We discuss a five dimensional inflationary scenario based on a supersymmetric $SO(10)$ model compactified on $S^1/(Z_2 \times Z_2')$. Inflation is implemented through scalar potentials on four dimensional branes, and a brane-localized Einstein-Hilbert term is essential to make both brane vacuum energies positive during inflation. The orbifold boundary conditions break the $SO(10)$ gauge symmetry to $SU(4)_c \times SU(2)_L \times SU(2)_R (\equiv H)$. The inflationary scenario yields $\delta T/T \propto (M/M_{\text{Planck}})^2$, which fixes $M$, the symmetry breaking scale of $H$ to be close to the SUSY GUT scale of $10^{16}$ GeV. The scalar spectral index $n$ is $0.98 - 0.99$, and the tensor to scalar ratio $r$ is $< 10^{-4}$. The inflaton decay into the lightest right handed neutrinos yields the observed baryon asymmetry via leptogenesis.

1 Introduction

Supersymmetric grand unified theories (SUSY GUTs) provide a prominent framework for physics beyond the standard model, and it is therefore natural to ask if there exists in this framework an intimate connection with inflation. Indeed, a class of realistic SUSY inflationary models elegantly addresses the question. In a promising SUSY inflationary model based on the $SO(10)$ subgroup $SU(4)_c \times SU(2)_L \times SU(2)_R (\equiv H)$, the scalar spectral index $n$ in the model has a value very close to unity (typically $n \approx 0.98 - 0.99$) in excellent agreement with a variety of observations including the recent WMAP data. In particular, the quadrupole microwave anisotropy is proportional to $(M/M_{\text{Planck}})^2$, where $M$ denotes the gauge symmetry breaking scale of $H$, and $M_{\text{Planck}} = 1.2 \times 10^{19}$ GeV. Thus, $M$ is expected to be of order $10^{16}$ GeV, which is quite close to the supersymmetric grand unification scale inferred from the evolution of the minimal supersymmetric standard model (MSSM) gauge couplings. The vacuum energy density during inflation is of order $10^{14}$ GeV, so that the gravitational contribution to the quadrupole anisotropy is essentially negligible. The inflaton field in this scenario eventually decays into right handed neutrinos, whose out of equilibrium decays lead to leptogenesis. However, a straightforward extension to the full $SO(10)$ model is obstructed by the notorious doublet-triplet splitting problem.

Orbifold symmetry breaking in higher dimensional GUTs have recently attracted a great deal of attention because the two particularly pressing problems encountered in four dimensional (4D) SUSY GUTs, namely, the doublet-triplet splitting problem and the dimension five proton decay problem are
easily circumvented without fine-tuning of parameters. The existence of the orbifold dimension can readily break a grand unified symmetry such as $SO(10)$ to its maximal subgroup $H$. Our objective here is to take advantage of recent orbifold constructions of five dimensional (5D) supersymmetric $SO(10)$, and provide a 5D framework which can be merged with the four dimensional (4D) supersymmetric inflationary scenario based on $H$. Because of $N = 2$ SUSY (in 4D sense) in 5D bulk, the F-term inflaton potential is allowed only on the 4D orbifold fixed points (branes), where only $N = 1$ SUSY is preserved.

2 F-term Inflation

The four dimensional inflationary model is best illustrated by considering the following superpotential which allows the breaking of some gauge symmetry $G$ down to $SU(3)_c \times SU(2)_L \times U(1)_Y$, keeping supersymmetry (SUSY) intact.

$$ W_{\text{infl}} = \kappa S (\phi \bar{\phi} - M^2) . $$

(1)

