Exact nonlinear dust kinetic Alfvén waves in a dust–ion plasma

Arshad M Mirza, M Ansar Mahmood and G Murtaza
Theoretical Plasma Physics Group, Department of Physics, Quaid-i-Azam University, Islamabad 45320, Pakistan
E-mail: a.m.mirza@yahoo.com

New Journal of Physics 5 (2003) 116.1–116.11 (http://www.njp.org/)
Received 27 April 2003
Published 26 September 2003

Abstract. The properties of nonlinear dust kinetic Alfvén waves in a collisionless, low (but finite)-β, dust–ion plasma are investigated by employing the pseudo-potential approach, which is valid for arbitrary amplitude solitary waves. It is shown both analytically and numerically that a dusty plasma model can support solitary waves consisting of density humps or dips. Furthermore, the properties of these solitary waves are found to be significantly modified by the speed and obliqueness of the wave propagation. The findings of the present investigation should be useful in understanding the formation of coherent nonlinear soliton structures in space and astrophysical dusty plasmas, such as in planetary rings and cometary tails.

Contents

1 Introduction 2
2 Mathematical formulation 3
3 Localized solutions 5
4 Summary 10
   Acknowledgments 10
   References 10

1 Permanent address: Salam Chair in Physics, Government College University, Lahore 54000, Pakistan.
1. Introduction

There has been a great deal of interest in studying kinetic Alfvén waves (KAWs) propagating obliquely to an external magnetic field in order to explain electromagnetic fluctuations and nonlinear structures in space and laboratory plasmas. Solitary kinetic Alfvén waves (SKAWs) arise when the dispersion relation of ordinary Alfvén waves is modified by the finite Larmor radius effect or simply by the finite electron inertial effect [1]. Recent data from the Freja satellite [2, 3] showed that the auroral low-frequency turbulence is dominated by strong electromagnetic spikes, which resemble solitary structures and can be interpreted as SKAWs. Most recent data from the Freja and FAST satellites [4] also reveal a clear signature of solitary Alfvénic structures (i.e. density cavities) in the auroral zone of the Earth’s ionosphere. Sagdeev [5] was the first who studied the arbitrary amplitude nonlinear wave in a simple unmagnetized plasma in the form of an integral equation analogue to a classical particle oscillation in a potential well derived from the basic governing equations. Since then the soliton structures and their applications in plasmas have been studied [6]–[8] in great detail following the Sagdeev potential approach. However, the nonlinear structures of KAWs were first studied by Hasegawa and Mima [9]. They presented an exact solution for an arbitrary amplitude solitary Alfvén wave by considering a collisionless low-$\beta$ ($m_e/m_i \ll \beta \ll 1$, where $\beta = 4\pi n_i T / B_0^2$, $m_e (m_i)$ is the electron (ion) mass, $n_i$ is the ion number density and $T$ is the effective temperature of the plasma) electron–ion plasma. Later, several other investigations on the nonlinear Alfvén waves [10]–[14] in e–i plasmas were carried out. For example, Seyler and Lysac [13] have extended the work of Hasegawa and Mima [9] and Shukla et al [11] by considering the nonlinear propagation of low-frequency ($\omega < \omega_{ce}$, where $\omega_{ce}$ is the electron cyclotron frequency) dispersive inertial and KAWs and by retaining the linear and nonlinear perpendicular ion inertial and ion Lorentz forces. They showed that the planar dispersive Alfvénic solitary structures cannot exist. Later, Shukla and Stenflo [14] re-examined the said work and pointed out the shortcomings of Seyler and Lysac’s work in some detail. It is worthwhile mentioning here that special care would be needed to interpret the experimental data of the Freja and Fast satellites while developing analytical models [14].

