Correlations in Random Ising Chains at Zero Temperature

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Abstract: We present a general method to calculate the connected correlation function of random Ising chains at zero temperature. This quantity is shown to relate to the surviving probability of some one-dimensional, adsorbing random walker on a finite interval, the size of which is controlled by the strength of the randomness. For different random field and random bond distributions the correlation length is exactly calculated.

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1 Introduction:

The perfect crystal is the result of physical abstraction in real materials one should always reckon on the existence of impurities and different types of lattice defects. There are important problems in the field of random systems such as localization in a disordered medium, the spin glass behaviour, diluted magnets etc. In order to obtain a theoretical understanding of these phenomena magnetic models with quenched disorder have been introduced and studied by different methods (for recent reviews see Refs.[1-3]). The random magnetic models have unusual low-temperature properties. These are a consequence of the complex structure of low energy metastable states generated by quenched disorder and frustration, which are believed the main ingredients of spin-glass behaviour[4]. The simplest system in this field is the random Ising model, which can be experimentally realised as diluted antiferromagnets[5]. The model already in one dimension shows interesting features, although its physical quantities are singular only at zero temperature.

The one-dimensional random Ising model - inspite of its low dimension - is exactly soluble only for a few specific form of randomness. One of those is the random bond Ising chain in a uniform field (RBIM) defined by the Hamiltonian:

\[ \mathcal{H}_{RB} = -\sum J_{i,i+1}\sigma_i\sigma_{i+1} - h\sum\sigma_i \]

Here \( \sigma_i = \pm 1 \) and the exchange integral \( J_{i,i+1} \) is equal to \( J > 0 \), with probability \( p \) and \( -J \) with probability \( q = 1 - p \), and the bond disorder is quenched. For this model the ground state energy, the zero point entropy and magnetization have been calculated by Derrida and co-workers[6] (see also in Ref[7]).

Another type of random system is the ferromagnetic Ising model in a random field (RFIM) with the Hamiltonian:

\[ \mathcal{H}_{RF} = -J\sum\sigma_i\sigma_{i+1} - \sum h_i\sigma_i \]

At \( T = 0 \) the thermodynamic properties of this model have been calculated for the binary distribution of the random fields[6]:

\[ P(h) = p\delta(h - H) + q\delta(h + H) \]

as well as for the asymmetric binary distribution[8]:

\[ P(h) = \frac{1}{2}\delta(h - H_1 - H_0) + \frac{1}{2}\delta(h + H_1 - H_0) \]
The first exact results at $T \neq 0$ have been obtained by Grinstein and Mukamel[9] with the diluted symmetric binary distribution:

$$P(h) = \frac{P}{2} [\delta(h - H) + \delta(h + H)] + q\delta(h)$$

in the limit $H \to \infty$. Very recently the non-linear and higher order susceptibilities of the model have also been determined[10].

A series of RFIM-s with continuous random field distribution has been studied by Luck and Nieuwenhuizen[11-13] at arbitrary temperatures. One of these models is characterised by the diluted symmetric exponential distribution $R(x)dx$ as:

$$R(x) = \frac{P}{2} e^{-x} + q\delta(x)$$

with $h_i = \tilde{H}x_i$, $\tilde{H} > 0$ and $-\infty < x_i < \infty$.

As far as the correlation functions of random Ising models are concerned only a few results are available. Derrida[14] has pointed out that the spin-spin correlation function is not a self-averaging quantity, therefore its average differs from the most probable value[15]. Exact results in the whole temperature range are available for the Grinstein-Mukamel model[9] as well as for the diluted symmetric exponential distribution in eq(6)[12]. Most recently Farhi and Gutmann[16] have calculated the zero temperature correlation function of the RFIM with the binary distribution in eq(3). We also mention a related exact study of the pair-correlation function of a one-dimensional lattice gas model in a random potential at zero temperature[17].

In this paper we study the connected correlation function of random Ising chains defined as:

$$\chi(l) = [\langle \sigma_i\sigma_{i+l} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+l} \rangle]_{av}$$

where $\langle ... \rangle$ denotes the thermodynamic average and $[...]_{av}$ stands for the quenched average over the random variables. The thermodynamic average in eq(7) could be non-zero even at $T = 0$ provided the ground state of the system is highly degenerate. This can be seen in two-dimensional frustrated models without randomness, in which the zero temperature correlations either decay as a power law[18-21] or exponentially[22].

