RATCHET EFFECTS IN LUTTINGER LIQUIDS

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Abstract We investigate a one-dimensional electron liquid with two point scatterers of different strength. In the presence of electron interactions, the nonlinear conductance is shown to depend on the current direction. The resulting asymmetry of the transport characteristic gives rise to a ratchet effect, i.e., the rectification of a dc current for an applied ac voltage. In the case of strong repulsive interactions, the ratchet current grows in a wide voltage interval with decreasing ac voltage. In the regime of weak interaction the current-voltage curve exhibits oscillatory behavior. Our results apply to single-band quantum wires and to tunneling between quantum Hall edges.

Keywords: ratchets, Luttinger liquids, conductance, current rectification, bosonization, impurity scattering, Hartree approximation

1. Introduction

Systems with asymmetric transport characteristics are also known as ratchets and Brownian motors since they can be used to generate dc currents from an ac or noisy voltage. This mechanism has important
applications in physics and biology [1]. As specific examples, we mention diodes and photovoltaic current rectifiers. Here we address the possibility of realizing such effects in Luttinger liquids, which is of interest in connection with nanostructured devices. Motivated by recent experiments on quantum dots [2], carbon nanotubes [3], and quantum Hall systems [4], we examine ratchet effects caused by two unequal constrictions as point scatterers.

2. Model

We consider a one-dimensional spinless electron liquid at zero temperature subject to an impurity potential \( V(x) \) and a pair interaction \( W(x) \) with the Hamiltonian

\[
H = -\frac{\hbar^2}{2m} \int dx \: \Psi^\dagger(x) \partial_x^2 \Psi(x) + \int dx \: V(x) \Psi^\dagger(x) \Psi(x) + \frac{1}{2} \int dx \int dx' \: \Psi^\dagger(x) \Psi^\dagger(x') W(x-x') \Psi(x') \Psi(x). \tag{1}
\]

The impurity potential \( V(x) \) is chosen to describe two unequal point scatterers at a distance \( a \). In the absence of interactions, the application of a voltage \( U \) leads to a current \( I \) with a finite conductance \( G = I/U \). The conductance is proportional to \( \int_{E_F-E/2}^{E_F+E/2} dE \: T(E) \) with the transmission probability \( T(E) \) for electrons with incident energy \( E \) in the vicinity of the Fermi energy \( E_F \). As a consequence of time reversal symmetry, \( T(E) \) does not depend on the direction of the incoming momentum and therefore noninteracting electrons have a symmetric transport characteristic with an odd function \( I(U) \). Therefore, the inclusion of interactions is mandatory for the analysis of ratchet effects.

3. Bosonization

For the inclusion of interactions, the bosonized representation of the model is particularly convenient. The bosonization technique [5] maps the quantum dynamics of the electron liquid onto a path integral for a bosonic field \( \vartheta \) which essentially describes collective displacements of the electron liquid. In terms of this field, the particle density of the electrons reads

\[
\rho(x) = \frac{1}{\sqrt{\pi}} \partial_x \vartheta(x) + \frac{1}{2\pi\alpha} \left( e^{i[2k_F x + 2\sqrt{\pi} \vartheta(x)]} + \text{h.c.} \right). \tag{2}
\]

Here, \( k_F \) is the Fermi wave vector and \( \alpha \) is a microscopic cutoff length scale of the order of \( k_F^{-1} \). The particle current is given by \( j = -\frac{1}{\sqrt{\pi}} \langle \partial_t \vartheta \rangle \), and the charge current by \( I = ej \).
In terms of the field $\vartheta$, the relevant contributions to the action corresponding to the Hamiltonian (1) read

$$S = \int dt dx \left\{ \frac{\hbar}{2v_F} (\partial_t \vartheta)^2 - \frac{\hbar v_F}{2g^2} (\partial_x \vartheta)^2 - \frac{1}{2\pi \alpha} V(x) [e^{i[2k_Fx + 2\sqrt{\pi} \vartheta(x)]} + \text{h.c.}] - \frac{W_0}{\pi^{3/2} \alpha} \partial_x \vartheta(x) [e^{i[2k_Fx + 2\sqrt{\pi} \vartheta(x)]} + \text{h.c.}] \right\}. \tag{3}$$

