Dynamics of skyrmions and edge states in the resistive regime of mesoscopic $p$-wave superconductors

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Abstract

In a mesoscopic sample of a chiral $p$-wave superconductor, novel states comprising skyrmions and edge states have been stabilized in out-of-plane applied magnetic field. Using the time-dependent Ginzburg-Landau equations we shed light on the dynamic response of such states to an external applied current. Three different regimes are obtained, namely, the superconducting (stationary), resistive (non-stationary) and normal regime, similarly to conventional $s$-wave superconductors. However, in the resistive regime and depending on the external current, we found that moving skyrmions and the edge state behave distinctly different from the conventional kinematic vortex, thereby providing new fingerprints for identification of $p$-wave superconductivity.

Keywords: $p$-wave superconductivity, Ginzburg-Landau, Mesoscopic superconductors

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1. Introduction

Edge states, appearing where the condensate homogeneity is broken, and domain walls, separating regions with different chiralities, are the main characteristics of chiral $p$-wave superconductivity \[1,2\]. They arise as a consequence of breaking the time-reversal symmetry in an order parameter with two components, i.e. $\Psi = (\psi_+, \psi_-)^T$ \[3\]. Besides the edge states and the domain walls another topological entity (the skyrmion) has recently emerged in chiral $p$-wave superconductivity \[4\]. Unlike the Abrikosov vortex that has a core due to the discontinuity of its phase, the skyrmion is coreless and defined by a loop domain wall \[5\].

Chiral $p$-wave superconductivity is realized in spin-triplet superconductors. In such materials two electrons pair up forming a triplet rather than a singlet as in conventional superconductivity. In order to fulfill the Pauli principle, the orbital part in spin-triplet superconductors has odd parity, i.e. angular momentum $L = 1$ ($p$-wave). As a consequence of the spin of the electronic pairs, another topological entity, the half-quantum vortex (HQV), arises in these materials. HQVs are expected to be unscreened by the Meissner effect due to their spin currents, i.e. they are likely to be found at the lateral borders of the sample \[6\].

Substantial evidence has been provided over the years that strontium ruthenate, Sr$_2$RuO$_4$ (SRO), is a chiral $p$-wave superconductor \[7,8,9\]. However, the lack of direct observation of states carrying spontaneous currents around space homogeneities undermines the candidacy of SRO to the $p$-wave class of superconducting materials \[10,11\]. In this work we study the electrical response of skyrmions and edges states of a mesoscopic chiral $p$-wave superconductor sample when an external current is applied to the sample. Three different regimes are expected, as in conventional superconductivity, namely superconducting, resistive and normal regime. However, the temporal evolution of the two-component superconducting order parameter ($\Psi$) is found to provide rich physics, and depending on the magnitude of applied-current, the skyrmionic and edge states must present different behavior from kinematic vortices in conventional superconductors \[12,13,14\]. This in turn provides new possibilities for resistive stages in the sample behavior, and indirect means to identify $p$-wave superconductivity.
2. Theoretical Formalism

Within the weak-coupling limit and considering a cylindrical Fermi surface, the dimensionless time-dependent Ginzburg-Landau (TDGL) equations for the two component order parameter \( \Psi = (\psi_+ , \psi_-)^T \) and the vector potential \( A \), in chiral p-wave superconductors reads:

\[
\left( \frac{\partial}{\partial t} + i \varphi \right) \Psi = \frac{2}{3} \left[ \tilde{D}^2 - \Pi_0^2 \right] \left( \begin{array}{c} \psi_+ \\ \psi_- \end{array} \right) + \Psi \left( 1 - \frac{1+\tau}{2} |\Psi|^2 \pm \frac{\tau}{2} \Psi^\dagger \sigma_2 \Psi \right), \tag{1}
\]

\[
\kappa^2 \nabla \times (\nabla \times \tilde{A}) + \left( \nabla \varphi + \frac{\partial \tilde{A}}{\partial t} \right) = \tilde{J}_s, \tag{2}
\]

where \( \varphi \) is the electrostatic potential, \( \tilde{D} = (\nabla - i \tilde{A}) \) is the covariant derivative, \( \Pi_{1(-)} \) is the Landau level creation (annihilation) operator, \( \tau \) is a phenomenological parameter, \( \sigma_2 \) is a Pauli matrix, \( \kappa \) is the GL parameter, and \( \tilde{J}_s \) is the superconducting current density,

