Numerical simulation results

The simulations are performed with 1D PIC code, which is a newly extended version based on LAPINE [31]. In order to simulate laser–matter interactions with large scale pre-plasmas, the weighted particle technique is applied in the numerical simulations, which has proven to be more efficient than uniform weighted particles in large density gradient calculations [32]. In addition, a 4th order particle cloud and 4th order FDTD method are applied in our simulations, because these features make it suitable for simulating laser–solid-density-plasma interactions at relativistic intensities [32]. The laser is of intensity $10^{20}$ W cm$^{-2}$ or normalized amplitude $a = 8.54$ (with laser wavelength 1 $\mu$m), entering the simulation box from the left boundary, where the laser amplitude rises over 33 fs to $a = 8.54$ and then remains constant. To confirm that the rising amplitude is not the dominant factor, we also run simulations with rising duration of 100 fs, keeping other parameters the same; the results show exactly the same physics. The initial plasma density profile is taken as $n_e = n_{\text{solid}}/(1 + \exp[-2(z - z_0)/L_p])$, where $n_{\text{solid}} = 50n_i$ is the solid plasma density and $L_p$ is the pre-plasma scale length. The ions are deliberately treated as immobile in the simulation to prevent gradual pre-plasma scale length variation and eliminate effects associated with ion mobility. As the charge separation electric field calculation is of great importance in our following analysis and the non-neutrality of total electric charge in the simulation box would lead to artificial overestimation of the charge separation electric field, we choose to use a large simulation box instead of the reduced one to guarantee ‘all’ particles are confined in the simulation box without escaping. Furthermore, choosing a large simulation box can also avoid boundary effects caused by electrons recoiling due to the artificial electrostatic field on the boundaries, which could otherwise interrupt the physics we are studying. In the PIC simulations, the simulation box is of size 400 $\mu$m, which is divided into 40 000 cells, with each cell containing energetic electrons with kinetic energy greatly exceeding the corresponding laser ponderomotive energy’, are answered. A two-stage acceleration model is proposed to identify the source of super-high energetic electrons. The first stage is synergetic acceleration by longitudinal electric field and two counter-propagating laser pulses; a scaling law is obtained theoretically, with its efficiency depending on the pre-plasma scale length and laser intensity. The second stage is related to the intense electrostatic potential building in front of the target and the accompanying reflection of electrons by this electrostatic potential barrier; the potential building and electron energy enhancement are figured out analytically and confirmed through electrostatic PIC simulations.
1000 electrons and 1000 ions. In our simulations, the region $0 < z < 100 \mu m$ is left as vacuum, $L_p$ varies from $L_p = 1 \mu m$, $5 \mu m$, $10 \mu m$ to $15 \mu m$, $z_0$ is fixed as $180 \mu m$ and the minimum plasma density is set as $0.001n_e$ for all simulation cases. In order to analyse the electron energy distributions in detail, we have placed two diagnostic planes to temporally record the electrons passing through. As shown in figure 1(a), the first diagnostic plane is located at $z = 100 \mu m$ to record the electron going through in the $z$-direction, and the other one is located at $z = 300 \mu m$, recording the electron passing through in the $z$-direction.

To ensure the accuracy of the simulation, as we have done previously [33], we record the energy history of laser flux entering the simulation box, electromagnetic field energy in the simulation box, and particle kinetic energy in the simulation box. The total simulation time is set to be $400T_0$, to avoid the electron recoil effect. Furthermore, we also loosen the simulation resolution by two, four and eight folds to confirm convergence of the simulation results. We find that the resolution demand is at least 25 cells per wavelength, provided that 4th order particle cloud, 4th order FDTD schemes and 1000 particles per cell are used.

The fast electron energy spectra obtained for different pre-plasma scale lengths ($L_p = 1 \mu m$, $5 \mu m$, $10 \mu m$ and $15 \mu m$) while keeping laser intensity fixed at $10^{20}$ W cm$^{-2}$, are presented in figure 2. Solid lines in figure 2 record the energy spectra of electrons passing through the diagnostic plane located at $z = 300 \mu m$. As we can see, there is a clear relation between cut-off kinetic energy and pre-plasma scale length—the larger the scale length the higher the cut-off kinetic energy—which is in agreement with earlier published works [25]. We have also found that the cut-off electron kinetic energy greatly exceeds the corresponding laser ponderomotive energy, which is about $3.8 \text{ MeV}$ at intensity $10^{20}$ W cm$^{-2}$. For pre-plasma of scale length $1 \mu m$, $5 \mu m$, $10 \mu m$ and $15 \mu m$, the corresponding cut-off energies are $30 \text{ MeV}$, $60 \text{ MeV}$, $100 \text{ MeV}$ and exceeding $120 \text{ MeV}$ respectively. The dashed lines in figure 2 record the energy spectra of electrons passing through the diagnostic plane located at $z = 100 \mu m$. By comparing the two energy spectra recorded by two different diagnostic planes, we can find that the cut-off energy recorded at $z = 300 \mu m$ is several times larger than that recorded at $z = 100 \mu m$. The black, red, green and blue lines show the energy spectra for pre-plasma of scale length 1, 5, 10, and $15 \mu m$ respectively. The laser is of intensity $10^{20}$ W cm$^{-2}$, and laser wavelength is $1 \mu m$.

