Giant magnons in $AdS_4/CFT_3$: dispersion, quantization and finite-size corrections

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Abstract

We study giant magnon solutions in $AdS_4 \times CP^3$. We compute quantum corrections to their dispersion relation. We find out that the one-loop correction vanishes in infinite volume. This implies that the interpolating function $h(\lambda)$ between strong and weak coupling regimes does not have a constant term $\lambda^0$ at strong coupling. We also compute first nonvanishing finite volume correction to the one-loop expression. When compared to the Lüsher formula, our results could provide a nontrivial check of the $AdS_4 \times CP^3$ S–matrix proposed recently in arXiv:0807.1924.
1 Introduction

Nice example of integrable gauge theory is high–energy QCD [1, 2, 3]. Recently integrability was discovered for $AdS_5 \times S^5$ string theory and gauge theories [4, 5, 6, 7]. Many new applications of integrability arised together with famous Maldacena’s $AdS/CFT$ duality [8].

Recently Aharony, Bergman, Jafferis and Maldacena [9] proposed duality of $AdS_4 \times CP^3$ and $CFT_3$ theories. On conformal side of duality they considered a superconformal Chern–Simons theory with $SU(N) \times SU(N)$ symmetry and level $k$. One can find coupling constant in string theory taking the limit $k, N \to \infty$ and holding fixed

$$\lambda = \frac{N}{k} \equiv 8g^2.$$  \hspace{1cm} (1)

Integrability on the string side of the story was shown in [10, 11] in the strong coupling regime, and algebraic curve for string was proposed in [12]. Minahan and Zarembo [13] proposed two–loop Bethe ansatz for the $SU(4)$ sector of $N = 6$ Chern–Simons theory, and in the paper of Gromov and Vieira [14] all–loop Bethe ansatz was conjectured. Also one should mention works [15, 16, 17], where both sides of this $AdS_4/CFT_3$ duality were studied.

Minahan and Zarembo show an integrable spin chain with alternating spins which corresponds to operators in Chern–Simons theory. Elementary excitations of this chain are magnons. Magnons can form bound states [18, 19]. In string theory they correspond to “dyonic giant magnons” [20]. In fact, they are semi–classical string solutions which one can map to operators with large energy and momenta. For the giant magnons problem of determining spectrum from both sides of duality becomes easier.

There is an exact dispersion relation for a giant magnon in infinite volume. This relation is believed to be exact for all $\lambda$:

$$\Delta = \sqrt{\frac{Q^2}{4} + 4h^2(\lambda) \sin^2\left(\frac{p}{2}\right)}.$$  \hspace{1cm} (2)

where $Q$ is the number of magnons. Asymptotics of $h(\lambda)$ looks like follows:

$$h(\lambda) = \begin{cases} \sqrt{\frac{\lambda}{2}}, & \lambda \gg 1 \\ \lambda, & \lambda \ll 1 \end{cases}. $$  \hspace{1cm} (3)

In infinite volume this law can be fixed by symmetry except for the explicit form of $h(\lambda)$ [21]. We can evaluate the dispersion relation at one–loop and restrict $h(\lambda)$ at strong coupling. Also we want to compute all spectrum of the theory. So it is highly important to calculate finite volume corrections to the energy.

In this paper we use the algebraic curve technique (see, for example, [4, 5]) to compute magnon’s dispersion law and finite volume corrections the to one–loop expression of ground state energy. The paper is organised as follows. In section 2 we quantize the giant magnon (GM) solution using the algebraic curve technology. During this procedure we have to define twists with the help of usual orbifold treatment [22]. We introduce quasimomenta, which are obtained only from known analytical properties and asymptotics. Matching the asymptotics of quasimomenta we obtain the dispersion law (for infinite volume)

$$\Delta_s = \sqrt{\frac{Q^2}{4} + 2\lambda \sin^2\left(\frac{p}{2}\right)} \quad \text{(GM)}, \quad \Delta_b = \sqrt{Q^2 + 8\lambda \sin^2\left(\frac{p}{4}\right)} \quad \text{("big" GM)}. $$  \hspace{1cm} (4)
This expression coincides with one in [23], and it confirms validity of our technique in $AdS_4 \times CP^3$.

Then, in section 3 we calculate quantum fluctuations around the classical solutions, perturbing quasimomenta adding the extra poles (for first appearance of this method see [24]). In this way we obtain semi-classical spectrum of fluctuations around the GM solution, taking into account different polarizations of excitations. It occurs that all fluctuation energies are given by the same function.

