Atom Interferometry in a Vertical Optical Lattice

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Abstract

We have studied the interference of degenerate quantum gases in a vertical optical lattice. The coherence of the atoms leads to an interference pattern when the atoms are released from the lattice. This has been shown for a Bose-Einstein condensate in early experiments. Here we demonstrate that also for fermions an interference pattern can be observed provided that the momentum distribution smaller then the recoil momentum of the lattice. Special attention is given to the role of interactions which wash out the interference pattern for a condensate but do not affect a spin polarized Fermi gas, where collisions at ultra cold temperatures are forbidden. Comparing the interference of the two quantum gases we find a clear superiority of fermions for trapped atom interferometry.

1 Introduction

Atom interferometry lies at the heart of quantum mechanics because it directly reveals the wave nature of massive particles. It is also important with respect to technical applications because it can be used for precision measurements of forces and rotations. To observe the interference, an atomic beam or a trapped atomic sample is subjected to a periodic scatterer — often realized with an optical lattice. The lattice probes the coherence of the atoms between neighboring lattice sites, thus leading to constructive or destructive interference, depending on the relative phase between the different paths. On a distant screen, an interference pattern is observed if the width of the initial momentum spread of the atoms in the direction of the lattice is smaller than the recoil momentum of the lattice $\pi/d$, where $d$ is the lattice spacing. For a thermal beam this condition can be achieved by using apertures which filter out a coherent part of the beam. For a trapped sample however, the temperature of the atoms must be reduced below the recoil temperature. An optical lattice created by two counter propagating laser beams has a lattice spacing of $d = \lambda/2$, where $\lambda$ is the wavelength of the laser. The recoil temperature which is associated with the recoil momentum is in general below 1 µK. Consequently, the experimental conditions for observing interference with trapped atoms are similar to the requirements for achieving Bose-Einstein condensation. It is for this reason, that the first experimental observation of the interference of trapped atoms was made with a Bose-Einstein condensate. Being a completely coherent sample a condensate shows high contrast and good visibility. However the required momentum condition can also be achieved with other atomic samples such as
Figure 1: Dispersion relation: energy of the quasi momentum states of a periodic potential, presented in an extended-zone scheme. The bandgap $E_{\text{gap}}$ and the band width $E_{\text{bw}}$ are indicated on the right. A particle that is initially prepared at rest (position 1, $q = 0$) changes its quasi momentum under a constant external force $F$: $\dot{q} = F/\hbar$. The quasi momentum increases linearly until it reaches the Bragg momentum $q_B = \pi/d = 2\pi/\lambda$ (position 2). For a sufficiently high lattice it is Bragg reflected and reappears at the opposite side of the Brillouin zone (position 3, $-q_B$). It then scans anew the first Brillouin zone, thus performing a periodic motion in quasi momentum and in real space. In case of a shallow lattice there is a finite probability to undergo a Zener tunnelling transition from position 3 to position 4 corresponding to a band transition from the first to the second band. The particle is not reflected at the band edge and gains more quasi momentum (position 5). Subsequent tunnelling into higher bands is more and more likely and the particles motion can follow the external force finally being released from the lattice.

thermal clouds [6] or degenerate Fermi gases. Indeed our experimental results presented in this work prove that the observation of atomic interference within a trapped sample is not restricted to Bose-Einstein condensates. On the contrary atomic Fermi gases exhibit an even superior performance because the initially good visibility of the interference pattern of a condensate is counterbalanced by a washing out of the fringes for longer holding times.

The work is organized as follows: in the first part we introduce the theoretical concepts to describe particles in a periodic potential which is subjected to an external force. In the second part we introduce our experimental setup, the loading procedure of the atoms in the lattice and show how the interference pattern can be detected. In the last part we present our results and discuss possible extensions of our work for future applications in precision interferometry.
2 Theoretical Concepts

Particles in a homogeneous periodic potential are described with Bloch states. They extend over the whole lattice and are characterized by the quasi momentum \( q \) and the band index \( n \). The dispersion relation of the Bloch states for a sinusoidally potential is shown in Fig. 1 in an extended-zone-scheme: it shows the typical band structure with the band gap \( E_{\text{gap}} \) at the recoil energy \( E_r = \hbar^2 (2\pi/\lambda)^2 / 2m \), where \( m \) is the mass of the particle. For simplicity we only consider one dimension. Without an external force, a particle which is prepared in a quasi momentum state \( q \) and a band \( n \) will remain in this state and propagate with a velocity which is given by the first derivative of the dispersion relation

\[
v = \frac{1}{\hbar} \frac{\partial E(q)}{\partial q}.
\]

