A note on 4d $\mathcal{N} = 3$ from little string theory

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**ABSTRACT:** We study little string theory (LST) compactified on $T^2$, partially breaking supersymmetry by a discrete T-duality twist acting on both the Kähler and the complex structure of the torus. This setup gives raise to 4d $\mathcal{N} = 3$ models and it can be performed in both the type IIA and type IIB LSTs. We comment on the relation with other constructions proposed in the literature.
1 Introduction

The study of supersymmetric non-Lagrangian theories has been an intense field of research in the last decade, after the breakthrough of [1]. It has been shown that they are ubiquitous, and they have played a relevant role in the analysis of non-perturbative phenomena, in their connection with localization, integrability and compactification. An important consequence of the existence of non-Lagrangian theories is that they can evade some QFT exact results that follow from the Lagrangian description.

For example it has been recently observed that theories with twelve supercharges can exist in 4d, but that they must necessarily be non-Lagrangian. Indeed such models have not been investigated in the past because they can only correspond to strongly coupled isolated fixed points and they cannot have a perturbative regime. In other words perturbative definitions of 4d models with \( \mathcal{N} = 3 \) supersymmetry necessarily enhance to \( \mathcal{N} = 4 \) [2]. Nevertheless purely \( \mathcal{N} = 3 \) theories have been obtained in [3–5] (see also [6–8] for alternative constructions in terms of the discrete gauging of a subgroup of the global symmetry of \( \mathcal{N} = 4 \) SYM.). These construction required a higher dimensional stringy background and a twist in terms of a discrete symmetry related to the S-duality group. In the stringy inspired constructions the S-duality group is associated to other dualities and symmetries of the gravitational description. Further checks and generalizations of these proposals have been made in [9–17].

In the constructions of [3] the authors considered an F-theory setup, reduced it to 4d \( \mathcal{N} = 4 \) SYM and at the same time they applied an opportune twist to mod out some of the supercharges. This twist involved a discrete subgroup of the R-symmetry
group combined with a twist by the S-duality symmetry of the 4d theory, that had a geometric origin in the higher dimensional stringy description. The two effects did not completely cancel out, giving rise to the possibility of constructing theories with a lower amount of supercharges, precisely twelve.

The S-duality twist boils down to introduce a non-perturbative object in type IIB string theory, denoted as S-fold in [18], that generalizes the orientifold projection. In a subsequent paper [5] the authors provided an M-theory construction and related this to an opportune reduction of 6d \( \mathcal{N} = (2,0) \). They considered M-theory on \( \mathbb{R}^{1,5} \times \mathbb{C} \times T^3 \), by wrapping the M5 branes on a \( T^2 \) inside \( T^3 \) [5]. By carefully analyzing the T-duality structure of the theory one can reconstruct the twist leading to twelve supercharges in 4d.

In this short note we propose an alternative field theoretical mechanism, starting from a 6d non-local field theory, in which gravity is completely decoupled, but T-duality remains as a symmetry. These theories are known as little string theories (LST) [19–21] (for review see [22, 23]) and they correspond to 6d theories with maximal supersymmetry, either \( \mathcal{N} = (1,1) \) or \( \mathcal{N} = (2,0) \).

Once these 6d theories are compactified on a 2-torus, the \( O(2,2;\mathbb{Z}) \) T-duality group contains two \( SL(2,\mathbb{Z}) \) factors. One of them acts on the complex structure and the other on the Kähler structure of the 2-torus\(^1\). Depending on the supersymmetry we started with, \( \mathcal{N} = (1,1) \) or \( \mathcal{N} = (2,0) \), one of these two \( SL(2,\mathbb{Z}) \) symmetries becomes the S-duality group of the 4d theory. As we will see, the other \( SL(2,\mathbb{Z}) \) factor contains a discrete group that has to be considered in the discrete twist (the so called R-twist) in order to preserve some supercharges. This last discrete group is necessary because the little string theory has only an \( SO(4)_R \) symmetry group, while the full \( SO(6)_R \) is manifest only in the limit of vanishing torus. The full discrete symmetry subgroup of the 4d theory is here reconstructed in terms of discrete subgroups of \( SO(4) \) and of \( SL(2,\mathbb{Z}) \). Using this construction we obtain \( \mathcal{N} = 3 \) 4d theories starting from 6d theories with \( A_n \) type simply laced gauge groups.

