Achieving precision cosmology with gravitational-wave bright sirens from SKA-era pulsar timing arrays

Ling-Feng Wang,1 Guang-Peng Zhang,1 Yue Shao,1 and Xin Zhang2,1,3
1Department of Physics, College of Sciences, Northeastern University, Shenyang 110819, China
2MOE Key Laboratory of Data Analytics and Optimization for Smart Industry, Northeastern University, Shenyang 110819, China

Nanohertz gravitational waves (GWs) generated by the individual inspiraling supermassive black hole binaries (SMBHBs) in the centers of galaxies may be detected by pulsar timing arrays (PTAs) in the future. The GW signals from individual SMBHBs can be employed as standard sirens to measure absolute cosmic distances, and further provide constraints on cosmological parameters via the distance–redshift relation. Namely, these SMBHBs with known redshifts can serve as bright sirens. In this paper, we analyze the ability of SKA-era PTAs to detect existing SMBHB candidates by means of simulating the timing residuals of pulsar signals, and use the mock data of SMBHB bright sirens to perform cosmological parameter estimations. We find that once the root mean square of timing residuals (σt) could be reduced to 20 ns, about only 100 millisecond pulsars are needed to provide a tight constraint on the Hubble constant (H0) with the precision close to 1%, which meets the criterion of precision cosmology. We further show that the SMBHB bright sirens can effectively break the cosmological parameter degeneracies inherent in the CMB data, and thus improve the constraint precision of the equation of state of dark energy (w) to 3.5%, which is comparable with the result of Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO. We conclude that the bright sirens from SKA-era PTAs will play an important role in breaking the cosmological parameter degeneracies and exploring the nature of dark energy.

I. INTRODUCTION

Gravitational waves (GWs) are ripples in the fabric of spacetime, produced when large masses accelerate. The detection of GW150914 [1], the first GW event of binary black hole (BBH) coalescence, has marked the beginning of the era of GW astronomy. The luminosity distances of GW sources, encoded in the amplitudes of GW waveforms, can be inferred from GW measurements, usually referred to as “standard sirens” [2–4]. If the electromagnetic (EM) counterparts of GW events are detected, the distance–redshift relation can be established to constrain cosmological parameters [5–8]. These GW events with associated EM counterparts are referred to as “bright sirens” [9, 10].

Typical sources of bright sirens are provided by the events of binary neutron star (BNS) mergers. This is because BNS mergers are accompanied by productions of kilonovae and short gamma-ray bursts, which can serve as EM counterparts of GW events [5, 6, 11, 12]. GW170817, the first detected BNS coalescence event [13–15] and the only available bright siren till now, gives a ∼14% measurement of the Hubble constant H0 [16]. In the future, the third-generation ground-based GW detectors (the Einstein Telescope [17, 18] and the Cosmic Explorer [19, 20]) enable ones to acquire hundreds of available BNS bright sirens with a 10-year observation [21–25].

In addition, there are another kind of bright sirens served by the supermassive black hole binaries (SMBHBs), because they can produce observable EM emissions due to their surrounding gas-rich environments and external magnetic fields [26–33]. Actually, the observations of SMBHB bright sirens are extremely challenging, but it is still expected that such GW-EM observations can be realized in the forthcoming decades. The detections of SMBHBs with masses of ∼108–109 M⊙ can be realized by the planned space-borne GW observatories, e.g., the Laser Interferometer Space Antenna [34–39], Taiji [40–44], and TianQin [45–49]. The detections of SMBHBs with masses of ∼108–1010 M⊙ require some natural galactic-scale detector comprised of an array of millisecond pulsars (MSPs), usually referred to as “Pulsar Timing Array” (PTA). There have been a series of works discussing the SMBHB bright sirens based on space-borne observatories [50–58], but using PTAs to observe SMBHB bright sirens still deserves further investigation.

When GWs pass between pulsars and the Earth, the paths of the pulsar signals change, thus affecting the time of arrivals (ToAs) of radio pulses. Nanohertz GWs from the individual inspiraling SMBHBs could be detected by monitoring the spatially correlated fluctuations of ToAs induced by GWs. With the concept proposed decades ago [59–62], there are three major PTA projects, namely, the Parkes Pulsar Timing Array [63, 64], the European Pulsar Timing Array [65], and the North American Nanohertz Observatory for Gravitational Waves [66]. They have also been combined to form the International Pulsar Timing Array [67, 68] aimed at significantly enhancing sensitivities. So far, most of the efforts have been devoted to detecting the stochastic gravitational wave background (SGWB) [69–71]. The detection of individ-
The ToA data are obtained via monitoring the pulses from MSPs over a decade or longer with typical cadence of biweekly to monthly. The GW signals are detected in the timing residuals of MSPs by removing model-predicted ToAs from the observational ToA data. The root mean square (rms) of timing residuals reflects the stability of pulsars and the quality of the ToA data.