We have found that the cut-off kinetic energy of electrons increases with the increase of the pre-plasma scale length. Meanwhile, we have also noticed that the cut-off electron kinetic energy recorded by the diagnostic plane located at $z = 300 \mu m$ is several times larger than that recorded at $z = 100 \mu m$. The aim of this work is to uncover the mysteries, (i) the increase in the generation efficiency of energetic electrons with the increase of pre-plasma scale length and (ii) the source of super-high energetic electrons with energy greatly exceeding the corresponding laser ponderomotive energy. In order to understand the underlying physics of the observed phenomena, we now turn to analysing the trace of a particular electron as a function of time and the $z$-$p_z$ phase-space dynamics. Figure 3 shows the trace of a particular electron initially located at $z = 130 \mu m$, with laser of intensity $10^{20}$ W cm$^{-2}$ and pre-plasma of scale length $10 \mu m$. Figure 4 describes the phase-space patterns of laser–pre-plasma interactions with laser of intensity $10^{20}$ W cm$^{-2}$ and pre-plasma of scale length $10 \mu m$ respectively. The phase-space density $F(n_z)$ gives a value proportional to the number of electrons found between $z$ and $z + dz$ having longitudinal momentum ranged between $p_z$ and $p_z + dp_z$. The normalized electrostatic potential, $-e\phi/m_ec^2$, due to the longitudinal charge separation field $E_z$, is shown in black curves covered on phase plots. The blue lines are the $E_z$ components of the superposition of incoming and reflected laser pulses. The electron longitudinal momentum $p_z$ is normalized by $m_ec$ and $z$ is in the units of laser wavelength, which is $1 \mu m$.

As shown in figure 3, in the very early stages of laser propagation in under-dense preformed plasma, some electrons are swept away in the forward direction by the laser ponderomotive force, leaving behind immobile ions. The electric field $E_z$...
due to charge separation within the under-dense plasma region tries to pull the electrons in the backward direction. When the laser arrives at the critical density surface and is reflected back, the ponderomotive force of the reflected laser pulse can further accelerate the electrons in the backward direction. Actually, the first stage acceleration is due to the synergetic effects by this longitudinal charge separation field \( Ez \) and the ponderomotive force of the reflected laser pulse. From the Woodward–Lawson theorem [34], we know that a single electron in vacuum, oscillating coherently with a propagating plane laser pulse would gain zero cycle averaged energy since the electron energy gain in one half cycle is exactly equal to the energy loss in the next half cycle. In the presence of plasma, the phase velocity of the light is decreasing/increasing with the decrease/increase of plasma density. Therefore, electrons can never continuously stay in phase with the laser beam. However, when there exists an external electric field [25, 35–37], even though this field is very weak, the Woodward–Lawson theorem can be broken and the electron can obtain non-zero energy from the synergetic effects of the external electric field and the laser pulse. If the extension of the external electric field were infinite, the electron would always stay in phase with the laser and be accelerated to infinite energy.

When the incident laser arrives at the critical density surface and is reflected back, due to the formation of the steep interface of electron density [38], a strong delta-like charge separation field or step-like electrostatic potential, as shown in figure 4, is built up therein, and is strong enough to drive electrons to very high velocity within very short time and short distance. Imagine we are standing on the frame of a backward propagating electron, we will find that the incident laser pulse is oscillating very fast, and its only contribution to the motion of the electron is to increase its mass by a factor \( \gamma = (1 + \alpha^2/2)^{1/2} \) in an average way (appendix A); however the reflected laser pulse is oscillating so slowly that this electron can be captured and continually accelerated backward by its ponderomotive force. Actually the first stage acceleration strongly depends on the pre-plasma scale length. As clearly demonstrated in figure 4(b) (10 \( \mu \)m), the first stage acceleration is stronger than that in figure 4(a) (5 \( \mu \)m). According to the Woodward–Lawson theorem [34], a single electron cannot gain non-zero cycle-averaged energy from one plane wave. However, in our case, there exists an external electric field \( Ez \) due to the charge separation in the under-dense pre-plasma region. Actually, as we shall analyse in the next section, the pre-plasma scale length determines the space extension of the energetic electrons off the potential barrier. The laser is of intensity \( 10^{30} \) W cm\(^{-2}\), and laser wavelength is 1 \( \mu \)m.

Figure 3. Trace of a particular particle initially located at \( z = 130 \) \( \mu \)m. (a) is the particle’s position \( z \) as a function of time, (b) is the particle’s momentum \( p \) as a function of time, (c) is its \( \gamma \) as a function of time, and (d) is the \( E_z \) experienced by the particle as a function of time. The laser is of intensity \( 10^{20} \) W cm\(^{-2}\); laser wavelength is 1 \( \mu \)m and pre-plasma scale length is 10 \( \mu \)m.

