More Transverse Polarization Signatures of Extra Dimensions at Linear Colliders

Thomas G. Rizzo

Stanford Linear Accelerator Center
Stanford University
Stanford CA 94309, USA

Abstract

Polarization of both electron and positron beams at a future linear collider (LC) allows for the measurement of transverse polarization asymmetries. These asymmetries have been shown to be particularly sensitive to graviton or other spin-2, $s$–channel exchanges in the process $e^+e^- \rightarrow f\bar{f} \ (f \neq e)$ which allows for a doubling of the usual search reach. A question then arises as to whether other $e^+e^-$ processes also show comparable sensitivity. Here we extend our previous analysis to the set of final states $e^+e^-, W^+W^-, 2\gamma$ and $2Z$ as well as to the Møller scattering process $e^-e^- \rightarrow e^-e^-$. We demonstrate that these reactions yield transverse polarization asymmetries which are somewhat less sensitive to graviton exchange than are those obtained in our earlier analysis for $e^+e^- \rightarrow f\bar{f}$.

*Work supported by the Department of Energy, Contract DE-AC03-76SF00515
1 Introduction

The existence of new physics (NP) beyond the Standard Model (SM) at or near the TeV scale is anticipated on rather general grounds. Future colliders, the LHC and/or a Linear Collider (LC), may be above threshold for the production of new particle states, such as SUSY, in which case the NP will be observed directly. In such a scenario the detailed analysis of the NP will be relatively straightforward though it may take some time and the combined data from both colliders to accomplish. Alternatively, experiments may uncover new reactions with small rates which are forbidden in the SM, thus pointing at NP; it may be very difficult in such cases to access the details of the underlying theory without observing the direct production of the new particles inducing these processes. The appearance of NP may, however, be even more subtle than either of these scenarios. We can imagine that collider data begin to show small deviations from the SM predictions for various observables, e.g., cross sections and asymmetries, which grow with increasing energy. A set of such observations signals the existence of NP beyond the collider’s kinematic reach which is manifesting itself in the form of higher dimensional operators, i.e., generalized contact interactions. In the more complete theory at higher energies these operators arise from the exchanges of new particles, which are too massive to be directly produced at the collider. These particles may occur with different spins and in various channels depending upon the particular theory.

The literature contains a rather long list of potential NP scenarios of this type that lead to contact interactions once the heavy fields are integrated out: a $Z'$, scalar or vector leptoquarks,$^1\,^2$, $R$-parity violating sneutrino($\tilde{\nu}$) exchange$^3$, scalar or vector bileptons$^4$, graviton Kaluza-Klein (KK) towers$^5\,^6\,^7$ in extra dimensional models$^8\,^9$, gauge boson KK towers$^{10}\,^{12}$, and even string excitations$^{11}$. It is this type of observation of NP that we will discuss in this paper.

It is clear that it is important to develop techniques that will help in distinguishing
among the many possible sources of contact interactions. In a previous paper\cite{12} we have considered one such possibility at the LC: transverse polarization (TP) and the associated asymmetries\cite{13} that can be formed through its use. Provided both the $e^-$ and $e^+$ beams can be initially longitudinally polarized, spin rotators can then be used to convert the longitudinal polarization to transverse polarization with near to 100% efficiencies. As the reader may recall and as we will see below, double beam polarization is necessary to generate the TP asymmetries. While historically the possible use of TP as a tool for new physics searches has not gotten much attention\cite{13}, our earlier analysis of TP asymmetries for the process $e^+e^- \rightarrow f \bar{f}$ ($f \neq e$) found them to be a unique probe for the $s$-channel exchange of spin-2 fields, especially when we sum over all of the accessible final states, $f$. Currently, we associate such exchanges with the Kaluza-Klein graviton towers of the Arkani-Hamed, Dimopoulos and Dvali(ADD)\cite{8} or Randall-Sundrum(RS)\cite{9} scenarios.

The purpose of the present paper is to extend the previous analysis to other final states which are also accessible in $e^+e^-$ collisions: $e^+e^-$, $W^+W^-$, $2\gamma$ and $2Z$; for completeness we also include the Möller scattering process $e^-e^- \rightarrow e^-e^-$. Here we wish to explore whether any of these final states lead to TP asymmetries which are also particularly sensitive to spin-2/graviton exchanges. Unfortunately, we will show that this is not the case. Though the discovery reach for each of these processes is significant, it is always somewhat less than that found for the sum over the $f \bar{f}$ final states obtained in our earlier analysis.

