Virtual Air Temperature Function for Room Thermostats

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Abstract. Room thermostats and indoor controllers are currently based on the air temperature set, which is a regulated quantity. The aim of the control is to keep the air temperature on its setpoint. The purpose is, however, to achieve and maintain thermal comfort in the indoor environment. Even so, thermal comfort is dependent on more parameters than simply air temperature. The thermal comfort function has been known since the 1970s, and has since been verified many times. This paper introduces a novel virtual air temperature function that can be used in indoor thermostats, that takes into account all important inputs for thermal comfort assessment, such as average radiant temperature and air velocity, as well as clothing, activity and user expectations. The method for obtaining listed input parameters is described and does not require more complex technical equipment than a conventional thermostat. In addition, the paper describes the actual function of the virtual air temperature that determines the comfort setpoint for the room thermostat. This function is verified in several operating states by both quantification and calculation. The results are compared against normal recommendations for determining the setpoint, and the difference in thermal comfort is discussed. Finally, the applicability of the equation to normal room thermostats is verified.

1. Introduction

Energy saving requirements nowadays lead toward an effort to reduce energy consumption for heating, ventilation, cooling, lighting and other building services. Energy consumption in buildings comprises approximately 40% of total energy consumption [1]. It is important to determine optimal indoor environmental parameters to reach thermal comfort, and to minimalize energy demands of buildings to create a healthy indoor environment.

In the field of heating operation, room thermostats and indoor controllers are currently based on a set air temperature, which is a regulated quantity. Market overview [2] shows that there is no controller on the market that operates in reference to thermal comfort. The aim of the control nowadays is merely to maintain the chosen air temperature on its setpoint. Even so, the purpose of the regulation is to achieve and maintain thermal comfort in the indoor environment.

Thermal comfort is dependent on more parameters than simply air temperature. Its function has been known since the 1970s. Fanger [3] has long dealt with thermal comfort issues and in 1970 specified the basic and complementary factors that influence thermal comfort (as defined by PMV and PPD indices). His model of thermal comfort has come to be a standard and until now has been verified many times [2].

Gagge et al. [4] opted for a different approach to assessing thermal comfort. He used his physiological model to determine SET, the standard effective temperature recognized by ASHRAE in 1986. The SET
calculation is similar to calculation of the PMV index, as it is also based on the thermal balance equation, which includes such personal factors as clothing and metabolism.

The basic difference between the PMV index and the SET temperature is that SET is based on a prediction of the mean skin and skin moisture temperature (TSENS thermal sensitivity index and TDISC thermal discomfort index). Fiala [5] defined the Dynamic Thermal Sensitivity Index (DTS) as an equivalent index to the PMV index. The DTS index can also be used for time-variable conditions and represents overall personal thermal sensations, which are dependent on mean skin temperature, change over time and body core temperature. The latest models of thermal comfort include the Berkeley model [6] which describes the psychological aspects of the thermal comfort perception.

Those results lead authors to formulate the hypothesis that different heating system uses would require different temperature controls because occupants would sense heat variably under the thermal conditions created by different heating systems. Because this hypothesis has been proven [7], a further scientific effort was devoted to the development of the "virtual temperature" equation, which takes into account not only air temperature, but also other aspects that affect the indoor environment.

This paper presents an equation of virtual temperature derived from the Fanger thermal comfort equation, and its method of obtaining inputs for calculation. The equation is verified in several operating states by its quantification and calculation, and also compared with traditional thermostats in simulation.

### 2. Nomenclature

| Symbol | Dimension | Meaning |
|--------|-----------|---------|
| C      | m²/W      | Sensibility of human feeling |
| E_d    | W/m²      | Heat flux of molecular diffusion through skin |
| E_sw   | W/m²      | Heat flux of sweating |
| E_LRE  | W/m²      | Heat flux by respiration – latent part |
| E_DRE  | W/m²      | Heat flux by respiration – sensible part |
| E_R    | W/m²      | Radiant heat flux |
| E_C    | W/m²      | Convective heat flux |
| f_cl   | -         | Surface clothing factor, defined as the ratio of the surface of the exposed person to the naked human skin |
| h_c    | W/m²K     | Convection heat transfer coefficient |
| L      | W/m²      | Imbalance of thermal comfort equation |
| M      | W/m²      | Human metabolic production |
| p_a    | Pa        | Partial pressure of water vapor |
| PMV    | -         | Predicted Mean Vote |
| t_a    | °C        | Indoor air temperature |
| t_e    | °C        | Outdoor air temperature |
| t_c1   | °C        | Surface temperature of clothing |
| t_r    | °C        | Mean radiant temperature |
| t_VIRT | °C        | Virtual air temperature |
| v_var  | m/s       | Relative velocity of air |
| W      | W/m²      | Human effective mechanical output |