Figure 4. Comparisons of \( z-p \) phase-space plots with different pre-plasma scale length: (a) 5 \( \mu \)m and (b) 10 \( \mu \)m. The black curves superimposed on the phase-space plots are the electrostatic potential curves \( \left( \int E_z dz \right) \), normalized by \(-e\alpha/mc^2\), and the blue lines are the \( E_z(\times0.25) \) components of the superposition of incoming and reflected laser pulses. The first stage, as indicated schematically by the bottom straight black line in (b), is due to the synergetic acceleration by the longitudinal electric field \( E_z \) and the ponderomotive force of the reflected laser pulse, and the second stage, as indicated schematically by the top straight black line in (b), is attributed to the intense electrostatic potential building and the accompanying reflection of the energetic electrons off the potential barrier. The laser is of intensity \( 10^{30} \) W cm\(^{-2}\), and laser wavelength is 1 \( \mu \)m.
continuously emitting electron beams or separated multi-electron bunches, we find that some electrons can arrive at positions where the potential energies are still large as long as their initial kinetic energies. When these electrons are reflected back to their original positions, the kinetic energies obtained will increase to $E_{\text{kin}}(t) = N \times E_{\text{kin}}$ with $N > 1$. Although it seems impossible, this process conserves the total energy of the system, and $\sum n_a E_{\text{kin}} = \sum n_t E_{\text{inf}}$ is always satisfied, with $E_{\text{inf}}$ obeying $E_{\text{inf}} < E_{\text{kin}} < E_{\text{kin}}$. In the next section, solid interpretations are presented, including mathematical analysis and electrostatic numerical simulations, for the building process of electrostatic potential and the accompanying electron kinetic enhancement by reflection off this potential barrier.

### 3. Two-stage acceleration model

The synergetic acceleration by longitudinal electric field and laser ponderomotive force—We consider the relativistic electron dynamics in the presence of two counter-propagating plane laser waves with vector potential $a_+ \ldots a_-$ and longitudinal field $E_z$, as shown in figure 5. $a_+ \ldots a_-$ means the laser pulse is propagating in the same direction as an electron in the presence of electric field $E_z$. Note that in this section—analyzing synergetic accelerating—simplicity the $z$-direction is taken along the backward direction, i.e. the opposite direction to the PIC simulation configurations. Considering the electron propagates with high velocity along the $z$-direction, the only contribution of the incident wave $a_-$ is to increase the electron mass in an averaged way (appendix A). The $z$-momentum and energy equation, in normalized units, can be written as

$$\frac{d\gamma}{dr} = -\frac{1}{2\gamma} \frac{\partial a_z^2}{\partial z} + E_z. \quad (1)$$

where $\gamma$ is the electron velocity component along the $z$-direction and the relativistic factor $\gamma$ defined as $\gamma = \gamma_\text{rel}$ with $\gamma_\text{rel} = (1 + a^2/2 + \Delta^2/2)^{1/2}$, $a^2/2$ is the average mass increase due to the incident laser wave of the form $a_+ = a \sin(t + \chi)$, and $\gamma_\text{rel} = 1/(1 - v^2/2)$.

For a reflecting plane wave of the form $a_+ = a_+ \sin(t - z)$, from equations (1) and (2), we find

$$\frac{d\gamma}{dr} \gamma_\text{rel}(1 - v_z) = -E_z(1 - v_z). \quad (3)$$

Assuming electric field $E_z$ to be constant, equation (3) can be integrated and we have

$$\gamma_\text{rel}(1 - v_z) = \sigma_{\text{ref}} - E_z(t - t_0 - z - z_0). \quad (4)$$

where $t_0$ is the time at which the electron crosses $z = z_0$ and $\sigma_{\text{ref}} = \gamma_\text{rel}(1 - v_z)|_{t=t_0, z=z_0}$. Note for the highly relativistic case, we have $\sigma_{\text{ref}} \sim (1/2)(\gamma_\text{rel}(\gamma_\text{rel})) \ll 1$.

The trajectory of the electron $z$ can be found by introducing a local time $\tau = t - z$, in which $d\tau/dr = dr/d\tau - dz/d\tau$ and $dr/d\tau = (dz/dr)(dr/d\tau)/v_z$, as

$$\frac{dz}{d\tau} = \frac{v_z}{1 - v_z}. \quad (5)$$

Using $v_z$ from equation (4), $dz/d\tau$ can be found to be

$$\frac{dz}{d\tau} = \frac{1}{2}[f^2(\tau) - 1], \quad (6)$$

where $f(\tau) = \gamma_\text{ref}(\tau + t_0)(\sigma_{\text{ref}} - E_z\tau)$.

The change in the electron energy only due to the contribution of laser waves is given by $\Delta E(\tau) = \gamma_\text{ref}(\tau + t_0)\gamma_\text{rel}(\tau + t_0) - \gamma_\text{ref}(\tau + t_0)\gamma_\text{rel}(\tau + t_0) - E_z(\tau + t_0 - z(\tau + t_0))$.

