The application of a Morlet wavelets bandpass filter in the fault diagnosis of rolling bearings

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Abstract. An improved demodulation approach is proposed for the construction of a bandpass filter and determination of its parameters. An envelope analysis of the relationship among the central frequency, bandwidth and the system’s damping coefficient was conducted and it was demonstrated that to extract the fault features effectively, the quality factor of the bandpass filter should remain constant and the bandwidth should be determined according to the damping coefficient of the system, the central frequency of the filter, and the fault frequency of the bearing. A combined wavelet bandpass filtering function was constructed based on the Morlet wavelet function. The capture of fault signals from the rolling bearing was optimized in virtue of excellent bandpass filtering properties of the combined wavelet’s better response and the strong sensitivity of the correlated kurtosis to periodic pulse signals. The experimental test data and the simulation results demonstrated that the proposed approach is an effective method for fault diagnosis of rolling bearings.

1. Introduction

As a key component of rotating machinery, bearings have been utilized widely in various types of industrial equipment and the early diagnosis of bearing faults can avoid catastrophic failure, with consequent equipment damage, casualties or even fatalities. In the diagnosis of bearing faults, fault features are extracted from vibration signals and analyzed to identify the condition of the bearing. Hence, the extraction of bearing fault features is the core part of fault diagnosis.

In presence of bearing faults, the impulse signals of faults tend to be concentrated in the resonance frequency band of the system and fault features can be extracted by envelope demodulation. In envelope demodulation, the selection of the bandpass filter parameters directly affects the envelope filter and is a key step in fault feature extraction [1]. Shi et al proposed a filtration method based on wavelets and kurtosis in reference [2]. Several filter groups were established, based on the number of filters corresponding to each octave, and filtration analysis of the frequency domain was achieved based on all filter groups. The signal with maximum kurtosis is defined as the optimized filtration signal. Tse et al proposed a method that achieves the automatic selection of an optimized wavelet filter in reference [3]. Herein, the optimized central frequency of the filter was identified using genetic algorithms. Yu et al proposed a fault feature extraction approach based on the empirical mode decomposition and the Hilbert transform and demonstrated that this approach was superior to conventional envelope analysis methods.
in reference [4]. Lin and Qu proposed a fault feature extraction approach based on Morlet wavelet noise elimination in reference [5]. This approach can achieve an improved signal-to-noise ratio. Jerome and Randall demonstrated that kurtosis can be used as an effective parameter for fault diagnosis in reference [6]. Also, it has been demonstrated that the wavelet-based Hilbert transform can achieve fault envelopment analysis very effectively [7-11].

All of the approaches mentioned above have obtained optimized parameters of bearing fault features with kurtosis of the bearing impulse signals as an objective function. Determination of the parameters for envelope demodulation of a bandpass filter has not yet been studied intensively. In the present investigation, filter bandwidths were determined based on an analysis of the central frequency and the bandwidth of the bandpass filter and the damping coefficient of the system. A combined wavelet bandpass filtration function was established using the Morlet wavelet function and the corrected kurtosis was employed as the criterion for the selection of an optimized filter, instead of kurtosis, to accurately reflect the periodic pulse signals, thus effectively extracting bearing fault information directly from the test signals. Additionally, optimized frequency band filtration signals were obtained by scanning filtration of the analysis frequency domain, based on analysis of the wavelets. The simulation and experimental test results revealed that the proposed approach can identify precisely optimized filter parameters and can achieve effective extraction of fault features.

2. Combined wavelet-based envelope analysis

The core part of envelope analysis is the selection of the bandpass filter and the determination of its parameters. Currently, various bandpass filters are available, but the selection of a bandpass filter for effective extraction of fault impulse signals of rolling bearings has not been clearly understood. For the selection of filter parameters, Sawalhi et al proposed optimization with the kurtosis of signals after filtration or envelope demodulation as the objective function in reference [12], although the mechanism was not fully described. The principles of bearing fault feature extraction, based on combined wavelet envelope analysis, are as follows: by establishing a combined wavelet-based envelope analysis model, the optimized central frequency and the bandwidth of the bandpass filter in the envelope solutions were identified according to the corrected kurtosis values of combined wavelet envelope analysis signal and the fault features of the bearing were obtained, based on the envelope analysis.

