An individual-centric probabilistic extension for OWL: Modelling the Uncertainness
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Abstract
The theoretical benefits of semantics as well as their potential impact on IT are well known concepts, extensively discussed in literature. As more and more systems are currently using or referring semantic technologies, the challenging third version of the web (Semantic Web or Web 3.0) is progressively taking shape. On the other hand, apart from the relatively limited capabilities in terms of expressiveness characterizing current concrete semantic technologies, theoretical models and research prototypes are actually overlooking a significant number of practical issues including, among others, consolidated mechanisms to manage and maintain vocabularies, shared notations systems and support to high scale systems (Big Data). Focusing on the OWL model as the current reference technology to specify web semantics, in this paper we will discuss the problem of approaching the knowledge engineering exclusively according to a deterministic model and excluding a priori any kind of probabilistic semantic. Those limitations determine that most knowledge ecosystems including, at some level, probabilistic information are not well suited inside OWL environments. Therefore, despite the big potential of OWL, a consistent number of applications are still using more classic data models or unnatural hybrid environments. But OWL, even with its intrinsic limitations, reflects a model flexible enough to support extensions and integrations. In this work we propose a simple statistical extension for the model that can significantly spread the expressiveness and the purpose of OWL.

Keywords: Semantic Computing, Knowledge Modelling, Ontology Engineering, OWL

1 Introduction
During the past years semantic technologies have progressively emerged as a response to the challenges of a new generation of systems reflecting a novel perspective of the information, extremely demanding in terms of interoperability [1]. The theoretical benefits of semantics, as well as their potential impact on IT, are well known concepts, extensively discussed in literature. As more and more systems are currently using or referring semantic technologies, the challenging third version of the web (Semantic Web or Web 3.0 [2]) is progressively taking shape. A semantic approach is currently in use into a wide range of application domains to model and
process complex data ecosystems. Common examples include (but evidently are not limited to) the next generation of services (semantic services [3]) and a set of techniques and solutions that apply semantic reasoning as well as specific semantic support to enhance the interoperability among systems on a large scale. On the other hand, apart from the relatively limited capabilities in terms of expressiveness characterizing current concrete semantic technologies, theoretical models and research prototypes are actually overlooking a significant number of practical issues including, among others, consolidated mechanisms to manage and maintain vocabularies, shared notations systems and support to high scale systems (Big Data [4]). Therefore, while the role of semantics inside modern IT is actually a completely consolidated concept, semantic technologies are expected to evolve in the next future to cover current lacks and to provide a more consistent support to practical applications. While the technology has progressed to embrace significant semantics, now semantic technologies have to evolve consequently and have to exhaustively cope larger sets of requirements from multiple domains. Focusing on the OWL model [5] as the current reference technology to specify web semantics, in this paper we will discuss the problem of approaching the knowledge engineering exclusively according to a deterministic model and excluding a priori any kind of probabilistic semantic. Indeed, in the era of Big Data entropy plays a key role in many contexts and, consequently, the uncertainty is more and more part of most modern models and processes. Modelling uncertainty is a big challenge. In the context of this paper, uncertainty is approached from a semantic perspective, assuming the Ontology schema as a deterministic asset inside the data model and the Ontology population as a probabilistic component of the information. Even though without exhaustively covering the modelling of uncertainty at an overall level, this vision is very consistent at a practical level but, unfortunately, not supported by current models and technologies. Those limitations determine that most knowledge ecosystems including, at some level, probabilistic information are not well suited inside OWL environments. Therefore, despite the big potential of OWL, a consistent number of applications are still using more classic data models or unnatural hybrid environments. Because of its ubiquity and its intrinsic relation to real world actions, processes and phenomena, modelling uncertainty and its contextual processing are extremely important. Addressing uncertainty according to an ontologic approach guarantees a consistent semantic model and increased reasoning capabilities. OWL, even considering its intrinsic limitations, reflects a model flexible enough to support extensions and integrations. In this work a simple statistical extension for the model that can significantly spread the expressiveness and the purpose of OWL is proposed.

This paper is structured according to a classical schema: the introductory part is completed by the next section that provides an overview of the related work focusing on the main differences between this work and similar ones; then the core part of the paper is proposed; finally, as common, the paper ends with a conclusions section. The core part is composed of three different sub-sections that respectively deal with the representation of probabilistic semantics on OWL according to an individual-centric perspective, with its computation and with the description of a complex study case.

