EXPERIMENTAL STUDY OF LIQUID SLOSHING FORCE CHARACTERISTICS IN RECTANGULAR TANK OF SPRAYER UNDER HARMONIC EXCITATION

简谐激励下喷雾机矩形药箱内液体晃动力特性实验研究

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ABSTRACT
The stability of the boom system of sprayers is easily affected by the liquid sloshing force and the uniformity of droplet deposition deteriorates. Therefore, the liquid sloshing forces in the rectangular tank were measured through experiments. Three main factors affecting the sloshing forces were examined. Experimental results reveal that the sloshing forces measured fit well with the theoretical curve and the maximum sloshing force is independent of the excitation amplitude for a violent sloshing. Based on these characteristics, a practical method was proposed which can approximately calculate the maximum sloshing force based on the linear model, and can be used for the sprayer chassis design and active and passive control of boom attitude.

INTRODUCTION
Sloshing is fluctuation of a free liquid surface. The distribution of the liquid in a tank will change if the sloshing occurs. At the same time, the liquid applies a large impact pressure on the vessel wall. The sloshing force of the liquid is the result of these two factors. Sloshing may cause bad effects such as fatigue, damage, and instability of the container and external structure. Therefore, sloshing has been studied in many engineering fields, involving aerospace (Miao Li and Wang, 2017; Chiba and Magata, 2017), ships (Zhao et al., 2011; Vieira et al., 2018), transportation (Kolaei, Rakheja and Richard, 2014; Toumi, Bouazara and Richard, 2009), water conservancy (Li Di and Gong, 2012; Li and Wang, 2012) and so on. Apart from theoretical analysis and numerical simulation, experiment is also an important means to research the problem of sloshing (Ishikawa et al., 2016; Yan, Rakheja and Siddiqui, 2009; Cavalaghi et al., 2017).

With the development of the agricultural mechanization, boom sprayers and plant protection UAVs are widely used today. The weight of the liquid in tanks tends to be 1/3 of the total or more in sprayers. Usually, the tank is placed on the upper part of the sprayer and a high chassis is used in order not to damage crop. Therefore, the centre of gravity is high and the tread is narrowly relative to the vehicle height. When the sprayer drives on the uneven ground, vibration in the vertical direction and sway in the longitudinal and lateral directions will occur because of the fluctuation of land. Moreover, the attitude of sprayers is also influenced by the driving conditions such as start, stop, acceleration, deceleration and turning. The liquid sloshing is easily induced by these factors. The liquid sloshing force whose value and direction are constantly changing directly acts on the sprayer, and the attitude of the sprayer will be affected in turn. Not only does the stability of the sprayer deteriorate, but also the balance of the boom is influenced, which thereby reduces the application effect of the pesticide. So, the liquid sloshing in the tank has begun to attract...
attention (Jeon, 2003). The stability and safety of the crop protection UAV are also affected by the liquid sloshing caused by the change of attitude (Li et al., 2017).

The change of the sprayer's driving state during operation has great uncertainty, and the ground unevenness is also a kind of low-frequency large-amplitude random excitation. Therefore, the change of the attitude of the tank is also random. The posture of the tank randomly changes with time during the whole operation process, and the attitude of the tank may be any combination of translation and rotation at a certain moment. Obviously, it is difficult to establish a correspondence between the liquid sloshing and such a random excitation, so the sloshing mechanism and the control strategy cannot be discussed further.

Although pure harmonic excitation is less likely to occur than periodic or other types of excitation, understanding the behaviour of a system undergoing harmonic excitation is essential in order to comprehend how the system will respond to more general types of excitation (Thomson and Dahleh, 2013). Therefore, the rectangular tank commonly used in sprayers was chosen in this paper, and the harmonic excitation with known frequency and amplitude was applied to study the sloshing characteristics of the liquid in the tank. The lateral force generated by liquid sloshing was measured by force sensors, and the factors that influence the sloshing force were analysed.

