COLLECTIVE MODES IN COLOR SUPERCONDUCTING MATTER

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The properties of plasmons, Nambu-Goldstone bosons and gapless Carlson-Goldman collective modes in color-flavor locked phase of color superconducting dense quark matter at finite temperature are reviewed. A possibility of a kaon condensation with an abnormal number of the NG bosons is also discussed.

1. Introduction

In this talk I review the approach and results of recent studies on collective modes in the color-flavor locked (CFL) phase of dense quark matter at finite temperature. Also, I am going to mention about an interesting possibility of a kaon condensation appearing on top of the CFL phase.

Let me start by noticing that there is a growing belief in the literature that the cores of some compact stars might be made of a color superconducting quark matter. Although this was not proved in any reliable way, such a possibility at least seems to be likely. Indeed, different theoretical studies suggest that the value of the superconducting order parameter could be as large as 10 to 100 MeV at densities just a few times larger than the density of the ordinary nuclear matter. If this is really so, the observed properties of compact stars might show up an indication of a color superconducting state.

The CFL phase of dense quark matter has very interesting properties. Most of them have already been recognized in the original paper. Thus, I am not going to describe them here. For the purposes of this presentation, it would suffice to mention only a few key facts that play a significant role in the analysis.

First of all, it would be appropriate to recall the color-flavor structure of the order parameter. In general, it contains an antitriplet-antitriplet ($\bar{3}, \bar{3}$) and a sextet-sextet ($6, 6$) contribution:

\[
\Delta_{ij}^{ab} = \Delta_{\bar{3}, \bar{3}}^{\alpha \beta} (\delta_i^\alpha \delta_j^\beta - \delta_i^\beta \delta_j^\alpha) + \Delta_{6, 6}^{\alpha \beta} (\delta_i^\alpha \delta_j^\beta + \delta_i^\beta \delta_j^\alpha),
\]

(1)

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where the flavor ($i, j = 1, 2, 3$) and color ($a, b = 1, 2, 3$) indices are explicitly displayed. The dominant contribution to the order parameter is $\Delta_{\bar{3},\bar{3}}$, but a nonzero although small admixture of $\Delta_{6,6}$ always appears.

In the color superconducting phase, there are two types of quark quasiparticles which form octet and singlet representations under the vacuum symmetry group $SU(3)_c + L_R$. Their one-particle spectra develop different energy gaps around the Fermi surface. The values of the gaps are $\Delta_1 = 2(\Delta_{\bar{3},\bar{3}} + 2\Delta_{6,6})$ and $\Delta_8 = -(\Delta_{\bar{3},\bar{3}} - \Delta_{6,6})$, respectively. In the case of a pure $(\bar{3},\bar{3})$ order parameter, these gaps are related as follows: $\Delta_1 = -2\Delta_8$. Without losing generality (unless, of course, $\Delta_1 = \pm \Delta_8$), I consider this relation to hold approximately at both zero and nonzero temperatures, i.e., $\Delta_1 \approx -2\Delta_8 = -2|\Delta_T|$.

2. Strategy

The aim of this study is to reveal the whole class of collective modes in the CFL phase that couple to either vector or axial color currents. This ambitious task is solved (at large densities where the color interaction becomes weak) by studying the locations of poles in the corresponding current-current correlation functions,

\[
\int d^4x e^{iqx} \langle T j^\mu A(x)j^{\nu B}(0) \rangle \equiv \langle j^{\nu B} j^\mu A \rangle_q \sim \sum_n \frac{(\ldots)^{\mu \nu, AB}}{q_0^2 - \Pi^{(n)}(q)},
\]

\[
\int d^4x e^{iqx} \langle T j_5^\mu A(x)j_5^{\nu B}(0) \rangle \equiv \langle j_5^{\nu B} j_5^\mu A \rangle_q \sim \sum_n \frac{(\ldots)^{\mu \nu, AB}}{q_0^2 - \Pi_5^{(n)}(q)},
\]

where the sum runs over all poles. In order to derive such correlation functions it is sufficient to know only their irreducible parts, i.e., the polarization tensors.

