Influence of gauge-field fluctuations on composite fermions near the half-filled state

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ABSTRACT

Taking into account the transverse gauge field fluctuations, which interact with composite fermions, we examine the finite temperature compressibility of the fermions as a function of an effective magnetic field \( \Delta B = B - 2n_ehc/e \) (\( n_e \) is the density of electrons) near the half-filled state. It is shown that, after including the lowest order gauge field correction, the compressibility goes as

\[
\frac{\partial n}{\partial \mu} \propto e^{-\Delta \omega_c/2T} \left( 1 + \frac{A(\eta) (\Delta \omega_c)^{1+\eta}}{\eta-1} \right) \text{ for } T \ll \Delta \omega_c,
\]

where \( \Delta \omega_c = e\Delta B_{mc} \). Here we assume that the interaction between the fermions is given by

\[
v(q) = \frac{V_0}{q^{2-\eta}} \quad (1 \leq \eta \leq 2),
\]

where \( A(\eta) \) is a \( \eta \) dependent constant. This result can be interpreted as a divergent correction to the activation energy gap and is consistent with the divergent renormalization of the effective mass of the composite fermions.

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I. INTRODUCTION

In 1989 Jiang et al. [1] observed that a two dimensional electron gas in the fractional quantum Hall (FQH) regime, at the filling fraction $\nu = 1/2$, forms a metallic state even at very low temperatures. At that time the only known quantum metallic state at zero temperature was Landau’s Fermi liquid state in absence of a magnetic field. Thus the experiment suggests that the electrons at $\nu = 1/2$ may form a new quantum metallic state at zero temperature. The possibility of a new metallic state at $\nu = 1/2$ has attracted a lot of attention [2-3].

On the theoretical side, Jain [4] has introduced the composite fermion approach, which successfully explains the stability of the sequence of the filling fractions $\nu = p/(2p \pm 1)$ ($p$ is an integer). Halperin, Lee, and Read (HLR) [5] observed that this sequence is reminiscent of the Shubnikov-de Haas oscillation of the conventional Fermi liquid in the presence of a weak magnetic field, which indicates the possible existence of a Fermi surface at $\nu = 1/2$. This important observation and a set of new experiments [2,3] suggest a strong connection between the Fermi liquid at zero magnetic field and the new metallic state at $\nu = 1/2$. Using the Chern-Simons gauge field theory formulation of the composite fermion approach, HLR realized these ideas and developed a theory that describes the new metallic state [5].

A composite fermion is obtained by attaching an even number ($2n$) of flux quanta to an electron and the transformation can be formally realized by introducing an appropriate Chern-Simons gauge field [5-7]. At the mean field level, the FQH state with the filling factor $\nu = p/(2np+1)$ can be described as the integral quantum Hall (IQH) state of the composite fermions with $p$ Landau levels occupied in an effective magnetic field $\Delta B = B - B_{1/2n}$, where $B_{1/2n} = 2nn_ehc/e$ and $n_e$ is the density of electrons. Note that $\Delta B = 0$ for $\nu = 1/2n$ states so that, within the mean field approximation, the composite fermions can be described by the conventional Fermi liquid theory at these filling factors [5,7]. In particular, the main sequence ($n = 1$) of the hierarchical structure of the FQH states [4-6] can be viewed as the IQH effect of the composite fermions, which leads to the analogue of
the Shubnikov-de Haas effect near $\nu = 1/2$.

HLR had also gone beyond the mean field approximation by including gauge field fluctuations within the random phase approximation (RPA). This allowed them to construct a modified Fermi-liquid-theory description of the $\nu = 1/2$ state. They found that the gauge field fluctuations give rise to a singular contribution to the self-energy in the one-particle Green’s function of the composite fermions [5,8]. Later various kinds of methods were used to go beyond the perturbation theory [9-16], which were motivated by the fact that the singular self-energy correction leads to a divergent effective mass of the composite fermions [5]. If one applied this divergent effective mass to the Shubnikov-de Haas effect near $\nu = 1/2$, one would find that the gap $\Delta_p$ of $\nu = p/(2p \pm 1)$ FQH state goes down faster than $1/p$ as $p \to \infty$:

$$p \Delta_p \to 0 \quad \text{as} \quad p \to \infty.$$  \hspace{2cm} (1)

However, the one-particle Green’s function of the composite fermions is not gauge invariant. Therefore, it is not clear whether the divergent effective mass in the one-particle Green’s function is related to the above energy gap $\Delta_p$ which is measurable in real experiments.

There have been several studies of the two-particle correlation functions which concentrated on the non-renormalization of the gauge field propagator [11-14,16]. Recently three of us with Furusaki have examined several gauge invariant two-particle correlation functions for all ratios of $\omega$ and $q$ [17]. We found that, at low energies and in the long-wavelength limit, the gauge field fluctuations do not cause any divergent correction (up to two-loop level), and the two-particle correlation functions have the Fermi-liquid forms with a finite effective mass if one assumes a non-singular Fermi-liquid-parameter-function $f_{pp'} [17]$. Fermi liquid form of the density-density correlation function in the small $q$ and $\omega$ limit was also found in the eikonal approximation [11] even though the result is not the same as that of the two-loop perturbative calculation [17]. Altshuler, Ioffe, and Millis also examined the two-particle correlation functions and especially found peculiar behaviors near $q = 2k_F$ [18].
We would like to mention that Fermi-liquid theory with a finite effective mass is not the conclusive interpretation of the behaviors of the density-density correlation function in the long wavelength and the low frequency limit. That is, it is still possible that the effect of the divergent effective mass may be cancelled by a contribution from a singular Fermi-liquid-parameter-function \( f_{pp'} \) so that the density-density correlation function for the long wavelength and the low energy limit behaves as if the effective mass is finite \[19\]. Indeed, Stern and Halperin \[19\] calculated the energy gap of the system from the one-particle Green’s function of the composite fermions in a finite effective magnetic field \( \Delta B \). They argued that even though the one-particle Green’s function is not gauge-invariant, the edge of the spectral function at zero temperature, across which the spectral function vanishes, should be gauge-invariant. By identifying the region where the spectral function vanishes, they found an energy gap which is in agreement with the previous self-consistency treatment \[5\]. In view of the complexity of the problem, we feel that it is important to investigate whether the effect of the large enhancement of the effective mass will show up in some gauge-invariant response functions. In this paper, we calculate the lowest order correction (due to the gauge field) to the finite temperature compressibility as a function of an effective cyclotron frequency \( \Delta \omega_c = \frac{e \Delta B}{m c} \) (where \( m \) is the bare mass of the fermions) in the limit of large \( p \), i.e., near \( \nu = 1/2 \). We find that when a chemical potential \( \mu \) lies exactly at the middle of the successive effective Landau levels, for \( T \ll \Delta \omega_c \), the compressibility behaves as

