Phenomenon of electrization caused by gravitation of massive body

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The value of excess charge in the kernel of massive body (and the opposite in sign excess charge at the surface) caused by the influence of gravitational forces is determined.

Up to now an old problem of connecting rotototy and magnetic characteristic heavenly bodies has been of gread interest. One of the aspects of the above problem is a question concerning quasi-neutrality disturbance of the substance of the body. (neutrality which on average in scale is greater than the lattice parameter or Debye radius because of gravitational forces. In addition to condensing the substance of the body these forces also create an excess of a positive charge in the center of a heavenly body and an excess of a negative charge on periphery of a body. (gravitational forces affect more strongly heavy nuclei than light electrons). The electric field caused by this effect results in the compression of the electric component of the substance of a heavenly body.

Rotating around its own axis that body owing to redistribution of its charge must obtain magnetic moment. Thus far the scientists have tried to explain (by the mechanism under discussion) the magnetic properties of heavenly bodies. The aim of this paper is to prove hopelessness of such attempts due to a negligible value of the effect of charge redistribution which corresponds to parameter \( \alpha \):

\[
\alpha = \frac{G m_p^2}{e^2} \approx 10^{-36},
\]

(1)

Here \( G \) - a gravitation constant, \( m_p \) and \( e \) - the mass and charge of a proton.

1. Let’s start with elementary evaluations directly concerning a solid (crystalline) state of a substance. In this case redistribution of a charge is the displacement of a nucleus relative to the center of ”Vigner-Zeitc cell” and
leads to the appearance of dipole moment and polarization [1]. It is obvious that both the electric and gravitational forces which act on the displacement of a nucleus must be in balance. Hence

\[ ZeE = -GM^2 \nabla \int dx' n_p(x')/|x - x'|, \]  

(2)

Here \( Ze \) and \( M \) - a charge and mass of a nucleus, \( E \) - the vector of an electric field, \( n_p \) - the concentration of nuclei. Taking into account equation \( \text{div} E = 4\pi \delta \rho \) (\( \delta \rho \) - an excess of a charge), one may find:

\[ \delta \rho = \left( \frac{A}{Z} \right)^2 \alpha \rho_p \sim \alpha \rho_p. \]  

(3)

Here \( A \) - the mass of a nucleus divided by the mass of a proton, \( \rho_p \) - the density of an electric charge of nuclei.

Simple dimension considerations are in favour of evaluation (3).

Dimensionless relation \( \delta \rho/\rho_p \) must be proportional to \( G \) (or more exactly to \( \alpha \)) which follows from the application of perturbation theory on the gravitation interaction. Corresponding coefficient of proportionality may depend on dimensionless parameters of substance - \( z \), the ratio of masses of a proton and an electron; the ratio of Coulomb energy of a single particle to the largest of the following values: \( E_F \) and \( T \) (\( E_F \) - Ferme energy and \( T \) - temperature). Now you can see for yourself that for astronomic bodies the above parameters differ from 1 not more than 100 - 1000 times. Therefore they can’t considerably influence eq. (3), (because \( \delta \rho/\rho_p \), depends on \( G \) linearly). The negative charge, which compensate (3) and is localized on the surface of a body, equals

\[ Q = - \int dx \delta \rho \sim \alpha Q_p, \]  

(3')

Here \( Q_p \) - the total charge of the nuclei of a body. The polarization \( P = Zen_p \delta \) (\( \delta \) - the shift of a nucleus) is equal to \(-E/4\pi\), because induction \( D = 0 \) owing to the absence of external charges. As a result the elementary calculation using eq. (3) gives the ratio of the magnetic moment of a body to its mechanical moment.

\[ \sim -\alpha \frac{e}{m_pc}. \]  

(4)

Owing to the infinitesimal of parameter \( \alpha \) (see (1)) the above mentioned values are very small and the effect of charge redistribution under discussion can’t have direct observing demonstrations.
Suffice it to say, that for the Earth (the mass $\sim 10^{27}g.$, the radius $\sim 10^9sm$) the magnitude of the surface charge ($3'$) corresponds to one electron per 1 $m^2$ of the surface.

