Effect of Strongly Coupled Plasma on the Spectra of Hydrogenlike Carbon, Aluminium and Argon

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April 11, 2007

Abstract

A detailed study has been performed for estimating the orbital energies, positions and shifts of the Lyman lines of C\textsuperscript{5+}, Al\textsuperscript{12+} and Ar\textsuperscript{17+} under strongly coupled plasma with a view to understand such line positions and shifts obtained in laser produced plasma experiments. The effect of strongly coupled plasma has been treated within the Ion Sphere (IS) model. Both non-relativistic and relativistic methods have been used for estimating the spectral properties. Theoretical estimates with IS model of the plasma are in conformity with the results of laser plasma experiments on these highly stripped ions. The experimental data for the systems have also been compared with the theoretical estimates using Debye screening model of the plasma with spatial confinements which gives additional restrictions to the wave functions at finite boundaries.
1 Introduction

The spectral properties of atomic systems are modified considerably under external con-
finements [1, 2, 3]. Of particular interest, is the effect of a surrounding plasma of different
coupling strengths $\Gamma$, defined as the ratio of average Coulomb potential energy between
pairs of particles and their kinetic energy. $\Gamma < 1$ for weakly coupled plasma, one can apply
the standard Debye screening model [4] in which the potential energy between charged
particles is represented by a screened Coulomb potential. The condition $\Gamma \geq 1$ refers
to strongly coupled plasma in which the potential energy function, though simple, is of
completely different nature than in a Debye screening model [5]. Such plasma conditions
prevail in, highly evolved stars, the interior of Jovian planets, explosive shock tubes, two
dimensional states of electrons trapped in surface states of liquid helium, laser produced
and inertial confinement fusion plasmas [5, 6]. Recent experimental observations using
laser produced plasmas [7, 8, 9, 10, 11, 12] open up an interesting field for the theoretical
investigations along this line. Such high density plasmas are of particular interest
in astrophysics and inertial confinement fusion processes. The X-ray opacity of matter
under stellar interior conditions and the X-ray diagnostics of ICF plasmas can be achieved
from such a study [11]. Effect of dense plasma on the ionization potential, collision and
photo absorption cross sections, fine structure splitting and spectral line shifts have been
investigated earlier by Stewart and Pyatt [13], Rozsnai [14], Ray [15], Jung [16], Griem
[17], Siedel et al. [18] and Skupski [19]. Applications of density functional approach along
this line was reviewed by Gupta and Rajagopal [20].

In the current context, we will focus our attention to the experimental findings based
on time and space resolved extreme ultraviolet spectra of Carbon plasmas with 100 $fs$
laser pulses [10], inertially confined laser imploded Ar plasma [11] and ultrashort laser
produced Al plasma [12]. For such laser produced plasmas $\Gamma > 1$ and one can apply
strongly coupled plasma model to investigate the spectral properties of isoelectronic ions
of Hydrogen. In this communication we would like to investigate in detail the effect
of strongly coupled plasma on the Lyman lines of highly stripped Carbon, Aluminium
and Argon. Ion Sphere (IS) model of the plasma [5] has been utilized for such a study.
Our motivation is to investigate how the simple IS model is effective in obtaining results
which can be compared favourably with the experimentally observed values. In addition
we would also like to investigate the applicability of the Debye plasma model with a
spherical confinement on the spectral line positions and shifts of the Lyman lines under the
laser plasma experimental conditions [10, 11, 12] and to estimate the shifts in ionization
potentials. Such studies have been done earlier for Hydrogen [21, 22] and Helium like
systems [23, 24] to understand the behavior of the structural properties of one and two
electron systems under weak as well as strongly coupled plasmas. A brief outline of the
theory is given in Section 2 and a discussion of the results follow in Section 3.

