Big rip avoidance via black holes production

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Abstract

We consider a cosmological scenario in which the expansion of the Universe is dominated by phantom dark energy and black holes which condense out of the latter component. The mass of black holes decreases via Hawking evaporation and by accretion of phantom fluid but new black holes arise continuously whence the overall evolution can be rather complex. We study the corresponding dynamical system to unravel this evolution and single out scenarios where the big rip singularity does not occur.
I. INTRODUCTION

Phantom dark energy fields are characterized by violating the dominant energy condition, $ρ + p > 0$. Thereby the conservation equation, $\dot{ρ} + 3H(ρ + p) = 0$, has the striking consequence that the energy density increases with expansion [1, 2]. In the simplest case of a constant ratio $w \equiv p/ρ$ one has $ρ \propto a^{-3(1+w)}$, where $1 + w < 0$, while the scale factor obeys $a(t) \propto (t^* - t)^{-n}$ with $n = -2/[3(1 + w)]$ and $t \leq t^*$, being $t^*$ the “big rip” time (at which the scale factor diverges). However, as we shall show below, the big rip may be avoided if black holes are produced out of the phantom fluid at a sufficiently high rate. One may think of three different mechanisms by which phantom energy goes to produce black holes.

(i) In phantom dominated universes more and more energy is continuously pumped into any arbitrary spatial three-volume of size $R$. Hence, the latter will increase as $a$ while the mass, $M$, inside it will go up as as $a^{m+3}$ -with $m = 3|1 + w|$. Therefore, the ratio $M/R$ will augment with expansion whenever $m > -2$ (i.e., $w < -1/3$, dark energy in general). As a consequence of the energy being pumped faster than the volume can expand, the latter will eventually contain enough energy to become a black hole. This is bound to occur as soon as $ρ R^3 \geq R/(2G)$ (i.e., when $R \geq (2G ρ)^{-1/2}$).

(ii) As statistical mechanics tells us, equilibrium thermal fluctuations obey $< (δE)^2 > = k_B C_V T^2$, where $δE = E - < E >$ is the energy fluctuation at any given point of the system around the mean value $< E >$, $C_V$ the system heat capacity at constant volume, $k_B$ Boltzmann’s constant, and $< ... >$ denotes ensemble average [3]. As demonstrated in [4], unlike normal matter, dark energy gets hotter with expansion according to the law $T \propto a^{-3w}$. The latter follows from integrating the temperature evolution equation $\dot{T}/T = -3H(\partial p/\partial ρ)_n$ which, on its turn, can be derived from Gibbs’ equation $TdS = d(ρ/n) + p d(1/n)$ and the condition that the entropy be a state function, i.e., $\partial^2 S/(\partial T \partial n) = \partial^2 S/(\partial n \partial T)$ -see [4] for details. Therefore, it is natural to expect that the phantom fluid will be very hot at the time when it starts to dominate the expansion. Since the phantom temperature grows unbounded so it will the energy fluctuations as well -a straightforward calculation yields $< (δE)^2 > \propto a^{-3(1+2w)}$. Eventually they will be big enough to collapse and fall within their Schwarzschild radius. (This simple mechanism was applied to hot thermal radiation well before phantom energy was introduced [5]).

(iii) As demonstrated by Gross et al. [6] thermal radiation can give rise to a copious
production of black holes of mass $T^{-1}$ whenever $T$ is sufficiently high. In this case, black holes nucleate because the small perturbations around the Schwarzschild instanton involve a negative mode. This produces an imaginary part of the free energy that can be interpreted as an instability of the hot radiation to quantum tunnel into black holes. A simpler (but less rigorous) treatment can be found in Ref. [7]. One may speculate that black holes might also come into existence by quantum tunnelling of hot phantom fluid similarly as hot radiation did in the very early Universe.

Notice that mechanisms (i) and (iii) differ from the conventional gravitational collapse at zero temperature -the latter does not produce black holes when the fluid has $w < -1/3$, see e.g. [8].

Obviously, one may wonder whether if phantom can produce particles other than black holes. In actual fact, there is no reason why dark energy in general (not just phantom) should not be coupled to other forms of matter. However, its coupling to baryonic matter is highly constrained by measurements of local gravity [9] but not so to dark matter. In any case, we will not consider such possibility because it would introduce an additional variable in our system of equations below (Eqs. (6)- (9)) and would increase greatly the complexity of the analysis.

