Hadron Spectroscopy from the Lattice

Chris Michael

Division of Theoretical Physics, Department of Mathematical Sciences,
University of Liverpool, Liverpool L69 3BX, UK.

Abstract.

Lattice QCD determinations appropriate to hadron spectroscopy are reviewed with emphasis on the glueball and hybrid meson states in the quenched approximation. Hybrids are discussed for heavy and for light quarks. The effects of sea quarks (unquenching) are explored. Heavy-light systems are presented - particularly excited B mesons.

INTRODUCTION

Quantum Chromodynamics is generally acknowledged to be the theory of hadronic interactions. Its perturbative features are well understood and provided the main motivation for its adoption. To give a complete description of hadronic physics, it is essential to develop non-perturbative methods to handle QCD. QCD is a quantum field theory which needs to be regulated in order to have a well defined mathematical approach. Inevitably such a regularisation will destroy some of the symmetries of QCD. For example dimensional regularisation is often used in perturbative studies and it breaks the 4-dimensional Lorentz invariance. Similarly lattice regularisation as proposed by Wilson [1] breaks Lorentz invariance since a hypercubic lattice of space-time points is invoked. The key feature of Wilson’s proposal is that gauge invariance is exactly retained. Then the approach to the continuum limit (as the lattice spacing $a$ is reduced to zero) can be shown to be well defined. Using Monte Carlo methods to explore lattice QCD, reliable predictions can now be made for continuum quantities as I shall discuss. Moreover, in the continuum limit, the Lorentz invariance is found to be fully restored.

Lattice QCD needs as input the quark masses and an overall scale (conventionally given by $\Lambda_{QCD}$). Then any Green function can be evaluated by taking an average of suitable combinations of the lattice fields in the vacuum samples. This allows masses to be studied easily and matrix elements (particularly those of weak or electromagnetic currents) can be extracted straightforwardly. Scattering and hadronic decays are only accessible in a rather limited way.

Unlike experiment, lattice QCD can vary the quark masses and can also explore different boundary conditions and sources. This allows a wide range of studies
which can be used to diagnose the health of phenomenological models as well as casting light on experimental data.

One very special case is of considerable interest: this is quenched QCD where the sea-quark masses are taken as infinite. This suppresses quark loops in the vacuum completely, leaving just the full non-perturbative gluonic interactions. This gluonic vacuum turns out to reproduce most of the salient features of QCD. It is also a very convenient approximation to use for comparison with phenomenological models. Quenched QCD is computationally rather easy to study and the precise results allow the continuum limit to be extracted reliably. Studies with sea quark effects included (known as dynamical fermion studies) are computationally much more demanding. I discuss the current situation in this area in the last section.

GLUEBALLS

Glueballs are defined to be hadronic states made primarily from gluons. The full non-perturbative gluonic interaction is included in quenched QCD. In the quenched approximation, there is no mixing between such glueballs and quark-antiquark mesons. A study of the glueball spectrum in quenched QCD is thus of great value. This will allow experimental searches to be guided as well as providing calibration for models of glueballs.

In principle, lattice QCD can study the meson spectrum as the sea quark mass is decreased towards experimental values. This will allow the unambiguous glueball states in the quenched approximation to be tracked as the sea quark effects are increased. It may indeed turn out that no meson in the physical spectrum is primarily a glueball - all states are mixtures of glue, $q\bar{q}$, $qq\bar{q}$, etc. Studies conducted so far show no significant change of the glueball spectrum as dynamical quark effects are added - but the sea quark masses used are still rather large [2].

In lattice studies, dimensionless ratios of quantities are obtained. To explore the glueball masses, it is appropriate to combine them with another very accurately measured quantity to have a dimensionless observable. Since the potential between static quarks is very accurately measured from the lattice (see the next section for more details), it is now conventional to use $r_0$ for this comparison. Here $r_0$ is implicitly defined by $r^2dV(r)/dr = 1.65$ at $r = r_0$. In practice $r_0$ may be related to the string tension $\sigma$ by $r_0\sqrt{\sigma} = 1.18$.

