Recasting $H_0$ tension as $\Omega_m$ tension at low $z$

Eoin Ó Colgáin$^{a,b}$

$^a$ Asia Pacific Center for Theoretical Physics, Postech, Pohang 37673, Korea

$^b$ Department of Physics, Postech, Pohang 37673, Korea

Abstract

Inspired by the recent observation that local measurements of the Hubble constant $H_0$ and the Planck CMB value based on $\Lambda$CDM show a discrepancy at $4.4\sigma$ [1], we study $\Lambda$CDM at low redshift. Concretely, we expand $\Lambda$CDM perturbatively at small $z$ and perform a two-parameter fit of the distance modulus to Pantheon data for a running cut-off $z_{\text{max}} \leq 0.3$. Moving beyond the Hubble constant $H_0$, we shift focus to matter density $\Omega_m$, noting foremost that its best-fit value is sensitive to the cut-off. For $z_{\text{max}} > 0.1$, the uncertainties in $\Omega_m$ decrease and the difference with the Planck value $\Omega_m = 0.315 \pm 0.007$ becomes noticeable. In particular, in the range $0.1 \leq z_{\text{max}} < 0.16$, the best-fit value is negative and the discrepancy with the Planck value approaches $4\sigma$. Restricting to $z_{\text{max}}$ where the best-fit value is positive and physical, the discrepancy is reduced to $3.1\sigma$. For high-energy theorists, the analysis appears to support the de Sitter Swampland conjecture.
1 Introduction

Ever since the seminal Riess, Macri et al. local determination of the Hubble constant $H_0$ to 2.4% uncertainty [2], the tension with the Planck value based on $\Lambda$CDM [3], dubbed $H_0$ tension, has been difficult to ignore. Starting with a difference of 3.4$\sigma$, we have witnessed a steady growth in the discrepancy to the point that the statistical significance is now 4.4$\sigma$ [1]. Despite the immense success of $\Lambda$CDM - built on the assumption that our Universe is described by a cosmological constant and cold dark matter - this brings us ever closer to a point in time when the standard model of cosmology may be due a potential rewrite. That being said, in spite of the tension, the fact that measurements of $H_0$ based on radically different datasets at different redshift agree so well is truly remarkable.

In line with steadily more accurate determinations of $H_0$ over recent years, we have witnessed a tangible migration of theorists. Going beyond the assumption of a cosmological constant, there is now no shortage of dynamical dark energy models on the market [4]. At one end of the spectrum (of speculation), one finds the de Sitter Swampland conjecture [5,6], which boldly claims that de Sitter vacua belong to the “Swampland” [7] of inconsistent low-energy theories coupled to gravity, and for this reason, de Sitter vacua are ruled out. Bearing in mind that de Sitter is an attractor for $\Lambda$CDM, the conjecture is also in tension with $\Lambda$CDM. The de Sitter Swampland conjecture is controversial (see [8] for a Swampland review), but the implication for $\Lambda$CDM appears clear.

Our goal in this short note is to quantify tension between $\Lambda$CDM and data in light of $H_0$ tension and the de Sitter Swampland. There are a number of tensions between $\Lambda$CDM and existing datasets, for example [9–11], but the most striking clash can be found in a local measurement of $H_0$ [1]. Inspired by $H_0$ tension, we will perform an analysis of $\Lambda$CDM at low redshift $z$, which we will fit to the Pantheon compilation of type Ia supernovae [12] (see [13] for an alternative perspective). It is worth stressing that in contrast to other cosmological studies, e. g. [14–16], here we will adopt a minimal approach and work within a single dataset. The Pantheon dataset consists of 1048 supernovae in the redshift range 0.01 - 2.26 and the idea is simply to impose a cut-off at $z_{\text{max}}$ and restrict the analysis to supernovae below this value [1].