Most of the studies on SKAWs, discussed above, have dealt with electron–ion plasmas. During the last decade, however, the most alluring issue has been to understand the various collective processes in dusty plasmas (plasmas with extremely massive and negatively charged dust grains) because of their vital role in the study of laboratory, space and astrophysical plasma environments, such as cometary tails, asteroid zones, planetary rings, interstellar medium, the Earth’s environment, etc [15]–[18]. Dusty plasmas are also ubiquitous in low-temperature laboratory systems such as in plasma processing (for example, coating and etching of thin films [19]), radio-frequency plasma discharges [20], plasma crystals [21], etc. It has been found both theoretically and experimentally that the massive charged dust grains not only change the properties of the existing plasma, but can also introduce several new types of modes [22]–[24]. A number of authors have studied coherent structures in unmagnetized [25, 26] and magnetized [27, 28] dusty plasmas. It was found that very low-frequency fluctuations on very long dust timescales can be excited. For example, Farid et al [28] have studied nonlinear electrostatic waves in a magnetized dust–ion plasma and found the existence of solitary waves with a negative potential. Recently, Yinhua et al [29] have investigated nonlinear dust KAWs (DKAWs) in a low-$\beta_d$ ($m_i/m_d \ll \beta_d \ll 1/Z_d$, where $\beta_d = 4\pi n_{d0} T / B_0^2$, $m_i$ and $m_d$ are the ion and dust masses, $Z_d$ represents the number of charges residing on the dust grain surface, $n_{d0}$ is the equilibrium dust density, $B_0$ is the external magnetic field and $T$ is the effective temperature)
collisionless plasma, which are Alfvén-like waves driven by the polarization drift of the dust and bending of the magnetic field lines. According to their analysis solitons involving smooth density dips and cusped density humps can coexist. Later, Mahmood et al [30] investigated how cusped density humps become smoother when finite-\(\beta\) effects are taken into account. They also found the existence of kink-type solitons [31] in dust-contaminated plasmas.

In this paper, we consider a dust–ion magnetoplasma and study the nonlinear propagation of dust KAWs by retaining full dust nonlinearity and by taking into account the effects of small but finite-\(\beta\), such that \(m_i/m_d \ll \beta_d < 1/\text{Z}_d\). We have used the Sagdeev potential approach which is valid for arbitrary amplitudes. A set of nonlinear equations has been developed which can be reduced to an energy integral and analysed both analytically as well as numerically so as to investigate the properties of the localized structures. We found the existence of sub-Alfvénic solitons with density humps (or compressive solitary wave solutions) and density dips (or rarefactive solitary wave solutions). Furthermore, at super-Alfvénic speeds, only compressive solitary wave solutions are found to exist. We have also found that the properties (such as width and amplitude) of these solitary waves are significantly modified by the speed as well as the obliqueness of the wave propagation. These results would be useful in understanding the formation of coherent nonlinear soliton structures in space and astrophysical plasmas. Since we are considering very low-frequency waves on the dust timescale, the ions are fully relaxed and in thermodynamic equilibrium. Thus, the ions obey a Boltzmann density distribution. The dust is cold and its motion is strongly affected by the external magnetic field. The dispersion of the wave perpendicular to the external magnetic field is provided by the averaged dust Larmor radius caused by the ion pressure via a self-consistent electrostatic field. The balance of the wave dispersive and nonlinear effects can give rise to localized solitary structures.

This paper is organized in the following way. In section 2, we present the governing equations and the linear dispersion relations. In section 3, we reduce the governing equations to the energy integral equation and discuss arbitrary amplitude solitary wave structures both analytically as well as numerically. Finally, in section 4, we summarize and discuss our main results.

