EFFICIENT CLASSICAL SIMULATION OF MEASUREMENTS IN OPTICAL QUANTUM INFORMATION

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We present conditions for the efficient simulation of a broad class of optical quantum circuits on a classical machine: this class includes unitary transformations, amplification, noise, and measurements. Various proposed schemes for universal quantum computation using optics are assessed against these conditions, and we consider the minimum resource requirements needed in any optical scheme to generate optical nonlinear processes and perform universal quantum computation.

Information processing using the rules of quantum mechanics may allow tasks that cannot be performed using classical laws. Of the many possible realizations of quantum information processes, optical realizations have the advantage of negligible decoherence. Both qubit and continuous-variable schemes offer significant potential for optical quantum information processing, especially if efficient processes can be performed that are not efficient on any classical device.

Advanced techniques in linear optics and squeezing are known to be insufficient to perform universal quantum computation; in particular, optical nonlinear processes (such as a Kerr nonlinearity) have been identified as a necessary requirement. (Schemes that employ only linear optics are not scalable, in that they require resources that grow exponentially in the number of qubits.) However, Kerr nonlinearities suffer either from weak strengths or high losses, and the lack of appropriate nonlinear materials greatly restricts the type of processes that can be performed in practice.

Recently, nonunitary processes such as measurement have been identified as a means to implement nonlinear operations. Proposals for optical quantum computation by Knill, Laflamme and Milburn (KLM) and Gottesman, Kitaev and Preskill (GKP) employ photon counting to induce nonlinear transformations in optical systems. Photon counting is an important example of a process that can be used to achieve nonlinear transformations via feedforward of measurement results. Such a nonunitary transformation appears to enable impressive capabilities equivalent to nonlinear transformations.

It is imperative to determine what type of processes (unitary transformations, projective measurements, interaction with a reservoir, etc.) can be used to implement nonlinear transformations and thus perform universal quantum computation. One approach is to identify classes of processes that can be efficiently simulated on a classical computer. Under the assumption that universal quantum computation is not efficiently simulatable classically, such
processes are also insufficient to implement optical nonlinear transformations. The Gottesman-Knill (GK) theorem, the continuous-variable classical simulatability theorem of Bartlett et al. (BSBN), and the general optical classical simulatability theorem of Bartlett and Sanders (BS) allow us to investigate the classical complexity of a quantum process using particular classes of initial states, unitary operations and measurements.

Here, we consider the implications of the BS optical classical simulatability theorem on various proposals for quantum computation using optics. This theorem employs the powerful formalism of Gaussian completely positive (CP) maps to describe efficiently simulatable operations on Gaussian states.

**Theorem for efficient classical simulatability (BS):** Any quantum information process that initiates in a Gaussian state and that performs only Gaussian CP maps can be efficiently simulated using a classical computer. These maps include (i) the unitary transformations corresponding to linear optics and squeezing, (ii) linear amplification (including phase-insensitive and phase-sensitive amplification and optimal cloning), linear loss mechanisms or additive noise, (iii) measurements that are Gaussian CP maps including, but not limited to, projective measurements in the position/momentum eigenstate basis or coherent/squeezed state basis, with finite losses, and (iv) any of the above Gaussian CP maps conditioned on classical numbers or the outcomes of prior Gaussian CP measurements (classical feedforward).

Our theorem for efficient classical simulation provides a powerful tool in assessing whether a given optical process can enhance linear optics to perform nonlinear transformations or allow quantum processes that are exponentially faster than classical ones. Algorithms or circuits employing Gaussian CP maps can be efficiently simulated on a classical computer, and thus do not provide any sort of quantum exponential speedup.

We now consider some of the key new results of this theorem in terms of known processes.

**Corollary 1:** Linear optics or squeezing transformations conditioned on the measurement outcome of homodyne detection with finite losses using Gaussian states cannot induce a nonlinearity.

Thus, initiating with Gaussian states, it is not possible to use homodyne measurements and feedforward of measurement results to induce a (possibly nondeterministic) optical nonlinearity in the way that photon counting allows in the KLM scheme. In terms of optical implementations of quantum computing, this theorem reveals why all previous schemes either propose some form of optical nonlinearity, use other forms of measurement such as photon counting, or are not efficiently scalable.

This theorem also places severe constraints on the use of photodetection to perform nonlinear transformations in realizations of optical quantum
Table 1. Efficient classical simulatability for schemes employing various initial states, unitary gates, and measurements.

| Initial States | Unitary Gates                  | Measurements                  | Efficiently simulatable? |
|----------------|--------------------------------|-------------------------------|-------------------------|
| Vacua          | Linear optics, squeezing       | Gaussian CP (i.e., homodyne)  | Yes                     |
| Vacua          | Linear optics, squeezing,      | Homodyne                      | No (Lloyd & Braunstein) |
| Single photons | Linear optics only             | Photon counting               | No (KLM)                |
| Vacua          | Linear optics, squeezing       | Photon counting & homodyne    | No (GKP)                |
| Single photons | Linear optics, squeezing       | Homodyne                      | ???                     |

computing. For a threshold photodetector with perfect efficiency, the POVM is given by two elements, corresponding to “absorption” and “no-absorption” of light. Photon counters are effectively constructed as arrays of such detectors. The vacuum projection describes the non-absorption measurement, and the corresponding map describing this measurement result is Gaussian CP. However, the absorption outcome is not.

Corollary 2: Gaussian CP maps conditioned on the no-absorption outcome of a photodetection measurement are also Gaussian CP, whereas transformations conditioned on the absorption outcome are not.

Note that the same result holds for finite-efficiency photodetectors: such detectors can be modelled as unit efficiency photodetectors with a linear loss mechanism describable using Gaussian CP maps. Thus, the absorption outcome of photodetection and the feedforward of this measurement result is a key resource for optical quantum information processing. This corollary also proves that any nonlinear gate employing linear optics and photon counting must be nondeterministic; a photon counting measurement of a Gaussian state could possibly result in an outcome of zero photons, and such a result corresponds to an efficiently classically simulatable process. (Note that nonlinear optics, in contrast, can be deterministic.)

Our classical simulatability theorem may be useful in assessing the “minimum” requirements for universal quantum computation with optics. Table 1 presents various classes of initial states, unitary gates, and measurements (that can be used for classical feedforward) and their classical simulatability according to our theorem. Employing only Gaussian states and Gaussian CP maps results in an efficiently simulatable circuit; one can now consider supplementing this set with various “resources” that may allow for universal quantum computation. As shown by Lloyd and Braunstein, the addition of a Kerr nonlinearity or any higher-order transformation on a single mode
results in universal quantum computation. The schemes of KLM and GKP reveal that photon counting is also a resource that allows for universal quantum computation. The KLM scheme also requires single photon Fock states “on demand” as ancilla inputs to their nondeterministic nonlinear gates; such states lie outside the domain of our theorem (they are not Gaussian) and may serve as a resource for performing nonlinear operations.

It is interesting to consider, then, if single photons on demand are by themselves sufficient to bestow Gaussian CP maps with the power to perform nonlinear operations and thus universal quantum computation. Considering the recent progress in creating single photon turnstile devices\(^1\) (with low probability of producing zero or two photons by accident), a scheme that requires single photons but otherwise employs only linear optics, squeezing, and high-efficiency homodyne detection would obviate the need for ultra-high efficiency photon counters\(^4\).

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