2 Theory

In presence of an external plasma environment the potential energy is modified and the
non-relativistic Hamiltonian of a Hydrogen like atomic system [a.u. is used throughout]
can be represented by

\[ H_0 = -\frac{1}{2} \nabla^2 + V_{\text{eff}}(r) \quad (1) \]

where the structure of the one body effective potential depends on the type of the coupling of the plasma with the atomic charge cloud. For the relativistic treatment appropriate modification of the Hamiltonian is done through the introduction of the Dirac operators. Currently we are interested in the case of strongly coupled plasma for which \( \Gamma \geq 1 \). In case of such a homogeneous one component plasma surrounding an ion of nuclear charge \( Z \) having one valence electron, one can define a sphere of radius \( R \) (usually referred to as the Wigner-Seitz radius) such that the plasma electrons with density \( n \) together with the valance electron completely neutralize the central positive charge; thus maintaining the overall charge neutrality of the system \([5, 13, 20]\). In such a situation the Wigner-Seitz radius \( R \) is given by

\[ R = \left[ \frac{(Z - 1)}{4\pi n} \right]^{\frac{1}{3}} \quad (2) \]

From classical electrostatics one can easily obtain

\[ V_{\text{eff}}(r) = -\frac{Z}{r} + \frac{(Z - 1)}{2R} \left[ 3 - \left( \frac{r}{R} \right)^2 \right] \quad (3) \]

In order to analyze the energy of the system for different coupling strengths of the plasma reflected in \( R \), one has to solve the appropriate Schrödinger equation

\[ H_0 \psi = E_0 \psi \quad (4) \]

subject to the normalization constant

\[ \langle \psi | \psi \rangle = 1 \quad (5) \]

For the relativistic case the corresponding Dirac equation is to be solved. It is assumed that no electron current takes place at the boundary surface defined by the Wigner-Seitz radius \( R \) and the wave function should satisfy the boundary condition

\[ \psi(r) = 0 \quad \text{at} \quad r = R \quad (6) \]

Such boundary conditions can always be satisfied by choosing the basis sets appropriately. We represent the radial part of the orbital
\[ \psi(r) = (R - r)\chi(r) \]  
(7)

where \( \chi(r) \) is a linear combination of Slater type orbitals (STO)

\[ \chi(r) = \sum_i C_i r^{n_i} e^{-\rho_i r} \]  
(8)

Since the analytical solution of Hydrogen like problem in a plasma is difficult we adopt the basis set expansion technique for obtaining the energy of the ground state in a plasma environment. The non linear parameters \( n_i \) and \( \rho_i \) here are preassigned and the linear coefficients are determined from the solution of the generalized eigenvalue equation

\[ H_0 C = E_0 S C \]  
(9)

which yields the ground state energy at different plasma coupling strengths which are functions of the plasma parameters. All the integrals are to be evaluated at finite domain radius \( R \). For the relativistic case a numerical evaluation of the energies is sought using Dirac Hamiltonian and standard relativistic program package as developed by Fritzsche et al. [25].

In addition to evaluation of the ground state energies at different plasma coupling strengths we have adopted the applications of linear response theory under an external time dependent perturbation [21, 22, 23, 24] for estimating the low lying excitation energies with a view to calculate the spectral line positions under plasma environment.

To be more specific we apply a harmonic perturbation on the system

\[ H'(r, t) = g(r)e^{-i\omega t} + g^\dagger(r)e^{i\omega t} \]  
(10)

where \( g(r) \) is an one particle perturbation, currently of dipolar form. The external perturbation changes the ground state wave function \( \psi \) and the perturbed wave functions can be evaluated through the optimization of a variational functional [26]

\[ J(\phi) = \frac{1}{T} \int_0^T dt \frac{\langle \phi | H_0 + H' - i \frac{\partial}{\partial t} | \phi \rangle}{\langle \phi | \phi \rangle} \]  
(11)

with

\[ \delta J(\phi) = 0 \]  
(12)

The optimization is carried out with respect to linear variation parameters introduced in function \( \phi \). The basis sets for the perturbed functions are similar to that given by Equations (7) and (8) with different linear and non linear parameters. The functional has poles at certain frequency \( \omega \), the positions of which indicate the singly excited states of the system. One can extract the transition properties from a study of the pole positions [21]. A discussion of the results is given in the next section.
3 Results and Discussions

