Circuit Effect On The Current-Voltage Characteristics Of Ultrasmall Tunnel Junctions

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We have used the method of generating functional in imaginary time to derive the current-voltage characteristics of a tunnel junction with arbitrary tunneling conductance, connected in series with an external impedance and a voltage source. We have shown that via the renormalized charging energy and the renormalized environment conductance, our nonperturbative expressions of the total action can be mapped onto the corresponding perturbative formulas. This provides a straightforward way to go beyond the perturbation theory. For the impedance being a pure resistance, we have calculated the conductance for various voltages and temperatures, and the results agree very well with experiments.

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1. INTRODUCTION

By decreasing the capacitance \( C \) of a double tunnel junction structure, the role of Coulomb charging energy \( e^2/2C \) becomes more important. For example, if \( C \) is of the order 1 \( fF \) and the temperature \( T \) is lower than 1 \( K \), the elementary Coulomb charging energy \( e^2/2C \) dominates the charge transport through a tunnel junction system, provided that the tunneling resistance \( R_T \) is larger than the resistance quantum \( R_K = h/e^2 \). The technology nowadays can fabricate tunnel junctions with \( C \) less than 1 \( fF \), and hence phenomena related to Coulomb blockade (CB) have been extensively studied in recent years.

For a tunnel junction system with \( R_T \) much larger than \( R_K = h/e^2 \), the perturbation theory of CB has been well established, where one performs low order perturbation expansion in terms of the tunneling Hamiltonian \( H_t \). The so-obtained theoretical results are in good agreement with experimental observations in multijunction structures. On the other hand, for a single tunnel junction, one has to take into account the effect of circuit elements which are attached to the single junction in order to perform measurements. If the circuit impedance is much smaller than the quantum resistance \( R_K \), for example, the free space impedance \( Z_f \approx 377 \) \( \Omega \), the CB effect can hardly be detected. However, the CB phenomena emerge when a large impedance is inserted into the circuit in series connection with the single junction. In a multijunction system, the effect of circuit elements is secondary because there are more than one large junction resistance. The influence of circuit elements is a specific type of electromagnetic environment (EME) effect.

In the weak tunneling regime, Devoret et al. have used the Fermi’s golden rule to treat the tunneling Hamiltonian, and have developed a so-called \( P(E) \) theory to study the Coulomb charging effect of a single tunnel junction surrounded by an EME. Girvin et al. have used the Green’s function technique to reach the same theory. They studied a tunnel junction connected in series with an external impedance and a voltage source. The Hamiltonian of the device consists of four parts

\[
H = H_c + H_{el} + H_t + H_{ex}.  \tag{1}
\]

When a charge \( Q \) is added to the junction with junction capacitance \( C \), the Coulomb charging energy is

\[
H_c = Q^2 / 2C.  \tag{2}
\]

The quasi-particle states in the left and the right electrode are labeled by \( \sigma \) for the spin and the transverse motion. The longitudinal wave vectors in the right electrode are represented by \( k \), and in the left electrode by \( q \). In standard quasi-particle notations, the Hamiltonian \( H_{el} \) for the electrodes has the form

\[
H_{el} = \sum_{k\sigma} \varepsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \sum_{q\sigma} \varepsilon_{q\sigma} c_{q\sigma}^\dagger c_{q\sigma},  \tag{3}
\]

Since \( \sigma \) is conserved during the tunneling through a junction, we use \( \sigma \) to specify the tunneling channel. For a fixed tunneling channel, let \( t_{kq\sigma} \) be the matrix element for tunneling from the state \( q\sigma \) in the left electrode to the state \( k\sigma \) in the right electrode. Then, the tunneling Hamiltonian is expressed as

\[
H_t = \sum_{kq\sigma} [t_{kq\sigma} c_{k\sigma}^\dagger c_{q\sigma} \exp(-i\varphi) + H.c.],  \tag{4}
\]

where the operator \( \varphi \) is conjugate to \( Q \) via the commutation relation \([\varphi, Q] = i\epsilon\). We should mention that here we have neglected the influences of the tunneling time, which is appropriate for metallic tunnel junctions. Since the energies of tunneling electrons are near the Fermi energy, \( t_{kq\sigma} \) can be well approximated by a constant value \( t \). In the presence of a bias voltage \( V \), the external impedance is simulated by a set of LC circuits, and is expressed as

\[
H_{ex} = \sum_{m=1}^{\infty} \left[ \frac{g_m^2}{2C_m} + \frac{(eVt - \varphi - \varphi_m)^2}{2e^2L_m} \right],  \tag{5}
\]
where the operators $\varphi_m$ and $q_n$ obey the commutation relation $[\varphi_m, q_n] = i e \delta_{m,n}$.

