On cosmologically induced hierarchies in string theory

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Abstract

We propose, within a perturbative string theory example, a cosmological way to generate a large hierarchy between the observed Planck mass and the fundamental string scale. Time evolution results in three large space dimensions, one additional dimension transverse to our world and five small internal dimensions with a very slow time evolution. The evolution of the string coupling and internal space generate a large Planck mass. However due to an exact compensation between the time evolution of the internal space and that of the string coupling, the gauge and Yukawa couplings on our Universe are time independent.
1 Introduction

Long time ago, Dirac initiated an ambitious program to relate very large numbers to the age of the universe [1]. This had as an immediate consequence, the variation in time of the fundamental constants on which stringent experimental limits exist [2], ruling out many of the proposed models including the simple model Dirac had himself proposed. In addition to the well-known hierarchy between the Planck scale on one hand and the electroweak scale, and the cosmological constant on the other, string theory faces another similar problem which amounts to the explanation of the smallness of the extra dimensions.

Here, we report on a cosmological solution, obtained within an effective field theory approximation, which allows to obtain, à la Dirac, a large hierarchy between the Planck scale and the string scale as well as three large space dimensions, a small compact five dimensional space and one large dimension transverse to our world. The model has the additional desirable feature of having a perturbative string coupling while maintaining a Yang-Mills coupling of order one. The model is to be thought of as a toy model allowing to obtain initial conditions in the primordial universe with the required hierarchies. So, as inflation models, it is not intended to be realistic until the present era. We do not address the difficult problem of explaining why the hierarchies are maintained after this era. Neither do we explain why we started with the particular string model with the given supersymmetry breaking we consider.

This Letter is organised as follows. In Section 2 we present the string model as well as the cosmological background, obtained by compactifying to four dimensions the solution found in [3]. Section 3 is devoted to the evaluation of the various scales and couplings as functions of time. Section 4 discusses briefly the stability of the model and also shows that scalar fields describing the position of branes in our cosmological background get scalar potentials becoming naturally flat by the time evolution and are therefore potential candidates for quintessential models of dark energy.

2 A cosmological solution in string theory

We are considering here a class of orientifold string models [4, 5] containing D8 branes and non-dynamical orientifold O8 planes, whose specific charges will be discussed in two explicit models at the end of this section. Supersymmetry is mainly broken on the D8/O8 system [6], while in the closed (bulk) sector is either exactly supersymmetric or has softly broken supersymmetry in the large radius limit [7]. The corresponding bosonic effective action is

\[
S = \frac{M_8^8}{2} \int d^{10}x \sqrt{-g} \left[ e^{-2\Phi} (R + 4(\partial\Phi)^2) - \frac{1}{2 \times 10!} F_{10}^2 \right] \\
- \int_{y=0} d^9x (T_0 \sqrt{-\gamma} e^{-\Phi} + q_0 A_9) - \int_{y=\pi R} d^9x \sqrt{-\gamma} \left[ e^{-\Phi} (T_1 + \frac{1}{g^2} tr F^2) + q_1 A_9 \right], \tag{1}
\]
where $M_s$ is the string scale, $\Phi$ is the dilaton, $A_9$ the RR nine-form coupled to D8 branes and O8 planes, $\gamma$ is the induced metric and $(1/g^2) \sim M_5^5$ is related to the 9d Yang-Mills dimensionful coupling. A certain number of D8 branes of NS-NS and RR charges $(T_0, q_0)$ were placed at the origin $y = 0$ and the rest of them, containing gauge fields $F$, were placed at $y = \pi R$ of a compact coordinate $y$ of radius $R$. An important fact for the following is that whereas the model has no Ramond-Ramond tadpoles by consistency reasons, it has dilaton tadpoles [6]. In the generic case

$$T_0^2 \geq q_0^2, \quad T_1^2 \geq q_1^2,$$

it was shown in [3] that the field equations of (1) result in a space-time metric and string coupling

$$ds^2 = \left[ G_0 + \frac{3|q_0|\kappa^2}{2\lambda} e^{\lambda \gamma(y)} \right]^{-\frac{1}{2}} \left[ \delta_{\mu\nu} dx^\mu dx^\nu + e^{2\lambda}( - dt^2 + dy^2 ) \right],$$

$$e^\Phi = \left[ G_0 + \frac{3|q_0|\kappa^2}{2\lambda} e^{\lambda \gamma(y)} \right]^{-\frac{1}{6}},$$

whereas the Einstein frame metric $ds^2 = \exp(\Phi/2)ds^2_E$ is

$$ds^2_E = \left[ G_0 + \frac{3|q_0|\kappa^2}{2\lambda} e^{\lambda \gamma(y)} \right]^{\frac{1}{12}} \left[ \delta_{\mu\nu} dx^\mu dx^\nu + e^{2\lambda}( - dt^2 + dy^2 ) \right].$$