Following equation (4) and making use of the inequality ($\sigma_{\text{ref}} \ll 1, \sigma_{\text{ref}} + v_z \ll 1$ and $E_z \ll 1$), $\Delta E(\tau)$ can then be rewritten as

$$\Delta E(\tau) = \frac{1}{2} \int_0^\tau f(\tau)^2(\tau + t_0)d\tau \quad (7)$$

Through equations (6) and (7), we can find the maximal possible energy gain within the limited longitudinal scale length $L$ and the maximal in-phase time $\tau = \pi/2$,

$$L = \frac{1}{2E_z^2} \left[ \frac{\gamma_\text{ref}^2(\pi/2 + t_0)}{\sigma_{\text{ref}} - \pi/2} - \frac{\gamma_\text{ref}^2(\tau_0)}{\sigma_{\text{ref}} - \pi/2} + \frac{\pi^2}{4} \right]. \quad (8)$$

$$\Delta E(\pi/2) = \frac{\sigma_{\text{ref}}^2}{2E_z} f(\sigma_{\text{ref}}), \quad (9)$$

where we define $\sigma_{\text{ref}} = \sigma_{\text{ref}}/E_z \ll \pi/2$, and

$$f(\sigma_{\text{ref}}) = \int_0^{\pi/2} \sin(\chi) d\chi. \quad (10)$$

As $\tau_0$ is just an arbitrary initial local time, for simplicity we set $\tau_0 = 0$ in the following expressions. Assuming $a \gg 1$,
$L \gg 1$ and $a^2 = Ra^2$, where $R$ is the reflection rate, based on equation (8) we can obtain,

$$E_z = \frac{a}{L^{1/2}} \left[ \frac{R_0 + \pi/4}{2\sigma_0 (\sigma_0 - \pi/2)} - \frac{R}{2 g(\sigma_0)} \right]^{1/2}. \quad (11)$$

Combining equation (9) and equation (11), the maximal-possible electron kinetic energy gain within the limited longitudinal length $L$ from the laser of incident amplitude $a$ and reflection rate $R$ can be expressed as

$$\Delta \varepsilon = \eta a L^{1/2} = \frac{R f(\sigma_0)}{2 g(\sigma_0) \sigma_0^{1/2}}, \quad (12)$$

with $g^2(\sigma_0) = (R + \pi/4)/[2\sigma_0 (\sigma_0 - \pi/2)] - Rf(\sigma_0)/2$.

In equation (12), the coefficient $\eta$ is a function of $R$ and $\sigma_0$. From figure 6, for the typical reflection rate $R = 0.9$, $\sigma_0$ almost saturates at 0.5 for a large range of $\sigma_0$. Finally, we give a scaling law which describes the maximal-possible electron energy gain for the synergetic acceleration process, where the laser intensity $I$ is normalized by $1.37 \times 10^{18} \text{ W cm}^{-2}$ and the longitudinal length $L \sim \beta L_p$ is normalized by $\mu m$,

$$\varepsilon [\text{MeV}] = 0.64 \times \beta^{1/2} \times I^{1/2} \times L_p^{1/2}. \quad (13)$$

In equation (13), we assume that the longitudinal length is determined by the pre-plasma scale length with $L \sim \beta L_p$. Coefficient $\beta$ can be obtained by the direct comparisons of equation (13) and values from PIC simulations, where the pre-plasma is assumed to be of the form $1[1 + \exp(-2z/L_p)]$ in the PIC simulations. According to the scaling law of equation (13), we can see that the first stage acceleration, or the synergetic acceleration by longitudinal electric field $E_z$, and the ponderomotive force of the reflected laser, depends on both the incident laser intensity and the pre-plasma scale length.

In order to validate the PIC results with theory, a direct comparison of maximum electron energy observed in PIC simulation and theoretical scaling law is present in figure 7. The electron energy spectra are collected by the diagnostic planes located at 100 $\mu m$, which could record the maximum electron energy accelerated in the first stage. Figure 7(a) corresponds to the cases with fixed pre-plasma scale length $L_p = 10 \mu m$, where the black line is for incident laser of intensity $10^{20} \text{ W cm}^{-2}$, blue line is for $10^{20} \text{ W cm}^{-2}$, and red line for $10^{21} \text{ W cm}^{-2}$. Figure 7(c) corresponds to the cases with fixed incident laser intensity $10^{20} \text{ W cm}^{-2}$ and varying pre-plasma scale length, where black line is for pre-plasma scale length $L_p = 1 \mu m$, blue line is for $L_p = 5 \mu m$, and red line for $L_p = 10 \mu m$. (b) and (d) are the comparisons between theoretical scaling law and PIC simulations, where black lines are the plot of equation (13) and red squares are the results from PIC simulations.

**Figure 6.** Dimensionless coefficient $\eta$ as function of $\sigma_0$ and $R$, where $\sigma_0 = \sigma_{00}/E_z$, $\eta = f(\sigma_0)/2g(\sigma_0)$, and $R \equiv a^2/\sigma_0^2$ is the reflection rate of the incident laser beam. See text for explanations of $\sigma_0$, $f(\sigma_0)$ and $g(\sigma_0)$.

