Isoscaling in the Lattice Gas Model

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The isoscaling behavior is investigated using the isotopic/isobaric yields from the equilibrated thermal source which is prepared by the lattice gas model (LGM) for lighter systems with A = 36. The isoscaling parameters \( \alpha \) and \( -\beta \) are observed to drop with temperature for the LGM with the asymmetric nucleon-nucleon potential. However, the isoscaling parameters do not show temperature dependence for the LGM with the symmetric nucleon-nucleon potential. The relative neutron or proton density shows a nearly linear relation with the N/Z (neutron to proton ratio) of system.

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Isoscaling has been observed in a variety of reactions under the conditions of statistical emission and equal temperature recently by Tsang et al. This kind of scaling means that the ratio \( R_{21}(N,Z) \) of the yields of a given fragment \((N,Z)\) exhibits an exponential dependence on \( N \) and \( Z \) when these fragments are produced in two reactions with different isospin asymmetry, but at the same temperature. Experimentally the isoscaling has been explored in various reaction mechanisms, ranging from the evaporation, fission and deep inelastic decay can be, within grand-canonical approach, characterized by a yield of fragments with neutron and proton numbers \( N \) and \( Z \):

\[
Y(N,Z) = F(N,Z) \exp \left( \frac{B(N,Z)}{T} - \frac{N\mu_n}{T} - \frac{Z\mu_p}{T} \right) \tag{1}
\]

where \( F(N,Z) \) represents contribution due to the secondary decay from particle stable and unstable states to the ground state; \( \mu_n \) and \( \mu_p \) are the free neutron and free proton chemical potentials; \( B(N,Z) \) is the ground state binding energy of the fragment, and \( T \) is the temperature.

Typically, the investigations of isoscaling focused on yields of light fragments with \( Z=2-8 \) originating from de-excitation of massive hot systems produced using reactions of mass symmetric projectile and target at intermediate energies, such as \( ^{112,124}Sn + ^{112,124}Sn \) in Michigan State University (MSU) data or by reactions of high-energy light particle with massive target nucleus \( ^{11,13}C \). In a recent article, the isoscaling using the heavy projectile residue from the reactions of 25 MeV/nucleon \(^{86}Kr\) projectiles with \(^{124}Sn\), \(^{112}Sn\) and \(^{58}Ni\) targets which was performed at Texas A&M University (TAMU) and the isoscaling phenomenon on the full sample of fragments emitted by the hot thermally equilibrated quasi-projectiles with mass \( A = 20-30 \) are also reported.

In this study, we present an isoscaling analysis for the light fragments from thermal sources which are produced by the lattice gas model. Instead of the fixed charge number of the reaction system in MSU data or TAMU data, here we fixed the source mass for lighter system, but changing the charge number and neutron number. The isospin fractionation is also observed via the relative free neutron and free proton density which is obtained by the isoscaling parameters.

A thermally equilibrated system undergoing statistical decay can be, within grand-canonical approach, characterized by a yield of fragments with neutron and proton numbers \( N \) and \( Z \):

\[
R_{21}(N,Z) = Y_2(N,Z)/Y_1(N,Z) = C \exp(\alpha N + \beta Z) \tag{2}
\]

with \( \alpha = \Delta\mu_n/T \) and \( \beta = \Delta\mu_p/T \), with \( \Delta\mu_n \) and \( \Delta\mu_p \) being the differences in the free neutron and proton chemical potentials of the fragmenting systems. \( C \) is an overall normalization constant.

The tool we will use here is the isospin dependent lattice gas model (LGM). The lattice gas model was developed to describe the liquid-gas phase transition for atomic system by Lee and Yang. The same model has already been applied to nuclear physics for isospin symmetrical systems in the grand-canonical ensemble with a sampling of the canonical ensemble.