\[
\frac{\partial n}{\partial \mu} \propto e^{-\Delta \omega_c/2T} \left( 1 + \frac{A(\eta)}{\eta - 1} \left( \frac{\Delta \omega_c}{T} \right)^{1+\frac{\eta}{\eta - 1}} \right),
\]

where \( A(\eta) \) is a \( \eta \)-dependent positive dimensionful constant. Here, we assume that the interaction between the fermions has the form: \( v(q) = V_0 / q^{2-\eta} \) (1 \( \leq \eta \leq 2 \)). If we interpret the activation energy as a renormalized energy gap \( \Delta \omega_c^* \), i.e., \( \frac{\partial n}{\partial \mu} \propto e^{-\Delta \omega_c^*/2T} \), it is given by \( \Delta \omega_c^* \approx \Delta \omega_c \left( 1 - \frac{2A(\eta)}{\eta - 1} \left( \frac{\Delta \omega_c}{T} \right)^{-\frac{\eta}{\eta - 1}} \right) \). If we write \( \Delta \omega_c^* = \frac{e \Delta B}{m^* c} \), the above result is consistent with a divergent correction to the effective mass \( m^*/m \approx 1 + \frac{2A(\eta)}{\eta - 1} \left( \frac{\Delta \omega_c}{T} \right)^{-\frac{\eta}{\eta - 1}} \), because \( A \) should be proportional to a small expansion parameter, which is \( 1/N \) for a
large $N$ generalized model. In particular, for the Coulomb interaction ($\eta = 1$), $m^*/m \approx 1 + 2A(\eta = 1) \ln (\epsilon_F/\Delta\omega_c)$ ($\epsilon_F$ is the Fermi energy) as predicted in terms of a self-consistent argument [5,19].

We would like to remark that a comparison with the recent experimental measurements [3,20] of the energy gap is complicated by the large impurity effects. The disordered potential due to the impurities causes a spatial fluctuation of the fermion density distribution, which is equivalent to a large spatial fluctuation of the Chern-Simons magnetic flux or $\Delta B$. This means that there is a range of $\Delta B$ controlled by the degree of disorder around the filling factor $\nu = 1/2$, where impurity effects are very important. In reality, this is the region where the gap measurement is not possible due to the suppression of the amplitude of the Shubnikov-de Haas effect. We feel that a deeper understanding of the impurity effects is necessary before a recent experimental report [20] of an increase in the effective mass near the boundary of the disorder dominated region can be properly interpreted.

Before the main discussion, we would like to point out that there is a gauge-invariant (for the Chern-Simons gauge field) one-particle Green’s function — the Green’s function of the physical electrons, which does not have a Fermi-liquid form [21] even though the two-particle Green’s functions are similar to those of the Fermi liquid with a finite or divergent effective mass. In the first place, the electrons see a strong magnetic field and the electron Green’s function does not have any singularity at $k_F$. Secondly the spectral weight of the electron Green’s function is exponentially small at low energies even for the Coulomb interaction, which is very different from the Fermi liquid result [21]. Thus the $\nu = 1/2$ state really represents a new class of metallic state.

The remainder of the paper is organized as follows. In section II, we introduce the model and describe a method to calculate the lowest order correction to the compressibility $\frac{\partial n}{\partial \mu}$, where $n$ is the density of the composite fermions. In section III, the compressibility of the fermions is calculated for $T \ll \Delta\omega_c \ll \mu$ when the chemical potential $\mu$ lies exactly
at the middle of the two successive effective Landau levels. In section IV, we discuss and contrast two different methods of evaluating the compressibility and emphasize the gauge-invariant nature of the method used in this paper. We discuss and interpret our results in section V.

II. THE MODEL AND THE COMPRESSIBILITY

Let us consider the model for the composite fermions in which a statistical gauge field or a Chern-Simons gauge field has been introduced. The model is given by [5,6]

\[ Z = \int D\psi \, D\psi^* \, Da_\mu \, e^{i \int dt \, d^2 r \, \mathcal{L}}, \]  

where the Lagrangian density \( \mathcal{L} \) is

\[ \mathcal{L} = \psi^*(\partial_0 + ia_0 - \mu)\psi - \frac{1}{2m} \psi^*(\partial_i - ia_i + ieA_i)^2\psi \]

\[ - i \frac{a_0}{2\pi} \epsilon^{ij} \partial_i a_j + \frac{1}{2} \int d^2 r' \psi^*(r)\psi(r)v(r - r')\psi^*(r')\psi(r'), \]  

where \( \psi \) represents the fermion field and \( \phi \) is an even number \( 2n \) which is the number of flux quanta attached to an electron, and \( v(r) \propto V_0/r^\eta \) is the Fourier transform of \( v(q) = V_0/q^{2-\eta} \) \( (1 \leq \eta \leq 2) \) which denotes the interaction between the fermions. We choose the Coulomb gauge \( \nabla \cdot a = 0 \). Note that the integration over \( a_0 \) enforces the following constraint [5,6]:

\[ \nabla \times a = 2\pi \tilde{\phi} \psi^*(r)\psi(r). \]  

The saddle point of the above action is given by the following conditions [5,6]:

\[ \nabla \times a = 2\pi \tilde{\phi} \psi^*(r)\psi(r) \equiv eB_{1/2n} \quad \text{and} \quad a_0 = 0. \]  

Therefore, at the mean field level, the fermions see an effective magnetic field \( \Delta A \equiv A - a/e \):

\[ \Delta B = \nabla \times \Delta A = B - B_{1/2n}, \]  

6
which becomes zero at the Landau level filling factor $\nu = 1/2n$. The IQH effect of the fermions may appear when the effective Landau level filling factor $p = \frac{n_{e\hbar c}}{e\Delta B}$ becomes an integer. This implies that the real external magnetic field is given by $B = B_{1/2n} + \Delta B = \frac{n_{e\hbar c}}{e} \left( \frac{2np+1}{p} \right)$ which corresponds to a FQH state of electrons with the filling factor $\nu = \frac{p}{2np+1} [5,6]$.

After integrating out the fermions and including gauge field fluctuations within the random phase approximation, the effective action of the gauge field can be obtained [5]

$$S_{\text{eff}} = \frac{1}{2} \int \frac{d^2q}{(2\pi)^2} \frac{d\omega}{2\pi} \delta a^*(q,\omega) D_{\mu\nu}^{-1}(q,\omega,\Delta\omega_c) \delta a_{\nu}(q,\omega),$$

(8)

where $D_{\mu\nu}^{-1}(q,\omega,\Delta\omega_c)$ was calculated by several authors [5,6,22-24]. For our purpose, the $2 \times 2$ matrix form for $D_{\mu\nu}^{-1}$ is sufficient so that $\mu, \nu = 0, 1$ and 1 represents the direction that is perpendicular to $q$ [5].

The compressibility of the fermions $\frac{\partial n}{\partial \mu}(\mu, \Delta\omega_c)$ as a function of chemical potential $\mu$ and an effective cyclotron frequency $\Delta\omega_c = \frac{e\Delta B}{mc}$ can be obtained from $n(\mu, \Delta\omega_c) = -\frac{\partial \Omega}{\partial \mu}$ (n is the density of the fermions), i.e., $\frac{\partial n}{\partial \mu} = -\frac{\partial^2 \Omega}{\partial \mu^2}$. The density of the free fermions $n_0(\mu, \Delta\omega_c)$ and the lowest order correction $n_1(\mu, \Delta\omega_c)$ due to the transverse part of the gauge field fluctuations are given by the diagrams in Fig.1 (a) and (b) respectively. These contributions can be obtained from the relations $n_0(\mu, \Delta\omega_c) = -\frac{\partial \Omega_0}{\partial \mu}$ and $n_1(\mu, \Delta\omega_c) = -\frac{\partial \Omega_1}{\partial \mu}$, where $\Omega_0$ and $\Omega_1$ are the thermodynamic potential of the free fermions and the lowest order correction to the thermodynamic potential given by the diagrams in Fig.3 (a) and (b) respectively.

The density of the free fermions $n_0(\mu, \Delta\omega_c)$ at finite temperatures can be written as

$$n_0(\mu, \Delta\omega_c) = \frac{m(\Delta\omega_c)}{2\pi} \sum_l n_F(\xi_l),$$

(9)

where $\xi_l = (l + 1/2)(\Delta\omega_c) - \mu$ and $n_F(x) = \frac{1}{e^{x/T} + 1}$. Thus the compressibility of the free fermions is given by

$$\frac{\partial n_0}{\partial \mu} = \frac{m}{2\pi} \frac{\Delta\omega_c}{T} \sum_l n_F(\xi_l)(1 - n_F(\xi_l)).$$

(10)
The lowest order correction (due to the transverse part of the gauge field) to the density of the fermions can be obtained from

\[ n_1(\mu, \Delta \omega_c) = T \sum_{i\nu_n} \sum_{\mathbf{q}} D_{11}(\mathbf{q}, i\nu_n) \frac{\partial}{\partial \mu} \Pi_{11}(\mathbf{q}, i\nu_n), \quad (11) \]

where \( \nu_n = 2\pi n T \) is the Matsubara frequency. Here \( \Pi_{11} \) is the transverse part of the fermion polarization bubble:

\[ \Pi_{11}(\mathbf{q}, i\nu_n) = -\sum_{lm} |M_{lm}(\mathbf{q})|^2 \frac{n_F(\xi_l) - n_F(\xi_m)}{i\nu_n - \xi_m + \xi_l} - \frac{1}{m} \left( \frac{m\Delta \omega_c}{2\pi} \sum_l n_F(\xi_l) \right), \quad (12) \]

where \( |M_{lm}(\mathbf{q})|^2 \) comes from the form of the current-current vertex and is calculated by several authors [6,22,23]. After analytic continuation \( i\nu_n \rightarrow \nu + i0^+ \), one gets the real part and the imaginary part of the retarded polarization function:

\[ \Pi'_{11}(\mathbf{q}, \nu) = -\sum_{lm} |M_{lm}(\mathbf{q})|^2 \frac{n_F(\xi_l) - n_F(\xi_m)}{\nu - \xi_m + \xi_l} - \frac{1}{m} \left( \frac{m\Delta \omega_c}{2\pi} \sum_l n_F(\xi_l) \right), \]
\[ \Pi''_{11}(\mathbf{q}, \nu) = \pi \sum_{lm} |M_{lm}(\mathbf{q})|^2 \left[ n_F(\xi_l) - n_F(\xi_m) \right] \delta(\nu - \xi_m + \xi_l). \quad (13) \]

Here we use the convention that \( A' \) and \( A'' \) represent the real and the imaginary parts of a quantity \( A \). Now the correction to the compressibility can be obtained as

\[ \frac{\partial n_1}{\partial \mu} = T \sum_{i\nu_n} \sum_{\mathbf{q}} \left[ D_{11}(\mathbf{q}, i\nu_n) \frac{\partial^2}{\partial \mu^2} \Pi_{11}(\mathbf{q}, i\nu_n) + \frac{\partial}{\partial \mu} D_{11}(\mathbf{q}, i\nu_n) \frac{\partial}{\partial \mu} \Pi_{11}(\mathbf{q}, i\nu_n) \right]. \quad (14) \]

For calculational convenience, we introduce \( \tilde{D}_{11}(\mathbf{q}, i\nu_n) \) which does not depend on \( \mu \). Then the correction to the physical fermion density \( n_1(\mu, \Delta \omega_c) \) can be obtained from

\[ n_1(\mu, \Delta \omega_c) = -\frac{\partial \Omega_{\text{toy}}}{\partial \mu}, \quad \text{where} \]

\[ \Omega_{\text{toy}} = T \sum_{i\nu_n} \sum_{\mathbf{q}} \tilde{D}_{11}(\mathbf{q}, i\nu_n) \Pi_{11}(\mathbf{q}, i\nu_n), \quad (15) \]

and replace \( \tilde{D}_{11}(\mathbf{q}, i\nu_n) \) by \( D_{11}(\mathbf{q}, i\nu_n) \) after taking the derivative with respect to \( \mu \). Using
the spectral representation, one can write $\Omega_{\text{toy}}$ as

$$
\Omega_{\text{toy}} = \Omega_a + \Omega_b ,
$$

$$
\Omega_a = \sum_q \int_{-\infty}^{\infty} \frac{dx}{\pi} n_B(x) \tilde{D}_{11}'(q, x) \Pi_{11}''(q, x) ,
$$

$$
\Omega_b = \sum_q \int_{-\infty}^{\infty} \frac{dx}{\pi} n_B(x) \tilde{D}_{11}''(q, x) \Pi_{11}'(q, x) ,
$$

where $n_B(x) = \frac{1}{e^{x/T} - 1}$. After taking the derivative with respect to $\mu$ and replacing $\tilde{D}_{11}$ by $D_{11}$, we get the lowest order correction to the density of the fermions:

$$
n_1 = n_a + n_b ,
$$

$$
n_a = - \sum_q \int_{-\infty}^{\infty} \frac{dx}{\pi} n_B(x) D_{11}'(q, x) \frac{\partial}{\partial \mu} \Pi_{11}''(q, x) ,
$$

$$
n_b = - \sum_q \int_{-\infty}^{\infty} \frac{dx}{\pi} n_B(x) D_{11}''(q, x) \frac{\partial}{\partial \mu} \Pi_{11}'(q, x) .
$$

For the lowest order correction $\frac{\partial n_1}{\partial \mu}$ to the compressibility, the derivative with respect to $\mu$ should be taken for both $D_{11}$ and $\Pi_{11}$. Thus $\frac{\partial n_1}{\partial \mu}$ can be written as

$$
\frac{\partial n_1}{\partial \mu} = \frac{\partial n_a}{\partial \mu} + \frac{\partial n_b}{\partial \mu} ,
$$

$$
\frac{\partial n_a}{\partial \mu} = - \sum_q \int_{-\infty}^{\infty} \frac{dx}{\pi} n_B(x) \left[ D_{11}' \frac{\partial^2 \Pi_{11}''}{\partial \mu^2} + \frac{\partial D_{11}'}{\partial \mu} \frac{\partial \Pi_{11}''}{\partial \mu} \right] ,
$$

$$
\frac{\partial n_b}{\partial \mu} = - \sum_q \int_{-\infty}^{\infty} \frac{dx}{\pi} n_B(x) \left[ D_{11}'' \frac{\partial^2 \Pi_{11}'}{\partial \mu^2} + \frac{\partial D_{11}''}{\partial \mu} \frac{\partial \Pi_{11}'}{\partial \mu} \right] .
$$

Note that Eq.(18) is equivalent to Eq.(14). This procedure generates the diagrams for the compressibility, which are shown in Fig.4. In the next section, we evaluate the expressions for the compressibility.

**III. THE FINITE TEMPERATURE COMPRESSIBILITY FOR $T < \Delta \omega_c \ll \mu$**

In this section, we calculate the compressibility of the fermions as a function of $\Delta \omega_c$ and $T$ in the limit $T < \Delta \omega_c$. First we would like to give a general discussion of the interaction effects on the compressibility. For free fermions at zero temperature and finite
magnetic field, \( \frac{dn}{d\mu} = \sum_m \delta(\mu - (n + \frac{1}{2}))\Delta \omega_c \) is the density of states. Each \( \delta \)-function corresponds to a degenerate effective Landau level. The interaction has two kinds of effects on the compressibility \( \frac{dn}{d\mu} \). First, the interaction effects split the degeneracy of the states in each effective Landau level (when the effective Landau level is partially filled). This effect spreads the \( \delta \)-function in the free fermion compressibility into broadened peaks. The width of the peak (defined as the width of the region where \( \frac{dn}{d\mu} \neq 0 \)) can be viewed as the width of the effective Landau bands (i.e., the broadened effective Landau levels).

Second, the interaction effects may shift the center of the effective Landau bands. However, since the average compressibility over many effective Landau levels is not changed by the transverse gauge field interaction, we expect that such an interaction can only cause a uniform shift of the center of the effective Landau bands, as one can see later in our explicit calculations. The activation energy gap measured in the transport experiments is given by the gap between the effective Landau bands. Thus the uniform shift is not important for the calculation of the experimentally measurable activation energy gap.

In the following calculations, we will assume that the chemical potential \( \mu \) lies exactly at the middle of the two successive effective Landau levels, and investigate the activated behavior of the compressibility. In this case, the uniform shift of the center of the effective Landau bands is cancelled out and does not appear in the compressibility.

Let \( p \) be the number of filled effective Landau levels. For the free fermions, when \( T \ll \Delta \omega_c \), we can expect that the compressibility shows a thermally activated behavior. In fact, from Eq.(10) and for \( T \ll \Delta \omega_c \), it can be shown that at finite temperatures the compressibility of the free fermions can be written as

\[
\frac{\partial n_0}{\partial \mu} = \frac{m}{2\pi} \frac{\Delta \omega_c}{T} \left( e^{-|\xi_p|/T} + e^{-\xi_{p+1}/T} \right) + \mathcal{O}(e^{-2|\xi_p|/T}).
\]

(19)

Note that it becomes

\[
\frac{\partial n_0}{\partial \mu} = \frac{m\Delta \omega_c}{\pi T} e^{-\Delta \omega_c/2T} + \mathcal{O}(e^{-\Delta \omega_c/T})
\]

(20)
for a chemical potential lying exactly at the middle of the Landau levels labeled by \( p \) and \( p + 1 \). Our aim is to calculate the lowest order correction (due to the gauge field fluctuations) to the above free fermion result.

In order to calculate the lowest order correction \( \frac{\partial n}{\partial \mu} \), we consider first \( \Omega_{toy} = \Omega_a + \Omega_b \). Substituting Eq.(13) to Eq.(16), we get

\[
\Omega_a = \Omega_{a1} + \Omega_{a2},
\]

\[
\Omega_{a1} = \sum_{q} \sum_{l} |M_{ll}(q)|^2 \tilde{D}'_{11}(q,0) n_F(\xi_l)(1 - n_F(\xi_l)),
\]

\[
\Omega_{a2} = \sum_{q} \sum_{l \neq m} |M_{lm}(q)|^2 \tilde{D}'_{11}(q,\xi_m - \xi_l) n_F(\xi_m)(1 - n_F(\xi_l)),
\]

and

\[
\Omega_b = \Omega_{b1} + \Omega_{b2},
\]

\[
\Omega_{b1} = \sum_{q} \int_{0}^{\infty} \frac{dx}{\pi} (1 + 2n_B(x)) \tilde{D}''_{11}(q,x) \left[ -\sum_{lm} |M_{lm}(q)|^2 \frac{n_F(\xi_l) - n_F(\xi_m)}{x - \xi_m + \xi_l} \right],
\]

\[
\Omega_{b2} = \sum_{q} \int_{0}^{\infty} \frac{dx}{\pi} (1 + 2n_B(x)) \tilde{D}''_{11}(q,x) \left[ -\frac{\Delta\omega_c}{2\pi} \sum_{l} n_F(\xi_l) \right].
\]

Now some explanations for each contribution are in order. \( \Omega_{a1} \) and \( \Omega_{a2} \) are contributions from the exchange interaction via the gauge field and represent the effect of the intra-Landau level and the inter-Landau level particle-hole excitations respectively. \( \Omega_{b1} \) and \( \Omega_{b2} \) are due to the thermal and the quantum (represented by \( n_B(x) \) and 1 in the factor \( 1 + 2n_B(x) \)) fluctuations of the gauge field. Note that the quantum contribution survives in the \( T \to 0 \) limit. In particular, \( \Omega_{b2} \) comes from the diamagnetic coupling between the fermions and the gauge field. We also note that the intra-Landau level terms (with \( l = m \)) are associated with the splitting of the degenerate states in each Landau-level, and contribute to the spreading of the Landau-levels. On the other hand, the inter-Landau level terms (with \( l \neq m \)) will contribute to the shift of the center of the Landau bands.

The corresponding contributions to the density of the fermions are defined as \( n_a = -\frac{\partial \Omega_a}{\partial \mu} \) and \( n_b = -\frac{\partial \Omega_b}{\partial \mu} \). Thus the correction to the density of the fermions \( \frac{\partial n}{\partial \mu} \) is given
Thus \( \frac{\partial n_1}{\partial \mu} = \frac{\partial n_a}{\partial \mu} + \frac{\partial n_b}{\partial \mu} \). Now we are going to find the contributions which are order of \( e^{-|\xi_p|/T} \) or \( e^{-|\xi_{p+1}|/T} \). Note that \( \frac{\partial n_a}{\partial \mu} = \frac{\partial n_{a1}}{\partial \mu} + \frac{\partial n_{a2}}{\partial \mu} \), where \( n_{a1} = -\frac{\partial n_{a1}}{\partial \mu} \) and \( n_{a2} = -\frac{\partial n_{a2}}{\partial \mu} \).

In the appendix, we show that \( \frac{\partial n_{a1}}{\partial \mu} \) is order of \( e^{-2|\xi_p|/T} \) which is exponentially smaller than \( e^{-|\xi_p|/T} \) or \( e^{-|\xi_{p+1}|/T} \). It is also shown that \( \frac{\partial n_{a2}}{\partial \mu} \) is order of \( e^{-2|\xi_p|/T} \) after a partial cancellation between \( \Omega_{b1} \) and \( \Omega_{b2} \) by the f-sum rule.

Now let us look at \( \frac{\partial n_{a1}}{\partial \mu} \) for which a detailed expression is given in the appendix. As mentioned before, we assume that we are very close to the half-filled state, \( i.e., \mu/\Delta \omega_c \gg 1 \), which also corresponds to the large \( p \) limit. In this case, it can be shown that

\[
\frac{\partial n_{a1}}{\partial \mu} \approx -\frac{1}{T^2} \sum_q \left[ e^{-\xi_{p+1}/T} \left| M_{p+1p+1}(q) \right|^2 D'_{11}(q, 0) + e^{-|\xi_p|/T} \left| M_{pp}(q) \right|^2 D'_{11}(q, 0) \right] + \mathcal{O}(e^{-2|\xi_p|/T}) \\
\approx -\frac{1}{T^2} \left[ e^{-\xi_{p+1}/T} + e^{-|\xi_p|/T} \right] \sum_q \left| M_{pp}(q) \right|^2 D'_{11}(q, 0) + \mathcal{O}(e^{-2|\xi_p|/T}) .
\]

(23)

For \( \xi_{p+1} = |\xi_p| = \Delta \omega_c/2 \), we get

\[
\frac{\partial n_{a1}}{\partial \mu} \approx -\frac{2}{T^2} e^{-\Delta \omega_c/2T} \sum_q \left| M_{pp}(q) \right|^2 D'_{11}(q, 0) + \mathcal{O}(e^{-\Delta \omega_c/T}) .
\]

(24)

Thus \( \frac{\partial n_{a1}}{\partial \mu} = \frac{\partial n_{a2}}{\partial \mu} + \mathcal{O}(e^{-\Delta \omega_c/T}) \).

Now let us evaluate the following quantity.

\[
I = \sum_q \left| M_{pp}(q) \right|^2 D'_{11}(q, 0) = 2 \sum_q \left| M_{pp}(q) \right|^2 \int_0^\infty \frac{dy}{\pi} \frac{D''_{11}(q, y)}{y} .
\]

(25)

Note that the matrix element \( \left| M_{pp}(q) \right|^2 \) comes from the vertex of the paramagnetic part of the current-current correlation function. For the large \( p \) limit or \( \mu/\Delta \omega_c \gg 1 \), we may use a semiclassical approximation \( j \approx v_F \rho \), where \( j \) and \( \rho \) are the current and the density of the fermions. Thus \( \left| M_{pp}(q) \right|^2 \) can be approximated as \( \left| M_{pp}(q) \right|^2 \approx v_F^2 \left| M_{pp}^{00}(q) \right|^2 \), where \( \left| M_{pp}^{00}(q) \right|^2 \) is the corresponding matrix element for the density-density correlation function [6,22-24]. Using the above approximation, we get

\[
\left| M_{pp}(q) \right|^2 \approx \frac{v_F^2}{2\pi T_c^2} e^{-X} \left[ L_0^0(X) \right]^2 ,
\]

(26)
where $l_c^2 = \frac{hc}{e^2}$, $X = \frac{1}{2} q^2 l_c^2$, and $L_p^\alpha(X)$ is a Laguerre polynomial. For the large $p$ limit, $L_p^\alpha(X)$ can be approximated as [25]
\begin{equation}
L_p^\alpha(X) \approx \frac{1}{\pi} e^{X/2} X^{-\alpha/4} p^{\alpha/4} \cos \left( 2 \sqrt{pX} - \frac{\alpha\pi}{2} - \frac{\pi}{4} \right). \tag{27}
\end{equation}
We use $p \approx \mu/\Delta\omega_c$ and the above results to get
\begin{equation}
|M_{pp}(q)|^2 \approx \frac{m v_F}{\pi^3} \frac{(\Delta\omega_c)^2}{q} \cos^2 \left( \sqrt{2pql_c} - \frac{\pi}{4} \right). \tag{28}
\end{equation}
Note that $D_{11}(q,y)$ consists of two contributions coming from the intra-Landau-level and the inter-Landau-level processes respectively. That is, in the particle-hole bubbles appearing in the $1/N$ expansion (or the RPA approximation) of the gauge field propagator, the particle line and the hole line may carry the same effective-Landau-level index or different indices. For the inter-Landau-level process, there is an excitation gap which is the order of $\Delta\omega_c$. Thus, for $y < \Delta\omega_c$, the intra-Landau-level process is the only contribution to $D_{11}(q,y)$. As shown before, the intra-Landau-level contribution to a particle-hole bubble gives rise to the $n_F(\xi_t)(1 - n_F(\xi_t))$ factor in the gauge field propagator, which becomes exponentially small for $T \ll \Delta\omega_c$. This suggests that $D_{11}(q,y)$ becomes exponentially small for $y < \Delta\omega_c$ and $T \ll \Delta\omega_c$ so that we can ignore the contribution coming from $y < \Delta\omega_c$ for our purpose. Thus we consider only the contribution coming from the inter-Landau-level process, which appears only above the gap $\Delta\omega_c$. For $\mu/\Delta\omega_c \gg 1$ or the large $p$ approximation, one may argue that the smearing of the discrete spectral function $D_{11}(q,y)$ of the gauge field propagator, which comes from the Landau-level structure, does not cause any significant change in the global behavior of the response functions. Therefore, we use $D_{11}''(q,y)$ for $\Delta B = 0$ instead of $D_{11}''(q,y)$ for finite $\Delta B$, but a lower cutoff $\Delta\omega_c$ is introduced in the $y$ integral in Eq.(25) to mimic the gap in $D_{11}''(q,y)$. Since the precise value of the gap is not known, the numerical coefficient of the final answer to the response function is unreliable, but the functional dependence on $\Delta\omega_c$ is not affected.

The transverse gauge field propagator $D_{11}(q,\omega)$ for $\Delta B = 0$ is given by $1/(-i\gamma \frac{\chi}{q} + \chi q^\eta)$ [5], where $\gamma = \frac{2n_{eF}}{k_F}$, $\chi = \frac{1}{24\pi m} + \frac{V_0}{(2\pi\phi)^2}$ for $\eta = 2$, and $\chi = \frac{V_0}{(2\pi\phi)^2}$ for $\eta \neq 2$. For the large
The evaluation of the $q$ integral in Eq. (25) gives us
\[
\int \frac{d^2q}{(2\pi)^2} |M_{pp}(q)|^2 D''_{11}(q, y) \approx -\frac{mv_F}{8\pi^3} \frac{1}{1+\eta} \frac{1}{\sin \left(\frac{\pi}{1+\eta}\right)} \gamma^{-\frac{\eta-1}{\eta+1}} \chi^{-\frac{2}{\eta+1}} y^{-\frac{\eta-1}{\eta+1}} (\Delta \omega_c)^2.
\]

(29)

Now we can perform the $y$ integral, yielding
\[
I \approx 2 \int_{\frac{\Delta \omega_c}{2}}^{\infty} \frac{dy}{\pi} \sum_q |M_{pp}(q)|^2 \frac{D''_{11}(q, y)}{y} = -\frac{mv_F}{4\pi^4} \frac{1}{\eta-1} \frac{1}{\sin \left(\frac{\pi}{1+\eta}\right)} \gamma^{-\frac{\eta-1}{\eta+1}} \chi^{-\frac{2}{\eta+1}} (\Delta \omega_c)^{\frac{\eta+3}{\eta+1}}.
\]

(30)

Therefore, for $\xi_{p+1} = |\xi_p| = \Delta \omega_c/2$, we get
\[
\frac{\partial n}{\partial \mu} \approx \frac{A(\eta)}{\eta-1} \frac{m}{\pi} \frac{\Delta \omega_c^{\frac{\eta+3}{\eta+1}}}{T^2},
\]

(31)

where
\[
A(\eta) = \frac{v_F}{2\pi^3} \frac{1}{\sin \left(\frac{\pi}{1+\eta}\right)} \gamma^{-\frac{\eta-1}{\eta+1}} \chi^{-\frac{2}{\eta+1}}.
\]

(32)

Combining the result of Eq. (31) and that of the free fermions given by Eq. (20), we get
\[
\frac{\partial n}{\partial \mu} \approx \frac{m(\Delta \omega_c)}{\pi T} e^{-\Delta \omega_c/2T} \left(1 + \frac{A(\eta)}{\eta-1} \frac{\Delta \omega_c^{\frac{3}{T}}}{T^2}\right).
\]

(33)

This is the central result of this paper.

Note that $A(\eta)$ should be proportional to a small expansion parameter, for example, $1/N$ in a large $N$ generalized model. Thus $1 + \frac{A(\eta)}{\eta-1} \frac{\Delta \omega_c^{\frac{3}{T}}}{T^2} \approx e^{\frac{A(\eta)}{\eta-1} \frac{\Delta \omega_c^{\frac{3}{T}}}{T^2}}$, so that the result of Eq. (33) is consistent with the renormalized energy gap $\Delta \omega_c^* \approx \Delta \omega_c \left(1 - \frac{2A(\eta)}{\eta-1} (\Delta \omega_c)^{-\frac{\eta-1}{\eta+1}}\right)$ if we write $\partial n/\partial \mu \propto e^{-\Delta \omega_c^*/2T}$. This implies that $m^*/m \approx 1 + \frac{2A(\eta)}{\eta-1} (\Delta \omega_c)^{-\frac{\eta-1}{\eta+1}}$ from $\Delta \omega_c^* = \frac{eA_B}{m^* c}$. In particular, for the Coulomb interaction ($\eta = 1$), $\Delta \omega_c^* \approx \Delta \omega_c \left(1 - 2A(\eta = 1) \ln \left(\frac{\epsilon_F}{\Delta \omega_c}\right)\right)$ and $m^*/m \approx 1 + 2A(\eta = 1) \ln \left(\frac{\epsilon_F}{\Delta \omega_c}\right)$. These results were predicted by HLR in terms of a self-consistency argument [5] and are also consistent with the recent work of Stern and Halperin [19].
IV. POLARIZATION BUBBLE VERSUS SELF-ENERGY

In the previous sections, we used Eq.(16) and the subsequent derivatives of $\Omega_{\text{toy}}$ to get the correction to the compressibility. There is an alternative way to express $\Omega_{\text{toy}}$, which involves the use of the self-energy. That is, Eq.(15) can be written as

$$\Omega_{\text{toy}} = -T \sum_{i\omega_n} \sum_{l} \frac{m\Delta \omega_c}{2\pi} \Sigma(\xi_l, i\omega_n) \ G(\xi_l, i\omega_n) \ ,$$

where $\omega_n = (2n + 1)\pi T$ is the Matsubara frequency and $G(\xi_l, i\omega_n) = \frac{1}{i\omega_n - \xi_l}$. $\Sigma(\xi_l, i\omega_n)$ is the one-loop self-energy correction dressed by the gauge field $\tilde{D}_{11}$ and is given by the diagrams in Fig.5. We note that $\Omega_{\text{toy}}$ is finite, whereas $\Sigma$ is known to be infinite (for $\eta = 2$) at finite temperatures and $\Delta \omega_c = 0$. In this section, we wish to clarify how this apparent difficulty is resolved. Using the spectral representation, we can rewrite Eq.(34) as

$$\Omega_{\text{toy}} = \Omega_c + \Omega_d \ ,$$

$$\Omega_c = \frac{m\Delta \omega_c}{2\pi} \sum_{l} \int_{-\infty}^{\infty} \frac{dx}{\pi} \ n_F(x) \ \Sigma''(\xi_l, x) \ G'(\xi_l, x) \ ,$$

$$\Omega_d = \frac{m\Delta \omega_c}{2\pi} \sum_{l} \int_{-\infty}^{\infty} \frac{dx}{\pi} \ n_F(x) \ \Sigma'(\xi_l, x) \ G''(\xi_l, x) \ .$$

Now we would like to compare two ways of calculating $\Omega_{\text{toy}}$. First let us discuss the case of $\Delta \omega_c = 0$. If we use Eq.(16), one can show that $\Omega_a$ is finite by using $\Pi''_{11}(q, x) \approx -\gamma x/q$. Suppose that we are going to use only the first diagram of the transverse part of the polarization bubble in Fig.2 (b) to calculate $\Omega_b$. Since the leading contribution of the first diagram to $\Pi'$ is given by $n_0/m$ where $n_0$ is the density of the free fermions, it can be shown that $\Omega_b$ diverges in this case. However, the second diagram also contributes $-n_0/m$ which cancels the constant term of the first diagram. This cancellation is required by the gauge-invariance. As a result, $\Pi' \approx \chi_0 q^2$ with $\chi_0 = \frac{1}{24\pi m}$ so that $\Omega_b$ becomes finite. In particular, for the short range interaction ($\eta = 2$), $\Omega_a$ and $\Omega_b$ give rise to the same contributions with different coefficients.

Next we examine what happens if we use Eq.(35) which expresses $\Omega_{\text{toy}}$ in terms of the
self-energy. For $\Delta \omega_c = 0$, Eq.(35) can be rewritten as

$$\Omega_{\text{toy}} = \Omega_c + \Omega_d,$$

$$\Omega_c = \sum_k \int_{-\infty}^{\infty} \frac{dx}{\pi} n_F(x) \Sigma''(\xi_k, x) G'(\xi_k, x),$$

$$\Omega_d = \sum_k \int_{-\infty}^{\infty} \frac{dx}{\pi} n_F(x) \Sigma'(\xi_k, x) G''(\xi_k, x).$$

As a well known result [8], $\Sigma''(\xi_k, x)$ diverges for $T \neq 0$. Thus we may conclude that $\Omega_c$ diverges and this divergence must be cancelled by a similar term in $\Omega_d$. Now one may wonder whether there is any cancellation at the self-energy level especially between the first and the second diagrams in Fig.5 as in the case of the polarization bubbles. Since the second diagram generates only the real part, there is no cancellation in $\Sigma''$. For $\Sigma'$, both of the two diagrams contribute. However, one can see that there is no cancellation between the two contributions because of the presence of the additional fermion propagator in the first diagram. We believe that these are the symptoms of the gauge non-invariant nature of the self-energy. In the previous sections, we consider first the polarization bubbles which are gauge-invariant objects. Note that there is an explicit cancellation in this gauge-invariant combination. Therefore, we think that using the polarization bubble makes the gauge-invariance manifest.

Armed with these arguments, we can investigate the $\Delta \omega_c \neq 0$ case. Recalling that the first and the second terms of Eq.(12) correspond to the first and the second diagrams of Fig.5, we may anticipate a similar cancellation between these two terms as the case of $\Delta \omega_c = 0$. Indeed the f-sum rule, which is given by

$$\sum_{lm} |M_{lm}(q \to 0)|^2 \frac{n_F(\xi_l) - n_F(\xi_m)}{\xi_m - \xi_l} = \frac{1}{m} \left( \frac{m \Delta \omega_c}{2\pi} \sum_l n_F(\xi_l) \right) = \frac{n_0}{m},$$

allows a cancellation between the first term and the second term in the $q \to 0, i\nu_n \to 0$ limit.

We use this result in the appendix to estimate various contributions to the compressibility.

V. CONCLUSION

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In the previous paper [17], we showed that the density-density correlation function has a Fermi-liquid form as far as the long wavelength and the low frequency limits are concerned. An important issue is whether this result is compatible with the previous self-consistency treatment based on the one-loop self-energy correction [5] and the present calculation of the energy gap, which are in favor of a divergent effective mass at the half-filling. For a class of Fermi-liquid interaction parameters $f_{pp'}$, which gives a finite angular average $f_{0s}$, three of us with Furusaki demonstrated that the effective mass is finite if we want to fit the result of the density-density correlation function to the usual Fermi-liquid theory framework [17]. However, it is still possible that the effect of the divergent effective mass is cancelled by a contribution from a singular $f_{pp'}$, which gives a divergent $f_{0s}$, in the density-density correlation function [19]. From the previous paper [17], it is clear that this scenario is possible only for $f_{pp'} \propto \frac{1}{|\hat{p}-\hat{p}'|}$ or $f_{pp'} = \zeta \delta(\hat{p} - \hat{p}')$ with a divergent $\zeta$ (Note that an $f_{pp'}$ of the delta function type can be absorbed into the definition of the finite Fermi velocity if $\zeta$ is finite [17]). Even though this possibility is quite plausible, it is still not clear whether we are allowed to interpret all physical measurements in terms of the conventional Fermi-liquid theory. Thus we are still at the stage of collecting necessary informations for the ultimate understanding of the effect of the gauge field fluctuations on the composite fermions.

Recently Stern and Halperin [19] calculated the energy gap of the system from the one-particle Green’s function of the composite fermions in a finite effective magnetic field $\Delta B$. They identified the region where the spectral function vanishes at zero temperature, which is argued to be gauge-invariant, and found an energy gap which is in agreement with the previous self-consistency treatment [5] and the present calculation. The advantage of our calculation is that we directly evaluated the gauge-invariant two particle Green’s function, and we could consider the finite temperature situation. We would like to mention that the present perturbative calculation suggests that the perturbation theory for the compressibility breaks down for sufficiently small $\Delta \omega_c$ in the sense that the correction to
the energy gap becomes larger than the bare energy gap. Thus one needs a truly gauge-invariant non-perturbative treatment in order to understand this peculiar system.

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Appendix

In this appendix, we show that \( \frac{\partial n_{a2}}{\partial \mu} \) and \( \frac{\partial n_{b}}{\partial \mu} \) are exponentially smaller than \( \frac{\partial n_{a1}}{\partial \mu} \) which is calculated in the main text. As discussed in section IV, there is a partial cancellation between \( \Omega_{b1} \) and \( \Omega_{b2} \) in Eq.(22) due to the f-sum rule given by Eq.(37). As a result, \( \Omega_{b} \) can be rewritten as

\[
\Omega_{b} \approx \sum_{q} \int_{0}^{\infty} \frac{dx}{\pi} (1 + 2n_{B}(x)) \tilde{D}_{11}''(q, x) \left[ -\sum_{lm} |M_{lm}(q)|^2 \right. \\
\times \left( \frac{n_{F}(\xi_{l}) - n_{F}(\xi_{m})}{x - \xi_{m} + \xi_{l}} - \frac{n_{F}(\xi_{l}) - n_{F}(\xi_{m})}{\xi_{l} - \xi_{m}} \right) \\
= \sum_{q} \int_{0}^{\infty} \frac{dx}{\pi} (1 + 2n_{B}(x)) x \tilde{D}_{11}''(q, x) \left[ -\sum_{lm} |M_{lm}(q)|^2 \frac{n_{F}(\xi_{l}) - n_{F}(\xi_{m})}{(x - \xi_{m} + \xi_{l}) (\xi_{l} - \xi_{m})} \right].
\]

From Eq.(21) and Eq.(A.1), we get the lowest order correction to the density of the fermions \( n_{1} = n_{a} + n_{b} \) as follows.

