The pure annihilation type $B_c \rightarrow M_2M_3$ decays in the perturbative QCD approach

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Abstract

In the standard model the two-body charmless hadronic $B_c$ meson decays can occur via annihilation diagrams only. In this work, we studied the $B_c \rightarrow PP, PV/VP, VV$ decays by employing the perturbative QCD (pQCD) factorization approach. From our calculations, we find that (a) the pQCD predictions for the branching ratios of the considered $B_c$ decays are in the range of $10^{-6}$ to $10^{-8}$; (b) for $B_c \rightarrow PV/VP, VV$ decays, the branching ratios of $\Delta S = 0$ decays are much larger than those of $\Delta S = 1$ ones because the different Cabibbo-Kobayashi-Maskawa (CKM) factors are involved; (c) analogous to $B \rightarrow K\eta/(\eta')$ decays, we find $Br(B_c \rightarrow K^{+}\eta^{'}) \sim 10 \times Br(B_c \rightarrow K^{+}\eta)$, which can be understood by the destructive and constructive interference between the $\eta_q$ and $\eta_s$ contribution to the $B_c \rightarrow K^{+}\eta$ and $B_c \rightarrow K^{+}\eta'$ decay; (d) the longitudinal polarization fractions of $B_c \rightarrow VV$ decays are in the range of 86% − 95% and play the dominant role; and (e) there is no CP-violating asymmetries for the considered $B_c$ decays because only one type tree operators involved.

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I. INTRODUCTION

In 1998, a new stage of $B_c$ physics began because of the first observation of the meson $B_c$ at Tevatron [1]. For $B_c$ meson, one can study the two heavy flavors $b$ and $c$ in a meson simultaneously. From an experimental point of view, more detailed information about $B_c$ meson can be obtained at the Large Hadron Collider (LHC) experiment. The LHC is scheduled to start to run in this month, where the $B_c$ meson could be produced abundantly. The $B_c$ meson decays may provide windows for testing the predictions of the standard model (SM) and can shed light on new physics (NP) scenarios beyond the SM.

From a theoretical point of view [2], the non-leptonic decays of $B_c$ meson are the most complicated decays due to its heavy-heavy nature and the participation of strong interaction, which complicate the extraction of parameters in SM, but they also provide great opportunities to study the perturbative and nonperturbative QCD, final state interactions, etc. The non-leptonic $B_c$ weak decays have been widely studied for example in Refs. [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32] by employing the Naive factorization approach (NFA) [33], the QCD factorization approach (QCDF) [34], the perturbative QCD (pQCD) approach [35, 36, 37] and other approaches and/or methods.

In this paper we focus on the two-body non-leptonic charmless decays $B_c \to PP, PV/VP, VV$ (here $P$ and $V$ stands for the light pseudo-scalar and vector mesons), which can occur through the weak annihilation diagrams only. The size of annihilation contributions is an important issue in $B$ physics. Indeed, the two-body charmless $B_c$ decays considered here are rather different from those $B_c \to J/\psi P(V)$ decays where the initial $c$ quark behaves as a spectator.

Recently, the two-body non-leptonic charmless $B_c \to M_2M_3$ [1] decays have been studied by using the $SU(3)$ flavor symmetry or by employing the QCDF factorization approach [38]. The authors in Ref. [38] provided two different estimates for non-leptonic charmless $B_c$ decays. But their predictions for the branching ratios of $B_c \to \phi K^+, \bar{K}^{*0}K^+$ decays in the QCDF are much smaller (a factor of 10) than those obtained by using the $SU(3)$ flavor symmetry. So large discrepancies among the theoretical predictions for the branching ratios indicate clearly that it is very necessary to make more studies for these kinds of $B_c$ decays by employing other different approaches, in order to understand these decays better and provide the theoretical support for the related experimental studies.

In this paper, we will calculate the branching ratios and the polarization fractions of thirty $B_c \to PP, PV/VP, VV$ decays by employing the low energy effective Hamiltonian [39] and the pQCD factorization approach. By keeping the transverse momentum $k_T$ of the quarks, the pQCD approach is free of endpoint singularity and the Sudakov formalism makes it more self-consistent. It is worth of mentioning that one can do the quantitative calculations of the annihilation type diagrams in the pQCD approach. The importance of annihilation contributions has already been tested in the previous predictions of branching ratios of pure annihilation $B \to D_sK$ decays [40], direct CP asymmetries of $B^0 \to \pi^+\pi^-$, $K^+\pi^-$ decays [35, 36, 41] and in the explanation of $B \to \phi K^*$ polarization problem [42, 43], which indicate that the pQCD approach is a reliable method.

1 For the sake of simplicity, we will use $M_2$ and $M_3$ to denote the two final state light mesons respectively, unless otherwise stated.
to deal with annihilation diagrams.

The paper is organized as follows. In Sec. II we present the formalism and wave functions of the considered $B_c$ meson decays. Then we perform the perturbative calculations for considered decay channels with pQCD approach in Sec. III. The numerical results and phenomenological analysis are given in Sec. IV. Finally, Sec. V contains the main conclusions and a short summary.

II. FORMALISM AND WAVE FUNCTIONS

A. Formalism

Since the $b$ quark is rather heavy, we work in the frame with the $B_c$ meson at rest, i.e., with the $B_c$ meson momentum $P_1 = \frac{m_B}{\sqrt{2}} (1, 1, 0_T)$ in the light-cone coordinates. For the non-leptonic charmless $B_c \rightarrow M_2 M_3$ decays, assuming that the $M_2$ ($M_3$) meson moves in the plus (minus) $z$ direction carrying the momentum $P_2$ ($P_3$) and the polarization vector $\epsilon_2$ ($\epsilon_3$) (if $M_2(3)$ are the vector mesons). Then the two final state meson momenta can be written as

$$P_2 = \frac{m_{B_c}}{\sqrt{2}} (1 - r_2^2, r_2^2, 0_T), \quad P_3 = \frac{m_{B_c}}{\sqrt{2}} (r_3^2, 1 - r_3^2, 0_T),$$

respectively, where $r_2 = m_{M_2}/m_B$ and $r_3 = m_{M_3}/m_B$. When $M_2, M_3$ are the vector mesons, the longitudinal polarization vectors, $\epsilon_2^L$ and $\epsilon_3^L$, can be given by

$$\epsilon_2^L = \frac{m_{B_c}}{\sqrt{2m_{M_2}}}(1 - r_2^2, -r_2^2, 0_T), \quad \epsilon_3^L = \frac{m_{B_c}}{\sqrt{2m_{M_3}}}(-r_3^2, 1 - r_3^2, 0_T).$$

The transverse ones are parameterized as $\epsilon_2^T = (0, 0, 1_T)$, and $\epsilon_3^T = (0, 0, 1_T)$. Putting the (light-) quark momenta in $B_c$, $M_2$ and $M_3$ mesons as $k_1$, $k_2$, and $k_3$, respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, k_{1T}), \quad k_2 = (x_2 P_2^+, 0, k_{2T}), \quad k_3 = (0, x_3 P_3^-, k_{3T}).$$

Then, for $B_c \rightarrow M_2 M_3$ decays, the integration over $k_1^-, k_2^-$, and $k_3^+$ will conceptually lead to the decay amplitudes in the pQCD approach,

$$A(B_c \rightarrow M_2 M_3) \sim \int dx_1 dx_2 dx_3 db_1 b_2 b_3 \cdot \text{Tr} \left[ C(t) \Phi_{B_c}(x_1, b_1) \Phi_{M_2}(x_2, b_2) \Phi_{M_3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right].$$

where $b_i$ is the conjugate space coordinate of $k_{iT}$, and $t$ is the largest energy scale in function $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. The large double logarithms ($\ln^2 x_i$) are summed by the threshold resummation [44], and they lead to $S_t(x_i)$ which smears the end-point singularities on $x_i$. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively [45]. Thus it makes the perturbative calculation of the hard part $H$ applicable at intermediate scale, i.e., $m_{B_c}$ scale. We will calculate analytically the function $H(x_i, b_i, t)$ for the considered
decays at leading order (LO) in $\alpha_s$ expansion and give the convoluted amplitudes in next section.

For these considered decays, the related weak effective Hamiltonian $H_{\text{eff}}$ can be written as

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ V_{cb}^* V_{ud} (C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)) \right],$$

(5)

with the single tree operators,

$$O_1 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\alpha \bar{c}_\gamma (1 - \gamma_5) b_\alpha,$$
$$O_2 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\beta \bar{c}_\gamma (1 - \gamma_5) b_\alpha,$$

(6)

where $V_{cb}, V_{ud}$ are the CKM matrix elements, ”D” denotes the light down quark $d$ or $s$ and $C_1,2(\mu)$ are Wilson coefficients at the renormalization scale $\mu$. For the Wilson coefficients $C_{1,2}(\mu)$, we will also use the leading order (LO) expressions, although the next-to-leading order calculations already exist in the literature [39]. This is the consistent way to cancel the explicit $\mu$ dependence in the theoretical formulae. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulae as given in Ref. [36] directly.

### B. Wave Functions

In order to calculate the decay amplitude, we should choose the proper wave functions of the heavy $B_c$ and light mesons. In principle there are two Lorentz structures in the $B_{u,d,s}$ or $B_c$ meson wave function. One should consider both of them in calculations. However, since the contribution induced by one Lorentz structure is numerically small [46, 47] and can be neglected approximately, we only consider the contribution from the first Lorentz structure.

