Exciting dark matter in the galactic center

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(Dated: November 11, 2009)

We reconsider the proposal of excited dark matter (DM) as an explanation for excess 511 keV gamma rays from positrons in the galactic center. We quantitatively compute the cross section for DM annihilation to nearby excited states, mediated by exchange of a new light gauge boson with off-diagonal couplings to the DM states. In models where both excited states must be heavy enough to decay into $e^+e^-$ and the ground state, the predicted rate of positron production is never large enough to agree with observations, unless one makes extreme assumptions about the local circular velocity in the Milky Way, or alternatively if there exists a metastable population of DM states which can be excited through a mass gap of less than 650 keV, before decaying into electrons and positrons.

Dedicated to the memory of Lev Kofman

PACS numbers: 98.80.Cq, 98.70.Rc, 95.35.+d, 12.60Cn

I. INTRODUCTION

There is presently much discussion about whether several anomalous observations in galactic gamma ray and cosmic ray astronomy are better explained by dark matter models or by more conventional astrophysics. One such example is the 511 keV gamma ray excess from the galactic center, which has been observed for 40 years, most recently by the SPI spectrometer on the INTEGRAL satellite [1]; for a recent review see ref. [2]. Pulsars [3], gamma ray bursts [4], supernovae [5], low-mass x-ray binaries [6], the galactic black hole [7] and anisotropic propagation effects [8] have been suggested as sources of positrons whose annihilation could explain the observations, but there is no consensus within the astrophysical community as to which of these might be the right explanation.

There have also been many attempts to explain the 511 keV signal using particle physics models, including annihilations of MeV scale dark matter (DM) [9] or millicharged DM [10], emission from cosmic strings [11], decays of sterile neutrinos [12], axinos [13], moduli (or modulinos) [14], WIMPs [15, 16], light photinos [17], or emission from composite objects [18]. Excited dark matter (XDM) [19] is a particularly appealing example, in which heavy DM particles in their ground state scatter into excited states, $\chi_0 \rightarrow \chi_1$, with mass difference $\delta M = M_1 - M_0$. The excited states subsequently decay into $e^+e^-$, where the leptons are approximately non-relativistic due to the mass difference being close to the threshold value $\Delta M \gtrsim 2m_e$. If the different DM states are members of a multiplet of a new hidden gauge symmetry, this framework has the advantage of being able to explain the small mass splittings naturally through quantum loop effects [20]; moreover the same class of theories can potentially explain excess positrons seen by higher energy experiments including ATIC [21], PAMELA [22], and the Fermi Large Area Telescope [23].

The XDM proposal requires that the excitation cross section be large, in fact close to the unitarity bound, in at least a few (and possibly many) partial waves [13, 25]. Ref. [19] made a first attempt to achieve such large values in a model where the excitation was mediated by exchange of a light scalar $\phi$ with $m_\phi$ in the MeV–GeV range. It was recognized that one must resum ladder diagrams with multiple $\phi$ exchanges when the DM particles are scattering at low velocity, but a quantitatively reliable way of doing so was not yet appreciated at the time of this work.

Subsequently ref. [20] showed how the calculation can be set up in the framework of nonrelativistic quantum mechanics of a two-state system. It provides an example of the Sommerfeld enhancement [26, 27], which in the last few years has been widely studied in the context of galactic dark matter annihilations. However, unlike the simpler one-state system where the enhancement can be approximated analytically, in the two-state system no analytic results for the excitation cross section are known (however see ref. [27] for recent progress in the annihilation cross section for multi-state DM). Ref. [28] made a first attempt to numerically solve the two-state system and to perform a preliminary scan of the parameter space. Technical challenges limited that analysis to relatively small coupling strengths of the exchanged boson to the dark matter. One goal in the present work is to extend these results to stronger couplings and to explore more widely the range of possibilities, including the dependence on the mass of the exchanged boson. A second is to explore the dependence of the predicted rate on parameters of the dark matter density and velocity profiles in the galaxy.

In section [11] we review the nonrelativistic quantum mechanical formulation of the problem and give a classical estimate of the number of partial waves that can be expected to significantly contribute to the excitation cross section. In section [111] we discuss the numerical method for computing partial wave amplitudes $f_i$ as a function
of velocity, and present sample results for $f_t$, as well as a survey of the range of boost factors which arise from a broad exploration of parameter space. In section IV we show how these results go into the calculation of the rate of positron production in the galactic bulge; our choice of different DM density and velocity dispersion profiles is explained there. Section V presents the results of our scan of the XDM model’s parameter space, consisting of the interaction strength and the mass of the exchanged particle, and the mass splitting between ground and excited state. We at first hold the DM mass particle, and the mass splitting between ground and excited state. We at first hold the DM mass at a TeV \cite{22}, but then explore the dependence on $M_0$ and galactic DM distribution parameters at some optimal values of the microscopic parameters.

Our results show that a large enough rate to match observations cannot be achieved in the most straightforward implementation of the XDM scenario, but in section VI we review a modified version which can overcome this deficit. In this version of XDM, dark matter has at least three states; the middle state is stable or metastable and can excite over a smaller gap to the highest state. In section VII we compare the predictions for the angular distribution of $511$ keV gamma rays with the observed signal. Conclusions are given in section VIII. Appendix A derives the Born approximation for the excitation cross section, and B reviews the derivation of the radial dependence of the DM escape velocity.

II. EXCITATION CROSS SECTION

A key realization in ref. \cite{22} is that the DM particles in the galaxy are highly nonrelativistic, and so it is not necessary to use the full apparatus of quantum field theory to analyze their scattering. Instead one can use the low energy effective theory, which is quantum mechanics. This enormously simplifies the problem, getting around the need to resum a perturbative expansion in ladder diagrams to find an effect which is nonperturbative, and enhanced by $1/v$ (or $1/v^2$), the inverse velocity of the DM particles.

