An Analysis of LOS Coverage in Vehicular Networks with Roadside Units and Relays
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Abstract—This paper analyzes the use of vehicular relays as a means to extend the Line-of-Sight (LOS) coverage from roadside units (RSUs) toward users on the streets in mmWave or visible light communications. In this paper, we consider the scenario where RSUs select vehicles within their LOS coverage as relays. As a result, the LOS coverage of those RSUs is extended by the LOS coverage newly provided by the vehicular relays. To account for the spatial relationship between vehicles and RSUs, we use Cox point processes. We assume that the LOS distances from RSUs or relays are independent and exponentially distributed. To address the spatial interactions between RSU LOS coverage and relay LOS coverage, we use the notion of mean area fraction to evaluate the LOS coverage.

I. INTRODUCTION

A. Motivation and Background

Vehicular networking is one of the most promising use cases for modern wireless communications. It facilitates various applications such as cooperative driving, sensor data sharing, vehicle and pedestrian positioning, pedestrian safety, and Internet-of-Things (IoT) data sharing [1], [2]. To support the diverse use cases requiring high data rates and reliability, vehicles are envisioned to communicate with nearby vehicles, pedestrians on streets, base stations, or roadside units (RSUs) by leveraging line-of-sight (LOS) mmWave or visible light communications.

Nevertheless, due to the high penetration loss caused by buildings, vehicles or pedestrians on streets are not always in the RSU’s LOS. Random obstacles on urban roads may restrict the LOS coverage of those RSUs, undermining the reliability of mmWave or visible light communications. One of the most well-studied approaches to counteract such random blockages is to utilize vehicles as relays wirelessly associated with the RSUs [3], [4]. In this context, the relay is designed to increase the probability that vehicles or pedestrians will be in LOS w.r.t. any transmitter, by overcoming the geometric limitations of vehicular networks. For instance, [5] numerically showed that the use of relays expands the LOS coverage region. However, it is challenging to calculate the amount of increment in the LOS coverage because of the spatial correlation between RSUs and vehicles [6]–[8]. In this paper, we use stochastic geometry and random sets to analyze the size of the LOS coverage region in vehicular networks with vehicular relays.

B. Contributions

To model the LOS coverage region of vehicular networks, we first model the RSUs and vehicles as Cox point processes on Poisson lines representing the roads. Then, we assume that RSUs choose vehicles within their LOS coverage regions as relays. We model the RSU coverage and relay coverage as random rectangle sets meant to cover the vehicular and pedestrian users along the roads. The spatial dependency between RSUs and relays results in the overlap of LOS coverages. The proposed model accurately characterizes the LOS coverage regions jointly covered by the RSUs and their relays.

To derive the size of the LOS coverage region that is generated by the union of random rectangles, we use the notion of mean area fraction. It evaluates the average relative area of the random set w.r.t. that of the entire plane. Specifically, we first analyze the mean area fraction of the RSU coverage and then evaluate the mean area fraction of the union of the RSU coverage combined with the relay coverage (RSU-plus-relay coverage). Unlike the mere summation of the coverage regions—which does not account for the overlap of RSU coverage and relay coverage, our analysis accurately quantifies the amounts by which the LOS coverage increased by the relays. The framework provided in this paper can be used to assess the use of vehicular relays in the LOS-critical vehicular network applications, such as mmWave V2X communications or positioning of vehicles based on LOS time-difference-of-arrival [9].

II. SYSTEM MODEL

A. Spatial Model

We model the set of roads as an isotropic Poisson line process \( \Phi_l \) [10], [11]. The latter is generated by a Poisson point process \( \Xi \) of intensity \( \lambda_l / \pi \) on the cylinder \( \mathcal{C} = \mathbb{R} \times [0, \pi) \). Each point \( (r, \theta) \) of \( \Xi \) corresponds to an undirected line \( l(r, \theta) \) on the plane \( \mathbb{R}^2 \). Here, \( r \) is the distance from the origin to the line \( l(r, \theta) \) and \( \theta \) is the angle between the line \( l(r, \theta) \) and the \( x \)-axis, measured in the counterclockwise direction. These lines form a Poisson line process \( \Phi_l \).