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2}N_c} \left[ (P + M_{B_c}) \gamma_5 \phi_{B_c}(x) \right]_{\alpha\beta}.$$  

(7)

Since $B_c$ meson consists of two heavy quarks and $m_{B_c} \approx m_b + m_c$, the distribution amplitude $\phi_{B_c}$ would be close to $\delta(x - m_c/m_{B_c})$ in the non-relativistic limit. We therefore adopt the non-relativistic approximation form of $\phi_{B_c}$ as [19, 28],

$$\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2}N_c} \delta(x - m_c/m_{B_c}),$$

(8)

where $f_{B_c}$ and $N_c$ are the decay constant of $B_c$ meson and the color number, respectively.

For the pseudoscalar meson(P), the wave function can generally be defined as,

$$\Phi_P(x) = \frac{i}{\sqrt{2}N_c} \gamma_5 \left\{ P\phi_P^A(x) + m_0^P \phi_P^P(x) + m_0^P (\not{p} - 1) \phi_P^T(x) \right\}_{\alpha\beta}$$

(9)

where $\phi_P^{A,P,T}$ and $m_0^P$ are the distribution amplitudes and chiral scale parameter of the pseudoscalar mesons respectively, while $x$ denotes the momentum fraction carried by
quark in the meson, and $n = (1, 0, 0_T)$ and $v = (0, 1, 0_T)$ are dimensionless light-like unit vectors.

For the wave functions of vector mesons, one longitudinal (L) and two transverse (T) polarizations are involved, and can be written as,

\[
\Phi^L_V(x) = \frac{1}{\sqrt{2N_c}} \left\{ M_V \epsilon^L_V \phi^L_V(x) + \epsilon^L_V \phi^T_V(x) + M_V \phi^s_V(x) \right\}_{\alpha\beta},
\]

(10)

\[
\Phi^T_V(x) = \frac{1}{\sqrt{2N_c}} \left\{ M_V \epsilon^T_V \phi^V(x) + \epsilon^T_V \phi^T_V(x) + M_V i\epsilon_{\mu\nu\rho\sigma} \gamma^5 \gamma^\mu \epsilon^\nu_T n^\rho v^\sigma \phi^a_V(x) \right\}_{\alpha\beta},
\]

(11)

where $\epsilon^L_V (T)$ denotes the longitudinal (transverse) polarization vector of vector mesons, satisfying $P \cdot \epsilon = 0$ in each polarization. We here adopt the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor $\epsilon^{\mu\nu\alpha\beta}$. For the distribution amplitudes of pseudoscalar $\phi^A_P$, $\phi^A_V$ and longitudinal and transverse polarization, $\phi^{T,a}_V$ and $\phi^{V,T,a}_V$, which will be presented in Appendix A.

### III. Perturbative Calculations in PQCD

From the effective Hamiltonian (5), there are 4 types of diagrams contributing to the $B_c \to M_2 M_3$ decays as illustrated in Fig. 11, which result in the Feynman decay amplitudes $F^a_{fa}$ and $M^na_{na}$, where the subscripts $fa$ and $na$ are the abbreviations of factorizable and non-factorizable annihilation contributions, respectively. Operators $O_{1,2}$ are $(V - A)(V - A)$ currents, we therefore can combine all contributions from these diagrams and obtain the total decay amplitude as,

\[
\mathcal{A}(B_c \to M_2 M_3) = V^*_{cb} V_{ud} \left\{ f_{fa} F^a_{fa} a_1 + M^na_{na} C_1 \right\},
\]

(12)

where $a_1 = C_1/3 + C_2$. In the next three subsections we will give the explicit expressions of $F^a_{fa}$, $M^na_{na}$ and the decay amplitude $\mathcal{A}(B_c \to M_2 M_3)$ for $B_c \to M_2 M_3$ decays: including eight $B_c \to PP$, fifteen $B_c \to PV$ or $B_c \to VP$, and seven $B_c \to VV$ decay modes.

#### A. $B_c \to PP$ decays

In this section, we will present the factorization formulae for eight non-leptonic charmless $B_c \to PP$ decays. From the first two diagrams of Fig. 11 i.e., (a) and (b), by perturbative QCD calculations, we obtain the decay amplitude for factorizable annihilation
The three input parameters from various related experiments \[48, 49\] contributions as follows,

\[
F_{fa}^{PP} = -8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
\times \left\{ h_{fa}(1 - x_3, x_2, b_1, b_2)E_{fa}(t_0) \left[ x_2 \phi_2^A(x_2)\phi_3^A(x_3) + 2r_0^2 r_0^3 \phi_3^P(x_3) \right] \right. \\
\times \left. [(x_2 + 1)\phi_2^P(x_2) + (x_2 - 1)\phi_3^P(x_2)] + h_{fa}(x_2, 1 - x_3, b_2, b_3)E_{fa}(t_0) \right. \\
\times \left. [(x_3 - 1)\phi_2^A(x_2)\phi_3^A(x_3) + 2r_0^2 r_0^3 \phi_3^P(x_2) \left( (x_3 - 2)\phi_3^P(x_3) - x_3\phi_3^T(x_3) \right) \right] \right\} \tag{13}
\]

where \(\phi_2^{(3)}\) corresponding to the distribution amplitudes of mesons \(M_{2(3)}\), \(r_0^{2(3)} = m_0^{M_{2(3)}}/m_{B_c}\), and \(C_F = 4/3\) is a color factor. In Eq. \(\tag{13}\), the terms proportional to \((r_0^{2(3)})^2\) have been neglected because they are small indeed, \(\max(r_0^{2(3)})^2 \leq 7\%\). The function \(h_{fa}\), the scales \(t_i\) and \(E_{fa}(t)\) can be found in Appendix \[3\].

For the non-factorizable diagrams (c) and (d), all three meson wave functions are involved. The integration of \(b_2\) can be performed using \(\delta\) function \(\delta(b_3 - b_2)\), leaving only integration of \(b_1\) and \(b_2\). The corresponding decay amplitude is

\[
M_{na}^{PP} = -16\sqrt{6}\frac{\sqrt{3}}{3} \pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \\
\times \left\{ h_{na}(x_2, x_3, b_1, b_2)E_{na}(t_d) \left[ (r_c - x_3 + 1)\phi_2^A(x_2)\phi_3^A(x_3) + r_0^2 r_0^3 \phi_3^P(x_2) \right] \\
\times \left. (3r_c + x_2 - x_3 + 1)\phi_2^P(x_3) - (r_c - x_2 - x_3 + 1)\phi_3^P(x_3) + \phi_2^T(x_2) \right. \\
\times \left. (r_c - x_2 + x_3 - 1)\phi_3^P(x_3) + (r_c - x_2 + x_3 - 1)\phi_3^T(x_3) \right] \right\} \tag{14}
\]

where \(r_{bc} = m_{b(c)}/m_{B_c}\).

For the \(\eta - \eta'\) system, there exist two popular mixing basis: the octet-singlet basis and the quark-flavor basis \[48, 49\]. Here we use the quark-flavor basis \[48\] and define

\[
\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}, \quad \eta_s = s\bar{s}. \tag{15}
\]

The physical states \(\eta\) and \(\eta'\) are related to \(\eta_q\) and \(\eta_s\) through a single mixing angle \(\phi\),

\[
\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = U(\phi) \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}. \tag{16}
\]

We assume that the distribution amplitudes of \(\eta_q\) and \(\eta_s\) are the same as the distribution amplitudes of \(\pi\), except for the different decay constants and the chiral scale parameters. The three input parameters \(f_q, f_s\) and \(\phi\) in the quark-flavor basis have been extracted from various related experiments \[48, 49\]

\[
f_q = (1.07 \pm 0.02)f_\pi, \quad f_s = (1.34 \pm 0.06)f_\pi, \quad \phi = 39.3^\circ \pm 1.0^\circ. \tag{17}
\]

The chiral enhancement factors are chosen as

\[
m_0^{\eta_q} \equiv m_0^{\eta_q} \equiv \frac{m_0^{\eta_q}}{2m_q} = \frac{1}{2m_q} \left[ m_0^{\eta_q} \cos^2 \phi + m_0^{\eta_q} \sin^2 \phi - \frac{\sqrt{2}f_s}{f_q}(m_0^{\eta_q} - m_0^{\eta_s}) \cos \phi \sin \phi \right], \tag{18}
\]

\[
m_0^{\eta_s} \equiv m_0^{\eta_s} \equiv \frac{m_0^{\eta_s}}{2m_s} = \frac{1}{2m_s} \left[ m_0^{\eta_q} \cos^2 \phi + m_0^{\eta_q} \sin^2 \phi - \frac{f_q}{\sqrt{2}f_s}(m_0^{\eta_q} - m_0^{\eta_s}) \cos \phi \sin \phi \right]. \tag{19}
\]
In the numerical calculations, we will use these mixing parameters as inputs. It is worth of mentioning that the effects of possible gluonic component of $\eta'$ meson will not considered here since it is small in size [50, 51, 52].