The effect of strongly coupled plasma on the orbital energy and low lying excited states \( C^{5+}, Al^{12+} \) and \( Ar^{17+} \) has been analyzed in details using IS model within non relativistic as well as relativistic theory. The particular ions have been chosen as laser produced plasma experiments in such systems exist [10, 11, 12] and spectral lines of Lyman lines originating in plasma environments have been reported. Our aim is to see the reliability of the IS model of the plasma in predicting the experimentally observed lines of the Lyman series. The shifts can always be estimated from the free line positions. The orbital energies for different plasma coupling strengths have been obtained from the solution of the generalized eigenvalue Equation (10) with respect to a limited basis set composed of linear combination of STO’s. For \( C^{5+} \) ion we have chosen only a two parameter representation for the ground orbital and its reliability has been tested by comparing the eigen energy for the free systems. For \( Al^{12+} \) and \( Ar^{17+} \) we have chosen four parameter representation for the same. To study the excitation energies and transition wavelengths under plasma we used a twelve parameter representation of the first order perturbed orbitals for \( C^{5+} \) while an 8 parameter representation was adopted for \( Al^{12+} \) and \( Ar^{17+} \). For the case of \( Al^{12+} \) and \( Ar^{17+} \) the results for our detailed investigations using IS model with different electron densities have been displayed in Tables 1 and 3. We have considered the behavior of the ground state orbital energy and the transition energy to first three dipole allowed excited states \( 2p, 3p \) and \( 4p \). The energy shifts have been calculated for \( Al^{12+} \) while for \( Ar^{17+} \), the wavelengths for the free as well as those in presence of plasma have been reported. This is because the data on the laser produced experiments on plasma for \( Al^{12+} \) [10] and \( Ar^{17+} \) [12] have been given accordingly. We wish to have an overall idea also about how the energy levels behave in case of Debye type plasma with spherical confinement. Here the effective potential is given by [4]

\[
V_{\text{eff}}(r) = -\frac{Ze^{-\mu r}}{r}
\]  

(13)

where \( Z \) is the nuclear charge and \( \mu \) is the Debye screening parameter given by

\[
\mu = \left[ \frac{4\pi(1+Z)n}{\kappa T} \right]^{\frac{1}{2}}
\]

(14)

\( \mu \) is a function of the temperature \( T \) and number density \( n \) of the plasma electrons. One can simulate a large number of plasma conditions by properly choosing \( n \) and \( T \). Using the potential function given by Equation (13) with a given parameter \( \mu \), one can proceed in the same way as is being done in the strongly coupled plasma model to study the behavior of orbital energies and excitation properties. In such calculations we have chosen the plasma temperature \( T \) as reported in the experimental papers [11, 12] and varies the electron density \( n \) to get the screening parameters \( \mu \). For each \( \mu \) value we have chosen the radius of confinement as \( R = \frac{1}{\mu} \) which effectively gives the Debye sphere of influence. The spatial confinement with respect to the Debye radius is incorporated in
the numerical calculations in exactly the same way as is being done for the Ion Sphere (IS) model. Such results have been displayed in Tables 2 and 4 for the respective cases of $Al^{12+}$ and $Ar^{17+}$. The number of parameters for the ground and excited state functions are identical in the Debye plasma and in the IS models. In Tables 1 to 4 the transition energies from the $1s \rightarrow 2p$, $3p$ and $4p$ states have been reported for the cases only in which the excited state is bound. As soon as the transition energy exceeds that of the ionization energy for increased plasma strength, it goes in the continuum and such cases have not been displayed in the Tables. Experimental shift for the Lyman $\alpha$ ($Ly_\alpha$) line for $Al^{12+}$ with estimated electron density $n \sim (5 - 10) \times 10^{23}/cc$ and temperature $T \sim 300$ eV is given by $3.7 \pm 0.7$ eV [12]. Our calculation using IS model at $n = 2.5 \times 10^{24}/cc$ yields a value 3.41 eV whereas a quantum mechanical calculations of Nguyen et al. [27] based on collision theory yields a value 3.5 eV at $n = 8 \times 10^{23}/cc$ and $T \sim 300$ eV. Figure 1 shows the general trend of the transition energy $1s \rightarrow 2p$ for $Al^{12+}$ against the Ion Sphere radius $R$ with non relativistic and relativistic models. For the relativistic case weighted average of the $p_2$ and $p_1$ state energies have been reported all throughout. It appears that the relativistic results differ only at higher plasma electron densities. In Figure 2 we plotted the non relativistic and relativistic transition wavelengths $1s \rightarrow 2p$, $3p$ and $4p$ against IS plasma density for $Ar^{17+}$. The relativistic effects are little more pronounced here as the nuclear charge $Z$ is larger. Figure 3 displays a comparison of our calculated results for the transition wavelengths for $Ar^{17+}$ using non relativistic as well as relativistic methods within Ion Sphere (IS) model and spatially confined Debye screening model with the laser plasma experimental data. The experimental data are in reasonable agreement with the calculated theoretical results. The laser plasma experiment by Nantel et al. [10] yields data on Hydrogen and Helium like spectra of $C$ under strong plasma with estimated density of $n = 1.5 \times 10^{21}/cc$ and temperature 48 eV. We have performed non relativistic and relativistic estimates of the positions of Lyman lines of $C^{5+}$ using the Ion Sphere (IS) model at experimental density and spatially confined Debye plasma model at the same density and temperature. In Figure 4 we displayed our results along with those obtained by Nantel et al. [10]. We observed very reasonable fitting with the experimental lines positions for the Lyman transitions $1s \rightarrow 2p$, $3p$, $4p$, $5p$ and $6p$. It appears that with IS model non relativistic and relativistic estimates agree very well while there are little variations with confined Debye plasma model.

