Branes and the Gauge Hierarchy

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Abstract

If the fundamental type-I string scale is of the order of few TeV, the problem of the
gauge hierarchy is that of understanding why some dimensions transverse to our brane-
world are so large. The technical aspect of this problem, as usually formulated, is ‘why
quantum corrections do not modify drastically the masses and other parameters of the
Standard Model’. We argue that within type-I perturbation theory, the technical hierarchy
problem is solved (a) if all massless tadpoles cancel locally over distances of order the
string length in the transverse space, or (b) if the massless fields with uncancelled local
tadpoles propagate ‘effectively’ in \( d_\perp \geq 2 \) large transverse dimensions. These restrictions
ensure that loop corrections to the Standard Model parameters decouple from the four-
dimensional Planck scale, except when there are uncancelled tadpoles in \( d_\perp = 2 \) in which
case the dependence on \( M_P \) is logarithmic. This latter case is thus singled out as the
only one in which the origin of the hierarchy would not be attributed entirely to ‘out of
this world’ bulk physics. The role of the renormalization group equations in summing the
leading large logs is replaced by the classical 2d supergravity equations in the transverse
space.

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The effective Lagrangian describing low-energy physics in our world is

\[ S_{\text{eff}} = \int d^4x \sqrt{g} \left( \Lambda + M_P^2 R + \mathcal{L}_{SM} \right), \quad (1) \]

where \( \mathcal{L}_{SM} \) is the Lagrangian for the gauge, matter and Higgs fields of the Standard Model. The gauge hierarchy and cosmological constant problems are the questions of why, compared to the typical Standard Model scale around the TeV, the two scales of gravity, \( M_P \sim 10^{15} \text{ TeV} \) and \( \Lambda^{1/4} \lesssim 10^{-15} \text{ TeV} \) respectively, are so hierarchically different. There are of course some extra hierarchies in the parameters of the Standard Model per se, mainly in connection to fermion masses. We will however adopt the point of view that at least the most severe one – the apparent smallness of neutrino masses – is intimately related to the basic gauge hierarchy, and will prove to be natural once this latter has been explained.

The technical aspect of the gauge-hierarchy problem is a question of more limited scope. The question is why radiative corrections do not destabilize the masses and other parameters of the Standard Model. In the context of renormalizable local field theory, spontaneously or softly-broken supersymmetry solves this problem by allowing only logarithmic dependence on the ultraviolet cutoff, usually taken at or near \( M_P \).

More recently it has been proposed that the fundamental scale \( M_s = 1/l_s \) of string theory could be in the TeV region \( 1 \), with the Standard Model fields living on what looks at distances larger than \( l_s \) as a three-brane \( 2 \) \[3\]. The weakness of gravity would then be attributed to the existence of large transverse dimensions, in the submillimeter to the subfermi region, in which only the gravitational sector propagates \( 4 \). This possibility has a perturbative description within type-I string theory, which necessarily implies some submillimeter dimensions when the fundamental tension is near the TeV \( 4 \) \[3\]. Furthermore, explicit model building looks promising \( 9 \), while this radical proposal appears to be a priori compatible with all experimental constraints if the number of large transverse dimensions is larger than two, and marginally compatible if it is equal to two \( 10 \).

One would think that in such a scenario the technical aspect of the gauge hierarchy problem is automatically solved, since the ultraviolet cutoff \( M_s \) is of the same order as the electroweak scale \( 4 \). It was pointed out, however, in ref. \( 11 \) that within type-I perturbation theory gauge couplings and other parameters of the world-brane theory could receive corrections growing linearly or logarithmically with the size \( R \) of the transverse space, and hence a posteriori with \( M_P \). The precise rate of growth was attributed to the ‘effective dimensionality’ of the transverse space. Since such corrections would modify

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\( 1 \) Earlier efforts to lower the compactification \( 6 \) or string scale \( 7 \) near the TeV region have been made in the context of the heterotic string, while the possibility of lowering the unification scale has been discussed more recently in \( 8 \).
the parameters of the Standard Model at the electroweak scale, controlling them can be considered as the novel guise of the technical problem associated with the gauge hierarchy.

