Does God So Love the Multiverse? *

Don N. Page †
Institute for Theoretical Physics
Department of Physics, University of Alberta
Room 238 CEB, 11322 – 89 Avenue
Edmonton, Alberta, Canada T6G 2G7
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Abstract

Monotheistic religions such as Judaism and Christianity affirm that God loves all humans and created them in His image. However, we have learned from Darwin that we were not created separately from other life on earth. Some Christians opposed Darwinian evolution because it undercut certain design arguments for the existence of God. Today there is the growing idea that the fine-tuned constants of physics might be explained by a multiverse with very many different sets of constants of physics. Some Christians oppose the multiverse for similarly undercutting other design arguments for the existence of God. However, undercutting one argument does not disprove its conclusion. Here I argue that multiverse ideas, though not automatically a solution to the problems of physics, deserve serious consideration and are not in conflict with Christian theology as I see it.

Although this paper as a whole is addressed primarily to Christians in cosmology and others interested in the relation between the multiverse and theism, it should be of interest to a wider audience. Proper subsets of this paper are addressed to other Christians, to other theists, to other cosmologists, to other scientists, and to others interested in the multiverse and theism.

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†Internet address: don@phys.ualberta.ca
Preface Added to the Original

This paper was originally written as an expansion, for the proceedings, of lectures given at Shandong University in Jinan, China, to students and other academics who were interested in the relation between science and religion. Therefore, it employs a variety of philosophical and theological argumentations that go beyond the usual scientific reasoning of physics papers. However, I believe that this paper may also be of sufficient interest for many physicists to be included in the usual physics eprint archives, despite the differences in style. (Those who are not interested in Christian theology may wish to skip the Bible verses and stories, though they illustrate the claim of the breadth of God’s love. Similarly, those not interested in arithmetic may wish to skip the example of the large finite set of pili-increasers and the offer of a prize of perhaps $1051 for the first member to be found of this set of 10^{154} or so members, though this set illustrates how evidence for a multiverse may be contained within a single universe.)

The written version of this paper has many different purposes. One primary purpose is to persuade my fellow Christians that multiverse ideas are not necessarily opposed to theism and to Christianity. A secondary purpose, and one of the main reasons for posting it on the physics eprint archives, is to show other scientists that not all Christians are opposed to multiverse ideas, just as not all Christians are opposed to evolutionary ideas, even though some Christians have opposed both.

I have long been open to multiverse ideas and remember asking Steven Weinberg around 1983 what he thought of the idea that the constants of physics might vary from one component of the quantum state of the universe to another. At that time, Weinberg said, “I would find that very depressing.” However, Weinberg has come around to champion the idea that the cosmological ‘constant’ might vary across a multiverse\(^1\).

Even more recently, Weinberg has made the following argument\(^2\):

> “Finally, I have heard the objection that, in trying to explain why the

\(^{1}\)Steven Weinberg, “Anthropic Bound on the Cosmological Constant,” Physical Review Letters 59, 2607-2610 (1987); Hugo Martel, Paul R. Shapiro, and Steven Weinberg, “Likely Values of the Cosmological Constant,” The Astrophysical Journal 492, 29-40, astro-ph/9701099, <http://arxiv.org/abs/astro-ph/9701099>.

\(^{2}\)Steven Weinberg, “Living in the Multiverse,” in Bernard Carr, ed., Universe or Multiverse? (Cambridge University Press, Cambridge, 2007), pp. 29-42, hep-th/0511037, <http://arxiv.org/abs/hep-th/0511037>.
laws of nature are so well suited for the appearance and evolution of life, anthropic arguments take on some of the flavor of religion. I think that just the opposite is the case. Just as Darwin and Wallace explained how the wonderful adaptations of living forms could arise without supernatural intervention, so the string landscape may explain how the constants of nature that we observe can take values suitable for life without being fine-tuned by a benevolent creator. I found this parallel well understood in a surprising place, a New York Times op-ed article by Christoph Schönborn, Cardinal Archbishop of Vienna. His article concludes as follows:

‘Now, at the beginning of the 21st century, faced with scientific claims like neo-Darwinism and the multiverse hypothesis in cosmology invented to avoid the overwhelming evidence for purpose and design found in modern science, the Catholic Church will again defend human nature by proclaiming that the immanent design evident in nature is real. Scientific theories that try to explain away the appearance of design as the result of ‘chance and necessity’ are not scientific at all, but, as John Paul put it, an abdication of human intelligence.’

“It’s nice to see work in cosmology get some of the attention given these days to evolution, but of course it is not religious preconceptions like these that can decide any issues in science.”

Here I wish to go on record as a Christian who respectfully differs from Cardinal Schönborn’s opinion that the multiverse idea was “invented to avoid the overwhelming evidence for purpose and design found in modern science” and that it is “an abdication of human intelligence.” Different multiverse theories were invented for different reasons, and there are intelligent reasons for investigating them. Furthermore, while I agree with Weinberg that multiverse theories would not require that the ‘constants’ of nature be fine tuned, I would also insist that they do not preclude the possibility that the entire multiverse was designed at a higher level, say by a benevolent Creator.

