MAGNETOHYDRODYNAMIC SHOCK–CLUMP EVOLUTION WITH SELF-CONTAINED MAGNETIC FIELDS

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ABSTRACT

We study the interaction of strong shock waves with magnetized clumps. Previous numerical work focused on a simplified scenario in which shocked clumps are immersed in a globally uniform magnetic field that extends through both the clump and the ambient medium. Here, we consider the complementary circumstance in which the field is completely self-contained within the clumps. This situation could arise naturally during clump formation via dynamical or thermal instabilities, for example, as a magnetic field pinches off from the ambient medium. Using our adaptive mesh refinement magnetohydrodynamics code AstroBEAR, we carry out a series of simulations with magnetized clumps that have different self-contained magnetic field configurations. We find that the clump and magnetic evolution are sensitive to the fraction of the magnetic field aligned with, or perpendicular to, the shock normal. The relative strength of magnetic pressure and tension in the different field configurations allows us to analytically understand the different cases of post-shock evolution. We also show that turbulence and the mixing it implies depends on the initial field configuration and suggest ways in which the observed shock–clump morphology may be used as a proxy for identifying internal field topologies a posteriori.

Key words: ISM: jets and outflows – ISM: magnetic fields – ISM: supernova remnants – magnetohydrodynamics (MHD) – planetary nebulae: general

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1. INTRODUCTION

The distribution of matter on virtually all astrophysi- cally relevant scales is non-uniform. Within our own Galaxy, matter overabundances are found in molecular clouds; within these clouds, matter is further distributed unevenly in the star-forming regions known as molecular cloud cores. Clumps of material exist on smaller scales as well. This heterogeneous distribution of matter is required, of course, for star and planet formation. On the other hand, energetic sources such as young stellar objects (YSOs), planetary nebulae, and supernovae inject kinetic energy back into their environments in the form of winds, jets, and shocks. On larger cosmological scales, galaxies are clustered, implying that the early evolution of the universe involved heterogeneous or “clumpy” flows as well. The central regions of active galaxies with their supermassive black holes are involved heterogeneous or “clumpy” flows as well. The central regions of active galaxies with their supermassive black holes are also expected to be home to extensive regions of heterogeneous density distributions with strong incident winds and shocks. Thus, understanding how clumps interact with winds, jets, and shocks remains a central problem in astrophysics. Since dynamically significant magnetic fields are expected to thread much of the plasma in the interstellar and intergalactic medium, the role of magnetic forces on shock–clump interactions is also of considerable interest.

Early analytic studies of single clump/shock interactions focused on the early stages of the hydrodynamic interaction, where the solution remained amenable to linear approximations. The evolution late in time, when the behavior becomes highly nonlinear, remained intractable from a purely analytic standpoint. Analyses at late times have therefore benefited greatly from numerical investigation—a review of the pioneering literature may be found in Klein et al. (1994, hereafter KMC94), or Poludnenko et al. (2002). Illustrating the maturity of the field, a variety of physics has now been included in the studies. KMC94 systematically discussed the evolution of a single, adiabatic, non-magnetized, non-thermally conducting shocked clump overrun by a planar shock in axisymmetry (“2.5D”). Similar simulations were carried out in three dimensions (3D) by Stone & Norman (1992). The role of radiative cooling (e.g., Mellema et al. 2002; Fragile et al. 2004), smooth cloud boundaries (e.g., Nakamura et al. 2006), and systems of clumps (e.g., Poludnenko et al. 2002) have all been studied. A similar problem involving clump–clump collisions has also received attention (e.g., Miniati et al. 1999; Klein & Woods 1998). Most studies predominantly use an Eulerian mesh with a single- or two-fluid method to solve the inviscid Euler equations. One notable exception is the study of Pittard et al. (2008): these authors used a “k + ε” model to explicitly handle the turbulent viscosity.

Although the list of papers described above shows there have been many studies of hydrodynamic shock–clump interactions, fewer numerical studies have focused on magnetohydrodynamics (MHD) shock–clump interactions. Of particular note are the early studies by Mac Low et al. (1994), Jones et al. (1996), and Gregori et al. (2000), which articulated the basic evolutionary paths of a shocked clump with an embedded magnetic field. Further studies at higher resolution (Shin et al. 2008) or including other physical processes such as radiative cooling (Fragile et al. 2005) or heat conduction (Orlando et al. 2005) have also been carried out. In all these studies, however, the magnetic field was restricted to uniform geometries in which the field extended throughout the entire volume including the clump, ambient, and incident shocked gas. Thus, \( B_0 = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \), where \( B_x, B_y, \) and \( B_z \) were constants.

Throughout these studies, the role of fields could be traced to the relative importance of components perpendicular or parallel to the shock normal. The results can be summarized as follows: (1) when the field is parallel to the shock direction, the magnetic field is amplified at the head of and behind the clump. The top of the shocked clump is streamlined but there is no significant suppression of the fragmentation of the clump even for low initial magnetic \( \beta \), the ratio of thermal pressure to magnetic pressure. (2) When the magnetic field is perpendicular to the...
shock normal, the field wraps around the clump and becomes significantly amplified due to stretching driven by the shocked flow. In these cases, the shocked clump becomes streamlined due to field tension and its fragmentation via instabilities can be suppressed even for high initial $\beta$ cases. Adding radiative cooling into the simulation can further change the shocked behavior as more thin fragments and confined boundary flows are formed (Fragile et al. 2005). There were also studies in recent years focusing on the multi-physics aspect of the problem by incorporating processes like thermal diffusion, etc. into the MHD simulations (Orlando et al. 2008).

Thus, these studies with uniform fields have shown the importance of the initial field geometry on the evolution of MHD shocked clumps. The assumption of uniform fields is, however, an over-simplification of real environments in which clumps most likely have some internal distribution of fields that may, or may not, be isolated from the surrounding environment. The creation of an interior field would likely be linked to the ways clumps can be formed. For example, shells of magnetized gas can be swept up via winds or blast waves. If these shells break up via dynamic modes such as Rayleigh–Taylor (hereafter RT) or Kelvin–Helmholtz (hereafter KH) instabilities, then the clumps that form are likely to develop complex internal field topologies. While these fields may stretch into the surrounding medium, reconnection can lead to topological isolation. Numerical studies of MHD RT unstable layers relevant to supernova blast waves confirm the development of internal fields (e.g., Jun et al. 1995). Numerical and high energy density laboratory plasma experiments have also shown how collimated MHD jets can break up into clumps via kink mode instabilities (e.g., Lebedev et al. 2005). The clumps that form via instabilities have been shown to have complex internal fields.

Another example arises when a cold shell embedded in a hot environment attempts to evolve toward thermal equilibrium via thermal conduction. If the shell contains an initially tangled field, then some of the shell material will be captured in the tangled field region and become disconnected from the background field via anisotropic thermal conduction (Li et al. 2012).