2. A more rigorous derivation of eq. (2) for a solid state of a substance is based on the selection of the part system from the total energy depending on the shift of nuclei $\delta_k$ (k - the number of a nucleus) and minimization of this part over $\delta_k$ accompanied by a change of: $\delta_k \rightarrow p_k/(Ze_n)$. Besides the sums for the grate may be replaced by integrals:

$$\sum_k \rightarrow \int dx n_p, \ p_k \rightarrow P(x).$$

Let’s start with gravitational energy for nuclear interaction:

$$E_{gr} = -\frac{GM^2}{2} \sum_{k,k'} |x_k - x_{k'}|^{-1}$$

Substituting $x_k \rightarrow x_k + \delta_k$ and factorizing over $\delta$ up to the first order inclusive, one finds:

$$E_{gr} = -\frac{GM^2}{Ze} \int dx \{P(x) \nabla \int \frac{dx' n_p(x')}{|x - x'|}\}. \quad (5)$$

Coulomb energy of interaction of nuclei and electrons can be written as:

$$E_c = \frac{Ze^2}{2} \sum_{k,k'} |x_k - x_{k'}|^{-1} - Ze^2 \int dx n(x) \sum_k |x_k - x|^{-1} + \frac{e^2}{2} \int \int \frac{dxdx' n(x)n(x')}{|x - x'|},$$

here $n \approx Zn_p$ - the concentration of electrons. In repeating the same considerations which resulted in (5) we must have in mind two circumstances:

1. Now we must factorize $\delta$ (or $P$) to the second degree inclusive, (as $\alpha$ is very small, see (1)).

2. Factorization with respect to $\delta$ is impossible in case of interaction with electrons of the same cell because of the necessity to take into account the contribution of region $r < \delta$ and this case must be considered exactly.

It is represented in the second term of $E_c$, in which one must select an integral over the cell volume:

$$-Ze^2 \int \frac{dx n(x)}{|x - \delta|},$$

multiplied by the total number of nuclei. It leads to the following expression for the part depending on $P$:

$$E_c^{(1)} = \frac{2\pi}{3} \int dx P^2.$$
In the remaining part of $E_c$ after factorizing over $\delta$ an ordinary term of dipole-dipole interaction arises, the interaction of different cells being reduced to it:

$$E_c^{(2)} = \int \int dxdx' \left( \frac{P(x)P(x')}{2|x - x'|^3} - 3 \frac{(P(x), P(x - x'))(P(x'), (x - x'))}{2|x - x'|^5} \right)$$

(Terms of $E_c$ which are linear with respect to $P$ disappear owing to the neutrality of a system). The equation for $E_c^{(2)}$ may be written as (see Enclosure):

$$E_c^{(2)} = \frac{2\pi}{3} \int dx \left[ P^2(x) - 3(P(x), \nabla) \frac{(\nabla P(x))}{\Delta} \right]$$

For that reason the sum of $E_c^{(1)}$ and $E_c^{(2)}$ can be written as:

$$E_c = 2\pi \int dx P_1^2, \quad (6)$$

here $P_1 = \frac{\nabla \text{div} P}{\Delta}$ - the longitudinal part of vector $P$ (it is in fact in (5)). Minimization of sums (5) and (6) over $P_1$ (if $E = -4\pi P$) brings us back to eq. (2).

Let’s emphasize that in the last equation there is no correction for the distinction of the functioning field from the average one [1]. That correction would appear if we placed the nucleus considered - for which the balance of forces is written - into empty space (2). In fact we have the interaction of a nucleus with electrons of its cell (cavity). It is described by $E_c$ and the required contribution $-4\pi P/3$ to intensity because $\frac{\delta E}{\delta P} = -E$. To complete the proof of eq. (2) one should make sure that the parts of the system energy not considered above (to be more exact their parts depending on $P$) do not influence the result. First of all it is true for the electronic component of energy - kinetic, exchange, correlation [2].

In case of a strongly compressed substance - this case is more interesting for heavenly bodies having a solid state substance - kinetic energy of free electron gas is an important factor (other components of energy are very small compared with Coulomb energy taken into account above).

