Effects of CP violation on Event Rates in the Direct Detection of Dark Matter

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Abstract

A full analytic analysis of the effects of CP violating phases on the event rates in the direct detection of dark matter in the scattering of neutralinos from nuclear targets is given. The analysis includes CP violating phases in softly broken supersymmetry in the framework of the minimal supersymmetric standard model (MSSM) when generational mixings are ignored. A numerical analysis shows that large CP violating phases including the constraints from the experimental limits on the neutron and the electron electric dipole moment (EDM) can produce substantial effects on the event rates in dark matter detectors.

Supersymmetric theories with R parity conservation imply the existence of a lowest mass supersymmetric particle (LSP) which is absolutely stable\textsuperscript{1}. In supergravity models\textsuperscript{2} with R parity invariance one finds that over a majority of the parameter space the LSP is the lightest neutralino, and thus the neutralino becomes a candidate for cold dark matter (CDM) in the universe. A great deal of dark matter exists in the halo of our galaxy and estimates of the density of dark matter in our solar neighborhood give densities of $0.3\text{GeV/cm}^3$ and particle velocities of $\approx 320\text{km/s}$. One of the interesting suggestions regarding the detection of dark matter is that of direct detection via scattering of neutralinos from target nuclei in terrestrial detectors\textsuperscript{3}. Considerable progress has been made in both technology of detection\textsuperscript{4}. 

\textsuperscript{1}Supersymmetric theories with R parity conservation imply the existence of a lowest mass supersymmetric particle (LSP) which is absolutely stable. In supergravity models with R parity invariance one finds that over a majority of the parameter space the LSP is the lightest neutralino, and thus the neutralino becomes a candidate for cold dark matter (CDM) in the universe. A great deal of dark matter exists in the halo of our galaxy and estimates of the density of dark matter in our solar neighborhood give densities of $0.3\text{GeV/cm}^3$ and particle velocities of $\approx 320\text{km/s}$. One of the interesting suggestions regarding the detection of dark matter is that of direct detection via scattering of neutralinos from target nuclei in terrestrial detectors. Considerable progress has been made in both technology of detection.
as well as in accurate theoretical predictions of the expected event rates in detectors such as Ge, NaI, CaF$_2$, and Xe.

In this paper we discuss the effects of CP violating phases in supersymmetric theories on event rates in the scattering of neutralinos off nuclei in terrestrial detectors. Such effects are negligible if the CP violating phases are small. Indeed the stringent experimental constraints on the EDM of the neutron and of the electron would seem to require either small CP violating phases or a heavy supersymmetric particle spectrum, in the range of several TeV, to satisfy the experimental limits on the EDMs. However, a heavy sparticle spectrum also constitutes fine tuning and further a heavy spectrum in the range of several TeV will put the supersymmetric particles beyond the reach of even the LHC. Recently a new possibility was proposed, i.e., that of internal cancellations among the various components contributing to the EDMs which allows for the existence of large CP violating phases, with a SUSY spectrum which is not excessively heavy and is thus accessible at colliders in the near future. CP violating phases $O(1)$ are attractive because they circumvent the naturalness problem associated with small phases or a heavy SUSY spectrum. The EDM analysis of Ref. was for the minimal supergravity model which has only two CP violating phases. The analysis of Ref. was extended in Ref. to take account of all allowed CP violating phases in the Minimal Supersymmetric Standard Model (MSSM) with no generational mixing. This extension gives the possibility of the cancellation mechanism to occur over a much larger region of the parameter space allowing for large CP violating phases over this region. Indeed a general numerical analysis bears this out.

