A Fast Analysis Method of Mechanical Characteristics for Railway Vehicle Traction Motor Hanging Leaf Spring Based on Analytical Calculation

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Abstract. In order to analyse the mechanical characteristics of the traction motor hanging leaf spring of railway vehicles quickly and effectively, by establishing the vertical and the lateral force model of the hanging leaf spring, the mathematical model for analysing the stiffness characteristics and the stress characteristics of the hanging leaf spring were established based on analytical calculation, and the reliability of the mathematical model was verified by the loading-unloading test. The test results show that the analysis method for analysing the mechanical characteristics of the traction motor hanging leaf spring established is reliable. This research can provide a certain popularization and application value for the analysis and design of the railway vehicle traction motor hanging leaf spring.

1. Introduction

During the running of railway vehicles, track excitation will be transmitted to the traction motor through the wheel-set, the gear box, and the other transmission components, and then affect the vibration characteristics of the motor [1, 2]. Therefore, in the design of the railway vehicle power bogie system, it must ensure that the traction motor has a good working conditions, so as to prevent the vibration of the traction motor from being too severe and influencing it’s service life and reliability [3, 4]. Many research results have shown that, traction motor mounted on the frame by hanging leaf spring can effectively mitigate the impact from the track irregularity shock, and can prolong the service life of the motor [5, 6]. However, there is no convenient analysis method for hanging leaf spring at present. Most of them are analyzed by the means of the finite element simulation analysis [7, 8]. It can be seen that, the finite element simulation analysis method needs to establish different simulation models for different vehicles, which leads to a longer development and analysis cycle, high analysis cost and slow analysis speed [9]. This paper starts from the angle of theoretical analysis, by establishing the vertical and the lateral force model of the hanging leaf spring, and a fast analysis method for analyzing the stiffness characteristics and the stress characteristics of the hanging leaf spring was established based on analytical calculation, which can provide an effective reference for the analysis of the traction motor hanging leaf spring.
2. Force model of the hanging leaf spring

The widely used structure of a power bogie system for a traction motor mounted on the frame is shown in Figure 1. In the bogie system, two traction motors are installed on a same bogie through the suspension form, and each motor is fixed together by two leaf springs and a bogie frame, that is to say, two motors are hung by four leaf springs on each frame. Here, the leaf spring and the motor bracket are connected together through a rubber sleeve. The rubber sleeve is installed in the bottom of the leaf spring eye, and the lateral damper is installed on both sides of the motor bracket to reduce the vibration between the motor and the bogie frame.

![Figure 1. Power bogie system of traction motor mounted on the frame.](image)

Analyzing the working situation of the hanging leaf spring, it can be seen that, the leaf spring is mainly subjected to the vertical and the lateral action force. Therefore, in order to simplify the analysis process, it can be analyzed from two aspects of its vertical force and lateral force. Taking the fixed connection end of the hanging leaf spring and the bogie frame as the coordinate origin O, and setting the thickness center of the leaf spring as the coordinate axis x, the force model of the leaf spring under the effect of different loads can be obtained, as shown in figure 2.

![Figure 2. Force model of the leaf spring under the effect of different loads.](image)

In Figure 2, \( h_3 \) is the root thickness, \( h_2 \) is the end thickness, \( h_1 \) is the maximum thickness of the leaf spring eye, \( h_{x2} \) is the thickness of the leaf spring middle oblique section, \( h_{x1} \) is the thickness of the eye end slanting section of the leaf spring; \( L_3 \) is the length of the leaf spring root straight section, \( L_2 \) is the distance from the root to the end of the leaf spring, \( L_1 \) is the distance from the root of the leaf spring to the center of the eye; \( F \) is vertical load, \( P \) is the lateral load. And \( h_{x2}, h_{x1} \) can be expressed as follows

\[
\begin{align*}
    h_{x2} &= \frac{h_2 - h_1}{L_2 - L_3}x + \frac{h_3 L_2 - h_2 L_3}{L_2 - L_3}, \quad L_3 \leq x < L_2, \\
    h_{x1} &= \frac{h_1 - h_2}{L_1 - L_2}x + \frac{h_3 L_3 - h_2 L_2}{L_1 - L_2}, \quad L_2 \leq x < L_1
\end{align*}
\]  

