Horizon problem in a closed universe
dominated by fluid with negative pressure

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Abstract

We discuss the horizon problem in a universe dominated by fluid with negative pressure. We show that for generally accepted value of nonrelativistic matter energy density parameter $\Omega_{m0} < 1$, the horizon problem can be solved only if the fluid influencing negative pressure (the so-called “X” component) violates the point-wise strong energy condition and if its energy density is sufficiently large ($\Omega_{X0} > 1$). The calculated value of the $\Omega_{X0}$ parameter allowing for the solution of the horizon problem is confronted with some recent observational data. Assuming that $p_{X}/\rho_{X} < -0.6$ we find that the required amount of the “X” component is not ruled out by the supernova limits. Since the value of energy density parameter $\Omega_{\Lambda 0}$ for cosmological constant larger than 1 is excluded by gravitational lensing observations the value of the ratio $p_{X}/\rho_{X}$ should lie between the values $-1$ and $-0.6$ if the model has to be free of the horizon problem beeing at the same time consistent with observations. The value of $\Omega_{X0} + \Omega_{m0}$ in the model is consistent with the constraints $0.2 < \Omega_{\text{tot}} < 1.5$ following from cosmic microwave background observations provided that

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$\Omega_{m0}$ is low ($< 0.2$).

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I. HORIZON PROBLEM IN STANDARD COSMOLOGY

According to the standard scenario, about 300 000 years after the Big Bang the Universe cooled down to the level that atoms could form. Electrons were captured by nuclei and photons, which until that time were in thermal equilibrium with plasma, lost charged partners for interaction and started their free travel across the Universe. These photons are detected today and form the so-called cosmic microwave background (CMB). The radiation has a black-body spectrum and corresponds to the temperature \( T = 2.73 \pm 0.01 \) K, independently on the direction it comes from. Extreme isotropy of the radiation is its most puzzling property because the regions from which two hitting us today antipodal photons come, could never communicate with each other. Hence the same temperature of these regions cannot be explained in the standard scenario. This is called a horizon problem. Its inevitability in the standard cosmological model is usually illustrated by non-intersection of two past light cones of these regions at the recombination epoch, i.e. at the epoch when the relic radiation appeared (Fig. I). This non-intersection means that there were no events in the history of the universe which could influence the recombination process at the points \( A \) and \( B \) simultaneously.

II. SOME ATTEMPTS TO SOLVE THE HORIZON PROBLEM

One of the first conclusions which follow even from superficial analysis of the picture (Fig. I) is that the horizon problem exists because the recombination of atoms took place so early comparing to the age of the universe \( t_R/t_0 < 10^{-4} \). If \( t_R \) was large enough the appropriate light cones would intersect and the problem would not appear. Unfortunately in the framework of the standard scenario the recombination epoch cannot be shifted significantly into the future. What we can do instead is to assume that the last scattering of relic photons did not take place at \( t_R \) but more recently, for example, due to the Compton scattering on free electrons filling up the whole universe [1]. Detailed calculations show,
however, that for generally accepted values of cosmological parameters the horizon problem still exists [2]. In other words shifting the last scattering surface into the future does not help to solve the problem.

Careful glance at the picture suggests another solution to the problem: an appropriate bending of past light cones of the points A and B (concave form instead of convex one) can cause that the horizons would overlap (Fig. 2). This is realized in an inflationary scenario [3] where a large cosmological term gives the required form of the light cones. Inspite of the fact that there is no unique physical model of the inflation the idea of rapid exponential or power law expansion of the early universe is regarded as very attractive because it solves additionally several other problems of the standard model. We shall not discuss this topic now. In inflationary scenarios the horizon problem does not appear because the past light cones of points A and B bend radically towards the time axis (Fig. 2) due to the accelerating expansion.