In section 4 we discuss ground state energy around classical solutions, i.e. one–loop energy shift. Like in [25] and [26] (see also [27] for similar methods) it is useful to rewrite it as contour integral, which can be calculate in GM case. Using the saddle–point method, we obtain asymptotics in powers of $\sqrt{\lambda}$. We hope that this computation will be extremely helpful for checking $S$–matrix recently obtained in [28].

Note added. While this paper was in preparation we received the paper [29] with some overlapping results concerning giant magnon classical solutions.

2 Magnons: algebraic curve, quasimomenta and dispersion

GM was introduced in [20] in the context of gauge/string correspondence. Precisely, they suggested the relation between the spin chain magnon states and specific rotating semiclassical string states on $\mathbb{R} \times S^2$. In $AdS_4/CFT_3$ theory it was discussed in [30] (for similar computation in $AdS_5/CFT_4$ see [31]). It is a classical soliton solution on worldsheet, which corresponds to the $Q = 1$ fundamental excitation of the gauge theory. One should mention that there are two types of GM (see [32]). First type magnon lies in $CP^1 \approx S^2$ space, second type — in $RP^2 \subset CP^3$. We should call GM as “big” GM, if it consists of two arbitrary GM’s. We have a condition that verify our dispersion (1):

$$\Delta_b(P) = 2\Delta_s(p) = 2\Delta_s(P/2),$$  \hfill (5)

where $P$ and $p$ — momenta of “big” GM and small GM consequently. The infinite volume dispersion is exact, non–relativistic and looks like in (4), where $p$ is the magnon momentum. For consequences of the symmetry see for example [21].

2.1 Giant magnon

The algebraic curve [12] formalism maps solutions in $AdS_4 \times CP^3$ to a set of quasimomenta $\{q_1(x)\ldots q_{10}(x)\}$. This map looks like follows. Consider a diagonalization of monodromy matrix

$$\Omega(x) = P \exp \int d\sigma J_\sigma(x),$$  \hfill (6)

where $J(x)$ is a flat connection on equations of motion. Exactly,

$$J(x) = j + x \ast j \frac{x}{x^2 - 1}.$$  \hfill (7)

Here $j$ encapsulates equations of motion for fields in a very nice form:

$$d \ast j = 0.$$  \hfill (8)
One can see that $\Omega(x)$ is analytic on $\mathbb{C}$ except $x = \pm 1$. The eigenvalues of this matrix (i.e. quasi-momenta) form Riemann surface (with poles). In our case the logarithms of these quasimomenta can be organised in Riemann surface with 10 branches. These quasimomenta are not independent:

$$\{q_1, q_2, q_3, q_4, q_5\} = -\{q_{10}, q_9, q_8, q_7, q_6\}.$$  \tag{9}

Let us describe the analytic structure of quasimomenta more precisely. The algebraic curve for magnon solution looks as shown in Figure 1.

In terms of variables $X^+, X^-$ (in fact, they are conjugated), the magnon momentum is

$$p = \frac{1}{i} \log \frac{X^+}{X^-},$$  \tag{10}

and it is constrained by the charge

$$Q = \frac{2g}{i} \left( X^+ + \frac{1}{X^+} - X^- - \frac{1}{X^-} \right).$$  \tag{11}

As in [33], we can work with the log cut description of the GM solution. We have an inversion symmetry for quasimomenta:

$$q_1(1/x) = -q_1(x), \quad q_3(1/x) = -q_4(x),$$

$$q_2(1/x) = -q_2(x), \quad q_5(1/x) = q_5(x).$$  \tag{12}

and some asymptotic rules:

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{pmatrix} \xrightarrow{x \to \infty} \frac{1}{2gx} \begin{pmatrix} L + E + S \\ L + E - S \\ L - M_r \\ L + M_r - M_u - M_v \\ M_v - M - u \end{pmatrix}, \quad \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{pmatrix} \xrightarrow{x \to \pm 1} \frac{1}{2(x \pm 1)} \begin{pmatrix} \alpha \\ \alpha \\ \alpha \\ \alpha \\ 0 \end{pmatrix}.$$  \tag{13}

where $M_u, M_v, M_r$ are quantum numbers characterizing the $SU(4)$ state, belongs to representation with Dynkin labels $(L - 2M_u + M_r, M_u + M_v - 2M_r, L - 2M_v + M_r)$. It turns out that in Bethe Ansatz Equations $Q$ corresponds to number of roots in one Bethe string. The density of these
Bethe roots is roughly constant and it introduces logarithmic cut on Riemann surface (see for more details [18, 19]). These rules fix the charges uniquely:

\[ \begin{align*}
q_1(x) &= -q_{10}(x) = \frac{\alpha x}{x^2 - 1} \\
q_2(x) &= -q_9(x) = \frac{\alpha x}{x^2 - 1} \\
q_3(x) &= -q_8(x) = \frac{\alpha x}{x^2 - 1} + \frac{1}{i} \log \frac{x - 1/X^-}{x - 1/X^+} + \phi_1 \\
q_4(x) &= -q_7(x) = \frac{\alpha x}{x^2 - 1} + \frac{1}{i} \log \frac{x - X^+}{x - X^-} + \phi_1 \\
q_5(x) &= -q_6(x) = \frac{1}{i} \left( \log \frac{x - X^+}{x - X^-} \right) + \phi_2 \\
q_6(x) &= 0
\end{align*} \]

(14)

In these equations \( \phi_1 \) is a twist introduced in [34] (see Appendix A for computation details). It should be equal to \( -p/2 \) to satisfy boundary conditions. One can check that asymptotics of \( q_5(x) \) as \( x \to \infty \) is as it should:

\[ q_5(x) = -\frac{Q}{2gx} + O \left( \frac{1}{x^2} \right). \]

(15)

Asymptotics of quasimomenta shouldn’t contain energy. So we can extract the dispersion law from this condition. Let us write down the asymptotics for \( q_3(x) \) and \( q_4(x) \) (asymptotics for \( q_1(x) \) and \( q_2(x) \) are rather trivial):

\[ \begin{align*}
q_3(x) &= \frac{\alpha + \frac{i}{x} - \frac{i}{X^+}}{x} + O \left( \frac{1}{x^2} \right), \\
q_4(x) &= \frac{\alpha - i(X^- - X^+)}{x} + O \left( \frac{1}{x^2} \right).
\end{align*} \]

(16)

In our case \( 2g\alpha = E + L = \Delta - Q/2 \), so we obtain (4). Let us note that this is in perfect agree with [30] at strong coupling. Moreover, we obtain expression for the dyonic GM with arbitrary \( Q \). For \( Q = 1 \) dyonic magnon reduces to the simple GM from introduction.

2.2 “Big” giant magnon

Here we have \( p = \frac{2}{i} \log \frac{X^+}{X^-} \) because there are two type of excitations \( (u \text{ and } v) \). The set of charges looks as follows (see Figure 1):

\[ \begin{align*}
q_1(x) &= -q_{10}(x) = \frac{\alpha x}{x^2 - 1} \\
q_2(x) &= -q_9(x) = \frac{\alpha x}{x^2 - 1} \\
q_3(x) &= -q_8(x) = \frac{\alpha x}{x^2 - 1} + \frac{1}{i} \log \frac{x - X^-}{x - X^+} + \frac{1}{i} \log \frac{x - 1/X^-}{x - 1/X^+} + \phi_2 \\
q_4(x) &= -q_7(x) = -q_8(x) = \frac{\alpha x}{x^2 - 1} + \frac{1}{i} \log \frac{x - X^-}{x - X^+} + \frac{1}{i} \log \frac{x - 1/X^-}{x - 1/X^+} + \phi_2 \\
q_5(x) &= -q_6(x) = 0,
\end{align*} \]

(17)

where \( \phi_2 \) is the twist for this case. As before let us write down the asymptotics for \( q_3 \) (asymptotics for \( q_1 \) and \( q_2 \) are trivial and \( q_4 = q_3 \)):

\[ q_3(x) = \frac{\alpha - \frac{1}{x} \left( \frac{1}{X^+} - \frac{1}{X^-} - X^+ + X^- \right)}{x} + O \left( \frac{1}{x^2} \right), \]

(18)
If $Q$ is the number of roots of type $u$ we expect $q_1, q_2 \simeq (L + E)/(2gx)$, $q_5 \simeq 0$ and $q_4 = q_3 \simeq (L - Q)/(2gx)$. Thus $2g\alpha = E + L$, and therefore we obtain $\Delta_b$ as in (4).

