Study on Urban Rainstorms Design Based on Multivariate Secondary Return Period

Jinping Zhang1,2 · Hang Zhang1 · Hongyuan Fang1

Received: 10 December 2021 / Accepted: 4 April 2022 / Published online: 12 April 2022
© The Author(s), under exclusive licence to Springer Nature B.V. 2022

Abstract
With the rapid urbanization, waterlogging losses caused by rainstorm are becoming increasingly severe. In order to reveal the correlations between rainstorm characteristic elements, and make the calculation of rainstorm return period more reasonable and objective, this study established the joint distribution models of rainstorm elements by using copula theory based on the rainfall data in a Chinese megacity, Zhengzhou. Then their combined design values of primary return period (PRP) and secondary return period (SRP) are derived by the maximum probability method and the same frequency method. Finally, the rainstorm pattern was acquired associated with Pilgrim & Cordery method (PC). The results indicate that the calculation of rainstorm return period (RRP) with SRP is more reasonable than PRP. For same RRP, the rainstorm volume (RV) of “Or” return period type is largest, while the “And” return period’s is smallest, and the RVs of Kendall return period and survival Kendall return period are between them. Concerning Kendall return period, the RVs calculated by the maximum probability method and the same frequency method are pretty close, and their relative deviations are from -5.84% to 4.69%. Compared to “Or” return period, the rainstorm patterns of Kendall return period can reduce the magnitude and investment of the stormwater infrastructure. Moreover, the rainfall with designed rainstorm pattern of survival Kendall return period mainly concentrated before the rain peak in contrast with Kendall return period.

Keywords Urban rainstorm · Primary return period · Secondary return period · Copula theory · Rainstorm pattern design

1 Introduction

In recent years, rainstorms are becoming more frequently affected by climate changes and human activities, which have led to serious urban waterlogging disasters and become the focus of public’s concern worldwide (Bae and Chang 2019; Bertilsson et al. 2019; Liao

* Hang Zhang
zh_strive1997@gs.zzu.edu.cn

1 School of Water Conservancy Engineering, Zhengzhou University, Zhengzhou 450001, China
2 State Key Laboratory of Severe Weather, Chinese Academy of Meteorological Sciences, Beijing 100081, China
et al. 2019; Lohani et al. 2004; Loke et al. 2021; Yao et al. 2021). Zhengzhou city, a super-large central city of China, also suffers from urban flooding. A typical extreme rainstorm happened in Zhengzhou city was on 7/20/2021, which resulting in 292 deaths and 1.736 million people affected. Thus, the risk assessment of rainstorm event is significant for the urban flood control and disaster reduction.

Rainstorm return period (RRP) was widely used to evaluate the occurrence risk of rainstorm event in previous studies (Zhang et al. 2012; Chin 2017; Singh et al. 2020). RRP refers to the average time interval of a specific rainstorm event. However, traditional calculation of RRP only considers the rainstorm volume, but for other rainstorm characteristic elements as peak rainfall intensity (PR), pre-peak cumulative rainfall, peak appearance time, and rainstorm duration are not involved, thus the correlations of these characteristic elements are ignored. Actually, the multivariable return period are now gradually applied to the hydrology field, such as drought (Chen et al. 2013; Tu et al. 2016; Van de Vyver and Van den Bergh 2018; Zhou et al. 2019), sea wave (Chen et al. 2017) and flood events (Chen et al. 2018; Li et al. 2014; Requena et al. 2016), but it is still rare for rainstorm. RV and PR are two mainly considerate elements for a rainstorm event, but rainfall after peak (RAP) can also influence the potential inundation process. Therefore, it is crucial to include RV, PR and RAP in rainstorm risk assessment.

“Or” return period (ORP) as well as “And” return period (ARP) are two common used multivariate return period types and also called as primary return period, but they cannot identify the disaster events’ hazard area accurately (Salvadori and De Michele 2004; Salvadori et al. 2011; Huang and Chen 2015). To overcome this shortcoming, (Salvadori et al. 2011, 2013) proposed KRP and SKRP in 2011 and 2013 respectively. Compared with ORP and ARP, the theoretical identification of dangerous domain is improved by KRP and SKRP, thus their calculations are more reasonable. In previous research, most concerns are limited to KRP, while SKRP is less, even be ignored, which results in an incomplete study on SRP. In view of this, we first redefined SRP, that is, both KRP and SKRP were included in SRP, and then combined with copula theory, maximum probability method, a new rainstorm pattern was derived, this is also the innovation of this study.

