FedMGDA+: Federated Learning meets Multi-objective Optimization

Zeou Hu¹,², Kiarash Shaloudegi², Guojun Zhang¹, Yaoliang Yu¹

University of Waterloo and Vector Institute¹
{zeou.hu, guojun.zhang, yaoliang.yu}@uwaterloo.ca

Huawei Noah’s Ark Lab²
kiarash.shaloudegi@huawei.com

June 23, 2020

Abstract

Federated learning has emerged as a promising, massively distributed way to train a joint deep model over large amounts of edge devices while keeping private user data strictly on device. In this work, motivated from ensuring fairness among users and robustness against malicious adversaries, we formulate federated learning as multi-objective optimization and propose a new algorithm FedMGDA+ that is guaranteed to converge to Pareto stationary solutions. FedMGDA+ is simple to implement, has fewer hyperparameters to tune, and refrains from sacrificing the performance of any participating user. We establish the convergence properties of FedMGDA+ and point out its connections to existing approaches. Extensive experiments on a variety of datasets confirm that FedMGDA+ compares favorably against state-of-the-art.

1 Introduction

Deep learning has achieved impressive successes on a number of domain applications, thanks largely to innovations on algorithmic and architectural design, and equally importantly to the tremendous amount of computational power one can harness through GPUs, computer clusters and dedicated software and hardware. Edge devices, such as smart phones, tablets, routers, car devices, home sensors, etc., due to their ubiquity and moderate computational power, impose new opportunities and challenges for deep learning. On the one hand, edge devices have direct access to privacy sensitive data that users may be reluctant to share (with say data centers), and they are much more powerful than their predecessors, capable of conducting a significant amount of on-device computations. On the other hand, edge devices are largely heterogeneous in terms of capacity, power, data, availability, communication, memory, etc., posing new challenges beyond conventional in-house training of machine learning models. Thus, a new paradigm, known as federated learning (FL) [1] that aims at harvesting the prospects of edge devices, has recently emerged. Developing new FL algorithms and systems on edge devices has since become a hot research topic in machine learning.

From the beginning of its birth, FL has close ties to conventional distributed optimization. However, FL emerged from the pressing need to address news challenges in the mobile era that existing distributed optimization algorithms were not designed for per se. We mention the following characteristics of FL that are most relevant to our work, and refer to the excellent surveys [2, 3, 4] and the references therein for more challenges and applications in FL.

• Non-IID: Each user’s data can be distinctively different from every other user’s, violating the standard iid assumption in statistical learning and posing significant difficulty in formulating the goal in precise mathematical terms [3]. The distribution of user data is often severely unbalanced.
• **Limited communication:** Communication between each user and a central server is constrained by network bandwidth, device status, user participation incentive, etc., demanding a thoughtful balance between computation (on each user device) and communication.

• **Privacy:** Protecting user (data) privacy is of utmost importance in FL. It is thus not possible to share user data (even to a cloud arbitrator), which adds another layer of difficulty in addressing the previous two challenges.

• **Fairness:** As argued forcibly in recent works (e.g., [5, 6]), ensuring fairness among users has become another serious goal in FL, as it largely determines users’ willingness to participate and ensures some degree of robustness against malicious user manipulations.

• **Robustness:** FL algorithms are eventually deployed in the wild hence subject to malicious attacks. Indeed, adversarial attacks (e.g., [7, 8, 9]) have been constructed recently to reveal vulnerabilities of FL systems against malicious manipulations at the user side.

In this work, motivated from the above challenges, we propose a new algorithm FedMGDA+ that complements and improves existing FL systems. FedMGDA+ is based on multi-objective optimization and is guaranteed to converge to Pareto stationary solutions. FedMGDA+ is simple to implement, has fewer hyperparameters to tune, and most importantly refrains from sacrificing the performance of any participating user. We demonstrate the superior performance of FedMGDA+ under a variety of metrics including accuracy, fairness, and robustness. We summarize our contributions below:

• In §2 we provide a novel, unifying and revealing interpretation of existing FL practices;

• In §3 we propose FedMGDA+ that complements existing FL systems and we prove its convergence properties under stochastic sampling;

• In §5 we perform extensive experiments to validate the competitiveness of FedMGDA+ using a variety of desirable metrics.

We conclude in §6 with some future directions and defer discussions on related work to Section 4.

2 Problem Setup

In this section, we recall the federated learning (FL) framework of McMahan et al. [1] and point out a simple interpretation that seemingly unifies different implementation practices.

We consider FL with $m$ users (edge devices), where the $i$-th user is interested in minimizing a function $f_i : \mathbb{R}^d \rightarrow \mathbb{R}, i = 1, \ldots, m$, defined on a shared model parameter $w \in \mathbb{R}^d$. Typically, each user function $f_i$ also depends on the respective user’s local (private) data $D_i$. The main goal in FL is to collectively and efficiently optimize individual objectives $\{f_i\}$ while meeting the challenges mentioned in §1.

McMahan et al. [1] proposed FedAvg to optimize arithmetic average of individual user functions:

$$\min_{w \in \mathbb{R}^d} A_f^\lambda(w), \quad \text{where} \quad A_f^\lambda(w) := \sum_{i=1}^{m} \lambda_i f_i(w). \quad (1)$$

The weights $\lambda_i$ need to be specified beforehand. Typical choices include the dataset size at each user, the “importance” of each user, or simply uniform, i.e., $\lambda_i \equiv 1/m$. FedAvg works as follows: At each round, a (random) subset of users is selected, each of which then performs $k$ epochs of local (full or minibatch) gradient descent:

$$\text{for all } i \text{ in parallel, } \quad w^i \leftarrow w^i - \eta \nabla f_i(w^i), \quad (2)$$

and then the weights are averaged at the server side: $w \leftarrow \sum_i \lambda_i w^i$, which is then broadcast to the users in the next round. The number of local epochs $k$ turns out to be a key factor. Setting $k = 1$ amounts to solving (1) by the usual gradient descent algorithm, while setting $k = \infty$ (and assuming convergence for each local function $f_i$) amounts to
(repeatedly) averaging the respective minimizers of \( f_i \)'s. We now give a new interpretation of FedAvg that hopefully yields insights on what it optimizes with an intermediate \( k \).

Our interpretation is based on the proximal average \[ 10 \]. Recall that the Moreau envelope and proximal map of a convex function \( f \) is defined respectively as:

\[
M_f^\eta(w) = \min_x \frac{1}{2\eta} \| x - w \|_2^2 + f(x), \quad P_f^\eta(w) = \arg\min_x \frac{1}{2\eta} \| x - w \|_2^2 + f(x).
\]  

(3)

Given a set of convex functions \( f = (f_1, \ldots, f_m) \) and positive weights \( \lambda = (\lambda_1, \ldots, \lambda_m) \) that sum to 1, we define the proximal average as the unique function \( A_{f, \lambda}^\eta \) such that \( P_{A_{f, \lambda}^\eta} = \sum \lambda_i P_{f_i}^\eta \). In other words, the proximal map of the proximal average is the average of proximal maps. It was proved in \[ 10 \] that:

\[
A_{f, \lambda}^0(w) := \lim_{\eta \to 0^+} A_{f, \lambda}^\eta(w) = \sum_i \lambda_i f_i(w),
\]

(4)

\[
A_{f, \lambda}^\infty(w) := \lim_{\eta \to \infty} A_{f, \lambda}^\eta(w) = \min_{\sum_i \lambda_i w_i = w} \sum_i \lambda_i f_i(w_i).
\]

(5)

It is then clear that setting \( k = 1 \) in FedAvg amounts to minimizing \( A_{f, \lambda}^0 \) while setting \( k = \infty \) in FedAvg amounts to minimizing \( A_{f, \lambda}^\infty \). This motivates us to interpret FedAvg with an intermediate \( k \) as minimizing \( A_{f, \lambda}^\eta \) with an intermediate \( \eta \). More interestingly, if we apply the PA-PG algorithm in \[ 12 \] Algo. 2] to minimize \( A_{f, \lambda}^\eta \), we obtain the simple update rule

\[
w \leftarrow \sum_i \lambda_i P_{f_i}^\eta(w),
\]

(6)

where the proximal maps are computed in parallel at the user’s side. We note that the recent FedProx algorithm of \[ 13 \] amounts to a randomized version of \[ 6 \]. The difference between the proximal average \( A_{f, \lambda}^0 \) and the arithmetic average \( A_{f, \lambda}^\infty \) can be uniformly bounded using the Lipschitz constant of each function \( f_i \) \[ 12 \]. Thus, for small step size \( \eta \), FedAvg (with any finite \( k \)) and FedProx all minimize some approximate form of the arithmetic average in \[ 1 \].

How to set the weights \( \lambda \) in FedAvg has been a major challenge. Recall that in FL data is distributed in a highly non-iid and unbalanced fashion, so it is not clear if some chosen arithmetic average in \[ 1 \] would really satisfy one’s actual intention. A second issue with the arithmetic average in \[ 1 \] is its well-known non-robustness against malicious manipulations, which has been exploited in recent adversarial attacks \[ 8 \]. Instead, Agnostic FL (AFL \[ 5 \]) aims to optimize the worst-case loss:

\[
\min_w \max_{\lambda \in \Lambda} A_{f, \lambda}^0(w),
\]

(7)

where the set \( \Lambda \) might cover reality better than any specific \( \lambda \) and provide some guarantee for any specific user (hence achieving some form of fairness). On the other hand, the worst-case loss in \[ 7 \] is perhaps even more non-robust against adversarial attacks. For instance, artificially adding a positive constant to some loss \( f_i \) can make it dominate the entire optimization process. q–FedAvg \[ 6 \] provided an interpolation between FedAvg and AFL using the \( \ell_p \) norm.

## 3 Federated Learning as Multi-objective Minimization

In this section, we formulate FL as multi-objective minimization (MoM), extend the multiple gradient descent algorithm \[ 14, 15, 16 \] to FL, draw connections to existing FL algorithms, and prove convergence properties of our extended algorithm FedMGDA+. Basic concepts in MoM, including the notion of Pareto optimality and stationarity, are recalled in Appendix A.

