Study on stability of roll over control of aircraft

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Abstract. In this paper, studies the problem of rolling out of flight in air flight that may be out of control. First, the corresponding mathematical model is established through dynamic analysis. Secondly, the mathematical model is studied by using Lyapunov function method. Finally, the stability of the model is demonstrated in theory. The effectiveness and safety of the controller are designed.

1. Introduction

The problem of stability is the key problem in the operation of the aircraft. It is an important problem which must be considered first. We begin to discuss the control of the elevator and the throttle. A fixed wing aircraft has a minimum speed that is called a stall speed. When it is below that speed, the aircraft will fail due to the inability to get enough lift, and the aircraft is out of control. We can get the aircraft's current airspeed in real time through the airspeed sensor of the aircraft. When the airspeed is reduced, it is necessary to increase the airspeed by increasing the throttle or push rod to reduce the altitude of the aircraft. Therefore, there are two different control modes for fixed wing aircraft, which are chosen according to the actual situation.

We usually have two control modes. The first control mode is based on the set target airspeed. When the actual airspeed is higher than the target speed, the elevator rod is controlled and the push rod is controlled. The height of the air speed is affected, the altitude of the aircraft is controlled by the throttle. When the altitude is higher than the height of the target, the throttle is reduced. Add the throttle. Therefore, when the aircraft is flying, if it is lower than the target height, control the throttle increases, resulting in an increase in airspeed, and then the flight control control pull rod, so the aircraft rises; when the height of the aircraft is higher than the target height, the control throttle decreases, resulting in a decrease in the airspeed, so the flight control controls the push rod to reduce the height. The advantage of this control method is that the aircraft is always controlled by the air speed as a first factor, so it can ensure the safety of the flight, especially when the engine is out of fire, so that the aircraft can continue to remain safe until the altitude is reduced to the ground. The shortcoming of this way is that the height control is indirect control. Therefore, height control may be delayed or fluctuated.

The second mode of control is to set the angle of attack when the plane is flat. When the altitude is higher or lower than the target height, the climbing angle of the limit is set on the basis of the difference between height and target height on the basis of the plane flying angle of attack. The elevator plane is controlled by the deviation of the current pitch angle and the climbing angle of the aircraft. Make the aircraft reach this climbing angle quickly. Finish the elimination of height deviation. However, the height or decrease of the aircraft will inevitably result in a change in the airspeed. Therefore, the throttle is used to control the
airspeed of the aircraft, that is, when the airspeed is lower than the target airspeed, the throttle is added on the basis of the current throttle. The current airspeed is lower than the target airspeed, and the throttle is reduced on the basis of the current accelerator. The advantage of this control method is that it can react for the first time to the height of the change. Therefore, the height control is better. The disadvantage is that when the throttle is invalid, such as the engine flameout, the height of the flight control will keep the plane at the maximum elevation angle of the limit and eventually cause the stall due to the lack of power. Therefore, these two control modes are selected according to the actual situation. We chose second control modes, and added when the speed of airspeed was lower than a certain speed, it was considered abnormal and immediately turned into the first control mode to ensure the safety of the aircraft.

Adaptive sliding mode highly maneuverable flight control problem is studied based on backstepping process [1]. The position and attitude tracking control problem of four rotor UAV is studied [2]. The global problem of a class of predator-prey systems with nonmonotonic functional response is studied [3]. The asymptotic behavior of a stage structuc model is studied [4]. The problem of vibration stability for semi linear functional differential equations with damping is studied [5]. The stability and Hopf bifurcation of a class of nonlinear systems is studied [6]. The global stability of a class of nonlinear systems is studied [7]. The problems related to the stability of the aircraft are studied [8-20].

The stability problem is very important in the aircraft. On the basis of the literature [1-20], this paper studies the roll over control problem of the aircraft. We have established the corresponding mathematical model. The Lyapunov function method is used to study the asymptotic stability of the model.

2. Roll Control Model

In this paper, Based on the existing literature, according to the principle of dynamics, we set up the following mathematical model of aircraft motion.

$$\begin{align*}
\dot{x}_1 &= (a_3 - a_2)x_2x_3 = k_1x_1 \\
\dot{x}_2 &= (a_1 - a_3)x_1x_3 = k_2x_2 \\
\dot{x}_3 &= (a_2 - a_1)x_1x_2 = k_3x_3
\end{align*}$$  \(1\)

where

(i) \(x_1, x_2, x_3\) is the projection component of the angular velocity on three spindles;

(ii) \(a_1, a_2, a_3\) is the moment of inertia around three inertial axes;

(iii) \(k_1x_1, k_2x_2, k_3x_3\) is the control torque around the three spindles, which is used to counteract the rollover. The normal posture of the aircraft is \(x_1 = x_2 = x_3 = 0\);

(iv) \(a_1, a_2, a_3, k_1, k_2, k_3\) is a constant, \(a_1 > 0, a_2 > 0, a_3 > 0, k_1 < 0, k_2 < 0, k_3 < 0\).

3. Stability Analysis

In order to facilitate the research of system (1), using energy function constructed \(V(x_1, x_2, x_3)\) function. Constructing Lyapunov function as follow.

$$V(x_1, x_2, x_3) = a_1x_1^2 + a_2x_2^2 + a_3x_3^2$$

by conditions (iv), we have

$$V(x_1, x_2, x_3) = a_1x_1^2 + a_2x_2^2 + a_3x_3^2 < 0, \quad (x_1, x_2, x_3) \neq (0,0,0)$$

Therefore, \(V(x, y)\) is positive definite function.

We consider the derivative \(\frac{dV}{dt}\) of system (2), and use conditions (iv), we have
\[
\frac{dV}{dt} = \frac{\partial V}{\partial x_1} \frac{dx_1}{dt} + \frac{\partial V}{\partial x_2} \frac{dx_2}{dt} + \frac{\partial V}{\partial x_3} \frac{dx_3}{dt}
\]

\[
= 2a_1x_1[-(a_1 - a_2)x_1x_1 + k_1x_1] + 2a_2x_2[-(a_1 - a_3)x_1x_2 + k_2x_2] \\
+ 2a_3x_3[-(a_2 - a_3)x_1x_2 + k_3x_3] \\
= (-2a_1a_3 + 2a_2a_3 - 2a_2a_1 + 2a_2a_3 - 2a_3a_2 + 2a_3a_1)x_1x_2x_3 \\
+ 2k_1a_1x_1^2 + 2k_2a_2x_2^2 + 2k_3a_3x_3^2 \\
= 2(k_1a_1x_1^2 + k_2a_2x_2^2 + k_3a_3x_3^2) \\
\leq 2kV(x_1, x_2, x_3) \\
= 2kV(x_1, x_2, x_3)
\]

Where

\[k = \max\{k_1, k_2, k_3\} < 0\]

Therefore, \(\frac{dV}{dt}\) is a definite negative, the zero solution of the Mechanical control system (1) is asymptotically stable.

4. Conclusion

In this paper, the problem of rolling out of control in flight of aircraft is studied. First, the corresponding mathematical model is established through the principle of dynamics. Secondly, the model is studied by using the Lyapunov function method. Finally, the stability of the model is demonstrated. The stability and safety of the aircraft are proved.

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