In the present paper we discuss the possibility of solving the horizon problem without very effective inflationary epoch in the early universe and without shifting the last scattering surface into the future. The possibility arises in the closed universe, in which relic photons may come from geometrical antipode. This is of course trivial fact, but straightforward calculation shows that without fluid with negative pressure such universe should be extremely dense, what cannot be accepted. Inspite of the fact that many observations favor open or flat universe, the closed one is still not ruled out [4]. Since the universe in our model, apart from the relativistic and nonrelativistic matter, is also filled with the fluid influencing negative pressure (simulating repulsive force) the evolution of resulting closed cosmological models do not end up with the Big Crunch. Contrary, the universe is ever expanding reaching the size consistent with observations. Solution of the horizon problem in such models was discussed by several authors. Especially two kinds of fluid, in the context of the horizon problem, were in the past taken into account: the so-called string-like fluid, described by the equation of state \( p_s = -\rho_s/3 \) [5–7], and the cosmological constant [4]. It seems that the first kind of fluid (string-like one) is already ruled out by supernova observations [8] which indicate
that the ratio $\alpha_X$ of the pressure $p_X$ to the energy density $\rho_X$ of the unknown, the so-called “X” component of the universe must be less than $-0.6$ (95% confidence). As regards the second kind of fluid, most of recent astronomical observations suggest existence of positive cosmological constant [9–11], but the upper limit for its density is slightly too low in order to solve the horizon problem, especially if the nonrelativistic matter content of the universe is large [12].

The purpose of the present paper is discussion of the horizon problem in a universe dominated by a more general kind of fluid influencing negative pressure.

In the next section we shortly motivate dealing with the fluid with negative pressure.

In Section IV we discuss the horizon problem in the FRW model with such fluid.

In Section V we show that the horizon problem does not appear if the fluid violates the point-wise strong energy condition and if its contribution to the total energy density of the universe is predominant, closing the universe. Two special cases: vacuum and string-like matter dominated models, are discussed in more detail. In the last section we compare the models with observations and summarize the results.

III. MOTIVATION FOR DEALING WITH FLUID WITH NEGATIVE PRESSURE

Introduction of fluid with negative pressure in cosmology has a long history. Einstein introduced it in form of the $\Lambda$-term in order to get static cosmological model by compensating gravitational attraction with repulsive affect of the cosmological constant. In recent years cosmological models with $\Lambda$-term have been intensively investigated in the context of large scale structure formation [13] as well as in the context of the age-of-the-universe problem [14,15]. In both cases vacuum energy forms a smooth component of the dark matter. Visser considered the age-of-the-universe problem by treating it, as far as possibly, in a model independent way, without assuming any particular equation of state, and he showed that if the Hubble parameter is high enough, in order to solve the problem, the point-wise strong energy condition (SEC) must be violated between the epoch of galaxy formation and the
present [16]. Another reason for dealing with the “X” component follows from high redshift supernovae observations which indicate that the universe is accelerating [17,18]. The simplest way to explain this is to assume that the universe is dominated by some fluid with negative pressure. Moreover the observed number of gravitational lensing events cannot be explained without existence of such form of matter [11].

Physical interpretation can be attached to the fluid with negative pressure according to different physical models. The cosmological constant interpreted as an energy density of physical vacuum (satisfying the equation of state $p_v = -\rho_v$) is the most obvious example. The equation of state $p_s = -\rho_s/3$ corresponds to the string-like matter, which can be regarded as a network of cosmic strings conformally stretched by the expansion [19,1], global texture [3] or decaying cosmological constant [20]. Intermediate equations of state different from $p_v = -\rho_v$ and $p_s = -\rho_s/3$ can also be achieved in models with fundamental fields (scalar, vector, or tensor) forming on average some “not normal” fluid [21].

In the present paper we consider fluid with negative pressure by assuming the equation of state

$$p_X = \alpha_X \rho_X,$$

(1)

where

$$-1 \leq \alpha_X < -\frac{1}{3},$$

(2)

hence violating SEC. Cosmological constant corresponds to $\alpha_v = -1$ and is, in some sense, an extreme possibility. For the purposes of the present paper we admit, however, the upper bound of the interval ($\alpha_s = -1/3$) as well. Summarizing, we assume that the pressure of the fluid fulfills the inequality

$$- \rho_X \leq p_X \leq -\frac{1}{3} \rho_X.$$  

(3)

