On the Mathematics of the Fraternal Birth Order Effect and the Genetics of Homosexuality

Tanya Khovanova

Abstract
Mathematicians have always been attracted to the field of genetics. The mathematical aspects of research on homosexuality are especially interesting. Certain studies show that male homosexuality may have a genetic component that is correlated with female fertility. Other studies show the existence of the fraternal birth order effect, that is, the correlation of homosexuality with the number of older brothers. This article is devoted to the mathematical aspects of how these two phenomena are interconnected. In particular, we show that the fraternal birth order effect implies a correlation between homosexuality and maternal fecundity. Vice versa, we show that the correlation between homosexuality and female fecundity implies the increase in the probability of the younger brothers being homosexual.

Keywords Fraternal birth order effect · Male homosexuality · Fecundity · Genetics · Sexual orientation

Introduction
According to the study by Blanchard and Bogaert (1996): 
"Each additional older brother increased the odds of [male] homosexuality by 34%" (see also Blanchard [2004], Bogaert [2006], Bogaert et al. [2018], and a recent survey by Blanchard [2018]). The current explanation is that carrying a boy to term changes their mother’s uterine environment. Male fetuses produce H–Y antigens which may be responsible for this environmental change for future fetuses.

The research into a genetic component of male gayness shows that there might be some genes in the X chromosome that influence male homosexuality. It also shows that the same genes might be responsible for increased fertility in females (see Ciani, Cermelli, & Zanzotto [2008] and Iemmola & Ciani [2009]).

In this article, we compare two mathematical models. In these mathematical models, we disregard girls for the sake of clarity and simplicity.

The first mathematical model of the Fraternal Birth Order Effect (FBOE), which we denote FBOE-model, assumes that each next-born son becomes homosexual with increased probability. This probability is independent of any other factor.

The second mathematical model of Female Fecundity (FF), which we denote FF-model, assumes that a son becomes homosexual with probability depending on the total number of children and nothing else.

We show mathematically how FBOE-model implies correlation with family size and FF-model implies correlation with birth order. That means these two models are mathematically intertwined.

We also propose the Brother Effect. Brothers share a lot of the same genes. It is not surprising that brothers are more probable to share traits. With respect to homosexuality, we call the correlation that homosexuals are more probable to have a homosexual brother than a non-homosexual the Brother Effect. The existence of genes that increase predisposition to homosexuality implies the Brother Effect. The connection between the FBOE-model and the Brother Effect is more complicated.

We also discuss how to separate FBOE and FF in the data.

The “Extreme Examples” section contains extreme mathematical examples that amplify the results of this article. The “FBOE-model and the family size” section shows how FBOE-model implies the correlation with family size. The “FF-model implies birth order correlation” section shows how FF-model implies the correlation with birth order. In the “Brothers” section, we discuss the connection between FBOE-model and the Brother Effect. In the “Separating Birth Order and Female Fertility” section, we discuss how to separate the effects.

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Fecundity” section, we discuss how to separate the birth order from the family size.

Extreme Examples

First, consider extreme theoretical examples. In the first two examples, suppose mothers only give birth to sons and only to one or two sons.

First Extreme Example

This is an extreme variation in FBOE-model. Suppose the first son has a zero probability of being gay (which means that first sons are never gay) and the second son has probability one of being gay (which means he is always gay). Then, all mothers of one son will have a straight son. All mothers with two sons will have one gay and one straight son. Homosexuals appear only in two-son families and never in one-son families. Therefore, FBOE-model implies the correlation with family size.

Second Extreme Example

This is an extreme variation in FF-model. Suppose mothers with one son have probability zero of having a gay son. Suppose mothers with two sons have two homosexual sons with probability one. The first born is sometimes gay and sometimes straight, but the second son is always gay. Hence, it is more probable that the second son is gay. Therefore, FF-model implies the correlation with birth order.

These extreme variations in the FBOE-model and FF-model show that these two models are intertwined.

The next two sections explain this in more detail.

FBOE-Model and the Family Size

Let us build a model with variables for numbers that correspond to the FBOE-model. In this simple model, we assume that the probability of a child being gay depends only on birth order and nothing else.

FBOE-Model

Let us assume that mothers have either one or two boys. Let \( a \) be the probability of a woman having one boy, and correspondingly, \( 1 - a \) of having two boys. Suppose \( N \) is the total number of women in consideration. Suppose \( p_1 \) is the probability that the first boy is homosexual and \( p_2 \) is the probability that the second boy is homosexual. The fraternal birth order effect means that \( p_2 > p_1 \).

Now, we produce the results of such a model.

Let us first estimate the total number of boys \( T \):

\[
T = aN + 2(1-a)N = (2-a)N.
\]

The number of homosexuals in the one-son families is expected to be \( ap_1N \). The expected number of homosexual first-born sons in two-son families is \((1-a)p_1N\) and the expected number of homosexual second-born sons is \((1-a)p_2N\). The total expected number of homosexuals \( H \) is the sum:

\[
H = p_1N + (1-a)p_2N.
\]

The probability that a randomly chosen boy is a homosexual is

\[
\frac{H}{T} = \frac{p_1 + (1-a)p_2}{2-a}.
\]

Let us see what happens with fecundity. Suppose we pick a mother randomly, then pick her son randomly. If there is only one son, then he is the one we have to pick. The probability that we pick a gay son, given that we picked the mother with one child is \( p_1 \). The probability that we pick a gay son, given that we picked the mother of two children is \((p_1 + p_2)/2 > p_1\). This difference is the source of the correlation with fecundity.

