A Methodology for the Enhancement of the Energy Flexibility and Contingency Response of a Building through Predictive Control of Passive and Active Storage

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Abstract: Optimal management of thermal energy storage in a building is essential to provide predictable energy flexibility to a smart grid. Active technologies such as Electric Thermal Storage (ETS) can assist in building heating load management and can complement the building’s passive thermal storage capacity. The presented paper outlines a methodology that utilizes the concept of Building Energy Flexibility Index (BEFI) and shows that implementing Model Predictive Control (MPC) with dedicated thermal storage can provide predictable energy flexibility to the grid during critical times. When the utility notifies the customer 12 h before a Demand Response (DR) event, a BEFI up to 65 kW (100% reduction) can be achieved. A dynamic rate structure as the objective function is shown to be successful in reducing the peak demand, while a greater reduction in energy consumption in a 24-hour period is seen with a rate structure with a demand charge. Contingency reserve participation was also studied and strategies included reducing the zone temperature setpoint by 2°C for 3 h or using the stored thermal energy by discharging the device for 3 h. Favourable results were found for both options, where a BEFI of up to 47 kW (96%) is achieved. The proposed methodology for modeling and evaluation of control strategies is suitable for other similar convectively conditioned buildings equipped with active and passive storage.

Keywords: energy flexibility; predictive control; thermal storage; active storage; passive storage; contingency reserve; BEFI; model-based control

1. Introduction

As the global focus to decrease Greenhouse Gas (GHG) emissions continues, the demand for cleaner electricity is increasing; however, this promising development puts a strain on electric grids. Recently, utility grids have been incorporating clean renewable energy generating resources such as photovoltaics (PV) or wind turbines. However, this increase in intermittent production results in a supply side that has a variable output, creating new challenges for utilities and electricity customers.

One major challenge linked to integrating distributed renewable energy sources into utility grids is that times of high consumption from buildings rarely correspond with the power generation from these intermittent renewable sources. Periods of peak consumption from buildings (morning and evening for residential and afternoon for commercial) do not match the time of greatest solar generation, which occurs midday, as depicted by the well-known illustration of the “duck curve” [1]. Similar supply–demand mismatch issues are observed elsewhere, although for different reasons (for instance in heating dominated regions).

Demand side management and Demand Response (DR) programs are popular approaches to incentivize electricity customers—often building owner/operators—to alter their electricity demand during critical times for the grid.
The concept of energy flexibility has received significant attention in recent years. It is useful to estimate the amount of energy flexibility that a building can provide to the grid, since activation of this flexibility can help alleviate the grid’s strain when the demand approaches exceed what the grid can safely supply. The International Energy Agency Energy in Buildings and Communities Programme’s (IEA EBC) Annex 67 introduced the notion of “Energy Flexible Building”, defined by the Annex as “a building able to manage its demand and generation in accordance with local climate conditions, user needs, and grid requirements” [2].

Building control can take advantage of thermal mass to shift power consumption from one critical period to another. Different end-uses can be rescheduled before or after a specific period without adverse impact, such as a reduction of thermal comfort. We can think also of specific systems within Heating, Ventilation, and Air Conditioning (HVAC) (active thermal storage or batteries) or embedded in the building—such as heavy radiant floors—that can be used to shift energy consumption without affecting occupant comfort. When coordinating these different systems, future or expected needs, availabilities, and constraints that may depend on occupant activity schedule, weather, grid state, day in week, etc. must be considered. Model predictive control is an optimal tool to achieve this.

The most relevant application of energy flexibility is demand response during peak periods of the grid. A Building Energy Flexibility Index (BEFI) could be used to quantify potential participation of a customer for such a demand response event. A preliminary introduction and description of the concept of a BEFI with case studies can be found in two previous conference papers on the topic [3,4].

Over 99.8% (37.3 GW) of the electric power is generated by hydroelectric plants in Québec, Canada [5], and commercial buildings in this region often use electricity as their main or only energy source. An abundance of stable generation has resulted in low electricity rates and elevated fuel prices with limited gas distribution in certain areas. During very cold winter days, a heavy demand on the grid is observed and is partially due to space heating peak loads. About 9% of the province’s winter peak load is associated with heating in commercial and institutional (C & I) buildings [6]; thus, C & I buildings represent a considerable amount of the electric load in the province. Due to these mentioned issues related to peak demand, there is increasing activity in quantifying the energy flexibility of buildings and increasing participation demand response programs.

It is possible (and worthwhile) to consider optimization of the entire electric energy system, not only the grid. This electric system spans from the generation units to the final customer end-uses. Buildings are part of this system as end-uses and can contribute to the energy flexibility of the electric system. Building science and advanced controls must be applied to help make this energy flexibility available to the grid. This study incorporates the concept of an energy flexible building and the use of Model Predictive Control (MPC) for buildings in cold climate regions with thermal storage as a dispatchable storage medium.

A notable feature of some Québec commercial customer rates is that each monthly bill throughout the year can be affected by the building’s highest winter demand. The minimum monthly electric billing demand is set at 65% of the peak power reported during the winter months. Thus, appropriate consideration should be given to the demand management strategies of a building during winter since it can affect the bills over the year.

Other fields such as electrical engineering and chemical processing have a well-established MPC domain for control techniques [7], while MPC has been shown to be a promising strategy for improved operation in buildings [8–10]. It has yet to be adopted for widespread implementation in real buildings, though it has received rising attention in buildings research.

MPC is a multi-variable control method that incorporates several additional aspects with the controller: a dynamic model of the system to be controlled, forecasts of future disturbances such as weather, a cost function that is minimized over a prediction horizon, and sometimes a history of past control sequences. The basis of MPC in buildings is
that better control of the building energy systems and planning of resources is possible due to insights into forecast weather, expected occupancy, and the thermal response of the building. For example, better management could be achieved through MPC of thermal storage capabilities through better planning of when to store or use the stored thermal energy.

Optimization of building control sequences can become complicated due to the number of variables and constraints that considered. The construction of a suitable building control model is a central step for successful MPC. As each thermal model is often designed for a specific building, the level of effort needed to create a model can be significant and difficult to determine in advance. Expert engineering judgment and heat transfer knowledge are paramount for the development of adequate building thermal models for MPC, such as an understanding of what physical details are important to include or omit in the model.

Several modeling software are appropriate for studying MPC in buildings. Some advantages and disadvantages have been discussed by [11], where they compare Modelica and MATLAB. However, one missing piece in the area of MPC and building controls is a practical way to visualize and quantify the effects that advanced controls have on building operation and energy use, which hampers efficient transfer of research developments to a wider audience.

