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Wenjie Kong, Hai Du, Qinlin Zhang, Qixuan Li, Xinyue Lv, Lianbin Zhou, and Weiguo Zhang

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Wenjie Kong,1,2 Hai Du,1,2,a) Qinlin Zhang,1 Qixuan Li,1 Xinyue Lv,1 Lianbin Zhou,1 and Weiguo Zhang3

AFFILIATIONS
1 School of Energy and Power Engineering, Xihua University, Chengdu 610039, China
2 Key Laboratory of Fluid and Power Machinery, Ministry of Education, Xihua University, Chengdu 610039, China
3 Rotor Aerodynamics Key Laboratory, China Aerodynamics Research and Development Center, Mianyang, Sichuan 621000, China

a) Author to whom correspondence should be addressed: duhai@mail.xhu.edu.cn

ABSTRACT
The flow field structure of a rotorcraft is complex; specifically, the rotor tip vortex structure has a great influence on the rotor performance. Therefore, in this paper, the evolution characteristics of rotor tip vortices and the dynamic mode decomposition (DMD) of rotor tip vortices in a rotor hovering state are studied. Through a time-resolved particle image velocimetry experiment, a comparative study of the blade tip vortex flow field at a fixed rotation speed (1500 rpm) and a collective pitch of 6° and 9° was performed. The method of DMD is used for the reduced-order analysis of the vorticity field of the blade tip vortex in the hovering state. By this method, these important vortex structures are extracted and discussed; meanwhile, the future flow field is also reconstructed. The results of flow visualization indicate that the trajectory of the blade tip vortex is moving down the axis, while moving toward the hub in the radial direction in the hovering state. The results of DMD analysis show that during the evolution of the blade tip vortex, different modes have different contributions to the rotor as a whole. In addition, the larger the collective pitch, the larger the modal coefficient amplitude and the slower the stabilization speed.

I. INTRODUCTION
Generally, the aerodynamic performance of a rotorcraft is more complicated than a fixed-wing aircraft, and its complexity comes from the unique flow field structure around the rotor. The rotor blade velocity increases along the direction of the wing-tip and generates most of the aerodynamic force at the wing-tip. In addition to the vortex sheet generated by the blade movement, strong blade tip vortices will appear in the wake of each blade tip. These vortices are always close to the rotor, creating a complex three-dimensional induced flow field, which affects the aerodynamic load of the rotor, aerodynamic performance, vibration level of the rotor, aeroelasticity, and acoustic performance. For example, rotor blades can interact with vortices to generate unsteady aerodynamic loads, thus causing problems such as high-intensity rotor vibration and pulsating noise.

By far, regarding the research methods of rotorcraft wake, whether it is a wind tunnel experiment (particle image velocimetry (PIV) or laser Doppler velocimetry (LDV)) or numerical simulation, the number of data samples obtained is extremely huge. How to use these beneficial data and quickly perform data classification and processing to extract the vortex structures of the rotorcraft is important, which is essential to understand the complex flow characteristics of the rotorcraft. In addition, low-dimensional reduction of these high-dimensional flow field data could help further our understanding of the generation, movement, evolution, instability, and decay processes of the vortex structure.

In fact, with the development of related research on flow dynamic systems, low-dimensional reduction order technology is increasingly applied to the analysis of complex fluid flows. Proper original decomposition (POD) and dynamic modal decomposition (DMD) are two of the most representative methods. The POD method is a reduced-order approximation of an infinite-dimensional nonlinear system, which is represents the transient velocity field as a linear superposition of a set of mutually orthogonal pod modes, using each mode to reflect the different...
structural characteristics of the original transient flow field. The DMD method is an approximation to the Koopman operator. When analyzing nonlinear differential equations, Koopman proposed that a linear infinite-dimensional system can be used to represent a finite-dimensional nonlinear system, which can avoid the problems encountered in the study of nonlinear systems. Schmid et al. first proposed the DMD method to analyze large datasets obtained from experiments or numerical simulations in fluid mechanics research, and to extract dynamic related flow characteristics of the flow field.

