NUMERICAL EXPERIMENTS ON THE DETAILED ENERGY CONVERSION AND SPECTRUM STUDIES IN A CORONA CURRENT SHEET

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ABSTRACT

In this paper, we study the energy conversion and spectra in a corona current sheet (CS) by 2.5 dimensional MHD numerical simulations. Numerical results show that many Petschek-like fine structures with slow-mode shocks mediated by plasmoid instabilities develop during the magnetic reconnection process. The termination shocks can also be formed above the primary magnetic island and at the head of secondary islands. These shocks play important roles in generating thermal energy in a corona CS. For a numerical simulation with initial conditions close to the solar corona environment, the ratio of the generated thermal energy to the total dissipated magnetic energy is around 1/5 before secondary islands appear. After secondary islands appear, the generated thermal energy starts to increase sharply and this ratio can reach a value of about 3/5. In an environment with a relatively lower plasma density and plasma β, the plasma can be heated to a much higher temperature. After secondary islands appear, the one-dimensional energy spectra along the CS do not behave as a simple power law and the spectrum index increases with the wave number. The average spectrum index for the magnetic energy spectrum along the CS is about 1.8. The two-dimensional spectra intuitively show that part of the high energy is cascaded to large kx and ky space after secondary islands appear. The plasmoid distribution function calculated from numerical simulations behaves as a power law closer to \( f(\psi) \sim \psi^{-5} \) in the intermediate \( \psi \) regime. By using \( \eta_{\text{eff}} = \frac{V_{\text{inflow}}}{L} \), the effective magnetic diffusivity is estimated to be about \( 10^{11} \sim 10^{12} \text{m}^2\text{s}^{-1} \).

Key words: magnetic reconnection – methods: numerical – magnetohydrodynamics (MHD) – shock waves – Sun: corona

1. INTRODUCTION

An elongated current sheet (CS) attached above the flare loop top (Sui & Holman 2003; Lin et al. 2005; Liu et al. 2010) is usually observed in an eruptive solar flare. Magnetic reconnection inside CSs plays an important role in releasing and converting the magnetic energy to plasma thermal and kinetic energy. Reconnection inflows and high speed outflows in CSs have been recognized by a lot of observations (Wang et al. 2007; Savage et al. 2012; Takasao et al. 2012). The speed of outflows ranges from 100 to 1000 km s\(^{-1}\). Many studies show that the outflow speed is around 100–450 km s\(^{-1}\) (Asai et al. 2004; Savage & McKenzie 2011; Takasao et al. 2012; Liu et al. 2013; Zhang & Ji 2014). The maximum outflow velocity can reach 1000 km s\(^{-1}\) (Innes et al. 2003; Liu et al. 2013), which is close to the Alfvén velocity in the solar corona. The temperature of the plasmas related to a magnetic reconnection process can be around 3 MK ~ 20 MK. The 3 MK plasma was observed above the post-flare loops by Landi et al. (2012) in EUV with Hinode/Extreme-ultraviolet Imaging Spectrometer spectra. Using the Solar and Heliospheric Observatory Ultraviolet Coronagraph Spectrometer data, Ciaravella & Raymond (2008) observed plasmas with temperature around 6 MK inside the CS region. The bright blob with hot plasma in the Atmospheric Imaging Assembly 131 Å passband was seen with peak temperature ~11 MK (Cheng et al. 2011). Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI) X-ray spectra and images simultaneously show that the plasma has been heated to >10 MK in a solar flare in the paper by Su et al. (2013), and Susino et al. (2013) even observed the hot loop plasmas with peak temperature reaching ~22 MK in RHESSI hard X-ray spectra. In the paper by Sun et al. (2014), the RHESSI hard X-ray emissions in Figure 6(b) indicate that some local heating is located at the reconnection site. The local heating regions possibly correspond to some fine structures such as magnetic islands within the CS. They also predict that the sharp change of temperature across the CS in Figure 7(f) in their paper is probably a signature of a slow shock. The generation mechanism of the high temperature emission observed by solar telescopes is still not understood well. Susino et al. (2013) proposed that both Petschek-like reconnection and turbulent reconnection can possibly explain the hot CS plasma.

The Petschek-like reconnection model can be used to explain some observational evidence of white-light, UV, and X-ray emissions (Ko et al. 2010). However, a non-uniform resistivity, which should be enhanced at the X-point, is required to produce steady-state Petschek reconnection (see Kulsrud 2001 and references therein). The physical mechanism of the origin of the enhanced resistivity is still unknown. Birn et al. (2009) studied the energy conversion mechanism by the three-dimensional MHD magnetic reconnection model; they concluded that Petschek reconnection accelerates plasma to convert magnetic energy to bulk kinetic energy, and then the accelerated plasma slows down and the bulk kinetic energy is transformed to heat. The hottest plasma can reach around 40 MK in their simulation. However, they assumed a non-uniform resistivity, and the corresponding Lundquist number at the X-point is only 200, which is much smaller than that in the solar corona (\( \gtrsim 10^{12} \)). Both radiative cooling and heat conduction effects are not included in their model.

Fast magnetic reconnection that develops in a turbulent plasma is expected to produce hot plasma as observed by UCVS and X-ray telescope images (Bemporad 2008). Several types of reconnection models in turbulent plasma are proposed, e.g., stochastic reconnection (Lazarian & Vishniac 1999), fractal reconnection (Shibata & Tanuma 2001), or plasmoid instabilities...
These models have been studied analytically and numerically over the past few years, and some common characteristics were found. In a high Lundquist number environment, numerous observations (Lin et al. 2008; Milligan et al. 2010; Takasaki et al. 2012; Kumar & Cho 2013; Liu 2013), numerical simulations (Bhattacharjee et al. 2009; Samtaney et al. 2009; Mei et al. 2012; Ni et al. 2012a), and even laboratory experiments (Dong et al. 2012) show that multiple levels of plasmoids can always occur in an unstable magnetic reconnection process. Numerical simulations have demonstrated that the reconnection rate $\gamma$ can be greatly increased to a high value ($\sim 0.01$) (Huang & Bhattacharjee 2010) after multiple levels of plasmoid instabilities appear, and $\gamma$ weakly depends on the Lundquist number. This value is very close to the reconnection rate measured by observations (Ko et al. 2010). However, few numerical simulations focus on the generation of hot plasma inside the CS during plasmoid instabilities. In the paper by Ni et al. (2012a), the plasma density ($\sim 10^{18}/m^3$) is set much higher than that in the corona environment, and the temperature only increases from 1 MK to around 2 MK.

