The effects of a multiple solution method in mathematics learning for secondary schools

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Abstract. Solving problems in mathematics does not involve a single method only. Using a multiple solution method can also assist students in seeking a solution. A multiple solution method is a method that can be applied in the process of teaching and learning in which students will learn a variety of methods to be applied in finding solutions for a single question. This paper reports a study conducted to review students’ views of the effects of the use of a multiple solution method in learning mathematics. The study used two approaches: the quantitative approach (Section A, B and C) and qualitative approach (Section D), in a questionnaire for collecting data. 100 students from a secondary school located in Klang, Selangor were selected as respondents in this study. The questionnaire focused on the perceptions of students in using a multiple solution method and consequently the effect towards students’ level of self-regulation. The Likert scale ranging from 1 (strongly disagree) to 4 (strongly agree) was used. All respondents were also instructed to indicate the challenges faced in the use of a multiple solution method in the open-ended questions. The results showed that students have a good perception on the use of a multiple solution method in mathematics learning. A multiple solution method can also affect students’ self-regulation. The findings of the open-ended questions show that which are the challenges faced by students when use multiple solution method are get confused because it has many methods, it is hard because students lack mathematics skill, and also involve time constraints because they need to memorize many methods.

1. Introduction

Solving mathematical problems varies according to the situation such as in learning mathematics or in a real-life. Students can use a multiple solution method in finding the solution whether it is one or more problems. However, what is a multiple solution method? Generally, the multiple solution method means arriving at a solution by using more than one method. In mathematics learning, a learner needs to present a variety of methods in order to solve a problem. He or she can present the solution using an equation, graph, table, and many more [1]. The multiple solution method actually can improve students’ mathematical knowledge. When students try to compare the solution with their friends who come out with different ways to solve the problem, students actually increase their understanding [2] and by
comparing different solution methods for solving the same problem also will improve mathematics learning [3]. There are many ways to solve the problem but sometimes when students are only on one way solution, students do not have another option to solve the problem. Most of the problems happen when the students present a short solution or only a few steps of the solution, so the selection of student work is generally straightforward [4]. When students are focussed on a single method, students lack flexibility. It is because students cannot present any ideas that are related to find the solution of the problem [5]. The education system is not to be blamed on the one solution method in solving the problem, but if students have the opportunity to solve in many ways, it can help students to be more flexible. How do they become more flexible in finding the solution? If students use multiple solution method, will it give a good effect in their learning process? These issues will affect the learning skills of the students in learning mathematics.

2. Literature review

The perception of multiple solution is defined as students’ feedback about the use of multiple solution method in learning mathematics. Each person has his or her own perception towards multiple solution method. In using the multiple solution in teaching and learning process, it can increase the students’ motivation [6]. By applying the multiple solution method, students were motivated to develop more than one solution on the same problem where students are not too focus only in single solution. Besides that, students get motivated in their learning because there are some favourable conditions for students when they use multiple solutions in the classroom [7]. When the teacher has a deep understanding of the contents by topic they are teaching, it can grab students’ attention [8]. Learning from multiple solution methods fosters the understanding of solution methods, especially when the learning is supported by self-explanation or by the instructional explanation which compare different solutions [9]. According to Schukajlow and Krug in 2012, not all students are using the multiple solution method in the learning. 58% apply two or more solutions and it showed that if the teacher encourages their students to apply more than one method, as most of the students will do it. Students promote higher self-regulation of their learning when they prompted to develop multiple solution method [10]. There are positive effects of encouraging students to find various methods of solution and competence between their friends during the learning process as well as the students’ interest [11].

Students’ self-regulation is a contributor of mathematics success learning because it relates to the factor of study environment regulation and seeking of help from friends and teacher [12]. Those students who can regulate their learning are proposed to gain the most out of education because their actions are motivated by learning rather than external rewards [13]. The same goes with other research [14] where the researchers also found that when students work with others, it contribute to students’ level of self-regulation. Based on the previous study in 2017, the study stated that there was no difference in students’ self-regulatory from the different group where the researcher just pick two group classes with an equal number of students [15]. It can be proved by Leidinger and Perels who found that the achievement from students in grade four increased [16].

