Using variational regularization method of electrical impedance tomography to reduce the position error of human arm

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Abstract. This paper investigates how to reconstruct human arm images using electrical impedance tomography. A simplified arm model is first established. The comparison between the Tikhonov regularized reconstruction algorithm and the Variation regularized reconstruction algorithm is then made. The results show that the Variation regularized reconstruction algorithm has the lesser position error on the organizational junction edges and is basically overlap with the target area of the model. How to eliminate the artefact of the reconstructed image needs to be further investigated.

1. Introduction

Electrical impedance tomography (EIT) is a general technique for computing the imaging of the internal electrical properties of living organisms [1]. The technique is non-invasive, harmless, inexpensive, simple to operate. Because it is functional imaging, not structure imaging as X-CT imaging it is, it can detect the physiological characteristics of the internal tissues of the human. Therefore, it can be used to monitor the changes of the diseased tissue because of its non-invasive characteristic [2]. Based on the electrical characteristics of an object, EIT works by injecting a safe current into the electrodes placed on the surface of the object and measuring the electrical signal on the surface to estimate the distribution and changes of the electrical characteristics of the object [2-3]. Studies show that the EIT has a broad application prospect in the area of medical [4]. However, EIT has a disadvantage of low spatial resolution as well as low detection accuracy, which hinder its application in the clinical. In addition, the EIT image reconstruction is a non-linear ill-conditioned inverse problem, and its solution is severely ill-posed. The weak interference or measurement error in the EIT measurement process will lead to a completely different conductivity distribution image [5]. Therefore, in order to obtain a stable solution for the EIT inverse problem, a regularization algorithm is adapted into the image reconstruction algorithm for overcoming the ill-conditioned characteristics of the imaging [6].

The electrical conductivity for different tissues of an organism is different, which is manifested as a clear boundary between different tissues. However, the reconstructed image obtained by the Tikhonov regularization algorithm tends to be smooth, and the boundary between the target area and the background area is blurred, which lead to the image has a low contrast and sharpness and is far from it really is. In reference [7], He et al, improved the Tikhonov regularization algorithm and introduced the variation function as the penalty function term of the regularization algorithm to construct the variation regularization algorithm, which not only overcomes the ill-conditioned characteristics for the EIT inverse problem but also makes the reconstructed image maintain a high contrast and sharpness.
In this article, we first construct a relatively simple arm model that gives a rigorous theoretical derivation of the EIT problem. Then, we use the finite element method to solve the forward problem, and present two different regularization algorithms to overcome the ill-conditioned characteristic for the inverse problem. A proper solution was obtained, and finally the normal arm and the diseased arm images were reconstructed. The results show that the Variation regularized reconstruction algorithm has the lesser position error on the organizational junction edges and is basically overlap with the target area of the model.

2. Mathematical theory and model

We have established a relatively simple arm model. Actually, the human arm model is more complicated that includes skin, fat, muscle, bone cortical and bone marrow. In the simulation model, only three tissues are considered, including muscle, cortical bone and bone marrow \(^8\). The simplified model and the conductivity distributions for these three types of tissues are shown by the left picture in Figure 1. Using the full electrode excitation method, each electrode is evenly placed around the boundary of the 2D model. The target conductivity area is an annular area surrounded by an outer ring with a diameter of 1.5 cm and an inner ring with a diameter of 1 cm. The red area is bone marrow with a conductivity of 0.003 S/m, the blue area is bone cortical with a conductivity of 0.03 S/m, and the rest is muscle tissue with a conductivity of 0.55 S/m \(^9-10\).

The EIT problem can be simplified into a quasi-static electromagnetic field problem, and the quasi-static electromagnetic field satisfies the Maxwell equations

\[ \begin{align*}
J &= \sigma \cdot E \\
\nabla \cdot J &= 0 \\
E &= -\nabla \varphi
\end{align*} \tag{1} \]

Where \( J \) is the current density, \( \sigma \) is the conductivity, \( E \) is the electric field strength, and \( \varphi \) is the electric potential distribution.

Derive the electric potential governing equation from the above formulas

\[ \nabla \cdot [\sigma(x, y)\nabla \varphi(x, y)] = 0 \quad (x, y) \in \Omega \tag{2} \]

Formula (2) satisfies the following two types of boundary conditions.

Dirichlet boundary condition: the boundary electric potential is a known function, namely

\[ \varphi_{\partial \Omega} = \varphi(x, y) = f(x, y) \quad (x, y) \in \partial \Omega \tag{3} \]

Where \( \partial \Omega \) is the boundary of the measurement field \( \Omega \). \( f(x, y) \) is the electric potential function on the boundary.

Neumann boundary condition: the normal component of the boundary current density is known, namely

\[ \sigma(x, y) \frac{\partial \varphi(x, y)}{\partial \vec{v}} \bigg|_{\partial \Omega} = j(x, y) \quad (x, y) \in \partial \Omega \tag{4} \]

In the formula, \( j(x, y) \) represents the function of the known current density, and \( \vec{v} \) represents the external unit normal vector of the field \( \Omega \).
3. Forward and inverse problem

3.1 Forward problem
The forward problem of EIT is to calculate the potential distribution \( \varphi \) with the knowledge of the conductivity distribution \( \sigma \) and the Neumann boundary condition. In general, the geometric models of the solution domain are irregular and conductivity distribution cannot be expressed in an analytical formula [11]. It is hard to get the precise solution of Equation (2). The solution of forward problem can be obtained using the finite element method (FEM). Reference [12] details the FEM.