The organization of this paper is as follows. After our introduction, we will provide a brief overview and review of TP and the associated asymmetries in $e^+e^-$ collisions generalizing the formalism from our previous discussion of the $f \bar{f}$ final states to accommodate those that are considered here. In section 3 we analyze in turn the TP asymmetries for each of the final states $e^+e^-, W^+W^-, 2\gamma$ and $2Z$ as well as $e^-e^- \rightarrow e^-e^-$ in both the SM and the ADD model. In particular we show how the SM predictions for TP asymmetries
are modified by the presence of spin-2 graviton exchange. Subsequently, the search reaches arising from the deviations in the TP asymmetries for the ADD model are obtained for each of these final states. The use of these final states for uniquely identifying graviton exchange is also analyzed. A discussion and our conclusions can be found in section 4.

2 Transverse Polarization Asymmetries: Background

Much of the formalism regarding TP and the associated asymmetries can be found in our earlier work[12]. Here we will provide only a quick overview and the necessary background required to follow the analysis we present below.

Consider the set of processes \( e^+e^- \rightarrow X\bar{X} \) with the both electron and positron beams polarized. Taking the initial \( e^\pm \) beam momenta along the \( \mp z \)-axis we denote the longitudinal and transverse polarizations of the \( e^- (e^+) \) by their cartesian components \( P_{x,y,z} (\bar{P}_{x,y,z}) \). For the moment we allow these two polarization vectors to be arbitrarily oriented. To proceed, we will follow a modified version of the notation used by Hikasa[13] and denote the corresponding helicity amplitudes for this process by \( T_{h'h'}^{h\bar{h}} \) where \( h(\bar{h}) \) represent the \( \pm \) helicity of the initial \( e^- (e^+) \) and \( h'(\bar{h}') \) is the corresponding helicity of \( X(\bar{X}) \). Considering the cases of interest, \( X = f, e, \gamma, W, Z \), we find that many of the products of these amplitudes cancel when summed over final helicities even when graviton exchange is included. For example, in the case of fermion pairs in the final state, we obtain

\[
\begin{align*}
\sum_{ij} T_{+-}^{*ij} T_{++}^{ij} &= 0 \\
\sum_{ij} T_{\pm\pm}^{*ij} T_{-+}^{ij} &= 0 \\
\sum_{ij} T_{++}^{*ij} T_{--}^{ij} &= 0.
\end{align*}
\] (1)
Similar equalities hold for the case of gauge bosons in the final state and, due to crossing symmetry, for the Møller scattering process $e^-e^- \rightarrow e^-e^-$. When these conditions hold, terms in the spin-averaged matrix element proportional to either $P_{x,y}$, $\bar{P}_{x,y}$ individually or the products $P_z\bar{P}_{x,y}$ and $P_{x,y}\bar{P}_z$ are all seen to vanish. In this case the spin-averaged matrix element for this class of processes can be symbolically written as

$$|\tilde{M}|^2 = \frac{1}{4}\left[ (|T_{+-}|^2 + |T_{-+}|^2 + |T_{++}|^2 + |T_{--}|^2) + P_z(|T_{+-}|^2 - |T_{-+}|^2 + |T_{++}|^2 - |T_{--}|^2) \\
+ \bar{P}_z(|T_{+-}|^2 - |T_{-+}|^2 - |T_{++}|^2 + |T_{--}|^2) \\
+ P_z\bar{P}_z(|T_{+-}|^2 + |T_{-+}|^2 - |T_{++}|^2 - |T_{--}|^2) \\
+ 2(P_z\bar{P}_x - P_y\bar{P}_y) \Re(T^*_{+-}T_{-+}) \cos 2\phi + 2(P_z\bar{P}_y + P_y\bar{P}_x) \Im(T^*_{+-}T_{-+}) \sin 2\phi \right],$$

(2)

where $\phi$ is the azimuthal angle and the implied summations over the final state helicities in each product of amplitudes, $ij$, are suppressed. As in the work of Hikasa[13], the helicity amplitudes in the expression above are now defined with the angle $\phi$ set to zero. Note that the $T_{\pm\pm}$ amplitudes only appear quadratically. We observe from this expression the important fact that the $\phi$-dependent pieces are only accessible if both beams are simultaneously transversely polarized. Thus we are reminded that to have azimuthal transverse polarization asymmetries at a LC we must begin with both beams longitudinally polarized and employ spin rotators; this differs from the case of the usual left-right (longitudinal) polarization asymmetry, $A_{LR}$, which requires only single $e^-$ beam polarization, i.e., the term above linear in $P_z$.

