Abstract
Production technology planning requires information on tool life $T$ and its relation to cutting speed $v$. These technological parameters fundamentally affect the economics of the operation of the entire production system. The aim of the present study is to investigate the $v$-$T$ relationship under technological conditions where this relationship cannot be described by the methods used so far – typically the Taylor formula. As the Taylor formula often cannot be linearized on a log-log scale, a general tool life function has been developed for describing a $v$-$T$ function with a convex-concave part. The function was validated by the boring of hardened ball bearing rings with the technological data $D = 45$, 75 and 100 mm, $a_v = 0.14$ mm, $f_z = 0.005$ mm/rev. The general $v$-$T$ tool life function has been transformed into a quadratic polynomial known in catastrophe theory as cusp catastrophe and allows the description of abrupt changes in the system under study. Using catastrophe theory, an analogy is established between the general tool life function and the cusp catastrophe, allowing topological mapping
of the general v-T function. Results were verified by machinability tests in the turning of C35 and C60 conventional and specially deoxidized C-steels during steelmaking. Cutting tests were performed on specimens with a diameter of \( D_{\text{spec}} = 76 \text{ mm} \) and a length of \( L = 700 \text{ mm} \) at a speed of \( v = 100, 150, 200 \text{ m/min} \) and a carbide tool of category P20. It was found that in the convex-concave section of this function, 2–3 cutting speeds can be selected for a given tool life, which is advantageous for harmonizing tool changes in multi-operation technology.

**Keywords**
General tool life function, catastrophe-theory, turning, non-metallic deposition on tool

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**Highlights**
- The tool life \( T \) of the cutting tool – which is depending on the speed \( v \) – is an important cost factor in machining;
- The general v-T function can also have a convex-concave section that is experimentally validated;
- Catastrophe theory can be used to describe sudden changes in the cutting speed-tool life relationship;
- Technological experiments prove the usefulness of catastrophe theory in judging machinability.

**Introduction**
Among the technological costs of the mechanical engineering industry, the cost of tools represents a significant proportion. Therefore, it is natural that the wear of tools has long been a central theme in the theory of cutting. Over 60 years ago Finnie\(^1\) reviewed the previous 100-year history of cutting theory, paying special attention to tool wear and tool life. Interest in this field has persisted, as shown by, for instance, Aydin et al.\(^2\) for their study of wear on saw blades or Tatar and Svenningson\(^3\) for wear tests recently performed on milling. Since the classic Taylor article on cutting theory in 1907, tool life has remained a central theme in cutting theory. Researchers have long and extensively investigated the wear and life of cutting tools.

Cui et al.\(^4\) experimentally investigated the development of tool wear and the achievable tool life at different cutting speeds for high-speed face milling of AISI H13 steel. Dogra et al.\(^5\) performed hard turning experiments on continuous, interrupted, and fully interrupted workpiece surfaces using coated and CBN tools. Their aim was to investigate under which tool life and surface integrity parameters (roughness, hardness, residual stress) coated carbide tools could be used instead of expensive CBN inserts. Sagar et al.\(^6\) investigated the development of flank wear during turning of tungsten heavy alloys and developed an analytical model to model wear and tool life. Dureja et al.\(^7\) dealt with a comprehensive analysis of different modeling and optimization procedures for hard turning. Amaitik et al.\(^8\) used multi-linear regression analysis to investigate the relationship between tool life and cutting speed and feed at milling. Researchers are increasingly using special methods to describe tool life depending on different cutting parameters as well as on the workpiece. Xue et al.\(^9\) developed an improved particle swarm optimization-back-propagation neural network model for this task.

The application of catastrophe theory in machining has already produced a number of promising results. This is due to the fact that several aspects of chip removal may be characterized by a sudden transformation of the gradual change into an abrupt/transient motion. On the one hand, as known, structural metals are characterized by the property of increasing their resistance to deformation during plastic deformation. On the other hand, the increase in temperature due to the thermal action of the deformation work reduces this resistance, that is, it is negative feedback from energetic point of view. A consequence may be thermal instability leading to the formation of so-called lamellar chips. Following Recht's work,\(^10\) this has been extensively studied by several researchers, including Komanduri and von Turkovich.\(^11\) Catastrophe theory can also be used to study forming with high deformation. Klamecki\(^12\) used this possibility in the shear zone by applying the principle of energy minimum, demonstrating that the phenomena he studied in chip formation can be described by the properties of Thom’s cusp catastrophe.\(^13\) The minimum energy surface is folded/back-bending and the system state changes along it. This can lead to sudden large changes. A similar finding was made by Miernik,\(^14\) who described the deformations in the chip root by a third- and fifth-degree polynomial using the principle of minimum energy. The former is a peak disaster with one degree of freedom, and the latter is a “butterfly catastrophe” with two degrees of freedom, with a double recoil on its energy surface.