Based on Eqs. (12) and (13,14), we can write down the total decay amplitudes for eight $B_c \to PP$ decays easily,

$$\mathcal{A}(B_c \to \pi^+ \pi^0) = V_{cb}^* V_{ud} \left\{ f_{B_c} F_{a}^{\pi^+\pi^0} a_1 + M_{na}^{\pi^+\pi^0} C_1 \right\} \left[ -f_{B_c} F_{a}^{\pi^+} a_1 + M_{na}^{\pi^+} C_1 \right] = 0 ,$$

$$\mathcal{A}(B_c \to \pi^+ \eta) = V_{cb}^* V_{ud} \left\{ f_{B_c} F_{a}^{\pi^+\eta} a_1 + M_{na}^{\pi^+\eta} C_1 \right\} \left[ +f_{B_c} F_{a}^{\eta} a_1 + M_{na}^{\eta} C_1 \right] \cos \phi ,$$

$$\mathcal{A}(B_c \to \pi^+ \eta') = V_{cb}^* V_{ud} \left\{ f_{B_c} F_{a}^{\pi^+\eta'} a_1 + M_{na}^{\pi^+\eta'} C_1 \right\} \left[ +f_{B_c} F_{a}^{\eta'} a_1 + M_{na}^{\eta'} C_1 \right] \sin \phi ,$$

$$\mathcal{A}(B_c \to K^0 K^+) = V_{cb}^* V_{ud} \left\{ f_{B_c} F_{a}^{K^0K^+} a_1 + M_{na}^{K^0K^+} C_1 \right\} ,$$

$$\mathcal{A}(B_c \to K^+ \pi^0) = V_{cb}^* V_{us} \left\{ f_{B_c} F_{a}^{K^+\pi^0} a_1 + M_{na}^{K^+\pi^0} C_1 \right\} ,$$

$$\mathcal{A}(B_c \to K^0 \pi^+) = \sqrt{2} \mathcal{A}(B_c \to K^+ \pi^0) ,$$

$$\mathcal{A}(B_c \to K^+ \eta) = V_{cb}^* V_{us} \left\{ f_{B_c} \left[ F_{a}^{K^+\eta} \cos \phi - F_{a}^{\eta K^+} \sin \phi \right] a_1 + M_{na}^{K^+\eta} \cos \phi - M_{na}^{\eta K^+} \sin \phi \right\} C_1 \right\} ,$$

$$\mathcal{A}(B_c \to K^+ \eta') = V_{cb}^* V_{us} \left\{ f_{B_c} \left[ F_{a}^{K^+\eta'} \sin \phi + F_{a}^{\eta' K^+} \cos \phi \right] a_1 + M_{na}^{K^+\eta'} \sin \phi + M_{na}^{\eta' K^+} \cos \phi \right\} C_1 \right\} .$$
By following the same procedure as stated in the above subsection, we can obtain the analytic decay amplitudes for $B_c \to PV, VP$ decays,

$$
F_{fa}^{PV} = 8\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_1 b_3 db_3 \\
\times \left\{ h_{fa}(1 - x_3, x_2, b_3, b_2) E_{fa}(t_a) \left[ x_2 \phi_2^A(x_2) \phi_3(x_3) - 2r_0^2 r_3 \phi_3^T(x_3) \right] + h_{fa}(x_2, 1 - x_3, b_3, b_2) E_{fa}(t_b) \right\},
$$

$$
M_{na}^{PV} = \frac{16\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \\
\times \left\{ h_{na}(x_2, x_3, b_1, b_2) E_{na}(t_c) \left[ (r_c - x_3 + 1) \phi_2^A(x_2) \phi_3(x_3) - r_0^2 r_3 \phi_2^P(x_2) \right] + h_{na}(x_2, 1 - x_3, b_1, b_2) E_{fa}(t_b) \right\}.
$$

$$
F_{fa}^{VP} = 8\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \\
\times \left\{ h_{fa}(1 - x_3, x_2, b_3, b_2) E_{fa}(t_a) \left[ x_2 \phi_2(x_2) \phi_3^A(x_3) + 2r_0^2 r_3 \phi_3^P(x_3) \right] + h_{fa}(x_2, 1 - x_3, b_2, b_3) E_{fa}(t_b) \right\},
$$

$$
M_{na}^{VP} = \frac{16\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \\
\times \left\{ h_{na}(x_2, x_3, b_1, b_2) E_{na}(t_c) \left[ (r_c - x_3 + 1) \phi_2(x_2) \phi_3^A(x_3) + r_0^3 r_3 \phi_2^P(x_2) \right] + h_{na}(x_2, 1 - x_3, b_1, b_2) E_{na}(t_d) \right\}.
$$

The total decay amplitudes of the fifteen $B_c \to PV, VP$ decays can therefore be written
\[
\mathcal{A}(B_c \to \pi^+ \rho^0) = V_{cb}^* V_{ud} \left\{ [f_{B_c} F_{f_a}^{\pi^+ \rho^0_{du}} a_1 + M_{na}^{\pi^+ \rho^0_{u}} C_1] - [f_{B_c} F_{f_a}^{\rho^0_{du}} a_1 + M_{na}^{\rho^0_{du}} C_1] \right\},
\]
\[
\mathcal{A}(B_c \to \pi^+ \omega) = V_{cb}^* V_{ud} \left\{ [f_{B_c} F_{f_a}^{\pi^+ \omega_{du}} a_1 + M_{na}^{\pi^+ \omega_{u}} C_1] + [f_{B_c} F_{f_a}^{\omega_{du}} a_1 + M_{na}^{\omega_{du}} C_1] \right\},
\]
\[
\mathcal{A}(B_c \to \overline{K}^0 K^{*+}) = V_{cb}^* V_{ud} \left\{ f_{B_c} F_{f_a}^{\overline{K}^0} a_1 + M_{na}^{\overline{K}^0 K^{*+}} C_1 \right\},
\]
\[
\mathcal{A}(B_c \to K^+ \rho^0) = V_{cb}^* V_{us} \left\{ f_{B_c} F_{f_a}^{K^+ \rho^0} a_1 + M_{na}^{K^+ \rho^0} C_1 \right\},
\]
\[
\mathcal{A}(B_c \to K^0 \rho^+) = \sqrt{2} \mathcal{A}(B_c \to K^+ \rho^0),
\]
\[
\mathcal{A}(B_c \to K^+ \omega) = V_{cb}^* V_{us} \left\{ f_{B_c} F_{f_a}^{K^+ \omega} a_1 + M_{na}^{K^+ \omega} C_1 \right\},
\]
\[
\mathcal{A}(B_c \to \rho^+ \pi^0) = V_{cb}^* V_{ud} \left\{ [f_{B_c} F_{f_a}^{\rho^+ \pi^0_{du}} a_1 + M_{na}^{\rho^+ \pi^0_{u}} C_1] - [f_{B_c} F_{f_a}^{\rho^+ \pi^0_{du}} a_1 + M_{na}^{\rho^+ \pi^0_{u}} C_1] \right\},
\]
\[
\mathcal{A}(B_c \to \rho^+ \eta) = V_{cb}^* V_{ud} \left\{ [f_{B_c} F_{f_a}^{\rho^+ \eta_{du}} a_1 + M_{na}^{\rho^+ \eta_{u}} C_1] + [f_{B_c} F_{f_a}^{\eta_{du}} a_1 + M_{na}^{\eta_{du}} C_1] \right\} \cos \phi,
\]
\[
\mathcal{A}(B_c \to \rho^+ \eta') = V_{cb}^* V_{ud} \left\{ [f_{B_c} F_{f_a}^{\rho^+ \eta_{du}} a_1 + M_{na}^{\rho^+ \eta_{u}} C_1] + [f_{B_c} F_{f_a}^{\eta_{du}} a_1 + M_{na}^{\eta_{du}} C_1] \right\} \sin \phi,
\]
\[
\mathcal{A}(B_c \to \overline{K}^0 K^+) = V_{cb}^* V_{ud} \left\{ f_{B_c} F_{f_a}^{\overline{K}^0 K^+} a_1 + M_{na}^{\overline{K}^0 K^+} C_1 \right\},
\]
\[
\mathcal{A}(B_c \to K^{*+} \pi^0) = V_{cb}^* V_{us} \left\{ f_{B_c} F_{f_a}^{K^{*+} \pi^0} a_1 + M_{na}^{K^{*+} \pi^0} C_1 \right\},
\]
\[
\mathcal{A}(B_c \to K^{*0} \pi^+) = \sqrt{2} \mathcal{A}(B_c \to K^{*+} \pi^0),
\]
\[
\mathcal{A}(B_c \to K^{*+} \eta) = V_{cb}^* V_{us} \left\{ f_{B_c} \left[ F_{f_a}^{K^{*+} \eta} \cos \phi - F_{f_a}^{\eta_{K^{*+}}} \sin \phi \right] a_1 + \left[ M_{na}^{K^{*+} \eta} \cos \phi - M_{na}^{\eta_{K^{*+}}} \sin \phi \right] C_1 \right\},
\]
\[
\mathcal{A}(B_c \to K^{*+} \eta') = V_{cb}^* V_{us} \left\{ f_{B_c} \left[ F_{f_a}^{K^{*+} \eta} \sin \phi + F_{f_a}^{\eta_{K^{*+}}} \cos \phi \right] a_1 + \left[ M_{na}^{K^{*+} \eta} \sin \phi + M_{na}^{\eta_{K^{*+}}} \cos \phi \right] C_1 \right\},
\]
\[
\mathcal{A}(B_c \to \phi K^+) = V_{cb}^* V_{us} \left\{ f_{B_c} F_{f_a}^{\phi K^+} a_1 + M_{na}^{\phi K^+} C_1 \right\}. 
\]
C. $B_c \rightarrow VV$ decays

