Quantum entanglement of dark matter

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We suggest that dark matter in the universe has quantum entanglement among dark matter particles if the dark matter is a Bose-Einstein condensation of ultra-light scalar particles. In this theory any two regions of a galaxy are quantum entangled due to the quantum nature of the condensate. We calculate the entanglement entropy of a typical galactic halo, which turns out to be at least $O(ln(M/m))$ where $M$ is the mass of the halo and $m$ is the dark matter particle mass.

Keywords: dark matter, BEC, Entanglement, galactic halos
Dark matter (DM) is one of the greatest mysteries in physics. Despite decade-long efforts DM defies a successful explanation. The flatness of the galactic rotation curves implies the presence of invisible DM in galactic halos [1], and any successful DM theory should explain the curves. The cold dark matter (CDM) with the cosmological constant (ΛCDM) model is very successful in explaining the large scale structure of the universe, but it encounters many difficulties at the galactic scale. For example, numerical simulations with ΛCDM model predict a cusped central density and too many subhalos compared to observations [2,3]. Therefore, we need an alternative for CDM playing the role of CDM at the super-galactic scale and suppressing sub-galactic structures. The Bose-Einstein condensate (BEC) DM or the scalar field dark matter (SFDM) [6,7] can be a good candidate. In this model DM is a BEC of Ultra-light Scalar particles with mass $m \simeq 10^{-22}$ eV, which implies a large DM Compton wavelength. This prevents a formation of structures smaller than a galaxy. Beyond this scale the coherent nature of BEC DM makes it behave like CDM, and hence solves the problems of CDM.

The idea that DM is a ultra-light boson has a long history (See Ref. [8–13] for a review.). Baldeschi et al. [14] considered galactic halos of self-gravitating bosons, and Membrado et al. [15] obtained rotation curves for the self-gravitating boson sphere. Sin [6] suggested that galactic halos are like gigantic atoms made of ultra-light BEC DM such as pseudo Nambu-Goldstone bosons (PNGBs). In this model, boson DM particles are described by a single macroscopic wave function and the quantum uncertainty principle prevents halos from self-gravitational collapse. In the context of field theory and general relativity Lee and Koh [7,16] extended Sin’s BEC DM model and suggested that dark matter in a halo composes a single giant boson star described by Einstein-Klein-Gordon Equations (EKG). Widrow et al. [17] suggested a new numerical method using the Schrödinger equation for collisionless matter. Similar ideas have been developed by many authors using various potentials and fields [18–35]. It has been shown that these models could explain the observed rotation curves [20,36–38], the large scale structures of the universe [39], the cosmic background radiation, and spiral arms and bars [40]. Therefore, scalar field dark matter can be a good alternative for CDM.

On the other hand, quantum entanglement (a nonlocal quantum correlation) [41] is an important physical resource for quantum information processing such as quantum key distribution and quantum computing. Recently, it was shown that entanglement is a key to understand gravity [42] and even space time itself [43]. Furthermore, entanglement is considered as a new order parameter for condensed matter systems such as BEC.

In this paper, we show that DM in the universe has entanglement among DM particles. In BEC DM model [6] the galactic DM halo is described with a wave function $\psi(\mathbf{r})$ of the non-linear Schrödinger equation (the Gross-Pitaevskii equation (GPE))

$$i\hbar \partial_t \psi(\mathbf{r}) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}) + m \Phi \psi(\mathbf{r})$$

(1)

with a self-gravitation potential $\Phi$;

$$\Phi = \int_0^r dr' \frac{1}{r'^2} \int_0^r dr'' 4\pi r''^2 (GmM|\psi(r')|^2 + \rho_v),$$

(2)

where $m$ is the individual DM particle mass, $M$ is the mass of the halo, and $\rho_v$ is the mass density of visible matter. $\Phi$ plays a role of a trap potential for the galactic BEC. For simplicity, here we consider a spherical symmetric case and ignore $\rho_v$. It is a good approximation for DM dominated galaxies. GPE can be obtained from the mean field approximation of a many body BEC Hamiltonian or the Newtonian approximation of SFDM Lagrangian [7].

