Hard Scattering in the M-Theory Dual for the QCD String *

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November 1, 2018

Abstract

Conventional superstring amplitudes in flat space exhibits exponential fall off at wide angle in contrast to the power law behavior found in QCD. It has recently been argued by Polchinski and Strassler [1] that this conflict can be resolved via String/Gauge duality. They carried out their analysis in terms of strings in a deformed $AdS^5$ background. On the other hand, an equally valid approach to the String/Gauge duality for 4-d QCD is based on M-theory in a specific Black Hole deformation of $AdS^7 \times S^4$. We show that a very natural extension to this phenomenologically interesting M-theory background also gives the correct hard scattering power laws. In the Regge limit we extend the analysis to show the co-existence of both the hard BFKL-like Pomeron and the soft Pomeron Regge pole.

*HET–1311: This work was supported in part by the Department of Energy under Contracts No. DE-FG02-91ER40676 and No. DE-FG02-91ER40688
1 Introduction

It is generally acknowledged that ’t Hooft’s $1/N_c$ expansion for QCD perturbation theory should map order by order onto the topological expansion for some kind of “QCD string”. Assuming confinement, the spectrum for the leading term at infinite $N$ must consist of an infinite sequence of stable glueballs (e.g., closed string excitations) and the general form of unitarity corrections at higher genus as well as non-perturbative effects, such as instantons, skyrmions, have been studied extensively. However, until recently with Maldacena’s explicit examples of String/Gauge duality, any direct relationship between a “QCD string” and the fundamental superstring was missing.

Most problematic for such a relationship is the stark contrast between the exponentially soft properties of superstrings scattering at high energies (in flat space) and the requirement from the leading large $N$ diagrams of QCD of hard partonic behavior at short distance. Thus the super string appears to be in conflict with a large number of QCD phenomena related to asymptotic freedom, scaling in deep inelastic scattering, power law fall-off of form factors and wide angle scattering to name a few. However in a most interesting paper, Polchinski and Strassler may have begun to resolve this fundamental difficulty. They have suggested a mechanism for hard wide angle string scattering in the context of String/Gauge duality and this has been extended by Polchinski and Susskind to show how form factors for strings may also exhibit power law behavior at large $Q^2$.

They argue that as strings move in the extra (radial) direction, they exhibit both soft (IR) and hard (UV) aspects by appearing to the Yang Mills observers as alternately “fat” and “thin” respectively. In particular Polchinski and Strassler consider glueball scattering in the dual description of IIB strings scattering in an $AdS^5 \times X_5$ background with a suitable IR cut-off (or deformation) in the warped “radial” coordinate. They then relate the power counting rules of Brodsky et al. for wide angle scattering and form factors directly to the conformal scaling of the glueball wave functions in the UV. However there is another approach to 4d QCD, based of M-theory (or IIA strings in an $AdS^7 \times S^4$ black hole), which to date yields the most concrete results for QCD glueballs masses. Since the conformal scaling in $AdS^7$ is different from the $AdS^5$ analysis, there is a question on how this scenario can also agree with the parton counting rules for $QCD_4$.

Here we show that if proper account is taken of the mapping that relates the membrane in 11d M-theory to the 10d IIA string theory, new scaling factors for the effective string parameters result which correct conformal scaling powers to again reproduce the parton result.
While this result is certainly expected on the basis of String/Gauge duality, it is useful to check it and to understand how it comes about. Section 2 explains how the parton scaling for glueball operators in $\text{AdS}^5$ (IIB string) and in $\text{AdS}^7$ (M-theory) are reconciled. In Section 3 the full form for the wide angle scattering amplitude in M-theory is derived and the cross sections are compared with the weak coupling parton \[5\] and the strong coupling $\text{AdS}^5$ results. In Section 4 we consider Regge behavior in the near-forward limit in order to clarify the relation between hard BFKL Pomeron \[8\] and the soft Pomeron responsible for the Reggeized glueball exchange process at large $N$. We end in Section 5 with a brief comment on issues related to the high energy Froissart bound.

2 Wide Angle Glueball Scattering in the M-theory

In QCD wide angle scattering exhibits a power law fall-off in energy up to small logarithmic corrections due to asymptotic freedom. For example the 2-2 glueball amplitude scales as

$$A_{\text{qcd}}(s, t) \sim \left( \sqrt{\alpha_{\text{qcd}}^\prime} p \right)^{4-n}$$

at large $p = \sqrt{s}$ and fixed $-t/s$. The power is determined by $n = \sum_i n_i$, where $n_i$ is the number of “partons” for the external bound state or more precisely lowest twist $\tau_i = d_i - s_i$ for the interpolating fields. Here we focus on this power behavior for each external line to explain how M-theory (or $\text{AdS}^7$/gauge duality) manages in the end to give the same power as that found earlier by Polchinski and Strassler \[1\] for $\text{AdS}^5$ String/Gauge duality for $QCD_4$, postponing to the next section the full derivation of the scattering cross section.