There are three kinds of polarizations of a vector meson, namely, longitudinal (L), normal (N), and transverse (T). The amplitudes for a $B_c$ meson decay to two vector mesons are also characterized by the polarization states of these vector mesons. The decay amplitudes $\mathcal{M}(\sigma)$ in terms of helicities, for $B_c \rightarrow V(P_2, \epsilon_2^+)V(P_3, \epsilon_3^+)$ decays, can be generally described by

$$\mathcal{M}(\sigma) = \epsilon_{2\mu}^*(\sigma) \epsilon_{3\nu}^*(\sigma) \left[ a g^{\mu\nu} + \frac{b}{m_{M_1} m_{M_3}} P_1^{\mu} P_1^{\nu} + i \frac{c}{m_{M_2} m_{M_3}} \epsilon^{\mu\alpha\beta} P_{2\alpha} P_{3\beta} \right],$$

$$\equiv m_{B_c}^2 \mathcal{M}_L + m_{B_c}^2 \mathcal{M}_N \epsilon_2^*(\sigma) \cdot \epsilon_3^*(\sigma = T) + i m_{B_c^*} \epsilon^{\alpha\beta\gamma\rho} \epsilon_{2\alpha}^*(\sigma) \epsilon_{3\beta}^*(\sigma) P_2^\gamma P_3^\rho,$$

where the superscript $\sigma$ denotes the helicity states of the two vector mesons with $L(T)$ standing for the longitudinal (transverse) component. And the definitions of the amplitudes $\mathcal{M}_i$ ($i = L, N, T$) in terms of the Lorentz-invariant amplitudes $a$, $b$, and $c$ are

$$m_{B_c}^2 \mathcal{M}_L = a \epsilon_2^*(L) \cdot \epsilon_3^*(L) + \frac{b}{m_{M_2} m_{M_3}} \epsilon_2^*(L) \cdot P_3 \epsilon_3^*(L) \cdot P_2,$$

$$m_{B_c}^2 \mathcal{M}_N = a,$$

$$m_{B_c}^2 \mathcal{M}_T = \frac{c}{r_2 r_3}.$$

We therefore will evaluate the helicity amplitudes $\mathcal{M}_L, \mathcal{M}_N, \mathcal{M}_T$ based on the pQCD factorization approach, respectively.

For every component of the polarization, the corresponding Feynman amplitude can be written in the following form,

$$F_{fa}^L = 8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \times \left\{ [x_2 \phi_2(x_2) \phi_3(x_3) - 2 r_2 r_3 ((x_2 + 1) \phi_2^*(x_2) + (x_2 - 1) \phi_2^0(x_2)) + \phi_2^0(x_3)] E_{fa}(t_a) h_{fa}(1 - x_2, x_3, b_3) + E_{fa}(t_b) h_{fa}(2 - x_3, b_2, b_3) \right\},$$

$$M_{na}^L = \frac{16\sqrt{6}}{\pi C_F m_{B_c}^2} \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \times \left\{ E_{na}(t_c) \left[ (r_3 - x_3 + 1) \phi_2(x_2) \phi_3(x_3) - 2 r_2 r_3 \phi_2^0(x_2)((3 r_3 + x_2 - x_3 + 1) \phi_2^0(x_2) - (r_3 + x_2 - x_3 + 1) \phi_3^0(x_3) + (r_3 - x_2 - x_3 - 1) \phi_2^0(x_3)) \right] \right\},$$

$$+ \left\{ E_{na}(t_c) \left[ (r_3 + x_2 - x_3 - 1) \phi_2(x_2) \phi_3(x_3) - 2 r_2 r_3 \phi_2^0(x_2)((4 r_3 + x_2 + x_3 - 1) \phi_2^0(x_2) - (r_3 + x_2 + x_3 - 1) \phi_3^0(x_3) \right] \right\}.$$
\[ F_{fa}^N = 8\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 r_2 r_3 \]
\[ \times \left\{ h_{fa}(1 - x_3, x_2, b_3, b_2) E_{fa}(t_a) \left[ (x_2 + 1)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) + (x_2 - 1)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) \right] \right. \\
\[ + E_{fa}(t_b) h_{fa}(x_2, 1 - x_3, b_2, b_3) \times [(x_3 - 2)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) - x_3 (\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3))] \right\} \}
\[ M_{na}^N = \frac{32\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 r_2 r_3 \]
\[ \times \left\{ r_c [\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)] E_{na}(t_c) h_{na}(x_2, x_3, b_1, b_2) \right. \\
\[ - \left. r_b [\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)] E_{na}(t_d) h_{na}(x_2, x_3, b_1, b_2) \right\} , \] (52)
\[ F_{fa}^T = 16\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 r_2 r_3 \]
\[ \times \left\{ h_{fa}(1 - x_3, x_2, b_3, b_2) E_{fa}(t_a) \left[ (x_2 + 1)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) + (x_2 - 1)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) \right] \right. \\
\[ + E_{fa}(t_b) h_{fa}(x_2, 1 - x_3, b_2, b_3) \times [(x_3 - 2)(\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)) - x_3 (\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3))] \right\} \}
\[ M_{na}^T = \frac{64\sqrt{6}}{3} \pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 r_2 r_3 \]
\[ \times \left\{ r_c [\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)] E_{na}(t_c) h_{na}(x_2, x_3, b_1, b_2) \right. \\
\[ - \left. r_b [\phi_2^a(x_2)\phi_3^a(x_3) + \phi_2^a(x_2)\phi_3^a(x_3)] E_{na}(t_d) h_{na}(x_2, x_3, b_1, b_2) \right\} . \] (53)

For seven $B_c \to VV$ decays, considering all the polarization ($H = L, N, T$) contributions and the Feynman decay amplitudes as shown in Eqs. (49-54), the total decay amplitude of these channels can be obtained directly,
\[ \mathcal{M}^H(B_c \to \rho^+ \rho^0) = V_{cb} V_{ud} \left\{ [f_{B_c} F_{fa:H}^{\rho^+ \rho^0} a_1 + M_{na:H}^{\rho^+ \rho^0} C_1] \right. \\
\[ - [f_{B_c} F_{fa:H}^{\rho^+ \rho^0} a_1 + M_{na:H}^{\rho^+ \rho^0} C_1] \right\} = 0 , \] (55)
\[ \mathcal{M}^H(B_c \to \rho^+ \omega) = V_{cb} V_{ud} \left\{ [f_{B_c} F_{fa:H}^{\rho^+ \omega} a_1 + M_{na:H}^{\rho^+ \omega}] + [f_{B_c} F_{fa:H}^{\omega \rho^+} a_1 + M_{na:H}^{\omega \rho^+} C_1] \right\} , \] (56)
\[ \mathcal{M}^H(B_c \to K^{*0} K^{*+}) = V_{cb} V_{ud} \left\{ f_{B_c} F_{fa:H}^{K^{*0} K^{*+}} a_1 + M_{na:H}^{K^{*0} K^{*+}} C_1 \right\} , \] (57)
\[ \mathcal{M}^H(B_c \to \phi K^{*+}) = V_{cb} V_{us} \left\{ f_{B_c} F_{fa:H}^{\phi K^{*+}} a_1 + M_{na:H}^{\phi K^{*+}} C_1 \right\} , \] (58)
\[ \mathcal{M}^H(B_c \to K^{*+} \rho^0) = V_{cb} V_{us} \left\{ f_{B_c} F_{fa:H}^{K^{*+} \rho^0} a_1 + M_{na:H}^{K^{*+} \rho^0} C_1 \right\} , \] (59)
\[ \mathcal{M}^H(B_c \to K^{*0} \rho^+) = \sqrt{2} \mathcal{M}^H(B_c \to K^{*+} \rho^0) , \] (60)
\[ \mathcal{M}^H(B_c \to K^{*+} \omega) = V_{cb} V_{us} \left\{ f_{B_c} F_{fa:H}^{K^{*+} \omega} a_1 + M_{na:H}^{K^{*+} \omega} C_1 \right\} . \] (61)
IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate the branching ratios (and polarization fractions, relative phases) for those considered thirty $B_c \to M_2M_3$ decay modes. The input parameters and the wave functions to be used are given in Appendix $\mathbb{A}$. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

| Decay Modes $(\Delta S = 0)$ | $BR's(10^{-8})$ | Decay Modes $(\Delta S = 1)$ | $BR's(10^{-8})$ |
|--------------------------------|-----------------|--------------------------------|-----------------|
| $B_c \to \pi^+\pi^0$          | 0               | $B_c \to \pi^+K^0$             | $4.0^{+1.9}_{-0.6}(m_c)^{+2.3}_{-1.6}(a_i)^{+0.5}_{-0.3}(m_0)$ |
| $B_c \to \pi^+\eta$           | $22.8^{+6.9}_{-4.6}(m_c)^{+7.2}_{-4.5}(a_i)^{+3.4}_{-2.0}(m_0)$ | $B_c \to K^+\eta$ | $0.6^{+0.6}_{-0.5}(m_c)^{+0.6}_{-0.5}(a_i)^{+0.2}_{-0.1}(m_0)$ |
| $B_c \to \pi^+\eta'$          | $15.3^{+4.6}_{-3.1}(m_c)^{+4.8}_{-3.0}(a_i)^{+2.2}_{-1.4}(m_0)$ | $B_c \to K^+\eta'$ | $5.7^{+0.9}_{-1.0}(m_c)^{+1.0}_{-1.6}(a_i)^{+0.0}_{-0.3}(m_0)$ |
| $B_c \to K^+\bar{K}^0$        | $24.0^{+2.4}_{-0.6}(m_c)^{+7.3}_{-6.0}(a_i)^{+6.8}_{-5.5}(m_0)$ | $B_c \to K^+\pi^0$ | $2.0^{+0.5}_{-0.3}(m_c)^{+1.2}_{-0.8}(a_i)^{+0.3}_{-0.1}(m_0)$ |