For random Ising chains the correlation function in eq(7) is usually calculated in the transfer matrix formalism[9,12]. At $T = 0$, however, one may use another approach based on an analysis of the degenerate ground state configurations. For a given quenched disorder, due to frustration, there are spins in the system which are ”loose”, i.e. they are
free to point in any direction. These loose spins may form a domain, and the connected correlation function is non-zero, only if both spins considered belong to the same domain of loose spins. At this point calculation of the correlation function at $T = 0$ is essentially reduced to an investigation of the size distribution of domains of loose spins, which in turn is equivalent to a one-dimensional random walker problem on a finite interval.

The setup of the paper is the following. In Section 2 we present our method to calculate the connected correlation function for random Ising chains at $T = 0$. In Section 3 and 4 correlation lengths are calculated for RFIM and RBIM, respectively. Finally, the results are discussed in Section 5.

2 Connected correlations and their relation with random walkers

In this Section we develop a formalism to calculate the connected correlation function for random field Ising models at $T = 0$. It will be shown in Section 4 how these results can be applied for the random bond problem.

The random fields we consider have a discrete distribution, furthermore the possible values of $h_i$ are integer multiples of a unit, denoted by $H$, i.e. $h_i = m \times H$. For the binary distribution in eq(3) $m = \pm 1$, while for the diluted symmetric distribution in eq(5) $m = 0, \pm 1$. Continuous distributions, like in eq(6) can also be discretised, an example is shown in Section 3.3. In this way one can also treat the asymmetric binary distribution in eq(4) provided the ratio of the parameters $H_1$ and $H_0$ is rational.

The structure of ground state configurations of a RFIM with discrete randomness is thoroughly analysed in the literature (see c.f. in Refs[2,3]). In the weak-coupling limit, when $2J < \min\{|h_i|\} = h_{min}$ the spins are frozen to the direction of the local fields. For stronger couplings, so that $2J > h_{min}$ there is a tendency for neighbouring spins to align parallel with each other, so that domains of parallel spins are formed. With increasing value of the coupling the average size of a domain increases, but the ground state never consists of one single domain. It can be understood, since the necessary energy to create a domain wall - $2J$ - can be accumulated from fluctuations of the random field, even if the local field is arbitrarily small.

The size of random field fluctuations is characterised by the integrated random field function, defined as: $H(k) = \sum_{i=1}^{k} h_i$. For an illustration we draw this function on Fig 1 for a given random field distribution together with the corresponding ground state config-
uration of the system at some value of the coupling $J$. As is seen on this Figure the first domain wall is located between spins 3 and 4, at a local maximum of $H(k)$. Indeed, the sum of random field energies $H(6) - H(3) < -2J$ covers the cost of creation a domain wall.

The position of the second domain wall, however is not unique: it can be at any of the three degenerate local minima located at $(6,7), (9,10)$ or $(11,12)$. Energetically there is no difference between these configurations, since the corresponding random field energies are the same: $H(6) = H(9) = H(11)$. As a consequence in the interval $(7,11)$ the position of the spins in the ground state is not fixed. Such a region will be called as a domain of "loose" spins (DLS).

Based on this example we can easily postulate the properties of a DLS. First let us restrict ourselves to a DLS which separates a $\downarrow \ldots \downarrow$ and a $\uparrow \ldots \uparrow$ domain, as shown in our example on Fig 1. Such a DLS is bounded by two degenerate local minima. Inside the DLS the integrated random field function relative to its value at the boundaries - $\Delta H(k)$ - does not exceed $2J$, thus the energy necessary to create a domain wall can not be accumulated from the random field. On the other hand in both directions outside of the DLS $\Delta H(k)$ exceeds $2J$, before crossing zero. This last condition ensures the existence of $\downarrow \ldots \downarrow$ and $\uparrow \ldots \uparrow$ ferromagnetic domains at two sides of the DLS.

The other type of DLS separating $\uparrow \ldots \uparrow$ and $\downarrow \ldots \downarrow$ domains can be characterised similarly. In this case after the transformation $\sigma_i \rightarrow -\sigma_i$, $h_i \rightarrow -h_i$ the previous considerations can be applied for $-H(k)$. We mention, if $2J/H$ is an integer the degeneracy of the ground state is higher, than for a slightly larger or smaller value of the coupling. It is connected to the fact that in this case conditions both for $\uparrow \downarrow$ and $\downarrow \uparrow$ DLS can be satisfied at the same time. In the following we are not going to deal with such situations, thus our considerations apply for $2J/H \neq \text{integer}$.

