The ongoing COVID-19 pandemic has posed a tremendous threat to the public and health authorities. Wuhan, as one of the cities experiencing the earliest COVID-19 outbreak, has successfully tackled the epidemic finally. The main reason is the implementing of Fangcang shelter hospitals, which rapidly and massively scale the health system’s capacity to treat COVID-19 confirmed cases with mild symptoms. To give insights on what degree Fangcang shelter hospitals have contained COVID-19 in Wuhan, we proposed a piecewise smooth model regarding the patient triage scheme and the bed capacities of Fangcang shelter hospitals and designated hospitals. We used data on the cumulative number of confirmed cases, recovered cases, deaths, and data on the number of hospitalized individuals in Fangcang shelter hospitals and designated hospitals in Wuhan to parameterize the targeted model. Our results showed that diminishing the bed capacity or delaying the opening time of Fangcang shelter hospitals, both would result in worsening the epidemic by increasing the total number of infectives and hospitalized individuals and the effective reproduction number $R_e(t)$. The findings demonstrated that Fangcang shelter hospitals avoided 17,013 critical infections and 17,823 total infections while it saved 7 days during the process of controlling the effective reproduction number $R_e(t) < 1$. Our study highlighted the critical role of Fangcang shelter hospitals in curbing and eventually stopping COVID-19 outbreak in Wuhan, China. These findings may provide a valuable reference for decision-makers in regarding ramping up the health system capacity to isolate groups of people with mild symptoms in areas of widespread infection.

KEYWORDS
COVID-19 outbreak, effective reproduction number, Fangcang shelter hospitals, sensitivity analysis, transmission model

MSC CLASSIFICATION
92B05, 92D30

1 | INTRODUCTION

The ongoing novel coronavirus disease 2019 (COVID-19) has attracted the world wide attention and brought unprecedented threat to the global health and economy since it was firstly reported in December 2019. Although many stringent
control measures including travel restrictions, contact tracing and quarantine, and lockdown of entire cities are carried out to combat the transmission of COVID-19, the surge of infections has placed a huge pressure on the medical system, especially for those developing countries, such as India. As May of 1, 2022, a total of 511,252,681 confirmed cases with 6,238,149 deaths have been reported worldwide. One of the main reasons for the continuous climbing of the number of confirmed cases and deaths was the shortage of medical resources. The health systems’ capacity to respond to the require of patients, who seeking hospital beds for necessary treatment, was challenged by the influx of patients. The intensive care units were overburdened by those patients with severe and critical symptoms. Thus, exploring how the medical resources affect the control of COVID-19 is of utmost importance.

Wuhan, the former epicenter of COVID-19 pandemic in China, faced resource shortages initially. No beds were available in Wuhan for COVID-19 confirmed cases at the beginning of February though the capacity to accept COVID-19 confirmed cases increased with several hospitals being designated as COVID-19 hospitals. The situation did not improve until a patient triage scheme was proposed by Chinese government and Fangcang shelter hospitals, which serve as temporary hospitals to provide services such as isolation, treatment, and disease monitoring, were opened. Based on the clinical manifestations, all confirmed COVID-19 cases were classified into mild, moderate, severe, and critical groups in terms of COVID-19 diagnosis and treatment guideline in China. A total of 16 Fangcang shelter hospitals were opened since the first three Fangcang shelter hospitals were opened on February 5, 2020, which massively scaled the health system’s capacity. They were rapidly built by converting the exhibition centers and stadiums to isolate and care for the confirmed cases with mild and moderate conditions who didn’t need intensive care, while those confirmed COVID-19 cases with severe and critical symptoms were admitted to designated hospitals where critical care was available. The 16 Fangcang shelter hospitals contained about 13,000 beds and provided care to approximately 12,000 patients. Such sufficient medical resources resulted in no patient being unattended or untreated and ultimately containing the outbreak of COVID-19 in Wuhan. Previous studies introduced the key features, essential functions and patient management of Fangcang shelter hospitals and emphasized its vital role in response to the emergency of COVID-19 in China. However, only a few work focused on how the massively expanding of the health system’s capacity in time by Fangcang shelter hospitals help combating the COVID-19 in Wuhan.

Many researchers have worked on mathematical modeling of the COVID-19. They focused on exploring the transmission dynamics of the COVID-19 pandemic and seeking suggestions on how to tackle it from different perspectives. Different mathematical models have been proposed to investigate the spread and dynamics of COVID-19 since the outbreak of COVID-19. Laxminarayan et al. revealed the transmission pathways and characteristics of COVID-19 and identified the key factors which affect the contact pattern and the total number of infections. Previous studies highlighted the critical contribution of non-pharmaceutical interventions (NPIs) in containing the transmission of COVID-19 by mimicking the unfolding epidemic with the laboratory surveillance data. Previous studies estimated the size of the COVID-19 epidemic in Wuhan with the initial data, which has done valuable attempts on modeling and forecasting the tread of COVID-19. Previous studies designed compartment models with two or three phases to mimic the data on the COVID-19 epidemic in Wuhan, which provided a detailed view into the vital role of the lockdown strategy and the hospital bed capacity in helping curb and eventually stop the outbreak in Wuhan. Zhou et al. formulated piecewise smooth models to explore how the hospital bed shortages affect the containment of COVID-19 in certain regions. However, the effects of Fangcang shelter hospitals and the lockdown strategy on minimizing the number of infected individuals and ultimately stopping the transmission of COVID-19 in Wuhan have not yet been well evaluated.

