Multisubband transport and magnetic deflection of Fermi electron trajectories in three terminal junctions and rings

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Abstract
We study the electron transport in three terminal junctions and quantum rings looking for the classical deflection of electron trajectories in the presence of intersubband scattering. We indicate that although the Aharonov–Bohm oscillations and the Lorentz force effects co-exist in the low subband transport, for higher Fermi energies a simultaneous observation of both effects is difficult and calls for carefully formed structures. In particular, in quantum rings with channels wider than the input lead the Lorentz force is well resolved but the Aharonov–Bohm periodicity is lost in chaotic scattering events. In quantum rings with equal lengths of the channels and T-shaped junctions the Aharonov–Bohm oscillations are distinctly periodic but the Lorentz force effects are not well pronounced. We find that systems with wedge-shaped junctions allow for observation of both the periodic Aharonov–Bohm oscillations and the magnetic deflection.

(Some figures may appear in colour only in the online journal)

1. Introduction
External magnetic field ($B$) applied perpendicular to the plane of confinement of a two-dimensional electron gas (2DEG) deflects the trajectories of the electrons carrying the current flow. The deflection by the Lorentz force—responsible for the Hall effect—occurs also at the nanoscale and imprints classical features on the quantum transport phenomena. The action of the Lorentz force was first detected in mesoscopic cross junctions [1]. Cyclotron deflection of the electron trajectory was predicted for coherent ballistic electron injection through a quantum point contact (QPC) [2]. This deflection was later observed [3] by spatially resolved scanning gate microscopy [4]. Semiclassical electron orbits [5, 6] are observed in magnetic focusing experiments [7] which detect peaks of conductance ($G$) between two QPCs for $B$ values which make the distance between the contacts equal to an integer multiple of the cyclotron diameter. A theoretical description of the magnetic injection in T-shaped and wedge junctions was given in [8]. Recently, magnetic deflection in multiterminal quantum billiards was also studied [9].

For open quantum rings the classical deflection of the electron trajectories by the Lorentz force competes with quantum interference effects [10], which are strong provided that the electron wavefunction passes with equal amplitude through both the arms of the ring. The preferential electron injection to one of the arms of the quantum ring by the Lorentz force reduces the visibility of the Aharonov–Bohm (AB) conductance oscillations in high $B$ [10]. Since low visibility of the AB oscillations may also result from decoherence, a conclusive experiment on the Lorentz force effect was proposed for a three terminal quantum ring [11]. According to the calculations [11] the vanishing AB conductance oscillation at high $B$ should be accompanied by an imbalance of the electron transport probability to the left and right output leads. This behavior was indeed observed in subsequent conductance measurements [12] for a three terminal quantum ring.

The theoretical description of the Lorentz force for quantum rings [10, 11, 13, 14] and the experiment [12] studied the transport in the lowest subband of the transverse quantization. However, most experiments on the AB effects in quantum rings [15] correspond to multisubband transport. The
wave packet dynamics in the second subband was described in [16], where, however, a single-output-lead system with very narrow channels was considered, in which the Lorentz force effects are negligible. The purpose of this paper is to describe the effects of the Lorentz force in a three terminal system when the stationary current flow at the Fermi level goes through several subbands. A basic motivation for this study is the properties of the asymptotic states within the channels (see below). The charge density shift in the first subband is shifted to the edge of the channel which is preferred by the Lorentz force. However, the charge density shift in the second subband is just the opposite. Therefore, the Lorentz force effect in multiband transport needs to be clarified.

Below, we solve the scattering problem for several three terminal systems using the wavefunction picture [17] of the current flow and evaluate the linear conductance in the second subband is just the opposite. Therefore, the Lorentz force deflection leaves its signature in the low frequency part of the transform. We discuss the role of the width of the channels and the types of junctions that allow the classical Lorentz force and the AB oscillations to co-exist.