### 3. Methods

Virtual temperature is artificially calculated target air temperature, which must be reached to ensure the requested level of thermal comfort. This temperature must replace traditional set-points in a room thermostat, usually set as hard value between 20 and 26 °C. The function of virtual temperature is derived
from the thermal comfort equation [8]. Source equation is focused on calculation of Predicted Mean Vote (PMV), which represents predicted subjective thermal feeling (1):

\[
PMV = \left[0.303 \times e^{(-0.036 \times M)} + 0.028\right] \times \\
\left\{ \begin{array}{l}
(M - W) - 3.05 \times 10^{-3} [5733 - 6.99(M - W) - p_a] - 0.42[(M - W) - 58,15] \\
-1.7 \times 10^{-5}M(5867 - p_a) - 0.0014M(34 - t_a)
\end{array} \right.
\]

\[
-3.96 \times 10^{-8} f_{cl}[(t_{cl} + 273)^4 - (t_r + 273)^4] - f_{cl} h_{cl}(t_{cl} - t_a)
\]

where

\[
t_{cl} = 35.7 - 0.028(M - W) - l_{cl}[3.96 \times 10^{-8} f_{cl}[(t_{cl} + 273)^4 - (t_r + 273)^4] + f_{cl} h_{cl}(t_{cl} - t_a)]
\]

\[
h_{cl} = \begin{cases} 
2.38 \times |t_{cl} - t_a|^{0.25} & \text{for } 2.38 \times |t_{cl} - t_a|^{0.25} > 12.1 \times \sqrt{v_{ar}} \\
12.1 \times \sqrt{v_{ar}} & \text{for } 2.38 \times |t_{cl} - t_a|^{0.25} < 12.1 \times \sqrt{v_{ar}}
\end{cases}
\]

\[
f_{cl} = \begin{cases} 
1.00 + 1.290 l_{cl} & \text{for } l_{cl} \leq 0.078 \text{ m}^2 \cdot K/W \\
1.05 + 0.645 l_{cl} & \text{for } l_{cl} \geq 0.078 \text{ m}^2 \cdot K/W
\end{cases}
\]

Using the given equation faces two main problems. The first is that room thermostats measure only air temperature and contain no information regarding other measures, such as metabolic production, clothing, radiant temperature, and others. The second problem is that the practical solution of \( t_{cl} \) is iterative and non-convergent. The result for this value is extremely sensitive.

In addition, even when we know all the inputs, we can only calculate the PMV index by this equation. The method of regulation is to set up a requested value of sensor signals and its comparison with measured values, tending to keep the regulatory deviation as low as possible. Thus, the equation has to be rearranged in order to express air temperature on the left side.

3.1. Obtaining inputs

To calculate set point for virtual temperature, a wide list of inputs must be obtained in order to calculate the thermal comfort equation. Those inputs are divided into user-dependent and physical-dependent.

### Table 1. A meaning of button values from Figure 1

| \( l_{cl} \) (m²·K/W) | \( M \) (W/m²) | PMV (-) |
|---------------------|----------------|--------|
| 0.0465 (underwear)  | 66.9 (sitting, light activity) | ± 0.4 (strict comfort req.) |
| 0.0775 (light clothing) | 90.2 (house work, cooking) | ± 0.8 |
| 0.1085 (sweater, long trousers) | 131 (dancing, ongoing party) | ± 1.2 |
| 0.3255 (bed, duvet) | 46.6 (sleeping) | ± 1.5 (energy saving) |

3.1.1. User-dependent inputs. Inputs related to users are metabolic activity, clothing and requested levels of thermal comfort. Those measures are to be user-set. For setting, the front panel of the thermostat has been designed and is shown in Figure 1. To make this decision more user-friendly, discrete steps
have been determined for user choice. Users are expected to press a button that best describes their current situation in three dimensions, rather than setting a single desired temperature. Meanings of buttons are in Table 1.

3.1.2. Mean radiant temperature. Mean radiant temperature is a mean temperature of surfaces, dependent on the surface spatial angle of its observation and emissivity. Mean radiant temperature is directly unmeasurable and usually calculated from air temperature, globe temperature and air velocity. Measurement of these quantities is beyond the scope of a conventional thermostat.