Here, we proposed a piecewise smooth compartment model to investigate the spread of COVID-19 in Wuhan and quantify how the Fangcang shelter hospitals and patient triage scheme controlled the epidemic. Previous modeling studies on the hospital bed capacity either focused on the theoretical analysis of the model or did not categorize the patients on the basis of their condition. To acquire a better understanding of the roles played by Fangcang shelter hospitals in the infection dynamics of COVID-19 in Wuhan, China, we investigated how the variation of bed capacities in Fangcang shelter hospitals and designated hospitals affect the transmission. We introduced nonsmooth functions dependent on the number of hospitalized individuals with mild-to-moderate symptoms in designated hospitals and Fangcang shelter hospitals, respectively, while a piecewise-defined function was adopted to describe the daily number of hospitalized individuals with severe-to-critical symptoms in designated hospitals. Our study may provide a reference for COVID-19 control in other countries now and in the future.
2 | METHODS

2.1 | Data collection and analysis

The epidemic of COVID-19 in Wuhan led to 50,008 infected cases as of April 16, 2020, 47,300 infected individuals had recovered, and 2579 people died of the virus. We obtained the data of COVID-19 in Wuhan on the reported cumulative number of confirmed cases, recovered cases, and deaths from January 10 to April 16, 2020, from the National Health Commission of the People’s Republic of China, Health Commission of Hubei Province, and Wuhan Municipal Health Commission, as shown in Figure 1A–C. The number of hospital beds during the COVID-19 outbreak in Wuhan was also collected. Two designated hospitals with about 800 beds were put in use to treat the virus before January 23, 2020, when no beds were available for COVID-19 patients in the designated hospitals. With such a shortage of hospital beds, rapidly and massively scaling the capacity to isolate those mild to moderate COVID-19 patients is extremely urgent. On February 5, 2020, three Fangcang shelter hospitals, which provided about 4000 beds, were established by converting the stadiums and exhibition centers. Over the following days, a total of 13 Fangcang shelter hospitals besides Wuhan Huoshenshan Hospital and Wuhan Leishenshan Hospital were opened with the hospital beds being expropriated for COVID-19 patients gradually. The 16 Fangcang shelter hospitals provided about 13,000 beds and admitted more than 12,000 patients in total. As the bed occupancy approaching zero and the subsiding of the epidemic in Wuhan, the first Fangcang shelter hospital was suspended on March 1, 2020, thereafter all the Fangcang shelter hospitals were closed gradually with the last one being closed on March 10, 2020. The detailed information on the hospital beds during the COVID-19 outbreak in Wuhan was shown in Figure 1D–F. The number of hospital beds and the number of patients in the designated hospitals were collected from Wuhan Municipal Health Commission, while the number of patients and the number of hospital beds in Fangcang shelter hospitals from February 5 to March 10 were extracted from Chen et al.

2.2 | The model

According to the COVID-19 diagnosis and treatment guideline in China, all confirmed individuals are categorized into four groups, that is, mild, moderate, severe, and critical, dependent on their clinical manifestations. For simplicity, we call those confirmed individuals with mild to moderate symptoms (resp. moderate to critical symptoms) mild infected individuals (resp. critical infected individuals) in the rest of this work. Based on the clinical progression of the disease,
intervention measures, and epidemiological status of the individuals, we stratify the population into eight compartments: susceptible (S), exposed (E), asymptomatic infected (I_\text{A}), mild infected (I_m), critical infected (I_c), hospitalized in Fangcang shelter hospitals (H_m), hospitalized in designated hospitals (H_c), and recovered (R), as shown in Figure 2. During the early stage of COVID-19 outbreak in Wuhan, medical resource, which is described in terms of numbers of hospital beds in this work, is limited. As a result, the numbers of hospitalized individuals in Fangcang shelter hospitals and designated hospitals are affected by the available hospital beds in these hospitals, respectively. In addition, the policy of one patient one bed was put in place, which suggested one hospitalized individual can only take one hospital bed, so the number of available hospital beds was just the number of patients who can be hospitalized. The critical infecteds were admitted into the designated hospitals. Most of the mild infecteds were admitted into Fangcang shelter hospitals, and a small number of them were admitted into designated hospitals if there were available beds in designated hospitals. Based on the flowcharts of the epidemic in Figure 2, we formulated model Equations (1) accordingly.

\[
\begin{align*}
\frac{dS}{dt} &= \frac{\beta c(t)S(\theta_A I_A + \theta_m I_m + I_c)}{N}, \\
\frac{dE}{dt} &= \frac{\beta c(t)S(\theta_A A + \theta_m I_m + I_c)}{N} - \sigma E, \\
\frac{dI_A}{dt} &= p\sigma E - \gamma_A I_A, \\
\frac{dI_m}{dt} &= (1 - p)q\sigma E - \xi_I I_m - \eta_{m1}(t)I_m - \eta_{m2}(t)I_m, \\
\frac{dI_c}{dt} &= (1 - p)(1 - q)\sigma E + \xi_I I_m - \eta_c(t)I_c - \alpha_I I_c, \\
\frac{dH_m}{dt} &= \eta_{m2}(t)I_m - \gamma_m H_m - \alpha_m H_m - \xi_I H_m, \\
\frac{dH_c}{dt} &= \eta_{m1}(t)I_m + \eta_c(t)I_c + \xi_I H_m - \gamma_c(t)H_c - \alpha_c(t)H_c, \\
\frac{dR}{dt} &= \gamma_A I_A + \gamma_m H_m + \gamma_c(t)H_c.
\end{align*}
\]

In model (1), we denote the contact rate as c(t), the transmission probability per contact as \(\beta\), and the incubation period as \(1/\sigma\). The functions \(\eta_{m1}(t)\), \(\eta_{m2}(t)\), and \(\eta_c(t)\) represent the progressing rate of \(I_m\) to \(H_m\), \(I_m\) to \(H_m\), and \(I_c\) to \(H_c\), respectively; \(\xi_I\) and \(\xi_I\) are the progressing rate of \(I_m\) to \(I_c\) and \(H_m\) to \(H_c\), respectively; \(\gamma_A\), \(\gamma_m\), and \(\gamma_c(t)\) are the recovery rates for \(I_A\), \(H_m\), and \(H_c\) respectively; and \(\alpha_m\) and \(\alpha_c(t)\) stand for the disease-induced death rates for \(H_m\) and \(H_c\), respectively. Note that due to the shortage of beds in Fangcang shelter hospitals and designated hospitals, confirmed individuals may not be admitted to the hospitals as soon as they are diagnosed. Whether the confirmed individuals can be hospitalized depends on the daily number of empty hospital beds in Fangcang hospitals and designated hospitals. The total hospital beds and empty hospital beds in Fangcang shelter hospitals and designated hospitals vary daily. Denote the number of beds in the designated hospitals and Fangcang shelter hospitals as \(B_c(t)\) and \(B_m(t)\), respectively, see more detailed definitions and values of variables and parameters in Table 1. Denote \(T_1\) as the date January 23, 2020, when the lockdown strategy began to implemented, \(T_2\) as the date February 5, 2020, when the beds in Fangcang shelter hospitals were put into use to treat
| Variables | Description                                      | Initial value | Resource |
|-----------|--------------------------------------------------|---------------|----------|
| $S$       | Susceptible population                           | 11,081,000    | 27       |
| $E$       | Exposed population                               | 40            | LS       |
| $I_A$     | Asymptomatic infected population                 | 28            | LS       |
| $I_m$     | Mild infected population                          | 88            | LS       |
| $I_c$     | Critical infected population                      | 3             | LS       |
| $H_m$     | Hospitalized population in Fangcang shelter hospitals | 0             | Data     |
| $H_c$     | Hospitalized population in designated hospitals   | 38            | Data     |
| $R$       | Recovered population                              | 2             | Data     |

