Gradient Estimation for Federated Learning 
over Massive MIMO Communication Systems

Yo-Seb Jeon, Mohammad Mohammadi Amiri, Jun Li, and H. Vincent Poor

Abstract

Federated learning is a communication-efficient and privacy-preserving solution to train a global model through the collaboration of multiple devices each with its own local training data set. In this paper, we consider federated learning over massive multiple-input multiple-output (MIMO) communication systems in which wireless devices train a global model with the aid of a central server equipped with a massive antenna array. One major challenge is to design a reception technique at the central server to accurately estimate local gradient vectors sent from the wireless devices. To overcome this challenge, we propose a novel gradient-estimation algorithm that exploits the sparsity of the local gradient vectors. Inspired by the orthogonal matching pursuit algorithm in compressive sensing, the proposed algorithm iteratively finds the devices with non-zero gradient values while estimating the transmitted signal based on the linear minimum-mean-square-error (LMMSE) method. Meanwhile, the stopping criterion of the proposed algorithm is designed by deriving an analytical threshold for the estimation error of the transmitted signal. We also analyze the computational complexity reduction of the proposed algorithm over a simple LMMSE method. Simulation results demonstrate that the proposed algorithm performs very close to centralized learning, while providing a better performance-complexity tradeoff than linear beamforming methods.

Index Terms

Federated learning, distributed machine learning, massive multiple-input multiple-output (MIMO), compressive sensing, orthogonal matching pursuit (OMP), receive beamforming.

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I. INTRODUCTION

Machine learning has attracted significant interest as a breakthrough for emerging applications of wireless communications [1]–[6]. The fundamental idea of machine learning for wireless communications is to learn a model (e.g., input-output relation) based on large amounts of data and computing power. It has been demonstrated that the learned model can be exploited either to improve the performance of conventional model-based techniques (e.g., [3], [4]) or to describe an unknown input-output relation whose characterization was otherwise challenging due to mathematical intractability (e.g., [5], [6]). The most common and widely adopted form of machine learning is centralized learning in which a central server equipped with sufficient storage and computing power has full access to the entire data set. Unfortunately, centralized learning is infeasible in many applications of wireless communications. The major reason is that data sets are usually generated at wireless devices, but transmitting them to the central server is highly limited by both the amount of radio resources and communication latency allowed in the applications. Particularly, this problem becomes more severe as the data size and the model complexity increase. In addition, sending the data may not be allowed in privacy-sensitive applications such as social networking, e-health, and financial services.

Recently, federated learning has drawn increasing attention as a viable solution to overcome the limitations of centralized learning [7]–[10], in which a global model at a central server is collaboratively trained by multiple devices each with its own local data set. The major advantage of federated learning is a significant reduction in communication overhead as the devices only send an updated model instead of the whole data set. In addition, this approach preserves the privacy of the devices because the data is kept where it is generated while a central server has no direct access to the local data sets. Distributed machine learning also provides similar advantages, but federated learning focuses on a more practical setting which may include unbalanced and non-identically-distributed data sets, unreliable communication, and massively distributed data [7]. Thanks to the advantages and the practicality of federated learning, it has been adopted as a key enabler for emerging applications of wireless communications [11]–[13]. For example, in [12], federated learning is applied to learn the statistical properties of vehicular users in wireless vehicular networks. Another example is introduced in [13] which applies federated learning to learn the locations and orientations of the users in wireless virtual reality networks.

There also exist recent studies that seek to optimize federated learning over wireless com-
munication systems [14–26]. In these studies, the physical characteristics of wireless networks are considered to improve the performance of federated learning under a practical scenario. A device scheduling problem is studied in [14–18] based on various scheduling criteria, while a joint resource allocation and user scheduling problem is tackled in [19]. Transmission and reception techniques for federated learning over wireless channels are developed for a simple Gaussian multiple access channel (MAC) in [20] and for a fading MAC in [21]. A transmission technique for the fading MAC is also proposed in [22] jointly with a device scheduling method. The capacity of wireless channels can be significantly improved by employing multiple antennas at a receiver along with a proper reception technique [27]. To exploit this advantage, several existing works have studied reception techniques at the central server equipped with multiple antennas in federated learning [23–25]. For example, in [23], a joint optimization framework for device selection and receive beamforming is proposed to maximize the number of selected devices while guaranteeing the target mean square error (MSE) requirement. In this framework, effective channel information formed by the receive beamforming is assumed to be known at the wireless devices, but this assumption may not be feasible in practical communication systems. A more realistic scenario is considered in [25] by establishing an optimization framework based on a practical channel estimation method. A linear receive beamforming method that does not require the channel information at the devices is also developed in [24], but the effectiveness of this method is only shown for the asymptotic regime in which the number of the antennas at the server is infinite. Furthermore, both works in [25] and [24] adopt a simple linear beamforming method which is not optimal in terms of the estimation error in general. Therefore, further investigation on the reception technique at the server is still needed to minimize the estimation error at a multi-antenna central server in federated learning over wireless communication systems.

In this work, we study federated learning over massive multiple-input multiple-output (MIMO) systems, in which a central server equipped with a massive number of antennas trains a global model by collaborating with multiple wireless devices each with its own local training data set. In this scenario, each device computes and sends a local gradient vector based on its local data set, while the central server estimates the local gradient vectors to update a global model. Under this scenario, we develop a novel gradient-estimation method that effectively reduces the estimation error at the central server by exploiting the sparsity of the local gradient vector. The major contributions of this paper are summarized as follows:

- We introduce a transmission strategy for wireless devices in federated learning over massive
MIMO systems. The key process of this strategy is to permute the local gradient vectors using different patterns across the wireless devices. We also explain that by applying our transmission strategy, the sparsity of the local gradient vector can still be preserved in a transmitted signal vector even if multiple local gradient vectors are simultaneously transmitted using the same radio resource.

- We introduce three linear beamforming methods, namely maximal ratio combining (MRC), zero forcing (ZF), and linear minimum mean square error (LMMSE), by applying conventional beamforming methods in massive MIMO systems. Particularly to design the LMMSE method, we establish a statistical model for the transmitted signal according to our transmission strategy. We also explain both the advantages and the limitations of these methods under the considered federated learning scenario.

- We propose a gradient-estimation algorithm inspired by the orthogonal matching pursuit (OMP) algorithm in compressive sensing. The key idea of the proposed algorithm is to exploit the sparsity of the local gradient vector, which implies that at each radio resource element, only a small number of devices transmit non-zero gradient values. Based on this idea, the proposed algorithm iteratively finds the devices with the non-zero gradient values at each resource element while estimating the transmitted signal according to these devices. To improve the estimation performance of the proposed algorithm, we modify the LMMSE method by introducing a more accurate statistical model for the transmitted signal. We also design a proper stopping criterion for the proposed algorithm by deriving an analytical threshold for the norm of a residual vector which represents the estimation error at each iteration.

- We analyze and compare the computational complexity of the MRC method, the LMMSE method, and the proposed algorithm. To this end, we characterize the number of real multiplications required by each method. Our key finding is that the complexity reduction achieved by the proposed algorithm compared to the LMMSE method increases as the number of the server’s antennas increases when the transmitted signal is very sparse. Therefore, by analysis, we demonstrate that the proposed algorithm is computationally more efficient than the LMMSE method in the considered federated learning scenario.

- Using simulations, we evaluate the performance gain of the proposed gradient-estimation algorithm for an image classification task using the MNIST dataset [28]. In these simulations, we compare the classification accuracy of the proposed algorithm with those of
Fig. 1. Federated learning over a TDD massive MIMO communication system in which a central
centralized learning and the linear beamforming methods. Simulation results demonstrate
that the performance of the proposed algorithm is very close to the optimal performance
achieved by the centralized learning, while it outperforms other linear beamforming meth-
ods. Meanwhile, it is also shown that the proposed algorithm reduces more than 70%
of the computational complexity of the LMMSE method. Therefore, using simulations,
we show that the proposed algorithm significantly improves the performance-complexity
tradeoff achievable in federated learning over massive MIMO systems, compared to the
LMMSE method.

