Pairing gaps in atomic gases at the BCS–BEC crossover

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New Journal of Physics 6 (2004) 137
Received 29 June 2004
Published 21 October 2004
Online at http://www.njp.org/
doi:10.1088/1367-2630/6/1/137

Abstract. Strong evidence for pairing and superfluidity has recently been found in atomic Fermi gases at the BCS–BEC crossover both in collective modes and radio frequency (RF) excitation energies. It is argued that the scale for the effective pairing gaps measured in RF experiments is set by the lowest quasiparticle in-gap excitation energies. These are calculated at the BCS–BEC crossover from semiclassical solutions to the Bogoliubov–deGennes equations. The strong damping of the radial breathing mode observed in the BCS limit occurs when the lowest quasiparticle excitation energies coincide with the radial frequency, which indicates that a coupling between them takes place.

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1. Introduction

The recent developments in cold Fermi gases mark another milestone in a century of marvellous discoveries within the field of superfluidity and superconductivity. Experiments have established the stability of cold Fermi gases with strong attractive interactions [1]–[5] and proven the unitarity limit near Feshbach resonances. A smooth BCS–BEC crossover is found in which the Fermi atoms gradually bind into (Bose) molecules near Feshbach resonances as predicted in [6]–[8]. The collective modes provide strong evidence for superfluidity [9, 10]. Radio frequency (RF) spectroscopy [11] clearly shows a peak in the response function that depends on interaction strength, density and temperature as expected from resonance superfluidity theory [12, 13].

The advantage of the trapped atomic gases is that we can tune the interactions, densities, temperatures and other trap parameters in a controlled way and thus explore superfluidity in detail as well as in general. This insight has already been exploited to describe some features of pairing in nuclei and neutron stars [14, 15]. Furthermore, the pairing gaps $\Delta$ are of the order of the Fermi energy $E_F$ around the BCS–BEC crossover and therefore $\Delta / E_F$ is an order of magnitude larger than in high-temperature superconductors and superfluid $^3$He, and two orders of magnitude larger than in standard superconductivity (see e.g., [16]). Just a few years ago pairing gaps as large as the Fermi energy was not considered possible by most of the condensed matter community. In a uniform system, the gap is $\Delta = 0.54E_F$ in the unitarity limit according to Monte Carlo calculations [14]. The observable transition frequencies are, however, related to the quasiparticle energies, which are considerably smaller than the pairing field in the centre of the trap and therefore in-gap surface excitations.

The purpose of this work is to compare the recent experimental results on RF spectroscopy and collective modes with theoretical calculations. After a general introduction to pairing and superfluidity in uniform systems in section 2, we address pairing in traps in section 3 with special emphasis on the in-gap quasiparticle excitations at the BCS–BEC crossover, and how they compare to the data of Chin et al [11] in section 4. In section 5, we discuss the collective mode frequencies and damping and the evidence for superfluidity, and finally give a summary.

2. Pairing in uniform systems

The quasiparticle excitations in traps are usually very different from those in uniform systems. Yet the bulk pairing field is important for understanding the BCS–BEC crossover, and it is very useful as a basis for pairing in traps with many particles within the Thomas–Fermi (TF) approximation.

Solving the gap equation at zero temperature for a Fermi gas interacting through an attractive scattering length $a < 0$ gives a pairing gap in the dilute limit, $|a|k_F \ll 1$,

$$
\Delta = \kappa E_F \exp \left[ \frac{\pi}{2ak_F} \right].
$$

(1)

Here, $\kappa = 8/e^2$ in standard BCS. Gorkov included induced interactions which reduces the gap by a factor $\sim 2.2$ to $\kappa = (2/e)^{7/3}$ [17]. In the unitarity limit $k_F |a| \gtrsim 1$, which can be reached around Feshbach resonances, the gap is of order the Fermi energy [7, 15, 16]. Extrapolating (1)
to $ak_F \to \pm \infty$ gives a number close to that found from odd–even staggering binding energies $\Delta = 0.49E_F$ calculated by Monte Carlo [14].

