Microlensing with the space interferometer

Radioastron

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ABSTRACT

It is well-known that gravitational lensing is a powerful tool to investigate matter distributions including DM. Typical angular distances between images and typical time scales depend on gravitational lens masses. For microlensing case angular distances between images or typical astrometric shifts due to microlensing are about $10^{-5} - 10^{-6}$ µas. Such an angular resolution will be reached with the space–ground interferometer Radioastron. The basic targets for microlensing searches should be bright point-like radio sources at cosmological distances. In this case, an analysis of their variability and a solid determination of microlensing could lead to an estimation of their cosmological mass density, moreover, in this case one could not exclude a possibility that non-baryonic dark matter also form microlenses if the corresponding optical depth will be high enough. To search for microlensing the most perspective objects are gravitational lensed systems as usually, like CLASS gravitational lens B1600+434, for instance. However, for direct resolving these images and direct detection of apparent motion of the knots, a Radioastron sensitivity have to be improved, since an estimated flux density is too low and to observe the phenomena one should improve sensitivity in 10 times at 6 cm wavelength, for instance, otherwise, it is necessary to increase an integration time (assuming that a radio source has a typical core – jet structure and microlensing phenomenon is caused superluminal apparent motion of knots). In the case of a confirmation (or a disproval) of claims about microlensing in gravitational lens systems one can speculate about a microlens contribution into the gravi-
tational lens mass. Astrometric microlensing due Galactic MACHOs actions is not very important because of low optical depths and long typical time scales. Therefore, a launch of space interferometer Radioastron will give new excellent facilities to investigate microlensing in radio band, since in this case there is a possibility not only to resolve microimages but also observe astrometric microlensing.

Key words: Gravitational Lenses, Quasars, Dark Matter.

1 INTRODUCTION. MICROLENSING FOR DISTANT QUASARS.

Gravitational microlensing effect was predicted by Byalko (1969); Paczynski (1986) (if sources are stars in Milky Way or Large Magellanic Cloud discovered by MACHO, EROS and OGLE collaborations (Aldock et al. 1993; Aubourg et al. 1993; Udalski et al. 1994) discussed in details later in a number of papers (see, for example, Zakharov (1997); Zakharov & Sazhin (1998); Zakharov (2003, 2005); Kerins (2001); Grieß (2002); Evans (2003); Evans & Belokurov (2003, 2004)). However, microlensing for distant quasars was considered by Gott (1981) (soon after the first gravitational lens discovery by Walsh, Carswell & Wevman (1993)) and discovered by Irwin et al. (1989) in gravitational lenses systems since an optical depth for such systems are highest.

For cosmological locations of gravitational lenses and stellar masses, typical angles between images are about \( \sim 10^{-6} \) sec (Wambsganss 1990, 1993, 2001), or more precisely

\[
\theta_E = \frac{R_E}{D_S} \approx 2.36 \times 10^{-6} h_{65}^{-1/2} \sqrt{\frac{M}{M_\odot}} \text{ arcsec},
\]

(1)

where \( R_E \) is the Einstein – Chwolson radius, \( D_S \) is an angular diameter distance between a source and an observer, \( h_{65} = \frac{H_0}{(65 \text{ km/(c \cdot Mpc))}} \). \( H_0 \) is the Hubble constant.

Theoretical studies of microlensing in gravitational lens systems started since Chang & Refsdal (1979) paper. Unfortunately, till now it is impossible to resolve microimages, however in this case there is a chance to observe temporal variations of observed fluxes, or so called photometric microlensing.

In principle the gravitational lens effect is achromatic, but sizes and locations for different spectral bands could be different and in this case we could observe chromatic effect (Wambsganss & Paczynski 1991).

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2 ASTROMETRIC MICROLENSING

Astrometric microlensing was discussed in number of papers (Hog, Novikov & Polnarev 1993; Walker 1994; Miyamoto & Yoshii 1995; Sazhin 1996; Sazhin et al. 1998; Paczynski 1998; Boden, Shao & Van Buren 1998; Tadros, Warren & Hewett 1993; Honma 2001; Honma & Kuravama 2002; Asada 2002; Takahashi 2002; Totani 1993; Inoue & Chiba 2003), but actually that is signature of well-known light bending in the gravitational field and at the first time light bending by gravitational field was discussed by Newton (1718), the first published derivation of light bending for light was given by Soldner (1804) in the framework of Newtonian theory of gravitation. In the framework of general relativity light bending was calculated by Einstein (1915) and his prediction was confirmed in 1919 (Dyson, Eddington & Davidson 1920). Actually such an astrometric displacement of distant image due to light bending by gravitational field of microlenses is called astrometric microlensing and the effect could be detectable with optical astrometric mission like SIM (Space Interferometry Mission, see http://sim.jpl.nasa.gov), GAIA (Global Astrometric Interferometer for Astrophysics, see http://sci.esa.int/gaia) and radio projects like VERA (VLBI Exploration of Radio Astrometry) and Radioastron.

