On the Dispersion Relation of Magnetoplasmons in a Planar Graphene-Based Superlattice

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The dispersion relation for magnetoplasmons in a planar superlattice with periodically alternating regions of gapless and gapped modifications of graphene has been derived within the frame of the random-phase approximation. The contribution of virtual transitions between the lower electron miniband and the upper hole miniband to the polarization operator has been taken into account in addition to the contribution of virtual intra-miniband transitions.

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1. INTRODUCTION

Collective excitations in gapless graphene in a magnetic field were studied theoretically in a number of works [1, 2]. However, less attention was paid to collective excitations in a gapped modification of graphene.

In this work, magnetoplasmons in a planar superlattice based on gapless graphene and its gapped modification are considered (Fig. 1). We use a model that allows finding analytically the dispersion relation for charge carriers in such a superlattice [3]. The dispersion relation of plasmons in this superlattice was found in [4].

The extrema of the bands of both the gapped and gapless modifications of graphene in the $k$ space lie at the $K$ and $K'$ points of the Brillouin zone. The band structure of the gapped modifications of graphene also contains two valleys. The basic difference from gapless graphene is the presence of the energy gap between the extrema of the conduction and valence bands. In the general case, the center of the gap is displaced in energy from the position of the $K$ and $K'$ points of gapless graphene (the level $E = 0$) by the work function.

Alternating strips of gapless and gapped graphene provide modulation in the space of the energy gap. This is equivalent to the application of a one-dimensional periodic potential. The energy spectrum of the systems splits into smaller bands called minibands separated by minigaps.

In this work, we suggest a model for the description of the superlattice under consideration, in which the charge carriers in different valleys interact identically with the magnetic field. In the general case, such a valley degeneracy can be broken, e.g., by a uniaxial strain in the graphene plane [5].

2. WAVEFUNCTIONS AND THE SINGLE-PARTICLE ENERGY SPECTRUM OF CHARGE CARRIERS

In our previous work [4], we introduced the effective Hamiltonian of charge carriers in the valley of the $K$ point of a graphene-based superlattice in zero magnetic field. Two case were considered: (i) a quasi-one-dimensional case (the Fermi level falls within a miniband) and (ii) a quasi-two-dimensional case (the Fermi level is located within the miniband).

In this work, we will assume that a more general quasi-two-dimensional case takes place in zero magnetic field. In the presence of the magnetic field, we replace the momentum operator $\hat{p} = -i\nabla$ (in units of $\hbar = 1$) in the effective Hamiltonian of charge carriers in the graphene superlattice by the operator $\hat{\pi} = \hat{p} - \frac{e}{c}A$,

where $e$ is the charge of the particle, $c$ is the speed of light in vacuum, and $A = (0, Bx, 0)$ is the vector potential.
tential of the magnetic field $B = (0, 0, B)$. The $(x, y)$ plane coincides with the superlattice plane (see Fig. 1). In this case, the effective Hamiltonian of charge carriers in the valley of the $K'$ point has the form

$$
\hat{H}_{\text{eff}}^{K'} = v_\perp \sigma_x \hat{\pi}_x + v_\parallel \sigma_y \hat{\pi}_y - \sigma_z \Delta_{\text{eff}} + V_{\text{eff}}. \tag{1}
$$

where $v_\perp$ and $v_\parallel$ are the effective Fermi velocities across and along the strips, respectively, of gapless and gapped graphene and, additionally, $v_\parallel \approx v_F$ ($v_F$ is the Fermi velocity in gapless graphene), whereas $v_\perp \ll v_\parallel$ owing to a low probability of tunneling of charge carriers through the regions of gapped graphene. The velocities $v_\perp$ and $v_\parallel$ were chosen such that the dispersion relation of charge carriers in zero magnetic field would agree with the dispersion relation for the superlattice [4].

The Pauli matrices $\sigma_x$, $\sigma_y$, and $\sigma_z$ act in the space of two sublattices of the hexagonal lattice of graphene and its gapped modification.

For simplicity, we consider here the lower electron and upper hole minibands. They are separated by the miniband $2\Delta_{\text{eff}}$. If the work function of the gapped modifications is nonzero, the center of this miniband is displaced in energy with respect to the $K$ and $K'$ points of gapless graphene by the effective work function $V_{\text{eff}}$.

