Development of random thinking: Reflections and considerations

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Abstract. This article discloses some considerations and reflections in the teaching of random thinking and data systems based on a documentary review. A documentary sample was used consisting of 40 documentary sources (articles, postgraduate dissertations and books), published in indexed journals, about random thinking. The research was based on a qualitative content analysis method which used the coding and categories saturation process. The results let us establish conceptual relationships considering three dimensions linked to the random thinking teaching, they are: epistemological corpus, theoretical corpus and the social function. The results allow to base new proposals of curricular, didactic and evaluative type for the teaching of this thinking, at the same time, it allows to carry out processes of self-assessment of the teaching practice and of the need that currently exists before the research in the classroom and the contextualization of mathematical knowledge.

1. Introduction
Mathematical education and mathematical knowledge [1] are part of the curricular structures of the integral formation of students at all levels of schooling, from their initial formation to their university education, which articulates the mathematical processes, context and basic knowledge, the latter also known as mathematical thoughts [2].

One of the basic knowledge is precisely random thinking and data systems, which is based directly on concepts and procedures of probability theory and inferential statistics, and indirectly on descriptive statistics and on combinatorial [3].

Addressing the curricular phenomenon and the teaching and learning processes of mathematics intrinsically implies, understanding the didactics of science and more specifically, the teaching of mathematics. Therefore, it is necessary that every professional who teaches mathematics, moreover to evidencing a mathematical knowledge, should evidence a didactic and pedagogical knowledge, where working on the conditions of the situations created for the teaching process, therefore, determining specific conditions for the teaching of mathematical thoughts is a task of the formative subjects [4].

In this way, the paper presents certain reflections and considerations regarding the teaching and learning processes related to random thinking and data systems, the product of a documentary review and a qualitative content analysis, where it was interesting to find the relationships among the categories
that formed the concept of random thinking from the different primary sources (articles published in indexed journals) addressed and systematized.

Three dimensions, resulting from the content analysis process, were addressed at a general level. The results in terms of dimensions were epistemological corpus of random thinking, theoretical corpus of random thinking and, social function of random thinking.

2. Materials and methods
The study was addressed from a qualitative approach and a method of content analysis, that replace the subjective dimensions of the study of documents, which does not seek to understand the style of the text, but the words, the phrases and topics that are addressed there [5].

In this way, it was sought with this technique, to understand how from the different research experiences, the didactic process for the development of random thinking has been approached and how it is seen from the mathematics teaching community.

For the collection of the information, there was a theoretical sample composed by a total of 40 documentary sources (articles, postgraduate dissertations and books), which were systematized in an analytic matrix designed in Excel and whose coding process was based on the proposal of [6]. Open, axial and selective coding was necessary to structure this information thought the creation of semantic networks, with the help of the Atlas Ti software, to finally carry out the contrast process [7]. This process generated some reflections and considerations in the development and teaching processes of random thinking and data systems.

3. Results
The results found after performing the content analysis to the different primary sources are shown below. The relationships and saturations of the emerging categories are shown in three semantic networks constructed with the Atlas Ti software, which are presented by each of the dimensions established as axial categories in the saturation process: Dimension 1. Epistemological Corpus of random thinking; Dimension 2. Theoretical Corpus of random thinking and, Dimension 3. Social function of random thinking.

3.1. Dimension 1. Epistemological corpus of random thinking
In this dimension three subcategories emerged, the epistemological dimension of random thinking, the pedagogical dimension in the teaching of this thought and of course, the curricular and didactic dimension of random thinking.

One of the fundamental aspects in teacher training is didactic training [8], in order to generate transposition and didactic processes [9] and meet the needs of the educational context, which, as a social phenomenon, is found in constant change [10] and also requires disciplinary knowledge, didactic knowledge [11]. Now, one of the elements of knowledge construction in random thinking is precisely the conceptual field related to statistics that [12], from a curricular [13] and formative vision [14] has been considered since the beginning of the educational process [15] as a basis for experimentation and scientific research, in the sense that, it is about collecting, systematizing, analyzing and interpreting data that explain various phenomena and find answers to questions and hypotheses raised, all enclosed in the development of mathematical thinking and the different processes [16].

Based on the above, in [17] the conceptual, didactic [18] and perhaps epistemological gaps [19] are shown, around the teaching and therefore learning processes that occur in institutions and students at the time to address the development of random thinking and data systems, given that, almost always their teaching is left at the end of the math courses together with the little importance that the statistical component has in some institutions (see Figure 1).