The timing residuals induced by a single GW source, measured at time $t$ on the Earth, can be written as

$$s(t, \hat{\Omega}) = F_+(\hat{\Omega}) \Delta A_+ (t) + F_\times(\hat{\Omega}) \Delta A_\times (t),$$  \hspace{1cm} (1)

where $\hat{\Omega}$ is a unit vector pointing from the GW source to the observer. The two functions $F_+(\hat{\Omega})$ and $F_\times(\hat{\Omega})$ are the geometric factors,

$$F_+(\hat{\Omega}) = \frac{1}{4(1 - \cos \theta)} \left\{ (1 + \sin^2 \beta) \cos^2 \beta_p \cos [2(\alpha - \alpha_p)] - \sin 2\beta \sin 2\beta_p \cos (\alpha - \alpha_p) + \cos^2 \beta (2 - 3 \cos^2 \beta_p) \right\},$$

$$F_\times(\hat{\Omega}) = \frac{1}{2(1 - \cos \theta)} \left\{ \cos \beta \sin 2\beta_p \sin (\alpha - \alpha_p) - \sin \beta \cos^2 \beta_p \sin [2(\alpha - \alpha_p)] \right\}. \hspace{1cm} (2)$$

$F_+(\hat{\Omega})$ and $F_\times(\hat{\Omega})$ are equivalent to the antenna pattern functions used in the context of laser interferometric GW detection [97]. $(\alpha, \beta)$ and $(\alpha_p, \beta_p)$ are the sky locations of GW sources and pulsars, respectively. $\theta$ is the angle between the GW source and the pulsar with respect to the observer,

$$\cos \theta = \cos \beta \cos \beta_p \cos (\alpha - \alpha_p) + \sin \beta \sin \beta_p. \hspace{1cm} (3)$$

In Eq. (1), $\Delta A_+ (t)$ and $\Delta A_\times (t)$ include two terms, i.e., the Earth term $A_{(+,\times)} (t)$ and the pulsar term $A_{(+,\times)} (t_p)$,

$$\Delta A_{(+,\times)} (t) = A_{(+,\times)} (t) - A_{(+,\times)} (t_p), \hspace{1cm} (4)$$

$$t_p = t - d_p (1 - \cos \theta)/c, \hspace{1cm} (5)$$

where $d_p$ represents the pulsar distance, and $t_p$ is the time at which GW passes the MSP. The specific forms of the Earth term and the pulsar term depend on the types of GW sources. SMBHBs in circular orbits are expected to generate pulsar timing signals with the following forms,

$$A_+ (t) = \frac{h_0(t)}{2 \pi f(t)} \left\{ (1 + \cos^2 \iota) \cos 2\psi \sin[\phi(t) + \phi_0] \right\}, \hspace{1cm} (6)$$

$$A_\times (t) = \frac{h_0(t)}{2 \pi f(t)} \left\{ (1 + \cos^2 \iota) \sin 2\psi \sin[\phi(t) + \phi_0] \right\}. \hspace{1cm} (7)$$

Here, $\iota$ is the inclination angle of the binary orbit, $\psi$ is the GW polarization angle, $\phi_0$ is a phase constant, and the GW strain amplitude $h_0$ is given by

$$h_0 = 2 \left( \frac{G M_2}{c^4 (\pi f)^{2/3} d_\perp} \right)^{5/3}. \hspace{1cm} (8)$$

Here, $M_2 = M_c(1 + z)$ represents the redshifted chirp mass and $M_c$ is the chirp mass defined as $M_c = \eta^{3/5} M$. $M = m_1 + m_2$ is the total mass of binary system with
component masses $m_1$ and $m_2$, and $\eta = m_1 m_2 / (m_1 + m_2)^2$ is the symmetric mass ratio. $d_l$ represents the luminosity distance to the GW source.

