Adiabatic Quantum Transport in a Spin Chain with a Moving Potential

Vinitha Balachandran and Jiangbin Gong

Department of Physics and Centre for Computational Science and Engineering, National University of Singapore, 117542, Republic of Singapore

(Dated: February 2, 2008)

Many schemes to realize quantum state transfer in spin chains are not robust to random fluctuations in the spin-spin coupling strength. In efforts to achieve robust quantum state transfer, an adiabatic quantum population transfer scheme is proposed in this study. The proposed scheme makes use of a slowly moving external parabolic potential and is qualitatively explained in terms of the adiabatic following of a quantum state with a moving separatrix structure in the classical phase space of a pendulum analogy. Detailed aspects of our adiabatic population transfer scheme, including its robustness, is studied computationally. Applications of our adiabatic scheme in quantum information transfer are also discussed, with emphasis placed on the usage of a dual spin chain to encode quantum phases. The results should also be useful for the control of electron tunneling in an array of quantum dots.

PACS numbers: 03.67.Hk, 32.80.Qk, 73.63.Kv

I. INTRODUCTION

Efficiently transferring quantum information is an important challenge for the practical realization of quantum computers. Optical fibers, where quantum information is transmitted by mobile carriers like photons, are most desirable for long-distance communication. However, the need to interface with solid state quantum computer components considerably restricts the experimental feasibility of using optical systems as quantum channels. One promising alternative is to use condensed matter systems where quantum information transfer and on the quantum control in spin chains using nuclear magnetic resonance techniques have also been reported [11].

Despite many fruitful studies of spin chains from a quantum information perspective, many theoretical problems are still open. To be more specific let us consider first Bose's original proposal [1]. Therein the fidelity of quantum information transfer is gradually degraded by the dispersion effects associated with the quantum propagation. Furthermore, particular external magnetic field should be designed to ensure a correct quantum phase at the receiver's site. These issues and others motivated a series of sophisticated protocols to achieve better quantum information transfer. One noteworthy approach was to pre-engineer the nearest-neighbor couplings of a spin chain or even a spin network [3, 4, 5, 6, 7, 8]. A second approach exploits the mirror symmetry of a spin chain [8, 10]. Another approach suggests to use Gaussian wavepackets with slow dispersion [11] to encode the quantum information to be transferred along an unmodulated spin chain. Unfortunately, these pioneering approaches rely upon specific analytical forms of the involved Hamiltonian and hence are not robust to imperfection or physical fluctuations in the spin-spin coupling strength. One encounters the same situation when applying other more subtle techniques [12, 13, 14].

To have the desired robustness that may be necessary for any type of quantum information transfer, two quantum control schemes based on adiabatically varying the coupling strength in a spin chain have also been suggested [15, 16]. Note however, these schemes require individual addressing of the nearest-neighbor coupling and hence present new experimental challenges. Another novel and quite robust quantum transfer protocol in spin chains is the dual spin chain scheme [17, 18, 19, 20]. Therein the quantum information to be transferred is encoded in two parallel spin sub-chains (initially assumed to be identical, but even this condition may be lifted under certain conditions). Thanks to the encoding with two spin sub-chains, the quantum transfer can be very robust to static disorder. Nevertheless, even this promising scheme is not perfect, because (i) it may need too many quantum measurements (or too many steps of “trial and error”), (ii) it may not operate well in the presence of time-dependent disorder, and (iii) the effects of nonideal measurements are still under investigation [21].

Hence, it remains an open question as to which quantum transfer scheme will ultimately be adopted experimentally, with high fidelity and low cost. It is our belief that in the end a combination of several techniques may...
be able to offer the most powerful protocol for quantum information transfer in solid state systems.

In this paper, we introduce an adiabatic transport scheme assisted by a slowly moving external field applied to a spin chain. Thanks to a pendulum analogy, the central idea can be understood in a very simple manner. The essence is that when an external field is moving slowly, the spin excitation may adiabatically follow the field under certain conditions. During this process the quantum population of spin excitation is transferred from one end of the spin chain to the other end. As we show below in detail, this adiabatic scheme offers a number of advantages: (i) it is highly robust, (ii) it can be operated rather fast (in the absence of disorder, it may be as fast as one tenth of the natural propagation speed of the spin chain), (iii) the required field strength can be decreased by using wavepackets as initial states, (iv) the transfer can be easily stopped and relaunched, (v) the time of arrival of the quantum population transfer to the last spin with high probability can be easily predicted, and (vi) it can offer a promising means of quantum information transfer when combined with the above-mentioned dual spin chain scheme.

