Laser-Doppler vibrometer microscope with variable heterodyne carrier

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Abstract. The generation of a heterodyne carrier frequency for coherent detection via offset-locking in an optical phase-locked loop was demonstrated in communication, spectroscopy and other fields. In state-of-the-art laser-Doppler vibrometers (LDV), common acoustooptic devices (Bragg cells) allow efficient heterodyning with a static frequency. Moving mechanical parts vibrate in many technical applications, e.g. moving cars, trains, robots, etc. In such applications, a large laser-Doppler-shift generated by the movement impairs the measurement of the relevant vibrations. A variation of the frequency offset of the interfering laser beams can suppress the Doppler effect of high-speed movements with slow speed variations and adjusts the signal spectrum to the optimal value. Thus, the electronic bandwidth can be set to an optimal value within the modulation bandwidth of the relevant vibrations. However, Bragg cells are not well applicable to shift laser frequencies without affecting the interferometer adjustment. In this paper, we propose offset locking of two independent lasers with an optical phase-locked loop to exploit the possibility of flexible heterodyne-carrier generation for laser-Doppler vibrometry. We demonstrate a LDV setup with offset locked visible DBR semiconductor lasers for a flexible heterodyning up to 200MHz and present first measurements.

1. Introduction
Laser-Doppler vibrometers (LDV) are a group of indispensable instruments to measure vibrational information without contact in many applications in research, development and production [1, 2]. LDVs exploit the laser-Doppler effect which linearly transduces a motion into a phase modulation of the radiation, which is scattered back by the moving or vibrating object [1]. Therefore, a fairly-stable laser frequency (or wavelength) as the standard is required which is usually provided by most laser sources. The phase modulation at the laser frequency (as the fundamental carrier at hundreds of THz) cannot be measured directly. Thus, it is down-mixed by interference with a second laser beam and coherently detected with a fast photodetector. In heterodyne interferometers, the laser radiation for down-mixing comprises a frequency-shifted copy of the original laser radiation. Therefore, the vibration-induced phase modulation (Bessel spectrum) emerges around a heterodyne carrier frequency in the spectrum of the photodetector signal. Especially in field applications (air-born, vehicle-born or hand-held applications), a superposed high velocity with slow variations can generate large laser-Doppler frequency shifts which shifts the complete Bessel spectrum [2]. This effect can also impair tracking-LDV measurements on large rotating structures (e.g. wind turbines). Typical heterodyne LDVs incorporate a fixed frequency shift up to several hundreds of MHz between the
interfering beams by acoustooptic (AO) devices [1, 2]. Carrier-tracking demodulation by PLL [3] or post-processing can eliminate the disturbing motions [4]. Therefore, the complete vibration spectrum must remain within the electronic bandwidth to enable distortionless reconstruction of the underlying vibration. However, the motion-induced frequency shift may shift the Bessel spectrum out of measurement bandwidth or to regimes where a higher noise level or spurious lines impair the sensitivity of the LDV. For the full exploitation of the measurement bandwidth for vibration detection, a variable and stabilized difference frequency in coherent detection can compensate the motion-induced frequency shift. The goal is to enable a variable frequency difference between the interfering laser beams, to maintain the central frequency of the Bessel spectrum at the center of the measurement bandwidth. In conventional AO devices, the diffraction angle is directly dependent on the drive frequency [5]. Thus, with these devices a flexible change of frequency shift is not possible without undesirable misalignment.

In this work, we pursue the generation of a variable heterodyne carrier frequency for LDV by frequency-offset locking of two semiconductor lasers in an optical phase-locked loop. In general, heterodyning by offset-locking in an optical phase-locked loop (OPLL) was shown in the 1960s for remote sensing and communication [6, 7] and the principle of coherent receiver (with CO$_2$ laser) was patented in the mid-1970s for optical communication systems [8]. Hitherto, the generation of a well-defined frequency and phase relation between two independent laser sources was demonstrated in many applications [9–18]. With the emergence of semiconductor lasers with intra-cavity wavelength selection, reliable laser sources become available with linewidths of kHz to MHz. Especially at visible wavelengths, only recent developments provide distributed Bragg reflector (DBR) semiconductor lasers with fast tuning by injection current and temperature over a wide wavelength range [19].

In this paper, we discuss the advantages and challenges of a flexible generation of heterodyne carrier frequency for laser-Doppler vibrometry by OPLL. This technique provides the capability of eliminating dominant motions of the vibrating sample and adapting the heterodyne frequency carrier to the measurement problem. Decisive parameters of the OPLL are presented in conjunction with the application to laser-Doppler vibrometry. As an application, we demonstrate our LDV microscope setup with two visible DBR semiconductor lasers at 632 nm which is capable to provide a variable heterodyne carrier frequency up to 200 MHz by offset-locking via OPLL. Further, we present first measurements of resonances of a piezoelectric transducer to show the capability and limitations of the novel application to laser-Doppler vibrometry. Finally, we discuss the vibration sensitivity.