This paper analyzed the characteristic elements of rainstorm events firstly, including RV, PR, RAP, and then using Copula method, the ternary joint distribution model of rainstorm elements was constructed. Moreover, combined rainstorm elements with PRP and SRP are displayed to show their differences. Finally, applied with maximum probability method (MP) and same frequency method (SF), a new rainstorm pattern was derived based on designed rainstorm elements. Overall framework of this paper is shown in Fig. 1.

2 Methodology

2.1 Study Area and Data Sources

Zhengzhou city is located in the south of the North China Plain and the lower reaches of the Yellow River, and it lies between 112°42’ E-114°14’ E and 34°16’ N-34°58’ N. There is little precipitation in spring and winter, but more in hot summer season, and its average annual precipitation is about 625 mm. The downtown area of Zhengzhou City is classified into five regions: Huiji District, Zhongyuan District, Jinshui District, Erqi District and Guancheng Ethnic Minority District. Nowadays, concentrated administrative,
commercial and residential buildings exist in these regions, once urban flooding occurs, it will produce huge economy losses and public safety will also be threatened.

In this study, the observed rainstorm data of 13 rain gauge stations from 2011 to 2018 are collected from the Zhengzhou Meteorological Bureau. The location of rain gauge stations is shown in Fig. 2.

2.2 Methods

2.2.1 Copula Theory

It is not easy to establish a joint distribution model with multiple variables if variables are not independent of each other. The Copula function is a successful tool for constructing multi-dimensional joint distribution when the marginal distribution of each variable is known.

Copula theory was firstly proposed by Sklar (Sklar 1959), which can connect the multivariate marginal distributions with a joint distribution. Four commonly used Copula functions in hydrology field are selected to construct the ternary joint distribution of rainstorm elements in this study, including Gumbel Copula, Clayton Copula, Frank Copula and Gaussian Copula. The formulas of Gumbel Copula, Clayton Copula, Frank Copula and Gaussian Copula are shown respectively as follows:
where $u$, $v$ and $w$ are marginal distribution of variables respectively, $\theta$ is a coefficient representing the dependence of variates, $\Phi$ is the standard normal distribution function, $\Sigma$ is the correlation coefficient matrix of Gaussian Copula. $W$ is the integral variable matrix.

$$C(u, v, w) = \exp \left\{ -\left[ (-\ln u)^\theta + (-\ln v)^\theta + (-\ln w)^\theta \right]^{1/\theta} \right\}$$  \hspace{1cm} (1)

$$C(u, v, w) = \left( u^{-\theta} + v^{-\theta} + w^{-\theta} \right)^{-1/\theta}$$  \hspace{1cm} (2)

$$C(u, v, w) = -\frac{1}{\theta} \ln \left[ 1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)(e^{-\theta w} - 1)}{(e^{-\theta} - 1)^2} \right]$$  \hspace{1cm} (3)

$$C(u, v, w; \sum) = \int_{-\infty}^{\Phi^{-1}_u} \int_{-\infty}^{\Phi^{-1}_v} \int_{-\infty}^{\Phi^{-1}_w} \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}} \exp \left\{ -\frac{1}{2} W^T \Sigma^{-1} W \right\} dW \hspace{1cm} (4)$$

where $u$, $v$ and $w$ are marginal distribution of variables respectively, $\theta$ is a coefficient representing the dependence of variates, $\Phi$ is the standard normal distribution function, $\Sigma$ is the correlation coefficient matrix of Gaussian Copula. $W$ is the integral variable matrix.

Fig. 2 Location of rainfall stations in Zhengzhou downtown area
2.2.2 Secondary Return Period

Supposing a two-dimensional joint distribution \((X, Y)\), its return period is traditionally defined as the following two types (Shiau 2006): 1) if \(X\) or \(Y\) exceeds the setting thresholds, the calculated return period is called ORP; 2) if \(X\) and \(Y\) both exceed the setting thresholds, the calculated return period is called ARP. The ORP and ARP were computed based on formula (5) and (6) respectively, and their schematic diagram is shown in Fig. 3.

\[
ORP = \frac{1}{P(X \geq x \lor Y \geq y \lor Z \geq z)} = \frac{1}{1 - C(u, v, w)} \quad (5)
\]

\[
ARP = \frac{1}{P(X \geq x \land Y \geq y \land Z \geq z)} = \frac{1}{1 - u - v - w + C(u, v) + C(u, w) + C(v, w) - C(u, v, w)} \quad (6)
\]

where \(u\), \(v\) and \(w\) are the marginal distribution of \(X\), \(Y\), \(Z\), respectively; \(C\) is the joint distribution value.