Theoretical analysis indicates that for the quenched approximation the dimensionless ratio $mr_0$ will differ from the continuum limit value by corrections of order $a^2$. Thus in Fig. 1 the masses are plotted versus the lattice spacing $a^2$ for the $J^{PC}=0^{++}$ and $2^{++}$ glueballs. The straight lines then show the continuum limit obtained by extrapolating to $a = 0$. As can be seen, there is essentially no need for data at even smaller $a$-values to further fix the continuum value. The values shown correspond to $m(0^{++})r_0 = 4.33(5)$ and $m(2^{++})r_0 = 6.0(6)$. Since several lattice groups [3–6] have measured these quantities, it is reassuring to see that the purely lattice observables are in excellent agreement. The publicised difference of quoted
FIGURE 1. The value of mass of the $J^{PC} = 0^{++}$ and $2^{++}$ glueball states from refs [3–6] in units of $r_0$ where $r_0 \approx 0.5$ fm. The $T_2$ and $E$ representations are shown by octagons and diamonds respectively and their agreement indicates the restoration of rotational invariance for the $2^{++}$ state. The straight lines show fits describing the approach to the continuum limit as $a \to 0$.

$m(0^{++})$ from UKQCD [5] and GF11 [6] comes entirely from relating quenched lattice measurements to values in GeV as I now discuss.

In the quenched approximation, different hadronic observables differ from experiment by factors of up to 10%. Thus using one quantity or another to set the scale, gives an overall systematic error. Here I choose to set the scale by taking the conventional value of the string tension, $\sqrt{\sigma} = 0.44$GeV, which then corresponds to $r_0^{-1} = 373$ MeV. An overall systematic error of 10% is then to be included to any extracted mass. This yields $m(0^{++}) = 1611(30)(160)$ MeV and $m(2^{++}) = 2232(220)(220)$ MeV where the second error is the systematic scale error. Note that these are glueball masses in the quenched approximation - in the real world significant mixing with $q\bar{q}$ states could modify these values substantially.

Recently a lattice approach using a large spatial lattice spacing with an improved action and a small time spacing has been used to study glueball masses. The results [7] are that $r_0 m(0^{++}) = 3.98(15)$, $r_0 m(2^{++}) = 5.85(2)$, $r_0 m(1^{+-}) = 7.21(2)$ and $r_0 m'(2^{++}) = 8.11(4)$. There remains a small discrepancy with the result for
FIGURE 2. The mass of the glueball states with quantum numbers $J^{PC}$ from ref [5]. The scale is set by $\sqrt{\sigma} \approx 440$ MeV which yields the right hand scale in MeV. The solid circles represent mass determinations whereas the open circles are upper limits.

I have focussed on the scalar and tensor glueball results because these are the lightest and best measured states in lattice studies. The glueball spectrum has been extracted for all $J^{PC}$ values [4,5]. Results are shown in Fig. 2. One signal of great interest would be a glueball with $J^{PC}$ not allowed for $q\bar{q}$ - a spin-exotic glueball or oddball. These states are shown in Fig. 2 to be high lying: at least above $2m(0^{++})$. Thus they are likely to be in a region very difficult to access unambiguously by experiment.

The only other candidate for a relatively light glueball is the pseudoscalar. Values quoted of $r_0 m(0^{-+}) = 5.6(6), 7.1(1.1)$ and $5.3(6)$ from refs [4,5] suggest an average of $6.0(1.0)$, not appreciably lighter than the tensor glueball. This is confirmed by preliminary results from the group of ref [7] that the pseudoscalar is heavier than the tensor glueball.

Within the quenched approximation, even though the $0^{++}$ glueball width is zero, it is possible in principle to measure the decay matrix element between a $0^{++}$ glueball and two pseudoscalar $q\bar{q}$ mesons. This then allows an estimate of the
HEAVY QUARK INTERACTIONS

In the limit $m_Q \to \infty$, the heavy quark effective theory describes a universal behaviour. For finite $m_Q$, corrections of order $1/m_Q$ are expected. The simplest way to study the heavy quark limit on a lattice is to use static quarks. The potential energy $V(R)$ between a static quark and antiquark at separation $R$ is readily obtained. Then for heavy quarks, one may solve for the spectra in this potential using the Schrödinger equation in the adiabatic approximation. The quenched lattice potential is well measured and is found to have a form parametrised by

\[
V(R) = V_0 - \frac{e}{R} + \sigma R
\]  

where $e$ is the coefficient of the Coulomb term and $\sigma$ is the string tension. This expression shows that the potential continues to increase as $R$ is increased - this is confinement.