Conditional on \( \Phi_l \), the locations of RSUs are modeled as a Poisson point process \( \Phi(\mathbb{R} \times [0, \pi)) \) of intensity \( \mu_l \) on each line of \( \Phi_l \) and the locations of vehicles are modeled as an independent Poisson point process \( \Psi(\mathbb{R} \times [0, \pi)) \) of intensity \( \mu_v \) on each line of \( \Phi_l \). Therefore, the locations of RSUs and the vehicles form Cox point processes denoted by \( \Phi \) and \( \Phi_v \), respectively [10], [11]. Fig. 1 shows the RSUs and vehicles jointly created under the conditional structure of the Cox point process. We assume

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Finally, the RSU LOS coverage set—the region where users are in LOS w.r.t. at least one RSU on roads—is defined as follows:

\[ C_{\text{RSU}} = \bigcup_{X_i \in \Phi} C_{X_i} = \bigcup_{X_i \in \Phi} (S_{X_i} \times [-\eta/2, \eta/2]), \tag{3} \]

where we use the union to combine the LOS coverage of RSUs since when RSUs are close to each other, their respective LOS coverage regions may overlap in space.

In this paper, we consider the case where each RSU selects one vehicle inside its LOS segment as a relay. For tractability, we ignore the case where the same vehicle is selected by more than two RSUs and where there is no vehicle in the RSU LOS coverage domain. The proposed modeling is justified by the fact that in mmWave or visible light communications, only those vehicles in LOS w.r.t. the RSUs are capable of extending the LOS coverage associated to their RSUs. We also assume there exist LOS-blocking obstacles from the selected relays at random distances. Therefore, the locations of relays, their LOS segments, and the relay LOS coverage set are respectively defined as follows:

\[ \Phi' = \sum_{X_i \in \Phi} \text{Unif}[\Phi_v(S_{X_i})] = \sum_{Y_i} \delta_{Y_i}, \tag{4} \]

\[ S_{Y_i} = \begin{cases} \{x \in \text{left}(l(r, \theta); X_i) \ s.t. \|x - X_i\| < W'_i\} & \text{if } l(r, \theta) \text{ is the right side of } l(r, \theta) \text{ w.r.t. } X_i, \\ \{x \in \text{right}(l(r, \theta); X_i) \ s.t. \|x - X_i\| < V'_i\} & \text{otherwise}, \end{cases} \tag{5} \]

\[ C_{\text{Relay}} = \bigcup_{Y_i \in \Phi'} (S_{Y_i} \times [-\eta/2, \eta/2]), \tag{6} \]

where \text{Unif}[\Phi_v(S_{X_i})] is a uniformly selected point from the point process \( \Phi_v(S_{X_i}) \) and \( W'_i \) and \( V'_i \) are i.i.d. exponential random variables. When \( \mu_v \gg \mu \), we use the fact that the uniform selection of one point in the Poisson point process in the segment \( S_{X_i} \) is in fact a point uniformly distributed in the segment \( S_{X_i} \). As a result, the locations of relays are given by

\[ \Phi' = \sum_{X_i \in \Phi} \text{Unif}(S_{X_i}), \tag{7} \]

where, with a slight abuse of notation, \( \text{Unif}(S_{X_i}) \) is a point uniformly and independently selected in the segment \( S_{X_i} \). In other words, the locations of the selected relays are given by i.i.d. uniformly distributed points within each of the LOS segments.

