Indirect influence in social networks as an induced percolation phenomenon

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Motivation- behaviors can be contagious in social networks
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Publishing behavior in collaboration networks

Indirect influence

- publish in complex networks
- publish in complex networks
- publish in complex networks

direct influence

- publish in complex networks
- publish in phase transition
- publish in phase transition

complex networks OR phase transition?

Motivation- Indirect influence in behavior contagion

- Indirect influence in behavior contagion from social experiments
  - Obesity experiment lasting 32 years
    - person-to-person spread of obesity
    - extended to three degrees of separation
  - Repeatedly found in behaviors of happiness, smoking, drug, alcohol, loneliness, among others.

[Christakis et al. New Engl. J. Medicine 2007, PNAS 2010, Annals internal medicine 2010]
Motivation- Indirect influence in behavior contagion

- Indirect influence observed in ecological trait evolution.

[Guimaraes et al. Nature, 2017]

Motivation- Indirect influence in behavior contagion

- Indirect influence observed in scientific collaboration
Research question

What are the potential underpinning mechanisms for indirect influence in behavior contagion,

so that we can propose models to mimic the contagion process?

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Indirect influence as an induced percolation

- **Model of Induced percolation:**

  Induced index $m_i$: Max indirect (second) neighbors in state 1 of those Direct neighbors in state 1

  
  \[
  m_i = \begin{cases} 
  \text{Max indirect (second) neighbors in state 1} \\
  \text{of those Direct neighbors in state 1}
  \end{cases}
  \]

  
  $m_i = 3$  $k_i = 2, d_i = 6$

  Direct influence as comparison
  k-core index $k_i$: directed neighbors in state 1
  degree index $d_i$: second neighbors in state 1

Indirect influence as an induced percolation

- **Model of Induced percolation:**

  Induced index $m = 2$

  
  \[
  m = \begin{cases} 
  \text{Induced neighbors in state 1}
  \end{cases}
  \]

  
  0 1 1 0

  1

  1

  maintain

  Order parameter GOUT: corresponds to the largest spreading coverage.
Indirect influence as an induced percolation

• Model of Induced percolation:

Induced index \( m = 2 \)

\[
x = \sum_{k_{in}, k_{out}}^{+\infty} k_{out}P\left(\frac{k_{in}, k_{out}}{k}\right) \left[1 - (1 - y)^{k_{in}}\right]
\]

\[
y = \sum_{k_{in}, k_{out}}^{+\infty} k_{out}P\left(\frac{k_{in}, k_{out}}{k}\right) \sum_{k_{in}}^{+\infty} \left(\frac{k_{in}}{k}\right)^{y} \left(1 - x\right)^{k_{in}} \left[1 - \left(1 - \frac{x}{y}\right)^{k_{in}}\right]
\]

\[
P_{\infty} = \sum_{k_{in}, k_{out}}^{+\infty} P\left(\frac{k_{in}, k_{out}}{k}\right) \left[1 - (1 - y)^{k_{in}}\right]
\]
Rich critical behaviors induced by indirect influence

- Order parameter
  GOUT on directed nets
  - $\langle k \rangle$: average degree
  - $m$: induced index

- GOUT on undirected nets:

J. Xie, X. Wang, L. Fang, J.H. Zhao, Y. Moreno, Y. Hu, Induced Percolation on Networked Systems, PNAS, 119(9), 2022.
Rich critical behaviors induced by indirect influence

- GOUT on mixed nets:
  - $\langle k \rangle$: average degree
  - $m$: induced index
  - $p$: proportion of directed links

Rich critical behaviors induced by indirect influence

- Size distribution $P(s)$ of small clusters at the critical point of induced percolation ($m = 4$) on undirected networks.
Rich critical behaviors induced by indirect influence

- Size distribution $P(s)$ of small clusters at the critical point of induced percolation ($m = 4$) on undirected networks.

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Take-away message

- Indirect influence in social networks as an induced percolation phenomenon

- Induced percolation leads to rich critical behaviors depending on a single network parameter.

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