Thus, the next level of studies of MHD shock–clump interactions is the exploration of more realistic magnetic fields. Since all studies to date have initialized their simulations with uniform fields, in this work we begin with only interior fields. Our simulation campaign is designed to explore the following question: how do more complex field topologies within the clump alter the evolution of shocked clumps? In an effort to isolate important physical processes, we choose to use relatively simple interior fields, i.e., purely toroidal and purely poloidal fields, both with different alignments with the direction of shock propagation. While we have also carried out simulations with random fields, we will report the results of those studies in a subsequent paper.

In Sections 2 and 3, we describe the numerical method and model. In Section 4, we report our results. In Section 5, we provide an analytic model for the evolution field energy that allows us to correctly order the different initial cases. Finally, in Section 6, we summarize our results and provide our conclusions.

2. MHD EQUATIONS WITH RADIATIVE COOLING

For the simulations, we employed the AstroBEAR code using a 3D computational grid. The AstroBEAR code is a parallel adaptive mesh refinement (AMR) Eulerian hydrodynamics code with capabilities for MHD in 2D and 3D. There are several schemes of varying order available for the user. Details on AstroBEAR may be found in Cunningham et al. (2009), Carroll-Nellenback et al. (2013), and at https://clover.pas.rochester.edu/trac/astrobear. While the code can treat multiple atomic, ionic, and molecular species, in this work we assume the gas has solar abundance and a uniform atomic mass $\mu_A = 1.3$. The MHD equations that we solve numerically are as follows:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + \left( p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\mathbf{B} \cdot \mathbf{B}}{4\pi} \right] = 0, \tag{2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B}) = 0, \tag{3}
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot \left[ \mathbf{v} \left( E + p + \frac{B^2}{8\pi} \right) - \frac{\mathbf{B} \cdot \mathbf{B}}{8\pi} \right] - \Lambda(n, T) = 0, \tag{4}
\]

where $\rho$, $n$, $\mathbf{v}$, $\mathbf{B}$, and $p$ are the density, particle number density, velocity, magnetic field, and pressure, respectively, and $E$ denotes the total energy density given by:

\[
E = \epsilon + \frac{p}{\gamma} + \frac{\mathbf{B} \cdot \mathbf{B}}{8\pi}, \tag{5}
\]

where the internal energy $\epsilon$ is given by:

\[
\epsilon = \frac{p}{\gamma - 1} \tag{6}
\]

and $\gamma = 5/3$.

We denote the radiative cooling by a function of number density and temperature: $\Lambda(n, T)$. In our simulations, we implement the Dalgarno McCray cooling table as it is more realistic compared to simple analytic cooling functions (Dalgarno & McCray 1972). The gas is allowed to cool to a floor temperature of 50 K and then cooling is turned off. We define our parameter regime as “weakly cooling” so that the region inside the clump can cool and hold up together but the dynamics will mostly come from the interactions between the incoming shock and the self-contained magnetic field. This result means that we require the cooling timescale behind the transmitted shock to be smaller than the clump crushing timescale by a factor of less than 10. As we are more interested in the dynamics of the interaction mentioned above, the employment of a different cooling table or cooling floor temperature will result in similar conclusions if the “weakly cooling” assumption is maintained.

The above MHD equations were solved with the Monotone Upstream-centered Schemes for Conservation Laws primitive method with total variation diminishing preserving the Runge–Kutta temporal interpolation. The resistivity is ignored in our calculation so that the dissipation of the magnetic field is numerical only. Small artificial viscosity and diffusion are implemented in order to achieve the required stability and symmetry and prevent carbuncles that can occur when strong cooling and shocks are coupled. AstroBEAR has shown excellent scaling up to $10^4$ processors (Carroll-Nellenback et al. 2013) and the simulations presented in this paper were carried out on 1024 cores of an IBM BlueGene P machine at the University of Rochester’s Center for Integrated Computational Research. In the simulations presented here, numerical considerations with the BlueGene machine led us to turn off AMR, although we carried out AMR versions of the runs on other machines.
its axis aligned with the stripes on the clump surface denote a self-contained toroidal magnetic field with its axis aligned with the x axis inside the clump.

3. PROBLEM DESCRIPTION AND SIMULATION SETUP

The initial conditions for the simulations presented in this paper are all based on the same clump/shock/ambient medium conditions, i.e., the clump, ambient, and shock conditions are the same. The only variable we explored was the internal magnetic field initial condition is illustrated in Figure 1.

We chose conditions that were astrophysically relevant with a focus on clumps occurring in interstellar environments. We note, however, that the behavior seen in our model will scale with the appropriate dimensionless numbers. We denote the shock speed by $v_s$, the ambient sound speed by $c$, the clump density by $\rho_c$, the ambient density by $\rho_a$, the clump thermal pressure by $P_{th}$, the self-contained magnetic field pressure by $P_B$, the clump radius by $r_c$, and the radiative cooling length by $r_r$. Then, as long as the Mach number $M = v_s/c$, clump density ratio $\xi = \rho_c/\rho_a$, plasma beta $\beta = P_{th}/P_B$, and cooling parameter $\chi = r_r/r_c$ are the same between two runs, the solutions should be independent of absolute scales for the input parameters.

Thus, we chose an ambient gas that is non-magnetized and isothermal, with a particle number density of $1 \text{cc}^{-1}$ and a temperature of $10^4 \text{K}$. Our clump starts with a radius of $r_c = 150 \text{AU}$ and is in thermal pressure equilibrium with the ambient medium. The clump has a density contrast of $\xi = 100$, i.e., the particle number density of $100 \text{cc}^{-1}$ and a temperature of $100 \text{K}$. The domain is a box with dimensions $2400 \text{AU} \times 60 \text{AU} \times 60 \text{AU}$, with a resolution of $1296 \times 324 \times 324$, which gives 54 cells per clump radius. We use outflow boundary conditions on the six sides of the box. We are thus able to follow the evolution for approximately 16 clump radii.

The magnetic fields in our clumps were chosen to allow for self-contained geometries. We use $\beta_{\text{avg}}$ to denote the ratio of thermal pressure to averaged magnetic pressure across the entire clump:

$$\beta_{\text{avg}} = \frac{P_{\text{th}}}{P_{B,\text{avg}}},$$

(7)

where $P_{B,\text{avg}}$ denotes the average magnetic field pressure inside the clump. The detailed setup of the self-contained magnetic field is described in Appendix A.

To better characterize the initial magnetic field configuration, we used a dimensionless number $\eta$ to define the fraction of magnetic energy of the field component that is perpendicular to the shock propagation direction. If the average magnetic field energy density for the initial setup is $B_0^2/8\pi$, then the perpendicular component has an average magnetic field energy density of $\eta B_0^2/8\pi$, while the parallel component has an average magnetic field energy density of $(1 - \eta)B_0^2/8\pi$. $\eta$ for different initial magnetic field setups is summarized in Table 1.

