Minimal Lepton Flavor Violation and Renormalization Group
Evolution of Lepton Masses and Mixing

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Abstract

We study the renormalization group equations (RGEs) of the neutrino parameters in models of Minimal Lepton Flavor Violation. In such models, the RGEs can be described in terms of flavor spurions, such that only the coefficients depend on the specific model. We explicitly demonstrate this method for the SM and MSSM for both Type-I and Type-III seesaw models. For that purpose, the RGEs of neutrino parameters in the MSSM Type-III seesaw have been computed. We have extended this method to get the evolution equations at second order. The implications for leptogenesis are also discussed.

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I. INTRODUCTION

The data from past and ongoing neutrino oscillation experiments, as well as from cosmology and astrophysics, have now confirmed that neutrinos have distinct masses and that the three neutrino flavors $\nu_e$, $\nu_\mu$, and $\nu_\tau$ mix among themselves to form the three mass eigenstates. The fact that the neutrinos are massive and mix implies non-conservation of lepton flavor. Hence, lepton flavor violating processes are expected in the lepton sector just as quark flavor violating processes arise in the quark sector.

In the quark sector of the Standard Model (SM), flavor violation is induced by the Yukawa matrices such that baryon number remains an exact symmetry. The fact that flavor changing neutral currents (FCNCs) are heavily suppressed puts stringent constraints on the possible structure of new degrees of freedom carrying flavor quantum numbers. These constraints can be satisfied either if new particles are very heavy or if flavor symmetries suppress the flavor changing couplings. One of the most predictive and restrictive symmetry principles that can be used is Minimal Flavor Violation (MFV) [1]. The MFV framework is the assumption that in the quark sector the only sources of flavor symmetry breaking are the Yukawa couplings.

While the idea of MFV has a straightforward and unique realization in the quark sector, the situation is different in the lepton sector. The reason is that the neutrinos can be Majorana particles, in which case total lepton number is no longer a symmetry of the theory. Due to this complication, the Minimal Lepton Flavor Violation (MLFV) hypothesis is not uniquely defined; there are two ways to define it [2]. In the first case, known as MLFV with minimal field content, we do not add any new field to the theory, and treat the neutrino mass terms as non-renormalizable terms. The only irreducible sources of lepton flavor violation are the charged-lepton Yukawa matrix and the effective left-handed Majorana mass matrix. The breaking of total Lepton Number (LN) is independent of the flavor violation and happens at some very high scale.

The other possibility, called MLFV with extended field contents (MLFV-ex), is to introduce new fields to the SM. In particular, three heavy right-handed neutrinos are added to the SM. Their Majorana mass term, which is assumed to be flavor universal, explicitly breaks LN. In this scenario, the two Yukawa matrices act as the only irreducible sources of flavor violation. In the MLFV-ex scenario, the low energy observables depend on the high energy parameters of the theory. For example, the FCNC constraints in the leptonic sector affects leptogenesis. This has been studied in [3] with the mass-splitting of the right-handed neutrinos, required for successful leptogenesis, being introduced from flavor symmetry considerations only. To have a complete understanding of the relation between the high energy parameters and the low energy observables, one needs to study the complete renormalization group (RG) evolution effects in this context. RG evolution has already been shown to have strong effects on leptogenesis [4]. Ref. [5] shows the stability of the MLFV under
RG evolution in the context of soft masses in the Minimal Supersymmetric Standard Model (MSSM). While [6] takes into account the RG evolution effects in the context of $\mu \to 3e$ and $\tau \to 3\ell$ decays, a general analysis of RG evolution of lepton masses and mixing parameters in the MLFV framework is still lacking.

In this paper, we consider the RG evolution of lepton masses and mixing parameters in the MLFV-ex scenario, with the SM as the low energy effective theory. The basic idea is that the RGEs can be written in terms of spurions that depend only on the Yukawa matrices. The coefficient of each term can be model dependent. Moreover, we assume that the universality of the Majorana masses is broken slightly, and hence treat the Majorana mass matrix as a spurion of our theory, and we get the RGE for this spurion as well. This is, in fact, a natural assumption as the universality is automatically broken in course of RG evolution. We show explicitly how one can write the RGEs for the SM and MSSM in both Type-I and Type-III seesaw models. The advantage of the spurion formalism is that it shows how each combination enters and can be used as a check for any MLFV model.

II. THE MODEL: $\nu$MSM AND MLFV WITH EXTENDED FIELD CONTENT

We consider the SM extended by three right-handed neutrinos, which are singlets under the SM gauge group. This model is referred to as the $\nu$MSM [7]. We also consider the case where they are triplet under the SU(2)$_L$ group later in this section.

We begin by considering the model excluding all mass terms of the leptons and gradually introduce mass terms to study their effect on the flavor symmetries of the theory, at different energy scales $\mu$. In the massless lepton limit, at high scale $\mu > M_R$, the $\nu$MSM enjoys a flavor symmetry $G^0_{\text{LF}}$, similar to that of the quark sector, given by

$$G^0_{\text{LF}} = \text{SU}(3)_{l_L} \otimes \text{SU}(3)_{e_R} \otimes \text{SU}(3)_{\nu_R}.$$  \hfill (2.1)

Here we consider only the non-Abelian part of the flavor symmetry group. This sector is also invariant under U(1) of hypercharge ($Y$), total lepton number (LN), as well as U(1)$_E$ (or U(1)$_\nu$), which corresponds to a rotation of the $e_R$ (or $\nu_R$) fields.

The presence of Majorana mass term for the right-handed neutrinos reduces the symmetry. Let us denote the right-handed neutrinos by $\nu^i_R$, $i \in \{1, 2, 3\}$. The only source of LN violation in this model is the Majorana mass term of these right-handed neutrinos given by

$$\mathcal{L}_{\text{Maj}} = -\frac{1}{2} \bar{\nu}^C_R M_\nu \nu_R + \text{h.c.},$$  \hfill (2.2)

where $C$ denotes charge conjugation. The right-handed Majorana mass matrix $M_\nu$ is symmetric, $M_\nu = M_\nu^T$. Furthermore, without any loss of generality, we can choose $M_\nu$ to be real by re-definition of the phases of $\nu^i_R$. (The $\nu$MSM was originally defined [7] in the basis where the charged lepton mass matrix and the Majorana mass matrix are real and diagonal.) In
general $M_{\nu}$ breaks SU(3)$_{\nu R}$ completely. For a universal mass matrix, however, the breaking is into an O(3) group. In this case, the Majorana mass matrix is given by

$$(M_{\nu})_{ij} = M_R \delta_{ij},$$

and the flavor symmetry group becomes

$$G_{\text{LF}}^0 \rightarrow G_{\text{LF}} = \text{SU}(3)_{l_L} \otimes \text{SU}(3)_{\epsilon_R} \otimes \text{O}(3)_{\nu_R}. \quad (2.4)$$

The two Yukawas $Y_e$ and $Y_{\nu}$ are given by

$$\mathcal{L}_{\text{Yukawa}} = -\bar{e}_R Y_e \phi^\dagger l_L - \bar{\nu}_R Y_{\nu} \tilde{\phi}^\dagger l_L + \text{h.c.}, \quad (2.5)$$

where $\phi$ is the SM Higgs doublet and $\tilde{\phi} = i\sigma^2 \phi^*$, $\sigma^2$ being the second Pauli matrix.

It is customary to treat $G_{\text{LF}}$ as an unbroken symmetry of the underlying theory which can be achieved by treating the Yukawa matrices as spurion fields with non-trivial quantum numbers under $G_{\text{LF}}$

$$Y_e \sim (\bar{3}, 3, 1), \quad Y_{\nu} \sim (\bar{3}, 1, 3). \quad (2.6)$$

The $\nu$MSM in the massless lepton limit and with universal right-handed Majorana masses enjoys the flavor symmetry $G_{\text{LF}}$ and this is the MLFV hypothesis with extended field content (MLFV-ex) [2]. Going beyond the MLFV-ex hypothesis, in this paper we choose the universality of $M_{\nu}$ to be slightly broken, which happens also as a result of RG evolution. We thus treat $M_{\nu}$ as a spurion transforming, under $G_{\text{LF}}$, as

$$M_{\nu} \sim (1, 1, 6). \quad (2.7)$$

The spurions have the following transformation properties:

$$Y_e \rightarrow U_R Y_e U_L^\dagger, \quad Y_{\nu} \rightarrow O_{\nu} Y_{\nu} U_L^\dagger, \quad M_{\nu} \rightarrow O_{\nu} M_{\nu} O_{\nu}^T, \quad (2.8)$$

where $U_L \in \text{SU}(3)_{l_L}$, $U_R \in \text{SU}(3)_{\epsilon_R}$ and $O_{\nu} \in \text{O}(3)_{\nu_R}$. This technique is known as spurion analysis.

Finally, the heavy fields generate small neutrino masses via the seesaw relation [8]

$$m_{\nu} = \frac{v^2}{2} Y_{\nu}^T M_{\nu}^{-1} Y_{\nu}, \quad (2.9)$$

where the vacuum expectation value of the SM Higgs is defined as $\langle \phi \rangle = (0, v/\sqrt{2})^T$. In the MLFV-ex model, the left-handed neutrino mass matrix is given by

$$m_{\nu}|_{\text{MLFV-ex}} = \frac{v^2}{2M_R} Y_{\nu}^T Y_{\nu}. \quad (2.10)$$

Note that in general $Y_{\nu}^T Y_{\nu}$ and $Y_{\nu}^\dagger Y_{\nu}$ are two different sources of $G_{\text{LF}}$ breaking. Only in the limit where $Y_{\nu}$ is real are they the same [2]. We do, however, expect to have CP violation in
the theory and thus we do not concentrate on the case of real $Y_\nu$. We consider the MLFV-ex model for $\mu > M_R$ energy regime for the rest of the paper, with the exception that the universality of $M_\nu$ is assumed to be slightly broken. We consider the case where $M_R$ is large compared to the electroweak symmetry breaking scale. This ensures that $U(1)_{LN}$ is broken at some high scale, and that, in general, the breaking of LN by the Majorana mass term is independent of $G_{LF}$-violation.

Next, we discuss the effective theory below $M_R$, or equivalently below the scale of the lightest of the heavy right-handed neutrinos, when universality is broken. In this regime, all the three heavy right-handed neutrinos get integrated out, and as a result the flavor symmetry group reduces to

$$G_{LF} \rightarrow G'_{LF} = SU(3)_L \otimes SU(3)_e_R.$$  \hspace{1cm} (2.11)

In this energy region, the dimension-5 non-renormalizable term in the Lagrangian responsible for the LN-violating left-handed Majorana neutrino masses is of the form

$$\mathcal{L} \sim l_L C L m_\nu l_L \phi \phi.$$  \hspace{1cm} (2.12)

There are two sources of $G'_{LF}$ breaking in this case. The charged lepton Yukawa $Y_e$ and the left-handed neutrino mass $m_\nu$ that transform as

$$Y_e \sim (\bar{3}, 3), \quad m_\nu \sim (6, 1).$$  \hspace{1cm} (2.13)

Thus, the model becomes equivalent to the MLFV hypothesis with minimal field content \cite{2}. In this case, $m_\nu$ remains the only relevant quantity that contains the high energy information of the neutrino parameters, which in turn can be extracted by the measurement of the neutrino masses and mixing parameters. Hence, the effect of RG evolution becomes an important factor to be taken into account, which we will be studying in the following sections.

In the framework of spurion analysis, $G_{LF}$ is broken by the background values of the spurions. We consider the background values of $Y_{e,\nu}$ to be small, the largest one being experimentally measured to be $Y_\tau \sim 0.01$ at the scale $M_Z$. Thus, we can use perturbation theory and consider only the leading order corrections. To first order, the operators responsible for the breaking of $G_{LF}$ are combinations of two Yukawa matrices, that is, working at one loop is equivalent of considering spurions with two couplings. There are several combinations of couplings that can appear in the result. These couplings and their transformation properties are given in Table \text{II}. As can be seen, $M_\nu$ appears only when we consider the evolution of $M_\nu$ itself.

The flavor symmetry structure is more complicated when the heavy neutrinos are not exactly degenerate. A breaking of the universality of $M_\nu$, however small, is also necessary for leptogenesis as has been shown in \cite{3}. In that paper, the degeneracy is broken by appropriate combinations of spurions in the MLFV-ex scenario. Our assumption is that the
Combination of spurions | Transformation  
--- | ---  
$Y_e^\dagger Y_e$ | $(8 \oplus 1, 1, 1)$  
$Y_e Y_e^\dagger$ | $(1, 8 \oplus 1, 1)$  
$Y_e^\dagger Y_\nu$ | $(8 \oplus 1, 1, 1)$  
$Y_\nu^\dagger Y_\nu$ | $(1, 1, 8 \oplus 1)$  
$\text{Tr}[Y_e^\dagger Y_e] = \text{Tr}[Y_e^\dagger Y_e]$ | $(1, 1, 1)$  
$\text{Tr}[Y_\nu^\dagger Y_\nu] = \text{Tr}[Y_\nu^\dagger Y_\nu]$ | $(1, 1, 1)$  
$T_e \equiv Y_e^\dagger Y_e - \frac{1}{3} \text{Tr}[Y_e^\dagger Y_e] I_3$ | $(8, 1, 1)$  
$T_e' \equiv Y_e Y_e^\dagger - \frac{1}{3} \text{Tr}[Y_e Y_e^\dagger] I_3$ | $(1, 8, 1)$  
$T_\nu \equiv Y_\nu^\dagger Y_\nu - \frac{1}{3} \text{Tr}[Y_\nu^\dagger Y_\nu] I_3$ | $(8, 1, 1)$  
$T_\nu' \equiv Y_\nu Y_\nu^\dagger - \frac{1}{3} \text{Tr}[Y_\nu Y_\nu^\dagger] I_3$ | $(1, 1, 8)$

**TABLE I:** Transformations of combinations of two spurion fields under $G_{LF}$. We have used the SU(3) algebra $3 \otimes \overline{3} = 8 \oplus 1$.

amount of non-degeneracy is small and $G_{LF}$ is still the flavor symmetry of the underlying theory. The effect of the breaking is due to the fact that running at the scale in between the three masses is not described by any of the two regions we discussed above. Yet, if the breaking is small this running is not significant and integrating out all the neutrinos together is a good approximation. Moreover, if the degeneracy is lifted due to RG evolution, then taking it into account is formally a higher order effect.

