Topology Optimization and Design for Additive Manufactured Supporting Structure of Vehicle Rudder

Jingyi Chen1,*, Kehao Xin1, Xiaolong Gu1 and Hongjuan Ji1
1Beijing Institute of Space Long March Vehicle, China Aerospace Science and Technology Co., Beijing, 100076, China

*E-mail: chenjy@14.calt.casc

Abstract. The object investigated in this work is a design method for the supporting structure of vehicle rudders, which are the crucial components in flight control system. As a practical application of topology optimization for additive manufacture, the design process was presented in this article. To obtain the optimal material distribution, topology optimization based on SIMP method was applied, with compliance minimization as the optimization objective. A numerical optimization model was built and calculated in Inspire, and the result in an equivalent load scheme of 30 percent point load and 70 percent uniform pressure was adopted to be the initial geometry for the subsequent design. Concentrating on realizing the optimized geometry, this design took a series of technical constrains for additive manufacture and mass requirements into consideration. Notably, the FEM simulation for the supporting structure is necessary, which consist of stress analysis and mode analysis. And the technical feasibility was verified in its practical manufacturing process. Finally, by the experiment about dynamic characteristic, it is verified that the lightweight rudder designed in this work can satisfy all the requirements for flight control.

1. Introduction

As a key component in flight control, traditional rudders of the vehicle commonly consist of the metal frame skins, and insulation composite layers or coating for thermal protection, as fig.1. Generally, to reinforce the integral stiffness, radial and circular ribs, centered on the rudder shaft axis, are used in the design of frames. Considering the mechanical properties in high temperature, the frame is a precision casting of titanium alloy (Ti-alloy). With Ti-alloy skins welded on it, the main supporting structure is similar to the sandwich panel. The manufacture involves a series of complicated processes, such as Investment Casting and Vacuum Brazing, that is a crucial factor to influence the cost of rudders.

With the development of additive manufacture, mechanical properties of Ti-alloy products have been close to Ti-alloy forgings and the precision have been raised above the level of Investment Casting. SLM (Selective Laser Melting), a branch of additive manufacture, may provide a new approach to manufacture the rudder supporting structure, due to its advantage of accommodating highly complex geometries and inner void structures to mold the rudder frame and skins integrally, instead of welding. In particular, SLM provides an opportunity to make components close to their theoretically optimal geometry. In order to generate a robust support-free structure there are some technical constrains which must be satisfied, such as the minimum feature size, the bridging distance, the manufacturable inclination angle [1], the traditional shape of rudder supporting structure has not been completely appropriate for additive manufacture anymore. Consequently, this work focus on a design method for the supporting structure of vehicle rudder with combining topology optimization.
and practical experience. The proposed method is required to be appropriate for the metal additive manufacture. Besides, due to the design of rudders must meet the strength and stiffness tolerance requirement in flight, the finite-element method is applied to analyze the structure stress, modes and nature frequency that will be verified by the subsequent test.

**Figure 1.** Structure and compositions of traditional vehicle rudder.

2. **Topology optimizing model**

In this work, SIMP is employed for the rudder supporting structure optimization in order to obtain an adequate strength, high stiffness and light weight geography.

Rossow and Taylor demonstrated the finite element representation of potential energy for the first time, to seek the optimal distribution of material for sheets [2]. Based on the homogenization theory, the continuum structure is discretized into finite units, and the variable \( \rho_e \) about density is introduced for each unit.

