Single-shot realization of nonadiabatic holonomic quantum gates in decoherence-free subspaces

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Nonadiabatic holonomic quantum computation in decoherence-free subspaces has attracted increasing attention recently, as it allows for high-speed implementation and combines both the robustness of holonomic gates and the coherence stabilization of decoherence-free subspaces. Since the first protocol of nonadiabatic holonomic quantum computation in decoherence-free subspaces, a number of schemes for its physical implementation have been put forward. However, all previous schemes require two noncommuting gates to realize an arbitrary one-qubit gate, which doubles the exposure time of gates to error sources as well as the resource expenditure. In this paper, we propose an alternative protocol for nonadiabatic holonomic quantum computation in decoherence-free subspaces, in which an arbitrary one-qubit gate in decoherence-free subspaces is realized by a single-shot implementation. The present protocol not only maintains the merits of the original protocol, but also avoids the extra work of combining two gates to implement an arbitrary one-qubit gate and thereby reduces the exposure time to various error sources.

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I. INTRODUCTION

Quantum computers exploit the fundamental principles of coherent superposition and quantum entanglement to provide an efficient solution to certain computational tasks, such as factoring large integers [1] and searching unsorted databases [2]. The implementation of circuit-based quantum computation relies on the ability to realize a universal set of accurately controllable building blocks, including arbitrary one-qubit gates and a nontrivial two-qubit gate [3]. However, control errors that accumulate in the operation processing pose a serious challenge in realizing quantum computation. To tackle this, adiabatic and nonadiabatic holonomic quantum computations have been proposed. Holonomic gates depend only on evolution paths but not on evolution details, making them robust against control errors.

Berry phases [4] were first exploited to realize quantum computation, known as adiabatic geometric quantum computation [5]. Such an idea was then generalized to adiabatic non-Abelian geometric phases [6], based on which adiabatic holonomic quantum computation was proposed [7–9]. Adiabatic Abelian and non-Abelian geometric phases provide useful tools to quantum gates that are robust against control errors. However, a challenge for these implementations is the long run time needed for adiabatic evolution, which makes the gates vulnerable to environment-induced decoherence and thereby hinders experimental realization. To resolve this problem, nonadiabatic geometric quantum computation [10, 11] based on nonadiabatic Abelian geometric phases [12] has been put forward, and later generalized to nonadiabatic holonomic quantum computation [13, 14] based on non-Abelian geometric phases [15]. Since nonadiabatic holonomic quantum computation has the merits of both short runtime and robustness against control errors [16, 17], it has received increasing attention recently. Up to now, nonadiabatic holonomic quantum computation has been well-developed in both theory [18–35] and experiment [36–39].

Apart from control errors, decoherence caused by the interaction between a quantum system and its environment is another important challenge to realize quantum computation. To obtain quantum gates that are robust against both control errors and decoherence, the combination of nonadiabatic holonomic quantum computation [13] and decoherence-free subspaces [40–42] is a promising strategy. Yet, such a combination is nontrivial since only the decoherence-free subspaces that are compatible with the conditions of nonadiabatic holonomic gates can be used to protect nonadiabatic holonomic quantum computation. After a great effort, the first protocol of nonadiabatic holonomic quantum computation in decoherence-free subspaces has been developed in Ref. [14], and a number of implementation schemes in various physical systems, such as trapped ions [22], nitrogen-vacancy centers [23] and superconducting circuits [24], have been proposed recently. Nonadiabatic holonomic quantum computation in decoherence-free subspaces has the merits of short runtime and resilience to both control errors and decoherence.

However, all previous schemes of nonadiabatic holonomic quantum computation in decoherence-free subspaces require two noncommuting one-qubit gates to realize an arbitrary one-qubit gate. It doubles the exposure time of the gates to errors, as well as the resource expenditure needed for combining two holonomic gates to achieve an arbitrary one-qubit gate. This motivates us to consider a revised protocol to further improve the efficiency of nonadiabatic holonomic quantum computation in decoherence-free subspaces. Noting that the single-shot implementation approach has been used to realize nonadiabatic holonomic gates in a closed quantum system [29, 30], we find that a similar approach can be also applied to nonadiabatic holonomic quantum computation in decoherence-free subspaces.

