Singlet-octet-glueball mixing of scalar mesons

E. Klempt\textsuperscript{a}, A.V. Sarantsev\textsuperscript{a,b}

\textsuperscript{a}Helmholtz-Institut für Strahlen- und Kernphysik der Universität Bonn, Nussallee 14-16, 53115 Bonn, Germany
\textsuperscript{b}NRC “Kurchatov Institute”, PNPI, Gatchina 188300, Russia

Abstract

The mixing angles between scalar isoscalar resonances and a scalar glueball are determined from their decays into two pseudoscalar mesons. For $f_0(1370)$ and $f_0(1500)$, at most a small glueball component is admitted by the data. The decay modes of $f_0(1170)$, $f_0(1770)$, $f_0(2020)$, and $f_0(2100)$ require significant glueball fractions. Above this mass, the errors in the decay frequencies become too large to extract a glueball component. The summation of all observed glueball fractions up to $2100$ MeV yields $(78\pm 18)$%. The glueball fractions as function of the mass are consistent with a scalar glueball at $1865$ MeV and a width of $370$ MeV as suggested by a measurement of the yield of scalar isoscalar mesons in radiative $J/\psi$ decays.

1. Introduction

Light mesons with identical spin and parity are observed in meson nonets that can be decomposed into a singlet and an octet \cite{1}. Well known are the nonets of pseudoscalar mesons housing four kaons, three pions, the singlet and an octet \cite{1}. At higher masses close-by pairs of resonances were reported \cite{3}. A recent analysis of the data on $J/\psi$ radiative decays \cite{1500}, in radiative decays - without constraints from further data like $\pi\pi$ elastic and inelastic scattering - confirmed the $K\bar{K}$, $\eta\eta$ and $\eta\eta'$ mix but the masses of $\eta_s$ and $\eta_b$ are too large to play a role for the low-mass mesons. Except for the pseudoscalar mesons, most meson nonets show nearly ideal mixing: the lighter isoscalar meson in a nonet is mainly composed of up and down quarks only while the heavier meson can be described as mainly $s\bar{s}$ state. In an analysis of radiative $J/\psi$ decays constrained by a large number of further data, ten scalar isoscalar resonances were reported \cite{3}. A recent analysis of the data on radiative $J/\psi$ decays - without constraints from further data like $\pi\pi$ elastic and inelastic scattering - confirmed four of them \cite{3}. The flavor content of the two lightest isoscalar mesons was studied by Oller \cite{5} (see also \cite{9}) by a fit to the two-meson residues. The $f_0(500)$ resonance was found to be $\sim(n\bar{n}+s\bar{s})$ (singlet-like), the $f_0(980)$ to be $\sim(n\bar{n}+s\bar{s})$ (octet-like). No gluonic contribution was required. The interference of the $f_0(1370)$ resonance with $f_0(1500)$ in radiative $J/\psi$ decays into $\pi\pi$ and $K\bar{K}$ - constructive in $\pi\pi$ and destructive in $K\bar{K}$ - identified the former state as mainly singlet, the latter one as mainly octet \cite{3}. At higher masses close-by pairs of resonances were seen that were interpreted as states with octet and singlet $q\bar{q}$ components. Octet states should not be produced in radiative $J/\psi$ decays. Their production was ascribed to gluonic components in their wave functions. The gluonic contribution to singlet states was identified by the enhanced production in radiative $J/\psi$ decays. The scalar glueball was shown to extend over a wide mass range and to be part of several scalar mesons. Its mass and width were determined to $M = (1865 \pm 25)$ MeV, $\Gamma = (370^{+30}_{-26})$ MeV. This mass is just compatible with the result from unquenched lattice calculations which predict a scalar glueball mass of $(1795 \pm 60)$ MeV \cite{7}.

In this Letter we determine the fractional glueball contents and the mixing angles for these and higher-mass scalar mesons. In Section 2 we give the relations used to determine mixing angles. In Section 3 we determine the scalar mixing angle from a fit to $f_0(1370)$ and $f_0(1500)$ decays, search for a glueball fraction in their wave function, and discuss if they form, together with $a_0(1450)$ and $K_0^*(1430)$, a valuable nonet. In Sections 4 - 6 we discuss mixing angles and glueball fractions of $f_0(1710)/f_0(1770)$, $f_0(2020)/f_0(2100)$, and of $f_0(2200)/f_0(2330)$, and possible nonet assignments. The Letter ends with a discussion of the results and a short summary (Section 7).

2. SU(3) relations

The mixing angle of scalar mesons can be derived from their decays exploiting SU(3) relations \cite{8,10}. A singlet isoscalar meson may decay into $\pi\pi$, $KK$, $\eta\eta$ and $\eta\eta'$ with squared coupling constants proportional to 3 : 4 : 1 : 0. An octet isoscalar meson has the corresponding squared coupling constants 3 : 1 : 1 : 4. These relations hold true when $\eta$ and $\eta'$ are pure octet and singlet states. However, singlet and octet isoscalar mesons mix, and the ratios for decays into $\eta$ and $\eta'$ are modified due to a finite pseudoscalar
mixing angle. Singlet and octet mixing also occurs for scalar isoscalar mesons, and the decay coupling constants depend on the scalar mixing angle. Further, decays into $K\bar{K}$ are suppressed by a suppression factor $\lambda$. When these complications are taken into account, the SU(3) relations governing the decays of $q\bar{q}$ mesons and glueballs into two pseudoscalar mesons can be cast into a form presented in Table 1 and, in graphical form, in Figure 1. We call the higher-mass state $f^H$. For the lower-mass state $f^L$, orthogonal in SU(3), cos $\phi^*$ is substituted by sin $\phi^*$, and $-\sin \phi^*$ by cos $\phi^*$.

