Intermediate inflation on the brane and warped DGP models

Ramón Herrera, Marco Olivares and Nelson Videla

Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile.

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Abstract

Brane and warped Dvali-Gabadadze-Porrati inflationary universe models in the context of intermediate inflation are studied. In both models we consider that the energy density is a standard scalar field and we discuss the evolution of the universe during the slow-roll inflation. Also, we describe the conditions for these models. The corresponding Wilkinson Microwave Anisotropy Probe seven year data are utilized to fix some parameters in our models. Our results are compared to those found in General Relativity.

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*Electronic address: ramon.herrera@ucv.cl
†Electronic address: marco.olivares@ucv.cl
‡Electronic address: nelson.videla@ucv.cl
I. INTRODUCTION

It is well known that the inflationary model is to date the most compelling solution to many long-standing problems of the standard Big Bang model (horizon, flatness, monopoles, etc.)[1, 2]. The most significant feature of the inflationary universe model is that it supplies a causal interpretation of the origin of the observed anisotropy of the cosmic microwave background radiation (CMB) and the distribution of large scale structure in the universe[3, 4].

As regards exact solutions, the acceleration of the universe due to an exponential potential is often called power-law inflation, since the scale factor has an evolution power law type[5]. In addition, de-Sitter inflationary universe is produced by a constant scalar potential, see Ref.[1]. Following Ref.[6], exact solutions can also be found for the scenario of intermediate inflation. In this inflationary model the scale factor, \(a(t)\), increments as
\[
a = \exp[A t^f],
\]
where \(A\) and \(f\) are two constants; \(A > 0\) and \(0 < f < 1\)[6]. The expansion of this scale factor is slower than de-Sitter inflation, but faster than power law inflation, this is the reason why it is known as "intermediate". It is well known that the intermediate inflationary model was originally formulated as an exact solution, but it may be best developed from the slow-roll approximation. Considering this approximation, it is possible to obtain a spectral index \(n_s \sim 1\). In particular, the value \(n_s = 1\) (Harrison-Zel’dovich spectrum) is found for the value of the parameter \(f = 2/3\)[7], but this value of the spectral index is disfavored by the current Wilkinson Microwave Anisotropy Probe (WMAP) observational data[3, 4]. Also, an important observational quantity in the intermediate model, is the tensor to scalar ratio \(r\), which is significantly \(r \neq 0\)[8, 9] for values of \(f \neq 2/3\). Other motivation to consider this type of the expansion comes from string/M-theory, which appears to be relevant in the low-energy string effective action[10, 11] (see also, Refs.[12–15]). These theories can be utilized to solve the initial singularity and describe the present acceleration in the universe, among others[16]. In this way, the intermediate inflation model may be derived from an effective theory at low dimensions of a fundamental string theory.

On the other hand, implications of string/M-theory to Friedmann-Robertson-Walker (FRW) cosmological models have currently attracted a great deal of attention; specifically some were concerned with braneantibrane configurations such as space-like branes and the
implications to cosmology\cite{17}. In this configuration the standard model of particles is confined to the brane, while gravitation propagates into the bulk space-time. Here, the effect of extra dimensions induces extra terms in the Friedmann equation \cite{18,20}. In particular, the cosmological Randall-Sundrum (RS) type II scenario has received great attention in the last years\cite{21}. This alternative to Einstein’s general relativity cosmological models is called brane-world cosmology. For a comprehensible review of brane-cosmology, see e.g. Refs.\cite{22–24}.

In the Dvali-Gabadadze-Porrati (DGP) model \cite{25} the induced gravity brane world was presented as a replacement to the RS one-brane model \cite{21}. Here, general relativity was recuperated, also despite an infinite extra dimension, but without warping in 5-dimensional Minkowski space-time. In this model, the gravitational actions on the brane are controlled by the rival between the 5D curvature scalar in the bulk and the 4D curvature scalar on the brane. In comparison to the RS case with high energy modifications to general relativity, the DGP model produced a low energy modification. In the DGP brane, in relation to the embedding of the brane in the bulk, there are present two branches of background solutions, because there are two different ways to embed the 4-dimensional brane into the 5-dimensional space-time. For a comprehensible review of the phenomenology of DGP model, see\cite{26}; and inflation models and reheating in this scenario were studied in Refs.\cite{27–31}.

The target of this work is to present two models of inflationary universes in the context of intermediate inflation, namely, (i) when the scalar field is confined to the brane and (ii) when the scalar field is confined on a warped DGP. We shall resort to the seven-year data of the Wilkinson Microwave Anisotropy Probe (WMAP) to restrict the coefficients in both models. In particular, we find constraints on the fundamental parameters.

