New Results on the Stability and $L_2$-$L_\infty$ Control of Itô Stochastic Systems With Sawtooth-Like Input Delay

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ABSTRACT This paper considers the stability and $L_2$-$L_\infty$ control problem for Itô stochastic systems with the sawtooth-like input delay. Compared with previously-known results on systems with the sawtooth-like input delay, a new auxiliary system is introduced to reduce the conservatism of the system. Besides, the influence of stochastic noises is investigated. For the delayed stochastic system without the disturbance, an LMI type mean-square asymptotical stability criterion is derived by using the Bessel-Legendre stochastic inequality and a modified finite-interval quadratic polynomial inequality. Then a linear feedback controller is proposed to realize the stability of the system with a prescribed $L_2$-$L_\infty$ performance under the influence of disturbances, where the controller gain matrix is designed by introducing slack variables. To be pointed out that the augmented Lyapunov-Krasovskii functional (LKF) used in this paper is new, which is constructed by introducing an auxiliary system to increase the delay-dependent function terms in the discriminant. Finally, two numerical examples are given to show the validness and effectiveness of the theoretical results.

INDEX TERMS Itô stochastic system, Sawtooth-like input delay, mean-square asymptotical stability, $L_2$-$L_\infty$ control, quadratic polynomial inequality.

I. INTRODUCTION

In the past few decades, there remains an increasing interest in the study of time delay, since it appears in many practical systems, such as chemical processes, power systems, biology systems, networked control systems (NCSs), and so on. As we all know, the time delay often causes undesirable and poor performance or even instability of systems, so in order to ensure the stability of time-delay systems, a lot of research has been done, see [1]–[4]. Since NCSs bring great advantages such as low cost, reduced weight and power requirements, and can deal with all the continuous, discrete, and hybrid control asynchronous process and support topologies, they have been studied by many researchers, see [5]–[8]. The time delay in NCSs is called the sawtooth-like input delay in the form of $D(t) = d(t - t_k + \tau)$, which is piecewise continuous. Liu, et al. [9] proposed a novel input-output approach via a Wirtinger-type inequality and extended piecewise continuous (in time) Lyapunov functionals to the general sawtooth delay. Sun, et al. [10] considered the predictor-based controller and assumed that there exists an instant where the input delay is zero to obtain a stability criterion for the stability of linear systems with the sawtooth delay. There are also many other research papers discussing the influence of the sawtooth delay, where many researchers put their main attention on the sampled-data delay (the general sawtooth delay with $d = 1$) [11]–[15]. However, it should be pointed out that the literatures mentioned above are about the influence of the sawtooth-like input delay in deterministic systems.

On the other hand, random phenomena are ubiquitous in nature, and a large number of significant results have been given for the stochastic time-delay systems, among which the study on the stability and control is the important part, see [16]–[21], $L_2$-$L_\infty$ [29]–[31] and $H_2/H_\infty$ [32]–[35] control are common ways to solve the control problems. Gong, et al. [16] created a new stochastic integral inequality called Bessel-Legendre stochastic inequality...
to guarantee the filtering error system to be asymptotically mean-square stable with a prescribed $L_2$-$L_\infty$ performance level. Hou, et al. [29] combined the disturbance observer based on disturbance-observer-based control and $L_2$-$L_\infty$ control, which can attenuate and reject different types of disturbances. [30] investigated the $L_2$-$L_\infty$ control for leader-following coordination problems with the external disturbance. It should be noted that $L_2$-$L_\infty$ control design has caused wide concern mainly because it is insensitive to the exact knowledge of the statistics of the noise signals. Such a control procedure can ensure that the $L_2$-$L_\infty$ gain from the noise input signals to the controlled output will be less than a prescribed level, where the noise input is an arbitrary energy-bounded signal [21]. From this viewpoint, to study the stability and $L_2$-$L_\infty$ control for Itô stochastic systems with the sawtooth delay is of practical significance. Recently, in [22], the sliding mode control design was studied for a class of stochastic switching systems subject to semi-Markov process via an adaptive event-triggered mechanism. Qi, et al. [23] addressed the finite-time event-triggered control problem for nonlinear semi-Markovian systems. [24] investigated the attack-resilient control problem for Markov jump systems (MJSs) with additive attacks. [25] discussed $H_\infty$ finite-time realization for a class of uncertain Markovian jump systems (MJSs) with unmeasurable state via sliding mode control (SMC) method. Clearly, these papers put forward some significant results on the control of Markov jump systems, which will be helpful to the development of related theories. Different from [22]–[25], we mainly study the delayed systems with Brown motions and aim to present some new stability conditions. In recent years, researchers have paid some attention on the consensus problem of multiagent systems. [26] investigated the consensus tracking problem for a class of continuous switched stochastic nonlinear multiagent systems with an event-triggered control strategy. [27] addressed the event-triggered consensus tracking problem for a class of higher order stochastic nonlinear multiagent systems.

To the best of our knowledge, there are few papers on the stability and $L_2$-$L_\infty$ control of Itô stochastic systems with the sawtooth delay. Because of this, we will investigate the stability and $L_2$-$L_\infty$ control of system (1). Two cases of the system are discussed: one is to find the maximum delay time $h_M$ when $r(t) = 0$ and the other one is to design an appropriate controller and get a minimum performance level to make the system stable with given $h_M$ when $r(t) \neq 0$. In the study process, we have the following important contributions and main advantages: (1) The sawtooth delay is first studied to combine Itô stochastic systems with the disturbance input; (2) We introduce an auxiliary system and create a new Lyapunov-Krasovskii functional by adding two integrals to the classical one which includes the auxiliary system state value at the time $t + \varphi(t)$, which can change the number of the delay-dependent function terms to reduce the conservatism since $t + \varphi(t)$ may not belong to the range $(t \in [t_k, t_{k+1}))$.