The outline of the paper is as follows. In Sec. II, we introduce an adiabatic transport scheme assisted by a slowly moving external field applied to a spin chain. Detailed computational results are presented in Sec. III. The robustness of our scheme is studied in Sec. IV. In Sec. V, we discuss how our adiabatic population transfer scheme, which does not yet take care of the quantum phases (also essential for quantum information transfer), can be combined with the dual spin chain scheme to offer a potentially powerful approach for quantum information transfer. We conclude this work in Sec. VI.

II. ADIABATIC QUANTUM TRANSPORT IN SPIN CHAINS: A PENDULUM PERSPECTIVE

Consider a one-dimensional Heisenberg chain of \( N + 1 \) spins subject to an external parabolic magnetic field. The associated Hamiltonian is given by

\[
H_s = -\frac{J}{2} \sum_{n=0}^{N-1} \sigma_n \cdot \sigma_{n+1} + \sum_{n=0}^{N} C (n - n_0)^2 \sigma_n^z, \tag{1}
\]

where \( \sigma \equiv (\sigma^x, \sigma^y, \sigma^z) \) are the Pauli matrices, \( J \) the coupling strength between nearest neighbor spins, and \( C \) is proportional to the magnetic dipole of the spins and the amplitude of the parabolic field whose minimum is at site \( n_0 \). Note that \( n_0 \) will be time-dependent in our control scheme. Below we assume all the system parameters have been appropriately scaled and take dimensionless values, with \( J = 1, \ h = 1 \) throughout. As such, the energy scale (e.g., the parameter \( C \)) should be understood with respect to \( \hbar / J \). Because the spin chain Hamiltonian \( H_s \) commutes with the total polarization \( S_z \equiv \sum_{n=1}^{N} \sigma_n^z \), the dynamics of the spin chain preserves the total polarization. Here we restrict our analysis to the subspace of \( S_z = 1 - N \), where in total only one spin is flipped. In this subspace the total state of the chain can be written as

\[
|\Psi(t)\rangle = \sum_{m=0}^{N} c_m(t) |m\rangle, \tag{2}
\]

where \( |m\rangle \) represents one basis state with a spin up at the \( m \)-th site and all other spins down. The complex coefficients \( c_m(t) \) are the probability amplitude. Below we also shift the zero of the energy scale such that if one spin is down, its interaction with the external magnetic field contributes zero to the total energy.

To understand the essence of the spin chain dynamics from a semiclassical perspective, we now consider the large \( N \) limit of the spin chain. Denote \( k \) as the quasi-momentum of a plane spin wave and the \( |k\rangle \) the associated eigenstate of the quasi-momentum. Then the Hamiltonian in Eq. (1) can be rewritten as

\[
H_s = -J \int_0^{2\pi} \cos(k) |k\rangle \langle k| + \frac{C}{2} \sum_n |n\rangle \langle n| (n - n_0)^2. \tag{3}
\]

This form can be further simplified in an operator form, i.e.,

\[
H_s = -J \cos(\hat{k}) + \frac{C}{2} (\hat{n} - n_0)^2, \tag{4}
\]

where \( \hat{k} |k\rangle = k |k\rangle; \; \hat{n} |n\rangle = n |n\rangle; \; [\cos(\hat{k}), \hat{n}] = -i \sin(\hat{k}), \) and \( [\sin(\hat{k}), \hat{n}] = i \cos(\hat{k}) \). The Hamiltonian in Eq. (4) can now be easily recognized to be the Hamiltonian of a quantum pendulum \( H_p \), with an effective Planck constant \( \sqrt{C} \). Specifically, with the mapping \( \hat{k} \rightarrow \hat{x}, \sqrt{C} \hat{n} \rightarrow \hat{p} \), the spin chain Hamiltonian \( H_s \) is mapped to

\[
H_p = -J \cos(\hat{x}) + \frac{1}{2} (\hat{p} - p_0)^2, \tag{5}
\]

where \( p_0 = \sqrt{C} n_0 \) and \( [\cos(\hat{x}), \hat{p}] = -i \sqrt{C} \sin(\hat{x}) \). The semiclassical Hamiltonian for this quantum pendulum, i.e., \( H_p = -J \cos(x) + \frac{1}{2} (\hat{p} - p_0)^2 \) is obtained by replacing \( \hat{x} \) and \( \hat{p} \) with \( x \) and \( p \).

Many quantum transport features of the spin chain can now be understood in terms of the semiclassical dynamics of the pendulum analogy thus obtained. In particular, the quantum transport in the momentum space of the pendulum is now in parallel with the transfer of spin excitation from one site to another. Hence, the issue of robust quantum transport of spin excitation along the spin chain now reduces to the design of a control scenario that enables robust transport of the pendulum state in its momentum space.
An adiabatic scheme for robust quantum population transfer along a spin chain can now be proposed. The key observation is the existence of a motional separatrix in the classical phase space of the pendulum. This separatrix is located at $-J \cos(x) + (1/2)(p-p_0)^2 = J$ [see Fig. 1]. If we now slowly move up the separatrix along the momentum space by increasing $p_0$, then a quantum state initially trapped inside the separatrix cannot penetrate this separatrix and is expected to adiabatically follow the movement of the separatrix, giving rise to adiabatic transport in the momentum space. Translating this pendulum language back to the spin chain case, one anticipates that a slowly moving parabolic magnetic field (with slowly increasing $n_0$) should result in a robust scenario for transferring quantum population along the spin chain. During this process the dispersion of the spin wave should also be bounded by the separatrix structure, i.e., a moving but non-spreading wavepacket [22] of the spin wave can be expected. The main remaining task of this paper is devoted to detailed aspects of this adiabatic control scheme.

It is also interesting to note that the dynamics of the spin chain can be mapped to that of a tight-binding model. To see this consider first the associated Schrödinger equation,

$$\frac{idc_n}{dt} = \frac{J}{2} (c_{n-1} + c_{n+1}) + \frac{C}{2} (n-n_0)^2 c_n, \quad 0 < n < N,$$

$$\frac{idc_N}{dt} = \frac{J}{2} c_{N-1} + \frac{C}{2} (N-n_0)^2 c_N. \quad (6)$$

Consider next a tight-binding Hamiltonian $H_t$ describing, for example, an array of $(N+1)$ identical quantum dots subject to an external parabolic field, with one electron tunneling between the quantum dots. Then $H_t$ assumes the following form,

$$H_t = -\frac{J}{2} \sum_{n=0}^{N-1} (a_n^\dagger a_{n+1} + a_n a_{n+1}^\dagger) + \sum_{n=0}^{N} C_2 (n-n_0)^2 a_n^\dagger a_n. \quad (7)$$

where $J$ represents the constant tunneling rate between the nearest-neighbor quantum dots, and $a_n^\dagger$ and $a_n$ represent the creation and annihilation operators. Because the total number of electrons is already assumed to be one, the system wavefunction can also be written as $|\Psi(t)\rangle = \sum_{m=0}^{N} c_m(t) |m\rangle$, where $|m\rangle$ denotes the state with an electron in the $m$th quantum dot and $c_m(t)$ denotes the associated quantum amplitude. In this representation, one immediately finds that the evolution of this tight-binding system takes the same form as Eq. (6). Thus, the above pendulum analogy is also applicable to a tight-binding system and is hence very useful for consideration of adiabatic quantum transport in quantum dot arrays [23]. Note also that one may start from Eq. (6) to have an alternative derivation of the pendulum analogy [24].

To end this section, we stress that the proposed control scheme is based upon a semiclassical perspective afforded by the pendulum analogy. What is not addressed is the issue of transferring the quantum phase along the spin chain. As such, although the pendulum analogy helps design our scheme for the robust transport of quantum excitation along a spin chain or the robust transport of an electron in a quantum dot array, the issue of quantum information transfer is only partially touched. Indeed, the introduction of an external field will change the energy of the spin chain system and hence will necessarily introduce extra dynamical phases to the evolving quantum system. This makes it clear that transporting quantum phases along the spin chain requires additional considerations. This quantum phase issue will be considered in detail in Sec. V.

III. ADIABATIC TRANSPORT BY A MOVING POTENTIAL: COMPUTATIONAL RESULTS

In this section, we illustrate our adiabatic quantum population transfer scheme with detailed computational...
The quantum state of the spin chain at a later time is given by $|\Phi\rangle = \sum_{m=0}^{N} c_m(0)|m\rangle$. Two types of $c_m(0)$ will be considered below. In the first case only the $m = 0$th spin is excited, with $c_m(0) = \delta_{m0}$. In the second case, the initial state is a Gaussian wavepacket truncated to three sites only, with $c_m(0) \propto \exp[-(m-1)^2/2\sigma_0^2]$ for $m = 0 - 2$ and $\sigma_0 = 0.707$ being the width of the Gaussian wavepacket. In either case a parabolic magnetic field first centered on the $n = 0$th site is applied and then slowly moved to the regime of larger $n$. This is realized by introducing the time dependence of $q_0$ via $q_0 = 0 + St$, where $S$ is the moving speed. Note that a static parabolic field was previously introduced to induce a quasi harmonic lower energy spectrum such that good transfer of Gaussian wavepackets [10] may be realized. By contrast, our moving potential scenario is more active and effective in controlling the quantum transport and is in principle applicable to cases where the shape of the external potential is not parabolic.