The note is organized as follows. In section 2 we review the basic aspects of LST on \( T^2 \), focusing on the geometric origin of the S-duality arising from T-duality of the 6d setup. In section 3 we study the discrete twist that gives origin to the 4d models with 12 supercharges, We discuss both the type IIA and the type IIB constructions. In section 4 we compare our construction with the one of [5]. In section 5 we conclude discussing further generalizations of our construction.

\(^1\)The \( SL(2,\mathbb{Z}) \) transformation on the Kähler structure is a TsT transformation in string theory. Indeed one can first T-dualize on a circle, then perform an \( SL(2,\mathbb{Z}) \) transformation on the complex structure and then perform another T-duality. Such TsT corresponds to an \( SL(2,\mathbb{Z}) \) on the Kähler structure [24].
2 LST on $T^2$

Little string theories are non-local field theories in 5D and 6D [19]. They can be constructed considering $k$ parallel and overlapping NS 5-branes and then sending the string coupling $g_s \to 0$. Another way to obtain these theories consists of considering $k$ M5-branes with a transverse circle of radius $R$ in the limit $R \to 0$, $M_p \to \infty$ and $RM_p^3 = M_s^2$ constant. After compactification on a $T^d$, LSTs exhibit a $O(d,d;\mathbb{Z})$. The type IIA and IIB LSTs are T-dual upon compactification on a circle.

The compactification of LSTs on a $T^2$, with radii $r_1$ and $r_2$, gives raise to $\mathcal{N} = 4$.

Here we turn on also a B-field with flux given by

$$B = \frac{\theta \alpha'}{r_1 r_2}. \quad (2.1)$$

As a consequence of the presence of the B-field one finds that the Kähler structure parameter of the torus is

$$\rho = \frac{i r_1 r_2}{\alpha'} + \theta. \quad (2.2)$$

Following [25] and [26] we observe that that T-duality on the torus corresponds to the S-duality of $\mathcal{N} = 4$ SYM acting on the holomorphic gauge coupling $\tau_{gauge} = \frac{\theta}{2\pi} + \frac{4\pi i}{g_{SYM}^2}$ under the group $SL(2,\mathbb{Z})$. This last acts as

$$\tau_{gauge} \mapsto \tau_{gauge}' = \frac{a \tau_{gauge} + b}{c \tau_{gauge} + d} \quad (2.3)$$

with the $SL(2,\mathbb{Z})$ matrix of the form $(a \ b \ c \ d)$ and $a, b, c, d$ integers such that $ad - bc = 1$.

To avoid any confusion, we referred here to the holomorphic gauge coupling of $\mathcal{N} = 4$ SYM as $\tau_{gauge}$ and we distinguished it from the complex structure to the torus, identified by $\tau$. Furthermore, T-duality on $S^1 \subset T^2$ can be used to exchange $\tau$ and $\rho$. In this way we will be able to either associate the Kähler (in type IIB) or the complex structure (in type IIA) to the holomorphic gauge coupling.

Here we study the type IIB case, where the identification is $\tau_{gauge} = \rho$ and we perform a T-duality on the torus considered above. This can be done after fixing the metric and the B-field. The metric on the torus can be taken to be

$$g_{\mu\nu} = \begin{pmatrix} \frac{r_1^2}{\alpha'} & 0 \\ 0 & \frac{r_2^2}{\alpha'} \end{pmatrix} \quad (2.4)$$

where we have rescaled the radii of the torus as $r_{1,2} \mapsto r_{1,2}/\sqrt{\alpha'}$. Furthermore, observing that the Kähler structure parameter has the form $\rho = \rho_1 + i \rho_2 = B_{21} + i \sqrt{\det g}$ and that the field $B_{\mu\nu}$ is antisymmetric, we have

$$B_{\mu\nu} = \theta \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (2.5)$$

