Influence of Volume Fracturing on Casing Stress in Horizontal Wells

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Abstract: In horizontal wells, the casing string is affected by the gravity effect, temperature effect, swelling effect, bending effect, friction effect and other mechanical effects. In view of this situation, the mathematical models of casing swelling effect and temperature effect caused by volume fracturing are established. The case analysis shows that the length of the unsealed section in the vertical section has a great influence on the axial shortening of the casing during fracturing. With the increase of the unsealed section length, the axial shortening of the casing increases gradually under the same wellhead pressure. In the process of fracturing, repeated squeezing and pressurization lead to periodic changes of the wellhead pressure, casing deformation and load, which leads to fatigue damage and even fracture of casing. At the same time, a large amount of fracturing fluid is continuously injected through the casing during the fracturing process, which makes the wellbore temperature change greatly. The additional stress caused by the temperature change reduces the casing strength, which has an important impact on the wellbore integrity. The mathematical model of temperature stress and its effect on the casing strength during volume fracturing is established. With the increase of the temperature stress acting on the casing, the casing collapse strength decreases gradually. When the temperature stress reaches 200 MPa, the casing collapse strength decreases to 84% of the original. The research results can provide a reference for the casing integrity design and control in the horizontal well fracturing process.

Keywords: casing deformation; bulging effect; temperature effect; volume fracturing; casing strength

1. Introduction

Since the 1940s, hydraulic fracturing has been considered as one of the prefer-able technologies in petroleum industries for several decades for the purpose of hydrocarbon well stimulation to recover a huge volume of oil and gas. Therefore, the oilfield vigorously promotes the application of hydraulic fracturing technology to improve oil recovery. Further, the process of hydraulic fracturing, which is as a key stimulation technology with horizontal wells, has been widely used in order to improve the reservoir productivity [1–3]. The temperature change is caused by cementing quality problems and the fracturing fluid injection. The calculation of the bending force cannot be simply regarded as a pure bending mechanical problem of the beam. A bending force calculation model considering the amplification effect is urgently needed. Volume fracturing technology can effectively transform unconventional reservoirs. However, at the same time, it also causes new problems of casing deformation and failure. [4–7]. In Quebec, Canada, 28 wells have experienced varying degrees of casing damage after the completion of the large volume fracture [8]. The same thing happened in the Marcellus shale gas field in the United States. Casing problems are also serious in China. More than a dozen wells have experienced similar problems during fracturing operations, which have affected the development of reservoir reconstruction volume [9–12]. This problem will make it difficult to insert subsequent tools.
In serious cases, multiple well segments have to be abandoned or even the whole well has to be abandoned before the completion of all fracturing grades of hydraulic fracturing operations, thus causing huge economic losses [13–15]. Based on the above situation, the casing strength checking calculation and failure mechanism during the large volume fracturing are discussed below [16,17]. In the process of the downhole operation, the construction pipe string is subject to mechanical influences such as the gravity effect, temperature effect, swelling effect, bending effect and friction effect [18–20].

During the construction of underground work of several reworks or engineering quality accidents, the failure reasons are mostly caused by more than a few effects in addition to the man-made factors. The authors established a ballooning effect and temperature effect model based on the horizontal well fracturing engineering volume. After that, the calculations of the different vertical wells were analyzed without the cementing segment length of the casing axial length and the additional axial force changes. The fracturing volume has also been established in the process of hydraulic fracturing, and the mathematical model of temperature stress and its effects on the casing strength were implemented. Additionally, the integrity analysis was done for different wall thicknesses of the casing strength and the influence of the temperature stress of the outer crowded. The research results can therefore be used as a reference for the wellbore integrity design and for control during use of the volume fracturing operations through the horizontal well.

