Spectrum of Cosmic Microwave Fluctuations and the Formation of Galaxies in a Modified Gravity Theory

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Abstract

A modified gravity (MOG) possesses a light, neutral vector particle called a “phion” associated with a vector field $\phi^\mu$, which forms a cold fluid of Bose-Einstein condensates before recombination with zero pressure and zero shear viscosity. The energy density associated with this Bose-Einstein condensate fluid dominates the energy density before recombination and produces a density parameter, $\Omega_\phi \sim 0.3$, that together with the fractional baryon density $\Omega_b \sim 0.04$, and a cosmological constant parameter $\Omega_\Lambda \sim 0.7$ yields an approximate fit to the data for the acoustical oscillations in the CMB power spectrum. The quantum phion condensate fluid is abundant well before recombination and can clump and form the primordial structure for galaxies. At late times in the expanding universe, in local bound systems such as galaxies ordinary baryonic matter dominates the matter density. For galactic systems in the present epoch, the modified Newtonian acceleration law determined by MOG describes well galaxy rotation curve data and X-ray cluster mass profile data.

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1 Introduction

A relativistic modified gravity (MOG) theory [1, 2] has been proposed to explain the rotational velocity curves of galaxies and the X-ray data for clusters of galaxies with a modified Newtonian acceleration law, without non-baryonic dark matter. A fitting routine for galaxy rotation curves has been used to fit a large number of galaxy rotational velocity curve data, including low surface brightness (LSB), high surface brightness (HSB), dwarf galaxies and elliptical galaxies with both photometric data and a two-parameter core model without non-baryonic dark matter [2, 3]. The fits to
the data are remarkably good and for the photometric data only the one parameter, the mass-to-light ratio $\langle M/L \rangle$, is used for the fitting, once two parameters $M_0$ and $r_0$ are universally fixed for galaxies and dwarf galaxies. The fitting results for galaxies are consistent with the data for the Tully-Fisher relation. A large sample of X-ray mass profile cluster data has also been fitted. The gravity theory also fits the anomalous acceleration data observed during the Pioneer 10-11 spacecraft missions, and is consistent with gravity observations in the solar system and the binary pulsar PSR 1913+16.

The MOG requires that Newton’s constant $G$, the coupling constant $\omega$ that measures the strength of the coupling of a skew field to matter and the mass $\mu$ of the skew field, vary with distance and time, so that agreement with the solar system and the binary pulsar PSR 1913+16 data can be achieved, as well as fits to galaxy rotation curve data and galaxy cluster data. The variation of $G$ leads to a consistent description of relativistic lensing effects for galaxies without non-baryonic dark matter. In ref. [2, 7], the variation of these constants was based on a renormalization group (RG) flow description of quantum gravity theory formulated in terms of an effective classical action. Large infrared renormalization effects can cause the effective $G$, $\omega$, $\mu$ and the cosmological constant $\Lambda$ to run with momentum $k$ and a cutoff procedure leads to a space and time varying $G$, $\omega$ and $\mu$, where $\mu = 1/r_0$ and $r_0$ is the effective range of the skew symmetric field. In the MOG theory [2], the action contains a contribution that leads to effective field equations that describe the variations of $G$, $\omega$ and $\mu$. In principle, we can solve for the complete set of field equations and determine the dynamical behavior of all the fields. However, in practice we make approximations allowing us to obtain partial solutions to the equations, yielding predictions for the various physical systems considered.

A modified gravity theory must be able to explain the following cosmological data and phenomena:

1. The cosmic microwave background (CMB) data, including the power spectrum for both large and small angular scales on the sky;
2. The formation of galaxies before and after recombination;
3. Gravitational lensing;
4. N-body simulations of large-scale galaxy surveys;
5. The accelerating expansion of the universe.

The standard explanation for these phenomena is the cold dark matter (CDM) scenario and some form of uniform dark energy that begins to dominate in the present epoch of the universe. The problem with this scenario is that 96% of the universe’s matter and energy budget is invisible. It remains undetected in ground-based laboratory experiments, and has yet to be given a satisfactory physical interpretation. An attempt to explain the CMB power spectrum at the surface
of last scattering about 370,000 years after the big bang in terms of the visible baryon number density \( \Omega_b \) leads to a satisfactory fit to the WMAP and Archeops data \[11, 12, 13\] for the first two peaks in the power spectrum at small angles with the baryon density fraction \( \Omega_b \sim 0.04 \) and the cosmological constant density fraction \( \Omega_\Lambda \sim 0.95 \). This scenario requires a neutrino mass contribution with \( m_\nu \sim 1 - 2 \) eV. The baryon density parameter \( \Omega_b \sim 0.04 \) is in agreement with recent estimates for this quantity at big bang nucleosynthesis. The problem with this pure baryon-neutrino scenario is that a third putative peak in the angular power spectrum is erased by baryon drag. A problem with neutrinos describing dark matter is that they become non-relativistic at a late-time epoch in the expanding universe, and their fluctuations are erased in the power spectrum.

The most recent balloon-born Boomerang data \[13\] indicate the existence of a third peak in the power spectrum, which is erased by baryon drag without cold dark matter \[10\]. If future data confirms that a significant third peak in the angular power spectrum exists, then this strongly supports that some form of dark, non-baryonic matter should exist at the surface of last scattering. All of this suggests that some kind of non-relativistic dark matter should be accounted for in cosmology. On the other hand, the MOG fits to the galaxy rotation curve data and the X-ray galaxy cluster data \[3, 5\] without non-baryonic dark matter are remarkably good and avoid problematic issues related to CDM fits to the galaxy data.

It is also necessary to provide a satisfactory explanation for the early universe growth of structure and the eventual emergence of galaxies and clusters of galaxies. Large computer N-body simulations show that some kind of non-relativistic dark matter has to constitute the proto-galaxies about 200 million years after the big bang \[14, 15\]. The data suggest that early universe density fluctuations were a Gaussian random field described by a cold dark matter (CDM). The standard assumption is that the CDM is made of elementary particles that interact only gravitationally. Baryons and neutrinos alone fail to produce a realistic description of large-scale galaxies and clusters surveys.

An important extra-degree of freedom in MOG is a light, electrically uncharged vector particle called a “phion”. In the early universe at a temperature \( T < T_c \), where \( T_c \) is a critical temperature, the phions become a Bose-Einstein condensate (BEC) fluid. The phion condensates couple weakly with gravitational strength to ordinary baryonic matter. This cold fluid of phion condensates dominates the density of matter at cosmological scales and, because of its clumping due to gravitational collapse, allows the formation of structure and galaxies at sub-horizon scales well before recombination. We do not postulate the existence of cold dark matter in the form of heavy, new stable particles such as supersymmetric WIMPS.

In the following, we shall calculate the power spectrum for values \( l \gg 1 \) obtaining the positions and heights of the acoustical peaks from cosmological parameters. We shall use the analytical formula derived by Mukhanov \[16\] and earlier work by Hu and Sugijama \[17\], Frampton, Ng and Rohm \[18\], Weinberg \[19\] and Dodelson \[20\]. The approximate analytical formula yields results for the small angular
scale acoustical oscillations that agree with reasonable accuracy with the WMAP, Archeops and Boomerang data [11, 12, 13]. Since the phion condensates do not couple to radiation and only couple with weak gravitational strength to baryons, their density perturbations are not subject to dissipation like photon and neutrino density perturbations before recombination.

