A Task-Space Form-Finding Algorithm for Tensegrity Robots

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ABSTRACT

A modified dynamic relaxation algorithm for the form-finding of tensegrity robots in task space is proposed in this paper. There are some form-finding problems of tensegrity structures that are hard to be handled by the node-based dynamic relaxation algorithm. For tensegrity structures with multiple interconnected rods or rigid rods, the node-based dynamic relaxation algorithm is usually difficult to perform this type of form-finding. The node-based dynamic relaxation algorithm becomes rather cumbersome if several different ratios between rod lengths and cable lengths are desired, which restricts its applicability to less regular structural forms. The performance of the algorithm is affected by the relative difference in stiffness between members (cables and rods). To solve these problems, we extend the general dynamic relaxation algorithm from node space to task space. The proposed algorithm considers multiple interconnected rigid rods as an entire rigid body to derive its motion in six degrees of freedom. We simulated the motion process of the rigid body approaching its self-equilibrium position under the force exerted by all surrounding cables. In the form-finding process, the relative location relationship of all interconnected rods on a complex rigid body is maintained. The algorithm is valid for the form-finding of tensegrity structures which are with several different ratios between rod lengths and cable lengths. Besides, the modified algorithm is not affected by the stiffness of interconnected rods on rigid bodies. To demonstrate the capability of the method, four examples, including a two-segment spine robot, a bio-inspired tensegrity arm, a multi-segment spine robot, a tensegrity robot with disconnected and interconnected rigid rods, are presented in this paper.

INDEX TERMS

Tensegrity robots, tensegrity structures, task-space form-finding, self-equilibrium.

I. INTRODUCTION

Tensegrity structures are composed of compressive members and tensile members [1]. In 1948, Snelson invented a tensegrity structure named X-Piece, while the term “tensegrity” was coined by Fuller [2], [3]. A tensegrity configuration that has no contacts between its rigid bodies is called a class-1 tensegrity system, and a tensegrity system with as many as k rigid bodies in contact is a class-k tensegrity system [1], [4]. This structure has attracted the attention of researchers in multiple fields such as architecture [5], [6], aerospace [7], civil engineering [8], biology [9], [10], and robotics [11]–[17]. Tensegrity is a type of self-equilibrium structure that is stabilized by prestress [18]. The self-equilibrium configuration of tensegrity structures can be found by using various form-finding methods. This study focuses on form-finding problems of robots with tensegrity structures.

Several methods have been proposed to solve form-finding problems. Tibert and Pellegrino reviewed and illustrated these methods. They classified form-finding methods into static and kinematic categories [19]. The dynamic relaxation method is one of the kinematic form-finding categories. The method was put forward by Motro and Belkacem as a general form-finding method for tensegrity structures [19]. It converts a static problem into a pseudodynamic problem for form-finding. Motro handled the form-finding problem of tensegrity prism by the dynamic relaxation method [20]. Belkacem used the method to analyze the triangular and square tensegrity prisms [21]. This algorithm is efficient for structures with a few nodes. However, the speed of convergence decreases with an increase in the number of nodes [22]. The dynamic relaxation method can be used for the form-finding of nonregular tensegrity systems [23]. A modified algorithm extended dynamic relaxation algorithm to accommodate clustered tensegrity structures. It is shown that the method scales up to larger structures more efficiently [24].
The dynamic relaxation method mentioned above focuses mainly on the form-finding of tensegrity structures in node space. Li et al. described the node and task space of tensegrity structures in the paper [25]. In this paper, we extend “task space” from the robotic area to describe the position and orientation of rigid bodies in tensegrity structures. Each rigid body is regarded as an end effector. The position and orientation of all rigid bodies can be represented in task space. These tensegrity structures approach a self-equilibrium position by updating the position of nodes gradually. Due to two points that can uniquely determine a member (cable or rod), the node-based method is effective for the tensegrity structure with disconnected rods as their compression members. However, these rods are not necessarily disconnected but may be interconnected. Compression members of tensegrity structures can be rigid bodies with multiple interconnected rods like Snelson’s ‘X-piece’ [2]. From the engineering point of view, these rigid bodies connected by tensile members and stabilized by prestressing are called tensegrity configurations [1]. In this case, the shape of a rigid body is determined by multiple nodes (more than two nodes) of the rigid body. In the process of node-based dynamic relaxation form-finding, the coordinates of nodes change constantly. The shape of a rigid body which is composed of multiple interconnected rods is easy to change. Thus, it is difficult to perform form-finding of tensegrity robots with multiple interconnected rods by the node-based dynamic relaxation algorithm. However, for the actual tensegrity structures, the shapes of rigid bodies on tensegrity structures need to maintain. The dynamic relaxation method becomes rather cumbersome if several different ratios between rod lengths and cable lengths are desired, which restricts its applicability to less regular structural forms [22]. Besides, the performance of the algorithm is affected by the relative difference in stiffness between members (cables and rods). The stiffness property assignment is crucial to the relaxation procedure. Experiments show that the least successful procedures are those in which the relative difference in stiffness between the members is the greatest [26].

