Investigations on the charmless decays of $X(3872)$ in intermediate meson loops model

Yan Wang, Qi Wu, Gang Li, Wen-Hua Qin, Xiao-Hai Liu, Chun-Sheng An, and Ju-Jun Xie

1College of Physics and Engineering, Qufu Normal University, Qufu 273165, China
2School of Physics and Center of High Energy Physics, Peking University, Beijing 100871, China
3Department of Physics, School of Science, Tianjin University, Tianjin 300350, China
4School of Physical Science and Technology, Southwest University, Chongqing 400715, China
5Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
6School of Nuclear Science and Technology, University of Chinese Academy of Sciences, Beijing 101408, China
7School of Physics and Microelectronics, Zhengzhou University, Zhengzhou, Henan 450001, China

The charmless decay processes of $X(3872)$ provide us a good platform to study the nature and the decay mechanism of $X(3872)$. Based on a molecular nature of $X(3872)$ as a $DD^*$ bound state, we have investigated the charmless decays $X(3872) \rightarrow VV$ and $VP$ via intermediate $D^0\bar{D}^0 + c.c.$ meson loops, where $V$ and $P$ stand for light vector and pseudoscalar mesons, respectively. We discuss three cases, i.e., pure neutral components ($\theta = 0$), isospin singlet ($\theta = \pi/4$) and neutral components dominant ($\theta$ is a phase angle describing the proportion of neutral and charged constituents. The proportion of neutral and charged constituent have an influence on the decay widths of $X(3872) \rightarrow VV$ and $VP$. With the coupling constant of $X(3872)$ to the $DD^*$ channel obtained under the molecule ansatz of $X(3872)$ resonance, the predicted decay widths of $X(3872) \rightarrow VV$ are about tens of keVs, while the decay width can reach a few hundreds of keVs for $X(3872) \rightarrow VP$. The dependence of these ratios between different decay modes of $X(3872)$ to $VV$ and $X(3872) \rightarrow VP$ to the mixing angle $\theta$ is also investigated. It is expected that the theoretical calculations here can be tested by future experiments.

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I. INTRODUCTION

In 2003, the $X(3872)$ state was first observed by the Belle Collaboration in the $J/\psi\pi^+\pi^-$ invariant mass spectrum of the $B \rightarrow KX(3872) \rightarrow K\pi^+\pi^-J/\psi$ decay [1]. Then, it was confirmed in the $J/\psi\pi^+\pi^-$ channel from $p\bar{p}$ collisions by CDF and D0 Collaborations [2, 3], and $e^+e^-$ collisions by BABAR Collaboration [4, 5]. Its quantum numbers were determined to be $I^G(J^{PC}) = 0^+(1^{++})$ by LHCb Collaboration [6]. There are two salient features of $X(3872)$, one is that it has very narrow width ($\Gamma_X < 1.2$ MeV), the other one is that its mass is extremely close to the mass threshold of $D^0\bar{D}^0$ channel.

The interpretation of the nature of $X(3872)$ is still an open question. Since its quantum numbers are $J^{PC} = 1^{++}$ and its mass is very close to the $D^0\bar{D}^0$ threshold, one naturel explanation is that it is a $D\bar{D}$* hadronic molecule as discussed in Refs. [7, 34]. In general, a hadronic molecule can couple to other components which have the same quantum numbers. For instance, the possibility of a charmonium $c\bar{c}$ excited state admixture was investigated in Refs. [35, 36]. It was also pointed out that the $D^{*+}D^{*0}$ and $D^{*+}_{s}D_{s}^{*0}$ components are necessary to explain the branching ratio of $X(3872)$ to $J/\psi\rho$ and $J/\psi\omega$ [37, 39]. On the other hand, the $X(3872)$ is also considered as a tetraquark state [40–43]. However, searching for the charged partners of $X(3872)$ shows negative results [44]. Besides, the $X(3872)$ was also viewed as a conventional charmonium state [45–47].