Here $\phi$ and $\bar{\phi}$ represent superfields whose scalar components acquire non-zero vacuum expectation values (VEVs). For the particular example of $G = H$ above, they belong to the $(4, 1, 2)$ and $(\bar{4}, 1, 2)$ representations of $H$. The $\phi, \bar{\phi}$ VEVs break $H$ to the MSSM gauge group. The singlet superfield $S$ provides the scalar field that drives inflation. Note that by invoking a suitable $R$ symmetry $U(1)_R$, the form of $W$ is unique at the renormalizable level, and it is gratifying to realize that $R$ symmetries naturally occur in (higher dimensional) supersymmetric theories and can be appropriately exploited. From $W$, it is straightforward to show that the supersymmetric minimum corresponds to non-zero (and equal in magnitude) VEVs for $\phi$ and $\bar{\phi}$, while $\langle S \rangle = 0$. (After SUSY breaking à la $N = 1$ supergravity (SUGRA), $\langle S \rangle$ acquires a VEV of order $m_{3/2}$ (gravitino mass)).

An inflationary scenario is realized in the early universe with both $\phi, \bar{\phi}$ and $S$ displaced from their present day minima. Thus, for $S$ values in excess of the symmetry breaking scale $M$, the fields $\phi, \bar{\phi}$ both vanish, the gauge symmetry is restored, and a potential energy density $\kappa^2 M^4 (\equiv V_0)$ dominates the universe. With SUSY thus broken, there are radiative corrections from the $\phi - \bar{\phi}$ supermultiplets that provide logarithmic corrections to the potential which drives inflation. In one loop approximation, the

$$ V \approx V_0 \left[ 1 + \frac{\kappa^2 N}{32 \pi^2} \left( 4 \ln \frac{\kappa |S|}{\Lambda} + (z + 1)^2 \ln (1 + z^{-1}) + (z - 1)^2 \ln (1 - z^{-1}) \right) \right] . $$

(2)

where $z = x^2 = |S|^2 / M^2$, $\Lambda$ denotes a renormalization mass scale and $N$ denotes the dimensionality of the $\phi, \bar{\phi}$ representations. From Eq. (2) the
quadrupole anisotropy is found to be\textsuperscript{11}

\[
\left( \frac{\delta T}{T} \right)_Q \approx \frac{8\pi}{\sqrt{N}} \left( \frac{N_Q}{45} \right)^{1/2} \left( \frac{M}{M_{\text{Planck}}} \right)^2 x_Q^{-1} y_Q^{-1} f(x_Q^2)^{-1} . \tag{3}
\]

The subscript $Q$ is there to emphasize the epoch of horizon crossing, $y_Q \approx x_Q(1 - 7/12x_Q^2 + \cdots)$, $f(x_Q^2)^{-1} \approx 1/x_Q^2$, for $S_Q$ sufficiently larger than $M$, and $N_Q \approx 50 - 60$ denotes the e-foldings needed to resolve the horizon and flatness problems. From the expression for $\delta T/T$ in Eq. (3) and comparison with the COBE result $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$ \textsuperscript{4}, it follows that the gauge symmetry breaking scale $M$ is close to $10^{16}$ GeV. Note that $M$ is associated in our $SO(10)$ example with the breaking scale of $H$ (in particular the $B-L$ breaking scale), which need not exactly coincide with the SUSY GUT scale. We will be more specific about $M$ later.

The relative flatness of the potential ensures that the primordial density fluctuations are essentially scale invariant. Thus, the scalar spectral index $n$ is 0.98 for the simplest example based on $W$ in Eq. (1).

Several comments are in order:

- The 50-60 e-foldings required to solve the horizon and flatness problems occur when the inflaton field $S$ is relatively close (to within a factor of order 1-10) to the GUT scale. Thus, Planck scale corrections can be safely ignored.
- For the case of minimal Kähler potential, the SUGRA corrections do not affect the scenario at all, which is a non-trivial result\textsuperscript{11}. More often than not, supersymmetric inflationary scenarios fail to work in the presence of SUGRA corrections which tend to spoil the flatness of the potential needed to realize inflation.
- Turning to the subgroup $H$ of $SO(10)$, one needs to take into account the fact that the spontaneous breaking of $H$ produces magnetic monopoles that carry two quanta of Dirac magnetic charge\textsuperscript{12}. An overproduction of these monopoles at or near the end of inflation is easily avoided, say by introducing an additional (non-renormalizable) term $S(\phi\bar{\phi})^2$ in $W$, which is permitted by the $U(1)_R$ symmetry. The presence of this term ensures the absence of monopoles as explained in Ref.\textsuperscript{12}. Note that the monopole problem is also avoided by choosing a different subgroup of $SO(10)$.
- At the end of inflation the scalar fields $\phi$, $\bar{\phi}$, and $S$ oscillate about their respective minima. Since the $\phi$, $\bar{\phi}$ belong respectively to the $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ of $SU(4)_c \times SU(2)_L \times SU(2)_R$, they decay exclusively into right handed neutrinos via the superpotential couplings,

