Strongly Coupled Condensate of High Density Matter

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Abstract

Arguments are summarized, that neutral matter made of helium, carbon, etc., should form a quantum liquid at the above-atomic but below-nuclear densities for which the charged spin-0 nuclei can condense. The resulting substance has distinctive features, such as a mass gap in the bosonic sector and a gap-less spectrum of quasifermions, which determine its thermodynamic properties. I discuss an effective field theory description of this substance, and as an example, consider its application to calculation of a static potential between heavy charged impurities. The potential exhibits a long-range oscillatory behavior in which both the fermionic and bosonic low-energy degree of freedom contribute. Observational consequences of the condensate for cooling of helium-core white dwarf stars are briefly discussed.

Based on a talk given at the international workshop “Crossing the boundaries: Gauge dynamics at strong coupling” honoring the 60th birthday of M.A. Shifman

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Foreword

Like many in the audience, I first met Misha on the pages of journal publications, before meeting him in person. While working on an undergraduate thesis at Moscow University, I came across Misha’s review paper “Anomalies and Low-Energy Theorems of Quantum Chromodynamics” [1]. Impressions of that work were very distinct—a clear exposition of subtle field theory aspects of the quantum anomalies, culminating in creative applications to low-energy hadron phenomenology. The work stood out by its originality, depth, inspiration and balance of the formalism and applications—the remarkable signatures of Misha’s enormous contribution to theoretical physics at the forefront of both field theory and particle phenomenology.

I met Misha in person in Minneapolis in 1998. The discussion with him was very inspiring. Soon, in Aspen, we started to work on a project. A bit later I ceased the opportunity to get exposed to two years of a unique FTPI experience. We continued to work on and off on various projects since then. I value those works very highly, and feel privileged, as I’m sure many of you do too, for having such a collaborator.

Happy 60th Birthday Misha!

Description of charged condensate

Consider a neutral system of a large number of nuclei each having charge \( Z \), and neutralizing electrons. If average inter-particle separations in this system are much smaller than the atomic scale, \( \sim 10^{-8} \text{ cm} \), while being much larger than the nuclear scale, \( \sim 10^{-13} \text{ cm} \), neither the atomic nor nuclear effects will play any significant role. Moreover, the nuclei can also be treated as point-like particles.

In what follows we focus on spin-0 nuclei with \( Z \leq 8 \) (helium, carbon, oxygen), and consider the electron number-density in the interval \( J_0 \simeq (0.1 - 5 \text{ MeV})^3 \). Then the electron Fermi energy will exceed the electron-electron and electron-nucleus Coulomb interaction energy. Moreover, at temperatures below \( \sim 10^7 \text{ K} \), which are of interest here, the system of electron represents a degenerate Fermi gas.

Since the nuclei (we also call them ions below) are heavier, temperature at which they’ll start to exhibit quantum properties will be lower. Let us define the “critical” temperature \( T_c \), at which the de Broglie wavelengths of the ions begin to overlap

\[
T_c \simeq \frac{4\pi^2}{3m_Hd^2}, \quad d \equiv \left( \frac{3Z^3}{4\pi J_0} \right)^{1/3},
\]

where, \( m_H \) denotes the mass of the ion (the subscript "H" stands for heavy), and \( d \) denotes the average separation between the ions\(^1\).

\(^1\)The de Broglie wavelength above is defined as \( \lambda_{dB} = 2\pi/|k| \), where \( k^2/2m_H = 3k_B T/2 \). We define \( T_c \) as the temperature at which \( \lambda_{dB} \simeq d \). Note that this differs by a numerical factor of \( \sqrt{2\pi/3} \) from the standard definition of the thermal de Broglie wavelength, \( \Lambda \equiv \sqrt{2\pi/mk_B T} \), that appears in the partition function of an ideal gas of number-density \( n \) in the dimensionless combination \( \Lambda^3 n \).
Somewhat below $T_c$ quantum-mechanical uncertainties in the ion positions become greater than an average inter-ion separation. Hence the latter concept loses its meaning as a microscopic characteristic of the system; the ions enter a quantum-mechanical regime of indistinguishability. Then, the many-body wavefunction of the spin-0 ions should be symmetrized, and this would unavoidably lead to probabilistic “attraction” of the bosons to condense, i.e., to occupy one and the same quantum state. We refer the system of condensed nuclei and electrons as charged condensate.

