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COLLECTIONS

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Illuminating the complex role of the added mass during vortex induced vibration

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ABSTRACT

The role of the added mass coefficient in vortex induced vibration (VIV) of the bluff body is complex and elusive. It is certain that decoding the relationship between the added mass and the vibration pattern will benefit the prediction and prevention of VIV. We present a study on VIV of a long flexible cylinder and forced vibration of a rigid cylinder, in a combination of experimental optical measurements and high-fidelity numerical simulation. We focus on uniform flow over a uniform cylinder at a fixed Reynolds number, $R_e = 900$, but systematically varied the motion amplitude in the in-line (IL) and cross-flow direction (CF), as well as the phase angle ($\theta$) between the motions. We show that $\theta \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ is associated with negative added mass coefficients in the cross-flow direction ($C_{my} < 0$), and there is a strong correlation between the vortex shedding mode of "2P" or "P+S" and $C_{my} < 0$.

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I. INTRODUCTION

It is well known that the vortices shedding from bluff bodies generate oscillatory forces, which may induce the vibration of the bluff body, if it is flexibly mounted or deformable as a string.\textsuperscript{1} This particular fluid-structure interaction (FSI) problem is better known as vortex induced vibration (VIV). VIV has significant importance in everyday life and physical applications in the real world, such as a cable in the blowing wind or a riser in the ocean current. When VIV happens, due to the interaction of the vortex and the body (the relative motion varies between the vortex and the body), the effective added mass $C_m$ can vary significantly from a negative value to a large positive value,\textsuperscript{2-5} and $C_m$ plays a complex role in determining the system natural and vibration frequencies.\textsuperscript{6}

A rigid circular cylinder oscillating only in the cross-flow (CF) direction is one of the simplest models of VIV. It has been established that over a broad range of incoming velocities, different regimes of response can be classified as initial, upper, and lower branches for the rigid cylinder CF-only free vibration, and distinct vortex shedding modes are identified.\textsuperscript{1} Furthermore, by imposing prescribed forced motion and measuring the corresponding fluid forces, the lift coefficients in a phase with velocity and added mass coefficients are both strongly correlated with the true reduced velocity $V_r = \frac{U}{f}$ and non-dimensional CF amplitude $\frac{d}{D}$, where $U$ is the incoming velocity, $f$ is the vibration frequency, $d$ is the cylinder diameter, and $A_v$ is the prescribed CF vibration amplitude.\textsuperscript{5}

Detailed flow visualization study\textsuperscript{9} established a correlation between the wake pattern and hydrodynamic force, where a slight change in the vibration frequency may result in a sharp change in the magnitude of the oscillatory lift force and the phase between the fluid force and cylinder motion (namely, the sign of $C_{my}$), accompanied by a vortical wake mode switch.

If the cylinder is allowed to move in combined in-line (IL) and CF response,\textsuperscript{10} considerable differences will be observed in fluid forces, structural responses, and wake patterns. In particular, from both rigid cylinder free and forced vibration experiments, the phase $\theta$ between the IL and the CF motions is found to have a strong influence on the fluid forces on the oscillating cylinder. Dahl et al.\textsuperscript{11} found that positive $C_{my}$ is strongly associated with a phase angle $\theta$ corresponding to a counterclockwise trajectory. Moreover, the occurrence of high harmonics (third harmonics) in the CF direction is also found to be associated with specific values of $\theta$.\textsuperscript{11} Although previous studies have highlighted that the IL motion has significant impact on CF motion and hydrodynamic forces, to the best of the authors' knowledge, no systematic study has been conducted on the correlation (mapping) between the
wake pattern and hydrodynamic force, especially $C_{my}$ for the combined-IL-and-CF rigid cylinder vibration yet.