2. Theory

In this paper we consider systems with three leads attached [11, 12]—see figure 1. The input channel is the one at the bottom of the figure. In multiterminal devices, the phase and visibility of AB oscillations are affected by the current/voltage measurement configurations (see the discussion in [19]). The configuration that we consider in this work is compatible with the experiment of [12], in which the input lead is biased with respect to both the output terminals kept at the same potential. In [12] the current was measured as a function of the small bias to extract the linear conductance. In the Landauer–Büttiker approach the current flowing through terminal $p$ is given by [20]

$$I_p = \int dE i_p (E), \quad (1)$$

where $i_p (E) = \frac{2e}{\hbar} \sum_q T^{pq} (E) [f_q (E) - f_p (E)]$, and $T^{pq}$ is the summed (see below) transfer probability for each subband of terminal $q$ to each subband of terminal $p$, and $f_p$ is the Fermi distribution function $f_p (E) = [\exp (E - E_F^p / k_B T) + 1]^{-1}$. For the Fermi energy kept equal in the left and right contacts (figure 1) the electron transfer probability from the left to the right lead is irrelevant for the description of the flowing currents. Consequently we have $i_l (E) = \frac{2e}{\hbar} T^l (E) [f_l (E) - f_r (E)]$, where $T^l$ is the summed electron transfer probability from the input to the output lead, and the factor of 2 accounts for the spin degeneracy of the Fermi level for the neglected spin Zeeman effect. In the limit of small bias $E_F - E_F^l = eV$ one develops the Fermi distribution functions in the Taylor series $f_l = f_0 + \frac{\partial f_0}{\partial E} |_{E=E_F} (E - E_F) + \cdots$, where $f_0$ is the distribution with Fermi energy $E_F$. Upon subtraction one obtains the current as a linear function of bias

$$I_l = \frac{2e^2}{\hbar} V \int dE T^l (E) \left(-\frac{\partial f_0}{\partial E}\right). \quad (2)$$

In the $T = 0$ K limit, which is mostly considered below, one obtains

$$I_l = \frac{2e^2}{\hbar} T^l (E_F) V, \quad (3)$$

i.e. the linear conductance of the left lead $g_l = I_l / V$ is a Fermi level property given by the summed transfer probabilities from all the channels of the input lead to the channels of the left output lead.

In order to determine the transfer probabilities below we solve the scattering problem for a given Fermi energy $E_F$. For this purpose we consider the effective mass Schrödinger equation

$$\left[\frac{1}{2m^*} (\mathbf{p} + e\mathbf{A})^2 + W (\mathbf{r})\right] \Psi (\mathbf{r}) = E_F \Psi (\mathbf{r}), \quad (4)$$

with the assumption that the electron comes to the scattering region from a given subband $\mu$ of the input channel. In equation (4) $m^*$ and $W$ are the electron effective mass and the confinement potential, respectively. We apply $m^* = 0.067 m_0$ for GaAs and for W we take 0 inside the channels and 200 meV outside, which corresponds to GaAs/AlGaAs structure.

For the strong confinement in the growth ($z$) direction that is present in 2DEG all the electrons occupy the same state of the vertical quantization. The scattering problem can then be solved using a two-dimensional model, which we apply below. In order to set the boundary conditions we need first to solve the Hamiltonian eigenequation in each of the leads. For the Lorentz gauge $\mathbf{A} = (A_x, A_y, 0) = (0, Bx, 0)$, the electron eigenfunctions in the leads are separable into products of
we discuss the electron transfer probability to the left or subband. The eigenequation for the transverse wavefunction transverse \((x)\) and longitudinal \((y)\) wavefunctions \(\psi_{k_\mu} = \exp(\imath k_\mu y)\Psi_{k_\mu}(x)\), where \(k_\mu\) is the wavevector of the \(\mu\) th subband. The eigenequation for the transverse wavefunction reads
\[
\left[ \frac{1}{2m^*} \left( -\hbar^2 \frac{\partial^2}{\partial x^2} + (eBx + \hbar k)^2 \right) + W(x) \right] \Psi(x) = E_F \Psi(x).
\]

The dispersion relation calculated for a channel of width 64 nm is displayed in figure 2(b). A given Fermi energy fixes the number of subbands participating in the transport along with the wavevectors \(k_\mu\). Application of the external magnetic field breaks the parity symmetry of the transverse wavefunctions (cf. figures 2(c) and (d)). For \(B > 0\) the term \(2eB\hbar k\) of the transverse Hamiltonian shifts the lowest subband solution to the left of the channel for the electron moving up the channel \((k > 0)\), in agreement with the orientation of the classical Lorentz force (see figure 2(d)).