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3C. Schönborn, N. Y. Times, 7 July 2005, p. A23.
1 God’s Love for All Humans

A central point of Judaism and Christianity is that God loves everyone. For Christians, one of the most famous verses in the Bible is in the Gospel of John, John 3:16: “For God so loved the world that He gave His only begotten Son, that whoever believes in Him should not perish but have everlasting life” [1]. However, this idea starts way back in the Old Testament (the first part of the Bible, the part accepted by both Jews and Christians).

In Genesis, the first book of the Old Testament, God began a revelation through the family of Abraham, Isaac, and Jacob (who became the Israelites, Hebrews, and/or Jews). In God’s original call to Abraham in Genesis 12:1-3, He said, “I will make you a great nation; I will bless you and make your name great; ... and in you all the families of the earth shall be blessed.” As a personal example of this, my family is not Jewish, and yet academically I have been marvelously blessed by the remarkable contributions of a vast number of brilliant Jewish scientists, and spiritually I have been immeasurably blessed by the teachings, life, death, and resurrection of the Jewish rabbi Yeshua, known in English as Jesus, who we Christians believe to be the Christ or Messiah, the anointed Son of God and Savior.

Later in the Old Testament, in the book of Ruth, God’s love was extended beyond the Israelites to Ruth, a woman of the foreign country of Moab who married an Israelite who had settled there during a famine in Israel. After both Ruth and her mother-in-law Naomi were widowed, Ruth moved with Naomi to Israel and became an ancestor to Israel’s greatest king, David, as well as an ancestor of Jesus.

Jonah is another book in the Old Testament stressing the extension of God’s love beyond the Israelites. This famous short story tells of the prophet Jonah who was sent by God to preach against the evils of Nineveh, a great city of ancient Assyria, on the eastern bank of the Tigris River at the location of the present city of Mosul, Iraq. As an Israelite enemy of Nineveh, Jonah wanted Nineveh to remain unrepentant and be destroyed by God. Therefore, initially he refused to go warn that evil city but rather fled in the opposite direction. As Jonah later admitted to God, “Therefore I fled previously to Tarshish; for I know that You are a gracious and merciful God, slow to anger and abundant in lovingkindness, One who relents from doing harm.” However, through a remarkable event God got Jonah to obey Him, and the Ninevites repented and were spared, still to the bitter anger of Jonah.
The book closes with God’s emphasizing His love for the foreign city: “And should I not pity Nineveh, that great city, in which are more than one hundred and twenty thousand persons who cannot discern between their right hand and their left—and much livestock?”

In the New Testament, God sent His Son Jesus Christ to live, die on the cross for our sins, and defeat death by His resurrection, to bring forgiveness and salvation to all of us who cannot meet God’s standards as set down by the laws God had given to the Israelites in the Old Testament. As the Apostle Paul expressed it in Romans 3:23-24: “For all have sinned and fall short of the glory of God, being justified freely by His grace through the redemption that is in Christ Jesus.” This Gospel message of salvation by faith was offered to all people, not just the Jews. The last recorded words of Jesus Christ, in the Gospel of Matthew 28:18-20, are these: “All authority has been given to Me in heaven and on earth. Go therefore and make disciples of all the nations, baptizing them in the name of the Father and of the Son and of the Holy Spirit, teaching them to observe all things that I have commanded you; and lo, I am with you always, even to the end of the age.”

The first book of the history of Christianity, after the Gospel stories of the life, death, and resurrection of Jesus, is the Acts of the Apostles. It particularly records how the Apostles Peter and Paul began the work of extending the Gospel message beyond the Israelites to all nations.

God not only loves all people, but He also said that He created all people in His own image. As it is written in the first book of the Bible, in Genesis 1:27, “So God created man in His own image; in the image of God He created him; male and female He created them.”

The question arises as to how unique does that make us. The Bible certainly emphasizes that the image of God extends to all humans. But are we created entirely separately from the rest of creation?

Some have taken the image of God for humans to imply that God created us individually and separately from other living beings. However, Darwin’s theory of evolution suggests that we are related to the rest of life. It also suggests that we humans were not separately created by an individual act, independent of the creation of the remainder of the earth’s biosphere.
2 Parallels Between Evolution and Multiverse Ideas

When Darwin proposed evolution, many conservative Christians accepted it. One famous example was Benjamin B. Warfield (1851-1921), the conservative Christian theologian and principal of Princeton Seminary from 1887 to 1921. Warfield wrote the chapter on “The Deity of Christ” in The Fundamentals, from which the term Fundamentalism arose. Thus one of the most famous original Fundamentalists accepted Darwinian evolution, writing [2], “I am free to say, for myself, that I do not think that there is any general statement in the Bible or any part of the account of creation, either as given in Genesis 1 and 2 or elsewhere alluded to, that need be opposed to evolution.”

However, many Christians later came to oppose evolution, perhaps most famously some other Fundamentalists. Although there were many reasons for this, which I cannot get into here, one possible reason is that evolution did remove one particular design argument for the existence of God, that all of the marvelously many different species of living things on earth had been separately designed and created by God. Nevertheless, evolution did not disprove the existence of God or of some overall design. Indeed, there are many leading theologians and scientists today that accept both evolution and creation by God, such as Francis Collins, the head of the Human Genome Project [3].