2. Signal in heterodyne vibrometry

2.1. Coherent detection of laser-Doppler shift

LDVs contain a typical laser interferometer with a broadband acquisition of the light phase difference [1]. For the interference, the radiation of usually the same coherent light source is split into two copies generating a reference and measurement beam. The measurement beam contains phase changes or fluctuations associated with the measured quantity. In LDVs, these phase changes $\varphi_{\text{vib}}(t)$ over time $t$ due to laser-Doppler effect are principally caused by changes of the geometric length of the measurement beam (reference path length remains constant). For a displacement $s(t)$ of the sample (parallel to the incident measurement beam) relative to the LDV (containing sender and receiver), the observed phase modulation is [2]

$$\varphi_{\text{vib}}(t) \approx \frac{4\pi}{\lambda} s(t)$$

with the wavelength $\lambda$ of the light. The approximation in Eq. 1 is sufficient for estimating the laser-Doppler effect even at high modulation frequencies [20].
The fundamental carrier of this phase modulation (by laser-Doppler effect) is the light frequency \( f_m \) (430 to 770 THz at visible light) which cannot be measured directly. This phase modulation of the measurement beam is linearly shifted to the measurement bandwidth of the LDV via interference (coherent detection) with the reference beam at the frequency \( f_r \). The resulting current at the photodetector (PD) is

\[
    i_{PD}(t) = 2\chi K_{PD} \sqrt{P_m P_r} \cos \left[ 2\pi (f_m - f_r) t + \Delta \varphi(t) \right]
\]

with the interference efficiency \( \chi \), the PD sensitivity \( K_{PD} \), the differential phase fluctuation \( \Delta \varphi \), and the power \( P_m \) of the measurement beam at the photodetector (\( P_r \) respectively for reference beam). Thus, any differential phase modulation is linearly shifted to the difference frequency \( f_m - f_r \). In heterodyne LDV, a non-zero, static difference frequency (intermediate frequency) of the interfering laser beams is applied. In analogy to communication technology, this intermediate frequency is denominated as heterodyne carrier frequency \( f_c = |f_m - f_r| \) \[2\]. The advantages of heterodyning for LDV are the choice of a low noise spectral range and the unambiguous discrimination of the direction of the vibration or motion \[2\].

### 2.2. Bandwidth requirement and tracking of low-frequency motions

A decisive specification of heterodyne LDVs is the necessary measurement bandwidth which is dependent on the vibration-signal bandwidth induced by laser-Doppler effect. For the estimation, a sample is moving (in direction of the LDV) at high-speed with slow variations with the motion \( s(t) \) superposed by an harmonic vibration with frequency \( f_vib \) giving

\[
    s(t) = s_{lf}(t) + \hat{s} \cos(2\pi f_vib t + \varphi_{vib})
\]

with the vibration amplitude \( \hat{s} \) and the vibration phase \( \varphi_{vib} \). The interferometer signal according to Eqs. 1, 2, and 3 can be expanded with the Jacobi-Anger identity \[21\]. Thus, the typical Bessel spectrum emerges in the spectrum of the vibrometer signal \[2\] of the form

\[
    i_{PD}(t) = 2\chi K_{PD} \sqrt{P_m P_r} \sum_{n=-\infty}^{\infty} J_n \left( \frac{4\pi \hat{s}}{\lambda} \right) \cos \left[ 2\pi \left( f_c + \frac{2}{\lambda} \frac{ds_{lf}(t)}{dt} + nf_{vib} \right) t + n\varphi_{vib} + \varphi_0 \right]
\]

with the Bessel function of first kind \( J_n \) and the (integer) order \( n \) and constant phase \( \varphi_0 \). The amplitudes of the equidistant spectral lines are determined by the modulation index \( M_{vib} = 4\pi \hat{s}/\lambda \) and the frequency spacing corresponds to the vibration frequency \( f_{vib} \). According to Carson’s rule \[22\] the required measurement bandwidth, which contains > 99% of the spectral power (of the harmonic vibration), is \( B_{vib} > 2(M_{vib} + 1)f_{vib} \).

In Eq. 4, the central frequency \( f_{j0} \) of the Bessel spectrum \( n = 0 \) observes an instantaneous frequency shift \( \Delta f(t) = 2ds_{lf}(t)/\lambda dt \) = \( 2v_{lf}(t)/\lambda \) in respect to the heterodyne carrier frequency \( f_c \) with the (low-frequency) speed \( v_{lf} \). For an undisturbed reconstruction of the motion, the heterodyne carrier frequency must be chosen, that (for the maximum values of \( v_{lf} \), \( \hat{s} \) and \( f_{vib} \)) no overlap of the spectra occurs \( (f_c > B_{vib}/2) \). Therefore, it follows

