Ensemble kalman filter and particle filter-based state estimation on electrical power systems

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Abstract. In this article, a referential study of the sequential importance sampling particle filter with a systematic resampling and the ensemble Kalman filter is provided to estimate the dynamic states of several synchronous machines connected to a modified 14-bus test case, when a balanced three-phase fault is applied at a bus bar near one of the generators. Both are supported by Monte Carlo simulations with practical noise and model uncertainty considerations. Such simulations were carried out in MATLAB by the Power System Toolbox, whereas the evaluation of the Particle Filter and the Ensemble Kalman Filter by script files developed inside the toolbox. The results obtained show that the particle filter has higher accuracy and more robustness to measurement and model noise than the ensemble Kalman filter, which helps support the feasibility of the method for dynamic state estimation applications.

1. Introduction

By definition, dynamic state estimation (DSE) applied to synchronous machines is the dynamic tracking and estimation of the states of a synchronous generator or condenser and related electrical units, given the mathematical model of the system dynamics supported by measurements of electrical or physical variables. The study of estimation of dynamic states is very important due to transition towards the smart grids, where management of electrical power systems requires faster control systems. In this context, it is necessary to know the accuracy of the different state estimation methods.

Most monitoring systems are based on static modeling of the electrical power systems, however, due to variability of the loads connected to the system and the distributed energy resources (DER’s) incorporated into the generation side, the power system no operates in a steady-state [1]. The static state estimation does no capture these dynamics operations, while DSE can do it [2], [3], [4], [5].

Typically, DSE is assisted by phasor measurement units (PMUs) which provide important data for an accurate estimation at very high sampling rate (30 to 60 samples per second). Besides, DSE permits synchronizing with the global positioning system (GPS) clock, and it can capture electromechanical dynamic response.

The methods used to study state estimation on electrical power system (EPS) are: Kalman filters (KF) [6] used for the analysis of dynamic linear systems with normally distributed noise. Extended Kalman filter (EKF) [7], often applied for nonlinear systems, relies on accuracy of the linearization, approximating nonlinear function. Unscented Kalman filter (UKF) [8] that
improves the accuracy of the EKF [9]. Cubature Kalman filter (CKF) [10], ensemble Kalman filter [11] are others KF-based algorithms. The EKF, UKF, CKF, and EnKF have been the most used in the DSE of EPS [12].

The ensemble Kalman filter (EnKF) is a recursive filter suitable to resolve problems with a large number of variables and originally was widely used in weather forecasting where the models are of high order and extremely nonlinear [13].

Particle filters (PF) [14, 15, 16] and other stochastic methods are an alternative to Kalman filters because they are probability-based estimators. PF are robust to nonlinearities and has shown to give better results than EnKF. In addition, advances in computers speed performance and highly trustable measurements from PMUs help to improve performance of the PF.

In [12], four Kalman filter-based algorithm, EKF, UKF, CKF, and EnKF, are compared on the basis of Gaussian distribution and shown that the estimation accuracy of CKF and UKF are better than of the EKF and EnKF. In this work, the main contributions is to show that Sequential Importance Sampling (SIS) PF with Systematic Resampling (SysR) method is suitable for state estimation of a synchronous machine connected to the IEEE modified 14-bus test case system as seen in [17] and provide superior performance compared to EnKF. We consider a multi-machine setting that include transient and subtransient reactances in the dynamic model for synchronous machine, dynamic model for excitation system type IEEE DC1A (IEEE Standard 421.5 -2005), currently accepted by IEEE standard for excitation systems models for power systems stability studies [17], that correspond to the IEEE Type 1 and a dynamic model for turbine governor TG-P Type 1. The estimation accuracy of the filters, PF and EnKF are compared for a balanced three phase fault.

\[ x_k = f_k(x_{k-1}, u_{k-1}, n_{k-1}) \] (1)

The system state \( x_k \) at step \( k \) is a function, represented by \( f_k \), of the previous state \( x_{k-1} \), system input \( u_{k-1} \) and the system process noise \( n_{k-1} \) that is independent and identically distributed (i.i.d.). The function \( f_k \) can be nonlinear. The filter has the objective to estimate recursively the state \( x_k \) considering the system representation based on measurement function, given by

\[ y_k = h_k(x_k, u_k, m_k) \] (2)