A. Quantum mechanical analysis

We assume that the DM is Majorana and that its multiplicity arises from it being in a nontrivial representation of a nonabelian gauge symmetry. This immediately implies that any two members of the multiplet, say $\chi_1$ and $\chi_2$, can only interact via off-diagonal couplings to the gauge boson. Furthermore the gauge symmetry should be spontaneously broken so that $\chi_1$ and $\chi_2$ can be non-degenerate; then the exchanged vector boson gets a mass $\mu$. This leads to an attractive Yukawa potential, but only as an off-diagonal term in the matrix elements of the potential for the two-state system. It is important to note that each state consists of two particles,

$$|1\rangle = |\chi_1, \chi_1\rangle, \quad |2\rangle = |\chi_2, \chi_2\rangle$$  \hspace{1cm} (1)

by virtue of the fact that whenever a gauge boson is exchanged, the color of both DM particles has to change. There also exist the states $|\chi_1, \chi_2\rangle$ and $|\chi_2, \chi_1\rangle$ which live in their own superselection sector, but we do not need to consider them because $\chi_2$ is presumed to not be present in the initial state. The diagonal terms have no interactions; this part of the Hamiltonian consists only of the mass-energies of the two particles. Subtracting the mass of the ground state, $2M_0$, the matrix potential in the basis $|\Phi\rangle$ is

$$V_{ij} = \left( \begin{array}{cc} 0 & -\alpha_\mu e^{-\mu r}/2 \delta M \\ -\alpha_\mu e^{-\mu r}/2 \delta M & 0 \end{array} \right)$$  \hspace{1cm} (2)

where $\alpha_\mu = g^2/4\pi$ is the fine structure constant for the SU(N) gauge coupling $g$. The wave function for the two-state system (with components labeled by index $i$) in the CM frame is $\Psi^i = \sum_k P_l(\cos \theta) R_k(r)$, where $k$ is the initial momentum. Defining $\Phi_{l,i}(r) = R_{l,i}/r$, the Schrödinger equation is

$$-\frac{1}{M_0} \Phi''_{l,i} + \left( \frac{l(l+1)}{M_0 v^2} \delta_{ij} + V_{ij} \right) \Phi_{l,i} = \frac{k^2}{M_0} \Phi_{l,i}$$  \hspace{1cm} (3)

For the numerical solution it is useful to rescale $r = (\alpha_\mu/2\delta M) x$ and define the dimensionless variables

$$\Gamma = M_0 \alpha_\mu^2 \frac{e^\mu}{2\delta M}, \quad \Delta = \frac{k^2}{2M_0\delta M} = \frac{v^2}{v_f^2}, \quad \eta = \frac{\alpha_\mu \mu}{2\delta M}$$  \hspace{1cm} (4)

where $v_1 = \sqrt{2\delta M/M_0}$ is the threshold velocity for producing the excited state. Then the Schrödinger equation takes the form

$$-\Phi'' + \left( \frac{l(l+1)}{x^2} + \Gamma(\hat{\Phi} - \Delta) \right) \Phi = 0$$  \hspace{1cm} (5)

with the dimensionless potential $\hat{\Phi}$. We require $\Delta > 1$ for the initial state to have enough energy to produce the heavier $|\chi_2, \chi_2\rangle$ final state.

To extract the scattering amplitudes, we decompose the numerical solution into incoming and outgoing waves, $\Phi^i_{l,11}$, $\Phi^i_{l,12}$ and $\Phi^i_{l,2}$. Partial wave unitarity implies the conservation of flux, $k|\Phi^i_{l,11}|^2 = k|\Phi^i_{l,12}|^2 + k|\Phi^i_{l,2}|^2$ (where $k^2 \equiv k^2 - 2M_0\delta M$), which we use as a check on our numerics. The fraction of incoming $|\chi_1, \chi_1\rangle$ states which gets converted to the $|\chi_2, \chi_2\rangle$ final state is thus

$$f_l = \frac{k' |\Phi^i_{l,2}|^2}{k |\Phi^i_{l,11}|^2}$$  \hspace{1cm} (7)

in the $l$th partial wave. $f_l$ provides a measure of the extent to which a given partial wave can saturate the unitarity bound $f_l \leq 1$ for $\sigma_l$, its contribution to the cross section. The partial wave cross section is

$$\sigma_l = \frac{\pi (2l + 1)}{M_0^2 v^2} f_l(v)$$  \hspace{1cm} (8)

and the total cross section is $\sigma = \sum_l \sigma_l$. 
B. Classical treatment

To get additional insight, it is useful to think about the classical version of the problem, in the limit where the scattering is elastic. The standard method for solving the central potential problem is to change variables to \( u = 1/r \) and to solve for \( \theta(u) \) instead of \( r(t) \), where \( \theta \) is the polar angle in the scattering plane (see for example section 4.5 of \[8\]). Take \( M = \frac{1}{2} M_0 \) to be the reduced mass. Then it is straightforward to solve the first integral of the motion for \( \theta(u) \) with initial conditions where the particles have velocity \( v = v_{\text{rel}}/2 \) in the center-of-mass frame and impact parameter \( b \). Taking \( w = bu \), the scattering angle is

\[
\theta_s = \pi - 2 \int_0^{w_0} dw \left( 1 - w^2 + \frac{2 \alpha_g}{M b v_{\text{rel}}^2} e^{-\mu b/w} \right)^{-1/2} \tag{9}
\]

where \( w_0 \approx 1 \) is the turning point of the effective potential, the point where the integrand diverges. Notice that \( \theta_s = 0 \) in the limit where the scattering potential vanishes. The result \[10\] shows the classical origin of the Sommerfeld enhancement, where the effect of the potential is strengthened by the factor \( 1/v^2 \) at low velocities, since the particle has more time to be influenced when it is moving slowly.

We can estimate the maximum angular momentum \( l = M_0 v b \) which gives significant scattering by demanding that \( (2 \alpha_g/M b v_{\text{rel}}^2) e^{-\mu b} \gtrsim 1 \). Solving for the argument of the exponential gives

\[
l_{\text{max}} \sim \frac{M_0}{\mu} v \ln \left( \frac{\alpha_g}{v l_{\text{max}}} \right) \sim \frac{M_0}{\mu} v \ln \left( \frac{\alpha_g \mu}{M_0 v^2} \right) \tag{10}\]

The prefactor \( M_0 v/\mu \) is what one would have obtained by assuming the range of the force is \( 1/\mu \). Substituting this estimate for \( l_{\text{max}} \) (without the logarithm) into the formula for the cross section \[8\] and assuming \( f_1 = 1 \) for all \( l < l_{\text{max}} \) results in the geometrical value of the cross section, \( \sigma = \pi / \mu^2 \). The extra \( \ln (\alpha_g \mu / M_0 v^2) \) factor is the origin of the Sommerfeld enhancement, which boosts the cross section by the square of the logarithm for low-velocity scattering.

For the inelastic process, these classical insights need to be modified, since there is a threshold \( v > v_1 \) for production of the excited state. Nevertheless they provide some idea as to how many partial waves one may expect to be important in the quantum mechanical cross section \[7\].

