Generating Compact Geometric Track-Maps for Train Positioning Applications

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Abstract—In this paper we present a method to generate compact geometric track-maps for train-borne localization applications. We first give a brief overview on the role of track maps and it becomes apparent that there are hardly any adequate methods to generate suitable geometric track-maps. Therefore, we present a novel map generation procedure that uses an optimization formulation to find the continuous sequence of track geometries that fits the available measurement data best. The optimization is initialized with the results from a localization filter developed in our previous work. The filter also provides the required information for shape identification and measurement association. The approach will be evaluated using simulated data in comparison to the typically used data-point based maps.

I. INTRODUCTION

For safety reasons trains currently only operate strictly signal-based. Therefore, each track is divided in multiple block sections delimited by signals. The signals are coordinated by a central safety logic which guarantees that only one train can occupy such a block section at the same time. The necessary train-position information is gathered by sensors installed at the tracks itself. Although this system has proven itself to be safe and reliable it suffers either from high costs for the huge amount of sensors or a low track capacity due to longer block sections [1]. To overcome this bad trade-off, trains have to become intelligent vehicles (IV) which are able to localize themselves continuously without any track-side installations.

The challenge when developing such a train-borne localization system is to fulfill the high demands in terms of reliability, availability, maintainability, and safety (RAMS) in the sense of EN 50126 [2]. Although the development of train-borne localization systems has gained of interest in recent years, there is currently no sensor configuration available fulfilling all demands [3].

In this paper we want to focus on the role of digital track maps in train-borne localization systems and how they can help to fulfill the RAMS demands in the near future. Therefore, we will investigate how track maps are utilized to improve the positioning accuracy, availability and integrity of train-borne localization systems. After that we will give a brief overview on the methods commonly used to generate track maps. From this overview it will become apparent that there are hardly any adequate methods to generate suitable track maps for the purposes described above. Therefore, we will present a new approach to generate compact geometric track-maps based on the results of the localization filter we presented earlier in [4] motivated by [5].

II. TRACK MAPS IN TRAIN-BORNE LOCALIZATION

We start with a brief overview on the different types of track maps and the methods used to generate them. Afterwards we briefly discuss the shortcomings of these methods, which motivated us to come up with a novel approach to generate compact geometric track-maps.

A. Map Types

There are three different categories of track maps used for train-borne localization:

1) Topological Track-Maps: This is the most basic track-map type. It only stores the topology and mileage of the railway network. This is possible due to the fact that the position $p$ of a train can unambiguously be defined in railway coordinates by $p = \{t, s\}$, where $t$ represents a unique track ID and $s$ being a continuous track-length parameter. These maps are widely used in the railway system today since an additional absolute position information is not needed for its safe operation.

Theoretically it is possible to realize a train-borne localization system with these maps. Therefore, it has to be assumed that the start point and the pre-set route of a specific train is known. Then a train can localize itself by measuring its traveled distance relative to its start point and relating this information according to the pre-set route within the map [6]. Unfortunately, the pre-set route is normally not known on the train itself. This makes the localization result ambiguous because after a switch the position can no longer be clearly determined. To solve this ambiguity maps holding additional information have been introduced as described in the following second category.

2) Topographic and Geometric Track-Maps: Compared to the topological track-maps described before, topographic track-maps additionally store the track-course in absolute coordinates. Furthermore, if they hold track-characteristic information like the specific track element type (straight, circular arc or transitional arc), orientation, curvature, or something similar, they can be additionally named geometric track-maps. The additional information stored in these maps allows to apply different map-matching techniques which creates the possibility to implement more track-selective localization approaches compared to topological track-maps.

The map-matching approaches vary depending on the used sensor configuration. Many approaches utilize global

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navigation satellite system (GNSS) data and inertial measurement unit (IMU) data together with course and curvature information from a track map to realize track-selective map-matching approaches [7]–[11]. However, the additional map information can not only be used to improve the localization accuracy. It can also be used to increase the availability and integrity of the localization system itself as can be seen for example in [12]–[14].

