CONTRAINS FROM THE DAMPING TAIL

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ABSTRACT

The detection of anisotropy in the cosmic microwave background (CMB) on arcminute scales by the Cosmic Background Imager (CBI) provides us with our first measurement of the damping tail and closes one chapter in the CMB story. We now have experimental verification for all of the features in the temperature anisotropy spectrum predicted theoretically two decades ago. The CBI result allows us to constrain both parameterized models based on the inflationary cold dark matter paradigm and to examine model-independent constraints on the matter content, the distance to last scattering, and the thickness of the last scattering surface. In particular, we show that recombination had to proceed “slowly,” with the surface of last scattering having a width \( \Delta \alpha \gtrsim 50 \). This provides strong constraints on nonstandard recombination scenarios. By providing a lower limit on the duration of recombination, it implies a lower limit on the polarization of the subdegree scale anisotropy, which is close to current experimental upper limits.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

Recently the Cosmic Background Imager (CBI) team announced the first detection of anisotropy in the cosmic microwave background (CMB) on angular scales of several arcminutes (Padin et al. 2001). They quote two “band powers,” with amplitudes \( [l(l+1)C_l/(2\pi)]^{1/2} = 59^{+7}_{-6.3} \) and \( 29.7^{+4.8}_{-4.2} \) \( \mu K \), corresponding to window functions centered at \( l = 603_{-166}^{+180} \) and \( 1190_{-224}^{+261} \), respectively. (Here \( l \) is the spherical harmonic index in a multipole expansion of the temperature fluctuation on the sky.) This detection is important not only for the specific constraints that it places on theories of the anisotropy spectrum, to which we shall return shortly, but also because it closes one phase of anisotropy research.

Theoretical models of CMB anisotropy, based on the growth of structure through gravitational instability in a dark matter–dominated universe, predict that the angular power spectrum of the temperature anisotropy should have three distinct parts, separated by two important physical scales (see, e.g., Bond 1998). With the 1992 discovery of anisotropy by the COBE experiment (Smoot et al. 1992), we obtained experimental verification of the first part: the plateau in the spectrum at large angular scales (low \( l \)), which is generated as photons lose energy climbing out of potentials on the last scattering surface (Sachs & Wolfe 1967). Numerous experiments (most recently, Miller et al. 1999; de Bernardis et al. 2000; Hanany et al. 2000; see Tegmark, Zaldarriaga, & Hamilton 2000 for a more complete list) have now reported detections of an acoustic peak in the power spectrum on degree scales (\( l \sim 200 \)). The second part of the anisotropy spectrum, this provides us with a snapshot of sound waves in the surface of last scattering (Peebles & Yu 1970; Doroshkevich, Zeldovich, & Sunyaev 1978) and encodes a wealth of information about cosmology and our model for structure formation. Now the detection of fluctuations on scales of several arcminutes (\( l \sim 10^3 \)) provides for the first time experimental verification of the final piece of the spectrum: the diffusion damping tail (Silk 1968). Although the important, predicted polarization is yet to be detected and much work remains to refine our knowledge of the spectrum, the current experimental situation is in remarkable agreement with theoretical predictions made two decades ago (Wilson & Silk 1981; Silk & Wilson 1981; Vittorio & Silk 1984; Bond & Elsathiou 1984, 1987), enhancing our faith in our paradigm for structure formation in the universe.

For many years theorists have been describing what we may learn from “future” measurements of the damping tail. The CBI measurement provides us with the opportunity to finally begin to implement these claims. In the following we do so, identifying some constraints on cosmological models and models for structure formation arising from the CBI data. Padin et al. (2001) have already stated limits on a subset of the popular theoretical models of structure formation; here we point out some additional constraints on the general paradigm. These will strengthen considerably as CBI reports data covering more sky and binned more finely in \( l \).

2. THE BIG PICTURE

Another detection on smaller angular scales (\( l \sim 5600 \); Dawson et al. 2000) at a lower amplitude is consistent with Sunyaev-Zeldovich (SZ) fluctuations (Sunyaev & Zeldovich 1980) expected in popular models of structure formation. These models predict that the SZ effect is the dominant secondary anisotropy on arcminute scales, that the bulk of the signal comes from clusters of galaxies spread over a range of redshifts, and that the signal falls off rapidly toward the larger angular scales probed by CBI. The CBI detection is of sufficient amplitude that it is highly unlikely to be secondary anisotropy—we shall assume from now on that CBI is constraining the primordial anisotropy from last scattering.