The pairing gap can qualitatively be followed in the crossover model of Leggett [7] by solving the gap equation

$$
1 = \frac{2\pi\hbar a}{m} \sum_k \left[ \frac{1}{\epsilon_k} - \frac{1}{E_k} \right].
$$

Here, the quasiparticle energy is $E_k = \sqrt{(\epsilon_k - \mu)^2 + \Delta^2}$ with $\epsilon_k = \hbar^2 k^2 / 2m$. The chemical potential $\mu$ follows from conservation of particle number density

$$
n = \sum_k \left[ 1 - \frac{\epsilon_k - \mu}{E_k} \right].
$$

In the dilute (BCS) limit the gap equation leads to the standard BCS gap of (1) not including the Gorkov correction (see figure 1). The chemical potential is $\mu = E_F$ and does not include the standard mean-field Hartree–Fock correction $2\pi an / m$ of a dilute gas.

The advantage of the crossover model is that it extends to the strongly interacting and molecular BEC limit. Here, the pairing gap approaches $\Delta = 4E_F / \sqrt{3\pi ak_F}$. The chemical potential approaches half of the molecular binding energy $E_b = -\hbar^2 / ma^2$ with a BEC mean
field corresponding to a molecular scattering length of $a_m = 2a$. Four-body [18], Monte Carlo calculations [19] and experiments [4] do, however, indicate that $a_m \simeq 0.6a$.

On the BCS side ($a_{kF} < 0$), the minimum quasiparticle energy is $\Delta$ and occurs when $k = k_F$.

On the BEC side ($a_{kF} > 0$), the chemical potential is negative and the minimum quasiparticle excitation energy is the quasiparticle energy for $k = 0$.

$$E = \sqrt{\mu^2 + \Delta^2}.$$ \hspace{1cm} (4)

The chemical potential is observed in the spin excitation response function [20].

Monte Carlo calculations [14, 19] of binding energies, equation of states and pairing gaps agree qualitatively with the crossover model. The pairing gap is $\Delta / E_F = 0.54$ in the unitarity limit [14], which is quite close to the extrapolation of the Gorkov gap $\Delta^{Gorkov} / E_F = (2/e)^{7/3} \simeq 0.49$ but somewhat smaller than that of the crossover model $\Delta^{Leggett} / E_F = 0.69$.

In the BCS limit the critical temperature for the BCS transition is $T_c = (eC/\pi) / \Delta_1$, where $C = 0.577 \ldots$ is the Euler constant. In the crossover model, there are two transition temperatures in the BEC limit [8], namely the molecular BEC temperature at $T_{c} = [n/2(3/2)\pi]^{1/3} / m = 0.218E_F$ and the molecule dissociation temperature $T_{c}^{dissoc} = E_b / \ln(E_b / E_F)^{3/2}$.

3. Pairing in harmonic oscillator (h.o.) traps

The various regions of pairing were described in [21] for a dilute system of $N$ atoms with Hamiltonian

$$H = \sum_{i=1}^{N} H_0(r_i) + 4\pi \hbar^2 a_{m} / m \sum_{i<j} \delta^3(r_i - r_j),$$ \hspace{1cm} (5)

in a h.o. potential $H_0(r) = p^2/2m + \sum_{k=1}^{3} m\omega_k^2 r_k^2 / 2$. We shall mention a few relevant results before we investigate the strongly interacting limit.

At least two dimensionless parameters are required to describe this system even in the spherically symmetric case: $\omega_1 = \omega_2 = \omega_3 \equiv \omega_0$. These are, e.g., the number of particles $N$ and the interaction strength $a$ (when energies are measured in units of $\hbar\omega_0$ and lengths in $a_{osc} = \sqrt{\hbar / m\omega_0}$). Several interesting pairing regions or ‘phases’ appear versus $N$ and $a$. In contrast, the binding energies and pairing in an uniform gas at zero temperature are functions of one parameter only, e.g., $ak_F$.

