Heavy quarks in the presence of higher derivative corrections from AdS/CFT

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Abstract

We use the gauge-string duality to study heavy quarks in the presence of higher derivative corrections. These corrections correspond to the finite coupling corrections on the properties of heavy quarks in a hot plasma. In particular, we study the effects of these corrections on the energy loss and the dissociation length of a quark-antiquark pair. We show that the calculated energy loss of heavy quarks through the plasma increases. We also find in general that the dissociation length becomes shorter with the increase of coupling parameters of higher curvature terms.
1 Introduction

The experiments of Relativistic Heavy Ion Collisions (RHIC) have produced a strongly-coupled quark–gluon plasma (QGP) \cite{1}. There are no known quantitative methods to study strong coupling phenomena in QCD which are not visible in perturbation theory (except by lattice simulation). A new method for studying different aspects of QGP is the AdS/CFT correspondence \cite{2,3,4,5}. This method has yielded many important insights into the dynamics of strongly-coupled gauge theories. It has been used to investigate hydrodynamical transport quantities in various interesting strongly-coupled gauge theories where perturbation theory is not applicable \cite{6}. Methods based on AdS/CFT relate gravity in AdS space to a conformal field theory on the four-dimensional boundary. It was shown that an AdS space with a black brane is dual to a conformal field theory at finite temperature.

The universality of the ratio of shear viscosity $\eta$ to entropy density $s$ \cite{7,8,9,10} for all gauge theories with Einstein gravity dual raised the tantalizing prospect of a connection between string theory and RHIC. The results were obtained for a class of gauge theories whose holographic duals are dictated by classical Einstein gravity. Recently, $\frac{2}{3}$ has been studied for a class of CFTs in flat space with higher derivative corrections \cite{11,12,13,14,15,16,17}. In these studies, the effects of $R^2$ corrections to the gravitational action in AdS space have been computed and it was shown that the conjecture lower bound on the $\frac{2}{3}$ can be violated. For example, in the Reissner–Nordström–AdS black
brane solution in Gauss–Bonnet gravity, the $\frac{2}{3}$ bound is violated and the Maxwell charge slightly reduces the deviation \cite{16}. Regarding this study and motivated by the vastness of the string landscape \cite{18}, we explored the modification of the jet quenching parameter and drag force on a moving heavy quark in the strongly-coupled plasma in \cite{19}.

Recently, a new higher derivative theory of gravity in five-dimensional spacetime which contains not only the Gauss-Bonnet term but also a curvature-cubed interaction introduced \cite{33,34}. This theory is known as quasi-topological gravity theory which is thought to be dual to the large $N$ limit of some conformal field theory without supersymmetry. Unlike Lovelock gravity, this cubic term is not purely topological. Therefore it would be useful to consider curvature-cubed terms as the higher derivative corrections and investigate behavior of the heavy quarks by means of the $AdS/CFT$ correspondence. Holographic investigation of Quasi-topological gravity have been done in \cite{35}. Also it was shown that the lower bound of the ratio of shear viscosity to density entropy can be violated in this background \cite{36}.

In this paper we use the $AdS/CFT$ correspondence to study effect of higher derivative corrections to the properties of the heavy quarks.\footnote{In general, we do not know about forms of higher derivative corrections in string theory, but it is known that due to the string landscape one expects that generic corrections can occur.} One should notice that string theory contains higher derivative corrections from stringy or quantum effects, and such corrections correspond to $1/\lambda$ and $1/N$ corrections. In the case of $\mathcal{N} = 4$ super Yang–Mills theory, the dual corresponds to the type $\Pi B$ string theory on $AdS_5 \times S^5$ background. The leading order corrections in $1/\lambda$ arise from stringy corrections to the low energy effective action of type $\Pi B$ supergravity, $\alpha'^2 R^4$.

