Investigation of $B_{u,d} \rightarrow (\pi,K)\pi$ decays within unparticle physics

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A B S T R A C T

We investigate the implication of unparticle physics on the $B_{u,d} \rightarrow (\pi,K)\pi$ decays under the constraints of the $B_{d,s}$-$B_{d,s}$ mixing. We found that not only the unparticle parameters that belong to the flavor changing neutral current (FCNC) processes but also scaling dimension $d_{d}$ could be constrained by the $B_{d,s}$-$B_{d,s}$ mixing phenomenon. Employing the minimum $\chi^2$ analysis to the $B_{u,d} \rightarrow (\pi,K)\pi$ decays with the constraints of $B_{d,s}$ mixing, we find that the puzzle of large branching ratio for $B_d \rightarrow \pi^0\pi^0$ and the discrepancy between the standard model estimation and data for the direct CP asymmetry of $B^+ \rightarrow K^+\pi^0$ and $B_d \rightarrow \pi^+\pi^-$ can be resolved well. However, the mixing induced CP asymmetry of $B_d \rightarrow K_S\pi^0$ could not be well accommodated by the unparticle contributions.

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1. Introduction

Recently some incomprehensible phenomena at B factories have been explored, especially $B_{d,s} \rightarrow (\pi,K)\pi$ decays. Firstly, the observations on the large branching ratio (BR) for $B_d \rightarrow \pi^0\pi^0$ decay with the world average $\mathcal{B}(B_d \rightarrow \pi^0\pi^0) = (1.31 \pm 0.21) \times 10^{-6}$ and the direct CP asymmetry for $B_d \rightarrow \pi^+\pi^-$ with $\mathcal{A}_{CP}(B_d \rightarrow \pi^+\pi^-) = 0.38 \pm 0.07$ [1] are inconsistent with the theoretical estimations of around $0.5 \times 10^{-6}$ and $10\%-20\%$, respectively. Secondly, a disagreement in the CP asymmetries (CPAs) for $B_d \rightarrow K^+\pi^-$ and $B^+ \rightarrow K^+\pi^0$ has been observed to be $-0.097 \pm 0.012$ and $0.050 \pm 0.025$ [1], respectively, while the naive estimation is $\Delta_{CP} = \mathcal{A}_{CP}(B^+ \rightarrow K^+\pi^0) - \mathcal{A}_{CP}(B_d \rightarrow K^+\pi^-) \sim 0$. Although many theoretical calculations based on QCDF [2], PQCD [3] and SCET [4] have been tried to produce the consistencies with data in the framework of standard model (SM), however, the results have not been conclusive yet [5]. For instance, the recent PQCD result for the $\Delta_{CP}$ is 0.08 \pm 0.09, which is actually consistent with the data. However, the PQCD prediction $\mathcal{A}_{CP}(B^+ \rightarrow K^+\pi^0)_{PQCD} = -0.01 \pm 0.03$ still has a 1.4$\sigma$ difference from the current experimental data [6]. In addition, the difference between $(\sin 2\beta)_{K_{S}\pi^0}$ and $(\sin 2\beta)_{B_{d,s}}$ in the mixing-induced CPA from the PQCD prediction is 0.065 \pm 0.04, which shows about $2\sigma$ off the data $-0.30 \pm 0.19$. Hence, the inconsistencies between data and theoretical predictions provide a strong indication to investigate the new physics beyond SM.

There introduced many extensions of the SM, and enormous studies have been done on searching some specific models beyond SM, e.g. on supersymmetric model [7], extra-dimension model [8], left-right symmetric model [9] and flavor-changing $Z'$ model [10]. Although new physics effects will be introduced, however, in phenomenological sense we just bring more particles and their related interactions to our system. Recently, Georgi proposed completely different stuff and suggested that an invisible sector, dictated by the scale invariance and coupled weakly to the particles of the SM, may exist in our universe [11,12]. Unlike the concept of particles in the SM or its normal extensions where the particles own the definite mass, the scale invariant stuff cannot have a definite mass unless it is zero. Therefore, if the peculiar stuff exists, it should be made of unparticles [11]. Furthermore, in terms of the two-point function with the scale invariance, it is found that the unparticle with the scaling dimension $d_u$ behaves like a non-integral number $d_u$ of invisible particles [11]. Based on Georgi’s proposal, the phenomenology of unparticle physics has been extensively studied in Refs. [11–16]. For illustration, some examples such as $t \rightarrow eU$ and $e^+e^- \rightarrow \mu^+\mu^-$ have been introduced to display the unparticle properties. In addition, it is also suggested that the unparticle production in high energy colliders might be detected by searching for the missing energy and momentum distributions [11–13]. Nevertheless, we have to point out that flavor factories with high luminosities, such as SuperKEKB [17], SuperB [18] and LHCb [19], etc., should also provide good environments to search for the unparticle effects in indirect way.

