Molecular Collective Vibrations in the Ternary Neutronless Fission of $^{252}$Cf

Ș. Mișicu$^{1,2}$*, P.O.Hess$^3$, A. Sândulescu$^{1,2}$ and W. Greiner$^2$

$^1$ National Institute for Nuclear Physics, Bucharest-Magurele, P.O.Box MG-6, Romania
$^2$ Institut für Theoretische Physik der J.W.Goethe Universität, Frankfurt am Main, Germany
$^3$ Instituto de Ciencias Nuclerares, UNAM, Circuito Exterior, C.U., A.P. 70-543, 04510 México, D.F., Mexico

Abstract

Based on a recent experimental finding which may suggest the existence of a tri-nuclear molecular structure before the cold ternary fragmentation of $^{252}$Cf takes place, we solved the eigenvalue problem of a certain class of vibrations which are very likely to occur in these molecules. These oscillations are the result of the joined action of rotations of the heavier fragments and the transversal vibrations of the lighter spherical cluster with respect to the fission axis. In the calculation of the interaction between the heavier fragments we took into account higher multipole deformations, including the hexadecupole one, and introduced a repulsive nuclear part to insure the creation of a potential pocket in which a few molecular states can be accommodated. The possibility to observe the de-excitation of such states is discussed in connection with the molecular life-time.

PACS number : 21.60.Gx,24.75.+i,25.85.Ca;

keywords : giant molecule, cold fission, collective motion, heavy-ion interaction

*misicu@theor1.theory.nipne.ro
The scope of this letter is to extend our recent investigations on the molecular configurations in the binary cold fission\cite{1,2} and the ternary cold fission\cite{3}. Growing interest aroused in the last year due to the experimental indication of a long-living ($\geq 10^{-13}$s) structure in the $^{10}$Be accompanied ternary cold fission\cite{4}. Very recently a molecular structure in which $^{12}$C plays the role of the light accompanying particle has been reported\cite{5}.

In this letter we make the suggestion that the ternary cold fission of $^{252}$Cf is a process consisting of two main stages: in the preformation stage a quasi-bound molecular structure is formed in which the heavier fragments are almost co-linear and the light particle (e.g. $\alpha$, $^3$Li, $^{10}$Be and $^{12}$C) which is responsible for the molecular bonding, is orbiting in the equatorial region. This is similar to the case encountered in Molecular Physics, where in a linear or nonlinear chain of three atoms, the central atom ensures two bondings with the eccentric atoms\cite{6}. In the second stage the quasi-molecular state is decaying\cite{7}. Our aim is to study the collective vibrations of the system before the decay takes place.

Recently it has been advocated by us\cite{1}, based on the concept of nuclear molecule\cite{8}, that for fragments emitted in the binary cold fission, with almost no excitation energy, a collective vibrational spectrum will show up as a consequence of small non-axial fluctuations at scission. Such a molecular spectrum can be achieved if the interplay between the Coulomb and the repulsive nuclear core on one hand and the attractive nuclear part on the other hand will produce a pocket in the interaction potential between the fragments\cite{8}. In the case of di-nuclear systems it was shown that possible molecular collective modes can be associated to the elongation variable and rotational vibrations taking place perpendicularly to the fission axis\cite{10}. The last type of modes, e.g. butterfly (bending) and anti-butterfly (wriggling) is also believed to be responsible for the formation of angular momenta in fragments emerging in binary spontaneous fission\cite{2,11,12}.

In a previous paper the classical expression of the tri-nuclear Hamiltonian has been worked out for the case of the $^{96}$Sr+$^{10}$Be+$^{146}$Ba molecule in terms of the Jacobi variables $\mathbf{R}, \mathbf{\xi}$ and the angular velocities $\mathbf{\omega}'$ of the molecular frame\cite{3}. The equilibrium configuration was that of three aligned clusters, with the lighter in-between. In such a configuration the
interaction between the heavy fragments is almost entirely given by the Coulomb term. However, the interaction between the lighter fragment and the heavy fragments consists also of a noticeable nuclear component, which in fact is responsible for the nuclear bond. Like in the case of binary molecules, butterfly modes can occur, in which the fragments rotate in phase while the lighter fragment is approximately preserving its pole-pole configuration with the heavier fragments.

