Negative refractive index in cubic noncentrosymmetric superconductors

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We study the negative refractive index in cubic noncentrosymmetric superconductors. We consider the Maxwell equations under the Meissner effect, and the magnetoelectric effect arising due to inversion-symmetry breaking. We derive the dispersion relations of electromagnetic waves and show that the refractive index becomes negative at frequencies just below the plasma gap. We find that the chiral mechanism of the negative refractive index, which is usually discussed in chiral materials with negative permittivity, can be directly applied to noncentrosymmetric superconductors with positive permittivity. A rough estimation from experimental values of the penetration depth of LiPt$_3$B indicates that a negative refractive index may appear in UV regions.

1. Introduction

The magnetoelectric effect has caught the attention of researchers in studies of condensed-matter systems with time-reversal or inversion-symmetry breaking [1–4]. In such materials, cross polarization/magnetization is induced by magnetic/electric fields, and leads to optical phenomena such as optical activity [5], or to transport phenomena such as the gyrotropic current induced by AC magnetic fields in noncentrosymmetric conductors [6] ($j = -\kappa \partial_t B$). Such an AC current was rediscovered in the study of the chiral magnetic effect [7] in Weyl semimetals with inversion-symmetry breaking [8,9], and is sometimes referred to as the gyrotropic magnetic effect or dynamic chiral magnetic effect.

The magnetoelectric effect plays an essential role in the realization of the negative refractive index in chiral materials [10–13] or topological metals [14]. In the original route proposed by Veselago [15], the negative refractive index is realized via double negative permittivity and permeability, which are unlikely in natural materials, and realized only in artificial metamaterials [16]. However, via the magnetoelectric effect, the negative refractive index can be realized in a broader range of materials with time-reversal or inversion-symmetry breaking. Thus, the chiral mechanism may provide a chance for broad applications. This mechanism is based on two effects: (I) The degenerate gap of transverse waves originating from plasma screening. (II) The helical lift of the degeneracy in momentum space originating from the magnetoelectric effect [10–13] or the chiral magnetic effect [14], which is analogous to the lift of the spin degeneracy due to the Rashba spin–orbit coupling [17].
Now we address the question of whether other types of materials realize the chiral mechanism or not. (I) It is well known that photons acquire the mass (gap) in superconductors via the Meissner effect or the Anderson–Higgs mechanism. (II) It is known that the magnetoelectric effect arises in noncentrosymmetric superconductors [18,19]. Thus, we expect that noncentrosymmetric superconductors exhibit a negative refractive index.

In this paper, we discuss the negative refractive index in cubic noncentrosymmetric superconductors. We study the Maxwell equations under the Meissner and magnetoelectric effects. By computing the dispersion relations of electromagnetic waves, we show that the refractive index indeed becomes negative for either of the circular polarizations at frequencies below the plasma frequency. We roughly estimate the plasma frequency from the penetration depth observed in experiments, which indicates that the negative refractive index may occur in UV regions. Finally, the reflection and transmission coefficients in an insulator–superconductor–insulator junction are computed as an experimental implication.

2. Gyrotropic magnetic effect

We start with a computation of electric currents in cubic noncentrosymmetric superconductors under electromagnetic fields on the basis of the Abelian–Higgs model with corrections from inversion-symmetry breaking. The Abelian–Higgs model can be used as a low-energy effective theory of nonrelativistic superconductors with emergent particle–hole symmetry [20,21] as well as relativistic superconductors. The action is given as

\[ S = S_{EM} + S_s + S_{LI} \]

\[ = \int dt d^3x \left( L_{EM} + L_s + L_{LI} \right), \]  

(1)

\[ L_{EM} = \frac{1}{8\pi} \left( E^2 - B^2 \right), \]  

(2)

\[ L_s = \frac{1}{2} \left| \left( \frac{\hbar}{ic} \frac{\partial}{\partial ct} + e^s A_0 \right) \psi \right|^2 - \frac{1}{2} \left| \left( \frac{\hbar}{i} \nabla + e^s A \right) \psi \right|^2 \]