2.1 Microlenses in our Galaxy

Let us remind basic definitions and their relations. We consider a point size lens. A distance between source and an observer is $D_s$, a distance between a gravitational lens and observer is $D_d$, a distance between a gravitational lens and a source is $D_{ds}$. Thus, we obtain gravitational lens equation (Schneider et al. 1992)

$$\eta = D_s \xi / D_d + D_{ds} \Theta(\xi),$$

(2)

where vectors $\eta, \xi$ define coordinates in the source and lens planes correspondingly, but the angle is determines by the relation

$$\Theta(\xi) = 4GM\xi / c^2 \xi^2.$$  

(3)

If the right hand side (3) is equal to zero, we obtain the conditions when a source, a lens and an observer are located on the same line ($\eta=0$). The corresponding length $\xi_0 = \sqrt{4GM D_d D_{ds} / (c^2 D_s)}$ is called Einstein – Chwolson radius. One could calculate also Einstein – Chwolson angle $\theta_0 = \xi_0 / D_d$.

If we write gravitational lens equation in dimensionless variables, then we obtain
\[ x = \xi / \xi_0, \quad y = D_s \eta / (\xi_0 D_d), \quad \alpha = \Theta D_{ds} D_d / (D_s \xi_0), \]

and the gravitational lens equation has the following form:

\[ y = x - \alpha(x) \quad \text{or} \quad y = x - x/x^2. \]  

Solving the equation \( x \), we obtain

\[ x^\pm = y [1/2 \pm \sqrt{1/4 + 1/y^2}]. \]  

Then we calculate distance between images:

\[ x^+ = y \left[ \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y^2}} \right], \quad x^- = y \left[ -\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{y^2}} \right], \]

\[ l = x^+ + x^- = 2y \sqrt{\frac{1}{4} + \frac{1}{y^2}}. \]

**2.2 Typical time scales for astrometric microlensing in our Galaxy**

Let us consider asymptotic for \( x^+ \) and \( y \to \infty \), then \( x^+ \to y + \frac{1}{y} \) and angular distance between real image position and image position in Einstein – Chwolson angles \( \Delta = x^+ - y \sim \frac{1}{y} \) (the angle describes an astrometric microlensing).

Let us remind typical scales for lengths, time and angles. Let us consider the Galactic case if a gravitational lens has stellar mass \( \sim M_\odot \) and is located at 10 kpc, then

\[ \xi_0 := \left[ \left( \frac{4G M}{c^2} \right) \left( \frac{D_d (D_s - D_d)}{D_s} \right) \right]^{1/2} \]

\[ = 9.0 \text{ A.U.} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D_d}{10 \text{ kpc}} \right)^{1/2} \left( 1 - \frac{D_d}{D_s} \right)^{1/2}. \]  

Thus, we have for Einstein – Chwolson angle

\[ \theta_0 := \left[ \left( \frac{4G M}{c^2} \right) \left( \frac{D_s - D_d}{D_s D_d} \right) \right]^{1/2} \]

\[ = 0.902 \text{ mas} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{10 \text{ kpc}}{D_d} \right)^{1/2} \left( 1 - \frac{D_d}{D_s} \right)^{1/2}. \]  

It is known that a distance between images is about \( \sim 2\xi_0 \) for small \( y \), thus the angular distance about (mas). Due to a proper motion, we have

\[ \dot{r} = \frac{V}{D_d} = 4.22 \text{ mas} \cdot \text{ year}^{-1} \left( \frac{V}{200 \text{ km} \cdot \text{ c}^{-1}} \right) \left( \frac{10 \text{ kpc}}{D_d} \right), \]

where \( V \) is a transverse velocity of a lens. Using last two expressions, one calculates typical
time scale for microlensing, which a time to cross Einstein radius by a source due to a proper motion (all distance could be considered at a celestial sphere):

\[
t_0 := \frac{\theta_0}{r} = 0.214 \text{ year} \left( \frac{M}{M_\odot} \right)^{1/2} \left( \frac{D_d}{10 \text{ kpc}} \right)^{1/2} \times \left( 1 - \frac{D_d}{D_s} \right)^{1/2} \left( \frac{200 \text{ km} \cdot \text{c}^{-1}}{V} \right).
\]  

