Results of a theoretical and experimental research of a vertical hopper-feeding device with rollers for flat and close-to-equal-sized parts with implicit asymmetry

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Abstract. The paper considers the proposed design and a developed on the basis of an integrated approach mathematical model of the feed rate of a vertical hopper-feeding device with rollers for flat and close to equidimensional parts of the shape of bodies of rotation with asymmetry at the ends of two types: with a cylindrical stepped end and an end in the form of a truncated cone. The developed model, the adequacy and correctness of which is confirmed by the results of experimental studies, allows at the design stage with high precision for practice to determine the parameters of the feeding device, which will ensure the required feed rate and reliability of automatic machines and lines.

1. Introduction

Mechanical hopper-feeding devices are used to ensure reliable automatic feeding of parts weighing up to 0.07 kg into automatic machines and lines with the feed rate required for their uninterrupted operation. Therefore, over the years, in foreign and domestic works, the solution of problems of calculation and design of optimal designs of such devices is becoming increasingly important [1-4].

Mechanical hopper-feeding devices ensure the feed rate of their functions through the mechanical action of the elements of the device on the parts, the movement of which is carried out under the action of gravity, inertia and concentrated forces created by special mechanisms of the device (for example, turners, ejectors). The main functional elements of hopper-feeding devices are the grasping and orienting elements. The gripping elements ensure the capture of one part from the total number of parts poured into the hopper, and the orienting elements bring all the parts captured by the gripping element to a certain required position, in which they are directed to the technological equipment of the line. Therefore, the rational design of the gripping and orienting elements largely determines the feed rate and reliability of the machines and equipment of the entire line [5]. In most traditional hopper-feeding devices, gripping and orientation is performed by a single gripping-orienting element, that is why these functions are combined.

Over the past several decades, parts with an implicit asymmetry at the ends have begun to appear widely in various industries, which practically do not have a displacement of the center of mass relative to the center of the part, can be either solid or hollow, and their main geometric parameters are characterized by the following ranges of values:

\[
2 \leq \frac{l}{d_1} \leq 5 ; \quad 0.6 \leq \frac{d_2}{d_1} \leq 0.9 ; \quad 2\beta \leq 30^\circ ; \quad r \geq 0.5d_1 ; \quad 0.25 \leq \frac{h}{l} \leq 0.3 ,
\]

where \(l\) – is the length of the part; \(d_1, d_2\) – the diameters of the larger and smaller ends; \(\beta\) – the angle at the top of the end face of the part (for stepped parts and parts, one of the ends of which is made in the form of a truncated cone); \(r\) – the radius of the asymmetric end of the part (for a part with a spherical end); \(h\) – the height of the asymmetric end face of the part.
For such parts, the number of standard sizes of which is increasing every year, the design of the gripping and orienting elements of the hopper-feeding device should be very different from the traditional ones, since the combination of the processes of gripping and orienting parts will cause a significant decrease in the reliability and feed rate of both the device itself and the entire line.

To increase the reliability and feed rate when feeding bar parts with implicit asymmetry, with a ratio of $l$ to $d_1$ more than two, improved hopper-feeding devices were proposed with the implementation in their design of the principle of separating the functions of gripping parts and their orientation. When this principle is implemented, the gripping elements, made in the form of through pockets with the maximum possible value of the gap between the pocket and the part, ensure the capture of parts with the highest possible probability. And the organs providing orientation, repeating the profile of the asymmetric end face of the part along the external office, reliably orient the parts in a passive way. The results of theoretical and experimental studies have confirmed the reliability of their work and the ability to ensure the feed rate of feeding such parts into the line up to 300 parts/min [6-8].

For automatic feeding of close to equidimensional and flat parts with obvious asymmetry, various designs of hopper-feeding devices with a vertically located rotating disk and gripping-orienting elements made in the form of pockets, repeating the profile of the parts, have traditionally been used [9]. This allowed the parts that were incorrectly sunk into the pocket to fall out of it unhindered. If to use the traditional vertical hopper-feeding device for such parts, but with the absence of clearly pronounced orientation keys, then the decrease in the reliability of its operation will be due to a violation of the orientation process and jamming of parts in pockets. It will be the more intense, the less pronounced the asymmetry of the part, and the more the width of the pocket will already be. This will lead not only to a decrease in the feed rate of the hopper-feeding device, but also to a significant increase in the manufacturing labor intensity of the profile pockets of the hopper-feeding device for parts with implicit asymmetry.

