The effect of impact obliquity on shock heating in planetesimal collisions

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Abstract—Collisions between planetesimals in the early solar system were a common and fundamental process. Most collisions occurred at an oblique incidence angle, yet the influence of impact angle on heating in collisions is not fully understood. We have conducted a series of shock physics simulations to quantify oblique heating processes, and find that both impact angle and target curvature are important in quantifying the amount of heating in a collision. We find an expression to estimate the heating in an oblique collision compared to that in a vertical incidence collision. We have used this expression to quantify heating in the Rheasilsiv-forming impact on Vesta, and find that there is slightly more heating in a 45° impact than in a vertical impact. Finally, we apply these results to Monte Carlo simulations of collisional processes in the early solar system, and determine the overall effect of impact obliquity from the range of impacts that occurred on a meteorite parent body. For those bodies that survived 100 Myr without disruption, it is not necessary to account for the natural variation in impact angle, as the amount of heating was well approximated by a fixed impact angle of 45°. However, for disruptive impacts, this natural variation in impact angle should be accounted for, as around a quarter of bodies were globally heated by at least 100 K in a variable-angle model, an order of magnitude higher than under an assumption of a fixed angle of 45°.

INTRODUCTION

Collisions between planetesimals were common and fundamental events in the early solar system. Recent work has suggested that impact heating was an important complement to heating from short-lived radionuclide decay, especially in collisions between porous planetesimals (Davison et al. 2010). Almost all impacts occur with oblique incidence: The most common impact angle is 45° from the horizontal, and the probability of an impact occurring at an angle <70° from the horizontal is approximately 90% (Gilbert 1893; Shoemaker 1962). However, to date the effect of impact obliquity has not been accounted for in studies of shock heating in collisions between planetesimals; instead, numerical modeling studies have tended to assume a normal incidence angle. Scaling laws of melt production in impacts have tended to assume a normal incidence angle (e.g., Ahrens and O’Keefe 1977; Bjorkman and Holsapple 1987).

The effect of impact angle on crater shape is well documented: The crater size has been shown to scale with a dependence on sin(θ) for impacts in the gravity regime, and with sin²(θ) in the strength regime (Gault and Wedekind 1978). However, Pierazzo and Melosh (2000) showed that the volume of material heated in an impact depends on the transient crater volume, which scales with a dependence on approximately sin¹.₃(θ) for planetary impacts (Gault and Wedekind 1978; Schmidt and Housen 1987). In that pioneering study, the effects of porosity were not accounted for, and the results apply only for the case of an impact onto a planar
target surface. In addition, the dependence on the transient crater volume only seems to apply for impact angles $\geq 30^\circ$. Pierazzo and Melosh (2000) suggest that in more oblique impacts, the shock is weakened sufficiently that the heated volume cannot be normalized easily by the vertical incidence case. Other studies have simulated heating during impacts on curved surfaces, for example, during catastrophic collisions (Love and Ahrens 1996), hit-and-run collisions (Asphaug et al. 2006), and planetary-scale impacts on Mars (Marinova et al. 2008, 2011), but to date there has been no systematic study of the influence of target curvature on heating in oblique collisions between porous planetesimals.

3-D MODELING OF PLANETESIMAL COLLISIONS

In this study, the iSALE-3D shock physics model (Amsden and Ruppel 1981; Elbeshausen et al. 2009) was employed to investigate the effects of impact angle on heating in collisions between planetesimals, for a range of target curvatures and initial porosities (here, we define target curvature, $\chi$, as the ratio of the radii of the colliding planetesimals: $\chi = R_i/R_t$, where $R_i$ is the radius of the impactor, and $R_t$ is the radius of the target; see Fig. 1). iSALE-3D is a multimaterial, finite difference shock physics code which has been developed to simulate hypervelocity impact processes. A detailed description of the development history of iSALE-3D is presented in Elbeshausen et al. (2009). The code follows a similar approach to the 2-D model iSALE (Collins et al. 2004; Wünnemann et al. 2006), but has been adapted to run in three-dimensional Cartesian coordinates. Both iSALE-2D and iSALE-3D inherit much of their underlying structure from SALE/SALE3D (Amsden et al. 1980; Amsden and Ruppel 1981) and extensions of these two codes specifically developed for impact applications (Melosh et al. 1992; Ivanov et al. 1997; Ivanov and Artemieva 2002; Ivanov 2005). iSALE is well tested against laboratory experiments at low and high strain rates (Wünnemann et al. 2008); both codes have been benchmarked against other hydrocodes (Pierazzo et al. 2008), while iSALE-3D has been validated against impact experiments into ductile targets (Davison et al. 2011), and used to simulate a range of gravity and strength dominated craters (Elbeshausen et al. 2009, 2013). For this study, the $\varepsilon-\alpha$ porous compaction model (Wünnemann et al. 2006; Collins et al. 2011), which was previously available in the 2-D version of iSALE, was implemented in iSALE-3D.

To determine the mass of material heated in a collision, we followed the approach used in previous studies (Pierazzo and Melosh 2000; Davison et al. 2010): Lagrangian tracer particles were placed throughout the computational mesh and recorded the peak shock pressure experienced by the material they were assigned to at the beginning of the calculation. As they moved through iSALE-3D’s Eulerian grid the tracers recorded a full pressure-time history. This allowed us to perform two analyses in postprocessing of the simulations. First, we used the same approach as Davison et al. (2010) to convert the peak shock pressures into postshock temperatures (Table 1), and thus determine the mass of material heated to a given final temperature in the collision. Second, the original location of material shock heated to a given postshock temperature can be determined, similar to the approach used in Pierazzo and Melosh (2000).