(11)

Let us present rough estimates of an optical depth for astrometric microlensing using estimates for classic microlensing given by MACHO and EROS collaborations \( \tau_{\text{halo}} \sim 1. \times 10^{-7} \). Since image displacement for classic microlensing is about \( \theta_{\text{class}} \sim 1 \text{ mas} \), then an optical depth to have displacement \( \theta_{\text{threshold}} = 10 \mu\text{as} \) and \( \theta_{\text{threshold}} = \mu\text{as} \) is given by the expression

\[
\tau_{\text{astromet}} = \tau_{\text{halo}} \left( \frac{\theta_{\text{class}}}{\theta_{\text{threshold}}} \right)^2.
\]

(12)

So, for \( \theta_{\text{threshold}} = 10 \mu\text{as} \) an optical depth is about \( \tau_{\text{astromet}} \sim 1. \times 10^{-3} \) and for \( \theta_{\text{threshold}} = \mu\text{as} \) it is about \( \tau_{\text{astromet}} \sim 0.1 \), and since according to last estimates \( \tau_{\text{halo}} = 1.2 \times 10^{-7} \) \( \text{(Alcock et al. 2000b; Griest 2002)} \). An optical depth for classical microlensing toward Galactic bulge is about \( \sim 3 \times 10^{-6} \) \( \text{(Alcock et al. 2000a)} \), thus an optical depth for astrometric microlensing is higher.

We assume that typical time scale for astrometric microlensing is double time to change an image position displacement from \( \theta_{\text{threshold}} \) to maximal displacement \( \theta_{\text{max}} \). A typical maximal displacement is \( \theta_{\text{max}} := \sqrt{2} \theta_{\text{threshold}} \). Then typical time scales for astrometric microlensing (one could use other definitions but difference with the definition could be described by a factor \( \sim 1 \) )

\[
t_{\text{astromet}} = t_0 \frac{\theta_{\text{class}}}{\theta_{\text{threshold}}}.
\]

(13)

So, for \( \theta_{\text{threshold}} = 10 \mu\text{as} \) a typical time scale is about \( t_{\text{astromet}} \sim 20 \text{ years} \) and for \( \theta_{\text{threshold}} = \mu\text{as} \) it is about \( t_{\text{astromet}} \sim 200 \text{ years} \).

3 PROJECTED PARAMETERS OF THE SPACE INTERFEROMETER RADIOASTRON

According to the schedule of the Russian Space Agency the space radio telescope Radioastron will be launched in the next few years (see description of the project \( \text{[Kardashev 1997]} \)). This project was initiated by Astro Space Center (ASC) of the Lebedev Physical Institute
of the Russian Academy of Sciences (RAS) in collaboration with other institutions of RAS and Russian Space Agency. Scientists from 20 countries develop the scientific payload for the satellite and will provide a ground base support of the mission. The project was approved by RAS and Russian Space Agency. This space based 10-meter radio telescope will be used for space–ground VLBI measurements. For observations four wavelength bands will be used corresponding to \( \lambda = 1.35 \text{ cm} \), \( \lambda = 6.2 \text{ cm} \), \( \lambda = 18 \text{ cm} \), \( \lambda = 92 \text{ cm} \).

It will be not the first attempt to build a telescope with a size larger than the Earth size. In 1997 Institute of Space and Technology of Japan launched a HALCA satellite with 8 m radio telescope and as a result VLBI Space Observatory Programme (VSOP) was formed [Horiuchi et al. (2004)]. Since the apogee height for radiotelescope HALCA was 21,200 km, the apogee height for Radioastron should about 350,000 km (or even \( 3.5 \times 10^6 \text{ km} \) see below), and as a result the fringe size for the minimal wavelength will be smaller than 1-10\( \mu \text{as} \). The minimal correlated flux for space-ground VLBI should be about 100 mJy for the 1.35 cm wavelength at \( 8\sigma \) level (Kardashev 1997), therefore source fluxes should be higher than the threshold and about 24 mJy for the 6 cm wavelength.

An orbit for the satellite was chosen with high apogee and with period of satellite rotation around the Earth 9.5 days, which evolves as a result of weak gravitational perturbations from the Moon and the Sun. The perige is in a band from 10 to 70 thousand kilometers, the apogee is a band from 310 to 390 thousand kilometers. The basic orbit parameters will be the following: the orbital period is \( p = 9.5 \text{ days} \), the semi-major axis is \( a = 189,000 \text{ km} \), the eccentricity is \( e = 0.853 \), the perigee is \( H = 29,000 \text{ km} \).

