Super-Horizon modes and cosmic expansion

ENRIQUE GAZTAÑAGA and BENJAMÍN CAMACHO-QUEVEDO

1 Institute of Space Sciences (ICE, CSIC), 08193 Barcelona, Spain ; gazta@darkcosmos.com
2 Institut d’Estudis Espacials de Catalunya (IEEC), Barcelona, Spain

ABSTRACT

The large scale structures observed in cosmic maps correspond to super-horizon (causally disconnected) perturbations in the early Universe that are usually attributed to scale invariant adiabatic modes from cosmic Inflation. Here, we interpret discrepant measurements of the expansion rate \( H_0 \) (the so called Hubble tension) as super-horizon perturbations and show that they are neither adiabatic nor scale invariant. We argue that these measurements indicate that cosmic expansion originates from gravitational collapse and bounce, rather than from a singular Big Bang. This explains the observed cosmic acceleration and large scale structure without the need of Dark Energy or Inflation.

Keywords: Cosmology — Cosmic Microwave Background — Dark Energy — Dark Matter — General Relativity — Black Holes — Neutron stars

1. INTRODUCTION

The standard cosmological model (Dodelson 2003), also called \( \Lambda \)CDM, assumes that our Universe began in a hot Big Bang expansion at the very beginning of space-time. Such initial conditions violate energy conservation and are very unlikely (Dyson et al. 2002). Moreover, the assumption that everything we know started at \( \tau = 0 \) rises the well known Horizon Problem. This is a profound problem that challenges the Big Bang as a viable explanation. The \( \Lambda \)CDM model solves this by introducing Cosmic Inflation, a theory that is very hard to test because it is based on energies \( (10^{15} - 10^{19} \text{GeV}) \) that are far beyond what we can ever access with particle accelerators (Dodelson 2003). But there seems to be no viable alternative to Inflation within the known laws of Physics. Alternatives exist within Quantum Gravity and the Brane World (see e.g. Durrer et al. 2005 and references therein). Inflation also provides a prediction for the initial conditions for large scale structure. The simplest models of Inflation predict adiabatic scale invariant fluctuation in good agreement with current observations (Dodelson 2003).

Additional conceptual problems of \( \Lambda \)CDM are the need to include Dark Matter and Dark Energy, for which we have no direct evidence or fundamental understanding. With these fixes, the \( \Lambda \)CDM model seems to provide a very successful framework to understand most cosmological and astrophysical observations by fitting a few free cosmological parameters. But recent observations of anomalies in these fits (see Abdalla et al. 2022 for an extensive summary) show discrepancies with the \( \Lambda \)CDM predictions that are hard to explain. Here we will focus on variations of the expansion rate today, \( H_0 \), over super-horizon scales across the sky, but similar arguments apply to other cosmological parameters. We will assume that such variations are due to perturbations around a uniform model and show that they are inconsistent with the \( \Lambda \)CDM predictions. We will end by proposing an alternative explanation.

2. SUPER-HORIZON MODES

The \( \Lambda \)CDM model assumes that the whole Universe can be modeled as an infinite homogeneous and isotropic background space, given by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, where the physical radial distance is given by \( dr = a \, d\chi \) in terms of co-moving coordinates \( d\chi \). The adimensional scale factor, \( a = a(\tau) \), gives the expansion as a function of co-moving time \( \tau \). In the physical (or rest) frame: \( r = a \chi \) which implies \( \dot{r} = Hr \) where \( H \equiv \dot{a}/a \). The Hubble Horizon is \( r_H \equiv c/H \) (we use units of speed of light \( c = 1 \)), so that \( r > r_H \) implies \( \dot{r} > c \) and corresponds to super-horizon scales. For a perfect fluid with density \( \rho \) and pressure \( p \), the solution to the field equations in a flat space are well-known:

\[
H^2 = \frac{8\pi G}{3} \rho = \frac{H_0^2}{3} \left[ \Omega_m a^{-3} + \Omega_R a^{-4} + \Omega_\Lambda \right] \tag{1}
\]
with \( \Omega_X \equiv \frac{\rho_X}{\rho_c} \) and \( \rho_c \equiv \frac{3H^2}{8\pi G} \). Current \((a = 1)\) matter density is given by \( \Omega_m \), while radiation is given by \( \Omega_R \).

