Four-body baryonic decays of $B \to p\bar{p}\pi^+\pi^-(\pi^+K^-)$ and
$\Lambda\bar{p}\pi^+\pi^-(K^+K^-)$

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Abstract

We study the four-body baryonic $B \to B_1\bar{B}_2M_1M_2$ decays with $B_{1,2}$ ($M_{1,2}$) being charmless baryons (mesons). In accordance with the recent LHCb observations, each decay is considered to proceed through the $B \to M_1M_2$ transition together with the production of a baryon pair. We obtain that $\mathcal{B}(B^- \to \Lambda\bar{p}\pi^+\pi^-) = (3.7^{+1.5}_{-1.0}) \times 10^{-6}$ and $\mathcal{B}($ $B^0 \to p\bar{p}\pi^+\pi^-, p\bar{p}\pi^+K^-) = (3.0 \pm 0.9, 6.6 \pm 2.4) \times 10^{-6}$, in agreement with the data. We also predict $\mathcal{B}(B^- \to \Lambda\bar{p}K^+K^-) = (3.0^{+1.3}_{-0.9}) \times 10^{-6}$, which is accessible to the LHCb and BELLE experiments.
I. INTRODUCTION

One of the main purposes of the B factories and current LHCb is to study CP violation (CPV), which is important for us to understand the puzzle of the matter-antimatter asymmetry in the Universe. As the observables, the (in)direct CP-violating asymmetries (CPAs) require both weak and strong phases [1–3], whereas the T-violating triple momentum product correlations (TPCs), such as $\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$ in a four-body decay, do not necessarily need a strong phase [4, 5]. For example, the LHCb Collaboration has provided the first evidence for CPV from the TPCs in $\Lambda_b \rightarrow p\pi^-\pi^+\pi^-$ [6], and measured TPCs in $\Lambda_b \rightarrow pK^-\mu^+\mu^-$ [7].

As the similar baryonic cases, the four-body baryonic $B$ decays can also provide TPCs. For a long time, the $B^− \rightarrow \Lambda\bar{p}\pi^+\pi^−$ decay was the only observed decay mode in $B \rightarrow B_1\bar{B}_2M_1M_2$ [8]. Until very recently, more four-body baryonic $B$ decays have been observed by the LHCb [9], which motivate us to give theoretical estimations on the corresponding decay branching ratios. The experimental measurements for the branching ratios of $\bar{B}^0/B^− \rightarrow B_1\bar{B}_2M_1M_2$ at the level of $10^{-6}$ are given by [8, 9]

$$B(B^0 \rightarrow p\bar{p}\pi^+\pi^-) = (3.0 \pm 0.2 \pm 0.2 \pm 0.1) \times 10^{-6},$$
$$B(\bar{B}^0 \rightarrow p\bar{p}K^+\pi^-) = (6.6 \pm 0.3 \pm 0.3 \pm 0.3) \times 10^{-6},$$
$$B(B^- \rightarrow \Lambda\bar{p}\pi^+\pi^-) = (5.92^{+0.88}_{-0.84} \pm 0.69) \times 10^{-6},$$

(1)

where the resonant $B(B^- \rightarrow \Lambda\bar{p}\rho^0, f_2(1270) \rightarrow \pi^+\pi^-)$ have been excluded from the data [8]. In comparison with $B(\bar{B}^0 \rightarrow p\bar{p}K^+K^-) \simeq (1.3 \pm 0.3) \times 10^{-7}$ and $B(\bar{B}_s^0 \rightarrow p\bar{p}\pi^+\pi^-) < 7.3 \times 10^{-7}$ (90% C.L.) [9], the decays with $B \sim 10^{-6}$ in Eq. (1) are recognized to have the same theoretical correspondence, where $B^0/B^- \rightarrow B_1\bar{B}_2M_1M_2$ proceed through the $B \rightarrow M_1M_2$ transition along with the $B_1\bar{B}_2$ production, as depicted in Fig. [1]. Note that the $B_s^0$ decays of $B_s^0 \rightarrow p\bar{p}K^+\pi^\mp$ and $p\bar{p}K^+K^-$ with $\bar{s}$ being replaced by $\bar{d}$ in $B^0 \rightarrow p\bar{p}\pi^+\pi^-$ and $p\bar{p}K^+\pi^\pm$ have also been found with the branching ratios of order $10^{-6}$ [9], respectively.

