Balanced right/left-handed mixtures of quasi-planar chiral inclusions

R. Marqués¹ᵃ, F. Mesa², L. Jelinek¹ᵇ and J.D. Baena¹ᶜ

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¹ Dept. of Electronics and Electromagnetism, University of Seville (Spain), e-mail: (a) marques@us.es, (b) bish@atlas.cz, (c) juan_dbd@us.es
² Dept. of Applied Physics 1, University of Seville (Spain), e-mail: mesa@us.es

Abstract

Some novel quasi-planar chiral inclusions, feasible from standard photo-etching techniques, are proposed. It is shown that such inclusions can be designed in order to present balanced electric, magnetic and magneto-electric polarizabilities. Using these inclusions, random and periodic bi-isotropic artificial metamaterials exhibiting a balanced positive/negative refractive index can be build up. These metamaterials would exhibit reasonable bandwidths and excellent matching to free space.

1. Introduction

In spite of some proposal in such direction [1] – [9], the development of bulk isotropic metamaterials exhibiting negative refractive index (NRI) and good matching to free space is still a challenging issue. A promising approach to this problem could take advantage of the simultaneous electric and magnetic polarizability of chiral inclusions in order to obtain a mixture with simultaneously negative $\varepsilon$ and $\mu$. As far as we know, the first proposal in such direction was made in [10], and further developed in [11]. In these works racemic and chiral mixtures of chiral metallic inclusions were proposed as a practical way of designing left-handed metamaterials. Other proposals taking advantage of chirality for negative refractive artificial media design have also been made (see [12] and [13], for instance). In this contribution we will further develop this approach presenting some novel NRI bi-isotropic metamaterial designs that exhibit reasonable bandwidths and excellent matching to free space.

2. Balanced positive/negative refractive index (BPNRI) metamaterials

BPNRI metamaterials are defined as bi-isotropic media exhibiting balanced electric, $\chi_e$, and magnetic $\chi_m$ susceptibilities, that is

$$\chi_e(\omega) = \chi_m(\omega),$$

over a wide bandwidth which includes both positive and negative values of $\varepsilon_r = 1 + \chi_e$ and $\mu_r = 1 + \chi_m$. Such media are characterized by the following properties:

- Wide NRI pass-band for the positive and/or the negative circularly polarized TEM eigenwaves, defined by the condition $\sqrt{\varepsilon_r \mu_r} \pm \kappa < 0$ [14] where the negative sign for the square root must be chosen if $\varepsilon$ and $\mu$ are both negative.
• No forbidden bands, as it is deduced from the dispersion equation \( k^\pm = k_0 \left( \sqrt{\mu_\epsilon} \pm \kappa \right) \) with \( k_0 = \omega \sqrt{\epsilon_0 \mu_0} \).

• Transition between the NRI and PRI pass-bands through a zero phase velocity point (a behavior similar to that previously reported for some transmission line metamaterials [15]).

• Good matching (perfect matching for paraxial rays) to free space, due to the matching of TEM impedances \( \eta = \sqrt{\mu/\epsilon} = \eta_0 \).

In the following we will develop a design with \(|\kappa| \sim \chi_e, \chi_m\). In such case only one of the TEM circularly polarized eigenwaves will exhibit negative refraction and the NRI pass-band is defined by the condition \( \chi_e = \chi_m < -0.5 \) [16].

3. Balanced quasi-planar chiral inclusions

Figure 1 shows two quasi-planar chiral inclusions suitable for balanced metamaterial design: the chiral split ring resonator (Ch-SRR) and the chiral spiral resonator (Ch-SR). They are the broadside-coupled versions of the spiral resonator [17] and the NB-SRR [18] previously proposed by some of the authors. In both cases the frequency of resonance can be obtained from an \( LC \) circuit model: \( \omega_0 = 1/\sqrt{LC} \) where \( L \) and \( C \) are the effective inductance and capacitance of the inclusion (analytical expressions can be found in [19]). The polarizabilities can be obtained following the standard technique developed in [19] and in [17] – [18]. This calculation gives:

\[
\begin{align*}
\alpha_{zz}^{mm} &= \alpha_0^{mm} \frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega R/L} ; \\
\alpha_{zz}^{em} &= \pm \alpha_0^{em} \frac{\omega_0^2}{\omega_0^2 - \omega^2 + j\omega R/L} ; \\
\alpha_{zz}^{ee} &= \alpha_0^{ee} \left( \frac{\omega_0}{\omega} \right)^2 \frac{\omega^2}{\omega_0^2 - \omega^2 + j\omega R/L} ; \\
\alpha_{xx}^{ee} &= \alpha_{yy}^{ee} = \alpha_0 ; \\
\alpha_0 &= \frac{\pi^2 r^4}{L} ; \\
\alpha_0 &= \frac{t^2}{\omega_0 L} \left( \frac{C_0}{C} \right) ;
\end{align*}
\]

where \( n = 1 \) for the Ch-SR and \( n = 2 \) for the Ch-SRR (see [19] for more details on notation). From (2) – (4) it follows that

\[ \alpha_{zz}^{mm} \alpha_{zz}^{ee} + (\alpha_{zz}^{em})^2 = 0 , \]

which is a general property arising from the \( LC \) nature of the model [11]. In order to obtain a balanced design we will further impose

\[ \alpha_0^{ee} = \mu_0 \epsilon_0 \alpha_0^{mm} , \]

which is satisfied provided that

\[ t\lambda_0 = \frac{2}{n} \frac{C}{C_0} (\pi r)^2 \]

where \( \lambda_0 \) is the wavelength at resonance. Incidentally, since the frequency of resonance of the Ch-SRR is twice that of the Ch-SR, (8) provides exactly the same geometrical parameters for both inclusions. The accuracy of this expression for giving a balanced design has been shown in [16] by electromagnetic simulation.
4. Isotropic arrangement

