An improved optimisation method based on augmented constraint for electricity market clearing considering non-increasing stepwise bidding

Li-Min Cheng1,2 | Yu-Qing Bao1,2 | Chen Chen1,2

1NARI School of Electrical Engineering and Automation, Nanjing Normal University, Nanjing, Jiangsu, China
2Nanjing Institute of Intelligent High-End Equipment Industry Co., Nanjing, China

Abstract

The development of optimization methods for market clearing aims to improve the rationality of bidding and maximise social welfare. Existing electricity markets only allow a non-decreasing bidding curve for generating companies (GenCos) owing to the convex limitation of the optimization model. However, the marginal cost of GenCos is sometimes non-increasing, such as GenCos involved in deep peak regulation, which requires the non-increasing bidding of GenCos. Focussing on the non-increasing stepwise bidding curves of the GenCos, an improved optimization method is proposed that imposes augmented constraints on the bidding segments so that each bid segment can be allocated in order. In this way, the proper sequence of the bid quantity of each segment can be guaranteed. The numeral testing results verify the effectiveness of the proposed method.

1 | INTRODUCTION

The rapid adjustment of energy policies and the marketisation of electricity pose impacts to and risks on the economic efficiency of power system operations and the electricity market in China [1]. In an ideal electricity market with a uniform purchase price (UPP), bidding prices of the generating companies (GenCos) are little higher than the marginal cost of production and power generation should be scheduled efficiently [2]. In actual cases, maximisation of profit is still the dominant objective of GenCos, and unconstrained bidding may result in adverse consequences such as increased electricity market pricing and improper quantity settlement [3]. Thus, optimization methods for market clearing contribute to covering shortages in pricing and settlement mechanisms, improve the rationality of bidding strategies, and maximise social welfare.

Studies on optimization methods for market clearing described them as mathematical problems with equilibrium constraints (MPECs) [4]. Numerous non-linear programming approaches exist for solving MPECs, which are mostly employed to solve small-scale problems. Sequential quadratic programming methods have been adopted to solve the scheduling problem of a coupled system with hydro and wind energy in a day-ahead market [5]. Artificial intelligence approaches have been adopted to solve the problem of optimal participation in multiple electricity markets [6, 7]. Interior point methods have been adopted to find the global economic optimal solution of the whole system in a decentralised bilateral multitransitional-based market [8], whereas mixed integer linear programming (MILP) is suitable for solving large-scale problems [9, 10]. Compared with non-linear programming approaches, the MILP method possesses more advantages for market clearing problems in meeting new requirements flexibly, and efficiently providing proximity information about the optimal solution [11–13]. A risk-based unit commitment (RUC) model is proposed in Zhang et al. [11] that can identify underlying risks of scheduling imposed by wind power uncertainty, and the RUC model is reformulated as an MILP problem without adding variables. An MILP formulation is presented to solve payment cost minimisation problems in

DOI: 10.1049/es2.12017
Jamilzadeh et al. [12]. The market pricing scheme problems in Savelli et al. [13] are solved by the MILP method.

However, the bidding methods of each GenCo in the literature [11–13] are mostly based on a monotonically non-decreasing stepwise bidding function owing to the convex limitation of the optimization model [14]. In many cases, such as GenCos involved in deep peak regulation (DPR) [15], the marginal cost of GenCos is reduced in the beginning and then increased owing to the physical characteristic of the power generation of GenCos, and the bidding curves are therefore non-monotonic. A typical non-monotonic marginal cost curve and the bidding curves are illustrated in Figure 1 [16]. The optimization model may be developed as a non-convex model. To formulate the non-convex optimization models precisely, mixed-integer non-linear programming (MINLP) is often required. Based on existing algorithms for MINLP in practice, the finding of a global optimal solution cannot be guaranteed [17]. Methods related to the economic dispatch problem in the linear formulations [21] are applied to transform MINLP problems into MILP problems. The non-convexity of constraints is eliminated by substitution with convex constraints [18], and heuristic algorithms are employed in scheduling problems with renewable sources and demand response [19, 20]; the non-convex objective function is described by efficient piecewise-linear formulations [21]. Although these methods have many contributions to coping with the non-convexity of the optimization problems, the following limitations should be addressed.

Notwithstanding the contributions of these works, limitations exist that need to be improved:

• The substitution of convex constraints is skillful and the approximation is technically complicated in Chen et al. [18].
• The performance of heuristic algorithms in Abdi et al. [19] and Forstenbacher et al. [20] were found to be poor for larger power systems, and the heuristic algorithms have high computational costs.
• The non-convex optimization model with discontinuous non-decreasing cost curves is solved by piecewise-linear formulations in Geng et al. [21], but the non-monotonicity of the marginal cost curve was not considered. If non-monotonic stepwise bidding is allowed without improving the market clearing optimization model, improper quantity settlement for the GenCos may occur.