The frequencies and phases of GWs are given by

$$f(t) = \left[ f_0^{-8/3} \frac{256}{5} \pi^{8/3} \left( \frac{G M_z}{c^3} \right)^{5/3} t \right]^{-3/8},$$

$$\phi(t) = \frac{1}{16} \left( \frac{G M_z}{c^3} \right)^{-5/3} \left\{ (\pi f_0)^{-5/3} - [\pi f(t)]^{-5/3} \right\},$$

where $f_0$ is the GW frequency at the time of our first observation, with $f_0 = 2 f_{\text{orb}}$. Here, $f_{\text{orb}} = (2 \pi T)^{-1}$ is the orbit frequency, and $T$ is the orbital periods of SMBHB candidates taken from Refs. [88–90, 98–102].

We define the signal-to-noise ratio (SNR) of the GW signal detected by a PTA as

$$\rho^2 = \sum_{j=1}^{N_p} \sum_{i=1}^{N} \left[ \frac{s_j(t_i)}{\sigma_{t,j}} \right]^2,$$

where $N$ is the total number of data points for each MSP, $N_p$ is the number of MSPs, $s_j(t_i)$ is the timing residual of $j$-th MSP at time $t_i$ [see Eq. (1)], and $\sigma_{t,j}$ is the rms timing residual of $j$-th MSP.

In this work, we adopt the Fisher information matrix to estimate the parameters of GW sources. For a PTA including $N_p$ independent MSPs, the Fisher matrix is

$$\Gamma_{ab} = \sum_{j=1}^{N_p} \sum_{i=1}^{N} \frac{\partial s_j(t_i)}{\sigma_{t,j} \partial p_a} \frac{\partial s_j(t_i)}{\sigma_{t,j} \partial p_b},$$

where $p_a$ and $p_b$ denote the free parameters to be estimated. The error of the parameter $p_a$ can be estimated as $\Delta p_a = (\Gamma^{-1/2})_{aa}$, once the Fisher matrix $\Gamma_{ab}$ is calculated out. Here, nine parameters are taken into account in the Fisher matrix, including eight parameters of a GW source and the pulsar distance, i.e., $M_c$, $\alpha$, $\beta$, $t$, $\psi$, $\phi_0$, $f_0$, $d_l$, $d_p$.

We adopt the current available sample of SMBHB candidates (154) obtained from various characteristic signatures. Among these SMBHB candidates, most (149) are obtained via periodic variations in their light curves [89, 90, 98], and the others are Mrk 231 from Ref. [99], NGC 5548 from Ref. [100], OJ 287 from Ref. [88], SDSS J0159+0105 from Ref. [101], and Ark 120 from Ref. [102].

Fig. 1 shows the positions of the selected 500 MSPs on the sky. We select 500 pulsars within 3 kpc from the Earth obtained from the ATNF pulsar catalog [103].

For PTA observations, the capability of detecting GWs is affected by some dominating factors, including the number of MSPs, the rms timing residual, the cadence of monitoring ToAs, and the observation time span. For the number of MSPs used in the PTA observations, we are still uncertain about how many stable MSPs could be available in the SKA-era PTAs, and therefore in the current literature [87, 104] actually different MSP numbers are assumed. In this paper, we select 100, 200, and 500 MSPs within 3 kpc from the Earth, obtained from the Australia Telescope National Facility (ATNF) pulsar catalog [103], to construct PTAs. Fig. 2 shows the positions of the selected 500 MSPs on the sky. For the rms timing residual, it consists of white noise and red noise. The red noise can be ignored by assuming that it can be mitigated to a low noise level or it does not influence single GW detection using special techniques [104].
The main white noise includes radiometer noise and jitter noise. According to the analysis for SKA in Ref. [105], the jitter noise will dominate over the radiometer noise for the majority of bright pulsars, and the total white noises of these pulsars are around 10 – 50 ns. In this paper, we set $\sigma_t = 20$ ns for SKA-era PTAs, considering that the red noise is ignored and the white noise only consists of the radiometer noise and the jitter noise, consistent with Ref. [104]. We also consider another case of $\sigma_t = 100$ ns for comparison. Following Ref. [87], we assume that the ToA data are obtained via monitoring the pulses from MSPs with typical cadence of two weeks and the observation span is 10 years.

