Spin Profile of Galactic Halos and Disk Formation

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Abstract. We summarize recent developments in the study of the origin of halo spin profiles and preliminary implications on disk formation. The specific angular-momentum distributions within halos in N-body simulations match a universal shape, $M(<j) \propto j/(j_0+j)$. It is characterized by a power law over most of the mass, and one shape parameter in addition to the spin parameter $\lambda$. The angular momentum tends to be aligned throughout the halo and of cylindrical symmetry. Even if angular momentum is conserved during baryonic inflow, the resultant disk density profile is predicted to deviate from exponential, with a denser core and an extended tail. A slightly corrected version of the scaling relation due to linear tidal-torque theory is used to explain the origin of a typical power-law profile in shells, $j(M) \propto M^s$ with $s \gtrsim 1$. While linear theory crudely predicts the amplitudes of halo spins, it is not a good predictor of their directions. Independently, mergers of halos are found to produce a similar profile due to $j$ transfer from the orbit to the product halo via dynamical friction and tidal stripping. The halo spin is correlated with having a recent major merger, though this correlation is weakened by mass loss. These two effects are due to a correlation between the spins of neighboring halos and their orbit, leading to prograde mergers.

1. Introduction

Disk galaxies that form in cosmological hydro simulations are surprisingly smaller and of lower spin than observed disks, posing an “angular-momentum problem” (e.g. Navarro & Steinmetz 1997, 2000). A key ingredient in the origin of galactic rotation, which can be studied in detail, is the distribution of angular momentum in the dark halo within which baryonic inflow is assumed to form the disk. We describe here several advances in the investigation of the origin of halo angular momentum and its distribution, and preliminary implications on disk formation, based on high-resolution N-body simulations and analytic approximations. The universal $j$ profile (§2) and the implied disk structure (§3) are presented in Bullock et al. (2000, BDK+). The issues of tidal-torque theory (§4) are studied by Porciani, Dekel, & Hoffman (2000, PDH) and the scaling relation is by Porciani & Dekel (2000, PD). The $j$ profile due to mergers (§5) is by Dekel & Burkert
2. A Universal Halo Spin Profile

We use a high-resolution N-body simulation of the ΛCDM cosmology ($\Omega_m = 0.3$) inside a $60 h^{-1}$Mpc box, with particles of $10^9 M_\odot$ and force resolution of $2 h^{-1}$kpc (ART code, Kravtsov et al. 1997). Spherical halos are identified around density maxima using an NFW density profile; they contain a mass $M$ inside a virial radius $R$, defined such that it encompasses a mean overdensity of 340. We focus here on 500 isolated halos with $M > 10^{12} h^{-1} M_\odot$.

We find that the mass distribution of specific angular momentum $j$ is well fit by a universal profile:

$$M(< j) = \mu M j/(j_0 + j), \quad \mu > 1.$$  \hspace{1cm} (1)

This is roughly a power law for $j < j_0$, which flattens off if $j > j_0$. The shape parameter $\mu$ correlates with the fraction of mass in the power-law regime, which always contains more than half the mass.

One can replace $j_0$ with a robust spin parameter $\lambda' \equiv J/(\sqrt{2} M V_c R)$, where $V_c^2 = GM/R$. It coincides with the standard spin parameter $\lambda$ for an isothermal sphere and for an NFW profile with concentration $C \sim 10$. The pairs $(\mu, j_0)$ and $(\mu, \lambda')$ are related via

$$j_0 b(\mu) = \sqrt{2} V_c R \lambda', \quad b(\mu) \equiv -\mu \ln(1 - \mu^{-1}) - 1.$$ \hspace{1cm} (2)

Either pair can fully characterize the $j$ profile. Fig. 1 shows a few examples. Fig. 2 (left) shows a certain correlation between $\mu$ and $\lambda'$, with a correlation coefficient $r = 0.23$, meaning that high-spin halos tend to have power-law profiles.
We find that the direction of the angular-momentum vector tends to be aligned throughout the halo; for more than half the mass, $j$ and $j_z$ typically differ by less than a factor of 2. The spatial symmetry of $j$ tends to be more cylindrical than spherical; in 80% of the halos, the mean value of $j$ (over longitude) decreases by less than a factor of 2 when moving away from the equatorial plane along lines parallel to the spin axis.