As we mentioned before, in the regime of weak tunneling, perturbation theory has been applied to the Hamiltonian Eq. (5), to calculate the current as a function of the bias voltage $V$. Beyond the weak tunneling regime, following the pioneering work of Ambegaokar et al., the nonperturbative approach of path integral has been used to investigate the statistical properties and the transport behavior in the linear-response regime of tunnel junctions (also see Ref. 3 for the current-bias case). Recently, using the path integral approach formulated in imaginary time, we found that the strong tunneling processes renormalize both the charging energy and the external impedance. Furthermore, within this imaginary time formalism, we have shown that at sufficiently high temperatures, the action of the tunnel junction can be approximated by the action of the corresponding resistance, and so an explicit expression of the high temperature quantum conductance can be derived. Along the line of Feynman’s formalism for polaron, the Coulomb charging effect has been investigated with the Ohmic approximation, the nonequilibrium Green’s function method, and the mean-field theory. The mean-field theory also demonstrates the renormalization of the charging energy and the external impedance, and the so-calculated zero-bias conductance agrees with experiment as long as $k_B T$ is not much less than $e^2/2C$ and $R_T$ is not much smaller than $R_K$.

Nonperturbative approach has so far produced concrete results only for the situation of zero-bias. In this paper we will consider finite bias and so investigate the full current-voltage (I-V) characteristics of a voltage-biased tunnel junction connected in series with an external impedance. The derivation of the path integral representation for the generating functional in imaginary time will be outlined in Sec. II, leaving the complete mathematical manipulations in the Appendix. With the generating functional we obtain the tunneling current as a function of the bias voltage. Specific cases suitable for comparing with experiments will be studied in details in Sec. III, showing very good agreement with experiments. Finally, in Sec. IV we will prove that via the renormalized charging energy and the renormalized environment conductance, our nonperturbative expressions of the total action can be mapped onto the well-established perturbative results. This provides a straightforward way to go beyond the perturbation theory.

II. GENERAL FORMULATION OF THE I-V CHARACTERISTICS

To study the physical system described by Eqs. (1)-(4), one needs to deal with the electron tunneling and the effect of external impedance. While the EME has been treated nonperturbatively by many authors, in this paper we will analyze both the tunneling part and the EME part with a nonperturbative approach. To achieve this goal, we will start from the generating functional for a tunneling junction connected in series with an external impedance and a voltage source.

The effect of the bias voltage $V$ can be incorporated into the Hamiltonian not explicitly depending on the time variable via a time-dependent unitary transformation defined as

$$U(t) = \exp \left[ i e V t \left( \sum_{k,s} c_{k,s}^\dagger c_{k,s} \right) \right].$$

It transforms the Hamiltonian in Eq. (1) to

$$\mathcal{H} = Q^2/2C + \sum_{k,\sigma} (\varepsilon_{k\sigma} + eV)c_{k\sigma}^\dagger c_{k\sigma} + \sum_{q,\sigma} \varepsilon_{q\sigma} c_{q\sigma}^\dagger c_{q\sigma} + \sum_{kq\sigma} \left( t_{kq\sigma} c_{k\sigma}^\dagger c_{q\sigma} \exp (i\phi) + H.c. \right),$$

where $Q=Q-CV$ reflects the quantum fluctuations of the charge on the tunnel junction, and $\phi = eVt - \varphi$ serves as the phase of the external impedance. These two new canonical variables satisfy the commutation relation $[\phi, Q] = i e$.

The corresponding current operator has the form

$$I_T = -i e \sum_{kq\sigma} t_{kq\sigma} c_{k\sigma}^\dagger c_{q\sigma} \exp (i\phi) - H.c..$$

The generating functional in imaginary time is defined as

$$Z_V[\eta] = \text{tr} \left\{ \hat{T}_\tau \exp \left\{ - \int_0^\beta d\tau [\mathcal{H} - I_T \eta(\tau)] \right\} \right\},$$

where $\hat{T}_\tau$ is the time-ordering operator in imaginary time. Substituting
In the total action

$$S[\phi, \eta] = S_c[\phi] + S_{\text{ex}}[\phi] + S_t[\phi, \eta],$$

the first term on the right hand of the above equation of the Trotter product. Next, the charge numbers are summed over. Such mathematical manipulations are performed in the Appendix, leading to the expression

$$Z_V[\eta] = N^\infty \sum_{l=0}^\infty (-1)^l \int_0^{\beta} d\tau_1 \int_0^{\tau_1} d\tau_{l-1} \cdots \int_0^{\tau_2} d\tau_1 \prod_{m=1}^\infty \int D\varphi_m \int d\phi_0 \prod_{i=1}^M \int d\phi_i \sum_{k_1,q_1,\sigma_1,\xi_1} \cdots \sum_{k_l,q_l,\sigma_l,\xi_l} \exp \left\{ -\sum_{i=0}^M \varepsilon \phi_i^2 / 4E_c + C_m \varphi_i^2 / 2e^2 + (\varphi_i - \phi_i)^2 / 2e^2 L_m \right\} \times l_1 \xi_2 \cdots \xi_l \left[ 1 + i\epsilon \xi_1(\tau_1) \right] \cdots \left[ 1 + i\epsilon \xi_l(\tau_1) \right] \prod_{\sigma} \text{tr}\varphi \ e^{-\beta h_\sigma \ h_{k_1,q_1,\sigma_1,\xi_1}(\tau) \cdots h_{k_l,q_l,\sigma_l,\xi_l}(\tau_1)},$$

