THE I MPLICATIONS OF DIRECT REDSHIFT MEASUREMENT OF GAMMA-RAY BURSTS

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ABSTRACT

The recent discoveries of X-ray and optical counterparts for γ-ray bursts (GRBs), and the possible discovery of a host galaxy, imply that direct measurement of the redshift of some GRB host galaxies is imminent. We discuss the implications of such measurements. These measurements could enable us to determine the GRB luminosity distribution and the variation of the rate of GRBs with cosmic time and even, under favorable circumstances, to estimate Ω. Using GRB 970508 alone, assuming standard candles, and assuming the GRB source to be at the redshift of the absorption line observed in the optical transient spectrum, we constrain the intrinsic GRB evolution to ρ(z) = (1 + z)^{-0.5 ± 0.7}.

Subject headings: cosmology: observations — gamma rays: bursts

1. INTRODUCTION

The recent observations of γ-ray bursts (GRBs) by the Italian-Dutch satellite BeppoSAX (Costa et al. 1997), with error boxes of a few arcminutes across, enabled a follow-up with optical and radio observations and the discovery of X-ray, optical, and radio counterparts to GRBs (see, e.g., Bond et al. 1997; Galama et al. 1997; Heise et al. 1997; Piro et al. 1997; Sahu et al. 1997; van Paradijs et al. 1997). The optical observations provided, for the first time, independent estimates of the distances to GRB sources, using absorption lines or association with host galaxies (Metzger et al. 1997), and demonstrated beyond doubt the cosmological origin of GRBs. It is highly possible that in the months to come several GRBs will have independent redshift estimates. We could use these to obtain estimates of the luminosity diversity and the intrinsic evolution of the GRB rate with cosmic time. Under favorable conditions we might even be able to use GRBs as cosmic probes for estimating cosmological parameters.

Given a group of GRBs with measured fluxes and redshifts, we should first calculate the luminosity distribution of the sources. Then, using this luminosity function, we should estimate the theoretical cosmological peak-flux distribution. Comparison of this distribution with the observed one would yield a direct estimate of the cosmic evolution of GRBs—that is, the variability of the GRB rate per unit comoving volume per unit comoving time. Depending on the width of the luminosity function, the null assumption of no cosmic evolution could be proved or ruled out with a few dozen bursts. Cosmological parameters like the closure parameter, Ω, and the cosmological constant, Λ, influence only weakly the peak-flux distribution (Cohen & Piran 1995); hence, this analysis can be done safely assuming that Ω = 1 and Λ = 0. However, once the luminosity function is known, we might be able to use the peak-flux redshift relation to obtain a direct measure of Ω, provided that the luminosity function is narrow enough.

2. THE LUMINOSITY FUNCTION

A measurement of a redshift of the optical counterpart of a GRB or the association of a host galaxy with a GRB provides us with the redshift of the burst, z. In addition, we have the usual peak photon flux parameter, p (photons cm^{-2} s^{-1}), which is transferred to apparent luminosity, L (ergs cm^{-2} s^{-1}), using bursts’ spectra. The peak photon flux is related to the redshift and source luminosity as

$$p = \frac{N[p_1(1+z) \cdots p_2(1+z)]}{4\pi(1+z)(2cH_0)[1 - (1 + z)^{-1/2}]}.$$  (1)

The detector boundaries $p_1$, $p_2$ are 150 keV and 300 keV, respectively, for the BATSE detector. (In order to maintain a uniform catalog, we use BATSE data for all bursts, including those detected by BeppoSAX.) The Hubble distance is $cH_0$, and $N[p_1, p_2]$ is the number of photons emitted in the range $[p_1, p_2]$. We have used $Ω = 1$ and $Λ = 0$ in equation (1). The effect of $Ω$ and $Λ$ on the luminosity is not large, and we will discuss it later. The luminosity depends on the Hubble constant only via the scale factor $h_3 = [H_0/(75 \text{ km s}^{-1} \text{ Mpc}^{-1})]^2$.