2. Stress Governing Equation of Fracturing Process

2.1. Swelling Effect Model in the Fracturing Process

During fracturing, different internal and external pressures of the unsealed casing will lead to a casing bulging effect, and the increase of the casing pressure during fracturing will also lead to a casing bulging effect, resulting in axial deformation of the unsealed casing and additional force. However, after the completion of a fracturing, the internal pressure reduction is bound to be small due to the pressure relief inside the casing, so the casing is prone to fatigue and initial cracks under multiple cycles [21–23]. Considering the variation of the average internal and external pressure of the casing at the well head and the bottom, the mathematical model of the bulging force can be expressed as:

\[
\Delta F_e = 0.6A_i(\Delta P_i) - 0.6A_o(\Delta P_o)
\] (1)

Since the bulging effect acts on the whole casing string, the shape variables generated by the bulging effect of the whole string are the same as the sum of the form variables generated by all levels [24,25]. Assuming that the string is composed of multiple micro-elements, the formula of the total shape variable caused by the bulging effect can be summarized as:

\[
\Delta L_3 = -\frac{\mu}{E} \frac{\Delta P_i - R^2 \Delta P_o - \frac{1+2\mu}{2\pi} \delta L^2}{R^2 - 1} - 2 \frac{\mu}{E} \frac{R^2 \Delta P_{o\delta} - \Delta P_{o\delta}}{R^2 - 1} L
\] (2)

The well trajectory of directional or horizontal wells with changes in inclination and azimuth can be expressed in the form of curve integral, and its deformation can be expressed as:

\[
\Delta L_3 = \frac{2\mu}{E} \int_0^1 P_o(z) \left( \frac{D^2 - P_i(z) d^2}{D^2 - d^2} \right) dz
\] (3)

where \(\Delta P_i\) is the variation of average pressure inside the tube, MPa; \(\Delta P_o\) is the variation of average pressure outside the pipe, MPa; \(A_i\) is the cross-sectional area of casing pipe, mm\(^2\); \(A_o\) is the cross-sectional area of casing, mm\(^2\); 0.6\(A_i(\Delta P_i)\) is the forward bulging force of casing, N; 0.6\(A_o(\Delta P_o)\) is the reverse bulging force of casing, N; \(E\) is the elastic modulus of casing, GPa; \(\mu\) is the casing Poisson’s ratio; \(\delta\) is the pressure drop per unit length caused by fluid flow, MPa; \(L\) is the casing length, m; \(D\) is the outer diameter of casing, mm; \(d\) is the inner diameter of casing, mm; \(P_o(z)\) is the casing external pressure at well depth \(z\), MPa; \(P_i(z)\) is pressure in casing at \(z\) depth, MPa; \(\Delta P_i\) is the change value of fluid density inside...
the casing, kg/m$^3$; $\Delta \rho_o$ is the change value of fluid density outside the casing, kg/m$^3$; $R$ is the ratio of outer and inner diameters of tubing; $\Delta P_{ls}$ is the variation value of wellhead oil pressure, MPa; $\Delta P_{os}$ is the change value of external wellhead pressure, MPa.

2.2. Model of Temperature Effect during Fracturing

The temperature effect is caused by the temperature of the fracturing fluid causing the casing material to expand or contract. The rest temperature in the well increases as the depth of the well increases [26,27]. As the string goes into the well, the temperature increases until it is equal to the fluid in the well. When the temperature in the well changes, such as by injecting cold fracturing fluid into the well, the string temperature will change accordingly.

2.2.1. Casing Radial Displacement

As the temperature difference changes, the casing, as a cylindric structure, will generate displacement in the radial direction. Its displacement formula can be expressed as:

$$u_r = \frac{1 + \mu \alpha}{1 - \mu} \int_a^r \Delta T \gamma dr$$  \hspace{1cm} (4)

$\Delta T$ takes temperature difference along the radial direction is constant, and the radial displacement of the casing outside surface caused by the temperature change itself. Thus, its radial displacement can be expressed as:

$$u_{b1} = a \Delta T \frac{1 + \mu b^2 - a^2}{1 - \mu}$$  \hspace{1cm} (5)

Due to the radial bulging of the casing, the annular volume reduction of the unsealed casing and the expansion of the annulus air liquid can be expressed as:

$$\Delta V_1 = \Delta x \pi \left[(b + \mu b_1)^2 - b^2\right] = \pi \left(2bu_{b1} + u_{b1}^2\right) \Delta x$$  \hspace{1cm} (6)

$$\Delta V_2 = \alpha_c \pi \left(a_1^2 - b^2\right) \Delta T \Delta x$$  \hspace{1cm} (7)

where $\alpha$ is the thermal expansion coefficient of the material, 1/°C; $\mu$ is poisson’s ratio of the material; $r$ is the distance from any point on the casing body of the oil layer to the center of the circle, m; $a$ is the inner radius of oil reservoir casing, m; $\Delta T$ is the variation of casing temperature in the reservoir, °C; $b$ is the outer radius of casing in the reservoir, m; $\Delta x$ is a microelement of casing length in the suspension section, m; $\alpha_c$ is the volume coefficient of drilling fluid, 1/°C; $a_1$ is the volume coefficient of drilling fluid, m.

