Leading Chiral Corrections to the Nucleon Generalized Parton Distributions

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Abstract

Using heavy baryon chiral perturbation theory we study the leading chiral corrections to the complete set of nucleon generalized parton distributions (GPDs). We compute the leading quark mass and momentum transfer dependence of the moments of nucleon GPDs through the nucleon off-forward twist-2 matrix elements. These results are then applied to get insight on the GPDs and their impact parameter space distributions.

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I. INTRODUCTION

Recently there have been great interests in generalized parton distributions (GPDs) of hadrons (see e.g. Refs. [1–3] for the original work, and Refs. [4–6] for recent reviews). GPDs relate quite different physical quantities, such as Feynman’s parton distribution functions (PDFs) and hadron form factors, in the same framework. By generalizing PDFs’ one dimensional parton distribution pictures, GPDs provide the three-dimensional pictures [7, 9]. Furthermore, GPDs also give information on highly desirable quantities such as the quark orbital angular momentum contribution to proton spin [2].

Useful constraints on the forms of the nucleon GPDs have been obtained at DESY [10] and Jefferson Lab [11]. Typically the GPDs contribute to experimental observables through convolutions; they are not directly measurable through those experiments. Thus inputs from theories are important. Valuable insights are gained through the impact parameter distribution interpretation [7] (see also [8]) and model computations of GPDs [see e.g. [6] for a review]. Recently, lattice techniques have first been applied to compute the moments of nucleon GPDs directly from QCD [12]. The latest unquenched lattice results are presented in [13]. However, due to the limitation of computing power, these calculations often employed $u$ and $d$ quark masses heavier than their physical values and employed big momentum transfer. Thus, extrapolations in quark mass and momentum transfer are required to obtain physical results. In general, the quark mass and momentum transfer dependence have non-analytic structures which should be incorporated in the parametrization of the extrapolation formulas. Fortunately, chiral perturbation theory ($\chi$PT) [14–17], which is an effective field theory of QCD, can be applied to extract those non-analytic structures in a systematic, model-independent way.

Recently $\chi$PT has first been applied to the computation of hadronic twist-2 matrix elements [18, 19], which is related to the moments of PDFs and GPDs through the operator product expansions. Many applications have been worked out, e.g., chiral extrapolations of lattice data [18–21] including (partially) quenching [22, 23] and finite volume [21] effects, GPDs for quark contribution to proton spin [24], gravitational form factors [25], pion GPDs [26, 27], large $N_C$ relations among nucleon and $\Delta$-resonance distributions [28] (see also earlier work in the large $N_C$ limit [29]), soft pion productions in deeply virtual Compton scattering [30–32], SU(3) symmetry breaking in the complete set of twist-2 [33] and twist-3
light cone distribution functions, pion-photon transition distributions and exclusive semileptonic B decays. The method is also generalized to the multi-nucleon case. There are also earlier results derived using the soft pion theorem.

In this paper, we use χPT to study the complete set of nucleon GPDs. We compute the leading quark mass and momentum transfer dependence of the moments of nucleon GPDs through computing the nucleon off-forward (meaning the initial and final nucleon momenta being different) twist-2 matrix elements. Then we apply the χPT results of the moments to get insight on the GPDs themselves and their impact parameter space distributions.

II. NUCLEON GPDS

The eight nucleon GPDs, \( H, E, \bar{H}, \bar{E}, H_T, \bar{H}_T, E_T, \) and \( \bar{E}_T \), form the complete set of leading twist GPDs for quarks of flavor \( q \). They are defined through off-forward nucleon matrix elements of the vector, axial vector and tensor quark bilinears:

\[
\langle P' | \bar{q} \left( -\frac{z}{2} n \right) q \left( \frac{z}{2} n \right) | P \rangle = \int dx e^{-ixzn \cdot P} \overline{U}(P') \left[ H \gamma_\mu + E \frac{i \sigma^{\alpha \beta} n_\alpha \Delta_\beta}{2M} \right] U(P),
\]

\[
\langle P' | \bar{q} \left( -\frac{z}{2} n \right) \gamma_5 q \left( \frac{z}{2} n \right) | P \rangle = \int dx e^{-ixzn \cdot P} \overline{U}(P') \left[ \tilde{H} \gamma_5 + \tilde{E} \frac{\gamma_5 n \cdot \Delta}{2M} \right] U(P),
\]

\[
\langle P' | \bar{q} \left( -\frac{z}{2} n \right) i n_\alpha r_\beta \sigma^{\alpha \beta} q \left( \frac{z}{2} n \right) | P \rangle = \int dx e^{-ixzn \cdot P} \overline{U}(P') \left[ H_T i n_\alpha r_\beta \sigma^{\alpha \beta} + \tilde{H}_T \frac{(n \cdot P) (r \cdot \Delta) - (r \cdot P) (n \cdot \Delta)}{M^2} + E_T \frac{\gamma_\mu (r \cdot \Delta) - \gamma_\mu (n \cdot \Delta)}{2M} \right] U(P),
\]

where \( M \) is the nucleon mass, \( U \) is the nucleon Dirac spinor with normalization \( \overline{U}(P)U(P) = 2M \), \( P^\mu = (P + P')^\mu / 2 \), \( \Delta^\mu = P^\mu - P'^\mu \), \( n \) is a dimensionless light-like vector \( [n^2 = 0] \), \( \xi = -n \cdot \Delta / (2n \cdot \bar{P}) \) and \( t = \Delta^2 \). \( r \) is a transverse vector satisfying \( r \cdot n = r \cdot \bar{P} = 0 \). We have used \( \epsilon^{0123} = -1 \) and the notation \( \epsilon^{ABCD} = \epsilon^{\alpha \beta \gamma \delta} A_\alpha B_\beta C_\gamma D_\delta \). Here we have suppressed the Wilson lines connecting quark fields locating at \( -\frac{z}{2} n \) and \( \frac{z}{2} n \) to make the nonlocal quark operators gauge invariant. These GPDs are functions of \( x \), \( \xi \), and \( t \). Both \( x \) and \( \xi \) have support from \(-1\) to \(+1\).

The GPDs encode information of ordinary PDFs and nucleon form factors. In the
forward limit of $\Delta^\mu \to 0$, we have

$$H(x, 0, 0) = f_1(x), \quad \tilde{H}(x, 0, 0) = g_1(x), \quad H_T(x, 0, 0) = f_T(x),$$

(4)

where $f_1(x)$, $g_1(x)$ and $f_T(x)$ are spin-averaged, helicity and transversity PDFs, respectively. On the other hand, forming the first $x$ moments of the new distributions, one gets the following sum rules,

$$\int dx H(x, \xi, t) = F_1(t),$$

$$\int dx E(x, \xi, t) = F_2(t),$$

$$\int dx \tilde{H}(x, \xi, t) = G_A(t),$$

$$\int dx \tilde{E}(x, \xi, t) = G_P(t).$$

(5)

where $F_1$ and $F_2$ are the Dirac and Pauli form factors and $G_A$ and $G_P$ are the axial-vector and pseudo-scalar form factors. There are also tensor form factors associated with the first $x$ moments of $H_T$, $\tilde{H}_T$ and $E_T$, but not $\tilde{E}_T$, because time reversal invariance demands [42]

$$\int dx \tilde{E}_T(x, \xi, t) = 0.$$

(6)

The most interesting sum rule relevant to the nucleon spin is [2],

$$J_q = \frac{1}{2} \int dx x [H(x, \xi, 0) + E(x, \xi, 0)],$$

(7)

where $J_q$ is the $q$ quark contribution to proton spin in a frame in which the proton has a definite helicity. The $\xi$ dependence in the sum rule has dropped out. Since $J_q$ can further be decomposed into quark helicity and orbital angular moment contributions, by measuring $J_q$ from experiments sensitive to GPDs and measuring the helicity contribution from polarized deep inelastic scattering, the quark orbital angular momentum contribution to proton spin in principle can be obtained.

There are also a set of eight gluon GPDs defined as the matrix elements of non-local gluon operators. They mix with the isoscalar combination of quark GPDs under renormalization scale and transform in the same way as isoscalar quark GPDs under chiral transformation. We will first focus on the quark GPDs, and later come back to the gluon GPDs.
III. CHIRAL PERTURBATION THEORY

$\chi$PT is a low-energy effective field theory of QCD. $\chi$PT makes use of the symmetries and scale separation of QCD and allows a model independent description of physics below the chiral symmetry breaking scale $\mu_\chi(\sim 4\pi F_\pi \sim 1 \text{ GeV})$, where $F_\pi = 93$ MeV is the pion decay constant. In this paper, we will compute single nucleon matrix elements associated with nucleon GPDs; thus, the relevant scales below $\mu_\chi$ (light scales) include the pion mass $m_\pi \simeq 139$ MeV and the characteristic momentum $p$ in the problem. The nucleon mass $M$, which is numerically of the same size as $\mu_\chi$, is treated as a heavy scale as $\mu_\chi$. Thus the standard heavy baryon $\chi$PT [16] approach is used to systematically disentangle the light and heavy scales. Here the following four small expansion parameters are treated the same in the chiral expansion and denoted as

$$\varepsilon = \frac{p}{\mu_\chi}, \frac{m_\pi}{\mu_\chi}, \frac{p}{M}, \frac{m_\pi}{M}. \quad (8)$$

