Multi-objective optimization and decision-making of the combined control law of guide vane and pressure regulating valve for hydroelectric unit

Tianyu Zhang | Jianzhong Zhou | Xin Yang | Honghui Li

School of Civil and Hydraulic Engineering, Huazhong University of Science and Technology, Wuhan, China

Correspondence
Jianzhong Zhou, School of Civil and Hydraulic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China.
Email: jz.zhou@hust.edu.cn

Funding information
National Natural Science Foundation of China, Grant/Award Number: 51809099; Key Program of the National Natural Science Foundation of China, Grant/Award Number: U1865202

Abstract
This paper aimed to investigate the multi-objective optimization and decision-making of the combined control law of guide vane (CCLGV) and pressure regulating valve (PRV) for hydroelectric unit under load rejection. Firstly, the multi-objective optimization problem of CCLGV-PRV is formulated. Then, a new multi-objective gravitation search algorithm with non-dominated screening and chaos mutation (NMOGSA) is adopted to solve it. Moreover, an improved entropy weight method and relative objective proximity method are applied to select suitable solution. Finally, a comprehensive comparative study is carried out to reveal the mechanism between the CCLGV-PRV and several basic parameters. The results indicate that replacing surge tank with PRV can effectively solve the contradictory between water pressure at the bottom of volute and rotational speed under load rejection. The NMOGSA shows excellent performance in solving the multi-objective optimization problem of CCLGV-PRV and get a group of Pareto set. The improved entropy weight and relative objective proximity method make the final scheme clear. The rise rate of rotational speed is completely determined by the guide vane (GV), and the water pressure is jointly determined by the control law of the GV and PRV. The two-stage closing and two-stage opening (TCTO) is the final recommendation for CCLGV-PRV.

KEYWORDS
combined control law, hydroelectric unit, multi-objective optimization and decision-making, pressure regulating valve

1 | INTRODUCTION

With the increasing electrical power system based on renewables, the role of hydropower is becoming more and more important.\textsuperscript{1,2} The hydropower stations convert the potential energy of water into kinetic energy to generate electricity, which is sent to various types of loads via high-voltage transmission lines.\textsuperscript{3-5} The consumer’s electricity consumption changing makes the power balance of electric system destroyed during the transmission of electric energy, hydroelectric unit entering transient process. Transient process refers that the hydroelectric unit...
transits from one stable state to another due to the change of load during the operation of the electric system.\textsuperscript{6–8}

Load rejection is one of the most dangerous conditions in many transient processes. It is a situation that the generator circuit breaker disconnects accidently and the resistance moment of the unit instantly disappears for some reason. Under this condition, the water pressure in the pipeline especially at the bottom of volute and the rotation speed of the unit are much larger than those in normal operation,\textsuperscript{9} which will cause the penstock burst, speed runaway and major losses.

It is seeking a reasonable control strategy that is an important topic to reduce the investment in hydropower station while ensuring the safe operation of hydropower station.\textsuperscript{10,11} Only by altering the closing law of guide vane cannot be the smooth transition between different operating conditions in engineering be realized. Surge tank and pressure regulating valve (PRV) are commonly applied to improve the quality of transient process,\textsuperscript{12} reducing the water pressure and rotation speed within a reasonable range.

For surge tank, its construction requires comprehensive consideration of penstock layout, geology, topography, operating conditions, mechanical-electrical characteristics, and other factors.\textsuperscript{13} The surge tank will be far away from the plant in the case that mountain slope is relatively slow or is not suitable for excavation, increasing the investment in penstock. In order to avoid the contradiction between the site selection of surge tank and the investment of penstock for the complex geologic, the PRV can be considered to replace the surge tank after sufficient demonstration in medium and small hydropower stations to improve transient process quality. The construction cycle of PRV is short and the layout form is flexible, which can save about 90% investment compared with the surge tank.

For more than 40 years, experts, scholars, and practitioners in China's water conservancy and hydropower industry have continuously developed the “valve instead of well” technology in theory and practice. Up to now, about 13 hydropower stations in China have adopted the technology of “valve instead of well.” The basic situation of hydropower stations with “valve instead of well” is that the length of water diversion system is 1127–6800 m, and the single capacity is 1.6–12.5 MW. The length of the water diversion system of hydropower stations using the technology of “valve instead of well” abroad is about 7000 m and the single capacity is 30–50 MW. Its scale is larger than that in China.\textsuperscript{14}

Pressure regulating valve plays a role in limiting the water pressure at the bottom of volute under load rejection in hydropower station with long diversion system. Hydroelectric unit under load rejection, the guide vane closes and the PRV opens quickly. PRV begins to close slowly after the guide vane is completely closed. The water in the pipeline is discharged downstream through PRV, by which the water pressure in pipeline is released. The flow change in the water diversion pipeline is slowed down at the same time and the rise of water pressure is also controlled.

For hydropower stations equipped with PRV, the control problem of it arises, which plays a very key role in transient process. Therefore, there are two control devices of discharge in the diversion system together with guide vane (GV), which makes the problem more complicated. It is necessary to consider the combined control of GV and PRV for a better effectiveness. However, quite few researches on its combined control strategy are presented at home and abroad, ignoring three important issues. Firstly, the dynamic quality of transient process is improved only by changing the closing law of GV, without considering the combined control strategy between the GV and PRV. Secondly, the combined control law of guide vane and pressure regulating valve (CCLGV-PRV) optimized, several parameters of transient process are simply weighted and transformed into a single objective optimization problem. Thirdly, attention is paid to the multi-objective optimization solution set, but the decision-making of Pareto set is missing.

Specially, considering the rotational speed and water pressure, Zhang et al.\textsuperscript{15} designed a three-stage closing law with real-time and adaptive adjustment of inflection point position by incoming the feedback signal of GV. Liu et al.\textsuperscript{16} studied the influence of emergent closing laws of GV on transient process under load rejection and compared the single-stage and two-stage closing law with time delay. The time-delay closing law compared with the traditional one, Yu et al.\textsuperscript{17} makes an analysis about its superiority. Shu et al.\textsuperscript{18} introduced the closing law optimized by genetic algorithm that the opening and time of the closing law is selected as the optimization variables, and the rotational speed of hydroelectric unit and water pressure at the bottom of volute is weighted into the comprehensive objective function. Liu et al.\textsuperscript{19} developed a closing law of GV first delay and then straight shut down under load rejection, and optimized it to reduce the water pressure at the bottom of volute and rotational speed of hydroelectric unit. Zhou et al.\textsuperscript{20} converted into a comprehensive objective weighted the water pressure at the bottom of the volute, the rise rate of rotational speed and the water pressure at the entrance of draft tube, and optimized the closing law of GV by the simulated annealing algorithm. Lai et al.\textsuperscript{21} applied MOASA algorithm to optimize the three-stage closing law of GV, aiming at high-head pumped storage power station, and obtained the optimal solution set. Xu et al.\textsuperscript{22} used an enhanced multi-objective gravitational search algorithm to optimize the closing law of GV for pump turbine, but there is not much explanation...
how to filter the optimal solution from Pareto set. Having optimized the closing law of GV in pump storage power station used NSGA-III algorithm, Liu et al.\textsuperscript{23} analyzed the mechanism of key parameters under load rejection. Zhao et al.\textsuperscript{24} proposed the novel small signal model and co-optimization strategy is presented to coordinate the stability-tracking conflict, which have potential value in the stability of renewable energy systems with multiple hydropower units.