+ \alpha |\psi|^2 - \frac{\beta}{2} |\psi|^4, \]

(3)

\[ L_{LI} = -\frac{1}{2} \sum_{ij} d_{ij} B_i \cdot \text{Re} \left[ \psi^* \left( \frac{\hbar}{i} \nabla + \frac{e^s A}{c} \right) \psi \right], \]

(4)

where \(-e^s\) is the electric charge of the complex scalar condensate \(\psi\) and \(\hbar\) and \(c\) are the Planck constant and speed of light, respectively. \(A_0, A, E = -\nabla A_0 - \partial_t A/c, \) and \(B = \nabla \times A\) are the scalar potential, vector potentials, electric fields, and magnetic fields, respectively. We assume that the permittivity \(\epsilon\) and permeability \(\mu\) are isotropic and unity just for simplicity. \(L_{LI}\) is the so-called Lifshitz invariant [18,19]. It is invariant under the time-reversal operation, but not under the spatial-inversion operation, so that it characterizes the noncentrosymmetric nature of superconductors. Below we consider a cubic system, and take \(d_{ij} = d\delta_{ij}\).
From variation of the matter parts $S_s$ and $S_{LI}$ with respect to $A$, we obtain

$$j_{tot} = \frac{\delta (S_s + S_{LI})}{\delta A/c}$$

$$= -e^* \left( \hbar \nabla \theta + \frac{e^*}{c} A \right) |\psi|^2$$

$$- de^* B |\psi|^2 - dc \nabla \times \left( \left( \hbar \nabla \theta + \frac{e^*}{c} A \right) |\psi|^2 \right),$$

where $\psi = |\psi| e^{i\theta}$. The sum of the three terms on the right-hand side is the total electric current. The first term is the standard superconducting (diamagnetic) current. The last two terms express the effect of inversion-symmetry breaking. In noncentrosymmetric superconductors, an electric current is induced by magnetic fields via the magnetoelectric effect, in addition to the diamagnetic current. Below we consider the uniform state, in which $|\psi|^2 = \beta/\alpha = n_s$ (we assume $\alpha, \beta > 0$) and $\nabla \theta = 0$ by a gauge choice. Then we have

$$j_{tot} = -\left( \frac{e^*}{c} \right)^2 n_s A - 2dn_se^* B.$$

(6)

The second term in Eq. (6) has the same form as the gyrotropic magnetic effect in clean Weyl semimetals with inversion-symmetry breaking [8,9], and because of the formal similarity, the chiral mechanism in topological metals [14] can be applied to noncentrosymmetric superconductors. We note, however, that although we have assumed the absence of a phase gradient to derive Eq. (6), it strongly depends on the physical situation, and in some cases, such as the ground state of an open sample, the phase gradient cancels out the second term in Eq. (6) [22,23]. Since the second term plays an essential role in our chiral mechanism, the negative refractive index can be realized only by preparing special physical setups in which the cancellation does not occur; the second term becomes nonzero, e.g., in special geometry such as a thin film connected to external superconductor reservoirs of fixed phases [22,23], or in the uniform nonequilibrium (transient) state realized by suddenly applying electromagnetic fields to the uniform ground state in vanishing electromagnetic fields.

### 3. Negative refractive index

Now we study the dispersion relations of electromagnetic waves. From variation of the total action (1) with respect to $A_0$ and $A$, and from the Bianchi identities, we obtain the Maxwell equations as

$$\nabla \cdot E = -4\pi \left( \frac{e^*}{c} \right)^2 n_s A_0,$$

(7)

$$\nabla \times B = \frac{1}{c} \partial_t E + \frac{4\pi}{c} j_{tot},$$

(8)

$$\nabla \cdot B = 0,$$

(9)

$$\nabla \times E = -\frac{1}{c} \partial_t B.$$

(10)