The number black holes in the first generation (assuming that all those initially formed arise simultaneously) will be $N_i = \left(\frac{a}{R}\right)^3_i = C a^{3-9(1+w)/2}$ with $C = [2G\rho_0 a_0^{3(1+w)}]^{3/2}$ -here the subscript zero signals the instant at which phantom dark energy starts to overwhelmingly dominate all other forms of energy-, and we may, reasonably, expect that they will constitute a pressureless fluid. Later on, more black holes may be formed but if much of the phantom energy has gone into black holes, then the second generation will not come instantly (as there will be less available phantom energy than at $t = t_i$).

Further, the black holes will accrete phantom energy and lose mass at a rate $\dot{M} = -16\pi M^2 \dot{\phi}^2$ regardless of the phantom potential, $V(\phi)$ [10]. (Notice that for scalar phantom fields $\rho + p = -\dot{\phi}^2$). Additional mass will be lost to Hawking radiation [11].

The system of equations governing this complex scenario is

$$\dot{\rho}_{bh} + 3H\rho_{bh} = \Gamma \rho_x + 16\pi nM^2 (1 + w)\rho_x - n \frac{\alpha}{M^2},$$  \hspace{1cm} (1)

$$\dot{\rho}_x + 3(1 + w)H\rho_x = -\Gamma \rho_x - 16\pi nM^2 (1 + w)\rho_x,$$ \hspace{1cm} (2)
\begin{align}
\dot{\rho}_\gamma + 4H\rho_\gamma &= n\frac{\alpha}{M^2}, \\
3H^2 &= \rho_x + \rho_{bh} + \rho_\gamma, \\
\dot{M} &= 16\pi M^2(1 + w)\rho_x - \frac{\alpha}{M^2}.
\end{align}

(subscripts \(bh, x, \) and \(\gamma\) stand for black hole, phantom, and radiation components, respectively, and we use units in which \(8\pi G = c = \hbar = 1\)). Here, \(\Gamma = \text{constant} > 0\) denotes the rate of black hole formation, \(n = N/a^3\) the number density of black holes, \(\alpha\) a positive constant, and \(H = \dot{a}/a\) the Hubble function. The first term in (5) corresponds to the mass loss rate of a single black hole via phantom accretion; the second term to spontaneous Hawking radiation [12]. For simplicity, we assume that the black holes solely emit relativistic particles (i.e., a fluid with equation of state \(p_\gamma = \rho_\gamma/3\)). This explains Eq. (3). The second term on the right of Eq. (2) ensures the energy conservation of the overall phantom plus black hole fluid in the process of phantom accretion -the mass loss of black holes must go into phantom energy.

At first glance, depending on the values assumed by the different parameters, \(\Gamma, w, \) and \(\alpha\), two very different outcomes seem possible: (i) a big rip singularity, if the phantom energy eventually gets the upper hand; and (ii) a quasi-equilibrium situation between the black hole and phantom fluids, if none of them comes to dominate the expansion -in this case the big rip cannot be guaranteed since the overall equation of state will not be constant.

There are five equations and, seemingly, six unknowns, namely, \((\rho_{bh}, \rho_x, \rho_\gamma, H, n \) and \(M)\) but, in actual fact only five \((\rho_{bh}, \rho_x, \rho_\gamma, a, \) and \(M)\) as \(n\) can be written as \(\rho_{bh}/M\).

To make things easier, we shall further assume that the black holes are massive enough to safely neglect Hawking evaporation and that, initially, there was no radiation present. The latter assumption is not much unrealistic, the fast expansion redshifts away radiation and dust matter very quickly. (The black-hole fluid also has the equation of state of dust but, as said above, black holes are continuously created out of phantom). Thus, we can dispense with Eq. (3) and the last term on the right hand side of Eqs. (1) and (5).