As shown in Fig. 3, the curves of “\(P = P_1\)” and “\(P = P_2\)” represent contour lines of two return periods and \(P\) means the joint probability. For any point \((u_1, v_1)\) on the contour line, the corresponding \((x, y)\) can be obtained by \(x = F^{-1}(u)\) and \(y = F^{-1}(v)\). In addition, selecting different points from the same return period contour will result in different risk regions, that is, there are multiple combination events corresponding to the same return period. Moreover, the definitions of ORP and ARP are presented irrationally. For the variables \(X\) and \(Y\), the event of “\(X\) exceeding the threshold” or “\(Y\) exceeding the threshold” is considered as dangerous, larger return period usually means a smaller danger zone, and the dangerous events with larger return period should be involved in the events with smaller return period, but some events that do not satisfy this rule existed in ORP and ARP. Figure 3a is the sketch figure of ORP, “A” is a point on the line \(P = P_1\) and “B” is a point on the line \(P = P_2\). If \(P_1 < P_2\), the dangerous region corresponding to point A should be included in the region of point B, whereas, the existing C* area is obviously unreasonable, the same situation happened in Fig. 3b. This indicates that PRP has a theoretical defect in describing the return period of events (Salvadori and De Michele 2004; Huang and Chen 2015).

According to the Kendall distribution function defined by Nelson (Nelsen 2006), three categories of safe, critical and dangerous are classified by Salvadori (Salvadori et al. 2011)
through judging the joint distribution probability. In 2013, Salvadori (Salvadori et al. 2013) further divided the two-dimensional space into three parts using curves formed by pair \((x, y)\) satisfying \(\bar{F}(x, y) = t\). The KRP and SKRP remedied the PRP’s deficiency by improving the identification of dangerous zone \(\bar{F}(x)\), KRP and SKRP were computed as following:

\[
\bar{F}(x) = 1 - F(x)
\]

\[
KRP = \frac{1}{P[C(u, v, w) > t]} = \frac{1}{1 - K_C(t)}
\]

\[
SKRP = \frac{1}{P(X < x, Y < y, Z < z)} = \frac{1}{\hat{C}(\bar{F}(x), \bar{F}(y), \bar{F}(z))} = \frac{1}{1 - \overline{K_C}(t)}
\]

where \(K_C(t)\) is the Kendall function value with cumulative probability \(t\) (Graler et al. 2013), \(\overline{K}_C(t)\) is the survival Kendall function value (Salvadori et al. 2013). Since \(K_C(t)\) and \(\overline{K}_C(t)\) cannot be figured out by analytic formula, thus the Monte Carlo simulation method was used to generate rainstorm elements combinations here based on the joint distribution model, then the KRP and SKRP would be calculated with empirical frequency method.

3 Results and Discussions

3.1 Joint Distributions of Rainstorm Elements

3.1.1 Correlation Analysis

It is found that about 70 percent waterlogging incidents were caused by short-duration rainstorm events, whose lasting time less than 3 h, through python web crawler. According to the statistics, the rainfall events lasting for 1 h accounted for the largest proportion (45 percent), and in which 80 percent present the single-peak rain pattern. Therefore, the rainstorm events with one-peak of 1 h is focused in this paper.

Figure 4 shows the correlations of rainstorm element. And the correlation coefficients between RV and PR, RV and RAP, PR and RAP are 0.8623, 0.7446 and 0.5175, respectively, which indicating that there is a positive correlation among them.

3.1.2 Marginal Distribution

Pearson Type III Distribution (P-III), Gaussian Distribution, Gamma Distribution, Generalized Extreme Value Distribution and Weibull Distribution are employed to select the best fitting theoretical marginal distribution. Firstly, Kolmogorov–Smirnov (KS) test was used to proof which can pass the significance test at 0.05 level, and then the fitting results between theoretical and empirical frequency of univariable were calculated. Eventually, P-III distribution was chosen as the optimal type of RV and PR, and Gamma distribution as the best one for RAP. The fitting degree between theoretical and empirical distribution of RV, PR and RAP are 0.9769, 0.9836 and 0.9718, respectively.
3.1.3 Joint Distribution of Rainstorm Elements

The maximum likelihood estimation method (Xu et al. 2008) was applied to calculate the parameters of Gumbel, Frank, Clayton and Gaussian Copula. Furthermore, the square Euclidean distance ($d^2$) (Li et al. 2021) and OLS value (Kong et al. 2020) of different Copulas are shown in Table 1.