In this letter we further elucidate the origin of these corrections, and state the conditions under which the technical hierarchy problem can be solved. As we will explain, such effects will be completely absent (a) if all tadpoles of massless bulk fields cancel locally at scales \( l_s \) in transverse space, or (b) if bulk fields with uncanceled local tadpoles propagate ‘effectively’ in \( d_\perp > 2 \) large transverse dimensions. The limiting case \( d_\perp = 2 \) leads to logarithmic sensitivity of the world-brane parameters on \( R \) and \( M_P \), while for \( d_\perp = 1 \) the sensitivity is linear and one hits a strong-coupling singularity \([12]\)\([13]\) much before \( M_P \) is allowed to reach its experimentally-determined value. It is important to stress that \( d_\perp \) need not be the total number of superstringy dimensions for two reasons \([11]\): because linear divergences may persist when one transverse dimension grows much larger than all the others, and because some of the tadpoles may couple to fields living on higher branes that encompass our four-dimensional worldvolume.

Local tadpole cancellation constrains severely model building, and in particular the mechanisms of supersymmetry breaking. The case of uncanceled local tadpoles in \( d_\perp = 2 \), on the other hand, appears particularly interesting because of the logarithmic sensitivity of the Standard Model parameters on the size of the transverse space. This leaves open the possibility of dynamically determining the hierarchy by minimizing an effective potential on our world-brane. It is to this end crucial to know how to resum all the large logs – as in renormalizable field theory where they are absorbed in a finite number of renormalizable couplings. This may sound hopeless in our context, since there is no field-theory description beyond the TeV. Fortunately, we will argue that an equally-powerfull procedure is available: all large logs can be effectively absorbed into the values of finitely-many massless fields living in the bulk and evaluated at the (transverse) position of our world-brane. The role of the renormalization group equations is thus played by the classical supergravity equations in the effectively two-dimensional transverse space – with higher derivative terms being ignored because the variations of fields are logarithmic.

It is important to appreciate that in a non-supersymmetric string theory the bulk one-loop vacuum energy, \( \Lambda_{\text{bulk}} \sim M_s^{10} \), gives rise to a four-dimensional cosmological constant with quadratic sensitivity on the Planck scale, \( \Lambda \sim M_P^2 M_s^2 \) \([14]\). This is analogous to the quadratically-divergent one-loop contribution, proportional to \( \text{Str} M^2 \), in a softly-broken supersymmetric field theory. Since the effective potential on the world-brane has at best logarithmic sensitivity on \( M_P \), the hierarchy might be determined dynamically only if such quadratic corrections are absent \([13]\)\([14]\). In the framework of softly broken supersymmetry this condition appears to be adhoc, unless the breaking is induced by compactification in a way analogous to turning on finite temperature \([3]\)\([16]\). In the novel framework, on the other hand, the spontaneous breaking can be confined to the world-brane at the string scale, and then communicate very weakly to the bulk \([5]\). This can for instance be achieved by turning on internal magnetic fields, or in T-dual language by rotating the branes in the compact space \([17]\)\([18]\).
Ultraviolet versus Infrared

If all the Standard Model fields live on a set of D-branes, then the observed non-gravitational processes are given by amplitudes whose external legs are open strings with endpoints on these D-branes. The branes must of course extend along the four dimensions of our physical space-time. We assume for simplicity that the only other dimensions whose size is significantly larger than the string length are transverse to the common worldvolume of our branes. The possible soft (substringy) momenta in the theory have thus a four-dimensional longitudinal component ($p_\parallel$) and a transverse component ($p_\perp$). The quarks, leptons and gauge bosons of the Standard Model, i.e. all external legs in the amplitudes of interest, are not allowed to carry any transverse momentum.

Fig. 1: The (a) Möbius, (b) planar annulus, and (c) non-planar annulus diagrams contributing to a 4-point amplitude of world-brane fields. On top of every diagram we indicate the momentum flowing along the cylinder. The infrared divergences of diagrams (a) and (b) appear as ultraviolet quantum-gravity effects to an observer localized on the world-brane.

Consider now the one-loop contributions to such amplitudes, given by the three diagrams of figure 1: (a) Möbius, (b) planar annulus with all external legs on the same boundary, and (c) non-planar annulus with some external legs on each of the two boundaries. All potential divergences of these amplitudes are interpreted by the ten-dimensional string theorist as large infrared effects. They occur because some massless (or very light) intermediate open- or closed-string state propagates very nearly on-shell. The four-dimensional
Standard-Model physicist, on the other hand, may interpret these effects very differently. Some of them will look as genuine infrared effects, already incorporated in his effective low-energy action \( \mathcal{I} \), while others modify his parameters at the weak scale and appear intimately related to the ultraviolet structure of the fundamental theory!

