Dynamic analysis of the elasto-plastic behaviour of buildings and structures in the SCAD ++ software package

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Abstract. The problem formulation of nonlinear equations of motion obtained by the finite element method applying to the dynamic analysis of buildings and structures is presented. The elasto-plastic behaviour of reinforced concrete structural elements is described using the plastic flow theory, moreover, for concrete, the Drucker-Prager yield criterion is used for bending elements, and the yield surface, which coincides in shape with the Geniev strength surface, for compressed-bending elements. Concrete degradation due to crack opening is simulated by the descending branch of the $\sigma-\varepsilon$ diagram. The plastic flow theory with von Mises yield criterion describes the behaviour of reinforcement. Using the aforementioned constitutive models, a finite element library was developed, including plane shell quadrilateral and triangular finite elements based on the Mindlin-Reissner shear theory as well as a two-node spatial frame finite element based on the Timoshenko shear theory. The seismic analysis of the 3-D model of multi-storey building is considered as an example.

1. Introduction
The nonlinear dynamic analysis allows us to significantly approximate the behaviour of the design model to the real behaviour of buildings and structures, especially when dynamic loads cause damage and partial destruction of structural elements. In this article, we will limit ourselves to taking into account only physical nonlinearity and the impact of seismic loads. We will consider the elasto-plastic behaviour of reinforced concrete structures, and the degradation of concrete caused by the formation of cracks will be simulated by the descending branch of the $\sigma-\varepsilon$ diagram for concrete.

In many works, for example, in [1], [16], [17], [18], [22], [24], simplified nonlinear models are used. Most often, these are nonlinear hinges that realize lumped plasticity in one way or another, in which it is assumed that all other elements work linearly and elastically. In addition, a number of papers use nonlinear pushover analysis and/or a unimodal approximation of dynamic analysis, which can be correct if the first natural vibration mode contains a significant percentage of modal masses. However, for many design models, including design models of multi-storey buildings, eigenmodes with a small percentage of modal masses are in the lower part of the spectrum, therefore nonlinear pushover analysis, as well as a unimodal approximation of dynamic analysis, significantly underestimate the calculated values of forces in structural elements.

There are much fewer works in which each nonlinear finite element is considered in an elasto-plastic formulation (distributed plasticity) than works in which mentioned above simplified approaches are used. Without pretending to be a complete review, we give only some of them.
Using the MARC.MSC software package, in [5] a seismic analysis of the containment vessel was performed using quadrilateral finite elements of a thin-walled shell for itself shell and 8-node volumetric finite elements for supporting structures. The reinforcement in the containment vessel was modeled with quadrilateral membrane finite elements, and in support structures with 2-node rod finite elements, which works only on tension-compression. The behavior of concrete and reinforcement is described by the plastic flow theory; moreover, the Buyukozuturk yield criterion is used for concrete, and von Mises yield criterion is used for reinforcement.

An extensive report [19] for the reinforced concrete elements of ordinary bridges compares the results obtained by using lumped plasticity and distributed plasticity for concrete and reinforcement. It is noted that computational models created on the base of distributed plasticity require not only significantly greater computational efforts but also are much more difficult for the understanding of engineers than computational models using lumped plasticity.

In [20], the problem of the structure – soil interaction in a 3-D formulation is considered, and the soil is modeled by volumetric finite elements using the Drucker-Prager plastic flow theory, and concrete – by volumetric finite elements applying a microplane model. Volumetric finite elements and the plastic flow theory with the von Mises yield criterion are also used for reinforcement.

Given the computational complexity of seismic analysis based on the direct integration of nonlinear equations of motion, we tried to reasonably simplify the calculation model. On the one hand, we rejected the simplifications associated with the unimodal approximation, as well as with static nonlinear pushover analysis, which originally arose as a tool used to certify structures in the USA, but not as a type of strength analysis. In addition, we abandon the technique of nonlinear hinges and consider a complete elasto-plastic formulation for each nonlinear finite element, which we use both for the bar systems as well as for plates and shells for which the use of nonlinear hinges is extremely difficult. On the other hand, we do not consider multilevel models for concrete based on the interaction of macro-level and micro-level models, but restrict ourselves to applying to concrete the theory of plasticity with elements of degradation modeling a softening of concrete when cracking, and to reinforcement – the plastic flow theory using von Mises yield criterion.