According to the existing literature, the optimization of dynamic quality during transient process is limited to changing the closing law of GV. Meanwhile, not considering the contradictory relations among the objectives, most of current researches transformed it into single objective problem by means of weighting lineally. Moreover, in the researches of multi-objective optimization, the decision-making on the optimal solution set is ignored. Whereupon, this paper aimed to study multi-objective optimization and decision-making of CCLGV-PRV for hydroelectric unit under load rejection. The motivation and innovation are as follows:

1. The multi-objective optimization problem of CCLGV-PRV is proposed for the first time.
2. A new multi-objective gravitation search algorithm with non-dominated screening and chaos mutation (NCMOGSA) is adopted to solve the optimization problem of CCLGV-PRV.
3. A multi-objective decision-making method that improved entropy weight and relative objective proximity, is applied to select suitable solution for engineering requirement.
4. The mechanism between the CCLGV-PRV and several technical parameters is revealed based on a comprehensive comparative study.

The rest of the paper is organized as follows. In Section 2, the problem formulation of the CCLGV-PRV optimization is proposed, including establishment of numerical model of hydropower station, proposal of CCLGV-PRV, multi-objective function and constrain conditions. In Section 3, the basic concept of NCMOGSA, constraint handling method and the procedure of multi-objective optimization of CCLGV-PRV using NCMOGSA are introduced. In Section 4, the improved entropy weight and relative objective proximity method are presented. In Section 5, an experimental study and a comprehensive comparative analysis are provided. In Section 6, summarize the paper and the conclusions including outlook of this field are given.

2 | PROBLEM FORMULATION

As an indispensable part in the design stage of hydropower station, calculation of transient process plays an important role in the safe and stable operation of hydropower station where the water pressure at the bottom of volute and the rise rate of rotational speed is the core part. Load rejection occurring, the unit enters transient process, which is controlled by GV and PRV. Therefore, the optimization of CCLGV-PRV is a multiple objective optimization problem with several constraints. In this section, the mathematical model of hydropower station, strategy of CCLGV-PRV, optimization objectives, and constraints are put forward.

### 2.1 Mathematical model

A precise mathematical description of the various parts of the hydropower station is the basis for optimization. The mathematical model includes pipelines and a variety of boundary conditions. The mathematical models of pipeline, turbine, PRV and generator are presented in this section.

#### 2.1.1 Pipeline model

The non-constant flow in the penstock can be expressed by the motion and continuity equation.\textsuperscript{25} As is shown formula (1) and formula (2).

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial L} + \frac{g}{2D} \frac{\partial H}{\partial L} = 0
\]

\[
\frac{c^2}{g} \frac{\partial V}{\partial L} + V \left( \frac{\partial H}{\partial L} + \sin \alpha \right) + \frac{\partial H}{\partial t} = 0
\]

where \( V \) is the average flow velocity of pipeline, \( H \) is the piezometric water head of pipe centerline, \( d \) is the inner diameter of the pipes, \( f \) is the Darcy-Weisbach coefficient of friction, \( L \) is length along the pipe centerline, \( \alpha \) is the angle between pipe centerline and the horizontal, \( g \) is the gravitational constant, \( c \) is the water-hammer velocity in penstock.

The characteristic line method is adopted to the above partial differential equations.\textsuperscript{26} It is the positive(\( C^+ \)) and the negative(\( C^- \)) characteristic line equations that we can get in the basis of formula (1) and formula (2). As are shown in formula (3) and formula (4). The schematic diagram is illustrated in Figure 1.

\[
C^+: Q^+_p = C_p - C_a H_p^t
\]

\[
C^-: Q^-_p = C_n + C_a H_p^t
\]

where \( Q \) is the flow and \( H \) is water head, and the superscript represents the calculation time, and the subscript represents
the calculation position. The calculation formula of other parameters is defined as follows:

\[ C_p = Q_{A1} - \Delta t + C_a H_{A}^{t=\Delta t} - C_f \left| Q_{A}^{t+\Delta t} - Q_{A}^{t-\Delta t} \right|, \]
\[ C_n = Q_{B1} - C_d A_v \frac{Q_{B1}}{\sqrt{H_{prv}}} - C_f \left| Q_{B}^{t+\Delta t} - Q_{B}^{t-\Delta t} \right|, \]
\[ C_f = \frac{f \Delta t}{2DA} \]

\[ Q_{P1} = Q_{p2} + Q_{prv} \]  \hspace{1cm} (5)

Having been equivalent to a kind of orifice outflow, the characteristic equation of PRV can be described as follows:

\[ Q_{prv} = C_{pv} \tau_p \sqrt{2gH_{prv}} \]  \hspace{1cm} (6)

where \( Q_{p1}, Q_{p2} \) is upstream and downstream discharge of PRV. \( Q_{prv} \) is the discharge of the PRV. \( H_{prv} \) is the head acting on PRV, which is obtained from the water pressure before PRV minus the water pressure after PRV. \( C_{pv} = Q_{ov}/\sqrt{H_{prv}} \) in which \( Q_{ov} \) is flow through the fully open valve under a water pressure of \( H_{prv} \); \( \tau_p \) is effective valve opening at the end of time, \( \tau_p = (C_d A_v) / (C_d A_v)_{0} \); \( C_d \) is the coefficient of discharge; \( A_v \) is the area of valve opening; and the subscript 0 refers to steady-state conditions.

2.1.3 | Turbine model

The turbine has a strong nonlinearity among the multiple parameters and time variation of hydraulic turbine.

\[ n_{11} = f_1(n_{11}, y) \]  \hspace{1cm} (7)
\[ M_{11} = f_2(n_{11}, y) \]  \hspace{1cm} (8)

where \( y \) is the opening of GV, \( M_{11} \) is the unit torque, and \( n_{11} \) is the unit rotational speed, which can be calculated by \( n_{11} = n_{t-\Delta t} D_1 / \sqrt{H} \). \( n_{t-\Delta t} \) is the rotation speed of turbine at the last moment. Hence, the discharge and torque of turbine can be obtained through the following equations:

\[ Q_t = Q_{11} D_1^2 \sqrt{H} \]  \hspace{1cm} (9)
\[ M_t = M_{11} D_1^2 \sqrt{H} \]  \hspace{1cm} (10)

where \( Q_t \) is discharge and \( M_t \) is torque of turbine. \( H \) is denoted the working water head of turbine. \( D_1 \) is the nominal diameter of turbine.

