Simulation of Turbulent Flow with High Reynolds Number over an Elliptical Cylinder

M K Rawat
Assistant Professor, Department of Mechanical Engineering, GLA University, Mathura, Uttar Pradesh, INDIA-281001
manish.rawat@gla.ac.in

Abstract. A complex turbulent flow with high Reynolds number (Re= 5 × 10^5, 1 × 10^6, 2 × 10^6 and 3.6 × 10^6) over an elliptical cylinder, is investigated using 2D URANS equations with a standard k-ε turbulence model. The present study compares the effect of various upstream velocity profiles on the variations in flow characteristics along the surface of an elliptical cylinder having variable axis ratios. The comparison of results with the published data for a circular cylinder in cross flow shows the importance of axis ratio and other velocity profiles to reduce the drag force and variation in pressure distribution. Drag force over an elliptical cylinder is found to be lower than that of a circular cylinder. The drag force is seen to increase with increasing slenderness ratio. The drag coefficient of an elliptical cylinder with an axis ratio 0.6 is found to reduce by half as compared to that of a circular cylinder. The influence of slenderness ratio and velocity profile over back pressure recovery, flow separation and shear stress distribution is observed for an elliptical cylinder.

Keywords: Elliptical cylinder; URANS; Turbulent flow; Plain shear, Parabolic and triangular velocity profile etc.

1. Introduction: The flow separation and wakes formation phenomenon on bluff bodies is one of the most studied aspects in fluid mechanics due to its significance in fluid flow and its importance in practical aerodynamic and hydrodynamic applications. Most of the studies on unsteady flow have been conducted with symmetric flow condition over a circular cylinder. Some of them are Catalano et al. [1], Ong et al. [2], and Tutar et al. [3]. However in most of the real engineering applications like wings, submarines, missiles, and rotor blades, modeling flow as a flow over a circular cylinder with uniform velocity profile
may lead to large discrepancies. In these applications elliptical shape offers less resistance to the flow and higher structural strength as compared to circular cylinder. For such type of flows, modelling of flow over an elliptical cylinder would rather be more accurate in which parameter like axis ratio, velocity distribution at upstream side and Reynolds number can influence the pressure distribution, wake formation, skin friction and coefficient of drag.

The plain shear velocity, parabolic velocity and triangular velocity are various velocity profiles which can affect the flow parameter over the surface of elliptical cylinder and these profile form when flow takes place at high speed with complex flow conditions like movement of turbine blade at high speed with variable direction. Complicated nature of flow restricts the experimental and theoretical analysis to laminar flow (less Re). However, CFD provide a promising approach to analyze such type of problems at high Reynolds number. Fig. 1. Shows the different types of velocity profile

![Velocity profiles](image)

(a) parabolic velocity (b) triangular velocity (c) plain shear velocity

Flow simulations for elliptical cylinders with variation in axis ratio ranging from zero representing a flat plate to unity representing a circular cylinder provide a generic analysis model for a larger class of bluff bodies. Among the few numerical simulation results over elliptic cylinder reported in the literature are Y. Mochimaru [4], Z. Li et al.[5] and J. Moon et al. [6]. Y. Mochimaru [4] predicted the coefficient of drag and flow streamline with Reynolds number (10^5 ) and different axis ratio. Z. Li et al. [5] applied K ω-SST model to evaluate drag coefficient for Reynolds number up to 10^6 at different axis ratio. J. Moon et al [6], applied large eddy simulation to evaluate force coefficients and Nusselt number for the flow over an elliptical cylinder at Re = 3000. Catalano et al. [1] and Singh et al.[7] applied LES as well as URANS over a circular cylinder in highly turbulent flow. Many other researchers [12-18] also evaluate flow characteristics on different type of structures.

The present study aims to develop a generic model by introducing different velocity profiles with variable axis ratio as a key parameter which affects the flow behaviour over the external surface of a larger class of bluff bodies ranging from a flat plate to circular cylinder. In this investigation, elliptical cylinder is subjected to a cross flow with different velocity profile such as plain shear, parabolic and
triangular velocity at inlet preserving the same mass flow rate as that of uniform flow conditions at inlet. The predictions of various flow characteristics such as drag coefficient, pressure coefficient and separation angle are compared with others result. The validation of present model was done against the data available for a circular cylinder and agreement is found to be plausible.

2. Mathematical formulation

The rectangular computational domain along with its dimensions and the boundary conditions used are shown in Fig. 2. The inlet boundary is at a distance 8D (D is the diameter of cylinder) upstream while outlet boundary is located at distance 22D downstream from the centre of the cylinder to mitigate the far field effects from the main flow region over an elliptical cylinder. Two dimensional meshing of the model is used for conducting numerical simulations. 400 grid points are taken on the circumferential direction of the cylinder while 100 grid points are used in the direction normal to wall.