In the broken phase, the polarization tensor (related to the vector color current) contains two qualitatively different contributions. One of them is the ordinary one-loop quark contribution, while the other comes from the “would be Nambu-Goldstone (NG) bosons”. This latter contribution is subtle. It appears in all nonunitary gauges as a result of proper handling the gauge invariance of the model. A similar contribution also appears naturally in the effective theory. Diagrammatically, the full expression is presented as follows:

where the double line denotes the would be NG bosons. The corresponding analytical expression was derived based on arguments of gauge symmetry in our recent paper. It reads

\[
\Pi^{(full)}_{\mu \nu}(q) = \Pi_1(q)O_{\mu \nu}^{(1)}(q) + \left( \Pi_{2}(q) + \frac{[\Pi_{4}(q)]^2}{\Pi_{3}(q)} \right) O_{\mu \nu}^{(2)}(q),
\]
where the component functions $\Pi_n(q)$ are extracted from the expansion of the one-loop contribution of quarks to the polarization tensor,

$$\Pi_{\mu\nu}(q) = \Pi_1(q)O^{(1)}_{\mu\nu}(q) + \Pi_2(q)O^{(2)}_{\mu\nu}(q) + \Pi_3(q)O^{(3)}_{\mu\nu}(q) + \Pi_4(q)O^{(4)}_{\mu\nu}(q). \tag{5}$$

The tensors $O^{(n)}_{\mu\nu}(q)$ (two of which are not transverse) are defined as follows:

$$O^{(1)}_{\mu\nu}(q) = g_{\mu\nu} - u_\mu u_\nu + \frac{\vec{q}_{\mu}\vec{q}_{\nu}}{|q|^2}, \quad O^{(2)}_{\mu\nu}(q) = u_\mu u_\nu - \frac{\vec{q}_{\mu}\vec{q}_{\nu}}{|q|^2} - \frac{q_\mu q_\nu}{q^2}, \quad O^{(3)}_{\mu\nu}(q) = q_\mu q_\nu q^2, \quad O^{(4)}_{\mu\nu}(q) = O^{(2)}_{\mu\lambda} u_\lambda q_{\nu} \frac{|q|}{q^2} + \frac{q_\mu}{|q|} u^\lambda O^{(2)}_{\lambda\nu}. \tag{6}$$

with $u_\mu = (1,0,0,0)$ and $\vec{q}_\mu = q_\mu - (u \cdot q) u_\mu$. It is remarkable to notice that the contribution of the would be NG bosons [i.e., $\Pi_2/\Pi_3$ term in Eq. (5)] is defined in terms of the nontransverse components of the quark contribution itself.

The polarization tensor in Eq. (5) determines the (connected) correlation function

$$\langle j^A_{\mu} j_B^\nu \rangle_q = \delta^{AB} \left[ \frac{q^2 \Pi_1}{q^2 + \Pi_1} O^{(1)}_{\mu\nu}(q) + \frac{q^2 [\Pi_2 \Pi_3 + (\Pi_4)^2]}{q^2 + \Pi_2} \frac{(\Pi_4)^2}{O^{(2)}_{\mu\nu}(q)} \right]. \tag{8}$$

The locations of poles in this function define the dispersion relations (as well as screening effects) of the magnetic and electric type collective excitations,

$$q^2 + \Pi_1(q) = 0, \quad \text{“magnetic modes”}, \tag{9}$$

$$[q^2 + \Pi_2(q)] \Pi_3(q) + [\Pi_4(q)]^2 = 0, \quad \text{“electric modes”}. \tag{10}$$

In a similar way, one derives the dispersion relations of the pseudoscalars coupled to the axial color currents, $\Pi_3(q) = 0$. These pseudoscalars are nothing else but the NG bosons associated with breaking of the global chiral symmetry.

### 3. Plasmons and “light” plasmons

Dense quark matter is a non-Abelian plasma. Similarly to an ordinary plasma, the non-Abelian plasma supports collective oscillations of the (color) charge density. By analogy, it is natural to call the corresponding modes plasmons.