$\theta$ and $g_{SYM}$ are, respectively, a theta angle and the coupling constant of the four dimensional SYM theory.
At this point one can use the Buscher rules [27]. First we apply a T-duality along $r_1$ obtaining

$$\hat{g}_{\mu\nu} = \left( \begin{array}{cc} \alpha' r_1^2 & \theta \alpha' r_1^2 \\ \frac{\theta \alpha'}{r_1^2} & \frac{\theta \alpha'}{r_1^2} + \theta^2 \alpha'^2 \end{array} \right), \quad \hat{B}_{\mu\nu} = 0. \quad (2.6)$$

Next we perform the second T-duality in the $r_2$ direction. We find

$$\tilde{g}_{\mu\nu} = \left( \begin{array}{cc} \frac{\alpha' r_2^2}{r_1^2 r_2^2 + \theta^2 \alpha'^2} & 0 \\ 0 & \frac{\alpha' r_2^2}{r_1^2 r_2^2 + \theta^2 \alpha'^2} \end{array} \right), \quad \tilde{B}_{\mu\nu} = \left( \begin{array}{cc} 0 & \frac{\theta \alpha'}{r_1^2 r_2^2 + \theta^2 \alpha'^2} \\ \frac{\theta \alpha'}{r_1^2 r_2^2 + \theta^2 \alpha'^2} & 0 \end{array} \right). \quad (2.7)$$

Thus T-duality on gives the transformations

$$\frac{r_{1,2}^2}{\alpha'} \mapsto \frac{\alpha' r_{2,1}^2}{r_1^2 r_2^2 + \theta^2 \alpha'^2}, \quad \theta \mapsto -\frac{\theta \alpha'}{r_1^2 r_2^2 + \theta^2 \alpha'^2}. \quad (2.8)$$

The action of T-duality on $\rho$ is then

$$\rho = \frac{i \sqrt{r_1 r_2}}{\alpha'} + \theta \mapsto \rho' = \frac{\alpha'}{\theta \alpha' + i \sqrt{r_1 r_2}} = -\frac{1}{\rho}. \quad (2.9)$$

and it corresponds to the action of $S \in SL(2, \mathbb{Z})$ on the holomorphic gauge coupling $\tau_{gauge}$. The action of the generator $T$ is instead associated to a shift of the B-field. It this way we have recovered the action of $SL(2, \mathbb{Z})$ on the Kähler structure of the torus on which we have compactified the type IIB LST. This construction can be also extended to the type IIA case. The two descriptions are equivalent because they are related by T-duality on $S^1$.

## 3 $\mathcal{N} = 3$ from LST

In this section we construct 4d SCFTs with $\mathcal{N} = 3$ supersymmetry starting from LSTs on a torus. We consider first the $\mathcal{N} = (2,0)$ Type IIA case. In this case there is a global $SO(1,5) \times SO(4)_R$ symmetry and the supercharges are referred as $Q_{\alpha A}$ and $\bar{Q}_{\alpha \bar{A}}$ where $\alpha$ is the spinor index for $SO(1,5)$ and $A = 2, \bar{A} = \bar{2}$ are spinor indices for $SO(4)_R$. The supercharges we are considering correspond to $(4,2)$ and $(4,\bar{2})^5$. Once we compactify on a $T^2$ we find that the global symmetry becomes

$$SO(1,3) \times SO(4)_R \times U(1)_r \quad (3.1)$$

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3We used the symbols $\hat{g}_{\mu\nu}$ and $\hat{B}_{\mu\nu}$ to refer to quantities after T-duality was performed along the $r_1$ direction.

4The symbols $\tilde{g}_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ indicate quantities after T-duality was performed along the $r_2$ direction.

