New Classes of Cosmic Energy and Primordial Black-Hole Formation

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Abstract

It has recently been suggested that the formation of horizon-size primordial black hole (PBH) from pre-existing density fluctuations is effective during the cosmic QCD phase transition. In this Letter we discuss the dependence of PBH formation on effective relativistic degrees of freedom, $g_{\text{eff}}$, during the cosmic QCD phase transition. Our finding is important in the light of recent cosmological arguments of several new classes of cosmic energy that appear from universal neutrino degeneracy, quintessential inflation, and dark radiation in brane world cosmology. Extra-energy component from the standard value in these new cosmological theories is represented as an effective radiation in terms of $g_{\text{eff}}$. We conclude that the PBH formation during QCD phase transition becomes more efficient if negative extra-component of the cosmic energy is allowed because of the increase of the duration of the QCD phase transition, which leads to smaller mass scale of PBHs. This suggests larger probability of finding more PBHs if the dark radiation exists as allowed in the brane world cosmology.

Key words: Cosmology, Quark-gluon plasma, Black holes
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1 Introduction

First argument of the PBH formation was presented by Carr and Hawking \cite{1}, in which the key concept of the PBH formation is that the three different length scales of the particle horizon, the Schwarzschild radius, and the Jeans length of any overdense region at radiation-dominated epoch are of the same order of magnitude. Once an initially superhorizon-size density fluctuation crosses the particle horizon, competition between self gravity and centrifugal pressure force determines whether such a fluctuation collapses into black hole or not \cite{2,3}. If the self gravity overcomes the pressure force, then the fluctuation will form a black hole with the order of horizon mass. Jedamzik \cite{4} found that, during cosmic QCD phase transition, there is almost no pressure response on adiabatic compression of fluctuation in the mixed phase of quark-gluon plasma and hadron gas. This could occur because the enhanced energy density due to the compression of the fluctuations will turn into the high energy quark-gluon phase, instead of increasing centrifugal pressure force. As a result the overdense region from initially existing fluctuations tends to collapse efficiently to form the PBH during the QCD epoch when horizon mass scale is around $1 M_\odot$. PBH mass spectrum is most likely dominated by the horizon mass at the phase transition epoch \cite{5}. Green and Liddle \cite{6} verified that this speculation turns out to be a good approximation in their detailed studies of the PBH mass spectrum.

These theoretical speculations have been studied more quantitatively in 1D general-relativistic hydrodynamic calculations \cite{7}, where the required critical overdensity parameter, $\delta_{hc} \equiv \delta \rho / \rho$, was found to be $\delta_{hc} \sim 0.7$ for the PBH formation. Similar sophisticated calculations \cite{8} have also shown numerically that $\delta_{hc}$ is even as low as $\delta_{hc} \sim 0.54$ by taking account of the dynamics of the QCD phase transition in supercooling phase by the use of EOS in the MIT bag model for the mixed phase.

Recently an interesting cosmological proposal has been made that our universe is occupied fractionally by a new class of extra-energy component in addition to the ordinary matter, known as “quintessence”. The quintessential scalar field causes the accelerating universal expansion at present \cite{9,10} and it also has a possibility of making significant contribution to the total energy density at radiation dominated epoch \cite{11,12,13,14}. Such an extra-energy density is interpreted as to increase relativistic degrees of freedom, $g_{\text{eff}}$, and thereby causes more rapid expansion rate at the same temperature. Similar effect is expected in the neutrino-degenerate universe models \cite{15}, and the universal lepton asymmetry was critically studied in the context of constraining the primordial nucleosynthesis and cosmic microwave background anisotropies. On the other hand, another attempt has been made quite recently to understand the Einstein gravity in the light of the unified theory \cite{16,17,18,19}. Five
dimensional brane world cosmology leads to an effective theory of gravity, which is motivated by the superstring theory or M-theory, which is a strong candidate for the unified theory. In this scenario the “standard” Friedmann equation should be slightly modified by adding several new terms which arise from the extra dimensions. Several authors showed that there is a new class of effective energy among them which diminishes in proportion to \( a^{-4} \) called “dark radiation”, where \( a \) is the cosmic scale factor. The interesting point is that this dark radiation term could be even negative because it originates purely from fifth dimensional geometry. If this is the case, negative dark radiation causes the cosmic expansion slower. We can therefore interpret this effect as a decrease in effective degrees of freedom, \( g_{\text{eff}} \).

