Neutrino and gamma-ray production from proton-proton interactions in binary-driven hypernovae

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We estimate the neutrino emission from the decay chain of the π-meson and μ-lepton, produced by proton-proton inelastic scattering in energetic \( E_{\text{iso}} \gtrsim 10^{52} \text{erg} \) long gamma-ray bursts (GRBs), within the type I binary-driven hypernova (BdHN) model. The BdHN I progenitor is a binary system composed of a carbon-oxygen star (CO\(_{\text{core}}\)) and a neutron star (NS) companion. The CO\(_{\text{core}}\) explosion as supernova (SN) triggers a massive accretion process onto the NS. For short orbital periods of few minutes, the NS reaches the critical mass, hence forming a black hole (BH). Recent numerical simulations of the above scenario show that the SN ejecta becomes highly asymmetric, creating a cavity around the newborn BH site, due to the NS accretion and gravitational collapse. Therefore, the electron-positron (e\(^\pm\)) plasma created in the BH formation, during its isotropic and self-accelerating expansion, engulfs different amounts of ejecta baryons along different directions, leading to a direction-dependent Lorentz factor. The protons engulfed inside the high-density \( \sim 10^{23} \text{ particle/cm}^3 \) ejecta reach energies in the range \( 1.24 \lesssim E_p \lesssim 6.14 \text{ GeV} \) and interact with the unshocked protons in the ejecta. The protons engulfed from the low density region around the BH reach energies \( \sim 1 \text{ TeV} \) and interact with the low-density \( \sim 1 \text{ particle/cm}^3 \) protons of the interstellar medium (ISM). The above interactions give rise, respectively, to neutrino energies \( E_\nu \lesssim 2 \text{ GeV} \) and \( 10 \lesssim E_\nu \lesssim 10^3 \text{ GeV} \), and for both cases we calculate the spectra and luminosity.

I. INTRODUCTION

Multi-messenger astronomy is a fundamental technique to get complementary information about the physical processes, dynamics, evolution and structure behind the cosmic sources emission [1]. With the advent of new facilities generating high-quality data of cosmological energetic sources such as supernovae (SNe), gamma-ray bursts (GRBs) and active galactic nuclei (AGN), the analysis of the multi-messenger emission becomes a necessity. Our aim here is to present, for the case of long GRBs, the emission of the neutrino messenger from the process of proton-proton (\( pp \)) interactions occurring in the source. Many studies about neutrino emission from GRB have been made since the pioneering work of Waxman & Bahcall [2]. They studied the production of neutrinos of energies \( E_\nu \sim 10^{14} \text{ eV} \) coming from the photomeson process by the interaction between very-high energy accelerated protons \( (E_p \lesssim 10^{20} \text{ eV}) \) and photons emitted through synchrotron/IC radiation by accelerated electrons. Other works followed specifically studying the neutrino production in the traditional fireball model of GRBs (see, e.g., [3–6]). In these works, it is shown that within the fireball the \( \nu s \) are produced by the decay of pions and muons created by the internal shock, mainly by two dominant processes. 1) Photomeson production \( (p + \gamma \rightarrow \pi, \mu \rightarrow \nu_{\mu,\tau}) \) between very-high energy protons accelerated by the Fermi mechanism and synchrotron/IC photon \( (\sim 1 \text{ MeV}) \) from accelerated electrons. The neutrinos created by this channel are characterized by energies \( \sim 10^{14} \text{ eV} \) [3]. If the protons are accelerated up to \( \sim 10^{20} \text{ eV} \) and interact with photons of few eV (Optical-UV photons), neutrinos emerge with energies \( \sim 10^{18} \text{ eV} \) [4]. 2) \( \pi \) and \( \mu \) production via proton-neutron interaction between accelerated protons and coasting neutrons [5, 6], once these two components of the same expanding fireball are decoupled. This channel gives rise to \( \nu s \) with energies of \( 5–10 \text{ GeV} \).

Other two ways of neutrino production that have been investigated in the GRB literature are the \( n \) decay and \( \nu s \) associated to a stellar collapse or to a merging event which leading to a GRB (see Ref. [6] and references therein). Both of these ways are less efficient respect to the previous channels and produce neutrinos of energies \( 10 \lesssim E_\nu \lesssim 10^2 \text{ MeV} \), that would be difficult to detect due to the low values of \( \nu N \) cross-section for low \( E_\nu \).

In this work, we focus on the neutrino production by \( pp \) interaction following, instead of the fireball model, the BdHN model, whose characteristics are stated in the next subsections. The typical energies of the escaping \( \nu s \) are \( \sim \text{ GeV} \) and, as we will show here, the interaction dynamics is different with respect to the one of the fireball model.
A. BdHN I: from MeV to GeV and TeV neutrinos

There are two principal classes of GRBs (see Ref. [7] for the subdivision of long and short GRBs in seven different subclasses). The “short bursts” originate in the mergers of binaries composed of a neutron star (NS) accompanied either by another NS, or a white dwarf (WD) or a black hole (BH), or mergers of binary WDs. The “long bursts” split in four different subclasses, all of them originating from binaries which have been called binary-driven hypernovae (BdHNe) of type I, II, III and IV [8].

We are here interested in the BdHNI. The progenitor is composed of a pre-SN carbon-oxygen star (hereafter CO core) and a NS companion in close orbit [9–11]. The explosion of the CO core as SN forms a newborn NS (hereafter νNS) at its center and, at the same time, ejects material that triggers a hypercritical accretion process onto the NS companion. Depending on the binary parameters, the system leads to different subclasses. In compact binaries with orbital periods $\sim 5$ min, the accretion onto the NS is sufficient to bring it to the critical mass, forming a BH by gravitational collapse (see Figure 2 in Ref. [12]). These are the BdHNI, and they explain energetic long bursts with isotropic energy in gamma-rays $E_{iso} \gtrsim 10^{52}$ erg and peak energy $E_{p,i} \gtrsim 200$ keV. We refer the reader to Refs. [8, 13] for details on other BdHN classes that are not the subject of this work.

1. Formation channel and occurrence rate of BdHN

Then, BdHNI forms NS-BH binaries and the parameters for which the hypercritical accretion leads the NS to the critical mass, with consequent gravitational collapse forming a rotating BH, have been widely explored (see, e.g. Refs. [10–12]). This knowledge, in turns, has allowed to envisage possible evolutionary paths for these binaries.

The evolutionary path leading to BdHNI is thus expected to be closely related to the ones known for the formation of compact-object binaries, e.g. NS-NS or NS-BH, as introduced by the X-ray binary and SN communities. Thus, a plausible BdHNI formation channel starts with a binary composed of e.g. $\gtrsim 10-12 M_{\odot}$ zero-age main-sequence (ZAMS) components [9, 14]. After the core-collapse of the primary, the binary is formed by a newly born NS and the secondary, ordinary star. The system then evolves through mass-transfer episodes and possibly multiple common-envelope epochs [15]. These binary interactions lead to the ejection of the hydrogen and helium outermost layers of the secondary star. At this stage, the binary is composed of a CO core (or a helium star) and a NS companion. The X-ray binary and SN communities dubbed these systems “ultra-stripped” binaries [15]. These binaries are thought to produce the 0.1–1% of the SN population [16].

Therefore, the binary progenitors of the BdHNI are expected to be formed in an evolutionary path very similar to the one of ultra-stripped binaries. The majority of population synthesis simulations lead to ultra-stripped binaries with orbital periods $3 \times 10^3 - 3 \times 10^6$ s [15], which are longer than the ones of BdHNI. It is then clear that the BdHNI progenitors must be a small subset of the binary progenitors of ultra-stripped binaries. This conforms with the fact that GRBs are indeed a rare phenomenon, i.e. their density rate is much lower with respect to other astrophysical sources, therefore the population of binaries that end with the appropriate physical conditions to produce a BdHNI must be low [17].

The observed occurrence rate of BdHNI and II, are respectively $\sim 1$ Gpc$^{-3}$ y$^{-1}$ and $\sim 100$ Gpc$^{-3}$ y$^{-1}$ [7]. Therefore, they are 0.5% and 0.005% of the SNe Ibc rate, which is $2 \times 10^4$ Gpc$^{-3}$ y$^{-1}$ [18]. Since (0.1–1%) of the SN Ibc are expected to be produced by ultra-stripped binaries [16], their density rate is expected to be in the range (20–200) Gpc$^{-3}$ y$^{-1}$. This implies that $\lesssim 5\%$ of this population is enough to explain the BdHNI population. Interestingly, these estimates agree with traditional population synthesis analyses concluding that only $\sim 0.001–1\%$ of massive binaries lead to double compact-object binaries [19–21].

2. Baryonic content available for proton-proton interactions in a BdHNI

A copious neutrino emission is one of the crucial physical phenomena characterizing the BdHNI scenario [10]. Thanks to the neutrino-antineutrino flux, the accretion process onto the NS can proceed at hypercritical, super-Eddington rates, leading to neutrinos of energies 20–30 MeV with luminosities of up to $10^{51}$ erg s$^{-1}$ (see, e.g., Ref. [11]). Interestingly, neutrino flavour oscillations owing to neutrino self-interactions have been also shown to be relevant during this hypercritical accretion process [22].

In this article, we focus on the emission of neutrinos of higher energies than the aforementioned ones. Specifically, we show that BdHNI also produce neutrinos in the GeV and TeV energy domains via $pp$ interactions.

In order to set up the possible $pp$ interactions occurring in a BdHNI, we start by analyzing the structure of the baryonic matter present. For this task, we make use of recent three-dimensional simulations of this system (see, e.g., Ref. [11] and Figure 2 in Ref. [12]). The SN ejecta, although starts expanding in a spherically symmetric way, becomes highly asymmetric by the accretion process onto the NS [11, 12] and the BH formation [23]. Due to this morphology, the electron-positron ($e^+e^-$) plasma created in the process of BH formation, which expands isotropically from the newborn BH site, experiences a different dynamics along different directions due to the different amounts of baryonic matter encountered [24].

