A model of new information dissemination in the society: important area in the parameter space

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Abstract. A basic mathematical model of new information dissemination in the society is constructed and studied in some of the important cases. The suggested model has been described using the system of four ordinary differential equations with quadratic nonlinearity in the right parts. Two stationary solutions which provide a quite logical interpretation for this system were found. The important area with interesting properties of stationary solutions was separated in the parameters’ space of the system. The global properties of a phase portrait of the constructed dynamic system were investigated by qualitative methods of the differential equations theory. The obtained results allowed to find one of possible scenarios of new information dissemination in the society.

1. Introduction
In [1] the authors built up a mathematical model for the dissemination of new information in society:

\[
\begin{align*}
\frac{dN}{dt} &= \beta N - \gamma AN, \\
\frac{dC}{dt} &= \alpha AN - \mu(C - C_0), \\
\frac{dA}{dt} &= \rho C - \eta\gamma AN - \lambda A, \\
\frac{di}{dt} &= \sigma N - \omega i.
\end{align*}
\]

(1)

The proposed mathematical model, of course, is a very generalized and will require further detailing. But already in this form, it allows to link the factors selected for the promotion of news information into a certain system and can be useful for general picture studying. While constructing, it was considered that the main factors for the dissemination of new information were the following values, depending on the time \( t \):

– \( N(t) \) (from the News) is the amount of news information (messages of various kinds), contributing to the spread of a new concept in society (or a segment of society);

– \( C(t) \) (from the Censorship) is the number of bodies with their information resources in the structure of society (or a segment of society), interested in preserving previously adopted concepts;
– $A(t)$ (from the Alternative view) is the amount of information (messages of various kinds) that prevents the spread (including on behalf of the censorship bodies) of a new concept in society (or a segment of society);
– $i(t)$ (from the index) is the relative characteristic of the acceptance of a new concept at time $t$,

$$i = 1 - \frac{I'}{I},$$

where $I,\%$, is the characteristic of complete acceptance of established positions in society before the start of observations; $I',\%$, is the corresponding characteristic of the acceptance of established positions with spreading new views in the media.

Non-negative parameters $\beta, \gamma$ characterize the intensity of information dissemination through the media and the likelihood of neutralizing the effect of the message by presenting an alternative point of view, respectively. The non-negative parameter $\alpha$ characterizes the reaction to the intensity of alternative points of view confrontation, the positive parameter $\mu$ is the coefficient equal to the reciprocal of the time of functioning of the additionally created bodies (it is assumed that in the social environment the number of administrative resources is always used to support their concepts). The parameter $\rho \geq 0$ describes the average rate of news appearance from one source of information $C$, and $\eta \geq 0$ – the average amount of news information $A$ to neutralize the effect of the message $N$. The parameter $\lambda > 0$ – a coefficient inversely proportional to the time of forgetting information $A$. The parameter $\sigma > 0$ characterizes the rate of acceptance of a new idea, $\omega \geq 0$ – the coefficient of restoration of acceptance of the old concept.

Using [2-5], it is shown that system (1) possesses the properties of uniqueness, unbounded extensibility of solutions and their continuous dependence on parameters. Also, for this system, the invariance of the set is proved

$$R^4 = \{(N, C, A, i) \in R^4 : N \geq 0, C \geq 0, A \geq 0, i \geq 0\}.$$

Two stationary solutions were found that allow a completely logical interpretation [1]:

$$X_{1,i} = (N_{1,i}, C_{1,i}, A_{1,i}, i_{1,i}) = \left(0, C, \frac{\rho C}{\lambda}, 0\right),$$
$$X_{2,i} = (N_{2,i}, C_{2,i}, A_{2,i}, i_{2,i}),$$
$$N_{2,i} = \frac{\mu(\lambda \beta - \gamma P C_i)}{\beta(\alpha \rho - \mu \eta \gamma)}, C_{2,i} = \frac{a \lambda \beta - \eta \mu \gamma^2 C_i}{\gamma(\alpha \rho - \mu \eta \gamma)},$$
$$A_{2,i} = \frac{\beta}{\gamma}, i_{2,i} = \frac{\sigma \mu(\lambda \beta - \gamma P C_i)}{a \beta(\alpha \rho - \mu \eta \gamma)}.$$