In what follows we will for simplicity assume in our analysis that we are in an energy regime where the effects of the finite width of the $Z$ can be neglected; in addition we will assume that we can also safely neglect (as is usual) the small imaginary contributions to the amplitudes arising from graviton exchange[14] in the ADD model so that the $\Im$ terms
in the expression above can be dropped. In forming the TP asymmetries we will limit ourselves to the case where the beams are purely transversely polarized with the directions of polarization vectors being back-to-back. We can define $\phi$ to be the angle between the $e^\pm$ polarization directions and the plane of the momenta of the outgoing $X\bar{X}$ particles in the final state.

Given the squared matrix element we can now form as before the differential azimuthal asymmetry distribution which we symbolically define by

$$\frac{1}{N} \frac{dA}{dz} = \left[ \frac{\int_+ d\sigma}{\int d\sigma} - \frac{\int_- d\sigma}{\int d\sigma} \right],$$

(3)

where $\int_\pm$ are integrations over regions where $\cos 2\phi$ takes on $\pm$ values; integration over the full ranges of $z$ and $\phi$ occurs in the denominator, except for possible acceptance cuts or cuts employed to remove QED $t-$ and $u-$ channel poles. (Here, $z = \cos \theta$, is the usual scattering angle.) Note that the presence or absence of the ‘$\text{Im}$’ terms proportional to $\sin 2\phi$ do not influence the value of this asymmetry since they cancel in both the numerator and denominator. It is important to recall that we expect this differential asymmetry to take on rather small numerical values since it is normalized to the total cross section and not to the differential cross section at the same value of $z$ as is usually done. This isolates the important angular behavior in the numerator of the TP asymmetry where it can be much more easily studied. In the case of $e^+e^- \rightarrow f\bar{f}$ ($f \neq e$), we found that this asymmetry was proportional to $1 - z^2$ in the SM as well as in most of the SM extensions discussed above. Only in the case of spin-2/graviton exchange was there a significant distortion of the angular dependence of this asymmetry thus leading to a unique signature for this special kind of NP.
3 Analysis

In order to begin our analysis we need the full set of helicity amplitudes for the above processes including the contributions from spin-2 graviton exchange. These can be found in the literature, e.g., [16, 13] and through the use of crossing symmetry. In the ADD scenario, which we will concentrate on in what follows, the relative contribution of the spin-2 graviton to these amplitudes always appears with a suppression factor, \( f_g \); employing the convention of Hewett [6], this is given by

\[
f_g = \frac{\lambda s^2}{4\pi \alpha M_H^4}. \tag{4}
\]

where \( M_H \) represents the cutoff scale in the Kaluza-Klein (KK) graviton tower sum and \( \lambda = \pm 1 \). This factor clearly shows the dimension-8 origin of the gravitational \( \sim T_{\mu\nu}T^{\mu\nu} \) interaction induced in the ADD model after summing over the KK tower. In the RS model, to which our analysis is easily generalized, the corresponding expression can be obtained through the replacement

\[
\frac{\lambda}{M_H^4} \rightarrow \frac{-1}{8\Lambda^2} \sum_n \frac{1}{s - m_n^2 + im_n \Gamma_n}. \tag{5}
\]

where \( \Lambda_\pi \) is of order a few TeV and \( m_n(\Gamma_n) \) are the masses(widths) of the TeV scale graviton KK excitations.

With the amplitudes in hand we can directly proceed to calculate the azimuthal polarization asymmetries. For numerical purposes we will assume that \( P_{e^-} = 80\% \) and \( P_{e^+} = 60\% \) as in our previous analysis. We remind the reader that by combining the data for the various \( f \bar{f} \) final states, \( f = \mu, \tau, c, b, t \), search reaches, i.e., 95\% CL bounds, as large as \( M_H \sim 20\sqrt{s} \) were obtained in our earlier work using only the TP asymmetry in the fits. Similarly, the identification (ID) reach, the value of \( M_H \) for which a 5\( \sigma \) signal for spin-2
exchange is observed, was found to be $M_H \sim 10 \sqrt{s}$. These values are roughly a factor of 2 greater than those obtained from more conventional analyses [17]. It is to these values that we must compare the results we obtain below.

