The impact of $a_0^0(980) - f_0(980)$ mixing on the localized $CP$ violations of the $B^- \to K^- \pi^+ \pi^-$ decay

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Abstract

In the framework of the QCD factorization approach, we study the localized $CP$ violations of the $B^- \to K^- \pi^+ \pi^-$ decay with and without $a_0^0(980) - f_0(980)$ mixing mechanism, respectively, and find that the localized $CP$ violation can be enhanced by this mixing effect when the mass of the $\pi^+\pi^-$ pair is in the vicinity of the $f_0(980)$ resonance. The corresponding theoretical prediction results are $A_{CP}(B^- \to Kf_0 \to K^- \pi^+ \pi^-) = [0.24, 0.36]$ and $A_{CP}(B^- \to K^-f_0(a_0) \to K^- \pi^+ \pi^-) = [0.33, 0.52]$, respectively. Meanwhile, we also calculate the branching fraction of the $B^- \to K^- f_0(980) \to K^- \pi^+ \pi^-$ decay, which is consistent with the experimental results. We suggest that $a_0^0(980) - f_0(980)$ mixing mechanism should be considered when studying the $CP$ violation of the $B$ or $D$ mesons decays theoretically and experimentally.

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I. INTRODUCTION

$CP$ violation plays an important role for the test of the Standard Model (SM) and extractions of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The processes of nonleptonic decays of $B$ mesons provide us with opportunities for exploring $CP$ violation. In SM, $CP$ violation depends on the weak complex phase in the CKM matrix \cite{1,2}. The main uncertainties of $CP$ violation come from the insufficient understanding of strong interaction associated with the nonperturbative QCD. In the past few years, a large amount of experimental data have been collected for $CP$ violation of two body decays of the $B$ meson by $B$ factories, BABAR, Belle, and LHC experiments. The large $CP$ violations have been found by the LHCb Collaboration in the three-body decay channels of $B^\pm \rightarrow \pi^\pm \pi^+\pi^-$ and $B^\pm \rightarrow K^\pm \pi^+\pi^-$. Hence, the exploration of the theoretical mechanism for $CP$ violation becomes interesting in the two- and three-body decays of the $B$ meson.

The nature of the light scalar mesons has attracted much attention for decades since its discovery \cite{5–11}. Because of sharing the same quantum numbers, light scalar mesons play an important role to understand the QCD vacuum. The $a_0^0(980) - f_0(980)$ mixing mechanism has been a hot research topic because of its potential to help understand the structure of scalar mesons. In late 1970s, the $a_0^0(980) - f_0(980)$ mixing effect was first suggested theoretically \cite{12}. $a_0^0(980)$ and $f_0(980)$ have the same spin parity quantum numbers but different isospins. Because of the isospin breaking effect, when they decay into $K\bar{K}$ there exists a difference of 8 MeV between the charged and neutral kaon thresholds. Up to now, $a_0^0(980)$ and $f_0(980)$ mixing has been studied extensively in various processes and with respect to its different aspects \cite{13–33}. The signal of this effect was observed for the first time by the BESIII Collaboration in the $J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi \eta \pi^0$ and $\chi_{c1} \rightarrow a_0^0(980)\pi^0 \rightarrow f_0(980)\pi^0 \rightarrow \pi^+\pi^-\pi^0$ decays \cite{34}. Inspired by the fact that $\rho - \omega$ mixing (also due to isospin breaking effect) can induce large $CP$ violations when the invariant mass of the $\pi\pi$ pair is in the $\rho - \omega$ mixing effective area \cite{35–37}, we intend to study the $a_0^0(980) - f_0(980)$ mixing effect on the localized $CP$ violations in three-body decays of the $B$ meson.