The outline of the paper is as follows. The next section presents the brane-intermediate Inflationary scenario for our model and in subsection \textbf{II B} deals with the calculations of cosmological perturbations. In Section \textbf{III} we study the warped DGP model in the context of the intermediate inflation and in subsection \textbf{III B} we also determine the corresponding cosmological perturbation. Finally, in Sect.\textbf{IV} we summarize our finding. We chose units so that $c = \hbar = 1$. 
II. INTERMEDIATE INFLATION ON THE BRANE

A. The model and the basic equations

We consider the five-dimensional brane scenario, in which the flat Friedmann equation is given by\[19, 32\]
\[
H^2 = \kappa \rho \left[ 1 + \frac{\rho}{2\tau} \right] + \frac{\Lambda_4}{3} + \frac{\xi}{a^4},
\]
(2)
where \(H = \dot{a}/a\) denotes the Hubble parameter, \(a\) represents the scale factor and \(\rho\) is the matter field confined to the brane. \(\Lambda_4\) is the 4-dimensional cosmological constant and the constant \(\kappa = 8\pi G/3 = \frac{8\pi}{3m_p^2}\). Here, the term \(\xi/a^4\) denotes the influence of the bulk gravitons on the brane, where \(\xi\) is an integration constant. The parameter \(\tau\) is the brane tension and is relates the 4- and 5-dimensional Planck masses through \(m_p = \sqrt{3M_5^6/(4\pi\tau)}\). From nucleosynthesis, the brane tension is constrained by the value \(\tau > (1\text{MeV})^4 [33]\). Nevertheless, a stronger constraint for the brane tension results from current tests for deviation from Newton’s law, in which \(\tau \geq (10\text{ TeV})^4 [34, 35]\).

In the following, we will assume that the cosmological constant \(\Lambda_4 = 0\), and once inflation begins, the quantity \(\xi/a^4\) will rapidly become unimportant, so that the Friedmann Eq.(2) results in

\[
H^2 = \kappa \rho \left[ 1 + \frac{\rho}{2\tau} \right].
\]
(3)

In the following, we consider that the matter field \(\rho\) is a standard scalar field \(\phi\), in which the energy density \(\rho = \frac{\dot{\phi}^2}{2} + V(\phi)\), and the pressure \(P = \frac{\dot{\phi}^2}{2} - V(\phi)\), where \(V(\phi) = V\) is the scalar potential. We assume that the scalar field \(\phi\) is confined to the brane, so that its field equation has the standard form

\[
\dot{\rho} + 3H(\rho + P) = 0,
\]
(4)
or equivalently

\[
\ddot{\phi} + 3H\dot{\phi} + V' = 0,
\]
(5)
where \(V' = \partial V(\phi)/\partial\phi\) and the dots mean derivatives with respect to the cosmological time.

In the following, we will not consider Eq.(3) in the lower energy limit \((\rho \ll \tau)\) or in the high energy limit \((\rho \gg \tau)\) as our initiating point, instead we will study the full Eq.(3).
Combining Eqs. (3) and (4) the square of the velocity $\dot{\phi}^2$ of the scalar field becomes

$$\dot{\phi}^2 = \frac{2}{3\kappa} \frac{(-\dot{H})}{\sqrt{1 + \frac{2H^2}{\kappa\tau}}},$$

(6)

and the effective potential can be written

$$V = \tau \left( -1 + \sqrt{1 + \frac{2H^2}{\kappa\tau}} \right) - \frac{1}{3\kappa} \frac{(-\dot{H})}{\sqrt{1 + \frac{2H^2}{\kappa\tau}}},$$

(7)

Note that in the low-energy limit, $\rho \ll \tau$, the expressions for $\dot{\phi}^2$ and the scalar potential $V$ given by Eqs. (6) and (7), reduce to the standard inflation, where $\dot{\phi}^2 = 2(-\dot{H})/3\kappa$ and $V = (3H^2 + \dot{H})/3\kappa$, see Ref. [9]. Note also that in the high energy limit, $\rho \gg \tau$, the values of $\dot{\phi}^2$ and the scalar potential $V$ reduce to the brane world cosmology in which, $\dot{\phi}^2 = \sqrt{2\tau/\kappa} (-\dot{H})^{3/2}$ and $V = \sqrt{2\tau/\kappa} [H - \frac{\dot{H}}{3\Lambda}]^{3/2}$. [36].

The solution for the standard scalar field $\phi$ at late times can be found from Eqs. (1) and (6) as

$$\phi(t) - \phi_0 = \frac{B[t]}{K},$$

(8)

where $\phi(t = 0) = \phi_0$ is an integration constant, $K$ is a constant equal to

$$K = \left[ \sqrt{\frac{3}{1 - f}} \left( \frac{\kappa}{2} \right)^{2-f} \right]^{\frac{1}{f-1}},$$

and the function $B[t]$, represents the incomplete Beta function [37], given by

$$B[t] = B\left[ \frac{\kappa\tau t^{2(1-f)}}{2(Af)^2}, 4(f-1), \frac{3}{4} \right].$$

From Eq. (8), the Hubble parameter as a function of the scalar field, $\phi$, becomes $H(\phi) = Af(B^{-1}[K(\phi)])^{f-1}$, where $B^{-1}$ represents the inverse function of the incomplete Beta function and without loss of generality we have used $\phi_0 = 0$.

