Enhancement of non-equilibrium thermal quantum discord and entanglement of a three-spin XX chain by multi-spin interaction and external magnetic field

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Abstract. We investigate the non-equilibrium thermal quantum discord and entanglement of a three-spin chain whose two end spins are respectively coupled to two thermal reservoirs at different temperatures. In the three-spin chain, besides the XX-type nearest-neighbor two-spin interaction, a multi-spin interaction is also considered and a homogenous magnetic field is applied to each spin. We show that the extreme steady-state quantum discord and entanglement of the two end spins can always be created by holding both a large magnetic field and a strong multi-spin interaction. The results are explained by the thermal excitation depression due to switching a large energy gap between the ground state and the first excited state. The present investigation may provide a useful approach to control coupling between a quantum system and its environment.

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1. Introduction

Quantum entanglement was once considered as a unique resource that can be used in quantum information processing [1]. However, recent researches have shown that besides entanglement a composite quantum system may have other kinds of nonclassical correlations which can appear even in separable states [2, 3, 4, 5, 6]. In order to quantitatively describe such quantum correlations in a composite quantum system, many different measures have been proposed [6, 7, 8, 9]. Among them, quantum discord (QD), firstly introduced by Ollivier and Zurk [6], has received considerable attention [10, 11, 12, 13, 14, 15, 16]. Using an optical architecture, Lanyon et al. [4] experimentally showed that even fully separable states with quantum discord can be used to construct quantum computer.

In all real situations, quantum systems can not be completely isolated from their environments. Coupling of a quantum system to its surrounding unavoidably results in the destruction of quantum correlations of the system. The effect of environments to entanglement of bipartite quantum systems have intensively been investigated. It has been shown that entanglement undergoes sudden death due to the interaction of quantum systems with their reservoirs [17]. In recent years, the QD dynamics of open quantum systems has also attracted much interest in both theory and experiment [18, 19, 20, 21, 22, 23, 24, 25, 26, 27]. Werlang et al. [18] investigated the dynamics of both entanglement and QD in the Markovian environments, and showed that QD is more robust against decoherence than entanglement. Recently, an interesting dynamical feature of QD, named sudden transition, has been observed [21, 22]. It means that for certain initial states QD undergoes sudden change between a “classical decoherence” phase and a “quantum decoherence” phase [20, 21]. This sudden transition behavior can be explained in a geometrical way and has connection with the property of environment [20, 21]. Xu et al. [25, 26] experimentally investigated both the Markovian and non-Markovian dynamics of classical and quantum correlations and observed the sudden transition behavior of QD.

Apart from the situation in which a quantum system is coupled to a single environment, it may also be possible that a quantum system is simultaneously in contact with two different thermal baths. In semiconductor quantum dots nuclear spins and electronic spins consist of a composite quantum system for quantum information processing and quantum computing but the coupling manners of nuclear spins and electron spins to their surroundings are much different [28, 29, 30]. With the help of NMR and quantum optical techniques, one can create two reservoirs at different effective temperatures for nuclear spins and electron spins in quantum dots [31, 32]. For superconductor qubits, the two-different-thermal-bath coupling situation may directly be designed [33]. When interacting with two reservoirs at different temperatures, a quantum system may approach a steady state instead of a thermal equilibrium state. Thus, in general, the presence of heat/energy/mass currents passing through the quantum system in a steady-state may modify the quantum correlations.
In recent years, quantum correlations of coupled qubits in contact with two different thermal environments, i.e., the non-equilibrium thermal environment model, have received some attention [34, 35, 36, 37, 38]. Quiroga et al. [36] proposed a two-interacting spin-1/2 system in contact with two heat reservoirs at different temperature, and identified a nonequilibrium enhancement-suppression transition behavior of entanglement due to the presence of temperature gradient. Employing the same model, Sinaysky et al. [37] found that the spin system can converge to steady state and studied the dependence of the steady-state concurrence on the mean temperature and temperature difference of the reservoirs and the energy splits of the spins. Huang et al. [38] investigated the nonequilibrium thermal steady-state entanglement in a three spin-1/2 XX chain in contact with two heat reservoirs at different temperature and found that the temperature difference of the heat baths benefits the entanglement in the nonsymmetric coupling case. Spin chain models in contact with two reservoirs at two different temperatures have also been employed for studying heat current transfer [39, 40, 41]. Yan et al. [39] considered an interacting spin-1/2 chain connected to two phonon baths held at different temperatures and showed that heat transport through the spins systems can be controlled by applying an inhomogeneous magnetic field due to switching an energy gap.