Here $G_0$ is an integration constant determining the string coupling constant at the origin, $\kappa^2 = 1/M_8^8$ and $\lambda$ is determined below. The solution (4) is nonsingular in the compact $y$ coordinate and has big-bang type singularities at $t = \pm \infty$, separated by an infinite proper time. Notice that the string coupling is finite in the whole spacetime, including at the big-bang singularities, rendering such solutions attractive string frameworks for studying the pre-big-bang [9] and the epikyrotic scenarios. The boundary conditions at the position of the branes

$$T_0 = q_0, \quad \frac{T_1}{ch(\pi \lambda R)} = q_1$$

(5)

determine the parameter $\lambda$.

By making the change of variables

$$X_0 = \frac{1}{\lambda} e^{\lambda t} ch(\lambda y), \quad X = \frac{1}{\lambda} e^{\lambda t} sh(\lambda y),$$

(6)

we get the spacetime metric

$$ds^2 = \left[ G_0 + \frac{3|q_0|\kappa^2}{2} |X| \right]^{-\frac{1}{2}} \left[ \delta_{\mu\nu} dx^\mu dx^\nu - dX_0^2 + dX^2 \right],$$

(7)

when $y > 0$. The $Z_2$ identification $y \rightarrow -y$ is mapped in terms of the coordinates $(X_0, X)$ to the parity $\Pi_X$. This means that the orientifold operation acts in the $(X_0, X)$ plane as $\Omega' = \Omega \Pi_X$.  

\footnote{For previous classical solutions of nonsupersymmetric strings, see e.g. [8].}
In addition, the identification of points on the circle \( y = y + 2\pi R \) results in \((X_0, X)\) coordinates in the orbifold identification

\[
\begin{pmatrix} X_0 \\ X \end{pmatrix} \rightarrow \begin{pmatrix} ch(2\pi \lambda R) & sh(2\pi \lambda R) \\ sh(2\pi \lambda R) & ch(2\pi \lambda R) \end{pmatrix} \begin{pmatrix} X_0 \\ X \end{pmatrix},
\]

which is nothing but a two-dimensional boost \( \mathcal{K}_{2\pi \lambda R} \) with a velocity \( V = \text{th}(2\pi \lambda R) \) in the \((X_0, X)\) space\(^4\).

Notice that (7) coincides with the spacetime metric obtained in \([12]\) in the supersymmetric Type I’ string with \( N \) D8 branes at the origin \( X = 0 \) of a compact coordinate of radius \( R \) and \( 32 - N \) D8 branes at \( X = \pi R \).

The metric (4) and the identifications \( \Omega' \) and (8) allow a simple physical interpretation of our configuration in the \((X_0, X)\) coordinates. Indeed, the fixed points of the two orientifold operations are

\[
\Omega' : X = 0 \quad \text{and} \quad \Omega' \, \mathcal{K}_{2\pi \lambda R} : X = \text{th}(\pi \lambda R) \, X_0.
\]

Consequently, the branes (and orientifolds) located at the origin stay at the origin \( X = 0 \) in the static background (7), whereas the branes and the orientifold planes at \( y = \pi R \) move at a constant velocity \( v_1 = \text{th} (\pi \lambda R) \). Moreover, the boundary conditions (5) encode the dynamics of the two boundaries in the form

\[
T_0 = q_0 \quad \text{and} \quad T_1 \sqrt{1 - v_1^2} = q_1,
\]

whose interpretation is quite obvious, since \( T_1 \sqrt{1 - v_1^2} \) is the boosted tension of the branes/O-planes moving with the velocity \( v_1 \) in the static background (7). The RR charge conservation implies therefore the boosted version of the NS-NS tadpole conditions

\[
T_0 \sqrt{1 - v_0^2} + T_1 \sqrt{1 - v_1^2} = 0,
\]

where in our case \( v_0 = 0 \).

These time-dependent solutions can be used to find four-dimensional cosmological solutions if we consider that all dimensions, except the time and three noncompact space ones parallel to the branes, are compact. For this purpose we consider five space dimensions to span a five-torus and toroidally compactify our solution in order to have D3-O3 systems propagating into a five dimensional bulk space. We make the split \( M = (\alpha, m) \), where \( \alpha = 0 \cdots 4 \) are the noncompact spacetime coordinates plus the \( X \) coordinate and \( m = 5 \cdots 9 \) are compact coordinates parallel to the 8-branes/orientifold planes.