The multiple solution method also has challenges in implementation. When students try to apply the multiple solution method in their learning, they can get confused to apply the best method to the question. Students are stuck to apply the multiple solution method because they cannot understand the meaning of some terms of the questions, and it was also difficult to develop the mathematical model into the problem. When students cannot understand what the questions was about, not only multiple solution method but students cannot apply even single method when they get confused [10]. Students lacked mathematics skill such as problem solving, differentiate the question task are the difficulties students faced when learning mathematics. Furthermore, when students are afraid to ask, it can lead them to not attempt answering questions [17]. To add to this view, this study aims to identify students’ perception using multiple solution method in mathematics learning. Besides that, it determines the effect of multiple solution method towards self-regulation and the challenges faced by students.
3. Research methodology
This study involved both approaches which are the qualitative approach and quantitative approach. This study used a descriptive design for the quantitative approach. The quantitative approach subscribes to a specific empirical approach to knowledge, trusting that by measuring precisely enough we can make claims about the objective of study [18]. The quantitative approach was applied to identify students’ perception using the multiple solution methods in mathematics subject and to determine the effect of multiple solution method towards self-regulation. As for the qualitative approach, data were collected by using open-ended questions in order to examine the challenges of using the multiple solution method. The target population of this study were 100 respondents from a secondary school, located in Klang, Selangor.

This study analyses using a non-probability sampling which is convenience sampling to select the respondents. Before questionnaires had been distributed to respondents, the pilot study was tested with the 10 respondents. SPSS was used to check the reliability of the questionnaire. Table 1 shows the result of pilot study where the value of Cronbach’s Alpha is greater than 0.7. For Section A, it only focus on the demographic part consist of 2 items which are race and class of respondents. After questionnaire was distributed to respondents, they need to tick the box that are relevant to them. The aim for section B is to identify students’ perceptions using the multiple solution method in mathematics subject. In this part, it consists of nine related items. Next, Section C is to determine the effect of multiple solution methods toward self-regulation. It consist of 5 items. For Section B and Section C, the respondents need to rate honestly for each item in scale from 1 to 5 where 1= strongly disagree, 2= disagree, 3= neutral, 4= agree and 5= strongly agree. Lastly, Section D consists of the open-ended questions which was the respondents need to answer the question ‘What are the challenges on using the multiple solution methods? and it will classified the respondents answer into several categories to find the differences and similarities of patterns to answer the questions where the challenges faced by students when using the multiple solution method.

| Table 1. Pilot study results. |
|-----------------------------|
| Cronbach’s Alpha | N of Items |
| .948 | 16 |

4. Findings and discussion

4.1. Section A: the demographic part
The finding for section A, the demographic part consists of race and class as shown in table 2 and table 3.

| Table 2. The distribution of the respondents based on race. |
|----------------|
| Race | Frequency | Percentage (%) |
| Malay | 26 | 26.0 |
| Indian | 41 | 41.0 |
| Chinese | 33 | 33.0 |
| Total | 100 | 100.0 |

It can be seen that, out of 100 respondents, 26 (26%) respondents are Malay students, 41 (41%) are Indian students and 33 (33%) are Chinese students.
Table 3. The distribution of the respondents by class.

| Class        | Frequency | Percentage (%) |
|--------------|-----------|----------------|
| 4 Charismatic| 13        | 13.0           |
| 4 Optimistic | 20        | 20.0           |
| 4 Noble      | 9         | 9.0            |
| 4 Efficient  | 7         | 7.0            |
| 4 Nifty      | 36        | 36.0           |
| 4 Tolerance  | 15        | 15.0           |
| Total        | 100       | 100.0          |

It can be seen in table 3, out of 100 respondents, the highest respondents came from 4 Nifty which is 36 (36%) respondents, then followed by 4 Optimistic which is 20 (20%) respondents. Next, 15 (15%) respondents are from 4 Tolerance and 13 (13%) respondents are from 4 Charismatic. Furthermore, 9 (9%) respondents are from 4 Noble and the lowest numbers of respondents are from 4 Efficient which is 7 (7%) respondents.

4.2. Section B: students’ perception using multiple solution method

The finding for Section B is shown in table 4. The overall results for students’ perception using the multiple solution methods in mathematics subject (Mean=4.0078, SD= 0.64179). Most of the respondents think that using multiple solution method can increase their knowledge since it gives the highest good perception of using the multiple solution method in mathematics learning (Mean= 4.24, SD= 0.842). Other than that, they think it is good to use the multiple solution method than the single method because they can understand better about the solution and relate it with the problem as well as perform better when using the multiple solution method. In addition, they also think it is easy to find the answer using the multiple solution method. However, they have a moderate perception towards the presentation of the multiple solution method foster learning outcome compared to the presentation of single solution method and it give the lowest mean of respondents’ perception (Mean= 3.94, SD= 1.013).