In this report, we will calculate the four-body baryonic $B$ decays in accordance with the decaying processes in Fig. [1] with the extraction of the $B \rightarrow M_1M_2$ transition form factors from the $B \rightarrow D^{(s)}M_1M_2$ and $B \rightarrow M_1M_2M_3$ decays and the adoption of the timelike baryonic form factors from the two-body and three-body baryonic $B$ decays. Our theoretical approach will be useful for the estimations of TPCs in $B \rightarrow B_1\bar{B}_2M_1M_2$ to be compared to future measurements by the LHCb.
FIG. 1. Feynman diagrams for the charmless four-body baryonic $B$ decays, where (a,b,c) depict the $\bar{B}_s^0 \to p\bar{p}M_1M_2$ decays, while (d,e) the $B^- \to \Lambda\bar{p}M_1M_2$ decays.

II. FORMALISM

In terms of the quark-level effective Hamiltonion for the charmless $b \to q_1\bar{q}_2q_3$ transition, the amplitudes of the four-body baryonic $B$ decays by the generalized factorization approach are derived as [10]

\begin{align*}
A_1(\bar{B}_s^0 \to p\bar{p}M_1M_2) &= \frac{G_F}{\sqrt{2}} \left\{ \langle p\bar{p}|\alpha^q_+(\bar{u}u)_V - \alpha^q_-(\bar{u}u)_A|0\rangle + \langle p\bar{p}|\beta^q_+(\bar{d}d)_V - \beta^q_-(\bar{d}d)_A|0\rangle \\
&+ (\alpha^q_1 - \alpha^q_{10}/2) \langle p\bar{p}|(\bar{q}q)_{V-A}|0\rangle \langle M_1M_2|(\bar{q}b)_{V-A}|\bar{B}_s^0\rangle \\
&+ \alpha^q_6 \langle p\bar{p}|(\bar{q}q)_{S+P}|0\rangle \langle M_1M_2|(\bar{q}b)_{S-P}|\bar{B}_s^0\rangle \right\}, \\
A_2(B^- \to \Lambda\bar{p}M_1M_2) &= \frac{G_F}{\sqrt{2}} \left\{ (\alpha^s_1 + \alpha^s_0) \langle \Lambda\bar{p}|(\bar{s}u)_{V-A}|0\rangle \langle M_1M_2|(\bar{u}b)_{V-A}|B^-\rangle \\
&+ \alpha^s_6 \langle \Lambda\bar{p}|(\bar{s}u)_{S+P}|0\rangle \langle M_1M_2|(\bar{u}b)_{S-P}|B^-\rangle \right\},
\end{align*}

where $G_F$ is the Fermi constant, $V_{ij}$ are the CKM matrix elements, and $(\bar{q}_1q_2)_{V(A)}$ and $(\bar{q}_1q_2)_{S(P)}$ stand for $\bar{q}_1\gamma_\mu(\gamma_5)q_2$ and $\bar{q}_1(\gamma_5)q_2$, respectively. The parameters $\alpha^q_i$ and $\beta^q_i$ in
Eq. (2) are given by
\[
\alpha_\pm^q = \alpha_2^q + \alpha_3^q \pm \alpha_5^q + \alpha_6^q, \quad \beta_\pm^q = \alpha_3^q \pm \alpha_5^q - \alpha_6^q / 2,
\]
\[
\alpha_{1,2}^q = V_{ub}V_{td}^*a_{1,2}, \quad \alpha_j^q = -V_{tb}V_{td}^*a_j, \quad \alpha_6^q = V_{tb}V_{td}^*2a_6,
\]
with \( q = (d, s) \) and \( j = (3, 4, 5, 9, 10) \), where \( a_i \equiv c_i^{\text{eff}} + c_{i+1}^{\text{eff}} / N_c^{\text{eff}} \) for \( i = \text{odd (even) with} \) the effective color number \( N_c^{\text{eff}} \) and Wilson coefficients \( c_i^{\text{eff}} \) in Ref. \[10\]. From \( A_1(\bar{B}_s^0 \to p\bar{p}M_1M_2) \) and \( A_2(B^- \to \Lambda\bar{p}M_1M_2) \) in Eq. (2), the allowed decays are
\[
\begin{align*}
\bar{B}_s^0 &\to p\bar{p}\pi^+\pi^-, \quad \bar{B}_s^0 \to p\bar{p}K^+\pi^-, \quad (q=d) \\
\bar{B}_s^0 &\to p\bar{p}\pi^-K^-, \quad \bar{B}_s^0 \to p\bar{p}K^+K^-, \quad (q=s) \\
B^- &\to \Lambda p\pi^+\pi^-, \quad B^- \to \Lambda pK^+K^-.
\end{align*}
\]
Note that the \( \bar{B}_s^0 \to p\bar{p}\pi^+K^- \) and \( \bar{B}_s^0 \to p\bar{p}K^+K^- \) decays have the matrix elements of \( \langle p\bar{p}|(\bar{s}s)_{V,A,S,P}|0 \rangle \) with the \( \bar{s}s \) quark currents, which eventually cause the terms of \( \alpha_{4,6,10}^s \) to give nearly zero contributions due to the OZI suppression of \( \bar{s}s \to p\bar{p} \) \[11\].