When comparing bursts from different redshifts, one must recall that the observed peak flux is in a fixed energy range, which corresponds to different energy ranges at the sources. In order to discuss a single luminosity that classifies the bursts, we consider $L = \int_{50 \text{ keV}}^{300 \text{ keV}} L_\nu d\nu$ at the source. To convert from the observed peak flux to the intrinsic luminosity, we assume that the source spectral form is a power law $L_\nu = L_\nu(50 \text{ keV})^{-(2 - \alpha)/(\epsilon_2^0 - 1)}(50 \text{ keV})^{-\epsilon_2}$ in the energy range 50 keV < $\nu$ < 300 keV(1 + $z_{\text{max}}$), so that wherever the source is, the detector sees power-law spectra. We use $\alpha = 1.5$ for all bursts. This value is probably a good typical estimate (Band et al. 1993), even though the spectra are not the same for all bursts. If necessary, one can use the measured spectrum of each burst to estimate its intrinsic luminosity at the 50–300 keV band. However, at this stage this simple estimate is sufficient. Alternatively, one can view the variability in the spectral index as an additional random variable that simply widens the luminosity function. Using this spectral shape, we obtain

$$L = 7.7 \times 10^{30} \text{ ergs s}^{-1} \times \left(\frac{l}{10^{-7} \text{ ergs s}^{-1} \text{ cm}^{-2}}\right) \frac{[1 - (1 + z)^{-1/2}]^2}{(1 + z)^{-\epsilon_2 / 2}}.$$  (2)

Using equation (2), we determine the luminosity of each burst. Then we estimate the luminosity function using maximum likelihood or any other statistical method. To do so, we assume a functional shape of the luminosity function, determine
its parameters from the data, and then estimate the quality of the fit. Using the Cramér-Rao inequality (Porat 1993), we can estimate the statistical error in this procedure. If the luminosity function has a form of a normal distribution, with 20 bursts we will be able to estimate, with 95% confidence, the standard deviation to ±30% of its true value. For a power-law distribution, 20 bursts will enable us to determine the power-law index to ±0.5, again with 95% confidence. Recall that current data, and in particular the peak-flux statistics of GRBs, do not constrain the luminosity distribution of GRBs (Loredo & Wasserman 1995).

The luminosity distribution obtained in this way is for detected bursts only, and therefore it is biased. We define by \( V(L) \) the volume from which we can detect a burst of luminosity \( L \). Then the distribution we measure is \( \Phi(L)V(L) \), where \( \Phi(L) \) is the intrinsic luminosity function. By dividing the measured distribution by \( V(L) \), we remove this bias from our estimate. Clearly, if the subgroup of GRBs with optical counterparts is furthermore biased, this distribution estimate will be biased in the same way.

3. COSMOLOGICAL GRB EVOLUTIONS

One of the interesting features that might distinguish different cosmological GRB models is the rate at which GRBs occur per unit time per unit comoving volume: \( \rho(z) \). These new measurements could yield a direct estimate of this distribution. Once the GRB luminosity distribution is known, we can proceed and compare the theoretical peak-flux statistics (using the observed luminosity distribution) with the observed one. This distribution depends strongly on the intrinsic evolution of GRBs, that is, on variation of \( \rho(z) \). Following Cohen & Piran (1995), we characterize this dependence as \( \rho(z) = (1 + z)^{-b} \).

Comparison of the theoretical and observed distribution would limit \( b \).

Using the measured luminosity function, \( \Phi(L) \), which is based on bursts with measured redshifts, we calculate the theoretical peak-flux statistics, \( N_{\text{peak}}(p) \), for different values of the evolution parameter \( b \). For each \( b \) we use the BATSE peak-flux data to calculate the likelihood function log \( \Pi N_{\text{peak}}(p) \), assuming an evolution \( (1 + z)^{-b} \). Then we search for \( b \) with the maximum likelihood method and for \( b \) where the likelihood falls to 1% of this value. Finally, we use the K-S test to see if the best solution is consistent with the data.