When the cylinder aspect ratio $L/d$ ($L$ is the cylinder length) becomes larger, the rigid cylinder approximation is no longer held and the model becomes flexible. One of the key differences between rigid and flexible cylinders is that the flexible model obtains multiple structural natural frequencies and vibration modes.\(^\text{13}\) It was found that the VIV of the flexible cylinder could happen at a large range of incoming flow velocities and oscillate at different modal numbers. In addition, a large variation of the distribution of the added mass\(^\text{14}\) and different wake patterns\(^\text{1}\) were observed along the span. Some preliminary results have demonstrated the validity of the strip theory that the rigid cylinder can well capture the fluid force distribution along the flexible cylinder,\(^\text{16}\) but it is still not clear about the relationship between the wake pattern and hydrodynamic coefficients, especially the added mass, along the flexible cylinder.

In this paper, we will address the question of the relationship between the added mass coefficient $C_{my}$ and the vortical wake pattern behind the rigid and flexible cylinder undergoing VIVs, using high-fidelity numerical simulation and experimental results. We aim to shed some light on the complex role of the added mass coefficient $C_{my}$ in VIV and its correlation with the unique vortical wake patterns.

II. SIMULATION AND EXPERIMENT METHODS

We start with the simulation of the forced vibration of a rigid cylinder of aspect ratio $L/d = 4\pi$ and in a uniform oncoming flow, at Reynolds number\(^\text{15}\) ($Re_d = 900$). The motion of the rigid cylinder is prescribed as follows:

\[
\begin{align*}
\dot{y}(t) &= \frac{A_y}{d}\cos(\omega t), \\
x(t) &= \frac{A_x}{d}\cos(2\omega t + \theta),
\end{align*}
\]

where $A_x$ and $A_y$ are the cylinder IL and CF amplitudes, respectively. $\omega = 2\pi f$ is the cylinder vibration frequency, and $\theta$ is the phase between the IL and CF trajectories. In total 273 simulations have been carried out, which can be divided into three groups, namely, group 1, $\frac{A_x}{d} = 0.1$ and $\frac{A_y}{d} = 0.7$, group 2, $\frac{A_x}{d} = 0.1$ and $\frac{A_y}{d} = 0.5$, and group 3, $\frac{A_x}{d} = 0.15$ and $\frac{A_y}{d} = 0.5$. Moreover, $V_r$ is in the regime of $[4, 8]$, and $\theta$ is in the regime of $[0, 2\pi]$. Therefore, the added mass coefficient in the CF direction ($C_{my}$) and the lift coefficient in the phase with velocity ($C_{lv}$) for the rigid cylinder can be calculated as follows:

\[
C_{my} = -\frac{2U^2}{\pi d^2} \int_0^{T_s} \left( \frac{C_l(t)\dot{y}(t)dt}{J_r(j^2(t)dt)} \right),
\]

\[
C_{lv} = \frac{2}{T_s} \int_0^{T_s} \left( \frac{C_l(t)\dot{y}(t)dt}{J_r(j^2(t)dt)} \right),
\]

where $C_l$ is the instantaneous lift coefficient and $T_s$ is the one vibration cycle. For the flexible cylinder, the distribution of $C_{my}$ along the span, hence, can be calculated as follows:

\[
C_{my}(z) = -\frac{2U^2}{\pi d^2} \int_0^{T_s} \left( \frac{C_l(z,t)\dot{y}(z,t)dt}{\int_0^{T_s} (j^2(z,t)dt)} \right),
\]

\[
C_{lv}(z) = \frac{2}{T_s} \int_0^{T_s} \left( \frac{C_l(z,t)\dot{y}(z,t)dt}{\int_0^{T_s} (j^2(z,t)dt)} \right),
\]

where $z$ is the location along the flexible model.

In addition, experiments and simulations have been conducted on a tension (varies linearly along the span) dominated flexible beam (string) with a uniform circular cross section, pinned at both ends and free to move in both the IL ($x$) and CF ($y$) directions. The flexible cylinder is placed in a uniform oncoming flow with velocity $U$ parallel to the $x$ axis. It is worth noting that simulations and experiments have exactly same parameters: $L/d = 240$, the structure-to-fluid mass ratio ($m^*$) is 4.0, the damping ratio ($\zeta$) is 0.087, the nominal reduced velocity ($U/\omega$) is 17.22, and $Re = 900$.