Finally, we define the RSU-plus-relay coverage as the domain where users are in LOS w.r.t. at least one RSU or relay. Using Eqs. (4) and (7), we define the relay coverage set \( C_{\text{Relay}} \) and the RSU-plus-relay coverage set \( C_{\text{RSU+relay}} \) as follows:

\[ C_{\text{Relay}} = \bigcup_{X_i \in \Phi} \left(S_{\text{Unif}}(S_{X_i}) \times [-\eta/2, \eta/2]\right), \tag{8} \]

\[ C_{\text{RSU+relay}} = C_{\text{RSU}} \cup C_{\text{Relay}} = \bigcup_{X_i \in \Phi} \left(S_{X_i} \cup S_{\text{Unif}}(S_{X_i}) \times [-\eta/2, \eta/2]\right). \tag{9} \]

Fig. 3 illustrates the RSU-plus-relay coverage.
\section*{C. Performance Metric}

To compute the area of the LOS coverage domain, we use the mean area fraction of the RSU coverage set \cite{14}. It is defined as follows:

\[
\nu(C_{\text{RSU}}) = \mathbb{E} \left[ \int_{\mathbb{R}^2} \mathbb{I}_{C_{\text{RSU}}} \mathbb{I}_{C_0}(x) \, dx \right] = \lim_{r \to \infty} \frac{\int_{B_0(r)} \mathbb{I}_{C_{\text{RSU}}} \mathbb{I}_{C_0}(x) \, dx}{\int_{B_0(r)} \mathbb{I}_{C_0}(x) \, dx},
\]

where \(C_0\) is a unit square \(\mathbb{R}^2\) and \(B_0(r)\) is the ball of radius \(r\) centered at the origin. Similarly, the mean area fraction of RSU-plus-relay coverage set is given by

\[
\nu(C_{\text{RSU+relay}}) = \nu(C_{\text{RSU}} \cup C_{\text{Relay}}) = \mathbb{E} \left[ \int_{C_0} \mathbb{I}_{(C_{\text{RSU}} \cup C_{\text{Relay}}) \cap C_0}(x) \, dx \right] = \lim_{r \to \infty} \frac{\int_{B_0(r)} \mathbb{I}_{(C_{\text{RSU}} \cup C_{\text{Relay}}) \cap C_0}(x) \, dx}{\int_{B_0(r)} \mathbb{I}_{C_0}(x) \, dx}.
\]

### A. Stationarity of Coverage

\textbf{Proposition 1.} \(\text{The RSU coverage } C_{\text{RSU}}, \text{ the vehicle relay coverage } C_{\text{Relay}}, \text{ and the RSU-plus-relay coverage } C_{\text{RSU+relay}} \text{ are stationary particle models.}\)

\textbf{Proof:} The RSU coverage set \(C_{\text{RSU}}\) is the union of i.i.d. random sets centered on the RSU point process. Since the RSU point process is stationary \cite{13}, \(C_{\text{RSU}}\) is a stationary particle model \cite{14}.

Since \(\Phi'\) is stationary and \(C_{\text{Relay}}\) is the union of i.i.d. random sets, \(\Phi_{\text{Relay}}\) is a stationary particle model. Similarly, we conclude that the RSU-plus-relay coverage set \(C_{\text{RSU+relay}}\) is also a stationary particle model.

\textbf{Fact 1.} For a stationary particle model, its mean area fraction corresponds to the probability that the origin is contained by the set \cite{15}.

### B. RSU Coverage

\textbf{Theorem 1.} \(\text{The mean area fraction of RSU coverage is}\)

\[
\nu(C_{\text{RSU}}) = 1 - \exp \left( -2 \lambda \eta (1 - e^{-2\mu r}) \right). \tag{10}
\]

\textbf{Proof:} From the stationarity of RSU coverage, the mean area fraction is equal to the probability that the origin is contained in the RSU coverage set \cite{11}. Therefore, we have

\[
\nu(C_{\text{RSU}}) \equiv P(0 \in C_{\text{RSU}}) = 1 - P(0 \notin C_{\text{RSU}}).
\]