Let us begin the present analysis by considering the case of Bhabha scattering, i.e., $e^+e^- \to e^+e^-$. Extracting the overall electromagnetic coupling factors the helicity amplitudes are given by

$$T_{++} = -(1 + z)[1 + \frac{s}{t} + g_R^2(\frac{s}{s - M_Z^2} + \frac{s}{t - M_Z^2})] + f_g[2\frac{u}{s} + \frac{3}{4}(1 + \frac{t}{s})]$$

$$T_{+-} = -(1 + z)[1 + \frac{s}{t} + g_L^2(\frac{s}{s - M_Z^2} + \frac{s}{t - M_Z^2})] + f_g[2\frac{u}{s} + \frac{3}{4}(1 + \frac{t}{s})]$$

$$T_{+\bar{+}} = T_{-\bar{-}} = -(1 - z)[1 + g_R g_L(\frac{s}{s - M_Z^2})] - f_g(\frac{3}{4} + \frac{t}{s})$$

$$T_{++} = T_{--} = \left[-\frac{s}{t} + g_R g_L(\frac{s}{t - M_Z^2})\right] - f_g\left(1 + \frac{3}{4}\frac{s}{t}\right),$$

where $z = \cos \theta$, $t(u) = -s(1 \mp z)/2$, $g_L = (-1/2 + s_w^2)/(s_w c_w)$ and $g_R = s_w/c_w$ with $s_w^2 = \sin^2 \theta_w \simeq 0.23$ being the conventional weak mixing angle. We observe that, unlike the case of the $f \bar{f}$ final state, there are non-annihilation channel amplitudes present, i.e., $T_{\pm \bar{\pm}}$, corresponding to the $t$– channel diagrams. Note that the conditions of Eq. (1) hold for this set of amplitudes. Note further that the spin-2 exchange merely augments the amplitudes which are already present in the SM (though with different $\cos \theta$ dependencies), i.e., no new helicity amplitudes are generated by spin-2 over those already due to spin-1.

From the above amplitudes we can immediately obtain the bin-integrated TP asymmetry using Eqs. (2) and (3); a sample result is shown in the top panel of Fig. 1 for the case of a $\sqrt{s} = 500$ GeV LC assuming an integrated luminosity of 500 $fb^{-1}$. In this panel the SM prediction is compared to that obtained in the ADD model assuming $M_H = 3\sqrt{s} = 1.5$
Figure 1: (Top) Differential transverse polarization azimuthal asymmetry distribution for Bhabha scattering, $e^+e^- \rightarrow e^+e^-$, at a 500 GeV LC assuming a luminosity of 500 fb$^{-1}$. The histogram is the SM prediction while the data points are for the ADD model assuming $M_H = 1.5$ TeV. The errors shown are the quadratic sum of the separate statistical and systematic errors. (Bottom) 95% CL search reaches for $M_H$ as a function of the LC integrated luminosity arising from the transverse polarization asymmetry in $e^+e^- \rightarrow e^+e^-$. From bottom to top the curves correspond to center of mass energies of 0.5, 0.8, 1, 1.2 and 1.5 TeV, respectively.
Figure 2: Same as the previous figure but now for Møller scattering, $e^- e^- \rightarrow e^- e^-$. 
TeV. Due to the existence of the $t-$channel QED pole, an angular cut $z \leq 0.95$ has been applied. In the SM, for $s \gg M_Z^2$, the TP asymmetry scales\cite{13} as $\sim -(1 + z)^2$, which is a fair approximation to what we see here. For $z < 0$ we see that the ADD prediction lies somewhat above the SM but drops significantly below it once large positive $z$ values are reached. It is clear from this figure that large search reaches are not likely in this channel, an expectation borne out by the results shown in the lower panel. (Recall that the contribution of the graviton exchange terms scale approximately as $M^{-4}_H$.) Here we see that the 95% CL reach in the $\sqrt{s}=500$ GeV case is only about 2.5 TeV or $\sim 5\sqrt{s}$ with similar results holding for larger center of mass energies. Since it is likely that other forms of NP, such as a $Z'$, can also cause similar distortions in the shape of the TP asymmetry distribution, there is no unique graviton signature in this case.