Where, measurement \( y_k \) is a function, (possibly nonlinear, represented by \( h \)) of the state \( x_k \), input \( u_k \) and \( m_k \) that is an i.i.d. measurement process noise. According to Bayesian theory, the state \( x_k \) can be recursively estimated if measurements up to step \( k \) are available. The rest of this work is organized as follows: section 2 and section 3 show relevant theory of EnKF and PF, respectively. The main results and a brief discussion are presented in section 5 while section 6 presents the conclusions.

2. Ensemble Kalman filter method

The ensemble Kalman Filter (EnKF) is a recursive filter, where the forecast \((f)\) at time step \( k \) is carried out based on the information obtained from the analysis \((a)\) step \( k - 1 \). The EnKF methodology relies on the ensemble representation of the predictive and filtering probability density functions \( pdfs \) known as the forecast ensemble and the analysis ensemble, respectively. Thus the EnKF combines a stochastic prediction step with a stochastic analysis step [18].

An approach to EnKF used for state estimation of the IEEE modified 14-bus test case system is presented in Algorithm 1. Where, in the forecast step, \( x^f_{k,i} \) is the nonlinear forecast ensemble model and \( x^f_k \) is the mean of the approximated predictive probability density function \((pdf)\).

The sample mean of the modeled observation ensemble is \( y_k \). The cross covariance between the forecast ensemble and the modeled observation ensemble is \( P^xy_k \) while \( P^yy_k \) is the sample covariance of the modeled observation ensemble.
In the analysis step is not necessary to evaluate the covariances $P_{f_k}^k$ and $P_{a_k}^k$ to compute the Kalman gain. The Kalman gain is based on sample covariances, allows to approach the EnKF by the generation of surrogate observations $Y_{s_k}^i = y_{s_k,i}^i$, $i = 1, \ldots, N$ where $y_{s_k,i}^i$ are random samples with mean $y_k$ and covariance $R_k$. $x_{a_k,i}^i$ is the analysis ensemble obtained by replacing the observation by a set of surrogates observations.

This shows us that all ensemble members of a particular state are updated by the same Kalman gain but different innovation term. Due to possible sampling errors in the generation of the surrogate observations, the covariance matrix of the observation is replaced by the sample covariance, $R_{s_k}^i$, in the calculation of the Kalman gain [19].

**Algorithm 1** Ensemble Kalman Filter

For $k = 1$ to the number of time steps

1. Forecast step:
   
   $x_{f,k,i}^i = f_{k,k-1}(x_{f,k-1,i}^i)$, $i = 1, 2, \ldots, N$, \\
   $x_{f,k}^i = \frac{1}{N} \sum_{i=1}^{N} x_{f,k,i}^i$, \\
   $y_k = \frac{1}{N} \sum_{i=1}^{N} h_k x_{f,k,i}^i$, \\
   $P_{xy}^k = \frac{1}{N-1} \sum_{i=1}^{N} (x_{f,k,i}^i - x_{f,k}^i)(h_k x_{f,k,i}^i - y_k)^T$, \\
   $P_{yy}^k = \frac{1}{N-1} \sum_{i=1}^{N} (h_k x_{f,k,i}^i - y_k)(h_k x_{f,k,i}^i - y_k)^T$.

2. Analysis step:
   
   $y_{s,k,i}^i \sim \mathcal{N}(y_k ; y_k, R_k)$; \\
   $R_k = \frac{1}{N-1} \sum_{i=1}^{N} (y_{s,k,i}^i - y_k)(y_{s,k,i}^i - y_k)^T$, \\
   $K_k = P_{xy}^k (P_{yy}^k + R_k)^{-1}$, \\
   $x_{a,k,i}^i = x_{f,k}^i + K_k (y_{s,k,i}^i - h_k x_{f,k}^i)$, $i = 1, \ldots, N$.