III. PARTIAL WAVE AMPLITUDES

In this section we describe our method of numerical solution of the Schrödinger equation, leading to the partial wave amplitudes \( f_i(v) \). In each partial wave, one boundary condition is that there are only incoming waves of state \(|1\rangle\) from \( r = \infty \), whereas the outgoing waves are an admixture of \(|1\rangle\) and \(|2\rangle\). Specifically, \(|2\rangle\) must approach an eigenstate of radial momentum with positive eigenvalue,

\[-i \frac{\partial}{\partial r} \Phi_{l,2} = k' \Phi_{l,2} \tag{11}\]

where \( k' = \sqrt{k^2 - 2M_0 \delta M} \). This constitutes two conditions, one for the real part and one for the imaginary part of the wave function. At the origin, both components behave like \( r^{l+1} \), but their relative amplitudes are not known, so one must parametrize \( \Phi_0 \sim r^{l+1}(e^{i \alpha_0} + b \Phi_0) \) with \( b \) a complex number. We thus have a shooting problem: the real and imaginary parts of \( b \) must be adjusted so as to satisfy the two boundary conditions at infinity. Standard algorithms exist for this kind of problem; we adopt the routines in ref. \[31\].

Instead of shooting, one can alternatively use a simpler approach, which is to solve the Schrödinger equation with the correct \( r^{l+1} \) behavior near \( r = 0 \), but arbitrary relative amplitudes of \( \Phi_1 \) and \( \Phi_{l,2} \), to obtain some solution \( \Phi_0 \), which does not have the right behavior at large \( r \). Since the complex conjugate \( \Phi_0^* \) is also a solution, one can algebraically construct the \( b \) linear combination \( a \Phi_0 + b \Phi_0^* \) that has the desired behavior at large \( r \). This is much faster than shooting because no iteration is required.

In principle, the method is straightforward, but complications arise when one tries to consider parameters in the regime \( \Gamma \gg 1 \), that is, \( \alpha_g \gg \sqrt{2 \delta M / M_0} \). Notice that this need not be a particularly strong coupling for the application we have in mind, where \( \delta M \sim 2m_\chi \) and \( M \sim \text{TeV} \); then \( \alpha_g \gg 10^{-3} \). The algorithms break down except when \( \eta \), which determines the dark gauge boson mass, is sufficiently large. The problem arises from the need to consider very different scales \( 1/\Gamma \) and \( 1/\eta \); in particular the wave function oscillates many times in the interaction region \( x < 1/\eta \). Then (we suspect) the solution with the desired behavior at large \( r \) becomes an exponentially small component of the generic solution, and so its extraction gets lost in the numerical noise. Despite this limitation, we will be able to explore the parameter space widely enough to see how the results extrapolate to the difficult regions, and thus give a complete characterization of the solutions.

A. Results for \( f_i \)

We show two examples of the results for \( f_i \), eq. \[7\] in figure \[1\] as a function of \( \Delta - 1 = v^2/v_1^2 - 1 \); recall that \( v \) is the velocity of one of the particles in the center of mass frame, and \( v_1 \) is the threshold velocity for producing the excited DM states. These results look quite different from the usual single-state \( s \)-wave Sommerfeld enhancement, which has numerous sharp resonance bands at low velocity. Here, there can be at best a few oscillations in a given partial wave, but not strongly peaked. These
might be due to having an approximately integral number of wavelengths within the width of the potential well, which of course is not square and so one would not expect nearly bound resonances to be very sharp. It would be interesting to find an analytic approximation to help better understand these results. We tried to develop the WKB approximation in this context, but without success.\footnote{At least in some regions of parameter space, the WKB approximation fails due to the breakdown of matching conditions at the classical turning point. It is possible that more sophisticated approximate analytical techniques such as those used in \cite{27} ameliorate this problem.}

For clarity in figure\cite{1} we have chosen examples where the number of partial waves which contribute significantly are relatively small, but for our actual computations, there are examples (particularly for small values of $\eta \ll \sqrt{\Gamma}$) where hundreds of partial waves are relevant. Using the estimate \cite{10}, and the definitions \cite{4}, we find

$$l_{\text{max}} \sim \frac{\sqrt{\Gamma \Delta}}{\eta} \ln \frac{\eta}{\Delta} \quad (12)$$

We typically computed up to $l_{\text{max}} = 500$ to insure convergence of the sum over $l$ in the cross section, and we restrict ourselves to gauge boson masses, hence values of $\eta \sim \sqrt{\Gamma}$, which are large enough to justify neglecting higher values of $l$ (it also turns out that the interesting functional dependence of $\sigma$ on $\eta$ occurs in the region where $\eta \sim \sqrt{\Gamma}$). It will be apparent from the results that this is a mild restriction, in terms of understanding trends across the full parameter space of the model.

B. Boost factors

There is a large literature on the Sommerfeld enhancement for $s$-wave annihilation of DM at low velocities, where the effect is characterized by a boost factor, defined as the ratio of the enhanced cross section to the Born approximation value. Even though all our results for the scattering cross section are computed directly, without reference to the perturbative value, one might nevertheless be interested to know their ratio in the present context. To make the comparison, we need the perturbative expression for $\sigma$ (here computed using relativistic quantum field theory),

$$\sigma = \frac{\pi \alpha_s^2}{2 \Delta} \left\{ \frac{32}{(M_0 v_t^4(2\Delta - 1) + \mu^2) v_t^2} \ln \frac{1 + \epsilon}{1 - \epsilon} + \frac{128 M_0^2 \sqrt{\Delta(\Delta - 1)}}{\mu^4 + M_0^4 v_t^4 + 2 \mu^2 M_0^2 v_t^2(2\Delta - 1)} \right\} \quad (13)$$
where $\epsilon = 2M_0^2v_t^2\sqrt{\Delta(\Delta - 1)/(\mu^2 + M_0^2v_t^2(2\Delta - 1))}$, and we have assumed $\delta M \ll \mu \ll M_0$ to simplify the expression; see appendix A for details.