Besides the weird property of non-integer number of unparticles, the most astonished effect is that an unparticle could carry a peculiar CP conserving phase associated with its propagator in the time-like region [12,13]. It has been pointed out that the unparticle phase plays a role like a strong phase and has an important impact on direct CP violation (CPV) [14]. In this Letter, we will make detailed analysis to examine whether the puzzles in $B_{u,d} \rightarrow (\pi,K)\pi$
decays with the $B_{d,s} \rightarrow \bar{B}_{d,s}$ mixing constraints could be resolved when the invisible unparticle stuff is introduced to the SM.

In order to study the flavor physics associated with scale invariant stuff, we follow the scheme proposed in Ref. [11]. For the system with the scale invariance, there exist so-called Banks–Zaks (BZ) fields that have a nontrivial infrared fixed point at a very high energy scale [20]. Subsequently, with the dimensional transmutation at the $\Lambda_{\text{QCD}}$ scale, the BZ operators composed of BZ fields will match onto unparticle operators. We consider only vector unparticle operator in the following analysis. Then, the effective interactions for unparticle stuff and the particles of the SM are adopted to be

$$\frac{c_{L}^{q}}{\Lambda_{\text{QCD}}^{4}} i\gamma_{\mu} (1 - \gamma_{5}) q \tilde{C}_{L}^{q} \partial_{\mu} + \frac{c_{R}^{q}}{\Lambda_{\text{QCD}}^{4}} i\gamma_{\mu} (1 + \gamma_{5}) q \tilde{C}_{R}^{q} \partial_{\mu}, \tag{1}$$

where $c_{L,R}^{q}$ are effective coefficient functions and $\tilde{C}_{L,R}^{q}$ denotes the spin-1 unparticle operator with scaling dimension $d_{\text{U}}$ and is assumed to be hermitian and transverse $\tilde{C}_{L}^{q} = 0$. Since so far the theory for BZ fields and their interactions with SM particles is uncertain, here $c_{L,R}^{q}$ are regarded as free parameters. With scale invariance, the propagator of vector unparticle can be obtained by [12,13]

$$\int d^{4}x e^{ip x} (0| T(0_{U}^{L}(x) O_{L}^{U}(0)|0) = i \Delta_{U}(p^{2}) \left( -g^{\mu\nu} + \frac{p^{\mu}p^{\nu}}{p^{2}} \right) e^{-i\phi_{U}} , \tag{2}$$

with $\phi_{U} = (d_{U} - 2)\pi$ and

$$\Delta_{U}(p^{2}) = \frac{1}{2 \sin (d_{U}\pi/2)} \left( \frac{1}{(p^{2} + i\epsilon)^{2-d_{U}}} \right),$$

$$A_{d_{U}} = \frac{16\pi^{2}/2}{(2\pi)^{d_{U} - 1}} \frac{\Gamma(d_{U} + 1/2)}{(d_{U} - 1)\Gamma(2d_{U})} \tag{3}$$

where $\phi_{U}$ could be regarded as a CP conserving phase [12–14]. We note that when unparticle stuff is realized in the framework of conformal field theories, the propagators for vector and tensor unparticles should be modified [21]. Although conformal invariance typically implies scale invariance, however, in principle it is not necessary. Unparticle stuff with scalar invariance in 2D space-time has been investigated in Ref. [22]. Hence, in this work, we still concentrate on the stuff built out of scale invariance. For simplicity, we set the unknown scale factor $d_{U}$ to be $1$ TeV throughout the analysis.

2. Constraints of $B_{d,s} \rightarrow \bar{B}_{d,s}$ mixing

Since the tree level FCNC processes are allowed in the unparticle physics, it is expected that the $B_{d,s} \rightarrow \bar{B}_{d,s}$ mixings offer strong constraints on the unparticle parameters of $c_{L,R}^{d,s}$ and $c_{L,R}^{b}$. Moreover, it will be shown that the scaling dimension $d_{U}$ also can be constrained by $B_{d,s} \rightarrow \bar{B}_{d,s}$ mixings.