4 Conclusion

From the analysis of the calculated data by using IS and Debye models one can conclude that IS model, though simple, yields very reasonable theoretical estimates of spectral line positions and shifts of the spectral lines obtained from laser produced plasmas. It can be a viable method for the understanding of the experimental observations on strongly coupled plasmas obtained in laboratory and astrophysics.
5 Acknowledgement

The authors are thankful to AvH foundation for financial assistance towards mutual visits of Indian and German scientists. The financial assistance from the Department of Science and Technology (DST), Govt. of India under research grant no. SR/S2/LOP-05/2005 is gratefully acknowledged.

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Figure 1: Plot of the relativistic and non relativistic transition energy ($1s \rightarrow 2p$) (a.u.) obtained by using IS model against plasma electron density (/cc) for Al$^{12+}$. 
Figure 2: Plot of the relativistic (dotted line with symbols) as well as non relativistic transition (solid line with symbols) wavelength ($1s \rightarrow 2p, 3p, 4p$) (Å) obtained by using IS model against plasma electron density (/cc) for $Ar^{17+}$. 
Table 1: Relativistic & non-relativistic transition energy of Al$^{12+}$ for different Ion-Sphere (IS) radius.

| Ion-sphere Radius (a.u.) | Plasma Density $n_e$/c.c. | Orb Ener (a.u.) | Transition | Transition energy (a.u.) | Energy shift (eV) |
|--------------------------|---------------------------|----------------|------------|-------------------------|------------------|
|                          |                           | Rel | Non-Rel | Scheme | Rel | Non-Rel | Rel | Non-Rel | Rel | Non-Rel |
| $\infty$                |                           | 84.69 | 84.50   | 1s→2p  | 63.53747 | 63.37500 |      |          |      |          |
|                          |                           |      |         |        | 63.53715 | 63.37401 | 0.0087 | 0.0269   |      |          |
|                          |                           |      |         |        | 75.29858 | 75.11111 | 0.0280 | 0.1709   |      |          |
|                          |                           |      |         |        | 79.40327 | 79.21875 | 0.5649 | 0.5927   |      |          |
| 9.9                      | 1.99(+22)                 | 82.8729 | 82.6819 | 1s→2p  | 63.53747 | 63.37500 |      |          |      |          |
|                          |                           |      |         |        | 63.37401 | 63.37401 | 0.0087 | 0.0269   |      |          |
|                          |                           |      |         |        | 75.10482 | 75.10482 | 0.0280 | 0.1709   |      |          |
|                          |                           |      |         |        | 79.19697 | 79.19697 | 0.5649 | 0.5927   |      |          |
| 5.782                    | 1.0(+23)                  | 81.5785 | 81.3876 | 1s→2p  | 63.53199 | 63.37003 | 0.1165 | 0.1352   |      |          |
|                          |                           |      |         |        | 75.09233 | 75.09233 | 0.7489 | 0.5108   |      |          |
|                          |                           |      |         |        | 79.10701 | 79.10701 | 2.9418 | 3.0406   |      |          |
| 3.38146                  | 5.0(+23)                  | 79.3706 | 79.1796 | 1s→2p  | 63.51337 | 63.35014 | 0.6558 | 0.6765   |      |          |
|                          |                           |      |         |        | 74.98010 | 74.98010 | 4.3974 | 3.5647   |      |          |
|                          |                           |      |         |        | 78.70955 | 78.70955 | 14.7121 | 13.8540  |      |          |
| 3.18207                  | 6.0(+23)                  | 79.0376 | 78.8467 | 1s→2p  | 63.50841 | 63.34516 | 0.7908 | 0.8120   |      |          |
|                          |                           |      |         |        | 74.94203 | 74.94203 | 5.3253 | 4.6006   |      |          |
|                          |                           |      |         |        | 78.66120 | 78.66120 | 17.5805 | 15.1717  |      |          |
| 3.0227                   | 7.0(+23)                  | 78.7399 | 78.5489 | 1s→2p  | 63.50344 | 63.34018 | 0.9260 | 0.9475   |      |          |
|                          |                           |      |         |        | 74.90371 | 74.90371 | 6.2605 | 5.6434   |      |          |
|                          |                           |      |         |        | 78.85861 | 78.85861 | 8.1539 | 7.5351   |      |          |
| 2.89111                  | 8.0(+23)                  | 78.4694 | 78.2784 | 1s→2p  | 63.49847 | 63.33519 | 1.0612 | 1.0833   |      |          |
|                          |                           |      |         |        | 74.86516 | 74.86516 | 7.2026 | 6.6924   |      |          |
| 2.7798                   | 9.0(+23)                  | 78.2206 | 78.0297 | 1s→2p  | 63.49349 | 63.33019 | 1.1968 | 1.2193   |      |          |
|                          |                           |      |         |        | 74.89993 | 74.89993 | 8.1539 | 7.5351   |      |          |
| 2.68386                  | 1.0(+24)                  | 77.9897 | 77.7987 | 1s→2p  | 63.48851 | 63.32519 | 1.3323 | 1.3554   |      |          |
|                          |                           |      |         |        | 74.85436 | 74.78648 | 9.1145 | 8.8334   |      |          |
| 2.13018                  | 2.0(+24)                  | 76.2519 | 76.0610 | 1s→2p  | 63.43847 | 63.27495 | 2.6939 | 2.7225   |      |          |
|                          |                           |      |         |        | 74.58295 | 74.31376 | 19.2284 | 21.6967  |      |          |
| 1.97749                  | 2.5(+24)                  | 75.6022 | 75.4113 | 1s→2p  | 63.41328 | 63.24965 | 3.3794 | 3.4109   |      |          |
|                          |                           |      |         |        | 74.38536 | 74.03433 | 24.6051 | 29.3004  |      |          |
| 1.86089                  | 3.0(+24)                  | 75.0346 | 74.8437 | 1s→2p  | 63.38798 | 63.22420 | 4.0678 | 4.1035   |      |          |
|                          |                           |      |         |        | 74.18693 | 73.75165 | 30.0046 | 36.9925  |      |          |
| 1.76768                  | 3.5(+24)                  | 74.5273 | 74.3364 | 1s→2p  | 63.36256 | 63.19856 | 4.7595 | 4.8012   |      |          |
|                          |                           |      |         |        | 73.98516 | 73.47956 | 35.4951 | 44.3965  |      |          |
| 1.4770                   | 6.0(24)                   | 72.5370 | 72.3461 | 1s→2p  | 63.23364 | 63.06557 | 8.2676 | 8.4800   |      |          |
| 1.3419                   | 8.0(24)                   | 71.3210 | 71.1306 | 1s→2p  | 63.12816 | 62.94262 | 11.1379 | 11.5850  |      |          |
| 1.2457                   | 1.0(25)                   | 70.2961 | 70.1059 | 1s→2p  | 63.02800 | 62.82014 | 14.0593 | 15.0985  |      |          |
Table 2: Relativistic & non-relativistic transition energy of Al^{12+} for different Debye Screening parameter and box radius.