The physical pion fields $(\pi^0, \pi^+, \pi^-)$ enter the theory through the matrices

$$\Sigma = e^{\Pi/F_\pi}, \quad u = \sqrt{\Sigma}, \quad (9)$$

where

$$\Pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}. \quad (10)$$

The relevant terms in the chiral Lagrangian are

$$\mathcal{L} = \frac{F_\pi^2}{8} \text{Tr} \left[ \partial^\mu \Sigma \partial_\mu \Sigma^\dagger \right] + \lambda \text{Tr} \left[ m_q \Sigma^\dagger \right. \left. + \text{h.c.} \right] + iN^\dagger \nu \cdot DN + 2g_A N^\dagger S \cdot A N + \cdots, \quad (11)$$

where the quark mass matrix $m_q = \text{diag}(m_u, m_d) = m_q^\dagger$, and we will take the isospin symmetry limit $m_u = m_d = \overline{m}$. The nucleon field $N = (p, n)^T$, $\nu$ is the nucleon velocity and $S^\mu = i\sigma^{\mu\nu} \gamma_5 v_\nu$ is the nucleon spin vector. $\nu \cdot S = 0$. $g_A = 1.26$ is the axial-vector coupling constant. The pion-nucleon couplings arise in Eq. (11) through the vector and axial couplings

$$\mathcal{V}^\mu = \frac{1}{2} \left( u \partial^\mu u^\dagger + u^\dagger \partial^\mu u \right), \quad \mathcal{A}^\mu = \frac{i}{2} \left( u \partial^\mu u^\dagger - u^\dagger \partial^\mu u \right), \quad (12)$$

and the chiral covariant derivative

$$D^\mu = (\partial^\mu + \mathcal{V}^\mu). \quad (13)$$
Under a $SU(2)_L \otimes SU(2)_R$ chiral rotation, the hadronic fields in the chiral lagrangian transform as

$$\begin{align*}
\Sigma &\rightarrow L \Sigma R^\dagger, \\
m_j &\rightarrow L m_j R^\dagger, \\
N &\rightarrow U(x) N, \\
u &\rightarrow Lu \U^\dagger(x) = U(x) u R^\dagger, \\
\mathcal{A}^\mu &\rightarrow U(x) \mathcal{A}^\mu U^\dagger(x), \\
D^\mu N &\rightarrow U(x) D^\mu N,
\end{align*}$$

such that the chiral lagrangian in Eq. (11) stays invariant under chiral transformation.

IV. THE VECTOR OPERATORS

Instead of directly matching the non-local quark bilinear operators to hadronic operators in $\chi$PT, it is conceptually more straightforward to deal with matching of local operators. One can perform operator product expansions (OPEs) to convert the non-local operators to the sums of local twist-2 operators then do the matching [18, 19]. The Taylor-series expansion of Eq.(1) gives

$$\langle P'|O^m|P \rangle = (n \cdot \bar{P})^m \U(P') \left[ H_{m+1} + E_{m+1} \frac{i \sigma^\alpha n_\alpha \Delta^\beta}{2M} \right] U(P),$$

where

$$O^m = \bar{q} \eta \left( \bar{n} \cdot \overset{\leftrightarrow}{D} \right)^m q$$

is a twist-2 operator dotted by the $n^{\mu_0} n^{\mu_1} \cdots n^{\mu_m}$ tensor to project out the symmetric and traceless part. The gauge invariant covariant derivative $\overset{\leftrightarrow}{D}^\mu = (\overset{\leftarrow}{D}^\mu - \overset{\rightarrow}{D}^\mu)/2$. $H_{m+1}(\xi, t) = \int dx x^m H(x, \xi, t)$ and $E_{m+1}(\xi, t) = \int dx x^m E(x, \xi, t)$ are the $(m + 1)$-th moments in $x$ of the GPDs. The nucleon matrix element of $O^m$ has different form factor structures [2, 43, 44]. Using the notation of Ref. [44],

$$\begin{align*}
\langle P'|O^m|P \rangle = \U(P') & \left[ \sum_{j=0}^{m} \left\{ \frac{\eta}{2M} n \cdot \Delta^j (n \cdot \bar{P})^{m-j} A_{m+1,j}(t) \\
& - i \frac{\sigma^\alpha n_\mu \sigma^\beta n_\mu}{2M} (n \cdot \Delta)^j (n \cdot \bar{P})^{m-j} B_{m+1,j}(t) \right\} \\
& + \frac{1}{M} (n \cdot \Delta)^{m+1} C_{m+1}(t) \right|_{m \text{ odd}} \U(P) \right].
\end{align*}$$
The constraints on \( j \) and \( C_m \) are due to the requirement of time reversal invariance [2]. By comparing this equation with Eq.(15), we have

\[
H_{m+1}(\xi, t) = \int_{-1}^{1} dx x^m H(x, \xi, t) = \sum_{j=0}^{m} (-2\xi)^j A_{m+1,j}(t) + (-2\xi)^{m+1} C_{m+1}(t) |_{m \text{ odd}} ,
\]

\[
E_{m+1}(\xi, t) = \int_{-1}^{1} dx x^m E(x, \xi, t) = \sum_{j=0}^{m} (-2\xi)^j B_{m+1,j}(t) - (-2\xi)^{m+1} C_{m+1}(t) |_{m \text{ odd}} \tag{18}
\]

after using the Gordon decomposition. The time reversal invariance demands \( H_{m+1} \) and \( E_{m+1} \) to be even in \( \xi \):

\[
H_{m+1}(-\xi, t) = H_{m+1}(\xi, t) , \quad E_{m+1}(-\xi, t) = E_{m+1}(\xi, t) . \tag{19}
\]

To apply heavy baryon \( \chi PT \), we perform the \( 1/M \) expansion to Eq.(17) [16]. The leading terms are

\[
\langle P' | O^m | P \rangle = 2N \sum_{j=0}^{m} \left\{ (n \cdot \Delta)^j (Mn \cdot v)^{m-j+1} E_{m+1,j}(t) \right. \\
+ i\epsilon^{\mu\nu\Delta S} (n \cdot \Delta)^j (Mn \cdot v)^{m-j} M_{m+1,j}(t) \} \\
+ (n \cdot \Delta)^{m+1} C_{m+1}(t) |_{m \text{ odd}} N , \tag{20}
\]

where

\[
E_{m+1,j} = A_{m+1,j} + \frac{t}{4M^2} B_{m+1,j} , \quad M_{m+1,j} = A_{m+1,j} + B_{m+1,j} . \tag{21}
\]

For convenience, we will work in the Breit frame where \( v^\mu = (1, \vec{0}) = P^\mu/M + \mathcal{O}(1/M) \), \( S^\mu = (0, \vec{\sigma}/2) \) and \( \Delta^\mu = (0, \vec{\Delta}) \). The normalization of the Pauli spinor \( N \) is \( \overline{N}N = 1 + \mathcal{O}(1/M) \).

### A. Pionic Vector Operators

In a similar manner, the pion vector GPD is defined as

\[
\langle \pi^i(P'_\pi) | \overline{q} \left( \frac{-z}{2} n \right) \tau^\alpha q \left( \frac{z}{2} n \right) | \pi^j(P_\pi) \rangle \\
= \int dy e^{-iyzn \cdot \vec{P}_\pi} H_\pi^\alpha(y, \xi_\pi, t)n \cdot \vec{P}_\pi \text{tr} \left[ \tau^i \tau^\alpha \tau^j \right] , \tag{22}
\]

where the isospin operator \( \tau^a = (1, \vec{\tau}) \), and from now on \( q \) is an isospin multiplet [but note that the isoscalar quark also contains the s quark contribution]. There is no \( E_\pi \) GPD
because the pion is spinless. The Taylor-series expansion of the above equation gives

\[ \langle \pi^i(P'_\pi)|\mathcal{O}_\alpha^m|\pi^j(P_\pi)\rangle = H_{\pi,m+1}(\xi_\pi,t) (n \cdot \vec{P}_\pi)^{m+1} tr[\tau^i\tau^\alpha]\tau^j, \quad (23) \]

where \( \vec{P}_\pi = (P_\pi + P'_\pi)/2 \) and

\[ \mathcal{O}_\alpha^m = \bar{q} \tau^\alpha \eta (n \cdot \vec{iD})^m q, \quad (24) \]

and \( H_{\pi,m+1}(\xi_\pi,t) = \int dyy^m H_\pi(y,\xi_\pi,t) \).

The pion GPDs are strongly constrained by charge conjugation (C) and isospin symmetry. Under C,

\[ C\mathcal{O}_\alpha^m C^{-1} = (-1)^{m+1} \mathcal{O}_\alpha^m, \quad (25) \]

for \( \alpha = 0 \) and 3 [33]. For \( m \) even \( (m = 2k) \), the above equation implies

\[ \langle \pi^0(P'_\pi)|\mathcal{O}_0^{2k}|\pi^0(P_\pi)\rangle = 0, \quad \langle \pi^0(P'_\pi)|\mathcal{O}_3^{2k}|\pi^0(P_\pi)\rangle = 0, \quad (26) \]

because \( \pi^0 \) is C even. Furthermore, by isospin symmetry, \( \langle \pi^\pm(P'_\pi)|\mathcal{O}_0^{2k}|\pi^\pm(P_\pi)\rangle = 0 \). This implies

\[ H_{\pi,2k+1}^0(\xi_\pi,t) = 0, \quad (27) \]

or, equivalently,

\[ H_{\pi}(y,\xi_\pi,t) = H_{\pi}^0(-y,\xi_\pi,t). \quad (28) \]

For \( m \) odd \( (m = 2k - 1) \),

\[ \langle \pi^i(P'_\pi)|\mathcal{O}_0^{2k-1}|\pi^j(P_\pi)\rangle = 2\delta_{ij} \left\{ \sum_{l=0}^{k-1} (n \cdot \Delta)^{2l} (n \cdot \vec{P}_\pi)^{2k-2l} A_{2k,2l}^{\pi,0}(t) + (n \cdot \Delta)^{2k} C_{2k}^{\pi,0}(t) \right\}, \quad (29) \]

where \( P'_\pi - P_\pi = \Delta \) and \( \xi_\pi = -n \cdot \Delta / (n \cdot \vec{P}_\pi) \).