This study aims to evaluate the energy flexibility potential of a zone equipped with a dedicated thermal storage device. One aim is to match stored thermal energy in a dedicated storage device to the zone heating energy demand for the following day. To achieve this, an estimate of the heat requirements of the conditioned zone for the following day are needed and optimal controls through model predictive control (MPC) are evaluated. Prediction uncertainties due to model parameter identification and weather forecasts are also considered.

2. Literature Review—Enhanced Flexibility Through Optimal Control

Building load flexibility can be described as a building’s capability to modify its energy demand at a specific time by postponing or shifting consumption when compared to a reference scenario (business as usual). Incorporating the principles of building energy flexibility together with on-site energy storage devices and advanced or optimized control strategies is essential for optimizing energy consumption and matching demand with the availability of energy from the grid at critical times [12,13]. In recent years, various international studies on the quantification and utilization of building energy flexibility have been conducted, and there is now a breadth of published work in this relatively newly defined area [2].

The adjustability or flexibility of a building’s heating and/or cooling system has been the topic of many studies, some of which will be described in the following paragraphs. In general, these studies studied the impact of Demand Side Management (DSM) strategies at the building level or at the energy infrastructure level.

Several studies have looked at the energy flexibility potential of a single building. Le Dréau and Heiselberg [14] assessed the potential of residential buildings to regulate heating power and to identify strategies to capitalize on flexibility potential. Hurtado et al. [15] proposed a method to quantify the demand flexibility potential of individual buildings. They assessed the effects on demand flexibility from weather fluctuations, construction variations, and comfort consideration. Other researchers have concluded that specific technologies and parametric optimizations are needed to maximize energy flexibility in residential buildings, such as Weiß et al. [16], who found that, for buildings in Austria built after 1980, 50% of peak loads from space heating can be shifted to off-peak times.

Other studies have considered a group of buildings together and the potential aggregated energy flexibility. Vigna et al. [17] expanded the study of energy flexibility on a single building to a cluster of buildings. This approach capitalizes on the disparities in energy consumption patterns between different building types and management of load shifting. Foteinaki et al. [18] investigated the potential for flexibility of low-energy build-
ings and analyzed the thermal storage capacity present in the structural mass. The study showed that low-energy buildings can remain autonomous for several hours and that, when many buildings are aggregated together—rather than a standalone building—the flexibility becomes considerable.

Effective advanced control strategies harness the thermal inertia inherently present in the structural components of a building and coordinate the operation of different systems such as thermal storage, electrical storage, on-site renewables, and heat pumps [19–21]. The fluctuations in weather and occupancy directly affect the operation of a building, which can result in significant load variations between daytime and nighttime and thus large demand variations. To manage these fluctuations, proper energy management and solid knowledge of the dynamic behaviour of buildings is of the utmost importance.

2.1. Improving Energy Flexibility in Buildings

Energy flexible buildings, through smart DSM, smart DR, and the incorporation of efficient energy storage, are one of the most encouraging prospects to roll-out storage and renewable technologies on a large scale in existing electricity networks.

From field study data, Aduda et al. [22] demonstrated that office buildings can adequately provide power flexibility for the grid. However, estimation of the demand-side flexibility potential may be difficult due to variations in indoor air quality and thermal comfort in different zones within a building.

In a study by Sánchez Ramos et al. [23], demand management measures were analyzed by using the buildings' thermal mass as thermal storage. Cost savings of 3.2% for heating and 8.5% for cooling were observed after the implementation of new control strategies for demand management. Twice the amount of savings were seen when a building was previously refurbished. They concluded that low installation costs make these measures viable if regional electric pricing and user behaviour allow for adequate flexibility.

Perez et al. [24] investigated the ability of individual residential customers to lower peak demand at the community level. Their proposed integrated control and scheduling was found to minimize the peak demand for the neighborhood by taking advantage of individual preferences and physical differences between houses (i.e., insulation values, window type, how many floors, orientation, etc). A reduction of one-quarter of the daily peak load was achieved for a group of houses when compared with the reference case of individually controlled thermostats and appliances.

Sharma et al. [25] presented a model for optimal energy management of a residential building. They proposed centralized energy management system (CEMS), where optimized decisions were determined by considering realistic model parameter settings and customer preferences. The proposed CEMS reduced both the energy cost and energy consumption of the customers over a day.

Patteeuw et al. [26] investigated how the participation of residential heat pumps in load shifting could help reduce operational costs and CO$_2$ emissions and how homeowners with heat pumps could be motivated to achieve benefits from flexibility measures. Operation during dynamic time-of-use (TOU) pricing performed better; however, dynamic TOU showed poorer outcomes at high levels of residential heat pump penetration.

2.2. Predictive Building Load Management and Model Predictive Control (MPC)

Model predictive control and its potential to improve the operation of buildings has been abundantly illustrated over the last couple decades [27–32]. Often, MPC studies have concentrated on the operation of dedicated energy storage, mostly focusing on cooling applications and typically subjected to TOU rates [33]. Morošan et al. [34] looked at a multi-zone building and studied the efficacy of joining decentralized and centralized control operations. Information was exchanged at each time step, and good control performance and low computational times were achieved. Another demonstration of distributed rather than centralized MPC was found in Hou et al. [35], where it was implemented in a real building.
Huang et al. [36] investigated robust MPC, which integrates uncertainty to improve the stability of MPC but found that the additional computational burden impeded performance and practicality. Jorissen and Helsen [37] recognized the challenges of developing the required model of each building for an MPC controller and that significant expert knowledge is often required for proper building thermal model development. Jorissen and Helsen [37] suggested a “Linear Automated Toolchain for MPC” (LAT-MPC) that permits a high degree of automation for controller model development and running MPC.

Sturzenegger et al. [8] showed a successful case of implementing MPC for 3 months in a real office building located in Switzerland. Data showed that adequate comfort levels for occupants were maintained, and this suggested that the MPC performance was greater compared to the original strategy.

3. Methodology

Data-driven reduced-order thermal models (ROMs) for different archetype zones and system configurations are useful tools to identify strategies for developing and quantifying energy flexibility in the building–grid interaction. These models, which account for thermal mass and the inherent thermal delay, are typically first- to third-order Resistance–Capacitance (RC) thermal networks and can be calibrated with smart meter and weather data or more detailed data from building automation systems (BAS) and smart thermostats.

An MPC-based simulation was carried out to estimate the heat requirements of the conditioned zone for the following day and to identify optimal controls for the thermal storage device at a zone temperature setpoint schedule. Several simulation studies were carried out to further evaluate the versatility of the identified schedules, while different weather conditions, control horizons, and cost functions were evaluated.

This study was carried out in the following steps, as shown in Figure 1:

1. Real building measurement data were collected from the BAS. Data included variables such as building power kW, zone air temperature, weather data, and specific data related to the ETS in question. Data were collected at 15-minute intervals.