The classical expression of the kinetic energy of the three-body system, after removing the center of mass contribution, is expressed as a sum of translational and rotational degrees of freedom:

\[ T = \frac{1}{2} \mu_{12} \dot{R}^2 + \frac{1}{2} \mu_{(12)3} \dot{\xi}^2 + \frac{1}{2} \omega_1 \mathcal{J}_1 \omega_1 + \frac{1}{2} \omega_2 \mathcal{J}_2 \omega_2 + \frac{1}{2} \omega_3 \mathcal{J}_3 \omega_3 \]  

(1)

The first term describes the relative motion of the di-nuclear sub-system \((12)\) with reduced mass \(\mu_{12} = \frac{m_1 m_2}{m_1 + m_2}\), whereas the second one corresponds to the relative motion of the third cluster with respect to the heavier fragments center-of-mass with reduced mass \(\mu_{(12)3} = \frac{(m_1 + m_2)m_3}{m_1 + m_2 + m_3}\). The vectors \(\omega_{1,2,3}\) denote the angular velocities of the rotational motion of the three clusters, referred to the laboratory frame, \(\omega^T\) being the transpose of \(\omega\). In this paper we consider a spherical light cluster and hence the last term in eq. (1) disappears. The inertia tensors \(\mathcal{J}_i\) are defined in the intrinsic frame such that the only non-vanishing components are the first two diagonal terms, \((\mathcal{J}_i)_{11} = (\mathcal{J}_i)_{22} \equiv J_i\), the quantum rotation around the symmetry axis of any of the two heavier fragments being discarded.

In what follows we are interested in studying the collective spectrum which develops upon constraining the tri-nuclear molecule to perform an oscillatory motion similar to the valence angle bending in molecular physics and the butterfly(bending) modes in di-nuclear molecules, i.e. to perform small displacements from the equilibrium position which result in the decrease of the angle between the two valence bonds, \(\Phi = \pi - \varphi_1 - \varphi_2\), attached to the spherical light fragment 3 (\(\varphi_i\) is the angle between the axis joining the two heavier fragments and the line joining the heavy fragment \(i\) with the light cluster). In the same time, since the
nuclear proximity forces have the tendency to keep constant the reciprocal distances and orientations of the heavy fragments with the light one, we exclude possible bond stretching vibrations. If the bond stretching is absent, then there will be a corresponding decrease in the distance between the heavy nuclei 1 and 2, when the bending angles $\varphi_1$ and $\varphi_2$ are increasing. The quantitative translation of the above mentioned considerations provides us with a set of constraints between the variables of interest in this problem: $R$—the distance between the centers of the two heavier fragments, $\xi$—the distance between the light cluster 3 and the center-of-mass of the heavy fragments ensemble, and the small bending angles $\varphi_1$, $\varphi_2$. These last two variables are related between them, due to the assumption on the constancy of the pole-pole configuration between the light cluster 3 and the heavy fragments 1 and 2

$$\varphi_2 = \frac{R_1 + R_3}{R_2 + R_3} \varepsilon$$

where $\varepsilon = \varphi_1$. Consequently we obtain the following relations, which allows us to eliminate from the kinetic energy (1) the variables $R$ and $\xi$ in favor of $\varepsilon$

$$R = (R_1 + R_2 + 2R_3) \left(1 - \frac{1}{2} \frac{R_1 + R_3}{R_2 + R_3} \varepsilon^2\right)$$

$$\xi = \xi_0 + \frac{1}{2} \left(\frac{\partial^2 \xi}{\partial \varepsilon^2}\right)_{\varepsilon=0} \varepsilon^2$$

where

$$\xi_0 \equiv \xi(0) = \frac{A_1(R_1 + R_3) - A_2(R_2 + R_3)}{A_1 + A_2}$$

$$\left(\frac{\partial^2 \xi}{\partial \varepsilon^2}\right)_{\varepsilon=0} = \frac{A_1 A_2}{A_1 + A_2} \frac{(R_1 + R_3)(R_1 + R_2 + 2R_3)^2}{(R_2 + R_3)(A_1(R_1 + R_3) - A_2(R_2 + R_3))}$$

Note that the above expressions have been written in the second order in $\varepsilon$.

With the above choice the heavy fragments are constrained to rotate only around an axis perpendicularly to the axis joining their centers. This possibility is justified experimentally by the small forward anisotropy of the angular distribution of prompt $\gamma$ radiation.

