Demystifying the delayed-choice quantum eraser

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Abstract

The delayed-choice quantum eraser has long been a subject of controversy, and has been looked at as being incomprehensible to having retro-causal effect in time. Here the delayed-choice quantum eraser is theoretically analyzed using standard quantum mechanics. Employing a Mach–Zehnder interferometer, instead of a conventional two-slit interference, brings in surprising clarity. Some common mistakes in interpreting the experiment are pointed out. It is demonstrated that in the delayed mode there is no which-way information present after the particle is registered on the screen or the final detectors, contrary to popular belief. However, it is shown that another kind of path information is present even after the particle is registered in the final detectors. The registered particle can be used to predict the results of certain yet to be made measurements on the which-way detector. This novel correlation can be tested in a careful experiment. It is consequently argued that there is no big mystery in the experiment, and no retro-causal effect whatsoever.

Keywords: delayed choice, quantum eraser, two slit interference, quantum complementarity

(Some figures may appear in colour only in the online journal)

1. Introduction

Wave-particle duality, as it is understood today, is a concept grounded in the principle of complementarity formulated by Niels Bohr [1]. Quantum objects, which we refer to as quantons, can exhibit wave properties, akin to being ‘spread out’, or particle properties, akin to being localized. The two-slit interference experiment has become a testbed for probing these and related issues [2]. In an oft-considered thought experiment, there is a 1-bit (two state) quantum path detector sitting in the path of a quanton passing through a double-slit (see figure 1). The two states of this ‘which-way’ detector are correlated with the two paths of the quanton.
Reading the state of the which-way detector can provide information regarding which slit the quanton passed through. An interesting idea was advanced by Jaynes [3], according to which one may choose to examine those states of the which-way detector which do not distinguish between the two paths of the quanton, thus erasing the which-way information. This may enable bringing back interference. Scully and Drühl [4] also formulated such an idea, coining the term ‘quantum eraser’. Going further, they proposed that in a modified experiment, one could choose to delay the erasing of the which-way information until after the quanton is registered on the screen. This ‘delayed choice quantum eraser’, they showed, would also bring back interference. The delayed-choice quantum eraser experiment led to a lively debate which continues to this day [5–11]. A great deal of confusion prevailed relating to this proposed experiment, as to whether it implies making the quanton behave like a wave or a particle, once it has been registered on the screen. This apparent ‘retro-causality’ is still subject to discussion [9–11].

The quantum eraser has now been experimentally realized by various people using photons [14–24]. There have been some other proposals using neutral kaons [25], via a modified Stern–Gerlach setup [26, 27], and also using atoms in an optical Stern–Gerlach model [28]. The idea of the quantum eraser has also been generalized to three-path interference [29].

Here, we take a fresh look at the delayed choice quantum eraser, and analyze various issues which have been under debate.

2. Two-slit interference and the quantum eraser

Below, we briefly explain the basic idea behind the quantum eraser. Consider a quanton going through a double-slit, and let $|\psi\rangle$ be the state of the quanton when it emerges from the double-slit:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( |\psi_1\rangle + |\psi_2\rangle \right),$$

where $\psi_1, \psi_2$ are states localized at the location of slits 1 and 2, respectively. The states $\psi_1, \psi_2$ are orthogonal because of their spatial separation. The quanton travels to the screen, and the probability of it landing at a position $x$ is given by

$$|\langle x|\psi(t)\rangle|^2 = \frac{1}{2} \left[ |\psi_1(x,t)|^2 + |\psi_2(x,t)|^2 + \psi_1^*(x,t)\psi_2(x,t) + \psi_2^*(x,t)\psi_1(x,t) \right].$$

Figure 1. Schematic diagram of a two-slit interference experiment in the presence of a 1-bit which-way detector.
wherethe last two termsrepresent interference. In the subsequent discussion we will drop the label \( t \), and will just assume the state on the screen to be the time-evolved state.