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abatic holonomic quantum computation in decoherence-free subspaces by properly choosing the system Hamiltonian.

In this paper, we propose an alternative protocol of nonadiabatic holonomic quantum computation in decoherence-free subspaces, in which an arbitrary one-qubit gate is directly realized by a single-shot implementation. The present protocol not only maintains the merits of the original protocol, but also avoids the extra work of combining two gates to implement an arbitrary one-qubit gate and thereby reduces the exposure time to various error sources.

The paper is organized as follows. In Sec. II, nonadiabatic holonomic quantum computation in decoherence-free subspaces is briefly reviewed. In Sec. III, the present protocol is demonstrated by a physical model of a N-qubit system interacting with a dephasing environment. Section IV is the conclusion.

II. NONADIABATIC HOLONOMIC QUANTUM COMPUTATION IN DECOHERENCE-FREE SUBSPACES

We first recapitulate the main idea of nonadiabatic holonomic quantum computation in decoherence-free subspaces.

Any real quantum system inevitably interacts with its environment. The dynamics of an open quantum system cannot be described by a unitary operator due to the interaction, and the states of the system are affected by decoherence in general. However, if the interaction between the quantum system and its environment possesses some symmetry, there may exist subspaces that are immune to decoherence and the states in these decoherence-free subspaces evolve unitarily. Then, nonadiabatic holonomic quantum computation can be realized in the decoherence-free subspace if it contains a smaller subspace.

To describe this idea in more detail, we use $H$ to denote the Hamiltonian of the quantum system under consideration, which may be time dependent, and $H_I$ to denote the interaction between the quantum system and its environment. $H_I$ can be generally written as $H_I = \sum S_a \otimes B_a$, where $S_a$ and $B_a$ are operators acting on the system and the environment, respectively. If all $S_a$ have a common set of time-independent degenerate eigenvectors $(|\psi_1\rangle, |\psi_2\rangle, \cdots, |\psi_K\rangle)$, which comprise an invariant subspace of $H$, denoted as $S^D$, i.e., satisfying the conditions,

\begin{align}
(a) & \quad S_\alpha |\psi_k\rangle = \lambda_\alpha |\psi_k\rangle, \quad \text{for all } \alpha, \\
(b) & \quad H|\psi_k\rangle \in S^D, \quad k = 1, 2, \cdots, K,
\end{align}

where $\lambda_\alpha$ is a degenerate eigenvalue of $S_\alpha$, then $S^D$ defines a $K$-dimensional decoherence-free subspace. In this case, a quantum state initially prepared in the subspace $S^D$ will undergo a unitary evolution with evolution operator $U(t) = \exp \left(-i \int_0^t H(t) dt \right)$, and remain in the subspace for the whole evolution time. If there is a smaller subspace $S^L = \text{Span}(|\psi_{k1}\rangle, |\psi_{k2}\rangle, \cdots, |\psi_{k_K}\rangle) \subset \text{Span}(|\psi_1\rangle, |\psi_2\rangle, \cdots, |\psi_K\rangle)$, satisfying the additional two conditions,

\begin{align}
(c) & \quad U(t) \left( \sum_{i=1}^M |\psi_{ki}\rangle \langle \psi_{ki}| \right)U^\dagger(t) = \sum_{i=1}^M |\psi_{ki}\rangle \langle \psi_{ki}|, \\
(d) & \quad \langle \psi_{ki}|U(t)\langle H U(t)|\psi_{kj}\rangle = 0, \quad i, j = 1, 2, \ldots, M.
\end{align}

where $t$ is the evolution period, then nonadiabatic holonomic quantum computation in decoherence-free subspaces can be realized by encoding computational qubits into the $M$-dimensional subspace $S^D$ of the $K$-dimensional decoherence-free subspace $S^F$.

