Doubly heavy baryons at LHC

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The theoretical analysis of production, lifetime, and decays of doubly heavy baryons is presented. The lifetime of Ξ++ cc baryon recently measured by the LHCb Collaboration is used to estimate the lifetimes of other doubly heavy baryons. The production and the possibility of observation of Ξbc baryon at LHC are discussed.

I. INTRODUCTION

Doubly heavy baryons are extremely interesting objects that allow us to take a fresh look at the problems of the production and hadronization of heavy quarks. These baryons consist of two heavy and one light quarks and therefore, unlike ordinary heavy baryons, are characterized by several scales at once:

\[ m_{Q_1,2} \gg m_Q \cdot v, m_Q \cdot v \gg \Lambda_{QCD}, \]

where \( m_{Q_1,2} \) are masses of heavy quarks, and \( v \) is there velocity inside the quarkonium. For clarity, one can go to the coordinate representation and select a specific family of baryons. Thus, for a baryon \( \Xi_{bc} \) containing \( b \)- and \( c \)-quarks simultaneously, the scales are ordered as follows:

\[ \lambda_b : \lambda_c : r_{bc} : r_{QCD} \approx 1 : 3 : 9 : 27, \]

where \( \lambda_Q = 1/m_Q \) is a Compton length of quark, \( r_{bc} \approx 1/(v \cdot m_Q) \) is heavy quark size, \( r_{QCD} = \Lambda_{QCD} \) is a scale of nonperturbative confinement [1].

It is worth to mention, that a baryon with one heavy quark is characterized by only two scales, namely, the mass of the heavy quark and \( \Lambda_{QCD} \). In the limit \( m_{Q_1,2} \rightarrow \infty \) a heavy

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diquark interacts with a light quark as heavy anti-quark and, therefore, it is quite natural to subdivide calculating the characteristics of doubly heavy quarkonium in two stages: the calculation of the properties of the heavy diquark and the subsequent calculation of the properties of the system of quark-diquark \(^1\).

The problems of production and decays of such systems was of interest to researchers for many years. But the last year was special because it was marked by the discovery of the doubly charmed \(\Xi^{++}_{cc}\) baryon in the decay mode \(\Lambda^+ c K^−π^+\) \([4]\). The LHCb Collaboration observes hundreds of such particles. This discovery was confirmed by the observation of decay \(\Xi^{++}_{cc} \rightarrow \Xi^+_c π^+\) \([5]\). This circumstance greatly revived the research activities in this direction. In this article we discuss the perspectives of further research of doubly heavy baryon states: there decays, productions and possibility of observation of excited states.

The rest of the paper is organized as follows. In the next section production of doubly heavy baryons is considered. Section III is devoted to theoretical calculation of the lifetimes of the considered particles. Observation probability of these baryons is discussed in section IV and finally the Conclusion will be given.

II. DOUBLY HEAVY BARYON PRODUCTION

It is natural to use a two-step procedure to produce a doubly heavy baryon. In the first calculation step a doubly heavy diquark is produced perturbatively in the hard interaction. In the second step a doubly heavy diquark is transformed to the baryon within the soft hadronization process.

Our calculation of doubly heavy diquark production were done within the following approach:

1. the color singlet model for doubly heavy mesons and the color triplet model for doubly heavy baryons;

2. the contribution from scattering of sea heavy quark and gluon \((Q_1 g \rightarrow Q_1 + Q_2 + \bar{Q}_2)\) does not take into account to avoid double counting; \(^2\)

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\(^1\) An alternative approach based on the direct solution of the three-body problem is presented in \([2, 3]\).

\(^2\) Furthermore, accounting these process is questionable for LHCb kinematic region due to rather small transverse momenta of doubly heavy system which is comparable with a heavy quark mass.
3. the contribution of color sextet state to baryon production is neglected.

Quarks in color antitriplet $\bar{3}_c$ attract each other and their interaction can be described by the wave function in the framework of potential model, as well as the quark-antiquark interaction in quarkonium. By analogy with quarkonium one can write for the production amplitude of doubly heavy diquark:

$$A^{S_{J_J_z}} = \int T^{S_{S_z}}_{Q_1\bar{Q}_1Q_2\bar{Q}_2} (p_i, k(\bar{q})) \cdot \left( \Psi^{L_{J_z}}_{[Q_1\bar{Q}_2]3c} (\bar{q}) \right)^* \cdot C^{J_{J_z}}_{s_zl_z} \frac{d^3\bar{q}}{(2\pi)^3},$$

where $T^{S_{S_z}}_{Q_1\bar{Q}_1Q_2\bar{Q}_2}$ is an amplitude of the hard production of two heavy quark pairs;

$\Psi^{L_{J_z}}_{[Q_1\bar{Q}_2]3c}$ is the diquark wave function (color antitriplet);

$J$ and $J_z$ are the total angular momentum and its projection on $z$-axis in the $[Q_1Q_2]3c$ rest frame;

$L$ and $l_z$ are the orbital angular momentum of $bc$-diquark and its projection on $z$-axis;

$S$ and $s_z$ are $Q_1Q_2$-diquark spin and its projection;

$C^{J_{J_z}}_{s_zl_z}$ are Clebsh-Gordon coefficients;

$p_i$ are four momenta of diquark, $\bar{Q}_1$ quark and $\bar{Q}_2$ quark;

$\bar{q}$ is three momentum of $Q_1$-quark in the $Q_1Q_2$-diquark rest frame (in this frame $(0, \bar{q}) = k(\bar{q})$).

Under assumption of small dependence of $T^{S_{S_z}}_{b\bar{b}c\bar{c}}$ on $k(\bar{q})$

$$A \sim \int d^3\bar{q} \Psi^* (\bar{q}) \left\{ T(p_i, \bar{q})|_{\bar{q}=0} + \bar{q} \frac{\partial}{\partial \bar{q}} T(p_i, \bar{q})|_{\bar{q}=0} + \cdots \right\}$$

and, particularly, for the $S$-wave states

$$A \sim R_S(0) \cdot T_{Q_1\bar{Q}_1Q_2\bar{Q}_2} (p_i)|_{\bar{q}=0},$$

where $R_S(0)$ is a value of radial wave function at origin.

