Possible Effects of Quantum Mechanics Violation Induced by
Certain Quantum Gravity on Neutrino Oscillations

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Abstract

In this work we tried extensively to apply the EHNS postulation about the quantum mechanics violation effects induced by the quantum gravity of black holes to neutrino oscillations. The possibilities for observing such effects in the neutrino experiments (in progress and/or accessible in the near future) were discussed. Of them, an interesting one was outlined specially.

PACS: 14.60.Pq, 03.75.-b, 03.65.Bz
It is known that neutrino oscillation is a very possible solution for the long-standing puzzle about the ‘deficiency’ of solar neutrino [1] and the atmospheric neutrino problem [2]. The neutrino oscillation experiments are very difficult but people have been achieving progress steadily year to year. Especially, Super-Kamiokande has a high rate to collect the solar and atmospheric neutrino events. Recently they have reported some evidence for neutrino oscillations [3]. In addition, several long-base neutrino experiments for the oscillation in matter are in progress as planned.

We also know that many years ago Hawking, based on the principle of quantum mechanics and gravity, proposed a very interesting conjecture that the quantum gravity effects of black holes may cause to emit particles in thermal spectrum [4]. According to the conjecture, black holes may create particles in pairs and some of particles fall back into the black holes while some of the others escape ‘away’ thermodynamically, thus part of the information about the state of the system may be lost to the black holes. For a quantum mechanical system, due to the effects caused by the microscopic real and virtual black holes, the system which is in a pure quantum state may transmit to a mixed one, i.e. it manifests quantum mechanics violation (QMV). To describe a mixed quantum system from a pure state to a mixed one, instead of the wave function, density matrix description has to be adopted [5]. In such an evolution, where the QMV effects are involved, CP, and probably CPT, can be violated due to the non-local quantum gravity effects. Thus Hawking’s suggestion not only in a macroscopic configuration but also in a microscopic ‘elementary’ particle level has received careful considerations. First of all, more than 10 years ago Ellis, Hagelin, Nanopoulos and Srednicki (EHNS) [6], being motivated to track the ‘sources’ for CP and CPT violation, proposed to observe the QMV induced by the quantum gravity effects in $K^0 - \bar{K}^0$ system with additional reasonable assumptions, and then the authors of [7,8] reexamined and formulated the effects with more care, and gave refreshed bounds on the parameters of the effects for QMV. To play the same game, the authors of [9] discussed the possibility of observing the effects in $B^0 - \bar{B}^0$ system. The authors of [11] studied such effects on neutrino oscillation. As the kinetic energies and the masses stand on the same foot for gravity in the stress-energy
tensor, and no matter neutrino masses are zero or finite (a tiny nonzero mass at most) thus the energy of the neutrinos will play the roles in the stress-energy tensor when considering gravity. Considering the fact that the neutrino oscillation observations may take place in very different and long distances, even so long as the distance from the Sun to the Earth, it may have some advantage in looking for the QMV effects, at least, we should consider them quantitatively. In this paper, we will extend to apply the EHNS formulation to the neutrino cases, and discuss the effects of the QMV affecting various neutrino oscillation observations, relevant to the present and planned neutrino experiments.

To follow the notation of EHNS, let us repeat briefly their formulation so as to start the calculations and discussions here.

To describe such states, commonly instead of wave function, the density matrix is employed. The density matrix of a pure state can always be written as

$$\rho_{\text{pure}} = |\psi><\psi|,$$

while a mixed state then should be in the form

$$\rho_{\text{mix}} = \sum_a P_a |\psi_a><\psi_a|,$$

with \(\sum_a P_a = 1\),

where \(|\psi>\) and \(\psi_a>\) are the regular wave functions respecting the superposition rule and normalization \(<\psi|\psi> = 1\), \(<\psi_a|\psi_a> = 1\) (not to sum over \(a\)). Note that

$$\text{Tr}(\rho_{\text{pure}}) = \text{Tr}(\rho_{\text{mix}}) = 1,$$

but

$$\text{Tr}(\rho^2_{\text{pure}}) = \text{Tr}(\rho_{\text{pure}}) = 1, \quad \text{Tr}(\rho^2_{\text{mix}}) < 1.$$

