Triplet-like correlation symmetry of continuous variable entangled states

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Abstract. We report on a continuous variable analogue of the triplet two-qubit Bell states. We discuss the symmetry properties of entangled states of either kind, and theoretically and experimentally demonstrate a remarkable similarity of two-mode continuous variable entangled states with triplet Bell states with respect to their correlation patterns. Understanding the symmetry properties helps finding decoherence-free subspaces.

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1. Introduction

Entanglement has enjoyed a special role in physics ever since the famous discussion between Einstein and Bohr about their Gedankenexperiments [1, 2]: a pair of subsystems was postulated in which the ability to infer the values of two conjugate observables of the second subsystem based on observations of the first subsystem is less uncertain than the uncertainty allowed by quantum mechanics in the case of a single isolated subsystem. Such counterintuitive states exhibit correlations between two subsystems of a very non-classical nature and in turn lead to fundamentally new interactions and applications in the field of quantum information [3, 4]. Launched by this essential role in quantum information processing, a great number of experiments have investigated the production of entangled states of photons, both in discrete variable (DV) regime and in continuous variable (CV) regime.

For a long time DV and CV states were investigated separately from each other. Recently, the developments of quantum information applications such as quantum cryptography, quantum computations and others, exploit both ‘worlds’—discrete and continuous, as far as both have some advantages [5]–[10]. This stimulates further theoretical and experimental research aimed to unify the ‘worlds’.

We found an interesting similarity between DV and CV states, based on the rotational invariance of the bipartite states. It is well known that the symmetry pattern of rotational invariance of DV entangled states plays a very special role in the theoretical analysis of entanglement [11]–[15]. We show a continuous variable analogy of the triplet Bell states in the sense of their symmetry. We theoretically demonstrate a similarity between the correlation pattern of the triplet Bell states and a two-mode continuous variable entangled state. We investigate experimentally a two-mode CV polarization entangled source and the correlations between the two modes along different detection directions. The measured correlations show essentially the same symmetry with those of discrete triplet Bell states.

2. Correlation properties of two-mode states

To describe a mode of the electromagnetic field we use the creation and annihilation operators ($\hat{a}^\dagger$ and $\hat{a}$) and a pair of canonical conjugate quadratures ($\hat{X}$ and $\hat{P}$). We follow a notation where $\hat{a} = \hat{X} + i\hat{P}$. Let us consider two modes A and B mixed on a 50/50 beamsplitter with a relative phase of $\pi/2$. The resulting output modes (after the beamsplitter) are labelled as C and D, respectively. The beamsplitter transformation is $\hat{a}_{C,D} = (\hat{a}_A \pm i\hat{a}_B)/\sqrt{2}$, thus the output conjugate quadratures are

$$\hat{X}_{C,D} = (\hat{X}_A \mp \hat{P}_B)/\sqrt{2}, \quad \hat{P}_{C,D} = (\hat{P}_A \pm \hat{X}_B)/\sqrt{2}. \quad (1)$$

Now we consider a case when both input modes are squeezed in $\hat{X}$ direction in phase space: $\hat{X}_A^{(r)} = e^{-r}\hat{X}_A^{(0)} \to 0$ and $\hat{X}_B^{(r)} = e^{-r}\hat{X}_B^{(0)} \to 0$, where $r$ is a parameter of squeezing. Then the output modes (1) becomes individually noisy but conditionally quiet:

$$\hat{X}_C + \hat{X}_D = \sqrt{2}\hat{X}_A \to 0, \quad \hat{P}_C - \hat{P}_D = \sqrt{2}\hat{X}_B \to 0. \quad (2)$$

Let us define a quadrature with an arbitrary phase $\phi$ as $\hat{X}(\phi) = \cos(\phi)\hat{X} + \sin(\phi)\hat{P}$. Then the correlations (2) can be shortly rewritten as

$$\hat{X}(\phi) + \hat{X}(-\phi) = \sqrt{2}\cos(\phi)\hat{X}_A + \sqrt{2}\sin(\phi)\hat{X}_B \to 0. \quad (3)$$

