Abstract—The implementation of computational sensing strategies often faces calibration problems typically solved by means of multiple, accurately chosen training signals, an approach that can be resource-consuming and cumbersome. Conversely, blind calibration does not require any training, but corresponds to a bilinear inverse problem whose algorithmic solution is an open issue. We here address blind calibration as a non-convex problem for linear random sensing models, in which we aim to recover an unknown signal from its projections on sub-Gaussian random vectors each subject to an unknown multiplicative factor (gain). To solve this optimisation problem we resort to projected gradient descent starting from a suitable initialisation. An analysis of this algorithm allows us to show that it converges to the global optimum provided a sample complexity requirement is met, i.e., relating convergence to the amount of information collected during the sensing process. Finally, we present some numerical experiments in which our algorithm allows for a simple solution to blind calibration of sensor gains in computational sensing applications.

Index Terms—Blind calibration, non-convex optimisation, sample complexity, computational sensing, bilinear inverse problems.

I. INTRODUCTION

The problem of acquiring an unknown signal $x$ in the presence of sensing model errors is crucial for modern computational sensing strategies such as Compressed Sensing (CS), in which such errors inevitably affect physical implementations and can significantly degrade signal recovery [1]. Among the physical sources of such errors we may include: convolution kernels [2]–[4] as caused by lowpass optical elements, which affect the measured coefficients at the focal plane; attenuations and gains on the latter coefficients, e.g., pixel response non-uniformity [5], fixed-pattern noise or vignetting; complex-valued gain and phase errors in sensor arrays [6]–[8].

Assuming such errors remain stationary throughout the sensing process, the use of linear random operators in CS suggests that repeating the acquisition, i.e., taking several snapshots under new independent draws of a randomised sensing model could suffice to diversify the measurements and extract the information required to learn both the unknown signal and the model error. We here address the specific case of a single, unstructured vector $x \in \mathbb{R}^n$ that is sensed by collecting $p$ snapshots of $m$ random projections, i.e., our sensing model is

$$y_i = d A_i x, \quad d := \text{diag}(d) \in \mathbb{R}^{m \times m}, \quad l \in [p] := \{1, \ldots, p\},$$

where $y_i = (y_{i,1}, \ldots, y_{i,l})^\top \in \mathbb{R}^m$ is the $l$-th snapshot; $d = (d_1, \ldots, d_m)^\top \in \mathbb{R}^m$ is an unknown, positive and bounded gain vector that is identical throughout the $p$ snapshots; the random sensing matrices $A_l \in \mathbb{R}^{m \times n}$ are independent and identically distributed (i.i.d.) and each $A_l$ has i.i.d. rows, the $i$-th row $a_{i,l} \in \mathbb{R}^n$ being a centred isotropic (i.e., $E a_{i,l} = 0_n, E a_{i,l} a_{i,l}^\top = I_n$) sub-Gaussian random vector (for a formal definition, see [9, Section 5.2.5]). Note that $x$ is also assumed to remain identical throughout $p$ snapshots, e.g., a fixed scene being monitored by an imaging system.

The inverse problem corresponding to this bilinear sensing model is hereafter called blind calibration [8], [11]. In particular (1) relates to computational sensing applications in which unknown $d$ are associated to positive gains (see Figure 1) while $p$ random matrix instances can be applied on a source $x$ by a suitable (i.e., programmable) light modulation embodiment. This setup matches compressive imaging configurations [4], [10], [12]–[14] with an important difference in that the absence of a priori structure on $(x, d)$ in (1) implies an over-Nyquist sampling regime with respect to (w.r.t.) $n$, i.e., exceeding the number of unknowns as $m p \geq n + m$. When the effect of $d$ is critical (i.e., assuming $d \approx I_n$ would lead to

Figure 1. A randomised sensing model; blind calibration entails the joint recovery of the source $x$ and sensor gains $d$ by exploiting multiple random sensing matrices $A_l$ (e.g., programmable random masks in a random convolution setup [10]). The intensity of $d$ is represented in shades of red as a possible vignetting of the sensor array.

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an inaccurate recovery of \( x \) solving this problem justifies a possibly over-Nyquist sampling regime as long as \( (x, d) \) can both be recovered accurately (e.g., as an on-line calibration).