2. Problem formulation

We neglect the anisotropy of concrete compared with structural anisotropy caused by reinforcement.

Unlike many industrial FEA software that takes into account the work of reinforcement only in tension-compression, we also take into account the work of reinforcement in transverse shear. This makes it possible to significantly improve the stability of the numerical approach in those cases when in the finite elements the concrete has completely collapsed and the reinforcement has not yet [7], [8].

The S.P. Timoshenko shear theory is applied for bar finite element, and the Mindlin-Reissner one – for shell finite elements [7], [8]. It is assumed that the reinforcement does not slip in concrete, which agrees well with the taking into account the softening of concrete simulating the crack appearance and the kinematic hypotheses of Timoshenko and Mindlin-Reissner models.

It is known that the descending branch of the $\sigma - \varepsilon$ diagram gives rise to a mesh-dependent finite element solution [15]. In this work, in the presence of descending branches of the $\sigma - \varepsilon$ diagram, to ensure the stability of numerical results during mesh refinement, a simple engineering idea is used, consisting in the fact that the reinforcement, whose elastic modulus is a magnitude on order greater than the concrete deformation modulus, does not have a descending branch in the $\sigma - \varepsilon$ diagram and must regularize the numerical solutions. Thus, we consider only reinforced concrete structures in which the presence of reinforcement is mandatory. The condition under which the reinforcement stabilizes the numerical solution is determined by the length of the descending branch of the $\sigma - \varepsilon$ diagram for concrete and is given in [7], [8].

2.1 The finite element library

The finite element library of the SCAD ++ FEA software [14], taking into account physical nonlinearity, contains quadrilateral and triangular isoparametric finite elements, as well as a 2-node
finite element of the spatial frame. All presented finite elements can be used to model the behavior of homogeneous materials, such as steel. In this case, they do not contain inclusions. In this paper, we consider reinforced concrete structures for which each of the finite elements contains inclusions modeling reinforcement.

In the case of shell finite elements (figure 1), the reinforcement is smeared in the plane of the finite element because we assume that the spacing between the reinforcing rods is quite small. At the same time, the discreteness of reinforcement over the thickness of the element is maintained. So rebar layers are formed. Each rebar layer is formed by the same reinforcing rods located at the same spacing, and their axes are parallel to each other. The number of rebar layers is not limited. \( s_1, s_2, \ldots \) are the directions of the axes of the rebar layers, coinciding with the direction of the axes of the rods forming this layer.

![Figure 1. Quadrilateral and triangular finite elements](image)

Each of the axes \( s_1, s_2, \ldots \) can be rotated to the local coordinate axis Ox relative to the axis Oz on the arbitrary angle, which allows us to consider structures with a geometric shape of any complexity for any configuration of a finite element mesh.

The principle of virtual work is used to obtain the tangent stiffness matrix and the vector of internal forces:

\[
\iiint_V \sigma : \delta \varepsilon dV + \sum_s A_s h_s \iiint_\omega \left[ \sigma_s \delta \varepsilon_s \right] d\Omega - \delta A_{ext} = 0 ,
\]

(1)

where \( V \) is a volume of the finite element, \( \Omega \) is the domain of finite element in-plane, \( \sigma \) and \( \varepsilon \) is a stress and strain tensors for concrete, \( s \) sum \( s \) covers all rebar layers, \( A_s \) and \( h_s \) is the cross-section area and spacing of rods forming the rebar layer \( s \), \( \sigma_s, \tau_{xy}^s, \tau_{xz}^s \) and \( \varepsilon_s, \gamma_{xy}^s, \gamma_{xz}^s \) are the components of stress and strain tensors for rods of rebar layer \( s \), \( m_s = 0.66 \) – shear correction factor for the circular cross-section of reinforcement rod, \( \delta A_{ext} \) – virtual work of external forces. The expression in parentheses appeared due to taking into account the work of reinforcement on the transverse shear. When calculating the integrals over the volume of the finite element, the trapezoid method is used for integration over the thickness and the Gauss-Legendre method for integration over the domain \( \Omega \) according to the \( 2 \times 2 \) scheme for a quadrilateral finite element and with a single Gauss point for a triangular one – the finite element is divided into layers by thickness.

