New confining force solution of QCD domain wall problem

S. M. Barr
Department of Physics and Astronomy, Bartol Research Institute,
University of Delaware, Newark, DE 19716, USA

Jihn E. Kim
Department of Physics, Kyung Hee University, 26 Gyeongheedaero, Dongdaemun-Gu, Seoul 130-701, Korea

The serious cosmological problems created by the axion-string/axion-domain-wall system in standard axion models are alleviated by positing the existence of a new confining force. The instantons of this force can generate an axion potential that erases the axion strings long before QCD effects become important, thus preventing QCD-generated axion walls from ever appearing. Axion walls generated by the new confining force would decay so early as not to contribute significantly to the axion dark energy.

I. INTRODUCTION

The idea of axion fields \(^1\) arose in the context of the Peccei-Quinn (PQ) mechanism \(^2\) for solving the “Strong CP Problem”, i.e. explaining why QCD interactions are CP invariant to a very high degree. (For a review of the Strong CP Problem, see \([3]\).) It was soon realized, however, that axion fields have many interesting cosmological effects: (a) Coherent oscillations of the axion field \([\Phi]\) are a leading candidate for dark matter. (b) The axion field can be the inflaton: \([5]\). (c) Topological defects in the axion field, specifically “axion strings” and “axion domain walls”, form in the early universe and can affect cosmic evolution in various important ways. If these topological defects persist until the present, they can “overclose” the universe (i.e. dominate the present energy density in the universe) \([3]\); while if they were able to disappear in the early universe by radiating away their energy (predominantly into axion particles) they significantly increase the amount of axion dark matter, which tightens the observational constraints on axion models \([7–11]\).

It is this last point that we wish to address in this letter. We show that there is an interesting mechanism that can greatly suppress the contribution to dark matter coming from axions radiated by strings and domain walls, and thus alleviate the constraints on models where axions are the dark matter.

In a typical axion model one has a complex scalar field, \(\Phi\), with a zero-temperature potential of the “Mexican hat” form: \(1/4\lambda |\Phi|^2 - (F_a/2)^2|^2\). This form is dictated by the Peccei-Quinn symmetry, which is a global \(U(1)\) under which the phase of \(\Phi\) rotates. When the cosmic temperature falls well below \(F_a\), \(\Phi\) develops a vacuum expectation value (VEV): \(\Phi(x) = (F_a/\sqrt{2})e^{ia(x)/F_a}\). The value of \(a(x)\) varies randomly in space, so that cosmic strings form, around which \(a(x)/F_a\) winds by \(2\pi\). The PQ symmetry has a QCD anomaly, which means that at the quantum level it is violated by non-perturbative QCD effects (instantons). When the cosmic temperature falls to near the QCD scale \((\Lambda_{QCD} \sim 200\text{ GeV})\), the effects of QCD instantons “turn on” and lift the degeneracy of the axion field, giving it a potential of the form

\[
V(a) = c\Lambda^4_{QCD} \left( 1 - \cos \left( N \frac{a}{F_a} + \theta_{QCD} \right) \right) = c\Lambda^4_{QCD} \left( 1 - \cos \left( \frac{a}{f_a} + \theta_{QCD} \right) \right), \tag{1}
\]

where \(c\) is a dimensionless quantity; \(N\) is a model-dependent integer that gives the strength of the QCD anomaly; \(f_a = F_a/N\); and the angle \(\theta_{QCD}\) is a parameter appearing in the only CP-violating term in the QCD lagrangian. One sees that the axion field will find a minimum \(N \frac{a}{f_a} \equiv \frac{a}{f_a} = -\theta_{QCD}\) that cancels out \(\theta_{QCD}\). This is how axion models solve the Strong CP problem.

When the temperature in the early universe becomes low enough that the QCD-instanton potential for the axions “turns on”, the axion field typically finds itself misaligned, i.e. displaced from a minimum, and so starts to undergo coherent long-wavelength oscillations about a minimum \([4]\). These oscillations are equivalent to a gas of axion particles of low-momentum, and can play the role of cold dark matter. This is called the “misalignment” mechanism for producing axion dark matter.

On the other hand, and problematically, if \(N \neq 1\) the axion field has several degenerate minima \(a/f_a = \theta_{QCD} + 2k\pi\), \(k = 0, 1, 2, \cdots, N - 1\).
The regions of the universe where the axion (randomly) chooses different minima are separated by domain walls. These domain walls are attached to axion strings. As one goes around a string, $a/f_a$ winds by $2\pi$, so that the axion field passes through $N$ different minima, implying that each string has $N$ walls attached to it.