For $B_c \to PP, PV, VP$ decays, the decay rate can be written as

$$\Gamma = \frac{G_F^2 m_{B_c}^3}{32\pi} |A(B_c \to M_2M_3)|^2$$

(62)

where the corresponding decay amplitudes $A$ have been given explicitly in Eqs. (20,27) and Eqs. (32,46). Using the decay amplitudes obtained in last section, it is straightforward to calculate the branching ratios with uncertainties as presented in Tables (I,II,III).

| Decay Modes $(\Delta S = 0)$ | $BR's(10^{-7})$ | Decay Modes $(\Delta S = 1)$ | $BR's(10^{-8})$ |
|--------------------------------|-----------------|--------------------------------|-----------------|
| $B_c \to \pi^+\rho^0$          | $1.7^{+0.1}_{-0.0}(m_c)^{+0.1}_{-0.2}(a_i)^{+0.0}_{-0.3}(m_0)$ | $B_c \to K^+\rho^0$ | $3.1^{+0.6}_{-0.8}(m_c)^{+1.2}_{-1.5}(a_i)^{+0.1}_{-0.2}(m_0)$ |
| $B_c \to K^0\bar{K}^*\pm$     | $1.8^{+0.1}_{-0.1}(m_c)^{+0.1}_{-0.1}(a_i)^{+0.0}_{-0.0}(m_0)$ | $B_c \to K^0\rho^0$ | $6.1^{+1.3}_{-1.5}(m_c)^{+2.0}_{-2.0}(a_i)^{+0.2}_{-0.3}(m_0)$ |
| $B_c \to \pi^+\omega$         | $5.8^{+1.4}_{-2.2}(m_c)^{+1.1}_{-1.3}(a_i)^{+0.4}_{-1.0}(m_0)$ | $B_c \to K^+\omega$ | $2.3^{+1.1}_{-1.2}(m_c)^{+1.8}_{-1.2}(a_i)^{+0.1}_{-0.1}(m_0)$ |

For $B_c \to VV$ decays, the decay rate can be written explicitly as,

$$\Gamma = \frac{G_F^2|P_c|}{16\pi m_{B_c}^3} \sum_{\sigma=L,T} \mathcal{M}^{(\sigma)} \mathcal{M}^{(\sigma)}$$

(63)

where $|P_c| \equiv |P_{2z}| = |P_{3z}|$ is the momentum of either of the outgoing vector mesons.

Based on the helicity amplitudes (18), we can define the transversity amplitudes,

$$A_L = -\xi m^2_{B_c} \mathcal{M}_L, \quad A_\| = \xi \sqrt{2} m^2_{B_c} \mathcal{M}_N, \quad A_\perp = \xi m^2_{B_c} \sqrt{2(r^2 - 1)} \mathcal{M}_T.$$  

(64)
for the longitudinal, parallel, and perpendicular polarizations, respectively, with the normalization factor \( \xi = \sqrt{G_c^2 P_c}/(16\pi m^2_{B_c}) \) and the ratio \( r = P_2 \cdot P_3/(m_{M_2} \cdot m_{M_3}) \). These amplitudes satisfy the relation,

\[
|A_L|^2 + |A_||^2 + |A_\perp|^2 = 1
\]

following the summation in Eq. (63).

Since the transverse-helicity contributions manifest themselves in polarization observables, we therefore define two kinds of polarization observables, i.e., polarization fractions \((f_L, f_||, f_\perp)\) and relative phases \((\phi_||, \phi_\perp)\) as \([53]\),

\[
f_{L(||,\perp)} = \frac{|A_{L(||,\perp)}|^2}{|A_L|^2 + |A_|||^2 + |A_\perp|^2}, \quad \phi_{L(||,\perp)} = \text{arg} \frac{A_{L(||,\perp)}}{A_L};
\]

It should be noted that the final results of relative phases will plus one value, i.e., \( \pi \), due to an additional minus sign in the definition of \( A_L \).

We also define another two quantities reflecting the effects of CP-violating asymmetries indirectly \([53, 54]\),

\[
\Delta \phi_|| = \frac{\overline{\phi}_|| - \phi_||}{2}, \quad \Delta \phi_\perp = \frac{\overline{\phi}_\perp - \phi_\perp - \pi}{2},
\]

where \( \overline{\phi}_|| \) and \( \overline{\phi}_\perp \) are the CP-conjugated relative phases corresponding to \( \phi_|| \) and \( \phi_\perp \), respectively.

With the complete decay amplitudes, by employing Eq. (63) and the input parameters and wave functions as given in Appendix A, we will present the pQCD predictions for CP-averaged branching ratios, longitudinal polarization fractions and relative phases of the considered decays with errors as shown in Tables IV and V.

Based on the pQCD predictions as given in Tables I-V, we have the following remarks:

- Among considered pure annihilation \( B_c \to PV/VP, VV \) decays, the pQCD predictions for the CP-averaged branching ratios for those \( \Delta S = 0 \) processes are much larger than those of \( \Delta S = 1 \) channels ( one of the two final state mesons is the \( K^{(*)} \) meson ), which are mainly due to the large CKM factor \( |V_{ud}/V_{us}|^2 \sim 19 \). For \( B_c \to \pi^+\pi^0, \rho^+\rho^0 \) decays, the contributions from \( \bar{u}u \) and \( \bar{d}d \) components cancel each

| TABLE III: Same as Table II but for \( B_c \to VP \) modes. |

| Decay Modes | \( BR_s(10^{-7}) \) | Decay Modes | \( BR_s(10^{-8}) \) |
|-------------|-----------------|-------------|-----------------|
| \( \Delta S = 0 \) | \( \Delta S = 1 \) | \( \Delta S = 0 \) | \( \Delta S = 1 \) |
| \( B_c \to \rho^+\pi^0 \) | \( 0.5_{-0.3}^{+0.7} (m_c)_{-0.2}^{+0.3} (a_i)_{-0.3}^{+0.2} (m_0) \) | \( B_c \to K^{*0}\pi^+ \) | \( 3.3_{-0.2}^{+0.5} (m_c)_{-0.3}^{+0.4} (a_i)_{-0.1}^{+0.2} (m_0) \) |
| \( B_c \to \rho^+\eta \) | \( 3.6_{-0.8}^{+1.4} (m_c)_{-0.6}^{+0.9} (a_i)_{-0.0}^{+0.0} (m_0) \) | \( B_c \to K^{*+}\eta^0 \) | \( 0.9_{-0.0}^{+0.1} (m_c)_{-0.2}^{+0.6} (a_i)_{-0.0}^{+0.0} (m_0) \) |
| \( B_c \to \rho^+\eta' \) | \( 1.0_{-0.5}^{+0.7} (m_c)_{-0.3}^{+1.7} (a_i)_{-0.2}^{+0.0} (m_0) \) | \( B_c \to K^{*+}\eta' \) | \( 3.8_{-0.0}^{+0.0} (m_c)_{-0.0}^{+1.0} (a_i)_{-0.0}^{+0.0} (m_0) \) |
| \( B_c \to K^{0}\pi^+ \) | \( 5.6_{-0.0}^{+1.1} (m_c)_{-0.9}^{+1.2} (a_i)_{-0.3}^{+0.0} (m_0) \) | | |
TABLE IV: The pQCD predictions of branching ratios \((BR's)\) and longitudinal polarization fractions \((LPF's)\) for \(B_c \rightarrow VV\) modes.

| Decay Modes | \(BR's(10^{-7})\) | \(LPF's(\%)\) |
|-------------|----------------|---------------|
| \(B_c \rightarrow \rho^+ \rho^0\) | 0 | – |
| \(B_c \rightarrow \rho^+ \omega\) | \(10.6^{+3.2}_{-0.2}(m_c)^{+0.2}_{-0.2}(a_i)\) | \(92.9^{+1.0}_{-0.1}(m_c)^{+1.2}_{-0.1}(a_i)\) |
| \(B_c \rightarrow K^0 K^{*+}\) | \(10.0^{+0.6}_{-0.4}(m_c)^{+0.6}_{-0.4}(a_i)\) | \(92.0^{+0.5}_{-0.4}(m_c)^{+0.6}_{-0.4}(a_i)\) |
| \(B_c \rightarrow K^0 \rho^+\) | \(0.6^{+0.0}_{-0.0}(m_c)^{+0.2}_{-0.1}(a_i)\) | \(94.9^{+0.2}_{-0.2}(m_c)^{+2.0}_{-1.4}(a_i)\) |
| \(B_c \rightarrow K^{**} \rho^0\) | \(0.3^{+0.0}_{-0.0}(m_c)^{+0.1}_{-0.1}(a_i)\) | \(94.9^{+0.2}_{-0.2}(m_c)^{+1.3}_{-1.4}(a_i)\) |
| \(B_c \rightarrow K^{**} \omega\) | \(0.3^{+0.0}_{-0.0}(m_c)^{+0.0}_{-0.0}(a_i)\) | \(94.8^{+0.3}_{-0.2}(m_c)^{+1.1}_{-1.2}(a_i)\) |
| \(B_c \rightarrow \phi K^{**}\) | \(0.5^{+0.0}_{-0.0}(m_c)^{+0.1}_{-0.0}(a_i)\) | \(86.4^{+0.0}_{-0.0}(m_c)^{+4.9}_{-9.0}(a_i)\) |