A detailed calculation of the high-apogee evolving orbit can be done if the exact time of launch is known.

After several years of observations, it would be possible to move the spacecraft to a much higher orbit (with apogee radius about \( 3.2 \cdot 10^6 \text{ km} \)), by additional spacecraft maneuver using gravitational force of the Moon. In this case it would be necessary to use 64-70 m antennas for the spacecraft control, synchronizations and telemetry.\(^1\)

The fringe sizes (in micro arc seconds) for the apogee of the above-mentioned orbit and for all Radioastron bands are given in Table 3.

Thus, there are non-negligible chances to observe mirages (shadows) around the black

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\(^1\) http://www.asc.rssi.ru/radioastron/
Table 1. The fringe sizes (in micro arc seconds) for the standard and advanced apogees $B_{\text{max}}$ (350 000 and 3 200 000 km correspondingly).

| $B_{\text{max}}$ (km) | $\lambda$ (cm) | 92   | 18   | 6.2  | 1.35 |
|-----------------------|----------------|------|------|------|------|
| $3.5 \times 10^5$     | 540            | 106  | 37   | 8    |
| $3.2 \times 10^6$     | 59             | 12   | 4    | 0.9  |

hole at the Galactic Center and in nearby AGNs in the radio-band using Radioastron facilities (Zakharov et al. 2005a,b,c,d,e,f).

4  MICROIMAGE RESOLVING FOR DISTANT QUASARS

4.1  Microlens locations

If microlenses are located in our Galaxy, recent observations by MACHO, EROS, OGLE collaborations (and their theoretical interpretations) showed that an optical depth for Galactic microlens is about $10^{-6} - 10^{-7}$. In spite of the fact that for a selected source a probability for microlensing is very small and for the discovery one could monitor about $10^6$ background sources (like for microlensing in our Galaxy). That is a hard problem because we have not enough background point-like distant sources, however an angular distance between images is about $10^{-3}$ arcsec, therefore there is a possibility to resolve point-like quasar images with VLBI technique in radio bands, but unfortunately a sample of bright extragalactic sources is small to realize the program (there is also a chance to resolve the stellar images in IR band with the modern optical telescopes Delplancke, Gorski & Richichi (2001); Paczynski (2001)). It was shown that an optical depth for microlenses located in halo or (and) in quasar bulge is low (Zakharov, Popović & Jovanović 2004). We will not study the case because of the optical depth is low but also angular distance between images is much shorter than the Radioastron fringe size.

4.2  Cosmological distribution for microlenses

Let us consider cosmologically distributed microlenses since there is a hypothesis that variability of essential fraction of distant quasars is caused by microlensing. If it is, one can say that a probability (an optical depth) is high in radio band also.

To evaluate an optical depth we will assume that a source is located at a distance with cosmological redshift $z$. Calculations for different parameters are given by Zakharov, Popović & Jovanović.
We will remind of the results. In calculations we use a point-like source approximation (it means that as a result we obtain an upper limit for an optical depth).

An optical depth could be evaluated using approximations given by Turner (1984); Fukugita and Turner (1991)

\[
\tau_p^L = \frac{3}{2} \frac{\Omega_L}{\lambda(z)} \int_0^z dw \frac{(1+w)^3[\lambda(z) - \lambda(w)]\lambda(w)}{\sqrt{\Omega_0(1+w)^3 + \Omega_\Lambda}},
\]

where \(\Omega_L\) is compact lens density (in critical density units), \(\Omega_0\) is matter density, \(\Omega_\Lambda\) is a \(\Lambda\)-term density (or quintessence),

\[
\lambda(z) = \int_0^z \frac{dw}{(1+w)^2\sqrt{\Omega_0(1+w)^3 + \Omega_\Lambda}},
\]

is an affine parameter (in \(cH_0^{-1}\) units).

We use realistic cosmological parameters to evaluate integral (14). Observations of cosmological SN Ia and CMB anisotropy give the following parameters \(\Omega_\Lambda \approx 0.7, \Omega_0 \approx 0.3\) (or so-called concordance model parameters). For example, recent observations of the WMAP team gives for the best fit \(\Omega_\Lambda \approx 0.73, \Omega_0 \approx 0.27\) (Bennett et al. 2003; Spergel et al. 1993).

