Chondrule Accretion with a Growing Protoplanet

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Abstract

Chondrules are primitive materials in the solar system. They were formed in about the first 3 Myr of the solar system’s history. This timescale is longer than that of Mars formation, and it is conceivable that protoplanets, planetesimals, and chondrules might have existed simultaneously in the solar nebula. Due to protoplanets’ perturbation on the planetesimal dynamics and chondrule accretion on them, all the formed chondrules are unlikely to be accreted by the planetesimals. We investigate the amount of chondrules accreted by planetesimals in such a condition. We assume that a protoplanet is in oligarchic growth, and we perform analytical calculations of chondrule accretion by both a protoplanet and planetesimals. Through the oligarchic growth stage, planetesimals accrete about half of the formed chondrules. The smallest planetesimals get the largest amount of chondrules, compared with the amount accreted by more massive planetesimals. We perform a parameter study and find that this fraction is not greatly changed for a wide range of parameter sets.

Key words: meteorites, meteors, meteoroids – minor planets, asteroids: general – planets and satellites: formation – planets and satellites: terrestrial planets

1. Introduction

Chondrules are mm-sized spherical particles found in chondritic meteorites. Their properties suggest that their precursors were melted by flash-heating events in gas nebulae (e.g., Scott & Krot 2005; Scott 2007). They make up ~20%–80% of most chondrites’ volume, and their formation started at the time of the formation of Ca-Al rich inclusions (CAIs) and continued for at least ∼3 Myr (Connelly et al. 2012). This means that these heating events were common in the first 3 Myr of the solar system’s formation.

Several formation mechanisms for chondrules have been proposed (e.g., Desch et al. 2012). These include the X-wind model (e.g., Shu et al. 1996, 2001), nebular lightning model (e.g., Desch & Cuzzi 2000; Muranushi 2010), nebula shock model (e.g., Iida et al. 2001; Mann et al. 2016), and impact-jetting model (Johnson et al. 2015; Hasegawa et al. 2016a, 2016b). These models can reproduce some petrologic and geochemical aspects of chondrules (Rubin 2000). The models also must explain chondrule abundance.

The amount of chondrules that can be inferred from chondrites is not equal to the produced amount of chondrules. This is not only because the present asteroid-belt mass is much smaller than that of the primordial one, but also because it is unclear how the parent bodies of chondrites formed. If the currently available chondrites were generated as fragments of massive bodies (e.g., DeMeo et al. 2015), then one can think of the following possibility (Hasegawa et al. 2016a): even if planetesimals originally did not contain any chondritic materials, they could accrete chondrules as the chondrules formed over time. In this case, the planetesimals could have a chondrule-rich surface layer. Following the subsequent collisional cascade, such a surface layer could break into chondrites. In order to examine this possibility, it is important to investigate how chondrules were accreted by massive bodies such as planetesimals and protoplanets.

Recent studies (Ormel & Klahr 2010; Lambrechts & Johansen 2012) have investigated the accretion process of small particles onto massive bodies in laminar disk gas, known as pebble accretion. The particles that are strongly affected by gas drag, such as chondrules, boulders, or fragments of larger bodies, are efficiently accreted by massive planetesimals and protoplanets. Accretion of chondrules through pebble accretion was studied by Johansen et al. (2015). They considered the possibility that planetesimals are born and grow in an ocean of chondrules. However, chondrules were formed about 3 Myr after CAI formation, and it is conceivable that planet formation actively took place at that time. In fact, Dauphas & Pourmand (2011) suggested that the timescale of Mars formation is 1.8+0.3−0.9 Myr or less after CAI formation. If such a body is in the planetesimal swarm, which can be the parent bodies of chondrites, runaway and oligarchic growth of the body occurs (Wetherill & Stewart 1989; Kokubo & Ida 1996, 1998). It is therefore crucial to explore how chondrule formation and accretion occur simultaneously with the growth of protoplanets. The accretion efficiency of chondrules by planetesimals decreases when protoplanets affect the dynamics of the planetesimals (Levison et al. 2015). This is because planetesimals tend to be kicked out of the pebble sea due to the gravitational interaction with protoplanets, which increases both the eccentricity and the inclination of the planetesimals. Hasegawa et al. (2016a) studied the pebble accretion of chondrules by planetesimals, assuming that chondrules are formed by impact jetting. In this formation scenario, chondrule-forming impacts are realized when protoplanets are present in planetesimal disks (Johnson et al. 2015). Hasegawa et al. (2016a) found that there are certain ranges of parameters that satisfy the timescale of chondrule formation, the magnetic field strength estimated from the Semarkona ordinary chondrite (Fu et al. 2014), and the condition of efficient pebble accretion. However, the accretion efficiency of chondrules onto planetesimals and a protoplanet was not directly calculated in previous studies.

In this paper, we investigate chondrule accretion under the presence of a growing protoplanet. Since the timescale of
runaway growth is much smaller than that of chondrule formation, we consider that a protoplanet is already in the oligarchic and it is in a swarm of planetesimals. We adopt the impact-jetting model as a chondrule-forming process in the fiducial model. We calculate the growth of a protoplanet analytically. The chondrule accretion rates by a protoplanet and planetesimals are also calculated in each timestep as the protoplanet grows. Moreover, we obtain the mass of the accreted chondrules. Our model is described in detail in Section 2. In Section 3, we present the results, showing both the timescale of chondrule accretion by a protoplanet and planetesimals and the amount of chondrules accreted by them. In Section 4, we discuss the implications of chondrule accretion and physical processes that are not included in this paper. Finally, Section 5 contains our conclusions.

2. Model

Our models are constituted from the combination of a disk model, chondrule formation model, and chondrule accretion model. We consider the mass of the smallest planetesimals \( m_{pl,\text{min}} \), an orbital radius \( r \), the timescale of gas depletion \( \tau_g \), and the accretion enhancement factor \( f_{acc} \) as parameters. In our fiducial model, \( m_{pl,\text{min}} = 10^{23} \text{ g} \) planetesimals are located at \( r = 2 \text{ au} \), the gas density is constant with time \( \tau_g = \infty \), and \( f_{acc} = 1 \). This set of parameters is adopted because the timescale of chondrule formation by the impact-jetting process is consistent with data from chondrites (Hasegawa et al. 2016b) and chondrules can be accreted efficiently by planetesimals (Hasegawa et al. 2016a). While the size of \( 10^{23} \text{ g} \) planetesimals may be too large (about 230 km in radius with a material density of \( 2 \text{ g cm}^{-3} \)) for the present asteroids Morbidelli et al. (2009) showed that the size distribution of asteroids can be reproduced when the initial planetesimals are larger than 100 km in size. Table 1 summarizes the important physical quantities.

2.1. Disk Model

First, we introduce a disk model that consists of dust and gas. We adopt a power-law disk model similar to the minimum-mass solar nebula model (Hayashi 1981). Following Kokubo & Ida (2000) and Hasegawa et al. (2016b), we give the surface densities of dust \( \Sigma_d \) and gas \( \Sigma_g \) as

\[
\Sigma_d = 10 \times f_d \left( \frac{r}{1 \text{ au}} \right)^{-3/2} \text{ g cm}^{-2},
\]

\[
\Sigma_g = 2400 \times f_d \left( \frac{r}{1 \text{ au}} \right)^{-3/2} \text{ g cm}^{-2},
\]

where \( f_d \) is an increment factor. In this paper, \( f_d \) is a parameter. Reflecting the results of Hasegawa et al. (2016b), we consider a massive disk case, \( f_d = 3 \), in our fiducial model. The stellar mass is \( 1 M_\odot \). Under the optically thin limit, the disk temperature is given by

\[
T = 280 \left( \frac{r}{1 \text{ au}} \right)^{-1/2} \text{ K},
\]

and the sound speed \( c_s \), gas pressure scale height \( h_g \), and density of gas \( \rho_g \) are

\[
c_s = 1.1 \times 10^3 \left( \frac{r}{1 \text{ au}} \right)^{-1/4} \text{ cm s}^{-1},
\]

\[
h_g = 4.7 \times 10^{-2} \left( \frac{r}{1 \text{ au}} \right)^{5/4} \text{ au},
\]

\[
\rho_g = 2 \times 10^{-9} f_d \left( \frac{r}{1 \text{ au}} \right)^{-11/4} \text{ g cm}^{-3},
\]

respectively. In some calculations, we consider gas depletion. For these, the timescale of gas depletion \( \tau_g \) and \( \Sigma_g \) and \( \rho_g \) are multiplied by \( \exp(-t/\tau_g) \), where \( t \) is time (cf. Equations (2) and (6)). In disks, the gas component moves with a sub-Keplerian velocity. The velocity can be written as \( (1 - \eta) v_K \), where \( v_K \) is...

