Masses and widths of tetraquarks with hidden bottom

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Abstract

The relativistic four-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing of the meson-meson states with the four-quark states is considered. The four-quark amplitudes of the tetraquarks with hidden bottom, including $u$, $d$, $s$ and bottom quarks, are constructed. The poles of these amplitudes determine the masses and widths of tetraquarks.

Keywords: Tetraquarks with hidden bottom, coupled-channel formalism.

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I. Introduction.

The remarkable progress at the experimental side has opened up new challenges in the theoretical understanding of heavy flavor hadrons. The observation of $X(3872)$ resonance [1] has been confirmed by CDF [2], D0 [3] and BaBar Collaboration [4]. Belle Collaboration observed the $X(3940)$ in double-charmonium production in the reaction $e^+e^- \rightarrow J/\psi + X$ [5]. The state, designated as $X(4160)$, was reported by the Belle Collaboration in Ref. 6. The fact that the newly found states do not fit quark model calculations [7] has opened the discussion about the structure of such states. Maiani et al. advocate a tetraquark explanation for the $X(3872)$ [8, 9]. On the other hand, the mass of $X(3872)$ is very close to the threshold of $D^*D$ and, therefore, it can be interpreted as molecular state [10 – 15]. In our paper [16] the dynamical mixing between the meson-meson states and the four-quark states is considered. Taking the $X(3872)$ and $X(3940)$ as input [17, 18] we predicted the masses and widths of $S$-wave tetraquarks with open and hidden charm.

In the present paper the relativistic four-quark equations for the tetraquarks with hidden bottom in the framework of the dispersion relation technique are found. We searched for the approximate solution of these equations by taking into account two-particle, triangle and four-particle singularities, all the weaker ones being neglected. The masses and widths of the low-lying bottom tetraquarks are calculated.

After this introduction, we discuss the four-quark amplitudes, which contain the two bottom quarks (Section II). In Sect. III, we report our numerical results (Tables I and II).

II. Four-Quark Amplitudes for the Tetraquarks with Hidden Bottom.

We derive the relativistic four-quark equations in the framework of the dispersion relations technique.
The correct equations for the amplitude are obtained by taking into account subsystems with the smaller number of particles. Then one should represent a four-particle amplitude as a sum of six subamplitudes:

\[ A = A_{12} + A_{13} + A_{14} + A_{23} + A_{24} + A_{34}. \]  

(1)

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams.

We need to consider only one group of diagrams and the amplitude corresponding to them, for example \( A_{12} \). We shall consider the derivation of the relativistic generalization of the Faddeev-Yakubovsky approach [19, 20] for the tetraquark.

The four-quark amplitude of \( b\bar{b}u\bar{u} \) tetraquark includes the quark amplitudes with quantum numbers of \( 0^{--} \) and \( 1^{--} \) mesons. The set of diagrams associated with the amplitude \( A_{12} \) can further be broken down into five groups corresponding to subamplitudes: \( A_1(s, s_{12}, s_{34}), A_2(s, s_{23}, s_{14}), A_3(s, s_{34}, s_{123}), A_4(s, s_{34}, s_{234}), A_5(s, s_{12}, s_{123}) \), if we consider the tetraquark with the \( J^{PC} = 2^{++} \).

Here \( s_{ik} \) is the two-particle subenergy squared, \( s_{ijk} \) corresponds to the energy squared of particles \( i, j, k \) and \( s \) is the system total energy squared.