Let us take a closer look at these potential divergences. Those associated with light open strings – the photon and gluons in particular – occur at *exceptional* values of the external momenta and are the usual infrared effects of Yang-Mills theory. Likewise, the potential divergence in the non-planar amplitude (c) is to the eyes of our brane physicist a soft effect coming from the exchange of a gravitational particle. Indeed, for generic momenta of the external legs, the intermediate closed string in the transverse channel carries non-vanishing four-momentum \( p_\parallel \), which cuts off the putative infrared divergence.\(^2\) Strictly-speaking the effective theory \( \mathcal{I} \) does not, of course, account correctly for the fact that gravity is higher-dimensional above the Kaluza-Klein scale \( 1/R \), with \( R \) the typical radius of the large transverse dimensions. For instance, for a single large dimension the tree-level exchange amplitude is of the order of

\[
\mathcal{A} \sim \left( \frac{p_\parallel}{M_s} \right)^{2n} \mathcal{G}^{(2)}, \quad \text{where} \quad \mathcal{G}^{(2)} \sim \frac{1}{p_\parallel \coth(\pi p_\parallel R)}
\]

is the two-point function on the brane obtained by summing over the Kaluza-Klein modes, \( p_\parallel \) denotes here the modulus of the longitudinal momentum, and the vertex-suppression factor in the amplitude has \( n = 2 \) or 1 for gravitational or gauge-like couplings. For ultrasoft momenta \( p_\parallel \ll 1/R \), the two-point function \( \mathcal{G}^{(2)} \sim 1/\pi R p_\parallel^2 + \pi R/3 \) exhibits the standard inverse-square behaviour plus a large constant correction due to the extra dimension \([19]\). For \( p_\parallel \gg 1/R \) on the other hand, \( \mathcal{G}^{(2)} \sim 1/p_\parallel \) corresponding to Newton’s law in five dimensions. The basic point for our purposes here, in any case, is that at current accelerator energies and for non-exceptional values of momenta such gravitational exchanges remain suppressed, and all potential infrared divergences are regularized.

The Möbius and the planar-annulus diagrams, on the other hand, are very different. Since there is no 4d momentum flowing down the cylinder, their potential divergences persist for arbitrary external momenta at the weak scale. Such large effects will thus correct the tree-level parameters of the Standard Model, and must be kept under control to avoid destabilizing the gauge hierarchy. In order for such effects to be present, a massless (or light) closed string with soft transverse momentum \( p_\perp \ll M_s \) should be allowed to disappear into the vacuum. This can only happen if there are local tadpoles uncancelled at distances large compared to the string scale, \( l \sim 1/p_\perp \gg l_s \). Global \( (p_\perp = 0) \) tadpole cancellation in a compact space is of course required for topological consistency – our

\(^2\) Note that \( p_\parallel \) is conserved at each vertex, while \( p_\perp \) is not conserved since the branes break the translation invariance in transverse space.
perturbation theory would otherwise be truly divergent and hence sick. Local tadpoles, on the other hand, need not a priori destroy perturbativity, and their effects can as we will argue be effectively resummed. The simplest context in which to discuss these ideas is type-I or type-I’ theory with $2^d$ orientifold $(9-d)$-planes transverse to a $d$-dimensional torus $T^d$. Global consistency requires also $32$ D$(9-d)$-branes, which are free classically to move at arbitrary positions on the torus. Tadpoles will cancel locally only at one special point of moduli space – when the $32$ D-branes are split in groups of $2^{5-d}$, with each group sitting precisely at an orientifold plane \cite{12 20 21}. Another simple example is the $T^4/Z_2 \times T^d$ orbifold \cite{22}, in which only $T^d$ is considerably larger than string size. Global tadpole cancellation requires $32$ D$(9-d)$-branes and $32$ D$(5-d)$-branes, all free to move on the transverse torus. Local tadpole cancellation on $T^d$ is again achieved at a special point in moduli space, i.e. when the D-branes split in equal numbers, with each group sitting at one of the $2^d$ orientifolds \cite{23 21}.

