Search for $\alpha$-cluster states in even-even Cr isotopes

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The $\alpha$ + core structure is investigated in even-even Cr isotopes from the viewpoint of the local potential model. The comparison of $Q_\alpha/A$ values for even-even Cr isotopes and even-even $A = 46, 54, 56, 58$ isotobars indicates that $^{46}$Cr and $^{54}$Cr are the most favorable even-even Cr isotopes for $\alpha$-clustering. The ground state bands of the two Cr isotopes are calculated through a local $\alpha$ + core potential with two variable parameters. The calculated spectra give a very good description of most experimental $^{46}$Cr and $^{54}$Cr levels. The reduced $\alpha$-widths, rms intercluster separations and $B(E2)$ transition rates are determined for the ground state bands. The calculations reproduce the order of magnitude of the available experimental $B(E2)$ values without using effective charges and indicate that the first members of the ground state bands present a stronger $\alpha$-cluster character. The volume integral per nucleon pair and rms radius obtained for the $\alpha+^{50}$Ti potential are consistent with those reported previously in the analysis of $\alpha$ elastic scattering on $^{50}$Ti.

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I. INTRODUCTION

The $\alpha$-cluster structure is present in nuclei of different mass regions and is matter of discussions with many different approaches. A comprehensive review of this research theme, as well as the nuclear clustering in general, is found in Ref. [1], and recent advances are described in Refs. [1, 2]. The $\alpha$-cluster model has been successful in describing energy levels, electromagnetic transition strengths, $\alpha$-decay widths and $\alpha$-particle elastic scattering data. Previous studies [3, 4] have questioned whether the most likely cluster + core structures are determined strictly by the doubly closed shell for cluster and core. Our recent article [5] shows that the local potential model (LPM) with the $\alpha$ + core interpretation can be applied systematically in intermediate mass nuclei around the double shell closure at $^{90}$Zr, obtaining a good description of the energy levels and orders of magnitude of the $B(E2)$ transition rates without the use of effective charges. Additionally, Ref. [5] shows that the $\alpha$ + core potential employed in $^{94}$Mo and $^{96}$Ru are similar to the real parts of the optical potentials used in other studies for analysis of $\alpha+^{90}$Zr and $\alpha+^{92}$Mo elastic scattering, respectively.

In the $fp$-shell region, $^{44}$Ti is one of the most studied nuclei with the $\alpha +$ core interpretation, since it has the configuration of $\alpha$-particle plus $^{40}$Ca doubly magic core. Works on the $\alpha+^{40}$Ca structure in $^{44}$Ti (examples in Refs. [6, 10]) show good descriptions of the ground state band and $B(E2)$ transition rates. Neighboring nuclei as $^{43}$Sc, $^{43}$Ti, and others at the beginning of the $fp$-shell have been analyzed with the LPM [11, 12], however, considering the effect of noncentral forces that arise with non-zero spin cores. The $\alpha +$ core structure in the $fp$-shell region has also been examined in nuclei as $^{42}$Ca and $^{46}$Sc from the viewpoint of the orthogonality condition model [13, 14] and deformed-basis antisymmetrized molecular dynamics [17] with favorable results.

The motivation of this work is the investigation of the $\alpha +$ core structure in nuclei of the Cr isotopic chain, which is near the well-studied $^{44}$Ti and presents even-even nuclei without the $\alpha +$ {doubly closed shell core} configuration. The structure of the Cr isotopes has been analyzed in previous studies with different approaches, such as the shell-model and its variations [16–18], Hartree-Fock or Hartree-Fock-Bogoliubov models [13, 22], and proton-neutron interacting boson model [18]. Also, the $^{48}$Cr nucleus has been described by the $^{40}$Ca+$\alpha +\alpha$ structure with the use of the generator coordinate method [22] and orthogonality condition model [24]. Recently, T. Togashi et al. [17] investigated natural and unnatural-parity states in neutron-rich Cr and Fe isotopes using large-scale shell-model calculations, obtaining good agreement with experimental energy levels. K. Kaneko et al. [16] analyzed neutron-rich Cr nuclei using the spherical shell model; the results give a good general description of the experimental energy levels, and the experimental $B(E2; 2^+_1 \rightarrow 0^+_1)$ values for $^{52}$Cr and $^{54}$Cr are well reproduced by using effective charges $e_{\pi} = 1.5 \epsilon$ and $e_{\nu} = 0.5 \epsilon$.