In order to obtain a BPNRI metamaterial the inclusions shown in Fig.1 must be properly arranged. The main advantage of the Ch-SR is its small electrical size (balanced designs with sizes of about $\lambda_0/13$ can be obtained [16]). However, the Ch-SR has not enough symmetry to allow for the design of an isotropic cubic resonator, suitable for a periodic isotropic design [6]. This condition is fulfilled by the Ch-SRR which allows for the design of cubic arrangements satisfying the cubic T group of symmetry (in Schoenflies notation), which is enough to guarantee an isotropic metamaterial design [6]. Therefore, Ch-SRs are suitable for the design of random BPNRI media and the Ch-SRR is appropriate for the design of periodic BPNRI media. In both cases a Lorentz homogenization procedure provides the following relation between the macroscopic fields and volume polarizations

$$M = N \left\{ \mu_0 \hat{\alpha}_{mm} \left( H + \frac{M}{3} \right) - \hat{\alpha}_{em} \left( E + \frac{P}{3\varepsilon_0} \right) \right\}$$  \hspace{1cm} (9)

$$P = N \left\{ \hat{\alpha}_{ee} \left( E + \frac{P}{3\varepsilon_0} \right) + \mu_0 \hat{\alpha}_{em} \left( H + \frac{M}{3} \right) \right\}$$  \hspace{1cm} (10)

where $N$ is the number of particles per unit volume, and $\hat{\alpha}_{mm}, \hat{\alpha}_{ee}, \hat{\alpha}_{em}$ are some average polarizabilities given by $\hat{\alpha}_{mm} = \alpha_{zz}^{mm}/3, \hat{\alpha}_{ee} = \alpha_{zz}^{em}/3$ and $\hat{\alpha}_{ee} = (\alpha_{xx}^{ee} + \alpha_{yy}^{ee} + \alpha_{zz}^{ee})/3$.

For a balanced design of the inclusions, taking (8) into account, we finally find

$$\chi_e = \frac{N}{3\Lambda} \left\{ \frac{2\alpha_0}{\varepsilon_0} \left( \frac{\omega_0^2}{\omega^2} - 1 \right) + \mu_0 \alpha_0^{mm} \left( \frac{\omega_0^2}{\omega^2} - \frac{2N\alpha_0}{9\varepsilon_0} \right) \right\}$$ \hspace{1cm} (11)

$$\chi_m = \frac{N\mu_0\alpha_0^{mm}}{3\Lambda} \left( 1 - \frac{2N\alpha_0}{9\varepsilon_0} \right)$$ \hspace{1cm} (12)

$$\kappa = \pm \frac{\omega_0 N\mu_0\alpha_0^{mm}}{\omega} 3\Lambda$$ \hspace{1cm} (13)

where

$$\Lambda = K \left\{ \frac{\omega_0^2}{\omega^2} - 1 + \frac{N\mu_0\alpha_0^{mm}}{9} \left( 1 + \frac{\omega_0^2}{K\omega^2} \right) + j \frac{R}{\omega L} \right\} ; \quad K = \left( 1 - \frac{2N\alpha_0}{9\varepsilon_0} \right)$$ \hspace{1cm} (14)

These expressions satisfy (1) in the limit $\omega_0/\omega \rightarrow 1$. Since most resonant metamaterials have a moderate bandwidth ($\sim 10\%$ or less), this condition is approximately fulfilled inside the left-handed pass-band.

Figure 2 shows the propagation constants and impedances, $k^\pm = k_0 \left( \sqrt{\mu_r\varepsilon_r} \pm \kappa \right)$ and $\eta = \sqrt{\varepsilon/\mu}$, computed from (11) - (13) for the TEM eigenwaves in a compact arrangement of balanced...
Figure 2: Plots of $k^\pm/k_0 = \omega \sqrt{\varepsilon_0 \mu_0}$ (left) and $\eta/\eta_0$ against $\omega/\omega_0$ (right) for a cubic fcc lattice of balanced Ch-SRs with $c/r_{ext} = 0.2$, and $t/r_{ext} = 0.306$. Substrate is foam with $\varepsilon = \varepsilon_0$ and $N$ is calculated in the text. The NRI pass-band is marked in the figures.

Ch-SRs randomly oriented. $N$ corresponds to a cubic fcc lattice of balls with radius $a = 1.1 r_{ext}$ which contains the Ch-SRs, i.e. $N = 0.74 \left(\frac{4}{3} \pi (1.1 r_{ext})^3\right)^{-1}$. The remaining parameters are given in the caption. As it can be seen, a BPNRI behavior is quite approximately obtained.

5. Conclusion

Along this paper a fully analytical method for designing bi-isotropic balanced positive/negative refractive index metamaterials has been presented. Some new quasi-planar chiral inclusions have been proposed for this purpose. These inclusions can be easily manufactured by standard photoetching techniques and the condition for balanced design can be expressed in a quite simple way \[ S \]. Hopefully, the interesting properties of bi-isotropic BPNRI metamaterials, such as the smooth transition between the NRI and the PRI pass-bands and the excellent matching to free space, could find application in focusing devices and other applications.

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