The essential observations in Ref \[1\] are as follows. Glueball scattering amplitudes in the gravity description are given by the product of wave functions, $\phi_i(r, X) \exp[ixp_i]$ for each external leg and a local string scattering amplitude $A(p_i, r, X)$ in the bulk gravity theory. They take the bulk metric to be $\text{AdS}^5 \times X_5$,

$$ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2 + R^2 ds_X^2,$$

with $ds_X^2$ denoting an additional 5 compact dimensions. $R$ is the ads radius. By introducing an IR cut-off ($r > r_{\text{min}}$) the string states (aka glueballs) are discrete and massive. In a plane wave state ($\exp[ixp]$) at fixed external momentum $p$, the proper distance ($\Delta s \simeq (r/R)\Delta x$) is red shifted in the IR (small $r$). Alternatively one may view the local scattering to take place with an anti-red-shifted effective momentum,

$$\hat{p}_s(r) = \frac{R}{r}p.$$
At high string momentum \( (l_s p_s > 1) \) relative to the string scale, \( l_s = \sqrt{\alpha_s'} \), wide angle scattering is indeed exponentially suppressed because of the Hagedorn-like spectrum of soft modes at high energy. Consequently the wide angle scattering of the external glueballs is negligible except in a small region in the IR,

\[
r > r_{\text{scat}} = \sqrt{\alpha_s' R p},
\]

that shrinks as a function of momentum. The dominant contribution to the wide angle scattering therefore scales due to UV boundary condition on the wave function. This scaling rule is set by the conformal weight of the corresponding gauge operator dual to the string state.

\[
\phi_i(r_{\text{scat}}) \sim \left( \frac{r_{\text{scat}}}{r_{\min}} \right)^{-\Delta_4^{(i)}} \sim \left( \sqrt{\alpha_s' R p / r_{\min}} \right)^{-\Delta_4^{(i)}} \sim \left( \sqrt{\alpha_{\text{qcd}}'} P \right)^{-\Delta_4^{(i)}}
\]

For \( \text{AdS}^5 \), the conformal dimension \( \Delta_4^{(i)} \) is equivalent to the twist \( n^{(i)} \) required by correspondence to the power law fall-off for hard parton scattering. Thus it is a consequence of String/Gauge duality that the color singlet string description encodes the gauge theory parton constituent rule. The final step in Eq. \( \ref{eq:5} \), converting to the hadronic string scale, comes from the relation

\[
\alpha'_{\text{qcd}} \sim \left( R / r_{\min} \right)^2 \alpha_s'.
\]

The 2nd power in \( R / r_{\min} \) reflects the stabilization of minimal area at the IR cut-off for a world sheet spanning the Wilson loop.

How does this work in M-theory? One begins in M-theory with an 11d Black Hole deformation of \( \text{AdS}^7 \times S^4 \),

\[
ds^2 = \frac{r^2}{R^2} \eta_{\mu \nu} dx^\mu dx^\nu + \frac{R^2}{r^2} \left( 1 - \frac{r_{\min}^6}{r^6} \right)^{-1} dr^2 + R^2 ds_Y^2,
\]

which is essentially the same form as \( \text{AdS}^5 \times X^5 \) except for an explicit IR cut-off and an extra 6, instead of 5, compact coordinates. Again we treat wide angle scattering as a local event in the bulk with the same red-shift factor rescaling to the local scattering momenta, \( \hat{p}_s(r) = (R/r)p \), as before. However this alone would lead to the wrong parton scaling for hard scattering cross sections, because the \( \text{AdS}^7 \) asymptotic form for glueball wave functions, \( \phi_i(r) \sim C_i^2 (r/r_{\min})^{-\Delta_6^{(i)}} \), are not the same as conformal power for \( \text{AdS}^5 \) (e.g., a scalar glueball has \( \Delta_6 = 6 \) instead of \( \Delta_4 = 4 \)). To avoid this consequence, one must realize that from an M-theory perspective, strings are a consequence of membranes wrapping the “11th” dimension and that in \( \text{AdS}^7 \) this 11th dimension is warped just like another spatial coordinate (\( x^7 \)) with the proper size radius: \( \hat{R}_{11}(r) = (r/R)R_{11} \). To account for this effect one must introduce local values for the effective string length and coupling constant:

\[
\hat{l}_s^2(r) = \frac{R}{r} \left( l_p^3 / l_{11}^3 \right), \quad \text{and} \quad \hat{g}_s^2(r) = \frac{r^3}{R^3} \left( l_{11}^6 / l_p^3 \right).
\]
(Our convention is to scale the local variable relative to the value at the AdS radius, so that \( \hat{g}_s(R) = g_s \) and \( \hat{l}_s(R) = l_s = \sqrt{\alpha'} s \) at \( r = R \).) This additional deformation is precisely what is required\(^1\).