\[
n_{a} = n_{a1} + n_{a2},
\]

\[
n_{a1} = -\frac{1}{T} \sum_{q} \sum_{l} |M_{ll}(q)|^2 D_{11}'(q, 0) n_{F}(\xi_{l})(1 - n_{F}(\xi_{l}))(1 - 2n_{F}(\xi_{l})),
\]

\[
n_{a2} = -\frac{1}{T} \sum_{q} \sum_{l \neq m} |M_{lm}(q)|^2 D_{11}'(q, \xi_{m} - \xi_{l}) \left[ n_{F}(\xi_{m})(1 - n_{F}(\xi_{m}))(1 - n_{F}(\xi_{l})) \\
- n_{F}(\xi_{m})n_{F}(\xi_{l})(1 - n_{F}(\xi_{l})) \right],
\]

and

\[
n_{b} \approx \frac{1}{T} \sum_{q} \int_{0}^{\infty} \frac{dx}{\pi} (1 + 2n_{B}(x)) x D_{11}''(q, x) \\
\times \left[ \sum_{lm} |M_{lm}(q)|^2 \frac{n_{F}(\xi_{l})(1 - n_{F}(\xi_{l})) - n_{F}(\xi_{m})(1 - n_{F}(\xi_{m}))}{(x - \xi_{m} + \xi_{l}) (\xi_{l} - \xi_{m})} \right].
\]

These equations are equivalent to Eq.(17).

As shown in Eq.(18), in order to calculate the compressibility, one should take the derivative of both \( D_{11}(q, x) \) and \( \Pi_{11}(q, x) \). Note that \( \frac{\partial D_{11}}{\partial \mu} \sim D_{11}^{-2} \frac{\partial \Pi_{11}}{\partial \mu} \) and \( \frac{\partial \Pi_{11}}{\partial \mu} \) contains the factor \( n_{F}(\xi_{l})(1 - n_{F}(\xi_{l})) \). Thus \( \frac{\partial D_{11}}{\partial \mu} \) generates additional factors \( e^{-|\xi_{l}|/T} \) and \( e^{-|\xi_{l}+1|/T} \).
Since we want to keep only the terms which are proportional to $e^{-|\xi_p|/T}$ or $e^{-\xi_{p+1}/T}$, we can ignore the terms $\frac{\partial n_{11}}{\partial \mu} \frac{\partial D_{11}}{\partial \mu}$, which are of order $e^{-2|\xi_p|/T}$. Ignoring these terms in Eq.(18) which is equivalent to keeping only the $\mu$ dependence in $n_F$ in Eq.(A.2) and Eq.(A.3), the lowest order correction to the compressibility $\frac{\partial n_{1}}{\partial \mu}$ can be calculated as follows.

\[
\frac{\partial n_{a}}{\partial \mu} = \frac{\partial n_{a1}}{\partial \mu} + \frac{\partial n_{a2}}{\partial \mu},
\]

\[
\frac{\partial n_{a1}}{\partial \mu} \approx -\frac{1}{T^2} \sum_{q} \sum_{l} |M_{ll}(q)|^2 D_{11}'(q,0)
\times n_F(\xi_l)(1 - n_F(\xi_l)) \left[ 1 - 6 n_F(\xi_l)(1 - n_F(\xi_l)) \right],
\]

\[
\frac{\partial n_{a2}}{\partial \mu} \approx -\frac{1}{T^2} \sum_{q} \sum_{l \neq m} |M_{lm}(q)|^2 D_{11}'(q,\xi_m - \xi_l)
\times \left[ n_F(\xi_m)(1 - n_F(\xi_m))(1 - 2n_F(\xi_m))(1 - n_F(\xi_l))
\right.
\]
\[
\left. - n_F(\xi_m)n_F(\xi_l)(1 - n_F(\xi_l))(1 - 2n_F(\xi_l)) \right] \quad \text{(A.4)}
\]

and

\[
\frac{\partial n_{b}}{\partial \mu} \approx -\frac{1}{T^2} \sum_{q} \int_{0}^{\infty} \frac{dx}{\pi} (1 + 2n_B(x)) x D_{11}''(q,x) \left[ \sum_{lm} \frac{|M_{lm}(q)|^2}{(x - \xi_m + \xi_l)(\xi_l - \xi_m)} \right.
\times \left. n_F(\xi_l)(1 - n_F(\xi_l))(1 - 2n_F(\xi_l)) \right.
\]
\[
\left. - n_F(\xi_m)(1 - n_F(\xi_m))(1 - 2n_F(\xi_m)) \right]. \quad \text{(A.5)}
\]

Keeping only the terms that are proportional to $e^{-|\xi_p|/T}$ or $e^{-\xi_{p+1}/T}$, one can show that the contributions from $\frac{\partial n_{a1}}{\partial \mu}$ and $\frac{\partial n_{a2}}{\partial \mu}$ do not contain such terms that are proportional to $e^{-|\xi_p|/T}$ or $e^{-\xi_{p+1}/T}$. This result can be obtained as follows. In each case of $\frac{\partial n_{a1}}{\partial \mu}$ and $\frac{\partial n_{a2}}{\partial \mu}$, the first term and the second term inside the square bracket contain contributions proportional to $e^{-|\xi_p|/T}$ or $e^{-\xi_{p+1}/T}$. It can be seen that these contributions in the first term cancel each other when the chemical potential lies exactly at the middle of the successive effective Landau levels, and thus they correspond to a uniform shift in these Landau levels.

The same story applies to the second term in the square bracket. However, it turns out that the contributions from the first term and the second term cancel again each other so
that the contributions proportional to $e^{-|\xi_p|/T}$ or $e^{-\xi_{p+1}/T}$ do not exist in general. Thus

$$\frac{\partial n_{a2}}{\partial \mu} = \mathcal{O}(e^{-2|\xi_p|/T}) \quad \text{and} \quad \frac{\partial n_b}{\partial \mu} = \mathcal{O}(e^{-2|\xi_p|/T})$$

so that we can ignore these contributions compared to $e^{-|\xi_p|/T}$ or $e^{-\xi_{p+1}/T}$. 

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Figure captions

Fig.1 (a) The diagram that represents the density of the free fermions in an effective magnetic field $\Delta B$. (b) The lowest order correction to the density of the fermions due to the gauge field fluctuation. Here the solid line represents the bare electron propagator. The wavy line denotes the RPA gauge field propagator which is given by the diagram in Fig.2 (a).

Fig.2 (a) The wavy line denotes the RPA gauge field propagator and the dashed line is the bare gauge field propagator. Here the hatched bubble (b) represents the transverse part of the polarization bubble.

Fig.3 The diagrams that correspond to the thermodynamic potential of the free fermions (a) and the gauge field contribution (b) to the thermodynamic potential.

Fig.4 The diagrams that represent the lowest order correction to the compressibility of the fermions.

Fig.5 The diagrams that represent the lowest order correction to the self-energy of the fermions.