Mathematical and Computer Modelling as the Basis of Structural Health Monitoring

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Abstract. As it is known only instrumental monitoring system, based on results of mathematical and computer (numerical) modeling (for instance, based on finite element analysis) and comparison with corresponding measured data allows high-performance control of actual state and the prediction of building safe operation. Moreover, instrumental monitoring of unique buildings without corresponding correct and adequate so-called “monitoring-oriented” models has random nonsensical nature and therefore it is not of practical interest. This paper is devoted to theoretical methodology foundations of structural health monitoring (SHM). In order to review various corresponding approaches, development, modification and verification of parameterized finite building element models, so, significant stage of life cycle (the stages of construction and operation), original algorithm of adaptation (calibration) in accordance with measurements results, methodology of natural frequencies measurements and modal shapes and structural evaluation algorithm are under consideration for each approach. The main criterion for this adaptation is the correspondence of computed and measured spectrum of the natural frequencies and mode shapes in the entire frequency range, significant for the system-wide changes assessment and for possible defects localization.
1. Introduction. Problem state
Generally there are four basic methods of instrumental monitoring at the present time [1, 2]: geodetic measurements; geotechnical monitoring of soil foundation state; measuring of loads and strains in the foundation and superstructure; dynamic (seismometric) approach. Special attention should be paid to the seismometric method, which allows to survey the building as a whole and to identify significant changes in the load-bearing structures without instrumental actions and visual inspections of each structure. The experiments carried out on real objects confirmed high potential of this method, also revealed a number of problems. Besides, it is necessary to note that seismometric method is rather useful for structural health monitoring (SHM) of buildings with difficult real time access to load-bearing structures. It is convenient for high dimensional finite element models with variability of loads, masses and stiffnesses etc. Significant practical results dealing with development and applications of instrumental monitoring methods (including dynamic) of energy and civil engineering objects belong to the research groups of G.E. Shablinsky, A.V. Korgin, A.F. Emanov [3], M.A. Shahramanyan, A.M. Shahramanyan and others [4–7].

All recent years, both in Russian and foreign scientific periodicals, much attention has been paid to the development of adequate spatial models and structural analysis methods for the design stage, taking into account the actual state. We should note the following leaders of the corresponding Russian methodical, algorithmic and software developments for unique buildings and structures: A.M. Belostotsky [8, 9], A.S. Gorodetsky, S.I. Dubinsky, O.V. Kabantsyev [10], N.I. Karpenko [11–13], S.N. Karpenko [11, 12, 14], O.V. Mkrtychev [15–20], A.V. Perelmuter [10, 21–23], V.A. Semenov, V.I. Travush [14, 24–27], O.V. Trifonov, S.Yu. Fialko [28-30]. In particular, problems of mathematical modelling in the basis and as part of SHM systems were developed by A.M. Belostotsky [8, 9], N.K. Kapustyan, M.A. Korgina, Yu.I. Kudishin, S.F. Kuznetsov, R. Katzenbach [31–33], A. Schmitt and others. There is a representative set of universal and specialized finite element software (including those verified in the Russian Academy of Architecture and Construction Sciences (RAACS)), which are normally used for structural analysis of unique buildings all over the world.

So-called “monitoring-oriented” models (several models or parameterized one) have a number of specific differences from the conventional design models, which are normally used to justify design decisions: input of real (actual measured instead of design) physical and mechanical parameters of construction materials (steel, concrete, reinforcement etc.) and geometry of structures; input of real (actual measured instead of design) loads; inclusion of non-bearing structures (dividing walls, facades, etc.) in static and dynamic operation of structures under weak “background” loads; work modelling of several joints in accordance with schemes, different from design ones (for example, elastic restraint instead of hinge joint); adaptability (calibration, “learning”) of model in accordance with data obtained from instrumental measurements (including detected defects). Thus, identification problem of an object actual state with the use of measurement results of dynamic parameters (natural frequencies and modal shapes) is rather actual one. We should mention two main groups of approaches, which are used within adaptation of computational models: “intuitive engineering” approaches and mathematically formalized computer-oriented approaches.