Plasmons exist in the normal as well as in the superconducting phase. The gap in their spectrum, usually called plasma frequency ($\omega_p$), is directly related to the quark density in the system. Thus, by considering the limit $q_0 \gg |\Delta_T|$ (large energy) and $|\vec{q}| \to 0$ (long wavelength) in Eqs. (9) and (10), we derive the dispersion relations for both the magnetic and electric modes

$$q_0^2 = \omega_p^2, \quad \text{with} \quad \omega_p = \frac{g_\mu}{\sqrt{2\pi}} \gg |\Delta_T|. \tag{11}$$

As in the case of ordinary metals, the plasmon frequency is essentially unaffected by a small (compared to the chemical potential $\mu$) temperature of the system and by the presence of the superconducting gap. This should have been expected.
Much more interesting are the “light” plasmons that were also discovered in the CFL phase of dense quark matter. These light plasmon modes have no analogues in ordinary metals. Their appearance in the CFL phase seems to be directly related to the presence of two different types of quark quasiparticles having nonequal gaps in their spectra, $|\Delta_1| \neq |\Delta_8|$. In the long wavelength limit, the energy of the light plasmon is always smaller than the threshold of producing a pair of quasiparticles, $|q_0| < 2|\Delta_0|$. At zero temperature, in particular, one derives $q_0 = m_\Delta \approx 1.362|\Delta_0|$. The numerical calculation of the temperature dependence of the light plasmon mass (i.e., the gap in its spectrum) $m_\Delta(T)$ is also available. Qualitatively, $m_\Delta(T)$ is a monotonically decreasing function of temperature which approaches zero as $T \to T_c$. Besides that, in the whole range of temperatures $0 < T < T_c$, it remains larger than $1.362|\Delta_T|$ and smaller than the threshold $2|\Delta_T|$. It is clear, therefore, that light plasmon excitations are stable with respect to decays into quark quasiparticles. It should be mentioned, however, that other decay channels (for example, involving the NG bosons) might eventually lead to a nonzero width. Unfortunately, this question has not been studied yet in detail.

Before concluding this section, I should note that an indication of a light plasmon could also be seen in the derivative expansion. However, the derivative expansion approach (treating the ratios $|q_0|/|\Delta_T|$ and $|\vec{q}|/|\Delta_T|$ as small parameters) overestimates the value of the mass.

4. Nambu-Goldstone bosons

The global chiral symmetry is broken in the CFL phase of dense QCD. This means that an octet of pseudoscalar NG bosons should appear in the spectrum. The dispersion relation of the NG bosons reads: $\Pi_3(q) = 0$. This relation could be analytically studied at small temperatures and in the nearcritical region.

In the first case ($T \ll |\Delta_0|$), the dispersion relation of the NG bosons reads:

$$q_0 = \frac{|\vec{q}|}{\sqrt{3}} \left[ 1 - i \frac{5\sqrt{2}\pi}{4(21 - 8 \log 2)} e^{-\sqrt{3}2|\Delta_0|} \right].$$

Note that this relation reveals an exponentially small imaginary part when $T \to 0$. It is worthwhile to mention, however, that the width of the NG bosons might also get corrections due to the interactions of the NG bosons with one another. Since their decay constant is of the same order as the chemical potential $\mu$, such corrections are expected to be parametrically suppressed by some power of the ratio $T/\mu$.\(^{1}\)

When $T \to T_c - 0$, the dispersion relation reads

$$q_0 = \frac{|\Delta_T|}{T} (\pm x_n^{*} - iy_n^{*}) |\vec{q}|,$$

\(^{1}\)A simple estimate shows that the power low corrections to the imaginary part in Eq. (12) are negligible for $T \gtrsim 0.01|\Delta_0|$. This leaves a window of temperatures, $0.01|\Delta_0| \ll T \ll |\Delta_0|$, where the dispersion relation (12) is qualitatively correct.
where $x_{ng}^* \approx 0.215$ and $y_{ng}^* \approx 0.245$. This result shows that the NG bosons have a large width. This is hardly a surprise in the nearcritical region where the gap in the quasiparticle spectrum is vanishingly small. I would like to point out, however, that the numerical analysis reveals well pronounced peaks in the NG boson spectral density and the peak locations scale with momentum in accordance with a linear dispersion law.

