Fluctuation Conductivity in Unconventional Superconductors near Critical Disorder

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The fluctuation conductivity $\sigma_s$ in bulk superconductors with non $s$-wave pairing and with nonmagnetic disorder of strength $D$ is studied at low $T$ and within the Gaussian approximation. It is shown by assuming a quasi two-dimensional (2D) electronic state that, only if the gap function $d_\mu(p)$ is, as in a 2D $p$-wave pairing state, linear in the in-plane (relative) momentum $p_\perp$, the in-plane fluctuation conductivity on the line $D = D_c$ is weakly divergent in low $T$ limit. The present result may be useful in clarifying the true gap function of spin-triplet Sr$_2$RuO$_4$ through resistivity measurements.

KEYWORDS: Sr$_2$RuO$_4$, fluctuation conductivity, unconventional superconductivity, dirty superconductor

Conventionally, the pairing state of a superconducting (SC) phase is identified by examining microscopic properties deep in the SC phase or by directly finding a pairing mechanism consistent with the electronic state of the material. In contrast, it is usually difficult to obtain information on the pairing state through physical properties near a SC transition such as the fluctuation effects. For example, the fluctuation conductivity $\sigma_s$ accompanying a thermal transition depends only on the dimensionality of fluctuation except for a constant numerical prefactor of its expression, which is not easy to examine experimentally with high precision. But how about near a SC transition at zero temperature ($T = 0$)? From this point of view, we examine the nature of $\sigma_s$ in unconventional (non $s$-wave) superconductors near a quantum critical point, $(D, T) = (D_c, 0)$, which is the end point of disorder-driven transition line $T_c(D)$, where $D$ is the strength of microscopic (nonmagnetic) disorder. Particular attention is paid to $\sigma_s$ on the specific line $D = D_c$ in the $D - T$ phase diagram. We find in the Gaussian approximation that, only in the case of pairing of $p$-wave type, $\sigma_s$ on the $D = D_c$ line is weakly divergent in low $T$ limit.

This work was motivated by the unresolved issue of the pairing symmetry of Sr$_2$RuO$_4$. In spite of various proposals, the pairing state of this material has not been determined thus far. A key issue is to determine where on the Fermi surface the nodes of SC energy gap are present. Having Sr$_2$RuO$_4$ in mind, we assume that the electronic states are of quasi 2D character, while the
dimensionality of low energy modes such as the superconducting fluctuation is three-dimensional (3D). Further, all calculations will be carried out within the weak coupling approximation, and the effects of disorder on the fluctuation, as well as on the normal properties, will be treated within the Born approximation since the nature of fluctuation is determined by the electronic details in the normal phase. A key fact in the present analysis is that, when the gap function is of \( p \)-wave type (according to the classification defined in ref. 4), a part of the lowest order gradient terms in the Gaussian Ginzburg – Landau (GL) action is proportional to a \( \ln(1/T\tau) \)-factor arising from the diffusion propagator induced by the disorder, where \( \tau \) is the lifetime of quasiparticles, while for gap symmetries with higher harmonics (i.e., \( d \)-wave, \( f \)-wave types etc. or others) the corresponding \( \ln(1/T\tau) \)-factor appears only in higher gradient terms that are unnecessary for obtaining the leading term of \( \sigma_s \).

As a microscopic model, we start with the BCS Hamiltonian of a layered material with a random potential term

\[
\mathcal{H} = \sum_\sigma \sum_\mathbf{p} \xi_\mathbf{p} a^\dagger_\sigma(\mathbf{p}) a_\sigma(\mathbf{p}) + \sum_l \int d^2r_\perp u_l(\mathbf{r_\perp})a^\dagger_\sigma(\mathbf{r_\perp},l) a_\sigma(\mathbf{r_\perp},l)
\]

\[
-\frac{1}{2} \sum_{\sigma,\sigma'} \sum_{\mathbf{p,}p'q} V_{\text{BCS}}(\mathbf{p}, \mathbf{p'}) a^\dagger_\sigma(\mathbf{p}_+) a^\dagger_{\sigma'}(-\mathbf{p}_-) a_{\sigma'}(-\mathbf{p}_'),
\]

