Characteristics of ion acoustic modified Korteweg de Vries (KdV) solitons in multicomponent plasma with negative ions

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Abstract. Propagation characteristics of ion acoustic solitons in multicomponent plasma with negative ions have been investigated experimentally in a new double plasma device. At a critical concentration of negative ions $r_c$, simultaneous evolution of compressive/rarefactive solitons (modified KdV solitons) from an initial compressive/rarefactive density perturbation has been observed. The Mach velocity and width of the solitons are measured and compared with the solutions of the modified KdV equation.

1. Introduction
Solitons are nonlinear waves, which are formed by delicate balance between nonlinearity and dispersion. In plasmas, ion acoustic waves are able to propagate as solitons and are described by the well known KdV equation [1]. After the first experimental detection by Ikezi et al. [2], ion acoustic KdV solitons have been studied extensively over the years [3-5]. While propagating through plasma, the KdV solitons have velocity proportional to wave amplitude and square of its width is inversely proportional to amplitude [6,7]. These two properties are often used to identify a KdV soliton.

The KdV equation, which contains a quadratic nonlinear term, admits only compressive soliton solutions in normal two component plasma [1]. In multicomponent plasma with negative ions, the KdV equation possesses both compressive and rarefactive soliton solutions depending on the concentration of negative ions $r$ [8,9]. When the concentration of negative ions is below a critical value $r_c$, compressive solitons exist which are associated with density compression of positive ions. When concentration of negative ions exceeds the critical value, rarefactive solitons appear which correspond to density rarefaction of positive ions and density compression of negative ions. Both compressive and rarefactive solitons in multicomponent plasma with negative ions have been studied experimentally [10-12]. At the critical concentration of negative ions, the coefficient of the quadratic nonlinear term in the KdV equation becomes zero. Therefore, at $r = r_c$ a modified KdV (mKdV) equation has been derived considering higher order nonlinearity (cubic nonlinearity) [13,14].

The KdV equation, however, cannot fully interpret the compressive and rarefactive solitons in the vicinity of the critical concentration of negative ions. Hence, a nonlinear differential equation containing quadratic and cubic nonlinear terms has been derived to study ion acoustic solitons in this regime ($0 < r < r_c$ and $r > r_c$) [13]. Interestingly the solitons appearing as the solution of this nonlinear equation have characteristics different from the KdV and mKdV solitons [15]. These new kinds of solitons have been recently observed in a laboratory experiment [16]. In the limit $r = 0$ and $r = r_c$, the
nonlinear differential equation containing both quadratic and cubic nonlinear terms reduces to the KdV and mKdV equation respectively.

The mKdV equation has soliton solutions that are independent of the sign of the perturbation. As a result, the modified KdV equation allows simultaneous existence of both compressive and rarefactive solitons in multicomponent plasma when \( r = r_c \). The Mach velocity, defined as the ratio of soliton velocity to the ion acoustic velocity of the fast mode \( (C_s) \), of the mKdV soliton is proportional to the square of its amplitude and the width is inversely proportional to its amplitude, which are different from the KdV solitons.

In the present paper propagation characteristics of mKdV solitons are studied in multicomponent plasma with negative ions. The experimental setup and procedure are presented in Section 2 followed by a brief outline of the relevant theory in Section 3. Experimental results are discussed in Section 4 and Section 5 contains the conclusion.

2. Experimental setup and procedure

The experiment is carried out in a new Double Plasma Device of 110 cm in length and 55 cm in diameter. A schematic diagram of the experimental setup is shown in figure 1. Two magnetic cages with multidipole magnet arrangements are used to improve plasma density and uniformity. The chamber is divided into a source and a target section separated by a fine stainless steel mesh grid of 80% transparency. The base pressure of the chamber is 1.2x10^{-6} Torr. Ar plasma is produced independently in each section by dc discharge between tungsten filaments (1% thoriated) as cathode and magnetic cages as anode at Ar partial pressure 3 - 4 x 10^{-4} Torr. Discharge voltage and discharge current are maintained at 60 V and 50 – 100 mA respectively. Plasma parameters are measured with the help of a plane Langmuir probe made up of stainless steel disc of 6 mm diameter. A retarding potential analyzer (RPA) is used to measure the ion saturation current and ion temperature. The retarding potential analyzer consists of two fine nickel grids and a stainless steel collecting plate housed in a ceramic cage 0.3 cm thickness. The first grid is kept floating to repel the plasma electrons and the second grid is biased from – 45 V to + 45 V while the positive ion current is drawn at the collector plate biased at – 90 V. Typical plasma parameters are: electron density \( n \sim 10^{8} – 10^{9} \text{ cm}^{-3} \), electron temperature \( T_e \sim 1 -2 \text{ eV} \) and ion temperature \( T_i \sim 0.1 \text{ eV} \).

