Radiation model for spectral channels of Sentinel-2A and Landsat 8 radiometers

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Abstract. A description of a radiation model used for calculations of radiation transfer in a cloudless and cloudy atmosphere of the Earth is given. To parameterize functions of atmospheric transmission due to molecular absorption, exponential series are applied, with parameters determined on the basis of line-by-line calculations using modern spectroscopic databases. For effective parametrization, a simple and accurate method of taking into account the spectral dependence of the response function of recording devices has been developed. Examples are given for sensors installed on Sentinel-2A and Landsat 8 satellites.

1. Introduction

Monitoring the atmosphere and the Earth’s surface, and improving the quality of weather and climate predictions, both globally and regionally, requires more and more complex and accurate radiation models. This stimulates spectroscopic studies, design and updating of existing spectroscopic databases. For example, the latest version of the spectroscopic database HITRAN was released in 2018 [1]. Currently it contains 9163535 spectral lines. However, direct calculation methods based on these databases are unacceptable, due to computational cost. Currently developed approaches and software are not universal. As a rule, radiation models contain parameters calculated for specific versions of databases, and it is not possible to update spectroscopic information. For example, the 6SV model [2] used to interpret satellite observations in [3] is parametric, and its parameters have not been updated for a long time [4]. Therefore, new approaches and information systems are needed to process these increasing amounts of spectroscopic information, compress them and ensure high accuracy of calculations of radiation characteristics. Moreover, the problem is to ensure that these approaches are universal and applicable to any sensors installed on both satellites and ground-based photometers. Currently, libRadtran [5] is one of the popular applications, which allows calculations in the spectral range from 120 nm to 100 μm. This application is a library of radiation transfer procedures and programs. It was designed as a convenient and universal tool that provides many options to adjust and change the atmosphere with molecules, aerosol particles, water and ice clouds and the surface as a lower boundary. The radiative transfer code libRadtran is used for the atmospheric correction for different sensors. An example of its use for a satellite-based radiometer (HIRS) is described in detail in article [6]. NDVI index is widely used in Russian studies [7], the correct definition of which depends on the quality of atmospheric correction. One of the unique characteristic of this library is that it
includes several computer codes, including line-by-line calculations, as well as calculations for broadband radiation receivers based on exponential series. This program uses the k-distribution method (exponential series in the Russian press) to simulate measurements of a broadband infrared device.

Our team also develops similar software in the form of the Atmospheric Radiation Information system [8], which includes a block of shortwave radiation and a block of longwave radiation, and allows to perform calculations using the DISORT package. Moreover, we developed a method for decomposing parameters into series of exponents which is methodologically simpler, since it is based on a rigorous mathematical approach using Laplace transform. Also, this technique has been tested in the processing of data from a satellite radiometer HIRS [9]. The exact mathematical formulation of this problem allows us to obtain a low-parametric description and a simple software implementation of this method, which takes into account the instrument functions of the recording devices.

2. Approximation of the transmission function of exponential series
Consider an integral radiative transfer equation with a generalized kernel [10]:

\[ z(x) = \int_{x} k(x',x)z(x)dx + \Psi(x) \]  

(1)

\( z(x) \)-collision density. It is related to the radiative intensity by the relation

\[ I(\vec{r},\vec{n}) = \left| \mu_\phi \right| \sum_{\alpha} \frac{z(\vec{r},\vec{n})}{\alpha(\vec{r})} \]  

(2)

\( x \) – phase space of coordinates and directions \( x = (\vec{r},\vec{n}) \), \( \alpha = \alpha_\lambda + \alpha_\mu - \) extinction coefficient as the sum of the scattering and absorption coefficients. \( \mu_\phi = \cos \theta \), \( \theta \)-Sun's zenith angle, \( S_\phi \) solar constant. The generic kernel is described by the expression:

\[ k(x',x) = \frac{\omega(\vec{r})f(\vec{r},\vec{n},\vec{n}')\alpha(\vec{r})e^{\left( -\tau(\vec{r},\vec{r}') \right)}}{[\vec{r} - \vec{r}']} \]  

(3)

where \( \Psi(x) \) – the density distribution of sources:

\[ \Psi(x) = \sigma(\vec{r})e^{\left( -\tau(\vec{r},\vec{r}_m) \right)}\delta(\vec{n} - \vec{n}_\alpha) \]  

(4)

Equation (1) describes the standard problem. This problem considers the surface as a black body. The solution of this equation is easily generalized to the case of a diffusely reflecting surface [11].