In order to further study the complex flow of the rotor and its aerodynamic characteristics, it is urgent to develop corresponding rapid analysis techniques for the rotorcraft flow field and apply the dynamic modal dimension reduction technology to accurately obtain the complex reduced-order model of the vortex structure. In view of the above problems, this paper uses the TR-PIV experimental facilities to carry out proficiency vortex field measurements at the blade tip; then, the DMD method is used to carry out modal reduction and reconstruction of the flow field, which can further analyze the kinematic development process of the blade tip vortex in the complex wake of the rotor, and then further understand the unsteady and nonlinear complex aerodynamic mechanism of the blade tip vortex, providing a theoretical basis for the modeling and control of the wake vortex.

The structure of this article is as follows: Sec. II introduces the rotorcraft model, PIV experimental facility system, and DMD method; Sec. III analyzes the time-averaged and the unsteady vortex flow field future in the hovering state. At the same time, the motion characteristics of the blade tip vortex are also studied. In Sec. IV, the DMD method is used to reduce the order of the flow field, and the modal growth after reduction attenuation, modal characteristics, and reconstruction of the flow field; and Sec. V summarizes the work of this paper.

II. EXPERIMENTAL EQUIPMENT AND RESEARCH METHODS

A. Experimental system

The entire experimental system is shown in Fig. 1, which consists of a rotorcraft, a mounting bracket, and a PIV system. The rotorcraft model has two blades (the blades are made of carbon fiber), and the shape of the blade is shown in Fig. 2. The blade torque is 0, the solidity is 0.104, the blade length is 380 mm, and the basic chord length of the blade is 32 mm. The leaf aspect ratio is 11.875, and the airfoil is NACA 0015.

The rotorcraft model is fixed on a cylindrical mounting bracket, with a support rod diameter of 120 mm and a height of 1500 mm. When carrying out the PIV flow field test experiment, a smoke generator box is placed directly above the rotor, which can evenly eject the tracer particles downward. The laser’s optical path is from the bottom to the top, and the sheet light is perpendicular to the plane of the propeller disk. The thickness of the sheet light in the observation area is about 1 mm. The camera is placed horizontally and perpendicular to the laser plane. The rotation direction of the rotor is clockwise when viewed from top to bottom. There is no free flow during the experiment, which shows that the helicopter rotor is in a hovering state. In this research, the rotation speed of the blade is fixed at $n = 1500$ rpm, corresponding to the blade tip speed 59.69 m/s, two blade collective pitch angles are taken ($\theta = 6^\circ$ and $\theta = 9^\circ$) for flow field measurements.

B. Time-resolved PIV system

The TR-PIV system consists of a high-speed camera (PCO. dimax HS4, camera lens model is Nikon 50 D), a synchronizer (ILA-5150-data-synchronizer), a dual pulse laser (Vlite-Hi-527-30, wavelength 527 nm), workstations, tracer particle generator, and other components. The tracer particles spray smoke evenly downwards directly above the rotor through the smoke machine. It is 0.6 $\mu$m, has good followability and has a high scattering rate to the laser and negligible effect on the flow field. The black matte paint is sprayed on the surface of the blade model to eliminate the error caused by the reflection of the intense laser on the surface before the experiment.

During the experiment, the light source emitted by the laser from the bottom up is perpendicular to the blade, and the camera is
taken perpendicular to the laser surface. Computer-controlled synchronizer and camera work during the experiment. The laser emission frequency and camera sampling frequency are 1000 Hz, and the double exposure time interval is 200 μs. The camera pixel is 2000 × 2000 pix (during the experiments, the size of the field of view is 280 × 200 mm²), the exposure time is 100 μs, and the camera aperture is f/5.6.

In post-processing, the PIV view2C software is used for cross-correlation analysis of the captured images, and correlation is determined using the standard Fast Fourier Transformation (FFT) correlation algorithm, the Nyquist-frequency filtering, and the 9-point least-squares peak computing algorithm based on the Gauss fitting. The inquiry area has 64 × 64 pixels, and the overlap factor is 50%. The samples are 200 pairs of transient flow field maps, and the system error of the PIV is less than 4% (calibrated by hotline).

C. Dynamic mode decomposition

The method of dynamic mode decomposition (DMD) is a decomposition technique capable of reducing the dimensions of a dynamic system. Compared with the method of proper orthogonal decomposition (POD), the DMD can obtain more kinetic information about the original system, and through mode reduction, vortices can be accurately identified for tracking the development and evolution of vortex structures. In the DMD method, the

FIG. 3. Flow chart of data processing in the DMD method.

