R-Parity Violation and Scalar Leptons Production at Lepton-Antilepton Colliders

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Abstract

In the framework of models with $R$-parity violation, scalar neutrinos and scalar leptons production in the processes $l_i^+ l_j^- \rightarrow \tilde{\nu}_{k, L} Z^0$ and $l_i^+ l_j^- \rightarrow \tilde{l}_{k, L}^\pm W^\pm$ is considered. We also consider within the Minimal Supersymmetric Standard Model Higgs bosons production in the processes $\mu^+ \mu^- \rightarrow H^0 Z^0$, $\mu^+ \mu^- \rightarrow H^\pm W^\mp$.

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1. Introduction

As known, in theories with $R$-parity violation \cite{ref1}-\cite{ref7} supersymmetric particles may be produced singly as a result of their superpartners collision. In particularly, in \cite{ref8}-\cite{ref11} has been suggested scalar neutrino production virtually and in resonance at $e^+e^-$-collisions, in \cite{ref12} its production virtually and in resonance at $\mu^+\mu^-$-collisions \cite{ref1}.

However, the scalar neutrino mass is not fixed in theory and, thus, we don’t know which energies are necessary for scalar neutrinos production in resonance.

That is why it is necessary to study associated scalar neutrino production with gauge bosons.

In \cite{ref14} scalar neutrino production with photon in lepton-antilepton colliders has been considered:

$$l_i^+l_j^- \rightarrow \tilde{\nu}_k \gamma,$$  \hspace{1cm} (1)

Here we study scalar neutrinos and charged leptons production with $W^\pm$-boson and $Z^0$-boson :

$$l_i^+l_j^- \rightarrow \tilde{\nu}_{k,L} Z^0,$$ \hspace{1cm} (2)

$$l_i^+l_j^- \rightarrow \tilde{l}_{k,L} W^\pm.$$ \hspace{1cm} (3)

It is interesting to notice that the result received below for the processes (2),(3) are applicable to the tree processes of charged and pseudoscalar Higgs

\footnote{for references on $\mu^+\mu^-$ and $\mu^+\mu^-$-colliders see \cite{ref12} and \cite{ref13} respectively.}
bosons production in association with gauge bosons:

\[ \mu^+\mu^- \rightarrow H_3^0 Z^0, \]  
\[ \mu^+\mu^- \rightarrow H^\pm W^\mp, \]

in the framework of the Minimal Supersymmetric Standard Model (MSSM), see [15, 16] and references therein) at large \( \tan \beta \) limit.

We do not consider here the processes \( \mu^+\mu^- \rightarrow H_1^0 Z^0 \) because the main contribution to this processes comes from \( s \)-channel \( Z^0 \)-bosons exchange diagram of the Fig.2, however at large \( \tan \beta \) the \( s \)-channel \( Z^0 \)-boson exchange contribution is supressed as \( \frac{1}{\tan \beta} \), whereas other diagrams of the Fig.2 are enhanced as \( \tan \beta \). The \( t, u \)-channel contribution to the process \( \mu^+\mu^- \rightarrow H^0 Z^0 \) within the Standard Model has been considered in [18], for main \( s \)-channel \( Z^0 \)-boson exchange contribution see e.g. [20] references therein.

2. Results

Using \( R \)-parity violating interaction with electrons and neutrinos:

\[ \mathcal{L} = h_{ijk}(\bar{l}_i P_L l_j \tilde{u}_k + \bar{l}_i P_L \nu_j \tilde{l}_k) + H.c. \]  

we obtain the following amplitudes of processes (2),(3):

\[ M = \frac{g h_{ijk}}{\cos \theta_W} \bar{u}(k_1) i \left( a_L \frac{\hat{k}_4 \hat{Z}}{t} - a_R \frac{\hat{Z} \hat{k}_4}{u} - \frac{(k_4 Z)}{s - m_\nu^2} \right) P_L u(k_2), \]  

where \( a_L = -\frac{1}{2} + \sin^2 \theta_W, a_R = \sin^2 \theta_W, \)

\[ M = \frac{g h_{ijk}}{\sqrt{2}} \bar{u}(k_1) \left( \frac{\hat{k}_4 \hat{W}}{t} + 2\frac{(k_4 W)}{s - m_\nu^2 + i m_\nu \Gamma_\nu} \right) P_L u(k_2). \]  