\[
    f_c > \Delta f + \frac{B_{vib}}{2} = \frac{2v_{lf}}{\lambda} + \left( \frac{4\pi \hat{s}}{\lambda} + 1 \right) f_{vib}
\]

with the assumption of bidirectional (maximum) values \( v_{lf} \). Additionally, the measurement bandwidth \( B_{el} \) has to be chosen that the transfer function and the phase shift for all frequency components is flat. The heterodyne carrier frequency \( f_c \) is usually placed in the center of the measurement bandwidth, which has to be larger than twice the heterodyne carrier frequency \( B_{el} > 2f_c > B_{vib} \) consequently.
Thus, any instantaneous frequency shift $\Delta f(t)$ induced by (unwanted) low-frequency motion costs measurement bandwidth which cannot be utilized for detection of vibrations. Further, the vibration spectrum may be shifted to frequencies with high noise level or spurious lines. Therefore, a variable heterodyne carrier frequency $f_c(t)$ enables the flexible choice for the spectral region of the measurement bandwidth. Further, it allows to compensate frequency shift due to low-frequency motions of the sample (directly at the PD) to maintain a constant center frequency of the vibration spectrum ($f_{J0} = f_c(t) + 2v_{lf}(t)/\lambda \approx \text{const.}$).

2.3. Reconstruction of vibration

Standard FM decoders from communication technology (e.g. IQ demodulator) can be utilized to reconstruct the motion [2]. For a complete motion recovery, the local oscillator is driven with the heterodyne carrier frequency $f_c$. If compensation of low-frequency motions via a variable heterodyne carrier is applied, the stabilized center frequency $f_{J0}$ of the vibration spectrum has to be fed to the demodulator. Therefore, it is beneficial to synchronize both frequencies and to allow only discrete frequency steps for compensation [3].

In ISO 16063-41 the calibration with the raw vibration spectrum is recommended to avoid any influence of demodulation [23]. If the narrow-band approximation is applicable, the carrier-to-sideband ratio CSR directly delivers the modulation index $M_{vib}$ with the Bessel function of first kind $J_n$ and (integer) order $n$.

\[
M_{vib} \approx \frac{2 J_1 (M_{vib})}{J_0 (M_{vib})} = \frac{2}{\sqrt{\text{CSR}}}
\]

(6)

Further, the following relation from the magnitudes of adjacent Bessel lines can be deduced [2] with the derivation property of the Bessel function for broad-band phase modulation.

\[
M_{vib} = \frac{2n J_n (M_{vib})}{J_{n-1} (M_{vib}) + J_{n+1} (M_{vib})}
\]

(7)

2.4. Influence of phase noise on coherent detection

In LDVs, phase fluctuations of the interfering beams within the electronic bandwidth are transduced to the phase-modulation of the signal current (Eq. 2). If these phase fluctuations are correlated to a certain degree (of coherence), only the non-correlated difference of the phase fluctuation contributes to the phase noise in the detected vibrometer spectrum. Beyond the photodetector bandwidth, this differential phase fluctuations are integrated and degrade interference contrast ($\chi$ in Eq. 2). Regarding free-running laser sources, which are not derived from stable atomic transition (like HeNe), the laser emission frequency may fluctuate. However, with common lasers and good drive electronics, frequency fluctuations below several tens of MHz can be achieved, which changes the wavelength of the measurement beam (as reference length for the displacement) only insignificantly.

If the phase fluctuation (Eq. 4) can be split into laser-Doppler-induced phase modulation $\phi_{vib}(t)$ and differential phase noise $\Delta \phi_{N}(t)$, the signal current for a harmonic vibration $i_{PD}(t)$ becomes

\[
i_{PD}(t) \approx 2 \chi KPD V P_m P_r \sum_{n=-\infty}^{\infty} J_n (M_{vib}) \left\{ \cos [2\pi (f_c + nf_{vib}) t + n\phi_{vib}] \pm \frac{\Delta \phi_{N}}{2} \cos [2\pi (f_c + nf_{vib} \pm f_N) t + n\phi_{vib} \pm \phi_{N}] \right\}
\]

(8)

The relation shows that the each spectral line of the intended vibration spectrum gets two sidebands with the differential phase modulation of both laser sources. Therefore, it is obvious that small differential phase noise is desirable for an undisturbed vibration spectrum.
3. Generation of variable heterodyne frequency carrier

3.1. State of the art

There are multiple technical realizations for the introduction of a frequency shift between the reference and the measurement beam (derived a common laser source) with paratellurite Bragg cells, Lithium-Niobate frequency shifters or serrodyning [5, 24]. Further, a moving or vibrating reference mirror or direct frequency modulation of the laser (in an unbalanced interferometer) can achieve heterodyning [25].