The age-old question is, which slit did the quanton go through? To address this question, let us introduce a which-way detector at the double-slit, as shown in figure 1. Although which-way detection can be implemented in a variety of ways, we only consider a 1-bit detector, such as a quantum spin \(-1/2\), without assuming a specific form of it. The which-way detector becomes entangled with the states of the two paths, and the combined state of the quanton and which-way detector is given by

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} \left[ |\psi_1\rangle |\uparrow\rangle + |\psi_2\rangle |\downarrow\rangle \right],
\]

where \( |\uparrow\rangle \), \( |\downarrow\rangle \) denote certain orthonormal states of the which-way detector, such as the eigenstates of the \( z\)-component of a spin \(-1/2\). We will assume that the state of the quanton at the screen and the which-way detector continues to be given by (3), while remembering that \( \psi_1(x), \psi_2(x) \) at the screen would represent time-evolved states. Treating the explicit time evolution of states is not important for our purpose here, as what matters is that the entanglement in (3) is retained. Regarding the which-way detector states, as they resemble the states of a spin \(-1/2\), it can be assumed that there is no ‘free’ Hamiltonian, so the states do not change over time. This also makes it straightforward to decide whether one wants to look at the quanton registering on the screen before measuring the which-way detector state, or vice-versa. For example, if one wants to make a which-way measurement before the quanton hits the screen, one has to project the entangled state on a particular which-way detector state, and examine in which state the quanton emerges. On the other hand, if one wants to measure the which-way detector after the quanton hits the screen, one has only to project the entangled state for a particular position eigenstate at the screen, and note which state of the which-way detector is left behind. In both cases, the state of the quanton will be assumed to be the state when the quanton has reached the screen. One can now evaluate the probability density of the quanton falling on the screen at a position \( x \), namely \( |\langle x|\Psi\rangle|^2 \), as

\[
|\langle x|\Psi\rangle|^2 = \frac{1}{2} \left[ |\psi_1(x)|^2 + |\psi_2(x)|^2 + \psi_1^*(x)\psi_2(x)\langle \uparrow|\downarrow\rangle + \psi_2^*(x)\psi_1(x)\langle \downarrow|\uparrow\rangle \right].
\]

The cross terms in the above, which represent interference, have a factor proportional to \( |\langle \uparrow|\downarrow\rangle| \), which is equal to zero, thus destroying the interference of the quanton. The standard quantum lore is that since the which-way detector ‘carries’ the which-way information about the quanton (by virtue of the entangled state), the interference is destroyed.

A quantum eraser is introduced in the following manner: if \( |\uparrow\rangle, |\downarrow\rangle \) are orthonormal, one can then introduce another set of orthonormal states: \( |\pm\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle \pm |\downarrow\rangle) \), which resemble the eigenstates of the \( x\)-component of a spin \(-1/2\). The entangled state (3) can then be written as

\[
|\Psi\rangle = \frac{1}{2} (|\psi_1\rangle + |\psi_2\rangle)|\pm\rangle + \frac{1}{2} (|\psi_1\rangle - |\psi_2\rangle)|\mp\rangle.
\]

It is obvious that (5) shows no interference, as it is the same state as (3). However, if the quanton is detected in coincidence with the state \(|\pm\rangle\) of the which-way detector, it shows an interference which is exactly the same as that shown by (1). Alternately, if the quanton is detected in coincidence with the state \(|\mp\rangle\), it shows an interference which is slightly shifted. In this sense, the states \(|\pm\rangle\) may be called both-ways states. The two interferences may be
represented as
\[\langle + | \Psi(x) \rangle^2 = \frac{1}{4} \left( |\psi_1(x)|^2 + |\psi_2(x)|^2 + \psi_1^*(x)\psi_2(x) + \psi_2^*(x)\psi_1(x) \right),\]
\[\langle - | \Psi(x) \rangle^2 = \frac{1}{4} \left( |\psi_1(x)|^2 + |\psi_2(x)|^2 - \psi_1^*(x)\psi_2(x) - \psi_2^*(x)\psi_1(x) \right).\] (6)

The standard narrative states that which-way information, which was carried by the correlated state (3), is erased on obtaining a both-ways state \(|+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\). Because of the fact that coincident detection of the quanton with |±\rangle states restores the interference, the process is known as quantum erasure [4].