Linear shape functions are applied. To overcome the effect of shear locking, the MITC4 technique [2] is used for the quadrilateral finite element and the Discrete Shear Gap [3] – for the triangular one.
The two-node finite element of the spatial frame is shown in figure 2. To calculate the integrals over the volume of the finite element, the cross-section area is triangulated. At the centers of gravity of the triangles, the components of the stress and strain tensor are calculated and stored. Longitudinal reinforcing rods are discretely taken into account, and no binding of their centers of gravity to the vertices of triangulation is required.

The principle of virtual work is applied to obtain the tangent stiffness matrix as well as the vector of internal efforts:

\[
\int \int \int \sigma : \delta \mathrm{d}V + \sum_s \int \left[ \sigma_i \delta \varepsilon_i + m_s \left( \tau_{xy} \delta \gamma_{xy} + \tau_{xz} \delta \gamma_{xz} \right) \right] \mathrm{d}x - \delta A_{\text{ext}} = 0,
\]

where \(a\) is the length of the finite element, \(x\) is the coordinate along the longitudinal axis \(OX\), the sum over \(s\) covers all the rods of the longitudinal reinforcement, and all other designations correspond to the above.

The linear shape functions are used, and to avoid shear locking, the following expressions for the transverse shear deformations are assumed:

\[
\gamma_{xy}(x,z) = 0.5 \cdot \left[ \gamma_{xy}(0,z) + \gamma_{xy}(a,z) \right], \quad y \leftrightarrow z.
\]

Details for both shell finite elements and spatial frame finite elements are given in [8].

2.2 The constitutive relations

Since in this paper we consider the behaviour of a structure under the action of both a constant load in time and a seismic one, in which deformations in structural elements change cyclically, the application of the deformation theory of plasticity to such problems seems unreasonable. Therefore, we apply the plastic flow theory.

2.2.1 Concrete. For concrete, in the case of bending structural elements, such as frame crossbars or floor slabs, the Drucker-Prager yield criterion is used. However, in the case of significant compressive stresses, the Drucker-Prager model ceases to adequately describe the behaviour of concrete, therefore, for the columns and walls, a yield surface in the form of a circular or non-circular paraboloid is used. The initial shape of the paraboloid is taken as the strength surface proposed in [13], therefore, for brevity, we will call the corresponding yield criterion the Geniev criterion.

The yield surface equation is as follows:

\[
f = 3\alpha^2 b I_1 + 1.5 \cdot 3^{1/2} + \beta (a - b l_1) J_2^{3/2} + 3 J_2 - \sigma_0,
\]

where \(a = \sigma_c + \sigma_t, \quad b = \sigma_c + \sigma_t, \quad 0.531 < \alpha \leq 1/3^{1/2}, \quad \beta = 1 - 3 \alpha^2, \quad \sigma_0 = 3 a \alpha^2, \quad \sigma_c\) and \(\sigma_t\) are the compressive and tensile strength of concrete, respectively, \(I_1\) is the first invariant of the stress tensor, \(J_2\) and \(J_3\) are the second and third invariants of the deviator of stresses. The parameter \(\alpha\) defines the deviation of the paraboloid from a circular shape.
The $\sigma - \epsilon$ diagram is adopted as it is presented in figure 3. Section AA' corresponds to the linear work of the material (Hooke’s law), and sections AB and A’B’ correspond to descending branches in the compression and tension zones simulating concrete degradation during crack propagation. Parameter $E$ is the initial modulus of deformation of concrete, $E_c$ and $E_t$ are the softening modules in the compression and tension zones. The parameters $\alpha_c$ and $\alpha_t$ determine the residual strength of the concrete and are usually either 0 or $\alpha_c < 1$, $\alpha_t < 1$.

When the image point moves along the descending branch, the $\sigma_c$ or $\sigma_t$ decreases, which leads to compression and moving of the yield surface in the space of principal stresses, and the softening of concrete occurs. This relates both to the Drucker-Prager yield surface and to the Geniev one.