(1)
According to the actual structure of the leaf spring, it can be seen that, the leaf spring is a variable section form, which consists of three sections: the root clamping straight section III, the middle variable section II, and the end eye variable section I, that is, the thickness of the leaf spring varies with its position. Since the transverse section of the actual leaf spring has semi-circular arcs on both sides, the width of the leaf spring also changes with the position of the leaf spring. The shape of its cross section is semicircular arc, as shown in Figure 3, where, \( b_x \) is the leaf spring width at different positions \( x \), \( h_x \) is the leaf spring thickness at different positions \( x \), \( B \) is the maximum effective width of the leaf spring, \( r_x \) is the radius of the side arc of the leaf spring cross section, \( b_x = B - 2r_x \), \( r_x = 0.5h_x \).

\[ r_x = 0.5h_x. \]

**Figure 3.** Schematic diagram of the leaf spring cross section.

As can be seen from Figure 3, the semicircular arc section I can be regarded as consisting of a rectangular section ii and two semicircular sections iii. Based on this, the geometric properties of the cross section of the hanging leaf spring can be obtained, that is, the cross section area: \( A_i = h_x b_x + \pi r_x^2 \); the section inertia moment: \( I_i = \frac{h_x b_x^3}{12} + \frac{\pi r_x^4}{4} \); the section modulus in bending: \( W_i = \frac{I_i}{r_x} \).

### 3. The vertical stiffness analysis mathematical model of the hanging leaf spring

According to the vertical load model shown in Figure 2, using the elasticity mechanics theory \([10]\), the maximum vertical deformation of the hanging leaf spring at the end position can be obtained, that is

\[ f_c = \int_0^{L_3} \frac{F}{EA_{13}(x)} \, dx + \int_{L_3}^{L_2} \frac{F}{EA_{12}(x)} \, dx + \int_{L_2}^{L_4} \frac{F}{EA_{11}(x)} \, dx \]  

(2)

Where, \( E \) is the elastic modulus; \( A_{13}(x), A_{12}(x), A_{11}(x) \) are the cross section areas at different positions. Therefore, according to Eq. (2), the vertical stiffness of the hanging leaf spring can be obtained.

\[ K_c = \frac{1}{f_c} \int_0^{L_3} \frac{1}{EA_{13}(x)} \, dx + \int_{L_3}^{L_2} \frac{1}{EA_{12}(x)} \, dx + \int_{L_2}^{L_4} \frac{1}{EA_{11}(x)} \, dx \]  

(3)

### 4. The lateral stiffness analysis mathematical model of the hanging leaf spring

According to the lateral load model shown in Figure 2, using the Moire integral method \([10]\), the maximum deformation of the hanging leaf spring at the end position can be obtained, that is

\[ f_h = \int_0^{L_2} \frac{P(L_3 - x)^2}{EI_{13}(x)} \, dx + \int_{L_3}^{L_2} \frac{P(L_3 - x)^2}{EI_{12}(x)} \, dx + \int_{L_2}^{L_4} \frac{P(L_3 - x)^2}{EI_{11}(x)} \, dx \]  

(4)

Where, \( I_{13}(x), I_{12}(x), I_{11}(x) \) are the section inertia moments of the leaf spring at different positions. Therefore, according to the Eq. (4), the lateral stiffness of the leaf spring can be obtained by
5. The stress analysis mathematical model of the hanging leaf spring

According to the calculation method of the axial tension and bending stress, using the superposition principle, the stress of the leaf spring at different positions under the combined force can be obtained.

\[
\sigma(x) = \begin{cases} 
\frac{F}{A_3(x)} + \frac{P(L_1 - x)}{W_{i3}(x)}, & 0 \leq x < L_3 \\
\frac{F}{A_2(x)} + \frac{P(L_1 - x)}{W_{i2}(x)}, & L_3 \leq x < L_2 \\
\frac{F}{A_1(x)} + \frac{P(L_1 - x)}{W_{i1}(x)}, & L_2 \leq x \leq L_4
\end{cases}
\]  