In our early work [6] we discussed the similarity of the production mechanisms of doubly charmed baryons and the associative $J/\psi$ and the open charm in hadronic interactions. Indeed, both processes within a single parton scattering approach are described by the similar sets of diagrams, because both ones involve the production of four heavy quarks (see diagram examples in Fig. 1). However, the experimental data indicate the presence of contribution of double parton scattering (DPS), which dominates at LHC energies [7]. Within the DPS mechanism two $c\bar{c}$ pairs are produced independently in the different parton
interactions. Such mechanism can contribute to the associative $J/\psi + c$ production but one can hardly contribute to the process $\Xi_{cc}$ production, because to produce doubly charmed baryon $c$ charm quark from different pairs are needed.\footnote{However these is a research, where it was made an attempt to expand the DPS model to the case of $\Xi_{cc}$ production \cite{8} using quark-hadron duality approach.} Thus we currently tend to think, that DPS mechanism contributes only to $J/\psi + c$ production. This is why the yield of $\Xi_{cc}$ is essentially smaller, than the yield of the associative production of $J/\psi$-meson and open charm, whereas the yields of $B_c$ mesons and $\Xi_{bc}$ baryons should be comparable. Also it is worth to mention that $J/\psi + c$ cross section and $\Xi_{cc}$ cross section should have different dependence on the $pp$ interaction energy: DPS cross section increases faster than SPS.

It should be noted that the doubly heavy diquark production can not be described within the fragmentation model due to the large contribution of non-fragmentation diagrams, which can not be interpreted as $b$-quark production followed by the fusion of $b$-quark into $bc$-diquark. The same feature is inherent in the process of $B_c$-meson production. This is not surprising because the production processes of $bc$-diquark production and $B_c$ production are described by the same set of the diagrams. The difference comes from different color coefficients and different choice of values for $c$ and $b$ quark masses.

The dominant contribution to the production cross under LHCb kinematics conditions comes from gluonic interaction, as well as for the $B_c$ meson:

$$gg \rightarrow \Xi_{bc} + \bar{b}c.$$
Figure 2: $\Xi_{bc}$ $p_T$ distribution v.s. $B_c$ $p_T$ distribution for $\sqrt{s_{gg}} = 30$ GeV and $\sqrt{s_{gg}} = 60$ GeV, correspondingly. The same quark mass values are used for both estimations: $m_c = 1.5$ GeV and $m_b = 4.8$ GeV. Also, for convenience of comparison, we put $|R_{B_c}(0)|^2$ and $|R_{[bc]_3}(0)|^2$ equal.

determined by the difference of wave functions:

$$\frac{\sigma_{\Xi_{bc}}}{\sigma_{B_c}} \sim \frac{|R_{[bc]_3}(0)|^2}{|R_{B_c}(0)|^2} \quad (3)$$

Indeed, if one choose the same quark mass values for the subprocesses $gg \rightarrow [bc]_3 + \bar{b}\bar{c}$ and $gg \rightarrow B_c + \bar{b}\bar{c}$ and put $R_{[bc]_3}^2 = R_{B_c}^2$, one can see that this process have very similar behavior on transverse momenta of doubly heavy system, as it is shown in Fig. 2, where we put $|R_{B_c}(0)|^2$ and $|R_{[bc]_3}(0)|^2$ equal for convenience of comparison.

Of course, a color antitriplet of $bc$ system is not a $\Xi_{bc}$ yet. It should be somehow transformed to the $bcq$ baryon. The transverse momentum of light quark $q$ with mass $m_q$ is about $\frac{m_q \vec{p}_{\Xi_{bc}}}{m_{\Xi_{bc}} \vec{p}_{\Xi_{bc}}}$, where $\vec{p}_{\Xi_{bc}}$ is a transverse momentum of $\Xi_{bc}$. For LHCb kinematical conditions such quark always exits in the quark sea. This is why we assume, that a doubly heavy is hadronized by joining with a light quarks $u, d$ and $s$ in proportion $1 : 1 : 0$. We also assume that it is hadronized with probability equal 1. It is worth to note, that the latter assumption is pretty much a guess, because diquark has a color charge and therefore strongly interacts with its environment, that could lead to the diquark dissociation. Thus, (3) can be considered as an upper limit for ratio of yields of $\Xi_{bc}$ and $B_c$.

We estimate the ratio of yields $\Xi_{bc}$ and $B_c$ for hadronic interactions at $\sqrt{s} = 13$ TeV for several scales ($\mu_R = \mu_F = 10$ GeV, $\mu_R = \mu_F = E_{T_{\Xi_{bc}}}^{\Xi_{bc}}/2$, $\mu_R = \mu_F = E_{T_{B_c}}^{\Xi_{bc}}$, $\mu_R = \mu_F = 2E_{T_{B_c}}^{\Xi_{bc}}$) and find, that the dependence of this value on scale choice is unessential. The main uncertainties come from wave functions and from choice of mass values for $b$ and $c$ quarks.
Figure 3: The ratio of production yields of $\Xi_{bc}$ and $B_c$ for hadronic interaction at $\sqrt{s} = 13$ TeV in units of $|R_{[bc]_3}(0)|^2/|R_{B_c}(0)|^2$ for the similar quark masses ($m_b = 4.8$ GeV, $m_c = 1.5$ GeV, solid curve) and for the different quark masses ($m_b = 4.8$ GeV and $m_c = 1.5$ GeV for $B_c$ production, and $m_b = 4.9$ GeV and $m_c = 1.7$ GeV for $\Xi_{bc}$ production, dashed curve). The CT14LL parameterization [9] is used for PDFs.

In Fig. 3 we show the ratio of yields $\Xi_{bc}$ and $B_c$ in hadronic interactions as a function of $p_T$ at $\sqrt{s} = 13$ TeV, for the similar masses ($m_b = 4.8$ GeV, $m_c = 1.5$ GeV) and for different masses ($m_b = 4.8$ GeV and $m_c = 1.5$ GeV for $B_c$ production, and $m_b = 4.9$ GeV and $m_c = 1.7$ GeV for $\Xi_{bc}$ production). Here we also put $|R_{B_c}(0)|^2 = |R_{[bc]_3}(0)|^2$. One can see, that these distributions are approximately flat. Thus, one can conclude, that the estimation (3) is approximately valid for all transverse momenta.

There are many estimations for $R_{[bc]_3}(0)$ value, as well as for $R_{B_c}(0)$ (see, for example [1, 10–13]). However, to obtain the ratio, it is rational to use values extracted within the similar framework. From [1] and [10], where the non-relativistic model with Buchmüller-Tye wave function was used, we obtain that

$$\frac{|R_{[bc]_3}(0)|^2}{|R_{B_c}(0)|^2} = \frac{(0.71 \text{ GeV}^{3/2})^2}{(1.28 \text{ GeV}^{3/2})^2} \approx 0.31.$$  

From [12] and [11], where the relativistic potential model was applied and relativistic
correction have been accounted perturbatively, we obtain for the same ratio

\[
\frac{|R_{[bc]\bar{3}}(0)|^2}{|R_{B_c}(0)|^2} = \frac{(0.74 \text{ GeV}^{3/2})^2}{(1.46 \text{ GeV}^{3/2})^2} \approx 0.26.
\]