The new definition of the scattering region at wide angles,

\[
r > r_{scat} = \hat{l}_s(r_{scat}) R p = \sqrt{\alpha'} R^{\frac{1}{2}} r_{scat}^p \, ,
\]

leads to

\[
\phi_i(r) \sim \left( \frac{r_{scat}/r_{min}}{\Delta_0} \right)^{\Delta_i} \sim \left( \frac{\sqrt{\alpha'} p/\sqrt{r_{min}^3/R^3}}{\frac{2}{3} \Delta_0} \right)^{\frac{2}{3} \Delta_i} \sim \left( \sqrt{\alpha'_qcd} \right)^{-\frac{2}{3} \Delta_i} \tag{10}
\]

for each external line. For example for the 0++ scalar glueball corresponding to interpolating YM operator \( Tr[F^2] \), the factor of 2/3 exactly compensates for the shift in the conformal dimension from \( \Delta_4 = 4 \) for AdS\(^5\) to \( \Delta_6 = 6 \) for AdS\(^7\) as it should to give the parton results, \( n_i = 2 \Delta_6^{(i)}/3 \). This time, in converting to the hadronic scale in Eq. \(10\), we must realize the relationship of \( \alpha'_qcd \) to the string scale is

\[
\alpha'_qcd \sim \left( \frac{R}{r_{min}} \right)^3 \alpha'_s \, ,
\]

which differs from the AdS\(^5\) string relation \( \alpha'_s \). The 3rd power is a consequence of the fact that in M-theory the area law for the Wilson loop really comes from a minimal volume for a wrapped membrane world volume stabilized at \( r \approx r_{min} \) rather than a minimal world surface area for a string. In summary the requisite adjustment is essentially a multiplicative factor of 2/3 reflecting the difference between scattering strings with 2d world sheets by membranes with 3d world volumes.

### 3 Scattering Amplitude

We now look at the full scattering amplitude in more detail for the M-theory construction. Here we need the precise form AdS\(^7\) × \( S^4 \) Black Hole metric,

\[
ds^2 = \frac{r^2}{R^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu + dx_{11}^2 \right) + \frac{R^2}{r^2} \left( 1 - r_{min}^6/r^6 \right)^{-1} dr^2 + \frac{r^2}{R^2} \left( 1 - r_{min}^6/r^6 \right) d\tau^2 + \frac{1}{4} R^2 d^2 \Omega_4^2 \, ,
\]

where \( \frac{1}{2} Rd\Omega_4 \) are coordinates for \( S^4 \) with 1/2 the AdS radius (R), \( dx_{11} \) on an \( S^1 \) circle with radius \( R_{11} \) and \( d\tau \) on a “thermal” \( S^1 \) circle (\( \tau \leftrightarrow \tau + \beta \)). After these compactification, the boundary

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\(^1\)An alternative approach is to go directly to 10d IIA string theory in the string frame. Here one sees a rescaling by \( r^{3/2} \) for the local momenta, which is equivalent to the rescaling of \( l_s(r)p_s(r) \) in our 11d approach. We have found this correspondence less intuitive
(2, 0) CFT for $AdS^7$ reduces to 5d Yang-Mills theory at finite temperature, $\beta^{-1}$. Assuming anti-periodic boundary condition in $\tau$, all conformal and SUSY symmetries are broken with $\Lambda \equiv \beta^{-1}$ setting the scale for glueball masses in (strong coupling) 4-d QCD. (The requirement that the horizon is a coordinate singularity fixes $\beta = (2\pi/3) (R^2/r_{\min})$.)

The 2-to-$m$ glueball scattering amplitude $T(p_i)$ in a gravity-dual description, is expressed in terms of the bulk scattering amplitudes $A(p_i, r_i, Y_i)$ for the plane wave glueball states $\phi_j(r, Y) \exp[ix_j p_j]$ associated with each external line:

$$T(p_i) = \prod_j \int d\mu_j \phi_j(r_j, Y_j) A(p_1, r_1, Y_1, \ldots, p_{m+2}, r_{m+2}, Y_{m+2}) . \quad (13)$$