The rest of the paper is organized as follows: In the second part, we give some problem formulation and preliminaries. In the third part, we obtain a quadratic function parameterized by the sawtooth delay $D(t)$, where we prove the mean-square stability of Itô stochastic system when $r(t) = 0$ and get a controller gain matrix for the mean-square stability when $r(t) \neq 0$. In the fourth part, we propose two numerical examples to prove the effectiveness of the design scheme. Finally, the conclusion is given in Section VI.

**II. PROBLEM FORMULATION AND PRELIMINARIES**

Consider the following stochastic system:

$$
\begin{aligned}
dx(t) &= [A_1x(t) + B_1u(t) + C_1r(t)]dt + [A_2x(t) + B_2u(t) + C_2r(t)]dw(t) \\
z(t) &= A_3x(t) + B_3u(t) \\
x(t) &= \phi(t), \quad t \in [-h_M, 0].
\end{aligned}
$$

(1)

The control law with linear state feedback is $u(t) = Kx(t - D(t))$, where $K \in \mathbb{R}^{m \times n}$ denotes the controller gain matrix which will be discussed in Theorem 2. The sawtooth delay function $D(t)$ is assumed to be of the form

$$
D(t) = d(t - t_k + h_m), \quad t \in [t_k, t_{k+1}),
$$

(2)

satisfying $dh_m \leq D(t) \leq h_M$ and $\dot{D}(t) = d$, which is shown in Fig. 1, where $h_m, h_M, d$ are known positive constant scalars, $t_k$ is a time series satisfying $h_m < t_{k+1} - t_k$. In system (1), $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2$ are known constant matrix. $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathbb{R}^m$ is the input variable, $r(t) \in \mathbb{R}^p$ is the disturbance input belonging to $L_2[0, \infty)$, that is $\int_0^\infty \|r(t)\|^2 dt < \infty$, $w(t)$ is a standard Wiener process with suitable dimensions and it satisfies $E[dw(t)] = 0$ and $E[dw(t)dw(t)\| = dt$, $\phi(t)$ is the initial function of $x(t)$ which is continuously differentiable [16].

**Remark 1:** Let $V(t, x) \in C^{1,2}(\mathbb{R}_+, \mathbb{R}^n)$ be a scalar function and $V(t, x) > 0$, consider the following stochastic system [34]

$$
\dx(t) = \mathcal{L}V(t, x)dt + g(t)dw(t),
$$

(3)

the Itô formula of $V(t, x)$ is given as follows:

$$
\dx(V(t, x)) = \sum \frac{\partial^2 V(t, x)}{\partial x^2}g(t)dw(t)dt,
$$

(4)

where

$$
\mathcal{L}V(t, x) = \frac{\partial V(t, x)}{\partial t} + \sum \frac{\partial^2 V(t, x)}{\partial x^2}f(t) + \int_0^\infty [g(t)]^2 dt.
$$
Next, two definitions of the mean-square stability and some important lemmas are introduced which will be applied to the proof of the main results.

**Definition 1 [28]:** The system (1) with \( r(t) = 0 \) is said to be mean-square stable if for any \( \varepsilon > 0 \), there exists a \( \delta > 0 \) such that \( E[\{x(t)\}^2] < \varepsilon, t > 0 \) when \( \sup_{t \in \mathbb{R}} E[\{\phi(t)\}^2] < \delta \). Moreover, if \( \lim_{t \to +\infty} E[\{x(t)\}^2] = 0 \) holds for any initial condition, then the system is said to be asymptotically mean-square stable.

**Definition 2 [28]:** Given a scalar \( \gamma > 0 \), the system (1) is said to be asymptotically mean-square stable with a prescribed \( L_2-L_\infty \) performance level \( \gamma \) if it is asymptotically mean-square stable, and under zero initial condition, \( \|z(t)\|_\infty < \gamma \|r(t)\|_2 \) for all nonzero \( r(t) \in L_2[0, \infty) \), where
\[
\|z(t)\|_\infty = \sup_{t > 0} \sqrt{Ez^T(t)z(t)}
\]
\[
\|r(t)\|_2 = \sqrt{\int_0^\infty r^T(t)r(t)dt}.
\]

**Lemma 1 (Bessel-Legendre Stochastic Inequality [16]):** If there exist \( x(t) \in \mathbb{R}^n, f(t) \in \mathbb{R}^n, g(t) \in \mathbb{R}^n \) such that the integral \( \int_{-h}^0 f^T(s)Rf(s)ds \) is well defined, then for any matrix \( R \in \mathbb{R}^{n \times n} \) and \( R \geq 0 \) and scalar \( h > 0 \),
\[
\int_{-h}^0 f^T(s)Rf(s)ds \geq \frac{1}{h} \sum_{k=0}^N (2k + 1)(S^T_k R S_k - 2 \Psi_k),
\]
where
\[
\Psi_k = (\int_{-h}^0 L_k(u)g_k(u)dv_k(u))R(\int_{-h}^0 L_k(u)dv_k(u)),
\]
\[
\Phi_k = \sum_{i=0}^{k-1} \frac{2i + 1}{h} ((-1)^k i + 1)\Omega_k + x_k(0) - (-1)^k x_k(-h),
\]
\[
\Omega_k = \int_{-h}^0 L_k(u)x_k(u)du,
\]
\[
L_k(u) = (-1)^k \sum_{i=0}^k (-1)^i \begin{pmatrix} k+i \choose l \end{pmatrix} \begin{pmatrix} u + h \choose l \end{pmatrix} \begin{pmatrix} h \choose i \end{pmatrix},
\]
\[
k = 0, 1, \ldots, N.
\]

**Remark 1:** For any function \( x(t) : [-h, \infty) \to \mathbb{R}^n \), \( x_i(s) \) represents \( x(t + s) \) for all \( t \geq 0 \) and \( s \in [-h, 0] \). \( f_i(s) \) and \( g_i(s) \) are defined as \( x_i(s) \). When the interval \([-h, 0]\) is modified as the general form \([a, b]\), the inequality in Lemma 1 still holds.