It seems to me that there may be a parallel development occurring today. Before Darwin, some Christians took the marvels of humanity as evidence of separate and individual design. Now, some Christians take the marvels of the fine-tuning of the constants of physics as evidence of theism and often of separate and individual design of these constants by God [4, 5, 6, 7, 8, 9, 10, 11, 12, 3]. Here I wish to argue that this could be equally mistaken.

I have found that my views are rather similar to those of a minority of theists, notably John Leslie [13, 14], Stephen Barr [15], Robin Collins [16, 17, 18], Gerald Cleaver [19, 20], Klaas Kraay [21], and others that Kraay cites, who break tradition and argue that a multiverse could reveal an even more grand design of the universe, since the physical process that generates the multiverse would have to have suitable basic laws and initial conditions to produce any life at all (no matter what the constants of physics are, since often they seem to be fine tuned for several different reasons [13, 14]). The laws and initial conditions would apparently have to be even
more special to produce not just life, but life like ours observing the order we actually do see around us. Leslie, Barr, Collins, Cleaver, Kraay, and others claim that since God is infinitely creative, it makes sense to say that He might create a physical reality much larger than the single visible part of the universe or multiverse that we can observe directly.

3 Fine Tuning in Our Universe

Now it does seem to be true that we could not be here if many of the constants of physics were significantly different, so that in our part of the universe, the constants of physics do in fact seem to be fine tuned for our kind of life. This is generally agreed upon both by those who attempt to use this fine tuning to support theism, as in the references above, and by many scientists who are usually neutral or opposed to such an attempt [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. Of course no one knows what other forms of life might be possible if the constants of physics were significantly different, but the general consensus seems to be that if would be very difficult to imagine the possibility of any complex life at all existing if certain combinations of the constants of physics were greatly different.

For example, one of the most remarkable fine tunings is the value of the cosmological constant or energy density of the ‘dark energy’ responsible for the current acceleration of distant galaxies away from each other. (Even though a positive cosmological constant corresponds to positive energy density which normally would gravitate or tend to pull things together, it is also accompanied by tension or negative pressure that antigravitates by an amount that is larger by a factor of three, the number of dimensions of space. Thus the net observed effect of the positive cosmological constant or dark energy is antigravity, the gravitational repulsion of otherwise empty space that causes distant galaxies now to be accelerating apart.)

Measurements show that the cosmological constant is more than 120 orders of magnitude smaller than unity in certain natural units (called Planck units, obtained by setting to unity the speed of light, Planck’s quantum constant of action, and Newton’s gravitational constant). With the other constants kept fixed, it would be difficult to have a universe with gravitationally formed structures lasting long enough for life if the cosmological constant were even just a few orders of magnitude (powers of 10) larger than its observed value. But even if one tuned the other constants to
allow the possibility of such structures when the cosmological constant has a value many orders of magnitude larger than its observed value, one still seems to need it to be many orders smaller than unity. So one does not see how to avoid at least some significant amount of fine tuning of this parameter. (Basically, if the cosmological constant were of the order of unity in the natural Planck units, the spacetime of the universe would always have large quantum mechanical fluctuations, and no one knows any plausible way to have persisting complex structures that one could call life in such a case.)

Another constant that is many orders of magnitude away from unity, in this case about 36 orders of magnitude larger than unity, is the ratio of the electrostatic repulsion to the gravitational attraction between two protons (the nuclei at the centers of hydrogen atoms). With other constants kept fixed, it seems that one could not have the types of stars that appear to be necessary for life if this constant differed by much more than even one order of magnitude (factor of 10) from its actual value. Again one could try to imagine a universe hospitable to some other form of life when this constant is significantly different by also tuning other constants to an appropriate range, but again it seems that complex life of any form relying mainly on the electromagnetic and gravitational forces would be impossible if this constant were close to unity. (Then it seems that one could not have stars, planets, and living organisms with large numbers of atoms, since the number of atoms in such structures generally scales as a positive power of this constant and would approach some small number near unity if this constant were itself near unity [34, 35].)

Martin Rees [28] discusses in much more detail these two constants and four others in our universe that are crucial for its properties. Life as we know it would apparently be impossible if any one of them were greatly different (with the others held fixed). So although it might not be necessary for all of them to have their observed values, there are some combinations of them that apparently could not be very much different and yet give a universe with life, at least life at all similar to present life on earth.

4 Explanations for Fine Tuning

So there is a general consensus that there is at least some fine tuning of the constants of physics in our part of the universe, though not that all of the constants
had to have values close to what we observe. But what is the explanation for this phenomenon? There are three general types of explanations that are often put forward.

Some suggest that the fine tuning was done by a separate act of God to allow life. Others say that it is presumably an accidental fluke. And yet others propose that it arises from a huge multiverse of very many different possible constants of physics. It is also noted in several of the references that I have given, such as [13, 36], that the three explanations are not mutually exclusive, so that virtually any combination of them is logically possible. However, it is the multiverse explanation that is now rapidly growing in favor, though not without a lot of opposition from both theists and nontheists.