2.2.2. Casing Axial Displacement

Therefore, the borehole temperature is initially constant and the highest temperature change in values as appeared during calculating the parameters of the temperature effect due to fracturing. The average temperature change causes hence temperature force ($\Delta F_I$) and the length change ($\Delta L_5$) calculation can be represented as:

Temperature force caused by temperature:

$$\Delta F_I = -1.4743W \left(\frac{T_{he} + T_{bc}}{2} - \frac{T_{hs} + T_{bs}}{2}\right)$$  \hspace{1cm} (8)

Temperature induced length change:

$$\Delta L_5 = -\frac{\alpha_T L \Delta F_5}{58W}$$  \hspace{1cm} (9)
where $T_{hs}$ is the perennial average temperature at the wellhead, °C; $T_{bs}$ is the formation temperature at the bottom of the well, °C; $T_{be}$ is the wellhead fluid temperature, °C; $T_{te}$ is the bottom hole fluid temperature, °C; $W$ is the weight of casing pipe, N/m; $\alpha_T$ is the thermal expansion coefficient of the material, 1/°C.

2.3. Analysis of Temperature Stress and Casing Strength during Fracturing

In the process of horizontal well fracturing, continuous injection of a large amount of fracturing fluid through the casing causes a great change in wellbore temperature. The additional stress caused by temperature change reduces the casing strength and has an important impact on wellbore integrity [4].

Based on the characteristics of fracturing and wellbore temperature variation, it is assumed that the horizontal section inclination angle is constant at 90°. Because the radial thermal shrinkage deformation of the casing is small, the model only considers the axial thermal stress of the casing. The range of wellbore temperature variation in fracturing operation is usually within 100 °C. In the model, the casing yield strength, elastic modulus and linear thermal expansion coefficient do not consider the influence of temperature [5].

During the establishment of the calculation model, the calculation formula of the vertical well section refers to the temperature stress model of the bar under constrained conditions at both ends, and the temperature difference of the calculation model of the horizontal section takes the temperature difference between the landing point A and the toe point B for calculation. In summary, the fracture temperature stress calculation model can be expressed as:

$$\sigma = \lambda_T E \Delta T_{\text{max}}$$  (10)

Under the combined action of internal pressure and axial load conditions, the casing extrusion strength is considered to be the most dangerous case. In this calculation, the influence of temperature stress on the casing extrusion strength is mainly considered. The calculation formula of the casing extrusion strength under temperature stress can be expressed as:

$$P_{c'} = \left[ \sqrt{1 - 0.75 \left( \frac{P_i + \sigma}{Y_p} \right)^2} - 0.5 \left( \frac{P_i + \sigma}{Y_p} \right) \right] P_{co}$$  (11)

where $\sigma$ is the temperature stress generated by the casing during fracturing, MPa; $\lambda_T$ is the coefficient of thermal expansion, °C⁻¹; $E$ is the elastic modulus of casing, MPa; $\Delta T_{\text{max}}$ is the maximum temperature difference between two points in the horizontal segment, °C; $P_{i}$ is the internal pressure strength of the casing when axial force is considered, MPa; $P_{bo}$ is the original internal pressure strength of the casing, MPa; $P_{oc}$ is the original casing extrusion strength, MPa; $P_{i}$ is pressure in casing, MPa; $Y_p$ is the casing yield strength, MPa; $r_o$ is the outer radius of casing, mm; $r_i$ is the radius of the casing tube, mm, A and B are two targets in the horizontal section of horizontal well.