Therefore, for automatic feeding of close to equidimensional and flat parts, traditional vertical hopper-feeding devices cease to be effective, which forces enterprises in many cases to switch from automatic feeding of parts, providing feed rate from 150 to 350 parts/min, to manual feed, in which parts are fed with feed rate 30-50 parts/min. This requires the creation of new designs of vertical hopper-feeding devices, the rational parameters of which will be selected on the basis of the developed complex of mathematical models, algorithms and programs. At the same time, the adequacy and correctness of mathematical models describing the probabilistic processes of the functioning of the hopper-feeding device and the mutual influence of the parameters of the loaded parts and the hopper-feeding device on its feed rate of mathematical models should be confirmed by experimental studies.

2. Formulation of the problem
The need to improve the known design of the hopper-feeding device with a vertical rotating disk and profile pockets made along its circumference, used for parts with an asymmetric ratio $l/d_1 \leq 1$ at the ends, was due to the following circumstances. Firstly, when feeding parts with a high coefficient of friction, the probability of getting stuck in the profile pockets of the disk of the hopper-feeding device sharply increases. Secondly, not the versatility of the hopper-feeding device, since for parts of the same size range, but with different configurations, it was required to design a new disk with original profile pockets. This disadvantage is further exacerbated when feeding parts with implicit asymmetry.

The diagram of the developed structure is shown in Fig. 1. The improved device contains a disk 3 with freely rotating profile rollers 4 evenly spaced along its periphery, forming gripping bodies along the outer outline of the parts 6 loaded into the hopper 5. The disk is located on the shaft 2, which receives rotation from the motor-reducer 1.

Parts that are poured into the hopper of the device roll down its walls towards the gripping elements. When moving forward with a smaller end face, the part freely repeats the outer contour of the parts inserted into the pocket in this way. In the future, such parts are moved by a disk to a special side dispensing window, made in the body of the hopper-feeding device. When the part moves towards the profile pocket with its large end forward or side surface, only its partial sinking into the pocket occurs, in which the part rests on the surface of the pocket of the profile rollers, encountering an obstacle for full immersion into the pocket. In the course of further rotation of the disk, under the
action of gravity, such parts fall out of the profile pockets back into the hopper to the total mass of parts, mix with other parts in the total mass and, changing their orientation in space, subsequently completely sink into the profile pocket.

Figure 1. Diagram of the improved vertical hopper-feeding device: a – general sectional view; b – view A; c – layout diagram of the part with the end in the form of a truncated cone; d – layout diagram of a cylindrical part with a stepped end: 1 – gear motor; 2 – shaft; 3 – disk; 4 – roller; 5 – hopper wall; 6 – part

In contrast to the traditional design of the vertical hopper-feeding device with pockets repeating the profile of the part, the improved hopper-feeding device with rollers is designed in such a way that when feeding parts with a high value of the friction coefficient, the probability of their jamming in the pockets of the device is significantly reduced. In addition, the improved hopper-feeding device is universal, since it became possible for parts of a similar size range and various configurations, not to make a new rotating disk with original profile pockets, but to achieve the required size of the pockets by quickly replacing the rollers. This made it possible to significantly reduce the complexity and laboriousness of manufacturing pockets in comparison with the traditional design and, thereby, to reduce the cost of manufacturing the hopper-feeding device.

The developed vertical hopper-feeding device is intended for feeding flat and close to equidimensional parts and significantly expands the possibilities of using the traditional design, which is capable of providing only automatic feeding of hollow parts in the form of a cap, the height of which is 10-20% greater than the outer diameter. Enhanced hopper-feeding devices were originally designed for parts with implicit asymmetry, but can also be used for parts with explicit orientation keys.

The main characteristic of any feeding device is its feed rate, which depends on the design of the hopper-feeding device and the parts it loads. Known mathematical models of the feed rate of the traditional vertical hopper-feeding device and their visualization for parts in the form of a cap [9] and flat parts with asymmetry at the ends of two types (with an end in the shape of a truncated cone and a cylindrical stepped part) [11] with specific geometric parameters showed the possibility of this design to provide feed rate when feeding these parts into the line from 175 to 225 parts/min.

In order to determine the feed rate of the improved vertical hopper-feeding device with rollers for flat and close to equidimensional parts with sufficient accuracy for practice, it is necessary to develop mathematical models that describe the probabilistic process of capturing parts with an implicit asymmetry in the device, as well as conduct theoretical and experimental studies to confirm their adequacy and correctness.