In the condensate the scalars occupy a quantum state with zero momentum. Moreover, small fluctuations of the bosonic sector happen to have a mass gap, $m_\gamma = (Ze^2J_0/m_H)^{1/2}$, which exceeds $T_c$ by more than an order of magnitude. Therefore, once bosons are in the charged condensate, their phonons cannot be thermally excited. However, the gap-less fermionic degrees of freedom are thermally excited, and carry the most of the entropy of the entire system [2]-[5].

For further discussions it is useful to rewrite the expression for $T_c$ in terms of the mass density $\rho \equiv m_HJ_0$ measured in g/cm$^3$:

$$T_c = \rho^{2/3} \left( \frac{3.5 \cdot 10^2}{Z^{5/3}} \right) K,$$

where the baryon number of an ion was assumed to equal twice the number of protons, $A = 2Z$ (true for helium, carbon, oxygen...). Thus, for $\rho = 10^6$ g/cm$^3$ and helium-4 nuclei we get $T_c \simeq 10^6$ K, while for the carbon nuclei with the same mass density $T_c \simeq 2 \cdot 10^5$ K.

Temperature at which the condensation phase transition takes place, $T_{\text{condens}}$, need not coincide with $T_c$. Moreover, we would expect $T_{\text{condens}} \ll T_c$. Calculation of $T_{\text{condens}}$ from the fundamental principles of this theory is hard. However, we can obtain an interval in which $T_{\text{condens}}$ should fit. For this we introduce the following parametrization:

$$T_{\text{condens}} = \zeta T_c,$$

where $\zeta$ is an unknown dimensionless parameter that should depend on density more mildly than $T_c$ does. Numerically, however, this parameter should vary in the interval $0.1 \ll \zeta \ll 1$: The point $\zeta = 0.1$ would corresponds to the temperature of the Bose-Einstein (BE) condensation of a free gas for which, $T_{\text{condens}}^{BE} \simeq 1.3/m_Hd^2$, is known from the fundamental principles. The condensation temperature in our system should be higher than $T_{\text{condens}}^{BE}$ since the repulsion makes easier for the condensation to take place [6]. In our case, repulsive interactions between the bosons are strong – the Coulomb energy is at least an order of magnitude greater that any other energy scale in the system. Hence, we should expect $\zeta \gg 0.1$. On the other hand, given the definition of $T_c$, the parameter $\zeta$ cannot be greater than unity. In what follows we will retain $\zeta$ in our expressions, but use $\zeta \simeq 1$ when it comes to numerical estimates.

The condensation will take place after gradual cooling, only if $T_{\text{condens}}$ is greater than the temperature at which the substance could crystallize. A classical plasma
crystallizes when the Coulomb energy becomes about $\sim 180$ times greater than the average thermal energy per particle [7, 8, 9]. This gives the following crystallization temperature\(^2\)

$$T_{\text{cryst}} \simeq \rho^{1/3} \left(0.8 \cdot 10^3 Z^{5/3}\right) \, K.$$  \hspace{1cm} (4)

Note that the density dependence of $T_c$ is different from that of $T_{\text{cryst}}$—for higher densities $T_c$ grows faster, making condensation more and more favorable! One can define the “equality” density for which $T_{\text{condens}} = T_{\text{cryst}}$:

$$\rho_{\text{eq}} = \left(\frac{2.3}{\zeta}\right)^3 Z^{10} \, g/cm^3.$$  \hspace{1cm} (5)

For helium, $Z = 2$, and $\rho_{\text{eq}} \simeq 10^4 \, g/cm^3$; while for carbon, $Z = 6$, and $\rho_{\text{eq}} \simeq 10^9 \, g/cm^3$ (as mentioned above, we use $\zeta \simeq 1$). These results are very sensitive to the value of $\zeta$; for instance, $\rho_{\text{eq}}$ could be an order of magnitude higher if $\zeta \simeq 0.5$. Irrespective of this uncertainty, however, the obtained densities are in the right ballpark of average densities present in helium-core white dwarfs $\sim 10^6 \, g/cm^3$, (for carbon dwarfs, they’re closer to those expected in high density regions only [5].)

