Abstract:

The contribution deals with the negotiation process from a game theory perspective. On a negotiation model based on Nash bargaining problem, it demonstrates how to achieve greater utility and its division, or in a simplified form, division of a higher yield.

The graphical form of the model helps to understand the way negotiations takes place and some aspects of it. The problem of subjective assessment of reality can be largely addressed by the negotiation process.

Understanding the role of subjectivity in bargaining allows to improve your own bargaining skills and gain more in dividing the results achieved together. A proper set-up of the so-called point of disagreement is the key to the advantage of good preparation.

The findings are related to the structure of negotiation ensuing from the negotiation program at Harvard Law School.

Keywords: Bargaining, cooperative games, game theory, bargaining problem, point of disagreement.

JEL Code: C71, D21.

---

1 University of Finance and Administration (VSFS, www.vsfs.cz), e-mail: radim.valencik@vsfs.cz
2 University of Finance and Administration (VSFS, www.vsfs.cz), e-mail: jan.cervenka@hotmail.com
1. Introduction

In article titled “Analysis Tools of Connecting Investment Opportunities and Investment Means in the Area of Small and Medium-Sized Enterprises” (Valencík and Červenka, 2016) an apparatus is presented, which analyses the relations between owners of investment opportunities and owners of financial resources. This negotiation can be generalized and converted into the so-called Nash bargaining problem and, for further analysis, use the tools of this part of game theory. Understanding the division of patterns and factors that influence the size of the subjects’ share is a prerequisite for the subsequent practical use of this knowledge.

The aim of this contribution is to propose a negotiation model and to select the theoretical solution that best meets practical requirements. Next, it is to apply the model and then use it to demonstrate some aspects of negotiation that are not entirely clear and which can contribute to increasing the bargaining power of small and medium-sized business.

2. Literature review

In the area of involving game theory in bargaining, basic theoretical sources are based on the work of Nash (1950), who defined the negotiation problem in game theory, including axioms, and also contributed to the solution of this problem. Subsequently, other authors find other possible solutions that correspond to the changed axioms for the solution. These include Raiffa (1953), Kalai and Smorodinsky (1975) and Kalai (1977). Kıbrıs (2010) clearly processed the development of negotiation solutions.

The negotiation process has been alternatively studied at Harvard Law School, a faculty of law, since 1983. The basic principles of negotiation, on which the programme is built, are summarized in the well-known book “Getting to Yes” (Fisher and Ury, 1987). Ury and other authors then develop these basic principles for different situations, such as overcoming disagreement (Ury, 2015), and so on. An interesting criticism of this project is offered by a former FBI chief negotiator, who built his bargaining skills on the basis of many practical experiences. From his own experience and detailed analysis of bargaining, he concludes that academic studies are inadequate in real practice because of their excessive emphasis on rationality. He argues and practically demonstrates that emotions play an irreplaceable role in the negotiations, which ultimately have a decisive word in concluding agreements. (Voss and Raz, 2016).

The article builds on previous articles dealing with this field, especially “Analysis of Financial Markets Evolution by Utilizing the Theory of Cooperative Game” (Valenčík et al., 2015a), “The Fourth Industrial Revolution or the Economics of Productive Services?” (Valenčík et al., 2015b), “Phenomenon of a ”Snag“ in financial markets and its analysis via the cooperative game theory“ (Černík and
Valenčík, 2016), and “Analysis tools of connecting investment opportunities and investment means in the area of small and medium-sized enterprises” (Valenčík and Červenka, 2016), which deal with the negotiation between financial providers and owners of investment opportunities and related issues.

3. Material and Methods

The article uses an exploration method that is common in field of game theory. It uses the model of this situation/game to analyse the strategic situation. It examines how the model behaves under changed conditions and compares outputs with known theories and behaviour of negotiation participants. From these comparisons it then derives conclusions. In this contribution, Nash's bargaining problem is used as a model. The model is further used to analyse the impacts of an individual view of the bargaining situation and to determine the importance of setting the point of disagreement at the start of negotiations. The model conclusions relate to the theoretical framework of the Harvard negotiation programme.

