Abstract

Some of basic problems in neutrino physics such as new energy scales, the enormous gap between neutrino masses and the lightest charged fermion mass, possible existence of sterile neutrinos in eV mass range are studied in the local gauge group $SU_L(4) \times U(1)$ for electroweak unification, which does not contain fermions with exotic electric charges. It is shown that neutrino mass spectrum can be decoupled from that of the other fermions. Further normal seesaw mechanism for neutrinos, with right handed neutrino Majorana masses to be of order $M \gg M_{\text{weak}}$ as well a new eV-scale can be accommodated. The eV-scale seesaw may manifest itself in experiments like Liquid Scintillation Neutrino Detector (LSND) and MiniBooNE (MB) experimental results and future neutrino experiments.

In recent years enormous progress has been made in neutrino physics, which also has relevance to many fields other than the particle physics; in particular, nuclear physics, astrophysics and cosmology. This has been made
possible by the quantum mechanical phenomena of interferometry which provides sensitive method to explore extremely small effects. This has resulted in the discovery of neutrino oscillations which imply that they have tiny but finite masses against the prediction of the standard model of particle physics. Thus they provide evidence for new physics which goes beyond the standard model. New physics requires new energy scale beyond that provided by the standard model but such a scale has not yet been pinned down. Thus there is a need to consider extensions of the electro-weak group which would provide a new scale between the electroweak and grand unification. The neutrinos may also provide an understanding of the origin of matter (baryogenesis) through leptogenesis. For this purpose right handed neutrinos, which are seesaw partners of light neutrinos, with a mass scale of $10^{10} - 10^{11}$ GeV or even in TeV region may be needed. Further more one sees the enormous gap between neutrino masses, revealed by neutrino oscillations, and the lightest charged fermion mass ($m_e$) in contrast to that between $m_e$ and $m_t$ (top quark mass), which is populated by charged leptons and quarks. Further ($m_\nu$)$_{\text{max}} / m_e < 2 \times 10^{-6}$ which needs to be understood. This may be an indication of decoupling of neutrino mass spectrum from other fermions. Moreover while all neutrino data can be explained by flavor oscillations of three active neutrinos [1], the Liquid Scintillation Neutrino Detector (LSND) anomaly [2] stands out. This anomaly together with MiniBooNE (MB) experiment [3] may require at least two sterile neutrinos [4], [5] that mix with the active neutrinos. Another possibility is by a decaying sterile neutrino, again in eV range [6]. Their mass is in the range of electrovols. The purpose of this paper is a modest attempt towards understanding of some of the problems mentioned above.

We consider the extension of electroweak group $SU_L(2) \times U(1)$ to $SU_L(4) \times U_X(1)$ as such an extension can answer some of the questions raised above as we shall see. In particular, it is shown that in addition to normal seesaw mechanism for neutrino masses, where right handed Majorana mass $\gg M_{\text{weak}}$ a new eV-scale can be accommodated. The latter may be a manifestation of LSND and MB experiments. Further we show that neutrino mass spectrum can be decoupled from that of charged leptons.

If one restricts oneself to only $SU_L(4)[7]$, one can not only accommodate known leptons nicely, but also a right handed Majorana neutrino. However in order to accommodate the quarks, the group has to be extended to $SU_L(4) \times U_X(1)[8]$, [9], [10]. In the original minimal version, where leptons ($l^c, \nu^c_i, N_i, l$)$_R$ form an $SU(4)$ quartet, in order to cancel the anomalies, one
has to have quarks with exotic electric charges $\frac{4}{3}$ and $\frac{5}{3}$. In this paper we shall consider other versions, which do not involve quarks with exotic charges $[^{11}]$.