The CBI measurement shows that the spectrum has begun to damp significantly by \( l \sim 1000 \), as expected from photon diffusion during recombination (Silk 1968). A finer binning in \( l \) would be required to verify that the damping is (close to) exponential and to constrain the damping scale, \( l_D \), more precisely from the shape of the decline in power.

Within the standard cosmological model, on the scales probed by the CBI experiment, the CMB power spectrum depends mainly on the primordial power spectrum, the
physical matter density \( (\rho_{\text{mat}} \propto \omega_{\text{mat}} \equiv \Omega_{\text{mat}} h^2) \), the baryon density \( (\rho_b \propto \omega_b \equiv \Omega_b h^2) \), and the (comoving) angular diameter distance to last scattering \( r_c \). (There is a small correction to the spectrum arising from gravitational lensing, which we can safely ignore.) If we assume that the primordial power spectrum has no sharp features, we can isolate the other three important ingredients of the model: \( \omega_{\text{mat}}, \omega_b, \) and \( r_c \).

The new observational limit here is on the damping scale, which has the nice property that it depends primarily on the background cosmology\(^1\) and not on the assumed model of structure formation (inflation, defects, etc.). To a first approximation the damping scale is the geometric mean of the horizon and the photon mean free path just before recombination (Kaiser 1983; see Hu & White 1997 for numerical fitting functions). Thus, an increase in the matter density, which decreases the size of the horizon at last scattering, will shift the damping to smaller angular scales. For baryon densities consistent with big bang nucleosynthesis the damping scale is also shifted to smaller angular scales by a decrease in the mean free path (an increase in the baryon density).

Let us begin by considering general inferences drawn from the locations of the gross features (peaks and damping) in the spectrum. The first acoustic peak appears to lie at \( l_0 \sim 200 \) (Miller et al. 1999; de Bernardis et al. 2000; Hanany et al. 2000), providing us with a first rough measurement \( l_p/l_A \sim 5 \). This ratio can be interpreted as the Q of the sound "cavity" (fluid at last scattering), and \( Q \sim 5 \) is in accord with our theoretical expectations (Hu & White 1996). Theoretically, \( l_p/l_A \) is independent of distance to last scattering and only weakly dependent on the assumed energy content, e.g., \( \rho_{\text{mat}}/\rho_r \), and baryon content.\(^2\) A ratio in the range 4–6 is a strong indication that the fluctuations are adiabatic, such as are produced "uniquely" by inflation (Hu, Turner, & Weinberg 1994; Liddle 1995). Unfortunately, within the current uncertainties on \( l_p \) the constraints, while disfavoring a shift in \( l_p/l_A \) by a factor of 1.5 as predicted by isocurvature models, are not very tight.

Narrowing our attention to adiabatic models, if we combine data on the first peak with the highest \( l \) constraint from CBI, we can impose a lower limit on the matter density. Recall from above that the damping scale is moved to smaller angular scales as \( \omega_{\text{mat}} \) is increased. Holding \( \omega_b \) fixed at 0.02 the ratio of the amplitudes of the fourth to the first peak grows by a factor of 2 as we increase \( \omega_{\text{mat}} \) from 0.05 to 0.25. Although this ratio is affected by changes in the spectral index, it is not affected by changes in the normalization or a late epoch of reionization. Taking the band powers as measures of the power at their central \( l \), we estimate that observationally this ratio is close to \( 2 \). This corresponds to \( \omega_{\text{mat}} \sim 0.15 \) under our assumptions. Values of \( \omega_{\text{mat}} \sim \omega_b \) would have this ratio considerably lower, although experimental uncertainties are such that models of this type are not (yet) completely ruled out by the current CMB data. A full study of parameter space and accounting for experimental uncertainties would be necessary to draw firm conclusions. Such a study will be highly informative when additional data from CBI are released, particularly to higher \( l \).