\(^2\)The tree processes of Higgs bosons production in association with photons in \( \mu^+\mu^- \) collisions has been considered in [3, 4, 11, 12].
Here we neglect the lepton masses and use the following notations: \( Z_\mu, W_\mu \) is the polarization 4-vector of the \( Z^0 \) - and \( W^\pm_L \) bosons, \( s = (k_1 + k_2)^2, t = (k_1 - k_4)^2, u = (k_2 - k_4)^2 \), \( m_l, m_{\tilde{\nu}} \) are the masses of the scalar lepton and scalar neutrino respectively, \( \Gamma_{\tilde{\nu}} \) is the width of the scalar neutrino, in case of the process (3) we denote \( h_{ijk} = \sum_p h_{ipk} V_{pj} \) (we take into account that typically in SUSY models if we neglect leptons masses, the masses of scalar leptons of the all flavours are equal to each other).

For the differential cross sections of the processes (2),(3) we obtain the following results:

\[
\frac{d\sigma(l_i^+ l_j^- \rightarrow \tilde{\nu}_{k,L} Z^0)}{dt} = \frac{\alpha h_{ijk}^2}{8 \sin^2 \theta_W \cos^2 \theta_W s^2} \left( \left( \frac{a_l^2 + a_R^2}{u^2} \right) (tu - m_{\tilde{\nu}}^2 m_Z^2) + \frac{2a_L a_R (t - m_{\tilde{\nu}}^2)(u - m_{\tilde{\nu}}^2)}{tu} + \frac{sm_{\tilde{\nu}}^2}{(s - m_{\tilde{\nu}}^2)} \left( \frac{a_L}{t} - \frac{a_R}{u} \right) + \left( \frac{1}{8} m_Z^2 - \frac{1}{2} m_{\tilde{\nu}}^2 \right) \frac{s}{(s - m_{\tilde{\nu}}^2)^2} \right),
\]

(9)

\[
\frac{d\sigma(l_i^+ l_j^- \rightarrow l_{k,L}^+ W^\pm)}{dt} = \frac{\alpha h_{ijk}^2}{8 \sin^2 \theta_W s^2} \left( a \frac{1}{t} - m_W^2 m_{\tilde{\nu}}^2 \frac{1}{t^2} + b \right),
\]

(10)

\[
a = m_W^2 + m_l^2 - s - \frac{2s m_{\tilde{\nu}}^2}{(s - m_{\tilde{\nu}}^2)^2 + \Gamma_{\tilde{\nu}}^2 m_{\tilde{\nu}}^2} (s - m_{\tilde{\nu}})^2,
\]

(11)

\[
b = -1 - \frac{2m_{\tilde{\nu}}^2 s}{(s - m_{\tilde{\nu}}^2)^2 + m_Z^2 \Gamma_{\tilde{\nu}}^2} + \frac{s}{2m_W^2} \left| -1 + \frac{(s - m_l^2 - m_W^2)}{s - m_{\tilde{\nu}}^2 + im_{\tilde{\nu}} \Gamma_{\tilde{\nu}}} \right|^2,
\]

(12)

\[
t_- < t < t_+,
\]

(13)

where

\[
t_{\pm} = m_{\tilde{\nu},i}^2 + m_{Z,W}^2 - s \pm \sqrt{(m_{\tilde{\nu},i}^2 + m_{Z,W}^2 - s)^2 - 4m_{\tilde{\nu},i}^2 m_Z^2 m_{Z,W}^2}
\]

(14)

for the processes (2),(3) respectively.
After performing integration within the limits (14),(15) we obtain for the total cross sections the following result:

\[
\sigma(l_i^+ l_j^- \rightarrow \tilde{\nu}_k, L Z^0) = \alpha h_{ijk}^2 \frac{m_\tilde{\nu}^2}{8 \sin^2 \theta_W \cos^2 \theta_W s^2} (m_Z^2 + m_{\tilde{\nu}}^2 - s) + \\
\frac{4 a_L a_R m_{\tilde{\nu}}^2 (s - m_{\tilde{\nu}}^2)}{(m_Z^2 + m_{\tilde{\nu}}^2 - s)} - \frac{16 m_W^2}{2 (s - m_{\tilde{\nu}}^2)} \log \left( \frac{t_+}{t_-} \right) + \\
(t_+ - t_-)(2a_L a_R - 2(a_L^2 + a_R^2) + \frac{1}{8} s m_{\tilde{\nu}}^2 (m_Z^2 - 4m_{\tilde{\nu}}^2)) (s - m_{\tilde{\nu}}^2)),
\]