The technique described above to determine the final temperature of the material is dependent on the accuracy of the ANEOS equation of state and the choice of porous compaction parameters when converting peak shock pressures to postshock temperatures. As the ANEOS equation of state does not account for the latent heat of melting, ANEOS overestimates temperatures in excess of the melt temperature. To account for this source of inaccuracy, the peak shock pressures and entropy that correspond to the postshock temperatures that are used in this work are also presented in Table 1. As the ANEOS equations of state are improved in the future, these shock pressures and entropies can be used to amend the temperatures quoted in the remainder of this study.

Fig. 1. The impact angle, $\theta$, is measured from the tangential plane at the point of impact (long dashed line; this line is also equivalent to the target surface in planar target simulations, $\chi = 0$). Target curvature is defined as the ratio of the impactor radius to the target radius, $\chi = R_i/R_t$. 

- **Fig. 1.** The impact angle, $\theta$, is measured from the tangential plane at the point of impact (long dashed line; this line is also equivalent to the target surface in planar target simulations, $\chi = 0$). Target curvature is defined as the ratio of the impactor radius to the target radius, $\chi = R_i/R_t$. 

Table 1. Peak shock pressure, specific entropy, and specific internal energy associated with a given postshock temperature.

| Postshock temperature\(^a\) (K) | Peak shock pressure\(^b\) (GPa) | Specific entropy\(^b\) (J kg\(^{-1}\)) | Specific internal energy\(^b\) (J kg\(^{-1}\)) |
|---------------------------------|--------------------------------|--------------------------------|--------------------------------|
| 310                             | 17                            | 0.55                           | 2.4                            |
| 320                             | 21                            | 1.1                            | 3.5                            |
| 330                             | 24                            | 1.7                            | 4.5                            |
| 340                             | 27                            | 2.3                            | 5.3                            |
| 350                             | 29                            | 2.9                            | 6.1                            |
| 400                             | 37                            | 5.9                            | 9.5                            |
| 500                             | 49                            | 12                             | 15.0                           |
| 600                             | 58                            | 19                             | 21.0                           |
| 700                             | 66                            | 25                             | 28.0                           |
| 800                             | 81                            | 31                             | 36.0                           |
| 900                             | 85                            | 37                             | 44.0                           |
| 1000                            | 89                            | 42                             | 5.2                            |

\(^a\)From an initial temperature of 300 K.

\(^b\)Pressure, entropy, and energy are calculated by following the procedure of Davison et al. (2010), using the ANEOS equation of state for forsterite/dunite (Benz et al. 1989) and the \(\varepsilon-\alpha\) porous compaction model (Wünnemann et al. 2006; Collins et al. 2011).

However, as most of this study focuses on heating to temperatures below the melt temperature, this error is of little importance here.

**Material Model**

The material was simulated using the ANEOS equation of state for dunite/forsterite (Benz et al. 1989), which has been shown to be a reasonable analog for meteoritic material (Davison et al. 2010). Material was assigned a shear strength using the strength model described in Collins et al. (2004), with parameters appropriate for weak rock (Leinhardt and Stewart 2009).

Porosity is an important parameter controlling the amount of heating in planetesimal collisions. Recent experimental (Teiser and Wurm 2009), computational (Cuzzi et al. 2008), and observational studies (Bland et al. 2011) have shown that porosity would have been significant (>50%) in the earliest solid bodies to form in the solar system. For this work, we have implemented the \(\varepsilon-\alpha\) porous compaction model (Wünnemann et al. 2006; Collins et al. 2011) in iSALE-3D, to fully quantify the effects of impact obliquity on heating in collisions between analogs for early solar system materials. Based on previous studies (Wünnemann et al. 2008; Davison et al. 2010), the porous compaction parameter, \(\kappa\), was set to 0.98. The volumetric strain at the onset of plastic compaction is \(10^{-5}\), which is roughly equivalent to a stress of approximately 1 MPa, similar to the cohesive strength of the material. A limitation of the continuum approximation used in the \(\varepsilon-\alpha\) porous-compaction model is that it requires the scale of the pores to be smaller than the scale of the computational cells, and for the pores to be uniformly distributed throughout the material. Any heating and compaction are therefore averaged over the bulk material. In natural materials, pore space is often heterogeneously distributed; thus heating by shockwaves can lead to localized “hotspots” (Kieffer et al. 1976), on the scale of the pores, which cannot be resolved by this model. Mesoscale modeling (Güldemeister et al. 2013; Bland et al. 2014; Davison et al. 2014) and experimental studies (Neal and Collins 2013; Neal et al. 2014) are ongoing to help resolve these heterogeneous small-scale processes.