When the traps contain relatively few atoms, $N \lesssim 10^3$, which are weakly interacting, the mean field does not significantly lift the degenerate angular momentum states $l = n_F, n_F - 2, \ldots, 1$ or 0, where $n_F = E_F / \hbar\omega = (3N)^{1/3}$, due to the SU(3) symmetry of the spherical symmetric h.o. potential. Consequently, pairing takes place between all these states which leads to the supergap [21]

$$\Delta = G = \frac{32\sqrt{2n_F}}{15\pi^2} \frac{|a|}{a_{osc}} \hbar\omega_0 = \frac{32}{15\pi^2} k_F |a| \hbar\omega_0.$$ \hspace{1cm} (6)

Here, $k_F = \sqrt{2n_F} / a_{osc} \simeq 1.7N^{1/6} / a_{osc}$ is the Fermi wave number in the centre of the trap. For more particles in the trap the stronger mean field causes level splitting, which reduces pairing...
towards single-level pairing, $\Delta_{n_F,l}$, which displays a distinct shell structure with h.o. magic numbers. Pairing in nuclei has a similar shell structure on top of an average gap equal to the supergap which for constant density scales with the atomic mass number $A$ as $\Delta \simeq G \simeq 5.5\text{MeV}/A^{1/3}$. For very large nuclei, pairing approaches that in bulk matter $\Delta \simeq 1.1\text{MeV}$.

For stronger interactions, pairing also takes place between shells and the gap increases to \( \Delta \simeq G \simeq 5.5\text{MeV}/A^{1/3} \). For very large nuclei, pairing approaches that in bulk matter $\Delta \simeq 1.1\text{MeV}$.

This multi-shell pairing region extends up to $2G \ln(e^{\xi}/\Delta) \lesssim \hbar \omega_0$.

For even stronger interactions, the pairing field exceeds the h.o. energy in the centre of the trap and the coherence length, $\xi = k_F/\pi m \Delta$, is much smaller than the TF radius $R_{TF}$ of the cloud. This spatially inhomogeneous case with a strong pairing field $\Delta(r)$ can be solved by the Bogoliubov–deGennes equations

$$E_\eta u_\eta(r) = [H_0 + U(r) - \mu]u_\eta(r) + \Delta(r)v_\eta(r),$$

$$E_\eta v_\eta(r) = -[H_0 + U(r) - \mu]v_\eta(r) + \Delta(r)u_\eta(r).$$

Here, $E_\eta$ are the quasiparticle energies, $u_\eta$ and $v_\eta$ the Bogoliubov wave functions and $U(r)$ the mean field. The pairing field can be approximated by the Gorkov gap in the TF approximation [21].

Let us first study the spherically symmetric trap. The Bogoliubov wave function can be written in the form $u_\eta = u_{nl}(r)Y_{lm}(\theta, \phi)$, where $(l, m)$ are the angular momentum quantum numbers. The Bogoliubov–deGennes equations can be solved semiclassically [22] which leads to a WKB quantization condition

$$\left(n + \frac{1}{2}\right) \frac{\pi}{2} = m \int_{R_1}^{R_2} dr \sqrt{\frac{E_{n,l}^2 - \Delta(r)^2}{2m(\mu - U(r)) - r^2/\alpha_{osc}^2 - l(l+1)/r^4}},$$

independent of $l$ as long as $l \ll n_F$. In this expression we can use TF for the pairing field $\Delta(k_F(r))$ of (1), i.e. with $k_F(r) = \sqrt{2n_F(1 - r^2/R_{TF}^2)/\alpha_{osc}}$ and $E_F(r) = \hbar^2 k_F^2(r)/2m$. The
resulting quasiparticle energies $E_n$ [21] are given by inverting the relation

$$k_F|a|\hbar\omega_0 \simeq \frac{E_n}{(n + 1/2)} \left\{ \ln \left[ \frac{k(n + 1/2)^2\pi^2n_F}{4 E_n^3} \right] \right\}^{-1}.$$

(10)

It is valid above the multishell pairing regime of (7) and up to the dense limit $1/(ak_F) \lesssim -1$.

We can furthermore extend the semiclassical model to stronger interactions and the unitarity limit. As in the Leggett crossover model, we assume that the correlations are included in the pairing field and that the mean field can be included in $\mu$ and in an effective h.o. potential. The Bogoliubov–deGennes equations can then be applied—specifically the semiclassical quantization condition of (8). However, although the crossover model provides a simple and qualitatively description of the ground state around the BCS–BCS crossover, it differs from the ground state found by Monte Carlo calculations and four-body calculations for both the pairing field and the chemical potential as mentioned above. We will therefore include the effect of mean field by replacing the chemical potential by that calculated by Monte Carlo [14, 19]. We include the induced interactions by using the Gorkov result of (1) for the pairing field which is also a fair approximation around the unitarity limit according to Monte Carlo calculations [14].