One may wonder whether the black holes may coalesce leading to bigger black holes and produce a big amount of relativistic particles. We believe we can safely ignore this possibility since the fast expansion renders the chances of black holes encounters highly unlikely.
Accordingly, the system of equations reduces to

\[
\dot{\rho}_{bh} + 3H\rho_{bh} = \Gamma \rho_x + 16\pi (1 + w)M \rho_{bh} \rho_x ,
\]

(6)

\[
\dot{\rho}_x + 3(1 + w)H\rho_x = -\Gamma \rho_x - 16\pi M(1 + w)\rho_{bh} \rho_x ,
\]

(7)

\[
3H^2 = \rho_x + \rho_{bh},
\]

(8)

\[
\dot{M} = 16\pi M^2 (1 + w)\rho_x.
\]

(9)

Thus, we may choose the three unknowns, \(\rho_{bh}, \rho_x\) and \(M\) -since \(H\) is linked to \(\rho_{bh}\) and \(\rho_x\) by the constraint Eq. (8).

Let us assume a phantom dominated universe (i.e., no other energy component enters the picture initially). Sooner or later a first generation of black holes will arise; then, the question arises: “will new black holes condense out of phantom (second generation) before the first generation practically disappears eaten by the phantom fluid and consequently, the Universe become forever dominated by a mixture of phantom and black holes, or the black holes will disappear before they can dispute the phantom the energy dominance of cosmic expansion?”

To ascertain this we shall apply, in the next Section, the general theory of dynamical systems [13] to the above set of equations (6)–(9). We shall analyze the corresponding critical points at the finite region as well as at infinity. As we will see, due to the existence of a critical point in the finite region of the phase portrait (connected with the creation rate, \(\Gamma\), of new black holes), there exist solutions that instead of ending up at the big rip tend asymptotically to a Minkowski spacetime. Hence, in some cases, depending on the initial conditions, the big rip singularity can be avoided thanks to the formation of black holes out of the phantom fluid.

Before going any further, it is fair to say that, in reality, no one knows for certain if phantom fluids have a place in Nature: they may suffer from quantum instabilities [14], although certain phantom models based in low–energy effective string theory may avoid them [15]. On the other hand, observationally they are slightly more favored than otherwise -though, admittedly, this support has dwindled away in the last couple of years. In view of this unsettled situation, we believe worthwhile to explore the main possible consequences its actual existence may bring about on cosmic evolution.
II. DYNAMICAL STUDY

Here we apply the theory of dynamical systems to analyze the above set of differential equations (6)–(9). Using the total density $\rho_T = \rho_{bh} + \rho_x$, the system can be recast as

$$\dot{\rho}_x = -3(1 + w)\sqrt{\rho_T}\rho_x - \Gamma \rho_x - 16\pi(1 + w)M\rho_x(\rho_T - \rho_x),$$

$$\dot{\rho}_T = -3(\rho_T^{3/2} + w\rho_T^{1/2}\rho_x),$$

$$\dot{M} = 16\pi M^2(1 + w)\rho_x.$$  

Its critical points follow from setting $\dot{\rho}_x = \dot{\rho}_T = \dot{M} = 0$.

Three possible situations arise, namely:

1. $M = 0$ and $\rho_x = 0$;
2. $M = 0$ and $\rho_x \neq 0$;
3. $M \neq 0$ and $\rho_x = 0$.

In virtue of Eq. (11), the first one implies $\rho_T = 0$, i.e., the origin. The second one corresponds to

$$\rho_T = -w\rho_x \quad \text{and} \quad \sqrt{\rho_T} = -\frac{\Gamma}{3(1 + w)}.$$  

(13)

Since both densities must be semi-positive definite, we have that $w < -1$ which is consistent with the assumption of phantom fluid. For fluids satisfying the dominant energy condition this critical point does not exist. Finally, the third case also implies the origin as, by virtue of Eq. (11), $\rho_T = 0$.

A. The critical points at the finite region

In the finite region, the obvious critical point is the origin ($M = \rho_x = \rho_T = 0$), and, for the phantom case ($w < -1$), the point given by

$$\rho_x = -\frac{\Gamma^2}{9w(1 + w)^2}, \quad \rho_T = \frac{\Gamma^2}{9(1 + w)^2}, \quad M = 0.$$  