(15)

\[
\sigma(l_i^+ l_j^- \rightarrow \tilde{l}_k, L W^\pm) = \alpha h_{ijk}^2 \frac{m_{\tilde{l}}^2}{16 \sin^2 \theta_W s^2} (a \log \left( \frac{t_+}{t_-} \right) + (b - 1)(t_+ - t_-)).
\]

(16)

At \(\sqrt{s} \gg m_{\tilde{\nu}}, m_{\tilde{l}}, m_W\) the previous formulas are reduced and we have:

\[
\sigma(l_i^+ l_j^- \rightarrow \tilde{\nu}_{k,L} Z^0) = \alpha h_{ijk}^2 \frac{m_{\tilde{\nu}}^2}{4 \sin^2 \theta_W \cos^2 \theta_W s} ((a_L^2 + a_R^2) \log \left( \frac{s}{m_{\tilde{\nu}} m_Z} \right) + a_L a_R - (a_L^2 + a_R^2)).
\]

(17)

\[
\sigma(l_i^+ l_j^- \rightarrow \tilde{l}_{k,L} W^\pm) = \alpha h_{ijk}^2 \frac{m_{\tilde{l}}^2}{8 \sin^2 \theta_W s} (\log \left( \frac{s}{m_{\tilde{l}} m_W} \right) - 1).
\]

(18)

On Fig. 3.4 we present the number of events \(\tilde{\nu}_{k,L} Z^0\) and \(\tilde{l}_{k,L} W^-\) per year for the processes (2),(3) versus \(m_{\tilde{\nu}}, m_{\tilde{l}}\) at fixed \(\sqrt{s}\), at yearly luminosity \(L = 1000 fb^{-1}\). In our numerical results we suppose that scalar neutrino predominantly decay into \(W^\pm \tilde{l}^\mp\) pairs (i.e. \(\Gamma_{\tilde{\nu}} = \Gamma(\tilde{\nu} \rightarrow W \tilde{l})\)).

3. Comparison with other mechanisms of scalar leptons production

The processes [21] - [26]:

\[
e^+ e^- \rightarrow \tilde{\nu}^* \tilde{l}^\pm \tilde{l}^\mp
\]

(19)

which proceed through virtual neutral gauge bosons (and their superpartners exchanges) have a larger cross sections than the processes (2),(3) however, it becomes kinematically allowed at energies \(\sqrt{s} > 2m_{\tilde{\nu}}, (2m_{\tilde{l}})\), whereas
process (3),(4) is kinematically allowed at lower energies \( \sqrt{s} > m_{\tilde{\nu}} + m_Z, m_{\tilde{l}} + m_W \).

At LHC scalar leptons may be also produced in pairs (see [27] and references therein), however with increasing masses of the scalar leptons the cross sections are decreases faster than in case of the reactions (2),(3).

4. Higgs bosons production with gauge bosons

Using Higgs bosons interactions with leptons (A4)-(A6) we obtain in the large \( \tan \beta \) limit that the amplitudes of the processes (4),(5) are different only by coefficient from amplitudes of scalar lepton and neutrino production with \( W^{\pm} \)-boson and \( Z^0 \)-boson considered in formulas (7),(8) above:

\[
M(\mu^+\mu^- \rightarrow H^{\pm}W^{\mp}) = \frac{g^2}{2} \frac{m}{m_W} \tan \beta \bar{u}(k_1) \left( \frac{\hat{k}_4 W}{t} + 2 \frac{(k_4 W)}{s - m_H^2} \right) P_L u(k_2),
\]

\[
M(\mu^+\mu^- \rightarrow H^0 Z^0) = M_L - M_R,
\]

\[
M(\mu^+\mu^- \rightarrow H_{1,2}^0 Z^0) = M_L + M_R + M_0,
\]

where

\[
M_{L,R} = \frac{g^2}{2 \cos \theta_W} \frac{m}{m_W} \tan \beta \bar{u}(k_1) \left( a_{L,R} \frac{\hat{k}_4 Z}{t} - a_{R,L} \frac{\hat{k}_4}{u} + \frac{(k_4 Z)}{s - m_Z^2} \right) P_L u(k_2),
\]

where \( M_0 \) is the contribution from \( s \)-chanel \( Z^0 \)-bosons exchange.