3. Theoretical part
The mathematical model of the feed rate of the improved hopper-feeding device must be built on the basis of the proposed theoretical integrated approach, the methodology of which is described in [12], depending on the convenience of calculations in the form of one of the expressions:
\[ F = knE_{\text{max}} \left[ 1 - \left( \frac{n}{n_{\text{max}}} \right)^4 \right] = \frac{6\omega_0}{d_1 + \Delta + \delta} E_{\text{max}} \left[ 1 - \left( \frac{\nu}{\nu_{\text{max}}} \right)^4 \right], \quad (1) \]

in which \( k \) – is the number of gripping parts; \( n \), \( \nu \) and \( n_{\text{max}} \), \( \nu_{\text{max}} \) – the frequency of rotation of the disk and its peripheral speed along the axis of the pockets and their maximum values at which it is impossible to capture the part, respectively; \( E_{\text{max}} \) – the maximum value of the capture coefficient \( E \); \( \Delta \) – the gap between the part and the roller in the pocket; \( \delta \) – the minimum roller diameter.

Thus, the construction of the mathematical model of the feed rate of the improved hopper-feeding device with rollers for flat and close to equidimensional parts is reduced to the determination of the parameters \( E_{\text{max}} \) and \( n_{\text{max}} \) (or \( \nu_{\text{max}} \)) of the model (1). In this case, to find \( E_{\text{max}} \), which is the product of probabilities \( p_1 \), characterizing the position of the part on the way to the pocket, favorable for gripping, and \( p_2 \), characterizing the absence of interference with gripping from other parts, we use the expressions presented below:

\[ p_1 = (p_{k_1} + p_{k_{II}} + p_{k_{III}}p_{I_{III}}) p_m, \quad (2) \]

\[ p_2 = 1 - \frac{\arctan \theta_0}{\pi} \frac{0.9d_1 + 1.4l}{d_1 + 2l}, \quad (3) \]

where \( p_{k_1}, p_{k_{II}}, p_{k_{III}} \) – the probabilities that the part, when falling to the bottom of the hopper, will be on its surface, respectively, with larger and smaller bases and bottom; \( p_{I_{III}} \) – the probability of turning by the required end to the pocket when the part falls to the bottom of the hopper by the side surface; \( p_m \) – the likelihood of no interference with the specified turn from other parts; \( \mu_0 \) – coefficient of friction between parts.

The dependences of each of the probabilities included in expression (2) for the considered types of parts were obtained in the form of expressions

\[ p_{k_1} = \frac{1}{2} \frac{x_c}{\sqrt{4x_c^2 + d_1^2}}, \quad (4) \]

\[ p_{k_{II}} = \frac{1}{2} \frac{l - x_c}{\sqrt{4(l - x_c)^2 + d_2^2}}, \quad (5) \]

\[ p_{k_{III}} = 1 - p_{k_1} - p_{k_{II}}, \quad (6) \]

\[ p_{I_{III}} = \frac{1}{\pi} \frac{\arccos \frac{2x_c}{\sqrt{4x_c^2 + d_1^2}} - \arcsin \frac{\mu}{\tan \alpha}}{\arcsin \frac{l}{d_1} - \arctg \frac{l}{d_1} + \frac{\pi}{2}}, \quad (7) \]

\[ p_m = \frac{\arcsin \frac{l}{d_1}}{\arctg \frac{l}{d_1}}, \quad (8) \]

in which the following parameters of the part and the hopper-feeding device are used: \( \mu \) – is the coefficient of friction of the part against the structural elements of the improved hopper-feeding device; \( x_c \) – the distance from the end face with the diameter to the center of mass of the part along the axis of symmetry; \( \alpha \) – angle of inclination of the hopper base [11].
The distance $x_c$ for a part with a tapered end face will be determined by expression (9), and for a stepped cylindrical part – by expression (10)

$$x_c = \frac{d_1^2 h^2 + d_2^2 (l-h)(f+h)}{2d_1^2 h + 2d_2^2 (l-h)}.$$  \hspace{1cm} (9)

$$x_c = \frac{3d_1^2 h^2}{2} + \frac{\left(\frac{d_1^2 + d_2^2}{d_1^2 + d_2^2 + 2d_1d_2} + h\right) d_1^2 + d_2^2 + d_1d_2}{4} (l-h)^2 \frac{3d_1^2 h + (d_1^2 + d_2^2 + d_1d_2) (l-h)}{3d_1^2 h + (d_1^2 + d_2^2 + d_1d_2) (l-h)}.$$  \hspace{1cm} (10)