Thus, \(\Omega_0 = 0.3\) and \(\Omega_L = 0.05\) (\(\Omega_L = 0.01\)) could be adopted as realistic, if we assume that almost all baryonic matter form microlenses (\(\Omega_L = 0.05\)), or 20% baryonic matter forms microlenses (\(\Omega_L = 0.01\)). However, for \(z \sim 2.0\) optical depth could be about \(\sim 0.01 - 0.1\) (Zakharov, Popović & Jovanović 2004). If about 30% non-baryonic dark matter forms cosmologically distributed objects with stellar masses (such as neutralino stars suggested by Gurevich and Zybin (1995); Gurevich et al. 1996, 1997), parameter \(\Omega_L = 0.1\) could be adopted as realistic and in this case an optical depth could be about \(\sim 0.1\). Therefore, if 25% of baryonic matter form cosmologically distributed microlenses one could say that the Hawkins’s hypothesis that microlensing cause variability for essential fraction of all quasars should be ruled out, but in the case when 30% of non-baryonic dark matter form microlenses about 10% of distant quasars demonstrate these features.

4.3 Observed features of microlensing for quasars

More than 10 years ago Hawkins (1993, 1996, 2002, 2003) put forward the idea that nearly all quasars are being microlensed (however, based on photometric observations of sample about 25,000 quasars, Vanden Berk et al. (2004) claimed that microlensing model for an explanation of variabilities is unlikely).

As previous estimates show us that in the case if Hawkins hypothesis is correct, \(\Omega_L\)
should be about 1 and that is a contradiction for data of observational cosmology, but hypothesis could be correct in part and if observations would dictate that \( \Omega_L \) is larger than 0.05 we could conclude that non-baryonic matter form microlenses (they could be neutralino clouds or primordial black holes). If the Hawkins hypothesis is correct in part at least, in this case also an essential fraction of distant point like sources should demonstrate features of microlensing since the optical depth could be evaluated by Eq. (14) as well. No doubt that except microlensing there are other causes of variabilities, however one could use different techniques to separate different types of variabilities (see, Koopmans & de Bruyn (2000); Koopmans et al. (2000b), for example), since there is different dependence of modulation indices as a function of frequency for oscillations (scintillations) and for microlensing. However, resolving the microimages and measuring the centroid displacements for bright point-like sources in radio band will be a critical test to prove (or rule out) the Hawkins hypothesis about microlensing for point like sources at cosmological distances.

To prove the microlensing hypothesis for a distant quasar, the source have to have the following properties from a list of perspective targets of VSOP or Radioastron missions (or from its extended version):

a) A source should demonstrate signatures of microlensing which are different from typical features for scintillations at time scales \(< 3–5 \text{ years}\) (that is an estimated time of Radioastron mission);

b) A compact core for the source should have size \( \lesssim 40 \mu \text{as} \) and flux density should be higher than Radioastron thresholds \( \gtrsim 20 \text{ mJy at 6 cm wavelength} \) and \( \gtrsim 100 \text{ mJy at 1.35 cm wavelength} \).

In the case, if the Hawkins hypothesis is correct an essential fraction of all point like sources at cosmological distances should demonstrate signatures of photometric and therefore astrometric microlensing.

In the case, if the Hawkins hypothesis is incorrect and cosmologically distributed microlenses give a small contribution into critical density \( \Omega_{\text{tot}} \), but even for this case one could evaluate \( \Omega_L \) from an observed rate of microlensed sources satisfying condition b), since the observed rate gives an estimate for the optical depth.

According to Horiuchi et al. (2004) results about 14\% \pm 6\% of sources (from 344 ones) have core size \( \lesssim 40 \mu \text{as} \) and the angle corresponds to the fringe size at the 6 cm wavelength. This part of sources could be used for photometric monitoring and for a further analysis of a preferable explanation of variability. If the analysis would indicate that microlensing
is a preferable cause of variability the candidate could be selected as the first order one. But even in the case, if a source would demonstrate variability that could be explained by another cause (but not by microlensing), the source should be checked to search for image splitting or (and) astrometric image displacement since models for alternative explanation of variabilities could be not quite correct.

From theoretical point of view there is a possibility to detect microlensing for both core and bright knots. In this case the two situations will be characterized by different time scales.

