Asymptotically free scaling solutions in nonabelian Higgs models

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We construct asymptotically free renormalization group trajectories for the generic nonabelian Higgs model in four-dimensional spacetime. These ultraviolet-complete trajectories become visible by generalizing the renormalization/boundary conditions in the definition of the correlation functions of the theory. We identify a candidate three-parameter family of renormalization group trajectories interconnecting the asymptotically free ultraviolet regime with a Higgs phase in the low-energy limit. We provide estimates of their low-energy properties in the light of a possible application to the standard model Higgs sector. Finally, we find a two-parameter subclass of asymptotically free Coleman-Weinberg-type trajectories that do not suffer from a naturalness problem.

INTRODUCTION

While the naturalness problem has been a dominant paradigm for model building beyond the standard model of particle physics, the triviality problem of the Higgs sector conceptually appears much more severe as it inhibits a constructive ultraviolet (UV)-complete definition of the standard model as an interacting quantum field theory. The triviality of the standard model Higgs sector is expected to arise from the fundamental scalar degrees of freedom. For pure scalar theories, strong evidence for triviality – the fact that the continuum limit can only be taken for the noninteracting theory – has been accumulated by lattice simulations in $d = 4$ and analytic methods (see [1] for a rigorous proof in $d > 4$). For nonabelian Higgs models, Monte Carlo methods have found no indication for continuous phase transitions facilitating a nontrivial continuum limit. In practice, triviality arguments have been used to put upper bounds on the Higgs mass long before its discovery.

Since ATLAS and CMS have found a comparatively light scalar boson, the standard model appears to be in a “near-critical” regime, indicating that the Higgs self-interaction is small near the Planck scale. This would be natural if all standard-model interactions including the scalar self-interaction were asymptotically free (AF). [10] This is, however, not the case from the standard viewpoint of perturbative $\beta$ functions.

The construction of AF Yang-Mills-Higgs systems is in principle straightforward on the basis of a perturbative analysis. In particular, the problematic quartic scalar interaction $\lambda$ can be marginal-relevant (UV stable) or irrelevant (UV unstable), depending on the model and the choice of trajectories. AF trajectories can be built also for models including fermions, by imposing strict eigenvalue conditions on the system of beta-functions. UV-complete trajectories which emanate from the Gaussian fixed point can also be built by fixing the unstable marginal-irrelevant direction. In RG-improved perturbation theory, this scenario requires additional fermions as well as eigenvalue conditions to be satisfied. This implies a reduction of couplings, here effectively removing one parameter, as $\lambda$ is then purely induced, implying a prediction of the Higgs-to-W-boson mass ratio. To our knowledge none of such theories comes sufficiently close to the standard model. Alternatively, UV completion in Higgs models can be achieved via asymptotic safety, which also requires dynamical fermions.

In this Letter, we consider the construction of AF Yang-Mills-Higgs systems from a new viewpoint. Our central idea is that, in order for suitable AF nonabelian Higgs models to exist, the scalar potential needs to approach absolute flatness concurrently with the vanishing gauge coupling $g$. This permits large amplitude fluctuations of the scalar field controlled by the latter parameter. We thus suggest to consider gauge-rescaled scalar field variables $\phi \rightarrow g^{\mu} \phi$, with some power $P$, as the relevant measure for amplitudes. While $P$ at this point merely seems to be an unphysical rescaling parameter, we show that it parametrizes RG-scale-dependent boundary conditions for the effective potential. These in turn are equivalent to $g$-dependent renormalization and boundary conditions for the correlation functions of the theory. As a consequence, $P$ parametrizes a set of physically distinct RG flows, each one possessing a Gaussian FP and allowing for AF trajectories.

First signatures of such a trajectory have been found in a gauged Yukawa model in [10]. In the present work, we explore the general pattern to construct UV-complete trajectories for AF nonabelian Higgs models, including the physically relevant SU(2) model, for the first time.