In the direction pointing from the CO core to the accreting NS, outwards and lying on the orbital plane, the NS and the BH formation cave a region character-
ized by very poor baryon pollution, a cavity (see 1; see also Refs. [11, 12, 23]). The production of the $e^+ e^-$ plasma and its subsequent evolution and transparency leading to sub-MeV emission, overcoming the so-called GRB compactness problem, has been extensively studied in the theoretical framework of the fireshell model, which fully solves the hydrodynamic equations of motion of the plasma (see Refs. [25–28]). We refer to these references for details. The formation of the $e^+ e^-$ pair plasma is governed by quantum electrodynamics, i.e. $h/(m_ec^2) \sim 10^{-21}$ s, while the collapse of the NS into a BH occurs on a gravitational timescale $GM/c^3 \sim 10^{-6}$ s. No numerical relativity simulations are currently able to simultaneously follow such extremely different timescales. However, general relativistic effects can be considered since the $e^+ e^-$ pair creation occurs locally, in the exterior spacetime, and the energy density of pairs is low enough to disregard its feedback onto the spacetime. Therefore, the background metric can be considered as fixed in the estimation of the pair-creation rate and its evolution [26]. The verification of this model in the analysis of the prompt emission of specific GRB sources, can be found, e.g., in Refs. [29–32].

These studies have been specialized in the case where the $e^+ e^-$ plasma engulfs a limited amount of baryons, characterized by a baryon load parameter $B \lesssim 10^{-2}$. The baryon load is defined as $B \equiv M_b c^2 / E_{e^+ e^-}$, namely the ratio between the baryon rest-mass energy respect to the $e^+ e^-$ energy. Such low values of $B$ allow the plasma to reach transparency with high Lorentz factor $\Gamma \sim 1/B \gtrsim 10^2$, needed to explain the gamma-ray prompt emission of the GRB. We denote with $\gamma$ the Lorentz factor of a single particle, and with $\Gamma$ the one of bulk motion.

In the other directions along the orbital plane, the $e^+ e^-$ plasma penetrates inside the SN ejecta at $\sim 10^8$–$10^{10}$ cm, and evolves engulfing much larger amounts of baryons, finally reaching transparency at $10^{12}$ cm with $\Gamma \lesssim 4$. The theoretical description and numerical simulations of this system in which the $e^+ e^-$ plasma engulfs larger amounts of baryons ($B \sim 10^3$) have been presented in Ref. [24]. We recall that, therein, it has been also shown that the transparency of this plasma with such $\Gamma$ explains the observed flares in the X-rays at nearly 100 s (rest-frame time) in the early GRB afterglow.

B. Characterizing the $pp$ interactions in a BdHN I

From the above physical and geometrical description, we are ready to set up the properties of the incident and target protons. Therefore, at least two types of $pp$ interactions occur in a BdHN I:

1. Interaction of the protons with $\Gamma < 7$ within the self-accelerated $e^+ e^- p$ plasma that penetrates the SN ejecta, with the unshocked protons ahead the plasma expansion front, at rest inside the ejecta (see Figure 1). This situation occurs along directions of high density of SN ejecta leading to a baryon load parameter of order of 50 (see Figure 1 and, e.g., Figures 34, 35, and 37 in Ref. [24]).

2. Interaction of the protons with $\Gamma \sim 10^2$–$10^3$ engulfed in the self-accelerated $e^+ e^- p$ plasma in the direction of least baryon density around the newborn BH, with the protons at rest of the interstellar medium (ISM) (see Figure 1). We adopt the interaction with the ISM occurs at a distance $\sim 10^{16}$ cm from the system, as inferred from the time and value of $\Gamma$ at transparency (see e.g. Ref. [30] for details). This situation occurs along the directions of very-low density of SN ejecta leading to a baryon load parameter of order of $10^{-2}$–$10^{-3}$ (see Figure 1 and, e.g., Figures 34, 35 in Ref. [24]).

The scenario described above for the $pp$ interactions within the BdHN model of long GRBs is markedly different from previous works based on the traditional collapsar and the fireball model with relativistic shocks (see, e.g., Refs. [33–35], for details). In the latter, collimated jets accelerate protons and producing very energetic neutrinos from several hundreds of GeV to several TeV. In the BdHN model studied here, the plasma expands along all the directions, with a different value of the engulfed amount of matter from the SN ejecta, so with different energy of the secondary emerging particles. Therefore, in the scenario explored here there is no collimated jet, the protons are less energetic, and likewise the neutrinos that are produced which have energies of a few GeV.

We carry out the analysis of the neutrino from $pp$ interaction in the BdHN I scenario in detail in the following sections. In Section II, we describe the equations of the expansion of the $e^+ e^- \gamma$ plasma inside the SN ejecta expect the secondary emerging particles (Section II A) and compute the process of interaction during the initial stages of the expansion, namely the interaction 1). We describe how, from these simulations, we got the physical quantities necessary to compute the particles spectra (Section II B). We show the different neutrino spectra emerging from the interaction (Section II C). We consider for the cross-section of $pp$ inelastic scattering the parameterization of Blattng et al. [36]. We assume a monochromatic proton energy distribution derived from the value of the proton Lorentz factor at every radius of the shell expansion. In Section III, we focus on the second type of interaction, the case 2). Since the protons energies are greater than in the case 1), i.e. $E_p \sim 1$ TeV, we use the parameterization by Kelner et al. [37], both for the $pp$ cross-section and the emerging particle spectra, that is appropriate for protons energies until $10^5$ TeV. Finally, we discuss and summarize in Section IV the main results of this work and we try to give an estimate for the detection of the produced neutrino and gamma-rays.
Figure 1. Schematic figure of the \(pp\) interactions occurring in a BdHN. The interactions 1) and 2) as described in the text: 1) the \(e^+e^-\) plasma propagates in a fixed direction with high baryon load, e.g. \(B = 51.75\), reaching Lorentz factor of up to \(\Gamma \lesssim 7\) in their travel inside the ejecta. The engulfed protons have such \(\Gamma\) (see Section II B for details) and interact with the protons at rest, ahead of the plasma front, and deposited all of their energy. The dotted circular line represents the \(\nu\)NS-BH binary orbit; 2) protons engulfed by the \(e^+e^-\) plasma propagate in the direction where the cavity is open. This plasma is loaded with a relatively low baryon content (e.g. \(B \sim 10^{-3}\)), so the plasma reaches high Lorentz factor at transparency, \(\Gamma \sim 10^2 – 10^3\). The engulfed protons have such \(\Gamma\) factor and interact with the ISM protons at rest. We emphasize that the homogeneous and different colors of the shaded regions is just to highlight the different densities in the two target regions and to explain schematically the dynamics along the two directions where we follow the expansion of the plasma. Indeed, as explained in Section II B (see also Figure 2 in Ref. [12]), the distributions of matter in the ejecta is not uniform and the asymmetric evolution of the plasma inside the ejecta is direction dependent, with various values of baryon load along different directions of expansion.

II. PP INTERACTIONS INSIDE THE HIGH-DENSITY EJECTA

We analyze here the \(pp\) interaction that occurs when the \(e^+e^-\gamma\) plasma starts to engulf the baryons present in the SN ejecta, forming a \(e^+e^-\gamma p\) plasma (see Figure 1). These accelerated baryons interact with the target baryons ahead of them, still at rest, by the following process:

\[
p + p \rightarrow \Delta^{++} \rightarrow \begin{cases} p + n + \pi^+ \\ p + p + \pi^0 \\ p + p + \pi^+ + \pi^- \\ p + n + \pi^+ + \pi^+ + \pi^- \end{cases}
\]

with consequent decay of \(\pi^+ \rightarrow \mu^+ + \nu_\mu, \pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu, \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu\) and \(\pi^0 \rightarrow 2\gamma\). The different decay channels of the \(\Delta^{++}\) resonance depend on its energy. If it is a \(\Delta^{++}(1232)\), it decays mainly in a nucleon and a pion \((N + \pi)\) with branching ratio (B.R.)=99.4%; higher \(\Delta\) decays in a nucleon plus a pion \((N + \pi)\) with lower B.R. or in a \(\Delta(1232) + \pi\) with the consequent decay of the \(\Delta(1232)\) as before.

In order to study this phenomenon, we have performed relativistic hydrodynamic (RHD) simulations of the dynamics of the \(e^+e^-\) plasma expanding and swapping baryonic matter in the SN ejecta. We describe the RHD simulations in the next subsection. As we clarify below, the study of the expansion dynamics allows, within the BdHN scenario, to evaluate the Lorentz factor and energy of both the incident and target photons participating in the interactions.

A. RHD simulations

In this section, we summarize the equations that govern the expansion of the plasma inside the SN ejecta within the BdHN scenario. We have run RHD simulations (see Ref. [24], for additional details) performed with a one-dimensional implementation of the RHD mod-
uence of the PLUTO code [38]. Consequently, the code integrates a system of partial differential equations in distance and time. This allows to compute the evolution of the thermodynamical variables and dynamics of the $e^+e^−$ plasma which is carrying baryons from the ejecta, along one selected radial direction, at any time. We recall that the density of plasma is different along different directions (see point 4 below for details). The equations are those of ideal relativistic hydrodynamics (see Section 10 in Ref. [24]). The plasma and the set of equations satisfies the equation of state of an ideal relativistic gas with adiabatic index 4/3 (see Appendix B in Ref. [24]).

The simulation starts at the moment of BH formation, so the initial conditions are taken from the final configuration of the numerical simulations in Ref. [11]:

1. The SN remnant is obtained from the explosion of the CO$_{\text{core}}$ evolved from a zero-age main-sequence (ZAMS) star of mass $M_{\text{ZAMS}} = 30M_\odot$. This CO$_{\text{core}}$ has a total mass of 11.15 $M_\odot$, of which 2 $M_\odot$ conform the mass of the νNS (collapsed iron core) and 9.15 $M_\odot$ conform the total ejecta mass (envelope mass). At the SN explosion time, the ejecta profile follows a power-law profile $\rho \propto r^{-2.8}$ (see, e.g., Ref. [11]).