Numerically, the complex three-dimensional flow past the vibrating cylinder is obtained by large-eddy simulation (LES), which is based on the entropy viscosity method (EVM)\(^\text{19}\) developed and implemented on a spectral hp element code.\(^\text{20}\) The experiment is conducted at MIT Tow Tank Lab that employs an array of high speed cameras. Note that in the experiments, the time series of the structure displacements are measured at 51 points in both the IL and CF directions along the span, and the fluid force distribution is determined by the inverse force reconstruction method. (A complete description of the experimental apparatus and methods is given in Ref.\(^\text{14}\).) In this paper, the analyzed experimental and numerical results are over 20 stable CF oscillation cycles.

III. RESULTS

Figures 1(a)–1(c) show the added mass coefficient in the CF direction ($C_{my}$) as a function of $V_r$ (horizontal axis) and $\theta$ (vertical axis) in the three selected groups of $\frac{A_x}{d}$ and $\frac{A_y}{d}$. It is found that $C_{my}$ can be both positive and negative in the current $V_r \cdot \theta$ range, and the negative $C_{my}$ is consistently found strongly associated with the phase $\theta$, i.e., the cylinder orbits. Specifically, the negative $C_{my}$ favors $\theta \in \left(\frac{1}{2}, \frac{3}{2}\right)$. However, for the current selected amplitude combinations, the effect of $\frac{A_y}{d}$ and $\frac{A_x}{d}$ on $C_{my}$ is not as prominent as that of $V_r$ and $\theta$.

The value of the added mass coefficient reflects the relative motion between the oscillating cylinder and the shedding vortex in the near wake. As shown in the snapshot of the 3D vortical wake in Figs. 1(d) and 1(e) and the middle sliced 2D vortical wake in Figs. 1(f) and 1(g) for two selected cases [corresponding to the circles in Fig. 1(b)], different dominant vortex modes, both “P+S” (“2P”) and “2S” modes, are observed for different prescribed motions.

For every simulation, we have observed the wake variation over 20 oscillation cycles and classified the modes, which are highlighted in Figs. 1(a)–1(c). The red circle denotes the stable “2S” vortex mode, shown in Figs. 1(d) and 1(f), the blue cross denotes the stable “P+S” or “2P” vortex mode, shown in Figs. 1(e) and 1(g), and the yellow square denotes the unstable vortex shedding mode. In current selected IL and CF amplitude combinations, it is clear that there is a strong correlation between the sign of the negative $C_{my}$ and the cylinder vortical wake
FIG. 1. Simulation results of the forced vibration of a rigid cylinder at $Re = 900$. Figures of the first row, $C_{my}$ map of different amplitude groups: (a) group 1: $A_y/d = 0.7, A_x/d = 0.1$; (b) group 2, $A_y/d = 0.5, A_x/d = 0.1$; (c) group 2, $A_y/d = 0.5, A_x/d = 0.15$. The black bold dashed line highlights $C_{my} = 0$: red dots indicate the stable “2S” vortex mode; the blue crosses represent the stable “P+S” or “2P” vortex mode; and the yellow squares denote the unstable vortex mode over 20 cylinder vibration cycles found in the simulation. Figures of the second row, snapshot of the three dimensional vortices of group 2 [corresponding to the red diamonds shown in (h) and (i)]. (d) “2S” mode, $V_r = 5$, and $\theta = 0.25\pi$; (e) “P+S” mode, $V_r = 5$, and $\theta = \frac{3\pi}{2}$, corresponding to the two blank circles in (b). Figures of the third row, snapshot of the two-dimensional vorticity field at the spanwise location $z/L = 0.5$. (f) “2S” mode, a slice of the vorticity field in (d); (g) “P+S” mode, a slice of that in (e). Note that vortices in (d) and (e) are represented by iso-surfaces of $Q = 0.5$ and colored by $x_z$, and the black arrow denotes the acceleration direction of the cylinder. Figures of the fourth row: time history of the CF motion and lift coefficient: (h) case $V_r = 5$ and $\theta = 0.25\pi$; (i) $V_r = 5$ and $\theta = 1.25\pi$. The inset of figures shows the instantaneous pressure coefficient $C_p = \frac{p-p_0}{\frac{1}{2} \rho U^2}$ distribution along the cylinder, of which the pink color denotes the positive pressure region and the yellow color denotes the negative pressure region; the magnitude of the unit $C_p$ corresponds to the length of $d/8$. 
mode: when $C_{my}$ is negative or close to zero, the cylinder wake is dominated by either "P+S" or "2P" vortex mode; when $C_{my}$ is positive, classical "2S" von Kármán appears; close to $C_{my} = 0$, an unstable vortex mode (switch between "2S" and "P+S") can be found.