The electric charge operator can, in general be defined as a linear combination of diagonal generators of the group $[\hat{I},$ being the unit matrix]

$$ Q = \frac{1}{2}\left[\lambda_3 + \frac{b}{\sqrt{3}}\lambda_8 + \frac{2c}{\sqrt{6}}\lambda_{15}\right] + \frac{Y_X}{2}\hat{I} $$

$$ = \text{diag}\left[\frac{1}{2} + \frac{b}{6} + \frac{c}{6} + \frac{Y_X}{2}, -\frac{1}{2} + \frac{b}{6} + \frac{c}{6} + \frac{Y_X}{2}, -\frac{2b}{6} + \frac{c}{6} + \frac{Y_X}{2}, -\frac{c}{2} + \frac{Y_X}{2}\right] $$

$$ = \frac{1}{2}(\lambda_3 + \hat{Y}_1) + \frac{Y_X}{2}\hat{I} \quad (1) $$

where $Y_X$ is the hypercharge associated with $U_X$ and

$$ \hat{Y}_1 = \text{diag}\left[\frac{b + c}{3}, \frac{b + c}{3}, -\frac{2b + c}{3}, -c\right] \quad (2) $$

Now

$$ \frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \quad (4) $$

$$ \frac{1}{g'^2} = \frac{1}{g_1^2} + \frac{1}{g_X^2} \quad (5) $$

where in the $SU_L(4)$ limit

$$ g_1^2 = \frac{1}{C_1^2}g^2 $$

$$ C_1^2 = \frac{b^2 + 2c^2}{3} \quad (3) $$

This gives

$$ \frac{1}{g'^2} = \frac{b^2 + 2c}{3g^2} + \frac{1}{g_X^2} $$

Since $\frac{g'}{g} = \tan \theta_W$, one obtains

$$ \frac{g_X^2}{g^2} = \frac{\sin^2 \theta_W(m_X)}{1 - \frac{3+b^2+2c^2}{3}\sin^2 \theta_W(m_X)} \quad (4) $$

In the minimal version, $b = 1$, $c = 2$

$$ \frac{g_X^2}{g^2} = \frac{\sin^2 \theta_W(m_X)}{1 - 4\sin^2 \theta_W(m_X)} \quad (5) $$
and \( Q = (1, 0, 0, -1) + \frac{Y_X}{2}(1, 1, 1, 1) \), so that for leptons, \( Y_X = 0 \), where as in order to accomodate quarks we take, \( Y_X = -\frac{2}{3} \) for the first two generations of quarks and \( Y_X = -\frac{1}{3} \) for the third generation. This is because in order to cancel the anomalies, one generation is to be treated differently from the other two. In this case we have quarks with exotic electric charges \(-\frac{4}{3}\) and \(-\frac{5}{3}\) respectively.

In order to accomodate known isospin doublets of left-handed quarks and leptons in the two upper components of 4 and 4* (or 4*, 4) representations of \( SU(4) \) and to forbid exotic electrical charges, we must have \( \frac{b+c}{6} = \pm \frac{1}{4}, \; \frac{b+2c}{6} = -\frac{5}{2} \) so that \( b = 2c = \pm 1 \). We thus consider the version with \( b = 2c = 1 \) (the other choice is equivalent). This choice gives \( C_1^2 = \frac{1}{2} \)

\[
\frac{g_X^2}{g^2} = \frac{\sin^2 \theta_W(m_X)}{1 - \frac{3}{2} \sin^2 \theta_W(m_X)}
\]

A straight forward application of renormalization group equations gives \[8\] \[ \sin^2 \theta_W = \sin^2 \theta_W(m_Z) \]

\[
1 - (1 + C_1^2) \sin^2 \theta_W - \frac{\alpha^{-1}(m_Z)}{\alpha^{-1}(m_Z)} = \frac{2\alpha(m_Z)}{4\pi} \left[-C_1^2 \left(-\frac{22}{3} + \frac{4nf}{3} \right) + \frac{4nf}{3} C_1^2 \right] \ln \frac{m_X}{m_Z}
\]

\[
= \frac{\alpha(m_Z)}{4\pi} \frac{44}{3} C_1^2 \ln \frac{m_X}{m_Z}
\]

For our case \( C_1^2 = \frac{1}{2} \) and we obtain

\[
1 - \frac{3}{2} \sin^2 \theta_W - \frac{\alpha(m_Z)}{\alpha^{-1}(m_Z)} = \frac{22}{3} \frac{\alpha(m_Z)}{4\pi} \ln \frac{m_X}{m_Z}
\]

where \( \sin^2 \theta_W(m_Z) = 0.23122 \) and \( \alpha^{-1}(m_Z) = 128 \). The unification scale \( m_X \) is not very sensitive to \( \alpha_X \). For \( m_X = 10^3 \text{GeV}, 10^6 \text{GeV}, 10^{10} \text{GeV}, 10^{16} \text{GeV}, \alpha_X = 1.22 \times 10^{-2}, 1.28 \times 10^{-2}, 1.37 \times 10^{-2}, 1.54 \times 10^{-2} \) respectively. To put it in proper perspective, we note that the coupling \( \alpha' = \frac{\alpha}{\cos^2 \theta_W} \) associated with \( U(1) \) of the Standard Model is \( \approx 1.30\alpha \approx 1.02 \times 10^{-2} \). Before we give the anomaly free fermion content, we note that for our choice \( b = 2c = 1 \),

\[
Q = \text{diag}[\frac{3}{4} + \frac{Y_X}{2}, -\frac{1}{4} + \frac{Y_X}{2}, -\frac{1}{4} + \frac{Y_X}{2}, -\frac{1}{4} + \frac{Y_X}{2}]
\]

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We can have two possibilities; which are given in Table 1 below.

| SU(4) quartet: | SU(4) singlet: |
|----------------|----------------|
| $F_{L}^i \equiv \left( \begin{array}{c} \nu_i \\ e_i^- \\ E_i^- \\ F_i^- \end{array} \right)_{L, \ Y_X = -\frac{3}{2}}$ | $F_{R}^i \equiv \left( \begin{array}{ccc} N_i & e_i^- & F_i^- \end{array} \right)_{R}$ |
| $i = 1, 2, 3$ | $Y_X : \begin{pmatrix} 0 & -2 & -2 \end{pmatrix}$ |
| $F_{1R}^q \equiv \left( \begin{array}{c} u_1^a \\ d_1^a \\ D_1^a \\ H_1^a \end{array} \right)_{R, \ Y_X = -\frac{1}{2}}$ | $F_{1R}^q \equiv \left( \begin{array}{ccc} u_1^a & d_1^a & D_1^a & H_1^a \end{array} \right)_{R}$ |
| $F_{2R}^q \equiv \left( \begin{array}{c} \nu_i^{ac} \\ e_i^{ac} \\ U_i^{ac} \\ T_i^{ac} \end{array} \right)_{R, \ Y_X = -\frac{5}{2}}$ | $F_{2R}^q \equiv \left( \begin{array}{ccc} \nu_i^{ac} & e_i^{ac} & U_i^{ac} & T_i^{ac} \end{array} \right)_{R}$ |
| $i = 2, 3$ | $Y_X : \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} & -\frac{2}{3} \end{pmatrix}$ |

The second alternative is very attractive, as it can naturally accommodate more than one right handed neutrinos per generation, some of which can be identified with sterile neutrinos when the $SU_L(4) \times U_X(1)$ symmetry is suitably broken in the lepton sector.
We first note that $SU(4)$ lepton multiplet split as follows under the subgroup $SU_L(2) \times U_Y(1)$ of $SU(4)$ as a doublet

$$\left( \begin{array}{c} e_i^c \\ \nu_i^c \end{array} \right)_R, \quad Y_1 = \frac{1}{2}$$

and two singlets

$$\left( \begin{array}{c} N_i \\ N_i^s \end{array} \right)_R, \quad Y_1 = \frac{-1}{2}$$

After breaking the group $SU(4) \times U_X(1)$ to $SU_L(2) \times U_Y(1)$ of the standard model, we have a doublet $\left( \begin{array}{c} e_i^c \\ \nu_i^c \end{array} \right)_R$ with $Y = 1$, a singlet $e_i^c_L$ with $Y = 2$ and two singlets $\left( \begin{array}{c} N_i \\ N_i^s \end{array} \right)_R$ with $Y = 0$. It is clear that two extra neutrinos are decoupled from the group $SU_L(2) \times U_Y(1)$.

The interaction Lagrangian is given by (suppressing the generation index $i$)

$$L_I = -\frac{g}{\sqrt{2}}[\bar{e}_R^c \gamma^\mu \nu_R W^-_\mu + h.c.] - \frac{g}{\sqrt{2}}[\bar{e}_R^c \gamma^\mu e_R (W_{3\mu} + \frac{g_1}{2g} B_{1\mu} + \frac{g_X}{2g} V_\mu) + \bar{\nu}_R^c \gamma^\mu \nu_R^c (-W_{3\mu} + \frac{g_1}{2g} B_{1\mu} + \frac{g_X}{2g} V_\mu)]$$

$$+ \bar{N}_R \gamma^\mu N_R (U_{3\mu} - \frac{1}{2} \frac{g_1}{2g} B_{1\mu} + \frac{g_X}{2g} V_\mu) + \bar{N}_R \gamma^\mu N^s_R (-U_{3\mu} - \frac{1}{2} \frac{g_1}{2g} B_{1\mu} + \frac{g_X}{2g} V_\mu)]$$

$$- \frac{g}{\sqrt{2}}[\bar{e}_R^c \gamma^\mu N_{eR} X^-_\mu + \bar{\nu}_R^c \gamma^\mu N_{eR} X^0_\mu + \bar{e}_R^c \gamma^\mu N^s_{eR} Y^-_\mu + \bar{\nu}_R^c \gamma^\mu N^s_{eR} Y^0_\mu + \bar{N}_R \gamma^\mu N^s_R U) + h(\bar{a})]$$

where the vector boson $B_{1\mu} = \sqrt{\frac{2}{3}} W_{8\mu} + \sqrt{\frac{1}{3}} W_{15\mu}$ is coupled to $U_Y(1)$ and $U_{3\mu} = -\sqrt{\frac{1}{3}} W_{8\mu} + \sqrt{\frac{2}{3}} W_{15\mu}$. Note that in the symmetry limit $g_1 = \sqrt{2}g$.

Further we note that the vector boson $B_\mu$ corresponding to $U_Y(1)$ is given by

$$\frac{B_\mu}{g'} = \frac{B_{1\mu}}{g_1} + \frac{V_\mu}{g_X} \quad (8)$$

Thus

$$A_\mu = \frac{e}{g} W_{3\mu} + \frac{e}{g'} B_{1\mu} + \frac{e}{g_X} V_\mu$$

$$= \frac{e}{g} W_{3\mu} + \frac{e}{g'} B_\mu$$

$$Z_\mu = \frac{e}{g'} W_{3\mu} - \frac{e}{g} B_\mu \quad (9)$$
There are two more vector bosons, which we define as follows

\[ Z'_\mu = -\frac{g_1}{g} B_{1\mu} + \frac{g_2}{g} V_{\mu} \]
\[ Z''_{\mu} = U_{3\mu} \quad (10) \]

Hence rewriting the interaction Lagrangian in terms of vector bosons \( A_{\mu}, Z_{\mu}, Z'_\mu \) and \( Z''_{\mu} \) we have

\[ \mathcal{L}_{\text{neutral}} = -g \sin \theta [(\bar{e}_R^c \gamma^\mu e_R^c + \bar{e}_L^c \gamma^\mu e_L^c)] A_{\mu} - \frac{g}{2 \cos \theta_W} [(\bar{e}_R^c \gamma^\mu e_R^c - \bar{\nu}_R \gamma^\mu \nu_R^c)] Z_{\mu} \]
\[ -2 \sin^2 \theta_W (\bar{e}_R^c \gamma^\mu e_R^c + \bar{e}_L^c \gamma^\mu e_L^c)] Z_{\mu} \]
\[ -\frac{1}{2} g \left[ \frac{1}{2} (\bar{e}_R^c \gamma^\mu e_R^c + \bar{\nu}_R \gamma^\mu \nu_R^c) + \bar{N}_R \gamma^\mu N_R + \bar{N}_R^s \gamma^\mu N_R^s \right] \]
\[ -\frac{g^2}{g_1^2} \tan^2 \theta_W (\bar{e}_R^c \gamma^\mu e_R^c - 2 \bar{e}_L^c \gamma^\mu e_L^c + \bar{\nu}_R \gamma^\mu \nu_R^c)] Z'_\mu \]
\[ -\frac{1}{2} g [\bar{N}_R \gamma^\mu N_R - \bar{N}_R^s \gamma^\mu N_R^s] Z''_{\mu} \quad (11) \]

From Eqs. (7) and (11), it is clear that the doublet \( \left( \begin{array}{c} N_i \\ N_i^s \end{array} \right)_R \) belongs to fundamental representation of the \( U \)-spin \( SU(2) \) subgroup of \( SU(4) \) (the other subgroup being \( SU_L(2) \)) with vector bosons \( U, U, U_{3\mu}, Z''_{\mu} \) belonging to the adjoint representation of this group. To break the symmetry and at the same time give Dirac masses to fermions which have both left handed and right handed components viz charged leptons and quarks, the minimally required Higgs are given in Table 2:
The following comments are in order: \( \rho \) and \( \chi \) correspond to Higgs field \( \phi = \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) \), \( < \phi > = \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right) \), \( \tilde{\phi} = i \tau_2 \phi = \left( \begin{array}{c} \phi^0 \\ -\phi^- \end{array} \right) \), \( < \tilde{\phi} > = \left( \begin{array}{c} \frac{v}{\sqrt{2}} \\ 0 \end{array} \right) \). Thus \( < \rho > \) and \( < \chi > \) give masses to the vector bosons of the standard model gauge group and masses to the charged leptons and quarks of the standard model. For charged leptons only \( < \chi > \) contributes. The other Higgs are needed to break the group \( SU(4) \times U(1) \) so as to give superheavy masses to the extra vector bosons and extra quarks outside the standard model.

With the symmetry breaking pattern discussed above, the mass Lagrangian for vector bosons is given by

\[
\mathcal{L}_{mass}^W = \frac{1}{2} g^2 u^2 [2W^+W^- + \left( \frac{1}{\cos \theta_W} - Z + \frac{1}{2} \left( 1 - \frac{6 g^2}{g_1^2} \right) \tan^2 \theta_W Z^0 \right)^2 + 2X^+X^- + 2Y^+Y^-] \\
+ \frac{1}{2} g^2 u'^2 [2W^+W^- + \left( \frac{1}{\cos \theta_W} - Z + \frac{1}{2} \left( 1 - \frac{2 g^2}{g_1^2} \right) \tan^2 \theta_W Z^0 \right)^2 + 2X^0X^0 + 2Y^0Y^0] \\
+ \frac{1}{2} g V^2 [2X^+X^- + 2X^0X^0 + 2U^0U^0 + \left( \frac{1}{2} Z^0 + Z^0 \right)^2] \\
+ \frac{1}{2} g V'^2 [2Y^+Y^- + 2Y^0Y^0 + 2U^0U^0 + \left( \frac{1}{2} Z' - Z'' \right)^2] \quad (12)
\]
Since $V \approx V' \gg u \approx u'$, therefore neglecting the terms of order $\frac{u^2}{V}$, we can write

$$L_{\text{mass}}^W \approx \frac{1}{2} g^2 [(u^2 + u'^2)(2W^+W^- + \frac{1}{\cos^2 \theta_W} Z^2)] + \frac{1}{2} g^2 [V^2(2X^+X^- + 2X^0X^0 + 2U \bar{U})]$$

$$(\frac{1}{2}Z' + Z'')^2 + V'^2(((\frac{1}{2}Z' - Z'')^2 + 2Y^+Y^- + 2Y^0Y^0 + 2U \bar{U})]$$

This gives the gauge boson masses

$$m_W^2 = \frac{1}{2} g^2 (u^2 + u'^2) = \frac{m_Z^2}{\cos^2 \theta_W}$$

$$m_X^2 = \frac{1}{2} g^2 V^2, \quad m_Y^2 = \frac{1}{2} g^2 V'^2, \quad m_U^2 = m_X^2 + m_Y^2$$

$$m_{Z',Z''}^2 = \frac{1}{2} g^2 \left( \frac{V^2 + V'^2}{V^2 + V'^2} \right)$$

So far we have introduced essentially two energy scales, represented by the vacuum expectation values $u(\approx u')$ and $V(\approx V')$ the former corresponds to the SM scale ($\approx 175\text{GeV}$) and the latter although not fixed, but much higher, an interesting one would be an intermediate energy scale (between SM and GUT) $\approx 10^{10}\text{GeV}$.

Lagrangian (11) explicitly shows the decoupling of extra leptons from the standard model. Only connection with the standard model is through the mixing of $Z$ with $Z'$ (involving terms of order $\frac{u^2}{V}$) when the symmetry is broken to give masses to the gauge bosons. Since extra bosons other than the standard bosons are very heavy, the mixing term only give a negligible contribution to the standard model observables. Similarly $Z'$ being very heavy, its contribution to standard model observables is also negligible.

We now discuss $SU_L(4) \times U_X(1)$ Yukawa interaction in the charged lepton sector, which is

$$H_Y = h_{ij}(\overline{e}_{iL}X_j e_{iL}) + h.c.$$ so that

$$M_i = h_{ij}(\overline{e}_{iL}u_j e_{iR}) + h.c.$$ (14)

For simplicity we may take $u_e = u_\mu = u_\tau = u$. It is important to note that we have no new charged leptons, other than those of the standard model. As far as new quarks are concerned, their masses will be determined essentially by $h_{iQ}V$, $h_{iQ}$ being the corresponding Yukawa coupling constant, $Q$ stands
for U, T, D and H quarks. If Yukawa couplings are of order unity, masses of new quarks will be of same order as those of vector bosons X etc.

The above scalars do not give masses to neutrinos. In a way this is a great advantage as neutrinos have a completely different mass spectrum from other fermions.

To give Majorana masses to neutrinos, we introduce Higgs scalars $S_{\alpha\beta}$ [$\alpha, \beta$ are $SU(4)$ indices] belonging to the symmetric representation 10 of $SU(4)$ with $Y_X = 1$. The electric charge matrix for this representation is

$$\hat{Q} = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Thus charged leptons can not get any masses from the Higgs $S_{\alpha\beta}$ nor the quarks. In order to make $N$ heavy and $N^s$ light ($\sim$eV) we introduce $\bar{S}_{\alpha\beta}$, $\bar{S}'_{\alpha\beta}$, and $\bar{S}''_{\alpha\beta}$ with $Y_X = -1$

with their expectation values in Table 3.

| Table III | Vacuum expectation values of Higgs belonging to rep. 10 of SU(4) |
|-----------|----------------------------------------------------------------|
| $< \bar{S}_{\alpha\beta} >$ | $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa' & 0 & 0 \\ 0 & 0 & \kappa' & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ |
| $< \bar{S}'_{\alpha\beta} >$ | $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa'_s \\ 0 & 0 & 0 & \kappa'_s \\ 0 & 0 & 0 & \kappa_R \end{pmatrix}$ |
| $< \bar{S}''_{\alpha\beta} >$ | $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \kappa_R & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ |

where $\kappa_s, \kappa'_s \ll \kappa' \ll \kappa_R$. We wish to remark that we have selected three different Higgs scalars $S_{\beta\alpha}$ with the same $Y_X = -1$, just as we selected three Higgs quartets $\rho, \eta, \xi$ with $Y_X = -\frac{1}{2}$, for the reason that with one $S_{\beta\alpha}$ the vacuum expectation values $\kappa', \kappa_R, \kappa_s, \kappa'_s$ as different components of the same Higgs scalar would have been critically alligned. They, now belonging to three different Higgs scalars, are of course very hierarchical to accomodate different energy scales. However, the problem of hierarchy is there for models which go beyond the standard model and has no easy solution as is well known.
These additional Higgs give an extra contribution to $\mathcal{L}_{\text{mass}}^W$ in Eq. (13) [only $\kappa_R$ is important]

$$\frac{1}{2} g^2 \kappa^2_R (X^+ X^- + \overline{X^0} X^0 + \frac{1}{2} Z' + Z'')^2$$

so that only $X$ and $Z'$ and $Z''$ gauge bosons get extra contribution, giving further splitting between these and $Y$ and $U$ bosons.

The Yukawa couplings of these scalars to leptons are

$$\sum_b f_{ij}^{b} F_{i\alpha}^T C^{-1} F_{j\beta} S_{\beta\alpha}^{b}$$

(15)

where $b = \text{no prime, prime and double prime}$. Then the neutrino mass matrix is given by

$$M_N = f_{ij}^{b} (\kappa_R N_i^T C^{-1} N_j) + f_{ij}^{b} \kappa^\prime \nu_i N_j + f_{ij}^{b} (\kappa_s N_i^T C^{-1} N_j + \kappa^\prime_s \nu_i N_j^s) + \text{h.c}$$

(16)

Eq. (16) gives $9 \times 9$ neutrino mass matrix in the weak eigenstates basis:

$$M_{\nu N} = \begin{pmatrix} 0 & m_D & m_D^* \\ m_D^T & M & 0 \\ m_D^{*T} & 0 & m^s \end{pmatrix}$$

(17)

where $(m_D)_{ij} = f_{ij}^{i} \kappa^i$, $(M)_{ij} = f_{ij}^{i} \kappa_R, (m_s^s)_{ij} = f_{ij}^{i} \kappa_s^i$, $(m_s^s)_{ij} = f_{ij}^{i} \kappa_s^s$.

Since $\kappa_s, \kappa_s^\prime << \kappa^\prime, \ll \kappa_R$, therefore in the limit $m_s^s, m^s \to 0$, the diagonalization gives

$$M_{\nu N} = \begin{pmatrix} A & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(18)

where $A = -m_D M^{-1} m_D^T$. However, by introducing a unitary matrix $U$:

$$U = \begin{pmatrix} 1 & b & 0 \\ -b^T & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$b = -m_D M^{-1} b^T = -M^{-1} m_D^T$$

(19)

we obtain the mass matrix

$$U M_{\nu N} U^T = \begin{pmatrix} A & 0 & m_D^s \\ 0 & M & 0 \\ m_D^{*T} & 0 & m_s \end{pmatrix} + O\left(\frac{1}{M^2}\right)$$

$$= M \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & m_D^s \\ 0 & 0 & 0 \\ m_D^{*T} & 0 & m_s \end{pmatrix} + O\left(\frac{1}{M^2}\right)$$

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This gives the effective $6 \times 6$ light neutrino mass matrix

$$M_{\nu} = \begin{pmatrix} -m_D M^{-1} m_D^T & m_s^* \\ m_D^T & m_s \end{pmatrix}$$  \hspace{1cm} (21)

The phenomenology of this $6 \times 6$ matrix already exist in the literature [4,5,12,13]. In Eq. (21) $-m_D M^{-1} m_D^T$ gives the normal see saw mechanism. Here $M \gg m_D$ and may be of order $10^{10} - 10^{12}$GeV a scale which may be relevant for thermal, non-degenerate leptogenesis. Now a word about energy scales: $\kappa_R$ is of order of $10^{10}$GeV while $\kappa'$ is of order 1GeV so that if all Yukawa coupling constants in Eq.(16) are of order unity, active neutrinos mass is of order $0.1$eV to satisfy the constraint from WMAP data $\Sigma m_{\nu_i} < (0.4 - 0.7)$eV while atmospheric data give $m_{\nu} > 5 \times 10^{-2}$eV.$\kappa_s$ is of order a few eV so that sterile neutrinos are to be relevant for “short” baseline oscillation searches. Finally in order to have active-sterile mixing small($\sim 0.1$)[see ref 11, 12]$\kappa'_s \simeq 0.1 \kappa_s$. For other approaches for possible existence of sterile neutrinos in eV scale range whithin the framework of higher order gauge groups beyond the standard model., see ref [13,14,15].

In summary we have shown (i) intermediate mass scales between electroweak mass scale and that of grand unification can be accommodated, relevent for leptogeneses (ii) one can naturally accommodate more than one right handed neutrino per generation, some of which can be identified with sterile neutrinos that mix with active ones (iii) neutrinos mass spectrum is decoupled from other fermions (iv) both the normal seesaw mechanism with right handed neutrino Majorana mass $M \gg M_{\text{weak}}$, which may be relevant for leptogenesis and an eV-scale which may manifest itself in “short” baseline oscillation searches neutrino experiments.

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