Measurement of Differential Hardening under Biaxial Stress of Pure Titanium Sheet

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Abstract. Biaxial stress tests of a commercially pure titanium sheet (JIS #1) using the cruciform specimen and the servo-controlled biaxial testing machine have been carried out in order to elucidate its anisotropic plastic deformation behavior. The geometry of the cruciform specimen is identical to that regulated by the ISO 16842. Nine linear stress paths, \( \sigma_y : \sigma_x = 1:0, 4:1, 2:1, 4:3, 1:1, 3:4, 1:2, 1:4, \) and \( 0:1 \) in the first quadrant of the principal stress space are applied to the cruciform specimens. Contours of plastic work in the principal stress space and the directions of plastic strain rates at selected levels of plastic work have been precisely measured. The range of the equivalent plastic strain applied to the specimens is \( 0.002 \leq \varepsilon_p^e \leq 0.01 \). The shapes of the work contours significantly change with increasing \( \varepsilon_p^e \); the test material exhibits differential hardening (DH). Using the data of the work contours and the directions of plastic strain rates, the applicability of selected anisotropic yield functions to the accurate prediction of the plastic deformation behavior of the test material is examined.

1. Introduction

Pure titanium has high corrosion resistance and therefore has mainly been applied to industrial chemical equipment and heat exchangers. The demand for them has increased lately in recent years. However, since pure titanium has a low Young’s modulus and strong in-plane anisotropy, it does not lend itself well to press forming. In order to achieve time and cost effectiveness in the press forming of pure titanium sheet, it is vital to accurately predict potential defects in the resulting parts using finite element simulations and to determine optimum forming conditions. To do this, it is critical that an accurate constitutive model for pure titanium sheet be established [1].

The authors investigate the work hardening behavior of pure titanium sheet under many linear stress paths and measured the plastic work contours in the first, second and fourth quadrants of a principal stress space [2]. We found that: (i) the pure titanium showed significant in-plane anisotropy with respect
to the flow stresses and \( r \)-values; (ii) the plastic work contours show significant asymmetry with respect to the line of \( \sigma_t = \sigma_y \), and (iii) the material exhibited significant differential work hardening; the degree of asymmetry of successive work contours decreases with increasing plastic work. Moreover, we found that the conventional anisotropic yield functions are not able to perfectly reproduce the deformation behavior of the pure titanium under biaxial loading. Therefore, we utilize a method of constructing a yield locus piece by piece, proposed by Vegter and Van den Boogaard [3], to determine the approximation curves reproducing the observed plastic work contours. Recently, Hama et al. [4] performed a crystal-plasticity finite-element analysis to study the deformation behavior of a commercial pure titanium Grade 1 sheet upon different strain paths. They could qualitatively capture the abovementioned differential hardening behavior of the pure titanium sheet investigated by Ishiki et al. [2].

In this study, we apply the Yld2000-2d yield function [5] to reproduce the differential work hardening behavior of a pure titanium Grade 1 sheet. The material parameters and exponent of the Yld2000-2d yield function are determined to minimize the cost function proposed by Hakoyama and Kuwabara [6]. This method is useful to optimize the parameters of an assumed material model by minimizing the deviation of the shapes of the plastic work contours and the directions of the plastic strain rates from those observed in the biaxial tensile tests.

2. Experimental method

2.1. Test material

The test material used in this study was a 0.7mm-thick commercial pure titanium Grade 1 sheet (JIS #1). The mechanical properties of the material obtained from the uniaxial tensile tests along the rolling (RD), 45° (DD) and transverse (TD) directions are show in Table 1. The material had significant in-plane anisotropy in both tensile flow stresses and \( r \)-values. Figure 1 shows the variation of \( r \)-values at nominal strain \( \varepsilon_N = 0.1 \) and tensile flow stresses with tensile directions. The flow stresses were determined at plastic work per unit volume identical to that consumed during the uniaxial tensile test for RD up to the logarithmic plastic strain of \( \varepsilon_N^p = 0.002 \).