Compactification from 10d to 5d in the string frame asks for the relations

\[
g^{(10)}_{\alpha\beta} = \delta_{\alpha\beta} \quad \text{and} \quad g^{(5)}_{\alpha\beta} = g^{(5)}_{\alpha\beta},
\]

\(^4\)For the quantization of string models on lorentzian orbifolds, see e.g. [11].
where $V_5 \equiv v_5 \exp(5\tilde{\sigma})$ is the volume of the internal five-torus with $v_5 \equiv r_5^2$ the constant volume parameter and $\tilde{\sigma}$ is the breathing mode of the five-torus. In our case

$$\langle V_5 \rangle = v_5 \ e^{5<\tilde{\sigma}>} = v_5 \left[G_0 + \frac{3|q_0|\kappa^2}{2}|X| \right]^{-\frac{5}{2}},$$

where the five-dimensional metric takes the form

$$ds_5^2 = \left[G_0 + \frac{3|q_0|\kappa^2}{2}|X| \right]^{-\frac{5}{2}} \left[\delta_{\mu\nu}dx^\mu dx^\nu - dX_0^2 + dX^2 \right].$$

It is useful for later purposes to display also the results in the 5d Einstein frame. This can be achieved by the Weyl rescalings

$$V_5 = e^{\frac{5\Phi}{10}} V_{E,5}, \quad g_{\alpha\beta}^{(5)} = e^{\frac{5\Phi}{10}} g_{E,\alpha\beta}^{(5)}. \quad (15)$$

In the Einstein frame

$$\langle V_{E,5} \rangle = v_5 \left[G_0 + \frac{3|q_0|\kappa^2}{2}|X| \right]^{\frac{5}{2}},$$

and the five-dimensional metric takes the form

$$ds_{E,5}^2 = \left[G_0 + \frac{3|q_0|\kappa^2}{2}|X| \right]^{\frac{5}{2}} \left[\delta_{\mu\nu}dx^\mu dx^\nu - dX_0^2 + dX^2 \right].$$

This five-dimensional Einstein background is a classical solution of the five-dimensional lagrangian

$$S_5 = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[R^{(5)} - \frac{1}{2}(\partial\Phi)^2 - \frac{40}{3}(\partial\sigma)^2 - \frac{1}{2 \times 5!} e^{\frac{5\Phi}{10} - \frac{10\sigma}{3}} F_5^2 \right]$$

$$- \int_{X=0} d^4x \left[\sqrt{-\gamma} T_0 \ e^{\frac{5\Phi}{10} - \frac{5\sigma}{3}} + q_0 \ A_4 + \cdots \right]$$

$$- \int_{X=v_1 T} d^4x \left[\sqrt{-\gamma} \left(T_1 e^{\frac{5\Phi}{10} - \frac{5\sigma}{3}} + v_5 \frac{g}{\kappa^2} e^{\frac{5\Phi}{10} + 5\sigma} \text{tr}F^2 \right) + q_1 \ A_4 + \cdots \right],$$

(18)

where $\sigma = \tilde{\sigma} - \Phi/4$ and $(1/\kappa_5^2) = v_5 M_8^4$ is obtained by a straightforward dimensional reduction from the original 10d/9d one. In (13), $T_{0,1}$ $(q_{0,1})$ denote now the D3 branes tensions (RR charges) on the two boundaries and $\cdots$ denote contributions of the internal components of the gauge fields $F$, as well as the fields describing the fluctuations of the branes.

If the time evolution drives the internal five-space to a size smaller than the string scale, then we must perform T-duality in (13), (14) along the internal five-torus. The T-dual five volume reads $V_5' = 1/(V_5 M_8^{10})$ and the T-dual string coupling $\exp(\Phi') = \exp(\Phi)/(V_5 M_8^4)$ is constant. The T-dual solution describes smeared D3 branes along the five-torus. If the $X$ coordinate is much larger than the other five internal ones, there is a five dimensional T-dual lagrangian describing this solution, which reads

$$S_5 = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[R^{(5)} - \frac{1}{2}(\partial\Phi')^2 - \frac{40}{3}(\partial\sigma')^2 - \frac{1}{2 \times 5!} e^{\frac{40\Phi'}{3} - \frac{20\sigma'}{3}} F_5^2 \right]$$

$$- \int_{X=0} d^4x \left[\sqrt{-\gamma} T_0 \ e^{-\frac{20\sigma'}{3}} + q_0 \ A_4 + \cdots \right]$$

$$- \int_{X=v_1 T} d^4x \left[\sqrt{-\gamma} \left(T_1 e^{-\frac{20\sigma'}{3}} + v_5 \frac{g}{\kappa^2} e^{-\frac{40\Phi'}{3}} \text{tr}F^2 \right) + q_1 \ A_4 + \cdots \right].$$