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The physical essence of this expression is a clear correlation symmetry of two entangled output modes C and D. A quadrature of the mode C with a phase $\phi$ is correlated with a ‘mirror’ quadrature of the mode D with a phase $-\phi$. In other words, any measurement result of a quadrature $\hat{X}_C(\phi)$ is opposite (in the limit $r \to \infty$) to the measurement result of a ‘mirror’ quadrature $\hat{X}_D(-\phi)$. As we show below, this type of correlations is similar to the triplet Bell states, thus we call it ‘triplet-like’. It is worth noting that the rotation in two-dimensional phase space is only one kind of unitary transformation of the continuous variable states. This phase space rotation corresponds to the $z$-rotation on the Bloch sphere (see the appendix), which does not change the absolute value of the complex coefficients in the superposition of two qubit states. If one involves unitary transformations which change the absolute values of coefficients of the Fock state expansion of the state, then it might be possible to construct CV analogue of the singlet Bell state. We leave the last question, however, for a future study.

3. Correlation properties of two-qubit states

In the DV case, the basis of maximally entangled states consists of four Bell states—the fully isotropic singlet state and three triplet states

$$\Phi^{\pm} = ((|0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B)/\sqrt{2}, \quad \Psi^{\pm} = ((|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B)/\sqrt{2},$$

(4)

corresponding to the total spin $S = 0$ and $1$ manifold. The property of rotational invariance of the singlet Bell state $\Psi^{-}$ is widely known and used in various experiments with qubits. Other Bell states, so called triplet states $\Psi^{*}$, $\Phi^{\pm}$, do not have this property. Instead, they have a different symmetry which is also used in various quantum information tasks such as hidden-variable models, separability criterion, entanglement distillation and characterization [11]–[14].

As we show in the appendix, all triplet states follow certain $U \otimes U^*$ invariances, which means, that if one applies a unitary operation $U$ on the first subsystem and a unitary operation $U^*$ on the second subsystem, the joint state remains unchanged. Consequently, when measuring in a basis obtained by a $U \otimes U^*$ transformation of the standard computational basis, the observed correlations between local measurements on the subsystems are unchanged. As we show in the appendix, the relative symmetry between two perfectly correlated measurements is either central symmetry (for the singlet state) or mirror symmetry (for the triplet states). The latter case of mirror symmetry appears to be very similar to the ‘triplet-like’ symmetry of continuous variable states, which is described in the previous section.

The observed similarity is not exactly straightforward, as neither of three triplet states possesses full $U \otimes U^*$ or $U \otimes U^*$ invariance. However, in some cases it is sufficient to have lower symmetry, e.g. in the case of certain decoherence–free subspaces. For example, it has been shown how to realize decoherence–free quantum memory using encoding of a qubit in properly chosen Bell states [16]. The similarity we found can help us to identify decoherence-free subspaces also in the CV case. Referring to the note at the end of section 2, the similarity between the CV and the DV entangled case is most pronounced for the $\Psi^{-}$ and $\Phi^{\pm}$ states, which show $U \otimes U^*$ invariance for rotations around the $z$-axis.

4. Description of the experiment

In the experiment, we investigate the correlation pattern underlying the two-mode CV entangled states and demonstrate its triplet-like nature. Instead of the usual quadrature variables in single
mode phase space, we perform the experiment in two mode polarization space described by the Stokes variables $S_1$, $S_2$, $S_3$, which are convenient for multimode polarization entanglement characterization [17]. These polarization Stokes variables have the distinct experimental advantage over the quadrature variables [18]–[20] that the characterization of all relevant parameters of the polarization variables is possible by linear optical manipulations and direct detection [21]. Therefore, a highly efficient continuous variable polarization entangled source is used for the experimental demonstration [22].