For the matrix elements in Eq. (2), the baryon-pair productions from the quark currents are given by \[5, 12\]
\[
\begin{align*}
\langle B_1 \bar{B}_2|q_1\gamma_\mu q_2|0 \rangle &= \bar{u} \left[ F_1\gamma_\mu + \frac{F_2}{m_{B_1} + m_{B_2}} i\sigma_\mu q_\mu \right] v, \\
\langle B_1 \bar{B}_2|q_1\gamma_\mu\gamma_5 q_2|0 \rangle &= \bar{u} \left[ g_A\gamma_\mu + \frac{h_A}{m_{B_1} + m_{B_2}} q_\mu \right] \gamma_5 v, \\
\langle B_1 \bar{B}_2|q_1 q_2|0 \rangle &= f_S \bar{u}v, \quad \langle B_1 \bar{B}_2|q_1 q_2|0 \rangle = g_P \bar{u}v,
\end{align*}
\]
where \( q = p_{B_1} + p_{B_2}, \quad t \equiv q^2, \quad u(v) \) is the (anti-)baryon spinor, and \( (F_{1,2}, g_A, h_A, f_S, g_P) \) are the timelike baryonic form factors. On the other hand, the \( B \to M_1M_2 \) transition matrix elements are parameterized as \[13\]
\[
\langle M_1M_2|q_1\gamma_\mu(1 - \gamma_5)b|B \rangle = h\epsilon_{\mu\alpha\beta\gamma}p_B^\alpha(p_{M_2} - p_{M_1})^\beta + irq_\mu + iw_+p_\mu + iw_-(p_{M_2} - p_{M_1}),
\]
where \( p = p_{M_2} + p_{M_1} \) and \( (h, r, w_\pm) \) are the form factors. Subsequently, one can also get \( \langle M_1M_2|\bar{q}_1(\gamma_5)b|B \rangle \) from Eq. (6) based on equations of motion. In terms of the approach of pQCD counting rules, the momentum dependences for the \( 0 \to B_1 \bar{B}_2 \) and \( B \to M_1M_2 \) transition form factors are given by \[14 - 17\]
\[
F_1 = \frac{C_{F_1}}{t^2}, \quad g_A = \frac{C_{g_A}}{t^2}, \quad f_S = \frac{C_{f_S}}{t^2}, \quad g_P = \frac{C_{g_P}}{t^2},
\]
\[
h = \frac{C_h}{t^2}, \quad w_+ = \frac{D_{w_+}}{t^2}, \quad w_- = \frac{D_{w_-}}{t^2},
\]
(7)
FIG. 2. Three angles of \( \theta_B, \theta_M, \) and \( \phi \) in the phase space for the four-body \( B \rightarrow B_1 \bar{B}_2 M_1 M_2 \) decays.

where \( \tilde{C}_i = C_i[\ln(t/\Lambda_0^2)]^{-\gamma} \) with \( \gamma = 2.148 \) and \( \Lambda_0 = 0.3 \) GeV. We note that since \( F_2 \) is derived to be \( F_2 = F_1/(t \ln[t/\Lambda_0^2]) \) [18], which is much less than \( F_1 \), while the small value of \( \mathcal{B}(\bar{B}^0 \rightarrow p \bar{p}) = (1.5^{+0.7}_{-0.5}) \times 10^{-8} \) [19, 20] causes a tiny \( C_{hA} \) in \( h_A = C_{hA}/t^2 \), we may not consider the effects from \( F_2 \) and \( h_A \). In addition, by following Ref. [16], we have neglected the terms related to \( r \) and \( w_+ \) in Eq. (5) due to the wrong parity [22].

