CP violation in the decay mode $B \rightarrow \pi \gamma \gamma$

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Abstract

Within the framework of Standard Model, the exclusive decay mode $B \rightarrow \pi \gamma \gamma$ is studied. Although the usual short distance contribution is small compared to the similar $B \rightarrow K \gamma \gamma$ mode, the process offers the possibility of studying the CP violation, a feature absent in the $B \rightarrow K$ counterpart.

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1 Introduction

As has been emphasised time and again, because of very clean experimental signatures, radiative decays of the B-meson can serve to be very useful tools to test the underlying structure of the theory in question responsible for flavour changing neutral currents (FCNC). The radiative decays, in particular, are very sensitive to higher order QCD corrections and any new physics effects beyond the Standard Model (SM). The inclusive decay mode $B \rightarrow X_s \gamma$ as well as the exclusive mode $B \rightarrow K^{*-} \gamma$ have been experimentally measured and have invited a lot of theoretical attention. It is expected that the future B-factories should be able to make measurements of two photon modes. The quark level decay, $b \rightarrow s \gamma \gamma$ has been studied in [1]-[3]. The basic amplitude in question receives contributions from the irreducible triangle diagram and the reducible pieces (where the second photon is emitted by one of the external quark lines of a basic $b \rightarrow s \gamma$ amplitude). The quark level amplitude is appropriate for the
inclusive two photon process. For an exclusive channel, with some meson \( M \) in the final state, the reducible diagram is to be thought of as the one in which the second photon is attached to one of the external meson legs of the basic amplitude \( B \to M\gamma \) rather than the quark level process. It is then straightforward to note that this amplitude vanishes if the meson \( M \) in question is scalar or pseudoscalar. Therefore, for such cases, it suffices to consider the irreducible contribution only. Apart from these short distance contributions, one can have the long distances contributions as well. Of course, in the case of resonances contributing to any process, the resonant contribution is a reducible one. However, in the above, the demarcation between reducible and irreducible diagrams is more specific in that it is used for the usual short distance, non-resonant contribution only. The two photon process proceeding via \( b \to s \) transition has been recently studied \[4\] and it has been pointed out that the long distance contributions can be of the same order, in fact sometimes larger, compared to the short distance irreducible contributions.

The present study extends these ideas to the decay \( B \to \pi\gamma\gamma \) proceeding via the transition \( b \to d \). The amplitude for this mode suffers from CKM suppression when compared with the \( B \to K\gamma\gamma \) amplitude. However, in contrast to the \( B \to K \) process, \( B \to \pi \) channel offers the possibility of studying the CP asymmetry. We estimate the branching ratio and CP asymmetry for the process \( B \to \pi\gamma\gamma \) taking account of the irreducible contribution as well as all significant resonance contributions.

## 2 Effective Hamiltonian and Irreducible triangle contribution

The effective Hamiltonian relevant for a \( b \to d \) transition contributes to radiative processes through triangle diagrams with the photon being emitted by the quark loop and has the form \[5\]

\[
\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{td}^* V_{tb} \left[ \sum_i C_i(\mu) O_i(\mu) \right. \\
- \lambda_u [C_1(O_1^u(\mu) - O_1(\mu)) + C_2(O_2^u(\mu) - O_2(\mu))] 
\]

(1)

with

\[
O_1 = (\bar{d}_i c_j)_{V-A} (\bar{e}_j b_i)_{V-A},
O_2 = (\bar{d}_i c_i)_{V-A} (\bar{e}_j b_j)_{V-A},
O_3 = (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A},
O_4 = (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A},
O_5 = (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A},
\]