To calculate this properly, we need to choose a boy randomly and find the average fertility of the mother. The formula is given by the following equation:

\[
\frac{\# \text{ Number of single sons} + 2 \cdot \# \text{ Number of non-single sons}}{\# \text{ Number of sons}}.
\]

First, we calculate average maternal fertility per boy:

For a randomly chosen boy (including both homosexual and non-homosexual boys), there are \(aN \) mothers of one son and \((1-a)N \) mothers of two sons. Hence, a mother of a randomly chosen boy has on average

\[
\frac{aN + 2 \cdot (1-a)N}{(2-a)N}
\]

children, which is equal to

\[
a + \frac{4 - 4a}{2-a} = 2 - \frac{a}{2-a}.
\]

Let us see what happens with homosexual boys. We have \( ap_1N \) expected gay boys from one-son families and \((1-a)(p_1 + p_2)N \) expected gay boys from two-son families. Now, we plug this into Eq. (1) where we replace a randomly chosen boy with a gay boy to get
FF-Model Implies Birth Order Correlation

Let us build a mathematical model with variables instead of fixed numbers that correspond to the correlation of homosexuality with female fecundity. In this simple model, we assume that the probability of a child being gay depends only on the family size and nothing else.

**FF-Model**

Let us assume that mothers have either one or two boys. Let $a$ be the probability of a woman having one boy, and correspondingly, $1 - a$ of having two boys. Suppose $N$ is the total number of women in consideration. Suppose $q_1$ is the probability that a boy in a one-son family is homosexual and $q_2$ is the probability that a boy in a two-son family is homosexual. We assume that $q_2 > q_1$ to support the correlation of female fecundity with homosexuality.

Here are the results of such a model. In our notation, we use $f$ for first sons and $s$ for second sons.

Let us see what happens with birth order. We start with first sons. The total number of first sons $T_f$ is $N$:

$$T_f = N.$$  

The number of homosexuals in one-son families is expected to be $aq_1N$. The number of homosexual first sons in two-son families is expected to be $(1 - a)q_2N$. The total number of first sons that are homosexual, $H_f$, is expected to be

$$H_f = aq_1N + (1 - a)q_2N.$$  

The probability that the first-born is homosexual is

$$
\frac{H_f}{T_f} = aq_1 + (1 - a)q_2.
$$

Now, we do the same for the second-born sons. The expected total number of them $T_s$ is

$$T_s = (1 - a)N.$$  

The expected number of homosexuals among them $H_s$ is

$$H_s = (1 - a)q_2N.$$  

The probability that the second-born son is homosexual is

$$
\frac{H_s}{T_s} = q_2.
$$

The final mathematical step needs to show that the probability that the first born is homosexual is less than the probability that the second born is homosexual. It follows from the fact that $q_1 < q_2$. Indeed

$$
\frac{H_f}{T_f} = aq_1 + (1 - a)q_2 < aq_2 + (1 - a)q_2 = q_2 = \frac{H_s}{T_s}.
$$
If we denote by \( c \) the ratio \( q_2 / q_1 \), then the ratio of increase in the proportion of the homosexuals in later children, that is, \( \frac{H_2}{H_1} \) divided by \( \frac{H_1}{H_1} \) is

\[
\frac{q_2}{aq_1 + (1 - a)q_2} = \frac{c}{a + (1 - a)c}.
\]

It is useful to note that if \( c = 1 \), then there is no correlation with the birth order: the first-born sons and second-born sons are homosexuals with the same probability.

The impact of the FF-model on the birth order is stronger if we consider larger families. Suppose \( q_1 < q_2 < q_3 < \ldots \) are the probabilities of sons being homosexual in families of size 1, 2, and so on, respectively. Then, the average probability \( y_i \) of a child number \( i \) being gay depends on the distribution of family sizes. Suppose the number of families of size \( m \) is \( N_m \), then we can calculate \( y_i \) as

\[
y_i = \frac{q_iN_i + q_{i+1}N_{i+1} + q_{i+2}N_{i+2} + \cdots}{N_i + N_{i+1} + N_{i+2} + \cdots}.
\]

We can show that \( y_j > y_i \) when \( j > i \). Let us denote \( Q_1 = q_iN_i + q_{i+1}N_{i+1} + q_{i+2}N_{i+2} + \cdots + q_{j-1}N_{j-1} \) and \( Q_2 = q_jN_j + q_{j+1}N_{j+1} + q_{j+2}N_{j+2} + \cdots \). Further, let us denote \( M_1 = N_i + N_{i+1} + N_{i+2} + \cdots + N_{j-1} \) and \( M_2 = N_j + N_{j+1} + N_{j+2} + \cdots \). Then, \( y_i = \frac{Q_1}{M_1} \) and \( y_j = \frac{Q_2}{M_2} \).