where

$$h_\sigma = \sum_k (\varepsilon_{k\sigma} + eV) c_{k\sigma}^\dagger c_{k\sigma} + \sum_q \varepsilon_{q\sigma} c_{q\sigma}^\dagger c_{q\sigma},$$

and

$$h_{k,q,\sigma,\xi}(\tau) = e^{\tau h_\sigma} c_{k\sigma}^\xi c_{\sigma\xi} e^{-\tau h_\sigma} e^{i\xi \phi(\tau)}.$$
describes the contribution of tunneling processes. In Eq. (17), \( \alpha_{\text{ex}}(\omega) = R_{\text{ex}}K/4\pi Z_{\text{ex}}(-i|\omega|) \) and \( Z_{\text{ex}}(\omega) \) is the Fourier coefficient of the external impedance, where \( \omega = 2\pi\beta / \lambda \) is the Matsubara frequency. For a purely resistive impedance \( R_{\text{ex}} \), this expression is simplified to \( \alpha_{\text{ex}}(\omega) = -\alpha_{\text{ex}}|\omega|/4\pi \) with \( \alpha_{\text{ex}} = R_K / R_{\text{ex}} \). The \( S_\theta(\phi, \eta) \) in Eq. (18) contains both the bias voltage and the driving source of the generating functional. Here \( \alpha_\phi(\omega) = \alpha_\phi|\omega|/4\pi \) with \( \alpha_\phi = R_K / R_T \). It is important to notice that for \( V = 0 \) and \( \eta(\tau) = \eta(\tau') = 0 \), Eq. (14) reduces to the familiar form of the partition function of an unbiased device.

Before analyzing the current, we would like to clarify what is indicated by the equations of the generating functional and the related actions. In derivation of the generating functional Eq. (14), the technique that we used singles out the ground state, projecting out all excited states. At sufficiently low temperatures, our calculation should reproduce expectation values in the ground state. The ground state can be obtained from the non-equilibrium state at finite voltages by the transfer of a macroscopic number of electrons from one electrode to another. The exponential dependence on the voltage favors paths in which a large number of electrons move from one electrode to the other at the beginning of the path, as implicit in the action given by Eq. (13). The dc current \( I(V) \) is readily derived as the first order functional derivative of \( Z_V(\eta) \) with respect to \( \eta \),

\[
I(V) = 2e \int_0^\beta d\sigma \alpha_\sigma(\sigma) Z_V^{-1} \int D\phi e^{-S[\phi]} \times \frac{1}{2\pi} \left\{ \exp \left( eV\sigma \right) \exp \left( i[\phi(\sigma) - \phi(0)] \right) - \exp \left( -eV\sigma \right) \exp \left( -i[\phi(\sigma) - \phi(0)] \right) \right\},
\]

where \( Z_V \) is the path integral with the action \( S[\phi] = S_0[\phi] + S_{\text{ex}}[\phi] + S_{\theta}(\phi, \eta) = 0 \). The current autocorrelation function and high order correlation functions can be obtained as well with the corresponding high order functional derivatives of the generating functional \( Z_V(\eta) \).

Since in standard four-probe experiments, the tunneling current is measured as a function of the average voltage drop \( V_t \) across the tunnel junction, instead of the bias voltage \( V \), we should rewrite the above equation in a suitable form. This can be done by introducing the phase fluctuation of the external impedance, \( \theta(t) = \phi(t) - e(V - V_t)t \), into the path integral expression for the tunneling current. Then, Eq. (19) becomes

\[
I = 2e \int_0^\beta d\sigma \alpha_\sigma(\sigma) Z^{-1} \int D\theta e^{-S[\theta]} \times \frac{1}{2\pi} \left\{ \exp \left( eV_t\sigma \right) \exp \left( i[\theta(\sigma) - \theta(0)] \right) - \exp \left( -eV_t\sigma \right) \exp \left( -i[\theta(\sigma) - \theta(0)] \right) \right\},
\]

where \( Z \) is the partition function. The total action \( S[\theta] = S_0[\theta] + S_{\text{ex}}[\theta] + S_{\theta}[\theta] \) in the above equation is readily obtained from Eq. (14) by setting \( \eta = 0 \) and \( V = 0 \), but with \( \theta \) instead of \( \phi \) as the variable.

Before going further, we would like to point out that the above general expression reduces to the \( P(E) \) theory at the weak tunneling limit. To show this, we only need to neglect the contribution of the tunnel junction to the total action, and then to perform an analytical continuation from imaginary time to real time. Therefore, for tunnel junctions with tunnel resistances much larger than the quantum resistance, one can either use the \( P(E) \) theory formulated in real time, or use our Eq. (20) represented in imaginary time.