Following Visser [16] we shall call the fluid satisfying the inequality (3) – “abnormal”.
IV. HORIZON PROBLEM IN THE FRW MODEL WITH "ABNORMAL" FLUID

We assume that the universe is filled with relativistic matter (e.g. relic radiation), nonrelativistic matter (e.g. galaxies) and the “abnormal” fluid (e.g. cosmological constant, stringlike matter, etc.) – the fluid satisfying the inequality (3). Let us define (cf. Fig. 1):

\[ r_0 \equiv c \int_0^{t_0} \frac{dt}{R(t)} \] – comoving radius of the observer’s particle horizon,

\[ r_R \equiv c \int_0^{t_R} \frac{dt}{R(t)} \] – comoving radius of the particle horizon at the recombination epoch,

\[ \chi_R \equiv r_0 - r_R \equiv c \int_{t_R}^{t_0} \frac{dt}{R(t)} \] – comoving coordinate of the regions from which hitting us today relic photons were emitted,

\[ R(t) \] – scale factor.

It follows that the horizon problem does not appear if

\[ 2r_R > r_0, \quad \text{or equivalently if} \quad r_R > \chi_R. \] (4)

In other words comoving radius of the particle horizon at the recombination epoch must be larger than the present comoving distance to the scattering surface taking place at that epoch.

In the model under consideration the expressions for \( r_R \) and \( \chi_R \) may be rewritten in the form (c.f. [2])

\[ r_R = \left( \frac{\Omega_{r0} + \Omega_{m0} + \Omega_{X0} - 1}{k} \right)^{1/2} \int_{z_R+1}^\infty \left[ x^4 \left( \Omega_{r0} + \frac{\Omega_{m0}}{z_R + 1} \right) + (1 - \Omega_{r0} - \Omega_{m0} - \Omega_{X0})x^2 \right. \\
+ \left. \Omega_{X0}x^{3(\alpha+1)} \right]^{-1/2} dx, \] (5)

\[ \chi_R = \left( \frac{\Omega_{r0} + \Omega_{m0} + \Omega_{X0} - 1}{k} \right)^{1/2} \int_1^{z_R+1} \left[ \Omega_{r0}x^4 + \Omega_{m0}x^3 + (1 - \Omega_{r0} - \Omega_{m0} - \Omega_{X0})x^2 \right. \\
+ \left. \Omega_{X0}x^{3(\alpha+1)} \right]^{-1/2} dx, \] (6)

where \( z_R \) – redshift corresponding to the recombination epoch,
\( \Omega_{r0}, \Omega_{m0} \) – relativistic and nonrelativistic matter energy density parameters,
\( \Omega_{X0} \) – “abnormal” fluid energy density parameter.
Since the integrands in both cases are rapidly decreasing functions of $x$, and since $z_R$ is relatively large ($\approx 1200$) even rough estimate of numerical values of the above integrals leads to the conclusion that $r_R$ cannot be larger than $\chi_R$ for any reasonable values of $\Omega_{r0}$, $\Omega_{m0}$ and $\Omega_{X0}$. Numerical integration confirms this estimate. E.g. for $\Omega_{r0} = 0.00004$, $\Omega_{m0} = 0.3$ and $\Omega_{\varphi0} \approx 0.7$ ($\alpha_X = -1$ – nearly flat model with cosmological constant) we get the value $r_R/\chi_R \approx 0.0150$, much too low to solve the problem. For $\Omega_{m0} = 0.1$ and $\Omega_{\varphi0} = 0.9$ we get $r_R/\chi_R \approx 0.0153$, and the problem still remains. Considering other forms of “abnormal” fluid ($\alpha_X > -1$) does not change the ratio significantly. Hence the conclusion could be drawn that the horizon problem cannot be solved in the standard (noninflationary) cosmological scenario independently of whether the “abnormal” fluid is involved or not. In the next part of the paper I will show that this conclusion is not quite correct.