To formulate FL as an instance of MoM, we treat each user function \( f_i \) as a separate objective and aim to optimize them simultaneously. The notion of Pareto optimality and stationarity immediately enforces fairness among users, as\footnote{For nonconvex functions, similar results hold once we carefully deal with the multi-valuedness of the proximal map, see \[ 11 \].}.
we are not allowed to improve some user by sacrificing others. To further motivate our development, let us recall the objective in AFL [5]:

$$\min_w \max_{\lambda \in \Delta} \lambda^T f(w) \equiv \min_w \max_{i=1, \ldots, m} f_i(w),$$

(8)

where $\Delta$ denotes the simplex. By optimizing the worst loss than the average loss in FedAvg, AFL provides some guarantee to all users hence achieving some form of fairness. However, note that AFL’s objective (8) is not robust against adversarial attacks. In fact, if a malicious user artificially “inflates” its loss $f_1$ (e.g., by adding/multiplying a constant), it can completely dominate and mislead AFL to solely focus on optimizing its performance. The same issue applies to $q$-FedAvg [6] (albeit with a less dramatic effect if $q$ is small).

AFL’s objective (8) is very similar to the Chebyshev approach in MoM (see Appendix A for details), which inspires us to propose the following iterative algorithm:

$$\hat{w}_{t+1} = \arg\min_w \max_{\lambda \in \Delta} \lambda^T (f(w) - f(\hat{w}_t)), \tag{9}$$

where we adaptively “center” the user functions using function values from the previous iteration. When the functions $f$ are smooth, we can apply the quadratic bound to obtain:

$$w_{t+1} = \arg\min_w \max_{\lambda \in \Delta} \langle J_f(w_t)\lambda, w - w_t \rangle + \frac{1}{2\eta}||w - w_t||^2, \tag{10}$$

where $J_f = [\nabla f_1, \ldots, \nabla f_m] \in \mathbb{R}^{d \times m}$ is the Jacobian and $\eta > 0$ is the step size. Crucially, note that $f(w_t)$ does not appear in the above bound (10) since we subtracted it off in (9). Strong duality holds in (10) so we can swap min with max and obtain the dual:

$$\max_{\lambda \in \Delta} \min_w \lambda^T J_f^T (w_t) (w - w_t) + \frac{1}{2\eta}||w - w_t||^2. \tag{11}$$

Thus, iteratively we compute

$$w_{t+1} = w_t - \eta d_t, \quad d_t = J_f(w_t)\lambda_t^*, \quad \text{and} \quad \lambda_t^* = \arg\min_{\lambda \in \Delta} ||J_f(w_t)\lambda||^2. \tag{12}$$

Note that $d_t$ is precisely the minimum-norm element in the convex hull of the columns (i.e., gradients) in the Jacobian $J_f$, and finding $\lambda_t^*$ amounts to solving a simple quadratic program. The resulting iterative algorithm in (12) is known as multiple gradient descent algorithm (MGDA), which has been (re)discovered in [14, 15, 16] and recently applied to multitask learning in [17, 18] and to training GANs in [19]. Our concise derivation here reveals some new insights about MGDA, in particular its connection to AFL.

To adapt MGDA to the federated learning setting, we propose the following extensions.

**Balance between user average performance and fairness.** We observe that the MGDA update in (12) resembles FedAvg, with the crucial difference that MGDA automatically tunes the dual weighting variable $\lambda$ in each step while FedAvg pre-sets $\lambda$ based on *a priori* information about the user functions (or simply uniform in lack of such information). Importantly, the direction $d_t$ found in MGDA is a common descent direction for all participating objectives:

$$f(w_{t+1}) \leq f(w_t) + J_f^T (w_t) (w_{t+1} - w_t) + \frac{1}{2\eta}||w_{t+1} - w_t||^2 \leq f(w_t), \tag{13}$$

where the first inequality follows from familiar smoothness assumption on $f$ while the second inequality follows simply from plugging $w = w_t$ in (10) and noting that $w_{t+1}$ by definition can only decrease (10) even more. It is clear that equality is attained iff $d_t = J_f(w_t)\lambda_t^* = 0$, i.e., $w_t$ is Pareto-stationary (see Definition 11). In other words, MGDA never sacrifices any participating objective to trade for more sizable improvements over some other objective, something FedAvg with a fixed weighting $\lambda$ might attempt to do. On the other hand, FedAvg with a

\[\text{To be precise, AFL restricted $\lambda$ to a subset $\Lambda \subseteq \Delta$. We simply set $\Lambda = \Delta$ to ease the discussion.}\]
fixed weighting $\lambda$ may achieve higher \textit{average performance} under the weighting $\lambda$. It is thus natural to introduce the following trade-off between average performance and fairness:

$$w_{t+1} = w_t - \eta d_t, \quad d_t = J_f(w_t)\lambda^*_t, \quad \lambda^*_t = \arg\min_{\lambda \in \Delta, \|\lambda - \lambda_0\|_\infty \leq \epsilon} \|J_f(w_t)\lambda\|^2.$$ \hspace{1cm} (14)

Clearly, setting $\epsilon = 0$ recovers \texttt{FedAvg} with \textit{a priori} weighting $\lambda_0$ while setting $\epsilon = 1$ recovers MGDA where the weighting variable $\lambda$ is tuned without any restriction to achieve maximal fairness. In practice, with an intermediate $\epsilon \in (0, 1)$ we may strike a desirable balance between the two (sometimes) conflicting goals. Moreover, even with the uninformative weighting $\lambda_0 = 1/m$, using an intermediate $\epsilon$ allows us to upper bound the contribution of each user function to the common direction $d_t$ hence achieve some form of robustness against malicious manipulations.

\textbf{Robustness against malicious users through normalization.} Existing works \cite[e.g., 3, 24]{21, 22, 23} have demonstrated that the \textit{average} gradient in \texttt{FedAvg} can be easily manipulated by even a single malicious user. While more robust aggregation strategies are studied recently \cite[e.g., 21, 22, 23]{21, 22, 23}, here we propose to simply normalize the gradients from each user to unit length, based on the following considerations: (a) Normalizing the (sub)gradient is common for specialists in nonsmooth and stochastic optimization \cite{22} and sometimes eases step size tuning. (b) Solving the weighting variable $\lambda^*_t$ in \cite{22} with normalized gradients still guarantees fairness, \textit{i.e.}, the resulting direction $d_t$ is descending for all participating objectives (by a completely similar reasoning as the remark after \cite{23}). (c) Normalization restores robustness against multiplicative “inflation” from any malicious user, which, combined with MGDA’s built-in robustness against additive “inflation” \cite[see eq. (9)]{22}, offers reasonable robustness guarantees against adversarial attacks in practice.

\textbf{Balance between communication and on-device computation.} Communication between user devices and the central server is heavily constrained in FL due to a variety of reasons mentioned in \cite{21}. On the other hand, modern edge devices are capable of performing reasonable amount of on-device computations. Thus, we allow each user device to perform multiple local updates before communicating its update $g = w^0 - w^*$, namely the difference between the initial $w^0$ and the final $w^*$. to the central server. The server then calls the (extended) MGDA to perform a global update which will be broadcast to the next round of user devices. We note that similar strategy was already adopted in many existing FL systems \cite[e.g., 1, 3, 13]{1, 3, 13}.

\textbf{Subsampling user devices to alleviate non-iid and enhance throughput.} Due to the massive number of edge devices in FL, it is not realistic to expect most devices to participate at each or even most rounds. Consequently, the current practice in FL is to select a (different) subset of user devices to participate in each round \cite{1}. Moreover, randomly subsampling user devices can also help combat the non-iid distribution of user-specific data \cite[e.g., 1, 23]{1, 23}. Here we point out an important advantage of our MGDA-based algorithm: its update is along a common descending direction \cite[see (13)]{13}, meaning that the objective of any \textit{participating} user can only decrease. We believe this unique property of MGDA provides strong incentive for users to participate in FL. To our best knowledge, existing FL algorithms do not provide similar algorithmic incentives. Last but not the least, subsampling also solves a degeneracy issue in MGDA: when the number of participating users exceeds the dimension $d$, the Jacobian $J_f$ has full row-rank hence \cite{12} achieves Pareto-stationarity in a single iteration and stops making progress. Subsampling removes this undesirable effect and allows different subsets of users to be continuously optimized.

With the above extensions, we summarize our extended algorithm \texttt{FedMGDA+} in Algorithm \cite{1} and we prove the following convergence guarantees (precise statements and proofs in Appendix \cite{13}):

\textbf{Theorem 1.} \textit{Let each user function $f_i$ be $L$-Lipschitz smooth and $M$-Lipschitz continuous, and choose step size $\eta_t$ so that $\sum_t \eta_t = \infty$ and $\sum_t \sigma^2 \eta_t < \infty$, where $\sigma^2 = \text{the variance of (the stochastic) common direction} d_t$ under random sampling of the sets $I_t$. Then, with $r = k = 1$, we have}

$$\min_{t=0, \ldots, T} \mathbb{E}\|J_f(w_t)\lambda_t\|^2 \to 0, \quad \text{where} \quad \lambda_t = \arg\min_{\lambda \in \Delta} \|J_f(w_t)\lambda\|.$$ \hspace{1cm} (15)

Here $r$ is the number of local batches. The convergence rate depends on how quickly the variance term $\sigma_t$ diminishes (if at all). When the functions $f_i$ are convex, we can derive a finer result:
Alternative algorithm for FL provides one of the first convergence guarantees for FedMGDA+ recently in [26]. The stochastic case (such as what we consider here) is much more challenging and our theorems (such as the proximal average).

McMahan et al. [1] proposed the first FL algorithm which is called “FederatedAveraging” (a.k.a., FedAvg). The proposed algorithm is a synchronous update scheme that proceeds in several rounds. At each round, the central server sends the current global model to a subset of users, each of which then uses its respective local data to update the received model. Upon receiving the updated local models from users, the server performs aggregation, such as simple averaging, to update the global model. For more discussion on different averaging schemes, see [25]. FedAvg was extended in [13] to better deal with non-i.i.d. distribution of data. The new algorithm adds a “proximal regularizer” to the local loss functions, and minimizes the Moreau envelope function for each user:

$$\min_w f_i(w) + \frac{\lambda}{2} \|w - w_t^*\|^2$$

The resulting algorithm is called FedProx, and it reduces to FedAvg by setting $\mu = 0$, choosing the local solver to be SGD, and using a fixed number of local epochs across devices.