Attached to this teaching element, is the learning process, which is a little investigated by the teaching agents; The understanding of these processes is a problem of a curricular and pedagogical nature, as it is stated in [20], that in the teaching and learning processes it is important that teachers take into account learning styles [21], since each subject is united in their cognitive structure and this would allow...
addressing aspects and didactic type strategies [22] most relevant to the structural dimensions of students participating in a training experience in the mathematics field, understanding didactic strategies as a set of real actions aimed at the phenomenon of teaching that seeks to promote learning and understanding of science knowledge and also demonstrating an epistemological corpus of knowledge to establish specific and objective relationships in the practice of the curriculum, that is to say, the didactic [23].

Figure 1. Epistemological corpus of random thinking.

3.2. Dimension 2. Theoretical corpus of random thinking
In this dimension, all the theoretical - conceptual content in the teaching of random thinking and data systems will be found. Based on the need to organize and analyze a set of data, it is that this thinking has arisen, in order to interpret qualitatively the describing information a particular phenomenon and context [24], along with this, the field of probability as an element that completes the statistics and allows defining events from an inference process [25], studying this field, becomes a challenge for teachers [26] but also an opportunity to apply different teaching strategies and theories permitting meaningful learning [27] for students. Therefore, the applicability that this thinking has for decision-making in different contexts, such as the labor context, where according to [28], the presence of uncertainties in social and human phenomena make the study of probability become more and more significant in the teaching of random thinking.

In this context, the formation of all these concepts, procedures and theories that structure random thinking and its relationship with the integral formation of mathematical knowledge, according to [29] strengthens and facilitates the resolution of more complex and even dynamic problems that are results
based on mathematical knowledge. This fact can be represented and modeled mathematically as a way of approaching the reality of objects, of phenomena and being in the capacity of decision-making, and inferential elements of each problem situation addressed.

As expressed in [30], stating that random thinking and data systems help to make decisions of uncertainty, chance, risk or ambiguity, due to lack of reliable information in which it is not possible to predict with certainty what will happen. Later these situations and processes can be modeled through mathematical systems related to probability theory and statistics.

In Figure 2, the relationships and saturations made can be summarized from the content analysis. It is clear that the teaching of this thinking addresses a fairly structured theoretical corpus, statistical processes (descriptive and inferential), the study of stochastic processes [31], probability, chance and mathematical hope, are part of the conceptual constructions that should be developed in random thinking and data systems [32].

**Figure 2.** Theoretical corpus of random thinking.

### 3.3. Dimension 3: Social function of random thinking

Finally, the last dimension in this content analysis is presented, which refers to the social function of random thinking and the justification and importance that it has in the formation of students’ mathematical knowledge. From this context, [33] The study of random thinking can be justified from the social point of view, since addressing concrete and real problem situations from socio-historical, political and cultural environments, where predictable phenomena could become real events [34], that is, the development of decision-making capacity based on ethical and moral training in students who access a set of data to interpret, understand or predict a certain phenomenon.
Way of random thinking, implies that students understand human and social relationships from the transdisciplinary dimension [35], thereby allowing them to approach understanding from a different science to a phenomenon that is studied from the explanation of the data, while there is a formation in terms of tolerance and uncertainty by the student, that is, from values and principles that are characteristic of the social problem [36] studied from the aspects of statistics.

In Figure 3, the relationships found in the emerging codes of the content analysis performed can be observed in a summary way.

Figure 3. Social function of random thinking.

Training in contextualized mathematics implies work through problem situations, while problem solving [37] allows the approach of mathematical modeling and the development of skills, mathematical processes and of course, mathematical thoughts such as numerical thinking, in which the use of mathematical language to represent the world of life and the different phenomena approached from any science, evidence a logical semantic structure [38], as is the mathematical language [39].

Ultimately, it is sought with the teaching of mathematics and of course with the teaching of numerical thinking, that what the student studied, makes sense, it is not a matter of mathematization process, but an understanding of mathematical knowledge, where students develop skills, processes and mathematical thinking skills, in which the usefulness of mathematical knowledge and the importance that it has in solving a contextualized problem situation is understood [40].

4. Conclusions
According to the analysis carried out, it can be concluded that the teaching of random thinking is given from three dimensions at a general level: the epistemological corpus of random thinking, the theoretical corpus of random thinking and, from its social function when addressing problems, facts and/or contextualized phenomena. As for the epistemological corpus for the teaching of random thinking, three
sub-dimensions were observed: the first has to do with the epistemic dimension in its broad sense, the second with the pedagogical dimension (pedagogical knowledge) and the third dimension is related to the curricular and didactic aspect for teaching this thinking.