In this paper, we will investigate the localized $CP$ violation by $a_0^0(980) - f_0(980)$ mixing and the branching fraction of the $B^- \rightarrow K f_0 \rightarrow K^-\pi^+\pi^-$ decay in the QCDF approach. The remainder of this paper is organized as follows. In Sect. II, we present the formalism for $B$ decays in the QCDF approach. In Sect. III, we present the $a_0^0(980) - f_0(980)$ mixing mechanism, calculations of the localized $CP$ violation and the branching fraction of the $B^- \rightarrow K f_0 \rightarrow K^-\pi^+\pi^-$ decay. The numerical results are given in Sect. IV and we summarize and discuss our work in Sect V.
II. B DECA YS IN THE QCD FACTORIZA TION APPROACH

In the framework of the QCD factorization approach \cite{38, 39}, one can obtain the matrix element $B$ decaying to two mesons $M_1$ and $M_2$ by matching the effective weak Hamiltonian onto a transition operator, which is summarized as follow ($\lambda^D_p = V_{pd}^* V_{pD}$ with $D = d$ or $s$)

$$\langle M_1 M_2 | H_{\text{eff}} | B \rangle = \sum_{p=u,c} \lambda^D_p \langle M_1 M_2 | T^p_A + T^p_B | B \rangle,$$

where $T^p_A$ and $T^p_B$ describe the contributions from non-annihilation and annihilation topology amplitudes, respectively, which can be expressed in terms of the parameters $a^p_i$ and $b^p_i$, respectively, both of which are defined in detail in Ref. \cite{38}.

Concretely, $T^p_A$ contains the contributions from naive factorization, vertex correction, penguin amplitude and spectator scattering and can be expressed as

$$T^p_A = \delta_{pu} \alpha_1 (M_1 M_2) A([\bar{q}_s u][\bar{u}D]) + \delta_{pu} \alpha_2 (M_1 M_2) A([\bar{q}_s D][\bar{u} u])$$

$$+ \alpha^p_3 (M_1 M_2) \sum_q A([\bar{q}_s D][\bar{q} q]) + \alpha^p_4 (M_1 M_2) \sum_q A([\bar{q}_s q][\bar{q} D])$$

$$+ \alpha^i_{3,EW} (M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s D][\bar{q} q]) + \alpha^i_{4,EW} (M_1 M_2) \sum_q \frac{3}{2} e_q A([\bar{q}_s q][\bar{q} D]),$$

where the sums extend over $q = u, d, s$, and $\bar{q}_s (= \bar{u}, \bar{d}, \bar{s})$ denotes the spectator antiquark. The coefficients $\alpha^i_3 (M_1 M_2)$ and $\alpha^i_{3,EW} (M_1 M_2)$ contain all dynamical information and can be expressed in terms of the coefficients $a^i_1$.

As for the power-suppressed annihilation part, we can parameterize it into the following form:

$$T^p_B = \delta_{pu} b_1 (M_1 M_2) \sum_{q' q} B([\bar{u} q'][\bar{q}' u][\bar{D} b]) + \delta_{pu} b_2 (M_1 M_2) \sum_{q' q} B([\bar{u} q'][\bar{q}' D][\bar{u} b])$$

$$+ b^p_3 (M_1 M_2) \sum_{q' q} B([\bar{q} q'][\bar{q}' D][\bar{q} b]) + b^p_4 (M_1 M_2) \sum_{q' q} B([\bar{q} q'][\bar{q}' q][\bar{D} b])$$

$$+ b^p_{3,EW} (M_1 M_2) \sum_{q' q} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' D][\bar{q} b]) + b^p_{4,EW} (M_1 M_2) \sum_{q' q} \frac{3}{2} e_q B([\bar{q} q'][\bar{q}' q][\bar{D} b]),$$

where $q, q' = u, d, s$ and the sums extend over $q, q'$. The sum over $q'$ arises because a quark-antiquark pair must be created via $g \rightarrow q' q'$ after the spectator quark is annihilated.

III. $a^0(980) - f_0(980)$ MIXING MECHANISM, CALCULATION OF CP VIOLATION AND BRANCHING FRACTION

A. $a^0(980) - f_0(980)$ mixing mechanism

In the condition of turning on the $a^0(980) - f_0(980)$ mixing mechanism, we can get the propagator matrix of $a^0(980)$ and $f_0(980)$ by summing up all the contributions of $a^0(980) \rightarrow f_0(980) \rightarrow \cdots \rightarrow a^0(980)$
and \( f_0(980) \to a^0_0(980) \to \cdots \to f_0(980) \), respectively, which are expressed as \[33\]

\[
\begin{pmatrix}
P_{a_0}(s) & P_{a_0f_0}(s) \\
P_{f_0a_0}(s) & P_{f_0}(s)
\end{pmatrix} = \frac{1}{D_{a_0}(s)D_{f_0}(s) - |\Lambda(s)|^2} \begin{pmatrix} D_{a_0}(s) & \Lambda(s) \\ \Lambda(s) & D_{f_0}(s) \end{pmatrix},
\]