Since position of spins in a DLS is not fixed these regions are the source of non-zero ground state entropy in a RFIM. These regions are also responsible for non-vanishing value of the connected correlation functions at $T = 0$. It is easy to see that the thermal average of $\chi(l)$ in eq(7), which is now performed over the degenerate ground state configurations, is non-zero only if the two spins are in the same DLS. If one spin is in a ferromagnetic domain its value is the same for all ground state configurations, consequently the connected correlations are zero. On the other hand if two spins are in different DLS they are independent variables, thus again the connected correlations are vanishing.

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The above relationship between $\chi(l)$ and DLS makes it possible to calculate the leading behaviour of the connected correlation function in a RFIM in a simple way. To do this one should i) first consider DLS regions of length $\tilde{l} > l$ and determine the $W(\tilde{l})$ probability that a point of the line belongs to one of those DLS. ii) Then find the probability that the other reference point of $\chi(l)$ is also on the same DLS and iii) finally calculate the thermal and quenched averages in eq(7) with the condition that both endpoints of $\chi(l)$ are on the same DLS. In the following we show that the leading behaviour of $\chi(l)$ is determined by the probability $W(l)$, which has an exponential dependence on $l$, whereas the probabilities indicated in ii) and iii) are comparatively negligible, since they depend on $l$ as power laws.

To calculate the probability $W(\tilde{l})$ one can use a geometrical interpretation of a DLS as a one-dimensional random walker with steps $h_i/H$ on an intervall consisting of

$$L = \left\lceil \frac{2J}{H} \right\rceil + 1$$

points. Here $[x]$ denotes the integer part of $x$ and $x$ is non-integer. The walker starts at one endpoint of the intervall and after $\tilde{l}$ steps made on the strip returns to the same endpoint. For large $\tilde{l}$ the leading $\tilde{l}$ dependence of $W(\tilde{l})$ is exponential, and $W(\tilde{l})$ corresponds to the surviving probability of the adsorbing random walker:

$$W(\tilde{l}) \sim \exp \left[ -\frac{\tilde{l}}{\xi(L)} \right]$$

Next we show that the probabilities mentioned in ii) and iii) have weaker $l$ dependence than the surviving probability. It is clear from a simple geometrical consideration that the probability in ii) is at most $\sim 1/l$. On the other hand to estimate the conditional probability in iii) one should first notice that the different ground states are characterised by one parameter, the position of the domain wall. Thus an average over this parameter is equivalent to the thermal and quenched averages in eq(7). Using the fact that the number of possible positions of a domain wall in a DLS of length $l$ is proportional to $l^2$ one can estimate the conditional probability in iii) as $\sim l^{-2}$.

The connected correlation function $\chi(l)$ is then obtained by summing the probabilities for $\tilde{l} \geq l$ with the result in leading order:

$$\chi(l) \sim W(l) \sim \exp \left[ -\frac{l}{\xi(L)} \right]$$

where $L$ is given in eq(8). In the following Section the correlation length $\xi(L)$ will be calculated for different random field distributions.
3 Random field Ising models

The surviving probability in eq(10) can be most easily calculated in the transfer matrix formalism[23]. In this case the elements of the transfer matrix $T(n, m)$, $n, m = 1, 2, ..., L$ are given as the probability of a step from position $m$ to $n$. In the RFIM language $T(n, m) = P(h(n, m))$, where $h(n, m) = (n - m) \times H$. A matrix-element of $T$ is zero, whenever the corresponding $h(n, m)$ is not contained in the set of random fields of the model. The leading eigenvalue of the transfer matrix - $\lambda(L)$ - is connected to the surviving probability as:

$$W(l) \sim \lambda(L)^l$$

thus the correlation length in eq(10) is given by

$$\xi(L) = -\frac{1}{\log \lambda(L)}$$

For a general RFIM simple analytical results can be obtained for $2J/H < 1$ and in the strong coupling limit $2J/H \gg 1$. In the former case $L = 1$, the intervall of the walker consists of one single point:

$$\lambda(1) = P(0) \quad 2J/H < 1$$

Therefore non-vanishing correlations can only be present in diluted models (see cf. eqs.(5) and (6)). In the strong coupling limit, which corresponds to $L \gg 1$ we consider distributions with zero average $< h >= 0$. Then the finite size corrections to the leading eigenvalue are quadratic:

$$1 - \lambda(L) \sim L^{-2} \sim (H/J)^2 \quad H/J \ll 1$$

which follows from the Gaussian nature of the free random walk[23]. According to eqs.(12) and (14)

$$\xi(J/H) \sim (J/H)^2 \quad J/H \gg 1$$

For intermediate (non-integer) values of $2J/H$ the leading eigenvalue of the transfer matrix can be calculated numerically, so that - together with the asymptotic relation in eqs(15) - one can in principle obtain the correlation length of connected correlations for all types of RFIM-s. In the following we present three examples, in which the calculation can be performed analytically for all non-integer values of $2J/H$. 

