Interaction and Quantum Decoherence in Disordered Conductors

Andrei D. Zaikin\textsuperscript{a,1}, Dmitrii S. Golubev\textsuperscript{b}

\textsuperscript{a}Forschungszentrum Karlsruhe, Institut für Nanotechnologie, D-76021 Karlsruhe, Germany
\textsuperscript{b}I.E. Tamm Department of Theoretical Physics, P.N. Lebedev Physics Institute, Leninskii pr. 53, 117924 Moscow, Russia

Abstract

We present a nonperturbative approach which allows to evaluate the weak localization correction to the conductivity of disordered conductors in the presence of interactions. The effect of the electron-electron interaction on the magnetoconductance is described by the function $A(t) \exp(-f(t))$. The dephasing time is determined only by $f(t)$, and this time remains finite down to $T=0$ due to the electron-electron interactions. In order to establish the relation between our nonperturbative analysis and the perturbative results the effect of interaction on the pre-exponent $A(t)$ should be taken into account. The dephasing time cannot be unambiguously determined from a perturbative calculation.

Key words: Mesoscopic systems; weak localization; interaction and quantum decoherence.

1. Introduction

Recent experiments by Mohanty, Jariwala and Webb [1] attracted a lot of attention to the fundamental role of interactions in disordered mesoscopic systems. These experiments strongly indicate that the low temperature saturation of the electron decoherence time $\tau_\phi$ in disordered conductors has an intrinsic nature. Further analysis [4] allowed to rule out various experimental artifacts and several theoretical explanations based on extrinsic effects.

Can one expect the effect [1] theoretically? In [5] we developed a nonperturbative theoretical analysis of the above phenomenon. We demonstrated that the low temperature saturation of the decoherence rate $1/\tau_\phi$ in disordered mesoscopic systems can have an intrinsic nature: it can be caused by the electron-electron interaction. The results [5] are in a good agreement with experimental findings [1]. We also argued [6] that this interaction-induced decoherence at low $T$ has the same physical nature as in the case of a quantum particle interacting with a bath of harmonic oscillators [7–9].

In contrast, in [10,11] and in some other papers it was argued that interaction-induced dephasing of electrons at $T \to 0$ is not possible. Let us review the main arguments in favor of this conclusion [10,11].

One of such arguments is not necessarily related to electrons in a disordered metal. One can argue that a particle with energy $\sim T$ cannot excite harmonic oscillators with frequencies $\omega > T$ and, hence, the latter can only provide some renormalization effects. Within this scenario electron scat-
tering on such a static potential is purely elastic and cannot lead to dephasing.

It is easy to observe that the above argument explicitly contradicts to the exact results obtained e.g. within the Caldeira-Leggett model [7]. In this model even at $T = 0$ the off-diagonal elements of the particle density matrix decay at a finite length set by interaction with an effective environment. This effect is due to all high frequency modes, i.e. the picture is by no means “static”. Our results [5] demonstrate that also in disordered conductors the high frequency “quantum” modes with $\omega > T$ do contribute to electron dephasing.

One can also argue that the electronic system can behave differently from a bosonic one [7] because of the Pauli principle which restricts the electron ability to exchange energy at low $T$. Again, this argument contradicts to the well known results obtained for fermionic systems. E.g. tunneling electrons exchange energy with the effective environment (formed by other electrons in the leads) even at $T = 0$ [13,14]. This so-called “P(E)-theory” [13] yields measurable consequences and was verified in many experiments [15]. A close formal and physical similarity between the P(E)-theory and our analysis [5] is discussed in [16].

A more formal argument is based on a perturbative calculation [11]. The authors [11] argued that their results explicitly contradict to ours and, hence, the latter cannot be correct. A direct comparison between the above results is not quite simple. The main reason for that is that our calculation [5] is essentially nonperturbative. It was performed with the exponential accuracy which is sufficient to determine $\tau_\varphi$ and the weak localization correction to the conductance $\delta\sigma$ in the leading approximation for all values of the magnetic field. Aleiner et al. [11] considered the limit of strong magnetic fields $\tau_H \ll \tau_\varphi$ ($\tau_H$ is the decoherence time due to the magnetic field) in which one can calculate the subleading correction to $\delta\sigma$ perturbatively in the interaction. In this case the dephasing time $\tau_\varphi$ can be extracted from a perturbative expression for $\delta\sigma(H)$ only if one makes an additional assumption about the explicit form of the phase relaxation in time. No such assumption is needed within our analysis. In addition to that, the perturbative results [11] were presented only in the high temperature limit $T\tau_H \gg 1$ which, being combined with $\tau_H \ll \tau_\varphi$, is equivalent to a strong inequality $T\tau_\varphi \gg 1$. No such inequality was imposed in our calculation [5].