(2)
\[ O_6 = (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \]

\[ O_7 = \frac{e}{16\pi^2} \bar{d}_i \sigma^{\mu\nu} (m_d P_L + m_b P_R) b_i F_{\mu\nu}, \]

\[ O_8 = \frac{g}{16\pi^2} \bar{d}_i \sigma^{\mu\nu} (m_d P_L + m_b P_R) T_{ij}^a b_j C_{\mu\nu}^a, \]

and

\[ O_1^u = (\bar{d}_i u_j)_{V-A} (\bar{u}_j b_i)_{V-A} \]

\[ O_2^u = (\bar{d}_i u_i)_{V-A} (\bar{u}_j b_j)_{V-A}, \]

In writing the effective Hamiltonian, the unitarity of the CKM matrix has been used and the parameter \( \lambda_u \) is:

\[ \lambda_u = \frac{V_{us} V_{ub}^*}{V_{td} V_{tb}} \]

In the Wolfenstein parametrization of the CKM matrix, the above equation simply reads

\[ \lambda_u = \rho (1 - \rho) - \frac{\eta^2}{(1 - \rho)^2 + \eta^2} - \frac{\eta}{(1 - \rho)^2 + \eta^2} + \ldots \]

A simple Fierz rearrangement of \( O_1^u \) and \( O_2^u \) yields,

\[ O_1^u = -(\bar{d}_i b_i)_{V-A} (\bar{u}_j u_j)_{V-A} = -O_3 (q = u) \]

and

\[ O_2^u = -(\bar{d}_i b_j)_{V-A} (\bar{u}_j u_i)_{V-A} = -O_4 (q = u) \]

The effective Hamiltonian therefore becomes

\[ H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{td}^* V_{tb} \left[ C_1 O_1(\mu) [1 + \lambda_u] + C_2 O_2(\mu) [1 + \lambda_u] \right. \]

\[ + \ O_3 [C_3 + C_1 \lambda_u \delta_{uq}] + O_3 [C_4 + C_2 \lambda_u \delta_{uq}] \]

\[ + \sum_{i>4} C_i(\mu) O_i(\mu) \]

In the above equation, \( \delta_{uq} \) simply implies that this term contributes only for the u-quark and there is no summation over repeated indices.

The quark level transition amplitude for \( b \rightarrow d \gamma \gamma \) (with an incoming \( b \) line and an outgoing \( d \) line and the two photons being emitted by the quark loop) is

\[ M_{b \rightarrow d} = \left[ \frac{16\sqrt{2} \alpha G_F V_{td}^* V_{tb}}{9 \pi} \right] \bar{u}(p_d) \left\{ \sum_q A_q J(m_q^2) \gamma^\mu P_L R_{\mu\nu\rho} \right. \]

\[ + \ i B (m_d K(m_d^2) P_L + m_b K(m_b^2) P_R) T_{\mu\nu} \]

\[ + \ C(-m_d L(m_d^2) P_L + m_b L(m_b^2) P_R) \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \]

\[ \left. \sum \epsilon(\mu) \epsilon^\nu(k_1) \epsilon^\nu(k_2) \right\}, \]
where
\[ R_{\mu\nu\rho} = k_{1\nu}\epsilon_{\mu\rho\sigma}k_1^\sigma k_2^\lambda - k_{2\mu}\epsilon_{\nu\rho\sigma}k_1^\sigma k_2^\lambda + (k_{1\cdot}k_2)\epsilon_{\mu\nu\rho\sigma}(k_2 - k_1)^\sigma, \]
\[ T_{\mu\nu} = k_{2\mu}k_{1\nu} - (k_{1\cdot}k_2)g_{\mu\nu}, \]
\[ A_u = 3(C_3 + C_1\lambda_u - C_5) + (C_4 + C_2\lambda_u - C_6), \]
\[ A_d = \frac{1}{4}[3(C_3 - C_5) + (C_4 - C_6)], \]
\[ A_c = 3(C_1(1 + \lambda_u) + C_3 - C_5) + (C_2(1 + \lambda_u) + C_4 - C_6), \]
\[ A_s = A_b = \frac{1}{4}[3(C_1(1 + \lambda_u) + C_3 - C_5) + (C_2(1 + \lambda_u) + C_4 - C_6)], \]
and
\[ B = C = -\frac{1}{4}(3C_6 + C_5). \]