The experimental setup is shown in figure 1. The preparation of the highly entangled polarization state is accomplished in two steps: generating two independent polarization squeezed beams A and B with the same noise properties, and producing polarization entanglement by interference of the two beams on a 50/50 beam splitter (BS). In the inset of figure 1, the setup for the generation of polarization squeezed beam is shown: a home-made Cr$^{4+}$:YAG laser with a central wavelength of 1497 nm was used to produce soliton-shaped...
pulses at a repetition rate of 163 MHz with a duration of 140 fs. These pulses were measured to be shot noise limited at our measurement frequency (17.5 MHz) and thus can be assumed to be coherent. We exploit the method based on single pass of two orthogonally polarized light pulses through a birefringent fibre (13.2 m of 3M FS-PM-7811, mode field diameter 5.7 µm, beat length 1.67 mm) [23, 24], two quadrature squeezed states were then independently generated.

The two light pulses were adjusted to have the same optical powers after the fibre, the generated squeezed states thus have approximately the same quadrature noise property (the squeezed quadrature is skewed by angle \( \theta_{sq} \) from the amplitude quadrature). To generate the polarization squeezing, the two emerging pulses were overlapped with a \( \pi/2 \) relative phase shift. This was accomplished by using an active phase lock in the pre-compensation of the fibre birefringence which introduced a \( \delta \phi \) relative shift between the two polarization eigenmodes of the fibre (figure 1). This resulted in a circularly polarized beam at the fibre output, mathematically described by \( \langle \hat{S}_3 \rangle \neq 0 \) and \( \langle \hat{S}_1 \rangle = \langle \hat{S}_2 \rangle = 0 \). The conjugate polarization operators, which can exhibit polarization squeezing, are then found in the plane given by \( \hat{S}_1 - \hat{S}_2 \), referred to as the ‘dark plane’. We also define a quadrature \( S(\theta) = \cos(\theta)S_1 + \sin(\theta)S_2 \). Our polarization squeezing is derived from Kerr squeezed states in which the squeezed quadrature is skewed by \( \theta_{sq} \) from the amplitude direction. Thus the squeezed Stokes operator is given by \( \hat{S}(\theta_{sq}) \), and the orthogonal, anti-squeezed Stokes operator is \( \hat{S}(\theta_{sq} + \pi/2) \), which will be written as \( \hat{S}(\theta_{asq}) \) for convenience [21, 23]. These operators both have zero mean values, and they both commute with the bright \( \hat{S}_3 \) component of the optical field. Therefore, such polarization squeezing is mathematically equivalent to vacuum squeezing, furthermore we can have the correspondence between \( \hat{S}(\theta_{sq}) \) and \( \hat{S}(\theta_{sq} + \pi/2) \) on the one hand and the two single mode conjugate variables \( \hat{X} \) and \( \hat{P} \).

Two such polarization squeezed beams are then simultaneously generated and mixed on a 50 : 50 BS (figure 1). The two resulting intense beams, labelled C and D, are set via a phase lock to have equal intensity, i.e. the two inputs are set to have a \( \pi/2 \) relative phase shift. Thus the two outputs of the BS become entangled and they are also circularly polarized. The correlation pattern that the polarization entanglement follows between the two modes can be easily achieved according to correspondence between the quadrature variables and the polarization variables, which is expressed by \( \hat{S}_C(\theta_{sq} + \phi_1) + \hat{S}_D(\theta_{sq} + \phi_2) \rightarrow 0 \) and \( \hat{S}_C(\theta_{asq} + \phi_1) - \hat{S}_D(\theta_{asq} + \phi_2) \rightarrow 0 \) with \( \phi_1 = -\phi_2 \). These beams are then measured independently in two Stokes measurement apparatuses. Each one is composed of only a half-wave plate (\( \lambda/2 \)) followed by a polarizing BS (PBS). Rotation of the half-wave plate allows for the observation of arbitrary Stokes parameters in the dark \( \hat{S}_1 - \hat{S}_2 \) plane, and, therefore, allows for the direct observation of the triplet-like correlation pattern underlying the CV polarization entanglement. In each Stokes measurement the outputs of the PBS are detected by identical pairs of balanced photo-detectors based on custom made pin photo-diodes (98% quantum efficiency at dc and ac). The detection frequency of 17.5 MHz was chosen to avoid low frequency technical noise as well as the 163 MHz laser repetition rate, although in principle any frequency up to several THz is possible [25]. The four detected ac photocurrents are passively pairwise subtracted. The resulting subtracted pairs are added and monitored on a spectrum analyser (HP 8590E, 300 kHz resolution bandwidth, 30 Hz video bandwidth).
Figure 2. Measurement of the noise of the entangled beam pair along $\hat{S}(\theta_{sq})$ and $\hat{S}(\theta_{sq} + \pi/2)$. The noise of the individual beams $\hat{S}_{C,D}(\theta_{sq})$ and $\hat{S}_{C,D}(\theta_{sq} + \pi/2)$ is plotted on the left side, the correlations $\Delta^2(\hat{S}_C(\theta_{sq}) + \hat{S}_D(\theta_{sq}))$ and $\Delta^2(\hat{S}_C(\theta_{sq} + \pi/2) - \hat{S}_D(\theta_{sq} + \pi/2))$ are plotted on the right side. Note the difference in the level of correlations of the two signals, which is a consequence of a residual asymmetry of the splitting ratio of the entangling BS together with the high level of excess noise. The measurement frequency was 17.5 MHz, the resolution bandwidth was 300 kHz and the resolution bandwidth was 30 Hz.

