RECOVERY OF SPARSE MATRICES VIA MATRIX SKETCHING

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ABSTRACT

In this paper, we consider the problem of recovering an unknown sparse matrix \( X \) from the matrix sketch \( Y = AXB^T \). The dimension of \( Y \) is less than that of \( X \), and \( A \) and \( B \) are known matrices. This problem can be solved using standard compressive sensing (CS) theory after converting it to vector form using the Kronecker operation. In this case, the measurement matrix assumes a Kronecker product structure. However, as the matrix dimension increases the associated computational complexity makes its use prohibitive. We extend two algorithms, fast iterative shrinkage threshold algorithm (FISTA) and orthogonal matching pursuit (OMP) to solve this problem in matrix form without employing the Kronecker product. While both FISTA and OMP with matrix inputs are shown to be equivalent in performance to their vector counterparts with the Kronecker product, solving them in matrix form is shown to be computationally more efficient. We show that the computational gain achieved by FISTA with matrix inputs over its vector form is more significant compared to that achieved by OMP.

Index Terms— Compressive sensing, Sparse matrix recovery, \( l_1 \) norm minimization, FISTA, OMP

1. INTRODUCTION

We consider the problem of recovering an unknown matrix \( X \) from the following observation model

\[
Y = AXB^T \tag{1}
\]

where \( X \in \mathbb{R}^{N \times N} \), \( A \in \mathbb{R}^{M \times N} \), \( B \in \mathbb{R}^{L \times N} \) and \( A^T \) denotes the transpose of the matrix \( A \). This problem has been studied by many researchers in different contexts for arbitrary matrices \( X \) \([1,3]\).

In many applications dealing with high dimensional data, sparsity is one of the low dimensional structures widely observed. Most popular transforms applied to 2-dimensional signals are in the form of \((1)\) where compression is obtained by a transformation of rows followed by a transformation of columns of the data matrix \([4,7]\). With an arbitrarily distributed sparse matrix \( X \) in which each column/row has only a few non zeros, a natural question to ask is whether it is possible to design sensing matrices in the form of \((1)\) so that \( X \) can be uniquely recovered from \( Y \) when \( M, L < N \). Sparse signal recovery has attracted much attention in the recent literature in the context of compressive sensing (CS) \([8,10]\). In the standard CS framework, a commonly used mechanism is to stack the high dimensional data into vector form to recover the sparse vector uniquely from an underdetermined linear system \([8,9]\).

The observation model \((1)\) can be equivalently written in vector form using Kronecker products as:

\[
y = Cx \tag{2}
\]

where \( y = \text{vec}(Y) \in \mathbb{R}^{ML} \), \( C = B \otimes A \in \mathbb{R}^{ML \times N^2} \), \( x = \text{vec}(X) \in \mathbb{R}^{N^2} \), \( \otimes \) denotes the Kronecker operator and \( \text{vec}(X) \) is a column vector that vectorizes the matrix \( X \) (i.e. columns of \( X \) are stacked one after the other). The sensing matrix in \((2)\) has a special structure, i.e., it can be represented as a Kronecker product of two matrices \( A \) and \( B \). It has been shown \([11,14]\) that the sparse signal \( x \) from \((2)\) can be recovered by solving the following \( l_1 \) norm minimization problem

\[
\min ||x||_{1} \quad \text{s.t.} \quad Cx = y \tag{3}
\]

under certain conditions on the matrices \( A \) and \( B \) where \( ||x||_p \) denotes the \( l_p \) norm of \( x \). In particular, these results imply that the capability of recovering \( x \) based on \((2)\) is ultimately determined by the worst behavior of \( A \) or \( B \). Also, this approach is computationally complex especially when the matrix dimension \( N \) increases \([6,7]\).

Several recent papers addressed the problem of recovering a sparse \( X \) from \((1)\) without employing the Kronecker product. In \([7]\), it was shown that a unique solution for \( X \) can be found when \( X \) is distributed sparse under certain conditions on \( A \) and \( B \) by solving the following optimization problem:

\[
\min ||X||_1 \quad \text{s.t.} \quad AXB^T = Y \tag{4}
\]

where \( ||X||_1 \) is the \( l_1 \) norm of \( \text{vec}(X) \). The authors derive recovery conditions when the matrices \( A \) and \( B \) contain binary elements which are better than that obtained via the Kronecker product approach. In \([6]\), the authors discuss advantages in terms of computational, storage, calibration and implementation while solving \((3)\) in matrix form compared to that with vector form. However, no specific algorithm was developed to solve for \( X \). In \([5]\), a version of orthogonal matching pursuit (OMP) (dubbed 2D OMP) is presented to find a sparse \( X \) in the matrix form \((1)\) when \( A = B \).