The backscattered \((b)\) and transferred \((t)\) amplitudes are found via solution of the Schrödinger equation with boundary conditions (6) and (7). The solution employs the finite difference approach. The discretization of the equations allows for both the appearance of evanescent modes in the Fermi level wavefunction within the scattering region and the multisubband scattering.

The electron transfer probability from subband \(\mu\) of the input channel to subband \(j\) of the output lead is then given by
\[
T^{(t)}_{\mu,j} = \sum_{l|} \frac{\theta_{l|}}{\theta_{l}} |\bar{\Psi}_{l|,j}(x)|^2, 
\]
where the \(\theta\) s are the fluxes of the probability density currents integrated across the channels for the asymptotic Hamiltonian eigenstates. In this paper we discuss the electron transfer probability to the left or right output leads for an electron incident from subband \(\mu\), which is given by
\[
T^{(l)}_{\mu} = \sum_{j} T^{(l)}_{\mu,j}. 
\]
Figure 3. The electron densities calculated for $E_F = 15$ meV for $B = 0$ ((a), (c), (e)) and $B = 1$ T ((b), (d), (f)), for electrons incident from the lowest ((a), (b)), second ((c), (d)) and third ((e), (f)) subbands.

Transfer functions $T^{l/r} = \sum \mu T^{l/r}_{\mu}$, which are proportional to the conductance of both the output leads. The conductance can be then evaluated as $G^{l/r} = \frac{2e^2}{h} T^{l/r}$.

3. Results

Below, we consider systems with an input lead that is 64 nm wide, unless explicitly stated otherwise.

3.1. Wider output channels and wedge junctions

Let us first discuss a type of junction which corresponds to the calculations of [2]—see figure 2(a). The electrons are injected from the input lead of width 64 nm to a much wider perpendicular channel. The solutions of the scattering problem are plotted in figures 2(e) and (h). For $B \gg 0$ the electrons are directed to the left output lead (see also figure 3) independently of the number of subbands participating in the charge transport and for each subband from which the electron comes to the junction. Since for each incident subband at $B > 0$ the electron tends to pass to the left output channel, the overall slope of $T^l(B)$ increases with the number of occupied subbands.

The transfer probabilities for a quantum ring built in a similar manner (output channels and the ring wider than the input lead—figure 4(a)) are shown in figures 4(b), (d), (f) for one, two and three subbands appearing at the Fermi level. The transfer and backscattering probabilities for the ring additionally contain rapid oscillations which are due to the interference effects for the electron transfer amplitudes going through the left and right arms of the ring. However, the results—positions of the rapid oscillation features—do not exhibit any evident periodicity which is usually taken as the proof of the Aharonov–Bohm effect for coherent current flow.

We performed a Fourier analysis of the transfer probability to one of the output leads

$$f(v) = \int dB \left( T^l(B) - \langle T^l \rangle \right) \exp(-2\pi i B v),$$

where $\langle T^l \rangle$ is the average transfer probability within the considered $B$ range. The results for $F = |f|^2$ for the lowest transfer functions $T^{l/r} = \sum \mu T^{l/r}_{\mu}$, which are proportional to the conductance of both the output leads. The conductance can be then evaluated as $G^{l/r} = \frac{2e^2}{h} T^{l/r}$.

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subband transport are displayed in figure 4(c). The upper and lower panels of figure 4(c) display the low and high frequency ranges, respectively. The low frequency part describes mostly the deviation of $T'(B)$ from its average value, which is strong when the Lorentz force governs the electron flow. The high frequency parts of the plot cover the region where the peaks due to the AB oscillation should be expected to appear.