In this paper we establish a direct relation between perturbative and nonperturbative results in the problem of quantum dephasing in disordered conductors. We will demonstrate that (i) the perturbation theory in the interaction is insufficient for the problem in question and (ii) in order to determine $\tau_\varphi$ it is necessary to perform a nonperturbative analysis with the exponential accuracy. We analyze both the dephasing rate and the system magnetoconductance for various temperatures and the magnetic fields.

2. Insufficiency of the perturbation theory

Originally the dephasing time $\tau_\varphi$ of electrons in disordered conductors was introduced and evaluated by Altshuler, Aronov and Khmelnitskii within a phenomenological nonperturbative procedure [2]. Later Fukuyama and Abrahams [17] studied the problem perturbatively in the interaction. Since then the perturbative approach was frequently used in the analysis of quantum dephasing. We are going to demonstrate that this perturbative strategy is insufficient and does not allow to unambiguously determine the dephasing time even in those limits where the weak localization correction to the conductance can be calculated perturbatively in the interaction.

In what follows we will restrict our analysis to quasi-1d disordered systems. Extension of our results to higher dimensions is straightforward [18]. In the 1d case the weak localization correction to the wire conductance reads

$$
\delta\sigma_1(H) = -\frac{e^2\sqrt{D}}{\pi^{3/2}} \int_0^+ dt e^{-t/\tau_\varphi} F(t/\tau_\varphi). \tag{1}
$$
Here the function $F(t/\tau_\varphi)$ accounts for the electron-electron interaction ($F \equiv 1$ without interaction). Provided in the long time limit the function $F$ decays faster than $1/\sqrt{t}$ the integral (1) converges even for $1/\tau_H = 0$ and we get

$$\delta \sigma_1 = -a \frac{e^2}{\pi} \sqrt{D \tau_\varphi},$$  \hspace{1cm} (2)

where $a \sim 1$ depends on the particular form of the function $F$. This result is obviously nonperturbative in the interaction. E.g. if one formally defines the “interaction strength” $\lambda$ in the effective Hamiltonian, in certain limits one will obtain $1/\tau_\varphi \propto \lambda$ and, hence, $\delta \sigma_1 \propto 1/\sqrt{\lambda}$. For $\tau_H > \tau_\varphi$ any attempt to expand the conductance in powers of $\lambda$ is meaningless and may only yield divergences in all orders.

In order to avoid this problem the authors [11] suggested to consider the limit of strong magnetic fields $\tau_H \ll \tau_\varphi$. In this case the integral (1) is cut at times $t \sim \tau_H$ much shorter than $\tau_\varphi$ and the expansion of $\delta \sigma_1$ in powers of $\tau_H/\tau_\varphi \propto \lambda$ can be performed. Keeping only the leading terms of this expansion, from (1) one readily finds

$$\delta \sigma_1 - \delta \sigma_1^{(0)} \simeq -\frac{e^2 \sqrt{D \tau_H}}{2\pi} \frac{\tau_H}{\tau_\varphi} F'(0),$$  \hspace{1cm} (3)

where $\delta \sigma_1^{(0)} = -(e^2/\pi)\sqrt{D \tau_\varphi}$. We observe that a perturbative expansion in the interaction allows us to determine the combination $F'(0)/\tau_\varphi$ but not $\tau_\varphi$ itself. The full function $F(t/\tau_\varphi)$ (and thus $F'(0)$) will remain unknown. Although in the limit $\tau_H \ll \tau_\varphi$ the value $\delta \sigma_1(H)$ can be calculated perturbatively in the interaction, this would yield no information about the dephasing time $\tau_\varphi$. Such information can be extracted from the perturbation theory only if one assumes some particular form of the function $F(t/\tau_\varphi)$. But this form should be found rather than assumed. This task can be accomplished only if one goes beyond the perturbation theory.