2.4. Mathematical Model of the Influence of Bending Stress on Casing Strength

When the casing goes into the curved well section, it will bend along with the borehole bending. The bending of the casing mainly affects the casing strength by changing the stress distribution in the casing, leading to local deformation and significant increase of the stress, thus leading to the casing strength decline [19,20]. On the one hand, the casing bending will make casing lose roundness, which makes casing strength decrease. On the other hand, the inner side of the casing will generate compressive stress on the bending section, and the outer side will generate tensile stress. According to the biaxial stress circle theory, the compressive stress on the inner side of the bending casing will reduce the casing’s compressive strength, while the tensile stress on the outer side will reduce the casing’s extruding strength. Therefore, the reduction of the casing internal pressure and external extrusion strength under bending stress should be considered when verifying casing strength in the bending well section.
The casing will bend in the curved wellbore and generate bending force. According to the pure bending of the deformation condition (M = Me is constant), the bending crankshaft stress of the casing in the well with the curvature can be represented as:

$$\sigma = \frac{ED_0 \theta}{4} \frac{a_0 L}{\tanh(a_0 L/2)}$$  \hspace{1cm} (12)

$$a_0 = \sqrt{\frac{F_T}{E I}}$$  \hspace{1cm} (13)

In the axial direction of the casing pipe in the bending section, the casing pipe in the bending section is subjected to the axial stress caused by gravity on one hand and the bending stress on the other hand (positive on one side and negative on the other side). The superposition of the axial stress caused by gravity and the bending stress is the axial stress borne by the casing in the bending section. The formulas for calculating the casing’s internal pressure and external extrusion strength under axial load can be represented as [4]:

$$P_{b_{ax}} = \left[ \sqrt{1 - \frac{3r_i^4}{3r_o^4 + r_i^4} \left( \frac{\sigma_a}{Y_p} \right)^2} + \frac{r_i^2}{\sqrt{3r_o^4 + r_i^4}} \left( \frac{\sigma_a}{Y_p} \right) \right] P_{bo}$$  \hspace{1cm} (14)

$$P_{c_{ax}} = \left[ \sqrt{1 - 0.75 \left( \frac{\sigma_a}{Y_p} \right)^2} - 0.5 \left( \frac{\sigma_a}{Y_p} \right) \right] P_{co}$$  \hspace{1cm} (15)

where $\sigma$ is bending stress, Pa; $E$ is the elastic modulus, Pa; $D_o$ is the outer diameter of casing, m; $\theta$ is borehole curvature per 30 m, ($^\circ$); $L$ is the length of bending section, m; $I$ is the polar moment of inertia, m$^4$; $F_T$ is the casing weight of bending section, N; $P_{bo}$ is the internal pressure strength of casing under axial load, MPa; $P_{bo}$ is the original internal pressure strength of the casing, MPa; $P_{ca}$ is the casing extrusion strength under axial load, MPa; $P_{co}$ is the original casing extrusion strength, MPa; $\sigma_a$ is the axial stress of casing, MPa; $Y_p$ is the casing yield strength, MPa; $r_o$ is the outer radius of casing, mm; $r_i$ is the radius of the casing tube, mm.

3. Results and Discussion

3.1. Basic Date of a Horizontal Well

The oblique depth and vertical depth of horizontal well A on the site are 5128 m and 3099 m, respectively, and the casing length of the oil reservoir in the unsealed section is 1906 m, as shown in Figure 1. During fracturing, the wellhead pressure is 70 MPa and the fracturing fluid density is 1.05 g/cm$^3$. According to the previous swelling effect model formula, the swelling effect causes the casing length of the unsealed section to be shortened by about 1.13 m and the additional casing axial tension is 468 kN.

3.2. Analysis of Casing Axial Shortening and Additional Axial Force under Different Lengths of Unsealed Segments

When the cementing quality of vertical well segment is poor, the length of unsealed section may increase. Therefore, axial shortening and additional axial force of the casing under three conditions of the unsealed segment length of 1906, 2420 and 3104 m are studied, as shown in Figure 2a,b, as well as in Tables 1 and 2 of unsealed segment casing length of 1906 m. When the wellhead pressure is constant, with the increase of the length of the unsealed section, the axial shortening of the casing gradually increases, but the additional axial force does not change much. When the casing length of the unsealed section is fixed, the axial shortening amount and additional axial force of the casing increase linearly with the increase of wellhead pressure. During fracturing, repeated squeeze and pressure will cause periodic changes of wellhead pressure, which will also lead to periodic changes of the casing deformation and load, resulting in casing fatigue damage and even fracture.
The length of casing in the unsealed section is 1906 m, as shown in Figure 1. During fracturing, the wellhead pressure is 70 MPa and the fracturing fluid density is 1.05 g/cm³. According to the previous swelling effect formula, the swelling effect causes the casing length of the unsealed section to be shortened by about 1.13 m and the additional casing axial tension is 468 kN.