First of all, we separate the matrix element of $\Delta B = 2$ transition for the $B_{d}^{0} \rightarrow B_{s}^{0}$ mixing ($q = d, s$), which is denoted by $M_{12}^{q}$, into the SM and unparticle contribution as follows.

$$M_{12}^{q} = M_{12}^{q,\text{SM}} + M_{12}^{q,\text{NP}} = |M_{12}^{q,\text{SM}}| e^{i\phi_{q}^{\text{SM}}} + |M_{12}^{q,\text{NP}}| e^{i\phi_{q}^{\text{NP}}}, \tag{4}$$

where $\phi_{q}^{\text{SM}}$ and $\phi_{q}^{\text{NP}}$ represent the phases of mixing amplitudes. For the second term, we use the superscript ‘NP’ in order to represent general new physics (NP) contribution. Later on, we regard this $M_{12}^{q,\text{NP}}$ as the unparticle mixing amplitude.

As is well known, the magnitude of total mixing amplitude $|M_{12}^{q}|$ is given by the $B_{d}^{0} \rightarrow B_{s}^{0}$ oscillating frequency as follows:

$$\Delta M_{q} = 2 |M_{12}^{q}| \tag{5}$$

and the mixing phase $\phi_{q} = \arg M_{12}^{q}$ can be obtained from the mixing induced CP asymmetry of $b \rightarrow c \bar{s} s$ processes. We summarize current experimental data in Table 1.

The SM mixing amplitude reads

$$M_{12}^{q,\text{SM}} = \frac{G_{F}^{2} m_{t}^{2}}{12\pi^{2}} m_{B_{q}} f_{B_{q}} B_{B_{q}} \left( V_{tb} V_{tb}^{*} \right)^{2} \langle \bar{q} \gamma_{5} q \rangle^{*} S_{0}(\Delta t), \tag{6}$$

where $\langle \bar{q} \gamma_{5} q \rangle = 0.552$ is short distance QCD correction term [25], and $S_{0}(\Delta t) = 2.35 \pm 0.06$ is an Inami–Lim function for the $t$-quark exchange in the loop diagram [26]. The quantities of $f_{B_{q}}$ and $B_{B_{q}}$ are non-perturbative parameters which can be obtained from the lattice calculations. We follow the procedure given in Ref. [27] for dealing with these non-perturbative parameters. The procedure mainly employs the result of lattice calculations in two different ways. The one is to use the result of JLQCD Collaboration [28], and the other is to combine the results of JLQCD and HPQCD [29] Collaborations. We note that one can obtain the SM mixing phases from Eq. (6) as follows.

$$\phi_{q}^{\text{SM}} = 2\beta, \quad \phi_{q}^{\text{SM}} = -2\lambda^{2}\eta, \tag{7}$$

where $\beta$ is an angle of CKM unitarity triangle, $\lambda$ and $\eta$ are from the Wolfenstein parametrization [30]. We use the result of UT-fit [31] from the tree level processes for the $\beta$, $\eta_{1}$ and $\eta_{2}$. The value of $\eta_{1}$ is denoted by $|V_{cb}|$, and $\eta_{2}$ is the apex of the CKM unitary triangle. For the $|V_{cb}|$, we adopt the result of global fit to moment of inclusive distributions in $B \rightarrow X_{\ell} \ell$, which is performed in the framework of heavy quark expansions with kinetic scheme [32]. After putting all the SM input parameters into Eq. (6), the SM mixing amplitudes are obtained. And using the experimental data shown in the Table 1, the NP mixing amplitudes for the $B_{d} \rightarrow \bar{B}_{d}$ mixing could be gained through the Eq. (4). The numerical values are summarized in Table 2.

As for the unparticle contribution to the mixing amplitude, we begin with the effective Hamiltonian for $\Delta B = 2$ processes in unparticle sector such as

