Phase structure and chiral limit of compact lattice QED with Wilson fermions * †

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We study the phase structure and chiral limit of 4d compact lattice QED with Wilson fermions (both dynamical and quenched). We use the standard Wilson action (WA) and also the modified action (MA) with some lattice artifacts suppressed. We show that lattice artifacts influence the distributions of eigenvalues $\lambda_i$ of the fermionic matrix especially for small values of $\lambda_i$. Our main conclusion is that the chiral limit of compact QED can be efficiently located using different techniques.

1. Introduction

The lattice formulation of QED is not unique. One has to decide on a physical ground which version of QED is realised in nature if different lattice versions of QED do not belong to the same universality class. The old physics, i.e., known from experiment, has to be reproduced. When we consider QED as arising from a subgroup of some non–abelian (e.g., grand unified) gauge theory we are necessarily dealing with the compact version. Our choice in this work is a compact formulation of QED.

In the theory with Wilson’s fermions chiral symmetry is broken explicitly, and, presumably, can be only restored by fine-tuning the parameters in the continuum limit if we are dealing with a meaningful lattice discretization. What one can expect at nonzero spacing is that at some $\kappa_c \equiv \kappa_c(\beta)$ a so–called partial symmetry restoration takes place when the Wilson mass term and ordinary mass term cancel at zero momentum in certain vertex functions. If so one can approach continuum limit and chiral symmetry restoration along the line $\kappa_c(\beta)$. Another question is if in the continuum limit of our theory the chiral symmetry is realised explicitly or is spontaneously broken.

2. Actions and order parameters

The modified lattice action $S_{MA}(U, \bar{\psi}, \psi)$ for 4d $U(1)$ gauge theory (QED) is

$$S_{MA} = \beta \cdot S_G(U) + S_F(U, \bar{\psi}, \psi) + \delta S_G(U).$$

In eq.(1) $S_G(U)$ is the standard plaquette (Wilson) action for the pure gauge $U(1)$ theory, and the additional term $\delta S_G$ suppresses lattice artifacts (i.e., monopoles and negative plaquettes). The fermionic part of the action $S_F(U, \bar{\psi}, \psi)$ is

$$S_F = \sum_{f=1}^{N_f} \sum_{x,y} \sum_{f,s} \bar{\psi}_x^f \mathcal{M}^{xy}_{ss'} \psi_y^{f,s} \equiv \bar{\psi} \mathcal{M} \psi,$$

$$\mathcal{M} \equiv 1 + \kappa \cdot \hat{\mathcal{M}}(U),$$

where $\mathcal{M}$ is Wilson’s fermionic matrix, $N_f$ is the number of flavours and $\kappa$ is the hopping parameter. The first two terms in r.h.s of eq.(1) make up the standard Wilson action $S_{WA}$.

In our work we used both $S_{WA}$ and $S_{MA}$ with $N_f = 2$ for dynamical fermions. Apart from $\langle \bar{\psi} \psi \rangle$ and $\langle \bar{\psi} \gamma_5 \psi \rangle$ we calculated the pion norm (III)

$$\Pi(U) = \frac{1}{L^4} \sum_x \text{Tr} \left( \mathcal{M}^{-1} \gamma_5 \mathcal{M}^{-1} \gamma_5 \right).$$
where $L$ is the lattice size. Its advantage is that it appears to be a very sensitive quantity in the 'critical' region. Introducing eigenvectors $g_n(s, x)$ of $\gamma_5 M$ with eigenvalues $\mu_n$: $\gamma_5 M g_n = \mu_n g_n$, one can easily obtain a spectral representation

$$\Pi = \frac{1}{L^4} \sum_n \frac{1}{\mu_n} \sum_s |g_n(s, 0)|^2 \ .$$

Following the common practice we identify here the chiral transition with the appearance of zero or near to zero eigenvalues of the fermionic matrix $M$. Evidently, an eigenstate of $M$ with eigenvalue zero is also an eigenstate of $\gamma_5 M$. So, the presence of configurations which belong to zero eigenvalues of $M$ gives rise to poles in $\Pi$.

3. Conjugate gradient (cg–) method

To locate $\kappa_c(\beta)$ one can use the convergence rate of the cg-method. This is the iterative method of solving a system of linear equations $D \cdot X = \varphi$, where $D$ is a hermitian $n \times n$ matrix and $\varphi$ is an input vector ($D = M^\dagger M$ in our case). The convergence of the cg–method should be controlled by the condition number $\xi \equiv \lambda_{\text{max}}/\lambda_{\text{min}}$, where $\lambda_{\text{max}}$ and $\lambda_{\text{min}}$ are maximal and minimal eigenvalues of $D$. Close to $\kappa_c$ the minimal eigenvalue of $M^\dagger M$ is small and is supposed to be $\sim (1 - \kappa/\kappa_c)^2$.