Decomposition of this energy over the nuclear shift relative to the center of a cell:

$$\delta E_{\text{kin}} \sim \int dxn\delta U, \quad \delta U = Ze^2 \left( \frac{1}{r} - \frac{1}{|r + \delta|} \right)$$

leads to the zero result owing to the spherical symmetry $n$ and a known decomposition of Coulomb term into a series over Lezhandr polynomial. As regards the energy of nuclei in a solid state at low temperatures one should
consider only zero energy of nuclear oscillation which is equal to \( \frac{3}{2} \hbar w_0 \) for one nucleus \( (w_0 \text{ - the oscillation frequency}) \). Hence only part \( w_0 \) depending considerably on shift \( \delta \) or \( P \) could affect the proof given above. But in case of an exceptional small value of \( \delta \) \((\delta \sim \alpha R \text{ - see point 1, } R \text{ - the radius of a body})\) the shift of the oscillation center of a nucleus in the cells model does not influence the oscillation frequency at all. A change in energy in shifting a nucleus by \( \delta_r \) is equal to \( l^2 e^2 \delta_r^2/(2R^3) \) (where \( R \text{ - the radius of a cell} \)) and in substituting \( \delta_r \rightarrow \delta_r + \delta \) (the shift of the oscillation center) the square of frequency as the second derivative of energy with respect to \( \delta_r \) does not change at all.

3. Eq. (3) is in fact of universal nature and it is valid irrespective of the state of a substance. We will utilize a method of functional density \([3]\), writing down the free energy of a system (in general case temperature \( T \) is not equal to zero) as a function of the density of electrons \( n \) and nuclei \( n_p \).

\[
F\{n, n_p\} = F_0 + E_{sc} + F_{xc} - \mu \int dx n - \mu_p \int dx n_p, \tag{7}
\]

here the first term conforms to free electronic and nuclear gases, the second one - Coulomb and gravitation energy of their interaction in approximation of a self - consistent field, the third term corresponds to exchange and correlation effects. The latter two terms correspond to Lagranzh factors and show conservation of total numbers of electrons and nuclei. Minimum \( F \) for \( n \) and \( n_p \) defines equilibrium distributions of these quantities. Writing down

\[
E_{sc} = -\frac{GM^2}{2} \int \frac{dx dx'}{|x - x'|} n_p n_p' + \frac{e^2}{2} \int \frac{dx dx'}{|x - x'|} (n - zn_p)(n' - zn_p'),
\]

we obtain conditions of minimum (7) for \( n \) and \( n_p \):

\[
\Delta \frac{\delta(F_0 + F_{xc})}{\delta n(x)} = 4\pi e^2(n - zn_p) \tag{7'}
\]

\[
\Delta \frac{\delta(F_0 + F_{xc})}{\delta n_p(x)} = -4\pi ze^2(n - zn_p) - 4\pi GM^2 n_p. \tag{7''}
\]

If we could omit the left part of eq. (7''), then, taking into account that \( \delta \rho = e(zn_p - n), \rho_p = zen_p \), we would obtain eq. (3), and if (7'') is substituted into (7') and if appropriate simplifications are made then (7') will fit Chandrasekar equations which define equilibrium configuration of electrons and nuclei. Let’s emphasize that the left part of that equation is defined by the lightest particles - electrons, the right part - contains only gravitation
quantities after the above substitution of (7′′) into (7′) although gravitation does not act on the electrons directly. The electrons are affected by the electric field investigated in this paper, which only quantitatively (see (3)) coincides with a gravitational field.

Thus eq. (7′) and (7′′) are rewritten as:

\[ \delta \rho = \alpha \rho_p (1 + \sigma)^{-1}; \quad \sigma = \frac{\Delta F/\delta n_p}{z \Delta F/\delta n}, \quad (8) \]

where here and below \( F = F_0 + F_{xc} \).