The model used in nonadiabatic holonomic quantum computation in decoherence-free subspaces consists of $N$ physical qubits interacting collectively with a dephasing environment. By using three neighboring physical qubits undergoing collective dephasing to encode one logical qubit, a universal set of holonomic gates is obtained. However, all previous schemes [14, 22–25] of nonadiabatic holonomic quantum computation in decoherence-free subspaces require two noncommuting gates to realize an arbitrary one-qubit gate.

III. THE PROTOCOL

We now put forward an alternative protocol of nonadiabatic holonomic quantum computation in decoherence-free subspaces, in which an arbitrary one-qubit gate can be obtained by a single-shot implementation. The model used in the present protocol is $N$ physical qubits interacting collectively with a dephasing environment [9, 14]. For the dephasing environment, the interaction Hamiltonian is described by

$$H_I = S_N \otimes B,$$

where $S_N = \sum \sigma_z^i$ is a collective spin operator with $\sigma_z^i$ being the Pauli $z$ operator acting on the $i$th qubit, and $B$ is an arbitrary environment operator. The Hamiltonian used in the present protocol reads

$$H_N = \sum_{k<l} \left( J_{kl}^x R_{kl}^x + J_{kl}^y R_{kl}^y \right) + \sum m J_m^z \sigma_m^z,$$

where $J_{kl}^x$ and $J_{kl}^y$ are real-valued controllable coupling parameters defining the strengths of the $XY$ interaction $R_{kl}^x = \frac{1}{\sqrt{2}} (\sigma_k^x \sigma_l^x + \sigma_k^y \sigma_l^y)$ and the Dzialoshinski-Moriya interaction $R_{kl}^y = \frac{1}{\sqrt{2}} (\sigma_k^x \sigma_l^y - \sigma_k^y \sigma_l^x)$ [43, 44], respectively, and $J_m^z$ is a real-valued parameter describing the strength of a local field affecting the $m$th physical qubit. Here, $\sigma_m^z$ represents the Pauli $\nu$ operator ($\nu = x, y, z$) acting on the $m$th physical qubit. This kind of Hamiltonian has been widely used in literature [45–51]. Compared with the Hamiltonian used in the original protocol [14], the Hamiltonian used here includes the local field term $\sum m J_m^z \sigma_m^z$. This term plays an important role for realizing an arbitrary one-qubit gate by a single-shot implementation. It provides an off-resonant coupling between computational states and logical auxiliary states, which makes the rotation angle variable.
To realize nonadiabatic holonomic quantum computation in decoherence-free subspaces, a universal set of quantum gates is needed. In the following, we demonstrate how to realize an arbitrary one-qubit holonomic gate by a single-shot implementation and how to realize an entangling two-qubit holonomic gate with the Hamiltonian expressed by Eq. (4).

### A. One-qubit gates

To realize an arbitrary one-qubit holonomic gate in a decoherence-free subspace by a single-shot implementation, we need to consider the quantum system of three physical qubits interacting collectively with a common dephasing environment. In the case of \( N = 3 \), the interaction Hamiltonian and the system Hamiltonian are specified as \( H_I = S \otimes B \) and \( H_3 \), respectively. For this system, there is a three-dimensional subspace,

\[
S^D_1 = \operatorname{Span}\{|100\rangle, |010\rangle, |001\rangle\},
\]

which satisfies conditions (a) and (b), being a decoherence-free subspace. Here, \(|0\rangle\) and \(|1\rangle\) represent the eigenvectors of the Pauli \( z \) operator, corresponding to eigenvalues \(+1\) and \(-1\), respectively.

We choose \( H_3 \) to be time independent and express its nonzero parameters as

\[
J_{12}^x = \cos \theta \cos \varphi, \quad J_{12}^y = -J \cos \phi \sin \theta, \quad J_{13}^y = -J \cos \phi \cos \theta, \quad J_2^z = J_3^z = J \sin \phi,
\]