Assuming that $|V_{\text{eff}}| < \Delta_{\text{eff}} \ll \Delta_0$ ($\Delta_0$ and $V_0$ are the half-width of the band gap and the work function of gapless graphene by the effective work function $V_{\text{eff}}$, respectively), we find

$$
\Delta_{\text{eff}} = \frac{\pi v_F}{2d_1} \left[ 1 - \frac{v_F}{d_1} \Delta_0 \right], \tag{2}
$$

$$
V_{\text{eff}} = \frac{v_F}{d_1} \Delta_0 V_0,
$$

where $d_1$ is the width of gapless graphene strips with $v_F/d_1 < \Delta_0$. The additional condition of applicability of Eqs. (2) is the inequality $\Delta_0 > 2v_F/d_1$, where $d_1$ is the width of gapped graphene strips [3].

The effective Hamiltonian of charge carriers in the valley of the $K'$ point is

$$
\hat{H}_{\text{eff}}^{K'} = v_\perp \sigma_x \hat{\pi}_x + v_\parallel \sigma_y \hat{\pi}_y + \sigma_z \Delta_{\text{eff}} + V_{\text{eff}}. \tag{3}
$$

The mass term (the third one) is written with the opposite sign compared to Hamiltonian (1). This provides the unitary equivalence of $\hat{H}_{\text{eff}}^{K'}$ and $\hat{H}_{\text{eff}}^{K}$. They can be transformed to one another by the unitary transformation

$$
\hat{H}_{\text{eff}}^{K} = U \hat{H}_{\text{eff}}^{K'} U^\dagger, \quad U = \sigma_x. \tag{4}
$$

The immediate consequence of this fact is the equivalence of their energy spectra.

Let us introduce the dimensionless variable

$$
\xi = \sqrt{\frac{v_\parallel}{v_\perp}} \left( \frac{x}{l_B} + k_y \right), \tag{5}
$$

where $k_y$ is the $y$ component of the crystal momentum and

$$
l_B = \sqrt{\frac{e}{|e|B}}. \tag{6}
$$

is the magnetic length. The wavefunctions of Hamiltonian (1) for the Landau levels $n = 1, 2, \ldots$ of electrons and holes are

$$
\psi_{n\mu}(x, y) = C_n^{(\mu)} \begin{pmatrix} A_n^{(\mu)} \phi_n - (1) \xi \end{pmatrix} \frac{e^{ik_y y}}{\sqrt{L_y}}, \tag{7}
$$

$$
\psi_{n\mu}(x, y) = C_n^{(\mu)} \begin{pmatrix} -A_n^{(\mu)} \phi_n - (1) \xi \end{pmatrix} \frac{e^{-ik_y y}}{\sqrt{L_y}},
$$

where $L_y$ is the size of the system along the $y$ axis and the $+(-)$ sign corresponds to positive (negative)-frequency solutions. Here, for definiteness, we regard the positive-frequency solutions for the Landau level $n_0 = 0,

$$
\hat{\Psi}_{0\mu}(x, y) = \begin{pmatrix} 0 \phi_0(\xi) \end{pmatrix} \frac{e^{ik_y y}}{\sqrt{L_y}}, \tag{8}
$$

$$
\hat{\Psi}_{0\mu}(x, y) = \begin{pmatrix} \phi_0(\xi) 0 \end{pmatrix} \frac{e^{-ik_y y}}{\sqrt{L_y}},
$$

coincide with solution (7) for $n = 0$ if one takes into account that $A_0^{(+) = 0}$ and $C_0^{(+) = 1}$. However, the situation with the negative-frequency solutions for $n = 0$ is different: an uncertainty of the form $0/0$ emerges in the coefficient $A_0^{(-)}$. These solutions are

$$
\hat{\Psi}_{0\mu}(x, y) = C_0^{(-)} \begin{pmatrix} \phi_0(\xi) \phi_0(\xi) \end{pmatrix} \frac{e^{ik_y y}}{\sqrt{L_y}}, \tag{9}
$$

$$
\hat{\Psi}_{0\mu}(x, y) = C_0^{(-)} \begin{pmatrix} \phi_0(\xi) \phi_0(\xi) \end{pmatrix} \frac{e^{-ik_y y}}{\sqrt{L_y}},
$$

where

$$
\phi_0(\xi) = \frac{i \pi^{1/4} l_B^{1/2} \Delta_{\text{eff}}}{v_\perp} \left[ 1 - \Phi(\xi) \right] e^{\xi^2/2},
$$

$$
\Phi(\xi) = \frac{1}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{\xi^2/2} dt.
$$