**Notation:** Upper-case and lower-case boldface letters denote matrices and column vectors,
respectively. $\mathbb{E}[\cdot]$ is the statistical expectation, $\mathbb{P}(\cdot)$ is the probability, $(\cdot)^{\mathsf{T}}$ is the transpose, $(\cdot)^{\mathsf{H}}$ is the conjugate transpose, $\lceil \cdot \rceil$ is the ceiling function, $\lfloor \cdot \rfloor$ is the floor function, and $|\cdot|$ is the absolute value. $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote real and imaginary components, respectively. $|\mathcal{A}|$ is the cardinality of set $\mathcal{A}$. $(\mathbf{a})_i$ represents the $i$-th element of vector $\mathbf{a}$. $\|\mathbf{a}\| = \sqrt{\mathbf{a}\mathbf{a}^{\mathsf{H}}}$ is the Euclidean norm of vector $\mathbf{a}$. $\mathcal{CN}(\mu, \Gamma)$ represents the distribution of a circularly symmetric complex Gaussian random vector with mean vector $\mathbf{\mu}$ and covariance matrix $\Gamma$. $\mathbf{0}_n$ and $\mathbf{1}_n$ are an $n$-dimensional vectors whose elements are zero and one, respectively.
II. SYSTEM MODEL

We consider federated learning over a time-division-duplex (TDD) massive MIMO communication system in which a central server equipped with $M$ antennas trains a global model by collaborating with $K$ single-antenna wireless devices, as illustrated in Fig. 1. In this system, the server and the wireless devices share a global model (e.g., neural network) represented by a parameter vector $\mathbf{w} \in \mathbb{R}^{N_w}$, but only the devices have training data samples to train the global model. We denote the local data set available at device $k$ by $B_k$ for $k \in \mathcal{K} = \{1, \ldots, K\}$ which consists of $|B_k|$ training data samples. We model the wireless channel from the devices to the server by an $L$-tap channel impulse response (CIR) while assuming a block-fading channel in which the CIR taps remain constant within each communication round. We also assume that the channel is perfectly known at the server via an uplink channel training. We adopt an orthogonal frequency division multiplexing (OFDM) modulation with $N_{\text{sub}}$ subcarriers to deal with inter-symbol interference in the wireless channel.

We assume that the central server adopts a gradient-based iterative algorithm (e.g., stochastic gradient descent algorithm or Adam optimizer) to train the global model. Each iteration of the algorithm corresponds to one communication round that consists of uplink and downlink phases, as illustrated in Fig. 1. Let $T$ be the number of iterations and $\mathbf{w}_t$ be the parameter vector at iteration $t$ of the algorithm. During the downlink phase of communication round $t$, the server transmits $\mathbf{w}_t$ to all the wireless devices. Whereas, during the uplink phase, each device transmits its local gradient vector computed based on its local data set as well as the global model $\mathbf{w}_t$. In this work, we focus only on a transmission/reception strategy for the uplink phase, while assuming that the downlink transmission is error free, as in [20]–[24]. Under this assumption, all devices have a globally consistent parameter vector $\mathbf{w}_t$ for all $t \in \mathcal{T} = \{1, \ldots, T\}$. 

Fig. 2. A transmission strategy adopted by wireless devices to send local gradient vectors to a central server.
We now present a transmission strategy employed at each device during the uplink phase. To model the limited computing power of the wireless devices, we assume that a local gradient vector at each device is computed over only a small fraction of its local data set. Let $\mathcal{B}_k[t] \subset \mathcal{B}_k$ be a set of the samples selected by device $k$ to compute the local gradient vector at communication round $t$. Then the local gradient vector computed at device $k$ is given by

$$g_k[t] = \frac{1}{|\mathcal{B}_k[t]|} \sum_{b \in \mathcal{B}_k[t]} \nabla f(w_t, b), \quad (1)$$

where $\nabla f(\cdot, b)$ is the gradient of a loss function computed for the training data sample $b \in \mathcal{B}_k$ defined by the learning task. Note that the number of the selected samples at each round may vary across the devices as their computing power can be different. After computing the local gradient vector, device $k$ constructs a transmitted signal, namely $x_k[t]$, by applying the permutation and the scaling operations to $g_k[t]$: \(x_k[t] = \sqrt{\frac{N_w}{\|g_k[t]\|^2}} \mathbf{P}_k g_k[t] \quad (2)\)

where $\mathbf{P}_k \in \mathbb{R}^{N_w \times N_w}$ is a random permutation matrix employed at device $k$. The scaling operation in (2) is adopted to ensure that every device has the same transmit power of $\|x_k[t]\|^2 = N_w$ at each communication round. Also, the motivation and the advantages of using the permutation will be discussed in the sequel. Considering the OFDM modulation, the $n$-th element of $x_k[t]$, namely $x_k[t, n] \in \mathcal{R}$, is transmitted via the $n$-th resource element of an uplink OFDM block, which corresponds to the $f_n$-th subcarrier of the $t_n$-th OFDM symbol where $f_n = n - (t_n - 1)N_{\text{sub}}$ and $t_n = \lceil \frac{n}{N_{\text{sub}}} \rceil$. The number of the OFDM symbols is set as $\lceil \frac{N_w}{N_{\text{sub}}} \rceil$ to transmit $N_w$ elements. Our transmission strategy is illustrated in Fig. 2.

We also describe a reception strategy employed at the server during the uplink phase. The received signal associated with the $n$-th resource element of the uplink OFDM block is expressed as

$$y_c[t, n] = \sum_{k=1}^{K} h_{c, k}[t, n] x_k[t, n] + z_c[t, n], \quad (3)$$

where $h_{c, k}$ is the channel frequency response vector of device $k$ and $z_c[t, n] \in \mathbb{C}^M$ is the noise signal at the $f_n$-th subcarrier of the $t_n$-th OFDM symbol. We assume that the noise signals at
different radio resources are independent and identically distributed (i.i.d.) as $\mathcal{CN}(0, M, \sigma^2_I M)$. The real-domain equivalent representation of $y^c[t, n]$ is given by

$$y^c[t, n] = \sum_{k=1}^{K} h_k^c[t, n] x_k^c[t, n] + z[t, n],$$  \hspace{1cm} (4)$$

where

$$y^c[t, n] = [\text{Re}(y^c[t, n]), \text{Im}(y^c[t, n])]^T,$$

$$h_k^c[t, n] = [\text{Re}(h_k^c[t, n]), \text{Im}(h_k^c[t, n])]^T,$$

$$z[t, n] = [\text{Re}(z^c[t, n]), \text{Im}(z^c[t, n])]^T.$$ 

Note that $z[t, n] \sim \mathcal{CN}(0, 2M, \sigma^2_I 2M)$ where $\sigma^2 = \frac{\sigma^2_c}{2}$. The above representation can be rewritten as follows:

$$y^c[t, n] = H^c[t, n] x^c[t, n] + z[t, n],$$  \hspace{1cm} (5)$$

where

$$H^c[t, n] = [h_1^c[t, n], h_2^c[t, n], \ldots, h_K^c[t, n]],$$

$$x^c[t, n] = [x_1^c[t, n], x_2^c[t, n], \ldots, x_K^c[t, n]]^T.$$ 

Based on the received signals $\{y^c[t, n]\}_{n=1}^{N_w}$, the server estimates the transmitted signals $\{x^c[t, n]\}_{n=1}^{N_w}$ which provide information of the local gradient vectors sent from the wireless devices. The details of this estimation process will be discussed in Secs. III and IV. Let $\hat{x}_k[t, n]$ be the estimate of the transmitted signal sent from device $k$ at the $n$-th resource element of the uplink OFDM block. By aggregating the estimates of the transmitted signals associated with all resource elements, the server reconstructs the local gradient vector sent from device $k$ as follows:

$$\hat{g}_k[t] = \sqrt{\frac{\|g_k(t)\|^2}{N_w}} P_k^T \hat{x}_k[t],$$  \hspace{1cm} (6)$$

where $\hat{x}_k[t] = [\hat{x}_k[t, 1], \ldots, \hat{x}_k[t, N_w]]^T$. In (6), we assume that the permutation matrix $P_k$ and the norm of the local gradient vector $\|g_k(t)\|$ for all $k \in \mathcal{K}$ are known at the server.\footnote{Although each device needs to send one additional real value to convey the information of the norm of its local gradient vector, this has a negligible impact on the overall uplink transmission because the size of the local gradient vector $N_w$ is much larger than one.}
reconstructing all the local gradient vectors, the server aggregates these vectors to obtain the
global gradient vector defined as

$$\bar{g}[t] = \frac{1}{\sum_{j=1}^{K} |B_j[t]|} \sum_{k=1}^{K} |B_k[t]| \hat{g}_k[t].$$

(7)

The computing power of each device may not change during the training process, so we assume that \(\{ |B_k[t]| \}_{t \in T} \) is fixed and known at the central server. The global gradient vector in (7) is utilized to update the parameter vector \(w_t\). For example, if the central server adopts a gradient descent algorithm, the update of the parameter vector is expressed as

$$w_{t+1} \leftarrow w_t - \eta_t \bar{g}[t],$$

(8)

where \(\eta_t\) represents the learning rate at iteration \(t\).