3. Disk Structure: Core and Tail

A simple model can convert the halo $M(<j)$ into a disk density profile. Assume that the gas initially traces the $j$ distribution of the halo, and that $j$ is conserved during the gas contraction into a rotation-supported disk of mass $f M$. Then, using Eq. 1, the disk mass inside radius $r$ is $\propto j(r)/[j_0 + j(r)]$. Under the assumption of an isothermal halo with a flat rotation curve, one has $j(r) = r V_c$, and then the disk surface-density profile is

$$\Sigma_d = f \frac{\mu M}{2\pi} \frac{r_d}{r (r_d + r)^2} , \quad r_d = \frac{\sqrt{2\lambda R}}{b(\mu)} . \quad (3)$$

The characteristic length scale $r_d$ is based on Eq. 2.

A more accurate numerical calculation using an NFW profile, that self-consistently solves for the disk circular velocity at $r$ and takes into account the adiabatic contraction of the halo in response to the baryonic infall, leads to the typical profile shown in Fig. 2 (right). Compared to an exponential disk (which is similar to the profile obtained from a uniform, solid-body rotating halo), our resultant profile has a core and tail excess, with somewhat higher densities both at small and large radii. Since observed (stellar) disks are closer to exponential,
this result may either indicate that \( j \) is not exactly preserved during the collapse, or it may require some additional viscous processes (e.g. Lin & Pringle 1987). In some cases, the obtained slow-rotating core may be associated with a bulge. The extended disk, on the other hand, may be useful in explaining the damped Lyman-alpha systems observed at high redshift (Maller et al. 2000).

Our result for the limiting case of \( j \) conservation clearly sharpens the big "angular-momentum problem", because \( j \) transport from gas to dark matter and from inside out can only worsen any agreement with observed disks. It seems that one needs to appeal to some process of gas removal from subclumps before they merge into the halo center, and thus reduce \( j \) transport.

4. Spin Profile based on Tidal Torque Theory

We find in the simulations that the halo \( j \) profile in spherical shells is roughly a power law, \( j(M) \propto M^s \), with \( s \) distributed like a Gaussian: \( s = 1.3 \pm 0.3 \) (see an early indication by Barnes & Efstatiiou 1987). Here \( j \) is the specific angular momentum in a shell of radius \( r \), and \( M \) is the mass in the sphere encompassed by this shell. This profile can be related to the distribution \( M(< j) \) under the assumptions of spherical symmetry for the density and cylindrical symmetry for \( j \). Linear tidal-torque theory (TT) provides one hint for the origin of this power law, as follows.

TT (e.g. White 1984) implies that the \( j \) gained by a halo at time \( t \) before its turn-around is

\[
J_i(t) = a(t)^2 \hat{D}(t) \epsilon_{ijk} T_{jl} I_{lk},
\]

(4)

where the time growth is from some fiducial initial time \( t_i \), \( I_{lk} \) is the inertia tensor of the proto-halo at \( t_i \), and \( T_{jl} \) is the tidal tensor at the halo center, smoothed on the halo scale. This is based on assuming the Zel’dovich approximation for the velocities inside the proto-halo, and a 2nd-order Taylor expansion of the potential (see PDH). We show elsewhere (PD) that the standard scaling relation of TT should be slightly modified; it should read

\[
j \propto t_c \sigma(M) M^{2/3},
\]

(5)

where \( t_c \) is the turn-around time of the halo (assuming that the relevant cosmology at \( t_c \) is still EdS) and \( \sigma(M) \) is the rms density fluctuation on scale \( M \) at \( t_i \).

To obtain the spin profile we apply Eq. 5 shell by shell. The turn-around time of each shell is determined by \( \delta(M)D(t_c) \approx 1 \), where \( \delta(M) \) is the mean density inside \( M \). The densities within the different shells are correlated, such that, for a Gaussian field, the typical density fluctuation profile about a random point scales like \( \delta(r) \propto \xi(r) \), where \( \xi(r) \) is the linear two-point correlation function (Dekel 1981). This is accurate to a few percent also around a high peak (Bardeen et al. 1986, Fig. 8). Thus, for a power-law power spectrum \( P_k \propto k^n \) and a flat universe, we obtain for the \( j \) profile within each halo

\[
j(M) \propto M^{2/3+(3+n)/3}.
\]

(6)

This implies \( 1 < s < 4/3 \) for \( -2 < n < -1 \), the range appropriate for large galactic halos in a CDM spectrum, in pleasant agreement with our finding for
the simulated halos. A similar analysis, using the accretion rate into halos based on the Extended Press-Schechter approximation, also explains a weak anti-correlation detected in the simulations between \( s \) and halo mass (BDK+).