2. The orbital period is $P \approx 5$ min, i.e. a binary separation $a \approx 1.5 \times 10^{10}$ cm.

3. The ejecta have negligible pressure and is considered to be in homologous expansion, $v(r) \propto r$, spanning velocities from $10^8$ cm s$^{-1}$ of the innermost ejecta layer to $2 \times 10^9$ cm s$^{-1}$ of the outermost one. The velocity of the remnant is, however, not relevant in the dynamics of the $e^+e^-$ plasma since its velocity is much higher than the one of the remnant. Therefore, for practical purposes, the remnant can be considered at rest as seen from the plasma.

4. The baryon load of the $e^+e^-$ plasma is not isotropic: the baryon density is different along different directions; see, e.g., Figure 2 in Ref. [12]. According to the three-dimensional simulations of Ref. [11], the density profile of the ejecta along a given direction, at the BH formation time, decays with distance as a power-law, i.e. $\rho \propto (R_0 - r)^\alpha$ (see, e.g., Figures 34–35 of Ref. [24] that show the mass profiles along selected directions). The normalization, the constant $R_0$ and the parameter $2 < \alpha < 3$ depend on the emission angle, fixing a direction.

5. The total isotropic energy of the $e^+e^-$ plasma is set to $E_{e^+e^-} = 3.16 \times 10^{53}$ erg. Therefore, in the high density region, the baryon load parameter is $B = 51.75$, according to the above ejecta properties.

The evolution from these initial conditions leads to the formation of a shock and to its subsequent expansion until reaching the outermost part of the SN. The relevant region for the present work comprises the radial distances $\sim 10^9$–$10^{10}$ cm.

Since throughout this expansion baryons are continuously phagocytosed, the spectrum of the secondary particles, the proton energy distribution, and the baryon number density $n_B$ depend all on the radial position of the shock. Taking snapshots of this process, we obtain the relative spectrum for each secondary particle within a thin shell close to the shock. In the end, we integrate all these spectra over the radius to have an estimate of the released energy through the different channels.

Considering that the protons follow a Maxwell-Boltzmann energy distribution in the comoving frame, it can be seen that the energy distribution in the laboratory frame is peaked enough to be well-approximated by a delta-function. Hence, we consider a monochromatic proton energy distribution $J_p(E_p)$: namely, $J_p(E_p) \propto \delta(E_p - E_p^0)$. The value of $E_p^0$ depends on the Lorentz factor $\gamma(r)$. Due to momentum-energy conservation, $\gamma$ decreases rapidly with time. Therefore, we focus on the first stages of the expansion when protons have enough energy to interact. We estimate the interactions from a radius $r_i$ where the $\gamma e^+e^-$ plasma with engulfed protons has the maximum Lorentz factor, up to a final radius, $r_f$, over which the proton energy goes below the interaction threshold energy (see below). Despite the threshold radius is at $r_{th} = 4.79 \times 10^{10}$ cm, in order to have clear spectra with enough points, from our numerical simulation, we find that this region extends from $r_i = 9.59 \times 10^8$ cm to $r_f = 2.98 \times 10^{10}$ cm, so $\Delta r = r_f - r_i = 2.88 \times 10^{10}$ cm. This can be compared with the total extension of the SN ejecta that is of the order of $10^{12}$ cm [24].

In the following section, we describe how we extract from these simulations the physical quantities that we use to compute the particles spectra in Section II C: the protons Lorentz factor, the number density of the incident and target protons, each of them considered at every radius of the expansion of the shock inside the ejecta.

### B. Physical quantities for the $pp$ interaction

The baryons of the SN ejecta are incorporated time by time, at every radius, by the $e^±\gamma$ plasma. Therefore, the incident protons are the ones engulfed on the expanding shock front, which have the maximum $\gamma(r)$ factor. The target protons are the ones of the ejecta that has not been yet engulfed, and located closely to the shock front. These target protons can be safely assumed to be at rest with respect to the incident protons in the shock front. Having clarified this, we identify the physical quantities needed to calculate the spectra at each radius: the Lorentz factor of the protons in the shock front, $\gamma_p$, their energy, their density, $n_{sh}$, and the density of the unshocked protons, $n_t$. These quantities change at every radius as the plasma expands inside the ejecta. We refer to all quantities in the laboratory frame.

The procedure to compute the above quantities is as
follows. First, we obtain the position of the shock front from the simulation, i.e. $r_{\text{front}}$. This radius can be estimated as the radius at which the pressure inside the SN ejecta falls off abruptly since it separates the shocked and unshocked regions. It corresponds also to the radius at which the protons of the shock have their maximum Lorentz factor. Although the pressure at $r > r_{\text{front}}$ falls down fast, the extension of this region is smaller than the mean-free-path of the front protons, $\lambda_p$, defined by $\lambda_p^{-1} = \sigma_{pp}(E_p) \times n_p$, where $\sigma_{pp}$ is the $pp$ cross-section and $n_p$ their number density. Thus, in order to calculate the density of the incident and target protons, we make an average process to consider all the possible interacting protons at a given time. For the incident protons density, $\langle n_{\text{sh}} \rangle$, we average the radial density in the region $r_{\text{front}} - \lambda_p < r < r_{\text{front}}$. We can consider it as the incident density at an average radius $\langle r_{\text{front}} \rangle$. A similar averaging process is applied ahead the front, i.e. in the region $r_{\text{front}} < r < r_{\text{front}} + \lambda_p$, to obtain the density of the target protons, $\langle n_t \rangle$.

Then, we calculate the maximum value of $\gamma_p$ inside the shell and, correspondingly, the energy of the protons, $E_p(r) = \gamma_p(r) m_p c^2$. The protons Lorentz factor $\gamma_p$, at the generic radius $r_k$, is given by the value of the baryons velocity, $\beta_k$, at the shock front position $\langle r_{\text{front}} \rangle$. The profile of the maximum values of the Lorentz factor, at every front radius, is shown in Figure 2. We emphasize that each point in the curve of $\gamma_p$ in Figure 2 corresponds to the maximum value of the Lorentz factor in the front of the shell. The numerical simulations of the expansion of the plasma inside the ejecta give us a distribution of the particles velocity (see Figure 37 in Ref. [24]). From this velocity distribution, we extract the maximum Lorentz factor, consistently with the density average process explained above (see the peaks of the shell front in the same Figure 37 of Ref. [24]). From the discussion above, we recognize that the maximum protons Lorentz factor, in Figure 2, corresponds to the Lorentz factor of the shell bulk motion $\Gamma$, namely $\gamma_p = \Gamma(\langle r_{\text{front}} \rangle)$. The correspondence between the maximum protons Lorentz factor and the Lorentz factor of the shell bulk motion is justified mainly by two conditions. First, the expansion of the plasma is driven by the hydrodynamic equations of a self-accelerated, ideal relativistic gas (see Section 10 in Ref. [24]). Then, the particles engulfed by the expanding front shell, at each radius, are accelerated by the bulk motion itself. Secondly, since the region under consideration for the neutrino production (namely the region where the accelerated protons have enough energy to create the $\Delta$ resonance) is relatively small (see Figure 2 and below), which corresponds to a time interval of $\sim 1$ s (see Figure 10). Consequently, due to this short time interval, the self-acceleration of the expanding plasma and its bulk motion accelerate and drive the motion of the engulfed particles at every time.

From the above, we can see that the energy of protons is in the range $1.24 \leq E_p \leq 6.14$ GeV, which is high enough to produce secondary particles. The proton energy threshold to produce pions in the final state is, for the interaction $pp \rightarrow pn\pi^+$, $E_{p,\text{Th}} = 1228$ MeV and, for $pp \rightarrow pp\pi^0$, $E_{p,\text{Th}} = 1217$ MeV. The neutrino production at these low energies is dominated by the protons with the highest energy ($\gamma \sim 6$).

Figure 2 also shows four vertical lines at fixed front radii of reference: the first vertical line corresponds to the radius $r_1$ where the protons have their maximum energy, while the last line corresponds to the radius $r_f$ (there $\gamma_p = 1.878$). The intermediate radii have been chosen only the show the evolution of the particles spectrum during the expansion. In the following, we compute the particles spectra at these four specific radii (different values of the radii will be explicitly indicated).

In the region of the expansion of the shock, at every radius, the average number density of the target protons $\langle n_t \rangle$ in the remnant varies between $8 \times 10^{23}$ cm$^{-3}$ at $r_1$ to $\sim 5 \times 10^{23}$ cm$^{-3}$ at $r_{\text{end}} = 5.51 \times 10^{10}$ cm (the end point of the simulation). The protons number density at the front of the expanding shell, $\langle n_{\text{sh}} \rangle$, does not vary much either; it is in the range $(0.5 - 9) \times 10^{25}$ cm$^{-3}$ (the maximum value occurs in the region close to the initial radius $r_1$, and the lower value to the final radius $r_{\text{end}}$), as shown in Figure 3. Even in this last figure, the density is plotted as a function of the front radius (consistently with Figure 2). The decrease of $\langle n_{\text{sh}} \rangle$ can be explained by the following considerations. From the values of $n_t$ above, we see that, due to the geometry of the system (see Figure 1) the target density decreases moving towards the outermost regions. Then, during its expansion, time by time, the plasma in-
corporates less matter. Moreover, at the beginning of the expansion, since the target density is higher, the Lorentz factor of the protons starts to drop down. Now, remembering how we calculate the front number density (and, then, its definition) as we explained above, namely as a punctual density in a region close to the front of the expanding shell, we understand that the combination of the two factors of a lower plasma expansion rate and lower target density leads to a decrease of shell number density.

Having derived all the necessary physical quantities, we proceed next to the calculation of the spectra of the emerging particles.

C. Particles spectra

We turn now to the spectra for the emerging particles from the decay of the $\pi$ and the $\mu$. We need to consider the fact that the $\mu$ can be unpolarized and polarized.

In order to obtain the pions production rate, we have used the parameterization for the pion production cross-section presented in Ref. [36]. In their work, they provide a useful formula for the production of the three types of pions ($\pi^0, \pi^+, \pi^-$) as a function of the pion and incident proton energy, $d\sigma(E_\pi, E_p)/dE_\pi$, in two ranges of incident protons kinetic energy in the laboratory frame $T_{p,lab}$: $0.3 \leq T_{p,lab} \leq 2$ GeV and $2 \leq T_{p,lab} \leq 50$ GeV.