(19)
The $T$-dual fields $\Phi', \sigma'$ in the 5d Einstein frame are related to the ones before $T$-duality by $(\Phi', \sigma') = (-\Phi/4 - 5\sigma, (\sigma/4) - 3(\Phi/16))$.

### 2.1 Explicit string examples

Our first example is the model discussed in [3], which is an orientifold [4,5] of Type II strings on $M^9 \times S^1$ with broken supersymmetry [7]. The explicit construction starts from a freely-acting orbifold $g$ in the closed sector of the type II string. After a radius redefinition, the orbifold $g$ becomes a periodic identification $y = y + 2\pi R$ accompanied by the spacetime fermion number operation, imposing different boundary conditions for bosons and fermions and breaking therefore the supersymmetry. The orientifold operations $\Omega' = \Omega \Pi_y (-1)^{F_L}$ and $\Omega' g$ create O-planes of two different types. The fixed plane of $\Omega'$ sits at the origin $y = 0$ and is a standard $O_+$ plane, whereas the fixed plane of $\Omega' g$ sits at $y = \pi R$ and is an antiorientifold plane, due to the action of $(-1)^F$. More precisely, it is an $O_-$ plane due to the simultaneous action of $(-1)^F$ and $(-1)^{F_L}$ operations. The $O_+ - O_-$ has the additional interesting and important (for our purposes) property to eliminate the would be closed string tachyon, which is odd under the world-sheet fermion number operator $(-1)^{F_L}$. The model contains by consistency also 32 D8 branes and therefore no open string tachyons exist. The partition function for this non-tachyonic model and the standard consistency checks were displayed in the Appendix of [3]. The conditions (2) are satisfied in this explicit example\(^5\), since

$$\begin{align*}
T_0 = q_0 &= (N - 16) T_8, & T_1 &= (48 - N) T_8, & q_1 &= (16 - N) T_8.
\end{align*}$$

The second example starts with an orientifold of Type IIB superstring by $\Omega' = \Omega \delta$, where $\delta y = y + \pi R$ is a shift by half of the $S^1$ circle described by $y$. After T-duality on $y$ the shift operation generates $O8_+$ planes at $y = 0$ and $O8_-$ planes at $y = \pi R$. The total RR charge being zero, the orientifold does not ask for consistency any addition of D8 branes and is a well known example of orientifolds without open strings [13]. Let us now add, consistently with RR tadpole cancellation, $N$ D8 branes at $y = 0$ and an equal number $M = N$ of anti D8 branes at $y = \pi R$. In the large radius limit the would be open string tachyons stretched between branes and antibranes are very massive and do not play an important role in the dynamics of the system. Supersymmetry is broken at the string scale at $y = \pi R$, whereas at the lowest order it is exact in the bulk. There is an overall dilaton tadpole and the bosonic effective action is again of the form [11], with the obvious replacement $32 - N \rightarrow M$ for the $y = \pi R$ localized action. The various tensions and charges are in this case

$$\begin{align*}
T_0 = q_0 &= (N - 16) T_8, & T_1 &= (N + 16) T_8, & q_1 &= (16 - N) T_8.
\end{align*}$$

\(^5\)At the one-loop level a cosmological constant $\Lambda_1$ is generated in the bulk. In the large radius limit $RM_s >> 1$, relevant for our cosmological solution, $\Lambda_1$ is however parametrically very small and can be neglected.
If the number of brane-antibrane pairs is \( N < 16 \), then the conditions (2) are again fulfilled and the classical solution (3) is valid.

3 Cosmological hierarchies

One of the boundaries of spacetime in the cosmological solution (14), which will be identified in the following with our brane universe, is moving with a constant velocity \( X = v_1 T \). Again, due to the cosmological nature of our solution, it is important in the following to distinguish between the string and the Einstein frame.

