NONEXTENSIVE STATISTICAL EFFECTS ON NUCLEAR
ASTROPHYSICS AND MANY-BODY PROBLEMS

A. LAVAGNO AND P. QUARATI

Dipartimento di Fisica, Politecnico di Torino and INFN - Sezione di Torino e
di Cagliari, Italy

Density and temperature conditions in many stellar core (like the solar core) imply
the presence of nonideal plasma effects with memory and long-range interactions
between particles. This aspect suggests the possibility that the stellar core could
not be in a global thermodynamical equilibrium but satisfies the conditions of a
metastable state with a stationary (nonextensive) power law distribution function
among ions. The order of magnitude of the deviation from the standard Maxwell-
Boltzmann distribution can be derived microscopically by considering the pres-
ence of random electrical microfields in the stellar plasma. We show that such a
nonextensive statistical effect can be very relevant in many nuclear astrophysical
problems.

1. Introduction

The solar core is a neutral system of electron, protons, alpha particles
and other heavier nuclei, usually assumed as an ideal plasma in thermo-
dynamical equilibrium described by a Maxwellian ion velocity distribution.
Because the nuclear rates of the most important reactions in stellar core
are strongly affected by the high-energy tail of the ion velocity distribution,
let us start by reminding the meaning of ideal and non-ideal plasma. A
plasma is characterized by the value of the plasma parameter \( \Gamma \)
\[
\Gamma = \frac{\langle U \rangle_{\text{Coulomb}}}{\langle T \rangle_{\text{thermal}}},
\]
where \( \langle U \rangle_{\text{Coulomb}} \) is the mean Coulomb potential energy and \( \langle T \rangle_{\text{thermal}} \) is
the mean kinetic thermal energy. Depending on the value of the plasma
parameter we can distinguish three regimes:
- \( \Gamma \ll 1 \) - Dilute weakly interacting gas, the Debye screening length \( R_D \) is
  much greater than the average interparticle distance \( r_0 \approx n^{1/3} \), there is a
  large number of particles in the Debye sphere.
- \( \Gamma \approx 0.1 \div 1 \) - \( R_D \approx r_0 \), it is not possible to clearly separate individual and
collective degree of freedom and the plasma is a weakly non-ideal plasma.
- \( \Gamma \geq 1 \) - High-density/low-temperature plasma, Coulomb interaction and quantum effects dominate and determine the structure of the system.

In the solar interior the plasma parameter \( \Gamma_\odot \simeq 0.1 \) and the solar core can be considered as a weakly nonideal plasma. Similar behavior occurs in other astrophysical systems with \( 0.1 < \Gamma < 1 \), among the others we quote brown dwarfs, the Jupiter core, stellar atmospheres. Weakly nonideal conditions can influence how the stationary equilibrium can be reached within the plasma. In fact, in weakly nonideal astrophysical plasmas we have that the collision time is of the same order of magnitude of the mean time between collisions, therefore, several collisions are necessary before the particle loses memory of the initial state; collisions between quasi-particles (ion plus screening cloud) are inelastic and long-range interactions are present.

In the next section we will see how the presence of memory and long-range forces can influence the thermodynamical stability and the stationary distribution function inside the stellar core.

2. Metastable states of stellar electron-nuclear plasma

We can distinguish two kind of thermodynamical equilibrium state ¹:

- global thermodynamical equilibrium: the free energy density is minimized globally
- local thermodynamical equilibrium: free energy density is minimized only in a restricted space, not globally. In this case the system is in a metastable state.

Metastable states are always characterized by long-range interactions and/or fluctuations of intensive quantities (like inverse temperature \( \beta \), density, chemical potential) and the stationary distribution function can be different from the Maxwellian one. In fact, in many-body long-range-interacting systems, it has been recently observed the emergence of long-standing quasi stationary (metastable) states characterized by non-Gaussian velocity distributions, before the Boltzmann-Gibbs equilibrium is attained ²,³.

Considering the corrections to an ideal gas due to identity of particles and to inter-nuclear interaction and the black-body radiation emitted, by minimizing the free energy density of the electron-nuclear plasma, we have
obtained the following values \(^4\)

\[
n_\ast \simeq 2.74 \cdot 10^{-14} \text{fm}^{-3}, \quad k_B T_\ast \simeq 5 \cdot \text{keV} \quad \text{and} \quad R_\ast \approx 0.2R_\odot,
\]

with a typical stellar chemical composition \(\bar{Z} = 1.25\). States with different values of \(k_B T\) (lower) and \(n\) (higher) are **metastable** states that can be featured by temperature fluctuations or density fluctuations, by quasi-particle models or by the presence of self-generated magnetic fields or random microfields distributions.