The quark level amplitude when sandwiched between the hadronic states gives the appropriate matrix element describing the meson decay. We thus get,
\[
M_{\text{irred}}(B \to \pi\gamma\gamma) = \left(16\sqrt{\frac{2\alpha_G F V_{td} V_{tb}}{9\pi}}\right) \left[\frac{1}{2}(\pi|d\gamma^\rho b|B) \sum_q A_q J(m_q^2) R_{\mu\nu\rho}\right. \\
+ \frac{1}{2}(\pi|d\bar{b}|B) \left\{ t^B(m_d K(m_d) + m_b K(m_b))T_{\mu\nu} \right. \\
+ C(-m_d L(m_d) + m_b L(m_b))\epsilon_{\mu\nu\rho\sigma}k_1^\sigma k_2^\rho \left.<\bar{u}|\pi\gamma\gamma\pi|d]\right. \\
\left.\bar{d}|B\right> \right] \epsilon^\nu(k_1)\epsilon^\rho(k_2).
\]

In the above expressions we introduced the functions
\[ J(m^2) = I_{11}(m^2), \quad K(m^2) = 4I_{11}(m^2) - I_{00}(m^2), \quad L(m^2) = I_{00}(m^2), \]
where
\[ I_{pq}(m^2) = \int_0^1 dx \int_0^{1-x} dy \left(\frac{x^p y^q}{m^2 - 2(k_{1\cdot}k_2)xy - i\epsilon}\right)
\]
\[ (13) \]

We use the following parametrization for the matrix elements of the quark vector current:
\[ \langle \pi^-|p)|\bar{d}u\rangle \gamma_V - A|0\rangle = if_{\pi p}, \]
\[ (14) \]
\[ \langle \pi^-|d\gamma^\mu b|B\rangle = \left((p_B + p_\pi)_{\mu} - \frac{m_B^2 - m_\pi^2}{q^2}q_{\mu}\right) F_{1B}(q^2) \\
+ \left(\frac{m_B^2 - m_\pi^2}{q^2}\right) q_{\mu} F_{0B}(q^2), \]
\[ (15) \]
with \( q = p_b - p_d = k_1 + k_2 \). It then follows that
\[ \langle \pi^-|d\bar{b}|B\rangle = (m_b - m_d)^{-1}\left[q_{\mu}\langle \pi|d\gamma^\mu b|B\rangle \right] \\
= (m_b - m_d)^{-1}(m_B^2 - m_\pi^2) F_{0B}(q^2). \]
\[ (16) \]
Other matrix elements can be related to these using the isospin relations:

\[ \langle \pi^- (p) | (\bar{d}u)_{V-A} | 0 \rangle = \sqrt{2} \langle \pi^0 (p) | (\bar{u}u)_{V-A} | 0 \rangle = -\sqrt{2} \langle \pi^- (p) | (\bar{d}d)_{V-A} | 0 \rangle \]

and

\[ \langle \pi^- | \bar{d} \gamma_\mu b | B^- \rangle = \langle \pi^+ | \bar{u} \gamma_\mu b | B^0 \rangle = \sqrt{2} \langle \pi^0 | \bar{d} \gamma_\mu b | B^0 \rangle = \sqrt{2} \langle \pi^0 | \bar{u} \gamma_\mu b | B^- \rangle \]

We use the explicit numerical dependence of \( F(q^2) \) on \( q^2 \) given by Cheng et al [3].

## 3 Resonance contributions

### 3.1 The \( \eta_c, \eta \) and \( \eta' \) resonances

The \( \eta_c \) contribution comes via the decay \( B \to \pi \eta_c \), followed by \( \eta_c \) decaying into two photons. The amplitude for \( \eta_c \) to decay to two photons is parametrized as

\[ \langle \gamma \gamma | T | \eta_c \rangle = 2i B_{\eta_c} \epsilon^{\mu \nu \alpha \beta} \epsilon_{1 \mu}^* \epsilon_{2 \nu}^* k_{1 \alpha} k_{2 \beta}. \tag{17} \]

The coefficient \( B_{\eta_c} \) can be determined from the \( \eta_c \to \gamma \gamma \) decay rate.

\[ \Gamma(\eta_c \to \gamma \gamma) = \frac{|B_{\eta_c}|^2 m_{\eta_c}^3}{16\pi}. \]

The \( B \to \pi \eta_c \) amplitude

\[ \langle \pi \eta_c | T | B \rangle = -\langle \pi \eta_c | \mathcal{H}_{\text{eff}} | B \rangle, \]

can be got from the relevant piece of \( \mathcal{H}_{\text{eff}} \), which after Fierz transformation reads

\[ \mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} V^{*}_{cd} (C_1 + \frac{C_2}{3}) (\bar{c} \bar{c})_{V-A} (\bar{d} \bar{b})_{V-A}. \tag{18} \]

Using factorization and the following definitions

\[ \langle 0 | A^c_\mu | \eta_c \rangle \equiv i f_{\eta_c} q_\mu \quad A^c_\mu = \bar{c} \gamma_\mu \gamma_5 c \]

\[ q^\mu \langle \pi^- | \bar{d} \gamma_\mu b | B \rangle = F_0(m_{\eta_c}^2)(m_B^2 - m_\pi^2) \]

we can write

\[ \langle \pi^- \eta_c | T | B \rangle = -i \frac{G_F}{\sqrt{2}} V_{cb} V^{*}_{cd} (C_1 + \frac{C_2}{3}) \left( f_{\eta_c} F_0(m_{\eta_c}^2)(m_B^2 - m_\pi^2) - f_\pi F_0(m_\pi^2)(m_B^2 - m_{\eta_c}^2) \right) \tag{19} \]

where a dipole form of the form factor \( F_0(m^2) \) is used.

The \( \eta_c \) resonance contribution is thus,

\[ \mathcal{M}_{\eta_c} = 2B_{\eta_c} \frac{G_F}{\sqrt{2}} V_{cb} V^{*}_{cd} (C_1 + \frac{C_2}{3}) \]

\[ \left[ f_{\eta_c} F_0(m_{\eta_c}^2)(m_B^2 - m_\pi^2) - f_\pi F_0(m_\pi^2)(m_B^2 - m_{\eta_c}^2) \right] \]

\[ \epsilon_1^{\mu}(k_1) \epsilon_2^{\nu}(k_2) \epsilon_{\mu \alpha \beta} k_1^\alpha k_2^\beta \frac{1}{q^2 - m_{\eta_c}^2 + m_{\eta_c} \Gamma_{\text{total}}} \].
Analogous to \( \eta_c \), \( \eta' \) can also contribute via the effective Hamiltonian, eq.(15). However, unlike \( \eta_c \), \( \eta' \) has a very strong coupling to a two gluon state. A number of theoretical models for \( \eta' \) production in B-decays via two gluon states have been proposed [7,8,9]. But these theoretical models have their own uncertainties attached to them. Instead of relying on any such model, we directly use the experimental data for \( B \to \pi \eta' \) decay process [10] and parametrise the \( \eta' \) resonance contribution as \((i = -, 0)\):

\[
\mathcal{M}_{\eta'} = 2B_{\eta'} F(B^i \pi^i \eta') \epsilon^{* \mu}_1(k_1) \epsilon^{* \nu}_2(k_2) \epsilon_{\mu \nu \alpha \beta} k_1^\alpha k_2^\beta \frac{1}{q^2 - m_{\eta'}^2 + i m_{\gamma} \Gamma_{\eta'}^{\text{total}}} \tag{21}
\]

where \( B_{\eta'} \) is defined as for \( \eta_c \) with \( \eta_c \to \eta' \) and the factor \( F(B^i \pi^i \eta') \) is related to the decay rate \( \Gamma(B^i \to \pi^i \eta') \) as:

\[
\Gamma(B^i \to \pi^i \eta') = \frac{1}{16\pi m_B} |F(B^i \pi^i \eta')|^2 \lambda^\frac{1}{2}(1, \frac{m_{\pi}^2}{m_B^2}, \frac{m_{\eta'}^2}{m_B^2}) \tag{22}
\]

The branching ratio for \( B \to \pi \eta \) has also been measured [11] and is found to be larger than the \( \eta' \) branching ratio. Moreover, the branching ratio for \( \eta \) to go into two photons is roughly 20 times than that of \( \eta' \). Therefore, \( \eta \) channel is expected to contribute the largest.

Again, the amplitude can be directly read off from Eq.(21) by replacing the \( \eta' \) quantities by appropriate \( \eta \) values.

### 3.2 Contribution due to \( \rho \)

The \( \rho \) meson contributes to the process in the following way:

\[ B^i(p_B) \to \rho^i + \gamma(k_1), \quad i = \pm, 0 \]

followed by

\[ \rho^i \to \pi^i + \gamma(k_2), \]

and the process with \( k_1 \leftrightarrow k_2 \).

It has been emphasized [12] that the weak-annihilation contribution to \( B \to \rho \gamma \) can be significant and thus we take into account this effect also while writing the \( B \to \rho \gamma \) vertex. Therefore,

\[
\langle \rho(p_\rho) \gamma(k_1)|T|B(p_B) \rangle = -\frac{e G_F}{\sqrt{2}} V_{tb} V_{td}^{*} \epsilon^{* \mu}(k_1) \epsilon^{\mu \beta}(p_\rho) \left[ F_{PC}^{\text{Total}} \epsilon_{\mu \nu \alpha \beta} k_1^\mu P_\rho^\alpha + i F_{PV}^{\text{Total}} (g_{\nu \beta} p_B \cdot k_1 - p_{B \rho} k_1^\beta) \right] \tag{23}
\]

where, \( F_{PC}^{\text{Total}} \) and \( F_{PV}^{\text{Total}} \) are the parity conserving and parity violating form factors including the weak annihilation contributions as given in [12]. We parametrize the \( \rho^i \to \pi^i \gamma \) transition as

\[
\langle \pi(p_\pi) \gamma(k_2)|T|\rho(p_\rho) \rangle = g_{\rho \pi \gamma} \delta(k_2) \epsilon^{* \mu}(k_2) k_2^\mu P_\rho^{\lambda} \epsilon_{\kappa \sigma \lambda \delta} \tag{24}
\]
and $g^i_{\pi \gamma}$ is determined from the corresponding decay rate.

Therefore, the complete $T$ matrix element for the $\rho$ resonance can be written as

$$\langle \pi \gamma|T|B \rangle = \mathcal{M}_\rho$$

$$= -\frac{eG_F}{\sqrt{2}} v_{td}^* g^i_{\pi \gamma} \varepsilon^*_\mu(k_1) \varepsilon^*_\nu(k_2)$$

$$\left\{ \epsilon^{\alpha \nu \gamma \delta} k_{2\alpha}(p_B - k_1)_{\gamma} k_{1\beta} \left[ \frac{\left( g_{\delta \sigma'} - \frac{(p_B - k_1)_{\sigma'}(p_B - k_1)_{\sigma}}{m_{K^*}^2} \right)}{(p_B - k_1)^2 - m_{K^*}^2 + i m_{K^*} \Gamma_{K^*}^{\text{total}}} \right] \right.$$  

$$\left[ F_{PC}^{\text{Total}} \epsilon^{\beta' \sigma' \tau'} (p_B - k_1)_{\tau'} - F_{PV}^{\text{Total}} (g^{\mu \sigma'} (p_B - k_1)_{\beta'} - g^{\beta' \sigma'} (p_B - k_1)^\mu) \right]$$

$$+ \left( \begin{array}{c} \mu \leftrightarrow \nu \\ k_1 \leftrightarrow k_2 \end{array} \right) \}$$