Our goal in this paper is to develop algorithms to solve for sparse \( X \) from \((1)\) without the employment of Kronecker products. We extend fast iterative shrinkage threshold algorithm (FISTA) \([15,16]\) developed for the vector case to sparse matrix recovery with matrix inputs. We further consider a greedy based approach via OMP to find the sparse solution. We show that both algorithms with matrix inputs are equivalent to their vector counterparts obtained via Kronecker products in terms of performance. However, the computational complexity of the matrix approach is shown to be much less, especially with FISTA, compared to solving the problem in vector form.

2. SPARSE MATRIX RECOVERY VIA \( l_1 \) NORM MINIMIZATION

2.1. Vector formulation

While numerous algorithms have been proposed in the literature to solve \((3)\), in this paper we consider FISTA as discussed in \([15,16]\). We consider the noisy observation model so that FISTA with vector inputs as given in Algorithm \([10,16]\), is the solution of

\[
\min_x \left\{ \frac{1}{2}||y - Cx||^2_2 + \lambda||x||_1 \right\} \tag{5}
\]
where $\lambda$ is a regularization parameter. In Algorithm 1 $L_f = ||C||_2$ is the Lipschitz constant of $\nabla f(x)$ where $||C||_2$ denotes the spectral norm of $C$, $\nabla$ denotes the gradient operator, and $f(x) = \frac{1}{2}||y - CX||^2_2$ and

$$\text{soft}(u, a) = \text{sgn}(u_i)\left(|u_i| - a\right)_+ \quad (6)$$

for $i = 1, \cdots, N^2$ where $u_i$ is the $i$-th element of $u$, $x_+ = x$ if $x > 0$ and equals 0 otherwise.

Algorithm 1 FISTA for sparse signal recovery with vector inputs

Input: observation vector $y$, measurement matrix $C$

Output: estimate for signal, $\hat{x}$

1. Initialization: $x^0 = 0$, $x^1 = 0$, $t_0 = 1$, $t_1 = 1$, $k = 1$

2. while not converged do
3. $x^k = x^{k-1} + \frac{t_{k-1} - 1}{t_k}(x^k - x^{k-1})$
4. $u^k = x^k - \frac{1}{\beta \lambda} C^T (C x^k - y)$
5. $x^{k+1} = \text{soft}\left(u^k, \frac{\lambda}{\beta}\right)$
6. $t_{k+1} = \frac{1 + \sqrt{1 + 4t_k^2}}{2}$
7. $\lambda_{k+1} = \max(\beta \lambda_k, \lambda)$
8. $k = k + 1$
9. end while

$\hat{x} = x^k$

The computational complexity of FISTA is dominated by step 4 in Algorithm 1. The matrix-vector multiplications require $O(N^2 ML + N^4 + N^2 ML)$ computations. Since $M, L \leq N$, the complexity is in the order of $O(N^3 ML)$. Thus, FISTA is feasible only when $N, M, L$ are fairly small.

2.2. Matrix formulation

With the noisy version of (1), we aim to solve the following $l_1$ norm minimization problem:

$$\min_{X} \left\{ \frac{1}{2} ||Y - AXB^T||_F^2 + \lambda ||X||_1 \right\} \quad (7)$$

where $\lambda$ is a regularization parameter and $||A||_F$ is the Frobenius norm of $A$. In generalizing FISTA to solve (1), we follow a similar approach as discussed in [1]. Consider the more general unconstrained optimization problem:

$$\min_{X \in \mathbb{R}^{N \times N}} F(X) + \lambda G(X) \quad (8)$$

where $G(\cdot)$ is a proper, convex, lower semicontinuous function, and $F(\cdot)$ is a convex smooth (continuously differentiable) function on an open subset of $\mathbb{R}^{N \times N}$ containing $dom G = \{X | G(X) < \infty\}$. We assume that $dom G$ is closed and $\nabla F$ is Lipschitz continuous on $dom G$ with Lipschitz constant $L_f$; i.e.

$$||\nabla F(X) - \nabla F(Z)||_F \leq L_f ||X - Z||_F \quad (9)$$

When $F(X) = \frac{1}{2}||Y - AXB^T||^2_2$, it can be shown that $||\nabla F(X) - \nabla F(Z)||_F = ||f(x) - f(z)||_2$ and $||X - Z||_F = ||x - z||_2$ where $z = \text{vec}(Z)$, $f(x) = \frac{1}{2}||y - CX||^2_2$ and $C = B \otimes A$ as defined before. Thus, the Lipschitz constant of $\nabla F(X)$ is the same as $\nabla f(x)$, and we use the same notation $L_f$ as used in Algorithm 1.