The appearance of \( \sigma \) either keeps the magnitude of \( \delta \rho/\rho_p \) constant, or decreases this relation. The single case when it may increase considerably, represents exceptional nearness of \( \sigma \) and \(-1\). But this is practically unbelievable. Furthermore, we may think that \( \sigma << 1 \). Let’s illustrate it by using two examples [2]. For both examples it is supposed that the electron gas is strongly compressed and degenerated so that the corresponding contribution to \( F \) is \( \sim \frac{\hbar^2}{m} \int dxF \). In the first example nuclei are localized in nodes of a grate and the energy of their zero oscillations represent \( \delta F \sim \frac{Ze\hbar}{\sqrt{M}} \int dxF \). (see above). Then for \( \sigma \) in (8) we find:

\[ \sqrt{\frac{m}{M}} Z (a_0 n^{1/3})^{-1/2} << 1, \]

where \( m \) - the mass of the electron, \( a_0 = \frac{\hbar^2}{me^2} \) - it’s ”Bohr-radius”. This smallness is connected with inequalities: \( m/M << 1 \) and \( a_0 n^{1/3} >> 1 \) in a compressed substance.

The second example represents a weak non ideal nuclear ”Boltzman-system” for which \( F(n_p) \sim -e^2 \int dxF \) is ”Debay-Hukkel” - correlation correction. So for \( \sigma \) one finds:

\[ \epsilon \sqrt{\frac{n^{1/3}}{T}} (\sqrt{Za_0 n^{1/3}}) << 1 \]

because the condition of weak non-perfection is \( T >> e^2 n^{1/3} \).

4. In conclusion let’s go back to the question about the minimum of (7) in connection with disturbance of local electric neutrality of the system.

Note that the above violation is typical of the crystalline state of a substance in the absence of gravitation forces, which is evident and the electrons are not localized in contrast to nuclei which are localized at the point.

It is important that this violation is not described by minimum of \( F \), as a point in functional space in which its functional derivative disappears. In this case we deal with a boundary minimum reached when the parameter
which defines the length of nuclear localization tends to its limiting value which equals zero.

Let’s consider the second clear Coulomb component of $E_{sc}$ which should be broken into neutral and spherical on the whole "Vignier-Zeitc" cells with a nucleus in the center. In such a model the radius of the cell is $R$ and the nucleus is spread in sphere with radius $\rho$. This model represents energy:  
$$E_{sc} = -\frac{3}{10} Z^2 e^2 \frac{2x^3 + 4x^2 + 6x + 3}{(x^2 + x + 1)^2}$$

where $x = \rho/R$. At $x = 0$ this equation gives a well-known binding energy of the grate:  
$$-\frac{9Z^2 e^2}{10R}.$$  
It is evident that maximum of $|E_{sc}|$ is in fact reached on the boundary of permissible area (at $x = 0$ and fixed $R$).

\textit{Enclosure}

Initial expression for $E_c^{(2)}$ can be written as:

$$E_c^{(2)} = \int d\mathbf{x} P_i(\mathbf{x}) K_{ij}(\mathbf{k}) P_j(\mathbf{x}),$$

where $\mathbf{k} = -i\nabla$, $\nabla$ act on $P_j(\mathbf{x})$, and

$$K_{ij}(\mathbf{k}) = \frac{1}{2}\int \frac{d\lambda}{\lambda^3} (\delta_{ij} - 3\frac{\lambda_i\lambda_j}{\lambda^2}) e^{-i\mathbf{k}\mathbf{\lambda}}.$$

This integral satisfies evident condition $K_{ii} = 0$ and hence it may be represented as

$$K_{ij} = \frac{1}{2K^2} k_l k_m (\delta_{ij} - 3 \frac{k_l k_m}{k^2}).$$

The expression for $E_c^{(2)}$ mentioned in this paper gives the calculation that is not complicated but awkward.

In conclusion we would like to note critical debate on this questions with Vasilyev B.V., Grigoryev V.I. and Maximov V.I..

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\textit{LITERATURE}
1. Tamm I.E. The Theoretical Course on Electricity. Moscow, Science, 1989.

2. Landau L.D., Lifshite E.M.. Statistical physics, part 1, Moscow, Science, 1995.

3. Lundkvist S., March M., The Theory of Heterogeneous Electrons gas. Moscow, Mir, 1987.