Throughout the paper, we use $\beta_{\text{avg}}$ as a measure of dynamical importance of the self-contained magnetic field and we investigate the shocked behavior of situations where the self-contained field is either strong or weak. We refer to the simulations with $\beta_{\text{avg}} = 0.25$ as “strong” field cases and those with $\beta_{\text{avg}} = 1.0$ as “weak” field cases throughout the paper. The orientation of the magnetic field relative to the incident shock is another critical parameter, as was already seen in the uniform field simulations described in the Introduction. In our simulations, we focus on the cases when the self-contained magnetic field is either purely poloidal or purely toroidal. For these field configurations that possess axial symmetry, the orientation of the field axis $\mathbf{b}$ to the shock normal $\mathbf{n}$ will be the term that matters. For each configuration, we run both parallel ($\mathbf{b} \cdot \mathbf{n} = 1$) and perpendicular ($\mathbf{b} \cdot \mathbf{n} = 0$) cases. The complete set of runs presented in this study are described and coded in Table 1 and these orientations are presented visually in Figure 2.

We do not begin our simulations in a force-free state as it is not clear that this is the most generic astrophysical situation. Clumps created in dynamic environments subject to repeated incident flows may not have time to relax to force-free conditions. Thus, we expect the clumps will be deformed by the self-contained field on timescales of

$$\tau_B = \frac{r_c}{u_A} \approx 436 \text{ yr},$$

(8)

where $u_A$ is the Alfvén speed of the self-contained field calculated from the average magnetic energy density inside the clump. In our simulations, the clump evolution driven by the shock is always faster than, or comparable to, this timescale as we discuss below.

The incoming shock has a Mach number $M = 10$, which puts our simulations in the strong shock regime (KMC94). To understand the role of the magnetic fields, we identify the clump crushing timescale as:

$$\tau_{cc} = \frac{\sqrt{\pi} r_c}{v_s} \approx 95 \text{ yr}.$$

(9)

Thus, $\tau_{cc} < \tau_B$ and we expect that the strong shock dynamics driven by the transmitted wave propagating into the clump will...
Figure 2. Initial setup of the clump magnetic field. The actual domain is four times as long in the $x$ direction as in the $y$ and $z$ directions. The first letter denotes the field configuration: T for toroidal only; P for poloidal only. The second letter denotes the field orientation with respect to the shock propagation direction: A for aligned; P for perpendicular. The blue arrow denotes the shock direction.

dominate over any relaxation-driven effects from the internal magnetic field. To confirm this, we also define the energy parameters of the shock–clump interaction: $\sigma_{th} = K_s/E_{th}$ and $\sigma_B = K_s/E_B$. These are ratios between the shock kinetic energy density $\rho_s v_s^2$ and the thermal or average magnetic energy density contained in the clump, respectively. From parameters for our simulation, we then have $\sigma_{th} \approx 222$ and $\sigma_B \approx 50$. Thus, although the clump is initially magnetically dominated, the shock has higher energy densities than either the thermal or magnetic energy contained inside the clump. Given these conditions and our choice of $\tau_{cc} \ll \tau_B$, we expect that most of the simulation evolution will be driven by the shock and not internal relaxation.

We note that the cooling timescale for the transmitted shock, $\tau_{r} = E_r/E_s = kT_{ps}/n_A$, is below the clump crushing time to ensure noticeable cooling and is given by:

$$\tau_{r} \approx 48 \text{ yr} \ll \tau_{cc}. \quad \text{(10)}$$

Therefore, we are in the regime of “weakly cooling” inside the clump where the magnetic energy is concentrated, i.e., for the transmitted shock, the ratio of cooling time to crushing time $\chi = \tau_{r}/\tau_{cc} < 1$. The cooling length scale can be calculated as:

$$l_r = v_{ps} \tau_{r}, \quad \text{(11)}$$

where $v_{ps}$ is the post-shock sound speed:

$$v_{ps} = \sqrt{\gamma k_B T_{ps}/m_A}. \quad \text{(12)}$$

From the above equations, we can calculate the ratio of the clump radius to the cooling length behind the transmitted shock:

$$\chi_\star = r_c/l_r \approx 5.64. \quad \text{(13)}$$

Therefore, one clump radius contains approximately six cooling length scales. The bow shock in our simulations has a cooling time that is longer than the evolutionary timescale of the flow and remains adiabatic in its dynamics. Note that although the situation we consider here is freely scalable, the condition “weakly cooling” should always be satisfied. Since the cooling length scale does not depend on the size of the clump, it can become extremely small compared to the clump radius when the scale length is increased; it can therefore become a dominating process after applying such a scaling.
We begin with the simulations in which the internal self-contained magnetic field is relatively strong ($\beta_{\text{avg}} = 0.25$). Recall in what follows that the incident shock kinetic energy is dominant in the initial interaction even though the clump is magnetically dominated in terms of its own initial configuration. Figure 3 shows case TAS: i.e., the internal magnetic field is toroidal and aligned with the shock normal.

At early times, $t \ll \tau_{\text{cc}}$, the shocked clump evolution appears similar to that of the unmagnetized case (not shown). The usual pair of shocks form: a bow shock facing into the incoming flow and a transmitted shock that propagates into the clump. Note that the transmitted shock in our simulations is radiative, meaning that thermal energy gained at the shock transition is quickly radiated away. With the loss of thermal pressure support, the shock collapses back toward the contact discontinuity. In this regime, shock regions become thin and post-shock densities are high (Yirak et al. 2010). In our simulations, only the bow shock cools effectively, which is evident at the thin boundary flows.

The effect of the toroidal field becomes particularly apparent in the morphology after a crushing time. In the middle frame in Figure 3 ($2\tau_{\text{cc}}$), we see the clump collapsing toward the symmetry axis due to a pinch by the toroidal magnetic field. This behavior is in contrast to the hydrodynamic or MHD adiabatic case with parallel fields in which the shocked clump material expands laterally and is then torn apart by RT instabilities. Even in radiative hydrodynamic cases, the shocks tend to flatten the clump, which then breaks up into small fragments (Yirak et al. 2010). Only in uniform perpendicular field cases do we see situations where the flow becomes shielded from RT instabilities. The internal toroidal field simulations show something entirely different, however. Here, the tension force from the compressed internal toroidal field is strong enough to suppress the lateral expansion. This inward-directed tension controls the subsequent evolution.

The ongoing compression within the clump driven by the tension of the toroidal field restricts the downstream flow. Thus, only a limited turbulence wake forms. The compression of the clump and downstream flow into a narrow cone continues at later times, as can be seen in the frame corresponding to $t = 3.5\tau_{\text{cc}}$. By this time the shocked clump has become compressed into a very narrow conical feature resembling the “nose cone” observed in the MHD jet simulations (e.g., Frank et al. 1998; Lind et al. 1989). The development of a dense, streamlined clump by the end of the simulations indicates that for these configurations the long term evolution will be simply slow erosion of the clump without significant fragmentation.