In the system parameter space two areas are highlighted, in of which $X_{i,0} \in R^4, i = 1,2$, but at the same time have significantly different properties:

$$\Omega_1 : \left\{ \frac{\gamma P C_i > \lambda \beta}{\mu \eta \gamma > \alpha \rho} \right\}, \quad \Omega_2 : \left\{ \frac{\gamma P C_i < \lambda \beta}{\mu \eta \gamma < \alpha \rho} \right\}.$$

Using qualitative methods of differential equation theory, the global properties of the phase portrait of the constructed dynamic system were studied. This made it possible to identify several possible scenarios for the dissemination of new information in society.
This paper examines the properties of system (1) solutions in the field of parameters
\[ \Lambda_1 : \begin{cases} \gamma p_C < \lambda \beta \\ \mu \eta \gamma > \alpha \rho \end{cases} \]
which is of particular interest to the application. There is only one stationary solution in this parameter area in \( R^4 : X_{st} = (N_{st}, C_{st}, A_{st}, i_{st}) = \left(0, C_*, \frac{\rho C_*}{\lambda}, 0\right)\).

Note. It should be considered that the variable \( i(t) \) appears only in the last equation of system (1), so in the future it makes sense to conduct studies only for the system of three equations, which we will rewrite in a more convenient form for research:

\[
\begin{align*}
\frac{dC}{dt} &= \alpha AN - \mu (C - C_*), \\
\frac{dA}{dt} &= \rho C - (\lambda + \eta \gamma) A, \\
\frac{dN}{dt} &= (\beta - \gamma A) N, 
\end{align*}
\]

then apply the conclusions and results to the variable \( i(t) \).

It is easy to show that the set \( R^3 = \{(C, A, N) \in R^3 : C \geq 0, A \geq 0, N \geq 0\} \) for this system is invariant one and contains only one, and unstable, stationary-point \( X_{st} = (C_*, A_*, N_*) = \left(C_*, \frac{\rho C_*}{\lambda}, 0\right)\). \( X_{st} \) can be interpreted as the state of a society where a certain concept dominates (for example, ideological or technological). To support it, an administrative resource in quantity \( C_* \) is used with required amount of information \( \frac{\rho C_*}{\lambda} \) in mass media.

As the system (2) is autonomous, the initial hypothesis can be written as follows:

\[ C(0) = C_0 \geq 0, A(0) = A_0 \geq 0, N(0) = N_0 \geq 0. \] (3)

2. Model Analysis (2), (3) in Parameter Area \( \Lambda_1 \)

Studying a three-dimensional system (2), (3), the properties of an auxiliary two-dimensional system of differential equations are significantly used:

\[
\begin{align*}
\frac{dA}{dt} &= \rho C - (\lambda + \eta \gamma) A, \\
\frac{dN}{dt} &= (\beta - \gamma A) N, 
\end{align*}
\]

received from the system (2) with \( \alpha = 0 \) and \( C(t) = C_* \) with \( t \geq 0 \).

Interpretation. The system (4) simulates a situation when, with the information “stuffing” of a new idea into society, the information protection authorities do not respond to it, as they believe that the previously necessary amount of administrative resource is enough to support the usual provisions and neutralize the reaction to the appearance of new information in the media.

The system (4) in parameter area \( \Lambda_1 \) has in invariant set \( R^2 = \{(A, N) \in R^2 : A \geq 0, N \geq 0\} \) only one stationary solution \( X_{st} = (A_*, N_*) = \left(\frac{\rho C_*}{\lambda}, 0\right)\), that is saddle. Known methods of qualitative analysis of two-dimensional systems of differential equations [6] allow to build up a phase portrait and study...
the behavior of the trajectories of the system (4) (figure 1). As can be seen from the figure, all trajectories of the system (4) with initial conditions $A(0) = A_0 \geq 0, N(0) = N_o > 0$ with $t \to +\infty$ have similar behavior: $A(t) \to 0, N(t) \to +\infty$.