2. Numerical thermal building or device control-oriented models were developed. These models are physics-based ROM grey-box RC thermal networks. These models were then calibrated using the collected data from the first step. Critical parameters were identified using the gradient descent-based optimization function *fmincon* in MATLAB. A detailed explanation of the model development process for the ETS device is found in [4,38,39].

3. Predictive control strategies for very cold days were identified for optimized peak load management and building energy flexibility by quantifying the BEFI depending on when a signal from the utility is given to the building owner. A heuristic approach was compared to optimized control MPC using *fmincon* in MATLAB. See Appendix A for further details on the developed BEFI.

4. Shoulder season MPC studies were also performed with the goal of improving the operation of the ETS to reduce energy consumption by storing the appropriate amount of energy needed for a warmer shoulder season day. With current control methods, it is common for the device to become overcharged on warmer shoulder season days.

5. Accounting for model prediction uncertainty due to weather forecast and identified model parameters was done by evaluating various uncertainty scenarios and by plotting the uncertainty bounds for the duration of the identified operation schedule.

6. Contingency strategies were evaluated to quantify the energy flexibility available from the building to the grid at specific times. Two strategies were evaluated: (1) reducing the zone temperature setpoint for 3 h and (2) discharging the stored thermal energy in the ETS for 3 h.
3.1. Description of the Case Study Building and Electric Thermal Storage Device (ETS)

This study focuses on the energy flexibility potential of a building with an ETS storage system and using MPC as the control method. The controller’s objective is to minimize a cost function that may incorporate energy and/or power while maintaining the zone within reasonable comfort limits. This paper takes a step forward from Date et al. [39] to investigate MPC, the effects of uncertainties and predictions, and contingency reserve strategies. This paper further evaluates more complex control strategies, namely MPC with different objective functions, and the potential for this building to help resolve contingency reserve issues of the electric grid. The objectives in this paper include minimization of cost under different scenarios on very cold days, improvements in operation during warmer shoulder season days, accounting for prediction uncertainties due to weather forecasts and model parameters, and participation in contingency events.

3.1.1. Description of the Case Study Building Zone

the investigated building zone is the warehouse portion of a two-storey office building located in Sherbrooke, Canada. The building was built in 1989 and has a floor area of 9000 m$^2$, and the warehouse has a floor area of 1650 m$^2$. The HVAC system servicing the warehouse section is equipped with an air-based ETS device with a rated maximum charging input of 106 kW. Data collection was ongoing since 2014 at 15-minute intervals for several measurements points. The peak load of the building was measured in February 2015 at 600 kW, with a monthly consumption of 166 MWh. The on-site 106 kW ETS supplies hot air to a warehouse zone in the building.

Figure 2 depicts how heat is delivered to the warehouse zone by the HVAC system [39]. Sensors measuring air temperatures are located throughout the HVAC system, and four thermocouples are embedded in the ETS, which measure brick temperatures. During operation of the ETS, a portion of supply air is directed to the thermal storage device to provide the zone with increased heat from stored heat energy. The zone is modeled using a 1RC explicit finite difference modeling approach (Figure 3), with a heating coil model with proportional-integral control. The zone model was originally developed for a previous study [39] and was calibrated using several days of measured data from the real building. Statistical indices were found for a Root-Mean-Square Error (RMSE) of 0.75 °C and a Mean Absolute Error (MAE) of 0.66 °C. This simple model was used for demonstration of the overall methodology presented in this study and was sufficient for shorter control horizons, such as the ones evaluated here (1 or 2 days). C is the equivalent thermal capacitance of the zone, $T_{\text{air}}$ is the effective operative temperature of the zone, $R$ is the equivalent resistance.
of the zone, and \( q_{aux} \) is all heating sources to the zone air node. Solar gains had little influence on the behaviour of the warehouse zone, especially during the winter season. It is acknowledged that the building model is extremely simple; however, this does not make the proposed methodology less meaningful.

![Figure 2](image_url). Heating, Ventilation, and Air Conditioning (HVAC) system with Electric Thermal Storage (ETS) and temperature sensor locations [39].

![Figure 3](image_url). Simple 1R1C thermal model of warehouse zone.

The system control variables are shown in Figure 4. The control variables are as follows:

1. \( P_{\text{elec,ETS}} \): the electric power to the ETS providing heat to the brick storage medium,
2. \( \text{Fan}_{\text{ETS}} \): the fan operation (ON/OFF), which notifies when air is passed through the brick storage medium,
3. \( Q_{\text{aux}} \): the additional auxiliary heating supplied to the supply air.

\( T_{\text{inlet}} \) is the air temperature entering and bypassing the ETS; \( T_{\text{outlet,ETS}} \) is the ETS outlet air temperature; and \( T_{\text{supply,HVAC}} \) is the zone supply air temperature. \( T_{\text{room}} \) is the air temperature of the warehouse zone.

3.1.2. Description of Thermal Energy Storage Device

During times of reduced electricity prices or when demand is low on the grid, ETS systems store heat converted from electrical power. They provide heat from stored energy to the building during times of higher demand or high electric pricing [40]. They can deliver a noteworthy reduction in electricity bills when there is dynamic TOU tariffs or when the utility pricing structure has a demand charge [41,42]. They use specially designed bricks as a storage medium, and they release heat to circulating air or water and deliver
it to the building zone. The device considered here contains 3121 kg of magnesite bricks in an insulated heat storage tank. Heating elements made from electric wires are located between the rows of bricks. The bricks are rated for a maximum temperature of 871 °C. The total storage capacity is 640 kWh (2184 kBTU). A fan is used to send air to the device to recover heat. Specifications for the ETS are shown in Table 1, and the schematics are shown in Figures 5 and 6 [39].

![Figure 4. System schematic [39].](image)

Control of the device is based on several factors: (1) determining when to charge the storage medium, (2) control of the maximum temperature during the charging, and (3) control of total amount of stored heat. During the discharge cycle, a consistent outlet air temperature is internally controlled by the device.

The maximum brick temperature setting is based on a function related to the outdoor ambient temperature (depicted in Figure 7) and available power of the building. Once per day, a reading of the outdoor air temperature is used to set the maximum brick temperature.

Table 1. Electric Thermal Storage (ETS) device specifications [39].

| Parameter                        | Value            |
|----------------------------------|------------------|
| Electric Charging Input          | 106 kW           |
| Thermal Storage Capacity         | 640 kWh          |
| Storage Material                 | High-density ceramic bricks |
| Number of Heating Elements       | 24               |
| Maximum Brick Temperature        | 871 °C           |
| Weight of Bricks                 | 3121 kg          |
| Number of Bricks                 | 384              |

3.2. Control-Oriented Thermal Model

The control-oriented models are described below (first introduced in a precursor study [39]). The models were derived from energy conservation and heat conduction equations in the two-dimensional lumped parameter formulation. A grey-box modeling approach was employed, where physically meaningful parameters can be calibrated and/or identified using information of the building construction and measurement data.