When the toroidal axis is perpendicular to the shock normal, however, the evolution is quite different. In Figure 4, we show three snapshots of density for the run TPS. In this case, the field is attempting to pinch the clump onto the $z$ axis (a compression “inward” toward the clump axis along the $x$ and $y$ directions). The shock, however, only produces a compression along the $x$ axis. The differential forces on the clump do yield a transient period of flattening, as is seen in both the hydrodynamic and uniform field MHD simulations. However, the presence of the internal toroidal fields alters the internal distribution of stresses. The result is a differential aerodynamical resistance to the flow over the clump as it becomes immersed in the post-shock region. Note that the magnetized clump is easier to distort along the $z$ axis compared to the $y$ axis where tension forces are at work. Thus, at $t = \tau_{\text{cc}}$, we see the clump becoming ellipsoidal or football shaped. The structural coherence that the tension force provides in the $y$ direction during the compression phase continues to shape the subsequent flow evolution. By $t = 2\tau_{\text{cc}}$, the oblate clump, which continues to erode by the incoming wind, begins developing a concave morphology along the $z$ axis. The subsequent formation of a banana-shaped configuration tilts
Figure 4. Case of a strong toroidal field only, perpendicular to the shock propagation direction. Evolution of the clump material at 1, 2, and 3.5 times the clump crushing time is shown from the top to the bottom. The color indicates the concentration of the clump material, normalized by an initial value. (A color version of this figure is available in the online journal.)

We now turn to the poloidal strong field cases. Figure 5 shows the simulation of a shocked clump when the internal field is poloidal and aligned with the shock normal (case PAS). In this run, there is a strong concentration of fields at the clump axis, as well as a relatively weak field near the clump surface. When the axis is aligned with the shock normal, we can see that during the compression phase \( t = \tau_{cc} \) the clump is compressed radially as in the unmagnetized case. Note, however, that a depression develops along the clump axis as the incident flow’s ram pressure is relatively unimpeded there by the magnetic field. Because the field along the axis is aligned with the flow direction, the evolution resembles the global field parallel case (Mac Low et al. 1994). However, by \( t = 2\tau_{cc} \), the differential stresses of the internal self-contained poloidal field yield a different evolution.
compared to both our previous toroidal cases and the uniform field cases.

While the clump expands laterally as in the unmagnetized case, it then develops a hollow core. The initial phase of the axial core was already apparent at earlier times; now, however, we see that the outer regions corresponding to the domains closer to the clump surface with relatively strong magnetic fields (weaker than the field on the axis, but stronger than the region surrounding the $r_c/3$ point; see Appendix A) retain their coherence while the incident flow has evacuated the area surrounding the axial core. Thus, the poloidal field yields a coherence length associated with the curvature (and tension) of the field around its circumference. Regions closer to the axis with weak initial fields get distorted, compressed, and driven downstream while the regions with a strong field or a fully flow-aligned field better resist the compression.

The shaft-shaped feature surrounding the hollow core has a relatively low $\beta$ compared to the rest of the clump. It gradually deforms as a result of field line tension (squeezing outward toward the clump periphery away from the axis) on timescales of $t = \tau_B$, which for these runs is $2.8\tau_{cc}$. Consequently, we see in the last frame at $t = 3.5\tau_{cc}$ that the shaft disappears and the clump is fragmented into an array of cold, magnetized clumplets, similar to the TP case.

Figure 6 shows the simulation with a strong internal poloidal field oriented perpendicular to the shock normal (coded PPS). The influence of the different field orientation is already evident in the first frame at $t = \tau_{cc}$. The initial compression phase has produced an ellipsoidal clump distribution a similar to the toroidal perpendicular simulation (Figure 4). In this case, the internal stresses of the poloidal field change the orientation of the ellipse while also producing substructure due to the smaller scale of field loops ($R \sim 0.5r_c$ for the poloidal field rather than $R \sim r_c$ for the toroidal case). By $t = 2\tau_{cc}$, we see a shaft and a ring structure develop as in the PAS case, but now the smaller scale of the loops (radius of curvature) allow these structures to be partially eroded by the incoming shock. The shaft is then fragmented by the shock rather than the field pinch, and the ring leaves an extended U-shaped structure. As a result, two large clumplets located in the $y$–$z$ plane form at $3.5\tau_{cc}$. For configurations TA and PP, the initial setup is entirely axisymmetric.

4.2. Shocked Clumps with a Weak Self-contained Ordered Field

We now look at the results where the contained magnetic field is relatively weak compared with the previous cases ($\beta_{avg} = 1$). In this regime, we still expect to see the field exerting an influence over the shock–clump evolution but the final outcome of the flow, in terms of its global properties, may not be clearly differentiated between the initial field configurations.

Figure 7 shows the simulation of a shocked clump when the internal field is toroidal and aligned with the shock normal (coded TAW). Here, the most significance difference comparing to the TAS case is that the post-shock clump material does not collapse into a core. Instead, the ram pressure of the incident flow pushed through the clump axis after the initial compression phase $\tau_{cc} < t < 2\tau_{cc}$. This result indicates that the pinch force provided by the toroidal field no longer overwhelms the stresses produced by the flow, as it does in the case with a stronger initial field and a lower initial $\sigma_B$. By $3.5\tau_{cc}$, the clump evolves into a series of cold dense clumps as in the hydrodynamic case although the position of the clumps appears to reflect the original toroidal orientation of the field.

Figure 8 shows the case of a weak internal toroidal field with its axis perpendicular to the shock normal (coded TPW). Compared to the TPS case in the previous subsection, we can see that the clump opens up at $t = 2\tau_{cc}$ similar to the TAW case because of the lack of strong pinch forces. One can still see the effect of the field in the orientation of the two nascent clumps forming aligned with the $z$-axis. Indeed, by $3.5\tau_{cc}$, the clump material forms an array of clumplets with a stronger distribution along the $z$ axis than along the $x$ or $y$ axis, which is similar to the TPS case. Thus, like the TAW case, even a weaker self-contained magnetic field still yields an influence over the global flow evolution.

Figure 9 shows the simulation of a shocked clump when the internal field is poloidal and aligned with the shock normal
Figure 7. Case of a weak toroidal field only, perpendicular to the shock propagation direction. Evolution of the clump material at 1, 2, and 3.5 times the clump crushing time is shown from the top to the bottom. The color indicates the concentration of the clump material, normalized by an initial value. (A color version of this figure is available in the online journal.)

In Figure 10, we show the simulation with a weak internal poloidal field oriented perpendicular to the shock normal (case PPW). The evolution is comparable with the PPS case. Once again, the U-shaped feature that forms after the shock has passed through the entire clump is less pronounced due to reduced pinch forces. Note that we see that the final fragmentation produces two large clumplets at $3.5\tau_{cc}$.

The overall evolution of the weaker field cases shows the effect the field has in terms of the final spectrum of fragments produced by the shock–clump interactions. Unlike purely hydrodynamic cases, the fragmentation of the initial clump into smaller clumplets does depend on the initial field geometry and its orientation relative to the incident shock, at least for the evolutionary timescales considered in this study. Thus, even in cases where the field does not dominate the initial energy budget of the clump, the shock dynamics do depend on the details of the initial field. Note also that in all cases a nearly volume-filling
turbulent wake develops behind the clump at later evolutionary times. For the TA and PP configurations, the initial setup is axisymmetric. But as a result of numerical instabilities and a finite domain size, we can observe asymmetry at late frames in Figures 3, 5, 7 and 9.