2.2. Biaxial tensile testing method

Figure 2 shows the schematic diagram of the cruciform specimen used for biaxial tensile tests of the as-received sheet sample. The geometry of the specimen was the same as that proposed by Kuwabara et al. [7, 8], and was formally adopted as an international standard [9]. The specimen arms were parallel to the RD and TD of the material. Each arm of the specimen had seven slits 60 mm long and 0.2 to 0.3 mm wide at 7.5 mm intervals to remove the geometric constraint on the deformation of the 60 × 60 mm² square gauge area. The slits were fabricated by laser cutting.

| Tensile direction /° | \( E \)/GPa | \( \sigma_{0.2} \)/MPa | \( c \)/MPa | \( n \) | \( \alpha \) | \( r \)-value |
|----------------------|------------|-----------------|-----------|-----|-------|----------|
| 0                    | 107        | 138             | 418       | 0.165 | -0.0006 | 1.55     |
| 45                   | 110        | 171             | 396       | 0.151 | 0.0013 | 3.55     |
| 90                   | 117        | 208             | 396       | 0.115 | 0.0012 | 3.88     |

\( ^a \) Approximated using \( \sigma = c(\alpha + \varepsilon^p)^n \) for \( \varepsilon^p = 0.002-0.07 \)

\( ^b \) Measured at uniaxial nominal strain \( \varepsilon_N = 0.1 \)

The normal strain components \( (\varepsilon_x, \varepsilon_y) \) were measured using uniaxial strain gauges (YFLA-2, Tokyo Sokki Kenkyujo Co.) mounted at ±21 mm from the center along the maximum loading direction. According to the finite elemental analysis (FEA) of the cruciform specimen as shown in Figure 2, the
measurement error of stress was estimated to be less than 2 % [10, 11]. Details of the biaxial tensile testing apparatus and test method are given in Refs. [7] and [8].

Linear stress paths were applied to the cruciform specimens; the true stress ratios \( \sigma_x : \sigma_y \) were chosen to be 4:1, 2:1, 4:3, 1:1, 3:4, 1:2 and 1:4. Standard uniaxial tensile specimens (JIS 13 B-type) were used for the uniaxial tensile tests with \( \sigma_x : \sigma_y = 1:0 \) and 0:1. True stress increments were controlled and applied to the specimens so that the von Mises equivalent plastic strain rate became approximately constant at \((2–6) \times 10^{-4} \text{s}^{-1}\) for all stress paths. Two specimens were used for each stress path.

The concept of the plastic work contour in the stress space [12, 13] was introduced to evaluate the work hardening behavior of the test material under biaxial tension. The true stress-logarithmic plastic strain curve obtained from the uniaxial tensile test along the RD was selected as a reference datum for work hardening. First, we select a particular value of logarithmic tensile plastic strain \( \varepsilon_{0p} \) to determine the associated uniaxial tensile true stress \( \sigma_{0u} \) and the plastic work per unit volume \( W_0 \) consumed

![Figure 1](image1.png)

**Figure 1.** Variation of \( r \)-values and uniaxial tensile flow stresses with tensile directions. The flow stresses were measured at the reference logarithmic plastic strain of \( \varepsilon_{0p} = 0.002 \).

![Figure 2](image2.png)

**Figure 2.** Cruciform specimen used for the biaxial tensile tests (units in mm).
during the test up to $\varepsilon_0^p$. Next, the uniaxial true stress $\sigma_0^{\text{uniaxial}}$ measured from a tensile test in the TD and the biaxial true stress components $(\sigma_x, \sigma_y)$ measured from biaxial tensile tests were determined for the same plastic work as $W_0$. The stress points $(\sigma_0, 0), (0, \sigma_0^{\text{uniaxial}})$, and $(\sigma_x, \sigma_y)$ plotted in the principal stress space form a plastic work contour associated with $\varepsilon_0^p$. When $\varepsilon_0^p$ is taken to be sufficiently small, the corresponding work contour can be practically viewed as a yield locus.

3. Results of biaxial tensile tests

Figure 3 (a) shows the measured stress points forming the plastic work contours. Each stress point represents the average of two specimens’ data; the difference between the two was less than 2% of the flow stress for all data points. The maximum value of $\varepsilon_0^p$ for which the work contour has a full set of nine stress points measured using cruciform specimens was $\varepsilon_0^p = 0.01$.

Figure 3 (b) shows the stress points forming the work contours; the stress values are normalized by the $\sigma_0$ associated with each work contour. The maximum values of $\varepsilon_0^p$ were different among seven
stress passes. The shape of the work contours changed with work hardening or equivalently with $\varepsilon^p_0$; thus, the test material exhibited differential hardening. The work contours normalized by $\sigma_0$ showed shrinkage except for $\sigma_x: \sigma_y = 4:1$ and $2:1$.

