Non-perturbative tests of HQET in small-volume quenched QCD

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We quantitatively investigate the quark mass dependence of current matrix elements and energies, calculated over a wide range of quark masses in the continuum limit of small-volume quenched lattice QCD. By a precise comparison of these observables as functions of the heavy quark mass with the predictions of HQET we are able to verify that their large quark mass behaviour is described by the effective theory.

1. MOTIVATION

The simplification of the QCD dynamics in the limit of large masses of c– and b–quarks gives rise to the Heavy Quark Effective Theory (HQET) as a standard phenomenological tool to describe decays of heavy-light hadrons and their transitions in terms of hadronic matrix elements. (See e.g. \cite{1} for a review.) Starting from the HQET Lagrangian of a heavy quark, $\mathcal{L}_{\text{HQET}}$, that reads

$$\bar{\psi}_h \left[ D_0 + m - \frac{\omega_{\text{spin}}}{2m} \mathbf{D}^2 - \frac{\omega_{\text{kin}}}{2m} \mathbf{B} \cdot \mathbf{B} \right] \psi_h + O(\frac{1}{m^2}),$$

this effective theory provides an expansion of the QCD amplitudes in the inverse heavy quark mass, $1/m$, and is renormalizable at any finite order in $1/m$ according to power counting.

Making HQET an effective theory for QCD requires matching calculations to express the parameters $m, \omega_{\text{spin}}, \ldots$ in \cite{1} by those of QCD, and the agreement of different determinations of quantities such as $V_{\text{cb}}$ \cite{2}, which involve perturbative HQET, already reflect the success of the effective theory approach. Apart from these phenomenological tests of HQET, however, independent, non-perturbative ones are still desirable.

Though in principle the lattice achieves this by varying $m$, clean comparisons of QCD and HQET in the continuum limit demand $m \ll 1/a$ prior to $a \to 0$. Yet \cite{3} rather turned that restriction into an idea to connect them non-perturbatively: consider QCD in a small volume $L^3$, where the b can be simulated as relativistic fermion and HQET becomes an expansion of QCD in the variable $1/z \equiv 1/(ML)$, $M$: RGI heavy quark mass.

Here, we confront the large–z behaviour of correlation functions, computed with Schrödinger Functional boundary conditions \cite{4}, to the static theory and estimate the size of $1/m$–corrections.

This is also of practical relevance in the strategy to solve renormalization problems in HQET non-perturbatively by matching to QCD in finite volume \cite{5}. For a full report on our study (and a list of references to related work) consult \cite{5}.

2. LARGE-MASS ASYMPTOTICS

Our observables derive from relativistic heavy-light SF correlation functions. With $f_3$ a correlator between a heavy-light pseudoscalar boundary source and the axial current $A_0 = \bar{\psi}_l \gamma^0 \gamma^5 \psi_b$ in the bulk, $k_V$ its vector channel analogue and $f_1$ a boundary-to-boundary correlation, we define

$$Y_{PS}(L, M) \equiv \frac{\langle a(T/2) \rangle}{\sqrt{J}} , \quad R(L, M) \equiv -\frac{\langle a(T/2) \rangle}{k_V(T/2)} ,$$

which from the multiplicative renormalizability of the SF boundary fields follow to be finite quan-
tities provided that \( A_0, V_0 \) denote renormalized currents. Effective energies are constructed as
\[
\Gamma_{PS}(L, M) \equiv -f'_A(T/2)/f_A(T/2), \quad \Gamma_V : f_A \to k_V
\]
with spin-averaged sum and difference
\[
\Gamma_{av} \equiv \frac{1}{4} \left[ \Gamma_{PS} + 3 \Gamma_V \right], \quad \Delta \Gamma \equiv \Gamma_{PS} - \Gamma_V. \quad (3)
\]

Being (ratios of) matrix elements between low-energy heavy-light and vacuum-like states \([5]\), \( (2) \) and \( (3) \) should then be described by HQET, which we test by verifying their large–

z

behaviour of \((2)\) thus splits into RGIs
\[
\Gamma_{PS}(L, M) \equiv \frac{f_{\text{stat}}(T)}{\sqrt{f_{\text{stat}}(T)}} = \lim_{z \to \infty} Y_{PS}(L, M). \quad (4)
\]