Depending on which set of states of the which-way detector one chooses to look at, one may choose to retain or erase the which-way information. If one measures the z-states of the which-way detector before the quanton hits the screen, and finds, for example, |↓\rangle, one knows for sure that the quanton went through slit 2, and not through slit 1. One can repeat this procedure for many quantons, and for each of them one knows which slit they went through. However, those quantons will not form an interference pattern on the screen. Alternatively, one may decide to measure the x-state of the which-way detector before the quanton hits the screen. If one obtains |+\rangle, one knows that the state of the quanton is \(\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)\). This would mean that the quanton went through both slits, like a wave. If one obtains |−\rangle, it implies that the state of the quanton is \(\frac{1}{\sqrt{2}}(|\psi_1\rangle - |\psi_2\rangle)\). Here too, the quanton went through both slits, like a wave, but in a slightly different fashion. Naturally, in these two cases, one does obtain an interference pattern. Thus, one can force the quanton to behave like a particle, or a wave, by choosing which set of states of the which-way detector one measures.

A clarification may be in order here regarding a philosophical objection that has been raised recently [11]. What do we mean when we say that the which-way information about the quanton is carried? We simply mean that the entangled state (3) has the potential to yield which-way information, by means of an appropriate measurement. It is not implied that the quanton is actually going through the path, e.g. |\psi_1\rangle, and the which-way detector is |↑\rangle. In fact, one may equally say that the entangled state carries information about which of the two states, \(\frac{1}{\sqrt{2}}(|\psi_1\rangle \pm |\psi_2\rangle)\) the quanton may be found in. This again means that the correlated state has the potential to yield information on which of the two states \(\frac{1}{\sqrt{2}}(|\psi_1\rangle \pm |\psi_2\rangle)\) the quanton will be found in, if an appropriate measurement is performed on the which-way detector.

An apparently perplexing situation arises if one observes the which-way detector after the quanton has been registered on the screen. One may still (rather naively) try to correlate the measurement results of the which-way detector (on whichever basis) with the detection of the quanton on the screen. It should be noted that the probabilities given by (6) are independent of whether one looks at the which-way detector before or after the quanton hits the screen [10]. Therefore one would still see no interference in coincidence with the states |↑\rangle, |↓\rangle, but would see a ‘recovered’ interference in coincidence with either |−\rangle or |+\rangle, separately. If one continues to use the logic of the preceding discussion, it appears to imply that one can force the quanton to behave like a particle or a wave, after it has been registered on the screen. This inference has perplexed people and led many to debate whether quantum mechanics allows one to have backward in time influence.

3. Understanding quantum correlations

Let us first clearly understand the basis on which we infer the path followed by the quanton from the state of the which-way detector. This inference is a result of quantum entanglement,
and the resulting correlation between the two. For simplicity, consider two spin $-1/2$ particles 1 and 2, in an entangled state:

$$|\phi\rangle = \frac{1}{\sqrt{2}}[(|\uparrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2)].$$

(7)

where labels 1 and 2 refer to the two particles, and states $|\uparrow\rangle$, and $|\downarrow\rangle$, denote the eigenstates of the z-component of the spins. The same state can also be written as

$$|\phi\rangle = \frac{1}{\sqrt{2}}[(|+\rangle_1|+\rangle_2 + |−\rangle_1|−\rangle_2)].$$

(8)

where the state $|+\rangle_i$, $|−\rangle_i$ denotes the eigenstates of the x-component of the spins. Now if one measures the z-component of spin 1, and, let us suppose, finds it in the state $|\uparrow\rangle_1$, one immediately knows that the state of spin 2 is $|\uparrow\rangle_2$, because the measurement reduces the state $|\phi\rangle$ given by (7) to $|\uparrow\rangle_1|\uparrow\rangle_2$. On the other hand, if one measured the x-component of the spin of particle 1, and found e.g. $|−\rangle_1$, one would immediately know the state of particle 2 to be $|−\rangle_2$. This is because the measurement reduces the state $|\phi\rangle$ given by (8) to $|−\rangle_1|−\rangle_2$. Thus, because of the entangled state, there is a correlation between the z-components of the two spins. For the same reason there is a correlation between the x-components of the two spins. However, there is no correlation between e.g. the z-component of spin of particle 1 and the x-component of spin of particle 2. This can be simply verified as follows. Measurement of z-component of spin 1 and obtaining $|\downarrow\rangle_1$ leads to $1 \langle\downarrow|\psi\rangle = |\downarrow\rangle_2 = \frac{1}{\sqrt{2}}(|+\rangle_2 − |−\rangle_2)$. If one were now to measure the x-component of the spin of particle 2 on this reduced state, one is equally likely to get $|+\rangle_2$, or $|−\rangle_2$.

