Explicit SO(10) Supersymmetric Grand Unified Model
for the Higgs and Yukawa Sectors

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Abstract

A complete set of fermion and Higgs superfields is introduced with well-defined SO(10) properties and $U(1) \times Z_2 \times Z_2$ family charges from which the Higgs and Yukawa superpotentials are constructed. The structures derived for the four Dirac fermion and right-handed Majorana neutrino mass matrices coincide with those previously obtained from an effective operator approach. Ten mass matrix input parameters accurately yield the twenty masses and mixings of the quarks and leptons with the bimaximal atmospheric and solar neutrino vacuum solutions favored in this simplest version.

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In a series of recent papers [1] - [4] the authors have shown how fermion mass matrices can be constructed in an SO(10) supersymmetric grand unified framework by use of a minimal
Higgs structure which solves the doublet-triplet splitting problem \[5\]. The construction was carried out in an effective operator approach with phenomenological input, including the Georgi-Jarlskog relations \[6\]. Here we show how one can introduce a set of matter and Higgs \(SO(10)\) superfields with \(U(1) \times Z_2 \times Z_2\) family charges from which the derived Higgs and Yukawa superpotentials uniquely give the structure of the fermion mass matrices previously obtained. The quark and lepton mass and mixing data are reproduced remarkably well with the solar neutrino vacuum solution preferred, provided the up quark mass is not zero at the GUT scale – otherwise the small angle MSW solution \[7\] is obtained. The right-handed Majorana neutrino matrix arises from a Higgs field which couples pairs of superheavy conjugate neutrino singlets.

We begin with a listing in Table I. of the Higgs and matter superfields in the proposed model along with their

**Higgs Fields Needed to Solve the 2-3 Problem:**

- \(45_{B-L}: A(0)^{+-}\)
- \(16: C(\frac{3}{2})^{-+}, C'(\frac{3}{2} - p)^{++}\)
- \(\overline{16}: \bar{C}(-\frac{3}{2})^{++}, \bar{C}'(-\frac{3}{2} - p)^{-+}\)
- \(10: T_1(1)^{++}, T_2(-1)^{+-}\)
- \(1: X(0)^{++}, P(p)^{+-}, Z_1(p)^{++}, Z_2(p)^{++}\)

**Additional Higgs Fields for the Mass Matrices:**

- \(10: T_0(1 + p)^{+-}, T'_0(1 + 2p)^{+-}, T_0(-3 + p)^{-+}, T'_0(-1 - 3p)^{-+}\)
- \(1: Y(2)^{-+}, Y'(2)^{++}, S(2 - 2p)^{-+}, S'(2 - 3p)^{-+}, V_M(4 + 2p)^{++}\)

Table I. Higgs superfields in the proposed model.

family charges. As demonstrated in \[5\], in order to do all the symmetry breaking, one \(45\) adjoint Higgs with its VEV pointing in the \(B - L\) direction, a pair of \(16 + \overline{16}\) spinor Higgs,
plus a pair of $10$ vector Higgs and several Higgs singlets are required. In order to complete
the construction of the Dirac mass matrices, four more vector Higgs and four additional
singlets are needed. Finally, one Higgs singlet is introduced to generate the right-handed
Majorana mass matrix.

From the Higgs $SO(10)$ and family assignments, it is then possible to write down explicit-
lly the full Higgs superpotential, where we have written it as the sum of five terms:

$$W_{\text{Higgs}} = W_A + W_{CA} + W_{2/3} + W_{HD} + W_R$$

$$W_A = trA^4/M + M_AtrA^2$$

$$W_{CA} = X(\overline{CC})^2/M_C^2 + F(X)$$
$$+\overline{C}(PA/M_1 + Z_1)C + \overline{C}(PA/M_2 + Z_2)C'$$