The nuclei of the Cr region have been intensively studied in recent years, since changes in the nuclear shell structure have been observed with the increase of neutron number in the isotopic chain, raising a question about the persistence of the traditional magic numbers in neutron-rich nuclei of the $fp$-shell. A subshell closure at $N = 32$ is indicated by new measurements of nuclear masses, high $2^+_1$ energy levels, and low $B(E2; 0^+_1 \rightarrow 2^+_1)$ values in nuclei as $^{52}$Ca [25, 26], $^{54}$Ti [27, 28], and $^{56}$Ca [29, 30] compared with neighboring isotopes, and a subshell closure at $N = 34$ has been observed in $^{54}$Ca [31]. Different studies [16, 18, 32, 33] aim for an increase of collectivity for
neutron-rich nuclei in the \( fp \)-shell region near \( N = 40 \). In this context, the present work contributes to further discussions on the nuclear structure of this region.

The next section describes a criterion for selection of the preferential nuclei for \( \alpha \)-clustering in the set of even-even Cr isotopes. In Section III, the \( \alpha \)-cluster model and \( \alpha \) + core potential are described in detail. In Section IV, an analysis of the selected nuclei (\( ^{46}\text{Cr} \) and \( ^{54}\text{Cr} \)) is presented from the standpoint of the LPM. Conclusions are shown in Section V.

II. SELECTION OF PREFERENTIAL NUCLEI FOR \( \alpha \)-CLUSTERING

A preliminary question is the choice of a criterion to determine the most favorable nuclei for \( \alpha \)-clustering in a specified set of nuclei. We use the same criterion employed in our previous work \([5]\) which is based only on experimental data of binding energy \([34,35]\). An appropriate quantity for comparing different nuclei is the variation of average binding energy per nucleon of the system due to the \( \alpha \) + core decomposition. This value is given by

\[
\frac{Q_\alpha}{A_T} = \frac{B_\alpha + B_{core} - B_T}{A_T}, \tag{1}
\]

where \( Q_\alpha \) is the \( Q \)-value for \( \alpha \)-separation, \( A_T \) is the mass number of the total nucleus and \( B_\alpha, B_{core} \) and \( B_T \) are the experimental binding energies of the \( \alpha \)-cluster, the core and the total nucleus, respectively. Thus, an absolute (or local) maximum of \( Q_\alpha/A_T \) indicates the preferred nucleus for \( \alpha \)-clustering in comparison with the rest of (or neighbouring) nuclei in the set.

This study focuses mainly on the comparison of Cr isotopes. FIG. I shows graphically the values of \( Q_\alpha/A_T \) for even-even Cr isotopes, where there are two \( Q_\alpha/A_T \) peaks for \( ^{46}\text{Cr} \) (\( \approx -147.7 \text{ keV} \)) and \( ^{54}\text{Cr} \) (\( \approx -146.8 \text{ keV} \)). These nuclei correspond to \( ^{42}\text{Ti} \) and \( ^{50}\text{Ti} \) cores with magic numbers of neutrons \( N = 20 \) and \( N = 28 \), respectively. However, the \( ^{56}\text{Cr} \) and \( ^{58}\text{Cr} \) isotopes have \( Q_\alpha/A_T \) values (\( \approx -147.1 \text{ keV} \) and \( -149.4 \text{ keV} \), respectively) very close to those of \( ^{46}\text{Cr} \) and \( ^{54}\text{Cr} \). This feature suggests the influence of a neutron subshell closure above \( N = 28 \) for the core. In the case of \( ^{58}\text{Cr} \), there is a \( ^{54}\text{Ti} \) core with \( N_{core} = 32 \), which is related to a subshell closure as discussed in previous studies (see Section II).

FIG. II shows graphically the values of \( Q_\alpha/A_T \) for even-even \( A = 46, 54, 56, 58 \) isotobars. An absolute \( Q_\alpha/A_T \) peak is seen to \( ^{46}\text{Cr} \) in the \( A = 46 \) graph, and a local peak is seen for \( ^{54}\text{Cr} \) in the \( A = 54 \) graph. However, the \( ^{56}\text{Cr} \) and \( ^{58}\text{Cr} \) nuclei do not correspond to absolute or local maxima of \( Q_\alpha/A_T \) in the \( A = 56 \) and \( A = 58 \) graphs.