Figure 5. Geometry of a wedge junction (a). The upper panels in (b)–(d) show the transfer probabilities to the left $T_l$ and right $T_r$ output leads and the backscattering probability $R$ summed over the subbands the electron comes from. The lower panels in (b)–(d) show the subband-resolved transfer probability. Parts (b), (c) and (d) correspond to 2, 3 and 5 subbands at the Fermi level, respectively ($E_F = 6.5, 15$ and 34 meV).

Figure 6. The same as figure 4 but for a ring with equal length of the channels everywhere.
For the system of figure 4 the Fourier transform is entirely dominated by the low frequency part which results from the distinct growth of $T'$ from 0 to 1 within the studied range of $B$. This growth is exclusively due to the Lorentz force which dominates the $T(B)$ characteristics. The higher frequency part visible only under a strong zoom displays a peak near $\nu = 8 \ T^{-1}$, which corresponds to the magnetic field period of 0.125 T, which in turn would correspond to the AB period for a ring of radius 102 nm—somewhat smaller than the mean radius of the ring of 120 nm (see figure 4(a)). For $|B| > 0.75$ T, the $T$ oscillations that are due to the quantum interference...
eventually disappear (figure 4(b)), and then the transport is totally governed by the magnetic injection. The lower panels of figures 4(b), (d), (f) display the amplitudes of oscillations of $T'$ and $T''$ which are calculated for selected $B$ values (points in the plots) as the differences of the maximal and minimal $T$ values within the range $(B - \Delta B/2, B + \Delta B/2)$, where $\Delta B = 0.091$ T is the period of AB oscillations for a strictly one-dimensional ring of radius $R = 120$ nm, i.e. the value of the magnetic field for which a magnetic flux quantum $\Phi_0 = \pi(\Delta B)R^2 = \hbar/e$ threads the ring.

In figures 4(d) and (f) we can see that for two and three subbands at the Fermi level the oscillations of the transfer probabilities persist up to higher values of $B$. The Fourier transforms of $T'$ are dominated by the low frequency part due to the Lorentz force, and the higher frequency part does not exhibit any pronounced peak indicating any distinct period.

The studied system—with the ring and the output channels that are wider than the input lead—allows the Lorentz force to dominate the current flow. The increased width of the channels leaves space for magnetic deflection of their trajectories. On the other hand, a part of the lateral spatial quantization energy for the electron that enters the structure from the thinner input lead to the wider channels is transformed to the kinetic energy of progressive motion which enhances the elastic and intersubband scattering effects. Thus the system starts to resemble a cavity rather than a one-dimensional ring and the former is notorious for its chaotic transport properties [23].

3.2. Channels of equal length and wedge junctions

Let us now consider the type of junction in which the input and output channels have similar widths—see figure 5(a) for the geometry and figures 5(b)–(d) for the transfer probabilities for two-, three- and five-subband transport, respectively. We notice in figures 5(c) and (d) that for higher subbands the guiding role of the Lorentz force is distinctly less effective—which is due to the properties of the channel eigenstates discussed above. In consequence the overall growth of $T'$ summed over the subbands from $-1.2$ to $+1.2$ T only weakly depends on the number of subbands participating in the transport—in contrast to the system studied in the preceding subsection. Moreover, in figures 4(b)–(d) only the $T'_1$ probability is a monotonic function of the external magnetic field. For the second subband only at higher $B$ does the Lorentz force deflection of the trajectory win with the shifts of the asymptotic channel wavefunction that occur due to the orthogonality conditions (figure 2(d)).