Spectrum studies are important for understanding the energy conversion mechanism and dynamics of plasmoids during plasmoid instabilities. In the papers by Ni et al. (2012b, 2013), Shen et al. (2013), and Bárta et al. (2011b), one-dimensional magnetic and kinetic energy spectra along the CS have been studied in detail. The results show that the spectrum does not have a single-power-law form for both magnetic and kinetic energy—the spectral index increases with the wave number after secondary islands appear. The average spectral index for magnetic energy is around 2.0 and it is around 3.0 for kinetic energy in previous studies (Ni et al. 2012b, 2013). These values are larger than the Kolmogorov spectral index ($\sim 5/3$). The plasmoid distribution function has also been studied in detail in some previous papers, e.g., (Fermo et al. 2010; Uzdensky et al. 2010; Huang & Bhattacharjee 2012; Loureiro et al. 2012; Guo et al. 2013; Shen et al. 2013). Their results also demonstrate that the spectrum of the distribution function does not behave as a single power law.

In this work, we study the energy conversion during plasmoid instabilities in the solar corona CS. The physical parameters such as the initial temperature, density, and strength of the magnetic field in the simulations are close to those in the real corona environment. The evolution of the reconnection rate, temperature, and velocity of the plasma inside the CS region has been analyzed. However, the energy conversion process in a flare CS has already been studied in detail in some previous papers (e.g., Reeves et al. 2010). Compared with these previous studies, a more realistic temperature-dependent high Lundquist number ($\gtrsim 10^6$) and much higher resolutions are used in our models. Therefore, we can discover many fine structures related to the heating mechanism inside the CS region. Both the one- and two-dimensional spectra have also been studied carefully in this work. In Section 2, we present the numerical approach and initial states of our simulations. The results are presented in Section 3. We summarize our results and give discussions in Section 4.

2. FRAMEWORK OF NUMERICAL MODELS

The 2.5 dimensional one-fluid MHD model is used in this work. We only consider the fully ionized hydrogen gas and the plasma are composed with electrons and ions. The temperature of the two species is considered to be the same ($T_i = T_e = T$).

The MHD equations in our simulations are given by:

$$\partial_t \rho = - \nabla \cdot (\rho v),$$

$$\partial_t e = - \nabla \cdot \left[ (e + p + \frac{1}{2} |B|^2) v - \frac{1}{\mu_0} (v \cdot B) B \right] + \nabla \cdot \left( v \frac{\eta}{\mu_0} B \times (\nabla \times B) \right) + L_{\text{rad}} + H,$$

$$\partial_t (\rho v) = - \nabla \cdot \left( \rho v v + \frac{1}{2} |B|^2 \right) + \nabla \cdot \frac{\tau}{\rho},$$

$$\partial_t B = \nabla \times (v \times B - \eta \nabla \times B),$$

$$e = \frac{p}{\Gamma_0 - 1} + \frac{1}{2} \rho v^2 + \frac{1}{2} \mu_0 B^2,$$

$$\rho = \frac{2 \rho_i p_i T_i}{m_i},$$

$$\tau = \nu \left( \nabla v + (\nabla v)^T - \frac{2}{3} (\nabla \cdot v) I \right).$$

where $\rho$ is the plasma mass density, $v$ is the center of mass velocity, $e$ is the total energy density, $B$ is the magnetic field, $\tau$ is the stress tensor, $m_i$ is the mass of a hydrogen ion, $\eta$ is the magnetic diffusivity, $\nu$ is the dynamic viscosity coefficient, and $\rho$ is plasma thermal pressure. The ratio of specific heats $\Gamma_0$ is set to 5/3 (ideal gas). The magnetic permeability coefficient $\mu_0$ is set to $4 \pi \times 10^{-7}$. The radiative cooling $L_{\text{rad}}$ and heating $H$ for fully ionized high temperature plasma are analytically assumed to be (Nagai 1980):

$$L_{\text{rad}} = \begin{cases} 
2.23872 \times 10^{-27} \left( \frac{\rho}{m_i} \right)^2 & T^{-1.385} \\
2.5 \times 10^5 K < T < 10^6 K \\
4.64515 \times 10^{-32} \left( \frac{\rho}{m_i} \right)^2 & T^{-0.604} \\
10^6 K < T < 2 \times 10^7 K \\
1.73380 \times 10^{-39} \left( \frac{\rho}{m_i} \right)^2 & T^{0.413} \\
2 \times 10^7 K
\end{cases}$$

$$H = 4.64515 \times 10^{-32} \left( \frac{\rho}{m_i} \right)^2 T^{-0.604}$$

where $T_0 = 10^6 K$ is the initial temperature in the whole simulation domain at $t = 0$.

The simulation domain extends from $x = 0$ to $x = L_x$ in the $x$-direction and from $y = 0$ to $y = 2L_y$ in the $y$-direction, with $L_0 = 10^8$ m. Open boundary conditions are used in both $x$ and $y$ directions.

We use a force-free CS with a strong guide field in the center as the initial equilibrium distributions of magnetic fields:

$$B_{z0} = 0$$
\[ B_{x0} = b_0 \tanh \left( \frac{x - 0.5L_0}{0.05L_0} \right) \]

\[ B_{z0} = b_0 / \cosh \left( \frac{x - 0.5L_0}{0.05L_0} \right) \]

where \( b_0 = 0.001 \) T. The initial CS width is thus \( \delta_0 = 0.1L_0 \).

Due to the force-freeness and neglect of gravity, the initial equilibrium thermal pressure and plasma \( \beta \) is uniform. In this work, we have simulated two cases, \( \beta = 0.1 \) in case A and \( \beta = 0.05 \) in case B. These yield the initial plasma thermal pressure \( p_0 = 1/(8\pi) \) Pa in case A and \( p_0 = 1/(16\pi) \) Pa in case B. Since the initial equilibrium temperature is \( T_0 = 10^6 \) K in both cases, we find that the initial plasma density \( \rho_0 \approx 2.4 \times 10^{-12} \) kg m\(^{-3}\) and Alfvén velocity \( v_{A0} \approx 580 \) km s\(^{-1}\) in case A, \( \rho_0 \approx 1.2 \times 10^{-12} \) kg m\(^{-3}\) and \( v_{A0} \approx 820 \) km s\(^{-1}\) in case B. Therefore, case A could represent a lower height in the solar corona than that in case B.