In Table 1 the scalar mixing angle $\varphi^*$ is given in the quark basis

$$
\left( \begin{array}{c} f^H \\ f^L \end{array} \right) = \left( \begin{array}{cc} \cos \varphi^* & -\sin \varphi^* \\ \sin \varphi^* & \cos \varphi^* \end{array} \right) \left( \begin{array}{c} |nn \rangle \\ |ss \rangle \end{array} \right) \quad (1)
$$

In the singlet/octet basis, the mixing angle is given by

$$
\left( \begin{array}{c} f^H \\ f^L \end{array} \right) = \left( \begin{array}{cc} \cos \theta^p & -\sin \theta^p \\ \sin \theta^p & \cos \theta^p \end{array} \right) \left( \begin{array}{c} |8 \rangle \\ |1 \rangle \end{array} \right) \quad (2)
$$

where $f^H$ is the heavier isoscalar meson. The two angles are related by $\varphi^* = \theta^p + (90 - 35.3)^\circ$, $\theta_{\text{ideal}} = 35.3^\circ$ is the ideal mixing angle with $\tan \theta_{\text{ideal}} = 1/\sqrt{2}$. Fig. 1 is very similar to Fig. 15.2 in the Review of Particle Physics (RPP2020) [11].

The pseudoscalar mixing angle in the quark basis is given by

$$
\left( \begin{array}{c} \eta \\ \eta' \end{array} \right) = \left( \begin{array}{cc} \cos \phi^p & -\sin \phi^p \\ \sin \phi^p & \cos \phi^p \end{array} \right) \left( \begin{array}{c} |nn \rangle \\ |ss \rangle \end{array} \right) \quad (3)
$$

and in the singlet/octet basis by

$$
\left( \begin{array}{c} \eta \\ \eta' \end{array} \right) = \left( \begin{array}{cc} \cos \theta^p & -\sin \theta^p \\ \sin \theta^p & \cos \theta^p \end{array} \right) \left( \begin{array}{c} |8 \rangle \\ |1 \rangle \end{array} \right) \quad (4)
$$

The octet and singlet decay constants are different, and the mixing between the two isoscalar states is described by two mixing angles $\theta^p$ and $\theta^s$ [12]. For the study of scalar mesons into two pseudoscalar mesons, we use the quark basis. In this basis the two mixing angles $\phi^p$ and $\phi^s$ are very similar in magnitude [13]; we use $\phi^p = (39.3 \pm 1.0)^\circ$ [14]. In the singlet-octet basis, this mixing angle corresponds to $\theta^p = (39.3 + 35.3 - 90)^\circ = -(15.4 \pm 1.0)^\circ$.

The strangeness suppression factor $\lambda$ can be derived from $J/\psi$ decays into baryons. The partial decay widths are taken from the RPP2020 [11]. Corrected for the phase space they yield reduced widths $\Gamma'$.

$$
\lambda = \Gamma'_{J/\psi \rightarrow \Sigma^+ \Xi^-} / \Gamma'_{J/\psi \rightarrow pp} = 0.88 \pm 0.14
$$

$$
\lambda = \Gamma'_{J/\psi \rightarrow \Sigma^+ \Xi^0} / \Gamma'_{J/\psi \rightarrow nn} = 0.69 \pm 0.15
$$

$$
\lambda = \Gamma'_{J/\psi \rightarrow \Lambda \bar{\Lambda}} / \Gamma'_{J/\psi \rightarrow n\bar{n}} = 1.04 \pm 0.17
$$

$$
\lambda^2 = \Gamma'_{J/\psi \rightarrow \Xi^- \Xi^+} / \Gamma'_{J/\psi \rightarrow pp} = (0.70 \pm 0.06)^2
$$

This gives a mean value of

$$
\lambda = 0.84 \pm 0.04 \quad (5)
$$

Table 1: SU(3) structure constants for the decays of $(q\bar{q})$-mesons, $\gamma_{\bar{q}q}$, and glueballs, $\gamma_{\bar{q}q}^G$, into two pseudoscalar mesons given by quark combinatorics. See text for the definition of the mixing angles. The creation of an $s\bar{s}$ pair is suppressed by a factor $\lambda$.

