Possible New Interactions of Neutrino and the KATRIN Experiment

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Abstract

We analyse the possible role of new interactions of neutrino in the forthcoming tritium beta decay experiment KATRIN aimed at detecting the neutrino mass with the sensitivity of 0.3 - 0.2 eV.

It is shown that under certain circumstances the standard procedure of data analysis would have to be modified by the introduction of an extra parameter describing the strength of the new interactions.

Our model simulations show that the modified procedure may improve the quality of the fit compared with the standard case. Ignoring the possibility of new interactions may lead to a systematic error in the neutrino mass determination.

Keywords: Tritium beta decay, neutrino mass, new interactions

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I. INTRODUCTION

The compelling evidence for non-zero neutrino mass has been a recent triumph of modern science. Neutrino oscillation experiments give us information on squared mass differences between different types of neutrinos. However, the absolute values of neutrino masses remain unknown. One way to find the absolute mass is to study the electron spectrum in beta decay. As suggested by Fermi in the late thirties, the deviation of the linearised spectrum (the Curie plot) from a straight line near the end point is a signal of non-zero neutrino mass.

This idea has been implemented in a number of recent experiments \cite{1,2} with all results consistent with zero mass and thus providing an upper limit on the neutrino mass. The work on the next generation tritium beta decay experiment, KATRIN, is in progress \cite{3,4}.

Theoretically, the existence of neutrino mass and the existence of new neutrino interactions are closely related. This is because the neutrino interactions described by the Standard Model cannot generate the neutrino mass while the additional interactions can. Thus a question arises: what is the potential effect of new interactions in beta decay and how is the neutrino mass measurement influenced?

This question has a long history starting from the time before V-A theory was established. Obviously there was a need to analyse all possible types of neutrino interactions in order to choose the one that was consistent with experiment. More recently, the interest in this problem was revived in \cite{5,6,7} motivated, in particular, by an unexpected experimental finding that the best fit for the squared neutrino mass turned out to be negative.

It was shown that the account of possible new neutrino interactions, such as right-handed (vector and scalar) currents can significantly affect the measured value of neutrino mass. In particular, the new interactions can drive negative the value of \(m^2\) extracted from experiment whereas the physical value \(m^2\) must be positive.

In this paper we extend the analysis of \cite{5,6,7} by using the fact that, from the point of view of tritium beta decay experiments, the neutrino spectrum can be considered degenerate. This leads to the appearance in the electron spectrum of only one extra parameter describing the strength of the new interactions. The modified electron spectrum can be computed analytically.

Thus, we are able to simulate the observed spectrum assuming that new interaction are present and have strength allowed by the existing constraints. We then can fit the simulated data by the usual spectral function (i.e. as if there were no new interactions). In such a procedure, the difference between the input and output values of the neutrino mass will describe the effect of new interactions on the neutrino mass measurement.
II. TRITIUM BETA DECAY SPECTRUM IN THE PRESENCE OF NEW INTER-ACTIONS

As was shown in [6], the integral spectrum for tritium beta decay near the end point is given by the following expression (the meaning of the effective neutrino mass \( m \) has recently been discussed in [8]):

\[
N(E) = K \left\{ \frac{1}{3} (E^2 - m^2)^{3/2} + x m \left[ E \sqrt{E^2 - m^2} - m^2 \ln \left( \frac{E + \sqrt{E^2 - m^2}}{m} \right) \right] \right\}. \tag{2.1}
\]

Here, \( E \) is the maximum neutrino energy, \( x \) is the dimensionless parameter describing the strength of the new interactions (for consistency with previous works our notations follow that of [6, 7]):

\[
x = x_R + x_{SR}
\]

\[
x_R = -2 \rho_R \frac{m_e}{\langle E \rangle} \sum_i \cos \theta_i \cos \theta_{iR}
\]

\[
x_{SR} = -2 \rho_{SR} \left( \frac{g_V^2}{G_V^2 + 3 G_A^2} \right) \sum_i \cos \theta_i \cos \theta_{iSR}
\]