Figure 1. Phase portrait of the system (4) in $R^2 = \{(A, N) \in R^2 : A \geq 0, N \geq 0\}$

Let’s show, that in $R^3^* = \{(C, A, N) \in R^3 : C \geq 0, A \geq 0, N \geq 0\}$ the system (2), (3) has similar phase portrait in characteristics.

Let $R^* = \{(C, A, N) \in R^3 : N > 0\}, \partial R^* = \{(C, A, N) \in R^3 : N = 0\}$. Mark for arbitrary solution $X(t) = (C(t), A(t), N(t))$ of system (2), (3) $A^*(X) - \omega$ - limit set of the solution [7].

**Lemma.** For all trajectories of the system (2), starting in $R^+$, the set $A^* \cap \partial R^*$ is empty.

**Proof.** The set $\partial R^*$ – invariant due to the system (2), (3). In fact, if $X_o = (C_o, A_o, N_o) \in \partial R^+$, then the system (2) is determined by linear equations

$$
\frac{dC}{dt} = -\mu(C - C_o), \quad \frac{dA}{dt} = \rho C - \lambda A, \quad N(t, X_o) = 0,
$$

where the stationary point $X_{\omega}$ – globally uniformly asymptotically stable in $\partial R^+$. Suppose, that the set $A^* \cap \partial R^*$ is not empty. Then there is a trajectory $X(t, X_o)$ of the system (2) such that, if $X(t) \in R^*$ then it follows that $X(t, X_o) \to \partial R^*$. Based on the theorem about continuous dependency of the system (2), (3) solutions from initial data [8] $X(t, X_o) \to X_{\omega}$ with $t \to +\infty$. But it may not be, because $X_{\omega}$ – unstable permanent unit of the system (2), (3). Thus, the lemma is proved.

**Theorem 1** All trajectories of the system (2) starting in $R^+$ are unlimited.

**Proof.** Suppose the opposite. Let with $X_o \in R^*$ the trajectory $X(t, X_o)$ is limited. Then consider the Lyapunov function:

$$
V(X, t) = \gamma AN - \beta N - \int_0^t \dot{A} \cdot N \, d\tau.
$$

Its derivative due to system (2) effect is following:

$$
\frac{dV}{dt} = \gamma AN - \beta N - \int_0^t \dot{A} \cdot N \, d\tau.
$$

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\[
\dot{V}(X,t) = \gamma \dot{N}A + \gamma \dot{A}N - \beta \dot{N} - \gamma \dot{A}N = \dot{N}(\gamma A - \beta) = -(\beta - \gamma)^2 N \leq 0.
\]

Prove the limitations of the function \(V(X,t)\) from below. Due to the assumed limited trajectories of the system, the term \(\gamma AN - \beta N\) is limited from below. On a set where \(\dot{A} < 0\) the last term of the function \(V(X,t)\) is positive. On a set where \(\dot{A} > 0\) pointed term can be assessed in this way:

\[
-\gamma \int_0^\tau \dot{A} N d\tau \geq -\gamma \int_0^\tau \gamma N_{max} (A(t) - A(0)) \geq -\gamma N_{max} A_{max} + \gamma N_{max} A(0).
\]

Therefore, the function \(V(X,t)\) is bounded from below. Obviously, the derivative \(\dot{V}(X,t)\) will also be bounded from below. Thus, according to the statement of the VIII.4.7 in work [7], it can be argued that \(\dot{V}(X,t) \to 0\) with \(t \to +\infty\).