2 Related Work

Despite most researchers explicitly or implicitly address the need for probabilistic semantics, there is not, currently, a significantly extended and comprehensive documentation in literature, probably due to a fundamental lack of consensus about an unified view of probabilistic ontologies [6]. While theoretical studies on uncertainty appear too abstracted to cope requirements of applied semantics, most practical works (e.g.,[7]) seem to provide ad-hoc solutions for spe-
cific problems more than a comprehensive generic solution. Indeed, in a system dealing with probabilistic ecosystems, due to the intrinsic need for probabilistic schemas, OWL structures are usual to have a minor scope and cannot provide, generally, trustful inferring and reasoning. However it is actually possible to clearly identify several approaches to probabilistic semantics. [8], [6] and [9] proposes a Bayesian extension to OWL, explicitly aimed at the development of probabilistic ontologies maintaining, according to the authors, the compliance with the original model. Similar approaches integrates OWL with probabilistic statements. The natural probabilistic extension of rdf ([10]) is probably the most natural one but several alternatives based on other logics have been proposed during the last years. For instance, in [11] probabilistic Datalog is used to model uncertainness, as well as in [12] log-linear description logics are adopted. These models propose close approaches that cannot evidently cover the needs of a dynamic scenario as the Semantic Web [13], even considering their potential evolution [14]. The work proposed significantly differs from the cited ones because of the following interrelated points:

Practical approach. The objective of this work is not to fully cover the theoretical model of uncertainness and, certainly, not to provide a new semantic technology able to represent and process probabilistic ontologies in a wide contest such as the Semantic Web. On the contrary, this work has a practical focus and wants to explain how the current OWL model can support a probabilistic understanding of ontologies by using just minor extensions that are not altering the OWL philosophy.

Individual-centric perspective for uncertainness. The uncertainness is indeed modelled from an individual-centric perspective according to the OWL model [5]. In practice, the concept schema inside the ontology is assumed to be fully deterministic but the population isn’t. This approach assures a convergence between the semantic interoperability at the basis of semantic technologies and the requirements in terms of probabilistic modelling. This is coherent with the previous point and, as extensively discussed in the following sections of the paper, this solution cover a significant set of practical case in the context of popular domains (e.g. Data Mining and Machine Learning) where the ontology schema is well known (or needs to be well known) while its population is the result of complex processes that usually introduce entropy and elements of uncertainness.

Simplicity and Applicability. From a naive point of view, OWL is a consolidated data model that can be currently considered as a de facto standard for knowledge representation. However, the potential of OWL-like models are for the most unexplored. In the philosophy of this work, designers are expected to model their knowledge environments exactly as they are doing by using classic OWL. The uncertainness is modelled, where needed, in a natural way without introducing any logic external to OWL.

3 Probabilistic Semantics to model Uncertainness

An OWL Ontology is composed of four different correlated elements: the concept (or class or thing), the property (ObjectProperty and DataProperty) and the individual. Focusing on an individual-centric perspective (figure 1), an individual element of the ontology can be associated to one or more concepts or, equivalently, the individual element is an instantiation of one or more given classes (concepts). Furthermore an individual can be related to other individuals through ObjectProperties, as well as it can be related to other data types external to the ontology through DataProperties. The deterministic specification of the individuals inside the ontology makes the whole model a deterministic environment. On the contrary, a probabilistic
specification of individuals means, as explained in the next sub-section, a probabilistic model able to represent the uncertainty in practical cases.

### 3.1 Modelling Probabilistic Semantics

An individual inside an OWL Ontology (eq. 1) is an element associated with a given Domain represented by the Ontology itself. The association of an individual with a Domain is a fully deterministic relation in the classic OWL and in this model extension too.

\[
\text{Individual} = [\text{Domain, Name, Type, Assertions, ...}]
\]  

(1)

An individual, as any other element of the Ontology, is univocally identified inside the domain by its Name. The identifier is intrinsically deterministic. The characterization of an individual ("what is it?") is provided through the specification of its Type (Class Assertion) that is an association with one or more ontology concepts (also called things). This relation is similar to the instantiation (individual in this case) of a class (concept in this case) in object-oriented models. In OWL, the ontology schema is deterministic as well as the association of individuals with concepts meanings that the existence and the nature of individuals are fully deterministic assumptions. The proposed model maintains a deterministic schema of concepts but introduces a probabilistic association of individuals with concepts (eq. 2). This means that concepts and their hierarchies are always valid inside a given domain as well as specified individuals. But the association of an individual with a concept is valid with a given probability (eq. 2).

\[
\text{Type}_i = T : \begin{cases} 
  \text{i is } a(p=p_x) \Rightarrow \text{i is } a(x, \text{Thing}) \\
  \text{x is } a \Rightarrow \text{x is } a(i, x) [p=p_x] \\
\end{cases}
\]  