Using simulated halos (in GIF simulations), PDH find that TT predicts an amplitude for the halo spin in the right ball-park. This confirms the relevance of TT to the \( j \) distribution in halos. On the other hand, PDH find that the predicted spin direction is only weakly correlated with the true direction, with an average deviation of 53°. This reflects large inaccuracies in the smoothing of the tidal tensor and the truncation of the Taylor expansion of the potential. This relative failure of TT in predicting the spin direction should be born in mind when trying to use TT for the analysis of spin-spin correlations and weak-lensing phenomena.

5. Spin Profile due to Mergers

A simple toy calculation for how \( j \) is being deposited in the halo during a merger provides another hint for the origin of the \( j \) profile. Consider a fixed halo and an incoming satellite of mass profiles \( M(r) \) and \( m(\ell) \) respectively. Assume that as it moves in, the satellite is losing mass outside a tidal radius \( \ell_t \), determined at \( r \) by

\[
m(\ell_t)/\ell_t^2 = \ell_t \mu(r)/r^3 , \quad \mu(r) = [2M(r) - r \, dM/dr] .
\] (7)

For isothermal halos this implies \( m(r) \propto M(r) \propto r \). Now assume that the satellite is spiraling in due to dynamical friction in circular orbits, and that the mass and \( j \) are deposited locally. Then \( j(r) \) is obtained by averaging over shells,

\[
4\pi r^2 \rho(r) j(r) = m(r) \frac{d[V_c(r)]}{dr} + \frac{dm(r)}{dr} rV_c(r) .
\] (8)
The $j$ transfer is due to both the slowdown of the satellite by dynamical friction and the direct tidal stripping of mass. For an isothermal halo, we obtain

$$j(M) \propto M.$$  \hfill (9)

Fig. 3 (left) shows the $j$ profile for an NFW halo swallowing a satellite that is loosing mass either according to the recipe $m(r) \propto M(r)$ or based on Eq. 7 assuming that the satellite has an isothermal profile, compared to the case of no mass loss. The general shape of $j(M)$ in the outer decade of mass, when realistic mass loss is considered, is reminiscent of the profiles detected in the cosmological simulation.

Full $N$-body simulations of mergers, spanning a range of halo and collision parameters, confirm the robust production of such power-law like profiles (DB).

The resulting $j(M)$ from a sequence of minor mergers would be a sum of similar contributions, each projected onto the direction of the total net angular momentum (perhaps determined by the most major merger).

Since halo accretion histories typically reflect some combination of relatively quiescent mass accretion and more pronounced mergers, the $j$ profiles of individual halos may reflect a combination of the two processes explored above. We see that the profiles produced by each of the two processes are, in general, of similar shape to the profiles measured in the simulations.

6. Spin and Merger History

Further investigations of halos in the ART and GIF simulations (WBD+) reveal interesting correlations between halo spin and merger history. For example, halos that had a major merger in the last Gyr tend to have higher spins, $\langle \chi \rangle = 0.055$, in comparison with the average of 0.035 for all halos. This is hardly surprising because most of the orbits of merging halos are expected to be of large impact parameter and therefore to bring in much angular momentum.

On the other hand, we find that major mergers also result in effective loss of $j$, typically by a factor of ~2 of the original orbital $j$. This spin loss is associated with mass loss of the weakly bound outer layers of the halos which are preferentially $j$-rich, Eq. 1. This spin loss somewhat weakens the tendency for high spin in merger products.

These effects may be correlated with the fact that major mergers tend to be prograde, with the spins of the two merging halos oriented roughly parallel to each other and to the angular momentum of their mutual orbit. Fig. 3 (right) shows the average $\cos(\theta)$ between the two spins and between the spin and orbit for pairs of halos at a given separation $r$. There is a significant alignment at $r < 400 h^{-1}$kpc, growing as a power law towards smaller distances. Prograde encounters indeed explain both the high spin and the high spin loss. Being prograde they are more effective in transferring energy from the orbit to internal motions and thus making the typical impact parameter for mergers even larger. This, plus the coherency of progenitor spins and orbital $j$, results in a high spin. But at the same time a prograde encounter causes effective stripping of $j$-rich mass.

High spin due to major mergers seems not to go naturally with the common wisdom that such mergers produce low-spin bulges and elliptical galaxies. How-
ever, it is the halo properties at $z \sim 1$ (say) rather than today that are likely to
determine the $j$ content of the gas which later falls in to form the disk. Then,
it is the halo merger history prior to $z \sim 1$ that is more relevant for the galactic
spin, while later mergers govern the bulge-to-disk ratio. The high-spin halos at
$z \sim 1$, those that were assembled and virialized not long before $z \sim 1$, may be
the hosts of high-spin disks that are observed as low-surface-brightness galaxies
(Dekel et al. 2001).

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