A comparison from ref [9] of the spectrum in the quenched lattice potential with the Υ states is shown in Fig. 3. The lattice result is qualitatively similar to the experimental Υ spectrum. The main difference is that the Coulombic part ($e$) is effectively too small (0.28 rather than 0.48). This produces [9] a ratio of mass differences $(1P - 1S)/(2S - 1S)$ of 0.71 to be compared with the experimental ratio of 0.78. This difference in $e$ is understandable as a consequence of the Coulombic force at short distances which would be increased by $33/(33 - 2N_f)$ in perturbation theory in full QCD compared to quenched QCD. We will return to discuss this.

The situation of a static quark and antiquark is a very clear case in which to discuss hybrid mesons which have excited gluonic contributions. A discussion of the colour representation of the quark and antiquark is not useful since they are at different space positions and the combined colour is not gauge invariant. A better criterion is to focus on the spatial symmetry of the gluonic flux. As well as the symmetric ground state of the colour flux between two static quarks, there will be excited states with different symmetries. These were studied on a lattice [10] and the conclusion was that the $E_u$ symmetry (corresponding to flux states from an operator which is the difference of U-shaped paths from quark to antiquark of the form $\nabla - \nabla$) was the lowest lying gluonic excitation. Results for this potential are shown in Fig. 3.

This gluonic excitation corresponds to a component of angular momentum of one unit along the quark antiquark axis. Then one can solve for the spectrum of hybrid
FIGURE 3. Potentials $V(R)$ between static quarks at separation $R$ for the ground state (square and *) and for the $E_u$ symmetry which corresponds to the first excited state of the gluonic flux (octagons and diamonds). Results in lattice units ($a_6^{-1} = 2.02$ GeV) from the quenched calculations of ref [9] are shown by symbols corresponding to different lattice spacings. For the ground state potential the continuous curve is an interpolation of the lattice data while the dotted curve with enhanced Coulomb term fits the spectrum and yields the masses shown. The lightest hybrid level in the excited gluonic potential is also shown.

mesons using the Schrödinger equation in the adiabatic approximation. The spatial wave function necessarily has non zero angular momentum and the lightest states correspond to $L^{PC} = 1^{+-}$ and $1^{-+}$. Combining with the quark and antiquark spins then yields [10] a set of 8 degenerate hybrid states with $J^{PC} = 1^{--}, 0^{-+}, 1^{+-}, 2^{--}$ and $1^{++}, 0^{+-}, 1^{++}, 2^{+-}$ respectively. These contain the spin-exotic states with $J^{PC} = 1^{-+}, 0^{+-}$ and $2^{+-}$ which will be of special interest.

Since the lattice calculation of the ground state and hybrid masses allows a direct prediction for their difference, the result for this 8-fold degenerate hybrid level is illustrated in Fig. 3 and corresponds [9] to masses of 10.81(25) GeV for $b\bar{b}$ and 4.19(15) GeV for $c\bar{c}$. Here the errors take into account the uncertainty in setting the ground state mass using the quenched potential as discussed above. Recently a different lattice technique [11] has been used to explore the excited gluonic levels in the quenched approximation. The results above are confirmed and preliminary
values quoted for the lightest hybrid mesons are 10.83 and 4.25 GeV respectively for $b\bar{b}$ and $c\bar{c}$ with no error estimates given.

The quenched lattice results, after adjusting to take account of the measured $b\bar{b}$ spectrum, suggest that the lightest hybrid mesons lie above the open $B\bar{B}$ threshold by about 270 MeV. This can also be studied by comparing directly the lattice hybrid masses with twice the quenched lattice masses for the $B$ meson [12]. Using quenched results from the smallest lattice spacing ($\beta = 6.2$) available with clover-improved fermions [13] yields $E_{\text{hybrid}} - 2m_B \approx 140(80)$ MeV. This estimate is somewhat smaller than that obtained above. In both cases, however, the hybrid levels lie above the open threshold and are likely to be relatively wide resonances. Another consequence is that the very flat potential implies a very extended wavefunction: this has the implication that the wavefunction at the origin will be small, so hybrid vector states will be weakly produced from $e^+e^-$. It would be useful to explore the splitting among the 8 degenerate $J^{PC}$ values obtained. This could come from different excitation energies in the $L^{PC} = 1^{+-}$ (magnetic) and $1^{+-}$ (pseudo-electric) gluonic excitations, spin-orbit terms, as well as mixing between hybrid states and $Q\bar{Q}$ mesons with non-exotic spin. One way to study this on a lattice is to use the NRQCD formulation which describes non-static heavy quarks which propagate non-relativistically. Preliminary results for hybrid excitations from several groups [14–16] give at present similar results to those with the static approximation as described above, but with some additional evidence, namely that the magnetic excitations (which include the $1^{+-}$ spin exotic) are lighter than the pseudo-electric ones. Future NRQCD results may be more precise and able to establish the splittings among different states.