3. Particle filtering method

The particle filter implements Bayesian tracking using sequential Monte Carlo methods, the posterior density function is approximated by a set of weighted randomly generated samples obtaining a closer approximation to the true representation when the number the samples increases, that is known as optimal Bayesian solution. The density function is represented by:

$$p(x_{0:k} | y_{0:k}) \approx \sum_{i=1}^{N} w_{k,i} \Delta(x_{0:k} - x_{0:k,i})$$

(3)

Where, $x_{0:k}$ is the set of all states up to step $k$, $y_{0:k}$ the set of measurements up to step $k$, the delta function $\Delta$, $x_{0:k,i}^i (i = 1, \ldots, N)$ a set of particles, $N$ the number of particles and $w_{k,i}$ a set of weights for particles chosen by importance sampling (IS) normalized such that $\sum_{i=1}^{N} w_{k,i} = 1$ [9].

The Particle Filter is a purely probabilistic method. The methodology consists of sampling from a known proposal distribution, also referred to as the importance distribution, the sampling
from the proposal should be easy \( q(x_k|y_{1:k}) \). A brief overview of the importance sampling approach for the expectation \( E[g(x_{0:k})] \) is given in [16].

The methodology for approximating the posterior pdf consist in drawing particles from the proposal distribution, i.e. \( x_{k,i} \sim q(x_k|x_{0:k-1}, y_{1:k}), i = 1, \ldots, N \).

The expected value is the conditional mean of \( x_k \), i.e. \( E[x_k|y_{1:k}] \), with the approximation given by:

\[
E[x_k] = \frac{1}{N} \sum_{i=1}^{N} x_{k,i} \tilde{w}_{k,i} \tag{4}
\]

Where \( \{x_{k,i}; i = 1, \ldots, N\} \) is the set of particles which are drawn according to the proposal pdf.

The SIS presents a problem, has limitations in terms of performance after a few recursive steps, known as the degeneracy problem [20], which is observed by monitoring the importance weights at every time step on the model from the entire set of particles, it may be that only one particle of the normalized importance weights have weight close to 1 while the remaining particles have weights close to zero, resulting in a large number of particles have insignificant weights, this may lead to a wrong sample approximation of the posterior function. This problem is solved by using a resampling procedure, widely used in practice, the Systematic Resampling (SysR) algorithm [21]. We use this algorithm in this work. Local simulation studies were performed with 50 particles.

The PF developed is based on the SIS with a SysR step added in order to handle the degeneracy problem with minimum random resampling and low computational cost. The algorithm does not take into account the past values of the importance weights and the conditional mean of the particles is given by the sample mean. The procedure to apply the PF is shown in Algorithm 2.

**Algorithm 2** Particle Filter

For \( k = 1 \) to the number of time steps

1. Forecast step:
   \( x_{k,i} \sim p(x_k|x_{k-1,i}^*), \quad i = 1, 2, \ldots, N \),

2. Analysis step:
   \( w_k(x_{k,i}) = p(y_k|x_{k,i}), \quad i = 1, 2, \ldots, N \),
   \( \tilde{w}_k(x_{k,i}) = \frac{w_k(x_{k,i})}{\sum_{i=1}^{N} w_k(x_{k,i})} \)

Obtain the resample set \( x_{k,i}^* \) according to SysR

\[
E[x_k] = \frac{1}{N} \sum_{i=1}^{N} x_{k,i}^* \]

4. Dynamic models

The synchronous machine can be represented with the voltage behind subtransient reactance model [22].