Figure 4 shows the directions of the plastic strain rates, $\beta$, measured at selected levels of $\varepsilon^p_0$. $\beta$ was determined as $\tan^{-1}\left(\frac{\Delta \varepsilon^p_y}{\Delta \varepsilon^p_x}\right)$, where $\Delta \varepsilon^p_x$ and $\Delta \varepsilon^p_y$ are the logarithmic plastic strain increments in the RD and TD for a time interval of $\Delta t$ ($=10$ s), measured using the strain gauges put on the cruciform specimen, see Figure 2. $\beta$ showed a tendency of increase with the increase of $\varepsilon^p_0$ for the range of $0^\circ \leq \varphi \leq 45^\circ$. The stress ratios that provide the plane strain tension for RD ($\beta = 0^\circ$) and TD ($\beta = 90^\circ$) are approximately $\sigma_x: \sigma_y = 15:14$ to $15:12$ and $5:8$, respectively, for a strain range of $0.002 \leq \varepsilon^p_0 \leq 0.01$.

4. Material modelling and discussion

The Yld2000-2d yield function [4] was used to approximate the work contours for $\varepsilon^p_0 = 0.01$. The material parameters $\alpha_i$ ($i=1~8$) and exponent $M$ of the Yld2000-2d yield function were determined following methods I, II, III, and IV.

Method I: $r_0$, $r_{45}$, $r_90$, and $r_0$ and $\sigma_0/\sigma_0$, $\sigma_45/\sigma_45$, $\sigma_90/\sigma_90$, and $\sigma_0/\sigma_0$ were used, where $r_0$ and $\sigma_0$ are the $r$-value and tensile flow stress measured at an angle of $\vartheta$ from the RD, respectively, and $r_0$ and $\sigma_0$ are the plastic strain rate ratio $\dot{\varepsilon}_x^p/\dot{\varepsilon}_y^p$ and the flow stress in balanced biaxial tension, $\sigma_x: \sigma_y = 1:1$, respectively. The values of these parameters are shown in Table 2. An exponent $M$ was selected to minimize the root mean square error $\delta_a$ between the measured stress points and the calculated yield locus:

$$\delta_a = \sqrt{\frac{\sum_j (a_{j,c} - a_{j,M})^2}{N}}$$ (1)

Here, $N (= 9)$ is the number of data points forming the work contour, $a_{j,M}$ is the distance from the origin of the principal stress space to the $j$th stress point, and $a_{j,c}$ is the distance from the origin of the principal stress space to the calculated yield locus along the stress path connected to the $j$th stress point, see Figure 5.

| $\varepsilon^p_0$ | 0.01 |
|-------------------|------|
| $\sigma_0$ /MPa    | 197  |
| $\theta^\circ$     |      |
| $\sigma_0/\sigma_0$|      |
| $r_0$              |      |
| 0                  | 1.00 |
| 45                 | 1.00 |
| 90                 | 1.16 |
| equibiaxial        | 1.29 |

Methods II, III, and IV: The material parameters $\alpha_i$ and exponent $M$ were determined to minimize the cost function as given by
\[ F = \sum_{j=1}^{N} w_{j,\alpha} (a_{j,C} - a_{j,M})^2 + \sum_{j=1}^{N} w_{j,\beta} (\beta_{j,C} - \beta_{j,M})^2 \]  

with the weighted parameters \( w_{j,\alpha} = 1 \) and \( w_{j,\beta} = 0 \) for method II, \( w_{j,\alpha} = 0 \) and \( w_{j,\beta} = 1 \) for method III, and \( w_{j,\alpha} = 1 \) and \( w_{j,\beta} = 0.35 \) for method IV. Here, \( N = 10 \) is the number of stress points forming a work contour including the first quadrant of the principal stress space and the uniaxial tensile flow stress at the DD, \( \beta_{j,M} \) is the direction of the plastic strain rate measured for the \( j^{th} \) stress path, and \( \sigma^0_y \) the material yield stress.

![Schematics for the calculation of (a) the root mean square error \( \delta_{\alpha} \) of a calculated yield locus from the measured work contour and (b) the root mean square error \( \delta_{\beta} \) of the calculated directions of the plastic strain rates based on the normality flow rule for a calculated yield locus from those measured.](image)

**Figure 5.** Schematics for the calculation of (a) the root mean square error \( \delta_{\alpha} \) of a calculated yield locus from the measured work contour and (b) the root mean square error \( \delta_{\beta} \) of the calculated directions of the plastic strain rates based on the normality flow rule for a calculated yield locus from those measured.

