Time evolution of link length distribution in PRL collaboration network

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An important aspect of a Euclidean network is its link length distribution, studied in a few real networks so far. We compute the distribution of the link lengths between collaborators whose papers appear in the Physical Review Letters (PRL) in several years within a range of four decades. The distribution is non-monotonic; there is a peak at nearest neighbour distances followed by a sharp fall and a subsequent rise at larger distances. The behaviour of the statistical properties of the distribution with time indicates that collaborations might become distance independent in about thirty to forty years.

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Ever since the discovery of small world effect in a variety of networks [1], study of real world networks and their theoretical modelling have generated tremendous activity. A network is equivalent to a graph and is characterised by the links which connect pairs of nodes. Based on observations and theoretical arguments, it has been established that factors like preferential attachment, duplication, aging etc. are responsible in determining the connectivity in many real world networks [2].

In a Euclidean network, where the nodes are embedded on a Euclidean space, it can be expected that the distance between nodes will play an important role in determining whether a link will connect them. In several theoretical models of Euclidean network, the link length distribution has been assumed to have a power law decay [3].

Linking schemes in a few real world networks in which geographical distance plays an important role have been studied. These are the internet [4], transport [5], neural network [6] and some collaboration networks [7–10]. In this article, we report the study of a network of collaborators whose papers appear in Physical Review Letters. We also study this distribution at different times as it is a dynamic network and reflects the evolution of both communication and human interactions.

Scientific collaboration network is a social network [11,12] in which close personal encounters are essential to a large extent and it is expected that the existence of links between authors will depend on the distance separating them. Communication is the key factor in a collaboration and it has undergone revolutionary changes over the years. This effect will manifest in the time evolution of the link length distribution. We have therefore studied its behaviour over four decades.

To obtain the link length distribution, one should take the collaboration network and calculate the geographical distances separating the host institutes of the authors who share a link. However, this becomes a formidable task. We have obtained the distance distribution in an indirect way. Noting that the collaboration acts are the papers, the distance between the co-authors in a particular paper would also supply the necessary data. We have therefore taken sample papers (at least 200 for each year) from the Physical Review Letters (PRL) and calculated the geographical distance between each pair of authors in a coarse grained manner for nine different years between 1965 to 2005 and obtained the link-length distributions.

The pair-wise distances \( l \) gives the distribution \( P(l) \) of the distance between two collaborating authors. We have also defined a distance factor \( d \) for each paper where \( d \) is the average of the pair-wise distances of authors co-authoring that paper. The corresponding distribution \( Q(d) \) has also been computed. For example, let there be a paper authored by three scientists and let \( l_{12}, l_{13}, l_{23} \) be the pairwise distances. Then \( d = (l_{12} + l_{13} + l_{23})/3 \). Note that in \( P(l) \), the fact that \( l_{12}, l_{13} \) and \( l_{23} \) are obtained from a single collaboration act is missing. Hence, in a sense, \( Q(d) \) takes care of the correlation between the distances.

Let us call \( Q(d) \) the correlated distance distribution.

In principle, the actual geographical distances have to be computed which is non-trivial. We have coarse grained the distances in a convenient way. To author X in a paper we associate the indices \( x_1, x_2, x_3 \) and \( x_4 \) (\( x_i \)'s are integers) which represent the University/Institute, city, country and continent of X respectively. Similar indices \( y_1, y_2, y_3 \) and \( y_4 \) are defined for author Y. If, for example, authors X and Y belong to the same institute, \( x_i = y_i = 1 \) for all i. On the other hand, if they are from different countries but the from same continent, \( x_4 = y_4 \) but \( x_i \neq y_i \) for \( i < 4 \). We find out for what maximum value of \( k \), \( x_k \neq y_k \). The distance between X and Y is then \( l_{XY} = k + 1 \). If \( x_i = y_i \) for all values of i it means \( l_{XY} = 1 \) according to our definition. As an example, one may consider the paper PRL 64 2870 (1990), which features 4 authors. Here authors 1 and 2 are from the same institute in Calcutta, India, and are assigned the variables 1, 1, 1, 1. The 3rd author belongs to a different institute in Calcutta and therefore gets the indices 2, 1, 1, 1. The last author is from an institute in Bombay, India, and is assigned the variables 3, 2, 1, 1. Hence \( l_{12} = 1, l_{13} = l_{23} = 2, l_{14} = l_{24} = l_{34} = 3 \) and the average \( d = 2.333 \). Defining the distances in this way, the values
of $l$ are discrete while the $d$ values have a continuous variation. For papers with two authors, the two distributions are identical but will be different in general.