The objective of this paper is to extend the application range of the dynamic relaxation method in the form-finding of tensegrity robots. To solve these form-finding problems, we extend the node-based dynamic relaxation algorithm to handle form-finding problems of tensegrity structures in task space. The proposed algorithm considers multiple interconnected rigid rods as an entire rigid body to derive its motion into translation and rotation of task-space and simulate the motion process of the rigid body approaching its self-equilibrium position under the force exerted by all surrounding cables. In the form-finding process, the relative location relationship of all interconnected rods on a complex rigid body is maintained. The modified dynamic relaxation algorithm can also be easily applied to the form-finding of tensegrity structures which are with several different ratios between rod lengths and cable lengths. The performance of the modified dynamic relaxation algorithm is not affected by relative stiffness differences between members any more. Even if the rods are rigid, the algorithm still valid. The algorithm can be used for form-finding of tensegrity robots with rigid compression members of interconnected rigid rods, disconnected rods, or a mixture of both. By solving the above problems, the application range of the dynamic relaxation algorithm in the form-finding of tensegrity robots is extended.

II. SELF-EQUILIBRIUM EQUATIONS

A. TOPOLOGY OF TENSEGRITY STRUCTURE

In a three-dimensional tensegrity structure, there are multiple rigid bodies as compressive members. For a rigid body, there are \( r \) nodes on it and \( s \) cables connecting it. A connectivity matrix can be used to describe the topology of a tensegrity structure [27]. A cable connectivity matrix for a rigid body is represented by \( C(\in \mathbb{R}^{r\times s}) \). Suppose member \( k \) connects nodes \( i \) and \( j (i < j) \). Then, the \( i \)th and \( j \)th members of the \( k \)th row of \( C \) are set to 1 and -1, respectively. The expression is as follows:

\[
C_{(k,p)} = \begin{cases} 
1 & \text{for } p = i \\
-1 & \text{for } p = j \\
0 & \text{otherwise}
\end{cases}
\]

Let \( N(\in \mathbb{R}^{r\times 3}) \) denote the coordinate matrix of nodes on the rigid body. The initial coordinate matrix of the nodes is \( N_0 \). A tensegrity structure with rigid bodies as compression members is shown in Figure 1. In this tensegrity structure, a rigid body is composed of four interconnected rods.

B. FORMULATION OF FORCE-EQUILIBRIUM AND MOMENT-EQUILIBRIUM EQUATIONS

The force-equilibrium and moment-equilibrium equations of a rigid body can be represented by

\[
\sum_{i=1}^{r} F_i = 0 \quad (2)
\]

\[
\sum_{i=1}^{r} T_i = 0 \quad (3)
\]

where \( F_i(\in \mathbb{R}^{1\times 3}) \) denotes the resultant force generated by cables acting on the \( i \)th node of the rigid body. \( T_i(\in \mathbb{R}^{1\times 3}) \)
denotes the moment produced by $F_i$ acting on reference point $n_i(\in \mathbb{R}^{1 \times 3})$ of the rigid body.