On the other hand, for the isovector case, under C

\[ \langle \pi^+(P'_\pi)|\mathcal{O}_3^{2k-1}|\pi^+(P_\pi)\rangle \rightarrow \langle \pi^-(P'_\pi)|\mathcal{O}_3^{2k-1}|\pi^-(P_\pi)\rangle \]

\[ = -\langle \pi^+(P'_\pi)|\mathcal{O}_3^{2k-1}|\pi^+(P_\pi)\rangle, \quad (30) \]

where the equals sign is due to \( \langle \mathcal{O}_3^{2k-1} \rangle \propto \langle \tau_3 \rangle \). This, together with \( \langle \pi^0|\mathcal{O}_3^{2k-1}|\pi^0 \rangle = 0 \), implies

\[ H_{3,2k}^3(\xi_\pi,t) = 0. \quad (31) \]
Thus

$$\langle \pi^i(P'_\pi)|O_{\alpha}^{2k}|\pi^i(P_\pi)\rangle = 2ie^{\pm i\beta} \sum_{l=0}^{k} (n \cdot \Delta)^{2l} (n \cdot \vec{P})^{2k-2l+1} A^{\pi,3}_{2k+1,2l}(t) .$$

(32)

Again, time reversal invariance requires $H^\alpha_\pi(y, \xi_\pi, t)$ to be even in $\xi_\pi$:

$$H^\alpha_\pi(y, \xi_\pi, t) = H^\alpha_\pi(y, -\xi_\pi, t) ,$$

(33)

as shown in Eqs. (29) and (32).

In matching $O^m_\alpha$ to the hadronic operators in $\chi$PT, it is useful to write

$$O^m_\alpha = O^m_{\alpha,R} + O^m_{\alpha,L} ,$$

(34)

where

$$O^m_{\alpha,R} = \bar{q}_R \tau^\alpha_R (n \cdot i\vec{D})^m q_R ,$$

(35)

and similarly for $O^m_{\alpha,L}$. $q_{L,R} = [(1 \mp \gamma_5)/2]q$ is the left(right)-handed quark field. The distinction between $\tau^a_L$ and $\tau^a_R$ is only for bookkeeping purposes. We will set $\tau^a_L = \tau^a_R = \tau^a$ at the end. Under a global chiral $SU(2)_L \times SU(2)_R$ transformation, $q_R \rightarrow Rq_R$ and $q_L \rightarrow Lq_L$. If we demand

$$\tau^a_L \rightarrow L\tau^a_L , \quad \tau^a_R \rightarrow R\tau^a_R ,$$

(36)

then $O^m_{\alpha,R(L)}$ will be invariant under chiral transformation. Furthermore, under charge conjugation, $\Sigma \rightarrow \Sigma^T$, $m_q \rightarrow m_q^T$, and if we demand

$$\tau^a_L \rightarrow \tau^a_L^T , \quad \tau^a_R \rightarrow \tau^a_R^T ,$$

(37)

then Eq.(25) can be satisfied for any $\alpha$.

Using the symmetries mentioned above, we now match $O^m_\alpha$ to the most general combination of hadronic operators with the same symmetries,

$$O^m_\alpha \rightarrow O^m_{\alpha,\pi} + O^m_{\alpha,N} + \cdots ,$$

(38)

where $O^m_{\alpha,\pi}$ denotes hadronic operators made of purely pion fields and $O^m_{\alpha,N}$ denotes hadronic operators with nucleon number equal to one. The ellipse denotes operators which do not contribute to our $SU(2)$ calculations in single nucleon sectors, such as operators with nucleon number equal to two and above or operators with hyperon fields.
For a given \( m \), the leading pionic operators in the chiral expansion are

\[
O_{\alpha, \pi}^{m} = \frac{F_{\pi}^{2}}{4} \sum_{j=0}^{m} \mathcal{A}_{m+1,j}^{\tau, \alpha}(0) (-in \cdot \partial)^{j} \mathrm{tr} \left[ \tau_{L}^{\alpha} \Sigma \left( in \cdot \partial \right)^{m-j+1} \Sigma \right] + \cdots ,
\]

(39)

where \( \vec{\partial} \mu = (\vec{\partial} \mu - \vec{\partial} \mu) / 2 \) and the ellipse denotes higher order operators with more powers of derivatives or quark mass matrix. \( m = 0, 1, 2... \) There is no restriction on the value of \( m \) [18, 19]. For \( \alpha = 0 \) (the isoscalar case), it is easy to see that the contribution with \( m \) even vanishes—a consequence of charge conjugation. Using

\[
(-in \cdot \partial)^{2m} \left[ \Sigma \left( in \cdot \partial \right)^{2n} \Sigma \right] = (-in \cdot \partial)^{2m+2} \left[ \Sigma \left( in \cdot \partial \right)^{2n-2} \Sigma \right] - (-in \cdot \partial)^{2m} \left[ (in \cdot \partial \Sigma) \left( in \cdot \partial \right)^{2n-2} (in \cdot \partial \Sigma) \right],
\]

the operator in Eq.(39) can be rewritten as

\[
O_{0, \pi}^{2k-1} = \frac{F_{\pi}^{2}}{2} \sum_{l=0}^{k-1} \mathcal{A}_{2k,2l}^{\tau, 0}(0) (-in \cdot \partial)^{2l} \mathrm{tr} \left[ (in \cdot \partial \Sigma) \left( in \cdot \partial \right)^{2k-2l-2} (in \cdot \partial \Sigma) \right] + \cdots ,
\]

(40)

which is the operator constructed in Ref. [26]. The prefactors in Eq.(40) are chosen such that Eq.(29) is reproduced in the leading order in the chiral expansion subject to constraints that relate \( C_{2k}^{\tau, 0} \) to \( A_{2k,2l}^{\tau, 0} \). The constraints come from the \( (n \cdot P_{\pi})(n \cdot P_{\pi}) = (n \cdot \bar{P}_{\pi})^{2} \) factor in \( \langle \pi^{i}(P_{\pi})|O_{0,\pi}^{2k-1}\pi^{j}(P_{\pi})\rangle \) which makes \( H_{\pi}^{0}(y, \xi_{\pi} = \pm 1, 0) = 0 \) [this was also observed in Refs. [38, 39]]. This property, however, does not persist at higher orders. At the next-to-leading order (NLO), there several sets of counterterms:

\[
O_{0, \pi, 1}^{2k-1} = a_{2k}^{\pi, 0} (-in \cdot \partial)^{2k} \mathrm{tr} \left[ \Sigma m_{q}^{\dagger} + m_{q} \Sigma^{\dagger} \right] ,
\]

\[
O_{0, \pi, 2}^{2k-1} = \sum_{l=0}^{k-1} b_{2k,2l}^{\pi, 0} \mathrm{tr} \left[ \Sigma m_{q}^{\dagger} + m_{q} \Sigma^{\dagger} \right] (-in \cdot \partial)^{2l} \mathrm{tr} \left[ (in \cdot \partial \Sigma) \left( in \cdot \partial \right)^{2k-2l-2} (in \cdot \partial \Sigma) \right] ,
\]

\[
O_{0, \pi, 3}^{2k-1} = \sum_{l=0}^{k-1} c_{2k,2l}^{\pi, 0} (\Sigma m_{q}^{\dagger} + m_{q} \Sigma^{\dagger}) \left( (in \cdot \partial \Sigma) \left( in \cdot \partial \right)^{2k-2l-2} (in \cdot \partial \Sigma) \right) ,
\]

\[
O_{0, \pi, 4}^{2k-1} = \sum_{l=0}^{k-1} d_{2k,2l}^{\pi, 0} \partial^{2} (-in \cdot \partial)^{2l} \mathrm{tr} \left[ (in \cdot \partial \Sigma) \left( in \cdot \partial \right)^{2k-2l-2} (in \cdot \partial \Sigma) \right] .
\]

(41)

It is \( O_{0, \pi, 1}^{2k-1} \) that makes \( H_{\pi}^{0}(y, \pm 1, 0) \) non-vanishing.
B. Nucleon Vector Operators

In matching $O_{\alpha}^m$ to nucleon operators, it is convenient to define

$$ \tau_{u\pm}^a = \frac{1}{2} \left( u \tau_{R}^a u^\dagger \pm u^\dagger \tau_{L}^a u \right), \quad (42) $$

such that under a chiral transformation, $\tau_{u\pm}^a \rightarrow U(x) \tau_{u\pm}^a U(x)^\dagger$ with $\tau_{R(L)}^a$ transforms as in Eq.(36). The leading operators contributing to $H_{m+1}^\alpha$ and $E_{m+1}^\alpha$ are

$$ O_{\alpha,N}^m = 2M (Mn \cdot v)^m \mathcal{N} \left[ H_{m+1}^{\alpha(0)}(\xi,0)n \cdot v + E_{m+1}^{\alpha(0)}(\xi,0) \frac{i \epsilon_{m\Delta S}}{M} \right] \tau_{u+N}^\alpha $$

$$ + 2M (Mn \cdot v)^m \mathcal{N} \left[ C_{m+1}^{\alpha(0)}(\xi,0)n \cdot S + D_{m+1}^{\alpha(0)}(\xi,0) \frac{i \epsilon_{Sn\Delta S}}{2M} \right] \tau_{u-N}^\alpha + \cdots, \quad (43) $$

where $E_{m+1} = H_{m+1} + E_{m+1}$. The prefactors are chosen such that Eq.(15) is reproduced in the leading order in the chiral expansion and a superscript (0) denotes the chiral limit value. The $C_{m+1}$ and $D_{m+1}$ operators will not contribute at the order we are working (NLO). The $H_{m+1}$ and $E_{m+1}$ operators in Eq.(43) can be further written as

$$ O_{\alpha,N}^m = 2\mathcal{N} \sum_{j=0}^{m} \left\{ \begin{array}{c} (n \cdot \Delta)^j (Mn \cdot v)^{m-j+1} E_{m+1,j}^{\alpha(0)}(0) \\
+ i \epsilon_{m\Delta S} (n \cdot \Delta)^j (Mn \cdot v)^{m-j} M_{m+1,j}^{\alpha(0)}(0) \end{array} \right\} $$

$$ + (n \cdot \Delta)^{m+1} C_{m+1}^{\alpha(0)}(0) \left. \right|_{m \text{ odd}} \tau_{u+N}^\alpha + \cdots, \quad (44) $$

where we have replaced the total derivative operator $-in \cdot D$ by $n \cdot \Delta$. The operators with derivatives acting only on $\tau_{u+}^a$ or $\tau_{u-}^a$ do not contribute to nucleon GPDs by direct computation.