### III. RG EVOLUTION OF NEUTRINO PARAMETERS

We now study the effect of RG running. At energy scales above $M_R$, the quantities of interest are the Yukawa matrices $Y_{e,\nu}$, and the right-handed neutrino mass matrix $M_\nu$. Below, we see how they run.

In all our discussions, we consider only one loop running. In term of spurions, each loop add two Yukawa terms, and thus working at one loop is done by using only terms that have two Yukawa couplings more than the tree level one. The evolution equations at second order are discussed in Appendix B.

#### A. RG evolution of $Y_e$

We define

$$\dot{Y}_e \equiv \frac{dY_e}{dt}, \quad t \equiv \frac{\ln(\mu/\mu_0)}{16\pi^2}. \quad (3.1)$$
Here $\mu_0 (> M_R)$ is some high energy scale at which we start running and the factor $(16\pi^2)$ appears because of the fact that we consider radiative corrections at 1-loop.

Under the flavor symmetry group $G_{LF}$, $Y_e$ transforms as $(\bar{3}, 3, 1)$ and so does $\tilde{Y}_e$. Hence $\tilde{Y}_e$ can be expressed as appropriate combinations of the spurion fields transforming as $(\bar{3}, 3, 1)$. Table I shows the combinations of two spurion fields with their transformation properties. Using the SU(3) algebra

$$8 \otimes \bar{3} = \overline{15} \oplus 6 \oplus \bar{3}, \quad 8 \otimes 3 = 15 \oplus \bar{6} \oplus 3,$$

and we can write

$$Y_e T_e = (\bar{3}, 3, 1) \otimes (8, 1, 1) \ni (\bar{3}, 3, 1), \quad (3.3)$$

$$Y_\nu T_\nu = (\bar{3}, 3, 1) \otimes (8, 1, 1) \ni (\bar{3}, 3, 1), \quad (3.4)$$

$$Y_e \text{Tr}[Y_e^\dagger Y_e] = (\bar{3}, 3, 1) \otimes (1, 1, 1) = (\bar{3}, 3, 1), \quad (3.5)$$

$$Y_\nu \text{Tr}[Y_\nu^\dagger Y_\nu] = (\bar{3}, 3, 1) \otimes (1, 1, 1) = (\bar{3}, 3, 1). \quad (3.6)$$

The above combinations are the only terms, containing three spurion fields, allowed to appear on the right-hand side (RHS) of the RGE for $\tilde{Y}_e$. $T_e Y_e$ gives the same term as that given by $Y_e T_e$ and so has not been listed separately. Thus, at 1-loop, when terms up to combinations of three spurion fields are allowed, the most general form of $\tilde{Y}_e$ is given by

$$\tilde{Y}_e = \tilde{a}_1 Y_e T_e + \tilde{a}_2 Y_\nu T_\nu + \tilde{a}_3 Y_e \text{Tr}[Y_e^\dagger Y_e] + \tilde{a}_4 Y_e \text{Tr}[Y_\nu^\dagger Y_\nu] + a_5 Y_e$$

$$= Y_e (a_1 Y_e^\dagger Y_e + a_2 Y_\nu^\dagger Y_\nu + a_3 \text{Tr}[Y_e^\dagger Y_e] + a_4 \text{Tr}[Y_\nu^\dagger Y_\nu] + a_5 \mathbf{1}_3), \quad (3.7)$$

where $a_1$, $a_2$, $a_3$, $a_4$ and $a_5$ are expected to be numbers of $O(1)$ that can be determined by the calculation of the 1-loop diagrams in the theory.

The case of $a_5$ is a bit more involved since it is a function independent of spurion fields. Thus $a_5$ must contain combinations of other couplings in the theory that transform trivially under $G_{LF}$. The couplings that we have in the theory are the gauge couplings, $g_i$, the Higgs self-coupling, $\lambda$, and the quark Yukawas $Y_{U,D}$. Since leptons are singlets under SU(3)$_C$, $g_3$ cannot contribute. Moreover, at 1-loop the Higgs self-coupling cannot contribute either. Terms proportional to $g_1$ and $g_2$ contributing to $a_5$ must be of form

$$a_{g_1} g_1^2 + a_{g_2} g_2^2. \quad (3.8)$$

The singlet combination made of the quark Yukawas $Y_{U,D}$ is of the form $\text{Tr}[Y_i^\dagger Y_i]$, and the most general form of the quark Yukawa contributions to $a_5$ is

$$a_U \text{Tr}[Y_U^\dagger Y_U] + a_D \text{Tr}[Y_D^\dagger Y_D]. \quad (3.9)$$

Thus the general form of $a_5$ is given by

$$a_5 = a_{g_1} g_1^2 + a_{g_2} g_2^2 + a_U \text{Tr}[Y_U^\dagger Y_U] + a_D \text{Tr}[Y_D^\dagger Y_D] \quad (3.10)$$
FIG. 1: The self-energy diagram of the Higgs $\phi$ with complete fermion loop, where the fermion pair $\{f_L, f_R\}$ can be $\{l_L, e_R\}$, $\{q_L, u_R\}$ or $\{q_L, d_R\}$ producing contributions proportional to $\text{Tr}[Y^\dagger_e Y_e]$, $\text{Tr}[Y^\dagger_\nu Y_\nu]$, $\text{Tr}[Y^\dagger_U Y_U]$ and $\text{Tr}[Y^\dagger_D Y_D]$ respectively.

and the general form of $\dot{Y}_e$ becomes

$$
\dot{Y}_e = Y_e \left( a_1 Y_e^\dagger Y_e + a_2 Y_\nu^\dagger Y_\nu \right) + Y_e \left( a_{g_1} g_1^2 + a_{g_2} g_2^2 \right) \\
+ Y_e \left( a_3 \text{Tr}[Y^\dagger_e Y_e] + a_4 \text{Tr}[Y^\dagger_\nu Y_\nu] + a_U \text{Tr}[Y^\dagger_U Y_U] + a_D \text{Tr}[Y^\dagger_D Y_D] \right).
$$

Terms proportional to $\text{Tr}[Y^\dagger_x Y_x]$ ($x \in \{e, \nu, U, D\}$) arise from a complete fermion loop in the self-energy correction of the scalar Higgs boson, as shown in Fig. I. Since quarks come in three colors, one gets

$$
a_3 : a_4 : a_U : a_D = 1 : r : 3 : 3,
$$

where each of the three quark colors contributes equally. $r \equiv a_4/a_3$ is determined by the transformation properties of the right-handed neutrinos under the gauge group. As we discuss below, at Eq. (4.3), for singlets $r = 1$, while for triplets $r = 3$. We can now define a quantity

$$
T \equiv \text{Tr}[Y^\dagger_e Y_e] + r \text{Tr}[Y^\dagger_\nu Y_\nu] + 3 \text{Tr}[Y^\dagger_U Y_U] + 3 \text{Tr}[Y^\dagger_D Y_D],
$$

and write $\dot{Y}_e$ in a simpler form as

$$
\dot{Y}_e = Y_e \left( a_1 Y_e^\dagger Y_e + a_2 Y_\nu^\dagger Y_\nu \right) + Y_e \left( a_T T + a_{g_1} g_1^2 + a_{g_2} g_2^2 \right),
$$

where $a_T$, $a_{g_1}$, $a_{g_2}$, are expected to be of $\mathcal{O}(1)$.

### B. RG evolution of $Y_\nu$

Next, we discuss the running of $Y_\nu$. Since $Y_\nu$ transforms as $(\bar{3}, 1, 3)$ under $G_{\text{LF}}$, so must be $\dot{Y}_\nu$. From Table II and using Eq. (3.2) we obtain that the only allowed combinations of spurion fields at one loop order are

$$
Y_\nu T_e = (\bar{3}, 1, 3) \otimes (8, 1, 1) \ni (\bar{3}, 1, 3),
$$

$$
Y_\nu T_\nu = (\bar{3}, 1, 3) \otimes (8, 1, 1) \ni (\bar{3}, 1, 3),
$$

$$
Y_\nu \text{Tr}[Y^\dagger_e Y_e] = (\bar{3}, 1, 3) \otimes (1, 1, 1) = (\bar{3}, 1, 3),
$$

$$
Y_\nu \text{Tr}[Y^\dagger_\nu Y_\nu] = (\bar{3}, 1, 3) \otimes (1, 1, 1) = (\bar{3}, 1, 3).
$$
\( T'_{\nu} Y_{\nu} \) gives the same term as that given by \( Y_{\nu} T_{\nu} \) and so has not been written here. Finally, as with \( \dot{Y}_e \), we can write

\[
\dot{Y}_\nu = \tilde{b}_1 Y_e T_e + \tilde{b}_2 Y_\nu T_\nu + \tilde{b}_3 Y_e \text{Tr}[Y_\nu Y_e] + \tilde{b}_4 Y_\nu \text{Tr}[Y_\nu Y_\nu] + \tilde{b}_5 Y_\nu , \tag{3.19}
\]

which can be simplified, using a similar approach to that of the previous section, to get

\[
\dot{Y}_\nu = Y_\nu (b_1 Y_\nu T_e + b_2 Y_\nu Y_\nu) + Y_\nu (b_T T + b_{g_1} g_1^2 + b_{g_2} g_2^2) , \tag{3.20}
\]

where \( T \) is defined in Eq. \( 3.13 \), and \( b_1, b_2, b_T, b_{g_1}, b_{g_2} \), are expected to be of \( \mathcal{O}(1) \).

\section*{C. RG evolution of the heavy right-handed Majorana mass \( M_\nu \)}

Once we know the evolution of the Yukawa matrices, we can discuss the running of the physical masses. We consider the right handed neutrino mass term

\[
L_{\text{Maj}} = -\frac{1}{2} \left[ \overline{\nu}_R M_\nu \nu_R + \overline{\nu}_R M_\nu^i \nu_R^i \right] . \tag{3.21}
\]

We first discuss the evolution of \( M_\nu \) below and later consider \( M_\nu^i \).

As already stated, \( M_\nu \) transforms as \((1,1,6)\) under \( G_{\text{LF}} \) and thus is symmetric under \( O(3)_{\nu_R} \). Hence while considering the RG evolution of \( M_\nu \), the RHS must contain terms which has the same transformation properties under \( G_{\text{LF}} \). Using the transformation rules in Table II and the SU(3) algebra

\[
6 \otimes 8 = 24 \oplus 15 \oplus 6 \oplus \bar{3} , \tag{3.22}
\]

the allowed terms are obtained to be

\[
M_\nu T_\nu' = (1, 1, 6) \otimes (1, 1, 8) \ni (1, 1, 6) , \tag{3.23}
\]

\[
M_\nu \text{Tr}[Y_\nu Y_\nu] = (1, 1, 6) \otimes (1, 1, 1) = (1, 1, 6) , \tag{3.24}
\]

\[
M_\nu \text{Tr}[Y_\nu Y_\nu'] = (1, 1, 6) \otimes (1, 1, 1) = (1, 1, 6) . \tag{3.25}
\]

The quark Yukawas, \( Y_{U,D} \), are expected to have contributions of form \( \text{Tr}[Y_i Y_i^\dagger] \) \( (i \in \{ U, D \}) \). In general, there will also be terms containing \( g_i^2 \) and \( \lambda \).

To get the final form of the \( Y_\nu \) dependence of \( \dot{M}_\nu \) we have to take into account the fact that \( M_\nu \) is symmetric. Symmetrizing we obtain

\[
\frac{1}{2} \left[ (M_\nu Y_\nu Y_\nu^\dagger)^{\alpha\beta} + (M_\nu Y_\nu Y_\nu^\dagger)^{\beta\alpha} \right] + \frac{1}{2} \left[ (M_\nu)^{\alpha\beta} \text{Tr}[Y_\nu Y_\nu] + (M_\nu)^{\beta\alpha} \text{Tr}[Y_\nu Y_\nu^\dagger] \right] = \frac{1}{2} \left[ (M_\nu Y_\nu Y_\nu^\dagger)^{\alpha\beta} + ((Y_\nu Y_\nu^\dagger)^T M_\nu)^{\alpha\beta} \right] + (M_\nu)^{\alpha\beta} \text{Tr}[Y_\nu Y_\nu^\dagger] ,
\]
where $\alpha, \beta$ are $O(3)_{\nu R}$ indices. We can then write the most general form of the RG equation for $M_\nu$ as

$$
\dot{M}_\nu = \frac{q_1}{2} \left[ M_\nu (Y_\nu Y_\nu^\dagger) + (Y_\nu Y_\nu^\dagger)^T M_\nu \right] + M_\nu \left( q_T T + q_{g_1} g_1^2 + q_{g_2} g_2^2 + q_{g_3} g_3^2 + q_\lambda \lambda \right),
$$

(3.26)

where $T$ is given by Eq. (3.13). All of $q_i$s are expected to be of $O(1)$. As already discussed, the trace term $T$ can appear only through Higgs interactions, and so it cannot be present here since $M_\nu$ does not couple to Higgs rendering $q_T = 0$. Moreover, since the added lepton fields $\nu^i_R$ are singlets under $U(1)_Y$ and $SU(3)_C$, $q_{g_1} = q_{g_3} = 0$. At this order, $\lambda$ dependence cannot appear either making $q_\lambda = 0$. So we are left with

$$
\dot{M}_\nu = \frac{q_1}{2} \left[ M_\nu (Y_\nu Y_\nu^\dagger) + (Y_\nu Y_\nu^\dagger)^T M_\nu \right] + q_{g_2} g_2^2 M_\nu.
$$

(3.27)

Here, we keep the $g_2^2$ dependence to get the general form of $\dot{M}_\nu$ for right-handed neutrino extended models with $G_{\text{LF}}$ flavor symmetry. For MLFV-ex, where the right-handed neutrinos are singlets under $SU(2)_L$, we have $q_{g_2} = 0$. It should also be noted that if we use the universality of $M_\nu$ as an initial condition in Eq. (3.27), when $G_{\text{LF}}$ is broken by the small background values of the spurion field $Y_\nu$, the universality of the Majorana mass matrix is also broken as $Y_\nu$ has non-zero off-diagonal entries in general. However, the breaking is small and we can still consider $G_{\text{LF}}$ as the flavor symmetry of the theory in the massless lepton limit and perform the spurion analysis.

Let us now consider the term containing $M_\nu^\dagger$, that involves the left-handed fields. Writing the indices explicitly, for a general $M_\nu$ matrix, we get that the mass term associated with the left-handed fields is $(M_\nu^*)_{\alpha\beta}$, instead of $(M_\nu)^{\alpha\beta}$ for the right-handed fields, and thus the allowed terms are

$$
T_\nu^T M_\nu = (1, 1, 8) \otimes (1, 1, 6) \ni (1, 1, 6),
$$

(3.28)

$$
\text{Tr}[Y_\nu^T Y_\nu] M_\nu = (1, 1, 1) \otimes (1, 1, 6) = (1, 1, 6),
$$

(3.29)

$$
\text{Tr}[Y_\nu^T Y_\nu] M_\nu = (1, 1, 1) \otimes (1, 1, 6) = (1, 1, 6).
$$

(3.30)

Hence after symmetrization the evolution equation of the right-handed neutrino mass has a Dirac structure and is given by

$$
\dot{M}_\nu = \frac{q_1}{2} \left[ \left( M_\nu (Y_\nu Y_\nu^\dagger) + (Y_\nu Y_\nu^\dagger)^T M_\nu \right) P_R + \left( (Y_\nu Y_\nu^\dagger) M_\nu + M_\nu (Y_\nu Y_\nu^\dagger)^T \right) P_L \right] + q_{g_2} g_2^2 M_\nu.
$$

(3.31)

Eq. (3.31) is the most general form of $M_\nu$ evolution, as obtained by loop diagram calculations in [9, 10].

**D. RG evolution of the left-handed Majorana mass $m_\nu$.**

At energy scales above $M_R$, the light left-handed neutrino mass $m_\nu$ is generated through the seesaw relation and hence the RG evolution of $m_\nu$ will be obtained through that of $Y_\nu$
and $M_\nu$, as given in Eqs. (3.20) and (3.27). Using the seesaw relation given in Eq. (2.9) and considering the fact that $(M_\nu^{-1})^{\alpha}\gamma(M_\nu)_{\beta\gamma} = \delta^{\alpha}_{\beta}$, we see that to get the RG evolution equation for $(m_\nu)^{ij}$, $i, j$ being the SU(2)$_L$ indices, one needs the evolution of $(M_\nu)_{\alpha\beta}$, i.e. the left-chiral projection of the RG evolution of $M_\nu$, which can be read off from Eq. (3.31). Finally, the evolution equation for $m_\nu$ is given by

$$
\dot{m}_\nu = m_\nu P + P^T m_\nu + p m_\nu ,
$$

where

$$
P = b_1 Y_e^\dagger Y_e + \left(b_2 - \frac{q_1}{2}\right) Y_\nu^\dagger Y_\nu ,
$$

$$
p = 2 \left(b_T T + b_{g_1} g_1^2 + b_{g_2} g_2^2\right) - q_{g_1} g_2^2 .
$$

Note that the RHS of the equation is symmetric under SU(3)$_L\otimes$SU(3)$_L$, as required. All of $b_{1,2}, b_T, b_{g_1,2}$ and $q_1$ are given below for the cases of Type-I and Type-III seesaw in SM and MSSM.

E. RG evolution at energies below $M_R$

To complete the discussion of RG evolution of the different quantities that are needed in order to have a complete description of all leptonic parameters at all energy scales, we now construct the RG evolution equations for $\mu < M_R$. In this regime the flavor symmetry is

$$
G'_{LF} \equiv \text{SU}(3)_{T_L} \otimes \text{SU}(3)_{e_R} ,
$$

and the Yukawa coupling, $Y_e(\bar{3}, 3)$, and the left-handed Majorana mass, $m_\nu(6, 1)$, are the only spurion fields. The RG evolution equation for $Y_e$ can be obtained following the procedure given in the last subsection to be

$$
\dot{Y}_e = Y_e \left( a_1 Y_e^\dagger Y_e + a_T T' + a_{g_1} g_1^2 + a_{g_2} g_2^2 \right) ,
$$

where

$$
T' = \text{Tr}[Y_e^\dagger Y_e] + 3 \text{Tr}[Y_U^\dagger Y_U] + 3 \text{Tr}[Y_D^\dagger Y_D] ,
$$

and $a_s$ are expected to be of $\mathcal{O}(1)$ as before.

In the low energy regime $m_\nu$ is an effective neutrino mass operator and its RG evolution is not given by Eq. (3.32). To determine the structure of the RG evolution equation for the left-handed Majorana mass $m_\nu$, we proceed in the same way as in case of $M_\nu$, keeping in mind the change in the chirality. Table I and the transformation rule in Eq. (3.22) can be used to determine the allowed combinations of $m_\nu$ and $Y_e$ that can appear on the RHS of $\dot{m}_\nu$ and those are $m_\nu T_e$ and $m_\nu \text{Tr}[Y_e^\dagger Y_e]$. Symmetrized over the SU(3)$_L$ indices, the most general form of $\dot{m}_\nu$, keeping 1-loop spurion contributions, is

$$
\dot{m}_\nu = \frac{p}{2} \left( m_\nu T_e + (m_\nu T_e)^T \right) + p_e \text{Tr}[Y_e^\dagger Y_e] + m_\nu \left( p_U \text{Tr}[Y_U^\dagger Y_U] + p_D \text{Tr}[Y_D^\dagger Y_D] + p_{g_1} g_1^2 + p_{g_2} g_2^2 + p_{\lambda} \lambda \right) ,
$$

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which can be simplified to
\begin{equation}
\dot{m}_\nu = \frac{p_1}{2} \left( m_\nu Y_e^\dagger Y_e + (Y_e^\dagger Y_e)^T m_\nu \right) + m_\nu (p_T T' + p_{g_1} g_1^2 + p_{g_2} g_2^2 + p_\lambda \lambda),
\end{equation}
(3.39)
where $T'$ has been defined in Eq. (3.37). As before, we have considered the SU(3)$_C$ charges of the quarks in fixing $p_{U,D}$ and writing $T'$. Here $p_i$s are the $O(1)$ numbers and we have used the fact that $m_\nu$ is symmetric under SU(3)$_{l_L}$.

IV. RESULTS

To illustrate the RGEs obtained in Section III using spurion analysis, we compare the coefficients with the evolution equations obtained by exact calculations in four different models. These models are the extended SM and MSSM, where the right-handed neutrinos can be singlets (Type-I seesaw [9, 11, 12]) or triplets (Type-III seesaw [10]).

A. Right-handed neutrino extended SM

Let us first consider the case of the SM extended with three right-handed neutrinos. There can be only two possibilities: the first option is when the right-handed neutrinos are singlets under the gauge group which is known as Type-I seesaw. The other option, known as Type-III seesaw, is when the neutrinos are triplets under SU(2)$_L$ and singlet under the remaining SU(3)$_C \times U(1)_Y$. Note that for Type-II seesaw [13] as well as Inverse seesaw [14], the flavor group and the spurions present in the theory are not identical to the above cases and cannot be treated as a realization of the case discussed here.

In the general case of Type-I and Type-III seesaw, each of the right-handed neutrinos can be expressed as
\begin{equation}
\nu_R = \sum_{a=1}^{N} \nu_R^a G^a, \quad (4.1)
\end{equation}
with
\begin{align}
G^a &\equiv \mathbb{1}, \quad N = 1 \quad \text{for Type-I seesaw}, \\
G^a &\equiv \sigma^a, \quad N = 3 \quad \text{for Type-III seesaw},
\end{align}
(4.2)
where $\sigma^a$ represent the Pauli matrices. Note that we work in three different spaces. The flavor index, $f = e, \mu, \tau$, is suppressed. There is also the internal SU(2) index of the Pauli matrices that we suppress here and in the rest of the paper. In the following we often get quantities that are universal in that index. Last, the explicit index $a$ that runs from 1 to $N$.

With the above definition, we can write $r \equiv a_4/a_3$ in Eq. (3.13) as
\begin{equation}
r = \sum_{a=1}^{N} \epsilon^T G^a G^a \epsilon = (1, 3), \quad (4.3)
\end{equation}
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where \( \epsilon \equiv i \sigma^2 \). The two numbers in the parenthesis are the values in Type-I and Type-III seesaws, and are universal in the SU(2) spaces.

The quantities that appear in the coefficients of \( \dot{Y}_e, \dot{Y}_\nu \) and \( \dot{M}_\nu \), and depend on the representation of the right-handed neutrinos are

\[
\alpha_1 = \sum_{a=1}^{N} G^a G^a = (1, 3),
\]

\[
\alpha_2 = \text{Tr}[G^a G^a] = (2, 2),
\]

\[
\alpha_3 = \sum_{a=1}^{N} G^a \epsilon G^a \epsilon = (-1, 3),
\]

\[
\alpha_4 = (\epsilon^T G^a)^T (\epsilon^T G^a)^{-1} = (-1, 1),
\]

\[
\alpha_5 = \sum_{a,b=1}^{N} (i \epsilon^{bac} G^a T \epsilon G^b) (\epsilon^T G^c)^{-1} = (0, -2),
\]

where \( \epsilon^{bac} \) is the completely anti-symmetric tensor in SU(2) indices and no summation convention has been used.

Let us now discuss the origin of \( \alpha_i \)s. \( \alpha_1 \) comes from the self-energy correction of \( l_L \), while \( \alpha_2 \) appears in the self-energy correction of \( \nu_R \). \( \alpha_3 \) comes in the correction of the vertex containing \( Y_e \), while \( \alpha_4 \) is present in the correction of the \( Y_\nu \) vertex. \( \alpha_5 \) appears in the vertex correction of \( Y_\nu \) because of SU(2)_L interactions. In the case of right-handed neutrino extended SM, self-energy, mass and vertex corrections contribute to the running of the Yukawa couplings \( Y_e, \nu \). Hence, \( \alpha_1 \) is expected to contribute to both \( \dot{Y}_e \) and \( \dot{Y}_\nu \), while \( \dot{Y}_e \) should contain \( \alpha_3 \) as well. \( \alpha_4 \) and \( \alpha_5 \) must appear in \( \dot{Y}_\nu \). As already discussed, these quantities do not appear in \( \dot{m}_\nu \) in the regime \( \mu < M_R \), since the right-handed neutrinos are already decoupled.

Let us now consider the coefficients \( a_{1,2}, a_T \) and \( a_{g_1, g_2} \) arising in \( \dot{Y}_e \) in Eq. (3.14). Collecting all the contributions, we get the coefficients in Eq. (3.14) to be [9, 10]

\[
a_1 = \frac{3}{2}, \quad a_2 = \frac{\alpha_1}{2} + 2\alpha_3, \quad a_T = 1, \quad a_{g_1} = \left( -\frac{3}{4} - 3 \right) \times \frac{3}{5}, \quad a_{g_2} = -3 \times \frac{3}{4}. \quad (4.9)
\]

The first term in \( a_{g_1} \) arises through the self-energy correction of Higgs field \( \phi \), which also contributes to \( a_{g_2} \). Here we have used GUT normalization for U(1)_Y charges and hence a factor of \( (3/5) \) comes with \( g_1^2 \). The coefficients appearing in the RG evolution equation of \( Y_\nu \) in Eq. (3.20) can also be obtained in a similar way and we have [9, 10]

\[
b_1 = \frac{1}{2} + 2\alpha_4, \quad b_2 = \frac{1}{2} (\alpha_1 + \alpha_2), \quad b_T = 1, \quad b_{g_1} = -\frac{3}{4} \times \frac{3}{5}, \quad b_{g_2} = -3 \times \frac{3}{4} + 3\alpha_5. \quad (4.10)
\]

The values of \( a_i \) and \( b_i \) in Type-I and Type-III seesaw scenarios are tabulated in Table II. As can be seen from the table, for Type-I seesaw in the extended SM model the coefficients are
TABLE II: Coefficients appearing in the RG evolution of $Y_e$, $Y_\nu$, $M_\nu$ and $m_\nu$ in the SM and MSSM, in case of Type-I and Type-III seesaw \[9–12\]. For the extended MSSM, $a_T$ is the coefficient of $T_D$, while $b_T$ and $p_T$ are of $T_U$ and $T'_U$, respectively.

$O(1)$ numbers, as expected. In the case of the Type-III seesaw, we see that there are numbers which are larger than $O(1)$, for example $a_2$ and $b_{g_2}$. Let us now try to understand the origin of these large numbers. The largest contribution to $a_2$ comes from the $\alpha_3$ in Eq. (4.9), which arises through the vertex correction due to right-handed triplets and a factor of three is expected. Thus, the relevant number which we expect to be of $O(1)$ is $(a_2/3)$. Moreover, the right-handed neutrino triplets have interactions with the SU(2)$_L$ gauge bosons over the singlets, and so we expect $b_{g_2}$ in the Type-III case to have a factor of six over $b_{g_2}$ in Type-I.

Let us now discuss the coefficients $q_1$ and $q_{g_2}$ appearing in the running of $M_\nu$. The coefficients are given by

$$q_1 = \alpha_2, \quad q_{g_2} = (0, -12),$$

where $\alpha_2$ is defined in Eq. (4.5) and is of $O(1)$. For Type-I seesaw, the right-handed neutrinos are singlets of SU(2)$_L$ and so $q_{g_2} = 0$, while for Type-III seesaw one gets by exact calculations
\[ q_{g_2} = -12 \] and \( (q_{g_2}/6) \) is of \( \mathcal{O}(1) \), as discussed earlier.

Last, we consider the evolution of the effective left-handed Majorana neutrino mass \( m_\nu \) in the energy scales \( \mu < M_R \). In this energy regime, the evolution equations are the same for all the different seesaws, since we are considering an effective theory. However they will depend on the underlying theory, which is the SM in this case. The values of different \( p_i \)s are given in Table 1 and are of \( \mathcal{O}(1) \) as anticipated.

Note that explicit 1-loop calculations show that \( p_{g_1} = 0 \). We were unable to find an explanation based on symmetry considerations and hence we think it is accidental. We expect \( g_1^2 \) dependent terms to emerge at 2-loop.

### B. Right-handed neutrino extended MSSM

We now consider the case of the MSSM extended by three right-handed neutrinos. Our formalism is applicable in this case as well, since the flavor structure of the MSSM is identical to that of the SM. But the Higgs sector of MSSM is different. One of the Higgses, \( H_U \), couples to leptons through the Yukawa coupling \( Y_U \) to give rise to the up-type lepton masses, while the other Higgs, \( H_D \), is responsible for the down-type lepton masses through the Yukawa coupling \( Y_D \). Hence there are two types of trace terms. The first is \( T_U \) which is a combination of \( \text{Tr}[Y_\nu^\dagger Y_\nu] \) and \( \text{Tr}[Y_U^\dagger Y_U] \). The other one is \( T_D \), a combination of \( \text{Tr}[Y_e^\dagger Y_e] \) and \( \text{Tr}[Y_D^\dagger Y_D] \). We define the trace terms as

\[
T_U = r' \text{Tr}[Y_\nu^\dagger Y_\nu] + 3 \text{Tr}[Y_U^\dagger Y_U] ,
\]

\[
T_D = \text{Tr}[Y_e^\dagger Y_e] + 3 \text{Tr}[Y_D^\dagger Y_D] ,
\]

where

\[
r' \equiv \sum_{i=1}^{N} (\epsilon G_a)^* (\epsilon G_a)^T = (1, 3)
\]

is a quantity, similar to \( r \) defined in Eq. (4.3) in the SM, that depends on the transformation of the right-handed neutrinos under the gauge group. The two numbers in the parenthesis are the values in Type-I and Type-III seesaw scenarios. As before, \( r' \) is universal in SU(2) spaces and we write down the universality constant only.

Let us now define the quantities that contribute to the evolution of \( Y_e, Y_\nu \) and \( M_\nu \) in Type-I and Type-III seesaws and depend on the gauge group representations of the right-handed neutrinos:

\[
\alpha_1' = \sum_{a=1}^{N} (\epsilon G_a)^\dagger (\epsilon G_a) = (1, 3) ,
\]

\[
\alpha_2' = \text{Tr}[(\epsilon G_a)^\dagger (\epsilon G_a)] = (2, 2) ,
\]

\[
C_2 = (0, 2) .
\]
$C_2$ is the quadratic Casimir for the irreducible representation $\mathcal{R}$ of SU(2)$_L$ in which the right-handed neutrinos $\nu^R_i$ reside. For Type-I seesaw $C_2 = 0$, while for Type-III seesaw the right-handed fields are in the adjoint representation of SU(2)$_L$ and hence $C_2 = 2$. RG evolution of Yukawas and masses in Type-III seesaw with MSSM as the underlying theory has not been computed before. We give some details of the calculation in Appendix A.

Let us now write down the coefficients involved in $\dot{Y}_e$ in Eq. (3.14).

$$a_1 = 3, \quad a_2 = \alpha'_1, \quad a_T = 1, \quad a_{g_1} = -3 \times \frac{3}{5}, \quad a_{g_2} = -3.$$  \hspace{1cm} (4.18)

We see that in the MSSM, as in the case of the SM, only $a_2$, the coefficient of $Y_\nu^\dagger Y_\nu$, depends on whether the seesaw is Type-I or Type-III. For the case of $\dot{Y}_\nu$, the coefficients appearing in Eq. (3.20) are

$$b_1 = 1, \quad b_2 = \alpha'_1 + \alpha'_2, \quad b_T = 1, \quad b_{g_1} = -\frac{3}{5}, \quad b_{g_2} = -3 - 2C_2.$$  \hspace{1cm} (4.19)

Comparing the expressions of $b_1$ in the SM and the MSSM, in Eqs. (4.10) and (4.19), we see that in the SM $b_1$ receives a contribution that depends on the right-handed neutrinos, which is absent in MSSM. This is to be attributed to the non-renormalization theorem due to which only the wavefunction renormalizations are responsible for the RG evolution of the quantities in MSSM and the mass and vertex corrections do not contribute. The absence of any vertex renormalization contribution makes $b_1$ independent of the right-handed neutrino fields in MSSM. The values of $a_i$ and $b_i$ in the two seesaw types are given in Table II.

From Table II it is seen that for Type-I seesaw scenario, all the numbers are of $\mathcal{O}(1)$ and consistent with prediction from spurion analysis. However, for Type-III seesaw both $b_2$ and $b_{g_2}$ are large numbers, the large contribution emerging from the wavefunction renormalization of the superfields $l$ and $\nu$ respectively.

Next, we move to the case of the right-handed Majorana mass $M_\nu$. The coefficients are

$$q_1 = 2\alpha'_2, \quad q_{g_2} = -4C_2.$$  \hspace{1cm} (4.20)

where $\alpha'_2$ and $C_2$ have already been defined in Eqs. (4.16) and (4.17) respectively. Values of $q_1$ and $q_{g_2}$ in the two types of seesaw scenarios are listed in Table II. As expected, $q_{g_2} = 0$ and $q_1$ is of $\mathcal{O}(1)$ in Type-I seesaw, while for Type-III seesaw $q_1$ and $(q_{g_2}/6)$ are $\mathcal{O}(1)$ numbers.

For energies $\mu < M_R$, evolution of the left-handed neutrino mass $m_\nu$ is the same in both Type-I and Type-III seesaws and the values of the coefficients [11, 12] are quoted in Table II. Note that the accidental cancellation seen in the SM case, $p_{g_1} = 0$, does not happen in the MSSM. The trace term appearing in this case is $T'^T \rightarrow T'^T = \text{Tr}[3Y_\nu^\dagger Y_\nu]$, since in the high energy theory only $H_U$ interacts with $\nu$. The Higgs self-coupling term with coefficient $p_\lambda$ does not exist in this scenario.

The above comparison shows that the method of spurion analysis gives the form of the RG evolution equations. Of course, working in a generic effective field theory we never
expect to get the exact values of the $O(1)$ numbers, which depend on the specific details of the model. One can use this same technique to get the evolution equations at second order. Calculation of evolution equations at 2-loop and comparison with the existing results obtained by loop calculations is given in Appendix B.

V. BREAKING DEGENERACY OF $M_\nu$ AND LEPTOGENESIS

In this section, we study effects related to the breaking of the universality of $M_\nu$. This breaking is important in the context of leptogenesis. It has been studied in detail in [3] where the mass degeneracy is removed by appropriate combinations of spurions transforming as $(1,1,6)$ under $G_{LF}$. Here we compare their results of explicit breaking with the effects generated through RG evolution.

We start with the case of degeneracy breaking by RG evolution. For this purpose, writing down the evolution equation for a component of $M_\nu$ from Eq. (3.27) we get

$$\left(\dot{M}_\nu\right)_{ij} = \frac{q_1}{2} \left[ (M_\nu)_{ik} (Y_\nu Y_\nu^\dagger)_{kj} + (Y_\nu Y_\nu^\dagger)_{ki} (M_\nu)_{kj} \right] + q_{g_2} g_2 (M_\nu)_{ij}.$$  \hspace{1cm} (5.1)$$

Using universal-mass initial condition, $(M_\nu)_{ij} = M_R \delta_{ij}$, one gets the final eigen-values of $M_\nu$ after RG running to be non-degenerate. The specific value of breaking depends on the values of $q_1$, $q_{g_2}$ as well as the RG evolution of the spurion field $Y_\nu$ and its background value, and thus on the underlying theory considered.

Next, we study degeneracy breaking at the high scale using spurion techniques. To the lowest order in the spurion fields $Y_e, Y_\nu$, the final Majorana mass matrix $M_F^{\nu}$ is written as

$$M_F^{\nu} = M_\nu + \sum_n c_n \delta M^{(n)}_\nu,$$  \hspace{1cm} (5.2)

where $M_\nu = M_R I$ is the universal mass matrix given in Eq. (2.3) and

$$
\begin{align*}
\delta M^{(1)}_\nu &= M_R \left( Y_\nu Y_\nu^\dagger + (Y_\nu Y_\nu^\dagger)^T \right), \\
\delta M^{(2)}_\nu &= M_R \left( Y_\nu Y_\nu^\dagger Y_e Y_e^\dagger + (Y_\nu Y_\nu^\dagger Y_e Y_e^\dagger)^T \right), \\
\delta M^{(22)}_\nu &= M_R \left( Y_\nu Y_\nu^\dagger (Y_\nu Y_\nu^\dagger)^T \right), \\
\delta M^{(23)}_\nu &= M_R \left( (Y_\nu Y_\nu^\dagger)^T Y_e Y_e^\dagger \right), \\
\delta M^{(24)}_\nu &= M_R \left( Y_\nu Y_e^\dagger Y_e Y_e^\dagger + (Y_e Y_e^\dagger Y_e Y_e^\dagger)^T \right),
\end{align*}$$  \hspace{1cm} (5.3)

considering terms containing up to four spurions. As discussed in [3], values of $c_n$ depends on dynamical properties: if the Yukawa corrections are generated within a perturbative regime, as is the case for RG evolution, $c_n$ decreases according to the power of Yukawa matrices, for example, in a standard loop-expansion one should have $c_{11} \sim g_2^2/(4\pi)^2$ and then $c_{2i} \sim c_{11}^2$ and so on. One cannot exclude a priori a strong-interaction regime where $c_n \sim O(1)$, for
all \( n \). But even in the case of strong-interaction, the series in Eq. (5.2) is expected to be dominated by the first few terms as the background values of the spurions \( Y_{e,\nu} \) are small. In this paper, we consider the perturbative regime of explicit breaking only.

In Ref. [3] it is shown that the amount of mass degeneracy breaking is important in the context of leptogenesis. In the rest of this section we consider the two sources of breaking and study the pattern of mass universality breaking and its effect on leptogenesis. We briefly describe the parametrization of the Yukawa \( Y_{\nu} \) following [3]. We choose to work in the basis where \( Y_{e} \) is diagonal. Then the neutrino mass matrix is given as

\[
m_{\nu} = U_{\text{PMNS}}^{*} m_{\nu}^{\text{diag}} U_{\text{PMNS}}^{\dagger},
\]

where

\[
m_{\nu}^{\text{diag}} = \text{diag}(m_{1}, m_{2}, m_{3})
\]

and \( U_{\text{PMNS}} \) is the unitary matrix that diagonalizes \( m_{\nu} \). In this basis, the most general form of \( Y_{\nu} \) is given by the Casas-Ibarra parametrization [15]:

\[
Y_{\nu} = \frac{1}{v} M_{\nu}^{1/2} R (m_{\nu}^{\text{diag}})^{1/2} U_{\text{PMNS}}^{\dagger} = \frac{\sqrt{M_{\nu}}}{v} R (m_{\nu}^{\text{diag}})^{1/2} U_{\text{PMNS}}^{\dagger},
\]

where \( R \) is a complex orthogonal matrix parametrized by six real quantities. We write \( R = OH \), where \( O \) is a real orthogonal matrix and \( H \) is complex orthogonal hermitian matrix and thus each \( O \) and \( H \) contains three real parameters. Since \( O \in O(3)_{\nu R} \) and \( O(3)_{\nu R} \) is a symmetry of the theory independent of any assumption on CP properties, we can choose \( O \equiv I \) to get \( R = H \). Thus finally

\[
Y_{\nu} = \frac{\sqrt{M_{\nu}}}{v} H (m_{\nu}^{\text{diag}})^{1/2} U_{\text{PMNS}}^{\dagger}.
\]

In the CP conserving limit, \( H = I \). The CP violating nature of \( H \) is clear in the following parametrization [16]:

\[
H = e^{i\Phi} = I - \frac{\cosh \rho - 1}{\rho^{2}} \Phi^{2} + i \frac{\sinh \rho}{\rho} \Phi,
\]

where

\[
\rho = \sqrt{\varphi_{1}^{2} + \varphi_{2}^{2} + \varphi_{3}^{2}}, \quad \text{and} \quad \Phi = \begin{pmatrix}
0 & \varphi_{1} & \varphi_{2} \\
-\varphi_{1} & 0 & \varphi_{3} \\
-\varphi_{2} & -\varphi_{3} & 0
\end{pmatrix}.
\]

Let us now proceed to the numeric example. In the generic case of [3], the breaking depends on the choice of \( c_{n}s \), while in case of RG evolution we need to specify the underlying theory (for example SM or MSSM and also Type-I or Type-III). In both cases, the mass-splitting of the right-handed neutrinos depends on \( \Phi \) as well as the neutrino masses and
FIG. 2: Majorana mass splittings as a function of $|\varphi|$ for normal neutrino mass hierarchy. We have defined $\Delta M_{ii} = M_i - M_1$. The green (light gray in black and white) dots show ‘SM Gen.’, and dark gray dots are for ‘MLFV’. The red (lower) and blue (upper) dotted lines correspond to SM and MSSM respectively.

mixing parameters through $Y_\nu$. For the purpose of illustration we choose $M_R = 10^{13}$ GeV, and $\varphi_1 = \varphi_2 = \varphi_3 = \varphi$, and then consider the range $10^{-3} \leq |\varphi| \leq 1$. The neutrino mass-squared differences are set to the central experimental values: $|\Delta m_{32}^2| = |m_3^2 - m_2^2| = 2.4 \times 10^{-3}$ eV$^2$ and $\Delta m_{21}^2 = m_2^2 - m_1^2 = 7.65 \times 10^{-5}$ eV$^2$. The lightest neutrino mass is chosen to be in the range $\{10^{-4}, 10^{-2}\}$ eV. The mixing angles have been fixed to tribimaximal values. Finally, points satisfying $|Y_{\nu})_{ij}| \leq 1$ are considered. For the MSSM, we take $\tan \beta = 20$.

To illustrate the mass-splitting generated through RG evolution, we consider the case of Type-I seesaw and show the results when the theory is extended SM, extended MSSM and also any generic theory with the same underlying symmetry as extended SM (referred to as ‘SM Gen.’). All these cases together are referred to as ‘Type-I RG’. In case of ‘SM Gen.’, we choose the coefficients appearing in the evolution of $Y_e$, $Y_\nu$ and $M_\nu$, given in Eqs. (3.14),
For ‘Type-I RG’, the high scale is chosen to be $\mu_0 = 10^{16}$ GeV, while the value of the mass-splitting is evaluated at $\mu = M_R$. For the general MLFV scenario [3] (referred to as ‘MLFV’), we consider the case when

$$c_{11} = c \quad \text{and} \quad c_{21} = c_{24} = c^2,$$

with all other $c_n$s set to zero. The value of $c$ is varied over a few orders of magnitude, $c \in \{10^{-2}, 1\}$, as can be seen in Figs. 2 and 3. In the ‘MLFV’ scenario, the mass-splitting does not depend on the energy scale.

Fig. 2 shows the plots for normal neutrino hierarchy ($\Delta m^2_{32} > 0$), while Fig. 3 shows that for the inverted case ($\Delta m^2_{32} < 0$). From the figures one can make the following observations:
• For both the cases of ‘MLFV’ and ‘Type-I RG’, the nature of variation of $\Delta M_{31} = M_3 - M_1$ with $|\varphi|$ is the same, for the whole range of $|\varphi| \in \{0.001, 1.0\}$, with either neutrino mass hierarchy. The generic variation trends are different for $\Delta M_{21} = M_2 - M_1$.

• In case of ‘Type-I RG’, for inverted hierarchy $\Delta M_{21}$ varies about two orders of magnitude as $|\varphi|$ is varied in the range 0.1 – 1.0. For normal hierarchy, the variation is small for $|\varphi| \lesssim 0.5$. For ‘MLFV’, variation of $\Delta M_{21}$ is quite small for inverted hierarchy.

• There is an overlap of $\Delta M_{21}$ generated in ‘MLFV’ for $c \in \{0.01, 0.1\}$ with that in ‘SM Gen.’ for $|\varphi| > 0.3(0.15)$ with normal(inverted) hierarchy. For higher $c$ values, $c \in \{0.1, 1.0\}$, the ‘MLFV’ can resemble the RG effect for the whole range of $|\varphi|$ for normal hierarchy, while for inverted hierarchy the same is accomplished for $|\varphi| > 0.2$.

• $\Delta M_{31}$ generated in ‘MLFV’ overlaps that in ‘SM Gen.’ for the whole range of $|\varphi|$ with both the hierarchies and for all $c \in \{0.001, 1.0\}$.

The above example shows a consistent treatment of the splitting that include both the generic splittings from spurion technique and the RG evolution. The result obtained in the case of a general splitting with spurions is different from what we get when RG effects are included. However, there is an overlap for some region of the parameter space.

Next, we discuss the effect of including RG evolution on leptogenesis, and compare it to the result obtained with the generic splitting \[3\]. The baryon asymmetry $\eta_B$ can be expressed as

$$\eta_B = 9.6 \times 10^{-3} \sum_i \epsilon_i d_i,$$  \hspace{1cm} (5.12)

where $d_i$ are the washout factors, and the $\epsilon_i$ are the CP asymmetries defined as \[17–19\]

$$\epsilon_i = \frac{\sum_k [\Gamma(\nu_R^i \to l_k \phi^*) - \Gamma(\nu^i_R \to \bar{l}_k \phi)]}{\sum_k [\Gamma(\nu_R^i \to l_k \phi^*) + \Gamma(\nu^i_R \to \bar{l}_k \phi)]}. \hspace{1cm} (5.13)$$

To determine $d_i$, we consider the strong washout regime and use the same approximations as in \[3,20\].

Values of $\eta_B$ obtained as a function of $|\varphi|$ is shown in Fig.4. The black (dashed) horizontal line shows the current experimental value of the baryon asymmetry \[21\]

$$\eta_B = (6.23 \pm 0.17) \times 10^{-10}, \hspace{1cm} (5.14)$$

at 1$\sigma$. It can be seen from Fig.4 that in case of generic mass splitting with spurion techniques \[3\] the correct value of $\eta_B$ can be achieved for $0.1 \lesssim |\varphi| \lesssim 0.4$, for the given choice of other parameters, with both the neutrino hierarchies and $c \in \{0.01, 0.1\}$. For other values of $|\varphi|$, the baryon asymmetry is lower than the current experimental value. For higher $c$ values, $c \in \{0.1, 1.0\}$, the correct $\eta_B$ is obtained for a small region around $|\varphi| \sim 0.1$ and
$|\varphi| \sim 0.4$. However, if one considers ‘Type-I RG’, the correct baryon asymmetry is achieved for the whole $|\varphi|$ range and for both hierarchies. The results obtained in the two cases are different, with a small overlap in the allowed parameter space. Hence, while relating the low energy effects with the high energy phenomena, one must include the complete RG evolution of parameters, rather than considering a generic mass splitting to mimic the effect.

VI. CONCLUSION

Neutrino physics provides a window to the physics of very high scale. In order to learn about high energy physics, one need to use RGEs to connect the low and high energy scales. In this paper, we study models of MLFV and write the RGEs in terms of spurions that capture the whole effect. It is only the coefficient of each term that varies between models.

Our results serve as a check on the existing calculations. For example, we find that both in
the SM and MSSM, the difference between the right-handed neutrino representations enters only in one term, when we consider the evolution of the Yukawa matrix \( Y_e \). For the purpose of illustration of our results, we have also computed the RGEs of Yukawas and masses in case of MSSM Type-III seesaw scenario, for the first time. If needed, this spurion analysis method to determine the RG evolution can be extended to two loop order, as has been done here, in which case we can check where the difference between Type-I and Type-III models resides. Our results can also be extended to other models. For example, in Type-II seesaw and Inverse seesaw, we have more sources of lepton flavor breaking. We can include them in the analysis in order to get more insight about where the running effects are coming from.

One implication of our results has to do with leptogenesis. Degenerate right-handed neutrinos cannot give the required baryon asymmetry of the Universe. Thus, they must be split. The splitting can be accomplished in two ways: explicitly with allowed spurion combinations from symmetry consideration, as is done in [3], or by considering RG evolution of different parameters consistently. We show that the effect of RG running can significantly change the allowed region of parameter space for successful leptogenesis compared to the explicit breaking, and hence should be taken into account.

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Appendix A: Calculation of RG evolution in MSSM Type-III seesaw

In this section we consider the MSSM extended by the addition of three right-handed triplet superfields \( \nu \). This is the only model out of the four we considered where explicit calculation does not exist in the literature, and thus we present it here.

The Yukawa part of the superpotential is given by

\[
\mathcal{W}_{\text{Yukawa}} = (Y_\nu)_{gf} \nu^c g H_U l_l^f + (Y_e)_{gf} e^c g H_D l_l^f + (Y_U)_{gf} u^c g H_U Q_l^f + (Y_D)_{gf} d^c g H_D Q_l^f ,
\]  

(A1)

where the first line corresponds to the Yukawa interactions for the lepton superfields, while the second line shows the Yukawa interactions for the quark superfields. The superfields \( e, u \) and \( d \) contain the SU(2)_L-singlet charged leptons, down-type quarks and up-type quarks, while \( l \) and \( Q \) contain the SU(2)_L lepton and quark doublets, respectively. Superpotential
corresponding to the Majorana mass term for triplet neutrino superfields is

$$W_{\text{Maj}} = \frac{1}{2} \nu^C g \left( M_\nu \right)_{gf} \nu^C f .$$  \hfill (A2)

$W_{\text{Maj}}$ is important for the seesaw mechanism, but it does not take part in the RG evolution of different quantities.

1. Wavefunction renormalization constants

Let us consider a general supersymmetric gauge theory containing $N_\Phi$ superfields $\Phi^{(i)}$ that transform under the irreducible representations $\mathcal{R}^{(i)}_1 \times \cdots \times \mathcal{R}^{(i)}_K$ of the gauge group $G_1 \otimes \cdots \otimes G_K$. The renormalizable part of the superpotential is given as

$$W_{\text{renorm}} = \frac{1}{6} \sum_{i,j,k=1}^{N_\Phi} \lambda^{(ijk)} \Phi^{(i)} \Phi^{(j)} \Phi^{(k)} ,$$  \hfill (A3)

where $(ijk)$ implies symmetrization over the indices. Due to the non-renormalization theorem, the RG evolution equations for different operators of the superpotential are governed only by the wavefunction renormalization constants for the superfields $\Phi^{(i)}$, given as

$$Z_{ij} = I_{ij} + \delta Z_{ij} .$$  \hfill (A4)

The bare and renormalized superfields, $\Phi^{(i)}_B$ and $\Phi^{(i)}$, are then related as

$$\Phi^{(i)}_B = \sum_{j=1}^{N_\Phi} Z_{ij}^{\frac{1}{2}} \Phi^{(j)} .$$  \hfill (A5)

Using dimensional regularization via dimensional reduction, the wavefunction renormalization constants, in $d = 4 - \varepsilon$ dimensions, at 1-loop are obtained as \cite{22, 23}

$$\delta Z_{ij}^{(1)} = - \frac{1}{16\pi^2 \varepsilon} \left[ \sum_{k,l=1}^{N_\Phi} \lambda^{*}_{k,l} \lambda_{jkl} - 4 \sum_{n=1}^{K} g^2_n C_2(\mathcal{R}^{(i)}_n) \delta_{ij} \right] ,$$  \hfill (A6)

where $C_2(\mathcal{R}^{(i)}_n)$ is the quadratic Casimir for the representation $\mathcal{R}^{(i)}_n$ of the gauge group $G_n$.

Comparing the superpotentials in Eqs. (A3) and (A1), and using Eq. (A6), we get the $1/\varepsilon$ coefficients of the wavefunction renormalization constants, for different lepton and Higgs
superfields, to be

\[ -(4\pi)^2 \delta Z_l = 2Y_e^\dagger Y_e + 2 \left( \sum_a (\epsilon G^a)^\dagger (\epsilon G^a) \right) Y^\dagger_\nu Y_\nu - \frac{3}{5} g_1^2 - 3g_2^2, \]  
(A7)

\[ -(4\pi)^2 \delta Z_{eC} = 4Y_e^* Y_e^T - \frac{12}{5} g_1^2, \]  
(A8)

\[ -(4\pi)^2 \delta Z_{\nu C} = 2 \text{Tr}[(\epsilon G^a)^\dagger \epsilon G^a] Y^\dagger_\nu Y_\nu - 4 C_2(\mathcal{R}_{SU(2)_L}) g_2^2, \]  
(A9)

\[ -(4\pi)^2 \delta Z_{H^U} = 2 \left( \sum_a (\epsilon G^a)^* (\epsilon G^a)^T \right) \text{Tr}[Y^\dagger_\nu Y_\nu] + 6 \text{Tr}[Y^\dagger_\nu Y_\nu] - \frac{3}{5} g_1^2 - 3g_2^2, \]  
(A10)

\[ -(4\pi)^2 \delta Z_{H^D} = 2\text{Tr}[Y^\dagger_\nu Y_\nu] + 6\text{Tr}[Y^\dagger_D Y_D] - \frac{3}{5} g_1^2 - 3g_2^2. \]  
(A11)

It must be noted that the wavefunction renormalization constants, given in Eqs. (A7) – (A9), are in general forms applicable to both Type-I and Type-III see-saw when we use appropriate forms of $G^a$, as given in Eq. (4.2). Thus the quantities, which depend on the transformation properties of the right-handed neutrino superfields, are

\[ r' = \sum_a (\epsilon G^a)^* (\epsilon G^a)^T = (1, 3), \]  
(A12)

\[ \alpha'_1 = \sum_a (\epsilon G^a)^\dagger (\epsilon G^a) = (1, 3), \]  
(A13)

\[ \alpha'_2 = \text{Tr}[(\epsilon G^a)^\dagger \epsilon G^a] = (2, 2). \]  
(A14)

Here the numbers in the parenthesis are the values in Type-I and Type-III see-saw scenarios, and are universal in the SU(2) space, as defined in Eqs. (4.14) – (4.16). We do not use any summation convention here. $C_2(\mathcal{R}_{SU(2)_L})$ in Eq. (A9) is the quadratic Casimir for the superfield $\nu$ under SU(2)$_L$ and hence, as given in Eq. (4.17), $C_2(\mathcal{R}_{SU(2)_L}) = 0$ for Type-I see-saw, and $C_2(\mathcal{R}_{SU(2)_L}) = 2$ for Type-III see-saw. In Section IV and in the remainder of the appendix we use $C_2 \equiv C_2(\mathcal{R}_{SU(2)_L})$.