1 or 0 denote the density of the unit is one or void. \( \Omega^{\text{mat}} \) denotes the one material domain, \( \Omega \) denotes the reference design domain.

\[
\rho_e = \begin{cases} 
1 & e \in \Omega^{\text{mat}} \\
0 & e \in \Omega \setminus \Omega^{\text{mat}}
\end{cases}
\]  

(1)

In order to avoid ‘grey’ material units (intermediate densities), Bendsoe and Zhou [3,4,5], proposed the SIMP (Solid isotropic material with penalization) method. The interpolation scheme and the relation between the density variable \( \rho_e \) and the young’s modulus \( E^0 \) for a unit is represented as follows

\[
E = \rho^n E^0, \; n > 1.
\]  

(2)

Where \( n \) is defined as the penalty parameter. With intermediate densities (grey) being penalized, the number of ‘grey’ is reduced effectively, that helps obtain a true “0-1” design. With the finite element method, the minimum compliance problem for elastic structure under loads and the fixed volume conditions is as follows:

\[
\text{min } c = U^T K U \\
\text{s.t. } K(\rho_e)U = P \\
\sum_{e=1}^{N} \rho_e v_e - V \leq 0 \\
0 < \rho_{\text{min}} \leq \rho_e \leq 1, \; e = 1, ..., N
\]  

(3)

Here, \( N \) denotes the total amount of units, \( K \) denotes the stiffness matrix, \( U \) and \( P \) denote the compliance and load vector, \( v_e \) denotes the volume of the \( e \)th cell, and \( V \) denotes the total amount of material volume.

In this work, the numerical model of topology optimization for supporting structure of vehicle rudder is built in Altair Inspire, which is a FEM analysis and optimization platform with rapid simulation capabilities. The initial CAD model consist of external composite layers and the internal metal supporting structure, and material properties are shown in Table 1.

To simplify the optimization model of rudder, the composite layer, which does not belong to the design target, is simplified as an isotropic material in the preprocessing. According to practical experience in rudders design, the joint region between the rudder and shaft, the zone in a distance of
6mm around the rudder edges, and skins are the nonoptimized domain, defined as the external boundaries, other internal areas are the optimized domain, defined as the design space of this model.

Table 1. Material properties of the model.

| Material properties | Yong’s modulus (E) | Poisson ratio (μ) | Density (ρ) | Yield strength (σ_{0.2}) |
|---------------------|-------------------|------------------|------------|--------------------------|
| Titanium alloy (TC4) of SLM | 109 GPa | 0.34 | 4.5×10³ kg/m³ | 830 MPa |
| Composite layers | 5 GPa | 0.07 | 1.7×10³ kg/m³ | 100 MPa |

In this model, thickness constrains are defined to control the wall thickness. The minimum thickness is 3mm, and the maximum thickness is 5mm. In general, thickness constrains also affect the element size, as the minimum thickness is 3 times the average element size.

A Support in all directions is applied on the joint region, to simulate the constrain of shaft. For rudders, normal pressure with variable distribution is the essential load case. Thereby, in this work, the pressure load in the typical flight condition (Table 2) of the vehicle is applied on the rudder surface. However, due to it is not available to import the aerodynamic loads field in Inspire, point loads and uniform pressures of zones have been applied on the surface as an equivalent of the aerodynamic loads in this optimization model. The surface of the rudder is divided into eight zones, and the details about equivalent loads in different zones are shown in Table 3.

Table 2. The typical load of rudder in flight.

| Mach Number Ma (/) | Dynamic pressure Q(Pa) | Normal force F(N) | Torque T(Nm) | Bending MM(Nm) |
|--------------------|-------------------------|-------------------|--------------|----------------|
| 4.19261            | 730884.50994            | -1012.73884       | -19.21413    | 45.78004       |

Table 3. The equivalent loads and pressure centers of zones.

| Section No. | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|-------------|----|----|----|----|----|----|----|----|
| range of the section(mm) | 25.0 | 70.0 | 130.0 | 180.0 | 236.0 | 280.0 | 340.0 | 410.0 |
| Pressure Center(X) | -10.3 | -45.5 | -113.2 | -139.4 | -200.7 | -157.4 | -171.8 | -179.6 |
| Pressure Center(Y) | 32.1 | 51.7 | 105.0 | 158.3 | 211.2 | 258.2 | 311.2 | 374.0 |
| Total normal force of the section | 11.5 | 14.0 | 23.8 | 35.0 | 45.2 | 55.3 | 53.1 | 53.4 |

3. The optimistic result

Fig. 2 shows the optimal geometric result with equivalent point loads applied on the pressure centers of zones. Fig. 3 shows the optimal geometric result with equivalent uniform pressures applied on zones. Combining the point loads and pressures in proportion, the result with synthesized loads is showed in Fig. 4. In various equivalent schemes, stiffness maximization is still selected to be the optimization objective, while the mass target of 30 percent is specified for the first two cases, and 20 percent for the last.