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Observables & Values & Note \\
\hline
$\Delta M_{d}$ & $(0.507 \pm 0.004)$ ps$^{-1}$ & HFAG [1] \\
$\Delta M_{s}$ & $(17.77 \pm 0.12)$ ps$^{-1}$ & CDF [24] \\
$\phi_{b}$ & $43^{\circ} \pm 2^{\circ}$ & HFAG [1] \\
\hline
\end{tabular}
\caption{Experimental values for the $B_{d,s} \rightarrow \bar{B}_{d,s}$ mixings.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
SM parameters & Values & NP parameters & Values \\
\hline
$2|M_{12}^{d,\text{SM}}|$ & $0.75^{+0.20}_{-0.26}$ ps$^{-1}$ & $2|M_{12}^{d,\text{NP}}| = 0.25 \pm 0.26$ ps$^{-1}$ \hspace{1cm} (a) \\
$0.97 \pm 0.29$ ps$^{-1}$ & $0.46 \pm 0.29$ ps$^{-1}$ \hspace{1cm} (b) \\
$\phi_{q}^{\text{SM}}$ & $45^{\circ} \pm 5^{\circ}$ & $\phi_{q}^{\text{NP}}$ & $-130^{\circ} \pm 180^{\circ}$ \hspace{1cm} (a) \\
& & $-132^{\circ} \pm 12^{\circ}$ \hspace{1cm} (b) \\
$2|M_{12}^{s,\text{SM}}|$ & $16.4 \pm 2.8$ ps$^{-1}$ & $2|M_{12}^{s,\text{NP}}|$ & $23.8 \pm 5.9$ ps$^{-1}$ \hspace{1cm} (b) \\
$23.8 \pm 5.9$ ps$^{-1}$ & - & - & - \\
$\phi_{q}^{\text{SM}}$ & $-2.3^{\circ} \pm 0.2^{\circ}$ & $\phi_{q}^{\text{NP}}$ & - \\
\hline
\end{tabular}
\caption{Numerical values for the SM $B_{d,s} \rightarrow \bar{B}_{d,s}$ mixing amplitudes. The values of the NP $B_{d,s} \rightarrow \bar{B}_{d,s}$ mixing amplitudes are obtained from the experimental data and the Eq. (4). The case (a) denotes JLQCD, while the case (b) denotes (HP+JLQCD). $M_{12}^{q,\text{NP}} = M_{12}^{q,\text{SM}}$ should be considered ($q = d, s$).}
\end{table}
\[ H^{d_{UL}} = 2 \frac{1}{4} \left( \frac{p^2}{A_{UL}^2} \right) d_{UL} - 1 \frac{1}{p^2} A_{UL} e^{-i\phi_{UL}} \]
\[ \times \left[ -\bar{q} \gamma^\mu (C_L^{qb} (1 - \gamma_5) + C_R^{qb} (1 + \gamma_5)) \right] \]
\[ \times b_d \gamma^\nu (C_L^{qb} (1 + \gamma_5) + C_R^{qb} (1 - \gamma_5)) b + \frac{1}{p^2} \bar{q} \gamma^\nu (C_L^{qb} (1 - \gamma_5) + C_R^{qb} (1 + \gamma_5)) b \]
\[ \times \bar{q} \gamma^\nu (C_L^{qb} (1 - \gamma_5) + C_R^{qb} (1 + \gamma_5)) b. \] (8)

The factor 2 is from the fact that there are s- and t-channel which give same result, and the factor 1/4 is due to the Wick contraction factor [33]. From this effective Hamiltonian, the transition matrix elements can be shown as
\[ M_{d UL}^{d_{UL}} = - \frac{\Delta_{UL} (p^2)}{(A_{UL}^2)^{d_{UL} - 1}} m_{b_d} f_{b_d} b_\delta a_{d_{UL}}^{d_{UL}}, \] (9)
where the \( a_{d_{UL}}^{d_{UL}} \) is defined by
\[ a_{d_{UL}}^{d_{UL}} = \left[ (C_L^{qb})^2 + (C_R^{qb})^2 \right] \frac{2}{3} - \frac{5}{12} \frac{m_{b_d}^2}{p^2} + \frac{5}{3} \frac{m_{b_d}^2}{6} \frac{m_{b_d}^2}{p^2}. \] (10)

It is interesting that \( \text{arg}(M_{d_{UL}}^{d_{UL}}) \) takes only positive value as displayed in Fig. 1. Therefore, as we can see from Table 2, (HP+)+JLQCD case cannot give a solution of \( d_{UL} \) while all value of \( \text{arg}(M_{d_{UL}}^{d_{UL}}) \) is possible in JLQCD case. So, we take only the JLQCD case. It depends on the scaling dimension \( d_{UL} \) through the \( e^{-i\phi_{UL}} \) term and the sign of \( \sin(d_{UL}) \) in the \( \Delta_{UL} (p^2) \). The plot of \( \text{arg}(M_{d_{UL}}^{d_{UL}}) \) versus \( d_{UL} \) is shown in Fig. 1.

