Time-Varying Input and State Delay Compensation for Uncertain Nonlinear Systems

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Abstract—A robust controller is developed for uncertain, second-order nonlinear systems subject to simultaneous unknown, time-varying state delays and known, time-varying input delays in addition to additive, sufficiently smooth disturbances. An integral term composed of previous control values facilitates a delay-free open-loop error system and the development of the feedback control structure. A stability analysis based on Lyapunov-Krasovskii (LK) functionals guarantees uniformly ultimately bounded tracking under the assumption that the delays are bounded and slowly varying.

I. INTRODUCTION

Numerous control techniques exist for linear systems with constant input delays (cf. [11]–[3] and references therein). Many of these results are extensions of classic Smith predictors [4], Artstein model reduction [5], or finite spectrum assignment [6]. Results that focus on simultaneous constant state and input delays for linear systems are provided in [7]–[9]. Extensions of linear control techniques to time-varying input delays are also available [10]–[13].

For nonlinear systems, controllers considering constant [16]–[23] and time-varying [20], [24]–[33] state delays have been recently developed. However, linear results considering delayed inputs are far less prevalent, especially for systems with model uncertainties and/or disturbances. Examples of these include constant input delay results in [34]–[46] and time-varying input delay results based on LMI conditions [47], backstepping [49]–[51] and other robust techniques [52]. Even more unique are results that consider both state and input delays in nonlinear systems. Recently in [51], the predictor-based techniques in [7] were extended to nonlinear systems with time-varying delays in the state and/or the input utilizing a backstepping transformation to construct a predictor-based compensator. The development in [51] requires knowledge of the plant dynamics and assumes that the plant is disturbance-free.

In this paper, we expand our previous time-varying input delay result [52] in two directions: a) Utilizing techniques for constant input-delayed systems first introduced in [46], we consider time-varying input delays in a nonlinear plant, and b) we add the ability to compensate for simultaneous arbitrarily large unknown time-varying state delays based on the techniques in [33]. Robust control methods are developed to compensate for the unknown time-varying state delays. To compensate for the input delay, an integral term composed of previous control values is used to yield a delay-free open-loop system. A Lyapunov-based stability analysis motivated by Lyapunov-Krasovskii (LK) functionals demonstrates the ability to achieve uniformly ultimately bounded tracking in the presence of model uncertainty, additive sufficiently smooth disturbances, and simultaneous time-varying state and input delays. The result is based on the assumption that the unknown state delay is bounded and slowly varying. Improving on the result in [52], we relax previous sufficient conditions on the control gains that required knowledge of the second derivative of the input delay.

II. DYNAMIC SYSTEM

Consider a class of second-order (Euler-Lagrange-like) nonlinear systems given by:

$$\ddot{x} = f(x, \dot{x}, t) + g(x(t - \tau_s), \dot{x}(t - \tau_s), t) + d + u_{\tau_1},$$  \hspace{1cm} (1)

where \(x, \dot{x} \in \mathbb{R}^n\) are the system states, \(u \in \mathbb{R}^n\) is the control input, \(f : \mathbb{R}^n \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^n\) is an unknown function, uniformly bounded in \(t\), \(g : \mathbb{R}^n \times \mathbb{R}^n \times [0, \infty) \rightarrow \mathbb{R}^n\) is an unknown function with delayed internal state, uniformly bounded in \(t\), \(d \in \mathbb{R}^n\) denotes a sufficiently smooth disturbance (e.g., unmodeled effects), and \(\tau_s, \tau_i \in [0, \infty)\) denote time-varying, non-negative input and state delays, respectively.