Analysing FedAvg has proved to be a challenging task due to its flexible updating scheme, partial user participation, and non-iid distribution of user-specific data [13]. The first theoretical analysis of FedAvg for strongly convex and smooth problems with non-iid data appeared in [25]. In this work, the effect of different sampling and averaging schemes on the convergence rate of FedAvg was investigated, leading to the conclusion that such effect becomes particularly important when the dataset is unbalanced and non-iid distributed. In [28], FedAvg was analyzed for non-convex problems. Huo et al. [28] formulated FedAvg as a stochastic gradient-based algorithm with biased gradients.

Theorem 2. Suppose each user function $f_i$ is convex and $M$-Lipschitz continuous. Suppose at each round FedMGDA+ includes a strongly convex user function whose weight is bounded away from 0. Then, with the choice $\eta_t = \frac{2}{c(t+2)}$ and $r = k = 1$, we have

$$E\|w_t - w_t^*\|^2 \leq \frac{4M^2}{c^2(t+3)}$$

and $w_t - w_t^* \to 0$ almost surely, (16)

where $w_t^*$ is the projection of $w_t$ to the Pareto stationary set of $\mathcal{X}$ and $c$ is some constant.

A slightly stronger result where we also allow some user functions to be nonconvex can be found in Appendix B. The same results hold if the gradient normalization is bounded away from 0 (otherwise we are already close to Pareto stationarity). For $r, k > 1$, using a similar argument as in [2] we expect FedMGDA+ to optimize some proxy problem (such as the proximal average).

We remark that convergence rate for MGDA, even when restricted to the deterministic case, was only derived in [13]. For strongly convex problems. Huo et al. [28] formulated FedAvg as a stochastic gradient-based algorithm with biased gradients.
and proved the convergence of FedAvg with decaying step sizes to stationary points. Further, Huo et al. [28] proposed FedMom, which is a server-side accelerated algorithm based on Nesterov’s momentum idea [29], and proved again its convergence to stationary points.

Recently, the interesting work [30] demonstrated theoretically that fixed points reached by FedAvg and FedProx (if at all) need not be stationary points of the original optimization problem, even in convex settings and with deterministic updates. To address this issue, [31] proposed FedSplit which restores the correct fixed points and enjoys convergence guarantees to them under convexity assumptions. It still remains open, though, if FedSplit can still converge to the correct fixed points under asynchronous and stochastic user updates, both of which are widely adopted in practice (and studied in our main paper).

Ensuring fairness among users has become a serious goal in FL since it largely determines users’ willingness to participate in the training process. Mohri et al. [5] argued that existing FL algorithms can lead to federated models that are biased toward different users. To solve this issue, [6] proposed agnostic federated learning (AFL) to improve fairness among users. AFL considers the target distribution as a weighted combination of the user distributions, i.e., \( D_\lambda = \sum_{i=1}^{m} \lambda_i D_i \) for some \( \lambda \in \Delta \), and optimizes the centralized model for the worst-case realization of \( \lambda \), leading to a saddle-point optimization problem which was solved by a fast stochastic optimization algorithm. On the other hand, q-fair federated learning (q-FFL) was proposed in [6] based on some fair resource allocation ideas from the wireless network literature. q-FFL’s goal is to achieve more uniform test accuracy across users. [6] further proposed q-FedAvg as a communication efficient algorithm to solve q-FFL.

In many FL applications, it is desirable to learn a personalized model for each user, geared more towards the respective user-specific data but also exploiting complementary data from other users [31]. Personalization also improves the performance of each individual user-specific model and encourages fairness among users. Mansour et al. [31] proposed a learning-theoretic approach to study model personalization in FL and analyzed user clustering, data interpolation, and model interpolation as possible approaches for personalization, with computation and communication efficient algorithms developed for each of these approaches.

FedAvg [1] relies on a coordinate-wise averaging of local models to update the global model. According to [32], in neural network (NN) based models, such coordinate-wise averaging might lead to sub-optimal results due to the permutation invariance of NN parameters. To address this issue, probabilistic federated neural matching (PFNM) was proposed in [33]. PFNM matches the neurons of received NN models from the users before averaging them. In case of poor matching (i.e., mismatching), PFNM applies Bayesian non-parametric methods to create new parameters in the global model. However, PFNM is only applicable to fully connected feed-forward networks. The recent work [32] proposed federated matched averaging (FedMA) as a layer-wise extension of PFNM to accommodate CNNs and LSTMs. Similar to PFNM, FedMA employs a Bayesian non-parametric mechanism to adjust the size of the central model to the heterogeneity of data distribution. On the other hand, the Bayesian non-parametric mechanism in PFNM and FedMA is particularly vulnerable to model poisoning attack [7, 9], since an adversary can easily trick the system to expand the global model in order to accommodate any poisoned local model.

FL systems are highly vulnerable to adversarial attacks as the central server by design cannot check how users’ updates are generated [7]. Model poisoning attack was shown in [7, 9] to be significantly more effective than data poisoning attack on FL systems, and it can easily evade common FL defence mechanisms such as coordinate-wise median, Krum [21], and geometric median. An adversary performs model poisoning attack on the central model by optimizing its local model for both the main task and a targeted task while explicit boosting the part of the model responsible for the latter. The explicit boosting helps to cancel out the effect of averaging with other users and makes sure the global model still achieves good performance on the main and targeted tasks. The first successful defence against model poisoning attack appeared in [8], where it was shown on the MNIST dataset that a combination of norm-clipping and differential privacy (DP) prevents simple model poisoning attacks. We note that the proposed algorithm was originally designed for learning differentially private language models in [34].

Manual inspection of raw data is an essential part of developing machine learning models for real-world applications, since it helps with identifying problems in the data such as labeling or preprocessing errors, and developing new modeling hypotheses [35]. In FL, data is distributed across large amounts of user devices, and is often private in nature; therefore, manual inspection of individual user data is not possible. To deal with this issue, Augenstein et al. [35] proposed to train a generative model with differential privacy guarantees from private user data in the FL way, and explored the idea to inspect the synthetic data generated from such a model as a viable surrogate for real private data.
To verify this empirically, we run (14) with different $\epsilon$.

Recovering to Appendix C. Due to space limits we only report some representative results, and defer the rest

Our experimental setups are given in Appendix C.1 and Appendix C.2. All experiments are run using a wide range of non-iid distributions of data (for results on F-MNIST, see Appendix C.4). These results confirm that changing fairness by tuning the $\epsilon$-constraint in eq. (14). Setting $\epsilon = 0$ recovers FedAvg while setting $\epsilon = 1$ recovers FedMGDA.

They developed differentially private federated GANs and RNNs for image and text applications, respectively, and showed that these generative models can help with debugging machine learning models during inference and training time.

## 5 Experiments

We evaluate our algorithm FedMGDA+ on several public datasets: CIFAR-10 [36], F-MNIST [37], and Adult [38], and compare to existing FL systems including FedAvg [1], FedProx [13], q-FedAvg [6], and AFL [5]. In addition, from the discussions in §3, one can envision several potential extensions of existing algorithms to improve their performance. So, we also compare to the following extensions: FedAvg-n which is FedAvg with gradient normalization, and MGDA-Prox which is FedMGDA+ with a proximal regularizer added to each user’s loss function. We make a distinction between FedMGDA+ and FedMGDA which is a vanilla extension of the MGDA idea to FL. The details of our experimental setups are given in Appendix C.1 and Appendix C.2. All experiments are run using a wide range of hyperparameters, see Appendix C.3. Due to space limits we only report some representative results, and defer the rest

**Recovering FedAvg.** As mentioned in §3 we can control the balance between the user average performance and fairness by tuning the $\epsilon$-constraint in eq. (14). Setting $\epsilon = 0$ recovers FedAvg while setting $\epsilon = 1$ recovers FedMGDA. To verify this empirically, we run (14) with different $\epsilon$, and report results on CIFAR-10 in Figure 1 for both iid and non-iid distributions of data (for results on F-MNIST, see Appendix C.4). These results confirm that changing $\epsilon$ from 0 to 1 yields an interpolation between FedAvg and FedMGDA, as expected. Since FedAvg essentially optimizes the (uniformly) averaged training loss, it naturally performs the best under this metric. Nevertheless, it is interesting to note that some intermediate $\epsilon$ values actually lead to better user accuracy than FedAvg in the non-iid setting (as shown in Figure 1(a)).

**Robustness.** We discussed earlier in §3 that the gradient normalization and MGDA’s built-in robustness allow FedMGDA+ to combat against certain adversarial attacks in practical FL deployment. We now empirically evaluate the robustness of FedMGDA+ against these attacks. We run various FL algorithms in the presence of a single malicious user who aims to manipulate the system by inflating its loss. We consider an adversarial setting where the attacker participates in each communication round and inflates its loss function by (i) adding a bias to it, or (ii) multiplying it by a scaling factor, termed the bias and scaling attack, resp. In the first experiment, we simulate a bias attack on Adult dataset by adding a constant bias to the underrepresented user, i.e. the PhD domain, since it’s more natural for such a user to cheat the system. In this setup, the worst performance we can get is bounded by training the model

---

1 One can also apply the gradient normalization idea to q-FedAvg; however, we observed from our experiments that the resulting algorithm is unstable particularly for big $q$ values.
using PhD data only. Results under the bias attack are presented in Figure 2 (Left); also see Appendix C.5 for more results. We observe that AFL and q-FedAvg perform slightly better than FedMGDA+ without the attack; however, their performances deteriorate to a level close to the worst case scenario under the attack. In contrast, FedMGDA+ is not affected by the attack with any bias, which empirically supports our claim in §3. Figure 2 (Right) shows the results of different algorithms on CIFAR-10 with and without an adversarial scaling. As mentioned earlier, q-FedAvg with gradient normalization is highly unstable particularly under the scaling attack, so we did not include its result here.