The transverse volume element in the bulk is

$$\int d\mu = \int_{r_{\min}}^{\infty} dr \int_0^{R_{11}} dx_1 \int_0^\beta d\tau \int d\Omega \sqrt{-g},$$

with $\sqrt{-g} = r^5/(16R)$. At wide angle we approximate the scattering by a local “form factor” $A(p_i)$ which is exponentially suppressed for momentum large with respect to the local string scale. Thus we can approximate the scattering amplitude by

$$T(p_i) = \frac{R_{11} \beta}{R} \int_{r_{\min}}^{\infty} dr r^5 A(p_i) \prod_{i=1}^{m+2} \phi_i(r), \quad (14)$$

in the region where there is appreciable wide angle scattering,

$$l_s(r) \hat{p}_s(r) \leq O(1) \quad \text{or} \quad r^3 \geq r_{\text{scat}}^3 = R^3 \alpha_s' p^2 . \quad (15)$$

Obviously one would like to be able to justify this approximation with more detailed calculations.

The membrane scattering amplitude at large $r$ is a constant which by dimensional analysis can only be a power of $l_p$,

$$A = Z_0 \frac{l_p^{(D-2)m}}{l_s^2} \frac{1}{l_P^2} . \quad (16)$$

Each of the $m$ additional external legs carries a factor $l_p^2 = \sqrt{R_{11} g_s \alpha_s'^2}$, consistent with the scattering of 10d strings. In flat space we know that the reduction from 11d M-theory to 10d strings requires an additional factor $Z_0 = l_p^2/l_s^2$. In $AdS^7$ the local conformal factor is

$$\frac{l_p^2}{l_s^2} \sim \exp\left[\frac{2\phi_0(r)}{3}\right] ,$$

which takes you to the string frame. For glueball scattering this factor is characterized by its value at $r_{GB} \sim r_{\min}$ where the glueball wave functions have there largest support,

$$Z_0 = \frac{l_p^2}{l_s^2(r_{\min})} = (g_{YM}^2 N)/N^{2/3} . \quad (17)$$
The standard normalization condition in 11d fixes the parametric dependence of the normalization constant in the asymptotic behavior of the glueball wave functions, $\phi_i(r) \simeq C(r_{\text{scat}}/r_{\text{min}})^{-\Delta_6^{(i)}}$, to be $C^{-2} = R_{11} \beta R r_{\text{min}}^4 = R_{11} (R^3 \Lambda)^3$.

Thus assembling all factors, the contribution from the large $r$ region is

$$T(p) = (Z_0 R_{11} \beta / R l_{p}^{11}) (p^0 C^2)^{m+2}/2 \int_{r_{\text{scat}}}^{\infty} dr r^5 \left( \frac{r_{\text{min}}}{r} \right)^{\Delta_6},$$

where $\Delta_6 \equiv \sum_{i=1}^{m+2} \Delta_6^{(i)}$, and the $r$-integration region has also been restricted to $r_{\text{scat}} < r$. After performing the integral the result is

$$T(p_i) = \left( \frac{\sqrt{\alpha'_\text{qcd}}}{N m} \right)^{m-2} \left( \frac{\sqrt{\alpha'_\text{qcd}} p}{(\sqrt{\alpha'_\text{qcd}} p)^{1/3} \Delta_6 - 4} \right),$$

expressed entirely in terms of the hadronic scale $\alpha'_\text{qcd}$. As we mentioned above for scalar glueballs $2/3 \Delta_6 = 4$, the number of constituent partons in the lowest dimension interpolating fields (e.g. $Tr[\epsilon^2]$), in general this is the “twist” $\tau_i$. We have checked explicitly that this works for all the tensor and vector glueball states identified in strong coupling in Brower, Mathur, and Tan [7]. One must make use of the general formula for conformal dimensions, $\Delta_6^{(p)} = d/2 + \sqrt{(d/2)^2 + m^2} - p$, for tensor fields in $AdS^{d+1}$ to check that the rescaling of the exponent by $2/3$ from $AdS^5$ to $AdS^7$ is a general feature.

Note that the first two factors correctly reproduce the leading $N$ behavior, $N^{-m}$, and the proper dimensional dependence, $\Lambda^{-m+2}$. Note we never needed the explicit expression for the $\alpha'_\text{qcd}$. In the strong coupling limit the actual value is

$$\alpha'_\text{qcd} \equiv \frac{27}{32 \pi^2} (g_{YM}^2 N \Lambda^2)^{-1},$$

where we have use the definition of the Yang Mills coupling $g_{YM}^2 = g_s l_s \Lambda$ and the expression for the $R^3 = N l_p^3$ at large $N$.