This leads to what is called the “axion domain wall problem” [6], which is that for $N \neq 1$ the network of strings and walls persists for all time, and comes to completely dominate the energy of the universe. This disaster could be avoided if the spontaneous breaking of the PQ $U(1)$ symmetry happened before or during inflation. Then the density of strings (and thus of the walls that eventually attach to them) could be so diluted by inflation as to be harmless. However, recent analyses based on BICEP2 data suggest that this possibility is excluded [12, 13]. The point is that in such a scenario the axion field would have been massless during inflation, and its fluctuations would have been encoded in the BICEP2 data.

That seems to leave only one way to avoid axion strings and walls “over closing” the universe, and that is to assume that $N = 1$ [14]. If $N = 1$, walls would still have formed, but with every string having just one wall attached to it. As pointed out by Vilenkin and Everett [15], with $N = 1$, the walls would quickly have gotten chopped up into finite sized areas bounded by closed strings. These finite patches of walls would have decayed away, by oscillating and radiating their energy into axion particles. It should be noted that a “generic” axion model would not have avoided if the spontaneous breaking of the PQ radiating their energy into axion particles. It should be noted that a “generic” axion model would not have avoided if the spontaneous breaking of the PQ since there may be other significant contributions to the dark matter density. Suppose we introduce a new non-abelian gauge interaction based on the group $G_h$. The three families of known quarks typically contribute 6 to $N$. If there exist additional heavy quarks it is possible that $N = 1$ can arise by cancellation. A more subtle way that $N$ can effectively be 1 was pointed out by Lazarides and Shafi [16], Choi and Kim [17] and in string models [14, 18], though these also require non-trivial conditions on the quark content and PQ charges of a model.

If $N = 1$, and the strings and walls can decay into axion particles, this would augment the density of axion dark matter. Let us denote by $(\alpha^{\text{dec}} + 1)$ the ratio of the total density of axion dark matter, including that produced by the decay of strings and walls, to that which would have been produced by misalignment alone. The factor $\alpha^{\text{dec}}$ is the subject of a long-running controversy. $\alpha^{\text{dec}}$ has been calculated by different groups with widely varying results. [6] obtained $\sim 0.19$; [8] obtained 6.9 $\pm$ 3.5; and [10, 11] obtained $\sim 186$.

This highly uncertain factor $\alpha^{\text{dec}}$ enters in an important way into bounds on the axion mass $m_a$ and the axion decay constant $f_a$. Visinelli and Gondolo [12] have recently found these to be

$$f_a = (8.7 \pm 0.2) \times 10^{10} \text{GeV}(\alpha^{\text{dec}} + 1)^{-6/7}, \quad m_a = (71 \pm 2) \mu\text{eV}(\alpha^{\text{dec}} + 1)^{6/7},$$

if axions make up all the dark matter in the universe. Since there may be other significant contributions to the dark matter density, these would be an upper bound for $f_a$ and a lower bound for $m_a$.

II. ELIMINATION OF STRINGS AND WALLS BY NEW CONFINING FORCE

We now propose a mechanism that would greatly suppress the contribution of decaying axion strings and walls to the dark matter density. Suppose we introduce a new non-abelian gauge interaction based on the group $G_h$. Let there be a single flavor of fermion that transforms only under $G_h$, which will be called the $G_h$-quark and denoted by $Q$. The Standard Model quarks transform only under the Standard Model gauge group and will be denoted by $u_i, d_i$, with $i = 1, 2, 3$. Suppose that the field we called $\Phi$ gives mass to $Q$ and also couples to the Higgs fields that give mass to the $u_i$ and $d_i$. The relevant terms in the lagrangian for our discussion are

$$\sum_{i=1}^3 Y_u^i \bar{u}_{R,i} u_L, \phi_1 + \sum_{i=1}^3 Y_d^i \bar{d}_{R,i} d_L, \phi_2^* + Y_Q \bar{Q}_{R} Q_L \Phi + g \phi_1^* \phi_2 \Phi + H.c.$$  

Thus both ordinary quarks and the $G_h$-quark $Q$ are charged under the Peccei-Quinn symmetry, which therefore has both a QCD and a $G_h$ anomaly. As before, let us write $\langle \Phi \rangle = (f/\sqrt{2}) e^{ia/f}$. If we assume that $f \gg v_1, v_2$, where $v_i \equiv \langle \phi_i \rangle$, then we may write $F_a = \sqrt{f^2 + v_1^2 + v_2^2} \equiv f$. And the zero-temperature potential for the axion field can be written

$$V(a) = V_{QCD}(a) + V_h(a) = c\Lambda_{QCD}^4 \left(1 - \cos \left( N_{QCD} \frac{a}{F_a} + \theta_{QCD} \right) \right) + c'\Lambda_h^4 \left(1 - \cos \left( N_h \frac{a}{F_a} + \theta_h \right) \right).$$

