Approximate Joint MAP Detection of Co-Channel Signals

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Abstract

In this article a novel equalization & multiuser detection algorithm is described. The proposed algorithm is motivated by a factor graph-based model of the received signal. The goal is to provide a low-complexity alternative to the maximum \textit{a posteriori} probability (MAP) symbol detector (i.e., the BCJR algorithm).

I. INTRODUCTION

A. System Model

In this work we consider a multiple access channel with $U$ users. Let the information bits, coded bits, and symbols of the $u$th user be denoted by column vectors $b^{(u)}$, $c^{(u)}$, and $x^{(u)}$, respectively. We define the collection of these terms for all users as

\[
B = [b^{(1)}, \ldots, b^{(U)}] \\
C = [c^{(1)}, \ldots, c^{(U)}] \\
X = [x^{(1)}, \ldots, x^{(U)}].
\]

The complex channel coefficients for the multipath channel between the $u$th transmitter and the receiver is denoted $h^{(u)} = [h_0^{(u)}, h_1^{(u)}, \ldots, h_{L-1}^{(u)}]^T$. The channel response is assumed to also include the transmit pulse. The $n$th sample of the received signal is given by

\[
r_n = \sum_{u=1}^{U} \sum_{l=0}^{L-1} h_l^{(u)} x_{n-l}^{(u)} + w_n,
\]

(1)

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where \( w_n \) is additive white Gaussian noise and \( L \) is the number of channel taps. The collection of all received samples is denoted \( r = [r_0, \ldots, r_{N-1}] \).

**B. MAP Detection**

The goal of the receiver is to detect all information bits \( B \) given observation \( r \). Because of the complexity of sequence detection of \( B \), we desire to perform MAP symbol-by-symbol (in our case, bit-by-bit) detection. The detector for the \( k \)th bit of user \( u \) is given by

\[
\hat{b}_k^{(u)} = \arg \max_{b_k^{(u)}} \sum_{\sim\{b_k^{(u)}\}} p(B|r), \tag{2}
\]

where the marginal is computed for \( b_k^{(u)} \) (i.e., \( \sum_{\sim\{b_k^{(u)}\}} \) denotes summation over the domain of all variables except \( b_k^{(u)} \)). According to Bayes Rule, \( (2) \) is equivalent to

\[
\hat{b}_k^{(u)} = \arg \max_{b_k^{(u)}} \sum_{\sim\{b_k^{(u)}\}} f(r, B), \tag{3}
\]

where the term \( 1/f(r) \) is a constant which has been removed. By the Total Probability Theorem, \( (3) \) can further be expressed as a marginalization over the full joint distribution as given by

\[
\hat{b}_k^{(u)} = \arg \max_{b_k^{(u)}} \sum_{\sim\{b_k^{(u)}\}} f(r, X, C, B). \tag{4}
\]

The marginalization in \( (4) \) cannot be performed directly, but an iterative implementation of the sum-product algorithm is well suited for this task.

**C. Probability Distribution**

Taking into account conditional independence of the variables, the joint distribution is given by

\[
f(r, X, C, B) = \prod_{n=0}^{N-1} f(r_n|x^{(1)}_n, \ldots, x^{(U)}_n) \prod_{u=1}^U p(x^{(u)}|c^{(u)})p(c^{(u)}|b^{(u)})p(b^{(u)}). \tag{5}
\]

Factorizations of the modulation \( p(x^{(u)}|c^{(u)}) \) and code \( p(c^{(u)}|b^{(u)}) \) constraints have been explored in literature [1], [2]. From \( (1) \) the likelihood function for each term \( r_n \) is dependent on a finite subset of the symbols. We define, \( x^{(u)}_{[n]} = [x^{(u)}_{n-L+1}, \ldots, x^{(u)}_n] \) to denote the symbols from user \( u \) which have components in the \( r_n \) sample. The distribution is then given by

\[
f(r, X, C, B) = \prod_{n=0}^{N-1} f(r_n|x^{(1)}_{[n]}, \ldots, x^{(U)}_{[n]}) \prod_{u=1}^U p(x^{(u)}|c^{(u)})p(c^{(u)}|b^{(u)})p(b^{(u)}). \tag{6}
\]
D. Factor Graph Model