Amplitudes \( M_{L,R,0} \) do not interfere with each other and for differential cross section of the process (4) we obtain:

\[
\frac{d\sigma(\mu^+\mu^- \rightarrow H_{1,2}^0 Z^0)}{dt} = \frac{\pi \alpha^2}{8 \sin^4 \theta_W \cos^2 \theta_W s^2 m_W^2} \tan^2 \beta ((a_L^2 + a_R^2)
\]

\[
\left( \frac{1}{u^2} + \frac{1}{t^2} \right) (tu - m_3^2 m_Z^2) + \frac{4a_L a_R (t - m_3^2) (u - m_3^2)}{tu} +
\]

\[
\frac{sm_3^2}{s - m_3^2} (a_L + a_R) \left( \frac{1}{t} - \frac{1}{u} \right)
\]
Differential cross section of the process (5) and total cross section of processes (4),(5) may be obtained using formulas (10)-(18) by replacements:

$$h_{ijk} \rightarrow \frac{gm}{\sqrt{2}m_W} \tan \beta, m_\nu \rightarrow m_3, m_i \rightarrow m_4$$ and $\Gamma_\nu = 0$.

For instance, far from threshold we obtain:

$$\sigma(\mu^+ \mu^- \rightarrow H_3^0 Z^0) = \frac{\pi\alpha^2}{2 \sin^4 \theta_W \cos^2 \theta_W \tan^2 \beta((a_L^2+a_R^2)(\log(\frac{s}{m_3 m_Z})-1)+a_L a_R).}$$  \hspace{1cm} (25)$$

$$\sigma(\mu^+ \mu^- \rightarrow H^\mp W^\pm) = \frac{\pi\alpha^2}{4 \sin^4 \theta_W \tan^2 \beta(\log(\frac{s}{m_4 m_W})-1).}$$  \hspace{1cm} (26)$$

At $m_4 > m_t, m_3 \approx m_4$ with high accuracy and consequently for the processes (4),(5) we can use numerical results depicted on the Fig.3,4 for $h_{ijk} = 10^{-2}$ which are the same as for the processes (4),(5) at $\tan \beta = 17.5$ (in accordance with the above-mentioned replacements).

Besides the contribution to the processes (2),(3) from the tree diagrams of the Fig.1 there is also a contribution from the loops with virtual $W^\pm$- and $t$-quarks and with other heavy particles in various extensions of the Standard Model such as contributions from squarks, charged Higgs bosons, chargino. Some of such contributions (the $ZW^\mp H^\pm$ vertex) have been calculated previously, see [29] and references therein. On the Fig.5 we depicted some of the box diagrams which also describe the loop contribution to the process (5). Naively the loop contribution is of order $\sigma \sim \frac{\alpha^4}{s}$, it is possible that various amplitudes from various contributions are partly compensate each other at some parameters (a similar situation takes place in the loop contribution [28] to the process $\mu^+ \mu^- \rightarrow H_i^0 \gamma$) and at these parameters the cross section may be essentially lower and the tree contribution will dominate over the loop contribution especially at $\tan \beta \gg 1$.

It must be noted also, at $\sqrt{s} \gg M$ (where $M$- is the mass of the heaviest
particles in loop that the loop contribution also decreases faster than the tree contribution, because loop integrals contain the additional degree of $M^2$.

Using result of the [14] for the process $\mu^+\mu^- \rightarrow H_1^0\gamma$:

$$\sigma(\mu^+\mu^- \rightarrow H_3^0\gamma) = \frac{\pi\alpha^2}{2\sin^2\theta_W}\frac{m^2}{m_W^2}\tan^2\beta\frac{1}{s-m^2_H}(1+\frac{m^4_H}{s^2})\log(\frac{s}{m^2_\mu})-\frac{2m^2_H}{s},$$

we see that near threshold the cross section of the tree process $\mu^+\mu^- \rightarrow H_1^0\gamma$ exceed the cross section of process (5), far from threshold, as seen from comparision of the (25) with (27), the cross section of the process of the process $\mu^+\mu^- \rightarrow H_1^0\gamma$ exceed the cross section of the process (5) in several times.

