OUTLINE OF A SUPERUNIFICATION MODEL

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ABSTRACT. Tensor products of SM bosons and fermions form a spin-2 self-realization of SU(5), and because every quark can be interpreted as a lepton that has coupled to an appropriate element of this adjoint representation, and inversely, SUGRA GUT interactions between proposed spin-2 elements and baryons of spin-(3/2) can exist as residual manifestations of quark-lepton transitions that occur within baryonic domains of asymptotic freedom. Such interactions preserve baryonic structure and continually re-establish a locally SUSY version of broken SU(5), indicating a recurring inflation event that addresses the large-scale.

1. SUGRA GUT Hypothesis

Einstein’s commitment to the a priori was reflected in many comments; e.g. "... but the creative principle lies with mathematics. In a certain sense therefore, I hold it true that pure thought can grasp reality as the ancients dreamed." This philosophy is not antithetical to the current aspirations of physics. General relativity is the Noether principle that preservation of GL(4) by the equations of physics is equivalent to the conservation of 4-momentum by classical gravity and the elusive SUGRA GUT theory is probably a Noether principle which identifies a spin-2 adjoint representation of SU(5) with all tensor products of standard model bosons and fermions that produce spin-2 composites. Specifically if three generations of fermions are incorporated, and if generation is preserved by gravitational GUT interactions, then there are $5^2 - 1$ such composites:

$$
(1.1) \sum_{\mu, \nu=0}^{3} T^{A}_{\alpha B} g^{\alpha}_{\mu \nu} dx^\mu \otimes dx^\nu : A, B = 1, ..., 5; \alpha = 1, ..., 5^2 - 1,
$$

To demonstrate that this adjoint representation is more than a formal result, it is shown that the above elements are components of couplings that form a locally super-symmetric, adjoint representation of SU(5). To establish this, it is observed that every quark can be interpreted as a lepton that has coupled with an appropriate spin-2 composite from the proposed adjoint representation, and inversely; and that interactions between spin-2 fields and spin-(3/2) baryons can therefore be interpreted as residual manifestations of interactions between spin-2 fields and quarks within baryonic domains of asymptotic freedom; i.e. manifestations of quark-lepton transitions within baryonic domains of asymptotic freedom. It can be demonstrated that there are $5^2 - 1$ classes of SUGRA couplings (the number required for a SUGRA self-realization of SU(5)) if and only if interactions between spin-2 composites and spin-(3/2) baryons are restricted to those that correspond to productions of leptons from quarks and inversely. Baryons of spin-(1/2) can also be included in the discussion provided that a photon is absorbed and radiated together with each spin-2 anti-field (to preserve local super-symmetry).
It will now be demonstrated that SUGRA GUT interactions within baryonic domains of asymptotic freedom produce quark-lepton transitions that preserve baryonic structure, provided that the four diagonal (preserved) generators of SU(5) are identified with quantum numbers $I_3$, $Y$ (strong hypercharge), $F$ (fermion number) and $\Delta Q$, where $\Delta Q$ describes the difference between the charges of a quark and lepton that share the same values of the other three quantum numbers.

2. Sub-Baryonic Interactions

The eight composites that correspond to the light fermionic generation are as follows:

(2.1) \[ \gamma \otimes \gamma \otimes e^{-}_L \otimes D_R \]

(2.2) \[ \gamma \otimes \gamma \otimes D_L \otimes e^{+}_R \]

(2.3) \[ \gamma \otimes \gamma \otimes e^{-}_R \otimes \overline{D}_L \]

(2.4) \[ \gamma \otimes \gamma \otimes D_R \otimes e^{+}_L \]

(2.5) \[ \gamma \otimes \gamma \otimes \nu^{-}_L \otimes \overline{U}_R \]

(2.6) \[ \gamma \otimes \gamma \otimes U_L \otimes \nu^{+}_R \]

(2.7) \[ \gamma \otimes \gamma \otimes \nu^{-}_R \otimes \overline{U}_L \]

(2.8) \[ \gamma \otimes \gamma \otimes U_R \otimes \nu^{+}_L . \]