The crucial point is the following: since one knows that measuring z-component of spin 1 will tell one about the z-component of spin-2, one might be tempted to make it an always-holds-true rule. Next, one first measures the x-component of spin of 2, and finds e.g. $|+\rangle$. Then one decides to ask, what was the z-component of spin 2 before one measured its x-component? One may naively use the above-mentioned always-holds-true rule, and measure the z-component of spin 1, to find (say) $|\uparrow\rangle_1$. One might now feel able to claim that this means that the z-component of spin 2, before one measured its x-component, was $|\uparrow\rangle_2$. This is incorrect, however, simply because the correlation between the z-components of the two spins is based on the entangled state (7), but this entangled state is already destroyed when one measured the x-component of spin 2. This example will now help in identifying where the flaw in the delayed-choice argument lies.

Suppose that spin 2 plays the role of the which-way detector in the quantum eraser experiment, and spin 1 plays the role of possible paths of the quanton in the following way: $|\uparrow\rangle_1 \rightarrow |\psi_1\rangle, |\downarrow\rangle_1 \rightarrow |\psi_2\rangle$, and $|+\rangle_1 \rightarrow \frac{1}{\sqrt{2}}(|\psi_1\rangle ± |\psi_2\rangle)$. Just as for the case of two spins, there is a correlation between the states $|\uparrow\rangle, |\downarrow\rangle$, and $|\psi_1\rangle, |\psi_2\rangle$. Also, there is a correlation between $|+\rangle, |−\rangle,$ and $\frac{1}{\sqrt{2}}(|\psi_1\rangle ± |\psi_2\rangle)$. This is by virtue of the entangled state given by (3) and (5). But now let us suppose that the quanton registers on the screen at a position $x_0$. The entangled state (3) is reduced to

$$\langle x_0|\Psi\rangle = \frac{1}{\sqrt{2}} \left[ \langle x_0|\psi_1\rangle|\uparrow\rangle + \langle x_0|\psi_2\rangle|\downarrow\rangle \right].$$

(9)

Since the entangled state is gone, one cannot use (9) to measure the z-state of the which-way detector, and then infer from the result whether the state of the quanton was $|\psi_1\rangle$ or $|\psi_2\rangle$ before it landed at position $x_0$. This point will be elaborated upon in the next section. The analogy between a quantum eraser setup and entangled spins is well-known [5]. In fact, Kastner has...
used a similar argument, by identifying the $\frac{1}{\sqrt{2}}[|\psi_1\rangle \pm |\psi_2\rangle]$ states with the $x$-basis of a spin $-1/2$, to emphasize that once the quanton registers on the screen, it yields no information regarding the path followed by the quanton [11–13].

4. Quantum eraser using a Mach–Zehnder interferometer

Although the analogy between entangled spins and the quantum eraser setup is apparent, one may not be fully convinced, because the quanton involves a continuous variable, i.e. position, and thus is not like a spin $-1/2$. In order to make the analogy genuinely one to one, we consider a Mach–Zehnder interferometer, with a 1-bit which-way detector, as shown in figure 2 (see also reference [30]). A Mach–Zehnder interferometer can be analyzed using quantum mechanics in the following way [31]. An incoming quanton in the state $|S\rangle$, is split by the first beam-splitter BS1 into a spatially separated superposition $\frac{1}{\sqrt{2}}(|T\rangle - |R\rangle)$, where the $-$ sign represents a $\pi$ phase shift due to reflection. The two components evolve, having reflected from the mirrors, passed through the beam-splitter BS2, to a final state at the detectors as follows:

$$\begin{align*}
T & \rightarrow \frac{1}{\sqrt{2}} (|D_1\rangle - |D_2\rangle) \\
R & \rightarrow \frac{1}{\sqrt{2}} (|D_1\rangle - |D_2\rangle),
\end{align*}$$

where $|D_1\rangle, |D_2\rangle$ are the states at detectors $D_1$ and $D_2$, respectively. To make the initial state appear the same as (1), one can redefine the states $|T\rangle, |R\rangle$ as $|\psi_1\rangle, |\psi_2\rangle$ by absorbing certain phase factors, to write the initial state after the first beam splitter as