The contribution of these local tadpoles to the world-brane amplitudes can be estimated easily as follows. Assuming $d$ large transverse dimensions $x^i \in [0, 2\pi R_i]$ we find the following estimate for what looks to a brane physicist as a large ‘ultraviolet’ contribution:

$$uv(A) \sim \frac{1}{V_\perp} \sum_{|p_\perp| < M_s} \frac{1}{p_\perp^2} F(p_\perp),$$

(3)

where $V_\perp = \prod_i R_i$ is the volume of the transverse space, $p_\perp = (n_1/R_1, \ldots, n_d/R_d)$ is the transverse momentum carried away by the massless closed string, and $F(p_\perp)$ are the local tadpoles, Fourier-transformed to lattice-momentum space. The tadpoles arise from the distribution of the D-branes and the orientifolds which act as classical point-like sources in the transverse space. In the specific examples discussed above, the tadpoles have the generic form

$$F(p_\perp) \sim \left( 2^{5-d} \prod_{i=1}^d (1 + (-)^{n_i}) - 2 \sum_{a=1}^{16} \cos(p_\perp \bar{x}_a) \right),$$

(4)

where the orientifolds are located at the corners of the cell $\prod_i [0, \pi R_i]$, and $\pm \bar{x}_a$ are the transverse positions of the $32$ D-branes. These positions correspond to the Wilson lines of the T-dual picture. The momentum sum in equation (3) is cutoff effectively at $M_s$ because of the form-factor suppressing exponentially the amplitude at higher scales. Heuristically, this is because the external open strings are localized on the world-brane with an uncertainty $o(l_s)$ – this is indeed the size of a D-brane as seen by a fundamental-string probe \cite{24}.

For generic positions of the D-branes, and for roughly equal radii $R_i \sim R \gg l_s$, the above expression has the following behaviour in the decompactification limit:

$$uv(A) \sim \begin{cases} o(R) & \text{for } d = 1 \\ o(\log R) & \text{for } d = 2 \\ \text{finite} & \text{for } d > 2 \end{cases}$$

(5)
This result is clearly dictated by the large-distance behaviour of the two-point function in the $d$-dimensional transverse space. The conclusion is that the radius $R$, and hence also the four-dimensional Planck scale $M_P \sim M_s (M_s R)^{d/2}$, decouples from the loop corrections to world-brane parameters for $d > 2$ large transverse dimensions. It also of course decouples for $d = 1, 2$ if the tadpoles cancel exactly locally. In either case the technical problem of the gauge hierarchy is solved, but there is little room for understanding its dynamical origin from the world-brane viewpoint. When $d = 1$ there are large linear corrections and, as we will explain in a minute, one hits quickly a strong-coupling singularity forbidding the expansion of the transverse space. Thus, the marginal case $d = 2$ is singled out as the most promising candidate for generating and solving the hierarchy.

Our discussion can be generalized easily to anisotropic compactifications, and/or in the presence of higher branes which extend partially into the ten-dimensional bulk. The precise criterion for solving the technical gauge-hierarchy problem is that all massless bulk fields coupling to our Standard Model have long-distance Green functions in the transverse space which are at most logarithmically large. We may define an ‘effective dimensionality’ $d_\perp$ in which the bulk fields with uncancelled local tadpoles propagate. For fields confined for example to a $3+p$-brane, $d_\perp$ is at most equal to $p$. If the transverse space is anisotropic, with $n$ large dimensions of size $r$ and one even larger of size $R$, then the Green function would exhibit linear growth when $R \gg r^n$ in string units, in which case the effective dimensionality is $d_\perp = 1$. Such possibilities have been discussed in ref. [11]. The condition for solving the technical problem associated with the gauge hierarchy is that for all massless bulk fields with uncancelled tadpoles $d_\perp \geq 2$. Logarithmic sensitivity is possible even if all six dimensions become large, provided the effective dimensionality of some bulk fields is two.