Employing numerical methods, we investigate energy loss and dissociation length of heavy quarks in section 2 and 3, respectively. One finds that the energy loss of heavy quarks increases by increasing higher derivative corrections. Also the dissociation length becomes shorter with the increase of coupling parameters of higher curvature terms. We summarize the effects of these corrections in the last section. In the appendix, we give a brief review of \cite{33}. 
2 Energy loss of heavy quark at finite coupling

In this section, we investigate the finite-coupling corrections to the energy loss of a moving heavy quark in the Super Yang-Mills plasma using the AdS/CFT. These corrections are related to the curvature corrections to the AdS black brane solution.

The effect of curvature-squared corrections to the drag force on a moving heavy quark in the Super Yang-Mills plasma is investigated in [39]. It is shown that the corrections to the drag force depend on the velocity of heavy quark. This dependance is such that for $v > v_c$, including the corrections increase the drag force. This means that at the critical velocity $v_c$, the curvature-squared corrections have the minimum effect on the drag force. For the particular case of Gauss-Bonnet gravity, we do not expect a critical velocity [39]. Also in this background, the drag force is larger than the $\mathcal{N} = 4$ case if $\lambda$ (Gauss-Bonnet gravity constant) is positive while it is smaller than the $\mathcal{N} = 4$ case if $\lambda$ is negative.

Now we continue with considering curvature-cubic corrections. Our purpose is finding a general rule for considering higher derivative terms. We use the proposal of [33, 34] and study new higher derivative theory of gravity in five-dimensional spacetime which contains not only the Gauss-Bonnet term but also a curvature-cubed interaction.

We should emphasize that in the case of these corrections, one can not predict a result for $\mathcal{N} = 4$ SYM because the first higher derivative correction in weakly curved type IIB backgrounds enters at order $\mathcal{R}^4$. These corrections on the drag force have been studied in [40] and it was found that the drag force for a heavy quark moving through $\mathcal{N} = 4$ SYM plasma is generally enhanced by the leading correction due to finite 't Hooft coupling. We will compare our results with this observation and interestingly find a general rule for curvature corrections.

2.1 Set up of calculations

In the framework of $AdS/CFT$, an external quark is represented as a string dangling from the boundary of $AdS_5$—Schwarzschild and a dynamical quark is represented as a string ending on flavor D7-brane and extending down to some finite radius in $AdS$ black brane background. We consider the $AdS$
black hole solution in quasi-topological gravity as

\[ ds^2 = r^2 \left(-N^2 f(r)dt^2 + d\vec{x}^2\right) + \frac{dr^2}{f(r)}, \]  

(1)

notice that we work in units where the radius of AdS is one. Here \( r \) denotes the radial coordinate of the black brane geometry and \( t, \vec{x} \) label the directions along the boundary at the spatial infinity. In these coordinates the event horizon is located at \( r_h \) and it is found by solving \( f(r_h) = 0 \) equation. The boundary is located at infinity and the geometry will be as asymptotically AdS. The constant \( N^2 \) specifies the speed of light of the boundary gauge theory and one can choose it to be unity. We name \( f(r) \) at the boundary where \( r \to \infty \), as \( f_\infty \) and one finds that

\[ N^2 = \frac{1}{f_\infty}, \]  

(2)

The temperature of the hot plasma is given by the Hawking temperature of the black hole

\[ T = \frac{N r_h}{\pi}. \]  

(3)

The relevant string dynamics is captured by the Nambu-Goto action

\[ S = -\frac{1}{2\pi \alpha'} \int d\tau d\sigma \sqrt{-\det g_{ab}}, \]  

(4)

where the coordinates \((\sigma, \tau)\) parameterize the induced metric \( g_{ab} \) on the string world-sheet and \( X^\mu(\sigma, \tau) \) is a map from the string world-sheet into the space-time. Defining \( \dot{X} = \partial_\tau X \), \( X' = \partial_\sigma X \), and \( V \cdot W = V^\mu W^\nu G_{\mu\nu} \) where \( G_{\mu\nu} \) is the AdS black hole solution in Quasi-topological gravity (1), then

\[ -\det g_{ab} = (\dot{X} \cdot X')^2 - (X')^2 (\dot{X})^2. \]  