The parameterization of the cross-section in Ref. [36] is appropriate for our calculations since it is accurate in the energy region of the present interest, namely $E_p < 7$ GeV.

Then the pions production rate can be computed as

$$Q_\pi(E_\pi) = c n_p \int_{E_\pi}^{E_{p,\text{max}}} J_p(E_p) \frac{d\sigma(E_\pi, E_p)}{dE_\pi} dE_p,$$

where $J_p(E_p)$ is the proton energy distribution, $n_p$ the number density of the target protons in the remnant, $c$ the speed of light and $E_{p,\text{max}}$ is the maximum energy of the protons in the system. Since we consider a fixed value for the proton energy, $E_p$, at the front of each spherical shell, we assume $J_p(E_p) = A \delta(E_p - E_p^0)$, where $A$ is the baryon number density at front of the shell.

With this $J_p(E_p)$, the equation for the production rate $Q_\pi$ becomes

$$Q_\pi(E_\pi) = c n_p A \frac{d\sigma(E_\pi, E_p^0)}{dE_\pi} \theta(E_p^0 - E_\pi) \theta(E_p^\text{max} - E_p^0).$$

With Eq. (3) for the $\pi$ production rate, we can compute the spectra for all the particles. Because the cross-section for neutral, negative and positive pions are different, we need to distinguish between emerging particles from $\pi^0$ decay in two photons, $\pi^- \to \nu_\mu + \bar{\nu}_\mu$ and $\pi^- \to e^- + \nu_e + \nu_\mu$ and from $\pi^+$ decay: $\pi^+ \to \mu^+ + \nu_\mu$ and $\pi^+ \to e^+ + \nu_e + \nu_\mu$ decay. In each of the following paragraphs will be shown the spectra for each specific particle from the three mesons.

We denote the spectrum of the produced particle $a$ as $\Phi_a = dN_a/dE_a$, where we indicate with $N_a$ the particle number density per unit time.

Throughout the article we denote as $\nu_{\mu(1)}$ the muonic neutrino/antineutrino from the direct pion decay, $\pi \to \mu \nu$, and $\nu_{\mu(2)}$ the neutrino/antineutrino from the consequent muon decay, $\mu \to e \nu_{\mu} \bar{\nu}_{\mu}$.

$\gamma$ spectrum

The spectrum of photons emerging from $\pi^0$ decay is given by

$$\Phi_{\pi^0 \to \gamma \gamma}(E_\gamma) = 2 \int_{E_{\pi,\text{min}}}^{E_{\pi,\text{max}}} Q_\pi(E_\pi) \sqrt{\frac{E_\pi^2 - m_\pi^2 c^4}{E_\pi}} dE_\pi,$$

where $E_{\pi,\text{min}} = E_\gamma + m_\pi^2 c^2/(4E_\pi)$ can be derived by the kinematics of the process. The factor 2 takes into account the two produced photons, while $Q_\pi(E_\pi)$ is given by Eq. (3), with the corresponding pion spectral distribution for $\pi^0$ (see Ref. [36]).

The photon emissivity (in erg/cm$^3$/s) is shown in Figure 4, and the total energy (in erg), integrated over all photons energies and calculated via Eq. (17) (see later Section II D), in the emissivity region, is given in Table I.

$\nu_{\mu(1)}$ spectrum

The spectrum of neutrino from direct pion decay $\pi \to \mu \nu$ can be calculated as follow

$$\Phi_{\pi \to \mu \nu}(E_{\nu_\mu}) = \frac{1}{\lambda} \int_{E_{\pi,\text{min}}}^{E_{\pi,\text{max}}} \frac{Q_\pi(E_\pi) \theta(\lambda - E_{\nu_\mu})}{\sqrt{E_\pi^2 - m_\pi^2 c^4}} dE_\pi,$$

(5)
where the values of $E_{\pi}^{\text{max}}$ and $E_{\pi}^{\text{min}} (E_{\nu}) = E_{\nu}/\lambda + (\lambda/4) \left( m_{\pi}^2 c^4/E_{\nu} \right)$, are derived from the kinematic of the process: $\lambda = 1 - r$, with $r = (m_{\mu}/m_{\pi})^2$, is the maximum energy fraction that the neutrino emerging from the direct decay can take from the pion.

The value of $E_{\pi}^{\text{max}}$ depends on $E_{\nu}$, hence it changes at every radius of integration in the expansion of the shell inside the ejecta. In order to derive $E_{\pi}^{\text{max}}$, we need to move to the centre-of-mass (CM) frame of the interaction. The invariant mass is

$$s(E_{\nu}) = \sqrt{2m_{\pi}c^2 + 2E_{\nu}m_{\mu}c^2},$$

the Lorentz factor of the CM is

$$\gamma_{\text{CM}}(E_{\nu}) = \frac{E_{\nu} + m_{\mu}c^2}{\sqrt{s}},$$

and its momentum

$$p_{\nu}^{\text{CM}}(E_{\nu}) = \frac{1}{2\sqrt{s(E_{\nu})}} \left[ \left( s(E_{\nu}) - (m_{\pi}c^2 - 2m_{\mu}c^2)^2 \right) \times \left( s(E_{\nu}) - (m_{\pi}c^2 + 2m_{\mu}c^2)^2 \right) \right]^{1/2}.$$  \hspace{1cm} (7)

The pion energy in the CM frame is $E_{\pi}^{\text{CM}} = \sqrt{p_{\nu}^{2} + (m_{\pi}c^2)^2}$. Then, the maximum pion energy $E_{\pi}^{\text{max}} (E_{\nu})$ is

$$E_{\pi}^{\text{max}} (E_{\nu}) = \gamma_{\text{CM}}(E_{\nu}) \left[ E_{\pi}^{\text{CM}}(E_{\nu}) + \beta_{\text{CM}}(E_{\nu}) p_{\nu}^{\text{CM}}(E_{\nu}) \right].$$  \hspace{1cm} (8)

The spectra derived by Eq. (5) must be calculated via Eq. (2) using the parameterization of the cross-section for $\pi^{-}$: $d\sigma_{\pi^{-}}(E_{\pi}; E_{\nu})/dE_{\pi}$, and for $\pi^{+}$: $d\sigma_{\pi^{+}}(E_{\pi}; E_{\nu})/dE_{\pi}$, given in Ref. [36]. The $\nu_{\mu(\tau)}$ emissivities (in erg/cm$^3$/s), for both mesons, are shown in Figure 5. The total energy (erg), integrated over the whole region of emissivity, is given in Table I.
**νµ(2) and νe spectra**

The neutrino spectra from the decay chain $\pi \rightarrow \mu \rightarrow \nu$ can be calculated as:

$$
\Phi_{\pi \rightarrow \mu \rightarrow \nu} (E_{\nu}) = \int_{E_{\min} (E_{\nu})}^{E_{\max} (E_{\nu})} \frac{Q_\pi (E_{\pi})}{\sqrt{E_{\pi}^2 - m_\mu^2 c^4}} g \left( \frac{E_{\nu}}{E_{\pi}} \right) dE_{\pi}.
$$

Here the pion production rate $Q_\pi$ is given in Eq. (3). The functions $g(z)$ represent the $\nu$ spectra after the decay chain ($\pi \rightarrow \mu \rightarrow \nu$). We use the ultrarelativistic limit ($\beta_{\pi} \rightarrow 1$, $\beta_{\mu} \rightarrow 1$), presented in Appendix A3 of Ref. [39].

The function $g(z)$ can be decomposed as the sum of an unpolarized spectrum, $g^0(z)$, plus a polarized one, $g^{pol}(z)$, $g(z) = g^0(z) + g^{pol}(z)$. The functions $G(z)$, polarized and unpolarized, can be found in the Appendix of Ref. [39]. Here the limit $\beta_{\pi} \rightarrow 1$ is well satisfied. Indeed, from the kinematics, we obtain that the Lorentz factor of the pions lies in the range $4.5 \leq \gamma_{\pi} \leq 34.5$.

**a. Without polarization.** In order to have an expression for the spectrum of the particles coming from the $\mu$-decay, we have to insert Eq. (3) into Eq. (9). The $G(z)$ equations for $\nu_{\mu(2)}$ and for $\nu_{e}$ (for unpolarized muon) are given in Eqs. (102)-(103) of Appendix A.3 in Ref. [39]. The minimum integration value $E_{\min}^\text{min}$ derive from the kinematic and is the same for $\nu_{\mu}$ and $\nu_{e}$ $E_{\min}^\text{min} (E_{\nu}) = E_{\nu} + m_\mu^2 c^4 / (4 E_{\nu})$.

The emissivities for $\nu_{\mu(2)}$ and $\nu_{e}$ are shown, respectively, in Figure 6 (for the particles from $\pi^-$ decay) and Figure 7 (for the particles from $\pi^+$ decay).

Let us consider now the case with polarization.

**b. Within polarization.** Since the muon can be polarized ($\mu^+$ has on average negative helicity and $\mu^-$ a positive helicity), the neutrinos produced by these muons depend on this polarization [39]. In order to get the spectrum of emerging particles, we need to add, to the unpolarized equations, similar functions of the polarized spectrum. The formula for the polarized spectrum are given in Eqs. (104)-(105) of Ref. [39].

The emissivity of $\nu_{\mu(2)}$ and $\nu_{e}$, from $\pi^-$ decay, are shown in Figure 8, while the ones for $\bar{\nu}_{\mu(2)}$ and $\nu_{e}$, from $\pi^+$ decay, are shown in Figure 9.

**D. Total luminosity and total energy release**

As we have seen from the above formulation we can obtain the particle spectra at every radius $r_i$, which we denote hereafter as $\Phi_{a}^i (E_{a})$. Thus, the particle emissivity at every radius, $\epsilon_{a}^i$, is given by

$$
\epsilon_{a}^i = \int_{0.3 \text{ GeV}}^{E_{\pi}^\text{max}} \Phi_{a}^i (E_{a}) E_{a} dE_{a},
$$

where $E_{\pi}^\text{max}$ is the maximum pion energy derived from the kinematic of the process.