**Initial and Boundary Conditions**

Simulations were performed over a large parameter space. Parameters studied include the initial porosity \((\phi = 0\text{--}50\%)\), the impact angle \((\theta = 90^\circ\text{--}15^\circ)\) measured from the tangent plane to the target surface; see Fig. 1), and the target curvature. In the simulations in this work, we modeled collisions in the range \(\chi = 0\text{--}0.2\), where \(\chi = 0\) is a planar target. Monte Carlo simulations (Davison et al. 2013) show that impacts with a low \(\chi\) were the most common type of impact on a meteorite parent body, and thus higher \(\chi\) collisions were not considered here. The range of \(\chi = 0\text{--}0.2\) encompasses >99.98% of all impacts expected on 100 km radius parent bodies during the first 100 Myr of solar system evolution. Rare, but energetically important collisions with greater \(\chi\) also require greater computation resources, and will be investigated in a future study.

To reduce the size of the parameter space explored, the initial temperature of the material was kept constant throughout all of the impacts presented in this work. The initial temperature of the material was a constant 300 K throughout both the impactor and target.
small differences in initial temperature, the results here
will hold, but for much higher temperatures (e.g., near
the solidus), further simulations are required to
determine the effect of impact angle. In most of the
simulations presented below, the impact velocity was
4 km s\(^{-1}\) (a typical collision velocity for planetesimals
in the early solar system, e.g., Bottke et al. 1994;
O’Brien et al. 2007; Davison et al. 2013). In some of the
simulations discussed, a velocity of 20 km s\(^{-1}\) was
chosen to compare to simulations in previous work
(Pierazzo and Melosh 2000). Future studies will explore
the combined effects of impact velocity, initial
temperature, and impact angle on impact heating.

The computational mesh was constructed as a half
space, so that only half of the domain needed to be
modeled, saving on computational resources and
allowing higher resolution simulations to be run. The
boundary of the mesh that included the impact velocity
vector and the line connecting the centers of the
impactor and target therefore acted as a symmetry plane.
The boundary condition on this face was a free-slip
condition, where material was allowed to move along the
boundary, but velocities normal to the face were set to
zero. All other boundaries allowed continuous outflow of
material—any impact ejecta that traveled quickly away
from the impact site was lost from the calculation. To
represent curved surfaces on a Cartesian mesh, any cell
that had at least half of its vertices within one radius of
the center of the body was assigned the appropriate
material properties for that body (projectile or target).
Time \(t = 0\) was defined as the instant that the projectile
made first contact with the target body.

Resolution

Three different model geometries were used in this
study, which can be defined by the target curvature,
relative to the size of the impactor. In all simulations
presented in the Results section, the impactor radius
was resolved by 20 computational cells, which
corresponded to 16,280 lagrangian tracer particles in the
impactor. For simulations with \(\chi = 0.1\), the target had
200 cells across its radius, and for \(\chi = 0.2\), the target
had 100 cells across its radius. In all cases, the impactor
radius was held constant at 50 km. However, as only
the shock and release stages of the collision were
modeled, gravity and strength did not affect the
outcome of the simulations, and therefore the results of
this study are independent of planetesimal size
(provided the assumption that pore-spaces are small
compared to the finest mesh size still holds); thus, all
results are presented in dimensionless units.

To test the dependence of impact heating on the
resolution of the computational mesh, a series of
simulations were run at a range of resolutions, between
5 cells per projectile radius (cppr) and 24 cppr for an
impact at 4 km s\(^{-1}\) into a target plane. In Fig. 2(a), the
heated mass from each simulation is normalized by the
mass in the equivalent simulation (same impact angle)
at the maximum resolution modeled (24 cells per projectile
radius), resulting in a range of resolutions up to 48 cppr.

![Resolution study. a) The mass of material heated by at
least 100 K (i.e., heated above 400 K) in a suite of iSALE-3D
simulations over a range of impact angles (30°–90°), for an
impact at 4 km s\(^{-1}\) into a target plane. Heated masses are
normalized by the equivalent simulation (same impact angle)
at the highest resolution modeled (24 cells per projectile
radius). b) iSALE-2D and iSALE-3D simulations of vertical
incidence angle impact simulations equivalent to those in (a),
for a range of resolutions up to 48 cppr.](image)
show that we are indeed approaching the asymptote. 20 cppr was chosen as a compromise between accuracy and model run time.

RESULTS: IMPACTS INTO PLANAR TARGETS

Benchmark Test: \( v_i = 20 \text{ km s}^{-1} \)

To verify that the model could produce results consistent with previous studies, a set of simulations similar to those in Pierazzo and Melosh (2000) were performed: This suite of simulations modeled impacts at 20 km s\(^{-1}\) into a target plane. For a close comparison with the previous models, one set of impact simulations used 0% porous dunite, and to determine the effect of porosity on those results, we also ran the same simulations with 50% porous dunite. In Fig. 3, the mass of material shocked to 50 GPa in the Pierazzo and Melosh (2000) simulations is plotted as a function of impact angle. The mass of material shocked to 50 GPa in the simulations with no porosity (equivalent to a final temperature of 510 K) is also plotted, along with the mass of material from the simulations with 50% porosity shocked to 1.6 GPa (chosen to give the same postshock temperature as the nonporous simulations). To allow a comparison between the two suites of simulations with different initial porosities, and with the Pierazzo and Melosh (2000) simulations, the heated mass was normalized to the mass heated in the vertical incidence impact in each suite of simulations.

