Resistive MHD modelling of the quasi-single helicity state in the KTX regimes

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Abstract
The potential formation of a quasi-single-helicity (QSH) state in the Keda Torus eXperiment (KTX) is investigated in resistive MHD simulations using the NIMROD code. We focus on the effects of finite resistivity on the mode structure and characteristics of the dominant linear and nonlinear resistive tearing-mode in a finite $\beta$, cylindrical configuration of a reversed field pinch model for KTX. In the typical resistive regimes of KTX where the Lundquist number $S = 5 \times 10^4$, the plasma transitions to a steady QSH state after evolving through an initial transient phase with multiple helicities. The dominant mode of the QSH state develops from the dominant linear tearing mode instability. In the lower $\beta$ regime, the QSH state is intermittent and short in duration; in the higher $\beta$ regime, the QSH state persists for a longer time and should be more easily observed in experiments.

Keywords: RFP, KTX, quasi-single helicity, MHD simulation, NIMROD

(Some figures may appear in colour only in the online journal)

1. Introduction

The reversed-field pinch (RFP) is a toroidal magnetic confinement device. Its main difference from tokamak is its toroidal field $B_\phi$, which is of the same order of magnitude as the poloidal field $B_\theta$ and becomes reversed near plasma edge [1]. Since the safety factor $q(r)$ in RFP is always less than one, the RFP plasma can easily become unstable to resistive-kink modes.

During the last two decades, both intermittent and stationary QSH states have been discovered in various RFP devices [2–6], including RFX-mod in Padova [7], MST at the University of Wisconsin–Madison [8], EXTRAP T2R in Stockholm [9] and RELAX in Kyoto [10]. Significant efforts within the RFP community have been devoted to the study of quasi-single helicity (QSH) states ever since [11–13]. The main characteristic of the QSH state is the formation of 3D helical flux surfaces in the plasma core, and the presence of an inner resonant dominant mode with poloidal periodicity $m = 1$ and toroidal periodicity $n_D > 3R_0/2a$ [12]. Its amplitude is typically several times larger than those of secondary modes. QSH states with either a double axis (DAx) or single helical axis (SHAx) have been found in experiments.
The appearance of the QSH state reduces magnetic fluctuations and improves the RFP confinement, especially for the SHAx state, which has an inner electron temperature transport barrier and relatively wide hot helical core [6, 15, 16]. The RFX-mod experimental statistics shows that the QSH state properties correlate with the Lundquist number \( S \), and higher plasma current leads to longer persistence (up to about 90\%) of the QSH state [6, 16, 17]. Lately, the helical magnetic perturbation from the external active coils has been used to excite QSH with specific dominant modes in RFX-mod [18].

The proposition that RFP plasma could exist in a pure single helicity SH state was put forward in 1983, based on 2D numerical simulations [19, 20]. The 2D simulations of pure (SH) states show the existence of two topologies for the corresponding magnetic surfaces: with or without a magnetic separatrix [13]. Later, numerical simulations of the RFP using 3D resistive magnetohydrodynamics codes SpeCyl [21, 22] also find MH and QSH states. Those simulations indicate that the transition from the MH to QSH state is controlled by the Hartmann number \( H \propto \frac{1}{\sqrt{\eta \nu}} \) [23], where \( \eta \) is resistivity, and \( \nu \) is viscosity. If \( H \) is large, the system is in the MH state; when \( H < 3000 \), the system displays temporal intermittency with a laminar phase of the QSH state [24, 25]. The scheme of using the helical boundary condition generated by active coils to excite and control the specific dominant modes of the QSH state in RFP has been studied using nonlinear 3D MHD simulations [18, 26–28].