From now on, we assume a monochromatic electromagnetic wave of the form $E = \tilde{E} e^{-i\omega t + ip \cdot x}$ and $B = \tilde{B} e^{-i\omega t + ip \cdot x}$ with frequency $\omega$ and wavevectors $p$. Then the Maxwell equations are reduced to
where \( \lambda = \sqrt{c^2/(4\pi n_0(e^*)^2)} \) and \( \delta = cd/e^* \) are the penetration depth and the effective length characterizing the inversion-symmetry breaking, and \( ip \times E = (t \cdot p)E, \) with \( t \) being \( 3 \times 3 \) matrices: \([t_{ij}]_{jk} = -i\epsilon_{ijk} \) (\( \epsilon_{ijk} \) are totally anti-symmetric tensors with \( \epsilon_{123} = 1 \)). The Maxwell equations (11) and (12) are \( 3 \times 3 \) matrix equations, and diagonal in circular polarizations satisfying

\[
i p \times E_{\pm} = t \cdot p E_{\pm} = \pm p E_{\pm},
\]

\[
i p \times B_{\pm} = t \cdot p B_{\pm} = \pm p B_{\pm}.
\]

The dispersion relations of the circular lights read

\[
\omega_{\chi \pm} = \pm c \sqrt{\left( p_{\chi} + \chi \frac{\delta}{\lambda^2} \right)^2 + \frac{1}{\lambda^2},}
\]

for the right-handed (\( \chi = + \)) (left-handed (\( \chi = - \)) circular polarization, and \( \tilde{\lambda} = \lambda/\sqrt{1 - \delta^2/\lambda^2} \) is the effective penetration depth [24]. The dispersion relations (15) have the same form as in Ref. [14], so that the analysis in Ref. [14] can be directly applied if we replace the plasma gap \( \omega_p \) in Ref. [14] by it due to the Anderson–Higgs mechanism \( c/\lambda \). As will be discussed in detail, the dispersion relations (15) show different characteristic behaviors in three regimes: (a) \( \delta = 0 \), (b) \( |\delta| < \lambda \), and (c) \( |\delta| > \lambda \). When the system is centrosymmetric, i.e., \( \delta = 0 \), two transverse modes are degenerate with gapped dispersion relations. The electromagnetic fields are exponentially expelled from superconductors, which is known as the Meissner effect. In noncentrosymmetric superconductors with nonzero \( \delta \), the helical degeneracy is lifted, and either of them has a negative refractive index just below the plasma gap \( c/\lambda \).

Now we discuss the negative refractive index. We first study the negative sign of the group velocity \( v_{g\chi} \) compared to the phase velocity \( v_{p\chi} \) [11,14]. These are defined as

\[
v_{g\chi} = \frac{\partial \omega_{\chi \pm}}{\partial p_{\chi}} = \frac{p_{\chi} + \chi \frac{\delta}{\lambda^2}}{\sqrt{\left( p_{\chi} + \chi \frac{\delta}{\lambda^2} \right)^2 + \frac{1}{\lambda^2},}}
\]

\[
v_{p\chi} = \frac{\omega_{\chi \pm}}{p_{\chi}} = \frac{\sqrt{\left( p_{\chi} + \chi \frac{\delta}{\lambda^2} \right)^2 + \frac{1}{\lambda^2}}}{p_{\chi}}.
\]

We consider the regime (b) \( |\delta| < \lambda \). Then the argument of the square root is positive. \( v_{g\chi} \) has a negative sign compared to \( v_{p\chi} \) when \( |p_{\chi}| < |\delta|/\lambda^2 \), i.e., when \( c/\tilde{\lambda} < \omega_{\chi} < c/\lambda \) for either of the circular polarizations. If \( \omega_{\chi} < c/\tilde{\lambda} \), both of them are expelled from superconductors, so that \( \tilde{\lambda} \) works as the effective penetration depth. Next we consider the regime (c) \( |\delta| > \lambda \). Then the dispersion relations (15) become pure imaginary at some momentum range and indicate unstable modes analogous to chiral plasma instabilities [25]. However, when we discuss transmission of electromagnetic waves, we need only the refractive index \( n(\omega) = cp(\omega)/\omega \) with positive-real \( \omega \) as information on the dispersion relations, so we do not encounter unstable modes. It would be
interesting to study the real-time dynamics in the regime (c), where an analogue of chiral plasma instabilities may affect the dynamic transition between the magnetic helicity and (quantized) vorticity of quantum vortices. Anyway, we consider positive $\omega$, and then a gap appears at $|p_\chi + \chi \delta/\lambda^2| < \sqrt{(\delta^2 - \lambda^2)/\lambda^2}$. Below the gap, the argument is still positive, and $v_{g\chi}$ has a negative sign compared to $v_{p\chi}$ at all frequencies lower than $c/\lambda$.