3.1 Nash's bargaining problem

For a deeper analysis of the negotiating situation, two entities with different interests, leading to the negotiation of agreement, are considered. It is intended to provide both parties with some benefits and the subsequent distribution thereof. A cooperative game apparatus, namely the Nash's bargaining problem is used (Nash, 1950). As Nash states, virtually every bargaining is a form of achievement and distribution of benefits. Such a negotiation situation can be viewed as a set of possible combinations of the division of the jointly obtained benefits from all possible arrangement of the subjects, with the point of disagreement \( d \), which determines the subset of the set \( S \), within which the solution will be sought, see Figure 1:

Figure 1: Negotiation as a Nash’s bargaining problem

Source: Authors, based on Nash (1950).
The point of disagreement identifies a solution, on which both negotiating parties agree without further negotiation - this is the result of the first proposal. Alternatively, the negotiation is the result of alternatives available to both parties - hence it is not worth a bargain for a lower benefit than that defined by point (d). The point of disagreement may also be located at the intersection of axes \((x, y)\) if both sides do not agree and have no alternative options, i.e., even small improvements are beneficial for both entities.

The curve bounded by the set \(S\) represents utility-possibility frontier, a set of Pareto efficient combinations, where one can no longer improve one subject without the second subject doing worse. It is worth pointing out that the model works with the concept of utility, which is a subjectively perceived benefit. It includes not only quantifiable incomes, but also their subjective importance and a number of other factors that affect bargaining, such as emotions. This knowledge is very important for understanding of the negotiation process - even a seemingly mutually beneficial agreement can be utterly frustrated if discourteous behaviour occurs during the negotiations whereby one party feels offended. Subjectively perceived damage may be higher than the subjective benefit of a financial gain. On the contrary, good personal relationships and a professional approach, for example, towards cultural differences of negotiation, can increase the benefit of the agreement. Because of the difficulty in quantifying utility its usually replaced by more easily quantifiable variables such as revenue, which will also be the quantity further used in this work, for a better idea of the outcome of the negotiation. It is still good to keep in mind the possibility of an inaccurate interpretation of the situation.

Nash’s solution to the bargaining problem is an exact solution based on various, in practice, unrealistic assumptions. These include, for example, perfect information, equal bargaining skills, knowledge of the power of negotiation, etc. In his “The Bargaining Problem” (Nash, 1950), J. Nash proposes a solution that lies at the so-called Nash product, which can be found according to the formula:

\[
\max \left[ u_1(x^*) - u_1(x^0) \right] \left[ u_2(x^*) - u_2(x^0) \right]
\]

where \(u_1\) and \(u_2\) are utilities of the first and second subject, point \(x^0\) corresponds to the benefit at the point of disagreement and the point \(x^*\) at the point of interest. Thus, the formula represents the maximization of the benefit that entities can obtain.

Another described approach comprises the dictatorial approach where one entity gains all the benefits. In practice, it is not easy to achieve this outcome in the negotiations. In a free environment, even an egalitarian solution is not always rational, proposing division equally (Kalai, 1977), regardless of the contribution of the parties to the outcome. The last of the commonly cited solutions is the Kalai-Smorodinsky approach (Kalai and Smorodinsky, 1975). From the viewpoint of application in the corporate sphere it seems to be the most usable - it can be
interpreted as a division taking into account the relative contribution of individual subjects to the joint result, as graphically illustrated in Figure 2.

**Figure 2. Kalai-Smorodinski solution to bargaining problem**

![Graphical illustration of the Kalai-Smorodinski solution](image)

*Source: Authors, based on (Kalai and Smorodinsky, 1975).*

where \(y(1)\) and \(y(2)\) represent the proceeds of the first and the second subject, a point KS of the Kalai-Smorodinski solution with revenues \(y_{KS}(1)\) and \(y_{KS}(2)\) for both subjects.

Kalai-Smorodinski solution corresponds to the generally understood model of a fair division according to the merits, corresponds to the common division within agreements between entities and therefore we will use it further for the graphic demonstration of some aspects of the negotiations.