The corresponding amplitudes for Møller scattering can be obtained by crossing and directly lead to the corresponding TP asymmetry results in this case as is shown in Fig. 2. Here a cut of $|z| \leq 0.95$ has been applied to remove the $u-$ and $t-$channel QED poles. In the central $z$ region the TP asymmetry is predicted to be very close to the SM in the ADD scenario differing only once $|z| \geq 0.5$ or so. Following the same procedures as in the case of Bhabha scattering we obtain the search reaches shown in the lower panel of Fig. 2. Here we see that somewhat larger reaches are obtained, i.e., $7 - 8\sqrt{s}$, but these are still smaller than those obtained in our earlier work. The increased reach in Møller vs. Bhabha scattering using TP asymmetries is similar to what happens in the case of conventional contact interaction searches employing longitudinal polarization\cite{15}. As in the case of Bhabha scattering there is no unique signature for graviton exchange in this process.

Let us now turn to the case of gauge boson pairs in the final state: $W^+W^-, 2\gamma$ and $2Z$. For these cases our labor is relatively easy: all of the helicity amplitudes for these processes in the ADD model can be obtained directly from the work of Agashe and Deshpande\cite{16}.
Figure 3: Same as the previous figure but now for the process $e^+e^- \rightarrow W^+W^-$. 
A quick analysis shows that the set of conditions analogous to Eq.(1) are satisfied for all of these final states.

We consider the $W^+W^−$ final state first; the helicity amplitudes for this process are rather complicated and visually uninformative so we will not reproduce them here. A sample TP asymmetry in this case is shown in Fig. 3. Note that the SM shape is close to $1 − z^2$ but deviates in detail from this in a $z$ asymmetric manner. In the ADD case, the TP asymmetry lies close to the SM in the backwards direction but falls significantly below it once $z > −0.5$ is reached. We also see that the distortion in the asymmetry due to graviton exchange is quite significant in the case where $M_H = 1.5$ TeV. As one might expect from this observation, the search reach for the ADD model in this case is somewhat larger than those obtained for the two processes above, $\simeq 8\sqrt{s}$ for a 500 GeV LC, as can be seen from the lower panel in Fig. 3. As in the cases above there is no obviously unique signature for spin-2/graviton exchange from this process.

The next possibility we consider is the pure QED process $e^+e^- → 2γ$. In the SM the TP asymmetry is predicted to be $z$-independent as is shown in Fig. 4. Here, the existence of rather conventional sources of contact interactions, such as a $Z'$ or bilepton, cannot alter the SM predictions for this reaction. The lowest dimension operators that can contribute here are of the form $\sim \bar{e}eF_{\mu\nu}F^{\mu\nu}$, which might be induced by compositeness, and $\sim T_{\mu\nu}T^{\mu\nu}$, as can be induced by compositeness or graviton exchange. Clearly a unique signature for graviton exchange is not possible using this process. A short analysis shows that the ADD contributions simply adds to the SM TP=constant term by a relative factor of $\sim f_g(1 − z^2)$ as can also seen in Fig.4. We can see this immediately from the helicity amplitudes for this process which are particularly simple:

$$T^{++}_{+−} = −T^{−+}_{−+} = \frac{−2(1 + z)}{(1 − z^2)^{1/2}}(1 − f_g ut/s^2)$$
Figure 4: Same as the previous figure but now for the process $e^+e^- \rightarrow 2\gamma$. 

\[ T_{++}^{+} = -T_{++}^{-} = -2 \frac{(1 - z)}{(1 - z^2)^{1/2}} \left( 1 - \frac{u_t}{s^2} \right), \]  

(7)

where an overall factor of \( e^2 \) has been scaled out as before. What are the reaches for this process? Note that as in the calculations above we will employ an angular cut, \(|z| \leq 0.95\), to remove the SM poles. Fig. 4 shows the search reach for the ADD model using this TP asymmetry; it is quite respectable in comparison to the others we have found above \( \sim 7\sqrt{s} \) for the case of a 500 GeV LC.