One must quickly point out that each of these three explanations really stands for a class of explanations, so that one should actually compare specific proposals taken from these classes rather than the classes themselves. Theists of different theological convictions might propose different ideas of how God would choose the constants. Those saying that the fine tuning is a fluke might say that the constants are determined by any number of different mathematical structures that just happened to give biophilic values, or they might propose that there is truly some random process determining the constants in some way not derivable from any simple mathematical structure. And of course there are a huge number of possible multiverse theories.

Some multiverse theories seem to me to be too general to be plausible, such as the idea of David Lewis [37] that all logical possibilities actually exist, or the original idea of Max Tegmark [38] that all mathematical structures have physical reality. These would seem to leave it unexplained why what we see has the order that it does, whereas a random possibility from all logical possibilities or from all mathematical structures would surely be far more chaotic [13, 14, 15, 36] (though I now admit that my objection to Tegmark’s original Level IV multiverse as logically inconsistent was wrong, since I misinterpreted each mathematical structure to be a description of reality rather than being simply a part of reality). However, there might be other multiverse theories that are more explanatory of the order that we do observe, perhaps arising naturally out of elegant but specific laws of nature. For example, Tegmark’s more recent ideas [39] that physical reality might be restricted to the mathematical structures of computable functions seems to me more hopeful,
though I would disagree with his view that physical reality is just mathematical structure or syntax and instead believe that it includes at least the semantics of consciousness.

One natural way to get a multiverse is to have a universe so large that highly varied conditions occur somewhere. Another is from Everett many-worlds [40], that all the quantum possibilities are actually realized. However, those possibilities do not necessarily give varying constants of physics.

One scenario that seems more hopeful is to get multiverses from inflation [41, 42, 43], which is a very rapid exponential expansion of the early universe that may make the universe enormously larger than what we can observe of it. If the inflationary scenario can include phase transitions, and if the constants of physics can differ across phase transitions, inflation tends to produce all such possibilities.

Recently it has been realized that string/M theory apparently leads to a huge multiverse of $10^{500}$ or so different vacua or sets of constants [44]. This would apparently be enough for the constants we see to occur somewhere (maybe once per $10^{200}$ vacua or so). Then perhaps $10^{300}$ or so vacua would fit what we see.

If only one universe in $10^M$ could fit our observations, but if $10^N$ different universes exist in the multiverse, then it might not be surprising that what we observe exists if $N > M$. E.g., in the previous paragraph, I was saying that perhaps $N$ is around 500 and $M$ is around 200, so then indeed $N > M$. However, the actual numbers are known very poorly [44]. We really don’t yet know whether $N > M$ in string/M theory, but that seems plausible. Then what we see could be explained without its having to be individually selected.

One might still ask whether the multiverse explanation always works, assuming that it has enough universes (e.g., $N > M$). Is it sufficient to explain what we see by a multiverse theory in which there are enough different conditions that ours necessarily occurs somewhere?

I would say no, but rather that there is the further requirement that the conditions we observe should not be too rare out of all the conditions that are observed over the entire multiverse. A theory making our observations too rare should not be considered a good theory.

Good theories should be both intrinsically plausible and fit observations. Intrinsic plausibility is quantified by what is called the $a$ priori probability of the theory,
the probability that one might assign to it from purely theoretical background knowledge, without considering any observations. The fit to observations is quantified by the conditional probability of the observation given the theory, what is called the likelihood. Then the probability of the theory after taking into consideration the observation, what is called the a posteriori probability of the theory, is given by Bayes’ theorem as being proportional to the product of the a priori probability and the likelihood [45].

I take the a priori probabilities of theories (intrinsic plausibilities before considering the observations) to be subjective but to be generally assigned higher values for simpler theories, by the principle that is called Occam’s razor. One problem with this is that David Deutsch [46, 47] notes that simplicity depends on one’s background knowledge that itself depends on the laws of physics.

The likelihood of a theory is itself neither the a priori or the a posteriori probability of the theory, but rather the conditional probability, not of the theory, but of the observation given the theory. A theory that uniquely gives one’s observation would have unit likelihood but might have very low a priori probability.

For example, consider an extreme solipsistic theory that only one’s actual momentary observation exists, not anyone’s else’s or even any of one’s own in either the past or the future, and perhaps not even that an external world exists at all. This theory would predict that observation with certainty if it were correct. (If the theory were true, certainly the observation would be that single one predicted by the theory.) Therefore, for that observation the likelihood is unity. However, such an extreme solipsistic theory, giving all the details of one’s observation or conscious perception without an external world giving other observations, would surely be highly complex and so would be viewed as extremely implausible, much more implausible than an alternate theory in which the observation resulted from the existence of an external world that also gives other observations. Therefore, this extreme solipsistic theory would be assigned very low a priori probability.

At the other extreme, consider the simple theory that predicts all possible observations equally (arguably a consequence of something like the modal realism of David Lewis [37]). Since this theory is so simple, it might be assigned a high a priori probability, but then because of the enormous number of observations it predicts with equal probability, it would give very low likelihood.
5 Applying Bayes’ Theorem

Let us consider a simple example with three possible theories and use Bayes’ theorem to calculate the resulting a posteriori probabilities. Suppose that we have theory $T_1$ with a priori probability 0.000 000 1 that would give probability 1 for what we see (unit likelihood; the observation would be certain if this theory were true), theory $T_2$ with a priori probability 0.001 that would give probability 0.01 for what we see (1% likelihood), and theory $T_3$ with a priori probability 0.998 999 9 that would give probability 0.000 000 1 for what we see (0.000 01% likelihood). Assume for simplicity that these three theories exhaust all possible theories.