(4)

where \( P_{a_0}(s) \) and \( P_{f_0}(s) \) are the propagators of \( a_0 \) and \( f_0 \), respectively, \( P_{a_0f_0}(s) \), \( P_{f_0a_0}(s) \) and \( \Lambda(s) \) arise due to the \( a^0_0(980) - f_0(980) \) mixing effect, and \( D_{a_0}(s) \) and \( D_{f_0}(s) \) are the denominators for the propagators of \( a_0 \) and \( f_0 \) when the \( a^0_0(980) - f_0(980) \) mixing effect is absent, respectively, which can be expressed as follows in the Flatté parametrization:

\[
D_{a_0}(s) = m_{a_0}^2 - s - i\sqrt{s}[\Gamma_{a_0}^{\pi\pi}(s) + \Gamma_{a_0}^{a_0K}(s)],
\]

(5)

\[
D_{f_0}(s) = m_{f_0}^2 - s - i\sqrt{s}[\Gamma_{f_0}^{\pi\pi}(s) + \Gamma_{f_0}^{K\bar{K}}(s)],
\]

where \( m_{a_0} \) and \( m_{f_0} \) are the masses of the \( a_0 \) and \( f_0 \) mesons, with the decay width \( \Gamma_{bc}^{a} \) can being presented as

\[
\Gamma_{bc}^{a}(s) = \frac{g_{abc}^2}{16\pi\sqrt{s}}\rho_{bc}(s) \quad \text{with} \quad \rho_{bc}(s) = \sqrt{1 - \frac{(m_b - m_c)^2}{s}}\left[1 + \frac{(m_b - m_c)^2}{s}\right].
\]

(6)

It was pointed out that the contribution from the amplitude of \( a^0_0(980) - f_0(980) \) mixing is convergent and can be written as an expansion in the \( K\bar{K} \) phase space when only \( K\bar{K} \) loop contributions are considered \[12, 40\],

\[
\Lambda(s)_{K\bar{K}} = \frac{g_{a_0K+K\bar{K}} - g_{f_0K+K\bar{K}}}{16\pi} \left\{ i \left[ \rho_{K+K\bar{K}} - \rho_{K^0K^0} \right] - O(\rho_{K+K\bar{K}}^2 - \rho_{K^0K^0}^2) \right\},
\]

(7)

where \( g_{a_0K+K\bar{K}} \) and \( g_{f_0K+K\bar{K}} \) are the effective coupling constants. Since the mixing mainly comes from the \( K\bar{K} \) loops, we can adopt \( \Lambda(s) \approx \Lambda_{K\bar{K}}(s) \).

B. Decay amplitudes, localizd CP violation and branching fraction

With the \( a^0_0(980) - f_0(980) \) mixing being considered, the process of the \( B^- \to K^-\pi^+\pi^- \) decay is shown in Fig. \( \text{I} \) and the amplitude can be expressed as

\[
\mathcal{M} = \langle K^-\pi^+\pi^- | \mathcal{H}^T | B^- \rangle + \langle K^-\pi^+\pi^- | \mathcal{H}^P | B^- \rangle,
\]

(8)

in which \( \mathcal{H}^T \) and \( \mathcal{H}^P \) are the tree and penguin operators, respectively, and we have

\[
\langle K^-\pi^+\pi^- | \mathcal{H}^T | B^- \rangle = \frac{g_{f_0\pi\pi}T_{f_0}}{D_{f_0}} + \frac{g_{f_0\pi\pi}T_{a_0}}{D_{a_0}D_{f_0} - \Lambda^2},
\]

\[
\langle K^-\pi^+\pi^- | \mathcal{H}^P | B^- \rangle = \frac{g_{f_0\pi\pi}P_{f_0}}{D_{f_0}} + \frac{g_{f_0\pi\pi}P_{a_0}}{D_{a_0}D_{f_0} - \Lambda^2},
\]