To fill these gaps, the improved optimization method for market clearing considering GenCos with non-monotonic stepwise bidding is proposed. The contributions of the study are as follows:

1. According to the current study, GenCos stepwise bidding has non-monotonic characteristics when the units participate in flexible power system scheduling such as DPR.
2. An improved optimization method was proposed to consider market clearing with GenCos non-monotonic stepwise bidding. Augmented market clearing constraints are imposed on the bidding segments, and then each bid segment can be allocated in order.
3. Based on the proposed method, the proper sequence of the bid quantity of each segment can be guaranteed. Compared with existing market clearing methods, the proposed method can solve the improper quantity settlement problem caused by non-increasing bidding, so that the non-increasing bidding curves of GenCos, which reflect the marginal cost characteristics of GenCos, can be allowed in the electricity market and result in more effective market clearing.

The remainder of this work is organised as following: Section 2 gives the mathematical formulations of the electricity market clearing problem. The novel optimise bidding method is proposed in Section 3. Section 4 consists of case studies. Finally, the conclusions are summarised in Section 5.

2 | MATHEMATICAL FORMULATIONS OF ELECTRICITY MARKET CLEARING PROBLEM

In this section, mathematical formulations of the electricity market clearing problem are formulated.

2.1 | Objective function

The market clearing problem is solved by maximising social welfare [22, 23]. The objective function can be formulated as:

\[
\max CS - PS
\]

(1)

where \( PS \) and \( CS \) are producer surplus and consumer surplus, respectively. \( PS \) and \( CS \) are expressed as

\[
CS = \sum_{t=1}^{N_T} \sum_{r=1}^{N_R} I_{r,t}
\]

(2)

\[
PS = \sum_{t=1}^{N_T} \sum_{r=1}^{N_R} \left( F_{r,t} + f_{i,t} \right)
\]

(3)

where \( I_{r,t} \) is the integral of the demand curve of retailer \( r \) in the \( t \)-th period. \( F_{r,t} \) and \( f_{i,t} \) are the integral of the bidding function and the fixed cost of GenCo \( i \) in the \( t \)-th period, respectively.

Fixed cost \( f_{i,t} \) of a traditional GenCos is described as

\[
f_{i,t} = f_{\min,i,t} + SU_{i,t} + SD_{i,t}
\]

(4)

\[
SU_{i,t} = c_{SU,i} \cdot x_{i,t}
\]

(5)
monotonic stepwise bidding can be described as

In uniform price auction markets, GenCos bid according to their marginal cost (MC), which is calculated by cost and power generation. The continuous form of the bidding function can be expressed as

\[ MC_{Gi,t} = \frac{dC_{Gi,t}}{dP_{Gi,t}} = f''(P_{Gi,t}) \] (9)

\[ P_{Gi,t} = k_{Gi} \cdot \frac{dC_{Gi,t}}{dP_{Gi,t}} = k_{Gi}(2 \cdot a_t \cdot P_{Gi,t} + b_t) \] (10)

According to cost function (Equation 8), fixed cost \( f_{\text{min},i,t} \) can be described as

\[ f_{\text{min},i,t} = a_i \cdot P_{\text{Gin},i,t}^2 + b_i \cdot P_{\text{Gmin},i,t} + c_i \] (11)

In the actual case, according to the adjustment range of the units, GenCos divide their power generation into multiple bidding segments as price-quantity bids. Power generation \( P_{Gi,t} \) is thus transformed into Equation (12) as

\[ P_{Gi,t} = u_{i,t} \cdot P_{Gmin,i,t} + \sum_{k=1}^{N_{gi}} g_{i,t,k} \] (12)

\[ 0 \leq g_{i,t,k} \leq g_{\text{max},i,t,k} \] (13)

Furthermore, Equation (10) can be transformed into a discrete form function as

\[ p_{Gi,t,k} = k_{Gi}(a_i \cdot (P_{Gi,t,k-1} + P_{Gi,t,k}) + b_i) \] (14)

\[ P_{Gi,t,k-1} = \begin{cases} u_{i,t} \cdot P_{Gmin,i,t} & k = 1 \\ u_{i,t} \cdot P_{Gmin,i,t} + \sum_{j=1}^{k-1} g_{\text{max},i,t,j} & k \geq 2 \end{cases} \] (15)

\[ P_{Gi,t,k} = u_{i,t} \cdot P_{Gmin,i,t} + \sum_{j=1}^{k} g_{\text{max},i,t,j} \] (16)

Based on \( g_{i,t,k} \) and \( p_{Gi,t,k} \), bidding function \( F_{i,t} \) of PS is expressed as

\[ F_{i,t} = \sum_{k=1}^{N_{gi}} g_{i,t,k} p_{Gi,t,k} \] (17)

2.2 | Stepwise bidding model of a generating company

The cost function of a GenCos can be expressed as

\[ c_{Gi,t} = a_i \cdot P_{Gi,t}^2 + b_i \cdot P_{Gi,t} + c_i \] (8)

In uniform price auction markets, GenCos bid according to their marginal cost (MC), which is calculated by cost and power generation. The continuous form of the bidding function can be expressed as

\[ M_{Ci,t} = \frac{dC_{Gi,t}}{dP_{Gi,t}} = f''(P_{Gi,t}) \] (9)

\[ P_{Gi,t} = k_{Gi} \cdot \frac{dC_{Gi,t}}{dP_{Gi,t}} = k_{Gi}(2 \cdot a_i \cdot P_{Gi,t} + b_i) \] (10)