Then the product of the a priori probability and of the likelihood for the first theory, $T_1$, is 0.000 000 1, for $T_2$ is 0.000 01, and for $T_3$ is nearly 0.000 000 1. The a posteriori probabilities of these three theories, by Bayes’ theorem, is proportional to these products, so all we need to do is to normalize the products by dividing each of them by their sum, which is nearly 0.000 010 2. After dividing by this, we then get that to two-digit accuracy, the approximate a posteriori probability of $T_1$ is 0.01, of $T_2$ is 0.98, and of $T_3$ is 0.01. That is, after the observation, we would think that theories $T_1$ and $T_3$ each have only about 1% probability of being correct, whereas in the end theory $T_2$ is seen to be 98% probable.

In this case, neither $T_1$ with its unit likelihood, nor $T_3$ with its nearly unit a priori probability, gains the highest a posteriori probability, which instead goes to the compromise theory $T_2$, which had both its a priori probability (0.001) and its likelihood (0.01) rather low. Of course, there is no guarantee that a compromise theory will gain the greatest a posteriori probability, since the result depends on the particular a priori probabilities and likelihoods of the theories being tested. However, if the theories were ranked in order from most specifically predictive, say giving unit likelihood or unit probability for the specific observation, to the simplest theory with the highest a priori probability, it would seem unusual for either of the two extreme theories to end up with the greatest a posteriori probability. In this way I would be surprised if the theory for the universe or multiverse with the greatest a posteriori probability turned out to be either the most specifically predictive theory (giving precisely one’s own observation with certainty, and no probability of any other possible observation one might have had instead) or the simplest (say predicting all possible observations with equal likelihood).
Suppose that as a theist one wants to assign \textit{a priori} probabilities to different theories of what kind of universe or multiverse God might want to create. One might guess that God might want to create the simplest or most elegant universe or multiverse that has beings like us that He considers to be in His image or that can have fellowship with Him or that has some other moral good in it. Then the theist might try to guess whether it would be simpler for God to create just one universe with a single set of constants of physics, or whether it would be simpler for Him to create a multiverse with a variety of sets of constants.

If there were a simple principle for choosing the particular constants that we observe (e.g., simple laws of physics that give these uniquely), then one might suppose that God would prefer just using those laws and creating a unique set of constants. However, we don’t see any reason why the constants we observe should be uniquely preferred over other possibilities. Of course, this could easily just be a failure of our imagination and knowledge, so we might continue looking. Even nontheistic scientists like David Gross might prefer a single-universe theory with a single set of constants of physics and persist in looking for simple laws of physics that would give this unique result, following Winston Churchill in saying, “Never, never, never, never, never give up” [48].

On the other hand, not finding a simple principle from which one can deduce uniquely the constants of physics we observe, and having some potential theories like string/M theory that strongly suggest a multiverse instead, might lead many, both theists and nontheists, to look for a multiverse theory instead. From a theistic perspective, it might seem simpler (or better in other regards) for God to choose a variety of the sets of constants of physics, a multiverse rather than a single universe.

The situation is reminiscent of that facing Johannes Kepler, who attempted to explain both the number of planets and their orbits. He was remarkably successful in describing the shape of the orbits, but his attempted explanation that the number was six failed, as was directly shown by the discovery of more planets. Now we know that different solar systems can have different numbers of planets, so there would not have been a way for Kepler successfully to explain a unique value. We are faced with a similar situation with the constants of physics, in that at present we do not know whether they are like the orbital shapes, which can be explained by simple principles (e.g., Newton’s law of universal gravitation) or whether they are like the
number of planets, which can have different values for different systems. I do not see
that it ultimately makes a significant difference theoretically which way the answer
turns out, but as seekers of the truth we would like to find what is actually the case,
or in theistic terms, what God actually did. Given our present knowledge, to me it
currently seems simpler to hypothesize that God created a multiverse, and I would
argue that that is a theologically acceptable option for Christians and other theists
to consider.

6 Toy Multiverse Model from Arithmetic

For those of you who are mathematically inclined, the following example from
arithmetic might be a helpful analogue, illustrating how observations within just
one universe can give evidence of whether or not other universes also exist.

Suppose that we imagine possible universes to be analogous to positive integers,
and one’s observed universe to be analogous to a particular integer. For example,
suppose that there were a simple prescription for translating the constants of physics
into a single integer (e.g., some binary string encoding the values of the constants).
Then we might seek the simplest explanation for the integer corresponding to the
observed universe or to the observed constants of physics in that universe.

If the integer itself were simple, such as $2^{2^{2^2}}$, which is very large (19729 decimal
digits) but quite simple to state, one might suppose that the simplest explanation
would be an explanation that postulates just the existence of one universe and hence
just that one simple integer. On the other hand, if the integer were a very complex
member of a simple set of integers, then it might be simpler to postulate that the
entire set exists.