Large CP violating phases can affect significantly dark matter analyses and other phenomena at low energy. A detailed analysis in Ref. shows that large CP violating phases consistent with the cosmological relic density constraints and the EDM constraints using the cancellation mechanism are indeed possible. CP violating phases affect event rates in dark matter detectors and a partial analysis of these effects with two CP phases and without the EDM constraint was given in Ref. Thus, currently there are no analyses where the effect of large CP violating phases on event rates and the simultaneous satisfaction of
the EDM constraints via the cancellation mechanism are discussed. Further, the previous analyses are all limited to two CP violating phases while supergravity models with non-universalities and MSSM can possess many more phases\textsuperscript{15}. In this paper we give the first complete analytic analysis of the effects of CP violation on event rates with the inclusion of all CP violating phases allowed in the minimal supersymmetric standard model (MSSM) when intergenerational mixings are ignored. We then give a numerical analysis of the CP violating effects on event rates with the inclusion of the EDM constraints. It is shown that while the effects of CP violating phases on event rates are significant with the inclusion of the EDM constraints, they are not enormous as for the case when the EDM constraints are ignored.

We discuss now the details of the analysis in MSSM. The nature of the LSP at the electro-weak scale is determined by the neutralino mass matrix which in the basis $(\tilde{B}, \tilde{W}, \tilde{H}_1, \tilde{H}_2)$, where $\tilde{B}$ and $\tilde{W}$ are the U(1) and the neutral SU(2) gauginos, is given by

$$
\begin{pmatrix}
|m_1|e^{i\xi_1} & 0 & -M_Z \sin \theta_W \cos \beta e^{-i\chi_1} & M_Z \sin \theta_W \sin \beta e^{-i\chi_2} \\
0 & |m_2|e^{i\xi_2} & M_Z \cos \theta_W \cos \beta e^{-i\chi_1} & -M_Z \cos \theta_W \sin \beta e^{-i\chi_2} \\
-M_Z \sin \theta_W \cos \beta e^{-i\chi_1} & M_Z \cos \theta_W \cos \beta e^{-i\chi_2} & 0 & -|\mu|e^{i\theta_\mu} \\
M_Z \sin \theta_W \sin \beta e^{-i\chi_1} & -M_Z \cos \theta_W \sin \beta e^{-i\chi_2} & -|\mu|e^{i\theta_\mu} & 0
\end{pmatrix}
$$

(1)

Here $M_Z$ is the Z boson mass, $\theta_W$ is the weak angle, $\tan \beta = |v_2/v_1|$ where $v_i = <H_i>$ = $|v_i| e^{i\chi_i}$ (i=1,2) where $H_2$ is the Higgs that gives mass to the up quarks and $H_1$ is the Higgs that gives mass to the down quarks and the leptons, $\mu$ is the Higgs mixing parameter (i.e. it appears in the superpotential as the term $\mu H_1 H_2$), $m_1$ and $m_2$ are the masses of the U(1) and SU(2) gauginos at the electro-weak scale with $\xi_1$ and $\xi_2$ being their phases.

The neutralino mass matrix of Eq. (1) contains several phases. However, it can be shown\textsuperscript{15} that the eigenvalues and the eigenvectors of the neutralino mass matrix depend on only two combinations: $\theta = \xi_1 + \xi_2 + \chi_1 + \chi_2 + \theta_\mu$ and $\Delta \xi = (\xi_1 - \xi_2)$. The neutralino matrix can be diagonalized by a unitary matrix $X$ such that

$$
X^T M^a \chi X = \text{diag}(\tilde{m}_{\chi_1}^a, \tilde{m}_{\chi_2}^a, \tilde{m}_{\chi_3}^a, \tilde{m}_{\chi_4}^a)
$$

(2)
We shall denote the LSP by the index 0 so that

$$\chi^0 = X_{10}^* \tilde{B} + X_{20}^* \tilde{W} + X_{30}^* \tilde{H}_1 + X_{40}^* \tilde{H}_2$$

(3)

The basic interactions that enter in the scattering of the LSP from nuclei are the neutralino-squark-quark interactions in the s channel and the neutralino-neutralino-Z(Higgs) interactions in the cross channel. The squark mass matrix $M^2_\tilde{q}$ involves both the phases of $\mu$ and of the trilinear couplings as given below

$$
\begin{pmatrix}
M^2_\tilde{Q} + m_q^2 + M^2_\tilde{z} (\frac{1}{2} - Q_q \sin^2 \theta_W) \cos 2\beta & m_q (A^*_q m_0 - \mu R_q) \\
m_q (A_q m_0 - \mu^* R_q) & M^2_U + m_q^2 + M^2_\tilde{z} Q_q \sin^2 \theta_W \cos 2\beta
\end{pmatrix}
$$