The simulation code of the models were developed using MATLAB and can be conveniently modified to adjust the model order. The model order was automatically adjusted from very simple (1 node) to more detailed by defining the two-dimensional grid of rows and columns of brick nodes. The number of rows multiplied by the number of columns was thus the total number of brick capacitance nodes for that given model.
order. Easy and quick modifications of the model are thus possible without rewriting the simulation code.

Figure 5 depicts the 1-capacitance model as an example, and further details on the modeling methodology can be found in [39]. The network was comprised of one row along the x-axis, one column along the y-axis, and the convective conductance term connected with the zone air node.

![Figure 5. Top view of the ETS 1-capacitance thermal model [39].](image1)

The bricks were heated up by the electric wires, and thermal energy was transferred to the bricks. The lumped parameter finite difference method used in this study was based on the material discretized into control volumes, where a node is located at the centroid of the control volume (Equation (1)).

![Figure 6. Side view schematic of the brick discharging model [39].](image2)
\[ T_{i}^{p+1} = T_{i}^{p} + \frac{\Delta t}{C_{i}} \left[ Q_{i}^{p} + \sum_{j=1}^{n} \frac{T_{j}^{p} - T_{i}^{p}}{R_{ij}} \right] \]

where the node temperature is \( T \), \( R_{ij} \) is the thermal resistance between nodes, \( Q_{i} \) is the heat generated/received at a node, and \( C \) is the thermal capacitance (\( C = \rho c_{p} A dx \)). \( n \) is the number of nodes connected to \( i \). Figure 5 shows the first-order model, and the brick temperature is calculated in Equation (2). The 1RIC zone model in Figure 3 was also solved using an equation similar to (1).

\[ T_{bricks}^{p+1} = T_{bricks}^{p} + \frac{\Delta t}{C_{bricks}} \left[ Q_{source}^{p} - Q_{conv}^{p} + 4 \cdot \frac{1}{R_{bricks}} (T_{ins}^{p} - T_{bricks}^{p}) + \frac{1}{R_{inf}} (T_{room}^{p} - T_{bricks}^{p}) \right] \]

where \( T_{bricks} \) is the temperature of all the bricks, \( C_{bricks} \) is the thermal capacitance for all bricks, \( Q_{source} \) is the heating power input, \( Q_{conv} \) is the heat transferred from the bricks to the air in the air channel, \( R_{bricks} \) represents 1/4 of total brick resistance, \( T_{ins} \) is the insulation layer surface temperature, \( R_{inf} \) is the infiltration to the mechanical room, and \( T_{room} \) is the mechanical room air temperature. In a previous paper investigating model complexity, it was found that adequate predictions were achieved using a slightly modified simple model. The reader is referred to Appendix B and the study by Date et al. [39] for further details on the ETS model.

### 3.3. Quebec Commercial Customer Electricity Rates

In the province of Quebec, Canada, there are utility rates that depend on the energy consumption and peak power demand of a building (or group of buildings) owned by the customer. Rate M, which is for the large commercial building sector, has a demand charge and two energy prices [43], as outlined in Table 2. A unique feature of this rate is that, at any given month, the minimum demand charge applied is set to 65% of the peak winter load. This means that appropriate attention should be given to the operation strategies over the winter period where peak demand is highest due to large space heating loads.

| Table 2. Pricing structure of utility rate M and rate flex M. |
|----------------------------------------------------------|
| **Rate M** |
| Large Commercial Building Sector |
| Demand Charge | $14.58/kW |
| First 210,000 kWh energy consumed | 5.03¢/kW |
| Remaining energy consumed | 3.73¢/kW |
| **Rate Flex M** |
| Large Commercial Building Sector |
| Winter 1 Dec.–31 Mar. |
| Demand Charge | $14.58/kW |
| Price of energy consumed outside peak events | 3.17¢/kW |
| Price of energy consumed during peak events | 50.00¢/kW |
| Summer 1 Apr.–30 Nov. |
| Demand Charge | $14.58/kW |
| First 210,000 kWh energy consumed | 5.03¢/kW |
| Remaining energy consumed | 3.73¢/kW |
3.4. Predictive Load Management Scenarios

Currently, control of the maximum power charging input to the ETS is determined based on a linear scale function relating outdoor temperature to the power input, as shown in Table 3 and Figure 7.

Table 3. Control of ETS power input: scale function.

| $T_{\text{ext}}$ | $P_{\text{ETS}}$ | Brick Temperature |
|------------------|------------------|-------------------|
| 0 °C             | 0%               | 93 °C             |
| -18 °C           | 100%             | 871 °C            |

By incorporating predictions of occupancy schedules and weather over the next day or two into the control sequence, optimal values of the maximum ETS charging input at each hour are identified that minimize electricity costs while maintaining the desired space conditioning of the warehouse zone. Simultaneously, the zone temperature profile is identified, which minimizes the following cost functions.

![Figure 7. Thermal storage power input depending on outdoor temperature.](image)

Three cost functions incorporating the two aforementioned utility rate structures and BEFI were implemented, and their results were compared to the typical manual control currently in operation.

1. Cost function using rate M: formulation of the first optimization problem is shown in Equation (3). The utility rate M in Table 2 is used as the cost function.

$$\min_{T_{\text{SP}}, P_{\text{ETS,maxSP}}} J_{PH} = \left( \sum_{i=1}^{N} P_i \Delta t \right) \cdot (\text{Cost}_{\text{Energy}}) + \max(P) \cdot (\text{Cost}_{\text{Demand}})$$

subject to 

$$T_{\text{SP,min}} \leq T_{\text{SP}} \leq T_{\text{SP,max}}$$

$$0 \leq P \leq P_{\text{max}}$$

$$0 \leq P_{\text{ETS,maxSP}} \leq P_{\text{ETS,max}}$$

(3)

