Proton Decay with a Light Gravitino or Axino

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Abstract

We consider the proton decay in supersymmetric models with a gravitino or axino lighter than the proton. This consideration leads to a stringent limit on the $R$ parity and $B$ violating Yukawa coupling of the superpotential operator $U_i^c D_j^c D_k^c$ as $\lambda''_{112} \leq 10^{-15}(m_3/2/eV)$ for a light gravitino, and $\lambda''_{112} \leq 10^{-15}(F_a/10^{10} \text{ GeV})$ for a light Dine-Fischler-Srednicki- Zhitnitskii axino. For hadronic axino, the constraint is weakened by the factor of $10^3$. 

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Proton stability strongly constrains the baryon \((B)\) and lepton \((L)\) number violating couplings. Since all known fermions lighter than the proton carry a nonzero lepton number, the couplings (or the combinations of couplings) relevant for the proton decay should conserve \(B-L\). However if there is a lighter fermion which does \textit{not} carry any lepton number, proton decay may be induced by a \(B\) violating but \(L\) conserving interaction alone \([1]\). There are in fact very interesting class of models which predict such a light fermion. In supersymmetric models in which supersymmetry (SUSY) breaking is mediated by gauge interactions, the squark and/or gaugino masses, i.e. the soft masses in the supersymmetric standard model (SSM) sector, are given by \(m_{\text{soft}} \simeq (\alpha/\pi)^n \Lambda_S\) where \(n\) is a model-dependent positive integer and \(\Lambda_S\) corresponds to the scale of spontaneous SUSY breaking \([2]\). In such models, in order for \(m_{\text{soft}}\) to be of order the weak scale, \(\Lambda_S\) is assumed to be \(10 \sim 1000\) TeV, leading to the gravitino mass \(m_{3/2} = \Lambda_S^2/M_P \leq 1\) keV far below the proton mass. If a global \(U(1)_{PQ}\) symmetry is introduced in a gauge-mediated SUSY breaking model to solve the strong CP problem by the axion mechanism \([3]\), SUSY breaking in the axion sector is mediated also by some gauge interactions. The axino mass in such models is given by \(m_{\tilde{a}} \simeq (\alpha/\pi)^m \Lambda_S^2/F_a\) where \(m\) is again a model-dependent (but typically bigger than \(n\)) positive integer and \(F_a\) denotes the scale of spontaneous \(U(1)_{PQ}\) breaking \([4]\). Obviously then the axino is lighter than the proton for a phenomenologically allowed \(F_a \geq 10^{10}\) GeV. In other type of models in which SUSY breaking is transmitted by supergravity interactions, the gravitino mass is fixed to be of order the weak scale, however there is still a room for an axino lighter than the proton \([5]\). As was pointed out in Ref. \([5]\), some supergravity-mediated models lead to \(m_{\tilde{a}} \simeq m_{3/2}(m_{3/2}/M_P)^{1/2} \simeq 1\) keV for which the axino would be a good warm dark matter candidate \([6]\). In this paper, we wish to examine the proton decay involving a light gravitino or axino to derive a constraint on the superpotential interaction \(\lambda_{ijk}^\nu U^c_i D^c_j D^c_k\) which violates \(R\) parity and \(B\), while conserving \(L\).

Let us first consider the proton decay involving a light gravitino, more precisely the helicity \(\pm 1/2\) Goldstino component. Our starting point is the effective lagrangian below the
scale $\Lambda_S = \sqrt{m_{3/2}M_P}$ but above the weak scale soft mass $m_{\text{soft}}$:

$$ \mathcal{L} = \mathcal{L}_{\text{SSM}} + \mathcal{L}_G, $$

where $\mathcal{L}_{\text{SSM}}$ denotes the lagrangian density of the SSM fields and the Goldstino lagrangian $\mathcal{L}_G$ is given by