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3.1 Binary distribution

The transfer matrix corresponding to the distribution in eq(3) is tridiagonal and given as:

\[ T_1 = \begin{pmatrix} 0 & p & 0 & p & 0 & p & \cdots & p \\ q & 0 & p & 0 & p & 0 & \cdots & p \\ q & 0 & p & 0 & p & 0 & \cdots & p \\ q & 0 & p & 0 & p & 0 & \cdots & p \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ q & 0 & p & 0 & p & 0 & \cdots & q \end{pmatrix} \] \quad (16)

The (non-normalized) leading right eigenvector of this matrix is

\[ \Phi_1(l) = \left(\frac{q}{p}\right)^{1/2} \sin\left(l \frac{\pi}{L+1}\right) \] \quad (17)

and the corresponding leading eigenvalue

\[ \lambda_1(L) = 2\sqrt{pq} \cos\left(\frac{\pi}{L+1}\right) \] \quad (18)

The correlation length is then can be obtained from eqs(12) and (8). This result agrees with that of Ref[16].

3.2 Diluted symmetric binary distribution

The transfer matrix corresponding to the distribution in eq(5) is symmetric and tridiagonal:

\[ T_2 = \begin{pmatrix} q & p/2 & p/2 & \cdots & \cdots & \cdots & \cdots & \cdots \\ p/2 & q & p/2 & \cdots & \cdots & \cdots & \cdots & \cdots \\ p/2 & p/2 & q & p/2 & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & q \end{pmatrix} \] \quad (19)

The leading eigenvector of \( T_2 \) is the same as in eq(17), however with \( q/p = 1 \) and the leading eigenvalue is given as:

\[ \lambda_2(L) = q + p \cos\left(\frac{\pi}{L+1}\right) \] \quad (20)

For \( L = 1 \), i.e. \( 2J/H < 1 \), \( \lambda_2(1) = q \) and the general relation in eq(13) is recovered, which agrees also with the result of Grinstein and Mukamel[9] as \( 2J/H \to 0 \).
3.3 Diluted symmetric exponential distribution

We consider the discretised version of the distribution in eq(6) when the allowed values of the random field are \( h_i = H \times i, \ i = 0, \pm 1, \pm 2, \ldots \) and the probability distribution is given as:

\[
P(h) = c \exp \left( -\alpha \left| \frac{h}{H} \right| \right) \delta(h - Hi) + c_0 \delta(h) \tag{21}
\]

Here \( \alpha > 0 \) and the distribution is normalised with \( c = p/2(\exp \alpha - 1), \ c_0 = q - c \) and \( p + q = 1 \). Taking the limit

\[
\alpha \to 0^+, \quad H \to 0, \quad \tilde{H} = H/(\exp \alpha - 1) = \text{finite} \tag{22}
\]

one arrives to a continuum description in the variables \( h_i = \tilde{H} x_i, -\infty < x_i < \infty \), with the probability distribution \( R(x)dx \) given in eq(6).

We write the transfer matrix of the problem in terms of the variables \( \omega = \exp(-\alpha), \ \omega_0 = q \) and \( \kappa = c/q \) as:

\[
T_3 = \omega_0 \begin{pmatrix}
1 & \kappa \omega & \kappa \omega^2 & \ldots & \kappa \omega^{L-1} \\
\kappa \omega & 1 & \kappa \omega & \ldots & \kappa \omega^{L-2} \\
\kappa \omega^2 & \kappa \omega & 1 & \ldots & \kappa \omega^{L-3} \\
\vdots & \vdots & \ddots & \ddots & \kappa \omega \\
\kappa \omega^{L-1} & \ldots & \ldots & 1 & \kappa \omega^{L-2}
\end{pmatrix} \tag{23}
\]

We mention that the same transfer matrix belongs to a directed polymer[23] on a strip of width \( L \) on the square lattice. In this case \( \omega_0 \) and \( \omega \) are the monomer fugacities for steps along and perpendicular to the strip, respectively and \( \kappa \) denotes the statistical weight corresponding to a turn of the chain by \( 90^\circ \).