where \( J, \phi, \theta, \) and \( \varphi \) are time-independent parameters. Here, a key point is the choice of local fields, two of which should be the same while the third is put to zero. By inserting Eq. (6) into Eq. (4), and using the Pauli operators \( \sigma_{\mu}^x = |1\rangle \langle 0| + |0\rangle \langle 1| \), \( \sigma_{\mu}^y = i(|1\rangle \langle 0| - |0\rangle \langle 1|) \), and \( \sigma_{\mu}^z = |0\rangle \langle 0| - |1\rangle \langle 1| \) pertaining to each of the physical qubits, we obtain an explicit expression of Hamiltonian \( H_3 \). For simplicity, we let \(|a\rangle = |100\rangle, |0\rangle_L = |010\rangle, \) and \(|1\rangle_L = |001\rangle \). We then have

\[
H_3 = J \cos \phi \left( \sin \frac{\theta}{2} e^{i \varphi} |a\rangle_L \langle 0| - \cos \frac{\theta}{2} |a\rangle_L \langle 1| + \text{H.c.} \right),
\]

It can be further rewritten as

\[
H_3 = J \cos \phi (|a\rangle \langle b| + |b\rangle \langle a|) + 2J \sin \phi |a\rangle \langle a|,
\]

where

\[
|b\rangle = \sin \frac{\theta}{2} e^{-i \varphi} |0\rangle_L - \cos \frac{\theta}{2} |1\rangle_L.
\]

We use \(|d\rangle\) to donate the dark state, i.e., the zero-energy eigenstate of \( H_3 \)

\[
|d\rangle = \cos \frac{\theta}{2} |0\rangle_L + \sin \frac{\theta}{2} e^{i \varphi} |1\rangle_L,
\]

which is orthogonal to \(|a\rangle\) and \(|b\rangle\).

It is easy to verify that the smaller subspace,

\[
S^U_1 = \operatorname{Span}\{|b\rangle, |d\rangle\} = \operatorname{Span}\{|0\rangle_L, |1\rangle_L\},
\]

satisfies conditions (c) and (d) if the evolution period \( \tau_1 \) is taken as

\[
J \tau_1 = \pi.
\]

Indeed, since \( H_3 \) is time independent, implying \( U(t) = H_3 \), it is straightforward to have \(|p|U(t)H_3U(t)q\rangle = \langle p|H_3|q\rangle\), \( p, q \in \{a, b\} \), i.e., condition (d) is satisfied. One will soon see that condition (c) is satisfied too. In this case, the subspace \( S^U_1 \) can be used as the computational space, and the logic qubit is encoded in it, while \(|a\rangle\) acts as an ancillary state.

The evolution operator in the decoherence-free subspace \( S^D_1 \) can be expressed as \( U(t) = \exp(-iH_3t) \). For \( t = \tau_1 \), there is \( U_1(\tau_1) = e^{-iH_3\tau_1} \). By using Eq. (12) and the identity \( 2|a\rangle\langle a| = (|a\rangle \langle a| + |b\rangle \langle b|) + (|a\rangle \langle a| - |b\rangle \langle b|) \), we find \( U_1(\tau_1) = \exp[-i \sin \phi (|a\rangle \langle a| + |b\rangle \langle b|) - iA] \) with \( A = \cos \phi |a\rangle \langle b| + |b\rangle \langle a| + \sin \phi |a\rangle \langle a| - |b\rangle \langle b| \). We see that \( A, |a\rangle \langle a| + |b\rangle \langle b| = 0, \) and \( A^{2n} = |a\rangle \langle a| + |b\rangle \langle b|, A^{2n+1} = A, \) for \( n = 1, 2, ..., \) which implies

\[
U_1(\tau_1) = e^{-i \sin \phi (|a\rangle \langle a| + |b\rangle \langle b|) - iA} = e^{-i(\pi + \gamma_1) \sin \phi}(|a\rangle \langle a| + |b\rangle \langle b|) + |d\rangle \langle d|.
\]

Clearly, \( U_1(\tau_1) \) maps states in the subspace \( S^U_1 \) into the subspace, i.e., condition (c) is indeed satisfied.