The parameters $n_{\text{max}}$, $u_{\text{max}}$ are determined from the conditions for the ejection of the part from the pocket when the part collides with its walls according to expressions (11) and (12), respectively

$$u_{\text{max}} = \sqrt{0.2g \left[ (4\Delta + 5d_1) + \sqrt{(4\Delta + 5d_1)^2 - 20(d_1 + \Delta)^2} \right]}.$$  \hspace{1cm} (11)

$$n_{\text{max}} = \frac{30}{\pi R} \left[ (4\Delta + 5d_1) + \sqrt{(4\Delta + 5d_1)^2 - 20(d_1 + \Delta)^2} \right].$$  \hspace{1cm} (12)

where $R$ – is the radius of the disk along the axis of rollers evenly spaced on its surface.

Then, the mathematical model of the feed rate of the improved vertical hopper-feeding device with rollers for parts, one end face of which is a truncated cone, will be represented by expressions (1) – (9), (11), (12), and for cylindrical parts – by expressions (1) – (8), (10) – (12).

We will carry out a theoretical study of the developed feed rate models for flat parts, which are characterized by $0.4 \leq l/d_1 < 0.75$, and close to equidimensional parts with $0.8 \leq l/d_1 \leq 1$ of two types: with an end face in the form of a truncated cone and stepped cylindrical parts. The graphs show the dependence of the feed rate of the improved hopper-feeding device on the rotational speed $n$ of the disk and the coefficient of friction $\mu$. Each of the figures shows graphs of two surfaces, reflecting the change in feed rate at two values of the part diameter $d_1$ equal to 0.01 m (graph 1) and 0.02 m (graph 2). Under points a and b of each figure, graphs of feed rate changes are indicated for parts, one of the ends face of which is made in the form of a truncated cone, and under points c and d – for stepped cylindrical parts. At the same time, on the graphs for flat details, under points a and c, feed rate models for the ratio $l/d_1 = 0.4$ are presented, and under points b and e – for the ratio $l/d_1 = 0.7$ (see Fig. 2). On the graphs for details close to equidimensional, under points a and c, feed rate models are presented for the ratio $l/d_1 = 0.8$, and under points b and d, for the ratio $l/d_1 = 1$ (see Fig. 3). The coefficient of friction varies in the range from 0.2 to 0.5, and the rotational speed of the disk – to the value $n_{\text{max}}$. 


Figure 2. Graphs of the dependence of the feed rate of the improved device on the rotational speed of the disk and the coefficient of friction for flat parts with an end in the form of a truncated cone (a, b) and cylindrical stepped (c, d).

For flat parts with an end face in the form of a truncated cone, with the ratio \( l/d_1 = 0.4 \) the maximum feed rate values of the device, depending on the coefficient of friction, the values are from 130 parts/min (at \( \mu = 0.5 \)) to 160 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.01 \) m and from 120 parts/min (at \( \mu = 0.5 \)) up to 145 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.02 \) m (Fig. 2, a). With the ratio \( l/d_1 = 0.7 \) the maximum values of device feed rate, depending on the coefficient of friction, the values are from 200 parts/min (at \( \mu = 0.5 \)) to 285 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.01 \) m and from 175 parts/min (at \( \mu = 0.5 \)) to 240 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.02 \) m (Fig. 2, b).