In order to represent the subamplitudes \( A_1(s, s_{12}, s_{34}), A_2(s, s_{23}, s_{14}), A_3(s, s_{34}, s_{123}), A_4(s, s_{34}, s_{234}), A_5(s, s_{12}, s_{123}) \) in the form of dispersion relations it is necessary to define the amplitudes of quark-antiquark interaction \( a_n(s_{ik}) \). The pair quarks amplitudes \( q\bar{q} \to q\bar{q} \) are calculated in the framework of the dispersion \( N/D \) method with the input four-fermion interaction [21 – 23] with quantum numbers of the gluon [24]. The regularizion of the dispersion integral for the \( D \)-function is carried out with the cutoff parameter \( \Lambda \). The four-quark interaction is considered as an input [24]:

\[ g_V(q\lambda I f \gamma_\mu q)^2 + g_V^{(s)}(q\lambda I f \gamma_\mu q)(\bar{s}\lambda \gamma_\mu s) + g_V^{(ss)}(\bar{s}\lambda \gamma_\mu s)^2. \]  

(2)

Here \( I_f \) is the unity matrix in the flavor space \( (u, d) \). \( \lambda \) are the color Gell-Mann matrices. Dimensional constants of the four-fermion interaction \( g_V, g_V^{(s)} \) and \( g_V^{(ss)} \) are the parameters of the model. At \( g_V = g_V^{(s)} = g_V^{(ss)} \) the flavor \( SU(3)_f \) symmetry occurs. The strange quark violates the flavor \( SU(3)_f \) symmetry. In order to avoid an additional violation parameters, we introduce the scale shift of the dimensional parameters [24]:

\[ g = \frac{m^2}{\pi^2} g_V = \frac{(m + m_s)^2}{4\pi^2} g_V^{(s)} = \frac{m^2_s}{\pi^2} g_V^{(ss)}. \]  

(3)

\[ \Lambda = \frac{4\Lambda(ik)}{(m_i + m_k)^2}. \]  

(4)

Here \( m_i \) and \( m_k \) are the quark masses in the intermediate state of the quark loop. Dimensionless parameters \( g \) and \( \Lambda \) are supposed to be constants which are independent of the quark interaction type. The applicability of Eq. (2) is verified by the success of De Rujula-Georgi-Glashow quark model [25], where only the short-range part of Breit potential connected with the gluon exchange is responsible for the mass splitting in hadron multiplets.

We use the results of our relativistic quark model [24] and write down the pair quarks amplitude in the form:

\[ a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})}, \]  

(5)
\[
B_n(s_{ik}) = \frac{(m_i + m_k)^2 \Lambda}{\pi} \int_{(m_i + m_k)^2} ds'_{ik} \rho_n(s'_{ik}) G_n^2(s'_{ik}),
\]
(6)

Here \(G_n(s_{ik})\) are the quark-antiquark vertex functions. The vertex functions satisfy the Fierz relations. All of these vertex functions are generated from \(g_V, g_V^{(s)}\) and \(g_V^{(ss)}\). \(B_n(s_{ik}), \rho_n(s_{ik})\) are the Chew-Mandelstam functions with cutoff \(\Lambda\) and the phase spaces, respectively.

In the case in question, the interacting quarks do not produce a bound state; therefore, the integration in Eqs. (7) – (11) is carried out from the threshold \((m_i + m_k)^2\) to the cutoff \(\Lambda(ik)\). The coupled integral equation systems (the meson state \(J^{pc} = 2^{++}\) for the \(b\bar{b}u\bar{u}\)) can be described as:

\[
A_1(s, s_{12}, s_{34}) = \frac{\lambda_1 B_1(s_{12}) B_1(s_{34})}{[1 - B_1(s_{12})][1 - B_1(s_{34})]} + 4 \hat{J}_2(s_{12}, s_{34}, 1, 1) A_3(s, s'_{23}, s'_{123}) ,
\]
(7)

\[
A_2(s, s_{23}, s_{14}) = \frac{\lambda_2 B_1(s_{23}) B_1(s_{14})}{[1 - B_1(s_{23})][1 - B_1(s_{14})]} + 2 \hat{J}_2(s_{23}, s_{14}, 1, 1) A_4(s, s'_{34}, s'_{234})
+ 2 \hat{J}_2(s_{23}, s_{14}, 1, 1) A_5(s, s'_{12}, s'_{123}),
\]
(8)