Luo\(^15\) reported an improvement in turning and end milling aimed at predicting tool wear through continuous monitoring of its condition. In evaluating the experimental results, the tools of catastrophe theory were utilized in such a way that the measurement data were processed with a cubic polynomial with variable parameters. The monitoring system thus designed made it possible to predict tool wear with more than 95% confidence.
Catastrophe theory has been successfully applied by several researchers in the case of non-free cutting. Here, the combined effect of the two shear zones prevails, and Shi et al.16 investigated the direction of chip ejection using catastrophe theory. It was found that the chip ejection angle differs if this event occurs as a result of decreasing or increasing the a/f chip ratio. With a more detailed analysis, Cui et al.17 and Zhu et al.18 came to the same conclusion.

This report describes the work aimed at analyzing the general lifetime function of cutting tools using the tools of catastrophe theory. To achieve this goal, the following steps have been taken:

- Investigation of the method used to detect specific v-T value pairs, which are sometimes multi-valued, in the general tool life function extended to the convex-concave section of the v-T cutting speed–tool life relationship commonly used in technology;
- Study of the properties of a third-degree polynomial obtained by mathematical rearrangement of the general tool life function using the tools of catastrophe theory;
- Comparative cutting tests by turning steels where a convex-concave v-T section can be detected in one version;
- Analysis of the results of the measurements obtained in the cutting test using catastrophe theory;
- Discussion of the experimentally validated theoretical v-T analysis.

It has been proved that there is an analogy between the phenomena studied by the catastrophe theory and the technological dependence of the wear and tool life of the cutting tool, which can be successfully used for the qualitative analysis of cutting theory and practical manufacturing technology.

Analysis of the general tool life function

The Taylor formula is commonly used to calculate the tool life of cutting tools. This, as is known, is an empirical fractional hyperbola which, by default, describes the relationship between cutting speed v and tool life T, preferably as a linear function on lg-lg scale diagrams (see the T\text{trad} in Figure 1). But not always. Chip removal is essentially a very complex technological process, in strong interaction with the properties of the machined material and the tool. Of particular importance in this respect is the empirical fact that the load conditions of the tool and the abrasive effect on the surface change significantly under certain technological conditions. The consequence of this is that in such cases the v-T relationship is described by the Taylor formula only within a limited interval. Johansson et al.19 presented a valuable review article on the generally used tool life models, where it was also concluded that the field of application of the Taylor formula is limited to smaller chip thickness ranges.

Cutting speed-tool life (v-T) relationship

Figure 2 shows tool life characteristics curves\textsuperscript{20} that show either (1) a monotonic (continuous) decrease or (2) a much more common curve with local extremes. Over the years, many proposals have been made to simplify the function describing the relationship between cutting data and tool life using approximation curves.
The authors of the current study have given several brief summaries of these in their previous works, most recently in Kundrak et al.\textsuperscript{21}

Several proposals have been made to solve this problem. The type of function proposed by Kundrak\textsuperscript{22} proved to be advantageous for theoretical analysis and practical application:

\[ T = \frac{C_1}{v^3} + C_2v^2 + C_3v \]  

(1)

An important feature of this function is that there is a well-utilized relationship between its two local extremes and the constant \( C_2, C_3^{20-23} \):

\[ v_{\text{min, max}} = \frac{1}{3} \left( C_2 \pm \sqrt{C_2^2 - 3C_3} \right) \]  

(2)

\[ C_2 = \frac{3}{2} (v_{\text{min}} - v_{\text{max}}) \]  

(3)

\[ C_3 = 3v_{\text{min}}v_{\text{max}} \]  

(4)

The section of the general tool life function, the nature of which is very different from the Taylor function, deserves special attention from a theoretical and technological point of view. This can be followed on function (1) by the expedient choice of the constants \( C_i \) \((i = 1, 2, 3)\). To illustrate this, Figure 3 shows the effect of \( C_2 \). Of course, the \( C_3 \) constant has a similar effect.