A major bottleneck that limits the performance of federated learning over the massive MIMO system is the mismatch between the local gradient vectors sent from the wireless devices and their estimates obtained at the server (i.e., \(\hat{g}_k[t] \neq g_k[t]\)). The main cause of this mismatch is inter-user interference (IUI) caused by simultaneous transmission of multiple devices, in addition to channel fading and the noise signal that distorts and corrupts the transmitted signal, respectively. Such mismatch may harm both the learning accuracy and the convergence rate of federated learning. Therefore, to resolve this problem, it is essential to develop an efficient local gradient estimation method that can accurately estimate all the local gradient vectors computed at the wireless devices.

### III. Linear Beamforming Approach

One simple solution for the local gradient estimation problem discussed in Sec. II is to apply conventional linear beamforming methods developed to solve a MIMO data detection problem \[30\]. This approach is possible because the received signal in (5) has an equivalent form with the received signal in the MIMO detection problem. Inspired by this observation, in this section, we introduce three linear beamforming methods that can be applied to solve the local gradient estimation problem in federated learning over massive MIMO systems. We then discuss the advantages and the limitations of these methods. Since the receive beamforming is separately applied to the received signal at each resource element of each communication round, in this section, we omit the indexes \(t\) and \(n\) to simplify the notation.
A. Maximal Ratio Combining (MRC)

The MRC method aims to maximize the power of the desired signal by aligning the direction of the receive beamforming into the channel direction. The receive beamforming matrix of the MRC method is given by

\[
F_{\text{MRC}} = \text{diag} \left( \frac{1}{\|h_1\|^2}, \ldots, \frac{1}{\|h_K\|^2} \right) H^T, \quad (9)
\]

where \( \text{diag}(a_1, \ldots, a_N) \) is an \( N \times N \) diagonal matrix whose \( i \)-th diagonal element is \( a_i \). Let \( \hat{x}_{\text{MRC}} = F_{\text{MRC}}y \) be the estimate of the transmitted signal obtained from the MRC method. Then the \( k \)-th element of \( \hat{x}_{\text{MRC}} \) is expressed as

\[
\hat{x}_{\text{MRC},k} = x_k + \frac{1}{\|h_k\|^2} \sum_{j \neq k} h_k^T h_j x_j + \frac{1}{\|h_k\|^2} h_k^T z, \quad (10)
\]

where \( x_k \) is the \( k \)-th element of \( x \). As can be seen in (10), the obtained estimate consists of not only the desired signal \( x_k \), but also an IUI signal and an effective noise signal corresponding to the second and the third terms in the right hand side (RHS) of (10). Fortunately, both the IUI and the noise signals vanish as the number of the server’s antennas increases by the central limit theorem [31]. Therefore, any non-zero gradient vector can be accurately estimated at the server by applying the MRC method when \( M \) is sufficiently large. This advantage, however, is not attained for a low-to-moderate number of the server’s antennas, so in this case, the MRC method is suboptimal in terms of the estimation performance.

B. Zero Forcing (ZF)

Unlike the MRC method, the basic idea of the ZF method is to minimize the power of the IUI signal by aligning the direction of the receive beamforming into the direction of the null space of the IUI signal. Because of the null-space constraint, the ZF method is applicable only when the number of the server’s antennas is not smaller than the number of the wireless devices (i.e., \( M \geq K \)). The receive beamforming matrix of the ZF method is given by

\[
F_{\text{ZF}} = (H^T H)^{-1} H^T. \quad (11)
\]

From (11), the estimate of the transmitted signal obtained from the ZF method is expressed as

\[
\hat{x}_{\text{ZF}} = F_{\text{ZF}}y = x + (H^T H)^{-1} H^T z. \quad (12)
\]
As can be seen in (12), the ZF method perfectly eliminates the IUI signal, so this method outperforms the MRC method in high signal-to-noise-ratio (SNR) regime. However, the ZF method also boosts the power of the effective noise signal whose expected power is given by

$$\mathbb{E}[\|(H^TH)^{-1}H^Tz\|^2] = \sigma^2\text{Tr}\left((H^TH)^{-1}\right).$$

(13)

Therefore, in low-to-moderate SNR regime, the ZF method may perform worse than the MRC method depending on the orthogonality of the channel directions of the wireless devices. The performance difference between the ZF and the MRC methods becomes marginal as the number of the server’s antennas increases.

C. Linear Minimum Mean Square Error (LMMSE)

The LMMSE method is an optimal linear beamforming method that minimizes the MSE between the transmitted signal and its estimate. Under the premise that $\mathbb{E}[x] = 0_K$ and $\mathbb{E}[xx^T] = I_K$, the receive beamforming matrix of the LMMSE method is given by

$$F_{\text{LMMSE}} = H^T(HH^T + \sigma^2I_{2M})^{-1}. \quad (14)$$

Then the MSE of the estimated transmitted signal obtained from the LMMSE method is obtained as

$$\mathbb{E}[\|x - \hat{x}_{\text{LMMSE}}\|^2] = \text{Tr}\left(I_K - H^T(HH^T + \sigma^2I_{2M})^{-1}H\right)$$

$$= \text{Tr}\left(\left(\frac{1}{\sigma^2HH^T + I_K}\right)^{-1}\right), \quad (15)$$

where $\hat{x}_{\text{LMMSE}} = F_{\text{LMMSE}}y$. Note that when using the ZF method, the MSE of the estimated transmitted signal is given by

$$\mathbb{E}[\|x - \hat{x}_{\text{ZF}}\|^2] = \sigma^2\text{Tr}\left((H^TH)^{-1}\right). \quad (16)$$

The comparison between (15) and (16) shows that the LMMSE method outperforms the ZF method in low-SNR regime, while the MSE difference between them reduces as the SNR increases.

It is important to notice that to make the LMMSE method applicable, the information of the first-order and the second-order statistics of the transmitted signal is required at the server. Unfortunately, it is difficult to characterize the statistical behavior of the gradient vector in the problem under consideration. As an alternative approach, we approximate the LMMSE method by
modeling the mean vector and the covariance matrix of the transmitted signal as $E[x] = 0_K$ and $E[xx^T] = I_K$, respectively. In this modeling, we assume that each element of the local gradient vector computed at device $k$ has the same expected power (i.e., $E[|g_k[t,n]|^2] = \frac{\|g_k[t]\|^2}{N_w}$) and also that its mean value is zero (i.e., $E[g_k[t,n]] = 0$). We adopt these assumptions because any biased assumption on the distribution of the local gradient values may increase the mismatch between the modeled and the true distributions. In addition, we use the fact that the local gradient values of different devices are statistically uncorrelated because of the random permutation matrix in (2) employed at each device. Note that this fact holds even when all the devices have the same local data set.

D. Discussion

As we already discussed, the common advantage of the above beamforming methods is that they enable the server to accurately estimate the local gradient vectors when both the number of the server’s antennas and the SNR are sufficiently large. In this special case, federated learning over the massive MIMO system may achieve the same convergence rate with centralized learning without suffering from any performance degradation. In general, however, the performance of both the MRC and the ZF methods is degraded by the use of non-optimal design criterion, while the LMMSE method suffers from the mismatch in the statistical information. In addition, although the LMMSE method is expected to perform better than the MRC and the ZF methods, it requires high computational complexity that increases with the number of the server’s antennas, so this method may not be affordable in practical massive MIMO systems. Considering these limitations, in what follows, we will develop a novel reception technique for federated learning over massive MIMO systems that can significantly improve the performance-complexity achieved by the linear beamforming methods.