The integration over the phase space of the four-body \( B(p_B) \rightarrow B_1(p_{B_1}) \bar{B}_2(p_{\bar{B}_2}) M_1(p_{M_1}) M_2(p_{M_2}) \) decay relies on the five kinematic variables, that is, \( s \equiv p^2, \) \( t \) and the three angles of \( \theta_B, \theta_M \) and \( \phi \). In Fig. 2 the angle \( \theta_{B(M)} \) is between \( \vec{p}_{B_1} (\vec{p}_{M_1}) \) of the \( B_1 \bar{B}_2 \) \( (M_1 M_2) \) rest frame and the line of flight of the \( B_1 \bar{B}_2 \) \( (M_1 M_2) \) system in the \( B \) meson rest frame, while the angle \( \phi \) is from the \( B_1 \bar{B}_2 \) plane to the \( M_1 M_2 \) plane, defined by the momenta of the \( B_1 \bar{B}_2 \) and \( M_1 M_2 \) pairs in the \( B \) rest frame, respectively. The partial decay width reads [23, 24]

\[
d\Gamma = \frac{|\tilde{A}|^2}{4(4\pi)^6m_B^3} X \alpha_B \alpha_M ds dt d\cos\theta_B d\cos\theta_M d\phi ,
\]

where \( X, \alpha_B \) and \( \alpha_M \) are given by

\[
X = \left[ \frac{1}{4} (m_B^2 - s - t)^2 - st \right]^{1/2} ,
\]

\[
\alpha_B = \frac{1}{t} \lambda^{1/2}(t, m_{B_1}^2, m_{\bar{B}_2}^2) ,
\]

\[
\alpha_M = \frac{1}{s} \lambda^{1/2}(s, m_{M_1}^2, m_{M_2}^2) ,
\]

respectively, with \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca \), while the allowed ranges of the five variables are given by

\[
(m_{M_1} + m_{M_2})^2 \leq s \leq (m_B - \sqrt{t})^2 , \quad (m_{B_1} + m_{\bar{B}_2})^2 \leq t \leq (m_B - m_{M_1} - m_{M_2})^2 ,
\]

\[
0 \leq \theta_B, \theta_M \leq \pi , \quad 0 \leq \phi \leq 2\pi .
\]

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III. NUMERICAL RESULTS AND DISCUSSIONS

For the numerical analysis, the CKM matrix elements in the Wolfenstein parameterization are presented as

\[ (V_{ub}, V_{tb}) = (A\lambda^3(\rho - i\eta), 1), \]
\[ (V_{ud}, V_{td}) = (1 - \lambda^2/2, A\lambda^3), \]
\[ (V_{us}, V_{ts}) = (\lambda, -A\lambda^2), \]  \hspace{1cm} (11)

with \((\lambda, A, \rho, \eta) = (0.225, 0.814, 0.120 \pm 0.022, 0.362 \pm 0.013)\) \[20\]. To estimate the non-factorizable effects in the generalized factorization approach \[10\], \(N_{c}^{\text{eff}}\) ranges from 2 to \(\infty\).

In Table II we show the values of \(a_i\) for the \(b \to d\) and \(b \to s\) transitions with \(N_{c}^{\text{eff}} = (2, 3, \infty)\), respectively.

According to the extractions of \((C_h, C_{w_\ast})\) in Refs. \[16, 17\], we fit the \(B \to \pi\pi\) transition form factors with the branching ratios of \(\bar{B}^0 \to D^{(\ast)}0\pi^0\pi^0\), \(B^- \to \pi^-\pi^0\pi^0\) and \(B^- \to K^-\pi^0\pi^0\), and the \(B \to (K, K\pi, K \bar{K})\) ones with those of \(B^- \to D^{(\ast)}0K^-K^0\), \(\bar{B}^0 \to D^0K^-\pi^+\) and \(B^- \to K^-K^+K^0\). Note that the contributions from the resonant \(B \to D^{(\ast)}(M_{0} \to )M_1M_2\), and \(B^- \to K^-K^+(M_{0} \to )M_1M_2\) decays with \(\rho^0, f^0_2(1270) \to \pi^+\pi^-\) or \(\phi \to K^+K^-\) have been excluded from the data. Unfortunately, the current observations of \(\mathcal{B}(B_s^0 \to \)