Table 4. The distribution of the respondents perception using multiple solution method in mathematics learning.

| Items                                                      | N   | Mean | Std. Deviation |
|------------------------------------------------------------|-----|------|----------------|
| I like to learn using multiple solution method.             | 100 | 4.05 | .914           |
| I found it is easy to find the solution using multiple solution method. | 100 | 4.03 | .937           |
| I feel confident when I use multiple solution method in my learning. | 100 | 4.10 | .990           |
| I can perform better when I use multiple solution method.   | 100 | 4.05 | .821           |
| I understand more when I learn multiple solution method than one solution method. | 100 | 4.12 | .879           |
| I prefer to use multiple solution methods than single solution method. | 100 | 4.03 | .948           |
| It increase my knowledge when I learned multiple solution method. | 100 | 4.24 | .842           |
Learning with multiple solution method is effective than single solution method. The presentation of multiple solution method foster learning outcome compared to the presentation of only one single solution method. Overall

Next, to determine the significant difference between classes on perceptions of using multiple solution methods in mathematics subject. The hypothesis to be tested is:

$H_0$: There is no significant different between classes on perceptions of using the multiple solution methods in mathematics subject.

$H_1$: There is significant different between classes on perceptions of using the multiple solution methods in mathematics subject.

As shown in figure 1, students from 4 Noble (Mean= 4.321) show a good perception of using the multiple solution method. Then, follow by 4 Charismatic (Mean= 4.214), 4 Efficient (Mean = 4.190) and 4 Nifty (Mean= 4.167). Then, other two classes give a moderate perception of using the multiple solution methods which are 4 Optimistic (Mean=3.878), 4 Tolerance (Mean= 3.815). Therefore, ANOVA test was conducted to determine the significant different.

**Table 5.** The ANOVA test of the perceptions of using multiple solution methods in mathematics subject by class.

|                      | Sum of Squares | df | Mean Square | F     | Sig. |
|----------------------|----------------|----|-------------|-------|------|
| Between Groups       | 2.983          | 5  | .597        | 1.484 | .202 |
| Within Groups        | 37.795         | 94 | .402        |       |      |
| Total                | 40.778         | 99 |             |       |      |
The F test as shown in table 5 showed the significant difference in the perceptions of using the multiple solution methods in mathematics subject by class of respondents (F(5, 94)=1.484, p-value=0.202 > 0.05). Thus, this simply indicates that we accept H₀, there is no significant difference between classes on perceptions of using the multiple solution methods in mathematics subject.

4.3. Section C: the effect of multiple solution methods toward self-regulation

The finding for Section C is shown in table 6. The overall results for the effects of the multiple solution methods (Mean=3.7260, SD=0.60879) in table 6 indicate a moderate mean towards self-regulation. Most of the respondents give a moderate scale of the effect of the multiple solution method towards self-regulation. The respondents accept that having a flexibility in solving the problems in mathematics are the most effect of multiple solution method since it gives the highest mean towards self-regulation (Mean=3.92), followed by reflection on various solution method help them be more flexible and effective. Other than that, the respondents also have a moderate mean on constructing the multiple solution method can improve self-regulation. Last but not least, not all of the respondents compare their work with friends, and it give the lowest mean of effect of this method towards self-regulation.

| Table 6. The distribution of effects of multiple solution method towards self-regulation. |
|---------------------------------|------|------|
| Items                           | N    | Mean | Std. Deviation |
| I have my own way to solve the problems in mathematics | 100  | 3.70 | .980 |
| I feel more flexibility when I use multiple solution method in solving the problems in mathematics | 100  | 3.92 | .825 |
| Reflecting on various solution method help me to apply multiple solution method more flexibly and effectively | 100  | 3.87 | .981 |
| I will compare my work with my friend whose use different method | 100  | 3.30 | 1.251 |
| Constructing multiple solution method improve my self-regulation | 100  | 3.84 | .982 |
| Overall                         | 100  | 3.7260 | .60879 |

Next, to determine the significant difference between Classes on the effects of the multiple solution methods towards self-regulation in mathematics subject. The hypothesis to be tested is:

H₀: There is no significant difference between Classes on the effects of the multiple solution method towards self-regulation in mathematics subject.

H₁: There is significant difference between Classes on the effects of the multiple solution method towards self-regulation in mathematics subject.
Figure 2. The descriptive statistics on the effects of multiple solution method towards self-regulation.

As shown in figure 2, the highest mean of respondents on the effects of the multiple solution method towards self-regulation by class was 4 Noble (Mean= 4.067) where it shows a good mean from 4 Noble students. Then, follow by 4 Efficient (Mean = 4.029) also gives a good response on the effects of multiple solution method towards self-regulation. Then, other four classes give a moderate perception of using multiple solution methods which are 4 Optimistic (Mean=3.88), 4 Charismatic (Mean= 3.785), 4 Tolerance (Mean = 3.64), and the mean for 4 Nifty (Mean= 3.511) was the lowest mean. Therefore, ANOVA test was conducted to determine the significant difference.