The effective cosmological constant, \( \Omega_\Lambda \), derives from:

\[ \rho_\Lambda \equiv \frac{\Lambda}{8\pi G}. \]

At any time, the expansion/collapse rate \( H^2 \) is given by \( \rho \). Energy–mass conservation requires that \( \rho \propto a^{-3(1+\omega)} \), where \( \omega = p/\rho \) is the equation of state of the different components: \( \omega = 0 \) for pressureless matter (or dust), \( \omega = 1/3 \) for radiation, and \( \omega = -1 \) for \( \rho_\Lambda \).

Cosmic acceleration is defined as \( q \equiv (\dot{a}/a)H^{-2} = -\frac{1}{2}(1 + 3\omega) \). For regular matter, we have \( \omega > 0 \) so we expect the expansion to decelerate \((q < 0)\) because of gravity. However, the latest concordant measurements from a Type Ia supernovae (SNe), galaxy clustering, and CMB all agree with \( \omega = -1.03 \pm 0.03 \) (DES Collaboration 2022); or \( q \simeq 1 \) in the future.

Note how \( q = 1 \) (or \( \omega = -1 \)) implies that \( \rho \) and \( H \) become constant, \( H = H_\Lambda \equiv c/r_\Lambda. \) Constant velocity is equivalent to no velocity in the rest frame. In the physical (or rest) frame such expansion is not accelerating, but is asymptotically static. This is important because it shows that we are trapped inside an Event Horizon. This frame duality can be understood as a Lorentz contraction \( \gamma = 1/\sqrt{1 - \dot{r}^2} \) where \( \dot{r} = Hr \). An observer in the rest frame, not moving with the fluid, sees the moving fluid element \( ad\gamma \) contracted by the Lorentz factor \( \gamma \). So the FLRW metric becomes de-Sitter like:

\[
a^2 d^2 \chi = \gamma^2 d^2 r = \frac{d^2 r}{1 - r^2 / r_\Lambda^2} = \frac{d^2 r}{1 + 2\Phi}, \tag{2}
\]

where \( \Phi \) is the gravitational potential which can also be interpreted as a metric perturbation. This radial element corresponds the metric of a hypersphere of radius \( r_H \) that expands towards a constant event horizon \( r_\Lambda \). In this limit, the FLRW metric reproduces the static de-Sitter metric: \( 2\Phi = -r^2 H_\Lambda^2 \), where the expansion becomes static (see Mitra 2012 for further discussion).

Structures larger than the Hubble Horizon \( r_H \) are not in causal contact, because the time that a perturbation takes to travel such distance is larger than the expansion time. As \( r_H \) increases with \( \tau \), the structures we observe today were not in causal contact in the early Universe (e.g. in the CMB). This is the horizon problem. In the \( \Lambda CDM \) model, the problem is solved by Cosmic Inflation, a period of exponential expansion that happened right at the beginning of time. Inflation solves the horizon problem, but leaves the universe empty. We need a mechanism to stop Inflation and to create the matter and radiation that we observe today.

If we could see the light from the Big Bang \((\tau = 0)\), it would come from a very distant spherical shell in the sky (red circle in the bottom right of Fig.1). The furthest we can actually see is the CMB shell (dashed circle \( a_\ast \simeq 10^{-3} \)), which is quite close to \( \tau = 0 \) \((a = 0)\). This means that \( r_H \propto c \tau \) subtends a very small angle in the sky: \( \theta = r_H/(a_\ast a_\tau) \approx 1 \) deg., where \( a_\ast \) is the co-moving angular diameter distance to the CMB. Larger scales are super-horizon. How is then possible that the CMB temperature is so uniform on larger scales (as shown by top panel in Fig.1)? Inflation can solve this puzzle but a collapsing phase, before the Big Bang, can also do it.