TABLE V: The pQCD predictions of relative phases for \(B_c \rightarrow VV\) modes.

| Decay Modes | \(\phi_{||}(\text{rad})\) | \(\phi_{\perp}(\text{rad})\) | \(\Delta \phi_{||}\) | \(\Delta \phi_{\perp}\) |
|-------------|----------------|----------------|----------------|----------------|
| \(B_c \rightarrow \rho^+ \rho^0\) | – | – | – | – |
| \(B_c \rightarrow \rho^+ \omega\) | \(3.86^{+0.31}_{-0.26}(m_c)^{+0.25}_{-0.19}(a_i)\) | \(4.43^{+0.16}_{-0.17}(m_c)^{+0.25}_{-0.19}(a_i)\) | 0 | –π/2 |
| \(B_c \rightarrow K^0 K^{*+}\) | \(3.68^{+0.18}_{-0.13}(m_c)^{+0.48}_{-0.21}(a_i)\) | \(3.76^{+0.16}_{-0.00}(m_c)^{+0.48}_{-0.20}(a_i)\) | 0 | –π/2 |
| \(B_c \rightarrow K^0 \rho^+\) | \(4.11^{+0.17}_{-0.20}(m_c)^{+0.30}_{-0.23}(a_i)\) | \(4.20^{+0.14}_{-0.05}(m_c)^{+0.30}_{-0.21}(a_i)\) | 0 | –π/2 |
| \(B_c \rightarrow K^{**} \rho^0\) | \(4.11^{+0.17}_{-0.20}(m_c)^{+0.30}_{-0.23}(a_i)\) | \(4.20^{+0.14}_{-0.05}(m_c)^{+0.30}_{-0.21}(a_i)\) | 0 | –π/2 |
| \(B_c \rightarrow K^{**} \omega\) | \(4.15^{+0.13}_{-0.25}(m_c)^{+0.25}_{-0.25}(a_i)\) | \(4.23^{+0.11}_{-0.09}(m_c)^{+0.25}_{-0.24}(a_i)\) | 0 | –π/2 |
| \(B_c \rightarrow \phi K^{**}\) | \(3.80^{+0.25}_{-0.34}(m_c)^{+0.44}_{-0.26}(a_i)\) | \(3.89^{+0.22}_{-0.19}(m_c)^{+0.43}_{-0.21}(a_i)\) | 0 | –π/2 |

other exactly and result in the zero branching ratios. In fact, these two channels are forbidden, even if with final state interactions. Simply, two pions can not form an s wave isospin 1 state, because of Bose-Einstein statics. Any other nonzero data for these two channels may indicate the effects of exotic new physics.

- There is no CP violation for all these decays within the standard model, since there is only one kind of tree operators involved in the decay amplitude of all considered \(B_c\) decays, which can be seen from Eq. (12).
- The pQCD predictions for the branching ratios of considered \(B_c\) decays vary in the range of \(10^{-6}\) (for \(B_c \rightarrow \overline{K}^0 K^+, \overline{K}^{*0} K^{**}\) and \(\rho^+ \omega\) decays) to \(10^{-8}\) (for most \(\Delta S = 1\) \(B_c\) decays). The \(B_s\) decays with the branching ratio of \(10^{-6}\) can be measured at the LHC experiment [38].
- As mentioned in the introduction, the authors of Ref. [38] studied many pure annihilation \(B_c\) decays by employing the SU(3) flavor symmetry and the OGE model respectively, and presented their numerical estimates for the branching ratios of \(B_c \rightarrow \phi K^+, \overline{K}^0 K^+, \overline{K}^{*0} K^+\) and \(\overline{K}^{*0} K^{**}\) decays. As a comparison, we show in Table VI the pQCD predictions and the results as given in Ref. [38] for relevant
channels. From Table VI, one can see easily that the pQCD predictions basically agree with the results obtained based on the $SU(3)$ flavor symmetry.

- For $B_c \to (\pi^+, \rho^+)(\eta, \eta')$ decays, the relevant final state mesons contain the same component $\bar{u}u + \bar{d}d$, therefore they have the similar branching ratios. The small differences among their branching ratios mainly come from the different mixing coefficients, i.e., $\cos \phi$ and $\sin \phi$.

- For $B_c \to K^+\eta^{(')}$ decays, however, one finds that $Br(B_c \to K^+\eta') \sim 10 \times Br(B_c \to K^+\eta)$, which is rather different from the pattern of $Br(B_c \to \pi^+\eta) \sim Br(B_c \to \pi^+\eta')$ and $Br(B_c \to \rho^+\eta) \sim Br(B_c \to \rho^+\eta')$. This large difference can be understood as follows: For the $\Delta S = 1$ processes, both $\eta_0$ and $\eta_s$ will contribute to $B_c \to K^+\eta$ and $K^+\eta'$ decays but with an opposite sign for $\eta_0$ and $\eta_s$ term, as well as different coefficients. Which results in a destructive interference between $\eta_0$ component for $B_c \to K^+\eta$, but a constructive interference for $B_c \to K^+\eta'$. This situation is very similar with that for the $B \to K\eta$ and $K\eta'$ decays \[55, 56, 57\].

- Unlike $B_c \to K^+\eta^{(')}$ decays, $Br(B_c \to K^+\eta') \approx 4Br(B_c \to K^{*+}\eta) \approx 3.8 \times 10^{-8}$. The reason is that both of them are mainly determined by the factorizable contributions of $\eta_s$ term.

- For $B_c \to VV$ decays, we can find that (a) the branching ratios are in order of $\mathcal{O}(10^{-8} \sim 10^{-7})$ except for $Br(B_c \to \bar{K}^0K^{**})$ and $Br(B_c \to \rho^+\omega) \sim 10^{-6}$; (b) the longitudinal polarization fractions are around 95% within the theoretical errors except for $B_c \to \phi K^{*+}$ ($\sim 86\%$) and play the dominant role.

- According to the discussions in Ref. \[38\], there are some simple relations among some decay channels in the limit of exact $SU(3)$ flavor symmetry. For $B_c \to PP$ decays, such relations are

$$A(B_c \to K^0\pi^+) = \sqrt{2}A(B_c \to K^+\pi^0) = \lambda A(B_c \to K^{+}\bar{K}^0),$$

where $\lambda = V_{us}/V_{ud} \approx 0.2$. For $B_c \to VP/PV$ and $B_c \to VV$ decays, the relations
read

\[ A(B_c \to K^{*0}\pi^+) = \sqrt{2}A(B_c \to K^{*+}\pi^0) = \lambda A(B_c \to \bar{K}^0K^+), \]
\[ A(B_c \to \rho^+K^0) = \sqrt{2}A(B_c \to \rho^0K^+) = \lambda A(B_c \to K^{*+}\bar{K}^0), \]
\[ (-1)^\ell A(B_c^+ \to \rho^+K^{*0}) = (-1)^\ell\sqrt{2}A(B_c^+ \to \rho^0K^{*+}) = \lambda A(B_c \to K^{*+}\bar{K}^0) \]

where \( \ell = 0, 1, 2 \). From our pQCD calculations, we notice that the first equality of each of the above relations (68-71) are valid in isospin symmetry. They hold exactly in our numerical calculations. The second equality of each relations are only valid at exact SU(3) symmetry thus they are violated at the order of SU(3) breaking effect in our calculations.

- Since the LHC experiment can measure the \( B_c \) decays with a branching ratio at \( 10^{-6} \) level, our pQCD predictions for the branching ratios of \( B_c \to \bar{K}^0K^+ \) and other decays with a decay rate at \( 10^{-6} \) or larger could be measured at the LHC experiments.

- For most considered pure annihilation \( B_c \) decays, it is hard to observe them even in LHC due to their tiny decay rate. Their observation at LHC, however, would mean a large non-perturbative contribution or a signal for new physics beyond the SM.

- It is worth of stressing that the theoretical predictions in the pQCD approach still have large theoretical errors induced by the still large uncertainties of many input parameters. Any progress in reducing the error of input parameters, such as the Gegenbauer moments \( a_i \) and the charm quark mass \( m_c \), will help us to improve the precision of the pQCD predictions.

V. SUMMARY

In short, we studied the two-body charmless hadronic \( B_c \to PP, PV/VP, VV \) decays by employing the pQCD factorization approach based on the \( k_T \) factorization theorem. These considered decay channels can occur only via the annihilation diagram and they will provide an important testing ground for the magnitude of the annihilation contribution.

The pQCD predictions for CP-averaged branching ratios, longitudinal polarization fractions and relative phases are displayed in Tables (I-V). From our numerical evaluations and phenomenological analysis, we found the following results:

- The pQCD predictions for the branching ratios vary in the range of \( 10^{-6} \) to \( 10^{-8} \), basically agree with the predictions obtained by using the exact SU(3) flavor symmetry. The \( B_c \to \bar{K}^0K^+ \) and other decays with a decay rate at \( 10^{-6} \) or larger could be measured at the LHC experiment.