2. Calculation of RG evolution equations

Let us now compute the $\beta$-functions. The RG evolution of $Y_e$ is given by

\[ \mu \frac{dY_e}{d\mu} = -\frac{1}{2} \left( Y_e \delta Z_l + Y_e \delta Z_{eC} + \delta Z_{eC}^* Y_e \right), \]  
(A15)

which reduces to

\[ \dot{Y}_e = Y_e \left[ 3Y_e^\dagger Y_e + \alpha'_1 Y_\nu^\dagger Y_\nu + \left( \text{Tr}[Y_e^\dagger Y_e] + 3\text{Tr}[Y_D^\dagger Y_D] \right) - \frac{9}{5} g_1^2 - 3g_2^2 \right] \]  
(A16)
where
\[ T_D = \text{Tr}[Y_e^\dagger Y_e] + 3\text{Tr}[Y_D^\dagger Y_D] . \] (A17)

Similarly, the evolution equation for \( Y_\nu \) is given by
\[
\dot{Y}_\nu = Y_\nu \left[ Y_e^\dagger Y_e + (\alpha_1' + \alpha_2') Y_\nu^\dagger Y_\nu + \left( r'Tr[Y_\nu^\dagger Y_\nu] + 3\text{Tr}[Y_\nu^\dagger Y_U] \right) - \frac{3}{5}g_1^2 - (3 + 2C_2)g_2^2 \right]
\]
\[ = Y_\nu \left[ Y_e^\dagger Y_e + (\alpha_1' + \alpha_2') Y_\nu^\dagger Y_\nu + T_U - \frac{3}{5}g_1^2 - (3 + 2C_2)g_2^2 \right] \] (A18)

where
\[ T_U = r'Tr[Y_\nu^\dagger Y_\nu] + 3\text{Tr}[Y_U^\dagger Y_U] . \] (A19)

The evolution equation of the right-handed neutrino mass \( M_\nu \) is given by
\[
\mu \frac{dM_\nu}{d\mu} = -\frac{1}{2} \left( \delta Z_{\nu\nu}^T M_\nu + M_\nu \delta Z_{\nu\nu} \right) , \] (A20)

which reduces to
\[
\dot{M}_\nu = \alpha_2' \left[ (Y_\nu Y_\nu^\dagger) M_\nu + M_\nu (Y_\nu Y_\nu^\dagger)^T \right] - 4C_2g_2^2M_\nu . \] (A21)

Appendix B: RG evolution equations at 2-loop

In Section III of the main part of the paper, we have considered the first order contribution of the spurion fields. Here, we study the second order terms in the RGEs of the Yukawas and the masses using the same technique.

1. 2-loop running of \( Y_e \)

In this section, we consider the evolution of \( Y_e \). The new contributions at 2-loop will consist of five spurion fields transforming as \((\bar{3}, 3, 1)\) under \( G_{LF} \). Any combination of three spurion fields with \((\bar{3}, 3, 1)\) and two other couplings in the theory transforming trivially under \( G_{LF} \) is also a valid term at this order. There must also be terms proportional to a single spurion field and four other couplings.

Using Table I, the SU(3)-algebra
\[ 8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1 , \] (B1)

and those given in Eq. (3.2), and the transformation properties
\[
\text{Tr}[Y_e^\dagger Y_e^\dagger Y_e] = (1, 1, 1) , \quad \text{Tr}[Y_\nu^\dagger Y_\nu^\dagger Y_\nu] = (1, 1, 1) , \quad \text{Tr}[Y_e^\dagger Y_e^\dagger Y_e] = (1, 1, 1) , \quad (B2)
\]
we get that

\[ Y_e T_e T_e = (3, 3, 1) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B3) \]
\[ Y_e T_e T_\nu = (3, 3, 1) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B4) \]
\[ Y_e T_\nu T_e = (3, 3, 1) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B5) \]
\[ Y_e T_\nu T_\nu = (3, 3, 1) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B6) \]
\[ Y_e \text{Tr}[Y_e^\dagger Y_e] T_e = (3, 3, 1) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B7) \]
\[ Y_e \text{Tr}[Y_e^\dagger Y_\nu] T_e = (3, 3, 1) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B8) \]
\[ Y_e \text{Tr}[Y_\nu^\dagger Y_e] T_\nu = (3, 3, 1) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B9) \]
\[ Y_e \text{Tr}[Y_\nu^\dagger Y_\nu] T_\nu = (3, 3, 1) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 3, 1) , \quad (B10) \]
\[ Y_e \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] = (3, 3, 1) \otimes (1, 1, 1) = (3, 3, 1) , \quad (B11) \]
\[ Y_e \text{Tr}[Y_e^\dagger Y_\nu Y_\nu^\dagger Y_\nu] = (3, 3, 1) \otimes (1, 1, 1) = (3, 3, 1) , \quad (B12) \]
and \[ Y_e \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_\nu] = (3, 3, 1) \otimes (1, 1, 1) = (3, 3, 1) \quad (B13) \]
are the only allowed combinations of five spurion fields that can appear on the RHS of \( \hat{Y}_e \) at second order. Hence we can write the most general form of the second order contributions to \( \hat{Y}_e \) as

\[ (4\pi)^2 \hat{Y}_e \bigg|_{\text{2-loop}} \sim Y_e \left( d_1 T_e T_e + d_2 T_e T_\nu + d_3 T_\nu T_e + d_4 T_\nu T_\nu \right) + Y_e \left( d_5 \text{Tr}[Y_e^\dagger Y_e] T_e + d_6 \text{Tr}[Y_e^\dagger Y_\nu] T_e + d_7 \text{Tr}[Y_\nu^\dagger Y_e] T_\nu + d_8 \text{Tr}[Y_\nu^\dagger Y_\nu] T_\nu \right) + Y_e \left( d_9 \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + d_{10} \text{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu] + d_{11} \text{Tr}[Y_\nu^\dagger Y_e Y_\nu^\dagger Y_\nu] \right) + Y_e \left( d_{12} T_e + d_{13} T_\nu + d_{14} \text{Tr}[Y_e^\dagger Y_e] + d_{15} \text{Tr}[Y_\nu^\dagger Y_\nu] \right) + d_{16} Y_e . \quad (B14) \]

The extra factor of \((4\pi)^2\) is there since we are considering 2-loop contributions. We rewrite Eq. (B14), using the definitions of \( T_e, T_\nu \) from Table [I] as

\[ (4\pi)^2 \hat{Y}_e \bigg|_{\text{2-loop}} = Y_e \left( d_1 Y_e^\dagger Y_e Y_e^\dagger Y_e + d_2 Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu + d_3 Y_\nu^\dagger Y_e Y_e^\dagger Y_\nu + d_4 Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu \right) + Y_e \left( d_5 Y_e^\dagger Y_e \text{Tr}[Y_e^\dagger Y_e] + d_6 Y_e^\dagger Y_e \text{Tr}[Y_e^\dagger Y_\nu] + d_7 Y_\nu^\dagger Y_e \text{Tr}[Y_\nu^\dagger Y_e] + d_8 Y_\nu^\dagger Y_\nu \text{Tr}[Y_\nu^\dagger Y_\nu] \right) + Y_e \left( d_9 \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + d_{10} \text{Tr}[Y_e^\dagger Y_e Y_\nu^\dagger Y_\nu] + d_{11} \text{Tr}[Y_\nu^\dagger Y_e Y_\nu^\dagger Y_\nu] \right) + Y_e \left( d_{12} Y_e^\dagger Y_e + d_{13} Y_\nu^\dagger Y_\nu + d_{14} \text{Tr}[Y_e^\dagger Y_e] + d_{15} \text{Tr}[Y_\nu^\dagger Y_\nu] \right) + d_{16} Y_e . \quad (B15) \]
where \( d_1, \ldots, d_{11} \) are expected to be \( \mathcal{O}(1) \) numbers. We have not written the terms of the form \( \text{Tr}[Y_i^\dagger Y_j] \cdot \text{Tr}[Y_j^\dagger Y_i], i, j \in \{e, \nu\} \), since such terms cannot be generated at 2-loop order. Each of \( d_{12}, d_{13} \) is expected to be a linear function of \( g^2_1, \lambda, \text{Tr}[Y_i^\dagger Y_i], \text{Tr}[Y_i^\dagger Y_j] \) and can be written, in general, as

\[
d_i = d^0_i g^2_1 + d^g_2 g^2_2 + d^\lambda_i \lambda + d^{Y_i^\dagger Y_i}_i + d^{Y_i^\dagger Y_j}_i \quad (i \in \{12, 13\}) .
\]

Unlike the 1-loop case, Higgs self-coupling can appear at 2-loop order via diagrams like the one shown in Fig. 5. Since the leptons are singlets under SU(3)_C, \( g^3_2 \) cannot be present in \( d_{12} \) and \( d_{13} \). As before, \( d^2_{12}, d^2_{13} \) are expected to be of \( \mathcal{O}(1) \). \( d_{14}, d_{15} \) must originate from a diagram containing complete lepton loop in Higgs self-energy correction and hence cannot contain \( \lambda \) or \( g^2_2 \). Hence we write

\[
d_i = d^0_1 g^2_1 + d^g_2 g^2_2 \quad (i \in \{14, 15\}) ,
\]

where \( d^2_{14}, d^2_{15} \) are to be of \( \mathcal{O}(1) \). \( \text{Tr}[Y_i^\dagger Y_i] \) or \( \text{Tr}[Y_i^\dagger Y_j] \) cannot be present in \( d_{14} \) and \( d_{15} \).

Let us now consider the quantity \( d_{16} \), which is independent of the spurion fields and must be a function linear in \( \text{Tr}[Y_i^\dagger Y_i], \text{Tr}[Y_i^\dagger Y_j], \text{Tr}[Y_i^\dagger Y_j], \text{Tr}[Y_i^\dagger Y_j], \text{Tr}[Y_i^\dagger Y_j] \) and quadratic in \( g^2_1 \), \( \text{Tr}[Y_i^\dagger Y_i], \text{Tr}[Y_i^\dagger Y_j] \) and \( \lambda \). In its most general form, it can be expressed as

\[
d_{16} = d^{U}_{16} \text{Tr}[Y_i^\dagger Y_i Y_i^\dagger Y_i] + d^{D}_{16} \text{Tr}[Y_i^\dagger Y_j Y_i^\dagger Y_j] + d^{U}_{16} \text{Tr}[Y_i^\dagger Y_i Y_i^\dagger Y_j] + \lambda \text{Tr}[Y_i^\dagger Y_i Y_i^\dagger Y_i] \quad (i \in \{16\})
\]

where all the coefficients \( d^U_{16} \) are expected to be \( \mathcal{O}(1) \) numbers. Unlike the case of first order evolution equation, here \( g^2_3 \) can appear at 2-loop since quarks have color charges. For example, diagrams shown in Fig. 6 will contribute terms proportional to \( g^2_3 \text{Tr}[Y_i^\dagger Y_i] \) and \( g^2_3 \text{Tr}[Y_i^\dagger Y_i] \). However, terms proportional to \( g^4_3 \) cannot be present. As can be checked, here we cannot have terms proportional to \( \lambda \text{Tr}[Y_i^\dagger Y_i] \) or \( \lambda \text{Tr}[Y_i^\dagger Y_i] \), while terms containing \( \lambda g^2_3, \lambda g^2_3 \) can contribute. Examples of diagrams giving rise to such terms are shown in Fig. 7.

There cannot exist any term proportional to \( \lambda g^2_3 \) or \( \lambda^2 \) in this case.

Having written the most general form of second order contributions to \( \hat{Y}_e \), we consider the fact that \( \text{Tr}[Y_i^\dagger Y_i] (i \in \{e, \nu, U, D\}) \) can only come from a complete fermion loop in the

FIG. 6: Example of diagrams at 2-loop with gluon contributions, leading to terms proportional to \( g^2_3 \text{Tr}[Y_i^\dagger Y_i] \) and \( g^2_3 \text{Tr}[Y_i^\dagger Y_i] \) in \( d_{16} \).
FIG. 7: Example of diagrams at 2-loop giving rise to $\lambda g_1^2, \lambda g_2^2$ terms in $d_{16}$.