where \( a_\sigma(\mathbf{r_\perp},l) \) is the annihilation operator of electron with spin \( \sigma \) and at the in-plane coordinate \( \mathbf{r_\perp} \) on the \( l \)-th plane, the attractive interaction \( V_{\text{BCS}}(\mathbf{p}, \mathbf{p'}) \) is assumed to be, as usual, separable like \( |g|w(\mathbf{p})w(\mathbf{p'}) \), and \( \mathbf{p_\perp} \) implies \( \mathbf{p} \pm \mathbf{q}/2 \). The random potential \( u_l(\mathbf{r_\perp}) \) obeys the Gaussian ensemble:

\[
\overline{u_l(\mathbf{r_\perp})} = 0; \overline{u_l(\mathbf{r_\perp})u_l(\mathbf{r_\perp}')} = (2\pi N_2(0)\tau)^{-1}\delta(\mathbf{r_\perp} - \mathbf{r_\perp}')\delta_{l,l'},
\]

where the overbar denotes the random average, and \( N_2(0) \) is the 2D density of states at the Fermi surface. The impurity scattering was assumed at this stage to be isotropic. We have verified that an inclusion of \( \tau \)-wave scattering rate \( \tau^{-1}_l \) does not affect the main features in eq. (3) becoming relevant to our derivation of \( \sigma_s(D = D_c) \).

The random-averaged Green’s function is, as usual, given by

\[
\mathcal{G}(\mathbf{p}, \varepsilon_n) = \frac{1}{i\varepsilon_n(1 + (2\pi|\varepsilon_n|)^{-1}) - \xi_\mathbf{p}},
\]

with a fermionic Matsubara frequency \( \varepsilon_n \). The dispersion in eq. (2) takes the quasi 2D form

\[
\xi_\mathbf{p} = \varepsilon_\mathbf{p_\perp} - E_F + t \cos(p_z s),
\]

where \( s \) is the interlayer distance and \( \mathbf{p} = (\mathbf{p_\perp}, p_z) \). Throughout this paper we take units \( \hbar = k_B = 1 \). Also we neglect the localization effect on noninteracting electrons and treat the disorder effect within the Born approximation.

To illustrate the points essential to our results, let us first explain the analysis of a case of \( d \)-wave type defined by \( V_{\text{BCS}} = |g|w_d(\mathbf{p})w_\mathbf{q}(\mathbf{p'}) \) with \( w_d(\mathbf{p}) = (p_x^2 - p_y^2)h_d(p_z s) \). Here, \( h_d(p_z s) \) is assumed to satisfy \( \langle h_d \rangle = s \int_{p_z} h_d(p_z s) > 0 \) so that \( h_d \) may be \( p_z \)-independent, reflecting the quasi 2D dispersion (3).
Fig. 1. Feynman diagrams defining the renormalized vertex \( W_q(p) \) (the hatched region), where the double line, the solid line with arrow, and the dashed line denote, respectively, the pair-field, the electron propagator, and the impurity line carrying \((2\pi \tau N_s(0))^{-1}\). The first term of r.h.s. implies the bare vertex \( w_d(p) \).

Following the standard procedure\(^7\) to decouple the BCS interaction in terms of the pair-field \( \Delta_\mu \) which is related to the \( d \)-vector as

\[
d_\mu(p) = w_d(p) \Delta_\mu(q, \omega_\nu),
\]

where \( \mu \) is the spin index, the Gaussian GL action takes the form

\[
S = \sum_\mu \sum_{q, \omega_\nu} K(Q, \omega_\nu) \Delta_\mu(q, \omega_\nu)^2,
\]

(4)

where \( K^{-1} = (|g|N_2(0))^{-1} - \pi(q, \omega_\nu) \), and

\[
\pi(q, \omega_\nu) = T \sum_{\varepsilon_n} \int \frac{d(p_z s)}{2\pi} \frac{\omega_\nu}{\omega} w_d^*(p) W_q(p) \mathcal{G}(p_+\varepsilon_n+) \mathcal{G}(-p_-, -\varepsilon_n-),
\]