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle).$$

(10)

Beyond the second beam-splitter, the two components $|\psi_1\rangle, |\psi_2\rangle$ evolve to

$$\begin{align*}
U_{BS2}|\psi_1\rangle & = \frac{1}{\sqrt{2}}(|D_1\rangle + |D_2\rangle) \\
U_{BS2}|\psi_2\rangle & = \frac{1}{\sqrt{2}}(|D_1\rangle - |D_2\rangle),
\end{align*}$$

(11)

where $U_{BS2}$ represents the unitary evolution due to the mirrors, and the second beam-splitter BS2, $|D_1\rangle, |D_2\rangle$ are the states at detectors $D_1$, and $D_2$, respectively. It might be interesting to see which state (10) will result at the final detectors. The final state, just before the quanton hits the detectors, is given by

$$|\psi_F\rangle = U_{BS2}|\psi_1\rangle = \frac{1}{\sqrt{2}}(U_{BS2}|\psi_1\rangle + U_{BS2}|\psi_2\rangle).$$

(12)

Using (11), it is straightforward to see that the probability of the quanton to end up at detector $D_1$ is $|\langle D_1|\psi_F\rangle|^2 = 1$, and the probability for it to end up at detector $D_2$ is $|\langle D_2|\psi_F\rangle|^2 = 0$. This represents interference, as detector $D_1$ registers a bright fringe (all quantons landing there), whereas detector $D_2$ registers a dark fringe (no quanton landing there).

Next, we consider the effect of introducing a 1-bit which-way detector in the path of the quanton. The combined state of the quanton and the which-way detector, after it passes through the first beam-splitter, and interacts with the which-way detector, is given by

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}} [(|\psi_1\rangle|\uparrow\rangle + |\psi_2\rangle|\downarrow\rangle).$$

(13)

After passing through BS2, the combined state is $|\Psi_F\rangle = U_{BS2}|\Psi_1\rangle$:

$$|\Psi_F\rangle = \frac{1}{2} [(|D_1\rangle + |D_2\rangle)|\uparrow\rangle + (|D_1\rangle - |D_2\rangle)|\downarrow\rangle].$$

(14)
Figure 2. A schematic diagram of a quantum eraser setup using a Mach–Zehnder interferometer. There is a 1-bit quantum which-way detector in the path of the quanton.

If one finds the which-way detector in the state $|\uparrow\rangle$, the quanton state is $\sqrt{2}(|D_1\rangle + |D_2\rangle)$, which means $D_1$ and $D_2$ are equally likely to click. If one finds the which-way detector in the state $|\downarrow\rangle$, the quanton state is $\sqrt{2}(|D_1\rangle - |D_2\rangle)$, which again means that $D_1$ and $D_2$ are equally likely to click. Thus, there is no ‘dark fringe’, hence no interference.

Next we look at the case where $D_1$ or $D_2$ register the quanton first, and much later, one chooses to look at a particular basis of the which-way detector. If e.g. $D_1$ clicks (quantum state is $|D_1\rangle$), equation (14) tells us that the which-way detector is now in the state $\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle)$.

Measuring the which-way detector in the $z$-basis, one is equally likely to find $|\uparrow\rangle$ or $|\downarrow\rangle$, which yields no which-way information. Thus we see that as soon as the quanton registers at a detector, all which-way information is lost. This is not surprising because equation (14) states that $|D_1\rangle, |D_2\rangle$ are not correlated to $|\uparrow\rangle, |\downarrow\rangle$, but rather to $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ and $\frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$. This shows that registering the quanton at $D_1$ or $D_2$ destroys the which-way information. In the delayed mode, even though there is no interference, clicks of $D_1$ and $D_2$ do not yield any which-way information. This aspect has not been understood in any previous analysis, and has been a source of confusion in interpreting delayed-choice experiments.