Equation (8) can now be solved in the TF approximation. For example, in the unitarity limit $\Delta(r) = \kappa E_F(r)$ and the attractive mean field is $U = \beta E_F(r)$, where $\beta \equiv E_{int}/E_{kin}$ is an universal parameter in the unitarity limit [15] and is directly related to the chemical potential at zero temperature. It has been measured directly from expansion energies [1, 2, 4, 5] and confirms Monte Carlo calculations $\beta \simeq -0.56$ in the unitarity limit [14, 19]. The integral in (8) can be expanded for radii near the surface and calculated analytically:

$$E_n = \left[ \left( n + \frac{1}{2} \right) \frac{\Gamma(7/4)}{\Gamma(5/4)} \hbar\omega_0 \right]^{2/3} \frac{(\pi\kappa E_F)^{1/3}}{\sqrt{1 + \beta}}.$$

(11)

Note that the form of the mean field in the unitarity limit is such that it can be included in the chemical potential and h.o. potential and ‘renormalizes’ these quantities by factors of $(1 + \beta)$ to some power. The mean field decreases the TF trap size $R_{TF} \propto (1 + \beta)^{1/4}$ as well as the chemical potential but increases the central density. As a result, the quasiparticle energies increase slightly. In (11) the Fermi energy is defined as $E_F = n_F\hbar\omega_0$ without mean field corrections as also defined in the experiment of [11].

The finite size of the system is manifest because $E_n/E_F$ is not only a function of $ak_F$ but also depends on $n_F = E_F/\hbar\omega_0$. For example, $E_n/E_F$ scales with particle number as $\sim N^{-2/9}$ in the unitarity limit.

Around the unitarity limit the semiclassical quantization condition is solved numerically with the TF Gorkov gap and a mean-field correction $\beta$ averaged over densities in the trap. On the BEC side the chemical potential $\mu$ should be included in excitations as in (4). For the lowest quasiparticle excitations near the surface region, where the density is low, this chemical potential is, within TF, simply the molecular binding energy, $\mu = E_b/2 = -\hbar^2/2ma^2$. The quasiparticle energy $E_n$ is therefore replaced by $\sqrt{E_n^2 + (E_b/2)^2}$. The lowest quasiparticle energies $E_0$ and $E_1$ including this molecular binding energy correction are shown in figure 1 as a function of $1/(ak_F)$. They reduce to (10) and (11) in the BCS and unitarity limits, respectively. On the BEC side the pairing becomes negligible as compared to the molecular binding energy and $E \simeq |E_b|/2$. The quasiparticle energies are much smaller than the pairing field in the centre of the trap and are therefore in-gap surface excitations.

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The Bogoliubov–deGennes equations and semiclassical solutions can in principle be extended to deformed traps. The quasiparticle energy degeneracy for \( l = 0, 1, \ldots \) will be split and depend on \( \omega_i \). However, due to energy-weighted sumrules we expect that the average quasiparticle energies can be approximated by (10) and (11) when \( \omega_0 \) is replaced by \( \bar{\omega} = (\omega_1 \omega_2 \omega_3)^{1/3} \). This in concordance with the analysis of the resonance superfluidity theory which is independent of the symmetry of the cloud [13].

4. RF excitation spectrum

For detailed calculations of RF response functions for trapped Fermi atoms and at finite temperature, we refer to recent calculations based on resonance superfluidity theory [13]. Their Bose–fermion model divides the system of strongly correlated atoms into molecular bosons and atoms and introduces effective couplings as well as cutoffs between these constituents. They can qualitatively explain the RF response functions at the various interaction strengths, densities and temperatures but exceed the effective pairing gaps quantitatively. We shall now attempt to understand the RF excitation spectrum in terms of the quasiparticle energies calculated above at zero temperature.