Based on this effective Hamiltonian, we study the unparticle contributions to the decay amplitudes for \( B_{u,d} \rightarrow (\pi, K) \pi \). It is known that the most uncertain theoretical calculations for two-body exclusive decays are the QCD hadronic transition matrix elements. To deal with the hadronic matrix elements, we adopt recent perturbative QCD (PQCD) calculations for the SM amplitudes and naive factorization (NF) approach for the amplitudes of unparticle contributions. The SM amplitudes for \( B_{u,d} \rightarrow (\pi, K) \pi \) decays can be parameterized in the context of quark diagram approach (QDA) [34] as follows:
\[ \sqrt{2} A_{SM}(B^+ \rightarrow \pi^+ \pi^0) = -T e^{i\gamma} - C e^{i\gamma} - P_{EW} e^{-i\phi}. \]
\[ A_{SM}(B^+ \rightarrow K^+ \pi^-) = -T e^{i\gamma} + P_{EW} e^{-i\phi}. \]
\[ \sqrt{2} A_{SM}(B^0 \rightarrow \pi^0 \pi^0) = -C e^{i\gamma} + P_{EW} e^{-i\phi}. \]
\[ A_{SM}(B^0 \rightarrow K^0 \pi^+) = P^* \gamma. \]
\[ \sqrt{2} A_{SM}(B^0 \rightarrow K^+ \pi^-) = -P^* - T e^{i\gamma}. \]
\[ \sqrt{2} A_{SM}(B^+ \rightarrow K^+ \pi^0) = -P^* - T e^{i\gamma} - C e^{i\gamma} - P_{EW}. \]
\[ \sqrt{2} A_{SM}(B^0 \rightarrow K^0 \pi^0) = P^* - C e^{i\gamma} - P_{EW}. \]
(15)
(16)
(17)
(18)
(19)
(20)
(21)

where \( T \) and \( C \) denote the color-suppressed amplitudes for \( B_{u,d} \rightarrow (\pi, K) \pi \), respectively, while \( P^* \) (\( P_{EW} \)) is gluonic (electroweak) penguin amplitude. All CP-conserving phases are included in these parameters. The phase \( \gamma (\beta) \) is the CP violating phase in the SM and from \( V_{ud} (V_{td}) \).

Table 3 shows the recent PQCD result for the values of each topological parameters [6,35].

For deriving the unparticle contributions, the definitions for relevant decay constants and form factors are given by

Table 3

| Topology | Abs | Arg | Topology | Abs | Arg |
|----------|-----|-----|----------|-----|-----|
| \( P^* \) | 43.6 \[10.8 \] \[8.9 \] | 2.9 \[1.7 \] \[2.2 \] | \( T \) | 23.2 \[0.0 \] \[0.0 \] | 0.0 \[0.0 \] \[0.0 \] |
| \( T^* \) | 6.5 \[1.4 \] \[1.4 \] | 0.1 \[0.0 \] \[0.0 \] | \( P \) | 5.6 \[0.0 \] \[0.0 \] | –0.4 \[0.0 \] \[0.0 \] |
| \( P_{EW} \) | 5.4 \[1.0 \] \[1.0 \] | –1.3 \[0.0 \] \[0.0 \] | \( C \) | 4.3 \[1.5 \] \[1.5 \] | –1.1 \[0.0 \] \[0.0 \] |
| \( C^* \) | 1.7 \[0.6 \] \[0.6 \] | –3.0 \[0.0 \] \[0.0 \] | \( P_{EW} \) | 0.7 \[0.2 \] \[0.2 \] | –0.1 \[0.0 \] \[0.0 \] |

Table 3: Recent PQCD predictions for the topological parameters of \( B_{u,d} \rightarrow (\pi, K) \pi \) in unit of \( 10^{-5} \) GeV. The phases are indicating strong phases of the parameters in radian unit. The predictions include NLO calculation.
\begin{align}
    \langle P(p_1) | \bar{q} \gamma_\mu q \rangle u(0) &= i f \gamma_\mu p_\mu,
    \\
    \langle P(p) | \bar{q} \gamma_\mu q \rangle u(0) &= i f m_0 \rho,
    \\
    \langle P(p) | \bar{q} \gamma_\mu b | \bar{b}(p_B) \rangle &= \left[ (p_B + p)_\mu - \frac{m_B^2}{q^2} q_\mu \right] F_{B \pi}^{B_0}(q^2) + \frac{m_B^2}{q^2} q_\mu F_{B \pi}^{B_0}(q^2),
\end{align}

with \( P = (\pi, K) \), \( q = p_B - p \) and \( m_0 = m_B^2 / (m_B + m_q) \). Here, due to \( m_0 \ll m_B \), we have neglected the \( m_0^2 \) effects in \( B \rightarrow P \) transition matrix element. Subsequently, by considering various flavor diagrams in which the typical diagrams mediated by unparticle are illustrated in Fig. 2, the unparticle amplitudes for \( B_{u,d} \rightarrow (\pi, K) \pi \) decays within NF approach are obtained to be

\begin{align}
    A^I(B \rightarrow \pi^+ \pi^-) &= C_L(q_1^2) f_{\pi} m_{B}^2 F_{0}^{B \pi}(m_{B}^2) d_{\text{dec}}^{I, \pi \pi} m_{\pi}^2,
    \\
    A^I(B \rightarrow K^+ \pi^-) &= C_L(q_1^2) f_{\pi} m_{B}^2 F_{0}^{B \pi}(m_{B}^2) d_{\text{dec}}^{I, \pi \pi} m_{K}^2,
\end{align}

where the coefficients \( d_{\text{dec}}^{I, \pi \pi} \) are defined in \( N_c = 3 \) is the number of colors, \( q_1^2 = m_{\pi}^2 \), and the chiral enhanced factor \( r_{1,2}^T \) and \( r_{1,2}^K \) are defined by

\begin{align}
    r_{1,2}^T &= \frac{m_{\pi}^2}{m_B(m_B + m_\pi)}, \\
    r_{1,2}^K &= \frac{m_{\pi}^2}{m_K(m_K + m_\pi)}, \\
    r_{1,2}^L &= \frac{m_{\pi}^2}{m_B(m_B + m_\pi)}, \\
    r_{1,2}^L &= \frac{m_{\pi}^2}{m_K(m_K + m_\pi)}.
\end{align}

We note that since \( q_1 \) in Fig. 2(a) involves different mesons, the estimation of \( q_1^2 \) should have ambiguity. To understand the typical value of \( q_1^2 \), we write the \( q_1 = p_B - k_2 - k_3 \) with \( k_2, k_3 \) being the momenta of valence quarks inside the light mesons. In terms of momentum fraction of valence quark and light-cone coordinates and by neglecting the transverse momentum, one can get \( k_2 = (0, m_B k_2 / \sqrt{2}, 0, \pm 1) \) and \( k_3 = (m_B k_3 / \sqrt{2}, 0, 0, \pm 1) \). As a result, we have \( q_1^2 = m_B^2 (1 - x_2) - x_3 / 2 \). According to the behavior of leading twist wave function of light meson, \( \phi^{\text{tw}} \propto x(1 - x) \) which is calculated by QCD sum rules [37], it is known that the maxima of \( x_{2,3} \) occur at \( x_2 = x_3 \approx 1 / 2 \). Therefore, for numerical estimations, the value for momentum transfer could be roughly taken as \( q_1^2 \approx m_B^2 / 4 \) with \( m_B = 5.28 \text{ GeV} \). Since the contributions of Fig. 2(a) are color-suppressed, we emphasize that the corresponding results are insensitive to the adopted value of \( q_1^2 \). We discard irrelevant factor \( 1 \) in the NF and match the sign with the QDA parametrization; then, the total amplitude is

\begin{align}
    A(B \rightarrow f) &= \kappa_f A^{3D}(B \rightarrow f) + A^{I}(B \rightarrow f).
\end{align}

Here, the \( \kappa_f \) is the ratio of phase space factor coming from the difference of notation of decay amplitude between NF and PQCD group and is defined by

\begin{align}
    \kappa_f &= \frac{G_{F}^{2} m_{B}^{2}}{128 \pi} \left( \frac{p_{f}}{8 \pi m_{B}^{2}} \right) = 1.15 \times 10^{-4}.
\end{align}

From Table 4, we see that besides \( d_{I4} \) and \( \Lambda_{I4} \), the introduced new free parameters are

\begin{align}
    C_{\mu}^{\text{db}}, \quad C_{\mu}^{\text{d}}, \quad C_{\mu}^{\text{b}}, \quad C_{\mu}^{\text{db}}, \quad C_{\mu}^{\text{d}}, \quad C_{\mu}^{\text{b}}.
\end{align}

which denote the couplings of unparticle to SM particles. First four parameters are strongly correlated by \( B_{d} \rightarrow B_{s} \) mixing phenomena as shown in Eq. (12). If we regard all these parameters to be real number and set \( \Lambda_{42} = 1 \text{ TeV} \) and \( d_{42} = 1.5 \) as we do in the \( B_{d} \rightarrow B_{s} \) analysis, 8 free parameters are involved for \( B_{u,d} \rightarrow (\pi, K) \pi \) in the unparticle physics.

In order to fit the data for the \( B_{u,d} \rightarrow (\pi, K) \pi \) decays with these 8 parameters, we perform the minimum \( \chi^2 \) analysis. According to current experimental observations, the amount of available data for \( B_{u,d} \rightarrow (\pi, K) \pi \) decays is 17 as their world averages are displayed in Table 5. Besides the BRS, the important quantities to display the new physics effects are the direct and mixing induced CPAs, where they could be briefly defined through

\begin{align}
    A_{f} &= \frac{\lvert \lambda_f \rvert^2 - 1}{1 + \lvert \lambda_f \rvert^2}, \\
    S_{f} &= \frac{2 \text{Im}(\lambda_f)}{1 + \lvert \lambda_f \rvert^2}.
\end{align}

with \( \lambda_f = e^{i\theta_{f}} \tilde{A}_{f} / A_{f} \), respectively. For direct CPA, \( f \) could be any possible final states; however, for mixing induced CPA, \( f \) could only be CP eigenstates. Since the PQCD prediction for the SM decay amplitudes has sizable error as shown in Table 3, it should be considered for the minimum \( \chi^2 \) analysis. Therefore, we define the \( \chi^2 \) to be

\begin{align}
    \chi^2 = \sum_{i=1}^{n} \frac{(t_i - e_i)^2}{\sigma_{i}^{\text{exp}}^2 + \sigma_{i}^{\text{th}}^2}.
\end{align}

\( e_i \) and \( t_i \) denote the experimental data for \( i \)th observable and its theoretical prediction within unparticle contribution, respectively. \( \sigma_{i}^{\text{exp}} \) is the experimental error of ith observable, while \( \sigma_{i}^{\text{th}} \) is theoretical error propagated from the errors of topological parameters obtained from PQCD. Since there are 8 free parameters involved in our analysis, the degree of freedom (d.o.f) for the fitting is \( 17 - 8 = 9 \). As for the angle \( \gamma \), we use the values of \( \gamma = (63_{-15}^{+15})^\circ \) from the PDG 2006 [36]. Consequently, by imposing the mixing constraints of \( C_{\mu}^{\text{db}}(B) \) and \( C_{\mu}^{\text{d}}(B) \) displayed in Eq. (12), we find the optimized values of unparticle parameters as follows:

\begin{align}
    C_{\mu}^{\text{db}} &= 3.3 \times 10^{-4}, \quad C_{\mu}^{\text{d}} = 4.6 \times 10^{-4}, \\
    C_{\mu}^{\text{b}} &= 7.6 \times 10^{-4}, \quad C_{\mu}^{\text{db}} = 11.2 \times 10^{-4}.
\end{align}
The experimental data for $B_{d,s} \to (\pi, K)\pi$ decays and comparison between the experimental data and the theoretical predictions with and without unparticle contribution. The BRs are ordered of $10^{-5}$. The data is updated by September 2007. ‘w/o’ means without unparticle contribution. χ² values of each contributions are shown.

| Observables | Data | Theory (w/o) | Theory | $\chi^2$ (w/o) | $\chi^2$ |
|-------------|------|--------------|--------|----------------|--------|
| $B(K^0\pi^0)$ | 23.1 ± 1.0 | 23.5 ± 1.2 | 23.1 ± 1.1 | 0.001 | 0.00 |
| $B(K^+\pi^-)$ | 12.9 ± 0.6 | 13.0 ± 0.2 | 12.7 ± 0.6 | 0.001 | 0.00 |
| $B(K^-\pi^+)$ | 19.4 ± 0.6 | 19.7 ± 0.1 | 20.3 ± 0.1 | 0.001 | 0.007 |
| $B(K^0\bar{K}^0)$ | 9.9 ± 0.6 | 8.8 ± 0.4 | 9.5 ± 5.1 | 0.046 | 0.006 |
| $A_{C}(K^0\pi^0)$ | 0.009 ± 0.025 | 0.00 ± 0.0 | 0.00 ± 0.0 | 0.13 | 0.13 |
| $A_{C}(K^+\pi^-)$ | 0.050 ± 0.025 | -0.017 ± 0.068 | 0.074 ± 0.068 | 0.84 | 0.11 |
| $A_{C}(K^-\pi^+)$ | -0.097 ± 0.012 | -0.099 ± 0.073 | -0.11 ± 0.075 | 0.001 | 0.03 |
| $A_{C}(K^0\bar{K}^0)$ | -0.14 ± 0.11 | -0.065 ± 0.040 | -0.058 ± 0.036 | 0.41 | 0.50 |

Since the unparticle contributions do not carry any $\Delta f_{d,s}$ processes, we find that the curvature in future would give strong constraint on $\Delta f_{d,s}$. Interestingly, after we fix $d_A = 1.5$ for the specific unparticle scaling dimension, we find that the unsolved problem of large BR for $B_d \to \pi^+\pi^-\pi^0$ could be explained excellently in the framework of unparticle physics. Moreover, the discrepancy between the standard model estimation and data for the direct CPA of $B^+ \to K^-\pi^+\pi^0$ and $B_d \to \pi^+\pi^-\pi^0$ could be reconciled very well. However, the puzzle of the mixing induced CPA of $B_d \to K\pi\pi$ could not be resolved well in unparticle physics.