In figure 2 we plot the resulting boost factors as a function of center-of-mass velocity of the incoming particles, for the same range of parameters that we explore in the next section as being relevant for positron production, namely $\Gamma = 10, \ldots, 10^5$ and $\eta \sim \text{few} \times \sqrt{\Gamma}$, for $\delta M = 0.2, 0.4, \ldots, 1 \text{ MeV}$, and assuming a DM mass of $M_0 = 1 \text{ TeV}$. From the figure it is apparent that the “boost factor” is an enhancement only for the smaller values of $\Gamma$, and is actually a suppression factor for $\Gamma > 100$, in the range of velocities where we have computed. One can see the onset of resonant enhancement at low velocity in the cases of $\Gamma = 10$ and $100$, where the boost factor suddenly increases as a function of the dimensionless gauge boson mass parameter $\eta$. For fixed parameters $\Gamma$, $\eta$, $\Delta$, we see from each family of curves that smallest values of $\delta M$ give the largest cross section. This can be understood as being due to Sommerfeld enhancement, since for fixed $\Delta$ decreasing $\delta M$ corresponds to decreasing the particle velocities.

One check on these results is provided by considering when the Born approximation for our potential should be valid. By demanding that the scattered wave be small compared to the incoming wave in nonrelativistic quantum mechanics, and working in the limit of low velocity, one obtains the constraint $2M_0|\int_0^{\infty} rV(r)dr| \ll 1$, which implies $2\alpha g M_0 \ll \mu$, or in terms of the parameters (4), $2\Gamma \ll \eta$. This is violated everywhere in the region of parameter space we have considered, so there is no contradiction between our results and expectations based on the Born approximation.

### IV. RATE OF POSITRON PRODUCTION

The rate of positron production within a radius $r_c$ of the galactic center is given by

$$R_{e^+} = \frac{1}{2} \int_0^{r_c} \langle \sigma v_{\text{rel}} \rangle n^2(r) 4\pi r^2 dr$$

(14)

where $n(r) = \rho(r)/M_0$ is the DM number density and $\rho(r)$ is its mass density. The factor of $\frac{1}{2}$ is to avoid
double-counting, and we have assumed that the DM is distributed with spherical symmetry. Since the INTEGRAL signal has a full-width at half-maximum of 8°, we take \( r_0 \) to be the radius which subtends 8° at our distance of \( R_0 = 8 \) kpc from the galactic center, giving \( r_c = 1 \) kpc. We will discuss the uncertainty in \( R_0 \) and its impact on our results below. The predicted rate is to be compared to the observed one (see ref. \[19\] for a detailed discussion) of \( R_{obs} = 3.4 \times 10^{10} \) for the component coming from the galactic bulge \[13\].

The cross section in \[13\] is averaged over the velocities of the DM particles, using the Maxwell-Boltzmann distribution function
\[
f(v, r) = \begin{cases} v^2 \exp \left(-v^2/2\sigma_v(r)^2\right), & v < v_{esc} \\ 0, & v \geq v_{esc}, \end{cases}
\]
with a cutoff at the escape velocity for galactic DM. The normalization factor is given by
\[
N^{-1}(r) = 2^{5/2} \pi \sigma_v^3(r) \left( \frac{\sqrt{\pi}}{2} \text{erf}(X) - X e^{-X^2} \right)
\]
where \( X = v_{esc}/(\sqrt{2}\sigma_v(r)) \). We follow ref. \[14\] in using the \( r \)-dependent escape velocity (see appendix \[3\] for derivation),
\[
v_{esc} = \begin{cases} v_c \sqrt{2 \left[ 1 - \ln \left( \frac{r}{r_{2}} \right) \right]}, & r \leq r_{-2} \\ v_c \sqrt{2 \frac{r_{-2}}{r}}, & r > r_{-2}.
\end{cases}
\]
\( v_c \) is the circular velocity of DM, assumed to be approximately constant at radii near the characteristic scale \( r_{-2} \), which is defined below. We take the fiducial value for \( v_c \),
\[
v_c = 220 \text{ km/s}
\]
as in ref. \[19\] and many other references, but below we will also explore the sensitivity of our predictions to changes in this value. To average over the cross section, one integrates over both DM particle velocities,
\[
\langle \sigma v_{rel} \rangle(r) = 8\pi^2 \int_0^\infty dv_1 \int_0^\infty dv_2 \int_{-1}^1 d(\cos \theta) f(v_1, r) f(v_2, r) \sigma v_{rel},
\]
where \( v_{rel} = (v_1^2 + v_2^2 + 2v_1 v_2 \cos \theta)^{1/2} \), and the cross section is taken to vanish if \( \frac{1}{2} v_{rel} \) is less than the threshold velocity defined in eq. \[4\]. We perform the integrals numerically.

### A. Pure DM radial distributions

The \( r \) dependence of the velocity dispersion \( \sigma_v(r) \) and also the escape velocity \( v_{esc}(r) \) appearing in \[15\] depend on the shape of the DM density profile \( \rho(r) \). We will consider several hypotheses for the form of \( \rho(r) \), inferred from \( N \)-body simulations, some based on pure DM (PDM), and others which contain a baryonic component in addition to the DM (BDM). The PDM Aquarius simulation \[32\] finds the relation \( \sigma^2 \propto r^{1.875} \rho(r) \), where \( \rho \) is the DM density. Thus one needs to specify the density profile in order to fix \( \sigma_v(r) \).

We consider two widely-used density profiles, the NFW form
\[
\rho(r) = 4\rho_{-2} \left( \frac{r}{r_{-2}} \right)^{-1} \left( 1 + \frac{r}{r_{-2}} \right)^{-2},
\]
and the Einasto form
\[
\rho(r) = \rho_{-2} \exp \left[ -2 \left( \left( \frac{r}{r_{-2}} \right)^{\alpha} - 1 \right) \right],
\]
where \( r_{-2} \) is the radius at which the logarithmic slope of the density is \(-2 \), the normalization is defined by \( \rho_{-2} = \rho(r_{-2}) \), and \( \alpha \) is the shape parameter. A fit of the NFW form to the Milky Way galaxy in ref. \[34\] gives \( r_{-2} = 14.1 (h/0.7)^{-2/3} \) kpc and \( \rho_{-2} = 0.13 (h/0.7)^2 \text{ GeV/cm}^3 \) where \( h \) is the Hubble parameter. These are close to the best-fit values from the Aquarius simulation for the AQ-A-1 galaxy, which was the highest resolution simulation considered, given in tables 1-2 of ref. \[33\].