According to cost function (Equation 8), fixed cost \( f_{\text{min},i,t} \) can be described as

\[ f_{\text{min},i,t} = a_i \cdot P_{\text{Gin},i,t}^2 + b_i \cdot P_{\text{Gmin},i,t} + c_i \] (11)

2.3 | Stepwise bidding model of retailers

The marginal revenue of the depends on the electricity consumption price and demand of users. The relation between the users’ electricity consumption price and demand \( D_{r,t} \) can be expressed by the price elasticity of the demand model [24]:

\[ D_{r,t} = A_{r,t} \cdot \left( \frac{P_{Dr,t}}{A_{r,t}} \right)^{\alpha_{r,t}} \] (18)

Equation (18) can be converted to an equivalent form as

\[ P_{Dr,t} = \left( \frac{D_{r,t}}{A_{r,t}} \right)^{1/\alpha_{r,t}} \] (19)
Based on $p_{Dri}$ and $k_{Rr}$, the continuous form of the demand bids of retailer $r$ in the $t$-th period can be expressed as

$$p_{Rr,t} = k_{Rr} \cdot p_{Dri} = k_{Rr} \left( \frac{D_{r,t}}{A_{r,t}} \right)$$  \hspace{1cm} (20)$$

Furthermore, the discrete form function of the price-demand quantity is expressed as

$$p_{Rr,t,k} = k_{Rr} \left( \frac{D_{r,t,k-1} + D_{r,t,k}}{2A_{r,t}} \right)$$  \hspace{1cm} (21)$$

The demand of retailer $r$ (Equation [18]) is thus transformed into Equations (22) and (23) as

$$D_{r,t} = D_{\min,r} + \sum_{k=1}^{N_{Tk}} b_{r,t,k}$$  \hspace{1cm} (22)$$

$$0 \leq b_{r,t,k} \leq b_{\max,r,t,k}$$  \hspace{1cm} (23)$$

Based on $b_{r,t,k}$ and $p_{Rr,t,k}$, the $I_{r,t}$ of CS is expressed as

$$I_{r,t} = \sum_{k=1}^{N_{Tk}} b_{r,t,k} p_{Rr,t,k}$$  \hspace{1cm} (24)$$

### 2.4 Electric power system constraints

A market clearing problem with the aim of maximising social welfare is subject to constraints (25–29):

$$\sum_{i=1}^{N_{Gi}} P_{Gi,t} + \sum_{w=1}^{N_{Wi}} P_{Wi,t} = \sum_{r=1}^{N_{Rr}} D_{r,t}$$  \hspace{1cm} (25)$$

$$u_{i,t} \cdot P_{Gmi} \leq P_{Gi,t} \leq u_{i,t} \cdot P_{Gmxi}$$  \hspace{1cm} (26)$$

$$x_{i,t} - y_{i,t} = u_{i,t} - u_{i,t-1}$$  \hspace{1cm} (27)$$

$$x_{i,t} + y_{i,t} \leq 1$$  \hspace{1cm} (28)$$

$$-RD_{i,t} \leq P_{Gi,t} - P_{Gi,t-1} \leq RU_{i,t}$$  \hspace{1cm} (29)$$

where Equation (25) is the power balance constraint, Equation (26) is the traditional GenCos capacity limit constraint, Equation (27) and Inequation (28) are the state transition constraints, and Equation (29) is the ramp rate limit constraints.

### 3 AUGMENTED BIDDING SEGMENT CONSTRAINTS OF GENERATING COMPANIES IN ELECTRICITY MARKET CLEARING

This section proposes augmented bidding segment constraints of GenCos with non-monotonic stepwise bidding.

Considering the case in which the stepwise bidding segments of GenCo $i$ are non-monotonic (the pricing of $P_{Gi,t,1}$, $P_{Gi,t,2}$, ..., $P_{Gi,t,k}$ are not arranged from low to high), the constraints are set to ensure each supply quantity $g_{i,t,k}$ of GenCo $i$ is allocated in the order $g_{i,t,1}$, $g_{i,t,2}$, ..., $g_{i,t,k}$. The quantity settlement range constraints are established by Big-M method [25], expressed as

$$-M(1 - \delta_{i,t,k}) \leq P_{Gi,t,k} - P_{Gi,t}$$
$$-M(1 - \delta_{i,t,k}) \leq P_{Gi,t} - P_{Gi,t,k-1}$$  \hspace{1cm} (30)$$

where $M$ is a big real number and the solution is insensitive to the actual value of $M$. $M$ can be conservatively selected as $M = \max\{P_{Gmax,i}\} + 1$, which is a sufficient but not necessary condition. Moreover, in practice, $M$ should not be selected to be extremely big. Otherwise, it may cause numerical instability. $\delta_{i,t,k}$ is a binary variable related to $g_{i,t,k}$ with the function of judging whether $P_{Gi,t}$ is in the $k$-th segment.

Binary variable $\delta_{i,t,k}$ is equal to 1 if Inequation (30) is equivalent to Inequation (31) as

$$0 \leq P_{Gi,t,k} - P_{Gi,t}$$
$$0 \leq P_{Gi,t} - P_{Gi,t,k-1}$$  \hspace{1cm} (31)$$

Inequation (31) restricts power generation $P_{Gi,t}$ of GenCo $i$ to the range of the $k$-th segment in the $t$-th period ($P_{Gi,t,k-1} \leq P_{Gi,t} \leq P_{Gi,t,k}$).