All suites of simulations shown in Fig. 3 display a similar qualitative dependence on impact angle. The nonporous simulations have a slightly stronger dependence on impact angle compared to the porous simulations. The simulations in Pierazzo and Melosh (2000) used a target composition to match the preimpact stratigraphy of the Chicxulub impact site, composed of layers of water, calcite, and dunite. In the simulations in this study, because we are interested in collisions between planetesimals, the material used for the target was a single layer of dunite; thus, the results are not expected to match the previous work exactly, but they do show that the dependence of impact heating on impact angle is consistent between the different studies. Two fits are shown in Fig. 3: one to the porous simulations (dashed line) and another to the nonporous simulations (solid line). These fits used the function:

\[
\frac{M( > P_{sh}, \chi, \theta)}{M( > P_{sh}, \chi = 0, \theta = \perp)} = \alpha \sin^b \theta
\]

in the simulations with no porosity (equivalent to a final temperature of 510 K) is also plotted, along with the mass of material from the simulations with 50% porosity shocked to 1.6 GPa (chosen to give the same postshock temperature as the nonporous simulations). To allow a comparison between the two suites of simulations with different initial porosities, and with the Pierazzo and Melosh (2000) simulations, the heated mass was normalized to the mass heated in the vertical incidence impact in each suite of simulations.

Table 2. Fitting parameters\(^a\) for impacts into planar targets.

| Simulations          | \( \alpha \) | \( \beta \) | \( R^2 \) |
|----------------------|--------------|--------------|----------|
| This work\(^b\), porous, 20 km s\(^{-1}\) | 1.05         | 0.86         | 0.95     |
| This work\(^b\), nonporous, 20 km s\(^{-1}\) | 1.04         | 1.11         | 0.97     |
| P&M 2000\(^c\), nonporous, 20 km s\(^{-1}\) | 1.08         | 1.09         | 0.93     |
| This work\(^d\), porous, 4 km s\(^{-1}\) | 1.06         | 0.82         | 0.92     |
| This work\(^d\), nonporous, 4 km s\(^{-1}\) | 0.99         | 9.22         | 0.99     |

\(^a\)From Equation (1).
\(^b\)Benchmark tests (20 km s\(^{-1}\)) shown in Fig. 3.
\(^c\)Pierazzo and Melosh (2000).
\(^d\)4 km s\(^{-1}\) simulations presented in Fig. 4.

Fig. 3. Benchmarking comparison of impact heating in simulations from Pierazzo and Melosh (2000) and simulations in this study. The impact velocity was 20 km s\(^{-1}\). For the Pierazzo and Melosh (2000) simulations, the heated mass was taken to be all material shocked above \( P_{sh} = 50 \text{ GPa} \). In the nonporous simulations of this study, \( P_{sh} = 50 \text{ GPa} \) was used, equivalent to a final temperature of 510 K (Table 1). In the porous simulations of this study, \( P_{sh} = 1.6 \text{ GPa} \) was used, also equivalent to a final temperature of 510 K. Shock heated masses are normalized by the heated mass in the equivalent simulation with a normal incidence impact angle. The dashed line is a fit to the porous simulations, and the solid line is a fit to the nonporous simulations. See the text for details.
exponent $\beta$ is related to the exponent, $\gamma$, from the $\pi$-group scaling law by $\beta = 2\gamma$ (Chapman and McKinnon 1986). The results shown here are in qualitative agreement with Schmidt and Housen (1987), that $\gamma$ for nonporous material is greater than $\gamma$ for porous material. In the Schmidt and Housen (1987) experiments, $\gamma = 0.65$ for competent rock, and $\gamma = 0.51$ for dry sand (compared to $\gamma = 0.56$ and 0.43, respectively, from the simulations described above). For comparison, the results from the Pierazzo and Melosh (2000) simulations are also shown in Fig. 3 and Table 2; the fit to those data yield $\gamma = 0.54$, similar to the nonporous simulations from this work.

**Lower Velocity Results:** $v_i = 4$ km s$^{-1}$

In Fig. 4, the mass of material heated by at least 100 K is shown for a range of impact angles ($\theta = 15^\circ$–$90^\circ$), from simulations of impacts into a planar target ($\chi = 0^\circ$) at 4 km s$^{-1}$, for both porous and nonporous dunite. The heated mass is normalized by the mass of material heated by 100 K in the simulation with $\theta = 90^\circ$. The fitting parameters are shown in Table 2. For the porous simulations, $\gamma = 0.43$ ($R^2 = 0.92$). However, for the nonporous simulations, the $\gamma$ required to fit the data is much higher ($\gamma = 4.6$). This is because at this low velocity, the volume of material heated is small compared to the impactor volume, and thus the approximation of the impact as a point source of momentum and energy no longer applies. To illustrate this, the heated mass normalized by the impactor mass for planar target impacts at both 20 km s$^{-1}$ and 4 km s$^{-1}$ into porous and nonporous materials has been plotted against $v_i^2/E_T$, where $E_T$ is the specific internal energy of the shock state required to produce a final (postrelease) temperature, $T$ (Fig. 5). In the literature (e.g. Pierazzo and Melosh 2000; Wünnemann et al. 2008), $v_i^2/E_T$ for the specific case where $T$ is the melt temperature is often referred to as the “melt number.”