From Figure 2 (Right) it is immediate to see that (i) the scaling attack affects all algorithms that do not employ gradient normalization; (ii) q-FedAvg is the most affected under this attack; (iii) surprisingly, FedMGDA+ and, to a lesser extent, MGDA-Prox actually converge to slightly better Pareto solutions. The above results empirically verify the robustness of FedMGDA+ under perhaps the most common bias and scaling attacks.

Fairness. Lastly, we compare FedMGDA+ with existing FL algorithms using different notions of fairness on CIFAR-10. For the first experiment, we adopt the same fairness metric as [6], and measure fairness by calculating the variance of users’ test error. We run each algorithm with different hyperparameters, and among the results, we pick the best ones in terms of average accuracy to be shown in Figure 3; full table of results can be found in Appendix C.6. From this figure, we observe that (i) FedMGDA+ achieves the best average accuracy while its standard deviation is comparable with that of q-FedAvg; (ii) FedMGDA+ significantly outperforms FedMGDA, which clearly justifies our proposed modifications in Algorithm 1 to the vanilla MGDA; and (iii) FedMGDA+ outperforms FedAvg-n, which uses the same normalization step as FedMGDA+, in terms of average accuracy and standard deviation. These observations confirm the effectiveness of FedMGDA+ in inducing fairness.

In the next experiment, we show that FedMGDA+ not only yields a fair final solution but also maintains fairness during the entire training process in the sense that, in each round, it refrains from sacrificing the performance of any participating user for the sake of improving the overall performance. To the best of our knowledge, “fairness during training” has not been investigated before, in spite of having great practical implications—it encourages user participation. To examine this fairness, we run several experiments on CIFAR-10 and measure the percentage of improved participants in each communication round. Specifically, we measure the training loss before and after each round for all participating users, and report the percentage of those improved or stay unchanged.

Formally, the percentage of improved users at time $t$ is defined as $\sum_{i \in I_t} \{f_i(w_{t+1}) \leq f_i(w_t)\}/|I_t|$, where $I_t$ is the set of selected users.
FedAvg  FedProx  FedMGDA  FedMGDA+  MGDA-Prox  FedAvg-n  q-FedAvg

Figure 3: Distribution of the user test accuracy on CIFAR-10: (Left) the algorithms are run for 2000 communication rounds and $b = 10$. The hyperparameters are: $\mu = 0.01$ for FedProx; $\eta = 1.5$ and decay = 1/10 for FedMGDA+ and FedAvg; $\eta = 1.0$ and decay = 1/10 for MGDA-Prox; $q = 0.5$ and $L = 1.0$ for q-FedAvg. (Right) the algorithms are run for 3000 communication rounds and $b = 400$. The hyperparameters are: $\mu = 0.1$ for FedProx; $\eta = 1.0$ and decay = 1/40 for FedMGDA+, MGDA-Prox, and FedAvg; $q = 0.1$ and $L = 0.1$ for q-FedAvg. The reported statistics are averaged across 4 runs with different random seeds.

Figure 4: The percentage of improved users in terms of training loss vs communication rounds on the CIFAR-10 dataset. Two representative cases are shown: (Left) the local batch size $b = 10$, and (Right) the local batch size $b = 400$. The results are averaged across 4 runs with different random seeds.

shows the percentage of improved participating users in each communication round in terms of training loss for two representative cases; see Appendix C.7 for full results using different hyperparameters.

We can see that FedMGDA+ consistently outperforms other algorithms in terms of percentage of improved users, which means that by using FedMGDA+, fewer users’ performances get worse after each participation. Furthermore, we notice from Figure 4 (Left) that, with local batch size $b = 10$, the percentage of improved users is less than 100%, which can be explained as follows: for small batch sizes (i.e., $b < |D|$ where $D$ represents a local dataset), the received updates from users are not the true gradients of users’ losses given the global model (i.e., $g_i \neq \nabla f_i(w)$); they are noisy estimates of the true gradients. Consequently, the common descent direction calculated by MGDA is noisy and may not always work for all participating users. To remove the effect of this noise, we set $b = |D|$ which allows us to recover the true gradients from the users. The results are presented in Figure 4 (Right), which confirms that, when step size decays (less overshooting), the percentage of improved users for FedMGDA+ reaches towards 100% during training, as is expected.

at time $t$, and $I\{A\}$ is the indicator function of an event $A$. 

10
6 Conclusion

We have proposed a novel algorithm FedMGDA+ for federated learning. FedMGDA+ is based on multi-objective optimization and aims to converge to Pareto stationary solutions. FedMGDA+ is simple to implement, has fewer hyperparameters to tune, and complements existing FL systems nicely. Most importantly, FedMGDA+ is robust against additive and multiplicative adversarial manipulations and ensures fairness among all participating users. We established preliminary convergence guarantees for FedMGDA+, pointed out its connections to recent FL algorithms, and conducted extensive experiments to verify its effectiveness. In the future we plan to formally quantify the tradeoff induced by multiple local updates and to establish some privacy guarantee for FedMGDA+.

Acknowledgements

Resources used in preparing this research were provided, in part, by the Province of Ontario, the Government of Canada through CIFAR, and companies sponsoring the Vector Institute. We gratefully acknowledge funding support from NSERC, the Canada CIFAR AI Chairs Program, and Waterloo-Huawei Joint Innovation Lab. We thank NVIDIA Corporation (the data science grant) for donating two Titan V GPUs that enabled in part the computation in this work.

References

[1] Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Agüera y Arcas. “Communication-Efficient Learning of Deep Networks from Decentralized Data”. In: AISTATS. 2017.
[2] Tian Li, Anit Kumar Sahu, Ameet Talwalkar, and Virginia Smith. “Federated learning: Challenges, methods, and future directions”. arXiv:1908.07873. 2019.
[3] Qiang Yang, Yang Liu, Tianjian Chen, and Yongxin Tong. “Federated Machine Learning: Concept and Applications”. ACM Transactions on Intelligent Systems and Technology, vol. 10, no. 2 (2019).
[4] Peter Kairouz et al. “Advances and Open Problems in Federated Learning”. arXiv:1912.04977. 2019.
[5] Mehryar Mohri, Gary Sivek, and Ananda Theertha Suresh. “Agnostic Federated Learning”. In: ICML. 2019.
[6] Tian Li, Maziar Sanjabi, Ahmad Beirami, and Virginia Smith. “Fair Resource Allocation in Federated Learning”. In: ICLR. 2020.
[7] Eugene Bagdasaryan, Andreas Veit, Yiqing Hua, Deborah Estrin, and Vitaly Shmatikov. “How To Backdoor Federated Learning”. arXiv:1807.00459. 2018.
[8] Ziteng Sun, Peter Kairouz, Ananda Theertha Suresh, and H Brendan McMahan. “Can You Really Backdoor Federated Learning?” arXiv:1911.07963. 2019.
[9] Arjun Nitin Bhagoji, Supriyo Chakraborty, Prateek Mittal, and Seraphin Calo. “Analyzing Federated Learning through an Adversarial Lens”. In: ICML. 2019.
[10] Heinz H. Bauschke, Rafal Goebel, Yves Lucet, and Xianfu Wang. “The Proximal Average: Basic Theory”. SIAM Journal on Optimization, vol. 19, no. 2 (2008), pp. 766–785.
[11] Yaoliang Yu, Xun Zheng, Micol Marchetti-Bowick, and Eric P. Xing. “Minimizing Nonconvex Non-Separable Functions”. In: AISTATS. 2015.
[12] Yaoliang Yu. “Better Approximation and Faster Algorithm Using the Proximal Average”. In: NIPS. 2013.
[13] Tian Li et al. “Federated Optimization for Heterogeneous Networks”. arXiv:1812.06127. 2018.
[14] Hiroaki Mukai. “Algorithms for multicriterion optimization”. IEEE Transactions on Automatic Control, vol. 25, no. 2 (1980), pp. 177–186.
[15] Jörg Fliege and Benar Fux Svaiter. “Steepest descent methods for multicriteria optimization”. Mathematical Methods of Operations Research, vol. 51, no. 3 (2000), pp. 479–494.
[16] Jean-Antoine Désidéri. “Multiple-gradient descent algorithm (MGDA) for multiobjective optimization”. Comptes Rendus Mathematique, vol. 350, no. 5 (2012), pp. 313–318.
[17] Ozan Sener and Vladlen Koltun. “Multi-Task Learning as Multi-Objective Optimization”. In: NIPS. 2018.

[18] Xi Lin, Hui-Ling Zhen, Zhenhua Li, Qing-Fu Zhang, and Sam Kwong. “Pareto Multi-Task Learning”. In: NIPS. 2019.

[19] Isabela Albuquerque et al. “Multi-objective training of Generative Adversarial Networks with multiple discriminators”. In: ICML. 2019.

[20] Cong Xie, Sanmi Koyejo, and Indranil Gupta. “Fall of Empires: Breaking Byzantine-tolerant SGD by Inner Product Manipulation”. In: UAI. 2019.

[21] Peva Blanchard, El Mahdi El Mhamdi, Rachid Guerraoui, and Julien Stainer. “Machine Learning with Adversaries: Byzantine Tolerant Gradient Descent”. In: NIPS. 2017.

[22] Dong Yin, Yudong Chen, Ramchandran Kannan, and Peter Bartlett. “Byzantine-Robust Distributed Learning: Towards Optimal Statistical Rates”. In: ICML. 2018.

[23] Ilias Diakonikolas et al. “Sever: A Robust Meta-Algorithm for Stochastic Optimization”. In: ICML. 2019.

[24] Kurt M Anstreicher and Laurence A Wolsey. “Two “well-known” properties of subgradient optimization”. Mathematical Programming, vol. 120, no. 1 (2009), pp. 213–220.

[25] Xiang Li, Kaixuan Huang, Wenhai Yang, Shusen Wang, and Zhihua Zhang. “On the Convergence of FedAvg on Non-IID Data”. In: ICLR. 2020.

[26] Jörg Fliege, A. I. F. Vaz, and L. N. Vicente. “Complexity of gradient descent for multiobjective optimization”. Optimization Methods and Software, vol. 34, no. 5 (2019), pp. 949–959.