2. Cost function using rate flex M: the second optimization problem in consideration is shown in Equation (4), where the objective is the minimization of the variable electricity cost shown in Figure 8. Equation (4) is a special case of Equation (3) but has an added variable energy cost over the day, indicated by subindex $i$, with higher cost during peak hours (Table 2).
\[
\min_{T_{SP}, P_{ETS,\text{maxSP}}} I_{PH} = \left( \sum_{i=1}^{N} P_i \Delta t \right) \cdot (\text{Cost}_{\text{Energy},i}) + \max(P) \cdot (\text{Cost}_{\text{Demand}})
\]

subject to
\[
T_{SP,\text{min}} \leq T_{SP} \leq T_{SP,\text{max}}
\]
\[
0 \leq P \leq P_{\text{max}}
\]
\[
0 \leq P_{ETS,\text{maxSP}} \leq P_{ETS,\text{max}}
\]

where \(N\) is the number of time steps over the prediction horizon \(PH\) (24, 36, 48 h, etc.), \(P_i\) is the power demand at time \(i\), and \(\Delta t\) is the time step. The objective is to identify two variables that minimize the cost associated with the utility rate charge: (1) an optimized setpoint schedule for the room temperature \(T_{SP}\) and (2) the maximum charging power input to the ETS, \(P_{ETS,\text{maxSP}}\). The setpoint is constrained by a lower \((T_{SP,\text{min}})\) and upper \((T_{SP,\text{max}})\) bound to maintain a level of thermal comfort for the zone occupants. The demand due to space heating \(P\) is constrained by the size of the heating equipment \(P_{\text{max}}\). Similarly, the maximum charging power input to the ETS is constrained by the specifications of the device, \(P_{ETS,\text{max}}\). Prediction control simulations are run at 5-minute intervals for higher time granularity and optimized values are identified at 1-hour intervals to speed up schedule identification.

3. Cost function using BEFI maximization during peak demand: the last optimization problem considered is using BEFI as the cost function. A BEFI is maximized during a specified period of the peak demand event.

\[
\max_{T_{SP}, P_{ETS,\text{maxSP}}} I_{PH} = \text{avg}[P_{ref} - P_{flex}]\text{during DR event}
\]

subject to
\[
T_{SP,\text{min}} \leq T_{SP} \leq T_{SP,\text{max}}
\]
\[
0 \leq P \leq P_{\text{max}}
\]
\[
0 \leq P_{ETS,\text{maxSP}} \leq P_{ETS,\text{max}}
\]

Equation (6) shows a more formal representation of the BEFI [3].

\[
\text{BEFI}(t, \Delta t, t_{\text{notice}}) = \frac{\int_{t}^{t+\Delta t} P_{\text{ref}} dt - \int_{t}^{t+\Delta t} P_{\text{flex}} dt}{\Delta t}
\]
where BEFI is the Building Energy Flexibility Index, \( t \) is the start time of event, \( \Delta t \) is the duration of event, \( t_{\text{notice}} \) is time of notification for the event, \( P_{\text{ref}} \) (kW) is the power demand from the reference scenario during an event, and \( P_{\text{flex}} \) (kW) is the power demand from a flexibility case during an event. The BEFI is the average difference between the power demand of the reference case, \( P_{\text{ref}} \), and the power demand of the alternative “flexibility scenario”, \( P_{\text{flex}} \), for the given event duration \( \Delta t \), shown in equation 6. BEFI could also be represented as a percentage by dividing it by the value of \( P_{\text{ref}} \).

3.5. Real-Time Optimizing Algorithm

The MATLAB function \textit{fmincon} uses a sequential quadratic programming algorithm and a line search method to find the minimum of a constrained nonlinear multi-variable function. The optimization algorithm in this paper identifies an hourly zone temperature setpoint schedule and maximum power input to the ETS.

3.6. Prediction and Control Horizons

Typically, in MPC, the optimal control problem is solved at each defined control step by looking ahead at forecast weather and occupancy schedules over the prediction horizon, \( PH \). The prediction horizon is a time period where we have reasonably reliable information, ranging from a few hours to a couple of days. Using data available from the prediction horizon period, an optimization routine is solved and an optimal sequence of control moves is identified through the implementation of MPC. The identified schedules and control moves are applied to the building over a “control horizon”, which can be the same length or be shorter than the prediction horizon. Once the current control horizon has ended, the optimization exercise is performed again for the following prediction horizon. This process is repeated until the end of the simulation time (e.g., one day or one year). For the sake of simplicity, in this particular case, the prediction and control horizon are the same length of time and MPC was only initiated with a notification signal: either 30 h for a 12-hour ahead notification at 6 p.m. or 22 h for a 4-hour ahead notification at 2 a.m. (for a DR event at 6 a.m.). Further studies could be carried out on performance related to the control horizon length; however, Date et al. [44] found that longer control horizons (12 h or more) were favourable for a similar building in a similar climate.

4. Results and Discussion

4.1. Weather Conditions—Cold Winter Day

MPC simulations were carried out over 10 days, with a focus on the last 48-hour period, with outdoor temperature conditions ranging from \(-20 \, ^\circ C\) to \(3 \, ^\circ C\) (Figure 9). A business as usual (BAU) indoor temperature setpoint profile, with a nighttime setpoint of \(18 \, ^\circ C\) and a daytime setpoint of \(22 \, ^\circ C\), was created as a benchmark, which is shown in the top graph of Figure 10 as a dashed black line. For the MPC studies, the zone temperature setpoint was constrained by a lower bound \( T_{\text{SP, min}} \) (\(17 \, ^\circ C\) at night and \(19 \, ^\circ C\) during the day) and an upper bound \( T_{\text{SP, max}} \) (\(24 \, ^\circ C\) at all times) in order to maintain a level of thermal comfort for the zone occupants. To demonstrate the methodology, typical known occupancy schedules for the warehouse (7 a.m.–6 p.m.) and weather data were used for this MPC study; however, an existing weather forecast tool such as CanMETEO [45] could be incorporated into eventual implementation within the building automation system in a real building.
4.2. Demand Response with 12- and 4-Hour Notifications

A scenario with a notification of demand response event given 12 h prior was investigated. The notification is given to the building owner at 6 p.m., indicating a DR event the following day at 6 a.m. and ending at 9 a.m. (3 h duration). The notification triggers an MPC algorithm that determines (a) an optimized zone temperature setpoint profile and (b) an optimized maximum allowable power input for charging the ETS device at each timestep. A second approach is conducted with a notification of 4 h ahead of the DR event at 2 a.m.