(9)
FIG. 1: The Feynman diagram for the $B^- \to K^- \pi^+ \pi^-$ decay with the $a_0^0(980) - f_0(980)$ mixing mechanism.

where $T_{a_0(f_0)}$ and $P_{a_0(f_0)}$ represent the tree and penguin diagram amplitudes for $B \to K a_0(f_0)$ decay, respectively. Substituting Eq. (9) into Eq. (8), the total amplitude of the decay $B^- \to K^- f_0(a_0)$ can be written as

$$M(B^- \to K^- \pi^+ \pi^-) = \frac{g_{f_0 \pi \pi}}{D_{f_0}} M(B^- \to K^- f_0) + \frac{g_{f_0 \pi \pi}}{D_{a_0} D_{f_0} - \Lambda^2} M(B^- \to K^- a_0).$$

In the QCD factorization approach, we derive the amplitudes of the $B^- \to K^- f_0$ and $B^- \to K^- a_0$ decays, which are

$$M(B^- \to K^- f_0) = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_{p}^{(s)} \left\{ (\delta_{p} a_{1} + a_{4}^{p} - r^{K}_{\chi} a_{6}^{p} + a_{10}^{p} - r^{K}_{\chi} a_{8}^{p}) f_{K} f_{0}^{B f_{0}^{\pi}} (m_{K}^{2}) ight. $$

$$- (\delta_{p} a_{2} + 2 a_{5}^{p} + 2 a_{7}^{p} + \frac{1}{2} a_{9}^{p} + \frac{1}{2} a_{7}^{p}) K f_{0}^{f_{0}^{B K}} (m_{K}^{2}) f_{0}^{F_{B K} f_{0}^{B K}} (m_{f_0}^{2})$$

$$- (a_{3}^{p} + a_{5}^{p} + a_{4}^{p} - r_{\chi}^{f} a_{6}^{p} - \frac{1}{2} a_{9}^{p} - \frac{1}{2} a_{10}^{p} + \frac{1}{2} r_{\chi}^{f} a_{8}^{p}) K f_{0} (m_{K}^{2}) f_{0}^{B K} (m_{f_0}^{2})$$

$$+ (\delta_{p} b_{2} + b_{3}^{p} + b_{3, EW}) K f_{0} f_{B} f_{0}^{f_{0}^{B K}} f_{K} + (\delta_{p} u_{b_{2}} + b_{3}^{p} - \frac{1}{2} b_{3, EW}) K f_{0} f_{B} f_{0}^{f_{0}^{B K}} f_{K},$$

$$M(B^- \to K^- a_0) = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_{p}^{(s)} \left\{ (\delta_{p} a_{1} + a_{4}^{p} - r^{K}_{\chi} a_{6}^{p} + a_{10}^{p} - r^{K}_{\chi} a_{8}^{p}) f_{K} f_{0}^{B f_{0}^{a_0}} (m_{K}^{2}) ight. $$

$$- (\delta_{p} a_{2} + 2 a_{5}^{p} + 2 a_{7}^{p} + \frac{1}{2} a_{9}^{p} + \frac{1}{2} a_{7}^{p}) K f_{0}^{f_{0}^{B K}} (m_{K}^{2}) f_{0}^{F_{B K} f_{0}^{B K}} (m_{f_0}^{2})$$

$$- (a_{3}^{p} + a_{5}^{p} + a_{4}^{p} - r_{\chi}^{f} a_{6}^{p} - \frac{1}{2} a_{9}^{p} - \frac{1}{2} a_{10}^{p} + \frac{1}{2} r_{\chi}^{f} a_{8}^{p}) K f_{0} (m_{K}^{2}) f_{0}^{B K} (m_{f_0}^{2})$$

$$+ (\delta_{p} b_{2} + b_{3}^{p} + b_{3, EW}) K f_{0} f_{B} f_{0}^{f_{0}^{B K}} f_{K} + (\delta_{p} u_{b_{2}} + b_{3}^{p} - \frac{1}{2} b_{3, EW}) K f_{0} f_{B} f_{0}^{f_{0}^{B K}} f_{K},$$