(5)

where the renormalized vertex \( W_q(p) \) is defined in terms of the bare vertex \( w_d(p) \) as indicated by Fig. 1. In general, features of crystal structure appear in the dispersion \( \varepsilon_p \), which may make the derivations of eq. (5) and \( W_q(p) \) complicated. If an isotropic Fermi surface is assumed, for simplicity, \( W_q(p) \) is given by

\[
W_q(p) = w_d(p) \frac{p_z}{2} \frac{\langle h_d \rangle (q_x^2 - q_y^2)}{4} \Gamma_q (2\varepsilon_n + \omega_\nu),
\]

(6)

where \( \Gamma_q (2\varepsilon_n + \omega_\nu) = (\tau|2\varepsilon_n + \omega_\nu| + l^2 q_1^2/2 + 2l^2 \tau^2 \sin^2(q_z s/2))^{-1} \) is the diffusion propagator, and \( \omega_\nu \) is a bosonic Matsubara frequency. Since the \( q \)-dependence in the second term [i.e., \( q_x^2 - q_y^2 \) in eq. (6)] is essentially determined by the \( p \)-dependence of \( w_d \), the effect of crystal structure is, except for its role leading to the form of \( w_d(p) \), unimportant in deriving the renormalized part corresponding to the second term of eq. (6). An effect of crystal structure on the GL gradient terms unaccompanied by the diffusion propagator will be commented on later together with an effect of the GL quartic term neglected here.

It is easily seen that the last term of eq. (6) leads to higher gradient terms like \( \sim (q_x^2 - q_y^2)^2 \) in GL action. A \( q_z \)-dependence neglected above, which may appear in \( \langle h_d \rangle \) of \( W_q \), is reflected only in a still higher gradient term like \( (q_x^2 - q_y^2)^2 q_z^2 \). Since such higher gradient terms are irrelevant to the
leading terms of interest in fluctuation conductivities, we can omit the last term of eq. (8) at this stage. Then, \( \mathcal{K}(\mathbf{q}, \omega) \) simply becomes

\[
\mathcal{K}^{-1}(\mathbf{q}, \omega) = \mu_0 + \gamma_1|\omega| + \xi_0^2 q^2 + 4 \left( \frac{\xi_0}{s} \right)^2 \sin^2(qs/2).
\]  

(7)

Here \( \mu_0 = 2\pi^2(T\tau)^2/3 + (D - D_c)/D_c \) with \( D/D_c = 1.78/(\pi Tc0\tau) \), \( \gamma_1 \simeq \tau_1 \), \( \xi_0^2 = l^2/2 \) with \( l = \nu\gamma \tau, \xi_0^2 = (\nu\tau s)^2/2 \), and \( Tc0 \) is \( T_c \) in clean limit \( (D = 0) \). Note that the fact \( \int_{\mathbf{q}} w_d(\mathbf{p}) = 0 \) is essential for obtaining the \( T_c \)-reduction induced by disorder. Hereafter, we only take the dissipative frequency terms into account. Inclusion of other dynamical terms is found not to change our main conclusions. The above-mentioned feature that the contribution accompanied by the diffusion propagator is reflected only in higher gradient terms of GL action is common to all pairing states including states of \( f \)-wave type.

Next, let us calculate the fluctuation conductivity \( \sigma_{s,\perp} \) for an in-plane current using the Gaussian GL action and the Kubo formula. As is well known, this is nothing but the Aslamasov – Larkin (AL) fluctuation conductivity. After the analytical continuation, it is expressed by

\[
\sigma_{s,\perp} = \frac{2\pi \xi_0^4}{RQ T} \int \frac{1}{\omega \sin^2(\pi \nu \omega / 2\nu \omega)} \int_{-\infty}^{\infty} \frac{2q^2}{(2\pi)^2 q^2} \int_{-\pi/d}^{\pi/d} dq_z \frac{2\pi}{2\pi} \left[ \text{Im} K^R(\mathbf{q}, \omega) \right]^2
\]

\[
= \frac{16\pi \gamma_1 T_{c0}}{3RQ \xi_0 \sqrt{\mu_0}} \int \frac{f(\mu_0 \omega / (2\gamma_1 T))}{\omega} \int \frac{d^3 q}{(2\pi)^3} \left[ (1 + q^2 + \omega^2)^2 \right]^{-1/2},
\]

(8)

where \( RQ = \pi h / 2c^2 = 6.45 \) (kΩ), \( K^R(\mathbf{q}, \omega) = [\mathcal{K}^{-1}(\mathbf{q}, 0) - i\gamma_1 \omega]^{-1} \) is the retarded fluctuation propagator, and, in moving to the last line, a 3D-type behavior of fluctuation was assumed. This assumption is valid when \( (0 < \sqrt{\mu_0} < 2\gamma_1 \omega) \). The weight of quantum fluctuation in \( \sigma_s \)-expressions is measured by the function \( f(x) = x^2 / \sin(x^2) \).