(14)
After linearizing the system of equations (10)-(12), we get

\[
\dot{\delta \rho_x} = - \left[ \frac{3}{2}(1 + w) \frac{\rho_x}{\sqrt{\rho_T}} + 16\pi M (1 + w) \rho_x \right] \delta \rho_T \\
- \left[ 3(1 + w) \sqrt{\rho_T} + \Gamma + 16\pi M (1 + w) (\rho_T - 2\rho_x) \right] \delta \rho_x \\
- 16\pi (1 + w) (\rho_T - \rho_x) \rho_x \delta M , \\
\delta \dot{\rho_T} = - \frac{3}{2} \left[ 3\sqrt{\rho_T} + w \frac{\rho_x}{\sqrt{\rho_T}} \right] \delta \rho_T - 3w \sqrt{\rho_T} \delta \rho_x , \\
\delta \dot{M} = 32\pi M (1 + w) \rho_x \delta M + 16\pi M^2 (1 + w) \delta \rho_x .
\]

(15)

1. **The critical point at the origin**

In this case, the system reduces to

\[
\dot{\delta \rho_x} = - \Gamma \delta \rho_x , \\
\dot{\delta \rho_T} = \delta \dot{M} = 0 .
\]

(18)

Here we have made the reasonable assumption that \( \rho_x / \sqrt{\rho_T} = 0 \) as \( \rho_x, \rho_T \to 0 \). This critical point is an attractor since its sole eigenvalue is negative.

2. **The finite critical point**

By imposing Eqs. (14) on (15)-(17) and linearizing, we get

\[
\dot{\delta \rho_x} = - \frac{\Gamma}{2w} \delta \rho_T + \frac{16\pi \Gamma^4}{81(1 + w)^2 w^2} \delta M , \\
\dot{\delta \rho_T} = \frac{\Gamma}{1 + w} \delta \rho_T + \frac{\Gamma w}{1 + w} \delta \rho_x , \\
\delta \dot{M} = 0 .
\]

(19)

(20)

(21)

The roots of the characteristic equation of this system are:

\[
\lambda_{\pm} = \frac{\Gamma}{2(1 + w)} [1 \pm \sqrt{-1 - 2w}] .
\]

(22)

In view that \( w < -1 \), both eigenvalues are real, one positive and the other negative, i.e., a saddle point.
B. The critical point at infinity

A full analysis, in the original three-dimensional system, of the critical point at infinity, is considerably hard since one has to embed the system in a four-dimensional space of difficult visualization. We will consider, instead, the three two-dimensional systems resulting from projecting the original one upon three mutually orthogonal and complementary planes.

1. The critical point at infinity in the plane \((ρ_x, ρ_T)\)

By setting \(M = 0\) the system (10)-(12) gets projected onto the plane \((ρ_x, ρ_T)\), resulting

\[
\begin{align*}
\dot{x} &= -3(1 + w)\sqrt{yx} - Γx = X(x, y), \quad (23) \\
\dot{y} &= -3y^{3/2} - 3w\sqrt{yx} = Y(x, y), \quad (24)
\end{align*}
\]

where \(x = ρ_x\) and \(y = ρ_T\). This plane corresponds to the situation that all black holes are formed with the same mass which does not vary with time.

Let us introduce a new, ancillary, coordinate \(z\), to deal with points at infinity, and consider the unit sphere in the three–dimensional space \((x, y, z)\). Then, by defining new coordinates, \(u\) and \(v\), by

\[
x = \frac{u}{z}, \quad y = \frac{v}{z}, \quad u^2 + v^2 + z^2 = 1, \quad (25)
\]

we can write,

\[
\begin{align*}
X(x, y) &= -3(1 + w)\sqrt{yx} - Γx \\
&= \frac{1}{z^{3/2}} \left[ -3(1 + w)\sqrt{vu} - Γu \right] \left(\frac{z}{z^{1/2}}\right) = \frac{1}{z^{3/2}} P(u, v, z), \quad (26)
\end{align*}
\]

\[
\begin{align*}
Y(x, y) &= -3y^{3/2} - 3w\sqrt{yx} \\
&= \frac{1}{z^{3/2}} \left[ -3v^{3/2} - 3wv^{1/2}u \right] \left(\frac{z}{z^{1/2}}\right) = \frac{1}{z^{3/2}} Q(u, v, z). \quad (27)
\end{align*}
\]

For the two–dimensional system (23)-(24) one follows

\[-Ydx + Xdy = 0. \quad (28)\]