Figure 1. Structure of horizontal well A.

### Table 1. Axial shortening of casing under different length of unsealed section.

| Wellhead Pressure (MPa) | Axial Shortening (m) |
|-------------------------|----------------------|
|                         | Length of Unsealed Section 1906 m | Length of Unsealed Section 2420 m | Length of Unsealed Section 3104 m |
| 20                      | 0.29                  | 0.36                  | 0.43                  |
| 30                      | 0.46                  | 0.57                  | 0.70                  |
| 40                      | 0.63                  | 0.78                  | 0.98                  |
| 50                      | 0.79                  | 0.99                  | 1.25                  |
| 60                      | 0.96                  | 1.21                  | 1.52                  |
| 70                      | 1.13                  | 1.42                  | 1.80                  |
| 80                      | 1.30                  | 1.63                  | 2.07                  |
| 90                      | 1.46                  | 1.84                  | 2.34                  |
| 100                     | 1.63                  | 2.06                  | 2.61                  |

### Table 2. Additional axial force of casing running in different length of unsealed section.

| Wellhead Pressure (MPa) | Additional Axial Force (kN) |
|-------------------------|----------------------------|
|                         | Length of Unsealed Section 1906 m | Length of Unsealed Section 2420 m | Length of Unsealed Section 3104 m |
| 20                      | 120.90                     | 116.09                     | 109.69                     |
| 30                      | 190.27                     | 185.46                     | 179.06                     |
| 40                      | 259.64                     | 253.83                     | 248.43                     |
| 50                      | 329.01                     | 323.20                     | 317.80                     |
| 60                      | 398.38                     | 393.57                     | 387.17                     |
| 70                      | 467.75                     | 462.94                     | 456.54                     |
| 80                      | 537.12                     | 532.31                     | 525.91                     |
| 90                      | 606.49                     | 601.68                     | 595.28                     |
| 100                     | 675.86                     | 671.05                     | 663.65                     |
3.3. Analysis of Influence of Different Fracturing Fluid Density on Casing Axial Force

Figure 3 shows the additional axial force of casing swelling effect under different fracturing fluid densities. With the increase of the fracturing fluid density, the additional axial force of swelling effect gradually increases, but the increase is very small, indicating that the fracturing fluid density has little influence on the additional axial force of swelling effect and is mainly affected by wellhead pressure.
Table 1. Axial shortening of casing under different length of unsealed section.

| Wellhead Pressure (MPa) | Axial Shortening (m) |
|------------------------|----------------------|
| Length of Unsealed Section 1906 m |
| 20 | 0.29 |
| 30 | 0.46 |
| 40 | 0.63 |
| 50 | 0.79 |
| 60 | 0.96 |
| 70 | 1.13 |
| 80 | 1.30 |
| 90 | 1.46 |
| 100 | 1.63 |

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| 30 | 190.27 |
| 40 | 259.64 |
| 50 | 329.01 |
| 60 | 398.38 |
| 70 | 467.75 |
| 80 | 537.12 |
| 90 | 606.49 |
| 100 | 675.86 |

3.4. The Effect of Different Temperature Difference on Casing Stress during Fracturing

According to Formula (5), the radial displacement of the casing under the action of different temperature differences (temperature reduction) during fracturing is studied, as shown in Figure 4. As can be seen from the figure, with the increase of temperature difference, the radial displacement of the casing gradually increases.