According to the comparison of these optimal results, the optimal material distributions in various equivalent schemes are extremely close, while some differences is noticeable in detail. Compared to
the pure pressure case, the optimization result in the point load case is more sensitive to the location of pressure centers. Furthermore, with the stiffness of the composite layer increasing, the tendency will be more apparent. Therefore, by comparing other results in various proportions of the point load to uniform pressure, the result in 30 percent of point load and 70 percent of uniform pressure is selected to be the basic topological geometry for further design.

![Figure 2](image1.png) The optimal geometric result with equivalent point loads, 30% mass target.

![Figure 3](image2.png) The optimal geometric result with equivalent uniform pressures, 30% mass target.

![Figure 4](image3.png) The optimal geometric result with synthesized loads, 20% mass target.

4. Reconstruction and product design
With the benefit of the development of additive manufacture, it is feasible to reconstruct the geometric result of topology optimization with minimum information loss, that due to its less limit in geometry than traditional subtractive manufacture. According to the optimal topology obtained in section 3, the rudder supporting structure is rebuild by the fitting method of NURBS curves and designed in detail. Even if the additive manufacture is labeled as a less limit and dimension free technology, there are a series of constrains must be satisfied in order to improve the feasibility in technology, especially for the closed shape which lead to a support-free design being employed, such as the rudder. For SLM, the factor sought be considered in design are mainly the direction of 3D-print, bridging distance, minimum feature size, and inclination angle. In order to reduce the cost and time in manufacture, the supporting structure was finally built in span direction. The CAD model of the ultimate design is shown in fig. 5, and A Ti-alloy product of the supporting structure of rudder is as fig. 6.

![Figure 5](image4.png) The ultimate CAD model of the rudder supporting structure.

In the design of aerospace vehicle rudder, there is a particular requirement about mass properties of the whole construct: the mass center need to be located at the axis of rudder shaft. Therefore a high-density (Wu-alloy) component was inserted into the tip of the rudder to balance the weight by laser cladding technology. From practical experiences, the inclination angles of ribs are mostly limited in 60 deg. Nevertheless, there are some parts having difficulty to attain it, where three-dimensional lattice structures are used. And as a kind of less density structure, it is not only an approach to remedy the inclination angle problem, but also provides an effective path to improve the local stiffness with less weight. Fig. 7 show the inner 3D lattice strictures of the product by X-rays inspection.
5. Stress and mode analysis

In order to verify the strength of the supporting structure of rudder designed in section 4, FE stress analysis by Abaqus is conducted for the entire rudder, with the composite layers included. In the analysis, to avoid the repeat of load cases, maximum normal loads in the extreme flight is applied on the rudder surface in the form of the original pressure field, instead of the equivalent loads. The iso-contours of von Mises stress are plotted in fig. 8. With the safety factor of 1.5, the zone of maximum stress is located nearby the connecting section, where the stress is equal to 322MPa. According to the assessment for the strength of whole structure designed above, the remaining strength factor is approximately 2.6, which means that the design in section 4 is still relatively conservative, and there is certain room for further optimization.

Figure 8. Von Mises stress of the rudder supporting structure.

In the mode analysis, a full-scale assembled model of rudder described above is created with a shaft model merged in, which is used to apply the elastic restraint to simulate the realistic constraints of driving motor and bearings. Two SPRING1 elements with displacement degree of freedom associated are used to simulate the bearings, and a torsion SPRING1 element (associated with rotational degrees of freedom) is used to simulate the driving motor. The result of the mode analysis is listed in Table 4. It is apparent that in comparison with traditional rudders there is a certain improvement in the bending mode frequency of the optimized rudder, which is associated with the stiffness of the supporting structure.