3.1 String frame

By defining the proper time on our brane universe

\[
\tau = \frac{4\sqrt{1 - v_1^2}}{5v_1|q_0|^2\kappa^2} \left( G_0 + \frac{3|q_0|\kappa^2}{2}v_1X_0 \right) \frac{5}{6},
\]

the induced metric, the overall radius of the five-torus and the string coupling on our brane have a time dependence governed by

\[
\begin{align*}
\frac{ds}{4}^2 &= \frac{(5v_1|q_0|^2\kappa^2)}{4\sqrt{1 - v_1^2}} \tau^{-\frac{2}{5}}\delta_{\mu\nu}dx^\mu dx^\nu - d\tau^2, \\
R_c &= \frac{(5v_1|q_0|^2\kappa^2)}{4\sqrt{1 - v_1^2}} \tau^{-\frac{1}{5}}r_c, \\
e^\Phi &= \frac{(5v_1|q_0|^2\kappa^2)}{4\sqrt{1 - v_1^2}} \tau^{-1}.
\end{align*}
\]

In the string frame therefore the time evolution describes a contracting universe. Notice that the four-dimensional Yang-Mills coupling in (15), given by

\[
\frac{1}{g_{YM}^2} = e^{-\Phi} \frac{R_c^5}{g^2} \sim v_5,
\]

where \( v_5 \) from now on denotes the dimensionless five-volume in string units, is time-independent due to the correlation between the time variation of the string coupling and that of the internal space.

On the other hand, the four-dimensional Planck mass \( M_P \) and the effective size \( R_X \) of the \( X \) coordinate are given by

\[
\begin{align*}
M_P^2 &= v_5M_s^3 \int_0^{v_1X_0} dX \ e^{-2\Phi(G_0 + \frac{3|q_0|\kappa^2}{2}|X|)^{-\frac{4}{5}}} = \frac{v_5M_s^3}{5|q_0|\kappa^2} \left[ (G_0 + \frac{3|q_0|\kappa^2}{2}v_1X_0)^{\frac{4}{5}} - G_0^4 \right], \\
R_X &= \int_0^{v_1X_0} dX \ (G_0 + \frac{3|q_0|\kappa^2}{2}|X|)^{-\frac{4}{5}} = \frac{4}{5|q_0|\kappa^2} \left[ (G_0 + \frac{3|q_0|\kappa^2}{2}v_1X_0)^{\frac{4}{5}} - G_0^4 \right].
\end{align*}
\]

As the internal five-torus shrinks, there are two possibilities which must be discussed separately:
i) $r_c >> M_s^{-1}$. In this case, the internal volume starts from large values and shrinks. We will discuss the hierarchy driven by the evolution until the internal radii become of the order the fundamental string length. In this case, the gauge couplings are tiny and time independent. This case corresponds to the scenario proposed in [14].

ii) $r_c \sim M_s^{-1}$. In this case, T-dualities along the five-torus coordinates must be performed. The T-dual time dependent solution corresponds to an expanding internal five-torus perpendicular to the D3 branes. The T-dual string coupling $\exp(\Phi') = 1/v_5$ is of order one and fixes also the gauge couplings $(1/g^2_{YM}) \sim \exp(-\Phi')$. Tree-level Yukawa couplings and one-loop generated masses are also time independent, even if the internal moduli are time dependent.

### 3.2 Einstein frame

In the Einstein frame, the proper time on our brane universe is defined by

$$\tau_E = \frac{3\sqrt{1 - v_1^2}}{5v_1|q_0|\kappa^2} \left( G_0 + \frac{3|q_0|\kappa^2}{2v_1X_0} \right)^{\frac{10}{9}}, \quad (26)$$

whereas the induced metric, the overall radius of the five-torus and the string coupling on our brane have a time-dependence governed by

$$\begin{align*}
\mathrm{d}s_4^2 &= \left( \frac{5v_1|q_0|\kappa^2}{3\sqrt{1 - v_1^2}} \tau_E \right)^\frac{4}{3} \delta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}\tau_E^2, \\
R_c &= \left( \frac{5v_1|q_0|\kappa^2}{3\sqrt{1 - v_1^2}} \tau_E \right)^\frac{4}{3} r_c, \\
e^\phi &= \left( \frac{5v_1|q_0|\kappa^2}{3\sqrt{1 - v_1^2}} \tau_E \right)^{-\frac{4}{3}}. \quad (27)
\end{align*}$$

The spacetime (27) describes an expanding FRW universe with an equation of state and Hubble expansion parameter given by

$$p = \frac{17}{3} \rho, \quad H = \frac{\dot{a}}{a} = \frac{1}{10\tau_E}. \quad (28)$$