Considering the slow-roll approximation, only the first term of the effective potential given by Eq. (7) predominates at large values of $\phi$. In this way, combining Eqs. (7) and (8), the effective potential as a function of the scalar field, $\phi$, results in

$$V(\phi) \simeq \tau \left( -1 + \sqrt{1 + \frac{2(Af)^2(B^{-1}[K(\phi)])^{-2(1-f)}}{\kappa\tau}} \right).$$

(9)

Here, we noted that we would have obtained the same effective potential represented by Eq. (9), considering the slow-roll conditions, in which $\dot{\phi}^2 \ll V(\phi)$ and $\ddot{\phi} \ll 3H\dot{\phi}$. Also, we
noted that in low-energy limit \( V(\phi) \propto \phi^{-4(f-1)} \) \cite{9} and in the high energy limit \( V(\phi) \propto \phi^{2(f-1)/3} \), see Ref. \cite{36}.

The dimensionless slow-roll parameters can be expressed as
\[
\varepsilon \equiv -\frac{\dot{H}}{H^2} = \frac{1-f}{Af[1-K(\phi)]^f}
\]
and
\[
\eta \equiv -\frac{\ddot{H}}{H \dot{H}} = \frac{2-f}{Af[1-K(\phi)]^f},
\]
respectively. The inflationary phase takes place whenever \( \varepsilon < 1 \) (or analogously \( \ddot{a} > 0 \)). Consequently, the condition for inflation to occur is satisfied when
\[
\phi > \frac{1}{\mathcal{R}} \mathcal{B} \left[ \left( \frac{1-f}{Af} \right)^{1/f} \right].
\]
Also, considering that inflation begins at the earliest possible phase (when \( \varepsilon = 1 \)) see Ref. \cite{9}, then \( \phi_1 \), can be expressed as
\[
\phi_1 = \frac{1}{\mathcal{R}} \mathcal{B} \left[ \left( \frac{1-f}{Af} \right)^{1/f} \right].
\]

Finally, the number of e-folds \( N \) between two values of cosmological times \( t_1 \) and \( t_2 \) or analogously between two different values \( \phi_1 \) and \( \phi_2 \) becomes
\[
N = \int_{t_1}^{t_2} H \, dt = A \left[ (t_2)^f - (t_1)^f \right] = A \left[ (\mathcal{B}^{-1}[K(\phi_2)])^f - (\mathcal{B}^{-1}[K(\phi_1)])^f \right]. \tag{10}
\]
Here, we have used Eq.\((8)\).

### B. Cosmological perturbations

In this subsection we will describe scalar and tensor perturbations, for our model. For a standard scalar field \( \phi \) the power spectrum \( \mathcal{P}_R \) of the curvature perturbations is given in the slow-roll approximation by the following expression \cite{22}:
\[
\mathcal{P}_R \simeq \left( \frac{H^2}{2\pi} \right)^2 = \frac{3\kappa}{8\pi^2} H^4 \left( -\dot{H} \right)^{-1} \sqrt{1 + \frac{2H^2}{\kappa \tau}}. \tag{11}
\]
Here, we have used Eq.\((6)\). The power spectrum can be written equivalently in terms of the standard scalar field \( \phi \) as
\[
\mathcal{P}_R = \frac{3\kappa}{8\pi^2} \frac{(Af)^3}{1-f} (\mathcal{B}^{-1}[K(\phi)])^{-(2-3f)} \sqrt{1 + \frac{2(Af)^2(\mathcal{B}^{-1}[K(\phi)])^{-2(1-f)}}{\kappa \tau}}. \tag{12}
\]
Combining Eqs.\((10)\) and \((12)\), \( \mathcal{P}_R \) can be expressed in terms of the number of e-folds \( N \), to give
\[
\mathcal{P}_R = \frac{3\kappa}{8\pi^2} \frac{(Af)^3}{1-f} \left[ \frac{Af}{1 + f(N-1)} \right]^{\frac{2-3f}{f}} \sqrt{1 + \frac{2(Af)^2 \left[ \frac{Af}{1+f(N-1)} \right]^{2(1-f)}}{\kappa \tau}}. \tag{13}
\]
Numerically from Eq.\((13)\), we obtained a constraint for the parameter \( A \). In fact, we can obtain the value of the parameter \( A \) for given values of \( f \) and the brane tension \( \tau \) when number \( N \) and the power spectrum \( \mathcal{P}_R \) are given. In particular, for the values \( \mathcal{P}_R =
$2.4 \times 10^{-9}$, $N = 60$, $f = 1/2$ and $m_p = 1$, we obtained for the brane tension: $\tau = 10^{-6}$, which corresponds to the parameter $A \simeq 3.694 \times 10^{-2}$, $\tau = 10^{-8}$, which corresponds to $A \simeq 3.692 \times 10^{-2}$, and $\tau = 10^{-10}$, which corresponds to $A \simeq 3.587 \times 10^{-2}$.