**Lemma 2 (Schur Complement Formula, [37]):** For a given symmetric matrix \( S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \), where \( S \) is \( r \times r \) dimensional matrix. The following two conditions are equivalent to \( S < 0 \):

1. \( S_{11} < 0, \quad S_{22} - S_{12}S_{11}^{-1}S_{12} < 0 \);
2. \( S_{22} < 0, \quad S_{11} - S_{12}S_{22}^{-1}S_{12} < 0 \).

**Lemma 3 [43]:** Let \( R_1 \in \mathbb{R}^{q \times q} \) be positive definite and a constant \( \alpha \in (0, 1) \). If there exist four matrices \( X_i \in \mathbb{R}^{q \times q} \) with \( X_i = X_i^T \) and \( Y_i \in \mathbb{R}^{q \times q} \) \((i = 1, 2)\) such that
\[
\begin{bmatrix} R_1 - X_1 & Y_1 \\ * & R_2 \end{bmatrix} \geq \begin{bmatrix} 0 \end{bmatrix}, \quad \begin{bmatrix} R_1 & Y_2 \\ * & R_2 - X_2 \end{bmatrix} \geq \begin{bmatrix} 0 \end{bmatrix},
\]
then the following inequality holds for \( \xi_i \in \mathbb{R}^q \):(i = 1, 2)
\[
-\frac{1}{\alpha} \xi_1^T R_1 \xi_1 - \frac{1}{1 - \alpha} \xi_2^T R_2 \xi_2 \leq -\xi_1^T [R_1 + (1 - \alpha)X_1] \xi_1
\]
\[
- \xi_2^T (R_2 + \alpha X_2) \xi_2 - 2\xi_1^T (\alpha Y_1 + (1 - \alpha)Y_2) \xi_2.
\]

**Lemma 4 [44]:** Let \( f(h) = a_0 + a_1 h + a_2 h^2 \) and \( h \in [dh_m, h_M] \), where \( a_0, a_1, a_2 \in \mathbb{R} \). If
\[
(1) \quad f(dh_m) < 0, \quad (2) \quad f(h_M) < 0,
\]
\[
(3) \quad a_0 + \frac{1}{2} (dh_m + h_M a_1 + dh_m h_M a_2) < 0,
\]
then \( f(h) < 0 \) for all \( h \in [dh_m, h_M] \).

**Notations:** In this paper, the used notations are standard. \( A \) is any positive integer; \( \dot{A} \) denotes the transpose of the matrix \( A \); \( \|x(t)\|^2 \) denotes \( x^T(t)x(t) \); \( A > 0 \) represents that \( A \) is a positive definite(positive semi-definite) matrix; \( S_{11}^{-1} \) denotes the set of \( n \times n \) real positive symmetric matrices; a block-column is denoted by \( \vert \cdots \cdots \vert \); the expression \( He[A] \) denotes \( A + AT \); \( E[\cdot] \) denotes the expectation operator.

**III. MAIN RESULTS**

First, we set \( f(t) = A_1 x(t) + B_1 u(t) + C_1 r(t) \), \( g(t) = A_2 x(t) + B_2 u(t) + C_2 r(t) \), the first equality in system (1) can be expressed by (3). Since system (1) only applies to a certain range \((t \in [t_k, t_{k+1})\), we introduce an auxiliary system:}
\[
\begin{align*}
\dot{\tilde{x}}(t) &= \tilde{f}(t)dt + \tilde{g}(t)dw(t), \quad t \geq t_k, \\
\tilde{x}(t) &= x(t), \quad t < t_k.
\end{align*}
\]
with \( \tilde{f}(t) = A_1 \tilde{x}(t) + B_1 \tilde{u}(t) + C_1 r(t) \), \( \tilde{g}(t) = A_2 \tilde{x}(t) + B_2 \tilde{u}(t) + C_2 r(t) \), \( \tilde{u}(t) = K \tilde{x}(t) - D(t) \). Clearly, \( \tilde{x}(t) = x(t) \) when \( t \in [t_k, t_{k+1}) \). According to (3) and (5), we find a function \( \phi(t) \) such that \( \tilde{x}(t + \phi(t) - D(t + \phi(t))) = x(t) \), then
\[
\phi(t) = d(t - t_k + dh_m) = \frac{D(t)}{1 - d}, \quad t + \phi(t) \geq t_k + \frac{dh_m}{1 - d},
\]
For the formal simplicity, some nomenclature are defined as given below:

$$\begin{align*}
\xi^T(t) &= [x^T(t), x^T(t - D(t)), x^T(t - h_M), x^T(t + \varphi(t))], \\
\eta_1^T(t) &= [x^T(t), x^T(t - D(t)), \int_{t-D(t)}^{t} x^T(s)ds], \\
\eta_2^T(t) &= [x^T(t - D(t)), x^T(t - h_M), \int_{t-h_M}^{t} x^T(s)ds], \\
\eta_3^T(t) &= [x^T(t + \varphi(t)), x^T(t), \int_{t}^{t+\varphi(t)} x^T(s)ds], \\
\eta_4^T(t, s) &= [f^T(s), x^T(s), \xi^T(t), \int_{s}^{t} x^T(\theta)d\theta], \\
\eta_5^T(t, s) &= [f^T(s), x^T(s), \xi^T(t), \int_{s}^{t-D(t)} x^T(\theta)d\theta], \\
\eta_6^T(t, s) &= [f^T(s), x^T(s), \xi^T(t), \int_{s}^{t+\varphi(t)} x^T(\theta)d\theta].
\end{align*}$$

$$\begin{align*}
\rho_1(t) &= \int_{t-D(t)}^{t} \frac{x(s)}{D(t)} ds, \\
\rho_2(t) &= \int_{t-D(t)}^{t} \frac{(t-s)x(s)}{D^2(t)} ds, \\
\rho_3(t) &= \int_{h_M}^{t-D(t)} \frac{x(s)}{h_M - D(t)} ds, \\
\rho_4(t) &= \int_{h_M}^{t-D(t)} \frac{t - D(t) - s}{(h_M - D(t))^2} ds, \\
\rho_5(t) &= \int_{t-D(t)}^{t+\varphi(t)} \frac{\xi(s)}{\varphi(t)} ds, \\
\rho_6(t) &= \int_{t}^{t+\varphi(t)} \frac{(t-s)\xi(s)}{(\varphi(t))^2} ds.
\end{align*}$$