Let us justify why this is an interesting exercise. First, at late times or low redshift, $\Lambda$CDM is expected to be described by an analytic solution to the Friedmann equation,

$$H(z) = H_0 \sqrt{1 - \Omega_m + \Omega_m(1+z)^3},$$

(1.1)

where the Hubble constant $H_0$ and the matter density $\Omega_m$ are the only free constant parameters. The expression is valid for a FLRW metric that is three-flat, so we have neglected spatial curvature. A connected observation is that $H_0$ is an overall factor that is insensitive to $z$ and can be determined at $z = 0$, so when Riess et al. determine $H_0$ they can do so in principle without assuming a cosmology. However, here the cosmology, namely $\Lambda$CDM,

\footnote{Given a cosmology, this is the natural way to fit data since one integrates the luminosity distance up to a given value of redshift.}
is captured by the constant $\Omega_m$. Secondly, given that (1.1) is analytic, this means that if one expands at small $z$, it is inevitable one picks up information about $\Omega_m$, so that one can starting probing $\Lambda$CDM through the value of $\Omega_m$. Of course, if the dataset is too sparse, there will be considerable uncertainty in the best-fit value of $\Omega_m$, but Pantheon has 630, 832, and 1025 supernovae below $z = 0.3$, $z = 0.5$ and $z = 1$, respectively. Third, it is insightful to see how the best-fit value of the cosmological parameters changes if we only had access to supernovae below a given redshift. A good model is one where the parameters are robust to such changes, especially within a 1 $\sigma$ confidence window.

Concretely, in this note we impose a series of cut-offs $z_{\text{max}}$, where we can expand the cosmology in $z$ and achieve reasonable results. We confine our attention to $z \leq 0.3$, so we still have access to sufficient supernovae, and importantly perturbation theory holds. For each $z_{\text{max}}$ we peform a two-parameter fit. From the constant term, assuming a value for the absolute magnitude $M$ of type Ia supernovae, this determines $H_0$\footnote{A proper determination of $H_0$ requires a knowledge of the absolute magnitude $M$ of type Ia supernovae, and this is not possible without using Cepheids to break the degeneracy between $H_0$ and $M$. See \cite{17, 18} for earlier studies in this direction.} while from the linear term one can extract $\Omega_m$. Since $\Omega_m$ is the only term we can properly determine, we focus on it. For $z_{\text{max}} < 0.1$, our analysis shows that there are too few supernovae to have confidence in the best-fit value of $\Omega_m$ (the 1 $\sigma$ confidence intervals are large), but at higher $z_{\text{max}}$, the confidence intervals contract and the discrepancy with the Planck value $\Omega_m = 0.315 \pm 0.007$ becomes apparent and approaches 4 $\sigma$ in the range $0.14 < z_{\text{max}} < 0.15$. From $z_{\text{max}} = 0.16$ onwards, where the best-fit value of $\Omega_m$ is positive, and thus physical, the discrepancy with the Planck value is approximately 3.1$\sigma$, but decreases with increasing $z_{\text{max}}$.

## 2 Data Fitting

In this section we introduce our assumptions and proceed to fit the $\Lambda$CDM model to the Pantheon supernovae data \cite{12} at low $z$. Our first input is that $\Lambda$CDM is described by only two parameters at late times through the Hubble parameter (1.1). By definition, the luminosity distance is

$$d_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')}$$ \hspace{1cm} (2.1)

where $c$ is the speed of light, and this serves as an input in the distance modulus

$$\mu = m_B - M = 5 \log_{10} \left( \frac{d_L}{10 \text{pc}} \right) = 25 + 5 \log_{10} \left( \frac{d_L}{\text{Mpc}} \right).$$ \hspace{1cm} (2.2)

Here $m_B$ denotes the apparent magnitude and $M$ is the absolute magnitude. In the last equality we have converted between parsecs and megaparsecs, so that we have the right units to describe $H_0$. 
The Pantheon supernovae dataset \cite{12} includes observations up to redshift \( z \leq 2.26 \), but understandably becomes sparse at higher redshift. For this reason, in confronting the model \( (1.1) \) with the data it is sufficient to focus on low redshift where there is still enough supernovae present to make concrete statements with meaningful statistics. Naturally, we could choose to integrate the luminosity distance \( (2.1) \) numerically, but by exploiting perturbation theory and Taylor expansion, our methods are accessible to anyone with a high-school education in mathematics: over-complicating the matter seems rather pointless.