5. Carlson-Goldman collective modes

The so-called Carlson-Goldman (CG) gapless modes were experimentally discovered by Carlson and Goldman about a three decades ago. One of the most intriguing interpretation connects such modes with a revival of the NG bosons in the superconducting phase where the Anderson-Higgs mechanism should commonly take place. In the two fluid description, the CG modes are seen as oscillations of the superfluid and the normal components in opposite directions. The local charge density remains zero in such oscillations, providing favorable conditions for gapless modes.

The CG modes can only appear in the vicinity of the critical temperature (in the broken phase), where a large number of thermally excited quasiparticles leads to partial screening of the Coulomb interaction, and the Anderson-Higgs mechanism becomes inefficient. As a result, such modes could exist only in a finite (possibly very small) vicinity of the critical temperature. Because of the Landau damping induced by the quasiparticles, however, the CG modes could easily become overdamped in the nearcritical region. In order to make them observable, one should use dirty systems in which scattering of quasiparticles on impurities tends to reduce the damaging effect of Landau damping.

In the nearcritical region, the dispersion relation of the CG collective modes in the CFL phase could be determined analytically. It is remarkable that the result closely resembles the dispersion relation of the NG bosons in Eq. (13),

$$q_0 = \frac{|\Delta T|}{T} (\pm x_{cg}^* - iy_{cg}^*) |q|, \quad (14)$$

where $x_{cg}^* \approx 0.193$ and $y_{cg}^* \approx 0.316$. As in the case of the NG bosons, the width of the CG modes is quite large. It should be emphasized here that the above result was obtained in the clean (no impurities) limit of dense quark matter. Notice that some impurities might come to game if the phase transition in temperature is a weak first order one. In this case, the bubbles of the new phase may play the role of impurities. Of course, this assumes that the transition is sufficiently weak, so that the value of the gap would have a chance to get quite small on the broken side (say, less than about 0.05$T_c$).

As in the case of the NG bosons, the numerical analysis of the electric gluon spectral density reveals distinguishable (although less pronounced than in the case of the NG bosons) maxima whose locations scale with momenta in accordance with a linear dispersion law.
6. Nambu-Goldstone bosons in phase with kaon condensation

While the CFL phase is the ground state of the three flavor dense quark matter in the chiral limit, this may not be the case when the quarks have nonzero masses. By making use of an auxiliary gauge symmetry, it was recently suggested that the originally derived low-energy action should be modified. In Minkowski space, the corresponding Lagrangian density should read

\[ L_{\text{eff}} = \frac{f^2}{4} \text{Tr} \left[ \nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2_\pi \partial_0 \Sigma \partial_0 \Sigma^\dagger \right] + \frac{1}{2} \left[ (\partial_0 \eta')^2 - v^2_\eta' (\partial_0 \eta')^2 \right] \]

\[ + 2c \left[ \det(M) \text{Tr} \left( M^{-1} \Sigma e^{\sqrt{\frac{2}{3} i f^2_\pi}} \right) + h.c. \right], \quad (15) \]

where, by definition, \( I \) is a unit matrix in the flavor space, and \( \Sigma \) is a 3 \times 3 unitary matrix field which describes an octet of the NG bosons. In the action, we took into account the \( \eta' \) field which couples to the NG octet when the quark masses are nonzero. At the same time, the NG boson, related to breaking the baryon number, was omitted in Eq. (15). Its dynamics is not affected much by the quark masses. The covariant time derivative in Eq. (15) is defined in terms of the quark mass matrix as follows:

\[ \nabla_0 \Sigma = \partial_0 \Sigma + \frac{1}{2} M \Sigma M^\dagger \Sigma - i \Sigma M^\dagger M, \quad \text{with} \quad M = \text{diag}(m_u, m_d, m_s). \quad (16) \]