In [14, 15] the corrections to the relativistic potential model predictions had been taken into account non-perturbatively, that leads to the noticeable difference of wave function values for different spin states. However the cross section ratio value remains the same:

\[
\frac{\sigma_{\Xi_{bc}}}{\sigma_{B_c}} = \frac{\sigma_{\Xi_{bc}}(1S_0) + \sigma_{\Xi_{bc}}(1S_1)}{\sigma_{B_c}(1S_0) + \sigma_{B_c}(1S_1)} \approx \frac{|R_{[bc]\bar{3}}(1S_0)(0)|^2 + 3 \cdot |R_{[bc]\bar{3}}(1S_1)(0)|^2}{|R_{B_c}(1S_0)(0)|^2 + 2.5 \cdot |R_{B_c}(1S_1)(0)|^2} \approx 0.32
\]

Therefore, one can conclude that

\[
\frac{\sigma_{\Xi_{bc}}}{\sigma_{B_c}} \leq \frac{1}{3}. \quad (4)
\]

It is worth to note that both the numerator and the denominator in (4) will be modified by the feed-down from excitations. However we believe, that in ratio these contributions will approximately canceled out. The obtained ratio value \(\sigma_{\Xi_{bc}}/\sigma_{B_c}\) coincides with that used in talk [16].

To estimate the absolute cross section value of \(\Xi_{bc}\) baryon production at LHCb (\(\sqrt{s} = 13\) TeV, \(2.0 < y_{\Xi_{bc}} < 4.5\)) we use the quark mass values \(m_b = 4.9\) GeV and \(m_c = 1.7\) GeV, the value of diquark wave function at origin \(R_{[bc]\bar{3}}(0) = 0.71\) GeV\(^{3/2}\) [1] and CT14LL parton density parameterization [9]. Varying scales from \(\mu_R = \mu_F = E_{\Xi_{bc}}/2\) to \(\mu_R = \mu_F = 2E_{\Xi_{bc}}\) we obtain, that the cross section value of \(bc\) baryons with \(1S\) wave state of doubly heavy diquark at LHCb is about \(10 \div 25\) nb depending on scale values. The feed-down from excitations can be estimated as 20-30%.

As it was mentioned before an analogous ratio can not be valid for \(J/\psi + c\) and \(\Xi_{cc}\) due to the large contribution of DPS to the associative \(J/\psi\) and \(c\) production.
III. DOUBLY HEAVY BARYON DECAYS WITHIN OPE METHOD

A. Method description

In accordance with Operator Product Expansion (OPE) and optic theorem the life time of doubly heavy baryon $B$ can be represented as

$$\Gamma_B = \frac{1}{2M_B} \langle B | \mathcal{T} | B \rangle,$$

(5)

where operator $\mathcal{T}$ is

$$\mathcal{T} = \text{Im} \int d^4x \left\{ \bar{T} H_{\text{eff}}(x) H_{\text{eff}}(0) \right\},$$

(6)

with

$$H_{\text{eff}} = \frac{G_F}{2\sqrt{2}} V_{q_3q_4} V_{q_1q_2}^* \left[ C_+(\mu) O_+ + C_-(\mu) O_- \right],$$

(7)

In the above expression Wilson coefficients $C_{\pm} (\mu)$ equal

$$C_+(\mu) = \left[ \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{\frac{6}{33-2n_f}}, \quad C_-(\mu) = \left[ \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{-\frac{12}{33-2n_f}},$$

(8)

where $\alpha_s(\mu)$ is a running strong coupling constant calculated within two-loop approximation and $n_f$ is a number of active flavors. The operators $O_{\pm}$ in (7) are determined as follows:

$$O_{\pm} = [\bar{q}_1 \gamma_\nu (1 - \gamma_5) q_2, [\bar{q}_3 \gamma^\nu (1 - \gamma_5) q_4] \left( \delta_{\alpha\beta} \delta_{\gamma\delta} \pm \delta_{\alpha\delta} \delta_{\gamma\beta} \right),$$

(9)

where $\alpha, \beta, \gamma, \delta$ are color indices of quarks.

For large energy of heavy quark decay one can represent $\mathcal{T}$ (6) a set of local operators ordered by increasing of their dimension. The contribution of high dimension term are suppressed by inverse powers of heavy quark mass $m_Q$, and therefore only several first terms contribute to the decay value. This method was broadly used for the calculation of lifetimes of heavy hadrons [6, 17–23], as well as doubly heavy hadrons [24, 25]. It was shown in the cited papers the operators of dimension 3 and 5

$$O_{QQ} = (\bar{Q}Q), \quad O_{QG} = (\bar{Q} \sigma_{\mu\nu} G^{\mu\nu} Q),$$

(10)

correspond to the spectator decay of heavy quark and give the main contribution to the value (5). The following operator of dimension 6 can also give noticeable contribution to the decay process:

$$O_{2Q2q} = (\bar{Q} \Gamma q)(\bar{q} \gamma Q).$$

(11)
Typical Feynman diagrams for the discussed processes are shown in Fig. 4. In accordance with OPE method the following mechanisms can contribute to the total decay width:

- Spectator mechanism (the operator (10) and the diagram 4(a)),
- Weak scattering, WS (the operator (11) and the diagram 4(b)),
- Pauli-interference, PI (the operator (11) and the diagrams 4(c), (d)),

B. Lifetimes of doubly charmed baryons $\Xi^{++}_{cc}$, $\Xi^{+}_{cc}$, $\Omega^{+}_{cc}$

The decay amplitudes for doubly charmed baryons $\Xi^{++}_{cc}$ and $\Xi^{+}_{cc}$ can be performed as follows:

\[
\mathcal{T}_{\Xi^{++}_{cc}} = 2\mathcal{T}_{35c} + \mathcal{T}_{\text{PI}}^{(\Xi^{++}_{cc})},
\]

\[
\mathcal{T}_{\Xi^{+}_{cc}} = 2\mathcal{T}_{35c} + \mathcal{T}_{\text{WS}}^{(\Xi^{+}_{cc})},
\]

\[
\mathcal{T}_{\Omega^{+}_{cc}} = 2\mathcal{T}_{35c} + \mathcal{T}_{\text{PI}}^{(\Omega^{+}_{cc})}.
\]

In these equations the contribution of operators with dimension 3 and 5 can be determined
as follows:

$$\mathcal{T}_{35c} = \Gamma_{c,spec}(\bar{cc}) - \frac{\Gamma_{0c}}{m_c^2} (2 + K_{0c}) P_{s1} + K_{2c} P_{s2} \Omega_{Ge},$$

(12)

where

$$\Gamma_{0c} = \frac{G_F m_c^5}{192 \pi^3}, \quad K_{0Q} = C_+^2 + 2 C_+^2, \quad K_{2Q} = 2(C_+^2 - C_+^2)$$

$$P_{c1} = (1 - y)^4, \quad P_{c2} = (1 - y)^3, \quad y = \frac{m_c^2}{m_c^2}, \quad r = \frac{m_r^2}{m_c^2}$$

$$P_{c1} = \sqrt{1 - 2(r + y) + (r - y)^2}[1 - 3(r + y) + 3(r^2 + y^2) - r^3 - y^3 - 4ry + 7ry(r + y)] + 12r^2 y^2 \ln \left( \frac{1 - r - y + \sqrt{1 - 2(r + y) + (r - y)^2}}{4ry} \right),$$

$$P_{c2} = \sqrt{1 - 4y(1 - 6y + 2y^2 + 12y^3)24y^4} \ln \frac{1 + \sqrt{1 - 4y}}{1 - \sqrt{1 - 4y}},$$

and the width of spectator mechanism was estimated in papers [24, 26–31].