For large-scale cosmology, the phion condensate density on the average dominates the matter density, while for locally bound galactic systems and clusters of galaxies visible baryon density dominates the matter density, and the MOG field equations and the quantum BEC equations lead to an effective modification of the Newtonian acceleration law for weak fields that yields an excellent fit to the galaxy rotation curve data and the X-ray cluster mass profile data [3, 5].

2 Classical Field Equations and Test Particle Motion

The relativistic gravitational field equations in MOG are given by (in units with $c = 1$) [2]:

$$G_{\mu\nu} - g_{\mu\nu}\Lambda + Q_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1}$$

$$\nabla_\nu B^{\mu\nu} + \frac{\partial V(\phi)}{\partial \phi_\mu} + \frac{1}{\omega} \nabla_\nu \omega B^{\mu\nu} = -\frac{1}{\omega} J^\mu, \tag{2}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, $\Lambda$ is the cosmological constant and $\nabla_\mu$ denotes covariant differentiation with respect to the metric $g_{\mu\nu}$. We have

$$Q_{\mu\nu} = G(\nabla^\alpha \nabla_\alpha \Theta g_{\mu\nu} - \nabla_\mu \nabla_\nu \Theta), \tag{3}$$

where $G(x)$ is the scalar field spacetime dependent gravitational “constant” and $\Theta(x) = 1/G(x)$. Moreover,

$$B_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu, \tag{4}$$

and $J^\mu$ denotes a matter current. The potential $V(\phi)$ has the form

$$V(\phi) = -\frac{1}{2} \mu^2 \phi^\mu \phi_\mu + W(\phi), \tag{5}$$

where $\mu(x)$ is the effective spacetime dependent mass of the vector field $\phi^\mu$ and $W(\phi)$ is a $\phi^\mu$ self-interaction potential. The coupling constant $\omega(x)$ is the scalar spacetime dependent coupling of the skew field $B^{\mu\nu}$ to matter.

The total energy-momentum tensor is given by

$$T_{\mu\nu} = T_{M\mu\nu} + T_{\phi\mu\nu} + T_{S\mu\nu}, \tag{6}$$

where $T_{M\mu\nu}, T_{\phi\mu\nu}$ and $T_{S\mu\nu}$ denote the energy-momentum contributions from the matter fields, the vector field $\phi^\mu(x)$, and the scalar fields $G(x), \omega(x)$ and $\mu(x)$, respectively.
From the Bianchi identities
\[ \nabla_\nu G^\mu\nu = 0, \tag{7} \]
and from the field equations (1), we obtain the conservation law:
\[ \nabla_\nu T^\mu\nu + \frac{1}{G} \nabla_\nu GT^\mu\nu - \frac{1}{8\pi G} \nabla_\nu Q^\mu\nu = 0. \tag{8} \]

The effective gravitational "constant" \( G(x) \) satisfies the field equations:
\[ \nabla_\alpha \nabla^\alpha G + \frac{\partial V(G)}{\partial G} + N = \frac{1}{2} G^2 \left( T + \frac{\Lambda}{4\pi G} \right), \tag{9} \]
where
\[ N = -3\Theta \left( \frac{1}{2} \nabla_\alpha G \nabla^\alpha G + V(G) \right) + G \left( \frac{1}{2} \nabla_\alpha \omega \nabla^\alpha \omega - V(\omega) \right) + \frac{G}{\mu^2} \left( \frac{1}{2} \nabla_\alpha \mu \nabla^\alpha \mu - V(\mu) \right) + \frac{3G^2}{16\pi} \nabla_\alpha \nabla^\alpha \Theta, \tag{10} \]
and \( T = g^{\mu\nu} T_{\mu\nu} \). The scalar field \( \omega(x) \) obeys the field equations
\[ \nabla_\alpha \nabla^\alpha \omega + \frac{\partial V(\omega)}{\partial \omega} + F = 0, \tag{11} \]
where
\[ F = -\Theta \nabla_\alpha G \nabla^\alpha \omega + G \left( \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + V(\phi) \right). \tag{12} \]
The field \( \mu(x) \) satisfies the equations
\[ \nabla_\alpha \nabla^\alpha \mu + \frac{\partial V(\mu)}{\partial \mu} + P = 0, \tag{13} \]
where
\[ P = - \left[ \Theta \nabla^\alpha G \nabla_\alpha \mu + \frac{2}{\mu} \nabla^\alpha \mu \nabla_\alpha \mu + \omega \mu^2 G \frac{\partial V(\phi)}{\partial \mu} \right], \tag{14} \]
and the last term arises from the \( \mu \) dependence of \( V(\phi) \).

The equation of motion for a test particle of mass \( m \) and charge \( \lambda \) when \( \omega = \omega_{\text{ren}} = \text{constant} \), where \( \omega_{\text{ren}} \) is the renormalized value of \( \omega \), is given by
\[ \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = f^\mu, \tag{15} \]
where \( \tau \) is the proper time along the test particle trajectory and
\[ f^\mu = \frac{\lambda \omega}{m} B^\mu_\nu \frac{dx^\nu}{d\tau}. \tag{16} \]
For massless photons the test charge \( \lambda \) is zero and photons move along null geodesics:
\[ \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0. \tag{17} \]
3 Bose-Einstein Condensate of Phions

An important prediction of quantum statistical mechanics is that of a phase tran-
sition in an ideal gas of identical bosons when the thermal de Broglie wavelength
exceeds the mean spacing between bosons [21]. The bosons in the lowest energy
state are stimulated by the presence of other bosons to occupy the same state, re-
sulting in a macroscopic occupation of a single quantum state that constitutes a
macroscopic quantum-mechanical system [22, 23].

Well before recombination, the boson-phion particle can collapse into the lowest
energy quantum ground state when the temperature of the phion fluid is below a
critical temperature, \( T < T_c \), and form a degenerate Bose-Einstein fluid that does
not couple to photons and is not dissipated by photon pressure [24, 25, 26, 27, 28].
The fluid has zero viscosity and pressure and forms the dominant energy density of
matter in the universe at large cosmological scales greater than the scales of galaxies
and clusters of galaxies. The phion Bose-Einstein condensates (BEC) are the dark
matter contribution to the cosmological density, which dominates the baryon density,
\( \Omega_\phi > \Omega_b \) where \( \Omega_\phi \) denotes the fractional BEC density, at cosmological scales and
couple weakly with gravitational strength to baryon matter.

The phions undergo a second-order phase transition through a spontaneous sym-
mmetry breaking mechanism below a critical temperature \( T < T_c \). This mechanism
can generate the effective phion mass \( m_\phi \) and the BEC exhibits a broken symmetry
and a phase coherence. The condensate wave function is non-vanishing and the
ground state (vacuum) depends coherently on the phases of the phions, such that
they are spatially correlated throughout the system. For a quantum state of light
phions, it is possible to form a non-relativistic quantum BEC of phions in a second
order phase transition.