(5)

We follow \[37, 38\] and focus on the dual configuration of the external quark moving in the \( x \) direction on the plasma. The string in this case, trails behind its boundary endpoint as it moves at constant speed \( v \) in the \( x \) direction

\[ x(r, t) = vt + \xi(r), \quad y = 0, \quad z = 0. \]  

(6)

\[ ^{2}\text{one finds a quick review of this background in the appendix.} \]
One finds the lagrangian in the static gauge \((\sigma = r, \tau = t)\) as follows

\[
\mathcal{L} = \sqrt{-\det g_{ab}} = N^2 + r^4 N^2 f(r) x'^2 - \frac{\dot{x}^2}{f(r)},
\]

(7)

The equation of motion for \(\xi\) implies that \(\frac{\partial \mathcal{L}}{\partial \xi'}\) is a constant. We name this constant as \(\Pi_\xi\) and solve this relation for \(\xi'\), the result is

\[
\xi'^2 = \frac{\left(\frac{\Pi_\xi^2}{f(r)}\right) (-N^2 f(r) + \nu^2)}{r^4 N^2 f(r) \left(-r^4 N^2 f(r) + \Pi_\xi^2\right)},
\]

(8)

We are interested in a string that stretches from the boundary to the horizon. In such a string, \(\xi'^2\) remains positive everywhere on the string. Hence both numerator and denominator change sign at the same point and with this condition, one finds the constant of motion \(\Pi_\xi\) in terms of the critical value of \(r_c\) as follows

\[
\Pi_\xi = \nu r_c^2,
\]

(9)

The drag force that is experienced by the heavy quark is calculated by the current density for momentum along \(x^1\) direction. After straightforward calculations, the drag force is easily simplified in terms of \(\Pi_\xi\)

\[
F = -\frac{1}{2\pi \alpha'} \Pi_\xi.
\]

(10)

As a result, to find the drag force one should find the constant of motion, \(\Pi_\xi\) from (9). Numerator and denominator in (8) change sign at \(r_c\) and it can be found by solving this equation

\[
f(r_c) - \frac{\nu^2}{N^2} = 0.
\]

(11)

As it is clear in the appendix, Gauss-Bonnet coupling and curvature-cubed interaction constant are \(\lambda\) and \(\mu\), respectively and the precise form of \(f(r)\) depends on \(\lambda\) and \(\mu\). It was found that there are three different \(AdS\) black hole solutions in quasi-topological gravity which are determined by \(f_1(r), f_2(r)\) and \(f_3(r)\) in (32). Then for different values of coupling constant \(\lambda\) and \(\mu\), one should choose appropriate form of \(f(r)\) from (32) and solve (11). However (11) is complicated one can solve it numerically. Then, we assume different values for \(\mu\) and \(\lambda\) and discuss behavior of the drag force in terms of these coupling constants.
2.2 Positive couplings

We assume both coupling parameters $\mu$ and $\lambda$ are positive. As pointed out in [33], for this case, only $f_3(r)$ in (32) leads to a stable AdS black hole solution. The drag force versus the velocity of the heavy quark has been plotted in Fig. 1. In the right and left plots of this figure, Gauss-Bonnet coupling constant is $\lambda = 0.01$ and $\lambda = 0.20$, respectively. Also different values of cubic-curvature coupling interaction are assumed.

As one finds from [40], by increasing $\lambda$ the value of the drag force increases. This behavior of the drag force is clearly seen in these plots. One finds that by increasing Gauss-Bonnet coupling constant from $\lambda = 0.01$ to $\lambda = 0.20$, the drag force increases. In the plots of Fig. 1, one finds that by increasing $\mu$ the value of the drag force also increases. Though at the small velocities, the cubic-curvature interactions have minimum effect on the drag force. As a result, the main effect of increasing cubic-curvature coupling constant is increasing the drag force value. This is the same as the case of $R^2$ and $R^4$ case [39, 40]. We should check this result in the case of non-positive $\mu$ and $\lambda$. 

