Triton Binding Energy and Minimal Relativity

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Abstract

For relativistic three-body calculations, essentially two different approaches are in use: field theory and relativistic direct interactions. Results for relativistic corrections of the triton binding energy obtained from the two approaches differ even in their sign, which is rather puzzling. In this paper, we discuss the origin of such discrepancy. We show that the use of an invariant two-body amplitude, as done in the field-theoretic approach, increases the triton binding energy by about 0.30 MeV. This may explain a large part of the discrepancy.
In recent years, many high-precision, charge-dependent calculations of the triton binding energy with realistic nucleon-nucleon (NN) interactions have been performed. Employing local two-nucleon potentials of high quality, in a 34-channel charge-dependent non-relativistic Faddeev calculation, the triton binding energy is predicted to be 7.62(1) MeV [1]. With a recent, high-precision version of the (non-local) Bonn potential (CD-Bonn), the same type of Faddeev calculation predicts 8.00 MeV for the triton binding energy [2], the difference in the predictions originating from variations in the off-shell behaviour of the potentials.

However, due to the presence of high momentum components in the triton wave function, it is important to go beyond the non-relativistic approximation. Moreover, even if relativistic effects turn out to be small, they may still be large as compared to the discrepancy with the experimental value, which is 8.48 MeV. Therefore, the present situation calls for an accurate knowledge of the effect of relativity on the three-nucleon bound-state.

Let us briefly review the current status of the relativistic three-body bound-state problem. Essentially, one may distinguish between two major theoretical frameworks:
1. field theory,

2. relativistic direct interactions.

Within the first scheme, Rupp and Tjon have used Bethe-Salpeter (BS) equations for two and three particles. They constructed covariant multirank separable interactions from phenomenological as well as meson-exchange potentials (Paris and Bonn). They predict 0.29-0.38 MeV more binding with respect to non-relativistic results.

The field-theoretic approach has also been pursued in our own work, in which we apply the relativistic three-dimensional version of the Bethe-Salpeter equation proposed by Blankenbecler and Sugar (BbS), together with relativistic one-boson exchange potentials. We find an increase of the triton binding energy due to relativistic effects of 0.19 MeV using the Bonn B potential, and obtain the same result with the more recent CD-Bonn.

Gross and collaborators have recently reported results for the triton binding energy very close to the experimental value, using a relativistic field-theoretic framework.

Alternative to field theory, other approaches have been pursued, in which the attempt is made to combine quantum mechanics with the requirements
of relativistic invariance. Within the scheme of relativistic direct interactions, relativistic Hamiltonians are defined as the sum of relativistic one-body kinetic energies, two- and many-body interactions and their boost corrections. The latter is derived from commutation relations of the Poincaré group [6, 7, 8, 9]. Recent work within this approach [10] reports that relativistic effects reduce the triton binding energy by 0.34 MeV.

In summary, even the sign of the relativistic correction is controversial. This is rather worrisome, and calls for further investigation of the many facets of the problem. It is the purpose of this note to discuss some aspects involved.

Within the framework of relativistic quantum field theory, there are essentially three sources of relativistic effects in the three-body system:

- The use of an invariant two-nucleon amplitude as input to the three-body calculation,
- relativistic kinematics,
- relativistic Faddeev equations/propagators.

For best transparency of the investigation, it helps to single out each of these effects, with the first one being the focus of this note.
To investigate this point, we use a simple prescription, known as *minimal relativity* [11]. A relativistic (invariant) two-body $t$-matrix, $t_{\text{rel}}$, can be related to the non-relativistic $t$-matrix, $t_{\text{NR}}$ (solution of the Lippman-Schwinger equation), by

$$t_{\text{rel}}(q', q) = \sqrt{E'/m} t_{\text{NR}}(q', q) \sqrt{E/m}$$  \hspace{1cm} (1)

with $E = \sqrt{m^2 + q^2}$ and $E' = \sqrt{m^2 + q'^2}$ ($q \equiv |q|$, $q' \equiv |q'|$). When $t_{\text{rel}}$ of Eq. (1) is inserted into the relativistic BbS equation (together with an analogous expression relating $V_{\text{rel}}$ and $V_{\text{NR}}$), the usual non-relativistic Lippmann-Schwinger equation for $t_{\text{NR}}$ is obtained.