\[
A_3(s, s_{23}, s_{123}) = \frac{\lambda_3 B_2(s_{23})}{[1 - B_2(s_{23})]} + 2 \hat{J}_3(s_{23}, 2) A_1(s, s'_{12}, s'_{34})
+ \hat{J}_1(s_{23}, 2) A_4(s, s'_{34}, s'_{234}) + \hat{J}_1(s_{23}, 2) A_5(s, s'_{12}, s'_{123}),
\]
(9)

\[
A_4(s, s_{34}, s_{234}) = \frac{\lambda_4 B_2(s_{34})}{[1 - B_2(s_{34})]} + 2 \hat{J}_3(s_{34}, 2) A_2(s, s'_{23}, s'_{14})
+ 2 \hat{J}_1(s_{34}, 2) A_3(s, s'_{23}, s'_{234}),
\]
(10)

\[
A_5(s, s_{12}, s_{123}) = \frac{\lambda_5 B_2(s_{12})}{[1 - B_2(s_{12})]} + 2 \hat{J}_3(s_{12}, 2) A_2(s, s'_{23}, s'_{14})
+ 2 \hat{J}_1(s_{12}, 2) A_3(s, s'_{23}, s'_{123}),
\]
(11)

where \(\lambda_i, \ i = 1, 2, 3, 4, 5\) are the current constants. They do not affect the mass spectrum of tetraquarks. Here \(n = 1\) corresponds to a \(q\bar{q}\)-pair with \(J^{pc} = 1^{--}\) in the \(1_c\) color state, and \(n = 2\) defines the \(q\bar{q}\)-pairs corresponding to tetraquarks with quantum numbers: \(J^{pc} = 0^{++}, 1^{++}, 2^{++}\). We introduce the integral operators:

\[
\hat{J}_1(s_{12}, l) = \frac{G_l(s_{12})}{[1 - B_l(s_{12})]} \int \frac{ds'_{12} G_l(s'_{12}) \rho_l(s'_{12})}{s'_{12} - s_{12}} \int \frac{dz_1}{2},
\]
(12)

\[
\hat{J}_2(s_{12}, s_{34}, l, p) = \frac{G_l(s_{12}) G_p(s_{34})}{[1 - B_l(s_{12})][1 - B_p(s_{34})]} \int \frac{ds'_{12} G_l(s'_{12}) \rho_l(s'_{12})}{s'_{12} - s_{12}}.
\]
where zeros of $D$ determinants define the masses of bound states of tetraquarks. $F_i(s, \lambda_i)$ determine the contributions of subamplitudes to the tetraquark amplitude.

III. Calculation results.

The model in consideration has only two parameters: the cutoff $\Lambda = 7.63$ and the gluon coupling constant $g = 1.53$. The experimental mass values of tetraquarks with the hidden bottom are absent. Therefore these parameters are determined by fixing the tetraquark masses for the $J^{PC} = 1^{++} X_b(10300)$ and $J^{PC} = 2^{++} X_b(10340)$ in the paper [26]. The widths of the tetraquarks with the hidden bottom are fitted by the fixing width $\Gamma_{2^{++}} = (39 \pm 26) MeV$ [27] for the $S$-wave tetraquark with the hidden charm $X(3940)$. The quark masses of model $m_{u,d} = 385 MeV$ and $m_s = 510 MeV$ coincide with the ordinary meson ones in our model [24]. In order to fix anyhow $m_b = 4787 MeV$, we use the tetraquark mass for the $J^{PC} = 2^{++} X_b(10340)$. The masses of meson-meson states with the spin-parity $J^{PC} = 0^{++}, 1^{++}, 2^{++}$ are considered in Table I. The contributions of the subamplitudes to the tetraquark amplitudes (hidden bottom) in the Table I are given. The contributions of the four-quark states $Q\bar{q}Qq$ are about $20\% - 50\%$ for the meson-meson states. The functions $F_i(s, \lambda_i)$ (Eq. (15)) allow us to obtain the overlap factors $f$ and the phase spaces $\rho$ for the reaction $X_b \rightarrow M_1M_2$.
We calculated the widths of the tetraquarks with hidden bottom (Table II). We used the formula $\Gamma \sim f^2 \times \rho$ [28].