In this work, we use the 3D full MHD code NIMROD [29] to evaluate the possibilities of achieving the QSH state in a newly constructed RFP device, KTX, which is a middle-sized torus, with major radius \( R_0 = 1.4 \) m, minor radius \( a = 0.4 \) m, and a designed maximum plasma current of 1 MA [30]. Currently, the plasma current is up to 205 kA and the maximum discharge length is 21 ms [31]. With 96 saddle coils installed outside of its copper shell, KTX is mainly designed to explore novel magnetic confinement regimes and advanced feedback control schemes for MHD instabilities, along with 3D physics. For the first time, our NIMROD simulations have demonstrated the potential formation of a QSH state in the RFP configuration of KTX regimes. In particular, we find the connection between the QSH state and the dominant linear tearing mode on KTX. We further investigate the condition of QSH state formation and find a new parameter dependence of QSH, which is different from previous studies.

The rest of the paper is organized as follows. In section 2, the single-fluid MHD model and a typical KTX equilibrium are described; in section 3, the linear MHD instability of KTX is analyzed including the growth rate and mode structure; in section 4, the 3D QSH state is demonstrated in a nonlinear simulation, where a saturated magnetic island is observed to form and persist. Finally, section 5 is devoted to a summary and discussion.

### 2. MHD model and equilibrium

The numerical simulations in this work are performed with a single-fluid MHD model implemented in the NIMROD code. The single-fluid MHD equations can be written as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]

\[
\rho (\frac{\partial \mathbf{u}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{u}) = \mathbf{J} \times \mathbf{B} - \nabla p + \rho \nu \nabla^2 \mathbf{u}
\]

\[
\frac{N}{(\gamma - 1)} (\frac{\partial \mathbf{T}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{T}) = -\rho \nabla \cdot \mathbf{v} - \nabla \cdot \mathbf{q}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J})
\]
∇ × B = μ₀J \quad (5)

q = -N(χ∥∇∥T + χ⊥∇⊥T) \quad (6)

where \( ρ, N, v, p, B, q, γ, η, ν, \) and \( χ∥(χ⊥) \) are the plasma mass density, number density, velocity, pressure, current density, magnetic field, heat flux, specify heat ratio, resistivity, viscosity, and parallel (perpendicular) thermal conductivity, respectively. The Lundquist number is defined as \( S = τ_A/τ_R \) with Alfvén time \( τ_A = a/V_A \) (\( V_A = B_0/\sqrt{\mu_0}\rho \)), resistivity diffusion time \( τ_R = μ_0a^2/ν \), and \( B_0 \) is the initial magnetic field strength on the magnetic axis.

In this work, we consider the low-\( \beta \) regime of KTX plasma, in contrast to the zero \( \beta \) regime considered in the SpeCyl simulations [22, 25]. Its initial configuration can be approximated as a cylindrical force-free RFP equilibrium, which can be specified using the following \( q \) profile (figure 1) [32]:

\[
q(r) = q_0[1 - 1.8748(\frac{r}{a})^2 + 0.83232(\frac{r}{a})^4].
\]

The above RFP equilibrium has been commonly adopted in previous studies [21, 33, 34]. For the KTX device, the equilibrium parameters at the magnetic axis are \( q_0 = 2a/3R_0 = 0.18 \), \( B_0 = 1.0 \) T, \( N = 1.2 \times 10^{19} \) m\(^{-3} \), \( J_{θφ} = 6.15 \times 10^3 \) MA m\(^{-2} \),
\[
\chi_r = 400 \, \text{m}^2 \, \text{s}^{-1}, \quad \chi_\perp = 1.4 \times 10^{-7} \, \text{m}^2 \, \text{s}^{-1}, \quad \text{resistivity } \eta \text{ and viscosity } \nu \text{ share the same normalized radial profile } \frac{\eta(r)}{\eta_0} = \left[1 + \left(r/a\right)^{20}\right]^2. \]

For the initial equilibrium state, the reversal parameter \( F = -0.0428 \), pinch parameter \( \Theta = 1.4905 \). In our simulations, \( 3 \times 10^{-7} < \eta_0 < 3 \times 10^{-4}, 1 \times 10^3 < S < 1 \times 10^7, \nu_0 = 1 \times 10^{-3}, \text{ and } 10^3 < H < 7 \times 10^4 \). For an arbitrary initial perturbation, the MHD equations (1)–(6) are numerically solved in the cylindrical geometry, along with the periodic boundary conditions in the axial direction and the ideal wall boundary conditions in the radial direction.