The scalar spectral index $n_s$ is given by $n_s - 1 = \frac{d\ln \mathcal{P}_g}{d\ln k}$, where the interval in wave number is related to the number of e-folds $N$ by the relation $d\ln k(\phi) = -dN(\phi)$. From Eq. (12) the scalar spectral index can be written as

$$n_s = 1 - \frac{(2 - 3f)}{Af} (\mathcal{B}^{-1}[K(\phi)])^{-f} - \frac{2Af(1-f)}{\kappa\tau} \frac{(\mathcal{B}^{-1}[K(\phi)])^{-(2-f)}}{1 + \frac{2(Af)^2(\mathcal{B}^{-1}[K(\phi)])^{-2(1-f)}}{\kappa\tau}}.$$  \hspace{1cm} (14)

Here, we noted from Eq. (14) that $n_s \neq 1$, for $f = 2/3$ (recall that $1 > f > 0$). However, as occurs in the standard intermediate model, $n_s = 1$ for the value $f = 2/3$, where the scale factor increases as $a(t) \sim e^{t^{2/3}}$, see Ref. [9].

On the other hand, the tensor-perturbation during inflation would produce gravitational waves. This perturbation in cosmology is more involved in our case, since in the brane-world gravitons propagate in the bulk. The amplitude of the tensor perturbations was calculated in Ref. [38], where

$$\mathcal{P}_g = 24\kappa \left(\frac{H}{2\pi}\right)^2 F^2(x).$$  \hspace{1cm} (15)

In our model we get

$$\mathcal{P}_g = \frac{6\kappa(Af)^2}{\pi^2} (\mathcal{B}^{-1}[K(\phi)])^{-(2(1-f))} F^2(x),$$  \hspace{1cm} (16)

where $x = Hm_p\sqrt{3/(4\pi\tau)} = (\mathcal{B}^{-1}[K(\phi)])^{-(1-f)} \sqrt{3(Af)^2m_p^2/4\pi\tau}$ and the function $F(x)$ is given by

$$F(x) = \left[\sqrt{1 + x^2} - x^2 \sinh^{-1}(1/x)\right]^{-1/2},$$  \hspace{1cm} (17)

here, the function $F(x)$, appeared from the normalization of a zero-mode [38].

On the other hand, an important observational quantity is the tensor to scalar ratio $r$, which is defined as $r = \frac{\mathcal{P}_g}{\mathcal{P}_R}$. Combining Eqs. (12) and (16), the tensor-scalar ratio, $r$, is given by

$$r = \frac{\mathcal{P}_g}{\mathcal{P}_R} = \frac{16(1-f)}{Af} \frac{F^2(\phi)}{\sqrt{1 + \frac{2(Af)^2(\mathcal{B}^{-1}[K(\phi)])^{-2(1-f)}}{\kappa\tau}}}.$$  \hspace{1cm} (18)

also, the relation between the tensor-scalar ratio $r$ and the number e-folds $N$ can be written as

$$r(N) = \frac{16(1-f)}{Af} \frac{\left[\frac{Af}{1+f(N-1)}\right]}{\sqrt{1 + \frac{2(Af)^2(\mathcal{B}^{-1}[K(\phi)])^{-2(1-f)}}{\kappa\tau}}} F^2(N).$$  \hspace{1cm} (19)
FIG. 1: The upper panel shows the evolution of the scalar potential $V$ versus the scalar field $\phi$. The lower panel shows the contour plot for the parameter $r$ as a function of the $n_s$. In both panels we considered different values of the brane tension $\tau$. From left to right, the dot-dashed, dashed, dotted and solid lines are for the brane tension $\tau = 10^{-10}$, $\tau = 10^{-8}$, $\tau = 10^{-6}$, and the standard intermediate model ($\tau \rightarrow \infty$), respectively. In both panels we have taken the values $f = 1/2$, $m_p = 1$, and $A \simeq 3.587 \times 10^{-2}; 3.692 \times 10^{-2}; 3.694 \times 10^{-2}$, respectively.

In Fig. (I), the upper panel shows the evolution of the effective potential $V(\phi)$ versus the scalar field $\phi$, and the lower panel shows the contour plot for the tensor-scalar ratio $r$ as a function of the $n_s$. In both panels we studied different values of the brane tension $\tau$. In particular, the dot-dashed, dashed, dotted, and solid lines are for the brane tension; $\tau = 10^{-10}$, $\tau = 10^{-8}$, $\tau = 10^{-6}$, and the standard intermediate model ($\tau \rightarrow \infty$), respectively. From the upper panel, we noted that, when we increased the brane tension $\tau$, the effective
potential graphs present a small displacement with respect to the scalar field $\phi$, when compared to the results of the potential obtained in the standard intermediate model in which $V(\phi) \propto \dot{\phi}^{-4(f-1)-1}$.