| Parameters | Definition                                                                 | Mean value | Resource |
|------------|---------------------------------------------------------------------------|------------|----------|
| $c(t)$    | Contact rate before travel ban (per person per day)                       | 24.88      | LS       |
| $c_0$     | Minimum contact rate (per person per day)                                 | 0.86       | LS       |
| $c_1$     | Exponential decreasing rate of the contact rate (per day)                 | 0.068      | LS       |
| $r_c$     | Probability of transmission from $I$ to $S$ per contact                   | 0.1911     | 29,30    |
| $\beta$  | Effective contact ratio of $I_A$ with $S$ to $I_c$ with $S$               | 0.0019     | LS       |
| $\theta_A$| Effective contact ratio of $I_m$ with $S$ to $I_c$ with $S$              | 0.0071     | LS       |
| $p$       | Ratio of asymptomatic infectives in infectives                           | 0.187      | LS       |
| $q$       | Ratio of mild infectives in symptomatic infectives                        | 0.5367     | LS       |
| $\sigma$ | Progression rate of exposed individuals to infectives (per day)          | 1/5.2      | LS       |
| $\xi_H$  | Progressing rate of hospitalized individuals in Fangcang shelter hospitals to the ones in designated hospitals (per day) | 0.0452     | Data     |
| $\xi_I$  | Progressing rate of mild infectives to critical infectives (per day)     | 0.009      | LS       |
| $\eta_m1$| Hospitalization rate of mild infectives in designated hospitals (per day) | 0.0232     | LS       |
| $\eta_m2$| Hospitalization rate of mild infectives in Fangcang shelter hospitals (per day) | 0.9476     | LS       |
| $\eta_I(t)$| Hospitalization rate of critical infectives in designated hospitals before February 5 (per day) | 0.4984     | LS       |
| $\eta_c1$| Maximum hospitalization rate of critical infectives in designated hospitals (per day) | 0.9581     | LS       |
| $r_I$     | Exponential increasing rate of hospitalization rate of critical infectives in designated hospitals (per day) | 0.3191     | LS       |
| $\gamma_A$| Recovery rate of asymptomatic infectives (per day)                       | 0.1238     | LS       |
| $\gamma_m$| Recovery rate of hospitalized individuals in Fangcang shelter hospitals (per day) | 0.0663     | LS       |
| $\gamma_I(t)$| Recovery rate of hospitalized individuals in designated hospitals before February 5 (per day) | 0.0231     | LS       |
| $\gamma_c1$| Maximum recovery rate of hospitalized individuals in designated hospitals (per day) | 0.0691     | LS       |
| $r_r$     | Exponential increasing rate of recovery rate of hospitalized individuals in designated hospitals (per day) | 0.9701     | LS       |
| $\alpha_1$| Disease-induced death rate of critical infectives (per day)              | 8.208 × 10⁻⁴| LS       |
| $\alpha_m$| Disease-induced death rate of hospitalized individuals in Fangcang shelter hospitals (per day) | 0.0031     | LS       |
| $\alpha_c1(t)$| Disease-induced death rate of hospitalized individuals in designated hospital before February 5 (per day) | 0.0161     | LS       |
| $\alpha_c1$| Maximum disease-induced death rate of hospitalized individuals in designated hospitals (per day) | 0.0012     | LS       |
| $r_r$     | Exponential decreasing rate of disease-induced death rate of hospitalized individuals in designated hospitals (per day) | 0.0531     | LS       |
the virus, $T_2$ as the date February 22, 2020, when no new beds in Fangcang shelter hospitals were installed and more and more beds in designated hospitals were available, and $T_3$ as the date March 10 when Fangcang shelter hospitals were suspended. Since the number of infected individuals inside Fangcang shelter hospitals is very small, we haven’t considered the infection in Fangcang shelter hospitals.

Fangcang shelter hospitals in Wuhan, China, saved hospital beds in designated hospitals by isolating and treating the infected individuals with mild symptoms. Since then, that is, February 5, 2020, an increasing number of critical infecteds were admitted into the designated hospitals for treatment, which resulted in an increasing function $\eta_c(t)$ of the hospitalization rate of critical infecteds in designated hospitals. And $\eta_c(t)$ remained as a constant after the day, that is, February 22, 2020, when the number of hospitalized individuals in Fangcang shelter hospitals began to decline. Thus, $\eta_c(t)$ is set as a piecewise-defined function, which takes the following form:

$$\eta_c(t) = \begin{cases} \eta_{c0}, & t \leq T_1, \\ \eta_{c0} + (\eta_{c1} - \eta_{c0}) \exp (-r_c(T_2 - t)), & T_1 < t \leq T_2, \\ \eta_{c1}, & t > T_2, \end{cases}$$

(2)

where $\eta_{c0}$ is the hospitalization rate of critical infecteds before February 5 and $\eta_{c1}$ is the hospitalization rate after February 22. Here, $T_1 = 26$ and $T_2 = 43$ since the data we used for model fitting begins at January 10, 2020.