IV. COMPRESSIVE SENSING APPROACH

Compressive sensing is a well-known technique to estimate an unknown sparse signal when its linear measurement is only known [33]. In this section, we propose a new local gradient estimation method inspired by compressive sensing in order to overcome the limitations of the linear beamforming methods discussed in Sec. III-D. We then analyze and compare the computational complexity of the proposed algorithm and the linear beamforming methods. To simplify the notation, we omit the indexes $t$ and $n$, as done in the previous section.
A. Sparsity of Local Gradient Vectors

The key motivation of the proposed algorithm comes from the sparsity of the local gradient vectors. The intuition behind this property is that the gradient w.r.t. a single training data sample may change only a small portion of a global model (e.g., neural network) as the training algorithm repeats provided that the global model is properly chosen. Since the wireless devices participating in the federated learning framework have a limited computing power and/or a stringent latency constraint, in most cases, each device computes the gradient w.r.t. a single data sample (e.g., a stochastic setting) or a small number of data samples (e.g., a mini-batch setting). In such cases, the local gradient vector sent from each device is likely to be sparse. Particularly when employing a ReLU activation function, which is the most popular activation function in deep learning, this sparsity becomes even higher since the gradient of the ReLU function is zero for any negative-valued input. Furthermore, even if the local gradient vectors are not sparse, they can be made sparse explicitly by applying some sparsification strategies (e.g., [10], [32]) developed to reduce the communication overhead in federated learning. Considering all these facts, we shall assume that the local gradient vectors sent from the wireless devices are sparse, i.e., \( \left| \{ g_k[t, n] \neq 0 \mid n \in \{1, \ldots, N_w \} \} \right| \ll N_w \) for \( k \in K \).

It is also important to notice that although the local gradient vectors are sparse, the transmitted signal \( x[t, n] \) in (5) may not be sparse if all the devices have the same sparsity pattern. Particularly this problem may happen in the i.i.d. data distribution case, in which all the local training data sets have an identical distribution. Fortunately, even in such extreme case, it is possible to enforce different devices to have different sparsity patterns by adopting random permutation matrices before the transmission of the gradient vectors as in (2). Therefore, the transmitted signal can also be made sparse by applying our transmission strategy introduced in Sec. II, regardless of the choice of the data distribution.

Motivated by the sparsity assumption discussed above, we rewrite the received signal in (5) as

\[
y = \sum_{k \in K_{\text{true}}} h_k x_k + z, \tag{17}
\]

\(^2\)The global model can be assumed to be properly chosen, if the model is complex enough to reduce all the empirical losses w.r.t. all the training samples, not a single sample only.
where $\mathcal{K}_{\text{true}}$ is a true support set representing the set of device indices that have non-zero gradient values, defined as

$$
\mathcal{K}_{\text{true}} = \{ x_k \neq 0 \mid k \in \mathcal{K} \}.
$$

(18)

Then by the sparsity, the size of the true support set is expected to be much smaller than the number of the wireless device (i.e., $|\mathcal{K}_{\text{true}}| \ll K$).

**B. OMP-LMMSE Algorithm**

The basic idea of the proposed algorithm is to exploit the sparsity of the local gradient vectors by properly modifying the orthogonal matching pursuit (OMP) algorithm in compressive sensing. OMP is an iterative greedy algorithm that selects an index with the maximum correlation to the residual part of the output signal at each iteration, while updating the residual by subtracting the effect of the selected indices [33]–[35]. Due to its fair performance with low computational complexity, OMP has been widely adopted in many applications of compressive sensing [33]–[35]. In this work, inspired by the OMP algorithm, we design an iterative greedy algorithm to find the true support set $\mathcal{K}_{\text{true}}$ and the corresponding estimate of the transmitted signal. The major differences of the proposed algorithm to the conventional OMP algorithm in [34], [35] are as follows:

- The proposed algorithm adopts the LMMSE method when estimating the transmitted signal for a given support set, while the conventional algorithm adopts a simple least-squares method. The LMMSE method is shown to generalize the least-squares method by considering both the second-order statistic of the transmitted signal and the effect of the noise signal. Particularly, the LMMSE method is properly modified in the proposed algorithm by establishing an accurate statistical model based on our transmission strategy.

- The proposed algorithm adopts a stopping criterion by setting a threshold on the norm of the residual vector. Although the same form of the stopping criterion has been widely adopted in the conventional algorithm, the proposed algorithm employs a different threshold which is analytically derived in accordance with the LMMSE estimate of the transmitted signal. In this derivation, the physical characteristic of the massive MIMO channel is also exploited.

We refer to the proposed algorithm as **OMP-LMMSE** since it combines the conventional OMP algorithm with the LMMSE method.
1) Definitions: A support set at iteration $i$, denoted by $S_i \subset K$, is a set of device indices that have been selected as the members of the true support set until iteration $i$. An estimated transmitted signal at iteration $i$, denoted by $\hat{x}^{(i)} \in \mathbb{R}^{|S_i|}$, is the estimate of the transmitted signal when assuming $K_{true} = S_i$. A residual vector at iteration $i$ is defined as $r_i = y - H^{(i)} \hat{x}^{(i)}$ which is a residual part of the received signal after subtracting the effect of the estimated transmitted signal at iteration $i$, where $H^{(i)}$ is defined as

$$H^{(i)} = [h_{S_i(1)}, \cdots, h_{S_i(i)}].$$

(19)

2) Support Set Update: At iteration $i$, the OMP-LMMSE algorithm selects the index of the device whose normalized channel has the maximum correlation with the previous residual vector $r_{i-1}$, by using the following criterion:

$$k_i^* = \arg\max_{k \in S_{i-1}} |\tilde{h}_k^T r_{i-1}|,$$

(20)

where $\tilde{h}_k = \frac{1}{||h_k||} h_k$. Once the best index is selected, the support set $S_{i-1}$ is updated by adding the selected index, i.e.,

$$S_i \leftarrow S_{i-1} \cup \{k_i^*\}.$$

(21)

The promising feature of the index selection criterion in (20) is that it correctly finds the device index with the maximum effective SNR, defined as $\rho_k = \frac{1}{\sigma^2} ||h_k||^2 |x_k|^2$, when the number of the server’s antennas is sufficiently large. To see this, we characterize the correlation between the channel vector of device $k$ and the residual vector as

$$\frac{1}{M} h_k^T r_{i-1} = \frac{1}{M} h_k^T (y - H^{(i)} \hat{x}^{(i)})$$

$$= \frac{||h_k||^2}{M} x_k + \sum_{s \in S_{i-1} \setminus \{k\}} \frac{h_k^T h_s}{M} x_s + \sum_{s \in S_{i-1}} \frac{h_k^T h_s}{M} (x_s - \hat{x}^{(i)}_s) + \frac{h_k^T z}{M}.$$ 

(22)

By the law of large numbers, the correlation in (22) approaches $x_k$ as $M$ increases; thereby, the metric in (20) can be approximated as

$$|\tilde{h}_k^T r_{i-1}| \approx ||h_k|| |x_k| \propto \sqrt{\rho_k},$$

(23)

for $M \gg 1$. By the above reason, the support set of the OMP-LMMSE algorithm at iteration $i$ is expected to find the $i$-th dominant element in the true support set $K_{true}$ in the massive MIMO system.
3) Transmitted Signal Estimation: After updating the support set at iteration $i$, the OMP-LMMSE algorithm estimates the transmitted signal associated with the current support set under the assumption of $K_{\text{true}} = S_i$. For this step, we adopt the LMMSE method in Sec. III-C, but based on a more accurate statistical model than the original LMMSE method. More precisely, we model the second-order statistic of the transmitted signal from device $k^*_i$ as

$$\mathbb{E}[|x_{k^*_i}|^2] = \alpha_{k^*_i} = \frac{1}{\|h_{k^*_i}\|_2^2} |\tilde{h}_{k^*_i}^T r_{i-1}|^2.$$  

(24)

This modeling can be more accurate than the modeling of $\mathbb{E}[|x_k|^2] = 1$ adopted in the LMMSE method, because $\alpha_{k^*_i}$ in (24) approaches $|x_k|^2$ as the number of the server’s antenna increases from (23).

Using this strategy, the estimate of the transmitted signal obtained at iteration $i$ is expressed as

$$\hat{x}^{(i)} = R_x^{(i)} (H^{(i)})^T \left( H^{(i)} R_x^{(i)} (H^{(i)})^T + \sigma^2 I_{2M} \right)^{-1} y,$$  

(25)

where $R_x^{(i)} = \text{diag}(\alpha_{S_1(i)}, \ldots, \alpha_{S_i(i)})$. To further reduce the computational complexity, we rewrite the estimate in (25) as

$$\hat{x}^{(i)} = R_x^{(i)} (H^{(i)})^T \Omega_i y,$$  

(26)

where

$$\Omega_i = \left( H^{(i)} R_x^{(i)} (H^{(i)})^T + \sigma^2 I_{2M} \right)^{-1}.$$  

(27)

Then $\Omega_i$ in (27) can be computed in a recursive manner:

$$\Omega_i = \left( \Omega_{i-1}^{-1} + \alpha_{k^*_i} h_{k^*_i}^T h_{k^*_i} \right)^{-1} = \Omega_{i-1}^{-1} - \frac{\alpha_{k^*_i} \Omega_{i-1} h_{k^*_i} h_{k^*_i}^T \Omega_{i-1}}{1 + \alpha_{k^*_i} h_{k^*_i}^T \Omega_{i-1} h_{k^*_i}}.$$  

(28)

Therefore, the OMP-LMMSE algorithm reduces the computational complexity required for estimating the transmitted signal, by recursively updating $\Omega_i$ at each iteration.