\[
\dot{\delta} = \omega - \omega_s \tag{5a}
\]

\[
\dot{\omega} = \frac{\pi f}{H} (-D(\omega - \omega_s) + T_m - P_e) \tag{5b}
\]
behind a subtransient reactance: \( E_{q}' \), subtransient voltage, \( E_{d}' \) voltage behind a transient reactance: \( E_{q}'' \), transient voltage behind \( \delta \), rotor angle, \( \omega \) is the rotor speed, \( \omega_s \) is the nominal rotor speed, \( f \) is the frequency, \( E_{q}' \) is the q-axis transient voltage, \( E_{d}' \) is the d-axis transient voltage \( (E_q', E_d') \) are internal voltage behind a transient reactance: \( E' = V_s + j X_d I_s \), \( X_d \) is the d-axis synchronous reactance, \( X_q' \) is the q-axis transient reactance, \( E_q'' \) is the q-axis subtransient voltage, \( E_d'' \) is the d-axis subtransient voltage \( (E_q'', E_d'') \) are internal voltage behind a subtransient reactance: \( E'' = V_s + j X_q'I_s \), \( X_q'' \) is the q-axis subtransient reactance, \( X_d'' \) is the d-axis subtransient reactance, \( E_{fd} \) is the field voltage, \( I_q \) is the q-axis current, \( I_d \) is the d-axis current, \( T_m \) is the mechanical torque, \( P_e \) is the active power, \( V_s \) is the terminal voltage and \( I_s \) is the stator current.

The state variables are rotor angle \( (\delta) \), rotor speed \( (\omega) \), the q-axis transient voltage \( (E_q') \), and the d-axis transient voltage \( (E_d') \) from each generator and compensator, which in total add up to 20 state variables, four states per each synchronous machine. The measurements are electrical power \( P_{elect} \), field voltage \( E_{fd} \), and mechanical torque \( T_m \), from each generator and compensator, which are assumed to be accessible through PMUs.

In this work, we simulate the system using the Power System Toolbox (PST) a MATLAB-based power system simulation software. The scripts for the EnKF and the PF were written inside the source code known of the PST as functions to perform the dynamic state estimation of the system under study. The function of PST used to model a synchronous machine with the voltage behind subtransient reactance model was ssimu - mac sub [23].

5. Simulation and analysis results
The performance of the EnKF and PF are tested and evaluated on a modified IEEE 14-bus system were a disturbance by a three phase fault was introduced. IEEE 14-bus system was modified to comply with bus-voltage level of other similar systems with different voltages level at buses connections of synchronous machines. In these systems the machines are connected through an ideal transformer to the original bus and the topology of the system is not changed as in figure 1. The IEEE 14-bus test system after the modification is shown in figure 2. The generators are equipped with governor models and exciter models.

Measurements, were contaminated with a random additive Gaussian distributed noise of 1% with a mean equal to zero and a low level of variability to consider the uncertainty of measurements. Each synchronous machine contributed with 4 state variables: rotor angle \( \delta \), rotor angular velocity \( \omega \), the d-axis transient voltage \( E_d' \) and the q-axis transient voltage \( E_q' \), which yields a total of 20 state variables for the entire system.
The sampling rate of the DSE was discretized by $\Delta t = 0.005\,[s]$. An ensemble set size of $N = 50$ was used in EnKF. The number of particles for the PF was set to $N = 50$. These settings have shown good results in [15]. The EnKF and PF were initialized with steady state values obtained from pre-fault conditions of the system. Two commonly known values are defined as constant: $f_0 = 60\,[Hz]$, $\omega_0 = 1.0\,[pu]$. The parameters of each synchronous machine, exciters and governors were taken from [24], table 1 to table 3.

A balanced three phase fault is applied on the electrical grid on line connecting bus 1 and bus 2, at time $t = 0.1\,s$. The near end of the fault is bus 1, and the far end of the fault is bus 2. The fault is cleared in the near end $0.01\,s$ after the three phase fault (6.6 cycles later), and it is cleared in the far end after $0.002\,s$ of clearing the fault on the near end (6.72 cycles later).