The eigenvalue problem of \( T_3 \) in eq(23) is similar to that of the unrestricted directed polymer[24]. The leading eigenvector is given as

\[
\Phi_3(l) = \cos \left[ \left( l - \frac{L + 1}{2} \right) \phi \right] \tag{24}
\]

where \( \phi \) is the smallest root of the equation:

\[
\cot \left( \frac{(L - 1) \phi}{2} \right) = \frac{\sin \phi}{\cos \phi - \omega} \tag{25}
\]

The leading eigenvalue is then:

\[
\lambda_3(L) = \frac{(1 - \omega^2)\kappa \omega_0}{1 - 2\omega \cos \phi + \omega^2} + (1 - \kappa)\omega_0 \tag{26}
\]
The correlation length can be obtained from eq(12) using the correspondences between $\omega, \omega_0, \kappa$ and the original parameters of the distribution in eq(21).

Now we evaluate our results in eqs(25) and (26) in the continuum limit of eq(22), which reads as $\omega \to 1^-$. The smallest root of eq(25) is proportional to $1 - \omega$, so that written it in the form of $\phi = \gamma (1 - \omega)$ one gets the following relation from eq(25):

$$\tan^{-1} \left( \frac{1}{\gamma} \right) = \frac{J}{H} \gamma$$

Then the leading eigenvalue is given by

$$\lambda_3 = \frac{1 + q \gamma^2}{1 + \gamma^2}$$

in the continuum limit. These results are identical to those obtained by Luck and Nieuwenhuizen[12] using the continuous distribution in eq(6) and taking the non-trivial limit $T \to 0^+$. We note that the correlation function calculated with the continuous distribution is discontinuous at $T = 0$, thus the limits $H \to 0$ and $T \to 0$ can not be interchanged.

4 Random bond Ising model

The formalism developed in Section 2 can also be applied to calculate $\chi(l)$ for the RBIM with the Hamiltonian in eq(1). It is easy to see that after the transformation $\sigma_i \to \sigma_i h/h_i$ in eq(2) one arrives to a RBIM with $J_{i,i+1} = J/h_i h_{i+1}$. The inverse transformation is given by

$$\sigma_i \to \sigma_i \prod_{k=1}^{i-1} \left( \frac{J_{k,k+1}}{J} \right)$$

thus the random fields can be expressed through the random bonds as

$$h_i = h \prod_{k=1}^{i-1} \left( \frac{J_{k,k+1}}{J} \right)$$

The random fields in eq(30) can take the values $\pm h$ like to the binary distribution in eq(3), however these $h_i$-s are correlated in different sites, since

$$< h_i h_{i+n} > = (p - q)^n$$

Thus the symmetric distribution with $p = q = 1/2$ is exceptional, in which case the RBIM is equivalent to a RFIM with the symmetric binary distribution in eq(3).
For \( p \neq q \) the probability is connected with bond variables, thus with the sign of the product of two consecutive random fields:

\[
P(h_{i-1}h_i) = \begin{cases} 
    p & h_{i-1}h_i > 0 \\
    q & h_{i-1}h_i < 0 
\end{cases} \quad (32)
\]

To write down the transfer matrix of this problem we work with the bond variable \( \bar{H}(i) = (H(i-1) + H(i))/2 \), which may take \( L - 1 \) different values: \( 1/2, 3/2, ..., L - 1/2 \). The one-step transfer matrix \( T_{RB}(i, i+1) \) is different if the number of the step \( i \) is even or odd. Therefore one should consider the two-step transfer matrix defined as:

\[
T_{RB}(i, i+2) = \begin{pmatrix}
    q^2 & qp & p^2 \\
    qp & q^2 & qp \\
    q^2 & qp & p^2 \\
    qp & q^2 & qp \\
    p^2 & qp & q^2 \\
    qp & q^2 & qp \\
    \vdots & \vdots & \vdots
\end{pmatrix} \quad (33)
\]

The transfer matrix connecting the states in steps \( i + 1 \) and \( i + 3 \) is the transpose of \( T_{RB}(i, i+2) \), thus both have the same eigenvalue spectrum. Writing the leading eigenvalue as \( \lambda^2_{RB} \) the correlation length is then obtained from the logarithm of \( \lambda_{RB} \) through eq(12). Since the transfer matrix in eq(33) is non-symmetric and non-tridiagonal we could solve its eigenvalue problem analytically only in a few special cases.