| Symbol | Meaning                              | Value       |
|--------|--------------------------------------|-------------|
| \( \rho_g \) | Gas volume density at the disk midplane | ...         |
| \( f_d \) | Increment factor of \( \rho_g \) and \( \Sigma_d \) | ...         |
| \( h_g \) | Gas pressure scale height            | ...         |
| \( \tau_g \) | Timescale of disk gas depletion      | ...         |
| \( r \)  | Orbital radius                      | ...         |
| \( T_K \) | Orbital period                      | ...         |
| \( M \)  | Mass of the protoplanet             | ...         |
| \( \tau_{pr} \) | Timescale of protoplanet growth     | ...         |
| \( t_{iso} \) | Time until the protoplanet reaches the isolation mass | ... |
| \( M_{iso} \) | Isolation mass of the protoplanet   | ...         |
| \( M_{ac} \) | Mass of the protoplanet when impact velocities exceed 2.5 km s\(^{-1}\) | ... |
| \( n_{pl} \) | Mass of the protoplanet when oligarchic growth begins | ... |
| \( m_{pl} \) | Mass of the planetesimals           | ...         |
| \( R_{pl} \) | Radius of the planetesimals         | ...         |
| \( \rho_{pl} \) | Material density of the planetesimals | ... |
| \( \epsilon_{pl} \) | Eccentricity of the planetesimals in oligarchic growth | ... |
| \( n_{pl} \) | Number of planetesimals             | ...         |
| \( M_{ch} \) | Mass of the field chondrules        | ...         |
| \( r_{ch} \) | Characteristic size of chondrules   | 1 mm        |
| \( \rho_c \) | Bulk density of the chondrules      | 3.3 g cm\(^{-3}\) |
| \( \rho_{ch} \) | Spatial density of the chondrules in the protoplanetary disk | ... |
| \( h_{ch} \) | Scale height of the chondrules      | ...         |
| \( \tau_{stop} \) | Timescale of gas drag on the chondrules | ... |
| \( F_{ch} \) | Mass fraction of planetesimals that can eventually generate chondrules via impact jetting | 10\(^{-2}\) |
| \( r_H \) | Hill radius                         | ...         |
| \( r_B \) | Bondi radius                        | ...         |
| \( M_t \) | Transition mass                     | ...         |
| \( f_{acc} \) | Increment factor for chondrule accretion by planetesimals | ... |
| \( M_{acc} \) | Mass of the accreted chondrules     | ...         |
| \( \tau_{acc} \) | Timescale of chondrule accretion    | ...         |
| \( \tau_n \) | Timescale of chondrules across \( r_B \) | ... |
| \( f_{i,i} \) | Mass fraction of the chondrules accreted by the planetesimals in the \( i \)th mass bin | ... |
| \( f_{m,ch} \) | Mass fraction of the chondrules with respect to an accreting planetesimal | ... |
| \( \Delta R_{ch} \) | Thickness of the chondrule layer on a planetesimal | ... |

\[
\rho_g = 2 \times 10^{-9} f_d \left( \frac{r}{1 \text{ au}} \right)^{-11/4} \text{ g cm}^{-3},
\]
the Keplerian velocity, and

\[ \eta \simeq 1.8 \times 10^{-3} \left( \frac{r}{1 \text{ au}} \right)^{1/2} \]  

(Nakagawa et al. 1986).

The velocities of chondrules are determined by the degree of coupling with gas. In this paper, we adopt 1 mm as the chondrule size, which is a typical value for the chondrules found in chondrites (Scott & Krot 2005; Scott 2007). Provided that chondrules are subjected to the Epstein drag force, their stopping time \( \tau_{\text{stop}} \) is given by

\[ \tau_{\text{stop}} = \frac{\rho_{\text{ch}} r_{\text{ch}}}{c_{\text{s}} \rho_{\text{pl}}} \approx 5.0 \times 10^{-5} f_{\text{d}}^{-1} \left( \frac{r_{\text{ch}}}{1 \text{ mm}} \right) \left( \frac{\rho_{\text{ch}}}{3.3 \text{ g cm}^{-3}} \right) \times \left( \frac{r}{1 \text{ au}} \right)^{3/2} T_{K}, \]  

where \( r_{\text{ch}} \) is the radius of chondrules, \( \rho_{\text{ch}} \) is their material density (Adachi et al. 1976; Weidenschilling 1977), and \( T_{K} \) is the orbital period, \( T_{K} = 2\pi /\Omega_{K} \), where \( \Omega_{K} \) is a Kepler frequency. Since the stopping time is much shorter than the orbital period, chondrules are well coupled with the disk gas and are on circular orbits. This indicates that, when chondrules were formed by impact jetting, they could go out of the feeding zone of a protoplanet along with the gas motion there.

The vertical scale height of chondrules (\( h_{\text{ch}} \)) is important for their accretion (Levison et al. 2015). Since the vertical diffusion of chondrules is affected by turbulence, \( h_{\text{ch}} \) is determined by the strength of the turbulence and \( \tau_{\text{stop}} \). We use the \( \alpha_{\text{eff}} \) parameter to describe the strength of the turbulence (Shakura & Sunyaev 1973). As suggested for protoplanetary disks, magnetic fields and the resultant disk turbulence probably played an important role in the evolution of the solar nebula. For this case, \( \alpha_{\text{eff}} \) can be written as a function of the magnetic fields (e.g., Wardle 2007), where \( B, B_{\text{r}}, B_{\theta} \) are the strength, radial component, and azimuthal component of the magnetic fields of the solar nebula around the chondrule-forming region, respectively. Once the value of \( \alpha_{\text{eff}} \) is given, the scale height of chondrules can be given as (Dubrulle et al. 1995)

\[ h_{\text{ch}} = \frac{H}{\sqrt{1 + H^2 h_{\text{g}}}}, \]  

where \( H \) is a quantity derived from the condition that turbulent vertical diffusion (\( \alpha_{\text{eff}} \)) balances out with dust settling toward the midplane, which is characterized by \( \tau_{\text{stop}} \). In the actual formula, \( H \) can be written as

\[ H = \left( \frac{1}{1 + \gamma_{\text{turb}}} \right)^{1/4} \left( \frac{\alpha_{\text{eff}}}{\tau_{\text{stop}} \Omega_{K}} \right)^{1/2}, \]  

\[ = 0.29 \left( \frac{3}{1 + 2\gamma_{\text{turb}}/2} \right)^{1/4} \left( \frac{\langle B \rangle}{50 \text{ mG}} \right) \times \left( \frac{\rho_{\text{ch}}}{3.3 \text{ g cm}^{-3}} \right)^{-1/2} \left( \frac{r_{\text{ch}}}{1 \text{ mm}} \right)^{-1/2} \left( \frac{r}{1 \text{ au}} \right)^{7/8}, \]  

where \( \gamma_{\text{turb}} \) is a quantity related to the nature of the turbulence. Based on the experimental results obtained from the Semarkona ordinary chondrite, the typical value of \( \langle B \rangle \) is \( \langle B \rangle \simeq 50–540 \text{ mG} \) for the solar nebula (Fu et al. 2014).

2.2. Growth of a Protoplanet

We use the same model of protoplanetary growth as that used in Hasegawa et al. (2016b; see their Section 2). We put a protoplanet in a planetesimal swarm. The initial mass of the protoplanet is defined as

\[ M_{\text{init}} = 50 m_{\text{min}} \left( \frac{m_{\text{plan}}}{10^{-23} \text{ g}} \right)^{-2/5} \left( \frac{\Sigma_{d}}{10 \text{ g cm}^{-2}} \right)^{3/5} \times \left( \frac{r}{1 \text{ au}} \right)^{6/5}, \]  

where \( m_{\text{min}} \) is the mass of the smallest planetesimals. When a protoplanet exceeds this mass, oligarchic growth begins (Ida & Makino 1993; Kokubo & Ida 1998). The accretion rate of a protoplanet (\( dM/dt \)) is given by

\[ \frac{dM}{dt} = C_{\pi} \Sigma_{d} \left( \frac{e_{\text{pl}}^2}{r_{\text{pl}}} \right) \frac{2GM_{\text{pl}}}{r_{\text{K}}}, \]  

where \( C \) is the accretion acceleration factor, \( C = 2 \), \( R \) is the radius of the protoplanet, and \( \langle e_{\text{pl}}^2 \rangle^{1/2} \) is the root mean square equilibrium eccentricity of the planetesimals. The radius of the protoplanet is calculated with \( \rho_{\text{pl}} = 2 \text{ g cm}^{-3} \), where \( \rho_{\text{pl}} \) is the material density of the protoplanet. The equilibrium eccentricity in the oligarchic growth stage is

\[ \langle e_{\text{pl}}^2 \rangle^{1/2} \simeq 5.6 \times 10^{-2} \left( \frac{m_{\text{pl}}}{10^{-23} \text{ g}} \right)^{1/15} \left( \frac{\rho_{\text{pl}}}{2 \text{ g cm}^{-3}} \right)^{2/15} \times \left( \frac{\rho_{\text{g}}}{2 \times 10^{-9} \text{ g cm}^{-3}} \right)^{-1/5} \left( \frac{r}{1 \text{ au}} \right)^{-1/5} \times \left( \frac{M}{M_{\odot}} \right)^{1/3}, \]  

where \( \rho_{\text{pl}} \) is the material density of the planetesimals. Note that laminar disks are assumed in order to obtain Equation (14) (Kokubo & Ida 2002). We also assume that the feeding zone of the protoplanet is 10 \( r_{\text{K}} \). The growth of the protoplanet continues until its mass reaches the isolation mass (\( M_{\text{iso}} \); e.g.,
\[ M_{\text{iso}} = 0.16 M_\odot \left( \frac{\Sigma_d}{10^4 \, \text{g cm}^{-2}} \right)^{3/2} \left( \frac{r}{1 \, \text{au}} \right)^3. \] (15)

### 2.3. Chondrule Formation

In our calculations, we normally adopt the impact-jetting model as a chondrule formation model. When the impact velocity of planetesimals exceeds 2.5 km s\(^{-1}\), chondrules are formed (Johnson et al. 2015; Wakita et al. 2016, 2017). The impact velocity \(v_{\text{imp}}\) is given by \(v_{\text{imp}} = \sqrt{v_{\text{esc}}^2 + \left( (e_{\text{pl}}^2)^{1/2} v_K^2 \right)^2}\), where \(v_{\text{esc}}\) is the escape velocity. We consider protoplanet–planetesimal collisions as chondrule-forming impacts. This is because planetesimal–planetesimal collisions are much less effective in generating chondrules than protoplanet–planetesimal collisions (Hasegawa et al. 2016b). In this situation, the mass of the chondrules produced during \(dt\) becomes \(F_{\text{ch}} dM\), where \(F_{\text{ch}}\) is the mass fraction of the chondrules generated by a jetting collision. When we consider protoplanet–planetesimal collisions and a threshold velocity for chondrule-forming impacts as 2.5 km s\(^{-1}\), \(F_{\text{ch}} \approx 0.01\) (Johnson et al. 2015; Wakita et al. 2016, 2017), which is adopted in our calculations. When the mass of the protoplanet reaches the isolation mass, the mass of the cumulative formed chondrules is \(\approx 0.01 M_{\text{iso}}\).