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### References

[1] E. Barberio, et al., Heavy Flavor Averaging Group (HFAG) Collaboration, arXiv: 0704.3575 [hep-ex], online update at http://www.slac.stanford.edu/xorg/hfag.
[2] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 606 (2001) 245, hep-ph/0104110;
[3] M. Beneke, G. Buchalla, M. Neubert, Nucl. Phys. B 675 (2003) 333, hep-ph/0308039;
[4] X.-q. Li, Y.-d. Yang, Phys. Rev. D 72 (2005) 074007, hep-ph/0508079.
[5] H.-n. Li, H.-l. Yu, Phys. Rev. D 53 (1996) 2480, hep-ph/9411308;
[6] Y.Y. Keum, H.-n. Li, A.I. Sanda, Phys. Lett. B 504 (2001) 6, hep-ph/0004004;
[7] Y.Y. Keum, H.-n. Li, A.I. Sanda, Phys. Rev. D 63 (2001) 054008, hep-ph/0004173;
[8] H.-n. Li, S. Mishima, A.I. Sanda, Phys. Rev. D 72 (2005) 114005, hep-ph/0508041.
[9] C.W. Bauer, D. Pirjol, I.Z. Rothstein, I.W. Stewart, Phys. Rev. D 70 (2004) 054015, hep-ph/0401188;
[10] C.W. Bauer, I.Z. Rothstein, I.W. Stewart, Phys. Rev. D 74 (2006) 034010, hep-ph/0510241;
[11] A.R. Williamson, J. Zupan, Phys. Rev. D 74 (2006) 014003, hep-ph/0601214;
[12] A.R. Williamson, J. Zupan, Phys. Rev. D 74 (2006) 03901.
[13] A. Jain, I.Z. Rothstein, I.W. Stewart, arXiv: 0706.3395;
[14] H.N. Li, S. Mishima, A.I. Sanda, Phys. Rev. D 72 (2005) 114005, hep-ph/0508041.
[15] C.H. Chen, C.Q. Geng, Phys. Rev. D 66 (2002) 014007, hep-ph/0205306;
[16] S. Khalil, Phys. Rev. D 72 (2005) 035007, hep-ph/0505151;
[17] R. Arnowitt, B. Dutta, B. Hu, S. Oh, Phys. Lett. B 633 (2006) 748, hep-ph/0509233;
[18] R. Arnowitt, B. Dutta, B. Hu, S. Oh, Phys. Lett. B 641 (2006) 305, hep-ph/0606130;
[19] C.H. Chen, C.Q. Geng, JHEP 0610 (2006) 053, hep-ph/0608166;
[20] K. Cheung, S.K. Kang, C.S. Kim, J. Lee, Phys. Lett. B 652 (2007) 319, hep-ph/0702050.
[21] K. Agashe, G. Perez, A. Soni, Phys. Rev. D 71 (2005) 016002, hep-ph/0408134;
[22] K. Chang, S.C. Kim, J. Song, JHEP 0702 (2007) 087, hep-ph/0607313;
[23] R.N. Mohapatra, G. Senjanovic, Phys. Lett. B 79 (1978) 283.
[24] P. Langacker, M. Plumberch, Phys. Rev. D 62 (2000) 013006;
[25] C.W. Bauer, C.Q. Geng, Phys. Lett. B 588 (2004) 218, hep-ph/0406126;
[26] D.A. Demir, G.L. Kane, T.T. Wang, Phys. Rev. D 72 (2005) 015012, hep-ph/0503290;
[27] C.H. Chen, H. Hatanaka, Phys. Rev. D 73 (2006) 075003, hep-ph/0602140;
[28] S. Baek, J.I. Jeon, C.S. Kim, Phys. Lett. B 641 (2006) 183, hep-ph/0607113.
[29] H. Georgi, Phys. Rev. Lett. 98 (2007) 221601, hep-ph/0703260.
[30] H. Georgi, Phys. Lett. B 650 (2007) 275, arXiv: 0704.2457 [hep-ph];
[31] K. Cheung, W.Y. Keung, T.C. Yuan, Phys. Rev. Lett. 99 (2007) 051803, arXiv: 0704.2588 [hep-ph];
[32] K. Cheung, W.Y. Keung, T.C. Yuan, arXiv: 0706.3155 [hep-ph].