We can directly discuss the negative refractive index. By rewriting Eq. (15) into the $\omega$-dependence of $p$, the refractive index reads

$$n_\chi = \frac{c p_\chi + \chi \delta}{\omega \chi} = -\frac{\chi c \delta}{\lambda^2} + \frac{\sqrt{\omega^2 c^2/\lambda^2}}{\omega \chi}$$

from the positive sign solution of the quadratic equation on $p$. In the regime (b) $|\delta| < \lambda$, we have $0 < \sqrt{\omega^2 c^2/\lambda^2} < c|\delta|/\lambda^2$ at $c/\lambda < \omega \chi < c/\lambda$, so that the refractive index (18) is negative for either of the circular polarizations. Next, in the regime (c) $|\delta| > \lambda$, we have $c \sqrt{\delta^2 - \lambda^2}/\lambda < \sqrt{\omega^2 c^2/\lambda^2} < c|\delta|/\lambda^2$ at $0 < \omega \chi < c/\lambda$, so that the refractive index (18) is negative for either of the circular polarizations. The plasma gap vanishes in the regime (c), which means that electromagnetic waves can propagate inside superconductors even though they experience the Meissner effect. This can be understood from the fact that the effective penetration depth $\tilde{\lambda}$ becomes pure imaginary, and can explicitly be seen from the refractive index (18). In the regime (a) or (b), the refractive index has an imaginary part if $\omega \chi < c/\tilde{\lambda}$, and electromagnetic waves become evanescent. On the other hand, the argument of the square root in Eq. (18) is always positive in the regime (c). No evanescent wave appears, and even a standing wave with $\omega \chi = 0$ is possible although it is spatially inhomogeneous [24].

Two remarks are in order: (I) In this analysis, we consider only the Lifshitz invariant (4) as the effect of the inversion-symmetry breaking [18,19]. This may implicitly assume that $\delta$ is small compared to $\lambda$. Practically, we may need to take higher-order inversion-symmetry breaking terms into account to study the regime (c). We note that in the first order of $\delta/\lambda$, the negative refractive index disappears since $\tilde{\lambda} = \lambda$. (II) We neglected the oscillation of the amplitude of the condensate, i.e., the oscillation of the Higgs mode caused by applying electromagnetic fields. This approximation is valid if the applied electromagnetic fields are off-resonance [20]. We can correctly treat these effects by computing the dispersion relations of lights on the basis of a microscopic theory such as a Bardeen–Cooper–Schrieffer-type Hamiltonian. However, this is beyond the scope of this paper.

The representative materials of cubic noncentrosymmetric superconductors are Li(Pd$_{1-x}$Pt$_x$)$_3$B (We also need the absence of mirror symmetry to have a nonvanishing magnetoelectric effect) [18,19]. The penetration depth of these materials is 190–364 nm at zero temperature [26] and is less than 400 nm at low temperatures [27], so that a negative refractive index may occur in UV regions, i.e., wavelengths shorter than any other material observed so far. Among them, LiPt$_3$B is expected to have the large inversion-symmetry breaking [27], and is most likely to exhibit a negative refractive index.

4. Transmission coefficients

We here compute the reflection and transmission coefficients of electromagnetic waves in an insulator–superconductor–insulator junction (the thickness of the superconductors is $l$). We consider a right- or left-handed electromagnetic wave normally incident from a nonmagnetic insulator.
(dielectric) with the refractive index $n_0 (> 0)$ as shown in Fig. 1. The transmission coefficients can be used to experimentally retrieve the negative refractive index [28].