The ‘effective’ pairing gaps reported in [11] are defined as the maximum of the RF response function. The excitations energies include the breaking of the pair in states |1\rangle + |2\rangle to an in-gap or above gap quasiparticle in state |1\rangle and an unpaired quasiparticle in the new spin state |3\rangle. The latter energy is, however, subtracted on average as it is related to the thermal peak. The RF excitation spectrum is therefore distributed around the lowest quasiparticle excitation energies. As discussed above we shall assume that the scale of the RF spectral weight in a deformed trap is given by the lowest quasiparticle excitations as given by (10) and (11) replacing \( \omega_0 \) with \( \bar{\omega} \). The distribution is also smeared by the spectrum of final h.o. states |3\rangle and by finite temperature effects and damping. The spectral response function is a sum over quasiparticle excitation energies weighted with transition matrix elements. These typically decrease rapidly with transition energy [20] and, therefore, the maximum of the response function is expected to peak at or near the lowest quasiparticle energies. Also the overlap of the quasiparticle wavefunctions with the unpaired particle wavefunctions, as well as the density of states, contribute to the shape of the RF spectrum.

In figure 1, we compare the two lowest quasiparticle excitations \( E_0 \) and \( E_1 \) to the effective pairing gaps \( h\nu/2 \) of Chin et al [11] for values of \( ak_F \) around the unitarity limit. \( E_n \) are calculated from (8), \( k_F \) from the central trap densities, and \( a \) is calculated assuming that the Feshbach resonances reside at \( B = 834 \text{G} \). We observe that the effective pairing gaps lie between the lowest quasiparticle energies \( E_0 \) and \( E_1 \) as argued above—except in the dilute BCS limit. The latter departure may be a finite temperature effect. In the BCS limit the pairing gap and thus \( T_c \) decrease towards the temperature of the gas leading to quenching of the gap. Unfortunately, the temperature cannot be presently measured accurately in traps.

Theoretically, the quasiparticle energies and pairing gaps in units of the Fermi energy are universal function of \( ak_F \) around the unitarity limit when \( n_F \) is unchanged. The effective pairing gaps at three different Fermi energies obey this scaling to a good approximation. It should be
noted that the exact position of the Feshbach resonance is important otherwise the scaling is destroyed. Best scaling is obtained by placing the resonance in $^6\text{Li}$ around $B \simeq 845$ G.

We emphasize that the pairing gaps are calculated from the semiclassical quantization condition of (8), which is based on the Bogoliubov–deGennes equations derived from the gap equation in the crossover mode. However, although the crossover model provides a simple and qualitative description of the ground state around the BCS–BCS crossover, it differs from the ground state found by Monte Carlo calculations and four-body calculations for both the pairing field and the chemical potential as mentioned above. The effect of mean fields was included by replacing the chemical potential with that calculated by Monte Carlo. Induced interactions were included by using the Gorkov result of (1) for the pairing field which also is a fair approximation around the unitarity limit according to Monte Carlo calculations [14].

### 5. Collective modes

The hydrodynamic and superfluid collective mode frequencies are unfortunately identical and we have to rely on other observables to be able to distinguish them and prove, e.g., superfluidity. We shall first discuss the collective modes for a very large number of particles in a trap and/or with strong interactions. Subsequently, we discuss the effect of a finite number of particles in a trap and damping with and without superfluidity.

The hydrodynamic and superfluid collective modes can be calculated in general for a polytropic equation of state, $P \propto n^{1+\gamma}$. Here, the polytropic power is $\gamma = 1$ in a dilute interaction dominated BEC, whereas an ideal Bose gas in the normal state has $\gamma = 2/3$ under adiabatic conditions. A dilute gas of Fermi atoms also has $\gamma = 2/3$ in both the hydrodynamic and superfluid limits. Both a Fermi gas [15] and a BEC [23] have $\gamma = 2/3$ in the strongly interacting (unitarity) limit. The effective power $\gamma$ has been calculated at the BCS–BEC crossover for the Leggett model [24, 25] and by Monte Carlo [14] and varies between $\gamma \sim 0.5$–1.3.

A spherical symmetric h.o. trap with a polytropic equation of state has collective mode frequencies [24] $\omega^2 = \omega_0^2 (l + 2n(\gamma(n + l + 1/2) + 1))$, where $l$ is the angular momentum and $n$ the number of radial nodes. In comparison, the collective modes in the collisionless limit are those of a free particle: $\omega/\omega_0 = 2n + l$, when its mean free path exceeds the size of the cloud.

