Modeling and computation of stretch-forming technological process rational parameters

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Abstract. Mathematical models and numerical methods for solving optimal control problems in the shaping of monolithic panels by stretching on the punch have been developed. As criteria, residual displacements and damage in plasticity are considered. The algorithms implemented in MSC.Marc allow us to calculate the optimal parameters of the stretch forming press work.

1. Introduction
In the aircraft industry, the stretch forming process is one of the main ways of obtaining shells. Existing software stretch forming presses provide the implementation of various schemes of forming single and double curvature sheeting. Various methods of stretch forming on these presses are proposed, for example, taking into account the movements of the workpiece relative to the punch [1, 2], and a certain sequence of the kinematic scheme of the stretch forming process with unloading [3, 4].

Automated shaping of parts on the FET transversal press requires the development of a control program and an electronic model of a stretch forming punch. Software for this equipment allows to obtain the following data on the basis of simplified models: the recommended position of the punch on the press table; program for the work press (the trajectory of the clips); number of transitions; pre-calculated maximum strain in the workpiece. The development of CAE-systems allows to simulate various schemes of stretch forming processes and to choose the best ones [5-8], but for a complete analysis it is necessary to develop an optimization method. For complex forms, intermediate heat treatments are used, which are also simulated using the finite element method [9].

The technological capabilities of the stretch forming process, the accuracy of the resulting part shape depend on the accuracy of the calculated and performed the trajectory of the sheet edges movement and mold tooling, which specifies the anticipate shape of the panel. The anticipate form of the panel should provide the specified residual curvature of the panel after it is release from the force tool. Methods for solving such inverse problems are presented in [10, 11]. In this work, the residual deviations of the unloaded workpiece after the deformation process from the required shape and the work of dissipation in plasticity are considered as a criterion for choosing the optimal trajectory of movement [12].
2. Formulation and method for solving problem

Let $V \subset R^3$ be a bounded domain with a sufficiently regular boundary $S$. The contact surface of die with deformable body is designated through $S_c$. The contact surface of die with deformable body is designated through $cS$. The surface of the workpiece in contact with the clamping devices is indicated by $bS$. Denote by $u = (u_1, u_2, u_3)$, $\bar{u} = (\bar{u_1}, \bar{u_2}, \bar{u_3})$, $\bar{u} = (\bar{u_1}, \bar{u_2}, \bar{u_3})$ - the vectors of current, residual displacements of deformable body and the vectors displacements of the die surface points; $u, \bar{u} \in [W_1^1(Q)]$, $Q = V \times \{0 < t \leq T\}$, $\bar{u} \in [W_1^1(Q)]$, $Q_c = S_c \times \{0 < t \leq T\}$. Here $t$ is the parameter of deformation.

It is assumed that some form of punch is known. In this case, the problem of kinematic shaping will include the problem of deformation in plasticity and elastic unloading. The problems of mechanics are formulated by variational principles with functionals:

$$J(\hat{u}) = W_c + a(\hat{u}, \hat{u}) \quad \text{при} \quad \hat{u} |_{t_0} = \hat{u}^*,$$  