First, let us comment on the strong disorder side \( D > D_c \) in which \( \mu_0 \) remains positive in \( T \to 0 \) limit. It is easily seen that \( \sigma_{s,\perp} \) vanishes like \( T^2 \) in \( T \to 0 \) limit in this case. This is the simplest example of the insulating fluctuation conductivity at \( T = 0 \). On the other hand, on the weak disorder side in which \( D < D_c, \mu_0 \) vanishes at a positive temperature \( T_c(D) \) on cooling. Near \( T_c(D) \) where \( \mu_0 \ll 2\gamma_1 T_c(D) \), the familiar divergent (superconducting) behavior \( \sigma_{s,\perp} \sim (T - T_c(D))^{-1/2} \) is found. Now, let us consider \( \sigma_{s,\perp} \) on the line \( D = D_c \) where \( \mu_0 = 2\pi^2(\tau s)^2/3 \). The ratio \( \mu_0/2\gamma_1 \) in this case also tends to vanish on cooling and becomes less than unity in \( T < T_{cr}^{(1)} \approx 0.2Tc0 \), and thus the same classical limit \( (f(x) \to 1) \) as in \( D < D_c \) dominates at \( D = D_c \) in low \( T \) limit. Through a detailed analysis, we find the remaining quantum correction in \( \sigma_{s,\perp} \) to be smaller by the factor \( \sim \sqrt{T/Tc0} \). Further, the condition of the 3D approximation, \( \sqrt{\mu_0} < 2\gamma_1 \omega/s \), is rewritten in this case as \( T < T_{cr}^{(2)} \approx \sqrt{3T}/(2\pi) \). Then, in \( T \ll \min(T_{cr}^{(1)} , T_{cr}^{(2)}) \), \( \sigma_{s,\perp}(D = D_c) \) approaches a
in terms of the isotropic dispersion. The resulting Gaussian action is merely given here:

\[ \sigma_{s\perp}(D = D_c) = \frac{1}{4\pi} \sqrt{\frac{3}{2}} \frac{R_Q^{-1}}{\xi_0(D = D_c)} \simeq \frac{\sqrt{3}}{7R_Q s} \frac{Tc0}{t} = \sigma_s^* \]  

on cooling. By combining this with the \( D \neq D_c \) results mentioned above, one will notice that the situation around \( D = D_c \) in the present 3D case is similar to that of the 2D insulator-superconductor transition[4]. A similar result is obtained for the out-of-plane conductivity \( \sigma_{s\parallel} \) of which the value at \( D = D_c \) is given by

\[ \sigma_{s\parallel}(D = D_c) = \frac{\xi_0}{\xi_0} \bigg|_{D=D_c} = \frac{t_s}{v_F} \sigma_{s\perp}(D = D_c). \]  

Now, we consider the cases of pairing of \( p \)-wave type[3]. Let us assume the attractive interaction to have a general form \( V_{BCS}(\mathbf{p}, \mathbf{p'}) = |g| \sum_{j=x,y} w_j(\mathbf{p}) w_j(\mathbf{p'}) \), where \( w_j = p_j h_j(p_s) \) with \( \int_{p_s} h_j(p_s) > 0 \). Then, the \( d \)-vector has the form \( d_{ij}(\mathbf{p}) = \sum_{j=x,y} p_j h_j(p_s) \Delta_{\mu,j} \). This includes the candidates for \( \text{Sr}_{2}\text{RuO}_4 \) (triplet) pairing state[3]:

\[ d_{ij}(\mathbf{p}) = \sum_{\rho=\pm1} \Delta_{\rho}(p_x + i p_y)(c - \cos p_s). \]  