In an axial symmetric trap, $\omega_1 = \omega_2 \equiv \omega_\perp$ and $\omega_3 = \lambda \omega_\perp$, the resulting breathing modes are the coupled monopole and quadrupole $m = 0$ modes [26]. For the very cigar-shaped traps $\lambda \ll 1$, used in [9, 10], the coupled modes become the radial

$$\omega_{\text{rad}} = \sqrt{2(\gamma + 1)} \omega_\perp,$$

and axial

$$\omega_{\text{ax}} = \sqrt{3 - (\gamma + 1)^{-1}} \omega_3$$

modes. Taking the effective power $\gamma$ from the Leggett model at the BCS–BEC crossover, good agreement is found [24, 25] with the experiments of Kinast et al [9] and Bartenstein et al [10] for the axial and radial modes in [9]. The radial mode in [10] is different and a transition or ‘break’ is observed around the break point $B \simeq 910$ G which is accompanied by strong damping.

In the unitarity limit $x = 1/(ak_F) = 0$ scaling predicts that $\gamma = 2/3$ as is also found in the axial and radial modes in [9] and in the axial mode in [10]. Furthermore, it follows from (12)
and (13) that as function of $x = 1/(ak_F)$ their slopes are

$$\omega'_\text{rad} = \sqrt{\frac{3}{10}} \omega'_{\perp}$$

and

$$\omega'_\text{ax} = \frac{1}{4} \left(\frac{3}{4}\right)^{3/2} \gamma' \omega_3$$

in the unitarity limit. The slopes of $\gamma$ and $\beta$ can be related, at $x = 0$, as

$$\gamma' = \frac{\beta'}{6(1 + \beta)}.$$  \hspace{1cm} (16)

By the Monte Carlo calculations [14, 19], the crossover model and the LOCV model [14, 15] we have $\beta'(x = 0) \simeq -0.10$ and $\beta(x = 0) \simeq -0.56$ (the crossover model does, however, give $\beta(x = 0) = -0.42$ which probably is due to the lack of Hartree–Fock energy corrections). Thus we obtain $\gamma'(x = 0) \simeq 0.40$. The resulting slopes of the collective frequencies $\omega'_\text{rad} \simeq 0.22\omega_{\perp}$ and $\omega'_\text{ax} \simeq 0.046\omega_3$ are also compatible with experiment [9, 10] in the unitarity limit.

The measured damping of the modes is not compatible with hydrodynamics. As pointed out in [9] the damping rate increase with increasing temperature, whereas the opposite is expected in hydrodynamics. In a superfluid, the condensate is gradually depleted as the temperature increases and coupling between the normal and superfluid components increase damping as observed. The damping in [10] peaks at the transition which, together with the abrupt transition in frequency, indicates a superfluid to collisionless transition rather than a smooth hydrodynamic to collisionless transition.

The damping rate from collisional damping in a normal gas can be estimated from the semi-quantitative form for the transition between collisionless and hydrodynamic limit [27, 28],

$$\omega^2 = \omega^2_C - \frac{\omega^2_C - \omega^2_H}{1 - i\omega \tau_{\text{coll}}}. \hspace{1cm} (17)$$

For the radial mode, the hydrodynamic frequency is $\omega_H \simeq \sqrt{10/3}\omega_{\perp}$ whereas the collisionless is $\omega_C \simeq 2\omega_{\perp}$. The relaxation processes are expressed in terms of a collision time $\tau_{\text{coll}}$. The maximal damping of the collective mode $\text{Im}(\omega)$ occurs between the hydrodynamic and collisionless limits for $\omega \tau_{\text{coll}} = 1$ and is $\text{Im}(\omega) \simeq 0.09\omega_{\perp}$, for the radial mode. The damping rates in [10, 29] are considerably larger around the break points, and cannot be caused by collisional damping alone.