**Figure 7.** (a) and (c) Electron energy spectra recorded at $z = 100 \mu m$ at the final time of simulations. (a) corresponds to the cases with fixed pre-plasma scale length $L_p = 10 \mu m$, where the black line is for incident laser of intensity $10^{20} \text{ W cm}^{-2}$, blue line is for $10^{20} \text{ W cm}^{-2}$, and red line for $10^{21} \text{ W cm}^{-2}$. (c) corresponds to the cases with fixed incident laser intensity $10^{20} \text{ W cm}^{-2}$, and red line for $L_p = 5 \mu m$, blue line is for $L_p = 1 \mu m$, blue line is for $L_p = 10 \mu m$. (b) and (d) are the comparisons between theoretical scaling law and PIC simulations, where black lines are the plot of equation (13) and red squares are the results from PIC simulations.

Electrostatic potential building and the accompanying electron reflection—To get insights on both (i) the possibility of the formation of the electrostatic potential barrier with the maximal value significantly larger than electron kinetic energy, and (ii) the role of the potential barrier in electron acceleration, let us consider a 1D model problem. Assume that at $t = 0$ we have a bunch of electrons with density $n_0$ and momentum $p_0 > 0$ occupying region $0 < z < z_0$ ($z_0 \ll L_p$) and a bunch of immobile ions, located at $z < 0$, such that total electron and ion charges compensate each other. We consider dynamics of electron bunch expansion assuming that the electrons, which come back to their original positions, do not move any more. Since we are considering the 1D geometry, the electric field acting on an electron solely depends on its original position at $t = 0$ and does not vary in time. Therefore, for an electron having $z(t = 0) = z_0 < z_b$ we have the following equation of motion,

$$\frac{d}{dt} \frac{p}{\sqrt{1 - p^2}} = -E_z(z_0), \quad (14)$$

where $E_z(z_0)$ is the electric field in the pre-plasma region.
where \( E_z(z_0) \) is the original electric field, normalized by \( e/m_e c \). From equation (14) we find the time dependence of the position \( z(t, z_0) \) of the electron initially located at \( z_0 \) as

\[
z(t, z_0) = z_0 + \int_0^t \frac{p_0 - E_z(z_0)\alpha}{\sqrt{1 + [p_0 - E_z(z_0)\alpha]^2}} \, dt',
\]

\[
= z_0 - \frac{1}{E_z(z_0)} \left\{ \sqrt{1 + [p_0 - E_z(z_0)\alpha]^2} - \sqrt{1 + p_0^2} \right\},
\]

(15)

where \( p_0 = p(t = 0) \). From equations (14) and (15) one can easily see that within the setting of the problem the electron coming back to its original position has \( p = -p_0 \) and, therefore, returns to the original energy.

The original increase of the normalized electrostatic potential within the electron bunch, \( b\phi \), can be easily found from Poisson equation,

\[
b\phi_0 = \frac{1}{2} \frac{\omega_{pe}^2 b}{c^2},
\]

(16)

where \( \omega_{pe}^2 = 4\pi e^2 n_0 m_e \). Now we will analyse time variation of the electrostatic potential at relatively large time \( t > p_0/E_z(z_0) \), when the majority of electrons have already returned to their original positions. Estimating the magnitude of \( E_z(z_0) \) from the Poisson equation, we can re-write this inequality as

\[
t > \tau_0 = \frac{p_0 c}{\omega_{pe} b}.
\]

(17)

Then the difference of the normalized electrostatic potential, \( \Delta \phi(t) \), between the head of the expanding electron bunch, \( z_b(t) = z(t, z_b) \), and the coordinate \( z_c(t) \) with \( z_c = z(t, z_c) \) of electrons returning to their original position at time \( t \), can be written as follows,

\[
\Delta \phi(t) = \int_{z(t)}^{z_b(t)} E_z(z) dz,
\]

(18)

or

\[
\Delta \phi(t) = -\int_0^{E[z(t)]} E(z) \frac{dz(t, z_0)}{dE(z_0)} \frac{dE}{dz}
\]

\[
= -\frac{1}{2} \frac{\omega_{pe}^2}{c^2} \left[ z_c^2 - z_b^2(t) \right] + \int_0^{2p_0} \frac{1}{\sqrt{1 + p_0^2} - \sqrt{1 + (p_0 - \xi)^2}} d\xi.
\]

(19)

Since we are considering the time \( t \gg \tau_0 \), where \( z_b(t) \to z_b \), we find the following asymptotic expression, \( \Delta \phi_{\infty} = \Delta \phi(t \to \infty) \),

\[
\Delta \phi_{\infty} = \int_0^{2p_0} \left[ \sqrt{1 + p_0^2} - \sqrt{1 + (p_0 - \xi)^2} \right] d\xi.
\]

(20)

From equation (20) we derive \( \Delta \phi_{\infty} \sim p_0^2 \) for \( p_0 \ll 1 \) and \( \Delta \phi_{\infty} \sim 2\ln(2)p_0^2 \) for \( p_0 \gg 1 \). In other words, for the non-relativistic case \( \Delta \phi_{\infty} \) is twice the initial electron kinetic energy \( E_{kin} \), while for a super-relativistic case \( \Delta \phi_{\infty} \sim 2\ln(2)E_{kin} \sim 1.4E_{kin} \).

