Four-dimensional lattice results on the MSSM electroweak phase transition†

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Abstract

We present the results of our large scale 4-dimensional (4d) lattice simulations for the MSSM electroweak phase transition (EWPT). We carried out infinite volume and continuum limit extrapolations and found a transition whose strength agrees well with perturbation theory. We determined the properties of the bubble wall that are important for a successful baryogenesis.

1 Introduction

The visible Universe is made up of matter. This statement is mainly based on observations of the cosmic diffuse γ-ray background, which would be larger than the present limits if boundaries between “worlds” and “anti-worlds” existed [1]. The observed baryon asymmetry of the universe was eventually determined at the EWPT [2]. This phase transition was the last instance when baryon asymmetry could have been generated around $T \approx 100-200$ GeV. Also at these temperatures any $B+L$ asymmetry could have been washed out. The possibility of baryogenesis at the EWPT is particularly attractive.

Perturbation theory (PT) does not give reliable EWPT predictions for larger Higgs boson masses [3, 4] in the standard model (SM). Large scale numerical simulations both on 4d and 3d lattices were needed to analyze the nature of the transition [5, 6]. They predict an end point for the first order EWPT at Higgs boson mass $72.1 \pm 1.4$ GeV [7, 8]. The present experimental lower limit of the SM Higgs boson mass, which is well above 100 GeV, excludes any EWPT in the SM. In order to explain the observed baryon asymmetry, extended versions of the SM are necessary. According to perturbative predictions the EWPT could be much stronger in the minimal supersymmetric extension of the standard model (MSSM) than in the SM [9], in particular if the stop mass is smaller than the top mass [10] and at the two-loop level. A reduced 3d version of the MSSM has recently been studied on the lattice [11]. The results show that the EWPT can be strong enough, i.e. $v/T_c > 1$, up to $m_0 \approx 105$ GeV and $m_{\tilde{t}} \approx 165$ GeV. The possibility of spontaneous CP violation for a successful baryogenesis is also addressed [12]. In this talk we review our study [13] of the EWPT in the MSSM on 4d lattices.

2 MSSM 4d simulation

Except for the U(1) sector and scalars with small Yukawa couplings, the whole bosonic sector of the MSSM is kept. Fermions, owing to their heavy Matsubara modes, are included perturbatively in the final result. Our work extends the 3d study [11] in several ways:

a) We use 4d lattices instead of 3d. Note, that due to very soft modes—close to the end point in the SM—much more CPU time is needed in 4d than in 3d. However, this difficulty does not appear in the MSSM because the phase transition is strong and the dominant correlation lengths are not that large in units of $T^{-1}_c$. Using

†Note, that the perfect agreement between the 3d and 4d results is a nonperturbative indication that the dimensional reduction program is correct for hard Matsubara modes.
unimproved lattice actions the leading corrections due to the finite lattice spacings are proportional to \( a \) in 3d and only to \( a^2 \) in 4d.

b) We have direct control over zero temperature renormalization effects.

c) We include both Higgs doublets not only the light renormalization effects.

According to standard baryogenesis scenarios (see e.g. [14]) the generated baryon number is directly proportional to the change of \( \beta \) through the bubble wall: \( \Delta \beta \). (\( \tan \beta = v_2/v_1 \), where \( v_1,2 \) are the expectation values of the two Higgses.)

The continuum lagrangian of the above theory reads

\[
\mathcal{L} = \mathcal{L}_g + \mathcal{L}_k + \mathcal{L}_V + \mathcal{L}_{sm} + \mathcal{L}_w + \mathcal{L}_s.
\]

The various terms correspond to the gauge part, kinetic part for Higgisses and third generation squarks, Higgs potential, squark mass terms , Yukawa couplings and quartic coupling proportional to the weak and QCD gauge coupling squares. The scalar trilinear couplings have been omitted for simplicity. It is straightforward to obtain the lattice action, for which we used the standard Wilson plaquette, hopping and site terms.

The parameter space of the above Lagrangian is many-dimensional. The experimental values were taken for the weak, strong and Yukawa couplings, and \( \tan \beta = 6 \) was used. For the bare soft breaking masses our choice was \( m_{Q,D} = 250 \text{ GeV} \), \( m_U = 0 \text{ GeV} \).

Our simulation techniques are similar to those used for the SU(2)-Higgs model [8] (overrelaxation and heat-bath algorithms are used for each scalar and gauge field). For a given lattice size we fix all parameters of the Lagrangian except the ratio of the coefficients of the \( |H|_2^2 \) and \( -(H_1^2 \bar{H}_2 + h.c.) \) terms in the Higgs potential. We tune this parameter to the transition point, where we determine the jump of the Higgs field, the shape of the bubble wall, and the change of \( \beta \) through the phase boundary. Next we perform \( T = 0 \) simulations with the same parameters and determine the masses (Higgses and W) and couplings (weak and strong). Extrapolations to infinite volumes and continuum are based on simulations at various lattice volumes and temporal extensions 2,3,4,5, respectively. Approaching the continuum limit, we move on an approximate line of constant physics (LCP). Our theory is bosonic, therefore the leading corrections due to finite lattice spacings are assumed to be proportional to \( a^2 \). This lattice spacing dependence is assumed for physical quantities in \( a \rightarrow 0 \) extrapolations.