The Schrödinger equation for the density matrix is accordingly written as

$$i\frac{\partial}{\partial t} \rho = [H, \rho],$$

where \(\rho\) can be either \(\rho_{\text{pure}}\) or \(\rho_{\text{mix}}\). Indeed so far it is exactly equivalent to the regular form of the Schrödinger equation for the wave functions. It is easy to prove that with Eq.(3) one
has
\[ \frac{d}{dt} \text{Tr}(\rho^2) = 0, \]
namely, pure and mixed states never interchange. However, as EHNS suggested [6], the Hawking’s quantum gravity effects at vicinity of real and virtual black holes may violate quantum mechanics i.e. modify the Schrödinger equation significantly. For simplicity, we derive all formulae for a two-energy-level system as an illustration. Generalizing the Schrödinger equation of a two-energy-level system, one can expand the $2 \times 2$ matrix form of $\rho$ and $H$ in terms of $\sigma_0$ and $\sigma_i$, where $\sigma_0$ is a $2 \times 2$ unit matrix and $\sigma_i (i = 1, 2, 3)$ are the well-known Pauli matrices, i.e.

\[ \rho = \rho_0 \sigma_0 + \rho_i \sigma_i, \quad H = H_0 \sigma_0 + H_i \sigma_i. \]

(6)

Thus besides the trivial $\rho_0$ component, Eq.(5) can be recast into a tensor form as [7]

\[ i \frac{d}{dt} \rho = 2 \epsilon^{ijk} H^i \rho^j \sigma^k, \quad (i, j, k = 1, 2, 3). \]

(7)

Due to the QMV effects being included by the concerned quantum gravity, EHNS introduced a non-hermitian piece to Eq.(7), which modifies the Schrödinger equation Eq.(5) greatly. The newly additional non-hermitian term is

\[ i \delta H \rho = -h^{0j} \rho^j \sigma_0 - h^j \rho^0 \sigma_j - h^{ij} \sigma_i \rho^j. \]

(8)

Since probability is conserved, and its entropy should not decrease, it is required

\[ h^{0j} = h^{j0} = 0. \]

EHNS [6] and the authors of ref. [7] applied this modified Schrödinger equation to the $K^0 - \bar{K}^0$ system. By enforcing different conservation laws on the effects, $h^{ij}$ would be constrained. If a physical quantity is conserved, its corresponding operator $O$ must commute with the Hamiltonian and requires $d/dt(\text{Tr}O\rho) = 0$. Hence

\[ \text{Tr}(O \delta H \rho) = 0. \]
EHNS and the authors of [7] assumed $O = \sigma_1$ which corresponds to strangeness being conserved ($\Delta S = 0$) in the neutral kaon system:

$$< K^0 | \sigma_1 | K^0 > = -1, \quad \text{while} \quad < \bar{K}^0 | \sigma_1 | \bar{K}^0 > = +1.$$  

Then $h_{\mu\nu}$ of Eq.(8) can be written as a $4 \times 4$ matrix:

$$h_{\mu\nu} = 2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha & -\beta \\ 0 & 0 & -\beta & -\gamma \end{pmatrix}. \quad (9)$$  

Whereas EHNS also proposed an alternative parameter set by assuming the conservation operator is $O = \sigma_3$, and it is the case that energy and the other quantum numbers such as leptonic number etc are conserved. Thus the matrix $h_{\mu\nu}$ reads

$$h_{\mu\nu} = 2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\alpha & -\beta & 0 \\ 0 & -\beta & -\gamma & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (10)$$  

Note that here we have added an extra factor 2 in front of the matrix at Eq.(10) which is a different parametrization from the notation given in [6], the reason is to make the form similar to that in Eq.(4) where the authors of [6] had put a factor 2 (see Eqs.(2.31) and (3.15) in [6]).

For the parametrization of Eq.(10), to avoid the states with complex entropy, $Tr \rho^2$ can never exceed unity, so it requires

$$\rho_\alpha H_{\alpha\beta} \rho_\beta \leq 0,$$

thus

$$\alpha > 0, \quad \gamma > 0, \quad \alpha \gamma \geq \beta^2. \quad (11)$$  

By fitting data of $\epsilon$ and the semileptonic decays of the K-system, EHNS obtained [6]

$$\alpha + \gamma \leq 2 \times 10^{-21} \text{ GeV},$$

5
while Huet and Peskin \[7\] updated the values as

\[ \beta = (3.2 \pm 2.9) \times 10^{-19} \text{ GeV}, \quad \gamma = (-0.2 \pm 2.2) \times 10^{-21} \text{ GeV}. \]

and recently Ellis et al. gave a further estimate as \[8\]

\[ \alpha \leq 4 \times 10^{-17} \text{ GeV}, \quad |\beta| \leq 3 \times 10^{-19} \text{ GeV}, \quad \gamma \leq 7 \times 10^{-21} \text{ GeV}. \quad (12) \]

Indeed, the only important issue quoted here is the order of magnitudes of \((\alpha, \beta, \gamma)\) and the concrete coefficients are not much of significance.