As we mentioned before, electrons, being finally reflected back by potential, will come to their original positions and obtain their original kinetic energy. Thus in the process of launching just one electron bunch, there is no possibility of increasing electron energy. However, the situation changes drastically when we launch a few electron bunches separated by a dwell time \( \tau_{bw} \). To get an insight into the electron acceleration mechanism, consider the case of two bunches. The first bunch, launched at \( t = 0 \) will expand as was discussed before. At time \( t = \tau_{bw} \) the second bunch starts launching. At that time, the first bunch has formed a ‘potential barrier’ between the head of the first bunch and the launch point, with \( \phi_{bar} = \Delta \phi_{\infty} \). However, almost all electrons of the first bunch have already come back to their original position and the electric field within the ‘potential barrier’ becomes very small, with \( E \sim \Delta \phi_{\infty}/\tau_{bw} \ll E_z(z_0) \). As a result, the second bunch also expands virtually into vacuum and at time \( t = 2\tau_{bw} \), the cumulative contribution of the first and second bunches will create a ‘potential barrier’ with \( \phi_{bar} = 2 \times \Delta \phi_{\infty} \). In addition, a relatively small number of electrons at the head of the first bunch turning back after the expansion of the second bunch will finally acquire not only their initial kinetic energy but also potential energy created by the second bunch. As a result, their total kinetic energy as they reach the launching location will double. Their additional energy comes at the expense of electron energy from the second bunch, which are decelerated somewhat as the bunches pass through each other.

We can consider the injection of many identical electron bunches with the dwell time between them such that the previous bunches do not impact the dynamics of later ones. One can easily find that the number of such bunches is limited by \( N_b \sim \ln(t/\tau_0) \). Therefore, maximum kinetic energy, acquired by the returned electrons of the very first bunch, after being accelerated by the electric field of all bunches can be estimated as \( E_{kin,max} \sim N_b \times E_{kin} \), which, nonetheless, can be significantly larger than \( E_{kin} \).

We can consider also continuous injection of electrons into a half-space taking the time-dependent distribution function of launching electrons \( f(t, v) \). Considering the non-relativistic case we take

\[
f(t, v) = \frac{n_0 \delta(v - v_0)}{1 - \alpha \omega_{pe}(n_0)},
\]

(21)

where \( \alpha \ll 1 \). This temporal evolution of electron launch, limited by \( \alpha \omega_{pe}(n_0) \ll 1 \), resembles the rate of bunch launches. The final energy by electric field of all bunches can be estimated as \( E_{kin,max} \sim N_b \times E_{kin} \), which, nonetheless, can also be significantly larger than \( E_{kin} \).

In order to confirm the above theoretical analysis, we also run a series of 1D electrostatic PIC simulations, which are solved by an energy conserving method (appendix B). The electrostatic PIC simulations solve the following equations,

\[
\frac{\partial \delta}{\partial t} + \frac{\partial \delta}{\partial z} + \frac{eE_z}{m_e} \frac{\partial \delta}{\partial v} = 0,
\]

(22)

\[
\frac{\partial E_z}{\partial z} = 4\pi e \int f \, dv,
\]

(23)

\[
f(t = 0) = n_0 \delta(v - v_0),
\]

(24)
with $\omega_{pe0} = 4\pi n_0 e^2/m_e$, $v = v[f(t)]$, $\tau = [1/\omega_{pe0}]$, $z = \xi/c/\omega_{pe0}$, $E_z = E_0/[\omega_{pe0}(\varepsilon e)]$, $\phi = \phi[m_e c^2/\varepsilon]$, $\omega_{pe} = \omega_{pe0}[\omega_{pe0}]$, $n_e = \omega_{pe0}^2[n_0]$ and $f = f[n_0/c]$. We define a reference density $n_0$, corresponding to a reference plasma frequency $\omega_{pe0}$. $\omega_{pe0}$ define the time scale in simulation, $c/\omega_{pe0}$ define the length scale and $c$ is the speed of light. We can change the plasma density in simulation by adjusting $\omega_{pe}$. If $\omega_{pe} = 1$, the plasma density used in simulation is exactly $n_0$. If $\omega_{pe} = 0.5$, the corresponding plasma density in simulation is $0.5 \times 0.5 \times n_0$.