- For \( B_c \to PV/VP, VV \) decays, the branching ratios of \( \Delta S = 0 \) processes are basically larger than those of \( \Delta S = 1 \) ones. Such differences are mainly induced by the CKM factors involved: \( V_{ud} \sim 1 \) for the former decays while \( V_{us} \sim 0.22 \) for the latter ones.

\[ \text{Here, since the longitudinal contributions dominate the } B_c \to K^{*0}\rho^+ \text{ decay, we use its longitudinal part (i.e., } \ell = 0 \text{) to compare with the decay amplitude of } B_c \to K^{*+}\bar{K}^0 \text{ decay.} \]
• Analogous to $B \rightarrow K\eta^{(')}$ decays, we find $Br(B_c \rightarrow K^+\eta') \sim 10 \times Br(B_c \rightarrow K^+\eta)$. This large difference can be understood by the destructive and constructive interference between the $\eta_q$ and $\eta_s$ contribution to the $B_c \rightarrow K^+\eta$ and $B_c \rightarrow K^+\eta'$ decay.

• For $B_c \rightarrow VV$ decays, the longitudinal polarization fractions are around 95% except for $B_c \rightarrow \phi K^{*+}$ ($f_L \sim 86\%$) and play the dominant role.

• Because only tree operators are involved, the CP-violating asymmetries for these considered $B_c$ decays are absent naturally.

• The pQCD predictions still have large theoretical uncertainties, induced by the uncertainties of input parameters.

• We here calculated the branching ratios and other physical observables of the pure annihilation $B_c$ decays by employing the pQCD approach. We do not consider the possible long-distance (LD) contributions, such as the re-scattering effects, although they may be large and affect the theoretical predictions. It is beyond the scope of this work.

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APPENDIX A: INPUT PARAMETERS AND DISTRIBUTION AMPLITUDES

The masses (GeV), decay constants (GeV), QCD scale (GeV) and $B$ meson lifetime are

$$\Lambda_{\text{MS}}^{(f=4)} = 0.250, \quad m_W = 80.41, \quad m_{B_c} = 6.286, \quad f_{B_c} = 0.489,$$

$$m_{\phi} = 1.02, \quad f_{\phi} = 0.231, \quad f_{\phi}^{T} = 0.200, \quad m_{K^{*+}} = 0.892,$$

$$f_{K^{*+}} = 0.217, \quad f_{K^{*+}}^{T} = 0.185, \quad m_{\rho} = 0.770, \quad f_{\rho} = 0.209,$$

$$f_{\rho}^{T} = 0.165, \quad m_{\omega} = 0.782, \quad f_{\omega} = 0.195, \quad f_{\omega}^{T} = 0.145,$$

$$m_{\pi^{0}}^{*} = 1.4, \quad m_{K^{0}}^{*} = 1.6, \quad m_{\eta^{(')}} \approx 1.08, \quad m_{\eta^{(')}_{s}} \approx 1.92,$$

$$m_{b} = 4.8, \quad f_{\pi} = 0.131, \quad f_{K} = 0.16, \quad \tau_{B_{c}^{+}} = 0.46\;\text{ps}. \quad (A1)$$

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take $A = 0.814$ and $\lambda = 0.2257$, $\bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$.
The twist-2 pseudoscalar meson distribution amplitude $\phi_P^A(P = \pi, K)$, and the twist-3 ones $\phi_P^P$ and $\phi_P^T$ have been parametrized as \[58, 59, 60,\]

\[
\phi_P^A(x) = \frac{f_P}{2\sqrt{2}N_c} 6x(1-x) \left[ 1 + a_1^PC_1^{3/2}(2x-1) + a_2^PC_2^{3/2}(2x-1) + a_4^PC_4^{3/2}(2x-1) \right],
\]

\[
\phi_P^P(x) = \frac{f_P}{2\sqrt{2}N_c} \left[ 1 + \left( 30\eta_3 - \frac{5}{2}\rho_2^P \right) C_2^{1/2}(2x-1) \right.
\]

\[
- 3 \left\{ \eta_3\omega_3 \rho + \frac{9}{20}\rho_2^P(1 + 6a_2^P) \right\} C_4^{1/2}(2x-1) \right] ,
\]

\[
\phi_P^T(x) = \frac{f_P}{2\sqrt{2}N_c} (1-2x) \left[ 1 + 6 \left( 5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_2^P - \frac{3}{5}\rho_2^P(a_2^P) \right) (1-10x + 10x^2) \right].
\]

with the Gegenbauer moments $a_1^T_0 = 0, a_1^K = 0.17 \pm 0.17, a_2^K = 0.115 \pm 0.115, a_4^K = -0.015,$ the mass ratio $\rho_{\pi(K)} = m_{\pi(K)}/m_{0\pi(K)}$ and $\rho_{\eta(s)} = 2m_{q(s)}/m_{qq(ss)}$, and the Gegenbauer polynomials $C_n^\nu(t),$

\[
C_2^{1/2}(t) = \frac{1}{2} (3t^2 - 1) , C_4^{1/2}(t) = \frac{1}{8} (3 - 30t^2 + 35t^4),
\]

\[
C_1^{3/2}(t) = 3t , C_2^{3/2}(t) = \frac{3}{2} (5t^2 - 1) , C_4^{3/2}(t) = \frac{15}{8} (1 - 14t^2 + 21t^4) .
\]

In the above distribution amplitudes for kaon, the momentum fraction $x$ is carried by the $s$ quark. For both the pion and kaon, we choose $\eta_3 = 0.015$ and $\omega_3 = -3 \ [58, 59].$

The twist-2 distribution amplitudes for the longitudinally and tranversely polarized vector meson can be parameterized as:

\[
\phi_V(x) = \frac{3f_V}{\sqrt{6}} x(1-x) \left[ 1 + a_{1V}C_1^{3/2}(2x-1) + a_{2V}C_2^{3/2}(2x-1) \right] ,
\]

\[
\phi_V^T(x) = \frac{3f_V^T}{\sqrt{6}} x(1-x) \left[ 1 + a_{1V}C_1^{3/2}(2x-1) + a_{2V}C_2^{3/2}(2x-1) \right],
\]

Here $f_V$ and $f_V^T$ are the decay constants of the vector meson with longitudinal and tranverse polarization, respectively. The Gegenbauer moments have been studied extensively in the literatures \[61, 62\], here we adopt the following values from the recent updates \[63, 64, 65\]:

\[
a_{1K^*} = 0.03 \pm 0.02, a_{2\rho} = a_{2\omega} = 0.15 \pm 0.07, a_{2K^*} = 0.11 \pm 0.09, a_{4\rho} = 0.18 \pm 0.08\]
\[
a_{1K^*} = 0.04 \pm 0.03, a_{2\rho} = a_{2\omega} = 0.14 \pm 0.06, a_{2K^*} = 0.10 \pm 0.08, a_{4\rho} = 0.14 \pm 0.07\]

The asymptotic forms of the twist-3 distribution amplitudes $\phi_V^{t,s}$ and $\phi_V^{v,a}$ are \[42\]:

\[
\phi_V^t(x) = \frac{3f_V^T}{2\sqrt{6}} (2x-1)^2, \quad \phi_V^s(x) = -\frac{3f_V^T}{2\sqrt{6}} (2x-1),
\]

\[
\phi_V^v(x) = \frac{3f_V}{8\sqrt{6}} (1 + (2x-1)^2), \quad \phi_V^a(x) = -\frac{3f_V}{4\sqrt{6}} (2x-1). \]

\[18\]
APPENDIX B: RELATED HARD FUNCTIONS

In this section, we group the functions which appear in the factorization formulae. The functions $h$ in the decay amplitudes consist of two parts: one is the jet function $S_t(x_i)$ derived by the threshold re-summation\cite{44}, the other is the propagator of virtual quark and gluon. They are defined by

$$h_f(x_3, x_2, b_3, b_2) = \left(\frac{i\pi}{2}\right)^2 S_t(x_2) \left[ \theta(b_3 - b_2) H_0^{(1)}(\sqrt{x_2 M_{B_c} b_3}) J_0(\sqrt{x_2 M_{B_c} b_2}) + \theta(b_2 - b_3) H_0^{(1)}(\sqrt{x_2 M_{B_c} b_2}) J_0(\sqrt{x_2 M_{B_c} b_3}) \right] H_0^{(1)}(\sqrt{x_2 x_3 M_{B_c} b_1})$$

(B1)

$$h_{n\alpha}^{(d)}(x_2, x_3, b_1, b_2) = \left(\frac{i\pi}{2}\right)^2 S_t(x_2) \left[ \theta(b_1 - b_2) H_0^{(1)}(\sqrt{x_2 (1 - x_3) M_{B_c} b_1}) J_0(\sqrt{x_2 (1 - x_3) M_{B_c} b_2}) + \theta(b_2 - b_1) H_0^{(1)}(\sqrt{x_2 (1 - x_3) M_{B_c} b_2}) J_0(\sqrt{x_2 (1 - x_3) M_{B_c} b_1}) \right]$$

\times \left\{ \begin{array}{ll}
\frac{i\pi}{2} H_0^{(1)}(\sqrt{F_c^2 M_{B_c} b_1}), & F_c < 0 \\
J_0(\sqrt{F_c^2 M_{B_c} b_1}), & F_c > 0,
\end{array} \right.