Since the QMV effects are caused by quantum gravity, it is suggested that \(\alpha, \beta, \gamma\) be proportional to \(M^2/M_{pl}\) where \(M_{pl}\) is the Plank mass and \(M\) is an energy or mass scale of the concerned physical process occurring in our quantum system (neutral kaon or neutrino under consideration).

Now let us turn to the case for neutrino oscillations.

The quantum gravity effects must play similar roles in all the quantum systems as that in the \(K^0 - \bar{K}^0\) system, but the crucial problem is if they are observable or not. The neutrino oscillations among different species neutrinos will be affected by the concerned QMV effects, and might be observable. Because we may observe the oscillations at very different distances in ‘vacuum’ and in matter as well, one may expect to have more advantages for observing such effects in neutrino oscillation systems than in the \(K^0 - \bar{K}^0\) system or else. In the two-generation neutrino oscillation case, the \(h_{\mu\nu}\) has just the form as Eq.(6) in the basis of Pauli matrices, whereas, in three-generation neutrino oscillation case, it becomes more complicated while the Pauli matrices will be replaced by the \(SU(3)\) Gell-Mann matrices \[10\].

In this work, as stated above, for simplicity we will restrain ourselves only to formulate the two-generation neutrino case. Namely we will only consider the form of \(\delta H\) given in Eq.(9) and Eq.(11). It certainly is interesting to note here that besides those expected effects, the lepton number is allowed to violate\[1\] even if the neutrinos are massless. Furthermore, one

\[1\]Here the lepton number is violated due to the interaction of the black holes.
will see that the effects themselves may induce oscillations so they are observable in present or accessible neutrino oscillation experiments.

The physical picture may be imagined as the following. As neutrinos $\nu_i$ interact with the heavy object, a ‘micro black hole’, due to the quantum effects, the black hole creates a pair of neutrino-antineutrino of certain species, the neutrino-antineutrino pair interacts with the coming neutrino in a certain (coherent or incoherent) manner, afterwards, a neutrino and a antineutrino fall into the black hole but one neutrino may escape away to respect the coming neutrino. Whereas we should note that the escaping neutrino does not need to be the same as the coming one. Which one escapes, only depending on its coupling to the micro black hole via gravity, namely the two species (of course may be the same) neutrinos have different couplings to the black holes (if $a \neq b$). If $\alpha \neq \gamma$ in the QMV terms, $h_{ij}$ of Eq.(9) and Eq.(10) indeed reflects such facts.

Thus we may try to apply this scenario to neutrino oscillation [11]. In order to have some idea about magnitude order of the effects for definiteness and comparison with the $K^0 - \bar{K}^0$ system as possible as one can, we further try to assume the corresponding parameters for neutrino systems as the follows:

$$\alpha_\nu \leq 4 \times 10^{-17} \left(\frac{E_\nu}{0.5}\right)^2, \quad |\beta_\nu| \leq 3 \times 10^{-19} \left(\frac{E_\nu}{0.5}\right)^2, \quad \gamma_\nu \leq 7 \times 10^{-21} \left(\frac{E_\nu}{0.5}\right)^2, \quad (13)$$

where $E_\nu$ is the energy of the emitted neutrino, and every quantities in the above are in GeV. Here 0.5 corresponds to the mass of kaon.

Note again that the parameters can be very different from that listed above, but we just assume them as a reference for later discussions. If assuming the solar neutrino deficit is due to neutrino oscillation [13], the parameter set Eq.(13) will be restricted by data. Later we will show that the solar neutrino and other neutrino experiments on the Earth may set some substantial constraints on the parameters.

Now let us discuss the meaning of the solutions obtained from Eq.(9) and Eq.(10).

I. The asymptotic behavior of QMV evolution of the neutrino system

It is easy to realize that the expressions Eq.(9) and Eq.(10) would lead to different
behaviors for the neutrino oscillations.

a). With the form of $\delta H$ given in Eq.(3), one has the solution that a probability for $\nu_e$ transition to another $\nu_x$ in vacuum ($x$ can be $\mu, \tau$ or a sterile neutrino flavor),

$$P_{\nu_e \rightarrow \nu_x} = \frac{1}{2} - \frac{1}{2} e^{-\gamma L} \cos^2 \theta_v - \frac{1}{2} e^{-\alpha L} \sin^2 \theta_v \cos \left( \frac{\Delta}{2 E_v} L \right),$$

(14)

where $\Delta \equiv |m_{\nu_1}^2 - m_{\nu_2}^2|$, $L$ is the distance from the production spot of $\nu_e$ to the detector and $\theta_v$ is the mixing angle of $\nu_e$ and $\nu_\mu$ in vacuum. To obtain the above formula, one should assume $\beta \ll \alpha, \gamma$, in fact this approximation is not necessary, but here only for demonstration convenience, otherwise the formula would become tedious. In the work [11], more precise numerical results were given.