3) Feature Track-Maps: This type of track-map also stores information on features or landmarks along the track. Features directly used for train-borne localization are for example ferromagnetic inhomogeneities of the rails [15] or characteristic distortions of the earth magnetic-field along the railway track [16]. Other features may be characteristic infrastructure elements like bridges, tunnels or stations, as suggested in [17] which can help to increase the accuracy and availability of GNSS positioning results.

B. Generation Methods

There are basically four main approaches to create digital track-maps [17]:

- Extraction from existing site plans, available as paper drawings, Computer Aided Design (CAD) plans, or Geographic Information Systems (GIS) databases,
- Direct surveying of the tracks, e. g. by GNSS measurements or the application of tachymetry,
- Analysis of orthophotos, or
- Application of simultaneous localization and mapping (SLAM) methods.

Although track-maps are indispensable for train-borne localization, it is often not described in detail how the necessary maps are created. The probably most commonly used maps are simply previously recorded data points which are available from the localization sensors anyway. If additional geometric track information is needed, it is mostly referred to the possibility to extract this data from existing site plans. There are also two simultaneous localization and mapping (SLAM) approaches available, especially designed for railway vehicles [18], [19]. Both methods also create data-point based track maps.

C. Discussion

Based on the explanations in Sec. II-A it becomes obvious how important track-maps are for train-borne localization. They help to increase the positioning accuracy, availability and integrity of the localization system. Thus, track maps act like an additional passive sensor helping to meet the RAMS requirements. This especially applies to geometric and feature track-maps. However, the usability of map information for localization purposes is largely influenced by the map representation and the map quality. For a map to be suitable for train-borne localization it has to fulfill at least two basic requirements:

- Track-length accuracy: It is essential to consistently assign all stored information with respect to the track-length \( s \) since all localization algorithms somehow rely on this assignment.
- Compactness: All information must be accessible in a computationally efficient way, as the map is often directly used in the localization algorithm itself, which has to run in real-time. Furthermore, it is advantageous if the map consumes as little memory as possible in order to be easily transferable.

All current generation methods directly utilizing measurement data store the map in a data-point format. Between neighboring data points interpolation techniques are applied. To avoid large interpolation errors the tracks are normally densely sampled, i. e. with a sample distance between 1 m and 30 m. Due to the necessary interpolation, such maps are not computationally efficient and the resulting map representation is neither easy accessible nor memory saving. Thus, these maps are not optimal in the sense of the compactness requirement mentioned above [22]. A more suitable track map representation would be a direct description of the geometric properties of each track element in a list. This would result in geometric track-maps easily fulfilling the compactness requirement. Those maps may be extracted from existing site plans. However, these site plans can differ significantly from the real track situation [23]. Two possible ways to create compact geometric track-maps based on measurement data are presented in [23], [24]. An alternative mapping approach, which advantageously incorporates the results of our previously published localization filter [4], will be presented in the remainder of this paper.

III. MAP GENERATION

The aim of our map generation procedure is to create compact geometric track-maps like the example listed in Tab.I. This table fully represents the track shown in Fig. 1. The compactness results from the fact that railway tracks always consist of a continuous sequence of well described geometric shapes (straight, transitional arc, and circular arc) [25]. Therefore, a railway track can unambiguously be described by a single starting point, the direction of the track at the starting point, the sequence of geometric shapes, and the geometric parameters for each shape (c. f. Tab.I).

| Track ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|---|---|---|---|---|---|---|---|
| Shape*  | st| ta| ca| ta| st| ta| ca| ta| st|
| Length \( L \) in m | 1000 | 231 | 476 | 231 | 1000 | 108 | 206 | 108 | 1000 |
| Radius \( r \) in m | \( \infty \) | \( -900 \) | \( -900 \) | \( -900 \) | \( \infty \) | 300 | 300 | 300 | \( \infty \) |
| Start point \( \xi_0 \) in m | 0 | --- | --- | --- | --- | --- | --- | --- | --- |
| Start point \( \eta_0 \) in m | 0 | --- | --- | --- | --- | --- | --- | --- | --- |
| Direction \( \phi_0 \) in ° | 10 | --- | --- | --- | --- | --- | --- | --- | --- |