Let \(0_1\) denote the point where the line \((l(r_i, \theta_i))\) is closest to the origin. Then, for a given line \((l(r_i, \theta_i))\) the typical point at the origin is not contained in the LOS coverage of those RSUs if (i) \(w_j < |x_j - 0_1|\) for all the RSUs \(X_j\) on the right side of \(0_1\) and (ii) \(v_j < |x_j - 0_1|\) for all the RSUs \(X_j\) on the left side of \(0_1\). Therefore, we have

\[
\mathbb{E} \left[ \sum_{r_i, \theta_i \in \mathbb{E}} \mathbb{E} \left[ \mathbb{I}_{X_j, \text{on l.h.s.}} \mathbb{I}_{X_j \notin \Phi_{l(r_i, \theta_i)}} \mathbb{I}_{w_j < |x_j - 0_1|} \Phi_{l(r_i, \theta_i)} \phi \right] \right] = \mathbb{E} \left[ \sum_{r_i, \theta_i \in \mathbb{E}} \mathbb{E} \left[ \mathbb{I}_{X_j, \text{on r.h.s.}} \mathbb{I}_{X_j \notin \Phi_{l(r_i, \theta_i)}} \mathbb{I}_{v_j < |x_j - 0_1|} \Phi_{l(r_i, \theta_i)} \phi \right] \right],
\]

where we use the Poisson property of \(\Phi_{l(r_i, \theta_i)}\) to obtain

\[
\sum_{X_j \notin \Phi_{l(r_i, \theta_i)}} \delta_{X_j - 0_1} \delta_{T_{j, \theta_i}} = \sum_{T_{j, \theta_i} \in \phi} \delta_{T_{j, \theta_i}},
\]

where \(\phi\) is a Poisson point process of intensity \(\mu\) on the real axis.

In addition, for the lines \((l(r_i, \theta_i))\), the coverage of RSUs on those lines does not contain the origin if and only if the distances from the origin to those lines are greater than \(\eta/2\). Then, we use the facts that the lines of the Poisson line process are independent, that \(W_j\) and \(V_j\) are i.i.d., and that the locations of RSUs on each line is a Poisson point process.
of intensity $\mu$. As a result, the mean area fraction of RSU coverage is

$$P(0 \notin C_{RSU}) = E_{\mu} \prod_{r_i, \theta_i \in \mathbb{Z}} \prod_{T_j \in \phi} \left[ T_j > 0 \right] E_{\phi} \left[ \prod_{T_j \in \phi} \left( 1 - e^{-\frac{T_j}{\mu}} \right) \right]$$

$$= E_{\mu} \prod_{r_i, \theta_i \in \mathbb{Z}} \prod_{T_j \in \phi} \left[ T_j < 0 \right] E_{\phi} \left[ \prod_{T_j \in \phi} \left( 1 - e^{-\frac{T_j}{\mu}} \right) \right]$$

$$= E_{\mu} \prod_{r_i, \theta_i \in \mathbb{Z}} \prod_{T_j \in \phi} \left[ T_j < 0 \right] E_{\phi} \left[ \prod_{T_j \in \phi} \left( 1 - e^{-\frac{T_j}{\mu}} \right) \right]$$

$$= \exp \left( -\lambda \eta \left( 1 - e^{-2\mu} \right) \right), \quad (11)$$

where we use the total independence property of the Poisson point processes $\phi$ and $\Xi$, and their probability generating functionals [14].

**Fig. 4** gives the mean area fraction of the RSU coverage with $\mu = 4$/km and $\eta = 100$ meters as the mean LOS distance $\gamma$ increases. The figure shows that the derived mean area fraction of RSU coverage in Eq. (10) accurately matches the simulation results. To get the simulation results, we measure the mean area fraction by a Monte Carlo method, namely counting the event that the typical point at the origin is covered the simulated RSU coverage in a very large disk. Note the increment of $\gamma$ has a diminishing impact on the mean area fraction, as the mean area fraction of any coverage is bounded above by the mean area fraction of the set for all streets present in the network. Note the set for all streets is depicted by the white strips in Fig. 1 [13] showed that it is given by $1 - e^{-\lambda \eta}$.  