FIG. 4. Time-average vorticity and velocity vector diagram. (a) n = 1500 rpm, θ = 6° and (b) n = 1500 rpm, θ = 9°.

FIG. 5. Schematic diagram of different azimuth angles (rotor rotates clockwise).
evolution of flow is treated as a linear dynamic process, and through
eigenstructure analysis of the snapshots of the flow field, the low-
order modes and their corresponding eigenvalues are obtained,
which contains information about the variations of the flow field
in the whole process.15–20 Each mode obtained with the DMD
method has a single frequency and growth rate, which are conve-
nient for the analysis of linear and periodic dynamic flows. This
method could provide abundant information for the study of coher-
ent structures and their effects at different scales in complex flow
fields.9,10,21–24

Figure 3 shows the flow chart of data processing in the DMD
method applied in this paper. First, two snapshot matrices \(X_{m-1} = \{x_1, x_2, \ldots, x_{m-1}\}\) and \(X_m = \{x_2, x_3 \ldots x_m\}\), in which \(x_i\) represent
the PIV snapshots at times with fixed time interval \(\Delta t\) are con-
structed based on the PIV snapshots of the flow field from time 1 to
"m." Assume that the flow field \(\mathbf{x}_{i+1}\) can be represented by a linear
mapping of \(x_i\), and then \(X_m = AX_{m-1}\), where \(A\) is the system matrix
of the high-dimensional flow field. Through singular value decom-
position \((Q, R)\), \(A\) can be approximated as \(A = A = R^{-1}Q^TX_m\).
Finally, the corresponding modal coefficients, DMD modes of the
flow field, and reconstructed flow fields are obtained.

III. FLOW FIELD TEST RESULTS AND DISCUSSION

A. Time-average vorticity field

The time-averaged vorticity field can accurately describe the
trajectory of the blade tip vortex in a hovering state. Mathematically,
the vorticity is defined as the curl of the fluid velocity vector. The
calculation formula is

\[
\mathbf{\omega} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \mathbf{k},
\]

FIG. 6. Vorticity contours at different azimuth angles at the rotor rotation speed of 1500 rpm and the collective pitch of 6°.

FIG. 7. Vorticity contours at different azimuth angles at the rotor rotation speed of 1500 rpm and the collective pitch of 9°.
TABLE I. Variations of blade tip vortex of blade 1 under different collective pitches.

| θ° | ψ° | ω_(z) (s^-1) |
|----|----|---------------|
| 6  | 22.5 | -515          |
|    | 54  | -497          |
|    | 85.5| -472          |
|    | 117 | -440          |
|    | 148.5| -376         |
|    | 180 | -300          |
| 9  | 22.5 | -735          |
|    | 54  | -710          |
|    | 85.5| -663          |
|    | 117 | -594          |
|    | 148.5| -509         |
|    | 180 | -405          |

where \( u, v, \) and \( w \) represent the velocity components of the three orthogonal directions in Cartesian coordinates. Under two-dimensional conditions, the vorticity can be simplified to \( \omega_z = \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \). Figures 4(a) and 4(b) are time-averaged vorticity and velocity vector diagram under the condition of the rotation speed of 1500 rpm and the collective pitch of 6° and rotation speed of 1500 rpm and the collective pitch of 9°. It can be seen that, from the time-averaged perspective, the strength of the blade tip vortex has weakened from top to bottom, and the vortex trajectory trend is to move down the axis, while gradually moving inward along the radial direction of the propeller. Comparing Figs. 4(a) and 4(b), it can be found that, under this experimental model and experimental conditions, the blade rotation speed is constant. The vorticity amplitude of the collective pitch of 9° is larger than the collective pitch of 6°, meanwhile, the radial movement is more obvious (the pull force of the rotor blade increases with the increase of the collective pitch).

FIG. 8. Radial and axial displacements of the vortex center of the rotor blade tip with azimuth. (a) Radial movement trajectory, (b) axial movement trajectory.