C. Form factor results of vector twist-2 matrix elements

Now we present the leading chiral corrections to the form factors of the vector twist-2 operators defined in Eq.(20). We will insert powers of $\epsilon$ to keep track of the chiral expansion. One should set $\epsilon = 1$ when using these results. The results of $E_{m+1,0}^\alpha(0)$ reproduce those of the forward twist-2 matrix elements in Refs. [18, 19, 28], while $E_{1,0}^\alpha(t)$ and $M_{1,0}^\alpha(t)$ agree with the electric and magnetic form factor results in $\chi$PT [17].
For the isoscalar \((\alpha = 0)\) form factors\(^1\),

\[
E_{m+1,2k}^0(t) = E_{m+1,2k}^{0(0)}(t) + \varepsilon^2 (1 - \delta_{m,0}\delta_{k,0}) \alpha_{m+1,2k}^0 m_\pi^2 + \varepsilon^2 \beta_{m+1,2k}^0 t + \mathcal{O}(\varepsilon^3),
\]

\[
M_{m+1,2k}^0(t) = \left(1 + \varepsilon^2 C_M^0\right) M_{m+1,2k}^{0(0)}(t) + \varepsilon^2 \delta_{m,2k+1} D_{2k+2,2k}^{0,M}(t)
+ \varepsilon^2 \gamma_{m+1,2k}^0 m_\pi^2 + \varepsilon^2 \zeta_{m+1,2k}^0 t + \mathcal{O}(\varepsilon^3),
\]

\[
C_{m+1}^0(t) = C_{m+1}^{0(0)}(t) + \varepsilon \delta_{m,2k+1} D_{2k+2}^{C}(t) + \mathcal{O}(\varepsilon^2),
\]

(45)

where \(k = 0, 1, 2 \ldots\) The \(\mathcal{C}\) contribution is from the wave function renormalization (Fig.1(f)) and the loop diagrams with one insertion of the nucleon operators in Eq. (44) (Figs. 1(b) and (c)). The \(\mathcal{D}\) contributions are from the loop diagrams with one insertion of the pionic operators in Eq. (44) (Figs. 1 (d) and (e)).

For the \(E_{m+1,2k}^0(t)\) form factor, the one-loop contributions from Figs. 1(b) and (f) cancel each other while the diagrams in Figs. 1(d) and (e) are of higher order and Fig. 1(c)

\(^1\) There is no restriction on the range of \(m\) in this ChPT calculation. We are only interested in the \(m_\pi\) and \(t\) dependence of the form factors, thus we keep track of the expansion in \(m_\pi\) and \(t\) using the parameter \(\varepsilon\) and count \(x = \mathcal{O}(\varepsilon^0)\). There is no intrinsic difference between operators of different \(m\)—they all transform in the same way under a chiral rotation [18, 19].
vanishes. At $O(\varepsilon^2)$, $E^{0}_{m+1,2k}$ also receives analytic contributions proportional to $m_\pi^2$ or $t$ from counterterms (Fig. 1(a)) except for the charge operator $E^{0}_{1,0}(0)$. $\alpha^{0}_{m+1,2k}$ and $\beta^{0}_{m+1,2k}$ are independent of $m_\pi$, $t$ and the renormalization scale $\mu$.

For the $M^{0}_{m+1,2k}$ form factor,

$$C^{0}_{M} = -\frac{3g^{2}_{A}m^{2}_{\pi}}{(4\pi F^{2}_{\pi})^{2}} \ln \frac{m^{2}_{\pi}}{\mu^{2}},$$

(46)

and

$$D^{0,M}_{2k+2,2k2}(t) = \frac{3g^{2}_{A}}{16\pi^{2}F^{2}_{\pi}} \sum_{l=0}^{k} A^{\pi,0}_{2k+2,2k-2l}(0) \int_{0}^{1} dy (l+1)(2l+1) \left( \frac{1}{2} - y \right)^{2l} m(y)^{2} \log \frac{m(y)^{2}}{\mu^{2}},$$

(47)

where the integration variable $y$ arises from the Feynman parametrization and $m(y) = \sqrt{m_\pi^2 - y(1-y)\Delta^2}$. The counterterm contributions $\gamma^{0}_{m+1,2k}$ and $\zeta^{0}_{m+1,2k}$ depend on $\mu$ but not on $m_\pi$ and $t$. The $\mu$ dependence of $\gamma^{0}_{m+1,2k}$ and $\zeta^{0}_{m+1,2k}$ cancels the $\mu$ dependence from $C^{0}_{M}$ and $D^{0,M}_{2k+2,2k2}$.

As for the $C^{0}_{m+1}$ form factor, it receives non-analytic contributions from the Fig. 1(d) diagram at $O(\varepsilon)$ and no contribution from analytic counterterms until $O(\varepsilon^2)$.

$$D^{C}_{2k+2}(t) = \frac{3g^{2}_{A}}{32\pi F^{2}_{\pi}} \left\{ \sum_{l=0}^{k} A^{\pi,0}_{2k+2,2k-2l}(0) \int_{0}^{1} dy \left[ \frac{1}{2} - y \right]^{2l+1} \left[ \frac{m^{2}_{\pi}}{m(y)} - 4(l+2) m(y) \right] \right\} + C^{\pi,0}_{2k+2}(0) \int_{0}^{1} dy \left[ \frac{m^{2}_{\pi}}{m(y)} - 4 m(y) \right] \right\}.$$

(48)

By setting $(m,k) = (1,0)$ and using $A^{\pi,0}_{2,0}(0) = -4C^{\pi,0}_{2}(0) = \langle x \rangle$, our results in Eq.(45) reproduce those of Refs. [24] and [25].

The leading chiral corrections for the isovector ($\alpha = 3$) form factors are

$$E^{3}_{m+1,2k}(t) = (1 + \varepsilon^{2}C^{3}_{E}) E^{3(0)}_{m+1,2k}(0) + \varepsilon^{2}\delta_{m,2k}^{3,E}D^{3,E}_{2k+1,2k2}(t) + \varepsilon^{2}(1 - \delta_{m,0}\delta_{k,0}) \alpha^{3}_{m+1,2k}m^{2}_{\pi} + \varepsilon^{2}\beta^{3}_{m+1,2k}t + O(\varepsilon^{3}),$$

$$M^{3}_{m+1,2k}(t) = M^{3(0)}_{m+1,2k}(0) + \varepsilon^{2}\delta_{m,2k}D^{3,M}_{2k+1,2k2}(t) + O(\varepsilon^{2}),$$

$$C^{3}_{m+1}(t) = (1 + \varepsilon^{2}C^{3}_{C}) C^{3(0)}_{m+1}(0) + \varepsilon^{2}\eta^{3}_{m+1}m^{2}_{\pi} + \varepsilon^{2}\theta^{3}_{m+1}t + O(\varepsilon^{3}).$$

(49)
Here
\[ C_E^3 = -\frac{(3g_A^2 + 1)m_\pi^2}{(4\pi F)^2} \ln \frac{m_\pi^2}{\mu^2}, \]
\[ C_C^3 = -\frac{(3g_A^2 + 1)m_\pi^2}{(4\pi F)^2} \ln \frac{m_\pi^2}{\mu^2}, \] (50)
and
\[ D_{2k+1,2k}^{3,E}(t) = \sum_{l=0}^{k} A_{2k+1,2k-2l}^{*3} \frac{(1+2l)}{32\pi^2 F^2} \]
\[ \times \int_0^1 dy \left( \frac{1}{2} - y \right)^{2l} \left\{ g_A^2 \left[ -2m_\pi^2 + (5 + 4l)(m(y))^2 \right] + m(y)^2 \right\} \log \frac{m(y)^2}{\mu^2}, \]
\[ D_{2k+1,2k}^{3,M}(t) = -\sum_{l=0}^{k} A_{2k+1,2k-2l}^{*3} \frac{Mg_A^2}{8\pi F^2} \int_0^1 dy \left\{ (1+2l) \left( \frac{1}{2} - y \right)^{2l} m(y) \right\}. \] (51)

Here \( A_{1,0}^{*3}(0) \) is the number of \( u \) quark minus the number of \( d \) quark in a \( \pi^+ \) meson, \( A_{1,0}^{*3}(0) = \langle 1 \rangle_{u-d} = 2 \); thus the charge operator \( E_{3,0}^{3}(0) \) is not renormalized. Unlike the isoscalar case, the chiral corrections to \( M_{m+1,2k}^3 \) and \( C_{m+1}^3 \) start at \( \mathcal{O}(\varepsilon) \) and \( \mathcal{O}(\varepsilon^2) \), respectively.