2. Mathematical formulation

Consider a collisionless two-component dusty plasma consisting of positively charged ions and negatively charged very massive dust grains. This type of two-component dusty plasma corresponds to a situation when most of the electrons from the ambient plasma are attached to the dust grain surface so that we may have \(n_{e0} \ll \text{Z}_d n_{d0}\), where \(n_{e0}\) (\(n_{d0}\)) is the unperturbed electron (dust particle) number density and \(\text{Z}_d\) is the number of electrons residing on the dust grain surface. This model is relevant to planetary ring systems, such as in Saturn’s rings [17, 25, 28], and in comets (e.g. Halley’s comet [27, 32]). This model is valid because, for such a situation \((n_{e0} \ll \text{Z}_d n_{d0})\), we have \(n_{e0}/n_{d0} \simeq (m_e/m_i)^{1/2}\), where \(n_{d0}\) is the unperturbed ion number density. Hence, at equilibrium we have \(n_{i0} \simeq \text{Z}_d n_{d0}\). Let us assume that the plasma is embedded in a homogeneous external magnetic field of strength \(B_0\) pointing along the \(z\) axis. We also assume that \(v_{Ad} > c_d\), where \(c_d = (\text{Z}_d T/m_d)^{1/2}\) and \(v_{Ad} = (B_0^2/4\pi n_{d0} m_d)^{1/2}\) are the dust acoustic and dust Alfvén speeds. In terms of \(\beta\) this assumption can also be written as \(\beta_d \equiv 4\pi n_{d0} T/B_0^2 < 1/\text{Z}_d\). Here, \(\text{Z}_d \beta_d\) can be of the same order as \(\beta\). We also assume that the spatial scales of the problem under consideration as well as the inter-grain distance are much larger than the grain size, so that the effects of dust charging in a magnetized plasma can be ignored and that the dust grains can be assumed to be of uniform mass and charge. Furthermore,
for very low-frequency waves on the dust timescale, we assume that \( v_\theta \gg v_{Ad} \) (i.e. \( \beta_d \gg m_i/m_d \)) and \( \max\{v_{\theta d}, v_{\theta d}\} \ll k v_{Ad} \), where \( v_\theta \) is the ion thermal speed and \( v_{\theta j}(j = i, d) \) is the dust collision frequency. This means that dust–ion and dust–dust collisions, which are much less frequent than ion collisions, can be neglected. Thus, for the wave motion, the ions are fully relaxed and in local thermodynamic equilibrium, obeying Boltzmann density distributions, whereas the dust ion collisions, can be neglected. Thus, for the wave motion, the ions are fully relaxed and in local thermodynamic equilibrium, obeying Boltzmann density distributions, whereas the dust fluid remains cold and the ion skin-depth effects can be ignored [29]. The latter may modify the wave dispersion.

The governing equations which describe the dynamics of a nonlinear wave mode in such a two-component (dust–ion) plasma with \( m_i/m_d \ll \beta_d < 1/Z_d \) are [30]

\[
\frac{\partial^4}{\partial x^2 \partial z^2}(\phi - \psi) = \frac{4\pi}{c^2} \frac{\partial^2 j_z}{\partial t \partial z},
\tag{1}
\]

\[
n_i \simeq n_i(0) \exp(-e\psi/T_i),
\tag{2}
\]

\[
D_i n_d + n_d \left( \frac{\partial v_{dx}}{\partial x} + \frac{\partial v_{dz}}{\partial z} \right) = 0,
\tag{3}
\]

\[
D_i v_{dx} = \frac{Z_d e}{m_d} \frac{\partial \phi}{\partial x} - \Omega_d v_{dy},
\tag{4a}
\]

\[
D_i v_{dy} = \Omega_d v_{dx},
\tag{4b}
\]

\[
D_i v_{dz} = \frac{Z_d e}{m_d} \frac{\partial \psi}{\partial z}.
\tag{4c}
\]

where \( D_i = \partial/\partial t + v_{dx} \partial/\partial x + v_{dz} \partial/\partial z \), and \( v_{dx}(v_{dz}) \) is the dust fluid speed in the \( x(z) \) direction, \( n_d(n_i) \) is the dust (ion) number velocity, \( \Omega_d = Z_d e B_0/m_d c \) is the dust cyclotron frequency, \( c \) is the speed of light, \( e \) is the magnitude of the electronic charge, \( j_z \) is the current density in the \( z \) direction and \( T_i \) is the ion temperature in energy units. In the derivation of the above equations, two-potential theory [33], for which \( E_x = -\partial_x \phi \) and \( E_z = -\partial_z \psi = -\partial_z (\phi - c^{-1} \partial_z A_z) \), has been used. Equation (1) is derived from the Maxwell equations whereas equation (2) gives the density of the thermal ions [29]. Equations (3)–(4c) describe the dynamics of cold and magnetized dust grains [30]. Finally, to close the system of equations (1)–(4c), we use the following charge neutrality and current continuity equations:

\[
Z_d n_d \simeq n_i,
\tag{5}
\]

\[
\frac{\partial j_z}{\partial z} = (-Z_d e) \left[ \frac{\partial n_d}{\partial t} + \frac{\partial}{\partial z} (n_d v_{dz}) \right].
\tag{6}
\]

In deriving equation (6), we have also used equation (3). The system of equations (1)–(6) governs the dynamics of a nonlinear wave propagating obliquely to an external magnetic field in a two-component (dust–ion) magnetoplasma.