One should notice that the dispersion law for usual GM differs from the one for the “big” GM.

### 2.3 Local charges

We know the explicit expression for quasimomenta so we can compute local conserved charges. As mentioned in [14],

$$q_1 + q_2 - q_3 - q_4 = Q_4^s + G_4 + G_4^1 - G_4^2$$

where bar means $1/x$ instead of $x$ in the argument. $G_4 + G_4$ is generating function for all conserved charges. Let us consider the small magnon case. Computation gives

$$G_4 + G_4 = \frac{1}{i} \log \left( \frac{1/x - XP}{1/x - XM} \right) = -\sum_{n=0} Q_{n+1}^s x^n,$$

where

$$Q_{n+1}^s = \frac{i}{n+1} \left( \left( \frac{1}{X^+} \right)^{n+1} - \left( \frac{1}{X^-} \right)^{n+1} \right)$$

For the “big” GM one can check that $Q_{n+1}^b = 2Q_{n+1}^s$.

### 3 GM quantization

Now we will quantize the classical solutions from section 2. We will calculate explicitly only the “big” GM case, for the usual GM the procedure is the same. So let us perturb the quasimomenta, introduced in subsection 2.2 by $\delta q$. One should notice that there are a number of different polarizations of excitations $N_{ij}$, like in [35]:

$$\delta q_i(x) \sim \eta_i N_{ij}^n \frac{\alpha(x_{ij}^n)}{x - x_{ij}^n},$$

where $\eta_i$ are signs of the residues and

$$\alpha(x) = \frac{x^2}{2g(x^2 - 1)}.$$ 

Different possible choices of charges correspond to different string polarizations:

- $CP^3$ : $(3, 5), (3, 6), (3, 7), (4, 5), (4, 6)$
- $AdS$ : $(1, 9), (2, 9), (1, 10)$
- Fermions : $(1, 5), (1, 6), (2, 5), (2, 6); (1, 7), (2, 7), (1, 8), (2, 8)$

When adding the poles we should satisfy several conditions:

- fluctuations $\delta q_i(x)$ can have poles at $x = \pm 1$, but those must be synchronized on different sheets of Riemann surface;
• final expression should satisfy $x \to 1/x$ symmetry.
• there should be a “feedback” of original solutions to inserting these fluctuations, so we have to shift the log cut. Close to $X^\pm$, the quasimomenta should behave as

$$\delta q(x) \sim \partial \log(x - X^\pm) = \frac{1}{x - X^\pm}.$$ (25)

For example $q_5(x) = 0$, so in $\delta q_5(x)$ we should add only poles at $x_n$ and $1/x_n$ (for symmetry $x \to 1/x$). In $q_3(x)$ and $q_4(x)$ we have logs with branch points at $X^\pm$ and $1/X^\pm$, so we have to add these points as poles in $\delta q_{3,4}(x)$.

These analytic conditions fix the asymptotic, which on the other hand is fixed by the global charges of the theory. Let us write down only half of charges, because other half is symmetric:

$$\delta \left( \begin{array}{c}
q_1 \\
q_2 \\
q_3 \\
q_4 \\
q_5
\end{array} \right) \simeq \frac{1}{2gx} \begin{pmatrix}
\delta E \\
\delta E \\
-N_{45} - N_{46} \\
-N_{45} - N_{46} \\
N_{15} + N_{16}
\end{pmatrix} \begin{pmatrix}
-N_{35} - N_{36} - N_{37} \\
N_{17} \\
N_{18}
\end{pmatrix} + \frac{1}{2gx} \begin{pmatrix}
N_{25} & N_{26} \\
-N_{25} & -N_{16}
\end{pmatrix} \begin{pmatrix}
2N_{29} & N_{19} & 2N_{1,10} \\
N_{27} & -N_{28} & -N_{18}
\end{pmatrix} \equiv \frac{1}{2gx} V (26)
$$