Regarding the theoretical corpus, that is, the mathematical knowledge on the part of the professors, the theories of probability, inferential and descriptive statistics, chance, combinatorial and the use of stochastic and mathematical models were observed.

Finally, there is a justification from the social, when addressing the real, contextualized, socio-cultural and coexistence problems, all of them possible to be seen from an exploratory and investigative logic.

**References**

[1] Emmanuelle D, Rodí F and Vernazza C 2018 Revista Acta Latinoamericana de Matemática Educativa 31(2) 1077A
[2] Niño J 1998 Serie Lineamientos Curriculares (Colombia: Ministerio de Educación Nacional)
[3] Molina F 2014 La reorganizacin cognitiva en el desarrollo de pensamiento aleatorio y sistemas de datos en estudiantes de grado quinto (Manizales: Universidad Nacional de Colombia)
[4] D Amore B 2014 Revista Paradigma 35 199
[5] López F 2002 Revista de Educación 4 167
[6] Corbin J and Strauss A 2007 Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory (Thousand Oaks: SAGE Publications)
[7] Martínez M 2006 Revista IIPSI Facultad de Psicología 9 123
[8] Gómez M 2005 Revista Latinoamericana de Estudios Educativos 1(1) 83
[9] Ramírez R 2005 Revista Folios 21 5
[10] Cantor M and Guerrero M 2016 El pensamiento aleatorio como fundamento para el desarrollo del pensamiento matemático y sus componentes (Colombia: Universidad de los Andes)
[11] Trigueros M 2005 Educación Matemática 17(1) 5
[12] López J Ojeda A and Salcedo J 2018 Revista de Investigación Educativa de la REDIECH 9(17) 87
[13] Hernandez I, Luna J and Cadena M 2017 Revista Historia de la Educación Latinoamericana 19 149
[14] Torres E 2016 Estadística y probabilidad en el currículo colombiano para educación básica y media (Bogotá: Universidad Nacional de Colombia)
[15] Merino M and Reyes K 2013 Revista de Didáctica de la Estadística 2 137
[16] Cano M and Zapata D 2016 Análisis del pensamiento aleatorio desde las representaciones semióticas presentes en las pruebas Saber grado quinto (Medellín: Universidad de Medellín)
[17] Fuentes D 2016 Secuencia didáctica para estudiantes de grado 5 relacionada con la comprensión de situaciones problema del pensamiento aleatorio (Bogotá: Universidad Nacional de Colombia)
[18] Perafán G 2015 Revista Folios 42 55
[19] Córdova D 2016 Propuesta de formación para docentes del grado primero, basada en enseñanza para la comprensión, como estrategia didáctica para el desarrollo del pensamiento aleatorio, en la institución educativa Villa del Socorro (Medellín: Universidad Nacional de Colombia)
[20] Emmanuelle D, Rodí F and Vernazza C 2018 Revista Acta Latinoamericana de Matemática Educativa 31(2) 1077A
[21] Niño J 1998 Serie Lineamientos Curriculares (Colombia: Ministerio de Educación Nacional)
[22] Molina F 2014 La reorganizacin cognitiva en el desarrollo de pensamiento aleatorio y sistemas de datos en estudiantes de grado quinto (Manizales: Universidad Nacional de Colombia)
[23] D Amore B 2014 Revista Paradigma 35 199
[24] López F 2002 Revista de Educación 4 167
[25] Corbin J and Strauss A 2007 Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory (Thousand Oaks: SAGE Publications)
[26] Martínez M 2006 Revista IIPSI Facultad de Psicología 9 123
[7] Gómez M 2005 Revista Latinoamericana de Estudios Educativos 1(1) 83
[9] Ramírez R 2005 Revista Folios 21 5
[10] Cantor M and Guerrero M 2016 El pensamiento aleatorio como fundamento para el desarrollo del pensamiento matemático y sus componentes (Colombia: Universidad de los Andes)
[11] Trigueros M 2005 Educación Matemática 17(1) 5
[12] López J Ojeda A and Salcedo J 2018 Revista de Investigación Educativa de la REDIECH 9(17) 87
[13] Hernandez I, Luna J and Cadena M 2017 Revista Historia de la Educación Latinoamericana 19 149
[14] Torres E 2016 Estadística y probabilidad en el currículo colombiano para educación básica y media (Bogotá: Universidad Nacional de Colombia)
[15] Merino M and Reyes K 2013 Revista de Didáctica de la Estadística 2 137