3 Strength of the phase transition

We compare our simulation results with perturbation theory (PT). We used one-loop PT without applying high temperature expansion (HTE). A specific feature was a careful treatment of finite renormalization effects. This type of one-loop perturbation theory is also applied to correct the measured data to some fixed LCP quantities, which are defined as the averages of results at different lattice spacings, (i.e. our reference point, for which the most important quantity is the lightest Higgs mass, \( m_h \approx 45 \text{ GeV} \)).

Fig. 1. shows the phase diagram in the \( m^2 \)-\( T \) plane. One identifies three phases. The phase on the left (large negative \( m^2 \) and small stop mass) is the “color-breaking” (CB) phase. The phase in the upper right part is the “symmetric” phase, whereas the “Higgs” phase can be found in the lower right part. The line separating the symmetric and Higgs phases is obtained from \( L_\tau = 3 \) simulations, whereas the lines between these phases and the CB one are determined by keeping the lattice spacing fixed while increasing and decreasing the temperature by changing \( L_\tau \) to 2 and 4, respectively. The shaded regions indicate the uncertainty in the critical temperatures. The phase transition to the CB phase is observed to be much stronger than that between the symmetric and Higgs phases. The qualitative features of this picture are in complete agreement with perturbative and 3d lattice results [4, 10, 11]; however, our choice of parameters does not correspond to a two-stage symmetric-Higgs phase transition. In the two-stage scenario there is a phase transition from the symmetric to the CB phase at some \( T_1 \) and another phase transition occurs at \( T_2 < T_1 \) from the CB to the Higgs phase. It has been argued [12] that in the early universe no two-stage phase transition took place.

The bare squark mass parameters \( m_{Q,D}^2, m_U^2, m_{D}^2 \) receive quadratic renormalization corrections. As it is well known, one-loop lattice PT is not sufficient to reliably determine these corrections. Therefore, we first determine the position of the non-perturbative CB phase transitions in the bare quantities (e.g. the triple point or the \( T=0 \) transition for \( m_2^2 \) in Fig. 1). These quantities are compared with the prediction of the continuum PT, which gives the renormalized mass parameters on the lattice.

Fig. 2. shows the continuum limit extrapolation for the normalized jump of the order parameter (\( v/T_c \): upper data) and the critical temperature (\( T_c/m_W \): lower data). The shaded regions are the perturbative predictions at our reference point (see above) in the continuum. Results obtained on the lattice and in PT agree reasonably within the estimated uncertainties.
4 Properties of the bubble wall

In order to produce the observed baryon asymmetry, a strong first order phase transition is not enough. According to standard MSSM baryogenesis scenarios \cite{14} the generated baryon asymmetry is directly proportional to the variation of $\beta$ through the bubble wall separating the Higgs and symmetric phases. By using elongated lattices $(2 \cdot L^2 \cdot 192)$, $L=8,12,16$ at the transition point we study the properties of the wall. In our simulation procedure we restrict the length of one of the Higgs fields to a small interval between its values in the bulk phases. As a consequence, the system fluctuates around a configuration with two bulk phases and two walls between them. In order to have the smallest possible free energy, the wall is perpendicular to the long direction. We eliminate the effect of the remaining zero mode by shifting the wall of each configuration to some fixed position. Fig. 3 gives the bubble wall profiles for both Higgs fields. The measured width of the wall is $[A+B \cdot \log(aLT_c)]/T_c$, $A=10.8 \pm 0.1$ and $B=2.1 \pm 0.1$. This behavior indicates that the bubble wall is rough and without a pinning force of finite size its width diverges very slowly (logarithmically) \cite{16}. For the same bosonic theory the perturbative approach predicts $(11.2 \pm 1.5)/T_c$ for the width.

Transforming the data of Fig. 3 to $|H_2|^2$ as a function of $|H_1|^2$, we obtain $\Delta \beta = 0.0061 \pm 0.0003$. The perturbative prediction is $0.0046 \pm 0.0010$. 

Figure 1: The phase diagram of the bosonic theory obtained by lattice simulations.

Figure 2: The normalized jump and the critical temperature in the continuum limit.
5 Conclusions

In this talk, I have briefly discussed our work in [3]. We presented 4d lattice results on the EWPT in the MSSM. Our simulations were carried out in the bosonic sector of the MSSM. We found quite a good agreement between lattice results and our one-loop perturbative predictions. We determined the phase structure of the MSSM and identified the three possible phases (Higgs, symmetric and colour broken). We analyzed the bubble wall profile separating the Higgs and symmetric phases. The width of the wall and the change in $\beta$ is in fairly good agreement with perturbative predictions for typical bubble sizes. Both the strength of the phase transition and the smallness of $\Delta \beta$ indicate that experiments allow just a small window for MSSM baryogenesis. Our results could be further checked on larger lattices, which is possible on a machine like PMS [17].

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