It is noted that in the basis of mass, because $|\nu_e> = (\cos \theta_v |\nu_1> + \sin \theta_v |\nu_2>)$ and $|\nu_\mu> = (-\sin \theta_v |\nu_1> + \cos \theta_v |\nu_2>)$, so

$$<\nu_e|\sigma_1|\nu_e> = 2 \sin \theta_v \cos \theta_v, \quad <\nu_\mu|\sigma_1|\nu_\mu> = -2 \sin \theta_v \cos \theta_v.$$

As the case of $K^0 - \bar{K}^0$ system, the conservation of $\sigma_1$ should mean that flavor conserves and there would be no transition among different flavors. At the first glimpse, $\delta H$ seems cause a flavor transition. In fact, if the original Hamiltonian does mix the flavors for massive neutrinos, the $\delta H \rho$ term does not cause it further, but strengthens or weakens the transition caused by the original Hamiltonian only. One can see that in the case an exponential factor exists in front of the harmonic oscillation, which is our familiar expression of neutrino oscillation in vacuum. Thus this extra factor changes the oscillation behavior, but does not cause it.

When the neutrinos are massless, it is another story. Then the mixing disappears, i.e. $\theta_v = 0$, then $<\nu_e|\sigma_1|\nu_e> = <\nu_\mu|\sigma_1|\nu_\mu> = 0$. It implies that the two states are degenerate in the regular QM framework. But as long as there are extra terms such as the QMV, the degeneracy is broken and an oscillation can occur due to the new effects. Hence in this case, the $\sigma_1$ conservation does not forbid such a transition between the different flavors, (because expectation value of $\sigma_1$ is zero for all flavors).
In fact, if neutrinos are massless, the oscillation is still expected. Namely if considering Eq. (7) only, we have \( m_\nu = 0 \), and \( \nu_1, \nu_2 \) are exactly \( \nu_e, \nu_\mu \), but with the extra \( \delta H \rho \) Eq. (8) in the evolution equation, and different couplings being indicated as \( h_{\mu \nu} \) in Eq. (9), the difference for different ‘flavor’ neutrinos manifests. Thus \( \nu_1 \) and \( \nu_2 \) have different behaviors as they propagate in an environment full with the micro black holes and ‘oscillations’ between them appear.

b). With expression Eq. (10), we have the solution [14]:

\[
P_{\nu_e \rightarrow \nu_x} = \frac{1}{2} \sin^2 2\theta_\nu \left( 1 - e^{-[(\alpha + \gamma)L] \cos(\frac{\Delta L}{2E_\nu})} \right). \tag{15}
\]

Note that to obtain the above result, we assumed that \( (\alpha, \beta, \gamma)_\nu \ll \frac{\Delta E_\nu}{2} \).

In fact, the exact result depends on the fact if the factor \( \kappa^2 \) is greater, equal or smaller than zero with the definition

\[
\kappa^2 \equiv 4 \left[ (\alpha - \gamma)^2 + 4\beta^2 - \frac{\Delta^2}{4E_\nu^2} \right]. \tag{16}
\]

If \( \kappa^2 \) is less than zero, the oscillating form of Eq. (15) is resulted in, only when \( \kappa^2 \) is greater or equal to zero, the expression turns into a purely damping solution. The precise version of Eq. (15) is

\[
P_{\nu_e \rightarrow \nu_x} = \frac{1}{2} \sin^2 2\theta_\nu \left\{ 1 - e^{-[(\alpha + \gamma)L] \left[ \frac{\alpha - \gamma}{\kappa} (e^{\kappa L/2} - e^{-\kappa L/2}) + \frac{1}{2} (e^{\kappa L/2} + e^{-\kappa L/2}) \right]} \right\}. \tag{17}
\]

Indeed when \( \kappa^2 < 0 \), \( \kappa \) is imaginary, the solution contains an oscillatory factor, otherwise attenuative.

Let us discuss the phenomenological significance of Eq. (14) and Eq. (15) in the below.

