Time Truncated Life Test for New Two-Sided Group Chain Sampling Plan (NTSGChSP-1) using Minimum Angle Method

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Abstract. This study constructs new two-sided group chain acceptance sampling plans (NTSGChSP-1) for generalized exponential distribution using minimum angle method. The minimum angle method is slightly advantageous over the previous methods of minimizing consumer’s and procedure’s risk, as it considers the optimal number of groups, g associated with the smallest angle, θ. In theory, an operating characteristics (OC) curve created with the smallest angle, θ closely resembles the ideal OC curve as the OC curve protects both parties, producer and consumer. The findings show that the optimal number of groups, g depends on the combinations of the design parameters. Plus, there is also a pattern for the optimal number of groups, g, where the optimal number of groups, g decreases as the specified constant, a increases. The two significant findings eventually assist the experimenter in designing the experiment on how many number of items should be selected for the inspection before making a final decision.

Keywords: Consumer’s risk, Generalized exponential distribution, Minimum angle method, New two-sided group chain acceptance sampling plans (NTSGChSP-1), Producer’s risk.

1. Introduction
Acceptance sampling involves sample inspection and the final decision is always to accept or to reject a lot. The sample inspection is a vital approach since it reduces the inspection time compared to inspection of the entire lot. Apart from that, a lot of items usually has thousands in quantities, therefore there is almost impossible to inspect every single item, one by one. Since the inspection is done solely on the sample, therefore there is possibility of making incorrect decision known as risk.

There are two risks in acceptance sampling, which are producer’s risk and consumer’s risk. The producer’s risk stands for the probability of rejecting a good lot while the consumer’s risk is interpreted as the probability of accepting a bad lot [1]. Both producer and consumer always look for an acceptance sampling plan that protects them, or in other words, an acceptance sampling plan with low risk for them.

In order to meet both parties’ requirements, researchers have been developing acceptance sampling plans with three different methods such as (i) minimizing the consumer’s risk, (ii) minimum sum of risks, and (iii) minimum angle method. The first method considers the consumer only and ignores the producer and without hesitation, the consumer would prefer the method as it protects them. This method has been studied religiously by several researchers including Kantam et al. [2], Rosaiah & Kantam [3], Tsai & Wu [4] and Teh et al. [5].
The second method concerning minimum sum of risks was suggested by Govindaraju and Subramani [6] as they realized that the established acceptance sampling plans only took into account the consumer’s risk and paid no attention to the producer’s risk. As indicated by the name, this method considers both parties (producer and consumer) when developing any acceptance sampling plan. Govindaraju and Subramani has applied this method to the single acceptance sampling plans (SSP) [6], the double acceptance sampling plans (DSP) [7] and the chain acceptance sampling plans (ChSP-1) [8].

Besides the minimum sum of risks method, minimum angle method is also another approach where both risks (producer and consumer) are considered. The minimum angle method has a slight advantage as it recognizes the smallest angle in determining the optimal solution while the minimum sum of risks method does not. The optimal solution does depend on the sampling plans: sample size, \( n \) when dealing with the SSP or number of group, \( g \) if related to the group acceptance sampling plans. Besides that, the smallest angle created in the operating characteristics (OC) curve leads to close resemblance of the ideal OC curve, which perfectly protects both parties (producer and consumer). This method has been studied by Soundararajan & Christina [9], Fallah Nezhad [10], Suresh & Usha [11] and Teh et al. [12].

In this study, new two-sided group chain acceptance sampling plans (NTSGChSP-1) has been proposed in order to balance the performances between the GChSP-1 and MGChSP-1. The NTSGChSP-1 has been studied by considering the consumer’s risk only and ignored the producer’s risk. Therefore, this study develops the NTSGChSP-1 for the generalized exponential distribution using the minimum angle method. This is motivated by the fact that no researchers have developed the NTSGChSP-1 for the minimum angle method. The main objective of this study is to obtain the optimal number of groups, \( g \) at different values of the shape and design parameters.