Another point to note is that if one chooses to forget about the which-way detector completely, and only calculates the probability of the quanton to hit $D_1$ or $D_2$, one finds that $|\langle D_1 |\psi_F\rangle|^2 = |\langle D_2 |\psi_F\rangle|^2 = 1/2$. This implies no interference as $D_1$ and $D_2$ will register an equal number of quanta. Although the which-way information is lost as soon as the quanton hits the detectors, the interference is also lost. Contrast this with the case where the quanton is not entangled with any which-way detector, namely, equation (12). In that case, the probability of the quanton hitting $D_1$ is 1, and that of hitting $D_2$ is 0. Entanglement with the which-way detector suffices to destroy the interference.

Interference is recovered, based on the usual quantum eraser experiments, by correlating the clicks in the detectors $D_1$, and $D_2$, with the $x$-states $|+\rangle, |-\rangle$ of the which-way detector. equation (14), when written in terms of these states, has the following form:

$$|\psi_F\rangle = \frac{1}{\sqrt{2}} [D_1]|+\rangle + |D_2|-\rangle.$$ (15)

If the which-way detector is first looked at in the $x$-basis, and one finds, for example $|+\rangle$, this means that $D_1$ will detect the quanton, and not $D_2$. When this happens for many quanta, it implies interference where the ‘bright fringe’ is at $D_1$ and the ‘dark fringe’ at $D_2$. Note that by reading the which-way detector in the $x$-basis $|\pm\rangle$, we choose to erase the which-way information. This is in agreement with Bohr’s complementarity principle, because, when interference
is observed, there is no which-way information. Similarly, if the which-way detector is found in the state $|−⟩$, it implies that $D_2$ will definitely detect the quanton, and not $D_1$. Many such quantons constitute interference, where the ‘bright fringe’ is now at $D_2$, instead of $D_1$. This interference is complementary to that seen in correlation with $|+⟩$, and, taken together, the two imply no interference.

If this part of the experiment is carried out in the delayed mode, the scenario becomes more interesting. As shown in the preceding discussion, in the delayed mode, as soon as the quanton registers at $D_1$ or $D_2$, the which-way information is erased. However, that is not sufficient to restore interference. In addition, one has to measure the which-way detector, and correlate each detected quanton with $|+⟩$ or $|−⟩$, in order to obtain two interferences which are ‘shifted’ with respect to each other. The bright fringe of one is the dark fringe of the other.

However, there is another aspect of this which has not been recognized in previous studies of delayed-choice experiments. Note that (15) implies that by looking at which detector the quanton has landed in, one can now predict which of the two which-way detector states, $|+⟩$ or $|−⟩$, will be surely obtained in a measurement. Therefore, even though the which-way information is erased after the quanton is registered in a detector, the quanton retains another kind of information about the which-way detector. This can easily be tested in correlated measurements in a delayed-choice quantum eraser experiment. So it is not true that in the delayed mode, the which-way information is erased only after the which-way detector is examined in the $x$-basis $|±⟩$, as is widely believed. Not only does the quanton registering at a detector erase the which-way information, it additionally retains information about precisely how it is erased, as $\frac{1}{\sqrt{2}}(|↑⟩ + |↓⟩)$, or as $\frac{1}{\sqrt{2}}(|↑⟩ − |↓⟩)$.

To summarize the conclusions of this section, when the which-way detector is measured before the quanton hits the final detectors, one can choose to either obtain the which-way information by reading $z$-basis states $|↑⟩, |↓⟩$ or erase it by reading $x$-basis states $|±⟩$. If the quanton hits the final detectors before the which-way detector is measured, the which-way information is erased, always. Reading out $z$-basis states $|↑⟩, |↓⟩$ does not yield any which-way information. However, reading out $x$-basis states $|±⟩$ allows one to recover two complementary interference patterns. More interestingly, every registered quanton can be used to predict which of the states $|+⟩, |−⟩$ will be obtained if one measures the which-way detector after a delay!