**The New Guise of the Renormalization Group**

According to the traditional viewpoint the physics between the electroweak and Planck scales is described by renormalizable four-dimensional field theory. This means that all large quantum corrections involving ultraviolet degrees of freedom can be absorbed into a finite number of parameters – the masses and renormalizable couplings of the Standard Model measured at the electroweak scale $M_Z$. If the theory beyond $M_Z$ is a string theory, this traditional logic is not valid. Nevertheless, as we will now explain, in the particular case $d_\perp = 2$ all large quantum corrections involving the transverse volume, and through it the four-dimensional Planck scale, can be similarly absorbed into a finite number of parameters of the brane theory. These parameters correspond to the values of the massless bulk background fields on the four-dimensional world-brane. The classical

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3 Proposals for stabilizing the hierarchy were discussed in refs. [25].
bulk equations of supergravity play the role of the renormalization group equations, in that they allow a resummation of the leading logarithmic corrections.

Let us explain this important point in some more detail, in the context of toroidal compactifications of type-I theory. The classical background has constant dilaton, metric and Ramond-Ramond (RR) gauge fields, as well as a number of δ-function sources representing the D-branes and the orientifolds. The number of these sources is being fixed by global tadpole cancellation – or else compactification of the transverse space would be obstructed topologically. Tadpoles need not however cancel locally, since the D-branes can be placed at arbitrary positions. These local sources modify the background fields, an effect that is classically invisible because the strength of the sources is controlled by the string coupling constant $g_s$. In the full quantum theory, on the other hand, we have to take into account the modification of the backgrounds.

Consider now a very large (in string units) transverse space, and focus on some particular set of D-branes on which live our Standard Model gauge and matter fields. Their effective Lagrangian depends on the dynamics of the bulk only through the values of the supergravity backgrounds, evaluated at the position of these D-branes. If the number of large transverse dimensions $d_\perp > 2$, the effects of the distant sources die out and the bulk fields can be approximated by their constant classical background values. The size and other macroscopic moduli of the transverse space decouple from the quantum-corrected brane theory, in the same way that the ultraviolet cutoff decouples from a super-renormalizable field theory. Examples of such models with $d_\perp > 2$ can be provided by T-duals of type-I compactifications with only D9-branes, in which $d_\perp$ can reach its maximum value of six, or with at most one set of D5-branes in which case the maximum $d_\perp$ equals four.

In the case $d_\perp = 2$ the effect of the distant sources grows at most logarithmically with size, so that the gradients of bulk fields away from these sources stay small. It can be argued that the classical supergravity equations in the transverse space can be used to resum all large corrections by solving for the spatial variation of the background fields. Indeed, $\alpha'$ corrections are negligible because the gradients of the background fields stay small. In what concerns supergravity (closed string) loops, these are higher dimensional and hence, by power counting, lead to higher derivative corrections which are suppressed. An independent argument follows from supersymmetry in the bulk which is at least six dimensional; this ensures that the two-derivative supergravity action receives no corrections. As a result, the quantum-corrected Lagrangian of the brane theory is obtained by evaluating these bulk backgrounds at the position of our world-brane. Note that this position is only defined within $\sim o(l_s)$, since at shorter distances the classical supergravity equations don’t make sense [20]. The main message is that when $d_\perp = 2$ we have a method for resumming the large logs which is equally-powerful as the renormalization-group equations in the case of renormalizable quantum field theory.
In the final case \( d_\perp = 1 \) the effect of the distant sources grows linearly, and one expects in general that some brane theory will become strongly coupled very rapidly as the value of the radius is being increased beyond string length. For instance in semi-realistic compactifications with non-maximal supersymmetry \((N = 2 \text{ or } N = 1)\), gauge couplings will in general acquire large threshold corrections, that grow linearly with the radius. If for some gauge group factor these are negative, the corresponding gauge coupling will hit a strong coupling singularity soon after the radius moves away from the string length. This leads to an upper bound for the size of the transverse dimension, similar to the one obtained in the compactification of M-theory on Calabi-Yau×\( S^1/Z_2 \) with standard embedding [13].

The phenomenon can be also understood as a consequence of the non-factorization of the internal manifold, due to a position dependence of the Calabi-Yau volume along the line-segment. One can of course avoid such singularities if tadpoles cancel exactly locally, or if there existed some special models in which for instance all linear thresholds happen to be positive, so that gauge couplings become infinitesimal in the large-radius limit. Such cases are however exceptional and too restrictive for model building. In the generic case bulk-field variations cannot be controlled by the supergravity equations, since in particular quantum gravity effects will become at least locally strong. The situation is therefore here analogous to that of non-renormalizable field theories.