The same magnitude (~0.1) of initial perturbations of both magnetic field and velocity are applied at \( t = 0 \) in the two cases to trigger the reconnection process. The forms of perturbations are listed as below:

\[ b_{x\text{pert}} = -\text{pert} \cdot b_0 \cdot \cos \left( \frac{2\pi x}{L_0} \right) \cdot \sin \left( \frac{2\pi (y - 0.5L_0)}{L_0} \right) \]

\[ b_{z\text{pert}} = \text{pert} \cdot b_0 \cdot \sin \left( \frac{2\pi x}{L_0} \right) \cdot \cos \left( \frac{2\pi (y - 0.5L_0)}{L_0} \right) \]

\[ v_{x\text{pert}} = \text{pert} \cdot v_{A0} \cdot \sin \left( \frac{2\pi x}{L_0} \right) \cdot \sin \left( \frac{\pi y}{2L_0} \right) \]

\[ v_{z\text{pert}} = -\text{pert} \cdot v_{A0} \cdot \sin \left( \frac{8\pi y}{L_0} \right) \cdot \frac{\text{random}_n}{\text{Max}([\text{random}_n])} \]

where \( \text{pert} = 0.1, v_{A0} \) is the initial Alfvén velocity as presented in the above paragraph, \( \text{random}_n \) is the random noise function in our code, and \( \text{Max}([\text{random}_n]) \) is the maximum of the absolute value of the random noise function. The perturbations result in a thinning of the CS in two sections between a set of three primary islands, whose midpoints are located at \( y = 0, L_0, \) and \( 2L_0 \) (see Figure 1). In this paper, we will focus only on the section in the domain \( L_0 < y < 2L_0, \) i.e., the bottom half of the box is used as an auxiliary part of the computation only. Its function is to generate a stationary primary plasmoid at the bottom of the height range of interest, which is not influenced by any effects of numerical boundary; the primary plasmoid acts like a line-tied bottom of the CS of interest.

The magnetic diffusivity in the two cases are both assumed to be \( \eta = 8 \times 10^5(T_0^3/T)^{1.5} \). Since the initial temperature is \( 10^6 \) K, the initial magnetic diffusivity is calculated as \( \eta_0 = 8 \times 10^8 \) m\(^2\) s\(^{-1}\). The Lundquist number based on this magnetic diffusivity, \( v_{A0}, \) and \( L_0 \), which corresponds to the “global scale” of the CS in the upper part of the box, is \( \delta_0 = 7.2 \times 10^5 \) in case A and \( \delta_0 = 1.0 \times 10^6 \) in case B. As the plasmoid instabilities develop, the highest temperature at the main X-point can reach around 10 MK in our simulations in case B, and the corresponding Lundquist number is around \( 3.3 \times 10^7 \). Such a Lundquist number is already very high compared with all the previous solar corona magnetic reconnection simulations, even though it is still lower than the Lundquist number in the real solar corona (\( \gtrsim 10^{12} \)). The dynamic viscosity coefficient is assumed to be a constant, \( \nu = 10^{-5} \) kg m\(^{-1}\) s\(^{-1}\). The initial \( \rho_0\eta_0 \) is around \( \sim 10^{-4} \) kg m\(^{-1}\) s\(^{-1}\).

The computations are performed by using the MHD code NIRVANA (version 36; Ziegler 2011). Adaptive mesh refinement is applied. The derivatives-based mesh refinement criterion is used for the mesh refinement. The gradient-based/second-derivatives-based criterion is given by:

\[ \left\{ \begin{array}{l} \alpha \frac{\| \delta U \|_2}{\| U \| + U_{\text{ref}}} + (1 - \alpha) \frac{\| \delta^2 U \|_2}{\| \delta U \|_2 + \text{FIL} \cdot \| (U + U_{\text{ref}}) \|} \\
\quad \times \frac{\delta x^{(U)}}{\delta x^{(U)}} \geq 0.8 \varepsilon_U \quad \forall U \text{ derefinement} \\
\quad \varepsilon_U \geq 0.8 \varepsilon_U \quad \forall U \text{ derefinement} \end{array} \right. \]

where \( U \) is a set of primary variables that the criterion is applied to. The mass density, momentum densities, energy density, and magnetic field can all be chosen as the variable \( U \) to set the criterion in the NIRVANA code. Undivided first (\( \delta U \)) and second (\( \delta^2 U \)) differences of \( U \) are computed is some sort of 2-norm. \( \alpha \in [0, 1] \) (switch between a purely gradient-based criterion when \( \alpha = 1 \) and second-derivatives-based criterion when \( \alpha = 0 \)), \( U_{\text{ref}} \) (reference values), \( \varepsilon_U \) (thresholds, the typical range is \([0.1, 0.5]\)), and \( \xi \) (level dependence) are user-controllable parameters. The criterion is checked on a generic block octant-wise including a two-cell wide buffer zone around the octant. FIL (preset to \( 10^{-2} \)) is a filter to suppress refinement at small-scale wiggles. For the results presented in this manuscript, we choose the magnetic field to set the criterion.
The threshold parameter $\varepsilon_U$ for the magnetic field is set to equal 0.38, and the reference value $U_{ref}$ is $2 \times 10^{-5}$ T, $\alpha = 0.6$.

The time integrator for MHD equations we have used in this code is the third-order accurate Runge–Kutta method. The second-order version of the central-upwind scheme is applied to the Euler equations with the Lorentz-force term combined with a CT scheme for the induction equation. The electric field is computed from a genuinely 2D central-upwind procedure (CCT) based on the evolution-projection method. The divergence-free condition of the magnetic field is a built-in property of the scheme by virtue of a constrained-transport ansatz for the induction function. The relative divergence of the magnetic field, which has been tested, is normally smaller than $10^{-6}$. The detailed descriptions of this kind of scheme are presented in the paper by Ziegler (2011). In that paper, numerical experiments illustrate the overall robustness and performance of the scheme for some tests.