For flat parts with a cylindrical stepped end, with the ratio \( l/d_1 = 0.4 \) the maximum feed rate values of the device, depending on the coefficient of friction, the values are from 130 parts/min (at \( \mu = 0.5 \)) to 160 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.01 \) m and from 120 parts/min (at \( \mu = 0.5 \)) up to 145 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.02 \) m (Fig. 2, c). With the ratio \( l/d_1 = 0.7 \) the maximum values of device feed rate, depending on the coefficient of friction, the values are from 210 parts/min (at \( \mu = 0.5 \)) to 285 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.01 \) m and from 180 parts/min (at \( \mu = 0.5 \)) to 250 parts/min (at \( \mu = 0.2 \)) at \( d_1 = 0.02 \) m (Fig. 2, d).
For parts close to equidimensional with an end face in the form of a truncated cone, with the ratio $l/d_1 = 0.8$ the maximum feed rate values of the device, depending on the coefficient of friction, are values from 220 parts/min (at $\mu = 0.5$) to 330 parts/min (at $\mu = 0.2$) at $d_1 = 0.01$ m and from 190 parts/min (at $\mu = 0.5$) up to 290 parts/min (at $\mu = 0.2$) at $d_1 = 0.02$ m (Fig. 3, a). With the ratio $l/d_1 = 1$ the maximum values of device feed rate, depending on the coefficient of friction, the values are from 270 parts/min (at $\mu = 0.5$) to 460 parts/min (at $\mu = 0.2$) at $d_1 = 0.01$ m and from 230 parts/min (at $\mu = 0.5$) to 430 parts/min (at $\mu = 0.2$) at $d_1 = 0.02$ m (Fig. 3, b).

For parts close to equidimensional with a cylindrical stepped end face, with the ratio $l/d_1 = 0.8$ the maximum feed rate values of the device, depending on the coefficient of friction, are values from 230 parts/min (at $\mu = 0.5$) to 340 parts/min (at $\mu = 0.2$) at $d_1 = 0.01$ m and from 200 parts/min (at $\mu = 0.5$) up to 295 parts/min (at $\mu = 0.2$) at $d_1 = 0.02$ m (Fig. 3, c). With the ratio $l/d_1 = 1$ the maximum values of device feed rate, depending on the coefficient of friction, the values are from 290 parts/min (at $\mu = 0.5$) to 490 parts/min (at $\mu = 0.2$) at $d_1 = 0.01$ m and from 250 parts/min (at $\mu = 0.5$) to 420 parts/min (at $\mu = 0.2$) at $d_1 = 0.02$ m (Fig. 3, d).
Thus, the graphs show that, depending on the type of parts loaded by the device with implicit asymmetry, the feed rate of the improved hopper-feeding device is on average from 120 parts/min to 490 parts/min.

4. Experimental results

To assess the adequacy and correctness of the developed feed rate model, two types of experiments were carried out for parts with an end in the form of a truncated cone (ratio \( l/d_1 = 0.53 \), diameter of a cylindrical end face \( d_1 = 17 \text{ mm} \), diameter of a conical end face \( d_2 = 12 \text{ mm} \), total length \( l = 12 \text{ mm} \), length of a cylindrical part \( h = 9 \text{ mm} \)).

The first stage of the experiments was carried out to assess the reliability of the theoretical values of probabilities \( P_{kI}, P_{kII}, P_{kIII} \) obtained from expressions (4) – (6) with their average experimental values obtained in each series of experiments \( N = 10 \), ten in each. Each experiment consisted of overturning a certain number of parts placed in a container and counting the number of parts that were located on the surface after overturning with a large, smaller base or side surface, respectively. The values of theoretical and experimental probabilities are shown in Table 1.

| Probability | Theoretical | Experimental for each series of experiments |
|-------------|-------------|-------------------------------------------|
|             | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \( P_{kI} \) | 0.223 | 0.2 | 0.3 | 0.2 | 0.1 | 0.3 | 0.2 | 0.3 | 0.2 | 0.2 |
| \( P_{kII} \) | 0.201 | 0.2 | 0.1 | 0.3 | 0.2 | 0.1 | 0.2 | 0.1 | 0.1 | 0.2 |
| \( P_{kIII} \) | 0.576 | 0.6 | 0.6 | 0.5 | 0.7 | 0.5 | 0.7 | 0.5 | 0.7 | 0.6 |

The second stage of the experiments was carried out on a mock-up of the improved vertical hopper-feeding device with rollers, the geometric parameters of which were optimally selected in order to ensure reliable gripping and orientation of the parts under consideration, and the rollers themselves were installed with a pitch of 30 mm around the circumference of the rotating disk with a diameter of 225 mm (Fig. 4).