| Decay | Coupling constants $\gamma_{\bar{q}q}$ |
|-------|-------------------------------------|
| $f^H \rightarrow \pi\pi$ | $\sqrt{3} \cos \varphi^*$ |
| $f^H \rightarrow K\bar{K}$ | $-(\sqrt{2} \sin \varphi^* + \sqrt{\lambda} \cos \varphi^*)$ |
| $f^H \rightarrow \eta\eta$ | $(\cos^2 \phi^p - \sin^2 \phi^s) \sin \varphi^* - \sqrt{\lambda} \cos^2 \phi^p \sin \varphi^*$ |
| $f^L \rightarrow \pi\pi$ | $\sqrt{2} \sin \varphi^*$ |
| $f^L \rightarrow K\bar{K}$ | $(\sqrt{2} \cos \varphi^* + \sqrt{\lambda} \sin \varphi^*)$ |
| $f^L \rightarrow \eta\eta$ | $(\cos^2 \phi^p - \sin^2 \phi^s) \sin \varphi^* + \sqrt{\lambda} \sin^2 \phi^p \cos \varphi^*$ |
| $a \rightarrow \eta\pi$ | $\sqrt{2} \cos \phi^p \sin \varphi^*$ |
| $a \rightarrow \eta'\pi$ | $\sqrt{2} \sin \phi^p$ |
| $a \rightarrow K\bar{K}$ | $\sqrt{\lambda}$ |
| $K \rightarrow K\pi$ | $\sqrt{3/\lambda}$ |
| $K \rightarrow K\eta$ | $\sqrt{2} (\cos \phi^p - \sqrt{\lambda} \sin \phi^p)$ |
| $K \rightarrow K\eta'$ | $\sqrt{2} (\sin \phi^p + \sqrt{\lambda} \cos \phi^p)$ |

This value is consistent with $\lambda = 0.77 \pm 0.10$ obtained from an analysis of the decay of tensor mesons [8]. The suppression factor is valid over a wide momentum range.

The strangeness suppression factor is a measure of SU(3) violation. Note that the strangeness suppression factor in exclusive two-body decays is different from the strangeness suppression in fragmentation. Here, we compare, e.g., $a_2(1320) \rightarrow K\bar{K}$ with $a_2(1320) \rightarrow \pi\eta$; in fragmentation one compares $a_2(1320) \rightarrow K\bar{K}$ with all $a_2(1320)$ decays including, e.g., $a_2(1320) \rightarrow \rho\pi$.

We write the wave function of a scalar states in the

![Figure 1: The SU(3) structure constants as functions of the mixing angle $\varphi$. For $\varphi = 0$, the meson is a $nn$ state, it is a $s\bar{s}$ state. Singlet and octet configurations are indicated.](image-url)
form
\[
\begin{align*}
    f_0^{\text{HH}}(xxx) &= (n\cos \phi_n^\alpha - s\sin \phi_n^\alpha) \cos \phi_\text{HH}^G + G \sin \phi_\text{HH}^G, \\
    f_0^{\text{LL}}(xxx) &= (n\sin \phi_n^\alpha + s\cos \phi_n^\alpha) \cos \phi_\text{LL}^G + G \sin \phi_\text{LL}^G.
\end{align*}
\]

\(\phi_n^\alpha\) is the scalar mixing angle, \(\phi_\text{HH}^G\) and \(\phi_\text{LL}^G\) are the meson-gluonball mixing angles of the high-mass state \(H\) and of the low-mass state \(L\) in the \(n\)th nonet. The fractional glueball content of a meson is given by \(\sin^2 \phi_\text{HH}^G\) or \(\sin^2 \phi_\text{LL}^G\).

The \(q\bar{q}\) component of a scalar meson couples to the final states with the SU(3) structure constant \(\gamma_\alpha\) and with a decay coupling constant \(c_\alpha\). The \(\gamma_\alpha\) depend on the SU(3) nonet, the constants \(c_\alpha\) depend on the SU(3) nonet: the decays of \(f_0(1500)\), \(f_0(1370)\), \(a_0(1450)\), and \(K^*(1430)\) should be described by the constant \(c_1\), \(f_0(1770)\) / \(f_0(1710)\) require a different value \(c_2\). The SU(3) structure constants \(\gamma_\alpha\) of a \(q\bar{q}\) singlet and of a glueball are identical. There is one coupling constant \(c_G\) for the glueball contents of all scalar mesons.

The coupling of a meson in nonet \(n\) to the final state \(\alpha\) can be written as
\[
g_n^\alpha = c_n \gamma_n^\alpha + c_G \gamma_\alpha^G.
\]
The fit to the data described in Ref. [3] returns the squared coupling constants \((g_n^\alpha)^2\). From these coupling constants, the partial decay widths were derived using the expression
\[
M^{\gamma n}_\alpha = \int_{\text{threshold}}^{\infty} \frac{ds}{\pi} \left( \frac{(g_n^\alpha)^2 \rho_n^G(s)}{M^2 - s + i\Gamma_n^G(s)}(s)^2 \right),
\]
where \(s\) is the two-meson invariant mass, \(\rho_n^G(s)\) the phase space, \(M\) is the nominal meson mass. The partial decay widths are given in Table 3 of Ref. [3]. For \(a_0(1450)\) and \(K^*_0(1430)\) we determine the coupling constants from their partial decay width into a final state \(\alpha\).