Prior approaches to blind calibration entail solving convex or alternating optimisation algorithms \([8, 11, 15]\) aided by multiple input signals (e.g., \( x_i, l \in [p] \)) instead of taking new draws of the sensing operator itself. However, such methods lack formal recovery guarantees and require the training signals to be as independent as possible, or to lie in a low-dimensional subspace (possibly known \textit{a priori}). More recently, \textit{lifting} approaches \([16]\) have been proposed to jointly recover \( (x, d) \) in (1) (as well as for more general \textit{blind deconvolution} problems \([2, 3, 4, 7]\)). Their main limitation is in that a semidefinite program is solved to recover a large-scale rank-one matrix \( xd^T \); this approach becomes computationally inefficient and unaffordable quite rapidly as \( m \) and \( n \) exceed a few hundreds.

Inspired by recent results on fast, provably convergent non-convex algorithms for phase retrieval \([17, 18, 19]\) we address the recovery of \( (x, d) \) by solving a non-convex problem presented in Section III, as naturally defined by (1). In particular, the use of \textit{mp} measurements in the model allows us to find an unbiased estimator of \( x \) as \( p \to \infty \); for \( p < \infty \) we will initialise our algorithm with this estimator and run projected gradient descent to obtain a recovery of \( (x, d) \). In Section III, the properties of the gradient and initialisation under random sensing vectors \( a_{l,l} \) will allow us to find a proof of convergence for this descent algorithm once a \textit{sample complexity} requirement on the amount of collected measurements is met. In Section IV we provide numerical evidence on the algorithm’s empirical phase transition; this is followed by a practical application of our method in a realistic computational sensing context.

II. A NON-CONVEX APPROACH TO BLIND CALIBRATION

A. Problem Statement

The formulation of an inverse problem for (1) is quite natural by means of a Euclidean data fidelity objective function

\[
\mathcal{F}(x, \gamma) := \frac{1}{mp} \sum_{l=1}^{m} \| \gamma A_l x - y_l \|^2,
\]

and

\[
\gamma A_l x - d_l A_l x,
\]

which we assume (with the orthogonal complement \( 1 \leq m \in \mathbb{R}^m \) and \( \rho \in \mathbb{R}^m \)). If we specify \( d^* = 1_m + \omega \) for \( \omega \in \mathbb{R}^m \) providing that the algorithm to study (3) will not grant convexity in the domain of (2), we will require a notion of distance. To this end, we could adopt

\[
\mathcal{F}(x, \gamma) := \frac{1}{m} \| \gamma A_l x - d_l A_l x \|^2,
\]

and

\[
\gamma A_l x - \omega A_l x.
\]

Thus, we can specify \( d^* = 1_m + \omega \) for \( \omega \in \mathbb{R}^m \) providing that the algorithm to study (3) will not grant convexity in the domain of (2), we will require a notion of distance. To this end, we could adopt

\[
\Delta \mathcal{F}(x, \gamma) := \frac{1}{m} \| \gamma A_l x - d_l A_l x \|^2 - \mathbb{E} \mathcal{F}(x, \gamma),
\]
the last equivalence being immediate from Table I. While this is a naturally balanced definition, proofs with it are slightly more cumbersome, so we resort to
\[
\Delta(\xi, \gamma) := \|\xi - x^*\|_2^2 + \frac{\|x^*\|_m^2}{m} \|\gamma - d^*\|_2^2.
\]
which for \((\xi, \gamma) \in \mathbb{R}^n \times C_\rho, \rho \in (0, 1)\) can be shown to verify \((1 - \rho)\Delta(\xi, \gamma) \leq \Delta R(\xi, \gamma) \leq (1 + 2\rho)\Delta(\xi, \gamma)\). With this we may define a neighbourhood of \((x^*, d^*)\) as
\[
D_{\kappa, \rho} := \{(\xi, \gamma) \in \mathbb{R}^n \times C_\rho : \Delta(\xi, \gamma) \leq \kappa^2\|x^*\|_2^2\}, \rho \in (0, 1),
\]
that is the intersection of an ellipsoid in \(\mathbb{R}^n \times \mathbb{R}^m\), as defined by \(\Delta(\xi, \gamma) \leq \kappa^2\|x^*\|_2^2\) with \(\mathbb{R}^n \times C_\rho\). We anticipate that local convexity can be shown on \(D_{\kappa, \rho}\) when testing the (projected) Hessian matrix against the direction \((\xi - x^*, \gamma - d^*)\), similarly to [18] Theorem 2.3. This argument is not explicitly used here (its proof will be reported in an upcoming journal paper [21]). In fact, a first-order analysis suffices to prove our main results in Section III.