RG FLOW OF THE MODEL

A key building block of the standard model of electroweak interactions is the nonabelian Higgs model with a fundamental scalar $\phi$, we consider gauge groups SU($N$), using the standard model SU(2) for concrete examples. This model includes a Yang-Mills sector $\mathcal{L}_{YM} = \text{tr} F_{\mu\nu} F^{\mu\nu}/2$ with the field strength $F_{\mu\nu}$ derived from the vector potential $W_{\mu}$, and a minimally coupled scalar sector with a scalar potential that depends on the invariant $\rho := \phi^4$. In this work, we analyze the RG flow not...
only restricted to the set of perturbatively renormalizable operators, but include a full scalar potential. Even if the higher operators turn out to be irrelevant and strongly suppressed along asymptotically free trajectories, it is crucial for the UV construction of these trajectories to go beyond the single-coupling analysis. We study the flow of a scale-dependent effective action

\[ \Gamma_k = \int Z_W \mathcal{L}_{YM} + Z_\phi (D^\mu \phi)^*(D_\mu \phi) + U(\rho), \]

where \( D_\mu = \partial_\mu - i g W_\mu \). Here, all wave function renormalizations \( Z_\phi W \), the coupling \( \tilde{g} \), and the potential \( U \) depend on a RG scale \( k \). The RG \( \beta \) function(s) for these quantities have been computed in [19], using the Wetterich equation [20, 21]. This formulation of the functional RG is useful as it makes no assumptions about the magnitude of the running masses and couplings, and incorporates dynamically generated thresholds. The relevance of the latter for UV completeness has first been studied in [22].

Using the background-field formalism, the running of the renormalized gauge coupling \( g^2 = \frac{\beta^2}{\omega} \) is linked to that of the wave function renormalization [23].

\[ \partial_t g^2 = \eta_W g^2, \quad \partial_t \log Z_W, \quad t = \log k. \]

The present ansatz for the effective action yields the standard one-loop running, amended by threshold effects owing to gauge bosons and the Higgs scalar acquiring masses in the broken regime. Similarly, the scalar anomalous dimension \( \eta_\phi = -\partial_t \log Z_\phi \) exhibits a standard one-loop form including threshold effects [19].

Our search strategy for asymptotic freedom generalizes the usual perturbative analysis, since we look for trajectories such that the \( \phi^4 \) coupling vanishes as \( \lambda \sim g^{4P} \) in the UV, with arbitrary power \( P > 0 \). The nontrivial asymptotic value for \( \lambda/g^{4P} \) can be observed by rescaling the scalar field

\[ x = g^{2P} \bar{x} = g^{2P} \frac{Z_\phi}{k^2 \rho}, \quad \rho = \phi^{1/\phi}, \]

such that \( x \) plays the role of a natural renormalized dimensionless field. For the full scalar potential, we demand that higher couplings vanish in the UV with corresponding or higher powers of \( g \). The dimensionless effective potential

\[ f(x) = u(\bar{x}) \bigg|_{\bar{x}=g^{-2P}x} = k^{-4} U(\rho) \bigg|_{\rho=g^{-2P}Z_\phi^{-1}k^2 x}, \]

should then stay finite and non-vanishing in the far UV (the dimensionless quantities \( u \) and \( \bar{x} \) are often used in the literature).

The flow equation for this rescaled effective potential reads [19],

\[ \partial_t f = \beta_f \equiv -4 f + (2 + \eta_\phi - P \eta_W) x f' \]

\[ + \frac{1}{16\pi^2} \left\{ 3 \sum_{i=1}^{N-1} l_i^{(G)^4} \left( g^{2(1-P)} \omega_i^{(W)}(x) \right) \right. \]

\[ + (2N - 1) l_{0}^{(B)^4} (g^{2P} f' + l_0^{(B)^4} (g^{2P} (f' + 2x f''))) \}, \]

where the scheme-dependent threshold functions \( l \) encode the decoupling of massive modes. Using the linear regulator [24], we have \( l_0^{(B)^4}(w) = \frac{1}{2} \left( \frac{1}{1 + \omega} - \frac{\omega}{6} \right) \) and analogously for \( l_0^{(G)^4}(w) \) upon replacing \( \eta_\phi \) by \( \eta_W \). The gauge-boson mass parameters \( \omega_i(W) \) arise from the eigenvalues of \( (g^{2P} Z_\phi/k^2) \phi \{ T_i, T_j \} \phi \), e.g., \( \omega_i^{(W)}(x) = x/2 \) for SU(2) for any \( i = 1, 2, 3 \).

