On accelerated Universe expansion

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(Dated: February 10, 2022)

It is shown that observed peculiarities of the Universe expansion are an inevitable consequence of the gravitational force properties following from gauge-invariant gravitation equations considered in detail in an author’s paper in Annalen der Physik, v.17, 28 (2008).

PACS numbers: 04.50+h; 98.80.-k

Numerous data testifying that the most distant galaxies move away from us with acceleration where obtained for the last 8 years [1]. This fact poses serious problems both for fundamental physics and astrophysics [2]. In the present paper it is shown that the available observational data are an inevitable consequence of properties of the gravitational force implying from gauge-invariant gravitation equations [3]. These equations were tested successfully by binary pulsar PSR 1913 + 16 [4].

In Minkowski’s space-time the radial component of the gravitational force of a point mass \( M \) affecting the free-falling particle of mass \( m \) is \( m \ddot{r} \) where the acceleration \( \ddot{r} = d^2r/dt^2 \) must be found from the gravitational equations in use. According to [3] the force is given by

\[
F = -m \left[ c^2 C/2A + (A'/2A - 2C'/2C)c^2 \right],
\]

where

\[
A = f^2/C, \quad C = 1 - r_g/f, \quad f = (r_g^3 + r^3)^{1/3}, \quad f' = df/dt.
\]

In this equation the dot denotes the derivative with respect to \( t \), \( r_g = 2GM/c^2 \), \( G \) is the gravitational constant, \( c \) is the speed of light at infinity, the prime denotes the derivative with respect to \( r \).

For particles at rest (\( \dot{r} = 0 \))

\[
F = -\frac{GmM}{r^2} \left[ 1 - \frac{r_g}{(r^3 + r_g^3)^{1/3}} \right].
\]

Fig. 1 shows the force \( F \) affecting a particle at rest and a particle free falling from infinity as the function of the distance \( r = r/r_g \) from the centre.

It follows from Fig. 1 that the gravitational force affecting free-falling particles changes its sign at \( r \approx 2r_g \). Although we have never yet observed particles motion at distances of the order of \( r_g \), we can verify this result for very distant objects in the Universe, at large cosmological redshifts, because it is well-known that the radius of the observed region of the Universe is of the order of its Schwarzschild radius.

A magnitude which is related with observations in the expanding Universe is the relative velocity of distant star objects with respect to an observer. The radial velocity \( v = \dot{R} = dR/dt \) of particles on the surface of a selfgravitating expanding homogeneous sphere of a radius \( R \) can be obtained from equations of the motion of a test particle [3]:

\[
v = \frac{Cf^2}{R^2} \sqrt{1 - \frac{C}{E}},
\]

where \( C \) are the functions of the distance \( R \), \( r_g = (8/3)\pi c^{-2} G \rho R^3 \) is Schwarzshild’s radius of homogeneous matter inside of the sphere and \( \rho \) is the matter density. The parameter \( E \) is the constant total energy of a particle divided by \( mc^2 \).

Fig. 2 shows the radial acceleration \( \ddot{R} = v' \dot{v} \) of a particle on the surface of the sphere of the radius \( R \) in flat space-time. Two conclusions can be made from this figure.

1. At some distance from the observer the relative acceleration changes its sign. If the \( R < 2 \cdot 10^{27} cm \), the radial acceleration of particles is negative. If \( R > 2 \cdot 10^{27} cm \), the acceleration is positive. Hence, for sufficiently large distances the gravitational force affecting particles is repulsive and gives rise to a relative radial acceleration of particles with respect to any observer.

2. The gravitational force, affecting the particles, tends to zero when \( R \) tends to infinity. (The same fact takes place as regards the force acting on particles in the

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The matter density is equal to $10^{-29}g\,cm^{-3}$.

Therefore, the difference in two local level $E_1$ and $E_2$ of an atom energy in the field is $\Delta E = (E_2 - E_1)\sqrt{C}$, so that the local frequency $\nu_0$ at the distance $R$ from an observer are related with the observed frequency $\nu$ by equality

$$\nu = \nu_0 \sqrt{C},$$

where we take into account that for the observer location $\sqrt{C} = 1$. It follows from (7) and (5) that the relationship between frequency $\nu$ as measured by the observer and the proper frequency $\nu_0$ of the moving source in the gravitational field takes the form

$$\frac{\nu}{\nu_0} = \sqrt{\frac{1 - v/c}{1 + v/c}} \quad (8)$$

Equation (8) yields the quantity $z$ as a function of $R$. By solving this equation numerically we obtain the dependence $R = R(z)$ of the measured distance $R$ as a function of the redshift. Therefore the distance modulus $\mu$ to a star object is given by

$$\mu = 5 \log_{10}[R(z) (z + 1)] - 5 \quad (9)$$

where $R(1 + z)$ is a bolometric distance (in pc) to the object.

If (4) is a correct equation for the radial relative velocity of distant star objects in the expansive Universe, it must to lead to the Hubble law at small distances $R$. Under this condition the Schwarzschild radius $r_g = (8/3)\pi G\rho R^3$ of the matter inside of the sphere is very small compared with $R$. For this reason $f \approx r$, and $C = 1 - r_g/r$. Therefore, at $E = 1$, we obtain from (4) that

$$v = HR, \quad (10)$$

where

$$H = \sqrt{(8/3)\pi G\rho}. \quad (11)$$

If $E \neq 1$ equation (4) does not lead to the Hubble law, since $v$ does not tend to zero when $R \to 0$. For this reason we set $E = 1$ and look for the value of the density at which a good accordance with observation data can be obtained.

The fig. (3) show the Hubble diagram based on eq. (9) compared with observations data [11]. It follows from this figure that the model under consideration are in a good accordance with observation data.

For the value of the density $\rho = 4.5 \cdot 10^{-30}g\,cm^{-3}$ we obtain from (11) that

$$H = 1.59 \cdot 10^{-18}c^{-1} = 49\,km\,c^{-1}\,Mpc. \quad (12)$$

Fig. 4 shows the dependence of the radial velocity $v$ on the redshift. It follows from this figure that at $z > 1$ the Universe expands with an acceleration. At $R \to \infty$ the velocity and acceleration tend to zero.
FIG. 3: The distance modulus $\mu$ vs. the redshift $z$ for the density $\rho = 4.5 \cdot 10^{-30} \text{g cm}^{-3}$. Small squares denote the observation data according to Riess et al.

FIG. 4: The radial velocity vs. redshift $z$ for the density $\rho = 4.5 \cdot 10^{-30} \text{g cm}^{-3}$

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