The gravitational collapse (clumping) of the phion condensate fluid to form grav-
itational potential wells before recombination, allows the early formation of struc-
tures at small horizon scales of the order of proto-galaxy sizes. These structures
eventually lead to the formation of clusters of galaxies in a hierarchical scheme of
structure growth. For this to happen, the density of non-relativistic BEC phions
\( \rho_\phi \) dominates the clumping fluid with a sub-horizon size of order of the size of a
proto-galaxy and with a suitable value of the coherence length \( R_c \).

A Jean’s scale \( \lambda_J \) separates gravitationally stable and gravitationally non-stable
modes. The time scale for gravitational collapse is given by \( \tau_{grav} \sim 1/\sqrt{G_{\text{ren}}\rho_\phi} \).
On the other hand, the time scale for fluid pressure to dynamically respond is
\( \tau_{press} \sim \lambda / c_s \), where \( \lambda \) is the size of the condensate in the fluid and \( c_s \) is the sound
velocity. In units with \( \hbar \) and \( c \) not equal to unity, for quantum non-relativistic
BEC matter, the velocity \( v_c \) in the condensate fluid is determined by the de Broglie
wavelength \( \lambda_B = h/m_\phi v_c \). The fraction of phions in the coherent BEC ground
state is about 100% before decoupling at a redshift \( z_d \sim 1100 \), and after decoupling
baryonic structures can begin to form and the quantum Jeans scale \( \lambda_{J\text{quant}} \) can
become the size of proto-galaxies.
Three characteristic lengths define the system: the Compton wavelength, \(\lambda_c = h/m_\phi c\), the average distance between particles, \(\lambda_d = n_\phi^{-1/3}\) where \(n_\phi\) denotes the number density of phions, and the thermal wavelength \(\lambda_T = h/p\). The thermal de Broglie wavelength is

\[
\lambda_T = \left(\frac{2\pi h^2}{m_\phi k_B T}\right)^{1/2},
\]

(18)

where \(k_B\) is Boltzmann’s constant. The non-relativistic limit corresponds to \(\lambda_T \gg \lambda_c\), while the relativistic limit holds for \(\lambda_T \ll \lambda_c\).

For an ideal non-relativistic Bose gas, the number of phions is given by

\[
N_\phi \equiv \sum_{\epsilon_\phi} (n_{\epsilon_\phi}) = \sum_{\epsilon_\phi} \frac{1}{\epsilon_\phi - 1 - \exp(\epsilon_\phi/k_B T) - 1},
\]

(19)

where \(\epsilon_\phi\) denotes the phion energy spectrum and \(z\) is the fugacity, related to the chemical potential \(\mu_{\text{chem}}\) by \(z = \exp(\mu_{\text{chem}}/k_B T)\). The fugacity \(z\) is solved in terms of the equation

\[
n_\phi = \frac{g_3/2(z)}{\lambda_T^3} + \frac{1}{V} \frac{z}{1 - z},
\]

(20)

where \(V\) is a characteristic volume. We can solve this equation to obtain

\[
n_\phi = \frac{g_3/2(z)}{\lambda_T^3} + \frac{1}{V} \frac{z}{1 - z},
\]

(21)

where

\[
g_n(z) = \frac{1}{\Gamma(n)} \int dx \frac{x^{n-1}}{z - 1 - \exp(x) - 1}.
\]

(22)

We can rewrite (21) as

\[
\lambda_T^3 \frac{\langle n_{\phi\phi}\rangle}{V} = \lambda_T^3 n_\phi - g_{3/2}(z),
\]

(23)

which implies that \(\langle n_{\phi\phi}\rangle/V > 0\) when the temperature \(T\) and \(n_\phi\) are such that

\[
\lambda_T^3 n_\phi > \zeta(3/2) = 2.612...,\]

(24)

where \(g_{3/2}(1) = \zeta(3/2)\) and \(\zeta(x)\) is the Riemann-zeta function. A finite number of phion bosons occupy the ground state level with \(\epsilon_\phi = 0\). This defines the critical temperature for phion condensation:

\[
T_c = \frac{2\pi h^2 n_\phi^{2/3}}{k_B m_\phi \zeta^{2/3}(3/2)},
\]

(25)

and the critical phion number density

\[
n_{c\phi} = \frac{\zeta(3/2)}{\lambda_T^3}.
\]

(26)
Thus, for $T < T_c$ a large number of phions occupy the ground state $\epsilon_\phi = 0$.

We are required in a relativistic cosmology to take into account the expansion of the universe. The solution for the case of an ideal boson gas in a Friedmann-Robertson-Walker (FRW) universe is complicated, so authors 31, 32, 33, 34, 35 have considered a boson gas in a closed Einstein universe. In the spatial geometry of a 3-sphere, the metric takes the form:

$$ds^2 = c^2 dt^2 - a_E^2 [d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2)],$$ \hspace{1cm} (27)

where $\chi$ and $\theta$ run from 0 to $2\pi$. The radius of the Einstein universe $a_E$ is constant.

The solution for the spin 1 phion discrete energy spectrum is given by ($\hbar = c = 1$):

$$\epsilon_n = \frac{1}{a_E} (n^2 + m_\phi^2 a_E^2)^{1/2}$$ \hspace{1cm} (28)

with degeneracy $d_n = 2(n^2 - 1)$ where $n = 2, 3, \ldots$.

Let us take the mean phion density at large scale galactic distances to be of the order of the critical density, $\rho_\phi \sim 9.52 \times 10^{-30} \text{ g cm}^{-3}$. From (25), we get in the condensation region

$$n_\phi \leq \left( \frac{k_B T_c \rho_\phi}{2\pi \hbar^2 \zeta^{2/5}(3/2)} \right)^{3/5}.$$ \hspace{1cm} (29)

We obtain for the phion condensate density that is comparable to the cosmological critical density for a critical temperature at recombination $T \sim 10^5 \text{ K}$:

$$n_\phi \leq 6.3 \times 10^7 \text{ cm}^{-3}.$$ \hspace{1cm} (30)

We get from $m_\phi = \rho_\phi/n_\phi$ for the effective phion condensate mass at the epoch of recombination:

$$m_\phi \sim 8.5 \times 10^{-5} \text{ eV/c}^2.$$ \hspace{1cm} (31)

## 4 Bose-Einstein Gas and the Ground State

A Bose-Einstein state was experimentally verified by the Joint Institute for Laboratory Astrophysics (JILA) in 1995. Anderson et al. 36 produced a condensate of spin-polarized $^{87}RB$ atoms in a confining magnetic trap. Densities and temperatures of order $10^{11} \text{ cm}^{-3}$ and tens of micro-Kelvin, respectively, were achieved.