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In the lattice gas model, $A (= N + Z)$ nucleons with an occupation number $s$ which is defined $s = 1$ (-1) for a proton (neutron) or $s = 0$ for a vacancy, are placed on the $L$ sites of lattice. Nucleons in the nearest neighboring sites interact with an energy $\epsilon_{s_i s_j}$. The Hamiltonian is written as

$$E = \sum_{i=1}^{A} \frac{p_i^2}{2m} - \sum_{i<j} \epsilon_{s_i s_j} s_i s_j,$$

(3)

In order to investigate the symmetrical term of nuclear potential in this model, we use two sets of parameters: one is an attractive potential constant $\epsilon_{s_i s_j}$ between the neutron and protons but no interaction between like nucleon, i.e. proton and proton or neutron and neutron, namely

$$\epsilon_{nn} = \epsilon_{pp} = 0. MeV, \quad \epsilon_{pn} = -5.33 MeV.$$  

(4)

This potential results in an asymmetrical potential among different kind of nucleons, hence it is an isospin dependent potential. For simplicity we call the calculation with this potential as isoLGM thereafter. Another set is the same interaction constant between like nucleons or unlike nucleons, i.e.

$$\epsilon_{pn} = \epsilon_{nn} = \epsilon_{pp} = -5.33 MeV.$$  

(5)

In this case, the nucleon potential is symmetrical among all nucleons, i.e. isospin independent potential. For simplicity, we call the calculation with Eq. (5) as noisoLGM thereafter. In this work, mostly we use isoLGM to explore isoscaling behavior, but we will also use noisoLGM to compare the isoscaling results.

In the LGM simulation, a three-dimension cubic lattice with $L$ sites is used. The freeze-out density of disassembling system is assumed to be $\rho_f = \frac{A}{V} \rho_0$, where $\rho_0$ is the normal nuclear density. The disassembly of the system is to be calculated at $\rho_f$, beyond which nucleons are too far apart to interact. Nucleons are put into lattice by Monte Carlo Metropolis sampling. Once the nucleons have been placed we also ascribe to each of them a momentum by Monte Carlo samplings of Maxwell-Boltzmann distribution.

Once this is done the LGM immediately gives the cluster distribution using the rule that two nucleons are part of the same cluster if $P_i^2/2\mu - \epsilon_{s_i s_j} s_i s_j < 0$. This method is similar to the Coniglio-Klein’s prescription in condensed matter physics.

In this paper we choose the small size nuclei with $A = 36$ as emission sources. Four isotonic sources, namely $^{36}Ca$, $^{36}Ar$, $^{36}S$ and $^{36}Si$, corresponding to $N/Z = 0.8$, 1.0, 1.25 and 1.57, respectively, are simulated. In all cases, the freeze-out density $\rho_f$ is chosen to be 0.563 $\rho_0$, which corresponds to $4^3$ cubic lattice is used. 10000 events were simulated for each $T$ which ensures good statistics for results.

As an example, in Fig. 1 we present the isotopic scaling (the upper panel) and the isobaric scaling (the lower panel) from the isospin dependent lattice gas model for $^{36}S$ to $^{36}Ca$, respectively, at $T = 5.0$ MeV. The charged particles with $Z \leq 8$ and $2 \leq N \leq 9$ have been accumulated to perform the ratios. There exist good linear behavior in semi-log plot. In order to extract the isoscaling parameters $\alpha$ and $\beta$, we use $R_{21}(N) = C \exp(\alpha N)$ to obtain $\alpha$ for a given $Z$ (Fig. 1a) and use $R_{21}(Z) = C \exp(\beta Z)$ to obtain $\beta$ for a given $N$ (Fig. 1b), respectively. Fig. 2
shows the extracted $\alpha$ or $\beta$ versus $Z$ or $N$. Obviously, $\alpha$ or $\beta$ keeps the same value in the wide $Z$ or $N$ range and their absolute values are almost the same which is due to the absence of Coulomb interaction in the lattice gas model.