(2)

In the same way, a set of P Assertions (eq. 3) defines the relations of an individual \(i\) with other individuals as well as with information external to the Ontology, respectively through assertions on ObjectProperties (\(OP\_Assertions\)) and on DataProperties (\(DP\_Assertions\)).

\[
\text{Assertions}_i^P = [\text{OP}^P \_\text{Assertions}_i^P \cup \text{DP}^P \_\text{Assertions}_i^P]
\]  

(3)
In the classic OWL model, assertions are understood as deterministic relations and, therefore, they are always valid like deterministic facts. Addressing probabilistic assertions is equivalent to establish a relation through a given property with a well-defined probability (eq. 4 and eq. 5).

\[
OP_{\text{Assertions}}^P = \begin{cases} 
  i \ \text{has}_{[p=p_x]} \ x \ \text{with} - \text{value} \ A \\
  x \ \text{is} - a \ \text{ObjectProperty} \\
  A \ \text{is} - a_{[p=p_A]} \ y \\
  y \ \text{is} - a \ \text{Thing} \\
  \Leftrightarrow \begin{cases} 
    \text{has}(i, x, A)_{[p=p_x]} \\
    \text{is} - a(x, \text{ObjectProperty}) \\
    \text{is} - a(A, y)_{[p=p_A]} \\
    \text{is} - a(y, \text{Thing}) 
  \end{cases}
\end{cases}
\]

\[
DP_{\text{Assertions}}^P = \begin{cases} 
  i \ \text{has}_{[p=p_x]} \ x \ \text{with} - \text{value} \ A \\
  x \ \text{is} - a \ \text{DataProperty} \\
  A \ \text{is} - a_{[p=p_A]} \ \text{some_data} \\
  \Leftrightarrow \begin{cases} 
    \text{has}(i, x, A)_{[p=p_x]} \\
    \text{is} - a(x, \text{ObjectProperty}) \\
    \text{is} - a(A, \text{some_data})_{[p=p_A]} 
  \end{cases}
\end{cases}
\]

Probabilistic features are not extended to negative property assertions that provide a unique opportunity to make statements where we know something that is not true. This kind of information is particularly important in OWL where the default stance is that anything is possible until you say otherwise. The encoding of those theoretical individual-centric probabilistic extensions in an OWL implementation is pretty immediate and can consist of XML properties in the corresponding XML tags to associate given probabilities. A possible syntax is showed in the code example 1 where an individual \( i \) is defined as part of an Ontology and is associated with a concept \( a \). Property assertions relate a given data (value) through the dataproperty \( Dp1 \) and an ontology concept \( k \) through the objectproperty \( Op1 \). As showed in the example, an added XML property provides a further semantic expressing the probabilities for the considered relations.

Code example 1: Use of extended OWL codification.

```xml
<owl:NamedIndividual rdf:about="&Ontology;i">
  <rdf:type rdf:resource="&Ontology;a" statOWL:probability="pa"/>
  <statOWL:probability="pDp1">value</statOWL:probability>
  <Ontology:Op1 rdf:resource="&Ontology;k" statOWL:probability="pOp1"/>
</owl:NamedIndividual>
```

3.2 Inferring Probabilistic Semantics

The extensions provided are semantically relevant even though, as showed, they just require minor syntactic notations. Those extended semantics are, without any doubt, significant at a level of knowledge representation as they allow to dynamically define probabilistic types and relations in the context of a formal data model. Furthermore, a fundamental and effective compliance with the original model and its philosophy is maintained. On the other hand, the
The most appreciated OWL feature is surely the capability to infer information from facts by defining logic rules. Inferred concepts make Ontology a rich data model where complex relations can be expressed in a declarative way, embedded in the schema and computed in an interoperable context. The OWL inference model doesn’t need significant extensions to work on probabilistic data. The key concept is that an inference rule, that involves probabilistic data, returns a probabilistic outcome. The probability associated with the inferred concept is a function of the probabilities related to the input elements. In the context of this work, the elements of the ontology are assumed to be statistically independent, so the probability of an inferred concept can be calculated as the product of the probabilities associated with the input elements. The probability $P_{inf}^k$ of a concept inferred by the rule $k$ involving $n$ ontology elements is given by the product of the probabilities associated with the involved elements (eq. 6). A simple example of OWL inference rule is defined by the code example 2: a concept $c$ is inferred by the rule $k$ assuming that the object property $Op1$ is associated with some element of type $b$.

$$P_{inf}^k \left( \bigcap_{i=1}^{n} e_i \right) = \prod_{i=1}^{n} P(e_i)$$ (6)