In the presence of a constant external force \( F_{\text{ext}} \), the dynamics of the particle can be described within a semiclassical approximation \[7\] and the quasi momentum evolves according to

\[
\dot{q} = \frac{F_{\text{ext}}}{\hbar}.
\]

When the quasi momentum of a particle which is prepared in the lowest band equals the Bragg momentum \( q_B = \pi/d = 2\pi/\lambda \) there are two options: if the band gap is large it stays confined within its band and is Bragg reflected from the lattice (Fig. 1). It reappears on the opposite site of the first Brillouin zone \((\pm q_B)\) and scans anew the Brillouin zone. The temporal evolution of the quasi momentum is hence a periodic sawtooth function. These dynamics are known as Bloch oscillations \[6\]. The angular frequency \( \omega_{\text{BO}} \) and the amplitude \( a \) are given by

\[
\omega_{\text{BO}} = \frac{F_{\text{ext}} \lambda}{2\hbar} = \frac{mg \lambda}{2\hbar},
\]

\[
a = \frac{E_{\text{bw}}}{2F_{\text{ext}}} = \frac{E_{\text{bw}}}{2mg},
\]

where \( E_{\text{bw}} \) is the band width. The second identity gives the explicit expression for the gravitational force \( F_{\text{ext}} = mg \). The second option is a Zener tunnelling process into the second band, whose probability depends strongly on the chosen parameters of the potential and can be tuned by changing the lattice depth.

3 Experimental Setup

In the experiment we employ a mixture of bosonic \(^{87}\text{Rb}\) and fermionic \(^{40}\text{K}\) atoms. Both species are prepared in a magneto-optical trap and subsequently transferred in a magnetic trapping potential with trapping frequencies of \( \omega_a = 2\pi \times 16 \text{ Hz} \) and \( \omega_r = 2\pi \times 200 \text{ Hz} \) in the axial and radial direction (for \(^{87}\text{Rb}\)). During the forced evaporation of \(^{87}\text{Rb}\) the fermions are sympathetically cooled and we can bring both species simultaneously to quantum degeneracy \[9\].
removing all bosons at the end of the evaporation ramp we can produce a pure Fermi gas of about $3 \times 10^4$ atoms spin polarized in the $F = 9/2, m_F = 9/2$ state. The typical temperature is $T = 0.3T_F$, where $T_F = 330$ nK is the Fermi temperature. A pure Bose-Einstein condensate can be produced with about $10^5$ atoms in the $F = 2, m_F = 2$ state. We then switch on adiabatically a lattice created by a retroreflected laser beam aligned along the vertical direction.

The wavelength of the lattice laser is far detuned to the red of the optical atomic transitions ($\lambda = 873$ nm) to avoid photon scattering. The depth of the potential can be adjusted in the range of $U_K = 1 - 4E_r$ for the fermions and $U_{RB} = 1 - 10E_r$ for the bosons. Note, that the recoil energy $E_r = \hbar^2 k^2 / 2m$ differs by factor of 2 for the two species due to the different mass. Because both, the Fermi temperature and the critical temperature for Bose-Einstein condensation are comparable to the recoil energy divided by the Boltzmann constant, the atoms are loaded mostly in the first band of the lattice potential. The Gaussian profile of the lattice beams provides the confinement in the horizontal plane. Using different intensities for the two beams we obtain a radial depth of $\ldots$
the optical potential of about 10 $E_r$, with a typical trap frequency of $2\pi \times 30$ s$^{-1}$. Due to the harmonic radial confinement every quasi momentum state has an additional harmonic oscillator ladder for each of the two radial dimensions. However, the Hamiltonian of the 3D system can be separated in its three coordinates \[^{[11]}\] and the motion in the direction of the lattice is decoupled from the motion in the radial direction. Therefore the three dimensional nature of our setup does not change the equation of motion \[^{[2]}\). A sketch of the experimental setup is shown in Fig.\[^{[2]}\].