The timescale of protoplanet growth \((\tau_{\text{pr}})\) is

\[
\tau_{\text{pr}} = f_r \frac{M}{dM/dt} = 2.7 \times 10^5 \times f_r \frac{m_{\text{pl,min}}}{\rho_{\text{pl}}} \left( \frac{M}{0.1 M_\odot} \right)^{1/3} \times \frac{r}{1 \, \text{au}} \frac{10^{24} \text{g}}{M_{\odot}} \times \frac{1}{2 \, \text{g cm}^{-3}} \times \frac{1}{\text{yr}},
\]

where \(f_r\) is a correcting factor, \(f_r = 3\) (Hasegawa et al. 2016b).

In the impact-jetting model, the timescale of chondrule formation is between when the mass of the protoplanet reaches \(M = M_{\text{esc}} \approx 0.018 M_\odot\), which is the mass at which the escape velocity becomes equal to 2.5 km s\(^{-1}\), and \(M = M_{\text{iso}}\).

Figure 1 shows the time evolutions of the mass of the protoplanet (\(M\)), the eccentricity of the smallest planetesimals \((e_{\text{pl,min}})\), and the mass of the cumulative formed chondrules \((M_{\text{ch,cum}})\) in our fiducial model. Since the eccentricities of planetesimals follow the Rayleigh distribution (Ida & Makino 1993), \(e_{\text{pl}} \approx (e_{\text{pl}}^2)^{1/2}\). The collision velocity exceeds 2.5 km s\(^{-1}\) at 3.3 \(\times 10^5\) yr. The protoplanet reaches the isolation mass, which is 1.4 \(M_\odot\), at a time of \(t_{\text{iso}} = 2.4 \times 10^6\) yr. Chondrules are formed during a span of 2 \(\times 10^6\) yr, which is consistent with the formation timescale of chondrules suggested from chondrites. The mass of the cumulative formed chondrules is 0.99 \(\times 10^{-2} M_{\text{iso}} \approx F_{\text{ch}} M_{\text{iso}}\).

We also consider two different models for chondrule formation. In these models, the production rate of chondrules is different from that of the impact-jetting model. In the first model, we assume that chondrules are formed at a constant rate during \(M_{\text{esc}} \leq M < M_{\text{iso}}\). This model is hereafter referred to as the constant production rate model. In the second model, we assume that the production rate decreases linearly with time. This model is hereafter called the decreasing production rate model. Since the production rate of the impact-jetting model increases with time \((F_{\text{ch}} dM)\), we can examine all three models for chondrule formation. Note that the total mass of the chondrules formed in all the models is about 0.01 \(M_{\text{iso}}\); in the constant production rate model, \(\approx F_{\text{ch}} M_{\text{iso}}/2 \times 10^6\) yr \(\approx 4 \times 10^{19}\) g of chondrules are formed per year, while in the decreasing production rate model, the mass of chondrules formed at \(M = M_{\text{esc}}\) is \(\approx 7 \times 10^{19}\) g, which is 10 times larger than that at \(M = M_{\text{iso}}\).

### 2.4. Chondrule Accretion

In the following, we describe how a protoplanet and planetesimals accrete chondrules. Our estimation is based on Lambrechts & Johansen (2012). In order to explicitly compare the accretion efficiencies of chondrules by a protoplanet with that by planetesimals, we assume that these massive objects are exposed to the same amount of chondrules. In other words, we independently estimate the accretion timescales of chondrules by a protoplanet and by planetesimals. The chondrule masses accreted by a protoplanet and by planetesimals are derived from these timescales.

#### 2.4.1. Protoplanet

The relative velocity \((\Delta \nu)\) between an accreting body and chondrules is important for estimating chondrule accretion. The relative velocity is caused by the eccentricity of the body, gas drag, and Keplerian shear. In our simulations, \(\Delta \nu\) between the protoplanet and the chondrules is written as \(\eta K\). The eccentricity of the protoplanet is \(\approx (m_{\text{pl,min}}/M) e_{\text{pl,min}}\) by energy equipartition. In our parameter range, \(\eta\) is larger than the eccentricity of the protoplanet and Keplerian shear. If we consider larger pebbles or a larger protoplanet, \(\Delta \nu\) is determined by Keplerian shear, as in the case of the estimation done by Lambrechts & Johansen (2012). Disk turbulence excites the eccentricity of a protoplanet (Iida et al. 2008). However, the turbulence is weak \((4 \times 10^{-3} \leq \alpha_{\text{eff}} \leq 5 \times 10^{-3}\) when 50 mG \(\leq (B) \leq 540\) mG in the solar nebula (Fu et al. 2014). In addition, a longer time is needed for a protoplanet to experience eccentricity pump-up by disk turbulence.
than to undergo eccentricity damping by dynamical friction from planetesimals. We do not consider the effect of turbulence on the protoplanet; hence, $\Delta \nu = \eta \nu_K$.

There are two modes when a protoplanet accretes chondrules (Lambrechts & Johansen 2012): the drift accretion mode and the Hill accretion mode. These two modes are divided by a transition mass ($M_0$). Comparing the Bondi radius $\eta_B = GM/\Delta \nu^2$ with the Hill radius $\eta_H = (M/3M_0)^{3/2}$, we get the transition mass ($M_0$),

$$
M_0 = \frac{\Delta \nu^2}{\sqrt{3} G \Omega_K} = 1.1 \times 10^{-3} \left( \frac{r}{1 \text{ au}} \right)^{1.2} \ M_{\odot},
$$

(17)

which is the mass at which $r_B = r_H$. Since $M_{\text{esc}} > M_0$, the protoplanet is in the Hill accretion mode (Lambrechts & Johansen 2012). Given that chondrules are well coupled with the disk gas (see Equation (8)), the accretion radius ($r_{\text{acc}}$) of chondrules by a protoplanet in the Hill mode is determined as follows. Chondrule accretion by a protoplanet can be achieved when the timescale on which the chondrules’ orbits become comparable to the stopping time of the chondrules:

$$
\frac{\Delta \nu}{GM/r_{\text{acc}}^3} = \tau_{\text{stop}}
$$

which becomes

$$
r_{\text{acc}} = \frac{GM}{\eta \Omega_K} = 7.2 \times 10^{-2} \left( \frac{r_{\text{ch}}}{1 \text{ mm}} \right)^{1/2}
\times \left( \frac{\rho_s}{3.3 \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{r}{1 \text{ au}} \right)^{1/2} \left( \frac{M}{M_{\odot}} \right)^{1/6} r_{\text{H}}.
$$

(18)

Substituting $r_{\text{H}} = 1.0 \times 10^{-2}(M/M_{\odot})^{1/3} \ r$, $r_{\text{acc}} = 7.2 \times 10^{-4}(M/M_{\odot})^{1/2}(r/1 \text{ au})^{1/2} \text{ au}$. The chondrule accretion radius by the protoplanet ($M_{\text{acc,pr}}$) is $M_{\text{acc,pr}} = \pi r_{\text{acc}}^2 \Delta \nu$, where $r_{\text{ch}}$ is the spatial density of chondrules. The density of chondrules can be given as $\rho_{\text{ch}} = M_{\text{ch}}/(2\pi r_{\text{ch}}^2 \Delta r_{\text{ch}})$, where $\Delta r$ is the orbital width that chondrules are distributed in; we give $\Delta r = h_{\text{ch}}$. Note that a specific choice of $\Delta r$ does not affect our conclusions, because the accretion timescales of chondrules by both a protoplanet and planetesimals have the same dependence on $r_{\text{ch}}$ (see below).

Now we derive the accretion rate ($M_{\text{acc,pr}}$) of chondrules accreted by a protoplanet. Considering the protoplanet at 2 au and $H = 0.53$, $M_{\text{acc,pr}}$ becomes

$$
M_{\text{acc,pr}} = \pi r_{\text{ch}}^2 \Delta \nu
\times \left( \frac{M_{\text{ch}}}{2\pi r_{\text{ch}}^2} \right)^{3/2} \left( \frac{H}{2} \right)^{-1}
\times \left( \frac{r}{2 \text{ au}} \right)
\times \left( \frac{r_{\text{ch}}}{1 \text{ mm}} \right)^{1/2} \left( \frac{\rho_s}{3.3 \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{M}{M_{\odot}} \right)^{1/6} T_K^{-1} M_{\text{ch}}.
$$

(19)

The timescale of chondrule accretion by the protoplanet is determined by $\tau_{\text{acc,pr}} = M_{\text{ch}}/M_{\text{acc,pr}}$,

$$
\tau_{\text{acc,pr}} = 0.91 \times 10^6 \left( \frac{L_3}{3} \right) \left( \frac{H^2/(1 + H^2)}{0.25} \right)^{1/2} \left( \frac{r}{2 \text{ au}} \right)
\times \left( \frac{r_{\text{ch}}}{1 \text{ mm}} \right)^{-1} \left( \frac{\rho_s}{3.3 \text{ g cm}^{-3}} \right)^{-1} \left( \frac{M}{M_{\odot}} \right)^{-1} \text{ yr}.
$$

(20)

Since the chondrule accretion radius becomes larger with increasing $M$ (see Equation (18)), $\tau_{\text{acc,pr}}$ decreases with increasing $M$.