![Distance distribution](image1)

**FIG. 1.** Distance distribution $P(l)$ as function of distance $l$ for different years.

![Correlated distance distribution](image2)

**FIG. 2.** Correlated distance distribution $Q(d)$ vs distance $d$ plot for different years are shown.

We have made exception for USA authors since it is a big country comparable in size to Europe which consists of many countries. Thus two authors belonging to, say, Kentucky and Maryland will have different country indices, i.e., $x_3 \neq y_3$.

Some papers like the experimental high energy physics ones typically involve many authors and many institutes. We have considered an upper bound, equal to 20, to the number of institutes and no bounds for the number of authors. In case of multiple addresses, only the first one has been considered.

Both the distributions $P(l)$ and $Q(d)$ have the following features:

1. A peak at $l$ or $d = 1$
2. A sharp fall at around $l$ or $d = 2$ and a subsequent rise. The fall becomes less steep in time.
3. Even for the most recent data, the peak at nearest neighbour distances is quite dominant. However, with the passage of time, the peak value at nearest neighbour distances shrinks while the probability at larger distances increases.

In Figs. 1 and 2, the distributions $P(l)$ and $Q(d)$ are shown. The two distributions have similar features but differ in magnitude, more so in recent years, when the number of authors is significantly different from two in many papers. The data for $Q(d)$ apparently has an oscillatory nature for larger values of $d$. However, we believe that these oscillations are due to the coarse graining of the data and it is more likely that the peak at the nearest neighbour distances is followed by a crest and a gentle hump at larger distances. The hump grows in size with time while the peak value at nearest neighbour distances diminishes.

![Mean value and standard deviation of distances](image3)

**FIG. 3.** The mean value and standard deviation of distances $d$ increase with time while the roughness of the distance distribution $Q(d)$ shows a steady decrease.

We make a detailed analysis of $Q(d)$, the correlated distance distribution. In Fig. 3, we present the results. The mean increases appreciably in consistency with our idea that with the progress of time there will be more collaborations involving people working at a distance. The fluctuation also shows an increase, although its increase is not that remarkable since the total range of interaction remains fixed in our convention. If collaborations were really distance independent, the distributions $Q(d)$ and $P(l)$ would have looked flat. We have estimated the deviation of $Q(d)$ from a flat distribution by calculating its “roughness” $R_Q$ defined as \( \sqrt{\langle (Q(d) - \bar{Q})^2 \rangle} \) where $\bar{Q}(d)$ is the mean value of $Q(d)$. $R_Q$ shows a decrease with time which is approximately linear.

The above results imply that even with the communication revolution, most collaborations take place among nearest geographical neighbours. The drop near $d = 2$ may be justified from the fact that in most cities one has only one university/institute and when one collaborates with an outsider, she or he belongs to some other city or country in most cases. There is some indication that in the not too distant future collaborations will become almost distance independent as in Fig. 3, $R_Q$ seems to vanish at around 2040 when extrapolated. It may also
happen that $R_Q$ saturates to a finite value in the coming years, and perhaps it is too early to predict anything definite.

What is the nature of the distribution when the real distances are considered? We notice that there is a sharp decrease of $Q(d)$ with $d$ initially which may be assumed to be exponential in nature. The way we have defined $l$ (or $d$), it maybe assumed that the true distances $d_{\text{real}}$ scale roughly as $\exp(\alpha d^a)$ where $a$ is a number of the order of unity. In that case, the initial exponential decay of $Q(d)$ with $d$ corresponds to a power law decrease with $d_{\text{real}}$. The subsequent rise of the distribution with $d$ should also show up against $d_{\text{real}}$.

In summary, we have studied the link length distributions in the Euclidean network of collaborators of PRL papers. Unlike the other features of a network, e.g., degree distribution or aging, we do not find a conventional power law or exponential decay but rather a non-monotonic behaviour. The data over different times shows that the communication revolution has indeed influenced long distance collaborations to a considerable extent.

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