For example, any large positive integer, no matter how complex, is a member of
the set of all positive integers, which is a very simple set. If the observed universe
corresponded to such a very complex integer (not given by a simple algorithm such as
the example $2^{2^{2^2}}$ above), one might suppose that it would be simpler to suppose that
the much simpler set of all possible positive integers existed. (Note that although this
set is infinite, it does not encompass negative integers, rational numbers, algebraic
numbers, real numbers, complex numbers, quaternions, or any number of other
mathematical structures, so that even an infinite multiverse need not encompass all
logical possibilities.)
However, one might worry that if the set of possibilities is infinite (even the highly limited infinity of the positive integers), then the probability of getting the particular observed integer would be zero, so the hypothesis or theory that the multiverse corresponded to the simple set of all possible positive integers would be assigned zero likelihood. (One way to avoid this would be to put a simple normalizable weight upon all the positive integers, so that the total weight is finite, allowing one to get nonzero probabilities for all finite positive integers, but for now let us try something else.)

To allow equal weights for all positive integers corresponding to universes in a multiverse, one might hypothesize that the multiverse corresponds to some finite set of positive integers. As an example of a large but finite simple set, consider the following set of what I call pili-increasers: positive integers \( n \) for which \( \pi(n)/\text{li}(n) \) is both greater than one and greater than that of any smaller \( n \). Here \( \pi(n) \) is the number of primes not greater than \( n \), and \( \text{li}(n) \) is the Cauchy principal value of the integral of the reciprocal of the natural logarithm of \( x \) from 0 to \( n \), which by the prime number theorem gives a good asymptotic estimate for \( \pi(n) \) but which oscillates around \( \pi(n) \) an infinite number of times as \( n \) increases [49].

This set of pili-increasers is a fairly simple set to define, and yet it apparently has more members than particles in the observable universe (the part of our universe we can see from light emitted after most of the electrons and ions combined in the early universe to make it transparent, which contains roughly \( 10^{90} \) particles). Each member of this set of pili-increasers likely has more than a thousand binary digits, so almost all individual members of this set are almost certainly much more complex than the entire set. (Imagine trying to memorize one of these thousand-binary-digit numbers, in comparison with trying to memorize my short definition of the set.)

The smallest member of this set of pili-increasers has been estimated to be about \( 1.398 \times 10^{316} \) [50]. Smaller candidate pili-increasers, for example near \( 10^{190} \), have not yet been rigorously ruled out, although such smaller numbers appear unlikely actually to be pili-increasers. (Ref. [50] only gives rigorous upper bounds on the smallest pili-increaser, the smallest \( x \geq 2 \) for which \( \pi(x) > \text{li}(x) \).)

Less is known about the largest pili-increaser, which I call the pili-maximizer, the positive integer \( n \) that maximizes the ratio of \( \pi(n) \) to \( \text{li}(n) \). However, from Figure 2b of [50], it appears that it might be about \( 10^{311} \) larger than the most likely
candidate for the smallest pili-increaser (i.e., larger by roughly one part in 140,000 of the smallest pili-increaser that is likely to be near $1.4 \times 10^{316}$), and give a maximum value of $\pi(n)/\text{li}(n)$ approximately $1 + 10^{-154}$. The number of pili-increasers would be expected to be about twice the value of $\pi(n) - \text{li}(n)$ at the pili-maximizer, which from Figure 2b of [50] appears to be nearly $10^{154}$. Thus one might expect somewhat more than $10^{154}$ pili-maximizers, but spread over a range of roughly $10^{311}$ from the smallest to the largest, so that if one picked a number at random within that range, the probability might be of the order of $10^{-157}$, or one part in more than thirteen trillions multiplied together, of picking a pili-increaser.

In fact, no one knows the precise value of any of the individual members of this set of pili-increasers. I am hereby offering to pay $1$ U.S. for each of the binary digits (probably 1051, if the estimates above are indeed applicable for all the members and are not just upper bounds for the smallest member) of the first member of this set that is found explicitly as a binary integer and is proved to be a member of the set. I am also offering a more attainable prize of $1$ U.S. for each binary digit proved to be correct for the logarithm to base 2 of the pili-maximizer (up to as many binary digits of this logarithm as are needed to give the integer precisely, perhaps 1061; I won’t pay any more for the arbitrarily large number of additional digits that can be found trivially if and when the exact pili-maximizer integer is found).

Although there is thus apparently an enormous number of members of this simple finite set, they are also apparently extremely rare among all integers in the same range. If one chose a random integer between the smallest and the largest member of this set, the chance that it would be in the set would apparently be smaller than the chance of choosing one particular particle out of all those in the observable universe. Therefore, if in some hypothetical universe some observer found the constants of the physics translated into one member of that extremely sparse set, it would certainly be strong evidence of fine tuning, strong evidence against a multiverse corresponding to all possible positive integers and even against a finite multiverse corresponding to all positive integers within one part in 140,000 of the smallest pili-increaser.