[27] Kaisa M. Miettinen. Nonlinear Multiobjective Optimization. Springer, 1998.

[28] Zhouyuan Huo, Qian Yang, Bin Gu, Lawrence Carin, and Heng Huang. “Faster On-Device Training Using New Federated Momentum Algorithm”. arXiv:2002.02090. 2020.

[29] Yurii Nesterov. “A method for unconstrained convex minimization problem with the rate of convergence $O(1/k^2)$”. Doklady AN USSR, vol. 269 (1983), pp. 543–547.

[30] Reese Pathak and Martin J. Wainwright. “FedSplit: An algorithmic framework for fast federated optimization”. arXiv:2005.05238. 2020.

[31] Yishay Mansour, Mehryar Mohri, Jae Ro, and Ananda Theertha Suresh. “Three Approaches for Personalization with Applications to Federated Learning”. arXiv:2002.10619. 2020.

[32] Hongyi Wang, Mikhail Yurochkin, Yuekai Sun, Dimitris Papailiopoulos, and Yasaman Khazaeni. “Federated Learning with Matched Averaging”. In: ICLR. 2020.

[33] Mikhail Yurochkin et al. “Bayesian Nonparametric Federated Learning of Neural Networks”. In: ICML. 2019.

[34] H. Brendan McMahan, Daniel Ramage, Kunal Talwar, and Li Zhang. “Learning Differentially Private Recurrent Language Models”. In: ICLR. 2018.

[35] Sean Augenstein et al. “Generative Models for Effective ML on Private, Decentralized Datasets”. In: ICLR. 2020.

[36] Alex Krizhevsky. Learning Multiple Layers of Features from Tiny Images. Tech. rep. University of Toronto, 2009.

[37] Han Xiao, Kashif Rasul, and Roland Vollgraf. “Fashion-MNIST: a Novel Image Dataset for Benchmarking Machine Learning Algorithms”. arXiv:1708.07747. 2017.

[38] Dheeru Dua and Casey Graff. UCI Machine Learning Repository. 2017.

[39] Quentin Mercier, Fabrice Poirion, and Jean-Antoine Désidéri. “A stochastic multiple gradient descent algorithm”. European Journal of Operational Research, vol. 271, no. 3 (2018), pp. 808–817.
A Multi-objective Minimization (MoM)

Multi-objective minimization (MoM) refers to the setting where multiple scalar objective functions, possibly incompatible with each other, need to be minimized simultaneously. It is also called vector optimization because the objective functions can be combined into a single vector-valued function. In mathematical symbols, MoM can be written as

$$\min_{w \in \mathbb{R}^d} f(w) := (f_1(w), f_2(w), \ldots, f_m(w)),$$

where the minimum is defined wrt the partial ordering:

$$f(w) \leq f(z) \iff \forall i = 1, \ldots, m, f_i(w) \leq f_i(z).$$

Unlike single objective optimization, with multiple objectives it is possible that more than one optimal solution exists. A Pareto optimal solution is one in which it is not possible to improve any component objective without compromising some other objective. Similarly, we call w∗ a weakly Pareto optimal solution if there does not exist any w such that f(w) < f(w∗), i.e., it is not possible to improve all component objectives in f(w∗). Clearly, any Pareto optimal solution is also weakly Pareto optimal but the converse may not hold.

We point out that the optimal solutions in MoM are usually a set (in general of infinite cardinality) and without additional subjective preference information, all Pareto optimal solutions are considered equally good (as they are not comparable against each other). This is fundamentally different from the single objective case.

From now on, for simplicity we assume all objective functions are continuously differentiable but not necessarily convex (to accommodate deep models). Finding a (weakly) Pareto optimal solution in this setting is quite challenging (already in the single objective case). Instead, we will contend with Pareto stationary solutions, namely those that satisfy an intuitive first order necessary condition:

Definition 1 (Pareto-stationary). We call w∗ Pareto-stationary iff there exists some convex combination of the gradients \(\{\nabla f_i(w^*)\}\) that equals zero.

Lemma 1 (14). Any Pareto optimal solution is Pareto stationary. Conversely, if all functions are convex, then any Pareto stationary solution is weakly Pareto optimal.

Needless to say, the above results reduce to the familiar ones for the single objective case (m = 1).

There exist many algorithms for finding Pareto stationary solutions. We briefly review three popular ones that are relevant for us, and refer the reader to the excellent monograph [27] for more details.

**Weighted approach.** Let \(\lambda \in \Delta\) (the simplex) and consider the following single weighted objective:

$$\min_w \sum_{i=1}^m \lambda_i f_i(w).$$

It is easily seen that any (global) minimizer of \(20\) is weakly Pareto optimal, which can be strengthened to Pareto optimal if all weights \(\lambda_i\) are positive. From Definition 1 it is clear that any stationary solution of the weighted scalar problem \(20\) is a Pareto stationary solution of the original MoM \(17\). Note that the scalarization weights \(\lambda\), once chosen, are fixed throughout. Different \(\lambda\) leads to different Pareto stationary solutions.

**\(\epsilon\)-constraint.** Let \(\epsilon \in \mathbb{R}^m, \iota \in \{1, \ldots, m\}\) and consider the following constrained scalar problem:

$$\min_w f_\iota(w)$$

subject to:

$$f_i(w) \leq \epsilon_i, \forall i \neq \iota.$$
Chebyshev approach. Let $s \in \mathbb{R}^m$ and consider the minimax problem:

$$\min_w \max_{\lambda \in \Delta} \lambda^T (f(w) - s).$$

(23)

Again, any (global) minimizer is weakly Pareto optimal. Here $s$ is a fixed vector that ideally lower bounds $f$. 
B Proofs

Theorem 3. Suppose each user function $f_i$ is $L$-smooth (i.e., $\nabla^2 f_i \preceq L$) and $M$-Lipschitz continuous. Then, with step size $\eta_t \in (0, \frac{1}{2M}]$ we have

$$
\min_{t=0,\ldots,T} \mathbb{E}[\|J_f(w_t)\lambda_t\|] \leq \frac{2E[f(0) - Ef(w_{T+1}) + \sum_{t=0}^{T} \eta_t (M \sigma_t + L \eta_t \sigma_t^2)]}{\sum_{t=0}^{T} \eta_t},
$$

where $\sigma_t^2 := \mathbb{E}\|J_f(w_t)\lambda_t - \hat{J}_f(w_t)\hat{\lambda}_t\|^2$ is the variance of the stochastic common direction. Moreover, if some user function $f_i$ is bounded from below, and it is possible to choose $\eta_t$ so that $\sum_t \eta_t = \infty$, $\sum_t \eta_t \sigma_t < \infty$, then the left-hand side in (24) converges to 0.

Proof. Let $\xi_t := J_f(w_t)\lambda_t - \hat{J}_f(w_t)\hat{\lambda}_t$, where

$$
\lambda_t = \arg\min_{\lambda \in \Delta} \| J_f(w_t)\lambda \|, \quad \hat{\lambda}_t = \arg\min_{\lambda \in \Delta} \| \hat{J}_f(w_t)\lambda \|.
$$

Then, applying the quadratic bound and the update rule:

$$
f(w_{t+1}) \leq f(w_t) - \eta_t J_f^T(w_t)\hat{J}_f(w_t)\hat{\lambda}_t + \frac{L \eta_t^2}{2} \| \hat{J}_f(w_t)\hat{\lambda}_t \|^2
$$

$$
= f(w_t) - \eta_t J_f^T(w_t)J_f(w_t)\lambda_t + L \eta_t^2 \| J_f(w_t)\lambda_t \|^2 + \eta_t J_f^T(w_t)\xi_t + L \eta_t^2 \| \xi_t \|^2
$$

$$
\leq f(w_t) - \eta_t (1 - L \eta_t) \| J_f(w_t)\lambda_t \|^2 + \eta_t M \| \xi_t \|^2 + L \eta_t^2 \| \xi_t \|^2,
$$

where we used the optimality of $\lambda_t$ so that

$$
\forall \lambda \in \Delta, \langle \lambda, J_f^T(w_t)J_f(w_t)\lambda \rangle \geq \langle \lambda_t, J_f^T(w_t)J_f(w_t)\lambda_t \rangle.
$$

Letting $\eta_t \leq \frac{1}{2M}$, taking expectations and rearranging we obtain

$$
\min_{t=0,\ldots,T} \mathbb{E}[\|J_f(w_t)\lambda_t\|] \leq \frac{2E[f(0) - Ef(w_{T+1}) + \sum_{t=0}^{T} \eta_t (M \sigma_t + L \eta_t \sigma_t^2)]}{\sum_{t=0}^{T} \eta_t},
$$

where $\sigma_t^2 := \mathbb{E}\|\xi_t\|^2$. \hfill \Box

Theorem 4. Suppose each user function $f_i$ is $\sigma$-convex (i.e., $\nabla^2 f_i \succeq \sigma$) and $M$-Lipschitz continuous. Suppose at each round $t$ FedMGDA includes some function $f_{vt}$ such that

$$
f_{vt}(w_t) - f_{vt}(w_t^*) \geq \frac{\ell_t}{2} \| w_t - w_t^* \|^2,
$$

where $w_t^*$ is the projection of $w_t$ to the Pareto stationary set $W^*$ of (17). Assume $E[|v_t|\ell_t + \sigma_t|w_t| \geq c > 0$, then

$$
E[\|w_{t+1} - w_t^*\|^2] \leq \pi_t (1 - c \eta_0) E[\|w_0 - w_0^*\|^2] + \sum_{s=0}^{t} \frac{\pi_s}{\pi_{s+1}} \|w_0 - w_0^*\|^2, \quad \pi_t = \prod_{s=1}^{t} \eta_s, \text{ and } \pi_0 = 1.
$$

where $\pi_t = \prod_{s=1}^{t} \eta_s$ and $\pi_0 = 1$. In particular,

- if $\sum_t \eta_t = \infty$, $\sum_\tau \eta^2 < \infty$, then $E[\|w_t - w_t^*\|^2] \to 0$ and $w_t$ converges to the Pareto stationarity set $W^*$ almost surely;

- with the choice $\eta_t = \frac{2}{c(t+1)}$ we have

$$
E[\|w_t - w_t^*\|^2] \leq \frac{4M^2}{c^2(t+3)},
$$

where

$$
\eta_t = \frac{2}{c(t+1)}
$$

we have

$$
E[\|w_t - w_t^*\|^2] \leq \frac{4M^2}{c^2(t+3)}.
$$

15
Proof. For each user $i$, let us define the function

$$\hat{f}_i(w) = I_i f_i(w),$$

(34)

where the random variable $I \in \{0, 1\}^m$ indicates which user participates at a particular round. Clearly, we have $E\hat{f}_i(w, I) = f_i(w)E I_i$. Therefore, our multi-objective minimization problem is equivalent as:

$$\min_w \left\{ E\hat{f}_1(w, I), \ldots, E\hat{f}_m(w, I) \right\},$$

(35)

since positive scaling does not change Pareto stationarity. (If one prefers, we can also normalize the stochastic functions $\hat{f}_i(w, I)$ so that the unbiasedness property $E\hat{f}_i(w, I) = f_i(w)$ holds.)