From figure 3 to figure 17, are shown the dynamic response of the state variables, $E_d'$, $E_q'$ and $\omega$, for the synchronous machines 1, 2, 3, 4 and 5 connected to buses 15, 16, 17, 19 and 18 respectively, on modified 14 bus test case systems, figure 2, for a three phase fault applied and cleared like was described before. The true states values from simulation results are given by solid lines while dashed lines represent EnKF and PF state estimations. The number of particles for PF and the ensemble set size of EnKF for each simulation of the state variables are 50. It can be seen that once the fault is applied, the rotor angle of each synchronous machine, starts a transient period.
Table 1. IEEE 14-bus modified test system synchronous machine data-100 MVA base.

| Type       | SyncG | SyncG | SyncC | SyncC |
|------------|-------|-------|-------|-------|
| obn(mbu)   | 1(15) | 2(16) | 3(17) | 6(19), 8(18) |
| Rated MVA  | 448   | 100   | 40    | 25    |
| Rated Kv   | 22    | 13.8  | 13.8  | 13.8  |
| H(s)       | 11.8989 | 4.985 | 0.6080 | 0.3000 |
| r_a (p.u.) | 0.00096 | 0.0035 | 0.000 | 0.0100 |
| x_d (p.u.) | 0.3728 | 1.180 | 5.9325 | 7.0760 |
| x_q (p.u.) | 0.3571 | 1.050 | 2.9300 | 3.4200 |
| x'_d (p.u.)| 0.0592 | 0.220 | 0.8575 | 1.2160 |
| x'_q (p.u.)| 0.1027 | 0.380 | 2.9300 | 2.3180 |
| x"_d (p.u.)| 0.0458 | 0.145 | 0.5775 | 0.8140 |
| x"_q (p.u.)| 0.0458 | 0.145 | 0.5775 | 0.8140 |
| x_l (p.u.) | 0.0335 | 0.075 | 0.33  | 0.4180 |
| T'_d0 (s)  | 0.5871 | 1.100 | 11.600 | 8.000 |
| T'_q0 (s)  | 0.1351 | 0.1086 | 0.1590 | 0.0080 |
| T"_d0 (s)  | 0.0248 | 0.0277 | 0.058  | 0.0525 |
| T"_q0 (s)  | 0.0267 | 0.0351 | 0.2010 | 0.0151 |
| S(1.0)     | 0.091 | 0.0933 | 0.295  | 0.304 |
| S(1.2)     | 0.400 | 0.4044 | 0.776  | 0.666 |

Table 2. IEEE 14-bus modified test system exciter data: IEEE DC1A of the IEEE Standard 421.5 (2005).

| Type       | IEEEET1 |
|------------|---------|
| obn(mbu)   | 1(15)   | 2(16)   | 3(17)   | 6(19), 8(18) |
| Rated MVA  | 448     | 100     | 40      | 25         |
| Rated Kv   | 22      | 13.8    | 13.8    | 13.8       |
| K_a (p.u.) | 50      | 25      | 400     | 400        |
| T_r (s)    | 0.0     | 0.060   | 0.0     | 0.0        |
| T_a (s)    | 0.060   | 0.200   | 0.050   | 0.050      |
| V_{Rmax} (p.u.) | 1.000 | 1.000  | 16.630  | 4.407      |
| V_{Rmin} (p.u.)  | -1.000 | -1.000 | -16.630 | -4.407     |
| K_e (p.u.)  | -0.0465 | -0.0582 | -0.170  | -0.170     |
| T_e (s)     | 0.520   | 0.6544  | 0.950   | 0.950      |
| K_f (p.u.)  | 0.0832  | 0.105   | 0.040   | 0.040      |
| T_f (s)     | 1.000   | 0.350   | 1.000   | 1.000      |
| E_1 (p.u.)  | 3.240   | 2.5785  | 6.375   | 4.2375     |
| SE(E_1)     | 0.072   | 0.0889  | 0.2174  | 0.2174     |
| E_2 (p.u.)  | 4.320   | 3.438   | 8.500   | 5.650      |
| SE(E_2)     | 0.2821  | 0.3468  | 0.9388  | 0.9386     |
Table 3. IEEE 14-bus modified test system governor data: Turbine - TG-P Type 1.