$$\begin{align*}
\alpha_{11} &= \text{col}\{e_1, e_2, e_0\}, \\
\alpha_{12} &= \text{col}\{e_0, e_0, e_1\}, \\
\alpha_2 &= \text{col}\{N_1, (1 - d)N_2, e_1 - (1 - d)e_2\}, \\
\alpha_{31} &= \text{col}\{e_2, e_3, h_M e_9\}, \\
\alpha_{32} &= \text{col}\{e_0, e_0, -e_0\}, \\
\alpha_4 &= \text{col}\{(1 - d)N_2, e_1, (1 - d)e_2 - e_3\}, \\
\alpha_5 &= \text{col}\{e_4, e_1, 0\}, \\
\alpha_{52} &= \text{col}\{e_0, e_0, e_0, -e_0\}, \\
\alpha_6 &= \text{col}\{\frac{1}{1 - d}N_4, N_1, \frac{1}{1 - d} e_4 - e_1\}, \\
\beta_1 &= \text{col}\{N_1, e_1, e_1, e_2, e_3, e_4, e_0\}, \\
\beta_{21} &= \text{col}\{N_2, e_2, e_2, e_3, e_4, e_0\}, \\
\beta_{22} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_1\}, \\
\beta_{31} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_2\}, \\
\beta_{32} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_3\}, \\
\beta_{33} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_4\}, \\
\beta_{34} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_5\}, \\
\beta_{35} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_6\}, \\
\beta_{36} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_7\}, \\
\beta_{41} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_8\}, \\
\beta_{42} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_9\}, \\
\beta_{51} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_10\}, \\
\beta_{61} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_11\}, \\
\beta_{62} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_0, e_10\}, \\
\beta_{71} &= \text{col}\{N_4, e_4, e_1, e_2, e_3, e_4, e_0\}, \\
\beta_{72} &= \text{col}\{N_1, e_1, e_1, e_2, e_3, e_4, e_0\}, \\
\beta_{73} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_0, e_11\}, \\
\beta_{90} &= \text{col}\{e_0, e_11, e_1, e_2, e_3, e_4, e_0\}, \\
\beta_{91} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_0, e_0\}, \\
\beta_{92} &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_0, -e_12\}, \\
\nu_1 &= \text{col}\{e_0, e_0, e_0, e_0, e_0, e_1\}, \\
\nu_2 &= \text{col}\{e_0, e_0, e_0, e_0, e_0, (1 - d)e_2\}, \\
\nu_3 &= \text{col}\{e_0, e_0, e_0, e_0, e_0, \frac{e_4}{1 - d}\}, \\
\kappa_1 &= \text{col}\{M_1, M_2, e_0\}, \\
\kappa_2 &= \text{col}\{M_4, M_1, e_0\}, \\
E_1 &= [I_{n \times n}, I_{n \times n}, 0_{n \times n}, \ldots, 0_{n \times n}], \\
E_2 &= [0_{n \times n}, I_{n \times n}, I_{n \times n}, \ldots, 0_{n \times n}], \\
E_3 &= [I_{n \times n}, 0_{n \times n}, 0_{n \times n}, \ldots, 0_{n \times n}].
\end{align*}$$

$$\tilde{A} = \begin{bmatrix}
a_{11} & 0 & 0 \\
0 & a_{22} & 0 \\
0 & 0 & 0
\end{bmatrix}.$$

A. Stability Criterion Under r(t)=0

Now, we present the following theorem to ensure the mean-square asymptotically stability for Itô stochastic system (1) with $r(t) = 0$.

Theorem 1: Consider the Itô stochastic system (1) where $K$ is a known matrix and $B_1 = B_1K$ ($l = 1, 2$), for given scalars $d \in (0, 1), h_m$ and $h_M$, then system (1) is mean-square asymptotically stable if there exist $P_{11} \in \mathbb{S}^{3n}, P_{12} \in \mathbb{S}^{3n}, Q_l \in \mathbb{S}^{3n}, R_l \in \mathbb{S}^{3n}, J_l \in \mathbb{R}^n, Y_l \in \mathbb{R}^n (l = 1, 2, 3; j = 1, 2, 3, 4)$ and $S_j(i = 1, 2, 3)$ with appropriate dimensions such that

$$\begin{align*}
dh_mP_{11} + P_{12} &> 0, \\
h_mP_{11} + P_{12} &> 0, \\
h_M &> \frac{d}{dh_m}P_{21} + P_{22} > 0, \\
P_{22} &> 0.
\end{align*}$$

$$\begin{align*}
2R_1 - X_1 &\geq 0, \\
Y_1 &\leq 0,
\end{align*}$$

and

$$\begin{align*}
D^2(t)\Phi_2(d) &= (D(t)\Phi_1(d) + \Phi_0(d)) < 0,
\end{align*}$$

where

$$\begin{align*}
\Phi_i(d) &= \Lambda_{i1} + \Lambda_{i2}, \\
i &= 0, 1, 2
\end{align*}$$

with

$$\begin{align*}
\Lambda_{01} &= d(\alpha_{11}^T P_{11} \alpha_{11} - \alpha_{31}^T P_{21} \alpha_{31} + \frac{\alpha_{51}^T P_{31} \alpha_{51}}{1 - d}), \\
+ 2H_e[\alpha_{11}^T P_{12} \alpha_2 + \alpha_{31}^T (h_M P_{21} + P_{22}) \alpha_4 + \frac{\alpha_{51}^T P_{22} \alpha_6}{1 - d}]
\end{align*}$$
\[
\alpha_{12} = \eta_1(t)P_1(t)\eta_1(t) + \kappa_1^T P_1(t)\eta_1(t) + \kappa_2^T P_2(t)\eta_2(t) + \frac{1}{1-d} \kappa_3^T P_3(t)\eta_3(t)
\]