\[ T = \frac{C_1}{v^3} + C_2v^2 + C_3v \]  

Figure 3. Characteristics of the general tool life function (1) at three characteristic values of the constant \( C_2 \) \((C_1 = 2,330,000, C_3 = 1755, a_p = 0.15, \text{ and } f_z = 0.05 \text{ mm}, \text{ material: } 100\text{Cr}6, \text{ HRC}62, \text{ tool: Compozit 10})\).

\[ T = \frac{C_1}{v^3} + C_2v^2 + C_3v \]  

Figure 4. General tool life function for hard turning of a hardened ball bearing ring\textsuperscript{20-23} \((a_p = 0.15 \text{ mm}, f_z = 0.05 \text{ mm/rev}, \text{ tool: Composite 10})\).

\[ T = \frac{C_1}{v^3} + C_2v^2 + C_3v \]  

Figure 5. Fitting of the general tool life function to a third-degree polynomial in the velocity range \( v = 4-50 \text{ m/min} \) (see Figure 4, \( D = 75 \text{ mm})\).

**Analysis of the general tool life function based on experiments with a CBN tool**

As an example, the general tool life curve for hard turning of the inner cylindrical surface of a hardened ball bearing ring of three diameters is shown in Figure 4.\textsuperscript{22}

An important feature of function (1) is that a third degree polynomial can be fitted to the middle section of Figure 4, which illustrates the effect of the constant \( C_2 \).

This fit is shown in Figure 5 for the velocity interval \( v = 4-50 \text{ m/min} \) for the curve corresponding to the hole turning of diameter \( D = 75 \text{ mm} \) shown in Figure 4.

Good fit can also be established by visual inspection, which in the present case is also shown by the Pearson number \( R^2 = 1 \). A more detailed study also supports this finding, as shown in Figure 6.

Function (1) is also useful in the application scope of the Taylor formula. In the section of the v-T curve where the Taylor formula describes the velocity–tool
life relationship well. Function (1) can also be applied with good agreement. Figure 7 shows this in this case at a constant $C_2 = 0$. Figure 8 illustrates how the correlation between Function (1) and the linear $\lg v - \lg T$ function can be increased by changing this constant $C_2$.

In the section of the function curves $v$-T where the cutting speed $v_{\max} < v$ (see e.g. Figure 4), in principle the Taylor formula could also be applied. The two functions can be compared using the exponent $-k$ of the Taylor formula. As is known,

$$vT^{-k} = C$$  \hfill (5)

In the velocity range $v \to v + \Delta v$, a change in tool life $T \to T + \Delta T$ occurs. With these, the exponent $-k$ can be calculated as a function of the speed $v$ by the following formula:

$$-k(v) = \frac{\lg \frac{T}{T + \Delta T}}{\lg \frac{v}{v + \Delta v}}$$  \hfill (6)

The $k(v)$ curves determined from the data in Table 1 are shown in Figure 9. This curve apparently has a limit, $v_{\lim} \approx 3.2$.

These findings prove valid across a wide range of technological versions. Given that catastrophe-theory analyses often use third-order polynomials, the tools of this theory offer useful help in analyzing the characteristics of the general tool life function.

**Catastrophe-theory analogy of the general tool life function based on cutting experiments**

In this section, previously performed cutting experiments are used to examine whether the catastrophe-theory analogy of the general tool life function can be successfully used for qualitative analysis.
For cutting with CBN tools

In practice, it may be necessary to optimize time-consuming tool changes in sub-operations of a multi-stage technological task when harmonization of tool life is required. In such cases, the correct choice of cutting speed is the solution, that is, the inverse of Figure 3 can be used.