The subsequent development is based on the assumption that \(x\) and \(\dot{x}\) are measurable outputs. Throughout the paper, a time-dependent delayed function is denoted as

$$\zeta_{\tau_i} \triangleq \begin{cases} \zeta(t - \tau) & t - \tau > t_0 \\ 0 & t - \tau \leq t_0 \end{cases},$$

where \(t_0\) denotes the initial time. Thus, \(u_{\tau_1}\) is defined by

$$u_{\tau_1} \triangleq \begin{cases} u(t - \tau_1) & t - \tau_1 > t_0 \\ 0 & t - \tau_1 \leq t_0 \end{cases}.$$
Additionally, $\|\cdot\|$ denotes the Euclidean norm of a vector and the following assumptions will be exploited.

**Assumption 1.** Each of the functions $f$ and $g$, along with their first and second partial derivatives, is bounded on each subset of its domain of the form $K \times [0, \infty)$, where $K \subset \mathbb{R}^n \times \mathbb{R}^n$ is compact. Furthermore, given such compact $K$, the corresponding bound is known.

**Assumption 2.** The nonlinear disturbance term and its time derivative are bounded by known constants.

**Assumption 3.** The desired trajectory $x_d \in \mathbb{R}^n$ is designed such that $x_d^{(i)} \in \mathbb{R}^n$, $\forall i = 0, 1, \ldots, 3$ exist and are bounded by known positive constants, where the superscript $(i)$ denotes the $i^{th}$ time derivative.

**Assumption 4.** The input and state delays are bounded such that $0 \leq \tau_i \leq \varphi_{i1}$ and $0 \leq \tau_i \leq \varphi_{i2}$, and the rate of change of the delays are bounded such that $|\dot{\tau}_i| \leq \varphi_{i2} < 1$ and $|\dot{\tau}_s| \leq \varphi_{s2} < 1$ where $\varphi_j \in \mathbb{R}^+ \forall j = i_1, i_2, s_1, s_2$ are known constants. Furthermore, the bounds on the input delay satisfy $\varphi_{i1} + \varphi_{i2} < 1$. The state delay is assumed to be unknown, while the input delay is assumed to be known.

**Remark 1.** In Assumption 4 the slowly time-varying constraint (i.e., $|\dot{\tau}_{i,s}| \leq \varphi_{i2,s2} < 1$) is common to results which utilize classical LK functionals to compensate for time-varying time-delays [13]. Knowledge of the state delays in the system is not required; however, the input delays present a more significant challenge. Although the controller requires the input delay to be known so that the interval of past control values can be properly sized, numerical simulations illustrate robustness to uncertainties in the input delay.

### III. CONTROL OBJECTIVE

The objective is to design a controller that will ensure the system state $x$ of the system in (1) tracks a desired state trajectory. To quantify the control objective, a tracking error, denoted by $e_1 \in \mathbb{R}^n$, is defined as

$$e_1 \triangleq x_d - x.$$  

(2)

To facilitate the subsequent analysis, two auxiliary tracking errors $e_2, r \in \mathbb{R}^n$ are defined as [6]

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1,$$  

(3)$$r \triangleq \dot{e}_2 + \alpha_2 e_2 + e_u,$$  

(4)

where $\alpha_1, \alpha_2 \in \mathbb{R}$ denote constant positive control gains, and $e_u \in \mathbb{R}^n$ denotes the mismatch between the delayed control input and the computed control input, defined as [4]

$$e_u \triangleq u_{\tau_i} - u.$$  

(5)

$\triangleright$ Many practical disturbance terms are continuous including friction (see [53, 54]), wind disturbances, wave/ocean disturbances, etc.

$\triangleright$ Many guidance and navigation applications utilize smooth, high-order differentiable desired trajectories. Curve fitting methods can also be used to generate sufficiently smooth time-varying trajectories.

$\triangleright$ Let $h \triangleq \max(t_0, t - \tau)$. Then, $h : [0, \infty) \rightarrow [0, \infty)$ is continuous. Further, since $u(t_0) = 0$, $u_{\tau_i} = u(h)$, and $e_u = u(h) - u$. Hence, $e_u$ is a continuous function of time if $u$ is a continuous function of time, and $e_u(t_0) = 0$.