All of these solutions to the negotiation problem in the original version are a mathematical discipline with many assumptions difficult to obtain in practice. However, the bargaining model can also be successfully used in its graphical form to illustrate the negotiating situation and to demonstrate various factors that affect the process and outcome of negotiations.

### 3.1 Harvard negotiation programme

The Negotiation Center, which has been involved in research on the negotiation process since 1983, is situated at Harvard Law School. The research is based on the basic principles described by Fisher and Ury, co-founder of the Harvard Negotiation Center, in their book “Getting to yes” (1987). It is mainly about concentrating on solving the problem as the opposite of addressing the problem, i.e., instead of describing what is wrong, the focus moves on to finding solutions and finding mutually beneficial solutions. The specific basic recommendations are subsequently:

1. Separate the people from the problem;
2. Focus on interests, not positions;
3. Invent options for mutual gain;
4. Insist on objective criteria (Fisher and Ury, 1987).

Another important idea that Fisher and Ury (1987) introduced comprises the role of the Best Alternative To a Negotiated Agreement (BATNA). BATNA is related to the preparation for negotiations. It means clarification of the alternative options of the negotiating entity, i.e., what is the minimum outcome of bargaining that overcomes these alternatives. It can be said that the Harvard negotiation programme today by its principles represents the mainstream of bargaining tactics.

Another valid perspective and criticism of the Harvard project in 2006 is provided by Chris Voss, the former FBI chief negotiator. He has built his bargaining skills on the basis of many practical experiences. After negotiating surprisingly successfully in the bargaining role plays against the experienced lecturers of the project and their students, he literally states: “... no matter how we dress up our negotiations in mathematical theories, we are always an animal, always acting and reacting first and foremost from our deeply held but mostly invisible and inchoate fears, needs, perceptions, and desires. That’s not how these folks at Harvard learned it, though. Their theories and techniques all had to do with intellectual power, logic ... They had a script to follow, a predetermined sequence of actions, offers, and counteroffers designed in a specific order to bring about a particular outcome. It was as if they were dealing with a robot ...” (Voss and Raz, 2016). This is, in other words, a confirmation of the fact that if we simplify the negotiating task and start using the yield or other obvious value instead of the utility, we can overlook the significant intangible benefit for some of the entities.

3.2 Incomplete information and subjectivity in information evaluation

During practical negotiations, in business practice, we encounter a fundamental problem - how to get information, how to evaluate them, how and at which stage of negotiation can we use them. Many information is unavailable or are out-dated, incomplete, unreliable, inaccurate, etc. Moreover, each of the negotiators has only a limited view of the current situation, interpreting the information available differently. Fall in the stock exchange is a reason for someone to leave the market, while others see it as an opportunity for cheaper purchases. It is clear that the evaluation of future revenues will, to a large extent, be individual and subjective even in case of known actual variables (Thalassinos et al., 2012a; 2012b). They will depend on expectations of future developments and other influences such as availability of investment funds, alternative investment options, etc. The same is true of the assessment of investment projects in the SME environment.

Another individual factor that needs to be considered comprises synergy effects, i.e., consideration of the impact of the agreement on the future negotiating position and the possibilities of the entity. In business practice, there is a situation where it is rational to conclude even a loss-making agreement, provided this agreement provides some, though at that moment financially uncharted, benefits (Bondarenko
et al., 2017. An example might be to gain a customer, who will enable to enter a new market, provide an important reference, etc. A narrowly conceived yield may be negative at that time, but the subjectively perceived benefit will be positive.