Is the charged condensate a ground state of the system at hand? For the higher values of the density interval considered, the crystal would not exist due to strong zero-point oscillations. At lower densities, the crystalline state has lower free energy (at least near zero temperature) due to more favorable Coulomb binding. Hence, the condensate can only be a metastable state. The question arises whether after condensation at $\sim T_{\text{condens}}$ the system could transition at lower temperatures $\sim T_{\text{cryst}}$ to the crystal state, as soon as the latter becomes available.

In the condensate, the boson positions are entirely uncertain while their momenta equal to zero. In order for such a system to crystallize later on, each of the bosons should acquire energy of the zero-point oscillations of crystal ions. As long as this energy, $\sim (Ze^2 J_0/m_H)^{1/2}$, is much greater than $T_{\text{cryst}}$, no thermal fluctuations can excite the condensed bosons to transition to the crystalline state. The latter condition is well-satisfied for all the densities considered in this work. There could, however, exist a spontaneous transition of a region of size $R_c$ to the crystallized state via tunneling. The value of $R_c$, and the rate of this transition, will be determined, among other things, by tension of the interface between the condensate and crystal state, which is hard to evaluate. However, for estimates the following qualitative arguments should suffice: the height of the barrier for each particle is $(Ze^2 J_0/m_H)^{1/2} = m_{\gamma}$, while the number of bosons in the $R_c$ region $\sim R_c^3 J_0/Z$. Hence, the transition rate should scale as $\exp(-m_{\gamma} J_0 R_c^4/Z)$. Since we expect that $R_c > 1/m_{\gamma}$, the rate is strongly suppressed for the parameters at hand.

\(^2\)The presented formula for the crystallization temperature is entirely classical. The temperature scale that determines the classical versus quantum nature of the crystallization transition is the Debye temperature $\theta_D \simeq 4 \cdot 10^3 \rho^{1/2} \, K$. Often, $\theta_D$ may significantly exceed $T_{\text{cryst}}$ [10]. In such cases, quantum zero-point oscillations should be taken into account. This seems to delay the formation of quantum crystal, lowering $T_{\text{cryst}}$ from its classical value at most by about $\sim 10\%$ [11]. Since this is a small change, we will ignore it in our estimates.
Effective field theory description

We use a low-energy effective field theory description to study the charged condensate. Even though realistic temperatures in the system may be well above zero, we focus on the zero-temperature limit. The relevance of this limit is justified \textit{a posteriori} and goes as follows: the spin-0 nuclei undergo the condensation to the zero-momentum state; their phonons cannot be excited since their gap, $m_\gamma$, is greater than $T_c$. On the other hand, gap-less near-the-Fermi-surface quasielectrons will be excited. Therefore, all the thermal fluctuations will end up being stored in the fermionic quasiparticles. For the latter, however, the finite temperature effects aren’t significant since their Fermi energy is so much higher, $T/J_0^{1/3} \ll 10^{-2}$. We note that the finite temperature effects, in a general setup with condensed bosons, were calculated in Refs. [12, 13].

We begin at scales that are well below the heavy mass scale $m_H$, but somewhat above the scale set by $\max[\mu_f, m_e]$, where $\mu_f$ and $m_e$ are the electron chemical potential and mass respectively. Hence the electrons are described by their Dirac Lagrangian, while for the description of the nuclei we will use a charged scalar order parameter $\Phi(x)$. As it was shown in [4], in a non-relativistic approximation for the nuclei, the effective Lagrangian proposed by Greiter, Wilczek and Witten (GWW) [14] in a context of superconductivity, is also applicable here, given that an appropriate reinterpretation of its variables and parameters is made.