In experiments, it is convenient to use Cr films near the tunnel junction as well-controllable EME. Consequently, from now on we will focus our attention on the case of purely resistive EME. Via a series expansion in \( \beta eV \), we will derive the required linear and nonlinear response functions. We must point out that our results are not accurate if \( \beta eV \) is large. Hence, when we compare our theory with experiments in the next section, we have checked that the theoretical results are valid under the conditions the experiments were performed.

### III. RESULTS COMPARED WITH EXPERIMENTS

In this section, we will derive from Eq. (20) some explicit results suitable for comparing with experiments. For this purpose we will follow the approach of equivalent circuits, where the total action of the EME including the tunnel junction is replaced by the actions of an effective resistance and an effective capacitance. We will at first calculate the effective circuit parameters, and then use them to present the full I-V characteristics of the tunnel junction.

We need to evaluate the partition function \( Z \) in Eq. (20). Let us first write down the well-established partition function in the weak tunneling limit

\[
Z_0 = \prod_{i=1}^{\infty} (\beta \lambda_i^{(0)})^{-1}
\]

where

\[
\lambda_i^{(0)} = \frac{\omega_i^2}{2E_c} + \frac{\alpha_{\text{ex}} \omega_i^2}{2\pi}
\]

are eigenvalues with respect to the eigenfunctions of the action

\[
S_0[\theta] = S_{\text{ex}}[\theta] + S_{\text{ex}}[\theta] = \sum_{i=1}^{\infty} \lambda_i^{(0)} (\theta_i^2 + \theta_i'^2).
\]

Beyond the weak tunneling regime, the partition function, although more complicated, can still be derived in the same manner. To the first order in \( \alpha_\phi \), we have
\[ Z = \prod_{n=1}^{\infty} \frac{1 + \int_0^\beta d\tau' \int_0^\beta d\tau \alpha_i(\tau - \tau') e^{f(\tau - \tau')}}{\beta \lambda_i^{(0)}}, \]  
\tag{23}

with the function \( f(\tau) \) is defined as
\[ f(\tau) = \frac{D\theta \exp \{-S_0[\theta(\tau)] \} \pm i \{ \theta(\tau) - \theta(0) \}}{\int D\theta' \exp \{-S_0[\theta(\tau)] \}}. \]  
\tag{24}

At sufficiently high temperatures, with a Taylor series expansion of \( \exp \{ f(\tau - \tau') \} \), the partition function \( Z \) can be expressed in the same form as \( Z_0 \) in Eq. (21), but with the eigenvalues
\[ \lambda_i = \frac{\omega^2}{2E_c} + \frac{\alpha^* \omega_i}{2\pi}. \]  
\tag{25}

Here the effective dimensionless conductance \( \alpha^* = \alpha_{\text{ex}} + \alpha_t + O(\beta E_c) \) contains the contributions from both the external resistance and the tunnel junction. The effective charging energy \( E_{c*} \), which is determined from
\[ \frac{1}{\beta E_{c*}} = \frac{1}{\beta E_c} + \frac{7.2\alpha_i^* E_c}{8\pi^4} + O[(\beta E_c)^3], \]  
\tag{26}

depends not only on the bare charging energy \( E_c \), but also on the junction conductance \( \alpha_t \). The effect of the total EME is then embedded in the effective capacitance \( c^2/2E_{c*} \) and the effective environmental resistance \( R_c/\alpha^* \). Consequently, from Eq. (20) the transport properties can be investigated analytically, and the total conductance
\[ G(V_t) = G(0) + \delta G(V_t) \]  
\tag{27}

has a voltage-independent term
\[ G(0) = \frac{1}{R_f} \left\{ 1 - \frac{\beta E_{c*}^2}{3} + \left[ \frac{1}{15} + \frac{7.2\alpha^*}{4\pi^4} \right] (\beta E_{c*})^2 \right\} + O[(\beta E_{c*})^3] \]  
\tag{28}

and a low-voltage correction
\[ \delta G(V_t) = \frac{1}{R_f} \left\{ \frac{\beta E_{c*}^2 (\beta eV_t)^2}{45} + O[(\beta eV_t)^3] \right\} + O[(\beta eV_t)^4]. \]  
\tag{29}

The dots in Fig. 1 are the measured normalized conductance taken from the Fig. 2a of Ref. (17). Using the sample parameter values \( \alpha_t=1.04 \) and \( \alpha_{\text{ex}}=8.09 \), as well as the experimental temperature \( T=4.2 \) K, our calculated normalized conductance as a function of the voltage is plotted in Fig. 1 as the solid curve. Our analytical result is in very good agreement with the measurement not only for the zero-voltage conductance, but also for the entire curve. We notice that the experimental data exhibits conductance-step structures with sharp changes at voltages \( V_{\pm 1} \approx \pm 0.2 \text{mV} \) and \( V_{\pm 2} \approx \pm 0.4 \text{mV} \), a phenomenon which was not discussed in the original experimental paper Ref. (17). Our conjecture is that such conductance steps are originated from the resonances of the EME modes. Using a single \( LC \) mode as the EME and in the weak tunneling regime, Ingold and Nazarov have predicted the conductance step structures, which are sharp at low temperature but are smeared out at higher temperatures. If we estimate the inductance \( L \) from the step width \( |eV_{\pm 1}|=\hbar/\sqrt{LC} \) with the capacitance in the range of \( fF \), we found \( L \) in the range of \( pF \), which is in agreement with the experimental values. These resonant EME modes are not included in the purely Ohmic EME, but can be investigated if we use the frequency-dependent external impedances to model the EME.