2. Minimum Angle Method

In this study, the main purpose is to determine the optimal number of groups, \( g \) associated with the smallest angle, \( \theta \). The smallest angle, \( \theta \) is calculated by the following formula

\[
\tan \theta = \frac{p_2 - p_1}{L(p_1) - L(p_2)},
\]

where \( p_1 \) and \( p_2 \) are the fraction defective at the acceptable quality level (AQL) and the limiting quality level (LQL), respectively. \( L(p_1) \) and \( L(p_2) \) denote as the probability of lot acceptance corresponding to the \( p_1 \) and \( p_2 \), respectively. All the symbols are shown in figure 1.

![Figure 1. The minimum angle method on the operating characteristic (OC) curve](image-url)
3. Material and Methods
The NTSGChSP-1 is constructed in the following four phases:

**Phase I: Identifying the Design Parameters**
For the NTSGChSP-1, the main objective is to obtain the optimal number of groups, $g$, at different values of the design parameters. The design parameters involved for the NTSGChSP-1 are specified constant, $a$, number of items, $r$, number of preceding lots, $i$, number of succeeding lots, $j$, and shape parameters, $\lambda$. Table 1 lists all the values for the design parameters used in this study.

| Design Parameters       | Specified constant ($a$) | 0.25 | 0.5 | 0.75 | 1.0 | 1.25 | 1.5 | 1.75 | 2.0 |
|------------------------|--------------------------|------|-----|------|-----|------|-----|------|-----|
| Number of preceding lots ($i$) | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| Number of succeeding lots ($j$) | 2 | 3 | 4 | 5 | 1 | 2 | 3 |  |

**Phase II: Developing the Operating Procedure**
Algorithm 1 describes the operating procedure for the NTSGChSP-1.

**Algorithm 1 : NTSGChSP-1**

- **Step 1**: Find the optimal number of groups ($g$) by minimizing $\alpha \leq 0.10$ and $\beta \leq 0.10$.
- **Step 2**: Allocate number of items ($r$) to each group. The sample size is given by $n = g \times r$.
- **Step 3**: Count the number of defectives ($d$) during test termination time ($t_0$).
- **Step 4**: Accept the current lot if $d = 0$ given that $d_i + d_j \leq 1$. $d_i$ is the number of defectives in the preceding lots and $d_j$ is the number of defectives in the succeeding lots.
  The current lot is also accepted if $d = 1$ provided that $d_i = 0$ and $d_j = 0$. Reject the current lot if $d > 1$.

**Phase III: Deriving the Probability of Lot Acceptance**
A tree diagram, as shown in Figure 2 is drawn in order to derive the probability of lot acceptance for the NTSGChSP-1, where $D$ and $\overline{D}$ stand for defective and non-defective respectively.

![Figure 2. A tree diagram for the NTSGChSP-1](image)
Based on figure 2 and upon simplification, the probability of lot acceptance, $L(p)$ for the NTSGChSP-1 is given by

$$L(p) = P_0 2^i [P_0 + (2i + 1)P_1].$$

where $P_0$ is the probability of zero defective, $P_1$ is the probability of one defective and $i$ is the number of preceding lots.

In this study, binomial distribution is used to derive $P_0$ and $P_1$. Upon simplification, the probability of lot acceptance, $L(p)$ is rewritten as

$$L(p) = (1 - p)^g r^{i+1} \left[ 1 + \frac{(2i+1)(gr)^2}{1-p} \right],$$

where $p$ is the fraction defective, $g$ is the optimal number of groups and $r$ is the number of items.

The fraction defective, $p$ is derived by using the cumulative distribution function (CDF) for the generalized exponential distribution, the mean ($\mu = \sigma$) and the test termination time ($t_0 = a \mu_0$). Upon simplification, the fraction defective, $p^*$ is stated as

$$p^* = \left[ 1 - \exp \left( -a \left( \frac{e}{\mu_0} \right) \right) \right]^{1/\lambda},$$

where $a$ is the specified constant, $\frac{\mu}{\mu_0}$ is the mean ratio and $\lambda$ is the shape parameter.

Since this study involves two parties (producer and consumer), there is slight difference in calculating the fraction defective, $p^*$. For $p_1$, it is calculated at different values of mean ratio, $\frac{\mu}{\mu_0}$, starting from 2 up to 12 with 2 increment. On the other hand, $p_2$ is obtained when the value of the mean ratio, $\frac{\mu}{\mu_0}$ is 1 because that is the lowest quality of an item that a customer is willing to accept at the LQL.