We shall assume that the phion boson inter-particle interactions are weak, so that at low temperatures in cosmology the de Broglie wavelengths of the phions are large compared to the inter-phion separation. The phion-phion interactions are dominated by elastic S-wave scattering and we can consider for weak fields of gravitational strength only two-body collisions. The scattering length can be positive, $a > 0$, for a repulsive interaction and negative, $a < 0$, for an attractive interaction. The phion self-interaction potential is given by

$$U(x - x') = g \delta(x - x').$$ \hspace{1cm} (32)
Here, the strength of the interaction is
\[ g = \frac{4\pi\hbar^2 a}{m_{\phi}}. \] (33)

The Hamiltonian for weakly interacting phions in an external gravitational potential \( V_{\text{grav}} \) is of the form
\[ H = \int d^3 x \psi^\dagger(x) \left[ -\frac{\hbar^2}{2m_{\phi}} \vec{\nabla}^2 + V_{\text{grav}}(x) \right] \psi(x) \]
\[ + \frac{1}{2} \int d^3 x \int d^3 x' \psi^\dagger(x) \psi^\dagger(x') U(x-x') \psi(x') \psi(x), \] (34)
where \( \psi(x) \) and \( \psi^\dagger(x) \) are the phion field annihilation and creation operators, respectively.

A necessary condition for the applicability of a calculation of the BEC gas is that \( a^3 \rho_{\phi} \ll 1 \). The equation of motion for the wavefunction takes the form
\[ i\hbar \frac{\partial \psi(x, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m_{\phi}} \vec{\nabla}^2 + V_{\text{grav}}(x) + g \psi^\dagger(x) \psi(x) \psi(x) \right] \psi(x, t). \] (35)

To solve this equation, a mean-field approach is adopted in which
\[ \psi(x, t) = \overline{\psi}(x, t) + \delta \psi(x, t), \] (36)
where \( \overline{\psi} = \langle \psi \rangle \) is the phion condensate wave function and \( \delta \psi \) is the thermal quantum fluctuation around the mean value. The quantity \( \overline{\psi} \) is an “order parameter” that describes a broken gauge symmetry; the Abelian gauge symmetry occurs in the classical MOG action for a massless phion field, \( m_{\phi} = 0 \). Moreover, we have that \( \langle \delta \psi \rangle = 0 \) and for small \( \delta \psi \) the temperature is assumed to be \( T \sim 0 \).

The basic equation for the condensate phion system is the Gross-Pitaevskii equation \[37, 38, 39\]:
\[ i\hbar \frac{\partial \overline{\psi}(x, t)}{\partial t} = \left[ -\frac{\hbar^2}{2m_{\phi}} \vec{\nabla}^2 + V_{\text{grav}}(x) + N_{\phi} g \overline{\psi}(x, t)^2 \right] \overline{\psi}(x, t). \] (37)

A stationary solution for the phion wave function can be obtained from
\[ \psi(x, t) = \exp(-i\mu_{\text{chem}} t/\hbar) \tilde{\psi}(x). \] (38)

This leads to the time-independent equation
\[ \left[ -\frac{\hbar^2}{2m_{\phi}} \vec{\nabla}^2 + V_{\text{grav}}(x) + N_{\phi} g \overline{\psi}(x, t)^2 \right] \overline{\psi}(x) = \mu_{\text{chem}} \tilde{\psi}(x). \] (39)

The phion condensate gas before recombination can become unstable for a negative scattering length, \( a < 0 \), corresponding to an attractive inter-phion potential.
A stable local minimum for the energy exists only up to a maximum number of phion condensates, and above this number the kinetic energy can no longer stabilize the condensate gas against collapse. This instability describes the early formation of proto-galaxies before recombination and growth of structure after recombination. In practise, the energy per phion condensate has three contributions: the kinetic energy, the phion potential energy and the potential energy due to gravity. An application of the virial theorem to the proto-galaxy condensate gas gives

\[ E_{\text{kin}} = E_\phi - \frac{3}{2} E_{\text{grav}}, \]  

where \( E_{\text{kin}}, E_\phi \) and \( E_{\text{grav}} \) denote the kinetic energy, the phion potential energy and the gravitational potential energy, respectively. As the number \( N_\phi \) of phion condensates increases, a critical point is reached when the condensate gas collapses and clumping can begin and form a proto-galaxy. The repulsive phion-phion forces can take over at a certain stage of the gravitational collapse of the phion condensate gas for a scattering length \( a > 0 \), preventing the formation of numerous unobserved compact sub-structures in N-body simulations.

The critical chemical potential for a transition is given by

\[ \mu_{\text{chem}} = \frac{8\pi a h^2}{m_\phi \lambda^3} \zeta(3/2), \]  

where \( a \) is the scattering length. For a galaxy-sized object, we can choose \( a \sim \lambda_G \sim 14 \text{ kpc} \) and \( m_\phi \sim 10^{-24} \text{ eV}/\text{c}^2 \) to give

\[ \frac{\mu_{\text{chem}}}{\hbar} \sim 10^{-14} \text{ s}^{-1}, \]  

corresponding to a period \( t_p \sim 10^{14} \text{ s} \) which is roughly the time it takes for a light signal to cross a galaxy, \( R_G/c \sim 10^{12} \text{ s} \).

## 5 Phion BEC and Evolution of the Universe

In the late-time universe, the non-relativistic phion matter \textit{locally} in bound systems such as stars, galaxies and clusters of galaxies is dominated by baryon matter and neutral hydrogen and helium gases. The quantum non-relativistic, phion condensate is subjected to only the weak fifth force with baryon matter, so that as the universe expands there is little or no decoherence of the phion condensate gas. However, during the evolution of galaxies ordinary baryonic matter begins to dominate inside the cores of galaxies and traces light. This predicts that \textit{cold dark matter halos do not exist around the cores of galaxies or in clusters of galaxies}. On the other hand, at inter-galactic cosmological scales the neutral BEC phions dominate the density of matter with \( \Omega_b < \Omega_\phi \). The spontaneous symmetry breaking leading to the phion BEC is relaxed inside the cores of galaxies, so that the effective mass \( m_\phi \)
of the phions is significantly reduced, $m_\phi \sim 10^{-24}$ eV, corresponding to a Compton wavelength $\lambda_c = 1/\mu_\phi = \hbar/m_\phi c \sim 14$ kpc.

Locally, inside the bound galaxy systems well after decoupling and the formation of galaxies, the spherically symmetric, static solution of the classical MOG field equations, together with the variation and renormalization of Newton’s gravitational constant $G$, leads at late times to the effective modification of Newtonian acceleration for weak fields in galaxies [2]. The expression for the modified acceleration is obtained from the static spherically symmetric solution of the equation valid outside the matter source

$$\phi''_0 + \frac{2}{r} \phi'_0 - \mu_\phi^2 \phi_0 = 0,$$

(43)

where $'$ denotes differentiation with respect to the radial coordinate $r$ and $\phi_0$ denotes the time component of the phion field $\phi_\mu$. The solution to (43) is the Yukawa potential and the modified acceleration law takes the form:

$$a(r) = -\frac{G_\infty M}{r^2} + K \frac{\exp(-\mu_\phi r)}{r^2} (1 + \mu_\phi r),$$

(44)

where

$$K = G_N \sqrt{M} \sqrt{M_0}.$$  

(45)

Here, $G_N$ denotes Newton’s gravitational constant, $M$ is the total mass of the galaxy and $M_0$ is a constant that measures the strength of the coupling of the skew field $B^{\mu\nu}$ to matter, expressed in units of the square-root of a mass. We choose

$$G_\infty = G_N (1 + \alpha),$$

(46)

where

$$\alpha = \sqrt{\frac{M_0}{M}}.$$  

(47)