In Inflation the spectrum of primordial super-horizon perturbations is scale invariant and adiabatic, so we expect to see temperature and metric perturbations of equal size at large scales. But there is an anomalous lack of the largest structures in the CMB sky temperature \( T \) with respect to the predictions of Inflation (see e.g. Gaztaña et al. 2003; Schwarz et al. 2016). This is apparent in Fig.2, which compares \( \delta T \) in the CMB sky
Figure 2. Comparison of CMB temperature $\delta T$ maps from data (Planck SMICA map, left) and a simulation of the best fit $\Lambda$CDM model (right), smoothed with a 30 deg. gaussian radius. Both maps have very similar amplitude in small scale angular modes ($C_\ell$, not shown), but there is a lack of power at the largest super-horizon scales ($\theta > 60$ deg.) in the real data. We also show the $H_0$ horizons displayed in Fig.1 as grey circles over the Planck map. The CMB dipole direction, shown as two red diamonds, does not seem to be related to the super-horizon perturbations.

with a $\Lambda$CDM simulation. The CMB isotropy scale can be measured with the homogeneity index, $H$, a fractal or Hausdorff dimension that is model-free and purely geometrical, independent of the amplitude of $\delta T$. Camacho-Quevedo & Gaztañaga (2022) find evidence of homogeneity ($H = 0$) for scales larger than $\theta_H = 65.9 \pm 9.2$ deg. on the CMB sky. This finding is at odds ($p < 10^{-5}$) with the predictions of Inflation.

A related anomaly is shown at the bottom of Fig.1. It displays a sky map of relative variations of $H_0$ from the best fit to the standard $\Lambda$CDM temperature spectrum $C_\ell$ (for $32 < \ell < 2000$) over large regions ($\theta > 30$ deg.) around each position in the sky (see Fosalba & Gaztañaga 2021 for details). The fit in each region agrees well with the predictions, but the fitted parameters vary across the sky. Here we interpret such variations as super-horizon perturbations.

There is a characteristic cut-off scale where $\Delta H_0$ vanished which are shown by grey circles labeled $H_1, H_2, H_3$. Same horizons are found for different cosmological parameters. They correspond to a cut-off in super-horizon perturbations from the $\tau = 0$ surface, indicating that the primordial spectrum is not scale invariant as predicted by the simplest models of Inflation.

In Fig.3 we compare the size of $H_1, H_2, H_3$ with the homogeneity scale $\theta_H$ as a function of the mean value (and dispersion) of $H_0$ measured in each region. The $\theta_H$ measurement corresponds to the full sky and is therefore assigned to the global Planck fit for $H_0$ (Planck Collaboration 2020). We can also place in the same plot the local type Ia SN measurement of $H_0$ from Riess et al. (2021) and the one from type II SN from de Jaeger et al. (2022). In this case the angle is taken to be 60 degrees as this is the angle in the CMB sky that corresponds to the

radial separation $\chi_*$ between the CMB surface and the SNe measurements. The angular spread is taken from the largest variance in the other measurements.

Note that the $\theta_H$ and $H_1, H_2, H_3$ measurements are independent from each other. The later are obtained by fitting the small scale $C_\ell$ power spectrum ($\leq 1$ deg.), while $\theta_H$ is measured from the scaling of the correlations at much larger scales ($\geq 30$ deg.). The SNe results are corrected to the CMB frame but this has a negligible impact on $H_0$ (Riess et al. 2021; de Jaeger et al. 2022).
We interpret these variations in $H_0$ as metric perturbations in Eq.2, where $2\Phi = -\dot{r}^2H^2$, so that:

$$\delta_\Phi \simeq 2\frac{\Delta H_0}{H_0} \simeq 0.2.$$  

Large scale metric perturbations do not evolve with time (Dodelson 2003) so we expect these variations both in nearby and CMB observations, in agreement with Fig.3. But the large values in Eq.3, are at odds with the small amplitude ($\delta_T \simeq 10^{-5}$) and location of radiation fluctuations in the CMB sky, as shown by comparing top and bottom panels in Fig.1. This indicates that, on the largest super-horizon scales, perturbations are not adiabatic, as predicted by Inflation (Dodelson 2003). Also note in Fig.2 and Fig.1, that the CMB dipole direction, shown as red diamonds, does not correlate with the note in Fig.2 and Fig.1, that the CMB dipole direction, as predicted by Inflation (Dodelson 2003). Also note in Fig.1, the CMB dipole direction, shown as red diamonds, does not correlate with the note in Fig.2 and Fig.1, that the CMB dipole direction.

## 3. THE BLACK HOLE UNIVERSE (BHU)

We next present an alternative to Inflation that could explain such anomalies. We call it the Black Hole (BH) Universe (BHU, Gaztañaga 2022a,b), where cosmic expansion originates from the gravitational collapse of a dust cloud. To understand this, we first notice that we live inside a BH. A BH is defined as an object with escape velocity $\dot{r} > c$, which is equivalent to a size $R \leq r_S$, where:

$$r_S = \frac{2GM}{c^2} = 2GM \simeq 2.9\text{Km} \frac{M}{M_\odot},$$  

is the Schwarzschild (SW) radius or BH event horizon. The mean density of a BH inside $r_S$ is always:

$$\rho_{BH} = \frac{3M}{4\pi r_S^3} \approx \frac{3r_S^2}{8\pi G} \simeq 10^{-2} \left[ \frac{M_\odot}{M} \right]^2 \frac{M_\odot}{\text{Km}^3}.$$  

compared to the atomic nuclear saturation density:

$$\rho_{NS} \simeq 2 \times 10^{-4} \frac{M_\odot}{\text{Km}^3},$$  

which corresponds to the density of a heavy nuclei. The later results from the Pauli Exclusion Principle applied to neutrons and protons. For a Neutron Star (NS) with $M \simeq 7M_\odot$, we have: $\rho_{BH} = \rho_{NS}$. This explains why we have never found a NS with $M \geq 7M_\odot$, as a collapsing cloud with such mass reaches BH density $\rho_{BH}$ before it reaches $\rho_{NS}$. Using more detailed considerations the maximum is $M \leq 3M_\odot$ (Özel & Freire 2016).

The density of a BH in Eq.5 is also the density of our Universe in Eq.1 inside $r_H$. Note from Eq.1 that $H^2$ tends toward a constant $\Lambda = H_0^2\Omega_\Lambda = \Lambda/3$. As explained before, the expansion becomes asymptotically static in the physical (rest) frame with a fixed radius: $r_H \rightarrow r_A$ in Eq.2. This $r_A$ corresponds to an event horizon (Gaztañaga 2022a,b) and the total mass-energy inside $r_A$ is given by: $M = r_A/2G$, which is the definition of a physical BH in Eq.4 for $r_S = r_A$. So our cosmic expansion happens inside a BH of finite size. For $5\Lambda \simeq 0.7$ and $H_0 \simeq 70\text{Km}/\text{s}/\text{Mpc}$ we have: $M \simeq 5.5 \times 10^{23}M_\odot$ or $r_S \simeq 1.6 \times 10^{23}\text{Km}$.

More generally, the FLRW solution can also describe a local spherical uniform cloud of variable radius $R$ and fix mass $M$, which collapses or expands in freefall. This is a well known concept in Newtonian Gravity which follows Eq.1 for arbitrary $R$. Based on Gauss law, each sphere $r < R$ collapses independent of what is outside $r > R$. This is also the case in General Relativity (GR), following Birkhoff’s theorem. As a consequence, the FLRW solution is also a solution for a local uniform cloud (Oppenheimer & Snyder 1939; Faraoni & Atieh 2020). The cloud physical radius $r = R$ follows a boundary condition: $-2\Phi = R^2/r^2_H = r_S/R$, which corresponds to a matching of FLRW in Eq.2 with a SW metric $2\Phi = -r_S/r$ outside $R$ (Gaztañaga 2022a,b):