The transmitted signal estimation in the OMP-LMMSE algorithm generalizes those in the conventional OMP algorithm in [34], [35] and the LMMSE beamforming method in Sec. III-C. First of all, the estimated transmitted signal of the conventional OMP algorithm at iteration $i$ is expressed as

$$\hat{x}^{(i)}_{\text{conv}} = \left( (H^{(i)})^T H^{(i)} \right)^{-1} (H^{(i)})^T y.$$  

(29)
The comparison between (25) and (29) shows that the estimate of the conventional OMP algorithm is the special case of that of the OMP-LMMSE algorithm when the noise power is zero and the expected power of the transmitted signal is equal to one. Also, the estimated transmitted signal of the LMMSE method in Sec. III-C is given by

\[ \hat{x}_{\text{LMMSE}}^{(i)} = (H^{(i)} \cdot (H^{(i)} + \sigma^2 I_{2M})^{-1} y. \] (30)

By comparing (30) with (25), it is also shown that the OMP-LMMSE algorithm generalizes the LMMSE method by capturing a more general case of \( R_x^{(i)} \neq I_{|S_i|} \).

4) Stopping Criterion: A perfect stopping criterion for the OMP-LMMSE algorithm is very difficult to derive without assuming perfect information of the true support set and the true transmitted signal at the server. As an alternative approach, we design a stopping criterion that is expected to act optimally under an ideal scenario, in which 1) all elements of the true support set are correctly selected during the first \(|K_{\text{true}}|\) iterations; and 2) the transmitted signal associated with the true support set follows the statistical model assumed in the proposed algorithm, i.e., \( E[x_k] = 0 \) and \( E[|x_k|^2] = \alpha_k \) for \( k \in K_{\text{true}} \). Although this scenario is ideal, it can also be realized in a massive MIMO system because both conditions hold from (23) and (24) when the number of the server’s antennas is sufficiently large.

For the ideal scenario discussed above, we design a stopping criterion by deriving an analytical threshold for the norm on the residual vector. In this scenario, the support set at iteration \( i \) belongs to one of the following cases:

- **Case 1:** The support set at iteration \( i \) is a subset of the true support set, but not equal to the true set, i.e., \( S_i \subseteq K_{\text{true}} \) and \( S_i \neq K_{\text{true}} \).
- **Case 2:** The support set at iteration \( i \) is equal to the true support set, i.e., \( S_i = K_{\text{true}} \).
- **Case 3:** The support set at iteration \( i \) includes the true support set and has one more element than the true set, i.e., \( S_i = K_{\text{true}} \cup \{ k_i^* \} \).

Clearly, the optimal decision for the OMP-LMMSE algorithm is to stop if the current set belongs to **Case 2**. Motivated by this, we derive a condition that determines the case to which the current support set belongs. To achieve this goal, we characterize the expected value of the norm squared of the residual vector in these three cases. The result of this characterization is given in the following proposition:
Proposition 1. If the support set at iteration $i$ belongs to Case $p$, the expected value of the norm squared of the current residual vector is given by

$$\mathbb{E}[\|r_i\|^2] = E_p^{(i)},$$

for $p \in \{1, 2, 3\}$, where

$$E_1^{(i)} = \sigma^4 \left[ \text{Tr}(\Omega_i) + \sum_{k \in K_{\text{true}} \setminus S_i} \alpha_k \|\Omega_i h_k\|^2 \right],$$

$$E_2^{(i)} = \sigma^4 \text{Tr}(\Omega_i),$$

$$E_3^{(i)} = \sigma^4 \left[ \text{Tr}(\Omega_i) - \frac{\text{Tr}(\Omega_{i-1}) - \text{Tr}(\Omega_i)}{1 + \alpha_k^*, h_k^* \Omega_{i-1} h_k^*} \right],$$

provided that $\mathbb{E}[x_k] = 0$ and $\mathbb{E}[\|x_k\|^2] = \alpha_k$ for $k \in K_{\text{true}}$.

Proof: See Appendix A.

Proposition 1 shows that $\mathbb{E}[\|r_i\|^2]$ decreases as the algorithm proceeds, while $E_1^{(i)} \geq E_2^{(i)} \geq E_3^{(i)}$. Therefore, the current support set is expected to belong to Case 3 if the norm squared of the residual vector is closer to $E_3^{(i)}$ than to $E_2^{(i)}$. Utilizing this observation, we set the stopping criterion of the OMP-LMMSE algorithm as $\|r_i\|^2 \leq E_{\text{th}}^{(i)}$, where

$$E_{\text{th}}^{(i)} = \frac{1}{2} (E_2^{(i)} + E_3^{(i)})$$

$$= \sigma^4 \left[ \text{Tr}(\Omega_i) - \frac{\text{Tr}(\Omega_{i-1}) - \text{Tr}(\Omega_i)}{2(1 + \alpha_k^*, h_k^* \Omega_{i-1} h_k^*)} \right].$$

The above criterion checks whether the support set at iteration $i$ belongs to Case 3 or not. This is equivalent to checking whether the support set at the previous iteration $i-1$ belongs to Case 2 or not. Therefore, after the algorithm stops by satisfying $\|r_i\|^2 \leq E_{\text{th}}^{(i)}$, the final estimate of the transmitted signal is set as the previous estimate obtained at iteration $i-1$, instead of the current estimate.

5) Summary: In Algorithm 1, we summarize the overall process of federated learning over the massive MIMO system when employing the proposed OMP-LMMSE algorithm. In this algorithm, Steps 3~4 and Steps 5~30 are associated with the downlink and uplink phases, respectively. Also, Steps 11~26 correspond to the OMP-LMMSE algorithm, which can be
Algorithm 1 Federated learning over the massive MIMO system with the OMP-LMMSE algorithm.

1: Initialize the weight vector $\mathbf{w}_1$.

2: for $t = 1$ to $T$ do

3: \hspace{1em} At the server:

4: \hspace{2em} Transmit $\mathbf{w}_t$ to $M$ wireless devices.

5: \hspace{1em} At device $k \in \mathcal{K}$:

6: \hspace{2em} Compute $\mathbf{g}_k[t]$ from (1).

7: \hspace{2em} Compute $\mathbf{x}_k[t]$ from (2).

8: \hspace{2em} Transmit $\mathbf{x}_k[t]$ to the server.

9: \hspace{1em} At the server:

10: \hspace{2em} for $n = 1$ to $N_w$ do

11: \hspace{3em} Set $\mathbf{h}_k = \mathbf{h}_k[t,n]$ and $\tilde{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ for $k \in \mathcal{K}$.

12: \hspace{3em} Set $\mathbf{S}_0 = \emptyset$, $\mathbf{r}_0 = \mathbf{y}[t,n]$, and $\mathbf{\Omega}_0 = \frac{1}{\sigma^2} \mathbf{I}_{2M}$.

13: \hspace{3em} for $i = 1$ to $K$ do

14: \hspace{4em} Find $k_i^* = \arg\max_{k \in \mathcal{S}_{i-1}} |\mathbf{h}_k^T \mathbf{r}_{i-1}|$.

15: \hspace{4em} Set $\mathbf{S}_i = \mathcal{S}_{i-1} \cup \{k_i^*\}$ and $\alpha_{k_i^*} = \frac{1}{\|\mathbf{h}_{k_i^*}\|} |\mathbf{h}_{k_i^*}^T \mathbf{r}_{i-1}|^2$.

16: \hspace{4em} Compute $\mathbf{\Omega}_i$ from (28).

17: \hspace{4em} Set $\mathbf{r}_i = \sigma^2 \mathbf{\Omega}_i \mathbf{y}[t,n]$.

18: \hspace{4em} Compute $E_{th}^{(i)}$ from (35).

19: \hspace{4em} if $\|\mathbf{r}_i\|^2 < E_{th}^{(i)}$ then

20: \hspace{5em} Update $i \leftarrow i - 1$.

21: \hspace{4em} Break the loop.

22: \hspace{4em} end if

23: \hspace{4em} end for

24: \hspace{4em} Compute $\hat{x}^{(i)} = \mathbf{R}_{x}^{(i)} (\mathbf{H}^{(i)})^T \mathbf{\Omega}_i \mathbf{y}[t,n]$.