### 3.1 Comparison with $AdS^5$ and QCD parton cross sections

It is interesting to compare the resulting M-theory fixed-angle differential cross sections with that from $AdS^5$ as well as that from lowest order perturbative QCD. Consider a 2-to-$m$ process, $p_1 + p_2 \to p_3 + \cdots + p_{m+2}$, in the high energy fixed-angle limit. In the CM frame, with $s \equiv (p_1 + p_2)^2 \to \infty$, all components of particle momenta are of the order $\sqrt{s}$ and all ratios $p_i \cdot p_j / s$ are fixed. For each particle in the final state, let $x_i = E_i / \sqrt{s}$ and denote the direction of its three-momentum by $\Omega_i$. Due to overall energy-momentum conservation, we only need to
consider independent variables \( \{ x_i, \Omega_i \}, \ i = 3, \ldots, m + 1 \). The resulting differential cross section can be written as \( \Delta \sigma_{2\to m} = d\sigma / dx_d\Omega_i \sim s^{m-3}|T|^2 \).

For QCD hard scattering in lowest order perturbation theory, this cross section is given by

\[
\Delta \sigma_{2\to m} \simeq \frac{1}{s} f \left( \frac{p_i \cdot p_j}{s} \right) \left( \frac{g_{YM}^2 N}{s} \right)^m \prod_i \left( \frac{\Lambda^2_{\text{qcd}}}{g_{YM}^2 N} \right)^{n_i-1},
\]

(21)

where \( f \) can in principle be calculated. This serves as the “Born” term which will be modified when asymptotic freedom is taken into account. The corresponding \( \text{AdS}^5 \) cross section for \( \text{AdS}^5 \) at strong coupling is

\[
\Delta \sigma_{2\to m} \simeq \frac{1}{s} f \left( \frac{p_i \cdot p_j}{s} \right) \left( \sqrt{g_{YM}^2 N} \right)^m \prod_i \left( \sqrt{\frac{g_{YM}^2 N}{s}} \right)^{n_i-1}.
\]

(22)

while for M-theory our result can be expressed as

\[
\Delta \sigma_{2\to m} \simeq \frac{1}{s} f \left( \frac{p_i \cdot p_j}{s} \right) \frac{1}{N^2m} \prod_i \left( \frac{1}{\alpha'_{\text{qcd}} s} \right)^{n_i-1}.
\]

(23)

The latter two strong coupling results have the same power law term if we make use of the appropriate expression for the QCD string tension at strong coupling,

\[
1/\alpha'_{\text{qcd}} \sim (g_{YM}^2 N)^{\frac{2}{3}} \Lambda^2_{\text{qcd}} \quad \text{for } \text{AdS}^5 \text{ IIB strings,}
\]

\[
1/\alpha'_{\text{qcd}} \sim g_{YM}^2 N \Lambda^2_{\text{qcd}} \quad \text{for } \text{AdS}^7 \text{ M-theory}.
\]

Of course the explicit dependence on \( g_{YM}^2 N \) for \( \text{AdS}^5 \) and \( \text{AdS}^7 \) need not agree with each other or with weak coupling QCD due to non-universal artifacts at strong coupling. Each of them can be “converted” into the perturbative result by the following substitution rules,

\[
g_{YM}^2 N \leftrightarrow \left( g_{YM}^2 N \right)^{\frac{1}{2}} \quad \text{for } \text{AdS}^5,
\]

\[
(\frac{g_{YM}^2 N}{s})^m \leftrightarrow 1 \quad \text{for } \text{AdS}^7,
\]

after \( \Delta \sigma \) is expressed in terms of the appropriate expression for \( \alpha'_{\text{qcd}} \). It was noted in Ref [1] that the substitution rule for \( \text{AdS}^5 \) works in several instances comparing strong and weak coupling results. We find it interesting that the \( \text{AdS}^7 \) cross section is naturally expressed entirely in term of the fundamental string tension for QCD. With dimensional transmutation this is also true of QCD itself at large \( N \). Perhaps this is also more general.

These results can also be interpreted probabilistically. The cross section is given by the product of a probability \( P_{m+2} \) of finding all partons within each hadron in a transverse area of order \( \Delta A_\perp \sim 1/s \) and an elementary “Rutherford” cross section \( \Delta \sigma_0 \),

\[
\Delta \sigma_{2\to m} = P_{m+2} \Delta \sigma_0.
\]
In all three cases, the probability $P_{m+2}$ is given geometrically as $\prod_i \left(1/\alpha'_{qcd}s\right)^{(n_i-1)}$. The Rutherford cross section, in additional to a factor of $N^{-2m}$, is again geometrical, $\Delta \sigma_0 \sim (C/s)f(P_cP_s/N)^{-2m}$. Only the non-universal prefactor distinguishes the three cases: For QCD, $AdS^5$ and $AdS^7$ the prefactors are $C = (g_{YM}^2N)^m$, $(g_{YM}^2N)^{m/2}$ and 1 respectively.