Further, for brevity, we will call the model of reinforced concrete using the Drucker-Prager yield criterion, CM2 (CM – Constitutive Model), and the model using the Geniev yield criterion, CM3. The designation CM1 refers to the deformation theory of plasticity, which is not used in this paper. Numerous numerical experiments have shown that the CM3 model describes well the behavior of compressed concrete, but it does not always successfully cope with the softening of concrete in the tensile zone [8]. For this reason, it is recommended to use the CM2 model for bending reinforced concrete elements.

2.2.2 Reinforcement. When using the plastic flow theory for concrete (CM2 and CM3), the plastic flow theory with the von Mises yield criterion, bilinear $\sigma - \epsilon$ diagram and kinematic hardening is also used for reinforcement.

2.3 Equations of motion. Nonlinear equations of motion are represented as the Cauchy problem:

$$
\begin{cases}
M\ddot{u} + Cu + N(u) + f_{\text{ext}}(t) + f_{\text{diss}}(t) = \Gamma \dot{u}(t), \\
\dot{u}(0) = u_0, \quad \ddot{u}(0) = 0.
\end{cases}
$$

where $M$ and $C$ are mass and dissipation matrices, $u$ is displacement vector, $N(u)$ is a nonlinear operator, returning a vector of internal forces, $\Gamma$ is the diagonal matrix, arising due to the application of the penalty function method [25] allowing us to take into account the imposed displacements $\bar{u}(t)$, $f_{\text{ext}}(t)$ is a vector of external forces. In this approach, the equations of motion are formulated in terms of absolute displacements, which allows us to naturally describe the asynchronous excitation of the supports during seismic action. In the case of linear equations of motion, this idea is presented in detail in [11]. The technique for choosing the penalty parameters is similar to that given in [6].

The dissipation matrix $C$ is accepted as follows:

$$
C(u) = \alpha M + \beta K_c(u),
$$

where $\alpha$ and $\beta$ are the proportionality coefficients. In contrast to the proportional damping in linear problem, the tangent stiffness matrix $K_c(u) = \partial N(u) / \partial u$ depends on the displacement vector $u(t)$, therefore, the dissipation matrix $C(u)$ also depends on time.

The external forces vector

$$
f_{\text{ext}}(t) = f_{\text{stat}}(t) + \Gamma \bar{u}_0 + f_{\text{dyn}}(t) + \Gamma \bar{u}(t),
$$

where $f_{\text{stat}}$ is a vector of static loads, $\bar{u}_0$ is a static imposed displacements, $f_{\text{dyn}}(t)$ is a vector of dynamic loads and $\bar{u}(t)$ is the dynamic imposed displacements.

Since the problem is nonlinear, we cannot separately solve the static problem from the action of a static load alone, then solve only the dynamics problem from the action of only a dynamic load, and then add these solutions [4]. We must consider the combined action of both static and dynamic loads since the principle of superposition for nonlinear problems is not fulfilled.
On the other hand, if, when integrating the Cauchy problem (5) under uniform initial conditions for displacements $u_0 = 0$, suddenly apply only one static load $f_{stat}$ or only imposed displacements $u_p$, then the system will oscillate, which does not correspond to the physical meaning of the problem. Therefore, at the first stage, a nonlinear static problem is solved

$$N(u_0) + \Gamma u_0 = f_{stat} + \Gamma u_p,$$

(8)

where we consider the action only statically applied loads and obtain static displacements $u_0$. At the second stage, static displacements $u_0$ is taken as non-uniform initial conditions for the Cauchy problem (5). If now suddenly we apply only those loads that caused the non-uniform initial conditions $u_0$ and set $C = 0$, then the system will not make any oscillations. Next, we numerically integrate the Cauchy problem (5) with given static and dynamic loads (7) and non-uniform initial displacements obtained from (8).

Typically, a seismic load is specified as an accelerogram. However, the proposed approach requires the seismic impact to be represented in the form of the seismogram $u(t)$. Therefore, the specified accelerogram must be integrated two times, for which the SCAD Office accelerogram editor is used [14]. Besides, it turned out that when imposed displacements are given, the use of linear interpolation of the time function leads to the appearance of spurious rapidly oscillating components of accelerations of nodes adjacent to nodes in which imposed displacements are applied. Therefore, we use cubic interpolation of the time function for imposed displacements. The details are in [11].