\[
\lambda_{02} = \beta_1^T Q_1\beta_1 + (1-d)(\beta_2^T Q_2\beta_2 - \beta_2^T Q_1\beta_2) - \beta_1^T Q_2\beta_2
\]

\[
\lambda_{11} = He\{d[a_1^T P_{11}a_2 - \alpha_3^T P_{31}a_3 + \frac{\alpha_3^T P_{31}a_3}{1-d}]
\]

\[
\lambda_{12} = He\{-d[\beta_1^T Q_1\beta_1 + \beta_2^T Q_2\beta_2 - \beta_1^T Q_2\beta_2 - \beta_1^T Q_2\beta_2 - \beta_1^T Q_2\beta_2]
\]

Proof: To begin with, we introduce a vector as

\[
\begin{align*}
V(t, x) &= V_1(t, x) + V_2(t, x) + h_m V_3(t, x), \\
V_1(t, x) &= \eta_1(t)P_1(t)\eta_1(t) + \eta_2^T(t)p_2(t)\eta_2(t) + \eta_3^T(t)P_3(t)\eta_3(t), \\
V_2(t, x) &= \int_{t-D(t)}^{t} \eta_1(t, s)P_1(t)\eta_1(t, s)ds
\end{align*}
\]
Based on the Bessel-Legendre stochastic inequality and

\[
\begin{align*}
\eta_0(t) &= (\beta_{21} + D(t)\beta_{22})\zeta(t),
\eta_5(t, t-D(t)) &= \beta_4 \zeta(t),
\eta_5(t, t-h_m) &= (\beta_{51} + D(t)\beta_{52})\zeta(t),
\eta_6(t, t+\varphi(t)) &= \beta_7 \zeta(t),
\eta_6(t, t) &= (\beta_{61} + D(t)\beta_{62})\zeta(t),
\int_{t-h_m}^{t-D(t)} \eta_3^2(t, s)dsQ_1 \frac{\partial \eta_4(t, s)}{\partial t}
\end{align*}
\]

\[
= \xi(t)^T [\hat{\beta}_{30} + D(t)\beta_{31} + D^2(t)\beta_{32}] T \mathbf{V}_1 \zeta(t),
\int_{t-h_m}^{t-D(t)} \eta_3^2(t, s)dsQ_2 \frac{\partial \eta_6(t, s)}{\partial t}
\]

\[
= \xi(t)^T [\beta_{60} + (h_m - D(t))\beta_{61} + (h_m - D(t))^2 \beta_{62}]^T \times Q_2 v_2 \zeta(t),
\int_{t-h_m}^{t-D(t)} \eta_6^2(t, s)dsQ_3 \frac{\partial \eta_6(t, s)}{\partial t}
\]

\[
= \xi(t)^T [\beta_{60} + D(t) + D(t)^2 \beta_{62}] T Q_3 v_3 \zeta(t).
\]

Based on the Bessel-Legendre stochastic inequality and Lemma 3, we can get the inequality

\[
\begin{align*}
h_M \left( \int_{t-h_m}^{t-D(t)} f^T(s)R_1 f(s)ds + \int_{t-h_m}^{t-D(t)} f^T(s)R_2 f(s)ds \right) + h_M \int_{t-h_m}^{t-D(t)} f^T(s)R_3 f(s)ds
\geq \xi(t)^T (h_M \mathbf{Y}_1 - h_M - D(t) + (1-d)h_M \mathbf{Y}_3) \zeta(t)
\geq \frac{1}{2} \xi(t)^T \Lambda_3 \zeta(t) \geq \frac{1}{2} \xi(t)^T \Lambda_4 \zeta(t),
\end{align*}
\]

with

\[
\begin{align*}
\Lambda_3 &= \frac{1}{\alpha} \left[ E_1^T (2R_1)E_1 + E_2^T (2(1-d)R_3)E_3 \right] + \frac{2}{1-\alpha} E_2^T R_2 E_2,
\Lambda_4 &= E_1^T (2R_1 + (1-\alpha)X_1)E_1 + E_2^T (R_2 + \alpha X_2)E_2
+ E_3^T (2(1-d)R_3 + (1-\alpha)X_3)E_3 + E_4^T (R_2 + \alpha X_2)E_2
+ 2E_1^T (\alpha Y_1 + (1-\alpha)Y_2)E_2 + 2E_2^T (\alpha Y_3 + (1-\alpha)Y_4)E_2,
\end{align*}
\]

where \( \alpha = D(t)/h_M, X_j, Y_j (j = 1, 2, 3, 4) \) satisfy (8) and

From (1), for any \( S_j \in \mathbb{R}^{n \times n}, i=1,2,3, \) it is evident that

\[
2 \xi^T(t)[\mathbf{N}_1^T S_1 + \mathbf{e}_1^T S_2 + \mathbf{e}_2^T S_3][A_1 \mathbf{e}_1 + B_1 K e_2 - N_1] \zeta(t) = 0.
\]

According to (11)-(13), we can get

\[
E \mathcal{L} V(t, x)
= E \xi^T(t)(D^2(t) \Phi_2(d) + D(t) \Phi_1(d) + \Phi_0(d)) \zeta(t) < 0,
\]

by the negative definition of (9), there exists \( \lambda_1 > 0 \) such that

\[
E \mathcal{L} V(t, x) \leq -\lambda_1 \xi^T(t) \zeta(t) \leq -\lambda_1 Ex^T(t) \zeta(t).
\]

Integrating from 0 to t on both sides of (4) and taking mathematical expectation

\[
EV(t, x) = E \mathcal{L} V(t, x) + EV(0, x(0)) \leq EV(0, x(0)) - \lambda_1 \int_0^t x^T(s) x(s)ds.
\]

