LSND anomaly from \textit{CPT} violation in four-neutrino models

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The LSND signal for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations has prompted supposition that there may be a fourth light neutrino or that \textit{CPT} is violated. Neither explanation provides a good fit to all existing neutrino data. We examine the even more speculative possibility that a \textit{four-neutrino model with CPT violation} can explain the LSND effect and remain consistent with all other data. We find that models with a $3 + 1$ mass structure in the neutrino sector are viable; a $2 + 2$ structure is permitted only in the antineutrino sector.

\section{I. INTRODUCTION}

The LSND experiment has found evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations at the 3.3$\sigma$ level $^1$$^2$, with indications for $\nu_\mu \rightarrow \nu_e$ oscillations at lesser significance $^2$$^3$.$^4$. The combination of the LSND data with the compelling evidence for oscillations in solar, atmospheric, accelerator, and reactor neutrino experiments cannot be adequately explained in the standard three-neutrino picture with \textit{CPT} conservation $^5$. Extensions to models with four light neutrinos (with the extra neutrino being sterile) $^4$ or \textit{CPT} violation $^6$$^7$$^8$ with three neutrinos have been proposed to accommodate all neutrino data. However, in both cases, recent analyses indicate that neither scenario provides a good description of the data $^7$$^8$$^9$.

The MiniBooNE experiment $^9$ is now taking data that will test the LSND oscillation parameters$^1$ in the $\nu_\mu \rightarrow \nu_e$ channel. A positive result in MiniBooNE will rule out current versions of \textit{CPT}-violating models, while a negative result will rule out four-neutrino models with \textit{CPT} conservation. In either case, the surviving models will still not give a good fit to all data.

In this letter we consider the very speculative possibility of \textit{CPT violation in four-neutrino models}. The \textit{CPT} violation is manifested as different mass matrices for neutrinos and antineutrinos$^2$. We find that such a scenario can accommodate the data only if the sterile neutrino is weakly coupled to active neutrinos. Thus, while $3 + 1$ models are viable, $2 + 2$ models (in which the sterile neutrino is strongly coupled to active neutrinos in solar and/or atmospheric oscillations), are not. A hybrid solution ($3 + 1$ for neutrinos and $2 + 2$ for antineutrinos) is also possible.

\section{II. FOUR NEUTRINOS OR \textit{CPT} VIOLATION?}

There are two types of four-neutrino models: (a) $3 + 1$, where active neutrinos have mass-squared differences and mixings similar to the standard three-neutrino model that describes solar and atmospheric data, and (b) $2 + 2$, where there are two pairs of closely spaced mass eigenstates, one of which accounts for the solar neutrino data and the other for the atmospheric neutrino data. The $3 + 1$ models with \textit{CPT} conservation are disfavored because the Bugey reactor $^11$ and CDHSW accelerator $^12$ experiments put constraints on oscillation amplitudes for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_\mu$ survival, respectively, which together imply an upper limit on the LSND oscillation amplitude that is below the experimental value $^7$$^17$$^10$. The $2 + 2$ models with \textit{CPT} conservation are ruled out because the combination of solar and atmospheric data do not allow enough room for a full sterile neutrino $^3$.

To resurrect the scenario in which sterile and active states are only weakly coupled, it has been shown that extending $3 + 1$ models by an extra sterile neutrino improves the fit to short-baseline data substantially $^17$. The constraints on $2 + 2$ models may be relaxed by including certain small neglected mixing angles in the analysis $^{18}$$^{19}$.

In \textit{CPT}-violating models with three neutrinos, the mass spectra, and hence the mass-squared differences, are different for neutrinos and antineutrinos$^3$. In the original versions of these models $^5$, the neutrino mass-squared differences accounted for the solar and atmospheric oscillations (but not the weak $\nu_\mu \rightarrow \nu_e$ signal in LSND), while the antineutrino mass-squared differences accounted for the LSND and atmospheric oscillations. With the addition of the KamLAND data indicating $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ oscillations at the solar $6\sigma$ scale, the antineutrino mass-squared differences were adjusted to account for the LSND and KamLAND oscillations (but not

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\textsuperscript{1} The bulk of the parameter region allowed by $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ data from LSND and KARMEN $^{12}$, and $\bar{\nu}_e \rightarrow \bar{\nu}_e$ data from the Bugey reactor experiment $^{11}$, lies in a narrow band in $(\sin^2 2\theta_L, |\delta m^2_L|)$ space along the line described approximately by $\sin^2 2\theta_L (|\delta m^2_L|)^{1/6} = 0.0025$ between $|\delta m^2_L| \sim 0.2$ and $1$ eV$^2$. A small allowed region near $|\delta m^2_L| \sim 7$ eV$^2$ and $\sin^2 2\theta_L = 0.004$ also exists $^{12}$.