Magnetic fields can be important in suppressing the instabilities associated with shocked clumps. According to Jones et al. (1996), the condition for the magnetic field to suppress the KH instability is that $\beta < 1$ for the boundary flows. The condition for the magnetic field to suppress the RT instability is that $\beta < \xi / M = 10$. For both strong and weak field cases presented in our paper, the $\beta$ at the boundary flows has a value between 1 and 10. Therefore, the KH instability is present in all of our cases, shredding the clump boundary flows and converting them into downstream turbulence. However, even for the weak self-contained field cases, the RT instability is suppressed. To demonstrate, we map the density and $\beta$ (presented by $1/\beta$ in Figure 11) for the TAW and PAW cases in Figure 11. We observe that the shocked clump material develops a streamlined shape in both cases. The region where density is concentrated has $1/\beta > 0.1$.

Finally, to illustrate the post-shock distribution of the magnetic field, we plot the density and field pressure by cutting through the $x$–$y$ midplane of the simulation box in Figure 12. This figure shows that the field follows the clump density distribution, as expected in our simulations where the diffusion is only numerical and weak.
Figure 11. Snapshot of shocked clumps cut through the center of the domain, at $t = 2.5 \tau_{cc}$, for the TAW and PAW cases. The upper panel corresponds to the density and the lower panel corresponds to $1/\beta$.

(A color version of this figure is available in the online journal.)

Figure 12. Snapshot of shocked clumps cut through the center of the domain, at $t = 2.5 \tau_{cc}$. The four panels correspond to the TA, TP, PA, PP cases from the top to the bottom, respectively. The upper half of each panel shows the clump density and the lower half shows the magnetic pressure in pseudocolor.

(A color version of this figure is available in the online journal.)
5. MATHEMATICAL MODEL AND ANALYSIS

Figures 13(a) and (b) show, for the strongly magnetized clump cases, the evolution of the kinetic energy and the total magnetic energy, respectively. Figure 14 shows the analogous plots for the weak field cases.

In Figure 13(a), we observe that prior to $\tau_{cc}$, the kinetic energy of the clump gained from the incoming shock is similar in all cases. Later, the curves begin to diverge, reach a peak, and then descend. The descending feature after $3\tau_{cc}$ is caused by clump material leaving the simulation box. The identical ascending behavior prior to $\tau_{cc}$ and the diverging behavior after for different field configurations will be explained in the next subsection. Similar trends can also be observed for the weak contained field cases in Figure 14(a).

In Figure 13(b), we observe that the total magnetic energy evolution for the four field configurations are different: the TAS case grows and has the highest magnetic energy at $\tau_{cc}$ and the PAS case fluctuates and has the lowest magnetic energy $\tau_{cc}$. After $\tau_{cc}$, the TAS curve begins to drop while the other two perpendicular cases continue to rise. At the end of $3\tau_{cc}$, the TPS case has the most magnetic energy, followed by the PPS case and then the PAS case. The TAS case dropped to the lowest. In Figure 14(b), the order of contained magnetic energy prior to $\tau_{cc}$ is the same as in Figure 13(a). However, the TAS curve does not drop afterward: it continues to rise and at the end of $3\tau_{cc}$, it ranked second in terms of total magnetic energy behind the TPS case. The other cases have similar features compared to their strong field counterparts. The magnetic field energy evolution is clearly related to the internal field configuration.

In summary, the kinetic energy transfer and the total magnetic field variation can be determined by the initial structure of the self-contained magnetic field. To account for the results exemplified in the figures, we propose that the shock–clump
interaction occurs in two phases, a compression phase and an expansion phase.

5.1. Modeling the Compression Phase

In the evolutionary phase of the shock–clump interaction, the transmitted shock passes through the clump and drives it to higher densities. After this compression phase, energy is then stored in the form of clump thermal pressure and increased magnetic field pressure. During this phase, the kinetic energy of the clump resides mostly in the form of linear bulk motion. Because of the incoming shock, this initial kinetic energy transfer to the clump is similar for all of the clump cases we have considered. The magnetic energy growth depends on the initial magnetic field geometry because the shock compression only directly amplifies the field components perpendicular to the shock normal.

We now develop a mathematical model that describes the magnetic field energy for the compression phase. We define \( l_{||} \) and \( l_{\perp} \) as the thicknesses of the clump along and perpendicular to the shock normal, respectively. We assume that the clumps are initially spherical, so \( l_{\perp,\alpha} = l_x = l_y = l_z = l_{||,\alpha} \), and that the shock propagates in the \( x \) direction. Subsequently, \( l_{||} \) corresponds to the \( x \) direction and \( l_{\perp} \) refers to the \( y \) and \( z \) directions, assuming that the compression is isotropic in the \( y-z \) plane.

Assuming that magnetic reconnection is slow on timescales of the compression phase, magnetic flux conservation can be used to estimate the magnetic energy increase from compression. The energy associated with a uniform field in the \( x-z \) plane increases \( \propto \left( l_3/l_{\perp,3}\right)^{2} \) whereas the energy of a uniform field in the \( x \) direction will increase \( \propto \left( l_3/l_{\perp,3}\right)^{2} \). Then, assuming that the initial field configuration has \( \eta \mathcal{B}_0^2/8\pi \) stored in the perpendicular component and \( (1-\eta)\mathcal{B}_0^2/8\pi \) stored in the parallel component, we obtain the magnetic energy density after compression:

\[
\epsilon_B = \frac{B^2}{8\pi} = \frac{1}{8\pi} \left[ \eta \mathcal{B}_0^2 (2r_c/l_{||})^2 (2r_c/l_{\perp})^2 + (1-\eta) \mathcal{B}_0^2 (2r_c/l_{||})^4 (2r_c/l_{\perp})^4 \right].
\]

(14)

where \( r_c \) is the initial clump radius. We use \( l_{||,h} \) and \( l_{\perp,\alpha} \) to denote the length of the two directions for the case where the clump does not contain any magnetic field, i.e., the hydrodynamic case. The magnetic energy density can then be rewritten as

\[
\epsilon_B = \frac{1}{8\pi} \left[ \eta \mathcal{B}_0^2 (2r_c/l_{||})^2 (l_{||,h}/l_{||})^4 (l_{\perp,\alpha}/l_{\perp})^2 + (1-\eta) \mathcal{B}_0^2 (2r_c/l_{||})^4 (l_{||,h}/l_{||})^4 (l_{\perp,\alpha}/l_{\perp})^4 \right].
\]

(15)

Assuming that the post compression clumps are self-similar (i.e., different in size, but with the same shape), then the ratio of perpendicular and parallel scale lengths is a constant during compression. This result allows us to define a constant shape factor \( e \), given by

\[
e = \left( l_{||}/l_{\perp} \right)^2 = \left( l_{||,h}/l_{||,h} \right)^2.
\]

(16)

To articulate the influence of the magnetic field compared to a purely hydrodynamic clump, we assume that the ratio of the magnetized to unmagnetized clump dimensions in a given direction after compression is inversely proportional to the ratio of forces incurred by the hydrodynamic and magnetized clumps, respectively. That is,

\[
l_{||,h}/l_{||} = \frac{F - f_B}{F} = 1 - f_B/F,
\]