Individual assessment of the situation should also be taken into account when negotiating. If the two sides begin to negotiate, their views on the possible returns from a concluded agreement may vary considerably. Everyone has their own perception of possible returns and their distribution, its own set $S$ (Figure 3):

**Figure 3. Different views on future possible returns of subjects**

![Figure 3](image)

*Source: Authors.*

Negotiation contributes to the exchange of information, influencing the perception of the other subject, convergence of opinions. Thus, the differences between the perceptions of the reality of both subjects diminish (Figure 4):

**Figure 4. Convergence of the realities of both parties by negotiation**

![Figure 4](image)

*Source: Authors.*

In case there is no unification of the views of both parties, different situations may occur. If both entities are rather pessimistic in anticipation of possible revenue distributions, i.e., they will perceive possible agreements rather more advantageous for the other one, the future agreement will lie in the penetration of individual $S$ sets
of both subjects. In other words, both parties will seek only an agreement that is realistic in their view and the limitation according to Figure 5:

**Figure 5. Penetration of sets of possible revenue distributions**

![Figure 5](image)

*Source: Authors.*

In case of an optimistic assessment of the situation, the subjects will be inclined to a broader perception of the possibilities and together they can develop a new common set $S(1,2)$ (Figure 6):

**Figure 6. New common set of possible revenue distributions**

![Figure 6](image)

*Source: Authors.*

In cases where the agreement is very attractive for one party, while the other party hesitates, doubts the advantage from its point of view, it is possible to use a combination of compensations or guarantees as a motivation. In this way, the agreed yield can also be secured for the questioning party and allow for an agreement.

The Harvard negotiation programme deals with the same issue in the way that it emphasizes the full understanding of the interests of the other party and the search for alternatives that bring the greatest benefit to both parties. Ideally, there is a blend of reality and extension of possibilities. From the experience of the authors of this work it is not easy for less experienced negotiators to understand the concept of
individual understanding of reality, and the approach described in this chapter could contribute to accepting different concepts of reality in others.

### 3.3 Meaning and setting up the point of disagreement

The commencement of the negotiation process is of great importance for negotiation; in Nash’s bargaining problem setting the point of disagreement. We demonstrate the importance of this setting on the bargaining model with a breakdown based on the benefit to the jointly achieved yield (Kalai-Smorodinského solution) see Figure 7.

*Figure 7. The significance of setting point of disagreement*

![Figure 7](image)

Where $KS_1$, $KS_2$ and $KS_3$ are agreements ensuing from points of disagreement $d_1$, $d_2$ and $d_3$. Points $y(1)_1$, $y(1)_2$ and $y(1)_3$ represent the revenues achieved by the first entity.

It is obvious that for the first subject it is preferable to reject any initial $d1$ conditions and negotiate from the basic parameters $d2$. Thereby, we reach higher proceeds $y(1)_2$. If he could convince the other entity during negotiation that his contribution is relatively lower and thus move the point of disagreement to point $d_3$, then he will achieve an even higher yield $y(1)_3$. In the bargaining practice this situation is related to the so-called anchoring technique. The negotiator starts with an unrealistic claim, which he then discards. For the other side, the finally negotiated agreement seems to be advantageous compared to the original unrealistic requirement.

Also, the BATNA (Best Alternative To Negotiated Agreement) concept is related to setting the point of objection. From the point of view of the other subject in the example above - if, on the basis of preparation, the entity knows that its BATNA corresponds to the proceeds from the $d_1$ disagreement, then it rejects the efforts to move the point of disagreement or terminate the negotiations altogether. Therefore,
it is important not to underestimate the preparation and examine the alternative options before the negotiations take place.

4. Results and Discussion

The text provides a way to clearly analyse bargaining between two entities and applies this model to bargaining to the corporate environment as well. As the most usable mathematical solution to the bargaining problem for the business area appears the Kalai-Smorodinski solution for its interpretation is based on merit distribution and easy application to a real environment. It clearly demonstrates the dependence of negotiations on the subjective assessment of both the current situation and the future development, as well as the possibility of influencing this perception through negotiation. At the same time, the text analyses the impact of the different points of disagreement and suggests options for its control.

What remains to be discussed is whether it is suitable to elaborate in detail the individual conclusions and to make some detailed, ideally mathematically based conclusions based on observation and evaluation of real bargaining situations in corporate practice. It is theoretically possible that because of the high degree of subjectivity and individuality of the assessment of well-known situations, an exact apparatus will not produce desired results in the terms of more accuracy. The authors of the article are convinced that the graphic form of the individual negotiating situations helps to understand the relationships between subjects and the dynamics of the negotiations.