The normalization factor $C_0^{(-)}$ is
where \( \tilde{\gamma} = (1 + i)\gamma \) is the charge conjugation operator and \( \hat{C} \) is the complex conjugation operator.

The wavefunctions of Hamiltonian (3) are found from wavefunctions of Hamiltonian (1) by transformation (4):

\[
\Psi_{nk_x}^{K',\pm}(x, y) = \sigma_2 \Psi_{nk_x}^{K,\pm}(x, y),
\tag{10}
\]

It is worth mentioning that the wavefunctions of electrons and holes in both valleys are coupled by the charge conjugation transformation

\[
\Psi_{nk_x}^{K,\pm}(x, y) = \hat{C} \Psi_{nk_x}^{K,\pm}(x, y),
\tag{11}
\]

where \( \hat{C} = \sigma_2 \hat{C} \) is the charge conjugation operator and \( \hat{C} \) is the complex conjugation operator.

The energy spectrum of Hamiltonians (1) and (3) is

\[
E_n^{h,\pm} = V_{\text{eff}} \pm \varepsilon_n,
\tag{12}
\]

where the plus (minus) sign stands for electrons (holes).

It should be emphasized that the description of the system under consideration by effective Hamiltonians (1) and (3) is applicable when the potential of the superlattice dominates over the Landau quantization: \( V \lesssim \sqrt{l_B^2 \Delta^2} \ll \Delta^2 \). This condition is always fulfilled at \( l_B \gg d \) (d = \( d_1 + d_1 \) is the superlattice period).

In the case of \( \Delta_{\text{eff}} = 0 \), the found wavefunctions (positive-frequency solutions) coincide with the wavefunctions of charge carriers in gapless graphene in a magnetic field (see [1] and references therein). In the case of \( \Delta_{\text{eff}} \neq 0 \), the zeroth Landau level in each valley is nondegenerate and all other levels are doubly degenerate.

3. GREEN’S FUNCTION

To find the Green’s function, we will need the Hamiltonian of a system of noninteracting single-particle excitations written in terms of secondary quantization operators:

\[
\hat{H}_0 = \sum_{n, k_x} E_n^{c} \hat{a}_{nk_x}^{\dagger} \hat{a}_{nk_x} + \sum_{n, k_y} E_n^{h} \hat{b}_{nk_y}^{\dagger} \hat{b}_{nk_y},
\tag{13}
\]

where \( E_n^{c} = E_n - V_{\text{eff}} = \varepsilon_n \) and \( E_n^{h} = E_n - V_{\text{eff}} = -\varepsilon_n \) (hereinafter, we measure the energy from the level \( E = V_{\text{eff}} \)), and \( \hat{a}_{nk_y} \) and \( \hat{b}_{nk_y} \) are the annihilation (creation) operators of an electron (hole) and the hole (electron), respectively.

\[
C_0(\gamma) = (1 + i)^{-1/2}, \quad I = \int_{-\infty}^{\infty} |\phi_0|^2 dx = \frac{\sqrt{\pi l_B^2 \Delta^2}}{v_0},
\]

\[
\tilde{I} = \int_{-\infty}^{\infty} e^{i\xi} |\Phi(\xi)| - 1|^2 d\xi \approx 0.78.
\]

Landau level with the \( y \) component \( k_y \) of the crystal momentum, respectively.