The predictor-corrector approach [12] is applied for the numerical integration of (5). The Newton-Raphson method is used to depress the residual vector during the corrector’s iterations. The $\alpha$-HHT method [21] considerably improves the numerical stability of our approach due to the damping of high-oscillated modes, which cannot be approximated well by accepted time step $\Delta t$ ($\omega \Delta t > 1$, where $\omega$ is the frequency of the considered vibration mode). The details are in [8].

3 Numeric results

The study was performed on a computer with a 12-core Intel Core i9 processor - 9920X 3.50 GHz. To reduce the analysis time, multithreaded parallelization of the main algorithms was produced, namely: the evaluation of internal forces procedure [9], the assembling of a consistent tangent stiffness matrix[9], solving the system of linear algebraic equations [10] and the procedure of forward and back substitutions.

The verification and validation of the developed finite elements for a single static loading are carried out based on comparison with the results of well-established experiments and the reliable numerical solutions published in peer-reviewed articles and is given in [8].

3.1 Test on cyclic loading.

Static cyclic loading experiments are much simpler to perform and interpret than a fully dynamic one. For this reason, static cyclic loading tests are often used to verify and validate numerical approaches intended to perform structural analysis for cyclic loading, both static and dynamic. The last type of load also includes seismic impact. The static cyclic loading test mainly checks how the nonlinear operator $N(u)$ works.

Figure 4 presents the scheme and loading schedule for an experimental study [23]. The design model uses 16 finite elements of the spatial frame. The location of the reinforcement, the mesh of triangulation and the section dimensions are shown in figure 5.a. A static force $P_{stat} = 157$ KN and the imposed displacements in accordance with the loading schedule presented in figure 4.b is applied to the upper end of the column.

The following physical and mechanical characteristics are used for concrete and reinforcement: $E = 18 \ 000$ MPa, $\sigma_c = 24.8$ MPa, $\sigma_t = 1.4$ MPa, $\nu = 0.2$, $E_c = -4000$ MPa, $E_r = -1700$ MPa, $\alpha_c = \alpha_r = 0.03$, $\overline{\sigma} = 0.54$ (figure 3), $E_s = 174 \ 710$ MPa, $\nu_s = 0.3$, $\sigma_s = 362$ MPa, $E_s' = 13 \ 381$ MPa. Here $E_c$, $\nu_c$, $\sigma_c$, and $E_c'$ are, respectively, the elastic modulus, Poisson's ratio, yield strength, and hardening modulus.
of reinforcement. Material models are adopted following CM3 – the plastic flow theory using the Geniev yield criterion for concrete and von Mises one for reinforcement.

![Figure 4](image)

**Figure 4.** The cyclic loading test: a – design model, b – loading schedule

![Figure 5](image)

**Figure 5.** The cross-section (a) and diagram shear force vs displacement in the top node (b)

Figure 5.b demonstrates the acceptable correspondence of the results obtained by numerical solution with the experimental results, which confirms the possibility of applying the finite element of the spatial frame proposed in this paper to solve the problems of cyclic loading and seismic impact with considerable plastic deformations of structural elements.

### 3.2 Multi-storey building under the action of seismic load

Figure 6 presents a calculation model of a multi-storey building containing 51 393 equations. The dimensions of the building in the plan are 35 m × 17 m, and the height is 33 m. The following physical and mechanical characteristics are accepted for concrete: $E = 30\,018$ MPa, $\nu = 0.2$, $\sigma_c = 18.5$ MPa, $\sigma_t = 1.55$ MPa, $a_c = a_t = 0$, $\alpha = 0.54$. In addition, for columns and walls $E_c = -1400$ MPa, $E_t = -2800$ MPa, and for floor slabs $E_c = E_t = -1400$ MPa. For reinforcement we take $E_s = 200\,000$ MPa, $\sigma_y = 400$ MPa, $\epsilon_s = 0.3$, $\sigma_t = 400$ MPa, $E_s' = 2\,000$ MPa. The reinforcement ratio for walls and floors is 0.02 and 0.003, respectively, and their thickness is 0.2 m. The cross-sectional dimensions for the outermost columns are 100 cm × 50 cm, and the middle ones are 50 cm × 50 cm. The reinforcement ratio for the outermost columns is 0.0201, and for medium ones – 0.0193. For floor slabs, the direction of the reinforcing rods is parallel to the OX and OY axes of the global coordinate system, and for walls is vertical and horizontal.