The points $v_i = f(T_i)$ of the curves shown in Figure 10 can be calculated from the following equation by rearranging (1):

$$v^3 + C_2 v^2 + C_3 v - C_1 T = 0$$

This equation can be written in the general form as:

$$v^3 + av^2 + bv + c = 0$$

Here $a = C_2$, $b = C_3$ and

$$c = -\frac{C_1}{T}$$

C is thus a system parameter that is a function of an arbitrarily selected tool life $T$ and determines the nature of the solution of equation (8). According to the Cardano method used to solve cubic equation, by expedient transformation equation (10) can be brought to the form

$$v^3 + pv^2 + qv = 0,$$

where:

$$p = C_3 - \frac{C_2^2}{3}$$

and

$$q = 2\left(\frac{C_2}{3}\right)^3 - \frac{C_2 C_3}{3} + c$$

(12)

Here $p$ and $q$ are system parameters. It can be seen from (12) that the parameter $q$ can be used to analyze the change of the cutting speed $v$ as a function of the tool life. The structure of the solutions of equation (10) using the discriminant

$$D_k = 4p^3 + 27q^2$$

(13)

of the quadratic equations is illustrated in Figure 11:

Here, in the p-q coordinate system, the boundary curves $B_1$ and $B_2$ divide the field into two parts. By rearranging

$$4p^3 + 27q^2 = 0$$

(14)

which is expressed from (13),

$$q_{lim} = \sqrt[3]{-\frac{4p^3}{27}}$$

(15)

is obtained, with which the boundary curves $B_1$ and $B_2$ can be calculated. In the case where the extreme values of the $v-T$ curve slip together, that is, $v_{min} = v_{max}$, then the discriminant of the quadratic equation in formula (2) is $D = 0$. This results in:

$$C_3 = \frac{C_2^2}{3}$$

(16)

In the case of $v_{min} = v_{max}$, given the situation $p = q = 0$, the function $T-v$ also has an inflection, which can be calculated from the second differential quotient of Function (1):
When $D > 0$, there is a real and a conjugate complex root, and when $D = 0$, $p \neq 0$, $q \neq 0$, then two real roots coincide. These are the bifurcation lines of the system, denoted by $B_1$ and $B_2$ in Figures 11 and 12. Here the boundary curves shown in the p-q plane are the projections of the array contour of v-curves. If $D = 0$, $p = q = 0$, then all three solutions coincide, and this is the origin of the p-q coordinate system. These characterize important features of the v-T tool life relationship of cutting tools. From the point of view of the roots of equation (10), according to the geometric interpretation and representation, this shows how the point $(p, q)$ is located in the plane p-q. The nature of curve $s_1$ in Figure 12 corresponds to the curve for constant $C_2 = 76.9$ and curve $s_2$ for the constant $C_2 = 69$. The origin of Figures 11 and 12 is a special point, as shown in Figures 10 and 12 by the curve $s$ for the constant $C_2 = -72.54$.

Figures 11 and 12 give a good overview of the mapping of v-T → p-q by the control parameters p and q. The mapping of the v-T functions shown in Figure 4 according to (11), (12) is illustrated in Figure 13 in the p-q coordinate system.

### Material build-up on the cutting tool

Sections 2 presented the analogy between tool life – which is important for both theory and practice – and technological parameters, primarily cutting speed $v$. While it is true that catastrophe theory deals essentially with the dynamic behavior of structures, it is sufficient to think only of Zeeman machines, which use concepts derived from a potential function. Such, of course, cannot be interpreted when examining the tool life. However, a strong analogy can be established, which makes the use of the tools of catastrophe theory usable in this field of technology as well.

This analogy is confirmed by studies aimed at determining the effect of occasional non-metallic or metallic material accumulation on the cutting tool. Particularly important in this respect are the material processes that take place on the tool surface within certain limits of the technological parameters, in particular the cutting speed $v$.

Two essentially similar phenomena are particularly worth highlighting. These are deposits of material on the tool that can destroy but also protect the surface of the tool. As a result, either harmful metallic (built-up edge) or useful non-metallic material deposition occurs on the tool surface. The type of function (1) shown in Figure 1 characterizes the tool life well.

In some cases a non-metallic material builds up on the tool. This is created in the material of the workpiece from deoxidation products remaining during steelmaking. The non-metallic layer can dramatically slow down tool wear. Figure 14(a) shows a picture of a conventional crater wear. Figure 14(b) shows a tool face after turning with the same time and the same technology, on which this special non-metallic layer has
formed. This non-metallic deposition also occurs on the flank of the tool (Figure 15). It can be seen that this phenomenon can greatly increase the tool life.