3. CONSTRAINTS ON MODELS

Let us now turn to more quantitative constraints on our models. With only two band powers we cannot provide limits on a large parameter space, so we shall here consider various effects in turn rather than varying all of them together. To avoid questions of the calibration uncertainty, the normalization of the spectrum, and any late (\( z \lesssim 10^5 \)) epoch of reionization, we shall focus our attention on the ratio of the band powers. Experimentally, this ratio is close to 2 and should lie between 1.3 and 3 at \( \pm 2 \sigma \). When computing theoretical predictions for this ratio we shall approximate the window functions as Gaussians centered on \( l = 603 \) and 1190 with \( \sigma = 104 \) and 146, respectively. Pearson et al. (2001) have shown a Gaussian is a good approximation for the visibility window function; we shall assume this holds for the band-power window function also.

The first limit we shall consider is on the (comoving) angular diameter distance to last scattering, which is a sensitive function of spatial curvature.

\[
r_c = \left| K \right|^{-1/2} \sinh \left[ \left| K \right|^{1/2}(\eta_o - \eta_s) \right],
\]

for \( K < 0 \) (for positive curvature replace \( \sinh \) with \( \sin \)). Here \( K = H_0^2(\Omega_m - 1) \) is the spatial curvature, \( \eta_o = \int dt/a \) is the conformal age of the universe today, \( \eta_s \) is the (conformal) age at last scattering, and we have set \( c = 1 \). At fixed \( \omega_{\text{mat}} \) and \( \omega_b \) the \( l \) of any feature in the spectrum depends linearly on this distance. The amount of power near the damping tail depends exponentially on \( l \), so this allows another test of curvature of the universe.

For fixed distance, \( r_c \), and matter density, \( \omega_{\text{mat}} \), the band-power ratio is quite insensitive to \( \omega_b \) so we shall here hold it fixed at 0.02 (O’Meara et al. 2001). We shall also assume that the underlying spectrum is scale-invariant for simplicity. Since the band powers are separated by a factor of 2 in angular scale, a deviation of \( \pm 0.2 \) in the spectral index translates roughly into a \( \pm 15\% \) change in the band-power ratio, which can be safely ignored. Our assumptions can be relaxed when band powers covering a wider range of \( l \) become available.

Figure 1 shows contours of the band-power ratio in the \( \omega_{\text{mat}}-r_c \) plane. Any upper limit to the distance is quite sensitive to our assumed lower limit on the band-power ratio. The lower limit on the distance is, however, reasonably robust: for any reasonable cosmological parameters \( r_c \gtrsim 6000 \) Mpc (comoving, with \( h = 0.65 \)). While this limit could be derived from considering, e.g., the first peak in the spectrum, the damping tail has the advantage of being less dependent on the assumption of a particular model for the calculation.

Our limit can be interpreted either as a constraint on late-time physics, which changes the distance-redshift relation while holding the redshift of last scattering (roughly) constant, or on more speculative physics, which modifies the redshift of recombination through, e.g., energy injection. The first case has been considered by Padin et al. (2001). To illustrate how the second may now be strongly constrained, we have calculated the anisotropies expected for the Ostriker & Steinhardt (1995) "concordance model"—a standard \( \Lambda \)CDM model with \( \Omega_m = 0.3, h = 0.67, \) and \( n = 1 \)—replacing the hydrogen-ionized fraction \( x_h \) by a Fermi function \( (1 + e^{-x})^{-1} \), where \( s \equiv (z - z_e)/\Delta z \). Standard

\(^1\) See, e.g., Hu & White (1997) for a discussion of technical caveats.

\(^2\) Since \( l_p \) is independent of \( \omega_b \) near \( \omega_b = 0.02 \), the ratio increases slowly with increasing \( \omega_b \). Current constraints on \( \omega_b \) (O’Meara et al. 2001) make this increase negligible.
recombination is well fit by $z_m \approx 1200$ and $\Delta z \approx 80$. A sampling of the spectra, computed using the code described in White & Scott (1996), are shown in Figure 2. Recall that our limit on $z_m$ will depend slightly on the particular cosmological parameters chosen, but should be very insensitive to the details of the model for structure formation (e.g., inflationary cold dark matter [CDM]).