Here $\Lambda_h$ is the scale at which the $G_h$ force becomes strong, which we assume to be $\gg \Lambda_{QCD}; \theta_h$ is the CP-violating phase of the $G_h$ interactions; and $N_{QCD}$ and $N_h$ are the integers giving the strength of the QCD and $G_h$ anomalies.
of the PQ $U(1)$ symmetry. In this simple model, $N_{\text{QCD}} = 6$ and $N_h = 1$. Note that it is trivial to have $N_h = 1$, as it is not connected with the number of families of Standard Model quarks.

When the cosmic temperature falls below $f_a$, strings form, as in standard axion models. When $T \sim \Lambda_h \gg \Lambda_{\text{QCD}}$, the $G_h$ instanton effects “turn on”, generating $V_h(a)$, but the QCD instanton effects have not yet turned on significantly so $V_{\text{QCD}}(a)$ can be ignored. $V_h(a)$ causes axion walls to form. Since $N_h = 1$, each string has only one wall attached to it, and the Vilenkin-Everett mechanism will operate: the walls will be chopped up into small patches bounded by closed strings, and will oscillate and radiate away their energy into axions. As will be seen, the contribution of these “decay axions” to the present dark matter density is negligible.

After the strings and the walls generated by $V_h(a)$ have disappeared, the axion field will settle to the value $a/f_a = -\theta_h$, with fluctuations of order $T^2/\Lambda_h^2$. Thus, the axion field is everywhere aligned and has a topologically trivial configuration. Consequently, when the temperature reaches $\Lambda_{\text{QCD}}$ and $V_{\text{QCD}}(a)$ turns on, no walls are formed (except for rare closed surfaces produced by thermal fluctuations).

One problem that is immediately apparent is that $V_h(a)$ would destroy the solution to the Strong CP Problem. It would freeze the axion field at the value $a/f_a = -\theta_h$ and prevent it from adjusting to cancel $\theta_{\text{QCD}}$. In fact, the effective strong CP phase would be $\bar{\theta} \equiv -N_{\text{QCD}}\theta_h + \theta_{\text{QCD}}$.

This difficulty would be avoided, however, if for some reason the potential $V_h(a)$ turned off again before the temperature fell to $\Lambda_{\text{QCD}}$. This can happen in the following way. If any $G_h$-quark has zero mass, then $G_h$ instanton effects would vanish, which would mean that $V_h(a)$ would also vanish. Suppose that in addition to the $G_h$-quark we are calling $Q$, which has non-zero Pececi-Quinn charge, we assume there is another $G_h$-quark $Q'$ that has vanishing Pececi-Quinn charge. The mass of $Q'$, therefore, does not come from the vacuum expectation value of $\Phi$ (whose phase is $a/f$), but from the vacuum expectation value of some other scalar field $X$ that does not transform under the Pececi-Quinn symmetry. Suppose that there is a phase transition at some critical temperature $T_\gamma$ where $\Lambda_{\text{QCD}} \ll T_\gamma \ll \Lambda_h$, such that $\langle X \rangle \neq 0$ above $T_\gamma$ but $\langle X \rangle = 0$ below $T_\gamma$. (Such inverted phase transitions are possible and have been studied in the past [22].) What would happen in this case is that when $T$ falls below $T_\gamma$ the axion dynamics and axion mass would be controlled by $V_{\text{QCD}}(a)$, allowing the standard axion solution of the Strong CP Problem to work.

Let us now return to the question of how much the “decay axions” contribute to the dark matter density. In our scenario, domain walls form when the potential $V_h$ turns on at a temperature of order $\Lambda_h$. These walls get chopped up by the Vilenkin-Everett mechanism, and the resulting finite loops of string and patches of domain wall radiate their energy into axions. However, when $T$ falls to $T_\gamma$ these decay axions become massless. At that point, as we will now see, their contribution to the energy density of the universe is very small compared to that of thermal energy in other massless particles and remains small.