| \(a_i\)  | \(N_{c}^{\text{eff}} = 2\) | \(N_{c}^{\text{eff}} = 3\) | \(N_{c}^{\text{eff}} = \infty\) |
|--------|----------------|----------------|----------------|
| \(a_1\) | —              | —              | —              |
| \(a_2\) | 0.22           | 0.02           | -0.37          |
| \(10^4 a_3\) | -10.4 - 6.9i  | 72.4           | 237.9 + 13.9i  |
| \(10^4 a_4\) | -377.6 - 34.7i | -417.2 - 37.0i | -496.5 - 41.6i |
| \(10^4 a_5\) | -171.4 - 6.9i  | -65.8          | 145.3 + 13.9i  |
| \(10^4 a_6\) | -560.7 - 34.7i | -584.9 - 37.0i | -633.4 - 41.6i |
| \(10^4 a_7\) | -93.3 - 1.4i   | -99.5 - 1.4i   | -112.0 - 1.4i  |
| \(10^4 a_8\) | -18.5 - 0.7i   | 0.18 - 0.46i   | 37.5           |

TABLE I. The parameters \(a_i\) with \(N_{c}^{\text{eff}} = 2, 3, \) and \(\infty\) to estimate the non-factorizable effects in the generalized factorization.
$M_1M_2M_3$) are not sufficient for us to extract the $B^0_s \to M_1M_2$ transition form factors. As a result, we obtain

$$(C_h, C_{w-})|_{B \to \pi\pi} = (3.6 \pm 0.3, 0.7 \pm 0.2) \text{ GeV}^3,$$

$$(C_h, C_{w-})|_{B \to K\overline{K}(K\pi)} = (-38.9 \pm 3.3, 14.2 \pm 2.3) \text{ GeV}^3. \quad (12)$$

The timelike baryonic form factors in Eq. (3) can be related with the $SU(3)$ flavor and $SU(2)$ spin symmetries, such that $(C_{F_1}, C_{g_A}, C_{f_S}, C_{g_P})$ are recombined by a new set of constant parameters as

$$C_{F_1} = \frac{5}{3} C_{\parallel} + \frac{1}{3} C_{\perp}, \quad C_{g_A} = \frac{5}{3} C_{\parallel}^* - \frac{1}{3} C_{\perp}^*, \quad (\text{for } \langle p\bar{p}|\bar{u}\gamma_\mu(\gamma_5)u|0}\rangle)$$

$$C_{F_1} = \frac{1}{3} C_{\parallel} + \frac{2}{3} C_{\perp}, \quad C_{g_A} = \frac{1}{3} C_{\parallel}^* - \frac{2}{3} C_{\perp}^*, \quad (\text{for } \langle p\bar{p}|\bar{d}\gamma_\mu(\gamma_5)d|0}\rangle)$$

$$C_{f_S} = \frac{1}{3} \bar{C}_{\parallel}, \quad C_{g_P} = \frac{1}{3} \bar{C}_{\parallel}^*, \quad (\text{for } \langle p\bar{p}|\bar{d}(\gamma_5)d|0}\rangle)$$

$$C_{F_1} = \sqrt{\frac{3}{2}} C_{\parallel}, \quad C_{g_A} = \sqrt{\frac{3}{2}} C_{\parallel}^*, \quad (\text{for } \langle \Lambda\bar{p}\bar{d}\gamma_\mu(\gamma_5)u|0}\rangle)$$

$$C_{f_S} = -\sqrt{\frac{3}{2}} \bar{C}_{\parallel}, \quad C_{g_P} = -\sqrt{\frac{3}{2}} \bar{C}_{\parallel}^*, \quad (\text{for } \langle \Lambda\bar{p}\bar{s}(\gamma_5)u|0}\rangle) \quad (13)$$

