A perfect monopole action for SU(2) QCD

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We found a quantum perfect lattice action in the 4-dimensional monopole current theory which is known as an effective theory in the infrared region of QCD. The perfect monopole action is transformed exactly into a lattice action of a string model. When the monopole interactions are weak as in the case of infrared SU(2) QCD, the string interactions are strong. The static potential and the string tension in this region can be estimated analytically by the use of the strong coupling expansion.

1. Introduction

We consider the following steps to obtain the renormalized trajectory as the continuum limit of lattice field theory. (I)Performing $n$ steps of block spin transformations from a-lattice to b-lattice. Here a-lattice is the lattice whose lattice constant is $a$, and b-lattice has the lattice constant $b = na$. (II)In order to go to the continuum limit keeping physical scale $b$ finite, we take the limit $n \to 0$ and $n \to \infty$ for fixed $b$. (III)One can find the perfect action and the perfect operator which reproduce the continuum value on the b-lattice.

In general, however, the above scenario is difficult to carry out. We suggest a simple but a non-trivial lattice model composed of monopole two-point interactions alone to derive analytically the renormalized trajectory and the perfect operator corresponding to a potential between static electric charges. This is similar to the blocking from the continuum theory as developed by Bietenholz and Wiese[1].

2. Perfect lattice action and operator for the monopole current theory

The renormalization group flow can be studied for arbitrary two-point interactions, but for simplicity we assume the following form of lattice monopole action.

$$S[k] = \sum_{s,s',\mu} k_\mu(s) D_0(s - s') k_\mu(s'),$$

$$D_0(s - s') = \beta \Delta^{-1}(s - s') + \alpha \delta_{s,s'} + \gamma \Delta(s - s').$$ (1)

The Wilson loop operator in the monopole current theory is given by the following operator[6–9] on the a-lattice:

$$W_m(C) = \exp\left\{2\pi i \sum_{s,\mu} N_\mu(s, S^I) k_\mu(s)\right\},$$

$$N_\mu(s, S_J) = \sum_{s'} \Delta^{-1}(s - s') \frac{1}{2} \epsilon_{\mu \alpha \beta \gamma} \partial_\alpha S^J_{\beta \gamma}(s' + \hat{\mu}),$$

$$\partial'_\beta S^J_{\beta \gamma}(s) \equiv J_\gamma(s).$$ (2)

Let us construct a blocked monopole current $K_\mu(s)$ on the coarse b-lattice satisfying the conservation law:

$$K_\mu(s) = \sum_{i,j,l=0}^{n-1} k_\mu(ns + (n - 1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma})$$

$$\equiv B_{k_\mu}(s).$$

It satisfies the conservation law $\sum_\mu \partial'_\mu K_\mu(s) = 0$.

The vacuum expectation value of the Wilson loop is

$$\langle W_m(C) \rangle = \sum_{K_\mu = -\infty}^{\infty} Z[K, J] \left/ \sum_{K_\mu = -\infty}^{\infty} Z[K, 0], \right.$$
where
\[ Z[K, J] = \sum_{k_{\mu}, k_{\nu} = -\infty}^{\infty} \exp \left\{ - \sum_{s, s', \mu} k_{\mu}(s)D_0(s - s')k_{\mu}(s') \right\} \]

\[ + 2\pi i \sum_{s, \mu} N_{\mu}(s)k_{\mu}(s) \delta \left( K_{\mu}(s^{(n)}) - B_{k_{\mu}}(s^{(n)}) \right). \]

Introducing auxiliary fields \( \phi \) and \( \gamma \), we rewrite the constraints \( \partial_{\mu}k_{\mu} = 0 \) and \( K_{\mu}(s^{(n)}) = B_{k_{\mu}}(s^{(n)}) \). Then we change the integral region of \( \gamma \) and \( \phi \) from the first Brillouin zone to the infinite region, since the monopole currents take integer values. Making use of the Poisson sum rule and recovering dimensional lattice constants \( a \) and \( b = na \), we carry out explicitly integrals with respect to \( F, \phi \), and \( \gamma \) and take the continuum limit \( (a \to 0, n \to \infty) \), for fixed \( b = na \). We obtain the perfect action and the perfect operator

\[
\langle W_m(C) \rangle = \exp \{ S_1[N] + S_2[B] \} \sum_{b^3K_{\mu}(s) = -\infty}^{b^3K_{\mu}(s) = -\infty} \exp \{ S_3[K] \}
\]

where \( S_1[N] \) is an interaction term between surface source \( N_{\mu}(x) \) in the continuum space time

\[
- \pi^2 \int_{-\infty}^{\infty} d^4d^3y \sum_{\mu} N_{\mu}(x)D_0^{-1}(x - y)N_{\mu}(y).
\]

\( S_2[B] \) is an interaction term between the surface source \( B_{\mu}(s) \) on the coarse lattice:

\[
\pi^2 b^8 \sum_{s, s', \mu, \nu} B_{\mu}(bs)D_{\mu\nu}(bs - bs')B_{\nu}(bs'),
\]

\[
B_{\mu}(bs^{(n)}) \equiv \lim_{n \to \infty} a^8 \sum_{s, s'} \Pi_{-\nu}(bs^{(n)} - as)
\]

\[ \times \left\{ \delta_{\mu\nu} - \frac{\partial_{\mu}\partial_{\nu}'}{\sum_{\rho} \partial_{\rho}\partial_{\rho}'} \right\} D_0^{-1}(as - as')N_{\nu}(as'). \]