(17)

where \( F \) is the force exerted by the transmitted shock and \( f_B \) is the “repelling” force exerted by the self-contained magnetic field (see Appendix B). The ratio of these two forces is proportional to the magnetic and kinetic energy densities, that is,

\[
f_B/F = \frac{\alpha B_0^2}{6\pi \rho_s v_t^2},
\]

(18)

where \( \alpha \) is a dimensionless number that depends on the magnetic field configuration and \( \rho_s \) and \( v_t \) are the density and velocity behind the transmitted shock, respectively. For example, if the repelling force is from the magnetic pressure gradient \( \nabla P_B \) only, and the magnetic field is distributed in a thin shell of radius \( r_c/\sqrt{3} \), then

\[
f_B = \frac{3 B_0^2}{r_c 8\pi}
\]

(19)

per unit volume. On the other hand, the ram pressure acting on the clump has

\[
F = \frac{\rho_s v_t^2 \pi r_c^2}{4\pi r_c^3/3}
\]

(20)

per unit volume. Therefore, from the above two expressions, we obtain that in the case considered, \( \alpha = 3 \).

Because the self-contained magnetic field is curved with a positive radius of curvature, a magnetic tension force in \( J \times B \) is present and can cancel some of the repelling force from the field pressure gradient. For instance, in the toroidal perpendicular case, the tension force along the \( x \) direction is \( \partial_y B_z^2/4\pi \). The tension force therefore reduces \( \alpha \) to \( \alpha = 1 \). We define \( \mu \) as the ratio of the initial averaged clump magnetic energy density and the external energy density driving the shock. We also assume \( \mu \ll 1 \) during the compression phase. Specifically,

\[
\mu \equiv \frac{B_0^2}{6\pi \rho_s v_t^2} \ll \frac{2}{3\sigma_B} \ll 1.
\]

(21)

We also define the hydrodynamic compression ratio

\[
C_h = (2r_c/l_{||,h})^4.
\]

(22)

Combining Equations (15), (17), and (22), the magnetic energy density after compression can then be written as

\[
\epsilon_B = (B_0^2 C_h/e/8\pi)(\eta + (1-\eta)e)(1-\alpha\mu)^4.
\]

(23)

Multiplying this total magnetic energy by the volume of the compressed clump gives the total magnetic energy,

\[
E_B = (B_0^2 C_h/8\pi)(\eta + (1-\eta)e^2)(1-\alpha\mu)^4 \pi l_{||} l_{\perp}.
\]

(24)

Assuming that all of the different clump field configuration cases evolve to similar shapes after compression (i.e., that \( e \) is constant), we then have

\[
E_B = e E_{\delta 0}(\eta + (1-\eta)e)(1-\alpha\mu) = \eta E_{\delta 0}(1 + (1-\eta)e)(1-\alpha\mu),
\]

(25)

where \( E_{\delta 0} = B_0^2 C_h l_{||,h}^3/8e \) is the total magnetic field energy in the absence of any repelling tension force from the self-contained field and \( E_h = e E_{\delta 0} \). Different initial field configurations lead to different strengths of the repelling force and
field amplification during the compression and therefore modify both $\alpha$ and $\eta$. Using the strong field case as an example, the $\eta$ parameter for the TA, TP, PA, and PP cases is 1, 0.5, 0.25, and 0.75, respectively. From the field gradient and the magnetic tension, we can use $\alpha$ for these four cases: 3, 1, 1, and 3, respectively (see Appendix B). Using $\mu \approx 0.013$ (from Section 2; $\sigma_B \approx 50$) and $\epsilon \approx 0.25$ (from the approximated ratio $l_\|/l_\perp \approx 0.5$), we find the total magnetic energy for the TA, TP, PA, and PP cases to be $0.96E_h$, $0.61E_h$, $0.43E_h$, and $0.78E_h$, respectively. Therefore, at the end of the compression phase, the total magnetic energy from highest to lowest is the TA, PP, TP, and PA cases. This theoretically-predicted ordering agrees with the line plots of Figure 13(b) from the simulations.

The simulations also justify the underlying assumption of Equation (17), namely that the energy transferred from the shock to the clump material is initially similar in all cases regardless of the initial field configurations because the field is weak with respect to the impinging flow. This assumption can be expressed as

$$ (F - f_B)l_\| \simeq Fl_\| $$

and is evidenced by the kinetic energy transfer plots (Figures 13(a) and 14(a)): during the compression phase, all clumps receive identical kinetic energy flux. Note that our model in the main text ignores differences in $\epsilon$. In Appendix C, we derive the corrections to Equation (25) when differences in $\epsilon$ are allowed.

5.2. Expansion Phase

Unlike the compression phase, in the expansion phase a large fraction of the kinetic energy of the clump comes from expansion motion parallel to the shock plane. However, the specific evolution of this phase depends on which two distinct circumstances arise at the end of the compression phase: either (1) the magnetic pressure gradient and tension force are small compared to the pressure force exerted by the shock or (2) the magnetic pressure gradient and tension force dominate over the shock.

If the shock is still dominant at the end of the compression phase (circumstance 1), the clump will expand similarly as in the hydrodynamic case. During this phase, the magnetic field inside the clump acts against this expansion: the clump material is doing work to the self-contained magnetic field (mainly via field stretching) in order to expand. Thus, in general, more magnetic energy at the end of the compression phase means a stronger force opposing the expansion. The kinetic energy in the expansion phase shows differences for the different field configurations: the higher the self-contained field energy at the end of the compression phase, the lower the kinetic energy transfer efficiency in the expansion phase. The ordering of kinetic energy transfer efficiency in the expansion phase from highest to lowest is then the PA, TP, PP, and TA cases. This again exactly agrees with our plots (Figures 13(a) and 14(a)).

In addition to circumstance (1), the expansion phase also sees a switch in the nature of the field amplification: the field is amplified according to how much kinetic energy is transferred into the expansion motion. Thus, the ordering of the magnetic field amplification in the expansion phase will be the same as the ordering for the kinetic energy transfer in that phase. In Figure 14(b), the weak field cases follow this pattern: the TAW, TPW, and PPW curves reverse their ordering when entering the expansion phase, giving them the same ordering as the kinetic energy transfer plot (Figure 14(a)). The PAW case does not conform with the prediction of the model because most of the field lines are parallel to the shock propagation direction so that they do not get amplifying by the stretching from the expansion motion in the $y-z$ plane.

If the shock is no longer dominant at the end of the compression phase (circumstance 2), then the clump evolves under the influence of a significant Lorentz force. The comparison between the TAS (Figure 13(b)) and TAW (Figure 14(b)) cases exhibits the transition and the distinction between circumstance (1) versus circumstance (2) evolution: at the end of the compression phase, the TAW case expands while the TAS case shrinks.