The solution of the equation (1) can be represented by Neumann series[10]:

\[ z = \sum_{i=1}^{N} K^i \Psi \]  

(5)

In the formulas (3) and (4), we select the factors \( \alpha \) and \( \tau_m \) in explicit form for the analysis of the solution that contains the characteristics of molecular absorption:

\[ k(x',x) = \alpha(r) \frac{\alpha_\lambda(r')}{\alpha(r')} e^{\left( -\tau_m(r,r') \right)}k_0(x',x) \]  

\[ \Psi(x) = \alpha(r)e^{\left( -\tau_m(r,r_m) \right)}\Psi_0(x) \]

Expressions for \( \Psi_0(x) \) and \( k_0(x,x') \) do not include molecular absorption characteristics. Consider the first two members of the series (5):

\[ K\Psi = \int_{x} k(x_1,x)\Psi(x_1)dx_1 = \alpha(x) \int_{x} k_0(x_1,x)\Psi_0(x_1)\alpha_\lambda(\eta) e^{\left( -\tau_m(\eta,\tau_m(r,r_m)) - \tau_m(\eta,\tau_m(r,r)) \right)}dx_1 \]  

(6)

\[ K^2\Psi = \int_{x} k(x_1,x)k(x_2,x)\Psi_0(x_2)dx_1dx_2 = \alpha(x) \int_{x} k_0(x_1,x)k_0(x_2,x)\Psi_0(x_2)\alpha_\lambda(\eta)\alpha_\mu(\tau_m(r,t))\times \]  

\[ \times e^{\left( -\tau_m(t,t_m) - \tau_m(t,r) - \tau_m(t,t_m) \right)}dx_1dx_2 \]  

(7)

It follows from the formulas (6)-(7) that the molecular absorption in the expression for the intensity of radiation enters only the optical thickness \( \tau_m \) in the exponent. Aerosol and molecular scattering can
be considered constant in the interval $\Delta \nu$ (where $\Delta \nu$ is an average spectral resolution $\Delta \nu$). But molecular absorption and the solar constant are quick-changing functions. This is demonstrated in Figure 1, where the spectral dependences of aerosol and molecular scattering are presented. The same figure shows the spectral dependence of the optical thickness of molecular absorption, averaged for the spectral interval 5cm$^{-1}$ wide. For a reference, the typical half-width of the absorption line is ~ 0.1 cm$^{-1}$.

Figure 1. a) Spectral dependences of optical depth ($\tau_a$ - aerosol scattering, $\tau_R$ - molecular scattering, $\tau_m$ - molecular absorption) for typical conditions of the Lower Volga region; b) Transmission in the spectral channels of Sentinel-2A.

The expression for the transmission function for a separate spectral channel can be obtained:

$$T = \int_{\Delta \nu} S_\phi(\nu) f(\nu) \exp(-\tau_m(\nu))d\nu / \int_{\Delta \nu} S_\phi(\nu) f(\nu)d\nu = \int_{\Delta \nu} G(\nu) \exp(-\tau_m(\nu))d\nu ,$$  \hspace{1cm} (8)

$G(\nu) = S_\phi(\nu) f(\nu) / \int_{\Delta \nu} S_\phi(\nu) f(\nu)d\nu$ - function normalized to one.

The representation of the transmission function as a series of exponents does not change the functional form of the solution.

$$T = \sum_{i=1}^{n} C_i \exp\left(-\tau_{m,i}\right)$$  \hspace{1cm} (9)

In order to make the transition from (8) to (9), the k-distribution method is used. This method is described in the article [12] on an example of a homogeneous path where the transmission function is described by the expression:

$$T = \frac{1}{\Delta \nu} \int_{\Delta \nu} \exp(-k(\nu)W)d\nu ,$$  \hspace{1cm} (10)

$W$ – absorbing mass, $k(\nu)$ – absorption coefficient which is function quick-changing functions. Applying Laplas transform to (10) the following can be obtained:

$$T = \int_{0}^{\nu_i} \exp(-k(g)W)dg ,$$  \hspace{1cm} (11)

$k(g)$ is the inverse function of $g(k)$, which is calculated by the formulas:

$$g(k) = \frac{1}{\Delta \nu} \int_{v_i}^{\nu} U(\nu)d\nu$$  \hspace{1cm} (12)
The function \( g(k) \), can be interpreted as the distribution function of the absorption coefficient. In particular, from formulas (12) and (13) it follows that the function \( g(k) \) is monotonically increasing and varies from 0 to 1. It will be shown below that a statistical approach to the description of the molecular absorption spectra is needed to clarify the meaning of the approximations introduced for nonuniform path. It can be shown that the formula for calculating the transmission function is valid for a nonuniform path

\[
T = \int_{\nu_1}^{\nu_2} \exp(-\tau(g))dg ,
\]

the function takes into account the instrument function [8]

\[
g(r) = \int G(\nu)U(\nu) d\nu .
\]