We observed that at large enough number of iteration steps $N_{cg}$ ($N_{cg} > N_0$) the average residue $\langle R \rangle \equiv \langle R \rangle(N_{cg})$ behaves as

$$\langle R \rangle = C \cdot \exp(-\alpha \cdot N_{cg}), \quad \alpha = \ln \frac{\sqrt{\xi} + 1}{\sqrt{\xi} - 1} \ (5)$$

independently of the distribution of eigenvalues $\lambda_i$ provided $n$ is large enough. To check it we generated $D$ with different (uniform, gaussian, double–peaked) distributions of eigenvalues and given $\lambda_{\text{min}}$ and $\lambda_{\text{max}}$. The components of the initial vector $X_0$ and of $\varphi$ were chosen every time randomly with gaussian distributions.

It follows from the above that, for the inversion of $M^\dagger M$, the $\langle N_{cg}^{-1} \rangle$ required for convergence to some small but fixed $R$ will behave as $\langle N_{cg}^{-1} \rangle \sim 1 - \kappa/\kappa_c$. In Fig.1 we show the dependence of $\langle N_{cg}^{-1} \rangle$ on $\kappa$ at $\beta = 0.8$ for quenched MA (qMA) for different $L$.

At $\kappa \sim \kappa_c$ the data fit nicely to straight lines giving reasonable estimation of $\kappa_c$. The volume dependence becomes rather weak for larger $L$. A similar picture was obtained for the case of dynamical fermions.

4. Phase diagram and chiral limit

For WA with dynamical fermions (dWA) at $\beta < \beta_0 \sim 1.0$ thermal cycles with respect to $\kappa$ or $\beta$ have a typical hysteresis behaviour for $\bar{\psi}\psi$ and plaquette $\Box$. Time histories (TH’s) of $\bar{\psi}\psi$ and plaquette $\Box$ for different starts show the existence of metastable states. So, we conclude that for dWA there is a 1st order phase transition (PT) line from $(\beta; \kappa) \simeq (1.0; 0)$ to $(\beta; \kappa) = (0.25)$ which is in agreement with $[3]$.

After the suppression of lattice artifacts (i.e., for dMA) this line disappears (see also $[4]$).

At $\beta > \beta_0$ and $\kappa < \kappa_c(\beta)$ the system with dWA is in the Coulomb phase. The photon correlator $\Gamma(\tau)$ is well consistent with that corresponding to a zero photon mass. For $\beta > \beta_0$ and $\kappa > \kappa_c(\beta)$ the correlator $\Gamma(\tau)$ shows a tachyonic–type behaviour ($m_\gamma^2 < 0$). Thus, we can conclude that there is a higher–$\kappa$ phase (or phases) differing from the Coulomb phase.

For quenched WA (qWA) at $\beta < \beta_0$ TH’s of $\Pi$ (as well as of $\bar{\psi}\psi$ and $\bar{\psi}\gamma_5\psi$) show very sharp peaks at $\kappa \sim \kappa_c(\beta)$ which means the appearance...
of small eigenvalues $\lambda_i$. It is worth noting that for WA at $\beta < \beta_0$ those peaks do not disappear at $\kappa > \kappa_c$ but instead become even more strong.

For WA at $\beta > \beta_0$ and for MA at any (positive) $\beta$ the dependence of TH’s on $\kappa$ changes drastically. We don’t find peaks of comparable amplitude ($\sim 10^4$) but, nevertheless, in some 'critical' region $\kappa \sim \kappa_c$ TH’s of $\Pi$ become much more rough than at smaller or larger values of $\kappa$. As far as for every configuration $\Pi$ is the arithmetic average of $4L^4$ terms corresponding to $4L^4$ different eigenvalues one can conclude that rather small $\lambda_i$ appeared.

The (renormalized) variance of the pion norm $\sigma^2(\Pi) \equiv L^4 \cdot \text{Var}(\Pi)$ appears to be a suitable 'order parameter'.

5. Conclusions

For the standard compact Wilson action we observe a presumably 1st order PT which disappears after suppressing lattice artifacts.

These lattice artifacts influence strongly the distribution of eigenvalues $\lambda_i$ of the fermionic matrix $\mathcal{M}$. This influence is especially pronounced for the near–to–zero values of $\lambda_i$.

After suppression of artifacts a chiral transition (i.e., appearance of near–to–zero eigenvalues of $\mathcal{M}$) is left on a 'horizontal' line $\kappa = \kappa_c(\beta)$.

As a preliminary conclusion, we have no sign for a qualitative change of the behaviour along the line $\kappa = \kappa_c(\beta)$.

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