Designated hospitals give priority to accepting the critical infecteds, and the mild infecteds are admitted into the designated hospitals only if there are beds available after receiving critical infecteds. So we adopt $\min \left\{ \eta_{m1}, \frac{B_c(t) - H_c(t) - \eta_c(t) I_c(t)}{I_m(t)} \right\}$ to describe the ratio of mild infecteds being hospitalized in the designated hospitals per day to maximize designated hospital utilization, which is a piecewise-defined function in term of the relationship between $\eta_{m1}$ and $\frac{B_c(t) - H_c(t) - \eta_c(t) I_c(t)}{I_m(t)}$. To be more detailed, when $\eta_{m1} \geq \frac{B_c(t) - H_c(t) - \eta_c(t) I_c(t)}{I_m(t)}$, namely, the beds in designated hospitals are sufficient, mild infecteds could be admitted into the designated hospitals with hospitalization rate $\eta_{m1}$; when $\eta_{m1} \leq \frac{B_c(t) - H_c(t) - \eta_c(t) I_c(t)}{I_m(t)}$, namely, the beds in designated hospitals are insufficient, only part of the mild infecteds could be admitted into the designated hospitals with the hospitalization rate $\frac{B_c(t) - H_c(t) - \eta_c(t) I_c(t)}{I_m(t)}$. As a result, the ratio of mild infecteds being hospitalized in the designated hospitals is defined as

$$\eta_{m1}(t) = \min \left\{ \eta_{m1}, \frac{B_c(t) - H_c(t) - \eta_c(t) I_c(t)}{I_m(t)} \right\}.$$

Fangcang shelter hospitals were officially opened on February 5 and thereafter, mild infecteds in Wuhan were admitted to Fangcang shelter hospitals for treatment. At the initial stage of the establishment of Fangcang shelter hospitals, not all mild infecteds could be hospitalized. We adopt the function $\min \left\{ \eta_{m2}, \frac{B_c(t) - H_c(t)}{I_m(t)} \right\}$ to describe the rate of mild infecteds being hospitalized by Fangcang shelter hospitals, which is also a piecewise-defined function in terms of the relationship between $\eta_{m2}$ and $\frac{B_c(t) - H_c(t)}{I_m(t)}$. When $\eta_{m2} \leq \frac{B_c(t) - H_c(t)}{I_m(t)}$, namely, the vacant beds in Fangcang hospitals are sufficient, mild infecteds could be admitted by Fangcang shelter hospitals with hospitalization rate $\eta_{m2}$; when $\eta_{m2} \geq \frac{B_c(t) - H_c(t)}{I_m(t)}$, namely, the vacant beds in Fangcang shelter hospitals are insufficient, only a part of the mild patients $\frac{B_c(t) - H_c(t)}{I_m(t)}$ could be admitted. As a result, the rate of mild infecteds being hospitalized by Fangcang shelter hospitals takes the form

$$\eta_{m2}(t) = \begin{cases} 0, & t \leq T_1 \text{ or } t \geq T_3 \\ \min \left\{ \frac{B_c(t) - H_c(t)}{I_m}, \eta_{m2} \right\}, & T_1 < t < T_3 \end{cases}.$$

(3)

With the fact that the medical resource, containment, and public awareness were improving and intensifying gradually after January 23, we assumed that the contact rate $c(t)$, the recovery rate of hospitalized individuals in designated hospitals $\gamma_c(t)$, and the disease-induced death rate in designated hospitals $\alpha_c(t)$ are time-dependent functions. To be detailed, the contact rate $c(t)$ is assumed to be a constant before January 23, while it is a decreasing function after January 23 with the lockdown strategies and raising of public awareness. As a result, $c(t)$ reads

$$c(t) = \begin{cases} c_0, & t \leq T_0, \\ (c_0 - c_1) \exp (-r_c(t - T_0)) + c_1, & t > T_0. \end{cases}$$

(4)
where \( c_0 \) stands for the contact rate without containment or awareness of COVID-19, \( c_1 < c_0 \) represents the minimum contact rate with the self-isolation besides intensifying interventions, and \( r_c \) is the reducing rate of the contact rate. Here, \( T_0 = 13 \) since we used data from January 10, 2020, onwards to fit the targeted model in this work.

As a large number of hospital beds and other medical supplies being added through Fangcang shelter hospitals and the improving of effectiveness of the control measures, the recovery rate \( \gamma_c(t) \) and disease-induced death rate \( \alpha_c(t) \) of the hospitalized in designated hospitals increased gradually. Thus, \( \gamma_c(t) \) and \( \alpha_c(t) \) take the following form:

\[
\gamma_c(t) = \begin{cases} 
\gamma_{c0}, & t \leq T_1, \\
(\gamma_{c1} - \gamma_{c0}) \exp(-r_c(T_2 - t)) + \gamma_{c0}, & T_1 < t \leq T_2, \\
\gamma_{c1}, & t > T_2, 
\end{cases}
\]

\[
\alpha_c(t) = \begin{cases} 
\alpha_{c0}, & t \leq T_1, \\
(\alpha_{c1} - \alpha_{c0}) \exp(-r_a(T_2 - t)) + \alpha_{c0}, & T_1 < t \leq T_2, \\
\alpha_{c1}, & t > T_2, 
\end{cases}
\]

where \( \gamma_{c0} \) (resp. \( \alpha_{c0} \)) is the recovery rate (resp. disease-induced death rate) before February 5, respectively, \( \gamma_{c1} > \gamma_{c0} \) and \( \alpha_{c1} > \alpha_{c0} \) are the maximum recovery rate and minimum disease-induced death rate in designated hospitals after February 22, and \( r_c \) (resp. \( r_a \)) are the increasing rate of the recovery rate (resp. decreasing rate of the disease-induced death rate).

3 | RESULTS

3.1 | Parameter estimation and model fitting

We set some of the initial values and parameters using data collected from Wuhan Municipal Health Commission of the People’s Republic of China from January 10 to April 16, 2020. During the initial stage of COVID-19 outbreak in Wuhan, China, almost everyone was susceptible to the virus, so we set \( S(0) = 11,081,000 \) according to the population of Wuhan. The average incubation period was computed as \( 1/\sigma = 5.32 \) days, while the average progressing rate of hospitalized individuals from Fangcang shelter hospitals to designated hospitals was computed as \( \xi_{H} = 0.0452 \). We derived other unknown parameter values by fitting our targeted model (1) to the collected multisource data, including the cumulative number of confirmed individuals, recovered individuals, and deaths (Figure 1A–C), number of hospitalized individuals in designated hospitals (Figure 1E) and Fangcang shelter hospitals (Figure 1F), and number of beds in designated hospitals and Fangcang shelter hospitals (Figure 1D), by using the nonlinear least-square method. The estimated initial and parameters are reported in Table 1.