According to Formulas (8) and (9), the axial length shortening and additional axial force of casing under different temperature differences during fracturing are studied, as shown in Figure 5a,b respectively, and the specific data are shown in Table 3. With the increase of the temperature difference, the casing axial length shortening amount and additional axial force increase gradually, and with the increase of the length of the unsealed section, the casing axial length shortening amount also increases gradually.
3.4. The Effect of Different Temperature Difference on Casing Stress during Fracturing

According to Formula (5), the radial displacement of the casing under the action of different temperature differences (temperature reduction) during fracturing is studied, as shown in Figure 4. As can be seen from the figure, with the increase of temperature difference, the radial displacement of the casing gradually increases.

![Figure 4: Radial displacement of casing under different temperature difference.](image)

According to Formulas (8) and (9), the axial length shortening and additional axial force of casing under different temperature differences during fracturing are studied, as shown in Figure 5a,b respectively, and the specific data are shown in Table 3. With the increase of the temperature difference, the casing axial length shortening amount and additional axial force increase gradually, and with the increase of the length of the unsealed section, the casing axial length shortening amount also increases gradually.

![Figure 5a: Axial length reduction.](image)

![Figure 5b: Additional axial force.](image)

**Figure 5.** Change of the casing axial length and additional axial force under different temperature difference.

**Table 3.** Casing deformation and additional stress under different temperature difference.

| Temperature Difference (°C) | Radial Displacement (mm) | Axial Length Reduction (m) | Additional Axial Force (kN) |
|-----------------------------|--------------------------|---------------------------|-----------------------------|
| 5                           | 0.0011                   | 0.1287                    | 2.20                        |
| 10                          | 0.0021                   | 0.2573                    | 3.39                        |
| 15                          | 0.0032                   | 0.3860                    | 6.59                        |
| 20                          | 0.0043                   | 0.5146                    | 8.78                        |
| 25                          | 0.0054                   | 0.6433                    | 10.98                       |
| 30                          | 0.0064                   | 0.7719                    | 13.18                       |
| 35                          | 0.0075                   | 0.9006                    | 15.37                       |
| 40                          | 0.0086                   | 1.0292                    | 17.57                       |
| 45                          | 0.0097                   | 1.1579                    | 19.76                       |
| 50                          | 0.0107                   | 1.2866                    | 21.96                       |
| 55                          | 0.0118                   | 1.4152                    | 23.16                       |
| 60                          | 0.0129                   | 1.5439                    | 26.35                       |
| 65                          | 0.0140                   | 1.6725                    | 28.55                       |
| 70                          | 0.0150                   | 1.8012                    | 30.74                       |

3.5. Analysis of the Influence of Temperature Stress on Casing Stress

The elastic modulus of the casing is 210 GPa, and the thermal expansion coefficient is $1.35 \times 10^{-5} \, ^\circ\!C^{-1}$. According to Formula (10), the temperature stress of the casing under the action of temperature difference in different horizontal sections is shown in Figure 6. As can be seen from Figure 6, with the increase of temperature difference between two points in the horizontal section, the additional temperature stress acting on the casing gradually increases. For volume fracturing, with the increase of fracturing discharge, the maximum temperature difference between the casing heel end and toe end increases continuously, and the additional temperature stress acting on the casing also increases continuously.
3.5. Analysis of the Influence of Temperature Stress on Casing Stress

The elastic modulus of the casing is 210 GPa, and the thermal expansion coefficient is $1.35 \times 10^{-5} \, ^\circ{}C^{-1}$. According to Formula (10), the temperature stress of the casing under the action of temperature difference in different horizontal sections is shown in Figure 6. As can be seen from Figure 6, with the increase of temperature difference between two points in the horizontal section, the additional temperature stress acting on the casing gradually increases. For volume fracturing, with the increase of fracturing discharge, the maximum temperature difference between the casing heel end and toe end increases continuously, and the additional temperature stress acting on the casing also increases continuously.

Figure 6. Temperature stress of casing under different temperature difference.

For well A, the casing extrusion strength of 139.7 mm casing (P110 steel grade) under different temperature stresses can be calculated by Formula (11), as shown in Figure 7. With the increase of the temperature stress on the casing, the outer extrusion strength of the three kinds of casing with thick wall decreases gradually. When the temperature stress reaches 200 MPa, the outer extrusion strength of the casing decreases to 84%.

