We study possible connection of the d+$^3$He backward elastic scattering cross section and the momentum distribution of the deuterons in $^3$He. With this aim the relativistic calculations in the framework of proton exchange are performed. We also consider possible reaction mechanisms beyond the one nucleon exchange.
1 Introduction

It was demonstrated [1], that dp elastic scattering at $\theta_{c.m.} = 180^\circ$ and inclusive (d,p) breakup with the proton registered at $\theta_p = 0^\circ$ are deeply connected. For example, the relativistic one-neutron-exchange mechanism of Figure 1a describes well energy dependence of differential cross section of the dp elastic scattering if empirical effective momentum distribution of the proton in the deuteron, extracted from exclusive (d,p) reaction data, is used. The details of extraction procedure from exclusive data, as well as comparison of the momentum distributions in the deuteron, extracted from different reactions see in Refs.[2, 3, 4, 5, 6]. Still, there are significant differences for spin-dependent observables in these two reactions.

The mechanism, similar to the one-neutron-exchange, was also assumed to be a basic one for proton–lightest-nuclei backward scattering [7, 8, 9, 10, 11, 12]. In this case the colliding proton and nuclei are exchanged by a group of $(A - 1)$ nucleons, where $A$ - is the atomic number. For example, the simplest reaction of such kind, $p^3$He backward elastic scattering, the two nucleon mechanism of Figure 1b should be dominant. Indeed it was shown, that in the framework of the dynamics at light cone and making use of the empirical momentum distributions of the proton and the deuteron in the $^3$He [14] one describes well the published experimental data [13] for cross section of this reaction [3]. Besides the two nucleon exchange mechanism of Figure 1b some other mechanisms were also discussed in the literature (see [12] and Refs. there). For example a $(NN)_1S_0 + N$-component of the $^3$He wave function was shown to be of a great importance besides a $d + p$-component.

In turn it was argued that in $d + ^3$He backward elastic scattering the main reaction mechanism is connected with the one-nucleon-exchange (ONE) of Figure 1c and only the $(NN)_1S_0 + N$-component should contribute [13]. The first data on differential cross section, $\frac{d\sigma}{d\Omega}$, and tensor analyzing power, $T_{20}$, were recently measured at RIKEN at $T_d = 140, 200$ and $270$ MeV [13].

The aim of this paper is to study how the $d + ^3$He backward elastic scattering is connected with exclusive $A(^3$He, d)$X$ reaction thorough the ONE mechanism and make estimation for the differential cross section at higher energy. Between possible reaction mechanisms beyond the ONE we draw attention to that with elastic deuteron-deuteron rescattering (ddS) of Figure 1d. Due to identity of the deuterons the dd elastic backward scattering is the same as the forward one and the contribution of this diagram should have the same energy dependence as that of ONE. Similar mechanism for the pd backward scattering was already considered in [16].

2 Minimal relativization of the $^3$He

According to [4] we will use so-called ”minimal relativization” scheme of the relativistic dynamics at infinite momentum frame (IMF) for the $^3$He wave function. This is done by introducing internal momentum between the deuteron and the proton in the
$^3$He and substituting this momentum in the nonrelativistic $^3$He wave function. For our purposes it is enough to consider the d+p channel of the $^3$He wave function.

Defining the IMF internal momentum, $\vec{k}$, we follow to our analysis of the $^3$He breakup published in Ref. [14, 17]. First one can define a fraction of the $^3$He momentum carried in by the deuteron in the IMF in longitudinal direction

$$\alpha = \frac{E_d}{E_d + E_p},$$

(1)

where $^3$He and deuteron momenta are assumed to be $p = (E_d, 0, 0, p_3)$ and $d = (E_d, \vec{d}_\perp, d_3)$, respectively. One can introduce invariant mass of virtual d+p pair

$$M_{d+p}^2 = \frac{M^2 + \vec{d}_\perp^2}{\alpha} + \frac{m^2 + \vec{d}_\perp^2}{1 - \alpha},$$

(2)

where $M$ and $m$ are the deuteron and proton mass. For simplicity we will put $M = 2m$. In terms of the invariant mass $M_{d+p}$ the internal momentum is defined to be

$$\vec{k} = (k_\perp, k_3), \quad \vec{k}_\perp = d_\perp,$$

(3)

$$k_3 = \pm \sqrt{\lambda(M_{d+p}^2, M^2, m^2)/(2M_{d+p}^2 - k_3^2)}.$$  

(4)

In (4) $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$. The signs ”+” and ”-” are chosen for $\alpha > \frac{2}{3}$ and $\alpha < \frac{2}{3}$, respectively.