Moreover,

\[
dx = \frac{du}{z} - \frac{u}{z^2}dz, \quad \text{and} \quad dy = \frac{dv}{z} - \frac{v}{z^2}dz. \quad (29)
\]
Hence

\[ Adu + Bdv + Cdz = 0, \quad (30) \]

where

\[ A = -zQ, \quad B = zP, \quad C = uQ - Pv. \quad (31) \]

Now, we construct a new three–dimensional system

\[ \dot{u} = Bz - Cv, \quad (32) \]
\[ \dot{v} = Cu - Az, \quad (33) \]
\[ \dot{z} = Av - Bu. \quad (34) \]

The region at infinity follows by setting \( z = 0 \). This implies

\[ uQ - vP = 0, \quad u^2 + v^2 = 1, \quad (35) \]

i.e.,

\[ u = 0, \quad v = 1, \quad \text{and} \quad u = v = \frac{\sqrt{2}}{2}. \quad (36) \]

These two are the critical points at infinity.

Let us begin by considering the first one, namely, \( u = 0, v = 1 \). To this end we perform the transformation,

\[ \xi = \frac{u}{v}, \quad \eta = \frac{z}{v}, \quad (37) \]

and obtain the following system,

\[ \dot{\xi}v + \xi\dot{v} = B\eta v - Cv, \quad (38) \]
\[ \dot{v} = C\xi v - A\eta v, \quad (39) \]
\[ \dot{\eta}v + \eta\dot{v} = Av - B\xi v. \quad (40) \]

Then, the two–dimensional system at infinity is,

\[ \dot{\xi} = -C(1 + \xi^2) + B\eta + A\eta\xi, \quad (41) \]
\[ \dot{\eta} = A(1 + \eta^2) - B\xi - C\xi\eta.\quad (42) \]
Upon linearizing around the critical point at infinity ($\xi = \eta = 0$, $v = 1$) we get,

\begin{align}
\dot{\xi} &= -3w\xi, \\
\dot{\eta} &= 3\eta.
\end{align}

(43) (44)

Altogether, this critical point at infinity is a saddle point for $w > 0$ and a repeller for $w \leq 0$.

To study the second critical point, $u = v = \sqrt{2}/2$, we perform a clockwise rotation so that the old $u$ axis comes to coincide with the new axis, $v'$. Proceeding as before, we obtain

\begin{align}
\dot{\xi}' &= 3w\sqrt{\frac{2}{2}} \xi' , \\
\dot{\eta}' &= 3(1 + w)\sqrt{\frac{2}{2}} \eta'.
\end{align}

(45) (46)

Clearly, this point is an attractor for $w < -1$, a saddle for $-1 < w < 0$, and a repeller for $0 < w$. Notice that the big rip singularity corresponds to this point when $w < -1$.

The top panel of Fig. 1 displays the phase portrait. All solutions start from $\rho_T = \rho_{bh} \to \infty$ (i.e., the top point on the vertical axis, $x = 0$). Some of them cannot avoid the big rip singularity (i.e., the common point to the circle and the straight line $x = y$). Those solutions that end up at the center of the circle (the Minkowski state, $\rho_x = \rho_{bh} = 0$) evade the big rip. We remark, by passing, that except for the particular case $\Gamma = 0$, no solution can go from the origin to the infinity along the straight line $\rho_x = \rho_T$.

For vanishing $\Gamma$ (bottom panel of Fig. 1) the finite critical point collapses to the critical point at the origin, which becomes a saddle point. This represents the usual scenario of a system composed of pressureless and phantom fluids in which is implicitly assumed that no black holes are produced.

The corresponding phase portraits when the dominant energy condition is satisfied are shown in the top ($0 > w > -1$) and bottom ($w > 0$) panels of Fig. 2.
FIG. 1: Phase portraits of the system (23)-(24) when the dark energy is of phantom type. The top panel corresponds to the case of a non-vanishing rate of black hole formation. The finite critical point (given by Eq. (14)), a saddle, acts a divider: trajectories at its left, and some passing through it, avoid the big rip; trajectories to its right, and some passing through it, end up at the big rip -see text. The bottom panel corresponds to the case of no black hole production. In this case, all trajectories end up at the big rip.
FIG. 2: Phase portrait of the system (23)-(24), with $\Gamma = 0$, when the dark energy obeys the dominant energy condition but fails to fulfill the strong energy condition (top panel). For the sake of completeness, the bottom panel displays the situation in which the fluid obeys both energy conditions. In both cases, because of the absence of phantom fluid, there is neither black hole production nor big rip.