MATERIALS AND METHODS

To measure the force generated by the sloshing, the liquid must be placed in a tank with certain shape and size. In other words, the force needs to be measured indirectly by means of the tank. However, under the external excitation, the inertial force of the box itself also acts on the force sensor at the same time. Therefore, it is essential to correctly measure and to distinguish the inertia force of the box from the liquid sloshing force in the experiment. A suspension system was designed in order to reduce the influence of friction, as shown in Fig. 1.

A frame like a door was fixed to the shake table and moves together with it. The tank made of transparent plexiglass was fixed to a steel plate, and the steel plate was suspended from the frame by thin steel wires. The uniaxial force sensor was placed horizontally with the steel plate and the frame attached at each end. Therefore, the weight of the steel plate, the tank and the liquid in the tank is borne by the wire ropes, and the force sensor measures only the force in the horizontal direction. In order to reduce the influence of the shake table's motion, the laser displacement sensor used for measuring the displacement of the tank was horizontally placed on the ground. The force and displacement signals from these sensors were recorded using a high-speed data acquisition instrument.

The horizontal displacement of the shake table is transmitted to the tank through the frame, the force sensor and the steel plate, causing the liquid in the tank to sway. The force generated by the sloshing of the liquid acts on the tank and is transmitted to the force sensor through the steel plate. In addition, during the reciprocating motion, the acceleration of the steel plate and the tank is not constant, and so the corresponding inertial force also acts on the force sensor. Therefore, the force sensor simultaneously measures the liquid sloshing force and the inertial force of the steel plate and the tank. In other words, the sloshing force should be the force measured by the force sensor minus the inertial force of the steel plate.

![Fig. 1 - Schematic diagram of experiment](image-url)

1. Frame 2. Steel wire 3. Tank 4. Load cell 5. Laser displacement sensor 6. Shake table

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and the tank. The inertial force can be calculated through the mass of the steel plate and the tank, the amplitude and frequency of shake table’s motion.

The complete test system is shown in Fig. 2. The horizontal displacement excitation is produced by a shake table whose frequency and amplitude can be adjusted according to the need. The frequency range is 0 - 2Hz and the frequency resolution is 0.02Hz. The force sensor is piezoelectric force sensor (CA-YD-303) and associated charge amplifier (YE5853), produced by Jiangsu Lianneng Electronic Technology Co., Ltd. The measurement range of the force sensor is 0-2kN. The laser displacement sensor (IL-300) is produced by Keyence Corporation with a measuring range of 300 mm. The data acquisition system is high-speed data acquisition card (4492) and the corresponding controller (8820) produced by National Instruments Corporation. The tank is made of plexiglass and has an internal dimension of 500x100x500 mm, corresponding to length, width and height, respectively. A large amount of water is used to dilute chemical pesticides during the application. In other words, majority of the liquid in the tank of sprayers is water. So, dyed water was used in the experiment.

The length and width of the tank have been fixed, so other factors that affect the sloshing force were considered, namely liquid depth, frequency and amplitude of the excitation. The experiment was performed in the following order. Firstly, the amplitude of excitation was kept constant, and harmonic displacement excitations with different frequencies were applied to the tank with a certain liquid depth, and then the above process was repeated by changing the liquid depth. Secondly, the liquid depth was kept constant, and harmonic displacement excitations with different frequencies were applied on the tank with the same amplitude, and then the process was repeated by increasing the amplitude. The values of each factor are shown in Tab. 1. Where, f is the first natural frequency determined by length and depth of liquid, [Hz].

### Table 1

| Parameters               | Ranges       | Increments |
|-------------------------|--------------|------------|
| Excitation frequency [Hz]| 0.5f - 1.8f  | 0.05f      |
| Excitation amplitude [mm]| 5 - 30      | 5          |
| Liquid depth [mm]       | 100 - 500   | 25         |

### RESULTS

**Influence of liquid depth on sloshing force**

The quantity of the liquid in the tank gradually reduced as the liquid is continuously consumed during the pesticide spraying operation, which can be reflected directly by the liquid depth for the rectangular tank.