Here, \( x_R \) and \( x_{SR} \) describe the strengths of right-handed and scalar right-handed interactions, respectively; they are expressed in terms of other two convenient parameters with the same physical meaning, \( \rho_R \) and \( \rho_{SR} \):

\[
\rho_R = \frac{g_R^2}{g^2} \frac{M_W^2}{M_R^2} (ME)_R
\]

\[
\rho_{SR} = \frac{g_{SR}^2}{g^2} \frac{M_W^2}{M_{SR}^2} (ME)_{SR},
\]

(2.5)

(2.6)

where the three sets \( (g, M_W); (g_R, M_R), \) and \( (g_{SR}, M_{SR}) \) refer to the coupling constants/boson mass values for the standard, right-handed, and the scalar right-handed interactions, correspondingly.

The factors \( (ME)_R \) and \( (ME)_{SR} \) account for the ratio of the hadronic matrix elements of the currents involved in tritium beta decay relative to those of the Standard Model. Each factor includes the elements of the quark CKM-type matrix generated by the appropriate non-standard interaction.

Further, there are 3 sets of angles in the above formulas: \( \theta_i, \theta_{iR}, \) and \( \theta_{iSR} \) where index \( i \) running over 1,2,3 refers to the neutrino mass eigenstates. The angles \( \theta_i \) belong to the Standard Model and arise because the standard weak interaction eigenstates are different from mass eigenstates. Similarly, if a new interaction is introduced, a new set of angles arises for the same reason. Although in general these new sets would be different from the Standard Model set (and from each other) it is usually assumed for simplicity that they are the same, i.e.

\[
\theta_{iR} = \theta_{iSR} = \theta_i.
\]

(2.7)
Under this assumption the existing experimental constraints \(^{9}\) on \(\rho_R\) and \(\rho_{SR}\) are \(^{1}\)

\[
\rho_R \leq 0.07 \quad \rho_{SR} \leq 0.1.
\]  

(2.8)

Finally, plugging these into formulas for \(x\) (and assuming that \(m_e/\langle E\rangle \sim 1\)) we obtain:

\[
|x_R| \leq 0.14 \quad |x_{SR}| \leq 0.035.
\]  

(2.9)

III. MODEL SIMULATIONS

In the context of the KATRIN experiment we have conducted a study of possible role of the new interactions by carrying out a number of simulations using Mathematica \(^{11}\) as our tool.

To generate out simulated data we used the theoretical formula (2.1) plus the “random error” term normally distributed around zero with the dispersion \(s\). The value of \(s\) was determined by a self-consistency requirement imposed by the KATRIN conditions: at the input value of neutrino mass \(m = 0.35\) eV the 1\(\sigma\) statistical error in the mass determination should be 0.07 eV\(^2\) \(^{4}\). Typically, we used as input values \(x = \pm 0.14\) and \(m = 0.35\) eV.

Regarding the energy, we assumed that \(E\) can take 20 values starting from \(E = 1\) eV through to \(E = 20\) eV with a step of 1 eV. We have tried several methods of extracting the neutrino mass from our simulated data.

In Method A we generated data according to Eq.(2.1) with non-zero \(x\) and \(m = 0.35\) eV, and then did an analysis assuming \(x = 0\) and finding the best fit for \(m\) or \(m^2\) based on Eq.(2.1) with \(x = 0\).

In Method B the data were generated in the same way as in Method A, but as our fitting formula we used Eq. (2.1) with 2 parameters (\(m\) and \(x\)) to be fitted.

In Method C the data generation method was again the same as above, but for the fitting purposes the parameters \(m\) and \(x\) were treated “asymmetrically”: the neutrino mass was considered as a fitting parameter while \(x\) took on different but fixed values.

Method A for \(x = -0.14\) yields \(m = 1\) eV (with negligible dispersion), i.e. it leads to a large (about 0.7 eV) systematic error in neutrino mass determination. The very small dispersion in \(m\) values is related to the fact that in our procedure values of \(m\) larger than 1 eV are not allowed because for \(E = 1\) eV they lead to negative values under the square roots in Eq. (2.1). Thus values we can investigate are squashed against this limit. In addition, the quality of the fit turns out to be bad (\(\chi^2/\text{d.o.f.} \simeq 6\)). For \(x = +0.14\) the method yields negative \(m^2\) values: \(m^2 = -1.39 \pm 0.08\), with \(\chi^2/\text{d.o.f.} \simeq 2.7\).