The stiffness of the cable members can be denoted as $k = (k_1, k_2, \ldots, k_s)$. Then, the force matrix $F(\in \mathbb{R}^{r \times 3})$ stacked by a combined force vector on each node of the rigid body can be expressed by

$$F = -C^T K \Delta L_s$$

(4)

where $K = \text{diag}(k)(\in \mathbb{R}^{s \times s})$.

The resultant force generated by cables acting on all nodes of a rigid body can be denoted as follows:

$$F = \begin{bmatrix} F_1 & \cdots & F_i & \cdots & F_r \end{bmatrix}^T$$

(5)

Similarly, the moment produced by cables acting on reference point $n_i(\in \mathbb{R}^{1 \times 3})$ of the rigid body can be denoted as follows:

$$M = \begin{bmatrix} T_1 \\ \vdots \\ T_i \\ \vdots \\ T_r \end{bmatrix} = \begin{bmatrix} (n_i - n_0) \times F_1 \\ \vdots \\ (n_i - n_0) \times F_i \\ \vdots \\ (n_i - n_0) \times F_r \end{bmatrix} \in \mathbb{R}^{r \times 3}$$

(6)

where $n_i(\in \mathbb{R}^{1 \times 3})$ denotes the coordinates of the $i$th node on the rigid body.

A matrix that contains the length and direction information of cables connecting the rigid body is defined as $L(\in \mathbb{R}^{s \times 3})$.

$$L = CL$$

(7)

The diagonal matrix $L_i(\in \mathbb{R}^{1 \times s})$ contains the current length of the cable members, which can be calculated as follows:

$$L_i = \text{diag} \left( \lVert L_{i1} \rVert_2 \cdots \lVert L_{is} \rVert_2 \cdots \lVert L_{is} \rVert_2 \right)$$

(8)

where $L_i(\in \mathbb{R}^{1 \times 3})$ denotes the $i$th row of $L$ and represents the length vector of the $i$th cable. In the process of form-finding, the variation matrix of the cable length $\Delta L_s(\in \mathbb{R}^{1 \times 3})$ can be defined as

$$\Delta L_s = (L_s - \text{diag}(L_0)) L_s^{-1} L_s$$

(9)

where $L_0(\in \mathbb{R}^{1 \times s})$ represents the initial length vector of the cables.

If all rigid bodies are in equilibrium under the force exerted by all surrounding cables, the self-equilibrium position of a tensegrity structure is found. Force and moment equilibrium equations are termination conditions for the iteration of a form-finding algorithm. The form-finding algorithm is presented in the next section.

### III. FORM-FINDING PROCESS

The dynamic relaxation method is executed by adding virtual mass and damping to nodes of members. In the process of form-finding, nodes are moved to the equilibrium position under the internal force of the tensegrity structure and converge to the equilibrium position under damping. The performance of the traditional node-based dynamic relaxation method is affected by relative difference in stiffness between members [26]. In addition, it is difficult for the method to find the self-equilibrium positions of tensegrity structures with interconnected rigid rods in node space. In this paper, we extended traditional node-based dynamic relaxation methods to handle form-finding problems of tensegrity structures with interconnected rigid rods in task space. For a certain rigid part of the structure fixed on the ground, the proposed algorithm is still valid.