The first group of approaches are normally used for complex multi-component systems including unique buildings and structures and leave a wide scope for interpreting the computed and measured dynamic parameters. We should mention the most rigorous and advanced approaches based on the numerical solution of non-correct inverse problems by the regularization method of A.N. Tikhonov from the second group. Among the scientists-mathematicians who conducted research in this area, it is also necessary to note M.M. Lavrentyev, V.Ya. Arsenin, V.A. Morozov, V.V. Vasin, A.G. Jagol, H. Engl, M. Hanke, A. Neubauer, L. Landweber, P.C. Hansen, C.R. Vogel et al. The notion of incorrectness with respect to inverse problems of the elasticity theory is considered, for example by A.O. Vatulyan [34–40], who provides methods for solving ill-posed problems, as well as mathematical formulations of physical problems including identification of linear dynamical systems. Mathematical majority formulations of direct static and dynamic problems of solid mechanics (structural mechanics) are correct. This assertion is incorrect in most cases for continual and discrete formulations of inverse
problems. Nowadays, leading specialists teams in the considering research area (in the UK – M.I. Friswell [41]; in the USA – D.C. Zimmerman [42, 43]; in Canada – B. Weber, P. Paulter, J. Proulx[44, 45]; in South Korea – I.H. Yeo, H.S. Lee et al.) developed competitive original algorithms and software systems for solution of incorrect inverse dynamical problems allowing to identify the actual state and localize defects for simple linear-elastic systems (a beam and a plate on a Winkler foundation, a truss, a frame) taking into account the measured natural frequencies and modal shapes.

As already noted, Interest in applying the approaches of the inverse problems theory has been increased, including regularization methods, for practically important solution problems of computational structural mechanics. Particularly research works of O.V. Vatulyan [40], V.G. Bazhenov [46], Ya.M. Parkhomovsky, V.A. Kostina, V.A. Postnov, A.I. Fimkin, A.M. Akhtyamov [47, 48], A.S. Semenov [49], J.E. Mottershead [50–56], M.I. Friswell [41], A. Constantinescu [57, 58], M. Bonnet, J. Chock, R. Kapania [59], T.S. Jang [60–63], C. Johnson, W. Ring et al. deal with application of regularization methods to solution of inverse problems of structural mechanics.

This paper is focused mainly on the development of adaptive mathematical models and numerical methods as a basis and SHM systems integral part of load-bearing structures of buildings and structures (including unique).

2. Theoretical foundations of SHM advanced methodology

Block diagram and content of developed computational and experimental methodology of structural health monitoring dealing with load-bearing structures of unique buildings, is presented in Figure 1 (notation system from [1, 2] is used).

“Design” finite element model is normally developed to study the load-bearing capacity of the current project version. Parameterized “monitoring-oriented” three-dimensional dynamic finite element model is constructed or modified for each significant stage of the building life cycle (the stages of construction and operation), verified and adapted in accordance with the measured data. The main criterion for the adaptation is the correspondence of calculated and measured spectrum of the natural frequencies and mode shapes in the entire frequency range, significant for the assessment of system-wide changes and for possible identification – localization defects.

Computational assessment of structures load-bearing capacity is carried out in accordance with design codes with the use of “design” and “monitoring-oriented” finite element models based on design and measured parameters of structures, foundation, loads etc.

Three-dimensional shell-beam finite element model (models) of coupled systems “foundation – building” are normally constructed for strain-stress state analysis and load-bearing capacity of actual design version. It is so-called “start” (“design”) model for subsequent parameterization and adaptation.

Vector of parameters of model has the form \( l \) is the number of stage of construction and operation \((l=1,2,...)\) with corresponding instrumental SHM

\[
\theta_l = \{\theta_1, \theta_2, \theta_3, ..., \theta_l\}.
\]

Vector (1) can contain the following measured parameters, which normally differ from design ones: \( \theta_1 \) is dynamic parameters of foundation; \( \theta_2 \) is physical and mechanical parameters of construction materials; \( \theta_3 \) is geometrical parameters of load-bearing structures (particularly eccentricities and inclinations of walls and columns; \( \theta_4 \) is measured loads and impacts; \( \theta_5 \) is stiffness and mass of nominally non-bearing structures (dividing walls, façade structures), included in dynamic operation of structures under weak “background” loads; \( \theta_6 \) is modelling of structural behavior of several joints in accordance with schemes, different from design ones.

Well-known construction methods of three-dimensional shell-beam dynamic finite element models [20, 21] with allowance for above mentioned factors are realized. Thus, the reduction of the concrete actual class in comparison with the project one are taken into account by corresponding reduction in the elasticity modulus, deviations of the columns geometric positions, walls and other load-bearing elements are taken into account by introduction of so-called “rigid inserts” allowing displacement of
the elements in the plan, and their inclination. The most problematic is the allowance for stiffness of dividing walls (especially located inside apartment) and facade structures included in the system dynamic operation for the operation stages, under weak background loads.

We can use “integrated” approach (a proportional stiffness increase of the vertical load-bearing structures), and introduction of each non-bearing structure with reduced dynamic stiffness in finite element model (this approach can substantially (at times) increase computational dimension of the model).