Then, the power ("luminosity") emitted in particles of type $a$, at the radius $r_i$, is

$$
L_{a}^i = \int_{V_i} \epsilon_{a}^i dV_i,
$$

where the integration is carried out over the volume $V_i$ of the emitting/interacting shell at the front position $r_i$.

The total emissivity and luminosity at the radius $r_i$ can be obtained as the sum of the contributions of all particles, i.e.:

$$
\epsilon_{\text{tot}}^i = \sum_{a} \epsilon_{a}^i, \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (12)
$$

$$
L_{\text{tot}}^i = \sum_{a} L_{a}^i, \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (13)
$$

The energy emitted in $a$-type particles is given by

$$
E_{a} = \int L_{a}^i (t) dt, \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (14)
$$

where the integration is carried out on the duration of the emission. Therefore, the total energy emitted in all...
the emission processes is

$$E = \sum_a E_a. \quad (15)$$

From the numerical simulation of the expanding shell inside the remnant, we know that the width of the shell is of the order of $\Delta r_{sh} \approx 3 \times 10^8$ cm. Since the mean free path of the interaction is much smaller than the shell width (see below), the interacting volume at the radius $i$ is, approximately, $V_i = 4 \pi r_i^2 \lambda_i$, where $\lambda_i$ is the mean-free path of the protons of energy $E_i$ in the shell front. The mean-free path is given by $\lambda_i = (\sigma_{\pi^{\pm, 0}} A)^{-1}$, where $A$ (as above) is the baryon number density at the front, and $\sigma_{\pi^{\pm, 0}}$ is the inclusive cross-section for $\pi^-$, $\pi^+$ and $\pi^0$ (see Section 3 in Ref. [36]). For $\pi^\pm$: $0.4 \leq \lambda_{\pi^\pm} \leq 11$ cm; for $\pi^0$: $1.18 \leq \lambda_{\pi^0} \leq 45$ cm; for $\pi^0$: $0.65 \leq \lambda_{\pi^0} \leq 50.4$ cm$^{-1}$.

\footnote{The mean-free path values, for each pion, are calculated at the initial and final radius showed in the several spectra in this section. Note that not all the spectra have the same final radius.}

Having defined this volume, we can calculate the luminosity $L_a$ at each radius following Eq. (11), i.e.

$$L_a^i \approx \epsilon_a V_i \approx \epsilon_a \times 4 \pi r_i^2 \lambda_i. \quad (16)$$

Figure 10 shows as an example the luminosity $L_a^i$ as a function of time, for $a = \bar{\nu}_\mu(2)$, within and without considering polarization effects.

Clearly, the luminosity is nonzero only in the region of the ejecta where $pp$ interaction leads to a nonzero production of secondary pions. Therefore, the emission occurs till the instant when the shell reaches the radius $r_f = r_4 \approx 4.79 \times 10^{10}$ cm, after which the proton energy is below the process energy threshold. Therefore, the emission time is very quick, from a fraction of a second to $\sim 1$ second. Indeed, the total precise emission time-interval is $\leq 1.5$ s, as it can be seen from Figure 10. The luminosity of each particle is very high (especially for the photons), but this does not violates the energy conservation (as can be seen in Table. I) since the majority of the emission occurs in the first fractions of second. For the photons this emission can be recognized as an initial spike in the spectrum (if we have a good resolu-
We can now estimate the total energy emitted in each particle type via Eq. (14). The time interval of the emission, $\Delta t$, is the time the shell spends to cover the distance between $r_{i-1}$ and $r_i$ with the velocity $\beta_i$, $\Delta t_i \approx \Delta r_i / (c \beta_i)$, namely

$$\mathcal{E}_a \approx \sum_{i=2}^{n} L^i_a \times \Delta t_i = \sum_{i=2}^{n} L^i_a \times \frac{\Delta r_i}{c \beta_i}, \quad (17)$$

where $L_a^i$ is given by Eq. (16). The total energy emitted in every particle type, in the whole emitting region, is summarized in Table I.

### III. TEV PROTONS INTERACTING WITH THE ISM

We now consider the interaction of incident protons engulfed by plasma of $\gamma e^{\pm}$ in the direction of the circumburst medium (CBM) of low baryon load $B < 10^{-2}$, with target protons of the ISM. Thus, the number density of the target is $n_{\text{ISM}} \sim 1 \text{ cm}^{-3}$. This expanding plasma reaches transparency far away from the BH site, with ultra-relativistic Lorentz factor of up to $\gamma_p = \Gamma \sim 10^3$. Therefore, we adopt here that the incident protons have energies $\sim 1 \text{ TeV}$.

For an isotropic energy of the plasma $E_{e^{\pm}} \sim 10^{53} \text{ erg}$ (see, e.g., Table 7 in Ref. [40] for examples of the energy released by BdHNe I), and a baryon load $B = 10^{-3}$, the total number of protons is given by

$$N_p = \frac{BE_{e^{\pm}}}{m_p c^2} = 6.65 \times 10^{52}. \quad (18)$$

We consider that the interaction with the ISM occurs in a spherical shell that we locate at a distance of between $10^{16} \leq r \leq 10^{17} \text{ cm}$ from the BH site (see e.g. Ref. [30]). In order to make the computation and obtain the spec-
A. Second approach

In this section, since we are going to work now with high-energetic \((E_p \geq 1 \text{ TeV})\) protons, we cannot use the same parameterization for the cross-section of inelastic \(pp\) interaction presented in Ref. [36] and used here for the high-density case. Indeed, the energy range of validity for that parameterization is between \((0.3-50) \text{ GeV}\); too low for our protons.

For these reasons, we now follow the approach described in Ref. [37] by Kelner et al., for the determination of the interaction cross-section and the spectra of the emerging particles. In this paper, they study the \(pp\) interaction using the \(SIBYLL\) [41] and \(QGSJET\) [42] codes. They divide their studies in two energy region:

1. for \(E_p \geq 0.1 \text{ TeV}\) and \(x = E_a/E_p \geq 10^{-3}\) (where \(E_a\) is the energy of the secondary product), they construct an analytical parametrization for the spectra of secondary particles, emerging from the decays of the \(\pi\) mesons and the \(\mu\) leptons, and an analytic formula for the energy distribution of pions (considering different fixed interacting proton energies);

2. for \(E_p \leq 0.1 \text{ TeV}\), they consider a different proton energy distribution, that covers a wide energy range, and develop a method, based on the so called "d–functional approximation" (see Ref. [43]), to integrate the equations and obtain the spectrum of the specific particle, until the proton energy threshold for the production of \(\pi\) is reached.

Since we are working with protons energies \(\geq 0.1 \text{ TeV}\), we focus only to the first method. Following Ref. [37], we denote the energy distribution of protons as \(J_p(E_p)\), in units \(\text{cm}^{-3} \text{ TeV}^{-1}\), which gives the number of protons per unit volume in the energy range between \(E_p\) and \(E_p + dE_p\). The secondary particles production rate in the energy interval \((E_a, E_a + dE_a)\), \(\Phi_a(E_a) = dN_a/dE_a\), for \(E_p \geq 0.1 \text{ TeV}\), is given by

\[
\Phi_a(E_a) = c n_p \int_{E_a}^{\infty} \sigma_{pp}^{\text{inel}}(E_p) J_p(E_p) F_a(x, E_p) \frac{dE_p}{E_p},
\]

where \(n_p\) is the density of the target protons (we assume it is \(1 \text{ particles}/\text{cm}^3\)), \(\sigma_{pp}^{\text{inel}}(E_p)\) is the inelastic \(pp\) cross-section, \(x \equiv E_a/E_p\), \(c\) the speed of light and \(F_a\) is the specific spectrum for the particle \(a\) that they derive, (and that we are going to use), with an accuracy better 10%.

The inelastic part of the total \(pp\) cross-section is given in Eq. (73) (for \(E_p > 0.1 \text{ TeV}\)) and Eq. (79) (for \(E_p \leq 0.1 \text{ TeV}\)) of Ref. [37]. In the parametrization of Ref. [37], it is considered both \(\pi^+\) and \(\pi^-\) derived by \(pp\) interaction, without distinguishing between electron and positron, as well as between neutrino and antineutrino. The reason for this is that the production of \(\pi^+\) exceeds only a little bit that of \(\pi^-\) and this effect is smaller than the accuracy of the approximations made in the analysis. Since now we follow their treatment, this implies that our calculations also include the contribution of antiparticles (e.g \(\pi^+\) and \(\pi^-\), \(\mu^+\) and \(\mu^-\) etc.).

1. Particles spectra

In order to get the emissivity of each specific particle mean Eq. (19), we need to specify our protons energy distribution \(J_p(E_p)\). We are considering only protons with fixed energy, then we can write it as \(J_p(E_p) = A \delta (E_p - E_p^0)\), where \(E_p^0\) is our proton fixed energy \((E_p^0 = 1 \text{ TeV})\). The constant \(A\) is the ratio between the number of interacting protons in the considered volume: \(A = N_p/V\) (in particle/cm\(^3\)). The volume is calculated as \(V = 4\pi (r_1^2 - r_2^2)/3\), with \(r_1 = 10^{16} \text{ cm}\) and \(r_2 = 10^{17} \text{ cm}\), instead the number of protons is derived in Eq. (18).

Since we are working at high energies, in the lower limit of the integral in Eq. (19) we can put easily \(E_{\gamma, 1} = E_{\nu_e(1)}, E_{\nu_{\mu(2)}}, E_{\nu_\tau}\), instead of the limits used in Section II C.