To summarize our conclusions in this section, we draw below a schematic correspondence between the dependence on the ultraviolet cutoff in quantum field theory, and the dependence on \( M_P \) in the brane picture.

| \( d_\perp \) | QFT analog          |
|---------|-------------------|
| 1       | non-renormalizable |
| 2       | renormalizable    |
| >2      | super-renormalizable |

An implication of our discussion is that the effective field-theory couplings of bulk fields to the brane [27] are generally modified. Firstly there can be bulk fields other than the graviton which can be emitted in the transverse space. Secondly, the local strength of the coupling on the brane can be enhanced or reduced relative to the four-dimensional Planck scale. As a result the bounds on the transverse size and the string scale obtained from exotic events with missing energy [28] may need to be further refined.
Two Examples

We will conclude our discussion with two simple examples that will help illustrate the main points of this letter. Consider first the $T^4/Z_2 \times T^2$ orbifold of type I theory, whose threshold corrections to the 4d gauge couplings were analyzed in refs. \cite{29}, \cite{11}. We take $T^2$ large and $T^4/Z_2$ of order the string size, and will use the type-I' description in which one has 32 D3-branes and 32 D7-branes, all transverse to the large $T^2$. Local tadpole cancellation in the large transverse space requires that exactly eight D3 branes and eight D7-branes are located in the vicinity (within $\sim l_s$) of each of the four orientifold planes. Our observable world would correspond to one such set. In this case the radii of $T^2$ decouple from quantum corrections to our effective world-brane action. This implies in particular that the threshold corrections to the 4d gauge couplings, at this point in moduli space, should not depend on the geometry of $T^2$.

To check this note first that because of the N=2 supersymmetry only charged BPS states contribute to the renormalization of the gauge couplings \cite{29}. The BPS open-string states can be most easily read off at the point of maximal $U(16) \times U(16)$ gauge symmetry, corresponding to all D-branes sitting at the same orientifold. Besides the adjoint vector multiplets, the massless states at this point are hypermultiplets in the antisymmetric $120$ and $\overline{120}$ representation of each gauge group, as well as a hypermultiplet in the representation $(16,16)$. The BPS states include all winding excitations on the two-torus. In the configuration in which the tadpoles cancel locally, the gauge group at each orientifold is $U(4) \times U(4)$, with massless hypermultiplets in the $6$ and $\overline{6}$ for each group factor, and one hypermultiplet in the $(4,4)$. Since all other states are hypermassive ($\sim R_i M_s^2$), our world-brane theory can decouple from the size of the torus only if its $\beta$-functions vanish identically. The $\beta$-function contributions to a $N=2$ $SU(n)$ theory are one-loop and proportional to

$$b_{SU(n)} = \begin{cases} 
2n & \text{adjoint} \\
-2 & \text{antisymmetric} \\
1 & \text{fundamental}
\end{cases}$$

(6)

It is thus straightforward to check that they vanish for the $SU(4)^2$ parts of the gauge group, thus confirming that the large logarithmic corrections are absent when the tadpoles cancel locally. Notice that the $U(1)^2$ factors become massive by mixing with bulk tensor hypermultiplets \cite{23}, so their apparent non-vanishing $\beta$-functions can be understood through the exchange of this hypermultiplet.

With the second example, we want to illustrate the general claim that all large effects in the decompactification limit of a 2d transverse space can be absorbed in a finite number of parameters, which correspond to the values of the bulk fields at the position of our world-brane. These fields have no longitudinal space dependence, and are therefore parameters of the brane theory. The simplest case is that of a maximally supersymmetric compactification of type-I theory on a two-torus. As before, we take $T^2$ very large and use
type I' language, so that there are 32 D7-branes and four orientifolds, all transverse to the two-torus. The world-brane action of the gauge fields in units of the fundamental string tension \(T_F = 1\) reads \[30\]

\[
\mathcal{L}_{brane} = \frac{1}{4} T_{(7)}^I \sqrt{g} e^{-\phi} \text{tr} \left( F^2 - \frac{1}{48} t_8 F^4 + \cdots + 2 C^{(4)} F \wedge F + \frac{1}{6} C^{(0)} \epsilon_8 F^4 \right)
\]