2.1. Principles of envelope analysis based on combined wavelet function

For a Morlet wavelet, the parent wavelet $g_i(t)$ is [7]:

$$g_i(t) = (e^{i2\pi ft} - e^{-2i\pi f_i})e^{-1/2(t)^2}$$

(1)

where $t$ denotes time.

The Fourier transform of $g_i(t)$ leads to:

$$\hat{g}_i(f) = 2\pi(e^{-2\pi^2(f-f_i)^2} - e^{-2\pi^2(j^2+f_i^2)})$$

(2)

where $f$ denotes frequency, $f_i$ denotes a specific frequency.

With $a$ as size parameters and $b$ as location parameters, the analysis wavelet established based on $g_i(t)$ is:

$$g_{i,a,b}(t) = \frac{1}{\sqrt{a}} g_i\left(\frac{t-b}{a}\right)$$

(3)

If $f_i > 1$, $e^{-2i\pi f_i}$ can be neglected and the conjugation of equation (3) is:

$$g^*_{i,a,0}(t) = \frac{1}{\sqrt{a}} e^{-1/2(t/a)^2 - j2\pi f_i(t/a)}$$

(4)
The combined wavelet $G_a^*(t)$ by linear superimposition of $f_L$-filter (values of $f_0$) is:

$$G_a^*(t) = \frac{1}{\sqrt{a}} \sum_{j=L}^{H} e^{-\frac{1}{2}t^2 - j2\pi(f_j t)}$$  \hspace{1cm} (5)

Simplifying equation (5) and defining $\Delta f = f_H - f_L$, $f_L' = \frac{f_L}{a}$, $f_H' = \frac{f_H}{a}$, $\Delta f' = \frac{\Delta f}{a}$, then:

$$G_a^*(t) = \frac{e^{-\frac{1}{2}(f_j t)^2} - e^{-\frac{1}{2}(f_j' t)^2}}{\sqrt{a}(1 - e^{-j\pi f_j t})}$$  \hspace{1cm} (6)

If equation (6) is regarded as a filtration function of the bandpass filter, the bandpass attenuation of the filter is positively related to $a$ and ripple oscillations in the bandpass are negatively related to $a$ and $\Delta f$. Indeed, the bandpass effect of the filter can be enhanced by optimizing $a$ and $\Delta f$. This filter exhibits advantages such as rapid convergence, constant bandpass gain and reduced phase loss. As proposed elsewhere [7], with the size parameters $a = 0.1652$, $\Delta f = 1$ Hz, the attenuation rate of the bandpass filter can be guaranteed, and the bandpass ripple can be controlled within $\pm$ 2%. In this case, equation (6) can be transformed to:

$$G_{3dB}(t) = \frac{2e^{-18.32tt^2} (e^{-j2\pi f_0' t} - e^{-j2\pi f_0 t})}{0.4064(1 - e^{-2j\pi t})}$$  \hspace{1cm} (7)

$f_L'$ and $f_H'$ can be regarded as upper and lower frequency for the bandwidth of the bandpass filter and the bandwidth ($B$) and central frequency ($f_0$) of different bandpass filters can be obtained by:

$$\begin{cases}B = f_H' - f_L' \\ f_0' = (f_L' + f_H')/2 \end{cases}$$  \hspace{1cm} (8)

For the signal $f(t)$, the wavelet transform can be defined as:

$$W_c(a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t)G_a^*(t)dt$$  \hspace{1cm} (9)

As its real part and imaginary part are orthogonal, $G_a^*(t)$ can be regarded as a Hilbert transform pair. For signal $f(t)$, the envelope can be defined as:

$$E_c(a) = 2\Delta f' a |W_c(a)|$$  \hspace{1cm} (10)

where $2\Delta f' a$ is a normalized coefficient to guarantee consistent amplitudes of signals before and after the envelope analysis. On one hand, combined wavelet-based envelope analysis facilitates the extraction of fault features as wavelet functions and the impulse signals of bearing faults have consistent shapes. On the other hand, combined wavelet-based filters exhibit superior performance over single wavelet-based ones, so that the amplitudes of signals before and after envelope analysis are consistent.