Let the temperature at which the axion field begins to oscillate due to $V_h$ be called $T_*$. One therefore has, roughly, that $m_a(T_*) \sim H(T_*) \sim T_*^2/M_p$. First, let us consider the axions in these coherent oscillations.\(^1\) For these coherent axions one has $\rho_a(T_*) \sim m_a(T_*)n_a(T_*) \sim m_a^2(T_*)f_a^2$, so that $\rho_a(T_*) \sim m_a(T_*)f_a^2 \sim T_*^2 f_a^2/M_p$. These coherent axions have typical wavelength $\lambda(T_*) \sim m_a^{-1}(T_*) \sim M_p/T_*^2$. When the temperature falls to $T_\gamma$, the (now massless) axions have number density and typical wavelength given by $n_a(T_\gamma) \sim (T_\gamma^2 f_a^2/M_p)(T_\gamma/T_*)^4 = (f_a^2 T_*^2)/(M_p T_\gamma)$ and $\lambda(T_\gamma) \sim (M_p/T_*^2)(T_\gamma/T_*) = M_p/(T_* T_\gamma)$. The energy density in these massless axions is thus $\rho_a(T_\gamma) \sim n_a(T_\gamma)\omega(T_\gamma) \sim (f_a^2/M_p)^2 T_\gamma^4 \ll T_\gamma^4$, i.e. much less than the thermal energy in other massless particles.

The amplitude of the coherent axion oscillations evolves in time in the following manner. When these oscillations begin (at $T = T_\gamma$), they have amplitude of order $f_a$ and the axion mass due to $V_h$ is large and $\bar{\theta}$ quickly settles to $-\theta_h$.

Next, when $T$ falls to $T_\gamma$, the axion potential due to $V_h$ is flat and the exponentially suppressed tiny QCD axion potential $V_{\text{QCD}}(a)$ becomes the only potential. From the second term of Eq. (4), the axion VEV is determined at $a/f_a = -\theta_h/N_h = -\theta_h$. From the first term of Eq. (4), $\rho_{\text{QCD}}(a) + \theta_{\text{QCD}} = \frac{1}{T_\gamma} + \theta_{\text{QCD}} \equiv a'/f_a$ is the axion field of the zero-temperature QCD, viz. $a/f_a = a'/f_a - \theta_{\text{QCD}} = -\theta_h$. Hence, the initial misalignment angle is $a'/f_a = \theta_{\text{QCD}} - \theta_h$, which is schematically shown for $\theta_{\text{QCD}} - \theta_h = \frac{3}{2}$ in Fig. (4). The temperature $T_1$ for the QCD axion oscillation to commence at $a'/f_a = \theta_1$ is about 1 GeV. From $T_\gamma$ down to $T_1$, axion is massless and $m_a(T_\gamma)n_a(T_\gamma) = m_a^2(T_\gamma)f_a^2$ is red shifted to $(T_1^2/M_p^2)f_a^2(T_1/T_\gamma)^4 = (f_a^2/M_p^2)T_1^4$ which is negligible even compared to the photon energy density at the commencement time of the current axion coherent-oscillation.

One sees, then, that the coherent axions produced when $V_h$ turns on have negligible effect on the present dark energy density. The analysis of the axions produced when strings and walls decay is very similar. When the domain walls form at $T_\gamma$, the horizon length is $\ell_\gamma = H^{-1}(T_\gamma) \sim M_p/T_*^2$. There is typically one horizon-length string per Hubble volume. A horizon-length string has mass $m_{\text{str}}(T_\gamma) \sim f_a^2\ell_\gamma$, and therefore the energy density in strings is of order $\rho_{\text{str}}(T_\gamma) \sim f_a^2\ell_\gamma^{-2}$. This energy is radiated into axions of typical wavelength $\ell_*$. Thus the number density of these

\(^1\) Above $T^*$, the axion mass is suppressed exponentially [21].
“decay axions” is of order $n_{\text{ax}}(T_c) \sim f_a^2 \ell_s^{-1} \sim T_c^2 f_a^2 / M_P$. One sees that this is of the same order as the number density of coherent axions. Moreover, their typical wavelengths are the same. Thus the analysis we made of the coherent axions applies to the decay axions as well. By the time the temperature falls to $\Lambda_{\text{QCD}}$, all the energy in axions coming from the strings, walls, and coherent oscillations that were due to $V_h$ have become negligible. And no new strings form when $T \sim \Lambda_{\text{QCD}}$, so that the axion dark energy present today is virtually entirely due to the coherent oscillations coming from $V_{\text{QCD}}$. In effect, then, we have made $\alpha_{\text{dec}}$ in Eq. (2) equal to zero.