We obtain the expression for the effective modified Newtonian acceleration law:

$$a(r) = -\frac{G_{\text{ren}} M}{r^2} [1 + \alpha (1 - \exp(-\mu_\phi r)(1 + \mu_\phi r))].$$

(48)

We can express the acceleration in terms of an effective Newtonian law in the form:

$$a(r) = -\frac{G_{\text{eff}}(r) M(r)}{r^2},$$

(49)

where

$$G_{\text{eff}}(r) = G_N [1 + \alpha (1 - \exp(-\mu_\phi r)(1 + \mu_\phi r))].$$

(50)

We note that the integration constant $K$, $\alpha$ and $G_{\text{eff}}$ depend on the inverse square root of the total mass $M$. This non-linear dependence on the mass $M$ cannot be interpreted in terms of a classical collection of point masses. However, our classical modification of Newton’s acceleration law is a consequence of an effective classical
limit of the non-linear Schrödinger equation for the phion BEC. Let us consider an application of Ehrenfest’s theorem:

$$\frac{d}{dt}\langle x \rangle = \langle \frac{p}{m} \rangle, \quad \frac{d}{dt}\langle \frac{p}{m} \rangle = \langle a \rangle. \quad (51)$$

The Hamiltonian operator obtained from Eq. (37) is given by

$$\hat{H} = -\frac{\hbar^2}{2m_\phi} \vec{\nabla}^2 + V_{\text{grav}}(x) + W(x), \quad (52)$$

where for a static potential

$$W(x) = N_\phi g |\tilde{\psi}(x)|^2, \quad (53)$$

and $-i\hbar \vec{\nabla} = \vec{p}$. We get

$$\frac{d}{dt}\langle p \rangle = \frac{i}{\hbar}\langle [H, p] \rangle = -\langle (\vec{\nabla} V_{\text{grav}}(x)) + \langle \vec{\nabla} W(x) \rangle \rangle, \quad (54)$$

where $-\langle \vec{\nabla} V_{\text{grav}} \rangle$ is the Newtonian force law and $-\langle \vec{\nabla} W \rangle$ is the quantum, BEC phion modification of the Newtonian force law corresponding to the modified acceleration law in MOG.

The effective modified acceleration law fits remarkably well the rotation curve data of galaxies and the mass profiles of X-ray clusters. This avoids the problem of standard cold dark matter models that predict density profiles within galactic cores that are too sharp and “cuspy”. The LSB galaxies have a small contribution from the baryonic mass component, so they are efficient tracers of rotation curves, and can help to distinguish dark matter halo models from the modified Newtonian acceleration law in MOG. The velocity rotation curves predicted by MOG fit well the inner cores of dwarf galaxies and LSBs in term of one parameter, the mass-to-light ratio $<M/L>$, using photometric data.

The MOG acceleration formula predicts that for satellite galaxies the rotational velocities and density profiles of the satellites have a Newtonian-Kepler behavior.

6 Weak Field Equations in Cosmology

In a homogeneous universe with small perturbations, the metric is given in a spatially flat universe in the conformal Newtonian-Poisson gauge by

$$ds^2 = a^2[(1 + 2\Phi) d\eta^2 - (1 - 2\Phi) \delta_{ik} dx^i dx^k], \quad (55)$$

where $\eta$ denotes the conformal time and $\Phi \ll 1$.

For weak gravitational fields the Poisson equation for the gravitational potential $\Phi$ takes the form:

$$\vec{\nabla}^2 \Phi(x, t) + a^2(t)Y(x, t) = 4\pi a^2(t)G(x, t)\delta \rho(x, t), \quad (56)$$

where $Y(x, t)$ is the quantum, BEC phion modification of the Newtonian force law.
where \( Y \) denotes the contributions obtained from \( \text{[3]} \), the factor \( a^2(t) \) is inserted to account for the difference between the FRW co-moving coordinate vector \( \mathbf{x} \) and the coordinate vector \( a(t)\mathbf{x} \) that measures the proper distances at time \( t \), and \( \delta \rho \) denotes the size of the density fluctuation. If we assume that for large cosmological scales, we can ignore the \( Y \) contributions and that \( G = G_{\text{ren}} = \text{constant} \), then we obtain \( \text{[19]} \):

\[
\Phi(\mathbf{x}, t) = -4\pi G_{\text{ren}} t^{2/3} a^2(t) \int d^3 q \frac{1}{q^2} \exp(i\mathbf{q} \cdot \mathbf{x}) N_\mathbf{q},
\]

where \( N_\mathbf{q} = \frac{\delta \mathbf{q}}{t^{2/3}} \), \( \delta \mathbf{q} = \delta \rho \mathbf{q}/\rho \mathbf{q} \). The co-moving wave number vector is denoted by \( \mathbf{q} \), which is related to the physical wave number at last scattering \( k = q/a(t_{LS}) \) where \( t_{LS} \) denotes the time of last scattering.

The equation for the vector field \( \phi_\mu \) for weak fields is given by \( \text{[2]} \):

\[
\vec{\nabla}^2 \phi_\mu(\mathbf{x}, t) - a^2(t) \mu_\phi^2 \phi_\mu(\mathbf{x}, t) - a^2(t) \partial W(\phi)(\mathbf{x}, t) = a^2(t) \frac{1}{\omega} J_\mu(\mathbf{x}, t),
\]

where we have assumed that the coupling constant \( \omega \) and the mass \( \mu_\phi \) have their renormalized values, and \( J^\mu = (q, J^i) \) \((i = 1, 2, 3)\) is a matter current.

### 7 Angular Distance and Cosmological Parameters

Before recombination the photons couple strongly to baryons, and after recombination hydrogen gas becomes neutral and the photons no longer interact with baryons. The free photons move along null geodesics and are described by the distribution function \( f \):

\[
dN = f(x^i, p_j, \eta) d^3 x d^4 p,
\]

where \( dN \) denotes the number of photons at time \( \eta \). In the absence of scattering, the distribution function \( f \) obeys the collisionless Boltzmann equation

\[
\frac{\partial f}{\partial \eta} + \frac{dx^i}{d\eta} \frac{\partial f}{\partial x^i} + \frac{dp^i}{d\eta} \frac{\partial f}{\partial p^i} = 0.
\]

The geodesic equations for the photons are

\[
\frac{dx^\mu}{d\tau} = p^\mu, \quad \frac{dp_\mu}{d\tau} = \frac{1}{2} \frac{\partial g_{\gamma\delta}}{\partial x^\mu} p^\gamma p^\delta.
\]

The Boltzmann equation takes the form

\[
\frac{\partial f}{\partial \eta} + n_i (1 + 2\Phi) \frac{\partial f}{\partial x^i} + 2p \frac{\partial \Phi}{\partial x^j} \frac{\partial f}{\partial p_j} = 0,
\]

where \( n_i = -p_i/p \) is a vector that determines the direction on the sky from which the photons reach an observer.
Under general assumptions, the fractional variation from the mean of the CMB temperature observed in the direction $\hat{n}$ takes the form \[19\]:

$$\frac{\Delta T}{T}(\hat{n}) = \int d^3k \delta \rho_k \exp(i d_A \hat{n} \cdot \mathbf{k}) \left[ F(k) + i \hat{n} \cdot \hat{k} G(k) \right],$$

(63)

where $d_A$ is the angular diameter distance from the surface of last scattering, and $k^2 \delta \rho_k$ is proportional to the Fourier transform of the fractional total energy perturbation. The form factor $F(k)$ arises from the Sachs-Wolfe effect and the intrinsic temperature fluctuations, while $G(k)$ arises from the Doppler effect.