3. \(f_0(1370)\) and \(f_0(1500)\): mixing angle and nonet

Figure 2 shows the \(\pi\pi\) and \(K\bar{K}\) invariant mass distributions produced in radiative \(J/\psi\) decays. In the \(\pi\pi\) spectrum, an enhancement is seen just below 1500 MeV falling down sharply above 1500 MeV. We assume that \(f_0(1370)\) is a mainly-singlet and \(f_0(1500)\) a mainly-octet state, as suggested in Ref. [3]. The two amplitudes then interfere constructively, creating the observed peak. In the \(K\bar{K}\) final state, the amplitudes for \(f_0(1370)\) and \(f_0(1500)\) interfere destructively, there is little intensity and a clear minimum at 1500 MeV. The constructive interference in \(\pi\pi\) and the destructive interference in \(K\bar{K}\) requires interference between octet and singlet amplitudes. The fit of Rodas et al. [3] to the data on radiative \(J/\psi\) decay into \(\pi\pi\) and \(K\bar{K}\) (without constraints from \(\pi\pi\) scattering) did not include \(f_0(1370)\). Consequently, the minimum at 1500 MeV in the \(K\bar{K}\) mass distribution was not well reproduced.

To be more quantitative, we have fit the data to determine the scalar mixing angle and to study a possible glueball component. Table 2 presents the resonances in the first nonet of scalar mesons above 1 GeV with their decay modes, and the fitted and the experimental squared coupling constants. The “experimental” squared coupling constants are taken from Ref. [3], those of the isovector and isodoublet mesons were calculated from the partial and total decay-widths given in the RPP2020 [11]. Only the isoscalar mesons are used for the fits described below. The coupling constants of isovector and isodoubet mesons are predictions.

In a first step, we assume that there is no glueball in the wave functions of \(f_0(1370)\) and \(f_0(1500)\) and fit the scalar mixing angle \(\phi_1^s\) and the coupling \(c_1\) only. Note that \(c_1\) is one constant for a full nonet. The fit to the eight branching ratios returns
\[
\begin{align*}
    \phi_1^s &= (56 \pm 8)\degree, \quad \theta_1^s = (1 \pm 8)\degree, \quad \chi^2/N_F = 19/(8 - 3) \quad (8)
\end{align*}
\]
Obviously, the \(f_0(1370)\) is compatible with a pure singlet, \(f_0(1500)\) with a pure octet state.

In a next step, we allow for mixing of the two isoscalar mesons with a glueball and impose \(\theta_1^s = 0\). This fit returns \(c_G = 0.34\) which we freeze for the subsequent fits: the glueball decay-coupling-constant is the same for all scalar mesons to which the glueball contributes. With \(c_G = 0.34\) fixed, the final fit returns mixing angles and a \(\chi^2:\)
\[
\begin{align*}
    \phi_1^s &= (64 \pm 12)\degree, \quad \theta_1^s = (9 \pm 12)\degree \quad (9)
    \phi_{1H}^G &= (5 \pm 8)\degree, \quad \phi_{1L}^G = (10 \pm 6)\degree, \quad \chi^2/N_F = 6.3/4
\end{align*}
\]
This is a significant improvement, and these two mesons may contain some glueball fraction. In any case, the glueball component given by \(\sin^2 \phi_{1H,1L}^G\) is small. It is estimated to 4.0 ± 2.5% for \(f_0(1370)\) and to be less than 5% in
Table 2: Coupling constants of decays of mesons in the first nonet of scalar mesons. The fit to the decays of the two isoscalar mesons yields $c_1 = 0.21 \pm 0.02$, $c_2 = 0.34$, $\varphi_1 = (64 \pm 12)^\circ$, $\varphi_{1f} = (5 \pm 8)^\circ$, $\varphi_{2f}^f = (10 \pm 6)^\circ$, $\chi^2 = 6.3$ for 8 data points.

| Decay          | $g_{fit}^2$ | $g_{exp}^2$ |
|----------------|-------------|-------------|
| $f_0(1500) \rightarrow \pi \pi$ | 0.037 | 0.034$\pm$0.007 |
| $K \bar{K}$   | 0.018 | 0.014$\pm$0.004 |
| $\eta \eta$   | 0.004 | 0.006$\pm$0.002 |
| $\eta \eta'$  | 0.050 | 0.061$\pm$0.014 |
| $f_0(1370) \rightarrow \pi \pi$ | 0.166 | 0.226$\pm$0.048 |
| $K \bar{K}$   | 0.151 | 0.116$\pm$0.048 |
| $\eta \eta$   | 0.042 | 0.040$\pm$0.011 |
| $\eta \eta'$  | 0.003 | 0.025$\pm$0.019 |
| $\alpha(1450) \rightarrow K \bar{K}$ | 0.038 | 0.048$\pm$0.016 |
| $\pi \pi$     | 0.053 | 0.047$\pm$0.010 |
| $\eta \eta'$  | 0.035 | 0.026$\pm$0.013 |
| $K^*_0(1430) \rightarrow K \pi$ | 0.066 | 0.450$\pm$0.048 |
| $K \eta$      | 0.002 | 0.045$\pm$0.020 |
| $K \eta'$     | 0.059 | - |

The fit assigns the $\eta \eta'$ to $f_0(1770)$ with the mixing angle $\varphi_{1f}^0 = (38 \pm 4)^\circ$, $\varphi_{2f}^0 = (30 \pm 6)^\circ$, $\varphi_{2f}^f = (20 \pm 5)^\circ$. The $\chi^2$ for $f_0(1770)$ is 44 for 8 data points.