Fig. 1: The drag force versus the velocity of the heavy quark for positive values of cubic-curvature coupling $\mu$ at fixed positive Gauss-Bonnet coupling constant.
2.3 non-positive couplings

Now we intend to study the effect of the non-positive coupling constants \((\lambda, \mu)\) to the drag force. Three distinct AdS black hole backgrounds are discussed in \((32)\). These solutions for different regimes of the parameter space of \((\lambda, \mu)\) are discussed in the table 1 of \([33]\). As it is explained in this table, to study the non-positive coupling constants, one needs \(f_1(r), f_2(r)\) and \(f_3(r)\) from \((32)\). In the case of positive \(\mu\) and negative \(\lambda\), only \(f_3(r)\) in \((32)\) leads to a stable AdS black hole solution. In Fig. 2, we assume \(\lambda = -0.2\) and \(\mu = +0.01, +0.02, +0.25\) and plot the drag force versus the velocity of the heavy quark. Also here, one finds that by increasing \(\mu\) the value of drag force increases. One should notice that at the small velocities, the cubic-curvature corrections have the minimum effects. Therefore we confirm the previous result. If one assumes negative \(\mu\) and positive \(\lambda\), also finds that for larger \(\mu\) the value of the drag force becomes larger.

2.4 analytic solution

Fortunately, we find an analytic result for the drag force in the special case of \(\mu = -\frac{\lambda^2}{\sqrt{3}}\) which corresponds to \(p = 0\) in \((34)\), as

\[
F_{R^2+R^3} = -\frac{1}{2\pi\alpha'} \left( \frac{\sqrt{3} \pi^2 T^2 v}{N^{\frac{3}{2}} \sqrt{-v^6 + 3v^4 \lambda N - 3v^2 N^2 + 3N^3}} \right),
\]

where \(N\) is defined in \((2)\).

It would be interesting to compare the drag force in the presence of higher derivative corrections with the case of \(\mathcal{N} = 4\) strongly-coupled SYM plasma \(F_{\mathcal{N}=4}\). The authors of \([38, 37]\) have obtained

\[
F_{\mathcal{N}=4} = -\left( \frac{\pi \sqrt{\tilde{\lambda}} T_0^2}{2} \right) \frac{v}{\sqrt{1-v^2}}.
\]

where \(\tilde{\lambda}\) is ‘t Hooft coupling\(^3\) and \(T_0\) is the temperature of AdS black hole solution without any corrections. Let us consider the case of \(\lambda \to 0\) in \((12)\). In this limit, one does not consider any correction in the action \((24)\) and finds that the drag force is nothing but the drag force in the case of \(\mathcal{N} = 4\) strongly-coupled SYM plasma \(F_{\mathcal{N}=4}\).

\(^3\) Notice that \(\alpha'^{-2} = \tilde{\lambda}\).
In this section we investigate the effect of the higher derivative terms to the dissociation length of quark-antiquark pair. In the usual fashion, the two endpoints of the classical open string at the boundary are seen as a quark and antiquark pair which may be considered as a meson [31]. Based on lattice results and experiments, it is found that the meson shows interesting behavior as the temperature of the plasma increases. It is known that heavy quark bound states can survive in a QGP to temperatures higher than the confinement/deconfinement transition [32]. Thermal properties of static quark-antiquark systems have been studied in [20, 21] in an AdS-Schwarzschild black hole setting using the AdS/CFT correspondence. In [29], a rotating quark-antiquark in the presence of higher...
derivative corrections is studied. In the case of Gauss-Bonnet corrections, it is shown that as the Gauss-Bonnet coupling constant $\lambda$ increases the string endpoints become less separated i.e. the radius of the rotating open string at the boundary decreases but the tip of the U-shaped string does not change considerably.