Table 1 $d^2$ and OLS of different Copulas

|                  | Gaussian Copula | Gumbel Copula | Clayton Copula | Frank Copula |
|------------------|-----------------|---------------|----------------|--------------|
| RV-PR            | $d^2$ 0.0625    | **0.0491**    | 0.1055         | 0.0522       |
|                  | OLS 0.0300      | **0.0254**    | 0.0373         | 0.0262       |
| RV-RAP           | $d^2$ 0.1518    | **0.1319**    | 0.2466         | 0.1841       |
|                  | OLS 0.0447      | **0.0417**    | 0.0570         | 0.0492       |
| PR-RAP           | $d^2$ **0.1390**| 0.7343        | 0.7315         | 0.7386       |
|                  | OLS **0.0428**  | 0.0983        | 0.0981         | 0.0986       |
| RV-PR-RAP        | $d^2$ **0.1403**| 1.0297        | 0.3982         | 1.4428       |
|                  | OLS **0.0430**  | 0.1164        | 0.0724         | 0.1387       |

* Representing the optimal
The results indicate that the Gumbel Copula performs best for the bivariate joint distributions model of RV-PR and RV-RAP, while for PR-RAP, the Gaussian Copula is the optimal. For the tri-variate joint distribution of RV-PR-RAP, the Gaussian Copula was chosen.

### 3.2 Rainstorm Return Period Calculation

The planning rainstorm return period in the central area of Zhengzhou city is generally within 1~5 years, and 10~20 years in the key areas. Therefore, the return periods of 2 years, 5 years, 10 years and 20 years were selected to calculate the recurrence period of rainstorm elements combination here. According to Sect. 2.2.2, the PRP and SRP calculation results of combined rainstorm elements are shown in Table 2.

According to the risk domain represented by each return period type as well as the non-diminishing property of Copula method, for a given return period, ORP should less than KRP and ARP ought greater than SKRP.

As shown in Table 2, for a given return period, ORP is the smallest and even smaller than the given RRP. Besides, the magnitude relationship between the primary return period and secondary return period is ORP<SKRP<KRP<ARP. As mentioned above, ORP’s and ARP’s identification of risk ranges are both unreasonable, thus PRP will result in lower or higher standards for flood control and drainage engineering, and further bring out waterlogging disasters or finical wasting. For safety reasons, it is rational to adopt KRP rather than ORP in the case of any rainstorm element exceeds the set standard, and SKRP could also be considered as a suitable candidate if all rainstorm elements exceed the set standard.

### 3.3 Design Values of Combined Rainstorm Elements

In three-dimension space, the combined rainstorm elements with same rainstorm return period will form a curve surface. According to (Salvadori et al. 2013; Tu et al. 2018), there exist a combination \((u_m, v_m, w_m)\) with the maximum joint probability density \(f(x, y, z)\). This means a combination, which most likely happen, could be found out at specific return period level based on the measured rainstorm data. Consequently, this rainstorm elements combination could be applied to the design of rainstorm pattern. The computed method of \((u_m, v_m, w_m)\) and \(f(x, y, z)\) are shown as follows:

\[
(u_m, v_m, w_m) = \arg \max f(x, y, z)
\]

\[
f(x, y, z) = c \left( u_x, v_y, w_z \right) \cdot f(x) \cdot f(y) \cdot f(z)
\]

| Return period | Frequency | Primary return period | Secondary return period |
|---------------|-----------|-----------------------|-------------------------|
|               | ORP | ARP | KRP | SKRP |
| 2             | 50%  | 1.39 | 3.58 | 2.43 | 1.70 |
| 5             | 80%  | 2.77 | 15.98 | 9.29 | 4.55 |
| 10            | 90%  | 4.99 | 50.1 | 26.54 | 10.91 |
| 20            | 95%  | 9.25 | 156.49 | 72.10 | 26.88 |

---

\(c\) Springer
where $u_m$, $v_m$ and $w_m$ are the univariate marginal distribution value of rainstorm elements with maximum occurrence probability.

In order to reveal the differences among various rainstorm return period (RRP) types as well as the impact of MP and SF methods, the univariate design values of rainstorm elements and their combined design values using MP and SF methods are shown in Table 3.