In the basis \(|a\rangle, |b\rangle, |d\rangle\), the unitary operator can be written as

\[
U_1(\tau_1) = \begin{pmatrix}
0 & e^{-i \gamma_1} & 0 \\
0 & 0 & 0 \\
e^{-i \gamma_1} & 0 & 0
\end{pmatrix},
\]

where \( \gamma_1 \) is given by

\[
\gamma_1 = \pi + \pi \sin \phi.
\]

Thus, the evolution operator projected onto the computational subspace \( S^U_1 \) reads

\[
U_{1U}(\tau_1) = |d\rangle \langle d| + e^{-i \gamma_1} |b\rangle \langle b|.
\]

The unitary operator \( U_{1U}(\tau_1) \) represents an arbitrary one-qubit gate, for which the rotation axis is determined by states \(|b\rangle\) and \(|d\rangle\), and the rotation angle is determined by phase \( \gamma_1 \).

Therefore, to achieve a desired nonadiabatic holonomic gate in the decoherence-free subspace \( S^D_1 \), one first calculates \( \phi, \theta, \) and \( \varphi \) by using Eqs. (9), (10), and (15), and then determines the parameters \( J_{12}^x, J_{12}^y, J_{13}^y, J^z_2 \), and \( J^z_3 \) by using Eq. (6). In this way, the Hamiltonian described by Eq. (8) is obtained, and the desired one-qubit gate can be realized by appropriately choosing the evolution period \( \tau_1 \) satisfying Eq. (12).

The above discussion shows that an arbitrary one-qubit gate can be realized by a single-shot implementation. Compared
with the original protocol of nonadiabatic holonomic quantum computation in decoherence-free subspaces as well as all the schemes of its physical implementation, the present implementation of one-qubit gates not only maintains the merits of the previous protocol, but also avoids the extra work of combining two gates to implement an arbitrary one-qubit gate and thereby reduces the exposure time to various error sources.

### B. Entangling two-qubit gate

To realize a universal set of nonadiabatic holonomic gates in decoherence-free subspaces, a nontrivial two-qubit holonomic gate is needed in addition to the one-qubit holonomic gates obtained above. Noting that our arbitrary one-qubit gates are compatible with the nonadiabatic holonomic two-qubit gates proposed in Refs. [14, 22–25], we may simply combine the present one-qubit gates with those previous two-qubit gates to form a universal set of quantum gates. Alternatively, we can also construct an entangling two-qubit holonomic gate by using the Hamiltonian in Eq. (4), leading to a wider class of two-qubit gates. We demonstrate how to obtain such generalized two-qubit gates by using the Hamiltonian expressed in Eq. (4). The scheme follows closely that of the holonomic one-qubit gates discussed above.

Consider six physical qubits interacting collectively with a common dephasing environment. In the case of $N = 6$, the interaction Hamiltonian and the system Hamiltonian are $H_I = S_6 \otimes B$ and $H_6$, respectively. For this system, there is a six-dimensional subspace,

$$S^D_2 = \text{Span}\{|001001, 010001, 001010, 001001, 011000, 000011\},$$

which satisfies conditions (a) and (b), being a decoherence-free subspace.

We choose $H_6$ to be time independent and express its nonzero parameters as

$$J_{16}^\xi = \lambda \cos \zeta \sin \frac{\alpha}{2} \cos \beta,$$

$$J_{15}^\xi = -\lambda \cos \zeta \sin \frac{\alpha}{2} \sin \beta,$$

$$J_{26}^\xi = -\lambda \cos \zeta \cos \frac{\alpha}{2},$$

$$J_3^\xi = J_0^\xi = \lambda \sin \zeta,$$

where $\lambda$, $\zeta$, $\alpha$, and $\beta$ are time-independent parameters.

By inserting Eq. (18) into Eq. (4) and using the Pauli operators $\sigma^x_\mu = |0\rangle_\mu \langle 1| + |1\rangle_\mu \langle 0|, \sigma^y_\mu = i |1\rangle_\mu \langle 0| - |0\rangle_\mu \langle 1|$, and $\sigma^z_\mu = |0\rangle_\mu \langle 0| - |1\rangle_\mu \langle 1|$, we can obtain an explicit expression for $H_6$. We let $|00\rangle_L = |001001\rangle, |01\rangle_L = |010001\rangle, |10\rangle_L = |001100\rangle, |11\rangle_L = |000111\rangle, |a_1\rangle = |010\rangle, \text{and} |a_2\rangle = |001\rangle$, where the first four states are consistent with the one-qubit code $|0\rangle_L = |010\rangle$ and $|1\rangle_L = |001\rangle$ used before.