4.3. Predictive Load Management Simulation

Table 4 show the simulation results for the different scenarios. The scenarios (1) BAU with ETS, (2) MPC rate M, (3) MPC rate flex M, and (4) MPC BEFI are compared to the “Business as Usual (BAU)” case without ETS.

|                        | BAU without Thermal Storage | BAU with Thermal Storage | MPC: Rate M | MPC: Rate Flex M | MPC: BEFI |
|------------------------|-----------------------------|--------------------------|-------------|------------------|-----------|
| Event Peak (kW) (6 a.m.–9 a.m.) | 73                          | 33                       | 31          | 0                | 0         |
| Wh/m² in PH            | 763                         | 1170                     | 905         | 1034             | 1321      |
| BEFI (kW) (6 a.m.–9 a.m.) | –                           | 36                       | 36          | 65               | 65        |
| BEFI (%) (6 a.m.–9 a.m.) | –                           | 55                       | 56          | 100              | 100       |

Figure 10 shows the MPC results for the case when a notification is given 12 h before a DR event, while Figure 11 shows the results for the case with 4 h notification time. Implementing MPC with active thermal storage can increase the value of BEFI and give energy flexibility to the grid during critical peak events; operation is also improved compared to BAU with ETS. As shown in Table 4, with a notification from the utility to the customer given at 6 p.m. (12 h ahead of an event at 6 a.m.) a BEFI of 55% to 100% is achieved. In other words, the peak demand during the critical event hours can be reduced by an average
of 36 kW (63%) up to 65 kW (8%) for 3 h, depending on the utility rate structure. There were two identified scenarios where zero power demand for heating was required during the critical morning hours of 6 a.m. to 9 a.m., giving a BEFI of 100%.

It was found that rate flex M is the most successful in reducing the peak demand, while a greater reduction in energy consumption on a 24-hour period is seen with rate M. A longer notification time also results in a higher BEFI during the critical times, as there is more time for the MPC to identify improved operation strategies or implement preheating. When the equation for BEFI is used as the objective function, favourable results for peak reduction are found and are comparable with the scenario of the rate flex M; however, one disadvantage is that there is no incentive to reduce energy, and this results in this scenario with a high energy consumption (13% more compared to BAU with thermal storage at 1321 Wh/m²).

Figure 10. MPC utility rate study: identified zone setpoint profiles and electric power use with notification 12 h ahead for a cold winter day.

Similarly, with a relatively short notification time of 4 h (given at 2 a.m.), MPC can help reduce the peak demand during the critical hours and can achieve a BEFI of up to 48 kW (74%) for 3 h with rate flex M (Table 4). The results show that implementing MPC...
with active thermal storage and a properly chosen objective function can increase BEFI and give energy flexibility to the grid during critical peak events.

![Zone Temperature Setpoint [°C]](image)

**Figure 11.** MPC utility rate study: identified zone setpoint profiles and electric power use with notification 4 h ahead for a cold winter day.

4.4. Shoulder Season Performance: Cold Day Followed by a Warm Day

Building energy performance during the shoulder seasons is also critical as it is easy to consume unnecessary energy on warmer days with an improper control strategy. The current practice for control strategies of the thermal storage device often use unnecessary energy by charging the bricks and by storing too much thermal energy when a warmer day is on the horizon. Typically, the ETS will look at the weather one time at 6 p.m. to determine the charging needed during the night for use of the stored energy the next day. Sometimes, when a cold day is followed by a warm day, this determined charging amount would be unnecessary and excess electricity would be used to store thermal energy when the needed thermal energy the next day is actually much less. Incorporating weather forecasts into the decision-making process for determining the charging input amount could improve energy use. An example of a cold day followed by a warm day is shown in Figure 12, where at 6 p.m., the outdoor temperature is below 0 °C, but the next day is warmer, going as high as 10 °C.
The results shown in Figure 13 and Table 5 correspond to shoulder season MPC simulation studies evaluating the performance using the three previously studied objective functions (rate M, rate flex M, and BEFI) with notifications at 12-hour and 4-hour ahead of an event. Even a warm shoulder season day can benefit from advanced MPC strategies, where energy consumption is lowered by up to 43% and a BEFI of up to 100% can be obtained. It is seen that both rate M and rate flex M give favourable results on a warm shoulder season day; however, an objective function of BEFI increases energy consumption and is not suitable for this type of day, where the more prevalent objective is to better manage (or reduce) energy use.

Table 5. MPC simulation results for improved operation of ETS and zone temperature setpoint on a warmer shoulder season day.

|                        | BAU without Thermal Storage | BAU with Thermal Storage | MPC: Rate M | MPC: Rate Flex M | MPC: BEFI |
|------------------------|-----------------------------|--------------------------|-------------|------------------|-----------|
| Event Peak (kW) (6 a.m.–9 a.m.) | 65                          | 31                       | 25          | 25               | 0         |
| Wh/m² in PH             | 490                         | 454                      | 389         | 386              | 614       |
| BEFI (kW) (6 a.m.–9 a.m.) | –                           | 25                       | 41          | 41               | 57        |
| BEFI (%) (6 a.m.–9 a.m.) | –                           | 55                       | 68          | 75               | 100       |

|                        | BAU without Thermal Storage | BAU with Thermal Storage | MPC: Rate M | MPC: Rate Flex M | MPC: BEFI |
|------------------------|-----------------------------|--------------------------|-------------|------------------|-----------|
| Event Peak (kW) (6 a.m.–9 a.m.) | 65                          | 31                       | 26          | 24               | 0         |
| Wh/m² in PH             | 456                         | 443                      | 259         | 328              | 395       |
| BEFI (kW) (6 a.m.–9 a.m.) | –                           | 25                       | 41          | 41               | 57        |
| BEFI (%) (6 a.m.–9 a.m.) | –                           | 55                       | 71          | 72               | 100       |
4.5. Accounting for Prediction Uncertainty

Presenting a range of reasonable operating predictions that take into account uncertainties related to weather forecasting and identified model parameter values can aid decision makers in assessing the risk in operation options available to them. Uncertainty in weather forecasts can be as high as 30%, and modeling uncertainty can be in that range as well.

Six scenarios of combinations incorporating ±10% deviation from the identified brick conductivity and ±3 °C deviation from the weather forecast [39] were evaluated and are presented in this section. These six scenarios were carried out for the three different cost functions (rate M, rate flex M, and BEFI) at the two notification times (12 h and 4 h), for a total of 36 cases.

From the Figures 14–16, one can make an informed decision about whether enabling a more advanced control strategy (in this case MPC) can be beneficial and whether the associated risk is appropriate. Figure 16 illustrates an example of how the net electric power prediction could vary depending on accuracy of the weather forecast and identified model parameter $k_{brick}$ when an identified operating schedule is obtained using an objective function of rate flex M. In other words, the grey area is a set of possible trajectories of the
Incorporating this type of evaluation and visualization in the decision-making process of building operators/owners and the utility grid would allow for a better understanding of the anticipated variations due to uncertainties in the problem. Future work could include incorporating an existing weather forecasting tool into the methodology, such as CanMETEO [45], to further evaluate the effects of weather uncertainty in MPC performance.

**Figure 14.** Uncertainty of Building Energy Flexibility Index (BEFI) predictions: 12-hour ahead notification for a cold winter day.

