Numerical modelling of water hammer with cavitation

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Abstract. A new numerical method for solving a system of one-dimensional nonstationary equations describing the flow with cavitation is presented. The method is based on the Godunov approach. The method is implemented taking into account the cavitation that occurs in the flow. Test calculations have shown the acceptable accuracy of the method and its ability to describe phase transitions in the flow.

1. Introduction
Simulation of compression and rarefaction wave propagation in pipeline systems filled with a weakly compressible liquid is a subject of considerable interest. From a practical point of view, it allows to calculate the operational modes of real pipeline systems with the necessary accuracy, that in turn helps to prevent such negative phenomena as water hammer and cavitation. From the point of view of theoretical studies, this problem is also of interest in terms of development of models for describing the complex of various interrelated and interdependent processes occurring over a wide range of spatial and temporal scales.

The flow of the liquid in the pipeline can be accompanied by the circulation of compression and rarefaction waves. In compression waves the liquid is loaded and the pressure increases, in the rarefaction waves the pressure drops. The loading pressure can increase significantly, but the decrease of the pressure will be limited. In particular, this is due to the fact that the pressure of the liquid cannot be less than the pressure of its saturated vapors. Cavitation occurs, and as a result the newly formed vapor does not allow the pressure to drop below a certain level. Cavitation also reduces the propagation celerity of waves in the pipe.

Thus, with the pressure decrease in the pipe, cavitation phenomena can occur and long vapor-liquid flow zones can form which can significantly change the flow pattern. So the development of new methods for solution of equations describing multiphase flows in pipelines is a very important problem, for example for industrial safety.

2. Equations describing the flow of an isothermal fluid in a pipe with cavitation
The flow of initially slightly compressible liquid through a pipeline with initially uniform diameter along the entire length is considered under the assumption of mechanical and thermal equilibrium of the phases and a negligibly small change in temperature.
In this paper, we consider only one mechanism of the gas-vapor phase volume growth: due to evaporation of the liquid as the system tends to “vapor-liquid” equilibrium. Such processes as growth of gas microbubbles initially existing in the liquid, as well as dissolved gases evolution with the pressure drop, are not considered in this work.

Then the system will be considered consisting of two components: the liquid phase \( l \) and the vapor phase of this liquid \( v \). Inclusions of other substances in the form of bubbles and dissolved gas are not considered. Accordingly, we assume that at each point of space there will exist a mixture of only these two components. The volume fraction of each substance will be \( \phi_l \), \( \phi_v \), respectively for liquid and vapor. Under the assumption of uniform mixing of liquid and vapor, the system can be characterized by an average density of this mixture \( \rho_{mix} \):

\[
\rho_{mix} = \phi_l \rho_l + \phi_v \rho_v,
\]

here \( \phi_l + \phi_v = 1 \), \( \rho_i \) is the individual liquid density and \( \rho_i \) is the individual vapor density.

In addition to the volume fraction \( \phi_i \), the composition of the mixture can be specified by the mass fraction of \( Y_i \):

\[
Y_i \rho_{cm} = \phi_i \rho_i.
\]

The equations of motion are as follows:

\[
\frac{\partial (\rho_{mix} \cdot A)}{\partial t} + \frac{\partial (\rho_{mix} \cdot w \cdot A)}{\partial x} = 0,
\]

\[
\frac{\partial (\rho_{mix} \cdot w \cdot A)}{\partial t} + \frac{\partial (\rho_{mix} \cdot w^2 \cdot A)}{\partial x} = - A \cdot \frac{\partial p}{\partial x} - A \cdot g \cdot \rho_{mix} \cdot \frac{\partial z}{\partial x} - \frac{\pi}{4} \cdot \lambda \cdot \rho_{mix} \cdot w \cdot |w| \cdot R,
\]

\[
\frac{\partial (\rho_v \cdot \phi_v \cdot A)}{\partial t} + \frac{\partial (\rho_v \cdot \phi_v \cdot w \cdot A)}{\partial x} = S_v,
\]

here \( x \) is the coordinate along the pipeline axis, \( t \) is the time, \( w \) is the velocity of the vapor-liquid mixture, \( p \) is the pressure in the vapor-liquid mixture, \( \lambda \) is the friction factor, \( R \) is the inner radius of the pipeline, \( z \) is the pipeline height, \( A \) is the area of the inner cross section of the tube, and \( S_v \) is the rate of change in the mass of the vapor.

