Deconfinement and Dissipation in Quantum Hall “Josephson” Tunneling

H.A. Fertig and Joseph P. Straley

Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506-0055

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The zero-bias tunneling resonance in quantum Hall bilayer systems is investigated via numerical simulations of the classical two-dimensional XY model with a symmetry-breaking field. Disorder is included in the model, and is shown to nucleate strings of overturned spins proliferated through the system, with unpaired vortices and antivortices at their endpoints. This string glass state supports low energy excitations which lead to anomalously large dissipation in tunneling, as observed in experiment. The effect of an in-plane magnetic field is discussed.

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Introduction. Bilayer quantum Hall systems have attracted much attention because, for small layer separations and vanishing interlayer tunneling, they support states with spontaneous interlayer coherence. This broken symmetry leads to a Goldstone mode [1] which may be described as a spin wave in an XY ferromagnet [2]. If one introduces interlayer tunneling, the system may also be described as a large area Josephson junction. In the XY language, the tunneling appears as a magnetic field tending to align the spins along the field direction—i.e. an in-plane field. While this symmetry-breaking field makes the physics of this system completely different than the heavily studied model without an in-plane field, vortex deconfinement does occur, albeit not via the usual Kosterlitz-Thouless mechanism [3,4]. The Josephson junction analogy suggests these systems might support a Josephson effect [5]. Recently, a sharp conductance resonance near zero interlayer voltage bias was detected [6,7] which is highly reminiscent of the DC Josephson effect. The width of this resonance, however, is surprisingly large, and perturbative treatments [8–10] have so far not been able to account for the dissipation associated with the vortices in the string glass state intrinsically has a weak temperature dependence, as illustrated in Fig. 1. Above a critical current scale $i_c$, we find that the strings depin and the resistance, as well as the voltage noise, is greatly enhanced. An observation of a peak in the noise near $i_c$ would give direct experimental evidence of the existence of the string glass state posited here. Finally, we show that an in-plane magnetic field $B_{\parallel}$ tends to align the strings along the field direction, and for large enough $B_{\parallel}$ the density of strings increases. At low temperature, the strings are pinned by disorder, so the zero-bias resonance remains in place. Again, this has been observed experimentally [7], but to our knowledge this is the first qualitative explanation of why this occurs.

Simulation Model. To conduct our simulations we place our system on an $L \times L$ square lattice, whose lattice constant $a_0$ should be understood as the underlying microscopic length scale of the system, the magnetic length $\ell_B = \sqrt{\hbar c/eB}$, with $B$ the perpendicular magnetic field. We impose periodic boundary conditions on this lattice. Our Hamiltonian is $H = H_{XY} + H_D$, with

$$H_{XY} = -K \sum_{<rr'>} \cos[\theta(r) - \theta(r')] - h \sum_i \cos \theta(r), \quad (1)$$

$$H_D = \sum_{<rr'>} A_{rr'} \sin[\theta(r) - \theta(r')]. \quad (2)$$

In Eq. 1, $\theta(r)$ represents the angle of a planar spin located at lattice site $r$, $\sum_{<rr'>}$ denotes a sum over nearest neighbors, and $h$ represents the effect of a magnetic
field tending to align spins along \( \theta(\mathbf{r}) = 0 \), whose underlying source is the interlayer tunneling matrix element of the electrons. In \( H_D \), \( A_{rr} \) is a random vector field residing on the bonds, which may be understood as follows. At long wavelengths, the state of the bilayer quantum Hall system may be specified by a three dimensional pseudospin field \( \mathbf{S}(\mathbf{r}) \). \( S_z \) here represents the difference in electron density between the two layers, and \( S_x, S_y \) are the components of the planar spin field. \( S_z \) ideally vanishes when the electric potential of the two layers is perfectly balanced; in practice, there are impurities which tend to push the electrons into one layer or the other. These same impurities will tend to locally lower or raise the total electron density, which can be accomplished by introducing a non-vanishing topological density \( q(\mathbf{r}) = \epsilon_{\mu\nu} \mathbf{S} \cdot [\partial_\mu \mathbf{S} \times \partial_\nu \mathbf{S}] / 8\pi \), which in the quantum Hall context is proportional to the real charge density \([12]\). The topological density contains a term of the form \( \vec{A} \cdot \vec{\nabla} \theta \) [9], with \( \vec{A} \) proportional to the induced \( S_z \). It is this contribution to the disorder we model, taking \( \partial_\theta \theta \to \sin[\theta(\mathbf{r}) - \theta(\mathbf{r}')] \), with \( < \mathbf{r} \mathbf{r}' > \) the nearest neighbor bond along the \( \mu \) direction, to account for the finite grid spacing in the simulation \([14]\). We take \( A_\mu(\mathbf{r}) = \sum_\nu e^{-|\mathbf{r} - \mathbf{r}'|^2 / 4\xi_\mu^2} v_\mu(\mathbf{r}') \), with \( v_\mu \) drawn from a uniform distribution satisfying \( -\Delta < v_\mu < \Delta \) \([15]\). Because disorder due to charged impurities in most quantum Hall systems varies slowly in space, the results we present are for relatively large values of \( \xi_\mu \). Our qualitative results however also hold for smaller values of \( \xi_\mu \).