| Ion        | Plasma Density (/c.c.) | Temp. (eV) | Debye Para (a.u.) | Debye Sh Rad (a.u.) | Orbital Energy -E(a.u.) | Transition Scheme | Transition Energy (a.u.) | Energy Shift (eV) |
|------------|------------------------|------------|-------------------|---------------------|-------------------------|---------------------|--------------------------|-------------------|
|            | 1s→2p                  |            |                   |                     |                         |                     |                          |                   |
| Al^{12+}   | 1.0(22)                | 300        | 0.154             | 6.50328             | 82.7066                 | Rel                 | 63.49789                 | 1.0770            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.33468                 | 1.0972            |
|            | 1.5(22)                | 300        | 0.188             | 5.30991             | 82.2731                 | Rel                 | 63.47852                 | 1.0641            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.31529                 | 1.0248            |
|            | 2.0(22)                | 300        | 0.217             | 4.59852             | 81.9048                 | Rel                 | 63.45912                 | 2.1320            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.29586                 | 2.1535            |
|            | 2.5(22)                | 300        | 0.243             | 4.11304             | 81.5756                 | Rel                 | 63.43951                 | 2.6656            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.27621                 | 2.6882            |
|            | 3.0(22)                | 300        | 0.266             | 3.75467             | 81.2852                 | Rel                 | 63.42043                 | 3.1848            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.25709                 | 3.2085            |
|            | 3.5(22)                | 300        | 0.288             | 3.47615             | 81.0081                 | Rel                 | 63.40068                 | 3.7222            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.23729                 | 3.7473            |
|            | 4.0(22)                | 300        | 0.308             | 3.25164             | 80.7568                 | Rel                 | 63.38146                 | 4.2453            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.21803                 | 4.2714            |
|            | 4.5(22)                | 300        | 0.326             | 3.06568             | 80.5311                 | Rel                 | 63.36315                 | 4.7435            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.19967                 | 4.7710            |
|            | 5.0(22)                | 300        | 0.344             | 2.90836             | 80.3059                 | Rel                 | 63.34388                 | 5.2679            |
|            |                        |            |                   |                     |                         | Non-Rel             | 63.18036                 | 5.2964            |
Table 3: Relativistic & non-relativistic transition energy of $\text{Ar}^{17+}$ for different Ion-Sphere (IS) radius.