As well as the above approximations we consider that the nuclei, building-up the molecule, are not performing $\beta$ or $\gamma$ vibrations.
We define a molecular frame whose z-axis coincides with the fission axis and the three-body plane is chosen to coincide with the x–z molecular plane (in this way we eliminate the y-component of the Jacobi coordinate ξ). Also, some assumptions have been made for the Euler angles of the interacting deformed fragments (χ₁, φ₁, φ₁). The angle φ₁ has already been defined above: it describes the angle between the fragment i symmetry axis and the molecular z-axis and it is expressed in terms of ε (see eq.(2)). The geometry of our problem, with the heavier fragments symmetry axes lying in the same plane, makes the Euler angles χ equal, i.e. χ₁ = χ₂. They are combined in the variable θ₃ = (χ₁ + χ₂)/2 which measures the rotation of the tri-nuclear aggregate with respect to the fission axis.

Following the standard procedures [8,13] the total kinetic energy (1) can be expressed as a sum of three parts, the rotational energy $T_{rot}$, the internal kinetic energy $T_{int}$ and the Coriolis coupling $T_{cor}$. In terms of the time derivatives of the Euler angles, $(\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)$, specifying the rotation of the molecular frame, the classical, rotational kinetic energy reads:

$$T_{rot} = \frac{1}{2} \sum_{ij} g_{ij}^{rot} \dot{\theta}_i \dot{\theta}_j$$

where the only non-vanishing components of the rotational metric tensor $g_{ij}^{rot}$ are given by:

$$g_{11}^{rot} = (J_0 + J_1 + J_2) \sin^2 \theta_2 - J_{13} \sin 2\theta_2 \cos \theta_3 + \left(J_1 + J_2 \frac{R_1 + R_3}{R_2 + R_3}\right) \varepsilon \sin 2\theta_2 \cos 2\theta_3$$

$$g_{22}^{rot} = J_0 + J_1 + J_2$$

$$g_{12}^{rot} = \left\{ J_{13} - 2\varepsilon \cos \theta_3 \left( J_1 + J_2 \frac{R_1 + R_3}{R_2 + R_3}\right) \right\} \sin \theta_3 \cos \theta_2$$

$$g_{23}^{rot} = \left\{ J_{13} - 2\varepsilon \cos \theta_3 \left( J_1 + J_2 \frac{R_1 + R_3}{R_2 + R_3}\right) \right\} \sin \theta_3$$

$$g_{13}^{rot} = - \left\{ J_{13} \cos \theta_3 - \varepsilon \cos 2\theta_3 \left( J_1 + J_2 \frac{R_1 + R_3}{R_2 + R_3}\right) \right\} \sin \theta_2$$

where $J_0 = \mu_{12}(R_1 + R_2 + 2R_3)^2 + \mu_{(12)3}\xi_0^2$ and $J_{13} = \mu_{(12)3}(R_1 + R_3)\xi_0$

The intrinsic kinetic energy will be comprised of ”ε”-vibrations and intrinsic rotations of the clusters:

$$T_{int} = \frac{1}{2} \left( J_{13} \frac{R_1 + R_3}{\xi_0} + J_1 + J_2 \left( R_1 + R_3 \right)^2 \right) \varepsilon^2 - J_{13} (\sin \theta_2 \sin \theta_3 \dot{\theta}_1 + \cos \theta_3 \dot{\theta}_2)\varepsilon^2$$

$$+ \frac{1}{2} \left( J_1 + J_2 \frac{R_1 + R_3}{R_2 + R_3}\right) \varepsilon^2 \dot{\theta}_3^2$$

(9)
There will be also a contribution from the Coriolis term

\[ T_{\text{cor}} = \left( J_1 + J_2 \frac{R_1 + R_3}{R_2 + R_3} \right) \left\{ \varepsilon (\sin \theta_2 \cos 2\theta_3 \dot{\theta}_1 - \sin 2\theta_3 \dot{\theta}_2) \dot{\theta}_3 + (\sin \theta_2 \sin 2\theta_3 \dot{\theta}_1 + \cos 2\theta_3 \dot{\theta}_2) \dot{\varepsilon} \right\} \]  

(10)