**FIXED POINTS AND SCALING SOLUTIONS**

Let us first search for scaling solutions, which correspond to FPs of the RG flow, representing candidates for asymptotic limits of AF trajectories. For this, we consider Eq. (5) in the limit \( g \to 0 \), but keeping \( x \) and \( f(x) \) finite. The latter facilitates to consider boundary conditions for the effective potential, and thus for correlation functions, which are unapparent in conventional perturbation theory. Since the scalar loops in the last line approach irrelevant constants for \( g \to 0 \), and the anomalous dimensions also approach zero asymptotically, the flow equation for \( f(x) \) becomes a first-order differential equation. The behavior of the gauge-boson-loop in the second line, depends on the value of \( P \). For \( P \neq 1 \), it approaches zero \( (P > 1) \) or an irrelevant constant \( (0 < P < 1) \) and hence can be ignored. Therefore, for any regulator and any SU(N), the FP solutions to the remaining part of the first line of Eq. (5) satisfying \( \partial_t f = 0 \) read

\[ f_*(x) = \xi x^2, \quad P \neq 1, \]

for a generic \( \xi \) (irrelevant constants in \( f(x) \) are ignored). For \( P = 1 \), the gauge loop contributes nontrivially to the effective potential. For SU(2), we find using the linear regulator

\[ f_*(x) = \xi x^2 - \left( \frac{3}{10\pi} \right)^2 2x + x^2 \log \left( \frac{x}{2 + x} \right), \quad P = 1, \]

with \( \xi \) arbitrary. The precise functional form is regulator dependent, but any regulator yields this Coleman-Weinberg-type shape. For \( \xi \geq 0 \), the potential is bounded from below and has a nontrivial minimum \( x_{\text{min}} \). For \( \xi = 0 \), the minimum is at infinity.

The FP potentials of Eqs. (6,7), once re-expressed in terms of the original fields \( \phi \), provide the simplest portrait of a two-parameter family of asymptotically free solutions. Different values of \( (\xi, P) \) correspond to different flows in coupling space. This translates into different
$g$-dependent boundary conditions for integrating the RG equation for $U(\rho)$. Near the FP, the trajectories differ from Eqs. [D4] by higher powers of the gauge coupling. The trajectories can systematically be constructed in a weak-coupling expansion by expanding $\beta_f$ in powers of $g^2$, and computing the potential $f(x)$ for which this approximate $\beta$ functional vanishes. This procedure is justified by the stability analysis given below.

For $P \in (0,1]$ the next-order approximation includes a linear term in the leading power of $g^2$. The leading power is $g^{2P}$ for $P \in (0,1/2]$, and $g^{2(1-P)}$ for $P \in [1/2,1]$. The corresponding effective potentials are in the regime of spontaneous symmetry breaking (SSB)

$$f(x) = \begin{cases} 
\xi x^2 - \xi \frac{3}{16 \pi} g^{2P} x & \text{for } P \in (0,1/2) \\
\xi x^2 - \frac{4 \pi}{12 \pi} g x & \text{for } P = 1/2 \\
\xi x^2 - \frac{4 \pi}{12 \pi} g^{2(1-P)} x & \text{for } P \in (1/2,1)
\end{cases}$$

For $P \in (0,1/2)$ or $P = 1/2$ the position of the minimum is $g^2$ independent ($\rho_{\min} = 3/32 \pi^2$ and $\rho_{\min} = 3(3+8\xi)/256\pi^2 \xi$ respectively), whereas for $P \in (1/2,1)$ it is proportional to $\xi^{-1} g^{2(1-2P)}$ and thus running to infinity in the UV. For $P = 1$, we solve the corresponding equation numerically. The resulting potential $u$ as a function of the unscaled field $\tilde{\rho}$ is shown in Fig. 1. Again, the minimum of $u$ approaches infinity $\sim 1/g^2$ in the UV, and the curvature at the minimum vanishes like $g^4$. For $P \in (1,2)$ with leading power $g^{2(P-1)}$, the expansion is more subtle due to logarithms. We evade these problems by retaining the whole gauge loop, resulting in

$$f(x) = \xi x^2 - \left(\frac{3}{16 \pi}\right)^2 \left[\frac{2 x}{g^{2(P-1)}} \log \left(\frac{x}{g^{2(P-1)} + x}\right)\right], \quad P \in (1,2).$$

