The QCD phase diagram in the presence of an external magnetic field: the role of the inverse magnetic catalysis

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The effect of an external magnetic field in QCD phase diagram, namely, in the location of the critical end point (CEP) is investigated. Using the 2+1 flavor Nambu-Jona-Lasinio model with Polyakov loop, it is shown that when an external magnetic field is applied its effect on the CEP depends on the strength of the coupling. If the coupling depends on the magnetic field, allowing for inverse magnetic catalysis, the CEP moves to lower chemical potentials eventually disappearing, and the chiral restoration phase transition is always of first order.

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1. Inverse magnetic catalysis in the PNJL model

The influence of strong external magnetic fields on the structure of the QCD phase diagram is a very important field of research due to its consequences on several physical phenomena: the measurements in heavy ion collisions at very high energies, the behavior of the first stages of the Universe and the understanding of compact astrophysical objects like magnetars.

The inclusion of a magnetic field in the Lagrangian density of the Nambu–Jona-Lasinio (NJL) model and of the Polyakov–Nambu–Jona-Lasinio (PNJL) model gives rise to the Magnetic Catalysis (MC) effect, i.e., the enhancement of the quark condensate due to the magnetic field [1, 2, 3], but fails to account for the Inverse Magnetic Catalysis (IMC) found in LQCD calculations [4, 5, 6] where the suppression of the quark condensate takes place due to the strong screening effect of the gluon interactions. In order to overcome this discrepancy, it was proposed, by using the SU(2) NJL model [7] and the SU(3) NJL/PNJL models [8], that the model coupling, $G_s$, can
Fig. 1. (Left panel) The renormalized critical temperatures of the chiral transition \((T_{\chi}^c/eB = 0) = 178\) MeV as a function of \(eB\) in the NJL model with a magnetic field dependent coupling \(G_s(eB)\) and a constant coupling \(G_0s\), and the lattice results [4]. (Right panel) The chiral \((T_{\chi}^c)\) and deconfinement \((T_{\Phi}^c)\) transitions temperatures as a function of \(eB\) in the PNJL, using \(G_s(eB)\) given by Eq. (1).

be seen as proportional to the running coupling, \(\alpha_s\), and consequently, a decreasing function of the magnetic field strength allowing to include the impact of \(\alpha_s(eB)\) in both models. Indeed, the strong screening effect of the gluon interactions in the region of low momenta weakens the interaction which is reflected into a decrease of the scalar coupling with the intensity of the magnetic field [9].

Since there is no LQCD data available for \(\alpha_s(eB)\), by using the NJL model we can fit \(G_s(eB)\) in order to reproduce the pseudocritical chiral transition temperatures, \(T_{\chi}^c(eB)\), obtained in LQCD calculations [4]. The resulting fit function that reproduces the \(T_{\chi}^c(eB)\) is

\[
G_s(\zeta) = G_s^0 \left( \frac{1 + a \zeta^2 + b \zeta^3}{1 + c \zeta^2 + d \zeta^4} \right)
\]

with \(a = 0.0108805\), \(b = -1.0133 \times 10^{-4}\), \(c = 0.02228\), and \(d = 1.84558 \times 10^{-4}\) and where \(\zeta = eB/\Lambda_{QCD}^2\). We also have used \(\Lambda_{QCD} = 300\) MeV.

In the NJL model, the renormalized pseudocritical chiral transition temperatures, \(T_{\chi}^c/T_{\chi}^c(eB = 0)\), are plotted in left panel of Fig. 1 as a function of \(eB\): with the magnetic field dependent coupling \(G_s(eB)\) (green line), given by Eq. (1); with LQCD results (red dots); and the usual constant coupling \(G_s = G_s^0\) (black dashed dot line), that shows magnetic catalyzes with increasing \(T_{\chi}^c/T_{\chi}^c(eB = 0)\) for all range of magnetic fields.