The angular diameter distance of the surface of last scattering is given by

$$d_A = \frac{1}{\Omega_c^{1/2} H_0 (1 + z_L)} \sinh \left[ \Omega_c^{1/2} \int_{1/(1 + z_L)}^1 \frac{dx}{(\Omega_\Lambda x^4 + \Omega_k x^2 + \Omega_m x)^{1/2}} \right],$$

(64)

where $z_L \sim 1100$ is the redshift at the surface of last scattering, $\Omega_k = 1 - \Omega_\Lambda - \Omega_m$ is the curvature parameter, and $\Omega_m$ and $\Omega_\Lambda$ denote

$$\Omega_m = \frac{8 \pi G_{\text{ren}} \rho_m}{3 H^2}, \quad \Omega_\Lambda = \frac{\Lambda}{3 H^2}.$$

(65)

Here, $G_{\text{ren}}$ is the renormalized value of the gravitational constant determined at some time $\bar{t}$ before the time of recombination $t \sim t_r$:

$$G_{\text{ren}} = G_0 (1 + Z),$$

(66)

where $G_0 \sim G_N$ is the “bare” Newtonian value of the gravitational constant. In our modified gravitational theory, the density of matter is given by

$$\Omega_m = \Omega_b + \Omega_\phi + \Omega_S,$$

(67)

where $\Omega_\phi$ and $\Omega_S$ are the densities of matter associated with the phion BEC density and the scalar fields, $G, \omega$ and $\mu$, respectively.

8 Baryon-Radiation Fluid and Phion BEC Fluid Before Recombination

The fluctuations of the background radiation depend upon the radiation energy density fluctuations, $\delta \rho_\gamma/\rho_\gamma$, and the gravitational potential $\Phi$. The medium consists of two components, the coupled baryon-radiation fluid and the non-relativistic phion BEC condensate fluid and scalar fields fluid. The light phion condensates are electrically neutral and do not couple to photons and couple to baryons with a gravitational strength. We shall assume that the phion BEC condensate density fluctuations $\delta_\phi = \delta \rho_\phi/\rho_\phi$ dominate i.e., $\delta_\phi \gg \delta_b$ and $\delta_\phi \gg \delta_S$, where $\delta_b = \delta \rho_b/\rho_b$ and $\delta_S = \delta \rho_S/\rho_S$ denote the baryon and scalar field fluctuations of the neutral scalar fields $G, \omega$ and $\mu$, respectively.
For an imperfect fluid with energy density $\rho$ and pressure $p$, the conservation law for the standard energy-momentum tensor is $\nabla_\alpha T^\alpha_\beta = 0$, which leads to the first order perturbation equation for the components $\nabla_\alpha T^\alpha_0 = 0$:

$$
\delta \rho' + 3H(\delta \rho + \delta p) - 3(\rho + p)\Phi' + a(\rho + p)\partial_i u^i = 0,
$$

(68)

where $'\,$denotes differentiation with respect to $\eta$, $u^i$ denotes the spatial velocity for $i = 1, 2, 3$. Another equation is obtained from the components of conservation, $\nabla_\alpha T^\alpha_i = 0$:

$$
\frac{1}{a^3}[a^5(\rho + p)\partial_i u^i]' - \frac{4}{3}\eta_{\text{vis}}\Delta \partial_i u^i + \Delta \delta p + (\rho + p)\Delta \Phi = 0,
$$

(69)

where $\eta_{\text{vis}}$ is the shear viscosity. These two equations hold separately for the baryon-radiation fluid and the neutral phion BEC fluid.

For the phion condensate density $\rho_\phi$, the classical pressure $p$ and the shear viscosity $\eta_{\text{vis}}$ are equal to zero. From (68) and $\rho_\phi a^3 = \text{const}$., we obtain

$$
(\delta \phi - 3\Phi)' + a\partial_i u^i = 0.
$$

(70)

The dissipation caused by photon pressure and shear viscosity $\eta_{\text{vis}}$ cannot be neglected for the baryon-radiation fluid and leads to Silk damping erasure of the baryon-radiation perturbations at recombination.

For non-relativistic baryons, the energy conservation law, $\nabla_\alpha T^\alpha_0 = 0$, reduces to a conservation of total baryon number and we obtain from (68):

$$
(\delta b - 3\Phi)' + a\partial_i u^i = 0.
$$

(71)

The equation for the perturbations in the radiation component, $\delta_\gamma = \delta \rho_\gamma / \rho_\gamma$, is given by

$$
(\delta_\gamma - 4\Phi)' + \frac{4}{3}a\partial_i u^i = 0.
$$

(72)

The tightly coupled baryon-photon fluid moves with a single velocity, and we get

$$
\frac{\delta s}{s} = \frac{3}{4}\delta_\gamma - \delta b = \text{const.},
$$

(73)

where $\delta s / s$ denotes the fractional entropy fluctuations in the fluid. Adiabatic perturbations give $\delta s = 0$ and $\delta b = (3/4)\delta_\gamma$.

The speed of sound in the baryon-photon fluid is given by

$$
c_s^2 \equiv \frac{\delta p}{\delta \rho} = \frac{\delta p_\gamma}{\delta \rho_\gamma + \delta \rho_b} = \frac{1}{3}\left(1 + \frac{3}{4}\frac{\rho_b}{\rho_\gamma}\right)^{-1}.
$$

(74)

The shear viscosity $\eta_{\text{vis}}$ is given by

$$
\eta_{\text{vis}} = \frac{4}{15}\rho_\gamma \tau_\gamma,
$$

(75)
where $\tau_\gamma$ is the photon mean-free time.

The dissipation scale for co-moving wave-number is

$$k_D(\eta) = \left(\frac{2}{5} \int_0^\eta d\eta c_s^2 \frac{\tau_\gamma}{a}\right)^{-1/2}. \quad (76)$$

We see from this equation that the viscosity damps the perturbations for co-moving scales $\lambda \leq 1/k_D$. For $c_s^2 = 1/3$ and assuming instantaneous recombination, the dissipation scale is given by [16]:

$$\frac{1}{k_D\eta_r} \sim 0.6(\Omega_\phi)^{1/4}(\Omega_b)^{-1/2}(z_r)^{-3/4}, \quad (77)$$

where $\eta_r$ and $z_r$ denote the time and redshift at recombination, respectively.