To study the interference we suddenly switch off the magnetic trap and let the atoms evolve in the combined lattice and gravitational potential. In our range of lattice depths only the atoms in the first band are trapped against gravity, while the small fraction in excited bands falls down freely. After a given evolution time we release the atoms from the trap and probe the cloud by absorption imaging after 8 ms (20 ms) of free expansion for the fermions (bosons). The interference of the atomic cloud can be made visible in different ways. The first method, where the potential height is lowered to a small value until the atoms drop out of the potential via Zener Tunnelling processes leads to a continuous output. For our setup the probability for such a process is given by $P = e^{-\lambda U_E^2g/\hbar^2g}$, where $g$ is the earth acceleration, and the band gap $E_{\text{gap}}$ can be derived from the lattice depth $U = sE_r$. Here the lattice depth is measured in units of the recoil energy (parameter $s$). Due to the exponential dependence on the square of the band gap, we can completely suppress Zener tunnelling ($s > 3$) or precisely tune it in order to realize a continuous output coupler ($0 < s < 2$). If the momentum spread of the atoms is smaller than the width of the first Brillouin zone a constant fraction of the atoms is released from the lattice and falls along gravity, every time the ensemble reaches the Bragg momentum. The output is expected to be pulsed. In the second method the lattice is switched off instantaneously (non-adiabatically) thus projecting the quasi momentum distribution on the momentum distribution. After a free expansion of the atoms, the spatial distribution of the atoms reveals the momentum distribution of the atoms inside the optical lattice. A similar result is achieved in a slightly modified method, where the optical lattice is switched off adiabatically. For these purposes the lattice depth is lowered to zero in about 50 $\mu$s, a time scale longer than the oscillation period of the atoms in each lattice site. Thereby, the quasi momentum distribution is exactly mapped (and not projected) to the momentum distribution. From the absorption image we can extract the momentum distribution of the atomic cloud. For interferometric applications one is interested in the time evolution of the quasi momentum. To these purposes we have to repeat the procedure for various trapping times and perform a frequency analysis of the data.

The interpretation of the atomic distribution after release from the trap as an interference pattern might not be obvious, especially when only a single peak in the momentum distribution is observed. The reason for this is that the wave nature of the atoms is only apparent in the structure of the Bloch states whereas the equations of motion (Eqns.\[^{[1]}\) and \[^{[2]}\) are classical. However,
in an pure quantum-mechanical description based on Wannier-Stark states the interpretation is straightforward [8]: the atoms occupy the stationary states of the Wannier-Stark ladder and after release, the outcoupled atoms of all states interfere during the expansion.

4 Results and Discussion

In Fig. 3a, we show the interference pattern of a Bose-Einstein condensate which is released continuously from the lattice directly after being loaded. It shows the pulsed output that is known for a coherent ensemble [5]. The distance between the pulses is defined by the Bloch oscillation period which amounts to $T_b = 1.1\, \text{ms}$ for the bosons and $T_f = 2.3\, \text{ms}$ for the fermions. The visibility of the interference pattern for the condensate is large. Note, that the vertical width of the pulses is much larger than the quasi momentum spread of the condensate in the lattice. This is due to the inter atomic repulsion that broadens...
the distribution during the expansion. In Fig. 3b we repeat the experiment with a degenerate Fermi gas. It shows the same structure however, because of the longer Bloch oscillation period we observe only three pulses. It is important to compare the visibility of the pattern with that of the condensate: initially, the fermions have a much broader quasi momentum distribution in the lattice. Its half-width $\delta q$ is determined by the Fermi momentum and for our parameters we find $\delta q = 0.75q_B$. This fulfills the requirement of a quasi momentum distribution which is narrower than the recoil momentum of the lattice. In contrast to the condensate, the momentum of the fermions does not change during the expansion because the Pauli principle forbids collisions between identical fermions at ultra low temperatures. As a result the interference patterns for both, the condensate and the Fermi gas show a similar contrast after the expansion (Fig. 3).

In a second experiment we have studied the evolution of the interference pattern for longer holding times. After loading we keep the atoms in the vertical lattice for several ms and repeat the continuous release as described in the previous paragraph. The results are shown in Fig. 4. While the distribution of the fermions is unchanged with respect to the previous measurement the interference pattern of the condensate has been washed out completely. This striking difference is due to the interaction of the atoms inside the condensate. A possible heating of the atoms in the lattice can be excluded because the radial momentum distribution does not change. We can give a qualitative explana-
Figure 5: Time resolved Bloch oscillations. The graph shows the evolution of the quasi momentum for 250 ms, corresponding to 110 periods. The data are fitted with a sawtooth function. Absorption images are shown for $t = 0$, corresponding to the initial preparation of the cloud at rest and at $t = T_{\text{Bloch}}/2$, where half of the cloud is Bragg reflected. The third image shows the cloud after 252 ms where the contrast has substantially decreased.