In the lower panel of Fig. (1), we show the dependence of the tensor-scalar ratio $r$ on the spectral index $n_s$. Following Ref. [3], we have 2-dimensional marginalized constraints corresponding to 68% and 95% confidence levels on inflationary parameters $r$ and $n_s$, the spectral index of fluctuations, defined at $k_0 = 0.002 \text{ Mpc}^{-1}$. In order to write down values that relate the ratio $r$ and the index $n_s$, we numerically solved Eqs. (14) and (18). Also, we have taken the values $f = 1/2$, $m_p = 1$ and for the parameter $A$ the values $A \simeq 3.587 \times 10^{-2}; 3.692 \times 10^{-2}; 3.694 \times 10^{-2}$, respectively. We noted that the values of the brane tension $\tau = 10^{-10}$ and $\tau = 10^{-8}$ enter the marginalized constraints corresponding to 68% confidence levels. However, the value $\tau = 10^{-6}$ only enters on 95% confidence level, as could be seen from the lower panel of Fig. (1). Also, for the value of the brane tension $\tau = 10^{-6}$, we numerically observed that the consistency relations, $n_s = n_s(r)$, present a small displacement in relation to the standard intermediate model. We noted that when we increase the value of the parameter $\tau$, for values of $\tau > 10^{-6}$, we observed that the contour plot in the $r - n_s$ plane becomes similar to the standard intermediate model. Also, we observed that the incorporation of the brane tension parameter gives us a freedom that allows us to modify the standard intermediate model by simply modifying the corresponding value of the parameter $\tau$, e.g., on the contour plot in the $r - n_s$ plane. In this form, our model is less restricted than the standard intermediate-model.

III. INTERMEDIATE INFLATION ON THE WARPED DGP BRANE

A. The model and the basic equations

We start by writing down the Friedmann equation on the warped DGP brane [39]

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[ \rho + \rho_0 (1 + \epsilon A(\rho, a)) \right],$$

where $k$ is the constant curvature of the three-space of the FRW metric and the $\mu$ parameter denotes the strength of the induced gravity term on the brane, with the parameter $\mu = 0$ giving the RS model. The $\epsilon$ parameter becomes either $+1$ or $-1$, corresponding to the two
branches of this model. We will denote $\epsilon = +1$ as the positive branch and the value $\epsilon = -1$ as the negative branch. The function $A(\rho, a)$ is given by

$$A(\rho, a) = \left[ A_0^2 + \frac{2\eta}{\rho_0} \left( \rho - \frac{\mu^2 E_0}{a^4} \right) \right]^{1/2},$$

where the constants $A_0$, $\rho_0$, and $\eta$, are defined as

$$A_0 = \sqrt{1 - 2\eta \mu^2 \Lambda / \rho_0}, \quad \rho_0 = \tau + 6 \frac{m_5^6}{\mu^2} \quad \text{and} \quad \eta = \frac{6m_5^6}{\rho_0 \mu^2} \quad (0 < \eta \leq 1). \quad (21)$$

Here, the constant $\Lambda$ is the effective 4-dimensional cosmological constant on the brane and is defined by $\Lambda = \frac{1}{2} (5\Lambda + \frac{1}{6} \kappa_5^4 \tau^2)$, in which $\kappa_5 = m_5^{-3}$ is the 5-dimensional gravitational constant, $(5)\Lambda$ is the 5-dimensional cosmological constant in the bulk, as before $\tau$ is the brane tension and $E_0$ is a constant related to Weyl radiation.

In the following, we will neglect the curvature term and the dark radiation term during inflation and the function $A(\rho, a) \approx A(\rho) \approx \left[ A_0^2 + \frac{2\eta \rho}{\rho_0} \right]^{1/2}$. In this way, the Friedmann Eq. (20) takes the form

$$H^2 = \frac{1}{3\mu^2} \left[ \rho + \rho_0 + \epsilon \left( A_0^2 + \frac{2\eta \rho}{\rho_0} \right)^{1/2} \right]. \quad (22)$$

In particular, for the DGP scenario, one has $A_0 = 1$ and the parameter $\eta = 1$.

Following Ref. [39], we noted that in the ultra high energy limit where $\rho \gg \rho_0 \gg \tau$ and $A_0 = 1$, the effective Friedmann Eq. (22) becomes $H^2 = \frac{1}{3\mu^2} \left( \rho + \epsilon \sqrt{2\rho \rho_0} \right)$. Also, in the intermediate energy region in which $\rho \ll \rho_0$ but $\rho \gg \tau$, for $\epsilon = -1$, the effective Friedmann equation can be rewritten by $H^2 = \frac{1}{18m_5^6} \left( \rho + \frac{\mu^2}{6m_5^2} \rho - \frac{\mu^4}{4m_5^4} \rho^2 \right)$. Finally, in low energy limit in which $\rho \ll \tau \ll \rho_0$ the Friedmann Eq. (22) becomes $H^2 = \frac{1}{3\mu^2} \left[ \rho + O \left( \frac{\rho}{\rho_0} \right)^2 \right]$, where the effective 4-dimensional Planck mass is defined by $\mu_p^2 = \mu^2 / (1 - \eta)$.