The effect of non-metallic inclusions formed in the steel material of the workpiece (this is also possible for other metals) was investigated for several versions of non-alloy C-steel marked C35 and C60. Important information for the evaluation of the results is that the steel was produced under large-scale conditions using oxygen-Siemens-Martin technology. It is known that the manufacturing process fundamentally determines the chemical composition of deoxidation products and inclusions remaining in the steel, and thus the strength and plasticity properties. Production batches weighed an average of 105 metric tons.

A cutting test of steel from three production batches of C60 grade batches is presented here. There were two experimental batches in which the standard deoxidation technology was changed. One had an Al overdose. The aim was to make the steel the cleanest available under operating conditions, with few inclusions remaining in the steel. In addition, the deoxidation products remaining in smaller quantities should be high Al content inclusions with high thermal strength. Turning of this steel certainly does not result in non-metallic deposits.

In the other experimental batch, a complex deoxidizing alloy was used, which caused the residual inclusions to be complex CaMgAl silicates, which are plastic under the conditions of warm forming. This steel is marked C60 HM (high machinability).

Table 2. Chemical composition of C60 grade experimental steel batches (%).

| Grade | Batch no. | C   | Mn | Si  | S   | P   | Al   | Cu |
|-------|-----------|-----|----|-----|-----|-----|------|----|
| C60   | 47,411    | 0.61| 0.64| 0.21| 0.024| 0.023| 0.192 | 0.23|
| C60 HM| 48,469    | 0.62| 0.77| 0.23| 0.025| 0.018| 0.01  | 0.21|
| C60 N | 77,332    | 0.57| 0.72| 0.22| 0.046| 0.027| –     | 0.19|

HM: high machinability.

a Over-deoxidized with Al.
b Deoxidized with complex SiCaMgAl alloy.
In addition to these two versions, one of the steel batches manufactured and tested under several normal operating conditions was selected whose machinability is among the results obtained for the two test materials. It is marked C60 N (in this case this does not indicate normalization of the steel, the machined steel samples were in a rolled state). Chemical compositions of the investigated materials are given in Table 2.

The results of the tool life tests are summarized in Figure 16 and Table 3. The standard representation of \( v-T \) functions is usually on an \( \lg-\lg \) scale, so in addition to the linear-scale in Figure 16(a), the figure also shows the standard \( \lg-\lg \) version (Figure 16(b)). The result of the \( v-T \rightarrow p-q \) transformation is shown in Figure 17.

The machinability of the steels was tested for 21 batches of regular operationally produced C35 grade C steel. The unalloyed C-steel was of C35 grade. Test specimens with a diameter of \( D = 76 \text{ mm} \) and a length of 700 mm were produced from rolled samples with a cross-section of \( 80 \times 80 \text{ mm}^2 \) taken from the production process. Machinability was tested by turning. Technological data were as follows: \( v = 100, 150, \) and \( 200 \text{ m/min} \), \( t = 5 \text{ min} \), tool: P20, \( f = 0.3 \text{ mm/rev.} \), \( a_p = 2 \text{ mm} \), \( \gamma = -6^\circ \), and \( \kappa = 70^\circ \). Machinability was assessed by measuring \( VB \) average flank wear. The results were summarized in five groups and are shown in Table 4.

Table 3. The constants of equation (5) for turning the steels given in Table 2.

| Steel  | \( C_1 \)       | \( C_2 \)   | \( C_3 \)   |
|--------|-----------------|-------------|-------------|
| C60    | 9,500,000       | -50         | 2178        |
| C60 HM | 76,900,000      | -240.75     | 17,923      |
| C60 N  | 76,900,000      | -228        | 17,923      |

Table 4. Results of the machinability test on samples taken from factory production at speeds \( v = 100, 150, \) and \( 200 \text{ m/min} \).

| Group number | Flank wear classification | Number of cases |
|--------------|---------------------------|-----------------|
| I.           | \( VB_{v=100} < VB_{v=150} < VB_{v=200} \) | 7               |
| II.          | \( VB_{v=100} > VB_{v=150} \)                | 4               |
| III.         | \( VB_{v=150} > VB_{v=200} \)                | 2               |
| IV.          | \( VB_{v=150} = VB_{v=200} \)                | 1               |
| V.           | Uninterpretable or non-metallic protective deposit | 7               |
Group I includes samples in which the flank wear increased; the tool life decreased monotonically as a function of cutting speed. In these cases, the Taylor formula can be used in the usual way. In Groups II and III there are essentially similar cases. They are characterized by the fact that the $v$-$T$ function does not monotonically decrease, so the general tool life function is required. The $v$-$T$ function includes both convex and concave curve segments, so the tool life $T$ has a range at which a selected $T_i$ value has two or three cutting speeds.