The Reynolds-averaged equations for mass and momentum conservation are given by

\[ \frac{\partial v_i}{\partial x_i} = 0 \]

\[ \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = -\frac{1}{\rho} \left( \frac{\partial p}{\partial x_i} \right) + \nu \frac{\partial^2 v_i}{\partial x_j^2} + \frac{\partial \bar{v}_i \bar{v}_j}{\partial x_j} \]

Where i, j =1, 2. Here horizontal and vertical directions are denoted by \( x_1 \) and \( x_2 \) respectively; Mean velocity components are \( v_1 \) and \( v_2 \); dynamic pressure, time and density are denoted by \( p \), \( t \) and \( \rho \) respectively.

The Reynolds stress component, \( \bar{v}_i \bar{v}_j \), is expressed by using the Boussinesq approximation,
\[-\bar{v}_i \bar{v}_j = \nu_T \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}\]

Where \( k \) and \( \delta_{ij} \) denotes turbulent kinetic energy and kronecker delta function respectively.

The \( k \)-\( \varepsilon \) Method is selected for turbulence modelling due to its simplicity, hardware requirement of computing machine and capability of prediction for highly turbulent flow. The \( k \) and \( \varepsilon \) equations are given by:

\[
\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_i} \left( \nu_T \frac{\partial k}{\partial x_i} \right) + \nu_T \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - \varepsilon
\]

\[
\frac{\partial \varepsilon}{\partial t} + u_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_i} \left( \nu_T \frac{\partial \varepsilon}{\partial x_i} \right) + C_1 \frac{\varepsilon}{k} \nu_T \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - C_2 \frac{\varepsilon^2}{k}
\]

Where \( \nu_T = C_\mu (k^2 / \varepsilon) \).

\( (C_\mu = 0.09, \sigma_\varepsilon = 1.3, \sigma_k = 1.0, C_1 = 1.44, C_2 = 1.92) \).

3. Results and discussion

In this investigation, the simulations are carried out with standard \( k \)-\( \varepsilon \) model for \( Re = 0.5 \times 10^6, 1 \times 10^6, 2 \times 10^6 \) and \( 3.6 \times 10^6 \) using commercial CFD code, Fluent. A second order finite volume method is used to discretize the computational domain and SIMPLE pressure-correction algorithm is employed to find the solution of problem numerically. The computations have been performed in order to evaluate the condition through which shear drag and pressure drag on a surface can be minimized to a suitable limit and to evaluate pressure distribution and skin friction coefficient for different axis ratio and velocity profile.

To validate our model, drag coefficient, pressure distribution and skin friction coefficient for a circular cylinder with uniform velocity at upstream side are calculated and compared with the results of other researchers. The overall drag coefficient is plotted against Reynolds number is depicted in Fig. 3.

Fig. 3: Variation in drag coefficient.

(—) Achenbach (1968); (○) LES; (■) URANS ( ■ ) present simulation.
The drag coefficient predictions for Reynolds Number between $5 \times 10^5$ and $2 \times 10^6$ agree with the other studies. At $3.6 \times 10^6$ the average drag coefficient computed by present simulation decreases slightly, whereas a slight increase was reported by Catalano et al. [1]. Table 1 shows the comparison of average drag coefficient at $Re = 1 \times 10^6$. The prediction of drag coefficient from this study shows the acceptability of present model to evaluate the drag coefficient with different inlet conditions and axis ratios of the immersed shape.

| Re. = $1 \times 10^6$ | $C_D$ |
|----------------------|------|
| Present study        | 0.39 |
| Ong et al. [2]       | 0.5174 |
| Catalano et al. [1] 3D LES & URANS | 0.31–0.35 & 0.41 |
| Experimental Result  | 0.21–0.63 |

The prediction of skin friction coefficient $C_f = \tau / (0.5 \rho U_e^2)$ on the surface of a circular cylinder with that of Catalano et al. [1] are shown in Fig.4 for $Re = 5 \times 10^5$, $1 \times 10^6$, $2 \times 10^6$ and $3.6 \times 10^6$. The distribution of $C_f$ around the cylinder surface obtained from present data exhibit a similar trend as that of Catalano et al. [1]; the peak magnitude of $C_f$ increase as the value of Reynolds number decreases but lower value of maximum magnitude of $C_f$ for same Reynolds number is observed as compared to 3D LES.
results. Further, the flow separation angle ($\theta_s$) is 118° at Re = $3.6 \times 10^6$, which is very close to the measured value of 115° as reported by Achenbach [10].