Higgs self-energy correction, as already stated in Section III A and shown in Fig. 1. Hence, we can write the ratios as

$$d_5 : d_6 : d_{12}^U : d_{12}^D = 1 : r : 3 : 3,$$
$$d_7 : d_8 : d_{13}^U : d_{13}^D = 1 : r : 3 : 3,$$  \quad (B19)

where $r$ for Type-I and Type-III seesaw is defined in Eq. (4.3) for SM and in Eq. (4.14) for MSSM. Hence, we can write

$$d_5 \text{Tr}[Y_e^\dagger Y_e] + d_6 \text{Tr}[Y_\nu^\dagger Y_\nu] + d_{12}^U \text{Tr}[Y_U^\dagger Y_U] + d_{12}^D \text{Tr}[Y_D^\dagger Y_D] \rightarrow d_{12}^T T ,$$
$$d_7 \text{Tr}[Y_e^\dagger Y_e] + d_8 \text{Tr}[Y_\nu^\dagger Y_\nu] + d_{13}^U \text{Tr}[Y_U^\dagger Y_U] + d_{13}^D \text{Tr}[Y_D^\dagger Y_D] \rightarrow d_{13}^T T ,$$

where $T$ is defined in Eq. (3.13) and $d_{13}^T, d_{14}^T$ are expected to be of $O(1)$. Thus the most general form of $\dot{Y}_e$ becomes

$$(4\pi)^2 \dot{Y}_e \bigg|_{\text{2-loop}} = Y_e \left( d_1 Y_e^\dagger Y_e Y_e^\dagger Y_e + d_2 Y_e^\dagger Y_e Y_e^\dagger Y_e + d_3 Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu + d_4 Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu \right)$$
$$+ Y_e \left( d_9 \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + d_{10} \text{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] + d_{11} \text{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] \right)$$
$$+ Y_e \left( d_{12}^U g_1^2 + d_{12}^D g_2^2 + d_{12}^U \lambda + d_{12}^D T \right) Y_e^\dagger Y_e + Y_e \left( d_{13}^U g_1^2 + d_{13}^D g_2^2 + d_{13}^U \lambda + d_{13}^D T \right) Y_\nu^\dagger Y_\nu$$
$$+ Y_e \left( d_{14} \text{Tr}[Y_e^\dagger Y_e] + d_{15} \text{Tr}[Y_\nu^\dagger Y_\nu] \right) + d_{16} Y_e . \quad (B20)$$

2. 2-loop running of $Y_\nu$

Let us now consider the second order terms arising in the RGE of $Y_\nu$. Considering Table II the transformation rules in Eqs. (3.2, B1), and the transformation properties in Eq. (B2),
we get that
\[
Y_\nu T_e T_\nu = (\bar{3}, 1, 3) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B21)
\]
\[
Y_\nu T_e T_\nu = (\bar{3}, 1, 3) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B22)
\]
\[
Y_\nu T_e T_\nu = (\bar{3}, 1, 3) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B23)
\]
\[
Y_\nu T_e T_\nu = (\bar{3}, 1, 3) \otimes (8, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B24)
\]
\[
Y_\nu \text{Tr}[Y_e^\dagger Y_e] T_e = (\bar{3}, 1, 3) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B25)
\]
\[
Y_\nu \text{Tr}[Y_e^\dagger Y_e] T_e = (\bar{3}, 1, 3) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B26)
\]
\[
Y_\nu \text{Tr}[Y_e^\dagger Y_e] T_e = (\bar{3}, 1, 3) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B27)
\]
\[
Y_\nu \text{Tr}[Y_e^\dagger Y_e] T_e = (\bar{3}, 1, 3) \otimes (1, 1, 1) \otimes (8, 1, 1) \ni (3, 1, 3), \quad (B28)
\]
\[
Y_\nu \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] = (\bar{3}, 1, 3) \otimes (1, 1) = (\bar{3}, 1, 3), \quad (B29)
\]
\[
Y_\nu \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] = (\bar{3}, 1, 3) \otimes (1, 1) = (\bar{3}, 1, 3), \quad (B30)
\]
and \[
Y_\nu \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] = (\bar{3}, 1, 3) \otimes (1, 1) = (\bar{3}, 1, 3) \quad (B31)
\]
are the only allowed combinations of five spurion fields that can appear on the RHS of $\tilde{Y}_e$ at second order. Hence, similar to $\tilde{Y}_e$, we can write the most general form of the second order contributions to $\tilde{Y}_\nu$ as
\[
(4\pi)^2 \tilde{Y}_\nu \bigg|_{\text{2-loop}} \sim Y_\nu \left( \tilde{f}_1 T_e T_\nu + \tilde{f}_2 T_e T_\nu + \tilde{f}_3 T_e T_\nu + \tilde{f}_4 T_e T_\nu \right)
+ Y_\nu \left( \tilde{f}_5 \text{Tr}[Y_e^\dagger Y_e] T_e + \tilde{f}_6 \text{Tr}[Y_e^\dagger Y_e] T_e + \tilde{f}_7 \text{Tr}[Y_e^\dagger Y_e] T_e + \tilde{f}_8 \text{Tr}[Y_e^\dagger Y_e] T_e \right)
+ Y_\nu \left( \tilde{f}_9 \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + \tilde{f}_{10} \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] + \tilde{f}_{11} \text{Tr}[Y_e^\dagger Y_e Y_e^\dagger Y_e] \right)
+ Y_\nu \left( \tilde{f}_{12} T_e + \tilde{f}_{13} T_\nu + \tilde{f}_{14} \text{Tr}[Y_e^\dagger Y_e] + \tilde{f}_{15} \text{Tr}[Y_e^\dagger Y_e] + \tilde{f}_{16} Y_e \right) \quad (B32)
\]
The above equation can be written in a simple form using the definitions of $T_e$, $T_\nu$ from Table II and the ratio of the coefficient of the traces, as done in case of $\tilde{Y}_e$, to give
\[
(4\pi)^2 \tilde{Y}_\nu \bigg|_{\text{2-loop}} = Y_\nu \left( f_{11} Y_e^\dagger Y_e Y_e^\dagger Y_e + f_{12} Y_e^\dagger Y_e Y_e^\dagger Y_e + f_{13} Y_e^\dagger Y_e Y_e^\dagger Y_e + f_{14} Y_e^\dagger Y_e Y_e^\dagger Y_e \right)
+ Y_\nu \left( f_{15} \text{Tr}[Y_e^\dagger Y_e] + f_{16} \text{Tr}[Y_e^\dagger Y_e] + f_{17} \text{Tr}[Y_e^\dagger Y_e] \right)
+ Y_\nu \left( f_{18} Y_e^\dagger Y_e + f_{19} Y_e^\dagger Y_e + f_{20} Y_e^\dagger Y_e \right)
+ Y_\nu \left( f_{21} Y_e^\dagger Y_e + f_{22} Y_e^\dagger Y_e + f_{23} Y_e^\dagger Y_e \right) Y_e^\dagger Y_e
+ Y_\nu \left( f_{24} Y_e^\dagger Y_e + f_{25} Y_e^\dagger Y_e + f_{26} Y_e^\dagger Y_e \right) \quad (B33)
\]
with $T$ defined in Eq. (313). Here, $f_{1,\ldots,4}$, $f_{9,\ldots,11}$, $f_{12}$ and $f_{13}$ are expected to be $O(1)$ numbers. $f_i$ ($i=14,15,16$) will have similar forms as $d_i$ ($i=14,15,16$), as given in Eqs. (B17) and (B18) respectively, with all $f_i$ ($i=14,15,16$) being $O(1)$ quantities. As before, we have not written the terms of the form $\text{Tr}[Y_i^\dagger Y_i] \cdot \text{Tr}[Y_j^\dagger Y_j]$, $i, j \in \{e, \nu, U, D\}$, since such terms cannot be generated at 2-loop.
3. 2-loop running of $M_\nu$

Next, we discuss the second order contribution to $\tilde{M}_\nu$. Using Table I and the SU(3) algebra given in Eqs. (3.22) and (B1), we obtain that

$$M_\nu T'_\nu T'_\nu = (1, 1, 6) \otimes (1, 1, 8) \otimes (1, 1, 8) \ni (1, 1, 6),$$  \hspace{1cm} (B34)

$$M_\nu (Y_\nu Y^\dagger_\nu Y_\nu Y^\dagger_\nu) = (1, 1, 6) \otimes (3, 1, 3) \otimes (8 \oplus 1, 1, 1) \otimes (3, 1, 3) \ni (1, 1, 6),$$ \hspace{1cm} (B35)

$$T'^T_\nu M_\nu T'_\nu = (1, 1, 8) \otimes (1, 1, 6) \otimes (1, 1, 8) \ni (1, 1, 6),$$ \hspace{1cm} (B36)

$$M_\nu \text{Tr}[Y^\dagger_\nu Y_\nu] T'_\nu = (1, 1, 6) \otimes (1, 1, 1) \otimes (1, 1, 8) \ni (1, 1, 6),$$ \hspace{1cm} (B37)

and

$$M_\nu \text{Tr}[Y^\dagger_\nu Y_\nu] T'_\nu = (1, 1, 6) \otimes (1, 1, 1) \otimes (1, 1, 8) \ni (1, 1, 6)$$  \hspace{1cm} (B38)

are the only combinations of five spurion fields that can contribute to $\tilde{M}_\nu$. The term in Eq. (B35), not present in Table I, is an allowed combination at second order. Here we have considered the fact that $M_\nu$ couples only to the right-handed neutrinos and hence $\tilde{M}_\nu$ cannot contain trace of four spurions at second order. Apart from the above terms, there will also be terms with three spurions and two other couplings in the theory, transforming trivially under $G_{\text{LF}}$. Terms containing one spurion and four other couplings are also allowed at this order. However, $M_\nu$ being coupled to right-handed neutrinos alone, $\tilde{M}_\nu$ will not contain terms proportional to trace of four $Y_{U,D}$ and also no $g_1^4$ or $\lambda^2$. If the right-handed neutrinos are singlets under the gauge group, as is the case for Type-I seesaw, they will not have any SU(2)$_L$ or SU(3)$_C$ charges and hence terms proportional to $g_2^4$, $g_3^4$ be absent. However, for Type-III seesaw scenario these are triplet under SU(2)$_L$ and hence $g_2^4$ contribution is expected to be there.

Finally, symmetrizing over the O(3)$_{\nu_R}$ indices, the most general form of the 2-loop contribution to $\tilde{M}_\nu$ can be written as

$$(4\pi)^2 \frac{\Delta M_\nu}{\text{2-loop}} = \frac{\tilde{h}_1}{2} \left( M_\nu (T'_\nu T'_\nu) + (T'_\nu T'_\nu)^T M_\nu \right) + \frac{\tilde{h}_2}{2} \left( M_\nu \left( Y_\nu Y^\dagger_e Y_e Y^\dagger_\nu \right) + \left( Y_\nu Y^\dagger_e Y_e Y^\dagger_\nu \right)^T M_\nu \right) \hspace{1cm} (B39)$$

$$+ \frac{\tilde{h}_3}{4} T'^T_\nu M_\nu T'_\nu + \frac{1}{2} \left( \tilde{h}_4 \text{Tr}[Y^\dagger_\nu Y_\nu] + \tilde{h}_4 \text{Tr}[Y^\dagger_\nu Y_\nu] \right) \left( M_\nu T'_\nu + T'^T_\nu M_\nu \right)$$

$$+ \frac{\tilde{h}_4}{2} \left( M_\nu T'_\nu + T'^T_\nu M_\nu \right) + \tilde{h}_5 g_2^4 M_\nu.$$ 

Eq. (B39) can be simplified using the definition of $T'_\nu$ and the fact that terms proportional to $\text{Tr}[Y^\dagger_\nu Y_\nu]$ ($i \in \{e, \nu, U, D\}$) appear only in the combination $T$, defined in Eq. (3.13), to get

$$(4\pi)^2 \frac{\Delta M_\nu}{\text{2-loop}} = h_1 \left( M_\nu (Y_\nu Y^\dagger_e Y_e Y^\dagger_\nu) + (Y_\nu Y^\dagger_e Y_e Y^\dagger_\nu)^T M_\nu \right)$$

$$+ h_2 \left( M_\nu Y_\nu Y^\dagger_e Y_e Y^\dagger_\nu + (Y_\nu Y^\dagger_e Y_e Y^\dagger_\nu)^T M_\nu \right) + h_3 \left( Y_\nu Y^\dagger_\nu \right)^T M_\nu \left( Y_\nu Y^\dagger_\nu \right)$$

$$+ h_4 \left( M_\nu (Y_\nu Y^\dagger_\nu) + (Y_\nu Y^\dagger_\nu)^T M_\nu \right) + h_5 g_2^4 M_\nu,$$ 

where $h_1$, $h_2$, $h_3$ and $h_5$ are expected to be $O(1)$ numbers in general. For Type-I seesaw, $h_5 = 0$. In writing Eq. (B40), we have considered the fact that terms with $\text{Tr}[Y^\dagger_i Y_i]$.
cannot arise at 2-loop. Finally, symmetrizing over the SU(3) $l$, $Y$, $M$, $g$, $h$ and we have

$$h_4 = h_4^{g_1} g_1^2 + h_4^{g_2} g_2^2 + h_4^\lambda + h_4^T T,$$  \( \text{(B41)} \)

where all $h_4^?$ must be of $O(1)$. In writing Eq. (B40) we have used the symmetry property of $M_\nu$: $M_\nu^T = M_\nu$. Leptons and Higgs, being singlets under SU(3)$_C$, $h_4$ will not involve $g_3^2$.

As before, we expect Eq. (B40) to give the right-handed projection of $\dot{M}_\nu$ only. The most general form of $\dot{M}_\nu$ will be given by

$$(4\pi)^2 \dot{M}_\nu \bigg|_{\text{2-loop}} = h_1 \left[ (M_\nu (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)^T M_\nu) P_R + (M_\nu (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)^T + (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)) M_\nu \right] P_L$$

$$+ h_2 \left[ (M_\nu (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)^T M_\nu) P_R + (M_\nu (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)^T + (Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger)) M_\nu \right] P_L$$

$$+ h_3 \left[ (Y_\nu Y_\nu^\dagger)^T M_\nu (Y_\nu Y_\nu^\dagger) P_R + (Y_\nu Y_\nu^\dagger) M_\nu (Y_\nu Y_\nu^\dagger)^T P_L \right]$$

$$+ h_4 \left[ (M_\nu (Y_\nu Y_\nu^\dagger) + (Y_\nu Y_\nu^\dagger)^T M_\nu) P_R + (M_\nu (Y_\nu Y_\nu^\dagger) + (Y_\nu Y_\nu^\dagger)) M_\nu \right] P_L ] + h_5 g_1^2 M_\nu . \quad \text{(B42)}$$

4. 2-loop running of the left-handed mass $m_\nu$ at $\mu < M_R$

At the energy scale $\mu < M_R$, the flavor symmetry group is $G'_{\text{LF}}$ and $Y_\nu (3, 3)$, $m_\nu (6, 1)$ are the only spurions in the theory. Let us first consider the running of $Y_\nu$ at this scale which can be obtained from Eq. (B20) simply by setting the coefficients of terms containing $Y_\nu$ to zero and we have

$$(4\pi)^2 Y_\nu \bigg|_{\text{2-loop}} = Y_\nu \left[ (d_1 Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger) + d_9 \text{Tr}[Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger] + (d_1^2 g_1^2 + d_2 g_2^2 + d_3^T Y_\nu Y_\nu^\dagger) \right]$$

$$+ Y_\nu \left[ (d_1^2 g_1^2 + d_2 g_2^2) \text{Tr}[Y_\nu Y_\nu^\dagger] + d_{16} \right] Y_\nu , \quad \text{(B43)}$$

where $d_{16}$ is given by Eq. (B18). $d_1$, $d_9$, $d_{12}$ and $d_{14}$ are expected to be $O(1)$ numbers.