$$R = \left[ r_H^2 r_S \right]^{1/3}.$$  

This equation requires that either $r_H > R > r_S$ or $r_H < R < r_S$. For a regular star we have $R > r_S$ which implies $R < r_H$: all scales are sub-horizon, as expected. But as discussed in Fig.1, in our Universe we observe super-horizon scales $R > r_H$ which means that $R < r_S$. Again showing that we are inside our own BH event horizon, as indicated by Fig.4.

### 3.1. The Big Bounce

Before it became a BH, the density of our FLRW cloud was so small that no interactions other than gravity could occurred. Radiation escapes the cloud, so that $p = 0$ ($\omega = 0$). Radial co-moving shells of matter are in free-fall (time-like geodesics of constant $\chi$) and continuously pass $R = r_S$ inside its own BH horizon, as illustrated in Fig.4.

Solving Eq.1 for a collapsing dust cloud, $H \propto -a^{-3/2}$, we find that the BH forms at time: $\tau_{BH} = -2r_S/3c \simeq -11\text{Gyrs}$ before $\tau = 0$ (the Big Bang) or $25\text{Gyr}$ ago. The collapse continuous inside until it reaches nuclear saturation (GeV) in Eq.6 and the situation is similar to the interior of a collapsing star ($R$ contains $10^{23}M_\odot$), but $r_H$ only has a few $M_\odot$. We conjecture that this leads to a Big Bounce because of Pauli’s Exclusion Principle. The collapse is halted causing the implosion to rebound (Baym & Pethick 1979) and expand. Diffuse baryons as well as compact stellar remnants (BH or NS) can result from such a Big Crunch and they could explain part or all of the Dark Matter (Carr & Kühnel 2020).
The bounce happens at energy densities (GeV) that are $10^{15} - 10^{19}$ orders of magnitudes smaller than Inflation or Planck energies. Thus, Quantum Gravity or Inflation are not needed to understand cosmic expansion. Because $R \propto r_H^2$ in Eq.7, $r_H \propto c t$ always expands or collapses faster than $R$. Once inside $r_S$, perturbations of size $r$ exit the horizon $R > r > r_H$ during collapse and re-enter $R > r_H > r$ as the expansion happens, see Fig.4, solving the horizon problem without Inflation.

3.2. Trapped Inside a Black Hole

Once the FLRW cloud becomes a BH, no events can escape $r_S$. This translates into an expansion that freezes out or becomes static in physical units when $R$ approaches $r_S$. As shown in Eq.2, this corresponds to cosmic acceleration, $\dot{H} = 0$, for the co-moving observer. Thus, $r_S$ behaves like a $\Lambda$ term ($\Lambda = 3 r_S^{-3}$), despite our choice of $\Lambda = 0$ in the background where the collapse occurred (Gaztañaga 2022c).

Such $r_S$ boundary imposes a cut-off in the spectrum of super-horizon perturbations generated during the collapse and bounce. During the collapse, $H \propto -a^{-3/2}$ ($\Lambda = k = p = 0$), the maximum radial comovil separation (and corresponding maximum transverse CMB angle) between two events is:

$$\chi_{max} = \int_{r_S}^{0} \frac{cd\alpha}{H a^2} = 2r_S ; \theta_{max} = \pi r_S / \chi_{\star},$$

where $\chi_\star$ is the angular diameter distance to the CMB, where we use the type Ia SN value of $H_0$ with $\Omega_m \simeq 0.25 \pm 0.05$ and $k = 0$ (i.e. an apparent $\Omega_\Lambda \simeq 0.75$), as measured today locally. Note that $\chi_{max}$ scales as $H_0^{-1}$. The good agreement of these predictions to the data in Fig.3, indicates that cosmic expansion originates from a gravitational collapse and bounce inside $r_S$ with $\Lambda = 0$. Inflation with a cut-off or phase transition (Barriga et al. 2001) still needs $\Lambda$ to account for $q$. 