The detection curves of SKA-era PTAs with six different settings of $N_p$ and $\sigma_t$ are plotted in Fig. 3, by using the hasasia package [106, 107]. Over the 10-year observational time span of PTAs, only a small early portion of the SMBHB inspiral could be detected. For example, according to our estimation by Eq. (9), the variation of the GW frequency of inspiraling SMBHB with $M = 10^6 M_\odot$ and $f_0 = 10^{-7}$ Hz in a 10-year observational time span is $4.36 \times 10^{-9}$ Hz. This variation is so tiny that the GW strain amplitude during the time span changes little, so we only plot the strain amplitudes when $f = f_0$, represented by the colored points. As $N_p$ increases and $\sigma_t$ decreases, the more sensitive detection curves enable ones to detect more SMBHB candidates. The dotted curves ($\sigma_t = 20$ ns) are obviously lower than the solid curves ($\sigma_t = 100$ ns), indicating that the rms timing residual has a dominating effect on the detections of SMBHBs. In the case of $N_p = 500$ and $\sigma_t = 20$ ns, about half of the SMBHB candidates are above the detection curves. It needs to be emphasized here that not all the SMBHB candidates above the detection curve could be detected certainly. Only those with high enough SNRs can be considered to be reliable GW events. Therefore, we need to further calculate SNR for each SMBHB candidate.

We calculate SNR for each SMBHB candidate according to Eq. (11). The SMBHB candidates with SNR $\rho > 10$ are considered to be detected by PTA and will be utilized to simulate the standard siren data. Fig. 4 shows the luminosity distance errors of the SMBHB candidates as a function of SNR. $\sigma_t$ is set to 100 ns in the upper panel and is set to 20 ns in the lower panel. The detected SMBHBs in the lower panel are much more than those in the upper panel, and even for the same SMBHB candidate, the lower panel shows higher SNR than the upper panel. It is shown that $\sigma_t$ has a significant impact on the detection of SMBHB. The specific numbers of the detected SMBHBs are listed in the third column of Table I. In the case of $\sigma_t = 100$ ns, only 4 and 12 SMBHBs are detected with 100 and 500 MSPs, respectively. In the case of $\sigma_t = 20$ ns, the number of detected SMBHBs are 20 and 37 with 100 and 500 MSPs, respectively, increasing a lot compared with the case of $\sigma_t = 100$ ns. Although the number of MSPs can also affect the detection number of SMBHBs, for example, the detection number increases from 4 ($N_p = 100$) to 12 ($N_p = 500$), its effect is not as obvious as $\sigma_t$. Actually, only a few dozen MSPs are available in the current PTA observations, because the requirements for the pulsars used for PTA observations are demanding. Our results show that too many MSPs may not be necessary for detecting individual SMBHBs, if the timing measurement could reach high enough precision.

Fig. 4 also shows the error of luminosity distance decreasing significantly with the increase of SNR. We find that increasing the number of MSPs can only reduce $\Delta d_L/d_L$ slightly. For example, in the case of $\sigma_t = 100$ ns, the lowest values of $\Delta d_L/d_L$ are close to $10^{-2}$ ($N_p = 100$) and $3 \times 10^{-2}$ ($N_p = 500$). A 5-fold increase in the number of MSPs (increase from 100 to 500) can only reduce $\Delta d_L/d_L$ by less than an order of magnitude. At the same time, we also find that lower $\sigma_t$ could evidently reduce $\Delta d_L/d_L$. In the case of $N_p = 100$, the values of $\Delta d_L/d_L$ are around $10^{-2}$ ($\sigma_t = 100$ ns) and $10^{-3}$ ($\sigma_t = 20$ ns). That is to say, $\sigma_t$ decreasing from 100 ns to 20 ns could reduce $\Delta d_L/d_L$ by an order of magnitude. This indicates that the rms timing residual is the most important factor for reducing the errors of luminosity distances. The reduction of $\Delta d_L/d_L$ will be finally reflected as the improvement of the constraint precision of cosmological parameters.

III. COSMOLOGICAL PARAMETER ESTIMATION

Now we construct the catalog of simulated bright sirens to constrain cosmological parameters. We change the values of $N_p$ and $\sigma_t$ to study their effects on cosmological parameter estimations. Six sets of standard siren cat-
FIG. 4: The measurement precision of luminosity distance ($\Delta d_L/d_L$) as a function of SNR. The data points represent the SMBHB candidates with SNR > 10, which are deemed to be detected by PTA. Upper panel and lower panel correspond to the cases of $\sigma_t = 100$ ns and $\sigma_t = 20$ ns, respectively. In each panel, $N_p$ increases in order of 100, 200, 500 from top to bottom. The impacts of $N_p$ and $\sigma_t$ on the SMBHB detections are distinctly revealed.