Notice that the four-dimensional Yang-Mills coupling is again time independent, since it is invariant under Weyl rescalings. The effective size $R_X$ of the X coordinate in the Einstein frame is

$$R_{E,X} = \int_0^{v_1X_0} \mathrm{d}X \left( G_0 + \frac{3|q_0|\kappa^2}{2}|X| \right)^{\frac{1}{2}} = \frac{3}{5|q_0|\kappa^2} \left[ \left( G_0 + \frac{3|q_0|\kappa^2}{2v_1X_0} \right)^{\frac{10}{9}} - G_0^{\frac{10}{9}} \right]. \quad (29)$$

Let us now assume that the time evolution is valid for a large cosmological time scale. At that time, some unknown dynamics must take over which stabilises the dilaton and the two relevant moduli fields $\sigma, g_{55}$ and produces a late time acceleration of the universe. Our main assumption is that this dynamics do not change in a significant way the hierarchies induced by the previous large cosmological evolution we are putting forward here. For computing the string coupling and the Planck mass we use the string frame, while we are careful in order
to distinguish between the string frame radius $R_X$ and the Einstein frame one, related by the Weyl rescaling (15). Since $\sigma$ and $\Phi$ are actually related by their $X$-dependence in (3) and (13) we can in an effective way relate the two length scales by powers of the string coupling as $R_X \sim \exp(\Phi/3)R_{E,X}$. Neglecting factors of order one, the relevant equations for our purposes are then

$$
R_{E,X} \sim v_1 \tau_E, \quad R_X \sim v_1 \tau,
M_P^2 \sim v_5 (v_1 X_0 M_s)^{4\over 3} M_s^2 \sim v_5 (R_X M_s)^{8\over 5} M_s^2,
\epsilon^\Phi \sim (v_1 \tau M_s)^{-1}, \quad R_c \sim r_c (v_1 \tau M_s)^{-1\over 5}.
$$

(30)

The velocity $v_1$ appearing in our solution in our explicit string example is computed from (11), (20) and is of order one. In this case, by using $M_P \sim 10^{19} GeV$, taking as an example $R_{E,X} \simeq 10^{-1} cm$ and by also using the relation $R_c \sim \exp(\Phi/4) R_{E,c}$, we find from (30)

$$
\tau_E \sim 3 \times 10^{-12} s, \quad \epsilon^\Phi \sim 10^{-15},
M_s \sim 10^7 GeV, \quad R_{E,c} \sim 10 r_c.
$$

(31)

Notice that the quantum gravity effects appear at the 10d Planck mass $M_* = \exp(-\Phi/4) M_s \sim 10^{11} GeV$. The time evolution gives rise to a four-dimensional Universe of a mm size, a new dimension $X$ with an effective mm size as well, five very small compact dimensions with a very slow time evolution, and a string coupling becoming tiny, of order $10^{-15}$. This conclusion only applies to the case i) of the previous subsection. This scenario is similar to the one proposed in [14]. In the case ii) of the previous subsection, even if in the Einstein frame all internal coordinates are expanding, the string frame analysis tells us that we must perform T-dualities along the five-torus in order to have a reliable low-energy effective action description. After T-duality, all internal coordinates are perpendicular to the D3 branes, the string coupling and gauge and Yukawa couplings are time independent and of order one. This case gives a cosmological realisation of the large extra dimension scenario [15], which has the additional interesting feature of keeping time-independence for brane observables by a time-independent string coupling.

There is a second logical possibility we would like to consider, even if not well motivated by our string examples. Namely, at the effective field theory level we can consider the velocity $v_1$ as a continuous parameter. Then by taking a very small velocity we can accommodate a much larger time-evolution, until a time prior but close to the nucleosynthesis. In this case, the model generates three space dimensions much larger than the others, one moderately large dimension transverse to our world $R_{E,X} \sim 1 mm$ and five small internal dimensions with a very slow time evolution. The numbers corresponding to case i) for this situation are

$$
\tau_E \sim 3 \text{ mins}, \quad v_1 \sim 10^{-14}, \quad \epsilon^\Phi \sim 10^{-15},
M_s \sim 10^7 GeV, \quad R_c \sim 10 r_c.
$$

(32)
Case ii) realises cosmologically, as before, the large extra dimension scenario [15]. Very small velocities $v_1 \sim 10^{-14}$ in a string context are unnatural and hard to get, nevertheless not impossible to realise.