To reveal the detailed flow physics of this phenomenon, from the simulations of group 2, we select the case of $V_r = 5$ and $\theta = \frac{\pi}{2}$ and plot in Figs. 1(d) and 1(f) the snapshots of the "2S" vortex mode. Figures 1(e) and 1(g) exhibit the "P+S" vortex mode corresponding to $V_r = 5$ and $\theta = \frac{3\pi}{2}$. The difference in relative motion between the cylinder and the vortex could clearly be observed: in Fig. 1(f), when the cylinder reaches $y(t_0) = 0.14 \, \frac{L}{d}$ from the balance position to the maximum positive position in the CF direction, the vortex (low pressure) appears at the back-side of the cylinder with respect to the cylinder acceleration direction. The time histories of the cylinder motion and the lift coefficient are shown in Fig. 1(h). It is found that the force of a large magnitude is mainly in-phase with the motion, resulting in a small negative $C_{my}$. As shown in Fig. 1(g) for the "P+S" vortex mode, when the cylinder arrives at position $y(t_0) = 0.14 \, \frac{L}{d}$ from the balance point to the maximum positive displacement in the CF direction, vortices appear in the front-side of the cylinder relative to the cylinder acceleration direction. In addition, four instantaneous pressure coefficient $C_p$ distributions along the cylinder are plotted for both cases, demonstrating large variation of the pressure around the cylinder when the vortex mode is different.

Unlike the forced vibration of a rigid cylinder, a uniform flexible cylinder in uniform flow may have different vibration patterns, i.e., different $\frac{d}{L}$ and $\theta$ along the span; hence, the hydrodynamic coefficients as well as the wake pattern may differ significantly at different locations along the flexible cylinder.

Figure 2(a) displays the experiment results of the IL and CF vibration amplitudes as well as $\theta$ distribution along the flexible cylinder, and Fig. 2(b) shows the corresponding simulation result. It could be identified that the CF vibration is a mixture of traveling and standing waves, while the IL vibration is dominated by the standing wave. Same as the observation in Ref. 20, $\theta$ varies continuously in half wavelength of the IL vibration mode and jumps abruptly at the IL nodes. Moreover, positive $C_{my}$ that implies a net positive energy transfer from