Therefore, an overall evaluation of the \( Q_\alpha/A_T \) values in FIGS. I and II implies that \( ^{46}\text{Cr} \) and \( ^{54}\text{Cr} \) are preferential nuclei for \( \alpha \)-clustering if they are compared with other even-even Cr isotopes and respective even-even isobars simultaneously. These two Cr isotopes are then selected for a more detailed analysis in next sections.

The existence of two \( Q_\alpha/A_T \) peaks with very close values in FIG. I indicates a transition from \( N_{core} = 20 \) to \( N_{core} = 28 \) as two preferential numbers for \( \alpha \)-clustering in this mass region. However, the variation of \( Q_\alpha/A_T \) is influenced by the changes in the nuclear shell structure for neutron-rich nuclei, and also the liquid drop behavior of the binding energy which is significant for many nuclei. For this reason, in other isotopic chains, the preferential number of neutrons of the core may vary in relation to the traditional magic numbers.

III. \( \alpha \)-CLUSTER MODEL

The properties of the nucleus are viewed in terms of a preformed \( \alpha \)-particle orbiting an inert core. Internal excitations of the \( \alpha \)-cluster and the core are not considered in the calculations. The \( \alpha \) + core interaction is described through a local phenomenological potential \( V(r) = V_C(r) + V_N(r) \) containing the Coulomb and nuclear terms. For the nuclear potential, we adopt the form

\[
V_N(r) = -V_0 \left[ 1 + \lambda \exp \left( -\frac{r^2}{\sigma^2} \right) \right] \left\{ \frac{b}{1 + \exp[(r-R)/a]} + \frac{1 - b}{1 + \exp[(r-R)/3a]} \right\}, \tag{2}
\]

where \( R \) and \( \sigma \) are free parameters and \( V_0, \lambda, a \) and \( b \) are fixed parameters. The Coulomb potential \( V_C(r) \) is taken to be that of an uniform spherical charge distribution of radius \( R_C = R \). The inclusion of the centrifugal term results in the effective potential...
FIG. 2. $Q_{\alpha}/A_T$ values obtained for the $\alpha$ + core decomposition of even-even $A = 46, 54, 56, 58$ isobars as a function of the total charge number $Z_T$. The $Q_{\alpha}/A_T$ values corresponding to $^{46}$Cr, $^{54}$Cr, $^{56}$Cr, and $^{58}$Cr are indicated.

TABLE I. Values of the parameters $R$ and $\sigma$ for $^{46}$Cr and $^{54}$Cr.

| Nucleus | System   | $R$ (fm) | $\sigma$ (fm) |
|---------|----------|----------|---------------|
| $^{46}$Cr | $\alpha + ^{42}$Ti | 4.658    | 0.248         |
| $^{54}$Cr | $\alpha + ^{50}$Ti | 4.674    | 0.210         |

$V_{\text{eff}}(r) = V(r) + \frac{L(L+1)\hbar^2}{2\mu r^2}$, \hspace{1cm} (3)

where $\mu$ is the reduced mass of the $\alpha +$ core system.

The shape employed in eq. (2) is a variation of the modified Woods-Saxon potential W.S.+W.S.\textsuperscript{3}. The factor of type $(1 + \text{Gaussian})$ allows the correct reproduction of the $0^+$ bandhead, which is described roughly with the original W.S.+W.S.\textsuperscript{3} potential in previous calculations for other nuclei [5, 7]. The effect of the $(1 + \text{Gaussian})$ factor in the effective potential with $L > 0$ is very weak; therefore, only the $0^+$ level is changed significantly in comparison with the spectrum produced by the simple W.S.+W.S.\textsuperscript{3} potential.

The ground state bands of $^{46}$Cr and $^{54}$Cr are calculated with the fixed values $V_0 = 220$ MeV, $a = 0.65$ fm, $b = 0.3$ and $\lambda = 0.14$, while $R$ and $\sigma$ are adjusted separately for each nucleus. The values of $V_0$, $a$ and $b$ are the same used in Refs. [5, 7] to describe the ground state bands of nuclei of different mass regions with the W.S.+W.S.\textsuperscript{3} nuclear potential. Firstly, the parameter $\lambda$ is fitted to reproduce the $0^+$ bandheads of the ground state bands of $^{20}$Ne, $^{44}$Ti, $^{94}$Mo and $^{212}$Po, using the corresponding $R$ values obtained from Ref. [5]. Then the parameters $\sigma$ and $R$ are fitted to reproduce the experimental $0^+$ and $4^+$ members of the ground state bands of $^{46}$Cr and $^{54}$Cr. (see TABLE I).