The requirement for these distinct evolutions to arise can be predicted using a dimensionless ratio calculated from the parameters of the initial field configuration. Assuming that the pressure from the expansion in the direction perpendicular to the toroidal field lines in the TA case is one-third of the total post shock ram pressure, the ratio between the total magnetic pressure and the pressure of the expansion motion is given by

$$ r_c = \frac{B_0^2C_h8\pi(1 - \eta)e(1 - \alpha\mu)^3}{\rho v_n^2/3}. $$

Using the parameters $\alpha = 3$, $\mu_{\text{strong}} = 0.013$, $\mu_{\text{weak}} = 0.005$, and a compression ratio $C_h = (2R/l_{\|,h})^3 \approx 3.5^3 \approx 150$, we find that $r_c \approx 1.42$ for the TAS (circumstance 2) case and $r_c \approx 0.44$ for the TAW case (circumstance 1), respectively. Intuitively, the threshold for the expansion of the toroidal configuration would require $r_c \leq 1$. Thus, in the TAS case, the field pinch is dominant at the end of the compression phase and the clump collapses down to the axis, whereas in the TAW case, the expansion is dominant and the clump behaves similar to the hydrodynamic case.

5.3. Mixing of Clump and Ambient Material

Figures 15(a) and (b) show the mixing ratio of wind and clump material at $t_c$ and $3t_c$ for the strong field cases. Figures 16(a) and (b) show the mixing ratio of wind and clump material at $t_c$ and $3t_c$ for the weak field cases. We define a wind–clump mixing ratio in a single computational cell as

$$ \nu = \frac{2\min(n_c, n_w)}{n_c + n_w}, $$

where $n_c$ and $n_w$ denote the clump and wind number densities, respectively. This definition shows that $\nu = 1$ means perfect mixing: there is equal number of clump and wind particles in the cell, while $\nu = 0$ means no mixing at all. In Figure 15, we see that the mixing ratios for the four strong self-contained field cases are almost identical at early times. This result is consistent with the fact that at early times the clump as a whole is in the processes of being accelerated along the shock propagation direction. The only mixing between the clump and the wind occurs at the edges of the clump from the interaction with the incoming shock. The strong field prevents strong mixing.

In the weak magnetic field cases, the toroidal configurations do not see a significant increase in the early time mixing ratio compared to the strong field case (Figure 16(a)). This result is because the toroidal case has most of its magnetic field concentrated at the edges of the clump (see Appendix A). Thus, the average plasma $\beta$ on the outer edge is still small enough to contain the clump material. In the weak poloidal configuration cases, however, the magnetic field is concentrated at the center.
of the clump and accordingly the PAW and PPW cases have the largest magnetic $\beta$ on the outer edge of clump, making them the most susceptible to early shock erosion. This effect explains the significant increase we see in the initial mixing ratio in the PAW and PPW cases (Figure 16(a)).

The late mixing ratio depends on how much kinetic energy is transferred from the wind to the clump. At late times, the PA configuration has the highest mixing ratio of the four studied cases. The PP and TP cases have intermediate mixing ratios, and the TA case has the lowest mixing ratio. This ordering agrees with the ordering of the kinetic energy transfer: the more force resisting compression from the self-contained magnetic field in the early phase, the less the kinetic energy transfer occurs in the expansion phase, and the less the mixing. The late mixing ratio also partially depends on the efficacy of enhanced turbulent mixing downstream. From the 3D images in the previous section (Figures 3–10), we can identify the downstream turbulence of the TA and PA cases as the least and most volume filling, respectively.

6. CONCLUSION

We have studied the evolution of clumps with initially self-contained magnetic fields subject to interactions with a strong shock using both numerical simulations and analytic theory. Our results show a new variety of features compared to previous work on shock–clump interactions with magnetic fields, which considered only cases in which the field threading the clumps was anchored externally (Jones et al. 1996; Gregori et al. 2000).

We find that the evolution of the total magnetic energy and kinetic energy of clumps depends primarily on the relative strength of the self-contained magnetic field, the incoming supersonic bulk kinetic energy (characterized by the $\mu$ parameter), and the geometry of the magnetic field (characterized by the $\eta$ and $\alpha$ parameters). We identified two phases in the clump evolution that we characterized as “compression” and “expansion” phases.

In general, we found strong distinctions in clump evolution depending on the relative fraction of the field in the clump aligned perpendicular to or parallel to the shock normal. This was demonstrated by considering distinct field configurations.
that we called “toroidal” and “poloidal.” For each case, we compared the shock–clump interactions when the symmetry axes were aligned with the shock normal and perpendicular to it. The evolution of the clump magnetic fields seen in our simulations can be described by the mathematical model culminating in Equation (25) during its compression phase.

The kinetic energy transfer from the supersonic flow to the clumps is similar in the compression phase for all of our cases considered. However, differences develop in the expansion phase depending on the initial field geometry and orientation, which in turn determines how much field amplification occurs in the compression phase. The evolution of the clump in the expansion phase depends on whether the shock or the magnetic field is dominant at the end of the compression phase.

The extent to which clump material mixes with wind material also depends primarily on the field orientation: in general, the more the initial field is aligned perpendicular to the shock normal, the better the clump can deflect the flow around the clump and the less effective the mixing. Equivalently, the better aligned the field is with the shock normal, the more effectively the clump material gets penetrated by the incoming supersonic flow. In this case, the clumps gain kinetic energy and expand and mixing is enhanced.

These simulations may provide morphological links to astrophysical clumpy environments. In our presented study, we use 150 AU clumps that are typical for YSOs. However, we also put emphasis on the “weakly cooling” condition that the cooling length as indicated by Equation (13) is not too small compared to the clump radius. For clumps with much higher clump densities, the ratio of clump radius to cooling length $\chi_*$ can be greatly increased. $\chi_*$ can also increase when one tries to scale the simulations to globules that are much larger in size. Therefore, in order to gain a full understanding of the studied subject, numerical studies in the parameter regime of “strongly cooling,” where $\chi_*$ is several orders of magnitude greater than its current value, are necessary in the future. Future studies may also include more realistic radiative cooling using more recently studied emission lines (Wolfire & Churchwell 1994) and equilibrium heating (van Loo et al. 2010), more realistic internal field geometry (for instance, a random field), more realistic multi-physical processes such as thermal conduction and resistivity, and more sophisticated mathematical models.