$$ \mathcal{L}_G = \frac{i}{2} \bar{G} \gamma^\mu \partial_\mu G + \frac{i}{4\sqrt{6}m_{3/2}M_P} \left( \bar{\lambda}^{a\rho} \sigma^{\mu\nu} \partial_\mu G F_{\mu\nu}^a + 2\sqrt{2} \bar{\psi}_I (1 - \gamma_5) \gamma^\mu \gamma^\nu \partial_\mu \phi_I^* \right) + \text{h.c.} $$

where $G$ denotes the four-component Majorana Goldstino field. Here $\mathcal{L}_{\text{SSM}}$ includes the terms associated with the $B$ violating superpotential interaction,

$$ W_{\text{SSM}} \ni \lambda''_{ijk} U_i^c D_j^c D_k^c, $$

and $(\phi_I, \psi_I)$ and $(\lambda^a, F_{\mu\nu}^a)$ stand for the left-handed chiral matter and gauge multiplets in the SSM sector. Note that the above form of Goldstino lagrangian is enough for the study of the process involving a single on-shell Goldstino obeying $i\gamma^\mu \partial_\mu G = m_{3/2}G$.

Integrating out all fields heavier than the scale of the QCD chiral symmetry breaking, i.e. $\Lambda_\chi \simeq 1$ GeV, we are left with an effective lagrangian of the light quarks, $q_\alpha$ ($\alpha = (u, d, s)$), and gluons together with the light Goldstino (of course also the light leptons and the photon which are not relevant for our discussion). The operators responsible for the proton decay in this effective lagrangian at $\Lambda_\chi$ are induced by the exchange of the $SU(2)_L$ singlet squarks as

$$ \mathcal{O}_{\text{eff}} = \frac{2i\lambda''_{112} y_{\alpha\beta\gamma}}{\sqrt{3}m_0^2 m_{3/2} M_P} (\bar{q}_\alpha (1 - \gamma_5) q^c_\beta) (\partial_\mu \bar{q}_\gamma (1 - \gamma_5) \partial^\mu G). $$

Here $m_0^2$ denotes the squark masses which are assumed to be (approximately) universal,

$$ y_{dSU} = y_{uds} = y_{usd} = 1, $$

and all other components of $y_{\alpha\beta\gamma}$ do vanish. Note that the above operator has $B = S = -1$, and thus the relevant proton decay mode is $p \to G + K^+$. For a generic non-universal squark mass matrix, $S = 0$ operator can be induced also to give rise to $p \to G + \pi^+$, however it is
suppressed by a small squark mixing. To arrive at the above interaction operator, we have
used the equation of motion of the on-shell Goldstino field and ignored the piece suppressed
by the small \(m_{3/2}\). Also ignored are the renormalization effects between the weak scale and
\(\Lambda_\chi\).

The hadronic matrix elements of the above \(B = S = -1\) operator would be described by
an effective chiral lagrangian including the Goldstino field. Let us consider a chiral operator
\(O_\chi\) which would induce \(p \to G + K^+\) as a low energy realization of the light quark operator
\(O_{\text{eff}}\) below \(\Lambda_\chi\). Obviously it can be written as \(O_\chi = Z^\mu (1 - \gamma_5) \partial^\mu G\) where \(Z^\mu\) is a fermionic \(B = S = -1\) operator including \(\bar{P}\) and \(K^+\). If \(Z^\mu\) does not include any spacetime derivative, \(O_\chi\) is suppressed by the small factor \(m_{3/2}/m_p\) (for on-shell Goldstino) where \(m_p\) denotes the proton mass. For \(Z^\mu\) containing a single spacetime derivative, we have

\[
O_\chi = \frac{2 \xi \Lambda''_{112}}{\sqrt{3} m_0^2 m_{3/2} M_p} (\bar{P}(1 - \gamma_5) \partial^\mu G) \partial^\mu K^+, \tag{6}
\]

where again the equations of motion are used together with \(m_{3/2} \ll m_p\). To estimate the
size of the hadronic coefficient \(\xi\), we use the naive dimensional analysis (NDA) rule of Ref. [8], yielding