The operators of the electron and hole fields for the valley of the \( K' \) point can be written as expansions in the secondary quantization operators:

\[
\hat{\Psi}_{nk_y}^{K,\pm}(x, y) = \sum_{n, k_x} \Psi_{nk_x}^{K,\pm}(x, y) \hat{a}_{nk_x},
\tag{14}
\]

\[
\hat{\Psi}_{nk_y}^{K,\pm}(x, y) = \sum_{n, k_x} \Psi_{nk_x}^{K,\pm}(x, y) \hat{b}_{nk_x},
\tag{15}
\]

Here, the arguments \( x \) and \( y \) are omitted for brevity. The expressions for the \( \Psi \) operators in the valley of the \( K' \) point are similar: they include the wavefunctions \( \Psi_{nk_x}^{K,\pm}(x, y) \) and \( \Psi_{nk_y}^{K,\pm}(x, y) \).

We define the operator of the difference between the numbers of electrons and holes as

\[
\hat{N}_0 = \sum_{n, k_y} a_{nk_y}^{\dagger} a_{nk_y} - \sum_{n, k_y} b_{nk_y}^{\dagger} b_{nk_y}.
\tag{16}
\]

Taking into account the commutation relations for the secondary quantization operators and omitting an insignificant constant, we write the operator

\[
\hat{H}'_0 = \hat{H}_0 - \hat{\mu} \hat{N}_0
\]

where \( \hat{\mu} = \mu - V_{\text{eff}} \) and \( \mu \) is the chemical potential.

Let us write the \( \Psi \) operators in the interaction representation [7]

\[
\hat{\Psi}_{nk_y}^{K,\pm}(x, y, t) = e^{i\hat{\mu}\hat{N}_0} \hat{\Psi}_{nk_y}^{K,\pm}(x, y) e^{-i\hat{\mu}\hat{N}_0} t
\]

\[
= \sum_{n, k_x} \Psi_{nk_x}^{K,\pm}(x, y) \hat{a}_{nk_x}(t) + \sum_{n, k_x} \Psi_{nk_x}^{K,\pm}(x, y) \hat{b}_{nk_x}(t),
\]

\[
\hat{\Psi}_{nk_y}^{K,\pm}(x, y, t) = e^{i\hat{\mu}\hat{N}_0} \hat{\Psi}_{nk_y}^{K,\pm}(x, y) e^{-i\hat{\mu}\hat{N}_0} t
\]

\[
= \sum_{n, k_x} \Psi_{nk_x}^{K,\pm}(x, y) \hat{b}_{nk_x}(t) + \sum_{n, k_x} \Psi_{nk_x}^{K,\pm}(x, y) \hat{a}_{nk_x}(t),
\]

where

\[
\hat{a}_{nk_x}(t) = \hat{a}_{nk_x} e^{-i(\varepsilon_n - \hat{\mu})t},
\]

\[
\hat{b}_{nk_x}(t) = \hat{b}_{nk_x} e^{-i(\varepsilon_n + \hat{\mu})t},
\tag{17}
\]

and \( \hat{a}_{nk_x}(t) \) and \( \hat{b}_{nk_x}(t) \) are the Hermitian conjugates of the above two operators.

The Green’s function of noninteracting particles is defined in a standard manner [8]:

\[
C_{0\alpha\beta}(x, x', y, y', t - t') = -i \langle T \hat{\Psi}_{\alpha\beta}(x, y, t) \hat{\Psi}_{\alpha\beta}^{K,\pm}(x', y', t') \rangle_{0},
\tag{18}
\]

where the angle brackets denote statistical averaging; \( T \) is the time ordering operator; \( \alpha, \beta = 1, 2 \) are pseudospin indices; \( \hat{\Psi}_{\alpha\beta}(x', y', t') = \hat{\Psi}_{\alpha\beta}(x', y', t')_{\gamma_0} \) is the Dirac conjugation spinor.