![Image](image-url)  
**Figure 4.** Model of the improved vertical hopper-feeding device (a) and a fragment of the rotating disk with rollers (b): 1 – rotating disk; 2 – rollers; 3 – part

The verification of the adequacy of the developed analytical model of feed rate was carried out by the method of video filming at disk rotation frequencies of 4, 6, 8, 9, 10 rpm; the setting of higher speeds was limited by the capabilities of the device drive. Delivery time of parts 60 s, controlled by an electronic stopwatch with an error of \( \pm 0.01\% \). The initial number of parts in the hopper of the vertical
hopper-feeding device is 150 parts; the coefficient of friction of parts against the elements of the vertical hopper-feeding device is 0.5. There were 10 single experiments at the indicated speeds of rotation of the disk. In each experiment, the parts were counted, issued from the vertical hopper-feeding device for 60 s of continuous operation, then the parts were returned back to the hopper-feeding devices hopper, and the experiment was repeated. Based on the number of parts issued in 60 s, the value of the minute actual feed rate of the vertical hopper-feeding device in a single experiment was calculated and then the arithmetic mean of the actual feed rate of the vertical hopper-feeding device and the variance of the average of 10 experiments were calculated. To assess the accuracy of the data obtained, well-known methods were used and a conclusion was made about the absence of sharply distinguished results and the homogeneity of the dispersions of the experiments carried out.

The results of the second stage of the experiment are shown in Table 2.

Table 2. Experimental values of the parameters of the improved device

| Experience number | 4   | 6   | 8   | 9   | 10  |
|-------------------|-----|-----|-----|-----|-----|
| $F_x$, parts/min  |     |     |     |     |     |
| 1                 | 34  | 44  | 58  | 73  | 80  |
| 2                 | 28  | 46  | 63  | 71  | 80  |
| 3                 | 32  | 48  | 61  | 69  | 74  |
| 4                 | 35  | 46  | 67  | 69  | 77  |
| 5                 | 31  | 47  | 64  | 71  | 82  |
| 6                 | 29  | 51  | 62  | 70  | 77  |
| 7                 | 34  | 48  | 64  | 73  | 78  |
| 8                 | 33  | 46  | 66  | 71  | 81  |
| 9                 | 29  | 49  | 62  | 68  | 78  |
| 10                | 31  | 48  | 60  | 69  | 80  |
| $F_{av}/F_{max}$, parts/min | 31.6/96 | 47.3/144 | 62.7/192 | 70.4/216 | 78.7/240 |
| $E$               | 0.3392 | 0.3285 | 0.3265 | 0.3259 | 0.3279 |

To assess the adequacy of the theoretical models developed, let us compare the results obtained.

5. Discussion of results
The results of comparison of theoretically and experimentally obtained values of probabilities $p_{k_1}$, $p_{k_2}$, $p_{k_3}$ are shown in Fig. 5. Each of the graphs shows the theoretical horizontal lines of the calculated probabilities $p_{k_1}$, $p_{k_2}$, $p_{k_3}$, their experimental mean values for each series of experiments in the form of points and the experimental curves obtained by approximating the mean values of each of the three probabilities.

Figure 5. Theoretical (1) and experimental (2) probability curves $p_{k_1}$ (a), $p_{k_2}$ (b), $p_{k_3}$ (c)

Comparison of the results of theory and experiment that deviations of experimental values of probabilities from theoretical ones are in the zone of permissible deviations. Therefore, the method of theoretical determination of probabilities $p_{k_1}$, $p_{k_2}$, $p_{k_3}$ reliably describes the influence of the geometric parameters of the parts on them.
The processing of the results of the second stage of the experiment in CurveExpert 1.4 made it possible to obtain the experimental dependence of the capture coefficient, substituting it into expression (1), we obtained the feed rate dependence of the form \[ F = kn \cdot 0.328 \cdot \left( l - 0.526 \cdot 10^{-6} \cdot n^4 \right). \]

At the same time, the theoretical mathematical model of feed rate developed in accordance with the proposed methodology for the specified parameters of the part and the improved vertical hopper-feeding device has the form \[ F = kn \cdot 0.323 \cdot \left( l - 0.331 \cdot 10^{-6} \cdot n^4 \right). \]

Fig. 6 shows comparative graphs of theoretical and experimental values of the grip coefficient \( E \) and feed rate \( F \) of the improved vertical hopper-feeding device for parts with an end face in the form of a truncated cone in the range of permissible disk rotation speeds (Fig. 6, a) and feed rate \( F \) in the area of the experiment performed (Fig. 6, b).

![Figure 6](image)

**Figure 6.** Comparative graphs of theoretical (1) and experimental (2) values grip coefficient and feed rate \( F \) in the range of permissible disk rotation speeds (a) and feed rate in the field of the experiment (b).