If one then further found that this integer were a special member of the set of pili-increasers, say the smallest or the largest, that would be such extreme fine tuning that it would be strong evidence for the hypothesis of a single value or a single universe (or at least for just a small number of integer values or universes).
On the other hand, if the evidence were strong that this number were not a special member of the set (though there is no algorithm for proving this), then that particular integer would apparently be much more complex than the simple set of all pili-increasers. In this case, I would say that the observer would be justified to postulate the existence of a multiverse giving the entire simple set, and not just the particular complex though fine-tuned example directly observed.

Although one might accuse this observer of being extravagant in postulating this enormous set of perhaps roughly $10^{154}$ multiverses, in reality the entire set would be much simpler than the individual member observed (at least if the individual member were in fact a random member of the set, as almost all members are, though there is no algorithm for proving that any particular member is). This is one sense in which an observation in one single universe can be more simply explained by postulating a large set of universes in a multiverse, though without needing to postulate that all possible universes exist.

This simple toy example from arithmetic illustrates the fact that in principle even from one single observation result (the one integer) in a single universe, one can gain strong evidence (though even here not a rigorous proof) for the much greater simplicity of either a single-universe or a multiverse hypothesis, depending on what is observed. Of course, in practice, even in this toy example it would be very difficult to calculate whether the integer were in the simple set of pili-increasers and then whether it were a special member such as the smallest or the largest. However, one might develop a suspicion that it might be a member if one found that the integer were a prime (as all pili-increasers are) in the relatively narrow range between the smallest member and the largest member, for which one can imagine fairly good approximate estimates being made in the foreseeable future. Then if one found that it were within a narrow uncertainty of the estimate for one of the endpoints of the range, one might suspect that it is at the endpoint and interpret that as evidence for the single-universe hypothesis, whereas if one found that it were not near an endpoint, one might take that as evidence for the multiverse hypothesis that the entire simple set existed.

Thus one can gain theoretical evidence for (or against) a multiverse even from observations restricted to one single universe, and in principle one might even gain evidence of how large the multiverse is. One’s limited observations within a single
universe are just a small sample of the whole, but if it is a sufficiently rich sample, it can give much information about the rest that is not directly observed.

7 The Growth of Our Knowledge of the Universe

Our whole growth of knowledge of the universe has been an expansion of its scope. As one grows as an infant, one rapidly grows beyond the view that one’s present observation is all that is real, as one develops memories about the past and anticipations of the future. One then goes beyond solipsism and gains an understanding that other persons or observers exist as well. In the early stage of human development, there was the focus on one’s family, which was then gradually extended to one’s tribe, one’s nation, one’s race, and, one might hope, to all humans. But then when one further considers what other conscious observations may be going on, one might well believe that consciousness extends to other creatures, such as other animals.

Of course, one’s direct observation never extends beyond one’s own immediate conscious perception, so one can never prove that there are past or future perceptions as well (and I know philosophers who do not believe the future exists). Similarly, one can never directly experience even the present conscious perceptions of another, which engenders the problem of other minds in philosophy. Nevertheless, most of us believe that we have fairly good indirect evidence for the existence of other conscious experiences, at least for other humans on earth with whom we can communicate, though it is logically possible that neither they nor any external world actually exists. (For me, I believe that it is much simpler to explain the details of my present observation or conscious perception or experience by assuming that an external world and other conscious experiences also exist, than by assuming that just my own momentary conscious perception exists.)

We may now extend the reasoning to suppose that if the universe is large enough, it will also include conscious extraterrestrials, even though we do not have even indirect evidence for them that is so nearly direct as our (inevitably still indirect) evidence for other conscious beings on earth. We can further theorize that if the universe is so large that there never will be any contact between its distant parts and our part, there still might be other conscious beings not in causal contact with us, so that we never could communicate with them to get, even in principle, the
indirect evidence of the same qualitative nature that we have for other humans here on earth with us.

A next step might be to postulate conscious beings and experiences in other universes totally disconnected from ours, so that even if one could imagine traveling faster than the speed of light, there would simply be no way to get there from here; the two parts would be in totally disconnected spacetimes. A similar situation would occur for putative conscious experiences in other branches of an Everett ‘many-worlds’ wavefunction or quantum state. From accepting the existence of such disconnected observers, it hardly seems like an excessive additional step to imagine observers in universes or parts of the multiverse with different constants of physics. One might even imagine observers in entirely different universes, not related to ours in the way an entire multiverse might be related by having one single over-arching set of natural laws.

So in this sense, the idea of a multiverse seems to be rather a natural extension of our usual ideas of accepting a reality beyond one’s immediate conscious perception, which is all the experience for which one has direct access. All the rest of one’s knowledge is purely theoretical, though one’s brain (assuming that one’s brain exists and not just the logical minimum of one’s immediate conscious awareness as a single disembodied entity with absolutely nothing else) is apparently constructed to bring this knowledge into one’s awareness without one’s having to be consciously aware of the details of why one seems to be aware of the existence of other conscious beings.

Despite the naturalness of the progression of ideas that leads to multiverse theories, there are various objections to it. However, none of the objections seems to me to be convincing, as there are highly plausible rebuttals to the objections.

