LETTER TO THE EDITOR

Enhanced phase sensitivity and soliton formation in an integrated BEC interferometer

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Received 21 October 2004
Published 22 November 2004
Online at stacks.iop.org/JPhysB/37/L385
doi:10.1088/0953-4075/37/23/L02

Abstract
We study the dynamics of Bose–Einstein condensates in time-dependent microtraps for the purpose of understanding the influence of the mean field interaction on the performance of interferometers. We identify conditions where the nonlinearity due to atom interactions increases the sensitivity of interferometers to a phase shift. This feature is connected with the adiabatic generation of a dark soliton. We analyse the robustness of this phenomenon with respect to thermal fluctuations, due to excited near fields in an electromagnetic surface trap.

There is currently a large interest in employing Bose–Einstein condensates (BECs) of dilute atomic gases in the field of matter wave interferometry [1, 2]. The advantages of atom interferometers compared to optical ones are well known [1, 3]: greater precision due to the large atomic mass, sensitivity to vibrations, inertial, and gravitational forces, access to quantum decoherence and to atomic scattering properties, to quote a few. A promising route towards compact applications are integrated devices, using for example miniaturized optical elements built near nanostructured surfaces [4, 5].

Atom–atom interactions that play a crucial role in BECs are usually considered as a drawback for matter wave interferometers because they introduce additional phase shifts and, more fundamentally, quantum fluctuations and diffusion of the relative phase between the parts of a spatially separated BEC, see, e.g., [6–13]. In this paper, we show that an operation mode for a BEC interferometer exists where atom interactions actually enhance the phase sensitivity. We study, in particular, the temporal scheme proposed in [4, 14] where a trapped atom sample is split and recombined by slowly deforming the trapping potential. A similar setting also describes approximately the flow of a condensate through an interferometer with spatially split arms [15–17]. Interference is usually looked for in the lowest vibrational modes of the recombined trap, whose relative populations depend on a phase difference imprinted, e.g.,
during the split phase (see, however, [17] for a multi-mode interferometer with non-interacting atoms). We show here that when a condensate is recombined, a grey soliton is formed and starts to oscillate with an amplitude controlled by the interferometer operation. It is quite interesting that conversely, the interference phase can also be read out by measuring the amplitude of either the soliton oscillation or the condensate dipole mode. In both cases, one achieves a better phase sensitivity with detection schemes that are relatively simple compared to projective measurements in the vibrational mode basis. Finally, the enhanced phase sensitivity of the condensate interferometer is illustrated by numerical simulations where the output signal is scrambled due to a fluctuating, random potential. This models thermally excited magnetic fields that occur in surface-mounted microtraps, and our results allow us to identify the noise level that can at most be tolerated.

In our model for the BEC interferometer, we focus on the quasi one-dimensional regime typical for elongated traps, assume zero temperature and describe the condensate dynamics along the loosely bound axis by a one-dimensional order parameter $\Phi(x, t)$ that solves the nonlinear Schrödinger or Gross–Pitaevskii equation (GPE) [18]

$$i\hbar \frac{\partial}{\partial t} \Phi(x, t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) + g|\Phi(x, t)|^2\right] \Phi(x, t).$$  \hspace{1cm} (1)

The potential $V(x, t)$ in equation (1) is harmonic for $t \leq 0$ and $t \geq \tau$. Similar to the setup proposed in [4, 14], a barrier is adiabatically and symmetrically raised and lowered during the operation time $\tau$, splitting and recombining the condensate. A relative phase $\Theta$ is imprinted at time $\tau/2$, when the two wells are sufficiently separated (negligible tunnelling). In the following, we use harmonic oscillator units defined by the initial trap frequency $\Omega$ and adopt $u = (\hbar/(m\Omega))^{1/2}$ as a unit of length. We normalize the order parameter to unity and get a dimensionless effective interaction constant $g = 2N(a_s/u)(\Omega_{\perp}/\Omega)(1 - 1.4603a_s/u_{\perp})^{-1}$ [19], where $N$ is the number of atoms, $a_s$ is the three-dimensional scattering length, $\Omega_{\perp}$ is the frequency of the transverse (radial) confinement and $u_{\perp}$ the corresponding ground state size.

We assume a constant, tight radial confinement and neglect the coupling between radial and axial excitations. The numerical integration of equation (1) is done with the split-operator algorithm [20] whose results we have checked for convergence using the conservation of total energy and the symmetry under time reversal. A typical density distribution is shown in figure 1 (left) with a trap potential $V(x, t) = \frac{1}{2} [x^2 - d(t)^2] / (x^2 + d(t)^2)$, where the well separation is parametrized by $d(t) = 2a \sin^2(\pi t/\tau)$ for $0 \leq t \leq \tau$. The imprinted phase shift is close to $\pi$ so that in the ideal gas case, one expects an output state close to the first excited trap eigenstate. One clearly sees that a small deviation from this phase shift gives a grey soliton that oscillates in the harmonic trap.