As we know, $\delta q$ has only poles (with known residues), so we can write down their explicit expression

$$\delta q(x) = V_0 \frac{\alpha(x)}{x - x_n} + MV_0 \frac{\alpha(1/x)}{1/x - x_n} + \left( \begin{array}{c}
1 \\
0 \\
0 \\
0
\end{array} \right) \frac{\delta E x}{2g(x^2 - 1)} + \left( \begin{array}{c}
0 \\
1 \\
1 \\
0
\end{array} \right) \frac{A^+ \alpha(x)/x}{x - X^+} +$$

$$+ \left( \begin{array}{c}
0 \\
1 \\
1 \\
0
\end{array} \right) \frac{A^- \alpha(x)/x}{x - X^-} + M \left( \begin{array}{c}
0 \\
0 \\
1 \\
0
\end{array} \right) \frac{A^+ \alpha(1/x)x}{1/x - X^+} + M \left( \begin{array}{c}
0 \\
0 \\
1 \\
0
\end{array} \right) \frac{A^- \alpha(1/x)x}{1/x - X^-},$$ (27)

where $V_0$ means $V$ with $\delta E = 0$ (because we want to find it from the explicit expression) and $M$ is a matrix which connect $\delta q(x)$ and $\delta q(1/x)$: $\delta q(x) = M \delta q(1/x)$.

We need to match poles on the different sheets of Riemann surface. $\delta q_1(x)$ and $\delta q_2(x)$, $\delta q_3(x)$ and $\delta q_4(x)$ have matching poles by pairs, so we need to fit the poles between $\delta q_2(x)$ and $\delta q_3(x)$:

$$\text{res}_{x = \pm 1} (\delta q_2(x) - \delta q_3(x)) = 0.$$ (28)

Due to the symmetry $x \to 1/x$ we should request $\delta q(0) = 0$ or:

$$\frac{A^+}{X^+} + \frac{A^-}{X^-} = 0.$$ (29)
Solving these equations we obtain $\delta E$:

$$
\delta E = \sum_{i,j \in E_1} N_{ij}^n \Omega_{ij}^n + 2 \sum_{i,j \in E_2} N_{ij}^n \Omega_{ij}^n,
$$

and $\Omega(x)$, $E_1$, $E_2$ are given by

$$
\Omega_{ij}(x) = \frac{1}{x^2 - 1} \left( 1 - \frac{X^+ + X^-}{X^+ X^- + 1} x \right),
$$

$$
E_1 = \{(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)\},
$$

$$
E_2 = \{(1, 7), (1, 8), (1, 9), (1, 10), (2, 7), (2, 8), (2, 9), (3, 7)\}.
$$

One should mention that all fluctuations are given by the same function (as it was in \[35\]), i.e. $\Omega_{ij}^n = \Omega(x_{ij}^n)$ for all $(ij)$. But the map $x \leftrightarrow n$ is given by the charges discontinuity

$$
q_i(x_n) - q_j(x_n) = 2\pi n,
$$

so fluctuations are not all the same.

Structure of perturbation is different for usual GM. In particular, there will be poles at $X^\pm$ and at $1/X^\pm$ in $q_5(x)$; $q_4(x)$ must have poles at $X^\pm$ and $q_3(x)$ — at $1/X^\pm$ (there are also obtained from the shifting the log cut). However, $\Omega(x)$ remains exactly the same.

### 4 Finite size GM: one-loop corrections

Let us study the one-loop energy corrections, i.e. the ground state energy around a classical solution:

$$
\delta \epsilon_{1-loop} = \frac{1}{2} \sum_{n \in \mathbb{Z}} \sum_{ij} (-1)^{F_{ij}} \gamma_{ij} \Omega_{ij}.
$$

This sum is graded, i.e. $F_{ij} = 0$ in case of fermion excitation and $F_{ij} = 1$ if bosonic one. Further calculation of $\delta \epsilon_{1-loop}$ goes along one in \[35\] and so we have

$$
\delta \epsilon_{1-loop} = - \sum_{ij} \gamma_{ij} (-1)^{F_{ij}} \int_{\mathbb{U}} \frac{dx}{4i} \frac{q_i' - q_j'}{2\pi} \cot \left( \frac{q_i - q_j}{2} \right) \Omega(x),
$$

where $\gamma_{ij} = 1$ or 2, according to coefficients in \[30\]. Expanding the cotangent when the quasimomenta are large we obtain

$$
\cot \left( \frac{q_i - q_j}{2} \right) = \pm i (1 + 2e^{\pm i(q_i - q_j)} + \ldots).
$$

and using that

$$
\sum_{ij} \gamma_{ij} (-1)^{F_{ij}} (q_i' - q_j') = 0,
$$

(similar to result in \[33\] and \[35\]) we can prove the cancellation of the leading term in the one-loop shift, so we have to calculate the leading correction, which corresponds to second term in the cotangent expansion. After a little algebra, we can find