To get the confidence intervals, we assumed that the cumulative number of confirmed cases, recovered cases, deaths, and number of hospitalized individuals in designated hospitals and Fangcang shelter hospitals follow Poisson distributions with the collected data on each day being their corresponding means. This allows us to randomly generate 500 samples of datasets that obey the Poisson distributions above. Then, we derived 500 groups of estimated values of state variables and parameters, which make it possible to compute the confidence intervals for the cumulative number of confirmed cases (Figure 3A), recovered cases (Figure 3B), deaths (Figure 3C), and for the number of hospitalized individuals in the designated hospitals (Figure 4A) and Fangcang shelter hospitals (Figure 4B), and for the effective reproduction number (Figure 4C). In Figures 3 and 4, the red circles represent the data from January 10 to April 16, 2020, the black curves are the fitting curves, and the gray regions are the 95% confidence intervals. The estimated values of cumulative number of confirmed individuals, recovered individuals, and deaths are plotted in black solid lines, as shown in Figure 3. The estimated values of the number of hospitalized individuals in designated hospitals and Fangcang shelter hospitals are also plotted in black solid lines, as shown Figure 4A,B. Figures 3 and 4 show that our targeted model (1) captures the data well. It is worth noting that the confidence intervals look relatively narrow. This is because the magnitude of the 500 randomly generated sets of data following the Poisson distributions is so large that the difference between the sets of data is relatively small compared to such a large magnitude. However, almost all the fitting data falls within the confidence intervals, which indicates the reasonability of the fitting results.
We calculated the basic reproduction number by using the next generation matrix method. Denoting \( Z \equiv (E, I_A, I_m, I_c, H_m, H_c) \), we have \( \dot{Z} = G(Z) \). Let \( G(Z) = P + Y \), where \( P \) represents the vector of new infections and \( Y \) represents the vector of all other transitions, respectively. And we get

\[
P = \begin{bmatrix}
\frac{\beta(t)S(t)I_A + \theta_m I_m + I_c}{N} \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}, \quad Y = \begin{bmatrix}
\sigma E \\
-p\sigma E + \gamma_A I_A \\
-(1-p)q\sigma E + \xi_m I_m + \eta_{m1}(t)I_m + \eta_{m2}(t)I_m \\
-(1-p)(1-q)\sigma E + \xi_m I_m + \eta_c(t)I_c + \alpha_c I_c \\
-\eta_m(t)I_m + \gamma_m H_m + \rho_m H_m + \xi_m H_m \\
-\eta_{m1}(t)I_m - \eta_c(t)I_c - \xi_m H_m + \gamma_c(t)H_c + \alpha_c(t)H_c
\end{bmatrix}.
\]
Differentiating $F$ and $V$ with respect to $Z$ and computing them at the initial state give

\[
F = \begin{bmatrix}
0 & \beta c(t)\theta_A & \beta c(t)\theta_m & \beta c(t) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
V = \begin{bmatrix}
\sigma & 0 & 0 & 0 & 0 & 0 \\
-\rho\sigma & \gamma_A & 0 & 0 & 0 & 0 \\
-(1-p)\rho\sigma & 0 & -\xi_I + \eta m_1(t) + \eta m_2(t) & 0 & 0 & 0 \\
0 & 0 & -\eta m_2(t) & \eta c(t) + \alpha_I & 0 & 0 \\
0 & 0 & -\eta m_1(t) & -\eta_c & -\xi_H & \gamma_c(t) + \alpha_c(t) \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}.
\]

Then, the effective reproduction number can be approximately defined as

\[
R_e(t) = \rho(FV^{-1}) = \frac{c(t)\beta \theta_A p}{\gamma_A} + \frac{c(t)\beta \theta_m(1-p)q}{\xi_I + \eta m_1(t) + \eta m_2(t)} + \frac{c(t)\beta (1-p)}{\eta c(t) + \alpha_I} \left[ \frac{q \xi_I}{\xi_I + \eta m_1(t) + \eta m_2(t)} + (1-q) \right]
\]
due to the fact that the ratio of the susceptible population to the total population is approximately 1 and some parameters are time varying. The effective reproduction number $R_e(t)$ was estimated in terms of the parameters reported in Table 1, as shown in Figure 4C. The shadowed region in Figure 4C is the 95% confidence interval of $R_e(t)$. It follows from Figure 4C that the effective reproduction number $R_e(t)$ falls below the threshold of 1 since February 12. This is mainly due to a change in the national COVID-19 testing criteria on February 12, in terms of which those with obviously clinical symptoms can be identified as confirmed cases.

### 3.2 Uncertainty and sensitivity analysis

Considering the uncertainty of the model parameters, we conducted sensitivity analysis of the effective reproduction number $R_e(t)$ with respect to various parameters over time by computing the partial rank correlation coefficients (PRCCs) based on Latin hypercube sampling.\(^{32}\) We chose the parameters closely related to the epidemic transmission and containment including the parameters related to the evolution of contact rate ($c_1, c_0$), ratio of asymptomatic infectives ($p$), progression rate of mild infectives to critical infectives ($\xi_I$), hospitalization rates in designated hospitals and Fangcang shelter hospitals ($\xi_{m1}, \xi_{m2}, \xi_0, \xi_c$), transmission probability ($\beta$), and effective contact ratios of $I_A$ and $I_m$ to $I$ ($\theta_A, \theta_m$), with other parameters fixed, as shown in Figure 5. The Latin hypercube sampling was done with 10,000 bins by varying these parameters based on their estimated values. The monotonic relationship between each parameter and the model outcomes was assessed over the entire time interval. The values of the PRCCs of each parameter over the time interval were calculated, which allowed us to determine whether the significance of one parameter occurred during the progression of the dynamics of our targeted model. Figure 5 showed that the parameters $\beta, c_0, c_1, \theta_A, \theta_m$ are positively correlated with $R_e(t)$ almost over the entire time interval, which indicates that declining $\beta, c_0, c_1, \theta_A, \theta_m$ can lead to a significant decreasing of the effective reproduction number $R_e(t)$ over the whole time interval. As a result, increasing the intensity of masks wearing (decreasing $\beta$), enhancing social distancing (decreasing $c_0, c_1$), strengthening awareness of self-protection (reducing $\theta_A$), and raising hospitalization rate of mild infectives (increasing $\theta_m$), all can achieve the goal of containing the epidemic.