**C. RSU-plus-relay Coverage**

This section presents the main result of the paper: the mean area fraction of the coverage created by RSUs and their associated vehicle relays.

**Theorem 2.** The mean area fraction of the RSU-plus-relay coverage is given by Eq. (12).

Proof: The mean area fraction of the RSU-plus-relay coverage is given by the probability that the origin is contained by the RSU-plus-relay coverage. Let $I_A$ denote an indicator function that takes one if the event $A$ occurs and zero if the event $A$ does not occur. The mean area fraction of the RSU-plus-relay coverage is

$$\nu(C_{RSU+relay}) = P(0 \in C_{RSU+relay}) = 1 - E[I_0 \notin C_{RSU+relay}].$$

Conditional on $\Phi_t$, we have

$$E[I_0 \notin C_{RSU+relay}] = E \left[ \prod_{r_i, \theta_i \in \Phi_t} \prod_{T_j \in \Phi_t} \left( 1 - e^{-\frac{T_j}{\mu}} \right) \right],$$

where we use the same technique as in the proof of Theorem 1. Furthermore, conditional on the LOS coverage distances of RSUs, namely $W_j$ and $Y_j$, the mean area fraction of the RSU-plus-relay coverage is

$$E \left[ \prod_{r_i, \theta_i \in \Phi_t} \prod_{T_j \in \Phi_t} \left( 1 - e^{-\frac{T_j}{\mu}} \right) \right] \left( 1 - e^{-\frac{T_j}{\mu}} \right) \left( 1 - e^{-\frac{T_j}{\mu}} \right)$$

where we drop the subscripts of the variables $W$ and $V$. Using the property of the indicator functions, we have

$$I_0 \notin C_{RSU+relay} = I_0 \notin C_{RSU} + I_0 \notin C_{relay} \cap C_{RSU}.$$  

Here, the first term is a measurable function of the random variables $\Phi_t, \phi, W, V$. Therefore, the mean area fraction is given by

$$E \left[ \prod_{r_i, \theta_i \in \Phi_t} \prod_{T_j \in \Phi_t} \left( 1 - e^{-\frac{T_j}{\mu}} \right) \right] \left( 1 - e^{-\frac{T_j}{\mu}} \right) \left( 1 - e^{-\frac{T_j}{\mu}} \right)$$

The origin is not contained in $C_{relay} \cap C_{RSU}$ if for all lines $l(r_i, \theta_i)$, neither the LOS segments centered on the left side of $0_t$, nor the LOS segments on the right side of $0_t$ contains the origin. Let $Y_j$ denote the location of the selected relay of the $j$-th RSU. As in the proof of Theorem 1 we have

$$(a) = E \left[ \prod_{r_i, \theta_i \in \Phi_t} \prod_{T_j \in \Phi_t} \left( 1 - e^{-\frac{T_j}{\mu}} \right) \right] \left( 1 - e^{-\frac{T_j}{\mu}} \right) \left( 1 - e^{-\frac{T_j}{\mu}} \right)$$

$$= E_{Y_j} \left[ 1 - e^{-\frac{y_j}{\mu}} \right] \left( 1 - e^{-\frac{y_j}{\mu}} \right) \left( 1 - e^{-\frac{y_j}{\mu}} \right)$$

$$= \int_{-w}^{w} 1 - \exp \left( -\frac{y}{v} \right) dy,$$
and mean area fraction of RSU-plus-relay coverage is given by
\[ 1 - \exp \left( -2\lambda \eta \left( 1 - \exp \left( -\mu_r \int_{-\infty}^{\infty} e^{-\frac{x+|w|}{\gamma}} dx \right) \right) \right). \]