Consider the following quadratic approximation of $F(\cdot)$ at $Z$ for any $Z \in dom G$:

$$Q_L(X, Z) := F(Z) + \frac{L_f}{2} ||X - Z||_F^2 + \lambda G(X) \quad (10)$$

where $\text{tr}(\cdot)$ denotes the trace of a matrix. We can rewrite (10) as,

$$Q_L(X, Z) = \frac{L_f}{2} ||X - U(Z)||_2^2 + \lambda \left||X\right||_1 + \tilde{F}(Z) \quad (11)$$

where $\tilde{F}(Z)$ is a function of only $Z$ and

$$U(Z) = Z - \frac{1}{L_f} \nabla F(Z) = Z - \frac{1}{L_f} A^T (AXB^T - Y) B. \quad (12)$$

Thus,

$$\arg \min_X Q_L(X, Z) = \arg \min_X \left\{ \frac{L_f}{2} ||X - U(Z)||_2^2 + \lambda \left||X\right||_1 \right\} \quad (13)$$

Since both terms are element wise separable, we have

$$\arg \min_Q Q(X, Z) = \text{soft} \left(U(Z), \frac{\lambda}{L_f}\right)$$

where $\text{soft}(U(Z), \frac{\lambda}{L_f})$ denotes an element wise operation with

$$\text{soft} \left(W, L_0\right) = \text{sgn}(W_{ij})\left(|W_{ij}| - L_0\right)_+ \quad \text{for all indices } i, j \text{ of the } N \times N \text{ matrix } W.$$

These steps lead to a generalization of FISTA with matrix inputs, as given in Algorithm 2.

Algorithm 2 FISTA for sparse matrix recovery with matrix inputs

Input: observation matrix $Y$, measurement matrices $A$ and $B$

Output: estimate for sparse signal matrix, $X$

1. Initialization: $X^0 = 0$, $X^1 = 0$, $t_0 = 1$, $t_1 = 1$, $k = 1$

2. while not converged do
3. $Z^k = X^k + \frac{t_{k-1} - 1}{t_k}(X^k - X^{k-1})$
4. $U^k = X^k - \frac{1}{\beta \lambda} A^T (AZB^T - Y) B$
5. $X^{k+1} = \text{soft} \left(U^k, \frac{\lambda}{\beta \lambda}\right)$
6. $t_{k+1} = \frac{1 + \sqrt{1 + 4t_k^2}}{2}$
7. $\lambda_{k+1} = \max(\beta \lambda_k, \lambda)$
8. $k = k + 1$
9. end while

$\hat{X} = X^k$

As in Algorithm 1 the computational complexity is dominated by step 4. The matrix-matrix multiplication at step 4 in Algorithm 4 is performed with $O(N^2 (N + M + 3L) + NML)$ computations. Since $M, L \leq N$, the worst case complexity is $O(N^3)$. Recall, that FISTA in vector form has worst case complexity of $O(N^4 ML)$. Thus, there is a $O(NML)$ gain in the matrix version compared to the vector approach.

2.3. Equivalence of Algorithms 1 and 2

It is easy to see that $x^k$ and $x^{k+1}$ computed in steps 3 and 5 in Algorithm 1 are the same as $\text{vec}(Z^k)$ and $\text{vec}(X^{k+1})$, respectively, if $u^k = \text{vec}(U^k)$ where $Z_k$, $U_k$ and $X^{k+1}$ are as computed at steps 3, 4 and 5 in Algorithm 2. Now,

$$\text{vec}(A^T AZB^T B - A^T Y B) = \left((B^T B) \otimes (A^T A)\right) \text{vec}(Z^k) - (B^T \otimes A^T) \text{vec}(Y)$$

where $A = B \otimes A$. Thus, $u^k$ computed at step 4 in Algorithm 1 is the vectorized version of $U^k$ of step 4 in Algorithm 2. We conclude that Algorithms 1 and 2 provide the same output, however, Algorithm 2 is more efficient.
The solution of (18) is given by
\[ x_t = D_t^{-1} d_t \] (15)
where \( D_t \) and \( d_t \) are as in (10).