The conservation laws that are introduced above permit three types of interactions. These three types, which will be designated Type A, Type B and Type C respectively preserve $I_3=-1/2$, $I_3=+1/2$ and $I_3=0$. A Type A interaction is exemplified by a locally SUSY vertex

(2.9) \[ U_L \otimes D_L \otimes D_L + \gamma_L \otimes \gamma_L \otimes U_R \otimes \nu^{-}_L \rightarrow \nu^{-}_L \otimes D_L \otimes D_L + \gamma_L \otimes \gamma_L, \]

and a complementary vertex

(2.10) \[ \nu^{-}_L \otimes D_L \otimes D_L + \gamma_L \otimes \gamma_L \otimes U_L \otimes \nu^{+}_R \rightarrow U_L \otimes D_L \otimes D_L + \gamma_L \otimes \gamma_L . \]

Clearly such interactions preserve baryonic structure, but do these interactions predict anything that could subject the proposed model to confirmation or disconfirmation? Possibly. Typically, each baryon consists of many triplets. Until now it was assumed that all triplets within a given baryon consist of the same three quark flavors. But the proposed model seems to indicate that at any given instant, some quarks may be replaced by dynamical configurations of leptons and spin-2 composites. Thus the proposed model seems to predict a fine-grained inhomogeneity of density for the interiors of baryons. In this context, the proposed theory may lend itself to confirmation.
3. The Standard Model in Terms of Type IIB Strings

AdS/CFT correspondence [E. D'Hoker, 2002] admits a model of the early universe in which a de Sitter sphere is equivalent to an event horizon that encloses a world volume of coincident D3-branes (a Type IIB string theory). If these D3 branes consist of three color branes, two left branes, a right brane and a leptonic brane, then the fermions of the standard model can be constructed by intersections of these branes with open strings. Due to the D3 nature of postulated branes moreover, it is argued that this structure permeates 4-spacetime. A 6-dimensional space is transverse to the world volume (this context parallels that described by Zweibach and others). The corresponding CFT model is in terms of a symmetry U(7), which is defined on the postulated seven D3-branes and implicitly then, on all of 4-spacetime. It is argued that Planck scale was introduced a priori onto 4-spacetime in the limit where radii of curvature of postulated 4-dimensional hyperspheres (Section 5) are very small [J. Towe, 2006].

Given a string background, every super Yang-Mills theory corresponds to a Kaluza-Klein theory on a (4+k)-dimensional product space, where k is the dimension of the space that is transverse to the event horizon and to the enclosed world volume. It is instructive to consider the Dirac operator

\[ i\Gamma^A D_A = i\gamma^\mu D_\mu(x) + i\gamma^m D_m(y), \]

which is defined on the (4+6)-dimensional manifold of the Kaluza-Klein theory that is relevant to the present discussion:

\[ \sqrt{\text{det} g_{AB} R_{AB}} = \sqrt{\text{det} g_{\mu\nu} \text{det} \gamma_{mn} \left[ R_4(x) + R_N(y) + \text{higherorderterms} \right]}. \]

Specifically, it is instructive to consider the eigenvalues of the Dirac operator on the 6-dimensional manifold that is transverse to the event horizon. In Planckian units, these eigenvalues are

\[ i\Gamma^A D_A = i\gamma^\mu D_\mu(x) + m M : m = 1, 2, 3, ..., \]

where M is the Planck mass [B. Zweibach, 2004]. Thus the proposed event horizon is regarded as enclosing an integral multiple of the Planck mass.