with $C_{\parallel(\perp)}^* \equiv C_{\parallel(\perp)} + \delta C_{\parallel(\perp)}$ and $\bar{C}_{\parallel}^* \equiv \bar{C}_{\parallel} + \delta \bar{C}_{\parallel}$, in which $\delta C_{\parallel(\perp)}$ and $\delta \bar{C}_{\parallel}$ have been added to explain the large and unexpected angular distributions in $B^0 \to \Lambda\bar{p}\pi^+$ and $B^- \to \Lambda\bar{p}\pi^0$ [20, 27], to account for the fact that the $SU(3)$ flavor and $SU(2)$ spin symmetries at large $t$ ($t \to \infty$) [14] should be broken at $t \simeq m_B^2$ [27]. The extractions of the form factors by the data of $B^0 \to n\bar{p}D^{*+}, B^0 \to \Lambda\bar{p}D^{(*)+}, B^0 \to \Lambda\bar{p}\pi^+, B^- \to \Lambda\bar{p}(\pi^0,\rho^0), B^0_{(s)} \to p\bar{p}$ and $B^- \to \Lambda\bar{p}$ give [28]

$$(C_{\parallel}, \delta C_{\parallel}) = (154.4 \pm 12.1, 19.3 \pm 21.6) \text{ GeV}^4,$$

$$(C_{\perp}, \delta C_{\perp}) = (18.1 \pm 72.2, -477.4 \pm 99.0) \text{ GeV}^4,$$

$$\bar{C}_{\parallel}, \delta \bar{C}_{\parallel}) = (537.6 \pm 28.7, -342.3 \pm 61.4) \text{ GeV}^4, \quad (14)$$

where the added constants for the broken effects have been approved by the excellent agreement for $B(B^0 \to \Lambda\bar{p}K^+ + \bar{\Lambda}\bar{p}K^-)$ [29]. Subsequently, we evaluate the branching ratios of $B \to B_1\overline{B}_2M_1M_2$ as shown in Table III and draw the distributions vs. $m_{B_1B_2}$ in Fig. 3.

As seen in Table III although the predicted result of $B(B^- \to \Lambda\bar{p}\pi^+\pi^-) = (3.7^{+1.5}_{-1.0}) \times 10^{-6}$ is a little lower, it is consistent with the data in Eq. (11) by taking the uncertainties into account. With the replacement of $B^- \to \pi^+\pi^-$ by $B^- \to K^+K^-$, the $B^- \to \Lambda\bar{p}\pi^+\pi^-$ and
\( \Lambda \bar{p} K^+K^- \) decays share the same decaying configuration. We hence predict that \( B(B^- \to \Lambda \bar{p} K^+K^-) = (3.0^{+1.3}_{-0.9}) \times 10^{-6} \), which is accessible to the LHCb and BELLE experiments. Unlike the \( B^- \to \Lambda \bar{p} M_1 M_2 \) decays, where \( a_{1,4,6} \) are stable by ranging \( N_c^{eff} \) from 2 to \( \infty \), the tree-level dominant \( B^0 \to p \bar{p} \pi^+ \pi^- \) decay has \( \alpha_{\pm}^d \simeq V_{ub} V^{*}_{us} a_2 \) in Eq. (2) to be sensitive to the non-factorizable effects. Since the non-factorizable effects are uncomputable, according to the data of \( B(B^0 \to p \bar{p} \pi^+ \pi^-) = (3.0 \pm 0.3) \times 10^{-6} \) in Eq. (11), we obtain \( B(B^0 \to p \bar{p} \pi^+ \pi^-) = (3.0 \pm 0.9) \times 10^{-6} \), where \( a_2 = 0.26 \pm 0.01 \) with the tiny value of \( \delta a_2 = 0.01 \) from the new data is compatible to \( \mathcal{O}(0.2 - 0.3) \) from the two-body \( B \) and \( \Lambda_b \) and three-body baryonic \( B \) decays [30–32]. For the measured branching ratio of \( B^0 \to p \bar{p} \pi^+K^- + p \bar{p} \pi^-K^+ \), it is found that the contribution is mainly from the penguin-level dominant \( B^0 \to p \bar{p} \pi^+K^- \) mode. Note that \( a_{3,5} \) from \( \alpha_{\pm}^s \simeq \beta_{\pm}^s = -V_{ub} V^{*}_{ts} (a_3 \pm a_5 + a_9) \) are also sensitive to the non-factorizable effects. With \( N_c^{eff} = 3 \), we obtain \( B(B^0 \to p \bar{p} \pi^+K^-) = (6.6 \pm 2.4) \times 10^{-6} \), which suggests that the decay is free from the non-factorizable effects. In Table (11) we have