The graphs show a high convergence of theoretical and experimental studies in terms of feed rate, grip coefficient, limiting disk rotation speed \( n_{\text{max}} \) and disk rotation frequency, at which the feed rate of the improved vertical hopper-feeding device with rollers when feeding parts with implicit asymmetry at the ends, one of which is made in the form of a truncated cone, is maximum.

### 6. Conclusions

The model, developed on the basis of an integrated approach, is a theoretical model of the feed rate of the vertical hopper-feeding device with rollers, which takes into account the effect of its parameters and the parameters of loaded parts with asymmetry along the ends on the feed rate of the vertical hopper-feeding device. It allows at the design stage to predict the actual feed rate and select the optimal parameters of the device when feeding flat and close to equidimensional parts with asymmetry at the ends, one of which can be cylindrical or made in the form of a truncated cone. The adequacy and correctness of the developed model is confirmed by the results of experimental studies on a model of the improved vertical hopper-feeding device for parts with an end in the form of a truncated cone.

Therefore, the mathematical model of feed rate can be used in the design with high accuracy of automatic feeding systems based on a vertical hopper-feeding device for feeding automatic machines and lines with parts with a capacity of 120 to 490 parts/min.

The improved vertical hopper-feeding device with rollers is most effective for feeding flat and close to equidimensional parts with asymmetry along the ends, one of which can be cylindrical or in the form of a truncated cone of parts with ratios of overall dimensions \( 0.4 \leq l/d_1 \leq 1 \), end face diameters \( 0.6 \leq d_2/d_1 \leq 0.9 \) and end face heights \( 0.5 \leq (l - h)/h \leq 0.6 \).
References

[1] I S Blyacherov 1990 Avtomaticheskaya zagruzka technologicheskikh mashin: Spravochnik / I S Blyacherov, G M Varyash i dr.; pod obtsch. red. I A Klusova (M.: Mashinostroenie) p 400
[2] G Boothroyd 2005 Assembly Automation and Product Design / Second Edition (CRC Press, Taylor & Francis Group) p 536
[3] S Ghosh, S P Singh 2011 Assembly Line – Theory and Practice  Rijeka, Croatia: InTech Europe 250 p (9. Optimizing Feeding Systems. pp 149-178).
[4] B Z Sandler 1999 Robotics: designing the mechanisms for automated machinery (San Diego, California, USA: ACADEMIC PRESS) p 433
[5] V V Preis 2003 Nadezhnost avtomaticheskikh rotorno-konveyernых liniy dlya sborki mnogoelementных izdeliy  Sborka v machinostroenii, priborostroenii № 10 pp. 17-22
[6] E V Davidova, V V Preis 2009 Analititcheskaya model proizvoditelnosti bunkernogo zagruzchnogo ustroystva s radialnymi gnezdami i koltsevym orientatorom  Sborka v machinostroenii, priborostroenii № 11 pp. 23-30
[7] E V Davidova, V V Preis, K N Provotorova 2014 Matematicheskaya model proizvoditelnosti diskovogo bunkernogo zagruzchnogo ustroystva s tangentsialnymi profilnymi gnezdami  Sborka v machinostroenii, priborostroenii № 10 pp. 7-10
[8] E V Pantyukhina, V V Preis, A V Khachaturian 2019 Feed rate evaluation of mechanical toothed hopper-feeding device with ring orientator for parts, asymmetric at the ends Journal of Physics: Conference Series Vol. 1260 pp 032032
[9] E V Davidova, V V Preis 2010 Analititcheskaya model i metodika rascheta proizvoditelnosti vertikalnogo bunkernogo zagruzchnogo ustroystva  Sborka v machinostroenii, priborostroenii № 9 pp. 27-31
[10] E V Davidova, V V Preis, A V Churochkin 2017 Bunkernoe zagruzchnoe ustroystvo Patent of the Russian Federation № 170000, 11.04.2017.
[11] E V Davidova, V V Preis, A V Churochkin 2016 Matematicheskaya model proizvoditelnosti vertikalnogo bunkernogo zagruzchnogo ustroystva dlya ploskih assimetricnyh predmetov obrabotki Progressivnie tehnologii i sistemi mashinostroeniya № 3 (54) pp. 36-40.
[12] E V Pantyukhina 2020 Integrated approach methodology for evaluating the feed rate of mechanical disk hopper-feeding devices Journal of Physics: Conference Series Vol. 1546. pp 012024