As it was mention above the contribution values of PI and WS mechanisms depend on the baryon composition. For example, it is clear from diagrams in Fig. 4 that for $\Xi_{cc}^{++} = (ccu)$ and $\Omega_{cc}^{++} = (ccs)$ the WS is forbidden and PI destructively contributes to the width. Contrary, for the $\Xi_{cc}^+$ the PI is forbidden. Taking this in mind one can perform the contributions of operators of 6 dimension as follows:

$$\mathcal{T}_{c}^{(\Xi_{cc})} = 2\mathcal{T}_{c}^{(\Xi_{cc}^{++})},$$

$$\mathcal{T}_{c}^{(\Omega_{cc}^{+})} = 2\mathcal{T}_{c}^{(\Omega_{cc}^{++})} + 2\sum_{l} \mathcal{T}_{c,\nu l}^{(\Omega_{cc}^{++})}$$

where (see, e.g., [24, 32–34])

$$\mathcal{T}_{c}^{(\Xi_{cc})} = \frac{G_F^2 m_c^2}{4\pi} \left(1 - \frac{m_u}{m_c}\right)^2 \left[ G_1(z_{-})(\bar{c}c)_{V-A}^{ij}(\bar{u}u)_{V-A}^{ij} + G_2(z_{-})(\bar{c}c)_{V-A}^{ij}(\bar{u}u)_{V-A}^{ij} \right] \left[ F_3 + \frac{1}{3}(1 - k^2) F_4 \right] +$$

$$\mathcal{T}_{c}^{(\Omega_{cc}^{+})} = \frac{G_F^2 m_c^2}{4\pi} \left(1 - \frac{m_d}{m_c}\right)^2 \left[ (F_6 + \frac{1}{3}(1 - k^2) F_5)(\bar{c}c)_{V-A}^{ij}(\bar{d}d)_{V-A}^{ij} \right] k^2 F_4, (13)$$

$$\mathcal{T}_{c}^{(\Xi_{cc})} = \frac{G_F^2 m_c^2}{4\pi} \left(1 + \frac{m_d}{m_c}\right)^2 \left[ (F_6 + \frac{1}{3}(1 - k^2) F_5)(\bar{c}c)_{V-A}^{ij}(\bar{d}d)_{V-A}^{ij} \right] k^2 F_4, (14)$$
In these relations we also introduce the notations

The hadronic matrix elements are determined as follows:

\[
\langle \bar{\xi} V_{-} \bar{c} (\bar{c} c)_{V -} (\bar{s} s)_{V -} \rangle = \left[ F_1 + \frac{1}{3} (1 - k^2) F_2 \right] + \frac{1}{4} (\bar{c} c)^{ij} (\bar{s} s)^{ij} \xi V_{-} A \]

\[
T^{c}_{Pl_{ud}} = -\frac{G_F^2}{4\pi} m_c^2 \left( 1 - \frac{m_s}{m_c} \right)^2 \left\{ \frac{1}{4} (\bar{c} c)^{ij} (\bar{s} s)^{ij} V_{-} A + \frac{1}{6} (\bar{c} c)^{ij} (\bar{s} s)^{ij} V_{-} A \right\},
\]

\[
T^{c}_{Pl_{\nu \tau}} = -\frac{G_F^2}{\pi} m_c^2 (1 - \frac{m_s}{m_c})^2 \left[ G_1 (z_\tau) (\bar{c} c)^{ij} (\bar{s} s)^{ij} V_{-} A + G_2 (z_\tau) (\bar{c} c)^{ij} (\bar{s} s)^{ij} V_{-} A \right],
\]

\[
T^{c}_{Pl_{\nu \mu \bar{\mu}}} = T^{c}_{Pl_{\nu \tau}} \quad (z_\tau \rightarrow 0)
\]

where

\[
(\bar{q} q)^{ij} = (\bar{q}^i \gamma_\alpha (1 - \gamma_5) q^j), \quad (\bar{q} q)^{ij} = (\bar{q}^i \gamma_\alpha (1 - \gamma_5) q^j).
\]

The hadronic matrix elements are determined as follows:

\[
\langle \Xi_{QQ} | (\bar{Q} \gamma_\mu (1 - \gamma_5) Q)(\bar{q} \gamma_\mu (1 - \gamma_5) q) | \Xi_{QQ} \rangle = 12 (m_Q + m_q) \cdot |\Psi^{d^0}(0)|^2,
\]

\[
\langle \Xi_{QQ} | (\bar{Q} \gamma_\mu \gamma_5 Q)(\bar{q} \gamma_\mu (1 - \gamma_5) q) | \Xi_{QQ} \rangle = 8 (m_Q + m_q) \cdot |\Psi^{d^0}(0)|^2,
\]

\[
\langle \Omega_{QQ} | (\bar{Q} \gamma_\mu (1 - \gamma_5) Q)(\bar{s} \gamma_\mu (1 - \gamma_5) s) | \Omega_{QQ} \rangle = 12 (m_Q + m_s) \cdot |\Psi^{d^0}(0)|^2,
\]

\[
\langle \Omega_{QQ} | (\bar{Q} \gamma_\mu \gamma_5 Q)(\bar{s} \gamma_\mu (1 - \gamma_5) s) | \Omega_{QQ} \rangle = 8 (m_Q + m_s) \cdot |\Psi^{d^0}(0)|^2,
\]

where \(Q = c, b\) is a heavy quark, \(q = u, d\) is light quark, and \(|\Psi^{d^0}(0)|^2\) is a wave function at origin. The wave function structure leads to the following relation:

\[
\langle \Xi_{QQ}^* | (\bar{Q} T^l \mu Q_k) (\bar{q} \gamma_\mu (1 - \gamma_5) q_i) | \Xi_{QQ}^* \rangle = -\langle \Xi_{QQ}^* | (\bar{Q} T^l \mu Q) (\bar{q} \gamma_\mu (1 - \gamma_5) q) | \Xi_{QQ}^* \rangle,
\]

where \(T^l \mu\) is an arbitrary spinor matrix.
C. Lifetimes of doubly beauty baryons $\Xi_{bb}^0$, $\Xi_{bb}^-$, $\Omega_{bb}^-$