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APPENDIX A
GEOMETRICAL SETUP OF A SELF-CONTAINED MAGNETIC FIELD

In order to ensure $\nabla \cdot \mathbf{B} = 0$, the self-contained magnetic field is set up by first choosing a vector potential distribution and then taking its curl. The geometry of the toroidal field is best demonstrated using cylindrical coordinates. The vector potential $\mathbf{A}$ has the following distribution:

$$A_r = 0 \quad (A1)$$

$$A_\theta = \begin{cases} B_{0,\text{tor}} \frac{\sqrt{r^2 - z^2}}{r r_c}, & \text{if } r \leq f \sqrt{r_c^2 - z^2} \\ B_{0,\text{tor}} \frac{\sqrt{r^2 - z^2}}{2(1-f)r_c}, & \text{if } r > f \sqrt{r_c^2 - z^2}. \end{cases} \quad (A3)$$

where $B_{0,\text{tor}}$ is the desired peak magnetic field intensity and $r$, $\theta$, and $z$ take their usual meanings in a cylindrical coordinate system: $r$ is the distance to the $z$-axis, $\theta$ is the azimuthal angle, and $z$ is the distance to the $x$-$y$ plane. $f < 1$ is an attenuation factor to cut off the magnetic field when $\sqrt{r^2 + z^2} > r_c$, i.e., outside the clump. This vector potential distribution gives the following $\mathbf{B}$ distribution upon taking the curl:

$$B_r = 0 \quad (A4)$$

$$B_\theta = \begin{cases} B_{0,\text{tor}} \frac{r}{r r_c}, & \text{if } r \leq f \sqrt{r_c^2 - z^2} \\ B_{0,\text{tor}} \frac{\sqrt{r^2 - z^2}}{2(1-f)r_c}, & \text{if } r > f \sqrt{r_c^2 - z^2}. \end{cases} \quad (A5)$$

$$B_z = 0. \quad (A6)$$

For any given $z$, the magnetic field intensity peaks at $f \sqrt{r_c^2 - z^2}$. If $f$ is close to 1, the field will be concentrated near the outer edge of the clump. In the present simulations, we take $f = 0.9$.

The poloidal field is best demonstrated using spherical coordinates. It has a vector potential distribution of

$$A_r = 0 \quad (A7)$$

$$A_\theta = -\frac{B_{0,\text{pol}}(r_c - r)^2 r \sin \theta}{2 r_c^2} \quad (A8)$$

$$A_\phi = 0, \quad (A9)$$

where $B_{0,\text{pol}}$ is the desired peak magnetic field intensity and $r$, $\theta$, and $\phi$ are the distance to the origin, the polar angle, and the azimuthal angle, respectively. Notice here that $r$ and $\theta$ are defined differently compared to cylindrical coordinates. The curl of this vector field is

$$\mathbf{B}_r = 0 \quad (A10)$$

$$\mathbf{B}_\theta = 0 \quad (A11)$$

$$\mathbf{B}_\phi = -\frac{B_{0,\text{pol}}(r_c - r)(r_c - 3r) \sin \theta}{r_c}. \quad (A12)$$

We observe that the magnetic field energy density $\mathbf{B}^2$ peaks at the center $r = 0$ and has a weaker secondary maximum at $r = 2r_c/3$. The field attenuates to zero at the outer edge of the clump $r = r_c$. There is another zero point in between $r = 0$ and $r = r_c$: $r = r_c/3$. The toroidal and poloidal field setup are orthogonal to each other and can be combined into a more general self-contained magnetic field distribution. The cases presented in our paper form the basis for understanding more complex self-contained magnetic field configurations.
Appendix B

The Geometrical Factor of the Magnetic Repelling Force \( \alpha \)

In the paper, we worked out the magnetic repelling force for the TA case:

\[
F_B = \frac{3}{r_c} \frac{B_0^2}{8\pi},
\]

which gives the parameter \( \alpha = 3 \). For the TP case, the magnetic tension force is pointing inward with

\[
F_T = \frac{1}{r_c} \frac{B_0^2}{4\pi},
\]

assuming that the radius of curvature of the magnetic field lines is \( R \). This tension force cancels some of the gradient force, which brings \( \alpha \) to 1.

For the PA case, the repelling force from the field gradient remains the same, since the average self-contained field pressure is an invariant for the four “strong field” cases. But the curved magnetic field on the outer edge of the clump has an average energy of \( B_0^2/2 \). The tension force is thus

\[
F_T = \frac{1}{r_c/2} \frac{B_0^2}{4\pi},
\]

where the field loop’s radius of curvature is \( r_c/2 \). This tension force also brings \( \alpha \) down to \( \alpha = 1 \).

For the PPS case, the tension force from the outer edge of the clump can be canceled by the tension force from the center of the clump so that their net contribution to the total repelling force is zero. Therefore, we obtain roughly the same \( \alpha \) as in the TA case.

Appendix C

Correction to the Shape Factor \( e \)

In deriving Equation (25), we used an assumption that no matter what the self-contained field configuration is, the clump is always compressed to a self-similar shape if the hydrodynamic setup is unchanged. However, we know that when the self-contained field is ordered, the force it exerts on the clump is inhomogeneous depending on the geometry. The difference in the repelling force there results in a difference in the shape factor \( e \) introduced in Section 5.1. We now look at how large this correction is for the four studied simulations.

Let us go back to Equation (15). Assume that the force exerted by the shock on the clump is different in the perpendicular and parallel directions: the force in the perpendicular direction is only a portion of that in the parallel direction, and this portion is fixed for all cases with the same hydrodynamic setup:

\[
F_y = \gamma F_x,
\]

where \( \gamma \) is fixed. Then, following the same procedure as in Section 5.1, we have

\[
\epsilon_B = \epsilon_h \left( \eta (1 - \alpha_x \mu) + (1 - \eta) \frac{(1 - \alpha_y \mu/\gamma)^2}{1 - \alpha_x \mu} \right),
\]

where \( \alpha_x \) and \( \alpha_y \) denote different repelling forces from the self-contained field on the \( x \) and \( y \) directions, respectively.

As in Section 5.1, \( \alpha_x \) for the simulated cases TAS, TPS, PAS, and PPS is 3, 1, 1, and 3, respectively. Since the perpendicular \( \alpha_x \) is just the aligned \( \alpha_x \) and vice versa, we know that the \( \alpha_y \) for these four cases is 1, 3, 3, and 1, respectively. We use the same parameters as in Section 5.1: \( \mu = 0.013 \). We assume the incoming shock engulfs a spherical sector of the clump with a cone angle \( 2\theta_e \). Then, the compression force applied in the \( y \) direction is a fraction of that of the initial incoming shock. This fraction is \( 2\pi \int_0^{\theta_e} \sin^2 2\theta \, d\theta = 0.125 \), (C3)

where the integration in the number calculates the fraction of the average pressure applied in the perpendicular direction when the compressed part of the clump is a spherical cone with cone angle \( \theta_e \). The global integration calculates the average over the compression process where \( \theta_e \) varies from \( 0 \) to \( \pi/2 \). The factor of two results from the fact that the perpendicular compression happens in both the \( +y \) and \( -y \) directions.

We can calculate the corrected compressed magnetic field energy for the TAS, TPS, PAS, and PPS cases. The results are 0.96\( E_h \), 0.73\( E_h \), 0.6\( E_h \), and 0.93\( E_h \), respectively. Comparing to the results presented in Section 5.1 (0.96\( E_h \), 0.61\( E_h \), 0.43\( E_h \), and 0.78\( E_h \)), we find there is a positive correction to the cases with \( \eta < 1 \). The ordering of the field amplification factor remains unchanged. Further sophisticated modeling is possible by taking into consideration the dependence of the Lorentz force on the compression ratio: the farther the compression, the smaller the magnetic field length scale and thus the stronger the repelling force. This technique results in a model with an integral equation, which we did not discuss in this paper.

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