(1)

$$W_{2/3} = T_AT_2 + Y'T_2^2$$

$$W_{HD} = T_1\overline{CC}Y'/M + T_0\overline{CC}' + \overline{T_0}(T_0S + T_0'S')$$

$$W_R = \overline{T_0}T_0V_M$$

The Higgs singlets are all assumed to develop VEV’s at the GUT scale. $W_A$ fixes $\langle A \rangle$
through the $F_A = 0$ condition where one solution is $\langle A \rangle \propto B - L$, the Dimopoulos-Wilczek
solution $\mathbb{R}$. $W_{CA}$ gives a GUT-scale VEV to $\overline{C}$ and $C$ by the $F_X = 0$ condition and also
couples the adjoint $A$ to the spinors $C$, $\overline{C}$, $C'$ and $\overline{C}'$ without destabilizing the Dimopoulos-
Wilczek solution or giving Goldstone modes. $W_{2/3}$ gives the doublet-triplet splitting by the
Dimopoulos-Wilczek mechanism. $W_{HD}$ mixes the $(1, 2, -1/2)$ doublet in $T_1$ with those in
$C'$ (by $F_{\overline{C}} = 0$), and in $T_0$ and $T_0'$ (by $F_{\overline{T_0}} = 0$). To fill out the model, we specify the
$SO(10) \times U(1) \times Z_2 \times Z_2$ quantum numbers of the various matter fields in Table II. We
require three chiral spinor fields $16_i$, one for each light family, two vector-like pairs of $16 + \overline{10}$
spinors which can get superheavy, a pair of superheavy $10$ fields in the vector representation,
and three pairs of superheavy $1 - 1^c$ fermion singlets.

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In terms of these fermion fields and the Higgs fields previously introduced, one can then spell out all the terms in the Yukawa superpotential which follow from their $SO(10)$ and $U(1) \times Z_2 \times Z_2$ assignments:

$$W_{Yukawa} = 16_3 \cdot 16_3 \cdot T_1 + 16_2 \cdot 16 \cdot T_1 + 16' \cdot 16' \cdot T_1$$
$$+ 16_3 \cdot 16_1 \cdot T'_0 + 16_2 \cdot 16_1 \cdot T_0 + 16_3 \cdot 3 \cdot A$$
$$+ 16_1 \cdot 3' \cdot Y' + 16 \cdot 3' \cdot P + 16' \cdot 3' \cdot S$$
$$+ 16_3 \cdot 10_2 \cdot C' + 16_2 \cdot 10_1 \cdot C + 10_1 \cdot 10_2 \cdot Y$$
$$+ 16_3 \cdot 1_3 \cdot C + 16_2 \cdot 1_2 \cdot \overline{C} + 16_1 \cdot 1_1 \cdot \overline{C}$$
$$+ 1_3 \cdot 1_3^c \cdot Z + 1_2 \cdot 1_2^c \cdot P + 1_1 \cdot 1_1^c \cdot X$$
$$+ 1_3^c \cdot 1_3 \cdot V_M + 1_2^c \cdot 1_2 \cdot V_M$$

where the coupling parameters have been suppressed. To obtain the GUT scale structure for the fermion mass matrix elements, all but the three chiral spinor fields in the first line of Table II. will be integrated out. The right-handed Majorana matrix elements will all be generated through the Majorana couplings of the $V_M$ Higgs field with conjugate singlet fermions as given above.

With R-parity conserved, $d = 4$ proton decay operators are forbidden. The $d = 5$ proton decay operators induced by colored-Higgsino exchange that are generally present in unified models are present here but are not dangerous. It can be shown that the family charge assignments prevent any new and dangerous proton decay operators from arising.
The procedure for deriving the Dirac mass matrices $U, D, L,$ and $N$ is the following. For each type of fermion $f$, where $f = u_L, u_c, d_L, d_c, \ell_L, \ell^+_L, \nu_L$ and $\nu^c_L$, the superheavy mass matrix connecting the $f$ to the $SU(3) \times SU(2) \times U(1)$-conjugate representation $\overline{f}$ is first found from Eq. (2) by setting the weak-scale VEV’s and the intermediate-scale VEV, $V_M$, to zero. This will give three zero mass eigenstates for each type of $f$, corresponding to the three light families. Then the terms in Eq. (2) involving $\langle T_1 \rangle$, $\langle C'_+ \rangle$, $\langle T_0 \rangle$, and $\langle T'_0 \rangle$ give rise to the $3 \times 3$ Dirac mass matrices coupling $u_L$ to $u_c^c_L$, etc. This procedure is spelled out explicitly in [9].