\[
\tilde{J}_s = \frac{1}{3} \text{Im} \left\{ \psi^\dagger \tilde{D} \psi_+ + \psi_- \tilde{D} \psi_+ \right\} + \frac{1}{3\sqrt{2}} \text{Im} \left\{ \Psi^\dagger \left[ \Pi_+ \sigma_+ + \Pi_- \sigma_- \right] \Psi \right\} i + i \Psi^\dagger \left[ \Pi_+ \sigma_+ - \Pi_- \sigma_- \right] \Psi \tilde{j}, \tag{3}
\]

where \( \sigma_{\pm} = (\sigma_x \pm i \sigma_y)/2 \), and \( \{i,j\} \) is the canonical base in Cartesian coordinates. In Eqs. (1)-(3) distances are scaled to the superconducting coherence length \( \xi \), time to the GL time \( t_0 \), and the vector and electrostatic potentials to \( A_0 = hc/2e\xi \) and \( \varphi_0 = A_0/ct_0 \), respectively. Similarly, the order parameter is scaled to its bulk zero-field value \( |\Psi(A=0)| \), and the current density to \( J_0 = (eh/m\xi)|\Psi_0|^2 \). In order to study the dynamical properties of mesoscopic chiral p-wave superconductors, we adopt the Coulomb gauge, i.e. \( \tilde{A} \) is divergence-free at all times, since it provides the equation for the electrostatic potential,

\[
\nabla^2 \varphi = \nabla \cdot \tilde{J}_s. \tag{4}
\]

For the vector potential and because of an out-of-plane applied magnetic field, we choose \( \tilde{A} = -(\tilde{r} \times \tilde{H})/2 \). The boundary conditions imposed at the superconductor-vacuum and superconductor-normal-metal interfaces are,

\[
\begin{aligned}
\Psi &= 0 \quad \text{at N and S sides,} \\
\partial_y \varphi + j &= 0 \quad \text{at N and S sides,} \\
\tilde{D}_x \psi_+ - D_x \psi_- &= 0 \quad \text{at E and W sides,} \\
\partial_x \varphi &= 0 \quad \text{at E and W sides,}
\end{aligned}
\tag{5}
\]

respectively. N, S, E and W stand for the cardinal points. Current \( j \) is applied at contacts located at N and S. Eq. (5) completes the TDGL equations for chiral p-wave superconductors which we solve using the finite-difference technique.

3. Results

In this work we stabilize skyrmions and edge states in a mesoscopic sample of size 10\( \xi \times 12\xi \) by applying an external magnetic field \( H = 0.8H_c \) out of plane of the sample. Although skyrmions can be stabilized also in bulk samples [3], the edge states containing one vortex just in component \( \psi_+ \), i.e. a half-quantum vortex for the system, appear only where the space homogeneity is broken, thus are characteristic of mesoscopic samples [3]. In what follows, we examine the response of such states to applied current. In our study, the external current density \( j \) is increased adiabatically from zero up to a certain value \( j_f \), streaming from the north to the south side of the sample.

The plot of voltage against current for a mesoscopic chiral p-wave superconductor is shown in Fig. 1 with the voltage defined as: \( V = \phi |y_f - \phi|y_i \), where the bar over the electrostatic potential denotes average, and \( y_i = 1.5\xi \) and \( y_f = 10.5\xi \). To date, for chiral p-wave superconductors only the stationary GL equations have been derived either phenomenologically or microscopically [3, 2, 15]. The TDGL equations (1) and (2), obtained as an extension of the stationary ones after imposing full gauge invariance are conceived for gapless superconductors, but are expected to capture the evolution of static and dynamic states in the here studied cases.