Up to this point we have been discussing a scenario where there is a new confining force that generates a potential $V_h$ and gives the axion a mass when the temperature is high compared to the QCD scale, thereby allowing the Vilenkin-Everett mechanism to get rid of all the strings before the QCD instanton effects turn on. However, we can see that the potential $V_h$ could be generated in a different way, without a new confining force. Suppose, for example, that (just as in the previous discussion) there are fields $\Phi$ and $X$, where $\Phi$ breaks the Peccei-Quinn symmetry spontaneously at a scale $f_a$ and $X$ has an inverted phase transition at some scale $T_c < f_a$. And suppose that these fields have a tree-level potential $V(\Phi, X) = \frac{1}{2} \lambda (|\Phi|^2 - f_a^2)^2 + \frac{1}{2} g X^2 (\Phi^* \Phi + \Phi \Phi^*) + V(X)$.

When $T < f_a$, but $T > T_c$, one has $\langle X \rangle \neq 0$, and the axion has a potential $V_h(a) = \frac{1}{2} g \langle X \rangle^2 (f_a e^{ia} + f_a e^{-ia}) = g \langle X \rangle^2 f_a \cos(a/f_a) \approx \frac{1}{2} (g \langle X \rangle^2 / f_a) a^2$. This gives a unique minimum (i.e. $N = 1$). Thus the walls that are produced will lead to the destruction of the string-wall system by the Vilenkin-Everett mechanism. When $T$ falls to $T_c$, the vacuum expectation value of $X$ disappears and the tree-level $V_h$ disappears. The scenario is thus very similar to the one discussed before. One difference is that there are one-loop diagrams involving the coupling $g$ that give a small mass to the axion for low temperatures. This potential has the form $V_h^\prime(a) \sim \frac{1}{16 \pi^2} g^2 (\Phi^2 + \Phi^* \Phi)^2 = \frac{1}{16 \pi^2} g^2 a^2$. One must have $g$ small enough so that this loop-induced potential does not interfere with the axion solution to the Strong CP Problem, but large enough that the domain walls produced by $V_h$ can eliminate the string-wall system by the Vilenkin-Everett mechanism.

The energy/length of a string is of order $f_a^2$. The surface tension of the walls produced by $V_h$ is of order $f_a^2 m_a \sim f_a^2 (g \langle X \rangle^2 / f_a)^{1/2}$. The condition that the walls are chopped up by the Vilenkin-Everett mechanism by the time temperature of the universe is $T_c$ is roughly that $g \langle X \rangle^2 / f_a > T_c^2 / M_P$. If we take $T_c \sim \langle X \rangle$, then this gives roughly $g > \langle X \rangle^2 f_a / M_P^2$. On the other hand, in order for the potential $V_h^\prime(a)$ not to prevent the PQ solution to the String CP Problem, one must have $g^2 < \sqrt{\theta} (\Lambda_{\text{QCD}}^4 / f_a^2)$. Combining these two limits on $g$, one may obtain $\langle X \rangle < \sqrt{\theta} M_P \Lambda_{\text{QCD}} / f_a$. If we take $f_a \sim 10^{12}$ GeV, these conditions can be satisfied with $\langle X \rangle \sim 1$ TeV, and $g \sim 10^{-18}$ GeV, which suggests that the term $g X^2 \Phi$ comes from a Planck-suppressed higher-dimension operator.

This last model is not meant as a fully realistic one, but only to illustrate that the $V_h$ required to eliminate strings by our mechanism can arise in another way than from a new confining force.
III. CONCLUSION

It seems that our present Universe seems to have experienced a GUT scale energy density, leaving an imprint of the fluctuation information of massless fields of that time. (Even if the final value of $r$ is reduced from the BICEP2 value, say by a factor of 4, still the energy scale is a GUT scale, $(1.18 \times 10^{16} \text{ GeV})^4$.) Since the QCD axion is massless at that time, if $f_a > 10^{14} \text{ GeV}$ and actively participated in the inflation process then its fluctuation might have been encoded in the Planck and BICEP2 data already and $f_a > 10^{12} \text{ GeV}$ is excluded [12, 13]. If the PQ phase transition occurred after the end phase of inflation, as these studies imply, the QCD axion domain wall number $N$ must be one [6, 14] in order that the system of axion strings and domain walls can decay by the Vilenkin-Everett mechanism. In that case, however, the axions radiated in the decay of the string-wall system might have given a significant contribution to the density of axion dark matter [7–11]. Numerical studies of this have had difficulty reaching consensus. In this regards, it is of utmost importance whether there was a need for the Vilenkin-Everett mechanism at the QCD phase transition or not. If there were no horizon scale strings left at the time of QCD phase transition, the domain wall problem in axion models [6] is no longer a cosmological problem because it is very improbable to create a horizon scale string-wall system at the time of QCD phase transition. Here, we have investigated a possibility for washing out horizon scale strings by the Vilenkin-Everett mechanism, using a new confining force.

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