Similar to the deuteron (see, e.g., [18]) one can connect the d+p-channel wave function of the $^3$He, $\Psi(\nu, \sigma_d, \sigma_p; \alpha, \vec{k}_\perp)$, with $^3$He→dp vertex function,

$$\Psi(\nu, \sigma_d, \sigma_p; \alpha, \vec{k}_\perp) = \frac{\Gamma(\nu, \sigma_d, \sigma_p; \alpha, \vec{k}_\perp)}{M_{d+p}^2 - M_r^2},$$

(5)

where $\nu, \sigma_d, \sigma_p$ and $L_3$ are magnetic quantum numbers for the $^3$He, the deuteron, the proton and orbital motion and $M_r$ is the $^3$He mass. In turn the d+p-channel wave function of the $^3$He can be written in terms of S and D wave components $u(k)$ and $w(k)$

$$\Psi(\nu, \sigma_d, \sigma_p; \alpha, \vec{k}_\perp) = \sqrt{\frac{1}{4\pi}} u(k) \left\langle \frac{1}{2} \sigma_d \sigma_p | \frac{1}{2} \nu \right\rangle +$$

$$w(k) \sum_{L_3, \mu} \left\langle \frac{1}{2} \sigma_d \sigma_p | \frac{3}{2} \mu \right\rangle \left\langle \frac{3}{2} \frac{3}{2} \frac{1}{2} \mu | \frac{1}{2} \nu \right\rangle Y_{2L_3}(\hat{k}).$$

(6)

In the framework of the dynamics in IMF the wave function (5) is assumed to be normalized as

$$\frac{1}{2} \int_0^1 \frac{d\alpha}{\alpha(1 - \alpha)} \int d^2k_\perp \frac{\varepsilon_p(k)\varepsilon_d(k)}{\varepsilon_p(k) + \varepsilon_d(k)} \sum_{\nu, \sigma_d, \sigma_p} |\Psi(\nu, \sigma_d, \sigma_p; \alpha, \vec{k}_\perp)|^2 = 1,$$

(7)

where

$$\varepsilon_p(k) = \sqrt{m^2 + k^2}, \quad \varepsilon_d = \sqrt{M^2 + k^2}.$$  

(8)
3 The cross section in the relativistic ONE approximation

According to the perturbation theory in IMF one gets the following expression for the matrix element in the ONE approximation:

\[ T_{\nu\sigma d',\sigma d} = 2S_{dp} (4\pi)^2 \frac{\varepsilon_p(k)\varepsilon_d(k)}{\varepsilon_p(k) + \varepsilon_d(k)} \times \]

\[ \times \sum_{\sigma_p} \Gamma^{\dagger}(\nu, \sigma_d, \sigma_{p}; \alpha, \vec{k}_\perp) \frac{1}{(1 - \alpha)(M_{dp}^2 - M_{\tau}^2)} \Gamma(\nu, \sigma_d, \sigma_p; \alpha, \vec{k}_\perp), \quad (9) \]

where 2 is the combinatorial factor due to Bose statistics for deuterons and \( S_{dp} \) is the spectroscopic factor in the \(^3\)He. Using the connection between the wave function and the vertex function (5) one can rewrite Eq.(9) in terms of the wave function (6)

\[ T_{\nu\sigma d',\sigma d} = 2S_{dp} (4\pi)^2 \frac{\varepsilon_p(k)\varepsilon_d(k)}{\varepsilon_p(k) + \varepsilon_d(k)} \times \]

\[ \times \sum_{\sigma_p} \Psi^{\dagger}(\nu, \sigma_d, \sigma_{p}; \alpha, \vec{k}_\perp) \frac{M_{dp}^2 - M_{\tau}^2}{1 - \alpha} \Psi(\nu, \sigma_d, \sigma_p; \alpha, \vec{k}_\perp). \quad (10) \]