Stimulated by the previous investigations, we here study how to control the steady-state QD and entanglement of spin systems. We consider a three-spin-1/2 chain in which besides the XX-type nearest-neighbor two-spin interaction a three-spin interaction is included and an external magnetic field is homogeneously applied to each spin, and meanwhile the two end spins are coupled to two thermal environments at different temperatures. We show that the coupling of the spin system to the thermal reservoirs can be controlled and the thermal excitation can be greatly depressed by the three-spin interaction and the magnetic field. As a result, the extreme steady-state QD and entanglement in the two end spins can be created.

This paper is organized as follows. In Sec. II, we describe the model and introduce the calculation method. In Sec. III, definitions on quantum discord and concurrence are briefly reviewed. In Sec. IV, numerical results, discussion and physical explanations are presented. Finally, a summary is given in Sec. V.

2. Model And Master Equation

The model under investigation is described in Fig. 1. We consider a three-spin-1/2 chain which Hamiltonian reads

\[
H_S = J \sum_{i=1}^{2} \left( \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y \right) + h \sum_{i=1}^{3} \sigma_i^z + k \left( \sigma_1^z \sigma_2^z \sigma_3^z + \sigma_1^y \sigma_2^y \sigma_3^y \right),
\]

where \( \sigma_i^\alpha (\alpha = x, y, z) \) are the Pauli matrices for the ith spin, \( J \) is the coupling constant between the nearest-neighbor spins, and \( h \) is the external magnetic field strength, homogeneously applied to each spin. Besides the two-spin interaction, the three-spin interaction [42, 43, 44] is also included, which strength is denoted by \( k \).
Figure 1. A schematic representation of a three-spin chain coupled to two thermal baths at different temperatures, $T_1$ and $T_3$.

As shown in Fig. 1, two end spins 1 and 3 are in contact with two phonon baths at different temperatures, $T_1$ and $T_3$, respectively. In the interaction picture, the Hamiltonian describing the interaction between the $j$th spin and its phonon bath is given by

$$H_{SBj} = \sigma_j^x \left( \sum_n g_j^{(n)} e^{-i\omega_n j t} b_{nj} + g_j^{(n)*} e^{i\omega_n j t} b_{nj}^\dagger \right) \equiv \sigma_j^x \otimes B_j, (j = 1, 3), \quad (2)$$

where $b_{nj}^\dagger (b_{nj})$ is the creation (annihilation) operator for the $n$th mode of thermal bath $j$, and $g_j^{(n)}$ is the coupling constant between the $j$th spin and the $n$th bath mode.

Let us first consider the eigenvalue problem of the Hamiltonian (1)

$$H_S |\phi_l\rangle = \varepsilon_l |\phi_l\rangle, (l = 1, 2, 3, \ldots, 8). \quad (3)$$

The spin-up and spin-down states of spin $i$ are represented by state-vectors $|1\rangle_i$ and $|0\rangle_i$, respectively. In the presentation spanned by the uncoupled basis $|n_1 n_2 n_3\rangle = |n_1\rangle_1 \otimes |n_2\rangle_2 \otimes |n_3\rangle_3$ with $n_i = 0, 1$, we can easily work out the eigenstates of Eq. (3) as follows

$$|\phi_1\rangle = |000\rangle, \quad (4)$$

$$|\phi_2\rangle = |111\rangle, \quad (5)$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}} (- |110\rangle + |011\rangle), \quad (6)$$

$$|\phi_4\rangle = \frac{1}{\sqrt{2}} (- |100\rangle + |001\rangle), \quad (7)$$

$$|\phi_5\rangle = \frac{1}{\sqrt{2}} \sin \alpha_1 |100\rangle + \cos \alpha_1 |010\rangle + \frac{1}{\sqrt{2}} \sin \alpha_1 |001\rangle, \quad (8)$$
of eigenstates (4)-(11) into Eq. (13), we find the following non-zero transition operators