Computing averages of the pairs of operators (17) and passing from the time \( t - t' \) to the frequency \( \omega \) and
from the coordinate $y - y'$ to the crystal momentum $k_y$, we find the Green’s function in the mixed $x - k_y$ representation

$$G_{0\beta}^{K}(x, x'; k_y, \omega) = \sum_{n} \frac{\psi_{K\alpha}(x)\overline{\psi}_{K\beta}(x')}{\omega - \varepsilon_n + \mu - i\delta \text{sgn}(\mu - \varepsilon_n)}$$

$$+ \sum_{n} \frac{\psi_{K\alpha}(x)\overline{\psi}_{K\beta}(x')}{\omega + \varepsilon_n + \mu - i\delta \text{sgn}(\mu + \varepsilon_n)}$$

Hereinafter, we take for brevity the wavefunctions without the factors $e^{\pm ik_y y'/\sqrt{L_y}}$. 

It should be mentioned that Green’s function (19) formally coincides with the Green’s function of a relativistic electron found with the use of $\Psi$ operators in the Furry representation if one sets $\mu = 0$ [9].

The expression for the Green’s function of a hole is similar to Eq. (21): the numerators of the fractions include the respective products of the hole wavefunctions and $\mu$ stands in the denominator with the opposite sign. Taking into account relation (10) between the solutions for different valleys, we find the Green’s functions of an electron and a hole in the valley of the $K'$ point

$$G_{0}^{K'\epsilon,h}(x, x'; k_y, \omega) = -\sigma_x G_{0}^{K'\epsilon,h}(x, x'; k_y, \omega)$$

4. POLARIZATION OPERATOR

The polarization operator is given by the loop diagram (Fig. 2). In the case of electrons, it reads [4]

$$\Pi^{e}(x, x'; k_y, \omega) = -igd \int \frac{dp_y}{2\pi} \int \frac{d\varepsilon}{2\pi} \times \text{Tr} \left\{ \gamma_0 G_{0}^{K}(x, x'; p_y, \varepsilon)\gamma_0 G_{0}^{K}(x, x'; p_y + k_y, \varepsilon + \omega) \right\}$$

where $g = g_s g_v$ is the degeneracy ($g_s = 2$ is the spin degeneracy and $g_v = 2$ is the valley degeneracy).

Here, for definiteness, we wrote the electron Green’s functions in the valley of the $K$ point. It is easily verified using relation (20) that Eq. (21) coincides with the expression for $\Pi^{e}(x, x'; k_y, \omega)$ written in terms of the electron Green’s functions in the valley of the $K'$ point.

The polarization operator $\Pi^{h}(x, x'; k_y, \omega)$ for holes can be written in terms of the hole Green’s function similar to Eq. (21). It is easily seen from the form of the hole Green’s function and relation (11) between the hole and electron solutions that $\Pi^{h}(x, x'; k_y, \omega)$ differs from $\Pi^{e}(x, x'; k_y, \omega)$ only in the sign in front of $\mu$. The polarization operator of holes is also identical for both valleys.

As in the case of zero magnetic field [4], polarization operator (21) must be renormalized, since it does not vanish in the absence of charge carriers, when $|\mu| < \Delta_{\text{eff}}$. We impose the renormalization condition

$$\Pi^{\text{Ren}}_{\text{e}}(x, x'; k_y, \omega) = \Pi^{e}(x, x'; k_y, \omega) - \Pi^{e}(x, x'; k_y, \omega)|_{|\mu|<\Delta_{\text{eff}}}. \quad (22)$$

The polarization operator has the form

$$\Pi^{e}_{\text{Ren}}(x, x'; k_y, \omega) = gd \int \frac{dp_y}{2\pi} F(\xi, \xi'; \eta, \eta'), \quad (23)$$

where

$$\xi = \frac{\sqrt{\parallel} (x + l_B p_y)}{B},$$

$$\xi' = \frac{\sqrt{\parallel} (x' + l_B p_y)}{B},$$

and the variables $\eta$ and $\eta'$ differ from $\xi$ and $\xi'$, respectively, in the replacement $p_y \rightarrow p_y + k_y$.

The function $F(\xi, \xi'; \eta, \eta')$ is the simplest if only the zeroth Landau level is filled:

$$F(\xi, \xi'; \eta, \eta') = \sum_{s=\pm} \int_{\xi}^{\infty} \frac{2(\xi - s\xi_0)^2 + s\delta \text{sgn} \omega}{|C_n(s)|^2 \phi_n(\xi)\phi_n(\xi')\phi_0(\eta)\phi_0(\eta')}$$

Here, the sum at $s = +$ is the contribution of positive-frequency solutions (virtual intra-miniband transitions), whereas the sum at $s = -$ is the contribution of negative-frequency solutions (virtual inter-miniband transitions).