RV was selected as the major characteristic element and been analyzed here. The results demonstrate that:

1. For the RRP from 2 to 20 years, the RVs of SRP figured by the MP method are smaller than its univariate value, and the RVs of different RRP is sorted as $RV_{ARP} < RV_{SKRP} < RV_{KRP} < RV_{univariate} < RV_{ORP}$. Furthermore, the relative deviations of $RV_{univariate}$ with $RV_{ORP}$, $RV_{ARP}$, $RV_{KRP}$ and $RV_{SKRP}$ are 21.46% to 63.18%, -36.36% to -28.10%, -19.31% to -11.89% and -25.81% to -20.26% respectively.

2. The RVs of ORP and SKRP computed by the SF method are bigger than the univariate value, whereas the RVs of ARP and KRP are just opposite, the relative deviations of $RV_{univariate}$ with $RV_{ORP}$, $RV_{ARP}$, $RV_{KRP}$ and $RV_{SKRP}$ are 20.35% to 29.96%, -25.00% to -18.50%, -22.92% to -6.61% and 13.90% to 24.23% respectively.

3. The RVs of ARP and SKRP computed by SF method are larger than MP’s, yet the RVs of ORP figured by the SF method are less than MP’s. As for KRP, the situation is quite different. For the RRP of 2 and 5 years, the RVs computed by SF method are bigger than MP’s, while the results are reversed for 10 and 20 years. And the reason for this result may be the characteristics of the local rainstorm, the rainstorm of different magnitude has different manifestations. Furthermore, the relative deviations of RVs calculated by SF method and MP method are 47.72% to 65.64% (ORP), 9.24% to 21.02% (ARP), -22.99% to -0.14% (KRP) and -4.48% to 6.20% (SKRP) respectively.

Table 3 The designed rainstorm elements computed by the MP and SF method at different return periods level

| Type | 2a | 5a | 10a | 20a |
|------|----|----|-----|-----|
|      | MP | SF | MP | SF | MP | SF | MP | SF |
| RV   | 22.7 | 22.7 | 35.2 | 35.2 | 45.2 | 45.2 | 55.4 | 55.4 |
| PR   | 9.6  | 9.6  | 14.9 | 14.9 | 18.7 | 18.7 | 22.3 | 22.3 |
| RAP  | 8.7  | 7.5  | 15.7 | 15.7 | 20.5 | 20.5 | 25.1 | 25.1 |
| RV   | 29.6 | 29.5 | 48.5 | 44.1 | 54.9 | 54.4 | 90.4 | 69.6 |
| ORP  | PR  | 12.2 | 12.4 | 17.5 | 18.6 | 21.2 | 21.9 | 33.2 | 27.7 |
| RAP  | 13.1 | 13.0 | 21.3 | 21.0 | 29.4 | 29.4 | 25.9 | 29.2 |
| RV   | 15.3 | 18.5 | 22.4 | 26.4 | 32.5 | 35.5 | 38.4 | 44.1 |
| ARP  | PR  | 4.6  | 7.4  | 7.7  | 11.5 | 11.7 | 15.0 | 19.8 |
| RAP  | 8.6  | 5.6  | 14.2 | 11.0 | 18.4 | 15.7 | 6.4  | 20.4 |
| RV   | 20.0 | 21.2 | 28.6 | 29.3 | 36.5 | 35.1 | 44.7 | 42.7 |
| KRP  | PR  | 8.7  | 8.7  | 12.0 | 12.6 | 14.7 | 15.2 | 16.1 | 18.9 |
| RAP  | 8.7  | 7.5  | 13.8 | 12.6 | 16.1 | 16.2 | 21.5 | 18.4 |
| RV   | 18.1 | 28.2 | 26.2 | 43.4 | 35.3 | 52.2 | 41.1 | 63.1 |
| SKRP | PR  | 7.0  | 12.1 | 10.6 | 17.9 | 12.4 | 21.4 | 14.0 | 24.8 |
| RAP  | 4.8  | 12.0 | 10.0 | 19.6 | 12.4 | 24.0 | 14.6 | 28.1 |
The above studied results show that, the designed rainstorm values of a single rainstorm element ignored the correlation between rainstorm variables. Besides, the designed values of combined rainstorm elements with PRP may be lower or higher. Therefore, it would be a good choice to use SRP combined with MP method for rainstorm design.