The heavy quark potential in the presence of curvature-squared corrections is calculated in [41]. It is shown that the potential can be calculated as a power series in $LT << 1$, where $T$ is the temperature of the hot plasma. One finds that at fixed temperature, as the Gauss-Bonnet coupling constant $\lambda$ increases the interquark distance $L$ decreases. It would be interesting to investigate this observation in the case of higher derivative corrections. To do this, we consider Quasi-topological gravity and study effect of curvature-cubed corrections to the dissociation length. Because of the complicated feature in this background, we use numerical methods.

### 3.1 dissociation length from AdS/CFT

To find the dissociation length, one should study the heavy quark potential, $V_{qq}(L)$\textsuperscript{4} where $L$ is the distance between two quarks [28]. One finds that the heavy quark potential can be negative, positive or zero. If $V_{qq}(L) < 0$, the dominant string configuration becomes the one for the U-shaped string which can be interpreted as a heavy meson. When $V_{qq}(L) > 0$, the heavy meson dissociates to two free quarks and the string configuration changes. This phenomena happens at special length $L = d$ which is obtained from $V_{qq}(L = d) = 0$. We call $d$ as a dissociation length. Thus by studying the heavy quark potential in the quasi-topological gravity, we will find the effect of higher derivative terms to this quantity.

The heavy quark potential is given by the expectation value of the following static Wilson loop

$$W(C) = \frac{1}{N} Tr P e^{i \int A_\mu dx^\mu},$$

(14)

where $C$ denotes a closed loop in spacetime and the trace is over the fundamental representation of $SU(N)$ group. We consider a rectangular loop along the time coordinate $t$ and spatial extension $L$. The static heavy quark potential is related to the expectation value of this rectangular Wilson loop

\textsuperscript{4} We call quark-antiquark potential in (15) as "heavy quark potential".
in the limit of $t \to \infty$,
\[
\langle W(C) \rangle \sim e^{-t V_{q\bar{q}}(L)},
\] (15)

This expectation value can be calculated from $AdS/CFT$ correspondence [20, 21]. In this setup, one should consider an infinitely massive quark in the fundamental representation of $SU(N)$ group in $\mathcal{N} = 4$ Yang-Mills gauge theory. This quark is dual to a classical string hanging down to the horizon from a probe brane at the boundary. The classical string hanging in the bulk space and connecting two endpoints has a characteristic U-shaped. We name $r_*$ as the tip of the U-shaped string and we let it to define the nearest point between the string and the horizon of the black hole; i.e. $r_* > r_h$. Let us emphasize that for non-physical states we would have $r_* < r_h$ [20].

The dynamics of the U-shaped string is given by the Euclidean version of the Nambu-Goto action in (4). To calculate the heavy quark potential, one has to subtract the infinite self-energy of two independent heavy quarks and from the $AdS/CFT$ correspondence. These massive quarks are dual to two straight strings that extend from the probe brane at the boundary to the horizon. The regularized action is shown by $\Delta S$ and it is related to the expectation value of Wilson loop in (15) by this equation
\[
\langle W(C) \rangle \sim e^{-\Delta S},
\] (16)

As a result, the heavy quark potential is
\[
V_{q\bar{q}}(L) = \frac{\Delta S}{t}.
\] (17)

The heavy quark potential in the vacuum and in the strongly coupled $N = 4$ SYM gauge theory was found in [20]
\[
V_{q\bar{q}}(L) = -\frac{4\pi^2}{\Gamma(1/4)^2} \left( \frac{1}{L} \right) R.
\] (18)

We consider $X^\mu = (t, x, 0, 0, r(x))$ for the coordinates of U-shaped string in the static gauge $\sigma = x$, $\tau = t$. As a result, the Euclidean version of Nambu-Goto action in (14) can be found as
\[
S = \frac{N t}{2\pi\alpha'} \int dx \sqrt{r^4 f(r) + r'^2},
\] (19)
3 dissociation length of quark-antiquark pair at finite coupling