In order to illustrate this conclusion let us consider several different functions $F(t/\tau_\varphi)$. Perhaps the most frequent choice of this function is

$$F_1(t/\tau_\varphi) = \exp(-t/\tau_\varphi).$$  \hspace{1cm} (4)

This assumption was also adopted in [11]. Performing a perturbative calculation in the high temperature limit $T \tau_H > 1$ the authors [11] arrived at the result $\delta \sigma_1 - \delta \sigma_1^{(0)} \propto T \tau_H^2$ which, being combined with (3) and (4), yields the dephasing rate

$$1/\tau_{\varphi1} \propto T \sqrt{\tau_H}.$$

Another possible choice of the function $F$ can be

$$F_2(t/\tau_\varphi) = \exp(-t/\tau_\varphi)^{3/2}.$$  \hspace{1cm} (5)

For $\tau_H > \tau_\varphi$ this function – as well as (4) – yields eq. (2). However, in the limit $\tau_H \ll \tau_\varphi$ from (1) and (5) one finds

$$\delta \sigma_1 - \delta \sigma_1^{(0)} \simeq \frac{e^2 \sqrt{D \tau_H}}{\pi \tau_\varphi^3} \left( \frac{\tau_H}{\tau_\varphi} \right)^{3/2}.$$  \hspace{1cm} (6)

Clearly, this result is incompatible with eq. (3) because $F'(0)$ should not depend on $\tau_H$ and $\tau_\varphi$. At the same time from (5) one has $F'(0) = 0$, and again the contradiction between (3) and (6) is obvious. Also, combining (6) with the perturbative results [11] one arrives at $1/\tau_{\varphi2} \propto T^{2/3}$ in agreement with [2,3] but in a clear disagreement with $1/\tau_{\varphi1}$ found in [11].

Finally, let us choose the trial function $F$ in the following form:

$$F_3(t/\tau_\varphi) = \frac{e^{-t/\tau_\varphi} \sqrt{b t}}{\sqrt{\tau_\varphi(1 - e^{-b t/\tau_\varphi})}}.$$  \hspace{1cm} (7)

where $b \sim 1$. For $\tau_H > \tau_\varphi$ the result (2) is recovered again, while in the limit $\tau_H \ll \tau_\varphi$ one arrives at eq. (3) with $F'(0) = b/4 - 1$. Combining this result with those of [11] one finds $\tau_{\varphi3} \propto (4 - b)/(T \tau_H^{1/2})$. For $b < 4$, $b = 4$ and $b > 4$ this result would imply respectively positive, infinite and even negative dephasing rates, which is an obvious nonsense.

The above examples clearly demonstrate that a perturbative procedure is principally insufficient for our problem because it leads to completely ambiguous results for $\tau_\varphi$. At the same time the nonperturbative eq. (1) yields practically indistinguishable magnetoconductance curves (see Fig. 1 of [18]) and the same dephasing time $\tau_\varphi$ (up to a prefactor $a \sim 1$ which can be absorbed in $\tau_\varphi$ anyway) for all the trial functions $F_1$, $F_2$ and $F_3$. 

3
In order to obtain correct information about $\tau_\varphi$ one should determine the function $F$ at times $t \sim \tau_\varphi$. This can be done only by means of a nonperturbative calculation simply because for $t \sim \tau_\varphi$ there exists no small parameter in the problem. Choosing the limit $\tau_H < \tau_\varphi$ enables one to find $\delta \sigma_1 (H)$ but not $\tau_\varphi$. We can also add that the phenomenological procedure [2] allows to non-perturbatively treat only the contribution of “classical” modes $\omega < T$, whereas the effect of “quantum” modes $\omega > T$ is accounted for by means of the nonperturbative analysis [5]. This analysis will be extended further in the next section.

3. Nonperturbative results

Let us express the function $F$ (1) in the form

$$F (t) = A (t) \exp (- f (t)) \quad (8)$$

Without interaction one has $A (t) \equiv 1$ and $f (t) \equiv 0$. For a complete description of the effect of interaction on the weak localization correction (1) it is necessary to evaluate both functions $f (t)$ and $A (t)$. An important observation is, however, that information about the effect of interaction on $A (t)$ is not needed to correctly evaluate the dephasing time $\tau_\varphi$, it is sufficient to find only the function $f (t)$ which describes the decay of correlations in time and provides an effective cutoff for the integral (1) at $t \sim \tau_\varphi$. The role of the pre-exponent $A (t)$ is merely to establish an exact numerical prefactor. Since $\tau_\varphi$ is defined up to a numerical prefactor of order one anyway, it is clear that only the function $f (t)$ — and not $A (t)$ — is really important.