**Figure 15.** Uncertainty of BEFI predictions: 4-hour ahead notification for a cold winter day.

**Figure 16.** Uncertainty of BEFI predictions: net electric power prediction for scenario with MPC rate flex M for a cold winter day.
4.6. Contingency Event Evaluation

The contingency reserve is an amount of power that the utility may call when needed to face a loss of a generation unit or other unexpected load unbalances. To address this need, real-time thermal load flexibility must be predicted ahead of time or calculated continuously and available at short notice (e.g., 10 min) over a time duration of about an hour or a few hours.

This energy flexibility can be enabled by actions in response to a signal from the grid. This section calculates such a BEFI (kW and %) in order to quantify real-time thermal load flexibility of the building. A preliminary contingency study on a school building was introduced in [3]. Two strategies were analyzed: (1) reducing the zone temperature setpoint for a duration of 3 h and (2) discharging the thermal storage device for a duration of 3 h. An example of strategy (1) is depicted in Figure 17.

The available energy flexibility may be quantified using the developed model to determine the power demand difference between the reference case (BAU, no ETS) with the two contingency strategies outlined in the previous paragraph.

In strategy 1, shown in Figure 18 where the zone temperature setpoint is reduced by 2 °C for 3 h, the BEFI varies from 0 up to 47 kW (or 0 up to 100%). The BEFI in this scenario is dependent on the time of day due to the reference case nighttime setback zone temperature schedule.

![Figure 17. Contingency strategy 1: reduce the zone temperature setpoint for a duration of 3 h at each hour—BEFI at hourly intervals](image)

For the second strategy of discharging heat from the thermal storage device for durations of 3 h, the results are shown in Figure 19. A BEFI of 0 to 41 kW (or 0 to 100%) is achievable with this contingency strategy. Favourable results were found for both of these options, where a BEFI of up to 47 kW (97%) is achieved for a duration of 3 h. The highest BEFI values are seen towards the start of the day as that is the time of biggest potential to reduce peaks due to the existing abrupt setpoint change from night setback to day time comfort levels. In the evening, a zero value for BEFI is seen due to the setpoint change back to the lowered night time value. There is zero heating demand and thus cannot be lowered any further, and the zone air temperature is free floating until the lower night time value is reached.
Figure 18. Contingency strategy 1: reduce the zone temperature setpoint for a duration of 3 h at each hour—BEFI at hourly intervals.

Figure 19. Contingency strategy 2: discharge the ETS for a duration of 3 h at each hour—BEFI at hourly intervals.

5. Conclusions

This paper presented a methodology for the implementation of MPC strategies to a building equipped with a dedicated active thermal storage device in order to predict and maximize the building energy flexibility that the building could provide to the electric grid by evaluating the BEFI related to the different strategies. Three MPC cost functions were studied: (1) minimization of the electricity cost subject to a utility rate with peak demand charge (rate M), (2) minimization of the electricity cost subject to a utility rate with dynamic pricing (rate flex M), and (3) maximization of BEFI during a critical DR event.

Given a notification 4 or 12 h ahead of a DR event with a given duration, MPC was implemented at an hourly time step to identify an optimized zone temperature setpoint profile and an optimized dynamic maximum allowable power input for charging the thermal storage device. The results show that implementing MPC with thermal storage can increase BEFI, can provide energy flexibility to the grid during critical peak events, and is better than manual control. It is seen not only that optimizing the zone profile is important but also that optimizing (limiting) the maximum allowable power to the thermal storage device aids in reducing both peak demand and energy consumption of the building.

It should be acknowledged that thermal comfort conditions for the different scenarios are different from that of the BAU case, which should be considered when choosing a strategy. Since the zone in this study is a warehouse, more flexibility in the comfort limits could be assumed when compared to an office or residential building.

Future work could include using this methodology to design optimal utility pricing structures rather than the design optimal control strategies. Though the scope of this study was limited by data availability, the developed methodology could be transposed to other similar convectively conditioned buildings and clusters of buildings for participation in community-scale energy aggregator events. A greater focus on occupancy modeling could eventually be incorporated into this methodology. There is still work to be done in terms of implementation of the methodology into real building building energy management systems.
Author Contributions: conceptualization, J.D. and J.A.C.; methodology, J.D.; software, J.D.; validation, J.D.; formal analysis, J.D.; investigation, J.D.; resources, J.A.C. and A.K.A.; data curation, J.D.; writing—original draft preparation, J.D.; writing—review and editing, J.A.C. and A.K.A.; visualization, J.D.; supervision, J.A.C. and A.K.A.; project administration, J.A.C. and A.K.A.; funding acquisition, J.A.C. and A.K.A. All authors have read and agreed to the published version of the manuscript.

Funding: Financial contributions from NSERC-Hydro-Québec Industrial Research Chair and Discovery Grant # RGPIN-04986-2016 are greatly acknowledged. Partially funding for PhD candidate Jennifer Date by FRQNT (Fonds de recherche Nature et technologies) and ASHRAE Graduate Student Grant-in-Aid Award are also recognized.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: The authors appreciate Hydro-Québec LTE researchers for their rewarding collaboration. Internal reviews conducted by CanmetENERGY-Varennes employees are also appreciated and recognized.

Conflicts of Interest: The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

Nomenclature

\[ \Delta t \] Time step (seconds) or duration of flexibility event hours

\[ C \] Thermal capacitance (J/K)

\[ C_{\text{bricks}} \] Thermal capacitance of brick storage medium (J/K)

\[ c_p \] Specific heat capacity (kJ/(kg·K))

\[ DH \] Hydraulic diameter (m)

\[ h \] Heat transfer coefficient (W/(m²·K))

\[ J \] Objective function

\[ k \] Thermal conductivity (W/(m·K))

\[ L \] Length of air channel (m)

\[ \dot{m} \] Mass flow rate (kg/s)

\[ M \] Volumetric flow rate (m³/s)

\[ N \] Number of time steps

\[ Nu \] Nusselt number

\[ P \] Air channel perimeter (m)

\[ \text{PH} \] Prediction Horizon

\[ P_{\text{ETS}} \] Electrical heat input to storage medium (%)

\[ \text{P}_{\text{max}} \] Maximum input of ETS (W)

\[ \text{P}_{\text{ref}} \] Reference power demand (kW)

\[ \text{P}_{\text{flex}} \] Power demand of flexibility scenario (kW)

\[ Q \] Power (W)

\[ Q_{\text{source}} \] Electrical heat input to storage medium (W)

\[ Q_{\text{conv}} \] Heat extracted from storage medium to air in channel (W)

\[ q \] Energy (Wh)

\[ q_{\text{conv}} \] Convective heat flux (W/m²)

\[ q_{\text{rad}} \] Radiative heat flux (W/m²)

\[ R \] Thermal resistance (K/W)
Appendix A. Concept of Building Energy Flexibility Index (BEFI)

BEFI is an informative concept to identify relevant actions undertaken to provide predictable energy flexibility of a building. Optimization routines could be performed to maximize this index during a specific period. BEFI may become a valuable consideration to support decisions regarding relevant equipment or systems installed in a building.

