ABSTRACT

Chiral symmetry and heavy quark symmetry constrain the interactions of mesons containing a single heavy quark with low-momentum pions. Chiral corrections to the Isgur-Wise function for $\bar{B}_s \to D_s^{(*)}$ versus $\bar{B} \to D^{(*)}$ semileptonic decay, the ratio of heavy meson decay constants $f_{D_s}/f_D$ or $f_{B_s}/f_B$, the amplitudes for $B_s - \bar{B}_s$ versus $B^0 - \bar{B}^0$ mixing, and the heavy meson mass splittings are calculated in chiral perturbation theory.

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It is at present impossible to calculate the properties of hadrons directly from QCD. It is possible, however, to obtain model-independent results by exploiting two global approximate symmetries of QCD. The first symmetry, chiral symmetry, becomes manifest in the limit of vanishing quark masses, \( m_q \to 0 \). This limit of QCD is applicable for the three lightest quarks \( u, d, \) and \( s \), and leads to the well-known \( SU(3) \) flavor symmetry amongst the light quarks. The second symmetry, heavy quark symmetry, is relatively new\(^1\). This global approximate symmetry corresponds to the limit of infinite quark masses, \( m_Q \to \infty \), and is manifest only if one constructs an effective field theory, the Heavy Quark Effective Theory (HQET). Heavy quark symmetry is a \( SU(2N_h) \) spin-flavor symmetry amongst \( N_h \) spin-1/2 heavy quarks with the same velocity in the HQET. This formulation of QCD is relevant for the charm and bottom quarks.

The heavy quark symmetry limit can be understood as follows. In a meson or baryon containing a single heavy quark \( Q \), the light degrees of freedom of the hadron (everything in the hadron except \( Q \)) is in the same non-perturbative state independent of the heavy quark mass \( m_Q \) and spin \( S_Q \)\(^2\). In other words, the heavy quark acts as a static color source, with its spin decoupled from the rest of the degrees of freedom of the hadron. Thus, in the heavy quark symmetry limit, the light degrees of freedom of the \( B^*(\ast) \) meson are the same as for a \( D^{(\ast)} \) meson because these mesons are related by spin-flavor symmetry transformations on the heavy quark. Note that one cannot calculate anything about the actual state of the light degrees of freedom; one only knows that it is identical for mesons related by heavy quark symmetry.

Both of these two global approximate symmetries have recently been employed in the description of the low energy interactions of hadrons containing a single heavy quark with pions\(^3\),\(^4\). To describe the pion interactions of heavy quark mesons, one must construct a chiral Lagrangian which respects heavy quark symmetry. Consider the pseudoscalar and vector mesons \( P_Q \) and \( P_Q^* \) containing heavy quark \( Q \). For a charm quark, these are the \( D, D^* \), while for a bottom quark, these are the \( B, B^* \). The hyperfine mass splitting \( \Delta = P_Q^* - P_Q \sim 1/m_Q \) is a \( 1/m_Q \) effect and is small. (For the \( D \) mesons, \( \Delta \sim 140 \) MeV, while for the \( B \) mesons, \( \Delta \sim 45 \) MeV.) Thus, in the heavy quark limit, \( P_Q \) and \( P_Q^* \) are degenerate and should be included in the chiral Lagrangian as a single field\(^5\),\(^6\),

\[
H_a^{(Q)} = \frac{(1 + \not{v})}{2} \left[ P_a^{\ast(Q)} \gamma^\mu - P_a^{(Q)} \gamma_5 \right].
\]

This heavy meson field transforms as a \( 3 \) under \( SU(3) \) light quark flavor, a \( 2 \) under \( SU(2) \) heavy quark spin, and a \( 2 \) under \( SU(2) \) heavy quark flavor. The lowest order chiral Lagrangian is given by

\[
\mathcal{L}_v = -i \text{Tr} \overline{H}_v (v \cdot D) H_v + 2g \text{Tr} \overline{H}_v H_v (S_{\ell v} \cdot A)
\]
where \( A^\mu = \partial^\mu \pi/f + \ldots \) and the spin operator for the light degrees of freedom \( S^\mu \) is defined by \( \text{Tr} \bar{H}_v H_v \gamma^\mu \gamma_5 = 2 \text{Tr} \bar{H}_v H_v S^\mu \). Note that a coupling to the heavy quark spin \( \text{Tr} \bar{H}_v A \gamma_5 H_v = 2 \text{Tr} \bar{H}_v (S_Qv \cdot A) H_v \) is forbidden by heavy quark symmetry. Thus, the above Lagrangian depends on only a single coupling constant \( g \). Heavy quark and chiral symmetry imply that the \( DD^*\pi \) and \( D^*D^*\pi \) (and the \( B^{(*)} \)) couplings are related and are all given in terms of \( g \).