Fig. 5 shows the comparison of pressure distribution $C_p = (P - P_\infty)/0.5 \rho U_\infty^2$ at Re = $1 \times 10^6$ on a circular cylinder. The curve obtained by present study captures the trend of others results with a minor off-shoot at the back of the cylinder indicating a negative Cp. Therefore, it is fair to state that URANS with standard k-ε model is capable of predicting the flow parameters for a fully developed turbulent flow in supercritical and upper-transition flow regime.

![Graph of Coefficient of Pressure Distribution at Cylinder Surface at Re = 1 \times 10^6](image)

**Fig. 5. Coefficient of pressure distribution at cylinder surface at Re = 1 \times 10^6**

4. **Parametric study**

With a view to investigate the response of an oval shaped cylinder vis-a-vis a circular cylinder subjected to turbulent cross-flow with various conditions of oncoming flow, numerical computations are carried out to evaluate drag coefficient, pressure coefficient and skin friction coefficient for a range spanning from super-critical Reynolds number to Reynolds number in upper-critical range for an elliptical cylinder with an axis ratios of 0.4, 0.6 and 0.8 at $5 \times 10^5 \leq Re \leq 3.6 \times 10^6$ and results are analyzed with reference to a circular cylinder ($\lambda_o = 1$).
Fig. 5 shows variation in drag coefficient with discrete value of axis ratio and velocity profile. For streamlined shapes, drag coefficient, $C_D$ decreases with decrease in axis ratio. Significant reductions in drag coefficient can be achieved for $\lambda_o \leq 0.6$ as compared to that of the circular cylinder. For an elliptical cylinder with $\lambda_o = 0.6$ at $Re = 1 \times 10^6$, the 45% reduction in drag coefficient is obtained as compared to the circular cylinder. The drag coefficient has almost equal magnitude for plain shear and uniform velocity profile but very small in magnitude as compare to that obtained for parabolic velocity and triangular velocity profile. The coefficient of drag is maximum for triangular velocity profile at any value of axis ratio.

Fig. 6 shows the pressure coefficient over an elliptical cylinder. All the predictions shows the back pressure recovery with the decrement in axis ratio (the cylinder become slimmer). Elliptical cylinders with axis ratio equal to one is having the negative back pressure coefficient in the Reynolds
number range considered but streamlined cylinders with smaller axis ratio can obtain the positive back pressure coefficient when cylinder is exposed to a flow with very high value of turbulence. Uniformity in pressure distribution can be achieved for smaller axis ratios and high Reynolds number that will also result in the reduction in drag coefficient.

The effect of initial velocity profiles on pressure coefficient over the external surface of an elliptical cylinder is shown in Fig. 7. The coefficient of pressure at the back side of the cylinder is higher for plain shear velocity profile than that for remaining velocity profile. The plain shear velocity profile shows recovery in the back pressure coefficient due to the shifting of separation point and suppression of vortex shedding. In case of plain shear velocity profile, the velocity difference between upper and lower sides of the cylinder increased with the Reynolds number, this difference is responsible for shifting of separation point to downstream side of the result. As a result, suppression of vortex shedding takes place.
Fig. 8. variation in skin friction coefficient distribution with axis ratio at \( \text{Re} = 1 \times 10^6 \)

Fig. 8 shows the distribution of skin friction coefficient at the surface of an elliptical cylinder for various axis-ratios. Flow separation-angle increases as the cylinder axis ratio decreases. For e.g. separation angle increases from 110° to 157° as the axis ratio decrease from 1 to 0.4 for uniform velocity profile.

Fig. 9. skin friction coefficient variation with velocity profiles
Fig. 9 shows the influence of initial velocity profile on the skin friction coefficient of an elliptical cylinder. All the predictions show that the higher skin friction coefficient can be obtained for plain shear velocity profile than for the uniform velocity. The variation in flow separation angle subjected to the two types of velocity profiles of oncoming flow has also been analyzed. The predictions show that identical separation angles are observed on the two sides of the cylinder axis in case of uniform, triangular and parabolic velocity while a skewed separation angles are observed in case of plain shear velocity.

5. Conclusion

The present study demonstrated the effects of the change in running condition with axis ratios ranging from 0.4 to 1 over an elliptical cylinder in a cross-flow with various velocity profiles at inlet. Drag coefficient is very less for uniform and plain shear velocity profile as the axis ratio decreases from unity to 0.4.

The back pressure recovery with decreasing axis ratios and flow subjected to plain shear velocity profile at inlet for the range of Reynolds numbers considered is significant owing to its relation to reduction of drag force. The forward shift in separation angle with decreasing axis ratio is observed in all the predictions. The separation angles were found identical on the lower and upper side of the cylinder for uniform, parabolic and triangular velocity but for plain shear velocity the separation point moved downstream on the upper side, and moved upstream on the lower side of the cylinder. Overall the present model is very conducive for the analysis of highly complex flows past an elliptical cylinder in the supercritical and upper transition flow regimes.

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