Figure 8 shows the simulation results, in which an electron bunch of velocity $v_0 = 0.7$, thickness $L_0 = 0.2$ and plasma frequency $\omega_{pe} = 0.5$ is emitted from the surface $z = 0$. Figures 8(a)–(c) show the time-snap of $z$-$v_z$ phase-space, electric field and potential profile at $t = 0.5$, $t = 28$ and $t = 80$, which clearly demonstrates that at $t = 28$, the electrons in the rear start returning to the emitting point at $z = 0$, well consistent with the theoretical analysis, $\tau = (2/\omega_{pe})[(v_0/L_0) = 28]$. In our simulation, we include a numerical friction mechanism to stopping electrons when re-entering into the emitting point. Figure 8(d) shows the maximal electric field and potential evolution with time, and we find that the maximal potential almost keeps constant even when the back edge of the bunch returns to the emitting point, which is also consistent with theoretical prediction. As expected by theoretical analysis, the maximal electric field decrease with time as $\eta / t$ when $t > \eta$. The kinetic energy of the returning electron is exactly equal to the initial value, having $v = -v_0$, which is, nonetheless, consistent with the theoretical prediction.

Let us consider the situation of emitting multiple bunches. Figure 9(a) and (b) show the two bunch cases with the dwell time $\tau_{bw} = 280$ greatly larger than $\tau = 28$. We noticed that the maximal potential energy can be further increased by the emission of the second bunch, finally reaching four times the original kinetic energy. The velocity of the returned electron can be as high as $v = -0.99$ compared with the initial value $v_0 = 0.7$, confirming the theoretical prediction that the kinetic energy of the returning electron is doubled. Figure 9(c)–(f) are cases of three ($\tau_{bw1} = 200$ and $\tau_{bw2} = 100$) and four ($\tau_{bw1} = 200$, $\tau_{bw2} = 100$ and $\tau_{bw3} = 50$) bunches, the maximal potential and the returning electron kinetic energy can be further increased as expected. Limited to the computational ability of our simulation, if the dwell time is long enough, the final maximal potential energy will be close to the theoretically predicted value $E_{k\text{max}} \sim \ln(t/\tau_{bw})E_{k\text{in}}$.

As shown in figure 4, the emission of electrons is a continuous process. Here in figure 10, we show the simulation results for a continuous electron beam with constant velocity $v_0 = 0.4$ and density profile $\omega_{pe0}^2 \exp(t/\omega_{pe0})$, where $\omega_{pe0} = 0.125$. The simulation results, as shown in figures 10(a) and (b), indicate that the maximal potential energy is more than three times as large as the initial kinetic energy at $t = 40$ and is still increasing gradually with time. Note that the oscillation of maximal potential energy, with the oscillation frequency increasing with time, comes from the plasma intrinsic oscillations, with its frequency determined by the density of the emitted electron beam. With the increase of electron beam density, the maximal potential energy and oscillation frequency are also increasing with time. Figure 10(c) records the velocity spectra of the returning electrons collected at the emission point, indicating that the returning electrons actually span a large velocity range, from $-0.3$ to $-0.7$. This spanning of the velocity range is also consistent with the theoretical prediction, with some of the electrons having velocity higher than the initial value 0.4, and some having velocity smaller than.

\[ \omega_{pe0} = 4\pi n_0 e^2/m_e, \quad v = v[f(t)], \quad \tau = [1/\omega_{pe0}], \quad z = \xi/c/\omega_{pe0}, \quad E_z = E_0/[\omega_{pe0}(\varepsilon e)], \quad \phi = \phi[m_e c^2/\varepsilon], \quad \omega_{pe} = \omega_{pe0}[\omega_{pe0}], \quad n_e = \omega_{pe0}^2[n_0] \quad \text{and} \quad f = f[n_0/c]. \]
4. Conclusions and discussions

The generation of super-high energetic electrons influenced by pre-plasma in relativistic intensity laser–matter interaction is studied in a one-dimensional slab approximation with particle-in-cell simulations. Different pre-plasma scale lengths and incident laser intensity are considered, showing an increase in both particle number and cut-off energy of energetic electrons with the increase of the pre-plasma scale length and laser intensity, and the obtained cut-off energy of electrons greatly exceeding the corresponding laser ponderomotive energy. The two questions, (i) why is the generation efficiency of energetic electrons increasing with the increase of pre-plasma scale length and laser intensity?, and (ii) what is the underlying acceleration mechanism of super-high energetic electrons with kinetic energy greatly exceeding the ponderomotive energy?, are answered in this work.

Furthermore, a two-stage electron acceleration model is proposed to explain the underlying physics in detail. The first stage is attributed to the synergistic acceleration by the longitudinal charge separation electric field $E_z$ and the ponderomotive force of the laser beams. The efficiency of the first stage acceleration depends on both the pre-plasma scale length and the laser intensity. The maximal possible energy gain during the first stage acceleration is analysed and a scaling law is obtained by solving the relativistic electron motions in the presence of two counter-propagating plane laser waves and the external electric field due to the charge separation within limited space extension, which is on the order of the pre-plasma scale length. The maximal-possible energy gain in the first stage is estimated to be $\varepsilon (\text{MeV}) = 1.6 \times 10^{12} I^{1/2} / I_p^{1/2}$, where $I$ is laser intensity normalized by $1.37 \times 10^{14} \text{W cm}^{-2}$, and $I_p$ is pre-plasma scale length normalized by $\mu\text{m}$. The scaling law indicates that with the increase of pre-plasma scale length and incident laser intensity, the maximal-possible electron energy also increases, which agrees well with the simulation results.