4. DISCUSSION AND CONCLUSION

The large scale structures that we observe in Cosmic maps are measured to be adiabatic and scale invariant, as predicted by Harrison-Zeldovich-Peebles (Zel'Dovich 1970; Harrison 1970; Peebles & Yu 1970), long before Inflation. The seeds could originate during the BHU collapsing phase or during Inflation. But we find that the largest scales are neither scale invariant:

- there is a cut-off in the largest super-horizon modes as measured in $\delta T$ (Fig.3). This reflects in the homogeneity scale $\theta_H = 65.9 \pm 9.2$ deg.,
- a similar cut-off is measured in values of $H_0$ (or other cosmological parameters) fit at different positions in the CMB sky (Fig.1),

or adiabatic:

- the amplitude of these super-horizon modes in radiation ($\delta_T \simeq 10^{-5}$) is much smaller that those measured in $H_0$ or $\rho$: $\delta_\rho \simeq 0.2$ (Eq.3) both within the CMB and between the CMB and SNe,
- the location of these super-horizon modes in radiation do not trace well the ones in $\delta_\rho$ as shown by the comparison of Fig.1.

Fig.3 shows a good agreement of these observation with the prediction of Eq.8. This supports the idea that the observe expansion results from a collapse and bounce, rather than from a singular creation. The collapse is halted by Quantum Mechanics: when the collapse reaches nuclear saturation in Eq.6, it bounces back like a core collapse supernova. Such Big Bounce could explain the large observed metric fluctuations in Eq.3. More work is needed to estimate the perturbations, composition and fraction of compact to diffuse remnants that resulted. This could explain some of the free parameters in the ΛCDM shown in Table 1.

Our FLRW cloud collapse happened in an existing background. We don’t know what else is out there or how it started, but we have assumed that it has the simplest topology: flat with $\Lambda = 0$, as in empty space. The observed accelerated expansion $q$ is usually attributed to a $\Lambda$ term in the background with $\Omega_\Lambda \simeq 0.75 \pm 0.05$ in Eq.1. During expansion, this is equivalent to the BHU event horizon $r_S$, where $\Lambda = 3 / r_S^2$ (Gaztañaga 2022c). We can only distinguish the effects of a true $\Lambda$ from $r_S$ during the collapse. Because of the Equivalence Principle, the dynamics of collapsing shells in free fall is not affected by $r_S$. But if $q$ originates from a true $\Lambda$ (rather than from $r_S$ with $\Lambda = 0$) it will change the collapse time, so that $\chi_{max} \simeq 3.2 r_S$ in Eq.8 (red lines in Fig.3) which is clearly ruled out by data, which favors $\Lambda = 0$. 

![Figure 4](https://example.com/figure4.png)

**Figure 4.** A uniform cloud of fixed mass $M$ and size $R = (r_H r_S)^{1/3}$ (red circle) collapses (left) to form a BH (middle) and bounce into expansion (right) inside $r_S = 2GM$ (black circle). The Hubble Horizon $r_H$ (blue dashed) moves faster than $R$, so that radial perturbations become super-horizon (yellow region) during collapse and re-enter $r_H$ during the expansion, solving the horizon problem without Inflation.
Table 1. Model comparison. Observations that require explanation.