Notice that $r$ depends on $x$. The Hamiltonian density of this action is constant and it is

$$H = -\frac{N t}{2\pi\alpha'} \frac{r^4 f(r)}{\sqrt{r^4 f(r) + r'^2}},$$

(20)

This constant is found at special point $r(0) = r_*$, where $r'_* = 0$, as

$$H = -\frac{N t}{2\pi\alpha'} \sqrt{r_*^4 f(r_*)}.$$  

(21)

Then it is possible to find $L$ as follows

$$\frac{L}{2} = \int_{r_*}^\infty dr \left( \frac{1}{r^4 f(r)} \left( \frac{r^4 f(r)}{r_*^4 f(r_*)} - 1 \right) \right)^{1/2}. $$

(22)

Finally, the heavy quark potential is given by

$$V_{\bar{q}q}(L) = \frac{N}{\pi\alpha'} \int_{r_*}^\infty dr \left( \left( \frac{r^4 f(r)}{r_*^4 f(r_*)} \right)^{\frac{3}{2}} - 1 \right) - \frac{N}{\pi\alpha'} \int_{r_h}^{r_*} dr. $$

(23)

We intend to study the effect of the higher derivative corrections to the heavy quark potential in (23) and the interquark distance in (22). For different values of coupling constants ($\lambda, \mu$), one should consider three distinct $AdS$ black hole backgrounds which are discussed in (32). However, we can not solve (23) and (22) analytically and we have to resort to numerical methods. Also the coefficient $\frac{N}{\pi\alpha'}$ does not play any role in our physical discussion.

### 3.2 Numerical Solutions

We illustrate behavior of $V_{\bar{q}q}(L)$ as a function of $L$ at fixed temperature ($r_h = 1$) in Fig. 3. It is clearly seen that there is a maximal interquark distance, $L_{max}$. It has been shown that for $L < L_{max}$ there are two kinds of strings; long strings and short strings [42, 43, 44, 45]. These strings correspond to the upper and lower parts of $V_{\bar{q}q}(L)$ in Fig. 3, respectively. The stability analysis has shown that short strings are favorable [42, 43, 44, 45]. One concludes that only the lower part is physical [21].

By analyzing Fig. 3., we investigate behavior of the dissociation length for different values for $\mu$ and $\lambda$. In this figure, the heavy quark potential (23)
Fig. 3: The heavy quark potential versus the interquark distance for different values of cubic-curvature coupling \( \mu \) at fixed Gauss-Bonnet coupling constant \( \lambda \). Left: \( \lambda = -0.2 \). Middle: \( \lambda = 0.01 \). Right: \( \lambda = 0.2 \).

is plotted versus the interquark distance \([22]\). We take that different values of cubic-curvature coupling \( \mu \) while the Gauss-Bonnet coupling constant \( \lambda \) is fixed in each frame. Notice that in this case the corresponding black hole backgrounds are specified by \( f_3 \). In this figure, from left to right the Gauss-Bonnet coupling constant \( \lambda \) is increasing, \( \lambda = -0.2, 0.01 \) and 0.2. By increasing Gauss-Bonnet coupling constant, the dissociation length of meson decreases. This phenomena has been found also in the case of a rotating meson \([29]\).

What is the effect of increasing cubic-curvature coupling \( \mu \) while Gauss-Bonnet coupling \( \lambda \) is fixed? In each plot of Fig. 3, \( \lambda \) is fixed and \( \mu \) is increasing. For example in the left plot of this figure \( \lambda = -0.20 \) and \( \mu = 0.01, 0.20, 0.25 \) and 0.28. One can see that the interquark distance decreases by increasing \( \mu \). This observation is clearly seen in the middle and right plots of Fig. 3, too. Therefore by increasing cubic-curvature coupling, the dissociation length of meson decreases.