Table 7. The ANOVA test of effects of multiple solution method towards self-regulation by classes.

| Sum of Squares | df | Mean Square | F       | Sig. |
|----------------|----|-------------|---------|------|
| Between Groups | 3.978 | 5 | .796 | 2.286 | .052 |
| Within Groups | 32.715 | 94 | .348 |       |      |
| Total          | 36.692 | 99 |       |      |      |

The F test as shown in table 7 showed significant difference effects of the multiple solution method towards self-regulation by classes of respondents [ F (5, 94) = 2.286, p-value = 0.052 > 0.05]. This simply indicates that we accept H0 which are there is no significant difference between Classes on the effects of the multiple solution methods towards self-regulation in mathematics subject. Although it is done in previous study on the effect of the multiple solution method towards self-regulation, but comparison on the effect between classes is still in its infancy.

4.4. Section D: open-ended questions ‘what are the challenges on using the multiple solution methods?’

In order to answer the research question in Section D, the challenges on using the multiple solution method, open-ended questions are conducted. 100 students are selected as respondents. The respondents
are represented as 1, 2, 3… and so on. Each respondent answered to 1 main question that relate with research question that represent the challenges that faced by respondent when applying multiple solution method in their learning. There are many responses from the respondents that researcher collected. Most of the respondents show different challenges such as confusing, hard, time constraint, and others. Below are the feedback the researcher gathered from the respondents.

4.4.1 Confusing
From the study, it can be founded that 32 (32%) of respondents think using the multiple solution in mathematics subject are complicated. This challenges are the main challenge in this study. The followings are the some of the feedback of respondents, based on their confusion:

“I somehow mix up all the formula”
(Respondent 4)

“Cannot remember the ways to solve and sometime we may confuse ourselves with other questions solution”
(Respondent 5)

“It might be wrong and confusing”
(Respondent 6)

“I will get confused easily …”
(Respondent 11)

“It can get confusing at times while learning”
(Respondent 20)

“Don’t know what type of formula to apply for which type of questions”
(Respondent 22)

“Sometimes it is very confusing for me to solve the problem in mathematics”
(Respondent 25)

“keliru dengan soalan pelbagai cara”
(Respondent 39)

“I can understand easily and I can choose any on the ways to solve my problem. Sometimes, it’s confusing and I have to remember a lot of ways “”
(Respondent 41)

“Cabaran yang saya hadapi ialah saya sering keliru akan kaedah pelbagai cara”
(Respondent 55)

“Will be very confusing at first …”
(Respondent 63)

“The challenges on using multiple solution method are to understand problems in a better way”
(Respondent 75)

“There are different types of answers in each multiple solution method so I usually get confused”
(Respondent 88)

“It is more effective but sometime I will get confused”
(Respondent 89)

In conclusion, respondents stated by applying the multiple solution method in mathematics subject makes them more confused regarding solving the problem, finding an appropriate way to apply. Therefore, the respondents can have misconceptions of the terms of the task.

4.4.2 Difficulty
24 (24%) respondents agreed that it was hard to apply the multiple solution method. The respondents blamed the question that it was hard and it was also hard to apply the multiple solution method. The followings are the some of the feedback of the respondents, based on their difficulties:

“Not easy to solve the problem in mathematics”
“Cabaran yang saya hadapi kebanyakannya adalah susah dan mencabar”
(Respondent 1)

“Amat susah sebab kaedah itu mempunyai pelbagai…”
(Respondent 27)

“Terlalu banyak soalan susah”
(Respondent 45)

“Some question are difficult so that I must refer the topic back…”
(Respondent 51)

“…to understand the problem in a better way”
(Respondent 73)

“Banyak soalan mencabar”
(Respondent 74)

“Usually I just use one solution only”
(Respondent 96)

As the conclusion, most of the respondents cannot differentiate the difficulty of questions and the
difficulty of multiple solution method. The multiple solution method was the technique to solve the
problem, but the difficulty of the question needs a solution to solve the problem. Therefore, the
respondents show that they lack in mathematics skills.