A special case is classified in Group IV, in which the measured wear is visibly unchanged in the speed range $v = 150–200$ m/min. This suggests that for this steel batch, the machinability may be a special limit case; in Figures 11 and 12 the curve $q$-$v$ passes through the origin (see curve $s$). For materials in Group I, $p > 0$, while for Groups II and III $p < 0$.

**Discussion**

One of the main advantages of catastrophe theory models is that they are able to predict the expected characteristics of the system over a wide range of interpretations, situations in which the cost and time required for total experimental measurement would be unfeasible in many cases. In the present case, the dynamics of the model does not appear directly in the relationship between the cutting speed $v$ and the tool life $T$, but in the change of the system parameters $p$ and $q$, which was derived from the $C_1$, $C_2$, $C_3$ constants of the general tool life function. Catastrophe theory is usually used to analyze the motion of dynamic systems. Here, however, the application possibilities of a cutting technology model are presented through a demonstrable mathematical and topological analogy.

In dynamic systems, the principle of energy minimum, as shown in Figure 12, can be represented as a back-folded surface in a topological approach. Quantities that describe the current state of a process can be considered as coordinates of a multidimensional space. The states of the studied system are represented in space by a hypersurface. The topological interpretation of the investigated system is a tool for the qualitative interpretation of the measurement results.

In the present case, of course, the principle of minimum energy does not apply. Yet the possibility of using the analogy has the potential to fit well the third-degree polynomial representing the cusp catastrophe to the convex-concave section of the general tool life function. The analogy therefore does not appear in that a gradual change suddenly becomes great. The drastic topographic change does not reflect a sudden change in state. In the present case, in relation to the cutting speed and the tool life – under definable conditions – a single current cutting speed value can have two or three tool life values. This can be seen in Figures 16 and 17, which summarize the cutting experiments for turning C60HM steel. This is a natural consequence of the occasionally appearing connected convex and concave sections on the $v$-$T$ curve. Thus, it may be the case that, under definable conditions, a technology can be selected in which a tool life can be set at two or three cutting speeds. In Figure 18, the area in which the function $T(v)$ is multivalent is delimited by points $A$, $B$, $C$, and $D$. For dynamic systems, the curve section $B$–$C$ shown in the figure is considered to be an unstable position in which the system cannot permanently remain. Structural stability is the guiding principle of catastrophe theory, introduced by Thom. This means that the behavior of the system is not affected by small perturbations and the system remains stable. This, of course, cannot be interpreted here. The analogy is therefore not valid in this respect because in the model discussed in this study, all three stages, that is, $A$–$B$, $B$–$C$, and $C$–$D$, are equivalent, and also stable in this sense. The actual work does not deal with sudden changes, but with the technological possibility that within definable limits, in this case in the field marked with points $A$–$D$ in Figure 18, it is possible to consider three technological versions. Thus, the tool life here is not the result of an investigation, but a parameter that helps to optimize the technological process. As a result, the application of catastrophe theory helps to optimize cutting technology by qualitatively describing the relationship between $v$-$T$ cutting speed and tool life, which is sometimes complicated.

Special attention is paid to the environment around the origins of catastrophe theory (Figure 12). This is also interesting in this analogical model. As can be seen in Figure 12, at the working point $v$-$T$ → $p$-$q$ close to the origin, practically any cutting speed can be selected for the tool life $T$ within an interval.
In the present case, the analogy of the cusp catastrophe was applied. There was no need to use higher order models. However, this would be possible, which would allow us to examine the interaction of several variables. For example, one of the cutting parameters (cutting speed \( v \), feed rate \( f \), or tool life \( T \)) can be arbitrarily selected as the main variable of the model, and the other two as system parameters that control its behavior. Such an analogy would require the use of a fifth-order polynomial instead of a third-order one, and its topography would lead to a double-curved, two-pointed, so-called nightingale model. The planned continuation of the work presented here is the theoretical and experimental examination of this assumption. From the application of the tools of catastrophe theory, it is expected that the parameters of cutting technology will be harmonized with a greater degree of freedom than at present.