Now we consider the running of the left-handed mass $m_\nu$. Using Table I the transformation rules in Eq. (B2) and the SU(3) algebra given in Eqs. (3.22) we get the second order contributions to $\dot{m}_\nu$ to contain the following combinations of five spurions:

$$m_\nu T_e T_e = (6, 1) \otimes (8, 1) \otimes (8, 1) \equiv (6, 1) , \quad \text{(B44)}$$

$$T_e^T m_\nu T_e = (8, 1) \otimes (6, 1) \otimes (8, 1) \equiv (6, 1) , \quad \text{(B45)}$$

$$m_\nu \text{Tr}[Y_\nu^\dagger Y_\nu] T_e = (6, 1) \otimes (1, 1) \otimes (8, 1) \equiv (6, 1) , \quad \text{(B46)}$$

and

$$m_\nu \text{Tr}[Y_\nu^\dagger Y_\nu Y_\nu^\dagger Y_\nu] = (6, 1) \otimes (1, 1) = (6, 1) , \quad \text{(B47)}$$

where the terms proportional to $\text{Tr}[Y_\nu^\dagger Y_\nu] \cdot \text{Tr}[Y_\nu^\dagger Y_\nu]$ are to be removed since such terms cannot arise at 2-loop. Finally, symmetrizing over the SU(3)$_{\text{L}}$ indices, we write down the
most general form of $\dot{m}_\nu$ at second order as

$$
(4\pi)^2 \dot{m}_\nu \big|_{\text{2-loop}} = r_1 \left( m_\nu \left( Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger \right) + (Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger)^T m_\nu \right) + r_2 \left( Y_{e}^\dagger Y_{e}^\dagger \right)^T m_\nu \left( Y_{e}^\dagger Y_{e}^\dagger \right) + r_3 \text{Tr}[Y_{e}^\dagger Y_{e}] \left( m_\nu \left( Y_{e}^\dagger Y_{e}^\dagger \right) + (Y_{e}^\dagger Y_{e}^\dagger)^T m_\nu \right) + r_4 \text{Tr}[Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger] m_\nu + r_5 \left( m_\nu \left( Y_{e}^\dagger Y_{e}^\dagger \right) + (Y_{e}^\dagger Y_{e}^\dagger)^T m_\nu \right) + r_6 m_\nu ,
$$

(B48)

where $r_1, r_2, r_3, r_4$ are expected to be of $\mathcal{O}(1)$, while the general forms of $r_5, r_6$ are

$$
r_5 = r_5^U \text{Tr}[Y_{U}^\dagger Y_{U}] + r_5^D \text{Tr}[Y_{D}^\dagger Y_{D}] + r_5^g g_1^2 + r_5^g g_2^2 + r_5^g \lambda ,
$$

(B49)

$$
r_6 = r_6^U \text{Tr}[Y_{U}^\dagger Y_{U} Y_{U}^\dagger Y_{U}] + r_6^D \text{Tr}[Y_{D}^\dagger Y_{D} Y_{D}^\dagger Y_{D}] + r_6^U \text{Tr}[Y_{U}^\dagger Y_{U} Y_{U}^\dagger Y_{U}] + \left( r_6^g g_1^2 + r_6^g g_2^2 + r_6^g g_3^2 \right) \text{Tr}[Y_{U}^\dagger Y_{U}] + \left( r_6^g g_1^2 + r_6^g g_2^2 + r_6^g g_3^2 \right) \text{Tr}[Y_{D}^\dagger Y_{D}] + \left( r_6^g g_1^2 + r_6^g g_2^2 \right) \lambda + r_6^g g_1^2 + r_6^g g_2^2 + r_6^g g_3^2 \lambda ,
$$

(B50)

with all $r_5^g, r_6^g$ being expected to be $\mathcal{O}(1)$ numbers. We can further simplify by considering the fact that terms proportional to $\text{Tr}[Y_{i}^\dagger Y_{i}](i \in \{e, U, D\})$ come through a complete fermion loop in Higgs self-energy corrections and hence we must have

$$
r_5 \text{Tr}[Y_{e}^\dagger Y_{e}] + r_5^U \text{Tr}[Y_{U}^\dagger Y_{U}] + r_5^D \text{Tr}[Y_{D}^\dagger Y_{D}] \rightarrow r_5^T T' ,
$$

where $T'$ is defined in Eq. (B37) and $r_5^T$ is of $\mathcal{O}(1)$. So the 2-loop contribution to $\dot{m}_\nu$ becomes

$$
(4\pi)^2 \dot{m}_\nu \big|_{\text{2-loop}} = r_1 \left( m_\nu \left( Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger \right) + (Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger)^T m_\nu \right) + r_2 \left( Y_{e}^\dagger Y_{e}^\dagger \right)^T m_\nu \left( Y_{e}^\dagger Y_{e}^\dagger \right) + r_4 \text{Tr}[Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger Y_{e}^\dagger] m_\nu + r_5^T \left( m_\nu \left( Y_{e}^\dagger Y_{e}^\dagger \right) + (Y_{e}^\dagger Y_{e}^\dagger)^T m_\nu \right) + r_6 m_\nu ,
$$

(B51)

with

$$
r_5^T = r_5^T T' + r_5^g g_1^2 + r_5^g g_2^2 + r_5^g \lambda .
$$

(B52)

5. Results

First, let us consider the case of the SM. Second order contributions to the RG evolution equations of $Y_e, Y_e, M_\nu$ or $m_\nu$ are not available in the literature for right-handed neutrino extended SM (Type-I or Type-III) in general. In Ref. [24], the contribution to $\dot{m}_\nu$ proportional to $r_2$ in Eq. (B51), for Type-I seesaw, is presented that gives

$$
r_2 = 2 .
$$

(B53)

Thus $r_2$ is of $\mathcal{O}(1)$, as expected. In the future, once a full calculation is done, it can be checked against our results.

Next, we move to the case of the MSSM. Unlike the case of SM, there are existing results for second order contributions in extended MSSM for Type-I seesaw [12], obtained from exact computations. In order to compare the results with the equations obtained above, we keep the following facts in mind:

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• Higgs self-coupling $\lambda$ is absent in MSSM, hence all terms proportional to $\lambda$ will vanish.

• Terms with $Y^i e Y$ can only contain $\Tr[Y^i Y]$ and $\Tr[Y_D^i Y_D]$, while terms with $Y_Y$ can only contain $\Tr[Y^i Y]$ and $\Tr[Y_D^i Y_U]$. Moreover they will appear only in the combinations $T_U$ and $T_D$, defined in Eqs. (4.12) and (4.13) respectively. Thus, the terms present will be $T_D Y^i e Y$ and $T_U Y^i Y_U$.

• $\dot Y$ cannot have terms proportional to $\Tr[Y_Y Y_Y]$ or $\Tr[Y_D^i Y_U Y_U]$. Similarly, $\dot Y$ cannot have terms proportional to $\Tr[Y^i Y e Y]$ and $\Tr[Y_D^i Y_D Y_D]$. Hence

$$d_{10} = 0, \quad d_{16}^{UU} = 0, \quad f_9 = 0, \quad f_{16}^{DD} = 0.$$  (B54)

• Since $Y$ couples to $H_U$ only, $\dot Y$ cannot contain terms $g_i^2 \Tr[Y^i Y]$ or $g_i^2 \Tr[Y^i Y_U]$. Similarly, $\dot Y$ cannot have terms $g_i^2 \Tr[Y^i Y]$ or $g_i^2 \Tr[Y^i Y_D]$. Thus,

$$d_{15}^{q_1} = d_{15}^{q_2} = 0, \quad d_{16}^{q_1} = d_{16}^{q_2} = d_{16}^{q_1} = 0$$

$$f_{14}^{q_1} = f_{14}^{q_2} = 0, \quad f_{16}^{q_1} = f_{16}^{q_2} = f_{16}^{q_3} = 0.$$  (B55)

• The right-handed Majorana mass $M_\nu$ couples only to the right-handed neutrinos which interacts with $H_U$, and not with $H_D$, and so $h_4$ in Eq. (B41) becomes

$$h_4 = h_4^{q_1} g_1^2 + h_4^{q_2} g_2^2 + h_4^T T_U.$$  (B56)

• Only $H_U$ is involved in the definition of the effective left-handed Majorana mass $m_\nu$, and hence we must have

$$T' = T_U = \Tr[Y^i Y] + 3 \Tr[Y^i Y_U],$$  (B57)

and $r_6$ in Eq. (B50) will have

$$r_6^{DD} = 0, \quad r_6^{q_1 D} = 0, \quad r_6^{q_2 D} = 0, \quad r_6^{q_3 D} = 0.$$  (B58)

Let us now compare the coefficients with the values obtained with exact computation [12]. For $Y$ evolution we get

$$d_1 = -4, \quad d_2 = 0, \quad d_3 = -2, \quad d_4 = -2, \quad d_9 = -3, \quad d_{11} = -1,$$

$$d_{12}^{p_1} = 0, \quad d_{12}^{p_2} = 6, \quad d_{12}^{T} = -3, \quad d_{13}^{p_1} = 0, \quad d_{13}^{p_2} = 0, \quad d_{13}^{T} = -1,$$

$$d_{14}^{q_1} = \frac{6}{5}, \quad d_{14}^{q_2} = 0, \quad d_{16}^{DD} = -9, \quad d_{16}^{U} = -3, \quad d_{16}^{q_1 D} = -\frac{2}{5},$$

$$d_{16}^{q_2 D} = 0, \quad d_{16}^{q_3 D} = 16, \quad d_{16}^{p_1} = \frac{27}{2}, \quad d_{16}^{p_2} = \frac{15}{2}, \quad d_{16}^{p_3} = \frac{9}{5}.$$  (B59)
Comparing the coefficients of $\dot{Y}_\nu$, we get:

\[
\begin{align*}
  f_1 &= -2, & f_2 &= -2, & f_3 &= 0, & f_4 &= -4, & f_{10} &= -3, & f_{11} &= -1, \\
  f_{12}^{g_1} &= \frac{6}{5}, & f_{12}^{g_2} &= 0, & f_{12}^T &= -1, & f_{13}^{g_1} &= \frac{6}{5}, & f_{13}^{g_2} &= 6, & f_{13}^T &= -3, \\
  f_{15} &= 0, & f_{15}^{g_2} &= 0, & f_{16}^{UU} &= -9, & f_{16}^{UD} &= -3, & f_{16}^{g_4} &= \frac{4}{5}, \\
  f_{16}^{g_4} &= 0, & f_{16}^{g_4} &= 0, & f_{16}^{g_4} &= \frac{207}{50}, & f_{16}^{g_2} &= \frac{15}{2}, & f_{16}^{g_4} &= \frac{9}{5}. 
\end{align*}
\] (B60)

Finally, comparing the evolution of $M_\nu$ and $m_\nu$ at second order, we find the values of $h_i$ and $r_i$ to be:

\[
\begin{align*}
  h_1 &= -2, & h_2 &= -2, & h_3 &= 0, & h_4^{g_1} &= \frac{6}{5}, & h_4^{g_2} &= 6, & h_4^T &= -2, \\
  r_1 &= -2, & r_2 &= 0, & r_4 &= 0, & r_5^{g_1} &= \frac{6}{5}, & r_5^{g_2} &= 0, & r_5^T &= -1, \\
  r_6^{UU} &= -18, & r_6^{UD} &= -6, & r_6^{g_4} &= \frac{8}{5}, & r_6^{g_4} &= 0, & r_6^{g_4} &= 32, \\
  r_6^{g_4} &= \frac{207}{25}, & r_6^{g_4} &= 15, & r_6^{g_4} &= \frac{18}{5}. 
\end{align*}
\] (B61)

As we can see from Eqs. (B59), (B60), and (B61), there are a few zeros. If supersymmetry is not broken, one has $r_2 = 0$ in MSSM Type-I seesaw [24]. However, the remaining zeros cannot be explained using spurion techniques. There are also some quantities which are not of $\mathcal{O}(1)$, namely $d_{12}^{g_2}, d_{16}^{UD}, d_{16}^{g_4}, d_{16}^{g_4}, f_{13}, f_{16}^{UU}, f_{16}^{g_4}, f_{16}^{g_4}, f_{16}^{g_4}, f_{16}^{g_4}, f_{16}^{g_4}, r_6^{UU}, r_6^{UD}, r_6^{g_4}, r_6^{g_4}$ and $r_6^{g_4}$. Of them, $x_{UU_i}, x_{UD_i}, x_{DD_i}, x_{g_4U_i}, x_{g_4D_i}$ can be large due to color factors, while the remaining become large because of the effect of gauge interactions.

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