2.4.2. Planetesimals

Next, we consider chondrule accretion by planetesimals. While we follow the basic formalism that has been developed by Lambrechts & Johansen (2012) and Ormel & Klahr (2010), the picture of chondrule accretion by planetesimals in our estimation is different from theirs. In oligarchic growth, random velocities and numbers of planetesimals are changed according to the mass growth of the protoplanet. These largely affect the chondrule accretion rate of planetesimals.

When the mass of planetesimals exceeds $M_0$, the accretion radius of chondrules is described in the same way as that of a protoplanet (see Equations (18) and (20)). In the following, we consider planetesimals that have smaller masses than $M_0$, i.e., in the drift accretion mode (Lambrechts & Johansen 2012). In this mode, the chondrule accretion radius is determined according to $r_B/\tau_{\text{stop}}$, where $\tau_B = r_B/\Delta \nu$ (Lambrechts & Johansen 2012). When $1 < r_B/\tau_{\text{stop}}$, chondrules are strongly affected by gas drag, and planetesimals cannot accrete them in whole $r_B$ this case corresponds to the settling regime in Ormel & Klahr (2010) (also see Section 2.4.1). For this case, the accretion radius is determined by the balance between the gravitational pull from a planetesimal and the gas drag acting on the chondrules. The accretion radius increases up to $r_B$ as $\tau_B/\tau_{\text{stop}}$ decreases. Since we consider chondrules (i.e., a constant value of $\tau_{\text{stop}}$), $\tau_B/\tau_{\text{stop}}$ decreases as $m_{\text{pl}}$ becomes smaller or $\Delta \nu$ larger. For the case in which $r_{\text{acc}} = r_B$, chondrule accretion becomes most efficient in the sense that all the chondrules in the Bondi radius will spiral toward the planetesimals. This arises because chondrules experience less gas drag as their orbits are deflected by the planetesimals. This settling regime continues until $\tau_B/\tau_{\text{stop}} \approx 0.25$, at which the gravitational focusing of a planetesimal regulates the dynamics of chondrules. For this case, the accretion radius is given by the gravitational focusing. This case is called the hyperbolic regime, and planetesimals are in this regime when $\tau_B/\tau_{\text{stop}} < 0.25$ (Ormel & Klahr 2010). In the hyperbolic regime, the orbit of a pebble is determined only by the gravitation interaction of a large body, while in the settling regime, the orbit is affected by both gas drag and the gravitational interaction, which is called pebble accretion in Lambrechts & Johansen (2012).

The relative velocity between planetesimals and chondrules is $\Delta \nu = e_p \nu_K$ for all three cases ($r_{\text{acc}} = (\tau B/\tau_{\text{stop}})^{-1} r_B$, $r_{\text{acc}} = r_B$, and $r_{\text{acc}} \sim R_p$). This is because $e_p$ is larger than $\eta$ and Keplerian shear. Since the eccentricities of planetesimals increase according to $M$ (Equation (14)), $\tau_B/\tau_{\text{stop}}$ is changed as

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4 In Ormel & Klahr (2010), this accretion process is called settling, since chondrules reside in the strong coupling regime.
the protoplanet mass ($M$) increases,

$$\frac{\tau_B}{\tau_{\text{stop}}} = 2.7 \times 10^{-2} \left( \frac{m_{\text{pl}}}{10^{23} \text{ g}} \right)^{4/5} \left( \frac{\rho_{\text{pl}}}{2 \text{ g cm}^{-3}} \right)^{-2/5} \times \left( \frac{2 \times 10^{-9} \text{ g cm}^{-3}}{r} \right)^{3/5} \left( \frac{2 \text{ au}}{r} \right)^{9/10} \times \left( \frac{M}{M_{\text{esc}}} \right)^{-1} \left( \frac{r_{\text{ch}}}{1 \text{ mm}} \right)^{1} \left( \frac{\rho_{t}}{3.5 \text{ g cm}^{-3}} \right)^{-1}. \quad (21)$$

As explicitly seen in Equation (21), $\Delta \nu$ increases and $\tau_B$ becomes smaller following the mass growth of protoplanets. Figure 2 shows $\tau_B/\tau_{\text{stop}}$ as a function of $M$ in our fiducial model. We set 20 bins between $m_{\text{pl,min}}$ and $M_{\text{ini}}$ (see Equation (12)). During chondrule formation ($M > M_{\text{esc}}$), the $\tau_B/\tau_{\text{stop}}$ of the smallest planetesimals ($p_{\text{min}}$) is always smaller than 0.25 (i.e., the hyperbolic regime). The median-mass planetesimals ($p_{\text{med}}$), which have $\sqrt{m_{\text{pl,min}} M_{\text{ini}}}$ mass, also spend most of the span of chondrule formation in the hyperbolic regime. In this figure, only the largest planetesimals, which have $m_{\text{pl,min}} M_{\text{ini}}^{1/20} M_{\text{ini}}^{1/20}$ mass, accrete chondrules via pebble accretion.

In our simulations, the size distribution of planetesimals is taken into account when the accretion timescale is estimated. The number of planetesimals is given by the power law, $n_{\text{pl}} = f_n (m_{\text{pl}}/M_{\text{ini}})^{-\eta}$, where $n_{\text{pl}}$ is the number of planetesimals in a bin (Kokubo & Ida 2000; Morishima et al. 2008). To keep the total mass of planetesimals ($\sum m_{\text{pl}} n_{\text{pl}}$) constant for all the simulations, $n_{\text{pl}}$ is multiplied by a factor $f_n$. This factor is approximately proportional to $m_{\text{pl,min}}^{-\eta}$. In our fiducial model, $f_n = 1$, and the total mass of planetesimals always corresponds to that in our fiducial model if $f_n = 3$. While the size distribution of planetesimals is included in our estimate, it is reasonable to assume that the planetesimals in each mass bin accrete chondrules from their whole $\tau_{\text{acc}}$. This is because the planetesimals’ cross sections of accretion are much smaller than $2\pi r_d \Delta t$. Then, the mass accretion rate of chondrules by the planetesimals in each bin is computed as the summation of that by each planetesimal. Since the protoplanet’s cross section of accretion is also much smaller than $2\pi r_d \Delta r$, we assume that a protoplanet and planetesimals do not compete in accreting chondrules. Also, to accurately estimate the accretion efficiency of chondrules by planetesimals only, the reduction of $n_{\text{pl}}$ due to protoplanet growth is neglected in our simulations. In other words, both a protoplanet and the planetesimals are exposed to the same amount of chondrules. This assumption may result in the total mass of the chondrules accreted by planetesimals being overestimated. Nonetheless, our estimate is useful in the sense that once the total amount of chondrules accreted by single planetesimals is obtained, we can readily calculate how many chondrules are eventually accreted by the planetesimals in each mass bin.

We now derive the accretion radius ($r_{\text{acc}}$) of the chondrules by planetesimals and its timescale ($\tau_{\text{acc,pl}}$). At first, we consider $1 < \tau_B/\tau_{\text{stop}}$. In this case, the chondrule accretion radius is (as in Section 2.4.1)

$$\frac{\Delta \nu}{\Gamma_{\text{pl}} / r_{\text{acc}}^2} = \tau_{\text{stop}} \Leftrightarrow r_{\text{acc}} = \left( \frac{\tau_B}{\tau_{\text{stop}}} \right)^{-1/2} r_B. \quad (22)$$

since the orbits of chondrules are affected by both the gas drag and the planetesimal gravity. Here, the Bondi radius of a planetesimal is $r_B = \Gamma_{\text{pl}} / \Delta \nu^2$. In this situation, the chondrule accretion rate by planetesimals ($M_{\text{acc,pl}}$) is $M_{\text{acc,pl}} = n_{\text{pl}} \pi \rho_{\text{ch}} \left( (\tau_B/\tau_{\text{stop}})^{-1/2} r_B \right)^2 \epsilon_{\text{pl}} V_K$. The timescale of chondrule accretion by planetesimals is

$$\tau_{\text{acc,pl}} = M_{\text{ch}} / M_{\text{acc,pl}}$$

$$= 1.7 \times 10^{3} f_n^{-1} \left( \frac{m_{\text{pl}}}{10^{23} \text{ g}} \right)^{1/2} \left( \frac{H^2 / (1 + H^2)}{0.25} \right) \times \left( \frac{\rho_{\text{pl}}}{2 \times 10^{-9} \text{ g cm}^{-3}} \right)^{8/5} \left( \frac{r}{2 \text{ au}} \right)^{11/10} \times \left( \frac{r_{\text{ch}}}{1 \text{ mm}} \right)^{-1} \left( \frac{\rho_{t}}{3.3 \text{ g cm}^{-3}} \right)^{-1} \text{ yr}. \quad (23)$$

When $0.25 < \tau_B/\tau_{\text{stop}} < 1$, planetesimals accrete chondrules from the whole Bondi radius, $r_{\text{acc}} = r_B$ (Ormel & Klahr 2010). In this case, $\tau_{\text{acc,pl}}$ becomes

$$\tau_{\text{acc,pl}} = 2.1 \times 10^{3} f_n^{-1} \left( \frac{m_{\text{pl}}}{10^{23} \text{ g}} \right)^{2} \left( \frac{H^2 / (1 + H^2)}{0.25} \right) \times \left( \frac{\rho_{\text{pl}}}{2 \times 10^{-9} \text{ g cm}^{-3}} \right)^{2/3} \left( \frac{r}{2 \text{ au}} \right)^{3} \times \left( \frac{\rho_{t}}{3.3 \text{ g cm}^{-3}} \right) \left( \frac{M}{M_{\text{esc}}} \right)^{1/2} \text{ yr}. \quad (24)$$