3.1. Exponent

The function $f (t)$ can be straightforwardly evaluated by means of the path integral formalism. This procedure amounts to calculating the path integral for the kernel of the evolution operator over the particle coordinate $r$ and the momentum $p$ [5]

$$J \sim \int D r \int D p \exp (i S_0 - i S'_0 - i S_R - S_I) \quad (9)$$

within the saddle point approximation on pairs of time reversed paths and to averaging over diffusive trajectories. Here $S_0$ and $S'_0$ represent the electron action on the two parts of the Keldysh contour, while $i S_R + S_I$ accounts for the interaction. The general expressions for $S_R$ and $S_I$ were derived in [5]. Within RPA these expressions contain the full information about the effect of interaction in all orders of the perturbation theory.

The saddle point approximation procedure was also described in details in [5]. One can demonstrate that the contribution of the real part $S_R$ of the action vanishes on any pair of diffusive time reversed paths. Note that such cancellation is a generic property of a wide class of influence functionals describing dissipative environments. E.g. the same cancellation is observed in the Caldeira-Leggett model [7] and in some other models [9]. Hence, the function $f (t)$ in the exponent (8) is determined only by the imaginary part of the action $S_I$ [5]. For the sake of generality here we will present the result for all dimensions. Evaluating $S_I$ on pairs of time reversed saddle point paths we find [18]

$$f (t) = \frac{4 e^2 D^{1-d/2}}{\sigma_d (2 \pi)^d} \int \frac{d^d x}{1 + x^2} \int \frac{d \omega \ d \omega'}{(2 \pi)^2} (1 - \cos \omega t)$$

$$\times \left[ \frac{|\omega'|^{d/2 - 2} (\omega - \omega') \coth \frac{\omega - \omega'}{2T}}{2 \omega} + \frac{|\omega'|^{d/2 - 2} \coth \frac{\omega - \omega'}{2T} + |\omega|^{d/2 - 2} \omega' \coth \frac{\omega'}{2T}}{\omega^2 - \omega'^2} \right] \quad (10)$$

Note that in 1d and 2d cases the integral of the first term over $\omega'$ diverges at $\omega' \to 0$. However, this divergence is exactly canceled by the second term and the whole integral is finite in any dimension.

Evaluating the result (10) for a 1d case in the quantum regime $\pi T t \ll 1$ we obtain

$$f (t) \simeq \frac{e^2}{\pi \sigma_1} \sqrt{2 D T \tau_\varphi} t + \frac{2 e^2}{\pi \sigma_1} \sqrt{\frac{D t}{\pi}} \left( \ln \frac{2 \pi t}{\tau_\varphi} - 6 \right) \quad (11)$$
where $\tau_e = v_F/l$ is the elastic mean free time. Note, that apart from the leading linear in time term there exists a smaller term $\propto \sqrt{\ln(t/\tau_e)}$, which also grows in time. In the opposite thermal limit $\pi T t \gg 1$ eq. (10) yields

$$f(t) \approx \frac{e^2}{\pi \sigma_1} \sqrt{\frac{2D}{\tau_e}} \left[ t + \frac{2\sqrt{2\pi \tau_e} \pi T t^{3/2}}{3} + \frac{\pi^{1/2} \zeta(1/2)}{\sqrt{2}} t\sqrt{T \tau_e} \right], \quad (12)$$

where $\zeta(x)$ is the dzeta-function. We observe that in both cases (11) and (12) there exists a linear in time temperature independent contribution to $f(t)$ which determines the dephasing time $\tau_\varphi$ at low temperatures [5,6]. Beside that at $T t \gg 1$ there exists another term $\propto T \tau_e^{3/2}$ which yields dominating contribution to $\tau_\varphi$ at high temperatures $T > T_q \sim 1/\sqrt{\sigma e}$, where the result [2] $\tau_\varphi \propto T^{-2/3}$ is recovered.