The friction factor \( \lambda \) should be determined taking into account possible appearance of a multiphase flow. In this paper, the Colebrook-White [1] ratio is used for the liquid, and the Lockhart-Martinelli method is used for the two-phase flow [2].

The Colebrook-White dependence shows the dependence of the friction factor \( \lambda \) on the Reynolds number \( \text{Re}=\nu D_0/\mu \) and the pipeline characteristics (diameter \( D \) and roughness \( k \)) [1]:

\[
\frac{1}{\sqrt{\lambda}} = -2 \log \left( \frac{2.51}{\text{Re} \sqrt{\lambda}} + \frac{k}{3.71D} \right).
\]

The following equations are used to calculate the pressure in a liquid and a vapor:

- equation of state of a weakly compressible isothermal liquid

\[
p = p_0 + c_i^2 (\rho_l - \rho_0) = p_0 + \frac{K}{\rho_0} (\rho_l - \rho_0),
\]

- equation of state of an ideal gas

\[
p = \frac{R}{\mu} \frac{T}{\rho_v}.
\]
here \( c_1 \) is the speed of sound in the liquid phase, \( K_p \) is the modulus of elasticity of the liquid, \( \rho_0 \) is the density of the liquid at a pressure \( p_0 \) (usually \( p_0 = 101325 \text{ Pa} \)), \( R \) is the universal gas constant, \( T \) is the temperature, and \( \mu \) is the molar mass of the vapor.

In this paper, to calculate the saturated vapor pressure \( p_s \), it is proposed to use the Clausius-Clapeyron equation in the form:

\[
    p_s = p_0 \exp \left( \frac{AH_b \mu}{T_b - T} \right) R, \tag{9}
\]

here \( AH_b \) is the heat of boiling, and \( T_b \) is the boiling point at a pressure \( p_0 \).

Then the vapor and liquid densities on the saturation line (\( \rho_v(p) \) and \( \rho_l(p) \)) can be calculated for the pressure \( p \), according to equations (7) - (8).

Accordingly, the simultaneous existence of vapor and liquid is possible for \( \rho_{\text{mix}} < \rho_l(p) \). In this case, equation (5) can be replaced by a simpler expression for determining \( \phi_v \) (instead of equation (5)) obtained from (1):

\[
    \phi_v = \frac{\left( \rho_{\text{mix}} - \rho_1(p_v) \right)}{\left( \rho_1(p_v) - \rho_1(p_s) \right)}. \tag{10}
\]

Finally, one more detail should be pointed out. The flow in the pipe is largely determined by the movement of its walls. In particular, the speed of wave propagation depends on the characteristics of the tube.

The state of the pipe is described by its diameter, which can be found from the following relationship [3]:

\[
    A = A_0 \left( 1 + \frac{D_p \left( 1 - v_p^2 \right)}{E \delta} \left( p - p_0 \right) \right), \tag{11}
\]

here \( A = 0.25 \pi D^2 \) is the cross-sectional area of the pipeline loaded with pressure \( p \), \( A_0 = 0.25 \pi D_0^2 \) is the sectional area of the unloaded pipeline at a pressure \( p_0 \) (\( p_0 = 101325 \text{ Pa} \)).

For the correct formulation of the problem, it is also necessary to specify the initial and boundary conditions. In this paper we use the same types of boundary conditions as in our articles [4]-[8]: the rigid wall condition (the velocity is zero) and the condition of constant pressure.

3. Numerical method

To solve the system of equations (3)–(11) Godunov-type method, described in detail previously in [4]-[7].