(7)

where \(T_{(7)}^I\) is the D7-brane tension, \(g\) is the induced metric on the worldvolume, \(t_8\) is the usual eight-index tensor that appears in the expansion of the Born-Infeld Lagrangian, \(\epsilon_8\) the totally-antisymmetric tensor, \(\phi\) is the ten-dimensional dilaton, and \(C^{(n)}\) the RR antisymmetric \(n\)-form fields. Assuming eight-dimensional Poincaré invariance sets \(C^{(4)} = 0\). The remaining bulk fields \(\phi\) and \(C^{(0)}\), together with the scale factor \(\omega\) of the 8d worldvolume flat space \(R^8\) and the three-component metric \(G_{ij}\) of the transverse space can be functions of the transverse coordinates \(\xi^i\), but constant along \(R^8\). Note that the ten-dimensional metric in the string frame used above reads

\[
ds^2 = \omega^2 dx^\mu dx^\nu \eta_{\mu\nu} + G_{ij} d\xi^i d\xi^j .
\]

(8)

These functions are non-trivial except at the special point in moduli space where there are eight D7-branes at each orientifold, so that all tadpoles cancel locally. The precise functions can be read off by considering this background as a compactification of F-theory on an elliptically-fibered K3 \[31\]. The transverse space is the base-space, and \(\tau = C^{(0)} + ie^{-\phi}\) is the complex structure of the torus fiber.

Without describing in detail these variations, let us just note that by evaluating these bulk fields at the position of our world-brane we can find the quantum-corrected parameters of the brane theory, for any given point in moduli space. Consider, in particular, the CP-even part of the gauge-field action, which reads

\[
\mathcal{L}_{brane} = \frac{1}{4} T_{(7)}^I e^{-\phi} \text{tr} \left( \omega^4 F^2 - \frac{1}{48} t_8 F^4 + \cdots \right) .
\]

(9)

To find the variation of the fields, one must consider the bulk supergravity equations reduced from ten down to the two transverse dimensions. The relevant action is

\[
S \propto \int d^2\xi \sqrt{G} e^{-2\phi_{(2)}} \left( \frac{1}{2} R^{(2)} - 2(\partial \phi_{(2)})^2 + 4 \frac{(\partial \omega)^2}{\omega^2} + \frac{e^{\phi_{(2)}\omega^4}}{16\pi^3\sqrt{G}} \sum_a \delta^{(2)}(\xi - \xi_a) \right)
\]

(10)

where \(\phi_{(2)}\) is the two-dimensional dilaton, \(e^{-\phi_{(2)}} = e^{-\phi} \omega^4\), while \(R^{(2)}\) is the 2d scalar curvature, and the \(\delta\)-function sources correspond to the D7-branes and the orientifolds.

Let us for instance explain in this context why the \(F^2\) term in \([2]\) receives no quantum corrections – either perturbative or non-perturbative. The coefficient of this term is precisely the exponential of the two-dimensional dilaton, which satisfies a free-field equation
without source obtained by varying the conformal factor of the two-dimensional metric in (10). As a result $\phi^{(2)}$ is a constant in transverse space, equal to its classical value, and independent of the positions of the D-branes as well as of the geometric torus moduli. This can be checked alternatively from the explicit form of the seven-brane solution [32] in which $e^{-\phi} \omega^4$ can be seen to be constant. The coefficient $e^{-\phi}$ of the $F^4$ term, on the other hand, has a variation which can be read off the F-theory solution. When one perturbs around the special configuration in which tadpoles cancel locally, one finds [31]

$$e^{-\phi} = 1 - \frac{g_s}{2\pi} \sum_{a=1}^{4} \log|1 - z_a / z|$$

(11)

where $z = \xi^1 + i\xi^2$ is a complex coordinate, $z = 0$ is the position of the orientifold and $z_a$ are the positions of the (pairs of) D7-branes. To evaluate this expression on the $b$th brane one must set $z = z_b + o(l_s)$, where $l_s$ plays the role of the ultraviolet cutoff analogous to the one imposed in equation (3). It is easy to see that these logarithms, which arise from the two-dimensional Green’s function, reproduce the one-loop logarithmic dependence of the $F^4$ term in this context [33]. Note that the splitting of the orientifold planes demonstrated by Sen [31] affects our discussion by corrections that are exponentially-suppressed at weak coupling.

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