In this example, we will assume \( |h_j(p_s)| \) to be independent of \( j \) for convenience of presentation. Under these apparatus, it is elementary to obtain the corresponding expressions to eqs. (8) and (9) again in terms of the isotropic dispersion. The resulting Gaussian action is merely given here:

\[ S^{(p)} = \frac{N_2(0)}{2} \sum_{i,j=x,y} \sum_{\mu,\nu} \Delta_{\mu,i}(\mathbf{q},\omega_{\nu}) [(\delta_{ij} - \hat{q}_i\hat{q}_j) K^{-1}_{T}(\mathbf{q},\omega_{\nu}) + \hat{q}_i\hat{q}_j K^{-1}_{L}(\mathbf{q},\omega_{\nu})] \Delta_{\mu,j}(\mathbf{q},\omega_{\nu}), \]  

where

\[ K^{-1}_{T}(\mathbf{q},\omega_{\nu}) = \mu_0 + \gamma_1 |\omega_{\nu}| + \xi_0^2 \hat{q}_j^2 + 4 \left( \frac{\xi_0^2}{s} \right) \sin^2(q_z s)/2, \]

\[ K^{-1}_{L}(\mathbf{q},\omega_{\nu}) = \xi_0^2 \left( C(q_\perp^2, T) \right) q_\perp^2, \]  

\( \hat{q}_j = q_j/|q_\perp| \), and \( C(q_\perp^2, T) = 2(1 + \ln(4\pi T + q_\perp^2 T^2))^{-1} \). The enhancement factor \( C(q_\perp^2, T) \) arising from the diffusion propagator appears only in the longitudinal part of the lowest order terms in the gradient because the term corresponding to the second term of eq. (8) is vectorial and proportional to \( \mathbf{q} \). An additional gradient term which may appear depending on the form of \( \varepsilon_{p_\perp} \) will be discussed later. The in-plane conductivity \( \sigma_{s\perp} \) in this case becomes

\[ \sigma_{s\perp}^{(p)}(D = D_c) = 3 \times \frac{2\pi \xi_0^4}{R_Q T} \int_{V} \frac{d^3q}{(2\pi)^3} \frac{q_\perp^2}{\sinh^2(\frac{q_\perp T}{2T})} \left( (\text{Im} K_{T}^R(\mathbf{q},\omega))^2 + (1 + C(q_\perp^2, T))^2 (\text{Im} K_{L}^R(\mathbf{q},\omega))^2 \right. \]

\[ + \left. \frac{(C(q_\perp^2, T))^2}{2} (\text{Im} K_{T}^L(\mathbf{q},\omega)) (\text{Im} K_{L}^L(\mathbf{q},\omega)) \right|_{D=D_c} \]

\[ \simeq 6\sigma_s^* (\ln C(T) + \text{const.}) \]  

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on the line $D = D_c$ and in low $T$ limit, where $C(T) = C(q_\perp^2 = 0, T) \simeq \ln(\tau T)^{-1}$, and the factor 3 in the first line is due to spin degeneracy in the spin-triplet channel. In eq. (13), we have neglected contributions associated with an additional term of supercurrent arising from the $q_\perp$-dependence of $C(q_\perp^2, T)$, which merely become, at most, of the order $\sigma_s^* C^{-1}(T) \ln C(T)$. On the other hand, the out-of-plane conductivity is given by

$$\sigma_s^{(p)}(D = D_c) \simeq 3\sigma_s^* \left(1 + C^{-1}(T) \right) \frac{\xi_0}{\xi_0} \bigg|_{D = D_c}. \quad (14)$$

The low $T$ behaviors of $\sigma_s^{(p)}$'s in $D \neq D_c$ are similar to those in the case of non $p$-wave type: $\sigma_s^{(p)}(D > D_c) \to T^2$, while $\sigma_s^{(p)}(D < D_c) \sim (T - T_c(D))^{-1/2}$ when $T \to T_c(D)^+$. As eq. (13) shows, however, $\sigma_s^{(p)}$ on the line $D = D_c$ is, in contrast to eq. (9), weakly divergent (superconducting) in $T \to 0$ limit.