In the case i) and if the solution is valid until nucleosynthesis there are possible effects coming from unstable but long lived closed string oscillators, which can decay into (standard model) fields. Standard Model degrees of freedom in this case must be realized nonperturbatively, see [14]. Lifetime of closed string oscillators is qualitatively of the order of magnitude $\tau \sim \exp(-2\Phi)M_s^{-1}$. For a string coupling smaller than $10^{-24}$ or so, they could in principle be relevant for the dark matter present in the universe. If the string coupling is however larger, as it is in our case, we must on the contrary insure that they decay before nucleosynthesis. This would impose in our case $\exp(\Phi) \geq 10^{-12}$. A more detailed analysis in this case is clearly needed in order to check the validity of these crude estimates. On the other hand, if the dilaton mass is generated by string loop effects, it will be naturally small and could eventually generate deviations from the equivalence principle [16].

4 Probe brane in the time dependent geometry

One of the important questions concerning the cosmological solution found in [3] and reviewed in Section 2 is its classical stability. One aspect of this is the dynamics of the branes which we assumed to be confined on the boundaries. Namely, we must check that a small perturbation on the moduli describing the positions of the branes will remain small with time.

Let us therefore consider the dynamics of a probe brane in the static bulk metric

$$ds^2 = \left[G_0 + \frac{3\kappa^2|q_0|}{2}X\right]^{\frac{1}{12}} \left[\delta_{\mu\nu}dx^\mu dx^\nu - dX_0^2 + dX^2\right].$$

(33)

In the following we define $\Omega = \left[G_0 + \frac{3\kappa^2|q_0|}{2}X\right]$. The background dilaton and RR fields are

$$\Phi = \Omega^{-\frac{5}{6}}, \quad A_9 = -\Omega^{-\frac{2}{3}}d^9x,$$

(34)

for $X > 0$, in a spacetime with boundaries $X = 0$ and $X = v_1X_0$. A probe brane of tension and RR charge $T_8, q_8$ and of position $X = X(X_0)$ in this geometry is described by the action

$$S = -\int_{X(X_0)} d^9x \Omega^{-\frac{5}{6}} \left(T_8\sqrt{1-X^2} - q_8\right).$$

(35)

The classical field equation for the position of a probe brane is then

$$\frac{d}{dX_0}\left(\Omega^{-\frac{5}{6}} \frac{T_8\dot{X}}{\sqrt{1-X^2}}\right) = |q_0|k^2 \Omega^{-\frac{5}{6}} \left(T_8\sqrt{1-X^2} - q_8\right).$$

(36)

The hamiltonian of the system described by the lagrangian (35) is

$$H = \Omega^{-\frac{5}{6}} \left(\frac{T_8}{\sqrt{1-X^2}} - q_8\right).$$

(37)
The equations of motion (36) are valid as long as the brane is in the bulk. When the brane arrives at the boundary located at $X = 0$ it gets reflected without changing its energy. When the brane arrives at the other moving boundary it is reflected and boosted by the operation $\Pi_X K_{2\pi LR}$ so that if its velocity just before the collision is $u \equiv th(\zeta)$, it becomes just after the collision

$$u' = \frac{V - u}{1 - uV} = th(2\pi \lambda R - \zeta) . \tag{38}$$

The energy of the brane $H = E$ is conserved between two collisions with the moving boundary. The energy changes at the collision as

$$\frac{E'}{E} = \frac{T_8 \text{ch}(2\pi \lambda R - \zeta) - q_8}{T_8 \text{ch}(\zeta) - q_8} . \tag{39}$$

Notice that the energy remains invariant for $\zeta = \pi \lambda R$, i.e. if the brane moves with the same velocity as the boundary.

A BPS probe brane at rest $\dot{X} = 0$ is a solution of the equations of motion if the boundaries are static, situation that describes the supersymmetric case. In our case, it also means that D-branes on top of the zero-velocity O-planes (corresponding to O-planes at $y = 0$ in the orientifold picture) will remain at rest if their initial velocity was zero.

Solutions of the classical field equations (36) have constant energy $H = E = \text{const.} > 0$ between two collisions with the moving boundary or, equivalently,

$$\frac{T_8}{\sqrt{1 - X^2} - q_8} = \Omega_\lambda^2 E . \tag{40}$$

Equation (40) can be analytically solved. For example, in the probe brane case $T_8 = q_8$, by defining the new variable

$$z = [2 + \frac{E}{T_8} \Omega_\lambda^2]^{\frac{1}{2}} , \tag{41}$$

the solution of (40) is given by the algebraic equation

$$z^3 - 3z = \left(\frac{E}{T_8}\right)^{\frac{1}{2}} \frac{3\kappa^2 |q_0|}{2} (X_0 - X_0^{(0)}) . \tag{42}$$

