Morphing of spatial objects in real time with interpolation by functions of radial and orthogonal basis

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Abstract. The article is devoted to visualization of spatial objects’ morphing described by the set of unordered reference points. A two-stage model construction is proposed to change object’s form in real time. The first (preliminary) stage is interpolation of the object’s surface by radial basis functions. Initial reference points are replaced by new spatially ordered ones. Reference points’ coordinates change patterns during the process of morphing are assigned. The second (real time) stage is surface reconstruction by blending functions of orthogonal basis. Finite differences formulas are applied to increase the productivity of calculations.

1. Introduction
Virtual environment interfaces (VEI) are applied to systems of various purposes. One of displaying tasks while designing VEI is transformation of an object with a non-analytical surface in real-time. A typical example is visualization of a beating heart. Complexity of this task is determined by application of a high-poly model (thousands of polygons) for visualization of a non-analytical surface. Vertices position control in real time requires high computational costs. It should be taken into account that modelling and visualization of an object is not only the task of the applied computing system. This task is implemented at the final stage of the work. Effective use of computing system’s resources along with high realistic visualization is actual.

2. Materials and methods
A two stage construction of the model of a non-analytical surface is proposed. At the first stage the surface is presented by a set of unordered reference points. The number of these points is several times less than the number of vertices of a polygonal model. This stage is preliminary. At the second stage the control of the position of reference points is implemented. Interjacent points are determined for every stage of morphing by interpolation. These interjacent points become vertices of the polyronal mesh. Finite difference formulas are applied for increasing the productivity of the system.

3. Ordering of the reference points of a non-analytic surface
Spatial objects are frequently described by a set of reference points located in the space of Cartesian coordinates $x, y, z$ in a random manner (scattered data). Interpolation is implemented to order the reference points. Radial basis functions (RBF) have good interpolation properties [1]. It is presumed that the surface of the object is closed in general case. For its’ interpolation in Cartesian coordinates the implicit form of interpolant $F(x,y,z)=0$ is frequently used [2, 3]. This form is inconvenient for visualization because determination of interjacent points requires application of the brute force
algorithm. At the same time application of explicit form $z = f(x, y)$, which is more convenient for visualization, is impossible, because interpolant in Cartesian coordinates is a multivalued function.

It is proposed to implement the interpolation using the parametrical form of interpolant and spherical coordinates as parameters:

$$c = f(\varphi, \theta), \quad c = x, y, z,$$

where $\varphi$ – azimuth angle (longitude), $\varphi = 0..2\pi$; $\theta$ – polar angle (colatitude), $\theta = 0..\pi$.

Firstly, the initial coordinate system is moved to the inner space of the object by parallel translation to geometrical center $(x_0, y_0, z_0)$:

$$x_i = x_i' - x_0, \quad y_i = y_i' - y_0, \quad z_i = z_i' - z_0,$$

where $x_i', y_i', z_i'$ – initial coordinates of $i$ reference point; $x_i, y_i, z_i$ – coordinates of $i$ reference point after translation.

Then spherical coordinates $\varphi_i, \theta_i$ are determined for every reference point $i$:

$$\rho_i = \sqrt{x_i'^2 + y_i'^2 + z_i'^2},$$
$$\varphi_i = \arctan\left(\frac{y_i'}{x_i'}\right),$$
$$\theta_i = \arccos\left(\frac{z_i'}{\rho_i}\right),$$

where $\rho_i$ – radius vector of $i$ reference point.

This translation to parametrical coordinate system (CS) allows finding the interpolant as single-valued function for most spatial objects. If the surface of the object has a complex form it could be divided into segments described by single valued function. Models of segments are constructed separately. The problem of joining these segments can be solved algorithmically.

RBF gives an interpolant of the following form:

$$x = \sum_{i=1}^{N} \lambda_{x i} \phi(\alpha_i),$$
$$y = \sum_{i=1}^{N} \lambda_{y i} \phi(\alpha_i),$$
$$z = \sum_{i=1}^{N} \lambda_{z i} \phi(\alpha_i),$$

where $\alpha_i$ – angular distance between the current point and $i$ reference point.

$\phi(\alpha_i)$ – RBF value that depends on angular distance $\alpha_i$;
$\lambda_{x i}, \lambda_{y i}, \lambda_{z i}$ – coefficients of influence of $i$ reference point on the current point in Cartesian coordinates;
$N$ – number of reference points of object’s surface.

Well-known kinds of RBF as functions of distance $\alpha$ can be applied to an interpolation task. The closed type of the surface allows decreasing the number of items in (4) because the reference point with the distance from the current point more than $|\alpha| > \pi/2$ can be discarded.