The purpose of the present Letter is to study how strongly the change of the relativistic degrees of freedom \( g_{\text{eff}} \) affects the PBH formation during QCD phase transition. The energy density of the universe gets larger for larger \( g_{\text{eff}} \) at the same temperature, but the duration of QCD phase transition becomes shorter. These two effects operate in opposite direction to form PBHs. Therefore, it is not a trivial subject since many physical quantities may be affected in a complicated manner by this parameter \( g_{\text{eff}} \). In order to clarify its effects we use a simple model for the numerical analysis instead of performing complicated hydrodynamic calculations. We then try to find the parameter dependence of the typical mass scale and the amount of PBHs which are formed during the QCD phase transition.

2 QCD Phase Transition

We first briefly review important quantities involved in the PBH formation during the QCD phase transition. Horizon mass at the QCD epoch is written as

\[
M_h(T) = 0.8M_\odot \left( \frac{T}{100\text{MeV}} \right)^{-2} \left( \frac{g_*}{51.25} \right)^{-\frac{1}{2}},
\]

where \( g_* \) is the number of relativistic degrees of freedom, and \( T \) is the cosmic temperature. This equation shows that the horizon mass at the QCD phase transition is compatible to typical mass scale of MACHOs.

Recent progress in lattice QCD calculations and accelerator experiments have provided with rich information of the QCD phase transition. We employ the MIT bag model to express the quark-gluon plasma phase. In this model the energy densities of quark-gluon plasma (\( \rho_{\text{qg}} \)) and hadron gas (\( \rho_{\text{h}} \)) and the
pressures \((p_{qg} \text{ and } p_h)\) respectively are given by

\[
\rho_{qg} = \frac{\pi^2}{30} g_{qg} T^4 + B, \quad \rho_h(T) = \frac{\pi^2}{30} g_h T^4, \quad (2)
\]

\[
p_{qg} = \frac{1}{3} \rho_{qg} - B, \quad p_h(T) = \frac{1}{3} \rho_h, \quad (3)
\]

where \(g_{qg}\) and \(g_h\) are the numbers of degrees of freedom of the quark-gluon plasma and hadron gas phases, and \(B\) is the Bag constant. The condition of pressure balance between the two phases, \(p_h(T_{QCD}) = p_{qg}(T_{QCD})\), for a first order phase transition yields

\[
B = \frac{\pi^2}{90} (g_{qg} - g_h) T_{QCD}^4, \quad (4)
\]

\[
L = \rho_{qg}(T_{QCD}) - \rho_h(T_{QCD}) = 4B, \quad (5)
\]

where \(L\) is the latent heat of the phase transition. This implies that the intensity of the first order QCD phase transition depends on the difference between the two degrees of freedom before \((g_{qg})\) and after \((g_h)\) the phase transition.

Duration of the phase transition is expressed by the change of the cosmic scale factor during the phase transition. From the conservation law of entropy we estimate the rate

\[
a_2 = a_1 \left(\frac{g_{qg}}{g_h}\right)^{\frac{1}{3}} \approx 1.44, \quad (6)
\]

where \(a_1\) and \(a_2\) are the scale factors at the beginning and the end of the QCD phase transition, respectively. We can learn from this that the duration of the phase transition is comparable to the Hubble time at the QCD epoch, and it is enough for fluctuations to collapse gravitationally.

3 Analysis

3.1 Theoretical Model

For the numerical analysis of the PBH formation we mainly follow the method developed by Cardall and Fuller (22). The condition for the gravitational collapse of PBH is that the self gravity is stronger than the centrifugal pressure
force, which is expressed as

$$G\rho^2 S_{\text{coll}}^5 \gtrsim pS_{\text{coll}}^3,$$

(7)

where \(\rho\) is energy density, \(p\) is pressure, and \(S_{\text{coll}}\) is the size of an overdense region at the turn around. This condition becomes

$$S_{\text{coll}} \gtrsim \sqrt{\frac{w}{G\rho}} \approx R_J$$

(8)

where \(w = \frac{p}{\rho} = \frac{1}{3}\) in the radiation dominated era and \(R_J\) is the Jeans length. Equation (8) is a well known condition for the gravitational collapse. This condition is also expressed, using the density contrast \(\delta_{hc}\) at horizon crossing (3), as

$$\delta_{hc} \gtrsim w.$$

(9)

This means that fluctuations having density contrast above \(\frac{1}{3}\) at the horizon crossing can collapse to form PBH at radiation dominated era.