| branching ratios | our results | data |
|-----------------|------------|------|
| \( 10^6 B(B^- \to \Lambda \bar{p} \pi^+ \pi^-) \) | 3.7^{+1.2}_{-0.5} \pm 0.1 \pm 0.9 | 5.9 \pm 1.1 |
| \( 10^6 B(B^- \to \Lambda \bar{p} K^+K^-) \) | 3.0^{+1.1}_{-0.5} \pm 0.1 \pm 0.7 | — |
| \( 10^6 B(B^0 \to p \bar{p} \pi^+ \pi^-) \) | 3.0^{+0.5}_{-0.3} \pm 0.3 \pm 0.7 | 3.0 \pm 0.3 |
| \( 10^6 B(B^0 \to p \bar{p} \pi^+K^+) \) | 6.6 \pm 0.5 \pm 0.0 \pm 2.3 | 6.6 \pm 0.5 |

FIG. 3. Invariant dibaryon mass spectra for \( B^- \to \Lambda \bar{p} M_1 M_2 \) (left panel) and \( B^0 \to p \bar{p} M_1 M_2 \) (right panel), respectively.
included the data to constrain the non-factorizable effects, which results in $\delta N_c^{eff} = 0.06$. We note that the two spectra in Fig. 3 for $B^- \to \Lambda \bar{p} M_1 M_2$ and $B^0 \to p \bar{p} M_1 M_2$ present the threshold effects as the peaks around the threshold areas of $m_{\Lambda \bar{p}} \simeq m_\Lambda + m_\bar{p}$ and $m_{p \bar{p}} \simeq m_p + m_{\bar{p}}$, respectively, which are commonly observed in the three and four-body baryonic $B$ decays [9, 26].

Finally, we remark that we cannot explain the data of $B(\bar{B}_s^0 \to p \bar{p} K^\mp \pi^\mp, p \bar{p} K^+ K^-) = (1.5 \pm 0.7, 4.6 \pm 0.6) \times 10^{-6}$ measured by the LHCb [9] due to the lack of the information for the transition form factors of $\bar{B}_s^0 \to (K^+ \pi^-, K^+ K^-)$. This calls for the theoretical and experimental studies of the three-body mesonic $\bar{B}_s^0$ decays that could proceed with the $\bar{B}_s^0 \to M_1 M_2$ transitions, such as the $\bar{B}_s^0 \to D_s^- \pi^+ K^0$, $\bar{B}_s^0 \to D^{*0} \pi^+ K^- (K^+ K^-)$ and $\bar{B}_s^0 \to \rho^- \pi^+ K^0$ decays with one of the mesons to be a vector one, in order to extract both $(h, w_-)$ in Eq. (6). On the other hand, the observed $\bar{B}_s^0 \to D^0 K^+ \pi^-$ and $\bar{B}_s^0 \to D^0 K^+ K^-$ decays [20] are also important as they relate to $w_-$. 

IV. CONCLUSIONS

In sum, we have studied the charmless four-body baryonic $B \to B_1 B_2 M_1 M_2$ decays, where the primary decaying processes are regarded as the $B \to M_1 M_2$ transitions along with the baryon-pair productions. According to the new extractions of the $B \to M_1 M_2$ transition form factors from the three-body $B \to D^{(*)} M_1 M_2$ and $B \to M_1 M_2 M_3$ decays, we have shown that $B(B^- \to \Lambda \bar{p} \pi^+ \pi^-) = (3.7^{+1.5}_{-1.0}) \times 10^{-6}$ and $B(\bar{B}^0 \to p \bar{p} \pi^+ \pi^-, p \bar{p} \pi^+ K^-) = (3.0 \pm 0.9, 6.6 \pm 2.4) \times 10^{-6}$, which agree with the data. We have also predicted $B(B^- \to \Lambda \bar{p} K^+ K^-) = (3.0^{+1.3}_{-0.9}) \times 10^{-6}$ to be accessible to the LHCb and BELLE experiments. The study of $B \to B_1 B_2 M_1 M_2$ benefits the future test of T violation, as the T-odd triple momentum product correlation of $\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$ can be directly constructed.

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