The energetic electrons pre-accelerated in the first stage could expand freely and build up an intense electrostatic potential barrier in front of the target, with potential energy several times as large as the electron kinetic energy. Some of the energetic electrons could be reflected by this potential and return back to the target, obtaining final kinetic energies several times as large as the initial values. The processes of potential building and the accompanying electron kinetic enhancement by this potential barrier are analysed theoretically and confirmed by electrostatic PIC simulations, in which the theoretical predictions and electrostatic PIC simulations are also in good agreement.

Note that the modulational (M) and Raman (RS) instabilities may also play roles in this process. It was shown that [39–41], for a relativistic laser with laser amplitude $a > 1$ and homogeneous density close to relativistic critical, the nonlinear stage of the instabilities will result in a strong heating of the electron distribution function. The existing analyses are restricted to homogeneous plasmas. While in our case, plasma is rather strongly inhomogeneous, their inhomogeneous thresholds will differ a lot [42], so that if the existing conclusions match our cases need further investigation.

In experiment, reflected light does not always retrace the path of the incident light. However, the backward going electrons do not necessarily have an additional acceleration by reflected light. For those electrons which fail to get accelerated by the first stage, their initial energy acquired at the critical surface is enough to build up large potential, provided that we have a large number of electron bunches or sustained emission of electron beam, as shown in figures 9 and 10.

Furthermore, other multidimensional effects of laser propagation through under-dense plasmas, such as beam self-focusing [43] and self-generated electromagnetic fields [44, 45], could also enhance the electron energy by breaking the Woodward–Lawson theorem. These multidimensional effects are ignored in the present work. The extension of our work to multi-dimensional configurations and addressing the effects of self-focusing and self-generated electromagnetic fields shall be presented in a following separate paper.

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Appendix A. Confirmation of the reduced model

We have studied the motion of a single electron in the field of $a_+, a_-, E_z$ by numerically solving the 1D-3V electron equation of motion with the standard Boris algorithm. Figure A1(a) shows the motion of a single electron in the fields of only $a_+$ and $E_z$. It indicates that when the Woodward–Lawson theorem is broken, electrons will be continuously
The majority of PIC schemes—including LAPINE, as we are using, have the property of exactly conserving the system total momentum, while not conserving the system total energy exactly. In fact, the typical PIC methods, which use explicit differentiation in time, i.e. explicit PIC schemes, tend to increase the total energy of the system by numerical heating. Conversely, the other categories of PIC methods, which use implicit differentiation in time, i.e. the so-called implicit PIC schemes, tend to decrease the total energy of the system by numerical cooling.

For large scale simulations, with \( L \gg \lambda_d \), both explicit PIC and implicit PIC can be applied; this does not affect the overall simulation picture, provided that the energy conservation is satisfied within an acceptable error, like \( \delta E/E \sim 1\% \). However in our case, where we need to resolve significantly fine structures, with \( L_o \ll \lambda_d \), the demand for energy conservation is of extremely high level.

To overcome this issue, we refer to the method introduced by Markidis and Lapenta [46], where a new PIC method, which conserves energy exactly, is used for the electrostatic PIC simulations, shown in figures 8–10. The equations of motion of particles and the Maxwell’s equations are differenced implicitly in time by the mid-point rule and solved concurrently by a Jacobian-free Newton Krylov (JFNK) solver. The particle average velocities and the electrostatic field are incurred implicitly in time by the mid-point rule and solved concurrently.

\[ \gamma u = (1 + \gamma^2 a^2)^{1/2} \]
\[ \gamma v = (1 + \gamma^2 a^2)^{1/2} \]

The results of full dynamics and reduced model are well fitted, which confirms our assumption in the article well.

**Appendix B. Simulation method of electrostatic PIC**

The majority of PIC schemes—including LAPINE, as we are using, have the property of exactly conserving the system total energy. The results by Cipiccia et al. [14], the demand for energy conservation is satisfied within an acceptable error, like \( \delta E/E \sim 1\% \). However in our case, where we need to resolve significantly fine structures, with \( L_o \ll \lambda_d \), the demand for energy conservation is of extremely high level.

To overcome this issue, we refer to the method introduced by Markidis and Lapenta [46], where a new PIC method, which conserves energy exactly, is used for the electrostatic PIC simulations, shown in figures 8–10. The equations of motion of particles and the Maxwell’s equations are differenced implicitly in time by the mid-point rule and solved concurrently by a Jacobian-free Newton Krylov (JFNK) solver. The particle average velocities and the electrostatic field are incurred implicitly in time by the mid-point rule and solved concurrently.

\[ \gamma u = (1 + \gamma^2 a^2)^{1/2} \]
\[ \gamma v = (1 + \gamma^2 a^2)^{1/2} \]

The results of full dynamics and reduced model are well fitted, which confirms our assumption in the article well.

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