(4)

Here $Q_u = 2/3(-1/3)$ for $q=u(d)$ and $R_q = v_1/v_2 (v_2/v_1)$ for $q=u(d)$, and $m_q$ is the quark mass. The squark matrix is diagonalized by $D_{qij}$ such that

$$D^\dagger_q M^2_\tilde{q} D_q = \text{diag}(M^2_{\tilde{q}1}, M^2_{\tilde{q}2})$$

(5)

As mentioned in the introduction the relative velocities of the LSP hitting the targets are small, and consequently we can approximate the effective interaction governing the neutralino-quark scattering by an effective four-fermi interaction. We give now the result of our analysis including all the relevant CP violating effects in a softly broken MSSM. The effective four fermi interaction is given by

$$L_{eff} = \bar{\chi} \gamma_\mu \gamma_5 \chi \bar{q} \gamma^\mu (A P_L + B P_R) q + C \bar{\chi} \chi m_q \bar{q} q + D \bar{\gamma}_5 \chi m_q \bar{q} \gamma q q$$

$$+ E \bar{\gamma}_5 \chi m_q \bar{q} q + F \bar{\chi} \chi m_q \bar{q} \gamma q q$$

(6)

where our choice of the metric is $\eta_{\mu\nu} = (1, -1, -1, -1)$. The deduction of Eq.(6) starting from the microscopic supergravity Lagrangian is given in Appendix A. Here we give the results. The first two terms ($A, B$) are spin-dependent interactions and arise from the $Z$ boson and the sfermion exchanges. For these our analysis gives

$$A = \frac{g^2}{4M^2_W} [\vert X_{30} \vert^2 - \vert X_{40} \vert^2] [T_{3q} - e_q \sin^2 \theta_W] - \frac{|C_q R|^2}{4(M^2_{q1} - M^2_\chi)} - \frac{|C'_q R|^2}{4(M^2_{q2} - M^2_\chi)}$$

(7)

$$B = -\frac{g^2}{4M^2_W} [\vert X_{30} \vert^2 - \vert X_{40} \vert^2] e_q \sin^2 \theta_W + \frac{|C_q L|^2}{4(M^2_{q1} - M^2_\chi)} + \frac{|C'_q L|^2}{4(M^2_{q2} - M^2_\chi)}$$

(8)
where

\[ C_{qL} = \sqrt{2}(\alpha q q_0 D_{q11} - \gamma q q_0 D_{q21}) \]  

\[ C_{qR} = \sqrt{2}(\beta q q_0 D_{q11} - \delta q q_0 D_{q21}) \]  

\[ C'_{qL} = \sqrt{2}(\alpha q q_0 D_{q12} - \gamma q q_0 D_{q22}) \]  

\[ C'_{qR} = \sqrt{2}(\beta q q_0 D_{q12} - \delta q q_0 D_{q22}) \]  

and where \( \alpha, \beta, \gamma, \) and \( \delta \) are given by

\[ \alpha_{u(d)j} = \frac{g m_{u(d)} X_{4(3)j}}{2 m_W \sin \beta (\cos \beta)} \]  

\[ \beta_{u(d)j} = e Q_{u(d)j} X'_{1j}^* + \frac{g}{\cos \theta_W} X'_{2j}^* (T_{3u(d)} - Q_{u(d)} \sin^2 \theta_W) \]  

\[ \gamma_{u(d)j} = e Q_{u(d)j} X'_{1j} - \frac{g Q_{u(d)} \sin^2 \theta_W}{\cos \theta_W} X'_{2j} \]  

\[ \delta_{u(d)j} = -\frac{g m_{u(d)} X'^*_{4(3)j}}{2 m_W \sin \beta (\cos \beta)} \]  

Here \( g \) is the \( SU(2)_L \) gauge coupling and

\[ X'_{1j} = X_{1j} \cos \theta_W + X_{2j} \sin \theta_W \]  

\[ X'_{2j} = -X_{1j} \sin \theta_W + X_{2j} \cos \theta_W \]  

The effect of the CP violating phases enter via the neutralino eigen-vector components \( X'_{ij} \) and via the matrix \( D_{qij} \) that diagonalizes the squark mass matrix.