\[
W = \frac{\gamma_i}{M_P} \bar{\phi}\bar{F}_i^c F_i^c , \tag{4}
\]

where the matter superfields $F_i^c$ belong to the $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ representation of $H$, and $M_P \equiv M_{\text{Planck}}/\sqrt{8\pi} = 2.44 \times 10^{18}$ GeV denotes the reduced Planck mass, and $\gamma_i$ are dimensionless coefficients.
3 Inflationary Solution and Brane Gravity

We consider 5D space-time \((x^\mu, y)\), \(\mu = 0, 1, 2, 3\), where the fifth dimension is compactified on an \(S^1/Z_2\) orbifold. The results in \(S^2/Z_2\) could be readily applied to the \(S^1/(Z_2 \times Z_2')\) case. The action is given by

\[
S = \int d^4x \int_{-y_c}^{y_c} dy \sqrt{|g_5|} \left[ \frac{M_5^3}{2} R_5 + \frac{\delta(y)}{\sqrt{g_5}} \left( \frac{M_4^2}{2} \tilde{R}_4 - \Lambda_1 \right) - \frac{\delta(y - y_c)}{\sqrt{g_5}} \Lambda_2 \right],
\]

where \(R_5 (\tilde{R}_4)\) is the 5 dimensional (4 dimensional) Einstein-Hilbert term\(^a\), and \(\Lambda_1, \Lambda_2\) are the brane cosmological constants. Note that the bulk cosmological constant is not introduced in the action. \(M_5\) and \(M_4\) are mass parameters.

The cosmological constants on the branes could be interpreted the vacuum expectation values of some scalar potentials from the particle physics sector. The brane curvature scalar (Ricci scalar) \(\tilde{R}_4(\tilde{g}_{\mu\nu})\) is defined with the induced metric of the bulk metric, \(\tilde{g}_{\mu\nu}(x) \equiv g_{\mu\nu}(x,y=0)\) \((\mu, \nu = 0, 1, 2, 3)\). For an inflationary solution, we take the metric ansatz,

\[
ds^2 = \beta^2(y)(-dt^2 + e^{2H_0 t} d\vec{x}^2) + dy^2,
\]

where \(H_0\) could be interpreted as the 4 dimensional Hubble constant. The non-vanishing components \((\mu, \mu)\) and \((5, 5)\) of the 5 dimensional Einstein equation derived from (5) gives

\[
3 \left[ \left( \frac{\beta'}{\beta} \right)^2 + \left( \frac{\beta''}{\beta} \right) - \left( \frac{H_0}{\beta} \right)^2 - \delta(y) \frac{M_4^2}{M_5^2} \left( \frac{H_0}{\beta} \right)^2 \right] = -\delta(y) \frac{\Lambda_1}{M_5^2} - \delta(y - y_c) \frac{\Lambda_2}{M_5^2},
\]

\[
6 \left[ \left( \frac{\beta'}{\beta} \right)^2 - \left( \frac{H_0}{\beta} \right)^2 \right] = 0,
\]

where primes denote derivatives with respect to \(y\). The last term in the left hand side in Eq. (7) arises from the brane scalar curvature term, and vanishes when \(H_0 = 0\).