In this case, for example, the expression for RBF of inverse multi-quadric has the following form:

$$\phi(\alpha_i) = \begin{cases} \frac{1}{\sqrt{1+(\alpha_i)^2}} & \text{if } \alpha_i < \frac{\pi}{2}, \\ 0 & \text{if } \alpha_i \geq \frac{\pi}{2}. \end{cases}$$

Determination of coefficients of influence $\lambda_{x i}, \lambda_{y i}, \lambda_{z i}$ is implemented in a usual manner from condition of exact passing the surface through the reference points – separately by coordinates $x, y, z$ [4]. For that purpose the system of $N$ equations is formed for every equation form (4). Each of these equations is
condition of passing the surface through the particular k reference point by its’ own coordinate:
\[
x_k = \sum_{i=1}^{N} \lambda_{xi} (a_{ki}), k = 1, \ldots, N, \\
y_k = \sum_{i=1}^{N} \lambda_{yi} (a_{ki}), \\
z_k = \sum_{i=1}^{N} \lambda_{zi} (a_{ki}),
\]

where \( a_{ki} \) – angular distance between \( i \) and \( k \) reference points.

The first system of equations in a matrix form has the following form:
\[
\begin{bmatrix}
\phi(a_{11}) & \phi(a_{12}) & \cdots & \phi(a_{1N}) \\
\phi(a_{21}) & \phi(a_{22}) & \cdots & \phi(a_{2N}) \\
\vdots & \vdots & \ddots & \vdots \\
\phi(a_{N1}) & \phi(a_{N2}) & \cdots & \phi(a_{NN})
\end{bmatrix}
\begin{bmatrix}
\lambda_{x1} \\
\lambda_{x2} \\
\vdots \\
\lambda_{xN}
\end{bmatrix}
= 
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_N
\end{bmatrix},
\]

or in a convolute form with evident notation – \( \Phi \cdot \Lambda \mathbf{X} = \mathbf{X} \), from which:
\[
\Lambda \mathbf{X} = \Phi^{-1} \cdot \mathbf{X}.
\]

Similarly, from the second and the third systems the other coefficients are determined. In case of trivial solve, functions (4) are supplemented with polynom of low degree, as it is usually implemented while RBF interpolation [5].

The next stage is determination of the new set of reference points of modelling surface located on it with regular step of arguments \( \phi, \theta \). For that argument \( \alpha \) in equation (4) is sequentially changed with constant increment \( \Delta \phi, \Delta \theta \) of parameters \( \phi, \theta \):
\[
\begin{align*}
\psi_{i+1} &= \psi_i + \Delta \phi, \quad \Delta \phi = \frac{2\pi}{N_\phi}, \\
\theta_{i+1} &= \theta_i + \Delta \theta, \quad \Delta \theta = \frac{2\pi}{N_\theta},
\end{align*}
\]
where \( N_\phi, N_\theta \) – numbers of reference points in directions of \( \phi \) and \( \theta \) coordiantes.

Choice of \( \Delta \phi, \Delta \theta \) influences the error of interpolation. As a result of regularization, the non-analytical surface obtains a new description which includes for every reference point coordinates \( x_i, y_i, z_i \) and coefficients of influence \( \lambda_{xi}, \lambda_{yi}, \lambda_{zi} \).

4. Application of blending functions of orthogonal basis for rapid visualization of non-analytical surface

At the stage of visualization, it is necessary to turn from reference points of surface to its polygonal mesh. It is algorithmically simple to construct a polygonal mesh on the base of ordered reference points. It is necessary to calculate interjacent points of surface with given step and to make these points vertices of polygonal mesh.

RBF interpolation can be applied for determination of interjacent points. However, the known RBF do not allow applying rapid computational algorithms. Value of RBF for every reference point depends on the distance between this reference point and the current point. The distance is measured along the straight line connecting these points and it is changed on every step of passing the surface.

At the stage of visualization in real time it is presumed to apply blending functions of the orthogonal basis (BFOB) in place of RBF. Their characteristic feature is that they depend on distances measured separately by directions of the first and the second arguments instead of straight line (radius).

General form of BFOB for this case:
\[
BF_i = bf(\psi_i) \cdot bf(\theta_i),
\]

where \( \psi, \theta \) – angular distances between \( i \) reference point and the current point of the surface, measured for arguments of parametrical coordinate system \( \psi, \theta \);
\( bf(\psi), bf(\theta) \) – blending functions with arguments \( \psi, \theta \).

With sequential passing, the surface one of angular distances \( \psi_i, \theta_i \) is changed at every step. It can
be implemented with finite difference formulas if proper blending functions in the form of power polynomials are chosen.

As an example, the following form of BFOB can be proposed:

\[
BF_i = \begin{cases} 
\left(1 - \left(\frac{2\varphi_i}{\pi}\right)^2\right) \left(1 - \left(\frac{2\theta_i}{\pi}\right)^2\right) & \text{if } |\varphi_i| < \frac{\pi}{2} \text{ and } |\theta_i| < \frac{\pi}{2}, \\
0 & \text{if } |\varphi_i| \geq \frac{\pi}{2} \text{ or } |\theta_i| \geq \frac{\pi}{2}.
\end{cases}
\]  

(11)

Blending functions are calculated by increments. Finite difference representation \(hf(\varphi)\) has the following form:

\[
bf(\varphi_{i+1}) = bf(\varphi_i) + \Delta_{\varphi_i}^1, \quad bf(0) = 1, \\
\Delta_{\varphi(i+1)}^1 = \Delta_{\varphi_i}^1 + \Delta_{\varphi_i}^2, \\
\Delta_{\varphi_i}^1 = -\frac{8}{\pi^2} \delta_{\varphi_i} = \text{const}, \\
\Delta_{\varphi_i}^2 = -\frac{8}{\pi^2} \delta_{\varphi_i} = \text{step of calculation}.
\]

(12)

where \(\Delta_{\varphi_i}^1, \Delta_{\varphi_i}^2\) – the first and the second finite differences of blending function; \(\delta_{\varphi}\) – step of calculation.