The next development of our considerations is facilitated by the peculiarities of the spontaneous fission of \(^{252}\text{Cf}\). Since the spin \( J \) of the mother nucleus is 0, the total helicity \( K \), i.e. the projection of the total angular momentum on the fission axis is zero for both binary and ternary fragmentations and therefore \( \dot{\theta}_3 = 0 \). To simplify even further one choose the molecular frame such that \( \theta_3 = 0 \). After quantizing the kinetic energy in three coordinates \((\varepsilon, \theta_1, \theta_2)\) and neglecting terms multiplied by the non-diagonal matrix-element \( J_1 + J_2 \frac{R_1 + R_3}{R_2 + R_3} - J_{13} \), which prove to be small in the resulting metric tensor, we arrive to a form of the kinetic energy in which the rotations are decoupled from the butterfly vibrations

\[ \hat{T} = -\frac{\hbar^2}{2(J_0 + J_1 + J_2)} \left( \frac{1}{\sin \theta_2} \frac{\partial^2}{\partial \theta_1^2} + \cot \theta_2 \frac{\partial}{\partial \theta_1} + \frac{\partial^2}{\partial \theta_2^2} \right) - \frac{\hbar^2}{2 J_\varepsilon} \frac{\partial^2}{\partial \varepsilon^2} \]  

(11)

where

\[ J_\varepsilon = J_1 + J_2 \left( \frac{R_1 + R_3}{R_2 + R_3} \right)^2 + J_{13} \frac{R_1 + R_3}{\xi_0} \]  

(12)

Next we turn our attention to the computation of the potential. The total interaction energy is given by the sum

\[ V = \sum_{i \neq j = 1}^{3} V_{ij}(R_{ij}) \]  

(13)

The interaction between two clusters composing the giant molecule can be calculated as the double folding integral of ground state one-body densities \( \rho_{1(2)}(r) \) of heavy ions:

\[ V(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \rho_1(\mathbf{r}_1) \rho_2(\mathbf{r}_2) v(s) \]  

(14)

We employ the M3Y \( NN \) effective interaction for the nuclear part of \( v \) as described in [14] to which we add a repulsive core in order to take into account two major factors - the density dependence of the \( NN \) interaction and the Pauli principle, which are important at distances...
corresponding to the overlap of the nuclear volumes. This choice is particularly useful for a molecular model in which the repulsive core prevent the re-absorption of the lighter fragment by the heavier one.

We consider that the nuclei composing the giant molecule are in their ground state with known quadrupole $\beta_2$, octupole $\beta_3$ and hexadecupole deformations $\beta_4$.

From what has been said above the interaction between the light cluster 3 and the heavy fragments remains unmodified when the bonding angle is decreased. The reciprocal distances and orientations between the light cluster and the heavier fragments being freezed. Therefore its contribution to the total Hamiltonian adds only a constant term. On the contrary, the interaction of the two heavy fragments 1 and 2 depends on the butterfly angle $\varepsilon$ as can be seen from the multipolar expansion of the potential

$$V_{12} = \sum_{\lambda_1, \lambda_2, \lambda_3, \mu} \frac{4\pi}{\sqrt{(2\lambda_1 + 1)(2\lambda_2 + 1)}} V_{\lambda_1, \lambda_2, \lambda_3}^{\mu, \mu_0}(R_{12}) Y_{\lambda_1, \mu}(\varepsilon, 0) Y_{\lambda_2, -\mu}(\frac{R_1 + R_3}{R_2 + R_3}, \varepsilon, 0)$$  \hspace{1cm} (15)

For small non-axial fluctuations (bendings), the potential in the neighborhood of the scission, or "molecular equilibrium" point $R_0 = R_1 + R_2 + 2R_3$, gets a simplified form, provided we keep terms up to the second power in angle:

$$V_{12} = V(R_0) + \frac{1}{2} C_\varepsilon \varepsilon^2$$  \hspace{1cm} (16)

where the stiffness parameter reads

$$C_\varepsilon = -\frac{1}{2} \sum_{\lambda_1, \lambda_2, \lambda_3} \left[ \lambda_1(\lambda_1 + 1) \frac{R_2 - R_1}{R_2 + R_3} + \left( \lambda_2(\lambda_2 + 1) \frac{R_1 - R_2}{R_2 + R_3} + \lambda_3(\lambda_3 + 1) \right) \frac{R_1 + R_3}{R_2 + R_3} \right] V_{\lambda_1, \lambda_2, \lambda_3}^{0, 0, 0}(R_0)$$