The solutions to the equations Eq. (7) and (8) is given by

\[
\beta(y) = \pm H_0 |y| + c
\]

where \(c (\approx 1)\) is an integration constant. The introduction of the brane scalar curvature term \(\tilde{R}_4\) does not affect the bulk solutions \((\mu, \mu)\), but it modifies the boundary conditions at \(y = 0\) and \(y = y_c\),

\[
\pm \frac{H_0}{c} - \frac{1}{2M_5^3} \frac{H_0^2}{c^2} = -\frac{\Lambda_1}{6M_5^3}, \\
\pm \frac{H_0}{c} \pm \frac{H_0}{c} y_c = \frac{\Lambda_2}{6M_5^3}.
\]

\(^a\)The importance of the brane-localized 4D Einstein-Hilbert term, especially for generating 4D gravity in a higher dimensional non-compact flat space was first noted in Ref. \[13\].
We note that $\Lambda_1$ and $\Lambda_2$ are related to the 4 dimensional Hubble constant $H_0$. While their non-zero values are responsible for the 3-space inflation, vanishing brane cosmological constants guarantee a 4 dimensional flat space-time. When $\Lambda_1 = 0$, $\Lambda_2$ must be also zero. Hence it is natural that the scalar field which controls inflation is introduced in the bulk.\footnote{Since SUSY is broken at low energies, the minima of the inflaton potentials on both branes should be fine-tuned to zero.} For $\Lambda_2 > 0$ and $c > H_0\kappa_c (\sim H_0/M_{\text{GUT}})$, ‘+’ is chosen in Eq. (9). From Eqs. (10)–(11), we also note that the brane cosmological constants $\Lambda_1$ and $\Lambda_2$ should have opposite signs in the absence of the brane curvature scalar contribution at $y = 0$. However, a suitably large value of $M_4/M_5$ can even make the sign of $\Lambda_1$ positive. Since the introduction of the brane curvature term does not conflict with any symmetry that may be present, there is no reason why such a term with a parameter $M_4$ that is large compared to $M_5$ is not allowed.\footnote{The condition for a positive brane cosmological constant on $B_1$ is found from (10) to be $(H_0/c)(M_2^2/M_5^2) > 2$. For $\kappa \sim 10^{-3}$, say, and $c \sim 1$, we have $H_0 \sim 10^{11}$ GeV and $M_5 \sim 10^{16}$ GeV (so that $M_4 \sim M_{\text{P}}$). Thus, there exists a hierarchy of order $10^2$ between the 5D bulk scale $M_5$ and the four dimensional brane mass scale $M_4$. One could construct a simple model to explain it.} Thus, $\Lambda_1$ and $\Lambda_2$ could both be positive and this fact will be exploited for implementing the inflationary scenario.

Our main task is to embed the 4D supersymmetric inflationary scenario in 5D space-time, employing the framework and solutions discussed above. In order to extend the setup to 5D SUGRA, a gravitino $\psi_M$ and a vector field $B_M$ should be appended to the graviton (fünfbein) $e_M^\alpha$. Through orbifolding, only $N = 1$ SUSY is preserved on the branes. The brane-localized Einstein-Hilbert term in Eq. (5) is still allowed, but should be accompanied by a brane gravitino kinetic term as well as other terms, which is clear in off-shell SUGRA formalism.\footnote{In a higher dimensional supersymmetric theory, a F-term scalar potential is allowed only on the 4 dimensional fixed points which preserve $N = 1$ SUSY. We require a formalism in which inflation and the Hubble constant $H_0$ are controlled only by the brane cosmological constants, such that during inflation the positive vacuum energy slowly decreases, and the minimum of the scalar potential corresponds to a flat 4D space-time. The boundary conditions (10) and (11) meet these requirements in the presence of the additional brane scalar curvature term at $y = 0$.} In a higher dimensional supersymmetric theory, a F-term scalar potential is allowed only on the 4 dimensional fixed points which preserve $N = 1$ SUSY. We require a formalism in which inflation and the Hubble constant $H_0$ are controlled only by the brane cosmological constants, such that during inflation the positive vacuum energy slowly decreases, and the minimum of the scalar potential corresponds to a flat 4D space-time. The boundary conditions (10) and (11) meet these requirements in the presence of the additional brane scalar curvature term at $y = 0$.}