...tion of this effect by looking at the total energy difference between atoms in neighboring lattice sites. In addition to the gravitational potential the bosons experience a phase shift due to the mean field energy. As the density profile of the condensate along the lattice is inhomogeneous the contribution from the mean field energy varies between neighboring lattice sites. This leads to a dephasing of the wave function in different lattice sites \[13\] and to a loss of phase coherence. In a spin polarized Fermi gas this effect is absent because collisions between the atoms are forbidden. Hence, the phase difference between atoms in neighboring lattice site is solely determined by the external force. An important conclusion can be drawn from this experiment: in interferometric applications of trapped atomic gases it is more favorable to employ a degenerate Fermi gas than a Bose condensed gas because the disadvantage of having a relatively broad momentum distribution is overcompensated by the absence of collisions.

After having shown the superity of fermions for trapped atom interferometry we now present an experiment where we have tested the limits of an interferometry with trapped Fermi gases. Using the same experimental sequence as...
described above we load a pure Fermi gas in the optical lattice and follow the evolution of the quasi momentum for many periods. At the end of each experimental run we now adiabatically switch off the optical lattice and image the cloud after 8 ms time of flight. From the absorption images which show the quasi momentum distribution we can extract the position of the main peak (Fig. 5). Indeed if we follow its motion we find the peculiar sawtooth shape expected for Bloch oscillations (see section 2). We can follow the oscillations for more than 250 ms corresponding to about 110 Bloch periods, and only at later times the contrast is degraded by a broadening of the momentum distribution. This is to our knowledge the longest lived Bloch oscillator observed so far in all kind of physical systems. The reduction of contrast can be seen from Fig. 5 where we show the absorption image of a data point at 252 ms. The atoms fill nearly homogeneously the first Brillouin zone and we cannot any longer determine the center of the quasi momentum distribution with high precision. From a fit to the present data we obtain a Bloch oscillation period of $T_B = 2.32789(22)$ ms. Assuming that the only uniform force acting on the atomic sample is gravity we determine a local gravitational acceleration from Eqn. 3 as $g = 9.7372(9)$ m/s$^2$.

The precision at the level of $10^{-4}$ obtained in this proof-of-principle experiment is promising for future application of this technique to accurate determination of forces. Note that the extension of the Fermi gas in the vertical direction is very small, $\Delta z = 55\mu$m, and in principle can be reduced down to the size of a few lattice sites without loosing the interference pattern. Such a sensor based on trapped fermions is therefore promising to extend the measurement of forces to length scales in the micrometer range, with applications to the study of forces close to surfaces, or to gravity at small scales [14].

5 Summary

In conclusion we have studied the interference of trapped bosons and fermions in an optical lattice aligned along gravity. For a condensate interference can be seen with high contrast. However the interference pattern is rapidly washed out for even short evolution times in the vertical lattice. The absence of interactions in a spin polarized fermionic sample prevents a reduction of the contrast for longer evolution times and proves the superiority of fermions for trapped atom interferometry. We have also demonstrated that interferometry with trapped fermions allows for a high precision measurement of forces on a micrometer length scale, with possible application to the study of fundamental phenomena.

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[10] In an infinite system these states are not square-integrable and are therefore often labeled as quasi stationary states or resonances.

[11] This is not strictly true because the lattice height varies across the beam diameter, thus coupling the motion along the lattice with the radial motion.

[12] Care has to be taken by calculating the momentum spread of a Fermi gas in the direction of the lattice. Because of the $\theta$-function shaped Fermi distribution the integration over the two radial directions leads to a momentum spread which is effectively smaller than the Fermi momentum.

[13] Because a tilted periodic potential does not posses an abolute potential minimum one cannot define a ground state of the system. Consequently a condensate which is loaded into such kind of potential is never in equilibrium and cannot be described with a single stationary wave function. In fact the experimental observation directly proves this non-stationarity.

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