The sloshing force with different liquid depths is shown in Fig. 3. When the liquid depth does not exceed 400 mm, there is no significant difference in the tendency of the sloshing force. So, only parts of
the results are shown for the sake of clarity, as shown in Figures (a) and (b). When the liquid depth is large, the sloshing force varies differently, so all the results are given, as shown in Figures (c) and (d). Please note that the excitation amplitude is 10mm in all of these cases.

If the excitation amplitude and the liquid response are small, the sloshing force for a rectangular tank under harmonic excitation \( x_0 \sin \omega t \) can be written as (Ibrahim, 2005)

\[
F = \rho Lwhx_0\omega^2 \sin \omega t \cdot \left[ 1 + \sum_{n=0}^{\infty} \frac{8 \tanh(k_n h)}{k_n^2 L^2 h} \left( \frac{\omega^2}{\omega_n^2 - \omega^2} \right) \right] \text{[N]} \tag{1}
\]

Where:
\( \rho \) is the density of the liquid, [kg.m\(^{-3}\)];
\( L, w, h \) is the length, width and depth of the liquid respectively, [mm];
\( x_0 \) is the amplitude of excitation, [mm];
\( \omega \) is the circle frequency of excitation, [rad/s];
\( \omega_n = \sqrt{gk_n \tanh(k_n h)} \) is the natural circle frequency of liquid sloshing, [rad/s];
\( k_n = (2n+1)\pi / L \), [m\(^{-1}\)];
\( g \) is the gravity acceleration, [ms\(^{-2}\)];
\( t \) is time, [s].

The sloshing force can be calculated according to given parameters mentioned above and was drawn in solid or dash line in the following figures.

![Figure 3: Liquid sloshing force with different liquid depth](image)

It can be seen that in the case of small liquid depth (i.e., 100mm), the experimental results are basically consistent with the theoretical ones over the entire frequency range. The biggest difference occurs at the natural frequency. The sloshing force at this frequency is infinity according to the formula (1), but the experimental result is a finite value. The reason lies in the fact that the liquid sloshing force expression (1) is based on the premise of the ideal fluid, without considering the viscosity of the fluid. For the actual fluid, there is friction between the fluid and the tank wall due to the presence of viscosity, which hinders the movement of the liquid. Therefore, the sloshing force is finite.
For the medium liquid depth (i.e., 200 and 300mm), the forces measured are slightly smaller than the theoretical ones when the excitation frequency is lower than the frequency where the maximum sloshing force occurs. Different from the small depth, the maximum sloshing force appears at a lower frequency (i.e., 0.95f). The sloshing forces at the following one or two frequencies successively reduce. Thus, the forces measured have a relatively large discrepancy with the theoretical results. Then, the differences are small again with the frequency increases. The frequency corresponding to the maximum sloshing force further lowers for the larger liquid depth (400mm), which is only 0.9f. The reason is analysed as follows. When the liquid depth is small, the free surface of liquid can keep as flat at almost whole frequency range, apart from the natural frequency, so the movement of liquid satisfies the condition of the linear model. The relatively flat surface can also be gotten at the lower frequency range for the medium or larger liquid depth. So, the discrepancy between the sloshing force measured and the theoretical one is not distinct. But, when the excitation frequency is larger than the natural frequency, the surface of liquid presents multiple standing waves or travelling waves with shorter wavelength for the medium or larger liquid depth. In other words, the surface of liquid is not flat but curved. More crests appear at the middle positions along the length direction and less liquid impacts on the side wall of the tank. So, the sloshing force measured is less than the theoretical one.

The frequency where the maximum sloshing force occurs for the 450 and 475mm liquid depth is 0.85f and 0.8f, respectively. For the latter, the sloshing forces decrease slowly and then gradually increase at the subsequent frequencies, showing relatively large discrepancies with the theoretical model. The reason is that the distance between the free surface of the liquid and the top cover of the tank is small (only 2 - 5cm), and the liquid impact not the side wall but the cover even if the sloshing is small. On the other hand, large amount of liquid in the lower part does not participate in the sloshing, but moves along with the tank. Therefore, inertial force plays a more important role than the sloshing force in this case. Based on this attribute, horizontal baffle can be placed on the side wall to reduce the liquid sloshing force.