The most popular realization is the frequency shift by paratellurite acoustooptic (AO) frequency shifters [2]. The AO design is usually optimized for efficiently diffracting into first order by operating in the Bragg regime (Bragg cell). Conservation of energy and momentum requires that the first-order beam is frequency shifted by acoustic drive frequency $f_c$ and diffracted with an angle $\theta_{\text{diff}}$. Therefore, the acoustic drive frequency corresponds to the heterodyne carrier frequency $f_c$ in most instruments with one Bragg cell. However, drive frequency and diffraction angle are directly coupled

$$\sin (\theta_{\text{diff}}) = \frac{\lambda}{2v_{ac}} f_c$$

with the specific sound velocity $v_{ac}$ of the AO material [5]. Therefore, any change in drive frequency results in an angular deviation which destroys the sensitive alignment of the LDV. Even for collinear designs, the drive frequency has to be tuned carefully to achieve energy and momentum conservation. Therefore, common acoustooptics systematically does not provide a reasonable capability of a variable frequency shift.

To overcome the problems of conventional AO frequency shifters, we incorporated two phase-locked semiconductor lasers which are stabilized to an offset frequency by an optical phase-locked loop (OPLL) for the generation of a variable frequency heterodyne carrier $f_c(t)$.

3.2. Heterodyning by optical phase-locked loop

A good interference requires the same transverse mode (spatial coherence) and polarization. Further, a well-defined phase relation (temporal coherence) between the interfering beams results in a high interference contrast [26]. These requirements can be met by two independent laser sources, where the mutual temporal coherence of both beams is established by an optical phase-locked loop (OPLL) (scheme in Fig. 1). Therefore, a photodetector (PD) detects the frequency difference $f_m - f_i$ between the (free-running) master laser (ML) and the tunable slave laser (SL). A phase-sensitive detector (PSD) compares this frequency difference to the frequency $f_{LO,el}$ of a local oscillator (LO). The resulting error is fed back to the tunable SL by a loop filter with the transfer function $F(s)$.

In lock, the SL inherits the coherence properties of the ML within the loop bandwidth $f_{ol}$ (coherence cloning). The loop bandwidth therefore has to be wider than the summed linewidth of the lasers [27], which is typically lower than several MHz for typical semiconductor lasers. In lock, the SL frequency matches the frequency of the ML (optical local oscillator LO_opt) shifted by the frequency of the electric local oscillator LO_el. Hence, the LO_el directly defines the frequency difference and, thus, the heterodyne carrier frequency (Eq. 2) which is

$$f_c = |f_m - f_i| = |f_{ML} - f_{SL}| = |f_{ML} - (f_{ML} \pm f_{LO,el})| = f_{LO,el}$$

with the frequency $f_{ML}$ of the ML, $f_{SL}$ of the SL, and $f_{LO,el}$ of the LO_el (absolute due to symmetry of cosine).
Figure 1. Scheme of an optical phase-locked loop (OPLL) with a frequency-offset lock of the tunable SL (current-controlled oscillator, CCO) to the frequency of the ML (optical local oscillator LO\textsubscript{opt}). The phase-sensitive detector (PSD) detects the phase error between the beat frequency and the LO\textsubscript{el} frequency. The error signal $\varphi_e(s)$ is fed back via the loop filter with transfer function $F(s)$ to the tunable SL. In lock, the frequency difference of the SL in respect to the ML is locked to the frequency of the LO\textsubscript{el} (BPF - bandpass filter). The optical part is shown dotted-red, the rest is electronics. Small-signal phase approximation in lock is blue.

3.3. Decisive parameters of optical phase-locked loop

The OPLL is a nonlinear control system, which can be linearized around the operating point in lock state for small phases $|\sin(\varphi) \approx \varphi|$ [28]. The decisive figures of merit are the open-loop gain $|G_{ol}(0)|$ and loop bandwidth $f_{ol}$. Within the bandwidth, the OPLL reduces differential phase noise and therefore generates phase relation (mutual coherence) between the two lasers in lock. For an open-loop transfer function $G_{ol}(s) = \varphi_o(s)/\varphi_e(s)$, the differential-phase-noise power spectral density (in lock state) can be estimated (for small phase noise) to [29]

$$S_{{2-s\text{-sided}}}^- (f) \approx \frac{1}{1 + G_{ol}(j2\pi f)} \left[ S_{\varphi}^{\text{ML,fr}} (f) + S_{\varphi}^{\text{SL,fr}} (f) \right]$$

with the phase-noise power spectral density of the free-running ML $S_{\varphi}^{\text{ML,fr}} (f)$, and the free-running SL $S_{\varphi}^{\text{SL,fr}} (f)$ (in steady-state $s = j2\pi f$). The contribution of the LO\textsubscript{el} is neglected since a signal generator with low phase noise is utilized.

Regarding variations of the heterodyne frequency carrier, the hold-in range $f_H$ defines the frequency range where the loop can track modulations of the local oscillators (ML or LO\textsubscript{el})

$$\Delta f_H = \pm \frac{1}{2\pi} \lim_{s \to 0} sG_{OL}(s) \approx \pm \frac{1}{2\pi} K_{\text{CCO}} K_{\text{PD}} 2\chi \sqrt{P_m P_i} K_{\text{PSD}} F(0)$$

with the DC gains $K_{\text{CCO}}$, $K_{\text{PD}}$, $K_{\text{PSD}}$ and the assumption of first-order-integrating CCO [28].

From Eqs. 11 and 12, it becomes clear that a high open-loop gain $|G_{ol}(0)|$ and wide bandwidth $f_{ol}$ are desirable, which results in a minimum differential-phase noise and large hold-in range in lock. These goals are, however, restricted by the stability criterion of a sufficient phase margin to 180° at loop gains > 1. Regarding intra-loop delays a wide loop bandwidth is demanding. Thus, any propagation length within the loop are to be reduced to a minimum to achieve a
broadband suppression of phase noise. Eq. 12 shows that a proper operation of the OPLL is dependent on the gains along the whole plant (SL → PD → PSD) and even the powers of the interfering beams. Additional to the requirement of modulation frequency within the hold-in range, the stable OPLL operation relies on modulation indices $M_{FM} < 2.4$ (first zero of $J_0$ Bessel line) [28]. At modulation frequencies near the loop bandwidth, error amplification (see servo bumps) occurs which impairs the stability of the OPLL. Thus, a wide loop bandwidth is beneficial (even for low-noise lasers) to be capable of a fast variations in the heterodyne carrier frequency for compensating dominant Doppler shifts.

For achieving the desired open-loop transfer behavior, the loop filter can be designed to correct the phase response by lead/lag compensation [28] (compare Fig. 2(a)). At MHz frequencies, the thermal cut-off frequency [30] in current-induced frequency variations of semiconductor lasers and further nonlinearities [27] impede wider loop bandwidths. Facing free-running lasers, a systematic approach of measuring the transfer function of the plant can be accomplished only with high efforts, since the instantaneous state is fairly near the set-point where linearization is possible. Thus, estimations of the open-loop transfer function $G_{OL}(s)$ in lock state have to be conducted.

3.4. Mutual coherence

The relation in Eq. 12 shows that the powers of the interfering beams are directly related to the open-loop transfer function $G_{OL}(s)$. Since the measurement power $P_m$ strongly varies with the reflectivity of the sample (typically over orders of magnitude), the laser-Doppler vibration signal on the beat PD would be filtered (within the OPLL bandwidth) or would induce loop instabilities. Thus, we propose a separate (vibrometer) PD to detect the phase fluctuations from the laser-Doppler effect at the vibrating sample. The measurement beam for the LDV is provided by the offset-locked slave laser beam. The master laser beam is utilized as reference beam.

The additional interference of the beams on the vibrometer PD has to consider the (generated) degree of mutual coherence between the (complex) EM waves of the locked lasers which is defined [26] over vibrometer delay $\tau_d = \Delta z/c$ as

$$g_{12}(\tau_d) = \frac{\langle E_m^* (t) E_r (t + \tau_d) \rangle}{\langle E_m (t) E_m^* (t) \rangle \langle E_r (t) E_r^* (t) \rangle} \propto \frac{\langle E_m^* (t) E_r (t + \tau_d) \rangle}{\sqrt{P_m P_r}}$$  \tag{13}

with the path difference $\Delta z$ between the position of the beat and the vibrometer detector. The ensemble average (symbolized by $\langle \cdot \rangle$) is equal to the temporal average for ergodic processes. The asterisk denotes the complex conjugate. The (complex) degree of mutual coherence is related to the heterodyne-frequency-carrier amplitude, since the interference efficiency (see Eq. 2) is

$$\chi \propto |g_{12}(\tau_d)|$$ \hspace{0.5cm} \tag{14}

if all other sources degradation of interference efficiency can be neglected (angular misalignment, spatial mutual coherence, state of mutual polarization etc.). The position of maximum temporal coherence is generated by the OPLL at the position of the beat PD. At this position, the maximum heterodyne-frequency-carrier amplitude is achieved. Any path difference $\Delta z$ to this position impairs the interference efficiency.

For many applications, a significant path difference between the PD positions might be inevitable (e.g. microscope). If the path difference exceeds certain limits, the degradation in temporal coherence leads to a collapse of the heterodyne carrier [31] to the summed linewidth of the (free-running) lasers. This effect of coherence collapse for LDV is subject to further research.
4. Setup of laser-Doppler vibrometer with heterodyning by OPLL

4.1. Realization of optical phase-locked loop

For our optical setup, we chose two visible distributed-Bragg-grating (DBR) GaAs semiconductor lasers (EYP-DBR-0633) which enable intrinsically modehop-free emission at of 632 nm with a wide tuning range of > 300 GHz. The tuning coefficient for injection current is approximately 700 MHz/mA and the injection-current modulation bandwidth is > 100 MHz. The free-running lasers show a summed linewidth of < 500 kHz. To achieve highest of optical output power, the set-point of both lasers was optimized with the knowledge of wavelength tuning behavior (over temperature and injection current) and optical output power. The wavelength mapping of both semiconductor lasers, was conducted with a wavelength meter (HighFinesse WS6-600). A commercial laser driver (Toptica SYS DC 100) stabilizes the current and temperature set-point of both semiconductor lasers which achieves reproducibility (after heat-up) of the frequency difference below ±100 MHz. For improved emission-wavelength stability, Faraday isolators protect each semiconductor laser from back-reflection from the experiment and fiber (Fig. 3).