$$(11)
and

\[ \mathcal{M}(B^- \to K^- a_0) = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda(s)^{1/2} \left\{ (\delta_{pa} a_1 + a_3^p - r^\pi a_6^p + a_1^p - r^K a_8^p)_{a_0 K} (m_B^2 - m_{a_0}^2) F_{0B}^{a_0}(m_K^2) f_K \\
- (\delta_{pa} a_2 + \frac{3}{2} a_0^p + \frac{3}{2} a_7^p)_{K a_0} (m_B^2 - m_{a_0}^2) F_{0}^{K \to K}(m_{a_0}^2) f_{a_0} \\
+ (\delta_{pa} b_2 + b_3^p + b_3^{a_0})_{a_0 K} f_B f_{a_0} \right\}, \]

(12)

respectively, where \( G_F \) represents the Fermi constant, \( f_B, f_K, \tilde{f}_0 \) and \( f_{a_0} \) are the decay constants of the \( B, K, f_0, \) and \( a_0, f_s = \frac{2m_s^2}{m_b(\mu)(m_a(\mu) + m_s(\mu))} \) (where \( \mu \) is the scale parameter), \( F_{0B}^{a_0}(m_K^2) \), \( F_{0}^{BK}(m_{f_0}^2) \) and \( F_{0}^{Bao}(m_{a_0}^2) \) are the form factors for the \( B \) to \( f_0, K \) and \( a_0 \) transitions, respectively.

By integrating the numerator and denominator of the differential \( CP \) asymmetry parameter, one can obtain the localized integrated \( CP \) asymmetry, which can be measured by experiments and takes the following form in the region \( R \):

\[ A_{CP}^R = \frac{\int_R ds d\Omega^*_1 |\mathcal{M}|^2 - |\tilde{\mathcal{M}}|^2}{\int_R ds d\Omega^*_1 |\mathcal{M}|^2 + |\tilde{\mathcal{M}}|^2}, \]

(13)

where \( s \) and \( s' \) are the invariant masses squared of \( \pi \pi \) or \( K \pi \) pair in our case, and \( \tilde{\mathcal{M}} \) is the decay amplitude of the \( CP \)-conjugate process.

Since the decay process \( B^- \to K^- \pi^+ \pi^- \) has a three-body final state, the branching fraction of this decay can be expressed as [41]

\[ B = \frac{\tau_B}{(2\pi)^5 16 m_B^2} \int ds |\mathbf{p}_1^*| |\mathbf{p}_3| \int d\Omega^*_1 \int d\Omega_3 |\mathcal{M}|, \]

(14)

in which \( \Omega^*_1 \) and \( \Omega_3 \) are the solid angles for the final \( \pi \) in the \( \pi \pi \) rest frame and for the final \( K \) in the \( B \) meson rest frame, respectively, \( |\mathbf{p}_1^*| \) and \( |\mathbf{p}_3| \) are the norms of the three-momenta of final-state \( \pi \) in the \( \pi \pi \) rest frame, and \( K \) in the \( B \) rest frame, respectively, which take the following forms:

\[ |\mathbf{p}_1^*| = \frac{\sqrt{\lambda(s, m_{\pi^+}^2, m_{\pi^-}^2)}}{2\sqrt{s}}, \]

\[ |\mathbf{p}_3| = \frac{\sqrt{\lambda(m_{\pi^+}^2, m_{\pi^-}^2, s)}}{2m_B}, \]

(15)

where \( \lambda(a, b, c) \) is the Kollén function and with the form \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc) \).

### IV. NUMERICAL RESULTS

When dealing with the contributions from the hard spectator and the weak annihilation, we encounter the singularity problem of infrared divergence \( X = \int_0^1 dx/(1 - x) \). One can adopt the method in Refs.