The quantum state of the spin chain at a later time can be directly calculated using the Schrödinger equation given above. In particular, the probability of transferring the quantum excitation to the last spin of the chain can be examined. If the performance of the population transfer is satisfactory, one should find $|c_N|^2 \approx 1.0$. Evidently, this condition of high transfer probability is already useful by itself for, e.g., transporting electrons in a quantum dot array in a controlled fashion. As shown in Sec. V, the phase of the quantum amplitude $c_N$ may be also taken care of by considering a dual spin chain.

We now discuss the feasibility of adiabatic quantum population transfer by taking advantage of the separatrix associated with the pendulum analogy. In the ideal case of adiabatic following, an initial quantum state enclosed by a separatrix will move with the slowly moving separatrix. Consider first an initial state localized exclusively at the $n = 0$th site. Then the associated $k$-distribution covers uniformly from 0 to $2\pi$. From a semiclassical perspective afforded by the pendulum analogy, such an initial state corresponds to an initial ensemble lying on the $(p - p_0) = 0$ axis of the classical phase space. This initial ensemble hence necessarily intersects with the separatrix (see Fig. 9). Because the motional period associated with the separatrix is infinity, those ensemble components that overlap with the separatrix will always regard the movement of the separatrix as “too fast to follow”. That is, as we slowly move the separatrix upwards in the classical phase space, some portion of the initial ensemble may break the adiabaticity and tunnel through the separatrix structure. Under such a situation adiabatic quantum population transfer is expected to partially break down. To reduce the degree of non-adiabaticity, one possible approach is to reduce the overlap of the initial state with the classical separatrix. This should be doable by increasing the effective Planck constant $\sqrt{C}$ (i.e., increasing the strength of the parabolic field) such that the separatrix regime supports less quantum states. This is indeed what we find computationally for a chain of 101 spins. In particular, Fig. 2 shows that for a field amplitude characterized by $C = 0.5$, the probability of transferring the initial excitation to the last spin is only 0.63. By increasing $C$ to 8.0, a transfer probability around 99% is observed. Figure 3 shows the actual excitation profile $|c_n|^2$ vs. $n$ for a spin chain subject to a parabolic potential moving at a constant speed of $S = 0.005$. At $t = 0$, the state is at site $n = 0$. At $t = 10000$, the quantum population is mainly at $n = 50$. Note that at that moment the excitation profile is slightly delocalized into three sites, but the peak probability is still as high as 0.97. This peak is propagated to site $n = 100$ at time $t = 20000$, with no further dispersion detected. This indicates that our moving potential scheme has the capacity to overcome the dispersion issue in quantum information transfer. Though the required field strength for large $|n - n_0|$ could be demanding experimentally, we point out that because the spin excitation is highly localized throughout the process, the moving parabolic magnetic field does not need to span over many spin sites (in our numerical experiments, we use a parabolic field that only spans 20 sites).

We have also examined the quantum dynamics for the second type of initial states, i.e., states with a Gaussian excitation profile at $t = 0$. Because such initial ensembles are localized in both $k$ and $n$, they can be naturally enclosed by the separatrix shown in Fig. 1. As such, if the shape of the initial excitation profile is appropriately adjusted, the initial state can be made not to intersect with the separatrix. This being the case, the adiabatic following should work better, probably requiring a much weaker parabolic field. This expectation is also confirmed computationally. In particular, Fig. 4 shows the transport of an initial Gaussian excitation profile, again for a chain of 101 spins. At $t = 0$, the excitation profile spans only the first three sites with a probability peak.
ferred to the peak of the spin excitation probability profile is transferred to the last spin, is as high as 0.997. Physically, this is due to the reflection process at the end of the spin chain. In some sense, the interplay of the parabolic field centered at the end of the spin chain and the reflection process acts as a lens refocusing the slightly dispersed profile, and the peak probability builds up on the last spin. Note also that in the absence of the control field, one may estimate $n_{\text{max}}$ from the width of the separatrix in the momentum space. Specifically, for a fixed value of the parameter $C$, $n_{\text{max}} \approx 4\sqrt{\frac{J}{C}} + 1$. For the numerical example in Fig. 4 one obtains $n_{\text{max}} \approx 4$. This estimate is quite consistent with the finding that during the population transfer, the moving wavepacket does not cover more than five sites. This result also implies that for weaker magnetic fields (smaller $C$), one can use more spins to form the wavepacket for analogous adiabatic population transfer.