**Summary**

The aim of this study was to summarize studies aimed at the functional analysis of the tool life of cutting tools. The widely known and used Taylor formula is unsuitable for describing cutting speed–tool life relationships that cannot be linearized on the Ig-Ig scale. Their graph also has a convex-concave section, for the description of which a commonly used tool life function has been developed. The applicability of the function has been demonstrated by turning hardened ball bearing rings.22

It was found that the convex-concave section of the function can be well approximated by a cubic polynomial analogous to the mathematical description of the cusp catastrophe known from catastrophe theory. This made it possible to topologically map the \( v \)-T tool life function and, on this basis, to analyze the properties of the function.

The analogy method was verified by turning C35 and C60 grade C steels. Two experimental steel batches weighing 105 tons were produced to study the \( v \)-T function relationship. One batch was over-deoxidized with Al to minimize the concentration of residual deoxidation products in the steel. These inclusions are brittle. The other experimental batch was treated with an alloy from which complex CuMgAl-silicate inclusions remained in the steel. These complex inclusions smear on the wear surfaces of the tool during cutting and reduce tool degradation. In this way, the Taylor curve obtained during the cutting of Al-deoxidized steel was distorted in the direction of the longer tool life, and a convex-concave section was formed on the \( v \)-T curve. The tests also included the turning of a batch of steel taken from factory production. The general tool life function fitted well to the \( v \)-T curve obtained from all three measurement series. The catastrophe theory analogy allowed for a detailed analysis of these. The results were also confirmed by factory routine machinability tests performed on C35 grade steel. Statistical processing of the turning of the 21 batches also confirmed the applicability of the catastrophe-theoretical analogy. It has been shown that in the convex-concave section of the general \( v \)-T curve, two or three cutting speeds can be selected for a currently desired tool life. This is advantageous for harmonizing tool change times in multi-operational processes. The planned continuation of the work presented in this paper is to extend the bivariate \( v \)-T cutting speed – tool life two-dimensional function analysis to \( v \)-f-T and \( v \)-ap-T three-dimensional analysis using catastrophe theory methods that presumably allow a greater degree of harmonization of cutting technology.

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Appendix

Notations

| Symbol | Description | Equation |
|--------|-------------|----------|
| \( a_p \) | Depth of cut | mm |
| \( a, b, c \) | Constants (equation (7)) |
| \( f_z \) | Feed per tooth (\( z \) is the number of edges of the milling tool) | mm/(rev, \( z \)) |
| \(-k\) | Exponent of the Taylor equation |
| \( p \) | System parameter (equation (10)) | m\(^2\)/min\(^2\) |
| \( q \) | System parameter (equation (10)) | m\(^3\)/min\(^3\) |
| \( q_{\text{limit}} \) | Boundary curve on the plane \( q-p \) (Figure 10, equation (15)) | m\(^3\)/min\(^3\) |
| \( t \) | Cutting time | min |
| \( v \) | Cutting speed | m/min |
| \( v_{\text{min}} \) | Minimum position of the curve \( v-T \) | m/min |
| \( v_{\text{max}} \) | Maximum position of the curve \( v-T \) | m/min |
| \( v_{\text{orig}} \) | Cutting speed for \( p = q = 0 \) | m/min |
| \( \gamma \) | Rake angle | (\(^\circ\)) |
| \( \kappa_r \) | Cutting edge angle | (\(^\circ\)) |
| \( C \) | Constant of the Taylor equation |
| \( C_1 = a \) | Constant of the general \( v-T \) tool life function (1) | m\(^3\)/min\(^2\) |
| \( C_2 = b \) | Constant of the general \( v-T \) tool life function (1) | m/min |
| \( C_3 \) | Constant of the general \( v-T \) tool life function (1) | m\(^2\)/min\(^2\) |
| \( D \) | Nominal cutter diameter | mm |
| \( D_{\text{exp}} \) | Diameter of the workpiece at cutting experiments | mm |
| \( D_k \) | Discriminant of equation (10) | m\(^6\)/min\(^6\) |
| \( R^2 \) | Pearson index |
| \( T \) | Tool life | min |
| \( T_{\text{calc}} \) | Calculated tool life | min |
| \( T_{\text{HM}} \) | Measured tool life for turning of high machinability steel | min |
| \( T_{\text{meas}} \) | Measured tool life | min |
| \( VB \) | Tool flank wear | mm |