The Pauli principle requirements for the $\alpha$ valence nucleons are introduced through the quantum number

$$G = 2N + L,$$ \hspace{1cm} (4)

where $N$ is the number of internal nodes in the radial wave function and $L$ is the orbital angular momentum.

The global quantum number $G$ identifies the bands of states. In this way, the restriction $G \geq G_{g.s.}$ is applied, where $G_{g.s.}$ corresponds to the ground state band. The value $G_{g.s.} = 12$ is employed for $^{46}$Cr and $^{54}$Cr. This value is obtained from the Wildermuth condition [36] considering the $(fp)^4$ configuration.

The energy levels and associated radial wave functions are calculated by solving the Schrödinger radial equation for the reduced mass of the $\alpha +$ core system.

IV. RESULTS

Using the $\alpha +$ core potential described in Section III we have calculated the ground state bands for $^{46}$Cr and
TABLE II. Calculated values of the volume integral per nucleon pair \( (J_R) \) and root-mean-square radius \( (r_{\text{rms, } R}) \) for the \( \alpha + \) core nuclear potentials employed for \( ^{46}\text{Cr} \) and \( ^{54}\text{Cr} \).

| Nucleus | System   | \( J_R \) (MeV fm\(^3\)) | \( r_{\text{rms, } R} \) (fm) |
|---------|----------|-----------------------------|-----------------------------|
| \( ^{46}\text{Cr} \) | \( \alpha + ^{42}\text{Ti} \) | 355.0 | 4.314 |
| \( ^{54}\text{Cr} \) | \( \alpha + ^{50}\text{Ti} \) | 301.1 | 4.323 |

\( ^{54}\text{Cr} \). The results are compared with the corresponding experimental energies in FIG. 3. The theoretical bands give a very good description of the experimental levels of \( ^{46}\text{Cr} \) from \( 0^+ \) to \( 10^+ \) and \( ^{54}\text{Cr} \) from \( 0^+ \) to \( 8^+ \) (uncertain assignments are indicated in FIG. 3), and a reasonable description of the higher spin levels, if we consider that the fixed parameters \( V_0, a, b \) and \( \lambda \) have been adjusted to reproduce the spectra of nuclei of different mass regions. It is gratifying that the mentioned results are obtained without a dependence on quantum number \( L \) in the \( \alpha + \) core potential.

According to Refs. [37, 38], the experimental \( 0^+, 2^+ \) and \( 4^+ \) levels of the \( ^{54}\text{Cr} \) g.s. band are populated in the \( ^{50}\text{Ti}(^{6}\text{Li},d)^{54}\text{Cr} \) and \( ^{50}\text{Ti}(^{16}\text{O},^{12}\text{C})^{54}\text{Cr} \) reactions. Such \( \alpha \)-transfer information reinforce the choice of these states for comparison with the calculated band. However, there is no mention that the other band members are populated in the same reactions. New \( \alpha \)-transfer experiments may be useful to confirm the spins and parities of the levels above \( 6^+ \) and verify if the levels above \( 4^+ \) are populated by these processes. It is important to observe that there are several experimental \( ^{54}\text{Cr} \) levels above \( E_x \approx 4.5 \text{ MeV} \) which are populated in the \( ^{50}\text{Ti}(^{16}\text{O},^{12}\text{C})^{54}\text{Cr} \) reaction and do not have definite assignments.

It is interesting to compare the \( \alpha + \) core potential of this work with optical potentials used to describe \( \alpha \) elastic scattering. The comparison may be done through the volume integral per nucleon pair

\[
J_R = \frac{4\pi}{A_\alpha A_{\text{core}}} \int_0^\infty V_N(r) r^2 dr ,
\]

and the root-mean-square (rms) radius associated with the potential

\[
r_{\text{rms, } R} = \left[ \frac{\int_0^\infty V_N(r) r^4 dr}{\int_0^\infty V_N(r) r^2 dr} \right]^{1/2}.
\]

Eqs. (5) and (6) refer specifically to the nuclear real part of the optical potential. TABLE II shows the \( J_R \) and \( r_{\text{rms, } R} \) values calculated for the nuclear \( \alpha + \) core potentials of \( ^{46}\text{Cr} \) and \( ^{54}\text{Cr} \). As usual, the negative sign of \( J_R \) is omitted.