From this, \( Ex^T(t) x(t) \) is bounded and then \( Ex^T(t) x(t) \) is uniformly continuous. Since \( EV(t, x) > 0, \) we have \( \lambda_1 E \int_0^\infty x^T(s) x(s)ds < E(0, x(0)) < \infty \) which means that the system is mean-square asymptotically stable when \( r(t) = 0. \)

Remark 2: In Theorem 1, \( \varphi(t) = \frac{D(t)}{h_M}. \) Since the denominator is nonzero, \( d = 1 \) can not be used in this paper, which is one of the main reasons for us to study the sawtooth delay with \( 0 < d < 1. \)

Remark 3: The LKF \( V(t, x) \) is inspired by [41]. Compared with the existing results, Theorem 1 introduces a new augmented LKF, since two integral terms \( \rho_0(t) = \int_t^{t+\varphi(t)} \xi(s)ds, \rho_0(t) = \int_t^{t+\varphi(t)} \xi(s)ds \) and \( \varphi(t) \) are included in the vector of the LKF. Besides, Lemma 1 and Lemma 3 are used to achieve tight estimations with three integral terms \( \int_t^{t-D(t)} f^T(s)R_1 f(s)ds, \int_t^{t-D(t)} f^T(s)R_2 f(s)ds, \int_t^{t-D(t)} f^T(s)R_3 f(s)ds, \)

which makes the result for system (1) to be less conservative.

Noting that it is difficult to evaluate the condition in (9) of Theorem 1 because \( D(t) \) is a function taking values in a finite interval. Now, we use Lemma 4 to obtain an LMI type conditions for the quadratic condition in (9), which is a convenient condition to realize.

Corollary 1: Let \( K \) be a known matrix and \( \hat{B}_1 = B_1 K \) \((l = 1, 2), \) for given scalars \( d, h_m \) and \( h_M, \) system (1) is mean-square asymptotically stable with \( r(t) = 0 \) if there exist matrices \( P_{i1} \in \mathbb{S}^{3n}, P_{i2} \in \mathbb{S}^{3n}, Q_i \in \mathbb{S}^{7n}, R_i \in \mathbb{S}^{6n} \)

\((i = 1, 2, 3), X_j \in \mathbb{R}^n, Y_j \in \mathbb{R}^n (j = 1, 2, 3, 4), S_1, S_2, S_3 \) with appropriate dimensions such that (7), (8) and the following conditions hold when \( 0 < d < 1, \)

\[
(dh_m)^2 \Phi_2(d) + dh_m \Phi_1(d) + \Phi_0(d) < 0,
\]

\[
h_M^2 \Phi_2(d) + h_M \Phi_1(d) + \Phi_0(d) < 0,
\]

\[
\Phi_0(d) + \frac{1}{2} (dh_m + h_M) \Phi_1(d) + dh_m h_M \Phi_2(d) < 0,
\]

where \( \Phi_0(d), \Phi_1(d), \Phi_2(d) \) are given in Theorem 1.
Remark 4: Adding inequalities or lemmas in [38], [39], [42] and [44] can give better results, but the complexity will also increase. On the basis of not affecting the structural integrity of the article and the complete display of innovative methods, in order to simplify the description, we adopt some inequalities and lemmas listed above.

B. L₂−L∞ CONTROL UNDER r(t) ≠ 0

Next, we discuss the stability of the system with a prescribed L₂-L∞ performance under r(t) ≠ 0, to begin with, let

\[ \tilde{\varphi}(t) = \text{col}(x(t), x(t-D(t)), x(t-h_M), x(t+\varphi(t)), x(t-D(t)-D(t-D(t)), x(t-h_M-D(t-h_M))), \]  
\[ \rho_1(t), \rho_2(t), \rho_3(t), \rho_4(t), \rho_5(t), \rho_6(t), r(t-D(t)), \]  
\[ r(t-h_M), r(t+\varphi(t)), \rho_7(t), \rho_8(t), \rho_9(t), \]  

with

\[ \rho_7(t) = \int_{t-h_M}^{t} x(s)ds, \]  
\[ \rho_8(t) = \int_{t-h_M}^{t} x(s)ds, \]  
\[ \rho_9(t) = \int_{t-h_M}^{t} \tilde{x}(s)ds, \]  

and

\[ \theta_1 = \{e_1, e_2, e_{17}\}, \theta_2 = \{e_2, e_3, e_{18}\}, \theta_3 = \{e_4, e_1, e_{19}\}, \]  
\[ \delta = \{e_3, e_4, \ldots, e_{19}\}. \]  

The terms given in the beginning of Section 3 do not change except N₁, N₂, N₅, N₄, M₁, M₂, M₃, M₄, which turn into the following expressions:

\[ N_1 = A_1e_1 + B_1Ke_2 + C_1e_{13}, \]  
\[ M_1 = A_2e_1 + B_2Ke_2 + C_2e_{13}, \]  
\[ N_2 = A_1e_2 + B_1Ke_5 + C_1e_{14}, \]  
\[ M_2 = A_2e_2 + B_2Ke_5 + C_2e_{14}, \]  
\[ N_3 = A_3e_1 + B_1Ke_6 + C_1e_{15}, \]  
\[ M_3 = A_3e_2 + B_2Ke_6 + C_2e_{15}, \]  
\[ N_4 = A_4e_1 + B_1Ke_1 + C_1e_{16}, \]  
\[ M_4 = A_4e_2 + B_2Ke_1 + C_2e_{16}. \]  

(15)

Let us choose S₂ = μ₁S₁, S₃ = μ₂S₁, where μ₁ and μ₂ are any scalar, and then

\[ \Delta_1 = (N_1^T + \mu_1e_1^T + \mu_2e_2^T)S_1, \]  
\[ \Delta_2 = A_1e_1 + B_1Ke_2 + C_1e_{13} - N_1. \]  