Binary variable $\delta_{i,t,k}$ is equal to 0 if Inequation (30) is equivalent to Inequation (32) as

$$-M \leq P_{Gi,t,k} - P_{Gi,t}$$
$$-M \leq P_{Gi,t} - P_{Gi,t,k-1}$$  \hspace{1cm} (32)$$

Inequation (32) is established for any value of $P_{Gi,t}$ and represents that power generation $P_{Gi,t}$ is outside the range of the $k$-th segment in the $t$-th period, and $P_{Gi,t}$ does not belong to the $k$-th segment ($P_{Gi,t,k-1} > P_{Gi,t}$ or $P_{Gi,t,k} < P_{Gi,t}$).

Because each segment is independent, power generation $P_{Gi,t}$ is unique, so $\delta_{i,t,k}$ needs to meet the quantity uniqueness constraint (33):

$$\sum_{k=1}^{K} \delta_{i,t,k} = 1$$  \hspace{1cm} (33)$$
Furthermore, considering the constraint of the unit commitment of GenCos, Equation (33) is modified as Equation (34):

$$
\sum_{k=1}^{N_{GC}} \delta_{i,k} = u_{i,t} \quad (34)
$$

Based on Equations (30) and (34), quantity bidding sequence binary variable $z_{i,t,k}$ is added, and the quantity bidding sequence constraint of each supply quantity $g_{i,t,k}$ is established as:

$$
z_{i,t,k} \cdot g_{\max i,t,k} \leq g_{i,t,k} \quad (35)
$$

Thus, $z_{i,t,k}$ indicates whether the quantity settlement of GenCo $i$ is the maximum quantity. Binary variable $z_{i,t,k}$ is equal to 1 if the quantity of the $k$-th segment of GenCo $i$ is the maximum quantity of the $k$-th segment in the $t$-th period; that is $g_{i,k} = g_{\max i,k}$. Meanwhile, $z_{i,t,k}$, $\delta_{i,t,k}$ and $g_{i,t,k}$ satisfy the relations as:

$$
(z_{i,t,k} + \delta_{i,t,k}) \cdot g_{\max i,t,k} \geq g_{i,t,k} \quad (36)
$$

$$
z_{i,t,k-1} - z_{i,t,k} = \delta_{i,t,k} \quad (37)
$$

The quantity bidding sequence constraint is supplemented by Equations (36) and (37), and then allocation of the quantities of segments before the $k$-th segment is guaranteed.

Inequations (30), (35), and (36) and Equations (34) and (37) are the constraints of the GenCos bidding segment in the case of non-monotonic stepwise bidding, and quantities of the segments are guaranteed to be allocated in order. The proposed method is named market clearing constraint-M (MCC-M).

A simplified flowchart of the improved optimization method for market clearing considering non-monotonic stepwise bidding is shown in Figure 2. The optimization method composed only of STAGE I and STAGE III is the traditional market clearing method [21]. According to the proposed method, MCC-M, STAGE II is added to the traditional market clearing method.

4 | TESTING EXAMPLES

The performance of the proposed method is investigated using the power market with a GenCos, wind power plants, and retailers. The power of bidding is divided into 10 segments [26] and the maximum power generation is equal to the maximum capacity registered by the GenCos. The pricing of the 10 bidding segments declared throughout the day is fixed within 24 h of the trading day.

A ten-piece piecewise linear function [27] is used to approximate the generation cost function. Then, the proposed method MCC-M is applied to the market clearing with the UPP of the 6-bus-3-gen system [23], and the market clearing result is compared with the traditional market clearing result. Furthermore, the proposed MCC-M method is applied to the market clearing with UPP of the 118-bus-54-gen system [28] and the effectiveness of the proposed method is analysed.

Finally, the algorithm is developed using MATLAB software with the Matpower, CPLEX, and YALMIP toolboxes. The hardware environment is a HUAWEI MateBook 14, CPU.
intel i7-8565U, with 8 GB RAM, a hard drive capacity of 512 GB, main frequency of 1.80 GHz, and a 64-bit operating system.

4.1 Verification of market clearing constraint-M method

In this subsection, the proposed MCC-M and traditional clearing methods [21] are tested and compared in the market clearing case of a 6-bus-3-gen system, considering a market structure with three GenCos, one wind power plant, and three retailers [23]. The operational time horizon is 24 h, and each time period is set to 1 h. The parameters of GenCos are shown in Table A1. The power output of wind power and the baseline load curve of retailers are shown in Figure 3.

![Figure 4](image.png)

**Figure 4** Stepwise bidding of the three generating companies

![Figure 5](image.png)

**Figure 5** Clearing results of two methods

The GenCos designs the price and quantity bids according to the MC, and the power generation is divided into segments according to the bidding function and range of adjustment. Considering the case in which the stepwise bidding segments of G1, G2 and G3 are non-monotonic, the stepwise bidding of the GenCos is shown in Figure 4. The proposed method MCC-M and the traditional method are used for market clearing; the clearing results of G2 are shown in Figure 5, which shows that compared with the clearing results based on the proposed method of MCC-M, the results of the traditional method have obvious errors in the third, fourth and fifth periods, and segments cannot be allocated in their original order. Furthermore, the clearing result of G2 in the fourth period is selected for analysis.

