A 1″ Telescope: The Optimal Approach to Bright-Star Planetary Transits

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ABSTRACT

Planetary transits of bright stars, $V < 10$, offer the best opportunity for detailed studies of extra-solar planets, such as are already being carried out for HD209458b. Since these stars are rare, they should be searched over the entire sky. In the limits of zero read-out time, zero sky noise, and perfect optics, the sensitivity of an all-sky survey is independent of telescope aperture: for fixed detector size and focal ratio, the greater light-gathering power of larger telescopes is exactly cancelled by their reduced field of view. Finite read-out times strongly favor smaller telescopes because exposures are longer so a smaller fraction of time is wasted on readout. However, if the aperture is too small, the sky noise in one pixel exceeds the stellar flux and the field of view becomes so large that optical distortions become unmanageable. We find that the optimal aperture is about 1″. A one-year survey using such a 1″ telescope could detect essentially all hot-jupiter transits of $V < 10$ stars observable from a given site.

Subject headings: techniques: photometric – surveys – planetary systems

1. Introduction

In Pepper, Gould & DePoy (astro-ph/0208042), we argued that all-sky surveys are the best way to find planets transiting bright $V < 10$ stars. Such transits offer the best opportunity for detailed studies of planets. We showed that the number of systems probed is given by

$$N_p = \frac{4}{3} \pi n \eta d_0^3 \left( \frac{R_0}{a_0} \right) \left( \frac{L}{L_0} \right)^{3/2} \left( \frac{R}{R_0} \right)^{-7/2} \left( \frac{a}{a_0} \right)^{-5/2} \left( \frac{r}{r_0} \right)^6,$$  (1)
where \( d_0 \) is the maximum distance at which an \( i = 90^\circ \) transit can be detected for a star of luminosity \( L_0 \), radius \( R_0 \), with a planet at semi-major axis \( a_0 \) and radius \( r_0 \), and where \( n \) is the local number density of such stars and \( \eta = 0.719 \) is a numerical factor.

We then showed that the sensitivity of a given survey will depend almost entirely on \( \gamma \), the number of photons collected from a fiducial star of some fixed designated magnitude, which we arbitrarily chose to be \( V = 10 \). That is, it will not depend on the details of the all-sky observing program. We then normalized equation (1) in terms of \( \gamma \),

\[
N_p = 600F(M_V) \left( \frac{a}{a_0} \right)^{-5/2} \left( \frac{r}{r_0} \right)^6 \left( \frac{\gamma}{\gamma_0} \right)^{3/2} \left( \frac{\Delta \chi^2_{\text{min}}}{36} \right)^{-3/2}
\]

where we adopted \( \gamma_0 = 2 \times 10^7 \), \( a_0 = 20 R_\odot \), \( r_0 = 0.10 R_\odot \), and where we have made our evaluation at \( M_V = 5 \) (i.e. \( R = 0.97 R_\odot \), \( V_{\text{max}} = 10 \), \( d_0 = 100 \text{ pc} \), and \( n = 0.004 \text{ pc}^{-3} \)). Here, \( F(M_V) \) is a function, which is shown in Figure 1, and \( \Delta \chi^2_{\text{min}} \) is the minimum \( \chi^2 \) improvement of a transit-model relative to a constant-flux model, which is set to avoid excessive false positives. Note that \( \gamma_0 = 2 \times 10^7 \) corresponds approximately to 1000 20-second exposures with a 5 cm telescope and a broadened \((V + R)\) type filter for one \( V = 10 \) mag fiducial star.

Our goal was to apply this formula to the problem of telescope design in a subsequent paper. However, in light of the referee report, we decided to append our work on telescope design as an additional chapter of the original paper. The following is that chapter.

2. Implications for Telescope Design

We now apply the general analysis of astro-ph/0208042 to the problem of optimizing telescope design for quickly locating a “large” number of bright transiting systems. Since only one such system is now known, we define “large” as \( \mathcal{O}(10) \). From equation (2) and the 0.75% frequency of hot jupiters measured from RV surveys, there are roughly five such systems to be discovered over the whole sky per magnitude for \( V_{\text{max}} = 10 \). Hence, from Figure 1, of order 15 are to be discovered from all spectral types. It would, of course, be possible to discover even more by going fainter, but setting this relatively bright limit is advisable for three reasons. First, as we argued in the introduction, the brightest transits are the most interesting scientifically, and most of the transits detected in any survey will be close to the magnitude limit. Second, as we discuss below, a wide range, \( \Delta V = V_{\text{max}} - V_{\text{min}} \), of survey sensitivity can only be achieved at considerable cost to the observing efficiency. Hence, if high efficiency is to be maintained, setting \( V_{\text{max}} \) fainter means eliminating the brightest (most interesting) systems. Third, at \( V_{\text{max}} = 10 \), we are already reaching distances of 100 pc for G stars. Hence the number of transits observed in fainter surveys will not
continue to grow as $d^3_{0}$ as in equation (1).

In previous sections, we ignored the loss of sensitivity to systems that are brighter than $V_{\text{min}}$, which is set by saturation of the detector (or more precisely, by the flux at which detector non-linearities can no longer be accurately calibrated). This fraction is $10^{-0.6\Delta V}$, or 6% for $\Delta V = 2$, which we therefore adopt as a sensible goal. That is, we wish to optimize the telescope design for,

$$8 = V_{\text{min}} < V < V_{\text{max}} = 10.$$  

(3)

(In any event, essentially all stars $V < 8$ have already been surveyed for XSPs using RV, and the problem of determining which among the planet-bearers have transits is trivial compared to the problem of conducting an all-sky photometric variability survey.)