In Ref. [18], the isospin violating decay process of $X(3872) \rightarrow J/\psi\rho$ was estimated using final state interactions (FSI) by consider intermediate $DD^*$ meson loop, where it was found that the contribution from FSI is tiny. The radiative decays $X(3872) \rightarrow \gamma J/\psi\rho'$ were investigated in Refs. [18, 32, 49], and the results support the molecular picture of $X(3872)$. While in Refs. [50, 51], the pionic transition from $X(3872)$ to $\chi_{cJ}$ was studied. In Ref. [50] it was concluded that these decay rates exhibit significantly different patterns depending on a pure charmonium or a multi-quark structure of $X(3872)$.

All these above theoretical studies of $X(3872)$ focus on its charmful decay modes. To better understand the nature of $X(3872)$, the study of its other decay modes is needed. For example, the charmless decays can also provide us a good platform to further study the nature of $X(3872)$. In this work, we have considered the charmless decay modes of $X(3872)$ to $VV$ and $VP$ ($V$ and $P$ stand for the vector meson and pseudoscalar meson) via intermediate charmless meson loops in an effective Lagrangian approach.

This article is organized as follows. In Sec. II based on a molecular nature of $X(3872)$ as a $DD^*$ bound state, we present the related decay amplitudes obtained with the effective Lagrangians constructed in the heavy quark limit and chiral symmetry. In Sec. III we show our nu-
As a result, the coupling constants appearing in Eq. (3) with its neutral and charged components, respectively. Mesons as in Refs. [55, 56], we obtain the mass difference of the following Lagrangian:

\[ g_{\text{eff}}^2 = 16\pi c_l^2(m_1 + m_2)^2 \sqrt{2c/\mu}, \quad (1) \]

where \( \mu = m_1m_2/(m_1 + m_2) \) is the reduced mass of \( m_1 \) and \( m_2 \).

Assuming that the \( X(3872) \) is a \( S \)-wave molecular state with quantum numbers \( J^{PC} = 1^{++} \) given by the superposition of \( D^0\bar{D}^{*0} \) and \( D^0D^{*+} \) hadronic configurations as

\[ |X(3872)\rangle = \frac{\cos \theta}{\sqrt{2}}(D^0\bar{D}^0 + D^0\bar{D}^{*0}) + \frac{\sin \theta}{\sqrt{2}}(D^{*+}D^- + D^-D^{*+}), \quad (2) \]

where \( \theta \) is a phase angle describing the proportion of neutral and charged constituents. For example, \( \theta = 0 \) stands for \( X(3872) \) as a pure \( D^0\bar{D}^0 / D^0\bar{D}^{*0} \), while \( \theta = \pi/4 \) and \( \theta = -\pi/4 \) correspond to the isospin singlet and isospin triplet states, respectively. Then, one can parameterize the coupling of \( X(3872) \) to the charmed mesons in terms of the following Lagrangian:

\[ \mathcal{L}_{X(3872)} = \frac{g_n}{\sqrt{2}} X_\mu^0(D^{0\mu}\bar{D}^0 + D^{0\mu}\bar{D}^{*0\mu}) + \frac{g_c}{\sqrt{2}} x^{\nu}_\mu(D^{*+}\mu D^- + D^{+}\mu D^{*+} - D^{*+}\mu D^-), \quad (\cdots) \]

where \( g_n \) and \( g_c \) are the coupling constants of \( X(3872) \) with its neutral and charged components, respectively.

Using the masses of the \( X(3872) \) and the charmed mesons as in Refs. [55, 56], we obtain the mass difference between the \( X(3872) \) and the \( D^0\bar{D}^0 / D^0\bar{D}^{*0} \) (neutral) and \( D^{*+}\bar{D}^0 / D^{+}\bar{D}^{*+} \) (charged) \( X(3872) \) to the charmed mesons in terms of the heavy amplitudes as follows \(^1\),

\[ g_n = |g_{\text{eff}}^n| \cos \theta, \quad g_c = |g_{\text{eff}}^c| \sin \theta. \quad (6) \]

**FIG. 1:** Diagrams contributing to the charmless decay \( X(3872) \to VV \) with \( DD^* + \text{c.c.} \) as intermediate states.