One class of interactions preserves gravitational equilibrium outside the horizon by introducing SUSY pairs of mass-less, closed strings into this domain. The proposed model now focuses upon the Lagrangians that represent this region and the mass within the space-like interior of the event horizon. It is argued that this mass is perceived as an imaginary (tachyonic) mass in the time-like sector which lies just outside the horizon, and that it produces vacua that do not preserve that symmetry of the gauge Lagrangians. If a Lagrangian undergoes a transformation however, so that it is expanded about the correct vacuum, the tachyonic mass becomes a real mass, and elements of the adjoint representation acquire this mass, breaking the gauge symmetry. The number of gauge bosons that acquire mass is (by the Nambu-Goldstone theorem) the difference between the number of elements in the adjoint representation of the broken symmetry and the number in the adjoint representation of the symmetry which characterizes the new symmetric domain. With reference to SU(5), which initially breaks down to SU(3)XSU(2)XU(1), this number is 12. Thus the X and Y particles that mediate the GUT interaction acquire mass, but the vector bosons of SU(2) do not. Subsequently however, SU(2) is broken by an analogous process. The hierarchy between the two symmetry-breaking events is maintained by renormalization theorems within super-symmetry. In the string
background, the above processes correspond to erosions of the event horizon, so that strings that were closed and mass-less again intersect with the branes from which they were temporarily isolated by the horizon.

The proposed model seems to argue that super-symmetry is never broken; e.g. that self-realizations of local super-symmetry are continually re-established by the influx of spin-2 fields into baryons; i.e. by interactions between spin-2 composites and valance quarks (quarks experiencing asymptotic freedom), which manifest themselves as residual SUGRA GUT interactions between spin-2 fields and spin-(3/2) baryons. And because mass-energy is preserved, and because symmetry-breaking events reduce the relevant symmetry to SU(3)XU(1), it is argued that each Planck mass becomes about $10^{19}$ baryons (1 GeV each). It is speculated that a resulting fusion of baryons produces a very dense star and that the resulting inflation event [A. Guth, 1981] produces the first galaxy.

\section*{4. Quark and Lepton Charges}

In the context of the above-described construction, the generators of flavor SU(3) are regarded as $I_3$ and strong hypercharge, $Y$. Moreover, $Y$ can be related to $I_3$ by the proposed SU(3) flavor symmetry (so that if $(-1/2) \leq I_3 \leq (+1/2)$, then $(-2/3) \leq Y \leq (+2/3)$). In this context, charges, $'Q'$, are assigned to the quarks by

$$Y = K(\pm I_3 + Q),$$

where $K$ is a proportionality constant to be determined, where in the absence of $Q$, $Y$ is proportional (by SU(3) symmetry) to both $+I_3$ and $-I_3$ and where the magnitude of the constant of proportionality is (by symmetry) 1.732. The $Y$-intercept is $Y=KQ$. If the $I_3$-$Y$ coordinate system upon which the triplet and anti-triplet are displayed is centered at the center of symmetry of two mutually inverted isosceles triangles, then the intercept occurs at $Y=-2/3$. Combining this equation with $Y=KQ$ produces

$$Q = \frac{-2}{3K}. \quad (4.2)$$

In the SU(3) symmetric context moreover, $I_3 = \pm 1/2$ when $Y=1/3$. Thus, one obtains a second equation

$$\frac{1}{3} = K(\pm \frac{1}{2}) + KQ. \quad (4.3)$$

Simultaneous solution of Equations 4.2 and 4.3 yields $K = \pm 2$. Given the above result, one can describe the isosceles nature of the triplet by choosing $K=+2$ for $I_3$ less than zero, and $K=-2$ for $I_3$ greater than zero. In this context one can calculate the charges of the quarks; e.g. Inserting a negative (positive) slope for a negative (positive) value of $I_3$ (dictated by the slopes of the sides of the isosceles triangle under consideration), one obtains:

$$\frac{1}{3} = -2(-\frac{1}{2}) + 2Q \quad (4.4)$$

and

$$\frac{1}{3} = +2(+\frac{1}{2}) + 2Q; \quad (4.5)$$

i.e. one obtains $Q=2/3$ and $Q=-1/3$ for the respective values $I_3=-1/2$ and $I_3=+1/2.$
The initial state of the postulated universe (determined in the proposed model by the seven coincident D3-branes that constitute the initial world volume) involves a chiral doublet and a singlet. There are two $I_3$ options for strings that begin on left branes and end on color branes. Such strings therefore occur as doublets:

$$\begin{pmatrix} U_L \\ D_L \end{pmatrix}.$$  

Similarly there are two $I_3$ options for a string that begins on a left brane and ends on a leptonic brane. Introducing labels $e^-_L$ and $\nu\bar{e}^-_L$ for such strings, one obtains doublets such as:

$$\begin{pmatrix} \nu\bar{e}^-_L \\ e^-_L \end{pmatrix}.$$  

On the other hand, there is only one $I_3$ option for a string that begins on a right brane and ends on a leptonic-brane. This is the option $I_3 = 0$. Describing this asymmetry in terms of what is called chirality, one states that since $e^-_L$ and $e^-_R$ are of the same charge, both cannot be included within a chiral system (defined as a system in which opposite helicities must also be of opposite charge). Thus $e^-_R$ is an $I_3$ singlet, which does not interact chirally (transforming $I_3$-up into $I_3$-down or inversely), so that $e^-_R$ is characterized by $I_3$=0. The same statement applies to $\nu\bar{e}^-_R$. In the same context, $U_R$ and $D_R$ are singlets of $I_3$=0.

The charges of both quarks and leptons can be determined by weak hypercharge, but the charges of quarks and leptons are determined here by flavor symmetry and strong hypercharge because this approach produces conservation laws in terms of which one can calibrate the above-described spin-2 adjoint representation of the symmetry SU(5). The conservation laws $\Delta I_3=0$, $\Delta Y = 0$, $\Delta F=0$ and $\Delta Q=0$ permit, and therefore mandate an interaction

$$\Sigma^- \to \Lambda^+ + b + \pi$$  

(which is confirmed by observation), where $\Sigma^-$ and $\Lambda$ are baryons, respectively characterized by charges -1 and 0 and by strangeness numbers -1 and -1, where $a$ denotes a non-baryonic fermion that is characterized by $I_3$=+1/2 and $b$ denotes a non-baryonic fermion of $I_3$=-1/2 and where the sum of the charges of $\pi$ and $b$ is -1. In this context one can formulate the following three equations:

$$Q(b) + Q(\pi) = -1,$$

$$Q(up) - \Delta Q = a$$  

and

$$Q(down) - \Delta Q = b.$$  

Simultaneous solution yields

$$\Delta Q = 2/3,$$

$$Q(b) = -1$$  

and

$$Q(a) = 0.$$
5. Large-Scale Structure

The proposed model can be extended to address large-scale structure. As postulated above, the influx of spin-2 fields into baryons continually re-establishes a recurring, broken version of SUSY SU(5), which indicates a recurring inflation event. It will be demonstrated that six Higgs-plus inflation cycles is equivalent to an iterative calculation that produces about $3.54 \times 10^{11}$ galaxies, which is roughly equivalent to the number indicated by observation. This result depends upon two considerations: one consists of boundary conditions determined by observations of local galactic clusters. The other consists of a replacement of what has traditionally been regarded as a cosmological constant by the relativistic 4-scaler:

$$\Lambda = \left[ \frac{\hbar}{\Delta \tau} \right]^2 : \Delta \tau = \Delta t \sqrt{1 - \frac{v^2}{c^2}},$$

where $\hbar$ represents Planck’s constant and $\tau$ denotes relativistic proper time. This cosmological parameter is applied to a model in which each inflation event produces a new group of 4-momentum states that are characterized by a common recessional speed; i.e. by a new irreducible representation of the Lorentz group SL(3,1). Elements of each irreducible representation are distinguished in terms of mass. The first inflation event is envisioned as producing a galaxy, the second a distribution of galaxies, the third a distribution of galactic clusters etc. It is observed that each element of each proposed irreducible representation corresponds to a Friedmann 4-distribution of mass-energy, and it is postulated that each flat 4-spacetime distribution is compact (including a boundary beyond which expansion of the flat distribution does not occur) and simply connected. Thus, it is postulated that each 4-spacetime distribution of mass-energy is a 4-hyper-sphere (feasibility argument is a 4-dimensional analogue of Poincare’s conjecture), enclosing a 5-dimensional space. Finally, the radius of curvature that is enclosed by each 4-hyper-sphere is regarded as proportional to the value of the proposed cosmological parameter, which is characteristic of the distribution (of the hyper-sphere) in question. Clearly, this radius of curvature is small—very close to zero—unless the recessional speed of the 4-spacetime distribution is near the speed of light. For these latter distributions however, the radii of curvature are significant, so that the hyper-spherical nature of the distributions is revealed. According to this model, one should observe, near the edge of the observable universe, the fragmentation of homogenous shock fronts into many smaller shock fronts that coincide with outer hemispheres of the postulated hyper-spheres. This structure is observed on scales of about $10^9$ ly and larger. It was first observed at Harvard-Smithsonian in 1984 [J. P. Huchra and M. Geller, 1984]. Let us now consider the iterative calculation that was described above.