Here \( \Pi_{-\mu}(nas^{(n)} - as) \) is defined as follows:

\[ \Pi_{-\mu}(nas^{(n)} - as) \equiv \frac{1}{n^3} \sum_i \delta \left( nas_i^{(n)} + (n - 1)a - as_{\mu} \right) \times \prod_{i \neq \mu} \left( \delta \left( nas_i^{(n)} + Ia - as_i \right) \right). \]

\( S_3[K] \) is a perfect monopole action on the coarse lattice

\[
- b^8 \sum_{s, s', \mu, \nu} K_{\mu}(bs)D_{\mu\nu}(bs - bs')K_{\nu}(bs').
\]

\( S_4[K, B] \) is an interaction term between the monopole current and the modified surface source on the b-lattice:

\[
2\pi ib^8 \sum_{s, s', \mu, \nu} B_{\mu}(bs)D_{\mu\nu}(bs - bs')K_{\nu}(bs').
\]

\( D_{\mu\nu}(bs^{(n)} - bs^{(n)'}) \) is the after gauge fixing inverse of the following operator:

\[
\left. \left\{ \delta_{\mu\nu} - \frac{\partial_{\mu}\partial_{\nu}'}{\sum_{\rho} \partial_{\rho}\partial_{\rho}'} \right\} D_0^{-1}(as - as'). \right.
\]

3. String representation of the monopole action

The quadratic monopole action can be transformed exactly into that of the string model using the BKT transformation and the dual transformation \([10, 13]\):

\[
\langle W_m(C) \rangle = \exp \{ S_1[N] \}
\]

\[
\times \sum_{\sigma_{\mu\nu}(s) = \infty}^{\text{finite}} \exp \{ S_5[\sigma] \} \left[ \sum_{\delta_{\mu\nu}(s) = \infty}^{\text{finite}} \exp \{ S_6[\sigma, B] \} \right].
\]

Here \( S_1[N] \) is the same as in Eq.\((1)\). \( S_5[\sigma] \) is a perfect action written by string variables \( \sigma_{\mu\nu}(s) \):

\[
- \pi^2 \sum_{s, s', \mu, \nu} \sum_{\sigma_{\alpha\beta}(s) \neq 0} \sigma_{\alpha\beta}(s)H_{\mu\alpha\nu\beta}(s - s')\sigma_{\nu\beta}(s'),
\]

where \( H_{\mu\alpha\nu\beta}(s - s') \) is

\[
\delta_{\alpha\beta} \sum_{s_1} D_{\mu\nu}^{-1}(s - s_1)\Delta_{s_1}^{-2}(s_1 - s').
\]
$S_b[\sigma, B]$ is the interaction term between $\sigma_{\mu\nu}(s)$ and $B_{\mu}(s)$:

$$-2\pi^2 \sum_{s,s'} \sum_{\mu,\nu} \sigma_{\mu\nu}(s) \partial^s_{\mu} A^{-1}_L(s-s') B_{\nu}(s').$$

Let us assume that the monopole action on the dual lattice is in the weak coupling region for large $b$ as realized in the infrared region of pure $SU(2)$ QCD. Then the string model on the original lattice is in the strong coupling region. The strong coupling expansion on the lattice can be performed easily and quantum fluctuations terms which include more plaquettes become small\cite{5,6}.

Then the vacuum expectation value for the Wilson loop is evaluated easily by using only the classical part of the above equation, that is, \exp \{ S_1[N] \}.

4. The rotational invariance

The static potential and the string tension can be calculated analytically. The plaquette variable $S_{\alpha\beta}$ in Eq.\ref{3} for the static potential $V(bI, 0, 0)$ is expressed by

$$S_{\alpha\beta}(z) = \delta_{\alpha 1} \delta_{\beta 4} \delta(2z) \delta(z_3) \theta(Ib - z_1) \theta(z_4) \theta(Tb - z_4).$$

Also the variable $S_{\alpha\beta}$ for the static potential $V(bI, I, b, 0)$ is given by

$$S_{\alpha\beta}(z) = (\delta_{\alpha 1} \delta_{\beta 4} + \delta_{\alpha 2} \delta_{\beta 3}) \delta(z_3) \theta(Tb - z_4) \times \theta(z_1) \theta(Ib - z_1) \theta(z_2) \theta(Ib - z_2) \delta(z_1 - z_2).$$

Then the static potentials $V(Ib, 0, 0)$ and $V(0b, Ib, 0)$ can be written as

$$V(Ib, 0, 0) = \frac{\pi \kappa Ib}{2} \ln \frac{m_1}{m_2},$$

$$V(0b, Ib, 0) = \frac{\sqrt{2}\pi \kappa Ib}{2} \ln \frac{m_1}{m_2},$$

where $m_1$ and $m_2$ are expressed by the original couplings in Eq.\ref{4} as $\kappa (m_1^2 - m_2^2) = \frac{1}{\gamma}$, $m_1^2 + m_2^2 = \frac{\alpha}{\gamma}$ and $m_1^2 m_2^2 = \frac{\beta}{\gamma}$. The potential takes only the linear form and the rotational invariance is recovered completely even for the nearest $I = 1$ sites. The string tension is evaluated as $\sigma = \pi \kappa \ln(m_1/m_2)/2$. This is consistent with the analytical results\cite{4} in the Type-2 superconductor. The two constants $m_1$ and $m_2$ may be regarded as the coherence and the penetration lengths.

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