\hspace{1cm} \hspace{1cm} - \sum_{\lambda_1, \lambda_2, \lambda_3} \frac{R_1 + R_3}{R_2 + R_3} \left( R \frac{\partial V_{\lambda_1, \lambda_2, \lambda_3}^{0, 0, 0}(R)}{\partial R} \right)_{R = R_0}$$  \hspace{1cm} (17)

Hence the quantized vibrational Hamiltonian of the giant tri-nuclear molecule acquires the form

$$H_{vib} = -\frac{\hbar^2}{2J_\varepsilon} \frac{\partial^2}{\partial \varepsilon^2} + \frac{1}{2} C_\varepsilon \varepsilon^2$$  \hspace{1cm} (18)

The spectrum of the butterfly vibrations is given then simply by
\[ E_\varepsilon = \left( n_\varepsilon + \frac{1}{2} \right) \hbar \omega_\varepsilon \]  

(19)

where \( \omega_\varepsilon = \sqrt{\frac{C_{\varepsilon}}{J_\varepsilon}} \).

As one can see from Table I, in the tri-nuclear case the values of \( \hbar \omega_\varepsilon \) are only slightly smaller if we take into account Nuclear+Coulomb forces compared to considering only Coulomb forces. This was to be expected since in this case the distance between the two heavier fragments is already some fm beyond the top of the barrier and therefore the Coulomb forces are clearly dominating the interaction. This has to be contrasted to the case when we remove the lighter cluster, i.e. we insert \( R_3 = 0 \) and \( A_3 = 0 \) in (17), and the two heavier fragments are brought in touch. In this case the nuclear contribution to the interaction significantly decreases the value of \( \hbar \omega_\varepsilon \). Notice also the variation with the type of nuclear collective flow reflected in the type of inertia moment, the smallest values for \( \hbar \omega_\varepsilon \) being obtained for rigid rotations.

To have an idea of the order of magnitude of tri-nuclear molecules life-times we evaluate the half-life of the corresponding di-nuclear configuration, when the lighter cluster is absent. Using the one-dimensional WKB formula

\[ \lambda = \frac{\omega_R}{2\pi} \exp \left\{ -2 \int_{R_{12}}^{R_{34}} \sqrt{\frac{2\mu_{12}}{\hbar^2} \left( D - \frac{\hbar \omega_R}{2} \right)} \right\} \]  

(20)

for the decay rate of a metastable state of energy

\[ \frac{\hbar \omega_R}{2} = \frac{1}{2} \hbar \sqrt{\frac{1}{\mu_{12}} \left( \frac{\partial V_{12}(R)}{\partial R^2} \right)_{R=R_{\min}}} \]  

(21)

in the potential pocket of depth \( D = V_{12}(R_{max}) - V_{12}(R_{min}) \) of the two heavier nuclei, we obtain the half-life:

\[ T_{1/2} = \frac{\ln 2}{\lambda} \]  

(22)

The computed life-times for di-nuclear molecules have large values(\( \gg 10^{-13} \)s, according to Table I) and since the light particle has the tendency to delay the penetration of the mutual barrier of the heavier fragments, due to the attractive interaction with the heavier fragments, it is then justified to expect even larger values of \( T_{1/2} \) for tri-nuclear molecules.
We chose in this paper a linear configuration for the giant tri-nuclear molecule. According to very recent calculations of penetrabilities in alpha ternary cold fission this configuration is reached at a certain step of the tunneling process. It could be possible that before the beginning of mutual penetration of the multi-dimensional barrier, the giant tri-nuclear molecule is found in a tri-angular quasi-equilibrium configuration. In a forthcoming paper we will present the calculations of the molecular collective spectra also for such a configuration but the general trends should be same: considering an adiabatic scenario for the decay, the collective molecular modes will adjust slowly to the value of the elongation variable. As we mentioned earlier, the life-times of such nuclear molecules are suspected to be larger than $10^{-13}$s, whereas the life-times of the first molecular states computed in this paper are expected to be $\tau \approx 1/\omega_\varepsilon > 10^{-22}$s. The gammas coming from the de-excitation of these states should be observed before tunneling ended and therefore they should not be pronouncedly Doppler-shifted.

ACKNOWLEDGEMENTS

One of the authors(S.M.) would like to acknowledge the financial support from DAAD-Germany.
REFERENCES

[1] S. Mišicu, A. Sândulescu and W. Greiner, Mod.Phys.Lett.A, 12 (1997) 1343.