To simulate dynamics in this model, we adopt a Langevin equation of motion,

\[
\Gamma \frac{d^2 \theta(\mathbf{r})}{dt^2} = \frac{\delta H}{\delta \theta(\mathbf{r})} + \zeta(\mathbf{r}) - \gamma \frac{d\theta(\mathbf{r})}{dt} + i_d \tag{3}
\]

\( \Gamma \) above represents an effective moment of inertia for an \( XY \) spin. Since \( \theta \) is conjugate to \( S_z \) in the full quantum theory, \( \Gamma \) originates from the energy cost for creating charge imbalance across the layers, and is proportional to the capacitance per unit area of the bilayer. The \( \zeta \) and \( \gamma \) terms are respectively a random torque and a uniform damping, introduced to model coupling to a heat bath. To satisfy the fluctuation-dissipation theorem, the random torques are drawn from a distribution satisfying \(< \zeta(\mathbf{r}, t) \zeta(\mathbf{r}', t') > = 2\gamma T \delta_{\mathbf{r}, \mathbf{r}'} \delta(t - t') \) with \( T \) the temperature of the system. Finally, the last term is a current which may be imposed to drive the system \([13]\).

**Results.** We begin by studying the resistance of the system near zero bias. This can be conveniently accomplished by examining the dynamics of the \( XY \) spins via the extended variables \( u(\mathbf{r}, t) = \int dt' \frac{d\mu(\mathbf{r}, t')}{d\mu(\mathbf{r}, t)} \). The variables \( u \) are identical to the original angles except they are unbounded. Their dynamics are analogous to those of particles in a washboard potential \([16]\), and in the absence of a drive current \( (i_d = 0) \) they diffuse. It is easily confirmed that \( < u^2(\mathbf{r}, t) > \propto t \) for large enough time \( t \), where \( < \cdot > \) denotes an average over sites and disorder configurations. We thus define a diffusion constant \( D = \lim_{t \to \infty} < u^2(\mathbf{r}, t) > / dt \). The "conductivity" of the \( u \)'s is then given by the Einstein relation \( \sigma = D / T \), which quantifies their linear response to the drive current. Since uniform average motion of the \( u \)'s generates a (Josephson) voltage, this means \( \sigma \) is proportional to the tunneling resistance of the system, \( R \).

![FIG. 1. Tunneling resistance averaged over 16 disorder realizations and $10^6$ Langevin sweeps for states in string glass state. Length, time and energy units chosen so that \( \Gamma = K = a_0 = 1 \); \( \hbar \xi^2 = 25 \), \( \gamma = 0.143 \) in these units. Inset: Corresponding resistance for no disorder.](image-url)
and so may be held in place by disorder. Because of
the quenched nature of the disordered state, we call this
a string glass. Low-energy, localized excitations are a
common feature of glasses [18], and their presence here
supports our characterization of this as a glassy state.

In the string glass state, a drive current acts as an ef-
fective force on the strings. In a typical run, at very low
drive the average “displacement” ∫ d2r u(r,t)/L2 for a
single disorder realization has a small slope. As i_d is in-
creased, one finds occasional short intervals for which this
slope is increased, occurring when string sections become
briefly depinned. With further increasing i_d these events
become more frequent, until a critical drive current i_c
is reached at which the strings essentially move freely.
In this situation, spins that were previously nearly static
now rotate much more frequently as strings pass through
them (inducing a Josephson voltage), and the dissipation
in the system is greatly enhanced [20]. This is apparent
in Fig. 3, which illustrates the current-voltage character-
istic for a single disorder configuration, with each data
point averaged over eight different initial seeds for the
random force. For each of these seeds we have averaged
over 2 × 10^6 Langevin sweeps, with the first 25% of these
thrown away for equilibration. The onset of “normal”
dissipation in this system is apparently a depinning phe-
nomenon. Because of the stick-slip motion in the vicinity
of i_c described above, there is considerable broadband
noise in the dissipation for i_d ~ i_c. This is typical of a
depinning phenomenon, and should generate a peak in
the noise spectrum of the power dissipated. An experi-
mental observation of such a noise peak would confirm
the string glass nature of the state, since it reflects the
depinning of the strings.