The function \( U(x) \) is defined by an expression similar to (13)

\[
U(\nu) = \begin{cases} 1, & \nu < \tau \\ 0, & \nu > \tau \end{cases}
\]

The method of calculating \( g(\tau) \) differs from the one for \( g(k) \) in that the summation of the intervals for which \( \tau(\nu) < \tau \) is carried out with the weight \( G(\nu) \). Numerical implementation of this scheme is a little more complicated than the one for \( g(k) \). The array of optical depth \( \tau(\nu_i) \), is calculated with a constant step and sorted. As a result, an increasing sequence \( \{\tau(\nu_i)\} \) is obtained. Simultaneously, permutations are made in the array \( G(\nu_i) \). This makes it possible to construct an iterative procedure

\[
g(\tau_{i+1}) = g(\tau_i) + G(\nu_i), g(0) = 0
\]

In the numerical implementation the inversion does not present any difficulties, it is enough to swap the argument and function. In the case of a nonuniform path, it is necessary to impose the same conditions as in frequency space to the space of cumulative frequencies \( g \). Optical thickness is associated with the absorption coefficient \( k(r) \) by the relation:

\[
\tau_m(\nu, \nu_1, \nu_2) = \int_{\nu_1}^{\nu_2} k(\nu, r) \rho(r) dr .
\]

Therefore, condition (6) is satisfied. It follows from it that the optical depth at a frequency \( \nu \) consists of the optical depths of two layers in the frequency space:

\[
\tau_m(\nu, r_1, r_2) = \tau_m(\nu, r_1, r_2) - \tau_m(\nu, r_1, r)
\]

The condition is satisfied in the cumulative space if the distribution functions are the same for the two layers, or the spectra correlate. In the event that the spectra of only one gas are located in a given spectral range, this approximation in the Earth’s atmosphere is well satisfied and we can write a relation called the k-correlation

\[
\tau_m(g, \nu_1, \nu_2) = \int_{\nu_1}^{\nu_2} k(g, r) \rho(r) dr .
\]

If condition (19) is satisfied, then the spectra of the absorption coefficients can be calculated by the line-by-line method for given heights. In order to take into account the fine structure of the spectra, the typical frequency step is equal to 0.001 cm\(^{-1}\). With a typical width of the instrumental function of \( \sim 10^5 \) cm\(^{-1}\), one implementation is \( \sim \times 10^5 \). Since the function \( k(g) \) is monotonously increasing, several quadratures are enough to calculate the integral (11). Experience shows that 5-7 Gaussian quadratures provide high accuracy of the transmission function calculation and the error is not higher than 1%. Thus, for wide spectral intervals \( \Delta \nu \), 5–7 terms of the series are sufficient to describe the transmission function. In relation (19), the optical thickness can be considered as an ordinary optical thickness at
some specially chosen frequency $v_r$. Based on the above, in order to take into account molecular absorption in the spectral range $\Delta v$ it is necessary to solve the transfer equation for 5-6 wavelengths, and then sum these solutions with weights $C_i$. Figure 2a shows an example of calculating the molecular absorption spectrum for the Earth’s atmosphere in channel of the Landsat 8 sensor.

![Figure 2](image)

**Figure 2.** a) Spectral dependence of the optical depth of molecular absorption for summer conditions. The path is vertical through the whole atmosphere. b) Spectral response in the OLI NIR band of Landsat 8.

Figure 2b shows the Spectral response for this sensor. Figure 3 shows the same absorption spectrum, but already transformed. The same figure shows the values of the function $\tau(g)$ in the nodes of Gaussian quadratures. In order to ensure fast computations, a database of effective absorption coefficients is created for given temperatures and pressures in the Earth’s atmosphere for each channel of the satellite sensor. During massive computations for the real temperature profile, pressures and gas concentrations in the Earth’s atmosphere the desired absorption coefficient profile is obtained by interpolation.

![Figure 3](image)

**Figure 3.** The spectral dependence of the optical depths (curve 1) in the space of cumulative frequencies. 2- The values of the function $\tau(g)$ in the nodes of Gaussian quadratures.
3. References

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