3.3 $\omega$, $\phi$ and $J/\psi$ contributions

The mesons $\omega$, $\phi$ and $J/\psi$ contribute only to the neutral decay mode. The matrix elements for the $\omega$ and $\phi$ can be easily got from that of $\rho$ by making appropriate changes to quantities corresponding to $\omega$ and $\phi$ and thus we don’t write the explicit expressions here.

The neutral mode also receives some contribution from $J/\psi$ via the annihilation diagram, which we expect to be small and do not include.

4 Results

It is worth mentioning that apart from the $\eta_c$ contribution, there is no handle for determining the relative sign of other matrix elements. This introduces some amount of uncertainty in the result for the total decay rate. The differential decay rate is given by

$$\frac{d\Gamma}{d\sqrt{s_{\gamma\gamma}}} = \left( \frac{1}{512 m_B^{3/2}} \right) \sqrt{s_{\gamma\gamma}} \left[ \left( 1 - \frac{s_{\gamma\gamma}}{m_B^2} + \frac{m_{\pi}^2}{m_B^2} \right)^2 - \frac{4m_{\pi}^2}{m_B^2} \right]^{1/2} \int_0^\pi d\theta \sin \theta |\mathcal{M}_{\text{tot}}|^2$$

where $\mathcal{M}_{\text{tot}}$ represents the complete matrix element for the decay process including the resonance terms, $\sqrt{s_{\gamma\gamma}}$ is the C.M. energy of the two photons and $\theta$ denotes the angle which the decaying B-meson makes with one of the two photons in the $\gamma\gamma$ C.M. frame. Figure 1 shows the differential decay rate as a function of the invariant mass of the two photons for the charged mode. The $\eta$, $\eta'$ and $\eta_c$ peaks show up at the corresponding mass values while the $\rho$ contribution gets spread over the whole range of $\sqrt{s_{\gamma\gamma}}$. Further, the interference effects between the resonant and the irreducible contributions are very small, with the result that the results are practically independent of the relative sign parameters that one may have introduced for each of the resonance terms.
The total decay rate, including all resonance contributions for this process is calculated to be:

\[ Br(B^- \to \pi^- \gamma \gamma) \sim 1.7 \times 10^{-6} \]  

(27)

We quote the individual contributions to the branching ratio:

\[ \eta_c \leftrightarrow 2 \times 10^{-9} \quad \rho \leftrightarrow 2 \times 10^{-9} \]  

(28)

Irreducible \( \leftrightarrow 3 \times 10^{-8} \quad \text{interference} \leftrightarrow 7 \times 10^{-9} \)

Irreducible \( \leftrightarrow 3 \times 10^{-8} \quad \text{interference} \leftrightarrow 7 \times 10^{-9} \)

Figure 1: Our result for the spectrum of \( B^- \to \pi^- \gamma \gamma \) plotted with logscale on y-axis. The parameters used are listed in the appendix.

The branching ratio for the neutral mode is also expected to be of similar magnitude. The neutral mode gets additional, although small, \((O \sim \rho)-\)contributions from the \( \omega \) and \( \phi \) modes as well. More accurate statements can be made about the neutral mode once \( B \to \pi \eta(\eta') \) is observed.