25: \hspace{4em} Set $\hat{x}_{\mathcal{S}_i(k)}[t,n] = \hat{x}^{(i)}_k$ for $k \in \{1, \ldots, |\mathcal{S}_i|\}$.

26: \hspace{4em} Set $\hat{x}_k[t,n] = 0$ for $m \notin \mathcal{S}_i$.

27: \hspace{4em} end for

28: \hspace{1em} Compute $\mathbf{g}_k[t]$ from (6) for $k \in \mathcal{K}$.

29: \hspace{1em} Compute $\bar{\mathbf{g}}[t]$ from (7).

30: \hspace{1em} Update $\mathbf{w}_{t+1}$ based on $\bar{\mathbf{g}}[t]$.

31: end for
changed according to the reception strategy adopted by the central server. In Step 17, the residual vector $r_i$ is determined using the following equality:

$$r_i = y[t, n] - H^{(i)}\hat{x}^{(i)}$$

$$= \left(I_{2M} - H^{(i)}R_x^{(i)}(H^{(i)})^T\Omega_i\right)y[t, n]$$

$$= \sigma^2\Omega_iy[t, n], \quad (36)$$

where the second equality is obtained from (26). By using the expression in (36), computing the estimated transmitted signal $\hat{x}^{(i)}$ is not required at each iteration. Instead, $\hat{x}^{(i)}$ is computed only once after the OMP-LMMSE algorithm ends (see Step 23). Using this strategy, the overall computational complexity of the OMP-LMMSE algorithm is further reduced.

### C. Computational Complexity Analysis

We analyze the computational complexity of three local gradient estimation methods: MRC in Sec. III-A, LMMSE in Sec. III-C and OMP-LMMSE in Sec. IV-B. To this end, we compute the number of real multiplications required by each method. The result is given in Table I, where $I_{stop}$ is the number of iterations in the OMP-LMMSE algorithm before it stops. In Table I we also present the complexity for a large-scale case with $M \gg 1$ and $K \gg 1$ which is the region of interest in massive MIMO systems. The complexity of the LMMSE method is determined by considering a recursive computation of the inverse matrix in (14), as given in (23).

Table I shows that the OMP-LMMSE algorithm has a significantly lower complexity compared to the LMMSE method, when the transmitted signal is very sparse. More precisely, in the large-
scale case, the ratio of the complexity of the OMP-LMMSE algorithm to that of the LMMSE method is obtained as

\[
\frac{C_{\text{large OMP}}}{C_{\text{large LMMSE}}} = \left( \frac{3I_{\text{stop}}}{2K} + \frac{1}{2K} \right) + \left( 1 - \frac{I_{\text{stop}}}{2K} \right) \frac{I_{\text{stop}}}{8M}. \tag{37}
\]

If the OMP-LMMSE algorithm properly stops with \( I_{\text{stop}} = |K_{\text{true}}| \), the complexity ratio in (37) becomes

\[
\frac{C_{\text{large OMP}}}{C_{\text{large LMMSE}}} = \left( \frac{3|K_{\text{true}}|}{2K} + \frac{1}{2K} \right) + \left( 1 - \frac{|K_{\text{true}}|}{2K} \right) \frac{|K_{\text{true}}|}{8M}, \tag{38}
\]

and consequently,

\[
\frac{C_{\text{large OMP}}}{C_{\text{large LMMSE}}} \rightarrow \frac{|K_{\text{true}}|}{8M} \quad \text{as} \quad \frac{|K_{\text{true}}|}{K} \rightarrow 0. \tag{39}
\]

The above result implies that if the size of the true support set is much smaller than the number of devices, the complexity reduction achieved by the OMP-LMMSE algorithm over the LMMSE method increases with the number of the server’s antennas. Therefore, in federated learning over the massive MIMO system, the OMP-LMMSE algorithm is significantly more beneficial than the LMMSE method in terms of the computational complexity. Note that although the MRC method achieves the lowest complexity among the three methods, it suffers from a performance degradation which will be shown numerically in Sec. V.

V. SIMULATION RESULTS

In this section, using simulations, we evaluate the performance and the complexity of various local gradient estimation methods in federated learning over a massive MIMO system. The wireless channel of the communication system is modeled by 10-tap CIR that follows uniform power delay profile, in which each CIR tap is distributed as \( \mathcal{CN}(0, 0.1) \). The number of subcarriers for OFDM signaling is set as \( N_{\text{sub}} = 1024 \), and the noise power is set as \( \sigma_c^2 = 1 \) (i.e., \( \sigma^2 = 0.5 \)).

In this simulation, we consider the task of image classification using the MNIST dataset [28] which consists of 10 classes corresponding to 10 digits with 60000 training and 10000 test data samples. In addition, we consider a non-IID setting for the distribution of local data sets, in which device \( k \) has the information of only a certain digit \( d_k \in \{0, \ldots, 9\} \) set as \( d_k = \left\lfloor \frac{k-1}{K/10} \right\rfloor \). Under this setting, the local data set of device \( k \) is determined as 1000 samples randomly selected from a partial set of training data samples labeled with digit \( d_k \). We also consider a stochastic setting, in which device \( k \) computes the local gradient vector based on one sample randomly
selected from its own local data set, i.e., $|B_k[t]| = 1$ for all $k \in \mathcal{K}$ and $t \in \mathcal{T}$. The classification accuracy is measured by computing the empirical losses w.r.t. the 10000 test data samples.

The classification task described above is learned by a single hidden-layer neural network with 784 input nodes, 20 hidden nodes, and 10 output nodes. The weights of the neural network correspond to the elements in the parameter vector of FL; thereby, the length of the parameter vector is given by $N_w = 15910$. The activation functions of the hidden layer and the output layer are set as the ReLU and the softmax functions, respectively. We assume that the central sever adopts the ADAM optimizer [29] to train the neural network with the cross-entropy loss function. The parameters of the ADAM optimizer are set as $\alpha = 0.01$, $\beta_1 = 0.9$, $\beta_2 = 0.999$, and $\epsilon = 10^{-8}$.

Fig. 3 compares the classification accuracies of the proposed OMP-LMMSE algorithm and the linear beamforming methods. As a performance benchmark, we also plot the optimal accuracy achieved by the centralized learning algorithm in which $\hat{g}_k[t] = g_k[t]$ for all $k \in \mathcal{K}$ and $t \in \mathcal{T}$. Both Figs. 3(a) and 3(b) show that the OMP-LMMSE algorithm outperforms other linear beamforming methods in terms of the classification accuracy. In other words, the OMP-LMMSE algorithm requires a less number of communication rounds than the linear beamforming methods, to achieve a certain level of the classification accuracy. Therefore, the proposed algorithm reduces the communication overhead and the latency in federated learning compared to other linear beamforming methods. Fig. 3(b) also shows that the OMP-LMMSE algorithm achieves almost the same performance with the centralized learning algorithm. In addition, the OMP-LMMSE algorithm even slightly outperforms the centralized algorithm in some regime, which can be due to an overfitting problem in the centralized approach. These results imply that the proposed algorithm with a large number of the server’s antennas effectively compensates for performance degradation in federated learning over wireless communications. Among the linear beamforming methods, the LMMSE method achieves the highest accuracy as it is designed to minimize the MSE of the local gradient vectors even if a naive statistical model is adopted.

Fig. 4 evaluates the complexity reduction of the proposed OMP-LMMSE algorithm over the LMMSE method for various $K$ and $M$. To this end, we first measure the average iteration number of the OMP-LMMSE algorithm, namely $I_{\text{avg}}$, using simulations. We then plot the ratio of the number of real multiplications of the OMP-LMMSE algorithm to that of the LMMSE
Fig. 3. Classification accuracies of various estimation methods with different $K$ and $M$.

The complexity reduction achieved by the proposed algorithm over the LMMSE method is more than 80% in most cases and more than 70% even in the worst case. This result also implies that the local gradient vectors are indeed sparse, as we expected in
Sec. IV-A, because the complexity of the OMP-LMMSE algorithm can be less than that of the LMMSE method only when $I_{stop}$ is much smaller than $K$. Another important observation is that this complexity reduction is achieved while the classification accuracy of the proposed algorithm is higher than that of the LMMSE method (see Fig. 3). Therefore, our simulation results show that the proposed algorithm achieves better performance-complexity tradeoff than the LMMSE method, by exploiting the sparsity of the local gradient vectors.