5. Conclusion

The so-called Nash’s negotiation problem can be used to graphically illustrate the negotiation process. For the practice of small and medium-sized enterprises, the Kalai-Smorodinski solution appears to be the most appropriate theoretical approach to the so-called negotiating problem. This solution can be interpreted in a clear way as a breakdown of revenues depending on the benefits of the acting entities, and in essence it is a reduction of bargaining for a revenue sharing agreement. This can form a basis of the agreement even at the moment of uncertain revenue. At the same time, the agreement can be enriched by some other provisions, such as different coverage of different input costs and other parameters that provide sufficient flexibility according to the required conditions.

One of the main challenges in real bargaining comprises ignorance of all information and subjective evaluation of information known to the public. This factually leads to the creation of individual models of negotiating reality. The process of negotiating will influence and probably converge both realities. Because of the individual models of reality, the setting of a good starting position for negotiation is important; therefore, it is important to prepare for negotiations with the identification of alternative options, both for themselves and for others, and to prepare a suitable
strategy. This way you can get an advantage right at the beginning of the negotiations. This way, the contribution has met its goals and with its practical focus it can be beneficial to the corporate sphere to improve both individual and team negotiation processes. Additional research will be focused on obtaining relevant data to confirm the above conclusions.

Acknowledgement:

Contribution supported by funds institutional support for the long-term conceptual development of the research organization of the University of Finance and Administration.

References:

Bondarenko, G.T., Isaeva, A.E., Orekhov, S.A., Soltakhanov, U.A. 2017. Optimization of the Company Strategic Management System in the Context of Economic Instability. European Research Studies Journal, 20(2B), 3-24.

Černík, O. and Valenčík, R. 2016. Phenomenon of a "Snag" in financial markets and its analysis via the cooperative game theory. Contributions to Game Theory and Management, 9, 102-117.

Fisher, R. and Ury, W. 1987. Getting to yes: Negotiating Agreement Without Giving in (Arrow ed.). London: Arrow Books Limited.

Kalai, E. 1977. Proportional solutions to bargaining situations: interpersonal utility comparisons: interpersonal utility comparisons. Econometrica: Journal of the Econometric Society, 45(7), 1623-1630.

Kalai, E. and Smorodinsky, M. 1975. Other solutions to Nash's bargaining problem. Econometrica: Journal of the Econometric Society, 43(3), 513-518.

Kıbrıs, Ö. 2010. Cooperative game theory approaches to negotiation. In Handbook of Group Decision and Negotiation, Springer, 151-166.

Nash, J.F. 1950. The bargaining problem. Econometrica: Journal of the Econometric Society, 18(2), 155-162.

Raiffa, H. 1953. Arbitration schemes for generalized two-person games. Annals of Mathematics Studies, (28), 361-387.

Thalassinos, I.E., Hanias, P.M. and Curtis, G.P. 2012a. Time series prediction with neural networks for the Athens Stock Exchange indicator. European Research Studies Journal, 15(2), 23-31.

Thalassinos, I.E., Ugurlu, E. and Muratoglu, Y. 2012b. Income Inequality and Inflation in the EU. European Research Studies Journal, 15(1), 127-140.

Valenčík, R., Černík, O., Červenka, J. and Mihola, J. 2015. Analysis of Financial Markets Evolution by Utilizing the Theory of Cooperative Games. In V. Pavlát and O. Schlossberger, Proceedings of the 7th International Conference on “Financial Markets within the Globalization of World Economy”, Prague, VŠFS EUPRESS, 36-46.

Valenčík, R. and Červenka, J. 2016. Analysis tools of connecting investment opportunities and investment means in the area of small and medium-sized enterprises. European Research Studies Journal, 19(4), 130-139.

Voss, C. and Raz, T. 2016. Never Split the Difference: Negotiating as if your Life Depended on it. New York, Harper Collins.