Three different regimes can be identified from the current-voltage characteristics of Fig. 1 namely the superconducting (stationary), resistive (non-stationary), and normal (ohmic) regime. At low currents the superconducting regime can exhibit weak resistance, consequence of the normal contacts (see the inset of Fig. 1). Fig. 2 shows the superconducting phase at \( j = 0 \) in contour plots of \( |\psi_+|^2 \). (a),
Figure 1: Measured voltage versus applied current in dimensionless units for a mesoscopic sample of size $10\xi \times 12\xi$ with normal contacts at north and south sides. An external magnetic field ($H = 0.8H_{c2}$) applied perpendicularly to the sample stabilizes domain walls, skyrmions and half-quantum vortices. Three different regimes can be identified, namely the superconducting (SC), resistive and normal regime (N). The resistive regime contains different states labelled here by (a) - (g).

$|\psi_-|^2$ (b), the current density $\vec{J}_x$ (c), and the cosine of the intercomponent phase difference $\cos(\theta_x - \theta_y)$ (d) [from now on simply call the phase difference]. The angular phases $\theta_x$ and $\theta_y$ are obtained from the redefined order parameters $\psi_x = (\psi_+ + \psi_-)/2$ and $\psi_y = (\psi_+ - \psi_-)/2i$, respectively. The superconducting state, according to panel (d) is composed of one skyrmion inside the sample and the edge state enclosing the sample and containing six HQVs at the borders, and four domain walls around the corners.

As one increases the external current the superconducting state of Fig. 2 shifts to the right due to the reduction of the superconducting currents in the east side compared to the west side (due to compensation of the Meissner currents with applied current, see e.g. Fig. 1). The resistive regime thus appears at currents where the flux motion drives the superconductor to a non-stationary state. From Fig. 1 one can see that such regime exhibits sequential jumps in the voltage as current is increased, which we attribute to different non-stationary states (labelled there by letters). In order to study the temporal evolution of the two-component superconducting order parameter in the resistive regime, we choose the state $a$ of Fig. 1 since it summarizes all the rich properties that a mesoscopic chiral $p$-wave superconductor presents. Animated data of the remaining states of Fig. 1 are therefore left for the supplementary section (a included).

The plot of Fig. 3(a) reveals that the voltage in state $a$ of Fig. 1 is a periodic function of time. Moreover, one can clearly see that there exist three distinct modes that correspond to a special flux motion. Contour plots of the phase difference show the superconducting state at these modes. From panel (●) to (■) one can distinguish three events: (i) the bottom skyrmion is heading towards the E side, (ii) one HQV at the E side left the sample at the south-east corner, and (iii) one HQV at the W side acquired a quantum of flux from component $\psi_-$ to form a full vortex. Next, the skyrmion having two quanta of flux broke into two HQVs and one of these went to the E side while the other fused with another quantum of flux to form a second full vortex [see panels (■) and (▲)]. Another mechanism of skyrmion annihilation, displayed in state $f$ (see the supplementary section), consist of one skyrmion losing its two quanta of flux in the form of two concentric HQVs. Finally, panels (▲) to (☆) show the fusion of two full vortices into a
skyrmion and the nucleation of a HQV at the W side from the west-south corner. There exists another mechanism of skyrmion creation, consisting of two quanta of flux being pumped inside the sample from the edge state, more precisely the W side (see the state \( f \) in the supplementary section).

The role of the normal contacts in the one dimensional movement of the HQVs is crucial. Owing to the superconducting-normal-metal interfaces the barrier for HQV exit/entry is cancelled on the N/S sides of the sample. Further, there also exists a barrier formed by Meissner currents on the E/W sides, that prevents the HQVs to leave the edge state or conversely that prevent the HQV to get in the sample. Altogether, the HQV at the E and W sides experience the easy direction for motion along the superconducting-vacuum interfaces.

4. Conclusions

In summary, using the time-dependent Ginzburg-Landau equations for chiral \( p \)-wave superconductors, we have shown some characteristic dynamics of skyrmions and the edge state in a mesoscopic \( p \)-wave superconductor. When an external current is applied to the sample, the resistive state shows much richer behavior compared to conventional \( s \)-wave superconductors. For example, depending on the strength of the external current, we found that the half-quantum vortices in the edge state can move along the direction of the applied current, contrary to standard kinematic vortices which always move perpendicularly to the current flow \([12, 13, 14]\). We also observe in the resistive regime that under the applied current skyrmions either nucleate the sample directly from the edge state or arise from the recombination of two full vortices. These findings combinatorially increase the possibilities for different resistive states in mesoscopic superconductors, worthy of further exploration.

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