5 MICROLENSING FOR GRAVITATIONAL LENSED SYSTEMS

Few years ago, Koopmans & de Bruyn (2000); Koopmans et al. (2000a) claimed that the most realistic explanation of short-term variability of a gravitational lens CLASS B1600+434 at 5 GHz and 8.5 GHz (variabilities and possible explanations of the phenomena were discussed by Koopmans et al. (2003); Winn (2004)). The authors considered different cases of variability such as scintillation due to scattering and microlensing. As a result they concluded, that microlensing phenomenon in radio band gives the natural fit for observational data. One could remind flux densities changed from 58(48) mJy in March 1994 to 29 (24) mJy in August 1995 for image A(B) (Koopmans, de Bruyn & Jackson 1998). Another decrease was found from 27(24) to 23(19) mJy and it was from February to October 1998 (Koopmans et al. (2000b); Koopmans & de Bruyn (2000). Strong variability was detected at 5 GHz, where flux density was about 34–37 mJy in 1987 Becker, White & Edwards (1991); Koopmans & de Bruyn (2000), but it was about 45(37) mJy for image A(B) (Koopmans, de Bruyn & Jackson 1998) and only 23 (18) mJy in June 1999 (Koopmans & de Bruyn 2000). Based on analysis of variabilities Koopmans & de Bruyn (2000) concluded that the variability is caused by superluminal motion of compact knots in jet (VLBA and 100-m Effelsberg telescope observations also found evidences for jet components in the CLASS gravitational lens B0128+437 (Biggs et al. 2004), but unfortunately their flux densities are too low to observe then with the Radioastron interferometer).

Let us remind that a typical threshold for Radioastron interferometer sensitivity at 5 GHz is about 23 mJy with an integration time 300 s (Kardashev 1997), therefore in principle, such density fluxes could be detected by Radioastron interferometer.

Treyer & Wambsganss (2004) concluded that for photometric fluctuations $\sim 0.5$ mag typical astrometric displacement should be about 20 – 40 $\mu$as (to evaluate photometric and
astrometric microlensing one could use numerical approaches and analytical asymptotical expansions near fold (Schneider and Weiss 1992) and cusp singularities (Zakharov 1995, 1997; Petters, Levine & Wambsganss 2001; Yonehara 2001). In principle such a displacement could be observed with Radioastron space interferometer at 6 cm and 1.35 cm wavelengths if flux densities for the object is high enough. For example, in the B1600+434 case the density flux is suitable for the core (at least, at 6 cm wavelength), but if the superluminal motion of knots is responsible for microlensing (as Koopmans & de Bruyn (2000) claimed) the sensitivity of Radioastron should be improved in 10 times at 6 cm wavelength to observe such a displacement of knots. At the 1.35 cm wavelength the Radioastron flux density threshold is probably too high to detect the displacement.

5.1 Typical time scales for microlensing

Let us remind that according to the standard model typical time scales for radio microlensing could be much smaller than typical time scale in optical band due to effects of special relativity and different geometry and locations of radiating regions in these bands, for example typical time scales in optical band are determined by a transverse velocity ($v_{\text{trans}}$), but in radioband time scales could be in $\beta_{\text{trans}}/v_{\text{trans}}$ times smaller (Koopmans & de Bruyn 2000) (all velocity are expressed in $c$ units).

Typical time scales is determined by a ratio typical sizes between caustics and an apparent velocity of the jet-component in the source plane (Blandford, McKee & Rees 1977; Blandford & König 1979; Koopmans & de Bruyn 2000). If jet-component moves with a relativistic bulk velocity $\beta_{\text{bulk}}$, then an apparent velocity $\beta_{\text{app}}$ is

$$\beta_{\text{app}} = \frac{\mathbf{n} \times (\beta_{\text{bulk}} \times \mathbf{n})}{1 - \beta_{\text{bulk}} \cdot \mathbf{n}} = \frac{\beta_{\text{bulk}} \sin(\psi)}{1 - |\beta_{\text{bulk}}| \cos(\psi)},$$  \hspace{1cm} (16)

where $\psi$ is the angle between the jet and a line of sight (Blandford, McKee & Rees 1977; Blandford & König 1979; Koopmans & de Bruyn 2000).

The apparent angular velocity of the jet component is (Koopmans & de Bruyn 2000)

$$\frac{d\theta_s}{dt} = \frac{\beta_{\text{app}} c}{(1 + \frac{z_s}{D_s}) D_s} = \frac{1.2 \cdot \beta_{\text{app}} \text{Gpc}}{(1 + z_s)} \frac{\mu \text{as}}{D_s \text{ week}},$$  \hspace{1cm} (17)

where $z_s$ and $D_s$ are the source redshift and the angular diameter distance to the stationary core, respectively. Using the estimate for observed source redshift $z_s$ (Fassnacht and Cohen 1998), Koopmans & de Bruyn (2000) concluded that angular velocity of B1600+434 should
be
\[ \frac{d\theta}{dt} = 0.34 \cdot \beta_{\text{app}} \frac{\mu\text{as}}{\text{week}}, \tag{18} \]
for a flat Friedmann universe with \( \Omega_m = 1 \) and \( H_0 = 65 \text{ km} \cdot \text{s}^{-1}\text{Mpc}^{-1} \). Based on observational data and simulations Koopmans & de Bruyn (2000) evaluated also a typical size of knots in jet in the source plane \( 2 < \Delta\theta_{\text{knot}} < 5 \mu\text{as} \) and an apparent velocity band \( 9 < \Delta\theta_{\text{app}} < 26.2 \). Therefore, apparent displacements for B1600+434 should be about about dozens \( \mu\text{as} \) and the displacement could be measured with the Radioastron interferometer at 6 cm wavelength.