As we pointed out, there are three distinct AdS black hole backgrounds which correspond to \( f_1(r) \), \( f_2(r) \) and \( f_3(r) \) in \([32]\). In the case of \( \lambda < 0 \) and \( \mu < 0 \) one should consider \( f_1(r) \). We show the heavy quark potential versus the interquark distance in the left plot of Fig. 4. In this plot, \( \lambda = -0.9 \) and from right to left curve \( \mu \) is increasing from \(-0.2, -0.1\) to \(-0.01\). As before, one finds that the dissociation length decreases by increasing the cubic-curvature constant \( \mu \). One should notice that the rate of decreasing is not so large. In the case of \( \lambda > 0 \) and \( \mu < 0 \), one should choose \( f_2(r) \)
Fig. 4: The heavy quark potential versus the interquark distance. Left: $\lambda = -0.9$ and from right to left $\mu = -0.2, -0.1, -0.01$. Right: $\lambda = 0.2$ and from right to left $\mu = -0.01, -0.0001$.

to investigate behavior of the heavy quark potential versus the interquark distance. We show the result in the right plot of Fig. 4. In this plot $\lambda = 0.20$ and $\mu$ is increasing from $-0.01$ to $-0.0001$. It is clearly seen that by increasing $\mu$, the dissociation length decreases. This observation is consistent with what we see in Fig. 3. One infers that as Gauss-Bonnet coupling constant $\lambda$ increases the interquark distance decreases. Also at fixed $\lambda$ by increasing cubic-curvature constant $\mu$ the interquark distant decreases. As a result, including the higher derivative corrections decrease the dissociation length.

4 Conclusion

The higher derivative corrections on the gravity side correspond to finite coupling corrections on the gauge theory side. The main motivation to consider these corrections comes from the fact that string theory contains higher derivative corrections arising from stringy effects. On the gauge theory side, computations are exactly valid when the ’t Hooft coupling constant goes to infinity ($\bar{\lambda} = g_Y^2 N \rightarrow \infty$). An understanding of how these computations are affected by finite $\lambda$ corrections may be essential for more precise theoretical
predictions.

Although AdS/CFT correspondence is not directly applicable to QCD, one expects that results obtained from closely related non-abelian gauge theories should shed qualitative (or even quantitative) insights into analogous questions in QCD. This has motivated much work devoted to study various properties of thermal SYM theories like the hydrodynamical transport quantities. In this paper we have studied the energy loss and the interquark-antiquark distance in the presence of higher derivative terms. We have considered the cubic-curvature terms which is known as quasi-topological gravity.

We calculated the energy loss of heavy quark and from numerical analysis, found that the drag force increases. Fortunately, we found an analytical result in (12) which confirms our result.

We introduced the heavy quark potential in (15). As it is seen from Fig. 3, the heavy quark potential is negative, positive or zero. By studying the zero case, we investigated effect of higher derivative terms in quasi-topological gravity on the dissociation length. We found that the interquark-antiquark distance becomes shorter with the increase of coupling parameters of higher curvature terms. This result is consistent with the case of rotating quark-antiquark pair in [29]. We can therefore conclude that the higher curvature corrections make the dissociation length shorter. Interestingly, the subleading term of the strong coupling expansion of the heavy quark potential in a $\mathcal{N} = 4$ SYM plasma is studied in [46]. It is also found that this correction reduces the magnitude of the heavy quark potential and leads to a smaller screening radius.