## LIGHT QUARK INTERACTIONS

Unlike very heavy quarks, light quark propagation in the gluonic vacuum sample is very computationally intensive - involving inversion of huge $(10^7 \times 10^7)$ sparse matrices. Current computer power is sufficient to study light quark physics thoroughly in the quenched approximation. The state of the art [17] is the Japanese CP-PACS Collaboration who are able to study a range of large lattices (up to about $64^4$) with a range of light quark masses. Qualitatively the meson and baryon spectrum of states made of light and strange quarks is reproduced with discrepancies of order 10% in the quenched approximation.

Here I will focus on hybrid mesons made from light quarks. There will be no mixing with $q\bar{q}$ mesons for spin-exotic hybrid mesons and these are of special interest. The first study of this area was by the UKQCD Collaboration [18] who used operators motivated by the heavy quark studies referred to above. Using non-local operators, they studied all 8 $J^{PC}$ values coming from $L^{PC} = 1^{+-}$ and $1^{+-}$ excitations. The resulting mass spectrum is shown in Fig. 4 where the $J^{PC} = 1^{+-}$ state is seen to be the lightest spin-exotic state with a statistical significance of 1 standard deviation. The statistical error on the mass of this lightest spin-exotic meson is 7%
FIGURE 4. The masses in lattice units (with $a^{-1}_{6,0} \approx 2$ GeV) of states of $J^{PC}$ built from hybrid operators with strange quarks, spin-exotic (•) and non-exotic (squares). The dot-dashed lines are the mass values found for $s\bar{s}$ operators. Results from ref [18].

but, to take account of systematic errors from the lattice determination, a mass of 2000(200) MeV is quoted for this hybrid meson with $s\bar{s}$ light quarks. Although not directly measured, the corresponding light quark hybrid meson would be expected to be around 120 MeV lighter. In view of the small statistical error, it seems unlikely that the $1^{-+}$ meson in the quenched approximation could lie as light as 1.4 GeV where there are experimental indications for such a state [19]. Beyond the quenched approximation, there will be mixing between such a hybrid meson and $q\bar{q}q\bar{q}$ states such as $\eta\pi$ and this may be significant in the experimental situation.

One feature clearly seen in Fig. 4 is that non spin-exotic mesons created by hybrid meson operators have masses which are very similar to those found when the states are created by $q\bar{q}$ operators. This suggests that there is quite strong coupling between hybrid and $q\bar{q}$ mesons even in the quenched approximation. This would imply that the $\pi(1800)$ is unlikely to be a pure hybrid, for example.

A second lattice group has also evaluated hybrid meson spectra from light quarks from quenched lattices. They obtain [20] masses of the $1^{-+}$ state with statistical and various systematic errors of 1970(90)(300) MeV, 2170(80)(100)(100) MeV and 4390(80)(200) MeV for $n\bar{n}$, $s\bar{s}$ and $c\bar{c}$ quarks respectively. For the $0^{+-}$ spin-exotic
state they have a noisier signal but evidence that it is heavier. They also explore mixing matrix elements between spin-exotic hybrid states and 4 quark operators.

HEAVY-LIGHT INTERACTIONS

Here we discuss the properties of $Q\bar{q}$ mesons and $Qqq$ baryons. The heavy quark effective theory indicates that many properties are independent of $m_Q$ to order $1/m_Q$. Many examples of this have been studied using lattice techniques: the Isgur-Wise function, the pseudoscalar coupling $f_Q\sqrt{m_Q}$ which yields $f_B$, the matrix element for $B\bar{B}$ mixing $B_B$, and meson and baryon spectra and decays. Here I focus on one aspect which is rather easy to describe and which has recently been studied: the spectrum of excited states of the B meson.