On the other hand, mean radiant temperature can be also calculated from the known internal surface temperatures of room walls. This temperature can be derived from internal air temperature, outdoor air temperature and wall thermal conductivity, with respect to thermal capacity and inertia.

Thermal conductivity and capacity does not change in time and can be set only once, when the thermostat is installed. Both internal and external air temperatures are easily measurable, so the method for calculating mean radiant temperatures is clear.

For initial settings, three questions are asked: first, the number of surfaces (walls, ceiling) exposed to outdoor conditions, selecting one, two or three surfaces. The second question, that of the heating system, offers three possibilities:

- Convective heating, such as convectors or fan-coils that heat by air
- Convective heating with small radiant components, such as radiators
- Heating with high radiant components, such as radiant panels or floor heating.

The third and last question, regarding the thermal resistance of building envelope, offers four choices:

- Historical buildings
- Buildings from the 90’s, without refurbishment
- Buildings meeting standard requirements
- Passive buildings

These questions, all with pre-defined answers lead to 36 combinations that have been pre-analyzed via building simulations, and after the dependence between mean radiant temperature, internal air temperature and outdoor temperature has been found. Mean radiant temperature in its steady state is expressed as:

\[ t_r = A \cdot (t_e - t_a) + t_a + B \]  

Where: A and B are coefficients, basically room-dependent.

A set of 36 simulations has been provided in Ansys Fluent, to determine those coefficients. The coefficient A is in Table 2 and coefficient B in Table 3.

For calculating \( t_r \), both measured air temperature and resulting virtual air temperature may be used, but with a different result. When using virtual air temperature, the result is a virtual \( t_r \), which could theoretically be measured in steady state. Computation then becomes iterative. When using actual measured temperature, the actual \( t_r \) is calculated, and so the resulting virtual air temperature is the temperature necessary to provide the requested thermal comfort.

Calculation is simple, because the measured value is always available. For calculation of virtual air temperature, use of sensor value is recommended.

3.1.3. Velocity of air. Air velocity in the interior is needed to calculate heat transfer from the occupants. This quantity is measurable. Unfortunately, velocity sensors are expensive and easily break, which makes them worthless in room thermostats. However, the air velocity does not change much in indoor environment and within a range of common values of 0–0.5 m/s, the predicted mean vote is not very sensitive to this parameter. Therefore, air velocity can be set as a fixed value corresponding to room parameters. For initial settings, one question and a sub question are asked. Those questions are in Table 4.
Table 2. Coefficient A for calculation of mean radiant temperature

|        | Convective heating | Convective + radiant | Radiative heating |
|--------|--------------------|----------------------|-------------------|
| 1 surface |                  |                      |                   |
| Historical building | -0.0654 | -0.0393 | -0.0263 |
| Building from 90’s | -0.0208 | -0.0012 | 0.0144 |
| Standard building | -0.0139 | 0.0017 | 0.0135 |
| Passive building | 0.0028 | -0.0104 | 0.0197 |
| 2 surfaces |                  |                      |                   |
| Historical building | -0.0982 | -0.0722 | -0.0591 |
| Building from 90’s | -0.0309 | -0.0114 | 0.0043 |
| Standard building | -0.0202 | -0.0046 | 0.0072 |
| Passive building | -0.0004 | -0.0124 | 0.0165 |
| 3 surfaces |                  |                      |                   |
| Historical building | -0.1311 | -0.1050 | -0.0920 |
| Building from 90’s | -0.0411 | -0.0215 | -0.0059 |
| Standard building | -0.0265 | -0.0108 | 0.0009 |
| Passive building | -0.0036 | -0.0145 | 0.0133 |

Table 3. Coefficient B for calculation of mean radiant temperature

|        | Convective heating | Convective + radiant | Radiative heating |
|--------|--------------------|----------------------|-------------------|
| Historical building | -0.757 | -0.858 | 0.117 |
| Building from 90’s | -0.268 | -0.373 | -0.031 |
| Standard building | -0.143 | -0.265 | -0.133 |
| Passive building | -0.026 | -0.166 | -0.328 |

Table 4. Initial determination of air velocity for calculation of heat transfer coefficient.

| How is this room ventilated? | Sub question                  | Air velocity |
|------------------------------|--------------------------------|--------------|
| Forced ventilation          | Supply air is cooler then indoor air | 0.2 m/s     |
|                             | Supply air is warmer then indoor air | 0.5 m/s     |
| Natural ventilation         | Tight windows                  | 0.1 m/s     |
|                             | Older windows with sealing      | 0.2 m/s     |
|                             | Leaking windows                 | 0.3 m/s     |

3.2. Expressing the virtual temperature

The thermal comfort equation (1) is designed to describe the predicted mean thermal sensation. For use in room thermostats, the equation’s purpose is different to find the air temperature to be set. Since the temperature $t_a$ is present in this system in the first and fourth powers and, in addition, in the non-convergent iterative calculation, one must understand the logic with which the equation is assembled.