V. HORIZON PROBLEM IN A CLOSED “ABNORMAL” FLUID-DOMINATED UNIVERSE

Almost constant and small value of the ratio $r_R/\chi_R \approx 0.015$ suggests that the horizon problem cannot be solved in the standard scenario even with some form of “abnormal” fluid. Very small value of this ratio means that the angle $\theta$ at which we observe today causally connected region at the recombination epoch – the so-called particle horizon [22] is of order of only few degrees ($\theta \lesssim 3^\circ$) [1]. It is known, however, that the above statement must be revised in a closed cosmological model.

Now let us assume that the universe is closed ($k = 1$), i.e. the total energy density of matter filling up the universe, including radiation, nonrelativistic matter and the “abnormal” fluid is larger than the critical density. In this case the expressions for $r_R$ and $\chi_R$ are

$$r_R = (\Omega_{r0} + \Omega_{m0} + \Omega_{X0} - 1)^{1/2} \int_{z_R+1}^{\infty} x^4 \left( \Omega_{r0} + \frac{\Omega_{m0}}{z_R + 1} \right) - (\Omega_{r0} + \Omega_{m0} + \Omega_{X0} - 1)x^2 + \Omega_{X0}x^{3(\alpha_X+1)} \right]^{-1/2} dx, \tag{7}$$

$$\chi_R = (\Omega_{r0} + \Omega_{m0} + \Omega_{X0} - 1)^{1/2} \int_{1}^{z_R+1} \left[ \Omega_{r0}x^4 + \Omega_{m0}x^3 - (\Omega_{r0} + \Omega_{m0} + \Omega_{X0} - 1)x^2$$

$$+ \frac{\Omega_{m0}}{z_R + 1} \right) dx.$$
\[ + \Omega_{X0} x^{3(\alpha_X + 1)} \]^{−1/2} dx, \quad (8)

and play role of angular coordinates (cf. Fig. 3).

As we know from the earlier discussion and also see from the picture the horizon problem appears (shaded regions do not overlap) if \( \chi_R > r_R \) what is always the case for reasonable values of the parameters \( \Omega_{r0}, \Omega_{m0} \) and \( \Omega_{X0} \). However, closeness of the universe gives chance to solve the horizon problem even if \( \chi_R \gg r_R \). Note that if \( \chi_R \) was very close to \( \pi \) then even for small value of \( r_R \) the shaded regions could overlap. Hence what we need is to fulfill the condition (Fig. 4)

\[ |\pi - \chi_R| < r_R. \quad (9) \]

Since \( \Omega_{r0} \) and \( \Omega_{m0} \) are more or less fixed (by observations) we can only vary the “abnormal” fluid energy density parameter \( \Omega_{X0} \). Even without performing explicit integrations we notice that only for

\[ -1 \leq \alpha_X \leq -\frac{1}{3} \quad (10) \]

\( r_R \) as well as \( \chi_R \) are growing functions of \( \Omega_{X0} \), what is necessary to approach \( \pi \) by \( \chi_R \) in order to fulfill the triangle inequality (10) since \( r_R \) is always relatively small. Numerical calculations show that the growth is relatively fast. Moreover, the fastest growth is achieved for cosmological constant (\( \alpha_v = -1 \)) which corresponds to the lower bound of the interval (10), and the slowest one for the string-like matter (\( \alpha_s = -1/3 \)) – upper bound of the interval. Since the cosmological constant acts more effectively let us focus on this case.

Starting from \( \chi_R \approx 0.02 \) and \( \chi_R \approx 0.0003 \) for \( \Omega_{m0} = 0.3 \) and \( \Omega_{v0} = 0.7 \) we reach \( \chi_R \approx 3.11 \) and \( r_R \approx 0.04 \) for \( \Omega_{v0} = 1.311 \). Note that in the latter case the condition (9) is fulfilled, and the horizon problem is solved. Further increasing \( \Omega_{v0} \) causes violation of the condition (9) starting from \( \Omega_{v0} = 1.327 \). In consequence horizon problem appears again.

Summarizing, horizon problem does not appear in the vacuum-dominated closed universe for \( \Omega_{r0} = 0.00004, \Omega_{m0} = 0.3, \) and \( \Omega_{v0} \) from rather narrow interval.
It is worth mentioning that the condition (10), up to the equality sign \( \alpha_s = -1/3 \), is just the violation of the strong energy condition. We remind that according to Visser \[14\] the same condition had to be violated in order to solve the age-of-the-universe problem.