**II. The equation Eq. (14) and Eq. (15) leads to completely different asymptotic limits as \( r \rightarrow \infty \) (or \( L \rightarrow \infty \))**

The exponentially damping term in Eq. (14) would wash out any information of neutrino mixing as long as the detector is placed far enough from the source. In that case, \( P_{\nu_e \rightarrow \nu_\mu} (t \rightarrow \infty) = \frac{1}{2} \) for two generations, and if generalizing the result to the n-generation structure [11]:

\[
P_{\nu_e \rightarrow \nu_x} (t \rightarrow \infty) = \frac{1}{n}, \quad P_{\nu_e \leftrightarrow \nu_\nu} (t \rightarrow \infty) = \frac{1}{n}, \quad P_{\nu_\tau \leftrightarrow \nu_\mu} (t \rightarrow \infty) = \frac{1}{n}.
\]
On the contrary, Eq. (15) would lead to a different consequence as
\[ P_{\nu_e \rightarrow \nu_\mu} (t \rightarrow \infty) = \frac{1}{2} \sin^2 2\theta_v, \]
i.e. the mixing angle between \( \nu_e \) and \( \nu_\mu \) is still there; for the 3-generation case we will have a similar result, only the simple ‘Cabibbo-like’ angle \( \theta_v \) should be replaced by the ‘KM-like’ entries.

All the above expressions can apply to the \( \nu_a \leftrightarrow \nu_b \) case with \( a, b \) being any pair of \( e, \mu, \tau \) as long as \( a \neq b \).

### III. The solar neutrino problem vs. the QMV effects

**a).** For \( \Delta \simeq 10^{-5} \text{eV}^2 \), i.e. MSW solution for the solar neutrino puzzle, one expects the averaged effect of oscillation term \( \cos(\frac{\Delta}{2E_\nu} L) \) vanishes. Therefore the transitional probability can be re-written as the follows:

In the case of Eq. (9)
\[ P(\nu_e \rightarrow \nu_e) = \frac{1}{2} \left[ 1 + (1 - X) e^{-\gamma L} \cos 2\tilde{\theta}_0 \cos 2\theta \right]. \] (18)

where \( \tilde{\theta}_0 \) is the neutrino mixing angle in the center of the Sun. \( X \) is the jumping probability from one neutrino mass eigenstate to another in the MSW resonant region. For the large angle solution \( X \simeq 0 \) and for the non-adiabatic solution it can be close to one. From Eq. (18) we may see that \( e^{-\gamma L} \gg 1 \) is not favored to fit the solar neutrino data besides violating the condition Eq. (11). Because in this case we obtain a constant suppression 0.5, which is disfavored [13]. As the result the bound \( \gamma L \leq 1 \) is enforced. If \( \gamma L \ll 1 \), the new violation effect is negligible. So only for \( \gamma L \sim O(1) \) the MSW solution for the solar neutrino problem should be modified. Here \( L \) is the distance between the Sun and the Earth. Generally we get \( \gamma \leq 6 \times 10^{-9} \text{km}^{-1} \).

In the case of Eq. (10), the new effects are averaged to be zero over the distance \( L \). The situation is exactly the same as the MSW solution without the QMV terms. In this case one cannot obtain any information about the QMV from fitting the solar neutrino data.

**b).** For the vacuum oscillation solution to the solar neutrino problem, \( \Delta \sim 10^{-10} \text{eV}^2 \). In this case the oscillation term is not averaged to be zero. The transitional probability is given
in Eq.(14), Eq.(13) and Eq.(17). Again we obtain the bound $\alpha L, \gamma L \leq 1$ in order to solve the solar neutrino puzzle. And only for $\alpha L, \gamma L \sim 1$ the parameter region of the vacuum oscillation solution should be modified.

IV. A very interesting feature indicated by Eq.(14)

Even if neutrinos are massless, the micro black hole effects can induce neutrino transition from one flavor state to another. For an $n$-flavor neutrino case, the oscillation probability could be simplified as

\[
\begin{align*}
P_{\nu_e \rightarrow \nu_x} &= \frac{1}{n} - \frac{1}{n}e^{-\gamma L} \\

P_{\nu_e \rightarrow \nu_e} &= \frac{1}{n} + \frac{n-1}{n}e^{-\gamma L}
\end{align*}
\]

(19)

where SU(n) should replace SU(2) for the case of two neutrino species.

Indeed it is interesting to ‘check’ if this oscillation probability alone is enough to solve the solar neutrino problem without requiring nonzero neutrino mass.