\[
\begin{align*}
\rho_{-2} &= 15 \text{ kpc} \left( \frac{h}{0.7} \right)^{-1}, & \rho_{-2} &= 0.14 \text{ GeV/cm}^3 \left( \frac{h}{0.7} \right)^2, \\
\alpha &= 0.17 \quad \text{(Aquarius-A-1 parameters)}
\end{align*}
\]

\[2\] In the notation of ref. \[34\], \( r_{-2} = r_s = r_v/c \) and \( 4\rho_{-2} = \delta_c \rho_c \), where \( r_v \) is the virial velocity, \( c \) is the concentration parameter, \( \delta_c = 100c^3/[3(\ln(1 + c) - c/(1 + c)) \rho_c \) is the present critical density of the universe, \( c \) is related to the virial mass \( M_v \) by
\[
r_v = \left( \frac{3M_v}{4\pi \cdot 100 \rho_c} \right)^{1/3}.
\]
Using their best fit values \( c = 18 \), \( M_v = 9.4 \times 10^{11} M_\odot \) (see erratum of \[34\]) leads to the values quoted for \( r_s \) and \( \rho_{-2} \).
We adopt these as our fiducial values. The resulting velocity dispersion is then

$$\sigma_v = \sigma_{v0} \left( \frac{r}{r_{-2}} \right)^{0.625} \left( \frac{\rho(r)}{\rho_{-2}} \right)^{1/3}$$  \hfill (23)

where the velocity scale $\sigma_{v0}$ (called $\sigma_{\text{max}}$ in Table 1 of ref. 33) is determined to be

$$\sigma_{v0} \approx 260 \text{ km/s}$$  \hfill (24)

The radial dependence of $\sigma_v$ is plotted in figure 4 showing that the Einasto profile leads to much higher velocities in the inner regions $r < r_{-2}$ than does the NFW profile.

B. Effect of baryons on distributions

The Aquarius simulation ignored the effects of baryons on the evolution of galaxies, but recent studies have included them and shown that their effect is to increase the velocity dispersion in the inner region $r \lesssim 10 \text{ kpc}$ (35-37). Fitting the results of figure 2 in ref. 35, we find the velocity dispersion

$$\sigma_v(r) = \sigma_{v0} \left( \frac{r}{r_{-2}} \right)^{-1/4}$$  \hfill (25)

where $\sigma_{v0}$ happens to take the same value as in (24). Its shape is plotted in figure 4. The rise in $v$ toward the galactic center can greatly boost the excitation rate of DM relative to the pure dark matter case, (23), and we will employ both for the purposes of comparison.

There is some disagreement between the different BDM simulations as to the impact of the baryons on the DM density profile. Ref. 35 finds that the cusp of the density distribution is softened (we refer to it as a "cored" profile), while ref. 37 obtains profiles that are consistent with the Einasto form. We will consider both possibilities in our computations of the positron production rate. To study the possible softening effect, we have digitized the BDM density profile at redshift $z = 0$ in fig. 1 of ref. 35 and fit its logarithm to a quartic polynomial,

$$\log_{10} \rho_{\text{cored}}(r) = \sum_{n=0}^{4} a_n \log_{10}(r/\text{kpc})$$  \hfill (26)

with coefficients $a_n = 1.7487$, 0.0395, $-2.537$, 1.459, $-0.3448$, respectively, for $n = 0, \ldots, 4$. This profile is plotted along with the NFW and Einasto forms (20)-(21) in figure 4 in the region $1 \text{ kpc} < r < 10 \text{ kpc}$. For larger $r$, the three shapes are in closer agreement, while for $r < 1 \text{ kpc}$ the BDM profile is taken to be constant.

Other BDM simulations do not find the erasure of the cusp; ref. 37 finds good fits to the Einasto profile with a range of $\alpha$ values that are consistent with (24), but that go as low as $\alpha = 0.145$. To illustrate the effect of the $\alpha$ parameter on the rate, we will consider this lower value in addition to the higher one $\alpha = 0.17$ in (22).

C. Consistency of velocity distributions

Strictly speaking, the velocity distributions may not be self-consistent when varying the dark matter density profile. A better approximation to the self-consistent distribution could be derived from the Jeans equation, following the technique of 38, for example. However, our results are much more sensitive to the shape of the density profile than the detailed velocity profile, so an improved treatment of the velocity profile should only lead to qualitatively small changes in our results. Figure 4, for example, shows that the rate of positron production is relatively insensitive to the scale of the BDM velocity distribution $v_0$ near or above the standard value (24).

We do however, find an enhancement in positron production in going from the PDM velocity dispersion profile (23) to the more cuspy BDM profile (24). We therefore might worry about overestimating the positron production rate if a self-consistent treatment removes the cusp in the profile. We believe it is more conservative to consider the cuspy profile (25), as we are interested in determining whether or not the XDM mechanism is viable for $\delta M \gtrsim 2m_e$. Even with the advantageous profile (25), we still find that the positron production rate is only sufficient for smaller $\delta M$.

V. SURVEY OF PARAMETER SPACE

There are four important dimensionless parameters that determine the rate of scatterings to produce the XDM states in the galaxy. Two have already been defined in eq. 4, $\Gamma$ and $\eta$, which depend only upon the microphysics, i.e., the DM mass, mass splitting, gauge...
boson mass, and interaction strength. The others depend on the ratio between the threshold velocity \( v_t \) or the escape velocity \( v_{\text{esc}} \) and the parameter \( v_0 \) (that controls the average speed of DM particles in the galaxy), motivating us to define

\[
\delta M = \frac{2\delta M}{M_0 v_0^2} = \frac{v_t^2}{v_0^2} \quad \gamma = \frac{v_{\text{esc}}}{v_0} \tag{27}
\]

FIG. 5: Contours of logarithm of predicted over observed rate of positron production in the plane of \( \eta - \delta M \) for several DM velocity and density profiles. (\( \eta \) is proportional to the exchanged gauge boson mass; see eq. (11).) Those labeled “0” (thickest contours) match the observed rate. The DM mass is assumed to be \( M_0 = 1 \) TeV.
dependence on $a, b$ arises from averaging the cross section over the velocity distribution of the DM. In ref. [13], a simpler estimate of the rate was made using the isothermal distribution function, $f(v) = Nv^2 e^{-v^2/v_0^2}$, giving

$$\langle \sigma v_{\text{rel}} \rangle = \frac{2\sqrt{\pi}}{M_0 v_0} \sum_l (2l + 1) \int_0^{v_{\text{esc}}^2} \frac{dv}{v_0^2} e^{-v^2/v_0^2} f_l(v)$$

(28)

where $v_{\text{esc}}$ is the escape velocity, also taken to be constant and in the range $\sim 500 - 600$ km/s. In the limit of large $a, b$, the rate is exponentially suppressed, $\sim (e^{-a} - e^{-b})$. Although we are calculating the rate more quantitatively in this paper, the simpler approach makes it clear how to expect the results to depend on $a, b$, at least semiquantitatively. In terms of dimensionful constants and the dimensionless ratios, one can parametrize the rate in the form

$$R_{e^+} = \frac{\alpha^2 e^{-2\beta}}{M_0^4 v_0} g(\Gamma, \eta, a, b)$$

(29)

where the function $g$ of the dimensionless variables has complicated dependence on $\Gamma$ and $\eta$, but roughly $(e^{-a} - e^{-b})$ dependence on $a, b$. The contours of $\log(e^{-a} - e^{-b})$ in the $a-b$ plane are roughly linear over some ranges, suggesting that there should be a quasilinear degeneracy between the parameters $\delta M$ and $v_{\text{esc}}^2$, which we will observe below.