Method B was used only for \(x = -0.14\). For \(x = +0.14\), based on results of Method A, one would expect that the best fit can yield negative \(m^2\) values, but the fitting formula,

\(^{1}\) In deriving these constraints the general approach (see e.g. \(^{10}\)) is followed and it is not assumed that the right-handed quark mixing angles are equal to the left-handed ones.
FIG. 1: Contours of equal $\chi^2$ in the plane ($m$, horizontal axis, vs. $x$, vertical axis). The $\chi^2$ values on the contours, moving out of the minimum valley, are $\chi^2/dof = 1.9, 2.9, 3.9$. The value of $m$ ranges between 0.2 and 0.4. The dot without error bars is the input value.

Eq. (2.1), contains not only $m^2$, but also $m$. Therefore it cannot be used in the case of negative $m^2$ without modifications.

Method B gives much better $\chi^2/d.o.f.$ values than method A ($\chi^2/d.o.f. \simeq 0.9$). We ran 10 simulations, with the results plotted in figure 1. These results give the average output values $m = 0.65 \pm 0.11 \text{ eV}$ and $x = -0.06 \pm 0.017$. It is disappointing that the mean values for our simulation are more than 3 standard deviations away from the input values. This can be related to the fact that the contours of equal $\chi^2$ in the plane ($x$, $m$) do not enclose a small region as can be seen in Fig. 1. Indeed, there seems to be a very long valley in the vicinity of the minimum.

Finally, Method C (see Table 1) seems to better reproduce the value of $m$; its unpleasant feature is that without additional information we do not know in advance the value of the $x$ parameter that should be plugged in before the fitting starts. Because of that, one can start with the largest (by modulus) value of $x$ allowed by the modern data and then gradually lower this value and see if the quality of the fit ($\chi^2$) has improved. However, from Table 1 we see that a good fit can be obtained for a range of $x$ values, and the outstanding problem is how to narrow this range down. The challenge will be to obtain independent measurements of, or limits on, $x$. Note also, that for the same reason as in Method B, only negative values
TABLE I: Method C with input values $x = -0.14$, $m = 0.35 \, eV$.

| Trial value of $x$ | Output value of $m$, $eV$ | $\chi^2$/d.o.f. |
|-------------------|-----------------------------|------------------|
| 0                 | 1                           | 6                |
| -0.10             | 0.46 ± 0.02                 | 0.9              |
| -0.15             | 0.33 ± 0.01                 | 0.9              |
| -0.20             | 0.25 ± 0.01                 | 0.9              |

of $x$ were used.

Therefore, if the standard procedure of KATRIN data analysis produces a fit that is not good enough and/or the best fit for the neutrino mass turns out to be unphysical then the hypothesis of new interactions should be tested as described above. We have performed additional simulations with the same input values of $m$ and $x$, but with smaller dispersions $s$. We find that, with a dispersion corresponding to a $1\sigma$ statistical error in the mass determination of 0.015 $eV^2$, reliable values of $x$ and $m$ can be extracted.

IV. CONCLUSIONS

We have analysed the possible role of new interactions of neutrinos in the forthcoming tritium beta decay experiment KATRIN aimed at detecting the neutrino mass with the sensitivity of 0.3 - 0.2 $eV$.

It is shown that under certain circumstances the standard procedure of data analysis would have to be modified by the introduction of an extra parameter describing the strength of the new interactions.

Our model simulations show that the modified procedure may improve the quality of the fit compared with the standard case. We find that it is possible for the new interactions, if present, to lead to a systematic error in the mass determination from an analysis which ignores this presence. However when new interactions are included in the analysis the mass determination may still be unreliable unless

(i) the strength of the new interaction can be determined by independent experiments and used as an input parameter in the analysis of the experiment, or

(ii) the statistical (and systematic) errors in the experiment can be reduced to 0.015 $eV^2$ in the mass at $1\sigma$.

We recognise that our simulations do not include all of the details necessary for a full simulation of these effects in the KATRIN experiment. But our results indicate that such a simulation could usefully be undertaken.
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