### 3. Linear MHD instability analysis

In the NIMROD code, the spatial domain of a cylinder can be represented in two different ways using numerical discretizations. One way is to use a 2D finite element mesh in a 'circular' shape for the \((r, \theta)\) plane and a finite Fourier series in the \(z\) direction (figure 2(a) of the revised manuscript). The other way is to use a 2D finite element mesh in 'rectangular' shape for the \((r, z)\) plane and a finite Fourier series in the \(\theta\) direction (figure 2(b) of the revised manuscript). Here \((r, \theta, z)\) are the cylindrical coordinates. The two numerical discretizations are referred to as the 'circular grid' and 'rectangular grid' respectively, for convenience. The NIMROD code can solve the MHD model equations using either of the two numerical discretizations of an entire cylindrical domain.

For benchmark purposes, both of the grids for cylindrical geometry, namely 'circular' and 'rectangular', are set up for the simulations (figure 2), and the results are compared to verify their correctness. For the same equilibrium physical parameters, the time evolution of the magnetic energies of the \(m = 1, n = 9\) mode from those two grids yield the same growth rate. The mode structures from the simulation results based on the two grids are shown to be consistent with each other (figure 3). In the rest of this paper, only the results from the 'circular grid' are shown to avoid redundancy.

The relationship between the growth rate \(\gamma\) and the toroidal model number \(n\) shown in figure 4(a) for a different Lundquist number \(S\), clearly indicates that the instability is located in plasma core, and the \(n = 8\) mode is the most unstable for this equilibrium. The normalized growth rate \(\gamma T_A\) as a function of \(S\) for the \(n = 8\) mode shown in figure 4(b), suggests that the instability is likely to be the resistive tearing mode in nature, since the asymptotic scaling \(\gamma T_A \propto S^{-0.5}\) obtained from our calculation, which is uniquely determined through data fitting using the least-squares method, agrees with the traditional theory for the resistive tearing mode in the small \(\Delta r\) regime [35, 36].

The characteristic perturbations and the structure of instability are further analyzed. Figure 5 shows the radial profiles of the normalized perturbation \(v_r\) and \(b_r\) along the mid-plane in the cross section of the \(n = 8\) mode for \(S = 1 \times 10^6\). Here \(v_r\) and \(b_r\) are the radial components of the perturbed velocity and magnetic field, respectively. The resonant surface locates at \(r = 0.163 \, \text{m}\), which is the location of the 1/8 resonant surface based on the \(q\) profile. Across that resonant surface, the radial profile of \(v_r\) is anti-symmetric, and that of \(b_r\) is symmetric, which is the same as the parity of the tearing-mode [37]. The 2D mode structure in the poloidal plane can be more clearly seen in figure 6, which indicates that the instability is dominated by the \(m = 1\) tearing mode.
During the instability mode evolves towards a saturated phase. Of 20 eV have been achieved, where the Lundquist number is of the order of $10^6$. In our simulations, the choice of the particular values of $\nu_0$ and $\nu_0$ is based on the Lundquist number range of both the designed phase-I and the recent initial experiment regimes of KTX. In particular, we use the parameter $\nu_0 = 6.0 \times 10^{-5} \Omega \text{m}$, $\nu_0 = 1 \times 10^{-3} \text{m}^2 \text{s}^{-1}$, density $N = 1.2 \times 10^{19} \text{m}^{-3}$, $B_0 = 1.0 \text{T}$, $H = 1.5 \times 10^3$, $\beta = 0.05$, and magnetic Lundquist number $S = 5.0 \times 10^4$ on the equilibrium magnetic axis. This would allow us to evaluate the possibility of QSH formation in the plasma regime of Lundquist number $S \sim 10^4$ and Hartmann number $H \sim 10^3$, which are more closely related to the current KTX experiments. This would also be the necessary step for investigating the scaling of QSH formation on $S$ towards higher Lundquist number regimes such as $S \sim 10^6$.