2.1.4 | Generator model

The generator separated from the grid can be regarded as a rotating rigid body under load rejection. Therefore, a first-order model is selected to calculate the rotational speed of hydroelectric unit. The equation is as follows:

\[ J \frac{\pi}{30} \frac{dn}{dt} = M_t - M_g \]  \hspace{1cm} (11)
where \( J \) is the moment of inertia and \( n \) is rotational speed. \( M_t \) is the shaft mechanical torque. \( M_g \) is the load torque whose value is 0 under load rejection.

### 2.2 CCLGV-PRV

Controlled by governor, the action of GV and PRV is generally designed as linear functions considering the flexibility of servo actuator.\(^{14,32}\) The closing law of GV (CLGV) under load rejection are mainly composed of two types, which is one-phase closing and two-phase closing, as shown in Figure 2. Meanwhile, the opening law of PRV (OLPRV) includes one-phase and two-phase opening as illustrated in Figure 3. Hence, there will be four kinds of CCLGV-PRV schemes for hydropower station equipped with PRV. In detail, “OCOO” refers to the guide vane is closed in one stage, and the pressure regulating valve is opened in one stage; “OCTO” means that the guide vane is closed in one stage and the pressure regulating valve is opened in two stages; “TCOO” means that the guide vane is closed in two stages and the pressure regulating valve is opened in one stage; “TCTO” means that the guide vane is closed in two stages and the pressure regulating valve is opened in two stages. Their diagram is depicted in Figure 4.

### 2.3 Objective function

#### 2.3.1 Water pressure objective

The hydroelectric unit under load rejection, the GV is closed quickly to ensure the safety of the unit, which will lead to a sharp rise of water pressure at the bottom of volute. The higher this value is, the greater construction investment is. As an essential index of guarantee calculation of regulation, the appreciation of water pressure at the bottom of the volute is set as objective-1, which is expressed as follows:

\[
\text{Min Obj}_{\text{pre}} = \max \left( H_{\text{volute}} \right) \tag{12}
\]

where \( H_{\text{volute}} \) is water pressure at the bottom of volute.

#### 2.3.2 Rotational speed objective

There is an imbalance between active torque and resistance torque under load rejection. It will contribute to the rapidly rise of the rational speed. High rotational speed makes the unit in runaway situation, which will seriously threaten the safety of the unit. Being an important measurement index, the rise rate of rotational speed is set as the objective-2, and the expression is as follows:

\[
\text{Min Obj}_{n} = \left( \max(n) - n_r \right) / n_r \tag{13}
\]

where \( n \) is the rational speed, \( n_r \) is its rating value.

### 2.4 Constraints

According to the design standards of hydropower station, the optimization parameters are restricted to ensure that all variables are within a reasonable range. Meanwhile, the convergence speed of the algorithm is accelerated, which improves the efficiency of optimization.

#### 2.4.1 Control time constraint

Aiming to limit to the control time within a reasonable range, this constraint is indicated by inequality (14):

\[
\begin{align*}
0 < t_{\gamma,i} &< t_{\gamma,\text{max}} & i = 1, 2, \ldots, m \\
0 < t_{\tau,i} &< t_{\tau,\text{max}} & i = 1, 2, \ldots, n
\end{align*} \tag{14}
\]

where \( t_{\gamma,i} \) is the time interval of the \( i \)-th phase of GV, and \( t_{\gamma,\text{max}} \) is the maximal closing time of GV. \( t_{\tau,i} \) is the time interval of the \( i \)-th phase of PRV and \( t_{\tau,\text{max}} \) is the maximal opening time of PRV. \( m, n \) is the number of phases in the CCLGV-PRV scheme.
2.4.2 | Total time constraint

This constraint bounds the total time of the GV and PRV within a certain range.

\[
\begin{align*}
0 < \sum_{i=1}^{m} t_{y,i} &< t_{y,\text{max}} \quad i = 1, 2, \ldots, m \\
0 < \sum_{i=1}^{n} t_{\tau,i} &< t_{\tau,\text{max}} \quad i = 1, 2, \ldots, n
\end{align*}
\]  

(15)

It is not until GV close completely that PRV close during the transient process. Therefore, the accumulated closing time of GV is equal to the total opening time of PRV, as expressed in formula (16):

\[
\sum_{i=1}^{m} t_{y,i} = \sum_{i=1}^{n} t_{\tau,i}
\]

(16)

2.4.3 | Maximum control rate constraint

This constraint is mainly concerned with the limitation of oil pressure in governor. The control accuracy of governor is required to be higher, the GV and PRV operated in a short time. Therefore, the limitation of oil pressure is converted into the acting rate of GV and PRV:

\[
\begin{align*}
k_{y,i} &< k_{y,\text{max}} \quad i = 1, 2, \ldots, m \\
k_{\tau,i} &< k_{\tau,\text{max}} \quad i = 1, 2, \ldots, n
\end{align*}
\]

(17)

where \( k_{y,i} \) is the closing rate of the \( i \)-th phase of GV, and \( k_{y,\text{max}} \) is the maximal closing rate of GV. \( k_{\tau,i} \) is the opening rate of the \( i \)-th phase of PRV, and \( k_{\tau,\text{max}} \) is the maximal opening rate of PRV.

2.4.4 | Control amplitude constraint

This constraint guarantees the opening of GV and PRV within an available range during each stage.

\[
\begin{align*}
0 < y_i &< y_{\text{max}} \quad i = 1, 2, \ldots, m \\
0 < \tau_i &< \tau_{\text{max}} \quad i = 1, 2, \ldots, n
\end{align*}
\]

(18)

where \( y_i \) is the closing amplitude of the \( i \)-th phase of guide vane, and \( y_{\text{max}} \) is the maximal closing amplitude of guide vane. \( \tau_i \) is the opening amplitude of the \( i \)-th phase of PRV, and \( \tau_{\text{max}} \) is the maximal opening amplitude of PRV.

2.4.5 | Total control amplitude constraint

This constraint is presented to ensure that the total opening amplitude of GV and PRV is controlled within the range of 0–100%.

\[
\begin{align*}
0 < \sum_{i=1}^{m} y_i &< y_{\text{max}} \\
0 < \sum_{i=1}^{n} \tau_i &< \tau_{\text{max}}
\end{align*}
\]

(19)

2.4.6 | Water pressure at the bottom of volute constraint

This constraint assures that the water pressure at the bottom of volute should meet regulation guarantee calculation requirement.

\[
H_c \leq H_{c,\text{max}}
\]

(20)
where \( H_c \) is the water pressure at the bottom of volute during the full load rejection, \( H_{c,\text{max}} \) is the limited value of it.

2.4.7 Water pressure at the entrance of draft tube constraint

By no means does the entrance of draft tube produce lower negative pressure. This constraint is described as in formula (21).

\[
H_d \geq H_{d,\text{min}} \tag{21}
\]

where \( H_d \) is the water pressure at entrance of draft tube during the full load rejection, \( H_{d,\text{min}} \) is the limited value of it.

3 OPTIMIZATION OF CCLGV-PRV USING NCMOGSA

Inspired by Newton’s law of universal gravitation, the NCMOGSA is put forward for multi-objective optimization problem, recently. It’s mainly made up of three steps: update velocity and position, non-dominated screening, and chaotic mutation, which exhibits high performance and good convergence ability. Hence, NCMOGSA is adopted to optimize the CCLGV-PRV. In this section, the basic concept of NCMOGSA, constraint handling method and calculation procedure of CCLGV-PRV optimization by NCMOGSA are introduced.

3.1 NCMOGSA

3.1.1 Update velocity and position

The moving speed and position of particle is updated according to formula (22) and formula (23).

\[
v^d_i(t + 1) = v^d_i(t) + r_1r_2(P^d_{\text{best}}(t) - P^d_i(t)) + c_2r_3(P^d_i(t) - P^d_{\text{best}}(t)) + a^d_i(t) \tag{22}
\]

\[
P^d_i(t + 1) = P^d_i(t) + v^d_i(t + 1) \tag{23}
\]

where \( v^d_i \) is the \( i \)-th particle’s velocity, \( a^d_i \) is the \( i \)-th acceleration of particle. \( r_1 \), \( r_2 \), \( r_3 \) are the random variables in the range \([0,1]\), \( c_1 \), \( c_2 \) are learning genes, \( P^d_i(t) \) is the particle’s position, \( P^d_{\text{best}}(t) \) is the best position that the \( i \)-th particle has ever appeared, \( G^d_{\text{best}}(t) \) is the global best position of all the particles during the optimization, \( d \) is one dimension of particle.

3.1.2 Non-dominated screening

Rapid sorting is adopted to determine non-domination of population. Having been labeled as rank, the particle with a smaller rank is defined as an excellent particle. The excellence of the particles can be judged by calculating the crowding distance. Meanwhile, we define that the larger the crowding distance is, the better the particle is. The fitness of particle is recalculated by rank according to formula (24).

\[
f_i(t) = \begin{cases} 
1, & \text{at boundaries & rank } = k \\
2, & \text{at center & rank } = 1 \\
k + 1, & \text{rank } = k 
\end{cases} \tag{24}
\]

Calculate the crowding distance for everyone in each rank in light of formula (25).

\[
Y_{[p]}^{\text{distance}} = \sum_{l=1}^{L} \left( \frac{f_{\text{max}}^l - f_{\text{min}}^l}{f_{\text{max}}^l - f_{\text{min}}^l} \right)^2 \quad p \in [2, N_r - 1] \\
Y_{[p]}^{\text{distance}} = \left\{ \begin{array}{ll} 
\sum_{l=1}^{L} \left( \frac{f_{\text{max}}^l - f_{\text{min}}^l}{f_{\text{max}}^l - f_{\text{min}}^l} \right)^2 & p = 1 \\
2 \left( \frac{f_{\text{max}}^l - f_{\text{min}}^l}{f_{\text{max}}^l - f_{\text{min}}^l} \right)^2 & p = N_r 
\end{array} \right. \tag{25}
\]

where \( f_i(t) \) is the \( i \)-th particle’s fitness, \( Y_{[p]}^{\text{distance}} \) is the crowding distance of individual \( P \) for each rank, \( f^l_P \) is the fitness of individual \( p \) in \( l \)-th objective, \( L \) is the number of objective, \( f^l_{\text{max}}, f^l_{\text{min}} \) are the maximum and minimum fitness for individuals in each rank of the \( L \) dimensional objective.

3.1.3 Chaotic mutation

It is by chaotic mutation that new particles are generated in the feasible region of the current particles. Chaotic mutation is shown as formula (26):

\[
\eta^d_i = \lambda \left( P^d_i(t) - P^d_{\text{min}}(t) \right) \left( 1 - \frac{P^d_i(t) - P^d_{\text{min}}(t)}{P^d_{\text{max}}(t) - P^d_{\text{min}}(t)} \right) \\
P^d_i(t) = \eta^d_i \left( P^d_{\text{max}}(t) - P^d_{\text{min}}(t) \right) + P^d_{\text{min}}(t) \tag{26}
\]

where \( \eta^d_i \) is the conversion value in chaotic mutation, \( \lambda \) is a constant, \( P^d_{\text{max}}(t), P^d_{\text{min}}(t) \) are the global maximum and
minimum values of the agents in the $t$-th iteration of the $d$-dimension.

3.2 Constraint handling

For multi-objective optimization problems with various constraints, the particles that violate the equality or inequality constraints of the algorithm during the optimization process are called infeasible solution. The reason for handling the infeasible solution is that it will interfere the evolution of the algorithm. Feasible solutions (SF) method\textsuperscript{34} is applied in this study, aiming to handle the constraints presented in formula (14)–(21). The modified fitness values are as follows:

$$f_m(P_i(t)) = \begin{cases} f_m^\text{w}(P_i(t)) & \text{if } P_i(t) \text{ is feasible} \\ f_m^\text{w}(P_i(t)) + v(P_i(t)) & \text{otherwise} \end{cases} \quad (27)$$

where $f_m(P_i(t))$ is the $m$-th objective of the $i$-th particle, $f_m^\text{w}$ is the maximum value of all feasible solutions in $m$-th objective, $v(P_i(t))$ is the global constraint violation degree. If no feasible solution exits in current particles, then $f_m^\text{w}(P_i(t)) = 0$.

$$v(P_i(t)) = \sum_{j=1}^{C} v_{j, \text{normalized}}(P_i(t))$$

$$v_{j, \text{normalized}}(P_i(t)) = \frac{v(P_i(t))}{v_{\text{max}}} \quad (28)$$

where $v_{j, \text{normalized}}(P_i(t))$ is the normalized violation of the $i$-th objective, $v(X(t))$ is the constraint violation of $X(t)$ for $j$-th objective, $v_{\text{max}}$ is maximum constraint violation for $j$-th objective among all $X_i(t)$, $C$ is the number of constraints.

3.3 Multi-objective optimization procedure

In this section, the NCMOGSA is adopted to the multi-objective optimization of CCLGV-PRV under load rejection. These are the main parts of the process that initializing the parameters of NCMOGSA, pre-process of constraints, post-process of constraints, determining the domination of the population, updating velocity and position of particles, picking out elite archive, conducting chaotic mutation. For more intuitively, the above process is shown in the form of framework in Figure 5.

4 MULTI-OBJECTIVE DECISION-MAKING METHOD

All particles in the Pareto optimal set are non-dominated solutions to each other. Operators need to make decisions according to the actual requirements. The improved entropy weight method\textsuperscript{35,36} is adopted to calculate the weight of each objective function, which facilitates decision makers to choose the optimal parameters of hydropower station. Finally, the distance between each particle in the Pareto and ideal point is calculated based on the relative objective proximity method.\textsuperscript{37} Having been sorted from the largest to the smallest distance, the particles with the highest ranking are selected as the recommended optimal operation parameters.

4.1 Improved entropy weight method

Entropy represents the degree of variation and the amount of information contained in the objective function. The smaller the entropy is, the greater the degree of variation and the amount of information contained in the objective function will be. The steps for solving entropy weights of each objective function are as follows.

Step 1: Establish the optimization system of non-dominated solutions. The decision matrix normalized, the relative superior degree matrix is obtained.

$$R_{ik} = \frac{X_{ik} - X_{\text{min},k}}{X_{\text{max},k} - X_{\text{min},k}} \quad (29)$$

Step 2: Calculate the entropy values of each decision attribute according to the relative superiority degree matrix and the definition of entropy.

$$H_j = -\frac{1}{\ln n} \left( \sum_{i=1}^{n} e_{ij} \ln e_{ij} \right) \quad (30)$$

$$e_{ij} = r_{ij} / \sum_{i=1}^{n} r_{ij} \quad (31)$$

where $0 \leq H_j \leq 1$, to make sense of $\ln e_{ij}$, it is assumed that when $e_{ij} = 0$ then $\ln e_{ij} = 0$.

Step 3: Calculate entropy weight of each objective function according to the improved entropy weight formula.

$$w_{hj} = \begin{cases} aH_j + b(1 - H_j), & H_j < 1 \\ 0, & H_j = 1 \end{cases} \quad (32)$$

$$a = \frac{1 - H_j}{\sum_{k=1,H_k \neq 1}^{m} (1 - H_k)}, \quad b = \frac{1 - H_j}{\sum_{k=1,H_k \neq 1}^{m} (1 - H_j)}$$
where $H_i$ is the same part of the entropy value of each index starting from the first decimal point.

4.2 Relative objective proximity method

Combined with the entropy weight of each objective function, it is defined that the normalized weighted distance between the objective vector of any feasible solution and the ideal point or negative ideal point. The method of relative target proximity between objective function and ideal point is introduced as follows.

Step 1: Standardized operation is performed on each particle in the Pareto.

$$
\bar{F}(X) = \left[\frac{f_1(X) - d_{j1}}{d_{j2} - d_{j1}}, \ldots, \frac{f_n(X) - d_{j1}}{d_{j2} - d_{j1}}\right] = (\bar{f}_1(X), \bar{f}_2(X), \ldots, \bar{f}_n(X))
$$

(33)

where $d_{j1}, d_{j2} (j = 1, 2, \ldots, n)$ is the minimum and maximum value of $j$-th objective function in Pareto.

Step 2: Define ideal point $F_{min} = [d_{11}, d_{21}, \ldots, d_{n1}]$. So, $F_{max} = [d_{12}, d_{22}, \ldots, d_{n2}]$ is stipulated as negative point. Furthermore, the ideal and negative ideal points are standardized to obtain $\bar{F}_{min} = [0, 0, \ldots, 0]$, $\bar{F}_{max} = [1, 1, \ldots, 1]$.

Step 3: Calculate the weight distance between each particle in Pareto and the ideal point, which is denoted by $g_1(X)$. Meanwhile, the distance between each particle and the negative ideal point is solved, as is expressed in $g_2(X)$.

$$
g_1(X) = \left\{ \sum_{j=1}^{n} \left[ w_{j} (f_j(X) - d_{j1}) \right] \right\}^{\frac{1}{2}}
$$

(34)

$$
g_2(X) = \left\{ \sum_{j=1}^{n} \left[ w_{j} (f_j(X) - d_{j2}) \right] \right\}^{\frac{1}{2}}
$$

(35)
Step 4: The relative target proximity $l(X)$ between the particle and the ideal point is defined as follows.

$$l(X) = \frac{g_2(X)}{g_1(X) + g_2(X)}$$  \hspace{1cm} (36)

Obviously, the larger the $l(X)$ is, the closer the particle is to the ideal point, and the higher the particle is in the order.

5 | EXPERIMENTAL STUDY

In this section, NCMOGSA is applied to optimize the four schemes for specific hydropower station. Next, the excellent scheme of CCLGV-PRV is determined according to the optimization results, which are screened according to the improved entropy weight method and relative objective proximity method. Finally, compared with scheme recommended by the manufacturer, a comprehensive analysis is carried out to reveal the mechanism between the CCLGV-PRV and several basic parameters.

5.1 | Study case

To verify the effectiveness of the multi-objective optimization of CCLGV-PRV, a hydropower station whose arrangement form is “one hole-two units” is selected as the research object. In front of each hydroelectric unit is installed PRV 28.35 m away from the upstream bifurcation point. And the outlet of PRV leads to downstream reservoir. The structure is shown in Figure 6. The parameters are given in Table 1. The working conditions is under load rejection simultaneously.

5.2 | Result and analysis on the multi-objective optimization of CCLGV-PRV

The basic NCMOGSA parameters are set as follows. The maximum iteration $MaxIt$ is 100, the population size $N_{pop}$ is 20, the elite archive is set to 20, the number of grids per dimension is 10, the inflation rate is 0.1, the deletion selection pressure is 2.0, the gravitational time constant is 2.0, the coefficient of chaos variation is 4.0, and the particle regeneration rate $c_1, c_2$ are 2.0.

The optimal solution set is obtained according to the process of 3.3, as shown in Figure 7. The corresponding specific values of each objective function in Figure 7 are given in Table A2 of the Appendix A. All the solutions being reasonable, the distribution of each particle in Pareto set is uniform and there is no phenomenon of falling into local optimal. It can be seen that NCMOGSA has an excellent performance.

What’s more, all the calculated results meet the requirements of the guarantee calculation of regulation given in Table A1 of the Appendix A. The water pressure at the entrance of draft tube for particles in Pareto is shown in Figure 8 and Figure 9. Among them, the results of TCOO and TCTO are obviously better than those of OCOO and
Therefore, TCOO and TCTO are selected to make further decision analysis according to the method mentioned in Section 4.

The entropy weight of each objective in TCOO and TCTO models are calculated according to the three steps in Section 4.1, and the results are given in Table 2. On this basis, the relative target proximity is calculated and sorted according to four steps in Section 4.2. The sorting result is shown in Figure 10, where $l_1(x)$ is the result of TCOO and $l_2(x)$ represents TCTO. The particle with the largest $l(x)$ is selected, whose parameters are shown in Table 3. Meanwhile, scheme recommended by manufacturer (SRM) is also given in Table 3. Next, the two schemes that have been optimized will be compared with the SRM, and finally give operational suggestions.

**FIGURE 7** Pareto solutions obtained through the NCMOGSA

**FIGURE 8** Water pressure at the entrance of draft tube under TCOO

**FIGURE 9** Water pressure at the entrance of draft tube under TCTO
5.3 Comparisons and discussion

In this section, having simulated the SRM and the excellent particles screened out in TCOO and TCTO, the opening of GV, the opening of PRV, the water pressure at the bottom of volute, the water pressure at the entrance of draft tube, and rotation speed are compared and analyzed. Meanwhile, the effect mechanism of each parameter to hydropower station is revealed. Finally, recommendations are put forward for the operation of the hydropower station.

Firstly, for the closing law of GV in Figure 11(A), the preferred particles of TCOO and TCTO as well as the SRM, are both in the form of two-stage. Among them, the closing rate of the first stage is basically the same, and the most important difference lies in the occurrence time of the turning point and the closing rate of the second stage. From the perspective of view of turning point, TCOO enters the closing law of the second stage at 6.86 s, and the closing time of the first stage is the shortest. The time of first stage of the TCTO is 7.69 s. Finally, the shutdown rate of SRM changing at 11.2 s means that the SRM takes 3.0 s to close the remaining 0.086. The TCOO closed the remaining 0.6 in 19.97 s, and TCTO closed the 0.28 left in 3.73 s. Obviously, the shutdown rate of second stage is largest in TCOO, while the minimum is SRM.

In terms of the opening law of the PRV, as is shown in Figure 11(B), the SRM and the TCOO open by a straight line. Different from them, the TCTO is a two-stage opening law. However, from the perspective of opening rate, the first stage of TCTO is the largest, while TCOO is slower, and the SRM is the slowest. In TCTO, the second stage starts at 7.76 s, and the opening rate don’t slow down until it opens completely.

From the perspective of the maximum water pressure at the bottom of the volute in Figure 11(C), the maximum value generated by the preferred particles in TCOO is the smallest, which is 211.68 m. However, it is the largest with the SRM, 218.75 m, and the calculation result of TCTO is between them. For the second wave peak, the value of TCOO is the largest, and the value of TCTO is the second, while it does not appear in the SRM. The root reason of this phenomenon is attributed to CCLGV-PRV. Firstly, the second wave peak is accompanied by a change in the rate of the two-stage closing law. Hence, the largest closing rate in the second stage of TCOO contributes to the second wave peak being the largest. The high frequency hydraulic oscillation generated by the full closure of PRV is almost the same.

The water pressure at the entrance of draft tube, as shown in Figure 11(D), the SRM is −1.0354 m, of which the largest is −1.048 m. The smallest is −1.04453 m in TCTO, which is located between the two schemes. So subtle are the differences between them that they can be completely ignored in engineering. This means that the three schemes have little differences on the water pressure at entrance of draft tube.

As far as the rise rate of rotation speed is concerned in Figure 11(E), the maximum value is 57.95% in SRM, while it is 54.52% in TCOO, and 52.51% in TCTO. After the maximum value, the SRM has the largest reduction rate, the TCOO is the smallest, and the TCTO is between them. However, the rotation speed of TCOO and TCTO almost coincides with each other around 27 s. In addition, the rotational speed of three schemes has no secondary oscillation.

In Figure 11(F), the order of the extreme value of reverse torque is: the reverse of torque of SRM is the largest, followed by TCOO, and TCTO is smallest. In the terms of the rate of decline, TCOO and TCTO are almost the same at the very beginning, while SRM is slower and TCTO is the first to show a turning point. But they are
FIGURE 11 The comparison of variables in the transient process under different schemes. (A) Guide vane opening. (B) Pressure regulating valve opening. (C) Water pressure at the bottom of volute. (D) Water pressure at the entrance of draft tube. (E) Rotation speed. (F) Torque.
almost completely recombined and tended to 0 at last. All these phenomena coincide with the rotational speed in the transient process of load rejection. As we all know, smaller reverse torque is good for the security of hydroelectric unit, which indicates the superiority of TCOO and TCTO.

Above all, firstly, dynamic quality calculated by the optimized particles are better than the result of SRM. It mainly shows that the water pressure at the bottom of volute and the rise of rotation speed are lower. Meanwhile, the negative torque is smaller, and water pressure at the entrance of draft tube have almost no difference. Secondly, the rise rate of rotational speed is completely determined by the guide vane, and the water pressure at the bottom of volute is jointly determined by the control law of the GV and PRV. The larger the speed and duration of GV closes for first stage are, the higher the extreme value of water pressure at the bottom of the volute will be. The second wave peak of water pressure will be determined by the second stage of closure with GV. During the whole process, the opening rate of the PRV will weaken the adverse effect that the rapid closure of GV leads to the rise of the water pressure at the bottom of the volute.

At last, both TCOO and TCTO can meet requirements of transient process but the rise rate of rotational speed has exceeded 50%, which makes the unit easy to enter runaway. However, the water pressure at the bottom of volute has a considerable margin of safety. Meanwhile, it is TCTO which opens fast at the very beginning and slow at the end that is conducive to the protection of the relay device. Therefore, the optimization result of the TCTO is recommended, whose detailed parameters is given in Table A2.

6 | SUMMARY AND CONCLUSIONS

There are few studies on multi-objective optimization and decision-making of CCLGV-PRV in the transient process, during which the minimization of the water pressure of volute and rise rate of rotational speed is the core problem. But, the two objectives are contradictory to each other. Therefore, this paper studies the multi-objective optimization problem of CCLGV-PRV to improve the quality of transient process. The numerical simulation model and multi-objective optimization of CCLGV-PRV are introduced. A recently proposed NCMOGSA algorithm is adopted to optimize CCLGV-PRV. Four schemes of CCLGV-PRV are proposed, and the optimal solution is decided by the improved entropy weight method and relative objective proximity. The CCLGV-PRV of hydroelectric unit in a practical project is optimized, which is compared with the SRM to show the superiority of the proposed multi-objective optimization decision-making strategy.

