Testing the initial-final mass relationship of white dwarfs

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Abstract. In this contribution we revisit the initial-final mass relationship of white dwarfs, which links the mass of a white dwarf with that of its progenitor in the main-sequence. Although this function is of paramount importance to several fields in modern astrophysics, it is still not well constrained either from the theoretical or the observational points of view. We present here a revision of the present semi-empirical initial-final mass relationship using all the available data and including our recent results obtained from studying white dwarfs in common proper motion pairs. We have also analyzed the results obtained so far to provide some clues on the dependence of this relationship on metallicity. Finally, we have also performed an indirect test of the initial-final mass relationship by studying its effect on the luminosity function and on the mass distribution of white dwarfs.

1. Introduction

The initial-final mass relationship of white dwarfs connects the mass of a white dwarf with that of its progenitor in the main-sequence. This function is of fundamental importance for several aspects of modern astrophysics since it is an input required for studying the chemical evolution of galaxies, the determination of the ages of globular clusters and their distances, and also to understand the properties of the Galactic population of white dwarfs. Despite its importance, an accurate measurement of this relationship is still not available and, thus, more efforts are needed from both the theoretical and the observational sides to improve it.

The first attempt to empirically map this relationship was carried out by Weidemann (1977), who has also provided a recent revision (Weidemann 2000). Although many improvements have been achieved in these 30 years, there are still some pieces missing in the puzzle. For instance, the dependence of this function on the different stellar parameters — like, for instance, the metallicity, magnetic field and angular momentum of the progenitor star — is still not clear. On the other hand, numerous works have dealt with the calculation of a theoretical initial-final mass relationship (Domínguez et al. 1999; Marigo 2001) but the differences in the evolutionary codes, such as the treatment of convection, the value of the assumed critical mass — which is the maximum mass of a white dwarf progenitor — or the mass loss prescriptions used lead to very different results.

From an observational perspective, most efforts up to now have focused on the observation of white dwarfs in open clusters, since this allows to infer the total age and the original metallicity.
of white dwarfs belonging to the cluster. Open clusters have made possible the derivation of a semi-empirical initial-final mass relationship using more than 50 white dwarfs, although only covering the initial mass range between 2.5 and 7.0 $M_\odot$ because stellar clusters are relatively young and, hence, the white dwarf progenitors in these clusters are generally massive. The extension of this mass range to smaller masses was done recently by Catalán et al. (2008a) studying white dwarfs belonging to common proper motion pairs. This was the first study that used these pairs with the purpose of improving the initial-final mass relationship. The stars studied in Catalán et al. (2008a) are at shorter distances in comparison with star clusters and this allows a better spectroscopic study of both members of the pair, obtaining their stellar parameters with accuracy. At the same time, the study of these pairs enables a wide age and metallicity coverage of the initial-final mass relationship. A parallel attempt to cover the low-mass domain of this relationship was done recently by Kalirai et al. (2008) studying white dwarfs in old open clusters.

In this contribution we obtain the initial-final mass relationship grouping all the available results in order to derive a semi-empirical relation, which has also been tested indirectly by computing the white dwarf luminosity function and mass distribution.

2. The initial-final mass relationship

In order to derive a new semi-empirical initial-final mass relationship we have considered our results obtained studying white dwarfs in common proper motion pairs (Catalán et al. 2008a) and completed the sample with the available data in the literature, which is mainly based on open clusters. We have carried out a re-analysis of the open cluster data starting from the white dwarf atmospheric parameters ($T_{\text{eff}}$ and $\log g$) given in the literature, as well as the ages and metallicities of the clusters used by the corresponding authors — see Catalán et al. (2008b) and references therein. The procedure followed consists basically in deriving the final mass ($M_f$) and the cooling time of each white dwarf from the atmospheric parameters and the cooling sequences of Salaris et al. (2000). These cooling tracks consider a CO core white dwarf with a larger abundance of O the center of the core and with a H thick envelope ontop of a He buffer. Their masses are $q(\text{H}) = M_\text{H}/M = 10^{-4}$ and $q(\text{He}) = M_\text{He}/M = 10^{-2}$, respectively. Since from the age of the cluster we know the total ages of these white dwarfs we derived the main-sequence lifetimes of the progenitors, and from these, their initial masses using the stellar tracks of Domínguez et al. (1999).

In Fig. 1 we present the final masses versus the initial masses obtained for white dwarfs in common proper motion pairs and open clusters. The observational data that can be used to define the semi-empirical initial-final mass relationship contains now 62 white dwarfs. It is important to emphasize that all the values below 2.5 $M_\odot$ correspond to the data obtained from common proper motion pairs and the recent data based in old open clusters (Kalirai et al. 2008). Before these studies no data for the low-mass domain were available, since white dwarfs in stellar clusters are usually more massive, especially if the clusters are young. The coverage of the low-mass end of the initial-final mass relationship is specially important since it is the most populated bin according to standard initial mass functions and at the same time it guarantees, according to the theory of stellar evolution, the study of white dwarfs with masses near the most common value, $M \sim 0.57 M_\odot$, which represent about 90% of the white dwarf population (Kepler et al. 2007). Thus, these new data increase considerably the statistical significance of the semi-empirical initial-final mass relationship. The results plotted in Fig. 1 reveal a clear dependence of the white dwarf masses on the masses of their progenitors, although a considerable scatter is present. The theoretical initial-final mass relationship of Domínguez et al. (1999) for solar composition has been also plotted in order to be consistent with the stellar tracks used to derive the initial masses. Although the distribution presents a large dispersion, which is larger than the error bars, a comparison of the observational data with these theoretical
relationships shows that they share the same trend. However, it should be noted that for each cluster the data presents an intrinsic spread in mass, which can be the result of a combination of different factors, which are discussed below.

Following closely recent works on this subject (Ferrario et al. 2005, Williams 2007), we assume that the initial-final mass relationship can be described by a linear function. As can be seen in Fig. 1 the semi-empirical data and the theoretical initial-final mass relationship (dotted line) can be divided in two different regions, both showing a linear trend. The threshold initial mass separating both regions is $2.7 \, M_\odot$. It is worth noticing that the low-mass region shows a shallower slope probably due to the smaller efficiency of mass loss. Taking this into account we have performed a weighted least-squares linear fit for each region, obtaining for $M_i < 2.7 \, M_\odot$:

$$ M_f = 0.096 M_i + 0.429 \quad \Delta M_f = 0.05 \, M_\odot $$  \hspace{1cm} (1)

whereas for $M_i > 2.7 \, M_\odot$ we obtain:

$$ M_f = 0.137 M_i + 0.318 \quad \Delta M_f = 0.12 \, M_\odot $$  \hspace{1cm} (2)

These expressions have been overplotted in Fig. 1 as two solid lines. Taking into account the scatter of the data and the values of the reduced $\chi^2$ of these fits (7.1 and 4.4, respectively) we have computed the dispersion of the derived final masses, obtaining 0.05 $M_\odot$ and 0.12 $M_\odot$, respectively. In past works, it was necessary to include in the fit a ficticious anchor point to represent the typical white dwarf mass, $M_f \sim 0.57 \, M_\odot$ (Kepler et al. 2007), since no data were available at the low-mass domain. In our case, this is not necessary since we are including new data that cover this typical mass value of white dwarfs.
The sample of white dwarfs studied here covers a range of metallicities from $Z = 0.006$ to 0.040. From a theoretical point of view it is well established that progenitors with large metallicity produce less massive white dwarfs. Thus, one should expect to see a dependence of the semi-empirical data on metallicity. We have performed a quantitative study of the correlation between the final masses and metallicity. We have computed the differences between the observed final masses and the final masses obtained using Eqs. (1) and (2). In order to quantify this correlation we have calculated the Spearman rank correlation coefficient obtaining $-0.002 \pm 0.128$. The error has been derived from a bootstrapping. Thus, we can conclude that the final masses and metallicities of this sample are not correlated, and that the scatter in the distribution in Fig. 1 is not due, at least not exclusively, to the effect of metallicity.

3. Main systematic uncertainties

The results obtained in this work are dependent on different assumptions and approaches that we have considered during the procedure followed to derive the final and initial masses, which we discuss in the following.

3.1. Thicknesses of the envelopes

As previously pointed out, the observed white dwarf masses in clusters scatter considerably in the same region of initial masses — see Fig. 1. One of the reasons that may explain this, besides the possible stochastic character of mass-loss processes, is the assumption of a given internal composition and of the outer layer stratification of the white dwarfs under study. The thicknesses of the H and He layers is a key factor in the evolution of white dwarfs, since they control the rate at which white dwarfs cool down. To derive the final masses we have used cooling sequences with fixed thicknesses of the envelopes, which might be more appropriate in some cases than in others. In fact, the exact masses that the layers of H and He may have is currently a matter of debate, being the subject of several studies. For instance, models reducing the thickness of the H envelope to $q(H) = 2.32 \times 10^{-6}$ were computed by Prada Moroni & Straniero (2002), obtaining cooling times shorter than those obtained in this work assuming a thick envelope H, $q(H) = 10^{-4}$. This stems from the fact that H has a larger opacity than He. In the case of an even thinner H envelope, $q(H) = 10^{-10}$, the cooling age could be reduced in 10% (1 Gyr) at $\log(L/L_\odot) = -5.5$. Thus, the uncertainty in the cooling times could be relevant in some cases, which would affect the estimates of the progenitor lifetimes and, in turn, the initial masses derived.

In order to estimate the effect that the thicknesses of H and He envelopes may have in the initial masses derived here, we have repeated the calculations using the cooling sequences of Fontaine et al. (2001) for a 50/50 CO core white dwarf with a standard He envelope, $q(He) = 10^{-2}$, and two different thicknesses for the H envelope, a thick one, $q(H) = 10^{-4}$, and a thin one, $q(H) = 10^{-10}$. We have verified that the initial masses are indeed sensitive to the cooling sequences used, as expected. We have obtained larger initial masses when considering a thin envelope, due to the longer cooling times obtained in this case. As previously pointed out, in principle it can be expected that the cooling time should be smaller for a thin H envelope model, but this assumption is only true at low enough luminosities (Prada Moroni & Straniero 2002). In fact, at intermediate luminosities a white dwarf with a thinner H envelope evolves slower than the thicker counterpart because it has an excess of energy to radiate away (Tassoul et al. 1990). The maximum difference in the initial masses ($\sim 1 M_\odot$) has been found to occur for high-mass progenitors ($M > 5 M_\odot$), while this value is one order of magnitude smaller for smaller masses ($\sim 0.1 M_\odot$). However, it should be noted that many other combinations are possible, for instance, with different thicknesses of the He envelope, which in this case has been kept fixed. However, since it is impossible to know which is the real chemical stratification of the outer layers of each individual white dwarf we have not formally introduced this error in
the calculations, since in some cases we would be overestimating the error of the initial masses derived here.

3.2. Composition of the core
The most commonly assumed composition for a typical white dwarf is a mixture of carbon and oxygen. However it should be taken into account that white dwarfs can have also other internal compositions. Those white dwarfs more massive than 1.05 \( M_\odot \) are thought to have a core made of ONe, while those with masses below 0.4 \( M_\odot \) have an He-core. ONe white dwarfs cool faster than CO or He white dwarfs because the heat capacity of O and Ne is smaller than that of C or He. On the contrary, He white dwarfs are the ones that cool slower. Thus, those white dwarfs studied here with masses near the limits between different core populations would be introducing an uncertainty in the cooling times obtained, since their cooling times are different depending on the adopted composition. For instance, a 1 \( M_\odot \) ONe white dwarf cools 1.5 times faster than a CO white dwarf of the same mass (Althaus et al. 2007). Thus, if an observed white dwarf has indeed an ONe core instead of the typical CO one, its progenitor lifetime would be underestimated in our analysis and, consequently, the initial mass derived would be larger than the real one. However, the exact impact of this depends also on the total age of the white dwarf. The smaller the total age, the higher the effect of considering a wrong internal composition.

3.3. Mass determinations when \( T_{\text{eff}} \leq 12000 \) K
Spectroscopic masses can be determined precisely if high signal-to-noise spectra is used, thus, accurate atmospheric parameters are obtained. However, it should be taken into account that the atmospheres of DA white dwarfs below 12 000 K could be enriched in He while preserving their DA spectral type (Bergeron et al. 1992). This He is thought to be brought to the surface as a consequence of the development of a H convection zone. Depending on the efficiency of convection the star could still show Balmer lines, instead of being considered as a non-DA white dwarf. This will affect considerably the determination of the spectroscopic mass, since a large abundance of He mimics the presence of a large surface gravity. As a consequence, the assumption of an unrealistic chemical composition could have a large impact on the cooling times estimated, although mainly at low effective temperatures. Nevertheless, a very large fraction of the stars in our sample (\( \sim 95\% \)) have temperatures well above this limit.

3.4. Total ages
The derived initial masses depend on the cooling times, the total ages, the metallicity and finally, on the stellar tracks used. Among these parameters the largest source of error is due to the uncertainty in the total ages of the white dwarfs. For white dwarfs in open clusters, the age can be usually derived with high accuracy from model fits to the turn-off location in a colour-magnitude diagram. The uncertainty on the age of a cluster is a systematic effect for stars belonging to the same cluster, since all the initial masses will be shifted together to larger or smaller masses in the final versus initial masses diagram. On the contrary, in the case of white dwarfs in common proper motion pairs, the accuracy in the total age depends on the evolutionary stage of the companion. The accuracy of the age using isochrone fitting could be large if the star is relatively evolved and located far away from the ZAMS. Using the X-ray luminosity method, the ages derived could also be quite precise — 8 to 20\% per cent, see Catalán et al. (2008a) — if the star is relatively young (\( t \leq 1 \) Gyr).
4. Luminosity function and mass distribution

We have computed a set of white dwarf luminosity functions considering an age of 11 Gyr for the Galactic disk and using bins of visual magnitude. In Fig. 2 we show from top to bottom the total luminosity function and the luminosity functions of white dwarfs with masses larger than 0.7 \( M_\odot \) and 1.0 \( M_\odot \), respectively. The total luminosity function (that is, considering the whole range of masses) was normalized to the bin corresponding to \( M_V = 11 \), and then this normalization factor was used for the luminosity functions of white dwarfs more massive than 0.7 \( M_\odot \) and 1.0 \( M_\odot \). We have used the stellar evolutionary inputs of Domínguez et al. (1999) — D99 — Marigo (2001) — M01 — Wood (1992) — W92 — and the semi-empirical initial-final mass relationship presented in Sect. 2. Circles, triangles and squares correspond to the observational data of the Palomar Green (PG) survey (Liebert et al. 2005). Comparing the different theoretical luminosity functions, it can be noted that the predicted number of massive white dwarfs is larger when using the inputs of Domínguez et al. (1999) and Marigo (2001) with our analytical initial-final mass relationship that in the case in which the expressions of Wood (1992) are used. This was expected since the latter favors the production of low-mass white dwarfs. If we consider only the first three results it can be noted that it is not possible to evaluate which initial-final mass relationship produces a theoretical luminosity function that best fits the observational data, since the error bars of the observational data are larger than the differences between the theoretical results. In any case, what it can be clearly seen is that all the theoretical relations predict more massive white dwarfs than the observations when a mass cut of 1.0 \( M_\odot \) is adopted, except in the case of Wood (1992). Nevertheless, conclusive results cannot be derived given the scarcity of observational data.

Following a similar procedure we have computed theoretical white dwarf mass distributions and compared our results with the recent data from the SDSS (DeGennaro et al. 2008). Since

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**Figure 2.** White dwarf luminosity functions versus visual magnitude using different initial-final mass relationships. Circles, triangles and squares correspond to the observational data of the PG Survey.
the accuracy on the mass determinations decreases considerably when white dwarfs are cooler than 12000 K, both the theoretical and the observational mass distributions only take into account white dwarfs with $T_{\text{eff}} \geq 12000$ K. We have considered the same initial-final mass relationships as in the previous calculation, and also the recent semi-empirical relationship derived by Kalirai et al. (2008) — K08. All the white dwarf distributions have been normalized to the total density obtained in each case. As it can be noted, there is a well defined peak in all the mass distributions, the location of which is defined mainly by the initial-final mass relationship considered. On the contrary, the height of the peak depends also on the lifetime of the progenitors. The central peak is shifted to larger masses when the initial-final mass relationship considered favors the production of more massive white dwarfs for the low mass progenitors. If we compare the theoretical mass distributions with the SDSS observational data it can be noted how the location and height of the central peak is best fitted by the predictions obtained when using our semi-empirical relationship.

5. Conclusions

In this work we have derived a new semi-empirical initial-final mass relationship by considering our results obtained studying white dwarfs in common proper motion pairs and re-evaluating the available data in the literature, mainly based on open clusters. With this study we have extended the observational data to the low-mass domain ($M < 2.5 \, M_\odot$), which was poorly studied until recently. The coverage of the low-mass end is of great importance since we are now studying the most populated region of initial masses according to the initial mass function, including in this way the well-established peak of the field white dwarf mass distribution in our study.
We have performed a weighted least-squares linear fit of the results, obtaining analytical expressions that connect the final and initial masses of progenitors of white dwarfs with masses ranging from 1 to 6.5 $M_\odot$. Since the sample of data collected for this study covers a wide range of ages and metallicities, we have performed an analysis to evaluate the dependence of the initial-final mass relationship on metallicity. The correlation factor obtained indicates that such dependence does not exist, or at least, it cannot be identified in our results. However, it should be taken into account that there are some other parameters that may play an important role (angular momentum, magnetism...), and that it is very difficult to study the effect of each of them separately.

We have also tested the initial-final mass relationship from an indirect point of view by computing the luminosity function and mass distribution of white dwarfs. For this purpose we have considered different stellar evolutionary inputs (stellar tracks and initial-final mass relationships). We have noted some differences between the theoretical luminosity functions, obtaining a clear dependence on the considered initial-final mass relationship. We have compared our results with the observational data from the PG Survey and we have always obtained a reasonable fit when the range of masses was constrained to $M > 0.7 M_\odot$, except when using the initial-final mass relationship of Wood (1992). However, given the presently available observational data, any attempt to discern which initial-final mass relationship better fits the data is not feasible. Nevertheless, our results favor an initial-final mass relationship that favors the production of more massive white dwarfs (Catalán et al. 2008b). In the case of the white dwarf mass distribution we have obtained more conclusive results. From a comparison of our predictions with the observational data of the SDSS we have noted that the theoretical mass distribution obtained when considering the semi-empirical initial-final mass relationship derived in this work is the one that best fits the central peak of the observational mass distribution, being in this way the one which is more representative of the characteristics of the white dwarf population.

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