The solutions with nonzero velocity satisfy the equation

$$\frac{T_8 \dot{X}}{(1 - X^2)^{\frac{1}{2}}} = |q_0| \kappa^2 \Omega^{-\frac{1}{2}} E > 0 . \tag{43}$$

The probe branes with nonzero initial velocity have a positive acceleration which will push them on top of the non-BPS system. In the orientifold picture, this means that branes at the origin $y = 0$, having zero initial velocity in the $(X_0, X)$ system, remain at the origin $y = 0$ (remain at rest in the $(X_0, X)$ system). Branes in any other point of the compact space have a net velocity and therefore will reach in a finite time the $y = \pi R$ nonsupersymmetric system. There they will be reflected and boosted and their fate will depend on the initial velocity they
had. One possibility, if their velocity is negative after the collision, is that they continue their trajectory until they reach the other boundary. The other possibility is that due to their positive acceleration they get reflected once again after some time on the moving brane. In particular, if initially the velocity was close enough to the one of the boundary \( X = v_1 X_0 \), the brane will leave the boundary for a short period of time and then due to its positive acceleration, it will collide with it again. After the second collision, we find by using (38) that the velocity becomes slightly smaller than the one of the boundary. Then it is accelerated, soon after will collide again the moving boundary and the process repeat itself. Let us denote by \( v_n \) the velocity of the probe brane immediately after the \( n^{th} \) collision, occurring at the time \( T_n \) and by \( v'_n \) the velocity immediately after the \((n+1)^{th}\) collision, occurring at the time \( T_n + \Delta T_n \). Then, if \( v_n = (1 - \eta_n) \theta h(\pi \lambda R) \) with \( \eta_n << 1 \), then between the two collisions the probe brane trajectory can be approximated with a constant acceleration trajectory provided that \( \Delta T_n << T_n \). The probe brane trajectory in this case is approximately given by

\[
X(t) \simeq \frac{1}{2} a_n (t - T_n)^2 + v_n (t - T_n) + \theta h(\pi \lambda R) T_n ,
\]

where the acceleration is given by \( a_n = \kappa^2 |q_0| (E/T_8) \Omega(T_n)^{-1/3} (ch(\pi \lambda R))^{-3} \). This intersects again the moving boundary at \( T_n + \Delta T_n \), which gives \( a_n \Delta T_n = 2 \eta_n \theta h(\pi \lambda R) \). Then it can be shown that \( v'_n = (1 + \eta_n) \theta h(\pi \lambda R) \) and that \( v_{n+1} = v_n \). This allows us also to estimate the effective time-dependence of the energy parameter \( E \) at late time \( T_n \) as \( E_n \sim T_n^{-2/3} \sim \Omega^{-2/3} \) which explain, by using (40) why the velocity of the probe brane can stay very close to the constant velocity of the boundary. By taking the limit \( \eta_n \to 0 \) we see that a probe brane on the top of the moving boundary will remain there, result that establish the stability of the initial configuration we assumed in (1). The analysis and this conclusion is also valid for probe antibranes. An interesting fact is that, once arrived on top of the moving O-planes \( X = v_1 T \), the probe brane experience a net force (non-zero acceleration) towards the O-plane and therefore undergoes oscillations around the O-plane trajectory \( X = v_1 T \). This is probably interpreted as energy radiated by the probe brane. A string theory description of these oscillations would be very interesting.

We end this letter with a comment on the possible role played by the probe brane position to late cosmology (see also [17] for a more detailed analysis of this possibility in brane-world models. For applications to inflation, see e.g. [18]). In the nonrelativistic limit \( \dot{X} << 1 \), the lagrangian (35) becomes

\[
L_p = \frac{1}{2} \dot{\chi}^2 - \frac{c}{\chi} , \quad \text{where} \quad \chi = \frac{\sqrt{T_8}}{\kappa^2 |q_0|} \Omega^2
\]

is the canonically normalized brane position and the constant \( c = (T_8 - q_8)(\sqrt{T_8}/\kappa^2 |q_0|) \) is different from zero for probe antibranes \( T_8 = -q_8 \). This inverse power-law potential is typical for scalar fields used in quintessence-type models of dark energy. In our case, this should happen at very late time and therefore after the stabilisation of the bulk moduli \( \Phi, \sigma \) and \( R_X \). Our
cosmological time evolution plays a useful role in this respect in ensuring naturally $\chi >> 1$ by time evolution and therefore providing a very flat $\chi$ scalar potential.

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