Under the assumption that the zero-mass states have their large components in the chiral representations $16_1$, $16_2$ and $16_3$, and all the other components are small, the Dirac mass matrices obtained have precisely the structure previously found in our studies by means of an effective operator approach:

$$U = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & \sigma + \epsilon/3 \\ \delta' e^{i\phi} - \epsilon/3 & 1 \end{pmatrix},$$

$$N = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \quad L = \begin{pmatrix} 0 & \delta & \delta' e^{i\phi} \\ \delta & 0 & -\epsilon \\ \delta' e^{i\phi} + \sigma + \epsilon & 1 \end{pmatrix},$$

(3)

with $U$ and $N$ in units of $M_U$ and $D$ and $L$ in units of $M_D$. The matrix parameters are identified with the Yukawa couplings and Higgs couplings and VEV’s as follows:

$$M_U = (t_3)_{5(10)}, \quad M_D = (t_3)_{5(10)},$$

$$\epsilon M_U = |3(a_q/p)(t_2)_{5(10)}|, \quad \epsilon M_D = |3(a_q/p)(t_2)_{5(10)}|,$$

$$\eta M_U = (y'/s'')^2(t')_{5(10)}, \quad \sigma M_D = -(c/y)(c')_{5(16)},$$

$$\delta M_D = t_0 t_0/s, \quad \delta M_D = t_0 t_0/s e^{-i\phi},$$

where the subscripts on $t_2$, $t_3$, $t'$ and $c'$ refer to the SU(5)[SO(10)] representation content of the VEV’s. The following shorthand notation has been introduced
The parameter $\eta$ is introduced to give a tiny non-zero mass to the up quark at the $\Lambda_G$ scale. Its appearance in $N$ will also play an important role in the determination of the type of solar neutrino solution. It should also appear in $D$ and $L$ but its effect is negligibly small there and of no consequence, so it is dropped. The only phase then appearing in the matrices is $\phi$ associated with $\delta'$, as other phases are unphysical and can be rotated away with the exception of that associated with $\epsilon$. It turns out, however, that the best fits to the data prefer a real $\epsilon$. Hence $\phi$ which can be identified with the complexity of the VEV of the $S'$ Higgs singlet is solely responsible for CP-violation in the quark sector. The structures of the matrix elements given in Eqs. (3), (4) and (5) can be understood in terms of simple Froggatt-Nielsen diagrams [10] given in [9].

Note that the 33 elements of the Dirac mass matrices are scaled by the VEV’s of the $10$, $T_1$. But the $F = 0$ conditions for the Higgs superpotential require that the pair of Higgs doublets which remain light down to the electroweak scale arise from $5(T_1)$, $\bar{5}(T_1)$, $\bar{5}(C')$ and, to a very small extent from $T_0$ and $T'_0$ terms, which are ignored here. In particular, we can write in terms of a mixing angle $\gamma$

$$H_U = 5(T_1), \quad H_D = \bar{5}(T_1) \cos \gamma + \overline{5}(C') \sin \gamma$$

whereas the orthogonal combination has become superheavy at the GUT scale. Thus the
ratio of the 33 mass matrix elements found from Eqs. (4) and (6) is given in terms of the VEV’s, $v_u$ and $v_d$ of $H_U$ and $H_D$, respectively, by

$$M_U/M_D = v_u/(v_d \cos \gamma) \equiv \tan \beta/\cos \gamma$$

(7)

Hence we find that the large $M_U/M_D$ ratio required for the top to bottom quark masses can be achieved with a moderate $\tan \beta$ provided $\cos \gamma$ is small.