Analogously, as before we will consider that the energy density $\rho$ is a standard scalar field $\phi$ satisfying the continuity equation given by Eq. (4). In this form, the square of the velocity $\dot{\phi}^2$ of the scalar field, considering Eqs. (4) and (22), becomes

$$\dot{\phi}^2 = 2\mu^2 (-\dot{H}) \left[ 1 - \epsilon \left( \alpha + \beta H^2 \right)^{-1/2} \right], \quad (23)$$

where the constants $\alpha$ and $\beta$ are defined by

$$\alpha = 1 + \frac{A_0^2}{\eta^2} - \frac{2}{\eta}, \quad \text{and} \quad \beta = \frac{6\mu^2}{\eta\rho_0}.$$
The effective potential as function of the $H$ and $\dot{H}$, can be obtained from Eqs. (22) and (23), and as a result is found

$$V = \frac{\eta \rho_0}{2} \left( \alpha + \beta H^2 \right) \left[ 1 - \epsilon \left( \alpha + \beta H^2 \right)^{-1/2} \right]^2 +$$

$$+ \mu^2 \dot{H} \left[ 1 - \epsilon \left( \alpha + \beta H^2 \right)^{-1/2} \right] - \frac{A_0^2 \rho_0}{2\eta}.$$  \hspace{1cm} (24)

Combining Eqs. (11) and (23) we found a relation between the scalar field and cosmological time given by

$$\phi(t) - \phi_0 = \frac{F[t]}{K},$$  \hspace{1cm} (25)

where $\phi(t = 0) = \phi_0$ is an integration constant and the constant $K$, is given by

$$K = \nu \left( \frac{1 - f}{2\mu^2 Af} \right)^{1/2} (\beta A^2 f^2)^{-\alpha f/2}.$$  

Here, the function $F[t]$ is the incomplete Lauricella function, see Ref. [40] and is defined as

$$F[t] = \left( \alpha + \frac{\beta}{t^{2(1-f)}} \right)^{-\nu} F_D^{(3)} \left[ \nu; 1 + \frac{\nu}{2}, 1 + \frac{\nu}{2}, -\frac{1}{2}, \nu + 1, \sqrt{\alpha}, -\sqrt{\alpha}, \epsilon \left( \alpha + \frac{\beta}{t^{2(1-f)}} \right)^{-\frac{1}{2}} \right],$$

where $\nu = f \left( 2(1 - f) \right)^{-1}$. The Hubble parameter as a function of the scalar field, $\phi$, results in $H(\phi) = Af (F^{-1}[K\phi])^{-f}$. Here, we have used $\phi_0 = 0$.

As before, considering the slow-roll approximation, the scalar potential given by Eq. (24) reduces to

$$V(\phi) \simeq \frac{\eta \rho_0}{2} \left( \alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}} \right) \left[ 1 - \epsilon \left( \alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}} \right)^{-1/2} \right]^2 - \frac{A_0^2 \rho_0}{2\eta}.$$  \hspace{1cm} (26)

Again, introducing the dimensionless slow-roll parameters $\varepsilon$ and $\eta$, we have $\varepsilon = -\frac{\dot{H}}{H^2} = \frac{1-f}{AF^{-1}[K\phi]}$, and $\eta = -\frac{H}{\dot{H}H} = \frac{2-f}{AF^{-1}[K\phi]}$. As before, if we consider that the inflationary scenario begins at the earliest possible stage, in which $\varepsilon = 1$, then the scalar field $\phi_1$, is given by $\phi_1 = \frac{1}{K} F \left[ \left( \frac{1-f}{AF} \right)^{1/f} \right]$. Also, the condition for inflation to occur is $\varepsilon < 1$ or equivalently when $\phi > \frac{1}{K} F \left[ \left( \frac{1-f}{AF} \right)^{1/f} \right]$.

For the number of e-folds between two different values $\phi_1$ and $\phi_2$ of the scalar field we have

$$N = \int_{\phi_2}^{\phi_1} H dt = A \left( (F^{-1}[K\phi_2])^f - (F^{-1}[K\phi_1])^f \right).$$  \hspace{1cm} (27)

Here, we have considered Eq. (25).
B. Cosmological perturbations

In the following, we will study the power spectra of scalar and tensor perturbations to the metric in our inflationary model. We consider the gauge invariant quantity \( \zeta = H + \frac{\delta \rho}{\rho} \) \cite{41, 42}. Here, \( \zeta \) defined on slices of uniform density and reduces to the curvature perturbation. A fundamental feature sign of \( \zeta \) is that it is nearly constant on super-horizon scales. This property, an effect of stress-energy conservation, does not depend on the gravitational dynamics \cite{43} (see also, Ref.\cite{44}). In this form, the power spectrum related to the curvature spectrum, could be written as
\[
P_R \simeq \langle \zeta^2 \rangle \quad \text{and it stays unchanged in the warped DGP model \cite{27, 45} (see also Ref.\cite{46}).}
\]
Therefore, for the spatially flat gauge, we have \( \zeta = H \frac{\delta \phi}{\phi} \), in which \( |\delta \phi| = H/2\pi \) \cite{2}.