[16] Cano M and Zapata D 2016 Análisis del pensamiento aleatorio desde las representaciones semióticas presentes en las pruebas Saber grado quinto (Medellín: Universidad de Medellín)
[17] Fuentes D 2016 Secuencia didáctica para estudiantes de grado 5 relacionada con la comprensión de situaciones problema del pensamiento aleatorio (Bogotá: Universidad Nacional de Colombia)
[18] Perafán G 2015 Revista Folios 42 55
[19] Córdova D 2016 Propuesta de formación para docentes del grado primero, basada en enseñanza para la comprensión, como estrategia didáctica para el desarrollo del pensamiento aleatorio, en la institución educativa Villa del Socorro (Medellín: Universidad Nacional de Colombia)
[20] Emmanuelle D, Rodí F and Vernazza C 2018 Revista Acta Latinoamericana de Matemática Educativa 31(2) 1077A
[21] Niño J 1998 Serie Lineamientos Curriculares (Colombia: Ministerio de Educación Nacional)
[22] Molina F 2014 La reorganizacin cognitiva en el desarrollo de pensamiento aleatorio y sistemas de datos en estudiantes de grado quinto (Manizales: Universidad Nacional de Colombia)
[23] D Amore B 2014 Revista Paradigma 35 199
[24] López F 2002 Revista de Educación 4 167
[25] Corbin J and Strauss A 2007 Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory (Thousand Oaks: SAGE Publications)
[26] Martínez M 2006 Revista IIPSI Facultad de Psicología 9 123
[7] Gómez M 2005 Revista Latinoamericana de Estudios Educativos 1(1) 83
[9] Ramírez R 2005 Revista Folios 21 5
[10] Cantor M and Guerrero M 2016 El pensamiento aleatorio como fundamento para el desarrollo del pensamiento matemático y sus componentes (Colombia: Universidad de los Andes)
[11] Trigueros M 2005 Educación Matemática 17(1) 5
[12] López J Ojeda A and Salcedo J 2018 Revista de Investigación Educativa de la REDIECH 9(17) 87
[13] Hernandez I, Luna J and Cadena M 2017 Revista Historia de la Educación Latinoamericana 19 149
[14] Torres E 2016 Estadística y probabilidad en el currículo colombiano para educación básica y media (Bogotá: Universidad Nacional de Colombia)
[15] Merino M and Reyes K 2013 Revista de Didáctica de la Estadística 2 137
[16] Cano M and Zapata D 2016 Análisis del pensamiento aleatorio desde las representaciones semióticas presentes en las pruebas Saber grado quinto (Medellín: Universidad de Medellín)
[17] Fuentes D 2016 Secuencia didáctica para estudiantes de grado 5 relacionada con la comprensión de situaciones problema del pensamiento aleatorio (Bogotá: Universidad Nacional de Colombia)
[18] Perafán G 2015 Revista Folios 42 55
[19] Córdova D 2016 Propuesta de formación para docentes del grado primero, basada en enseñanza para la comprensión, como estrategia didáctica para el desarrollo del pensamiento aleatorio, en la institución educativa Villa del Socorro (Medellín: Universidad Nacional de Colombia)
[30] Olaya R 2018 El pensamiento aleatorio en clave de comprensión de lo estocástico en la sociedad, en estudiantes de grado noveno, de la I. E. Tulio Arbeláez Zuluaga, Garzón-Huila (Manizales: Universidad Nacional de Colombia)

[31] Raffo E and Mejía M 2006 Revista Industrial Data 9(1) 1

[32] Carballo M and Ojeda Ana 2009 Revista Acta Latinoamericana de Matemática Educativa 22 1

[33] Guerrero, M 2018 Enfoque teórico para el desarrollo del pensamiento aleatorio y la formación ciudadana -dimensión de convivencia y paz, en aulas aceleradas III Encuentro Colombiano de Educación Estocástica (III ECEE) (Bogotá: Asociación Colombiana de Educación Estocástica)

[34] Cruz A Hernández E and Bernal J 2011 Scientia et Technica 3(49) 158

[35] Burbano V Valdivieso M and Aldana E 2017 Revista de Investigación, Desarrollo e Innovación 7(2) 269

[36] Sepúlveda A Medina C and Sepúlveda D 2009 Educación Matemática 21(2) 79

[37] Albarracín L and Gorgorió N 2013 Revista Latinoamericana de Investigación en Matemática Educativa 16(3) 289

[38] Peirce C 1974 La Ciencia de la Semiótica (Buenos aires: Nueva Visión)

[39] Puig L 1994 Revista Ectopias 51 3

[40] Inzunsa S 2010 Revista Mexicana de Investigación Educativa 15 42