We now proceed as in (39) and provide a slightly sharper analysis. Let us denote $w_t^*$ the projection of $w_t$ to the Pareto-stationary set $W^*$ of (35), i.e.,

$$w_t^* = \arg\min_{w \in W^*} \|w_t - w\|.$$

(36)

Then,

$$\|w_{t+1} - w_{t+1}^*\|^2 \leq \|w_{t+1} - w_t^*\|^2$$

(37)

$$= \|w_t - \eta_t d_t - w_t^*\|^2$$

(38)

$$= \|w_t - w_t^*\|^2 - 2\eta_t \langle w_t - w_t^*, d_t \rangle + \eta_t^2 \|d_t\|^2.$$  

(39)

To bound the middle term, we have from our assumption:

$$\exists \nu_t, \hat{f}_{\nu_t}(w_t, I_t) - \hat{f}_{\nu_t}(w_t^*, I_t) \geq \frac{\nu_t}{2} \|w_t - w_t^*\|^2,$$

(40)

$$\forall i, \hat{f}_i(w_t, I_t) - \hat{f}_i(w_t^*, I_t) \geq 0,$$

(41)

where the second inequality follows from the definition of $w_t^*$. Therefore,

$$(w_t - w_t^*, d_t) = \left<w_t - w_t^*, \sum_{i:t_i=1} \lambda_i \nabla f_i(w_t)\right>$$

(42)

$$\geq \sum_{i:t_i=1} \lambda_i (f_i(w_t) - f_i(w_t^*)) + \frac{\nu_t}{2} \|w_t - w_t^*\|^2$$

(43)

$$= \sum_{i} \lambda_i \left(\hat{f}_i(w_t, I_t) - \hat{f}_i(w_t^*, I_t)\right) + \frac{\nu_t}{2} \|w_t - w_t^*\|^2$$

(44)

$$\geq \frac{\lambda_i \nu_t + \sigma_t}{2} \|w_t - w_t^*\|^2.$$  

(45)

Continuing from (39) and taking conditional expectation:

$$E[\|w_{t+1} - w_{t+1}^*\|^2 | w_t] \leq (1 - c\eta_t) \|w_t - w_t^*\|^2 + \eta_t^2 M^2,$$

(46)

where $c_t := E[\lambda_i \nu_t + \sigma_t | w_t] \geq c > 0$. Taking expectation we obtain the familiar recursion:

$$E[\|w_{t+1} - w_{t+1}^*\|^2] \leq (1 - c\eta_t) E[\|w_{t} - w_{t}^*\|^2] + \eta_t^2 M^2,$$

(47)

from which we derive

$$E[\|w_{t+1} - w_{t+1}^*\|^2] \leq \pi_t (1 - c\eta_0) E[\|w_0 - w_0^*\|^2] + \sum_{s=0}^{t} \pi_s \eta_s^2 M^2,$$

(48)

where $\pi_t = \prod_{s=1}^{t} (1 - c\eta_s)$ and $\pi_0 = 1$. Since $\pi_t \to 0 \iff \sum_{t} \eta_t = \infty$, we know

$$E[\|w_{t+1} - w_{t+1}^*\|^2] \to 0$$

(49)

16
if \( \sum t \eta_t = \infty \) and \( \sum t \eta_t^2 < \infty \).

Setting \( \eta_t = \frac{2}{c(t+2)} \) we obtain \( \pi_t = \frac{2}{(t+2)(t+1)} \) and by induction

\[
\sum_{s=0}^t \frac{\pi_t \eta_s^2}{\pi_s} \leq \frac{4}{c^2(t+2)(t+1)} \sum_{s=0}^t \frac{s+1}{s+2} \leq \frac{4}{c^2(t+4)},
\]

whence

\[
\mathbb{E}[||w_{t+1} - w_{t+1}^*||^2] \leq \frac{4M^2}{c^2(t+4)}.
\]

Using a standard supermartingale argument we can also prove that

\[
w_t - w_t^* \to 0 \text{ almost surely.}
\]

The proof is well-known in stochastic optimization hence omitted (or see [39, Theorem 5] for details).
C  Full experimental results

In this section we provide more experimental details and results that are deferred from the main paper.

C.1  Experimental setup: CIFAR-10, and Fashion MNIST datasets

In order to create a non-i.i.d. dataset, we follow a similar procedure as in [1]: first we sort all data points according to their classes. Then, they are split into 500 shards, and each user is randomly assigned 5 shards of data. By considering 100 users, this procedure guarantees that no user receives data from more than 5 classes and the data distribution of each user is different from each other. The local datasets are balanced—all users have the same amount of training samples. The local data is split into train, validation, and test sets with percentage of 80%, 10%, and 10%, respectively. In this way, each user has 400 data points for training, 50 for test, and 50 for validation. We use a simple 2-layer CNN model on CIFAR-10 dataset. To update the local models at each user using its local data, we apply stochastic gradient descent (SGD) with local batch size \( b = 10 \), local epoch \( k = 1 \), and local learning rate \( \eta = 0.01 \), or \( b = 400 \), \( k = 1 \), and \( \eta = 0.1 \). To model the fact that not all users may participate in each communication round, we define parameter \( p \) to control the fraction of participating users: \( p = 0.1 \) is the default setting which means that only 10% of users participate in each communication round.

C.2  Experimental setup: Adult dataset

Following the setting in AFL [5], we split the dataset into two non-overlapping domains based on the education attribute—phd domain and non-phd domain. The resulting FL setting consists of two users each of which has data from one of the two domains. Further, data is pre-processed as in [6] to have 99 binary features. We use a logistic regression model for all FL algorithms mentioned in the main paper. Local data is split into train, validation, and test sets with percentage of 80%, 10%, and 10%, respectively. In each round, both users participate and the server aggregates their losses and gradients (or weights). Different algorithms have their unique hyper-parameters mentioned throughout the places they appear. Joint hyper-parameters that are shared by all algorithms include: total communication rounds \( = 500 \), local batch size \( b = 10 \), local epoch \( k = 1 \), local learning rate \( \eta = 0.01 \), and local optimizer being SGD without momentum, unless otherwise stated. One important note is that the phd domain has only 413 samples while the non-phd domain has 32, 148 samples, so the split is very unbalanced while training only on the phd domain yields inferior performance on all domains due to the insufficient sample size.

C.3  Hyper-parameters

We evaluated the performance of different algorithms with a wide range of hyper-parameters. Table 1 summarizes these hyper-parameters for each algorithm.

| Name       | Parameters                                                                 |
|------------|-----------------------------------------------------------------------------|
| AFL        | \( \gamma \in \{0.01, 0.1, 0.2, 0.5\}, \gamma \in \{0.01, 0.1\} \)        |
| q-FedAvg   | \( q \in \{0.001, 0.01, 0.1, 0.5, 1, 2, 5, 10\}, L \in \{0.1, 1, 10\} \)  |
| FedMGDA+   | \( \eta \in \{0.5, 1, 1.5, 2\}, \text{and} \text{ } \text{Decay} \in \{0, 10, 40, 100, 0.1, 0.5\} \) |
| FedAvg-n   | \( \eta \in \{0.5, 1, 1.5, 2\}, \text{and} \text{ } \text{Decay} \in \{0, 10, 40, 100, 0.1, 0.5\} \) |
| FedProx    | \( \mu \in \{0.001, 0.01, 0.1, 0.5, 1, 10\} \)                          |
| MGDA-Prox  | \( \mu = 0.1, \eta \in \{0.5, 1, 1.5, 2\}, \text{and} \text{ } \text{Decay} \in \{0, 10, 40, 100, 0.1, 0.5\} \) |

Table 1: Hyperparameters that are used in our experiments. “Decay” represents the total amount of step size decay for \( \eta \) over the course of training.
C.4 Recovering FedAvg full results: results on Fashion-MNIST and CIFAR-10

Complementary to the results shown in Figure 1, Figure 5 and Figure 6 summarize similar results on the F-MNIST dataset, while Figure 7 depicts the training losses on CIFAR-10 dataset in log-scale.

Figure 5: Interpolation between FedAvg and FedMGDA (F-MNIST, iid setting). (Left) Average user accuracy. (Right) Uniformly averaged training loss. Results are averaged over 5 runs with different random seeds.

Figure 6: Interpolation between FedAvg and FedMGDA (F-MNIST, non-iid setting). (Left) Average user accuracy. (Right) Uniformly averaged training loss. Results are averaged over 5 runs with different random seeds.