On the quantum level, the scale dependent renormalization of the effective theory implies logarithmic modifications, i.e., the axial current renormalization amounts
\[
X_{RGI}(L) = \lim_{\mu \to \infty} \left\{ \left[ 2b_0 g^2(\mu) \right]^{-2} X_{RGI}^{\mu}(L, \mu) \right\}, \quad (5)
\]
calculable in lattice QCD: \( X_{RGI} = Z_{RGI} X_{\text{bare}} \). The large–\( z \) behaviour of \((2)\) thus splits into RGIs

of the chromomagnetic operator \( \overline{\psi}_b \sigma \cdot B \psi_b \), whose anomalous dimension (AD) contributes to \( C_{\text{spin}} \).

Taking the entering ADs to best perturbative accuracy \([9]\), the conversion functions \( C \) in \((1)\)–\((4)\) are evaluated by solving RGEs \([5,6]\). As detailed there, their perturbative knowledge is adequate to allow for an investigation of the \( 1/z^n \)-corrections.

3. DISCUSSION OF THE RESULTS

Our quenched data refer to a volume of extent \( L = T \approx 0.2 \) fm, which admits to reach \( M > 2M_\text{th} \) while \( a \to 0 \) extrapolations are still well controlable. To account for the growing of the quark mass in lattice units at given \( a/L \) as \( z \) is increased, the coarsest resolutions that pass into the continuum extrapolations linear in \( (a/L)^2 \) are chosen by imposing a cut on \( aM \) \([5,8]\). Fig. 1 illustrates a typical case: the slopes are quite small, and the error in the continuum limit gets the larger the more \( a/L \) are to be discarded for increasing \( z \).

The main results and their polynomial fits in \( 1/z \) to quantify the deviations from the static limit are displayed in figs. 2–4; linear fits use only the heavier quark mass points.

3.1. Current matrix elements

Comparing the finite-mass matrix element of \( A_0 \) with the HQET prediction \( X_{RGI}(L) + O(1/z) \), we infer from fig. 2 that the perturbative \( C_{PS} \) reduces the mass dependence of \( Y_{PS} \) significantly and renders \( Y_{PS}(L, M)/C_{PS}(r) \) to be compatible with approaching the static result for \( X_{RGI} \).
Figure 2. Fits of $Y_{PS}/C_{PS}$ include $X_{RGI}$. The AD of the static axial current ($\gamma$) enters in $C_{PS}$.

Figure 3. Fits of $R/C_{PS/V}$, constrained to 1.

as $1/z \to 0$. Also the ratio $R$ of matrix elements of $A_0$ and $V_0$ in fig. 3 is consistent with the leading term in the $1/z$-expansion (fixed to 1 by the spin symmetry of the static theory, cf. [3]), if $C_{PS/V}$ is evaluated including at least the current’s two-loop ADs. In both cases, the coefficients of $1/z$-corrections are of order one and, therefore, the corrections are reasonably small.

3.2. Effective energies

In confirming the asymptotics [8], the smallness of $(1/z)^2$-terms found in the combination $L\Gamma_{av}/(zC_{mass})$ deserves particular emphasis regarding the static limit computation of the $b$-mass via $\Gamma_{av}$ [3,8], since it yields an estimate of the error to $M_b$, originating from the matching to QCD, of only $\approx 1\%$ [5]. The spin splitting $L\Delta_{\gamma}/C_{spin}$ (fig. 4) vanishes for $1/z \to 0$ as expected, exhibiting a rather large $1/z$-coefficient.

3.3. Summary

Our successful tests of HQET show the continuum limits of the non-perturbatively renormalized QCD observables at finite $z$ to meet the predictions of the effective theory. Only the functions $C$ relating them to the RGI’s of the latter induce perturbative uncertainties, but these are under control (except for $C_{spin}$ lacking the NNLO) and reveal the power corrections to dominate over perturbative ones in the accessible $z$-range. Finally, our results appear promising to determine $1/m_b$-corrections to B-physics matrix elements following [3] by extending the non-perturbative matching of HQET and QCD to subleading terms [5,10].

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