5Here the representations of $SO(1,3)$ are thought as representations of $SO(4) = SU(2) \times SU(2)$. Hence, we denote the representation $(2,1)$ as 2 and the $(1,2)$ as $\overline{2}$.

6The full $SO(6)_R$ R-symmetry group is recovered only when the size of the torus vanishes.
The supercharges transform with respect to this global symmetry group as
\begin{equation}
(2, 2)_1 \oplus (2, \bar{2})_{-1} \oplus (\bar{2}, 2)_1 \oplus (\bar{2}, \bar{2})_{-1}.
\end{equation}

The $U(1)_\tau$ symmetry is identified with the rotation on the torus wrapped by the NS 5-branes. The T-duality group has two $SL(2, \mathbb{Z})$ factors. One of the two factors is associated with $U(1)_\tau$, i.e. the module of the complex structure $\tau = \frac{g_{12}}{g_{22}} + i \sqrt{\det g}$, which is playing the role of the holomorphic gauge coupling. The other factor is associated to $U(1)_\rho$, i.e. the module of the Kähler structure of the torus $\rho = B + i \sqrt{\det g}$. In both cases we refer to $g$ as the metric on the 2-torus.

In order to associate a $U(1)$ bundle to an $SL(2, \mathbb{Z})$ bundle, the following procedure can be applied [28, 29]. We restrict ourselves to an $\mathcal{N} = 2$ sub-algebra of the supersymmetry algebra, which has a pair of right-handed supercharges $Q_{\dot{A}}^i, i = 1, 2$ and a single central charge $Z$. They satisfy
\begin{equation}
\{Q_{\dot{A}}^i, Q_{\dot{B}}^j \} = \epsilon_{\dot{A}\dot{B}} \epsilon^{ij} Z.
\end{equation}

Let’s look at the action of $SL(2, \mathbb{Z})$ on the supercharges to understand how $Z$ transforms under the S-duality group. The central charge is given by
\begin{equation}
Z = \sqrt{\frac{2}{Im \tau}} \left( \bar{m} \bar{n} \right) \tau \left( \begin{array}{c} 1 \\ \tau \end{array} \right).
\end{equation}

For simply-laced gauge groups, the $SL(2, \mathbb{Z})$ group acts as
\begin{align}
Im \tau & \mapsto |c \tau + d|^2 Im \tau \\
(\bar{m} \bar{n}) & \mapsto (\bar{m} \bar{n}) M^{-1} \\
\phi & \mapsto \overline{\phi} M^{-1} \\
\tau & \mapsto (a \tau + b)/(c \tau + d)
\end{align}

with $M = \left( \begin{array}{cc} a & b \\ c & d \end{array} \right)$, $a, b, c, d$ integers and $ad - bc = 1$. Moreover, $\overline{\phi}$ is the expectation value of the complex scalar which is the $\mathcal{N} = 2$ superpartner of the massless gauge fields and $\bar{m}, \bar{n}$ are the electric and magnetic charge respectively. Under the transformations (3.5) the central charge transforms as
\begin{equation}
Z \mapsto \frac{|c \tau + d|}{c \tau + d} Z.
\end{equation}

At this point one can define a $U(1)$ symmetry by requiring that the algebra is invariant under such transformation. Let’s work out the details. These symmetries, referred as $U(1)$ chiral rotations in [28], must act as $\exp(i \hat{\phi})$ on the $\bar{Q}$’s, and as $\exp(-i \hat{\phi})$ on the $Q$’s. Combining this fact with the transformation law of the central charge (3.6), one finds
\begin{equation}
\{\exp(-i \hat{\phi})Q, \exp(-i \hat{\phi})Q\} \simeq \frac{|c \tau + d|}{c \tau + d} Z.
\end{equation}
Thus, in order for the algebra to be invariant, we find that
\[
\exp(-i\hat{\phi}) = \left( \frac{c\tau + d}{c\tau + d} \right)^{\frac{1}{2}}.
\] (3.8)