As it might have been expected the age of the universe in this case is relatively large

\[
t_0 = 12.4 \times h^{-1} \times 10^9 \text{ years},
\]

where \( h \in (1/2, 1) \) is a normalized Hubble constant. For other values of \( \Omega_m \) (0.015, 0.1 and 0.2) corresponding minimum and maximum values of \( \Omega_v \), for which the horizon problem is solved, are given in Table I. The value \( \Omega_m = 0.015 \) in the table is justified by the paper of Hoell \textit{et al.} \[23\] in which it is argued that observations of absorption lines of the Lyman \( \alpha \) forests of quasars would suggest that \( \Omega_m \approx 0.014 \) and \( \Omega_v \approx 1.08 \). Note that the value \( \Omega_v \approx 1.06 \) solving the horizon problem in our model is close to the value of Hoell \textit{et al.}

Since the energy density parameters \( \Omega_r, \Omega_m \) and \( \Omega_v \) are not constant in time and the horizon problem is solved only for the value of \( \Omega_v \) belonging to some special interval, it follows that even if we now lived in the “isotropic era” (e.g. \( \Omega_m = 0.3, \Omega_v \approx 1.32 \)) it would not mean that the era would last forever. In order to see how the horizon problem evolves in time we evaluate the coordinate \( \chi_R(z) \) for a hypothetical observer living not at the present epoch, but at the epoch determined by the redshift \( z \). The coordinate reads

\[
\chi_R(z) = (\Omega_r + \Omega_m + \Omega_v - 1)^{1/2} \int_{z+1}^{z_R+1} \left[ \Omega_r x^4 + \Omega_m x^3 - (\Omega_r + \Omega_m + \Omega_v - 1) x^2 + \Omega_v \right]^{-1/2} dx,
\]

If we calculate this integral for different values of \( z \) starting with \( z_R \approx 1200 \) we realize that the horizon problem did not appear until \( z \approx 540 \) (for \( \Omega_m = 0.3 \)). Nonexistence of the horizon problem just after the recombination epoch is not surprising of course, because the observer detects relic photons coming from his closest neighbourhood. The problem arises some time after the recombination when the observer starts to detect photons coming from more distant regions, the regions that could not ever be in thermal equilibrium. And
this happens for $z \approx 540$, i.e. relatively soon after the recombination. In our model the recombination ($z_R \approx 1200$) takes place about $250 \times 10^3 \times h^{-1}$ years after the Big Bang and $z \approx 540$ corresponds to $880 \times 10^3 \times h^{-1}$ years. In the long interval of time between $z \approx 540$ and $z \approx 0.05$ the horizon problem exists. It dissapears again about $500 \times h^{-1}$ mln of years before the present epoch and will last for the next $500 \times h^{-1}$ mln of years ($z \approx -0.04$). Before entering the “isotropic era” we would observe vanishing of fluctuations of the microwave background radiation first at the smallest angular scale and then due to the expansion fluctuations at larger scales would die out. While leaving the “isotropic era” first fluctuations at largest angular scales ($\theta \approx 180^\circ$) would come into existence.

As we mentioned before effectiveness of the string-like fluid is lower than effectiveness of the $\Lambda$-term and larger value of $\Omega_{s0}$ is required in order to solve the horizon problem. In Table II minimum and maximum values of $\Omega_{s0}$ (for which the horizon problem is solved), for different contents of nonrelativistic matter are presented [6]. Note that the age of the universe in the string-like fluid dominated model is remarkably lower than in the vacuum dominated case. This might lead to the age-of-the-universe problem if it turned out that the Hubble constant is large.

The discussion presented so far concerned two extreme cases of the fluid with negative pressure solving the horizon problem in a closed cosmological model. These cases bound the interval of values of the $\alpha_X$ – parameter allowing for the solution of the horizon problem, from below ($\alpha_v = -1$ – cosmological constant) and from above ($\alpha_s = -1/3$ – string-like matter).