[2] S. Mišicu, A. Sândulescu, G.M. Ter-Akopian and W. Greiner, Phys.Rev.C 60 (1999) 034613.

[3] P.O. Hess, W. Scheid, W. Greiner and J.H. Hamilton, J.Phys.G 25, L139 (1999).

[4] A.V. Ramayya, J. K. Hwang, J. H. Hamilton, A. Sândulescu, A. Florescu, G. M. Ter-Akopian, A. V. Daniel, G. S. Popeko, W. Greiner, J. O. Rasmussen, M. A. Stoyer, J. D. Cole and GANDS Collaboration, Phys.Rev.Lett., 81 (1998) 947.

[5] J.K. Hwang, A.V. Ramayya, J.H. Hamilton and W. Greiner, to be published (1999).

[6] E.B. Wilson, J.C. Decius and P.C. Cross, Molecular Vibrations, (McGraw-Hill, New York, 1955)

[7] A. Sândulescu, S. Mišicu, F. Carstoiu and W. Greiner, Elem.Chast.i At.Yad. 30 (1999) 908.

[8] P.O. Hess and W. Greiner, Il Nuovo Cimento 83A (1984) 76; P.O. Hess, W. Greiner and W. T. Pinkston, Phys.Rev.Lett., 53 (1984) 1535.

[9] M. Seiwert, W. Greiner and W. T. Pinkston, J.Phys.G 11 (1985) L21.

[10] J.R. Nix and W.J. Swiatecki, Nucl.Phys. 71 (1965) 1.

[11] J.O. Rasmussen, W. Nörenberg and H.J. Mang, Nucl.Phys.A 136 (1969) 465.

[12] M. Zielinska-Pfabé and K. Dietrich, Phys.Lett. 49B (1974) 123.

[13] J.M. Eisenberg and W. Greiner, Nuclear Theory I: Nuclear Models, 3ed edition (North-Holland, Amsterdam, 1987).
[14] A. Săndulescu, Ş. Mişicu, F. Carstoiu, A. Florescu and W. Greiner, Phys. Rev. C 56 (1998) 2321.
TABLES

TABLE I. The quantum energy $\hbar \omega_\varepsilon$ of the butterfly mode in the case of tri-nuclear ($^{10}\text{Be}$ and $^{12}\text{C}$ accompanied ternary fission) and di-nuclear configurations. $N + C$ signify calculations with nuclear and Coulomb forces, whereas $C$ only with Coulomb forces. Three types of the inertia moments are considered: a) experimental , b) Irrotational Fluid and c) Rigid Rotator. On the last column we listed the half-life of the di-nuclear molecule.

| Inertia moment | $h\omega_\varepsilon^{N+C}$(KeV) | $h\omega_\varepsilon^C$(KeV) | $h\omega_\varepsilon^{N+C}$(KeV) | $h\omega_\varepsilon^C$(KeV) | $T_{1/2}(s)$ |
|----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-------------|
| $J = J_{exp}$  | $^{106}\text{Sr} + ^{10}\text{Be} + ^{146}\text{Ba}$| 1900.4                     | 2040.4                     | 1100.7                     | 3377.3       |
| $J = J_{IF}$   |                             | 1691.2                     | 1815.9                     | 880.9                     | 2702.8       | $\leq 5.5 \times 10^{-9}$ |
| $J = J_{RR}$   |                             | 1014.4                     | 1089.1                     | 440.4                     | 1351.4       |

| Inertia moment | $h\omega_\varepsilon^{N+C}$(KeV) | $h\omega_\varepsilon^C$(KeV) | $h\omega_\varepsilon^{N+C}$(KeV) | $h\omega_\varepsilon^C$(KeV) | $T_{1/2}(s)$ |
|----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-------------|
| $J = J_{exp}$  | $^{96}\text{Sr} + ^{12}\text{C} + ^{144}\text{Xe}$| 1799.7                     | 1917.3                     | 791.4                     | 3459.6       |
| $J = J_{IF}$   |                             | 1611.5                     | 1716.8                     | 625.5                     | 2734.5       | $\leq 9.6 \times 10^{-9}$ |
| $J = J_{RR}$   |                             | 996.0                      | 1061.1                     | 312.75                    | 1367.25      |