One could also evaluate linear sizes of knots through their angular diameter distances
\[ \Delta l = \frac{c}{H_0} \frac{\Delta\theta_{\text{knot}} [z_s - (1 + q_0)z_s^2/2]}{1 + z_s}, \tag{19} \]
where \( q_0 = 1.3 \cdot \Omega_m - 1 = -0.55 \) (for a flat universe and \( \Omega_m = 0.3 \)), therefore typical linear sizes of the knots should be \( \Delta l \in (5, 14) \times 10^{16} \text{ cm} \).

Typical scales for microlensing are discussed not only in books on gravitational lensing (Schneider et al. 1992, Petters et al. 2001), but in recent papers also (see, for example, Treyer & Wambsganss (2004)). Usually people discuss locations of microlenses in gravitational macrolenses because of an optical depth for microlensing is the highest in comparison with other possible locations of gravitational microlenses, but it is clear that the fact it was known quite well in advance. However, cases for microlens locations were considered, for example galactic clusters or extragalactic dark halos could have microlenses.

So, for example following to a recent paper by Treyer & Wambsganss (2004), we remind that typical length scale for microlensing and assuming concordance cosmological model parameters \( (\Omega_{\text{tot}} = 1, \Omega_{\text{matter}} = 0.3, \Omega_{\Lambda} = 0.7) \)
\[ R_E = \sqrt{2r_g \cdot \frac{D_s D_l}{D_t}} \approx 3.4 \cdot 10^{16} \sqrt{\frac{M}{M_\odot}} h_{65}^{-0.5} \text{ cm}, \tag{20} \]
where “typical” microlens and sources redshifts are assumed to be \( z_l = 0.5, z_s = 2 \) (similar to Treyer & Wambsganss (2004)), \( r_g = \frac{2GM}{c^2} \) is the Schwarzschild radius corresponding to microlens mass \( M \), \( h_{65} = H_0/(65 \text{ km/sec}/\text{Mpc}) \) is the dimensionless Hubble constant.

The corresponding angular scale is (Treyer & Wambsganss 2004)
\[ \theta_0 = \frac{R_E}{D_s} \approx 2.36 \cdot 10^{-6} \sqrt{\frac{M}{M_\odot}} h_{65}^{-0.5} \text{ arcsec}, \tag{21} \]
Using the length scale (20) and a velocity scale (say an apparent velocity $\beta_{\text{app}}$), one could calculate the standard time scale corresponding to the scale to cross the Einstein radius

$$
t_E = (1 + z_l) \frac{R_E}{v_\perp} = \begin{cases} 
\approx 2 \sqrt{\frac{M}{M_\odot}} \beta_{\text{app}}^{-1} h_{65}^{-0.5} \text{ weeks, if } v_\perp = c \beta_{\text{app}}, \\
\approx 27 \sqrt{\frac{M}{M_\odot}} v_{600}^{-1} h_{65}^{-0.5} \text{ years, if } v_\perp \sim 600 \text{ km/c,}
\end{cases}
$$

(22)

here we assume time scales are determined by an apparent velocity or a typical transverse velocity ($v_{600} = v_\perp / (600 \text{ km/c})$), respectively.

The time scale $t_E$ corresponding to the approximation of a point mass lens and small size of source in comparison with Einstein – Chwolson radius and probably the approximation and the time scale could be used if microlenses are distributed freely at cosmological distances and actually one Einstein – Chwolson angle is located far enough from another one.

If we use the simple caustic microlens model (like the straight fold caustic model), there are two time scales, namely it depends on sizes of ”caustic size” $r_{\text{caustic}}$ (if we use the following approximation for the magnification near the caustic $\mu = \frac{r_{\text{caustic}}}{y - y_c}$ ($y > y_c$ and $y$ is the perpendicular direction to the fold caustic)), thus $R > r_{\text{caustic}}$, then the relevant time scale is the ”crossing caustic time” [Treyer & Wambsganss 2004]

$$
t_{\text{cross}} = (1 + z_l) \frac{R_{\text{source}}}{v_\perp (D_s/D_l)} \\
\approx 0.62 R_{15} v_{600}^{-1} h_{65}^{-0.5} \text{ years} \\
\approx 226 R_{15} v_{600}^{-1} h_{65}^{-0.5} \text{ days,}
$$

(23)

(in the right hand side $D_l$ and $D_s$ correspond to $z_l = 0.5$ and $z_s = 2$ respectively and $R_{15} = R_{\text{source}}/10^{15} \text{ cm}$).