* st: straight, ta: transitional arc, ca: circular arc

1Some further conclusions on the requirements for digital track-maps as well as some modeling schemes can be found in [17], [20], [21].
A. Initial Situation

We assume to start with the results of the localization filter we presented in [4]. Along with the position solution this filter estimates some of the track’s geometric parameters which conveniently serve as initialization for the map generation process. Moreover, the filter allows to do an assignment between measurement data and identified track geometries which vastly simplifies the formulation of the mapping error that will be derived later in this section.

A summary of the already available parameters from the localization filter is listed in Tab. II. A visualization of the resulting initial track is shown in Fig. 2. The input measurements used for the filter have been generated by simulation. The used ground-truth track is shown in Fig. 1. The detailed simulation procedure and parameters are described in [4]. All further explanations are illustrated using this example data.

![Fig. 1. Exemplary track, consisting of the three standard track geometries: straight, transitional arc and circular arc. A compact geometric track-map representation of this track is given in Tab. I.](image)

![Fig. 2. Initially available track-elements resulting from the parameters given in Tab. II. The track has several gaps and is not continuous.](image)

![Fig. 3. Track map resulting from the simple concatenation of the track geometries estimated by the localization filter described in [4].](image)

![Fig. 3. Track map resulting from the simple concatenation of the track geometries estimated by the localization filter described in [4].](image)

B. Mapping Procedure

The initial track depicted in Fig. 2 is not usable for localization since it is not continuous. It has some gaps at the points where no track-geometry has been identified (c.f. Tab. II, track-IDs: 2, 4, 6, and 8). Therefore, the task for our map generation procedure is to connect the initially identified track-elements to a continuous track which also has to fit the available GNSS measurement data. To solve this task, first, missing track-geometries have to be identified. Afterwards, the geometric parameters of the tracks can be estimated.

1) Track Geometry Identification: We assume that the missing track-geometries can be concluded from the following knowledge [4], [25]: Railway tracks only consist of three basic geometric shapes, which are straights, transitional arcs and circular arcs. A straight can only be connected to a circular-arc with the help of a transitional-arc and vice versa. Since the preceding localization filter already identified straights and circular-arcs it can be concluded that the unknown geometries have to be transitional arcs.

2) Geometry Parameter Identification: After all track geometries have been identified, the corresponding geometric parameters have to be tuned such that the continuous concatenation of all track-elements fits best to the available GNSS measurement data.

Although, there is already a lot of information available from the preceding filter, it is necessary to tune all parameters together at the same time. This can be seen when we try to naively concatenate all identified tracks, where the parameters of the transitional arcs are inferred from the neighboring elements.2 The resulting track map for this

![Diagram](image)

naive approach is shown in Fig. 3. Obviously, it is a quite poor fit to the GNSS data. This is a result of the continuity constraint. Slight parameter inaccuracies of one track element are propagated on all succeeding elements. Therefore, it is necessary to tune all parameters together at once. To achieve this, we establish an optimization problem for the whole track by defining an appropriate error function that incorporates all given measurement information. In order to solve this optimization problem we furthermore have to reformulate the track parameters in a more suitable representation and have to choose an optimization method. All these essential aspects of the optimization are described in the following paragraphs.

a) Error-Function Definition: The error introduced by a track-element is given by the perpendicular distances between

![Diagram](image)