Our first goal is to expand the inverse of the Hubble parameter \( H(z) \) \( (1.1) \) to second order, thereby simplifying the integral in the luminosity distance,

\[
\frac{1}{H(z)}_{\text{quad}} = \frac{1}{H_0} \left( 1 - \frac{3\Omega_m}{2} z - \frac{3(4\Omega_m - 9\Omega_m^2)}{8} z^2 + \ldots \right),
\]

while at the same time convince the reader that this simple approximation is not overly crude in a range of redshift up to \( z = 0.3 \). To this end it is useful to define the difference

\[
\Delta(\Omega_m) \equiv \int_0^{z'=0.3} (H(z')^{-1} - H(z')^{-1}_{\text{quad}}) \, dz'.
\]

This allows us to record the difference for a range of \( \Omega_m \) and confirm that the difference between the exact expression and its quadratic approximation is not too large for redshifts up to \( z = 0.3 \) and matter density of the order \( \Omega_m \simeq 0.3 \), the expected Planck value.

| \( \Omega_m \) | \( H_0 \Delta(\Omega_m) \) |
|----------------|--------------------------|
| 0.01           | -8.5 \times 10^{-6}     |
| 0.1            | 3.4 \times 10^{-5}      |
| 0.2            | 2.3 \times 10^{-4}      |
| 0.3            | 4.5 \times 10^{-4}      |

Having established a degree of confidence in the approximation, one can expand in terms of \( z \) and integrate, finding that

\[
\mu(z) = 25 + 5 \log_{10} \left( \frac{cz}{H_0} \right) + \frac{5(4-3\Omega_m)}{4 \ln(10)} z - \frac{5(16+16\Omega_m-27\Omega_m^2)}{32 \ln(10)} z^2 + \ldots
\]

We will now simply fit \( (2.5) \) to the low redshift Pantheon data taking into account the apparent magnitude \( m_B \) and its error \( \delta m_B \), while using the nominal value \( M = -19.3 \) \cite{19}. As explained, only by assuming a value for \( M \) can we extract information about \( H_0 \). However, the value of \( H_0 \) will not be the primary focus of this note and we simply can check that we get a reasonable ball park figure.

Anyway, let us proceed and fit the distance modulus \( (2.5) \) against the data for a given maximum value of redshift \( z_{\text{max}} \). We employ \( \chi^2 \) fitting, where the quantity to be minimised is

\[
\chi^2 = \sum_{z_i \leq z_{\text{max}}} \left( \frac{\mu_i - \mu(z_i)}{\delta m_B(z_i)} \right)^2.
\]

\footnote{Alternatively, we could choose \( M = -19.23 \), which can be inferred from \( H_0 \) determined in \cite{2}.}
The sum is performed over all data points at redshift $z_i$ up to the cut-off and we have weighted the sum by the error in the apparent magnitude at each redshift $\delta m_B(z_i)$. In practice we use the Python package lmfit for curve-fitting and an estimation of the confidence levels.

Adopting the value $z_{\text{max}} = 0.15$ and assuming the constant value $M = -19.3$, the resulting best fit to the data can be found in Figure 1. To the naked eye this looks like a reasonable fit and from the covariance matrix returned by the fitting procedure one can make an estimate of the $1\sigma$ confidence interval:

$$H_0 = 73.199 \pm 0.438, \quad \Omega_m = -0.068 \pm 0.088.$$ \hfill (2.7)

**Figure 1:** Plot of best fit distance modulus $\mu(z)$ against data

In spite of our crude assumptions on $M$, this value is consistent with best estimate of $[\text{1}]$, $H_0 = 74.03 \pm 1.42$ km s$^{-1}$ Mpc$^{-1}$. However, this is not the main story here. The puzzling feature is not only that the best-fit value of $\Omega_m$ is negative, thereby making it unphysical, but it also deviates significantly from the Planck value, $\Omega_m = 0.315 \pm 0.007$. Recall that we have simply expanded the $\Lambda$CDM cosmology (1.1) perturbatively at small $z$, where it is valid to do so, and beyond the $\log_{10}(z)$ and constant term that describes $H_0$ or $M$, the expansion is guaranteed to pick up a value for $\Omega_m$. Moreover, as we can see from the $1\sigma$

\footnote{Note that $\mu = m_B - M$, so for us $\mu = m_B + 19.3$}
confidence interval at \( z_{\text{max}} = 0.15 \), we have enough supernovae to state that this disagrees with the Planck value and the difference certainly exceeds 1 \( \sigma \).