Turning to the right-handed Majorana mass matrix, we use the zero mass left-handed conjugate states that were found implicitly above for the Dirac matrix $N$ to form the basis for $M_R$. The right-handed Majorana neutrino matrix is then obtained from the last two terms in Eq. (2), and we find

$$M_R = \begin{pmatrix} 0 & A\epsilon^3 & 0 \\ A\epsilon^3 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Lambda_R$$

(8)

where

$$\Lambda_R = \lambda_{15}V_M (\bar{c}_3/z)^2,$$

$$A\epsilon^3 \Lambda_R = \lambda_{14}V_M (\bar{e}_1/x)(\bar{c}_2/p_{22})$$

(9)

Note that the whole right-handed Majorana mass matrix has been generated in this simple model by the one Majorana VEV coupling superheavy conjugate fermion singlets. By means of the seesaw formula [11], one can then compute the light neutrino mass matrix

$$M_\nu = N^T M_R^{-1} N = \begin{pmatrix} 0 & 0 & -\eta/\Lambda_R \\ 0 & \epsilon^2 & \epsilon \\ -\eta/\Lambda_R & \epsilon & 1 \end{pmatrix} M_\nu^2/\Lambda_R$$

(10)

We now address the predictions of the mass matrices. For this purpose it is convenient to replace the parameters $\delta$ and $\delta'$ by

$$t_L e^{i\theta} \equiv \frac{\delta - \sigma \delta' e^{i\phi}}{\sigma \epsilon/3}, \quad t_R \equiv \frac{\delta \sqrt{\sigma^2 + 1}}{\sigma \epsilon/3}$$

(11)

which are essentially the left-handed and right-handed Cabibbo angles. In terms of the dimensionless parameters $\epsilon$, $\sigma$, $t_L$, $t_R$, $e^{i\theta}$, $\eta$, $A$, and $M_U/M_D$, we then find at the GUT scale
\[ m_t^0/m_b^0 \approx (\sigma^2 + 1)^{-1/2} M_U/M_D, \quad m_u^0/m_t^0 \approx \eta, \]
\[ m_c^0/m_t^0 \approx \frac{\eta}{\sigma} \epsilon^2 \cdot [1 - \frac{2}{5} \epsilon^2], \quad m_b^0/m_t^0 \approx 1 - \frac{2}{3} \frac{\sigma}{\sigma^2 + 1} \epsilon, \]
\[ m_s^0/m_b^0 \approx \frac{1}{3} \epsilon^2 \cdot \left[ 1 + \frac{1}{3} \epsilon \frac{1 - \epsilon^2 - \epsilon \sigma^2/3}{\sigma \sigma^2 + 1} + \frac{1}{3} (t_L^2 + t_R^2) \right], \]
\[ m_d^0/m_s^0 \approx t_L t_R \cdot \left[ 1 - \frac{1}{3} \epsilon \frac{\sigma^2 + 2}{\sigma^2 + 1} + \frac{1}{18} (t_L^2 + t_R^2) \right] + (t_L^2 + t_L^2 t_R^2 + t_R^2), \]
\[ m_{\mu}^0/m_{\tau}^0 \approx \epsilon \frac{\sigma}{\sigma^2 + 1} \cdot \left[ 1 + \epsilon \frac{1 - \sigma^2}{\sigma^2 + 1} + \frac{1}{18} (t_L^2 + t_R^2) \right], \]
\[ m_{e_{\mu}}^0/m_{\mu}^0 \approx \frac{1}{3} \epsilon^2 \cdot \left[ 1 + \epsilon \frac{1 - \epsilon^2 - \epsilon \sigma^2/3}{\sigma \sigma^2 + 1} + \epsilon^2 \frac{\epsilon^4 + 9 \epsilon^2/2 + 3}{\sigma^2 (\sigma^2 + 1)^2} \right] - \frac{1}{3} \epsilon^2 (t_L^2 + t_R^2), \]
\[ V_{cb}^0 \approx \epsilon \frac{\sigma}{\sigma^2 + 1} \cdot \left[ 1 + \epsilon \frac{1}{3} \frac{1 - \epsilon^2 - \epsilon \sigma^2/3}{\sigma \sigma^2 + 1} \right], \]
\[ V_{ub}^0 \approx \frac{1}{3} \epsilon^2 \cdot \left[ 1 + \epsilon \frac{1 - \epsilon^2 - \epsilon \sigma^2/3}{\sigma \sigma^2 + 1} \right], \]
\[ \frac{V_{ub}^0}{V_{cb}^0} \approx \left[ 1 + \epsilon \frac{1 - \epsilon^2 - \epsilon \sigma^2/3}{\sigma \sigma^2 + 1} \right], \]
\[ m_2^0/m_3^0 \approx \left( \frac{\eta}{\sigma \sigma^2 + 1} \right) \left[ 1 + \frac{\eta}{\sigma \sigma^2 + 1} e^{-i \theta} \right], \]
\[ m_1^0/m_3^0 \approx \left( \frac{\eta}{\sigma \sigma^2 + 1} \right) \left[ 1 - \frac{\eta}{\sigma \sigma^2 + 1} e^{-i \theta} \right], \]
\[ U_{\mu_\nu}^0 \approx -\frac{1}{\sqrt{\sigma^2 + 1}} (\sigma - \epsilon) e^{i \theta}, \]
\[ U_{\nu_\tau}^0 \approx \frac{1}{\sqrt{\sigma^2 + 1}} (\sigma - \epsilon) (1 - \epsilon) t_R - \frac{\eta}{\sigma}, \]
\[ U_{e_\mu}^0 \approx -\frac{1}{\sqrt{\sigma^2 + 1}} (\sigma - \epsilon) e^{i \theta} \]
\[ + \frac{1}{\sqrt{\sigma^2 + 1}} (1 + \epsilon \sigma) t_R, \]
\[ U_{e_3}^0 \approx \frac{1}{3 \sqrt{\sigma^2 + 1}} (\sigma - \epsilon) t_R - \frac{\eta}{\sigma}. \]