B. Solution by Projected Gradient Descent

The solution of (2) is here obtained as summarised in Algorithm 1 and consists of an initialisation followed by projected gradient descent. Similarly to [17] we have chosen an initialisation \(\xi_0 \in C_\rho\) that is an unbiased estimator of the exact (signal) solution as \(p \to \infty\), i.e., \(\mathbb{E} \xi_0 \equiv \xi^*\); this is indicated in Table II. For \(p < \infty\) we will show in Proposition II that the initialisation lands in \((\xi_0, \gamma_0) \in D_{\kappa, \rho}\) for \(\rho \in (0, 1)\) with high probability, i.e., there exists a sample complexity \(mp\) ensuring \(\kappa\) can be made small.

As for the gains, we initialise \(\gamma_0 = 1_m\) \((\xi_0 = 0_m)\) and, since \(\rho < 1\) is small, we perform a few simplifications to devise our solver to (2). While generally we would need to project any \(\gamma\) on the simplex, we instead update the gains with the projected gradient \(\nabla f(\xi, \gamma) := P_{1/\rho} \nabla f(\xi, \gamma)\) (step 5) using the projection matrix \(P_{1/\rho} := \frac{1}{\rho} I_m - \frac{1}{\rho} I_m^T I_m\) (see Table I for its expression in terms of \(\epsilon, \omega\)). Then we apply \(P_{1/\rho}\), i.e., the projector on the convex set \(C_\rho\) (step 6). Actually, this step is just a formal requirement to ensure that each iterate \(\gamma_{k+1} \in C_\rho \subseteq \Pi^m_{\rho}\) in proving the convergence of Algorithm I to \((x^*, d^*)\); numerically, we have observed that step 6 can be omitted since \(\gamma_{k+1} \in C_\rho\) is always verified in our experiments.

Thus, Algorithm I is a descent with the projected gradient \(\nabla f(\xi, \gamma) := [(\nabla f(\xi, \gamma))^T (\nabla^2 f(\xi, \gamma))^T]^T\); the proposed version performs two line searches in 3: that can be solved in closed-form at each iteration as
\[
\mu_{\xi} := \frac{\sum_{l=1}^{m} (\nabla_{f}(\xi)_{l}^T \nabla_{\xi}(\xi_{l}, \gamma_{l}))}{\sum_{l=1}^{m} \|\nabla_{\xi}(\xi_{l}, \gamma_{l})\|_2^2}, \quad \mu_{\gamma} := \frac{\sum_{l=1}^{m} (\nabla_{f}(\xi)_{l}^T \nabla_{\gamma}(\xi_{l}, \gamma_{l}))}{\sum_{l=1}^{m} \|\nabla_{\gamma}(\xi_{l}, \gamma_{l})\|_2^2},
\]
and are simply introduced to improve the convergence rate. In the following we obtain the conditions that ensure convergence of this descent algorithm to the exact solution \((x^*, d^*)\) for some fixed step sizes \(\mu_{\xi}, \mu_{\gamma}\).

III. CONVERGENCE AND RECOVERY GUARANTEES

Recalling that all \(mp\) sensing vectors \(\alpha_{i,l}\) in I are i.i.d. sub-Gaussian, we now establish the convergence of Algorithm I in three steps: (i) the initialisation \((\xi_0, \gamma_0)\) is shown to lie in \(D_{\kappa, \rho}\) for \(\rho \in (0, 1)\) and small \(\kappa\) with high probability; (ii) \(\nabla f(\xi, \gamma)\) enjoys a condition by which, uniformly on \(D_{\kappa, \rho}\), a gradient descent update decreases the distance to \((x^*, d^*)\); (iii) by uniformity, applying this property to any \(k\)-th iterate \((\xi_k, \gamma_k)\) leads to finding fixed step values \(\mu_{\xi}, \mu_{\gamma}\) that grant convergence to \((x^*, d^*)\) as \(k \to \infty\). Proof sketches are provided after the main statements; the full arguments will be reported in an upcoming journal paper [21].