There is an additional significant simplification: the integrals of $\phi_n(\xi)\phi_n(\xi')\phi_0(\eta)\phi_0(\eta')$ over $p_y$ depend only on $x - x'$, as for charge carriers with a quadratic dispersion relation [10]. Calculating these integrals and performing the Fourier transform from $x - x'$ to $k_x$, we find the polarization operator in the momentum representation

$$\Pi^{\text{Ren}}_{\text{h}}(k, \omega) = \frac{gd}{2\pi l_B^2}$$

$$\times \sum_{s=\pm} \sum_{n=1}^{\infty} \frac{2(\xi_n - s\xi_0)^2 + s\delta \text{sgn} \omega}{|C_n(s)|^2 \left( \xi_n^2 \right)^{n!/2} e^{-\chi^2/2}},$$

where

$$\chi^2 = \frac{v_{\perp}^2 k_x^2 + v_{\parallel}^2 k_y^2}{v_{\perp} v_{\parallel}} l_B^2.$$
\[ 1 - V(k) \Pi_{\text{ren}}(k, \omega) = 0, \]  
(26)

where \( V(k) \) is the Coulomb interaction between the charge carriers in the superlattice.

In our case, it is the same as for semiconductor filaments arranged periodically in the same plane parallel to each other. The Coulomb interaction between charges in two filaments separated by the distance \( \nu d \) in such a system is \[11\]

\[ V(\nu, k_y) = 2e^2K_0(d|\nu k_y|), \]  
(27)

where \( d \) is the distance between the gapless graphene strips (coincides with the superlattice period); \( \nu \) is the strip (superlattice cell) number; \( \bar{\epsilon}^2 = \epsilon^2/\kappa_{\text{eff}} \), where \( \kappa_{\text{eff}} = (\kappa_1 + \kappa_2)/2 \) is the effective static dielectric constant determined by the static dielectric constants \( \kappa_1 \) and \( \kappa_2 \) of the media surrounding graphene, for example, vacuum and the substrate material; and \( K_0(x) \) is the modified Bessel function of the second kind.

Let us pass from the discrete variable of the strip number \( \nu \) to the transverse momentum \( k_x (-\pi/d \leq k_x \leq \pi/d) \) similar to \[11\]

\[ V(k) = \sum_{\nu=\infty}^{\infty} V(\nu, k_y)e^{i\nu k_x d} \]
\[ = 2e^2K_0 \left( \frac{d_1}{2} |k_y| \right) + 4\epsilon^2 \sum_{\nu=1}^{\infty} \cos(\nu k_x d) K_0(\nu d|k_y|), \]
(28)

where \( d_1 \) is the width of the gapless graphene strips.

Expression (28) is simplified in the case of interest of a small gapped graphene strip width \( d_1 \ll d_1 \) \[11\]

\[ V(k) = 2\epsilon^2 \ln \left( \frac{d}{\pi d_1} \right) + \left[ -2C - 2\psi \left( \frac{k_x d}{2\pi} + \frac{1}{2} \right) - \pi \tan \frac{k_x d}{2} \right]\bar{\epsilon}^2 + o(k_y d), \]
(29)

where \( C = 0.577\ldots \) is the Euler constant and \( \psi \) is the Euler \( \psi \) function. At the edges of the miniband \( (k_x = \pm \pi/d) \), we find from Eq. (28) similar to \[11\]

\[ V(k) = 2\epsilon^2 \ln \left( \frac{d}{\pi d_1} \right) + \frac{2\pi \epsilon^2}{|k_y| d} + o(k_y d). \]
(30)

6. NUMERICAL CALCULATION OF MAGNETOPLASMON FREQUENCIES

For numerical calculations, we take as an example a graphene–graphane superlattice formed by alternating strips of gapless graphene with the width \( d_1 = 8.52 \) nm and graphene strips with the width \( d_1 = 0.852 \) nm. The superlattice period is \( d = d_1 + d_1 = 9.372 \) nm.