### A. ITERATION ALGORITHM

Like the traditional dynamic relaxation method, we regard the form-finding of tensegrity with rigid bodies as a pseudodynamic process [26]. However, unlike the traditional dynamic relaxation method that deals with bars as nodes with virtual mass and damping, we view the rigid body as an entire system with six degrees of freedom by adding virtual mass/inertia and damping. $\Delta t$ is used to represent the iteration step size. The translation velocity and angular velocity of all rigid bodies at $t$ moment can be defined as

$$v_t = \frac{v_{t-\Delta t} + v_{t+\Delta t}}{2} (\in \mathbb{R}^{1 \times 3})$$

(10)

$$w_t = \frac{w_{t-\Delta t} + w_{t+\Delta t}}{2} (\in \mathbb{R}^{1 \times 3})$$

(11)

where $v_t$ and $w_t$ respectively denote the velocity and angular velocity of the rigid body at moment $t$. The acceleration and angular acceleration can be deduced by the central difference method as follows:

$$\dot{v}_t = \frac{v_{t+\Delta t} - v_{t-\Delta t}}{2 \Delta t} (\in \mathbb{R}^{1 \times 3})$$

(12)

$$\dot{w}_t = \frac{w_{t+\Delta t} - w_{t-\Delta t}}{2 \Delta t} (\in \mathbb{R}^{1 \times 3})$$

(13)

At moment $t$, the pseudodynamics equation of the structure is established as

$$F_t = m \dot{v}_t + c \ddot{v}_t (\in \mathbb{R}^{1 \times 3})$$

(14)

$$T_t = J \dot{w}_t + c \ddot{w}_t (\in \mathbb{R}^{1 \times 3})$$

(15)

where $m$, $c$, and $J$ denote the virtual mass, damping, and virtual inertia. Substituting (12) and (13) into (14) and (15), respectively, we obtain the translation velocity and angular velocity at the next moment as

$$v_{t+\Delta t} = \frac{m}{m + c} \cdot v_t + \frac{c}{m + c} \cdot \frac{J}{m + c} \cdot \dot{w}_t \in \mathbb{R}^{1 \times 3}$$

(16)

$$w_{t+\Delta t} = \frac{J}{J + c} \cdot w_t + \frac{c}{J + c} \cdot \dot{w}_t \in \mathbb{R}^{1 \times 3}$$

(17)

Within $\Delta t$, the translation of the reference point and rotation variation of the rigid body are defined as

$$\Delta n = \Delta t \cdot v_{t+\Delta t} (\in \mathbb{R}^{1 \times 3})$$

(18)

$$\Delta \Phi = \Delta t \cdot w_{t+\Delta t} (\in \mathbb{R}^{1 \times 3})$$

(19)
The modified algorithm is not affected by the stiffness of different ratios between rod lengths and cable lengths. Besides, form-finding of tensegrity structures which with several differences in the form-finding process and the algorithm is valid for the interconnected rods on a complex rigid body is maintained of freedom. Thus the relative location relationship of all rigid bodies are balanced. The procedure is thus time stepped using (10)-(23), until force-equilibrium and moment-equilibrium of a rigid body is achieved.

The above form-finding analysis is for a rigid body of a tensegrity structure. An entire rigid body moves to its equilibrium position step-by-step in the process of form-finding. The form-finding result for a two-segment tensegrity spine robot is shown in Figure 3. The coordinates of the rigid body at the next time step can be deduced by

\[
\tilde{N} = \begin{bmatrix}
(R(n_1 - n_r)^T + n_1^T + \Delta n^T)^T \\
(R(n_i - n_r)^T + n_i^T + \Delta n^T)^T \\
\vdots \\
(R(n_r - n_r)^T + n_r^T + \Delta n^T)^T
\end{bmatrix}
\]

The rotation axis \( \Psi \) can be denoted as follows:

\[
\Psi = \frac{\Delta \Phi}{\|\Delta \Phi\|_2} = \begin{bmatrix} \varphi_x & \varphi_y & \varphi_z \end{bmatrix} (\in \mathbb{R}^{1 \times 3}) \tag{20}
\]