This means that the trajectory of the system tends to its \(\omega\)-limit set 
\[
\Lambda^+ \subset M = \{(C, A, N) \in R^3 : A = \frac{\beta}{\lambda}, \quad N = 0\}.
\]

On property \(\omega\)-limit sets for autonomous systems, \(\Lambda^+\) is invariant due to the system (2). But on the plane \(A = \frac{\beta}{\gamma}\) is not in \(R^+\) invariant due to (2) sets. On the plane \(N = 0\), according to lemma, there can also be no points from \(\omega\)-limit set. We got a contradiction that proves the theorem.

Select in \(R^+\) two sets (figure 2):

\[
H_1 = \{(C, A, N) \in R^+ : A \leq \frac{\beta}{\lambda}\}, \quad H_2 = \{(C, A, N) \in R^+ : A > \frac{\beta}{\lambda}\}.
\]

Consider in \(H_1\) surfaces where the values \(\dot{C}(t), \dot{A}(t), \dot{N}(t)\) are respectively are zero:

\[
N = \frac{\mu(C - C_*)}{\alpha A}, \quad \text{(6)}
\]

\[
N = \frac{\beta C}{\eta \gamma A - \frac{\lambda}{\eta}}, \quad \text{(7)}
\]

\[
A = \frac{\beta}{\gamma}. \quad \text{(8)}
\]

Evaluate the mutual positions of the surfaces (6) and (7), having previously determined their projections on the plane \(N = 0\) (figure 2):

\[
C = C_* \text{ for (6) and } C = \frac{\lambda A}{\rho} \text{ for (7)}.
\]

At the intersection of these lines is a stationary solution \(X_* = (C_*, A_*, N_*) = \left(C_*, \frac{\beta C_*}{\lambda}, 0 \right)\).
Figure 2. $R_{1} = \{(C, A, N) \in R^{3} : C \geq 0, A \geq 0, N \geq 0\}$ for Parameter area $\Lambda_{1}$

On any section with plane $A = \tilde{A}$ the surfaces (6) and (7) respectively have the form:

\[
N = \frac{\mu C}{\alpha \tilde{A}} - \frac{\mu C_{c}}{\alpha \tilde{A}}, \quad (9)
\]
\[
N = \frac{\rho C}{\eta \gamma \tilde{A}} - \frac{\lambda}{\eta \gamma}. \quad (10)
\]

For parameters from $\Lambda_{1}$, the coefficient with $C$ of the line (9) is greater than the corresponding coefficient of the line (10), as from the inequality $\mu \eta \gamma > \alpha \rho$ after dividing for $\alpha \eta \gamma \tilde{A}$ it follows that

\[
\frac{\mu}{\alpha \tilde{A}} > \frac{\rho}{\eta \gamma \tilde{A}}.
\]

Note. Figure 3(a, b, c) allows visually evaluate the mutual positions of the surfaces (6), (7) in $H_{1}$. It will be used in the proof of the following theorem.
Figure 3 a. The surfaces (6), (7) with $0 < A < A_{n}$

Figure 3 b. The surfaces (6), (7) with $A = A_{n}$
Theorem 2. Let $X(t, X_0) = (C(t, X_0), A(t, X_0), N(t, X_0))$ – the solution of the system (2), (3) in parameter space $\Lambda_1$ in $R^+$. Then with $t \to +\infty$ component $A(t) \to 0$, and $N(t) \to +\infty$.

Proof. Demonstrate the proof in three steps.

The first step. Show that any solution $X(t) = (C(t), A(t), N(t))$ from $H_2$ a finite period of time falls into $H_1$. In fact, with $A > \frac{B}{\gamma}$ from the third equation of the system (2) it follows that $\hat{N}(t) < 0$. Then, remaining in $H_2$, the solution $X(t)$ in finite time falls into a fairly small neighborhood of the plane $N = 0$. But on this plane, all solutions of the system (2) tend to the stationary solution $X_{st}$, with $t \to +\infty$ for which $A_{st} = \frac{\rho C}{\lambda} < \frac{B}{\gamma}$ in $\Lambda_1$. Therefore, the theorem of continuous dependence on initial data [8] guarantees the getting of any solution of the system (2) from $H_2$ in $H_1$ for a finite period of time.