Appendix A

In the MSSM, the Higgs sector contains two doublets of Higgs bosons with opposite hypercharge ($Y=\pm 1$).

After spontaneous symmetry breaking the following physical states appear: charged Higgs bosons $H^\pm$, and three neutral ones, $H_1^0, H_2^0, H_3^0$.

At tree level the masses of charged Higgs bosons ($m_4$) and scalars $H_{1,2}^0$ and an angle $\alpha$ (which described the mixing of scalar states) are being expressed through the mass of pseudoscalar $H^0_3$ and $\tan\beta = \frac{v_2}{v_1}$ where $v_2, v_1$ are both doublets vacuum expectations by following relationships:

$$m_4^2 = m_3^2 + m_W^2$$

\hspace{1cm} (A.1)

$$m_{1,2}^2 = \frac{1}{2}\left[m_3^2 + m_Z^2 \pm ((m_3^2 + m_Z^2)^2 - 4m_Z^2m_3^2\cos^22\beta)^{1/2}\right]$$

\hspace{1cm} (A.2)
\[ \tan 2\alpha = \frac{m_2^2 + m_3^2}{m_3^2 - m_2^2} \tan 2\beta. \] (A.3)

It follows from (A1) that MSSM guarantees the existence of, at least, one light Higgs boson with \( m_2 < m_Z \).

Interactions of the Higgs bosons with muons are described by the lagrangian:

\[ \mathcal{L} = i \frac{g m}{2m_W} \cos \alpha \cos \beta \bar{\mu} H_1^0 + i \frac{g m}{2m_W} \sin \alpha \cos \beta \bar{\mu} H_2^0 + \]
\[ + \frac{g m}{2m_W} \tan \beta \bar{\mu} \gamma_5 H_3^0 + (i \frac{g m}{\sqrt{2}m_W} \tan \beta \bar{\mu} P_L \nu H^+ + h.c.) \] (A.4)

At \( \tan \beta \gg 1 \) the mass relation (A2),(A3) and formula (A4) are strongly reduced:

\[ m_2 = m_3, m_1 = m_Z, \frac{\sin \alpha}{\cos \beta} = \tan \beta \gg \frac{\cos \alpha}{\cos \beta} \text{ at } m_3 < m_Z, \] (A.5)

\[ m_2 = m_{H_Z}, m_1 = m_3, \frac{\cos \alpha}{\cos \beta} = \tan \beta \gg \frac{\sin \alpha}{\cos \beta} \text{ at } m_3 > m_Z. \] (A.6)

It must be noted, that radiative corrections \[30\]-\[33\] can strongly change relations (A1),(A2) however in the large \( \tan \beta \) limit and at \( m_3 < m_Z \) or at \( m_3 \gg m_Z \) formulas (A1),(A5),(A6) hold approximately true even after taking into account the radiative corrections.

**Figures Cuption**

Fig.1 Diagrams corresponding to the processes (2),(3).

Fig.2 Tree diagrams corresponding to the processes (4),(5).

Fig.3 Number of events \( \tilde{\nu}_{k,L} Z^0 \) per year \( (\sigma L) \) (at yearly luminosity \( L = 1000 fb^{-1} \)) produced in reaction (2) as a function of \( \sqrt{s} \) with \( h_{\mu k} = 10^{-2} \). Curves 1,2,3 correspond to the \( m_{\tilde{\nu}} = 0.1, 0.5, 0.7 TeV \) respectively.
Fig.4 Number of events $\tilde{l}_{k,L}W^\pm$ per year ($\sigma L$) (at yearly luminosity $L = 1000 fb^{-1}$) produced in reaction (3) as a function of $\sqrt{s}$ with $h_{\mu\nu k} = 10^{-2}$. Curves 1-5 correspond to the $m_{\tilde{l}} = m_{\tilde{\nu}} = 100 GeV; m_{\tilde{l}} = 300 GeV; m_{\tilde{\nu}} = m_{\tilde{l}} = 700 GeV; m_{\tilde{\nu}} = 500, m_{\tilde{l}} = 300 GeV; m_{\tilde{\nu}} = 700, m_{\tilde{l}} = 300 GeV$; respectively.

Fig.5 Loop diagrams corresponding to the processes (4),(5). Shaded ring correspond to the diagrams with $Z^0W^\pm H^\pm$ vertexes.

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Figure 1:
Figure 2:
Figure 5: