Hybrid paramagnetic-ferromagnetic quantum computer design based on electron spin arrays and a ferromagnetic nanostripe

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Abstract. Electron spins placed in a magnetic field gradient, interacting by dipolar magnetic couplings and manipulated by microwave pulses represent a possible architecture for a quantum computer. Here, a general design for the practical implementation of such nanodevice is presented on the example of electron spins in silicon carbide placed nearby a permalloy ferromagnetic nanostripe. Firstly, the confined spin wave resonance spectrum of the nanostripe and the properties of its magnetic field gradient are calculated. Then, I show how to avoid microwave driven electron spin decoherence. Then, I show that at temperatures requiring only nitrogen gas cooling, the silicon vacancy decoherence process due to ferromagnetic fluctuations is negligible compared to the one due to a two phonons Raman process, as long as the silicon vacancies spins are placed far enough from the ferromagnetic nanostripe. I then show that a nitrogen cooled hybrid quantum processor with 32 silicon vacancies spins qubits in SiC is theoretically possible. NV centers in diamond, N@C60 molecules, and radical molecules are three other good spin qubits candidates for implementing this hybrid quantum processor.

1 Introduction

During the last decades, the development of nanotechnologies has allowed the experimental demonstration of entanglement between the quantum states of different pairs of nanosystems [1–4], a key preliminary step towards quantum information processing. However, scaling those experiments up to at least ten coupled quantum systems, called quantum bits or qubits, on a single chip remains extremely challenging. This difficulty is linked to the need to couple quantum bits over nanoscale distances to perform conditional logical operations, except when some alternative strategy for long distance entanglement is available [5,6]. In the context of quantum bits encoded in electron spins in solids, the easiest solution to couple them is to use the dipolar magnetic coupling [3,4,7], which has however two drawbacks. Firstly, it becomes too weak beyond ten nanometers for efficient quantum state entanglement. Secondly, it is a permanent coupling, which requires methods to effectively switch those couplings on or off. The upscaling problem is thus directly related in this context to the difficulty to precisely position the electron spins on the chip and also to manipulate them with nanoscale resolution. State of art nanolithography and ion implantation methods to build silicon [8] or diamond [3,4,9] based small scale quantum devices with few coupled electron spins are still far from reaching the ten qubits scale. Building solid state arrays of dipolar coupled electron spin qubits thus requires an alternative nanotechnology and also a method and a nanodevice allowing to switch those dipolar couplings on or off. Transmission electron microscope (TEM) based nanofabrication methods with nanometer scale precision were recently demonstrated with carbon nanotubes [10] and graphene materials [11], and could provide the required alternative nanotechnology. Also, few years ago, it was shown that selective on/off switching of the dipolar coupling is possible using sequences of microwave pulses applied to dipolar coupled electron spins submitted to a magnetic field gradient [7]. The problem was thus shifted to the production of strong magnetic field gradient. In the field of quantum computing with nuclear spins, it was proposed to use dysprosium nanoferromagnet to produce a huge magnetic field gradient [12]. However, in the context of electron spin qubits, the nanoferromagnet also produces unwanted additional decoherence of the electron spins [13] due either to incoherent thermal excitation or to coherent microwave excitation of the spin waves [14] confined in the nanoferromagnet [15,16].

Here I present a general intermediate scale quantum register design containing a ferromagnetic nanostripe and nearby arrays of electron spins. To be more specific and without loss of generality, I consider electron spin qubits in silicon carbide (SiC) created nearby a permalloy ferromagnetic nanostripe. The spin qubits in SiC considered here are the silicon vacancies [17–21], which may be created...
in SiC by irradiation at the nanoscale using the 300 keV electrons of a TEM, TEM irradiation being a nanofabrication method with high spatial resolution under continuous development [10,11,18,21,22]. Using magnetostatic theory [12], the Landau Lifschitz equation of motion of magnetization in the nanoferromagnet [16], a one dimensional Schrödinger equation solved numerically by a transfer matrix method [23], and the density matrix theory of spin qubit decoherence [13], I show how to design simultaneously the magnetic field gradient and the confined spin waves spectrum of the nanoferromagnet in order to avoid additional decoherence. The remaining two phonon Raman decoherence process of silicon vacancies in SiC is then compared to the length of the required long coupling/refocusing microwave pulses sequences associated to this processor architecture [7]. This allows to show that a nitrogen cooled 32 spin qubits hybrid SiC-permalloy quantum processor is feasible. This hybrid quantum processor could thus be operated with microwave pulses [24] and read out with high sensitivity using ensemble optical measurements of the paramagnetic resonance of silicon vacancy electron spins in SiC [17,18].

2 Design and fabrication of a nanodevice with electron spin arrays nearby a ferromagnetic nanostripe

The general design proposed here for quantum information processing could be applied, in principle, to all kind of electron spin qubits like phosphorous donors in silicon [8,25], nitrogen donors or NV centers in diamonds [3,4,9], transition metals ions or shallow donors in zinc oxide [13,26–29], N@C60 molecules [30], paramagnetic defects in graphene [31], paramagnetic dangling bonds arrays on hydrogen passivated silicon surface [32], or supramolecular assemblies of paramagnetic molecules such as neutral radical molecules [33] or copper phthalocyanine molecules [34], as long as regular dense arrays of them could be produced at precise positions nearby a ferromagnetic nanostripe. Without loss of generality and to be more quantitative, I focus here on a nanodevice made with silicon carbide and permalloy materials and build with top-down methods. The successive steps of two possible nanofabrication processes allowing to build such a nanodevice are described in Figures 1a and 1b. The resulting nanodevice is shown in Figure 1c. Each of the three SiC polytypes, 4H, 6H, and 3C, has some paramagnetic defects related to silicon vacancies which could potentially encode a good electron spin quantum bit [17–21]. Some silicon vacancies defects in 6H-SiC have a spin $S = 3/2$ with an isotropic $g$ factor $g = 2.0032$ and a small zero field splitting [18]. Their spin state can also be prepared by optical alignment [18] and it can be readout by the highly sensitive ODMR method (optically detected magnetic resonance) [38]. It seems thus highly relevant to encode an electron spin qubit on this kind of silicon vacancy defect in 6H-SiC (for example at X band or at higher microwave frequency, using: $|S = 3/2, M_S = -3/2\rangle \equiv |0\rangle$ and $|S = 3/2, M_S = -1/2\rangle \equiv |1\rangle$). In the following calculations, I simplify the spin qubit model by using a simple pseudo spin $s = 1/2$ with a $g$ factor $g = 2$, without any loss of generality.

3 Dipolar magnetic field gradient outside the nanoferromagnet and effective Ising coupling between electron spin qubits

The practical implementation of single and two qubits quantum gates on chains of dipolar coupled electron spin qubits, as those shown in Figure 1, requires appropriate microwave pulse sequences and a strong magnetic field
gradient along the direction of the spin chains of the nanodevice to transform their dipolar coupling into an effective Ising-like coupling \([7]\). As here I also consider the possibility to perform ensemble measurements of the spin states of the spin qubits, for example using the highly sensitive ODMR method \([38]\), the magnetic field gradient produced by the permalloy ferromagnetic stripe shown in Figure 1 has not only to be strong enough along \(x\), but also to be enough homogeneous along \(y\) and \(z\). Strong enough along \(x\) means here that the energy difference between one qubit \(j\) (at position \(x_j\)) and the next one \(j+1\) (at position \(x_{j+1}\)) along the spin chain has to be much larger than the energy associated to the dipolar coupling between them, which depends on their relative distance \(l_{\text{intra}}\). In the nanodevice described in Figure 1, \(l_{\text{intra}}\) is thus also the periodicity of each spin chain. The condition for an effective Ising coupling translates into

\[
|g\mu_B (B_0 + B_z (x_{j+1})) - g\mu_B (B_0 + B_z (x_j))| \gg \frac{\mu_0 \mu^2 g^2}{4\pi l_{\text{intra}}^3},
\]

or approximately,

\[
|g\mu_B \frac{dB_z (x)}{dx}| \gg \frac{\mu_0 \mu^2 g^2}{4\pi l_{\text{intra}}^3},
\]

where \(B_z (x)\) is the value of the \(z\) component of the dipolar magnetic field produced by the ferromagnetic nanostripe at position \(x\), assuming that \(z \approx 0\) for each spin qubit, and that the \(y\) coordinate of each spin qubit does not matter for the value of \(B_z\) applied to the qubit. The irrelevance of the \(y\) coordinate is due to the approximate invariance by translation of the gradient produced by the long ferromagnetic nanostripe (infinite stripe model). This approximation is valid as long as the spin chains are created far from the edges of the ferromagnetic stripe along the \(y\) direction. Assuming a permalloy ferromagnetic stripe with a length \(L = 100 \mu m\) along the \(x\) axis, a width \(T = 100 \text{ nm}\) along the \(x\) axis, and a depth \(W = 800 \text{ nm}\) along the \(z\) axis, further assuming that the ferromagnetic stripe is fully magnetized along the \(z\) axis by the static applied magnetic field \(B_0\), one can calculate using magnetostatic theory \([12]\) the properties of the dipolar magnetic field produced by this nanoferromagnet, as shown in Figure 2.

Figure 2 shows that as long as the spin qubits encoded onto silicon vacancies in SiC are created far from the edges of the ferromagnetic stripe along the \(y\) direction and close enough to the \(z = 0\) plane, then the relevant \(z\) component of the dipolar magnetic field produced by the ferromagnetic nanostripe can be considered as only dependent on \(x\), and thus its gradient can also be approximately considered as being one dimensional along \(x\). One finds here \(B_z (x_{\text{optim}} = 230 \text{ nm}, 0) \approx -676 \text{ G}\) and \(\frac{dB_z (x_{\text{optim}} = 230 \text{ nm}, 0)}{dx} \approx 1.44 \text{ G/nm}\). From those results, one can thus estimate whether the condition given above to obtain an effective Ising coupling between the spin qubits is satisfied. Assuming that most of the spin qubits are created in SiC by TEM at \(x\) positions close to \(x_{\text{optim}} = 230 \text{ nm}\) and \(z = 0\), and further assuming a distance \(l_{\text{intra}} = 5 \text{ nm}\) between successive spin qubits along the \(x\) direction, one finds that \(|g\mu_B \frac{dB_z (x)}{dx}| \gg \frac{\mu_0 \mu^2 g^2}{4\pi l_{\text{intra}}^3}\) is more than hundred times larger than the dipolar coupling energy \(\frac{\mu_0 \mu^2 g^2}{4\pi l_{\text{intra}}^3}\). One could thus, at first sight, conclude that the proposed nanodevice described in Figure 1, satisfies the requirements for the practical implementation of quantum information processing with dipolar coupled electron spin qubits manipulated by appropriate sequences of microwave pulses \([7]\). However, in order to avoid
magnetization component \( z \) potential inside the nanostripe; (b) one dimensional eigenfunctions of the spin waves confined along the magnetization of permalloy at saturation was taken equal to spin waves \[15\] along the itself. This leads to the confinement of the ferromagnetic dipolar magnetic field inside the ferromagnetic nanostripe produce a very inhomogeneous magnetic field acting on the nearby electron spin qubits outside the nanoferromagnet will become fluctuating, leading to spin qubit decoherence \[13,24\]. As it can be seen in Figure 2a (in the case \( x = 0 \)) and in Figure 3a, all the spins contained inside the ferromagnetic nanostripe produce a very inhomogeneous dipolar magnetic field inside the ferromagnetic nanostripe itself. This leads to the confinement of the ferromagnetic spin waves \[15\] along the \( z \) axis, mainly on the edges of the ferromagnetic nanostripe. Here I have developed a method to calculate the confined spin wave resonance spectrum of a long ferromagnetic nanostripe fully magnetized. First, the Landau Lifsicht equation of motion of magnetization in the nanoferromagnet \[16\] is linearized. Then, the one dimensional eigenenergies and eigenfunctions of the spin waves confined in the inhomogeneous effective potential existing inside the ferromagnetic nanostripe along the direction \( z \) of the static magnetic field applied: (a) one dimensional eigenenergies of the spin waves along \( z \) axis (horizontal lines) represented on top of the inhomogeneous effective confining potential inside the nanostripe; (b) one dimensional eigenfunctions of the spin waves confined along \( z \) (the out of equilibrium magnetization component \( \delta m_z(z) \) has been plotted here). Here the permalloy ferromagnetic nanostripe has the dimensions: length \( L = 100 \, \mu \text{m} \) along the \( y \) axis, width \( T = 100 \, \text{nm} \) along the \( x \) axis, and depth \( W = 800 \, \text{nm} \) along the \( z \) axis. It was further assumed that the ferromagnetic stripe is fully magnetized along the \( z \) axis by the static magnetic field applied \( B_0 \), and the magnetization of permalloy at saturation was taken equal to \( \mu_0 M_{\text{sat},Py} = B_{\text{sat},Py} = 11,300 \, \text{G} \). Note also that for numerical calculations, the new variable \( z^* \) was used, defined by \( z^* = z + 450 \) (in nm).

4 Designing the field sweep confined spin wave resonance spectrum to avoid coherently driven electron spin qubit decoherence

One assumes here that the permalloy ferromagnetic nanostripe is almost fully magnetized at equilibrium along \( z \) at sufficiently low temperature and sufficiently high magnetic field (see Fig. 1). The excitation of a ferromagnetic spin wave implies that the magnetization inside the ferromagnetic nanostripe is no more uniform. In other words that means the dipolar magnetic field acting on the nearby electron spin qubits outside the nanoferromagnet will become fluctuating, leading to spin qubit decoherence \[13,24\]. As it can be seen in Figure 2a (in the case \( x = 0 \)) and in Figure 3a, all the spins contained inside the ferromagnetic nanostripe produce a very inhomogeneous dipolar magnetic field inside the ferromagnetic nanostripe itself. This leads to the confinement of the ferromagnetic spin waves \[15\] along the \( z \) axis, mainly on the edges of the ferromagnetic nanostripe. Here I have developed a method to calculate the confined spin wave resonance spectrum of a long ferromagnetic nanostripe fully magnetized. First, the Landau Lifsicht equation of motion of magnetization in the nanoferromagnet \[16\] is linearized. Then, the one dimensional eigenenergies and eigenfunctions of the spin waves confined in the inhomogeneous effective potential existing inside the ferromagnetic nanostripe along \( z \) are calculated, as it is shown, respectively, in Figures 3a and 3b, using an effective Schrödinger equation solved numerically by a transfer matrix method \[23\]. Then, the three dimensional eigenenergies of the confined spin waves are determined, and finally the resonance fields at which the microwave magnetic field of fixed frequency can excite a confined spin wave are determined, simply by writing the energy conservation rule. This rule says that the energy of the microwave photon absorbed should be equal to the three dimensional eigenenergy of the confined spin wave excited. It is important to note that the combination of the theoretical and of the experimental field sweep microwave absorption spectrum, provides an indirect method to determine the dipolar magnetic field produced by the ferromagnetic nanostripe on the spin qubits, because it is the same magnetization distribution that produces the dipolar magnetic field inside and outside the nanostripe. The full field sweep spectrum expected for the nanodevice described in Figure 1 is plotted in Figure 4a. It includes the field sweep spectrum expected for the ferromagnetic nanostripe (blue resonance lines with resonance magnetic fields occurring between 0.80 and 1.40 T). It also includes the field sweep spectrum expected for 16 electron spin qubits created at positions \( x \) close to \( x_{\text{optimal}} = 230 \, \text{nm} \), with \( x \) values comprised between \( x = 199 \, \text{nm} \) and \( x = 274 \, \text{nm} \) (ensemble C of 16 red resonance lines with resonance magnetic fields occurring around 1.28 T). In order to see more clearly that those electron spin qubits resonance lines (C) are shifted towards high magnetic field
values by the dipolar magnetic field $B_z(x,0)$ produced by the ferromagnetic nanostripe and previously calculated, I also added in Figure 4a the field sweep spectrum expected for an electron spin qubit created at infinite distance from the ferromagnetic stripe along the $x$ axis (single A red resonance line with resonance magnetic field occurring around 1.215 T) and also the field sweep spectrum expected for an electron spin qubit created at distance $x=500$ nm from the ferromagnetic stripe (single B red resonance line with resonance magnetic field occurring around 1.250 T). The most important result shown in Figure 4a and highlighted by Figure 4b, is the possibility to design the electron spin based quantum register proposed here such that no spectral overlap occur between the 16 resonances lines of the shifted electron spin qubits and the resonance lines of the two confined spin waves of the ferromagnetic nanostripe occurring at the highest magnetic field values, the so-called edge spin wave modes, occurring here at Q band, at 1.227 T and at 1.362 T. This provides a magnetic field interval of 1350 G free of any spin wave resonance. Further assuming an inhomogeneous linewidth below 1 G for each silicon vacancy electron spin qubit in nuclear spin free isotopically purified SiC, one could hope to build identical parallel arrays of 675 electron spin qubits along the $x$ axis. However, state of art pulse EPR spectrometers operating at Q band have a resonator bandwidth of around 1 GHz, corresponding here to a magnetic field interval of around 350 G, in which only around 175 electron spin qubits could be manipulated using selective microwave pulses [24]. It seems thus possible in principle to build a model quantum register of this kind having tens of electrons spins qubits with paramagnetic resonance lines not overlapping the confined spin wave resonance lines, thus avoiding any coherently driven spin decoherence.

5 Investigation of the spin qubit decoherence process due to incoherent magnetic fluctuations of the nearby nanoferromagnet

Till now, I have not considered the fact that the incoherent thermal excitation of ferromagnetic spin waves can produce some time dependent fluctuating dipolar magnetic field outside the ferromagnetic nanostripe, at the positions where the electron spin qubits are created. Those fluctuations always exist in a ferromagnetic nanostripe at equilibrium at a non zero temperature $T$. As it is also known from the standard density matrix theory of electron spin decoherence [13], any fluctuating magnetic field lead to electron spin decoherence. Thus here, one expects that the fluctuating dipolar magnetic field acting on electron spin qubits will produce a new spin decoherence process, which must be added to the other electron spin decoherence processes, which are intrinsic to the silicon vacancies in the silicon carbide matrix alone. This is a complicated problem not addressed to date.

To obtain this electron spin decoherence rate expression, one starts by writing in second quantization the expression of the fluctuating dynamic dipolar magnetic field produced by the nanoferromagnet which is coupled to a
given electron spin qubit located outside this ferromagnetic nanostripe. This can be done using the formalism of the dipolar Green function tensor and introducing the creation and annihilation operators in the expressions of the out of equilibrium transverse magnetization components associated to each spin wave mode. Then, the three correctly symmetrized quantum correlation functions [13] of the three components of this fluctuating dynamic dipolar magnetic field produced by the nanoferramagnet are calculated, as well as their time domain Fourier transforms. Those last ones are evaluated for the relevant frequencies and qubit positions, giving \( \zeta_{xx,\text{sym}}(\omega_{\text{shifted qubit}}, r_{\text{qubit}}, T) \), \( \zeta_{yy,\text{sym}}(\omega_{\text{shifted qubit}}, r_{\text{qubit}}, T) \), and \( \zeta_{zz,\text{sym}}(0, r_{\text{qubit}}, T) \). Those quantum correlation functions are finally used to compute the longitudinal and transverse (decoherence) relaxation rates for the electron spin qubit located nearby the ferromagnetic nanostripe, as a function of qubit position and temperature. Those electron spin decoherence-relaxation rates are formally given by the following expressions:

\[
\frac{1}{T_{1,e}}(r_{\text{qubit}}, T) = \frac{2}{h^2} \left( \frac{g_e \mu_B}{2} \right)^2 \times \left( \zeta_{xx,\text{sym}}(\omega_{\text{shifted qubit}}, r_{\text{qubit}}, T) + \zeta_{yy,\text{sym}}(\omega_{\text{shifted qubit}}, r_{\text{qubit}}, T) \right),
\]

\[
\frac{1}{T_{1,e}}(r_{\text{qubit}}, T) = \frac{2}{h^2} \left( \frac{g_e \mu_B}{2} \right)^2 \zeta_{zz,\text{sym}}(0, r_{\text{qubit}}, T),
\]

\[
\frac{1}{T_{2,e}}(r_{\text{qubit}}, T) = \frac{1}{2T_{1,e}} + \frac{1}{T_{\phi,e}}.
\]

After lengthy analytical calculations and several approximations (in particular, \( y \ll L, z \ll W \) and \( x > \frac{3a}{2} \)), one can obtain the following closed form expressions, which are relevant to the hybrid paramagnetic ferromagnetic quantum processor architecture presented here (more analytical details will be published elsewhere):

\[
\frac{1}{T_{1,e}}(r_{\text{qubit}}, T) \approx \frac{1}{h} \left( g_e \mu_B B_{\text{sat}} \right)^2 \times \left( \frac{\hbar \gamma_{\text{FMR}}}{(\hbar \omega_{\text{shifted qubit}} - \hbar \omega_{\text{FMR}})^2 + (\hbar \gamma_{\text{FMR}})^2} \right) \times \coth \left( \frac{\hbar \omega_{\text{FMR}}}{2k_B T} \right) \Psi(r_{\text{qubit}})
\]

and

\[
\frac{1}{T_{2,e}}(r_{\text{qubit}}, T) \approx \frac{1}{h^3} \left( g_e \mu_B B_{\text{sat}} \right)^2 \times \left( \frac{2\hbar \gamma_{\text{FMR}}}{(\hbar \omega_{\text{FMR}})^2 + (\hbar \gamma_{\text{FMR}})^2} \right) \times \coth \left( \frac{\hbar \omega_{\text{FMR}}}{2k_B T} \right) \Psi(r_{\text{qubit}})
\]

and with

\[
\Psi(r_{\text{qubit}}) = \frac{15}{64 \pi^4 S_F} \frac{a_F^3 T_{\text{sat}}}{(x_{\text{qubit}} - \frac{a_F}{2})^4},
\]

\( S_F \) being the spin on each site of the effective Heisenberg like ferromagnet assumed in this theoretical model, \( a_F \) being the ferromagnetic lattice parameter, and \( T_{\text{sat}} \) being the thickness of the nanostripe here (change of notation here, to make the difference with the temperature noted \( T \) above). The theory developed thus shows that this new electron spin decoherence process depends mainly on, on the square of the saturation magnetization of the nanoferramagnet, on the temperature, on the spectral detuning between the shifted paramagnetic resonance frequency of the electron spin qubit (\( \omega_{\text{shifted qubit}} \)) and the ferromagnetic resonance frequency of the nanostripe (\( \omega_{\text{FMR}} \)), on the frequency linewidth of the ferromagnetic resonance (\( \gamma_{\text{FMR}} \)) and also on the distance (\( x_{\text{qubit}} \)) of the qubit to the ferromagnetic nanostripe, as long as the qubit has a position assumed far from the edges of the ferromagnetic nanostripe along the \( y \) and \( z \) axis (\( y \ll L, z \ll W \) and \( x > \frac{3a}{2} \)). Numerical results of the calculation of the electron spin coherence time \( T_2 \) and of the electron spin longitudinal relaxation time \( T_1 \) of an electron spin qubit of the nanodevice shown in Figure 1 and operated at microwave Q band are presented in Figure 5. At helium temperature, at microwave Q band, and with an applied magnetic field of around 1.28 T, one finds: \( T_1(x = 230 \text{ nm}, 2 \text{ K}) = 3.4 \text{ s} \) and \( T_2(x = 230 \text{ nm}, 2 \text{ K}) = 6.4 \text{ s} \). Moreover, at room temperature and Q band, one finds \( T_1(x = 230 \text{ nm}, 300 \text{ K}) = 25 \text{ ms} \) and also \( T_2(x = 230 \text{ nm}, 300 \text{ K}) = 47 \text{ ms} \).

6 Main decoherence process in the silicon carbide-permalloy hybrid paramagnetic-ferromagnetic quantum processor

The nuclear spectral diffusion is expected to be very weak in SiC, just like in Si, where very few non zero nuclear spins are naturally present. If this is not the case, it is still possible, like for Si and diamond, to use isotopic purification of the SiC material to eliminate completely this decoherence process. On the experimental side, recent measurements at room temperature have shown that silicon vacancies in SiC exhibit coherent Rabi oscillations [18] over 80 \( \mu \text{s} \) and that they have a longitudinal spin relaxation time \( T_1 \approx 100 \mu \text{s} \) [39]. Those experiments being performed on samples with silicon vacancies spins concentration of around \( 10^{15} \text{ cm}^{-3} \), one expects for this concentration, a spin coherence time limited by instantaneous diffusion on the order of \( T_{2,\text{ID}} \approx 600 \mu \text{s} \), much longer than the \( T_1 \approx 100 \mu \text{s} \) measured at room temperature. One can thus conclude, that already at room temperature, and for this spin concentration, the spin decoherence time of silicon vacancies in silicon carbide is limited by phonon assisted processes. It must be noted that, to the best of my knowledge, no temperature study of \( T_1 \) and \( T_2 \) for silicon
vacancies in silicon carbide is available. One can however remark that finding a $T_2$ limited by a $T_1$ process at room temperature is a situation which is also found for two others spins qubits showing also room temperature spin coherence, the NV centers in diamond ($T_2$ mainly limited by an Orbach relaxation process \cite{40}) and the N@C$_{60}$ molecules ($T_2$ mainly limited by a two phonon Raman assisted relaxation process \cite{41}). In the case of the silicon vacancies in silicon carbide, sidebands in the photoluminescence spectrum indicate a local vibration mode around the defect in its excited state having a single quantum energy of around 80 meV \cite{39}. Assuming that this is also a good estimation for the single quantum energy of the ground state local vibration mode, it is then possible to evaluate the two phonon Raman process of spin relaxation which could control the spin coherence time of silicon vacancies spins in silicon carbide in the high temperature range, as in the case of N@C$_{60}$, using the following formula \cite{41} giving the associated relaxation/coherence rates, $T_{1,Raman}$ and $T_{2,Raman}$, in $\mu s^{-1}$:

$$
\frac{1}{T_{1,Raman}} \approx 10^{-10} \times \frac{1}{T_{2,Raman}} \approx 0.226 \times e^{-960/T} \left(1 - e^{-960/T}\right)^2
$$

( using $T_1 \approx 100 \mu s$ at 300 K, and $E_{eib} = 80$ meV). Using this formula, one further obtains $T_{2,Raman}$ (200 K) = 528 $\mu$s, $T_{2,Raman}$ (150 K) = 2.6 ms, and $T_{2,Raman}$ (100 K) = 65 ms. At lower temperature, other phonon assisted processes may become more and more relevant, such that this formula cannot be applied for an estimation of the decoherence at liquid helium temperature. Comparing those last estimations of phonon induced decoherence with the other theoretical estimations of spin wave induced decoherence (see Fig. 5b), providing $T_{2,spin\_wave}$ (100 K) $\approx$ 150 ms, one can conclude that in the high temperature range, between liquid nitrogen temperature and room temperature, the thermally induced spin decoherence resulting from crystal structure fluctuations (phonons bosonic bath) dominate slightly the thermally induced spin decoherence resulting from ferromagnetic nanostripe fluctuations (spin wave bosonic bath). Due to the approximations mades in the estimation of those two kinds of decoherence rate, one can keep in mind that they are on the same order of magnitude, of few tens of milliseconds at $T = 100$ K.

It is also important to note that specific inter-spins distances exist in the intermediate scale hybrid paramagnetic ferromagnetic quantum processor architecture based on SiC and permalloy proposed here. Along the direction $y$, which is transverse to the one of the spin chains, the inter-chain distance $l_{inter}$ is assumed sufficiently large such that instantaneous diffusion between the many copies of a same qubit, which all have the same given resonance frequency, can be neglected compared to the two phonon Raman process evaluated above for silicon vacancies in SiC. For $l_{inter} = 255$ nm, and using the formula for instantaneous diffusion in three dimension \cite{24} for estimating the instantaneous diffusion effect here, on obtains $T_{ID}$ ($l_{inter} = 255$ nm) = 10 ms. For $l_{inter} = 550$ nm, on obtains $T_{IQD}$ ($l_{inter} = 550$ nm) = 100 ms, which is sufficiently large to be neglected in the high temperature range. One has thus to choose $l_{inter} = 550$ nm for the proposed quantum processor architecture. Also, along the spin chain direction ($\times$), the inter-spin distance was...
assumed to be $l_{\text{intra}} = 5 \text{ nm}$ in the former calculation of the full electron spin resonance spectrum of the processor. However, in order to have the fastest Ising quantum gate between two successive qubits in the chain, while keeping the Ising coupling approximation valid, one finds that the optimal intra-chain distance is $l_{\text{intra,optim}} = 3.5 \text{ nm}$. At such a small nanoscale distance between successive electron spins in the spin chain, and despite the large frequency detuning between them, produced by the strong magnetic field gradient, itself produced by the nearby ferromagnetic nanostripe, it is clear that phonon assisted flip-flop processes could produce another source of spin decoherence. This is the thermally induced spectral decoherence resulting from the phonon modulation of the dipolar coupling between two spins. The theory of this decoherence process previously described for nuclear spins and adapted here to electron spins, shows [42] that the decoherence rate is inversely proportional to the sixth power of the distance between the two dipolar coupled spins, and is roughly given by the formula:

$$\frac{1}{T_{2,\text{flip flop}}(T)} \approx \frac{9\pi}{4} \left( \frac{\mu_0}{4\pi} \frac{g_e g_B}{I_{\text{intra}}} \right)^2 \times (S(S+1)) f(S, M_{S,i}, M_{S,j}) \frac{\omega_{\text{Debye}}}{(M^2)_{\text{Debye}}} \times \frac{T}{\Theta_{\text{Debye}}} \int_0^\infty dx \left( \frac{x^6}{\chi(x) - 1} \right),$$

where $v_s$ is the sound velocity in the material, $M$ is the average atomic mass in the material,

$$f(S, M_{S,i}, M_{S,j}) = \frac{[S(S+1) - M_{S,i}(M_{S,i}+1)] [S(S+1) - M_{S,j}(M_{S,j} - 1)]}{[S(S+1)]},$$

and $\Theta_{\text{Debye}}$ is the Debye temperature, defined by $k_B \Theta_{\text{Debye}} = \hbar \omega_{\text{Debye}}$, where $\omega_{\text{Debye}}$ is the Debye frequency of the solid material, here SiC. This rapid decay of the phonon assisted flip-flop decoherence rate with distance implies that only the two nearest neighbor spins of a given spin qubit will contribute to this rate in a first approximation. For the value $l_{\text{intra,optim}} = 3.5 \text{ nm}$ considered in the proposed quantum processor architecture, and considering the material parameters relevant for silicon carbide (6H-SiC), one finds, a $T_{2,\text{flip flop}}(T = 100 \text{ K}) \approx 10^6 \text{ s}$, which is orders of magnitude longer than the relevant decoherence time identified above. Thus, this thermally induced spectral diffusion process of decoherence, occurring inside a given spin chain, can be safely neglected at $T = 100 \text{ K}$ for $l_{\text{intra,optim}} = 3.5 \text{ nm}$ compared to the other decoherence processes relevant to silicon vacancies in SiC at $T = 100 \text{ K}$.

### 7 Scalability limit of the hybrid paramagnetic-ferromagnetic intermediate scale dipolar quantum computer

The optimal value $l_{\text{intra,optim}} = 3.5 \text{ nm}$ which was found above, implies a gating time for the Ising like dipolar quantum gate between two isolated silicon vacancies spin qubits of around $T_{\text{Ising, gate,2q}} = 207 \text{ ns}$. The single spin quantum gate duration is shorter for an isolated qubit. It corresponds to the standard microwave pulse duration for a Pi pulse, which is of around few tens of nanoseconds. The average gating time thus seems to be $207 \text{ ns}$ at first sight. However it was theoretically shown [7], that the refocusing and/or dynamical decoupling by microwave pulse sequences, which is required for the practical implementation of this dipolar quantum computer architecture, produces an increased gating time. It was more precisely shown that the total effective gating time increases linearly [7,43] with the number of spins qubits present in one single spin chain. Thus, the effective average gating time, in the presence of all the spin qubits of the spin chain is roughly given by: $T_{\text{gate, eff}}(N) \approx 207 \text{ ns} \times N$, where $N$ is the number of qubits per chain. One has now to check if error correction requirements [44] are compatible with this intermediate scale hybrid paramagnetic ferromagnetic quantum processor architecture. That means, we have to check that $T_{\text{gate, eff}}(N) \times 10^4 \leq T_2(T)$, which gives for $l_{\text{intra, optim}} = 3.5 \text{ nm}$ and $l_{\text{inter}} = 550 \text{ nm}$, the following condition: $2 \text{ ms} \times N \leq T_2(T)$. For the particular implementation of this intermediate scale hybrid paramagnetic ferromagnetic quantum processor with SiC and permalloy, one finds that at $150 \text{ K}$, where $T_2,\text{Raman}(150 \text{ K}) = 2.6 \text{ ms}$, and at all temperature above $150 \text{ K}$, this hybrid nanodevice cannot process quantum information over several qubits, despite the room temperature spin coherence of the silicon vacancy isolated spin qubit in SiC. However, at $100 \text{ K}$, where $T_2(100 \text{ K}) = 65 \text{ ns}$, this hybrid nanodevice can process quantum information over 32 electron spins qubits. It must be noted that no quantum information processing is possible at room temperature with this quantum processor architecture, whatever the kind of electron spin qubit chosen, even for NV centers in diamonds for which the Orbach process limited coherence time is of few milliseconds. However, it was shown above that a 32 qubits quantum computer based on silicon vacancies in SiC and on a permalloy nanostripe can be operated at microwave Q band and at $100 \text{ K}$, under an inexpensive nitrogen gaz flow, the spins states of the silicon vacancies being sensitively detected by photoluminescence methods [17,18]. Note that the same kind of hybrid quantum processor, but based on NV centers in diamond, could be operated in similar experimental conditions, using nitrogen cooling and ODMR methods at microwave Q band.

### 8 Discussion and conclusion

In this theoretical work, it was shown that a 32 qubits, intermediate scale, hybrid SiC-permalloy quantum processor, may be operated by microwave pulses and optical pulses at $100 \text{ K}$, above the liquid nitrogen temperature. Silicon carbide is a very attractive material for the practical implementation with a top-down approach of the general intermediate scale quantum register design proposed here, firstly due to the long electron spin coherence time of its paramagnetic silicon vacancies defects,
secondly because irradiation by low energy TEM electrons can produce silicon vacancies in SiC and because recent improvements of TEM nanofabrication methods reaching the nanometer scale spatial resolution suggest the possibility to create dense arrays or chains of silicon vacancies in SiC. Chains of N@C60 molecules placed in a carbon nanotube, separated along the chain by $l_{\text{intra,optim}} = 3.5$ nm would be a first alternative for a nitrogen cooled hybrid dipolar quantum processor. However, positioning or chemically anchoring those qubits on the nanodevice surface, as well as preparing and detecting sensitively their spin state remain important challenges to overcome. Chains of NV centers in diamonds, separated along the chain by $l_{\text{intra,optim}} = 3.5$ nm would be a second alternative. However, positioning those qubits on the nanodevice by ion implantation with such a high nanoscale resolution remains a great challenge. Self-assembled supramolecular linear chains of paramagnetic organic molecules would be another alternative. However, despite the fact that the nuclear spectral diffusion due to the many protons present in organic molecules may be fought by appropriate microwave pulse sequences, the need to prepare and detect sensitively their spin state, remain also in that case, important challenges to overcome. Note that a dysprosium ferromagnetic nanostructure would produce a three times bigger magnetic field gradient than in the case of permalloy. However, one expects with dysprosium an electron spin decoherence rate at least one order of magnitude larger than in the case of permalloy at cryogenic temperature due to its three times larger saturation magnetization and its much larger ferromagnetic resonance linewidth. In addition, dysprosium is no more ferromagnetic above 85 K, contrary to permalloy, which, altogether, makes not realistic quantum information processing with a nitrogen cooled nanodevice based on a dysprosium ferromagnetic nanostructure.

Thus the 32 qubits, nitrogen cooled, hybrid paramagnetic-ferromagnetic nanodevice presented here, based on the SiC material widely used for high temperature electronics and on the permalloy material widely used for magnetic elements in computers, seems to be a good choice for quantum information processing with dipolar coupled electron spins selectively and coherently manipulated by resonant microwave pulses. Future advances in the synthesis, in the organization, and in the sensitive detection of the spin states of supramolecular spin chains may however offer an interesting alternative. Also, NV centers spin chains in diamond may be another interesting alternative to implement this hybrid quantum processor, if ion implantation methods are sufficiently improved in the future in order to reach the nanoscale positioning required by this architecture.

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