Alternatively, the enhanced damping and break point could be caused by a superfluid to collisionless transition. However, in the centre of the trap the critical temperature $T_c = 0.28E_F \exp(\pi x/2)$ is at the break point $x \simeq -0.5$ much larger than the gas temperatures, $T \lesssim 0.03E_F$. Thus only the low-density surface layer is not in the superfluid phase. Estimates of the effective scattering cross-sections indicate that the gas is collisionless in the normal phase at the very low temperatures present in the experiments in [9, 10, 29].

It was pointed out in [10] that the lowest quasiparticle excitation energy is comparable to the collective energy $h\omega_{\perp}$ of the radial mode at their transition. An interesting coupling between the single-particle states and the collective mode may therefore take place which is special for a finite system. This coupling can be studied in the RPA equations which describe how the collective modes are build up of quasiparticle states. The collective modes are calculated for
dilute and spherical h.o. traps in [30] by solving the RPA equations using the quasiparticle states and energies as input. When interactions are weak such that $2G \ln(e^{F} n_{F}) \leq \hbar \omega_{0}$, the collective modes are dominated by the pairing gap and the h.o. shell structure. For stronger interactions, the collective modes can be calculated from RPA and they approach those of a superfluid.

Although the collective modes for both the dilute system and the unitarity regime typically are of the order of the h.o. frequency, the underlying quasiparticle spectrum is very different. In the dilute limit the pairing gap and the lowest excitation energies can be far below collective energies. This is similar to the situation in atomic nuclei where pairing energies are of the order 1 MeV whereas giant resonances lie around 10–20 MeV. In contrast, for trapped atoms in the unitarity regime the quasiparticle excitation energies $E_{n}$ lie well above $\hbar \omega$. Towards the BCS limit, however, $E_{0}$ approaches $\hbar \omega_{rad}$. It follows from the RPA equations that the collective frequency decreases and is damped with respect to a system with an infinite number of particles. This is in qualitative agreement with the experiment in [10] on the BCS side.

The observed transition or break in the radial frequency occurs at $B = 910$ G [10] for both $v_{\perp} = 750$ Hz and 2.4 kHz, i.e. for the same scattering length but for two different $k_{F}$ and radial frequencies. However, we find that in both cases the resulting radial frequency and lowest quasiparticle energy almost coincide at the break point, i.e.

$$\hbar \omega_{rad} \simeq 2E_{0}. \quad (18)$$

Note that $E_{F}$ and $k_{F}$ do not scale linearly with $v_{\perp}$ as does $\omega_{rad}$. It is thus a coincidence that (18) is fulfilled for the same scattering length and therefore at the same $B$ for the two different $v_{\perp}$. In the experiment of Kinast et al [9] both the trap frequencies are larger than in [10] and one can calculate that the break should occur around $B \simeq 980$ G according to the condition in (18). Preliminary data [29] find that damping does increase at such magnetic fields but that a possible break point is situated at even higher magnetic fields.

More precise measurements of $T_{c}$ and $T$ are required before the precise phase, damping mechanisms and break points can be determined.

6. Summary and outlook

We have argued that the recent experiments on RF spectroscopy [11] and collective modes [9, 10] give strong evidence for superfluidity in traps with Fermi atoms. The results compare very well with theoretical calculations [13, 24, 25] and the quasiparticle excitation energies discussed above as function of interaction strength, density and temperature.

There are, however, some details and deviations between theory and experiments that needs further investigation. The effective pairing gaps in [11] only partially obey the scaling with $E_{F}$. The radial mode collective frequency in [10] violate the scaling result in the unitarity limit and undergo a transition. Better control of the temperature and the exact position of the Feshbach resonance as well as experiments at several densities and temperatures are all desirable. These are necessary for a detailed check of the scaling with $ak_{F}$ around the unitarity limit and to pin down transitions and critical temperatures between superfluid and normal phases, collisionless and hydrodynamic dynamics.

In this respect, we can appreciate the great advantage of atomic gases, namely the large number of tunable parameters such as the number of particles, densities, interaction strengths, temperatures, trap deformation, number of spin states, etc. They therefore hold great promise for
a more general understanding of pairing phenomena not only in atomic Fermi gases but also in solids, metallic clusters, grains, nuclei, neutron stars, quark matter, etc.

Acknowledgments

Discussions with J Thomas and R Grimm are gratefully acknowledged.

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