With the above \( DD^* \) molecular picture for \( X(3872) \), these \( X(3872) \to VV \) and \( VP \) decays can proceed via \( X(3872) \to DD^* \to VV \) or \( VP \) through triangle loop diagrams, which are shown in Figs. 1 and 2, respectively.

**FIG. 2:** Diagrams contributing to the charmless decay \( X(3872) \to VP \) with \( DD^* + \text{c.c.} \) as intermediate states.

\(^1\) These coupling constants are assumed to be real.
quark limit and chiral symmetry,
\[
\mathcal{L} = -ig_{D^*} D\!\!\!/D\!\!\! \cdot P_{ij} D_{\mu j}^{*\mu} - D_{\mu i}^{*\mu} P_{ij} D_{\mu j}^{*\mu},
\]

\[
\frac{1}{2} g_{D^*} D\!\!\!/D\!\!\! \cdot P_{ij} D_{\mu j}^{*\mu} + i g_{D^*} D\!\!\!/D_{\mu j}^{*\mu} (\mathbb{V}^{\mu})_{ij}
\]

\[
-2 f_{D^*} D\!\!\!/D\!\!\! \cdot P_{ij} D_{\mu j}^{*\mu} (\mathbb{D}^{\mu})_{ij} + \mathbb{H}c.,
\]

(7)

with the convention \(\varepsilon_{0123} = 1\), where \(P\) and \(\mathbb{V}_\mu\) are 3 × 3 matrices for the octet pseudoscalar and nonet vector mesons, respectively,
\[
P = \begin{pmatrix}
\pi^0 a^0 & \pi^+ a^+ & K^+
\pi^- a^- & K^0
\end{pmatrix},
\]

\[
\mathbb{V} = \begin{pmatrix}
\rho^+ b^+ & K^{*+}
\rho^- b^- & K^{*0}
\phi^* & \phi
\end{pmatrix},
\]

(8)

(9)

In the heavy quark and chiral limits, the couplings of the charmed meson to the light vector mesons have the relationship [60, 61],
\[
g_{D^* D \!\!\! \cdot V} = g_{D^* \!\!\! \cdot V} = \frac{\beta g_{V}}{\sqrt{2}},
\]

\[
f_{D^* D \!\!\! \cdot V} = \frac{f_{D^* \!\!\! \cdot V}}{m_{D^*}^{*\mu}},
\]

\[
g_{D^* D^*} = 2 \frac{g}{f_{\pi}},
\]

\[
g_{D^* D^*} = \frac{g_{D^* D^*}}{m_{D^*}^{*\mu}},
\]

(10)

(11)

(12)

(13)

In this work, we take parameters \(\beta = 0.9\), \(\lambda = 0.56\) GeV\(^{-1}\), \(g = 0.59\), and \(g_{V} = m_{D}/f_{\pi}\) with \(f_{\pi} = 132\) MeV, as used in previous works [60, 62].

Then one can easily write the explicit transition amplitudes for \(X(3872)(p_1) \to [D^{(*)}(q_1)D^{(*)}(q_3)]D^{(*)}(q_2) \to V_1(p_2)V_2(p_3)\) shown in Fig. 2 as follows:

\[
\mathcal{M}_a = \int \frac{d^4q_2}{(2\pi)^4} 2f_{D^*} D\!\!\!/D\!\!\! \cdot V(q_1 - q_2)\mu \varepsilon_2^\mu
\]

\[
\times \frac{1}{q_1^2 - m_1^2} \left(\frac{g^{*\mu} - q_2^\mu q_3^\mu}{q_2^2 - m_2^2}\right) F(q^2),
\]

(14)

\[
\mathcal{M}_b = \int \frac{d^4q_2}{(2\pi)^4} 2f_{D^*} D\!\!\!/D\!\!\! \cdot V(q_1 - q_2)\mu \varepsilon_2^\mu
\]

\[
\times \frac{1}{q_1^2 - m_1^2} \left(\frac{g^{*\mu} - q_2^\mu q_3^\mu}{q_2^2 - m_2^2}\right) F(q^2),
\]