Because our quantum transport scheme is based upon the adiabatic following of the spin excitation profile with a moving external potential, it can stop and relaunch the excitation transfer at any time with great ease, by simply stopping and restarting the movement of the external parabolic potential. This is simpler than a recent approach using pulsed magnetic fields, and is also confirmed in our computational studies (not shown).

So we can further increase the moving speed of the control potential while still maintaining the adiabatic follow-

FIG. 4: Adiabatic quantum transport along a chain of 101 sites with an initial Gaussian excitation profile, at (a) $t = 0$, (b) $t = 10000$, (c) $t = 20000$, and (d) $t = 21000$. The amplitude of the parabolic field is given by $C = 2$, and the moving speed of the control field is given by $S = 0.005$. Note that the final excitation probability transferred to the last spin is as high as 0.997.
clusively localized at the 0th site along a chain of 101 spins. The amplitude of the moving parabolic potential is given by $C = 8$, and the moving speed is given by $S = 0.025$. Panels (a), (b) and (c) show the excitation profile at times $t = 1000$, 2600, and 4000.

FIG. 5: Adiabatic transfer of spin excitation for an initial state exclusively localized at the $n = 0$th site along a chain of 101 spins. The amplitude of the moving parabolic potential is given by $C = 8$, and the moving speed is given by $S = 0.025$. Panels (a), (b) and (c) show the excitation profile at times $t = 1000$, 2600, and 4000.

...ing and hence the adiabatic quantum population transfer? Our findings in this regard can be summarized as follows: (i) for large $C$ adiabatic quantum transport may survive for a moving speed around 10% of the coupling constant $J$. The smaller the field strength $C$ is, the lower the threshold moving speed will be; (ii) when the moving speed exceeds the threshold, the probability of successful population transfer gradually decreases, but can still be considerably large for a relatively short spin chain. For example, Fig. 5 shows the result for an initial state exclusively localized at the $n = 0$th site. The moving speed of the parabolic potential is $S = 0.025$. The peak value of the probability profile is 0.96 at time $t = 1000$. It reduces to about 0.95 at $t = 2600$ and 0.94 at $t = 4000$. Hence, in this case, only 2% reduction in the peak probability occurs when the moving speed $S$ increases by a factor of five. However, increasing the moving speed beyond this limit drastically reduces the probability of population transfer to the last spin. For a moving speed of $S = 0.30$, the probability maxima equals only 0.87 at $t = 2200$, and 0.84 at $t = 3400$. Analogous calculations are also carried out for the transport of an initial Gaussian excitation profile. As shown in Fig. 5 for a moving speed of 0.1, the peak value of probability remains around 0.76 during the transport process. As such, at the end (not shown) the adiabatic population transfer is also very successful for this high moving speed. But if the moving speed is further increased by several times, dispersion in the spin excitation profile will be considerable.

In short, our numerical experiments suggest that, to achieve adiabatic transport of spin excitation along a quite long spin chain using a moving parabolic potential, the associated moving speed can be as large as that of the natural propagation rate ($J$) of the system (without disorder).

IV. ROBUSTNESS OF ADIABATIC TRANSPORT

So far the majority of quantum state transfer schemes consider only idealized spin chains with no disorder in the spin-spin coupling strength. This suggests a gap between theoretical exploration and realistic situations in experiments. In particular, the effects of static and dynamic imperfections in spin chains are studied in very few cases [13, 18, 20, 23]. Here we computationally study the influence of static and dynamic disorder on our adiabatic population transfer scheme, by considering the model Hamiltonian in Eq. (1) with fluctuating spin-spin coupling constants. We hope to numerically confirm the robustness of our scheme as implied by its adiabatic nature.

The model Hamiltonian with disorder in the spin-spin coupling strength is given by

$$H_{sd} = \sum_{n=0}^{N-1} -\frac{(J + \delta_n)}{2}\sigma_n \cdot \sigma_{n+1} + \sum_{n=0}^{N} \frac{C}{2}(n - n_0)^2\sigma_n^z,$$

where $\delta_n$ are time-independent random numbers uni-
formally distributed in the interval $[-\Delta, \Delta]$, representing random fluctuations in $J$ with the amplitude $\Delta$. We call cases with such time-independent disorder as static disorder models. Note that specific results presented below refer to single disorder realizations. These results are very typical such that there is no need to average over many disorder realizations.