![Fig. 5. The mean area fraction with \( \mu = 4/\text{km} \) and \( \eta = 100 \text{ meters} \)](image1)

\[ \nu(C_{\text{RSU-plus-relay}}) = 1 - \mathcal{P}(0 \notin C_{\text{RSU-relay}}) \] and we have
\[
1 - \mathbb{E} \left[ \prod_{r_i \in \Phi_t} \prod_{t_j \in \phi} \left( 1 - e^{-\frac{|w|}{\gamma}} \right) + \int_0^{\infty} \int_0^{\infty} \left( \int_{-\infty}^{\infty} \frac{1 - e^{-\frac{|w|}{\gamma}}}{w + v} e^{-\frac{w + v}{\gamma^2}} dy dw dv \right) \right].
\]

We obtain the final result by using the probability generating functionals of the Poisson point process \( \phi \) and \( \Xi \).

Theorem 2 characterizes the amount of LOS coverage newly added to the system resulting from the use of vehicle relays. Figs. 5 and 6 show the simulated results on the mean area fractions of the RSU coverage and RSU-plus-relay coverage for various network parameters as \( \gamma \) increases. In general, the use of relays significantly increases the coverage when the mean area fraction of the RSU coverage is relatively low. When \( \gamma \) ranges from 0 to 150 meters, the RSU-plus-relay coverage is greater than the RSU coverage, by about 20 or 25 \%.

Note the analysis in this paper is conducted under the assumption that vehicles are uniformly selected out of the LOS coverage of the RSUs. Since those vehicles are equally likely to be selected, the analysis presented in this paper can be seen as the increase of the LOS coverage averaged across all potential relaying vehicles. Note that under the proposed model, a nonuniform relay selection principle may have a higher mean area fraction for the RSU-plus-relay coverage, but will incur additional overhead. The analysis on different relay selection techniques is left for future work.

Note also that we assume that the density of the vehicles is much larger than the density of the RSUs. This assumption ensures that each RSU is able to find at least one vehicle for relaying. On the other hand, if the densities of RSUs and vehicles are similar, one should consider the following two facts: (i) RSUs may not be able to find vehicles within their LOS coverage and (ii) relays could be simultaneously selected by more than one RSU. Combined with the complication due to the mobility of vehicles, this scenario would introduce an additional statistical dependence on the locations of the relays. Its analysis is left for future work.

### D. One-dimensional Coverage Approximation

Above, we computed the mean area fraction of the RSU coverage and RSU-plus-relay coverage set, by leveraging the stationarity of the coverage set and then by computing the probability that the coverage set contains the origin. Given that the number of LOS-accessible users is proportional to the size of the LOS coverage region, the presented mean fractions determine the average number of LOS-accessible users per unit space, including vehicles and pedestrian on the streets.

where we use the facts that (i) the probability density function of the exponential random variable is \( f(x) = 1 - \exp(-x/\gamma) \) and (ii) the location of the relay \( Y_j \) is uniformly distributed within \([t_j - w, t_j + v]\), where \( t_j \) is the location of the \( j \)-th RSU and \( w \) and \( v \) are the distance from the RSU to LOS-blocking obstacles. Here \( w \) and \( v \) denote the realizations of the i.i.d. exponential random variables.

As a result, by deconditioning w.r.t. \( W \) and \( V \), the mean area fraction of RSU-plus-relay coverage is given by
Fig. 7. Exact analysis of the mean area fraction of RSU coverage with $\gamma = 0.2$ and $\eta = 0.1$.

Fig. 8. 1-D additive approximation of the mean area fraction of RSU coverage with $\gamma = 0.2$ and $\eta = 0.1$.