If the tank is completely filled with liquid, they will move together. Sloshing will no longer occur due to the absence of free liquid surface. The sloshing force increases as the excitation frequency increases since it is actually the inertial force, as shown in Figure (d).

The experimental results near the natural frequency differ greatly from the theoretical curve, mainly due to the non-linearity of the large amplitude sloshing. The typical waveform of the free liquid surface is shown in Fig. 4 for better understanding.

![Waveform of liquid surface under different excitation frequency](image)

It can be seen that the liquid sloshing amplitude is small and the liquid surface is flat when the excitation frequency is far from the resonance frequency of the liquid (i.e., below 0.8f or above 1.15f), as shown in Figures (a) and (b). In this circumstance, the precondition of linearization can be basically satisfied, so the experimental result is in good agreement with the linear model. However, violent sloshing occurs near the natural frequency of the liquid. During the large amplitude sloshing, the liquid climbs rapidly along the tank wall, then tends to slow down, and finally falls quickly. At the same time, the rolling and breaking of the liquid surface often occurs. In addition, the movement of the liquid is also not limited to the excitation direction. Not only is the waveform a curved surface in the longitudinal direction, but also the surface is no longer flat in the width direction. The maximum wave height appears at a certain corner and then alternates among the four corners. This means that the motion of the liquid contains one or more waves that rotate around the vertical axis of the tank. Obviously, the liquid
sloshing amplitude does not satisfy the premise of the linear model, and so it is inevitable that the experimental results greatly deviate from the theoretical value.

**Influence of excitation amplitude on sloshing force**

Boom sprayers are widely used in the South China paddy field, where the height of hard bottom ranges from 58mm to -40mm (Zhu et al., 2016). Taking a certain boom sprayer as an example, the wheelbase is 1.5m and the distance between the bottom of the tank and the ground is 1.3m. Therefore, the maximum displacement of the tank in horizontal plane is about 40mm due to the ground unevenness excitation. The liquid in the tank is prone to sloshing violently under such large an excitation. So, it is necessary to discuss the effects of the excitation amplitude on the liquid sloshing force.

The sloshing force corresponding to different excitation amplitude is shown in Fig. 5. Please note that the liquid depth is 150mm.

**Fig. 5 - Liquid sloshing force with different excitation amplitude**

It can be seen that the sloshing force is in good agreement with the theoretical model in the lower frequency range, and the excitation amplitude has almost no influence. However, in the higher frequency range, the larger the excitation amplitude is, the more the sloshing force deviates from the theoretical value. In addition, the frequency at which the maximum sloshing force occurs is also affected by the excitation amplitude, that is, the larger the excitation amplitude is, the lower the frequency is. The reason is analysed as follows. The larger excitation amplitude is more likely to cause a violent sloshing of the liquid, so that the maximal sloshing force can be gotten at a lower frequency. Furthermore, the larger excitation amplitude is more likely to form a short-wavelength traveling wave in the high frequency range, and the impact pressure of the liquid is smaller. So, the sloshing force is less than the theoretical one.

Figure (b) shows the maximum sloshing force at different excitation amplitudes. It can be seen that when the excitation amplitude is equal to or more than 20 mm, the sloshing force is basically unchanged, showing a feature of saturation. The reason lies in the fact that the amplitude of the liquid sloshing has reached the limit and thus the sloshing force is not increased any longer. In other words, the sloshing amplitude cannot be further increased even if the excitation amplitude is increased.

**Approximate calculation of maximum sloshing force**

During the pesticide application operation of the sprayer, the continuous excitation from irregular ground and unstable driving may cause large amplitude sloshing of the liquid in the tank, and thus the sloshing force occurs.