The temperature dependence of the zero-voltage conductance has been measured thoroughly in Ref. (17). For the standard four-terminal setup, if the temperature is not too low, the inverse of the conductance dip at zero voltage is linear in temperature
\[ (\Delta G/G_T)^{-1} = G_T/(G_T - G(0)) = 3k_B T/E_c + \delta. \]  
\tag{30}

Our theory gives the offset
\[ \delta = 0.6 + 0.167(\alpha_{\text{ex}} + \alpha_t), \]  
\tag{31}

while the \( P(E) \) theory predicts
\[ \delta_{P(E)} = 0.6 + 0.167\alpha_{\text{ex}}. \]  
\tag{32}

Two sets of experimental data, taken from the Figs. 3a and 3b of Ref. (17), are plotted in Fig. 2 as dots, together with the best fitted dotted lines. The sample parameter values are \( \alpha_t=5.86, \alpha_{\text{ex}}=20.32 \) and \( C=1.99 \text{fF} \) for the upper set of data, and \( \alpha_t=3.02, \alpha_{\text{ex}}=1.50 \) and \( C=0.92 \text{fF} \) for the lower set of data. Using these parameter values, the analytical results of our theory are plotted in Fig. 2 as solid lines, and those of the \( P(E) \) theory are shown in dashed lines. While our theory agrees very well with the measurements, the deviation of the \( P(E) \) theory from the experiments increases when the tunnel conductance gets larger. We have also performed the self-consistent numerical calculations formulated in real time, and the results are indistinguishable from our analytical solution given by Eqs. (21) and (22).

**IV. ANALYTICAL SOLUTIONS BASED ON RENORMALIZED PARAMETERS**

In the previous section we have shown that in the regime of not too low temperatures, our strong tunneling formulas can be mapped onto the weak tunneling formulas by renormalizing the charging energy and the external impedance. Here we will analyze the situation of low temperatures. With large \( \beta \), the function \( f(\tau) \) in Eq. (24) can be written in a simple analytical form
\[ f(\tau) = -\frac{2}{\alpha_{\text{ex}}} \left( \gamma + \ln \frac{\alpha_{\text{ex}} E_c |\tau|}{\pi} \right), \]  
\tag{33}
where $\gamma$ is the Euler constant. With this expression we perform a similar mathematical manipulation as what we have done for the case of high temperature. For $\alpha_{ex} \gg 1$, the so-obtained eigenvalues have again the same form as given by Eq. (23), but with the effective dimensionless conductance $\alpha^* = \alpha_{ex} + O(1/\beta E_c)$, and the effective charging energy

$$E_c^* = \left[ 1 + \frac{\alpha_{ex} \exp(-\gamma) / \alpha_{ex}^{2/3}}{4 \pi \Gamma(2/3)} \right]^{-1} E_c. \quad (34)$$

In the above equation $\Gamma(x)$ is the Gamma function. At low temperatures the renormalization is expected to be weak, because the tunneling processes suffer strong Coulomb blockade. Using Eqs. (21) and (33), the I-V curves are derived analytically as

$$I(V) = \frac{V_i}{R_T} \frac{1}{\Gamma(2 + 2/\alpha_{ex})} \left[ \frac{\pi \exp(-\gamma) e V \xi_i}{\alpha_{ex} E_c^*} \right]^{2/\alpha_{ex}}. \quad (35)$$

Now we have derived the effective conductance $\alpha^*$ and the effective charging energy $E_c^*$ analytically for both low and high temperatures. Thus, we can map the nonperturbative expressions of the total action onto the perturbative results by replacing the bare parameters $\alpha_{ex}$ and $E_c$ with the effective parameters $\alpha^*$ and $E_c^*$, respectively. Consequently, we can calculate the tunneling current and the current-current correlation function beyond the perturbation theory. A very important feature of our renormalization theory is the appearance of the junction conductance in the effective charging energy $E_c^*$ as well as in the effective environmental conductance $\alpha^*$. Consequently, it is inappropriate to approximate the tunnel junction simply by an Ohmic element, although it is a good approximation at sufficiently high temperatures.

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**APPENDIX:**

In this Appendix we will derive Eq. (33) for the path integral representation of the generating functional of a voltage-biased tunnel junction connected in series with an external impedance. Since the cases we are interested in are not restricted to weak tunneling, we will use the nonperturbative resummation technique.

From Eqs. (5) and (6), we can expand the generating functional in a Taylor series of the tunneling processes

$$Z_V[\eta] = \sum_{n=1}^\infty \left\{ -\sum_{k,\sigma} (\xi_{k,\sigma} e^{i\eta} + \varepsilon V) c_{k,\sigma}^\dagger c_{k,\sigma} \right\} \exp \left( \frac{i\eta \phi - \varphi_n^2}{2c^2 L_n} \right), \quad (A1)$$

and

$$H_t = \sum_{k,\sigma} \{ t_{k,\sigma} [1 + i \eta n(\tau)] c_{k,\sigma}^\dagger c_{k,\sigma} \exp (i\phi) + H.c. \}. \quad (A3)$$

Since the EME is modeled by a set of harmonic oscillators, the trace over the EME modes can be expressed explicitly as path integrals. By dividing $\beta$ into $M$ segments of length $\varepsilon = \beta / (M+1) \rightarrow 0$ and then inserting eigenstates of $\phi$ and $Q$ between the elements of the Trotter product, we have

$$Z_V[\eta] = N^\varepsilon \sum_{l=0}^\infty (-1)^l \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_{l-1} \cdots \int_0^{\tau_2} d\tau_1 \int_0^{\tau_1} D\varphi_m \int d\phi_0 \int d\phi_1 \cdots \int d\phi_M \sum_{k_1, q, \sigma_1, \xi_1} \cdots \sum_{k_2, q, \sigma_2, \xi_2} \cdots \sum_{n_{M+1}} \exp \left\{ -\varepsilon [E_c n_1^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_1)^2 / 2c^2 L_m] \right\}$$

$$\times \exp \left\{ \varepsilon \varphi_1 \phi_0 - \phi_1 \right\} \exp \left\{ -\varepsilon [E_c n_2^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_2)^2 / 2c^2 L_m] \right\} \exp \left\{ i \varepsilon \varphi_2 \phi_1 - \phi_2 \right\} \cdots$$

$$\times \exp \left\{ -\varepsilon [E_c n_{M+1}^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_{M+1})^2 / 2c^2 L_m] \right\} \exp \left\{ i \varepsilon \varphi_{M+1} \phi_M - \phi_{M+1} \right\} \right.$$}

$$\times \xi \xi \cdots [1 + \varepsilon \xi \eta(\tau)] \cdots [1 + \varepsilon \xi \eta(\tau)] \sum_{\sigma} \exp e^{-\beta h_{k_1, q_1, \sigma_1, \xi_1} (\tau)} \cdots h_{k_1, q_1, \sigma_1, \xi_1} (\tau), \quad (A4)$$

$$\sum_{l=0}^\infty (-1)^l \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_{l-1} \cdots \int_0^{\tau_2} d\tau_1 \int_0^{\tau_1} D\varphi_m \int d\phi_0 \int d\phi_1 \cdots \int d\phi_M \sum_{k_1, q, \sigma_1, \xi_1} \cdots \sum_{k_2, q, \sigma_2, \xi_2} \cdots \sum_{n_{M+1}} \exp \left\{ -\varepsilon [E_c n_1^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_1)^2 / 2c^2 L_m] \right\}$$

$$\times \exp \left\{ \varepsilon \varphi_1 \phi_0 - \phi_1 \right\} \exp \left\{ -\varepsilon [E_c n_2^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_2)^2 / 2c^2 L_m] \right\} \exp \left\{ i \varepsilon \varphi_2 \phi_1 - \phi_2 \right\} \cdots$$

$$\times \exp \left\{ -\varepsilon [E_c n_{M+1}^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_{M+1})^2 / 2c^2 L_m] \right\} \exp \left\{ i \varepsilon \varphi_{M+1} \phi_M - \phi_{M+1} \right\} \right.$$}

$$\times \xi \xi \cdots [1 + \varepsilon \xi \eta(\tau)] \cdots [1 + \varepsilon \xi \eta(\tau)] \sum_{\sigma} \exp e^{-\beta h_{k_1, q_1, \sigma_1, \xi_1} (\tau)} \cdots h_{k_1, q_1, \sigma_1, \xi_1} (\tau), \quad (A4)$$

$$\sum_{l=0}^\infty (-1)^l \int_0^\beta d\tau_1 \int_0^{\tau_1} d\tau_{l-1} \cdots \int_0^{\tau_2} d\tau_1 \int_0^{\tau_1} D\varphi_m \int d\phi_0 \int d\phi_1 \cdots \int d\phi_M \sum_{k_1, q, \sigma_1, \xi_1} \cdots \sum_{k_2, q, \sigma_2, \xi_2} \cdots \sum_{n_{M+1}} \exp \left\{ -\varepsilon [E_c n_1^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_1)^2 / 2c^2 L_m] \right\}$$

$$\times \exp \left\{ \varepsilon \varphi_1 \phi_0 - \phi_1 \right\} \exp \left\{ -\varepsilon [E_c n_2^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_2)^2 / 2c^2 L_m] \right\} \exp \left\{ i \varepsilon \varphi_2 \phi_1 - \phi_2 \right\} \cdots$$

$$\times \exp \left\{ -\varepsilon [E_c n_{M+1}^2 + C_m \varphi_m^2 / 2c^2 + (\varphi_m - \phi_{M+1})^2 / 2c^2 L_m] \right\} \exp \left\{ i \varepsilon \varphi_{M+1} \phi_M - \phi_{M+1} \right\} \right.$$}

$$\times \xi \xi \cdots [1 + \varepsilon \xi \eta(\tau)] \cdots [1 + \varepsilon \xi \eta(\tau)] \sum_{\sigma} \exp e^{-\beta h_{k_1, q_1, \sigma_1, \xi_1} (\tau)} \cdots h_{k_1, q_1, \sigma_1, \xi_1} (\tau), \quad (A4)$$
where $\text{tr}_\sigma$ is the trace over all electron states in the channel $\sigma$, 
\[
h_\sigma = \sum_k (\varepsilon_{k\sigma} + eV) c_{k\sigma}^+ c_{k\sigma} + \sum_q \varepsilon_{q\sigma} c_{q\sigma}^+ c_{q\sigma} \tag{5}
\]
and
\[
h_{k,q,\sigma,\xi}(\tau) = e^{\tau \sigma} c_{k\sigma}^\xi c_{q\sigma} e^{-\tau \sigma} e^{\xi} \phi(\tau) . \tag{6}
\]
To simplify our mathematical expressions, here we have introduced the notations $c_{k\sigma}^\xi = c_{k\sigma}$ for $\xi = 1$, and $c_{k\sigma}^\xi = \xi$ for $\xi = -1$. The same notations apply to $c_{q\sigma}^\xi$.

Using the Poisson's resummation formula to sum over $n_i$, we obtain
\[
\begin{align*}
Z_V[\eta] &= \mathcal{N} \sum_{i=0}^{\infty} (-1)^i \int_0^\beta d\tau_i \int_0^{\tau_i} d\tau_{i-1} \cdots \int_0^{\tau_2} d\tau_1 \prod_{m=1}^M \int D\varphi_m \int d\phi_0 \prod_{i=1}^M \int d\phi_i \sum_{k_1,q_1,\sigma_1,\xi_1} \cdots \sum_{k_m,q_m,\sigma_m,\xi_m} \\
& \quad \times \prod_{\sigma} \text{tr}_\sigma e^{-\beta \sigma} h_{k_1,q_1,\sigma_1,\xi_1} \cdots h_{k_1,q_1,\sigma_1,\xi_1}(\tau_1).
\end{align*}
\]
Now we need to evaluate the trace over the electronic states in the above equation. This can be done by dividing the generating functional with a constant term
\[
Z_V[\eta] = \mathcal{N} \sum_{i=0}^{\infty} (-1)^i \int_0^\beta d\tau_i \int_0^{\tau_i} d\tau_{i-1} \cdots \int_0^{\tau_2} d\tau_1 \prod_{m=1}^M \int D\varphi_m \int d\phi_0 \prod_{i=1}^M \int d\phi_i \sum_{k_1,q_1,\sigma_1,\xi_1} \cdots \sum_{k_m,q_m,\sigma_m,\xi_m} \\
& \quad \times \prod_{\sigma} \text{tr}_\sigma e^{-\beta \sigma} h_{k_1,q_1,\sigma_1,\xi_1} \cdots h_{k_1,q_1,\sigma_1,\xi_1}(\tau_1).
\]
which can be absorbed in an irrelevant prefactor $\mathcal{N}$. In this way we have
\[
Z_V[\eta] = \sum_{i=0}^{\infty} (-1)^i \int_0^\beta d\tau_i \int_0^{\tau_i} d\tau_{i-1} \cdots \int_0^{\tau_2} d\tau_1 \prod_{m=1}^M \int D\varphi_m \int d\phi_0 \prod_{i=1}^M \int d\phi_i \sum_{k_1,q_1,\sigma_1,\xi_1} \cdots \sum_{k_m,q_m,\sigma_m,\xi_m} \\
& \quad \times \prod_{\sigma} \text{tr}_\sigma e^{-\beta \sigma} h_{k_1,q_1,\sigma_1,\xi_1} \cdots h_{k_1,q_1,\sigma_1,\xi_1}(\tau_1).
\]
where the symbol $\langle O \rangle_0$ denotes the quantum statistic average over free quasi-particles,
\[
\langle O \rangle_0 = \frac{\prod_{\sigma} \text{tr}_\sigma e^{-\beta (\sum_{k\sigma} \varepsilon_{k\sigma} c_{k\sigma}^+ c_{k\sigma} + \sum_q \varepsilon_{q\sigma} c_{q\sigma}^+ c_{q\sigma})} O}{\prod_{\sigma} \text{tr}_\sigma e^{-\beta (\sum_{k\sigma} \varepsilon_{k\sigma} c_{k\sigma}^+ c_{k\sigma} + \sum_q \varepsilon_{q\sigma} c_{q\sigma}^+ c_{q\sigma})}}. \tag{11}
\]
Since only the combinations $\langle c_{(k_\sigma,\sigma)}^\xi(\tau) c_{(k_\sigma,\sigma)}^\xi(\tau') \rangle_0$ and $\langle c_{(q_\sigma,\sigma)}^\xi(\tau) c_{(q_\sigma,\sigma)}^\xi(\tau') \rangle_0$ are nonzero, we use the Wick's theorem to obtain
\[
\begin{align*}
\sum_{k_1,q_1,\sigma_1,\xi_1} \cdots \sum_{k_m,q_m,\sigma_m,\xi_m} t^\xi_1 \xi_2 \cdots \xi_l c_{k_1,\sigma_1,\xi_1}(\tau_1)^c c_{q_1,\sigma_1,\xi_1}(\tau_1) \cdots c_{k_m,\sigma_m,\xi_m}(\tau_1)^c c_{q_m,\sigma_m,\xi_m}(\tau_1) & \\
& \times \langle c_{k_1,\sigma_1,\xi_1}(\tau_1) c_{q_1,\sigma_1,\xi_1}(\tau_1) \cdots c_{k_m,\sigma_m,\xi_m}(\tau_1) c_{q_m,\sigma_m,\xi_m}(\tau_1) \rangle_0.
\end{align*}
\]
In the limit $\varepsilon E_c \to 0$, only the $p=0$ term is relevant and all other terms are exponentially small. Then we have
\[
\lim_{\varepsilon E_c \to 0} \sum_{n_i=-\infty}^{\infty} e^{-\varepsilon E_i n_i^2 - \alpha n_i} \phi_i = \sqrt{\frac{\pi}{\varepsilon E_c}} e^{-\varepsilon \phi_i^2 / 4 E_c} . \tag{8}
\]
\[
\alpha(t) = \frac{t^2}{2} \sum_{k, q, k', q'} \langle \xi_{k, q}^\tau(\tau) \xi_{k', q'}^\tau(\tau) \rangle_0 \langle \xi_{q, q'}^\tau(\tau) \xi_{k, k'}^\tau(\tau) \rangle_0 \cdot (13)
\]
and the time-variables \(\tau_{p_1}\) and \(\tau_{p_2}\) are taken from the values \(\{\tau_1, \tau_2, \cdots, \tau_l\}\). We notice that the contributions of the terms with four or more channel indices equal are of order \(1/N\). In metallic tunnel junctions, \(N\) is very large and so such terms can be neglected. Then for nonzero \(\tau\), the kernel function \(\alpha(t)\) can be expressed as