As is clear from this definition, the quark masses produce effective chemical potentials for different flavors. In the case of a realistic hierarchy of the quark masses with \( m_s \gg m_u \approx m_d \), these chemical potentials may trigger a rearrangement of the CFL phase. In particular, when the value of strange quark mass is large enough so that the condition \( m_s > \sim (\Delta^2 m_u)^{1/3} \) is satisfied, the CFL vacuum becomes unstable with respect to a kaon condensation. The new ground state is determined by a “rotated” vacuum expectation value of the \( \Sigma \) field,

\[ \Sigma_\alpha \equiv \exp \left( i \alpha \lambda^6 \right) \exp \left( i \pi A \lambda^4 \right) \simeq \exp \left( i \alpha \lambda^6 \right) \left( 1 + \frac{i \pi A \lambda^4}{f_\pi} - \frac{\pi A \lambda^4}{2 f_\pi^2} + \ldots \right). \quad (17) \]

The value of the angle \( \alpha \) is determined from the condition that the vacuum expectation value in Eq. (17) corresponds to a global minimum of the potential energy,

\[ \cos \alpha = \frac{4 c p_f^2 m_u (m_s + m_d)}{f_\pi^2 (m_s^2 - m_d^2)} < 1, \quad (18) \]

where

\[ c = \frac{3 \Delta^2}{2 f^2_\pi}, \quad \text{and} \quad f^2_\pi = \frac{21 - 8 \ln 2}{36} \frac{\mu^2}{\pi^2}. \quad (19) \]

When the isospin symmetry is exact (i.e., \( m_u = m_d \)), the kaon condensation breaks the \( SU(2) \times U(1)_Y \) symmetry of the effective action down to \( U(1)' \). This symmetry breaking pattern (with three broken generators of a global symmetry)
Collective modes in color superconducting matter might suggest that three NG bosons should appear in the low energy spectrum of the system. The direct calculation shows, however, that only two NG bosons appear. In order to understand this seeming paradox better, we consider a toy model that closely mimics the dynamics of the kaon condensation. The Lagrangian density of the toy model reads

\[ \mathcal{L} = (\partial_0 + i\mu)\Phi^\dagger(\partial_0 - i\mu)\Phi - v^2\partial_i\Phi^\dagger\partial_i\Phi - m^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2, \]  

where \( \Phi^T = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2, \tilde{\phi}_1 + i\tilde{\phi}_2) \) is a complex doublet field, replacing four kaon degrees of freedom, and \( v \) is a velocity parameter (\( v < 1 \)). The parameter \( \mu \) corresponds to the effective chemical potential of strangeness in the realistic model.

It is straightforward to derive the spectrum of excitations in the toy model in the normal phase which is realized when \( \mu < m \). They are

\[ \omega_{K^0}(q) = \omega_{K^+}(q) = \sqrt{m^2 + v^2q^2} - \mu, \]  

\[ \omega_{K^0}(q) = \omega_{K^-}(q) = \sqrt{m^2 + v^2q^2} + \mu, \]  

where we explicitly identified the four degrees of freedom of \( \Phi \) with the kaons.

When the chemical potential becomes larger than the mass parameter, \( \mu > m \), the vacuum configuration with \( \langle \Phi \rangle = 0 \) is no longer a local minimum of the potential energy. This clearly indicates that the system develops an instability with respect to forming a condensate. In the true ground state, the field \( \Phi \) has a nonzero vacuum expectation value which, up to a global transformation, is

\[ \langle \Phi \rangle = (0, \phi_0)^T \text{ and } \phi_0^2 = \frac{\mu^2 - m^2}{2\lambda}. \]  

This condensate breaks the initial \( SU(2) \times U(1) \) symmetry group of the toy model down to \( U(1)' \). It is interesting to study the spectrum of excitations in this broken phase. The four degrees of freedom of the complex doublet \( \Phi \) now have the following dispersion relations:

\[ \omega_{1,2} = \sqrt{\mu^2 + v^2q^2} \pm \mu, \]  

and

\[ \tilde{\omega}_{1,2} = \sqrt{3\mu^2 - m^2 + v^2q^2} \pm \sqrt{(3\mu^2 - m^2)^2 + 4\mu^2v^2q^2}. \]  