\[
|\xi| \simeq 4 \pi f_\pi^2, \tag{7}
\]

where \(f_\pi = 93\) MeV is the pion decay constant. In fact, the NDA rule gives \(\Lambda_\chi = 4 \pi f_\pi\) and
then the typical energy in the proton decay, i.e. \(m_p\), is comparable to \(\Lambda_\chi\). This means that,
within the NDA rule, chiral operators with more spacetime derivatives are equally important
as the operator of Eq. (6). However for an order of magnitude estimate of the hadronic
matrix element, the consideration of \(Z^\mu\) with a single derivative would be enough. Then
applying the experimental limit on \(p \to K^+ + \nu\) for \(p \to K^+ + G\) induced by the interaction
of Eq. (6), we find the following constraint on the \(R\) parity and \(B\) violating coupling:

\[
\Lambda''_{112} \leq 5 \times 10^{-16} \left(\frac{m_0}{300\,\text{GeV}}\right)^2 \left(\frac{4 \pi f_\pi^2}{|\xi|}\right) \left(\frac{m_{3/2}}{1\,\text{eV}}\right), \tag{8}
\]

which is one of the main results of this paper.
Let us now consider the proton decay involving a light axino. Similarly to the case of a light gravitino, we start from the effective lagrangian at scales below the scale $F_a$ of $U(1)_{PQ}$ breaking but above $m_{soft}$:

$$L = L_{SSM} + L_A,$$

where the axino lagrangian $L_A$ can be read off from

$$
\int d^2 \theta d^2 \bar{\theta} \frac{c_L}{F_a} (A + A^\dagger) \Phi_I^\dagger \Phi_I + \left\{ \int d^2 \theta \frac{c_a}{16\pi^2 F_a} AW^a W^a + h.c \right\},
$$

where $A = (s + ia) + \sqrt{2} \theta \tilde{a} + F_A \theta_2$ is the axion superfield containing the axion $a$, the saxion $s$ and the axino $\tilde{a}$, while $\Phi_I$ and $W^a$ are the chiral superfields for the SSM matter and gauge multiplets $(\phi_I, \psi_I)$ and $(\lambda^a, F^a_{\mu\nu})$, respectively. Here $c_I$ and $c_a$ are dimensionless real coefficients. For $F_a$ defined as the scale of spontaneous $U(1)_{PQ}$ breaking, the coefficients $c_a$ of the axion coupling to the gauge multiplets are of order unity in general. However as we will discuss later, the size of the coefficients $c_I$ of the axion coupling to the matter multiplets is somewhat model-dependent. Note that the above lagrangian corresponds to the supersymmetric generalization of the conventional axion effective lagrangian $[9]$:

$$
L_a = \frac{2c_I}{F_a} \partial_\mu a \bar{\psi}_I \gamma^\mu \gamma_5 \psi_I + \frac{c_a}{32\pi^2 F_a} a F^{a\mu\nu} \tilde{F}_{\mu\nu}.
$$

Obviously it is manifestly invariant under the nonlinear $U(1)_{PQ}$ transformation, $A \rightarrow A + ic$ ($c = \text{real constant}$), up to the PQ anomaly. At any rate, the relevant axino lagrangian is given by

$$
L_A = \frac{1}{2} i \bar{a} \gamma^\mu \partial_\mu \bar{a} - \frac{c_I}{2F_a} (i \partial_\mu \bar{\psi}_I \gamma^\mu (1 + \gamma_5) \bar{a} \phi_I^* + h.c)
$$

$$
+ \frac{c_a}{32\sqrt{2}\pi^2 F_a} (\bar{\lambda}^a \gamma^\mu \gamma^\nu (1 - \gamma_5) \bar{a} F^a_{\mu\nu} + h.c),
$$

where $\tilde{a}$ denotes the four-component Majorana axino field. Again the exchange of the $SU(2)_L$ singlet squarks leads to the following $B = S = -1$ interaction in the effective lagrangian at $\Lambda_\chi$:

$$
O_{\text{eff}} = \frac{i \lambda''_{12} y_{\alpha\beta\gamma} c_\gamma}{m_0^2 F_a} (\bar{q}_\alpha (1 - \gamma_5) q_\beta) \partial_\mu \bar{q}_\gamma \gamma^\mu (1 + \gamma_5) \bar{a},
$$
where \(c_\gamma (\gamma = u, d, s)\) denotes the axino coupling to the supermultiplet containing the \(SU(2)_L\) singlet right-handed light quark \(q_{\gamma R}\) in Eq. (12) and the squark degeneracy is assumed also.