2This is possible because transitional arcs are executed as clothoids [25]. They are clearly defined by their length, their radius in the end point, and their orientation either in the start or end-point.
the track and the GNSS measurements related to this track. Let $x_t$ be a vector representing the parameters of track element $t$. Furthermore, let $z_{t,i}$ be the $i$-th GNSS position measurement assigned to this track element $t$ by the preceding filter. With $\hat{z}_{t,i}(x_t)$, the dropped perpendicular point of $z_{t,i}$ on the track element $t$ the error for this measurement is then defined as

$$e_{t,i}(x_t) = z_{t,i} - \hat{z}_{t,i}(x_t).$$

(1)

The sum over all this measurement errors for all track elements yields the total error of the whole track map, i.e.

$$F(\{x_1, \ldots, x_N\}) = \sum_{t \in T} \sum_{i \in C(t)} e_{t,i}^T \Omega_{t,i} e_{t,i},$$

(2)

where $C_t$ is the set of all measurements assigned to track $t$ and $\Omega_{t,i}$ is the information matrix corresponding to measurement $z_{t,i}$.

b) Parameter Representation: The parameterization of the track-map presented in Tab.I is not very suitable for an optimization. With this parameterization the whole track would be very sensitive to changes in specific parameters, e.g. small changes in $\varphi_0$ and $L$ would rotate, respectively move, major parts of the track. Therefore, an alternative representation is chosen with less sensitivity. All straights are now parameterized by their start and end point whereas transitional arcs and circular arcs are parameterized by a minimal set of geometric parameters. For our example track (Tab.II) this new parameterization is given in Tab.III and the corresponding parameter vector is

$$x = \{x_1, x_2, x_3, x_4, x_5, \ldots\}, \text{ with}$$

$$x_1 = \left[\xi_{0,1}, \eta_{0,1}, \xi_{e,1}, \eta_{e,1}\right]^T,$$

$$x_2 = L_2, \quad x_3 = \left[r_3, L_3\right]^T, \quad x_4 = L_4,$$

$$x_5 = \left[\xi_{0,5}, \eta_{0,5}, \xi_{e,5}, \eta_{e,5}\right]^T, \ldots$$

TABLE III

| Track ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|---|---|---|---|---|---|---|---|
| Shape*   | st | ta | ca | ta | st | ta | ca | ta | st |
| Length L in m | 278 | 415 | 206 | -- | 185 | 106 | 165 | -- | -- |
| Radius r in m | -- | -- | 882 | -- | -- | 297 | -- | -- | -- |
| Start point $\xi_{0,0}$ in m | 0 | -- | -- | -- | 1772 | -- | -- | -- | 2763 |
| Start point $\eta_{0,0}$ in m | 0 | -- | -- | -- | 683 | -- | -- | -- | 1660 |
| End point $\xi_{e,0}$ in m | 1019 | -- | -- | -- | 2338 | -- | -- | -- | 3722 |
| End point $\eta_{e,0}$ in m | 180 | -- | -- | -- | 1487 | -- | -- | -- | 1579 |

* st: straight, ta: transitional arc, ca: circular arc

c) Optimization Method: The goal of our optimization problem is to minimize the error function $F(x)$ given in (2). Although, the initial parameters given by our filter yield a poor track-map when concatenated (c. f. Fig. 3), they are still a good initial guess $x_0$ for the track map parameters. Therefore we can start the optimization with this initial parameters which are presumably close to the global optimum and it is sufficient to use the Levenberg-Marquardt algorithm [26] to find that optimum.

IV. Evaluation

The performance of the presented map generation method will be evaluated in this section. Therefore, the example described in Sec.III-A is used. For comparison a typically used data-point based track-map is used (c. f. Sec.II-C). It has been sampled from the GNSS data with a spacing of 1 m. Intermediate points are calculated by a linear interpolation.

A. Optimization Process

The progress of the residual $\|F(x)\|$ during the optimization is shown in Fig.4. It can be seen that the optimization converges very fast. After eight iterations the stopping criteria is reached. The biggest change in $\|F(x)\|$ occurs in the first iteration step of the optimization. This confirms our hypothesis from above that the initial parameters $x_0$, provided by the filter, are already a good guess and the Levenberg-Marquardt algorithm can quickly find the optimal solution.