4 Near-Forward Scattering and Regge Behavior

High energy hadron phenomenology has revealed that, in the near-forward limit, the elastic scattering amplitude contains both “hard” and “soft” components. Based on a perturbative QCD analysis, valid in the limit $s \gg |t| \gg \alpha'_{qcd}^{-1}$, contribution to high energy hadronic total cross section from “hard collisions” can be evaluated, leading to a “hard Pomeron” exchange, the celebrated BFKL Pomeron. On the other hand, phenomenological analysis, supported by large-N QCD string picture, demonstrates that scattering at large impact parameter is dominated by the exchange of a “soft” Pomeron Regge pole. The key distinguishing feature is their Regge slopes. The slope for the hard Pomeron is small; it is in fact a fixed branch point in the complex angular momentum plane if the running in QCD coupling is not taken into account. In contrast, the soft Pomeron is a factorizable Regge pole, having a normal hadronic slope with its first Regge recurrence at the $2^{++}$ tensor glueball. (Experimentally, one has $\alpha'_p \sim .2 \sim .3 \text{ GeV}^{-2}$ for the soft Pomeron.) We shall demonstrate next that both these components emerge naturally in our gravity-dual description for the leading large-N contribution to QCD.

To be specific, consider the 2-to-2 scattering, $p_1 + p_2 \rightarrow p_3 + p_4$, with Mandelstam variables $s \equiv (p_1 + p_2)^2$ and $t \equiv (p_3 - p_4)^2$ in the Regge limit $s \rightarrow \infty$ at $t$ fixed. The assumption of a single local scattering, Eq. [4] leads to $T(s,t) = \int_{r_{min}}^{\infty} dr \ K(r) \ A(s,t,r)$, where $A$ is a local four-point amplitude, and $K(r) \sim r^{5}\phi_1(r)\phi_2(r)\phi_3(r)\phi_4(r)$, up to a constant. Converting to local string parameters, amplitude $A(s,t,r)$ depends only on $\alpha'_s \hat{s}(r)$ and $\alpha'_t \hat{t}(r)$, which in the Regge limit becomes

$$T(s,t) = \int_{r_{min}}^{\infty} dr \ K(r) \beta(\hat{t})(\alpha'_s \hat{s})^{2+\alpha'_t \hat{t}},$$

(24)

with t-channel trajectory,

$$\alpha_s(\hat{t}) = 2 + \alpha'_s \hat{t}.$$  

(25)

The local Mandelstam invariants are defined by $\hat{s}(r) = (R/r)^3 s$ and $\hat{t}(r) = (R/r)^3 t$. The 3rd power can be understood from our earlier discussion by noting that $\alpha'_s \hat{s}(r) = \hat{t}^2(r)(\hat{p}_1(r)+\hat{p}_2(r))^2$. For $AdS^5$ only the momentum is rescaled, leading to 2 power of $(R/r)$. Note that the Regge slope is $\alpha'_s$ and its intercept, $\alpha_s(0) = 2$, would normally correspond at $t = 0$ to a spin-2 graviton exchange. However, due to the confinement mechanism giving rise to glueball masses, this...
intercept is shifted lower so that the trajectory intercept in less than 2. (More on this point shortly.)

Since \( \hat{t} \) is in the interval \([ (R/r_{\min})^3 t, 0] \), one finds that \( T(s, t) \) in the Regge limit is given by a linear superposition of power-behavior in \( s \) averaged over the radial coordinate, i.e., a cut in the J-plane. In the near-forward limit where \( |t| \) is small the cut is localized near \( j = 2 \) with the high energy behavior approximated by

\[
T(s, t) \sim f(t) s^{< \alpha_s(\hat{t})>},
\]

where \( < \alpha_s(\hat{t})> \approx \alpha_s(0) = 2 \). The precise result for the leading high energy behavior can be found by a saddle-point analysis or equivalently by studying the Mellin transform to the J-plane,

\[
T(j, t) = \int_{r_{\min}}^{\infty} dr K(r) \left( \frac{r_{\min}}{r} \right)^{(j+1)} \frac{\beta(\hat{t}(r))}{j - 2 + \alpha_s'(R/r)^3 t}.
\]

For \( t \neq 0 \), there is a pair of branch points due to end-point pinching. From the upper limit, one finds a branch point located at \( j = 2 \). The large r (UV) behavior for the product of wave functions leads to \( T(j, t) \sim f(t)(j - 2)^{\gamma} \), where \( \gamma = \Delta/3 - 2 > 0 \). From the lower limit (IR), since wave functions are \( O(1) \), one finds a logarithmic branch point at \( j = 2 + \alpha_s'(R/r_{\min})^3 t \).