6. A Theoretical Number of Galaxies

The boundary conditions indicated by observations of the local galactic clusters are that the number of galaxies in a typical local cluster is five and that separations of galaxies represent distances about ten times the galactic diameters; that separations of clusters represent distances about ten times the diameters of the clusters etc.

It is determined from observation that the typical galaxy is about $c t_0 = 10^5$ light years (ly) in diameter. The diameter $c t_N$ of the global state that has resulted from
the Nth inflation event is

\[ ct_N = ct_0 10^N : N = 0, 1, 2, 3, \ldots, \]

where the \( N=0 \) state corresponds to a single, typical galaxy.

Because five galaxies populate the typical basic cluster, and because up to a scale of about \( 10^9 \) ly, the number of galaxies can be enlisted as units in terms of which to express volume (up to this scale galaxies fill a volume rather than populate the surfaces of semi-spherical shells), one can establish the following equation to describe the \( N=1 \) cluster of galaxies:

\[ \frac{4}{3} \pi R_1^3 = 5, \]

which implies that

\[ R_1 = 1.06. \]

From equation 6.1, the radius of this galactic cluster in ly is about

\[ ct_1 = ct_0 10 = 10^5(10)ly = 10^6ly. \]

One now determines the approximate radius (as a number of galaxies) of the \( N=2 \) state. Given the counting device that is described by the second of the proposed boundary conditions, it is argued that the radius of the \( N=2 \) state (the separation of the \( N=1 \) cluster and a second cluster is ten times the diameter of the \( N=1 \) cluster), in terms of a number of galaxies is given by

\[ 10(diameter(localcluster)) = 10(2(1.06)) \approx 21. \]

Thus, the radius in light years of the \( N=2 \) state is (consulting (6.1))

\[ ct_2 = ct_0 10^2 = 10^5(10^2)ly = 10^7ly. \]

Proceeding in this way, one determines the radius (as an approximate number of galaxies) of the \( N=3 \) state, the \( N=4 \) state and the \( N=5 \) state. The radius of the \( N=5 \) state is (as an approximate number of galaxies) 168000. Thus, since the radius of the \( N=4 \) state in ly is about \( 10^9 \)ly, and since (by observation) galactic clusters populate the surfaces of semi-spheres on scales larger than \( 10^9 \)ly, the number of galaxies in the \( N=5 \) state is (summing areas of opposite spherical shells),

\[ 4\pi R_5^2 = 4(3.14)(168000)^2 = 3.54 \times 10^{11}. \]

The radius in light years of the \( N=5 \) state is, consulting equation 6.1,

\[ ct_5 = ct_0 10^5ly = 10^{10}ly, \]

which is thought to be the approximate radius of the universe in terms of Schwarzschild time [J. Towe, 2003, 2006].

7. Conclusion

One of the major problems that seems to stand in the way of physics beyond that standard model is that SUSY GUT theories predict a proton decay that has not been observed. The foregoing discussion addresses this by introducing a new SUSY GUT that preserves baryonic structure. In this model moreover, a broken version of the SUGRA GUT structure is continually re-established by the proposed interactions, so that multiple inflation events are predicted, which address the large-scale structure.
Specifically it was demonstrated that there are $5^2 - 1$ tensor products of bosons and fermions that are of spin 2; i.e. it was demonstrated that there is, formally, an adjoint representation of SU(5) in terms of spin-2 composites provided that one adopts three fermionic generations and the preservation of generation by the implicitly indicated GUT interactions. It was then shown that these composites are physically significant because they are components of couplings that constitute a locally super-symmetric, adjoint representation of SU(5).