As we are interested in a range of temperatures lower than the melt temperature, we refer to this number here as the heating number. For the four impact simulations represented on Fig. 5, each point represents the mass heated to a different final temperature listed in Table 1. If the point-source approximation applies, a power law can fit the data (typically, this is true for $v_i^2/E_T \gtrsim 30$). In the 20 km s$^{-1}$ impacts, all final temperatures shown for both porous and nonporous materials lie on a power law; the heated volumes are sufficiently large that the point-source approximation applies. For the 4 km s$^{-1}$ impact, a power law trend is observed for $T < 600$ K in porous material, but only for $T < 320$ K in nonporous material.
material. In this case, the mass heated to higher temperatures is so small that the point-source approximation is no longer valid. This explains the high γ required to describe the dependence of impact heated mass on impact angle for the nonporous scenario depicted in Fig. 4.

**RESULTS: THE EFFECT OF TARGET CURVATURE**

The results in this section are for impacts at 4 km s\(^{-1}\) into porous dunite targets. These parameters were chosen as typical conditions for a collision between planetesimals in the early solar system, when collision velocities were expected to be in the range 1–10 km s\(^{-1}\), with a mean of approximately 4 km s\(^{-1}\) (Bottke et al. 1994; O’Brien et al. 2007; Davison et al. 2013), and planetesimals were expected to retain some porosity.

**Preimpact Position of Heated Mass**

Figure 6 plots the initial location of each tracer particle that originated in the plane of impact (i.e., the plane perpendicular to the target plane that includes the impact trajectory). Each particle is colored by its postshock temperature, and plotted in its initial, preimpact position (following the scheme of Pierazzo and Melosh 2000). The top row of figures shows impacts into planar targets. The dependence of the position of the heated material on impact angle is qualitatively similar to the shock heated region in Pierazzo and Melosh (2000). The amount and location of heated material in impacts at 90° is not strongly affected as the target curvature increases (left-hand column of Fig. 6). However, at more oblique impact angles, the heated volume is smaller as the target surface becomes more curved (this is quantified in the following section). There are two reasons for this, related to the geometry of the impact (1) the material available to be heated in the downrange direction decreases as the curvature increases: the mass “missing” between the horizontal tangent to the target and the curved surface increases with increasing curvature; and (2) some of the mass of the impactor will go on to miss the target altogether in impacts with high impact angles and high target curvature (e.g., the bottom-right frame in Fig. 6)—thus, the so-called “interacting mass” of impactor is lower (Leinhardt and Stewart 2012). These two effects imply that at more oblique impact angles, the impact energy is not coupled to the target as effectively as in impacts into planar targets or at
vertical incidence angles. Both effects must be accounted for to fully quantify the effect of impact angle and target curvature on impact heating.

The Mass of Heated Material

The effect of target curvature on heated mass was studied for an impact heating scenario in which the point-source approximation does apply in the planar-target case \((v_i = 4 \text{ km s}^{-1}, \phi = 0.5, T_f = 400 \text{ K})\). In Fig. 7(a), the heated mass of material in three suites of impact simulations is shown, for \(\chi = 0, 0.1, \text{ and } 0.2\). This mass is normalized by the equivalent mass of material heated in a normal incidence angle impact onto a planar \((\chi = 0)\) target surface. As the curvature increases, the amount of heated material for a given impact angle decreases. To account for this decrease in heated mass due to the curvature, a modification needs to be made to Equation (1): The mass of material for any combination of \(\chi\) and \(\theta\) can be well fit by:

\[
M(> T, \chi, \theta) = a \sin^b(h \theta) / C_0 \cos^b(v \theta) / C_10 / C_17, (2)
\]

where \(a\) and \(b\) are the fitting parameters determined from the impact into a planar surface (Equation 1; Table 2; Fig. 4). In the case of impacts into a planar surface, the cosine term on the right-hand side tends to zero, and the fit is the same as the fit shown in Fig. (4) and Equation (1). For \(\chi = 0.1\), the coefficient of determination, \(R^2 = 0.99\), and for \(\chi = 0.2\), \(R^2 = 0.97\).

To visualize the goodness of fit, the normalized heated mass from the iSALE simulations was plotted against the normalized heated mass expected from Equation (2), assuming \(\alpha = 1.06\) and \(\beta = 0.82\) (Fig. 7b).

DETERMINING HEATING IN AN OBLIQUE COLLISION ON A CURVED SURFACE

For any given collision, the amount of material heated to a given temperature can be estimated (as long as the point-source approximation applies) using the following steps

1. Find the critical specific internal energy, \(E_T\), associated with heating the material from the ambient temperature to the required final temperature.
2. Use scaling relationships from the literature (e.g., Pierazzo et al. 1997) or from simple 2-D simulations to determine the amount of heated material in a normal incidence angle impact into a planar target.
3. Adjust the amount of heating in the impact using Equation (2) to account for impact angle and target curvature.