Only one of the relations in Eq. (24) and only one of the relations in Eq. (25) correspond to gapless excitations. In the far infrared region, the corresponding relations take the following forms:

\[ \omega_2 \simeq \frac{v^2q^2}{2\mu}, \quad \text{and} \quad \tilde{\omega}_2 \simeq \sqrt{\frac{\mu^2 - m^2}{3\mu^2 - m^2}}vq. \]  

Thus, there are only two gapless NG bosons in the low energy spectrum of the model. The other two excitations have gaps \( m_1 = 2\mu \) and \( \tilde{m}_1 = \sqrt{2(3\mu^2 - m^2)}. \)
Here one should notice that the existence of only two gapless states should have been expected based on a simple argument of continuity of the spectrum at the critical point $\mu = m$. By approaching this point from the side of the normal phase, we see from Eqs. (21) and (22) that only two out of the total four states become gapless as $\mu \to m - 0$.

To avoid a possible confusion, it is instructive to mention that the effective potential has three flat directions in the vicinity of the vacuum configuration, as it should in the case of the symmetry breaking $SU(2) \times U(1) \to U(1)'$. It is the first order derivative terms in the kinetic part of the action that prevent the appearance of the third gapless mode. The same first order derivative terms break the Lorentz invariance (and, maybe even more important, the discrete $C$, $CP$ and $CPT$ symmetries) of the model and the strong version of the Goldstone theorem cannot be applied. In non-relativistic physics, the known examples of systems with an abnormal number of the NG bosons are ferromagnets and the superfluid $^3$He in the so-called A-phase.

As is seen from Eq. (26), the two NG bosons have qualitatively different dispersion relations. One of them has a quadratic dispersion relation, while the other has a linear dispersion relation. The existence of the NG boson with a quadratic relation has an immediate implication — it prevents superfluidity in the system. Indeed, according to the Landau criterion, the superfluidity is possible only for flow velocities $v < v_c$, where $v_c = \min_i \omega_i(q)/q$ is the minimum taken over all excitation branches and all values of momentum. The presence of even a single branch with $\omega \sim q^2$ implies that $v_c = 0$ and, therefore, that there is no superfluidity.

The toy model illustrates a general phenomenon of spontaneous breaking of a global symmetry ($G \to H$) in systems with a broken Lorentz symmetry. In particular, the number of NG bosons can be smaller than the number of the generators $N_{G/H}$ in the coset space $G/H$. In the case of dense quark matter with the kaon condensation, in particular, half of the charged candidates to NG bosons have gaps in their energy spectra as a result of splitting produced by an effective chemical potential. The neutral NG boson candidates do not feel the chemical potential and, thus, remain gapless. This observation allows us to propose the following counting rule: the number of the gapless NG bosons equals to $N_{NG} = N_{G/H} - N_{ch}$, where $N_{ch}$ is the number of charged particle-antiparticle pairs among the NG boson candidates. In model (20), $N_{G/H} = 3$ and $N_{ch} = 1$, so that $N_{NG} = 2$.

Because the isospin symmetry is not exact in the real world ($m_u \neq m_d$), the effective action (17) has only $U(1) \times U(1)'$ symmetry. The kaon condensate breaks this symmetry down to $U(1)'$. As a result, only one NG boson (with a linear dispersion relation) appears in the low energy spectrum. The corresponding phase of matter is expected to be superfluid since there are no excitations with the quadratic dispersion relations. The approximate isospin symmetry could reveal itself only through the appearance of a light excitation, replacing the NG boson with the quadratic relation, see Eq. (26). When the difference between up and down quark masses is very small, this light excitation could considerably reduce the critical value.
of the superfluid velocity.

7. Conclusion and Outlook

The study of a large class of collective modes, reviewed in this talk, gives a rather complete description of physical degrees of freedom in the CFL phase of dense quark matter at zero and finite temperatures. Some of the modes (light plasmons and the gapless CG modes) have very interesting properties and have no analogues in other phases. Potentially this may play an important role in producing a key signal of a color superconducting phase in nature. With this in view, it would be of great interest to see, for example, whether the existence of the gapless scalar CG modes could affect any thermodynamical or transport properties of dense quark matter. And, in its turn, whether this could have any profound effect on the evolution of forming compact stars.