coupled reservoir-spin system can be written as

In Eq. (12), the summation \( \sum \) must be done over all possible differences between any two eigenenergies of the Hamiltonian (1). In Eq. (13), the summation \( \sum_{\epsilon_{i} - \epsilon_{j} = \omega} \) is over all the eigenvalues with a fixed difference \( \omega \). Obviously, \( A^\dagger (\omega) = A(-\omega) \). Upon substitution of eigenstates (4)-(11) into Eq. (13), we find the following nonzero transition operators

\[
|\phi_0\rangle = \frac{1}{\sqrt{2}} \sin \alpha_2 |110\rangle - \cos \alpha_2 |101\rangle + \frac{1}{\sqrt{2}} \sin \alpha_2 |011\rangle, \\
|\phi_7\rangle = \frac{1}{\sqrt{2}} \sin \alpha_2 |100\rangle + \cos \alpha_2 |010\rangle + \frac{1}{\sqrt{2}} \sin \alpha_2 |001\rangle, \\
|\phi_8\rangle = \frac{1}{\sqrt{2}} \sin \alpha_1 |110\rangle - \cos \alpha_1 |101\rangle + \frac{1}{\sqrt{2}} \sin \alpha_1 |011\rangle, \\
\]

with the corresponding eigenvalues \( \epsilon_1 = -3h, \epsilon_2 = 3h, \epsilon_3 = h - 2k, \epsilon_4 = -h + 2k, \epsilon_5 = -h - k - B, \epsilon_6 = h + k - B, \epsilon_7 = -h - k + B, \epsilon_8 = h + k + B, \) where \( B = \sqrt{8 + k^2}, \) \( \sin \alpha_1 = 2\sqrt{2}/\sqrt{8 + (k - B)^2}, \) \( \sin \alpha_2 = 2\sqrt{2}/\sqrt{8 + (k + B)^2} \) and \( \cos \alpha_2 = (k + B)/\sqrt{8 + (k + B)^2}. \)

In the representation spanned by eigenstates (4)-(11), the Hamiltonian of the coupled reservoir-spin system can be written as

\[
H = H_S + H_{SB1} + H_{SB3} = \sum_{i=1}^{8} \epsilon_i |\phi_i\rangle \langle \phi_i| + \sum_{j=1,3} A_j(\omega) \otimes B_j, \\
\]

where

\[
A_j(\omega) = \sum_{\epsilon_{i} - \epsilon_{j} = \omega} \langle \phi_i| \sigma_j^x |\phi_j\rangle \langle \phi_j| \langle \phi_j| \\
\]

In Eq. (12), the summation \( \sum_{\omega} \) must be done over all possible differences between any two eigenenergies of the Hamiltonian (1). In Eq. (13), the summation \( \sum_{\epsilon_{i} - \epsilon_{j} = \omega} \) is over all the eigenvalues with a fixed difference \( \omega \). Obviously, \( A^\dagger (\omega) = A(-\omega) \). Upon substitution of eigenstates (4)-(11) into Eq. (13), we find the following nonzero transition operators