FIG. 9. Distribution of eigenvalues in the complex plane. (a) \( n = 1500 \) rpm, \( \theta = 6° \) and (b) \( n = 1500 \) rpm, \( \theta = 9° \).
B. Unsteady vorticity field

When the rotor is in the hovering state, its aerodynamic characteristics can be considered as periodic changes; therefore, the flow field characteristic of one blade rotating at a 180° azimuth angle can be used to represent the vortex field characteristic of the blade tip. Figure 5 shows the schematic diagram of 6 different azimuth angles selected in this paper.

Figures 6 and 7 show the transient vorticity contours under the condition of the rotation speed 1500 rpm, collective pitch 6° and rotation speed 1500 rpm, collective pitch 9°. Both figures only show six different azimuth angles (22.5°, 54°, 85.5°, 117°, 148.5°, and 180°; the schematic diagram is shown in Fig. 5).

It can be seen from Fig. 6 that after the blade tip vortex is formed, with the increase of the azimuth angle, the vortices contract to the blade hub and moves down axially. At the same time, the vorticity amplitude of the blade tip gradually decreases with the increase of the azimuth angle, which reflects that the blade tip vortex dissipates continuously due to its viscosity during its motion. Comparing Figs. 6 and 7, it can be found that when the rotation speed is constant, under the same azimuth angle condition, the vorticity of the blade tip becomes larger with the increase of the collective pitch. It is shown that the blade tension increases with the increase of the collective pitch, and the tip induced vortex is stronger.

C. Development trajectory of the blade tip vortex

According to the vorticity center identification standard, Table I gives the values of the blade tip vortex intensity and vorticity center position generated by blade 1 with azimuth at a rotation speed of 1500 rpm and the collective pitch of 6° and 9°, respectively. It can be seen that at the same azimuth angle, the larger the collective pitch, the stronger blade tip vortex; at the same collective pitch, as the azimuth angle increases, the vorticity amplitude continuously decreases.

Figure 8 shows the dimensionless trajectory of the blade tip vortex (Vortex 1) generated by the blade 1 based on the vorticity criterion. It can be seen that as the azimuth angle increases, the vortex moves in the radial and axial directions at the same time. At a certain blade rotation speed, as the collective pitch increases, the blade tension increases, the down washing speed of the blade tip vortex is increased, and the radial contraction is more serious, at the same time, the axial displacement is also faster.

IV. DYNAMIC MODE DECOMPOSITION AND FLOW FIELD RECONSTRUCTION

In this section, the DMD method is used to decompose the dynamic modes of the vorticity field and obtain the time-space characteristics of the blade tip vortex coherent structure. In Sec. IV A, the eigenvalues are obtained after mode decomposition of the blade tip vortex, and the stability of different modes are analyzed. In Sec. IV B, the flow structure represented by different modes is discussed. In Sec. IV C, the modal coefficient development with time is analyzed. In Sec. IV D, the flow fields were reconstructed.

A. Eigenstructure analysis

Figure 9 shows the distribution of the first 15 orders of DMD eigenvalues in the complex plane. Figure 9(a) is at the rotation speed 1500 rpm, θ = 6°.
of 1500 rpm and the collective pitch of $6^\circ$, Fig. 9(b) is at the rotation speed of 1500 rpm and the collective pitch of $9^\circ$.

Figure 9(a) shows that the first-order mode (mode 1, black dot) is a quasi-static mode located on the unit circle and the imaginary part $\text{Re}(\mu_i)$ is zero, indicating that the flow field is quasi-steady, i.e., it does not change with time and does not grow nor decay. Mode 1 is close to the average flow field, and the flow structure is stable with time. The other modes are all located within the unit circle, indicating that they are all stable. Among them, the 2nd, 7th, 10th, and 13th modes (blue dot) are drifting modes because the imaginary parts of their eigenvalues are zero [$\text{im}(\mu_i) = 0$], suggesting that the time-averaged flow variables do not change with time during the linear development of the dynamic system. In addition, mode 3rd and 4th, 5th and 6th, 8th and 9th, 11th and 12th, 14th and 15th modes (red dot) are pairs of conjugate modes, and each pair has a different growth rate.