Figure 7 display one sampling calculation that is in parallel with the results in Fig. 4 but takes into account static disorder with the noise amplitude $\Delta = 0.5$. As demonstrated in Fig. 4 in the presence of such a high noise level, the quantum population of spin excitation is still successfully transferred to the last spin of the chain (peak probability around 99%), with the excitation profile almost unaltered as compared with the noiseless case studied in Fig. 5 For an even higher fluctuation level, e.g., $\Delta = 0.7$, the spin excitation profile is seen to gradually disperse as it is transported along the chain.

Figure 8 displays results for an initial Gaussian excitation profile also considered in Fig. 4 but in the presence of static disorder characterized by $\Delta = 0.5$. We find that the severe disorder can slightly change the shape of the spin excitation profile (though not so evident in Fig. 8) and hence the peak value of the probability profile slightly fluctuates during the controlled transport. Note however, the area enclosed by the main probability profile is found to be around 0.99 at all times. At time $t = 21500$, the peak probability of the spin excitation profile, still as large as 0.99, has been transferred to the 100th site as in the noiseless case of Fig. 4. In another sampling case for a $\Delta = 0.7$, the peak excitation probability that is transferred to the last spin decreases to 0.94. All these results clearly demonstrate the robustness of our adiabatic transport scheme to high-level static disorder.

We have also examined the robustness of our adiabatic transport scheme to dynamic disorder. To model time-dependent fluctuations in the spin-spin coupling strength, we now let each $\delta_n$ be given by the sum of ten oscillating functions, i.e., $\delta_n = \sum_{i=1}^{10} A \cos(\omega_i t + \phi_i)$, where $\omega_i$ are random frequencies distributed in $[0, \omega_{\text{max}}]$, and $\phi_i$ are random phases uniformly distributed in $[0, 2\pi]$.

Interestingly, our numerical experiments indicate that effects of dynamical disorder modeled above depend strongly on $\omega_{\text{max}}$, i.e., the cut-off frequency of the dynamic fluctuations. Introducing disorder more frequently, i.e., introducing a larger $\omega_{\text{max}}$, can lead to much decreased peak probability transferred to the last spin. In particular, we find that for $\omega_{\text{max}} \leq 0.1$, the effects of the dynamic disorder are essentially analogous to what is found for static disorder. For larger $\omega_{\text{max}}$, the deterioration of the adiabatic population transfer becomes considerable for the same noise amplitude $A$. Figure 9 displays the results for $A = 0.025$ and $\omega_{\text{max}} = 0.1$. Note that for $A = 0.025$, the amplitude of the noise is very large because the total fluctuation is a sum of ten functions oscillating at the same amplitude $A$. It is seen from Fig. 9 that for such a case of dynamic disorder, the spin
A simple and robust approach to transferring quantum population along a spin chain. This scheme requires a strong parabolic field if it is globally parabolic, but even this requirement can be greatly weakened if the initial spin excitation profile spans a few sites. Further, one does not really need a globally parabolic field to realize this adiabatic scheme: it suffices for the parabolic field profile to be wider than the spin excitation profile. With these considerations we may argue that the well-known dispersion issue in quantum information transfer along spin chains is essentially solved by our adiabatic scheme. Nevertheless, as also mentioned earlier, one important issue still remains open. That is, for the sake of quantum information transfer, how to take care of the quantum phase of a quantum state to be transported? Indeed, a moving external potential induces extra dynamical phase to the spin chain, and such a dynamical phase depends on the details of the control potential. These facts motivate us to seek an encoding approach that can protect a quantum state from the additional dynamical phases induced by the moving potential.

Fortunately, the idea of using a dual spin chain, first proposed to overcome the disorder and dispersion issues in quantum information transfer in spin chains [17, 20], offers a promising solution. Specifically, we propose to combine our adiabatic transport scheme with the dual spin chain scheme. Then, because each individual sub-chain acquires identical dynamical phases from the same external moving potential, the relative quantum phase between the two sub-chains is certain, and hence quantum information encoded in the dual spin chain can be transported without suffering from the extra uncertain...
dynamical phases.