For this interaction, it is meaningless to talk about “total luminosity” since the protons spend the time \(\Delta t = \Delta r/c\beta = 3 \times 10^6 \text{ seconds}\) (with \(\beta \approx 1\) because \(\gamma_p = 10^3\)) to cross the entire ISM region of width \(\Delta r = 9 \times 10^{16} \text{ cm}\). Since for the calculation of the horizon distance for neutrino (see Section IV) we need the luminosity, this can be obtained considering only the last interaction of the accelerated protons with the target protons of the ISM, namely the ISM shell between \(r_* = 10^{17} - \delta s \text{ cm}\) and \(r_2 = 10^{17} \text{ cm}\), where \(\delta s = 3 \times 10^{10} \text{ cm}\) is the distance covered by the accelerated protons in 1 second. Thus, the luminosity of the last emitting shell is given by \(L_a = \epsilon_a \times \Delta V_{\text{last shell}}\), where \(\epsilon_a\) the emissivity of the particle \(a\) calculated by Eq. (10) and \(\Delta V_{\text{last shell}} = 4/3 \pi (r_1^3 - r_2^3)\). The total emitted energy, for each particle \(a\), can be calculated as

\[
\mathcal{E}_{\text{tot, } a} = \Delta t \sum_i \epsilon_a^i \times \Delta V_i,
\]

where \(\Delta V_i = 4/3 \pi (r_i^3 - r_{i-1}^3)\), with \(r_i = r_{i-1} + \delta s\). Since \(\epsilon_a^i\) does not depend on the radius, thus \(\epsilon_a^i = \epsilon_a\), the total emitted energy can be get easily by

\[
\mathcal{E}_{\text{tot, } a} = \epsilon_a \Delta t \Delta V,
\]

with \(\Delta V = 4/3 \pi (r_1^3 - r_2^3)\).

\(\gamma\) from \(\pi^0\)

The photon emissivity from \(\pi^0\) decay is given by Eq. (19), with \(F_{\gamma}(x, E_p)\) derived by Ref. [37]. We need to emphasize that, in the analysis of Ref. [37], for the
parameterization of the photon spectrum, they consider also the photons produced by the different decay channels of $\gamma$ mesons.

The photons emissivity is shown in Figure 11. The total energy emitted through photons in all emitting region via Eq. (21) is $E_{\gamma} = 5.41 \times 10^{43}$ erg. The luminosity emitted in the last emitting shell of the ISM region, calculated as explained above, is $L_{\gamma} = 1.01 \times 10^{43}$ erg s$^{-1}$. Instead, the maximum photon energy corresponding to the maximum value of $E^2 dN_{\gamma}/dE$ is $E_{\gamma} = 91.62$ GeV.

$\nu_\mu$ from direct $\pi$ decay

The muon neutrino from direct pion decay ($\pi \rightarrow \mu \nu_\mu$) is given by the same Eq. (19), with $F_{\nu_\mu} = (E_{\nu_\mu}/E_p) - E_p$ derived in Ref. [37]. The respective emissivity is shown in Figure 12.

As one can see in Figure 12, the spectrum has a sharp cut-off at $x = 0.427$. This effect is due to the kinematics of the process since, at high energy, this neutrino can take only a factor $\lambda = 1 - r_{\pi} = 0.427$ of the pion energy. The total energy emitted in $\nu_\mu(1)$ inside the whole emitting region (calculated as before) is $E_{\nu_\mu(1)} = 1.60 \times 10^{43}$ erg, while the luminosity in the last emitting shell is $L_{\nu_\mu(1)} = 3.01 \times 10^{42}$ erg s$^{-1}$, whereas, the spectrum reaches its maximum value at the neutrino energy of $E_{\nu_\mu(1)} = 44.72$ GeV.

$\nu_\mu(2)$ and $\nu_e$ from muon decay

The muon neutrino luminosity from muon decay can be calculated from Eq. (19), with the specific $F_a = (x, E_p)$, for each particle derived in Ref. [37]. The spectrum of $\nu_\mu(2)$ and $\nu_e$ can be represented by the same function (with an accuracy less than 5% for $\nu_e$). The emissivity is shown in Figure 13. Differently from the $\nu_\mu(1)$, the energy of these particle can reach at maximum the energy of the $\mu$, which one has no constrain to reach a fixed specific quantities of the pion energy ($E_{\pi}^{\text{max}} = E_{\pi}$).

Comparing these plots with the one for the same particles obtained in the previous section, we can deduce the relevant differences. The first is in the emissivity. Between the two approaches there are 46 orders of magnitude of difference. This is principally because of the different values of the involved densities, since the physical conditions of the two interactions are quite different. Indeed we have:

1. in the first case, the constants $A$ and $n_p$ (respectively, the number density of the interacting and the target particles) are: $A \sim 10^{25}$ cm$^{-3}$ and $n_p \sim 10^{24}$ cm$^{-3}$. Then their product is of the order

2. in the second case, the same constants are $A \sim 10^{22}$ cm$^{-3}$ and $n_p \sim 10^{24}$ cm$^{-3}$. Then their product is much smaller.
of $10^{48}$ cm$^{-6}$.

2. in the second case, the same constants assume the values: $A \approx 15$ cm$^{-3}$ and $n_p = 1$ cm$^{-3}$. Then their product is of the order of 15 cm$^{-6}$.

We then deduce that the difference in the emissivity is due to density effects. Another difference between the two cases resides in the two types of cross-sections. Figure 14 shows the total inelastic cross-sections used in this paper. The cross-sections ($\sigma_{\pi^0}, \sigma_{\pi^+}, \sigma_{\pi^-}$) correspond to the energy integrated differential cross-sections used in section II, while $\sigma_{\text{inel}}$ to the one used in this section (see Ref. [37]). The order of magnitude of the two types of parameterization is almost the same: the [37] one dominate at $E_p \leq 10$ GeV, instead the [36] one at $E_p \geq 6$ GeV.  

We need also to note that the parameterization of Kelner et al. [37] takes into account the polarization effects and they do not distinguish between $\pi^-$ and $\pi^+$ or $\mu^-$ and $\mu^+$ (and, consequently, for the other particles and their antiparticles).

The total energy emitted in $\nu_{\mu}(2)$ in the whole emitting region results $E_{\nu_{\mu}(2)} = 1.98 \times 10^{43}$ erg, while the luminosity of the last emitting shell is $L_{\nu_{\mu}(2)} = 3.71 \times 10^{42}$ erg s$^{-1}$. The maximum of the spectrum is reached at the energy $E_{\nu_{\mu}(2)} = 63.9$ GeV.

IV. SUMMARY, DISCUSSION AND CONCLUSIONS

In this work, we have computed the photon and neutrino production via $pp$ interactions occurring within the BdHNe I scenario for energetic long GRBs, which we have recalled in Section I. From the dynamics of the BdHN I, it follows that the SN ejecta, due to the accretion and to the BH formation, becomes highly asymmetric around the newborn BH site. Therefore, the $e^\pm$ plasma created in the BH formation process, during its isotropic expansion and self-acceleration, engulfs different amounts of matter of the surrounding SN ejecta, depending on the direction (see Figure 1). This asymmetry leads to a direction-dependent Lorentz factor for the engulfed protons in the expanding shell. We followed the evolution of the expanding plasma inside the ejecta along a specific direction characterized by a fixed baryon load, $B$. Although the baryon load along other directions is different, the plasma follows the same dynamics, leading to analogous results but with a rescaled Lorentz factor of the bulk motion, the density of the matter inside the ejecta and, correspondingly, the emerging neutrino energy. The whole evolution of the expansion of the leptonic, first, and baryon-lepton, after, plasma is studied in [24].

From this scheme, we have here studied two different types of physical set-up for $pp$ interactions that cover the generality of the system. In the first part of the article, see Section II, we studied the $pp$ interactions inside the SN ejecta. For a quantitative estimate, we have adopted numerical values from recent hydrodynamical simulations [24] achieved through an implementation of the PLUTO code [38, 44]. The equations and the initial conditions for the integrations are summarized in Section II A. The $\gamma e^\pm$ expanding plasma engulfs protons of the ejecta with a baryon load parameter $B = 51.75$ and accelerates them up to energies of $\sim 7$ GeV ($\gamma_p \leq 7$). This region is characterized by a number of target protons (at rest) of $n_{\text{remn}} \sim 10^{23}$ cm$^{-3}$, while the number density of the front of the shell, in the whole considered region, is almost constant, $n_{\text{sh}} \approx 10^{25}$ cm$^{-3}$.

In order to calculate the emissivity of the emerging particles, and because of the low energetic interacting protons, we have used for this case the parametrization of the differential pion production cross-section presented in Ref. [36]. The obtained spectra show that, from this high-density region, the neutrinos and photons have energies $E_{\nu_{\mu}(1)} \leq 2$ GeV and $E_{\gamma}, E_{\nu_{\mu}(2)}, E_{\nu_e} < 5$ GeV. We have calculated that in the high-density region, the particle production occurs in the first $\sim 1.5$ s of the shell expansion (see Figure 10). At later times, the energy of the protons in the shell is below the threshold for any $pp$ interaction with the target protons in the remnant. From this plot, we see that the luminosity emitted in $\nu$ is very high $L_{\nu} = (2-3) \times 10^{51}$ erg s$^{-1}$ and, since the emission occurs in $1.5$ s, the associated total integrated energy is $\sim 10^{79} - 10^{81}$ erg (see Table I for the total energy of each particle). Summing up the total energy released in all the secondary $\nu$, we obtain that it corresponds to $3\%$ of the initial plasma energy, $E_{\nu e^\pm}$, while $14\%$ of the latter is carried off by photons.

In the calculation of the neutrino spectrum for the high-density region case, we have also considered the pos-
sibility that the polarization of the $\mu^\pm$ could affect the spectrum of the emerging $\nu$. Comparing the spectra in Figures. 6–7 with the ones in Figures. 8–9, we have found that the neutrinos emerging from the $\mu$ decay are not affected by the polarization of the parent $\mu$ or, at least, the polarization does not affect appreciably the final neutrino spectrum.

In the second part of the paper, see Section III, we have considered the expansion of the shell in the direction of low baryon load, where we adopted $B = 10^{-3}$ (see, e.g., Refs. [7, 24]). Here, the expanding $\gamma e\gamma$ plasma engulfs a small quantity of baryons in the cavity around the BH [23], allowing a self-acceleration that brings the engulfed protons to energies of up to $E_p \sim 1$ TeV ($\gamma_p \sim 10^3$). In order to obtain the final emissivity, here we use the parametrization of the cross-section and of the emerging particles spectra presented in Ref. [37]. In this case, we obtained a wider range of particles energies $1 \leq E_p \leq 10^3$ GeV (depends on different particles), with associated total luminosity of $L_\nu = 1.0135 \times 10^{45}$ erg s$^{-1}$, $L_{\nu(1)} = 3.01 \times 10^{42}$ erg s$^{-1}$, and $L_{\nu(2)} = 3.71 \times 10^{42}$ erg s$^{-1}$.