When $|\psi|^2$ changes by the step function, the surface Hall conduction arises from the last term in Eq. (5):

$$j_s = \pm \frac{ie de n_s}{\omega} \hat{z} \times \mathbf{E} \delta(z - z_s),$$

where the sign is $+ (−)$ for the left surface ($z_s = 0$) (right surface ($z_s = l$)). Due to the surface Hall conduction (19), the boundary condition on the tangential components of magnetic fields is modified as $\hat{z} \times (\mathbf{B}_{\text{Air}} - \mathbf{B}_{\text{SM}}) = -\hat{z} \times 4\pi ide n_s \mathbf{E}/\omega$, so that the tangential components of $\mathbf{B} = \mathbf{B} - ide n_s \mathbf{E}/\omega$ are continuous at boundaries instead of $\mathbf{B}$ itself, as well as the tangential components of $\mathbf{E}$.

The electric fields in the three regions in Fig. 1 are written as $\mathbf{E}_x = \mathbf{E}_x e^{-i\omega t} (\hat{x} + i\hat{y})$ with

$$\mathbf{E}_x = \begin{cases} e^{in_0 oz/c} - R_x e^{-in_0 oz/c} & z \leq 0 \\ B_x e^{in_x oz/c} - C_x e^{-in_x oz/c} & 0 \leq z \leq l, \\ T_x e^{in_0 (z-l)/c} & l \leq z \end{cases}$$

where $\omega$ is the frequency in an incident dielectric. We take the amplitude of the incident wave as unity, and then the reflection and transmission coefficients are nothing but $R_x$ and $T_x$. The amplitudes of the magnetic waves are obtained as $B_x = c\rho \times \mathbf{E}_x/\omega$. Then, from the continuity conditions at $z = 0$ and $z = l$ on $\mathbf{E}$ and $\mathbf{B}$, the reflection and transmission coefficients read

$$R_x = \frac{(n_r^2 - 1) \sin \kappa n_r}{2in_r \cos \kappa n_r + (1 + n_r^2) \sin \kappa n_r},$$

$$T_x = \frac{2in_r e^{-\chi \delta l/(n_0 \lambda^2)}}{2in_r \cos \kappa n_r + (1 + n_r^2) \sin \kappa n_r},$$

where $n_r = (n_+ + n_-)/(2n_0) = \sqrt{1 - c^2/\lambda^2} n_0$ and $\kappa = n_0 \omega l/c$. Equations (21) and (22) reproduce those in the gyrotropic medium [29]. The azimuth rotation and ellipticity of the transmitted waves are obtained from Eq. (22) as

$$\theta_T = (\text{Arg } T_+ - \text{Arg } T_-)/2 = -\delta l/(n_0 \lambda^2) \quad \text{and} \quad \eta_T = (1/2) \sin^{-1}(|T_+|^2 - |T_-|^2)/(|T_+|^2 + |T_-|^2) = 0.$$

Thus we can extract $\delta$ from $\theta_T$. $|T_+|$ or $|T_-|$ shows resonant perfect transmission if $\kappa n = \pi m$ with $m$ being some integer, which is the same as in conventional dielectric–metal–dielectric junctions. Similarly, the azimuth rotation and ellipticity of the reflected waves become $\theta_R = \eta_R = 0$, as expected from the time-reversal symmetry [29].
5. Summary

In this paper, we have discussed the negative refractive index in cubic noncentrosymmetric superconductors. We have studied the Maxwell equations under the Meissner and magnetoelectric effects. The latter arises due to the lack of inversion symmetry. We show that the chiral mechanism of the negative refractive index [10–14] can be directly applied to noncentrosymmetric superconductors. We have also computed the reflection and transmission coefficients in an insulator–superconductor–insulator junction as an experimental implication.

The candidate material to experimentally test our mechanism is Li(Pd$_{1-x}$Pt$_x$)$_3$B. In particular, large inversion-symmetry breaking is expected in LiPt$_3$B. A rough estimation from experimental values of the penetration depth indicates that LiPt$_3$B may exhibit a negative refractive index in UV regions.

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