We need modification of Eq.(9), taking account of the effects of the QCD phase transition. In order to proceed quantitative discussions, we introduce the quantity \(f\) and modify the above condition by

$$\delta_{hc} \gtrsim w(1 - f),$$

(10)

$$f = \frac{S_2^3 - S_1^3}{S_{\text{coll}}^3},$$

(11)

where \(S_2\) and \(S_1\) are the sizes of initially expanding fluctuation at the beginning and the end of the phase transition, and \(S_{\text{coll}}\) is the size at turn around (22). This quantity \(f\) works for softening the equation of state \(w\) and takes the value, \(0 \lesssim f \leq 1 - \left(\frac{S_1}{S_{\text{coll}}}\right)^3\). When \(f\) is the minimum value, \(f = 0\), the fluctuation is not affected by the QCD phase transition at all, and Eq.(10) turns out to be the simplest case of Eq.(9). When \(f\) is the maximum value, \(f = 1 - \frac{S_1}{S_{\text{coll}}}\), the fluctuation has been affected the most by the phase transition because the fluctuation starts collapsing early enough so that the overdense region includes the mixed phase of quark-gluon plasma and hadron gas. In the latter case the system does not feel any effective pressure response. \(S_2\) and \(S_1\) are calculated from the energy density of the background and the energy contrast of the fluctuation when the fluctuation crosses the particle horizon. We solve
the flat Friedmann equation for the background

$$\left( \frac{da}{dt} \right)^2 = \frac{8\pi G}{3} \bar{\rho} a^2, \quad (12)$$

and the closed Friedmann equation for the fluctuation

$$\left( \frac{dS}{d\tau} \right)^2 = \frac{8\pi G}{3} \rho S^2 - k, \quad (13)$$

$$= \frac{8\pi G}{3} \bar{\rho}_h R_h^3 (1 + w) \left( \frac{1 + \delta}{S^{1+3w}} - \frac{\delta}{R_h^{1+3w}} \right). \quad (14)$$

In these equations, we assume that the fluctuation has the top hat profile $\rho_h = \bar{\rho}_h (1 + \delta)$, where $\delta$ means the density contrast at horizon crossing. We normalize the density evolution and fix the gauge by setting $\tau_h = t_h$, $S_h = R_h$, and $(dS/d\tau)_h = (dR/dt)_h$ when the fluctuation crosses the horizon.

We show the quantity $w(1 - f)$ as the function of $\delta$ in Fig.1, from which we can show the effect of QCD phase transition on the PBH formation. Here, $x \equiv \bar{\rho}_h$, $\bar{\rho}_h$ is the mean energy density when the fluctuation crosses the horizon, and $\rho_1$ is the mean energy density above which the universe is in pure quark-gluon plasma phase. This variable $x$ indicates when the fluctuation under consideration crosses the particle horizon.

In order to connect the physics of the QCD phase transition with the relativistic number of degrees of freedom, we introduce effective background degrees of freedom $g_{\text{eff}} = g_{\text{bg}} + \Delta g$, where $g_{\text{bg}} \approx 14.25$ ($\gamma$, $e^\pm$, $\mu^\pm$, three kinds of neutrinos in the standard case), and $\Delta g$ is for the extra-component at the QCD phase transition epoch as to be discussed in the next section. The numbers of degrees of freedom of the hadron phase and quark-gluon phase are respectively given by, $g_h = g_{\text{eff}} + 3$ (pions), and $g_{\text{qg}} = g_{\text{eff}} + 37$ (21 for quarks and 16 for gluons). We thus obtain the relation between the parameter $x$ and the number of effective degrees of freedom $g_{\text{eff}}$ for horizon mass scale $M$,

$$x = \frac{\bar{\rho}_h}{\rho_1} = \frac{3g_{\text{eff}} + 145}{153.75} \left( \frac{M}{0.87 M_\odot} \right)^{-2}. \quad (15)$$