Ref. [42] analyzes the \( \alpha \)-particle elastic scattering at 140 MeV on \( ^{50}\text{Ti} \) using a double folding nuclear potential with DDM3Y interaction and three different shapes for the imaginary part of the optical potential; for this case, the values \( J_R = 286-290 \text{ MeV fm}^3 \) and \( r_{\text{rms, } R} = 4.482 \text{ fm} \) have been obtained. The \( J_R \) and \( r_{\text{rms, } R} \) values obtained in the present work for \( ^{54}\text{Cr} \) are close to the ones mentioned previously. This comparison shows there is no discrepancy between the \( \alpha + \) core potential applied in this work and the real part of the optical potential of Ref. [42]. The \( J_R \) and \( r_{\text{rms, } R} \) values shown in the TABLE II are also compatible with the ranges obtained for the same quantities in the analysis of the \( \alpha \) elastic scattering on \( ^{40}\text{Ca} \) at different \( E_{\alpha, \text{lab}} \) energies [43].

The radial wave functions of the states have been deter-
TABLE III. Calculated values for the rms intercluster separation \((\langle R^2 \rangle^{1/2})\), the reduced \(\alpha\)-width \((\gamma_{\alpha}^2)\) and the dimensionless reduced \(\alpha\)-width \((\theta_{\alpha}^2)\) for the members of the ground state bands of \(^{46}\)Cr and \(^{54}\)Cr. The channel radii used for the calculation of \(\gamma_{\alpha}^2\) and \(\theta_{\alpha}^2\) are obtained from eq. (9).

| \(J^n\) | \(\langle R^2 \rangle^{1/2}\) (fm) | \(\gamma_{\alpha}^2\) (keV) | \(\theta_{\alpha}^2\) \((10^{-3})\) |
|-------|--------------------------------|-----------------|-----------------|
| 0\(^+\) | 4.339                         | 2.179           | 6.916           |
| 2\(^+\) | 4.341                         | 2.213           | 7.024           |
| 4\(^+\) | 4.299                         | 1.690           | 5.364           |
| 6\(^+\) | 4.219                         | 0.935           | 2.967           |
| 8\(^+\) | 4.121                         | 0.374           | 1.189           |
| 10\(^+\) | 4.010                         | 0.084           | 0.268           |
| 12\(^+\) | 3.933                         | 0.010           | 0.032           |

| \(J^n\) | \(\langle R^2 \rangle^{1/2}\) (fm) | \(\gamma_{\alpha}^2\) (keV) | \(\theta_{\alpha}^2\) \((10^{-3})\) |
|-------|--------------------------------|-----------------|-----------------|
| 0\(^+\) | 4.290                         | 0.631           | 2.184           |
| 2\(^+\) | 4.290                         | 0.637           | 2.205           |
| 4\(^+\) | 4.249                         | 0.471           | 1.630           |
| 6\(^+\) | 4.181                         | 0.268           | 0.927           |
| 8\(^+\) | 4.089                         | 0.101           | 0.350           |
| 10\(^+\) | 4.003                         | 0.027           | 0.092           |
| 12\(^+\) | 3.933                         | 0.003           | 0.011           |

The root-mean-square (rms) intercluster separation is given by

\[
\langle R^2 \rangle_{G,J}^{1/2} = \left[ \int_0^{\infty} r^2 u_{G,J}^2(r) \, dr \right]^{1/2}, \tag{7}
\]

where \(u_{G,J}(r)\) is the normalized radial wave function of a \(|G,J\rangle\) state. The value of \(\langle R^2 \rangle^{1/2}\) is seen to decrease when one goes from the \(0^+\) state to the highest spin state of each band (see TABLE III). This antistretching effect is found in nuclei of other mass regions where the \(\alpha\)-cluster structure is studied, considering different local potential forms [5, 6, 44, 45]. Such a result shows that the inclusion of the (1 + Gaussian) factor in the nuclear potential does not significantly change this property in relation to the simple W.S.+W.S.\(^3\) potential.