Applying a variety of NN potentials, we find that the use of $t_{\text{rel}}$ as defined in Eq. (1) increases the triton binding energy by 0.25-0.37 MeV, see Table 1. Note that Eq. (1) allows to calculate the effect of minimal relativity for any potential (even non-relativistic ones) with no need to refit the nucleon-nucleon phase parameters [13].

Of course, the other aspects of relativity mentioned above also play a role, but they introduce uncertainties of a different nature. For instance, various three-dimensional relativistic three-body equations are available, which all have in common the elimination of the relative time component in the origi-
inal four-dimensional integration over internal momenta. This procedure is carried out in such a way as to preserve relativistic invariance and three-particle unitarity, but it is not unique. Also, the effect of relativistic kinematics is dependent on the way relative momenta of the interacting pair are defined [14]. Thus, in the field-theoretic approach, some ambiguities enter when a relativistic three-dimensional reduction of the BS equation is used. These require a separate investigation. On the other hand, the effect of using an invariant two-body amplitude in the Faddeev equations, which we have chosen to single out in this note, can be estimated independently by means of the minimal relativity prescription, Eq. (1). This effect is absent from approaches based on relativistic direct interactions, since they are not manifestly covariant. Therefore, it may shed some light on the discrepancy between the results from the two major theoretical frameworks.

The present situation can then be summarized as follows:

- Within a non-relativistic framework, and with local potentials, the triton binding energy is predicted to be 7.6 MeV, with no disagreement among different calculations.

- On the other hand, when relativity is included, one group [10] reports
less binding, while others find more \[3, 4, 5\]. The latter approaches are all based on relativistic quantum field-theory and obtain more binding energy regardless the dynamical input. Minimal relativity may explain the major reason for the extra binding.

Thus, a rather puzzling picture emerges: relativity increases drastically the uncertainty in the predictions, which now ranges from 7.3 to 8.2 MeV.

This state of affairs is unacceptable, and opens a much more complex and fundamental issue, namely, how to define a relativistic correction \[8\]. There is actually no unique way to define a relativistic correction, because there is no unique way to define a non-relativistic limit. Equivalent covariant theories may differ in their non-relativistic limit, since the latter depends on how such limit is taken \[8\]. So, strictly speaking, the only safe way to proceed is to start from a covariant theory.

The field-theoretic approaches we have mentioned above are consistently relativistic from the outset, while the calculations using direct interactions \[10\] are based on a non-relativistic baseline, with relativistic corrections applied in the form of a \(p/m\) expansion. As investigated by Glöckle and coworkers \[15\], such an expansion may be risky and hard to control and, therefore,
could be the source for the observed repulsion.

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[13] For an uncoupled partial wave, the phase shift \( \delta \) is related to the invariant scattering amplitude, \( t_{rel} \), by

\[
e^{i\delta} \sin \delta = -\frac{\pi}{2} q \frac{m^2}{E} t_{rel} ,
\]
while for the nonrelativistic $t$-matrix, $t_{NR}$, the relationship reads:

$$e^{i\delta} \sin \delta = -\frac{\pi}{2} q m t_{NR}.$$ 

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Table 1: **Effect of minimal relativity on the triton binding energy calculated for a variety of recent high-precision NN potential models.**

| NN Potential Model     | Increase of Triton Binding Energy (MeV) |
|------------------------|----------------------------------------|
| CD-Bonn [2]            | 0.25                                   |
| Nijmegen-I [12]        | 0.27                                   |
| Nijmegen-II [12]       | 0.37                                   |
| Reid’93 [12]           | 0.37                                   |