This SUSY GUT formalism was derived by observing that every quark can be interpreted as a lepton that is coupled with an appropriate element of the proposed spin-2 adjoint representation. The same was noted regarding every lepton. In this context it was demonstrated that SUGRA interactions between spin-2 composites and spin-(3/2) baryons can exist as residual manifestations of quark-lepton transitions within baryonic domains of asymptotic freedom. Finally it was shown that there are exactly $5^2 - 1$ SUGRA GUT couplings provided that one admits only those interactions that transform a quark into a lepton or inversely. The nature of baryons and fermions, as derived from the proposed Type IIB string theory, motivated association of the preserved generators of SU(5) with quantum numbers $I_3$, strong hypercharge, fermion number and $\Delta Q$ (the charge difference between a quark and lepton that share the same values of the other quantum numbers). It was noted that these conservation laws admit only three types of SUGRA GUT interactions: those that preserve $I_3=+1/2$, $I_3=+1/2$ and $I_3=-1/2$. These were designated Types A, B and C and examples were considered. It was demonstrated that all such interactions preserve baryon structure, departing from the SUSY GUT tradition that predicts proton decay. It was noted that all the interactions that were described, plus others that were implied can also occur within baryons of spin-(1/2), provided that a photon is absorbed and radiated together with each spin-2 field—this to preserve local super-symmetry. It was suggested that the proposed interactions may involve predictions that permit confirmation of the model. Specifically it was noted that the proposed interactions indicate that, at any given time, some triplets within a given baryon may involve three quarks, but that other triplets may involve replacements including leptons and spin-2 composites. Thus it was concluded that the proposed model seems to predict an small-scale inhomogeneity of density within baryons, and in this context, may lend itself to confirmation.

It was demonstrated that the proposed model can be extended to address large-scale structure. It was argued that the influx of spin-2 fields into baryons continually re-establishes a locally SUSY version of broken SU(5), and that these events produce recurring inflation events. It was demonstrated that six Higgs-plus-inflation cycles correspond to an iterative calculation that produces about $3.54 \times 10^{11}$ galaxies—a number which corresponds roughly to that indicated by observation.

It was argued that this process and result involve two considerations. One consists of boundary conditions that are determined by observations of local clusters. A second involves a new interpretation of what has traditionally been regarded as the cosmological constant. It was argued that the cosmological constant should be replaced by a relativistic 4-scaler: $\Lambda = \frac{\Lambda}{\Delta \tau}^2$. This was motivated by an intuition that each recurrence of inflation produced a new irreducible representation of the Lorentz group, which was characterized by a recessional speed, and the elements of which were distinguished in terms of mass. It was argued that the first inflation event produced a distribution of stars, the second a distribution of galaxies, etc.
Finally, because each distribution was transformed into a flat state by an inflation event, it was argued that each flat 4-distribution of mass-energy constitutes a compact 4-space (evidently, galaxies, galactic clusters etc. do not expand beyond the boundaries that are established at the conclusions of the inflation events that produce their common characteristic of flatness). This argument was extended by postulating that each 4-distribution is simply-connected. Finally then, it was postulated (parallel to Poincare’s conjecture regarding 3-spaces that are compact and simply connected) that each such 4-distribution constitutes a 4-spacetime hypersphere that encloses a 5-dimensional space. It was also argued that each 4-hypersphere encloses a radius of curvature that is proportional to the characteristic value of the proposed cosmological 4-scaler (which is determined by the recessional speed of the 4-momentum state in question).

Because the value of the cosmological parameter $\Lambda$ is near zero unless the characteristic recessional speed of a mass-energy distribution is near the speed of light, it was argued that the 5-dimensional character and the hyper-spherical nature of a 4-momentum state is revealed if and only if that state is receding at a speed near that of light. Thus it was predicted that shock fronts near the edge of the observable universe are observed as fragmented into smaller shock fronts that coincide with the outer hemispheres of predicted hyper-spheres. It was noted that this structure is indeed observed at scales of $10^9$ ly and larger, and that it was first observed in 1984 at Harvard-Smithsonian.

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