![Measured stress points forming plastic work contours (a)(c) and the directions of the plastic strain rates (b)(d) compared with those calculated using the selected yield functions.](image)

**Figure 6.** Measured stress points forming plastic work contours (a)(c) and the directions of the plastic strain rates (b)(d) compared with those calculated using the selected yield functions.
is the direction of the plastic strain rates calculated using the yield function and the associated flow rule for the $j$th stress path. Real-coded genetic algorithm was used to minimize the cost function and to avoid a local optimum solution. The value of $w_{i,\sigma}$ (for $\sigma_z : \sigma_y = 1:0$) was set to 100 so that the equivalent stress calculated using the yield function should coincide with the flow stress calculated using the strain hardening function determined for the RD.

Figure 6 (a) and (b) compares the calculated yield loci and the directions of the plastic strain rates, respectively, based on the von Mises [14], Hill’s quadratic [15] and method I with the observed ones. The material parameters of the Hill ’48 yield function were determined using $r_1$ and $r_9$ shown in Table 1. Figure 6 (c) and (d) compares the theoretical yield loci and the directions of the plastic strain rates based on the methods II, III and IV with the observed ones. Table 3 shows the values of the material parameters and exponents of the Yld2000-2d determined using methods I to IV.

To quantitatively evaluate the difference between the measured work contours and the shapes of the theoretical yield loci and between the measured directions of the plastic strain rates and those predicted using the selected yield functions, the root mean square error $\delta_a$ and $\delta_\beta$ were calculated using following equations, see Figure 5:

$$
\delta_a = \sqrt{\frac{\sum (a_{j,c} - a_{j,M})^2}{N}}
$$

$$
\delta_\beta = \sqrt{\frac{\sum (\beta_{j,c} - \beta_{j,M})^2}{N}}
$$

Figure 7 compares the values of $\delta_a$ and $\delta_\beta$ evaluated for the methods I, II, III, and IV. It is concluded that the Yld2000-2d yield function determined using the method IV gave the closest agreement with the experimental data for both the shapes of the work contours and the directions of the plastic strain rates.

### Table 3. Material parameters of the Yld2000-2d yield function determined using the methods I to IV.

|     | $M$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ |
|-----|-----|-------|-------|-------|-------|-------|-------|-------|-------|
| I   | 6.59| 1.145 | 0.790 | 0.741 | 0.787 | 0.871 | 0.576 | 1.060 | 1.026 |
| II  | 5.63| 0.805 | 0.996 | 0.925 | 0.794 | 0.872 | 0.246 | 1.099 | 0.845 |
| III | 4.68| 0.968 | 0.976 | 0.767 | 0.762 | 0.870 | 0.403 | 0.978 | 0.759 |
| IV  | 4.74| 1.097 | 0.809 | 0.761 | 0.802 | 0.877 | 0.512 | 1.090 | 0.939 |

**Figure 7.** Root mean square errors $\delta_a$ and $\delta_\beta$ evaluated for the methods I – IV.
In this study the material test data used to determine the material parameters of the Yld2000-2d yield function were those obtained from uniaxial and equibiaxial tension tests. Taking into account additional data points obtained from pure or simple shear tests [16, 17, 18], i.e., \( \sigma_y : \sigma_y = 1:-1 \) and \( -1:1 \), or \( \sigma_{DD} : \sigma_{DD} = 1:-1 \), where \( \sigma_{DD} \) is the normal stress in the DD of the test sample, would enhance the accuracy of the material model. Polycrystal analysis [19, 20] could substitute these time consuming experiments in future.

5. Conclusions

The plastic deformation behavior of a commercial pure titanium sheet (JIS #1) was investigated for linear stress paths using cruciform specimens [7, 9]. The conclusions obtained in this study are as follows.

1. Plastic work contours were measured by applying linear stress paths to the test material for a reference plastic strain range of \( 0.002 \leq \varepsilon_0^p \leq 0.01 \). The test material exhibited significant differential hardening in the low strain level. The measured work contours normalized by \( \sigma_0 \) associated with each work contour showed shrinkage along the stress paths except for \( \sigma_y : \sigma_y = 4:1 \) and 2:1.

2. The directions of the plastic strain rates showed a tendency of increase with the increase of \( \varepsilon_0^p \) for the stress ratios in the range of \( 0^\circ \leq \varphi \leq 45^\circ \), i.e., from the uniaxial tension in the RD to the equibiaxial tension.

3. The material parameters of the Yld2000-2d yield function were determined by applying the real-coded genetic algorithm developed in [6] to Eq. (2). The Yld2000-2d yield function determined using the method IV gave the closest agreement with the experimental data for both the shapes of the work contours and the directions of the plastic strain rates.

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