The results for a quantum ring based on this type of junction—with channels that are of similar width everywhere—are displayed in figure 6. We find an imbalance of the electron transfer due to the Lorentz force: $T'_1 > T'$ at $B > 0.75$ T. The oscillations of the transfer probabilities become more pronounced and distinctly more regular than in the system with wider output channels (compare the amplitudes of figures 6(b), (d), (f) with figures 4(b), (d), (f)). The Fourier spectra for one and two subbands (figures 6(c), (e)) exhibit well pronounced peaks near $\nu = 11$ T$^{-1}$.
corresponding to a magnetic period of 0.091 T—for a one-dimensional ring of radius 120 nm—equal to the average radius of the studied ring. A more complex structure is observed in the Fourier transform for the three subband case—the presence of a maximum is still evident—in contrast to the previously discussed structure (figure 4(g)).

3.3. Channels of equal length with $T$ junctions

The systems studied above contained wedge-shaped junctions. We find that the Lorentz force effects for these junctions are more evident than for a simpler $T$-shaped junctions. The results for a junction of this type are given in figure 7. Compared with figure 5 we notice that $T_\mu^l$ for $\mu \geq 2$ becomes independent of $B$ within the studied range of the magnetic field. Moreover, the plot for $T_2^l$ indicates that for the electron incident from the second subband at low magnetic field the current flows to the opposite output lead to the one which is preferred by the Lorentz force. For the ring (figure 8) with this type of junction we notice that the periodicity of the conductance oscillations remains more or less similar to that for wedge-shaped junctions (figure 6). On the other hand, the imbalance of the transfer probabilities to the two output channels is clearly introduced by the Lorentz force only in the lowest subband (figure 8(b)).

3.4. A ring with thin channels

The results for a ring with channels that are only 32 nm wide are displayed in figure 9. The results for the lowest subband channel do not exhibit any trace of the magnetic deflection (figure 9(b)); note in particular the missing low frequency part of the Fourier transform (figure 9(c)). For the two-subband and three-subband transport we notice an imbalance of the conductance to the two output leads which is non-classical—the current at $B > 0$ is directed to the right output. This system—close to a one-dimensional one—exhibits the clearest $AB$ periodicity of the systems studied in this paper.

3.5. Effect of the finite temperature

In order to estimate the effects of the finite temperature related to the widening of the Fermi level we performed an averaging of the transfer probability, calculating the integral of equation (2) for a temperature of 0.7 K. We considered the systems of figure 6 (wide channels, wedge junctions) and figure 9 (thin channels, abrupt junctions). A comparison of 0 and 0.7 K results is given in figure 10. The results for 0 K contain abrupt resonances which are sharp not only as a function of $B$ but also as a function of the energy. As the thermal averaging (equation (2)) is performed these rapid resonances disappear. For the single-subband transport at high $B$ the rapid drops of $T_l$ vanish and one finds that the electron is directed to the left lead for any high $B$. For transport above the single subband, these rapid conductance oscillations disappear, but without a pronounced enforcement of the classical deflection.

4. Summary and conclusions

We have studied the Lorentz force effects in three terminal junctions and quantum rings for both lowest subband and multisubband transport conditions, solving the stationary quantum scattering problem with a finite difference approach. For the lowest subband transport the shift of the asymptotic wavefunction within the channels is consistent with the orientation of the classical deflection. In consequence, for transport in the lowest subband a distinct effect of the Lorentz force and pronounced Aharonov–Bohm oscillations co-exist even for $T$-type junctions—in which the space for electron deflection is quite limited. Therefore, in quantum rings a clear Lorentz force effect and a pronounced $AB$ oscillation appear in the lowest subband transport as long as the channels are wide enough to allow for magnetic deflection. For higher Fermi energies corresponding to multiband transport usually more space is required for classical electron deflection than that present in a $T$ junction. The calculated transfer probabilities for multiband transport are distinctly
Aharonov–Bohm periodic but the effect of the Lorentz force is unclear, if there is any. One way to enhance the Lorentz force effects is to make the channels wider than the input lead. We demonstrated that this procedure—although successful for any subband—leaves signatures of chaotic transport on the conductance which does not exhibit any periodicity. Aharonov–Bohm oscillation and a distinct deflection of the electron trajectories can be obtained for wedge-shaped junctions.

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