As mentioned earlier, the process, although CKM suppressed, opens up the possibility of studying the CP asymmetry. We define the CP asymmetry as

\[ A_{CP} = \frac{d\Gamma}{d\sqrt{s}} \frac{d\Gamma}{d\sqrt{s}} - \frac{d\bar{\Gamma}}{d\sqrt{s}} + \frac{d\bar{\Gamma}}{d\sqrt{s}} \]  

(29)
with $\Gamma$ and $\bar{\Gamma}$ denoting the decay rates for $B^- \text{ and } B^+$ (or $B^0 \text{ and } \bar{B}^0$) respectively. The resonances by themselves do not contribute to CP asymmetry. The CKM factors for them are dominantly real and in any case will be overall multiplicative factors unchanged while going from $B \text{ to } \bar{B}$ when calculating the individual resonance contributions. However, there can be small contribution coming from the interference between the resonant and irreducible amplitudes. However, the interference terms being rather small have been neglected in this analysis, which in principle can contribute to the asymmetry.

Figure 2: The CP asymmetry for $B^- \rightarrow \pi^- \gamma\gamma$

Figure 2 shows the CP asymmetry, $A_{CP}$ as a function of $\sqrt{s_{\gamma\gamma}}$. Also, we present the integrated CP asymmetry value

$$A_{CP}|_{int} = \frac{\int \frac{d}{d\sqrt{s_{\gamma\gamma}}} \frac{d\Gamma}{d\sqrt{s_{\gamma\gamma}}} - \int \frac{d}{d\sqrt{s_{\gamma\gamma}}} \frac{d\bar{\Gamma}}{d\sqrt{s_{\gamma\gamma}}}}{\int \frac{d}{d\sqrt{s_{\gamma\gamma}}} \frac{d\Gamma}{d\sqrt{s_{\gamma\gamma}}} + \int \frac{d}{d\sqrt{s_{\gamma\gamma}}} \frac{d\bar{\Gamma}}{d\sqrt{s_{\gamma\gamma}}}}$$

$$= 1.739 \times 10^{-4}$$

Experimental observation of this decay can thus lead to a better understanding of the deeper structure of the effective Hamiltonian and is also expected to shed light on the possible determination of the relative signs between various interference terms. The current fluxes at the B-factories are of course too small for any possible measurement of these numbers, but hopefully in the future such measurements will be possible.
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Appendix

We give the input parameters used in the numerical calculations.

| $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7$ | $C_8$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| -0.222 | 1.09  | 0.010 | -0.023 | 0.007 | -0.028 | -0.301 | -0.144 |

Table 1: The approximate values of $C_i's$ at $\mu = m_b$

$m_b = 4.8 \, GeV \quad m_c = 1.5 \, GeV \quad m_t = 175 \, GeV$

$m_u = 0.15 \, GeV \quad m_d = 0$

$m_{B^0} = 5.2792 \, GeV \quad \Gamma^{B^0}_{\text{total}} = 4.22 \times 10^{-13} \, GeV$

$m_{B^+} = 5.2789 \, GeV \quad \Gamma^{B^+}_{\text{total}} = 4.21 \times 10^{-13} \, GeV$

$m_{\eta_c} = 3 \, GeV \quad B_{\eta_c} = 2.74 \times 10^{-3} \, GeV^{-1} \quad \Gamma^{\eta_c}_{\text{total}} = 1.3 \times 10^{-2} \, GeV \quad f_{\eta_c} = 0.35 \, GeV$

$m_\eta = 0.547 \, GeV \quad m_{\eta'} = 0.95778 \, GeV$

$B_\eta = 13.254 \times 10^{-3} \, GeV^{-1} \quad \Gamma^{\eta}_{\text{total}} = 1.18 \times 10^{-6} \, GeV \quad f_\eta = -2.4 \times 10^{-3} \, GeV$

We follow the Wolfenstein parametrization of the CKM matrix with

$A = 0.8 \quad \lambda = 0.22 \quad \eta = 0.34 \quad \rho = -0.07$

$V_{tb} \sim 1 \quad V_{ts} = -A\lambda^2$

$V_{cb} = A\lambda^2 \quad V_{cs} = 1 - \frac{\lambda^2}{2}$

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