2. The critical point at infinity in the plane $(\rho_x, M)$

The plane $\rho_T = 0$ does not belong to the physical region since it requires negative energy densities. Anyway, for completeness let us analyze this case. After setting $\rho_T = 0$, we get

$$\dot{x} = -\Gamma x + (1 + w) y x^2 = X(x, y), \quad (47)$$
$$\dot{y} = (1 + w) y^2 x = Y(x, y) \quad (48)$$

where $x = \rho_x$ and $y = 16\pi M$. 
Performing the transformations
\[ u = zx, \quad v = zy, \quad u^2 + v^2 + z^2 = 1, \quad (49) \]
it follows that,
\[ X(x, y) = \frac{1}{z^3} \left\{ -\Gamma z^2 u + (1 + w)u^2 v \right\} = \frac{1}{z^3} P(u, v, z), \quad (50) \]
\[ Y(x, y) = \frac{1}{z^3} (1 + w)v^2 u = \frac{1}{z^3} Q(u, v, z). \quad (51) \]
As before, the relation
\[ -Ydx + Xdy = 0 \quad (52) \]
implies
\[ -zQ du + zP dv + (uQ - vP) dz = 0. \quad (53) \]
At infinity, \( z = 0 \), the relationship \( uQ - vP = 0 \) is identically fulfilled. Hence, all points satisfying \( u^2 + v^2 = 1 \) are singular. This corresponds to a circle centered at the origin. Since the latter is an attractor, all the trajectories emanating from the infinity go to the origin to end there. So, the infinity is a repeller.

3. The plane \((\rho_T, M)\)

Upon setting \( \rho_x = 0 \) the original system reduces to
\[ \dot{\rho}_T = -3 \rho_T^{3/2}, \quad \dot{M} = 0. \quad (54) \]
which admits the simple solution, \( \rho_T \propto t^{-2} \). Hence, the solution comes from the infinity to the origin along the axis \( \rho_x = 0 \).

III. DISCUSSION

In this paper, we briefly raised the point that black holes may be produced in phantom dominated universes by three different mechanisms: (i) via energy accumulation in any given spatial three-volume, (ii) gravitational collapse of huge thermal fluctuations, and (iii) quantum tunnelling of very dense and hot phantom fluid [6]. In this regard, phantom fluids
might be viewed as “black holes factories”. However, while these processes look rather plausible the corresponding calculations are pending.

Nevertheless, after accepting that black holes may be produced by any of the sketched mechanisms, we studied the dynamical system associated to this scenario. Our main finding is that because of the existence of a critical saddle point in the finite region of the plane $(\rho_x, \rho_T)$ -given by Eqs. (14)- the big rip singularity (a generic feature of phantom-dominated universes) is no longer unavoidable. This critical point lies in the region of positive densities as $w < 0$. Moreover, since $\rho_T \geq \rho_x$ and $\rho_T/\rho_x = -w$, this point is located in the physical region ($\rho_T \geq \rho_x > 0$) when $w < -1$, that is, for phantom equations of state.

In the said plane $(\rho_x, \rho_T)$, there is an attractor at the origin, also the finite critical point described above (a saddle), and two critical points at infinity: one situated at the axis $\rho_T$ (a repeller), and another at the straight line $\rho_T = \rho_x$ (an attractor). The finite critical point, connected with $\Gamma > 0$, is a divider between the solutions that go from $\rho_x \to \infty$ to the origin and from $\rho_x \to \infty$ to the big rip. Those solutions that pass through this critical point, depending on the initial conditions, can either go to the origin (i.e., implying that the big rip is avoided), or to the critical point at infinity with $\rho_x = \rho_T$ (i.e., big rip) -see top panel of Fig. 1.