For the 2Z final state we would expect the TP asymmetry to behave similarly to the case of the 2\( \gamma \) final state apart from \( M_Z^2/s \) corrections which are most visible near the would-be forward and backward poles. The SM \( u- \) and \( t- \) channel poles encountered in the 2\( \gamma \) final state case are thus smoothed out by the finite \( Z \) boson mass so that no cuts are required. The helicity amplitudes for this process with the final states containing only transversely polarized \( Z \)'s are quite similar to those for the 2\( \gamma \) final state except for appropriate insertions of factors of \( \beta = (1 - 4M_Z^2/s)^{1/2} \). Additional amplitudes corresponding to \( Z \)'s with longitudinal polarization are also present, however. As for the 2\( \gamma \) final state, we know that deviations in the observables associated with the 2Z final state can arise from a number of higher dimensional operators (including in this case anomalous triple gauge boson couplings) that we cannot uniquely trace back to graviton exchange. Fig. 5 shows that the TP asymmetry for the 2Z final state is nearly \( z \)-independent except in the very forward and backward directions in the SM. ADD graviton exchange, as in the 2\( \gamma \) case, induces a \( \sim 1 - z^2 \)-like contribution to the asymmetry as is also shown here. Since the rate for the 2Z and 2\( \gamma \) final states are similar (after cuts), as are the SM and ADD induced shapes of the TP polarizations, we might expect comparable search reaches. These are shown for the 2Z case in the lower panel of Fig. 5 and we see that our expectations are essentially confirmed. For a 500 GeV LC a search reach of \( \sim 7\sqrt{s} \) is obtained, essentially the same as for the 2\( \gamma \)
Figure 5: Same as the previous figure but now for the process $e^+e^- \rightarrow 2Z$. 
4 Discussion and Conclusions

Disentangling the origin of contact interaction effects will require as many tools as possible. Transverse polarization asymmetries offer a special way to probe for NP in $e^+e^-$ processes and have been shown to be particularly sensitive to spin-2/graviton exchange for the case of the $f \bar{f}(f \neq e)$ set of final states. Not only does TP extend the conventional search reach but it also provides a means to uniquely identify spin-2 exchanges. Its utility for other final states has been, up to now, completely unknown.

In this paper we have extended our previous analysis of TP asymmetries to encompass the processes $e^+e^- \rightarrow e^+e^-, W^+W^-, 2\gamma$ and $2Z$ as well as $e^-e^- \rightarrow e^-e^-$ in order to access their sensitivity to graviton exchange within the context of the ADD model. The results of our analysis are twofold: (i) We have found that the various processes above lead to search reaches for the ADD cutoff scale, $M_H$, in the range of $(5 - 8)\sqrt{s}$ for a 500 GeV LC. These are very respectable reaches given that they are based on only a single observable and result from only a single final state. By contrast, the usual analyses which employ the total cross sections, angular distributions, tau polarization and the $A_{LR}^s$ for the various final states $f$ when combined have reaches that are only $\sim 10\sqrt{s}$. However, none of the final states we have studied here have search reaches as large as that obtained from the combined $f\bar{f}$ analysis. (ii) Our earlier analysis demonstrated that essentially only graviton exchange could shift the $1 - z^2$ shape of the TP asymmetry distribution. Though the $f\bar{f}$ final states asymmetries are uniquely modified by the presence of spin-2/graviton exchange, this is no longer true for any of the processes we have examined here. Clearly more detailed studies are needed to verify these results.
Hopefully signs of new physics will be observed soon after the turn on of future colliders.

Acknowledgements

The author would like to thank J.L. Hewett, G. Moortgat-Pick, K. Desch, J. Clendenin and M. Woods for discussion related to this work.

References

[1] J. L. Hewett and T. G. Rizzo, Phys. Rept. 183, 193 (1989).

[2] A. Leike, Phys. Rept. 317, 143 (1999) [arXiv:hep-ph/9805494]; M. Cvetic and S. Godfrey, [arXiv:hep-ph/9504216]

[3] W. Buchmuller, R. Ruckl and D. Wyler, Phys. Lett. B 191, 442 (1987) [Erratum-ibid. B 448, 320 (1999)]; J. L. Hewett and T. G. Rizzo, Phys. Rev. D 58, 055005 (1998) [arXiv:hep-ph/9708419]; Phys. Rev. D 56, 5709 (1997) [arXiv:hep-ph/9703337] and Phys. Rev. D 36, 3367 (1987).