3.2. Pre-exponent

A rigorous calculation of the pre-exponential function $\tilde{A}(t) = A(t)/\sqrt{T}$ in (1) in the presence of interactions is beyond the frames of the present paper. Fortunately the precise form of $\tilde{A}(t)$ for all times is not interesting for us here. Of importance is to qualitatively understand how the function $\tilde{A}(t)$ is modified in the presence of the electron-electron interaction.

It is well known [3,19] that without interactions the function $\tilde{A}(t)$ is related to the return probability of diffusive trajectories to the same point after the time $t$. In the presence of dissipation (described by the term $S_H$ in the effective action) the particle energy decreases and its diffusion slows down. This implies that at any given time $t$ the function $\tilde{A}(t)$ should exceed $1/\sqrt{t}$. On the other hand, at least if the interaction is sufficiently weak, diffusion will take place at all times and, hence, $\tilde{A}(t)$ will always decay as $1/\sqrt{t}$ or slower.

The latter – intuitively obvious – property of the pre-exponent was confirmed by our analysis [18]. The function $\tilde{A}(t)$ indeed decays at all times and no compensation of the exponential decay $\propto \exp(-f(t))$ by the pre-exponential function $\tilde{A}(t)$ can occur in the long time limit. Another important property [18] is that in the interesting limit of low temperatures the effect of interaction on the pre-exponent becomes important at $t \sim \tau_\varphi$, i.e. on the same time scale as for the function $f(t)$ in the exponent. These two properties allow to completely ignore the effect of interaction on the pre-exponent [5].

In the short time limit $t \ll \tau_\varphi$ the correction to the pre-exponent due to interaction is small and one can proceed perturbatively in the interaction. At $T \to 0$ in the leading approximation one finds [18]

$$A(t) \approx 1 + \frac{e^2}{\pi \sigma_1} \sqrt{\frac{2D}{\tau_e}} t, \quad (13)$$

This equation is important for deriving the perturbative expressions for the magnetoconductance in the limit $\tau_H \ll \tau_\varphi$. The corresponding results are presented below.

4. Perturbation theory for the conductance

Let us evaluate the weak localization correction (1) in the limit of strong magnetic fields $\tau_H \ll \tau_\varphi$. Performing the short time expansion of both $\exp(-f(t))$ and $A(t)$ to the first order and combining (1), (8) with (11), (13), in the limit $T \tau_H \ll 1$ we find

$$\delta \sigma_1 - \delta \sigma_1^{(0)} \approx \frac{e^2}{\pi} \frac{e^2}{\sigma_1} \frac{2D \tau_H}{\pi^2} \left( \ln \left( \frac{\tau_H}{\tau_e} \right) - 3.74 \right). \quad (14)$$

It is easy to observe that this result does not contain the zero temperature dephasing time at all. The linear in time terms in the expressions for $f(t)$ (11) and $A(t)$ (13) cancel each other exactly in the first order. The same cancellation occurs in the limit $T \tau_H \gg 1$. Again combining (1), (8) with (12), (13) and expanding $\exp(-f(t))$ to the first order in $f$, at $T \tau_H \gg 1$ we obtain

$$\delta \sigma_1 - \delta \sigma_1^{(0)} \approx \frac{e^2}{\pi} \frac{e^2}{\sigma_1} DT \tau_H^2 \left[ \frac{4}{3\pi} + \frac{\zeta(1/2)}{2\sqrt{2\pi T \tau_H}} \right]. \quad (15)$$
In order to establish the exact numerical prefactor in front of the last term in (15) we also took into account the \( T \)-dependent linear in time contribution to \( A(t) \) [18] omitted in eq. (13). We note that the result (15) agrees (up to a numerical prefactor of order one in front of the term \( \propto T \tau_H^2 \)) with the perturbative result [11] obtained in the same limit \( T \tau_H \gg 1 \). However, as it was already discussed above, no information about the dephasing time at low \( T \) can be extracted from (15).