Forward scattering by the interaction is included in the “free” bosonic theory via the Luttinger parameter $g := [1 + W_0/(\pi \hbar v_F)]^{-1/2}$ which is less than unity for repulsive interaction. Assuming that $W(x)$ is short ranged due to screening effects, only its weight $W_0 = \int dx W(x)$ is effective. The second and third lines of Eq. (3) describe backscattering off the impurity potential and by the interaction, respectively.

For a single point-like scatterer, $V(x) \simeq V_0 \delta(x)$, it was shown [6, 7, 8] that repulsive electron interactions $(g < 1)$ lead to a vanishing linear conductance. A weak scatterer was found to suppress the conductance according to

$$\Delta G \sim -V_0^2 U^{2g-2}, \tag{4}$$

where $\Delta G := G - G_0$ is conductance change due to the presence of the scatterer. On the other hand, for a strong scatterer, the conductance vanishes like $G \sim t^2 U^{2/g-2}$ with the amplitude $t$ for hopping across the scatterer. We subsequently focus on the weak-barrier case, deferring the strong currugation limit to Ref. [9].

4. Hartree picture

The suppression of the conductance in the presence of interactions can be attributed to the backscattering of electrons from a Hartree-type potential caused by Friedel oscillation in the vicinity of the impurity. In the framework of the bosonic action (3), this backscattering arises from the second-line contribution, and the third-line contribution is irrelevant for the asymptotics (4).

To illustrate the mechanism at work, it is instructive to briefly recall the scattering features of single electrons by a double barrier $V(x) = V_l \delta(x + a/2) + V_r \delta(x - a/2)$. We assume the amplitudes $V_l$ and $V_r$ to be positive. $a$ is the distance between the two barriers. Comparing the probability density $|\psi_\rightarrow(x)|^2$ for a particle incident from $x = -\infty$ with wave vector $k > 0$ to the density $|\psi_\leftarrow(x)|^2$ for a particle incident from $x = \infty$ with wave vector $-k$, one observes that the densities are not only different but also not related by reflection symmetry,
\[ |\psi_{\rightarrow}(x)|^2 \neq |\psi_{\leftarrow}(-x)|^2 \] (cf. Fig. 1). This implies that the contributions to the Hartree potential for both cases also will not display this symmetry. Therefore, in a current carrying state the effective scattering potential will depend on the current direction such that the net amount of scattering depends on the current direction. Therefore, one expects a ratchet effect from the interplay between asymmetric scattering potentials and interactions.

\[ \begin{array}{c|c}
\hline
x/a & \frac{|\psi_{\rightarrow}(x)|^2}{|\psi_{\leftarrow}(x)|^2} \\
\hline
-3 & 1 \\
-2 & 1 \\
-1 & 1 \\
0 & 1 \\
1 & 1 \\
2 & 1 \\
3 & 1 \\
\hline
\end{array} \]

\[ \begin{array}{c|c}
\hline
x/a & \frac{|\psi_{\leftarrow}(x)|^2}{|\psi_{\rightarrow}(-x)|^2} \\
\hline
-3 & 1 \\
-2 & 1 \\
-1 & 1 \\
0 & 1 \\
1 & 1 \\
2 & 1 \\
3 & 1 \\
\hline
\end{array} \]

Figure 1. Density profiles \(|\psi_{\rightarrow,-}(x)|^2\) averaged over a period \(\pi/k\) for a potential with \(V_l < V_r\). The averaged densities show drops at the positions \(x = \pm a/2\) of the scatterers. The amplitudes of the density drops depend on the direction of the incident wave.