Fig. 3 shows the temperature dependence of the absolute values of isoscaling parameters $\alpha$ and $\beta$. Here the scattering points are the calculation results for LGM with asymmetrical nucleon-nucleon potential and the lines represent the results for the LGM with the same nucleon-nucleon potential for unlike and like nucleon pairs. A decreasing trend of the values is clearly seen when the asymmetrical nucleon-nucleon potential is used in LGM, which indicates that the isospin dependence of the fragment yields becomes weak with increasing temperature. Considering that $\alpha = \Delta_\mu_n/T$ and $\beta = \Delta_\mu_p/T$, we can deduce the differences in free neutron and proton chemical potentials of the fragmenting systems which is shown in Fig. 4(a). Within the error bars, it looks like that $\Delta_\mu_n$ and $\Delta_\mu_p$ keep constant in the case of asymmetrical potential (Eq.(4)). However, a slight and wide valley can also be identified around $T = 5$ MeV as well as a slight kink shows in SMM model for Sn systems [11]. For our system, this valley may be related to the liquid gas phase change for the similar system around 5 MeV in the same model calculation as well as the data [27]. This valley becomes more obvious if we plot the values of $(\Delta_\mu_n - \Delta_\mu_p)/2$ as a function of temperature as suggested in Ref. [3,11]. Fig. 4(b) shows that a turning point seems to occur around 5 MeV. Of course, the error bars look larger for lower and higher temperatures, because of the poor statistics for diverse cluster species due to the evaporation mechanism in low $T$ and the vaporization mechanism in higher $T$. Around this point, an apparent critical behavior has been observed in the disassembly of hot nuclei with $A \sim 36$ in TAMU-NIMROD experimental data and model calculations [27] by a wide variety of observables: such as the maximal fluctuations, critical exponent analysis and fragment topological structure, namely nuclear Zipf law of Ma [28] and the heaviest and second heaviest correlation etc. Hence, this turning point of the difference of neutron and proton chemical potential might give an additional evidence of the chemical phase separation when the liquid gas phase transition occurs. Experimentally, this kind of turning point has been observed recently for the quasi-projectile from the peripheral collisions of $^{28}$Si $+ \ {^{112,124}}$Sn at 30 and 50 MeV/nucleon in another TAMU data and can be understood as a signal of the onset of separation into isospin asymmetric dilute and isospin symmetric dense phase in a recent paper by Veselsky et al [7].

Further, we shall investigate the effect of the asymmetrical nucleon-nucleon potential in the lattice gas model on the isoscaling behavior. We take the same potential between like nucleons and unlike nucleons in the lattice gas model (Eq.(5)) to compare with LGM with isospin asymmetrical nucleon-nucleon potential (Eq.(4)). In this case, we observe that temperature dependence of $\alpha$ and
Finally, we test the relative free neutron density and proton density, which can be deduced by the equations:

\[ \rho_n = \exp(\alpha'), \]
\[ \rho_p = \exp(\beta'). \]

in the lattice gas model. Here \( \alpha' \) and \( \beta' \) is the scaling parameters where we take \( Y_1(N,Z) \) from \(^{36}\text{Ar}\) instead of \(^{36}\text{Ca}\) as shown above. Principally, the association of a number density with the \( \exp(\Delta\mu/T) \) only is valid for a classical gas of free particles. The connection in this context of the LGM where the particles are interacting is not clear. However, as an attempt, we will still use the above equation to deduce the the relative free neutron density and proton density. In Fig. 5(a) we show the \( \rho_n \) (solid symbols) and \( \rho_p \) (open symbols) as a function of \( N/Z \) of the systems at \( T = 5 \text{ MeV} \) for the isoLGM case (squares) and the noisoLGM case (circles). In both cases, nearly linear relations of \( \rho_n \) and \( \rho_p \) have been observed with the increasing of \( N/Z \). The Fig. 5(b) shows the ratio \( \rho_n/\rho_p \). The solid (dashed) line represents the deduced \( \rho_n/\rho_p \) from solid and open squares (circles) of Fig. 5(a) with isoLGM (noisoLGM) calculation and the dotted line is the initial value of \( \rho_n/\rho_p \) which is calculated assuming neutrons and protons homogeneously distributed in a volume proportional to the nucleon number. In isoLGM case, the values of \( \rho_n/\rho_p \) for neutron-rich nuclei (\(^{36}\text{S} \) and \(^{36}\text{Si}\)) is much larger than the initial value and while the values of \( \rho_n/\rho_p \) for proton-rich nuclei (\(^{36}\text{Ca}\)) is much less than the initial value. While, in noisoLGM case, the values of \( \rho_n/\rho_p \) also increases with the initial \( N/Z \) value of sources, which is basically originated from the initial difference of isospin of hot emitters. The stronger enhancement of \( \rho_n/\rho_p \) in the isoLGM case may indicate of a neutron enrichment while a proton depletion in the nuclear gas phase. In this context, it may be consistent with the isospin fractionation effect which is a signal expected for the liquid gas phase transition in asymmetrical systems \(^{40}\text{Ca} \), \(^{40}\text{Ar} \), \(^{40}\text{S} \), \(^{40}\text{Si} \). However, this interpretation is not unique, since the larger \( \rho_n/\rho_p \) can be also directly attributable to the interaction and thus probably not an increase in neutron density.