Worked Example: Rheasilvia Crater on Vesta

In this section, we work through these steps for the Rheasilvia-forming impact on the asteroid Vesta. Recent numerical modeling has been able to reproduce a crater with similar size and morphology to Rheasilvia (Ivanov and Melosh 2013) and the deformation at the Rheasilvia antipode (Bowling et al. 2013). In these (vertical incidence) impact simulations,
Table 3. Melt scaling relationship fitting parameters from the literature for a range of geologic materials, for use in Equation 3.

| Material   | Porosity $\phi$ (%) | $T_{\text{final}}$ | $a$       | $\mu$       | $R^2$ | Point-source threshold | Reference          |
|------------|---------------------|---------------------|-----------|-------------|-------|------------------------|--------------------|
| Quartzite  | 0                   | Incipient melting   | $-0.867 \pm 0.054$ | $0.663 \pm 0.017$ | 0.998 | 30                     | Wünnewann et al. (2008) |
|            | 25                  |                     | $-0.567 \pm 0.063$ | $0.589 \pm 0.009$ | 0.999 | 30                     |                    |
|            | 50                  |                     | $-0.632 \pm 0.051$ | $0.556 \pm 0.003$ | 0.999 | 30                     |                    |
| Dunite     | 0                   | Incipient melting   | $-0.871 \pm 0.083$ | $0.662 \pm 0.018$ | 0.997 | 30                     |                    |
|            | 0                   | Complete melting    | $-0.972 \pm 0.064$ | $0.701 \pm 0.018$ | 0.998 | 30                     | Pierazzo et al. (1997) |
| Aluminum   | 0                   | Complete melting    | $-0.595 \pm 0.064$ | $0.667 \pm 0.017$ | 0.998 | 30                     |                    |
| Iron       | 0                   |                     | $-0.636 \pm 0.095$ | $0.699 \pm 0.026$ | 0.997 | 30                     |                    |
| Ice        | 0                   |                     | $-0.811 \pm 0.099$ | $0.708 \pm 0.029$ | 0.997 | 30                     |                    |
| Ice [150 K]| 0                   | Complete melting    | $-0.275 \pm 0.190$ | $0.554 \pm 0.08$  | n/a   | 80                     | Kraus et al. (2011)    |
|            | 25                  |                     | $-0.390 \pm 0.195$ | $0.572 \pm 0.09$  | n/a   | 80                     |                    |
|            | 50                  |                     | $-0.505 \pm 0.200$ | $0.589 \pm 0.13$  | n/a   | 80                     |                    |
| Dunite     | 0                   | 400 K               | $-0.47 \pm 0.06$   | $0.59 \pm 0.02$   | 0.994 | 100                    | This work$^b$         |
|            |                     | 700 K               | $-0.85 \pm 0.05$   | $0.65 \pm 0.02$   | 0.995 | 50                     |                    |
|            |                     | 1000 K              | $-0.89 \pm 0.06$   | $0.66 \pm 0.03$   | 0.993 | 40                     |                    |
|            | 10                  | Incipient melting   | $-0.86 \pm 0.05$   | $0.66 \pm 0.03$   | 0.995 | 30                     |                    |
|            |                     | 400 K               | $-0.30 \pm 0.03$   | $0.51 \pm 0.01$   | 0.999 | 100                    |                    |
|            |                     | 700 K               | $-0.45 \pm 0.03$   | $0.56 \pm 0.01$   | 0.998 | 70                     |                    |
|            |                     | 1000 K              | $-0.41 \pm 0.04$   | $0.55 \pm 0.02$   | 0.996 | 50                     |                    |
|            | 25                  | Incipient melting   | $-0.55 \pm 0.05$   | $0.60 \pm 0.02$   | 0.996 | 40                     |                    |
|            |                     | 400 K               | $-0.30 \pm 0.01$   | $0.47 \pm 0.01$   | 0.999 | 100                    |                    |
|            |                     | 700 K               | $-0.60 \pm 0.01$   | $0.54 \pm 0.01$   | 0.999 | 50                     |                    |
|            |                     | 1000 K              | $-0.47 \pm 0.02$   | $0.51 \pm 0.01$   | 0.999 | 40                     |                    |
|            | 50                  | Incipient melting   | $-0.43 \pm 0.02$   | $0.51 \pm 0.01$   | 0.997 | 40                     |                    |
|            |                     | 400 K               | $-0.51 \pm 0.02$   | $0.47 \pm 0.01$   | 0.998 | 40                     |                    |
|            |                     | 700 K               | $-0.60 \pm 0.01$   | $0.49 \pm 0.01$   | 0.999 | 40                     |                    |
|            |                     | 1000 K              | $-0.52 \pm 0.01$   | $0.48 \pm 0.01$   | 0.999 | 30                     |                    |
|            | 50                  | Incipient melting   | $-0.50 \pm 0.01$   | $0.49 \pm 0.01$   | 0.999 | 30                     |                    |

$^a$The point-source threshold is the minimum value of the melt/heating number ($v^2/E_T$) for which these scaling parameters ($a$, $\mu$) apply.

$^b$Scaling parameters for a range of porosities and final temperatures, calculated from two-dimensional iSALE simulations.

The impactor radius was estimated to be approximately 18.5 km, using the assumed mean impact velocity of 5.5 km s$^{-1}$ (see also Asphaug 1997; Jutzi and Asphaug 2011).

**Step 1**

Assuming the Vesta mantle is composed of forsterite/dunite, we can calculate the specific internal energy $E_T$ of the shocked state that results in a final (postshock) temperature $T_{\text{rel}}$ using the ANEOS equation of state for dunite (Benz et al. 1989) and the $\varepsilon$-$\alpha$ porous-compaction model (Wünnewann et al. 2008; Appendix A of Davison et al. 2010). Performing the above calculation for a suite of final temperatures yields the internal energies presented in Table 1, applicable for an initial temperature of 300 K. For a 10% porous dunite mantle, $E_T$ can be read from Table 1; $v^2/E_T = 112$ for $T = 400$ K. In this particular impact, for temperatures of 700 K and above, $v^2/E_T < 30$.