(6)

Where, \(W_{i3}(x), W_{i2}(x), W_{i1}(x)\) are the leaf spring section modulus in bending at different locations.

\[
A_3(x) = h_3b_{x3} + \pi r_{x3}^2, \quad I_{i3}(x) = \frac{b_{x3}h_3^2}{12} + \frac{\pi r_{x3}^4}{4}, \quad \frac{W_{i3}(x)}{r_{x3}}, \quad 0 \leq x < L_3,
\]

\[
A_2(x) = h_2b_{x2} + \pi r_{x2}^2, \quad I_{i2}(x) = \frac{b_{x2}h_2^2}{12} + \frac{\pi r_{x2}^4}{4}, \quad \frac{W_{i2}(x)}{r_{x2}}, \quad L_3 \leq x < L_2,
\]

\[
A_1(x) = h_1b_{x1} + \pi r_{x1}^2, \quad I_{i1}(x) = \frac{b_{x1}h_1^2}{12} + \frac{\pi r_{x1}^4}{4}, \quad \frac{W_{i1}(x)}{r_{x1}}, \quad L_2 \leq x \leq L_4
\]

6. Validation of the mechanical characteristics analysis model

In order to verify the reliability of the mechanical characteristics analysis method, taking a hanging leaf spring test specimen as an analysis example, the deformation and stress were tested by using the leaf spring tester. The parameters values of the leaf spring are as follows: \(h_3=18\) mm, \(h_2=8\) mm, \(h_1=16\) mm, \(L_3=135\) mm, \(L_2=420\) mm, \(L_1=530\) mm, \(B=80\) mm, \(E=206\) Gpa. Here, the test was carried out by gradually loading and unloading, the maximum vertical load applied was 40 000 N, the maximum lateral load applied was 1 435 N. The position of the strain gauge patch was located at the lower surface of the
leaf spring end. The leaf spring loading-unloading test is shown in Figure 4. The comparison results between the simulation value and the test value are shown in Table 1.

Table 1. Comparison between the simulation value and the test value.

| Parameter      | Simulation value | Test value | Relative deviation |
|----------------|------------------|------------|--------------------|
| Vertical deformation | 0.088 mm         | 0.087 mm  | 1.149%             |
| Lateral deformation    | 11.005 mm        | 10.786 mm | 2.030%             |
| Stress                  | 256.695 Mpa      | 264.259 Mpa| 2.862%             |

According to Figure 4 and Table 1, it can be seen that, the simulation analysis results of the vertical deformation, the lateral deformation, and the composite stress are similar to the test results. The relative deviations are only 1.149%, 2.030%, and 2.862%, respectively, which is in line with the engineering needs. In addition, the trend of the leaf spring deformation’s test curve is the same as that of the simulation analysis curve. The test results show that the analysis method established is reliable.

7. Analysis example of the hanging leaf spring mechanical characteristics

Taking the leaf spring shown in the previous section as an example, its mechanical characteristics are analyzed by using Eq. (3), Eq. (5), and Eq. (6). The analysis results are as follows: $K_c=453168$ N/mm, $K_h=130.40$ (N/mm), $\sigma_{\text{max}}=256.70$ Mpa. Figure 5 gives the load-deformation curve and the stress variation curve of the hanging leaf spring obtained by analysis. It can be seen that, the vertical stiffness of the leaf spring is several thousand times greater than the lateral stiffness. In addition, the change trend of the leaf spring stress is first decreased, then increased, and then decreased, that is, the variation range of the leaf spring stress can be regarded as consisting of three segments, and the maximum stress point appears at the end of the leaf spring.
8. Conclusion
The mathematical model for analyzing the stiffness characteristics and the stress characteristics of the traction motor hanging leaf spring was established, which can be used to analyze the mechanical characteristics of the hanging leaf spring quickly, and the reliability of the mathematical model was verified by the loading-unloading test. The research can provide a certain popularization and application value for the analysis and design of the railway vehicle traction motor hanging leaf spring.

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