2.2. Determination of bandwidth in envelope analysis

In envelope analysis, the bandwidth of the bandpass filter is a key parameter. Sawalhi et al discussed alteration of the bandwidth of a bandpass filter by deconvolution of a number of layers of wavelets in octaves [12]. For short bandwidths, the kurtosis of signals after envelope analysis is low, indicating that the impulse signals of the bearing faults were difficult to obtain at short bandwidths. In that circumstance, the optimized bandwidth that will provide effective extraction of the bearing fault impulse signals is considered. In the presence of bearing surface damage, periodic vibration signals with bearing
fault frequencies as the repeating frequency and the natural frequency of the bearing are observed as the free attenuation vibration frequency. Signals during the early stages of bearing faults are weak and usually are masked by instrument vibration (e.g., gear engagement) or other extraneous signals (e.g., asymmetric, out of alignment responses) and extraction of the fault features is extremely difficult. As a commonly used signal-processing method, envelope demodulation can achieve effective extraction of bearing fault signals. Therein, the core part is a determination of the appropriate analysis bandwidth of the signal.

To determine the bandwidth for signal analysis, the bandwidth of free attenuation vibration signals must be investigated first. The free attenuation vibration signals are defined as:

\[ h(t) = e^{-\xi^2 \pi^2 t} \sin(\sqrt{1-\xi^2} 2\pi f_n t) \]

(11)

where \( \xi \) refers to the damping coefficient of the system, and \( f_n \) refers to the natural vibration frequency of the system. If \( \xi \) is defined as 0.02, and \( f_n = 500 \text{ Hz} \), 1 000 Hz, 2 000 Hz, 3 000 Hz, 4 000 Hz, 5 000 Hz, 6 000 Hz, and 10 000 Hz, Table 1 shows the corresponding bandwidths. Table 2 summarizes the bandwidths at \( f_n = 5000 \text{ Hz} \) and \( \xi = 0.02, 0.03, 0.04, 0.05, 0.06, \) and 0.07. As observed, the bandwidth increases with \( f_n \) at constant \( \xi \) and their ratio \( Q \) remains constant. As shown in table 2, the bandwidth increases with \( \xi \) at constant \( f_n \).

| Natural frequency /Hz | 500 | 1000 | 2000 | 3000 | 4000 | 5000 | 6000 | 10000 |
|-----------------------|-----|------|------|------|------|------|------|-------|
| Bandwidth             | 20  | 40   | 80   | 120  | 160  | 200  | 241  | 402   |
| \( Q = f_n/B \)       | 25  | 25   | 25   | 25   | 25   | 25   | 24.89| 24.87 |

Table 1. The corresponding relationship between different natural frequencies and bandwidth \( (\xi = 0.02) \).

| Damping coefficient | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 |
|---------------------|------|------|------|------|------|------|
| Bandwidth           | 200  | 521  | 1008 | 2063 | 3254 | 4952 |

Table 2. The relationship between different damping coefficients and bandwidth \( (f_n = 5000 \text{ Hz}) \).

For the extraction of fault feature signals from rolling bearings by envelope demodulation, the bandwidth of the bandpass filter is determined according to the damping coefficient of the system and the central frequency of the bandpass filter in the envelope analysis and the ratio of the central frequency and the bandwidth will remain constant (specifically, it will have a constant quality factor). Additionally, the envelope spectrogram should contain the bearing’s fault feature frequency from first-order to third-order at least that will provide effective fault diagnosis for the bearing. Hence, the bandwidth of the bandpass filter should be no less than three times the fault feature frequency.

2.3. Selection of the central frequency in the envelope analysis

As mentioned above, the extraction of bearing fault signals requires not only an appropriate bandwidth of the bandpass filter but also an appropriate central frequency for the bandpass filter. Sawalhi et al optimized the selection of the central frequency of the envelope analysis using the kurtosis of the filtered signal as the objective function [12]. This approach was based on the fact that signals induced by the bearing faults are impulse signals whose kurtosis is larger than the central frequency of the bandpass filter. Nevertheless, incidents such as on/off of air valves or pitting faults on gear surfaces would induce such impulse signals, thus hindering the extraction of fault signals from a roller bearing. The corrected kurtosis (CK) is defined as follows:

\[ CK_M(T) = \frac{\sum_{i=1}^{N}(\prod_{m=0}^{M} y_{m+1})^2}{(\sum_{i=1}^{N} y_i^2)^{M+1}} \]

(12)
where \( y_i \) refers to the filtered signal, \( T \) refers to the period of pulse signals, and \( M \) refers to the quantity of periods with deviations. The corrected kurtosis value considers both the impacts and the periodicity of the impulse signals.

