Three-neutrino mixing: status and prospects

A Marrone¹, F Capozzi¹, E Lisi², D Montanino⁳, A Palazzo¹
¹ Dipartimento di Fisica and Sezione INFN di Bari, Via Amendola 173, 70126 Bari, Italy
² Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Via Orabona 4, 70126 Bari, Italy
³ Dipartimento di Matematica e Fisica and Sezione INFN di Lecce, Via Arnesano, 73100 Lecce, Italy
E-mail: antonio.marrone@ba.infn.it

Abstract. We discuss the present knowledge of the neutrino oscillation parameters. In a three-neutrino scenario, neutrino oscillations depend on six parameters, two squared mass differences ($\Delta m^2, \delta m^2$), three mixing angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and one phase $\delta$. While five out of these six parameters have been measured, the CP-violating phase $\delta$ remains unknown. Moreover, the octant of the mixing angle $\theta_{23}$ and the neutrino mass hierarchy are still undetermined. We update our previous analysis, by adding to the global fit the recent results of the antineutrino running of T2K, the first results of the NO$\nu$A experiment, the latest SuperKamiokande and IceCube atmospheric neutrino data.

1. Introduction
The three-neutrino mass-mixing framework can explain almost all $\nu$ oscillation data [1]. In this framework, the flavor eigenstates $\nu_\alpha (\alpha = e, \mu, \tau)$ are a superposition of the mass eigenstates $\nu_i (i = 1, 2, 3)$ through the three mixing angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and a CP-violating phase $\delta$. Neutrino oscillations depend on the neutrino masses $m_i$ through two quantities: $\delta m^2 = m_2^2 - m_1^2 > 0$ and $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$. While the sign of $\delta m^2$ is always positive, the sign of $\Delta m^2$ distinguishes two possible ordering of the mass eigenstates, the normal hierarchy (NH, $\Delta m^2 > 0$), and the inverted hierarchy (IH) in the opposite case [2]. Five out of the six above oscillation parameters have been measured by a number of experiments [3]. In particular, the two “solar” parameters ($\delta m^2, \theta_{12}$) have been measured by solar experiments in combination with KamLAND, $\theta_{13}$ by short-baseline experiments (SBL) and the “atmospheric” parameters ($\Delta m^2, \theta_{23}$) by atmospheric and long-baseline (LBL) experiments. However, it is still unknown if the mixing angle $\theta_{23}$ is close to maximal or not and which is eventually its octant. The precise value of the phase $\delta$ is also still unknown, even though very recent data begin to constrain its allowed range at 2-3 $\sigma$ level. Finally, present data, even in the global analysis, are only poorly sensitive to the hierarchy discrimination. Global analyses are a very useful tool to verify the consistency of all available oscillation data and can also, as in the past for the $\theta_{13}$ mixing angle, give some hints about parameters that are not well constrained by a single class of experiments. In this work we report about our present knowledge of the oscillation parameters, in view of the latest available experimental results. We will upgrade our previous analysis [4] by including in the global fit, besides the latest T2K [5] and NO$\nu$A [6], also the updated SuperKamiokande neutrino data [7] and the latest IceCube-Deep Core atmospheric neutrino results [8].
2. Methodology
Data samples are divided in different subsets and then combined in such a way to fully exploit parameter correlations. In particular, we divide our data in five subsets: LBL, KamLAND, solar, SBL, and atmospheric neutrino data. We include in our analysis of LBL data the experimental results of MINOS, T2K, NOvA and K2K and in the SBL analysis Daya Bay, RENO and Double Chooz data. Firstly, we combine solar and KamLAND data with LBL results. The solar parameters ($\delta m^2, \theta_{12}$) are well constrained by solar + KamLAND analysis, and can be essentially fixed at their best-fit values in the subsequent LBL analysis. The oscillation probability for LBL accelerator experiments depends mostly on the atmospheric parameters ($\Delta m^2, \theta_{23}$) in the $\nu_\mu \rightarrow \nu_\mu$ disappearance channel. It depends also on $\theta_{13}$ in the $\nu_\mu \rightarrow \nu_\tau$ appearance channel and, subdominantly, on the solar parameters and $\delta$. Therefore, we initially combine LBL, solar and KamLAND data. The best-fit value for $\theta_{13}$ from solar+KamLAND data ($\sin^2 \theta_{13} \sim 0.02$) is a bit larger than the one measured at SBL experiments. This weak preference for a non-zero $\theta_{13}$ is improved when adding LBL data, providing a significant measurement of $\theta_{13}$. However, the obtained best-fit value of $\theta_{13}$ is sensitive to the precise value of $\delta$ and $\theta_{23}$. Secondly, we add to our analysis the SBL results that provide a very precise value of $\theta_{13}$, independent on $\delta$ and $\theta_{23}$, and finally we add the SuperKamiokande results on atmospheric neutrinos. Atmospheric neutrino data are sensitive to the $\theta_{23}$ octant and also, although very weakly, to the phase $\delta$.