(16)

\[ e_i(i = 1, 2, \ldots, 19) \] is the ith n × 19n row-black vector such that \text{col}(e_1, e_2, \ldots, e_{19}) is a 19n × 19n identity matrix, e₀ = 0 < 19n. Theorem 2: Consider the Itô stochastic system (1) and let γ > 0 be a prescribed constant, then for given scalars d (0, 1), hₘ, h_M, μ₁ and μ₂, system (1) is mean-square asymptotically stable with a L₂-L∞ performance level γ if there exist P₁₁ ∈ Sₙ, P₁₂ ∈ Sₙ, Q₁ ∈ Sₙ, R₁ ∈ Sₙ (i = 1, 2, 3), X_j ∈ S^n, Y_j ∈ R^n (j = 1, 2, 3, 4), S₁, F with appropriate dimensions and F = S₁B₁K such that (7), (8) and the following conditions hold. Moreover, the L₂−L∞ controller gain matrix is given by K = (S₁B₁)^{-1}F.

\[ (dhₘ)^2\Phi_2(d) + dhₘ\Phi_1(d) + \Phi_0(d) - \gamma^2e_{22}^T e_{22} < 0, \]  
\[ h_M^2\Phi_2(d) + h_M\Phi_1(d) + \Phi_0(d) - \gamma^2e_{22}^T e_{22} < 0, \]  
\[ \Phi_0(d) - \gamma^2e_{22}^T e_{22} + \frac{1}{2}(dhₘ + h_M)\Phi_1(d) + dhₘh_M\Phi_2(d) < 0, \]  

(17)

where \#_0(d), \#_1(d), \#_2(d) are the results by modifying \#_0(d), \#_1(d), \#_2(d) which are given in Theorem 1 according to (15) and (16).

\[ \begin{bmatrix} \Theta_1 & MM \\ * & I \end{bmatrix} > 0, \quad \begin{bmatrix} \Theta_2 & MM \\ * & I \end{bmatrix} > 0, \]  

(18)

with

\[ MM = \left[ \begin{array}{ccc} A_3e_1 + B_3Ke_2 & M\delta \\ \end{array} \right]^T, \]
\[ \Theta_1 = \theta_1^T (dhₘP_{11} + P_{12}) \theta_1 + \theta_2^T ((h_M - dhₘ)P_{21} + P_{22}) \theta_2 \]
\[ + \theta_3^T (h_MP_{31} 1 - d + P_{32} 1 - d) \theta_3 < 0, \]
\[ \Theta_2 = \theta_1^T (h_MP_{11} + P_{12}) \theta_1 + \theta_3^T (h_MP_{31} 1 - d + P_{32} 1 - d) \theta_3 \]
\[ + \theta_2^T P_{22} \theta_2 < 0. \]

Proof: When r(t) ≠ 0, we assume V(0, x(0)) = 0, there exists matrix M with appropriate matrix such that the following inequality holds:

\[ \tilde{z}(t) ≤ \tilde{z}(t) \tilde{z}(t) + \delta^T M^T M \delta, \]

(19)

and since E[dvₜ(t)] = 0, we have

\[ G = Ez(T)(\tilde{z}(t) - \gamma^2 \int_{0}^{T} r(s)ds)ds \]
\[ = Ez(T)(\tilde{z}(t) - E[V(t, x) - V(0, x)]) \]
\[ + E \int_{0}^{T} \mathcal{L}[V(s, x(s))]ds - \gamma^2 \int_{0}^{T} r(s)ds)ds \]
\[ ≤ Ez(T)\tilde{z}(t) \chi_1 - EV_1(t, x) \tilde{z}(t) \]
\[ + E \int_{0}^{T} \tilde{z}(s) \chi_2 \tilde{z}(s)ds. \]

(20)

with

\[ \chi_1 = (A_3e_1 + B_3Ke_2)(A_3e_1 + B_3Ke_2) + \delta^T M^T M \delta, \]
\[ \chi_2 = D^2 \tilde{\Phi}_2(d) + D(t) \tilde{\Phi}_1(d) + \tilde{\Phi}_0(d) - \gamma^2 e_{22}^T e_{22}. \]

Because V₁, V₂, V₃ are positive definite, we get EV₁(t, x) ≥ EV₁(t, x), and

\[ V₁(t, x) = \tilde{z}(t) \theta_1^T (D(t)P_{11} + P_{12}) \theta_1 \]
\[ + \theta_2^T ((h_M - D(t)P_{21} + P_{22}) \theta_2 \]
\[ + \theta_3^T (D(t)P_{31} 1 - d + P_{32} 1 - d) \theta_3). \]

(21)

In (20), the first term and the second term on the right-hand side of the inequality are respectively the linear function of D(t) and the quadratic function of D(t). (17) and (18) can
ensure $G < 0$. Therefore, the $L_2-L_\infty$ performance $\|z(t)\|_\infty < \gamma \|r(t)\|_2$ is satisfied. Further, even $V(0, x(0)) \neq 0$, we can get

$$-\gamma^2 \|z\|^2_2 - V(0, x(0)) < G < E \int_0^t \dot{\zeta}^T(s) [D(t) \dot{\Phi}_2(d) + D(t) \dot{\Phi}_1(d) + \dot{\Phi}_0(d)] \zeta(s)ds < 0.$$  

Similar to (14), there exists $\lambda_2 > 0$ such that

$$-\gamma^2 \|z\|^2_2 - V(0, x(0)) < -\lambda_2 E \int_0^t \zeta^T(s)\zeta(s)ds \leq -\lambda_2 E \int_0^t x^T(s)x(s)ds,$$

then, we have $E \int_0^t x^T(s)x(s)ds < \infty$ which represents the system is mean-square asymptotically stable when $r(t) \neq 0$. This completes the proof.

Remark 5: Given a prescribed scalar $\gamma$, Theorem 2 discussed $L_2-L_\infty$ control of system (1) under zero initial condition. Similar to the proof of Theorem 1, the control problem is represented as the linear function of $D(t)$ and the quadratic function of $D(t)$. An interesting fact is that we prove the mean-square stability of system (1) and design a controller gain matrix even under $x(0) \neq 0$, which is more general than Definition 2.