Consider then a quantum channel consisting of two identical parallel spin chains subject to the same external parabolic potential,

\[
H_s^i = -\frac{J}{2} \sum_{n=0}^{N-1} \sigma_n^{(i)} \cdot \sigma_{n+1}^{(i)} + \sum_{n=0}^{N} C(n-n_0)^2 \sigma_n^{z(i)}, \tag{9}
\]

where \(i = 1, 2\) indices label the two sub-chains. Suppose the quantum state to be transferred is given by \(|\Psi(0)\rangle = \alpha|0\rangle + \beta|1\rangle\). Such a state can be encoded into the quantum channel prepared in the following entangled state,

\[
|\Psi(0)\rangle = \alpha|g(1)\rangle \otimes |0(2)\rangle + \beta|0(1)\rangle \otimes |g(2)\rangle \tag{10}
\]

as a superposition of two components: the \(n = 0\)th spin in the second (first) sub-chain being flipped and the first (second) sub-chain in its ground state denoted by \(|g\rangle\).

Note that for each sub-chain at most one spin is flipped and the associated dynamics will be restricted to the ground state or the subspace of one flipped spin.

This encoding can be extended to cases of entangled Gaussian wavepackets in a straightforward manner. However, for convenience here we discuss only cases arising from the initial state given by Eq. (10). After the independent evolution of the two sub-chains for a total duration of \(\tau\) under the action of the moving parabolic potential, the quantum state of the dual spin chain is given by

\[
|\Psi(\tau)\rangle = \sum_{n=0}^{N} c_n(\tau) |\Phi_n\rangle, \tag{11}
\]

where \(|\Phi_n\rangle \equiv \alpha|g(1)\rangle \otimes |n(2)\rangle + \beta|n(1)\rangle \otimes |g(2)\rangle\). Evidently, though each \(c_n\) contains the extra quantum phases induced by the external moving potential, this factor is identical for the two state components of \(|\Phi_n\rangle\). As already demonstrated in our numerical experiments using a single spin chain, the profile of \(|c_n|^2\) should also be highly localized, and the time of arrival of the peak value of \(|c_n|^2\) at the last spin can also be directly calculated from the moving speed of the parabolic potential.

Analogous to the original dual spin chain scheme, at the end of the adiabatic quantum transport the final state \(|\Psi(\tau)\rangle\) can be decoded by applying a CNOT operation to the last two \(N\)th spins of the dual chain. Upon this operation the final state is transformed to

\[
\sum_{n=0}^{N-1} c_n(\tau) |\Phi_n\rangle + c_N(\tau) \left[ \alpha|g(1)\rangle \otimes |N(1)\rangle + \beta|N(1)\rangle \otimes |g(2)\rangle \right] \tag{12}
\]

As such, by measuring the last spin of the second sub-chain, one gains important information about the transport. In particular, if the measurement outcome is spin up, then the initial state \(|\Phi\rangle = \alpha|0\rangle + \beta|1\rangle\) has been successfully transferred to the last spin of the first sub-chain, with probability \(|c_N|^2\); if the outcome is spin down, then the quantum state transfer is unsuccessful and one needs to wait for more time to perform additional measurements.

Significantly, because our adiabatic population transfer scheme can ensure a very high probability of excitation transfer to the last spin, the probability of spin-up measurements can be guaranteed to be very high (arbitrarily high if there were no restriction on the field strength). This hence overcomes, at least theoretically, one main disadvantage of previous dual spin chain schemes where too many measurements may be required for high fidelity quantum state transfer. Further, at the end of the adiabatic transport, the spin excitation is automatically localized at very few end spins. So it also becomes unnecessary to perform fast measurements at a particular time. Instead, one can choose measurement times at will so long as the moving parabolic potential has reached the last site of the spin chain. This makes it clear that our adiabatic scheme, when combined with quantum phase encoding schemes, can find important applications in quantum information transfer (in addition to quantum population transfer).

**VI. CONCLUSIONS**

In this work we have presented a simple and robust scheme to realize adiabatic population transfer in spin chains. The additional resource needed is a slowly moving external parabolic magnetic field. The basic mechanism is the adiabatic following of a quantum state with the movement of a separatrix structure in the classical phase space of a pendulum analogy. In particular, we have shown that our scheme can be used to transfer spin excitation from one end of a spin chain to the other end, with the initial excitation profile being a localized truncated Gaussian wavepacket or exclusively localized at a single spin site. It is found that much weaker external field is needed for adiabatic population transfer if the initial excitation profile covers a few spin sites. Effects of static and dynamical fluctuations in the spin-spin coupling strength are also computationally studied, confirming the robustness of our adiabatic population transfer scheme. 

Realizing the robust population transfer with small dispersion, we have also proposed to apply our approach to a dual spin chain such that robust quantum information transfer can be realized with important advantages. We hope that our theoretical scheme can motivate experiments using various implementations of a spin chain Hamiltonian, such as cold atoms in an optical lattice and electron tunneling in an array of quantum dots.