FIG. 5: Simulated standard-siren catalog from the SMBHB candidates. The luminosity distances of $d_L$ are calculated based on the $\Lambda$CDM model in which the fiducial values of cosmological parameters are set to be the Planck 2018 results. The error bars are the 1$\sigma$ (68.3% confidence level) uncertainties of $d_L$, obtained from the Fisher matrix.

catalogs are simulated, corresponding to different numbers of MSPs ($N_p = 100, 200, 500$) and different rms timing residuals ($\sigma_t = 100$ ns, 20 ns). Two representative catalogs of bright sirens are shown in Fig. 5. The numbers of bright sirens in the case of $\sigma_t = 20$ ns are much larger than those in the case of $\sigma_t = 100$ ns for the same number of MSPs. Improved SNRs of the GW events by decreasing $\sigma_t$ reduces the measurement errors of luminosity distances, which can be seen from the error bars of the data points. In the following, we shall use $\sigma(\xi)$ and $\varepsilon(\xi)$ to represent the absolute error and the relative error of the parameter $\xi$, respectively, with $\varepsilon(\xi) = \sigma(\xi)/\xi$. For the sake of simplification, the SMBHB bright siren data are hereafter called the PTA data.

We first take the base $\Lambda$CDM model as an example to discuss the constraint results under the six sets of catalogs. From Fig. 6, we see that the contours gradually become smaller with the increase of $N_p$ and the decrease of $\sigma_t$. The case of $N_p = 500$ and $\sigma_t = 20$ ns gives the tightest constraints with $\varepsilon(\Omega_m) = 8.0\%$ and $\varepsilon(H_0) = 0.9\%$. Actually, the precision of $H_0$ is already close to 1% in the case of $N_p = 100$ and $\sigma_t = 20$ ns, i.e. $\varepsilon(H_0) = 1.1\%$, which meets the standard of precision cosmology. That is to say, 100 MSPs are sufficient to precisely measure $H_0$ provided that $\sigma_t$ could reach 20 ns. If $\sigma_t$ can only reach 100 ns, at least 500 MSPs are needed to provide a 1.3% measurement on $H_0$.

After comparing the performances of the six catalogs in the $\Lambda$CDM model, we further show the capability of breaking the parameter degeneracy in the $w$CDM model. Fig. 7 shows that the CMB data and the PTA data have distinct orientations in the $w$–$H_0$ plane. The intersection of the contours leads to the breaking of parameter degeneracy. For the EoS parameter of dark energy ($w$), the CMB data cannot provide tight constraints, due to the fact that CMB encodes the information of the early universe, while dark energy becomes dominant in the late universe. Although the PTA data either cannot constrain $w$ well, it can provide tight constraints on $H_0$, thus breaking the degeneracy between the parameters $w$ and $H_0$. The constraint results are shown in Table II. In the case of $N_p = 100$ and $\sigma_t = 100$ ns, the combination of the CMB and PTA data gives the relative error $\varepsilon(w) = 4.8\%$. On this basis, we further increase the number of MSPs...
and reduce the rms timing residual. We find that decreasing \( \sigma_t \) from 100 ns to 20 ns or increasing \( N_p \) from 100 to 500 can make the relative error of \( w \) reach around 3.5\% – 3.7\%, which is comparable with the result of Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO [108].

In Table III, we list the constraint results of the best case with \( N_p = 500 \) and \( \sigma_t = 20 \) ns. The combination of the CMB and PTA data gives the relative error \( \varepsilon(w) = 3.4\% \). Compared to the case of using CMB data alone, the addition of the PTA data reduces the relative error of \( w \) by 84.5\%. In the \( w_0\omega_0\)CDM model with two dark-energy EoS parameters, the PTA data also show a strong capability of breaking the parameter degeneracies. Compared with the CMB data, the CMB+PTA data reduce the relative error of \( w_0 \) from 0.62 to 0.43, and make the absolute error of \( w_0 \) reach 0.3.