The square root means that the group that acts on the supercharges is a double cover of the duality group $SL(2, \mathbb{Z})$. The charge $q$ assigned to the supercharges is given by $e^{iq\arg(c\tau + d)}$. Therefore we see that $q = \pm 1$, i.e. $-1$ for the $Q$’s and $+1$ for the $\bar{Q}$’s. This additional symmetry, which acts as an accidental outer automorphism on the supercharges, characterizes the spectrum of $\mathcal{N} = 4$ SYM theory and is known in literature as Bonus Symmetry [30]. The $\exp(\mp i\hat{\phi})$ factor acts on the supercharges that transform under the 4 or $\bar{4}$ of $SO(1,5)$. Once we decompose the supercharges into 2’s or $\bar{2}$’s we end up with a $\exp(\mp i/2\hat{\phi})$ factor, where $\hat{\phi} = \arg(c\tau + d)$. The supercharges transform under $SO(1,3) \times SO(4)_{R} \times U(1)_{\tau} \times U(1)_{\rho}$ as
\[
(2, 2)_{1, -1} \oplus (\bar{2}, 2)_{-1, 1} \oplus (2, \bar{2})_{1, 1} \oplus (\bar{2}, \bar{2})_{-1, -1}.
\] (3.9)

Observe that the $SO(4)_{R} R$-symmetry combines with $U(1)_{\tau}$ and enhances to the $SO(6)_{R} R$-symmetry of 4d $\mathcal{N} = 4$ SYM.

We can now perform the discrete twist giving raise to the $\mathcal{N} = 3$ theory. At a geometric level we consider $(\mathbb{R}^{1,3} \times T^{2}_{LST})/\mathbb{Z}_{k}$ and observe the following identification:
\[
\tau = i \iff g = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = S
\]
\[
\tau = e^{i\pi/3} \iff g = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = TS
\]
\[
\tau = e^{i2\pi/3} \iff g = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} = (TS)^2
\]

The values of $\tau = i, e^{i\pi/3}$ are fixed points of the transformation $\tau \to \frac{a\tau + b}{c\tau + d}$. To each value of $\tau$ corresponds a different matrix of the S-duality group $SL(2, \mathbb{Z})$, which can be expressed in terms of the generators $S$ and $T$. For these specific values of $\tau$, there is an enhancement of the S-duality group $SL(2, \mathbb{Z})$, which becomes a symmetry group of the theory. The discrete subgroup associated to $\tau = i$ is $\mathbb{Z}_{4}$ while $\mathbb{Z}_{3}, \mathbb{Z}_{6}$ are associated to $\tau = e^{i\pi/3}$. Because of these enhancements, it is possible to quotient the $\mathcal{N} = 4$ theory by the $\mathbb{Z}_{k}$ discrete subgroups.$^{7}$ In order to clarify the procedure, we observe that the $\mathbb{Z}_{k}$ twist is generated by the following factors.

- The $\mathbb{Z}_{k}^{R}$ factor is a rotation generated by the matrix $R_{k} = \begin{pmatrix} \tilde{R}_{k}^{-1} & 0 \\ 0 & \tilde{R}_{k} \end{pmatrix}$ of the $SO(4)_{R}$ R-symmetry group.

- The $\mathbb{Z}_{k}^{T}$ factor is associated to rotation on the torus wrapped by NS 5-branes.

$^{7}$For the value $k = 2$ the discrete quotient corresponds to the usual orientifold projection which preserves 16 supercharges.
The $\mathbb{Z}_k^\rho$ factor is associated to thebonus symmetry group.

Each of these elements acts as a $e^{\pm i\pi/k}$ factor on the supercharges.

