Non-Abelian Gauge Symmetry and the Higgs Mechanism in F-Theory

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Abstract: Singular fiber resolution does not describe the spontaneous breaking of gauge symmetry in F-theory, as the corresponding branch of the moduli space does not exist in the theory. Accordingly, even non-abelian gauge theories have not been fully understood in global F-theory compactifications. We present a systematic discussion of using singularity deformation, which does describe the spontaneous breaking of gauge symmetry in F-theory, to study non-abelian gauge symmetry. Since this branch of the moduli space also exists in the defining M-theory compactification, it provides the only known description of gauge theory states that exists in both pictures; they are string junctions in F-theory. We discuss how global deformations give rise to local deformations, and also give examples where local deformation can be utilized even in models where a global deformation does not exist. Utilizing deformations, we study a number of new examples, including non-perturbative descriptions of $SU(3)$ and $SU(2)$ gauge theories on seven-branes which do not admit a weakly coupled type IIb description. It may be of phenomenological interest that these non-perturbative descriptions do not exist for higher rank $SU(N)$ theories.

1. Introduction

Much has been learned about the landscape of string vacua over the last fifteen years. On one hand, in weakly coupled corners of the landscape there are scenarios for controlled moduli stabilization that are giving rise to increasingly realistic global compactifications, and in many cases provide inspiration for new models of particle physics or early universe cosmology. On the other hand, there has been much formal progress in understanding the physics of compactifications at small volume or strong coupling. We have gained both a clearer view of well-known regions, and also a glimpse of relatively unexplored vistas.

The strongest example of the latter may be in compactifications of F-theory [1], which has been the subject of much study in recent years. In addition to enjoying [2,3]
certain model-building advantages for grand unification over their heterotic and type II string counterparts, F-theory compactifications provide a broad view of the landscape; much of their strongly coupled physics can be understood in terms of the geometry of elliptically fibered Calabi–Yau varieties, which can be explicitly constructed and studied as a function of their moduli. Recent works have made significant progress in understanding the physics of F-theory compactifications, including globally consistent models [4–19], $U(1)$ symmetries [20–27,27–33], instanton corrections [12,22,34–41], the physics of codimension two and three singularities [42–50], and chirality inducing $G_4$-flux [22,41,46,51–63]; there has also been progress in understanding the landscape of six-dimensional F-theory compactifications [24,64–67].

In this paper we study non-abelian gauge symmetry as it exists in global F-theory compactifications. There, non-abelian sectors (not arising from D3 branes) require a singular compactification geometry $X$. Lacking both the tools to deal directly with the singular geometry and also a fundamental quantization of M-theory, which defines any F-theory compactification, one must resort to studying the theory on a related smooth manifold $\tilde{X}$ and then inferring the physics as one takes the singular limit $\tilde{X} \to X$. Since a singular geometry is necessary for a non-abelian gauge sector and $\tilde{X}$ is smooth, the movement in moduli space $X \to \tilde{X}$ must describe spontaneous symmetry breaking via the Higgs mechanism. The Lie algebraic data of particle states in the broken gauge theory is encoded in the geometry of $\tilde{X}$.

As $X$ is a Calabi–Yau variety, the smoothing processes necessarily correspond to movement in the Kähler or complex structure moduli spaces of $X$. A common technique is to study F-theory via M-theory on a related singular variety $X_M$, where $X_M \to X$ is the limit of vanishing elliptic fiber, and then to blow-up $X_M$. This branch of the moduli space is often referred to as the M-theory Coulomb branch (which is a bit of a misnomer), but it does not exist in F-theory on $X$; as such, it does not describe the spontaneous breaking of any non-abelian gauge symmetry in F-theory. Instead, the breaking of gauge symmetry in F-theory is accomplished by singularity deformation via movement in complex structure. This description of spontaneous symmetry breaking exists both for M-theory on $X_M$ and F-theory on $X$.

This paper is organized as follows. First we will review F-theory, as well as the drawbacks of resolution and the advantages of deformation as a technique for its study. In Sect. 2 we present a simple example that demonstrates how the Higgs mechanism operates in global F-theory compactifications. In Sect. 3, we present a general discussion of complex structure deformations useful for studying F-theory, which applies even when the singularities cannot be globally Higgsed, as in the case of non-Higgsable clusters. In Sect. 4, we present new realizations of $\mathfrak{su}(3)$ and $\mathfrak{su}(2)$ gauge algebras on stacks of four and three seven branes; interestingly, these descriptions do not exist in the type IIb limit or for higher rank $\mathfrak{su}(N)$ algebras. Links to codes used to perform the computations in this paper can be found in Appendix A.

**F-theory and its defining M-theory compactification.**

In general, an F-theory compactification to $d$ dimensions on an elliptically fibered Calabi–Yau variety $X$ is defined to be the vanishing fiber limit of an M-theory compactification to $d−1$ dimensions on an elliptically fibered Calabi–Yau variety $X_M$; for us this will be a Weierstrass model. In taking this limit $X_M \to X$, one of the dimensions decompactifies (via a fiberwise T-duality), yielding a $d$-dimensional theory. This

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1 Actually, only a genus-one fibration is required, as explored recently in [68].