First of all, it is realized that $\gamma$ cannot be a constant, otherwise the $\nu_e$ suppression is energy independent which disagrees with the solar neutrino data [13]. By dimension analysis, one may assume $\gamma = \gamma_0 E_\nu^2 \sim \frac{E_\nu^2}{M_{pl}}$ for massless neutrino. With this assumption we see that the larger neutrino energy corresponds to larger suppression. So in the solar neutrino experiment the $^8$B neutrino is suppressed most, which is $\frac{1}{n}$. $^7$Be neutrino are suppressed less but very close to $\frac{1}{n}$. pp neutrino is suppressed least which is between $\frac{1}{n}$ and 1. After careful study we find that the solar neutrino data can be fitted best with $n = 3$. So in the following we will discuss three species neutrino case. We adopt the standard solar model (BP98) [16] for our discussions. The predicted neutrino flux for $H_2O$ experiment is: for $\nu_e \rightarrow \nu_\mu, \nu_\tau$ oscillation

\[
\Phi^{th}_{H_2O} = \begin{pmatrix}
2.21 +0.19 \\
-0.14
\end{pmatrix} \times 10^6 \text{cm}^{-2}\text{s}^{-1}
\]

(20)

for $\nu_e \rightarrow \nu_\mu, \nu_s$ oscillation ($\nu_s$ is a sterile neutrino)

\[
\Phi^{th}_{H_2O} = \begin{pmatrix}
2.01 +0.19 \\
-0.14
\end{pmatrix} \times 10^6 \text{cm}^{-2}\text{s}^{-1}
\]

(21)
The observed flux $\Phi_{\text{exp}}^{H_2O}$ is $2.42 \pm 0.06 \times 10^6 \text{cm}^{-2}\text{s}^{-1}$ from Super-Kamiokande \[3\] (it is $2.80 \pm 0.19 \pm 0.33 \times 10^6 \text{cm}^{-2}\text{s}^{-1}$ from Kamiokande). The ratio $\Phi_{\text{th}}^{H_2O}/\Phi_{\text{exp}}^{H_2O}$ is estimated to be $0.92 \pm 0.17$ and $0.84 \pm 0.17$ for $\nu_e \rightarrow \nu_\mu, \nu_\tau$, and $\nu_e \rightarrow \nu_\mu, \nu_s$ oscillation respectively. Theory agrees with the experiment within 1$\sigma$.

The neutrino capture rate in the chlorine experiments is obtained as

$$S_{\text{th}}^{\text{Cl}} = 2.6 \pm 0.4 \text{ SNU}$$ (22)

compared with the observed one $S_{\text{Cl}}^{\text{th}} = 2.55 \pm 0.25 \text{ SNU}$ \[7\]. The ratio is estimated as

$$S_{\text{th}}^{\text{Cl}}/S_{\text{Cl}}^{\text{exp}} = 1.0 \pm 0.1$$ (23)

So the theoretical expectation is in very good agreement with the experiment.

For the gallium experiments if the parameter $\gamma_0$ falls in the region $(1.5 - 3.7) \times 10^{-8}\text{MeV}^{-2}\text{km}^{-1}$, we obtain the capture rate as

$$S_{\text{th}}^{\text{Ga}} = (68 \sim 79) \text{ SNU}$$ (24)

This agrees with the experimental value $(73.4 \pm 5.7)\text{SNU}$ \[17\] at 1$\sigma$ level. At 2$\sigma$ level $\gamma_0$ can be taken a value from $(0.66 - 5.4) \times 10^{-8}\text{MeV}^{-2}\text{km}^{-1}$.

V. The scenario for the ‘atmospheric neutrino’ problem

With three-generation neutrinos and $\gamma_0$ given in IV, by fitting solar neutrino data at the level of 2$\sigma$ errors, we may estimate some of the observables further for the atmospheric neutrino observations.