The sum-product algorithm performs efficient marginalization by exploiting the factorization of the joint distribution \( f(r, X, C, B) \). As an example, consider the case of \( U = 2 \) and \( L = 4 \). The factor graph of the joint distribution is shown in Fig. 1 where \( r_n \) denotes the factor \( f(r_n|x_n^{(1)}, \ldots, x_n^{(U)}) \). The factors \( p(x^{(u)}|c^{(u)})p(c^{(u)}|b^{(u)})p(b^{(u)}) \) are further factored when implementing the sum-product algorithm. The factor nodes related to the observations \( r_n \) and the symbol variable nodes make up the “detection block” of the factor graph. The graph contains cycles within the detection block as well as between the symbol variables and the modulation/code factors. Converting the detection block into a state space model is a traditional approach to eliminating the cycles within the detection block [3], [4]. However, in our case complexity prohibits us from directly implementing the sum-product algorithm on the \( r_n \) factors of Fig. 1.

![Factor graph of \( f(r, X, C, B) \) for \( U = 2 \) and \( L = 4 \).]
E. Complexity

Although the distribution has been fully factored, the complexity of message passing algorithms associated with the factor nodes $f(r_n|\mathbf{x}_1^n, \ldots, \mathbf{x}_{U}^n)$ may be too complex. The complexity of the computations for the sum-product algorithm at each factor node is $O(M^{UL})$ where $M$ is the modulation order (assumed to be the same for each user). As an example, the complexity for QPSK, 4 users, and 4 channel taps (i.e., $M = 4$, $U = 4$, and $L = 4$) is $O(10^9)$.

Because of the problem of complexity with MAP multiuser detector, approaches with lower complexity have been considered for this problem.

- **Interference Cancellation**: Cancellation may be performed based on either hard or soft decisions. Detection is performed starting with the strongest signal and continuing to the weakest. Soft cancellation may be combined with iterative processing to iteratively improve the soft estimates.

- **Rake Gaussian**: This method was proposed in [5] for interleave-division multiple access. In this method, for the detection of symbol $x_k^{(u)}$ all other symbols are modeled as Gaussian random variables. This includes the symbols of all other users $u' \neq u$ and all other symbols of the desired user $k' \neq k$. The mean and variance of the Gaussian distribution are computed from the extrinsic symbol probabilities obtained from demodulation and decoding. The mean of the Gaussian random variable is analogous to a soft decision. The Gaussian model improves upon soft cancellation by also modeling the variance.

- **Concurrent MAP**: This method was proposed by [4] to improve upon the performance of the Rake Gaussian method. In this method, joint MAP detection of the desired user’s symbol is performed while interfering symbols of the other users are modeled as Gaussian random variables. Thus, the complexity of the method is linear in the number of users.

Visual comparisons of the Rake Gaussian and Concurrent MAP algorithms are given using factor graphs. The factor node $r_3$ from the example in Fig. 1 is used to show the Rake Gaussian and Concurrent MAP algorithms in Figs. 2 and 3 respectively. Green dashed lines represent connections where the message contains the mean and variance of the symbol’s Gaussian approximation.

The graphical models of Figs. 2 and 3 motivate a new approach which models the distribution of weaker terms in the signal component of $r_n$ as Gaussian random variables. Sum-product message passing is performed for the stronger terms in $r_n$. This hybrid approach has a complexity
equal to that of the Concurrent MAP algorithm and maintains a single, connected graph. The graphical model for the hybrid approach is shown in Fig. 4 where symbols $x_1^{(1)}$, $x_1^{(2)}$, and $x_2^{(2)}$ are the strongest component in $r_3$ for users 1 and 2. This model is further motivated by common pulse shapes such as the square root raised cosine pulse shown in Fig. 5 which contains the majority of its energy within the center of the pulse.

Fig. 2. Factor graph motivated representation of the Rake Gaussian method. Green dashed lines represent Gaussian mean and variances. As shown, this factor computes $f(r_3|x_3^{(2)})$.

Fig. 3. Factor graph motivated representation of the Concurrent MAP method. Green dashed lines represent Gaussian mean and variances. As shown, this factor is a slice of the overall graph to implement MAP detection of user 2 while modeling the interference from user 1.