In this way, the power spectrum, considering Eq.(23), is given by
\[
P_R \simeq \left( \frac{H^2}{2\pi} \right)^2 \frac{1}{8\pi^2 \mu^2} H^4 (-\dot{H})^{-1} \left[ 1 - \epsilon \left( \alpha + \beta H^2 \right)^{-1/2} \right]^{-1},
\]
or equivalently in terms of the standard scalar field \( \phi \)
\[
P_R = \frac{A^3 f^3 (1 - f)^{-1}}{8\pi^2 \mu^2 (F-1[K\phi])^2 - 3f} \left[ 1 - \epsilon \left( \alpha + \beta A^2 f^2 \left( F-1[K\phi] \right)^{-2(1-f)} \right)^{-1/2} \right]^{-1}.
\]

Analogously as before, the power spectrum, \( P_R \), also can be expressed in terms of the number of e-folds \( N \), as
\[
P_R = \frac{A^3 f^3}{8\pi^2 \mu^2 (1 - f)} \left[ \frac{Af}{1 + f(N - 1)} \right]^{2-3f} \times
\]
\[
\times \left[ 1 - \epsilon \left( \alpha + \beta A^2 f^2 \left( \frac{Af}{1 + f(N - 1)} \right)^{-2(1-f)} \right)^{-1/2} \right]^{-1}.
\]

Again, numerically from Eq.(30), we found a constraint for the parameter \( A \). Analogously, as in the case of the RS model, we can find the value of the parameter \( A \) for given values of \( f, m_5/\mu, \epsilon, \) and \( \eta \), when \( N \) and \( P_R \) are given. In the following, we will assume that the effective 4-dimensional cosmological constant \( \Lambda = 0 \), which implies that is \( \mathcal{A}_0 = 1 \), see Eq.(21), and the parameter \( \eta = 0.99 \), see Ref.\cite{45}.

In particular, for the values \( P_R = 2.4 \times 10^{-9}, \quad N = 60, \quad f = 1/2, \quad \epsilon = -1, \quad m_p = 1, \) and the ratio \( m_5/\mu = 0.02 \), see Ref.\cite{45}, we found for the parameter \( A \simeq 2.69 \times 10^{-2} \) and for the ratio \( m_5/\mu = 0.2 \), which corresponds to \( A \simeq 5.98 \times 10^{-2} \).
On the other hand, the scalar spectral index $n_s$, considering Eq. (29), we get

$$n_s = 1 - \frac{2 - 3f}{Af} (F^{-1}[K\phi])^{-f} + n_\epsilon,$$

(31)

where the correction, $n_\epsilon$, in the scalar spectral index is given by

$$n_\epsilon = \epsilon \beta Af (1 - f) \left( \alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}} \right)^{-3/2} \left[ 1 - \epsilon \left( \alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}} \right)^{-1/2} \right]^{-1}.$$

We noted numerically from Eq. (31) that the value of the spectral index is $n_s \gg 1$ for the positive branch $\epsilon = +1$, but this value of $n_s$ is disfavored from the observational data. In this form, intermediate inflation on a warped DGP cannot exit in the positive branch $\epsilon = +1$.

In the following, we will consider the negative branch $\epsilon = -1$.

Combining Eqs. (27) and (31), the scalar spectral index can be re-expressed in terms of the number of e-folds as

$$n_s(N) = 1 - \frac{2 - 3f}{1 + f(N - 1)} + n_\epsilon(N),$$

(32)

where the correction $n_\epsilon(N)$ becomes

$$n_\epsilon(N) = \epsilon \beta Af (1 - f) \frac{A}{1 + f(N - 1)} \left( \alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}} \right)^{-3/2} \left[ 1 - \epsilon \left( \alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^{2(1-f)}} \right)^{-1/2} \right]^{-1}.$$

Following Ref. [27] the amplitude of gravitational waves in our model is given by

$$P_g = \frac{64\pi}{m_p^2} \left( \frac{H}{2\pi} \right)^2 G_\gamma^2(x),$$

(33)

where the correction to standard 4D general relativity is $G_\gamma^{-2}(x) = \gamma + (1 - \gamma) F(x)^{-2}$ and the parameter $\gamma = \mu^2/m_p^2$. Here, the function $F(x)$ is defined by Eq. (17) and $x = H/\bar{\mu}$, where $\bar{\mu}$ is the energy scale associated with the bulk curvature [27]. In particular, for the brane cosmological constant to zero, $\kappa_4^2 = m_p^{-2} = \kappa_5^2 (1 - \gamma)$ [27, 39]. Note that the correction $G_\gamma^2$ coincides to the expression the RS case when $\gamma \rightarrow 0$, see Eq. (15). Also, when $x \rightarrow 0$, we get $G_\gamma^2 \rightarrow 1$ and it reduces to the standard 4D amplitude of gravitational waves in which $P_g = \frac{64\pi}{m_p^2} \left( \frac{H}{2\pi} \right)^2.$

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FIG. 2: The upper panel shows the evolution of the scalar spectrum index $n_s$ versus the number of e-folds $N$. The lower panel shows the contour plot for the parameter $r$ as a function of the $n_s$. Here, from WMAP seven-year data[3], we have 2-dimensional marginalized constraints (68% and 95% confidence levels) on the inflationary parameters $r$ and $n_s$. The dashed, dotted and solid lines are for the pairs $(A = 5.98 \times 10^{-2}, m_5/\mu = 0.2)$, $(A = 2.69 \times 10^{-2}, m_5/\mu = 0.02)$, and the standard intermediate model[9], respectively. In both panels we have taken the values $m_p = 1$, $\eta = 0.99$, $\epsilon = -1$, $A_0 = 1$, and $f = 1/2$.