\[
A_1^\dagger (\omega_1) = \frac{1}{\sqrt{2}} \left( \sin \alpha_1 |\phi_2\rangle \langle \phi_8| - \cos \alpha_2 |\phi_6\rangle \langle \phi_4| - \cos \alpha_2 |\phi_3\rangle \langle \phi_7| + \sin \alpha_1 |\phi_5\rangle \langle \phi_1| \right), \\
A_1^\dagger (\omega_2) = \frac{1}{\sqrt{2}} \left( \sin \alpha_2 |\phi_2\rangle \langle \phi_6| - \cos \alpha_1 |\phi_8\rangle \langle \phi_4| - \cos \alpha_1 |\phi_3\rangle \langle \phi_5| + \sin \alpha_2 |\phi_7\rangle \langle \phi_1| \right), \\
A_1^\dagger (\omega_3) = \frac{1}{\sqrt{2}} \left( |\phi_2\rangle \langle \phi_3| - \sin (\alpha_-) |\phi_6\rangle \langle \phi_5| + \sin (\alpha_-) |\phi_8\rangle \langle \phi_7| - |\phi_4\rangle \langle \phi_1| \right), \\
A_3^\dagger (\omega_1) = \frac{1}{\sqrt{2}} \left( \sin \alpha_1 |\phi_2\rangle \langle \phi_8| + \cos \alpha_2 |\phi_6\rangle \langle \phi_4| + \cos \alpha_2 |\phi_3\rangle \langle \phi_7| + \sin \alpha_1 |\phi_5\rangle \langle \phi_1| \right), \\
A_3^\dagger (\omega_2) = \frac{1}{\sqrt{2}} \left( \sin \alpha_2 |\phi_2\rangle \langle \phi_6| + \cos \alpha_1 |\phi_8\rangle \langle \phi_4| + \cos \alpha_1 |\phi_3\rangle \langle \phi_5| + \sin \alpha_2 |\phi_7\rangle \langle \phi_1| \right), \\
A_3^\dagger (\omega_3) = \frac{1}{\sqrt{2}} \left( |\phi_2\rangle \langle \phi_3| - \sin (\alpha_+) |\phi_6\rangle \langle \phi_5| - \sin (\alpha_+) |\phi_8\rangle \langle \phi_7| - |\phi_4\rangle \langle \phi_1| \right), \\
\]
where \( \omega_1 = 2h - k - B, \omega_2 = 2h - k + B, \omega_3 = 2(h + k) \), \( \alpha_+ = \alpha_1 + \alpha_2 \) and \( \alpha_- = \alpha_1 - \alpha_2 \).

By means of the general reservoir theory within the Born-Markov and rotating wave approximations [45, 46, 47], one can obtain the equation of motion for the reduced density matrix of the spin chain

\[
\frac{d\rho}{dt} = -i[H_S, \rho] + L_1(\rho) + L_3(\rho),
\]

(20)

where \( L_j(\rho) \) \((j = 1, 3)\) is the dissipative term due to the coupling of spin \( j \) to its thermal bath and is given by

\[
L_j(\rho) = \sum_{\omega_{\mu} > 0} \gamma_j(\omega_{\mu})(1 + n_j(\omega_{\mu})) \left( 2A_j^\dagger(\omega_{\mu}) \rho A_j(\omega_{\mu}) - \left\{ \rho, A_j^\dagger(\omega_{\mu}) A_j(\omega_{\mu}) \right\} \right)
+ \sum_{\omega_{\mu} > 0} \gamma_j(\omega_{\mu}) n_j(\omega_{\mu}) \left( 2A_j^\dagger(\omega_{\mu}) \rho A_j(\omega_{\mu}) - \left\{ \rho, A_j(\omega_{\mu}) A_j^\dagger(\omega_{\mu}) \right\} \right).
\]

(21)

In deriving out the master equation [20], we have assumed that the \( j \)th bath is always in a thermal equilibrium state at temperature \( T_j \). In Eq. (21), \( n_j(\omega_{\mu}) = 1/(\exp(\beta_j \omega_{\mu}) - 1) \) with \( \beta_j = 1/(T_j) \) is the mean thermal photon number of the \( j \)th bath at frequency \( \omega_{\mu} \) (taking the Boltzmann constant \( k_B = 1 \)), and \( \gamma_j(\omega_{\mu}) \) is defined through the integral \( \pi \sum_n |g^{(n)}|^2 (1 + b_{n_j}^\dagger b_{n_j}) = \int_0^\infty \gamma_j(\omega_{\mu})(1 + n_j(\omega_{\mu}))d\omega_{\mu} \).

Here, the Lamb shift has been omitted.

### 3. Quantum Discord and Concurrence

In this section, for convenience of discussions in the next section, we give a brief review on quantum discord (QD) and concurrence. QD is defined as the discrepancy between quantum extensions of two equivalent expressions for the classical mutual information [3]. In classical information theory (CIT), the total correlation between two random variables \( A \) and \( B \) can be described by either the mutual information \( I \) [48, 49]

\[
I_C(A; B) = H(A) + H(B) - H(A, B)
\]

(22)

or the equivalent expression

\[
J_C(A; B) = H(A) - H(A\|B),
\]

(23)

where \( H(X) = -\sum_x p_x \log_2 p_x \) \((X = A, B \text{ and } AB)\) is the Shannon entropy of the variable \( X \) with \( p_x \) being the probability of \( X \) assuming the value \( x \), and \( H(A\|B) = -\sum_{a,b} p_{ab} \log_2 p_{ab} = H(A, B) - H(B) \) \((p_{a|b} = p_{ab}/p_b)\) is the conditional entropy, which represents a weighted average of the entropies of \( A \) given the value of \( B \).

In the quantum information theory (QIT) [48, 49], the total correlation of a bipartite system consisting of subsystems \( A \) and \( B \) in a state described by the density matrix \( \rho_{AB} \) is defined as

\[
I_q(\rho_{A:B}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}),
\]

(24)

which is the straightforward extension of [22]. Here, \( S(\rho_{A(B)}) = -Tr(\rho_{A(B)} \log_2 \rho_{A(B)}) \) is the von Neumann entropy of the subsystem \( A(B) \), while \( S(\rho_{AB}) = -Tr(\rho_{AB} \log_2 \rho_{AB}) \) is the entropy of the composite system \( AB \).
The extension of (22) to the quantum realm is no longer straightforward since the value of $H(A\|B)$ is measurement dependence, and quantum measurement may fully destroy a quantum state. The counterpart of (23) in QIT may be defined as

$$J_q(\rho_{A:B}) = S(\rho_A) - S_{\{\Pi_j^B\}}(\rho_{A|B}),$$

(25)

where $\{\Pi_j^B\}$ are a set of projectors performed locally on subsystem $B$, and $S_{\{\Pi_j^B\}}(\rho_{A|B}) = \sum_j q_j S(\rho_A^j) = T_B(\Pi_j^B \rho_{A:B} \Pi_j^B)/q_j$ and the probability $q_j = T_{AB}(\Pi_j^B \rho_{A:B} \Pi_j^B)$. The project operator $\Pi_j^B = |\theta_j\rangle \langle \theta_j|$ with $|\theta_1\rangle = \cos \theta |0\rangle + e^{i\phi} \sin \theta |1\rangle$ and $|\theta_2\rangle = -\cos \theta |1\rangle + e^{-i\phi} \sin \theta |0\rangle$ ($0 \leq \theta \leq 2\pi$, $0 \leq \phi \leq 2\pi$). From (25), it is clear that different choices of $\{\Pi_j^B\}$ may lead to different values of $J_q(\rho_{A:B})$. The minimum difference between $I_q(\rho_{A:B})$ and $J_q(\rho_{A:B})$, called quantum discord (QD) [6], is used to describe the quantum correlation of a bipartite quantum system

$$D(\rho_{A:B}) = \min_{\{\Pi_j^B\}} [I_q(\rho_{A:B}) - J_q(\rho_{A:B})]$$

(26)

or equivalently

$$D(\rho_{A:B}) = I_q(\rho_{A:B}) - \max_{\{\Pi_j^B\}} [J_q(\rho_{A:B})].$$

(27)

From (24) and (27), the classical correlation contained in a quantum system is defined as [2]

$$C(\rho_{AB}) \equiv I_q(\rho_{A:B}) - D(\rho_{A:B}) = \max_{\{\Pi_j^B\}} \left[ S(\rho_A) - S_{\{\Pi_j^B\}}(\rho_{A|B}) \right].$$

(28)

In our investigation, entanglement is qualified by the Wootters concurrence [50]. For given density matrix $\rho_{AB}$ of a bipartite system $AB$, the concurrence is defined as $C = \max \{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \}$, where $\lambda_i$ ($i = 1, 2, 3, 4$) are the eigenvalues of the matrix $R = \rho_{AB} (\sigma_A^y \otimes \sigma_B^y) \rho_{AB}^* (\sigma_A^y \otimes \sigma_B^y)$, arranged in decreasing order of magnitude, $\rho_{AB}$ is the complex conjugate of $\rho_{AB}$ and $\sigma_A^y$ and $\sigma_B^y$ are the Pauli matrices for systems $A$ and $B$. The concurrence attains its maximum value 1 for maximally entangled states and 0 for separable states.

4. Results and Discussion

The master equation (20) can be easily solved by the fourth-order Runge-Kutta method in the representation spanned by the eigenstates of $H_S$. We take the evolution time long enough such that the final density matrix reaches steady state $\rho_{st}$. Then, according the definitions on QD and concurrence given in the preceding section, we can investigate the influence of the bath temperature, multi-spin interaction and external magnetic field on the QD and concurrence of the spin chain. In the calculation, we set the coupling constant $J = 1$. It means that all the interaction constants in the Hamiltonian are rescaled by the XX spin chain coupling strength. We also assume that the decay rate is spectrum independent, i.e. $\gamma(\omega) = \gamma$.
In Figs. 2 and 3, the steady-state QD and concurrence of spin pairs 13 and 23 are shown as a function of the magnetic field for various values of the temperature difference $\Delta T = T_1 - T_3$ and of the mean temperature $T_M = (T_1 + T_3)/2$. In these figures, we see that both the QD and concurrence first increase with increasing of the field, get maximal values and then decay to zero. As either the temperature difference or the mean temperature increases, in general, the QD and concurrence are diminished. Comparing Fig. 2 with Fig. 3, we notice that the mean temperature affects more strongly the concurrence than the temperature difference. The sudden death of concurrence as shown in Fig. 3 indicates that the QD is more robust against the mean temperature than the concurrence. From these figures, we come to the conclusion that the QD and concurrence can be enhanced by switching on the properly large magnetic field if both...
the mean temperature and temperature difference are not large.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Steady-state QD (green lines) and concurrence (red lines) as a function of the three-spin interaction strength $k$ with various values of the mean temperature and of the field strength. The figures (a) and (c), and (b) and (d) are for spin pairs 13 and 23, respectively. The other parameters are chosen to be $\gamma = 0.01$ and $\Delta T = 0.8$. The symbols shown in the inset of Fig. 4(b) are applicable to all the curves of Fig. 4.}
\end{figure}

Figures 4 and 5 show the steady-state QD and concurrence of spin pairs 13 and 23 as a function of the three-spin interaction strength $k$ for various values of the magnetic field strength. In these figures, we see that the QD and concurrence for spin pair 13 first increase with the three-spin interaction and then get a plateau. It is very interesting that the plateau can be raised to the maximum level by increasing the magnetic field strength. As shown in Figs. 4 and 5, this feature can be maintained even if the mean temperature is high and the temperature difference is large. As for the spin pair 23, figures 4 and 5 show that its QD and concurrence first increase with the three-spin interaction, get peaks and then decay to zero. In Fig. 6, the steady-state QD and concurrence of spin pairs 13 and 23 as a function of the magnetic field strength $h$ with various values of the three-spin interaction strength $k$. In Fig. 6(a), we observe that the QD and concurrence of spin pair 13 can get the maximum level plateau by increasing the field if the three-spin interaction is enough strong. The maximum plateau width is enlarged as the interaction strength increases. Thus, we can maintain the extreme QD and concurrence of the spin pair 13 by holding the strong interaction and magnetic field.

In order to find out the physical reasons for the observed phenomena, we first analyze the eigenvalues $\varepsilon_l \ (l = 1, \ldots, 8)$ of the Hamiltonian (1) as a function of the
Figure 5. Steady-state QD (green lines) and concurrence (red lines) as a function of the three-spin interaction strength $k$ with various values of the temperature difference and of the field strength. The figures (a) and (c), and (b) and (d) are for spin pairs 13 and 23, respectively. The other parameters are chosen to be $\gamma = 0.01$ and $T_M = 2$. The symbols shown in the inset of Fig. 5(b) are applicable to all the curves of Fig. 5.

Figure 6. Steady-state QD (green lines) and concurrence (red lines) as a function of the magnetic field strength $h$. The figures (a) and (b) are for spin pairs 13 and 23, respectively. The parameters are chosen to be $\gamma = 0.01$, $T_M = 1.2$ and $\Delta T = 0.8$. The symbols shown in the inset of Fig. 6(b) are applicable to the curves of Fig. 6 (a).
Figure 7. Eigenenergy $\varepsilon_l$ of the Hamiltonian (1) as a function of the three-spin interaction strength $k$ with various values of the field strength. The symbols shown in the subset of Fig. 7(a) are applicable to all the curves in the figures.