The construction of the GWW Lagrangian is based on the following fundamental principles: it is consistent with the translational, rotational, Galilean and the global $U(1)$ symmetries, preserves the algebraic relation between the charged current density and momentum density, gives the Schrödinger equation for the order parameter in the lowest order, and is gauge invariant [14]. Combined with the electron dynamics the GWW Lagrangian reads (we omit for simplicity the Maxwell term):

$$L_{\text{eff}} = \mathcal{P} \left( \frac{i}{2} (\Phi^* D_0 \Phi - (D_0 \Phi)^*) \Phi - \frac{|D_j \Phi|^2}{2m_H} \right) + \bar{\psi} (i\gamma^\mu D^\mu_f - m_f) \psi, \quad (6)$$

where we use the standard notations for covariant derivatives with the appropriate charge assignments: $D_0 \equiv (\partial_0 - iZeA_0)$, $D_j \equiv (\partial_j - iZeA_j)$, $D^\mu_f = \partial^\mu + ieA^\mu$, while $\mathcal{P}(x)$ stands for a general polynomial function of its argument. The coefficients of this polynomial, $\mathcal{P}(x) = \sum_{n=0}^{\infty} C_n x^n$, are dimensionful parameters that are inversely proportional to powers of a short-distance cutoff of the effective field theory.\footnote{In general one should also add to the Lagrangian terms $\mu_{NR} \Phi^* \Phi$, $\lambda (\Phi^* \Phi)^2/m^2_H$, $\lambda_1 (\Phi^* \Phi) \bar{\psi} \psi / (m_H J_0^{1/3})$, and other higher dimensional operators that are consistent with all the symmetries and conditions that lead to (6) (the Yukawa term is not). Here $\mu_{NR}$ denotes a non-relativistic chemical potential for the scalars. These terms are not important for the low-temperature spectrum of small perturbations we’re interested in, as long as $\lambda, \lambda_1 \lesssim 1$ and $J_0 \ll m_H^3$. However, near the phase transition point it is the sign of $\mu_{NR}$ that would distinguish between the broken and symmetric phases, so these terms should be included for the discussion of the symmetry restoration. We also note that the scalar part of (6) is somewhat similar to the Ginzburg-Landau...
Once the basic Lagrangian is fixed, we introduce the electron chemical potential term $\mu F = \epsilon_F = [(3\pi^2 J_0)^{2/3} + m^2_f]^{1/2}$. This is the only term that at the tree level sets a frame in which the electron total momentum is zero.

There exists a homogeneous solution of the equations of motion that follow from the effective Lagrangian (6) \[3\]:

$$Z|\Phi|^2 = J_0, \quad A_\mu = 0, \quad \mathcal{P}'(0) = 1. \quad (7)$$

(We use the unitary gauge $\Phi = |\Phi|$). The condition $\mathcal{P}'(0) = 1$ is satisfied by any polynomial functions $\mathcal{P}(x)$ for which the first coefficient is normalized to unity

$$\mathcal{P}(x) = x + C_2 x^2 + \ldots. \quad (8)$$

The above solution describes a neutral system of negatively charged electrons of charge density $-eJ_0$, and positively charged scalar condensate of charge density $Ze\Phi^+\Phi = eJ_0$ [4, 5].

Calculation of the spectrum of small perturbations is straightforward. The Lagrangian density for the fluctuations in the quadratic approximation reads [2]

$$\mathcal{L}_2 = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} m_0^2 A_0^2 - \frac{1}{2} m_\gamma^2 A_j^2 + \frac{1}{2} A_0 \left(\frac{2m_\gamma m_J}{-\Delta}\right) A_0, \quad (9)$$

where $\Delta$ denotes the Laplacian, and the last term emerged due to mixing of $A_0$ with the fluctuation of the $|\Phi|$, which we integrated out. As before,

$$m_\gamma^2 \equiv \frac{Ze^2 J_0}{m_H}, \quad (10)$$

and $m_0^2 = m_\gamma^2 + C_2 e^2 J_0^2$. At this stage we retained the fermionic fluctuations only in the Thomas-Fermi approximation [3]; an important refinement of this approximation, discussed in [4], will be included below.