When $\tau_B/\tau_{\text{stop}} < 0.25$, chondrule accretion is in the hyperbolic regime, and gravitational scattering plays the dominant role in accreting chondrules. Planetesimals can accrete chondrules only from the gravitationally enhanced cross section, $R_{\text{pl}} \sqrt{1 + (v_{\text{esc}} / \Delta \nu)^2}$, where $R_{\text{pl}}$ is the radius of planetesimals.
The timescale of chondrule accretion is

\[
\tau_{\text{acc,pl}} = 1.2 \times 10^7 r_{\text{pl}}^{-1} \left( \frac{m_{\text{pl}}/M_{\text{ini}}}{1/120} \right) \left( \frac{H^2/(1 + H^2)}{0.25} \right) \times \left( \frac{m_{\text{pl}}}{10^{23} \text{ g}} \right)^{-11/15} \left( \frac{\rho_{\text{pl}}}{2 \text{ g cm}^{-3}} \right)^{8/15} \left( \frac{r}{2 \text{ au}} \right)^{21/5} \times \left( \frac{\rho_{\text{pl}}}{2 \times 10^{-9} \text{ g cm}^{-3}} \right)^{1/5} \left( \frac{M}{M_{\text{esc}}} \right)^{-1/3} \times \left( 1 + \frac{\nu_{\text{esc}}}{\epsilon_{\text{pl}V_K}} \right)^{-2} \cdot \text{yr.} \quad (25)
\]

Orbital inclinations can also affect chondrule accretion (Levison et al. 2015). This quantity comes into play in our model, because planetesimals and a protoplanet coexist in the system. When the inclinations of planetesimals are larger than \( h_{\text{ch}}/r \), planetesimals cannot accrete chondrules in their whole orbits. We calibrate the effect of the inclination by computing the ratio of the orbital period to a time interval during which planetesimals reside within the height of \( h_{\text{ch}} \) from the midplane. The inclinations of planetesimals are given by \( i_{\text{pl}} = \epsilon_{\text{pl}}/2 \).

Using Hill’s equations (Nakazawa & Ida 1988), this ratio can be described as

\[
f_{\text{pl}} = \frac{4}{\pi} \text{asinh} \left( \frac{h_{\text{ch}}}{r_{\text{pl}}} \right) = \frac{2}{\pi} \text{asinh} \left( \frac{h_{\text{ch}}}{i_{\text{pl}}r} \right),
\]

by which \( M_{\text{acc}} \) is multiplied when \( r_{\text{pl}} > h_{\text{ch}} \). The derivation of \( f_{\text{pl}} \) is summarized in the Appendix.

2.4.3. The Resultant Timescale of Accreting Chondrules

The timescales of chondrule accretion in the fiducial model are shown in Figure 3. These timescales by a protoplanet and the planetesimals in each mass range are plotted as a function of \( M \). The timescale by the protoplanet decreases with increasing \( M \) (red solid line). This is because \( \tau_{\text{acc}} \) increases with increasing \( M \) (Equations (18) and (20)). The \( p_{\text{min}} \) planetesimals (blue dashed line) are in the hyperbolic regime, and \( \tau_{\text{acc,pl}} \) is given by Equation (25). In this regime, \( \tau_{\text{acc,pl}} \) depends on \( M \) only through \( \epsilon_{\text{pl}} \), which increases with increasing \( M \) (Equation (14)). As a result, the planetesimals can encounter more chondrules as the protoplanet becomes more massive. This is why \( \tau_{\text{acc,pl}} \) decreases gradually with increasing \( M \) when \( M < 0.7 M_{\odot} \). When \( M > 0.7 M_{\odot} \), the inclinations of the \( p_{\text{min}} \) planetesimals becomes larger than \( h_{\text{ch}}/r \). In this case, the planetesimals have less chance to accrete chondrules, simply because they can stay in the chondrule sea for a shorter time. Consequently, \( \tau_{\text{acc}} \) becomes longer. The effect of the inclination (Equation (26)) increases \( \tau_{\text{acc,pl}} \) as increasing \( M \). For the \( p_{\text{mid}} \) planetesimals (green dashed line), two similar features are seen in the behavior of \( \tau_{\text{acc}} \), compared with the \( p_{\text{min}} \) planetesimal case. The first is that the accretion timescale decreases slowly with increasing \( M \) when \( 0.025 < M/ M_{\odot} < 0.47 \). This is because the planetesimals are in the hyperbolic regime. The other feature is that \( \tau_{\text{acc}} \) increases with \( M \), which is caused by the inclination effect. Since the inclination of the \( p_{\text{mid}} \) planetesimals grows faster than that of the \( p_{\text{min}} \) planetesimals, the effect of \( f_{\text{pl}} \) becomes important when the protoplanet reaches \( 0.4 M_{\odot} \). There is another noticeable feature for the case of \( p_{\text{mid}} \): the accretion timescale jumps at \( M = 0.02 M_{\odot} \). This jump is caused by a discontinuous change of \( f_{\text{acc}} \) between the settling regime and the hyperbolic regime that occurs at \( \tau_{\text{acc}}/ \tau_{\text{stop}} \) at 0.25 (see Figure 2). The same jump is also seen in the \( \tau_{\text{acc,pl}} \) of the \( p_{\text{max}} \) planetesimals (purple dashed line). For the \( p_{\text{max}} \) planetesimals, \( \tau_{\text{acc,pl}} \) is constant when \( M < 0.03 M_{\odot} \) (Equation (23)). In this mass range, \( 1 < \tau_{\text{acc}}/ \tau_{\text{stop}} \); hence, the accretion radius is smaller than the Bondi radius (see Equation (22)). Since the relative velocity is determined by \( \epsilon_{\text{pl}} V_K \), \( \tau_{\text{acc}} \propto \epsilon_{\text{pl}}^{-1/2} \), it indicates that \( \tau_{\text{acc}} \) shrinks with increasing \( M \). At the same time, however, \( \tau_{\text{acc,pl}} \propto \tau_{\text{acc}}^{1/2} \), and the relative velocity dependence on \( \tau_{\text{acc,pl}} \) is not canceled out anymore. When \( M > 0.14 M_{\odot} \), \( \tau_{\text{acc,pl}} \) evolves according to Equation (25), that is, the hyperbolic regime. Figure 3 shows that \( \tau_{\text{acc,pr}} \) is shorter than any \( \tau_{\text{acc,pl}} \) when \( M > 0.04 M_{\odot} \). This suggests that the most chondrules would be accreted by a protoplanet. For planetesimals, the \( \tau_{\text{acc,pl}} \) of the \( p_{\text{min}} \) planetesimals is the smallest. While the timescale of chondrule accretion by a single planetesimal becomes shorter with increasing \( m_{\text{pl}} \), the \( \tau_{\text{acc,pl}} \) of the planetesimals in each mass bin becomes longer with increasing \( m_{\text{pl}} \). This is simply because the number of planetesimals is taken into account when computing \( \tau_{\text{acc,pl}} \).

In some of the following simulations, the effect of disk turbulence on chondrule accretion by planetesimals will be examined by multiplying \( \tau_{\text{acc,pl}} \) by a factor of \( f_{\text{acc}}^{-1} \). This is because a number of effects triggered by disk turbulence have been proposed. These include that chondrules can be concentrated by weak turbulence (e.g., Cuzzi et al. 2001), that the eccentricities of planetesimals are excited by turbulence (Ida et al. 2008), and that the collision probability between planetesimals and chondrules is changed by turbulence (Guillot et al. 2014). In this paper, we take into account only a turbulent effect that can change the collisional probability. This can be done by changing the value of \( f_{\text{acc}} \). The concentration process of chondrules by turbulence in oligarchic growth would be affected by protoplanets. While random torque arising from disk turbulence can pump up the planetesimals’ eccentricities, the eccentricity excitation by a protoplanet is likely to be more
important in our configuration (Hasegawa et al. 2016a). Thus, the concentration of chondrules and the eccentricity excitation by turbulence are not included in our simulations. Note that the estimation of \( h_{\text{ch}} \) includes the turbulent effect (Dubrulle et al. 1995).

3. Chondrule Formation and Accretion

We perform simulations of chondrule formation and accretion in which all the models are combined, following Section 2. In other words, the growth of a protoplanet, the formation of chondrules, and the accretion of both by the protoplanet and planetesimals are computed simultaneously. First, we discuss our procedures. Then, chondrule formation and accretion in our fiducial model are presented. We explore the parameter dependencies of \( M_{\text{acc}} \) and \( \tau_{\text{acc}} \). The parameter ranges in each model are summarized in Table 2.

### 3.1. Synthesis

Our simulations are composed of the growth of a protoplanet, chondrule formation, and chondrule accretion by the protoplanet and planetesimals. To synthesize these effects, we perform simulations based on the following procedure. The mass of a protoplanet is increased by \( \Delta M \), which is calculated by Equation (13) until its isolation mass, in a time interval \( dt \). After \( \Delta M \) reaches 2.5 km s\(^{-1}\), \( F_{\text{th},dM} \) chondrules are formed in \( dt \). These chondrules are handled as field chondrules. The mass of the field chondrules \( (M_{\text{ch}}) \) is the sum of the remaining field chondrules in the previous step and \( F_{\text{th},dM} \). The field chondrules are accreted by the protoplanet and planetesimals. The chondrule mass accreted by the protoplanet in \( dt \) \((M_{\text{acc,pr}}dt)\) is given by Equation (19). The chondrule mass accreted by the planetesimals \( (M_{\text{acc,pl}}dt) \) depends on the accretion mode of the planetesimals in each mass range (see Section 2.4.2). The mass of the remaining field chondrules is given by \( M_{\text{ch}} - (M_{\text{acc,pr}} + \sum M_{\text{acc,pl}})dt \). Then, a sequence of processes that can occur in a timestep \((dt)\) is ended. These processes are repeated until \( 3 \times 10^6 \) yr to assess chondrule formation and accretion.