5. Results

Based on the setup as depicted in figure 1, we implemented the polarization measurements. The results of the characterization of a two-mode CV polarization entangled source are presented in the following. In the results, the variances $\Delta^2_{\text{norm}}(\cdot)$ are normalized to the respective mean values of the $\hat{S}_3$ parameter corresponding to the shot noise reference. As a first step, the polarization squeezing of the two input modes A and B was measured. In order to characterize the initial squeezing of one input mode, we blocked the other input mode and measured the polarization squeezing of the output modes C and D. From the observed level of squeezing, we can infer the amount of squeezing in the input modes. Polarization squeezing of $-4.6 \pm 0.3$ dB was observed for the $\hat{S}_A(\theta_{sq})$ parameter of source A. Its canonical conjugate, $\hat{S}_A(\theta_{asq})$, was anti-squeezed by $+22.3 \pm 0.3$ dB. The second beam exhibited similar squeezing levels of $-4.5 \pm 0.3$ dB in $\hat{S}_B(\theta_{sq})$ and of $+22.2 \pm 0.3$ dB in $\hat{S}_B(\theta_{sq})$. These noise traces as well as those for the polarization entanglement are corrected for electronic noise. The individual squeezed beams A and B exhibited a total optical power of 9.4 mW, corresponding to an energy of 61 pJ per pulse. The squeezing angle $\theta_{sq}$ was $4^\circ$.

To generate the maximum polarization entanglement (figure 1), the interference visibility between the squeezed input modes A and B was optimized achieving $\geq 98\%$. Then we investigate the variances of the conjugate Stokes operators $\hat{S}(\theta_{sq})$ and $\hat{S}(\theta_{sq} + \pi/2)$ of the individual output modes C and D, as plotted in figure 2. Each individual mode is seen to exhibit a large excess independent of the angle $\theta$ (around 19 dB have been measured). The sum and difference signals of these two modes are $\Delta^2_{\text{norm}}(\hat{S}_C(\theta_{sq}) + \hat{S}_D(\theta_{sq})) = -3.4 \pm 0.3$ dB or $0.46 \pm 0.03$ and $\Delta^2_{\text{norm}}(\hat{S}_C(\theta_{sq} + \pi/2) - \hat{S}_D(\theta_{sq} + \pi/2)) = -2.9 \pm 0.3$ dB or $0.51 \pm 0.03$, respectively.
Figure 3. Measurement of the correlated variance of the entangled beam pair between the Stokes parameters $\hat{S}_C(\theta_{\text{asq}} + \phi_1)$ and $\hat{S}_D(\theta_{\text{asq}} + \phi_2)$ for the two cases: (a) the phase rotation angle $\phi_2$ was set to be 45° skewed from the anti-squeezing quadrature, while the $\phi_1$ on beam C side was scanned from 0 to $-90°$ relative to the anti-squeezing quadrature; (b) both $\phi_1$ and $\phi_2$ were rotated with the same angle but opposite directions for $90°$, that is, $-\phi_1 = \phi_2 = \phi \in (0, 90°)$.