The market clearing result with the traditional method in Figure 6a shows that total power generation $P_{G2,4}$ of G2 is located in the fifth segment, but the quantity of the first segment, $P_{G2,A,1}$, fails to be allocated completely, and the quantity of the fifth segment with a lower bidding price is...
completely allocated. From the result shown in Figure 6(b), we can observe that to comply with the principle of optimization, the traditional method prioritises to the quantity with low price, and the result violates the principle that each quantity of segments should be allocated in its original order.

In the same case, the MCC-M method with quantity settlement range constraints (30), quantity uniqueness constraints (34) and quantity bidding sequence constraints (35–37) is used during market clearing. The clearing result with proposed method MCC-M is shown in Figure 7. From the results shown in Figure 7 (a) and 7(b), total power generation $P_{G2,4}$ of G2 is also located in the fifth segment, and the power quantities of segments 1–5 ($P_{G2,4,1}$–$P_{G2,4,5}$) are allocated in their original order.

The situation is regarded as inaccurate quantity settlement in which power quantities of segments are allocated according to their bid price instead of their original order. A comparative analysis of the inaccurate quantity settlement rate for each GenCo of 24 periods is shown in Table 1, which demonstrates that in the case of non-monotonic stepwise bidding, the traditional method leads to an inaccurate quantity settlement in a total of nine periods, and each GenCo has some inaccurate settlement periods. The result of the traditional method is a lower cost, owing to the priority given to the low-price quantity instead of the original bidding order, resulting in an accurate rate of market clearing of only 62.5% for the correct periods. In contrast, the proposed method of MCC-M improves the accurate rate of market clearing significantly, and the power of all the GenCos are allocated from a small power segment to a large power segment in order. The problems of these traditional methods have been successfully solved by the proposed method MCC-M.

| Market clearing results     | Traditional method | Market clearing constraint-M method |
|-----------------------------|--------------------|------------------------------------|
| $C_{total}$ ($)             | 59,862.42          | 59,956.43                          |
| Inaccurate clearing of GenCos/IC$_{G1,2}$ |                     |                                    |
|                             | $G_{1,2}$          | None                               |
|                             | $G_{2,3}$, $G_{2,4}$, $G_{2,5}$ |                                    |
|                             | $G_{3,11}$, $G_{3,12}$, $G_{3,23}$, $G_{3,24}$ |                                    |
| Accurate rate of market clearing for 24 periods | 62.5%               | 100%                               |
| Processing time (s)         | 2.96               | 2.26                               |
| Memory requirement (Kb)     | 5956.00            | 6052.00                            |

**Figure 7** Uniform purchase price market clearing result with market clearing constraint-M of generating company 2 in the fourth period under the non-monotonic bidding case. (a) Total power generation. (b) The quantity settlement of every segment

**Figure 8** Wind power output and baseline loads of retailers

**Table 1** Comparative analysis of market clearing results with two methods in the 6-Bus-3-Gen case
TABLE 2 Comparative analysis of market clearing results with two methods in the 118-bus-54-gen case

| Market clearing results | Traditional method | Market clearing constraint-M method |
|-------------------------|--------------------|-------------------------------------|
| $C_{\text{final}}$ (\$) | 2,910,060.53        | 2,921,378.49                       |
| Inaccurate clearing of generating company/IC $G_{ij}$ | $G_{1,11-16}, G_{2,14-22}$ | $G_{1,11-16}, G_{2,14-22}$ |
|                        | $G_{3,11-23}, G_{4,11-23}$ | $G_{3,11-23}, G_{4,11-23}$ |
|                        | $G_{7,11-23}, G_{8,11-23}, G_{9,11-23}$ | $G_{7,11-23}, G_{8,11-23}, G_{9,11-23}$ |
|                        | $G_{10,11-23}$ | $G_{10,11-23}$ |
|                        | $G_{13,11-23}, G_{15,11-23}, G_{16,11-23}$ | $G_{13,11-23}$ |
|                        | $G_{17,11-23}$ | $G_{17,11-23}$ |
|                        | $G_{18,12-22}, G_{19,12-23}, G_{23,12-23}$ | $G_{18,12-22}, G_{19,12-23}, G_{23,12-23}$ |
|                        | $G_{24,11-23}, G_{27,11-23}, G_{31,11-23}$ | $G_{24,11-23}, G_{27,11-23}, G_{31,11-23}$ |
|                        | $G_{32,11-23}, G_{33,11-23}$ | $G_{32,11-23}, G_{33,11-23}$ |
|                        | $G_{34,12-22}, G_{35,12-23}, G_{36,11-23}$ | $G_{34,12-22}, G_{35,12-23}, G_{36,11-23}$ |
|                        | $G_{38,11-23}$ | $G_{38,11-23}$ |
|                        | $G_{41,11-23}, G_{42,11-23}, G_{43,11-23}$ | $G_{41,11-23}, G_{42,11-23}, G_{43,11-23}$ |
|                        | $G_{44,11-23}, G_{47,11-23}$ | $G_{44,11-23}, G_{47,11-23}$ |
|                        | $G_{48,11-23}, G_{49,11-16, 18-23}$ | $G_{48,11-23}, G_{49,11-16, 18-23}$ |
| $G_{67,11-16, 18-23}, G_{52,11-23}$ | $G_{67,11-16, 18-23}, G_{52,11-23}$ |
| $G_{53,11-23}, G_{54,11-23}$ | $G_{53,11-23}, G_{54,11-23}$ |

| Accurate rate of market clearing for 24 periods | 41.7\% | 100\% |
| Processing time (\$) | 5.45 | 6.31 |
| Memory requirement (Kb) | 12,632.00 | 12,804.00 |