That there are no pathologies in (9), such as ghost and/or tachyons, can be seen by calculating the Hamiltonian density:

$$\mathcal{H} = \frac{\pi_j^2}{2} + \frac{F_{0j}^2}{4} + \frac{1}{2} (\partial_j \pi_j) \left( m_0^2 + \frac{4M^4}{-\Delta} \right)^{-1} (\partial_j \pi_j) + \frac{1}{2} m_\gamma^2 A_j^2. \quad (11)$$

Here, $M^2 \equiv m_H m_\gamma$ and $\pi_j \equiv -F_{0j}$. The Hamiltonian is positive semi-definite. Moreover, the spectrum has a mass gap determined by $m_\gamma$ (10). There are two transverse polarizations of a massive photon, as well as the longitudinal mode, the phonon, with the same mass $m_\gamma$ [2].

(GL) Lagrangian for superconductivity. However, there are significant differences between them, one such difference being that the coherence length in the GL theory is many orders of magnitude greater than the average interelectron separation, while in the present case, the “size of the scalar” $\Phi$ is smaller that the average interparticle distance.
The massive bosonic collective excitations give rise to exponentially suppressed contributions to the value of specific heat of the charged condensate since typically \( m_{\gamma} \gg T_c \). The suppression scales as \( \exp(-m_{\gamma}/T) \), where \( T \ll T_c \). This is in contrast with the crystal, where the dominant contribution to the specific heat comes from a gap-less phonon, and scales with temperature as \( T^3 \).

As to the electrons, their behavior is similar in both crystal and condensate cases. At temperatures of interest they form a degenerate Fermi gas with gap-less excitations near the Fermi surface. Their contribution to the specific heat scales linearly with temperature. In the case of crystallized substance this is sub-dominant to the specific heat due to the crystal phonon. For the charged condensate, however, the (quasi)electron fluctuations are the dominant contributors to the specific heat.

To study the effects of collective bosonic and fermionic modes, as an interesting example, we look at a potential between two impurity nuclei (say hydrogen, or helium-3) of charge \( Q_1 \) and \( Q_2 \). The calculation of the propagator that involves the light collective modes (for relativistic fermions) gives the following result [4]:

\[
V_{\text{stat}} = \alpha_{\text{em}} Q_1 Q_2 \left( \frac{e^{-M r}}{r} \cos(M r) + \frac{4\alpha_{\text{em}} k_F^5 \sin(2k_F r)}{M^8 r^4} \right). \tag{12}
\]

The first, exponentially suppressed term modulated by a periodic function, is due to cancellation between the screened Coulomb potential and that of a phonon [4]. The fact of such a cancellation, and that it could give rise to the oscillatory behavior of the exponentially screened potential was pointed out before in Ref. [15] in the context of superconductivity\(^4\).

Most important, however, is the second term in (12) that has a long-range [4]. It dominates over the exponentially suppressed term in (12) for scales of physical interest, and exhibits the power-like behavior modulated by a periodic function.

The potential (12) is a generalization of the Friedel potential to the case when in addition to the fermionic excitations there are also collective modes due to the charged condensate. The long-range oscillating term in (12) is also a result of a subtraction between the conventional Friedel term and the long-range oscillating term due to a phonon. As a result, its magnitude is suppressed compared to what it would have been in a theory without the condensed charged bosons [4] (see, [16] for the discussion of the conventional Friedel potential, and Ref. [13] for its recent detailed study in the presence of the charged condensate at finite temperature.)\(^5\)

The potential (12) is not sign-definite. In particular, it can give rise to attraction between like charges; this attraction is due to collective excitations of both fermionic and bosonic degrees of freedom. This represents a generalization of the Kohn-Luttinger [18] effect to the case where on top of the fermionic excitations the collective modes of the charged condensate are also contributing\(^6\).