3.2.1. Understanding the PMV equation (1). Combining [3] and [8] provides a meaning of single addition in the bracket: the molecular diffusion of water vapour through skin (3), sweating (4), latent heat of respiration (5), sensible heat of respiration (6), radiative heat flux (7) and convective heat flux (8), all in W/m² of body surface.

$$E_d = 3.05 \times 10^{-3}[5733 - 6.99(M - W) - p_a] \quad (3)$$
\[
E_{sw} = 0.42[(M - W) - 58,15]
\]  
\[
E_{LRE} = 1.7 \times 10^{-5}M(5867 - p_a)
\]  
\[
E_{DRE} = 0.0014M(34 - t_a)
\]  
\[
E_R = 3.96 \times 10^{-8}f_{ci}(t_{cl} + 273)^4 - (t_r + 273)^4
\]  
\[
E_C = f_{ci}h_{cl}(t_{cl} - t_a)
\]

These additions were found by Fanger when he formulated the Thermal Comfort Equation (9). This equation says that a person with a neutral thermal sensation is in thermal equilibrium. The equation (1) works with this equation in the sense of a violation of its equality. Let us admit that a person, who does not have these components in balance, perceives a different thermal feeling than neutral. The imbalance can be described as (10), which tells us how far the thermal comfort equation is from equality.

\[
P - W = E_d + E_{sw} + E_{LRE} + E_{DRE} + E_R + E_C
\]  
\[
L = M - W - E_d - E_{sw} - E_{LRE} - E_{DRE} - E_R - E_C
\]

Then the first element of (1) expresses the dependence of the subjective thermal sensation on the objective thermal imbalances of the human body to which the body must adapt. Let us name it Sensibility of Human Feeling and express it as (11). Then, the predicted mean vote (1) can be substituted as (12).

\[
C = 0.303 \times e^{(-0.036 \times M)} + 0.028
\]  
\[
PMV = C \times L
\]

3.2.2. Finding a non-iterative method of calculation \( t_{cl} \). Problematic surface temperature of garment from (1) then can be described with (7) and (8) as:

\[
t_{cl} = 35.7 - 0.028(M - W) - l_{cl}(E_R + E_C)
\]

And after substitution using (10) and (12) the same equation is:

\[
t_{cl} = 35.7 - 0.028(M - W) - l_{cl}\left(M - W - E_d - E_{sw} - E_{LRE} - E_{DRE} - \frac{PMV}{C}\right)
\]

Thus, the calculation ceases to be iterative, and the air temperature is only present one time, in the first power in the \( E_{DRE} \) member.

3.2.3. Expressing the virtual air temperature. Equation (1) can then be rearranged to an equation of desired virtual temperature, which is necessary to reach the ensured requested thermal feeling with selected clothes and activity in a given room (15).

\[
t_{VRT} = \frac{1}{0.0014M + f_{ci}h_c} \times \left\{ \frac{PMV_{,0.303 \times e^{(-0.036 \times M)} + 0.028}}{0.303 \times e^{(-0.036 \times M)} + 0.028} - M + 3.05 \times 10^{-3}[5733 - 6.99(M - W) - p_a] \right. \\
\left. + 0.42[(M - W) - 58,15] + 1.7 \times 10^{-5}M(5867 - p_a) + 0.0476 \cdot M \\
+ 3.96 \times 10^{-8}f_{ci}(t_{cl} + 273)^4 - (t_r + 273)^4] + f_{ci} \cdot h_c \cdot t_{cl} \right\}
\]

\[
t_{cl} = 35.7 - 0.028(M - W) - l_{cl}\left(M - W - 3.05 \times 10^{-3}[5733 - 6.99(M - W) - p_a] - \\
0.42[(M - W) - 58,15] - 1.7 \times 10^{-5}M(5867 - p_a) - 0.0014M(34 - t_a) - \frac{PMV_{,0.303 \times e^{(-0.036 \times M)} + 0.028}}{0.303 \times e^{(-0.036 \times M)} + 0.028}\right)
\]

Where \( h_c \) and \( f_{ci} \) is calculated according to (1). Because \( t_{cl} \) is used to calculate radiative heat flux, related to actual mean radiant temperature, the actual measured air temperature is used to determine \( t_{cl} \). The air
temperature is still present in the equation (15) in the convective coefficient $h_c$ if it does not decide on air velocity. Change of this coefficient between using measured temperature and desired virtual temperature can be neglected, so measured air temperature can be used to calculate $h_c$.