For $P \geq 2$ the leading power is $g^2$. Still, we again include the full gauge loop as it gives a more accurate description of the effective potential than a mere leading-power approximation. This results in

$$f(x) = \xi x^2 - 2 \left(\frac{3}{16 \pi}\right)^2 \left[\frac{4 \pi}{\sin(4 \pi/x)} \left(\frac{x}{2 g^{2(P-1)}}\right)^{1/4} \sin(\pi/x)\right] - 2 F_1 \left(1; -\frac{4}{d_x}; 1 - \frac{4}{d_x}, -\frac{x}{2 g^{2(P-1)}}\right), \quad P \geq 2,$$

where $d_x = 2 + g^{2(P-1)/3}$ is the quantum dimension of the invariant $x$. Thus, also for $P > 1$ the vev diverges ($\rho_{\min} \sim g^{-(2/3)}$), whereas it vanishes asymptotically in terms of gauge-rescaled variables. Furthermore the potential is bounded from below for $\xi > 0$.

Let us now perform a stability analysis of these trajectories, taking advantage of their asymptotic description in terms of FPs of the RG flow of $f(x)$. For small gauge coupling, perturbations about these trajectories are translated in deviations from the FP, with components $\delta g^2 = g^2$ and $\delta f(x) = f(x) - f_\text{FP}(x)$. Since $\beta_{g^2}$ is proportional to $-g^4$, any eigenperturbation with non-vanishing gauge coupling must be marginal-relevant. Indeed the $g$-dependent potential $f$ determined above is by construction a parametrization of the marginal-relevant eigendirection, since its flow is frozen apart from the running of $g$. Conversely, any non-marginal eigenperturbation must have a vanishing $g^2$ component. At $g^2 = 0$, the eigenvalue problem simplifies to the Gaussian one, for which the eigenperturbations are simple powers, $\delta f \propto x^n$. This includes a relevant ($n = 1$) and a marginal direction ($n = 2$). Beyond the linear analysis, the $n = 2$ direction is actually marginal-irrelevant, as is familiar from perturbation theory. We emphasize that all our effective potentials are polynomially bounded, exhibit self-similar eigenperturbations and thus satisfy standard RG requirements [25].

To summarize, we have identified new AF trajectories in the nonabelian Higgs model. In addition to the standard mass-type relevant deformation, we have provided the approximate parametrization of one marginal-relevant eigenperturbation for each pair ($\xi,P$). For UV-complete trajectories, the marginal-irrelevant $\phi^4$-type perturbation is zero. Therefore, once a specific UV asymptotic behavior is determined by ($\xi,P$), only one physical parameter remains apart from an absolute scale. This is one parameter less than in usual perturbative scenarios. Yet, we gained the two positive parameters $\xi$ and $P$, labeling different AF trajectories.

**MASS SPECTRUM**

The preceding analysis investigated the UV behavior of the AF trajectories in the nonabelian Higgs model. In order to explore the long range mass spectrum, we have to integrate the flow of the effective potentials towards the IR. If the trajectories end in a SSB phase, a Fermi
scale $k_F$ and gauge boson and Higgs masses are generated. Trajectories emanating from a given fixed-point theory specified by $\xi$ and $P$ consist of the correspond-
ing marginal-relevant eigenperturbation (parametrized by the gauge coupling $g^2_{\text{CO}}$) possibly superimposed by a
finite component of the relevant direction (the $\delta f \sim x^{n=1}$
Gaußian perturbation) with some coefficient $c_{\text{A}}$ at a UV scale $\Lambda$. Inspired by the standard model hierarchy, we
assume $c_{\text{A}}$ to be very small, such that the system will spend a long RG time on top of the marginally relevant
trajectory, establishing a large hierarchy $k_F \ll \Lambda$. At a
cross-over (CO) scale $k_{\text{CO}}$ the relevant component sets in
and drives the system away from the marginal-relevant
trajectory. In practice, the initial conditions $c_{\text{A}}, g^2_{\text{CO}}$ at $\Lambda$
can be traded for $c_{\text{CO}}, g^2_{\text{CO}}$ to be specified at $k_{\text{CO}}$ (in the
standard model, $k_{\text{CO}} \sim O(1) \text{TeV}$).