Expressions (12) allow calculating the next value of blending function by two summing operations.

5. Determination of coefficients of influence of RBF for morphing in real time

The described sequence of operations gives the description of object of statical form. In case of morphing coordinates of reference points should be changed and it is equal to the appearance of a new form. Coefficients of influence obtained earlier don’t guarantee passing a surface through changed reference points. It is necessary to implement the described stages of modelling and obtain new values of coefficients \(\lambda_{x,y,z}\) to guarantee it. Solving of equations (8) in real time is difficult.

Closed surfaces are localized in space. Changes of their form have finite and often close limits (beating heart for example). It allows determining functional dependence of coefficients of influence of RBF on absolute values of coordinates of reference points. For that purpose values \(\lambda_{x,y,z}\) are determined for coordinates \(x,y,z\) changing in given range with given increment.

Then the obtained dependency is examined. This dependency is specific for every object of modelling. It can be described analytically with the help of spline interpolation or RBF interpolation for example. Table representation is possible, too. Consequently, huge calculations can be replaced with reading of ready results.

6. Experimental results

The proposed approach should be tested with a cognate analytical surface that allows determining the error of modelling easily. The surface of sphere is chosen. The window of modelling application with initial set of 100 points is shown in Figure 1. Figure 1b shows the result of ordering of initial reference points. New reference points are located in nodes of curvilinear coordinates. Initial reference points are marked with big marks. They belong to the surface. Figure 2a shows the result of surface reconstruction by BFOB interpolation. Resulting object is textured and shadowed (see Figure 2b).

Visual analysis shows good abilities of modelling of geometrical form of the object. Interpolation error estimation shows value 0.026 \%, that is fully acceptable for practical application.

7. Practical application of two-stage interpolation

One of possible applications of the proposed two-stage algorithm is visualization of a beating heart. In terms of modeling it is a complex dynamical non-analytical 3D object. Main aspects of its work are electrical and contractile. This task is actual for a wide range of applications: heart monitoring and diagnostic systems (of clinical or mobile usage[6]), simulation training systems for medical education and others. Examples of systems are ECGSim [7] – application that allows studying electrical activity of human heart ventricles, simulating changes of some electrical parameters of myocardium, visualizing the results on the static 3D model of the heart; software for topic diagnostics «DECARTO».
[8] – allows determining the parameters of electrical activity of a heart on the base of ECG. The task of visualization of a beating heart is not solved in these systems.

![Figure 1. Model of sphere surface: a - set of initial reference points, b - RBF interpolation of surface](image1)

The common approach to modeling of the electrical activity of a heart is usage of equivalent electrical generator of a heart. Multi dipole model [8] takes into account geometrical parameters of a model of a heart at every step of simulation, allows simulating electrical activity and heart beats simultaneously. Each dipole of the model reflects electrical activity of a particular area of a surface and its value depends on geometrical parameters such as angle and distance to the point of an ECG lead. The last two parameters are changed during cardio cycle.

3D model of a heart is closed mesh formed on the base of scattered points (see Figure 3a) and its’ beats are cyclic dynamical change of the form in a close range (see Figure 3a). Consequently, the task of its visualization can be solved by the proposed two-stage method. It can be effectively applied computer-aided simulation training suite “Virtual Surgeon” [9]. Endovascular surgery simulator displays the operation area including the beating heart and ECG in real time. It is necessary to prepare data sets of ECG and the corresponding beating heart images in advance because visualization in real time is resource-intensive in case of traditional methods. The proposed method can solve this problem. It allows visualizing a beating heart in real time with the possibility to change some parameters of modeling.

![Figure 2. Surface reconstructed by BFOB-interpolation: a – wire; b - textured](image2)

8. Conclusion
The obtained results of the study allow concluding that the proposed two-stage method of morphing is convenient for visualization of a complex spatial object in real time (two sum operations for step) with high quality (modeling error 0.026 %).

Modeling method includes a preliminary stage (interpolation of object’s surface by RBF with replacement of initial points with the new ones and assignment of coordinate change patterns during
the process of morphing) and a real time stage (surface reconstruction by blending functions of orthogonal basis with finite differences).

Figure 3. 3D model of a heart: a – initial form; b – after several steps of transformation.

Practical application of the proposed method is considered by the example of a beating heart that is a complex dynamical non-analytical 3D object, represented by a closed mesh formed, and its beats are cyclic dynamical changes of the form in a close range. It shows that the proposed method can be effective while solving practical tasks in a wide range of applications.

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