(15)

\[
\mathcal{M}_c = \int \frac{d^4q_2}{(2\pi)^4} 2f_{D^*} D\!\!\!/D\!\!\! \cdot V(q_1 - q_2)\mu \varepsilon_2^\mu
\]

\[
\times \frac{1}{q_1^2 - m_1^2} \left(\frac{g^{*\mu} - q_2^\mu q_3^\mu}{q_2^2 - m_2^2}\right) F(q^2),
\]

(16)

\[
\mathcal{M}_d = \int \frac{d^4q_2}{(2\pi)^4} 2f_{D^*} D\!\!\!/D\!\!\! \cdot V(q_1 - q_2)\mu \varepsilon_2^\mu
\]

\[
\times \frac{1}{q_1^2 - m_1^2} \left(\frac{g^{*\mu} - q_2^\mu q_3^\mu}{q_2^2 - m_2^2}\right) F(q^2),
\]

(17)

where \(p_1 (\varepsilon_1)\), \(p_2 (\varepsilon_2)\) and \(p_3 (\varepsilon_3)\) are the four-momenta (polarization vector) of the initial state \(X(3872)\), final state \(V_1\) and \(V_2\), respectively. \(q_1, q_2\) and \(q_3\) are the four-momenta of the up, right and down charmed mesons in the triangle loop, respectively.

The explicit transition amplitudes for \(X(3872)(p_1) \to [D^{(*)}(q_1)D^{(*)}(q_3)]D^{(*)}(q_2) \to V(p_2)V(p_3)\) shown in Fig. 2 are as follows:

\[
\mathcal{M}_a = \int \frac{d^4q_2}{(2\pi)^4} 2f_{D^*} D\!\!\!/D\!\!\! \cdot V(q_1 - q_2)\mu \varepsilon_2^\mu
\]

\[
\times \frac{1}{q_1^2 - m_1^2} \left(\frac{g^{*\mu} - q_2^\mu q_3^\mu}{q_2^2 - m_2^2}\right) F(q^2),
\]

(18)

\[
\mathcal{M}_b = \int \frac{d^4q_2}{(2\pi)^4} 2f_{D^*} D\!\!\!/D\!\!\! \cdot V(q_1 - q_2)\mu \varepsilon_2^\mu
\]

\[
\times \frac{1}{q_1^2 - m_1^2} \left(\frac{g^{*\mu} - q_2^\mu q_3^\mu}{q_2^2 - m_2^2}\right) F(q^2),
\]

(19)
\[ M_c = \int \frac{d^4q_2}{(2\pi)^4} [g_{\epsilon_1\epsilon_2}\xi][g_{D\gamma\gamma}(q_1 - q_2)\xi g_{\epsilon_1\epsilon_2}] \\
-4f_{\gamma\gamma}(p_{2\pi0}g_{\epsilon_1}\xi - p_{2\phi^0}g_{\epsilon_2}\xi)[g_{D\gamma\gamma}(p_{2\pi0}p_{2\phi^0})] \\
\times \left( q_1^2 q_2^2 / m_1^2 \right) \left( q_1^2 - q_2^2 q_2 / m_2^2 \right) \\
\times \frac{i}{q_2^2 - m_2^2} \mathcal{F}(q^2), \]  

where \( \mathcal{F}(q^2) \) is the form factor introduced to depict the off-shell effects of the exchanged mesons as well as the structure effects of the involved mesons. The form factor \( \mathcal{F}(q^2) \) is parameterized as

\[ \mathcal{F}(q^2) = \left( \frac{m^2 - \Lambda^2}{q^2 - \Lambda^2} \right)^n, \]