We start our simulations from a base-level grid of $160 \times 320$. The highest refinement level is 13, which corresponds to a grid resolution $\Delta x \approx 76$ m. It is around one magnitude higher than the ion inertial length in solar corona. Convergence studies have been carried out by repeating some simulations with a higher resolution for case B, with the highest refinement level limited to 14. The numerical results in the higher resolution case are very similar to the results presented in the next section. As we all know, numerical diffusion is inevitable in numerical experiments. To evaluate the numerical noise in our simulations, we use a similar method to that in the paper by Shen et al. (2011) to perform an estimate for case B as below. The magnetic induction Equation (4) can be written as:

$$\partial_t \psi = (v \times B)_x - \eta (\nabla \times B)_x.$$  \hspace{1cm} (16)

Here the flux function $\psi$ is defined through the relations $B_t = -\partial \psi / \partial y$, $B_x = \partial \psi / \partial x$. In the absence of numerical diffusion, both sides of Equation (16) should ideally balance each other. However, the two sides cannot exactly balance each other in realistic numerical simulations. We have estimated the numerical diffusivity around the main X-point within a short time for case B by using the following method. The main reconnection X-point is determined as the X-point that has the highest $\psi$ value of all X-points in the box. Suppose that the main X-point at $t = 113.85$ s is at position $(x_i, y_i)$. We define $a = \partial \psi / \partial y, b = [v(x_i, y_i) \times B(x_i, y_i)]_x$, and $c = \eta(x_i, y_i) (\nabla \times B(x_i, y_i))_x$. The values of $|(a - b + c)/|c|$ and $|(a - b + c)/(|a - b|)$ are calculated and presented in Figure 2. Since the plasma velocity and magnetic field in the x and y directions at the main X-point is near zero, the value of $|b|$ is much smaller than the value of $|c|$ in Figure 2. Therefore, $|(a - b + c)/|c| \approx (a + c)/|c|$ and $|(a - b + c)/(|a - b|) \approx (a + c)/|a|$ represent the ratio of $\eta_n/\eta_i$ and $\eta_n/\eta_i$, respectively. In Figure 2, one can find that the numerical diffusivity should be smaller than 20% of the physical one within such a time interval. We only choose a short time interval in Figure 2, the reason being that the position of the main X-point varies with time. The main X-point is no longer at around position $(x_i, y_i)$ after $t = 115.5$ s. Therefore, the value of $b$ gradually becomes larger than $c$ when the main X-point leaves position $(x_i, y_i)$. By using the same method, we cannot prove that the numerical diffusivity in the regions away from the reconnection X-point is also smaller than the physical diffusivity. Because numerical diffusion is mostly caused by the $v \times B$ term in this situation, such numerical diffusion could be larger than the physical $\eta \nabla \times B$ term.

3. NUMERICAL RESULTS

3.1. Current Sheet Dynamic Structures

Disturbed by initial perturbations, the CS section between the two primary islands as shown in Figure 1 starts to develop toward an increasingly thinner Sweet-Parker-like long CS. As the aspect ratio of the long CS exceeds a critical value, the CS is broken down to multiple secondary fragments and many small islands start to appear. These secondary magnetic islands start to grow bigger and move along the CS after they appear. Many newer and higher order islands also begin to develop. When we zoom in to small scales with higher resolutions as we did in Figure 2 in our previous paper (Ni et al. 2015), we find that some secondary CS fragments are broken down to thinner filaments and smaller third-order plasmoids are formed. As we continue to zoom in to smaller scales, the thinnest CS width is then found around 1000 m and the highest order of plasmoids in our simulations is of fourth-order. Since there are multiple reconnection X-points, some of the islands move upward and some of them move downward. The fast moving island can catch up with the slow island and the two islands close to each other moving with opposite directions will collide eventually. After the two islands collide, they can coalesce to form one bigger island. The above phenomena can be seen clearly in Figures 3, 5, and 7. From Figures 3 and 4, we can also find that the main X-point moves up with time in our simulations. The red crosses in Figures 4(a)–(c) stand for the main X-point. The main reconnection X-points at $t = 55.2$ s, $t = 138.8$ s, $t = 176.4$ s, $t = 210.1$ s, and $t = 265.3$ s are detected separately at $y = 1.499L_o$, $y = 1.519L_o$, $y = 1.582L_o$, $y = 1.614L_o$, and $y = 1.640L_o$. We use the same method as that in our previous papers (Ni et al. 2012a, 2012b, 2013) to detect the main X-point. Figures 3 and 5 demonstrate that the plasmoids above the main X-point eventually move out of the simulation domain and those below the main X-point collide with the primary big
island at the bottom. Figure 3 also shows that the maximum outflow velocity can reach around 1000 km s$^{-1}$ even before secondary islands appear. This value is the same as the observed maximum outflow velocity. After secondary islands appear, the outflow velocities above and below the main X-point acutely fluctuate.

Figures 3, 5, and 6 indicate that a termination shock is formed above the primary island. Figure 6(a) shows that the plasma velocity along the $y$-direction starts to decrease to a value smaller than the sound speed $c_s$ at around $y = 1.22L_0$ at $t = 265.3$ s. The entropy $S$ in Figure 6(b) and the magnetic field parallel to the shock front in Figure 6(c) suddenly jump to a much higher value at around $y = 1.22L_0$. This is exactly the behavior of a fast-mode shock. The termination shocks have also been found in the outflow regions of the multiple reconnection X-points in the plasmoid dominated regime. These fast-mode shocks may be related to the hard X-ray non-thermal emission above the soft X-ray flare arcades (e.g., Masuda et al. 1994; Tsuneta & Naito 1998; Krucker et al. 2010). However, the particle acceleration process by fast-mode shocks in kinetic scale is still not clear and beyond the scope of our present MHD work. In the MHD scale, the hot plasmas generated by fast-mode shocks in solar corona are expected to be observed (Habbal et al. 1979; Hsieh et al. 2009).
The slow-mode shocks are usually formed at the outflow regions of a Petschek-like CS or behind the moving magnetic islands (Tanuma et al. 2001). In our simulations, a lot of slow-mode shocks also appear at the edges of the plasmoids. The fifth contour plot for $t = 265.3$ in both Figures 3 and 5 clearly indicate that there is a pair of slow-mode shocks at the edges of the primary island in the down flow region below the main X-point. The slow-mode shocks are also formed at the edges of secondary islands. The bottom right panel of Figure 7(a) and bottom left panel of Figure 7(b) show an upward-moving magnetic island, which is formed by two coalescent islands; a pair of nearly symmetric slow-mode shocks is formed in front of such an island, and another pair of disturbed slow-mode shocks is behind it. Except for such special cases, the slow-mode shocks are usually formed behind the moving magnetic islands in our simulations. The shock angle of the two nearly symmetric shocks is around $\approx 3.7^\circ$. Figure 7(c) presents the magnetic field and the current density along a cut in the $x$-direction at $y = 1.697\ L_0$. Around $x = 0.5\ L_0$, the field component tangential to the shock, $B_j$, decreases rapidly toward the downstream side and the current density $J_z$ has a peak, but the component normal to the shock, $B_\perp$, stays nearly uniform along the cut.