The \( C \) term in Eq.(6) represents the scalar interaction which gives rise to coherent scattering. It receives contributions from the sfermion exchange, from the CP even light Higgs \( (h^0) \) exchange, and from the CP even heavy Higgs \( (H^0) \) exchange. Thus
\( C = C_f + C_{h^0} + C_{H^0} \)  

where

\[
C_f(u, d) = - \frac{1}{4m_q} \frac{1}{M^2_{q1} - M^2_x} \text{Re}[C_{qL}C_{qR}^*] - \frac{1}{4m_q} \frac{1}{M^2_{q2} - M^2_x} \text{Re}[C'_{qL}C'_{qR}^*] 
\]

\[
C_{h^0}(u, d) = -(+) \frac{g^2}{4M_W M^2_{h^0}} \frac{\cos \alpha (\sin \alpha)}{\sin \beta \cos \beta} \text{Re} \sigma 
\]

\[
C_{H^0}(u, d) = \frac{g^2}{4M_W M^2_{H^0}} \frac{\sin \alpha (\cos \alpha)}{\sin \beta \cos \beta} \text{Re} \rho 
\]

Here \( \alpha \) is the Higgs mixing angle, \((u,d)\) indicate the flavor of the quark involved in the scattering, and \( \sigma \) and \( \rho \) are defined by

\[
\sigma = X^*_40(X^*_20 - \tan \theta_W X^*_10) \cos \alpha + X^*_30(X^*_20 - \tan \theta_W X^*_10) \sin \alpha 
\]

\[
\rho = -X^*_40(X^*_20 - \tan \theta_W X^*_10) \sin \alpha + X^*_30(X^*_20 - \tan \theta_W X^*_10) \cos \alpha 
\]

The last three terms D, E and F in eq.(6) are given by

\[
D(u, d) = C_f(u, d) + \frac{g^2}{4M_W} \frac{\cot \beta (\tan \beta)}{m^2_{\Delta^0}} \text{Re} \omega 
\]

\[
E(u, d) = T_f(u, d) + \frac{g^2}{4M_W} [- (+) \frac{\cos \alpha (\sin \alpha)}{\sin \beta \cos \beta} \frac{\text{Im} \sigma}{m^2_{h^0}} + \frac{\sin \alpha (\cos \alpha)}{\sin \beta \cos \beta} \frac{\text{Im} \rho}{m^2_{H^0}}] 
\]

\[
F(u, d) = T_f(u, d) + \frac{g^2}{4M_W} \frac{\cot \beta (\tan \beta)}{m^2_{\Delta^0}} \text{Im} \omega 
\]

where \( \Delta^0 \) is the CP odd Higgs and where \( \omega \) is defined by

\[
\omega = -X^*_40(X^*_20 - \tan \theta_W X^*_10) \cos \beta + X^*_30(X^*_20 - \tan \theta_W X^*_10) \sin \beta 
\]

and

\[
T_f(q) = \frac{1}{4m_q} \frac{1}{M^2_{q1} - M^2_x} \text{Im}[C_{qL}C_{qR}^*] + \frac{1}{4m_q} \frac{1}{M^2_{q2} - M^2_x} \text{Im}[C'_{qL}C'_{qR}^*] 
\]
In the limit of vanishing CP violating phases our results of A, B and C limit to the result of reference \cite{1}. In that limit we have a difference of a minus sign in the Z-exchange terms of Ref.\cite{6} in their equations (2a, 2b, A1). Further, in the same limit of vanishing CP violating phases our results go to those of Ref.\cite{5} with an overall minus sign difference in the three exchange terms, i.e., the Z, the sfermions and the higgs terms. (see Appendix B for details).