Repeating the procedure for a range of \( z_{\text{max}} \), one can build up a better picture of how the \( \Omega_m \) and its 1 \( \sigma \) confidence interval varies with \( z_{\text{max}} \). The output of this exercise can be found in Figure 2. As is hopefully clear from the plot, beyond \( z_{\text{max}} = 0.1 \) the uncertainties in \( \Omega_m \), which were understandably quite large at lower redshift, have now contracted as is evident from the 1 \( \sigma \) confidence interval. It is clear that there is no constant value of \( \Omega_m \) that runs through all the confidence intervals, and even if it did, it marks a departure from the Planck value (solid line). Moreover, the fact that the model appears to favour unphysical values in the range 0.1 \( \leq z_{\text{max}} < 0.16 \) may be a cause for concern. Nevertheless, even if one restricts to \( z_{\text{max}} = 0.16 \), the best-fit value is \( \Omega_m = 0.0351 \) and the 3 \( \sigma \) confidence intervals are \((-0.202, 0.2936)\), which indicates that the Planck value is disfavoured by the data.

![Figure 2: Variation in \( \Omega_m \) with \( z_{\text{max}} \)](image)

### 3 Discussion

Planck CMB analysis based on \( \Lambda \)CDM is in conflict with local determinations of the Hubble constant due to Riess et al. and the discrepancy is now 4.4 \( \sigma \). Separately, arguments from high-energy theory through the Swampland program raise questions about a key assumption in the Planck analysis, namely the \( \Lambda \)CDM cosmological model. In this note, we shifted the focus of tension from \( H_0 \) to \( \Omega_m \) and based on the Pantheon supernovae dataset with simple analysis have demonstrated the existence of a similar tension in \( \Omega_m \) at low \( z \). It is worth stressing that perturbation theory holds at low \( z \) and in imposing a cut-off \( z_{\text{max}} \) we have

\footnote{Within our stated assumptions, the best-fit value of \( H_0/M \) appears to show running with \( z_{\text{max}} \) towards smaller values. In the light of \cite{20}, it would be interesting to study this more.}
committed no crime. The running of \( \Omega_m \) with \( z_{\text{max}} \), the unphysical best-fit values and the difference with the Planck value indicate that the model is breaking down at low \( z \).

One interesting feature of the discrepancy with Planck, which is evident from Figure 2, is that the difference appears to decrease with increasing redshift. Borrowing analysis from Pantheon [12], we know that if one utilises the full dataset the best-fit value of \( \Omega_m \) approaches the Planck value. Somewhat remarkably, although \( \Lambda \text{CDM} \) appears to break down locally at low redshift, in other words precisely where Riess et al. are performing their determination of \( H_0 \), as one goes to higher \( z \), \( \Lambda \text{CDM} \) becomes a better and better approximation. Reading between the lines, this suggests that the \( H_0 \) tension is simply an artifact of assuming \( \Lambda \text{CDM} \) is a good approximation to the Universe at late times and the tension will presumably be alleviated by the introduction of new physics, potentially dynamical dark energy.

This last point is worth stressing: our analysis appears to point to a late time solution to the \( H_0 \) tension. This of course has important implications for early time solutions, including ongoing attempts to alleviate the \( H_0 \) tension by incorporating an additional neutrino species [21]. Finally, as touched upon earlier, related cosmological tensions exist outside of \( H_0 \) and a discrepancy with Planck CMB has also been observed in the cosmic shear \( S_8 \equiv \sigma_8 (\Omega_m/0.3)^{0.5} \) [9–11], which tellingly depends on both the matter density and the amplitude of matter fluctuations \( \sigma_8 \). For the redshift range in the recent study [11], our analysis points to \( \Omega_m \) values that are at most 33% lower with respect to Planck values. For \( \alpha = 0.5 \), and assuming no change in \( \sigma_8 \), this translates into values of \( S_8 \) that are at most 18% lower and consistent with the observed lower values of \( S_8 \).

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