Table 4. The result of mode analysis.

| Mode NO. | Mode Frequency (Hz) |
|----------|---------------------|
|          | Optimized rudder    | Traditional rudder |
| 1        | 170.94              | 173.54             |
| 2        | 290.87              | 266.05             |

6. Experimental test

In order to verify the optimized rudder designed above can meet the requirement of control system, a dynamic characteristic experimental test has been finished. With a real vehicle cabin and servo motor as support boundaries and controller, the resonant frequency and transfer function of the flight control system were revised through the test. To obtain low order modes of the rudder for further flutter analysis, 11 displacement sensors were attached on rudder surface, and a vibration exciter was used to output the vibration force. Furthermore, for reducing the influence of gap nonlinearity, a normal load
of 30kg was applied on the tip of rudder. The mode parameters obtained in this test have been list in Table 5, and the mode shapes are presented in fig.9. Compared with the result of the mode analysis above, it shows a consistency in the shape of torsion mode. However, the torsion mode frequency in the test is relatively lower than the value in calculation, due to the difference of brace stiffness between actual boundaries and Spring elements in the simulation, which could be reduced through revising the spring stiffness associated or defining Dashpot elements. For the bending mode, the shape and frequency are both approximate to the results of the FEM simulation in section V, which depends on the individual structural stiffness of the rudder more.

**Table 5. Mode parameters in the test.**

| Mode NO. | Mode Frequency (Hz) | Damping ratio (%) | Mode mess (kg) | Load (kg) |
|----------|--------------------|------------------|----------------|-----------|
| 1        | 138.3              | 7.68             | 2.75           | 15kg      |
| 2        | 313.5              | 5.30             | 2.00           |           |

![Figure 9. Mode shapes obtained in the test.](image)

7. Conclusions
The application of topology optimization method in the design of vehicle rudder supporting structure has been presented in this article. With SIMP method, a high stiffness material distribution has been achieved. Based on the optimal geometry, a manufacturable CAD model has been rebuilt, and accomplished efficiently by additive manufacture, with a series of associated requirements to be satisfied, such as mass center location and geometric tolerances. This design is verified in the FEM analysis and dynamic characters experiment. The method could also be transferred to other rudders and wings. The conclusions are as follows:

a) In topology optimization with FEM software, it is turn out to be an effective approach to optimize with a typical load case and verify with extreme load case, considering the dynamic characters.

b) Due to optimal results are sensible to loads, original pressure distribution is still the best choice for topology optimizations of rudder supporting structures, even if the load approximation was employed in this work. And the influence of load schemes and thickness constrains on optimization results should be paid attention to.

c) With SLM technology, the use of 3D lattice structure could significantly improve the local stiffness with minimal increase in weight, especially for the large-sized inner void region. Furthermore, a secondary topology optimization for this optimized design model may lead to a more balanced result, if required.

**Acknowledgment**
The author would like to thank the Xian Bright Laser Technologies Company for its assistance in this study.

**References**
[1] Optimal topology for additive manufacture: a method for enabling additive manufacture of support-free optimal structures[J]. Materials and Design, 63(2014), 678-690
[2] Rossow M P, Taylor J E. A finite element method for the optimal design of variable thickness sheets [J]. AIAA, 1973, 11, 1556-1569
[3] Bendsoe M P, Sigmund O. Material interpolation schemes in topology optimization[J]. Archive of Applied Mechanics, 1999, 69(9), 635-654
[4] Rozvany G I N, Zhou M, Birker T. Generalized shape optimization without homogenization[J]. Structural and Multidisciplinary Optimization, 1992, 4(3), 250-252
[5] Rietz A. Sufficiency of a finite exponent in SIMP (power law) method[J]. Structural and Multidisciplinary Optimization, 2001, 21(2), 159-163. Electronic Publication: Digital Object Identifiers (DOIs)