\section{5D \textit{SO}(10) Model on $S^1/(Z_2 \times Z_2')$}

Let us consider the 4D $SU(4)_c \times SU(2)_L \times SU(2)_R(\equiv H)$ supersymmetric inflationary model.\footnote{An effective 4D theory with the gauge group $H$ is readily obtained from a 5D $SO(10)$ gauge theory if the fifth dimension is compactified on the orbifold $S^1/(Z_2 \times Z_2')$ where $Z_2$ reflects $y \rightarrow -y$, and $Z_2'$ reflects ...} An effective 4D theory with the gauge group $H$ is readily obtained from a 5D $SO(10)$ gauge theory if the fifth dimension is compactified on the orbifold $S^1/(Z_2 \times Z_2')$ where $Z_2$ reflects $y \rightarrow -y$, and $Z_2'$ reflects...
\( y' \to -y' \) with \( y' = y + y_c/2 \). There are two independent orbifold fixed points (branes) at \( y = 0 \) and \( y = y_c/2 \), with \( N = 1 \) SUSYs and gauge symmetries \( H \) and \( SO(10) \) respectively. The \( SO(10) \) gauge multiplet \((A_M, \lambda^1, \lambda^2, \Phi)\) decomposes under \( H \) as

\[
V_{45} \to V_{(15,1,1)} + V_{(1,3,1)} + V_{(1,1,3)} + V_{(6,2,2)} + \Sigma_{(15,1,1)} + \Sigma_{(1,3,1)} + \Sigma_{(1,1,3)} + \Sigma_{(6,2,2)},
\]

where \( V \) and \( \Sigma \) denote the vector multiplet \((A_{\mu}, \lambda^1)\) and the chiral multiplet \(((\Phi + iA_5)/\sqrt{2}, \lambda^2)\) respectively, and their \((Z_2, Z'_{2})\) parity assignments and KK masses are shown in Table I.

| Vector | \( V_{(15,1,1)} \) | \( V_{(1,3,1)} \) | \( V_{(1,1,3)} \) | \( V_{(6,2,2)} \) |
|--------|----------------|----------------|----------------|----------------|
| \((Z_2, Z'_{2})\) | \((+,+)\) | \((+,+)\) | \((+,+)\) | \((-,+\)) |
| Masses | \(2n\pi/y_c\) | \(2n\pi/y_c\) | \(2n\pi/y_c\) | \((2n+1)\pi/y_c\) |
| Chiral | \(\Sigma_{(15,1,1)}\) | \(\Sigma_{(1,3,1)}\) | \(\Sigma_{(1,1,3)}\) | \(\Sigma_{(6,2,2)}\) |
| \((Z_2, Z'_{2})\) | \((-,-\)) | \((-,-\)) | \((-,-\)) | \((+,-)\) |
| Masses | \((2n+2)\pi/y_c\) | \((2n+2)\pi/y_c\) | \((2n+2)\pi/y_c\) | \((2n+1)\pi/y_c\) |

Table I. \((Z_2, Z'_{2})\) parity assignments and Kaluza-Klein masses \((n = 0, 1, 2, \cdots)\) for the vector multiplet in \( N = 2 \) SUSY \( SO(10) \).

The parities of the chiral multiplets \( \Sigma \)'s are opposite to those of the vector multiplets \( V \)'s in Table I and hence, \( N = 2 \) SUSY explicitly breaks to \( N = 1 \) below the compactification scale \( \pi/y_c \). As shown in Table I, only the vector multiplets, \( V_{(15,1,1)} \), \( V_{(1,3,1)} \), and \( V_{(1,1,3)} \) contain massless modes, which means that the low energy effective 4D theory reduces to \( N = 1 \) supersymmetric \( SU(4)_c \times SU(2)_L \times SU(2)_R \). The parity assignments in Table I also show that the wave function of the vector multiplet \( V_{(6,2,2)} \) vanishes at the brane located at \( y = 0 \) (B1) because it is assigned an odd parity under \( Z_2 \), while the wave functions of all the vector multiplets should be the same at the \( y = y_c/2 \) brane (B2). Therefore, while the gauge symmetry at B2 is \( SO(10) \), only \( SU(4)_c \times SU(2)_L \times SU(2)_R \) is preserved at B1.