[4] J. Kalinowski, R. Ruckl, H. Spiesberger and P. M. Zerwas, Phys. Lett. B 414, 297 (1997) [arXiv:hep-ph/9708272]; J. Kalinowski, R. Ruckl, H. Spiesberger and P. M. Zerwas, Phys. Lett. B 406, 314 (1997) [arXiv:hep-ph/9703436]; T. G. Rizzo, Phys. Rev. D 59, 113004 (1999) [arXiv:hep-ph/9811440].

[5] F. Cuypers and S. Davidson, Eur. Phys. J. C 2, 503 (1998) [arXiv:hep-ph/9609487].

[6] J. L. Hewett, Phys. Rev. Lett. 82, 4765 (1999) [arXiv:hep-ph/9811356]; G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B 544, 3 (1999) [arXiv:hep-ph/9811291]; T. Han, J. D. Lykken and R. J. Zhang, Phys. Rev. D 59, 105006
(1999) [arXiv:hep-ph/9811350]; T. G. Rizzo, Phys. Rev. D 59, 115010 (1999) [arXiv:hep-ph/9901209].

[7] For an overview of RS phenomenology, see H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Rev. D 63, 075004 (2001) [arXiv:hep-ph/0006041]; Phys. Lett. B 473, 43 (2000) [arXiv:hep-ph/9911262]; Phys. Rev. Lett. 84, 2080 (2000) [arXiv:hep-ph/9909255]; J. L. Hewett, F. J. Petriello and T. G. Rizzo, JHEP 0209, 030 (2002) [arXiv:hep-ph/0203091].

[8] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) [arXiv:hep-ph/9803315]; I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 436, 257 (1998) [arXiv:hep-ph/9804398]; N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Rev. D 59, 086004 (1999) [arXiv:hep-ph/9807344].

[9] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999).

[10] See, for example, I. Antoniadis, Phys. Lett. B 246, 377 (1990); T. G. Rizzo and J. D. Wells, Phys. Rev. D 61, 016007 (2000) [arXiv:hep-ph/9906234]; M. Masip and A. Pomarol, Phys. Rev. D 60, 096005 (1999) [arXiv:hep-ph/9902467]; T. G. Rizzo, Phys. Rev. D 64, 015003 (2001) [arXiv:hep-ph/0101278].

[11] S. Cullen, M. Perelstein and M. E. Peskin, Phys. Rev. D 62, 055012 (2000) [arXiv:hep-ph/0001166].

[12] T. G. Rizzo, JHEP 0302, 008 (2003), [arXiv:hep-ph/0211374].

[13] R. Budny, Phys. Rev. D14, 2969 (1976); H.A. Olsen, P. Osland and I. Overbo, Phys. Lett. B97, 286 (1980); K. Hikasa, Phys. Rev. D33, 3203 (1986); C.P. Burgess and J.A. Robinson, Int. J. Mod. Phys. A6, 2709 (1991); A. Djouadi, F.M. Renard and C.
Verzegnassi, Phys. Lett. **B241**, 260 (1990); F.M. Renard, Z. Phys. **C44**, 75 (1989); J.L. Hewett and T.G. Rizzo, Z. Phys. **C36**, 209 (1987); J. Fleischer, K. Kolodziej and F. Jegerlehner, Phys. Rev. **D49**, 2174 (1994); for a recent discussion of this option at the LC, see K. Desch, talk given at the *International Workshop on the Linear Collider, LCWS2002*, Jeju Island, Korea, Aug. 2002.

[14] A. Datta, E. Gabrielli and B. Mele, Phys. Lett. B **552**, 237 (2003) [arXiv:hep-ph/0210318].

[15] See, for example, A. A. Pankov and N. Paver. [arXiv:hep-ph/0209058]

[16] K. Agashe and N. G. Deshpande, Phys. Lett. B **456**, 60 (1999) [arXiv:hep-ph/9902263]; G. Pasztor and M. Perelstein, *eConf* **C010630** P315 [hep-ph/0111471].

[17] See, for example, T. G. Rizzo, [arXiv:hep-ph/0303056]; J. A. Aguilar-Saavedra *et al.* [ECFA/DESY LC Physics Working Group Collaboration], “TESLA Technical Design Report Part III: Physics at an e+e- Linear Collider,” [arXiv:hep-ph/0106315]; S. Riemann, TESLA Linear Collider Note LC-TH-2001-007. For a review of the current and anticipated future collider reaches for extra dimensions, see J. Hewett and M. Spiropulu, “Particle physics probes of extra spacetime dimensions,” [arXiv:hep-ph/0205106].