The values obtained above are more than three times higher than the actual temperature of the solar interior and an electron density about half the actual value in the solar core. Therefore, the core of a star like the Sun can not exactly be considered in a global thermodynamical equilibrium state but can be better described as a metastable state and the stationary distribution function could be slightly different from the Maxwellian distribution.

In this context, it has been shown that when many-body long-range interactions are present, in many cases the system exhibits stationary metastable properties with power law distribution well described within the Tsallis nonextensive thermostatistics \(^5,6,7,8\).

### 3. Microscopic interpretation: random electrical microfield

In this section we want to investigate about a microscopic justification of a metastable power-law stationary distribution inside a stellar core. At this scope, let us start by observing that the time-spatial fluctuations in the particles positions produce specific fluctuations of the microscopic electric field (with energy density of the order of \(10^{-16} \text{MeV/fm}^3\)) in a given point of the plasma. These microfields have in general long-time and long-range correlations and can generate anomalous diffusion. The presence of the electric microfield average energy density, \(\langle E^2 \rangle\), modifies the stationary solution of the Fokker-Planck equation and the ion equilibrium distribution can be written as

\[
f(v) = C \exp \left\{ - \int_0^v \frac{mv \text{d}v}{kT(1 + \frac{\langle E^2 \rangle}{E_c^2})^\alpha} \right\},
\]

where \(E_c = \nu \sqrt{3xmkT/2e^2}\). In the solar core being \(E\) not too larger than \(E_c\), the distribution differs slightly from the Maxwellian. Crucial quantity is the elastic collision cross section is the enforced elastic Coulomb cross section \(\sigma_0 = 2\pi(\alpha r_0)^2\) where \(r_0\) is the inter-particle distance, \(\alpha\) is
related to the pair-correlation function $g(R, t)$. The stationary (metastable) distribution (2) for the solar interior can be written as a function of the kinetic energy $\epsilon_p$

$$f(\epsilon_p) = N \exp \left[ -\frac{\epsilon_p}{kT} - \delta \left( \frac{\epsilon_p}{kT} \right)^2 \right], \quad (3)$$

where the deformation parameter $\delta = (1 - q)/2$ can be written as

$$|\delta| \approx \frac{\sigma_0^2}{3(\sigma_0^2)} = 12 \alpha^4 \Gamma^2 \ll 1. \quad (4)$$

A reasonable evaluation of $\alpha$ gives: $\alpha = 0.55$, with $\Gamma \sim 0.1$ and we obtain $q = 0.990$ ($\delta = 0.005$). In the next section we will see as such a small deviation of the MB distribution can be very relevant in several nuclear astrophysical applications.

### 4. Signals in astrophysical problems

Let us illustrate few problems where we can find signals of the presence of deviations from the MB distribution. Their solutions can be achieved by means of modified (or generalized) rates calculated by means of deformed distributions. Among the others, we quote A) Solar neutrino fluxes; B) Jupiter energy production; C) Atomic radiative processes in electron nuclear plasmas; D) Abundance of Lithium; E) Temperature dependence of modified CNO nuclear reaction rates and resonant fusion reactions. For brevity we will discuss here the last point only. A detailed discussion of the other problems can be found in Ref.s 9,10.

#### 4.1. Temperature dependence of modified CNO nuclear reaction rates

The temperature dependence of CNO cycles nuclear rates is strongly affected by the presence of nonextensive effects in Sun like stars evolving towards white dwarfs ($10^7 \div 10^8$ K). Small deviations ($q = 0.991$) from MB distribution strongly increase the rates and can explain the presence of heavier elements (Fe, Mg) in final composition of white dwarfs, consistently with recent limit of the fraction of energy the Sun produces via the CNO fusion cycle (neutrino constraints). We obtain that 11 i) the luminosity yield of the pp chain is slightly affected by the deformed statistics, with respect to the luminosity yield of the CNO cycle; ii) the nonextensive CNO correction ranges from 37% to more than 53%; iii) above $T \approx 2 \cdot 10^7$ K,
the luminosity is mainly due to the CNO cycle only, thus confirming that CNO cycle always plays a crucial role in the stellar evolution, when the star grows hotter toward the white dwarf stage. Our results are reported in Fig. 1 and Fig. 2. In Fig. 1, we plot the dimensionless luminosity over temperature, for the pp chain and the CNO cycle. In Fig. 2, we report the dimensionless equilibrium concentrations of CNO nuclei over temperature.