\textsuperscript{2} Whether such a model can be constructed using nonlocality of the interactions without violating Lorentz invariance is still a matter of debate $^3$.

\textsuperscript{3} A comparison of solar and reactor neutrino data can constrain \textit{CPT} violation $^{10}$; a 90\% C. L. limit of $|\delta m^2_s - \delta m^2_l| < 1.3 \times 10^{-3}$ eV$^2$ was found in Ref. $^{22}$. 

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oscillations of atmospheric antineutrinos)\textsuperscript{3}. An analysis of the atmospheric data showed that the modified CPT-violating model did not give a good fit and is excluded at the 3σ level\textsuperscript{3}. Thus, neither four-neutrino models with CPT conservation nor three-neutrino models with CPT violation provides a consistent explanation of all the data including LSND.

### III. FOUR NEUTRINOS WITH CPT VIOLATION

The neutrino flavor states $\nu_\alpha (\alpha = e, \mu, \tau, s)$ are related to the mass eigenstates $\nu_i$ by a unitary matrix $U$, with

$$\nu_\alpha = \sum_{i=1}^{4} U_{\alpha i} \nu_i .$$

If CPT is not conserved, then the corresponding unitary matrix $\bar{U}$ for antineutrinos, given by

$$\bar{\nu}_\alpha = \sum_{i} \bar{U}_{\alpha i} \bar{\nu}_i ,$$

will not necessarily be equal to $U$. Furthermore, the neutrino and antineutrino eigenmasses will not necessarily be the same. We will assume that there are three different mass-squared difference scales for neutrinos, $\delta m^2_A \ll \delta m^2_\odot \ll \delta m^2_L$, that can explain the solar, atmospheric and LSND data, respectively, and that the corresponding mass-squared differences for antineutrinos are similar to those for neutrinos, i.e., $\bar{\delta} m^2_A \approx \delta m^2_\odot, \quad \bar{\delta} m^2_\odot \approx \delta m^2_L, \quad \text{and} \quad \bar{\delta} m^2_L \approx \delta m^2_L$.

#### A. 3 + 1 models

In 3 + 1 models there is one neutrino mass well-separated from the others by $\delta m^2_L$, and the sterile neutrino couples strongly only to the isolated state. There are four different mass spectra in 3 + 1 models, depending on whether the isolated state is above or below the others, and whether the other three neutrino states have a normal or inverted mass hierarchy. We consider the case with $m_4 > m_1, m_2, m_3$ and normal hierarchy, which implies $\bar{\delta} m^2_{12} \simeq \bar{\delta} m^2_{23} \simeq \bar{\delta} m_{13}^2 = \delta m^2_\odot \gg \bar{\delta} m^2_{12} = \delta m^2_\odot \gg \bar{\delta} m^2_{23} = \delta m^2_L$, with similar relations for $\delta m^2_A$ (the argument is similar for the other cases). Then the relevant oscillation probabilities for neutrinos are approximately,

$$P(\nu_e \rightarrow \nu_e)_{\text{solar}} \simeq 1 - 4|U_{e1}|^2|U_{e2}|^2 \sin^2 \Delta_s ,$$

$$P(\nu_\mu \rightarrow \nu_\mu)_{\text{atm}} \simeq 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2) \sin^2 \Delta_\alpha ,$$

$$P(\nu_\mu \rightarrow \nu_\tau)_{\text{MiniBooNE}} \simeq 4|U_{\tau 4}|^2|U_{\mu 4}|^2 \sin^2 \Delta_L ,$$