According to Rodrigues’ rotation formula [28], the transformation matrix for the rotation of a rigid body can be expressed as

\[
R = \cos((\Delta \Phi) \mathbf{I}_3) + (1 - \cos((\Delta \Phi) \mathbf{I}_3)) \hat{\Psi}^T \hat{\Psi} = \sin((\Delta \Phi) \mathbf{I}_3) \hat{\Psi} \in \mathbb{R}^{3 \times 3} \tag{21}
\]

where \( \mathbf{I}_3 \in \mathbb{R}^{3 \times 3} \) is an identity matrix, and \( \hat{\Psi} \in \mathbb{R}^{3 \times 3} \), which can be parameterized by the vector product matrix of the (thereby defined) angular velocity, is the skew symmetric matrix of \( \Psi \).

\[
\hat{\Psi} = \begin{bmatrix} 0 & -\varphi_z & \varphi_y \\ \varphi_z & 0 & -\varphi_x \\ -\varphi_y & \varphi_x & 0 \end{bmatrix} \tag{22}
\]

The motion of the \( i \)th node on the rigid body is shown in Figure 2. The coordinates of the rigid body at the next time \( N \in \mathbb{R}^{r \times 3} \) can be deduced by

\[
\sum_i F_i \quad \text{if} \quad i \neq 0 \quad \text{and} \quad \sum_i T_i \quad \text{if} \quad i \neq 0
\]

**IV. EXAMPLES**

The form-finding algorithm was described in the previous section. In this section, several tensegrity robots (a bioinspired tensegrity arm, a two-segment spine robot, a multi-segment spine robot) are presented to show the effectiveness of the form-finding algorithm. In these examples, the threshold of stopping iteration is set to \( 10^{-3} \). There are some estimation methods of mass and damping parameters to guarantee the stability and convergence of the iterative procedure [24], [29]. In this paper, we focus mainly on extending the application range of the dynamic relaxation algorithm in the form-finding of tensegrity robots.

A. TWO-SEGMENT TENSEGRITY SPINE ROBOT

A two-segment tensegrity spine robot is composed of two rigid bodies (thick lines) that are connected by four vertical cables and four saddle cables (thin lines). Each rigid body is composed of four interconnected rigid rods. As shown in Table 1, the initial self-stresses of one vertical cable is set more than others. In this form-finding example, the base of the tensegrity robots is fixed. For the cables, the stiffness is \( k = 352.005 \). The virtual mass, damping, and virtual inertia are set to 0.5, 10, and 50, respectively.

Because the initial self-stress of 1-5 vertical cable is higher than others, the top rigid body tilt to the 1-5 cable in form-finding.
TABLE 1. Member forces and lengths in initial and final states for two-segment spine.

| members   | Self-stresses | Lengths |
|-----------|---------------|---------|
|           | Initial value | Final value | Initial value | Final value |
| 1-5       | 17.616        | 10.321   | 0.074         | 0.053       |
| Vertical  |               |          |               |             |
| 2-6       | 5.181         | 6.786    | 0.074         | 0.078       |
| 3-7       | 5.181         | 2.466    | 0.074         | 0.066       |
| 4-8       | 5.181         | 2.466    | 0.074         | 0.066       |
| Saddle    |               |          |               |             |
| 3-5       | 7.718         | 10.683   | 0.110         | 0.118       |
| 3-6       | 7.718         | 7.977    | 0.110         | 0.110       |
| 4-5       | 7.718         | 10.683   | 0.110         | 0.118       |
| 4-6       | 7.718         | 7.977    | 0.110         | 0.110       |

During the form-finding process, the result of form-finding by numerical methods is shown in Table 1 and Figure 3. The initial and final node coordinates are listed in Table 2.

B. MULTI-SEGMENT TENSEGRITY SPINE ROBOT

In this example, form-finding of a nine-segment tensegrity spine robot shown in Figure 4 is presented. There are nine rigid bodies and sixty-four cables in this structure. To illustrate our form-finding methods, we present two cases. In the first case, the entire structure has no constraints (Figure 4). In the second case, we fixed its lowest segment on the ground (Figure 5).