Figure 7: Interpolation between FedAvg and FedMGDA (CIFAR-10). Both figures plot the uniformly averaged training loss in log-scale. (Left) non-iid setting. (Right) iid setting. Results are averaged over 5 runs with different random seeds.
C.5 Robustness full results: bias attack on Adult dataset

Table 2 shows the full results of the experiment presented in Figure 2 (Left).

| Name       | Bias | Uniform | PhD    | Non-PhD |
|------------|------|---------|--------|---------|
| AFL        | 0    | 83.26 ± 0.01 | 77.90 ± 0.00 | 83.32 ± 0.01 |
| AFL        | 0.01 | 83.28 ± 0.03 | 76.58 ± 0.27 | 83.36 ± 0.03 |
| AFL        | 0.1  | 82.30 ± 0.04 | 74.59 ± 0.00 | 82.39 ± 0.04 |
| AFL        | 1    | 81.86 ± 0.05 | 74.25 ± 0.57 | 81.94 ± 0.05 |
| q-FedAvg, q = 5 | 0    | 83.26 ± 0.18 | 78.60 ± 0.61 | 83.33 ± 0.19 |
| q-FedAvg, q = 5 | 1000 | 83.34 ± 0.04 | 76.57 ± 0.44 | 83.41 ± 0.04 |
| q-FedAvg, q = 5 | 5000 | 81.19 ± 0.03 | 74.14 ± 0.41 | 81.27 ± 0.03 |
| q-FedAvg, q = 5 | 10000 | 81.07 ± 0.03 | 73.48 ± 0.78 | 81.16 ± 0.02 |
| q-FedAvg, q = 2 | 0    | 83.30 ± 0.09 | 76.46 ± 0.56 | 83.38 ± 0.09 |
| q-FedAvg, q = 2 | 1000 | 83.33 ± 0.04 | 76.24 ± 0.00 | 83.41 ± 0.04 |
| q-FedAvg, q = 2 | 5000 | 83.11 ± 0.03 | 75.69 ± 0.00 | 83.20 ± 0.03 |
| q-FedAvg, q = 2 | 10000 | 82.50 ± 0.07 | 75.69 ± 0.00 | 82.58 ± 0.07 |
| q-FedAvg, q = 0.1 | 0    | 83.44 ± 0.06 | 76.46 ± 0.56 | 83.52 ± 0.07 |
| q-FedAvg, q = 0.1 | 1000 | 83.34 ± 0.03 | 76.35 ± 0.41 | 83.42 ± 0.02 |
| q-FedAvg, q = 0.1 | 5000 | 83.35 ± 0.03 | 76.57 ± 0.66 | 83.42 ± 0.03 |
| q-FedAvg, q = 0.1 | 10000 | 83.36 ± 0.05 | 76.80 ± 0.49 | 83.43 ± 0.05 |
| FedMGDA+    | Arbitrary | 83.24 ± 0.02 | 76.58 ± 0.27 | 83.32 ± 0.02 |
| Baseline_PhD |      | 81.05 ± 0.05 | 72.82 ± 0.95 | 81.14 ± 0.05 |

Table 2: Test accuracy of SOTA algorithms on Adult dataset with various scales of adversarial bias added to the domain loss of PhD; and compared to the baseline of training only on the PhD domain. The scale of bias for AFL is different from q-FedAvg since AFL uses averaged loss while q-FedAvg uses (non-averaged) total loss. The algorithms are run for 500 rounds, and the reported results are averaged across 5 runs with different random seeds.

The hyper-parameter setting for Figure 2: **Left**: step sizes $\gamma = 0.5, \gamma_w = 0.01$, and bias= 1 for AFL; $q = 5$, and $\gamma_w = 10000$ for q-FedAvg; $\eta = 1$, decay=1/3, and arbitrary bias for FedMGDA+. **Right**: $\mu = 0.01$ for FedProx; $q_1 = 0.5$ and $q_2 = 2.0$ for q-FedAvg; $\eta = 1$ and decay=1/10 for FedMGDA+ and FedAvg-n. The simulations are run with 20% user participation ($p = 0.2$) in each round which reduces the effectiveness of the adversary since it needs to participate in a bigger pool of users in comparison to our default setting $p = 0.1$. 


C.6 Fairness full results: first experiment

Tables C.3, C.4, C.5 and C.6 report the full results of the experiment presented in Figure 3 for different batch sizes and fractions of user participation.

| Algorithm  | Average (%) | Std. (%) | Worst 5% (%) | Best 5% (%) |
|------------|-------------|----------|--------------|-------------|
| Name       | η           | decay    |              |             |
| FedMGDA    | 1.0         | 0        | 67.59 ± 0.65 | 21.03 ± 2.40 | 22.95 ± 7.27 | 90.50 ± 0.87 |
|            | 1.0         | 1/10     | 69.06 ± 1.08 | 14.10 ± 1.61 | 44.38 ± 5.90 | 87.55 ± 0.84 |
|            | 1.5         | 1/10     | 69.87 ± 0.87 | 14.33 ± 0.61 | 42.42 ± 3.61 | 87.05 ± 0.95 |
|            | 1.0         | 1/40     | 71.15 ± 0.62 | 13.74 ± 0.49 | 44.48 ± 1.64 | 88.53 ± 0.85 |
| FedMGDA+   | 1.0         | 1/40     | 68.68 ± 1.25 | 17.23 ± 1.60 | 34.40 ± 6.23 | 88.07 ± 0.04 |
| FedMGDA+   | 1.5         | 1/40     | 71.05 ± 0.82 | 13.53 ± 0.77 | **46.50 ± 2.96** | 88.53 ± 0.85 |

| Name       | η           | decay    |              |             |
|------------|-------------|----------|--------------|-------------|
| MGDA-Prox  | 1.0         | 0        | 66.98 ± 1.52 | 15.46 ± 3.15 | 39.42 ± 10.35 | 87.60 ± 2.18 |
| MGDA-Prox  | 1.0         | 1/10     | 70.39 ± 0.96 | 13.70 ± 1.08 | 46.43 ± 2.17 | 87.50 ± 0.87 |
| MGDA-Prox  | 1.5         | 1/10     | 69.45 ± 0.77 | 14.98 ± 1.61 | 40.42 ± 5.88 | 87.05 ± 1.00 |
| MGDA-Prox  | 1.0         | 1/40     | 69.01 ± 0.51 | 16.24 ± 0.74 | 36.92 ± 4.12 | 88.53 ± 0.85 |
| MGDA-Prox  | 1.5         | 1/40     | 69.53 ± 0.70 | 15.90 ± 1.79 | 36.43 ± 7.42 | 87.55 ± 2.14 |

| Name       | η           | decay    |              |             |
|------------|-------------|----------|--------------|-------------|
| FedAvg     | 1.0         | 0        | 70.11 ± 1.27 | 13.63 ± 0.81 | 45.45 ± 2.21 | 88.00 ± 0.00 |
| FedAvg-n   | 1.0         | 1/10     | 67.69 ± 1.15 | 16.97 ± 2.33 | 37.98 ± 6.61 | 89.55 ± 2.61 |
|            | 1.0         | 1/10     | 69.66 ± 1.22 | 15.11 ± 1.14 | 40.42 ± 1.71 | 88.55 ± 0.84 |
|            | 1.5         | 1/10     | 70.62 ± 0.82 | 14.19 ± 0.49 | 43.48 ± 2.17 | 89.03 ± 1.03 |
|            | 1.0         | 1/40     | 70.31 ± 0.29 | 14.97 ± 0.96 | 42.48 ± 2.56 | 88.55 ± 2.15 |
|            | 1.5         | 1/40     | 70.47 ± 0.70 | 13.88 ± 0.96 | 44.95 ± 4.07 | 88.03 ± 0.04 |

| Name       | μ            |              |              |              |
|------------|--------------|--------------|--------------|--------------|
| FedProx    | 0.01         | 70.77 ± 0.70 | 13.12 ± 0.47 | 46.43 ± 2.95 | 88.50 ± 0.87 |
| FedProx    | 0.1          | 70.69 ± 0.58 | 13.42 ± 0.43 | 45.42 ± 2.14 | 87.55 ± 1.64 |
| FedProx    | 0.5          | 68.89 ± 0.83 | 14.10 ± 1.08 | 43.95 ± 4.52 | 88.00 ± 0.00 |

| Name       | q | L |              |              |              |              |
|------------|---|---|--------------|--------------|--------------|--------------|
| q-FedAvg   | 0.1| 0.1| 70.40 ± 0.41 | **12.43 ± 0.24** | 46.48 ± 2.14 | 87.50 ± 0.87 |
| q-FedAvg   | 0.5| 0.1| 70.58 ± 0.73 | 13.60 ± 0.47 | **46.50 ± 2.96** | 88.05 ± 1.38 |
| q-FedAvg   | 1.0| 0.1| 70.27 ± 0.61 | 13.31 ± 0.46 | 45.95 ± 1.38 | 87.55 ± 0.90 |
| q-FedAvg   | 0.1| 1.0| 70.95 ± 0.83 | 12.70 ± 0.74 | 46.45 ± 4.07 | 87.00 ± 1.00 |
| q-FedAvg   | 0.5| 1.0| 70.98 ± 0.52 | 12.96 ± 0.63 | 45.95 ± 1.45 | 88.00 ± 0.00 |
| q-FedAvg   | 1.0| 1.0| 69.98 ± 0.67 | 13.15 ± 1.12 | 45.95 ± 2.49 | 87.53 ± 0.82 |