Several conclusions can be drawn from this research:

1. PRV can effectively solve the contradictory problem between the water pressure at the bottom of volute and the rise of rotational speed under load rejection, and reduce the construction investment.
2. The NCMOGSA shows excellent performance in solving the multi-objective optimization problem of CCLGV-PRV, which can provide more choices for decision makers and operators.
3. The improved entropy weight and relative objective proximity method can make the final decision and screen of Pareto set. Dynamic quality calculated by the optimized particles are better than the result of SRM in terms of the water pressure at the bottom of volute and the rise rate of rotational speed.
4. The rise rate of rotational speed is completely determined by the GV, and the water pressure at the bottom of volute is jointly determined by the control law of the GV and PRV. TCTO is the final recommendation for CCLGV-PRV.

Although several findings are obtained from the present study, there are still several aspects that can be improved. In future work, we will focus on the study of the following topics.

1. For the status of the PRV in the stable operation of the hydropower station, whether there is self-excited vibration or not, it needs to further explore with the help of three-dimensional software.
2. At present, there are many intelligent optimization methods and decision-making methods. Further research can be done on the selection of optimization algorithm and the decision of pareto after optimization in the context of hydropower design.

ACKNOWLEDGMENTS

This work is supported by the Key Program of the National Natural Science Foundation of China (No. U1865202) and the National Natural Science Foundation of China (No. 51809099).

ORCID
Tianyu Zhang © https://orcid.org/0000-0003-3041-5576
REFERENCES

1. Yang WJ, Yang JD, Zeng W, et al. Experimental investigation of theoretical stability regions for ultra-low frequency oscillations of hydropower generating systems. *Energy*. 2019;186:135-171.

2. Zhu ZA, Wang X, Jiang CW, Wang LL, Gong K. Multi-objective optimal operation of pumped-hydro-solar hybrid system considering effective load carrying capability using improved NBI method. *Int J Electr Power Energy Syst*. 2021;129:106802.

3. Wei SP ed. Turbine regulating system. In: *Hydraulic Turbine Regulation*. Huazhong University of Science and Technology Press; 2009:1-28.

4. Zhao ZZ, Yang JD, Chung CY, Yang WJ, He XH, Man C. Performance enhancement of pumped storage units for system frequency support based on a novel small signal model. *Energy*. 2021;234:121207.

5. Zhang YN, Lian Z, Fu WL, Chen X. An ESR Quasi-Online Identification Method for the Fractional-Order Capacitor of Forward Converters Based on Variational Mode Decomposition. *IEEE Transactions on Power Electronics*. 2021;1. doi:10.1109/tpel.2021.3119966

6. Guo WC. A review of the hydraulic transient and dynamic behavior of hydropower plants with sloping ceiling tailrace tunnels. *Energies*. 2019;12(17):2-28.

7. Zhang H, Chen DY, Wu C, Wang X, Lee JM, Jung KH. Dynamic modeling and dynamic analysis of pump-turbines in S-shaped regions during run away operation. *Energy Convers Manag*. 2017;138:375-382.

8. Fu WL, Lu QP. Multiobjective optimal control of FOPID controller for hydraulic turbine governing systems based on reinforced multiobjective harris hawks optimization coupling with hybrid strategies. *Complexity*. 2020;2020:1-17.

9. Lai XJ, Li CS, Zhou JZ, Zhang YC, Li YG. A multi-objective optimization strategy for the optimal control scheme of pumped hydropower systems under successive load rejections. *Appl Energy*. 2020;261:1-19.

10. Leonard J, Kramer MA. Improvement of the backpropagation algorithm for training neural networks. *Comput Chem Eng*. 1990;14:337-341.

11. Wu RQ, Chen JZ eds. Optimization of regulating law. In: *Hydropower Hydraulic Transition Process*. China Water Power Press; 1997:84-92.

12. Zhao ZG, Yang JD, Yang WJ, Hu JH, Chen M. A coordinated optimization framework for flexible operation of pumped storage hydropower system: nonlinear modeling, strategy optimization and decision making. *Energy Convers Manag*. 2019;194:75-93.

13. Wang BB, Guo WC. Analytical solutions for determining extreme water levels in surge tank of hydropower station under combined operating conditions. *Commun Nonlinear Sci Numer Simulat*. 2017;47:394-406.

14. Sun Q. Study on transient process of long water diversion power station with pressure regulating valve. M.S. Thesis. Dept. WRH. Eng., Xian University of Technology, Xian, Japan; 2018.

15. Zhang J, Fang YT, Liu H, Zhou J. Study on the reversible pump-turbine closing law in pumped storage plant. *Fluid Mach*. 2004;12:14-18.

16. Liu XL, Zheng Y, Gao YN. Closing law of guide vane of reversible pump-turbine for pump storage power stations. *Water Resources and Power*. 2011;06:151-153.

17. Yu GL, Cai FL, Zhou JX. Influence of guide vane closing law on the hydraulic transients in a pumped storage station. *China Rural Water Hydropower*. 2016;05:189-192.

18. Shu SQ, Fan HG, Zhang BP. Application of genetic algorithm to the optimization of the closing of gates in pumped storage power stations. *J Tsinghua Univ (Sci&Tech)*. 2000;40:35-38.

19. Liu LZ, Fan HG, Chen NX. Optimization of distributor closing law in pumped storage stations. *J Tsinghua Univ (Sci&Tech)*. 2006;11:1892-1895.

20. Zhou TC, Zhang J, Yu XD. Optimizing closure law of wicket gates in hydraulic turbine based on simulated annealing. *J Drainage Irrigation Mach Eng*. 2018;36:320-326.

21. Lai XJ, Li CS, Zhou JZ, Zhang N. Multi-objective optimization of the closure law of guide vanes for pumped storage units. *Renew Energy*. 2019;139:302-312.

22. Zhou JZ, Xu YH, Zheng Y, Zhang YC. Optimization of guide vane closing schemes of pumped storage hydro unit using an enhanced multi-objective gravitational search algorithm. *Energies*. 2017;10(7):2-23.

23. Liu C, Zhou JZ, Lai XJ, Zhang TY. Optimization and mechanism of the wicket gate closing law for high-head pumped storage power stations. *IEEE Access*. 2021;09:11734-11749.

24. Zhao ZZ, Yang JD, Huang YF, Yang WJ, Ma WC. Improvement of regulation quality for hydro-dominated power system: quantifying oscillation characteristic and multi-objective optimization. *Renew Energy*. 2021;168:606-631.