Optimization means maximizing the photon collection rate, $\gamma/T$, where $T$ is the duration of the experiment and, again, $\gamma$ is the total number of photons collected from a fiducial $V = 10$ mag star. Explicitly,

$$\gamma = K \mathcal{E} T D^2 \frac{(\Delta \theta)^2}{4\pi},$$  

(4)

where $\Delta \theta$ is the angular size of the detector, $D$ is the diameter of the primary-optic, $\mathcal{E}$ is the fraction of the time actually spent exposing, and $K$ is a constant that depends on the telescope, filter, and detector throughput. For our calculations, we assume $K = K_{0} \equiv 40 e^{-} \text{cm}^{-2}\text{s}$, which is appropriate for a broad $(V + R)$ filter and the fiducial $V = 10$ mag star. The design problems are brought into sharper relief by noting that $\Delta \theta = \mathcal{L}/DF$, where $\mathcal{L}$ is the linear size of the detector and $F$ is the focal ratio, or $f/#$, of the optics. Equation (4) then becomes

$$\gamma = \frac{K \mathcal{E} \mathcal{L}^2 T}{4\pi F^2}.$$  

(5)

That is, almost regardless of other characteristics of the system, the camera should be made as fast as possible. We will adopt $F = 2$, beyond which it becomes substantially more difficult to design the optics. A more remarkable feature of equation (5) is that all explicit dependence on the size of the primary optic has vanished: a 1" telescope and an 8m telescope would appear equally good! Actually, as we now show, there is a hidden dependence of $\mathcal{E}$ on $D$, which favors small telescopes.

The global efficiency $\mathcal{E}$ can be broken down into two factors, $\mathcal{E} = \mathcal{E}_{0}\mathcal{E}_{S}$, where $\mathcal{E}_{0}$ is the fraction of time available for observing (i.e., during which the sky is dark, the weather is good, etc.), and $\mathcal{E}_{S}$ is the fraction of this available observing time that the shutter is actually open. The first factor is not affected by telescope design and so will be ignored for the moment. The second factor should be maximized. Since the readout time is fixed, the smaller is the telescope aperture, the longer can be the exposures before a $V_{\text{min}} = 8$ mag star
saturates, and so the smaller fraction of time is lost to read-out. For large-format detectors, the pixel size is typically $\Delta x_p = 15 \mu m$, for which significant non-linearities set in at about $60,000 \, e^-$. Hence, exposure times are

$$T_{\text{exp}} = 38 \, s \frac{K_0}{K} \left( \frac{D}{2.5 \, \text{cm}} \right)^{-2}$$

(6)

That is, the exposure time for a 1" “telescope” is already of the order of typical read-out times for large-format detectors. Clearly, smaller is better, but are there constraints from going too small?

One potential constraint comes from sky noise. To stay within the photon-noise limited regime, which has been assumed in all of our calculations, the sky inside one pixel should be at least one magnitude fainter than $V_{\text{max}}$, and preferably two mags fainter. Assuming the sky brightness in our broadening passband reaches a maximum of $V = 19.7 \, \text{arcsec}^{-2}$, and again assuming $\Delta x_p = 15 \mu m$ pixels, the sky in one pixel is

$$V_{\text{sky}} = 10.8 + 5 \log \left( \frac{D}{2.5 \, \text{cm}} \frac{F}{2} \right).$$

(7)

Hence, the sky is a bit bright for a 1" telescope, but appears quite satisfactory for a 2".

Finally, one must be careful that the field of view is not too large, or the focal-plane distortions at its edges will be difficult (and expensive) to correct. For example, for a $4k \times 4k$ detector with $\Delta x_p = 15 \mu m$ pixels, $\Delta \theta = 68^\circ(D/2.5 \, \text{cm})^{-1}(F/2)^{-1}$. At 1", this is probably too large to correct at reasonable expense. With this size detector, both sky-noise considerations, and problems in optics design argue for a 1.5" telescope. However, from equation (6), such a “large” telescope will most likely be dominated by read-out time. In summary, we conclude that a 1" to 2" telescope equipped with a $4k \times 4k$, $L = 6 \, \text{cm}$ detector and with $F = 2$ focal ratio is optimal for this observing problem. Among all existing transit programs of which we are aware, the WASP telescope ($D = 2.5'$ lens, $F = 2.8$ focal ratio, $2k \times 2k$, $L = 3 \, \text{cm}$ detector, Street et al. 2002) comes closest to meeting these design specifications.

Adopting $\gamma = \gamma_0$, $K = K_0$, $\mathcal{E}_0 = 20\%$, $\mathcal{E}_S = 50\%$, $L = 6 \, \text{cm}$, and $F = 2$, we find from equation (5) that the required duration of the experiment using our optimally-designed telescope is $T = 3 \, \text{months}$. Clearly, roughly a year is required just to get around the sky. The S/N acquired during such a year-long search would therefore roughly double the minimum requirements calculated here.

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REFERENCES

Street, R. A. et al. 2002, in ASP Conf. Ser., Scientific Frontiers in Research on Extrasolar Planets, eds. D. Deming and S. Seager, (San Francisco: ASP), in press

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Fig. 1.— The relative number of potential transiting systems $F(M_V)$ probed for fixed planetary radius $r$ and semi-major axis $a$ as a function of $M_V$. The solid line applies to a uniform distribution of stars – to model the immediate solar neighborhood. The dashed line applies to a thin disk – to model a search of a large portion of the Galactic disk. The distributions are arbitrarily scaled such that $F(M_V = 5) = 1$. 