The important observation is that

\[
\frac{Q_1}{M_1} \leq q_{i-1} < q_i < \frac{Q_2}{M_2}.
\]

Therefore,

\[
Q_1M_2 < Q_2M_1.
\]

It follows that

\[
Q_1M_2 + Q_2M_2 < Q_2M_1 + Q_2M_2.
\]

This implies

\[
y_i = \frac{Q_1 + Q_2}{M_1 + M_2} < \frac{Q_2}{M_2} = y_j.
\]

The results show that FF implies correlation with the birth order. The fraternal birth order effect was shown only for brothers and not for sisters. This means that the FBOE as a whole is not threatened by my examples. We will describe in “Separating birth order and female fecundity” section how to separate birth order and family size mathematically in the data.

**Brothers**

Siblings share a lot of genetic material. Not surprisingly, they have a lot of common traits. If a trait is genetic, then the probability that a sibling has it is higher than the probability that a randomly chosen person has it. Very often, the fact that siblings share traits with higher probability than random people share traits serves as a confirmation that the trait is genetic. That means, the existence of a homosexual gene would imply the higher probability that a gay person has a gay brother than the probability that a randomly chosen person has a gay brother. We call this correlation the Brother Effect.

Is there a mathematical way to connect the birth order with the Brother Effect? The answer: it is complicated.

Let us look at how the fraternal birth order effect influences the probability that a gay boy has a gay brother. The probability that a gay person has a gay brother depends on the number of boys in the family. If a boy does not have brothers, he cannot have a gay brother. If a boy has a million brothers, then with extremely high probability at least one of them will be gay.

Here are two extreme mathematical examples where we assume that mothers have only one or three sons.

**Third Extreme Example**

This is an extreme variation in the FBOE-model. Suppose the first son has zero probability of being gay and the second and third sons have probability one of being gay. That is, \( p_1 = 0 \) and \( p_2 = p_3 = 1 \). All gay boys in this model have a gay brother, while a randomly chosen boy sometimes has one and sometimes does not. With these particular probabilities, the FBOE-model implies the Brother Effect.

**Fourth Extreme Example**

This is an extreme variation in FBOE-model. Suppose the first and second sons have zero probability of being gay and the third son has probability one of being gay. That is, \( p_1 = 0 \) and \( p_2 = p_3 = 1 \). No gay boy in this model has a gay brother, while some randomly chosen boys have one. With these particular probabilities, FBOE-model contradicts the Brother Effect.

It follows that depending on the actual numbers FBOE might or might not imply the Brother Effect.
Separating Birth Order and Female Fecundity

Our simplistic models in the “FBOE-model and the family size” and “FF-model implies birth order correlation” sections showed that FBOE-model and FF-model imply each other. That means birth order and female fecundity are intertwined in the data. It is important to separate these two different models. To do it, we need to fix some variables.

Method 1

To show how the birth order works independently of female fecundity, we need to fix the family size. Suppose we consider only families of size 2. Then, the fertility does not play a role. In this case, according to the fraternal birth order effect, the second son is gay with higher probability than the first son. The corresponding probabilities derived from real data should confirm the FBOE-model without interference of the FF-model.

Method 2

To show how the female fecundity works independently of the fraternal birth order effect, we need to consider only the first sons. Then, the FBOE-model does not play a role. In this case, according to the FF-model, the first son in a larger family is gay with higher probability than the first son in a smaller family. The corresponding probabilities derived from real data should confirm the FF-model without the FBOE-model.

Consider the theoretical discussion of families of size one and two in “FBOE-model and the family size” and “FF-model implies birth order correlation” sections. Here is the joint mathematical model, which we call FBOE-FF-model.

FBOE-FF-Model

Let us consider only the case of women who have one or two boys. Let \( a \) be the probability of a woman having one boy, and correspondingly, \( 1 - a \) of having two boys. Suppose \( N \) is the total number of women in consideration. Suppose \( p_{11} \) is the probability that the first boy in a one-son family is homosexual, \( p_{12} \) is the probability that the first boy in a two-son family is homosexual, and \( p_{22} \) is the probability that the second boy in a two-son family is homosexual. The female fecundity means \( p_{12} > p_{11} \). The ratio \( \frac{p_{12}}{p_{11}} \) shows a contribution of FF independent of FBOE. The fraternal birth order effect means \( p_{22} > p_{12} \). The ratio \( \frac{p_{22}}{p_{12}} \) shows the contribution of FBOE independent of FF.

Conclusion

We showed mathematically that FBOE and FF are interrelated in the data: FBOE implies FF, and FF implies FBOE. We proposed data analysis that will separate these two contributions. To show the impact of FBOE without interference of FF, one has to look at families of the same size. To show the impact of FF without interference of FBOE, one has to look at the first-born sons.

Compliance with Ethical Standards

Conflict of interest The author declare that she has no conflict of interest.

Ethical Approval The research is purely mathematical and didn’t involve any animal or human subjects.

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