The central idea of this work, namely, using a slowly moving external potential to adiabatically transfer spin excitation, might be useful for other applications as well. For example, one may consider distributing entanglement
along a long spin chain in a controlled fashion, by use of a
control potential that has two components slowly moving
in opposite directions. Another interesting application is
related to studies of quantum signal amplification with
spin chain models. Recently, an interesting connection
between quantum state transfer and quantum state am-
plification is revealed [27]. In this regard, our adiabatic
scheme might also help design a new and useful approach
to controlled quantum amplification using a slowly mov-
ning external potential.

Acknowledgments

J.G. is supported by the start-up funding, (WBS grant
No. R-144-050-193-101 and No. R-144-050-193-133), Na-
tional University of Singapore, and the NUS “YIA” fund-
ing (WBS grant No. R-144-000-195-123) from the office
of Deputy President (Research & Technology), National
University of Singapore.

[1] S. Bose, Phys. Rev. Lett. 91, 207901 (2003).
[2] D. I. Tsomokos, M.J. Hartmann, S.F. Huelga, and M.B.
Plenio, New J. Phys. 9, 79 (2007); A. Romito, R. Fazio,
and C. Bruder, Phys. Rev. B 71, 100501(R) (2005).
[3] L. -M. Duan, E. Demler, and M.D. Lukin, Phys. Rev.
Lett. 91, 090402 (2003).
[4] P. Cappellaro, C. Ramanathan, and D.G. Cory, Phys.
Rev. A 76, 032317 (2007); J. Zhang et al., Phys. Rev. A
72, 012331 (2005).
[5] M. Christandl, N. Datta, A. Ekert, and A.J. Landahl,
Phys. Rev. Lett. 92, 187902 (2004); D.L. Feder, Phys.
Rev. Lett. 97, 180502 (2006).
[6] C. Facer, J. Twamley, and J. D. Cresser, e-print ArXiv
0706.3821v1.
[7] M.H. Yung and S. Bose, Phys. Rev. A 71, 032310 (2005).
[8] P. Karbach and J. Stolze, Phys. Rev. A 72, 030301
(2005).
[9] F.W. Strauch and C.J. Williams, e-print ArXiv:
0708.0577.
[10] T. Shi, Y. Li, Z. Song and C. P. Sun Phys. Rev. A 71,
032309 (2005).
[11] T.J. Osborne and N. Linden, Phys. Rev. A 69, 052315
(2004).
[12] A. Wojcik, T. Luczak, P. Kurzynski, A. Grudka, T.
Gdala, and M. Bednarska, Phys. Rev. A 72, 034303
(2005).
[13] A. Lyakhov and C. Bruder, Phys. Rev. B 74, 235303
(2006).
[14] H.L. Haselgrove, Phys. Rev. A 72, 062326 (2005).
[15] K. Eckert, O. Romero-Isart and A. Sanpera, New J. Phys.
9, 155 (2007).
[16] T. Ohshima, A. Ekert, D. K. L. Oi, D. Kaszlikowski, and
L. C. Kwek, e-print quant-ph/07020192v2
[17] D. Burgarth and S. Bose, Phys. Rev. A 71, 052315
(2005).
[18] D. Burgarth, V. Giovannetti and S. Bose, J. Phys. A 38,
6793 (2005).
[19] K. Shizume, K. Jacobs, D. Burgarth and S. Bose, Phys.
Rev. A 75, 062328 (2007).
[20] D. Burgarth and S. Bose, New J. Phys. 7, 135 (2005).
[21] A somewhat different but much related procedure was
recently used to map a kicked spin chain to a kicked ro-
tor model. For details see T. Boness, S. Bose, and T. S.
Monteiro, Phys. Rev. Lett. 96, 187201 (2006).
[22] A. Buchleitner, D.Delande, and J. Zakrzewski, Phys.
Rep. 368, 409 (2002).
[23] For a recent work on quantum control in quantum dot
arrays, see D. Petrosyan and P. Lambropoulos, e-print
quant-ph/07061478v1.
[24] J. Brand and A. R. Kolovsky, Eur. Phys. J. D. 41, No.
2, 331 (2007).
[25] J. Gong and P. Brumer, Phys. Rev. A 75, 032331 (2007).
[26] G. D. Chiara, D. Rossi, S. Montaner, and R. Fazio,
Phys. Rev. A 72, 012323 (2005).
[27] A. Kay, Phys. Rev. Lett. 98, 010501 (2007).