Note that the Georgi-Jarlskog relations hold, \( m_s^0 \approx \frac{1}{3} m_{\mu}^0 \) and \( m_\mu^0 \approx 3 m_e^0 \), emerge as required by design. The quark and charged lepton data are best fit at the low scale (see below) by assigning the following values to the model parameters: \( \epsilon = 0.145, \quad \sigma = 1.78, \quad t_L = 0.236, \quad t_R = 0.205, \quad \theta = 34^o, \) corresponding to \( \delta = 0.0086, \quad \delta' = 0.0079, \quad \phi = 54^o, \quad \eta = 8 \times 10^{-6}, \) and \( M_U/M_D \approx 113. \)

As noted earlier, in order to obtain the simple mass matrices in Eq. (3), we had to assume that the zero-mass states have their large components in the chiral representations \( 16_1, \ 16_2, \) and \( 16_3. \) The conditions on the state normalization factors are all satisfied provided the
following ratios are much less than unity:

\[
(a/p)^2, \quad (y'/s'')^2, \quad (c/y)^2, \quad (\bar{c}_1/x)^2, \quad (\bar{c}_2/p_{22})^2, \quad (\bar{c}_3/z)^2 \ll 1
\]

With the numerical choice of parameters given above and near equality of the various Higgs couplings, we find \((a/p)^2 \approx 0.02\) and \((y'/s'')^2 \sim 6 \times 10^{-6}\), so the first two conditions are easily satisfied. Requiring that \((c/y)^2 \ll 1\) and with the expression for \(\sigma\) obtained from Eqs. (4), we find

\[
\tan \gamma \equiv \frac{\langle \bar{5}(C') \rangle}{\langle \bar{5}(T_1) \rangle} \gg \sigma
\]

\[
\tan \beta \simeq \sqrt{\sigma^2 + 1}(\cos \gamma) m_t^0 / m_b^0 \ll m_t^0 / m_b^0
\]

in terms of the \(T_1 - C'\) mixing angle, \(\gamma\), in Eq. (6). With \(c/y \approx 0.1\), for example, \(\tan \gamma \approx 18\) which implies \(\tan \beta \approx 6\), a very reasonable mid-range value allowed by experiment. The others can also be satisfied \([9]\).