| Ion   | Plasma Density (/c.c.) | IS Radius (a.u.) | Orbital Energy -E(a.u.) | Transition Scheme | Transition Energy (a.u.) | Transition Wave length (Å) |
|-------|------------------------|------------------|-------------------------|------------------|-------------------------|--------------------------|
|       |                        |                  | Rel | Non-Rel | Rel | Non-Rel | Rel | Non-Rel |
| $\text{Ar}^{17+}$ 9.54(20) | 30.0 | 161.8549 | 161.1500 | 1s→2p | 122.10220 | 121.49997 | 3.7305 | 3.7490 |
|       |                       |                  |      |         | 3p  | 144.66130 | 143.99982 | 3.1488 | 3.1633 |
|       |                       |                  |      |         | 4p  | 152.56180 | 151.87442 | 2.9857 | 2.9992 |
| 2.58(22) 10.0 | 160.1549 | 159.4500 | 1s→2p | 122.10153 | 121.49929 | 3.7306 | 3.7491 |
|       |                       |                  |      |         | 3p  | 144.65685 | 143.99535 | 3.1489 | 3.1633 |
|       |                       |                  |      |         | 4p  | 152.54681 | 151.86477 | 2.9860 | 2.9994 |
| 1.0(23) 6.4941 | 158.7785 | 158.0736 | 1s→2p | 122.09966 | 121.49741 | 3.7306 | 3.7491 |
|       |                       |                  |      |         | 3p  | 144.64459 | 143.98306 | 3.1492 | 3.1636 |
|       |                       |                  |      |         | 4p  | 152.50553 | 151.81798 | 2.9868 | 3.0004 |
| 2.0(23) 5.1543 | 157.7581 | 157.0533 | 1s→2p | 122.09709 | 121.49482 | 3.7307 | 3.7492 |
|       |                       |                  |      |         | 3p  | 144.62769 | 143.96644 | 3.1495 | 3.1640 |
|       |                       |                  |      |         | 4p  | 152.44872 | 151.75973 | 2.9879 | 3.0015 |
| 3.0(23) 4.5027 | 157.0425 | 156.3376 | 1s→2p | 122.09452 | 121.49223 | 3.7308 | 3.7493 |
|       |                       |                  |      |         | 3p  | 144.61075 | 143.95059 | 3.1499 | 3.1643 |
|       |                       |                  |      |         | 4p  | 152.39194 | 151.70093 | 2.9891 | 3.0027 |
| 4.0(23) 4.0190 | 156.3612 | 155.7680 | 1s→2p | 122.09138 | 121.48964 | 3.7309 | 3.7494 |
|       |                       |                  |      |         | 3p  | 144.59007 | 143.93565 | 3.1503 | 3.1647 |
|       |                       |                  |      |         | 4p  | 152.32277 | 151.62842 | 2.9904 | 3.0041 |
| 5.0(23) 3.7978 | 155.9918 | 155.2869 | 1s→2p | 122.08937 | 121.48705 | 3.7309 | 3.7494 |
|       |                       |                  |      |         | 3p  | 144.57679 | 143.92126 | 3.1506 | 3.1650 |
|       |                       |                  |      |         | 4p  | 152.27847 | 151.58148 | 2.9913 | 3.0050 |
| 6.0(23) 3.5738 | 155.5713 | 154.8665 | 1s→2p | 122.08680 | 121.48447 | 3.7310 | 3.7495 |
|       |                       |                  |      |         | 3p  | 144.55975 | 143.90649 | 3.1510 | 3.1653 |
|       |                       |                  |      |         | 4p  | 152.22179 | 151.52097 | 2.9924 | 3.0062 |
| 7.0(23) 3.3948 | 155.1954 | 154.4906 | 1s→2p | 122.08422 | 121.48187 | 3.7311 | 3.7496 |
|       |                       |                  |      |         | 3p  | 144.54269 | 143.89068 | 3.1514 | 3.1657 |
|       |                       |                  |      |         | 4p  | 152.16513 | 151.46022 | 2.9935 | 3.0074 |
| Ion Density /c.c. | IS Radius (a.u.) | Orbital Energy -E(a.u.) | Transition Scheme | Transition Energy (a.u.) | Transition Wave length (Å) |
|------------------|------------------|------------------------|------------------|-------------------------|--------------------------|
|                  |                  |                        | Rel              | Non-Rel                 | Rel                      | Non-Rel                  |
| 8.0(23)          | 3.2470           | 154.8538               | 154.1490         | 1s→2p                   | 122.08165                | 121.47928                | 3.7312                   | 3.7497                   |
|                  |                  |                        |                  | 3p                      | 144.52559                | 143.87384                | 3.1517                   | 3.1660                   |