A more quantitative characterization is usually based on the final populations $p_{0,1}$ of the lowest trap eigenstates ($k = 0, 1$)

$$p_k = |\langle \phi_k | \Phi(\tau) \rangle|^2 = \left| \int dx \phi_k^* (x) \Phi(x, \tau) \right|^2,$$  \hspace{1cm} (2)

where $|\Phi(\tau)\rangle$ is the order parameter at time $t = \tau$. We choose here the convention that $|\phi_{0,1}\rangle$ are the ground state and the first excited state in the harmonic trap, including the nonlinearity. We illustrate below that this improves the accuracy of a two-mode approximation (see figure 2). Two alternative quantities, that appear simpler to be measured in practice, are the condensate mean position after recombination and the centre of the density dip characteristic for the grey soliton

$$\langle x \rangle_t = \int dx |\Phi(x, t)|^2 x, \quad q_t = \min_{\text{local}} |\Phi(x, t)|^2 \quad \text{for} \quad t > \tau.$$  \hspace{1cm} (3)
Figure 1. Left: recombining the interferometer arms after a phase shift close to π produces a grey soliton oscillating in the harmonic well (dark zigzag lines), as well as dipole and breathing oscillations of the background condensate (bright). Parameters chosen in the numerical simulation: g = 10, Θ = 0.9π, operation time τ = 70, maximum splitting 4a = 8. Right: evolution of the weights |c_0(t)|^2 and |c_1(t)|^2 for the first two eigenstates of the GPE in the instantaneous trapping potential V(x, t) (top and centre) and of the weights of higher modes (bottom).

Figure 2. Left: maximum oscillation amplitudes of the mean position of the whole condensate and of the soliton position amplitude, as a function of the phase shift Θ. For phase shifts very different from Θ = π, the grey soliton reaches the condensate border in the recombination stage and escapes; for these values, no results are plotted. Mean position amplitude (×6 for clarity): g = 0 (dash-dotted line), g = 5 (thin dashed line with triangles), g = 10 (thin solid line with squares). Soliton amplitude: g = 5 (thick dashed line with dots), g = 10 (thick solid line with stars). Right: final population p_0 of the trap ground state, computed by solving the full nonlinear Schrödinger equation (thin solid line with circles, g = 10), the two-mode model (solid line, g = 10) of equation (5), and the ideal Bose gas (dashed line, g = 0).

These quantities show oscillations whose amplitude depends on the imprinted phase shift Θ, as shown in figure 2 (left). The key observation is that around a complete swapping to the odd output state, the curves are much narrower compared to the non-interacting case where the dipole mode oscillation amplitude is |sin(Θ)|/√2. In this sense the nonlinearity increases the phase sensitivity of the interferometer.

4 We have checked that the soliton oscillation agrees with the effective equation of motion of [21] in the Thomas–Fermi limit, i.e. for interactions g ≫ 1. The difference with respect to [22] may be only apparent and related to the different scaling convention for the dimensionless parameters.
The populations $p_{0,1}$ show a similar behaviour which can be understood using a two-mode model similar to those studied in [13, 15, 16]. We consider an expansion of the order parameter

$$
\Phi(x, t) = \sum_k c_k(t) \phi_k(x; d(t)),
$$

where $\phi_k(x; d(t))$ are (real) eigenstates of the GPE in the double-well potential with $d(t)$ and $g$ fixed. In the adiabatic approximation and restricting the expansion to the lowest even and odd modes $k = 0, 1$, the equations for the coefficients are

$$
i \dot{c}_k = \mu_k c_k + g(2O_{01} - O_{kk})|c_{1-k}|^2 c_k + gO_{01} c_{1-k}^* c_k^*,
$$

where $\mu_k$ are the chemical potentials of the eigenstates. Due to parity conservation, the only nonzero interaction matrix elements are

$$O_{kl} = \int dx [\phi_k(x; d)\phi_l(x; d)]^2.
$$

Starting from the ground state $\phi_0(x; d(0))$, adiabatic evolution and phase imprint lead to the amplitudes $c_0(\tau/2 + 0) = \cos(\theta/2) c_0(\tau/2 - 0)$ and $c_1(\tau/2 + 0) = i \sin(\theta/2) c_0(\tau/2 - 0)$ up to small corrections due to tunnelling between the wells. With an exact $\pi$ phase shift, $c_0(t) \equiv 0$ for $t > \tau/2$ as in the non-interacting case. But for $\theta \neq \pi$, both modes are nonlinearly coupled. Solving equations (5) numerically, we obtain the results of figure 2 (right) for the final population $p_0$. Comparing to the full solution of the GPE, both curves show almost the same behaviour in the close vicinity of $\theta = \pi$, and agree qualitatively for other phase shifts: they both present a maximum and a minimum. This differs from the results of [16], probably because there the eigenstates of the linear Schrödinger equation were used for the expansion. Moreover, a more accurate agreement could not be expected because the two-mode approximation is valid only when the low energy states in the separated wells are weakly modified by the many-body interactions. Estimates available in the literature, e.g. [6], show that the two-mode approximation is limited to an interaction strength $g \ll 2^{3/2}\sqrt{\pi} \approx 2.98$, given our trap potential.