When it comes to image reconstruction, the inverse problem is determined by the forward problem, and we must first focus on the mesh generation of the solution domain. For a two-dimensional image, there are relatively mature mesh generators to complete regional subdivision. The measured voltage data on the electrodes in the simulation model is obtained by the software COMSOL Multiphysics.

The right picture in Figure 1 shows regional grid generated by the mesh generator, which is composed of 3264 subdivision regions. The regional grid is used to reconstruct the electrical conductivity distribution in the inverse problem.

After the meshing of the domain, the solution of Equation 2 can be converted into a discrete solution using the finite element method.

3.2 Inverse problem
The inverse problem is to estimate the internal conductivity distribution of the solution domain for which the boundary electrodes measured voltages are known. The non-linear relationship between \( \varphi \) and \( \sigma \) can be expressed in the following equation

\[
|h(\sigma)| = h(\sigma)
\]

The least square method is adopted to estimate the conductivity distribution. Newton algorithm is used to solve (5) iteratively [13-14]. The Jacobi matrix is ill-posed because of the existence of the large condition number in the iterative process. The next section tries to overcome the ill-conditioned characteristics for the Jacobi matrix, and to make the solution of the inverse problem well-posed.

4. Method

4.1 Tikhonov regularized algorithm
The Tikhonov regularized norm function of the nonlinear least squares inverse problem is

\[
\sigma(\alpha, L) = \arg \min \| h(\sigma) - d \|^2 + \alpha \| L(\sigma - \sigma_0) \|^2
\]

\( h \) is the nonlinear forward mapping; \( h(\sigma) \) is the calculated electrode potential; \( d \) is the measured electrode potential; \( \alpha \geq 0 \) is the regularization coefficient; \( L \) is the regularization matrix formed by a
specific continuous square function; $\sigma_0$ represents the estimation of the initial value for the model parameter $\sigma$.

4.2 Variation regularized algorithm

The variation regularization algorithm proposed in the reference [15], that is described as a specific form of

$$\sigma(\beta) = \arg \min \|h(\sigma) - d\|^2 + \beta V(\sigma)$$  \hspace{1cm} (7)

In the two-dimensional EIT problem, the field $\Omega \in R^2$ is composed of N elements. According to the definition, the variation function is expressed as the sum of the absolute values of the conductivity differences of all adjacent units. Its mathematical expression is in the form of

$$V(\sigma) = \begin{cases} \sum_{i,j=1}^{N} |\sigma(e_i) - \sigma(e_j)| & e_i, e_j \text{ is adjacent unit} \\ 0 & e_i, e_j \text{ is not adjacent unit} \end{cases}$$  \hspace{1cm} (8)

$V(\sigma)$ is the variation function, the topological structure of finite element division can be used. The element edge correlation matrix $M$ of adjacent elements can be expressed as

$$V(\sigma) = \sum_{i=1}^{N_{edge}} |M_{i}|$$  \hspace{1cm} (9)

This paper applies the modified Newton-Raphson algorithm to solve it. Formally

$$\sigma_{k+1} = \sigma_k + \delta\sigma_k$$

$$\delta\sigma_k = -\frac{\phi'(\sigma_k)}{\phi''(\sigma_k)}$$  \hspace{1cm} (10)

The first derivative and second derivative of norm function $\phi$ are

$$\phi'(\sigma) = J^T(h(\sigma) - d) + \beta M^T P^{-1} M \cdot \sigma$$

$$\phi''(\sigma) = J^T J + \beta \cdot M^T P^{-1} Q \cdot M$$  \hspace{1cm} (11)

$J$ is Jacobi matrix, $P$, $Q$ are diagonal matrices, the element values on the diagonal are

$$P_i = \sqrt{(M_{i}\sigma)^2 + \gamma}$$

$$Q_i = 1 - \frac{(M_{i}\sigma)^2}{(P_i)^2}$$  \hspace{1cm} (12)

$\gamma$ is the coefficient of variation, which tends to zero in the iterative process.

5. Experiments and results

This section presents the image reconstruction results using the introduced two regularized algorithms, namely the Tikhonov regularized algorithm and the Variation regularized algorithm. The normal arm simulation model and reconstructed images are in Figure 2. Figure 3 presents the diseased arm simulation model and reconstructed images.

As we can see, the results obtained using Tikhonov regularization algorithm are smoother on the organizational junction edges. Variation regularized algorithm performs better than Tikhonov regularization in organizational junction identification. Furthermore, the target area of Tikhonov regularization algorithm is deviating from target area of model about one centimetre. By contrast, The Variation regularized algorithm is basically overlap with the target area of the model.
Figure 2. Normal arm simulation model and reconstructed images by using Tikhonov, Variation regularization, respectively.

Figure 3. Diseased arm simulation model and reconstructed images by using Tikhonov, Variation regularization, respectively.

6. Conclusions
Regularization is an effective way to handle the ill-conditioned characteristics of the inverse problem of EIT. In order to improve the quality of reconstructed image, variation regularization algorithm was proposed. By comparison, variation regularization performs better than Tikhonov regularization in junction identification of different conductivity and have less position error. However, the variation regularization algorithm cannot well identify junction and eliminate the artefact.

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