In many body theory of BEC one usually decompose the field operator with the annihilation operator $a_\alpha$ as

$$\hat{\Psi}(\mathbf{r}) = \sum_\alpha \psi_\alpha(\mathbf{r}) a_\alpha,$$

(3)

where the single particle wave functions $\psi_\alpha(\mathbf{r})$ are orthonormal, i.e.,

$$\sum_\alpha \psi_\alpha(\mathbf{r}) \psi_\alpha^*(\mathbf{r}') = \delta(\mathbf{r} - \mathbf{r'}).$$

(4)

Then, $\hat{\Psi}(\mathbf{r})$ fulfills the commutation relation $[\hat{\Psi}(x), \hat{\Psi}^\dagger(y)] = \delta^{(3)}(x - y)\mathbf{1}$. BEC DM has a high phase transition temperature ($T_c > T_{\text{eV}}$), and the present temperature of DM is so low that we can treat DM state as a zero temperature BEC state. At zero temperature all boson particles are in a ground state, which is represented by

$$|\Psi_0\rangle = \frac{a_0^{\dagger N}}{\sqrt{N!}} |0\rangle,$$

(5)
where $N$ is the number of DM particles, $|0\rangle$ denotes the vacuum state, and $a_A^\dagger = \int d^3r \psi_0(r) \hat{\Psi}^\dagger(r)$. The lowest energy mode function $\psi_0(r)$ in a boson halo can be obtained by solving GPE (Eq. (11)).

Studies on entanglement of BEC usually focused on spinor BEC or multi-component BEC. However, for our purpose we need a formalism for a single component scalar BEC. Following Refs. 44 and 45, we consider the entanglement between two subregions of a galaxy, A and B. For example, the regions $A$ and $B$ can be an upper and a lower parts of a galaxy, respectively. To calculate the entanglement between two regions it is more convenient to define new annihilation operators $(a_A, a_B)$ such that

$$a_A^\dagger = \frac{1}{P_A} \int_{r \in A} d^3r \psi_0(r) \hat{\Psi}^\dagger(r),$$

$$a_B^\dagger = \frac{1}{P_B} \int_{r \in B} d^3r \psi_0(r) \hat{\Psi}^\dagger(r),$$

where $P_A = \int_{r \in A} d^3r |\psi_0(r)|^2 = 1 - P_B = 1 - \int_{r \in B} d^3r |\psi_0(r)|^2$. It means that $a_A^\dagger$ creates a DM particle at the subregion $A$, and the probability to find the particle there is $P_A$. Then, the ground state becomes

$$|\Psi_0\rangle = \frac{1}{N!} (\sqrt{P_A} a_A^\dagger + \sqrt{P_B} a_B^\dagger)^N |0\rangle$$

$$= \frac{1}{N!} \sum_{k=0}^{N} NC_k \sqrt{P_A^k P_B^{N-k}} a_A^k a_B^{N-k} |0\rangle$$

$$= \sum_{k=0}^{N} \sqrt{N!} C_k \sqrt{P_A^k P_B^{N-k}} |k\rangle_A |N-k\rangle_B$$

$$= \sum_{k=0}^{N} \sqrt{\lambda_k} |k\rangle_A |N-k\rangle_B,$$

where $\lambda_k = NC_k P_A^k P_B^{N-k}$. Note that we cannot make the state separable, thus any part of a galaxy is entangled with outside of the region in the galaxy.

For $P_A \ll P_B$ and $N \to \infty$ the state becomes a Poisson distribution with

$$\lambda_k \simeq e^{-N_A} \frac{N^k}{k!},$$

where $N_A = N P_A$ is the mean particle number in the subregion $A$. The measure of entanglement we choose is the entanglement entropy

$$S_E = - \sum_k \lambda_k \ln \lambda_k,$$

which is equivalent to the Poisson entropy in this case [46]. Unfortunately, there is no known closed form of the Poisson entropy. For large $N_A$ the entropy of the Poisson distribution can be approximated by [46]

$$S_E \simeq \frac{1}{2} \ln(2\pi e N_A) + O\left(\frac{1}{N_A}\right).$$

If $M_A$ is the mass of the region $A$, then $N_A$ is given by $M_A/m$. Therefore,