### A. Dependence on $\Gamma, \eta, \delta M$

For our initial exploration of parameter space, we computed the scattering rate for values $\Gamma = \alpha^2 M_0/2 \delta M = 10^3, 10^2, \ldots, 10^5$; this is the parameter that is most directly associated with the strength of the dark gauge interaction since it multiplies the potential in (16). For each value of $\Gamma$, we explore a range of $\eta = \alpha_2 \mu/2 \delta M$ (recall that $\eta$ determines the range of the interaction since $\mu$ is the gauge boson mass) that includes the region where the rate of excitations is maximized. For each value of $\Gamma$, we calculated contours of the positron production rate $R_{e^+}$ in the $\eta$-$\delta M$ plane. However to make the presentation more concrete, instead of using the dimensionless variable $a$, we momentarily assume that the model should also account for the positron excesses seen by PAMELA and Fermi/LAT, the latter of which suggests that $M_0 \cong 1$ TeV [29]. This enables us to plot the contours in the $\eta$-$\delta M$ plane. An important question is whether the rate can ever be large enough if $\delta M \geq 2 m_e$ since in the most natural models, each excited state must have enough energy to decay into an $e^+e^-$ pair.

In figure 6 we display results for three cases with the
BDM velocity dispersion \cite{25} and different assumptions for the density profile, and one example with the PDM dispersion \cite{23}. The largest rates are obtained in the former case, using cusp profiles, and $\Gamma = 10^4$. Even in this most favorable case however, the mass splitting cannot exceed 600 keV if the predicted rate is to match the observed one. The cored BDM density profile and the PDM velocity profile give much smaller rates, with correspondingly smaller upper limits on $\delta M$, 170 keV and 200 keV respectively.

**B. Dependence on other parameters**

Ideally one would like to find examples where the predicted rate can match observations for mass splittings $\delta M \gtrsim 2m_e$, since in the simplest models each excited DM state must decay into the ground state plus $e^+e^-$. Toward this end, we have explored the dependence on other parameters which we previously held fixed. For this purpose we fix instead the microscopic parameters at one of the highest-rate examples found in fig. 5. Namely we take $\Gamma = 10^4$, $\eta = 50$, using the BDM velocity profile, and Einasto density profile with $\alpha = 0.145$. We vary the DM mass $M_0$, the velocity dispersion parameter $v_0$, and the circular velocity parameter $v_c$. Recently ref. \cite{28} presented evidence, based on trigonometric parallaxes and proper motions of masers in star-forming regions of the Milky Way, favoring a surprisingly large value $v_c = 254 \pm 16$ km/s. The claim has been criticized in subsequent references \cite{41,41}; in particular ref. \cite{41} notes that the method used only constrains the ratio $v_c/R_0 \equiv 30$ km/s/kpc, where $R_0$ is the distance to the galactic center. We therefore also consider the variation of $v_c$ and $R_0$ together.

The results are shown in figure 6. The top left figure shows that there is an optimal value of $M_0 \equiv 600$ GeV for maximizing the rate, where $\delta M = 650$ keV gives the observed rate. This can be understood from the analytic approximation \cite{29} where for a fixed $\delta M$, the rate scales as $a^4 e^{-a} \sim M_0^{-4} e^{-c/M_0}$. The bottom left figure shows that the dependence on $v_0$, which controls the overall size of the DM velocity dispersion, is quite weak for values above the standard one $v_0 \equiv 220$ km/s (see footnote 8 for explanation). As expected, decreasing $v_0$ below this value only reduces the rate.

The right panels of figure 6 show the dependence on the circular velocity $v_c$, which controls the escape velocity of the DM. Keeping $R_0$ fixed, the upper right panel demonstrates that $v_c$ would have to be increased to 350 km/s to allow for $\delta M$ as large as $2m_e$. However increasing $R_0$ in a correlated way, as suggested by ref. \cite{41}, allows for a stronger effect, because the cutoff $r_c$ on the radial integration in eq. (44) increases proportionally with $R_0$. In this case, one would need $v_c = 280$ km/s and $R_0 = 9.4$ kpc. Such a large value of $v_c$ is $4\sigma$ away from the recent mean determination of $236 \pm 11$ km/s \cite{40} and that of $R_0$ is $2.5\sigma$ and $3.1\sigma$ outside the preferred values of refs. \cite{42,43} respectively.

We conclude that, by taking all parameters and distribution functions to their limits, it may be marginally possible to excite dark matter with a mass splitting as large as $2m_e$, although most practitioners would probably regard the required values of $v_c$ = 280 km/s and $R_0 = 9.4$ kpc as being unreasonably large.

VI. ALTERNATIVE XDM SCENARIOS AND TEV DARK MATTER

In ref. \cite{28} it was pointed out that a long-lived species $\chi_1$ with a mass splitting $\Delta M \sim 2m_e$ above the ground state $\chi_0$ could be excited to a third state $\chi_2$ with splitting $\Delta M \ll 2m_e$ in an “inverted mass hierarchy” scenario, whose spectrum is shown in fig. 7. Then the XDM mechanism can work via the excitations $\chi_1 \chi_1 \rightarrow \chi_2 \chi_2$ followed by the decays $\chi_2 \rightarrow \chi_0 e^+ e^-$. The reduced value of $\delta M$ allows the rate to be large enough to match observations without requiring extreme values for parameters of the galactic velocity profile.