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\(^4\)I’d like to thank Ki-Myeong Lee who recently brought the paper [15] to my attention.

\(^5\)Note that for spin-dependent interactions the same effects of the charged condensate would give a generalization of the Ruderman-Kittel-Kasuya-Yosida (RKKY) potential [17].

\(^6\)In the charged condensate Cooper pairs of electrons can also be formed, however, the cor-
Applications to White Dwarfs

The above described system of electrons and nuclei constitutes cores of white dwarf stars. Up to a factor of a few, these are roughly Earth’s size solar-mass objects; their mass density may range over $\sim (10^6 - 10^{10}) \text{ g/cm}^3$, most of them being near the lower edge of this interval. Since the dwarf stars exhausted thermonuclear fuel in their cores already, they evolve by cooling [19]; the ones that we consider in this work cool from $\sim 10^7 \text{ K}$ down to lower temperatures.

As a typical dwarf star cools down, the Coulomb interaction energy in a classical plasma of charged nuclei will significantly exceed their classical thermal energy, and the nuclei, in order to minimize energy, would organize themselves into a crystal lattice [20]. In most of these cases quantum effects of the nuclei should be negligible; for instance, the Debye temperature should be less than the temperature at which crystallization takes place, and the de Broglie wavelengths of the nuclei should be much smaller than the average internuclear separations. This indeed is the case in majority of white dwarf stars, the cores of which are composed of carbon and/or oxygen nuclei and span the interval of mass densities around $\sim (10^6 - 10^8) \text{ g/cm}^3$.

However, there exists a class of dwarf stars in which the nuclei enter the quantum regime before the classical crystallization process sets in [10, 11]. Among these, furthermore, there is a relatively small subclass of the dwarf stars for which the temperature $T_c$ is higher than the would-be crystallization temperature $T_{cryst}$ [5]. In such dwarf cores the charged condensation should be expected to take place.

White dwarfs composed of helium constitute a smaller sub-class of dwarf stars (see, [21, 22] are references therein); they exhibit best conditions for the charged condensation. Most of helium dwarfs are believed to be formed in binary systems, where the removal of the envelope off the dwarf progenitor red giant by its binary companion happened before helium ignition, producing a remnant that evolves to a white dwarf with a helium core. Helium dwarf masses range from $\sim 0.5 M_\odot$ down to as low as $(0.18 - 0.19) M_\odot$, while their envelopes are mainly composed of hydrogen.

Using the approach of [23], and following [5] we will consider an over-simplified model of a reference helium star of mass $M = 0.5 M_\odot$ with the atmospheric mass corresponding transition temperature, and the magnitude of the gap, are suppressed by a factor $\exp(-1/e_{eff}^2)$, where $e_{eff}^2$ is proportional to the value of the inter-electron potential that contains both screened Coulomb and phonon exchange. The fact that this potential has attractive domain, but is very small, is suggested by the static potential found in [4] (see also eq. (12) above); the latter is suppressed by a power of a large scale $M$. Furthermore, taking into account the frequency dependence of the propagator in the Eliashberg equation does not seem to change qualitatively the conclusion on a strong suppression of the Green’s function and pairing temperature.

Hence, even though the bosonic sector (condensed nuclei) is superconducting at reasonably high temperatures $\lesssim 10^6 \text{ K}$, interactions with gap-less fermions could dissipate the superconducting currents. Only at extremely low temperatures, exponentially close to the absolute zero, the electrons could also form a gap leading to superconductivity of the whole system. In the present work we consider temperatures at which electrons are not condensed into Cooper pairs, and ignore the finite temperature effects.
fractions of the hydrogen, and heavy elements (metallicity) respectively equal to

\[ X \simeq 0.99, \quad Z_m \simeq (0.0002 - 0.002) . \]  

(13)

The lower value of the metallicity \( Z_m \simeq 0.0002 \) is appropriate for the recently discovered 24 He WDs in NGC 6397 [22], but for completeness, we consider a wider range for this parameter.