If the left vertical cables have self-stresses identical to those of the right cables, the spine robot holds a straight symmetric shape, as shown in Figure 4(a). In the first case, the virtual mass, damping, and virtual inertia are set to 0.5, 10, and 50, respectively. For the cables, the stiffness is \( k = 852.005 \). As shown in Table 3, if the initial self-stresses of the left vertical cables are set higher than those of the right vertical cables, rigid bodies tilt to the left side in form-finding process, then the robot will arrive at a “C” bending shape, as shown in Figure 4(b).

For the fixed-base case, the virtual mass, damping, and virtual inertia are set to 0.5, 1, and 50, respectively. The stiffness of cables is \( k = 1000 \). Equilibrium problems involving only these unconstrained segments are taken into account in our algorithm. This is similar to an unconstrained configuration where each free rigid-body segment (thick lines) is equilibrated by the tensions of its surrounding cables (thin lines). As shown in Table 3, the self-stresses of the right vertical cables are set higher than those of the left vertical ones. Driven by the unbalanced tensions in the surrounding cables, the robot moves toward the final equilibrium configuration, which is a “C”-like shape. As can be seen in Figure 5,
TABLE 3. Member forces and lengths in initial states for multi-segment spine.

| Cases | Members | Stiffness | Initial self-stresses | Initial lengths |
|-------|---------|-----------|-----------------------|-----------------|
| Case1 | Vertical | 852.005   | Right 11.758           | 0.069           |
|       |         |           | Left 54.085            | 0.069           |
|       |         |           | Front 11.758           | 0.069           |
|       |         |           | Back 11.758            | 0.069           |
|       | Saddle  | 852.005   | Right 63.480           | 0.069           |
| Case2 | Vertical | 1000      | Left 13.800            | 0.069           |
|       |         |           | Front 13.800           | 0.069           |
|       |         |           | Back 13.800            | 0.069           |
|       | Saddle  | 1000      |                      | 21.981 0.110    |

TABLE 4. Member forces and lengths in initial and final states for tensegrity arm.

| Members | Self-stresses | Lengths |
|---------|---------------|---------|
|         | Initial value | Final value | Initial value | Final value |
| 1-7     | 52.8          | 22.562   | 0.755         | 0.669       |
| 2-5     | 17.6          | 31.105   | 0.200         | 0.238       |
| 2-6     | 52.8          | 38.175   | 0.200         | 0.158       |
| 3-5     | 17.6          | 24.038   | 0.200         | 0.218       |
| 3-6     | 52.8          | 21.968   | 0.200         | 0.112       |
| 4-5     | 17.6          | 13.299   | 0.200         | 0.188       |
| 4-6     | 17.6          | 31.003   | 0.200         | 0.238       |
| 4-7     | 17.6          | 28.287   | 0.200         | 0.230       |
| 5-13    | 35.2          | 7.318    | 1.449         | 1.570       |
| 6-13    | 35.2          | 7.515    | 1.449         | 1.570       |
| 8-12    | 17.6          | 22.636   | 0.259         | 0.273       |
| 9-13    | 35.2          | 5.559    | 0.222         | 0.138       |
| 10-12   | 17.6          | 20.796   | 0.457         | 0.467       |
| 10-13   | 17.6          | 24.282   | 0.358         | 0.377       |
| 10-15   | 35.2          | 3.593    | 0.325         | 0.235       |
| 11-12   | 17.6          | 20.205   | 0.457         | 0.464       |
| 11-13   | 17.6          | 23.522   | 0.358         | 0.375       |
| 11-14   | 17.6          | 2.623    | 0.325         | 0.282       |

the initial configuration is a vertical shape in dashed line and the final configuration is a bending shape in solid line.

In these two cases, tensegrity robots all arrive at “C” bending shapes in the final equilibrium position, these robots can also arrive at other equilibrium shapes (such as “S”, “O”). We can modify the initial self-stresses of different cables to obtain different equilibrium shapes of tensegrity structures. From these examples, we can see that the relative location relationship of all interconnected rods on a complex rigid body is maintained in the form-finding process.