In this way, considering Eqs.(29) and (33) we may write the tensor-scalar ratio as $r = \left( \frac{P_t}{P_R} \right)$ and this ratio as a function of the scalar field becomes

$$r = \frac{128\pi (1-f)}{A f (F^{-1}[K\phi])^3} \frac{\mu^2}{m_p^2} \left[ 1 - \epsilon \left( \alpha + \frac{\beta A^2 f^2}{(F^{-1}[K\phi])^2(1-f)} \right)^{-1/2} \right] G_x^2(\phi). \quad (34)$$

Analogously, as before we can write the tensor-scalar ratio as a function of the number of e-foldings as
\[
\begin{align*}
  r &= \frac{128\pi(1 - f)}{1 + f(N - 1)} \frac{\mu^2}{m_p^2} \left[ 1 - \epsilon \left( \alpha + \beta A^2 f^2 \left( \frac{Af}{1 + f(N - 1)} \right) \right)^{2(1 - f)} \right] \frac{G_2^2}{\gamma(N)}. \\
  \text{(35)}
\end{align*}
\]

In Fig.(2), the upper panel shows the evolution of the scalar spectrum index \( n_s \) versus the number of e-folds \( N \), and the lower panel shows the contour plot for the parameter \( r \) as a function of the \( n_s \), for different values of the parameters \( A \) and the ratio \( m_5/\mu \). In particular, the dashed, dotted, and solid lines are for the pairs \((A = 5.98 \times 10^{-2}, m_5/\mu = 0.2), (A = 2.69 \times 10^{-2}, m_5/\mu = 0.02, \text{see Ref.}[45]),\) and the standard intermediate model, respectively. Here, we have used the values \( m_p = 1, \eta = 0.99, \epsilon = -1, A_0 = 1 \) and \( f = 1/2 \).

From the upper panel, we observed that the number of e-folds \( N \) increases in the warped DGP scenarios when it is compared with the the case of the standard intermediate model, in which \( n_s = 1 - (N + 1)^{-1} \), see Ref.[9]. From the lower panel, we noted that for the \( r - n_s \) graphs, the ratio \( m_5/\mu = 0.02 \) presents a small displacement with respect to the results obtained in the standard intermediate model [9]. Also, we numerically found that when we decrease the value of the ratio \( m_5/\mu \), for lower values, \( m_5/\mu < 0.02 \) we observed that the new lines on the contour plot \( r - n_s \) becomes similar to the standard intermediate model.

IV. CONCLUSIONS

In this paper we have analyzed the intermediate inflationary model, for the cases of the brane model and the warped DGP model, respectively. We have found solutions of the Friedmann equations for a flat universe containing a standard scalar field \( \phi \). For both models, we considered the entire Eqs.(3) and (22), which show some new attractive features, during intermediate inflation. We have also obtained explicit expressions for the corresponding, scalar field, effective potential, power spectrum of the curvature perturbations \( P_R \), tensor-scalar ratio \( r \), and the scalar spectrum index \( n_s \). Also, in both cases, considering the WMAP seven year data, we have found constraints on the parameters for our models. Here, we have taken the constraint of the consistency relations, \( n_s = n_s(r) \).

For the brane model, we noted that for values of the brane tension \( \tau < 10^{-6} \), the model is well supported by the data as could be seen from Fig.(1). Here, we have used the values \( m_p = 1, A \simeq 3.694 \times 10^{-2}; 3.692 \times 10^{-2}; 3.587 \times 10^{-2}, \) and \( f = 1/2 \), respectively. We
numerically observed that for the value of the tension of the brane $\tau = 10^{-6}$, the consistency relations, $n_s = n_s(r)$, present a small displacement in relation to the standard intermediate model. In this way, we noted that when we increase the value of the parameter $\tau$, for values of the brane tension $\tau > 10^{-6}$, we observed that the contour plot in the $r - n_s$ plane becomes similar to the standard intermediate model.

For the warped DGP model, we noted numerically from Eq. (31) that the value of the spectral index is $n_s \gg 1$ for the positive branch, $\epsilon = +1$, but this value of $n_s$ is disfavored from the observational data and the warped DGP model does not work for this branch. From the upper panel in Fig. (2), we observed that the number of e-folds $N$ increases in the warped DGP model when it is compared with the standard intermediate model. From the lower panel we numerically noted that when we decrease the ratio $m_5/\mu$, for values $m_5/\mu < 0.02$, we observed that the new lines on the contour plot $r - n_s$ becomes similar to the standard intermediate model. Here, we have used the values $m_p = 1$, $\eta = 0.99$, $\epsilon = -1$, $A_0 = 1, A \simeq 5.98 \times 10^{-2}, 2.69 \times 10^{-2}$, and $f = 1/2$.

We have also found in both models that the incorporation of the additional term in Friedmann’s equation improves some of the characteristics of the intermediate inflation. In particular, this is emphasized in the consistency relations $n_s = n_s(r)$. Finally, from the effective potentials obtained in both models (not present minimum), we have not addressed the mechanism of reheating and subsequent connections to the standard Big-Bang model (see e.g., Ref. [47]). Calculation for the reheating temperature in these scenarios would produce new constraints on the brane parameters. We hope to return to this point in the future.

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