It is straightforward to find the following expression for the cooling time of a star in the classical regime [23]

\[ t_{He} = \frac{k_B}{CAm_a} \left[ \frac{3}{5} (T_f^{\frac{5}{2}} - T_0^{\frac{5}{2}}) + Z \frac{\pi^2 k_B}{3 E_F} (T_f^{\frac{5}{2}} - T_0^{\frac{5}{2}}) \right] , \]  

(14)

where \( T_f \) and \( T_0 \) denote the final and initial core temperatures. The first term in the bracket on the right hand side corresponds to cooling due to classical gas of the ions and the second term corresponds to the contribution coming from the Fermi sea. The latter is sub-dominant in the range of final temperatures we are interested in (the factor \( Z \) in front of this term is due to \( Z \) electrons per ion). Since \( T_f \ll T_0 \), the age of a dwarf star typically doesn’t depend on the initial temperature. Neglecting the fermion contribution, we find time that is needed to cool down to critical temperature \( T_f = T_c \)

\[ t_{He} = \frac{3}{5} \frac{k_B T_c M}{Am_a L(T_c)} \simeq (0.76 - 7.6) \text{ Gyr} . \]  

(15)

Where an order of magnitude interval in (15) is due to the interval in the envelope metallicity composition given in (13). We also find the corresponding luminosities

\[ L(T_c) \simeq (10^8 \text{ erg/s}) \frac{M}{M_\odot} \left( \frac{T_c}{K} \right)^{7/2} \simeq 1.5 \cdot (10^{-4} - 10^{-5}) L_\odot , \]  

(16)

which are in the range of observable luminosities (\( L_\odot \simeq 3.84 \cdot 10^{33} \text{ erg/s} \)).

After the condensation, specific heat of the system dramatically drops as the collective excitations of the condensed nuclei become massive and “get extinct”. A contribution from the Fermi sea, which is strongly suppressed by the value of Fermi energy, becomes the dominant one. The phase transition itself would take some time to complete, and the drop-off in specific heat will not be instantaneous.

In the zeroth approximation, we can regard the transition to be very fast, and retain only the fermion contribution to specific heat below \( T_c \). Then, the expression for the age of the star for \( T_f < T_c \), reads as follows

\[ t'_{He} = \frac{k_B}{CAm_a} \left[ \frac{3}{5} (T_c^{\frac{5}{2}} - T_0^{\frac{5}{2}}) + Z \frac{\pi^2 k_B}{3 E_F} (T_f^{\frac{5}{2}} - T_0^{\frac{5}{2}}) \right] . \]  

(17)

Notice the difference of (17) from (14) – in the former \( T_f < T_c \) and it is \( T_f \) that enters as final temperature in the fermionic part, while \( T_c \) should be taken as the final temperature in the bosonic part.
From the ratio of ages, \( \eta = \frac{t_{He}}{t'_{He}} \), for two identical helium dwarf stars, with and without the interior condensation, we deduce that the charged condensation substantially increases the rate of cooling—the age could be twenty times less than it would have been without the condensation phase [5].

The condensation of the core would induce significant deviations from the classical curve for helium white dwarfs. What is independent of the uncertainties involved in these discussions, is the fact that the luminosity function (LF) will experience a significant drop-off after the charged condensation phase transition is complete. This is due to the “extinction” of the bosonic quasiparticles below the phase transition point. In fact, the LF will drop by a factor of \( \sim 200 \). This may be relevant for an explanation of the observed termination of a sequence of the 24 He WD’s found in [22]. See Ref. [5] for more details.

Finally, the magnetic properties of the charged condensate, which are similar to those of type II superconductor, and in particular admit the presence of Abrikosov’s vortices, were studied in Ref. [24]. As was shown there, only very strong magnetic fields, \( \gtrsim 10^7 \text{ Gauss} \), will be able to penetrate the dwarf cores in the vortices, while weaker fields will be entirely expelled from it.

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