We first state a key result and its application to proving the properties of our initialisation for \(p < \infty\). As typically done when deriving sample complexity bounds, we will use some universal constants \(C, c > 0\) changing from line to line.

**Lemma 1 (Weighted Covariance Concentration Inequality).** Let \(\{\alpha_{i,l} \in \mathbb{R}^n : i \in [m], l \in [p]\}\) be a set of random vectors, each formed by \(m\) i.i.d. copies of a sub-Gaussian random variable \(X\) [9] Section 5.2.3] with \(\mathbb{E} X = 0, \mathbb{E} X^2 = 1\) and sub-Gaussian norm \(\|X\|_{\psi_2}\). For \(\delta \in (0, 1), t > 1\) we have, with probability exceeding
\[
1 - Ce^{-ct^2mp} - (mp)^{-t}
\]
for some \(C, c > 0\) depending only on \(\|X\|_{\psi_2}\), that
\[
\frac{1}{mp} \sum_{i=1}^{m} \sum_{l=1}^{p} \|\alpha_{i,l}^T \alpha_{i,l}^T - I_m\|_2 \leq \delta \|\theta\|_\infty \|X\|_{\psi_2}
\]
for all \(\theta = \{\theta_{i,l} \in \mathbb{R}^m\}^{m}_{i=1}\) provided \(n \geq t \log(mp)\) and \(mp \geq \delta^{-2}(n + m) \log(t^2)\).

**Proof sketch:** By defining the function
\[
S(u, \theta) := \frac{1}{mp} \sum_{i=1}^{m} \sum_{l=1}^{p} \theta_{i,l}^2 |u^T \alpha_{i,l}|^2 - \|u\|_2^2,
\]
the proof consists in bounding sup\(u \in S^m_{\rho}\); \(S(u, \theta)\) by a covering argument on \((u, \theta) \in S_{\rho}^{m-1} \times S_{\rho}^{m-1}\) \((\rho^{-1}\) is the \(\rho\)-p.sphere in \(\mathbb{R}^m\)), using the concentration and continuity of \(S(u, \theta)\).

**Proposition 1 (Initialisation Proximity).** Let \((\xi_0, \gamma_0)\) be as in Table II. For any \(\epsilon \in (0, 1)\) we have, with probability exceeding
\[
1 - Ce^{-ct^2mp} - (mp)^{-t}
\]
for some \(C, c > 0\), that \(\|\xi_0 - x^*\|_2 \leq \epsilon\|x^*\|_2\) provided \(n \geq t \log(mp)\) and \(mp \geq \epsilon^2(n + m) \log(\epsilon^{-1})\). Since \(\gamma_0 = 1_m\)
we also have $\|y_0 - d^*\|_\infty \leq \rho < 1$. Thus $(\xi_0, \gamma_0) \in D_{\kappa, \rho}$ with the same probability and $\kappa := \sqrt{\rho^2 + \rho^2}$.

**Proof sketch:** Since $\xi_0 = \frac{1}{mp} \sum_{i=1}^p (A_i)^T d A_i x$ we have

$$\|\xi_0 - x\| \leq \frac{1}{mp} \sum_{i=1}^p \sum_{j=1}^p d_{i,j} a_{i,j} x \|x\|_2.$$  

Applying Lemma 4 to the matrix norm at the right-hand side and assigning $\epsilon = \delta(1 + \rho)$ proves this Proposition.