For graphene, \( \Delta = 2.7 \) eV. For simplicity, we set \( V_{\text{eff}} = 0 \). Then we find from the calculation of the dispersion relation of charge carriers \( \Delta_{\text{eff}} = 98.56 \) meV, whereas \( E = 102.76 \) meV at the edge of the lower electron miniband.

\[ v_{\perp} \approx 0.19 \times 10^8 \text{ cm/s} \] and \( v_{\parallel} \approx \]

\[ v_F = 0.85 \times 10^8 \text{ cm/s near } E = \bar{\mu} \] \[13\]. The magnetic field is \( B = 5 \) T. The magnetic length amounts to \( l_B = 11.47 \) nm. Only the zeroth Landau level is occupied: \( \varepsilon_0 = \Delta_{\text{eff}}, \varepsilon_1 = 103.79 \) meV and \( \varepsilon_0 < \bar{\mu} < \varepsilon_1 \). The effective static dielectric constant is \( \kappa_{\text{eff}} = 5 \).

Let us calculate the dependence of the magnetoplasmon frequencies at the edge of the miniband, \( k_x = \pm \pi/d, \) on the crystal-momentum component \( k_y \). The results for five lower branches of the magnetoplasmon spectrum are shown in Fig. 3. Horizontal dashed lines mark the resonance frequencies corresponding to a non-zero imaginary part of the polarization operator: \( \omega_n = \varepsilon_n - \varepsilon_0 \) for \( n = 1, 2, \ldots, 6 \). The dispersion curves of magnetoplasmons do not intersect these horizontal lines, just tending to them asymptotically at \( k_y l_B \gg 1 \).

7. CONCLUSIONS

In this work, the problem of dispersion of magnetoplasmons in planar graphene-based superlattices has been studied analytically in the random-phase approximation. Since the lower electron miniband and the upper hole miniband are situated pretty close to one another in energy, the superlattice behaves like an anisotropic narrowband semiconductor. The Green’s function of charge carriers has been derived in a standard way. The polarization operator has been calculated from the found Green’s function in the zeroth approximation with respect to the interaction. Apart from the contribution of virtual intra-miniband transitions, the polarization operator includes the contribution of virtual inter-miniband transitions, which allows taking into account this contribution explicitly in the dispersion relation of magnetoplasmons in the medium under consideration.

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References

[1] Yu. E. Lozovik, S. P. Merkulova, and A. A. Sokolik, Phys. Usp. 51, 727 (2008).
[2] O. L. Berman, G. Gumbs, and Yu. E. Lozovik, Phys. Rev. B 78, 085401 (2008).
[3] P. V. Ratnikov, JETP Lett. 90, 469 (2009).
[4] P. V. Ratnikov and A. P. Silin, JETP Lett. 102, 713 (2015).
[5] G. L. Bir and G. E. Pikus, Symmetry and Stain-Induced Effects in Semiconductors (Nauka, Moscow, 1972; Wiley, New York, 1975).
[6] J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill, New York, 1964; Nauka, Moscow, 1978), Vol. 1.
[7] V. N. Tsytovich, Sov. Phys. JETP 13, 1249 (1961).
[8] J. González, F. Guinea, and M. A. H. Vozmediano, Nucl. Phys. B 424, 595 (1994).
[9] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, Course of Theoretical Physics, Vol. 4: Quantum Electrodynamics (Nauka, Moscow, 1989; Pergamon, Oxford, 1982).
[10] I. V. Lerner and Yu. E. Lozovik, Sov. Phys. JETP 47, 140 (1978).
[11] E. A. Andryushin and A. P. Silin, Phys. Solid State 35, 164 (1993).
[12] S. Lebège, M. Klintenberg, O. Eriksson, and M. I. Katsnelson, Phys. Rev. B 79, 245117 (2009).
[13] D. C. Elias, R. V. Gorbachev, A. S. Mayorov, S. V. Morozov, A. A. Zhukov, P. Blake, L. A. Ponomarenko, I. V. Grigorieva, K. S. Novoselov, F. Guinea, and A. K. Geim, Nat. Phys. 7, 701 (2011).

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