For the PSD and loop filter, we chose commercial analog electronics (Toptica mFALC) allowing a maximum difference frequencies and, thus, heterodyne carrier frequencies of 200 MHz. The loop filter enables lead/lag compensation to a 4th-order OPLL which was optimized in experiment (Bode plot in Fig. 2(a)). The derivative behavior of the loop filter was set to a phase lead of +22° at 1.7 MHz to enhance the loop bandwidth to higher frequencies. We observed stable operation with a loop bandwidth of maximum 3 MHz. Parallel to the loop filter, a slow (unlimited) integrator controls the SL set-point to compensate for drift between the semiconductor lasers.

![Figure 2](image)

**Figure 2.** (a) Bode plot of the loop filter $F(s)$ optimized for the stabilization of the two DFB semiconductor lasers. The maximum phase lag is −56.7°. The derivative behavior creates a weak gain maximum at 10 MHz and a phase lead of +22° (dots are measurements, line is fitted model). (b) Beat spectrum at the beat photodetector of the 50 MHz heterodyne frequency carrier at low (grey) and high gain (black). The phase noise is reduced by the OPLL to a pedestal around the heterodyne carrier frequency. At critically-stable gain, noise peaks (servo bumps) at a frequency offset corresponding to loop bandwidth (here at 3 MHz) are developed.
In accordance with Eq. 11, the differential phase noise is suppressed within the loop bandwidth (Fig. 2(b)). This reduces the Lorentzian line-shape of the free-running semiconductor lasers to a pedestal around the heterodyne carrier frequency. For high gains, excessive noise peaks (servo bumps) appear at frequencies where the phase shift is nearly 180° (low phase margin) which occurs around at the loop bandwidth for stable OPLLs [31].

The stabilized heterodyne carrier frequency can be varied or modulated by the local oscillator up to the loop bandwidth in the MHz range. We observed tracking (without losing lock) of the LO frequency-modulated with sinusoidal deviations of $\Delta f \geq 100$ kHz at $f_{\text{mod}} = 1$ MHz (modulation index $M_{\text{FM}} = \Delta f / f_{\text{mod}} = 0.1$). Which is consistent with the concept of the hold-in frequency $\Delta f_{\text{H}}$ (Sec. 3.3).

![Figure 3. Setup of the LDV microscope with generation of heterodyne carrier frequency via offset-lock in an optical phase-locked loop (OPLL). As reference vibration measurement system a Polytec OFV-353S is coupled into the measurement path. DBR is semiconductor laser, APP is anamorphic prism pair, FI is Faraday isolator, PM-SMF is polarization-maintaining single-mode fiber, FS is fiber splitter, HWP is half-wave plate, QWP is quarter-wave plate, PBS is polarizing beam splitter, BS is beam splitter, Beat PD is beat photodetector, PS is power splitter, LO is local oscillator, PSD is phase-sensitive detector, Vib PD is vibrometer photodetector, PH is pinhole, DF is dichroic filter, MO is microscope objective, CAM is overview camera, LED is light-emitting diode, and CT is cross table.](image)

4.2. Heterodyne laser-Doppler vibrometer microscope

With the realized OPLL-LDV microscope (see Fig. 3) the position of the vibrometer detector is inevitably different from the beat detector. The path difference accumulates to $\Delta z \approx 1.5$ m or a vibrometer delay $\tau_d = \Delta z / c \approx 5$ ns (Sec. 3.4). Despite the expected carrier collapse, the carrier-to-noise ratio at both detectors is similar which is probably due to the large OPLL bandwidth. For good OPLL stability, an additional Faraday isolator protects the beat PD from strong specular reflections from the vibrating sample.
A scanning-microscope setup is deployed to acquire the two-dimensional (out-of-plane) operating deflection shape (ODS) of microscopic samples with sufficient lateral resolution. The capability of the LDV microscope (with the commercial vibrometer) was already demonstrated with energy-harvesting MEMS [32]. For good resolution, the measurement spot size on the vibrating sample has to be smaller than the nodal distance of an ODS (Nyquist criterion). Since the spot size is directly proportional to wavelength, we chose visible lasers at 632 nm for our LDV microscope (Fig. 3). For spatial filtering of the elliptical beam shape of the semiconductor lasers, a polarization-maintaining single-mode fiber is applied for the reference beam. To efficiently exploit the emitted power, we circularized the beams with anamorphic prism pairs. Spatial filtering of the measurement beam by fiber was relinquished to avoid negative influence on the stabilization. However, a confocal pinhole in the beam expander optics allows to spatially filter the measurement beam. Further, the fundamental transverse mode of the reference beam spatially filters the measurement beam by interference on the vibrometer PD. For the future goal of RF-MEMS testing at high frequencies, the lateral resolution limit of conventional microscopy needs to be overcome systematically. Therefore, we previously proposed the application of absorbance modulation in thin photochromic layers on the sample to achieve the necessary resolution enhancement [33, 34].