Now, using \(G_s(eB)\) given in Eq. (1), we calculate the chiral and deconfinement transitions temperatures as a function of \(eB\) in the PNJL model. The results are shown in the right panel of Fig. 1 due to the existing coupling between the Polyakov loop field and quarks within the PNJL model, the \(G_s(eB)\) does not only affect the chiral transition but also the decon-
finement transition. Consequently, both temperatures transitions decrease with increasing magnetic field strength.

2. The influence of the inverse magnetic catalysis in the location of the critical end point

The nature of the phase transition and the existence of the critical end point (CEP) are open issues for theoretical studies about the QCD phase diagram [10]. From the experimental point of view the existence/location of the CEP is also a very timely topic. This renders important to know the conditions that can change the position of the CEP in the phase diagram, namely the presence of strong magnetic fields.

In the following, we will study two scenarios for the effect of a static external magnetic field on the location of the CEP when symmetric matter ($\mu_u = \mu_d = \mu_s$) is considered:

Case I – where we take the usual $G_s = G_s^0$ and no IMC effects are included;

Case II – where we will use $G_s(eB)$ given by Eq. (1) which will allow us to consider the IMC effects on the QCD phase diagram.

The results for Case I are plotted in the left panel of Fig. 2 and reproduce qualitatively the results previously obtained within the NJL model in [11]: as the intensity of the magnetic field increases, the transition temperature increases and the baryonic chemical potential decreases until the critical value $eB \sim 0.4 \text{ GeV}^2$. For stronger magnetic fields both $T$ and $\mu_B$ increase. In the right panel of Fig. 2 the CEP is given in a $T$ versus baryonic density plot. It is seen that when $eB$ increases from 0 to 1 GeV$^2$ the baryonic density at the CEP increases from $2\rho_0$ to $\sim 14\rho_0$ [12].

With respect to Case II the results for the CEP are presented in Fig. 3, red points. We clearly observe a different behavior when compared with...
Fig. 3. Location of the CEP on temperature vs baryonic chemical potential $\mu_B$ (left) and temperature vs baryonic density $\rho_B$ (right) diagrams, for both cases. The baryonic density $\rho_B$ is in units of nuclear saturation density, $\rho_0 = 0.16 \text{ fm}^{-3}$.

Case I (black points): at $B = 0$ both CEP’s coincide but, already for small values of $B$, the CEP is moved to lower temperatures and chemical potentials. Nevertheless, until $eB \sim 0.3 \text{ GeV}^2$ the pattern is similar for both Cases. However, for stronger magnetic fields the position of the CEP in Case II oscillates between $T \approx 169$ and $T \approx 177 \text{ MeV}$ while the chemical potential takes increasingly smaller values: a completely different behavior when compared with Case I, where both values of $T$ and $\mu_B$ for the CEP increase.

The reason of this behavior lies in fact that the restoration of chiral symmetry is stressed by the decreasing of the coupling $G_s(eB)$. The increasing of the magnetic filed is not sufficient to counteract this effect as can be seen if Fig. 4, where we plot the quarks masses ($M_u$-black line; $M_d$-red line; $M_s$-blue line) as function of $\mu_B$ for the respective temperature where the CEP occurs ($T_{CEP}^B$) at $eB = 0.1$ and $eB = 0.5 \text{ GeV}^2$. At $eB = 0.1 \text{ GeV}^2$ (left panel) $G_s$ is barely affected by the magnetic field, the values of the quark masses are very close to each other for both cases, and the CEP occurs at
smaller temperatures and at close, but smaller, chemical potentials. When $eB = 0.5 \text{ GeV}^2$, the quark masses in Case I are increased with respect to the $B = 0$ case (due to MC effect), being the restoration of chiral symmetry more difficult to achieve. However, when $G_s(eB)$, Case II, the masses of the quarks are already smaller than the $B = 0$ case (due to IMC effect) leading to an faster restoration of chiral symmetry at small temperatures and chemical potentials. Eventually, with the increase of $B$, the CEP would disappear in the temperature axis and the transition to the chiral restored phase is always of first order.

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