### 3.2 Present Day PBH Density

We consider the significance of the PBHs formed in the QCD phase transition in this sub-section. Present cosmological density parameter of PBHs formed
Fig. 1. Modification of collapsing condition for $x = \bar{\rho}_h/\rho_1 = 15$. $x = 15$ corresponds to a fluctuation with mass scale $M_{\text{PBH}} \sim 0.2M_\odot$ (22). $w(1 - f)$, for $w = \frac{1}{3}$, and $\delta$ are shown as indicated. Collapse region $\delta_a \leq \delta \leq \delta_b$ satisfies the collapsing condition of Eq.(10).

in the QCD phase transition is represented by

$$\Omega_{\text{PBH}} h^2 = 4.8 \times 10^6 g(T) \varepsilon(T) \left( \frac{T_0}{2.73 \text{K}} \right)^3 \left( \frac{T}{100\text{MeV}} \right),$$

where $h$ is the Hubble parameter in units of 100 km/s/Mpc, $g(T)$ is the relativistic degrees of freedom, and $\varepsilon(T)$ is the fraction of the radiation energy density which is converted into black holes at temperature $T$. $\varepsilon(T)$ is defined by

$$\varepsilon(T) = \int_{\delta_a}^{\delta_b} F(\delta, T) d\delta,$$

where $F(\delta, T)$ is the probability function of a horizon volume fluctuation to have overdensity parameter $\delta$ (23)

$$F(\delta, T) = \frac{1}{\sqrt{2\pi} \sigma(M)} \exp \left( -\frac{\delta^2}{2\sigma^2(M)} \right),$$
\[ \sigma(M) = 9.5 \times 10^{-5} \left( \frac{M}{10^{22} M_\odot} \right)^{(1-n)/4}. \] 

In Eq. (19) \( n \) is the spectral index of primordial density fluctuations. We consider the fluctuation which has overdensity larger than \( \delta_a \) [see Eq. (17)] and can collapse into black holes, so that it satisfies Eq.(10) for a given \( x \) (See Fig.1). It is to be noted that even a small fraction \( \varepsilon \) can result in the significant amount of cosmological energy density at present. From Eq.(15) we can estimate the mass of the PBH and the variance \( \sigma \) for a given \( x \) and \( n \). We can also calculate the efficiency \( \varepsilon \) and \( \Omega_{\text{PBH}} \) using Fig.1 and Eqs.(15)-(19).

**4 Results and Discussions**

If the universe is modeled to have a new class of extra-energy component, it leads to increase relativistic degrees of freedom, \( g(T) \). In the neutrino-degenerate universe models (15), additional degrees of freedom are written as

\[ \Delta g = \sum_\alpha \left( \frac{T_\gamma}{T_\alpha} \right)^4 \left[ \frac{30}{7} \left( \frac{\xi_\alpha}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi_\alpha}{\pi} \right)^4 \right], \] 

where \( \alpha \)'s correspond to \( e-, \mu- \) and \( \tau- \)neutrino species, and \( \xi \) is the neutrino chemical potential divided by the neutrino temperature, \( \xi_\alpha \equiv \mu_\alpha/T_\alpha \). We note that slightly non-zero chemical potential is favored in recent BBN analysis with better goodness of fit to the recent data of CMB anisotropies (15), which constrains \( \Delta g \lesssim 4.72 \).

Another class of extra-energy component appears in quintessential inflationary model (11; 12; 13; 14)

\[ \Delta g = g_Q = g_B \frac{\rho_Q}{\rho_B}, \] 

where \( \rho_B \) is the background photon energy density, \( \rho_B = \rho_\gamma (1+z)^4 (g_0/g(z))^{1/3} \), and \( \rho_Q \) is the energy density of quintessence field, \( \rho_Q = \dot{Q}_{\text{QCD}}^2/2 + V(Q_{\text{QCD}}) \). \( Q_{\text{QCD}} \) and \( \dot{Q}_{\text{QCD}} \) are the quintessence scalar field value and its time derivative at the QCD phase transition. \( \Delta g \) is constrained to be \( \lesssim 0.2 \) so as to satisfy the BBN and CMB analyses (14).