8 Objections to Multiverse Ideas

A scientific objection to a multiverse theory might be that the multiverse (beyond our observed part, which is within one single universe) is not observable or testable. But if one had precise theories for single universes and for multiverses that gave the distributions of different conditions, one could make statistical tests of our observations (likely or unlikely in each distribution). Unfortunately, no such realistic theory exists yet for either a single universe or a multiverse, so I would agree that at present we simply do not have any good theories for either to test.
Another objection is that a multiverse is not a clear consequence of any existing theory. Although it is beginning to appear to be a consequence of string/M theory, that is not yet certain, which is why there can be theorists like David Gross who are still holding out hope that string/M theory might turn out to be a single-universe theory after all, possibly enabling theorists (if they could perform the relevant calculations) to fulfill their wildest dreams of being able to calculate the constants of physics uniquely from some simple principles. One first needs to make string/M theory into a precise theory and calculate its consequences, whether single universe or multiverse. And if that theory gives predictions that do not give a good statistical fit to observations, one needs to find a better theory that does.

A philosophical objection to a multiverse theory is that it is extravagant to assume unfathomable numbers of unobservable universes. This is a variant upon the psychological gut reaction that surely a multiverse would be more complex than a single universe, and hence should be assigned a lower a priori probability. But this is not necessarily so, as I have explained above. The whole can be simpler than its parts, as the set of all integers is quite simple, certainly simpler than nearly all the (arbitrarily large) individual integers that form its parts. The mathematical example above of the simple set of pili-increasers also shows how even a very large finite set can be much simpler than almost all of its individual members.

As a further rebuttal of the accusation of extravagance, a theist can say that since God can do anything that is logically possible and that fits with His nature and purposes, then there is apparently no difficulty for Him to create as many universes as He pleases. He might prefer elegance in the principles by which He creates a vast multiverse over paucity of universes, i.e., economy of principles rather than economy of materials.

Another philosophical objection to multiverses is that they can be used to explain anything, and thereby explain nothing. I would strongly agree for this criticism of multiverse theories that are too vague or diffuse, which do not sufficiently restrict the measure on the set of observations to favor ordered ones such as what we observe. There is a genuine need for a multiverse theory not to spread out the probability measure for observations so thinly that it makes our observation too improbable. So this objection would be a valid objection to vast classes of possible multiverse theories, but I do not see that it is an objection in principle against a good multiverse
theory. Certainly not just any multiverse theory is acceptable, and even if simple single-universe theories do not work for explaining our observations, it will no doubt be quite a challenge to find a good multiverse theory that does succeed.

Most of the objections I have raised and attempted to answer so far would apply both to theistic and nontheistic scientists. However, if one is a theist, one might imagine that there are additional objections to multiverse theories, just as some theists had additional objections to Darwin’s theory of evolution beyond the scientific objections that were also raised when that theory had much less support.

For example, a theist might feel that a multiverse theory would undercut the fine-tuning argument for the existence of God. I shall not deny that it would undercut the argument at the level of the constants of physics (though I think there would still be such a design argument from the general apparently elegant structure of the full laws of nature once they are known). However, the loss of one argument does not mean that its conclusion is necessarily false.

I personally think it might be a theological mistake to look for fine tuning as a sign of the existence of God. I am reminded of the exchange between Jesus and the religious authorities recorded in the Gospel of Matthew 12:38-41: “Then some of the scribes and Pharisees answered, saying, ‘Teacher, we want to see a sign from You.’ But He answered and said to them, ‘An evil and adulterous generation seeks after a sign, and no sign will be given to it except the sign of the prophet Jonah. For as Jonah was three days and three nights in the belly of the great fish, so will the Son of Man be three days and three nights in the heart of the earth. The men of Nineveh will rise up in the judgment with this generation and condemn it, because they repented at the preaching of Jonah; and indeed a greater than Jonah is here.’” In other words, I regard the death and resurrection of Jesus as the sign given to us that He is indeed the Son of God and Savior He claimed to be, rather than needing signs from fine tuning.

Another theistic objection might be that with a multiverse explanation of the constants of physics, there is nothing left for God to design. But God could well have designed the entire multiverse, choosing elegant laws of nature by which to create the entire thing. In any case, whatever the design is, whether a logically rigid requirement, a simple free choice God made, or a complex free choice God made, theists would ascribe to God the task of creating the entire universe or multiverse.
according to this design.

A third more specifically Christian objection might be that if the multiverse (or even just our single part of the universe) is large enough for other civilizations to have sinned and needed Christ to come redeem them by something similar to His death on the Cross here on earth for our sins, then His death may not sound so unique as the Bible says in Romans 6:10: “For the death that He died, He died to sin once for all; but the life that He lives, He lives to God.” But the Bible was written for us humans here on earth, so it seems unreasonable to require it to describe what God may or may not do with other creatures He may have created elsewhere. We could just interpret the Bible to mean that Christ’s death here on earth is unique for our human civilization.

9 Conclusions

In conclusion, multiverses are serious ideas of present science, though certainly not yet proven. They can potentially explain fine-tuned constants of physics but are not an automatic panacea for solving all problems; only certain multiverse theories, of which we have none yet in complete form, would be successful in explaining our observations. Though multiverses should not be accepted uncritically as scientific explanations, I would argue that theists have no more reason to oppose them then they had to oppose Darwinian evolution when it was first proposed.

God might indeed so love the multiverse.

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