We obtain

$$H_6 = \lambda \cos \zeta \left( \sin \frac{\alpha}{2} e^{i\theta} |a_1\rangle_L \langle 00| - \cos \frac{\alpha}{2} |a_1\rangle_L \langle 01| + \text{H.c.} \right)$$

$$- \lambda \cos \zeta \left( \cos \frac{\alpha}{2} |a_2\rangle_L \langle 10| - \sin \frac{\alpha}{2} e^{i\theta} |a_2\rangle_L \langle 11| + \text{H.c.} \right) + 2 \lambda \sin \zeta \langle a_1| a_1 - |a_2\rangle \langle a_2|).$$

(19)

It can be further rewritten as

$$H_6 = \lambda \left[ \cos \zeta \langle a_1| b_1 \rangle + |b_1\rangle \langle a_1| + 2 \sin \zeta \langle a_1| a_1 \right]$$

$$- \lambda \left[ \cos \zeta \langle a_2| b_2 \rangle + |b_2\rangle \langle a_2| + 2 \sin \zeta \langle a_2| a_2 \rangle \right],$$

(20)

where

$$|b_1\rangle = \sin \frac{\alpha}{2} e^{-i\theta} |00\rangle_L - \cos \frac{\alpha}{2} |01\rangle_L,$$

$$|b_2\rangle = \cos \frac{\alpha}{2} |10\rangle_L - \sin \frac{\alpha}{2} e^{i\theta} |11\rangle_L.$$  

(21)

We use $|d_1\rangle$ and $|d_2\rangle$ to donate the dark states, i.e., the zero-energy eigenstates of $H_6$,

$$|d_1\rangle = \cos \frac{\alpha}{2} |00\rangle_L + \sin \frac{\alpha}{2} e^{-i\theta} |01\rangle_L,$$

$$|d_2\rangle = \sin \frac{\alpha}{2} e^{-i\theta} |10\rangle_L + \cos \frac{\alpha}{2} |11\rangle_L,$$

(22)

which are orthogonal to $|a_1\rangle$, $|a_2\rangle$, $|b_1\rangle$, and $|b_2\rangle$.

By following a line of argument similar to the one-qubit gates, it is easy to verify that the smaller subspace

$$S^D_2 = \text{Span} \{|b_1\rangle, |d_1\rangle, |d_2\rangle, |d_3\rangle\}$$

$$= \text{Span} \{|00\rangle_L, |01\rangle_L, |10\rangle_L, |11\rangle_L\},$$

$$= \text{Span} \{|001001\rangle, |010001\rangle, |001010\rangle, |001001\rangle\}$$

(23)

satisfies conditions (c) and (d) if the evolution period $\tau_2$ is taken as

$$\lambda \tau_2 = \pi.$$  

(24)

In this case, $S^D_2$ can be used as the computational space, and the logical qubits are encoded in it, while $|a_1\rangle$ and $|a_2\rangle$ are utilized as two ancillary states.

The evolution operator in the decoherence-free subspace $S^D_2$ can be expressed as $U_2(t) = \exp(-iH_6 t)$. By further pursuing the analogy of one-qubit gates, $U_2(t_2)$ can be expressed as $U_2(t_2) = \exp(-i\tau \sin \zeta \langle a_1| a_1 \rangle + |b_1\rangle \langle b_1| - i\tau A_1 \exp(i\tau \sin \zeta \langle a_2| a_2 \rangle + |b_2\rangle \langle b_2| + i\tau A_2) \text{with} A_i = \cos \zeta \langle a_i| b_i \rangle + |b_i\rangle \langle a_i| + \sin \zeta \langle a_i| a_i \rangle - |b_i\rangle \langle b_i| \rangle$. Noting that $|a_i\rangle, |a_1\rangle + |b_1\rangle = 0$, and $A_{2n} = \langle a_i| a_i \rangle + |b_i\rangle \langle b_i|$ and $A_{2n+1} = A_i$, for $n = 1, 2, ...$, we then obtain