25. Lohrasbi AR, Attarnejad R. Water hammer analysis by characteristic method. *Am J Eng Appl Sci*. 2008;4:287-294.

26. Lister M. The numerical solution of hyperbolic partial differential equations by the method of characteristics. *Math. Meth. Dig. Com*. 1960;1:165-179.

27. Chaudhry MH ed. Characteristics and finite-difference methods. In: *Applied Hydraulic Transients*. Springer-Verlag; 2014:105-108.

28. Xu YH, Zheng Y, Du Y, Yang W, Peng X, Li C. Adaptive condition predictive-fuzzy PID optimal control of start-up process for pumped storage unit at low head area. *Energy Convers Manage*. 2018;177:592-604.

29. Cheng YC, Zhang JB. Turbine Regulating Automatically. China Water Power Press; 2009.

30. Wang ZB, Li CS, Lai XJ, Zhang N, Xu YH, Hou JJ. An Integrated Start-Up Method for Pumped Storage Units Based on a Novel Artificial Sheep Algorithm. *Energies*. 2018;11(1):151. doi:10.3390/en11010151

31. Wang WJ, Yang JD, Guo WC. A mathematical model and its application for hydro power units under different operating conditions. *Energies*. 2015;8(9):10260-10275.

32. Hou JJ, Li CS, Guo WC & Fu WL. Optimal successive start-up strategy of two hydraulic coupling pumped storage units based on multi-objective control. *Int J Electr Power Energy Syst*. 2019;111:398-410. doi:10.1016/j.ijepes.2019.04.033

33. Yu H, Zhou JZ, Wang KS, Zhao ZG, Lai XJ, Xu YH. Closing law of guide vane of reversible pump-turbine for pump storage power stations. *Water Resources Power*. 2018;07:135-140.

34. Qu BY, Suganthan PN. Constrained multi-objective optimization algorithm with ensemble of constraint handling methods. *Eng Optim*. 2010;43(4):403-416.
35. Wan QZ, Yu Y. Power load pattern recognition algorithm based on characteristic index dimension reduction and improved entropy weight. *Energy Reports*. 2020;6:797-806.

36. Liu HB, Dong YJ, Wang FZ. Gas outburst prediction model using improved entropy weight grey correlation analysis and IPO-LSSVM. *Math Probl Eng*. 2020;2020:1-10.

37. Song HZ. A method based on objective adjacent scale for multi-objective decision-making and its application. *Math Pract Theory*. 2004;5:30-36.

**APPENDIX A**

**TABLE A1** Requirement limitation guarantee calculation of regulation

| Requirement limitation | Values |
|-------------------------|--------|
| Maximum rise of the rotational speed $n_{\text{max}}$ (%) | 65 |
| Maximum water pressure at the bottom of volute $H_{\text{volute, max}}$ (m) | 230 |
| Minimum water pressure at the entrance of draft tube $H_{\text{draft, min}}$ (m) | −6.0 |

**TABLE A2** Objective functions of each schemes

| Scenario | OCOO | OCTO | TCOO | TCTO |
|----------|------|------|------|------|
|          | Obj$_{\text{pre}}$ | Obj$_x$ | Obj$_{\text{pre}}$ | Obj$_x$ | Obj$_{\text{pre}}$ | Obj$_x$ | Obj$_{\text{pre}}$ | Obj$_x$ |
| 1        | 207.3390 | 0.6450 | 210.5763 | 0.6154 | 220.5256 | 0.5258 | 198.7961 | 0.5854 |
| 2        | 208.0250 | 0.6336 | 204.1508 | 0.6481 | 194.5593 | 0.6485 | 219.5827 | 0.5158 |
| 3        | 211.0398 | 0.6221 | 214.3631 | 0.5922 | 218.9914 | 0.5396 | 206.5694 | 0.5476 |
| 4        | 215.7258 | 0.6010 | 209.0802 | 0.6215 | 209.9604 | 0.5604 | 201.6143 | 0.5744 |
| 5        | 214.7072 | 0.6068 | 218.2309 | 0.5757 | 195.0245 | 0.6096 | 218.0366 | 0.5182 |
| 6        | 213.5579 | 0.6127 | 207.0677 | 0.6342 | 207.2070 | 0.5637 | 215.4549 | 0.5251 |
| 7        | 214.5606 | 0.6075 | 213.2823 | 0.6029 | 211.6812 | 0.5452 | 198.1555 | 0.5963 |
| 8        | 215.2628 | 0.6038 | 224.0972 | 0.5385 | 196.6297 | 0.6013 | 217.9950 | 0.5210 |
| 9        | 211.0246 | 0.6243 | 208.4481 | 0.6274 | 205.6372 | 0.5724 | 213.7476 | 0.5318 |
| 10       | 213.7781 | 0.6115 | 211.2167 | 0.6071 | 204.8286 | 0.5751 | 213.4278 | 0.5323 |
| 11       | 213.6508 | 0.6121 | 214.1622 | 0.5967 | 197.7004 | 0.5949 | 214.5552 | 0.5278 |
| 12       | 213.5618 | 0.6126 | 209.6570 | 0.6197 | 200.4868 | 0.5865 | 210.3641 | 0.5425 |
| 13       | 213.8510 | 0.6111 | 221.3386 | 0.5497 | 200.1473 | 0.5876 | 198.2366 | 0.5945 |
| 14       | 212.1604 | 0.6194 | 212.3682 | 0.6044 | 199.8047 | 0.5887 | 201.6197 | 0.5703 |
| 15       | 212.3476 | 0.6190 | 210.8482 | 0.6152 | 200.8077 | 0.5850 | 201.5595 | 0.5606 |
| 16       | 212.6126 | 0.6187 | 206.2310 | 0.6388 | 201.5637 | 0.5833 | 206.4968 | 0.5590 |
| 17       | 212.6151 | 0.6185 | 205.5079 | 0.6456 | 201.1423 | 0.5848 | 204.2933 | 0.5648 |
| 18       | 213.0022 | 0.6170 | 204.5538 | 0.6478 | 202.7056 | 0.5820 | 206.1840 | 0.5597 |
| 19       | 213.2293 | 0.6146 | 205.2355 | 0.6467 | 204.5156 | 0.5769 | 209.9725 | 0.5464 |
| 20       | 212.8675 | 0.6175 | 204.8831 | 0.6475 | 203.8086 | 0.5791 | 210.2420 | 0.5453 |

**How to cite this article:** Zhang T, Zhou J, Yang X, Li H. Multi-objective optimization and decision-making of the combined control law of guide vane and pressure regulating valve for hydroelectric unit. *Energy Sci Eng*. 2022;10:472–487. doi:10.1002/ese3.1038