In a recent theory of brane world cosmology (16; 17; 18; 19), completely a new class of cosmic energy, called dark radiation, emerges from extended Friedman
Fig. 2. The fraction of the radiation energy density converted into black holes in three cases of $g_{\text{eff}}$ values as a function of the mass of PBH. This function is steeply peaked for any $g_{\text{eff}}$, which suggests that PBH forms at a particular epoch for almost single horizon mass corresponding to a peaked $M$. In this figure we have tuned spectral index so that $\Omega_{\text{PBH}} \approx 1$ at present for the sake of illustration.

equation. Its extra-component is expressed by

$$\Delta g = g_{\text{DR}} = g_{B} \frac{\rho_{\text{DR}}}{\rho_{B}},$$

(22)

where $\rho_{\text{DR}}$ is the dark-radiation term, $\rho_{\text{DR}} = \frac{3\mu}{8\pi G}(1 + z)^4$, and $\mu$ is a constant which comes from the electric Coulomb part of the five-dimensional Weyl tensor (16). We note that the dark radiation can be negative depending on the sign of $\mu$. This is in remarkable contrast to Eqs.(20) and (21) which are always positive. The BBN and CMB constraints (24) on $\Delta g$ in the brane world cosmology is $-0.47 \lesssim \Delta g \lesssim 0.12$ (at the 2$\sigma$ C.L.). Note that only the BBN constraint allows $-4.65 \lesssim \Delta g$.

We show the calculated results for $g_{\text{eff}} = 9.60, 14.25$, and 20.00 in the Figs.2 and 3. $g_{\text{eff}} = 9.60$ corresponds to $\rho_{\text{extra}}/\rho_{B} = -0.27$ just after the QCD phase transition, which is the lowest value deduced from only the BBN constraint in brane world cosmology (24). $g_{\text{eff}} = 14.25$ is for the standard case, and $g_{\text{eff}} = 20.00$ corresponds to $\rho_{\text{extra}}/\rho_{B} = 0.33$, which is a likely case in the quintessential inflationary scenario (14). Refer to ref. (14) for more critical discussions of the observational constraints on $\rho_{\text{extra}}/\rho_{B}$ in quintessence scalar fields.
The key results are summarized as follows. Fine tuned Gaussian blue spectra fluctuation is needed with spectral index $n \approx 1.354 \pm 0.005, 1.377 \pm 0.005, 1.395 \pm 0.005$ in the cases of $g_{\text{eff}} = 9.60, 14.25, 20.00$ respectively, in order to have significant PBHs energy density $\Omega_{\text{PBH}}$ at present, i.e. $0.01 \leq \Omega_{\text{PBH}} \leq 1$.

Typical mass scales of PBH are $0.116 M_\odot$, $0.136 M_\odot$, and $0.159 M_\odot$ for $g_{\text{eff}} = 9.60, 14.25$ and 20.00, respectively, where three mass scales refer to maximum $\varepsilon(T)$’s. (Eq.(16) and Fig.2). The spectral indices as shown in Fig.3 are compatible with those inferred from the data of COBE observations, but larger than the best fit analysis of the combined different cosmological data sets (25). These indices are slightly larger than the values constrained from the evaporation of PBHs by Hawking radiation in much smaller mass scale (23). Increasing the effective number of degrees of freedom $g_{\text{eff}}$ causes the shorter duration of the QCD phase transition because the universal expansion obeys

$$\frac{a_2}{a_1} = \left(\frac{g_{\text{eq}}}{g_{\text{th}}}\right)^{1/3} = \left(\frac{g_{\text{eff}} + 37}{g_{\text{eff}} + 3}\right)^{1/3},$$

instead of Eq.(6). Accordingly, the duration of the mixed phase also becomes shorter, making less efficient PBH formation (Fig.3). The typical mass scale of the PBHs, however, increases with increasing $g_{\text{eff}}$ (Fig.2). This is because the fluctuations that are most effectively influenced by the QCD phase transition enter the horizon at later time with increasing number of degrees of freedom.
5 Conclusion

We studied the formation mechanism of PBHs during the QCD phase transition, being motivated by the cosmological interest in MACHOs as a dark matter and also new classes of cosmic extra-energy components for the degenerate neutrinos, dark energy, and dark radiation. In the present paper we take account of the effects of these extra-components by means of parameterizing their effect as an increase or decrease in effective relativistic degrees of freedom. We conclude that the PBH formation during QCD phase transition becomes more efficient with an negative extra-component because of the increase of the duration of the QCD phase transition. We also found that in such a case the typical mass scale of PBH becomes smaller. These results suggest larger probability of finding more PBHs that have typically sub-solar mass $M \sim 0.1M_\odot$ if the dark radiation exists as allowed in the brane world cosmology.

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