We found that the secondary particles spectra follow approximately a cutoff power-law function (see, for example, Figure 5 or 12), with spectral index $1 \leq \alpha < 3$ (depending on the considered particle and interaction). The power-law term usually derives from the spectral index of the primary interacting protons (see, for example, Ref. [45]). But, since we have considered a spherical expansion of the photon-lepton-baryon shell, and a fixed proton energy distribution (at each radius of the expansion), we deduce that the power-law term is intrinsic in the considered process. The exponential decay is explained by the kinematic of the process, since only a fraction of the initial incident proton energy is taken by the secondary neutrino (both from direct pion decay and muon decay). To be more specific, a fraction of the parent proton energy is taken by the pion; for the direct pion decay, the $\nu(1)$ can take, at maximum, a fraction $\lambda$ of the pion energy, while the muon can take, at maximum, the entire pion energy.

A precise estimate of the detection probability of these neutrinos is out of the scope of the present work and we plan to addressed it elsewhere. However, on the basis of the present results, we can express some considerations for the Earth’s neutrino detectors.

In general, the cosmological distances at which GRB occur make the neutrino detection very challenging because of the very low neutrino flux arriving to the Earth. As we shall show below, it is indeed the distance to the source the main problem for the current detection probability.

For the low energy neutrinos ($E_\nu \leq 2$ GeV, at the production site, i.e. in the source frame) coming from the high-density region, there are additional considerations:

- A lower energy neutrino has a lower probability to interact with a nucleon ($N$) (proton or neutron) in the detector material via the reaction $\nu + N \rightarrow \mu + N'$ (where $N'$ is another nucleon). Indeed, the cross-section for this reaction is $\sigma_{\nu N} \sim 10^{-39}$ cm$^2$ (see for example Refs. [46, 47]), for these low energy neutrinos (see below). In addition, the arrival neutrino energy is redshifted by a factor $1 + z$ with respect to the energy in the source frame;

The high energetic neutrinos coming from the interaction in the low-density region could be, in principle, more easily detected than the previous case. At these energies ($E_\nu \leq 10^3$ GeV), there is no background noise from atmospheric neutrinos or solar neutrinos and the cross-section $\sigma_{\nu N}$ is higher, i.e. $\sigma_{\nu N} \sim 10^{-37}$ cm$^2$ (see, e.g. Refs. [46, 47]). The cross-sections in Refs. [46, 47] consider the total charged current cross-sections including quasi-elastic scattering ($\nu + N \rightarrow l + N'$, with $l$ a lepton), single meson $m$ production ($\nu + N \rightarrow l + N' + m$) and deep-inelastic scattering ($\nu + N \rightarrow l + N' +$ hadrons). However, as we have shown, in the low density interaction the resulting energy released is much less, making the detection of these neutrinos with higher energy even much more difficult than the ones produced in the high density interaction (see below).

We can obtain order-of-magnitude estimate of the probability of detection of these neutrinos. We focus our attention on three detectors: SuperKamiokande, Hyper-Kamiokande and IceCube. The two Kamiokande detectors explore a wide energy range for neutrino (from a few MeV up to 100 PeV). The IceCube detector works principally on high-energy neutrinos ($\gtrsim$ PeV), but the core of the experiment (the Deep Core Detector) works until to energies of a few GeV. The only characteristic of these detectors that we need in our estimation is the effective detection volume: 22 kton for the SuperKamiokande [48], 560 kton for the HyperKamiokande [49], while for the IceCube effective detection volume we adopt 10 Mton for the $\nu_\mu$ from the high density region, 20 Mton for the $\nu_{\mu(1)}$ from the low density region; 30 Mton for the $\nu_{\mu(2)}$ from the low density region (see Ref. [50]), since the energies of these two $\nu$ are different and the detector reacts depending on the neutrinos energies.

We now estimate the detection horizon for the neutrinos studied here. For this purpose, we use the best experimental conditions, i.e. we use the peak neutrino luminosity and the corresponding neutrino energy. The number of neutrinos per-unit-time and per-unit-area that arrive to the detector can be estimated as

$$\frac{d^2N_\nu}{dSdt} = \frac{\mathcal{E}_\nu}{4\pi D^2 E_\nu^4},$$

where $\mathcal{E}_\nu$ is the total energy emitted during 1 s in neutrinos of energy $E_\nu$, $D$ is the luminosity distance to the
source, \(E_{\nu}^* = E_{\nu}/(1+z)\) is the redshifted neutrino energy, and \(z\) the source cosmological redshift.

Therefore, we can obtain the number of detectable neutrino, \(N_{\nu}^{\text{det}}\), as the number of neutrino per unit of time and area that arrives to the detector, given by Eq. (22), times the cross-section for the neutrino-nucleon interaction, \(\sigma_{\nu N}\), times the number of probable interacting baryon in the detector, \(N_b^{\text{det}}\), times the integration time of the detector \(T_{\text{int}}\):

\[
N_{\nu}^{\text{det}} = \frac{d^2\sigma_{\nu N}}{dT d\Omega} \times T_{\text{int}} \times \sigma_{\nu N} \times N_b^{\text{det}},
\]

This can be estimated as the interacting mass inside the detector multiplied by the Avogadro number \(N_A\). For the present three detectors, we have:

1. for the SuperKamiokande detector \(N_b^{\text{det}} = (22.5\ \text{kton}) \times 6.022 \times 10^{23} = 1.35 \times 10^{34}\ \text{baryons};\)
2. for the HyperKamiokande detector \(N_b^{\text{det}} = (560\ \text{kton}) \times N_A = 3.37 \times 10^{35}\ \text{baryons};\)
3. for the Deep Core Detector of IceCube, the respective effective volume for the different neutrino energies are: \(N_b^{\text{det}} = (10\ \text{Mton}) \times N_A = 6.022 \times 10^{36}\ \text{baryons}; N_b^{\text{det}} = (20\ \text{Mton}) \times N_A = 1.2044 \times 10^{37}\ \text{baryons};\)

Using the above estimates of \(N_b^{\text{det}}\), mean Eq. (23) and the Hubble-Lemaître law, \(cz = H_0 D_h\) (with \(H_0 = 72\ \text{km s}^{-1}\ \text{Mpc}^{-1}\)), we can obtain the neutrino-detection horizon, \(D_h\), i.e. the luminosity distance to the source for which we have \(N_{\nu}^{\text{det}} = 1:\)

\[
D_h = \frac{KH_0}{2c} + \frac{1}{2} \sqrt{\frac{K^2H_0^2}{c^2} + 4K},
\]

where \(K = \varepsilon_{\nu} \sigma_{\nu N} T_{\text{int}} N_b^{\text{det}}/(12\pi E_{\nu}).\)

Table II summarizes the value of \(D_h\) for \(\nu_{\mu}^{(1)}\) and \(\nu_{\mu}^{(2)}\), in the case of both the high and low density regions, for the three considered detectors. We consider an integration time of the detector of \(T_{\text{int}} = 1\ \text{s}\), in agreement with the time-interval of the emission inside the ejecta (see Figure 10). For the high density region, we can assume that the total luminosity of each specific neutrino, \(L_{\nu}\), corresponds to the total energy calculated via Eq. (17), \(\varepsilon_{\nu}\), which are summarized in Table I, divided by 1 s. For the neutrino emerging from TeV protons (low density case), the crossing time of the entire emitting region is \(3 \times 10^6\ \text{s}\), so we calculate \(L_{\nu}\) emerging from the last emitting shell of the ISM region (as explained in Section III A 1), which values are listed, for each specific particle, in Section III A 1.

We obtain the neutrino energy \(E_{\nu}\) under the following considerations:

- for the high density region, we consider the neutrino with the higher value of the emissivity at the outer radius \(r_1\). Neutrinos produced at the inner radii \((r_1, r_2, r_3)\) are not considered because the high particle density in the ejecta and the higher value of the neutrino energy enhance the probability of interaction with the baryons in that region;

The approach that we have made using the Hubble-Lemaître law is valid. We have compared the results from this law with the one obtained from the full definition of the luminosity distance as a function of the redshift \(z\), \(D_L (z)\), which derive from the Friedmann equation (for a flat Universe). Inserting Eq. (22) into Eq. (23), from the latter deriving \(D_L (z)\) and equating it with the definition of \(D_L (z)\), we derived the redshift that practically coincides with the one from Eq. (24).

This analysis suggests that only the IceCube detector might detect the neutrinos here analyzed, i.e. neutrinos produced by \(pp\) interactions in the context of BdHNe I, especially the one coming from the high-density region, even if all of them have the right energy range of sensitivity to detect our neutrinos. One could ask follow a similar approach searching for the right detector conditions in order to get one detection for our neutrinos. If we consider \(N_b^{\text{det}}\) as the unknown variable, we can get the necessary number of interacting baryons in the detector to have one detection. If we consider a distance \(z = 1\), it turns out that it is needed a detector with an interacting mass of \(\sim 10^{15}\ \text{Gton}\) and \(\sim 10^{23}\ \text{Gton}\), for the \(\nu\) from the high and low density region, respectively. Clearly, this option is not plausible.

We are still left with two possibilities for a direct detection of neutrinos from a BdHN: 1) the occurrence of a BdHN at a closer distance, or 2) to have neutrinos with a very high-energy \(E_{\nu}\), several orders of magnitude higher than the one analyzed in this work. The latter appears as a feasible possibility from the recent works on the so-called inner engine of the high-energy emission occurring in BdHN of type I, which predicts that along (or close to) the rotation axis of the BH, electrons can be accelerated to energies of up to \(10^{18}\ \text{eV}\), and protons up to \(10^{21}\ \text{eV}\) (see Refs. [51–53], for further details). This is an interesting subject for future research.