| Type          | TG - P | TG - P |
|---------------|--------|--------|
| obn(mbn)\textsuperscript{a} | 1(15)  | 2(16)  |
| Rated MVA     | 448    | 100    |
| Rated Kv      | 22     | 13.8   |
| 1/R (p.u.)    | 25     | 25     |
| $W_f$ (p.u.)  | 1.0    | 1.0    |
| $T_{max}$ (s) | 1.0    | 1.0    |
| $T_s$ (s)     | 0.1    | 0.1    |
| $T_c$ (s)     | 0.5    | 0.5    |
| $T_3$ (s)     | 0.0    | 0.0    |
| $T_4$ (s)     | 1.25   | 1.25   |
| $T_5$ (s)     | 10     | 0      |

and continues to decrease. At the beginning, oscillations occur in the state variables $\omega$, $E'_d$, and $E'_q$ and after 3 or 4 seconds this signals stabilize. The angular speed of synchronous machines shown oscillations of small amplitude and recover their nominal values.

![Synchronous machine #1](image.png)

**Figure 3.** Estimation and true values of Ed prima of synchronous machine 1

The EnKF presents an approaching noisier than PF to the true values. The root-mean square error (RMSE) is used to compare the estimated accuracy \[25\] of the EnKF and the SIS PF algorithms:

$$e_x = \sqrt{\frac{\sum_{i=1}^n \sum_{k=1}^N (x_{i,k}^{\text{est}} - x_{i,k}^{\text{true}})^2}{nN}} \quad (6)$$

Where, $n$ indicates the number of generators, $N$ is the number of times step, and $x_{i,k}^{\text{est}}$ and $x_{i,k}^{\text{true}}$ represent the estimated and true value of the $i$th generator at time step $k$, respectively. RMSE values are summarized in Table 4 and shown in figure 18. The accuracy of Particle filter is better than ensemble Kalman filter.
Figure 4. Estimation and true values of $E_d$ prima of synchronous machine 2 by EnKF and PF.

Figure 5. Estimation and true values of $E_d$ prima of synchronous machine 3 by EnKF and PF.

Figure 6. Estimation and true values of $E_d$ prima of synchronous machine 4 by EnKF and PF.
Figure 7. Estimation and true values of $E_d$ prima of synchronous machine 5 by EnKF and PF.

Figure 8. Estimation and true values of $E_q$ prima of synchronous machine 1 by EnKF and PF.

Figure 9. Estimation and true values of $E_q$ prima of synchronous machine 2 by EnKF and PF.
Figure 10. Estimation and true values of Eq prima of synchronous machine 3 by EnKF and PF.

Figure 11. Estimation and true values of Eq prima of synchronous machine 4 by EnKF and PF.

Figure 12. Estimation and true values of Eq prima of synchronous machine 5 by EnKF and PF.
Figure 13. Estimation and true values of angular speed of synchronous machine 1 by EnKF and PF.

Figure 14. Estimation and true values of angular speed of synchronous machine 2 by EnKF and PF.

Figure 15. Estimation and true values of angular speed of synchronous machine 3 by EnKF and PF.
Figure 16. Estimation and true values of angular speed of synchronous machine 4 by EnKF and PF.

Figure 17. Estimation and true values of angular speed of synchronous machine 5 by EnKF and PF.

Table 4. RMSE of estimation results for particle filter and ensemble Kalman filter.

| Filter type | State variables | $\delta$     | $\omega$     | $E_{q}'$  | $E_{d}'$   |
|-------------|-----------------|--------------|--------------|-----------|------------|
| Pf          | $2.573 \times 10^{-4}$ | $6.693 \times 10^{-5}$ | $3.605 \times 10^{-4}$ | 0.0015    |
| EnKf        | 0.0163          | 0.0031       | $3.849 \times 10^{-4}$ | 0.0032    |
6. Conclusion

This work uses the SIS PF with a SysR and the EnKF to estimate state variables of synchronous machines: generators and condensers, connected to a modified IEEE 14-bus test case. Based on the simulation results, it can be concluded that the PF attains a greater stability, higher estimation accuracy and better performance than EnKF for each state variable considered under contingencies, in this case a balanced three phase fault.

The PF provides effective performance from the first step on and requires no time for settling. It also can be concluded that the synchronous machines maintain synchronism between them after the occurrence of the contingency. The use of other electrical devices such as different exciters, governors, power system stabilizers as well as other types of filters are left to be investigated in future work.

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