These shock structures can be distorted as the reconnection outflow plasma becomes turbulent and the plasmoids collide with each other. The distorted slow-mode shock fronts which are formed behind an upward-moving magnetic island are presented in the upper right panels of Figures 7(a) and (b). The black arrows in the upper right panel of Figure 7(b) represent plasma velocity; one can see that the parallel shear flows with different velocities appear around the contact surface between the reconnection outflows and the ambient plasmas. However, the Kelvin–Helmholtz instabilities with multiple vorticities do not appear because of the strong aligned magnetic fields (Frank et al. 1996; Baty & Keppens 2002). These dynamic structures are only the secondary fragments of the CS which fluctuate along the shock fronts.

Figure 5 shows the temperature distributions at five different times for case B with $\beta = 0.05$. During the reconnection process, one can see that the temperature gradually increases with time by ohmic heating inside the CS region, which is similar to those previous work (e.g., Ugai 1992). However, as shown in Figures 4 and 5, the temperature distribution is not uniform at the CS region especially after secondary islands appear. Since significant heating takes place at the slow shocks attached to the plasmoids not at the X-points, the temperature inside the plasmoids is usually higher than that at the reconnection X-points as shown in Figure 4. The non-uniform behaviors of the temperature at the current regions in our simulations are in accordance with observation results on the CS above the cusp-shaped structure in the gradual phase by Sun et al. (2014). In Figure 7 in their paper, they also show the non-uniform temperature distribution along and vertical to the observed CS. They predict that the sharp change in both the temperature and the emission measure distribution curves could be evidence of a slow-mode shock produced by magnetic reconnection.
3.2. Reconnection Rate and Effective Diffusivity

We have used the same method as those in our previous papers to calculate the reconnection rate in both case A and case B. The reconnection rate is computed as the rate of change of the magnetic flux accumulated between the O-point in the primary island at $y = L_0$ and the main reconnection X-point (see Ni et al. 2012a, 2012b, 2013)

$$\gamma(t) = \frac{\partial \left( \psi_x(t) - \psi_0(t) \right)}{\partial t} \frac{1}{b_0 v_{AO}},$$

where $b_0$ and $v_{AO}$ are the initial magnetic field and Alfvén velocity at the inflow boundary presented in Section 2. One can note that the values of $v_{AO}$ are different between case A and case B. The magnetic reconnection rates varying with the normalized timescale in Figure 8(a) are very similar in the two cases. Secondary islands start to appear at around $0.87 t_{AOA}$ in case A and $0.88 t_{AOB}$ in case B, where $t_{AOA} \approx 173.66 s$ and $t_{AOB} \approx 122.80 s$ are the initial Alfvén time in case A and case B respectively. The reconnection rate can reach around 0.02 in both cases. However, as shown in Figure 8(b), the maximum temperature in case B can be more than two times higher than that in case A. Therefore, the plasma can be heated to a higher temperature in a reconnection layer with a lower plasma density. The hottest plasma in case B with $\beta = 0.05$ is around 30 MK, such high temperature plasmas can be observed by hard X-ray telescopes.

Lin et al. (2007) have measured the effective magnetic diffusivity $\eta_{eff} = v_i l$ by observations, where $v_i$ is the reconnect inflow velocity and $l$ is the half-thickness of the CS. Figure 9 presents the inflow velocity $v_i$ in the $x$-direction through the main X-point at $t = 55.2$ s and $l = 265.3$ s; in our simulations, here $v_i$ represents the inflow velocity $v_i$. In order to compare with observations, we measure the half-thickness of the CS $l$ from the position where $v_i$ starts to decrease to the position where $v_i = 0$ as indicated in Figure 9. At $t = 55.2$ s, the half-thickness is found to be $l = 2 \times 10^7$ m and the inflow velocity is around $9 \times 10^4$ m s$^{-1}$. At $t = 265.3$ s, $l$ decreases to $1.5 \times 10^7$ m and $v_i$ decreases to $3 \times 10^5$ m s$^{-1}$. The effective magnetic diffusivity is then obtained as $\eta_{eff} \approx 1.8 \times 10^{12}$ m$^2$ s$^{-1}$ at $t = 55.2$ s, and $\eta_{eff} \approx 4.5 \times 10^{11}$ m$^2$ s$^{-1}$ at $t = 265.3$ s. These values are close to those deduced by Lin et al. (2007) on the basis of observations. Though the resolution used in our simulations is much higher than that of the observational instruments. We should note that the refinement level for calculating the half-thickness of the CS in Figure 9 is zero and the resolution is around 625 km, which is close to the highest resolution of the solar space telescopes. The effective magnetic diffusivities at $t = 265.3$ s have also been measured through other X-points by using the same method as above. The measured values of $\eta_{eff}$ through these X-points are all on the order of $4.5 \times 10^{11}$ m$^2$ s$^{-1}$.