6
to parameterize the endpoint divergence as $X_{H,A} = (1 + \rho_{H,A} e^{i\phi_{H,A}}) \ln \frac{m_B}{\Lambda_h}$, with $\Lambda_h$ being a typical scale of order 0.5 GeV, $\rho_{H,A}$ an unknown real parameter and $\phi_{H,A}$ the free strong phase in the range $[0, 2\pi]$. For convenience, we use the notations $\rho = \rho_{H,A}$ and $\phi = \phi_{H,A}$. In our calculations, we adopt $\rho \in [0, 1]$ and $\phi \in [0, 2\pi]$ for the two-body $B^- \to K^- f_0$ and $B^- \to K^- a_0$ decays. The first term of Eq. (10) is the amplitude of the $B^- \to K^- f_0$ decay without the effect of the $a_0^0(980) - f_0(980)$ mixing when mass of the $\pi^+\pi^-$ pair is in the vicinity of the $f_0(980)$ resonance. Substituting this term into Eq. (13), we can get the localized $CP$ violation of the $B^- \to K^- f_0 \to K^- \pi^+\pi^-$ decay when we take the integration interval as $[m_{f_0} - \Gamma_{f_0}, m_{f_0} + \Gamma_{f_0}]$, which is $A_{CP}(B^- \to K f_0 \to K^- \pi^+\pi^-) = [0.24, 0.36]$ and shown in Fig. 2 (a). Substituting Eqs. (11) and (12) into Eq. (10), one can also get the total amplitude of the $B^- \to K^- f_0(a_0) \to K^- \pi^+\pi^-$ decay with the $a_0^0(980) - f_0(980)$ mixing mechanism. Then inserting it into Eq. (13), we can also get the result of the localized $CP$ violation in the presence of $a_0^0(980) - f_0(980)$ mixing by integrating the same integration interval as above. The predicted result is $A_{CP}(B^- \to K^- f_0(a_0) \to K^- \pi^+\pi^-) = [0.33, 0.52]$, which is plotted in Fig. 2 (b). Obviously, the $CP$ violating asymmetry in Fig. 2 (b) is significantly larger than that in Fig. 2 (a). Thus, we conclude that the $a_0^0(980) - f_0(980)$ mixing mechanism can induce larger localized $CP$ violation for the $B^- \to K^- \pi^+\pi^-$ decay. However, compared with the contribution from first term in Eq. (10), that from the second term is very small and even can be ignored when calculating the branching fraction, thus we have $B(B^- \to K^- f_0(a_0) \to K^- \pi^+\pi^-) \approx B(B^- \to K f_0 \to K^- \pi^+\pi^-)$. Then, we calculate the branching fraction of the $B^- \to K^- f_0 \to K^- \pi^+\pi^-$ decay combining the first term in Eq. (10), Eqs. (11) and (12), the theoretical result is $B(B^- \to K^- f_0 \to K^- \pi^+\pi^-) = [6.50, 15.0] \times 10^{-6}$ which is plotted in Fig. 3. This result is consistent with the experimental result $B(B^- \to K f_0 \to K^- \pi^+\pi^-) = (9.4^{+1.0}_{-1.2}) \times 10^{-6}$ when the divergence parameter ranges are taken as $\rho \in [0, 1]$ and $\phi \in [0, 2\pi]$.

V. SUMMARY AND DISCUSSION

In this work, we studied the localized integrated $CP$ violation of the $B^- \to K^- f_0(a_0) \to K^- \pi^+\pi^-$ decay considering the $a_0^0(980) - f_0(980)$ mixing mechanism in the QCD factorization approach. We found the localized integrated $CP$ violation is enlarged due to the $a_0^0(980) - f_0(980)$ mixing effect. Without the $a_0^0(980) - f_0(980)$ mixing, the localized $CP$ violation was found to be $A_{CP}(B^- \to K f_0 \to K^- \pi^+\pi^-) = [0.24, 0.36]$, while $A_{CP}(B^- \to K^- f_0(a_0) \to K^- \pi^+\pi^-) = [0.33, 0.52]$ when this mixing effect is considered. In addition, we also calculated the branching fraction of the $B^- \to K^- f_0 \to K^- \pi^+\pi^-$ decay, and obtained $B(B^- \to K f_0 \to K^- \pi^+\pi^-) = [6.50, 15.0] \times 10^{-6}$ as shown in Fig. 3 which agrees the experimental result $B(B^- \to K f_0 \to K^- \pi^+\pi^-) = 9.4^{+1.0}_{-1.2} \times 10^{-6}$ well. Since the mixing term is very small, while calculating the branching fraction we can take the approximation $B(B^- \to K^- f_0(a_0) \to K^- \pi^+\pi^-) \approx B(B^- \to K f_0 \to K^- \pi^+\pi^-$ by ignoring the $a_0^0(980) - f_0(980)$ mixing effect. However, for $CP$ violation, this mixing
FIG. 2: The localized CP violation of the $B^- \to K^- f_0 \to K^- \pi^+ \pi^-$ decay (a) without the $a_0^0(980) - f_0(980)$ mixing mechanism, (b) with the $a_0^0(980) - f_0(980)$ mixing mechanism.