Simple calculations now give the following expression for the differential cross section of the 180° elastic d+\(^3\)He scattering:

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{OP}} = (2S_{dp})^2 \frac{|T|^2}{64s} = \frac{\pi^2}{12s} \left[ \frac{\varepsilon_p\varepsilon_d (M_{dp}^2 - M_{\tau}^2)}{(\varepsilon_d + \varepsilon_p)(1 - \alpha)} \right]^2 n_d^2(k), \quad (11) \]

where \( \sqrt{s} \) is the total energy of colliding particles in the center-of-mass frame.

4 ddS mechanism

Now we will estimate the ddS of Figure 1d. Omitting the real part of the dd elastic cross section at zero angle one gets that appropriate amplitude will be pure imaginary. Because the ONE amplitude is pure real one concludes that interference between these mechanisms must be small:

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{ONE}} + \left( \frac{d\sigma}{d\Omega} \right)_{\text{dd-scatt.}}. \quad (12) \]

In turn the dd-scattering cross section is given by

\[ \left( \frac{d\sigma}{d\Omega} \right)_{\text{dd-scatt.}} = \frac{1}{(8\pi)^2 s} \left[ S_{dp} G_{\tau}(Q^2) M_{dd} \right]^2, \quad (13) \]

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where the formfactor $G_d^d(Q^2)$ is given by

$$G_d^d(Q^2) = \frac{1}{3} Q^2 F_000 + \frac{4}{3} Q^2 F_022,$$

$$F_{dLdL'}(Q) = \int_0^\infty dr j_l(Qr) U_L(r) U_{L'}(r).$$

In (14) $U_L(r)$ is radial wave function of the d+p configuration in the $^3\text{He}$ with orbital quantum number $L$ and $Q$ is deuteron transferred momentum.

5 Numerical calculations and discussion

In our calculations we use momentum distribution of deuterons in the $^3\text{He}$ extracted in framework of impulse approximation from $^{12}\text{C}(^3\text{He}, d)X$ breakup data at zero angle [14]. For spectroscopic number we use $S_{dp} = 1$.

To estimate contribution of the ddS we have to evaluate amplitude of elastic dd-scattering at $\theta = 0^\circ$ and expressed it as

$$M_{dd}(\theta = 0^\circ) = 2|p_{cm}^{dd}| \sqrt{s_{dd}} \sigma_{dd}^{tot},$$

where $p_{cm}^{dd}$ and $\sqrt{s_{dd}}$ are deuteron momentum and total energy of the dd-scattering in its center of mass frame. As a total dd cross section we use $\sigma_{dd}^{tot} = 2 \sigma_{pd}^{tot} \approx 150$ mb.

Results of our calculation are compared with RIKEN data at Figure 2. One sees that the ddS mechanism is not significant at this energy scale. In turn, as displayed at Figure 3 it becomes comparable with the ONE at $T_d \sim 1$ GeV.

One sees that ONE is a main mechanism of the reaction. Nevertheless few comments must be added:

- In the preset calculations we use maximal spectroscopic number $S_{dp} = 1$. But three-body calculations with realistic NN-potential give value $S_{dp} < 1$ (for example for Argon potential it was obtained $S_{dp} = 0.69$ [19]).

- We have estimated ddS in the framework of nonrelativistic calculations, but relativistic effect are seem to be as important in it, as in the ONE.

- We have used very naive assumptions about the amplitude of the elastic dd-scattering. More realistic amplitude should be critical for polarization observables.

- In the calculations presented here the "small components" of the $^3\text{He}$ wave function were neglected.

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Figure 1: One-neutron-exchange mechanism of the pd-backward scattering (a), Two-nucleon-exchange mechanism of the p$^3$He-backward scattering (b), One-nucleon-exchange (ONE) mechanism of the d$^3$He-backward scattering (c) and dd-scattering (ddS) mechanism (d).
Figure 2: Comparison of calculations for the differential cross section with experiment [RIKEN].
Figure 3: Behavior the differential cross section at energy scale of $T_d \sim 1$ GeV.