For \( t < 0 \), the leading asymptotic behavior at infinite \( s \) comes from the UV region near the AdS boundary, \( T(s, t) \sim f(t) s^{\alpha_s(0)}/(\log s)^{\gamma+1} \), as in the case of the fixed-angle scattering. So it should also be identified with hard gluon effects and therefore represents the physics of the BFKL hard Pomeron. Moreover, due to the local approximation in the bulk AdS string scattering, this does not lead to a factorizable t-channel exchange, another feature of the BFKL Pomeron. To properly isolate the hard processes one should introduce a cutoff, \( r_h \), where \( r_h >> r_{\min} \). The Born term of the BFKL hard Pomeron in a gravity-dual description therefore corresponds to the branch cut in the interval close to \( j=2 \), \([2 + \alpha_s'(R/r_h)^3 t, 2] \). Exactly at \( t = 0 \), these two branch points coincide and the hard process is give by pole, \( T(j, 0) \simeq 1/(j - 2) \). For small \( t \simeq 0 \), the branch cut acts effectively as a single pole, with a small effective slope:

\[
\alpha_{BFKL}'(0) \sim (R/r_h)^3 \alpha_s' = (r_{\min}/r_h)^3 \alpha_{qcd}' << \alpha_{qcd}'.
\]

Exchanging a BFKL-Pomeron naturally leads to a diffraction peak for the elastic differential cross section. Going to the coordinate space, one finds, for a hard process, the transverse size is given by

\[
< \vec{X}^2 > \sim (r_{\min}/r_h)^3 \alpha_{qcd}' \log s + \text{constant}.
\]

If the cutoff, \( r_h \), which characterizes a hard process, increases with \( s \), e.g. \( r_h^3 \sim \log s \) or faster, transverse spread would stop. In the language of a recent study by Polchinski and Susskind, this corresponds to “thin” string fluctuation.
Having identified the hard Pomeron, the question remains: where is the soft Pomeron? At moderate values of \( s \) we may imagine that the dominate scattering occurs in the IR region where the glueball wave-function are large. Thus in Eq. 24, a large contribution comes for the integrand with \( r_{GB} \simeq r_{min} \),

\[ T(s, t) \sim A(s, t, r_{GB}) \sim (\alpha'_{qcd}s)^{2+\alpha'_{qcd}t}/\log s. \] (30)

where we have made use of the fact that \( \alpha'_s s(r_{min}, s) = \alpha'_{qcd}s \). The effective Regge slope is typical of a soft Pomeron. Of course this is a rough characterization. Actually the integral does not yield a pure pole (i.e. power behavior), but more important it does not even factorize in the t-channel because of our local scattering approximation. In reality we know that at large \( N \), there is a infinite spectrum of stable glueballs that propagate on shell and on average gives the cut in the s-plane, \( (-s)^{\alpha_P(t)} \). In the old language of dual models, this is dual to the t-channel Regge pole. Again by continuing to integer \( J \) these are on-shell modes. All of this contradicts a local scattering picture for point like object in bulk AdS. One needs much more powerful methods to find the on-shell string spectrum to really understand these issues completely. Nonetheless consistent with the known spectrum of glueballs at strong coupling we can anticipate that the IR-region must give a factorizable Regge pole contribution,

\[ T(s, t) \sim A(s, t, r_{min}) \sim (\alpha'_{qcd}s)^{\alpha_P(t)}, \] (31)

where

\[ \alpha_P(t) = \alpha_P(0) + \alpha'_{qcd}t. \] (32)

Note that the slope of this Regge trajectory is \( \alpha'_{qcd} \) and the first physical particle lying on this trajectory is a tensor glueball, i.e., \( \alpha_P(m_T^2) = 2 \). For elastic scattering, this Regge trajectory should be identified with the “soft Pomeron”. That is a Pomeron at \( t = 0 \) which is a pole with slope renormalized from the string scale to a hadronic scale: \( \alpha'_s \rightarrow \alpha'_{qcd} \). From the diffraction peak for the elastic differential cross section, one finds that the transverse size increases as

\[ < \vec{X}^2 > \sim \alpha'_{qcd} \log s + \text{constant}. \] (33)

It is interesting to note that this “divergence” in hadron size, in the infinite \( s \) limit, corresponds precisely to the divergent “transverse size” in a string picture recently considered by Polchinski and Susskind. For the form factor this divergence must be removed by fluctuations into the thin string (UV) regime to avoid the traditional disaster of an infinite rms radius for hadronic form factors in the string description.