**V. THE AXIAL OPERATORS**

The Taylor-series expansion of Eq.(2) gives the form factors of off-forward nucleon axial twist-2 matrix elements
\[ \langle P' | \mathcal{O}_{5,\alpha}^m | P \rangle = \sum_{j=0}^{m} \bar{U}(P') \left[ \gamma_5 (n \cdot \Delta)^j (n \cdot \vec{P})^{m-j} \tilde{A}_{m+1,j}^\alpha(t) \right. \]
\[ + \left. \gamma_5 \frac{1}{2M} (n \cdot \Delta)^{j+1} (n \cdot \vec{P})^{m-j} \tilde{B}_{m+1,j}^\alpha(t) \right] \tau^\alpha U(P), \] (52)
where
\[ \mathcal{O}_{5,\alpha}^m = \bar{q} \tau^\alpha \gamma_5 (n \cdot \vec{iD})^m q. \] (53)

For \( m = 0 \), it reduces to the nucleon axial current matrix element
\[ \langle P' | \bar{q} \tau^\alpha \gamma_5 q | P \rangle = \bar{U}(P') \left[ \gamma_5 G_A(t) + \frac{\Delta_\mu}{2M} G_P(t) \right] \gamma_5 \tau^\alpha U(P), \] (54)
with $A_{1,0} = G_A$ and $B_{1,0} = G_P$. The form factors are related to the moments of axial GPDs as

\[
\tilde{H}_{m+1} = \int_{-1}^{1} dx x^m \tilde{H}(x, \xi, t) = \sum_{j=0}^{m} (-2\xi)^j \tilde{A}_{m+1,j}(t),
\]

\[
\tilde{E}_{m+1} = \int_{-1}^{1} dx x^m \tilde{E}(x, \xi, t) = \sum_{j=0}^{m} (-2\xi)^j \tilde{B}_{m+1,j}(t). \tag{55}
\]

Similar to the vector twist-2 operators, $O_{m,\alpha}^m$ is matched to pionic and nucleon operators

\[
O_{5,\alpha}^m \rightarrow O_{5,\alpha,\pi}^m + O_{5,\alpha,N}^m + \cdots. \tag{56}
\]

Since $O_{5,\alpha}^m = O_{\alpha,R}^m - O_{\alpha,L}^m$, the matching to leading pionic operators is similar to that of Eq. (39) with a different relative sign,

\[
O_{5,\alpha,\pi}^m = \frac{F_\pi^2}{4} \sum_{j=0}^{m} \tilde{A}_{m+1,j}^{\alpha}(0) (-in \cdot \partial)^j tr \left[ -\tau_L^\alpha \Sigma \left( in \cdot \frac{\partial}{\partial} \right)^{m-j+1} \Sigma^\dagger + \tau_R^\alpha \Sigma^\dagger \left( in \cdot \frac{\partial}{\partial} \right)^{m-j+1} \Sigma \right] + \cdots. \tag{57}
\]

Parity conservation governs that the axial operators match onto pionic operators with odd number of pions; thus diagrams in Figs. 1(d) and (e) could not contribute. Instead, the pion pole diagram in Fig. 2 gives the leading contribution from insertion of $O_{5,\alpha,\pi}^m$. In this diagram, a necessary input is the $\pi \rightarrow 0$ matrix element of $O_{5,\alpha,\pi}^m$ which is related to the $(m+1)$-th moment of pion light cone distribution function

\[
\langle \pi^n(\Delta) | O_{5,\alpha}^m | 0 \rangle = i \delta_{ab} F_\pi (-n \cdot \Delta)^{m+1} \langle z^m \rangle_\pi / 2^m. \tag{58}
\]

where $z = 1 - 2x'$,

\[
\langle z^m \rangle_\pi = \int_{0}^{1} dx' (1 - 2x')^m \phi_{\pi}(x') \tag{59}
\]

and where $\phi_{\pi}(x')$ is the pion light cone distribution function with the normalization $\langle z^0 \rangle_\pi = \int_{0}^{1} dx' \phi_{\pi}(x') = 1$. $\langle z^m \rangle_\pi$ vanishes for odd $m$ due to charge conjugation. In Ref. [33], it was shown that the leading non-analytic quark mass corrections to $\langle z^m \rangle_\pi$ can all be absorbed into $F_\pi$. Thus $\langle z^m \rangle_\pi$ is purely analytic at $O(\varepsilon^2)$ [33] but not analytic at higher orders [34]. Using the operators constructed in Eqs. (39) and (57), our leading-order (LO) result reproduces that of Ref. [38] derived from the soft pion theorem,

\[
H_{\pi}^{\alpha=3}(x, \xi_\pi = 1, t = 0) = \frac{1}{2} \phi_{\pi} \left( \frac{1 + x}{2} \right). \tag{60}
\]
In heavy baryon \( \chi \)PT, the leading axial nucleon operators in the matching are

\[
O_{5,\alpha,N}^m = \sum_{j=0 \text{ even}}^m 2M\mathcal{N}\left[ \frac{n \cdot (n \cdot \Delta)^j}{M^2} (Mn \cdot v)^{m-j} \tilde{E}^{\alpha(0)}_{m+1,j}(0) + \frac{(S \cdot \Delta)^{j+1}}{2M^2} (Mn \cdot v)^{m-j} \tilde{M}^{\alpha(0)}_{m+1,j}(0) \right] \tau^\alpha_{u+N} + \cdots , \tag{61}
\]

where

\[
\tilde{E}_{m+1,j} = \tilde{A}_{m+1,j} , \quad \tilde{M}_{m+1,j} = \frac{\tilde{A}_{m+1,j}}{1 + \sqrt{1 - \frac{t}{4M^2}}} + \tilde{B}_{m+1,j} . \tag{62}
\]

For the isoscalar case, the leading chiral corrections include diagrams of Figs. 1(a), (b), (f):

\[
\tilde{E}_{m+1,2k}^0(t) = \left( 1 + \varepsilon^2 C_M^0 \right) \tilde{E}_{m+1,2k}^{0(0)}(0) + \varepsilon^2 \tilde{\alpha}_{m+1,2k}^0 m^2_\pi + \varepsilon^2 \tilde{\beta}_{m+1,2k}^0 t + \mathcal{O}(\varepsilon^3) , \tag{63}
\]

\[
\tilde{M}_{m+1,2k}^0(t) = \left( 1 + \varepsilon^2 C_M^0 \right) \tilde{M}_{m+1,2k}^{0(0)}(0) + \varepsilon^2 \tilde{\gamma}_{m+1,2k}^0 m^2_\pi + \varepsilon^2 \tilde{\Delta}_{m+1,2k}^0 t + \mathcal{O}(\varepsilon^3) .
\]

The contribution of \( \tilde{E}_{m+1,2k}^0 \) to \( \tilde{M}_{m+1,2k}^0 \) is from \( 1/M^2 \) corrections of Fig. 1(b) type diagrams [45].

For the isovector case, the leading non-analytic contribution of \( \tilde{E}^3 \) is from Figs. 1(b), (c) and (f).

\[
\tilde{E}_{m+1,2k}^3(t) = \left( 1 + \varepsilon^2 C_M^3 \right) \tilde{E}_{m+1,2k}^{3(0)}(0) + \varepsilon^2 \tilde{\alpha}_{m+1,2k}^3 m^2_\pi + \varepsilon^2 \tilde{\beta}_{m+1,2k}^3 t + \mathcal{O}(\varepsilon^3) , \tag{64}
\]

where

\[
C_M^3 = -\frac{(2g_\pi^2 + 1)m^2_\pi}{(4\pi F_\pi^2)^2} \ln \frac{m^2_\pi}{\mu^2} . \tag{65}
\]

The \( \mu \) dependence in Eqs. (63) and (64) is absorbed by the counterterms \( \tilde{\alpha}_{m+1,2k}^0, \tilde{\gamma}_{m+1,2k}^0 \) and \( \tilde{\alpha}_{m+1,2k}^3 \).

In contrary, the leading contribution to \( \tilde{M}^3 \) is non-analytic \( \mathcal{O}(\varepsilon^{-2}) \) arising from the pion pole diagram shown in Fig. 2. At one loop, we obtain

\[
\tilde{M}_{m+1,2k}^3(t) = \delta_{m,2k} G_P(t) \left\langle z^{2k}_\pi \right\rangle_\pi / 2^{2k} + \tilde{M}_{m+1,2k}^{3(0)}(0) + \mathcal{O}(\varepsilon) , \tag{66}
\]
where $\langle z^{2k}\rangle_\pi$ is analytic at $O(\varepsilon^2)$ as mentioned above and the pseudo-scalar form factor [46],

$$G_P(t) = \frac{4g_AM^2}{m^2} - \frac{1}{2} \left( \frac{2\alpha_{18}}{g_A^2} \right) - \frac{2}{3} g_A M^2 \langle r_A^2 \rangle,$$

(67)

where $\alpha_{18}$ is a counterterm and $\langle r_A^2 \rangle$ is the square of nucleon axial charge radius.

**VI. THE TENSOR OPERATOR**

The tensor operator

$$\mathcal{O}_{T,\alpha}^m = \bar{q}_\tau r^\mu \sigma^{\mu\nu} (i \not D)^m q$$

(68)

has the nucleon matrix element [44]

$$\langle P' | \mathcal{O}_{T,\alpha}^m | P \rangle = \sum_{j=0}^m U(P') (n \cdot \Delta)^j (n \cdot \bar{P})^{m-j} \left[ i \bar{q}_\tau r^\mu \sigma^{\mu\nu} A_{m+1,j}^{T,\alpha}(t) \right]_{j \ even}$$

$$+ \left( \frac{1}{M^2} \right) \left[ (n \cdot \bar{P})(r \cdot \Delta) - (r \cdot \bar{P})(n \cdot \Delta) \right] A_{m+1,j}^{T,\alpha}(t) \right]_{j \ even}$$

$$+ \frac{\gamma \cdot (n \cdot \Delta) - \gamma \cdot (n \cdot \bar{P})}{2M} B_{m+1,j}^{T,\alpha}(t) \right]_{j \ even}$$

$$+ \frac{\gamma \cdot (n \cdot \bar{P}) - \gamma \cdot (n \cdot \Delta)}{M} B_{m+1,j}^{T,\alpha}(t) \right]_{j \ odd} \tau^\alpha U(P),$$

(69)

The form factors are related to the moments of nucleon tensor GPDs as

$$H_{T,m+1} = \int_{-1}^1 dx \bar{x}^m H_T(x, \xi, t) = \sum_{j=0, \ even}^m (-2\xi)^j A_{m+1,j}^{T,\alpha}(t),$$

$$E_{T,m+1} = \int_{-1}^1 dx \bar{x}^m E_T(x, \xi, t) = \sum_{j=0, \ even}^m (-2\xi)^j B_{m+1,j}^{T,\alpha}(t),$$

$$\bar{H}_{T,m+1} = \int_{-1}^1 dx \bar{x}^m \bar{H}_T(x, \xi, t) = \sum_{j=0, \ even}^m (-2\xi)^j \bar{A}_{m+1,j}^{T,\alpha}(t),$$

$$\bar{E}_{T,m+1} = \int_{-1}^1 dx \bar{x}^m \bar{E}_T(x, \xi, t) = \sum_{j=0, \ odd}^m (-2\xi)^j \bar{B}_{m+1,j}^{T,\alpha}(t).$$