Finally, let us derive the expression for the weak localization correction in the limit of high temperatures and weak magnetic fields \( \tau_H \gg \tau_\varphi \). In this limit the phase relaxation is determined by the term \( f_{cl} \propto T^{3/2} \) in (12). Keeping this term in the exponent, expanding the exponent in \( f(t) - f_{cl}(t) \) to the first order and making use of a short time expansion of \( A(t) \) we get

\[
\delta \sigma_1 \simeq -\left( \frac{2e^4\sigma_1 D}{9\pi^4 T} \right)^{1/3} \Gamma(1/3) + \zeta(1/2) e^2 \sqrt{D} \frac{\sqrt{T}}{(2\pi)^{3/2} \sqrt{T}}. \tag{16}
\]

Here \( \Gamma(x) \) is the Euler’s gamma-function. The first term in the right-hand side of this expression corresponds to the classical result [2], while the second term represents the first correction to this result. With decreasing temperature this correction grows faster than the absolute value of the first term. At the same time in the high temperature limit the second term in (16) always remains much smaller than the first one and the higher order terms should also be taken into account already at \( T > T_q \).

Note that the result (16) – as well as eqs. (14), (15) – does not depend on the zero temperature value of \( \tau_\varphi \), this value drops out due to the same cancellation of the linear in time \( T \)-independent terms from the exponent and the pre-exponent. Thus also the first order high temperature expansion of the weak localization correction to the conductance cannot provide correct information about the electron dephasing time at low temperatures.

In conclusion, we established an explicit relation between perturbative and nonperturbative results in the problem of quantum dephasing in disordered conductors. The dephasing time remains finite down to \( T = 0 \) due to the electron-electron interactions.

References

[1] P. Mohanty, E.M.Q. Jariwala, and R.A. Webb, Phys. Rev. Lett. 78, 3366 (1997); Fortsch. Phys. 46, 779 (1998).
[2] B.L. Altshuler, A.G. Aronov, and D.E. Khmelnitskii, J. Phys. C 15, 7367 (1982).
[3] B.L. Altshuler and A.G. Aronov, in Electron-Electron Interactions in Disordered Systems, edited by A.L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p.1.
[4] P. Mohanty, this volume and further Refs. therein.
[5] D.S. Golubev and A.D. Zaikin, Phys. Rev. Lett. 81, 1074 (1998); Phys. Rev. B 59, 9195 (1999).
[6] D.S. Golubev and A.D. Zaikin, Physica B 225, 164 (1998).
[7] A.O. Caldeira and A.J. Leggett, Physica A 121, 587 (1983); 130, 374 (1985).
[8] D. Loss and K. Mullen, Phys. Rev. B 43, 13252 (1991).
[9] U. Weiss, Quantum Dissipative Systems (World Scientific, Singapore, second enlarged edition, 1999).
[10] B.L. Altshuler, M.E. Gershenson, and I.L. Aleiner, Physica E 3, 58 (1998).
[11] I.L. Aleiner, B.L. Altshuler, and M.E. Gershenson, Phys. Rev. Lett. 82, 3190 (1999); cond-mat/9808053.
[12] D.S. Golubev and A.D. Zaikin, Phys. Rev. Lett. 82, 3191 (1999); cond-mat/9811118.
[13] Yu. V. Nazarov, Sov. Phys. JETP 68, 561 (1989); M.H. Devoret et al. Phys. Rev. Lett. 64, 1824 (1990); S.M. Girvin et al., ibid. 64, 1565 (1990).
[14] G. Schön and A.D. Zaikin, Phys. Rep. 198, 237 (1990).
[15] L.S. Kuzmin et al., Phys. Rev. Lett. 67, 1161 (1991); D.B. Haviland et al., Europhys. Lett. 16, 103 (1991); A.N. Cleland, J.M. Schmidt, and J. Clarke, Phys. Rev. B. 45, 2950 (1992); P. Joyez, D. Esteve, and M.H. Devoret, Phys. Rev. Lett. 80 1956 (1998).
[16] D.S. Golubev and A.D. Zaikin, to appear in: Quantum Physics at Mesoscopic Scale eds. D.C. Glattli, M. Sassetti, J. Tran Thanh Van (Frontieres, 1999); cond-mat/9907493.
[17] H. Fukuyama and E. Abrahams, Phys. Rev. B 27, 5976 (1983).
[18] D.S. Golubev and A.D. Zaikin, cond-mat/9907494.
[19] S. Chakravarty and A. Schmid, Phys. Rep. 140, 193 (1986).