3. SPARSE MATRIX RECOVERY VIA OMP

Next, we consider the extension of standard OMP to the matrix form. We can write the observation \( Y \) in (1) as a summation of \( N^2 \) matrices as given below:
\[ Y = \sum_{i,j} X_{ij} a_i b_j^T. \] (17)

When \( X \) is sparse with \( d \) nonzeros, the summation in (17) has only \( d \) terms. Let \( \Sigma_d \) denote the support of \( X \) so that \( X_{ij} \) is non zero for \( i,j \in \Sigma_d \). We can write (17) as
\[ Y = \sum_{(i,j) \in \Sigma_d} X_{ij} a_i b_j^T. \]

Our goal is to recover \( X_{ij} \) for \((i,j) \in \Sigma_d \) in a greedy manner. The proposed OMP version with matrix inputs is given in Algorithm 3. In Algorithm 3, \( \Lambda_t \) contains estimated \((i,j)\) pairs up to \( t\)-th iteration in which the \( m \)-th pair is denoted by \((\Lambda_t(m,1), \Lambda_t(m,2))\) for \( m = 1, \ldots, t \). Once \( \Lambda_t \) is updated as in step 3, the signal is estimated solving the following optimization problem:
\[ x_t = \arg\min_x \| Y - \sum_{m=1}^t x_m a_{\Lambda_t(m,1)} b_{\Lambda_t(m,2)}^T \|_F. \] (18)
The solution of (18) is given by
\[ x_t = D_t^{-1} d_t \] (19)
where \( D_t \) is a \( t \times t \) matrix in which the \((m,r)\)-th element is given by
\[ (D_t)_{m,r} = b_{\Lambda_t(r,2)}^T a_{\Lambda_t(m,2)} a_{\Lambda_t(m,1)} a_{\Lambda_t(r,1)} \] (20)
for \( m,r = 1, \ldots, t \) and
\[ d_t = [b_{\Lambda_t(1,2)}^T a_{\Lambda_t(1,1)} \cdots b_{\Lambda_t(t,2)}^T a_{\Lambda_t(t,1)}]^T \] (21)
is a \( t \times 1 \) vector. Then the new approximation at the \( t\)-th iteration is given by
\[ Q_t = \sum_{m=1}^t x_t(m) a_{\Lambda_t(m,1)} b_{\Lambda_t(m,2)}^T \] (22)
where \( x_t(m) \) denotes the \( m\)-th element of \( x_t \).

Algorithm 3 is a trivial extension of the standard OMP (and was also considered in [5] for \( A = B \)).

3.1. Computational complexity

As shown in [3] for \( A = B \), it can be easily verified that Algorithm 3 and the standard OMP in [15] with vector inputs in [2] provide the same performance at each iteration. However, the computational complexity of Algorithm 3 is less than that of its vector counterpart. Step 2 in Algorithm 3 can be implemented as a matrix multiplication \( A^T R_{t-1} B \). Thus, the computational complexity of this step is in the order of \( O(NML + N^2L) \). It is noted that, when implementing the standard OMP as in [15] with vector form, the equivalent step is computed with complexity of \( O(N^2ML) \). With respect to step 4 in Algorithm 3 the matrix \( D_t \) requires \( O(t^2(M + L)) \) computations at the \( t\)-th iteration. The vector \( d_t \) requires \( O(t(ML + M)) \) computations. Worst case complexity of the inverse operation is \( O(t^2) \). Matrix-vector multiplication in (18) requires \( O(t^3) \) computations. Thus, at a given iteration, worst case complexity of step 2 in Algorithm 3 is in the order of \( O(t^3ML) \). It can be shown that the worst case computational complexity of the equivalent step of standard OMP with Kronecker products to estimate the signal at \( t\)-th iteration, is in the order of \( O(t^2ML) \). Thus, steps 2 and 4 in Algorithm 3 provide us with a computational gain over the equivalent steps of the standard OMP with Kronecker products. Therefore, we conclude that Algorithm 3 is an efficient way to find sparse \( X \) from \( Y \) compared to its vector counterpart although both provide the same performance. It is further observed that this computational gain is not as significant as with FISTA.

4. NUMERICAL RESULTS

In this section, we demonstrate the capability of recovering sparse \( X \) from observation model (1) via different algorithms and provide insights into the computational gains achievable with the matrix version. First, we illustrate the performance of FISTA with different choices for \( A \) and \( B \). For numerical results, we assume that \( X \) is a distributed sparse matrix in which each column has a maximum of \( K \) nonzeros and the locations are generated uniformly. The values of nonzero entries are drawn from a uniform distribution in the range \([-250, 250] \cup [200, 250] \). We consider that the observation matrix \( Y \) in (1) is corrupted by additive noise and the elements of the noise matrix are assumed to be independently and identically distributed Gaussian random variables with mean zero and variance \( \sigma_v^2 \).