| Cosmic observation                          | Big Bang (ΛCDM) explanation | BHU explanation |
|--------------------------------------------|-----------------------------|-----------------|
| Expansion law                              | FLRW metric                 | FLRW metric     |
| Element abundance                          | Nucleosynthesis             | Nucleosynthesis |
| Cosmic Microwave Background (CMB)           | recombination               | recombination   |
| All sky CMB uniformity & LSS seeds         | Inflation                   | Uniform Big Bounce and perturbations |
| Cosmic acceleration, 14Gyr age, BAO & ISW  | Dark Energy                 | BH event horizon size $r_s$ |
| Rotational curves, lensing & Cosmic flows  | Dark Matter                 | compact stellar remnants of Big Bounce |
| CMB fluctuations $\delta T = 10^{-5}$      | free parameter              | perturbations during collapse |
| $\Omega_m/\Omega_B \simeq 4$               | free parameter              | fraction of compact to diffuse remnants |
| $M/\Omega_m \simeq 3$                      | free parameter              | time to de-Sitter phase |
| Large scales anomalies in CMB               | Cosmic Variance (bad luck) | super-horizon cut-off Eq.8 |
| anomalies in cosmological parameters       | Systematic effects          | super-horizon perturbations |
| flat universe $k = 0$                      | Inflation                   | topology of empty space |

The BH collapse time is proportional to $M$. A mass $M \approx 5.5 \times 10^{22} M_\odot$ is just the right one to allow enough time for galaxies and planets to form before de-Sitter phase dominates. This provides an anthropic explanation as to why we live inside such a large expanding BH. The BHU solution can also be used to model the interior of smaller BHs, but they will not have time to form regular galaxies or stars before they reach the asymptotically static de-Sitter phase.

Our expansion will become static inside a BH in a larger and older background, possibly containing other BHU. This provides another layer to the Copernican Principle and avoids the important conceptual problems of the standard Big Bang model: the singular creation, Inflation and the dark energy "mystery".

REFERENCES

Abdalla, E., Abellán, G. F., Aboubrahim, A., et al. 2022, arXiv. https://arxiv.org/abs/2203.06142
Barriga, J., Gaztañaga, E., Santos, M. G., & Sarkar, S. 2001, MNRAS, 324, 977
Baym, G., & Pethick, C. 1979, ARAA, 17, 415
Camacho-Quevedo, B., & Gaztañaga, E. 2022, JCAP, 4, 044. https://arxiv.org/abs/2106.14303
Carr, B., & Kühnel, F. 2020, ARNPS, 70, 355
de Jaeger, T., Galbany, L., Riess, A. G., et al. 2022, arXiv e-prints. https://arxiv.org/abs/2203.08974
DES Collaboration. 2022, PRD, 105, 023520
Dodelson, S. 2003, Modern cosmology, Academic Press, NY
Durrer, R., Kunz, M., & Sakellariadou, M. 2005, Physics Letters B, 614, 125
Dyson, L., Kleban, M., & Susskind, L. 2002, J.of High Energy Phy, 2002, 011
Faraoni, V., & Atieh, F. 2020, PRD, 102, 044020
Fosalba, P., & Gaztañaga, E. 2021, MNRAS, 504, 5840
Gaztañaga, E. 2022a, Physics of the Dark Universe, submitted. https://hal.archives-ouvertes.fr/hal-03344159
Gaztañaga, E. 2022b, Universe, 8, 257. https://www.mdpi.com/2218-1997/8/5/257
Gaztañaga, E. 2022c, Symmetry, 14, 300
Gaztañaga, E., Wagg, J., Multamäki, T., Montaña, A., & Hughes, D. H. 2003, MNRAS, 346, 47
Harrison, E. R. 1970, PRD, 1, 2726
Mitra, A. 2012, Nature Sci. Reports, 2, 923
Oppenheimer, J. R., & Snyder, H. 1939, Phy.Rev., 56, 455
Özel, F., & Freire, P. 2016, ARAA, 54, 401
Peebles, P. J. E., & Yu, J. T. 1970, ApJ, 162, 815
Planck Collaboration. 2020, A& A, 641, A6
Riess, A. G., Yuan, W., Macri, L. M., et al. 2021, arXiv e-prints. https://arxiv.org/abs/2112.04510
Schwarz, D. J., Copi, C. J., Huterer, D., & Starkman, G. D. 2016, CQGra, 33, 184001
Secrest, N. J., von Hausegger, S., Rameez, M., et al. 2021, ApJL, 908, L51
Zel’Dovich, Y. B. 1970, AAP, 500, 13