Simple models of nonabelian DM with hidden SU(2) gauge symmetry were constructed in ref. \cite{44} to realize this possibility. The scenario comes with the expense of needing a late-time nonthermal origin for the DM, as explained in ref. \cite{28} (see also ref. \cite{43}), since otherwise the same process needed for positron production in the galaxy will depopulate the $\chi_1$ states in the early universe. If the DM goes out of kinetic equilibrium early enough, a small remnant of its initial relic density can be maintained; ref. \cite{28} optimistically estimates that $\sim 1/10$ of the initially produced relic density can survive. In this case we would need parameters corresponding to a rate $R_{e^+}/R_{obs} = 100$ to compensate for the $\rho^2 \sim 0.01$ suppression in the rate, relative to the assumed case of a standard relic density. The upper right-hand panel of figure 5 (corresponding to BDM with a steep Einasto profile) reveals such examples (the contours labeled “2”) when $\Gamma \sim 10^3 - 10^4$ and $\delta M \sim 50$ keV.

In the case of nonthermally-produced intermediate states, in order for this component of the DM to dominate over the conventionally produced thermal component of the ground state, it is necessary to have an annihilation cross section that is stronger than the one needed for the standard thermal relic density, which was computed for

| $\chi_2$ | $\delta M \sim 100$ keV |
| $\chi_1$ | $\Delta M \geq 1$ MeV |
| $\chi_0$ | |

**FIG. 7:** Spectrum of states for inverted mass hierarchy.
SU(2) DM in ref. [44]. This puts a constraint on the dark gauge coupling of triplet DM,

$$\alpha_g > 0.03 \left( \frac{M_0}{1 \text{ TeV}} \right)$$  

(30)

If we assume that $M_0 \cong 1$ TeV, and $200$ MeV $< \mu < 1$ GeV, the preferred value for explaining excess leptons seen by the PAMELA and Fermi/LAT detectors, this puts constraints on the dimensionless parameters $\Gamma, \eta$:

$$4.6 < \Gamma \times \left( \frac{\delta M}{100 \text{ MeV}} \right) < 5100$$  

(31)

$$0.03 < \eta \times \left( \frac{\delta M}{100 \text{ MeV}} \right) < 5$$  

(32)

where we have also imposed that $\alpha_g < 1$. Referring to fig.5 we see that these can be satisfied for the top panels, which use Einasto profile and BDM velocity dispersion, even at $\Gamma = 10$. There thus seems to be considerable room in the parameter space for the XDM explanation of low-energy galactic center positrons, if one accepts the inverted mass hierarchy hypothesis and a relic density of intermediate mass states.

VII. ANGULAR DISTRIBUTIONS

Further constraints can be obtained by computing the expected angular profiles of the produced positrons. This gives one-sided bounds, because there is considerable uncertainty in the distance that a positron could propagate away from the galactic center before annihilating. But a model that predicts too wide an angular profile can be excluded since propagation effects will never make the true profile more narrow.

Neglecting propagation effects, the angular distribution of positrons is given by an integral similar to that in [44], but instead of integrating over the volume, one must integrate along the line of sight. We define a luminosity $L(\phi)$ by

$$L(\phi) = \int_0^\infty \langle \sigma v_{rel} \rangle(r) n^2(r) \, d\tilde{r}$$  

(33)

where $\tilde{r}$ is distance along the line of sight, and $r^2(\tilde{r}, \phi) = R_0^2 + \tilde{r}^2 - 2\tilde{r} R_0 \cos \phi$, as illustrated in fig.8. The observed profile is broadened by the SPI instrumental resolution of 3° full width at half maximum, so we smooth it using a gaussian,

$$L_{\text{obs}}(\phi) = N(c) \int d\phi_0 \, e^{-c(\phi-\phi_0)^2} L(\phi_0)$$  

(34)

where $c = \pi (3^\circ/180^\circ)/(2^{3/2} \ln 2)$.

Our results for the angular profile are shown in fig.9 for sample microphysics parameters $\Gamma = 100$ and $\eta = 3.5$. We find that the shapes are relatively insensitive to these values, and rather depend more strongly on the DM density and velocity profiles, and the parameters $a, b$ defined in (27). For the first five panels of this figure, we have ignored the overall rate and normalized all curves to match the observed profile at the galactic center, to better visualize the differences in widths of the predicted signals. The bottom right panel shows the real rate corresponding to each assumed density profile, as a function of $\delta M$ (assuming $M_0 = 1$ TeV as before).

One can see that for the larger values of $\delta M$, the pure dark matter velocity profiles (as well as cored BDM) produce too wide a profile to match the observations, while the cuspy BDM case gives a narrower result that can be consistent if the positrons travel a distance of order 1 kpc before annihilating, as has been argued is possible [40]. This indicates that the kinematic advantage of having higher central velocities in the BDM scenario is more important than the boost to the cross section that one would expect from the Sommerfeld enhancement at lower velocities. Thus not only are the cuspy BDM profiles more easily able to match the observed rate, but they are also more consistent with the observed angular distribution.

VIII. CONCLUSION

In this paper we have tried to give the most quantitative treatment to date of the excited dark matter mechanism for producing positrons at the galactic center. A main technical improvement was the numerical computation of the excitation scattering cross section, which is nonperturbative and guaranteed to satisfy unitarity constraints. A second difference relative to previous treatments is that we included the effects of baryons on the velocity dispersion of DM in the inner kpc of the galaxy, which can significantly boost the rate of excitations. In addition we considered NFW and Einasto profiles for the DM density, varying parameters determining the cusps of the distributions, as well as those affecting the velocity distributions.

Even making all the most optimistic assumptions for increasing the rate of DM excitations, we were not able to find realistic parameter values which yield a large enough rate, if the mass gap between the ground state $\chi_0$ and excited state $\chi_1$ of the DM is sufficient for the subsequent
decay $\chi_1 \rightarrow \chi_0 e^+e^-$ to produce the positron. The largest mass gap we found consistent with the observed rate was $\delta M \approx 650$ keV, assuming the standard values $v_c = 220$ km/s and $R_0 = 8$ kpc, respectively, for the circular velocity of the sun around and its distance to the galactic center. By pushing these values to $v_c = 280$ km/s and $R_0 = 9.4$ kpc, we can barely accommodate a mass gap of $\delta M = 2m_e$, but these choices are respectively 4$\sigma$ and 3$\sigma$ away from the mean values.

One way to ease the tension would be to consider models where the excited states are charged and thus able to decay into single electrons or positrons; this would lower the required mass gap to just $\delta M = m_e$. But this requires a great deal more model-building gymnastics than the case of a neutral excited state. A theoretically more appealing alternative is the case of three DM states where $\chi_0\chi_0 \rightarrow \chi_1\chi_2$, in which the mass gap $\delta M_{01}$ between $\chi_0$ and $\chi_1$ is much smaller than $\delta M_{02} > 2m_e$ [44, 47]. For example if $\delta M_{01} = 0$, then the effective $\delta M$ which determines the threshold velocity is half as large as in the models that produce two of the same excited state. This possibility can arise if the hidden sector gauge group is SU(2)$\times$U(1) and gets completely broken, for example. However to see if this class of models can really have a larger rate would require solving the Schrödinger equation in the three-state system, so while it is plausible that such asymmetric excitations could increase the rate, it is not proven by our analysis.