### IV. CONCLUSION

GW standard sirens are a late-universe cosmological probe with great potential to precisely measure absolute cosmological distances. The standard sirens with observed EM counterparts, referred to as bright sirens, can be utilized to establish the \( d_L-z \) relation, by which cosmological parameters can be constrained. GWs emitted from the individual inspiraling SMBHBs may be detected by PTAs in the nanohertz band. In this paper, we analyse the ability of SKA-era PTAs to detect some existing SMBHB candidates by means of simulating the timing residuals of pulsar signals. Combined with the measured redshifts of these SMBHB candidates, we use the MCMC method to show how the SMBHB bright sirens can measure the cosmological parameters and play a key role in

| \( N_p \) | \( \sigma_t \) [ns] | \( \varepsilon(\Omega_m) \) | \( \varepsilon(H_0) \) | \( \varepsilon(w) \) |
|---|---|---|---|---|
| CMB | - | - | 0.189 | 0.093 | 0.220 |
| PTA | 500 | 100 | 0.306 | 0.079 | 0.719 |
| CMB+PTA | 500 | 100 | 0.193 | 0.066 | 0.507 |

| \( N_p \) | \( \sigma_t \) [ns] | \( \varepsilon(\Omega_m) \) | \( \varepsilon(H_0) \) | \( \varepsilon(w) \) |
|---|---|---|---|---|
| CMB+PTA | 500 | 100 | 0.023 | 0.012 | 0.037 |
| PTA | 100 | 20 | 0.020 | 0.010 | 0.035 |
TABLE III: The absolute errors (1σ) and the relative errors of the cosmological parameters in the wCDM model using the CMB, PTA, and CMB+PTA data. The PTA data are simulated from a PTA with \( N_p = 500 \) and \( \sigma_t = 20 \) ns.

| Data  | \( \varepsilon(\Omega_m) \) | \( \varepsilon(H_0) \) | \( \varepsilon(w) \) | \( \varepsilon(\Omega_m) \) | \( \varepsilon(H_0) \) | \( \sigma(w_0) \) | \( \sigma(w_a) \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| CMB   | 0.027          | 0.009          | 0.189          | 0.093          | 0.220          | 0.184          | 0.095          | 0.62          | –              |
| PTA   | 0.080          | 0.010          | 0.136          | 0.043          | 0.387          | –              | –              | –              | –              |
| CMB+PTA | 0.017          | 0.006          | 0.019          | 0.010          | 0.034          | 0.086          | 0.042          | 0.43          | 0.3            |

breaking the cosmological parameter degeneracies inherent in the CMB data.

We select 100, 200, and 500 MSPs within 3 kpc from the Earth from the ATNF pulsar catalog to construct PTAs. We assume that the rms timing residual only consists of the white noise, and consider two cases of \( \sigma_t = 100 \) ns and \( \sigma_t = 20 \) ns. The observational time spans for all pulsars are 10 years with evenly spaced 2 weeks ToA measurements. We plot the detection curves of SKA-era PTAs for a 10-year observation, and show the strain amplitudes of SMBHBs, which shows that about half of the candidates may be detected by PTA with \( \sigma_t = 100 \) ns and most of the candidates may be detected by PTA with \( \sigma_t = 20 \) ns. There are two factors, i.e., \( N_p \) and \( \sigma_t \), mainly affecting the detection SNRs of SMBHBs. We find that increasing the number of MSPs can only reduce \( \Delta d_L/d_L \) slightly. A 5-fold increase in the number of MSPs (increasing from 100 to 500) can only reduce \( \Delta d_L/d_L \) by less than an order of magnitude. Lower \( \sigma_t \) could evidently reduce \( \Delta d_L/d_L \). \( \sigma_t \) decreasing from 100 ns to 20 ns could reduce \( \Delta d_L/d_L \) by an order of magnitude. This indicates that the rms timing residual \( \sigma_t \) is the most important factor for reducing the errors of luminosity distances.

We simulate some sets of the SMBHB bright siren data with different \( N_p \) and \( \sigma_t \) to constrain cosmological models. We find that, provided that \( \sigma_t \) could reach 20 ns, the precision of \( H_0 \) can be close to 1% in the \( \Lambda \)CDM model, even if PTA only includes 100 MSPs (\( N_p = 100 \)), which meets the standard of precision cosmology. We also find that the SMBHB bright sirens could effectively break the parameter degeneracies inherent in CMB. In the case of \( N_p = 100 \) and \( \sigma_t = 20 \) ns, the combination of the CMB and PTA data makes the constraint precision of \( w \) reach 3.5% in the wCDM model, which is comparable with the result of Planck 2018 TT,TE,EE+lowE+lensing+SNe+BAO [108].

Our results show that too many MSPs may not be necessary for measuring cosmological parameters, once a small enough \( \sigma_t \) can be realized in the SKA-era. We conclude that PTAs in the SKA-era will become a useful method for precisely measuring cosmological parameters and exploring the nature of dark energy.

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