3.4 Rainstorm Patterns

Pilgrim & Cordery (PC) method has been widely used to the designing of rainstorm process (Pilgrim and Cordery 1975; Yan et al. 2021). A new rainstorm pattern was introduced on the base of designed rainstorm elements in Sect. 3.3. With 10 min as a time interval, the rainstorm process of 1 h could be divided into six periods. And the implement steps of the new rainstorm pattern are as follows:

Step 1: Referring to the determination of the rain peak position with PC method, the rainfall amount in each period of the rainstorm event was sorted firstly, and the period with larger amount has the smaller serial number.
Step 2: Taking the smallest serial number as the rain peak position, and the peak rainfall of SRP calculated by the MP method listed in Table 3 would be placed at this period.
Step 3: Computing out the proportion of rainfall amount in each period, and then their mean proportions with all rainstorm samples could be figured out.
Step 4: Normalizing the proportions in the pre-peak period and post-peak period separately, then according to Sect. 3.3, the designed rainstorm elements values were successfully allocated to each period. Consequently, the new rainstorm pattern was successfully acquired.

Fig. 5  a) Rainstorm patterns of single rainstorm element, b) Rainstorm patterns of KRP with MP method and c) Rainstorm patterns of SKRP with MP method
The rainstorm patterns based on SKPR and MP method are shown as Fig. 5.

It can be seen from Fig. 5c that the RV is mainly concentrated in the pre-peak periods, besides, the rain intensity increases rapidly and the RV is accumulated quickly. By contrast, the rainstorm volume of KRP with MP method is mainly focused after the rain peak (Fig. 5b). Moreover, the rainstorm patterns based on the single rainstorm element with its marginal distribution (Fig. 5a) are similar to KRP’s.

Fig. 6 Comparison of rainstorm patterns of PRP and SRP with MP method: a) RRP of 2 years, b) RRP of 5 years c) RRP of 10 years, and d) RRP of 20 years
Although the rainstorm patterns of SRP are presented in Fig. 5, the differences between PRP and SRP are needed to be further exploited. In Fig. 6, (i) represents the rainstorm patterns of ORP and KRP, (ii) shows the rainstorm patterns of ARP and SKRP.

The (i) in Fig. 6 denotes that the rainstorm pattern of KRP is nearly involved in the ORP's. This indicates that the waterlogging disaster caused by the later rainstorm pattern is more severe than the former. Once the waterlogging control and drainage engineering is carried out according to ORP type, it is easier to lead to overestimation of rainstorm element value, thus resulting in unnecessary project scale.

The (ii) in Fig. 6 illustrates that the rainstorm volumes estimated by SKRP are mainly concentrated before the rain peak, while the ARPs’ are just the reverse at 2, 5 and 10 years return periods level. The situation is quite different from the 20 years', in which the rain peak is more prominent of ARP type.

In addition, Fig. 6 also provides a proof that a better regularity is existed in the rainstorm patterns of PRP and SRP of 2 and 5 years, but for the higher RRP of 10 and 20 years, the regularity is not obviously. For example, for the RRP of 10 years and 20 years, despite the rainstorm pattern of KRP are involved in the ORPs’, the excess RV of 10 years return period mainly distributed after the rain peak, while the 20 year’s is in reverse. This phenomenon may be explained by the regional rainstorm characteristics and further indicate that rainstorm event with various magnitudes will have their particular rainstorm patterns.

Therefore, compared with ORP, the KRP can decrease the scale of waterlogging control and drainage project by guaranteeing the reasonable return period, thus the project cost will be reduced moderately. The RV of SKRP type is slightly larger than that of ARP, and the RV of SKRP type is mainly concentrated before the peak, while the RV of ARP is in reverse.

Considering the limited rainstorm samples, we selected the rainstorm patterns derived by SRP and MP methods at the return period level of 5 years and 10 years as well the corresponding samples (Fig. 7), and their relative mean deviations are 0.22 and 0.27 separately. Moreover, since the designed combination has not yet occurred, these samples are only close from the RV, whereas there are larger deviations in PR and RAP, so the actual fitting effect could be better.
4 Conclusions

In this study, the concept of SRP was redefined firstly, and then using Copula theory and MP method, the combination values of rainstorm elements were calculated and the rainstorm patterns were designed successfully.

It is concluded that the extra-threshold risk (shown as Fig. 3) of rainstorm element can be described well by the designed value of combined rainstorm elements calculated by the SRP and MP method, and the designed RV of different return period types is sorted as $RV_{ARP} < RV_{SKRP} < RV_{KRP} < RV_{univariate} < RV_{ORP}$.