According to the analysis in the former section, the maximum sloshing force does not depend on the excitation frequency and amplitude any longer but tends to be fixed, once large amplitude sloshing occurs. This makes it possible to obtain the maximum sloshing force by means of the theoretical model. The sloshing force measured with different liquid depth in the experiment was carefully analysed and an obvious law can be found. If the entire frequency range is divided into two parts by the frequency at which the maximum sloshing force occurs, in the higher frequency range the discrepancies between the experimental results and the theory model are relatively large and depend on the liquid depth at a certain extent. However, in the lower frequency range the difference between the experimental results
and the theoretical ones is small and stable regardless of the change of the liquid depth. Therefore, the maximum sloshing force with different liquid depth may be approximately calculated by the formula (1).

When the liquid depth is small, the maximum sloshing force is obtained at 1.0f or 0.95f, so 0.95f is taken to calculate the sloshing force for the liquid depth of 100-175 mm. The maximum sloshing force generally appears at 0.95f as the liquid depth further increases, but it is slightly smaller than the corresponding theoretical result. So, 0.94f and 0.93f is taken to calculate the sloshing force for the liquid depth of 200 - 225 mm and 250 - 275 mm, respectively. The frequency can be further decreased to 0.92f for the liquid depth of 300-375 mm. The maximum sloshing force is basically the same as the theoretical value of the corresponding frequency for the liquid depth above 400 mm, and thus the frequency is taken as 0.9f, 0.85f and 0.8f, respectively. The theoretical results corresponding to the frequencies mentioned above are compared with the experimental ones, as shown in Fig. 6(a). The maximum relative error is equal to 6.4% within the whole liquid depth range.

![Fig. 6 - Maximum sloshing force tested with different liquid depth and theoretical value at different frequencies](image)

It can be seen from Fig. 3 and Fig. 5(a) that the experimental results of the sloshing force are always slightly smaller than the theoretical ones in the lower frequency range. It may originate from the force sensor itself because the measure range of the sensor is far larger than the actual sloshing force. Taking this fact into consideration, the frequency for calculating the theoretical value can be appropriately increased. That is, it is taken as 0.97f - 0.96f for the small liquid depth, 0.96f - 0.94f for the medium liquid depth, and 0.93f - 0.8f for the large liquid depth, respectively. In general, the greater the liquid depth is, the lower the frequency is.

As verification, the experimental results of a tank with 750 mm length and 100 mm width were compared with the theoretical values. The frequencies used to calculate the theoretical value are shown in Fig. 6(b).

It can be seen that the theoretical value calculated fits well with the experimental one. The maximum relative error is 23.3%, which occurs at the 100 mm liquid depth. But the actual difference is only 3.5N.

It should be pointed out that the frequency at which the maximum sloshing force occurs is 1.1f or 1.05f for the 750 mm length tank when the liquid depth is less than 175 mm, which is different from the 500 mm length tank. But the conclusions mentioned above are not affected by the changes of these frequencies. In other words, the conclusions can be applied to different length rectangular tank and have good robustness.

CONCLUSIONS

Harmonic displacement excitation was applied to the rectangular tank commonly used in sprayers in order to make the inside liquid sloshing. The effects of liquid depth, frequency and amplitude of the excitation on the liquid sloshing force were studied.

(1) When the liquid depth is relatively small and the excitation frequency is far from the natural frequency of the liquid sloshing, the experimental result of sloshing force is in good agreement with the
theoretical one. However, in the vicinity of the natural frequency, especially in the case of large liquid depth, a relatively large deviation between the test and the theory occurs.

(2) Larger excitation amplitude is more likely to cause violent sloshing of the liquid, but the maximum sloshing force does not increase linearly with the excitation amplitude, showing a characteristic of saturation.

(3) The maximum sloshing force can be approximately calculated by the linear model. It can be used for the sprayer dynamics analysis, chassis design and active and passive control of boom. It is also helpful for the flight control system design for plant protection UAVs.

(4) Based on the experiments and results in this paper, field tests can be better performed.

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