\[
\alpha(\tau) = \frac{1}{4\beta} \sin^2(\pi \tau/\beta),
\]
where \(\alpha = R_K/R_T = 4\pi^2|t|^2\mu_pN\) with \(\mu_p\) (or \(\mu_r\)) being the density of states in the left (or right) electrode. For Matsubara frequencies \(\omega_\nu = 2\pi\nu/\beta\), the relevant Fourier components reduce to the simple form \(\alpha(\omega_\nu) = -\alpha|\omega_\nu|/4\pi\).

It is obvious that the nonzero contributions to the sum over pairs in Eq. (12) come from those terms with \(\sum_{i=1}^l \xi_i = 0\) and even \(l\). We let \(l = 2r\) be such even integers, and then because

\[
\prod_{p=1}^r \sum_{\xi_p} \delta_{\xi_{p_1}, \xi_{p_2}} \alpha(t_{p_1} - t_{p_2})[1 + i\xi_{p_1} \eta(t_{p_1})][1 + i\xi_{p_2} \eta(t_{p_2})]
\]
and the sum over \(r\) pairs gives a factor \((2r - 1)!!\), we arrive at the path integral representation of the generating functional

\[
Z_V[\eta] = \left( \frac{\beta}{2\pi} \right)^{\frac{3r}{2}} \int D\phi_m \int D\phi_e \int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') e^{[(eV\tau + i\phi(\tau)) - (eV\tau' + i\phi(\tau'))][1 + i\eta(\tau)][1 - i\eta(\tau')]} \sum_{r=0}^{\infty} \frac{1}{r!} \left( \int_0^\beta d\tau \int_0^\beta d\tau' \alpha(\tau - \tau') \right)^r.
\]

This is Eq. (13) in Sec. II.

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FIG. 1. Normalized conductance of the tunnel junction as a function of the junction voltage for $\alpha_t=1.04$, $\alpha_{ex}=8.09$ and $\beta E_c=0.18$ at $T=4.2 \, K$. The solid curve is from our theory, and the dots are experimental data taken from Fig. 2a of Ref. (17).
FIG. 2. Inverse of the normalized conductance dip at zero voltage as a function of the temperature for $\alpha_t=5.86$, $\alpha_{ex}=20.32$ and $C=1.99\, fF$ (upper curves); and $\alpha_t=3.02$, $\alpha_{ex}=1.50$ and $C=0.92\, fF$ (lower curves). The dashed lines are calculated from the P(E) theory, while the solid lines are calculated from our present theory. The dots are the corresponding experimental data taken from Figs. 3a and 3b of Ref. (17), together with the best fitted dotted lines.