Note that while \( M_{\text{acc,pr}} \) and \( M_{\text{acc,pl}} \) are calculated independently, they are computed from the same amount of field chondrules. Some parameters affect either \( \tau_{\text{acc,pr}} \) or \( \tau_{\text{acc,pl}} \) but not both. In such cases, both \( M_{\text{acc,pr}} \) and \( M_{\text{acc,pl}} \) are changed, because \( M_{\text{ch}} \) is changed.

### 3.2. Fiducial Model

Figure 4 shows the mass of the cumulative formed chondrules \((M_{\text{ch,cum}})\) and those accreted by a protoplanet \((M_{\text{acc,pr}})\) and planetesimals \((M_{\text{acc,pl}})\) as a function of time. The protoplanet accretes the largest amount of chondrules, \( 5.0 \times 10^{-3} M_{\text{iso}} \), which is equal to 51\% of the formed chondrules (solid red line). The smallest-mass planetesimals have the second-largest amount of chondrules (blue dashed line), \( 1.2 \times 10^{-3} M_{\text{iso}} \), which is 12\% of the formed chondrules. The chondrule mass accreted by all the planetesimals in single mass bins becomes smaller as \( m_{\text{pl}} \) increases, since \( \tau_{\text{acc,pl}} \) becomes longer (Section 2.4.2). The total amount of chondrules that are accreted by all the planetesimals in all the mass bins is 44\% of the formed chondrules. The majority of the formed chondrules are accreted by the protoplanet and planetesimals (dotted-dashed line).

We find that chondrules are not accreted soon after they form. This is simply because the accretion timescale is \( \gtrsim 10^3 \) yr, which is much longer than the timescale of a collision, even for a protoplanet (see Figure 3). This feature can also be seen in Figure 4; for a given value of chondrule mass \((M_{\text{ch,cum}} \text{ and } M_{\text{acc}})\), there is a time lag for the mass of chondrules accreted by all bodies (dotted-dashed line) to catch up with the cumulative value (dotted line). This time lag roughly corresponds to the accretion timescale of the chondrules. Our results thus suggest that chondrules should have stayed in the solar nebula for 0.1–1 Myr. It is interesting that this time interval is roughly consistent with the isotope analysis of chondrules by Akai et al. (2007). In their study, the so-called compound chondrules, which are aggregates of two or more chondrules, were isotopically analyzed. They found that the secondary melting events occurred about 1 Myr after the primary melting happened. This infers that some of the chondrules stayed in the solar nebula for about 1 Myr.

#### 3.3. The Dependence on \( m_{\text{pl,min}} \)

In this section, we examine the effect of planetesimal mass on chondrule formation and accretion. We change the mass of the smallest planetesimals \( (m_{\text{pl,min}}) \) and then perform similar simulations. Figure 5 shows the \( M_{\text{acc}} \) and \( \tau_{\text{acc}} \) of a protoplanet and the \( m_{\text{pl,min}} \) planetesimals at \( 3 \times 10^6 \) yr as a function of \( m_{\text{pl,min}} \). The timescale of chondrule accretion by the protoplanet is constant with changing \( m_{\text{pl,min}} \) since it is independent of \( m_{\text{pl,min}} \) (Equation (20)). The timescale by the \( m_{\text{pl,min}} \) planetesimals increases with increasing \( m_{\text{pl,min}} \). Considering \( m_{\text{pl}} = m_{\text{pl,min}} \), Equation (25) is proportional to \( m_{\text{pl,min}}^{4/15} \). This dependence comes from the product of \( m_{\text{pl,min}}^{4/15} \), \( e_{\text{pl,min}} \), and \( m_{\text{pl,min}}^{1/15} \), and \( \tau_{\text{acc}} \sim R_{\text{pl,min}}^2 \times m_{\text{pl,min}}^{2/3} \). However, Figure 5 shows that the \( \tau_{\text{acc,pl}} \) of the \( m_{\text{pl,min}} \) planetesimals changes more rapidly than \( m_{\text{pl,min}}^{4/15} \). This arises because the accretion timescale is also affected by the

| Section | \( m_{\text{pl,min}} \) | \( f_d \) | \( r \) | \( \tau_g \) | \( f_{\text{acc}} \) | \( F_{\text{ch}} \) | Chondrule formation model |
|---------|----------------|-------|-----|-------|----------|----------|-------------------|
| Section 3.2 | \( 9 \times 10^3 \) g | 3 | 2 au | \( \infty \) | 1 | 0.01 | Impact jetting |
| Section 3.3 | \( 9 \times 10^3 \) g | 3 | 2 au | \( \infty \) | 1 | 0.01 | Impact jetting |
| Section 3.4 | \( 9 \times 10^3 \) g | 3 | 1–2 au | \( \infty \) | 1 | 0.01 | Impact jetting |
| Section 3.5 | \( 9 \times 10^3 \) g | 3 | 1–2.5 au | \( \infty \) | 1 | 0.01 | Impact jetting |
| Section 3.6 | \( 9 \times 10^3 \) g | 3 | 1–2 au | \( 10^3 \) yr–\( \infty \) | 1 | 0.01 | Impact jetting |
| Section 3.7 | \( 9 \times 10^3 \) g | 3 | 1–2 au | \( \infty \) | 0.3–10 | 0.01 | Impact jetting |
| Section 3.8 | \( 9 \times 10^3 \) g | 3 | 1–2 au | \( \infty \) | 1 | 0.01–0.10 | Impact jetting |

#### Table 2

Summary of Simulations

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inclination \( (f_d) \) when \( m_{pl,min} > 10^{21} \) g. In the case of \( m_{pl,min} < 10^{21} \) g, \( f_{pl,min} \) is smaller than \( h_{ch}/r \), even when \( M = M_{so} \), and the \( \tau_{acc,pl} \) of the \( pl_{min} \) planetesimals changes according to \( m_{pl,min}^{4/15} \).

Figure 5 also shows that \( M_{acc,pr} \) increases as \( m_{pl,min} \) increases when \( m_{pl,min} < 10^{24} \) g. This occurs because \( M_{acc,pl} \) decreases as \( m_{pl,min} \) increases. When \( M_{acc,pl} \) becomes smaller, more chondrules remain as field chondrules in a step. Because the mass of the field chondrules increases, the chondrule accretion rate by a protoplanet \( (M_{acc,pr} = M_{ch}/\tau_{acc,pr}) \) becomes larger at the subsequent times-steps. By contrast, the \( M_{acc,pr} \) at \( m_{pl,min} = 10^{24} \) g becomes smaller than that at \( m_{pl,min} = 10^{23} \) g. When \( m_{pl,min} > 10^{24} \) g, the mass of the protoplanet does not reach \( M_{so} \) within \( 3 \times 10^6 \) yr, since \( \tau_{pr} \) becomes larger due to a larger \( e_{pl,min} \) (see Equation (16)). Then, the cumulative formed chondrules have a mass smaller than \( F_{ch}M_{so} \). Because the total mass of the chondrules decreases, \( M_{acc,pl} \) also decreases. As \( m_{pl,min} \) increases, \( M_{acc,pl_{min}} \) decreases due to the increase of \( \tau_{acc,pl} \) (see Equation (25) and Figure 3). Except for \( m_{pl,min} = 10^{24} \) g, the protoplanet accretes 0.018 \( M_{so} - 0.050 \) \( M_{so} \), which is equal to 19%-50% of the formed chondrules. By contrast, planetesimals accrete 44%-81% of the formed chondrules. The smallest planetesimals get a larger amount of the formed chondrules, about 12%-28%.

### 3.4. The Dependence on \( f_d \)

Hasegawa et al. (2016a) showed that there are appropriate values of \( f_d \) and \( pl_{min} \) for chondrule formation and accretion by the impact-jetting process. In this section, we examine how the timescale of chondrule accretion and the amount of accreted chondrules depend on \( f_d \). We adopt \( f_d = 1, 2, 3 \) (fiducial), 5, and 10. Figure 6 shows \( M_{acc} \) and \( \tau_{acc} \) as a function of \( f_d \). Note that \( M_{so} \) is proportional to \( f_d^{3/2} \). As \( f_d \) increases, \( \tau_{acc,pr} \) and \( \tau_{acc,pl} \) become shorter. The protoplanet does not reach its isolation mass within \( 3 \times 10^6 \) yr, when \( f_d < 2.7 \) (Equation (16)). This is why the \( \tau_{acc} \) of the protoplanet and the \( pl_{min} \) planetesimals inflects around \( f_d = 3 \). Since \( \tau_{pr} \propto m_{pl,min}^{2/15} f_d^{-9/10} \), the protoplanet can get \( M_{so} \) if \( f_d = 1 \) and \( m_{pl,min} \leq 1.2 \times 10^{20} \) g.

When the protoplanet gets \( M_{so} \), the \( f_d \) dependence on \( \tau_{acc,pr} \) is caused by \( \tau_{acc,pl_{min}} \propto f_d M_{so}^{-1} \). The dependence on \( \tau_{acc,pl_{min}} \) is \( \tau_{acc,pl_{min}} \propto n_{pl}^{2} \tau_{acc} e_{pl} f_{pl}^{1} \propto f_d^{-13/10} \). The dependence of \( \tau_{acc,pl_{min}} \) on \( f_{pl,min} \) is stronger than that of \( \tau_{acc,pr} \). Then, \( M_{acc,pl_{min}}/M_{acc} \) becomes larger as \( f_d \) increases.