$$J(\hat{u}) = W_c + a(\hat{u}, \hat{u}),$$  

where $W_c$ - the contact potentials received by imposing of contact conditions on the equations of bodies motion by method of multipliers of Lagrange or by method of penal functions [13, 14] and differentiation on $t$; potential form are given by $a(\hat{u}, \hat{v}) = \int_{V} \frac{\partial E(\hat{u}_{ij})}{\partial \hat{u}_{ij}} \hat{v}_{ij} dV$, $a(\hat{u}, \hat{v}) = \int_{V} \left( \frac{\partial E(\hat{u}_{ij})}{\partial \hat{u}_{ij}} \right) \hat{v}_{ij} dV$, $E(\hat{u}_{ij}) = \frac{1}{2} c_{ijkl} \hat{u}_{ij} \hat{u}_{kl} - c_{ijkl} \hat{u}_{ij} \hat{u}_{kl} + \frac{1}{2} \sigma_{ijkl} \hat{u}_{ij} \hat{u}_{kl} + \frac{1}{2} \rho_{ijkl} \hat{u}_{ij} \hat{u}_{kl}$, $\sigma_{ijkl}$ - are the components of the elastic constant tensor; $\dot{\epsilon}_{ij}^p$ - are the plastic strain rates; $(\dot{\epsilon}_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}}$, $\lambda > 0$ - a function requiring a definition, $\sigma_{ij}$ - the vector space component of the stress tensor deviator, is directed normal to the yield surface, equation $f = 0$ defines the surface in space of the stress tensor deviator components [13]), $\dot{\epsilon}_{ij}$ - velocity components of the current and residual deformations of Green-Lagrange, $\sigma_{ij}$, $\rho_{ij}$ - components of the current and residual of the second stress tensor of Piola-Kirchhoff, $\dot{\epsilon}_{ij} = \frac{1}{2}(\dot{u}_{ij} + \dot{u}_{ji} + u_{kj} \dot{u}_{ki} + u_{ik} \dot{u}_{kj})$, $\dot{\epsilon}_{ij} = \frac{1}{2}(\dot{u}_{ij} + \dot{u}_{ji} + \dot{u}_{ij} \dot{u}_{ki} + \dot{u}_{ik} \dot{u}_{kj})$, $u_{ij} = \frac{\partial u_i}{\partial x_j}$, $i, j, k, l = 1, 2, 3$.

The optimal deformation problem is formulated in the following form: how to deform the panel in the stretch forming process on the punch, so that at the final moment of loading $t = T$ to obtain the specified values of plasticity deformation $\varepsilon_{ij}^p$ (values of residual displacements)? The process of deformation during stretch forming is defined by the trajectory of the panel edges $u |_{t_0} = u^*$.

The mathematical formulation of the optimal control problem includes the equations of the deformable solid mechanics, obtained from the stationary conditions (1), (2) and the optimization functional

$$J = \int_{S} \tilde{u}_n (T) dS \rightarrow \text{sup},$$  

where $\alpha$ - is the component of residual displacements, which determines the panel deflection.

An variant of the optimal control problem with a functional is given with the criterion
\[ J = \int_{0}^{T} \sigma_{ij} \dot{e}_{ij}^{p} dV dt \rightarrow \text{inf} . \] (4)

In the first case, the path at which maximal residual displacements are provided (closest to the contact surface of the punch) will be determined, and in the second case, the path with the minimum material damage in plasticity will be determined (the functional represents work of dissipation [12]).

Taking into account the discretization of variation (1), (2), finite element equations are formed to solve the contact problem [13, 14]

\[ i+\Delta t \mathbf{K}^{(i-1)} \Delta \mathbf{U}^{(i)} = i+\Delta t \mathbf{R}^{(i-1)}, \quad i+\Delta t \mathbf{R}^{(i-1)} \mathbf{U}^{(i)} = i+\Delta t \mathbf{R}^{(i-1)}, \] (5)

where \( i+\Delta t \mathbf{K}^{(i-1)}, \ i+\Delta t \mathbf{R}^{(i-1)} \) - are the tangent stiffness matrices (in matrices are already included the additional elements which are formed from contact restrictions), \( i+\Delta t \mathbf{R}^{(i-1)}, \ i+\Delta t \mathbf{R}^{(i-1)} \) - the vector of internal and external forces. The top indexes \( t + \Delta t \) of quantity indicate time for which it is calculated. The top indexes \( (i-1) \) indicate number of iteration at correction of the solution by Newton-Rafson's method. The solution of the following step is found on formulas \( i+\Delta t \mathbf{U} = \mathbf{U} + \Delta \mathbf{U}, \quad i+\Delta t \mathbf{U} = i \mathbf{U} + \Delta \mathbf{U} \). The second problem on the basis of data on initial stress and strain determines unloading movements. After that it is possible to find residual nodal movements \( \Delta \mathbf{U} = \mathbf{U} + \Delta \mathbf{U} \).