We should note that it is the sum of a fluctuation contribution $\sigma_s$ and a normal one $\sigma_n$ which is directly measured in experiments of the type performed in ref. (13). If the electronic structure is of 3D-type, the residual normal conductivity is of the order $R_Q^{-1} E_F \tau k_F$, implying that the ratio $\sigma_s^*/\sigma_n$ will be small, $O((T_c0/E_F)^2)$, for 3D-type Fermi surfaces. In the present case with a quasi 2D Fermi surface, however, this ratio is larger: since $\sigma_n \simeq R_Q^{-1} E_F \tau /2s$, we have $\sigma_s^*/\sigma_n \simeq \sqrt{s}/(8\pi E_F \tau \xi_0(D = D_c))$, which becomes $0.6\pi T_{c0}^2/(E_F t)$ in the present model. In a system with $\xi_0 \sim s$ (i.e., $t \sim T_{c0}$) such as Sr$_2$RuO$_4$, this ratio is $O(T_{c0}/E_F)$. Although the ratio $T_{c0}/E_F$ in Sr$_2$RuO$_4$ is small ($\sim 10^{-2}$), it will be possible to separate $\sigma_s$ from $\sigma_n$ by applying a magnetic field as long as $\sigma_n$ is insensitive to the field, because the fluctuation contribution $\sigma_s$, in particular near $D = D_c$, will be easily erased by applying a low magnetic field.

A few comments on the above derivation of $\sigma_s$ are in order. First, two main effects of crystalline anisotropy were neglected above. As one of them, the $\mathbf{p}$-dependence of the gap function $d_\mu(\mathbf{p})$ was linearized above, and, as an example, $w_j(\mathbf{p})$ should be proportional not to $p_j$ but to $\sin p_j$ in a real system. However, it is clear that the gradient terms, in particular its quadratic terms, of GL action do not essentially change due to this replacement. Next, the other gradient term which may appear in eq. (11) as a result of reduction of space symmetry should also be examined, which is typically given by

$$|\partial_x \Delta_{\mu,x}|^2 + |\partial_y \Delta_{\mu,y}|^2 \quad (15)$$

in zero field. The resulting GL gradient terms have three independent coefficients and hence, are essentially the same as those in the so-called 2D representation as long as the gauge field is spatially constant (note that a conductance for a uniform current in zero magnetic field is defined in terms of a spatially constant gauge field). However, the inclusion of eq. (13) does not qualitatively change our main results (9) and (13). A key point for reaching this conclusion is to notice that the contribution accompanied by the diffusion propagator leads not to the term (15) but only to
a longitudinal term $\sim |\text{div}\Delta|^2$ and that, as in clean limit, the term [13] can arise simply as a combined result of crystal anisotropy in the dispersion $\varepsilon_{\mathbf{p}_\perp}$ and of the $w_j(\mathbf{p})$-form. Then, it is straightforward to see, by power counting in $\mathbf{q}$-integrals, the appearance in cases of $p$-wave type of a $\ln\ln(T/T)$-divergence like that in eq. (13) even when the term [13] is present.

We have focused above on the AL term of Gaussian fluctuation conductivity. It is not difficult to examine other contributions, Maki–Thompson term $\delta\sigma_{MT}$ and the so-called DOS term $\delta\sigma_{DOS}$. We find that, in $T \rightarrow 0$ limit, they vanish on the $D = D_c$ line in the manners $\delta\sigma_{MT} \sim T_\tau$ and $\delta\sigma_{DOS} \sim (T\tau)^2$, and thus, we can neglect them as far as the behavior in low $T$ limit is concerned.

Fluctuation conductivity near the disorder-induced quantum critical point was also examined in ref.15, where the $p$-wave case does not seem to have been considered. The corresponding result to eq. (9) was concluded in ref. 15 to be rather proportional to $T^{1/4}$. This difference originates from the assumption implicit in ref. 15 that the mass renormalization $\delta\mu \sim T^{D/2}$ in $D$-dimensions in the one-loop order[4] is larger than the bare one $\mu_0 \sim T^2$. However, the former arises from the quartic (nonGaussian) term of GL action. In fact, it is verified that the above assumption $\delta\mu > \mu_0$ is valid in $T < T_{c0}\varepsilon_G^{(3)}$, where $\varepsilon_G^{(3)} \simeq (T_{c0} \varepsilon/(E_F \xi_c))^2$ is the 3D Ginzburg number measuring the temperature width $\Delta T/T_{c0}$ of the thermal critical region. Namely, if this temperature width of $O(T_{c0}^3/E_F^2)$ is negligible as in Sr$_2$RuO$_4$, the $T^{1/4}$-behavior of $\sigma_s(D = D_c)$ will not be observable at accessible temperatures, but rather, the present Gaussian results should be observed at low $T$.