A rigorous treatment of the Regge behavior in the leading large-N approximation should give a modified four-point function (or Veneziano amplitude) with both soft and hard Pomerons combined. The J-plane of the improved Veneziano amplitude must have contributions from scattering through configurations involving both thin and fat strings [4]. This is consistent with a partonic picture for which it has been suggested in the past that these contributions can
serve as “Born terms” of an iterative sum, leading to a single “Pomeron” which captures the physics of both hard scattering at short-distance as well as infrared physics of confinement at large distance [9]. On the other hand, phenomenologically it appears that for current accelerator energies it is sufficient to treat these contributions additively. Nevertheless, it is a major challenge to be able to really understand these effects entirely within the color singlet formulations of perturbative string field theory.

5 Discussion

Let us conclude with some comments on the nature of Pomeron as well as some speculative remarks on the constraint of unitary beyond the topological string expansion. Since the $1/N$ expansion is a perturbative solution to unitarity, at any finite order it needs not obey the non-linear unitarity bounds. It has been noted that the naive power law fall off for the wide angle 2-to-m cross sections of the parton model saturates, but does not violate, the unitarity bound. However in the Regge limit, our calculation gives both hard and soft Pomeron contributions with a common intercept greater than one,

$$\sigma_{\text{total}} \sim C \ s^\Delta,$$

for $\alpha_P(0) = 1 + \Delta$ in contradiction to the Froissart bound, $\sigma_{\text{total}} \leq C (\log s)^2$ asymptotically. Indeed this picture is supported experimentally by power law with an intercept, $\alpha_P(0) = 1 + \Delta$, where $\Delta \simeq 0.08$, while at presently available energies (e.g., $p\bar{p}$ scattering up to TeV range) the cross sections are still an order of magnitude smaller than the absolute unitarity bound.

From the large N perspective this power appears to be controlled by low energy dynamics, i.e., by the requirement that the trajectory interpolates the lightest tensor glueball, $\alpha_P(m_T^2) = 2$. We have previously pointed out [7], if one assumes a linear trajectory and the strong coupling approximation to the tensor glueball mass, one obtains

$$\Delta = \alpha_P(0) - 1 \simeq 1 - 0.66 \left( \frac{4\pi}{g_Y^2N} \right),$$

in contrast to $\Delta = 1 - O(1/\sqrt{g_Y^2N})$ in a deformed $AdS^5$ background [1, 10]. In both cases, Pomeron can be interpreted as “confined graviton”, with $\alpha_P(0) < 2$. If one also accepts fits to the empirical value of $\Delta$, this relation is consistent with the lattice estimate of the bare coupling constant at the weak to strong crossover. Thus a crude matching between weak and strong coupling seems to make sense. [7]

More importantly, with $\Delta > 0$, Pomeron-like power behavior leads to violation of
elastic unitarity for partial wave amplitudes. Traditionally one imagines restoring unitarity at high energies via an s-channel iterations such as the eikonal mechanism which requires summing higher genus contributions to all orders. In such a scheme, a “disk-like” picture emerges, with an effective hadronic radius

\[ R_{\text{eff}} \sim \sqrt{\alpha'_{\text{qcd}} \Delta \log s}, \]

obeying the Froissart bound.

More recently, Giddings [11] has also addressed the issue of unitarity violations at much higher energies where non-perturbative string interactions must be taken into account, e.g., the effect of Black Hole production. Curiously the production cross section for a single Black Hole leads to a cross section increasing with \( s \) as a power, \( \Delta = 1/(D - 3) \). With \( D > 4 \), this is consistent with our bound, \( \alpha_P(0) \leq 2 \). Giddings goes on to “unitarize” this perturbative black hole result to saturate the Froissart bound. It is then interesting to raise the question of whether the Pomeron intercept is constrained by a matching condition to the Black Hole production and what is the Gauge correspondence for this unitarization mechanism.

Much more can be said concerning scattering in the near-forward limit [9, 12] but we shall defer that to a future publication. In summary, in spite of these insights on high energy processes for a QCD string, a deeper microscopic understanding of these issues in terms of String/Gauge duality is lacking. In particular as emphasized in a recent paper [10], scaling in the deep inelastic limit requires a much more explicit connection to individual partons. Here these simple QCD-like models lead to structure functions but the scaling laws for the cross sections fail by powers relative to QCD. The recent analysis of the Penrose limit of \( AdS^5 \times S^5 \) type IIB string theory dual to \( \mathcal{N} = 4 \) SUSY Yang-Mills by Berenstein, Maldacena and Nastase [13] and high J configurations by Gubser, Klebanov and Polyakov [14] are promising developments that one may hope will lead to a rigorous derivation the String/Gauge duality for the partonic properties in this context.

Acknowledgement: We would like to thank A. Jevicki, D. Lowe, and M. Schvellinger for discussions and comments.
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