We recommend using Lanczos method for corresponding generalized and partial eigenvalue problems solution. Numerous computational experiments (including samples with “contrasting” ill-conditioned systems and systems with multiple eigenvalues) showed its reliability, efficiency and high computing speed of the eigenvalues given number and eigenvectors for practical problems.

As it follows from the common engineering sense and confirmed by formal above-mentioned mathematical manipulations, seismometric method of measurement should provide reasonable

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**Figure 1.** Block diagram and content of developed computational and experimental methodology of structural health monitoring dealing with load-bearing structures of unique buildings.
computing accuracy of lower total-system performance natural frequencies and mode shapes as well as natural frequencies and mode shapes corresponding to local deviations of structure state (including structural failures). Besides, efficiency and economic competitiveness are also provided. Analysis of available sources showed that so-called standing wave method, proposed by A.F. Emanov [3], is fully consistent with these principles and criteria.

Analysis of stress-strain state and load-bearing capacity of structures is carried out in accordance with the design codes and corresponding criteria with the use of finite element model comprising parameters of “monitoring-oriented” and design models.

Planning for measurement at SHM current stage should be done with the use of the previous stage results. Thus, the “suspicious” detection of natural frequencies and mode shapes requires installation of a sensors sufficient number for the measurements for the qualitative identification of these frequencies and shapes. However it should be noted that for most SHM applications, the available sensors number for monitoring is usually significantly less than the number of potential monitoring locations.