In Fig. 5 we present admissible values of “abnormal” fluid energy density parameter $\Omega_{X0}$ solving the horizon problem for various values of the $\alpha_X$ parameter. There are four pairs of lines corresponding to four values of the $\Omega_{m0}$ parameter. In each pair the lower line corresponds to the minimum value of $\Omega_{X0}$ parameter solving the horizon problem, while the upper line – to the maximum one. The left bound of the diagram ($\alpha_v = -1$) corresponds to the cosmological constant and the right bound ($\alpha_s = -1/3$) – to the string-like matter. Note that the interval of admissible values of $\Omega_{X0}$ grows with $\alpha_X$, i.e. it is narrow for the
VI. SUMMARY, CONSTRAINTS FROM OBSERVATIONS AND CONCLUSIONS

One of the possibilities of solving the horizon problem in the standard cosmological scenario (without inflation in the early universe) arises when we assume that the universe is closed. In such a model two relic photons reaching us from opposite directions could be in thermal equilibrium at the recombination epoch if since that time they travelled almost one-half of the circumference of the universe. Explicit calculation shows that in a matter-dominated model (without any form of “not normal” matter) with a last scattering surface of relic photons taking place at the recombination epoch \( z_R \approx 1200 \) this is possible only in an extremaly dense universe, which of course cannot be accepted. However, the situation drastically changes if we admit existence of some form of matter influencing negative pressure (e.g. cosmological constant, textures, strings, etc.). Using some estimations it was shown by Davies \([5]\) that in a closed universe with global texture the horizon problem in fact does not exist. It was also shown \([6]\) that not only textures but any form of the so-called string-like matter (satisfying the equation of state \( p_s = -\rho_s / 3 \)) solves the horizon problem. In the present paper we showed that any fluid violating the point-wise strong energy condition, hence described by the equation of state satisfying the inequality \(-\rho_X \leq p_X \leq -\rho_X / 3\), is relevant for solving the horizon problem as well. Peculiar attention was paid to the lower bound of the interval (cosmological constant). For four values of the matter energy density parameter \( \Omega_{m0} \) (0.015, 0.1, 0.2 and 0.3) and for the ratio \( p_X / \rho_X \) from the interval \((-1, -1/3)\) we calculated numerical values of the fluid energy density parameter \( \Omega_{X0} \) necessary to solve the problem.

The solution of the horizon problem in the model is not forever. It depends on the epoch of observations. For example the value \( \Omega_{v0} \approx 1.32 \) (for \( \Omega_{m0} = 0.3 \)) is necessary for observers living today. They would observe isotropic microwave background radiation for about \( 10^9 \times h^{-1} \) years. If \( \Omega_{v0} \) was larger the “isotropic era” would occur earlier; if smaller
it would occur in future.

So far we considered theoretical possibility of solving the horizon problem in a closed universe dominated by fluid with negative pressure not relating the values of $\Omega_{X0}$ and $\alpha_X$ to observations. Now we would like to discuss observational constraints on $\Omega_{X0}$ and $\alpha_X$. We remind that for our purposes the value of $\alpha_X$ must be less than $-1/3$, and the value of $\Omega_{X0}$ must be larger than 1. There are at least three various methods of measurements which in recent years are being applied for estimates of global curvature of the universe. These concern: high-redshift supernovae, gravitational lensing events and CMB. Especially the first two give strong evidence that the Universe is accelerating. The simplest way to explain this is assuming existence of some form of matter with negative pressure. A candidate which is being taken by many authors most seriously into account is cosmological constant. However, for our purposes this candidate is not the best one because gravitational lensing events in the Hubble Deep Field impose strong upper limit on $\Omega_{v0}$ which should be remarkably lower than 1 \[11\]. The value of this limit is also confirmed by quasar statistics \[24\]. Another candidate examined in this context few years ago was string-like matter \[5,6\], e.g. network of intercommuting cosmic strings, globally wound texture or decaying $\Lambda$-term. All of them are described by the equation of state $\alpha_s = -1/3$. Inspite of the fact that theoretically the string-like matter is relevant for solving the horizon problem, it is ruled out by high redshift supernovae observations which indicate that $\alpha_X < -0.6$ with 95% confidence \[8\].