For the double beauty baryons $\Xi_{bb}^0 = (bbu)$, $\Xi_{bb}^- = (bbd)$ and $\Omega_{bb}^- = (bbs)$ WS mechanism contributes only to the width of neutral states, whereas for charge states the PI mechanism contribution must be accounted for the charged states:

$$
\mathcal{T}_{\Xi_{bb}^0} = 2\mathcal{T}_{35b} + \mathcal{T}_{\text{WS}}^{(\Xi_{bb}^0)},
$$
$$
\mathcal{T}_{\Xi_{bb}^-} = 2\mathcal{T}_{35b} + \mathcal{T}_{\text{PI}}^{(\Xi_{bb}^-)},
$$
$$
\mathcal{T}_{\Omega_{bb}^-} = 2\mathcal{T}_{35b} + \mathcal{T}_{\text{PI}}^{(\Omega_{bb}^-)}.
$$

The spectator mechanism of $b$-quark decay is described by the following operators with dimensions 3 and 5:

$$
\mathcal{T}_{35b} = \Gamma_{b,\text{spec}}(\bar{b}b) - \frac{\Gamma_{bb}}{m_b^2} [2P_{c1} + P_{c2} + K_{bb}(P_{c1} + P_{cc1}) + K_{bb}(P_{c2} + P_{cc2})] O_{Gb},
$$

where

$$
\Gamma_{bb} = \frac{G_F^2 m_c^5}{192\pi^3},
$$

and the other functions are determined earlier. The operators of dimension 6 equal

$$
\mathcal{T}_{\text{WS}}^{(\Xi_{bb}^0)} = 2\mathcal{T}_{\text{WS},bu}, \quad \mathcal{T}_{\text{PI}}^{(\Xi_{bb}^-)} = 2\mathcal{T}_{\text{PI},d\bar{u}}, \quad \mathcal{T}_{\text{PI}}^{(\Omega_{bb}^-)} = 2\mathcal{T}_{\text{PI},s\bar{c}},
$$

where [35]

$$
\mathcal{T}_{\text{WS},bu} = \frac{G_F^2 |V_{ub}|^2}{4\pi} m_b^2 \left(1 + \frac{m_u}{m_b} \right)^2 \left(1 - z_+ \right)^2 \left[ F_6 + \frac{1}{3} (1 - k^2) F_5 \right](\bar{b}b)_{V-A}(\bar{u}u)_{V-A} + k^2 F_5 (\bar{b}b)_{V-A}(\bar{u}u)_{V-A},
$$

$$
\mathcal{T}_{\text{PI},d\bar{u}} = -\frac{G_F^2 |V_{ub}|^2}{4\pi} m_b^2 \left(1 - \frac{m_d}{m_b} \right)^2 \left[ G_1(z_-(\bar{b}b)_{V-A}(\bar{d}d)_{V-A}) + G_2(z_-(\bar{b}b)_{V-A}(\bar{d}d)_{V-A} \right]
$$

$$
\left[ F_3 + \frac{1}{3} (1 - k^2) F_4 \right] + \left[ G_1(z_-(\bar{b}b)_{V-A}(\bar{d}d)_{V-A}) + G_2(z_-(\bar{b}b)_{V-A}(\bar{d}d)_{V-A} \right] k^2 F_4 \right] + \left[ G_2(z_-(\bar{b}b)_{V-A}(\bar{d}d)_{V-A} \right] k^2 F_4 \right],
$$

$$
\mathcal{T}_{\text{PI},s\bar{c}} = -\frac{G_F^2 |V_{ub}|^2}{16\pi} m_b^2 (1 - \frac{m_s}{m_b})^2 \sqrt{(1 - 4z_+)} \left[ (1 - z_-(\bar{b}b)_{V-A}(\bar{s}s)_{V-A}) + \frac{2}{3} (1 + 2z_-(\bar{b}b)_{A}(\bar{s}s)_{V-A} \right] F_3 + \frac{1}{3} (1 - k^2) F_4 \right] + \left[ (1 - z_-(\bar{b}b)_{V-A}(\bar{s}s)_{V-A}) + \frac{2}{3} (1 + 2z_-(\bar{b}b)_{A}(\bar{s}s)_{V-A} \right] k^2 F_4 \right],
$$

$$
\mathcal{T}_{\text{PI},s\bar{c}} = -\frac{G_F^2 |V_{ub}|^2}{16\pi} m_b^2 (1 - \frac{m_s}{m_b})^2 \sqrt{(1 - 4z_+)} \left[ (1 - z_-(\bar{b}b)_{V-A}(\bar{s}s)_{V-A}) + \frac{2}{3} (1 + 2z_-(\bar{b}b)_{A}(\bar{s}s)_{V-A} \right] k^2 F_4 \right].
$$
where

(17) : \[ z_+ = \frac{m_c^2}{(m_b + m_d)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b + m_u)} \]
(18) : \[ z_- = \frac{m_c^2}{(m_b - m_d)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b - m_d)} \]
(19) : \[ z_- = \frac{m_c^2}{(m_b - m_s)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b - m_s)} \]

D. Lifetimes of \( \Xi_{bc}^+, \Xi_{bc}^0, \Omega_{bc}^0 \) baryons

It can be easily seen that in the case of \( \Xi_{bc}^+ = (bcu), \Xi_{bc}^0 = (bcd), \) and \( \Omega_{bc}^0 = (bcs) \) baryons both PI and WS channels are opened. As a result, the corresponding transition amplitudes are equal to

\[
\mathcal{T}_{\Xi_{bc}^+} = \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{35c}^{(\Xi_{bc}^+)} + \mathcal{T}_{WS}^{(\Xi_{bc}^+)}; \\
\mathcal{T}_{\Xi_{bc}^0} = \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{35c}^{(\Xi_{bc}^0)} + \mathcal{T}_{WS}^{(\Xi_{bc}^0)}; \\
\mathcal{T}_{\Omega_{bc}^0} = \mathcal{T}_{35b} + \mathcal{T}_{35c} + \mathcal{T}_{35c}^{(\Omega_{bc}^0)} + \mathcal{T}_{WS}^{(\Omega_{bc}^0)};
\]

where the contributions of \( c \) and \( b \) quarks’ spectator decays are given in the previous subsections and PI, WS amplitudes are equal to

\[
\mathcal{T}_{\Pi_{bc}}^{(\Xi_{bc}^+)} = \mathcal{T}_{\Pi_{bc}}^{(\Xi_{bc}^+)} + \mathcal{T}_{\Pi_{bc}}^{(\Xi_{bc}^0)} + \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)}; \\
\mathcal{T}_{\Pi_{bc}}^{(\Xi_{bc}^+)} = \mathcal{T}_{\Pi_{bc}}^{(\Xi_{bc}^+)} + \mathcal{T}_{\Pi_{bc}}^{(\Xi_{bc}^0)} + \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)}; \\
\mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)} = \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)} + \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)}; \\
\mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)} = \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)} + \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)}; \\
\mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)} = \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)} + \mathcal{T}_{\Pi_{bc}}^{(\Omega_{bc}^0)}.
\]