3.3. Energy Conversion in Current Sheet Regions Mediated by Plasmoids

Figure 10 presents the time-dependent energy conversion for $\beta = 0.1$ and $\beta = 0.05$ in a fixed region ($0.45 L_0 \leq x \leq 0.55 L_0$ and $L_0 \leq y \leq 2 L_0$). Since energy fluxes through the boundaries at $x_0 = 0.45 L_0$, $x_0 = 0.55 L_0$, $y_0 = L_0$ and $y_0 = 2L_0$ always exist, the magnetic, thermal, and kinetic energy flowing into this region through these boundaries from the beginning of the simulation ($t = 0$) to time $t$ is denoted by $E_{MI}(t)$, $E_{TI}(t)$, and $E_{KE}(t)$, respectively. (Note that these quantities may have negative signs if energy flows out of the region.) The magnetic, thermal, and kinetic energy confined to this region at time $t$ is denoted by $E_{ML}(t)$, $E_{TL}(t)$, and $E_{KL}(t)$, respectively. The initial magnetic, thermal, and kinetic energy at $t = 0$ is denoted by $E_{MI}$, $E_{TI}$, and $E_{KI}$, respectively. The total radiated thermal energy from beginning to time $t$ is denoted $E_{RAD}(t)$. In these notations, the dissipated magnetic energy in the region defined by $0.45 L_0 \leq x \leq 0.55 L_0$ and $L_0 \leq y \leq 2 L_0$ is given by $E_{MD}(t) = E_{MI} + E_{MF}(t) - E_{ML}(t)$. In the same region, the generated thermal energy is $E_{TG}(t) = E_{RAD}(t) + E_{TL}(t) - E_{TI}(t)$, and the generated kinetic energy is $E_{KG}(t) = E_{KL}(t) - E_{KE}(t)$, and $E_{KI}$. The detailed calculations of the above variables are similar to those in our previous paper (Ni et al. 2012a). However, there are some improvements in this work compared with our previous paper. We have used 2.5 dimensional MHD instead of the two-dimensional MHD equations, open instead of periodic boundaries are used at the top and bottom, and radiation cooling and heating terms are also included in this work. Therefore, the $z$ components of the magnetic field and velocity should be included to calculate the above variables; the energy...
flux $E_{MF}$, $E_{TF}$, and $E_{KF}$ at the boundaries and the radiated energy $E_{RAD}(t)$ also have to be included.

Figure 10 shows that the corresponding generated thermal and kinetic energy behave similarly to the dissipated magnetic energy. From the beginning to time $t$, the total dissipated magnetic energy exactly equals the generated thermal energy plus kinetic energy; the errors are under 0.1%. The magnetic energy is mostly converted to kinetic energy before secondary islands appear, e.g., the generated kinetic energy is around four times higher than the generated thermal energy from beginning to $t = 109 \, s \simeq 0.88 \, t_{A0B}$ in case B with $\beta = 0.05$. After secondary islands appear, the generated thermal energy grows much faster with time than the kinetic energy, and there is greater thermal energy generated than kinetic energy eventually. The small-scale slow-mode shocks attached to the edges of the multiple magnetic islands play important roles in generating thermal energy. Figures 5 and 7 clearly show that the highest temperature structures always appear at the shock front regions. In the previous papers by Kliem (1990) and Báráta et al. (2011b), they also pointed out that energy dissipation is
accomplished via many concurrent small-scale events appearing in multiple sites distributed in space. In order to inspect the effects of Joule heating, the generated thermal energy and Joule heating have been calculated in several fixed regions over a period. As long as the slow-mode shocks are included in these regions, we find that the generated thermal energy is always much larger than the Joule heating in a fixed period. Therefore, Joule heating is not the main reason for the sharply increasing thermal energy. Though the generated kinetic energy increases slower than thermal energy after secondary islands appear, there is still around 40% of the dissipated magnetic energy that has been converted to kinetic energy from the beginning to \( t = 330 \text{s} \simeq 2.7 \ t_{\text{A0}} \) in case B. This is different from the result in our previous paper (Ni et al. 2012a), where, after secondary islands appear, the generated kinetic energy quickly decreases to a small value and around 99% of the dissipated magnetic energy was transformed to thermal energy eventually in that paper. The periodic boundary conditions applied in the \( x \)-direction in the paper by Ni et al. (2012a) is the main reason for this difference. The outflow plasmas are confined inside the simulation domain by periodic boundary conditions; they collide with the primary island at the top and slow down eventually. The outflow plasmas in this present work can gradually escape from the top boundary because of the open boundary conditions. From Figure 10(a), one can also see that the magnetic energy is dissipated faster and more thermal energy is generated in case B with \( \beta = 0.05 \) than that in case A with \( \beta = 0.1 \) during a period after secondary islands appear. The reason is that the plasma in the lower \( \beta \) case can be compressed more strongly and the shock heating becomes more important in the energy conversion process.

The termination shocks at the head of plasmoids also contribute a small part of the generated thermal energy. Part of the kinetic energy of the reconnection outflows can be converted to thermal energy at the termination shocks. In Figure 5, one can see that the temperature increases at the termination shock above the primary island. However, the temperature of the heated plasma at the termination shock is much lower than the highest temperature at the slow-mode shock fronts.

At the fragment CS regions where the slow-mode shocks are not included, the heating by dynamic viscosity has been measured to compare with Joule heating. The Joule heating can be measured as \( Q_J = \eta (\nabla \times B)^2 / \mu_0 = \eta \mu_0 J^2 \); and the viscous heating is measured as \( Q_v = \nu (\nabla \cdot v)^2 / 3 \). At the small CS fragments, the current density \( J \) is around 0.1 A m\(^{-2}\), the maximum \( \nabla \cdot v \) is calculated to be around 10 s\(^{-1}\), and the magnetic diffusion \( \eta \) is around \( 10^6 \sim 10^7 \) m\(^2\) s\(^{-1}\) and \( \nu = 10^{-5} \) kg m\(^{-1}\) s\(^{-1}\). Then, we find that \( Q_v \) is around \( 30 \sim 300 \). Therefore, the heating by dynamic viscosity can be ignored at the fragment CS regions in this work.

3.4. Spectrum Studies

The one-dimensional energy spectra along the reconnection CS have been studied numerically in some papers (Bárta et al. 2011b; Ni et al. 2012b, 2013; Shen et al. 2013). After secondary islands appear, the numerical results in these previous papers demonstrate that the energy spectrum no longer behaves as a simple power law. The spectral index for both kinetic and magnetic energy varies with wave number; it usually increases to a higher value as the wave number increases. Before studying the two-dimensional spectra, we present the one-dimensional spectra along the CS for case B in Figure 11. The method similar to that in our previous papers (Ni et al. 2012b, 2013) has been used to obtain the spectral index for both kinetic and magnetic energy. First, each component of magnetic field, velocity, and density along the CS at \( x = 0.5L_0 \) has to be transformed to Fourier space. Then,
the magnetic energy spectrum is calculated as $E_{Bk} = \frac{\langle B^2 \rangle (ky)}{2\mu}$ and the kinetic energy spectrum is $E_{V_k} = \frac{\langle v^2 \rangle (ky)}{2\mu}$.