$$
\delta \epsilon_{1-loop} = \int_{\mathbb{U}} \frac{dx}{2\pi i} \partial_x \Omega(x) \sum_{ij} (-1)^{F_{ij}} e^{-i(q_i - q_j)}.
$$
While taking the of sum in this expression one should notice that all the terms without $q_5$ in exponent would be subleading to others. So let us calculate only terms with $q_5$. We consider a regime with $Q \ll g$ so for a giant magnon we have $1/X^+ = X^-$. In this approximation we obtain

\[ \sum_{ij} (-1)^{F_{ij}} e^{-i(q_i - q_j)} = 4 \exp \left( -\frac{ix\alpha}{x^2 - 1} \right) \frac{(x + 1)(1 - X^-)}{xX^- - 1}. \] (38)

Plugging this expression in the integral (37) and using the saddle–point method, we can write down asymptotics of the first quantum correction:

\[ \delta \epsilon_{1\text{-loop}} = \frac{8e^{-\alpha/2} (1 - \sec(p/2))}{\sqrt{\pi} \alpha} + O \left( \frac{1}{\alpha} \right) = \frac{8\sqrt{2g}}{E+L} \frac{e^{-\frac{E+L}{2g}} (1 - \sec(p/2))}{\sqrt{\pi}} + O \left( \frac{g}{E+L} \right). \] (39)

For a “big” GM calculations are similar, except for the sum in (38). One can obtain

\[ \sum_{ij} (-1)^{F_{ij}} e^{-i(q_i - q_j)} = 4 \exp \left( -\frac{ix\alpha}{x^2 - 1} \right) \frac{(1 - x^2)(X^-)^2 - 1}{(xX^- - 1)^2}, \] (40)

and the one–loop correction is

\[ \delta \epsilon_{1\text{-loop}} = -16\sqrt{2g} \frac{e^{-\frac{E+L}{2g}} tan^2 p}{\sqrt{\pi}} + O \left( \frac{g}{E+L} \right). \] (41)

5 Conclusion

We construct the algebraic curve for two classical solutions in $AdS_4 \times CP^3$, the so called giant magnons. There are two type of magnons in this theory, which live in different subspaces of $CP^3$. Using algebraic curve technique we identify the dispersion (4) relations and all local conserved charges (21) of the giant dyonic magnons.

Algebraic curve also allows to find out spectrum of quantum fluctuation and compute one–loop shift of all local conserved charges and in particular to the energy. Using this technique we find giant magnon excitations in $AdS_4 \times CP^3$. The results for $AdS_4 \times CP^3$ are quite similar to the ones in $AdS_5 \times S^5$, but nevertheless they have remarkable difference (see (30) and remark in the end of section 3).

Also we have obtained the one–loop finite-size correction to the magnon excitation. It turns out that these corrections differ from one in $AdS_5 \times S^5$. Corrections for usual GM and “big” GM are different (see (39) and (41)). These expressions can be useful for checking S–matrix, as it was done in $AdS_5 \times S^5$. It will be interesting to investigate such possibility in $AdS_4 \times CP^3$.

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## A Twists computation

Let us rewrite the explicit magnon solution from [23]:

$$Z = \frac{1}{\sqrt{2}} (e^{i\sigma p f(\tau, \sigma)}, g(\tau, \sigma), e^{-i\sigma p f(\tau, \sigma)}, g(\tau, \sigma)).$$  \hspace{1cm} (43)

In this expression $f(\tau, \sigma)$ and $g(\tau, \sigma)$ are periodic and well–defined functions on a world–sheet. We want to have a closed string, and twists are created for it. Let us consider the asymptotic $x \rightarrow \infty$. In this case the monodromy matrix (see [12]) gives us some information:

$$\Omega(x) = P \exp \int d\sigma J_\sigma(x) \sim P \exp \int d\sigma j_\sigma \sim h^{-1}(2\pi)h(0)$$  \hspace{1cm} (44)

Eigenvalues of $\Omega(x)$ are of type

$$\{e^{ip_1}, \ldots e^{ip_4}\},$$  \hspace{1cm} (45)

where $p_i$ — the twists, i.e. asymptotics of quasimomenta. In our case computation gives that there will be two non–zero diagonal elements, and the twists are equal to $-p/2$, where $p$ is a magnon momentum on the world–sheet.

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