II. NOVEL MULTIUSER DETECTION ALGORITHM

Consider a generic interference model (to represent intersymbol interference, multiuser interference, or both) in which $K$ signal components $x_1, x_2, \ldots, x_K$ are received with channel coefficients $h_1, h_2, \ldots, h_K$, respectively. The received signal is given by

$$y = \sum_{k=1}^{K} h_k x_k + w$$
where the noise $w$ is modeled as a zero-mean circularly symmetric complex Gaussian random variable with variance $\sigma^2$. The distribution on the received signal is given by

$$p(y|x_1, \ldots, x_K) \sim \mathcal{CN} \left( \sum_{k=1}^{K} h_k x_k, \sigma^2 \right)$$

where the channel coefficients and the noise power $\sigma^2$ are assumed to be known.

The message from factor node $Y$ to variable node $X_k$ is denoted $m_{Y \rightarrow X_k}$. Similarly, the message from variable node $X_k$ to factor node $Y$ is denoted $n_{X_k \rightarrow Y}$. According to the sum-product algorithm, the message $m_{Y \rightarrow X_i}$ is given by

$$m_{Y \rightarrow X_i}(x_i) = \sum_{\sim \{x_i\}} p(y|x_1, \ldots, x_K) \prod_{k \neq i} n_{X_k \rightarrow Y}(x_k). \quad (7)$$
The proposed algorithm modifies the sum-product algorithm computations as follows:

- The mean and variance of the input messages are computed according to

\[
\mu_{x_k} = \sum_{x_k} x_k n_{X_k \rightarrow Y}(x_k)
\]

\[
\sigma_{x_k}^2 = \sum_{x_k} (x_k - \mu_{x_k})^2 n_{X_k \rightarrow Y}(x_k)
\]

for all \( k = 1, \ldots, K \).

- For computation of the outgoing message \( m_{Y \rightarrow X_i}(x_i) \), the remaining variables for \( k \neq i \) are sorted by \( |h_k|^2 \sigma_{x_k}^2 \). Let the set \( A \) index the variables associated with the strongest components. These variables remain a part of the local marginalization as given in (7). The number of variables in the set \( A \) will depend on the acceptable complexity in implementation. The indices of the weaker components are included in the set \( B \) and the distributions of these variables are approximated by Gaussian random variables to eliminate the marginalization over these variables. Let the variables associated with sets \( A \) and \( B \) be given by \( x_A \) and \( x_B \), respectively.

- The message is computed with the following approximate sum-product computation:

\[
m_{Y \rightarrow X_i}(x_i) = \sum_{k \in A} \tilde{p}(y|x_i, x_A) \prod_{k \in A} n_{X_k \rightarrow Y}(x_k)
\]

where

\[
\tilde{p}(y|x_i, x_A) \sim \mathcal{N}\left(h_i x_i + \sum_{k \in A} h_k x_k + \sum_{l \in B} h_l \mu_{x_l}, \sigma^2 + \sum_{l \in B} |h_l|^2 \sigma_{x_l}^2\right).
\]

III. Preliminary Results

As an initial means of comparison, we simulate the performance for two users each employing a BPSK modulation and 1/2-rate turbo code. The selection of two users and BPSK keeps the complexity of the joint MAP factor graph reasonable and provides a point of comparison. The simulation parameters are summarized as follows:

- Code: 1/2-rate turbo code
- Modulation: BPSK
- Frame size: 500 symbols
- Pulse: Square root raised cosine with \( L = 4 \) and roll-off factor 0.35
- 15 iterations of message passing are performed
Fig. 6. FER comparison of the joint MAP (Joint MAP), the proposed novel approach (Hybrid FG), concurrent MAP (Concurrent MAP), and soft interference cancellation (Soft IC) algorithms as well as the single user performance. The FER of user 1 is provided. The power level of each user is equal (i.e., SIR=0 dB).

The FER performance of the joint MAP, the proposed novel approach, concurrent MAP, and soft interference cancellation algorithms is shown in Fig. 6. At a FER of $10^{-3}$ the proposed approach and Concurrent MAP methods demonstrate losses of 0.2 dB and 1.2 dB, respectively, compared to joint MAP detection. The Soft IC method becomes limited by interference.

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