A modification to the time or duration of recombination was discussed as a solution to the apparently missing second peak in the BOOMERANG data (de Bernardis et al. 2000; Hu 2000; White, Scott, & Pierpaoli 2000; Peebles, Seager, & Hu 2000; Tegmark & Zaldarriaga 2000). From Figure 2 detection of significant anisotropy at $l \approx 1000$ constrains the last scattering surface to lie at $z \gtrsim 800$ for the "concordance" cosmology.

Our picture of the ionization history of the universe is, thus, as follows: the visibility of flux shortward of Ly$\alpha$ in high-$z$ quasars (e.g., Fan et al. 2000) indicates that the universe is highly ionized back to $z \approx 6$. The detection of anisotropy on degree scales indicates that the universe was neutral above a redshift of $\sim 30$ (Tegmark et al. 2000). We have now been able to demonstrate that the universe reionized between $z \sim 800$ and $z \sim 1600$ when the temperature was $2000-4000$ K. This is, not surprisingly, in accord with our understanding of recombination physics (e.g., Seager, Sasselov, & Scott 1999) in the "standard" cosmology, and we expect the universe was ionized at all higher redshifts.

While the above limit is on the distance to the last scattering surface, we can also limit the thickness of the last scattering surface. A modification to the duration of recombination was discussed as a solution to the apparently missing second peak in the Boomerang data as described above. Because the power at $l \approx 10^3$ is "low," we know the duration of recombination cannot be too short. If recombination proceeded as quickly as the standard Saha theory (e.g., Lang 1980) would predict, for example, we would expect significantly more power at $l \approx 10^3$ than is observed (see Fig. 9 of Hu et al. 1995). To demonstrate the sense of this effect we have modified $\Delta z$ in our mock-up of recombination described above. A sampling of the spectra with $\Delta z$ ranging from 40 to 140 are shown in Figure 3. As a point weighted toward $l \approx 1500$ is not available, the unknown distance to last scattering introduces considerable uncertainty in the upper limit on $\Delta z$ using only the CBI data. We can see, however, that recombination cannot be much
shorter than $\Delta z \sim 50$—recombination to the ground state is inhibited by the recombination photons (Novikov & Zeldovich 1967).

Our lower limit on $\Delta z$ brings us nicely to the next fundamental CMB milestone—detection of polarization. There are extremely strong theoretical reasons to believe that the anisotropy is polarized at a low level, since the angular dependence of Thomson scattering is sensitive to polarization. To date, only upper limits have been reported (e.g., Staggs, Gundersen, & Church 2000; Hedman et al. 2001), but they remain above the theoretical predictions of popular models. Since polarization arises from scattering, it is generated “during” last scattering and possibly in a second, closer, scattering surface during reionization. The thicker the last scattering surface, the stronger the polarized signal, but also the larger the angular scale at which damping becomes effective. Our lower limit on the duration of recombination is thus a lower limit on the polarization of the subdegree scale anisotropy. This is shown in the lower panel of Figure 3, where the thicker last-scattering surfaces have enhanced power at $l \sim 300$ and a peak polarization signal shifted to lower $l$ by the increased damping.

4. CONCLUSIONS

Our theories suggest that the CMB power spectrum consists of three regions separated by two physical scales (the sound horizon and the damping length). With the detection of power on arcminute scales by the CBI experiment, all of the major parts of the temperature anisotropy spectrum have been observed: the low-$l$ plateau, the acoustic peaks, and the damping region. The results are strikingly similar to theoretical predictions made nearly two decades ago, lending further support to a model in which the large-scale structure grew through gravitational instability from small primordial perturbations in density (presumably laid down by inflation).

For many years theorists have been describing what we may learn from “future” measurements of the damping tail. Here we have begun to implement this program. A number of constraints on parameterized models in the CDM family have already been presented by Padin et al. (2001) on the basis of the data released to date. We have presented some different and more model-independent constraints, including constraints on the matter density, the distance to last scattering, and the duration of last scattering. Popular models based on inflationary CDM pass all of these constraints easily, while many nonstandard cosmological models fare less well. By providing a lower limit on the thickness of the last scattering surface, the CBI measurement implies a lower limit to the polarization of the subdegree scale anisotropy that is marginally lower than current upper limits. Further data from CBI and other experiments at these angular scales should enable us to extract more of the cosmological information contained in the damping tail.

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