It is assumed that at the moment \( t \) all the required values are determined. At each step \( \Delta t \) and at each iteration of the Newton-Raphson procedure, the overlap vector is checked to determine the penetration of a node of one body into another. When a contact surface is defined, a potential variation \( cW \) is added in (5) to calculate the contact forces preventing mutual penetration of the contacting bodies [13,14]. In the MSC.Marc in case of contact with a rigid body during sliding, the procedure of direct constraints is applied, in which the nodes of the deformable body accept the movements of the nodes of the rigid contact body \( \Delta \mathbf{U}_{\text{normal}} = \Delta \mathbf{U} \cdot \mathbf{n} \) on \( S_{c} \) [15].

For an approximate solution of the optimal control problem, the interval \([0, T]\) is divided into \( N \) parts: \( 0 = t_{0} < t_{1} < t_{2} < \ldots < t_{N} = T \). Using time-discrete equations of the step-by-step integration procedure (5), provided \( \Delta t \leq t_{k+1} - t_{k} \), stresses, strains and displacements are calculated. Functionals (3), (4) in this case can be represented as

\[ J = \sum_{k=0}^{N-1} \sum_{\alpha} \sum_{S} \Delta \mathbf{U}_{\alpha}(t) \rightarrow \text{sup}, \quad J = \sum_{k=0}^{N-1} \sum_{\alpha} \sum_{p} \sigma_{ij} \Delta e_{ij}^{p} \rightarrow \text{inf}, \] (6)

where \( \Delta \mathbf{U}_{\alpha}(t) \) - \( \alpha \)-components of the increment of residual displacements of nodes caused by the increment of current displacements.

Equations (5) and optimization criteria (6) form discrete optimal control problems. In such a formulation, the Bellman function is constructed and the problem is solved by the dynamic programming method [16].

3. Numerical results of the problem solution
The forming modeling of a double curvature panel with a thickness of 2 mm is carried out by the finite element method in MSC.Marc. The dimensions of the workpiece are 309x83 mm (figure 1). The workpiece has the properties of material 1163T. The material is isotropic and its characteristics are equal to the following values: Young's modulus \( E = 7454 \text{kg/mm}^2 \), Poisson's ratio \( \nu = 0.34 \), yield strength \( \sigma_{f} = 29.85 \text{kg/mm}^2 \), linear hardening modulus \( E_{f} = 200.75 \text{kg/mm}^2 \). The contact conditions are modeled slippage without friction.

Vector-function \( \mathbf{U}(t) \) of points panel displacements on the boundary \( S_{b} \) is specified components in the form \( \mathbf{U}_{i}(t) = f_{i}(t) \mathbf{U}_{i}^{*}, \quad \mathbf{U}_{j}(t) = f_{j}(t) \mathbf{U}_{j}^{*} \) (\( \mathbf{U}(t) = (\mathbf{U}_{1}(t), \mathbf{U}_{2}(t), \mathbf{U}_{3}(t)) \)), where \( \mathbf{U}_{i}^{*}, \mathbf{U}_{j}^{*} \) - some solution that provides the necessary residual shape of the panel, in which the deformation process is
performed under the condition of sheet thinning less than 20% in the study area. It is necessary to find the optimal functional dependence of the displacement components $U_3 = f(U_1)$.

**Figure 1.** Finite element model of the panel after stretch forming and unloading in comparison with the surface of the punch.

Figure 2 a, b shows the optimal deformation paths in plasticity in stretch forming process with different grids $N = 6$, $N = 10$ by criterion (3), in figure 2 c, d - by criterion (4). In figure 2, the horizontal line is the original panel, the lower curve is the shape of the punch, the movement functions provide wrap of the punch and stretching of the panel (the optimal paths are highlighted).

**Figure 2.** Possible options and the best way of deformation in plasticity by the technology stretch forming on the press

4. Conclusions
In the considered method, the original problem is reduced to a sequence of auxiliary simpler minimization problems. This method reduces the amount of computation compared to a simple
enumeration of various deformation paths, since in the process of calculating non-optimal trajectories are excluded.

As can be seen from the results, the preliminary stretching in the kinematic scheme of shaping by stretch forming, as suggested in [3], will not be optimal according to these criteria.

The developed numerical method implemented in the CAE system makes it possible to take into account the complex geometry of the model, the properties of the material and makes it possible, even at the preproduction stage, to optimize the shaping parameters in order to shorten the process development cycle.

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