|                  |                  |                        |                  | 4p                      | 152.10852                | 151.39962                | 2.9946                   | 3.0086                   |
| 9.0(23)          | 3.1220           | 154.5396               | 153.8348         | 1s→2p                   | 122.07908                | 121.47669                | 3.7313                   | 3.7498                   |
|                  |                  |                        |                  | 3p                      | 144.50846                | 143.85638                | 3.1521                   | 3.1664                   |
|                  |                  |                        |                  | 4p                      | 152.05193                | 151.33965                | 2.9957                   | 3.0098                   |
| 1.0(24)          | 3.0143           | 154.2480               | 153.5431         | 1s→2p                   | 122.07650                | 121.47410                | 3.7313                   | 3.7498                   |
|                  |                  |                        |                  | 3p                      | 144.49129                | 143.83861                | 3.1525                   | 3.1668                   |
|                  |                  |                        |                  | 4p                      | 151.99539                | 151.28082                | 2.9969                   | 3.0110                   |
| 1.5(24)          | 2.6332           | 153.0251               | 152.3203         | 1s→2p                   | 122.06361                | 121.46113                | 3.7317                   | 3.7502                   |
|                  |                  |                        |                  | 3p                      | 144.40493                | 143.74921                | 3.1544                   | 3.1688                   |
|                  |                  |                        |                  | 4p                      | 151.71395                | 151.01987                | 3.0024                   | 3.0162                   |
| 1.8(24)          | 2.4780           | 152.4192               | 151.7144         | 1s→2p                   | 122.05588                | 121.45334                | 3.7320                   | 3.7505                   |
|                  |                  |                        |                  | 3p                      | 144.35266                | 143.69573                | 3.1555                   | 3.1699                   |
|                  |                  |                        |                  | 4p                      | 151.54573                | 150.90012                | 3.0057                   | 3.0186                   |
| 2.0(24)          | 2.3924           | 152.0519               | 151.3471         | 1s→2p                   | 122.05071                | 121.44814                | 3.7321                   | 3.7506                   |
|                  |                  |                        |                  | 3p                      | 144.31763                | 143.66004                | 3.1563                   | 3.1707                   |
|                  |                  |                        |                  | 4p                      | 151.43565                | 150.83746                | 3.0079                   | 3.0199                   |
| 2.5(24)          | 2.2209           | 151.2303               | 150.5256         | 1s→2p                   | 122.03780                | 121.43514                | 3.7325                   | 3.7510                   |
|                  |                  |                        |                  | 3p                      | 144.22930                | 143.57026                | 3.1582                   | 3.1727                   |
|                  |                  |                        |                  | 4p                      | 151.15811                | 150.73922                | 3.0135                   | 3.0218                   |
Table 4: Relativistic & non-relativistic transition energy of Ar$^{17+}$ for different Debye Screening parameter and box radius.