**Phase IV: Measuring Performance**

The performance of the NTSGChSP-1 is measured based on the optimal number of groups, $g$ associated with the smallest theta, $\theta$ at the different values of the design parameters. Different values of design parameters have different optimal number of groups, $g$ which provide a range of $g$ values for industrial practitioners in selecting the suitable $g$.

**4. Results and Discussion**

Tables 2 to 4 show the optimal number of groups, $g$ for the generalized exponential distribution at the different values of shape ($\lambda = 1$ to $\lambda = 3$) and design parameters. There are lots of missing values indicated as (-) in the tables particularly in Table 2.

**Table 2. The optimal number of groups for generalized exponential distribution ($\lambda=1$)**

| Mean ratio | Specified constant, $a$ | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
|------------|-------------------------|------|-----|-----|---|-----|-----|-----|---|
| 1          | 2                       | -    | -   | -   | - | -   | -   | -   | - |
| 2          | 3                       | -    | -   | -   | - | -   | -   | -   | - |
| 3          | 4                       | -    | -   | -   | - | -   | -   | -   | - |
| 4          | 5                       | -    | -   | -   | - | -   | -   | -   | - |
Table 2 continued.

|   | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 4 |   |   |   |   |   |   |
| 1 |   |   |   |   |   |   |
| 2 |   |   |   |   |   |   |
| 3 |   |   |   |   |   |   |
| 4 |   |   |   |   |   |   |
| 5 |   |   |   |   |   |   |

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 6 |   |   |   |   |   |
| 1 |   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |
| 5 |   |   |   |   |   |
| 7 |   |   |   |   |   |

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 8 |   |   |   |   |   |
| 1 |   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |
| 5 |   |   |   |   |   |

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 10|   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |
| 5 |   |   |   |   |   |

|   | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 12|   |   |   |   |   |
| 2 |   |   |   |   |   |
| 3 |   |   |   |   |   |
| 4 |   |   |   |   |   |

Table 3. The optimal number of groups for generalized exponential distribution (\( \lambda = 2 \))

| Mean ratio | Specified constant, \( a \) | Generalized exponential distribution, \( \lambda = 2 \) |
|------------|-------------------------------|--------------------------------------------------|
|            |                               | 0.25    | 0.5     | 0.75    | 1       | 1.25    | 1.5     | 1.75    | 2       |
| 2          |                               |         |         |         |         |         |         |         |         |
| 1          |                               |         |         |         |         |         |         |         |         |
| 2          |                               |         |         |         |         |         |         |         |         |
| 3          |                               |         |         |         |         |         |         |         |         |
| 4          |                               |         |         |         |         |         |         |         |         |
| 5          |                               |         |         |         |         |         |         |         |         |

| 4          |                               | 19      | 6       | 3       |         |         |         |         |         |
| 1          |                               |         |         |         |         |         |         |         |         |
| 2          |                               |         |         |         |         |         |         |         |         |
| 3          |                               |         |         |         |         |         |         |         |         |
| 4          |                               |         |         |         |         |         |         |         |         |

| 1          |                               | (2.84352°) | (8.96397°) | (15.78105°) |         |         |         |         |         |
| 2          |                               |         |         |         |         |         |         |         |         |
| 3          |                               | (2.84680°) | (8.96411°) | (15.81887°) |         |         |         |         |         |
| 4          |                               |         |         |         |         |         |         |         |         |
| 3          |                               | (2.84350°) | (9.02370°) |         |         |         |         |         |         |
| 4          |                               |         |         |         |         |         |         |         |         |
| 5          |                               | (2.87140°) |         |         |         |         |         |         |         |
Table 3 continued.