The radial wave functions are also used for the calculation of the reduced \(\alpha\)-width [46, 47].

\[
\gamma_{\alpha}^2 = \frac{\hbar^2}{2\mu a_c} u^2(a_c) \left[ \int_0^{\infty} |u(r)|^2 \, dr \right]^{-1}, \tag{8}
\]

where \(\mu\) is the reduced mass of the system, \(u(r)\) is the radial wave function of the state and \(a_c\) is the channel radius. In this work, a procedure that avoids an arbitrary choice of channel radius is used. The value of \(a_c\) is given by the relation

\[
a_c = 1.295(A_{\alpha}^{1/3} + A_{core}^{1/3}) + 0.824 \, (\text{fm}), \tag{9}
\]

obtained from a linear fit [5] that considers other channel radii used for different \(\alpha\)+core systems in the literature. The dimensionless reduced \(\alpha\)-width \(\theta_{\alpha}^2\) is defined as the ratio of \(\gamma_{\alpha}^2\) to the Wigner limit, that is,

\[
\theta_{\alpha}^2 = \frac{2\mu a_c^2}{\hbar^2} \gamma_{\alpha}^2. \tag{10}
\]

Qualitatively, a large value of \(\theta_{\alpha}^2\) \((\approx 1)\) is interpreted as an evidence of a high degree of \(\alpha\)-clustering.

The g.s. bands of \(^{46}\)Cr and \(^{54}\)Cr show a rapid decrease of \(\gamma_{\alpha}^2\) with the increasing spin (see TABLE III). In agreement with the analysis of the rms intercluster separations, the behavior of \(\gamma_{\alpha}^2\) suggests a stronger \(\alpha\)-cluster character for the first members of these bands. For the two nuclei, the dimensionless reduced \(\alpha\)-widths \(\theta_{\alpha}^2\) present a small fraction of the Wigner limit, even for the first members of the band. This is an expected feature for strongly bound \(\alpha\)+core states, or states above the \(\alpha\)+core threshold and far below the top of the Coulomb barrier.

It is noted that the \(\theta_{\alpha}^2\) values for \(^{46}\)Cr are \(\approx 3\times\) higher than the respective values for \(^{54}\)Cr. The energy location of the \(\alpha\)+core threshold has an important influence on this difference. In the excitation energy scale, the \(\alpha\)+core threshold for \(^{54}\)Cr is \(\approx 1.1\) MeV higher than the corresponding threshold for \(^{46}\)Cr. Thus, the radial wave functions of the \(^{54}\)Cr states are less intense in the surface region and, consequently, these states have a lower degree of \(\alpha\)-clustering in comparison with \(^{46}\)Cr.

The model also allows the calculation of the \(B(E2)\) transition rates between the states of an \(\alpha\)-cluster band. In the case where the cluster and the core have zero spins, this quantity is given by

\[
B(E2; G, J \rightarrow J - 2) = \frac{15}{8\pi} \beta^2 J(J - 1) (2J + 1)(2J - 1) \langle r_{J,J-2}^2 \rangle^2, \tag{11}
\]

where

\[
\langle r_{J,J-2}^2 \rangle = \int_0^{\infty} r^2 u_{G,J}(r) u_{G,J-2}(r) \, dr, \tag{12}
\]
TABLE IV. Comparison of the calculated $B(E2)$ transition rates for the ground state bands of $^{46}$Cr and $^{54}$Cr with the corresponding experimental data [33, 48]. The calculated values have been obtained without effective charges.

| $^{46}$Cr ($\alpha+^{42}$Ti system) | $J^\pi$ | $B(E2; J \rightarrow J - 2)$ (W.u.) | $B(E2)_{\text{exp.}}$ (W.u.) |
|---|---|---|---|
| $2^+$ | 9.657 | 19(4)$^a$ |
| $4^+$ | 13.045 | — |
| $6^+$ | 12.454 | — |
| $8^+$ | 10.281 | — |
| $10^+$ | 7.057 | — |
| $12^+$ | 3.689 | — |

| $^{54}$Cr ($\alpha+^{50}$Ti system) | $J^\pi$ | $B(E2; J \rightarrow J - 2)$ (W.u.) | $B(E2)_{\text{exp.}}$ (W.u.) |
|---|---|---|---|
| $2^+$ | 7.456 | 14.4(6) |
| $4^+$ | 10.049 | 26(9) |
| $6^+$ | 9.688 | 18(5) |
| $8^+$ | 8.004 | 12.8(17) |
| $10^+$ | 5.729 | — |
| $12^+$ | 2.969 | — |

$^a$ Deduced from the experimental $B(E2; 0^+_g, s \rightarrow 2^+_i)$ value obtained by K. Yamada et al. [48].