Finally, we fit the spectrum to a power law $E_{Bk} \sim ky^{-\alpha_1}$ and $E_{V_k} \sim ky^{-\alpha_2}$ to obtain the energy spectrum index. Figure 11 shown that the spectra also do not behave as a simple power law after secondary islands appear. We only fit a line to get the spectrum index $\alpha_1$ within the region $10^{-8} \leq E_{Bk} \leq 10^{-3}$ and $\alpha_2$ within the region $10^{-10} \leq E_{V_k} \leq 10^{-3}$. Therefore, the spectral index we have obtained is an average value. The spectral index is larger before secondary islands appear for both the kinetic and magnetic energy spectrum. After secondary islands appear, the spectra become harder. The index for magnetic energy spectra decreases to around 1.8 and the one for kinetic energy spectra is around 2.9. These characteristics are very similar to those of one-dimensional energy spectra presented in the previous papers (Bárta et al. 2011b; Ni et al. 2012b, 2013; Shen et al. 2013). However, we should note that the values of these spectral indexes vary in a range—they are not precisely fixed. In the paper by Bárta et al. (2011b), the one-dimensional spectral index for the magnetic energy is 2.14 at $t = 316$. But such an index varies with time as shown in the papers by Ni et al. (2012b, 2013). It is normally in the range $1.5 < \alpha_1 < 2.5$ for the magnetic energy spectrum after secondary islands appear. We have chosen level 5 data to plot Figures 11(b) and (c). At $t = 55.2$ s, the CS is smooth and no secondary islands appear; the highest refinement level is 4. Therefore, level 4 data are used for plotting Figure 11(a).

For the first time, we have studied the two-dimensional energy spectra for both kinetic and magnetic energy. First, the magnetic field, velocity, and mass density in the region $0 \leq x \leq L_0$ and $0 \leq y \leq 2L_0$ are transformed to two-dimensional Fourier space. Then, we calculated the two-dimensional magnetic and kinetic energy as $E_{B_k} = \frac{\langle B^2 \rangle (kx, ky)}{2\mu}$ and $E_{V_k} = \frac{\langle v^2 \rangle (kx, ky)}{2\mu}$. There are $nx$ grids in the $kx$ direction and $ny$ grids in the $ky$ direction. $kx$ is defined as $0, 2\pi/L_x, 2\pi/L_x, 3\pi/L_x, ..., (nx-1)\times2\pi/L_x$, and $ky$ is defined as $0, 2\pi/L_y, 2\pi/L_y, 3\pi/L_y, ..., (ny-1)\times2\pi/L_y$. $L_x = L_y = L_0$ are the length scales we have selected in the $x$- and $y$-direction, respectively. The two-dimensional distributions of $lg(E_{B_k})$ for $t = 55.2$ s and $t = 138.8$ s are presented in Figure 12. Since the high energy parts of the two-dimensional spectra are mostly located at the positions with small $kx$ and $ky$. In order to see the distributions of the two-dimensional spectra more clearly, the imaginary parts of the spectra with negative coordinates are also presented in Figure 12. We have also zoomed into a smaller scale within $0 \leq kx \leq 160 \times 2\pi/L_0$ and $0 \leq ky \leq 160 \times 2\pi/L_0$. Then, one can clearly see that the high energy parts are located in the center of the plots. In Figure 12, we only need to focus on the first quadrant (the top right quadrant) with positive coordinates in the two panels. The dark red color represents the highest magnetic energy and the dark blue color represents the lowest one. At $t = 55.2$ s, before secondary islands appear, the distribution of magnetic energy in Fourier space is relatively smooth and the high magnetic energy is confined in the region near $ky = 0$. At $t = 265.3$ s after secondary islands appear, part of the high energy is obviously spread to the space with larger $kx$ and $ky$. As we know, the larger the wave number, the smaller the wave length. These evidences prove further that plasmoid instabilities can cause multiple levels of fine structures in the reconnection regions. The magnetic energy can be cascaded to smaller and smaller scales, and it is dissipated in these small scales eventually. Comparing with a one-dimensional spectrum, the two-dimensional ones show how asymmetric the spectra look like and give more detailed information. In the right panel of Figure 12, one can clearly see that the magnetic energy is not uniformly cascaded into Fourier space after secondary islands appear, and most of the magnetic energy is still confined in the scales with smaller $ky$. The level 4 data were used to derive Figure 12. The distributions of the two-dimensional kinetic energy spectra, which we do not show here, are very similar to those of the magnetic energy spectra shown in Figure 12.

The plasmoid distribution function is also important for revealing the statistical properties and understanding the dynamics of these plasmoids. Following the same method as Loureiro et al. (2012) and Shen et al. (2013), we have also calculated the plasmoid distribution function $f(\psi) = -dN(\psi)/d\psi$ numerically. $N(\psi)$ is the number of plasmoids with magnetic flux larger than $\psi$. The magnetic flux of a magnetic island is calculated using $|\psi_x - \psi_0|$, where $\psi_0$ is the magnetic flux at the O-point of the magnetic island and $\psi_x$ is the magnetic flux at the nearby X-point. For obtaining the plasmoid distribution function presented in Figure 13, plasmoids appearing in different snapshots with an interval of 0.048 s are accumulated during the evolution of the CS. Figure 13 shows that the plasmoid distribution function behaves as a power law closer to $f(\psi) \sim \psi^{-1}$ in the intermediate $\psi$ regime. In the large $\psi$ regime, the distribution function $f(\psi)$ gradually deviates from the power law $\sim \psi^{-1}$ to a more rapid falloff.
4. SUMMARY AND DISCUSSION

Studying the energy conversion and spectra of a corona CS is very important to understand the physical mechanisms of magnetic reconnection and to explain many observational features in flares. In this work we have simulated a 2.5 dimensional corona CS with more realistic physical parameters: the temperature-dependent high Lundquist number ($10^6 \sim 10^7$) has been used, and both radiation cooling and heating terms are included. Here is a summary of the main results.