In the next step, we set both $\lambda/2$ wave-plates in the two Stokes measurement setups along the anti-squeezing directions as the reference for the further rotations, and investigated the correlations between the Stokes parameters $\hat{S}_C(\theta_{\text{asq}} + \phi_1)$ and $\hat{S}_D(\theta_{\text{asq}} + \phi_2)$. To check the correlation dependence on the relevant rotation angles $\phi_1, \phi_2$, we first rotated the $\lambda/2$ wave-plate in mode D clockwise for 22.25°, that is, the Stokes parameter observed in beam D side was skewed by $\phi_2 = \pi/4$ from the anti-squeezing direction. Then we implemented the measurements of correlation between C and D by scanning $\phi_1$ of the beam C in the counterclockwise direction of from 0 to $-\pi/2(-90°)$ with the step of 5°. The results are plotted in figure 3(a), and we see from the results that the best correlator is achieved when $\phi_1 = -\phi_2 = -45°$, which corresponds to a measured correlation of $\Delta^2(\hat{S}_C(\theta_{\text{asq}} - 45°) - \hat{S}_D(\theta_{\text{asq}} + 45°)) = -3.1 \pm 0.3$ dB. In addition, we further verified the triplet-like correlation underlying modes C and D by scanning $\phi_1$ and $\phi_2$ in opposite directions with the same rotation step width, i.e. $-\phi_1 = \phi_2 = \phi$. The results for the observed correlation as a function of rotation angle $\phi$ from the anti-squeezing direction in the phase space are also plotted in figure 3(b). Therefore, the correlations between the Stokes parameters $\hat{S}_C(\theta_{\text{asq}} - \phi)$ and $\hat{S}_D(\theta_{\text{asq}} + \phi)$ were measured to be highly non-classical when $\phi$ was varied from 0 to $\pi/2$. The variation of the observed correlations was attributed to the slight asymmetry of the 50/50 BS (the actual transmittivity was measured as 0.49) [22] and the imperfect interference ($\geq98\%$ visibility) in the setup for producing the entanglement.

6. Summary
In summary, we found a remarkable similarity between two-mode continuous variable entangled states and the two-qubit Bell states. We demonstrate that even a mixed two-mode continuous variable entangled state with large excess noise in one quadrature shows this similarity. Furthermore, it is the first time that the triplet-like correlation pattern of continuous variable states has been discussed and demonstrated experimentally. Our result constitutes a
Figure A.1. Geometric picture of the general rotation (A.1) on the Bloch sphere.

step towards uniform consideration of discrete and continuous cases. However, the observed similarity is not exactly straightforward and should be further investigated. Particularly interesting would be to find an analogy of the singlet Bell state in the continuous variable case, which would come as close as possible to having full $U \otimes U$ invariance.

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Appendix. Symmetry properties of two-qubit Bell states

The general rotation $U$ on the Bloch sphere can be parametrized by three angles $\{\theta, \varphi, \alpha\}$ as

$$U(\theta, \varphi, \alpha) = \begin{pmatrix}
\cos(\alpha/2) & -\cos \theta \sin(\alpha/2) & -i \sin \theta \cos \varphi - i \sin \theta \sin \varphi \sin(\alpha/2) \\
-i \sin \theta \cos \varphi + i \sin \theta \sin \varphi \sin(\alpha/2) & \cos(\alpha/2) + i \cos \theta \sin(\alpha/2) & 0 \\
0 & 0 & 1
\end{pmatrix}.$$  

(A.1)

Geometrically, two angles $\{\theta, \varphi\}$ define an axis orientation, and the angle $\alpha$ defines rotation around this axis (see figure A.1).