In order to explore the exact impact of Fangcang shelter hospitals on the COVID-19 outbreak in Wuhan, China, we analyzed the sensitivity of the total number of infectives and hospitalized individuals to the bed capacity and opening time of Fangcang shelter hospitals. We quantified what would happen if the bed capacity of Fangcang shelter hospitals was declined, respectively, by 50% and 80%, or the Fangcang shelter hospitals opened 1 or 2 weeks earlier (January 29 or January 22), or opened 1 or 2 weeks later (February 12 or February 19), or no Fangcang shelter hospitals, as shown in Figure 6. We observed from Figure 6A–C and G–I that decreasing the bed capacity or delaying the opening time of Fangcang shelter hospitals would result in a significant increasing of the total number of infectives and hospitalized.
FIGURE 5  Partial rank correlation coefficients (PRCCs) of $R_e(t)$ for the key parameters $c_0, c_1, p, \xi, \eta_{m1}, \eta_{m2}, \eta_{m3}, \eta_{c1}, \beta, \theta_A, \theta_m$. The Latin hypercube sampling was done with 10,000 bins [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 6  Impact of the bed capacity and opening time of Fangcang shelter hospitals on the total number of infectives and hospitalized individuals, the cumulative number of critical infectives, and the effective reproduction number $R_e(t)$ in some period. And the time starting effective control of the epidemic will be delayed, which triggered severer outbreaks and longer length of uncontrollable time. Figure 6D–F showed that opening the Fangcang shelter hospitals in advance would effectively
reduce the total number of infectives and hospitalized individuals and the cumulative number of critical infectives, while the time starting effective control of the epidemic could not be advanced although the value of effective reproduction numbers would be diminished in some period. It was worth emphasizing that if no Fangcang shelter hospitals opened, a great increasing occurred in the total number of infectives and hospitalized individuals and the cumulative number of critical infectives, while the effective reproduction number also increased. The findings demonstrated that reducing the bed capacity and delaying the opening time of Fangcang shelter hospitals could trigger severer outbreaks of COVID-19 in Wuhan, China, while advancing the opening time would alleviate the outbreak of COVID-19, illustrating a critical impact of Fangcang shelter hospitals on combating the epidemic.

To study the effect of the opening time together with the bed capacity of Fangcang shelter hospitals on the COVID-19 epidemic in Wuhan, we addressed what would happen if delayed Fangcang shelter hospitals by 1 or 2 weeks (i.e., February 12 or February 19) with an increasing of the bed capacity by 1.5 times (i.e., $2.5B_m$) and 4 times (i.e., $5B_m$), or advance Fangcang shelter hospitals by 1 or 2 weeks with a decreasing of the bed capacity by 50% and 80%, as shown in Figure 7. Figure 7A (resp. Figure 7B) showed that postponing the opening time of Fangcang shelter hospitals by 1 week and raising the bed capacity by 1.5 times and 4 times wouldn’t result in much difference of the total number of infectives and hospitalized individuals (resp. the cumulative number of critical infectives) although both triggered severer outbreaks. A similar result can be derived when the opening time of Fangcang shelter hospitals is delayed by 2 weeks, as shown in Figure 7B,F. If the Fangcang shelter hospitals were advanced 1 or 2 weeks, keeping 50% bed capacity could finally alle-
violate the outbreak significantly, as shown in Figure 7C,D,G,H. However, keeping 20% bed capacity with 1 week earlier of the Fangcang shelter hospitals would not result in much difference from the baseline values on the cumulative number of critical infectives, which would be greatly diminished with 2 weeks earlier of the Fangcang shelter hospitals. From this point of view, the opening time of Fangcang shelter hospitals has a greater impact on the epidemic than the bed capacity.

To further reveal the vital role of Fangcang shelter hospitals in containing the epidemic in Wuhan, China, we differentiate the bed capacity in designated hospitals from Fangcang shelter hospitals and explored when the empty beds can accommodate the patients seeking beds, as shown in Figure 8. The bed capacity (Bc, blue curve), number of empty beds (Bc − Hc, pink curve), and patients seeking beds in designated hospitals (Im + ηmIm during the operation of Fangcang shelter hospitals and Im; otherwise, gray curve) are shown in Figure 8A. It can be seen from Figure 8A that the bed capacity in designated hospitals has been increasing before February 25, 2020. However, the number of empty beds in designated hospitals was less than the number of patients seeking beds in designated hospitals from January 20 to February 4 despite of the government’s continuous expansion of bed capacity. The reason was that the increasing of patients seeking beds in designated hospitals was much faster than the increasing of bed capacity, indicating that beds were urgently needed during this period. Therefore, Fangcang shelter hospitals has been continuously opened since February 5 to alleviate the shortage of hospital beds. Figure 8B showed the variation of bed capacity (Bm, blue curve), number of empty beds (Bm − Hm, pink curve), and patients seeking beds in Fangcang shelter hospitals (Im − ηmIm, gray curve). It follows from Figure 8B that the bed capacity in Fangcang shelter hospitals has been increasing before February 23, but the number of empty beds in Fangcang shelter hospitals was less than the number of patients seeking hospital beds till February 20, illustrating a shortage of beds in Fangcang shelter hospitals during this period. The number of empty beds in Fangcang shelter hospitals was increasing with the continuous decline of the number of patients seeking beds. The number of empty beds is approaching the bed capacity in Fangcang shelter hospitals on March 10, when all Fangcang shelter hospitals had been suspended. Figure 8C showed the total bed capacity (Bc + Bm, blue curve), total number of empty hospital beds (Bc + Bm − (Hc + Hm), pink curve), and total patients seeking hospital beds in Wuhan (Im + Im, gray curve). The results illustrated that the total bed capacity in Wuhan continued to increase from January 23 to February 25, but the number of empty beds was less than the number of patients seeking hospital beds from January 20 to February 16. It indicated insufficiency of hospital beds in Wuhan, which is due to the change in national test standards besides the climbing number of infected individuals.