$\beta_2$ is the recoil factor, given by

$$\beta_2 = \frac{Z_\alpha A_{\text{core}}^2 + Z_{\text{core}}A_\alpha^2}{(A_\alpha + A_{\text{core}})^2},$$

(13)

$u_{G,J}(r)$ and $u_{G,J-2}(r)$ are the radial wave functions of the initial $|G, J\rangle$ state and final $|G, J - 2\rangle$ state, respectively.

The calculated $B(E2)$ transition rates for the g.s. bands of $^{46}$Cr and $^{54}$Cr are presented in TABLE IV. A comparison of the calculated values and experimental data shows that the model can provide the correct order of magnitude of the experimental $B(E2)$ values without the use of effective charges. These results may be considered satisfactory since, in shell-model calculations for nuclei of this mass region [15, 18, 42, 50], substantial effective charges are necessary to reproduce the experimental data. Furthermore, it is shown that the calculated $B(E2)$ values for $^{54}$Cr reproduce nicely the increasing or decreasing trend of the experimental data between the $2^+ \rightarrow 0^+$ and $8^+ \rightarrow 6^+$ transitions.

There are few negative parity levels with definite assignments for $^{46}$Cr and $^{54}$Cr, and the clear identification of negative parity bands is not possible. Nevertheless, we have calculated the $3^-$ level of the $G = 13$ band for a comparison with experimental energy levels of the two nuclei, applying the depth $V_0 = 238$ MeV used in the simple W.S.+W.S. potential for the calculation of the negative parity bands of even-even nuclei around $^{94}$Mo [3]. The energies $E_{\text{calc}}(3^-) = 3.444$ MeV and 3.258 MeV are obtained for $^{46}$Cr and $^{54}$Cr, respectively, to be compared with the experimental excitation energies 3.1965 MeV (uncertain assignment) and 4.12705 MeV (definite assignment), respectively. A consistent analysis of the $\alpha$ + core negative parity bands depends on further experimental data.

V. CONCLUSIONS

The calculation of $Q_\alpha/A_T$ values for even-even Cr isotopes and even-even $A = 46, 54, 56, 58$ isotopes indicates that $^{46}$Cr and $^{54}$Cr are the preferential nuclei for $\alpha$-clustering when compared with their even-even isotopes and isobars simultaneously. The $\alpha$-cluster model gives a good account of the experimental ground state bands of these two nuclei through a local $\alpha +$ core potential with two variable parameters. The nuclear potential with $(1 + \text{Gaussian}) \times (\text{W.S.} + \text{W.S.})$ shape allows the correct reproduction of the $0^+$ bandheads and, additionally, describes the experimental higher spin levels of the two nuclei very well using a fixed set of four parameters which was successful in describing the ground state bands in nuclei of different mass regions.

The calculations of the volume integral per nucleon pair and rms radius show that the values for the $^{54}$Cr nuclear potential are close to those obtained for the real part of the optical potential for $\alpha-\text{Ti}$ elastic scattering at 140 MeV [42]. As the volume integral may vary with the $\alpha + \text{core}$ scattering energy, this comparison should not be seen as complete; however, it is shown that there is no discrepancy between the $\alpha-\text{Ti}$ optical potential and the $\alpha + \text{core}$ potential of this work.

The $B(E2)$ values obtained for $^{46}$Cr and $^{54}$Cr give the correct order of magnitude of the available experimental data without effective charges. The calculated intercluster rms radii and reduced $\alpha$-widths suggest that the $\alpha$-cluster character is stronger for the first members of the ground state bands of $^{46}$Cr and $^{54}$Cr. Therefore, the use of the $(1 + \text{Gaussian})$ factor in the $\alpha + \text{core}$ nuclear potential does not change significantly this feature as compared to other $\alpha + \text{core}$ calculations in different mass regions.

New experimental data, especially from $\alpha$-transfer reactions, will be important to complement the analysis of this work.

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