In figure 1 (right), we compare the evolution of the weights $|c_{0,1}(t)|^2$ between the full numerical solution and the two-mode approximation for a phase shift close to $\pi$. Again, the approximation reproduces the general features, with quantitative differences in the amplitudes and the frequencies of the oscillations. The curves also illustrate that for superpositions of instantaneous eigenstates, the adiabatic approximation fails due to the nonlinearity: the state after the phase imprint—it is almost identical to the lowest antisymmetric state—turns during the evolution into a state with a strong ground state admixture. This is due to the presence of an instability in the system. More precisely, the excitation spectrum of the antisymmetric state contains a soft mode [16, 23]. Solving the Bogoliubov–de Gennes equations, we have found that the first eigenmode acquires a purely imaginary frequency if the splitting $d$ between the wells exceeds some critical value that becomes smaller with increasing interactions. The imaginary excitation frequency leads to an exponential growth of the ground state, and this instability is rooted in the Josephson effect [24, 25]: putting a relative phase different from $\pi$ on the two arms of the interferometer, one introduces a tunnel current across the barrier that drives a population imbalance between the two wells. The interactions in each well subsequently enhance the phase difference, leading to a runaway effect for the antisymmetric state. The instability is weaker at large splitting because tunnelling becomes exponentially suppressed.

5 We were unable to reproduce quantitatively the imaginary eigenfrequency plotted in [16]. This may be related to the different ansatz for the perturbed order parameter used there.
This is consistent with the behaviour of the growth rate found from the Bogoliubov analysis (see also [16]). For the chosen parameters the timescale of the process is actually dominated by the merging of the two wells that increases Josephson critical current. As illustrated in figure 1 (right), the main population transfer occurs around the moment when the barrier between the two wells drops below the chemical potential.

The previous results suggest that in a realistic setting, the condensate interferometer is very sensitive to fluctuations of the trapping potentials. It has also become clear recently that electromagnetic field fluctuations are particularly relevant for integrated atom optics in the neighbourhood of material microstructures held at a finite temperature (see [5, 26, 27] and references therein). We have simulated these fluctuations in terms of a random potential that fluctuates in time and space with spectral characteristics similar to those of thermal magnetic near fields: white noise and a Lorentzian spatial correlation function [28]. We focus on the case that the correlation length is smaller than the distance between wells. This is realistic for microtraps whose splitting is larger than the distance to the underlying surface. Figure 3 shows the impact of increasing the noise strength for the interferometer output, averaged over a statistical sample of realizations. The steep structures around a $\pi$ phase shift are smoothed, as expected. It is surprising that this happens already for very weak noise: if the scattering rate off the noise potential is denoted $\gamma$, significant changes are seen for $\gamma \tau \sim 0.07$ already. The instability in the double-well potential thus makes the condensate more sensitive to thermal near fields. This differs from a non-split condensate in a single well, which is more robust in the presence of noise compared to an ideal gas, as shown in [29].

Throughout this paper, we have used the mean field description based on the Gross–Pitaevskii equation and neglected phase fluctuations. At fixed $g$, this can be justified for a sufficiently large number of atoms. The phase fluctuations in the initial condensate are still small even at finite temperatures $0 < T < T_g \approx N/(\Omega_{1//\Omega} + (3g/(4\sqrt{2}))^{2/3})$ (in our units) [11], using the regime of tight radial confinement. Similarly, the phase coherence of the split condensate [8, 13] is maintained for operation times $\tau < \tau_{\text{diff}} \approx (2\sqrt{3}/g)^{2/3}N^{1/2}$, following the procedure explained in [9].
Acknowledgments

A Negretti acknowledges the financial support of the European Union’s Human Potential Programme under contract HPRN-CT-2002-00304 (FASTNet). A Negretti thanks C Menotti, L P Pitaevskii, and M Wilkens for stimulating discussions, the Institut für Physik in Potsdam for the friendly hospitality, and the ECT* for giving the opportunity to use its facilities. We thank Th Busch and N P Proukakis for helpful suggestions to the manuscript. This work has been supported by the European Union’s IST Programme under contract IST-2001-38863 (ACQP).

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