Another way of getting around our no-go result is to assume the existence of a stable or metastable population of already excited states $\chi_1$, which need only be further excited to $\chi_2$ through a small mass gap $\delta M_{12}$, if $\delta M_{02}$ is assumed to be greater than $2m_e$ so that the decay $\chi_2 \rightarrow \chi_0 e^+e^-$ is allowed. We refer to this as the inverted mass hierarchy scenario. This idea comes with new complications, since it is difficult to prevent the excitation process from occurring earlier in the history of the universe, to maintain the population of $\chi_1$ to the present day. However there is an intriguing possibility to overcome this by assuming the present DM particles are products of the late decay of a much heavier predecessor, and thus were initially relativistic, at a time when they would normally have been nonrelativistic had they been produced through conventional freeze-out [28]. The higher velocity suppresses the Sommerfeld enhancement at early times, when it would have the undesirable effect of depleting the metastable intermediate DM states. We hope to examine this scenario more carefully in future work.

We have not tried to impose in detail the additional constraints on the model which arise if one would like it to also account for the PAMELA and Fermi/LAT excess electrons/positrons, although we did focus on TeV scale DM for that purpose. If the DM explanation of the high-energy leptons is not ruled out by upcoming analyses, this would be an interesting next step. It might also be worthwhile to investigate the effect of a nonspherically symmetric DM halo [48] on the rate of positron production.

Acknowledgment: We thank Gil Holder for valuable discussions about the galactic parameters.
Appendix A: Born approximation cross section

For comparison with the nonrelativistic excitation cross section from numerical solution of the Schrödinger equation, we here give the quantum field theoretic expression in the Born approximation. The spin-averaged squared matrix element is

\[
|M|^2 = 4g^4 \left\{ \frac{4}{(t-\mu^2)^2} \left[ 2(s^2 + u^2) + 4M_+^2 t - M_+^2 + M_-^4 - 6M_+^2 M_-^2 \right] + \frac{4}{(u-\mu^2)^2} \left[ 2(s^2 + t^2) + 4M_+^2 u - M_+^4 + M_-^4 - 6M_+^2 M_-^2 \right] + \frac{4}{(t-\mu^2)(u-\mu^2)} \left[ 4s^2 - 8M_+^2 s + 2M_-^2 s + 3(M_+^4 - M_-^4)^2 \right] \right\}
\]

(A1)

where \( g \) is the coupling constant, \( s, t, u \) are the Mandelstam variables, \( M_\pm = M_1 \pm M_0 \), and \( M_1, M_0 \) and \( \mu \) are the dark matter and gauge boson mass respectively.

In the center-of-mass frame,\n
\[
s = E_{cm}, t = -2p^2 (1 - \cos(\theta)), u = -2p^2 (1 - \cos(\theta))
\]

(A2)

so the cross section is

\[
\sigma = \frac{1}{E_{cm}^2 v_{rel}} \int \frac{d\Omega_{cm}}{4\pi} \frac{1}{8\pi E_{cm}} |M|^2
\]

\[
= \frac{g^4}{2\pi E_{cm}^3 v_{rel}} \left\{ \frac{8s^2 - 16M_+^2 s + 4M_-^2 s + 6(M_+^2 - M_-^2)^2}{pp'(\mu^2 + p^2 + p'^2)} \ln \left( \frac{\mu^2 + (p + p')^2}{\mu^2 + (p - p')^2} \right) - \frac{64\mu^4 + 192\mu^2 p^2 + 192\mu^2 p'^2 + 64\mu^2 M_+^2 + 160p^4 + 160p'^4 + 192p^2 p'^2 + 32s^2 - 16M_+^4 + 16M_-^4 - 96M_+^2 M_-^2}{\mu^2 + (p - p')^2)(\mu^2 + (p + p')^2)} \right. 
\]

\[
+ \left. \frac{16(2p^2 + 2p'^2 + M_+^2 + \mu^2)}{pp'} \ln \left( \frac{\mu^2 + (p + p')^2}{\mu^2 + (p - p')^2} \right) \right\}
\]

(A3)

We rewrite the kinematic variables using the dimensionless quantity \( \Delta \),\n
\[
v_{rel} = 2v_t \sqrt{\Delta}, \quad p = M_0 v_t \sqrt{\Delta}, \quad p' = M_0 v_t \sqrt{\Delta - 1}
\]

\[
E_{cm} = 2M_0 \sqrt{1 + v_t^2 \Delta}, \quad M_+ = 2M_0 \left( 1 + \frac{v_t^2}{4} \right), \quad M_- = \frac{M_0}{2} v_t^2
\]

(A4)

Using \( v_t \ll 1 \) and \( \delta M \ll \mu \ll M_0 \), we obtain the approximate expression (13) for the cross section.

Appendix B: Escape velocity

To be self-contained, we reproduce here the argument of ref. (13) for the \( r \)-dependence of the escape velocity. It is important to notice that dark matter does not dominate the mass of the inner galaxy. To account for the gravitational effect of the baryons, one can use the fact that the circular velocity \( v_c \) (the velocity of a test mass on a circular orbit around the galactic centre), is nearly constant out to radii or order \( r_{-2} \). We make the simplifying assumptions that there is no mass beyond \( r_{-2} \) and that the density is spherically symmetric.

The gravitational potential is

\[
\Phi = -G \int_r^\infty \frac{M(r')}{r'^2} dr'.
\]

(B1)

where \( M(r) \), the mass within radius \( r \), can be inferred from \( F = ma \) for a test particle of mass \( m \): \( mv^2/r = GmM(r)/r^2 \). Hence \( M(r) = rv^2_c^2/G \) for \( r < r_{-2} \) and \( M = r_{-2}v_c^2/G \) for \( r > r_{-2} \). The integral for \( \Phi \) becomes

\[
\Phi = v_c^2 \left\{ \ln \left( \frac{r_{-2}}{r} \right) - 1, \quad r < r_{-2} \right. 
\]

\[
- \frac{r_{-2}}{r}, \quad r < r_{-2}
\]

(B2)

The escape velocity is given by \( \frac{1}{2}v_{esc}^2 + \Phi = 0 \), leading to eq. (17).
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