In the case of nonzero quark masses, it is possible that the CFL phase is modified due to the appearance of the kaon condensate. The general properties of collective excitations in such an exotic phase were also briefly addressed in this presentation. This new state of dense matter gives an interesting example of a system in which the number of the NG bosons should not necessarily be equal to the number of the broken generators. Related to this is the possibility of excitations with (nearly) quadratic dispersion relations. The latter, if appear, may have interesting observable implications. Indeed, as is known from statistical physics, the thermodynamical quantities such as specific heat are quite sensitive to the details of the low energy spectrum.

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References

1. V.P. Gusynin and I.A. Shovkovy, Phys. Rev. D 64 (2001) 116005; hep-ph/0108173, to appear in Nucl. Phys. A. 700 (2002).
2. P. F. Bedaque and T. Schafer, Nucl. Phys. A 697 (2002) 802.
3. D. B. Kaplan and S. Reddy, hep-ph/0107265.
4. V.A. Miransky and I.A. Shovkovy, hep-ph/0108173.
5. T. Schafer, D. T. Son, M. A. Stephanov, D. Toublan and J. J. Verbaarschot, hep-ph/0108210.
6. M. Alford, K. Rajagopal, and F. Wilczek, Phys. Lett. B 422 (1998) 247; R. Rapp, T. Schafer, E.V. Shuryak, and M. Velkovsky, Phys. Rev. Lett. 81 (1998) 53.
7. R.D. Pisarski and D.H. Rischke, Phys. Rev. Lett. 83 (1999) 37.
1. A. Shovkovy

8. D.T. Son, Phys. Rev. D 59 (1999) 094019.
9. T. Schäfer and F. Wilczek, Phys. Rev. D 60 (1999) 114033; D.K. Hong, V.A. Miransky, I.A. Shovkovy, and L.C.R. Wijewardhana, Phys. Rev. D 61 (2000) 056001; R.D. Pisarski and D.H. Rischke, Phys. Rev. D 61 (2000) 074017.
10. M. Alford, J. A. Bowers, and K. Rajagopal, J. Phys. G 27 (2001) 541.
11. G.W. Carter and S. Reddy, Phys. Rev. D 62 (2000) 103002; D.K. Hong, S.D. Hsu and F. Sannino, Phys. Lett. B 516 (2001) 362.
12. M.G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B 537 (1999) 443.
13. I.A. Shovkovy and L.C. Wijewardhana, Phys. Lett. B 470 (1999) 189.
14. T. Schäfer, Nucl. Phys. B 575 (2000) 269.
15. V.A. Miransky, I.A. Shovkovy, and L.C.R. Wijewardhana, Phys. Rev. D 62 (2000) 085025; *ibid.* 63 (2001) 056005.
16. G.W. Carter and D. Diakonov, Nucl. Phys. B 582 (2000) 571.
17. R. Casalbuoni, R. Gatto and G. Nardulli, Phys. Lett. B 498 (2001) 179.
18. R.V. Carlson and A.M. Goldman, Phys. Rev. Lett. 34 (1975) 11; J. Low Tem. Phys. 25 (1976) 67.
19. A. Schmid and G. Schön, Phys. Rev. Lett. 34 (1975) 941.
20. S.N. Artemenko and A.F. Volkov, Sov. Phys. JETP 42 (1975) 1130; Sov. Phys. Usp. 22 (1979) 295.
21. Y. Ohashi and S. Takada, J. Phys. Soc. Japan 66 (1997) 2437; *ibid.* 67 (1998) 551.
22. R. Casalbuoni and R. Gatto, Phys. Lett. B 464, 111 (1999).
23. D. T. Son and M. A. Stephanov, Phys. Rev. D 61, 074012 (2000); *ibid.* 62, 059902 (2000).
24. J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. 127 (1962) 965.
25. C.P. Hofmann, Phys. Rev. B 60 (1999) 388.
26. G.E. Volovik, *Exotic properties of superfluid $^3$He,* (World Scientific, 1992) pp. 119-121.
27. E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part 2,* (Pergamon, 1980) pp. 88-93.