We can also seek for the detection of the gamma-rays produced through the \(pp\) interaction. We calculated the photon emission spectra produced by the \(n^0\) decay, and also from \(q\) decay (for the low density region), their total energy and luminosity. Since these photons have specific spectrum and peak energies, their possible detection probes, indirectly, this neutrino emission (since they come out from the same interaction) and, moreover, the whole astrophysical scenario. In Section I A 2, we recalled previous works in which the photon emission from the transparency of the \(e^+e^-\) plasma, loaded with baryons in different amounts, is shown to explain 1) with \(B \lesssim 10^{-2}\) the GRB (~ MeV) prompt emission [25–28], and with \(B \sim 100\) (along a different direction of expansion) the X-ray flares [24]. From our results, we obtain that:
The photons produced by the present mechanism in the region with a high density of baryons have energies of the order of a few GeV. Then, these photons are energetically different from the previously studied ones.

2. The photons produced in the region with a low density of baryons have energies from a few to hundreds of GeV (the peak of the spectrum is reached near 100 GeV). Even in this case, the energy range is different from the one of the GRB prompt emission.

3. The emission studied in Section II occurs in a very small timescale ($\lesssim 1$ s) and relatively near the BH site. The other two processes mentioned above are related to the transparency of the photon-lepton-baryon plasma and, then, occur at large distances from the BH site.

Therefore, these photons are in the GeV regime, and in the high density region they can be produced with large luminosity of $10^{51}$-$10^{52}$ erg s$^{-1}$ (see e.g. Table I). For a source at $z = 1$ ($D \approx 6.7$ Gpc), it would correspond to a flux at Earth of $10^{-7}$-$10^{-6}$ erg s$^{-1}$ cm$^{-2}$, a value sufficiently high to be detectable by the Large Area Telescope (LAT) of the Fermi satellite. Also the short timescale of this emission, $\lesssim 1$ s, can be detected easily by the aforementioned detector (see Ref. [54] for the second GRBs catalog by the Fermi-LAT detector). On the other hand, the emission radii of $\sim 10^{10}$ cm, together with the aforementioned high photon luminosity, would lead to a high opacity of the $\gamma + \gamma \rightarrow e^+e^-$ process and, then, to the impossibility for these photons to leave freely the system. We analyze this process in Appendix V B, where we derive the interaction length for this process.

The photons coming from the low density region have higher energies (hundreds of GeV, see Section IIIIA1), but a low luminosity $\sim 10^{41}$ erg s$^{-1}$ (see again Section IIIA1). Assuming the same distance of the GRB considered above, for these photons we get a flux on Earth of $10^{-15}$ erg s$^{-1}$ cm$^{-2}$, which is below the flux threshold of Fermi-LAT.

In Appendix V, we calculate the photons interaction lengths for any possible interactions occurring inside the ejecta and in the ISM region. The results of this analysis highlight the impossibility for the photons created inside the ejecta to escape from the system due to the high opacity for all the processes analyzed. However we emphasize that the most probable interaction, in this physical set up, is the pairs production by photon-photon interaction. As we expected, the photons produced through the interaction with the ISM are not affected by all the possible interaction with matter, but their low flux (as explained above) yield their detection hard.

From the above considerations, we conclude that a detection of photons with the above specified energies, flux and timescale, probe the production and emission mechanism of the $pp$ processes studied in this work and, in turn, of the associated neutrino emission, even if it is difficult to get because of the high opacity of the system.

### V. APPENDIX

In this appendix we weigh up the possibility to detect photons, by $\pi^0$-decay, for the two considered physical setups studied in Section II and Section III. The kernel of this section concerns the study of the possibility to detect the photons emerging from the $pp$ interaction inside the SN-ejecta (Section II). Since the particles density inside the ejecta is high, the photons can interact with matter and are not free to escape from the creation site. About the other case, namely when the photons are created in the interaction with the ISM by protons with energy $E_p = 10^3$ eV (Section III), the ISM particles number density is too low and, thus, we expect that photons are free to escape without interaction and can be detected on Earth. Thus, we concentrate our attention
on the first interaction.

There are several interaction mechanisms for photons in the considered region: 1) Photo-meson production: $\gamma + p \rightarrow h + n\pi$ (where $h$ is an hadron and $n$ the number of produced pions); 2) Photon-Proton pair-production: $\gamma + p \rightarrow p e^+ e^-$ (Bethe-Heitler process); 3) Compton scattering: $\gamma + p \rightarrow \gamma' p$ (where $\gamma'$ is the photon emerging with different energy in comparison with to the interacting one); 4) Pairs Compton scattering: $\gamma + e^\pm \rightarrow \gamma + e^\pm$; 5) Photons pair production: $\gamma + \gamma \rightarrow e^+ e^-$. In order to have an estimate for the photons detection, we calculate the interaction length $l_{\text{int}}$ for the necessary processes. The interaction length is defined as $l_{\text{int}} = (\sigma_{\text{int}} N_{\text{targ}})^{-1}$, where $\sigma_{\text{int}}$ is the cross-section for the specific considered interaction and $N_{\text{targ}}$ the number density of the target particles of the medium. Comparing $l_{\text{int}}$ with the extension of the specific region of interaction (ejecta or ISM) $\Delta L$, if $l_{\text{int}} > \Delta L$ the photon are free to escape and vice versa.

The main processes to consider in our physical system and that can affect the dynamics of the expanding photo-lepton-baryon-plasma are the pairs creation processes via $\gamma + \gamma$ and the Bethe-Heitler process. Indeed, notwithstanding at our high photons energies ($\lesssim 1$ GeV) also the photon-meson production has an important role, this process will create low energy $\nu$ and $\gamma$. These low energy photons interact with protons principally via Bethe-Heitler process, since their energy is lower than the parent one. The produced neutrino will have too low energy to be seen, via $\nu - N$ interaction. Then, in the following we concentrate only onto the 2) and 5) processes.

A. Photon-Proton pair-production

In order to calculate the interaction length for this process, we assume a photon energy $E_\gamma = 0.69$ GeV, corresponding to the peak of the higher photon spectrum (see the curve corresponding to the interaction radius $r_1$ in Figure (4)). The Bethe-Heitler pair production results as the dominant process for $E_\gamma > 10$ MeV. At $E_\gamma > 500$ MeV, the probability to have a pair production via this process is almost 1 (see Figure (34.17) in Ref. [46], where this probability is calculated for various absorbing elements, from hydrogen to lead).

The cross-section for this process is (at infinite photon energy)

$$\sigma_{pp}(\infty) = \frac{7}{9} \left( \frac{A}{X_0 N_A} \right),$$

with $X_0$ is the interaction length, $A$ the element molar mass and $N_A$ the Avogadro’s number (see Ref. [46]). At a finite photon energy $E_\gamma$, the cross-section becomes (see Ref. [55] for more details)

$$\sigma_{pp}(E_\gamma) = \sigma_{pp}(\infty) (1 - \xi),$$

with $\xi = [\sigma_{pp}(\infty) - \sigma_{pp}(E_\gamma)]/\sigma_{pp}(\infty)$. Considering hydrogen ($A = 1$ g/mol, $Z = 1$), the value of $\xi$ (for a photon energy of $E_\gamma \approx 0.7$ GeV) is $\xi = 0.174$ (see Ref. [55]). Then the cross-section becomes $\sigma_{pp} \approx 17.07$ mb. With a target particles mean number density of $\langle n \rangle \sim 6.5 \times 10^{23}$ particles/cm$^3$ (see Section. II B), we get a value for the interaction length of $l_{\text{int}} \approx 90.13$ cm.

As we expected from the consideration above, the pair production is a very efficient process for our photons energies and creates new pairs with mean energy of a few hundreds of MeV. Consequently, this process could affect the dynamics of the $e^\pm$-plasma.

B. Photon-Photon Pair Production

Concerning the pair production via photon-photon interaction, $\gamma + \gamma \rightarrow e^+ e^-$, since we guessed that also this process could be relevant for the calculation of the photon detection probability, instead of using mean values for the photon energy and for the density (as we have done for the previous interaction), in this case we calculate the interaction length at each step of the plasma expansion inside the ejecta. In this analysis, we consider only the photons produced by the interaction inside the ejecta because: 1) in Section IV we derived that are the ones with major detection probability due to their higher flux; 2) due to the creation of new $e^\pm$ pair, this could affect the others interacting processes analyzed in this appendix and the dynamics of the plasma itself.

We can get the interaction length $l_{\gamma \gamma}$ at the radius $i$ by mean of: $l_{\gamma \gamma}^i = (\sigma_{\gamma \gamma} n_i^\gamma)^{-1}$, with $n_i^\gamma$ the photon number density, while $\sigma_{\gamma \gamma}$ the pair production cross-section given in [56]. This cross-section is function of the interaction angle $\theta$ between the two photons. We emphasize that we assume an isotropic radiation field since, at every radius, the photons are produced with the same energy by the same mechanism. As photons energy, at each radius, we consider to the maximum point of the spectrum (see Figure 4).

The photons number density is given by:

$$n_i^\gamma = \frac{L_i^\gamma}{4\pi r_i^2 c^2},$$

where $L_i^\gamma$ is the photon luminosity, calculated at every radius as explained in Section II D. The result of these calculations is shown in Figure 15, where the interaction length is shown for two interactions angles $\theta = \pi$ (head-on collision) and $\theta = \pi/2$. This result could seems strange, since we expect that the cross section is higher for head-on collision, but can be understood looking at the plot in Figure 16, where the cross-section $\sigma_{\gamma \gamma}$ is shown as a function of the interaction angle $\theta$. It is clear the decrease of $\sigma_{\gamma \gamma}$ for $\theta \in (0; \pi)$.

From Figure (15) we understand that, in the considered region, the pair production is a very efficient process. Together with the Bethe-Heitler process, it results that our high energy photons are almost all transformed in pairs and cannot leave the system.
Figure 15. Interaction length for $\gamma\gamma$ pair production, for photons produced and interacting inside the ejecta, for two angles of interaction between the photons $\theta = \pi/2, \pi$.

Figure 16. Photon-Photon pair production cross-section as a function of the interaction angle $\theta$.

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