4.2 Market clearing of IEEE 118-bus system

The performance of the proposed MCC-M method is evaluated by a modified 118-bus-54-gen system [28] that contains 54 GenCos, three wind power plants, and six retailers. The pattern of the wind power output and the baseline load curve of each retailer are shown in Figure 8. The GenCos (G1, G2, ..., G54) are allowed to bid with non-monotonic stepwise segments. The stepwise bidding of the GenCos for clearing is shown in Figure 9, and the clearing results of the 14th period with the traditional method and the proposed method MCC-M are shown in Figure 10(a) and 10(b), respectively. Furthermore, a comparative analysis of market clearing for GenCos and the accurate rate of 24 periods are shown in Table 2.

In this 118-bus-54-gen system, the 37 GenCos have great flexibility with DPR capability. In DPR scenarios, the 37 GenCos bidding have non-monotonic characteristics. In Figure 9, there are 37 GenCos with non-monotonic stepwise bidding and 17 GenCos with monotonically non-decreasing stepwise bidding, and the lowest bidding price of each GenCo is shown by the height of the column in red.
In Figure 10a, each of the 37 GenCos with non-monotonic stepwise bidding has some inaccurate quantity settlements, and the power quantities of segments cannot be allocated in order because they prioritise the low-price quantity instead of the original bidding order. In Figure 10b, however, the proposed MCC-M method solves the inaccurate settlement problem and allows each power quantity of segments to be allocated in order. As can be seen in Table 2, the result of the traditional method is lower cost, but the accurate rate of market clearing is only 41.7%, whereas market clearing with MCC-M has no inaccurate clearing GenCos and improves the accurate rate of market clearing to 100%. Based on MCC-M, market clearing results may serve as an effective reference for power dispatchers to obtain robust and economic solutions, achieving a reasonable optimization goal of maximising social welfare.

Moreover, by comparing the processing time and the memory requirement in Tables 1 and 2, it can be seen that the processing time of the proposed method is competitive with the traditional method, and that the proposed algorithm responds quickly. However, the required memory of the proposed method is a little higher, owing to the additional auxiliary variables introduced in the optimization model.

5 | CONCLUSIONS

An improved optimization method MCC-M is proposed to support market clearing considering non-monotonic stepwise bidding. Augmented market clearing constraints are imposed on the bidding segments to guarantee the proper sequence of the bid quantity on each segment. In addition, the proposed MCC-M method in electricity markets (6-bus system and 118-bus system) is examined, and a comparative analysis shows that the proposed MCC-M method can guarantee orderly power quantity settlement of each bid segment. The proposed method is beneficial for power markets consisting of multiple heterogeneous competitive entities whose marginal cost is not monotonically non-decreasing.

In the future work, more efficiency can be considered, such as computational efficiency and market transaction efficiency.

ACKNOWLEDGEMENT

This work was supported in part by the Foundation of Jiangsu Key Laboratory of Smart Grid Technology and Equipment, and by the National Natural Science Foundation of China (Grant 51707099).