The structure is subject to the action of static loads in the form of dead load and operating load. Also, horizontal seismic load with synchronous excitations of the supports and with a specified accelerogram shown in figure 7 and reduced to magnitude 9 is applied in the horizontal direction along
the OY axis. The damping parameters $\alpha = 0.37$ and $\beta = 0.0018$ correspond to a modal damping of 5\% of the critical at reduction frequencies $\omega_1 = 4.02$ s\(^{-1}\) and $\omega_2 = 50$ s\(^{-1}\).

Figure 6. Design model of multi-storey building (51 393 equations)  

Figure 7. Accelerogram of input seismic motion

Figure 8 shows the chart of the transversal force $Q_x(t)$ directed along the global axis OY in the control element 1390 of the corner column (figure 6) at the time interval when $Q_x$ reaches extreme values. The solid line corresponds to the traditional linear solution when using finite elements for columns and slabs, in which reinforcement is not taken into account at all. The dashed line was obtained using the finite elements presented here and taking into account the reinforcement, with the linear $\sigma - \varepsilon$ diagrams for concrete and reinforcement. The dotted line corresponds to a nonlinear solution when the constitutive model CM2 is used for floor slabs, and CM3 is used for walls and columns.

Peaks of efforts obtained on the basis of traditional linear analysis (solid curve) turn out to be several times larger than with full nonlinear analysis (dotted curve). The efforts peaks obtained on the basis of linear analysis using finite elements that take into account reinforcement (dashed curve) turn out to be larger than in the case of full nonlinear analysis (dotted curve), but smaller than in traditional linear analysis (solid curve).

Damaged finite elements are shown in figure 9. In the marked columns, the concrete of the compressed zone in several or all fibers passed the limit point of the $\sigma - \varepsilon$ diagram and is at the descending path. The reinforcement is within elastic deformations. In the marked finite elements of walls in the several layers over the thickness concrete of the compressed zone also passed the limit point of the $\sigma - \varepsilon$ diagram, as well as in some or all of the reinforcement rods the yield stress is exceeded. We consider such elements destroyed. In the marked finite elements of the floor slabs, the
concrete of the stretched zone cracked and doesn't work, and the reinforcement of the stretched zone yields. It is possible the collapse of marked parts of the floor slabs.

It turned out that when performing the traditional linear analysis, the integration step $\Delta t$, at which satisfactory convergence of the numerical solution was obtained not only in displacements but also in efforts, is in ten times smaller ($\Delta t = 10^{-5}$ s) than when producing a fully nonlinear analysis ($\Delta t = 10^{-4}$ s). We explain this to the fact that nonlinear elastoplastic models are dissipative-type models in which the scattering of mechanical energy occurs with increasing plastic strains. In addition, hysteretic damping occurs during oscillations. Therefore, in elasto-plastic systems, in addition to vibration damping caused by viscous friction (term $C u$ in (5)), typical for elastic systems, there is also dissipation due to hysteresis effects. This leads to a more intense suppression of highly oscillating modes, which are not approximated sufficiently by the accepted integration step $\Delta t$ and contribute to the accumulation of computational error. In addition, we believe that this effect is one of the reasons that the elasto-plastic analysis leads, as a rule, to lower oscillation amplitudes than the elastic one. The time to solve the linear problem on the mentioned above computer was 13 hours 48 minutes 21 seconds, and the nonlinear problem – 36 hours 24 minutes 22 seconds.

4 Conclusions
The account of elasto-plastic behaviour of the material, type of the $\sigma - \varepsilon$ diagram, type of yield surface, the choice of the dissipation model, as well as the level of plastic deformations achieved under static loading, have a significant impact on response of the system on seismic impact.

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