For a simple estimate (blind to nonperturbative bound-
state effects [20]) of the mass spectrum, we initialize the
flow at $k_{\text{CO}}$ with a potential $f_{\text{CO}}$ that is equal to the
marginal perturbation plus a relevant component, $f_{\text{CO}} =
f(x; P; \xi; g^2_{\text{CO}}) + c_{\text{CO}} x$. We then evolve the full RG flow
from $k_{\text{CO}}$ down to $k_F$. At the Fermi scale, the gauge
coupling $g^2$ as well as the dimensionful conventionally-renormalized
vev $v$ and mass parameters $m^2_W$, $m^2_H$ in
$k_{\text{CO}}$-units, will depend only on $P$, $\xi$, $c_{\text{CO}}$ and $g^2_{\text{CO}}$
for sufficiently big $k_{\text{CO}}$ because of universality. We choose
$g^2_{\text{CO}}$ such that $g^2_{\text{CO}}$ acquires a standard-model-like value;
since the gauge running is logarithmically slow, $g^2_{\text{CO}}$ and
$g^2_{\text{F}}$ do not differ significantly. The parameter $c_{\text{CO}}$
should be chosen sufficiently small in order to justify that it is
ignored above $k_{\text{CO}}$, but also sufficiently large in order to
drive the system rapidly into the SSB regime; in practice,
c_{\text{CO}} = -0.01$ was used for our estimates. For the running
below $k_{\text{CO}}$, we approximate the full effective potential by a polynomial expansion about its minimum; order-$\phi^8$
polynomials turned out to be sufficient.

Figure 2 shows the Higgs-to-gauge boson mass ratio as
a function of the UV FP parameters $\xi$ and $P$ for $SU(N = 2)$. The mass ratio turns out to be an increasing linear
function of $\xi$. The slope depends on $P$ and decreases for
larger $P$. This suggests that any desired physical
value of the mass ratio corresponds to a one-dimensional
section through the $(\xi, P)$ plane, spanning the set of AF
theories. Comparing the IR results at the Fermi scale to
the initial values at $k_{\text{CO}}$, we find that the flow towards
the IR essentially preserves the mass ratio already set by
the initial condition at $k_{\text{CO}}$. In our scans we observed
an almost $(\xi, P)$-independent ratio $k_{\text{CO}}/k_F$ of about one
order of magnitude.

Let us finally explore the physical properties of
Coleman-Weinberg-like trajectories which are defined as
those with a zero relevant component [21]. We observe
that these trajectories can only end in the SSB phase in the
IR if the dimensionless effective potential $f(x)$ is suffi-
ciently deep inside the SSB regime in the UV. We find
that this is possible in the interval $P \in [1/2, 1]$ for suffi-
ciently small $\xi$ (our estimates for $P \geq 1$ are inconclusive).
This translates into a maximum value of the Higgs-to-
gauge boson mass parameter ratio, $m^2_H/4m^2_W \approx 0.0011$.

We emphasize that the measured value of the Higgs
boson mass can be understood as essentially driven by
top fluctuations [28, 31]. The observation of gener-
ically small Higgs masses (ignoring bound-state effects
[20]) in the pure nonabelian Higgs model along Coleman-
Weinberg trajectories thus appears to fit the require-
ments of a realistic model. These trajectories may also be useful to construct a natural large hierarchy in the
standard model via the Higgs portal [31]; in such a sce-
nario, our nonabelian Higgs model could play the role of a
UV-complete hidden sector.

In summary, we have discovered a three-parameter
family of AF nonabelian Higgs models. If usable in the
context of the full standard model or GUTs, our RG tra-
jectories do not suffer from $\phi^4$ triviality and thus are can-
didate building blocks for a UV-complete quantum field
theory. A two-parameter subset of Coleman-Weinberg-
like AF trajectories is even free from the naturalness
problem. We expect these trajectories to be directly ac-
cessible to lattice methods: simulations with bare po-
tentials along the marginal-relevant eigenperturbations
should lie on a line of constant physics. Still, rather large
lattices may be necessary to resolve the Fermi scale as
well as the crossover to the asymptotic regime.

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