In the linear limit, the dispersion relation is obtained by assuming that all perturbed quantities are proportional to \( \exp[i(k_x x + k_z z - \omega t)] \), where \( k_x, k_z \) are the wavenumbers along the \( x \) and \( z \) axes, respectively, and \( \omega \) is the wave frequency. We obtain the following biquadratic dispersion relation:

\[
\text{New Journal of Physics 5 (2003) 116.1–116.11 (http://www.njp.org/)}
\]
where $\beta = c_d^2/\nu_{Ad}^2$. In the derivation of the above dispersion relation, we have assumed $\omega \ll \Omega_d$. Equation (7) can further be solved to yield

$$\omega^2 = \omega_\pm^2 = \frac{1}{2} v_{Ad}^2 k_c^2 (1 + k_x^2 \rho_{sd}^2 + \beta) \pm \frac{1}{2} v_{Ad}^2 k_c^2 (1 + k_x^2 \rho_{sd}^2 + \beta) (1 - 4 \beta (1 + k_x^2 \rho_{sd}^2 + \beta)^{-2})^{1/2}. \quad (8)$$

Equation (8) represents similar dispersion relations to those obtained in [30]. However, in the present model we have completely omitted the concentration of electrons. In the limit $\beta \ll 1$, the above dispersion relation reduces to $\omega^2 = v_{Ad}^2 k_c^2 (1 + k_x^2 \rho_{sd}^2)$, which is the dispersion relation for the dust KAWs [29] in a dust-contaminated magnetoplasma depleted of electrons. It is evident from equation (8) that the dispersion is caused by the finite dust Larmor radius ($\rho_{sd}$) effect. When this dispersion balances the nonlinear effects, localized quasistationary wave structures are formed.

3. Localized solutions

We first normalize the variables as follows: $\tau = \Omega_d t$, $(\xi, \zeta) = (x, z)/\rho_{sd}$, $V = v/c_d$, $(\Phi, \Psi) = e(\varphi, \psi)/T_i$, $N_j = n_j/n_{jo}$ ($j = i, d$), where $c_d = (Z_d T_i/m_d)^{1/2}$ and $\rho_{sd} = (Z_d T_i/m_d \Omega_d^2)^{1/2}$ are the dust acoustic speed and dust gyroradius at $T_i$. Using the above normalized variables in equations (1) and (6), we obtain

$$\frac{\partial^4 (\Phi - \Psi)}{\partial \xi^2 \partial \zeta^2} = -\beta \left[ \frac{\partial N}{\partial \tau^2} + \frac{\partial^2}{\partial \tau \partial \zeta} (N V_d) \right]. \quad (9)$$

Similarly, equation (2) can be re-written in the following normalized form:

$$N = \exp(-\Psi). \quad (10)$$

In writing equations (9) and (10), we have used $N = N_1 = N$, which is obtained by using equation (5) and the charge neutrality condition at equilibrium. From equations (3)–(4c), we obtain the same normalized equations as equations (11)–(12c) of [30]. We now seek quasistationary localized solutions of these three normalized equations along with equations (9) and (10). For this purpose, we introduce the moving coordinate $\eta = \alpha \xi + \gamma \zeta - M_d \tau$, where $M_d = V/c_d$ is the dust Mach number defined in terms of the dust acoustic speed $c_d$ and soliton speed $V$, and $\alpha$ and $\gamma$ are the direction cosines. It may be noted here that our governing equations are only valid for $\alpha^2/\gamma^2 \gg 1$ so as to justify the use of two-potential theory and also $\gamma^2/\alpha^2 < \beta$ would allow us to take the low-frequency limit such that $\omega < \Omega_d$. For localized solutions, the following boundary conditions are used:

$$\Phi = \Psi = V = \frac{\partial \Phi}{\partial \eta} = \frac{\partial \Psi}{\partial \eta} = \frac{\partial N}{\partial \eta} = \frac{\partial V_d}{\partial \eta} = 0, \quad \text{and} \quad N = 1 \text{ at } \eta \rightarrow \pm \infty. \quad (11)$$