The nonlinear evolution of magnetic energy is dominated by $n = 7 - 12$ modes (figure 7(a)). At $t = 0.35$ ms, the instability mode evolves towards a saturated phase. During $t = 0.35 - 1.1$ ms, the modes $n = 7, 8, 9$ compete strongly, but at $t = 1.1$ ms, the energy of the perturbation begins to reside in the $n = 8$ mode. After $t = 2.0$ ms, with the decrease in the magnetic energy of the $n = 7$ and 9 modes, the plasma relaxes to the QSH state, which is dominated by the $n = 8$ mode. After $t = 5.6$ ms, the plasma transitions from the QSH to the MH state. It is customary to describe the width of the toroidal spectrum of $m = 1$ mode by using the $N_s$ parameter, which is defined as $N_s = [\sum_{n=1}^S(W_{1,s}/\sum_{n'=1}^S W_{1,n'})^{2}]^{-1}$, where $W_{1,n}$ is the magnetic energy of the $(m = 1, n)$ mode. A pure SH spectrum corresponds to $N_s = 1$; for the QSH state, $N_s$ may be less than 2. The time evolution of $N_s$ verifies that during those periods of QSH (as indicated from magnetic energy evolution of each in modes (figure 7(a)), the $N_s$ number is indeed less than 2 (figure 7(b)). The evolution of the $F$ and $\Theta$ parameters are shown in figures 7(c) and (d).

The emergence of QSH can be further demonstrated in the time history of the Poincaré plot in the poloidal plane (figure 8).
The formation mechanism for QSH has long been a subject of theory and experimental studies. SpeCyl code simulation indicates that the Hartmann number $H$ and helical magnetic perturbation can determine the QSH state [24–26]. Statistical results from the RFX-mod experiment indicate that plasma current and Lundquist number $S$ correlate to the presence of the QSH state. In contrast, our simulation suggests that $\beta$ is an important parameter for the formation of the QSH state in the KTX configuration and parameter regimes. The time dependence of the spectral spread $N_s$ for different $\beta$ shows clearly the correlation (figure 10). Indeed, the time for the first appearance of the QSH state, or the drop of $N_s$ value below 2, seems to increase with plasma $\beta$, as suggested in figures 7 and 10, where the appearance of the QSH state has been marked using red arrows. For lower $\beta$, the QSH state appears intermittently in a shorter persistence time; for higher $\beta$, the QSH state persists for a longer duration of time.

Figure 8. Snapshots of the Poincaré plots in the poloidal plane for the simulation case shown in figure 7: (a)–(f) correspond to $t = 0$ ms, 1.5 ms, 2.3 ms, 3.1 ms, 3.9 ms, and 6.2 ms, respectively.
of a QSH state.

5. Summary and discussion

In summary, for the first time, NIMROD simulations find the formation of a QSH state in KTX regimes with the Lundquist number $S = 5 \times 10^4$. The simulation starts from a MH state, which transitions to a QSH state when secondary modes decrease in amplitude. The simulation indicates that the dominant mode of the QSH state may develop from the linear mode that has the maximum growth rate. The plasma $\beta$ appears to be the key parameter that can substantially influence the emergence and duration of the QSH state. QSH form when $\beta$ is above a threshold and a higher $\beta$ leads to a longer duration of QSH.

An effective method to achieve and maintain the QSH state may be the auxiliary heating that enhances the electron temperature, even in regimes of low plasma current. However, in the transition between the MH phase and the QSH phase, it is not clear what mechanism suppresses the magnetic fluctuations of the secondary modes and maintains the dominant mode. In addition, how two-fluid effects and anomalous viscosity in high plasma current regimes may affect the QSH state should be also studied in future work.

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Figure 10. (a)–(e) Spectral spread in $N_e$ as a function of time for different $\beta$ with $S = 5.0 \times 10^4$ and Alfvén time $\tau_A = 6.08 \times 10^{-8}$. (f) The QSH persistence time as a function of $\beta$.

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