NOMENCLATURE

- \( t \): Index of periods
- \( i \): Index of GenCos
- \( r \): Index of retailers
- \( k \): Index of bidding segments
- \( N_t \): Number of periods
- \( N_G \): Number of GenCos
- \( N_R \): Number of retailers
- \( CS \): Consumer surplus
- \( PS \): Producer surplus
- \( f_{i,t} \): Integral of bidding function of GenCo \( i \) in the \( t \)-th period
- \( f_{i,t} \): Fixed cost of GenCo \( i \) in the \( t \)-th period
- \( f_{min,i,t} \): Cost of GenCo \( i \) to maintain minimum power generation in the \( t \)-th period
- \( SU_{i,t} \): Start-up cost of GenCo \( i \) in the \( t \)-th period
- \( SD_{i,t} \): Shut-down cost of GenCo \( i \) in the \( t \)-th period
- \( x_i \): Binary variable of startup
- \( y_i \): Binary variable of shutdown
- \( c_{SU,i} \): Cost of startup
- \( c_{SD_i} \): Cost of shutdown
- \( c_{Gi,t} \): Cost of GenCo \( i \) in the \( t \)-th period
- \( I_{r,t} \): Integral of the demand curve of retailer \( r \) in the \( t \)-th period
- \( P_{Gi,t} \): Power generation of GenCo \( i \) in the \( t \)-th period
- \( P_{Wi,w,t} \): Power generation of wind power generating unit \( w \) in the \( t \)-th period
- \( u_{i,t} \): Unit commitment of GenCo \( i \) in the \( t \)-th period
- \( P_{Gmini,i} / P_{Gmaxi,i} \): Minimum/maximum capacity of GenCo \( i \) in the \( t \)-th period
- \( RU_{i,t} / RD_{i,t} \): Ramp-up rate/ramp-down rate of GenCo \( i \) in the \( t \)-th period
- \( a_i / b_i / c_i \): Coefficient of the cost function for GenCo \( i \)
- \( MC_{Gi,t} \): Marginal cost of GenCo \( i \) in the \( t \)-th period
- \( k_{Gi} \): Coefficient of the bidding function for GenCo \( i \)
- \( g_{i,k} \): Supply quantities of GenCo \( i \) in the \( k \)-th segment of the \( t \)-th period
- \( g_{maxi,k} \): Size of the \( k \)-th segment of GenCo \( i \) in the \( t \)-th period
- \( N_{BG} \): Number of bidding segments of GenCo \( i \)
- \( P_{Gi,t} \): Continuous form of GenCo \( i \) bidding price in the \( t \)-th period
- \( P_{Gi,k} \): Discrete form bidding price of GenCo \( i \) in the \( k \)-th segment of the \( t \)-th period
- \( P_{Gi,t,k} / P_{Gi,t,k-1} \): Power generation limits of GenCo \( i \) in the \( k \)-th segment of the \( t \)-th period
- \( P_{DR,t} \): Users’ electricity consumption price of retailer \( r \) in the \( t \)-th period
- \( \alpha_r \): Coefficient of revenue function
- \( A_r \): Coefficient related to load size
- \( D_{r,t} \): Demand of retailer \( r \) in the \( t \)-th period
- \( k_{Rr} \): Coefficient of bidding function for retailer \( r \)
- \( P_{Kr,t} \): Continuous form bidding price of retailer \( r \) in the \( t \)-th period
- \( P_{Kr,r,k} \): Discrete form bidding price of retailer \( r \) in the \( k \)-th segment of the \( t \)-th period
- \( D_{r,t,k} / D_{r,t,k-1} \): Minimum/maximum demand of retailer \( r \) in the \( k \)-th segment of the \( t \)-th period
\[ D_{nir,r,k} \] Fixed demand of retailer \( r \) in the \( k \)-th segment of the \( t \)-th period

\[ b_{r,k} \] Demand quantities of retailer \( r \) in the \( k \)-th segment of the \( t \)-th period

\[ b_{\text{max},r,k} \] Size of the \( k \)-th segment of retailer \( r \) demand in the \( t \)-th period

\[ N_{bR} \] Number of bidding segments of retailer \( r \)

\[ M \] Bidding auxiliary coefficient of market clearing

\[ \delta_{i,t,k} \] Bidding judgement binary variable (0-1) of GenCo \( i \) in the \( k \)-th segment of the \( t \)-th period

\[ z_{i,t,k} \] Bidding sequence binary variable (0-1) of GenCo \( i \) in the \( k \)-th segment of the \( t \)-th period

\[ IG_{Gi,t} \] Inaccurate clearing of GenCo \( i \) in the \( t \)-th period

\[ AR \] Accurate rate of market clearing

**REFERENCES**

1. Xiao, Y., et al.: The coordinated development path of renewable energy and national economy in China considering risks of electricity market and energy policy. IEEE Trans. Ind. Inf. 13(5), 2566–2575 (2017)

2. Bahmani-Firouzi, B. et al.: Scenario-based optimal bidding strategies of GENCOs in the incomplete information electricity market using a new improved prey-predator optimization algorithm. IEEE Syst. J. 9(4), 1485–1495 (Dec. 2015)

3. Ye, Y., et al.: Multi-period and multi-spatial equilibrium analysis in imperfect electricity markets: a novel multi-agent deep reinforcement learning approach. IEEE Access. 7, 130515–130529 (2019)

4. Zhang, X.-P.: Restructured electric power systems and electricity markets. In: Restructured Electric Power Systems: Analysis of Electricity Markets with Equilibrium Models, 1st ed, pp. 53–97. Wiley-IEEE Press, Piscataway, NJ (2010)

5. Wim, A., Welington, O., Yongjia, S.: Adaptive partition-based level decomposition methods for solving two-stage stochastic programs with fixed recourse. Inf. J. Comput. 30(1), 57–70 (2018)

6. Faia, R., Pinto, T., Vale, Z.: GA optimization technique for portfolio optimization of electricity market participation. In: IEEE Symposium Series on Computational Intelligence (SSCI), Athens, pp. 6–9. (2016)

7. Laamiri, M.A., Makrizi, A., Essoufi, E.H.: Application of genetic algorithm for solving bilevel linear programming problems. In: Bioinspired Heuristics for Optimization, vol. 774, pp. 123–136. Springer, Cham, Switzerland (2018)

8. Gunnmaasankaraan, H., Viswanath, A., Mahata, K.: Transmission expansion for profit maximization of generators in a decentralized market structure. In: 2016 IEEE Electrical Power and Energy Conference (EPEC), pp. 1–6. IEEE, Ottawa (2016)

9. Carrion, M., Arroyo, J.M.: A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. IEEE Trans. Power Syst. 21(3), 1371–1378 (2006)

10. Morales-Espana, G., Latore, J.M., Ramos, A.: Tight and compact MILP formulation for the thermal unit commitment problem. IEEE Trans. Power Syst. 28(4), 4897–4908 (2013)