Assuming that all dependent variables are functions of $\eta$ only and using the boundary conditions given by equation (11), one can reduce [30] the normalized equations (9), (10) and equations (11)–(12c) of [30] to the following nonlinear differential equation for $N$:

$$\frac{d^2}{d\eta^2} \left[ (1 + \delta) \ln N + \frac{M_d^2}{2 N^2} \right] = (M_d^2 \gamma^2)^{-1} (1 - \beta M_d^2) (N - 1) (M_d^2 - N). \quad (12)$$
Integrating equation (12) and using the appropriate boundary conditions (equation (11)), we get the quadrature

$$\frac{1}{2} \left( \frac{dN}{d\eta} \right)^2 + U(N) = 0,$$

where the Sagdeev potential [5] $U(N)$ takes the following form:

$$U(N) = (2M_d^2 \gamma^2)^{-1}(1 - \beta M_d^2)((1 + \delta)/N - M_d^2/N^3)^{-\frac{1}{2}} \left[ \left( \frac{N - 1}{N} \right)^{M_d^4} - 2 \left( n - 1 \right) \left( 1 + \delta - \frac{1}{N} \right) - \delta \ln N \right] M_d^2 + (1 + \delta)(N - 1)^2. \tag{14}$$

Here, we have defined $M_d^\prime = M_d/\gamma$ and $\delta = \alpha/\gamma$. Equation (13) can be regarded as an ‘energy integral’ of an oscillating particle of unit mass, with a velocity $dN/d\eta$ and position $N$ in a potential $U(N)$. Equation (13) can be numerically integrated so as to obtain the solution.

We shall first investigate the behaviour of the Sagdeev potential $U(N)$ analytically and show that soliton solutions can exist. From equation (14), we have

$$U(N \simeq 1) \approx \beta(M_{Ad}^2 \gamma^2)^{-1}(1 - M_{Ad}^2)(M_{Ad}^2 - \beta)(M_{Ad}^2 - (1 + \delta)\beta)^{-1}(N - 1)^2. \tag{15}$$

Here, $M_{Ad} = \sqrt{\beta} M_d \equiv V/v_{dA}$ is the dust-\Alfvén Mach number. The effective dust-\Alfvén Mach number $M_{Ad}^\prime$ can be defined as $M_{Ad}^\prime = M_{Ad}/\alpha$. From equation (15), it is apparent that $N = 1$ is a double root, which is a necessary condition for the existence of the solitary wave. For localized solutions, the Sagdeev potential must be negative between the zeros at $N = 1$ and $N = N_m$, where $N_m$ corresponds to the maximum variation of $N$. We found that equation (15) satisfies the inequality $U(N) < 0$ provided that the following condition holds:

$$M_{Ad}^2 < 1, \quad \beta < M_{Ad}^2 < (1 + \delta)\beta, \quad (16a)$$

or

$$M_{Ad}^2 > 1, \quad M_{Ad}^2 > (1 + \delta)\beta. \quad (16b)$$

Since $N_m$ is an extremum for $N$, we can write $U(N = N_m) = 0$, which would relate the effective dust-\Alfvén Mach number $M_{Ad}^\prime$ to the amplitude $N_m$, such that

$$M_{Ad}^2 = \beta [b \pm (b^2 - c_0)^{1/2}], \tag{17}$$

where $b = N_m^2[(N_m - 1)(1 + \delta - 1/N_m) - \delta \ln N_m]/(N_m - 1)^2$ and $c_0 = N_m^2(1 + \delta)$.

To ensure the existence of the solitary wave solution, equations (16) and (17) have to be satisfied along with the following conditions such that

$$\frac{dU}{dN} \bigg|_{N = N_m} < 0 \quad \text{for } N_m < 1, \quad \text{and} \quad \frac{dU}{dN} \bigg|_{N = N_m} > 0 \quad \text{for } N_m > 1. \tag{18}$$

From equation (14), we get

$$\frac{dU}{dN} \bigg|_{N = N_m} = \beta N_m^3(M_{Ad}^2 \gamma^2)^{-1}(1 - M_{Ad}^2)(M_{Ad}^2 - \beta N_m)(M_{Ad}^2 - (1 + \delta)\beta N_m)^{-1}(N_m - 1). \tag{19}$$

Thus, the conditions given by equation (18) would be satisfied under the following inequalities:

(i) $M_{Ad}^2 < 1$ (sub-\Alfvén case),
The behaviour of the Sagdeev potential $U(N)$ (for $N > 1$) for $M_{\text{Ad}}^2 = 0.2$ (full curve), 0.25 (dotted curve) and 0.3 (broken curve). The parameters are $\beta = 0.05$, $\delta = 19$ and $\gamma^2 = 0.8$.

\begin{align}
M_{\text{Ad}}^2 &> \beta N_m, \quad (1 + \delta) \beta N_m, \quad \text{for } N_m < 1, \\
M_{\text{Ad}}^2 &< \beta N_m, \quad \text{for } N_m > 1,
\end{align}

or

\begin{align}
\beta N_m &< M_{\text{Ad}}^2 < (1 + \delta) \beta N_m, \quad \text{for } N_m < 1.
\end{align}

Figure 1. The behaviour of the Sagdeev potential $U(N)$ (for $N > 1$) for $M_{\text{Ad}}^2 = 0.2$ (full curve), 0.25 (dotted curve) and 0.3 (broken curve). The parameters are $\beta = 0.05$, $\delta = 19$ and $\gamma^2 = 0.8$.

From the above analysis, we found that solitary wave solutions can exist in sub-Alfvénic as well as super-Alfvénic ranges with a density hump or a density dip subjected to equation (17) along with the inequalities given by relations (16) and (20). From equations (16), (17) and (20), it is clear that the existence and amplitude of these solitons depend upon their propagation speed and direction and the value of $\beta$. This is also confirmed by our numerical results.

We shall now numerically examine the behaviour of the Sagdeev potential $U(N)$ for different values of $M_{\text{Ad}}$. The other arbitrary parameters are taken as $\beta = 0.05$, $\delta = 19$ and $\gamma^2 = 0.8$. Figure 1 gives the Sagdeev potential $U(N)$ (for $N > 1$) for $M_{\text{Ad}}^2 = 0.2$ (full curve), 0.25 (dotted curve) and 0.3 (broken curve), respectively. This figure shows that localized density humps can exist for sub-Alfvénic speeds. Figure 2 shows the plots of $U(N)$ (for $N < 1$) for the same parameters as in figure 1. In figure 2, we have not shown the complete curves of the Sagdeev potential, for convenience. However, we find that there exist singular points where the density gradient is infinite. These singular points are for those values of $N$ for which the denominator (the first term in square brackets) of equation (14) becomes zero. Since the Sagdeev potential is negative between a double zero and a simple zero, the corresponding solution [5, 6] is a localized density dip which can be obtained by numerically solving the ordinary differential equation (13). A similar type of behaviour of the Sagdeev potential $U(N)$ (for $N > 1$) has also been found (figure 3) for $M_{\text{Ad}}^2 > 1$, showing the possibility of a super-Alfvénic localized density hump. It is clear from figures 1–3 that, as the value of $M_{\text{Ad}}$ (the propagation speed) is increased, the amplitude of the density humps increases whereas the amplitude of the density dips decreases.

We have also numerically solved the ordinary differential equation (13) and obtained the said solitary wave solutions for the given arbitrary parameters. Figure 4 gives the profiles of sub-Alfvénic compressive solitary wave solutions, obtained by numerically solving equation (13) for

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The behaviour of the Sagdeev potential $U(N)$ (for $N > 1$) for $M_{\text{Ad}}^2 = 0.2$ (full curve), 0.25 (dotted curve) and 0.3 (broken curve). The parameters are $\beta = 0.05$, $\delta = 19$ and $\gamma^2 = 0.8$.}
\end{figure}
Figure 2. The plots of the Sagdeev potential $U(N)$ (for $N < 1$) for the same parameters as in figure 1.