In the limit \( p \to 0 \) the transfer matrix is tridiagonal in linear order of \( p \) and can be solved by the same eigenvector as \( T_2 \) in eq(19). The leading eigenvalue is given as

\[
\lambda_{RB} = 1 - \frac{p^2}{2} \left( 1 - \cos \frac{\pi}{L} \right) + O(p^2) \quad (34)
\]

In the symmetric distribution \( p = q = 1/2 \) the right eigenvector is given by

\[
\Phi_{RB}(2l) = \Phi_{RB}(2l+1) = \sin \left[ (2l + 1) \frac{\pi}{L+1} \right] \quad (35)
\]

for \( l = 0, 1, ..., L/2 - 1 \). The corresponding eigenvalue

\[
\lambda_{RB} = \cos \left( \frac{\pi}{L+1} \right) \quad (p = q = 1/2) \quad (36)
\]

is the same as for the RFIM with symmetric distribution in eq(18), which is in accord with our previous claim about the equivalence of the two problems.
Finally, we consider the limit $q \to 0$. Then the eigenvector is given in leading order as:

$$
\Psi(2l) = \Psi(L - 2l + 1) = q^{\frac{l-1}{L-1}}
$$

$$
\Psi(2l - 1) = \Psi(L - 2l + 2) = q^{1 - \frac{1}{L-1}}
$$

(37)

for $l = 1, 2, ..., (L - 1)/2$. The corresponding eigenvalue:

$$
\lambda_{RB} = q^{1/L} \quad q \ll 1
$$

(38)

Analysing the $p$-dependence of the correlation length one can see that it starts with zero in the ferromagnetic limit $p \to 1$, stays finite for non-zero concentration of ferromagnetic bonds and finally diverges in the antiferromagnetic limit $p \to 0$.

5 Discussion

In this paper the connected correlation function of random Ising chains is investigated at zero temperature. Our study is based on an analysis of the ground state configurations of these systems. Due to frustration there are spins in the ground state which are ”loose”, i.e. their position is not fixed by the interaction and the external field. These ”loose” spins form domains, and between two spins in the same domain there are non-vanishing correlations. It was shown then that the connected correlation function $\chi(l)$ is primarily determined by the probability of having a DLS of size $l$ in the system. Finally, this probability was calculated using an analogy with the surviving probability of some random walker on a finite intervall.

Considering different types of random field and random bond distributions we have calculated the correlation length using the transfer matrix method. Analysing these results one may observe two different behaviour in the weak disorder limit. The correlation length either vanishes as $\xi \sim 1/\log(1/p)$ or it is divergent like $\xi \sim 1/p$. The former behaviour is found in the RFIM with binary distribution in eq(18) as well as for the RBIM in the pure ferromagnetic limit $q \to 0$ in eq(38). On the other hand the correlation length is diverging in the diluted RFIM in eqs(20) and (28) as $p \to 0$. Similar $p$ dependence is observed in the RBIM in the antiferromagnetic limit, i.e. when $p \to 0$ in eq(34).

The exponential decay of correlations for random Ising chains is found as a general rule. One may find, however, a slower decay of correlations if the strength of randomness is smoothly position dependent. Let us consider a semi-infinite RFIM in which the strength
of the random field decays like $h_i \sim i^{-s}$ from the surface. Then the equivalent random walker has to be considered on an interval the size of which is increasing in time. Using results about random walkers[25] and directed polymers[24] inside a parabola, one can say that the decay of correlations in this inhomogeneously disordered system is of a stretched exponential form for $0 < s < 2$, whereas it can be described as a power law for $s \geq 2$.

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Figure Caption:

Fig.1. The integrated random field function $H(i)$ for some fixed values of the random field. The corresponding ground state of the system at a given value of the coupling $J$ is indicated below. Here vertical dashed lines denote the possible domain wall positions. State of spins in the region $\{7, 11\}$ is not fixed, they form a DLS (see text).
The integrated random field function $H(i)$ for some fixed values of the random field. The corresponding ground state of the system at a given value of the coupling $J$ is indicated below. Here vertical dashed lines denote the possible domain wall positions. State of spins in the region \{7, 11\} is not fixed, they form a DLS(see text).