In fact, this comparison can be done even with the current data and the assumption of a narrow luminosity distribution (standard candles). We can use GRB 970508 to constrain the evolution. Using a peak flux equal to \( 1.6 \times 10^{-3} \) ergs s\(^{-1}\) cm\(^{-2}\) (Kouveliotou et al. 1997), a redshift of the absorption lines of \( z = 0.835 \) (Metzger et al. 1997), which sets a lower limit of \( z > 0.835 \) for the burst, and the absence of prominent Ly\(\alpha\) forest in the spectrum that contains an upper limit of \( z < 2.1 \), we obtain \( b = -0.1 \pm 1.3 \) with a 99% confidence level. Assuming that the absorption line of GRB 970508 corresponds to its own redshift, we estimate that \( b = 0.5 \pm 0.7 \) with this confidence level (see Fig. 1). The simplest hypothesis of no evolution, \( b = 0 \), is consistent with the observations. A milder assumption of Gaussian luminosity distribution with \( \sigma_L = L_{\text{obs}}/2 \), instead of standard candles, yields a lower limit of \( b > -0.7 \) and no upper limit.

Without an independent estimate for the burst luminosity, one could fit the data with various models: (i) medium luminosity bursts with no evolution—the bursts are at an intermediate redshift, and the paucity of weak bursts originates from cosmological effects; (ii) low-luminosity bursts with a positive evolution—the bursts are at a low redshift, and the paucity of weak bursts originates from the evolution; and (iii) high-luminosity bursts with a negative evolution—the bursts are at a high redshift, and the evolution somewhat masks the cosmological effects. There is no way to distinguish, from the peak-flux statistics alone, among these possibilities. With an independent luminosity estimate, we can determine which of the above possibilities is the right one. This results in a better estimate of the evolution parameter.

Earlier estimates to estimate burst evolution without an independent luminosity estimate found only mild limits. Cohen & Piran (1995) found no limit on \( \beta \); Rutledge, Hui, & Lewin (1995) obtained a limit of \( \beta < 3 \); and Loredo & Wasserman (1996) obtained a limit of \( -2.75 < \beta < 1 \). Our preliminary limit, which is based on a single burst, restricts the limits regarding negative evolution significantly.

4. ESTIMATES OF COSMOLOGICAL PARAMETERS

Despite numerous attempts to estimate the cosmological closure parameter, \( \Omega \), its actual value is still unknown, and current estimates range from 0.2 to 1 (see, e.g., Peebles 1996). One may wonder whether GRBs would provide a meaningful independent estimate of \( \Omega \). Using GRB peak-flux statistics alone, \( \Omega \) could not be estimated from the current data (Cohen & Piran 1995). However, given a cosmological distribution of sources with measured redshifts, we can try to estimate \( \Omega \) in a manner similar to the attempts to estimate \( \Omega \) from Type I supernovae by Perlmutter et al. (1996). Perhaps GRBs will not be able to contribute meaningfully to the myriad of measurements before other methods become more precise, but it is likely that they provide a useful consistency check based on independent objects. Recall that GRBs are most likely farther than the observed Type I supernovae.

The observed peak flux depends on \( \Omega \) as

\[
I = I(L, \Omega)
= \frac{L}{64\pi} \frac{(H_0/c)^2 \Omega^4 (1 + z)^{3-n}}{[\Omega/2 + (\Omega/2 - 1)(\sqrt{\Omega z + 1} - 1)]}. \tag{3}
\]

Using the known parameters of each burst (peak flux and redshift), we obtain for each burst a function \( L = L(\Omega) \). [In the previous sections we have assumed that \( \Omega = 1 \) and obtained \( L(\Omega = 1) \)] For standard candles, all \( L \) must be equal. Given two sources, we have two equations, \( L = L_{1,2}(\Omega) \), with two variables, and we should be able to determine \( \Omega \).

A luminosity distribution will induce an uncertainty in this estimate that can be approximated by

\[
\sigma_{\Omega}(z) \approx \left. \frac{\partial \Omega}{\partial L} \right|_{\Omega=\Omega_{L}} \sigma_L
= \frac{1}{4 \sqrt{2z/3 - 2\sqrt{1 + z} + 3/2 + z / (4\sqrt{1 + z})}} \sigma_L \tag{4}
\]

For \( z = 1.5 \), we obtain \( \sigma_{\Omega} \approx 2\sigma_L/L \). Thus, assuming that \( \sigma_L/L = 1 \), we need 100 bursts with a measured \( z \) to estimate \( \Omega \) with an accuracy of \( \sigma_{\Omega} = 0.2 \). Such a goal could be achieved within several years. At present it is not known whether the GRB luminosity distribution is narrow enough and satisfies this
condition. However, as we have shown in § 2, the width of the luminosity distribution will be known to 30% when we have 20 bursts with measured redshift.