An in-plane component of the magnetic field leads to
further interesting physics. For an appropriate choice
gauge, the in-plane field introduces a phase in the
XY interactions produced by simulated annealing is a tendency for
the strings to align along Q. For larger n_Q, we find the
strings become narrower and the configurations appear
more ordered, resembling a disordered soliton lattice [21] – with the strings being essentially identical to solitons.

This qualitative transition is reflected in the critical
current i_c, as illustrated in Fig. 3. One can see i_c is
little changed from its n_Q = 0 value for n_Q = 1, 2,
whereas for n_Q = 3 and above, i_c becomes noticeably
suppressed. The existence of a parallel field scale above
which the depinning current decreases can be understood
in terms of collective pinning theory [22]. For large val-
ues of Q, the soliton lattice may be thought of as an
elastic medium, which is deformed and pinned by the
disorder. This deformation involves a length scale L_c for
a typical domain size, determined by balancing the pin-
ing and elastic distortion energies [22]. The length scale
is meaningful provided L_c L_Q > 2π; if this is not satisfied
then L_c is smaller than the average distance between soli-
tons and one enters a strong pinning regime. Following
this reasoning [23] we obtain an estimate for the criti-
cal wavevector above which one enters the weak pinning
regime, Q_c = hΔ_L/L_c. For large values of n_p, the density of
pinning centers. This implicitly defines a critical paral-
lel field scale B_c above which the depinning drive field i_c
will decrease with B_c. Below this, the strings are strongly
pinned by disorder, and B_c has little effect. This is con-
sistent with our simulation results, as illustrated in Fig.
3.

Our results show that the zero-bias tunneling reso-
nance is not shifted to higher voltage by an in-plane field,
consistent with experiment but in contrast to perturba-
tive theoretical treatments [8,9]. The latter results sug-
gest that a resonance should be seen at a voltage cor-
responding to the Goldstone collective mode frequency
hω_n(Q) = eV. A weak structure is observed obeying
this relation [7]. While the simulations reported here

FIG. 2. Snapshot from run with Γ = K = a_0 = 1, L = 79,
hξ_p^2 = .25, γ = 0.143, ξ_p/L = 0.13, Δξ_p^2 = 3.18. Only spins
with cos θ < 0 shown. Circles indicate most active spins,
which are localized near vortices (+) and antivortices (*)

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do not exhibit this behavior, it is likely that under different circumstances they would. Well above the depinning threshold, when sliding of the soliton lattice proceeds at a velocity such that a pinning center crosses a soliton once per period of a collective mode, the resulting commensuration acts as an increased friction that lowers the tunneling resistance of the system. A perturbative analysis [23] supports this, and suggests the width of such a resonance would be of the order $\Delta V \sim \sqrt{hV\gamma/T_c}$. For this to be observable in simulation, one needs very small values of $\gamma$, requiring impractically long equilibration times. Such a small effective $\gamma$ is consistent with experiment.

**Implications and Summary.** The picture that emerges from these simulations, and the string glass state they suggest, has several experimental implications. These include the noise peak near the depinning transition and the critical parallel field discussed above, both of which can in principle be observed. In addition, one would expect that small systems, for which the number of vortices generated by disorder is small, would generate strong sample to sample fluctuations in the tunneling resistance. Another signal would be the observation of a low temperature scale $T_F \approx \sqrt{K h \ell_0}$ below which the isolated excitations freeze out and the tunneling resistance drops rapidly [23,24]. $T_F$ is well below experimentally accessible temperatures for reported sample parameters [6], but might be observed in samples with large $h$ and $\ell_0$. Finally, since vortices carry charge $e/2$ [2] in this system and are deconfined, it is interesting to speculate that this fractional charge might be observed in shot noise experiments [25].

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