C. BIO-INSPIRED TENSEGRITY ARM

In this section, we consider the form-finding problem of a bio-inspired tensegrity arm proposed by Scarr [30] and Lessard et al. [31], [32]. The arm is composed of a shoulder tetrahedron, humerus, and forearm that are stabilized by corresponding cables. The virtual mass, damping, and virtual inertia are set to 0.5, 10, and 50, respectively. The member forces and lengths in the initial and final states of the tensegrity arm are listed in Table 4. The arm in the initial configuration (Figure 6(a)) is driven by the unbalanced tensions of these cables to the final equilibrium position (Figure 6(b)).

We can obtain different equilibrium shapes of bio-inspired tensegrity arm by setting different initial cable self-stresses. In this section, the arm is divided into three rigid parts to find equilibrium shape. In the form-finding process, three rigid bodies move to equilibrium position driven by unbalance force and moment in task space and the relative location relationship of all rods on a complex rigid body can be maintained. The traditional node-based dynamic relaxation method regards each rod as two nodes to form-finding, but it can’t maintain the relative location relationship of all rods on...
a rigid body during form-finding. Thus, the advantage of the form-finding algorithm proposed in this study is presented in these examples. From this example, we can see that the modified dynamic relaxation method can be easily applied to the form-finding of tensegrity robots which are with several different ratios between rod lengths and cable lengths.

**D. TENSEGRITY ROBOT WITH DISCONNECTED AND INTERCONNECTED RIGID RODS**

In this section, form-finding for a three-segment tensegrity robot with rigid bodies (composed of interconnected rigid rods) and rods is presented. Specifically, a disconnected rod is viewed as a single rigid body and moves in task space during the form-finding process. In this case, the virtual mass, damping, and virtual inertia are set to 0.5, 10, and 0.5, respectively. For the cables, the stiffness is \( k = 352.005 \). Initial coordinates of nodes and initial self-stresses of cables are set before form-finding. The three-segment spine is driven by the unbalanced tensions of cables to the final equilibrium position. The equilibrium shape of the tensegrity robot is illustrated in Figure 7(a).

For our algorithm, we don’t need to consider the stiffness of rods. The algorithm is still valid for tensegrity robots with rigid rods, because it derives motion of rigid bodies in task space rather than motion of nodes in node space. In these examples, the stiffness of both interconnected and disconnected rods is set high near to rigid. The maximum unbalanced force and moment are set as objective functions. The relation between the objective function and the number of iterations is illustrated in Figure 7(b). As can be seen from the curve, the algorithm is convergent. From this example, we can see that the performance of the method proposed in this paper is not affected by relative stiffness differences between members and the algorithm can be used for form-finding of tensegrity robots with interconnected rigid rods and disconnected rods.

**V. CONCLUSION**

In this paper, some problems of traditional node-based dynamic relaxation algorithm were listed. We modified the traditional form-finding algorithm to solve these problems in task space. By extending the traditional dynamic relaxation algorithm from node space to task space, this algorithm is valid for tensegrity robots with rigid bodies that are composed of interconnected rigid rods. In the form-finding process, the relative location relationship of all interconnected rods on a complex rigid body and the shapes of rigid bodies can be maintained. The modified dynamic relaxation algorithm can be easily applied to the form-finding of tensegrity structures which are with several different ratios between rod lengths and cable lengths. The performance of the modified dynamic relaxation method is not affected by relative stiffness differences between members. Even if the rods are rigid, the algorithm is still valid. The algorithm is valid for tensegrity robots with rigid compression members of interconnected rigid rods, disconnected rods, or a mixture of both. In this paper, we extended the application range of the dynamic relaxation method in the form-finding of robots with tensegrity structures. Several examples were presented to prove the validity of the algorithm, which is proposed in this paper.

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