The inflationary solution requires positive vacuum energies on both branes B1 and B2. While the scalar potential in Eq. 2 would be suitable for B1, an appropriate scalar potential on B2 is also required. Since the boundary conditions in Eq. 10 and 11 require \( \Lambda_1 \) and \( \Lambda_2 \) to simultaneously vanish, it is natural to require \( S \) to be a bulk field. Then, the VEVs of \( S \) on the two branes can be adjusted such that the boundary conditions are satisfied. As an example, consider the following superpotential on B2,

\[
W_{B2} = \kappa_1 S(Z\bar{Z} - M_1^2),
\]

where \( Z \) and \( \bar{Z} \) are \( SO(10) \) singlet superfields on the B1 brane with opposite \( U(1)_R \) charges.
5 Leptogenesis

After inflation is over, the oscillating system consists of the complex scalar fields \(\Phi = (\delta \phi + \delta \bar{\phi})\), where \(\delta \phi = \phi - M\) (\(\delta \bar{\phi} = \bar{\phi} - M\)), and \(S\), both with masses equal to \(m_{\text{infl}} = \sqrt{2\kappa M}\). Through the superpotential couplings in Eq. (4), these fields decay into a pair of right handed neutrinos and sneutrinos respectively, with an approximate decay width [2]

\[
\Gamma \sim \frac{m_{\text{infl}}}{8\pi} \left(\frac{M_i}{M}\right)^2,
\]

where \(M_i\) denotes the mass of the heaviest right handed neutrino with \(2M_i < m_{\text{infl}}\), so that the inflaton decay is possible. Assuming an MSSM spectrum below the GUT scale, the reheat temperature is given by [17]

\[
T_r \approx \frac{1}{3} \sqrt{\Gamma M_P} \approx \frac{1}{12} \left(\frac{55}{N_Q}\right)^{1/4} \sqrt{g_Q} M_i.
\]

For \(y_Q \sim\) unity (see below), and \(T_r \lesssim 10^{9.5}\) GeV from the gravitino constraint [18], we require \(M_i \lesssim 10^{10} - 10^{10.5}\) GeV.

In order to decide on which \(M_i\) is involved in the decay [19] let us start with atmospheric neutrino (\(\nu_\mu - \nu_\tau\)) oscillations and assume that the light neutrinos exhibit an hierarchical mass pattern with \(m_3 >> m_2 >> m_1\). Then \(\sqrt{\Delta m^2_{\text{atm}}} \approx m_3 \approx m^2_{D3}/M_3\), where \(m_{D3} (= m_i(M))\) denotes the third family Dirac mass which equals the asymptotic top quark mass due to \(SU(4)\)c. We also assume a mass hierarchy in the right handed sector, \(M_3 >> M_2 >> M_1\). The mass \(M_3\) arises from the superpotential coupling Eq. (4) and is given by \(M_3 = 2\gamma_3 M^2/M_P \sim 10^{14}\) GeV, for \(M \sim 10^{16}\) GeV and \(\gamma_3 \sim\) unity. This value of \(M_3\) is in the right ball park to generate an \(m_3 \sim 1 \text{ eV} (\sim \sqrt{\Delta m^2_{\text{atm}}})\), with \(m_i(M) \sim 110\) GeV [17]. It follows from [15] that \(M_i\) in [14] cannot be identified with the third family right handed neutrino mass \(M_3\). It should also not correspond to the second family neutrino mass \(M_2\) if we make the plausible assumption that the second generation Dirac mass should lie in the few GeV scale. The large mixing angle MSW solution of the solar neutrino problem requires that \(\sqrt{\Delta m^2_{\text{solar}}} \approx m_2 \sim \text{GeV}^2/M_2 \sim 1.6\) eV, so that \(M_2 \gtrsim 10^{11} - 10^{12}\) GeV. Thus, we are led to conclude [19] that the inflaton decays into the lightest (first family) right handed neutrino with mass

\[
M_1 \sim 10^{10} - 10^{10.5}\text{ GeV},
\]

such that \(2M_1 < m_{\text{infl}}\).