The auxiliary signal $e_u$ injects a delay-free control input into the error system development. In contrast to the development in [52], the term in (5) is embedded in a higher order derivative (i.e., $r$ instead of $e_2$). Functionally, $e_u$ still injects an integral of past control values into the open-loop system; however, the development introduces fewer cross-terms. The auxiliary signal $r$ is introduced to facilitate the stability analysis and is not used in the control design since the expression in (4) depends on the unmeasurable state $\hat{x}$. The structure of the error systems is motivated by the need to inject and cancel terms in the subsequent stability analysis as demonstrated in Section IV.

Using (1), (2), (3), and (5) to eliminate the delayed input term, (3) can be represented as

$$r = S_1 + S_2 - u,$$  

(6)

where the auxiliary functions $S_1 \in \mathbb{R}^n$ and $S_2 \in \mathbb{R}^n$ are defined as

$$S_1 \triangleq f(x_d, \dot{x}_d, t) - f(x, \dot{x}, t) + g(x_{d\tau_i}, \dot{x}_{d\tau_i}, t) - g(x_{\tau_i}, \dot{x}_{\tau_i}, t) + \alpha_1 \dot{e}_1 + \alpha_2 e_2,$$  

(7)$$S_2 \triangleq \ddot{x}_d - f(x_d, \dot{x}_d, t) - g(x_{d\tau_i}, \dot{x}_{d\tau_i}, t) - d.$$  

Based on (6) and the subsequent stability analysis,

$$u \triangleq (k_{s1} + 1) (e_2 - e_2(t_0)) + v,$$  

(8)

where $v \in \mathbb{R}^n$ is the solution to the following differential equation

$$\dot{v} = (k_{s1} + 1) (\alpha_2 e_2 + e_u), \quad v(t_0) = 0,$$  

(9)

and $k_{s1} \in \mathbb{R}$ is a positive constant control gain.

The closed-loop error system can be developed by taking the time derivative of (6) and substituting for (3) and the time derivative of (7) to yield

$$\dot{r} = \dot{\hat{N}} + N_d - e_2 - (k_{s1} + 1) r,$$  

(10)

where $\hat{N} \in \mathbb{N}^n$ and $N_d \in \mathbb{N}^n$ are defined as

$$\hat{N} \triangleq \hat{S}_1 + e_2,$$  

(11)$$N_d \triangleq \hat{S}_2.$$  

(12)

The control design in (7) and (8) is motivated by the desire to eliminate the delayed input, yielding the closed-loop error system in (9). The structure of (9) is advantageous because it facilitates the stability analysis by segregating terms that can be upper bounded by a state-dependent term and terms that can be upper bounded by constants. Based on Assumptions 2 and 5 the following inequalities can be developed from the expression in (11):

$$\|N_d\| \leq \zeta N_{d1},$$  

(12)

where $\zeta N_{d1} \in \mathbb{R}$, is a known positive constant. The Mean Value Theorem can be utilized to find an upper bound for the expression in (10) as [55] Appendix A

$$\|\hat{N}\| \leq \rho_1 (\|z\|) \|z\| + \rho_2 (\|z_{\tau_i}\|) \|z_{\tau_i}\|,$$  

(13)

where $z \in \mathbb{R}^{4n}$ denotes the vector

$$z \triangleq [e_1^T, e_2^T, r^T, e_u^T]^T.$$  

(14)
and the bounding terms \( \rho_1, \rho_2 : [0, \infty) \to [0, \infty) \) are positive, non-decreasing and radially unbounded functions.\(^6\) The upper bound for the auxiliary function \( \hat{N} \) is segregated into delay-free and delay-dependent bounding functions to eliminate the delayed terms with the use of an LK term in the stability analysis.