Figure 3. The behaviour of the Sagdeev potential $U(N)$ (for $N > 1$) for $M_{Ad}^2 = 3$ (full curve) and $M_{Ad}^2 = 4$ (broken curve). The other parameters are the same as in figure 1.

different values of $\gamma$ (at fixed $M_{Ad}$). The profiles of sub-Alfvénic rarefactive solitons, for the same parameters as in figure 4, are shown in figure 5. It is interesting to note that, regardless of the existence of the points (figure 2) at which the density gradient is infinite, the density itself is continuous (figure 5). However, it is clear from figure 6 that these solitons (density depressions) are bell-shaped [5, 30] due to the occurrence of points of infinite density gradient at their shoulders. A similar type of soliton with density compressions corresponding to figure 3 is shown in figure 6 for $\gamma^2 = 0.6$ (full curve) and 0.8 (broken curve). The density profiles have been plotted numerically by using the software Mathematica, which successfully handles the singularities. It is evident from figures 4–6 that, if the propagation direction of the solitary structures become more oblique, i.e. the direction cosine $\gamma$ becomes smaller, the amplitude of the density hump increases whereas the density dip decreases. Furthermore, super-Alfvénic bell-shaped density humps are found to be narrower compared to sub-Alfvénic density humps. Thus, we conclude that the amplitude as well as the width of the solitary waves depends upon their propagation speed (the dust-Alfvén Mach number) and their obliqueness (direction of propagation).
Figure 4. Density profiles (compressive solitons) for $\gamma^2 = 0.6$ (full curve), 0.7 (dotted curve) and 0.8 (broken curve). The other parameters are $\beta = 0.05$, $\delta = 19$ and $M_{Ad}^2 = 0.4$.

Figure 5. Density profiles (rarefactive solitons) for the same parameters as in figure 4.

Figure 6. Density profiles (compressive solitons) for $\gamma^2 = 0.6$ (full curve) and $\gamma^2 = 0.8$ (broken curve). The other parameters are $\beta = 0.05$, $\delta = 19$ and $M_{Ad}^2 = 3$. 
It may be pointed out here that we have used arbitrary parameters in order to check the properties of the solitary wave structures and found the coexistence of finite amplitude density humps and dips in the sub-Alfvénic region. The result may be useful in the interpretation of the SKAW data of the Freja satellite. However, there may be a possibility of similar controversies appearing in the dust–ion plasma as discussed [13, 14] for nonlinear Alfvén waves in electron–ion plasmas. The DKAW is basically a different (very slow) mode compared to the usual KAW and is a result of the dispersion caused by the dust gyroradius. We expect that our results have more relevance to the nonlinear structures involving solitary waves in space and astrophysical dusty plasmas [17, 25, 27, 28, 32], where negatively charged dust particulates and Boltzmann distributed ions are the dominant plasma species. It should be added here that we have discussed the properties of nonlinear Alfvén waves for a non-relativistic dust–ion plasma. However, some new interesting effects [34] may appear for a relativistic dust–ion plasma.

4. Summary

In this paper, we have examined the properties of nonlinear DKAWs in a magnetized dust–ion plasma. We have assumed that the dynamics of the cold and magnetized dust grains is governed by the continuity and momentum equations, while the thermal ions establish a Boltzmann density distribution. The two-component dusty plasma we have considered here is valid as long as \( n_e \ll Z_d n_d \). We reduced the basic set of nonlinear equations to an energy integral equation and analysed it both analytically as well as numerically. We found that sub-Alfvénic arbitrary amplitude compressive and rarefactive solitary waves can coexist. Further, our plasma model also supports super-Alfvénic solitary structures but only with a density hump (bell-shaped). The solitons are formed due to the balance between the wave dispersive and nonlinear effects. We have also found that the width and amplitude of the solitary waves depend on their propagation speeds (the dust-Alfvén Mach number) as well as their obliqueness (direction of propagation), along with other plasma parameters. Finally, we stress that the results of our present investigation should be useful in understanding the coherent nonlinear structures involving solitary waves in space and astrophysical dust-contaminated plasmas, particularly in planetary ring systems (namely Saturn’s F-ring), and in cometary environments (namely Halley’s comet), where negatively charged dust particulates and Boltzmann distributed ions are the dominant plasma species.

Acknowledgments

We are grateful to Professor P H Sakanaka for assisting us in the numerical simulations. This research work was partially supported by the Office of the External Activities of AS-ICTP, the Quaid-i-Azam University, Research Fund URF (2002–2003) and the Pakistan Science Foundation Project No PSF/Res/C-QU/Phys(130).