**Step 2**

Melt scaling relationships have been determined for vertical incidence impacts that relate the heated volume to $v^2/E_T$ (e.g., Ahrens and O’Keefe 1977; Bjorkman and Holsapple 1987):

$$\log \left( \frac{V_{\text{melt}}}{V_{\text{imp}}} \right) = a + \frac{3}{2} \mu \cdot \log \left( \frac{v^2}{E_T} \right)$$  \hspace{1cm} (3)$$

where the constants $a$ and $\mu$ have to be determined empirically. A list of values from the literature for a range of geologic materials is presented in Table 3. No constants for porous dunite have been determined to date; thus, here we performed some two-dimensional iSALE simulations of vertical impacts into a planar dunite target, over a range of impact velocities (4–30 km s$^{-1}$) and porosities (0–50%). The amount of material shock heated to a range of final temperatures was calculated using the Lagrangian tracer technique described earlier. As we are interested in temperatures
below the melt temperature, we have calculated the fit parameters $a$ and $\mu$ for a range of final temperatures, by a least-squares fit to Equation 3 for those impacts in the power-law (point-source) regime. These fit parameters, and the minimum heating number for which they can be applied (which increases with decreasing temperature), are also presented in Table 3. Our results for incipient melting of dunite are in good agreement with Wünne et al. (2008). While the uncertainties from the least-squares fit are presented in Table 3, perhaps a better gauge of the inherent uncertainty in these numbers can be gleaned from comparing the results from different studies for similar materials. For $T = 400$ K and $v_i = 5.5$ km s$^{-1}$, the heating number (112) is above the threshold determined from the fitting for 10% porous dunite, and thus the technique described above can be used to determine the amount of material heated to 400 K.

Using the scaling parameters from Table 3, we find that approximately 19 times the impactor volume is heated to 400 K (from an assumed starting temperature of 300 K). For $T = 700$ K, the heating number is 28 in the 5.5 km s$^{-1}$ Rheasilvia impact, which is below the threshold for the point-source approximation (70).

**Step 3**

Finally, we need to account for the effects of impact angle and target curvature. To account for impact angle, Collins et al. (2005) note that the crater diameter scales with $\sin^{-0.33}(\theta)$. Rearranging equation (21) from Collins et al. (2005) shows that the impactor radius will scale as $r_i(\theta) = r_i(90^\circ)/\sin^{0.43}(\theta)$: For example, for an impact angle of $45^\circ$, an impactor radius of 18.5/\sin^{0.43} (45°) = 21.5 km is required; impactor radii for a range of impact angles are presented in Table 4. By assuming Vesta was spherical before the Rheasilvia impact with a radius of 260 km, the target curvature, $\chi$, associated with each impactor can be determined. Then, using Equation 2, the amount of heated material determined in step 2 can be modified to account for the impact angle, impactor radius, and target curvature (Table 4). Here, we have used the values of $\alpha$ and $\beta$ determined above for porous dunite: $\alpha = 1.06$ and $\beta = 0.82$. At $\theta = 45^\circ$, the amount of material heated to 400 K is approximately 14.2 impactor volumes (compared to 19.7 at 90$^\circ$; Fig. 8a). However, if the heated volume is normalized by the volume of Vesta, we see that the total amount of heating is similar for all angles in the range $0^\circ$–90$^\circ$; the most heating occurs at $\theta = 45^\circ$, since the impactor required to form Rheasilvia is more massive at more oblique impact angles, which counters the reduction in heated mass at oblique angles.

**OBLIQUE INCIDENCE IMPACTS IN THE EARLY SOLAR SYSTEM**

To examine the importance of impact angle and target curvature on the total amount of heating in impacts between planetesimals, the parameterization described in this work was applied in a Monte Carlo simulation of impacts on meteorite parent bodies (Davison et al. 2013), which combines the results of collisional and dynamical models of the planetesimal population in the early solar system (O’Brien et al. 2006, 2007), scaling laws and hydrocode models to determine the range of plausible early impact histories of meteorite parent bodies.

The impact angle has to be accounted for in two places in the Monte Carlo calculation. First, the effective impact velocity, $v_{cr}$, used in calculating crater dimensions was taken to be the vertical component of the impact velocity ($v_{cr} = v_i \sin \theta$); and second, the amount of heated material was estimated using the scaling law developed above (Equation 2), with constants appropriate for porous dunite. The Monte Carlo simulation was run for 100 Myr on $10^5$ parent bodies with 100 km radius and a porosity of 0.5. Three Monte Carlo simulations were run to investigate the effect of impact angle on impact heating: (1) a constant impact angle of 90$^\circ$ to the target plane (analogous to the simulations of Davison et al. 2013) (2) a constant impact angle of 45$^\circ$ to the target plane (the most

| Angle, $\theta$ ($^\circ$) | Impactor radius$^a$, $r_i$ (km) | Curvature, $\chi$ | $V(>400 \text{ K})/V_i$ | $V(>400 \text{ K})/V_{Vesta}$ |
|---------------------------|-----------------------------|------------------|--------------------------|-----------------------------|
| 90                        | 18.5                        | 0.071            | 19.7                     | $7.1 \times 10^{-3}$       |
| 75                        | 18.8                        | 0.072            | 19.1                     | $7.2 \times 10^{-3}$       |
| 60                        | 19.7                        | 0.076            | 17.5                     | $7.6 \times 10^{-3}$       |
| 45                        | 21.5                        | 0.083            | 14.2                     | $8.0 \times 10^{-3}$       |
| 30                        | 24.9                        | 0.096            | 5.41                     | $4.7 \times 10^{-3}$       |
| 15                        | 33.0                        | 0.127            | 0.0                      | 0.0                         |