5. Discussion

One can now make a comparison of the quantum eraser experiment using the Mach–Zehnder setup with two entangled spins considered in section 3. Let us first write (15) in a slightly different form:

$$|Ψ_F⟩ = U_{BS2} |Ψ_I⟩ = \frac{1}{\sqrt{2}} [(|D_1⟩|+⟩ + |D_2⟩|−⟩)].$$

Using (11), one can write the above as

$$U_{BS2} |Ψ_I⟩ = \frac{1}{\sqrt{2}} U_{BS2} \left[ \frac{|ψ_1⟩ + |ψ_2⟩}{\sqrt{2}} |+⟩ + \frac{|ψ_1⟩ − |ψ_2⟩}{\sqrt{2}} |−⟩ \right]$$

$$|Ψ_I⟩ = \frac{1}{\sqrt{2}} \left[ \frac{|ψ_1⟩ + |ψ_2⟩}{\sqrt{2}} |+⟩ + \frac{|ψ_1⟩ − |ψ_2⟩}{\sqrt{2}} |−⟩ \right],$$

which means that the correlation between the clicks of $D_1, D_2$, and the state $|±⟩$, comes from the correlation between $\frac{|ψ_1⟩ + |ψ_2⟩}{\sqrt{2}}$, and $|±⟩$, contained in the initial entangled state. On the other
hand, $|\psi_1\rangle, |\psi_2\rangle$ are correlated with $|\uparrow\rangle, |\downarrow\rangle$ by virtue of (13). Now compare (13) and (17) with (7) and (8). Quantum states $|\psi_1\rangle, |\psi_2\rangle$ play the role of $|\uparrow\rangle_1, |\downarrow\rangle_1$, whereas $\frac{|\psi_1\rangle+|\psi_2\rangle}{\sqrt{2}}$ plays the role of $|\pm\rangle_1$. The which-way detector can be assumed to play the role of spin 2. For the entangled spins, measuring the z-component of spin 2 gives information about the possible outcome of the z-component of spin 1, and vice-versa. Measuring the x-component of spin 2 gives information about the possible outcome of the x-component of spin 1, and vice-versa. If the x-component of spin 1 is measured, it destroys all potential information about the result of a future measurement of the z-component of spin 2. In exactly the same way, as soon as $D_1$ clicks, the quanton state is $\frac{|\psi_1\rangle+|\psi_2\rangle}{\sqrt{2}}$, which implies that the both-ways state, when measured, will be $|+\rangle$. However, now one cannot get any information regarding the which-way states $|\uparrow\rangle, |\downarrow\rangle$, and hence no which-way information about the quanton.

One might wonder why it was not realized earlier that in the delayed mode, the quanton registered on the screen can give information about which of the states $|\pm\rangle$ one would obtain if the which-way detector is measured. A probable reason for this is that most analyses use two-slit interference instead of the Mach–Zehnder setup. While the Mach–Zehnder interferometer has only two output states, double-slit interference consists of a multitude of position states of the quanton. Although interference is most commonly studied in a two-slit experiment, the interference is not very ‘clean’ in the sense that the bright and dark fringes are not well separated. In a Mach–Zehnder experiment, the bright and dark fringes are well separated, and even registered on separate detectors. However, it is also possible to guess the outcome of the measurement of which-way states $|\pm\rangle$, by looking at the quantons hitting the screen in a two-slit interference experiment. Typical interference patterns, in a quantum-eraser two-slit experiment, are shown in figure 3. The red (solid) curve represents interference in coincidence with $|+\rangle$, whereas the blue (dashed) curve represents interference in coincidence with $|−\rangle$. Without any coincident detection, a quanton falling anywhere on the screen could belong to either the red curve or the blue curve. However, note that the peaks of the red curve are located exactly at the minima of the blue curve. Since one is aware that this is a two-slit interference whose parameters are known, one has information about the exact locations of the maxima, and the minima of the would be interference pattern. All of the quantons will fall on the green curve, which represents no interference. However, if a quanton falls on the position of a maximum of the red curve, it means it has zero probability of belonging to the blue curve. Such points are denoted in red in figure 2. Thus, it must actually belong to the red curve, and one can now predict that a measurement on the which-way detector will definitely yield a $|+\rangle$ state. On the other hand, if a quanton falls on the position of a maximum of the blue curve, this means it has zero probability of belonging to the red curve. Such points are denoted in blue in figure 2. Now one can predict with certainty that the which-way detector will yield $|−\rangle$. This effect is novel, and can be easily tested in a delayed-choice quantum eraser experiment. Needless to say, the conditions for sharp interference should be present for this to work. The effect is more stark in the Mach–Zehnder implementation of the delayed choice quantum eraser. As soon as the quanton is detected at $D_1$ ($D_2$), the state of the which-way detector changes to $|+\rangle_1 (|−\rangle_1$). It is trivial to see that if one tries to measure the z-states of the which way detector, one does get either $|\uparrow\rangle$ or $|\downarrow\rangle$, but this does not imply any which-way information.