The decay widths for these decay modes \( D^* \to D\pi, D^* \to D\gamma, \) and \( B^* \to B\gamma \) are calculable in terms of the coupling \( g \). Because \( \Delta \) is small, these are the dominant decay modes of \( D^* \) and \( B^* \). Note that because \( B^* \to B\pi \) (since \( m_\pi > \Delta \)), the \( B^*B\pi \) coupling cannot be measured directly. The coupling is given in terms of \( g \), however, since the \( B^*B\pi \) coupling equals the \( D^*D\pi \) coupling by heavy quark symmetry. The coupling \( g \) is constrained by experimental data \(7,8) \) on \( D^* \to D\pi \) and \( D^* \to D\gamma \), \( g^2 \lesssim 0.5^9) \). This bound is significantly lower than quark model predictions for \( g \).

In the heavy quark limit, \( \bar{B} \to D^{(*)} \) semileptonic decay is described by a single form factor, the Isgur-Wise function \( \xi(v \cdot v') \),

\[
\langle D(v') | \bar{c} \gamma_\mu b | \bar{B}(v) \rangle = \sqrt{m_D m_B} \xi(v \cdot v')(v + v')_\mu \\
\langle D^*(v', \epsilon) | \bar{c} \gamma_\mu \gamma_5 b | \bar{B}(v) \rangle = \sqrt{m_D m_B} \xi(v \cdot v')[(1 + v \cdot v') \epsilon_\mu - (\epsilon^* \cdot v)v'_\mu] \\
\langle D^*(v', \epsilon) | \bar{c} \gamma_\mu b | \bar{B}(v) \rangle = \sqrt{m_D m_B} \xi(v \cdot v') i\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v'^{*\alpha} v^\beta.
\]

The Isgur-Wise function \( \xi(v \cdot v') \) describes the overlap of the light degrees of freedom for mesons with velocity \( v \) and \( v' \). The momentum transfer to leptons in the semileptonic decay is given by \( q^\mu = m_B v^\mu - m_D v' \). At zero recoil where \( v = v' \) and the momentum transfer squared to leptons is at a maximum, \( q^2_{\text{max}} = (m_D - m_B)^2 \), the overlap of the light degrees of freedom is complete and the Isgur-Wise function is normalized, \( \xi(1) = 1 \). Because semileptonic \( \bar{B} \to D^{(*)} \) decay measures the matrix elements \( (3) \) times \( V_{cb} \), the normalization of the Isgur-Wise function at zero recoil implies that \( V_{cb} \) can be extracted at zero recoil. This procedure results in the best determination of \( V_{cb} \) to date \(10) \). Thus, it is important to investigate how chiral corrections affect the extraction of the Isgur-Wise function.

In the \( SU(3) \) limit, the Isgur-Wise function is independent of light quark flavor. \( SU(3) \)-violating pion loop corrections introduce light quark flavor dependence \(11) \). A one-loop calculation \( (m_s \neq 0) \) implies that the Isgur-Wise functions for \( \bar{B}_s \to D^{(*)}_s \) and \( \bar{B}_{u,d} \to D^{(*)}_{u,d} \) semileptonic decay are related as follows,

\[
\frac{\xi_s(v \cdot v')}{\xi_{u,d}(v \cdot v')} = 1 + \zeta(v \cdot v') \frac{M_K^2}{16\pi^2 f^2} \ln \left( \frac{M_K^2}{\mu^2} \right) \\
\text{where } f = 135 \text{ MeV and}
\]

\[
\zeta(v \cdot v') = \frac{30}{9} g^2 [r(v \cdot v') - 1] \\
r(v \cdot v') = \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \ln \left( v \cdot v' + \sqrt{(v \cdot v')^2 - 1} \right).
\]
The diagram responsible for this correction is depicted in fig. 1. Note that the chiral correction depends on \((v \cdot v')\). Because the heavy meson velocity changes due to the weak current, the full velocity-dependent field formulation\(^{12}\) of the heavy meson chiral Lagrangian is needed for this computation. A non-relativistic formulation of the heavy meson chiral Lagrangian does not suffice. At zero recoil, \(\zeta(1) = 0\) and the chiral correction vanishes. Thus, the normalization of \(\xi(1)\) is preserved as required by heavy quark symmetry. The chiral correction \(\Box\) is plotted as a function of \((v \cdot v')\) in fig. 2 for \(g^2 = 0.35\) and \(\mu = 1\) GeV. The correction is small; the largest correction is \(\approx 2.5\%\) at the kinematic point \(q^2 = 0\) or \((v \cdot v')_{\text{max}} = (m_B^2 + m_D^2)/2m_B m_D\). The size of the chiral correction is smaller than the anticipated size of \(1/m_c\) heavy quark symmetry-violating corrections. Given the present neglect of \(1/m_Q\) corrections in the extraction of \(V_{cb}\), \(SU(3)\) violation also can be neglected since it is an even smaller effect. Isospin-breaking in the Isgur-Wise function is expected to be even smaller than \(SU(3)\) violation.