References
[1] Belostotsky A M and Akimov P A 2016 Procedia Engineering 153 83–8
[2] Belostotsky A M, Akimov P A, Negrozov O A, Petryashev N O, Petryashev S O, Sherbina S V, Kalichava D K and Kaytukov T B 2018 Magazine of Civil Engineering 2 169–78
[3] Emanov A F, Selezev B C and Korshik N A Russian Geology and Geophysics 49 10 780–9
[4] Sidorov V N 2014 Procedia Engineering 91 58–2
[5] Feng D and Feng M Q 2018 Engineering Structures 156 105–117
[6] Masciiotta M G, Ramos L F, Vasta M and Lourenco P B 2017 Proced. Engineer. 199 2178–83
[7] Sartorato M, De Medeiros R, Vandelipitte D and Tita V 2017 Mechanical Systems and Signal Processing 84 A 445–61
[8] Belostotsky A M, Akimov P A, Afanasyeva I N and Kaytukov T B 2017 Intern. J Comp. Civil Struct. Engineer. 13 2 9–34
[9] Belostotsky A M, Akimov P A, Kaytukov T B, Petryashev N O, Petryashev SO and Negrozov O A 2016 Procedia Engineering 153 95–102
[10] Kabantsev O and Perelmuter A 2013 Proc. Engineer. 57 479–88
[11] Karpenko N I and Karpenko S N 2015 Proc. Engineer. 111 378–85
[12] Karpenko N I, Karpenko S N, Petrov A N, Voronin Z A and Evseeva A V 2015 Proc. Engineer. 111 386–9
[13] Karpenko N I, Mishina A V and Travush V I 2015 Proc. Engineer. 111 390–7
[14] Karpenko SN, Travush VI and Chepyzubov IG 2015 Proc. Engineer. 111 398–03
[15] Mkrtchyan O V and Busalova M S 2016 Proc. Engineer. 153 467–74
[16] Mkrtchyan O V and Busalova M S 2016 Proc. Engineer. 153 475–82
[17] Mkrtchyan O V, Dzhinchvelashvili G A and Bunov AA 2014 Proc. Engineer. 91 48–53
[18] Mkrtchyan O V, Dzhinchvelashvili G A and Busalova M S 2014 Proc. Engineer. 91 54–57
[19] Mkrtchyan O V, Dzhinchvelashvili G A and Busalova M S 2015 Proc. Engineer. 111 545–549
[20] Mkrtchyan O V, Dzhinchvelashvili G A and Busalova M S 2015 Proc. Engineer. 111 550–5
[21] Mikitarenko M A and Perelmuter A V 1998 Jour. Wind Engin. Indus.Aerodyn. 74–76 1091–100
[22] Perelmuter AV and Mikitarenko MA 1998 Jour. Construct. Steel Res.46 1–3 16–17
[23] Perelmuter A and Yurchenko V 2013 Proc. Engineer. 57 895–905
[24] Travush V, Emelianov S, Kolchunov V and Bulgakov A 2016 Proc. Engineer. 164 416–24
[25] Travush V I 1986 Jour. Appl. Math. Mech. 50 4 470–4
[26] Travush V I, Kashevaro G G, Martirosyan A S and Avhacheva I A 2016 Proc. Engineer. 153 766–72
[27] Travush V I, Martirosyan A S and Kashevaro G G 2016 Proc. Engineer. 153 773–80
[28] Fialko S 2010 Advan. Engineer. Software 41 12 1256–65
[29] Fialko S 2015 Computers & Mathematics with Applications 70 12 2968–87
[30] Fialko S Yu 2014 Archives of Civil and Mechanical Engineering 14 1 190–203
[31] Katzenbach R and Leppla S 2015 Procedia Engineering 117 162–71
[32] Katzenbach R, Leppla S, Vogler M, Seip M and Kurze S 2013 Proc. Engineer. 57 35–44
[33] Katzenbach R, Leppla S, Ramm H, Seip M and Kuttig H 2013 Proc. Engineer. 57 540–8
[34] Nedin R, Dudarev V and Vatulyan A 2017 Engineer. Struct. 15 391–405
[35] Nedin R, Nesterov S and Vatulyan A 2016 Appl. Math. Model. 40 4 2711–19
[36] Nedin R, Nesterov S and Vatulyan A 2016 Intern. Jour. Heat Mass Transr 102 213–18
[37] Nedin R, Nesterov S and Vatulyan A 2014 Intern. Jour. Solids Struct. 51 3–4 767–73
[38] Nedin R and Vatulyan A 2013 Intern. Jour. Solids Struct. 50 13 2107–14
[39] Nedin R D, Vatulyan A O, Dudarev V V and Bogachev IV 2018 Intern. Jour. Solids Struct. 139–40 121–28
[40] Vatulyan A O, Yavruyan O V and Bogachev IV 2014 Intern. Jour. Solids Struct. 51 11–12 2238–43
[41] Friswell M I, Mottershead J E and Ahmadian H 2001 Trans. Royal Soc. London, Series A, Special Issue on Experimental Modal Analysis 359 1778 169–86
[42] Zimmerman D and Kaouk M 1992 AIAA Jour. 30 7 1848–1855
[43] Zimmerman D and Kaouk M 1994 ASME Jour.Vibrat.Acous. 116 222–31
[44] Paulitre P, Weber B, Mousseau S and Proulx J 2016 Engineer. Struct. 114 209–25
[45] Weber B, Paulitre P and Proulx J 2009 Mech. Sys. Signal Process. 23 6 1965–85
[46] Bazhenov V G 2007 Physical Mesomechanics 10 5–6 2007 302–14
[47] Aitbayeva A A and Akhtyamov A M 2016 Jour. Appl. Math. Mech. 80 3 273–7
[48] Akhatov I Sh and Akhtyamov A M 2001 Jour. Appl. Math. Mech.65 2 283–90
[49] Belyaev A K, Polyaniskiy V A, Semenov A S, Tretyakov D A and Yakovlev Y A 2017 Proc. Struct. Integr. 6 201–07
[50] Au S-K, Brownjohn J M W and Mottershead J E 2018 Mech. Syst. Sign. Process. 102 139–57
[51] Chang Y-H, Wang W, Patterson E A, Chang J-Y and Mottershead J E 2017 Proc. Engineer. 199 423–8
[52] Jalali H, Ahmadian H and Mottershead J E 2007 Inter. Jour. Solids Struct. 44 25–26 8087–105
[53] Mottershead J E 1999 Mech. Syst. Signal Process. 13 3 367–74
[54] Prandina M, Mottershead J E and Bonisoli E 2009 Jour. Sound Vibrat. 323 3–5 662–76
[55] Wang W, Mottershead J E and Mares C 2009 Mech. Syst. Signal Process. 23 7 2088–2112
[56] Wei X and Mottershead J E 2016 Mech. Syst. Signal Process. 74 11–28
[57] Ballard P and Constantinescu A 1994 Jour. Mech. Phys. Solids 42 11 1994 1767–87
[58] Jalocha D, Constantinescu A and Neviere R 2015 Inter. Jour. Solids Struct. 67–68 169–81
[59] Haryadi S G, Kapania R K and Haftka R T 1998 Composites Part B: Engineer. 29 3 271–6
[60] Jang T S 2013 Computers & Structures 120 77–85
[61] Jang T S 2013 Mech. Syst. Signal Process. 25 4 1159–73
[62] Jang T S, Baek H, Han S L and Kinoshita T 2010 Mech. Syst. Signal Process. 24 6 1665–81
[63] Kwon S H, Kim C H and Jang T S 2007 Ocean Engineer. 34 10 1405–12