Similarly to the gravitino case, in order to estimate the proton decay rate from the above effective interaction, we consider a chiral operator of the form \(O_\chi = X^\mu \gamma^\mu (1 + \gamma^5) \tilde{a}\) where \(X^\mu\) is a \(B = S = -1\) fermionic current made of \(\bar{P}\) and \(K^+\) which are on mass-shell. For \(X^\mu \propto K^+ \bar{P} \gamma^\mu\), the chiral operator \(O_\chi\) with the smallest number of spacetime derivatives is given by

\[
\frac{\xi_\gamma c_\gamma \lambda_{112}''}{m_0^2 F_a} (\bar{P}(1 + \gamma^5)\tilde{a}) K^+,
\]

where the hadronic coefficients \(\xi_\gamma\) are again determined by the NDA rule as

\[
|\xi_\gamma| \simeq 16\pi^2 f_\pi^3.
\]

This then leads to the experimental bound on the \(R\) parity and \(B\) violating coupling as

\[
\lambda_{112}'' \leq 7 \times 10^{-16} \left( \frac{m_0}{300 \text{ GeV}} \right)^2 \left( \frac{16\pi^2 f_\pi^3}{c_\gamma |\xi_\gamma|} \right) \left( \frac{F_a}{10^{10} \text{ GeV}} \right),
\]

which is another result of this paper.

The above constraint from the proton decay involving a light axino depends upon the dimensionless coefficients \(c_\gamma\) describing the axino coupling to the supermultiplets of the \(SU(2)_L\) singlet quarks [see Eq. (12)], as well as the axion scale \(F_a\). In fact, the size of \(c_\gamma\) has a certain model-dependence. If the quark superfields carry a nonzero \(U(1)_{PQ}\) charge, which would be the case for the supersymmetric extension of the Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) axion model [10], the coefficients \(c_\gamma\) would be of order unity in general. However in hadronic axion models [11] in which all SSM fields have a vanishing \(U(1)_{PQ}\) charge, the coefficients \(c_\gamma\) are zero at tree level. However the axino-quark couplings are radiatively generated through the axino coupling to the gluon multiplet, yielding \(c_\gamma \simeq (\alpha_c/\pi)^2 \ln(F_a/m_{\text{soft}}) \simeq 10^{-3} \sim 10^{-2}\) [11]. Thus the constraint for hadronic axion models becomes weaker than that for DFSZ models by the factor of \(10^2 \sim 10^3\).

To conclude, we have considered the proton decay involving a gravitino or axino lighter than the proton. Generic models in which supersymmetry breaking is mediated by gauge
interactions contain such a light gravitino. Then the $R$ parity and $B$ violating coupling $\lambda''_{112}$ is strongly constrained by the proton stability [see Eq. (8)] to be less than about $10^{-15}(m_{3/2}/\text{eV})$. About the possibility of a light axino, gauge-mediated supersymmetry breaking models endowed with a global $U(1)_{PQ}$ symmetry generically predict an axino lighter than the proton. Also some supergravity-mediated models can give rise to a light axino, while the gravitino mass in such models is fixed to be the weak scale. We find that $\lambda''_{112}$ in models with a light axino is constrained [see Eq. (16)] to be less than about $10^{-15}(F_a/10^{10} \text{GeV})$ and $10^{-12}(F_a/10^{10} \text{GeV})$ for Dine-Fischler-Srednicki-Zhitnitskii axion models and hadronic axion models respectively.

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