Figure 9(b) shows the result of eigenstructure analysis for the blade wake flow field in the hovering state of the rotor with a rotation speed of 1500 rpm and the collective pitch of $9^\circ$. Mode 1 is a quasi-static mode, approximately representing the time-averaged flow field.
flow field. 2nd, 5th, 10th, 11th, 14th, and 15th modes are drifting modes. 3rd and 4th, 6th and 7th, 8th and 9th, 12th and 13th modes are pairs of conjugate modes.

B. Modal analysis

Section IV A shows that under this article’s experimental condition, the dynamic mode decomposition reveals that the flow field contains quasi-static modes, drifting modes, and conjugate modes. In order to compare and analyze the effects of different modes, the first 10 orders of modes are classified according to the mode type, and the modes under each category are reordered according to the magnitude, as shown in Table II.

Figures 10(a) and 10(b) show the vorticity field of the quasi-static model at the condition of $n = 1500$ rpm, $\theta = 6^\circ$, and $n = 1500$ rpm, $\theta = 9^\circ$, respectively. It can be seen from the quasi-static modes that the trajectory of the blade tip vortex moves radially along the hub and at the same time moves down in the axial direction, which is consistent with the original time-averaged flow field results in Fig. 4.

Figure 11 shows the drifting modes sorted according to the modal magnitude. Figure 11(a) shows the first, second, and third...
drifting modes at $n = 1500 \text{ rpm, } \theta = 6^\circ$ (mode 2, mode 7, mode 10, shown in Table II, respectively) and Fig. 11(b) shows the first, second, and third drifting modes at $n = 1500 \text{ rpm, } \theta = 6^\circ$ (mode 2, mode 5, mode 10, respectively). It can be seen that as the order of the drifting mode becomes higher, the dissipation of vorticity at the blade tip becomes more obvious. At the same time comparing the same modal order, it can be found that the vorticity of the blade tip vortex increases with the increase of the collective pitch.

Figure 12 shows the conjugate modes sorted according to the modal magnitude. Because the flow fields of a pair of conjugate
modes have identical structures, only one of them is shown in the figure. From the conjugate mode results, it can be seen that the vortex motion trajectory of the blade tip vortex shows an inward and downward movement trend.

Figure 12(a) shows the first, second, and third conjugate modes at a rotation speed of 1500 rpm and the collective pitch of 6°. Figure 12(b) shows the first, second, and third conjugate modes at a rotation speed of 1500 rpm and a collective pitch of 9°.

It can be found that the flow field structure is different in different modes, and the flow fields in different modes reflect vortex coherent structures with different strengths. At a certain rotation speed, the vorticity magnitude of conjugate modes increases as the collective pitch increases.

From comparative studies of conjugate modes with $\theta = 6^\circ$ and $\theta = 9^\circ$, it can be seen from the first conjugate mode (mode 3–4) that with the increment of the collective pitch, the vortex structure and size have changed significantly, the vortex core radius increases, and the vorticity also increases significantly. It can be seen from the second conjugate mode (mode 5–6 vs mode 6–7) that the vortex structure is a regular vortex that alternates positive and negative vortices at $\theta = 6^\circ$. However at $\theta = 9^\circ$, the vortex structure is irregular, and the vortex core radius changes very clearly. From the third conjugate mode (mode 8–9), it can be seen that under a collective pitch of 6°, the vortex strength is obviously weakened and began to break down, and at a collective pitch of 9°, the structure of the vortex is still clear.

C. Modal coefficient analysis

The real part of the modal coefficient reflects the rate of growth/decay of the corresponding mode, it will help to conduct flow stability analysis research. If the real part is positive, the corresponding mode grows; when it is negative, the corresponding mode decays. While the imaginary part contains the frequency information.

Since the imaginary parts of the modal coefficients of both quasi-static modes and drifting modes are zero, and the corresponding modes do not vary with time, Fig. 13 only shows the temporal variation of the conjugate mode modal coefficients. Figures 13(a), 13(c), and 13(e) are the modal coefficients of the first, second, and third conjugate modes, respectively.
and third conjugate modes of the blade tip vortex flow field at $n = 1500 \text{ rpm}, \theta = 6^\circ$. Figures 13(b), 13(d), and 13(f) are the modal coefficients of the first, second, and third conjugate modes at $n = 1500 \text{ rpm}, \theta = 9^\circ$.