### 9 Correlation Function and Multipoles

The correlation function for the temperature differences over the sky for a given angle $\theta$ is

$$C(\theta) = \left\langle \frac{\delta T}{T_0}(\hat{n}_1) \frac{\delta T}{T_0}(\hat{n}_2) \right\rangle, \quad (78)$$

where $\langle .. \rangle$ denotes the average over all $\hat{n}_1$ and $\hat{n}_2$ with $\hat{n}_1 \cdot \hat{n}_2 = \cos \theta$ and the monopole and dipole contributions have been subtracted. We write the correlation function in the form

$$C(\theta) = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l + 1) C_l P_l(\cos \theta), \quad (79)$$

where

$$C_l = \frac{2}{\pi} \int k^2 dk \left| \left( \frac{\Phi(\eta_r) + \delta_k(\eta_r)}{4} \right) j_l(k\eta_0) - \frac{3\delta_k'(\eta_r) j_l(k\eta_0)}{4k} \right|^2, \quad (80)$$

and $\eta_0$ denotes the present conformal time. The $P_l(\cos \theta)$ and $j_l(k\eta)$ are the Legendre polynomials and spherical Bessel functions, respectively. In terms of spherical harmonics $C_l = \langle |a_{lm}|^2 \rangle$ and for $\theta < 1$ the dominant contribution is for $l \sim 1/\theta$.

In the standard calculations of the CMB power spectrum, the results depend on the various cosmological parameters. The generic inflation prediction is, $|\Phi_k^2 k^3| = B k^{n_s - 1}$, with $1 - n_s \sim 0.03 - 0.08$. The amplitude $B$ is not predicted and has to be fitted to the observations and we assume a flat spectrum $n_s = 1$. The total density of matter is given in our modified gravity theory by

$$\Omega_m = \Omega_b + \Omega_\nu + \Omega_\phi + \Omega_S, \quad (81)$$

where

$$\Omega_i = \frac{8\pi G_{\text{ren}} \rho_i}{3H^2}. \quad (82)$$

Here, $i$ denotes the contributions from the baryons, neutrinos, phions and scalar field components. We have assumed that the gravitational constant has reached a constant renormalized value (66) before recombination.
10 Calculation of the Acoustic Oscillation Spectrum

Mukhanov [16] has obtained an analytical solution to the amplitude of fluctuations for $l \gg 1$:

\[ l(l + 1)C_{l} \sim \frac{B}{\pi} (O + N). \]  

(83)

Here, $O$ denotes the oscillating part of the spectrum, while the non-oscillating contribution can be written as the sum of three parts

\[ N = N_1 + N_2 + N_3. \]  

(84)

The oscillating contributions can be calculated from the formula

\[ O \sim \sqrt{\frac{\pi}{r_h l}} \left[ A_1 \cos \left( lr_p + \frac{\pi}{4} \right) + A_2 \cos \left( 2lr_p + \frac{\pi}{4} \right) \right] \exp \left( -\frac{(l/l_s)^2}{2} \right), \]  

(85)

where $r_h$ and $r_p$ are parameters that determine predominantly the heights and positions of the peaks, respectively. The $A_1$ and $A_2$ are constant coefficients given in the range $100 < l < 1200$ for $\Omega_b \ll \Omega_\phi$ by

\[ A_1 \sim 0.1 \xi \frac{(P - 0.78)^2 - 4.3}{(1 + \xi)^{1/4}} \exp \left( \frac{1}{2} (l_s^2 - l_f^2) \right), \]  

(86)

\[ A_2 \sim 0.14 \frac{(0.5 + 0.36P)^2}{(1 + \xi)^{1/2}}, \]  

(87)

where

\[ P = \ln \left( \frac{I}{200(\Omega_\phi)^{1/2}} \right), \]  

(88)

and $I$ is given by the ratio

\[ \frac{\eta_x}{\eta_0} \sim \frac{I}{z_x^{1/2}} = 3 \left( \frac{\Omega_\Lambda}{\Omega_\phi} \right)^{1/6} \left( \int_{\eta_0}^{\eta_x} \frac{dx}{(\sinh x)^{3/2}} \right)^{-1} \frac{1}{z_x^{1/2}}. \]  

(89)

Here, $\eta_x$ and $z_x$ denote a time and a redshift in the range $\eta_0 > \eta_x > \eta_r$ when radiation can be neglected and $y = \sinh^{-1}(\Omega_\Lambda/\Omega_\phi)^{1/2}$. To determine $\eta_x/\eta_0$, we use the exact solution for a flat dust-dominated universe with a cosmological constant $\Lambda$:

\[ a(t) = a_0 \left( \sinh \left( \frac{3}{2} H_0 t \right) \right)^{2/3}, \]  

(90)

where $a_0$ and $H_0$ denote the present values of $a$ and the Hubble parameter $H$. A numerical fitting formula gives [16]:

\[ P \sim \ln \left( \frac{l}{200(\Omega_\phi^{0.09})(\Omega_\phi)^{1/2}} \right), \quad r_p = \frac{1}{\eta_0} \int dp_c(\eta). \]  

(91)
Moreover,
\[ \xi = \frac{1}{3c_s^2} - 1 = \frac{3}{4} \left( \frac{\rho_b}{\rho_r} \right), \]  
(92)
where
\[ c_s(\eta) = \frac{1}{\sqrt{3}} \left[ 1 + \xi \left( \frac{a(\eta)}{a(\eta_r)} \right) \right]^{-1/2}. \]  
(93)
For the matter-radiation universe:
\[ a(\eta) = \bar{a} \left( \eta \eta_r \right)^2 + 2 \eta \eta_r, \]  
(94)
where for radiation-matter equality \( z = z_{eq} \):
\[ \frac{z_{eq}}{z_r} \sim \left( \frac{\eta_r}{\eta_s} \right)^2 + 2 \left( \frac{\eta_r}{\eta_s} \right), \]  
(95)
and \( \eta_{eq} = \eta_s(\sqrt{2} - 1) \) follows from \( \bar{a} = a(\eta_{eq}) \).

The speed of sound at recombination depends only on the baryon density, determining the deviation from its value in a purely ultra-relativistic medium, and it can be expressed as \( c_s^2 = 1/3(1 + \xi) \).

The \( l_f \) and \( l_s \) in (86) denote the finite thickness and Silk damping scales, respectively, given by
\[ l_f^2 = \frac{1}{2\sigma^2} \left( \frac{\eta_0}{\eta_r} \right)^2, \quad l_s^2 = \frac{1}{2(\sigma^2 + 1/(k_D \eta_r)^2)} \left( \frac{\eta_0}{\eta_r} \right)^2, \]  
(96)
where
\[ \sigma \sim 1.49 \times 10^{-2} \left[ 1 + \left( 1 + \frac{z_{eq}}{z_r} \right)^{-1/2} \right]. \]  
(97)
For the non-oscillating parts, we have
\[ N_1 \sim 0.063 \xi^2 \left[ P - 0.22(l/l_f)^{0.3} - 2.6 \right] \frac{1}{1 + 0.65(l/l_f)^{1.4}} \exp(-l/l_f)^2, \]  
(98)
\[ N_2 \sim \frac{0.037}{(1 + \xi)^{1/2}} \left[ P - 0.22(l/l_s)^{0.3} + 1.7 \right] \frac{1}{1 + 0.65(l/l_f)^{1.4}} \exp(-l/l_s)^2, \]  
(99)
\[ N_3 \sim \frac{0.033}{(1 + \xi)^{3/2}} \left[ P - 0.65(l/l_s)^{0.55} + 2.2 \right] \frac{1}{1 + 2(l/l_s)^2} \exp(-l/l_s)^2. \]  
(100)
Mukhanov's formula for the oscillating spectrum is given by
\[ C(l) \equiv \frac{l(l + 1)C_l}{[l(l + 1)C_l]_{low}} = \frac{100}{9} (O + N), \]  
(101)
where we have normalized the power spectrum by using for a flat spectrum with a constant amplitude \( B \):
\[ [l(l + 1)C_l]_{low} = \frac{9B}{100\pi}. \]  
(102)
We adopt the parameters
\[ \Omega_b \sim 0.04, \quad \Omega_\phi \sim 0.3, \quad \Omega_{\Lambda} \sim 0.7. \] (103)