In the heavy quark limit, this $Q\bar{q}$ meson will be the ‘hydrogen atom’ of QCD. Since the meson is made from non-identical quarks, charge conjugation is not a good quantum number. States can be labelled by $L_{\pm}$ where the coupling of the light quark spin to the orbital angular momentum gives $j = L \pm \frac{1}{2}$. In the heavy quark limit these states will be doubly degenerate since the heavy quark spin interaction can be neglected, so the $P_{-}$ state will have $J^P = 0^+, 1^+$ while $P_{+}$ has $J^P = 1^+, 2^+$, etc. A recent study [21] of this spectrum for $m_Q \to \infty$ gives the preliminary results shown in Fig. 5 for strange light quarks. Note that most of these states have not been experimentally established for excited $B$ mesons yet. A further study of systematic errors and an extrapolation to light quarks is in progress. It will be interesting to confirm the ordering of the $P_{-}$ and $P_{+}$ levels because it is possible that the long-range spin-orbit interaction could invert them, making the $P_{+}$ lighter.

Another lattice approach, NRQCD, has also been used to study this area and has reported [22] preliminary mass values for $b\bar{q}$ S and P-wave mesons and for $bqq$ baryons with light and strange quarks in qualitative agreement with experiment. This group has also given predictions for the $b\bar{c}$ meson [23].

For bound states of charm quarks, results for mesons [24] and baryons [25] have been reported. These quenched approximation studies give a reasonable description of known states and several predictions for new ones.

TOWARDS FULL QCD

Algorithms exist which allow lattice simulation of full QCD with sea quarks of mass $m_{\text{sea}}$. This study needs lots of CPU power since the sea quark loops in the vacuum are represented effectively as a long range interaction between the gluonic degrees of freedom. Most studies to date have been exploratory with sea quark masses above the strange quark mass. In this regime, very little change from the quenched approximation is seen in physical predictions from the lattice.

One area where specific changes are expected is for the potential between heavy quarks. At small separation $R$, the Coulombic term is expected to increase in strength and indeed some sign of this has been reported [26]. At larger $R$, signs
of string breaking are expected since a light quark antiquark pair can be produced from the vacuum to yield two $Q\bar{q}$ mesons with energy independent of $R$ at large separation. This has been explored by the SESAM and UKQCD collaborations and little sign of the effect is seen [27,26].

Since it has been very difficult to see unambiguous signs of sea quark effects in the spectrum, it is possible that such effects turn on non-linearly as the sea quark mass is reduced. As an example, in current studies the $\pi + \pi$ P-wave is heavier than the $\rho$ meson so the $\rho$ cannot decay. Further work is needed to reduce the sea quark mass and to increase the lattice size. Dedicated computing power of several hundreds of Gflops is available to lattice collaborations and progress in this area should now be possible.

**SUMMARY**

Quenched lattice QCD is well understood and accurate predictions in the continuum limit are increasingly becoming available. Glueball masses of $m(0^{++}) = 1611(30)(160)$ MeV; $m(2^{++}) = 2232(220)(220)$ MeV and $m(0^{-+}) = 2232(370)(220)$ MeV are predicted where the second error is an overall scale error. The quenched
approximation also gives information on quark-antiquark scalar mesons and on hadronic decay matrix elements of glueballs. This mixing with $q\bar{q}$ mesons may well result in no clear experimental glueball candidate.

For hybrid mesons, there will be no mixing with $q\bar{q}$ for spin-exotic states and these are the most useful predictions. The $J^{PC} = 1^{++}$ state is expected at $10.81(25)$ GeV for $b$ quarks; $4.19(15)$ GeV for $c$ quarks, $2.0(2)$ GeV for $s$ quarks and $1.9(2)$ GeV for $u$, $d$ quarks. Mixing of spin-exotic hybrids with $q\bar{q}q\bar{q}$ or equivalently with meson-meson is allowed and will modify the predictions from the quenched approximation.

The lattice has proved a valuable source of information on the heavy quark effective theory. It gives information on the link between a $b$ quark which enters the standard model and the $B$ meson of experiment. Matrix elements such as $f_B$ and $B_B$ have been measured. The spectra of $b\bar{q}$ and $bqq$ hadrons have been predicted.

Much activity is currently underway to explore the effects of sea quarks of ever decreasing mass. Future teraflops computing facilities will be essential to obtain quantitative results both for hadronic spectroscopy and for the study of quark-gluon plasma at non-zero temperature.

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