Table 3: Coupling constants of decays of mesons in the second nonet of scalar mesons. The fit yields $c_2 = 0.38 \pm 0.04$, $c^f = 0.34$, $\varphi_1 = (41 \pm 4)^\circ$, $\varphi_{1f}^f = (34 \pm 6)^\circ$, $\varphi_{2f}^f = (38 \pm 4)^\circ$. The $\chi^2$ for $f_0(1770)$ is 44 for 4 degrees of freedom.

| Decay          | $g_{fit}^2$ | $g_{exp}^2$ |
|----------------|-------------|-------------|
| $f_0(1770) \rightarrow \pi \pi$ | 0.036 | 0.042$\pm$0.012 |
| $K \bar{K}$   | 0.121 | 0.124$\pm$0.037 |
| $\eta \eta$   | 0.010 | 0.017$\pm$0.004 |
| $\eta \eta'$  | 0.130 | 0.030$\pm$0.018 |
| $f_0(1710) \rightarrow \pi \pi$ | 0.063 | 0.090$\pm$0.031 |
| $K \bar{K}$   | 0.170 | 0.186$\pm$0.043 |
| $\eta \eta$   | 0.036 | 0.145$\pm$0.051 |
| $\eta \eta'$  | 0.007 | 0.134$\pm$0.059 |

The $\chi^2$ for 44 for 4 degrees of freedom is unacceptably large. The large $\chi^2$ can be traced to the yields of $\eta \eta'$ decays. The fit assigns the $\eta \eta'$ intensity to $f_0(1710)$ that is interpreted here as mainly-singlet state. Little $\eta \eta'$ intensity is assigned to $f_0(1770)$. We do not exclude that this assignment by the fit is misleading: these two resonances are very close in mass and it could be difficult to separate contributions reliably. We emphasize that the quality of the different data sets is rather different: the $\pi \pi$ and $K \bar{K}$ data are the most reliable ones. Here, the $S$-wave has been extracted in a model-independent way. The $\eta \eta'$ S-wave contribution used here stems from an energy-dependent partial-wave analysis performed by the BESIII collaboration using Breit-Wigner representations for resonances. Unfortunately, the original data are not publicly available. Data on $\eta \eta'$ are poor.

4. $f_0(1710)$ and $f_0(1770)$: mixing angle and nonet

The two mesons $f_0(1710)$ and $f_0(1770)$ are rather close in mass. They produce a large peak in the $K \bar{K}$ invariant mass distribution (see Fig. 2), with much higher intensity than in the $\pi \pi$ spectrum. This could indicate a large $s \bar{s}$ component in the wave function in one of the two states. But the partial-wave analysis revealed considerably larger $K \bar{K}$ yields than $\pi \pi$ yields for both resonances. This is impossible for two states belonging to the same nonet.

Indeed, a fit to branching ratios of $f_0(1770)$ and $f_0(1710)$ (see Table 2) with an arbitrary mixing angle but with no glueball contribution fails. Hence we added a glueball component to the wave function and assumed a mainly-octet $q \bar{q}$ structure for $f_0(1770)$, a mainly-singlet $q \bar{q}$ structure for $f_0(1710)$ and constructive interference between glueball and $f_0(1770)$ in the $K \bar{K}$ decay amplitude. These assumptions entail for $f_0(1710) \rightarrow \pi \pi$ decays a constructive interference between the $q \bar{q}$ and glueball amplitudes, a destructive interference for $f_0(1770)$, and constructive interference between the $q \bar{q}$ and glueball amplitudes for both resonances in the $K \bar{K}$ decay mode. This pattern can be seen in Fig. 2: the $\pi \pi$ invariant mass distribution is rapidly falling down at 1750 MeV while the $K \bar{K}$ mass distribution exhibits a strong peak at this mass.
yields

$$\varphi_2^2 = (41 \pm 4)^{\circ}, \quad \vartheta_2 = -(14 \pm 4)^{\circ}$$

$$\phi_{2H}^G = -(29 \pm 6)^{\circ}, \quad \phi_{2L}^G = -(20 \pm 5)^{\circ}, \quad \chi^2/N_F = 44/4$$

The glueball content of $f_0(1710)$ is determined to $\sin^2 \phi_{2L}^G = (12\pm6)\%$ and of $f_0(1770)$ to $(25\pm10)\%$. The scalar mixing angle $\vartheta_2^G$ is not compatible with a simple singlet-octet configuration (with $\vartheta_2 = 0$).

In 1650 to 1850 MeV mass region an $a_0$ and a $K^*_0$ are both missing to complete a SU(3) nonet. Neither an $a_0$ nor a $K^*_0$ resonance was reported in the RPP2020 in this mass range. There are two possible interpretations.

First, the two resonances $f_0(1710)$ and $f_0(1770)$ could only be one single state - let us call it $f_0(1750)$ - and this could be the glueball. However, the squared masses of the five singlet and of the five octet scalar isoscalar mesons fall onto linear $(n, M^2)$ trajectories (see Fig. 3 in Ref. [8]). Clearly, its is difficult to take out the two states at 1700 MeV: a gap would be created. Further, the fit to the data deteriorates significantly when one of the two resonances is taken out.