Summarizing, if we want to solve the horizon problem with the aid of the fluid with negative pressure we must assume that the $\alpha_X$ parameter is more negative than $-0.6$ and less negative than $-1$. All values between them are so far consistent with observations. As regards upper constraints on $\Omega_{X0}$, evidences definitely ruling out the possibility $\Omega_{X0} > 1$ (necessary for solving the horizon problem) are not known to us. Gravitational lensing events provide such constraints but only in the case of cosmological constant. Recent estimate of the position of a Doppler peak in the angular power spectrum of CMB fluctuations indicates that $0.2 < \Omega_{\text{tot}} < 1.5$ \[25\], what is consistent with the presented model provided $\Omega_{m0}$ is not too large (because $\Omega_{X0} = \Omega_{\text{tot}} - \Omega_{m0}$ must be larger than 1). There is a hope that new
instruments exploring CMB such that VSA, MAP and Planck Surveyor satellite will provide more precise constraints on $\Omega_{\text{tot}}$ and $\Omega_X$ and will show whether the presented scenario can be considered as a realistic physical model.

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FIGURES

FIG. 1. Illustration of the horizon problem in the standard cosmological model. $t_0$ is the age of the universe and $t_R$ corresponds to the recombination epoch. $r$ is a comoving radial coordinate. Bending of photons world lines is due to the expansion.

FIG. 2. Solution of the horizon problem in an inflationary scenario. Due to the exponential or power law expansion of the very early universe the world lines of photons hitting points $A$ and $B$ bend in such a way that the light cones intersect.

FIG. 3. Illustration of angular coordinates $\chi_R$ and $r_R$ in a closed universe. Particle horizons of the points $A$ and $B$ at the recombination epoch do not overlap. Dotted lines represent relic photons paths from the last scattering surface to the observer.

FIG. 4. Solution of the horizon problem in a closed universe. Inspite of the fact that $\chi_R \gg r_R$ (as in a flat universe) particle horizons of the points $A$ and $B$ at the recombination do overlap and the horizon problem does not appear.

FIG. 5. Admissible values of the “abnormal” fluid energy density parameter $\Omega_{X0}$ solving the horizon problem for various values of the $\alpha_X$ parameter and for different values of the nonrelativistic matter energy density parameter $\Omega_{m0}$.
TABLES

TABLE I. Admissible values of $\Omega_v^0$ for which the horizon problem does not exist. $t_0$ is the age of the universe in units $10^9 \times h^{-1}$ years.

| $\Omega_m^0$ | $\Omega_v^0_{\text{min}}$ | $\Omega_v^0_{\text{max}}$ | $t_0$ |
|------------|-----------------|-----------------|-----|
| 0.015      | 1.0567          | 1.0572          | 22.5|
| 0.1        | 1.165           | 1.172           | 16.1|
| 0.2        | 1.246           | 1.259           | 13.9|
| 0.3        | 1.310           | 1.327           | 12.6|

TABLE II. Admissible values of $\Omega_s^0$ for which the horizon problem does not exist. $t_0$ is the age of the universe in units $10^9 \times h^{-1}$ years.

| $\Omega_m^0$ | $\Omega_s^0_{\text{min}}$ | $\Omega_s^0_{\text{max}}$ | $t_0$ |
|------------|-----------------|-----------------|-----|
| 0.015      | 1.379           | 1.418           | 9.5 |
| 0.1        | 1.623           | 1.685           | 8.8 |
| 0.2        | 1.798           | 1.875           | 8.3 |
| 0.3        | 1.933           | 2.022           | 8.0 |
\[ \Omega_{m0} = 0.3 \]

\[ \Omega_{X0} = \frac{1}{\alpha_X} \]

\[ \alpha_X \]

\[ -1.0 \quad -0.9 \quad -0.8 \quad -0.7 \quad -0.6 \quad -0.5 \quad -0.4 \quad -1/3 \]