We are now in conditions to fix more precisely the possible scenario described above. Let us consider the relations for the finite critical point, Eqs. (14). Bearing in mind that the Universe is nearly spatially flat ($\Omega_T = 1$), and conceding that this critical point may lie not very far from the present expansion era (the Universe began accelerating recently), we find $\Gamma = -3\sqrt{3} (1 + w)H_0$, and $\Omega_{x0} = -w^{-1}$. The Wilkinson microwave anisotropy probe (WMAP) [16] data are consistent with $w \simeq -1.1$. Hence, for the big rip to be attained the present dark energy density parameter must fulfill $\Omega_{x0} \gtrsim 0.9$. Since WMAP indicates $\Omega_{x0} \simeq 0.7$, our Universe may well avoid the big rip singularity (modulo $w$ is really a constant). This also implies $\Gamma \sim 0.5 H_0$.

Our system is a three–dimensional one and obviously there are other dimensions. In the plane $(\rho_x, M)$ all points at infinity are singular; projecting this onto the Poincaré sphere we obtain, in that plane, a singular circle around the origin. But, since the origin is an attractor, all the points in this circle are repellers.

As said above, some solutions can evade the big rip. Obviously, the latter become unavoidable if $\Gamma = 0$ (no black hole production), since in this case the critical point at the
finite region coincides with the origin.

There are also other situations in which the big rip can be avoided. For instance, when phantom dark energy corresponds to the generalized Chaplygin gas proposed in [17], or when wormholes intervene [18], or when the curvature scalar gets very large and quantum effects become dominant [19]. Nevertheless, to the best of our knowledge, the present scenario was never considered in the literature.

By contrast, as noted by Barrow [20], there are situations in which finite-time future singularities can arise even if the fluid filling the Universe obeys \( \rho > 0 \) and \( \rho + 3p > 0 \), i.e., under very mild conditions. We do not consider them here.

Admittedly, it can be argued that in view of the various simplifying assumptions, our treatment is not much realistic. In the first place, the rate \( \Gamma \) is not expected to be a constant, it will likely vary with expansion and depend on quantities like \( w \) and \( M \). Secondly, we have implicitly considered that all black holes are formed simultaneously with the same mass—a flat spectrum. It would be more natural to assume the number of black holes produced varies with mass and time. Further, as noted earlier, black hole spontaneous radiance should be included. Clearly, these features ought to be incorporated in future, more realistic, treatments. Nonetheless, we believe this small, first, step may lead the way to more ambitious undertakings.

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[1] R.R. Caldwell, Phys. Lett. B 545, 23 (2002).
[2] R.R. Caldwell et al., Phys. Rev. Lett. 91, 071301 (2003).
[3] See any standard textbook on statistical mechanics.

[4] D. Pavón and B. Wang, Gen. Relativ. Grav. 41, 1 (2009), arXiv:0712.0565.

[5] T. Piran and R.M. Wald, Phys. Lett. 90A, 20 (1982).

[6] D. Gross, M.J. Perry, and G.L. Yaffe, Phys. Rev. D 25, 330 (1982).

[7] J.I. Kapusta, Phys. Rev. D 30, 831 (1984).

[8] R.-G. Cai and A. Wang, Phys. Rev. D 73, 063005 (2006).

[9] P.J.E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003); K. Hagiwara et al., Phys. Rev. D 66, 010001 (2002).

[10] E. Babichev et al., Phys. Rev. Lett. 93, 021102 (2004).

[11] S.W. Hawking, Commun Math. Phys. 43, 199 (1975).

[12] D.N. Page, Phys. Rev. D 13, 198 (1976).

[13] G. Sansone and R. Conti, Equazioni Differenziali Non Lineari (Edizioni Cremonese, Rome, 1956).

[14] J.M. Cline, S.-Y. Jeon, and G.D. Moore, Phys. Rev. D 70, 043543.

[15] F. Piazza and S. Tsujikawa, JCAP 07(2004)004.

[16] E. Komatsu et al., “Five-years Wilkinson microwave anisotropy probe (WMAP) observations: cosmological interpretation”, arXiv:0803.0547.

[17] P.F. González-Díaz, Phys. Rev. D 68, 021303(R) (20063).

[18] J.A. Jiménez Madrid, Phys. Lett. B 634, 106 (2006).

[19] E. Elizalde, S. Nojiri, and S. Odintsov, Phys. Rev. D 70, 043539 (2004).

[20] J.D. Barrow, Class. Quantum Grav. 21, L79 (2004).