It is worth emphasizing that the designated hospitals can accommodate all the patients needing to be hospitalized in it since February 5 but this date for Fangcang shelter hospitals is February 21, which implies the opening of Fangcang shelter hospitals has saved at least 16 days for designated hospitals to treat the critical infectives. The number of patients needing to be hospitalized in designated hospitals declined greatly from February 4, although there was a slight rebound on February 12, when there were about 2000 patients seeking beds in designated hospitals. However, it was not until February 11 that the patients seeking beds in Fangcang shelter hospitals began to reduce gradually, and there were still up to about 2459 patients seeking beds in Fangcang shelter hospitals till February 20, from which day on, the beds provided by Fangcang shelter hospitals could meet the hospitalization requirement of patients. That demonstrates Fangcang shelter hospitals have saved more than 2400 beds for designated hospitals every day for at least 16 days. The findings further prove the vital role of Fangcang shelter hospitals in curbing the epidemic in Wuhan, China.

To assess how much the Fangcang shelter hospitals alleviated the epidemic in Wuhan quantitatively, we examined the impact of bed capacity and opening time of Fangcang shelter hospitals on the time when $R_e(t) = 1$, time when the daily number of confirmed cases less than 100, cumulative number of critical infectives, and final number of confirmed cases. More specifically, we computed these indexes by reducing the bed capacity $B_m$ to 0.8$B_m$, 0.5$B_m$, 0.2$B_m$, or delaying the opening time by 1, 2, and 3 weeks, respectively, or no Fangcang shelter hospitals at all, and summarized the results in Table 2. We can derive from Table 2 that the final number of confirmed cases would be increased by 1274 and 3691 cases (from 50,124 cases to 51,398 and 53,815 cases), respectively, if the bed capacity of Fangcang shelter hospitals was reduced by 20% or 50%. It would climb up to 55,877 cases with an increasing of 11.48% if the bed capacity of Fangcang shelter hospitals was diminished by 80%. If the Fangcang shelter hospitals were opened 1 or 2 weeks later, 4152 or 7048 more critical infections would occurred. It would increase by 9038 cases with 3 weeks later of Fangcang shelter hospitals. We should emphasize that the Fangcang shelter hospitals saved 7 and 62 days during the process of controlling the effective reproduction number $R_e(t) < 1$ and the daily number of confirmed cases less than 100, respectively, while it avoided 17,103 critical infections and 17,823 total infections, further illustrating the critical role of Fangcang shelter hospitals.
### TABLE 2  The impact of bed capacity and opening time of Fangcang shelter hospitals on the time when $R_e(t) = 1$, time when the daily number of confirmed cases less than 100, cumulative number of critical infectives, and final number of confirmed cases

| Parameters            | Time when $R_e(t) = 1$ | Time when the daily number of confirmed cases less than 100 | Cumulative number of critical infectives | Final number of confirmed cases |
|-----------------------|-------------------------|------------------------------------------------------------|------------------------------------------|---------------------------------|
| Baseline values       | February 11             | March 12                                                   | 24,574                                   | 50,124                          |
| Reducing bed capacity | 0.8$B_m$                | February 11                                               | March 12                                 | 25,147                          | 51,398                          |
|                       | 0.5$B_m$                | February 13                                              | March 21                                 | 26,629                          | 53,815                          |
|                       | 0.2$B_m$                | February 16                                              | April 13                                 | 31,266                          | 55,877                          |
| Delaying opening time | February 12             | February 13                                              | March 16                                 | 28,726                          | 52,823                          |
|                       | February 19             | February 18                                              | March 19                                 | 31,622                          | 56,378                          |
|                       | February 26             | February 18                                              | March 22                                 | 33,612                          | 61,458                          |
| No Fangcang           | February 18             | May 13                                                   | 41,677                                   | 67,947                          |
Fangcang shelter hospitals played a vital role in mitigating the COVID-19 outbreak in Wuhan, China, by providing high-quality medical care and treatment for more than 120,000 patients. Early evidence illustrated that Fangcang shelter hospitals were a major reason for successful controlling the outbreak of COVID-19 in Wuhan. In this work, we have done a retrospective study to mimic and reveal to what degree Fangcang shelter hospitals contained the epidemic in Wuhan. We have proposed a deterministic compartment model by differentiating the hospitalized individuals in Fangcang shelter hospitals $H_m$ and designated hospitals $H_c$. To mark the impact of bed capacity of Fangcang shelter hospitals and designated hospitals on the transmission dynamics of COVID-19 in Wuhan, we adopted piecewise-defined functions in terms of empty beds to describe the hospitalized individuals, which led to a piecewise smooth model. With the strengthening of prevention and control measures including the lockdown of the city on January 23, opening of the Fangcang shelter hospitals on February 5 and suspending of the Fangcang shelter hospitals on March 10, we assumed the control parameters are time dependent and piecewise defined.

It is worth noting that our targeted model highlights more on the critical role of the bed capacity in Fangcang shelter hospitals in the containment of the outbreak compared with the models presented in previous studies.\textsuperscript{8,9,21} We adopted one model with piecewise-defined control parameters to mimic the whole control process of the epidemic in Wuhan, while different models are adopted at different stages to distinguish the role of healthcare personnel (HCP) and the general population in the transmission of the epidemic in $\mathrm{i}$ et al.\textsuperscript{8} We have divided the infected individuals with symptoms before hospitalization into two classes, that is, the mild infectees and the critical infectees, dependent on their manifestations, while whether the infected individuals had symptoms and the severity of symptoms were not considered in Zhou et al\textsuperscript{9} and Wang et al.\textsuperscript{21} Moreover, our targeted model accurately characterized the numbers of hospitalized individuals in Fangcang shelter hospitals and those in designated hospitals affected by the bed capacity in Fangcang shelter hospitals and that in designated hospitals, while all the hospitalized individuals were classified into one compartment in Zhou et al\textsuperscript{9} and Wang et al.\textsuperscript{21}