To compare our results to those of reference \cite{21} we note that the analysis of reference \cite{21} was limited to the case of two CP violating phases and it gave the analytic results for only some of the co-efficients in the low energy expansion of Eq.(6). The terms E and F given by Eqs.(26) and (27) are new and vanish in the limit when CP is conserved. The term D given by Eq.(25) is non-vanishing in the limit when CP phases vanish. However, this term is mostly ignored in the literature as its contribution is suppressed because of the small velocity of the relic neutralinos. In fact the contributions of D,E and F are expected to be relatively small and we ignore them in our numerical analysis here.

For the computation of the event rates from nuclear targets in the direct detection of dark matter we follow the techniques discussed in Ref.\cite{8} and we refer the reader to it for details. We give now the numerical estimates of the CP violating effects on event rates. First we consider the case when the EDM constraint is not imposed. In Fig.1 we exhibit the ratio $R/R(0)$ for Ge where R is the event rate with CP violation arising from a non-vanishing $\theta_{\mu}$ and R(0) is the event rate in the absence of CP violation. The figure illustrates that the effect of the CP violating phase can be very large. In fact, as can be seen from Fig.1 the variations can be as large as 2-3 orders of magnitude. However, as noted above the EDM constraint was not imposed here. Next we give the analysis with inclusion of the EDM constraints. For this purpose we work in the parameter space where the cosmological relic density constraint and the EDM constraints are simultaneously satisfied and we compute the ratio $R/R(0)$ for Ge in this region. Specifically the satisfaction of the relic density and the EDM constraints is achieved by varying the parameters of the theory. The satisfaction of the EDM constraint is achieved through the cancellation mechanism discussed in Refs.\cite{14,15}. The result of this analysis is exhibited in Fig.2. Here we find the range of variation of $R/R(0)$
with $\theta_\mu$ to be much smaller although still substantial. Thus from Fig.2 we find that the variation of $R/R(0)$ has a range of up to a factor of 2 over most of the allowed parameter space satisfying the relic density and the EDM constraints in the regions of the parameter investigated. This variation is substantially smaller than the one observed in Fig.1 when the EDM constraints were not imposed.

In conclusion, we have given in this paper the first complete analytic analysis of the effects of CP violating phases on the event rates in the direct detection of dark matter within the framework of MSSM with no generational mixing. We find that the CP violating effects can generate variations in the event rates up to 2-3 orders of magnitude. However, with the inclusion of the EDM constraints the effects are much reduced although still significant in that the variations could be up to a factor $\sim 2$ as seen from the analysis over the region of the parameter space investigated. Of course the parameter space of MSSM is quite large and there may exist other regions of the parameter space of MSSM where the CP violating effects on event rates consistent with the EDM constraints are even larger.

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Fig. 1. Plot of $R/R(0)$ for Ge as a function of the CP violating phase $\theta_\mu$ in the MSSM case when $\tan\beta=2$, $m_0=100$ GeV, $|A_0|=1$ for the cases when the gluino mass is 500 GeV (solid curve), 600 GeV (dotted curve), and 700 GeV (dashed curve).

Fig. 2. Plot of $R/R(0)$ for Ge as a function of the CP violating phase $\theta_\mu$ for the parameter space discussed in the text. The parameter space spans regions satisfying the relic density and the EDM constraints obtained by varying other parameters in the theory.
1. Appendix A

In this appendix we give a derivation of the four fermi neutralino-quark effective Lagrangian with CP violating phases given in the text.

A. squark exchange terms

From the microscopic Lagrangian of quark-squark-neutralino

\[ -\mathcal{L} = \bar{q}[C_{qL}P_L + C_{qR}P_R]\chi \bar{q}_1 + \bar{q}[C'_{qL}P_L + C'_{qR}P_R]\chi \bar{q}_2 + H.c. \]  \hspace{1cm} (30)

the effective lagrangian for \( q - \chi \) scattering via the exchange of squarks is given by

\[ \mathcal{L}_{\text{eff}} = \frac{1}{M_{\tilde{q}_1}^2 - M_\chi^2}\bar{\chi}[C^*_{qL}P_R + C^*_{qR}P_L]q\bar{q}[C_{qL}P_L + C_{qR}P_R]\chi \]

\[ + \frac{1}{M_{\tilde{q}_2}^2 - M_\chi^2}\bar{\chi}[C'_{qL}P_R + C'_{qR}P_L]q\bar{q}[C'_{qL}P_L + C'_{qR}P_R]\chi \]  \hspace{1cm} (31)

We use Fierz reordering to write the Lagrangian in terms of the combinations \( \bar{\chi}\chi\bar{q}q \), \( \bar{\chi}\gamma^5\chi\bar{q}\gamma_q \), \( \bar{\chi}\gamma^\mu\gamma^\nu\chi\bar{q}_\mu\gamma_q \), \( \bar{\chi}\gamma^\mu\gamma^\nu\chi\bar{q}_\mu\gamma_q \), \( \bar{\chi}\gamma^\mu\gamma^\nu\chi\bar{q}_\mu\gamma^5_q \) and \( \bar{\chi}\chi\gamma^5\gamma_q \). For this purpose, we define the 16 matrices

\[ \Gamma^A = \{1, \gamma^0, i\gamma^i, i\gamma^0\gamma_5, \gamma^i\gamma_5, \gamma_5, i\sigma^{0i}, \sigma^{ij}\} : \ i, j = 1 - 3 \]  \hspace{1cm} (32)

with the normalization

\[ tr(\Gamma^A\Gamma^B) = 4\delta^{AB} \]  \hspace{1cm} (33)

The Fierz rearrangement formula with the above definitions and normalizations is

\[ (\bar{u}_1\Gamma^A u_2)(\bar{u}_3\Gamma^B u_4) = \sum_{C,D} F_{CD}^{AB}(\bar{u}_1\Gamma^C u_4)(\bar{u}_3\Gamma^D u_2) \]  \hspace{1cm} (34)

where \( u_j \) are Dirac or Majorana spinors and

\[ F_{CD}^{AB} = -(+) \frac{1}{16} tr(\Gamma^C\Gamma^A\Gamma^D\Gamma^B) \]  \hspace{1cm} (35)

and where the +ve sign is for commuting u spinors and the -ve sign is for the anticommuting u fields. In our case we have to use the -ve sign since we are dealing with quantum Majorana
and Dirac fields in the Lagrangian. We give below the Fierz rearrangement of the four combinations that appear in Eq.(31) above:

$$\bar{\chi}q\bar{q} = -\frac{1}{4}\bar{\chi}\chi q - \frac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}\gamma_5 q + \frac{1}{4}\bar{\chi}\gamma_\mu\gamma_5\chi\bar{q}\gamma_\mu\gamma_5 q$$

$$\bar{\chi}\gamma_5q\bar{q} = \frac{1}{4}\bar{\chi}\gamma_\mu\gamma_5\chi\bar{q}\gamma_\mu q - \frac{1}{4}\bar{\chi}\chi\bar{q}\gamma_5 q - \frac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}$$

$$\bar{\chi}q\gamma_5\chi = -\frac{1}{4}\bar{\chi}\gamma_\mu\gamma_5\chi\bar{q}\gamma_\mu q - \frac{1}{4}\bar{\chi}\chi\bar{q}\gamma_5 q - \frac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}$$

$$\bar{\chi}\gamma_5q\gamma_5\chi = -\frac{1}{4}\bar{\chi}\chi q - \frac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}\gamma_5 q - \frac{1}{4}\bar{\chi}\gamma_5\chi\bar{q}\gamma_5 q$$

In the above we have used the metric $\eta_{\mu\nu} = (1, -1, -1, -1)$, and we also used the fact that $\chi$ is a Majorana field so that $\bar{\chi}\gamma_\mu\chi = 0$ and $\bar{\chi}\sigma_{\mu\nu}\chi = 0$. By rearranging the terms to be in the form of $L_{eff}$ of Eq.(6) we can find out directly the squark contributions to $A, B, C, D, E$ and $F$ terms as given in the text of the paper.

B. Z boson exchange

From the $q - Z - q$ and $\chi - Z - \chi$ interactions in Eqs. (c62) and (c87a) of Ref. 22, we obtain the following effective Lagrangian for the $q - \chi$ scattering via $Z$-exchange:

$$L_{eff} = \frac{g^2}{4M_Z^2\cos^2\theta_W}\left[|X_{30}|^2 - |X_{40}|^2\right]\bar{q}\gamma_{\mu}[d_{qL}P_L + d_{qR}P_R]q\bar{\chi}\gamma_\mu\gamma_5\chi$$

where $d_{qL} = T_{3L} - e_q\sin^2\theta_W$ and $d_{qR} = -e_q\sin^2\theta_W$. From Eq.(37) we can read off the contribution to $A$ and $B$ from the $Z$ exchange. These contributions are given in Eqs.(7) and (8).

C. Higgs exchange terms

Higgs exchange will contribute to $C, D, E$ and $F$ terms. From the interaction Lagrangian of $L_{H\chi\chi}$ and $L_{Hq\bar{q}}$ in Eqs. (4.47) and (4.10) respectively of Ref. 23, one can get the effective Lagrangian for $q - \chi$ scattering via $h^0, H^0$ and $A^0$ exchanges. In our formalism we use $h^0, H^0$ and $A^0$ for the light, heavy and CP-odd neutral higgs. There are six contributions: three higgs exchange terms for the up flavor and three for the down flavor. To illustrate we choose
the up quark scattering with $\chi$ via the exchange of the heavy higgs $H^0$ ($H^0_1$ in the notation of Ref.\textsuperscript{23}):

$$\mathcal{L}_{\text{eff}} = \frac{1}{m^2_{H^0}} (J^1_{H^0} + J^2_{H^0}) I^u_{H^0}$$

(38)

where

$$J^1_{H^0} = -\frac{g}{2} \cos \alpha \bar{\chi} (Q''_{00}^* P_L + Q''_{00} P_R) \chi$$

$$J^2_{H^0} = \frac{g}{2} \sin \alpha \bar{\chi} (S''_{00}^* P_L + S''_{00} P_R) \chi$$

$$I^u_{H^0} = -\frac{g m_u \sin \alpha}{2 M_w \sin \beta} \bar{u} u$$

(39)

where $Q''_{00}$, $S''_{00}$ are as defined in Ref.\textsuperscript{23}. Defining $\rho$ by

$$\rho = Q''_{00} \cos \alpha - S''_{00} \sin \alpha$$

(40)

we get the $H^0$ contribution to $\mathcal{L}_{\text{eff}}$:

$$\mathcal{L}_{\text{eff}} = \frac{g^2 m_u \sin \alpha}{4 M_w m^2_{H^0} \sin \beta} Re(\rho) \bar{\chi} \chi \bar{u} u$$

$$+ \frac{i g^2 m_u \sin \alpha}{4 M_w m^2_{H^0} \sin \beta} Im(\rho) \bar{\chi} \gamma_5 \chi \bar{u} u$$

(41)

From Eq.(41) we can read off directly the contributions $C_{H^0}(u)$ and $E_{H^0}(u)$ as given by Eqs.(22) and (26).
2. Appendix B

Here we compare our results with those of Ref. [5] which is in the limit of no CP violation, no sfermion mixing and no heavy Higgs. In the limit of no CP violation and no sfermion mixing \(C_L, C'_L, C_R, C'_R\) given by Eqs.(9-12) in the text reduce to the following:

\[
\begin{align*}
C_L &= \sqrt{2} \alpha q_0, \quad C'_L = -\sqrt{2} \gamma q_0 \\
C_R &= \sqrt{2} \beta q_0, \quad C'_R = -\sqrt{2} \delta q_0
\end{align*}
\]

To express the above in the notation of Ref. [5] we set \(X^*_{10} = \beta\), \(X^*_{20} = \alpha\), \(X^*_{30} = \delta\), and \(X^*_{40} = \gamma\) keeping in mind that the \(H_1\) and \(H_2\) of Ref. [5] are defined oppositely to our notation.

Using the above along with Eqs.(13-16) we find for \(T_3 = \frac{1}{2}(-\frac{1}{2})\)

\[
\begin{align*}
|C_L|^2 &= \frac{m_a^2(m_d^2)\gamma^2(\delta^2)}{\nu_1^2(\nu_2^2)} \\
|C'_L|^2 &= 2(g_1 Y_R / 2 \beta)^2 \\
|C_R|^2 &= 2(\alpha g_2 T_3 + \beta g_1 Y_L / 2)^2 \\
|C'_R|^2 &= \frac{m_a^2(m_d^2)\gamma^2(\delta^2)}{\nu_1^2(\nu_2^2)}
\end{align*}
\]

where \(Y_L\) is the hypercharge, \(\nu_1 = < H_1 >\), and \(\nu_2 = < H_2 >\) and where \(H_1\) and \(H_2\) are in the notation of Ref. [5]. Further using the identity

\[
g_2^2 \frac{c_\theta W}{c_\theta W} (T_3 - e_q \sin^2 \theta W) = -(g_1 \sin \theta W + g_2 \cos \theta W)(\frac{1}{2} Y_L g_1 \sin \theta W - T_3 g_2 \cos \theta W)
\]

where the left hand side of Eq.(44) is written in the form used in Ref. [5], we can express A and B in the limit of no CP violation and no sfermion mixing as follows: for \(T_3 = \frac{1}{2}(-\frac{1}{2})\)

\[
\begin{align*}
A &= \frac{(\gamma^2 - \delta^2)}{4M_Z^2}(g_1 \sin \theta W + g_2 \cos \theta W)(\frac{1}{2} Y_L g_1 \sin \theta W - T_3 g_2 \cos \theta W) \\
&\quad - (\alpha g_2 T_3 + \beta g_1 Y_L / 2)^2 - \frac{m_a^2(m_d^2)\gamma^2(\delta^2)}{4(M_{4R}^2 - M_{4L}^2)\nu_1^2(\nu_2^2)}
\end{align*}
\]

\[
\begin{align*}
B &= \frac{(\gamma^2 - \delta^2)}{4M_Z^2}(g_1 \sin \theta W + g_2 \cos \theta W)(\frac{1}{2} Y_L g_1 \sin \theta W - T_3 g_2 \cos \theta W) \\
&\quad + \frac{(g_1 Y_R / 2 \beta)^2}{2(M_{4R}^2 - M_{4L}^2)} + \frac{m_a^2(m_d^2)\gamma^2(\delta^2)}{4(M_{4R}^2 - M_{4L}^2)\nu_1^2(\nu_2^2)}
\end{align*}
\]
To compare our $C$ term with that of Ref. 5 we again go to the limit of vanishing CP violating phases, assume no sfermion mixing, and in addition ignore the heavy Higgs exchange contribution (i.e., the term $C_{H^0}$ of Eq.(22) in the text). Then using similar notational changes as above we find that our $C$ under the approximations made in Ref. 5 is given by

$$C = \frac{g_2^2}{4M_Wm_{h^0}^2} \frac{(\cos \alpha)(\sin \alpha)}{(\sin \beta)(\cos \beta)} (\alpha - \beta \tan \theta_W)(\gamma \cos \alpha + \delta \sin \alpha)$$

$$- \frac{g_2}{4M_W} \left( \frac{\alpha g_2 T_3 + \beta g_1 Y_{1/2}}{M_{\tilde{q}_L}^2 - M_{\tilde{\chi}}^2} - \frac{\beta g_1 Y_{1/2}}{M_{\tilde{q}_R}^2 - M_{\tilde{\chi}}^2} \right) \frac{\gamma(\delta)}{\sin \beta(\cos \beta)}; \quad T_3 = \frac{1}{2}(-\frac{1}{2})$$

Comparing our results for A, B and C with those of Ref. 5 we find that our Z, sfermion and Higgs exchange terms have an overall minus sign relative to those of Ref. 5.
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16. The following corrections should be included in Refs. 14, 15: In the first paper of Ref. 14 a minus sign should be inserted on the right hand side of Eq.(16). In the expressions of $\eta_{qik}$ (or $\eta_{fik}$) in Refs. 14, 15 the first $\kappa_q$ (or $\kappa_f$) should be preceded by a minus sign.

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Eq.(5.5) of ref.23 which leads to the negative sign on the right hand side in Eq.(16).