The Hydraulic Jump in Liquid Helium

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Abstract. We present the results of some experiments on the circular hydraulic jump in normal and superfluid liquid helium. The radius of the jump and the depth of the liquid outside the jump are measured through optical means. Although the scale of the apparatus is rather small, the location of the jump is found to be consistent with the assumption that the jump can be treated as a shock, if the surface tension is taken into account. The radius of the jump does not change when going down in temperature through the lambda point; we think that the flow is supercritical. A remarkable feature of the experiment is the observation of stationary ripples within the jump when the liquid is superfluid.

When a jet of liquid falls on a horizontal surface, as one often observes in the kitchen sink, a discontinuity may occur in the depth of the out-flowing fluid: at a certain distance $R_j$ from the jet, there is an abrupt increase in the depth of the liquid, and a decrease in the average velocity of the liquid. This discontinuity is called the hydraulic jump, and was discussed in 1914 by Rayleigh. Although the hydraulic jump is a popular undergraduate experiment, the theory is challenging because of the free boundary, and it remains a problem of current theoretical interest. Many experimental measurements of the jump radius $R_j$ have been performed over the years (see the references cited in ref. [6]), mostly with fluids such as water or ethylene glycol. In this article, we report the results of observations of the hydraulic jump using liquid helium-4. Helium differs from the liquids that have previously been studied in having a much lower viscosity ($\nu \sim 2 \times 10^{-8} \text{ m}^2/\text{s}$), so jet Reynolds numbers up to $Re \sim 4 \times 10^4$ can be achieved. Another difference is the scale: typical values of $R_j$ in our apparatus are a few mm, an order or two smaller than in previous experiments on the hydraulic jump. This has the effect of increasing the importance of the surface tension.

The cell used in the experiment was mounted in a pumped helium optical cryostat, to permit direct imaging of the jump with a digital camera, as seen in Fig. 1. The jet was formed by admitting helium through a copper-nickel capillary tube of 100 $\mu$m inner diameter. The capillary was aligned so that the jet would be perpendicular to the surface of impact. The circular symmetry of the jump attested to the correctness of the alignment. The substrate surface was a sapphire disc (chosen for its high thermal conductivity), optically flat and aluminized.

The flux of liquid $Q$ was determined from the rate gas was admitted to the cell, measured with a flowmeter at room temperature. Four heat exchangers (copper capillaries 0.5 m long with 1 mm inner diameter) cooled, liquified and thermalized the gas. The resulting impedance limits the maximum flow rate in the experiment.

The radius of the jump $R_j$ was determined directly from the images taken by the digital camera. In order to permit measurement of the depth of the liquid, a horizontal wire was placed above the liquid. The image of the wire in the mirrored surface is displaced due to the index of refraction of the liquid on the surface. Measuring the displacement of the image relative to the wire thus permitted a measurement of the depth liquid outside the jump, $d$, to within a few $\mu$m.

Fig. 2 shows a comparison between the measurement of $R_j$ above the lambda transition and various models. To analyze the data, we have considered three models.
Two of the models, those of Watson and of Bush and Aristoff, follow Rayleigh in treating the jump as a shock discontinuity, where mass and momentum fluxes are conserved. They improve on Rayleigh in including the viscosity of the liquid. Near the jet impact, the fluid flow is modelled as a uniform flow near the free surface, and a growing boundary layer near the substrate; beyond the point where the boundary layer reaches the free surface, the flow is modelled with a similarity profile. (The matching is not exact, but close.) Bush and Aristoff’s model is similar to Watson’s, but includes the effect of the surface tension, whose curved surface exerts a force at the jump and must be included in the momentum flux conservation condition. The model of Bohr et al. differs in attempting to model the flow at the jump more accurately. They use a one-parameter polynomial model of the flow profile; both the profile parameter and the height of the surface are variables, which vary continuously at the jump. This model is capable of representing the roll which is known to develop beyond the jump.

In Fig. 2 it is seen that the predictions of Watson are slightly higher than the experimental results, and the predictions of Bush and Aristoff are slightly too low. It would appear that the effect of surface tension is important, but that the model of Bush and Aristoff is too crude, and overestimates the size of the effect. (Curiously, the experiments of Bush and Aristoff seem to indicate that their estimate of the effect of surface tension is not large enough.) The predictions of the model of Bohr et al. for our experiment are similar to those of Watson; this is not surprising in view of the fact that in practice, in all of our runs except at the highest temperatures and lowest flow rates, the jump is rather sharp.

Below the lambda transition, one might hope to observe a transition in the jump to the value predicted by Rayleigh in the inviscid case. However, we did not observe such a transition. We believe the fluid velocity in our experiment exceeds the critical velocity for superfluidity. For a fluid depth on the order of 10 µm, such as we expect within the jump, the critical velocity is on the order of 50 mm/s, corresponding to $Q$ less than 1 mm$^3$/s. Unfortunately, at such low fluxes, the jet becomes unstable and drips instead. However, even if the helium in our experiments is normal below the lambda point, the viscosity of the normal fluid drops rather abruptly as the temperature is lowered from 2.17 K to 1.5 K. In this regime, we start to observe stationary ripples inside the jump, as seen in Fig. 1. These can be explained as waves generated at the jump and travelling upstream at the same velocity the fluid flows downstream. The ripple wavelength can be used to estimate the depth of the liquid inside the jump by equating the phase velocity of shallow water capillary waves with the fluid velocity. The resulting estimate is in rough agreement with the prediction and is much smaller than the depth outside the jump. The temperature dependence of the decay length is not yet completely understood. Presumably, the lower viscosity below the lambda point allows the ripples to propagate further upstream from the jump, so they become more prominent. Analysis of this phenomenon is ongoing.

In summary, our experimental results agree with the shock model of the hydraulic jump. The effect of surface tension is important, but the model of Bush and Aristoff overestimates it. No abrupt change in the jump is observed at the lambda transition. The appearance of ripples inside the jump below the lambda point likely reflects the decreased viscosity of the normal fluid.

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