$$P(\nu_\mu \rightarrow \nu_\mu)_{\text{CDHSW}} \simeq 1 - 4|U_{\mu 4}|^2(1 - |U_{\mu 4}|^2) \sin^2 \Delta_L ,$$

where $\Delta_\alpha = \delta m^2_\odot/(4E_{\nu})$ is the usual oscillation argument for $x = s, a, \text{or} \ L$. The relevant oscillation probabilities for antineutrinos are approximately,

$$\bar{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\text{KamLAND}} \simeq 1 - 4|\bar{U}_{e1}|^2|\bar{U}_{e2}|^2 \sin^2 \bar{\Delta}_s ,$$

$$\bar{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)_{\text{atm}} \simeq 1 - 4|\bar{U}_{\mu 3}|^2(1 - |\bar{U}_{\mu 3}|^2 - |\bar{U}_{\mu 4}|^2) \sin^2 \bar{\Delta}_\alpha ,$$

$$\bar{P}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)_{\text{LSND}} \simeq 4|\bar{U}_{e4}|^2|\bar{U}_{\mu 4}|^2 \sin^2 \bar{\Delta}_L ,$$

$$\bar{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e)_{\text{Bugey}} \simeq 1 - 4|\bar{U}_{e4}|^2(1 - |\bar{U}_{e4}|^2) \sin^2 \bar{\Delta}_L ,$$

where $\bar{\Delta}_s = \delta m^2_\odot/(4E_{\nu})$. We note that LSND has already made a measurement for $P_{\text{MiniBooNE}}$, although at the 2σ level it is consistent with both $P_{\text{LSND}}$ and zero.

If CPT is not conserved\textsuperscript{4}, then in general $U \neq \bar{U}$. The Bugey+CDHSW bound on LSND and MiniBooNE can now be evaded since Bugey limits $|\bar{U}_{e4}|$ and CDHSW limits $|\bar{U}_{\mu 4}|$, but $|\bar{U}_{\mu 4}|$ and $|\bar{U}_{e4}|$ are no longer tightly constrained. The bounds on the amplitudes for $\bar{\nu}_e \rightarrow \bar{\nu}_e$ and $\nu_\mu \rightarrow \nu_e$ oscillations must be determined separately\textsuperscript{5}:

(i) The best constraint on $|\bar{U}_{e4}|$ comes from $\nu_e$ disappearance performed during GALLEX testing with a $^{51}$Cr neutrino source\textsuperscript{23}; it does not extend as low in $\delta m^2_\odot$ as the Bugey constraint and is about an order of magnitude weaker at high $\delta m^2_\odot$. (The GALLEX survival probability is given by Eq.\textsuperscript{10} with $\bar{U}_{e4}$ replaced by $U_{e4}$). When combined with the limit on $|\bar{U}_{\mu 4}|$ from CDHSW\textsuperscript{11}, a range of $\nu_\mu \rightarrow \nu_e$ oscillation amplitudes $4|U_{e4}|^2|U_{\mu 4}|^2$ is excluded; see Fig.\textsuperscript{4} A positive MiniBooNE result that exceeds the upper bound in Fig.\textsuperscript{4} will rule out 3 + 1 models with CPT violation, while a null or positive result that obeys the upper bound is easily accommodated. A MiniBooNE measurement consistent with the region allowed by