To facilitate the subsequent stability analysis, several auxiliary terms are introduced. Let \( \rho : [0, \infty) \to [0, \infty) \) be an auxiliary bounding function defined as

\[
\rho (||z||) = \sqrt{(\gamma_1 + 2\gamma_2 \varphi_{s1}) \rho_2^2 (||z||)} + 3\rho_1^2 (||z||),
\]

where \( \gamma_1 \) and \( \gamma_2 \) are positive adjustable constants, and let \( \zeta \in \mathbb{R}^{3n} \) be defined as

\[
\zeta \triangleq [e_1^T, e_2^T, r^T]^T.
\]

Auxiliary bounding constants \( \sigma, \delta \in \mathbb{R} \) are defined as

\[
\sigma \triangleq \frac{1}{2} \min \left\{ \frac{\alpha_1}{2}, \frac{\alpha_2}{2}, \omega (1 - \varphi_{s2}) \right\},
\]

\[
\delta \triangleq \frac{1}{2} \min \left\{ \sigma, \frac{\omega (1 - \varphi_{s2})}{2\varphi_{s1}}, \frac{\varphi_{s2} (1 - \varphi_{s2})}{\gamma_1} \right\},
\]

where \( \omega \in \mathbb{R} \) is a known, positive, adjustable constant.

Let

\[
\mathcal{D} \triangleq \left\{ \xi \in \mathbb{R}^{3n+1} \mid ||\xi|| < \inf \left\{ \rho^{-1} \left( \left[ \sqrt{2k_s} \sigma, \infty \right) \right) \right\} \right\},
\]

and

\[
\mathcal{S} \triangledown \triangleq \left\{ \xi \in \mathcal{D} \mid ||\xi|| < \sqrt{\frac{2}{k_s}} \inf \left\{ \rho^{-1} \left( \left[ \sqrt{2k_s} \sigma, \infty \right) \right) \right\} \right\},
\]

where, for a set \( A \subset \mathbb{R} \), the inverse image \( \rho^{-1} (A) \subset \mathbb{R} \) is defined as \( \rho^{-1} (A) \triangleq \{ a \in \mathbb{R} \mid |a| \in A \} \). Furthermore, let the functions \( P_{LK} : [0, \infty) \to [0, \infty) \), \( Q_{LK} : [0, \infty) \to [0, \infty) \), \( R_{LK} : [0, \infty) \to [0, \infty) \), and \( S_{LK} : [0, \infty) \to [0, \infty) \) be defined as

\[
P_{LK} \triangleq \varphi_{s1} \int_{t-\tau_i}^{t} \|\hat{u}(\theta)\|^2 d\theta,
\]

\[
Q_{LK} \triangleq \omega \int_{t-\tau_i}^{t} \left( \int_{s-\tau_i}^{s} \|\hat{u}(\theta)\|^2 d\theta \right) ds,
\]

\[
R_{LK} \triangleq \frac{\gamma_1}{2k_s} \int_{t-\tau_i}^{t} \rho_2^2 (||z(\sigma)||) ||z(\sigma)||^2 d\sigma,
\]

\[
S_{LK} \triangleq \frac{\gamma_2}{k_s} \int_{t-\tau_i}^{t} \left( \int_{s-\tau_i}^{s} \rho_2^2 (||z(\sigma)||) ||z(\sigma)||^2 ds \right) ds.
\]

Additionally, let \( y \in \mathbb{R}^{3n+4} \) be defined as

\[
y \triangleq \left[ \zeta^T \sqrt{P_{LK}} \quad \sqrt{Q_{LK}} \quad \sqrt{R_{LK}} \quad \sqrt{S_{LK}} \right]^T.
\]

\(^6\) For some classes of systems, the bounding functions \( \rho_1 \) and \( \rho_2 \) could be selected as constants. For these classes of systems, a global uniformly ultimately bounded result can be obtained as described in Remark \(^5\).

\(^7\) The construction of \( P_{LK}, Q_{LK}, R_{LK}, \) and \( S_{LK} \) is based on LK functionals. However, in this result, they are to be interpreted as time-varying signals that are a part of the system state.