IV. ALGORITHM

In this section, we use the LMI parser YALMIP and the SDP solver Sedumi in MATLAB to get our results. Inspired by [36], we propose two algorithms: one finds the maximum value of $h_M$, the other finds the minimum values of $\gamma$ and $K$.

Algorithm 1 The Maximum Value of $h_M$

Step 1: Given $d$, $K$, $h_m$, $h_M$.
Step 2: Using the linear search algorithm, if a series of $\alpha_t(t = 1, \ldots, n)$ can be found that ensure inequalities (7)-(9) have feasible solutions, then move to Step 3; otherwise, move to Step 6.
Step 3: Let $\tau = \frac{h_m + h_M}{h_m - h_M}$, when $i = 1$, we take $\alpha_t$, let $h_m = \tau$.
Step 4: If $h_M - h_m > 0$, let $i = i + 1$, and return to Step 3; otherwise, move to Step 5.
Step 5: There are solutions to this problem; print the maximum value of $h_M$ and then stop.
Step 6: There is no solution to this problem; stop.

Algorithm 2 The Minimum Value of $\gamma$ and $K$

Step 1: Given $d$, $h_m$, $h_M$.
Step 2: Using the linear search algorithm, if a series of $\alpha_t(t = 1, \ldots, n)$ can be found that ensure inequalities (7)-(8)-(17)-(18) have feasible solutions, then move to Step 3; otherwise, move to Step 6.
Step 3: When $i = 1$, we take $\alpha_t$, solve the following minimization problem: $\min_{\mu_2, \gamma} (7)-(8)-(17)-(18) \gamma^2$.
Step 4: If $i = i + 1$, if $i + 1 > n$, move to Step 5; otherwise, return to Step 3.
Step 5: There are solutions to this problem; print $\gamma$ and $K$, and then stop.
Step 6: There is no solution to this problem; stop.

V. NUMERICAL EXAMPLES

In this section, two examples are given to show the effectiveness of the proposed approach. For comparison, we select two examples in the existing literatures. Since there are a lot of results on the deterministic systems with sawtooth-like input delay, to illustrate the validity of our method, a deterministic model is first discussed in Example 1.

**Example 1:** Consider a deterministic system with the following parameters:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad A_2 = A_3 = B_2 = B_3 = C_1 = C_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$  

This example has been frequently used in other papers such as [38]–[43]. We use Example 1 to compare Corollary 1 with existing conclusions by setting $\tau = \frac{h_M}{d}$ [9]. $h_m = 0$, $x(0) = [1, -1]^T$. The results are shown in Table 1, we can get the maximum admissible upper bounds $h_M$ with given $d$. NoDVs represents the number of decision variables for LMIs. This paper gives less conservative results than other methods and the NoDVs are smaller than those. The state responses get the maximum admissible upper bounds $h_M$ with given $d$. The state responses of the proposed approach. For comparison, we select two examples in the existing literatures. Since there are a lot of results on the deterministic systems with sawtooth-like input delay, to illustrate the validity of our method, a deterministic model is first discussed in Example 1.

**Example 2:** Consider a Itô stochastic system with the following parameters [16]:

$$A_1 = \begin{bmatrix} -1.5 & 0.5 \\ 1.2 & -2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.8 & 0.3 \\ 0.2 & -0.4 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.8 & 0.2 \\ 0.6 & -0.5 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.5 & 0.3 \\ 0.6 & 1.0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.5 & 0.8 \\ 1.0 & 0.2 \end{bmatrix}, \quad B_2 = B_3 = C_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$  

In this example, we consider the system with sawtooth-like input delay: $h_M = 1.5$, $h_m = 0$, $x(0) = [1, -1]^T$, $\mu_1 = 1.3$, $\mu_2 = 0.7$ and $\tau(t) = [1, 5]^T \sin(t)e^{-2t}$. Table 2 lists the computed minimum value of $\gamma$ with various $d$. As we all know, the smaller the value of $\gamma$, the smaller the deviation of the system from the zero point. From Table 2, it’s clear that Theorem 2 gets a smaller $\gamma$ than that in [16], which implies that the proposed method gets better performance. Moreover, we can get the following state feedback controller gains:

$$K = \begin{bmatrix} -1.5629 & -1.1722 \\ -0.7814 & -3.1258 \end{bmatrix}.$$
Remark 6: In example 1, we choose to change the lower and the upper of $h_M$ with given $d$, keep bisecting it until we found the best solution $h_M$. In example 2, given $d$ and $h_M$, we calculated $K$ and the minimum $\gamma$. By using the design parameters, our results are less conservative than existing approaches.

VI. CONCLUSION

In this paper, we have addressed the stability and $L_2-L_{\infty}$ control problem of Itô stochastic system with the sawtooth-like input delay. The main contribution is that we make an auxiliary system to get the state value $x(t + \varphi(t))$, which increased the delay-dependent function terms in the discriminant for reducing the conservatism of the system. Based on the auxiliary system, we create a new LKF, which includes two new integrals different from the other literatures. Because of these, we obtain a quadratic function parameterized by $D(t)$, sufficient conditions are raised to ensure the mean-square stability, and the controller matrix is designed when $r(t) \neq 0$. Finally, two examples have been given to illustrate the validity of the proposed method. There are also some problems deserved to be addressed here. On the one hand, the system studied in this paper has the limitations on $d$ with $0 < d < 1$, and the topic is also very valuable for $d = 1$. On the other hand, there are a large number of literatures on the stability and control of neural networks, see [45], [46] for example, if we combine the method in existing literatures with our newly-constructed auxiliary system and the Bessel-Legendre stochastic inequality, it may provide better results on the stability of the system with less conservatism. We will further study these problems soon.

COMPETING INTERESTS

The authors declare that they have no competing interests, and approve the submission of this article.

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