In Fig. 1 we plot the normalized reconstruction error \( \|X - \hat{X}\|_F / \|X\|_F \) vs \( M \) when \( M = L \) averaging over 500 trials. We let \( M = 40 \), and \( \sigma_v^2 = 0.01 \). In Fig. 1 we illustrate two aspects. First, for given \( K \), different types of matrices \( A \) and \( B \) are examined. We consider independent random rows of the \( N \times N \) DCT matrix, Gaussian, and binary matrices. In the case of a Gaussian, elements are drawn from a normal ensemble and then orthogonalized. By binary matrix, we mean that the elements of the matrix can take values \( 1/0 \) or \( 0/1 \) with equal probability. Note that, random rows of DCT matrix and Gaussian matrix obey uniform uncertainty principle with good isometry constants in contrast to a binary matrix. When both matrices \( A \) and \( B \) are either random rows of DCT matrix or zero mean Gaussian, the recovery of the sparse matrix is guaranteed with less measurements compared to \( N \). When \( A \) and \( B \) are binary, the recovery is not so good, which is intuitive since binary matrices are not ”good” compressive sensing matrices. However, when \( A \) is binary and \( B \) is Gaussian, we see an improved performance compared to the case where both are binary. This provides an insight that even when one matrix does not obey uniform uncertainty principle with good isometry constant, the sparse matrix can be recovered reliably when the other matrix is a ”good compressive sensing” matrix.
In Table 1, we compare the average runtime with MATLAB (in Intel(R) Core(TM) i7-3770 CPU @ 3.40GHz processor with 12 GB RAM) for FISTA for matrix and vector versions as the sparse matrix dimension $N$ varies given that the number of iterations in both Algorithms 1 and 2 is fixed at the same value ($= 10000$). We let $K = N/20$ and $M = L = N/2$. Matrices $A$ and $B$ are assumed to be Gaussian. It reflects the computational efficiency of the matrix approach compared to the vector approach especially as $N$ increases although both algorithms provide the same performance.

To illustrate the performance of OMP with matrix inputs, we plot the fraction of the support correctly recovered with Algorithm 1 for different choices for $A$ and $B$ with $K = 1$ in Fig. 3. From Fig. 3, it is again observed that, although not with the same scale as with FISTA, the recovery capability can be improved when one projection matrix is binary and the other is Gaussian compared to the case where both $A$ and $B$ are binary. Another observation is that even with a Gaussian matrix, as $K$ increases the performance of OMP degrades significantly leaving OMP not a better choice when the sparsity level increases.

In this paper, we showed numerically that recovering $X$ based on (1) in its matrix form is more computationally efficient than solving it after converting to vector form via Kronecker products when $X$ is arbitrarily distributed sparse. We developed matrix versions of FISTA to solve $l_1$ norm minimization in (4) efficiently and OMP to solve for $X$ in a greedy manner. It has been shown that a significant computational gain is achieved by FISTA with matrix form compared to its vector counterpart. We further illustrated the recovery capability with different choices for projection operators. The results provide insight into the following. If a linear system of the form (2) can be put into matrix form (3) for different choices for $A$ and $B$, the recovery capability can be improved when one projection matrix is binary and the other is Gaussian compared to the case where both $A$ and $B$ are binary. Another observation is that even with a Gaussian matrix, as $K$ increases the performance of OMP degrades significantly leaving OMP not a better choice when the sparsity level increases.

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5. DISCUSSION

In this paper, we showed numerically that recovering $X$ based on (1) in its matrix form is more computationally efficient than solving it after converting to vector form via Kronecker products when $X$ is arbitrarily distributed sparse. We developed matrix versions of FISTA to solve $l_1$ norm minimization in (4) efficiently and OMP to solve for $X$ in a greedy manner. It has been shown that a significant computational gain is achieved by FISTA with matrix form compared to its vector counterpart. We further illustrated the recovery capability with different choices for projection operators. The results provide insight into the following. If a linear system of the form (2) can be converted into a matrix form as in (3), the problem can be solved more efficiently without losing performance with respect to the original vector form. Thus, it is worth investigating such scenarios where the matrix approach can be efficiently used to solve linear systems which are computationally demanding otherwise.
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