We emphasize that the result, eq. (13), in cases of $p$-wave type is valid only when the Fermi surface and thus, the gap function $d(\mathbf{p})$ are of quasi 2D-type. If the Fermi surface and hence, $d(\mathbf{p})$ is essentially 3D-type (e.g., $d(\mathbf{p}) = \sum_{j=x,y,z} \Delta_j p_j$), $\sigma_s(D = D_c; T \rightarrow 0)$ is nondivergent and is not useful in distinguishing the pairing states. By contrast, if $d(\mathbf{p})$ is 1D-type and, as its example, is approximated by $\sin(p_x)$, the fluctuation conductivity $\sigma_s x (D = D_c)$ in the $x$-direction is divergent like $(\ln(1/T\tau))^{1/2}$, while the corresponding ones in the $y$ and $z$ directions instead vanish like $(\ln(1/T\tau))^{-1/2}$ in low $T$ limit.

In conclusion, the fluctuation conductivities near a disorder-induced quantum critical point of a non $s$-wave bulk superconductor with quasi 2D-like electronic structure were examined in the Gaussian approximation and in zero field. When the dependence of the gap function on the in-plane momentum $\mathbf{p}_\perp$ is of a higher order, as in cases of $d$-wave or $f$-wave symmetry, $\sigma_s$ at the critical value of disorder approaches a finite value in low $T$ limit, just like the 2D nonGaussian result[10] while it is weakly divergent on cooling when the gap function is linear in $p_\perp$ or $\sin(p_\perp)$ (of a $p$-wave type). The present result may be useful for clarifying the form of pairing function of a material with spin-triplet pairing and a low dimensional Fermi surface and, for example, clarifying which of the $f$-wave gap function[13] $\propto p_x \pm ip_y$ and the function[14] $\propto p_x p_y (p_x \pm ip_y)$ is favorable as the pairing state of Sr$_2$RuO$_4$.

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[1] For a review, see W.J. Skocpol and M. Tinkham: Rep. Prog. Phys. 38 (1975) 1049.
[2] See, for example, data analysis in R. Ikeda, T.Ohmi and T. Tsuneto: J. Phys. Soc. Jpn. 58 (1989) 1377.
[3] V. J. Emery: J. Low. Temp. Phys. 22 (1975) 467.
[4] Through this paper, a state of p-wave type means that the gap function $d_\mu(p)$ is linear in the in-plane relative momentum $p_\perp$. Thus, $d_\mu(p) = (p_x \pm ip_y)g(p_zs)$ is one example of such states, while $d_\mu(p) = (p_x^2 - p_y^2)g_2(p_z)$ will be called a state of d-wave type.
[5] Y. Maeno et al.: Nature (London) 372 (1994) 532.
[6] A. A. Abrikosov, L. E. Gor’kov and I. E. Dzyaloshinskii: Methods of Quantum Field Theory in Statistical Physics (Prentice-Hall, N.J.) sect. 39.
[7] See, for example, J. Kurkijarvi, V. Ambegaokar and G. Eilenberger: Phys. Rev. B 5 (1972) 868.
[8] R. Ikeda: Int. J. Mod. Phys. B 10 (1996) 601.
[9] L. G. Aslamasov and A. I. Larkin: Sov. Phys. Solid State 10 (1968) 875.
[10] D. Dalidovich and P. Phillips: Phys. Rev. Lett. 84 (2000) 737.
[11] Y. Hasegawa, K. Machida and M. Ozaki: J. Phys. Soc. Jpn. 69 (2000) 336.
[12] M. E. Zhitomirsky and T. M. Rice: cond-mat/0102390.
[13] A. P. Mackenzie et al.: Phys. Rev. Lett. 80 (1998) 161.
[14] For a review, see J. A. Sauls: Adv. Phys. 43 (1994) 113.
[15] R. Ramazashvili and P. Coleman: Phys. Rev. Lett. 79 (1997) 3752.
[16] See §5 in A.J. Millis: Phys. Rev. B 48 (1993) 7183.
[17] M. J. Graf and A. V. Balatsky: Phys. Rev. B 62 (2000) 9697.