normalized to unity at \( q^2 = m^2 \) [61], where \( m \) and \( q \) are mass and momenta of the exchanged mesons. The cutoff \( \Lambda \) can be further reparameterized as \( \Lambda = m_{D\gamma\gamma} + \alpha A_{QCD} \) with \( A_{QCD} = 0.22 \text{ GeV} \). The model parameter \( \alpha \) is usually expected to be of order of unity [61, 63–66], but its concrete value cannot be estimated by the first principle. In practice, the value of \( \alpha \) is usually determined by comparing theoretical estimates with the corresponding experimental measurements. However, no charmless decay mode of \( X(3872) \) is known so far. For the rescattering processes studied in this work, it is found that the monopole form (\( n = 1 \)) or dipole form (\( n = 2 \)) for \( \mathcal{F}(q^2) \) is utilized, the numerical results are much sensitive to the values of parameter \( \alpha \), and we have to use a very small value, otherwise, these partial decay widths will be very large, even more than the total width of \( X(3872) \). In order to avoid too large dependence of the parameter \( \alpha \), we take \( n = 3 \) in the numerical calculations.

### III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will mainly discuss three cases where \( \theta = 0, \pi/6 \), and \( \pi/4 \). When \( \theta = 0 \), it indicates that \( X(3872) \) is a pure bound state with only neutral component. When \( \theta = \pi/4 \), the proportions of the neutral and charged components are the same. There are both neutral and charged components at \( \theta = \pi/6 \), but the proportion of the neutral component is dominant.

In Fig. 3(a) we plot the \( \alpha \)-dependence of the partial decay widths of \( X(3872) \rightarrow VV \) and \( X(3872) \rightarrow VP \) with \( \theta = 0 \), respectively. In the range of \( \alpha = 0.6 \sim 1.2 \), the predicted partial decay widths of \( X(3872) \rightarrow VV \) are about a few KeV, while the partial decay widths can reach a few tens of KeV for \( X(3872) \rightarrow VP \). Since the \( X(3872) \rightarrow K^{*0}K^{0} \) transition proceeds via \([D^+D^{*-}]D_s^{(*)}\) intermediate mesons, while the \( X(3872) \rightarrow K^{*+}K^{-} \) transition proceeds via \([D^0D^{*-}]D_s^{(*)}\) intermediate mesons. So in the case of \( \theta = 0 \), there is no neutral \( K^{*0}K^{0} \) channel as shown in Fig. 3(a). The same reason for \( X(3872) \rightarrow K^{*0} \) in Fig. 3(b). From Fig. 3(a), one can see that the partial decay width of \( X(3872) \rightarrow \rho \rho \) is larger than those of \( X(3872) \rightarrow K^{*+}K^{-} \) and \( \omega \omega \) decay modes. This is because both the charged \( \rho^+ \rho^- \) and neutron \( \rho^0 \rho^0 \) channels contribute to the \( \rho \rho \) channel. While for the \( X(3872) \rightarrow \rho^0 \rho^0 \) decay, its partial decay width is almost equal to the decay of \( X(3872) \rightarrow \omega \omega \). In addition, for the \( X(3872) \rightarrow K^{*+}K^{-} \) decays, there are only contributions from the exchanging of charged charm mesons. In the case of \( \theta = 0 \), only neutral charmed meson loops contribute to the isospin-violating channel \( X(3872) \rightarrow \rho \omega \). As a result, the obtained decay widths are almost the same as that of the channel \( X(3872) \rightarrow \omega \omega \).

In Fig. 3(b) we plot the \( \alpha \)-dependence of the partial decay widths of \( X(3872) \rightarrow VV \) and \( X(3872) \rightarrow VP \) with \( \theta = \pi/4 \). In the range of \( \alpha = 0.6 \sim 1.2 \), the predicted partial decay widths of \( X(3872) \rightarrow VV \) are about a few tens of KeV, while the partial decay widths can reach several hundred KeV for \( X(3872) \rightarrow VP \). The behavior is similar to that of Fig. 3. Since the case of \( \theta = \pi/4 \) corresponds to equal neutral and charged components in \( X(3872) \), so the channels \( X(3872) \rightarrow K^{*+}K^{-} \) and \( X(3872) \rightarrow K^{*0} \) have non-zero decay widths. The \( X(3872) \rightarrow K^{*0} \) transition proceeds via \([D^+D^{*-}]D_s^{(*)}\).
intermediate mesons, while the $X(3872) \to K^{*+}K^{*-}$ transition proceed via $[D^0\bar{D}^{*0}]D^+_s$ intermediate mesons. The mass of $X(3872)$ is much closer the mass threshold of $D^0\bar{D}^{*0}$ than $D^+D^{*-}$, so the threshold effects of $X(3872) \to K^{*+}K^{*-}$ will be larger than that of $X(3872) \to K^{*0}\bar{K}^{*0}$. However, the couplings constant values obtained from Eq. (6) have the relation $g_\pi < g_\rho$. Thus with the same value of $\alpha$, the obtained partial decay width of $X(3872) \to K^{*0}\bar{K}^{*0}$ is about several times larger than that of $X(3872) \to K^{*+}K^{*-}$. However, for the $X(3872) \to \rho\rho$ decay, there are contributions from the exchanging both charged charm mesons and neutral charm mesons, and these two contributions give the instructive interference of the decay amplitudes. A similar situation occurs in $X(3872) \to VP$ as shown in Fig. 4(b). A similar situation occurs in $X(3872) \to VP$ as shown in Fig. 5(b). In the case of $\theta = \pi/4$, the charged and neutral charmed meson loops should cancel out exactly in the isospin symmetry limit for the isospin-violating channel $X(3872) \to \rho^0\omega$. In other words, the mass difference between the $u$ and $d$ quark will lead to $m_{\rho^0} \neq m_{D^0\bar{D}^{*0}}$ due to the isospin symmetry breaking. As a result, the charged and neutral charmed meson loops cannot completely cancel out, and the residue part will contribute to the isospin-violating amplitudes. The partial widths of the isospin-violating channel $X(3872) \to \omega\rho^0$ as shown in Fig. 5(a) are suppressed.

Using the center value of the total decay width of $X(3872)$ that was reported recently by the LHCb Collaboration [67, 68], we obtain the branching ratio for $X(3872) \to VP$ in the cases of $\theta = 0, \pi/6$ and $\pi/4$, respectively. We take the range of $\alpha$ as $0.6 \sim 1.2$, then the numerical results are shown in the Table. Our theoretical numerical results show that with the increase of $\theta$, the partial decay widths of $K^{*+}K^{*-}$ and

FIG. 4: The $\alpha$-dependence of decay widths (in unit of keV) of $X(3872) \to VP$ with $\theta = \pi/4$.

FIG. 5: The $M_{X(3872)}$-dependence of the decay widths (in unit of keV) of $X(3872) \to VP$ with $\alpha = 1.0$. 
we have fixed the value of $X$ to $\cos \theta$ and we can define the following ratios in terms of the mass of $X$:

$$R_1 = \frac{\Gamma(X(3872) \to \rho \pi^0)}{\Gamma(X(3872) \to \omega \omega)},$$

$$R_2 = \frac{\Gamma(X(3872) \to \rho \rho)}{\Gamma(X(3872) \to \omega \omega)},$$

$$R_3 = \frac{\Gamma(X(3872) \to K^{*+}K^{-} + c.c.)}{\Gamma(X(3872) \to \omega \omega)},$$

$$R_4 = \frac{\Gamma(X(3872) \to K^{*0}K^{*0} + c.c.)}{\Gamma(X(3872) \to \omega \omega)}. \quad (22)$$

The ratios $R_1$ in terms of $\alpha$ are plotted in Fig. 6. The results of Fig. 6 show that the ratios are completely insensitive to this dependence. This stabilities of the ratios in terms of $\alpha$ indicate a reasonably controlled cutoff for each channel by the form factor to some extent. On the other hand, one can see that, in Fig. 6, there are extremely strong dependence of the ratio on the isospin mixing angle $\alpha$, which is of more fundamental significance than the parameter $\alpha$. This stability stimulate us to study the mixing angle $\theta$ dependence.