|   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|
| 1 | 2 | (2.79114) | (8.76326) | (15.51850) | (21.61552) | (26.93998) | (30.83203) | (33.51599) | (36.03540) |
| 2 | 3 | (2.79176) | (8.77551) | (15.80034) | (21.65168) | (27.68790) |       |       |       |
| 3 | 4 | (2.79097) | (8.88965) | (15.69116) |       |       |       |       |       |
| 4 | 5 | (2.79176) | (8.77551) |       |       |       |       |       |       |
|   |   | 11 | 3 | 2 | 1 | 1 |       |       |       |
| 1 | 2 | (2.78414) | (8.73827) | (15.42191) | (21.61744) | (26.66103) | (31.06636) | (33.70170) | (35.94415) |
| 2 | 3 | (2.78422) | (8.73953) | (15.49612) | (21.54629) | (26.92301) | (31.68964) |       |       |
| 3 | 4 | (2.78444) | (8.75995) | (15.46044) | (22.24616) |       |       |       |       |
| 4 | 5 | (2.78643) | (8.73953) | (15.89712) |       |       |       |       |       |
|   |   | 12 | 4 | 2 | 1 | 1 |       |       |       |
| 1 | 2 | (2.78526) | (8.74227) | (15.44162) | (21.58157) | (26.67298) | (30.85157) | (33.92800) | (36.09705) |
| 2 | 3 | (2.78526) | (8.75044) | (15.44786) | (21.59052) | (26.76741) | (31.14627) | (34.82756) | (38.00946) |
| 3 | 4 | (2.78577) | (8.74418) | (15.43627) | (21.85597) | (27.70522) |       |       |       |
| 4 | 5 | (2.78526) | (8.75365) | (15.62546) | (22.62185) |       |       |       |       |
|   |   | 13 | 4 | 2 | 1 | 1 |       |       |       |
| 1 | 2 | (2.78762) | (8.75029) | (15.45264) | (21.60483) | (26.72974) | (30.82628) | (34.10322) | (36.25519) |
| 2 | 3 | (2.78764) | (8.75130) | (15.45264) | (21.64644) | (26.75453) | (30.98056) | (34.41267) | (37.24418) |
| 3 | 4 | (2.78765) | (8.74988) | (15.45074) | (21.73740) | (27.25047) | (31.01666) | (36.23381) |       |
| 4 | 5 | (2.78802) | (8.77115) | (15.53961) | (22.14209) | (28.27908) |       |       |       |

Table 4. The optimal number of groups for generalized exponential distribution ($\lambda=3$)

| Specified constant, $a$ | Generalized exponential distribution, $\lambda = 3$ |
|------------------------|-----------------------------------------------|
| Mean ratio | $i$ | $r$ | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
| 1 | 2 | - | - | - | - | - | - | - | - | - |
| 2 | 3 | - | - | - | - | - | - | - | - | - |
| 3 | 4 | - | - | - | - | - | - | - | - | - |
| 4 | 5 | - | - | - | - | - | - | - | - | - |
Table 4 continued