$$U_2(t_2) = e^{-i\tau \sin \zeta \langle a_1| a_1 \rangle + |b_1\rangle \langle b_1|} e^{-i\tau A_1} e^{i\tau \sin \zeta \langle a_2| a_2 \rangle + |b_2\rangle \langle b_2|}$$

$$+ e^{-i\tau \sin \zeta \langle a_1| a_1 \rangle + |b_1\rangle \langle b_1|} e^{i\tau \sin \zeta \langle a_2| a_2 \rangle + |b_2\rangle \langle b_2|} + |d_1\rangle \langle d_1| + |d_2\rangle \langle d_2|.$$  

(25)

Clearly, $U_2(t_2)$ maps states in the subspace $S^D_2$ into the subspace.
In the basis $|a_1\rangle, |a_2\rangle, |b_1\rangle, |b_2\rangle, |d_1\rangle, |d_2\rangle$, the unitary operator takes the form

$$U_2(\tau_2) = \begin{pmatrix}
0 & 0 & e^{i\gamma_2} & 0 & 0 & 0 \\
0 & 0 & 0 & e^{i\gamma_2} & 0 & 0 \\
0 & 0 & 0 & 0 & e^{i\gamma_2} & 0 \\
0 & 0 & 0 & 0 & 0 & e^{i\gamma_2} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \quad (26)$$

where $\gamma_2$ is given by

$$\gamma_2 = \pi + \pi \sin \zeta. \quad (27)$$

The evolution operator projected onto the computational subspace $|b_1\rangle, |d_1\rangle, |b_2\rangle, |d_2\rangle$ reads

$$U_2^c(\tau_2) = |d_1\rangle\langle d_1| + e^{-i\gamma_2}|b_1\rangle\langle b_1| + |d_2\rangle\langle d_2| + e^{i\gamma_2}|b_2\rangle\langle b_2|. \quad (28)$$

$U_2^c(\tau_2)$ acts as an entangling two-qubit gate in the computational subspace $S_2^c$. To see this explicitly, we note that $U_2^c(\tau_2) = |0\rangle_L |0\rangle \otimes \exp(-i\gamma_2/2) \exp(i\gamma_2 n \cdot \sigma/2) + |1\rangle_L |1\rangle \otimes \exp(i\gamma_2/2) \exp(-i\gamma_2 n \cdot \sigma/2)$ with $n = (\sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha)$ and $m = (\sin \alpha \cos \beta, \sin \alpha \sin \beta, -\cos \alpha)$. Here, $\sigma = (\sigma^x, \sigma^y, \sigma^z)$ are Pauli operators, acting on $|0\rangle_L$ and $|1\rangle_L$. Since $\gamma_2$ in Eq. (28) can take any desired value by properly choosing $\zeta$, instead of being fixed to $\gamma_2 = \pi$ in all the previous schemes [14, 22–25], $U_2^c(\tau_2)$ covers a wider class of holonomic two-qubit gates in the decoherence-free subspace $S_2^d$.

The arbitrary one-qubit gate $U_1^c(\tau_1)$ given in Eq. (16) and the entangling two-qubit gate $U_2^c(\tau_2)$ given in Eq. (28) comprise a universal set of nonadiabatic holonomic gates in decoherence-free subspaces.

### IV. CONCLUSION

In conclusion, we have proposed an alternative protocol of nonadiabatic holonomic quantum computation in decoherence-free subspaces, in which an arbitrary one-qubit gate is directly realized by a single-shot implementation. The present protocol not only maintains the merits of the original protocol, but also avoids the extra work of combining two gates to implement an arbitrary one-qubit gate and thereby reduces the exposure time to various error sources. We hope that the present protocol will be useful to find new experimentally feasible settings that combine the ideas of holonomic quantum and decoherence-free subspaces for robust quantum computation.

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