\textsuperscript{4} The reason that 3 + 1 models with CPT conservation (where $U = \bar{U}$) are disfavored is that $|U_{\mu 4}|^2$ must be small from a combination of the strict CDHSW limit\textsuperscript{11} and the large mixing of atmospheric $\nu_\mu$, and $|U_{e4}|^2$ must be small from a combination of the strict Bugey limit\textsuperscript{13} and the large mixing of solar $\nu_e$ and KamLAND $\nu_e$, leading to an upper bound on the LSND amplitude $4|U_{e4}|^2|U_{\mu 4}|^2$\textsuperscript{13,14}. The CDHSW and Bugey limits are $\delta m^2_\odot$ dependent (as is the LSND allowed amplitude), but a comparison at all $\delta m^2_\odot$ shows that nowhere does the LSND+KARMEN 95% C. L. allowed region overlap the 95% C. L. allowed region from the other experiments\textsuperscript{23}. The most recent comprehensive analysis concludes that the 3 + 1 models have a goodness of fit of at most $5.6 \times 10^{-3}$\textsuperscript{2}. \textsuperscript{5} Although neutrino telescopes are capable of probing the LSND scale\textsuperscript{22}, they are unable to test CPT-violating schemes because neutrino and antineutrino oscillation probabilities are not measured separately.
trino allowed regions could be explained by
that might occur between the KamLAND and solar neutrino
dependent of the KamLAND fits. Thus, any differences
must agree, so that, e.g., antineutrinos (which could perhaps be measured sepa-
analysis of LSND and KARMEN data, both at the 90%
\(\nu\)
2 amplitude \(\delta m^2\)
(10% C. L. results are used in both cases). The dot-dashed line is the 90% C. L. upper bound from Bugey and CDHSW if CPT is conserved \[21\]. Also shown are the expected sensitivity (dashed) of the MiniBooNE experiment and, for comparison, the allowed region (within the dotted lines) for \(4|U_{e4}|^2|U_{\mu 4}|^2\) from a combined analysis of LSND and KARMEN data, both at the 90% C. L \[12\].

LSND and KARMEN would mean that the values of \(4|U_{e4}|^2|U_{\mu 4}|^2\) and \(4|U_{e4}|^2|U_{\mu 4}|^2\) are similar, but the Bugey+CDHSW bound demands \(\bar U \neq U\). In fact, a MiniBooNE measurement that lies below the GALLEX+CDHSW upper limit and above the Bugey+CDHSW limit, would place a lower bound on the amount of CPT violation.

(ii) The best accelerator constraint on \(|\bar U_{\mu 4}|\) comes from the CCFR experiment that searched for \(\bar \nu_\mu\) disappearance \[24\]; there is no limit for \(\delta m^2\) \(<\) 7 eV\(^2\). Although \(|\bar U_{\mu 4}|\) cannot be so large as to disturb the usual fits to atmospheric data (which indicate the dominant oscillation is at the \(\delta m^2\) scale), \(|\bar U_{\mu 4}|^2\) can probably be of order a few percent, which allows the LSND amplitude \(4|U_{e4}|^2|U_{\mu 4}|^2\) to be large enough to account for the LSND \(\bar \nu_\mu \rightarrow \bar \nu_e\) data, at least for the larger values of \(\delta m^2\) allowed by LSND (near 1 eV\(^2\) and 7 eV\(^2\)); see Fig. 2.

We note that fits for neutrinos and antineutrinos no longer must agree, so that, e.g., the solar fits are now independent of the KamLAND fits. Thus, any differences that might occur between the KamLAND and solar neutrino allowed regions could be explained by CPT violation. Similarly, results for atmospheric neutrinos and antineutrinos (which could perhaps be measured separately in MINOS \[25\]) could also be different and still be easily accommodated in the model. On the other hand, if there were no discernible difference between the neutrino and antineutrino fits to solar, KamLAND, and atmospheric data, the amount of CPT violation needed would be small, since then the only differences between neutrino and antineutrino parameters would occur in the small mixings between the sterile and the active states.

B. 2 + 2 models

Constraints on \(2 + 2\) models are different from those on the \(3 + 1\) models because they have different expressions for the oscillation probabilities. The difficulty with these models is that the sterile neutrino is strongly coupled to solar and/or atmospheric neutrino oscillations, and there are bounds on the amount of sterile content in each case. In \(2 + 2\) models, solar \(\nu_e\) oscillate predominantly to a linear combination of \(\nu_\tau\) and \(\nu_\mu\) and atmospheric \(\nu_\mu\) oscillate to the orthogonal combination \[10\]:

\[
\nu_e \rightarrow -\sin \alpha \nu_\tau + \cos \alpha \nu_\mu ,
\]

\[
\nu_\mu \rightarrow \cos \alpha \nu_\tau + \sin \alpha \nu_\mu .
\]