The second possibility is that an $a_0$ and a $K^*_0$ do exist. Indeed, the BABAR collaboration reported a fit to the Dalitz plots $\eta \rightarrow \eta' K^+ K^-$, $\eta' K^+ K^-$, $\eta K^+ K^-$ and identified a new isovector state $a_0(1700)$ with $M = (1704 \pm 5 \pm 2)$ MeV and $\Gamma = (110 \pm 15 \pm 11)$ MeV [17]. In an analysis of the $K \pi$ S-wave, the authors of Ref. [18] fit the data above 900 MeV and find a further pole slightly above 1800 MeV and a width of 200 to 260 MeV. The existence of a $K^*_0(1800)$ is at least not ruled out. However, the mass values of $a_0(1700)$, $K^*_0(1800)$, $f_0(1710)$, $f_0(1770)$ do not yield a consistent nonet: The GOM formula suggests $f_0(1770)$ to be a dominantly $n\bar{n}$ state even though it decays preferentially into $K\bar{K}$. Yet, the uncertainty in the mixing angle is substantial.

5. $f_0(2020)$ and $f_0(2100)$: mixing angle and nonet

Table 4 presents the results of our fit for the third pair of scalar mesons above 1 GeV. There is strong interference between these two resonances but also with both isoscalar mesons of the two neighboring nonets. Again, the couplings to $K\bar{K}$ of both isoscalar resonances in this nonet are larger than the couplings to $\pi\pi$. This is not possible without interference with another amplitude. Again, we assume the scalar glueball to make up a fraction of the mesonic wave functions of $f_0(2020)$ and $f_0(2100)$.

The data are again fit with an unacceptable $\chi^2$ that is due to the $f_0(2100) \rightarrow \eta\eta$ decay mode. The $\pi\pi$ and $K\bar{K}$ decay modes are described excellently. Therefore we think, we can extract the glueball content reliably. We find

$$\varphi_3 = (51 \pm 4)^{\circ}, \quad \vartheta_3 = -(4 \pm 4)^{\circ}$$

$$\phi_{3H}^G = -(23 \pm 7)^{\circ}, \quad \phi_{3L}^G = -(24 \pm 6)^{\circ}, \quad \chi^2/N_F = 15/3$$

and the glueball content of $f_0(2020)$ and $f_0(2100)$ is determined to $(16\pm9)\%$ and $(17\pm8)\%$. The nonet mixing angle is compatible with $f_0(2020)$ being a singlet and $f_0(2100)$ being an octet. The $K^*_0(1950)$ and $a_0(1950)$ could be the partners to form a full nonet. The Gell-Mann–Okubo mass formula does not constrain the mixing angle due to the large uncertainties in the masses.

6. $f_0(2200)$ and $f_0(2330)$

Too little is known about these two resonances. For an octet state, a $\pi\pi : K\bar{K} : \eta\eta$ ratio of $3 : 1 : 1$ is expected (for a pseudoscalar mixing angle $\vartheta_{ps} = 0$), not incompatible with the radiative decay rates (in units of $10^{-3}$) of $4 \pm 2 : 2.5 \pm 0.5 : 1.5 \pm 0.4$ observed for $J/\psi \rightarrow \gamma f_0(2330), f_0(2330) \rightarrow \pi\pi, K\bar{K}, \eta\eta$ even though some glueball admixture is certainly not excluded. For $f_0(2200)$ as singlet, we expect these ratios to be $3 : 4 : 1$ which are compatible with $5 \pm 2 : 5 \pm 2 : 1.5 \pm 0.4$. A glueball admixture is not required and not forbidden.

7. Discussion and Summary

The nonet containing $f_0(1370)$ and $f_0(1500)$ and the one containing $f_0(2020)$ and $f_0(2100)$ are shown to have mixing angles very close to the conjecture proposed in Ref. [8]: the lower-mass states are compatible with a pure singlet $qq$ component, the higher-mass state with a $qq$ component in an octet configuration. The two mesons $f_0(1710)$ and $f_0(1770)$ deviate from this conjecture. It is possible that the small mass difference between these two states leads to this unexpected mixing. Note that in the case of $f_0(2020)$ and $f_0(2100)$, only the nominal masses are very close. The measured mass difference is 150 MeV.

More important are the glueball contributions to the mesonic wave functions. The probability that the glueball mixes into one of these resonances is

$$f_0(1370), f_0(1500), f_0(1710), f_0(1770), f_0(2020), f_0(2100)$$

$$(5 \pm 4)\% < 5\% (12 \pm 6)\% (25 \pm 10)\% (16 \pm 9)\% (17 \pm 8)\%$$

The glueball is distributed, the sum of the fractional contribution is $(78 \pm 18)\%$. A small further contribution (of about 10%) can be expected from the two higher mass...
states $f_0(2200)$ and $f_0(2330)$. Figure 3 shows the fractional contribution of the scalar mesons to the glueball. The solid line is a Breit-Wigner function with mass and width $M = 1865\,\text{MeV}$, $\Gamma = 370\,\text{MeV}$, the area is normalized to one. The fractional glueball contributions to scalar mesons determined from the decays of scalar mesons are compatible with the Breit-Wigner resonance observed in production of scalar mesons in radiative $J/\psi$ decays. This is a remarkable verification of the interpretation of the bump observed in radiative $J/\psi$ decays as the scalar glueball of lowest mass.