From the amplitude of the modal coefficients, it can be clearly seen that when the rotor rotation speed is constant, as the collective pitch increases, the magnitudes of the mode coefficient increases significantly, meanwhile converge slowly. Comparing Figs. 13(a) and 13(b), it can be seen that in the first conjugate mode, the peak value of the modal coefficient is 1820 at $n = 1500 \text{ rpm}, \theta = 6^\circ$, and the peak value of the modal coefficient is 2150 at $n = 1500 \text{ rpm}, \theta = 9^\circ$. Comparing Figs. 13(c) and 13(d), it can be seen that in the second conjugate mode, the peak value of the modal coefficient is 530 at $n = 1500 \text{ rpm}, \theta = 6^\circ$, and it reaches stability by 0.055 s; however, at $n = 1500 \text{ rpm}, \theta = 9^\circ$, the peak value of the modal coefficient is 620, and it stabilizes by 0.07 s. Comparing Figs. 13(e) and 13(f), it can be seen that in the third conjugate mode, at $n = 1500 \text{ rpm}, \theta = 6^\circ$, the peak value of the modal coefficient is 43, it decays to 0 after 0.012 s, but at $n = 1500 \text{ rpm}, \theta = 9^\circ$, the modal coefficient peak value reached 145 and stable after 0.026 s.

The modal coefficient analysis research above shows that the flow field of the rotor blade tip vortex is stable under this experimental condition. However, the amplitude and stability time of the different conjugate mode coefficients are different. In addition, increasing the blade collective pitch angles, the amplitude of the corresponding modal coefficient will be heightened, thus increasing the modal energy. The main reason is that the collective pitch angles increase, and the induced velocity increases, resulting in an increase in vorticity intensity and slower vortex dissipation.

### D. Flowfield reconstruction

After modal analysis of the blade tip vortex by dynamic mode decomposition, the low-order eigenstructure, modal and modal coefficient can be obtained to analyze the characteristics of different modes. In addition, furthermore, the basic flow field can be reconstructed according to the low-order modes. The implementation method contains: After modal decomposition of the flow field, the eigenvalue $\lambda$, modal $\Phi$, and modal amplitude $b$ of different eigenfrequencies can be obtained [A shown in Sec. IV A; $\Phi$ and $b$ shown in Sec. IV B ($b = \Phi$)], then the approximate solution at all future times is given by

$$x(t) = \sum_{k=1}^{n} \Phi_k \exp \left( \frac{\lambda_k}{\Delta t} t \right) b_k.$$  

The flow field reconstruction was performed at $n = 1500 \text{ rpm}, \theta = 6^\circ$ (the azimuths were 22.5$^\circ$, 54$^\circ$, 85.5$^\circ$, 117$^\circ$, 148.5$^\circ$, and 180$^\circ$), which is the same condition as that of the PIV experiment. The original PIV vorticity field result is shown in Fig. 14(a), and the corresponding DMD reconstructed vorticity field is shown in Fig. 14(b).

It can be seen that the reconstructed flow field vortex structure and the original flow field vortex structure are consistent with time and space, indicating that the DMD modal reduction and reconstruction method is accurate in this paper. In this way, the reconstructed flow field can be used to predict the future flow field of the rotor blade tip vortex accurately.

### V. CONCLUSION

In this paper, the flow field of the rotorcraft blade tip vortexes is measured by TR-PIV equipment, and the dynamic mode decomposition method is conducted to research the blade tip vortex modal structure and evolution mechanism in the hovering state. The main conclusions are as follows.

From the results of the blade tip vortex motion characteristics, as the azimuth angle increases, the trajectory of the blade tip vortex moves down the axis, while gradually moving toward the hub in the radial direction. With the increase in the collective pitch, the moving distance increases.

From the dynamic mode decomposition results, different modes represent different flow field coherent structures, and the vortex structure and frequency are also different. This indicates that different modes contribute differently to the overall flow field of the rotorcraft. At a certain rotation speed, as the collective pitch increases, the amplitude of vorticity fluctuations increases significantly, and the mode coefficient converges slowly.

Through different low-order modal reconstruction obtained from the dynamic mode decomposition method in this paper, the approximate solution flow fields can be obtained and could be used to predict future flow fields.

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### DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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