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**5 Review of Quasi-topological gravity**

In this appendix we give a brief review of the quasi-topological gravity in five-dimensional spacetime [33]. The bulk action is given by

$$I = \frac{1}{16\pi G_5} \int dx^5 \sqrt{-g} \left( R - \Lambda + \frac{\lambda L^2}{2} \chi + \frac{7L^4 \mu}{8} Z_5 \right), \quad (24)$$
where $\lambda$ and $\mu$ are Gauss-Bonnet coupling and curvature-cubed interaction constant, respectively. The negative cosmological constant is related to radius of AdS space by $\Lambda = -\frac{12}{L^2}$. The curvature-squared interaction is given by $\chi_4$ as

$$\chi_4 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},$$

and $Z_5$ is the new curvature-cubed interaction

$$Z_5 = R_{\mu\nu}R_{\rho\sigma}^{\alpha\beta}R_{\alpha\beta}^{\mu\nu} + \frac{1}{14} (21R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} R - 120 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} R)$$

$$+ 144 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} R + 128 R_{\mu}^{\rho}R_{\nu}^{\rho}R_{\mu\nu}^{\mu}\mu - 108 R_{\mu}^{\nu}R_{\rho}^{\mu}R_{\rho\nu}^{\mu}R + 11 R^3)$$

The planar AdS black hole solutions for different values of the coupling constants were found in [33]. The solution, in units where the radius of AdS is one, is

$$ds^2 = r^2 \left(-N^2 f(r)dt^2 + d\vec{x}^2\right) + \frac{dr^2}{r^2 f(r)},$$

where $f(r)$ is determined by roots of the following equation

$$1 - f(r) + \lambda f(r)^2 + \mu f(r)^3 = \frac{r_h^4}{r^4}. \quad (28)$$

Here $r$ denotes the radial coordinate of the black brane geometry and $t$, $\vec{x}$ label the directions along the boundary at the spatial infinity. In these coordinates the event horizon is located at $f(r_h) = 0$ where $r_h$ is found by solving this equation. The boundary is located at infinity and the geometry will be as asymptotically AdS . The constant $N^2$ specifies the speed of light of the boundary gauge theory and one can choose it to be unity. We name $f(r)$ at the boundary where $r \to \infty$, as $f_\infty$ and one finds that

$$N^2 = \frac{1}{f_\infty}, \quad (29)$$

One also finds from (28) that $f_\infty$ satisfies

$$1 - f_\infty + \lambda f_\infty^2 + \mu f_\infty^3 = 0. \quad (30)$$

The temperature of the hot plasma is given by the Hawking temperature of the black hole

$$T = \frac{N r_h}{\pi}. \quad (31)$$
Authors in [33], solved (28) and found $f(r)$ for different values of coupling constants $\lambda$ and $\mu$. It is shown that there are three different solutions of (28) in the $\mu - \lambda$ plane:

$$f_1(r) = u + v - \frac{\lambda}{3\mu},$$

$$f_2(r) = -\frac{u + v}{2} + i \frac{\sqrt{3}}{2} (u - v) - \frac{\lambda}{3\mu},$$

$$f_3(r) = -\frac{u + v}{2} - i \frac{\sqrt{3}}{2} (u - v) - \frac{\lambda}{3\mu}.$$ (32)

where

$$u = (q + \sqrt{q^2 - p^3})^{\frac{1}{3}}, \quad v = (q - \sqrt{q^2 - p^3})^{\frac{1}{3}},$$ (33)

and

$$p = \frac{3\mu + \lambda^2}{9\mu^2}, \quad q = -\frac{2\lambda^3 + 9\mu \lambda + 27\mu^2 \left(1 - \frac{r^4}{r^4}\right)}{54\mu^3}.$$ (34)

There is a relation between Gauss-Bonnet coupling constant $\lambda$ and cubic-curvature coupling constant $\mu$ as follows

$$\mu = \frac{2}{27} - \frac{\lambda}{3} \pm \frac{2}{27} \left(1 - 3\lambda\right)^{\frac{3}{2}}.$$ (35)

which shows the upper and lower bound on the cubic-curvature interaction coupling. There is a special case $p = 0$ in (34) which corresponds to $\mu = -\frac{\lambda^2}{3}$. $f(r)$ is also found at this point.

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