In \([9]\) we have evolved the results in Eqs. (12) down to the low scales with a value for \(\tan \beta = 5\), \(\Lambda_G = 2 \times 10^{16} \text{ GeV}\), \(\Lambda_{SUSY} = m_t(m_t)\), \(\alpha_s(M_Z) = 0.118\), \(\alpha(M_Z) = 1/127.9\), and \(\sin^2 \theta_W = 0.2315\). With the quantities \(m_t(m_t) = 165 \text{ GeV}\), \(m_\tau = 1.777 \text{ GeV}\), \(m_\mu = 105.7 \text{ MeV}\), \(m_e = 0.511 \text{ MeV}\), \(m_u = 4.5 \text{ MeV}\), \(V_{us} = 0.220\), \(V_{cb} = 0.0395\), and \(\delta_{CP} = 64^\circ\) used to determine the input parameters, \(M_U \simeq 113 \text{ GeV}\), \(M_D \simeq 1 \text{ GeV}\), and \(\sigma, \epsilon, t_L, t_R, \theta\) and \(\eta\) given earlier, the following values are obtained compared with experiment \([12]\) in parentheses:

\[
m_c(m_c) = 1.23 \text{ GeV} \quad (1.27 \pm 0.1 \text{ GeV})
\]

\[
m_b(m_b) = 4.25 \text{ GeV} \quad (4.26 \pm 0.11 \text{ GeV})
\]

\[
m_s(1 \text{ GeV}) = 148 \text{ MeV} \quad (175 \pm 50 \text{ MeV})
\]

\[
m_d(1 \text{ GeV}) = 7.9 \text{ MeV} \quad (8.9 \pm 2.6 \text{ MeV})
\]

\[
|V_{ub}/V_{cb}| = 0.080 \quad (0.090 \pm 0.008)
\]

where finite SUSY loop corrections for \(m_b\) and \(m_s\) have been scaled to give \(m_b(m_b) \simeq 4.25\) GeV for \(\tan \beta = 5\).
The effective light neutrino mass matrix of Eq. (10) leads to bimaximal mixing with a large angle solution for atmospheric neutrino oscillations \[^{13}\] and the “just-so” vacuum solution \[^{14}\] involving two pseudo-Dirac neutrinos, if we set \(\Lambda_R = 2.4 \times 10^{14} \text{ GeV}\) and \(A = 0.05\). We then find

\[
\begin{align*}
m_3 &= 54.3 \text{ meV}, \quad m_2 = 59.6 \mu\text{eV}, \quad m_1 = 56.5 \mu\text{eV} \\
U_{e2} &= 0.733, \quad U_{e3} = 0.047, \quad U_{\mu3} = -0.818, \quad \delta'_{CP} = -0.2^\circ \sin^2 2\theta_{\text{atm}} = 0.89 \\
\Delta m^2_{23} &= 3.0 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{solar}} = 0.99 \\
\Delta m^2_{12} &= 3.6 \times 10^{-10} \text{ eV}^2, \quad \sin^2 2\theta_{\text{reac}} = 0.009 \\
\Delta m^2_{13} &= 3.0 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{reac}} = 0.009
\end{align*}
\] (16)

The effective scale of the right-handed Majorana mass contribution occurs two orders of magnitude lower than the SUSY GUT scale of \(\Lambda_G = 1.2 \times 10^{16} \text{ GeV}\). The effective two-component reactor mixing angle given above should be observable at a future neutrino factory, whereas the present limit from the CHOOZ experiment \[^{15}\] is approximately 0.1 for the above \(\Delta m^2_{23}\). In principle, the parameter \(A\) appearing in \(M_R\) can also be complex, but we find that in no case does the leptonic CP-violating phase, \(\delta'_{CP}\) exceed \(10^\circ\) in magnitude. Hence the model predicts leptonic CP-violation will be unobservable.

The vacuum solar solution depends critically on the appearance of the parameter \(\eta\) in the matrix \(N\), corresponding to the non-zero \(\eta\) entry in \(U\) which gives the up quark a mass at the GUT scale. Should we set \(\eta = 0\), only the small-angle MSW solution \[^{7}\] would be obtained for the solar neutrino oscillations. The large angle MSW solution is disfavored by the larger hierarchy, i.e., very small \(A\) value, required in \(M_R\).

In summary, we have constructed an explicit \(SO(10)\) supersymmetric grand unified model for the Higgs and Yukawa superpotentials which reproduces the fermion mass matrices previously obtained in an effective operator approach. All the quark and lepton mass and mixing data are fit remarkably well with a \(\tan \beta\) in the range of 5 - 10 with matrix parameters which are also quite reasonable.
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