Table 3: Test accuracy of users on CIFAR-10 with local batch size $b = 10$, fraction of users $p = 0.1$, local learning rate $\eta = 0.01$, total communication rounds 2000. The reported statistics are averaged across 4 runs with different random seeds.
| Algorithm       | Average (%) | Std. (%) | Worst 5% (%) | Best 5% (%) |
|-----------------|-------------|---------|--------------|-------------|
| Name            | η           | decay   |              |             |
| FedMGDA+        | 1.0 0       |         |              |             |
|                 | 66.50 ± 1.77| 23.22 ± 1.20| 19.48 ± 4.54| 91.53 ± 2.14|
| FedMGDA+        | 1.0 1/10    |         |              |             |
|                 | 70.64 ± 0.35| 16.23 ± 0.84| 35.95 ± 4.66| 87.55 ± 0.90|
| FedMGDA+        | 1.5 1/10    |         |              |             |
|                 | 69.29 ± 0.88| 13.52 ± 0.69| 44.45 ± 1.64| 86.55 ± 0.84|
| FedMGDA+        | 1.0 1/40    |         |              |             |
|                 | 68.47 ± 0.65| 18.07 ± 2.84| 32.95 ± 9.48| 88.55 ± 2.15|
| FedMGDA+        | 1.5 1/40    |         |              |             |
|                 | 68.76 ± 0.54| 17.21 ± 1.28| 34.43 ± 6.26| 87.53 ± 0.88|
| Name            | η           | decay   |              |             |
| MGDA-Prox       | 1.0 0       |         |              |             |
|                 | 70.06 ± 0.67| 13.69 ± 0.46| 42.43 ± 2.99| 87.03 ± 0.98|
| MGDA-Prox       | 1.0 1/10    |         |              |             |
|                 | 68.41 ± 0.88| 16.30 ± 1.84| 37.98 ± 3.15| 89.50 ± 1.66|
| MGDA-Prox       | 1.5 1/10    |         |              |             |
|                 | 68.19 ± 0.96| 19.25 ± 2.57| 28.90 ± 7.02| 88.55 ± 0.84|
| MGDA-Prox       | 1.0 1/40    |         |              |             |
|                 | 68.92 ± 0.78| 14.64 ± 0.59| 41.42 ± 2.98| 87.55 ± 1.61|
| MGDA-Prox       | 1.5 1/40    |         |              |             |
|                 | 68.87 ± 0.60| 17.47 ± 1.96| 31.48 ± 7.49| 89.00 ± 2.24|
| Name            | η           | decay   |              |             |
| FedAvg-n        | 1.0 0       |         |              |             |
|                 | 69.05 ± 0.94| 14.14 ± 1.53| 39.50 ± 6.22| 87.50 ± 0.87|
| FedAvg-n        | 1.0 1/10    |         |              |             |
|                 | 70.52 ± 1.18| 15.22 ± 1.74| 39.48 ± 7.90| 88.03 ± 0.04|
| FedAvg-n        | 1.5 1/10    |         |              |             |
|                 | 69.27 ± 0.97| 14.42 ± 0.85| 43.95 ± 3.76| 89.00 ± 1.00|
| FedAvg-n        | 1.0 1/40    |         |              |             |
|                 | 69.34 ± 1.75| 15.64 ± 2.90| 38.45 ± 8.50| 86.53 ± 0.85|
| FedAvg-n        | 1.5 1/40    |         |              |             |
|                 | 69.87 ± 0.59| 14.13 ± 0.14| 43.95 ± 1.42| 86.57 ± 1.64|
| Name            | μ           |         |              |             |
| FedProx         | 0.01        |         |              |             |
|                 | 69.74 ± 0.84| 13.26 ± 0.44| 47.90 ± 1.45| 87.50 ± 0.87|
| FedProx         | 0.1         |         |              |             |
|                 | 70.06 ± 0.67| 13.69 ± 0.46| 42.43 ± 2.99| 87.03 ± 0.98|
| FedProx         | 0.5         |         |              |             |
|                 | 69.64 ± 0.74| 13.55 ± 0.50| 44.90 ± 1.67| 88.00 ± 0.00|
| Name            | q           | L        |              |             |
| q-FedAvg        | 0.1 0.1     |         |              |             |
|                 | 70.21 ± 0.71| 13.23 ± 0.42| 46.98 ± 0.98| 87.03 ± 0.98|
| q-FedAvg        | 0.5 0.1     |         |              |             |
|                 | 70.34 ± 0.71| 13.05 ± 0.27| 47.43 ± 1.64| 88.00 ± 0.00|
| q-FedAvg        | 1.0 0.1     |         |              |             |
|                 | 70.19 ± 0.79| 12.79 ± 0.23| 48.42 ± 0.91| 88.00 ± 0.00|
| q-FedAvg        | 0.1 1.0     |         |              |             |
|                 | 70.18 ± 0.53| 13.04 ± 0.38| 47.45 ± 0.84| 88.05 ± 1.38|
| q-FedAvg        | 0.5 1.0     |         |              |             |
|                 | 70.30 ± 0.70| 13.28 ± 1.03| 45.45 ± 1.71| 87.55 ± 0.84|
| q-FedAvg        | 1.0 1.0     |         |              |             |
|                 | 69.39 ± 0.35| 13.75 ± 0.34| 43.98 ± 1.41| 87.08 ± 0.98|

Table 4: Test accuracy of users on CIFAR-10 with local batch size $b = 10$, fraction of users $\rho = 0.2$, local learning rate $\eta = 0.01$, total communication rounds 2000. The reported statistics are averaged across 4 runs with different random seeds.
Table 5: Test accuracy of users on CIFAR-10 with local batch size $b = 400$, fraction of users $p = 0.1$, local learning rate $\eta = 0.1$, total communication rounds 3000. The reported statistics are averaged across 4 runs with different random seeds.
| Algorithm      | Average (%) | Std. (%) | Worst 5% (%) | Best 5% (%) |
|---------------|-------------|----------|--------------|-------------|
| FedMGDA       | 65.18 ± 5.41 | 16.52 ± 4.70 | 33.95 ± 17.40 | 84.55 ± 1.67 |
| FedMGDA+ 1.0  | 6.50 ± 1.66  | 24.40 ± 3.10 | 0.00 ± 0.00  | 75.00 ± 43.30 |
| FedMGDA+ 1.0  | 70.14 ± 0.72 | 13.53 ± 1.28 | 44.93 ± 2.23 | 87.53 ± 0.88 |
| FedMGDA+ 1.5  | 69.57 ± 0.74 | 13.39 ± 1.09 | 46.42 ± 2.95 | 87.03 ± 1.03 |
| FedMGDA+ 1.0  | 68.09 ± 0.43 | 14.59 ± 0.64 | 41.98 ± 2.47 | 87.00 ± 1.00 |
| FedMGDA+ 1.5  | 69.29 ± 1.00 | 12.95 ± 0.44 | 46.48 ± 2.14 | 86.53 ± 0.85 |

| Name          | η   | decay |
|---------------|-----|-------|
| MGDA-Prox     | 1.0 | 0     | 6.75 ± 1.09 | 24.99 ± 1.96 | 0.00 ± 0.00 | 76.25 ± 41.14 |
| MGDA-Prox     | 1.0 | 1/10 | **70.73 ± 0.51** | 12.62 ± 0.73 | 48.00 ± 2.45 | 88.53 ± 0.85 |
| MGDA-Prox     | 1.5 | 1/10 | 68.79 ± 0.41 | 13.99 ± 1.03 | 42.95 ± 3.03 | 87.00 ± 1.00 |
| MGDA-Prox     | 1.0 | 1/40 | 68.38 ± 0.86 | 13.53 ± 0.69 | 43.98 ± 0.04 | 86.05 ± 1.42 |
| MGDA-Prox     | 1.5 | 1/40 | 70.14 ± 1.30 | **11.94 ± 0.79** | **48.98 ± 2.25** | 87.53 ± 0.88 |

| Name          | η   | decay |
|---------------|-----|-------|
| FedAvg        | 67.84 ± 3.24 | 13.83 ± 0.79 | 42.98 ± 4.62 | 86.00 ± 2.45 |
| FedAvg-n 1.0 | 66.11 ± 0.72 | 14.04 ± 0.70 | 40.93 ± 2.27 | 84.53 ± 1.62 |
| FedAvg-n 1.0 | 69.03 ± 1.02 | 14.17 ± 1.13 | 42.45 ± 5.14 | 87.03 ± 1.03 |
| FedAvg-n 1.5 | 64.08 ± 1.24 | 13.74 ± 0.43 | 40.38 ± 2.09 | 82.05 ± 1.34 |
| FedAvg-n 1.0 | 68.89 ± 0.81 | 13.43 ± 0.41 | 43.98 ± 1.38 | 85.57 ± 0.85 |
| FedAvg-n 1.5 | 65.66 ± 0.98 | 14.18 ± 0.62 | 40.90 ± 2.21 | 84.00 ± 0.00 |

| Name          | µ   |
|---------------|----|
| FedProx       | 69.14 ± 1.25 | 14.25 ± 1.13 | 41.95 ± 4.47 | 87.00 ± 1.00 |
| FedProx 0.1  | 70.15 ± 0.73 | 12.51 ± 1.44 | 47.90 ± 4.69 | 87.03 ± 1.75 |
| FedProx 0.5  | 69.44 ± 0.82 | 13.83 ± 1.84 | 42.38 ± 5.33 | 86.53 ± 0.85 |

| Name          | q | L |
|---------------|---|---|
| q-FedAvg 0.1 | 0.1 | 0.1 | 69.42 ± 1.10 | 14.34 ± 1.36 | 44.98 ± 4.35 | 87.55 ± 0.90 |
| q-FedAvg 0.5 | 0.1 | 0.1 | 69.73 ± 1.69 | 15.53 ± 4.18 | 36.45 ± 16.43 | 87.53 ± 0.88 |
| q-FedAvg 1.0 | 65.97 ± 3.01 | 17.29 ± 5.55 | 31.50 ± 16.02 | **90.00 ± 2.45** |
| q-FedAvg 0.1 | 68.20 ± 1.64 | 14.55 ± 3.59 | 40.45 ± 12.56 | 86.03 ± 1.38 |
| q-FedAvg 0.5 | 68.50 ± 2.62 | 16.98 ± 4.20 | 30.43 ± 16.90 | 88.05 ± 1.38 |
| q-FedAvg 1.0 | 65.89 ± 3.50 | 20.33 ± 4.07 | 24.92 ± 15.00 | 89.00 ± 1.73 |

Table 6: Test accuracy of users on CIFAR-10 with local batch size $b = 400$, fraction of users $p = 0.2$, local learning rate $\eta = 0.1$, total communication rounds 3000. The reported statistics are averaged across 4 runs with different random seeds.

### C.7 Fairness full results: second experiment

Figures 8 and 9 show the full results of the experiment presented in Figure 4 for different batch sizes.
Figure 8: The percentage of improved users in terms of training loss and the global test accuracy vs communication rounds on CIFAR-10 dataset with $p = 0.1$ and $b = 10$. Results are averaged across 4 runs with different random seeds.
Figure 9: The percentage of improved users in terms of training loss and the global test accuracy vs communication rounds on the CIFAR-10 dataset with $p = 0.1$ and $b = 400$. The results are averaged across 4 runs with different random seeds.