$^a$The impactor radius is scaled by $r_i(\theta) = r_i(90^\circ)/\sin^{0.43}(\theta)$.
frequent impact angle), and (3) a variable impact angle, selected by a random number, $\mathcal{R}$, and chosen so that the probability of an impact occurring at an angle greater than $\theta$ was $P(>\theta) = \cos^2\theta$ (Gilbert 1893; Shoemaker 1962); i.e., $\mathcal{R}$ was converted to the impact angle using $\theta = \arccos(\sqrt{\mathcal{R}})$, where $0 < \mathcal{R} < 1$.

**Monte Carlo results**

In the simulations with fixed impact angles of $90^\circ$ and $45^\circ$, 8.4% and 7.6% of parent bodies were catastrophically disrupted in the first 100 Myr, respectively: the difference between the two is due to some oblique impacts falling below the disruption threshold that would have caused a disruption if they were vertical incidence. In the simulation with a variable impact angle, 7.2% of parent bodies were disrupted. The slightly lower disruption rate for the variable-angle simulation is because some very oblique impacts are unable to disrupt the body. In all simulations, on parent bodies that were not disrupted within the first 100 Myr, there were on average $852 \pm 26$ collisions of impactors with a radius $>150$ m. Figure 9 details the amount of heating done by these impacts on the parent bodies. For each parent body simulated, the cumulative mass of material heated to at least 400 K from each of the approximately 850 impacts was calculated using relationships derived from hydrocode simulations (Davison et al. 2013; see also Table 3), and modified by Equation 2. The results in Fig. 9 are split into those bodies that survived for 100 Myr without experiencing a disruptive collision (Fig. 9a) and those that were disrupted before 100 Myr (Fig. 9b). For surviving parent bodies in the $90^\circ$ fixed-angle simulation, the fraction of the parent body heated to 400 K is log-normally distributed, with $\log_{10}(f_{\text{max}}/f_{\text{min}}) = 0.1$, where $f_{\text{max}}$ and $f_{\text{min}}$ are the upper and lower bounds of the fractional heating of a parent body in each bin. Not shown on this figure are the disrupted parent bodies that were globally heated to 400 K in the $\theta = 90^\circ$ case.
heating effects in nondisrupted parent bodies it is not essential to account for the natural variation in impact angle, as this is well approximated by the assumption of a constant impact angle of 45°.

For bodies that were disrupted before 100 Myr (Fig. 9b), the heated mass-frequency distributions are more complex than log-normal and there are significant differences between the distributions for the three simulations. In particular, there are a large number of parent bodies that have been globally heated (more than 90% of their volume) to at least 400 K in both the 90° fixed-angle simulation (around 65% of disrupted bodies) and the variable-angle simulation (around 26% of disrupted parent bodies). In the 45° fixed-angle simulation, however, these globally heated parent bodies are rare: just 2.4% of parent bodies. Of those parent bodies that are globally heated in the variable-angle simulation, the minimum impact angle required to achieve global heating in a disrupted collision was 45°, and 95% of those collisions occurred at angles steeper than 50°, explaining why so few bodies are globally heated when assuming a fixed angle of 45°. To estimate the heating in disruptive impact, it is important to account for the natural variation in impact angle; using a fixed 45° underestimates the amount of heating, and using a fixed 90° leads to an overestimation.

CONCLUSIONS

We have simulated hypervelocity collisions between planetesimals and determined the combined effect of impact angle and target curvature on collisional heating. In impacts for which the point-source approximation applies, the heated mass can be estimated using Equation (2) for any combination of impact angle and target curvature, once the heated mass in a planar impact at vertical incidence is known, and two material-specific constants have been determined. When applied to impact heating in the Rheasilvia impact on Vesta, this technique shows that impact angles of 45–90° would have yielded approximately the same total volume of heated material, with a maximum at θ = 45°. We also applied this calculation to Monte Carlo simulations of impact heating on meteorite parent bodies in the first 100 Myr of solar system evolution. For parent bodies that survived without a disruptive impact, the cumulative heated mass was approximately the same if the impact angle was assumed to be a constant 45° (the most common impact angle) or if the natural variation in impact angle (from vertical incidence to a grazing collision) was accounted for. However, the natural variation in impact angle has important consequences for disruptive parent body collisions: a much higher proportion of disrupted parent bodies are heated globally to 400 K under the assumption of variable impact angle (1 in 4) compared to the assumption of a fixed 45° impact angle (1 in 40). Further high-resolution numerical modelling of oblique incidence angle, catastrophic impact events is required to fully understand this phenomenon.

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Shock heating in oblique planetesimal collisions