The second step. Obviously, solutions $X(t)$ come from $H_2$ in $H_1$, where $\hat{N}(t) > 0$, through the part of the plane (8) $A = \frac{B}{\gamma}$, on which $\hat{A} < 0$ (figure 2). Now show that from the part of the set $H_1$, where $\hat{A} < 0$, with $t \to +\infty$ component $A(t) \to 0$, and $N(t) \to +\infty$.

Introduce two functions: $V_1(X) = \dot{A}$ and $V_2(X) = \dot{C}$. It is easy to check using the ratios (6), (7) that in $H_1$ on the surface $V_1(X) = 0$ $V_2(X) < 0$ if $A_{st} < A \leq \frac{B}{\gamma}$ (figure 3 b, c). Therefore, due to the system

(2) $\dot{V}_1(X)|_{X:V_1(X)=0} = \dot{A} = \rho \dot{C} - \eta \gamma \hat{N} A < 0$. Therefore, the solution of the system from the surface

Figure 3 c. The surfaces (6), (7) with $A_{st} < A \leq \frac{B}{\gamma}$.
\( \dot{A} = 0 \) falls into the area where \( \dot{A} < 0 \). Only at some values \( 0 < A < A_{\text{tr}} \), the solution can from the surface \( \dot{A} = 0 \) fall into the area where \( \dot{A} > 0 \) (bold line in figure 3a). Starting to increase, component \( A(t) \) will be able to increase to a value \( A = A_{\text{tr}} \) only provided that \( \dot{C} < 0 \) with \( A = A_{\text{tr}} \) (figure 3b). But for this, it is required to cross the surface \( V(X) = \dot{C} = 0 \). But on this surface due to the system (2) \[ \dot{V}(X) = \dot{C} = \alpha \dot{A} \dot{N} + \alpha NN' > 0 \]. Therefore, as \( \dot{N} > 0 \) in \( H_1 \), the component \( A(t) \) through the surface (7) will again fall into the area where \( \dot{A} < 0 \), and therefore will begin to decline. Thus, with \( t \to +\infty \) component \( A(t) \to 0 \), and \( \dot{N}(t) \to +\infty \).

The third step. The solution \( X(t, X_0) \), starting in the part \( H_1 \), where \( \dot{A} > 0 \) and \( \dot{C} < 0 \), either falls through the plane \( A = \frac{\beta}{\gamma} \) in \( H_2 \), or on the surface \( \dot{A} = 0 \), that is \( V(X) = 0 \). But in the first case, as shown in the previous steps, \( A(t) \to 0 \), and \( N(t) \to +\infty \) with \( t \to +\infty \). In the second case, due to the system (2) we have \[ \dot{V}(X) \bigg|_{X, V(X) = 0} = \dot{A} = \dot{\beta} \dot{C} - \eta \dot{N} N' A < 0 \]. Therefore, we get to the part of the area \( H_1 \), where \( \dot{A} < 0 \), due to the fact that \( \dot{N} > 0 \), again \( A(t) \to 0 \), and \( N(t) \to +\infty \) with \( t \to +\infty \). This completes the proof.

Interpretation. With this ratio of system parameters (2), the results of mathematical research make it possible to conclude that society is potentially ready to accept new ideas and views. Any appearance in the media of other views and opinions will find the support of the audience. In this case, the address audience completely changes the previously dominant concept.

3. Conclusion

The study allows us to formulate the following results.

1. The global properties of the phase portrait of the constructed dynamic system in the field of parameters \( \Lambda \) were studied using qualitative analysis methods.

2. An interpretation of the main result of the study is given as the readiness of society to completely change the dominant concept (for example, ideological or technological).

The authors consider the results obtained in this work to be a continuation of the systemic study set forth in the works [1, 9 – 11]. This research program is aimed at studying the media system as one of the most relevant and high-speed dynamic systems. The use of mathematical methods makes it possible to conduct media research more deeply and at a new scientific level. And turning to methods of nonlinear dynamics allows to study the structure and properties of processes in a system such as the media in details.

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