The meson decay constants \(f_D\) and \(f_{D_s}\) parametrize the amplitudes for the leptonic decays \(D_s^+ \to \mu^+ \nu_\mu\) and \(D^+ \to \mu^+ \nu_\mu\),

\[
\begin{align*}
\langle 0 | \bar{d} \gamma_m \gamma_5 c | D^+(v) \rangle &= i f_D m_D v_\mu \\
\langle 0 | \bar{s} \gamma_m \gamma_5 c | D_s(v) \rangle &= i f_{D_s} m_{D_s} v_\mu.
\end{align*}
\]

A chiral loop correction to the meson decay constants\(^{13}\) can be calculated by mapping the quark current onto the meson field current with the correct chiral and heavy quark symmetry transformation properties, \(\bar{\pi}_a \gamma^\mu (1 - \gamma_5) Q \to \frac{i}{2} f_D \sqrt{m_D} \text{ Tr} \left( \gamma^\mu (1 - \gamma_5) \left( H^{(Q)}(\xi^\dagger) \right)_a \right)\). The diagrams for the one-loop calculation are displayed in Ref. 13. For \(m_s \neq 0\), the chiral correction yields

\[
\frac{f_{D_s}}{f_D} = 1 - \frac{5}{6} \left( 1 + 3g^2 \right) \frac{M_K^2}{16 \pi^2 f_K^2} \ln \left( \frac{M_K^2}{\mu^2} \right),
\]

which implies \(f_{D_s}/f_D \approx 1.1\) for \(g^2 = 0.4\).

The amplitudes for \(B^0 - \bar{B}^0\) and \(B_s - \bar{B}_s\) mixing can also be related using chiral perturbation theory\(^{13}\). The mixing amplitudes are conventionally parametrized by the meson decay constants and \(B\) parameters,

\[
\begin{align*}
\frac{8}{3} f_B^2 m_B^2 B_B &= \langle \bar{B}(v) | \bar{b} \gamma_m (1 - \gamma_5) d \bar{b} \gamma^\mu (1 - \gamma_5) d | B(v) \rangle \\
\frac{8}{3} f_{B_s}^2 m_{B_s}^2 B_{B_s} &= \langle \bar{B}_s(v) | \bar{b} \gamma_m (1 - \gamma_5) d \bar{b} \gamma^\mu (1 - \gamma_5) d | B_s(v) \rangle.
\end{align*}
\]

The four-quark operators map onto the chiral Lagrangian operator

\[
\bar{b} \gamma_m (1 - \gamma_5) q^a \bar{b} \gamma^\mu (1 - \gamma_5) q^a \to \text{ Tr} \left( \left( \xi H^{(b)} \right)^a \gamma_m (1 - \gamma_5) \right) \text{ Tr} \left( \left( \xi H^{(b)} \right)^a \gamma^\mu (1 - \gamma_5) \right).
\]
Pion-loop graphs which involve just one of the above traces correspond to renormalization of the meson decay constant considered earlier. Thus, it is possible to separately compute the chiral loop renormalization of the $B$ parameter. The calculation yields

$$\frac{B_{B_s}}{B_B} = 1 - \frac{2}{3}(1 - 3g^2) \frac{M_K^2}{16\pi^2f_K^2} \ln \left( \frac{M_K^2}{\mu^2} \right).$$  \hspace{1cm} \text{(11)}$$

Eq. (8) and (11) imply

$$\frac{B_{B_s}f_{B_s}^2}{B_Bf_B^2} = 1 - \left( \frac{7}{3} + 3g^2 \right) \frac{M_K^2}{16\pi^2f_K^2} \ln \left( \frac{M_K^2}{\mu^2} \right),$$  \hspace{1cm} \text{(12)}$$

which $\approx 1.3$ for $g^2 = 0.4$.

Finally, chiral corrections to the heavy meson mass splittings can be calculated\textsuperscript{14),15),16),17). Unfortunately, the one-loop computation involves more unknown parameters than masses, so there are few definite predictions.
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