5. Results and discussion

5.1. Vibration measurements
To show the measurement capability of the OPLL-LDV, we conducted single-point vibration measurements on a piezoelectric disc (PI Ceramic PRYY+1119) at the radial and thickness resonance ($f_{rd} = 224$ kHz and $f_{th} = 11.4$ MHz). The theoretic thickness-resonance amplitude is $\hat{s}_{th} = 1.6$ nm (piezoelectric deformation coefficient $d_{33} = 0.4$ nm/V and excitation amplitude $u_{exc} = 4$ V at 50Ω). Since there are no vibration calibration objects in the MHz regime, a commercial LDV (Polytec OFV-353S) is coupled into the beam path of the OPLL-LDV as a reference vibrometric measurement system. The vibration signal directly at the vibrometer PD is analyzed with a spectrum analyzer (HP 8591E) assuming an ideal phase response in the observed frequency band. The raw-signal evaluation is further necessary, since the demodulation bandwidth of the associated decoder electronics (Polytec OFV-2500-2) is limited to 3.2 MHz. The frequency shift of the OFV-353S is 40 MHz. The OPLL-LDV was set to generate a static heterodyne frequency carrier at 50 MHz.

Since the wavelengths of both LDVs are similar (HeNe laser at 632.9 nm of OFV-353S) and the measurement points are aligned, the raw laser-Doppler signals after photodetection are comparable between the OPLL-LDV and the Polytec OFV. For the thickness resonance $f_{th}$, we measured a carrier-to-sideband ratio $CSR_{OFV} = (38 \pm 1)$ dBc and $CSR_{OPLL-LDV} = (40 \pm 2)$ dBc which corresponds to a vibration amplitude $\hat{s}_{th} = (1.2 \pm 0.2)$ nm (applying Eq. 6). The signal-to-noise ratio at this resonant frequency was approximately 48 dB (at 1 Hz resolution bandwidth) for the OPLL-LDV. The measured thickness-resonance amplitude at both LDVs showed a significant difference compared to theoretic value. The reason might be a lower piezo impedance, extra load of the solder drops, and/or slight frequency deviations from resonance.

For the vibration measurement of the radial resonance, a Bessel spectrum emerges around the heterodyne carrier frequency (Fig. 4(a)) in the photodetector spectrum of both LDVs. The calculated modulation index for the radial vibration at 224 kHz is $M_{vib}^{OFV} = 1.34 \pm 0.04$ and $M_{vib}^{OPLL-Vib} = 1.40 \pm 0.06$ (applying Eq. 7, averaged from eight Bessel lines). This modulation index correspond to a vibration amplitude of approximately $\hat{s}_{rd} \approx 70$ nm at the measurement point. The cause for the deviation between the LDV measurements is probably due to misalignment and different spatial resolution (measurement spot sizes).
Figure 4. (a) Vibration spectrum measured with OPLL-LDV of a piezoelectric disc vibrating at the radial resonance at $f_{rd} = 224$ kHz. The heterodyne carrier frequency of the OPLL-LDV was set to 50 MHz. (b) Spectral distribution of the (upper-sideband) relative noise level of the OPLL-LDV over frequency-offset from heterodyne carrier (inverse values correspond to SNR).

5.2. Vibration sensitivity
The differential phase noise of the heterodyne frequency carrier directly limits the vibration sensitivity of the OPLL-LDV (Eq. 11). Therefore, the vibration-amplitude sensitivity is usually expressed as noise-equivalent vibration amplitude $s_{ne}$ at 1 Hz resolution bandwidth (RBW). Applying narrow-band approximation, the noise-equivalent vibration amplitude (before demodulation) is deduced in analogy to Eq. 6 and the frequency offset from heterodyne carrier directly corresponds to the vibration frequency ($f_{vib} = f - f_c$), yielding

$$s'_{ne}(f - f_c) = \frac{\lambda}{2\pi\sqrt{\text{SNR}'(f - f_c)}}$$

with the signal-to-(single-sideband) noise ratio $\text{SNR}'$ at 1 Hz RBW. Note that this definition solely considers the unmodulated carrier which contains the same total power as any FM-signal with arbitrary modulation index $M_{vib}$.