References

[1] Hasegawa A and Uberoi C 1982 The Alfvén wave DOE Advances in Fusion Science and Engineering (Critical Review Series) (Washington, DC: US Department of Energy)

[2] Wahlund J E, Louarn P, Chust T, de Feraudy H, Roux A, Holback B, Dovner P O and Holmgren G 1994 Geophys. Res. Lett. 21 1831

[3] Louarn P, Wahlund J E, Chust T, de Feraudy H, Roux A, Holback B, Dovner P O, Eriksson A I and Holmgren G 1994 Geophys. Res. Lett. 21 1847

_new-journal-of-physics_ 5 (2003) 116.1–116.11 (http://www.njp.org/)
[4] Mäkelä J S, Mälkki A, Koskinen H, Boehm M, Holback B and Eliasson L 1998 J. Geophys. Res. Lett. 103 9391
Stasiewicz K, Holm gren G and Zanetti L 1998 J. Geophys. Res. Lett. 103 4251
[5] Sagdeev R Z 1966 Reviews of Plasma Physics vol 4, ed M A Leontovich (New York: Consultants Bureau) p 23
[6] Yu M Y 1976 Phys. Lett. A 59 361
Yu M Y 1978 J. Math. Phys. 19 816
[7] Bharuthram R and Yu M Y 1993 Astrophys. Space Sci. 207 197
[8] Vladimirov S V, Yu M Y and Stenflo L 1993 Phys. Lett. A 174 313
Vladimirov S V, Yu M Y and Tsytovich V N 1994 Phys. Rep. 241 1
[9] Hasegawa A and Mima K 1976 Phys. Rev. Lett. 37 690
[10] Buti B and Shukla P K 1979 Phys. Lett. A 74 409
[11] Shukla P K, Rahman H U and Sharma R P 1982 J. Plasma Phys. 28 125
[12] Wu D J, Wang D Y and Huang G L 1997 Phys. Plasmas 4 611 and references therein
[13] Seyler C E and Lysak R L 1999 Phys. Plasmas 6 4778
[14] Shukla P K and Stenflo L 1997 Phys. Plasmas 4 3445
Shukla P K and Stenflo L 2000 Phys. Plasmas 7 2747
[15] Horanyi M and Mendis D A 1985 Astrophys. J. 294 357
Horanyi M and Mendis D A 1986 Astrophys. J. 307 800
[16] Mendis D A and Rosenberg M 1992 IEEE Trans. Plasma Sci. 20 929
Mendis D A and Rosenberg M 1994 Ann. Rev. Astron. Astrophys. 32 419
[17] Goertz C K 1989 Rev. Geophys. 27 271
[18] Verheest F 1996 Space Sci. Rev. 77 267 and references therein
[19] Selwyn G H 1993 Japan. J. Appl. Phys. 32 3068
[20] Chu J H, Du J B and Lin I 1994 J. Phys. D: Appl. Phys. 27 296
[21] Thomas H et al 1994 Phys. Rev. Lett. 73 652
[22] de Angelis U, Bingham R and Tsytovich V N 1989 J. Plasma Phys. 42 445
[23] Rao N N, Shukla P K and Yu M Y 1990 Planet. Space Sci. 38 543
[24] Rao N N 1993 J. Plasma Phys. 49 375
Rao N N 1995 J. Plasma Phys. 53 317
[25] Shukla P K and Silin V P 1992 Phys. Scr. 45 508
[26] Mamun A A 1999 Astrophys. Space Sci. 268 443
[27] Ya Kotsarenko N, Koshevaya S V, Stewart G A and Maravilla D 1998 Planet. Space Sci. 46 429
[28] Farid T, Mamun A A, Shukla P K and Mirza A M 2001 Phys. Plasmas 8 1529 and references therein
[29] Chen Y, Lu W and Yu M Y 2000 Phys. Rev. E 61 809
[30] Mahmood M A, Mirza A M, Sakanaka P H and Murtaza G 2002 Phys. Plasmas 9 3794
[31] Drazin P G and Johnson R S 1990 Solitons: An Introduction (Cambridge: Cambridge University Press) p 25
[32] de Angelis U, Formisano V and Giordano M J 1988 Plasma Phys. Control. Fusion 40 399
[33] Kadomtsev B B 1965 Plasma Turbulence (New York: Academic) p 82
[34] Stenflo L and Shukla P K 2001 IEEE Trans. Plasma Sci. 29 208

New Journal of Physics 5 (2003) 116.1–116.11 (http://www.njp.org/)