Code example 2: Inferred concept involving an Individual Class and an Object Property.

```xml
<owl:Class rdf:about="&Ontology;c">
  <owl:equivalentClass>
    <owl:Restriction>
      <owl:onProperty rdf:resource="&Ontology;Op1"/>
      <owl:someValuesFrom rdf:resource="&Ontology;b"/>
    </owl:Restriction>
  </owl:equivalentClass>
</owl:Class>
```

The probability $P_c^k$ associated with the output of the rule $k$ can be obtained particularizing eq. 6 to an individual-centric probabilistic model. In the example, there are two different element of uncertainty: the probability $P(Op1)$ with whom $Op1$ is associated to its value and the probability $P(b)$ of the value to be an element of type $b$. Therefore, $P_c^k$ is defined by eq. 7.

$$P_c^k(Op1 \cap b) = P(Op1)P(b)$$ (7)

3.3 A case study

In this sub-section, a use case that points out the need for probabilistic semantics (and the benefits introduced by them) is shortly described. That is a complex case study which integrates common information retrieval techniques with abductive semantic reasoning on geographic space [15]. The data source is a general purpose social network (e.g. Facebook, Twitter, Google+) that implies a potential Big Data context with the consequent strong requirements for knowledge representation [16, 17, 18] and mining as well as for information geo-processing [19]. An outline of the process is showed in figure 2. It is composed of two sequential phases:

- **Information Retrieval.** The purpose of this phase is to collect contents from a social network matching a given pattern and to geo-localize them according to a semantic approach [17, 18]. It is based on the simultaneous processing of natural language and meta-data.
Information Processing. The input of this second sub-process is the information retrieved by the previous phase. That information is processed in its social and geographic context [15], modelled by an extensive set of semantic profiles reflecting spatial and social relations.

The reliability of the whole process depends on the accuracy of data retrieved during the first phase of the process. The key factor in this sense is the social network as an input. The most popular social platforms are evidently involved in complex business models and they are clearly evolving according to a commercial philosophy, along with a fundamental charm to attract millions of users. Even following a business-driven evolution, the last generation network is offering exciting possibilities from a scientific perspective because the explicit and implicit knowledge they can provide. Apart from the intrinsic and generic difficulty to manage extreme-scale dynamic information, retrieving information on social networks usually proposes further challenges in terms of reliability with respect to the retrieval on static contents (e.g. documents). First of all the (continuously changing) content itself. A multimedia content is not easy to "understand" if it is not considered in the proper context. Even for humans. Much more for a machine. Even though limiting the analysis to text contents, the common ambiguities characterizing natural languages have a stronger impact because of the shortness of social content that significantly limits the performance of retrieval techniques [20]. Meta-data commonly associated with content is a powerful integration for basic (but key) purposes (e.g. geo-localization and timing) as well as for more sophisticated scopes (normally aimed at establishing relations among contents and/or users). Meta-data can be different from platform to platform. Some meta-data is automatically managed by platforms, others by users. Designing "perfect" meta-data is hard. For example the famous hash-tag characterizing Twitter (and more recently appeared also on Facebook) is intrinsically understood as an user-centric mechanism: its completely syntactic character is very appreciated by users and suitable for a certain kind of quantitative analysis on large scale but it looks not too adequate for detailed analysis. Timing and geo-localization are intrinsically critical for social contents: there is a clear difference between the time and the place a content belong and the time and place of its publication. Many platforms simply doesn’t model that key difference. Others provide specific fields that are not always properly used by network members. It’s also common that platforms try approximations on geo-localization for commercial purposes that can have a bad impact on semantic spaces [17, 18]. The process represented in figure 2, like any other potentially addressing Big Data in a context-less ambiguous environment such as social networks, can be affected by a fundamental lack of reliability. But, if an individual-centric probabilistic model is adopted, then also the opportunity to model and process the uncertainties, the ambiguities and the entropy as a part of the knowledge in a semantic context is offered. A deterministic process is not managing any probabilistic information so, in most cases, there is no information about the reliability of processes as a function of the information they are processing. In some case, if the reliability
of the information retrieval process is known both with a correspondent error rate, the whole process is assumed to have the same reliability of the retrieval phase. This case study, just analysing overall statistics from low-scale experiments on real data and without the need to go throughout details, shows that common approximations are far away from the reality and that probabilistic semantics are a key factor to a reliable semantic computation in presence of uncertainness. Defining the correctness of a retrieval process on social networks is not obvious. In this work we have adopted an ad-hoc approach that assure a proper understanding of what is correct, what it is not and what is ambiguous. The information retrieval phase takes in input social objects and has to determine:

• **Content.** It is a filter. A given pattern (such as representing a topic of interest) determines the contents of interest. In this experiment, the pattern is defined by a set of generic keywords in a given domain. For example, the keyword *party* in the domain *lifestyle* is synonymous with "people having a party". Detecting "Tea Party" in an evident politic context is considered an error, even though the syntactic matching. Errors can be limited increasing the number of keywords but, in that case, the capability of detection is usually decreasing especially on short contents. Experiment at a relatively low scale on known dataset have shown a good reliability in the range 91-96% with an average error of 8% (see table 1). Cases in which there is a syntactic matching but the semantic matching cannot be determined are excluded by this statistic as well as contents that doesn’t have a syntactic matching are excluded a priori regardless from their meaning.

• **Localization.** In order to provide a contextual geo-processing, contents have to be associated with a location. The process under study adopts a semantic model for the geographic space [17, 18] that assumes the geographic space partitioned into correlated containers. Therefore, regardless from the level of detail, an error happens when a content is associated to a wrong container. The localization works on meta-data and, eventually, on the text content. Also the performance of localization (see table 1) are pretty good (average error around the 8%) but the error variance is relatively high. Anyway, most experiment are in a range of reliability between 83% and 94%. Non-localizable contents were not considered.

• **Timing.** Determining the correctness of the timing is pretty hard. Considering a relatively accurate time range (1 day) and only explicit information, performance looks good (see table 1) but those numbers are just a reference as an accurate validation is objectively hard.

As previously mentioned, the experiment about information retrieval were done on real set of data on relatively low scale. It’s hard to predict how those analytics could evolve as the scale significantly grows up. It is reasonable to assume a proportional increasing of the information entropy and results varying in a wide range as a function of the patterns and of the concrete network clusters that are analysed (assuming the processing of a whole network is unrealistic). According to the proposed approach, ontology are populated according to an individual-centric probabilistic model and, therefore, every query has its own probability associated with the result. Anyway, the semantic reasoner works on specific inferred concepts so it is possible to establish at least a kind of "query class" and assume the reasoning outcomes proportional to those classes. In order to provide a probabilistic overview of the process reliability, two different classes of reasoning are considered:

• **Quantitative analysis.** It assumes a contextual semantic geo-processing of the information based on a quantitative approach. In practice the filtering on contents is not relevant (all
Table 1: Probabilistic overview of the case study.

| Information Retrieval | Reliability(range) | Reliability(av.) | Error(range) | Error(av.) |
|-----------------------|--------------------|------------------|--------------|------------|
| Localization          | 83-94%             | 92%              | 6-17%        | 8%         |
| Timing                | 87-95%             | 91%              | 5-13%        | 9%         |
| Content               | 91-96%             | 92%              | 4-9%         | 8%         |

| Information Processing | Quantitative Analysis | Qualitative Analysis |
|------------------------|-----------------------|----------------------|
|                        | 72-89%                | 84%                  |
|                        | 66-86%                | 77%                  |
|                        |                       | 11-28%               |
|                        |                       | 14-34%               |
|                        |                       | 16%                  |
|                        |                       | 23%                  |

contents are considered) or very weak (entire domain(s) are considered). The performance of this kind of analysis are based on Localization and Timing (as previously defined). So the performances of any quantitative query are proportional to the performance showed in table 1.

- **Qualitative analysis.** It extends the quantitative analysis with a strong and critical dependence on the content. Consequently, inferred concepts will depend on the content (as previously defined). Performance of qualitative query can be considered proportional to the results shown in table 1.

This case study points out the importance, in terms of analysis reliability and capability, of probabilistic semantic in a context of complex reasoning in presence of uncertainness, ambiguity and entropy.

4 Conclusions

A probabilistic OWL extension was proposed in the paper. The individual-centric perspective assures a double-side consistency as, from one hand, it allows effectiveness in the solution of practical problems involving uncertainness and entropy by applying semantic technologies and, from another hand, it is fully compliant with the philosophy of the original model. This approach was experimentally used in different contexts. The case study described in the paper can be considered a significant test for the amount and, above all, the complexity of the target information. The use of a probabilistic model enabled a more consistent and reliable analysis, at a quantitative and a qualitative level, as well as further and improved analysis capabilities. An individual-centric probabilistic extension for OWL offers the opportunity to model and process the uncertainties and the entropies as a part of the knowledge in a semantic context. The practical approach, as well as the key requirement to maintain the model as simple as possible, has affected more than one theoretical issue but, in general, leads to a smart and intuitive approach at the time to reflect the uncertainness on the Ontology population covering a wide range of real use cases.

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