| Ion | Plasma Density (/c.c.) | Temp (eV) | Debye para (a.u.) | Debye Radius (a.u.) | Orbital Energy -E (a.u.) | Tran Sch | Transition Energy (a.u.) | Transition Wave length (Å) |
|-----|------------------------|-----------|-------------------|--------------------|--------------------------|----------|--------------------------|--------------------------|
| Ar$^{17+}$ | 1.0(23) | 1000 | 0.3103 | 3.2230 | 157.1904 | 156.4865 | 1s→2p | 121.94107 | 121.33840 | 3.7355 | 3.7540 |
|       |           |         | 3p | 144.17601 | 143.52411 | 3.1594 | 3.1594 | 3.1737 |
|       |           |         | 4p | 151.67941 | 145.0367 | 3.0031 | 3.0031 | 3.0031 |
| Ar$^{17+}$ | 5.0(23) | 1000 | 0.6938 | 1.4414 | 150.5664 | 149.8640 | 1s→2p | 121.33281 | 120.72769 | 3.7542 | 3.7730 |
|       |           |         | 3p | 142.49773 | 141.51795 | 3.1966 | 3.1966 | 3.2187 |
|       |           |         | 4p | 150.25318 | 145.0367 | 3.0316 | 3.0316 | 3.0316 |
| Ar$^{17+}$ | 1.0(24) | 1000 | 0.9812 | 1.0192 | 145.7360 | 145.0367 | 1s→2p | 120.61125 | 119.98983 | 3.7767 | 3.7962 |
|       |           |         | 3p | 141.17721 | 138.52859 | 3.2265 | 3.2265 | 3.2882 |
| Ar$^{17+}$ | 5.0(24) | 1000 | 2.1939 | 0.4558 | 126.5337 | 125.8566 | 1s→2p | 116.71007 | 111.28980 | 3.9029 | 4.0930 |
Figure 3: Comparison between the experimental results and that obtained theoretically by using Ion Sphere as well as Debye plasma model for $1s \rightarrow 2p, 3p, 4p$ transition wavelength (Å) of $Ar^{17+}$. The experimental figure has been taken from Ref. 11.
Figure 4: Comparison between the experimental results ($C^{4+}$ and $C^{5+}$) and that obtained theoretically by using Ion Sphere as well as Debye plasma model for $1s \rightarrow 2p, 3p, 4p, 5p, 6p$ transition wavelength (Å) of Hydrogen like Carbon. The experimental figure has been taken from Ref. 10.