In summary, the isoscaling is investigated using the fragment yield from the equilibrated thermal source with the same mass number but different \( N/Z \) which was prepared by the lattice gas model with the asymmetrical potential between like nucleons and unlike nucleons. The isotopic scaling and isobaric scaling are observed for light clusters and the isoscaling parameters \( \alpha \) and \( \beta \) are extracted from the ratios of \( R_{21}(N,Z) \) for fixed proton number or neutron number. It is found that \( \alpha \) and \( \beta \) are almost the same and they drop with the temperature. However, the difference of the neutron (\( \Delta\mu_n \)) and proton (\( \Delta\mu_p \)) chemical potential does not change much with temperature and a slight and wide valley for \( \Delta\mu_n = \Delta\mu_p \) is observed around \( T = 5 \text{ MeV} \) even though we have larger error bars for lower and higher temperatures, may indicate the onset of phase separation into isospin asymmetric dilute and isospin symmetric dense phase where \( \beta \) vanishes and their absolute values are smaller than the isoLGM case as shown by the lines in Fig. 8. The insensitivity of \( \alpha \) and \( \beta \) to temperature means that the isoscaling of fragment yields only stems from the initial \( N/Z \) of the system. This is possible since the nucleon-nucleon potential is the same for like nucleons and unlike nucleons in this noisoLGM case. When temperature increases, the yield of light clusters also increase, but their isospin population does not change between the different cluster species, which results that no dependence for the isotopic or isobaric ratio on temperature. In this case, the system looks like a single component fluid, the chemical phase separation can not occur. However, for the isoLGM, the nucleon-nucleon potential is asymmetrical between like pair and unlike pair, it will give an additional isoscaling contribution due to the isospin dependent equation of state except from the trivial contribution from the initial difference of the isospin of the source, which results in a bigger absolute value of \( \alpha \) and \( \beta \) than the noisoLGM case. In this context, the information on the equation of state of the asymmetric nucleon-nucleon potential could be extracted by the temperature dependence of isoscaling parameters.
the liquid gas phase change occurs. The relative free neutron density or proton density is attempted to be deduced from the isoscaling parameter and they reveal a nearly linear relation to the N/Z of the initial system. However, the values for neutron-rich source are much larger than the initial value of N/Z as well as the values for neutron-poor source are much less than the initial value of N/Z. This may be from the isospin fractionation effect, or can be directly attributable to the interaction.

In addition, in order to investigate the effect of the asymmetrical nucleon-nucleon potential in LGM, we also adopt the same potential between like nucleons and unlike nucleons in LGM. In this case, isoscaling still remains insensitive to the temperature dependence of isoscaling parameter (α and β) vanishes and their absolute values decrease. The insensitivity of the isoscaling parameter to temperature means that the isospin partition between different fragments is almost the same regardless the excitation extent of system and the chemical phase separation is absent in this case. However, for the isoLGM, the asymmetrical nucleon-nucleon potential between like pair and unlike pair gives an additional contribution from isospin dependent equation of state to the isoscaling behavior except from the trivial contribution from the initial difference of the isospin of the source, which results in a bigger absolute value of α and β than the noisoLGM case. In this context, information on the equation of state of the asymmetric nucleon-nucleon potential could be extracted by the temperature dependence of isoscaling parameters.

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