However, in the presence of a vapor phase in the corresponding cells (\( \phi_v > 0 \)), in the solution of the Riemann problem on the decay of a discontinuity, it is necessary to use the sound velocity in a two-phase medium. In the absence of vapors, the “effective” speed of sound in the liquid phase is used for this purpose. The speed of sound in the vapor-liquid mixture at a pressure \( p \) and temperature \( T \) with a volume fraction of the vapor \( \phi_v \) was taken in the form [9]:

\[
    c_{\text{mix}} = \sqrt{\frac{\rho_{\text{mix}} c_{\text{mix}}^2 \phi_v^2 + \frac{p^2}{T}}{\rho_{\text{mix}} c_{\text{mix}}^2}}, \quad \frac{1}{c_{\phi v}^2} = \frac{1}{c_{\mu v}^2} + \frac{1 - \phi_v}{\rho \phi_v c_{\phi v}^2}, \tag{12}
\]

here \( c_v \) is the speed of sound in a pure vapor, \( c_i \) is the speed of sound in a pure fluid, \( c_{\text{mix}} \) is the heat capacity of the mixture, determined through the heat capacities of the liquid \( c_v l \) and the vapor \( c_v v \), as follows:
\[ c_{vmix} = Y_v c_{vmix} + \left(1 - Y_v\right) c_{vl} \tag{13} \]

That is, in the formulas (17)-(20) as a \( c_{mix}^{n-1} \), we use a variable calculated from (37) using the known parameters \( T_i^n, P_i^n, \phi_i^n \) in the cell.

In addition, at the end of each time cycle, when the condition \( \rho_{mix} < \rho(p_s(T)) \) is satisfied, we calculate the volume fraction of the vapor \( \phi \), by formula (10).

Also it is necessary to use the Lockhart-Martinelli correlation for friction factor \( \lambda \) calculation in the case of the vapor appearance.

4. Test calculations using developed models

The proposed method has been implemented as a computational program for a personal computer.

Using the developed approach, a number of calculations have been performed.

To assess the accuracy of the model and the numerical method in the part of accounting for cavitation, the problem of a water hammer with the subsequent appearance of a strong rarefaction was considered. Such processes were often considered in the experiment and one of these experiments [10] was chosen as a benchmark for testing in this work.

In this formulation, a number of experiments were carried out, both without cavitation, and with cavitation of liquid at the closed valve. For comparison with the results of calculations based on the developed model with experimental data, the experiment from the series [10] was chosen.

In this experiment demineralized water was used as the working fluid. The pipe in which the experiments were performed had a length 37.2 m. It was made of cooper, its inner diameter was 22 mm, wall thickness 1.6 mm. At the inlet and outlet there were connected tanks with water. The pressure in the tanks was equal to 0.2 and 0.22 MPa. The pipe had a slight slope of 3.8 degrees. The water velocity was 0.71 m/s. The operating time of the valve was 5-10 ms, which is much shorter than the wave circulation time, which was equal to 28.2 ms (the effective time for shutting off the valve 2-4 ms). The propagation velocity of the wave in the pipe from the experimental measurements was 1319 m/s. In the experiments, the pressure was measured at several points along the length of the pipe: at the ends of the pipe at the tanks (at the valve), in the middle of the pipe and at a distance from the ends of the pipe at a quarter of its length.

In Figures 1-2 the calculated time dependences of pressure and velocity at two points are presented: at a small distance from the valve (halfsize of computational cell) and at the middle of the pipeline. The calculations were carried out on a uniform grid of 600 cells.

As one can see from these results, the flow pattern for the liquid without cavitation completely corresponds to the scheme described above. The sequence of circulating waves is clearly seen in Figures 1 b, 2 b (point in the middle of the pipeline). The first peak of pressure with a small plateau (line "Calculation without cavitation") corresponds to the passage of the first wave - actually a water hammer wave). The plateau after the peak at the level of 0.22 MPa corresponds to the passage of the second wave (from the input vessel to the closed end). In this wave the liquid is unloaded to the initial state, but the flow is reverse. The plateau at the minimum corresponds to the third wave (from the closed valve to the input vessel). Finally, the last stepped plateau at \( \approx 0.1 \) s is the result of the passage of the fourth wave, returning the system to its original state. Each of these four states corresponds to the length of time the wave travels along the entire length of the pipeline - 0.028 s. Similarly, the passage of waves can be seen on the speed profile.