(70)

After the $1/M$ expansions, Eq. (69) becomes

$$\langle P' | \mathcal{O}_{T,\alpha}^m | P \rangle = \sum_{j=0}^m 2\bar{x}N(P') (n \cdot \Delta)^j (Mn \cdot v)^{m-j} \left[ 2i \epsilon^{mun} M T_{m+1,j}^{T,\alpha}(t) \right]_{j \ even}$$

$$+ \left( \frac{n \cdot v}{M} \right) \left[ (n \cdot \Delta) E_{m+1,j}^{T,\alpha}(t) \right]_{j \ even} - \frac{i}{M} \left( \frac{n \cdot v}{M} \right) \left[ C_{m+1,j}^{T,\alpha}(t) \right]_{j \ odd}$$

$$+ \frac{i}{8M^2} \left[ \frac{\epsilon^{num} M W_{m+1,j}^{T,\alpha}(t) \left[ \bar{r} \right]_{j \ even} \right] \right] \tau^\alpha N(P),$$

(71)
with
\[ M_{m+1,j}^T = \left(1 - \frac{t}{16M^2}\right)A_{m+1,j}^T, \quad E_{m+1,j}^T = \frac{1}{2}A_{m+1,j}^T + \tilde{A}_{m+1,j}^T + \frac{1}{2}B_{m+1,j}^T, \]
\[ C_{m+1,j}^T = \tilde{B}_{m+1,j}^T, \quad W_{m+1,j}^T = -2A_{m+1,j}^T - 4B_{m+1,j}^T. \] (72)

Under charge conjugation,
\[ CO_{T,\alpha}^m C^{-1} = (-1)^{m+1} O_{T,\alpha}^m \quad (\alpha = 0, 3). \] (73)

By similar arguments to the vector operators, the moments of pionic tensor GPD can be parametrized as
\[ \langle \pi^i(P_\pi) | \bar{q} (\gamma_\mu r_\nu \sigma^{\mu\nu}) (\text{in} \cdot D)^{2k+1} q | \pi^j(P_\pi) \rangle = 2\delta_{ij} \sum_{l=0}^{k} \frac{1}{M} \left[ (n \cdot \mathbf{P}_\pi) (r \cdot \Delta) - (n \cdot \Delta) (r \cdot \mathbf{P}_\pi) \right] (n \cdot \mathbf{P}_\pi)^{2l} (n \cdot \mathbf{P}_\pi)^{2k-2l+1} E_{2k+2,2l}^{T,\pi,0}(t). \] (74)

\[ \langle \pi^i(P_\pi) | \bar{q} \tau^3 (\gamma_\mu r_\nu \sigma^{\mu\nu}) (\text{in} \cdot D)^{2k} q | \pi^j(P_\pi) \rangle = 2\tau^3 \sum_{l=1}^{k} \frac{1}{M} \left[ (n \cdot \mathbf{P}_\pi) (r \cdot \Delta) - (n \cdot \Delta) (r \cdot \mathbf{P}_\pi) \right] (n \cdot \mathbf{P}_\pi)^{2l} (n \cdot \mathbf{P}_\pi)^{2k-2l} E_{2k+1,2l}^{T,\pi,3}(t). \] (75)

where we have inserted a heavy scale \( M \) which is of order \( \mu_\chi \) to make \( E_{m,j}^T \) dimensionless.

The matching procedure is similar to the vector and axial vector cases which we will not repeat here. We just point out one main difference. \( O_{T,\alpha}^m \) can be decomposed as
\[ O_{T,\alpha}^m = \bar{q}_L \tau^\alpha_R \gamma_\mu r_\nu \sigma^{\mu\nu} (\text{in} \cdot D)^m q_R + \bar{q}_R \gamma^\alpha_R \gamma_\mu r_\nu \sigma^{\mu\nu} (\text{in} \cdot D)^m q_L. \] (66)

We will set \( \tau^\alpha_R = \tau^\alpha_R = \tau^\alpha \) at the end. \( O_{T,\alpha}^m \) will be invariant under a chiral rotation if we demand
\[ \tau^\alpha_R \rightarrow L \tau^\alpha_R R^\dagger, \quad \tau^\alpha_R \rightarrow R \tau^\alpha_R L^\dagger. \] (77)

Thus instead of using \( \tau_{a,\pm}^a \), we use \( \tau_{a,\pm}^a = \frac{1}{2} (u^a \tau_{a,\pm}^a u^a \pm u^a \tau_{a,\pm}^a u^a) \) which transforms as \( \tau^a_{\pm} \rightarrow U(x) \tau^a_{\pm} U(x)^\dagger \) under a chiral rotation to construct the nucleon operators [19].

For the isoscalar form factors,
\[ M_{m+1,2k}^{T,0}(t) = (1 + \varepsilon_2^0 \mathbf{C}_M^0) M_{m+1,2k}^{T,0(0)}(0) + \varepsilon_2^0 \gamma_{m+1,2k}^{T,0} + \varepsilon_2^0 \delta_{m+1,2k}^{T,0} t + O(\varepsilon^3), \]
\[ E_{m+1,2k}^{T,0}(t) = E_{m+1,2k}^{T,0(0)}(0) + \varepsilon_2^0 \gamma_{m+1,2k}^{T,0} + \varepsilon_2^0 \delta_{m+1,2k}^{T,0} t + O(\varepsilon^3), \]
\[ C_{m+1,2k+1}^{T,0}(t) = (1 + \varepsilon_2^0 \mathbf{C}_M^0) C_{m+1,2k+1}^{T,0(0)}(0) + \varepsilon_2^0 \delta_{m,2k+1}^{T,0} T_{2k+2,2k+1}^{0,C} + \varepsilon_2^0 \delta_{m+1,2k}^{T,0} t + O(\varepsilon^3), \]
\[ W_{m+1,2k}^{T,0}(t) = (1 + \varepsilon_2^0 \mathbf{C}_M^0) W_{m+1,2k}^{T,0(0)}(0) + \frac{\varepsilon_2^0}{3} \mathbf{C}_M^0 M_{m+1,2k}^{T,0(0)}(0) + \varepsilon_2^0 \delta_{m,2k+1}^{T,0} T_{2k+2,2k+1}^{0,W} + \varepsilon_2^0 \delta_{m+1,2k+2k}^{T,0} t + O(\varepsilon^3), \] (78)
are similar. They obey the relations,
\[
\mathcal{T}^{0,C}_{2k+2,2k+1}(t) = -\frac{3g_A^2}{32\pi^2 F_\pi^2} \sum_{l=0}^k E^{T,\pi,0}_{2k+2,2k-2l}(0) (2l + 1) \int_0^1 dy \left( \frac{1}{2} - y \right)^{2l} m(y)^2 \log \frac{m(y)^2}{\mu^2},
\]
\[
\mathcal{T}^{0,W}_{2k+2,2k}(t) = 16\mathcal{T}^{0,C}_{2k+2,2k+1}(t). \tag{79}
\]

Similar to Eq. (63), the contribution of \( M^{T,(0)(0)}_{m+1,2k} \) to \( W^{T,(0)(0)}_{m+1,2k} \) is from \( 1/M^2 \) corrections of Fig. 1(b) type diagrams.

Similarly, for the isovector form factors,

\[
M^{T,3}_{m+1,2k}(t) = \left( 1 + \varepsilon^2 C^{3,T}_M \right) M^{T,3(0)}_{m+1,2k}(0) + \varepsilon^2 \alpha^{T,3}_{m+1,2k} m_\pi^2 + \varepsilon^2 \beta^{T,3}_{m+1,2k} t + \mathcal{O}(\varepsilon^3),
\]
\[
E^{T,3}_{m+1,2k}(t) = \left( 1 + \varepsilon^2 C^{3,E}_M \right) E^{T,3(0)}_{m+1,2k}(0) + \varepsilon^2 \delta_{m,2k} \mathcal{T}^{3,E}_{2k+1,2k}(t) + \varepsilon^2 \gamma^{T,3}_{m+1,2k} m_\pi^2 + \varepsilon^2 \omega^{T,3}_{m+1,2k} t + \mathcal{O}(\varepsilon^3),
\]
\[
C^{T,3}_{m+1,2k+1}(t) = \left( 1 + \varepsilon^2 C^{3,T}_M \right) C^{T,3(0)}_{m+1,2k+1}(0) + \varepsilon^2 \eta^{T,3}_{m+1,2k} m_\pi^2 + \varepsilon^2 \phi^{T,3}_{m+1,2k} t + \mathcal{O}(\varepsilon^3),
\]
\[
W^{T,3}_{m+1,2k}(t) = W^{T,3(0)}_{m+1,2k}(0) + \varepsilon \delta_{m,2k} \mathcal{T}^{3,W}_{2k+1,2k}(t) + \mathcal{O}(\varepsilon^2), \tag{80}
\]

where

\[
C^{3,T}_M = -\frac{2g_A^2 + 1/2}{(4\pi F_\pi)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2}, \tag{81}
\]

and

\[
\mathcal{T}^{3,E}_{2k+1,2k}(t) = -\frac{1}{32\pi^2 F_\pi^2} \sum_{l=0}^k E^{T,\pi,3}_{2k+1,2k-2l}(0) \int_0^1 dy \left( \frac{1}{2} - y \right)^{2l}
\]
\[
\times \left\{ g_A^2 \left[ 2m^2 - (5 + 4l) m(y)^2 \right] - m(y)^2 \right\} \log \frac{m(y)^2}{\mu^2}
\]
\[
\mathcal{T}^{3,W}_{2k+1,2k}(t) = \frac{Mg_A^2}{\pi F_\pi^2} \sum_{l=0}^k E^{T,\pi,3}_{2k+1,2k-2l}(0) \int_0^1 dy \left( \frac{1}{2} - y \right)^{2l} m(y). \tag{82}
\]