Compared with ORP, the rainstorm patterns derived by KRP and MP method can achieve a proper magnitude of waterlogging control and drainage engineering, thus reduce the unnecessary investment. While compared with ARP, the RV of the designed rainstorm pattern based on SKRP is mainly concentrated before the rain peak, consequently, it would cause different urban waterlogging process and inundation regions. And the comparison between designed rainstorm pattern and samples also indicate the reliability of our research.

However, this study still has some disadvantages to be overcome. For example, the rainstorm series are not long enough, which may lead to less consideration of past rainstorm characteristics. Therefore, if the study is carried out in other areas, more sufficient rainstorm series will have more satisfying results.

Acknowledgements We would like to express appreciations to colleagues in the laboratory for their constructive suggestions. Also, we thank the anonymous reviewers and members of the editorial team for their constructive comments.

Authors Contributions All authors contributed to the study conception and design. Conceptualization: J.P. Zhang; Methodology, Data analysis and Writing-original draft preparation: H. Zhang; Writing-review and editing: H.Y Fang. All authors read and approved the final manuscript.

Funding This research was funded by the Scientific and Technologic Research Program of Henan Province (grant number 192102310508) and The Open Grants of the State Key Laboratory of Severe Weather (grant number 2021LASW-A15).

Availability of Data and Materials The data is not available for some reason, but the computing code will be available on request from the corresponding author.

Declarations

Ethical Approval Compliance with ethical standards.

Consent to Participate Not applicable.

Consent to Publish This manuscript will be published if accepted.

Competing Interests The authors declare that they have no conflict of interest.

References

Bae S, Chang HJ (2019) Urbanization and floods in the Seoul Metropolitan area of South Korea: what old maps tell us. Int J Disaster Risk Reduct 37:101186. https://doi.org/10.1016/j.ijdrr.2019.101186

Bertilsson L, Wiklund K, de Moura TI, Rezende OM, Veról AP, Miguez MG (2019) Urban flood resilience – A multi-criteria index to integrate flood resilience into urban planning. J Hydrol 573:970–982. https://doi.org/10.1016/j.jhydrol.2018.06.052
Chen YD, Zhang Q, Singh VP (2013) Evaluation of risk of hydrological droughts by the trivariate Plackett copula in the East River basin (China). Nat Hazards 68(2):529–547. https://doi.org/10.1007/s11069-013-0628-8

Chen ZX, Liu ZM, Zhao Q (2018) Comparative analysis on four recurrence levels of joint distribution of flood peak discharge and volume. Acta Sci Nat Univ Sunyatseni 57(1):130–135. https://doi.org/10.13471/j.cnki.acta.snsu.2018.01.017

Chen ZX, Lu JF, Yu JT (2017) Analysis on return levels of trivariate stormy waves based on asymmetric Archimedean copula function. Mar Sci Bull 36(6):631–637. https://doi.org/10.11840/j.issn.1001-6392.2017.06.004

Chin DA (2017) Designing bioretention areas for stormwater management. Environ Process 4:1–13. https://doi.org/10.1007/s40710-016-0200-0

Graler B, van den Berg MJ, Vandenberghe S, Petrozelli A, Grimaldi S, De Baets B, Verhoest NEC (2013) Multivariate return periods in hydrology: a critical and practical review focusing on synthetic design hydrograph estimation. Hydrol Earth Syst Sci 17:1281–1296. https://doi.org/10.5194/hess-17-1281-2013

Huang Q, Chen ZS (2015) Multivariate flood risk assessment based on the secondary return period. J Lake Sci 27(2):352–360. https://doi.org/10.18307/2015.0221

Kong XJ, Song LF, Xu B, Zou DG (2020) Correlation and distribution model for nonlinear strength parameters of rockfill based on Copula function. Chin J Geotech Eng 42(5):797–807. https://doi.org/10.11779/CJG202005001

Li TY, Guo SL, Liu ZJ, Li LP, Hong XJ (2014) Design flood estimation based on bivariate joint distribution of flood peak and volume. J Hydraul Eng 45(3):269–276. https://doi.org/10.13243/j.cnki.slxb.2014.03.003

Li YW, Xu FX, Guo YF (2021) Construction and application for a class of one-parameter perturbation of Copula functions. J Northwest Normal U 57(03):32–38. https://doi.org/10.16783/j.cnki.nwnuz.2021.03.006