The $\mathbb{Z}_k^{\tau,\rho}$ factors descend from the torus wrapped by the NS 5-branes. Indeed by following the procedure explained above we associate a $U(1)^{\tau,\rho}$ bundle to the discrete $SL(2,\mathbb{Z})^{\tau,\rho}$ groups. By acting explicitly on the supercharges with the $\mathbb{Z}_k^{\tau,\rho} \subset U(1)^{\tau,\rho}$ discrete symmetries, we can see that some supercharges are left invariant while some others are modded out.

\[
\begin{align*}
(2, +++)_{1,-1} & \mapsto (2, +++)_{1,-1} \\
(2, ---)_{1,-1} & \mapsto (2, ---)_{1,-1} \\
(\bar{2}, +++)_{-1,1} & \mapsto (\bar{2}, +++)_{-1,1} \\
(\bar{2}, ---)_{-1,1} & \mapsto (\bar{2}, ---)_{-1,1} \\
(2, +-)_{1,1} & \mapsto (2, +-)_{1,1} \\
(2, -+)_{1,1} & \mapsto e^{4\pi i/k} (2, -+)_{1,1} \\
(\bar{2}, -++)_{-1,-1} & \mapsto e^{-4\pi i/k} (\bar{2}, -++)_{-1,-1} \\
(\bar{2}, ---)_{-1,-1} & \mapsto (\bar{2}, ---)_{-1,-1}
\end{align*}
\]

From the above expression one can see that only 12 out of the original 16 supercharges survived the projection, giving rise to an $N = 3$ theory.

### 3.1 Type IIB Case

In this case the supercharges are $Q_{\alpha A}$ and $\bar{Q}_{\bar{\alpha} \bar{A}}$ corresponding to $(4, 2)$ and $(\bar{4}, \bar{2})$. As in the previous case, after compactification on a torus we have the following supercharges

\[
(2, 2)_{1} \oplus (\bar{2}, 2)_{-1} \oplus (2, \bar{2})_{-1} \oplus (\bar{2}, \bar{2})_{1},
\]

where the charge refers to the $U(1)_\rho$ symmetry in this case. Here the $SL(2,\mathbb{Z})_\rho$ is playing the role of S-duality because the module of the Kähler structure corresponds to the holomorphic gauge coupling of the 4d theory. We know that in this case there is also an $SL(2,\mathbb{Z})_\tau$, where that this factor descends from the $O(2, 2; \mathbb{Z})$ T-duality group. As a symmetry it corresponds to the maximal compact subgroup $U(1)_\tau$ of $SL(2, \mathbb{R})$. To find how the supercharges transform under $SO(1, 3) \times SO(4)_R \times U(1)_\rho \times U(1)_\tau$, one can follow the procedure outlined in the previous section. An alternative, but equivalent derivation consists of exchanging the $U(1)$ charges discussed in the previous type IIA case. This is because the two theories, considered on $S^1 \subset T^2$ are T-dual, and T-duality exchanges $\tau \leftrightarrow \rho$. With this procedure one would find the same

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8States of the representation 2 of $SO(4)$ are denoted as $(++)$, $(--)$. In a similar fashion states of the representation 2 are denoted as $(+-), (-+)$. 

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result up to an overall minus sign, which can be thought as a parity transformation
on the supercharges. In this case the supercharges are

\[(2, 2)_{1,-1} \oplus (\bar{2}, 2)_{-1,1} \oplus (2, \bar{2})_{-1,-1} \oplus (\bar{2}, \bar{2})_{1,1}. \tag{3.11}\]

The discrete twist by \( \mathbb{Z}_k \) is done by combining the discrete subgroups arising from
\( U(1)_{\tau}, U(1)_{\rho} \) and \( SO(4)_R \) as was done in the section above. The final result is still
that there are 12 supercharges left invariant by the projection.

4 Relations with the literature

So far we have obtained a 4d \( \mathcal{N} = 3 \) SCFT in terms of LST compactified on \( T^2 \). In
this section we compare our construction with the others appeared in the literature.
More precisely we compare our construction with the non-geometric one of [5] in
terms of M5 branes wrapping a \( T^2 \subset T^3 \).

Let us review the construction of [5]. The S-fold projection acts on a 4d theory
corresponding to a stack of D3 branes in type IIB. The discrete twist requires a
\( T^2 \) in
the transverse geometry, as shown in Figure 1. The torus breaks \( SO(6)_R \rightarrow SO(4)_R \),
and the full R-symmetry group is recovered only in the decompactification limit.
This picture can be T-dualized to type IIA and lift to M-theory as in Figure 1.