However, if the source radius $R_{\text{source}}$ is much smaller than the ”caustic size” $r_{\text{caustic}}$ $R_{\text{source}} \ll r_{\text{caustic}}$, one could used the ”caustic time”, namely the time when the source is located in the area near the caustic and the time scale corresponds to

$$
t_{\text{caustic}} = (1 + z_l) \frac{r_{\text{caustic}}}{v_\perp (D_s/D_l)} \\
\approx 0.62 r_{15} v_{600}^{-1} h_{65}^{-0.5} \text{ years} \\
\approx 226 r_{15} v_{600}^{-1} h_{65}^{-0.5} \text{ days,}
$$

(24)

where $r_{15} = r_{\text{caustic}}/10^{15} \text{ cm}$.

These time scales $t_{\text{cross}}$ and $t_{\text{caustic}}$ could be about days (or even hours) if $v_\perp$ is determined by an apparent motion of superluminal motion in jet.
Thus $t_{\text{cross}}$ could be used as a lower limit for typical time scales for the simple caustic microlens model, but since there are two length parameters in the problem and in general we do not know their values, we could not evaluate $R_{\text{source}}$ only from the time scales of microlensing because time scales could correspond to two different length scales. However, if we take into account variation amplitudes of luminosity, one could say that in general $t_{\text{cross}}$ corresponds to to smaller variation amplitudes than $t_{\text{caustic}}$, because if the source square is large there is a "smoothness" effect since only small fraction of source square is located in the high amplification region near the caustic.

6 CONCLUSIONS

First, one could point out that gravitational lensed systems are the most perspective objects to search for microlensing. Astrometric microlensing could be detected in the gravitational lens system such as B1600+434 in the case if a proper motion of source, lens and an observer are generated mostly by a superluminal motion of knots in jet (superluminal motion in jet was found with HALCA in the quasar PKS 1622-297 (Wajima 2005)). But in this case, based on density flux estimates done by Koopmans & de Bruyn (2000), one could say that a required sensitivity of the Radioastron interferometer should be improved in 10 times.

In the case if there is microlensing of core in the B1600+434 system for example, then astrometric microlensing in the system could be about should be about 20 – 40 $\mu$as (Trever & Wambsganss 2004) and the Radioastron interferometer will have enough sensitivity to detect such an astrometric displacement.

Second, in principle microlensing for distant sources could be the only tool to evaluate $\Omega_L$ from microlensing event rate. To solve this problem with the Radioastron interferometer one should analyze variabilities of compact sources with a core size $\lesssim 40 \mu$as and with high enough flux densities about $\gtrsim 20$ mJy at 6 cm wavelength and about $\gtrsim 100$ mJy at at 1.35 cm wavelength To fit the most reliable model for variabilities of the sources such as scintillations, microlensing etc. A fraction of the sources in the list of extragalactic targets for VSOP and Radioastron about 13% – 14 % (Moellenbrock et al. 1996; Hirabayashi et al. 2000; Scott et al. 2004; Kovalev et al. 2005). In the case, if the analysis would indicate that other explanations (such as scintillations) are preferable and future observations with Radioastron interferometer would show that there are no features for astrometric microlensing, one could conclude that Hawkins hypothesis should be ruled out. But if an essential fraction
of variability could be fitted by microlensing, the sources could be as the first order candidate to search for astrometric microlensing.

Therefore, one could say that astrometric microlensing (or direct image resolution with Radioastron interferometer) is the crucial test to confirm (or rule out) microlens hypothesis for gravitational lensed systems and for point like distant objects.

Astrometric microlensing due to MACHO action in our Galaxy is not very important for observations with the space interferometer Radioastron, since first, probabilities are not high; second, typical time scales are longer than estimated life time of the Radioastron space mission.

Therefore, just after the Radioastron launch it will be the first chance to detect microlensing by a direct way. So, the main goal of the paper to attract an attention to such a challenging possibility because, preflight time is very short now and perspective targets should be analyzed carefully by observational and theoretical ways in advance. A number of point like bright sources at cosmological distances and gravitational lensed systems with point like components demonstrating microlens signatures is not very high and the sources should be analyzed by the careful way to search for candidates where microlens model is preferable in comparison with alternative explanations of variabilities.

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