The constraint \(2M_2 > m_{\text{infl}}\) yields \(y_Q \lesssim 3.34\gamma_2\), where \(M_2 = 2\gamma_2 M^2/M_P\).

We will not provide here a comprehensive analysis of the allowed parameter space but will be content to present a specific example, namely

\[
M \approx 8 \times 10^{15}\text{ GeV}, \quad \kappa \approx 10^{-3}, \quad m_{\text{infl}} \sim 10^{13}\text{ GeV} (\sim M_2),
\]

(17)
with \( y_Q \approx 0.4 \) (corresponding to \( x_Q \) near unity, so that the inflaton \( S \) is quite close to \( M \) during the last 50–60 e-foldings).

Note that typically \( \kappa \) is of order \( 10^{-2} – \text{few} \times 10^{-4} \), so that the vacuum energy density during inflation is \( \sim 10^{-4} – 10^{-8} M_{\text{GUT}}^4 \). Thus, in this class of models the tensor to scalar ratio \( r \) is highly suppressed, \( r \lesssim 10^{-4} \). With \( \kappa \sim \text{few} \times 10^{-4} \), the scalar spectral index \( n \approx 0.99 \).

The decay of the (lightest) right handed neutrinos generates a lepton asymmetry which is given by

\[
\frac{n_L}{s} \approx \frac{10}{16\pi} \left( \frac{T_\gamma}{m_{\text{infl}}} \right) \left( \frac{M_1}{M_2} \right) \frac{\sin 2\delta}{|\langle h \rangle|^2} \frac{\sin^2 \theta}{\sin^2 \theta + \cos^2 \theta} \left( \frac{m_{D2}^2}{m_{D1}^2} \right)^2,
\]

where the VEV \( |\langle h \rangle| \approx 174 \text{ GeV} \) (for large \( \tan \beta \)), \( m_{D1,2} \) are the neutrino Dirac masses (in a basis in which they are diagonal and positive), and \( \cos \theta \equiv c_\theta \), \( \sin \theta \equiv s_\theta \), with \( \theta \) and \( \delta \) being the rotation angle and phase which diagonalize the Majorana mass matrix of the right handed neutrinos. Assuming \( c_\theta \) and \( s_\theta \) of comparable magnitude, taking \( m_{D2} >> m_{D1} \), and using (16) and (17), Eq. (18) reduces to

\[
\frac{n_L}{s} \approx 10^{-8.5} c_\theta^2 \sin 2\delta \left[ \frac{T_\gamma}{10^{8.5} \text{GeV}} \cdot \frac{M_1}{2 \cdot 10^{10.5} \text{GeV}} \cdot \frac{10^{13} \text{GeV}}{M_2} \cdot \frac{m_{D2}^2}{100 \text{GeV}} \right],
\]

which can be in the correct ball park to account for the observed baryon asymmetry \( n_B/s \approx -28/79 \frac{n_L}{s} \).

6 Conclusion

We have proposed a realistic model, which nicely blends together four particularly attractive ideas, namely supersymmetric grand unification, extra dimension, inflation and leptogenesis. To accommodate a 4D F-term inflationary model in 5D, a brane gravity term is necessary. The doublet-triplet problem is circumvented by utilizing orbifold breaking of \( SO(10) \), which may also help in suppressing dimension five proton decay. Concerning inflation, the scalar spectral index \( n \) lies very close to unity (\( \approx 0.98 – 0.99 \)), and the tensor to scalar ratio \( r \) is highly suppressed (\( \lesssim 10^{-4} \)). Finally, the inflaton decay produces heavy right handed Majorana neutrinos (in our case the lightest one), whose subsequent out of equilibrium decay leads to the baryon asymmetry via leptogenesis.

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