### IV. Stability Analysis

**Theorem 1.** Given the dynamics in (1), provided the control gains are selected based on the following sufficient conditions

\[
\alpha_1 > 1, \alpha_2 > 2, \gamma_1 > \frac{1}{(1 - \varphi_{s2})}, \omega > \frac{3\varphi_{s1}}{(1 - \varphi_{s2})},
\]

and the input delay is small enough so that there exists a gain \( k_s \) that satisfies

\[
\varphi_{s1} < \frac{k_s}{6 (\omega + 1) (k_s + 1)^2},
\]

the controller given in (7) and (8) ensures uniformly ultimately bounded tracking in the sense that \( \lim \sup_{t \to \infty} ||y|| \leq \frac{\sqrt{3c^2_{\delta}}}{{k_s}^{\omega}}, \) provided \( y(t_0) \in \mathcal{S}_\delta \).

**Proof:** Let \( \mathcal{D} \to \mathbb{R} \) be a candidate Lyapunov function defined as

\[
V \triangleq \frac{1}{2} e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} r^T r + P_{LK} + Q_{LK} + R_{LK} + S_{LK},
\]

which satisfies the following inequalities:

\[
\frac{1}{2} \|y\|^2 \leq V (y) \leq \|y\|^2.
\]

The time derivative of (27) can be found by applying the Leibniz Rule to (19), (20), (21) and (22), and by substituting (2-4), (7), and (9), yielding

\[
\dot{V} = e_1^T (e_2 - \alpha_1 e_1) + e_2^2 (r - \alpha_2 e_2 - e_u) + r^T \left( N + N_d - e_2 - (k_s + 1) r \right)
\]

\[
+ (\omega \tau_i + \varphi_{s1}) (k_s + 1) \|r\|^2 - \varphi_{s1} (1 - \tau_i) \|\hat{u}_{\tau_i}\|^2
\]

\[
- \omega (1 - \tau_i) \int_{t-\tau_i}^{t} \|\hat{u}(\theta)\|^2 d\theta
\]

\[
+ \left( \frac{\gamma_1}{2k_s} + \frac{\gamma_2}{k_s} \tau_i \right) r_2^2 (||z||) ||z||^2
\]

\[
- \frac{\gamma_1 (1 - \tau_i)}{2k_s} r_2^2 (||z_{\tau_i}||) ||z_{\tau_i}||^2
\]

\[
- \frac{\gamma_2}{k_s} (1 - \tau_i) \int_{t-\tau_i}^{t} \rho_2^2 (||z(\sigma)||) ||z(\sigma)||^2 d\sigma.
\]

Using (11), (21), (13), the inequality \( \tau_i < 1 \) and Young’s Inequality to show that

\[
\|e_1 e_2\| \leq \frac{1}{2} \|e_1\|^2 + \frac{1}{2} \|e_2\|^2,
\]

\[
\|e_2 e_u\| \leq \frac{1}{2} \|e_2\|^2 + \frac{1}{2} \|e_u\|^2 \quad \text{and} \quad \|r\| \|r_2 (||z_{\tau_i}||) ||z_{\tau_i}|| \leq \frac{k_s}{2} \|r\|^2 + \frac{1}{2} \|r_2 (||z_{\tau_i}||) ||z_{\tau_i}||^2.
\]

The expression in (29) can be upper bounded as

\[
\dot{V} \leq -\alpha_1 \|e_1\|^2 - \alpha_2 \|e_2\|^2 - \left( \frac{\gamma_1}{2} \right) \|r\|^2
\]

\[
+ \|e_1\|^2 + \|e_2\|^2 + \frac{1}{2} \|e_u\|^2 + \|r\| \|\rho_1 (||z||) ||z||\|
\]