B. Absolute Accuracy

The resulting track-map in its parameterized form is shown in Tab.IV. A visualization of the resulting track is shown in Fig.5 from which a good qualitative match with the GNSS measurement data and the underlying track becomes evident.

Figure 6 allows to investigate the maps quality in even more detail. This plot shows the absolute position deviation to the ground-truth track over the path length $l$. The deviation of our optimized map is on average 1.8 m. For comparison the deviation for the interpolated data-point based track-map is also shown. The error for this map varies drastically over the whole track length and the average deviation is 10.3 m. This value can be explained by the choice of the simulated measurement noise, which has a standard deviation of 10 m. Our approach clearly outperforms the simple data-point based approach since all measurements are jointly incorporated in the optimization step and the error induced by the measurements noise can be reduced significantly.

C. Geometric Accuracy

A often used metric to evaluate the geometric similarity of two paths is the fréchet distance [27]. The fréchet distance of our optimized map to the ground-truth track is 3.6 m. In comparison, the fréchet distance of the data-point based map is 26.9 m (c. f. Fig.7). Consequently, the generated geometric track-map is significantly better in representing the geometric characteristic of the track. Furthermore, the generated map
allows to efficiently access useful geometric information, e.g. the curvature at an arbitrary path length. For the data-point based map this is only possible with additional calculations.

D. Map Size

To compare the size of the maps, we express the storage demand as the number of needed data fields. For example, a position specification \( p = (\xi \eta)^T \) requires two data fields. The data-point based map consist of more than 8000 data fields, whereas the generated map only consists of 30 data fields. This clearly shows how compact geometric track-maps can be, compared to data-point based maps.

V. CONCLUSIONS

In this paper we presented an approach to generate geometric track-maps for train-borne localization systems. Based on a brief overview we showed how important these maps are for trains to become intelligent vehicles that are able to localize themselves in the near future. Furthermore, the overview revealed that there are hardly any adequate methods to generate suitable track maps for this purpose.

We presented an optimization method that finds the geometric parameters of an continuous track that fits the position measurements best. The method thereby uses information provided by a preceding localization filter for initialization, shape identification and data association.

Throughout simulations we demonstrated that the presented method is able to generate geometric track-maps which are more accurate than the typically used data-point based maps. Furthermore, the generated map provides additional geometric track information and all stored information can be accessed in a computational efficient way, due to the compact form of the map. In the future we also plan to evaluate the performance of the presented method on real sensor data.

TABLE IV

| Track ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|----------|---|---|---|---|---|---|---|---|---|
| Shape*  | st | ta | ca | ta | st | ca | ta | st | st |
| Length L in m | 200 | 210 | 210 | 210 | 210 | 210 | 210 | 210 | 210 |
| Radius r in m | 390 | 390 | 390 | 390 | 390 | 390 | 390 | 390 | 390 |
| Start point \( \xi_0 \) in m | 0 | -1 | 0 | -1 | 0 | -1 | 0 | -1 | 0 |
| Start point \( \eta_0 \) in m | 0 | -1 | 0 | -1 | 0 | -1 | 0 | -1 | 0 |
| End point \( \xi_e \) in m | 1007 | 1007 | 1007 | 1007 | 1007 | 1007 | 1007 | 1007 | 1007 |
| End point \( \eta_e \) in m | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 | 180 |

* st: straight, ta: transitional arc, ca: circular arc

Fig. 5. Final track-map resulting from the presented procedure.

Fig. 6. Absolute position error plotted against the track length \( l \). The results for our optimized geometric track-map and a typically used data-point based track-map (sample rate 1 m with linear interpolation) are shown.

Fig. 7. Comparison of Fréchet distances to the ground-truth track. The results for the optimized geometric track-map and a typically used data-point based track-map are shown.

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