![Graph showing the glueball content of scalar mesons.](image)

Figure 3: The glueball content of scalar mesons.

The pair of resonances $f_0(1710)$ and $f_0(1770)$ was mostly interpreted as one single resonance and was often identified with the scalar glueball [19]. The discovery of two scalar mesons, $f_0(1370)$ and $f_0(1500)$ (see [20] and references therein) stimulated Amsler and Close to propose a mixing scheme where two scalar $q\bar{q}$ mesons and the scalar glueball mix to create the three observed states $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ [21] [22]. This paper led to a large number of follow-up studies with different mixing schemes [23] [33]. These all have one property in common: they impose that the sum of the fractional glueball contributions from these three resonances adds up to one. This we do not impose. We find that the sum over six scalar resonances is $(78\pm18\%)$ and expect further $10\%$ from higher-mass resonances. Within errors, the full glueball is covered.

A second important difference is the nature of the glueball. In earlier papers, the glueball is seen as an intruder, as additional resonance entering the spectrum of scalar $q\bar{q}$ mesons (with possibly further tetraquarks or hadronic molecules). Glueball and scalar mesons mix but there is supernumerary of scalar states. We see the glueball as enhancement in the yields of ordinary scalar mesons. The decays of these scalar mesons show that their wave functions must contain a fraction of the glueball. But the glueball is not an additional meson, the glueball shows up only as fractional contribution to the wave functions of scalar mesons.

Summarizing, we have studied the decays of scalar isoscalar mesons. The decay couplings were fit with the assumption that their wave functions contain three components: $n\bar{n}$, $s\bar{s}$ and a glueball component. The $\pi\pi$ and $K\bar{K}$ decay modes are well described, the $\eta\eta'$ decay mode only partly, the $\eta\eta'$ decay mode is often at variance with the prediction. Since the glueball content is mostly determined by the $\pi\pi$ and $K\bar{K}$ decay modes, we extract the glueball component. It follows that the glueball is spread over a large number of resonances. The sum of all fractional contributions is close to one. The scalar glueball is thus not only identified in radiative decays of $J/\psi$ mesons but also by its fractional contributions to the wave functions of the scalar isoscalar mesons that were produced in radiative decays of $J/\psi$ mesons.

The masses of all eight scalar-isoscalar H and L states above 1 GeV discussed here fall onto two regular trajectories in a $M^2$, $n$ plot (see Fig. 3 in [3]). None of them seems irregular. Apparently, the scalar glueball does not enter the spectrum of scalar mesons as supernumerous state. It seems not to decay directly into two mesons but to decay only by mixing with regular scalar isoscalar mesons.

Acknowledgement

Funded by the NSFC and the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the funds provided to the Sino-German Collaborative Research Center TRR110 “Symmetries and the Emergence of Structure in QCD” (NSFC Grant No. 12070131001, DFG Project-ID 196253076 - TRR 110) and the Russian Science Foundation (RSF 16-12-10267).