Liao X, Xu W, Zhang J, Li Y, Tian Y (2019) Global exposure to rainstorms and the contribution rates of climate change and population change. Sci Total Environ 663:644–653. https://doi.org/10.1016/j.scitotenv.2019.01.290

Lohani AK, Ghosh NC, Chatterjee C (2004) Development of a management model for a surface waterlogged and drainage congested area. Water Resour Manag 18(5):497–518. https://doi.org/10.1007/s11269-004-0170-9

Loke AY, Guo C, Molassiotis A (2021) Development of disaster nursing education and training programs in the past 20 years (2000–2019): A systematic review. Nurse Educ Today. https://doi.org/10.1016/j.nedt.2021.104809

Nelsen RB (2006) An introduction to copulas (springer series in statistics). Springer, New York, p 216

Pilgrim DH, Cordery I (1975) Rainfall temporal patterns for design floods. J Hydraul Div 101(1):81–95. https://doi.org/10.1061/jyeceai.0004557

Requena AI, Chebana F, Mediero L (2016) A complete procedure for multivariate index-flood model application. J Hydrol 535:559–580. https://doi.org/10.1016/j.jhydrol.2016.02.004

Salvadori G, De Michele C (2004) Frequency analysis via copulas: Theoretical aspects and applications to hydrological events. Water Resour Res. https://doi.org/10.1029/2004WR003133

Salvadori G, De Michele C, Durante F (2011) On the return period and design in a multivariate framework. Hydrol Earth Syst Sci. https://doi.org/10.5194/hess-15-3293-2011

Salvadori G, Durante F, De Michele C (2013) Multivariate return period calculation via survival functions. Water Resour Res 49(4):2308–2311. https://doi.org/10.1002/wrcr.20204

Shiau JT (2006) Fitting drought duration and severity with two-dimensional copulas. Water Resour Manag 20(5):795–815. https://doi.org/10.1007/s11269-005-9008-9

Singh A, Sarma AK, Hack J (2020) Cost-effective optimization of nature-based solutions for reducing urban floods considering limited space availability. Environ Process 7:297–319. https://doi.org/10.1007/s40710-019-00420-8

Sklar A (1959) Fonctions de répartition à n dimensions et leurs marges. Publ Inst Statist Université Paris 8:229–231

Tu XJ, Chen XH, Zhao Y, Du YL, Ma MW, Li K (2016) Responses of hydrological drought properties and water shortage under changing environments in Dongjiang River basin. Adv Water Sci 27(6):810–821. https://doi.org/10.14042/j.cnki.32.1309.2016.06.003

Tu XJ, Du XX, Du YL, Chen XH, Li K (2018) Multivariate joint design of hydrological drought and impact of water reservoirs. J Lake Sci 30(02):509–518. https://doi.org/10.18307/2018.0222

Van de Vyver H, Van den Berg J (2018) The Gaussian copula model for the joint deficit index for droughts. J Hydrol 561:987–999. https://doi.org/10.1016/j.jhydrol.2018.03.064
Xu YP, Li J, Cao FF, Ran QH (2008) Application of Copula in hydrological extreme analysis. J Zhejiang U 42:1119–1122. https://doi.org/10.3785/j.issn.1008-973X.2008.07.005

Yan FJ, Li QF, Wang Y, Zhou ZM, Du Y, Peng F (2021) Medium and long duration design rainstorm calculation in Zhenjiang City. Water Resour Prot 37(2):108–111. https://doi.org/10.3880/j.issn.1004-6933.2021.02.017

Yao S, Chen N, Du W, Wang C, Chen C (2021) A cellular automata based rainfall-runoff model for urban inundation analysis under different land uses. Water Resour Manag 35:1991–2006. https://doi.org/10.1007/s11269-021-02826-2

Zhang X, Hu M, Chen G, Xu Y, Zhang X, Hu M, Chen G, Xu Y (2012) Urban rainwater utilization and its role in mitigating urban waterlogging problems—A case study in Nanjing, China. Water Resour Manag 26:3757–3766. https://doi.org/10.1007/s11269-012-0101-6

Zhou P, Zhou YL, Jin JL, Jiang SM, Wu CG (2019) Understanding of hydrological bivariate return periods and its application to drought analysis. Adv Water Sci 30(3):382–391. https://doi.org/10.14042/j.cnki.32.1309.2019.03.008

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.