In this case we have M-theory on \( \mathbb{R}^{1,3} \times T^3 \times \mathbb{C}^2 \) with a stack of M5 branes wrapping a \( T^2 \subset T^3 \).
The Kähler structure parameter is given by \( \rho = \int_{T^3} C + i \sqrt{\text{det} \, G} \) and it plays the role of the holomorphic gauge coupling
of the \( \mathcal{N} = 4 \) theory. We denoted with \( C \) the M-theory three form and \( G \) the metric
on \( T^3 \). The M-theory duality group in this background is \( SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})_{\rho} \). The authors showed that if the radii are constrained as \( r_{S^1} = r_{S^1} = r_{S^1}^{-1/2} = r \) (or \( r_{S^1} = \frac{2\sqrt{3}}{3} r_{S^1} = r_{S^1}^{-1/2} = r \) ) there is a discrete symmetry enhancement giving rise
to the S-duality twist.

Here we observe that our construction can be obtained from the one of [5] by
shrinking the M-theory circle \( r_{S^1} \). This limit correspond to \( r \rightarrow 0 \) while keeping either \( r_{S^1} \) or \( r_{S^1} \) equal to \( \sqrt{2} \). We are left with NS5 branes wrapping s \( T^2 \), where we kept
the same colors for the radii in order to make the relation explicit. In this case the
\( SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})_{\rho} \) duality group of M-theory becomes the usual T-duality group
that survives the decoupling of the gravitational degrees of freedom in LST. The
M-theory three form \( C_{\mu \nu \rho} \) becomes the two form \( B_{\mu \nu} \) in LST while the R-symmetry
group remains \( SO(4)_R \).

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As we discussed in the introduction, a previous attempt was proposed in [3], where the authors
obtained a geometric construction of 4d \( \mathcal{N} = 3 \) models, in terms of F-theory at terminal singularity.
Here we skip its review because it does not play a prominent role in our analysis.
Figure 1: Representation of the various constructions discussed in [5] giving raise to an $\mathcal{N} = 3$ model through the S-duality twist. The first figure represents the type IIB construction: there is a D3 brane and the transverse space is $\mathbb{C}^2 \times T^2$. This construction is related through T-duality to the second figure, that represents a D4 brane on $\mathbb{R}^{1,3} \times S^1$, with a transverse $\mathbb{C}^2 \times S^1$. This can be uplifted to M-theory, as shown in the third figure. The last figure represents our construction that can be obtained from the M-theory one by reducing along the red circle. The last picture represents an NS5 brane wrapping a $T^2$, and it is evident from the figure that all the informations necessary for the S-duality twist are in the 6d picture, i.e. in the 6d non-local field theory denoted as LST.

5 Conclusions

In this note we studied 4d $\mathcal{N} = 3$ SCFT from LST on a $T^2$, obtained by a twist of a discrete symmetry subgroup of the full T-duality group. This is an example of a purely field theoretical construction, possible because the T-duality action of the string theory background survives the decoupling of the gravitational degrees of freedom. We showed that the construction can be performed in both the type IIA and type IIB cases, either starting from the $\mathcal{N} = (2,0)$ or from the $\mathcal{N} = (1,1)$ case. We have also provided the relation with the other constructions proposed in the literature based on the S-fold projection.

There are many further questions that are of interest. For example it may be interesting to extend the analysis to the $\mathcal{N} = (1,0)$ LST classified in [31] and to connect the T-duality twist discussed here to the $\mathcal{N} = 2$ S-folds studied in [32].
Another possible extension of the analysis regards the study of the $D_n$ and the exceptional case. In these cases the S-duality group descends from the T-duality group in the same way at it does in the $A_n$ case, and it implies that the results extend straightforwardly. A more difficult problem regards the non-simply laced cases. In these cases the structure of the S-duality fixed point is different and the matter fields transforms under the S-duality group.

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