It is interesting that the leading chiral corrections of the tensor and vector matrix elements are similar. They obey the relations,

\[
\mathcal{T}^{0,C}_{2k+2,2k+1}(t) = -\frac{1}{2} \mathcal{D}^{0,M}_{2k+2,2k}(t) \left|_{(l+1)A^{\pi,0}_{2k+2,2k-2l} \rightarrow E^{T,\pi,0}_{2k+2,2k-2l}} \right.,
\]
\[
\mathcal{T}^{3,E}_{2k+1,2k}(t) = \mathcal{D}^{3,E}_{2k+1,2k}(t) \left|_{(2l+1)A^{\pi,3}_{2k+1,2k-2l} \rightarrow E^{T,\pi,3}_{2k+1,2k-2l}} \right.,
\]
\[
\mathcal{T}^{3,W}_{2k+1,2k}(t) = 8 \mathcal{D}^{3,M}_{2k+1,2k}(t) \left|_{(2l+1)A^{\pi,3}_{2k+1,2k-2l} \rightarrow E^{T,\pi,3}_{2k+1,2k-2l}} \right.. \tag{83}
\]

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The gluon GPDs are defined as [42, 47]

\[ n_\mu n_\nu \langle P' | F^{\mu \alpha} \left( -\frac{z}{2} n \right) F_\alpha \left( \frac{z}{2} n \right) | P \rangle = \int dx e^{-ixzn \cdot \bar{P}} U(P') n \cdot \bar{P} \left[ xH_g \theta + xE_g \frac{i\sigma^{\alpha \beta} n_\alpha \Delta_\beta}{2M} \right] U(P), \]

\[ n_\mu n_\nu \langle P' | F^{\mu \alpha} \left( -\frac{z}{2} n \right) i\bar{F}_\alpha \left( \frac{z}{2} n \right) | P \rangle = \int dx e^{-ixzn \cdot \bar{P}} U(P') n \cdot \bar{P} \left[ x\bar{H}_g \gamma_5 + x\bar{E}_g \frac{\gamma_5 n \cdot \Delta}{2M} \right] U(P), \]

\[ n_\mu n_\nu r_\alpha r_\beta \langle P' | F^{\mu \alpha} \left( -\frac{z}{2} n \right) F^{\nu \beta} \left( \frac{z}{2} n \right) | P \rangle = \int dx e^{-ixzn \cdot \bar{P}} \left[ (n \cdot \bar{P}) (r \cdot \Delta) - (n \cdot \Delta) (r \cdot \bar{P}) \right] \times U(P') \left[ xH_{Tg} in_\alpha r_\beta \sigma^{\alpha \beta} + x\bar{H}_{Tg} \frac{(n \cdot P) (r \cdot \Delta) - (r \cdot P) (n \cdot \Delta)}{M^2} \right. \right.

\left. \left. + xE_{Tg} \frac{\theta (r \cdot \Delta) - \theta (n \cdot \Delta)}{2M} + x\bar{E}_{Tg} \frac{\bar{\theta} (r \cdot \bar{P}) - \bar{\theta} (n \cdot P)}{2M} \right] U(P), \tag{84} \right. \]

where the Wilson lines are also suppressed. The Taylor-series expansion in z of the above equations gives rise to relations between the gluon twist-2 matrix elements and the moments of gluon GPDs:

\[ F_{g, m+1}(\xi, t) = \int dx x^m F_g(x, \xi, t), \tag{85} \]

where \( F_g(x, \xi, t) \) denotes a generic gluon GPD. Here, unlike the quark case, \( m = 1, 2, \ldots \) without \( m = 0 \). This is because the right-hand sides of Eq. (84) are of the form \( xF_g \) instead of \( F_g \). Therefore \( F_g \) can only be determined up to a function of the form \( \lambda(\xi, t)\delta(x) \) using the above definitions. However, the first moments in \( x \) of gluon GPDs can be probed by non-local gauge invariant operators as introduced in Ref. [48].

The local gluon twist-2 operators and isoscalar quark twist-2 operators all transform in the same way—as singlets—under chiral transformation. The gluon twist-2 operators match onto the same set of hadronic operators as the isoscalar quark twist-2 operators with different prefactors. Thus our previous results for the moments of isoscalar quark GPDs can be easily converted to moments of gluon GPDs.
VIII. χPT CONSTRAINTS ON THE NUCLEON GPDS

In this section we use the moments of nucleon GPDs calculated above to shed light on the GPDs themselves. We first pay attention to the non-analytic $O(\varepsilon)$ corrections before making transit to the impact parameter distributions of GPDs. There are no counterterms at $O(\varepsilon)$, therefore those corrections are clean predictions of χPT. Here are several interesting cases of this type:

1) At $O(\varepsilon)$, $E_0^0$ and $H_0^0$ only receive chiral corrections from the “D terms” which only depend on $x/\xi$ and $t$ [39],

$$\delta E_0^0(x, \xi, t) = -\delta H_0^0(x, \xi, t) = D(\frac{x}{\xi}, t)\theta \left(1 - \left|\frac{x}{\xi}\right|\right),$$

with $D(z, t) = -D(-z, t)$. From Eqs.(18), (45) and (48),

$$\delta H_{m+1}^0(\xi, t) = \int_{-1}^{+1} dx x^m \delta H_0^0(x, \xi, t) = (-2\xi)^{m+1} C_{m+1}(t),$$

where $C_{m+1}(t) = 0$ for $m$ even. Then one can prove $\delta E_0^0$ and $\delta H_0^0$ have the functional form in Eq.(86).

2) $E_3^3$, $E_3^T$ and $H_3^T$ receive $O(\varepsilon)$ contributions which are predictable. We will discuss their impact parameter distributions in the next subsection.

3) One can show that Eq.(66) implies

$$\tilde{E}_3^3(x, \xi, t) = \frac{\phi_\pi \left(\frac{\xi-x}{2\xi}\right)}{2|\xi|} G_P(t) + O(\varepsilon^0).$$

This result coincides with those derived in Refs. [40, 41] after using $\phi_\pi(x') = \phi_\pi(1-x')$.

A. Impact Parameter Distributions

In the limit of $\xi \to 0$ (and $t \to -\Delta_\perp^2$), the $\Delta_\perp$ Fourier transformation of the GPDs has the interpretation of simultaneous measurement of the longitudinal momentum and transverse position (impact parameter) of partons in the infinite momentum frame. The impact parameter dependent parton distribution for a generic nucleon GPD $F$ is

$$\mathcal{F}(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{ib_\perp \cdot \Delta_\perp} F(x, 0, -\Delta_\perp^2).$$
We are interested in the small $t$ ($t \ll \mu^2_{\chi}$), or large $b_{\perp} (b \gg 1/\mu_{\chi})$ region where $\chi$PT is reliable, so we expand $F$ in $t$,

$$F(x, 0, t = -\Delta^2_{\perp}) = F(x, 0, 0) - \partial_t F(x, 0, 0) \Delta^2_{\perp} + \cdots,$$  

(90)

which leads to

$$\mathcal{F}(x, b_{\perp}) = F(x, 0, 0) \delta^2 (b_{\perp}) + \partial_t F(x, 0, 0) \nabla^2_{\perp} \delta^2 (b_{\perp}) + \cdots.$$  

(91)

The $F$ term gives the $b_{\perp}$ integrated distribution

$$\int d^2 b_{\perp} \mathcal{F}(x, b_{\perp}) = F(x, 0, 0),$$  

(92)

while the $\partial_t F$ term is related to the averaged $b^2_{\perp}$ of $F$ as a function of $x$,

$$\langle b^2_{\perp} \rangle_F = \frac{\int d^2 b_{\perp} b^2_{\perp} \mathcal{F}(x, b_{\perp})}{\int d^2 b_{\perp} \mathcal{F}(x, b_{\perp})} = 4 \frac{\partial_t F(x, 0, 0)}{F(x, 0, 0)}. $$  

(93)

The terms with higher derivatives on the delta function give higher moments of $b^2_{\perp}$. In comparison, the charge radii of the electroweak form factors defined in Eq. (5) are

$$\langle r^2 \rangle_F = 6 \int dx \frac{\partial_t F(x, 0, 0)}{\int dx F(x, 0, 0)}.$$  

(94)

They also constrain the functional form of $F$.

In the following paragraphs, we will extract from the model-independent results of twist-2 matrix elements we obtained above to obtain $F(x, 0, 0)$ and $\langle b^2_{\perp} \rangle_F$. The GPD $\tilde{E}^3(x, \xi, t)$ has a special $\xi \to 0$ limit:

$$\tilde{E}^3(x, 0, t) = \delta(x) G_P(t) + \mathcal{O}(\xi^0)$$  

$$= \delta(x) \frac{4g_A M^2}{m^2_\pi} \frac{1}{t \varepsilon^2} + \mathcal{O}(\xi^0),$$  

(95)

where we have used Eqs. (88) and (67), $\phi_{\pi} (x') = 0$ for $x' < 0$ or $x' > 1$, and $\int_0^1 dx' \phi_{\pi} (x') = 1$. This yields $\langle b^2_{\perp} \rangle_\varepsilon = 4/m^2_\pi$ at $x = 0$.