We estimated the time-dependent hospitalization rate of critical infectees and mild infectees in designated hospitals and Fangcang shelter hospitals, contact rate, recovery rate, disease-induced death rate, and other model parameters, as reported in Table 1, by using the nonlinear least-square method to fit model (1). The fitting results shown in Figures 3 and 4 illustrated that the targeted model captured the data, including the number of hospitalized individuals in designated hospitals and Fangcang shelter hospitals, respectively, the cumulative number of confirmed cases, recovered cases, and death, well. The estimated effective reproduction number fell below the threshold of 1 after February 12, which was in line with the fact that the number of newly confirmed cases in Wuhan diminished since February 12, indicating the epidemic was controllable thereafter. The PRCCs of the effective reproduction number $R_e(t)$ with respect to control parameters over time demonstrated that decreasing the effective contact ratio of mild infectees $\theta_m$, which could be achieved by opening Fangcang shelter hospitals to isolate as many mild infectees as possible, could significantly decline $R_e(t)$.

To give a detailed understanding on how the rapidly and massively scaling of health system’s capacity by Fangcang shelter hospitals mitigated the outbreak, we explored what would happen as the bed capacity and opening time of Fangcang shelter hospitals varied, shown in Figures 6 and 7. The results shown in Figure 6 indicated that reducing the bed capacity and delaying the opening time of Fangcang shelter hospitals could trigger severer outbreaks by increasing the daily number of infectees and hospitalized individuals, the cumulative number of critical infectees, and the effective reproduction number in some period. Meanwhile, insufficient bed capacity and later opening of Fangcang shelter hospitals would postpone the time when the epidemic could be controllable. These findings indicated that even with rapid and stringent containments, such as traveling ban and contact tracing, the outbreak severity would be aggravated without sufficient bed capacity in time to hospitalize those mild infectees. The results shown in Figure 7 demonstrated that if delaying the Fangcang shelter hospitals by 1 or 2 weeks, enlarging the bed capacity $B_m$ to $2.5B_m$ and $5B_m$ didn’t result in obvious difference in the outbreak, although all could trigger severer outbreaks. This suggested the opening time of Fangcang shelter hospitals had a more significant impact on the outbreak than the bed capacity.

To further investigate the extent that Fangcang shelter hospitals alleviated the pressure of hospital bed shortage in Wuhan, we have examined the variation of bed capacity, empty beds, and number of patients seeking beds in designated hospitals, Fangcang shelter hospitals, and hospitals in Wuhan over time, which revealed when bed shortage occurred during the COVID-19 outbreak in Wuhan. We observed from Figure 8 that the designated hospitals, Fangcang shelter hospitals, and hospitals in Wuhan experienced a shortage of bed from January 20 to February 4, February 5 to February 20, and January 20 to February 6, respectively. The results indicated that the Fangcang shelter hospitals had saved at least
16 days for designated hospitals to treat critical infectives. The results listed in Table 2 demonstrated that without the Fangcang shelter hospitals, 17,103 more critical infections and 17,823 more total infections would occur, and the time when the epidemic was controllable (i.e., \( R_c(t) < 1 \)) would be delayed 7 days. Our study suggested that the effective group isolation of mild infectives in Fangcang shelter hospitals alleviated the outbreak, which led to the saving of hospital beds and treatment time for those critical infectives and finally successful stopping the COVID-19 outbreak in Wuhan.

We should emphasize that the impact of the bed capacity and opening time of Fangcang shelter hospitals on the containment of the epidemic has been quantitatively assessed in detail, which demonstrates the extent to which Fangcang shelter hospitals have relieved the outbreak. However, only the effect of hospital bed related parameters or the bed numbers on the cumulative number of confirmed cases was investigated in previous studies.\(^8,9,21\) The main findings in our work indicate that isolating and treating the mild infecteds by temporary hospitals play a key role in curbing and even stopping the epidemic. The main experience from Fangcang shelter hospitals can be extended to countries or regions facing the shortage of medical resources, where they could isolate and care groups of mild infecteds in schools, stadiums, and so on.

There is some limitation of our work. The key issue was scaling the hospital bed capacity when establishing Fangcang shelter hospitals. The cost of building and managing Fangcang shelter hospitals were not important compared with their effect on the control of the epidemic, so it was not considered in this work and we will study it in the future work. We have adopted some piecewise-defined functions to characterize the contact rate, hospitalization rate, death rate, and recovery rate. Taking the contact rate \( c(t) \) as an example, it is assumed to be a constant before January 23 and a decreasing function after January 23 when the lockdown strategy was implemented. In fact, the effect of lockdown on the contact rate should not begin since the implementation of lockdown, and it is more reasonable that the effect begins from some day after the lockdown. In addition, the contact tracing measure, the infectivity of exposed individuals during the late incubation period, and the infection inside the designated hospitals were not considered in this work. All of these limitations will affect our parameter estimates and the evaluation of the impact of control measures. A stochastic model or network model can be established to study the role of Fangcang shelter hospitals in the epidemic control in the future on the one hand. On the other hand, the bed capacity for a given epidemic scale may be presented by exploring the optimal strategies.

In conclusion, we have proposed a piecewise smooth compartment model dependent on the bed capacity and empty beds in designated hospitals and Fangcang shelter hospitals to reveal the critical role of Fangcang shelter hospitals in tackling the COVID-19 in Wuhan. The previous modeling studies of COVID-19 in Wuhan did not differentiate the bed capacity and patient in designated hospitals and Fangcang shelter hospitals, which was firstly mimicked and quantified in this work. Our findings demonstrated that the Fangcang shelter hospitals saved time and hospital beds to treat the critical infectives by isolating large number of mild infectives, which finally helped stop the epidemic in Wuhan. The results strongly support the importance of early preparation to ramp up the health system capacity that avoided a large number of infections. Our study provide a valuable reference for policy-makers in regions or countries facing shortage of medical resources.

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CONFLICT OF INTEREST

The authors declare that they have no conflict of interests.

ORCID

Aili Wang https://orcid.org/0000-0003-3372-899X

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