Applying the transformation $U \otimes U$ to the singlet state $\Psi^-$ one can check its invariance. The triplet states do not hold this property. Instead, we look at the lower symmetry with respect to the rotations around $x$-, $y$- and $z$-axis by any continuously varying angle $\alpha$. Specifying $x$, $y$ or $z$ as the rotation axis, $U(\theta, \varphi, \alpha)$ (equation A.1) turns into:

$$U_x = \begin{pmatrix}
\cos(\alpha/2) & -i \sin(\alpha/2) \\
-i \sin(\alpha/2) & \cos(\alpha/2)
\end{pmatrix}, \quad U_y = \begin{pmatrix}
\cos(\alpha/2) & -\sin(\alpha/2) \\
\sin(\alpha/2) & \cos(\alpha/2)
\end{pmatrix},$$

$$U_z = \begin{pmatrix}
e^{-i(\alpha/2)} & 0 \\
0 & e^{i(\alpha/2)}
\end{pmatrix}.$$  

(A.2)

By direct calculations of the transformations $U_i \otimes U_i$ and $U_i \otimes U_i^*$ we observe that the Bell states demonstrate a particular type of invariance, which is summarized in table A.1.

We can see that each of the Bell state is invariant under three rotations out of six. All triplet states are similar to each other in a sense that they are invariant under two $U \otimes U^*$ rotations out of three, and additionally have one complementary $U \otimes U$ symmetry. The transformations
Table A.1. Symmetric invariances of the Bell states under any continuous rotations around x-, y- and z-axis.

|   | $U_x \otimes U_x$ | $U_y \otimes U_y$ | $U_z \otimes U_z$ | $U_x \otimes U_y^*$ | $U_y \otimes U_z^*$ | $U_z \otimes U_x^*$ |
|---|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| $\Psi^-$ | Yes | Yes | Yes | No | No | No |
| $\Psi^+$ | No | No | Yes | Yes | No | No |
| $\Phi^-$ | Yes | No | No | Yes | Yes | Yes |
| $\Phi^+$ | No | Yes | No | Yes | Yes | Yes |

Figure A.2. Geometric picture of the correlation properties of the Bell states. The thick arrow represents a measurement result $\{\theta, \phi\}$ of the first subsystem on the Bloch sphere. The dashed arrows 1, 2, 3 and 4 represent perfectly correlated results $\{\theta', \phi'\}$ of a measurement of the second subsystem for the states $\Psi^-$, $\Psi^+$, $\Phi^-$ and $\Phi^+$, respectively.

$U^*$ which are conjugate to (A.2) can be rewritten as a reverse rotation by the same angle: $U^*(\alpha) = U(–\alpha)$. Here, we can clearly see the similarity of the triplet Bell states and CV entangled states described in section 2.

In certain cases it is important to have bipartite states which are invariant under a specific type of rotation. E.g. in experiments with decoherence-free quantum memory [16], the influence of the environment (i.e. small fluctuations of the large magnetic field oriented along the z-axis) results in uncontrollable rotations around the z-axis. By looking at table A.1, we can easily see, that the corresponding invariant states are $\Psi^-$ and $\Psi^+$. When dealing with different decoherence mechanisms one may consult table A.1 for picking the corresponding decoherence free subspace.

The general rotation (A.1) is parametrized by three angles $\{\theta, \phi, \alpha\}$, however, a point on the sphere can be specified by only two of them, e.g. $\{\theta, \phi\}$. A particular transformation from one point to another point can be achieved by two appropriate consecutive transformations (A.2), i.e. can be specified by two parameters $\{\theta, \phi\}$. Respectively, a complimentary operation $U^*$ can be specified by another pair of $\{\theta', \phi'\}$. With help of parameterization of $U$ by $\{\theta, \phi\}$ and $U^*$ by $\{\theta', \phi'\}$, we can visualize them geometrically on the Bloch sphere (figure A.2). Symmetry properties of the Bell states presented in table A.1 can be shown as symmetry properties of two points, or arrows, on the Bloch sphere.

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From this figure, we can explicitly see that the symmetry properties of the Bell states are of two distinctive types: singlet and triplet. The singlet state $\Psi^-$ follows inversion symmetry, i.e. perfectly correlated measurement results are reflection of each other at the centre of the sphere (the thick arrow and arrow 1 in figure A.2 are two correlated states, oriented in opposite directions). The triplet states follow mirror reflection symmetry, i.e. two perfectly correlated measurement results are mirror reflected in a certain plane ($xy$, $yz$ or $xz$).

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