![FIG. 2. Structural response and wake flow of the uniform flexible cylinder ($L/d = 240$) in the uniform current at $Re = 900$. (a) Experiment result: blue solid line, CF displacement; red solid line, IL displacement; black solid line, phase angle between IL and CF trajectories ($\theta$); black dashed line, $\theta = \pi$. (b) Simulation result: lines are equivalent to (a). (c) Distribution of the lift coefficient in the phase with velocity ($C_{lv}$): red solid line, experiment; red dotted line, simulation; black dashed line, $C_{lv} = 0$. (d) Distribution of the added mass coefficient in the CF direction ($C_{my}$): black solid line, experiment; black dotted line, simulation; black dashed line, $C_{my} = 0$; the blue shade highlights the region of the "P+S" vortex shedding mode. (e) Two-dimensional snapshots of the vorticity field at six spanwise locations highlighted by the red circles in (b): from top to bottom, $z/L = 0.9178$, $z/L = 0.7926$, $z/L = 0.5714$, $z/L = 0.4814$, $z/L = 0.2505$, and $z/L = 0.0842$. (f) A three-dimensional snapshot of the vortices. Note that the vortices are represented by iso-surfaces of $Q = 0.1$ and colored by $\omega_\theta$.](image-url)
the fluid to the structure in the CF direction is primarily associated with the CCW cylinder orbit, and with the finding in rigid and flexible cylinders.\[^{21}\] Note that in order to fully validate the simulation result, the comparison of $C_m$ and $C_{my}$ with that of the experiment is shown in Figs. 2(c) and 2(d), respectively. Interestingly, similarly to the forced vibration of a rigid cylinder, the negative $C_{my}$ distribution of the free vibration of the flexible cylinder is also associated with the $\theta \in [\frac{\pi}{2}, \frac{3\pi}{2}]$ region along the span. The local wake pattern behind the cylinder is shown in Fig. 2(e), which consists of snapshots of the cylinder motion and the shedding vortex pattern at six axial positions that are marked out by red dots in Fig. 2(b). It could be observed that negative $C_{my}$ is strongly correlated with the “P+S” vortex mode (no “2P” mode is found in current flexible cylinder simulation), while the “2S” vortex mode is found where $C_{my} > 0$. Furthermore, as shown in Fig. 2(f), accompanying the vortex pattern switching between “2S” and “P+S” along the span, the wake behind the cylinder exhibits two patterns, and it can be divided into six zones separated by the IL nodes. These two patterns correspond to one region of clear straight vortex tubes and the other one of wavy vortex tubes with strong stream-wise vortices. Note that the “P+S” vortex mode region is highlighted by the blue shade in Fig. 2(d).

IV. DISCUSSION AND CONCLUSION

One of the notable conclusions from previous research on the vortex induced vibrations of both the flexibly mounted rigid cylinder\[^{11}\] and the flexible cylinder\[^{21}\] is that the phase difference between the cylinder motion in-line and cross-flow $\theta$ alters the strength of the shedding vortices and its timing relative to the cylinder motion. Specifically, the cylinder CCW trajectory ($\theta \in [0, \pi]$) is found to favor positive energy-in from the ambient fluid to the oscillating structure. Our experimental measurements and numerical simulation of the flexible cylinder placed in the uniform inflow further demonstrate that such a phase difference for the cylinder orbit orientation affects the cross-flow added mass ($C_{my}$) as well. In general, $C_{my} < 0$ is a result of the out-of-phase between the cylinder motion and the vortex force and is found largely associated with $\theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$. In addition, strong correlation is observed between the vortex shedding mode of “2P” or “P+S” and $C_{my} < 0$, and therefore, a strong correlation between the cylinder orbit orientation and the wake mode exists as well. Such a relationship is also identified in the simulation of a forced vibrating rigid cylinder with prescribed in-line and cross-flow motions. In summary, by finding the strong correlation among the sign of the cross-flow added mass, the cylinder orbit orientation, and the vortex shedding mode from the rigid and flexible cylinders vibrate in the uniform inflow, our study helps us to illuminate the role of the added mass coefficient in bluff body VIVs. Nevertheless, as a fundamental fluid-structure interaction problem, there are many more new phenomena and mechanisms that need to be explored and explained. For example, it appears that the II motion plays an essential role to determine a large variation of hydrodynamic coefficients, even a small II amplitude may significantly change the instant separation point around the cylinder, and subsequently change the vortex shedding pattern. To this end, more systematic research are needed in the future to study the effect of different motion parameters on the hydrodynamic coefficients and vortical wake patterns of the bluff body undergoing VIVs.

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DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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