If CPT is violated, then there is a similar sterile mixing angle \(\bar \alpha\) for antineutrinos. The amount of sterile content is \(\cos^2 \bar \alpha\) in solar neutrino oscillations, \(\cos^2 \bar \alpha\) in KamLAND, \(\sin^2 \bar \alpha\) in atmospheric neutrino oscillations, and \(\sin^2 \bar \alpha\) in atmospheric antineutrino oscillations. Fits to
solar neutrino data give the 99% C. L. limit \[ \cos^2 \alpha \leq 0.45; \] (13)

note that there is no limit on \( \cos^2 \tilde{\alpha} \) from KamLAND since the short baseline has negligible matter effects and it therefore does not test the the sterile content. Fits to the atmospheric neutrino data (which do not distinguish between neutrinos and antineutrinos) give the 99% C. L. limit \[ \frac{2}{3} \sin^2 \alpha + \frac{1}{3} \sin^2 \tilde{\alpha} \leq 0.35, \] (14)
due to the lack of matter effects that would occur in \( \nu_\mu \rightarrow \nu_\tau \) oscillations \[28\]. In Eq. (14), neutrinos contribute with twice the strength of antineutrinos because of their larger interaction cross section.

The bounds on \( 2 + 2 \) models are shown in Fig. 3 versus \( \sin^2 \alpha \) and \( \sin^2 \tilde{\alpha} \). The \( CPT \)-conserving case \( \alpha = \tilde{\alpha} \) is indicated by the dotted line. A recent comprehensive analysis of \( 2 + 2 \) models with \( CPT \) conservation concludes that the goodness of fit to all data is only \( 1.6 \times 10^{-6} \) \[27\], which is worse than that of \( 3 + 1 \) models with \( CPT \) conservation\[6\]. When \( CPT \) is violated, \( i.e., \alpha \neq \tilde{\alpha} \), there is no region that obeys both the solar and atmospheric bounds. Thus even when \( CPT \) is violated, \( 2 + 2 \) models are strongly disfavored.

The analysis that lead to Eq. (13) used the standard solar model (SSM) \[27\] neutrino fluxes, including their theoretical uncertainties. If the \( ^8B \) neutrino flux is allowed to be free, this bound relaxes to \( \cos^2 \alpha \leq 0.61 \) \[28\]. Then the solar constraint in Fig. 3 is \( \sin^2 \alpha \geq 0.39 \), and there is a small allowed region with \( 0.39 \leq \sin^2 \alpha \leq 0.53 \) and small \( \sin^2 \tilde{\alpha} \). This can be understood qualitatively as follows: although there is room for a full sterile neutrino in the solar and atmospheric data when \( CPT \) is conserved, with \( CPT \) violation the fact that KamLAND does not test the sterile content means that oscillations of atmospheric antineutrinos can be largely to active neutrinos, which effectively dilutes the constraint on the sterile content in atmospheric neutrino oscillations. However, there is currently no reason to believe that the \( ^8B \) neutrino flux is not well-described by the SSM.

\section*{C. Hybrid models}

Since \( CPT \) is violated, in principle one can have a \( 3 + 1 \) model in the neutrino sector and a \( 2 + 2 \) model in the antineutrino sector, or vice versa. We now examine these two possibilities:

\section*{IV. SUMMARY}

We have argued that a four-neutrino model with \( CPT \) violation can provide an explanation of all neutrino oscillation data if the neutrino sector has a \( 3 + 1 \) structure and the antineutrino sector is either \( 3 + 1 \) or \( 2 + 2 \); a \( 2 + 2 \) structure is not allowed in the neutrino sector. If the antineutrino sector is \( 3 + 1 \), then \( \delta m^2_L \) must be near

\[\ldots\]

\[\ldots\]

\[\ldots\]
1 or 7 eV$^2$, while if it is 2 + 2, the entire range of $\delta m^2$ allowed by LSND, KARMEN and Bugey is possible.

A detection of $\nu_\mu \rightarrow \nu_e$ oscillations by MiniBooNE with parameters above the solid line in Fig. II will require $CPT$ violation to be too large and 3 + 1 models with $CPT$ violation will be excluded. However, if MiniBooNE has a null oscillation result or finds oscillation parameters that lie below the solid line, such models will remain viable. Even if the MiniBooNE allowed region for neutrinos falls within the LSND+KARMEN allowed region for antineutrinos, the Bugey+CDHSW bound on $CPT$ conserving 3 + 1 models (dot-dashed line in Fig. II) implies $CPT$ violation of the size of the LSND effect ($O(10^{-3})$ in terms of oscillation probabilities).

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