With the assumption that phase noise is dominant over amplitude noise, the achieved (phase) noise level is estimated from the beat signal with a spectrum analyzer (shown in Fig. 4) [35]. For our realization, the pedestal around the heterodyne carrier frequency defines the sensitivity of the instrument for vibration frequencies $f_{vib} \leq 50$ MHz (see Fig. 4(b)). With the assumption of a Lorentzian line-shape (of the summed linewidth of the lasers) the pedestal slope is approximately $-30$ dB per decade (at frequency offsets larger than the summed laser linewidth). At vibration frequencies larger than $f_{vib} > 50$ MHz the noise level due to relative intensity noise (RIN) of both semiconductor lasers (at $-110$ dB/Hz) limits the vibration sensitivity. Thus, for our OPLL-LDV the heterodyne carrier frequency must be set to $f_c \geq 50$ MHz to achieve comparable phase-noise level at both sidebands.

The pedestal noise level is $\text{SNR}' \geq 60$ dB at offset frequencies lower than the loop bandwidth, which corresponds to a noise-equivalent vibration amplitude of $s'_{ne}(f_{vib} < 3$ MHz) $\leq$
100 pm/√Hz (applying Eq. 15). For the first four Bessel lines of the observed radial resonance, we obtained SNR' > 20 dB. At the thickness resonance \( f_{th} = 11.4 \text{ MHz} \), the SNR' was 90 dB. Thus, the vibration-amplitude sensitivity at this frequency is approximately \( s'_{ne}(f_{th}) \approx 5 \text{ pm/√Hz} \). The achieved sensitivity is, therefore, 60 dB worse compared to theoretic shot noise level \( P_m \approx 150 \mu \text{W} \) scattered power from piezo-disc surface.

For the improvement of the vibration sensitivity of our OPLL-LDV at frequencies below 50 MHz, increased OPLL gain is necessary (see Eq. 11) which is ambitious due to the broad linewidth of the DBR semiconductor lasers. Typically single-frequency semiconductor lasers and fiber lasers with infrared emission show significantly narrower linewidths. This smaller inherent phase-noise requires a lower loop bandwidth for heterodyning, which helps to increase the OPLL gain (without loss of stability). However, the lower spatial resolution due to the wavelength may be disadvantageous for microscopy. On the other hand, heterodyning with visible gas or solid-state lasers (e.g. HeNe) is typically limited in difference frequency and tuning capability. Thus, capable LDV designs with variable heterodyning by OPLL have to make a trade-off between available narrow-linewidth lasers, fast tracking, and high spatial resolution (in microscopic samples).

6. Conclusion and outlook
We showed the benefits of a variable heterodyne frequency carrier for laser-Doppler vibrometry generated by an offset-lock of two lasers by an optical phase-lock loop. The frequency of the local oscillator directly defines the offset-lock frequency and, thus, the heterodyne carrier frequency. Any variation in the heterodyne carrier frequency does not impair the alignment of the instrument which is the problem for conventional acoustooptic frequency shifters. The heterodyne carrier frequency can be modulated to compensate dominant Doppler shifts due to relative motions between the sample and the instrument, e.g. for air-born, vehicle-born or handheld applications. In principle, the OPLL can follow modulation frequencies lower than its loop bandwidth with modulation indices < 2.4.

As a possible realization, we demonstrated a laser-Doppler-vibrometer microscope with a variable heterodyne frequency carrier. Therefore, we employed commercial electronics and visible DBR semiconductor lasers emitting at 632 nm with a summed linewidth of < 500 kHz. A stable offset-lock with a loop bandwidth up to 3 MHz was achieved for heterodyne frequency carrier up to 200 MHz. For a heterodyne carrier at 50 MHz, the capability of the laser-Doppler vibrometer setup was evaluated by measurements on a piezoelectric disc at its resonances. This vibration measurements were verified by a commercial laser-Doppler vibrometer. For vibration frequencies below 50 MHz the pedestal of the residual (differential) phase-noise limits the vibration sensitivity of our vibrometer. The achieved vibration sensitivity at vibration frequencies below the loop bandwidth of 3 MHz is 100 pm/√Hz. Higher loop gain could increase the vibration sensitivity (up to the loop bandwidth) to the cost of loop stability.

Further improvements in sensitivity require the choice of laser sources with inherently narrower linewidth. The achieved vibration sensitivity in the low MHz regime is far away from shot-noise level which is reached by heterodyne LDVs with an acoustooptic frequency shifter and a single narrow-linewidth laser source. However, in applications, where compensation of large motions is desirable, the concept of heterodyning by OPLL enables flexible wideband-tracking capability to the disadvantage of lower vibration sensitivity.

Beyond the phase-noise pedestal (> 50 MHz), the relative intensity noise of the semiconductor lasers currently limits the vibration sensitivity. Thus, in the next steps we plan to increase the capability of the vibrometer microscope for generation of heterodyne frequency carriers in the GHz regime, where common acoustooptic frequency shifters are inefficient. Such laser-Doppler vibrometers can enable vibration detection at high frequencies for RF-MEMS testing.
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