The up-down asymmetry of $\mu$-like and e-like events for $\cos \Theta > 0.2$ (down) and $\cos \Theta < -0.2$ (up), where $\Theta$ is zenith angle, are denoted by $Y_e, Y_\mu$. In the present scenario $Y_e$ is always close to 1, independent of the energy and the traveling distance of the neutrinos, it is in agreement with the data $Y_e(\text{sub} - \text{GeV}) = 1.13 \pm 0.08$ and $Y_e(\text{multi} - \text{GeV}) = 0.83 \pm 0.13$ \[3\]. However $Y_\mu$ is estimated as 0.62-0.98, 0.5 for multi-GeV and sub-GeV events respectively, i.e. at 1$\sigma$ level agrees with the measured $Y_\mu(\text{multi-GeV}) = 0.54 \pm 0.07$, but at 4$\sigma$ level with
the measured $Y_\mu$(sub-GeV) = 0.78 ± 0.06. It is a little more involved to calculate the ratio of the total $\mu$-like and e-like events. Here we only give a rough estimate on the double ratio by approximating that the down-going neutrino flux is almost unsuppressed, while the up-going and the horizontal neutrino flux is suppressed by a factor of 1/3. They are estimated to be 0.6 and 0.5-0.6 for sub-GeV and multi-GeV events respectively with the scenario. This is also not bad but in agreement with the measured ones $0.61 \pm 0.03 \pm 0.05$, $0.66 \pm 0.06 \pm 0.08$ [3]. We conclude here that our scenario with zero neutrino mass can fit all the measurements of the solar and atmospheric neutrino experiments, except $Y_\mu$(sub-GeV).

VI. An alternative mechanism for the solar and atmospheric neutrino flux ‘shortage’

As estimated in the previous sections, the QMV effects can serve as an alternative mechanism for the solar and atmospheric neutrino flux shortage, if assuming

$$ (\alpha, \beta, \gamma)_\nu = (\alpha, \beta, \gamma)_K \cdot \left(\frac{E_\nu}{M_K}\right)^2. \tag{25} $$

The factor $(E_\nu/M_K)^2$ appears here is due to that we think that there is some relation between neutrino oscillation and $K^0 - \bar{K}^0$ system for QMV induced by quantum gravity. Namely based on the ansatz proposed by EHNS, we will have $(\alpha, \beta, \gamma)_K \propto M^2/M_{pl}$ for the $K^0 - \bar{K}^0$ system, and a similar parametrization for neutrinos $(\alpha, \beta, \gamma)_\nu \propto E_\nu^2/M_{pl}$ thus the factor $(E_\nu/M_K)^2$ appears in Eq(25). In the solar neutrino case, $E_\nu \sim 0.3 - 10$ MeV, the factor suppresses $(\alpha, \beta, \gamma)_\nu$ by a factor of order $10^{-6} - 10^{-4}$. This postulation should be tested by experiments on the Earth.

The data on $\nu_\mu \rightarrow \nu_\tau$ oscillation by the CHARM II [18] collaboration claimed that no evidence of the neutrino flux change had been observed. In the experiment, $E_\nu \sim 27$ GeV and $L \sim 0.6$ km. Considering the errors, $|\alpha L|$ must be smaller than $10^{-3}$. This constraints requires

$$ (\alpha_0, \beta_0, \gamma_0)_\nu \leq 2 \times 10^{-12}\text{MeV}^{-2}\text{km}^{-1}. \tag{26} $$
Combining Eq. (26) with the enhancement factor \((\frac{\nu M}{M_K})^2 \approx 1.6 \times 10^3\), it indicates

\((\alpha, \beta, \gamma)_K \leq 4 \times 10^{-24}\) GeV.

This number is much below the upper bounds given by the authors of [7, 8]. If this is the case, the violation effects of the quantum mechanics would hardly influence the \(\epsilon\)–value in the neutral kaon system. This constraint also excludes the \(\nu_e, \nu_\mu, \nu_\tau\) oscillations discussed in IV. and V.. Hence a sterile neutrino must be introduced and \(\tau\) neutrino must be treated differently from the other species.

However, as pointed out above, the parameters for neutrino system do not need to be the same as that for neutral kaon system, so this comparison has qualitative meaning only.

**VII. The parameters \((\alpha, \beta, \gamma)_\nu\)**

If we assume the parameters obtained in the neutral kaon system can be generalized to the neutrino system through certain relation, then besides Eq. (25), we can have the following possibilities.

A possible adoption could be that

\[ (\alpha, \beta, \gamma)_\nu \propto \left(\frac{m_\nu M}{M_K}\right)^2, \]  

(27)

whereas if \(m_\nu\) is in the order of magnitude about a few tens of eV (for the \(\tau\) neutrino probably), it is easy to show that in neutrino systems such an ‘adoption’ would kill any possible observational effects of the QMV induced by the micro black holes.