In these expressions [25]

\[
\mathcal{T}_{\Pi_{bc}}^{(\Xi_{bc}^+)} = -\frac{G_F^2 |V_{cb}|^2}{4\pi} m_b^2 \left( 1 - \frac{m_c}{m_b} \right)^2 \left\{ [G_1(z_-)(\bar{b}b)^i_{ij}(\bar{c}c)^{ij}_{V-A} + G_2(z_-)(\bar{b}b)^i_{ij}(\bar{c}c)^{ij}_{V-A}] \times \right. \\
\left. \left[ F_1 + \frac{1}{3} (1 - k^2) F_2 \right] + [G_1(z_-)(\bar{b}b)^i_{ij}(\bar{c}c)^{ij}_{V-A} + G_2(z_-)(\bar{b}b)^i_{ij}(\bar{c}c)^{ij}_{V-A}] \times \right. \\
\left. k^2 F_2 \right\}, \quad (20)
\]
\[ T^b_{\Pi, d\bar{u}} = T^b_{\Pi, s\bar{c}} \ (z_- \to 0), \]
\[ T^b_{\Pi, \tau\bar{\nu}_\tau} = -\frac{G_F^2|V_{cb}|^2}{\pi} m_b^2 \left( 1 - \frac{m_c}{m_b} \right)^2 \left[ G_1(z_\tau)(\bar{b}b)^{ij}_{V-A}(\bar{c}c)^{ji}_{V-A} + G_2(z_\tau)(\bar{b}b)^{ij}_{A}(\bar{c}c)^{ji}_{V-A} \right], \tag{21} \]
\[ T^b_{WS, bc} = \frac{G_F^2|V_{cb}|^2}{4\pi} m_b^2 \left( 1 + \frac{m_c}{m_b} \right)^2 (1 - z_+)^2 \left[ F_0 + \frac{1}{3} (1 - k^2) F_5 (\bar{b}b)^{ij}_{V-A}(\bar{c}c)^{ji}_{V-A} + k^2 F_6 (\bar{b}b)^{ij}_{V-A}(\bar{c}c)^{ji}_{V-A} \right], \tag{22} \]
\[ T^b_{\Pi, e\bar{\nu}_e} = T^b_{\Pi, \mu\bar{\nu}_\mu} = T^b_{\Pi, \tau\bar{\nu}_\tau} \ (z_\tau \to 0), \]

where

\[ (20) : \quad z_- = \frac{m_c^2}{(m_b - m_c)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b - m_c)}, \]
\[ (21) : \quad z_\tau = \frac{m_\tau^2}{(m_b - m_c)^2}, \]
\[ (22) : \quad z_+ = \frac{m_c^2}{(m_b + m_c)^2}, \quad k = \frac{\alpha_s(\mu)}{\alpha_s(m_b + m_c)}. \]

The other functions are defined earlier.

### E. Numerical results

From presented above results it is clear that in OPE formalism theoretical predictions of doubly heavy baryons' lifetimes depend on such input parameters as quark masses, wave function at the origin, etc. In paper [35] the following values of these parameters were used:

\[ V_{cs} = 0.9745, \quad V_{cb} = 0.04, \tag{23} \]

\[ T = 0.4 \text{GeV}, \quad |\Psi^{di}(0)|^2 = (2.7 \pm 0.2) \times 10^{-3} \text{GeV}^3, \tag{24} \]

\[ m_s = 0.2 \text{GeV}, \quad m_c = 1.55 \text{GeV}, \quad m_b = 5.05 \text{GeV}. \tag{25} \]

This choice however leads to the following values of \( \Xi^{++}_{cc} \) baryon mass and lifetime:

\[ M_{\Xi^{++}_{cc}} = 3.478 \text{GeV}, \quad \tau_{\Xi^{++}_{cc}} = 0.44 \text{ps} \tag{26} \]

These results, unfortunately, disagree with resent experimental data [4, 36]

\[ M^{\exp}_{\Xi^{++}_{cc}} = (3621.40 \pm 0.72 \pm 0.27 \pm 0.14) \text{MeV}, \quad \tau^{\exp}_{\Xi^{++}_{cc}} = 0.256_{-0.022}^{+0.024} \pm 0.014 \text{ps}, \tag{27} \]

so some change of parameters is required. It should be noted that the values (25) correspond to constituent quark masses obtained from analysis of \( D \)-mesons' lifetimes. In papers [37, 38]
in Figure 5 we show model parameter dependence of $\Xi^{++}_{cc}$ lifetime, while Fig. 6a shows $m_c$ dependence of different channels that contribute to this lifetime. It can be seen from these figures that $\tau(\Xi^{++}_{cc})$ is most sensitive to change of $c$-quark mass. Our analysis shows that experimental value \((27)\) is restored with the following values:

$$m_c = 1.73 \pm 0.07 \text{ GeV}, \quad m_s = 0.35 \pm 0.2 \text{ GeV}. \quad (28)$$

With these masses we have $\tau(\Xi^{++}_{cc}) = 0.26 \pm 0.03$ ps. In the second column of table I we show calculated with these masses contributions of different decay channels to $\Xi^{++}_{cc}$ baryon lifetime in comparison with that presented in \([35]\). One can see from this table that, as it was
\[
\begin{array}{ccc}
\sum c \to s, \text{ ps}^{-1} & \Xi_{cc}^{++} & 5.1 \pm 0.5 \ (3.1) \\
\text{PI, ps}^{-1} & 5.1 \pm 0.5 \ (3.1) & 5.1 \pm 0.5 \ (3.1) \\
\text{WS, ps}^{-1} & -1.2 \pm 0.1 \ (-0.87) & 0.65 \pm 0.5 \ (0.62) \\
\tau, \text{ ps} & 0.26 \pm 0.03 \ (0.44) & 0.14 \pm 0.01 \ (0.2) & 0.18 \pm 0.02 \ (0.27)
\end{array}
\]

Table I: Lifetimes of doubly charmed baryons and different and partial contributions of different mechanisms (values in brackets correspond to [35]. Theoretical uncertainties are caused by \( m_{s,c} \) variation (28).

Figure 7: Lifetimes in ps for \( \Xi_{cc}^{++} \) (solid black curve), \( \Xi_{cc}^0 \) (blue dashed curve) and \( \Omega_{cc}^0 \) (red dotted curve) as a function of the model parameters. The results of [35] are shown by dots.

mentioned in the previous sections, the spectator decay channel gives the main contribution and it increases with the increase of charm quark mass. In addition, PI channel gives destructive contribution in this case, which leads to increase of the lifetime. As for weak scattering mechanism, it is forbidden for \( \Xi_{cc}^{++} \) decay.

Using the approach described above, it is easy to calculate also lifetimes of \( \Xi_{cc}^+ \) and \( \Omega_{cc}^+ \) baryons:

\[
\tau(\Xi_{cc}^+) = 0.14 \pm 0.01 \ \text{ps}, \quad \tau(\Omega_{cc}^+) = 0.18 \pm 0.02 \ \text{ps}.
\] (29)

Lifetime and decay width dependences on parameters are shown in figures 5, 6. The numerical estimations for parameter values (25) and (28) can be found in the third and fourth columns of table I. In the case of \( \Xi_{cc}^+ \) baryon the PI channel is forbidden, thus only the spectator decay and the weak scattering give contributions. For for \( \Omega_{cc}^+ \) baryon the spectator and PI channels are important. The contribution of the last one is positive. As a result theoretical predictions for the lifetimes of \( \Xi_{cc}^+ \) and \( \Omega_{cc}^+ \) are smaller than for \( \Xi_{cc}^{++} \) particle.
Figure 8: The partial widths for different operators for $bc$ baryons (in $\text{ps}^{-1}$): operators of dimension 3 and 5 corresponds to the spectator mechanism (dashed blue curve), operators of dimension 6 corresponding to the weak scattering and Pauly interference (red dotted and black dash-dotted curves respectively), the total width (black solid curve). The dots correspond to the predictions of [35].

|                  | $\Xi^+_bc$ | $\Xi^0_bc$ | $\Omega^0_bc$ |
|------------------|------------|------------|--------------|
| $\sum b \to c$, ps$^{-1}$ | $0.551 \pm 0.0311$ ($0.632$) | $0.551 \pm 0.0311$ ($0.632$) | $0.551 \pm 0.0311$ ($0.632$) |
| $\sum c \to s$, ps$^{-1}$ | $2.32 \pm 0.342$ ($1.51$) | $2.32 \pm 0.342$ ($1.51$) | $2.32 \pm 0.342$ ($1.51$) |
| PI, ps$^{-1}$     | $0.69 \pm 0.044$ ($0.81$) | $0.75 \pm 0.039$ ($0.86$) | $0.86 \pm 0.044$ ($0.98$) |
| WS, ps$^{-1}$    | $0.69 \pm 0.014$ ($0.65$) | $0.87 \pm 0.022$ ($0.79$) | $2. \pm 0.13$ ($1.7$) |
| $\tau$, ps       | $0.24 \pm 0.02$ ($0.28$) | $0.22 \pm 0.018$ ($0.26$) | $0.18 \pm 0.0088$ ($0.21$) |

Table II: Decay widths and lifetimes for $bc$-baryons. The meaning of symbols is the same as in Tab. I.

Let us now consider lifetimes of $bc$-baryons $\Xi^+_bc$, $\Xi^0_bc$, and $\Omega^0_bc$. The lifetime dependences on parameters are shown in Figure 7. In the following we will use constituent value $m_b = 5.05$ GeV for $b$-quark mass and (28) for $m_{c,s}$. In Figure 8 we show $m_c$ dependence of different channel contributions for these baryons. The predictions corresponding to parameter values (25) and (28) are given in table II. From presented results it is clear, that $c$-quark spectator decay is dominant for the considered baryons, while contributions of $b$-quark spectator decay is suppressed by $V_{cb}$ matrix element. As for dimension 6 operators PI and WS, their contributions are suppressed by large $b$-quark mass and are small. It is interesting to note, however, that, in contrast to $cc$ baryons, in the case of $bc$-baryons both PI and WS channels are not forbidden for all considered particles.

In the case of $bb$-baryons $\Xi^0_{bb}$, $\Xi^-_{bb}$, and $\Omega^-_{bb}$ spectator $b$-quark decay gives the dominant
Figure 9: Decay widths for $bb$ baryons. Designations as in Fig. 6

| $\sum b \to c$, ps$^{-1}$ | $\Xi^0_{bb}$ | $\Xi^-_{bb}$ | $\Omega^-_{bb}$ |
|---------------------------|-------------|-------------|---------------|
|                           | $1.9 \pm 0.0344$ (1.25) | $1.9 \pm 0.0344$ (1.25) | $1.9 \pm 0.0344$ (1.25) |

Table III: Decay widths and lifetimes for $bb$-baryons Designations as on Tab. I

contribution. As for dimension 6 operators, in complete agreement with OPE selection rules their contributions are suppressed by large quark mass. As a result, lifetime values presented in Table III are close to each other. It should be noted that, similar to $cc$ sector, different decay mechanisms are enabled for different baryons: WS is enabled only for neutral particle and PI is enabled only for charged ones. Parameter dependence of the lifetimes and decay widths of these baryons are shown in figures 9 and 10.

Figure 10: Lifetimes in ps for $\Xi^0_{bb}$ (solid black curve), $\Xi^-_{bb}$ (blue dashed curve) and $\Omega^-_{bb}$ (red dotted curve) as a function of the model parameters. The results of [35] are shown by dots.
F. Comparison with Other Works

One can find in the literature some other theoretical works devoted to analysis of doubly heavy baryons lifetimes. In the current subsection we will discuss these papers and compare presented there results with ours.

As it was mentioned above, in papers \([37, 38]\) it was assumed that quark masses used for doubly heavy baryons analysis could be a little bit different from constituent quark masses obtained from analysis of meson spectroscopy. In particular, in paper \([37]\) (\([\text{KR14}]\)) the following values were considered:

\[
\begin{align*}
m_q^{[\text{KR14}]} &= 363 \text{ MeV}, \\
m_s^{[\text{KR14}]} &= 538 \text{ MeV}, \\
m_c^{[\text{KR14}]} &= 1.7105 \text{ GeV},
\end{align*}
\]

that correspond to \(\Xi_{cc}^{++}\) mass and lifetime equal to

\[
\begin{align*}
M_{\Xi_{cc}^{[\text{KR14}]}^{++}} &= (3627 \pm 12) \text{ MeV}, \\
\tau_{\Xi_{cc}^{[\text{KR14}]}^{++}} &= 0.185 \text{ ps}.
\end{align*}
\]

One can see that the mass of the baryon is more close to the experimental value (27), while the lifetime is even smaller. We would like, however, make some comments considering the last result. Presented in \([37]\) analytical expression for \(\Xi_{cc}^{++}\) decay width reads

\[
\Gamma_{\text{tot}}^{[\text{KR14}]}(\Xi_{cc}^{++}) = 10 \frac{G_F^2 M_{\Xi_{cc}^{++}}^2}{192 \pi^3} f(x_{cc}),
\]

where

\[
x_{cc} = \frac{M_{\Xi_{cc}^{++}}^2}{M_{\Xi_{cc}^{++}}^2}.
\]

From this expression it is clear that in \([37]\) only spectator decays of the valence \(c\) quark contribute. Indeed, the prefactor \(10 = 2 \times (3 + 1 + 1)\) in relation (32) shows that only \(c \to sd, c \to se\nu\), and \(c \to \mu\nu_{\mu}\) channels were taken into account and the final result is doubled because of two valence quarks in \(\Xi_{cc}\) baryon Fock state. It seems to us, that such an approach is not reliable.