Coming back to the electron liquid in the bosonic representation, we find it important to include the third-line contribution in the action (3). In this term, the Hartree approximation amounts to replacing \(\partial_x \vartheta\) – which is proportional to the slowly varying part of the density, cf. Eq. (2) – by its average value \(\langle \partial_x \vartheta \rangle\). This average is readily calculated from the action perturbatively to second order in \(V\). Then, the third-line contribution can be absorbed into the second-line contribution by replacing the bare potential \(V(x)\) with the effective potential \(\bar{V}(x) = V(x) + \delta V(x)\) including the correction \(\delta V(x) = 2W_0 \langle \partial_x \vartheta(x) \rangle / \sqrt{\pi}\). In analogy to the analysis [6] yielding Eq. (4) for a single scatterer, one then can calculate the corrections to conductance to second order in this effective potential.

To estimate the strength of this effect, it is important to notice that, to leading order, this correction is proportional to the backscattering current off the impurities, i.e., one expects \(\delta V \propto V^2 U^{2g-1}\) corresponding to Eq. (4). The reinsertion of this correction into Eq. (4) suggests subleading ratchet corrections to the conductance of order \(\Delta G |_{\text{ratch}} \sim V^3 U^{4g-3}\).
Performing the systematic calculation outline above, we obtain an asymmetric ratchet contribution to the conductance

$$\Delta G_{\text{ratch}} \propto W_0 U^{4g-2} \sin(2k_F a) V_l V_r (V_l - V_r) \times H_g \left( \frac{ge U a}{\hbar v_F} \right) [1 - \cos(2k_F a) H_g \left( \frac{ge U a}{\hbar v_F} \right) ]$$

(5)

with the function

$$H_g(z) = \sqrt{\pi} \frac{\Gamma(2g) J_{g-1/2}(z)}{\Gamma(g) (2z)^{g-1/2}}$$

(6)

invoking the Bessel function $J_g$.

Equation (5) was obtained within the Hartree approximation. It becomes exact in a model of many bands $i$ interacting through the coupling interaction $\kappa_{ij} = \text{const.} \int \partial_x \phi_i \cos(2xk_F + 2\sqrt{\pi} \theta_j)$. However, the order-of-magnitude estimate of the current is valid in a more general case including our one-band model (3). This can be verified by analyzing the expression for the current in the forth order of the perturbation theory.

5. Conclusions

Equation (5) is our main result. Its proportionality to $W_0$ reflects the fact that the ratchet effect vanishes in the absence of interactions. It is valid provided the backscattering current can be obtained within perturbation theory. This is the case for weak scattering, more specifically for

$$V/E_F \ll (eU/E_F)^{1-g}.$$  

(7)

Within this limit, one can distinguish two regimes.

(i) At lower voltages $ge U a \ll \hbar v_F$, $H_g(z) \approx 1$ and the effect is in agreement with the above estimate. Additional oscillating factors reflect resonances due to quantum interferences [10]. The absolute value of the ratchet current grows with decreasing voltage for $g < 1/2$.

(ii) In the high-voltage regime $ge U a \gg \hbar v_F$, $H_g(z) \sim z^{-g} \cos(z - \pi g/2)$. Then $\Delta G_{\text{ratch}} \propto U^{3g-3}$. In this regime, the ratchet current grows with decreasing voltage for $g < 2/3$.

This result shows that the ratchet effect can be increasing with decreasing voltage. At low voltages beyond the point where the condition (7) breaks down and the total current vanishes with decreasing voltage, also the ratchet current has to vanish. Nevertheless, it can give a sizeable
contribution to the total current. This explains the pronounced asym-
metry observed experimentally in corrugated nanotubes [3].

Although the Hamiltonian (1) does not describe quantum Hall sys-
tems, the bosonized form (3) captures tunneling of electron or quasipar-
ticles between edges [11]. In a fractional quantum Hall state with filling
factor \( \nu \ll 1 \), the tunneling of quasiparticles corresponds to the case
\( g = \nu \). Thus, the interesting regime with small \( g \) is physically accessible.

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