The scenario described by Hawking is that the quantum effects cause the micro black holes to radiate particle pairs with one of the pair falling into the event horizon while the other escaping away. Considering this picture, an alternative postulation

\[ (\alpha, \beta, \gamma) \propto \frac{E_\nu \cdot m_\nu}{M_{pl}}, \]  

(28)

could be reasonable if there is a nonzero neutrino mass, because the escaping particle is moving relatively to the black holes.
Supposing $m_{\nu_e} \sim 10$ eV, with the condition of the CERN-SPS wide band neutrino beam (WBB) [18] the ansatz Eq. (28) results in

$$(\alpha, \beta, \gamma)_{\nu} \sim \frac{27 \times 10 \times 10^{-9}}{(0.5)^2} \times (\alpha, \beta, \gamma)_{K} \simeq 6 \times 10^{-27} - 4 \times 10^{-23}$ GeV,$$

in terms of the values given in [8,11]. If it is the case, the values of $(\alpha, \beta, \gamma)_{\nu}$ satisfy the condition Eq. (26) set by the CHARM II data. Similar to what we did in IV. and V. we can also fit the solar and atmospheric neutrino data in this case, most of the results are approximately the same except $\gamma_0 \simeq 10^{-9} \sim 10^{-10}$ MeV$^{-1}$km$^{-1}$. Here $\gamma \equiv \gamma_0 E$.

Therefore such phenomena may be accessible in the proposed long-baseline neutrino oscillation experiments where the neutrinos propagate sufficiently far to make the damping effects observable.

Let us give numerical estimation for the proposed long-baseline neutrino oscillation experiments.

**VIII. The scenario in long-baseline experiments**

In the planned long-baseline experiments, KEK-Super-Kamiokande (250 km), CERN-GranSasso (730 km) and Fermilab-Sudan II (730 km), the average energies of the $\nu_\mu$ beams are approximately 1 GeV, 6 GeV and 10 GeV [19]. Accordingly, the suppression factor $e^{-\gamma L}$ for the experiments should be

$$0.2, \sim 0.0, \sim 0.0, \quad (29)$$

respectively. These factors are estimated based on the value of $\gamma$ obtained above for the $E_\nu^2$ dependence postulation Eq. (25). So the QMV effects should be observable in these long-baseline experiments or the observation of the effects in the experiments will make more stringent constraints on the parameters.

For the $E_\nu$ dependence postulation Eq. (28) the damping effect is negligibly small, so that the planned long-baseline experiments are unable to observe the QMV effects.

**IX. Summary**

It is an interesting subject for both aspects: first, the conclusion would indicate, even indirectly, if there are the mysterious micro black hole effects, secondly, if this picture is valid,
their existence may be non-negligible in certain physical processes, especially, the neutrino oscillations would be affected and the resultant neutrino-flux attenuation may become observable at the planned long-baseline experiments.

Moreover, as we indicated above, if \((\alpha, \beta, \gamma)_{\nu} > \frac{\Delta_{4\nu}}{E_{\nu}}\) in Eq.(16), the harmonic oscillation form would turn into a pure exponential damping form.

The ansatz Eq.(9) and Eq.(10) lead to different asymptotic limits as \(t \to \infty\), for \(P(\nu_a \to \nu_b)\) with \(a \neq b\) being certain species of neutrinos. If the distance between detector and source is large enough, this difference of Eq.(1) and Eq.(10) is distinguishable.

If a sophisticated neutrino detector is located in Beijing to receive neutrino flux sent from KEK, CERN and Fermilab, as suggested by He and his collaborators [21], the distance is remarkably large, then according to the above analysis, observational prospect of such phenomena is optimistic.

In summary, neutrino oscillation experiments may put a stronger bound on the QMV effects than the K meson system if the \(E_{\nu}^2\) Eq.(25) dependence postulation is valid. Whereas in the case of the \(E_{\nu}\) dependence Eq.(28), K meson system would give stronger restrictions. Even if neutrinos are massless the QMV induced by micro black hole may ‘cause’ neutrino oscillation. We find that this oscillation has interesting prediction for solar neutrino and atmospheric behavior. Moreover long-baseline experiments on neutrino oscillation may provide us valuable information about the neutrinos and the QMV effects. Anyhow, the physical picture about the micro black holes has phenomenological significance, especially to the neutrino oscillation problem. It is worth further and deeper studies.

**Acknowledgment:** This work was supported in part by the National Natural Science Foundation of China (NNSFC) and Grant No. LWTZ-1298 of the Chinese Academy of Sciences. One the authors (C.-H. Chang) would like to thank JSPS for supporting him to visit Japan, and thank Prof. M. Kobayashi for warm hospitality during his visit of Institute of Particle and Nuclear Studies, KEK, Japan, as quite a lot of his part of the work is completed at KEK.
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