|    | 1     | 2     | 3     | 4     | 5     | 6     |
|----|-------|-------|-------|-------|-------|-------|
| 4  | 1     | 2     | 3     | 4     | 5     |       |
|    | (0.61661°) | (3.46669°) | (8.31151°) | (14.07898°) | (19.89826°) | (24.90914°) |
|    | 49    | 8     | 3     | 2     | 1     | 1     |
| 5  |       |       |       |       |       |       |
|    | (0.61661°) | (3.46669°) | (8.30816°) | (14.18459°) | (19.78793°) | (25.38892°) |
|    | 26    | 4     | 2     | 1     | -     | -     |
| 6  |       |       |       |       |       |       |
|    | (0.61661°) | (3.46870°) | (8.35326°) | (14.13174°) |       |       |
|    | 16    | 3     | 1     | 1     | -     | -     |
| 7  |       |       |       |       |       |       |
|    | (0.61662°) | (3.47179°) | (8.30816°) | (14.80780°) |       |       |
|    | 157   | 26    | 10    | 5     | 3     | 2     |
| 8  |       |       |       |       |       |       |
|    | (2.78414°) | (8.73827°) | (15.42191°) | (21.61744°) | (26.66103°) | (31.06636°) |
|    | 11    | 3     | 2     | 1     | 1     | -     |
| 9  |       |       |       |       |       |       |
|    | (2.78422°) | (8.73953°) | (15.49612°) | (21.54629°) | (26.92301°) | (31.68964°) |
|    | 6     | 2     | 1     | 1     | -     | -     |
| 10 |       |       |       |       |       |       |
|    | (2.78444°) | (8.75995°) | (15.46044°) | (22.24616°) |       |       |
|    | 4     | 1     | 1     | -     | -     | -     |
| 11 |       |       |       |       |       |       |
|    | (2.78643°) | (8.73953°) | (15.89712°) |       |       |       |
|    | 28    | 8     | 4     | 3     | 2     | 2     |
| 12 |       |       |       |       |       |       |
|    | (2.78526°) | (8.74227°) | (15.44162°) | (21.58157°) | (26.67298°) | (30.85157°) |
|    | 12    | 4     | 2     | 1     | 1     | 1     |
| 13 |       |       |       |       |       |       |
|    | (2.78526°) | (8.75044°) | (15.44786°) | (21.59052°) | (26.76741°) | (31.14627°) |
|    | 6     | 2     | 1     | 1     | 1     | -     |
| 14 |       |       |       |       |       |       |
|    | (2.78577°) | (8.74418°) | (15.43627°) | (21.85897°) | (27.70522°) |       |
|    | 4     | 1     | 1     | 1     | -     | -     |
| 15 |       |       |       |       |       |       |
|    | (2.78526°) | (8.75365°) | (15.62546°) | (22.62185°) |       |       |
|    | 32    | 9     | 5     | 3     | 2     | 2     |
| 16 |       |       |       |       |       |       |
|    | (2.78762°) | (8.75029°) | (15.45264°) | (21.60483°) | (26.72974°) | (30.82628°) |
|    | 13    | 4     | 2     | 1     | 1     | 1     |
| 17 |       |       |       |       |       |       |
|    | (2.78764°) | (8.75130°) | (15.45264°) | (21.64644°) | (26.75453°) | (30.98056°) |
|    | 7     | 2     | 1     | 1     | 1     | -     |
| 18 |       |       |       |       |       |       |
|    | (2.78765°) | (8.74988°) | (15.45074°) | (21.73740°) | (27.25047°) | (31.01666°) |
|    | 4     | 1     | 1     | 1     | -     | -     |
| 19 |       |       |       |       |       |       |
|    | (2.78802°) | (8.77115°) | (15.53961°) | (22.14209°) | (28.27908°) |       |
The (-) indicates that for the designated value of shape and design parameters, there is no optimal number of groups, \( g \) recorded. This is because the selected design parameters fail to meet the risks conditions, where the risks for both parties (producer and consumer) are set to be less than 0.1. Since the selected shape and design parameters fail to meet the conditions, therefore no optimal number of groups, \( g \) is recorded.

The second finding reveals that the optimal number of groups, \( g \) depends on the combination of the shape and design parameters. Different parameters contribute to the different optimal number of groups, \( g \). For instance, the optimal number of groups, \( g \) is 23 when the shape and design parameters are \( \left( \lambda, \frac{\mu}{\mu_0}, i, r, a \right) = (2,6,1,2,0.25) \). If the chosen parameters are \( \left( \lambda, \frac{\mu}{\mu_0}, i, r, a \right) = (3,4,2,3,0.5) \), then the optimal number of groups, \( g \) is 20.

The other finding shows that the optimal number of groups, \( g \) decreases as the specified constant, \( a \) increases. The optimal number of groups, \( g \) is 23 when the shape and design parameters are \( \left( \lambda, \frac{\mu}{\mu_0}, i, r, a \right) = (2,6,1,2,0.25) \). If the experimenter decides to test the items at a longer testing time, say two times longer than the stated mean life of an item, then the experimenter needs to have only one group, with each group has 2 items in it.

5. Conclusion
This study explored the NTSGChSP-1 for the generalized exponential distribution using the minimum angle method. The minimum angle method is chosen as it protects both parties (producer and consumer) while the established NTSGChSP-1 only cares for the consumer. The finding reveals that the optimal number of groups, \( g \) varies according to the designated shape and design parameters. Besides that, the finding also shows a pattern where the optimal number of groups, \( g \) decreases as the specified constant, \( a \) increases, which provide a range of \( g \) values for industrial practitioners in selecting the suitable \( g \).

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