Fig. 5 evaluates the impact of the number of devices $K$ on the classification accuracies of the proposed OMP-LMMSE algorithm and the LMMSE method when $M = 50$. Fig. 5 shows that the accuracy of both the OMP-LMMSE algorithm and the LMMSE method improves with the number of the wireless devices. This result corroborates our intuition as increasing the number of the devices leads to an increase in the number of the training samples utilized to train the neural network at each communication round. It is also shown that the accuracy improvement in the OMP-LMMSE algorithm is more significant than that in the LMMSE method. The major reason for this result is that when estimating the transmitted signal, the OMP-LMMSE algorithm considers only the devices with non-zero gradient values; whereas, the LMMSE method treats all the devices equally, even including the devices with zero gradient values. Therefore, for a fixed number of the server’s antennas, the performance gain of the proposed algorithm over the LMMSE method improves with the number of devices.
Fig. 5. The impact of the number of devices $K$ on the classification accuracies of the proposed OMP-LMMSE algorithm and the LMMSE method when $M = 50$.

Fig. 6. The impact of the number of the server’s antennas $M$ on the classification accuracies of the proposed OMP-LMMSE algorithm and the LMMSE method when $K = 200$.

Fig. 6 evaluates the impact of the number of the server’s antennas $M$ on the classification accuracies of the proposed OMP-LMMSE algorithm and the LMMSE method when $K = 200$. Fig. 6 shows that the accuracy of both the OMP-LMMSE algorithm and the LMMSE method improves with the number of the server’s antennas. This performance improvement is the consequence of exploiting the additional receive diversity. It is also shown that the performance...
gap between the proposed algorithm and the LMMSE method reduces as the number of the server’s antennas increases. This result demonstrates that the linear beamforming methods are also effective when the number of the server’s antennas is sufficiently large, which verifies our discussion in Sec. III.

VI. CONCLUSION

In this paper, we have presented a novel gradient-estimation algorithm for a central server in federated learning over massive MIMO systems. The key idea behind the presented algorithm is to exploit the sparsity of the local gradient vectors transmitted from the wireless devices, which implies that at each resource element, only a small number of devices transmit non-zero gradient values. We have deliberately modified the OMP algorithm in compressive sensing to effectively find the set of devices transmitting non-zero gradient values while estimating the transmitted signal based on the LMMSE method. From the computational complexity analysis, we have demonstrated that the proposed algorithm has a significantly lower complexity than the LMMSE method. Using simulations, we have also shown that the performance of the proposed algorithm is very close to the optimal performance achieved by the centralized learning, while providing a better performance-complexity tradeoff than other linear beamforming methods.

An important direction for future research is to extend the proposed algorithm by considering a proper scheduling algorithm. In this extension, a joint optimization of user scheduling and gradient estimation may further improve the performance of federated learning, particularly when the number of wireless devices is much larger than the number of antennas at the server. Another promising research direction is to investigate the performance gain of the proposed algorithm when channel state information at the server is imperfect, by applying a realistic channel estimation method developed for conventional massive MIMO systems.

APPENDIX A

PROOF OF PROPOSITION 1

Suppose that $\mathcal{S}_{i-1} \subset \mathcal{K}_{\text{true}}$ and also that $\mathbb{E}[x_k] = 0$ and $\mathbb{E}[|x_k|^2] = \alpha_k$ for $k \in \mathcal{K}_{\text{true}}$. Then the norm squared of the residual vector at iteration $i$ is expressed as

$$\mathbb{E}[\|r_i\|^2] = \text{Tr}(\mathbb{E}[r_i r_i^T]) = \sigma^2 \text{Tr}(\Omega_i \mathbb{E}[y_i y_i^T] \Omega_i),$$

(40)
where the second equality is obtained from (36). Thanks to the use of the permutation matrix in (2), we can ensure that $E[x_kx_j] = 0$ for $k \neq j$ with $k, j \in \mathcal{K}$, as discussed in Sec. III-C. Utilizing this along with (17), the covariance of the received signal is obtained as

$$E[yy^T] = \sum_{k \in \mathcal{K}_{true}} \alpha_k h_k h_k^T + \sigma^2 I_{2M}. \quad (41)$$

In what follows, we characterize $\Omega_i$ and then provide a closed-form expression for $E[\|r_i\|^2]$ for three cases discussed in Sec. IV-B.

A. **Case 1: $S_i \subset \mathcal{K}_{true}$ and $S_i \neq \mathcal{K}_{true}$**

In this case, $\Omega_i$ in (27) is expressed as

$$\Omega_i = \left( \sum_{k \in S_i} \alpha_k h_k h_k^T + \sigma^2 I_{2M} \right)^{-1}. \quad (42)$$

From (42), the covariance of the received signal in (41) is rewritten as

$$E[yy^T] = \Omega_i^{-1} + \sum_{k \in \mathcal{K}_{true \setminus S_i}} \alpha_k h_k h_k^T. \quad (43)$$

Then the norm squared of the residual vector when the support set belongs to Case 1 is obtained by applying (43) into (40):

$$E_1^{(i)} = \sigma^4 \left[ \text{Tr}(\Omega_i) + \sum_{k \in \mathcal{K}_{true \setminus S_i}} \alpha_k \|\Omega_i h_k\|^2 \right]. \quad (44)$$

B. **Case 2: $S_i = \mathcal{K}_{true}$**

In this case, $\Omega_i$ in (27) is expressed as

$$\Omega_i = \left( \sum_{k \in \mathcal{K}_{true}} \alpha_k h_k h_k^T + \sigma^2 I_{2M} \right)^{-1}. \quad (45)$$

From (45), the covariance of the received signal in (41) is rewritten as

$$E[yy^T] = \Omega_i^{-1}. \quad (46)$$

By applying (46) into (40), the norm squared of the residual vector when the support set belongs to Case 2 is given by

$$E_2^{(i)} = \sigma^4 \text{Tr}(\Omega_i). \quad (47)$$
C. Case 3: $\mathcal{S}_i = \mathcal{K}_{\text{true}} \cup \{k_i^*\}$

In this case, $\Omega_i$ in (27) is expressed as

$$\Omega_i = \left( \sum_{k \in \mathcal{K}_{\text{true}}} \alpha_k h_k h_k^T + \alpha_{k_i^*} h_{k_i^*} h_{k_i^*}^T + \sigma^2 I_{2M} \right)^{-1}. \quad (48)$$

From (48), the covariance of the received signal in (41) is rewritten as

$$\mathbb{E}[yy^T] = \Omega_i^{-1} - \alpha_{k_i^*} h_{k_i^*} h_{k_i^*}^T. \quad (49)$$

Applying (49) into (40) yields

$$E_3^{(i)} = \sigma^4 \left[ \text{Tr}(\Omega_i) - \alpha_{k_i^*} \|\Omega_i h_{k_i^*}\|^2 \right]. \quad (50)$$

From (28), the second term in the RHS of (50) is expressed as

$$\alpha_{k_i^*} \|\Omega_i h_{k_i^*}\|^2 = \frac{\alpha_{k_i^*} \|\Omega_{i-1} h_{k_i^*}\|^2}{1 + \alpha_{k_i^*} h_{k_i^*}^T \Omega_{i-1} h_{k_i^*}}. \quad (51)$$

Also, applying the trace function to (28) yields

$$\text{Tr}(\Omega_i) = \text{Tr}(\Omega_{i-1}) - \frac{\alpha_{k_i^*} \|\Omega_{i-1} h_{k_i^*}\|^2}{1 + \alpha_{k_i^*} h_{k_i^*}^T \Omega_{i-1} h_{k_i^*}}, \quad (52)$$

so we have

$$\alpha_{k_i^*} \|\Omega_{i-1} h_{k_i^*}\|^2 = \frac{\text{Tr}(\Omega_{i-1}) - \text{Tr}(\Omega_i)}{1 + \alpha_{k_i^*} h_{k_i^*}^T \Omega_{i-1} h_{k_i^*}}. \quad (53)$$

By applying (53) into (50), the norm squared of the residual vector when the support set belongs to Case 3 is given by

$$E_3^{(i)} = \sigma^4 \left[ \text{Tr}(\Omega_i) - \frac{\text{Tr}(\Omega_{i-1}) - \text{Tr}(\Omega_i)}{1 + \alpha_{k_i^*} h_{k_i^*}^T \Omega_{i-1} h_{k_i^*}} \right]. \quad (54)$$

REFERENCES

[1] C. Jiang, H. Zhang, Y. Ren, Z. Han, K.-C. Chen, and L. Hanzo, “Machine learning paradigms for next-generation wireless networks,” IEEE Wireless Commun., vol. 24, no. 2, pp. 98–105, Apr. 2017.