We shall not attempt to separate the degeneracy of the parameter \( h \) in the Hubble expansion parameter: \( H = 100 \, h \, \text{km/Mpc/sec} \) from the parameters \( \Omega_b, \Omega_\phi \) and \( \Omega_{\Lambda} \), for this requires very accurate CMB power spectrum data, which is not available at this time. We adopt in the following the value \( h \sim 0.71 \). Moreover, we shall not attempt accurate estimates of the parameters \( r_h \) and \( r_p \). It is important to emphasize that the positions and heights of the peaks depend sensitively on the values of the cosmological parameters \( \Omega_b, \Omega_\phi \) and \( \Omega_{\Lambda} \). Indeed, a fit to the CMB WMAP, Archeops and Boomerang data requires that \( \Omega_b < \Omega_m \sim \Omega_\phi < \Omega_{\Lambda} \) and that the universe is spatially flat with \( \Omega_b + \Omega_\phi + \Omega_{\Lambda} = 1 \) [11, 12, 13, 41, 42).

The oscillating power spectrum is obtained from the formula:
\[
C(l) = \frac{1}{l^{7/2}} [119.11(0.053((\ln(0.0096l) - 0.78)^2 - 4.3) \exp(2.13 \times 10^{-7}l^2) \cos(0.01l + \frac{\pi}{4})
+ 0.14(0.5 + 0.36 \ln(0.0096l))^2 \cos(0.021l + \frac{\pi}{4})) \exp(-8.26 \times 10^{-7}l^2]
+ \frac{0.25(\ln(0.0096l) - 0.024l^{0.3} - 2.6)^2 \exp(-4.01 \times 10^{-7}l^2)}{1 + 5.59 \times 10^{-5}l^{1.4}}
+ \frac{0.28(\ln(0.0096l) - 0.027l^{0.3} + 1.7)^2 \exp(-8.26 \times 10^{-7}l^2)}{1 + 5.59 \times 10^{-5}l^{1.4}}
+ \frac{0.18((\ln(0.0096l) - 0.011l^{0.55} + 2.2)^2 \exp(-8.26 \times 10^{-7}l^2)}{1 + 3.55 \times 10^{-6}l^2}. \] (104)

The fluctuation spectrum determined by the analytical formula, Eq.(104), is displayed in Fig. 1 for the choice of cosmological parameters given in (103).

Powerspectrum
Figure 1. The solid line shows the result of the calculation of the power spectrum acoustical oscillations: $C(l)$, and the $\odot$s correspond to the WMAP, Archeops and Boomerang data in units $\mu K^2 \times 10^{-3}$ [11,12,13].

Our predictions for the CMB power spectrum for large angular scales corresponding to $l < 100$ will involve the integrated Sachs-Wolfe contributions obtained from the gravitational potential. These predictions should be calculated using a computer code and will be investigated in a future article.

11 Conclusions

We have demonstrated that a modified gravity theory [2] can lead to a satisfactory fit to the acoustical oscillation spectrum obtained in the WMAP data [11] by employing the analytical formula for the fluctuation spectrum derived by Mukhanov [16]. The vector field $\phi^\mu$ (phion) and the scalar fields $G$, $\omega$ and $\mu$ occur as new degrees of freedom in the action of the MOG theory. The imperfect fluid before recombination consists of two components, the baryon-photon fluid and the neutral, non-relativistic phion BEC fluid. The latter low-temperature, quantum state condensate fluid has zero classical pressure and shear viscosity.

The analytical formula of Mukhanov [16] reproduces approximately the correct peak locations and peak heights in the power spectrum data for the parameters: $\Omega_b \sim 0.04$, $\Omega_m \sim \Omega_\phi \sim 0.3$, $\Omega_\Lambda \sim 0.7$ and $H = 71$ km/Mpc/sec. The varying constants $G$, $\omega$ and $\mu = 1/r_0 = 1/\lambda$ are assumed to have reached their renormalized constant values before and shortly after recombination. The analytical power spectrum fluctuations are accurate enough to verify that our MOG gives a reasonable description of the CMB spectrum for small angular values.

In the period before recombination and in cosmology at large inter-galactic scales the quantum BEC phion energy density dominates the matter density. The BEC fluid has zero classical pressure and zero viscosity, whereby the phion field perturbations as well as the scalar field perturbations are not subject to the Silk dissipation that erases the baryon-photon perturbations. The BEC fluid can describe a uniform density distribution that does not couple to radiation and can become unstable due to the attractive gravitational field and clump to form the seeds of galaxies well before recombination, allowing enough time as the universe expands to form clusters and super-clusters of galaxies.

An important problem to investigate is whether an N-body simulation calculation based on our phion BEC scenario can predict the observed large scale galaxy surveys. The formation of BEC proto-galaxy structure before and after the epoch of recombination and the growth of galaxies and clusters of galaxies at later times in the expansion of the universe has to be explained. The BEC condensates are not, perhaps, localizable as particles in the sense of heavy WIMPS or axions.
For local late-time bound systems such as galaxies and clusters of galaxies the symmetry breaking is relaxed and the phion condensates can become ultra-light and relativistic. Ordinary baryonic matter and neutral hydrogen and helium gases now constitute the dominant form of matter. The extra degree of freedom in MOG associated with the vector field $\phi^\mu$ and the spatial variation of Newton’s $G$ modifies for late-time bound systems the Newtonian acceleration law for weak gravitational fields. A calculation of the classical limit of the non-linear Gross-Pitaevskii equation for the condensate wave function leads to an effective, classical modified Newtonian acceleration law. The rotational velocity curves of spiral galaxies are flattened, because of the altered dynamics of the gravitational field at the outer regions of the galaxies and not because of the presence of a dominant dark matter halo.

The neutral phion BEC fluid dominates the cosmological density at large cosmological scales, and the phion condensate in cosmology has taken on the role of a light, cold “dark matter” fluid. This allows for a fitting of the acoustical fluctuation spectrum at small angular scales. According to our MOG scenario, we do not expect in the present universe to detect heavy WIMPS such as the supersymmetric neutralinos in accelerators or in underground experiments. The dual role played by the phion particle in describing galaxies and the large-scale structure of the universe is a generic feature of our MOG theory.

We have succeeded in fitting in a unified picture a large amount of data over 16 orders of magnitude in distance scale from Earth to the surface of last scattering some 13.7 billion years ago, using our modified gravitational theory. The data fitting ranges over four distance scales: the solar system, galaxies, clusters of galaxies and the CMB power spectrum data at the surface of last scattering.

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