Figure 8. The energy difference between $\varepsilon_3$ and $\varepsilon_5$ as a function of the field with a fixed value of the interaction, $k = 10$. 
interaction and magnetic field strengths. When the field is weak ($h << k$), the eigenstate $|\phi_5\rangle$ is the ground state of the spin chain since the eigenenergy $\varepsilon_5 = -h - k - \sqrt{8 + k^2}$ is the smallest one as shown in Fig. 7(a). It is noted that $\varepsilon_1 = -3h$. Thus, the eigenstate $|\phi_1\rangle$ becomes the ground state when $h > k$. The two states have the energy crossing around the point $h = k$ as shown in Figs. 7 (b)-7(d). Since then, the eigenstate $|\phi_5\rangle$ becomes the ground state. As the interaction strength $k$ further increases, the state $|\phi_3\rangle$ with the eigenenergy $\varepsilon_3 = -h - 2k$ crosses with the state $|\phi_1\rangle$ and becomes the first excited state of the spin chain. In Fig. 8, the energy difference between the eigenstates $|\phi_5\rangle$ and $|\phi_3\rangle$ is plotted as a function of the magnetic field. We see that the energy splitting linearly increases as the magnetic field increases. In fact, we have $\varepsilon_3 - \varepsilon_5 \approx 2h$ when $k$ is large.

![Figure 9](image_url)

**Figure 9.** Eigenstate occupation probabilities of the steady-state density matrix $\rho_{st}$ as a function of the three-spin interaction strength $k$ with various strength of the field. The other parameters are chosen to be $\gamma = 0.01$, $T_M = 1.2$ and $\Delta T = 0.8$. The symbols shown in the subset of Fig. 9(a) are applicable to all the curves of Figs. 9.

Figure 9 shows the eigenstate occupation probabilities which are defined as $P_l = tr(|\phi_l\rangle\langle\phi_l|\rho_{st}) = \langle\phi_l|\rho_{st}|\phi_l\rangle$ as a function of the three-spin interaction. We see that $|\phi_1\rangle$ is the most populated state when $k << h$, $|\phi_1\rangle$ and $|\phi_5\rangle$ cross and take the same probability around the point $k = h$ when $h$ is large, and then $|\phi_5\rangle$ becomes the most populated state and takes over all the occupation probability, as shown in Figs. 9(b)-9(d).

Therefore, when the magnetic field and three-spin interaction are strong, the possible thermal excited transition is one from $|\phi_5\rangle$ to $|\phi_3\rangle$, which is induced by the thermal resources interacting with the spins 1 and 3, respectively. However, $\varepsilon_3 - \varepsilon_5 \approx 2h$, as discussed above. Thus, the thermal excitation can be mostly depressed if $h \gg T_1, T_3$. 
In this case, the spin chain is nearly decoupled from the thermal resources. Thus, the QD and concurrence of the spin system are determined by the most populated eigenstate, i.e. the ground state of the spin system. These results mean that the eigenstate $|\phi_5\rangle$ makes the most contribution to the QD and concurrence of the spin pairs 13 and 23 when the magnetic field and three-spin interaction are strong enough. In Fig. 10, the QD and concurrence of $|\phi_5\rangle$ are plotted as a function of the three-spin interaction $k$. Comparing Fig. 10 with Fig. 4, we see that the above conclusion is really true.

5. Summary

We investigate the quantum discord and concurrence of the three-spin chain which ends are coupled to two thermal reservoirs at different temperatures. Besides the XX-type nearest-neighbor two-spin interaction a three-spin interaction is also included and a homogenous magnetic field is applied to each spin. For fixed temperatures of the thermal reservoirs, we find that the extreme steady-state QD and concurrence of the two end spins can always be created by raising the magnetic field strength with a strong multi-spin interaction. We show that the energy gap between the most populated ground state and the first excited state of the spin chain can become much larger than the thermal excitation energy when the magnetic field and multi-spin interaction are strong enough. In this way, the thermal excitation induced by the thermal reservoirs is nearly depressed and the spin chain is decoupled from the thermal environments. As a result, the QD and concurrence of the spin chain are totally determined by the most populated ground state of the spin chain. The present results may provide a useful approach to control coupling between a quantum system and its environment.
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