Figure 7. Casing collapse resistance under different temperature stress.
3.6. Analysis of Bending Stress and Casing Strength in Curved Hole

As calculated by Formulas (12) and (13), the bending stress of 139.7 mm casing with three wall thickness under different borehole curvature is shown in Table 4. In this table, the bending stress is the tensile stress of the outer side of the casing in the bending section, while the compressive stress of the inner side of the casing in the bending section is the inverse of these values. The bending stress of casing increases with the increase of borehole curvature, and the smaller the casing wall thickness is, the faster the bending stress increases.

Table 4. Casing bending stress under different borehole curvature.

| Borehole Curvature (°/30 m) | Wall Thickness 7.72 mm | Wall Thickness 9.17 mm | Wall Thickness 10.54 mm |
|-----------------------------|------------------------|------------------------|------------------------|
| 1                           | 8.06                   | 7.97                   | 7.89                   |
| 2                           | 16.12                  | 15.94                  | 15.77                  |
| 3                           | 23.17                  | 23.91                  | 23.66                  |
| 4                           | 32.23                  | 31.88                  | 31.54                  |
| 5                           | 40.29                  | 39.85                  | 39.43                  |
| 6                           | 48.35                  | 47.82                  | 47.32                  |
| 7                           | 56.41                  | 55.79                  | 55.20                  |
| 8                           | 63.46                  | 63.76                  | 63.09                  |
| 9                           | 72.52                  | 71.73                  | 70.97                  |
| 10                          | 80.58                  | 79.70                  | 78.86                  |

In order to analyze the decreasing extent of the casing strength in the curved hole, the casing bending stress values under different borehole curvature in Table 4 are substituted into Formulas (14) and (15), and the calculation results are shown in Figures 8 and 9. Where, in Formula (14), the bending stress is negative, and in Formula (15), the bending stress is positive. With the increase of the borehole curvature, the casing compressive strength and extruding strength decrease. For well A, the maximum bending stress of the 139.7 mm casing (P110 steel grade) is 67 MPa. The bending stress reduces the casing’s internal compression strength and external extrusion strength by 95%.

Figure 8. Casing compressive strength under different borehole curvature.
Figure 9. Casing extrusion strength under different borehole curvature.

4. Conclusions

According to the analysis of the casing stress and failure mechanism during fracturing, the following understandings can be obtained:

1. According to the formula of the swelling effect model and temperature effect model during fracturing, the cement should be returned to the wellhead as far as possible to reduce the impact of swelling effect.
2. The length of the unsealed section in a vertical well has a great influence on the axial shortening of the casing during fracturing. With the increase of the length of the unsealed section, the axial shortening of the casing gradually increases under the same wellhead pressure.
3. The mathematical model of temperature stress and its influence on casing strength in the process of volume fracturing is established. With the increase of temperature stress on the casing, the casing’s extrusion strength decreases gradually.
4. The mathematical model of the influence of bending stress on casing strength is established. With the increase of borehole curvature, the casing bending stress increases, and both the casing internal pressure strength and the casing external squeeze strength decrease. For well A, the maximum bending stress is 67 MPa, and the casing internal pressure strength and the casing external squeeze strength decrease to 95%.
5. The developed model increases the well integrity during production operations through checking the production casing integrity. Additionally, you can recommend that if the casing cannot stand the induced loads and forces resulting from the fracturing operations, the coiled tubing or drill pipe is recommended to perform the job, and so on.

Author Contributions: Conceptualization, Y.X. and J.W. (Jingpeng Wang); methodology, J.W. (Jiwei Wu); software, J.W. (Jiwei Wu); validation, J.W. (Jingpeng Wang), Y.X. and Z.L.; formal analysis, J.W. (Jingpeng Wang); investigation, J.W. (Jingpeng Wang); resources, J.W. (Jiwei Wu); data curation, J.W. (Jingpeng Wang); writing—original draft preparation, J.W. (Jingpeng Wang); writing—review and editing, J.W. (Jingpeng Wang); visualization, J.S.; supervision, Y.X.; project administration, J.W. (Jiwei Wu); funding acquisition, Z.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Not applicable.
Acknowledgments: The authors thank the Major Technology Field Test of Petrochina (No. 2019F-33) and the Major Science and Technology Projects for Rock Oil of Petrochina (No. 2019E-26), for their contributions to this paper.

Conflicts of Interest: The authors declare no conflict of interest.

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