(B2)

where

$$F_c = (r_c - x_2)(1 - x_3) + r_c^2, \quad F_d = r_b^2 - (1 - r_c - x_2)x_3,$$

and $H_0^{(1)}(z) = J_0(z) + i Y_0(z)$.

The hard scales are chosen as

$$t_a = \max\{\sqrt{x_2 M_{B_c}}, 1/b_2, 1/b_3\},$$

(B4)

$$t_b = \max\{\sqrt{1 - x_3 M_{B_c}}, 1/b_2, 1/b_3\},$$

(B5)

$$t_c = \max\{\sqrt{x_2 (1 - x_3) M_{B_c}}, \sqrt{|(r_c - x_2)(1 - x_3) + r_c^2| M_{B_c}, 1/b_1, 1/b_2}\},$$

(B6)

$$t_d = \max\{\sqrt{x_2 (1 - x_3) M_{B_c}}, \sqrt{|r_b^2 - (1 - r_c - x_2)x_3| M_{B_c}, 1/b_1, 1/b_2}\}.$$  

(B7)

They are given as the maximum energy scale appearing in each diagram to kill the large logarithmic radiative corrections.

The $S_t$ re-sums the threshold logarithms $\ln^2 x$ appearing in the hard kernels to all orders and it has been parameterized as

$$S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1+c)} [x(1-x)]^c,$$

(B8)

with $c = 0.4 \pm 0.1$. In the nonfactorizable contributions, $S_t(x)$ gives a very small numerical effect to the amplitude \cite{66}. Therefore, we drop $S_t(x)$ in $h_{n\alpha}$.

The evolution factors $E_{fa}$ and $E_{n\alpha}$ entering in the expressions for the matrix elements (see section III) are given by

$$E_{fa}(t) = \alpha_s(t) \exp[-S_2(t) - S_3(t)],$$

(B9)

$$E_{n\alpha}(t) = \alpha_s(t) \exp[-S_B(t) - S_2(t) - S_3(t)]|_{b_2 = b_3},$$

(B10)
in which the Sudakov exponents are defined as

\[
S_B(t) = s \left( r_c \frac{M_B c}{\sqrt{2}}, b_1 \right) + \frac{5}{3} \int_{t/b_1}^{t} \frac{d\bar{\mu}}{\mu} \gamma_q(\alpha_s(\bar{\mu})),
\]

(B11)

\[
S_2(t) = s \left( x_2 \frac{M_B c}{\sqrt{2}}, b_2 \right) + s \left( (1 - x_2) \frac{M_B c}{\sqrt{2}}, b_2 \right) + 2 \int_{t/b_2}^{t} \frac{d\bar{\mu}}{\mu} \gamma_q(\alpha_s(\bar{\mu})),
\]

(B12)

with the quark anomalous dimension \( \gamma_q = -\alpha_s/\pi \). Replacing the kinematic variables of \( M_2 \) to \( M_3 \) in \( S_2 \), we can get the expression for \( S_3 \). The explicit forms for the function \( s(Q, b) \) are defined in the Appendix A in Ref. [36].
[25] Y.S. Dai and D.S. Du, Eur. Phys. J. C 9, 557 (1999); D.S. Du and Z.T. Wei, Eur. Phys. J. C 5, 705 (1998).
[26] M. Masetti, Phys. Lett. B 286, 160 (1992).
[27] Q.P. Xu and A.N. Kamal, Phys. Rev. D 46, 3836 (1992).
[28] J. Sun, Y. Yang, W. Du and H. Ma, Phys. Rev. D 77, 074013 (2008); Phys. Rev. D 77, 114004 (2008); Eur. Phys. J. C 60, 107 (2009).
[29] M.A. Ivanov, J.G. Kö rner and O.N. Pakhomova, Phys. Lett. B 555, 189 (2003).
[30] F. Hussain and M.D. Scadron, Phys. Rev. D 30, 1492 (1984).
[31] J. Zhang and X.Q. Yu, Eur. Phys. J. C 63, 435 (2009).
[32] H.M. Choi and C.R. Ji, arXiv:0909.5028[hep-ph].
[33] J. Schwinger, Phys. Rev. Lett. 12, 630 (1964); M. Wirbel, B. Stech and M. Bauer, Z. Phys. C 29, 637(1985); M. Bauer, B. Stech and M. Wirbel, ibid. 34, 103(1987); L.L. Chau et al., Phys. Rev. D 43, 2176 (1991); 58, 019902(E) (1998).
[34] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B 591, 313 (2000).
[35] Y.Y. Keum, H.N. Li and A.I. Sanda, Phys. Lett. B 504, 6 (2001); Phys. Rev. D 63, 054008 (2001).
[36] C.D. Lü, K. Ukai and M.Z. Yang, Phys. Rev. D 63, 074009 (2001).
[37] H.N. Li, Prog. Part. & Nucl. Phys. 51, 85 (2003), and reference therein.
[38] S.D. Genon, J. He, E. Kou and P. Robbe, arXiv:0907.2256[hep-ph].
[39] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
[40] C.D. Lü and K. Ukai, Eur. Phys. J. C 28, 305 (2003).
[41] B.H. Hong and C.D. Lü, Sci. China G 49, 357 (2006).
[42] H.N. Li, and S. Mishima, Phys. Rev. D 71, 054025 (2005); H.N. Li, Phys. Lett. B 622, 63 (2005).
[43] A.V. Gritsan, eConf. C 070512, 001 (2007).
[44] H.N. Li, Phys. Rev. D 66, 094010 (2002).
[45] H.N. Li and B. Tseng, Phys. Rev. D 57, 443 (1998).
[46] C.D. Lü and M.Z. Yang, Eur. Phys. J. C 28, 515 (2003).
[47] A. Ali, G. Kramer, Y. Li, C.D. Lü, Y.L. Shen, W. Wang and Y.M. Wang, Phys. Rev. D 76, 074018 (2007).
[48] Th. Feldmann, P. Kroll and B. Stech, Phys. Rev. D 58, 114006 (1998).
[49] R. Escribano and J.M. Frere, J. High Energy Phys. 06 (2005) 029; J. Schechter, A. Subbaraman and H. Weigel, Phys. Rev. D 48, 339 (1993).
[50] X. Liu, H.S. Wang, Z.J. Xiao, L.B. Guo and C.D. Lü, Phys. Rev. D 73, 074002 (2006); H.S. Wang, X. Liu, Z.J. Xiao, L.B. Guo and C.D. Lü, Nucl. Phys. B 738, 243 (2006); Z.J. Xiao, X.F. Chen and D.Q. Guo, Eur. Phys. J. C 50, 363 (2007); Z.J. Xiao, D.Q Guo and X.F Chen, Phys. Rev. D 75, 014018 (2007); Z.J. Xiao, X. Liu and H.S. Wang, Phys. Rev. D 75, 034017 (2007); Z.J. Xiao, X.F. Chen and D.Q. Guo, arXiv:0701146[hep-ph].
[51] Y.Y. Charng, T. Kurimoto and H.N. Li, Phys. Rev. D 74, 074024 (2006).
[52] R. Escribano and J. Nadal, J. High Energy Phys. 05 (2007) 006.
[53] B. Aubert et al., (BaBar Collaboration), Phys. Rev. Lett. 99, 201802 (2007) .
[54] M.Beneke, J. Rohrer and D.S. Yang, Nucl. Phys. B 774, 64 (2007).
[55] C. Amsler et al., (Particle Data Group), Phys. Lett. B 667, 1 (2008).
[56] Heavy Flavor Averaging Group, E. Barberio et al., arXiv:0809.1297[hep-ex]; and online
update at http://www.slac.stanford.edu/xorg/hfag.

[57] Z.J. Xiao, Z.Q. Zhang, X. Liu and L.B. Guo, Phys. Rev. D 78, 114001 (2008).
[58] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112, 173 (1984); V.M. Braun and I.E. Filyanov, Z. Phys. C 44, 157 (1989); P. Ball, J. High Energy Phys. 09 (1998) 005; V.M. Braun and I.E. Filyanov, Z. Phys. C 48, 239 (1990); A.R. Zhitnisky, I.R. Zhitnitsky and V.L. Chernyak, Sov. J. Nucl. Phys. 41, 284 (1985), Yad. Fiz. 41, 445 (1985).
[59] P. Ball, J. High Energy Phys. 01 (1999) 010.
[60] V.M. Braun and A. Lenz, Phys. Rev. D 70, 074020 (2004); P. Ball and A. Talbot, J. High Energy Phys. 06 (2005) 063; P. Ball and R. Zwicky, Phys. Lett. B 633, 289 (2006); A. Khodjamirian, Th. Mannel and M. Melcher, Phys. Rev. D 70, 094002 (2004).
[61] P. Ball, V.M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. B 529, 323 (1998); P. Ball and V.M. Braun, Nucl. Phys. B 543, 201 (1999).
[62] P. Ball and V.M. Braun, Phys. Rev. D 54, 2182 (1996); P. Ball and R. Zwicky, J. High Energy Phys. 02 (2006) 034; P. Ball and M. Boglione, Phys. Rev. D 68, 094006 (2003).
[63] P. Ball and R. Zwicky, J. High Energy Phys. 04 (2006) 046.
[64] P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005).
[65] P. Ball and G.W. Jones, J. High Energy Phys. 03 (2007) 069.
[66] H.-n. Li and K. Ukai, Phys. Lett. B 555, 197 (2003).