References

[1] M. Gell-Mann, “Symmetries of baryons and mesons,” Phys. Rev. 125, 1067-1084 (1962).
[2] S. Okubo, “Note on unitary symmetry in strong interactions,” Prog. Theor. Phys. 27, 949-966 (1962).
[3] A. V. Sarantsev, I. Denissenko, U. Thoma and E. Klempt, “Scalar isoscalar mesons and the scalar glueball from radiative $J/\psi$ decays,” Phys. Lett. B 816, 136227 (2021).
[4] A. Rodas et al. [APAC], “Scalar and tensor resonance decays,” [arXiv:2109.00027 [hep-ph]].
[5] J. A. Oller, “The Mixing angle of the lightest scalar nonet,” Nucl. Phys. A 727, 353-369 (2003).
[6] E. Klempt, “Scalar mesons and the fragmented glueball,” Phys. Lett. B 820, 136512 (2021).
[7] E. Gregory, A. Irving, B. Lucini, C. McNeile, A. Rago, C. Richards and E. Rinaldi, “Towards the glueball spectrum from unquenched lattice QCD,” JHEP 10, 170 (2012).
[8] K. Peters and E. Klempt, “The Suppression of $s\bar{s}$ creation from tensor meson decays,” Phys. Lett. B 352, 467-471 (1995).
[9] A. V. Anisovich, V. A. Nikonov, A. V. Sarantsev, V. V. Anisovich, M. A. Matveev, T. O. Vulf, K. V. Nikonov and J. Nyiri, “Analysis of the meson-meson data in the framework of the dispersion D-matrix method,” Phys. Rev. D 84, 076001 (2011).
[10] C. Amsler, T. DeGrand and B. Krusch, “Quark model,” in: [11].
[11] P. A. Zyla et al. [Particle Data Group], PTEP 2020, no.8, 083C01 (2020).
[12] T. Feldmann, “Mixing and decay constants of pseudoscalar mesons: Octet singlet versus quark flavor basis,” Nucl. Phys. B Proc. Suppl. 74, 151-154 (1999).
[13] Y. H. Chen, Z. H. Guo and B. S. Zou, “Unified study of $J/\psi \rightarrow PV, P_8(1400)$ and light hadron radiative processes,” Phys. Rev. D 91, 014010 (2015).
[14] T. Feldmann, “Quark structure of pseudoscalar mesons,” Int. J. Mod. Phys. A 15, 159-207 (2000).
[15] M. Ablikim et al. [BESIII Collaboration], “Amplitude analysis of the $n^0\pi^0$ system produced in radiative $J/\psi$ decays,” Phys. Rev. D 92 no.5, 052003 (2015).
[16] M. Ablikim et al. [BESIII Collaboration], “Amplitude analysis of the $K_S^0 K_S^0$ system produced in radiative $J/\psi$ decays,” Phys. Rev. D 98 no.7, 072003 (2018).
[17] J. P. Lees et al. [BaBar], “Light meson spectroscopy from Dalitz plot analyses of $\eta_c$ decays to $\eta' K^+ K^-$, $\eta'\pi^+\pi^-$, and $\eta\pi\pi$ produced in two-photon interactions,” [arXiv:2106.05157 [hep-ex]].
[18] A. V. Anisovich and A. V. Sarantsev, “K matrix analysis of the $K\pi$ S-wave in the mass region 900 MeV - 2100 MeV and nonet classification of scalar $q\bar{q}$ states,” Phys. Lett. B 413, 137-146 (1997).
[19] J. Sexton, A. Vaccarino and D. Weingarten, “Numerical evidence for the observation of a scalar glueball,” Phys. Rev. Lett. 75, 4563-4566 (1995).
[20] C. Amsler et al., “High statistics study of $f_0(1500)$ decay into $\pi^0\pi^0$,” Phys. Lett. B 342, 433-439 (1995).
[21] C. Amsler and F. E. Close, “Evidence for a scalar glueball,” Phys. Lett. B 353, 385-390 (1995).
[22] C. Amsler and F. E. Close, “Is $f_0(1500)$ a scalar glueball?”, Phys. Rev. D 53, 295-311 (1996).
[23] L. Burakovsky and P. R. Page, “Scalar glueball mixing and decay,” Phys. Rev. D 59, 014022 (1999) [erratum: Phys. Rev. D 59, 079902 (1999)].
[24] W. J. Lee and D. Weingarten, “Scalar quarkonium masses and mixing with the lightest scalar glueball,” Phys. Rev. D 61, 014015 (2000).
[25] D. M. Li, H. Yu and Q. X. Shen, “Properties of the scalar mesons $f_0(1370), f_0(1500)$ and $f_0(1710)$,” Eur. Phys. J. C 19, 529-533 (2001).
[26] F. E. Close and A. Kirk, “Scalar glueball $q\bar{q}$ mixing above 1-GeV and implications for lattice QCD,” Eur. Phys. J. C 21, 531-543 (2001).
[27] F. Giacosa, T. Gutsche and A. Faessler, “A Covariant constituent quark / gluon model for the glueball-quarkonia content of scalar - isoscalar mesons,” Phys. Rev. C 71, 025202 (2005).
[28] F. E. Close and Q. Zhao, “Production of $f_0(1710), f_0(1500)$ and $f_0(1370)$ in $J/\psi$ hadronic decays,” Phys. Rev. D 71, 094022 (2005).
[29] F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, “Scalar nonet quarkonia and the scalar glueball: Mixing and decays in an effective chiral approach,” Phys. Rev. D 72, 094006 (2005).
[30] H. Y. Cheng, C. K. Chua and K. F. Liu, “Scalar glueball, scalar quarkonia, and their mixing,” Phys. Rev. D 74, 094005 (2006).
[31] J. Chen, L. Zhang and H. Xia, “The mixing of the scalar- isoscalar states $f_0(1370), f_0(1500)$ and $f_0(1710)$,” Mod. Phys. Lett. A 24, 1517-1531 (2009).
[32] L. C. Gui et al. [CLQCD], “Scalar Glueball in Radiative $J/\psi$ Decay on the Lattice,” Phys. Rev. Lett. 110, no.2, 021601 (2013).
[33] S. Janowski, F. Giacosa and D. H. Rischke, “Is $f_0(1710)$ a glueball?,” Phys. Rev. D 90, no.11, 114005 (2014).
[34] H. Y. Cheng, C. K. Chua and K. F. Liu, “Revisiting Scalar Glueballs,” Phys. Rev. D 92, no.9, 094006 (2015).
[35] J. M. Frère and J. Heeck, “Scalar glueballs: Constraints from the decays into $\eta$ or $\eta'$,” Phys. Rev. D 92, no.11, 114035 (2015).
[36] V. Vento, “Glueball-Meson Mixing,” Eur. Phys. J. A 52, no.1, 1 (2016).
[37] H. Noshad, S. Mohammad Zebardjad and S. Zarepour, “Mixing among lowest-lying scalar mesons and scalar glueball,” Nucl. Phys. B 934, 408-436 (2018).
[38] X. D. Guo, H. W. Ke, M. G. Zhao, L. Tang and X. Q. Li, “Revisiting the determining fraction of glueball component in $f_0$ mesons via radiative decays of $J/\psi$,” Chin. Phys. C 45, no.2, 023104 (2021).