Next, we turn to the dependence of these ratios defined in Eqs. (22) and (23) to the mixing angle $\theta$ with a fixed $\alpha$. In Fig. 7 we present the theoretical results of the ratio $R_i$ ($i = 1, 2, 3, 4$) defined in Eq. (22) and $r_i$ ($i = 1, 2$) defined in Eq. (23) as a function of the mixing angle $\theta$ with a fixed value $\alpha = 1.0$. It is interesting to note that the results of the ratio $R_2 = \frac{\Gamma(X(3872) \to \rho \rho)}{\Gamma(X(3872) \to \omega \omega)}$ are not dependent on the

\[ K^{*+}K^{-} + c.c. \]
and neutral components dominant ($\theta$ quark symmetry and chiral symmetry. We can see that charged constituents. We explore the rescattering mechanism to determine the value of the mixing angle. Based on a molecular nature of $X(3872)$, we have investigated the charmless decays of $X(3872) \rightarrow VV$ and $VP$. For $X(3872)$, we considered three cases, i.e., pure neutral components ($\theta = 0$), isospin singlet ($\theta = \pi/4$) and neutral components dominant ($\theta = \pi/6$), where $\theta$ is a phase angle describing the proportion of neutral and charged constituents. We explore the rescattering mechanism within the effective Lagrangian based on the heavy quark symmetry and chiral symmetry. We can see that although the decay widths increase with the increase of $\alpha$ when we fix the phase angle $\theta$, our theoretical results show that the cutoff parameter $\alpha$ dependence of the partial widths is not drastically sensitive, which indicates the dominant mechanism driven by the intermediate meson loops with a fairly good control of the ultraviolet contributions. When $X(3872)$ is a pure neutral bound state, the predicted partial decay widths of $X(3872) \rightarrow VV$ are about a few keV, while the partial decay widths can reach a few tens of $\text{keV}$ for $X(3872) \rightarrow VP$. When there are both neutral and charged components in $X(3872)$, the predicted decay widths of $X(3872) \rightarrow VV$ are about tens of $\text{keV}$, while the decay widths can reach a few hundreds of $\text{keV}$ for $X(3872) \rightarrow VP$.

Moreover, the dependence of these ratios between different charmless decay modes of $X(3872)$ to the charged and neutral mixing angle for the $X(3872)$ in the molecular picture is also investigated, which may be tested by future experiments and can be used to determine the value of the mixing angle.

**IV. SUMMARY**

Based on a molecular nature of $X(3872)$, we have investigated the charmless decays of $X(3872) \rightarrow VV$ and $VP$. For $X(3872)$, we considered three cases, i.e., pure neutral components ($\theta = 0$), isospin singlet ($\theta = \pi/4$) and neutral components dominant ($\theta = \pi/6$), where $\theta$ is a phase angle describing the proportion of neutral and charged constituents. We explore the rescattering mechanism within the effective Lagrangian based on the heavy quark symmetry and chiral symmetry. We can see that although the decay widths increase with the increase of $\alpha$ when we fix the phase angle $\theta$, our theoretical results show that the cutoff parameter $\alpha$ dependence of the partial widths is not drastically sensitive, which indicates the dominant mechanism driven by the intermediate meson loops with a fairly good control of the ultraviolet contributions. When $X(3872)$ is a pure neutral bound state, the predicted partial decay widths of $X(3872) \rightarrow VV$ are about a few keV, while the partial decay widths can reach a few tens of $\text{keV}$ for $X(3872) \rightarrow VP$. When there are both neutral and charged components in $X(3872)$, the predicted decay widths of $X(3872) \rightarrow VV$ are about tens of $\text{keV}$, while the decay widths can reach a few hundreds of $\text{keV}$ for $X(3872) \rightarrow VP$.

Moreover, the dependence of these ratios between different charmless decay modes of $X(3872)$ to the charged and neutral mixing angle for the $X(3872)$ in the molecular picture is also investigated, which may be tested by future experiments and can be used to determine the value of the mixing angle.

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FIG. 8: (a). The ratio \( R_i \) \((i = 1, 2, 3, 4)\) defined in Eq. (23) as a function of the mixing angle \( \theta \) with \( \alpha = 1.0 \). (b). The ratio \( r_i \) \((i = 1, 2)\) defined in Eq. (24) as a function of the mixing angle \( \theta \) with \( \alpha = 1.0 \).

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