[2] G. Zhu, D. Liu, Y. Du, C. You, J. Zhang, and K. Huang, “Toward an intelligent edge: Wireless communication meets machine learning,” IEEE Commun. Magazine, vol. 58, no. 1, pp. 19–25, Jan. 2020.

[3] H. He, S. Jin, C.-K. Wen, F. Gao, G. Y. Li, and Z. Xu, “Model-driven deep learning for physical layer communications,” IEEE Wireless Commun., vol. 26, no. 5, pp. 77–83, Oct. 2019.

[4] Y.-S. Jeon, N. Lee, and H. V. Poor, “Robust data detection for MIMO systems with one-bit ADCs: A reinforcement learning approach,” IEEE Trans. Wireless Commun., vol. 19, no. 3, pp. 1663–1676, Mar. 2020.

[5] T. OShea and J. Hoydis, “An introduction to deep learning for the physical layer,” IEEE Trans. Cognitive Commun. Netw., vol. 3, no. 4, pp. 563–575, Dec. 2017.
[6] Y.-S. Jeon, S.-N. Hong, and N. Lee, “Supervised-learning-aided communication framework for MIMO systems with low-resolution ADCs,” IEEE Trans. Veh. Tech., vol. 67, no. 8, pp. 7299–7313, Aug. 2018.

[7] J. Konečný, B. H. McMahan, and D. Ramage, “Federated optimization: Distributed optimization beyond the datacenter,” arXiv:1511.03575v1 [cs.LG], Nov. 2015 [Online]. Available: http://arxiv.org/abs/1511.03575

[8] J. Konečný, B. H. McMahan, F. X. Yu, P. Richtárik, A. T. Suresh, and D. Bacon, “Federated learning: Strategies for improving communication efficiency,” arXiv:1610.05492v2 [cs.LG], Oct. 2017 [Online]. Available: http://arxiv.org/abs/1610.05492

[9] H. B. McMahan, E. Moore, D. Ramage, S. Hampson, and B. A. Arcas “Communication-efficient learning of deep networks from decentralized data,” arXiv:1602.05629v3 [cs.LG], Feb. 2017. [Online] Available: http://arxiv.org/abs/1602.05629

[10] Y. Lin, S. Han, H. Mao, Y. Wang, and W. J. Dally, “Deep gradient compression: Reducing the communication bandwidth for distributed training,” in Proc. Int. Conf. Learn. Represent. (ICLR), Vancouver, BC, Canada, May 2018, pp. 1–13.

[11] S. Niknam, H. S. Dhillon, and J. H. Reed, “Federated learning for wireless communications: Motivation, opportunities and challenges,” arXiv:1908.06847v3 [eess.SP], Sep. 2019. [Online]. Available: http://arxiv.org/abs/1908.06847

[12] S. Samarakoon, M. Bennis, W. Saad, and M. Debbah, “Distributed federated learning for ultra-reliable low-latency vehicular communications,” arXiv:1807.08127v3 [cs.IT], Dec. 2019. [Online]. Available: http://arxiv.org/abs/1807.08127

[13] M. Chen, O. Semiari, W. Saad, X. Liu, and C. Yin, “Federated edge state learning for minimizing breaks in presence in wireless virtual reality networks,” arXiv:1812.01202v2 [cs.IT], Sep. 2019. [Online]. Available: http://arxiv.org/abs/1812.01202

[14] T. Nishio and R. Yonetani, “Client selection for federated learning with heterogeneous resources in mobile edge,” in Proc. IEEE Int. Conf. Commun. (ICC), Shanghai, China, May 2019, pp. 1–6.

[15] D. Liu, G. Zhu, J. Zhang, and K. Huang, “Data-importance aware user scheduling for communication-efficient edge machine learning,” arXiv:1910.02214v1 [cs.NI], Oct. 2019. [Online]. Available: http://arxiv.org/abs/1910.02214

[16] H. H. Yang, A. Arafa, T. Q. S. Quek, and H. V. Poor, “Age-based policy for federated learning in mobile edge networks,” arXiv:1910.14648v1 [cs.IT], Oct. 2019. [Online]. Available: http://arxiv.org/abs/1910.02214

[17] H. H. Yang, Z. Liu, T. Q. S. Quek, and H. V. Poor, “Scheduling policies for federated learning in wireless networks,” IEEE Trans. Commun., vol. 68, no. 1, pp. 317–333, Jan. 2020.

[18] M. M. Amiri, D. Gündüz, S. R. Kulkarni, and H. V. Poor, “Update aware device scheduling for federated learning at the wireless edge,” arXiv:2001.10402v1 [cs.IT], Jan. 2020. [Online]. Available: http://arxiv.org/abs/2001.10402

[19] M. Chen, Z. Yang, W. Saad, C. Yin, H. V. Poor, and S. Cui, “A joint learning and communications framework for federated learning over wireless networks,” arXiv:1909.07972v1 [cs.NI], Sep. 2019. [Online]. Available: http://arxiv.org/abs/1909.07972

[20] M. M. Amiri and D. Gündüz, “Machine learning at the wireless edge: Distributed stochastic gradient descent over-the-air,” to appear, IEEE Trans. Signal Process., [Online]. Available: http://arxiv.org/abs/1901.00844

[21] M. M. Amiri and D. Gündüz, Federated learning over wireless fading channels,” to appear, IEEE Trans. Wireless Commun., [Online]. Available: http://arxiv.org/abs/1907.09769

[22] G. Zhu, Y. Wang, and K. Huang, “Broadband analog aggregation for low-latency federated edge learning,” IEEE Trans. Wireless Commun., vol. 19, no. 1, pp. 491–506, Jan. 2020.

[23] K. Yang, T. Jiang, Y. Shi, and Z. Ding, “Federated learning via over-the-air computation,” arXiv:1812.11750v3 [cs.LG], Feb. 2019. [Online]. Available: http://arxiv.org/abs/1812.11750

[24] M. M. Amiri, T. M. Duman, and D. Gündüz, “Collaborative machine learning at the wireless edge with blind transmitters,” in Proc. IEEE Global Conf. Sig. Inf. Process. (GlobalSIP), Ottawa, ON, Canada, Nov. 2019, pp. 1–6.

[25] T. T. Vu, D. T. Ngo, N. H. Tran, H. Q. Ngo, M. N. Dao, and R. H. Middleton, “Cell-free massive MIMO for wireless federated learning,” arXiv:1909.12567v4 [eess.SP], Dec. 2019. [Online]. Available: http://arxiv.org/abs/1909.12567
[26] K. Wei, J. Li, M. Ding, C. Ma, H. H. Yang, F. Farhad, S. Jin, T. Q. S. Quek, and H. V. Poor, “Federated learning with differential privacy: Algorithms and performance analysis,” arXiv:1911.00222v2 [cs.LG], Nov. 2019. [Online]. Available: http://arxiv.org/abs/1911.00222

[27] E. Telatar, “Capacity of multi-antenna Gaussian channels,” European Trans. Telecommun., vol. 10, no. 6, pp. 585–595, 4Q 1999.

[28] Y. LeCun, C. Cortes, and C. Burges, “The MNIST database of handwritten digits,” [Online]. Available: http://yann.lecun.com/exdb/mnist/

[29] D. P. Kingma and J. Ba, “Adam: A method for stochastic optimization,” arXiv:1412.6980v9 [cs.LG], Jan. 2017. [Online]. Available: http://arxiv.org/abs/1412.6980

[30] D. Tse and P. Viswanath, Fundamentals of Wireless Communication, Cambridge, U.K.: Cambridge University Press, 2005.

[31] A. Papoulis and U. Pillai, Probability, Random Variables and Stochastic Processes, New York: McGraw-Hill, 2001.

[32] D. Alistarh, J. Li, R. Tomioka, and M. Vojnovic, “QSGD: Randomized quantization for communication-optimal stochastic gradient descent,” arXiv:1610.02132v4 [cs.LG], Dec. 2017. [Online]. Available: http://arxiv.org/abs/1610.02132

[33] Y. C. Eldar and G. Kutyniok, Compressed Sensing: Theory and Applications, Cambridge, U.K.: Cambridge University Press, 2012.

[34] J. A. Tropp, “Greed is good: Algorithmic results for sparse approximation,” IEEE Trans. Inf. Theory, vol. 50, no. 10, pp. 2231–2242, Oct. 2004.

[35] T. T. Cai and L. Wang, “Orthogonal matching pursuit for sparse signal recovery with noise,” IEEE Trans. Inf. Theory, vol. 57, no. 7, pp. 4680–4688, Jul. 2011.