It is important that the mass ratio of the accreted chondrules between a protoplanet and planetesimals does not change very much when \( f_d > 3 \). Even for the case of \( f_d < 3 \), the trend of our results does not change; most chondrules are accreted by a protoplanet. Thus, the results obtained from our fiducial case can be applicable for a wide range of disk masses.

### 3.5. The Dependence on \( r \)

The orbital radius varies the timescales of chondrule formation and accretion. We perform simulations with changing orbital radii from 1.0 to 2.5 au. The timescale of chondrule accretion becomes longer as \( r \) increases. Figure 7 shows the \( M_{acc} \) and \( \tau_{acc} \) of a protoplanet and the \( pl_{min} \) planetesimals at \( 3 \times 10^6 \) yr as a function of \( r \). The amount of chondrules accreted by both the protoplanet and the \( pl_{min} \) planetesimals drops at \( r = 2.5 \) au (top panel). This is because the protoplanet does not reach \( M_{so} \) within \( 3 \times 10^6 \) yr, as discussed in the previous section. In the following, we consider chondrule accretion at \( r < 2.5 \) au.

Based on the derivation in Section 2, \( \tau_{acc,pr} \propto r^{9/2} \) under the approximation of \( H^2/(1 + H^2) \sim H^2 \) (Equation (20)), while \( \tau_{acc,pl_{min}} \) changes more rapidly, which is given as \( \tau_{acc,pl_{min}} \propto r^{24/5} \) (Equation (25)). This indicates that, as \( r \) decreases, the \( \tau_{acc} \) of both the protoplanet and the \( pl_{min} \) planetesimals decreases. We find
dependence on $\tau_g$}

The above simulations are performed without gas depletion. When the gas density and surface density are changed with time, $\tau_{\text{stop}}$ and $\epsilon_{\text{pl}}$ also vary. Since we give gas depletion by $\exp(-t/\tau_g)$, $\tau_{\text{stop}}$ and $\epsilon_{\text{pl}}$ increase as gas disks evolve with time; $\tau_{\text{stop}}\propto \rho_g^{-1}\propto \exp(t/\tau_g)$ (Equation (8)), and $\epsilon_{\text{pl}}\propto \rho_g^{-1/5}\propto \exp(0.2t/\tau_g)$ (Equation (14)). This means that when $\tau_g\lesssim t_{\text{iso}}$, which is $2.4\times10^6$ yr in our fiducial model (Section 2.3), $\tau_{\text{stop}}$ and $\epsilon_{\text{pl}}$ are changed only by a factor of a few. Note that $H$ does not depend on $\tau_g$ because the $\tau_g$ dependence is canceled due to $H\propto (a_{\text{eff}}/\tau_{\text{stop}})^{1/2}\propto (\Sigma_g/\rho_g)^{-1/2}$ (Equations (9) and (11)).

We perform simulations with $\tau_g=10^6$ yr, $3\times10^6$ yr, $5\times10^6$ yr, and $10^7$ yr. Note that, while we consider the cases of $\tau_g=10^6$ yr and $3\times10^6$ yr only for completeness, the results for the case of $\tau\geq3$ Myr are more appropriate for chondrules found in chondrites. This is because chondrule formation likely continued until 3 Myr after CAI formation, and a gas disk would be needed for chondrule formation at that time (e.g., Hewins et al. 2005). Our fiducial model can be viewed as $\tau_g=\infty$. Figure 8 shows the resultant values of $M_{\text{acc}}$ and $\tau_{\text{acc}}$ for the protoplanet and the $p_{\text{min}}$ planetesimals at $3\times10^6$ yr. As $\tau_g$ increases, $\tau_{\text{acc,pr}}$ increases, and $\tau_{\text{acc,pl}}$ is hardly changed. The $\tau_g$ dependence on $\tau_{\text{acc}}$ arises from $r_{\text{acc}}^{-2}\Delta v^{-1}$. In the case of the protoplanet, $r_{\text{acc,pr}}^{-2}\Delta v^{-1}\propto \tau_{\text{stop}}^{-1}\propto \exp(-t/\tau_g)$ (Equations (8) and (18)). This is why $\tau_{\text{acc,pr}}$ increases with increasing $\tau_g$ under $\tau_g\gtrsim t_{\text{iso}}$. For the $p_{\text{min}}$ planetesimals, $\tau_{\text{acc,pl}}$ is multiplied by $f_{\text{pl}}^{-1}$ at $t=3\times10^6$ yr. Since $\tau_{\text{acc,pl}}\propto r_{\text{acc,pl}}^{-2}\Delta v^{-1}f_{\text{sp}}^{-1}$, which is approximately proportional to $i_{\text{pl}}/\epsilon_{\text{pl}}$, $\tau_{\text{acc,pl}}$ does not depend on $\tau_g$.

3.7. The Dependence on $f_{\text{acc}}$

In this paper, our model is based on the oligarchic growth model in laminar disks (Kokubo & Ida 2000). As described in Section 2.4.3, chondrule accretion can be affected by disk turbulence. In this section, we multiply $\tau_{\text{acc,pr}}$ by $f_{\text{acc}}$ to consider the case of more effective accretion of chondrules, which can be triggered by disk turbulence. We adopt $f_{\text{acc}}=0.3, 1$ (fiducial), 3, and 10. In these simulations, $\tau_{\text{acc,pl}}\propto f_{\text{acc}}^{-1}$, and $\tau_{\text{acc,pr}}$ is constant with changing $f_{\text{acc}}$ (see Figure 9). Our results show that the chondrule mass accreted by the $p_{\text{min}}$ planetesimals does not change in proportion to $f_{\text{acc}}$ (see Figure 9). As we see in Section 3.5, the dependence of $M_{\text{acc}}$ on $t_{\text{iso},p}$ is weak, since $M_{\text{acc}}=M_{\text{ch}}/\tau_{\text{acc}}$, and $M_{\text{ch}}$ becomes smaller when $\tau_{\text{acc,pr}}$ becomes small. As a result, the $M_{\text{acc}}$ dependence on $f_{\text{acc}}$ becomes small, and the $p_{\text{min}}$ planetesimals accrete 24% of the formed chondrules even when $f_{\text{acc}}=10$.

3.8. The Other Dependences

We also perform simulations with changing $F_{\text{ch}}$ and chondrule formation models. When we change $F_{\text{ch}}$, the mass of the formed chondrules is changed in proportion to $F_{\text{ch}}$. Since $\tau_{\text{acc}}$ does not depend on $F_{\text{ch}}$, the $M_{\text{acc}}$ of a protoplanet and planetesimals is proportional to $F_{\text{ch}}$.

When we change chondrule formation models, we fix the timescale of chondrule formation (i.e., $M_{\text{acc}}\lesssim M\lesssim M_{\text{esc}}$) and the total mass of the formed chondrules (see Section 2.3). We perform simulations with the constant production rate model and decreasing production rate model (Section 2.3). The chondrule mass accreted by the $p_{\text{min}}$ planetesimals increases in the following order: the impact-jetting model (fiducial), the constant production rate model, and the decreasing production rate model. This is because the $p_{\text{min}}$ planetesimals accrete more chondrules than the protoplanet when $M\approx M_{\text{esc}}$ (see
mass. This equation is seemingly proportional to \( m_{pl} \). However, since \( f_{i,t} \) decreases with increasing \( m_{pl} \) (see Section 3.2 and Figure 4), the value of \( f_{m,ch} \) remains small. The dependence of \( f_{i,t} \) on \( m_{pl} \) can be derived from \( \tau_{acc,pl} \). Considering that planetesimals are in the hyperbolic regime, \( f_{i,t} \propto m_{pl}^{-0.15/15} \) with the condition that \( f_{i,t} = 1 \) (see Equation (26)), and we obtain \( f_{m,ch} \propto m_{pl}^{-0.15/15} \). The small \( f_{m,ch} \) means that the accreted chondrules do not change the mass of the planetesimals.

By contrast, this fraction is too small to reproduce the fractional abundance of chondrules in chondrites (e.g., Scott & Krot 2005). In other words, if chondrites originated only from the fragmentation of planetesimals, our results suggest that fragments can satisfy the measured abundance of chondrules in chondrites. The chondrules accreted by planetesimals make a chondrule-rich layer on the surface region of the planetesimals. The thickness of this layer normalized by \( R_{pl} \) is computed as

\[
\frac{\Delta R_{ch}}{R_{pl}} = \frac{f_{m,ch} m_{pl}}{4 \pi R_{pl}^3 \rho_s} = 1.2 \times 10^{-2} f_{i,t} \left( \frac{\rho_{pl}}{2 \text{ g cm}^{-3}} \right) \left( \frac{\rho_s}{3.3 \text{ g cm}^{-3}} \right)^{-1} \times \left( \frac{m_{pl}}{10^{23} \text{ g}} \right)^{1/10} \left( \frac{\Sigma_d}{11 \text{ g cm}^{-2}} \right)^{1/5} \left( \frac{r}{2 \text{ au}} \right)^{3/5} \times \left( \frac{m_{pl,min}}{10^{23} \text{ g}} \right)^{-6/5}, \tag{28}
\]