11. Zhang, N., et al.: A convex model of risk-based unit commitment for day-ahead market clearing considering wind power uncertainty. IEEE Trans. Power Syst. 30(3), 1582–1592 (2015)

12. Jamali-zadeh, R., et al.: Payment cost minimisation auction for deregulated electricity market using mixed-integer linear programming approach. IET Gener. Transm. Distrib. 7(8), 907–918 (2013)

13. Savelli, I., et al.: An optimization model for the electricity market clearing problem with uniform purchase price and zonal selling prices. IEEE Trans. Power Syst. 33(3), 2864–2873 (2018)

14. Mohravipour, S.S., HaghiFam, M.R., Sheikh-El-Eslami, M.K.: Emergence of capacity withholding: an agent-based simulation of a double price cap electricity market. IET Gener. Transm. Distrib. 6(1), 69–78 (2012)

15. Yang, Y., Qin, C., Zeng, Y.: Interval optimization-based unit commitment for deep peak regulation of thermal units. Energies. 12(5), 1–21 (2019)

16. Qian, C., Bao, H.: The unscrambling of marginal cost algorithm in power market. In: 2010 Asia-Pacific Power and Energy Engineering Conference, Chengdu, China, pp. 1–4. IEEE (2010)

17. Wang, P., et al.: An efficient global optimization approach for solving mixed-integer nonlinear programming problems. In: The 40th International Conference on Computers & Industrial Engineering, Awaui, Japan, pp. 1–4. IEEE (2016)

18. Chen, Y., et al.: Robust unit commitment for large-scale wind generation and run-off-river hydropower. Csee. Ipes. 2(4), 66–75 (2016)

19. Abdi, H., Dehnavi, E., Mohammadi, F.: Dynamic economic dispatch problem integrated with demand response (DEDDCR) considering nonlinear responsive load models. IEEE Trans. Smart Grid. 7(6), 2586–2595 (2016)

20. Fortenbacher, P., Andersson, G., Mathieu, J.L.: Optimal real-time control of multiple battery sets for power system applications. In: 2015 IEEE Eindhoven PowerTech, Eindhoven, Netherlands, pp. 1–6. IEEE (2015)

21. Geng, Z., Conejo, A. J., Xia, Q.: Alternative linearisations for the operating cost function of UC problems. IET Gener. Transm. Distrib. 11(8), 1992–1996 (Jul. 2017)

22. Schwepp, E.C., et al.: Energy market transactions. In: Spot Pricing of Electricity, 1st ed, pp. 55–80. Springe, Boston, MA (1988)

23. Lin, J., Magnago, F.H.: Electricity markets: Theories and applications. In: Microeconomic Theories, 1st ed, pp. 57–95. Wiley-IEEE Press, Piscataway, NJ (2017)

24. Zhao, C., et al.: Multi-stage robust unit commitment considering wind and demand response uncertainties. IEEE Trans. Power Syst. 28(3), 2708–2717 (2013)

25. Ding, T., et al.: Big-M based MIQP method for economic dispatch with disjoint prohibited zones. IEEE Trans. Power Syst. 29(2), 976–987 (2014)

26. Stanton, P.J., et al.: Marketing strategies of Australian electricity distributors in an opening market. J. Bus Ind. Mark. 16(2), 81–93 (2001)

27. Wang, Q., Wang, J., Guan, Y.: Stochastic unit commitment with uncertain demand response. IEEE Trans. Power Syst. 28(1), 562–563 (2013)

28. Wood, A.J., Wollenberg, B.F., Sheble, G.B.: Transmission system effects. In: Spot Pricing of Electricity, 1st ed, pp. 55–80. Springer, Boston, MA (1988)

29. Carrio, M., Arroyo, J.M.: A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem. IEEE Trans. Power Syst. 21(3), 1371–1378 (2006)

30. Morales-Espana, G., Latore, J.M., Ramos, A.: Tight and compact MILP formulation for the thermal unit commitment problem. IEEE Trans. Power Syst. 28(4), 4897–4908 (2013)

How to cite this article: Cheng LM, Bao YQ, Chen C. An improved optimisation method based on augmented constraint for electricity market clearing considering non-increasing stepwise bidding. *IET Energy Syst. Integr.* 2021;3:202–212. https://doi.org/10.1049/esi2.12017
### APPENDIX

#### TABLE A1 Parameters of generators of the 6-bus-3-gen case

| Generators | Bus no. | a (MBtu/MW\(^3\)) | b (MBtu/MW) | c (MBtu) | \(P_{G_{\text{max}}}/P_{G_{\text{min}}}\) (MW) | RU/RD (MW/h) | \(c_{su}\) (MBtu) | Fuel price ($/MBtu) |
|------------|---------|------------------|-------------|----------|-----------------|-------------|---------------|------------------|
| G1         | 1       | 0.005            | 11.67       | 213.10   | 200/50          | 55          | 100           | 1                |
| G2         | 2       | 0.009            | 10.33       | 200.00   | 150/37.5        | 50          | 200           | 1                |
| G3         | 6       | 0.007            | 10.83       | 240.00   | 180/45          | 20          | 0             | 1                |

*Note: Values refer to Lin and Magraro [23].*