First of all, as it can be clearly seen from comparison with neutron’s total width, mentioned above factor 2 should be avoided. Indeed, since only one spectator decay \(d \to u\nu_{e}\) is possible in this case and there are two valence \(d\) quarks in the neutron, used in \([37]\) approach would give us the lifetime

\[
\tau_n = \left[\frac{2 G_F^2 m_n^5}{192 \pi^3} f\left(\frac{m_p^2}{m_n^2}\right)\right]^{-1} \approx 320 \text{ s},
\]

which is almost three times smaller than the experimental result \(\tau_n^{\exp} = 939 \text{ s}\). Without the factor 2 in relation (33) this disagreement is partially removed. In addition, in paper \([37]\)
contributions of any form factors are neglected. It is clear that the energy deposit in $\Xi_{cc}$ baryon decay is much larger than for neutron $\beta$-decay. It is well known, however, that even in the latter case $n \to p e^+ \nu_e$ such form factors are important (actually, the axial form factor helps us to obtain the experimental value of the considered lifetime), so it seems strange to forget about them in the case of $\Xi_{cc}$ lifetime.

The other point is that PI and WS contributions are completely ignored in [37]. As a result, one can expect that lifetimes of all $ccq$, $ccs$ baryons should be equal to each other. For some reason, however, the authors of paper [37] use completely different approach to calculate $\Xi_{cc}^+$ baryon lifetime and the value $\tau_{\Xi_{cc}^+} \approx \tau(\Xi_{cc}^{++})/2$ is given there. No detailed explanation for such difference in calculation methods is presented in [37].

If we use the presented in [KR14] values in described above OPE calculations, the lifetime of $\Xi_{cc}^{++}$ baryon is equal to 0.32ps, that is a little bit larger than the experimental result (27). In paper [38] ([KR18]) another set of quark masses was presented, that describe both meson and baryon masses:

$$m_{q_{[KR18]}} = 308.5 \text{ MeV}, \quad m_{s_{[KR18]}} = 482.2 \text{ MeV}, \quad m_{c_{[KR18]}} = 1655.6 \text{ GeV},$$

No predictions for the lifetimes can be found in this paper, but OPE approach gives the value $\tau(\Xi_{cc}^{++}) \approx 0.37 \text{ ps}$, which is also larger than the experimental one.

In a series of papers [33, 39–41] the lifetimes of heavy and doubly heavy baryons are considered in the framework of operator product expansion with PI and WS channels taken into account. The result of these works agrees qualitatively with ours (for example, the hierarchy of $cc$-baryons lifetimes is the same), but the numerical values of the lifetimes are somewhat larger. The reason for the difference is that used in these papers values of quark masses are smaller (for example, $m_c = 1.35 \text{ GeV}$ in these papers).

It should be noted that the mass of $c$ quark is not really large, so higher order contributions in operator product expansion could also give significant contributions. In the recent article [42] the authors show that the experimental value of $\Xi_{cc}^{++}$ baryon lifetime can be explained if contributions of higher dimension operators are taken into account. It is interesting to note, that the lifetimes of other doubly charmed baryons are changed in different way in comparison with our results: $\tau(\Xi_{cc}^+)$ decreases only slightly, while the lifetime of $\Omega_{cc}^+$ baryon increases and is comparable with $\tau(\Xi_{cc}^{++})$. It is clear that a detailed theoretical and experimental investigation of the lifetimes of these particles is highly desirable.
Here we briefly discuss the observation possibilities of doubly heavy baryons at LHC. As it was already mentioned the observation of Ξ_{cc}^{++} baryon has been done by the LHCb Collaboration in the decay mode Λ_{c}^{+}K^{-}π^{+}π^{+} [4] and confirmed in the decay mode Ξ_{cc}^{+}π^{+} [5].

The next step is the observation of Ξ_{cb} baryon. In spite of large number of theoretical predictions for branching fractions (see, for example, [1, 43–48] and Table IV), the "golden mode" is not found yet. Of course, the greater branching fraction value, the more chances for the decay mode to be observed. But the decay branchings of intermediate particles are also very important. In addition, as it is shown in [16], the possibility of the experiment also must be taken into account. For example, each extra track in final state decreases the registration efficiency. That is why understanding the experiment features is very important for searching the most promising decay modes. We share cautious optimism of [16] about the observation of particle in the LHCb data of Run I and Run II, and also think that in any case Ξ_{cb} will be observed in the LHCb data of Run III.

As for the observation of the Ξ_{bb}, we doubt its possibility at the LHC because of the very small production rate.


V. CONCLUSIONS

This article is devoted to theoretical study of total widths, production rates, and observation probabilities of the doubly heavy baryons.

We briefly discussed the production and the possibility of observation of $\Xi_{bc}$ baryon at LHC, and showed that the kinematical features of $\Xi_{bc}$ baryon production and $B_c$ meson production are very similar.

The main efforts were made to estimate the lifetimes of doubly heavy baryons in the framework of Operator Product Expansion (OPE). We studied the lifetime dependence on main parameters of this formalism, which are masses of $s$, $c$, and $b$ quarks and the value of the diquark wave function at the origin. We show, that the spectator heavy quark decays give the main contribution to the lifetimes of doubly heavy baryons. However, in the case of $\Xi_{cc}$ and $\Omega_{cc}$ baryons the contributions of the higher dimension terms, such as weak scattering and Pauli interference channels, are also important. For $bcq$ and $bbq$ baryons the higher dimension terms are suppressed by the large mass of the heavy quark and do not contribute essentially to the lifetime value.

The lifetime predictions for doubly heavy baryons are most sensitive to the charm quark mass. The knowledge of the experimental value of $\Xi_{cc}^{++}$ baryon lifetime allowed us to determine this parameter with pretty good accuracy and to make the lifetime predictions for other doubly heavy baryons.

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