4. Results

Virtual air temperature has been calculated for several operating states by quantifying the equation (15). Results of quantification are in Table 5 for transition states when measured air temperature is 21 °C ($T_{\text{VIRT}(ta=21^\circ\text{C})}$) and also for steady state, when measured air temperature is equal to virtual temperature.

| $t_e$ (°C) | Clothing       | Activity                  | Thermal preference | $T_{\text{VIRT}(ta=21^\circ\text{C})}$ (°C) | $T_{\text{VIRT}(\text{steady})}$ (°C) |
|------------|----------------|---------------------------|--------------------|------------------------------------------|---------------------------------------|
| -12        | light clothing | house work, cooking       | PMV                | -0.8                                     | 16.8                                  |
| -12        | light clothing | sitting, light activity   | PMV                | -0.8                                     | 25.2                                  |
| 5          | light clothing | sitting, light activity   | PMV                | -0.8                                     | 22.8                                  |
| 5          | sweater, trousers | sitting, light activity  | PMV                | -0.8                                     | 21.2                                  |
| 5          | underwear      | dancing, ongoing party    | PMV                | -0.4                                     | 14.2                                  |
| 5          | bed, duvet     | sleeping                  | PMV                | -0.4                                     | 18.5                                  |

Table 6. Saturday setting of room thermostat. Button PMV is set to -0.4 (strict comfort requirement), convective heating, 3 exposed surfaces.

| time       | clothes    | activity                  | description               |
|------------|------------|---------------------------|---------------------------|
| 00:00–08:00| bed, duvet | sleeping                  | sleeping overnight        |
| 08:00–08:30| underwear  | house work, cooking       | dressing, breakfast       |
| 08:30–09:30| light clothing | sitting, light activity  | breakfast                  |
| 09:30–13:00| light clothing | house work, cooking    | house work, preparing a meal |
| 13:00–16:00| sweater, long trousers | sitting, light activity | lunch, siesta, coffee     |
| 16:00–19:00| sweater, long trousers | house work, cooking    | house work, preparing a meal |
| 19:00–21:00| light clothing | sitting, light activity  | dinner with friends       |
| 21:00–00:00| underwear  | dancing, ongoing party    | party with friends        |

Comparison of transition and steady state shows that Virtual temperature is not only dependent on button settings, but much upon actual air temperature. This is caused by calculation of mean radiant temperature using actual air temperature. Virtual air temperature is a temperature to be met to reach the requested thermal sensation, with actual mean radiant temperature. In actuality, the change in air temperature causes a change in mean radiant temperature and therefore another change in virtual air temperature. The difference between actual virtual temperature and “final” steady state temperature always increases the regulation error, so in the case of PID regulation it accelerates the regulatory intervention.

To better understand the behavior of the function, a one day building simulation has been provided, in a TRNSYS Simulation Studio. The object of the simulation was BESTEST Case 600. The setting for the thermostat is shown in Table 6. In Figure 2, the temperature during the day is visible. Figure 3 shows a comparison of thermal sensations of the people doing the activity specified in Table 6, between using a standard thermostat (with constant set point and night setback) and with a thermostat using Virtual air temperature.
5. Conclusion
Virtual air temperature appears to be an interesting and novel way of managing indoor environments according to thermal comfort measures, rather than the common practice of regulation to a constant temperature setpoint. The human body is designed more for movement and change, than for sitting in uniform and constant environments.

A virtual air temperature approach can help to bring more dynamics into the indoor environment, with a strong emphasis on thermal comfort. A potential source of complication is the fact that setting the thermostat is slightly more complicated than setting a common thermostat. But due to non-iterative calculation, using only basic mathematic operations, the equation of virtual air temperature is easily implemented in any type of electronic device. Simulation verification proved that the equation developed and its use in room thermostats can lead to increased thermal comfort over using a common thermostat with constant set point.

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