As a local index, the corrected kurtosis value can reflect precisely the component features of specific periodic signals, which are difficult to obtain from kurtosis. Meanwhile, the corrected kurtosis value can reflect precisely the intensity of periodic pulse signals at constant \( T \). Therefore, the determination of the central frequency of the envelope analysis with corrected kurtosis as the objective function in analysis can effectively facilitate the extraction of bearing fault signals.

### 2.4. Extraction of bearing fault features

Besides the relationship between the system damping, central frequency and bandwidth of the bandpass filter, an envelope analysis program that can extract bearing fault signals effectively is also highly important. The principles of fault feature extraction based on combined wavelet envelope analysis are as follows:

- Set the selection of an appropriate bandwidth and central frequency for the bandpass filter in initial envelope analysis.
- The envelope analysis was applied on test signals based on equation (9), and the corrected kurtosis of signals was calculated.
- The bandwidth and central frequency of the new envelope analysis were determined with a specific scanning step according to the constant quality factor \( Q \), and new envelope analysis signals were obtained together with the corresponding corrected kurtosis.
- This process was repeated until the envelope analysis covered the entire frequency range of test signals. Finally, envelope signals with maximized corrected kurtosis were selected from the analysis as signals of bearing fault features and the fault features of the bearing were obtained by spectrogram analysis of the response.

### 3. Simulations

To verify the effectiveness of the approach, the simulation results were compared with those obtained using the conventional approach [13]. The vibration signal model of a rolling bearing was established by superimposition of fault impulse signals \( s(t) \), gear harmonic wave signals \( B(t) \), and white noise signals \( n(t) \):

$$
\begin{align*}
x(t) &= \sum_{i=1}^{k} A_i s(t - iT - \tau_i) + B(t) + n(t) \\
S(t) &= e^{-\xi t / \tau_i} \sin(2\pi f_s t + \phi_s) \\
A_i &= A_0 \cos(2\pi f_s t + \phi_s) + c_i \\
B(t) &= B_0 \sin(2\pi f_s t + \pi / 2)
\end{align*}
$$

(13)

where \( A_i \) refers to the amplitude modulation with period of \( 1/f_s \), \( f_s \) is the switching frequency, \( B(t) \) refers to the background harmonic component, \( f_s \) refers to the harmonic frequency, \( f_s \) refers to the natural frequency, \( S(t) \) refers to the exponential attenuation pulse, \( \phi_s \) and \( \phi_A \) refer to the initial phase, \( C_A \) refers to constant, \( T \) refers to the period between two adjacent pulses, \( \tau \) refers to the periodic delay of the \( i \)th pulse induced by sliding, \( \zeta \) refers to the damping coefficient of the system, and \( n(t) \) refers to the white noise. In this study, \( \zeta = 0.02, f_s = 40 \text{ Hz}, f_m = 1600 \text{ Hz}, T = 1/150 \text{ s}, f_n = 100 \text{ Hz}, \tau_i = 1, A_0 = 0, B_0 = 4, C_A = 0.5 \). The time domain of the simulation signals is illustrated in figure 1.
Simulation signals filtering was achieved using the combined wavelet. As the initial frequency should be larger than three times the maximum frequency, which is the frequency of the background harmonic component \( f_m \) in this study, the initial central frequency of the bandpass filter was set to be 500 Hz in the present study. As the scanning step should be smaller than the minimum fault frequency, which was 150 Hz in the present study, the step was set to be 100 Hz. According to requirements mentioned in Section 2.2, the initial bandwidth was determined to be 450 Hz and the quality factor of the bandpass filter was calculated to be 1.1. The time domain wave shapes of the simulation signals and envelope demodulation spectrum obtained by the proposed approach are presented in figure 1. The bandpass filter had a bandwidth of 1454.5 Hz and a central frequency of 1600 Hz. Figure 2 shows the fast kurtogram obtained using a conventional algorithm [12]. As can be observed, the sixth central frequency was 1024 Hz and the frequency band with a bandwidth of 93.75 Hz had the maximum kurtosis. The filtration signal corresponding to this frequency band is the optimized filtration signal, which spectrogram is shown in figure 3.