On the other hand, one may consider a one-dimensional (1-D)-based additive method. It approximates the area of the LOS coverage set as the product of the length of all LOS segments in a ball of radius $r$ and $\eta$. More precisely, in this approximation, the mean area fraction RSU coverage is approximated as follows:

$$\nu(C_{RSU}) \approx \lim_{r \to \infty} \frac{\eta \times \text{length}(B_0(r) \cap \bigcup_{X_i \in \Phi} S_{X_i})}{\text{area}(B_0(r))}.$$

To evaluate the right-hand side, we use the fact that the total length of the Poisson lines in a ball of radius $r$ is given by $\lambda_1 \times \pi r^2 = \pi \lambda r^2$ \(\square\). Thus, the numerator is given by

$$\eta \text{ length} \left( B_0(r) \cap \left( \bigcup_{X_i \in \Phi} S_{X_i} \right) \right) = \pi \eta \lambda r^2 \nu_1 \left( \bigcup_{X_i \in \Phi} S_{X_i} \right),$$

where $\nu_1 \left( \bigcup_{X_i \in \Phi} S_{X_i} \right)$ denotes the linear fraction of the union of LOS segments on a line and $\Phi$ is the RSU Poisson point process of intensity $\mu$ on that line. Since $\Phi$ is a stationary, the linear fraction is equal to the probability that a typical point is contained by the union of the LOS segments on that line. Therefore, the linear fraction in question is given by

$$\nu_1 \left( \bigcup_{X_i \in \Phi} S_{X_i} \right) = 1 - P \left( 0 \notin \bigcup_{X_i \in \Phi} S_{X_i} \right) = 1 - \exp \left( -2\mu \int_0^\infty \exp \left( \frac{-x}{\gamma} \right) \, dx \right) = 1 - \exp \left( -2\mu \gamma \right).$$

As a result, based on the 1-D-based additive approximation, we have

$$\nu(C_{RSU}) \approx \lim_{r \to \infty} \frac{\eta \pi \lambda r^2 (1 - \exp(-2\mu \gamma))}{\pi r^2} = \eta \lambda_1 (1 - \exp(-2\mu \gamma)).$$

Fig. 7 and 8 illustrate the difference between the exact analysis and the 1-D approximation.

Example 1. The error $\Gamma$ between the 1-D additive approximation and the exact analysis is given by

$$\Gamma = \left| \left( 1 - \exp \left( -2\lambda_1 \eta (1 - \exp(-2\mu \gamma)) \right) \right) - \left( \lambda \eta (1 - \exp(-2\mu \gamma)) \right) \right|.$$ 

Fig. 9 illustrates the error between the 1-D additive approximation and the exact analysis. For instance, when $\lambda_1 = 10/\text{km}$, $\mu = 7/\text{km}$, $\eta = 100$ meters, and $\gamma = 100$ meters, we have $\nu_1 \left( \bigcup_{X_i \in \Phi} S_{X_i} \right) = 1 - \exp\left( -2\lambda_1 \eta (1 - \exp(-2\mu \gamma)) \right) \approx 0.85$. On the other hand, based on the 1-D approximation, we have $\nu(C_{RSU}) \approx \lambda \eta \left( 1 - \exp(-2\mu \gamma) \right) \approx 0.95$. The 1-D additive approximation does not take into account for the fact that the LOS coverage regions may overlap in space. (See fig. 8). In addition, as the size of the LOS coverage region increases, the overlapping area also increases, amplifying the error.

IV. CONCLUSION

This paper provides an analytical framework to assess the impact of introducing relays in the LOS-critical vehicular applications such as mmWave V2X communications.
The proposed model accurately incorporates the geometric relationship between RSUs and vehicles in the modeling of their LOS coverage. Using the mean area fraction, we analyze the RSU coverage and the RSU-plus-relay coverage. We show, for practical values of network parameters, that even a random selection of the vehicle relay will increase the LOS region where any vehicular or pedestrian users are in LOS w.r.t. any transmitters including RSUs and relays.

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