![Figure 1: Schematic diagram of the experimental setup.](image-url)

RPA: Retarding Potential Analyzer, LP: Langmuir Probe, V_f: Filament Voltage, V_d: Discharge Voltage, S: Source Plasma, T: Target Plasma, F: Filament, G: Floating Grid, A: Anode, FG: Function Generator, V_s: Source bias voltage.
Negative ions are produced by adding SF$_6$ gas into the Ar plasma and the plasma is considered to be composed of Ar$^+$ ions, F$^-$ ions and electrons. The partial pressure of SF$_6$ is varied from $1 \times 10^{-5}$ to $5 \times 10^{-5}$ Torr using a double valve arrangement. The concentration of negative ion density defined by the ratio of negative ion to the positive ion density is measured with the help of Langmuir probe monitoring the decrease in the electron and ion saturation current [17]. Ion acoustic perturbations are excited by applying a positive or negative sinusoidal voltage pulse (~ 15 µsec duration) to the anode of the source plasma. The density perturbations are detected in the target plasma by the axially movable plane Langmuir probe which is biased positively with respect to the plasma potential. Signals are then recorded in a digital storage oscilloscope.

3. Theory

A nonlinear differential equation (containing quadratic and cubic nonlinear terms) for perturbations of small amplitude and long wavelength can be derived in multicomponent plasma with negative ions when $T=0$ and is written as [12]

$$
\frac{\partial \Psi}{\partial \tau} + A S \frac{\partial \Psi}{\partial \xi} + P \Psi^3 \frac{\partial \Psi}{\partial \xi} + \frac{S}{2} \frac{\partial^3 \Psi}{\partial \xi^3} = 0
$$

$$(1)$$

$$A = \frac{3}{2(1-r)S^4} \left(1 - \frac{r}{\mu^2}\right) - \frac{1}{2}$$

$$(2)$$

$$P = \left(\frac{15}{4(1-r)S^4}\right) \left(1 + \frac{r}{\mu^2}\right) - \frac{1}{4}$$

$$(3)$$

$$S = \left(\frac{1 + r / \mu}{1 - r}\right)^{1/2}$$

$$(4)$$

Here, $S$ is the ion-acoustic velocity of the fast mode ($C_s$) normalized to $C_0 (= (kT_e/M_e)^{1/2})$, $\tau = \omega_p t$, $\xi = x / \lambda_e$, $\omega_p = (4m/n \mu M_e)^{1/2}$, $\lambda_e = (kT_e/4m)^{1/2}$, $\Psi = e\phi/kT_e = \delta n / n$, where $\delta n$ is the perturbed electron density. $T = T_i/T_e$, $r = n_-/n_+$, $\mu = M_- / M_+$, where $n_-, n_+, M_-$, and $M_+$ are the densities and masses of negative and positive ions respectively.

In figure 2, the values of $A$, $P\Psi$ and $S$ when $\Psi' = 0.10$ are shown as a function of $r$ for $\mu = M_-^*/M_+^* = 0.476$. The normalized ion acoustic velocity $S$ increases with the increase in negative ion concentration. When the negative ion concentration $r$ is increased from 0 to 0.102, the value of $A$ decreases sharply and becomes zero when $r = 0.102 (r_c)$. When $r$ is increased further, the value of $A$ becomes negative, whereas $P$ and $S$ remain positive. For $\Psi' << 1$, $P\Psi'$ is negligibly small compared to $A$ when $r = 0$. In this case, equation (1) reduces to the KdV equation. When $r = r_c (= 0.102)$, as the value of coefficient $A = 0$, equation (1) reduces to the mKdV equation

$$
\frac{\partial \Psi}{\partial \tau} + P \Psi^3 \frac{\partial \Psi}{\partial \xi} + \frac{S}{2} \frac{\partial^3 \Psi}{\partial \xi^3} = 0
$$

$$(5)$$

The soliton solution of the mKdV equation is

$$\psi = \psi_0 \text{sech}\left[(P/3)^{1/2} \psi_0 (\xi - S\tau - PS\psi_0^2 \tau / 6)\right]$$

$$(6)$$

The velocity and width of the mKdV solitons are given by
\[ M = 1 + P \psi_0^2 / 6 \]  

\[ \left( \frac{D}{\lambda_c} \left| \psi_0 \right| \right) = \left( \frac{3}{P} \right)^{1/2} \]  