A well-designed index would help to quantify the flexibility available from a building, to improve its design, to increase its potential flexibility, to control the building in order to obtain maximum available flexibility when needed, and to compare different systems or designs. A depiction of a demand response event is shown in Figure A1 [3].

An example is illustrated in Figure A1. Suppose that a DR event occurs at time $t$ and that this event lasts for a duration $\Delta t$. The utility informs the building operators ahead of time at time $t_{\text{notice}}$ that the event will occur. The trajectory of the load follows
one trajectory, in blue, for the reference case and an adjusted trajectory, in orange, for the “flexibility-adjusted curve”.

Figure A1. Example: Building Energy Flexibility Index [4].

Appendix B. Control-Oriented Thermal Modeling of the Storage Device

The thermal models for control purposes of the storage device use the lumped parameter finite difference method with a fully explicit finite difference formulation. A heat balance calculation can be written for the control volume, resulting in this differential equation of a node:

\[ C_i \frac{dT_i}{dt} = Q_i + \sum_{j=1}^{n} \frac{(T_j - T_i)}{R_{i,j}} \]  

(A1)

Energy balance calculations are solved at each node using a fully explicit finite difference approach, where the current temperature is dependent on conditions at the previous time step. The time derivative term is discretized as follows:

\[ C_i \frac{dT_i}{dt} \approx C_i \Delta T_i \Delta t = C_i \left[ T_{p+1}^p - T_i^p \right] \]  

(A2)

General equations for cases with and without capacitance terms are derived (Equations (A3) and (A4)).

\[ T_{i+1}^p = T_i^p + \frac{\Delta t}{C_i} \left[ Q_i^p + \sum_{j=1}^{n} \frac{T_j^p - T_i^p}{R_{i,j}} \right] \]  

(A3)

\[ T_{i+1}^p = \frac{Q_i^p + \sum_{j=1}^{n} \frac{T_j^p}{R_{i,j}}}{\sum_{j=1}^{n} \frac{1}{R_{i,j}}} \]  

(A4)

It is assumed that \( R_{i,j} \) is the thermal resistance between nodes, \( Q_i \) is the heat source/generation at a node, \( T \) is the temperature of a node, and \( C \) is the thermal capacitance of a node (\( C = \rho c_p A dx \)).

Brick temperatures in the storage device can reach upwards of 871 °C; therefore, the heat lost to the mechanical room are significant and should be incorporated into the models. Convective and radiative heat losses are calculated using the heat transfer coefficient in Equations (A5) and (A6) from [46], where \( T_{surf} \) is the average of the wall surfaces in the mechanical room. \( T_{surf} \) is taken as the same temperature as zone air temperature, \( T_{room} \).

\[ h_{conv} = 1.26 \cdot |T_{int} - T_{room}|^{1/3} \]  

(A5)
\[ h_{\text{rad}} = \epsilon \sigma \cdot \left( T_{\text{int}}^2 + T_{\text{surf}}^2 \right) \cdot \left( T_{\text{int}} + T_{\text{surf}} \right) \]  

(A6)

Heat exchange in a channel can be calculated using Equation (A7). This was used to model the heat transfer between bricks, and airflow \( T_w \) is the temperature of the wall surface and is assumed to be the average brick temperature \( T_{\text{brick}} \). \( h \) is the convective heat transfer coefficient between air in the channel and channel surface, \( P \) is the channel perimeter, and \( L \) is the channel length. \( T_{\text{out}} \) can be replaced by \( T_{p(i)} \), and \( L \) can be replaced by \( Z(i) \); thus, the change in air temperature through the channel is given as a function of distance from the inlet \( x \). Temperatures at the exit of each control volume are calculated in Equation (A8).

\[
\frac{T_{\text{out}} - T_{\text{in}}}{T_w - T_{\text{in}}} = 1 - \exp \left( -\frac{hPL}{mc_p} \right) 
\]

(A7)

\[
T_{\text{out}}(p,i) = T_w(p) + \left[ T_{\text{in}}(p,i) - T_w(p) \right] \cdot e^{-\frac{2Z(i)}{a(p)}} 
\]

(A8)

where \( a(p) = \frac{M(p)\rho_c}{\nu_m \rho p} \) and \( \nu = \frac{Nu \cdot k}{DH} \)

where \( DH \) is the air channel hydraulic diameter and \( k \) is air conductivity. \( h \) may be a calibrated parameter as not all pertinent information for the device may be available. Equation (A9) is given for the energy taken from the bricks.

\[
Q_{\text{conv}} = 4 \cdot M(p)\rho_c p \left[ T_{\text{out}}(p, L) - T_{\text{in}}(p, 0) \right] 
\]

(A9)

It is assumed that the temperature at the exit of each section, \( T_{\text{out}}(p,i) \), is equal to the inlet temperature of the following section, \( T_{\text{in}}(p,i+1) \). A model with six sections was determined sufficient to predict accurate outlet air temperature results.

The model performance was evaluated using several statistical indices, including the root-mean-square error (RMSE) and the mean absolute error (MEA). Further details regarding model development, calibration methodology, and model performance are presented in [39].

Brick conductivity values were identified using an optimization routine. The RMSE for brick temperatures was chosen as the objective function to be minimized. The MATLAB function \textit{fmincon} was used; however, depending on the user’s preference, other suitable methods may replace this. The optimization problem is described in Equation (A10).

\[
\min_{k_{\text{brick}}} \sqrt{\frac{\sum_{i=1}^{N} (y_i - \hat{y_i})^2}{N}} 
\]

subject to \( k_{\text{brick,min}} \leq k_{\text{brick}} \leq k_{\text{brick,max}} \)  

(A10)

where \( y \) is measured sensor data and \( \hat{y} \) is the model predictions from the simulation. For each model with a prescribed number of capacitances (1 up to 140 capacitances), a suitable value for \( k \) is found that minimizes the RMSE. The identified “effective” brick conductivities over the amount of brick nodes are plotted in Figure A2.

A reduction of 34% in the RMSE was observed when a brick conductivity of 97.1 W/m-K was used for the case of a 1-capacitance model. The MAE decreased by 28%. Further details on the methodology can be found in [39].
Figure A2. Root-Mean-Square Error (RMSE) results of models with “effective” conductivity compared to real conductivity = 4.3 W/m K [39].

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