The results of calculations allow us to consider the tetraquarks with hidden bottom as the narrow resonances. The calculated width of $X_b(10300)$ tetraquark with the mass $M = 10303\, MeV$ and the spin-parity $J^{pc} = 1^{++}$ is about $\Gamma_{1^{++}} = 43\, MeV$. In our model we neglected with the mass distinction of the $u$ and the $d$ quarks. The width of $X_b(9940)$ tetraquark with the mass $M = 9936\, MeV$ and the spin-parity $J^{pc} = 0^{++}$ is equal to $\Gamma_{0^{++}} = 96\, MeV$. We calculated also the width of $X_b(10340)$ tetraquark $\Gamma_{2^{++}} = 80\, MeV$ and the width of $X_b(10550)$ tetraquark $(b\bar{b}\bar{s}s)\, \Gamma_{2^{++}} = 58\, MeV$. The tetraquarks with the spin-parity $J^{pc} = 0^{++}, 1^{++}$ ($s\bar{s}b\bar{b}$) have only the weak decays.

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Table I. Low-lying meson-meson state masses (MeV) of tetraquarks with hidden bottom and the contributions of subamplitudes to the tetraquark amplitudes (in percent).

Parameters of model: quark masses $m_u = 385\,\text{MeV}$, $m_s = 510\,\text{MeV}$, $m_b = 4787\,\text{MeV}$, cutoff parameter $\Lambda = 7.63$, gluon coupling constant $g = 1.531$.

| Tetraquark | $J^{pc} = 2^{++}$ | $J^{pc} = 1^{++}$ | $J^{pc} = 0^{++}$ |
|------------|-------------------|-------------------|-------------------|
| Masses:    | 10341 MeV         | 10303 MeV         | 9936 MeV          |
| $(b\bar{b})_{-}-(u\bar{u})_{-}$ | 54.54             | 44.64             | 17.00             |
| $(u\bar{b})_{-}-(b\bar{u})_{-}$ | 1.36              | 1.57              | 0.44              |
| $(b\bar{b})_{0}-(u\bar{u})_{0}$ | 0                 | 0                 | 66.12             |
| $(u\bar{b})_{0}-(b\bar{u})_{0}$ | 0                 | 0                 | 0.72              |

| Tetraquark | $J^{pc} = 2^{++}$ | $J^{pc} = 1^{++}$ | $J^{pc} = 0^{++}$ |
|------------|-------------------|-------------------|-------------------|
| Masses:    | 10552 MeV         | 10370 MeV         | 10094 MeV         |
| $(b\bar{b})_{-}-(s\bar{s})_{-}$ | 48.65             | 37.14             | 16.86             |
| $(s\bar{b})_{-}-(b\bar{s})_{-}$ | 2.42              | 2.62              | 0.68              |
| $(b\bar{b})_{0}-(s\bar{s})_{0}$ | 0                 | 0                 | 63.58             |
| $(s\bar{b})_{0}-(b\bar{s})_{0}$ | 0                 | 0                 | 1.12              |

Table II. The widths, overlap factors $f$ and phase spaces $\rho$ of tetraquarks with hidden bottom.

| Tetraquark (channels) | $J^{pc}$ | $f$  | $\rho$ | widths (MeV) |
|-----------------------|----------|------|--------|--------------|
| $X_6(9940)$ $\eta_{bb}$ | 0^{++}   | 0.66 | 0.0629 | 96           |
| $X_6(10300)$ $Y_{\rho}$ | 1^{++}   | 0.45 | 0.0617 | 43           |
| $X_6(10340)$ $Y_{\rho}$ | 2^{++}   | 0.55 | 0.0775 | 80           |
| $X_6(10550)$ $Y_{\varphi}$ | 2^{++}   | 0.49 | 0.0700 | 58           |
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