4.4.3 Time constraint
From the study, 12 (12%) respondents think using the multiple solution in mathematics subject have
time constraint. The followings are the some of the feedback of respondents, based on time constraint:

“If I want to understand more, I need to practice a lot to make me become more understand”
(Respondent 15)

“... Need to memorise and keep doing revision on it”
(Respondent 18)

“I need to do a lot of exercise”
(Respondent 43)

“…need more time to understand and do more exercise to improve”
(Respondent 62)

“Some questions are not straight forward questions, so we take longer time for answering the question
by using multiple solution method”
(Respondent 72)

“I need more time to really understand all the method as it is hard for me to memorize every method”
(Respondent 95)

In conclusion, the respondents stated that when applying a multiple solution method in the
mathematics subject, they need more time for memorizing the formula, understanding the problem and
applying a suitable method, and doing the exercise. Therefore, the respondents have time constraint of
the terms of the task.

4.4.4 Other
32(32%) of respondents think that using a multiple solution method makes the task easier to understand.
It was opposite of this study where the researcher want find the challenges faced by the respondents.
The followings are the some of the feedback of respondents, based on their feedback:

“...easier method to solve the problem”
(Respondent 10)
"We are able to use any methods during our examination if we tend to forget the other method. The more methods we learn, we are able to do our exam with more confident”

(Respondent 23)

“Saya berasa selesa dengan menggunakan kaedah ini”

(Respondent 29)

“Amat senang sebab ianya lebih mudah dari kaedah sebelum ini”

(Respondent 28)

“It is easy to get answer”

(Respondent 37)

“saya dapat meningkatkan skills matematik dan mudah menyelesaikan soalan matematik”

(Respondent 40)

“It is very easy and simple for me to answer”

(Respondent 46)

“It’s easy for me to learn this multiple solution method”

(Respondent 71)

“It is easy to get answer and to understand the method”

(Respondent 91)

Generally, the challenges that they faced are confusing, hard, time constraint and others like easy and understand better. Most students are still confused to apply a solution method to the set of questions. The major challenge that students faced in using multiple solution method in mathematics learning was what they perceived as confusing. This finding is aligned with Schukajlow & Krug’s study where students are confused to apply the multiple solution method because students cannot understand the meaning of some terms of the questions, and they found it difficult to develop the mathematical model into a problem [10]. Next, the finding also found that it is hard to apply multiple solution method because students lack skills in solving the problem. This finding is similar with Tambychik & Meerah, who discovered that students lacked in mathematics skill such as problem solving and differentiating the question task [17]. Other than that, respondents stated that when applying the multiple solution method in the mathematics subject, they need extra time for memorizing the formula, understanding the problem and applying the suitable method, and doing the exercise. Therefore, the respondents also face time constraint in terms of the finishing the task.

5. Conclusion

As a conclusion, the multiple solution method has potential use in mathematics learning. The respondents give a positive feedback about using it to solve mathematics problems. Other than that, students also give a positive feedback towards the effects of the method towards self-regulation. In addition, since classes do not affect perception and self-regulation, the method looks promising as a suitable way be applied to all classes. Furthermore, the findings from the open-ended questions revealed the challenges of using the multiple solution method. There are many challenges that students faced such as, confusion, difficulty in understanding, and time constraint. Therefore, the multiple solution method not only helps students to solve the problem, but it also revealed some challenges that students need to face when employing it.

In the teaching and learning process, it is suitable for teachers to apply a multiple solution method which consists of various solutions in solving one problem. Based on this study, it was found that students have a good perception toward the use of the multiple solution method. Therefore, by applying it in the teaching and learning process helps students to develop more than one solution in their learning. Furthermore, this study also found that using this method has benefits towards self-regulation as it helps students to be more flexible in their learning and to think more critically in developing the solutions, leading to developing higher order thinking skills (HOTS). Giving the opportunity to students to develop more than one solution can increase the level of achievement and therefore, the school’s overall performance also increases. This method offers the Ministry of Education a technique that it can suggest to every school to be applied in the teaching and learning process in order to increase the level of student
understanding. The Curriculum Department can recommend this technique to schools as a way to improve the quality of the curriculum because when the school administration tries to develop this technique among their students, the level of achievement will potentially increase because students understand better. Other than that, this technique can be one of the curriculum support material in the teaching and learning process.

6. Recommendation
This study focused on the effect of using multiple solution method in mathematics learning. For further study, researchers can conduct a qualitative research like having a pre-test and post-test of the study. It shows the comparison of the effects of the multiple solution method before and after applying the method and it can help the researcher to evaluate more comprehensively about the acceptance of the use multiple solution method in mathematics learning. Besides that, this study was conducted only at one school which is Sekolah Menengah Kebangsaan Convent Klang where all of the students are female students. For further study, the researcher suggests doing a study between two schools where all of the students of one school are male students. From that, researchers can identify which gender is most affected by the multiple solution method in mathematics learning.

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