FIG. 3: The branching fraction of the $B^- \to K^- f_0 \to K^- \pi^+ \pi^-$ decay.

effect does contribution a lot and cannot be neglected. The same situation is also expended for other $B$ or $D$ mesons decay channels. We thus suggest that $a_0^0(980) - f_0(980)$ mixing mechanism should be considered when studying the heavy meson decays both theoretically and experimentally when this mixing effect could exist.

Appendix A: THEORETICAL INPUT PARAMETERS

In the numerical calculations, we should input distribution amplitudes and the CKM matrix elements in the Wolfenstein parametrization. For the CKM matrix elements, which are determined from experiments,
we use the results in Ref. [41]:

\[ \bar{\rho} = 0.117 \pm 0.021, \quad \bar{\eta} = 0.353 \pm 0.013, \]
\[ \lambda = 0.225 \pm 0.00061, \quad A = 0.811^{+0.023}_{-0.024}, \] (A1)

where

\[ \bar{\rho} = \rho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}). \] (A2)

The effective Wilson coefficients used in our calculations are taken from Ref. [43]:

\[ c_1' = -0.3125, \quad c_2' = 1.1502, \]
\[ c_3' = 2.433 \times 10^{-2} + 1.543 \times 10^{-3}i, \quad c_4' = -5.808 \times 10^{-2} - 4.628 \times 10^{-3}i, \]
\[ c_5' = 1.733 \times 10^{-2} + 1.543 \times 10^{-3}i, \quad c_6' = -6.668 \times 10^{-2} - 4.628 \times 10^{-3}i, \] (A3)
\[ c_7' = -1.435 \times 10^{-4} - 2.963 \times 10^{-5}i, \quad c_8' = 3.839 \times 10^{-4}, \]
\[ c_9' = -1.023 \times 10^{-2} - 2.963 \times 10^{-5}i, \quad c_{10}' = 1.959 \times 10^{-3}. \]

For the masses appeared in \( B \) decays, we use the following values [41] (in units of GeV):

\[ m_u = m_d = 0.0035, \quad m_s = 0.119, \quad m_b = 4.2, \quad m_q = \frac{m_u + m_d}{2}, \quad m_{\pi^+} = 0.14, \]
\[ m_{B^-} = 5.279, \quad m_{K^-} = 0.494, \quad m_{f_0(980)} = 0.990, \quad m_{a_0(980)} = 0.980, \] (A4)

while for the widths we use (in units of GeV) [41]

\[ \Gamma_{f_0(980)} = 0.074, \quad \Gamma_{a_0(980)} = 0.092. \] (A5)

The following numerical values for the decay constants are used [5, 44, 45] (in units of GeV):

\[ f_{\pi^\pm} = 0.131, \quad f_{B^-} = 0.21 \pm 0.02, \quad f_{K^-} = 0.156 \pm 0.007, \]
\[ f_{f_0(980)} = 0.370 \pm 0.02, \quad f_{a_0(980)} = 0.365 \pm 0.02. \] (A6)

As for the form factors, we use [5]

\[ F_0^{B \to K}(0) = 0.35 \pm 0.04, \quad F_0^{B \to f_0(980)}(0) = 0.25, \quad F_0^{B \to a_0^0(980)}(0) = 0.25. \] (A7)

The values of Gegenbauer moments at \( \mu = 1\)GeV are taken from [5]:

\[ B_{1, f_0(980)} = -0.78 \pm 0.08, \quad B_{3, f_0(980)} = 0.02 \pm 0.07, \]
\[ B_{1, a_0^0(980)} = -0.93 \pm 0.10, \quad B_{3, a_0^0(980)} = 0.14 \pm 0.08. \] (A8)
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