For the other GPDs, it is convenient to express $F(x, 0, 0)$ and $\partial_t F(x, 0, 0)$ as (except $\tilde{E}^3$ which we will discuss later)

$$F(x, 0, 0) = a_F(x) + \varepsilon b_F(x) \frac{m_\pi}{\mu_{\chi}} + \varepsilon^2 c_F(x, \mu) \frac{m^2_\pi}{\mu^2_{\chi}} + \varepsilon^2 d_F(x) \frac{m^2_\pi}{\mu^2_{\chi}} \log \left( \frac{m^2_\pi}{\mu^2} \right) + \mathcal{O}(\varepsilon^3),$$  

$$\partial_t F(x, 0, 0) = \frac{1}{\mu_{\chi}} \left[ \varepsilon f_F(x, \mu) + g_F(x) \log \left( \frac{m^2_\pi}{\mu^2} \right) + \mathcal{O}(\varepsilon) \right],$$  

(96)
where all the prefactors are $m_{\pi}$ independent and the $\mu$ dependence of $c_F$ and $f_F$ cancel the $\mu$ dependence in the logarithms of the $d_F$ and $g_F$ terms, respectively.

We will show that those prefactors could have $\delta$ function structures. Note that instead of using $\delta$ functions with zero widths, one can use regularized delta functions with finite widths as well. For prefactors with odd powers of $\varepsilon$ (defined in Eq.(8)), the widths of the delta functions should be $\sim \varepsilon \sim m_{\pi}/\mu_\chi$ or smaller. For prefactors with even powers of $\varepsilon$, the widths of the delta functions should be $\sim \varepsilon^2 \sim (m_{\pi}/\mu_\chi)^2$ or smaller. To demonstrate this, we use the regularized delta function $\delta_{\Delta x}(x) = (\Delta x)^{-1} \theta(\Delta x - x)\theta(x)$. The $x^m$ moment of $\varepsilon^n\delta_{\Delta x}(x)$ gives

$$\langle x^m \rangle = \int_0^1 dx \varepsilon^n \delta_{\Delta x}(x)x^m = \frac{\varepsilon^n (\Delta x)^m}{m+1}. \quad (97)$$

When $m > 0$, the change of $\langle x^m \rangle$ due to regularization is $O(\varepsilon^n (\Delta x)^m)$. Thus the change to the parton distributions due to regularization is $O(\varepsilon^n \Delta x)$. These contributions should be absorbed by matrix elements of higher order operators in ChPT. However, in ChPT, the higher order operators are all analytic in light quark mass $m_q \propto m_{\pi}^2$. Thus, the regularization is sensible only when $\varepsilon^n (\Delta x)^m = \varepsilon$ to an even power.

1) For the isovector GPDs $E^3$, $E_3^\perp$ and $\tilde{H}_3^I$, their $O(\varepsilon)$ contributions are non-vanishing. From Eqs.(18), (21), and (49), the $x^m$ moment of $E^3(x,0,t)$ vanishes except for $m = 0$:

$$\delta E_{m+1}^3(0,t) = \varepsilon \delta_{m,0} D_{1,0}^{3,M}(t). \quad (98)$$

This implies

$$b_{E^3}(x) = \overline{b}_{E^3} \delta (x), \quad e_{E^3}(x) = \overline{e}_{E^3} \delta (x). \quad (99)$$

where $\overline{b}_{E^3} = -12 \pi \overline{E}_3 = -g_{\Delta}^2 M/F_\pi = -16.0$. Thus we have

$$\int d^2b_{\perp} E^3(x,b_{\perp}) = a_{E^3}(x) + \varepsilon \overline{b}_{E^3} \delta (x) \frac{m_{\pi}}{\mu_\chi} + O(\varepsilon^2),$$

$$\langle b_{\perp}^2 \rangle_{E^3} = \frac{4}{\varepsilon} \frac{\overline{E}_3 \delta (x)}{\mu_\chi m_{\pi} a_{E^3}(x)} + O(\varepsilon^0). \quad (100)$$

Note that $\int dx a_{E^3}(x)$ is the isovector nucleon anomalous magnetic moment in the chiral limit. The charge radius square of the isovector nucleon anomalous magnetic moment is

$$\langle r^2 \rangle_{E^3} = \frac{6}{\varepsilon} \frac{\overline{E}_3}{\mu_\chi m_{\pi} \int dx a_{E^3}(x)} + O(\varepsilon^0). \quad (101)$$

The behavior of $E_3^3(x,0,t)$ is similar to that of $E^3(x,0,t)$ with $b_{E_3^3}(x) \propto \delta (x)$, $e_{E_3^3}(x) \propto \delta (x)$, and so on. Also, $\tilde{H}_3^I(x,0,t) = -E_3^3(x,0,t)/2$ at $O(\varepsilon)$ from Eqs. (70), (72) and (80).
2) For the other GPDs, the $\mathcal{O}(\varepsilon)$ contribution to $F(x,0,t)$ vanishes such that $b_F(x) = e_F(x) = 0$. The non-analytic logarithmic terms $d_F(x)$ and $g_F(x)$ can be predicted in $\chi$PT. If $g_F(x)$ is non-vanishing at certain $x$, then the corresponding $\langle b_2^2 \rangle_F$ diverges in the chiral limit. Here we list the results based on the moment computations.

a) Spin-averaged quark GPDs: The first moment of $H(x,0,0)$ gives the total number of quarks of certain flavor in the nucleon which is independent of the quark mass. Thus $\int dx c_H(x) = \int dx d_H(x) = 0$. For the isoscalar combination $H^0$, there are no chiral logarithms at $\mathcal{O}(\varepsilon^2)$,

$$d_{H^0}(x) = g_{H^0}(x) = 0.$$ (102)

For the isovector combination $H^3$, there are chiral logarithms at $\mathcal{O}(\varepsilon^2)$ except for the first moment in $x$. This implies

$$d_{H^3}(x) = - (3g_A^2 + 1) \left[ a_{H^3}(x) - \delta(x) \int dx a_{H^3}(x) \right], \quad g_{H^3}(x) = 0.$$ (103)

The isovector GPD $E^3$ has been discussed in Eq.(100). The isoscalar combination $E^0 + H^0$, which satisfies the sum rule in Eq.(7), yields

$$d_{E^0}(x) + d_{H^0}(x) = -3g_A^2 \left[ a_{E^0}(x) + a_{H^0}(x) + \langle x \rangle_\pi \delta'(x) \right], \quad g_{E^0}(x) + g_{H^0}(x) \propto \delta'(x),$$ (104)

where $\delta'(x) = d\delta(x)/dx$. This result is derived from Eq.(45) and we have $d_{H^0}(x) = 0$ from Eq.(102).

b) Quark helicity GPDs: $\tilde{E}^3$ is not defined at $\xi = 0$. For $\tilde{E}^0$,

$$d_{\tilde{E}^0}(x) = -3g_A^2 a_{\tilde{E}^0}(x) - g_A^2 a_{\tilde{H}^0}(x), \quad g_{\tilde{E}^0}(x) = 0.$$ (105)

And for $\tilde{H}$,

$$d_{\tilde{H}^0}(x) = -3g_A^2 a_{\tilde{H}^0}(x), \quad g_{\tilde{H}^0}(x) = 0.$$ (106)

and

$$d_{\tilde{H}^3}(x) = - \left( 2g_A^2 + 1 \right) a_{\tilde{H}^3}(x), \quad g_{\tilde{H}^3}(x) = 0.$$ (107)

c) Quark transversity GPDs: $\tilde{E}_T = 0$ when $\xi = 0$. $\tilde{H}^3_T$ and $E^3_T(x,0,t)$ are already discussed following Eq.(101) above.

$$d_{H^0_T}(x) = -3g_A^2 a_{H^0_T}(x), \quad g_{H^0_T}(x) = 0,$$ (108)

$$d_{H^3_T}(x) = - \left( 2g_A^2 + \frac{1}{2} \right) a_{H^3_T}(x), \quad g_{H^3_T}(x) = 0.$$ (109)
\[
\begin{align*}
   d_{E^0_T}(x) &= -3g_A^2 \left[ a_{E^0_T}(x) + \frac{4}{3} a_{H^0_g}(x) + 2E^{T,0,0}_{2,0}(0)\delta'(x) \right], \quad g_{E^0_T}(x) \propto \delta'(x), \\
   d_{H^0_T}(x) &= -3g_A^2 \left[ a_{H^0_T}(x) - E^{T,0,0}_{2,0}(0)\delta'(x) \right], \quad g_{H^0_T}(x) = -\frac{g_{E^0_T}(x)}{2}.
\end{align*}
\]

3) In Ref. [49], Strikman and Weiss argued that the \( \langle b^2_{\perp} \rangle_{H_g} \) of the gluon GPD \( H_g(x, b_{\perp}) \) is proportional to \( 1/m^2_\pi \) at small \( x (x \lesssim m_\pi/M) \) and is proportional to \( 1/M^2 \) otherwise. While this behavior is qualitatively similar to \( E^3(x, b_{\perp}) \) [also \( E^3_2(x, b_{\perp}) \) and \( \tilde{H}^3_2(x, b_{\perp}) \)] described in Eq.(100), for \( H_g(x, b_{\perp}) \) the result is still inconclusive in our calculation. We confirm that \( \langle b^2_{\perp} \rangle_{H_g} = \mathcal{O}(1/M^2) \) or \( \mathcal{O}(1/\mu^2_\chi) \) when \( x \neq 0 \) from the result for the isoscalar moments in Eq.(45) but with \( m \geq 1 \). However, the possible \( \delta(x) \) contribution cannot be probed by the matrix element defined in Eq. (84). We will defer the investigation to a later publication.

Finally, Eqs.(104), (110) and (111) imply that \( x(H^0_g + \mathcal{E}^0_g), xE^0_Tg \) and \( x\tilde{E}^0_Tg \) have \( \langle b^2_{\perp} \rangle \propto \delta(x) \log (m^2_\pi) \) which diverges in the chiral limit.

IX. CONCLUSIONS

Using heavy baryon \( \chiPT \) we have studied the leading chiral corrections to the complete set of nucleon GPDs. We have computed the leading quark mass and momentum transfer dependence of the moments of nucleon GPDs through the nucleon off-forward twist-2 matrix elements. We have also applied these results to get insight on the GPDs and their impact parameter space distributions.

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