Figure 10 shows the results of \( \Delta R_{ch}/R_{pl} \) as a function of \( m_{pl} \), which are obtained from our calculations of the accreted chondrule mass (see Section 3.3). We find that, for the case of \( m_{pl,min} = 10^{21} \text{ g} \) (green dots), the results are well characterized by \( m_{pl}^{-0.15/15} \), while for the case of \( m_{pl,min} = 10^{19} \text{ g} \) (blue dots), they are well characterized by \( m_{pl}^{-0.15/15} \). These can be explained by the behavior of \( \Delta R_{ch}/R_{pl} \propto m_{pl}^{-0.15/15} \), for the former, \( \Delta R_{ch}/R_{pl} \propto m_{pl}^{-0.15/15} \) under the condition that \( f_{i,t} = 1 \). For the latter, \( \Delta R_{ch}/R_{pl} \propto m_{pl}^{-0.15/15} \) when \( f_{i,t} = 1 \) is given by Equation (26). Our results also show that, for the case of \( m_{pl,min} = 10^{23} \text{ g} \), the dependence of \( \Delta R_{ch}/R_{pl} \) on \( m_{pl} \) is weaker than \( m_{pl}^{-0.15/15} \), since the larger-mass planetesimals are in the settling regime. In the case of \( m_{pl,min} = m_{pl} = 10^{23} \text{ g} \), which are planetesimals with a radius of 230 km, the planetesimal has the 0.27 km chondrule layer on its surface. Wakita et al. (2017) showed that the majority of ejecta arise from a very thin surface layer, which is about 100 m from the surface. Then, the (high) abundance of chondrules in chondrites can be potentially explained by the chondrule layer if the original materials of the chondrites are in this layer. Based on a high fractional abundance of chondrules in chondrites, it can be expected that there was not a large amount of dust, which has a similar Stokes number to chondrules in the solar nebula at that time.

4.2. Other Effects

In our simulations, we assume that chondrules stay at their formed orbits. Theoretical studies suggested that chondrules migrate inward due to gas drag. This migration timescale is

\[
\tau_{migr} \propto \frac{m_{pl}}{\rho_{pl}} \frac{1}{\alpha},
\]

where \( \alpha \) is a gas-to-dust density ratio. However, the migration timescale is much longer than the chondrule formation timescale, as shown in Figure 10. Therefore, we can neglect the migration of chondrules in our simulations.
\[ m_{pl,\text{min}} = 10^{21} \text{ g} \] (green dots) and \[ 10^{-9} \text{ g} \] (blue dots).

Figure 10. Thickness of chondrule layers on planetesimals normalized by \( R_d \) as a function of \( m_{pl} \). The red dots represent \( \Delta R_c / R_d \) in our fiducial model \( (m_{pl,\text{min}} = 10^{23} \text{ g}) \). We also plot the results of models with \( m_{pl,\text{min}} = 10^{21} \text{ g} \) (green dots) and \( 10^{-9} \text{ g} \) (blue dots).

\[ \sim 10^5 \text{ yr} \] (Adachi et al. 1976; Weidenschilling 1977). This timescale is shorter than those of chondrule accretion (Figure 4), which indicates that chondrules would migrate inward before they were accreted by a protoplanet and planetesimals. By contrast, the isotopic measurement of compound chondrules suggested that the chondrules stayed in the solar nebula for 1 Myr (Akai et al. 2007). Some mechanism, such as a radial pressure bump (e.g., Taki et al. 2016) or vortices (e.g., Cuzzi et al. 2010; Fu et al. 2014), would be needed to keep chondrules from migration.

We consider only one protoplanet in our calculations. There is nonetheless a possibility that other protoplanets and even fully formed planets might have existed in the solar nebula at that time. The presence of other protoplanets would not change our results, since their orbital separation is \( \sim 10 \text{ au} \), which is larger than \( h_{ch} \). The chondrules produced by a protoplanet are accreted only by the protoplanet and surrounding planetesimals. Formation of giant planets affects the eccentricities of planetesimals. The perturbation from giant planets makes planetesimals dynamically hot. If the timescale of protoplanet growth becomes longer and the protoplanet does not get its isolation mass within a disk lifetime, the mass of the accreted chondrules will decrease, as we see in Section 3. In a subsequent paper, we will perform full N-body simulations of planetary growth under the existence of a giant planet and examine the eccentricities of planetesimals and the formation of chondrules by impact jetting (S. Oshino et al., in preparation).

We have not considered the space and velocity distribution of chondrules and planetesimals in our calculations. When planetesimals have larger eccentricities and inclinations due to the perturbations from giant planets or chondrules are spatially concentrated by a mechanism such as streaming instability (Youdin & Johansen 2007), the relative velocity and collisional probability between planetesimals and chondrules are largely changed in an orbit, especially for the vertical direction. Guillot et al. (2014) examined how disk turbulence affects the collisional probability between dust particles and planetesimals, including their 3D spatial distributions. However, the accretion of dust particles onto planetesimals, taking into account both their spatial and velocity distributions, remains to be explored. Meanwhile, our results are not largely changed as long as the picture of oligarchic growth in our fiducial model is not changed.

In Section 3.3, we perform simulations with \( m_{pl,\text{min}} < 10^{20} \text{ g} \) for completeness. However, the mass of planetesimals strongly affects the onset of runaway growth (Wetherill & Stewart 1989; Kokubo & Ida 1996; Kobayashi et al. 2016). When \( m_{pl,\text{min}} \) is smaller than a threshold value, planetesimals grow up orderly until certain conditions are satisfied such that runaway growth begins. Even if runaway growth occurs in a swarm of planetesimals that have \( m_{pl,\text{min}} < 10^{20} \text{ g} \), the mass distribution of planetesimals in oligarchic growth would be affected by \( m_{pl,\text{min}} \) (Morishima 2017).

It is also important to comment on the isolation mass, which regulates the end of chondrule formation in our simulations. In our fiducial model, the isolation mass of a protoplanet is \( 1.4 M_{\oplus} \). Even if \( f_a = 1 \), the final mass of the protoplanets is larger than the current mass of the asteroid belt. Such large bodies can be eliminated by the perturbations from giant planets or planetary migration. After the giant planets are formed, the protoplanets are scattered by their perturbations (e.g., Petit et al. 2002). In addition, type I migration becomes effective when the protoplanetary mass is larger than \( \sim 0.1 \sim 1 M_{\oplus} \) at \( 1 \sim 3 \text{ au} \) (see Section 4.3 in Hasegawa et al. 2016b).

While we have so far considered the possibility that chondrules formed via impact jetting will be accreted by their surrounding planetesimals, it might be interesting to discuss another possibility: the formation of planetesimals directly from chondrules ejected from planetesimal collisions. This possibility may work well to account for the currently existing meteoritic data (Alexander et al. 2008). Unless planetesimal formation from chondrules is not a dominant process, our results would not be largely changed, since these planetesimals also produce chondrules by impact jetting.

5. Conclusions

Investigating the process of chondrule accretion provides us with profound insights into the origins of our solar system, as well as the chondrule formation process. When a large number of massive planetesimals that can accrete chondrules are present, they grow up to be a protoplanet. The isotope measurement suggested that the timescale of Mars formation is less than the timescale of chondrule formation (Dauphas & Pourmand 2011). We have investigated chondrule accretion onto a protoplanet and planetesimals in oligarchic growth (Kokubo & Ida 1998) using the simple analytical approach. In our simulations, we have considered an impact-jetting model as the chondrule formation model. When the collision velocity exceeds \( 2.5 \text{ km s}^{-1} \), chondrules are formed via planetesimal collisions (Johnson et al. 2015; Wakita et al. 2016, 2017). The mass of the cumulative formed chondrules is about 1% of the protoplanet mass when planetesimal collisions transform about 1% of the impactor’s mass into (the progenitor of) chondrules. The protoplanet accretes about half of the formed chondrules. The other half are accreted by planetesimals. In our simulations, we divide planetesimals into 20 mass bins. The smallest planetesimal bin has the largest amount of chondrules, about 10% of the formed amount.

We have performed a number of similar simulations by changing the mass of the smallest planetesimals, the orbital radius, the timescale of gas depletion, the efficiency of chondrule accretion by planetesimals, the chondrule formation efficiency in the impact-jetting model, and the chondrule formation models. Under the condition that a protoplanet
reaches its isolation mass, the amount of chondrules accreted by the smallest planetesimals is about 10% of the formed chondrules for all the runs. This amount is not much changed by the chondrule formation models, since it is determined by the timescales of chondrule accretion. The mass of chondrules accreted by planetesimals is too small to explain the chondrule fraction in chondrites. Our results indicate that chondrules accreted by planetesimals make a layer on their surfaces. Only if chondrites come from this layer can their chondrule fraction be explained.

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Appendix

When a planetesimal stays in \( z \leq h_{ch} \), where \( z \) is the distance from the midplane, it can accrete chondrules. The motion of a planetesimal is given by Hill’s equation (Nakazawa & Ida 1988),

\[
\frac{d}{dt} (\Omega - \Omega_{pl}) = \frac{h_{ch}}{l_{pl}r} \sin \left( \frac{1}{\Omega_{pl}} \sin \left( \frac{h_{ch}}{l_{pl}r} \right) \right),
\]

where \( \Omega_{pl} \) is the longitude of the ascending node of a planetesimal. A planetesimal stays in \( z \leq h_{ch} \) until

\[
t \leq \frac{1}{\Omega_{pl}} \sin \left( \frac{h_{ch}}{l_{pl}r} \right),
\]

after this passed its ascending node. The fraction of the timescale in which a planetesimal stays in \( |z| \leq h_{ch} \) in an orbit \( f_{pl} \) is given as

\[
f_{pl} = \frac{\frac{d}{dt} (\Omega - \Omega_{pl})}{T_{K}} = \frac{2}{\pi} \sin \left( \frac{h_{ch}}{l_{pl}r} \right).
\]

This factor is defined when \( h_{ch} < l_{pl}r \).

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