4.2. Resonant reaction rates in astrophysical plasma

Cussons, Langanke and Liolios\textsuperscript{12} proposed, on the basis of experimental measurements at energy $E \sim 2.4\, \text{MeV}$, that the resonant behavior of the stellar $^{12}\text{C} + ^{12}\text{C}$ fusion cross section could continue down to the astrophysical energy range.

The reduction of the resonant rate due to resonant screening correction amounts to 11 orders of magnitude at the resonant energy of 400 keV, with important implications for hydrostatic burning in carbon white dwarfs.
Figure 2. Log-linear plot of dimensionless equilibrium concentrations of CNO nuclei over temperature. Classical statistics has been used. All curves are normalized with respect to the initial density \((^{14}\text{N})_0\) inside the Sun.

We have analytically derived two first-order formulae that can be used to express the non-extensive reaction rate as a product of the classical reaction rate times a suitable corrective factor for both narrow and wide resonances.

Concerning the fusion reactions between two medium-weighted nuclei, for example the \(^{12}\text{C} + ^{12}\text{C}\) reaction, our non-extensive factor, which can be formally defined as follows \(^{13}\)

\[
 f_{NE} = 1 + \frac{15}{4} \delta - \left( \frac{E_R}{k_B T} \right)^2 \delta ,
\]

gives rise to further correction beside the screening and the potential resonant screening

\[
 F = f_{NE} \cdot f_S \cdot f_{RS} ,
\]

where \(f_S\) and \(f_{RS}\) account for the Debye-Hückel screening and the resonant screening effect respectively.

We have applied our results to a physical model describing a carbon white dwarf’s plasma, with a temperature of \(T = 8 \cdot 10^8\) K and a mass density of \(\rho = 2 \cdot 10^9\) g/cm\(^3\) (the plasma parameter is, correspondingly,
Furthermore, we have set a deformation parameter $|\delta| = 10^{-3}$, regardless of its sign, and we have kept the energy of the possible resonance, $E_R$, as a free parameter. In Fig. 3 we plot our estimation of the effective total factor $F$ as a function of the resonance energy $E_R$.

\[ \Gamma \approx 5.6 \].

![Figure 3](image-url)

Figure 3. Linear plot of the effective factor $F$, defined in Eq.(5), against the resonance energy $E_R$. The dash-dotted (upper) line refers to super-extensivity, the dashed (lower) line to sub-extensivity, while the solid (middle) line describes the classical (MB) result.

All the plasma enhancements due to the presence of long-range many-body nuclear correlations and memory effects are in the direction of still more increasing the effective factor $F$ of nuclear rates of hydrostatic burning and white dwarfs environment.

5. Signals in high-energy nuclear collisions

In this section we want briefly to remark as nonextensive statistical effects can be also very relevant also in the phenomenological interpretation of the high-energy nuclear collisions data. In fact, the quark-gluon plasma close to the critical temperature is a strongly interacting system. For such a
system, the color-Coulomb coupling parameter of the QGP can be defined in analogy as
\[ \Gamma \approx C \frac{g^2}{r_0 T} > 1 \]

where \( C = 4/3 \) or 3 is the Casimir invariant for the quarks or gluons, respectively, and \( \alpha_s = g^2/(4\pi) = 0.2 \pm 0.5 \), \( r_0 \approx n^{1/3} \approx 0.5 \) fm.

Near the phase transition, the interaction range is much larger than the Debye screening length (small number of partons in the Debye sphere). In fact, \( \lambda_D = 1/\mu \leq 0.2 \) fm (using the non-perturbative estimate: \( \mu = 6T \)). The Coulomb radius for a thermal parton with energy \( 3T \) is given by \( < r > = Cg^2/3T = 1 \div 6 \) fm. Therefore one obtain \( < r > /\lambda_D = 5 \div 30 \).

Memory effects and long-range color interactions give rise to the presence of non–Markovian processes in the kinetic equation affecting the thermalization process toward equilibrium as well as the standard equilibrium distribution.

A complete description of the applicability of nonextensive statistical effects to high-energy heavy ion collisions lies out the scope of this contribution. However, we want to outline that this aspect has been recently studied by us in connection to a phenomenological interpretation of the SPS data \(^{14,15}\) and an analysis of the transverse pion momentum spectra and the net proton rapidity distribution measured at RHIC is under investigation.

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