$$S_E \simeq \frac{1}{2} \ln(N_A) + 1.41 = \frac{1}{2} \ln(M_A/m) + 1.41$$

For $m = 10^{-22}$eV, $S_E \simeq \frac{1}{2} \ln(M_A/M_\odot) + 102.41$. For a typical galaxy $M \simeq 10^{11} M_\odot$, so $S_E = O(10^2)$. This is not a big number, but it should be noticed that this value is just for the entanglement for a specific bisection of a galaxy. A galaxy can have arbitrary many ways of bisection, thus the total entanglement of the galaxy can be huge. Furthermore, there can be entanglement between other modes we did not consider. Therefore, $S_E$ in Eq. (11) should be treated as a lower bound for the entanglement of a galaxy. In BEC DM theory arbitrary two regions of a galaxy are entangled, because all DM particles are described by a single macroscopic state.
To see how entanglement scales we consider a bipartite system which consists of a spherical subregion (A) within the radial distance $r$ from the center of galaxy and outer region (B). The total DM mass within $r$ is given by

$$M_A(r) = 4\pi M \int_0^r dr' r'^2 |\psi_0(r')|^2.$$  \hspace{1cm} (12)

Using $M_A(r)$ the entanglement between A and B can be obtained by Eq. (11).

GPE for a small galaxy has an approximate analytic solution $\psi_0(r) = \psi_0(0)e^{-r^2/r_c^2}$ for the single particle ground state, where $r_c$ is the DM halo core radius. This approximate solution has no sharp boundary, and we arbitrary choose a spatial cut off at $3r_c$. Then, the normalization condition $4\pi \int_0^{3r_c} dr |\psi_0(r)|^2 = 1$ gives $\psi_0(0) \simeq \alpha r_c^{-3/2}$ with $\alpha = 0.713$. Thus,

$$\psi_0(r) = \frac{\alpha e^{-r^2/r_c^2}}{r_c^{3/2}}, \hspace{1cm} (13)$$

and inserting it into Eq. (12) gives

$$M_A(r) = \frac{\pi \alpha^2}{4} \left( \sqrt{2\pi} \text{erf} \left[ \frac{\sqrt{2}r}{r_c} \right] - 4e^{-\frac{2\sqrt{2}}{r_c^2}r} \right) M, \hspace{1cm} (14)$$

where erf denotes the error function. Now Eq. (11) gives

$$S_E(r) \simeq \frac{1}{2} \ln(M/m)$$

$$+ \frac{1}{2} \ln \left( \sqrt{2\pi} \text{erf} \left[ \frac{\sqrt{2}r}{r_c} \right] - 4e^{-\frac{2\sqrt{2}}{r_c^2}r} \right) + 0.95,$$  \hspace{1cm} (15)

FIG. 1. The dark matter density $\psi_0^2$ (solid line), and the mass inside $r$, $M_A(r)/M$ (dashed line) as functions of distance $r$ (in units of the halo core radius) from the halo center. The wave function is rescaled as $\psi_0(0) = 1$.

Fig. 1 and Fig. 2 show an example with $M = 10^7 M_\odot$ which can be a model of a dwarf galaxy. Note that the local dark matter density is proportional to $|\psi_0(r)|^2$. For a comparison, the wave function is rescaled and $M(r)$ is in units of $M$. In Fig. 2 $S_E(r)$ is a rapidly rising function for a small $r$ and approaches a maximum value. It should be noticed that our approximate formula for $S_E$ is valid only for $1 \ll N_A \ll N_B$, and hence invalid for $r \to 0$ or for too large $r$. $S_E$ is not small for even a small $r$, because the particle mass $m$ is extremely small compared to $M_A(r)$. Even a tiny region (say the solar system) has a gigantic number of DM particles which can be entangled with other regions.

The entanglement between a region and other parts of the current observable universe can be similarly estimated using the formula. A rough estimation gives $S_E \simeq \frac{1}{2} \ln(M_u/M_\odot) + 102.41 \simeq 127.73$ at least, where $M_u = O(10^{22}) M_\odot$ is the total DM mass of the universe. The inflation period might had provided chances for a quantum coherence among widely separated cosmic structures, however it is unclear whether there is entanglement among independent halos actually.

Since the BEC/SFDM with a ultra-light mass can be a good alternative for CDM, it is important to understand the difference between two models. BEC DM can have the macroscopic quantum nature, because DM can be robust against the decoherence, while CDM particles are usually thought as classical incoherent objects. Therefore, the
entanglement of DM can be a good criterion to distinguish two models. In the future we might utilize this inherent DM entanglement for a galactic scale quantum information processing such as quantum teleportation between two solar systems.

FIG. 2. Entanglement entropy between a central spherical region and the outside of the region as a function of $r$ up to $2r_c$. 

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