Until the cavitation of the flow the results of calculations based on the model taking into account cavitation completely coincide with the results of calculation of non-boiling liquid. However, after the appearance of vapor-liquid flows in the pipe, significant discrepancies arise. First of all, it concerns the time of the circulation of waves. Since the velocity of wave propagation through the vapor-liquid medium is much less than in the liquid medium, the second increase in pressure on the valve occurs with a significant delay: for non-boiling liquid growth begins on 0.118 s, and for boiling one on 0.293 s. In this case, the duration of the high-pressure pulse is practically the same (in the absence of
cavitation it is equal to 0.61 s, and 0.06 s when cavitation takes place). The amplitudes of the pressures attained are also close, although the profile itself is somewhat different, with the boiling of the liquid after a sharp pressure jump, there is a subsequent growth, and not an exit to the plateau. Just after the main impulse, there is still some peak of pressure. Another difference between the results obtained from these two models is that when allowance is made for the cavitation, pulsations of pressure of a lower amplitude appear in the calculations in the time interval between the passage of strong pulses (for the central sections of the pipe).

Figure 1. Calculated time dependencies of pressure: before the valve at the end of the pipeline (a), in the middle of the pipeline (b). (with and without cavitation).

But the greatest difference is observed in the flow velocity. With non-boiling fluids in regions between waves, the velocity gradient over the pipe is zero, this is reflected in the stepped velocity profile in Figure 2a. For boiling liquid a velocity gradient arises due to delays in cavitation in different sections of the pipe. The gradual increase in the velocity at a point in the middle of the pipe (see the figure 2b line «calculation with cavitation»), is precisely what indicates this.

Thus, there is a significant difference in the predictions of the flow characteristics, depending on whether the boiling is taken into account or not. It is clear that under such a situation, the cavitation should be taken into account in practically all problems associated with the water hammer.

It is also clear that this consideration should be conducted correctly and with maximum accuracy. In this connection, the question arises whether the model and the numerical method proposed in this paper are accurate.

In Figure 3 the time profiles of pressure, measured in the experiment and calculated according to the proposed approach, are given at two points. As one can see from Figure 3, there is a satisfactory agreement between calculation and experiment. All the qualitative features of the real flow are well described in the framework of the proposed approach quantitatively. The least accurately predicted time for the arrival of a second pressure pulse (the passage of the fourth wave) in calculations, the wave comes to 0.065 s with later than in the experiment. Most likely this is due to the insufficient accuracy of the numerical method - using the acoustic approximation. A more accurate numerical method is likely to yield a better match with experience, which can be done at subsequent stages of development of the proposed approach.

5. Conclusions
A new numerical method for solving a system of equations describing the motion of a one-dimensional weakly compressible isothermal liquid in a pipe with cavitation is proposed. This numerical method is constructed within the framework of Godunov's first-order accuracy method.

The proposed approach is applied to the case of the equilibrium vapor-liquid flow model.

It is shown that when there are areas with a boiling liquid, ignoring their appearance can lead to errors of hundreds of percent, primarily in determining the transit time of the waves. This allows us to conclude that for the correct description of the water hammer with cavitation, its description is a prerequisite for obtaining a correct and complete picture of the flow.
Figure 2. Calculated time dependencies of velocity: before the valve at the end of the pipeline (a), in the middle of the pipeline (b) (with and without cavitation).

Figure 3. Calculated and measured time dependencies of pressure: before the valve at the end of the pipeline (a), in the middle of the pipeline (b).

By comparing the calculated pressures with available experimental profiles, it is shown that the equilibrium model used and the proposed numerical method quite satisfactorily describe real physical processes, in particular, the amplitudes and duration of waves are described very accurately. However, there is some inaccuracy in the description of the arrival time of the waves. The error in the time of the onset of the second pressure increase at the closed valve is about 20-30%. This inaccuracy can be eliminated by further development of the proposed method.

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