\(^8\) Since \( \delta \) increases with increasing \( k_s \), the left-hand side of (28) decreases with increasing \( k_s \). Since \( \rho \) is a nondecreasing function, the right-hand side of (28) is nondecreasing with respect to \( k_s \). Hence, (28) can be satisfied for some \( k_s \). Furthermore, for any given \( k_s \), (29) is satisfied if the delay is small enough.
Completing the squares for \( r \), utilizing the inequalities

\[
\| e_u \|^2 \leq \frac{\alpha_2}{2} \| e_1 \|^2 - \frac{\alpha_2}{2} \| e_2 \|^2 - \| r \|^2 - \frac{\omega}{2(1-\hat{\tau}_i)} \| e_u \|^2
\]

\[
- \left( \frac{\alpha_2}{2} - 1 \right) \| e_1 \|^2 - \left( \frac{\alpha_2}{2} - 1 \right) \| e_2 \|^2
\]

\[
- \left( \frac{\omega}{2(1-\hat{\tau}_i)} \right) \| e_u \|^2
\]

\[
- \left( \frac{\omega}{2(1-\hat{\tau}_i)} \right) \| e_u \|^2
\]

\[
+ \frac{1}{2k_s} \left( 3\rho_2^2 (\| z_\tau \|) + \gamma_1 + 2\gamma_2 \rho_2 (\| z_\tau \|) \| z_\tau \| \right) \| z_\tau \|^2
\]

\[
- \frac{\omega}{3\rho_i} P_{L,K} - \left( \frac{1-\hat{\tau}_i}{3\tau_i} \right) Q_{L,K} - \frac{\gamma_2}{\gamma_1} \left( 1 - \hat{\tau}_s \right) R_{L,K}
\]

\[
- \frac{1}{2\tau_s} S_{L,K} + \frac{3\gamma_2^2 N_{sl}}{2k_s},
\]

and (26), (19) and (21), (30) can be rewritten as

\[
\dot{V} \leq -\sigma \| z \| - \frac{\omega}{3\rho_i} P_{L,K} - \left( \frac{1-\hat{\tau}_i}{3\tau_i} \right) Q_{L,K}
\]

\[
- \frac{\gamma_2}{\gamma_1} \left( 1 - \hat{\tau}_s \right) R_{L,K} - \frac{1}{2\tau_s} S_{L,K} + \frac{3\gamma_2^2 N_{sl}}{2k_s},
\]

\[
\leq -\delta \| y \|^2, \quad \forall \| y \| \geq \sqrt{\frac{3\gamma_2^2 N_{sl}}{2k_s}},
\]

provided \( y \in \mathcal{D} \), where \( \sigma (\| z \|) \), \( \sigma \), and \( \delta \) were introduced in (15), (17) and (13). Using (26), (28), and (32), Theorem 4.18 in [50] can be invoked to conclude that \( y \) is uniformly ultimately bounded in the sense that \( \limsup_{t \to \infty} \| y \| \leq \sqrt{\frac{3\gamma_2^2 N_{sl}}{2k_s}} \), and the result is global in the sense that \( \mathcal{D} = \mathcal{S}_{\mathcal{D}} = \mathbb{R}^{3n+4} \).

### V. Conclusion

This paper presents a robust controller for uncertain nonlinear systems which include simultaneous time-varying state and input delays, as well as sufficiently smooth additive bounded disturbances. The controller utilizes a robust design approach to compensate for the unknown state delays coupled with an error system structure that provides a delay-free open-loop error system. The controller and LK functionals guarantee uniformly ultimately bounded tracking provided the rates of the delays are sufficiently slow. The control development can be applied when there is uncertainty in the system dynamics and when the state delay is unknown; however, the controller is based on the assumption that the time-varying input delay is known. Simulation results point to the possibility that different control or analysis methods could be developed to eliminate the assumption that the input delay is known. That is, perhaps the interval of previous control values could somehow be designed big enough to provide predictive properties despite uncertainty in the input delay.

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