Figures 1 and 3 illustrate fault features of the bearing frequency and the rotation modulation frequency. As can be observed, the amplitude of the fault feature spectrogram in figure 1 was higher than that in figure 3, especially for high-resolution spectrogram lines, suggesting that the mechanism and the algorithm of the proposed new approach are suitable for the extraction of fault features.
4. Testing

4.1. Test data

The bearing data used in this study were extracted from the bearing database published by Case Western Reserve University. The bearings used were 6205-2RS JEM SKF rolling bearings with an outer diameter of 52 mm, an inner diameter of 25 mm, a diameter of the rolling elements of 7.94 mm, the quantity of rolling elements of 9, and a contact angle (α) of 0°. The sampling length was 12000 and the rotation speed of the rotor was 1797 r/min, which means that the switching frequency (fr) was 29.95 Hz and the sampling frequency (fs) was 12000. The faults were defined to be inner ring damage, and the fault feature frequency (fi) was calculated to be 162.185 Hz based on the geometric parameters and the switching frequency of the bearing. The time domain and frequency domain wave shapes of the faulty bearing are shown in figure 4.

![Figure 4. (a) Time domain diagram and (b) spectrogram of signals from a faulty bearing.](image)

4.2. Results

The proposed approach was compared to a conventional bearing condition monitoring approach that has been reported previously [12]. The initial frequency (600 Hz) of the envelope analysis bandpass filter was set to be three times the maximized frequency. As the scanning step should be smaller than the minimum frequency of any fault features (162.185 Hz), the scanning step was set to be 100 Hz. As mentioned in Section 3.2, the minimum bandwidth shall not be lower than 480 Hz. Hence, the quality factor (Q) was calculated to be 1.25.

![Figure 5. Spectrogram obtained by the proposed method.](image)
A spectrogram from the optimized signal is shown in figure 5 after the signal has been filtered by the proposed algorithm. The central frequency and bandwidth of the selected bandpass filter were 3400 Hz and 2720 Hz, respectively. The fast spectrogram is shown in figure 6. As can be observed, the secondary central frequency was 1500 Hz and the frequency band, with a bandwidth of 3000 Hz, had the maximum kurtosis. The envelope demodulation spectrogram of this frequency band is shown in figure 6.

![Figure 6. Fast kurtogram of the experimental signals.](image)

As can be observed, the spectrogram features of faults shown in figure 5 were significantly superior to those in figure 7, and the signal intensity in figure 5 was significantly higher than that in figure 7. The fault characteristic frequency (162Hz) base spectral line and its multi-order spectral lines (323Hz, 485Hz, …) obtained in figure 5 were more prominent than other characteristic spectral lines. Unlike the fault characteristic frequency spectral lines were mixed with other spectral lines in figure 7, which were difficult to be identified. Hence, the proposed new algorithm is more effective to identify incipient and developing roller bearing failures than is the conventional algorithm mentioned previously in reference [13].

5. Conclusions
For the establishment of a bandpass filter and the determination of its parameters for envelope demodulation of rolling bearings, a combined wavelet filtration-based approach for feature extraction from rolling bearings is proposed and a quantitative analysis was conducted of the correlation of the central frequency of the bandpass filter, the bandwidth, and the damping coefficient of the system.
For effective extraction of bearing fault signals, the quality factor of a bandpass filter should be constant, and the bandwidth should be determined according to the damping ratio of the system, the bandpass filter central frequency, and the bearing fault feature frequencies.

An optimized bandpass filter can be obtained by envelope scanning with corrected kurtosis as the objective function. Simulation and physical test results revealed that the principles and algorithm of the proposed envelope analysis approach were viable.

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