Secondly, we develop the requirements for convergence. Provided that the initialisation lands in $D_{\kappa, \rho}$, any update from $(\xi, \gamma) \in D_{\kappa, \rho}$ to some $\xi_+ := \xi - \mu_\gamma \nabla f(\xi, \gamma), \gamma_+ := \gamma - \mu_\gamma \nabla f(\xi, \gamma)$ has distance from the solution

$$\Delta(\xi_+, \gamma_+) = \Delta(\xi, \gamma)$$

- $2\mu(\|\nabla f(\xi, \gamma), \xi - x^*\| + \|\nabla f(\xi, \gamma), \gamma - d^*\|)$
- $\mu^2 \left( \|\nabla f(\xi, \gamma)\|_2^2 + \frac{m}{\|x\|^2} \|\nabla f(\xi, \gamma)\|_2^2 \right)$

where we have let $\mu_\gamma := \mu / (\|x\|^2)$ for some $\mu > 0$. To bound this we now verify a regularity condition on $\nabla^T f(\xi, \gamma)$ (analogous to [17], Condition 7.9) as a property holding uniformly on the neighbourhood with high probability.

**Proposition 2 (Regularity condition in $D_{\kappa, \rho}$).** For any $\delta \in (0, 1), \rho \in [0, 1]$ there exist a constant $\eta \in (0, 1), t > 1$ and a value $L < 3(1 + \kappa)(4 + \|x^*\|_2)$ such that, with probability exceeding

$$1 - C \left[ m e^{-\delta^3 \rho} + e^{-\delta^2 mp} + (mp)^{-t} \right]$$

for some $C, c > 0$, we have for all $(\xi, \gamma) \in D_{\kappa, \rho}$

$$\left\| \nabla^T f(\xi, \gamma) \right\|_2^2 \leq L^2 \Delta(\xi, \gamma)$$

(Lipschitz curvature)

$$\left\| \nabla^T f(\xi, \gamma) \right\|_2 \leq L \Delta(\xi, \gamma)$$

(Lipschitz gradient)

**Proof sketch:** The general argument template uses triangle and Cauchy-Schwarz inequalities to manipulate the left-hand sides of the bounded curvature and Lipschitz gradient conditions (all required components are developed in Table 1).

Then, Lemma 1 and [9] Lemma 5.17 allow us to bound terms of the type $\left\| \nabla^T f(\xi, \gamma) \right\|_2^2 \leq (1 + \delta)\|\theta\|_\infty$.

and $1 - \delta \leq \left\| \frac{1}{mp} \sum_{i=1}^m \sum_{j=1}^p \theta a_{i,j} a_{j}^T \right\|_2 \leq 1 + \delta$ respectively. In particular, the bounded curvature part of Proposition 2 is

$$\langle \nabla f(\xi, \gamma), \xi - x^* \rangle + \langle \nabla f(\xi, \gamma), \gamma - d^* \rangle$$

$$\geq (1 - 3\rho)(1 - \delta)\|\xi - x^*\|^2 + \frac{\rho}{\sqrt{m}} (1 - \delta)\|x^*\|^2 \|e - \omega\|^2$$

$$- (1 + 4\rho)(1 - \delta)\|x^*\|^2 \left\| \frac{\|e - \omega\|^2}{\sqrt{m}} \right\|$$

$$\geq \left( \frac{1}{\|x^*\|^2} \right) (\|\xi - x^*\|^2 + \|x^*\|^2 \|e - \omega\|^2),$$

reminding that $\gamma - d^* \equiv e - \omega$. Choosing $\eta \in (0, 1)$ so that $\eta < 1 - 14\rho - 3\delta$ yields the condition on $\rho$. As for the Lipschitz
which decreases if we let $\mu \in (0, \gamma/\mu^2)$. Moreover, since $P_{C_\mu}$ is contractive (since $C_\mu$ is a non-empty closed convex set) we have $\|\xi_{k+1} - d^*\|_2 \leq \|\xi_{k+1} - d^\star\|_2$ and $\Delta(\xi_{k+1}, \gamma_{k+1}) \leq \Delta(\xi_{k+1}, \gamma_{k+1})$. Applying recursively the inequalities for all $k > 0$ in Algorithm 1 by uniformity on $D_{\alpha, \rho}$ yields (7). We remark that the initialisation is critical to set the value of $\kappa$ for Proposition 1 with its initial value appearing in (7).