1. After the Sweet–Parker long CS breaks into multiple fragments, many Petschek-like fine structures with slow-mode shocks attached to the edges of the plasmoids are formed. Unlike the classical Petschek slow-mode shocks, these shock structures are not steady and they can be distorted by the colliding plasmoids and turbulent outflows. A lot of turbulent structures appear inside multiple plasmoids and in the downflow region. The termination shocks can also be formed above the primary

Figure 11. One-dimensional spectra along the current sheet at $x = 0.5L_0$ at three different times; (a)–(c) are for the magnetic energy; (d)–(f) are for the kinetic energy.

Figure 12. Distribution of logarithmic values of magnetic energy in two-dimensional Fourier space at two different times.
magnetic island and at the head of secondary islands. These shocks play important roles in generating thermal energy in a corona CS.

2. For a numerical simulation with initial conditions $\beta = 0.05$, $b_0 = 0.001$ T, and $\rho_0 = 1.2 \times 10^{-12}$ kg m$^{-3}$, about 80% of the dissipated magnetic energy is converted to kinetic energy before secondary islands appear. After multiple slow-mode shocks appear at the edges of the magnetic islands, the generated thermal energy increases sharply, and about 60% of the dissipated magnetic energy can be transformed to thermal energy eventually.

3. The one-dimensional energy spectra along the CS at $x = 0.5 L_0$ have been studied. After secondary islands appear, the average spectrum index for kinetic energy is around 2.9 and it is around 1.8 for the magnetic energy spectrum. These spectra do not behave as a simple power law and the spectrum index increases with the wave number, which are similar to the previous studies by Ni et al. (2012b, 2013), Shen et al. (2013), and Báta et al. (2011b). For the first time, we have studied the two-dimensional energy spectra of the corona CS. Comparing with the one-dimensional spectra, two-dimensional spectra intuitively show that part of the high energy is cascaded to larger $kx$ and $ky$ space after secondary islands appear. The spectra are asymmetric in the Fourier space — mostly the energy is always confined in the region with small $ky$.

4. The plasmoid distribution function has been calculated numerically by $f(\psi) = -dN(\psi)/d\psi$. It behaves as a power law closer to $f(\psi) \sim \psi^{-1}$ in the intermediate $\psi$ regime, which is the same as the result from Huang & Bhattacharjee (2012). In the large $\psi$ regime, the distribution function $f(\psi)$ gradually deviates from the power law $\sim \psi^{-1}$ to a more rapid falloff. However, the exponential tail in the large $\psi$ regime as presented in the paper by Huang & Bhattacharjee (2012) is not clearly identified from our data presented in Figure 13. It is probably because that we only use the snapshots before $t = 180$ s of case B data and the plasmoids we have identified are still relatively small.

5. By using $h_{\text{eff}} = v_{\text{inflow}} \cdot L$, the effective magnetic diffusivity is estimated to be about $10^{11} \sim 10^{12}$ m$^2$ s$^{-1}$. It is close to the results deduced by Lin et al. (2007) based on observations.

As we know, the effect of heat conduction can smooth out the temperature in solar corona magnetic reconnection events. The numerical simulations in the papers by Yokoyama & Shibata (1997, 2001) have proved this point. Our previous paper, Ni et al. (2012a), also studied the effect of anisotropic heat conduction on the magnetic reconnection process. Since the time step $dt$ for the explicit scheme used in the NIRVANA code is limited by the CFL condition, $dt \leq (dx)^2/2\kappa$, where $dx$ is the space step and $\kappa$ is the thermal conductivity coefficient, after secondary islands and smaller scale CS fragments appear, the extremely small $dx$ and large $\kappa$ cause the time step $dt$ to become too small to continue the simulations for including the heat conduction in both case A and case B. Therefore, the heat conduction terms are not included in this work. The characteristic timescales of heat conduction in directions parallel and perpendicular to magnetic fields have been analytically calculated to compare with the Alfvén crossing time. We find that the heat conduction effect could be very efficient at the high temperature regions in the direction parallel to magnetic fields, but it can be ignored in the direction perpendicular to the magnetic fields. As shown in Figures 5 and 7, much higher temperatures and steep temperature gradients are built up in the plasmoids and at the slow-mode shocks and X-points. Conductive energy transport from these regions into the inflow regions and into the big plasmoid at the bottom end of the CS is largely directed across the magnetic field. Therefore, even though anisotropic heat conduction is included, the high temperature plasmas can still be confined inside the plasmoids in 2.5 dimensional simulations. However, heat conduction can be efficient in the third direction and the maximum temperature in plasmoids (flux ropes) will become smaller in the full three-dimensional space than in 2.5 dimensional simulations.

As mentioned by Báta et al. (2011a, 2011b), the actual physical mechanism for energy transfer from the global scales, where the energy is accumulated, to the much smaller scales, where plasma-kinetic dissipation takes place, is an open issue. The spectrum studies presented in Figures 11–13 prove that the multiple cascading process is happening or has already happened in the CS region — both kinetic and magnetic energy are cascaded from large scales to small scales during the plasmoid cascading process. Therefore, such turbulent reconnection with multiple dynamic structures can explain well the energy transfer process in solar flare eruption. The HXR and radio observations (e.g., Karlický et al. 1996, 2000) also indicate that particle acceleration takes place via multiple concurrent small-scale events distributed turbulently in the flare volume, rather than by a single compact acceleration process hosted by a single diffusion region. Such observations are usually referred to as signatures of fragmented/chaotic energy release in flares. The spectra presented in Figures 11–13 cannot be directly compared with the observation spectra right now. In the future, we hope that the dynamic structures in the plasmoid instability process in our simulations can be used to study particle acceleration by the testing particle method (e.g., Li & Lin 2012). A huge number of particles can be placed separately in the dynamic structures with turbulent CS fragments, slow-mode shocks, or fast-mode shocks. Then we can analyze how these particles will be accelerated and calculate the particle energy spectrum distributions. According to these spectra, one can expect to find out the contributions of turbulent CS fragments, slow-mode shocks, and fast-mode shocks to particle...
accelerations separately. Finally, these particle energy and number density spectral distributions can be compared with observations.

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