3. Single parameter analysis
In this section the results on each of the six oscillation parameter is shown, for the three cases discussed above. Blu solid curves refer to NH, while red dashed ones are for IH. In Figure 1, LBL, Solar and KamLAND data are combined. Since Solar+KamLAND data are practically insensitive to the hierarchy, the curves for NH and IH basically coincide for $\Delta m^2$ and $\theta_{12}$. Bounds on all the parameters are obtained with the exception of $\delta$. However, the bounds from LBL data on $\theta_{13}$ (dominated by T2K $\nu_\mu \rightarrow \nu_\tau$ channel and now corroborated by T2K disappearance and NOvA recent results) induce an intriguing preference for $\delta \sim 1.5\pi$. It is worth noticing that T2K and NOvA data require require the maximization of the appearance probability and hence

![Figure 1. Combined analysis of LBL, Solar, and KamLAND data. Solid blu lines refer to NH, red dashed lines to IH.](image1.png)

![Figure 2. As in Figure 1, but adding SBL data.](image2.png)
Figure 3. Global fit of all data (NOνA LID). Figure 4. Global fit of all data (NOνA LEM).

$\sin \delta \sim -1$, dominating in the fit over the MINOS preference for $\sin \delta > 0$. With regard to the octant of $\theta_{23}$, slightly non-maximal $\theta_{23}$ mixing is preferred by MINOS disappearance data. In Figure 2, SBL data are added to the fit. Consequently, the uncertainty on $\theta_{13}$ is strongly reduced and the preference for negative $\sin \delta < 0$ slightly increased, especially for IH. The octant of $\theta_{23}$ is

Figure 5. Results of the analysis in the plane $(\sin^2 \theta_{23}, \sin^2 \theta_{13})$. 
swapped for NH. Finally, by adding the atmospheric data (Figure 3), the preference for $\sin \delta > 0$ is reinforced, even though with a slight low best-fit value and a more pronounced preference for non-maximal $\theta_{23}$ is found. The CP-conserving cases $\delta = 0, \pi$ are still allowed from the global fit at about $3\sigma$ level. The latest T2K and NO$\nu$A data give bounds on $\delta$ consistent with the previous analysis, disfavouring $\delta \sim 0.5\pi$ at about $3\sigma$ in both hierarchies. However, the NO$\nu$A collaboration presented two independent data analysis, that they call LID (the one used in Fig. 1–3) and LEM. In Figure 4 the global fit results with the NO$\nu$A LEM analysis are shown. In this case the preference for $\sin \delta \sim 1$ is more pronounced and $\delta \sim 0.5\pi$ is disfavoured at more than $3\sigma$. At present, the global analysis does not give statistically significant information on the hierarchy, with the NH weakly preferred ($\Delta \chi^2 \sim 0.4$ (NO$\nu$A LID) or $\sim 2.2$ (NO$\nu$A LEM)).

4. Two parameter covariances

In this section some of the correlation between the oscillation parameters are shown. Figure 5 shows the allowed regions in the $(\theta_{23}, \theta_{13})$ plane. The three columns refer to increasingly reach data set, for NH (top) and IH (bottom). In the first column it can be seen that there is a weak anticorrelation between the two mixing angles, coming from the LBL appearance data, because the oscillation probability contains a term proportional to the product $\sin^2 \theta_{13} \sin^2 \theta_{23}$. The strong appearance signals in T2K, both in the neutrino and antineutrino channels, and in NO$\nu$A require relatively higher $\theta_{13}$ values, while Solar+KamLAND prefer $\sin^2 \theta_{13} \sim 0.02$. This is the reason why the best fit for $\theta_{23}$ is in the second octant, for relatively low $\sin^2 \theta_{13}$, for both hierarchies. In the second column, when SBL results are included, the $\sin^2 \theta_{13}$ best-fit point moves to 0.023, and for NH this causes the swap of the $\theta_{23}$ octant. The inclusion of the atmospheric data (third column) does not change the preferred $\theta_{23}$ octant. Figure 6 shows the allowed regions in the $(\theta_{13}, \delta)$ plane. In the first panel it is evident how the preference for $\delta \sim 1.5\pi$ originates form a compromise between the relatively low value of $\theta_{13}$ from Solar+KamLAND data and the higher value required by the appearance signals of T2K and NO$\nu$A. With respect to our previous global analysis [3, 4], the allowed regions are reduced, and there is a range around $\delta \sim 0.5\pi$ excluded at $\sim 3\sigma$ for IH. When SBL results are added the preference for $\delta \sim 1.5\pi$ remains and the covariance between the two parameters is strongly reduced. The inclusion of the atmospheric data does not significatively alter this trend but slightly moves the best-fit point.
Figure 7. Results of the analysis in the plane \((\sin^2 \theta_{13}, \delta)\) with LEM NO\(\nu\)A data.

of \(\delta\) to lower values. Figure 7 shows the same correlations of Figure 6, but with the NO\(\nu\)A LEM data. In this case the size of the \(\delta\) allowed regions is reduced in both hierarchies.

5. Conclusions
In this work we presented an update of the global analysis of the available oscillation neutrino data. We have updated the \(\nu\sigma\) bounds on the known oscillation parameters and discussed the current information about the phase \(\delta\) and the octant of \(\theta_{23}\). We find an intriguing preference for \(\delta \sim 1.5\pi\) and in general for \(\sin \delta < 0\). The value \(\delta = \pi/2\) is now disfavoured at 3\(\sigma\). The octant of \(\theta_{23}\) is currently unstable, depending on the hierarchy and on the different data sets in the fit. There is currently no clear indication in favour of one hierarchy, even though a weak preference for NH emerges in the case of the NO\(\nu\)A LEM analysis. For the determination of the hierarchy there are very interesting but also very challenging experimental projects [9, 10, 11, 12, 13], able to achieve this goal on a time scale of five to ten years.

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