Finally, let us mention some extensions of this theory in which: (i) a stability result can be obtained for Algorithm 1 when (1) is affected by bounded additive noise, which degrades gracefully the recovery quality of $\hat{x}, d$; (ii) the signal $x = C s$ and gains $d = B g$ depend on some low-dimensional parameters $s, g$ with $\dim(s) = k \ll n$, $\dim(g) = h \ll m$, thus causing a reduction in the sample complexity; (iii) equation (1) is extended to a blind deconvolution problem compatible with our assumptions. These results will be presented in (21).

IV. NUMERICAL EXPERIMENTS

A. Empirical Phase Transition

To trace the empirical phase transition of Algorithm 1 we ran some extensive simulations by generating 144 random instances of (1), fixing $n = 2^a, m = 2^6$ and varying $p = \{2^2, \ldots, 2^{10}\}, \rho = \{10^{-3}, \ldots, 1\}$. Each instance was drawn with $x \in \mathbb{B}_2^n$, $a_{i,t} \sim N(0, I_n)$, $d = 1_m + \omega$ with $\omega \in 1_m \cap \rho S_m^{-1}$. Then, we solved (2) by our descent algorithm and evaluated $P_{C_\zeta} := P \left[ \max \left\{ \frac{d^* - d}{\|d^* - d\|_2}, \frac{x^* - x}{\|x^* - x\|_2} \right\} < \zeta \right]$ on the trials with $\zeta = -70$ dB (in accordance with the stop criterion at $f(\xi_k, \gamma_k) < 10^{-5}$). The results are reported in Figure 2 in which we highlight the contour levels of $P_{C_\zeta}$.

B. Blind Calibration of a Randomised Imaging System

To test our approach in a realistic context we assume that $x$ is a $n = 128 \times 128$ pixel colour image acquired by a sensing device that implements (1) in which its $m = 32 \times 32$ pixel sensor array suffers from a randomly drawn attenuation profile $d \in \Pi^m_m$ generated as before, fixing $p \approx 0.99$. We capture $p = 32$ (i.e., $mp = 2n$) snapshots with $a_{i,t} \sim N(0, I_n)$ (each colour channel is processed separately). By running Algorithm 1 we obtain the results depicted in Figure 3: the recovered $(\hat{x}, d) \equiv (x, d)$ by solving (2) attains $\max \left\{ \frac{d^* - d}{\|d^* - d\|_2}, \frac{x^* - x}{\|x^* - x\|_2} \right\} \approx -61.59$ dB in accordance with the stop criterion at $f(\xi_k, \gamma_k) < 10^{-6}$. Instead, by fixing $\gamma := 1_m$ and solving (2) only w.r.t. $\xi$, i.e., finding the least-squares (LS) solution $\hat{x}$ and fully suffering the model error, we obtain $\frac{d^* - d}{\|d^* - d\|_2} \approx -5.99$ dB. Moreover, Figure 4 reports the evolution of the distances $\Delta(\xi_k, \gamma_k)$ and $\Delta_F(\xi_k, \gamma_k)$ measured on two runs of this exemplary case with the same stop criterion, one with the line searches (3) (ending at $k = 369$) and one with a fixed step $\mu_{\xi} = \mu := 10^{-4}, \mu_{\gamma} := \mu \approx 0.88$ (ending at $k = 6301$). We observe that there is clearly a linear bound on the convergence rate and that (4) clearly achieves faster convergence than a fixed-step choice of $\mu_{\xi, \mu_{\gamma}}$. Finally we note that while the theory in Section III is developed for sub-Gaussian $A_1$ in (1), this experiment can be shown to run successfully when $A_1$ is implemented (e.g., optically) as a random convolution [10].

V. CONCLUSION

We presented and solved a non-convex formulation of blind calibration for linear random sensing models affected by unknown gains. In absence of a priori structure on the signal and gains, our solution achieves a linear rate $mp \gtrsim (n + m) \log(n)$. Future developments of this approach include the extension to complex gains (i.e., $d \in \mathbb{C}^m$), as well as modifying the algorithm to enforce the sparsity of $x$ (or $d$) by which a reduction of the sample complexity below $n + m$ (i.e., for actual CS) will be obtained.
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