5. FUTURE DETECTORS

In view of the promising avenues that these observations have opened, it is worthwhile to examine the effect of future, more sensitive detectors on these estimates. Cohen & Piran (1995) have estimated that BATSE is sensitive to long bursts up to \( z_{\text{max}} = 2.1^{+0.7} \). A more sensitive detector (by a factor of 10) will detect bursts up to \( z = 6.9 \). Even if \( \rho(z) \) is constant up to such a high value of \( z \), we find that the number of observed bursts will increase by a factor of only 2.1. The results are slightly better if the current \( z_{\text{max}} \) is smaller. For example, a detector that is 10-fold more sensitive will measure 2.6 times more bursts than BATSE if \( z_{\text{max}} = 1.5 \) and 3.5 times more bursts if \( z_{\text{max}} = 1 \).

At first sight these results might look discouraging. However, the rate of GRBs at high redshift is unknown and could be critical in determining the nature of GRBs. At present it is not known if there are bursts that originate from high redshift. Most models that are based on compact objects cannot produce sources at very early time, that is, before star formation. On the other hand, these models, and in particular the neutron star merger model, predict a high rate of GRBs that will follow any extended star formation activity with a time lag of approximately \( \approx 10^9 \) yr (Piran 1992). Neutron stars and binary neutron stars will form within \( 10^7 \) yr after star formation. From the known binary pulsars in the galaxy, we can estimate that \( \approx 10^9 \) yr will pass until the binary neutron stars lose angular momentum through gravitational radiation and merge (Narayan, Piran, & Shemi 1992). Nuclear abundance measurement indicates that heavy elements produced in supernovae began to be produced earlier than those produced in neutron star mergers (Cowan, Thielemann, & Truran 1991). It will be intriguing to see whether GRB rates follow the trend of supernovae or neutron star mergers.

Furthermore, one has to recall that the relevant question for our purpose is not how many bursts are observed but how many bursts are observed with measured redshifts. Currently, the rate of detection of bursts with counterparts is about one per month, and from those detected until now, only one has a measured redshift. Here there is an enormous potential for improvements. For example, systematic measurements of the redshift of all bursts observed by BATSE (\( \approx 300 \) per year) would yield an independent estimate of \( \Omega \), with \( \sigma_\Omega = 0.1 \), even if the luminosity function is wide (\( \sigma_L / L = 0.9 \)), within 1 yr. However, the 1\% systematic uncertainty in the BATSE localizations have made this task very difficult, if not impossible.

6. DISCUSSION AND CONCLUSIONS

As expected, the direct redshift measurement of GRB 970508 agrees well with estimates made previously with peak-flux count statistics (see, e.g., Fenimore et al. 1993; Loredo & Wasserman 1995; Rutledge et al. 1995; Cohen & Piran 1995). It is remarkable what might be done with several additional redshifts. An estimate of the bursts’ luminosity distribution to 30% accuracy can be obtained with 20 bursts.
This luminosity function combined with the observed peak-flux distribution would provide us immediately with an estimate of the cosmological evolution of the rate of GRBs.

It is generally accepted that a fireball (see, e.g., Piran 1996) is inevitable in any cosmological model. In this model, the observed $\gamma$-rays are produced during the conversion of a relativistic energy flow to radiation. However, the source that produces the flow remains unseen. The limits on cosmological evolution could shed light on the GRB mystery by distinguishing among different cosmological models. For example, the expected rate of merging neutron stars depends on the redshift (Piran 1992; Totani 1997) in a drastically different way than the evolution of active galactic nuclei (which seems to decay exponentially at low redshift) and is even different from the expected rate of supernovae as seen from nucleosynthesis evidence (Cowan et al. 1991).

The implications of these redshift measurements could extend even further. Detection of a significant group of GRBs with redshifts could enable us to utilize GRBs to study cosmology. Having 100 bursts with associated redshifts will enable us to estimate the cosmological parameter $\Omega$, even if the GRB luminosity distribution is relatively wide. This goal is not practical with the current detection rate of one burst per month obtained by BeppoSAX. However, if the luminosity distribution is narrow, or if a novel detection technique could be found that would yield a significantly higher rate of counterpart detection, we would be able to measure $\Omega$ using this method within several years.

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