What mental picture of the quanton traversing the two Mach–Zehnder paths should one construct? The mental picture consistent with the preceding analysis is that if the which-way detector is not measured before the quanton registers at the final detectors, the quanton does pass through both paths, like a wave, but the phase difference between the two paths is determined only when the quanton ends up at $D_1$ or $D_2$. To fix the phase difference between the two paths, one need not measure the x-states of the which-way detector. Each click of $D_1$ or
Figure 3. Probability density of quantons falling on the screen, as a function of position (green, dotted curve). These show no interference due to the presence of the which-way detector. Probability density of quantons in coincidence with which-way detector landing in $|+\rangle$ state (blue, solid curve). This shows an interference pattern, representing the erasure of which-way information. Probability density of quantons in coincidence with the which-way detector landing in $|-\rangle$ state (red, dashed curve). This too shows an interference pattern, but phase-shifted. See text for the meaning of red and blue circles.

$D_2$ uniquely determines the corresponding $x$-basis state of the which-way detector, and also the phase difference between the two paths. As the phase difference varies between two values from quanton to quanton, all of them taken together show no interference. On the other hand, if the which-way detector is measured in the $z$-basis, before the quanton hits the detectors $D_1$, $D_2$, the mental picture one may construct is that the quanton actually goes through only one of the two paths like a particle, and unlike a wave. Measuring the $z$-state kills one of the paths of the quanton, irrespective of how far away along the two paths the quanton has traveled before that measurement is made. Therefore, the bottom line is that which-way information about the quanton can only be obtained before the quanton registers at $D_1$ or $D_2$.

6. Conclusion

In conclusion, we have theoretically analyzed the delayed choice quantum eraser experiment in a two-slit interference setup, and also using a Mach–Zehnder setup. This is accomplished by introducing a 1-bit quantum which-way detector in the path of the quanton. We have first discussed the quantum correlations arising from entanglement, which form the basis on which one uses the which-way detector to obtain which-way information about the quanton. If the which-way detector is measured before the quanton is registered at the final detectors (or the screen), one can choose to either retrieve which-way information about the quanton by reading the which-way detector in the $z$-basis, or erase the which-way information by reading the which-way detector in the $x$-basis. In the latter case, two complementary interferences can be recovered by correlating the detected quantons with the $x$-basis states $|\pm\rangle$ of the which-way detector.

If the quanton is registered at the detectors (or the screen) before the which-way detector is measured, the which-way information is erased, and the state of the which-way detector is set by the process of registering of the quanton. This is proved by the fact that final detectors $D_1$, and $D_2$ can be used to predict which state of the $x$-basis of the which-way detector will emerge if a measurement is made on it after a delay. For example, when a quanton lands at detector $D_2$, according to the entangled state, the state of the which-way detector changes to
Two interferences can again be obtained by correlating with the \( x \)-basis states. Since the which-way \( x \)-basis state is already decided once the quanton lands at \( D_1 \) or \( D_2 \), it is obvious that if one chooses to measure the \( z \)-basis instead, it is not going to yield any which-way information. Not only that, by choosing to look at \( z \)-basis states, one loses the opportunity to recover the interference which is seen only in correlation with \( x \)-states. However, the loss of interference here does not imply that there is any which-way information present. It is not, contrary to popular belief.

In the two-slit implementation of the quantum eraser, the quanton, in the delayed mode, landing on the screen cannot always predict the \( x \)-state of the which-way detector simply because dark and bright fringes are not cleanly separated as in the Mach–Zehnder interferometer. However, there is no conceptual difference between the two. The quanton landing on certain specific positions can indeed predict the \( x \)-state of the which-way detector. This can be tested by means of careful experimentation. In the light of this analysis, there is no mystery in the delayed choice quantum eraser experiment, and no question of any retro-causality.

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