4. Results and discussions

Observed signals in Ar plasma for an initial positive pulse, \( V_{ex} = 1.0 \) V, at different distances from the grid are shown in figure 3. While propagating through plasma, the leading edge of the pulse steepens due to positive nonlinearity. Steepening makes the role of dispersion larger. First soliton is created (\( X = 4 \) cm) when nonlinearity balances dispersion. At the same time, the second peak begins to emerge which develops to a soliton later. The measured Mach velocity and width of the soliton are found to follow the prediction of the KdV equation.

**Figure 2**: Values of \( A, P\psi \) and \( S \) versus \( r \). \( \psi = 0.10, \mu = 0.476 \).

**Figure 3**: Detected signals at several distances from the grid in Ar plasma.

SF\(_6\) gas is introduced into the Ar plasma carefully in small incremental steps. Simultaneous excitation of compressive and rarefactive solitary waves (nKdV solitons) in multicomponent plasma with negative ions has been observed when the concentration of negative ions \( r = r_c = 0.15 \). The measured value of \( r_c \) is very close to the theoretical value (0.102). Variation of electron density perturbation for different amplitudes of initial compressive pulse when \( r_c = 0.15 \) is shown in figure 4(a). When the excitation voltage is small (\( V_{ex} < 0.5 \) V) the pulse propagates without steepening. Because, in this case, the coefficient of quadratic nonlinear term in equation (1) becomes zero and the higher order nonlinear term is negligibly small. As the excitation voltage increases (\( V_{ex} \geq 1.0 \) V) the leading edge of the pulse steepens leading to formation of compressive soliton. It is observed that with the increase in excitation voltage, velocity of the solitary wave increases and width decreases. At the same time, a rarefactive initial pulse is applied to the source anode and propagation characteristics are observed. Figure 4(b) shows the observed density perturbations for different excitation voltages of initial rarefactive pulse in multicomponent plasma when \( r_c = 0.15 \). When \( V_{ex} \geq -1.0 \) V, leading edge of the pulse steepens and develops to a rarefactive soliton. Behind the rarefactive soliton a slow mode is observed. It is noted that the amplitude of the rarefactive soliton (figure 4(b)) is smaller than that of the compressive soliton for a fixed \( V_{ex} \).
Figure 4: Detected signals at $X = 8$ cm from the grid for different excitation voltages when $r = r_c$. (a) Positive applied pulse (b) Negative applied pulse. Measured value of $r_c = 0.15$.

Measured velocities of compressive and rarefactive solitons at $r = r_c$ are plotted in figure 5(a) as a function of normalized wave amplitude. The velocities of solitons (compressive and rarefactive) are measured for various excitation voltages of the applied pulse from the time of flight data. Measured velocities are then normalized with $C_s$ to obtain the Mach velocity. The Mach velocity of solitons is found to increase with the increase in wave amplitude. The solid curve represents theoretical value obtained from equation (7) for $P = 3.04$.

Figure 5: Measured (a) Mach velocity and (b) Width of compressive (●) and rarefactive (■) solitons as a function of normalised wave amplitude when $r = r_c$. 
Measured widths of compressive and rarefactive solitons are plotted in figure 5(b) as a function of normalized wave amplitude for the same value of \( r \). The width \( D \) of the solitons (compressive and rarefactive) at different \( \delta n/n \) is obtained by multiplying the temporal width of the solitary wave with the corresponding wave velocity. The spatial width is then normalized with the Debye length (\( \lambda_e \)). It is observed that width of the solitons decreases with increasing wave amplitude. The theoretical curve is obtained from equation (8) for \( P = 3.04 \).

5. Conclusion
Evolution and propagation characteristics of ion acoustic solitary waves (mKdV solitons) at critical concentration of negative ions \( r_c \) have been studied experimentally in multicomponent plasma with negative ions. An initial positive and negative pulse evolve into compressive and rarefactive soliton respectively when \( r = r_c \). These solitons are described by the mKdV equation. The velocity and width of the soliton agree reasonably with the theory.

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