Uplink Sensing Using CSI Ratio in Perceptive Mobile Networks

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Abstract—Uplink sensing in perceptive mobile networks (PMNs), which uses uplink communication signals for sensing the environment around a base station, faces challenging issues of clock asynchronism and the requirement of a line-of-sight (LOS) path between transmitters and receivers. The channel state information (CSI) ratio has been applied to resolve these issues, however, current research on the CSI ratio is limited to Doppler estimation in a single dynamic path. This paper proposes an advanced parameter estimation scheme that can extract multiple dynamic parameters, including Doppler frequency, angle-of-arrival (AoA), and delay, in a communication uplink channel and completes the localization of multiple moving targets. Our scheme is based on the multi-element complex Taylor series of the CSI ratio that converts a nonlinear function of sensing parameters to linear forms and enables the applications of traditional sensing algorithms. Using the truncated Taylor series, we develop novel multiple-signal-classification grid searching algorithms for estimating Doppler frequencies and AoAs and use the least-square method to obtain delays. Both experimental and simulation results are provided, demonstrating that our proposed scheme can achieve good performances for sensing both single and multiple dynamic paths, without requiring the presence of a LOS path.

Index Terms—Integrated radar sensing and communication (ISAC), parameter extraction, perceptive mobile network, uplink sensing.

I. INTRODUCTION

P ERCEPTIVE Mobile Network (PMN) [1], [2] is a recently proposed next-generation mobile network based on joint radar-communication technology. The concept of PMN was proposed in [1] and then elaborated in [2]. In contrast to current communication-only mobile networks, PMNs are expected to serve as ubiquitous sensing networks while providing uncompromised mobile communication services. Integrated sensing and communication (ISAC) shows the prospect of realizing dual-function devices with reduced cost, packed size, smart functions, and uncompromised service quality. A key link facilitating this is that the communication channel state information (CSI) resembles the radar channel [3], [4].

As discussed in [2], there are three main types of sensing methods using the received communication signals in PMNs. They are named uplink sensing [5], [6], [7], [8], [9], downlink active sensing [10], [11], [12], and downlink passive sensing [13], [14]. In view of hardware cost and required facility changes, uplink sensing is the most viable way for realizing radar functions in PMNs in the near term, before the maturity of full duplex technologies.

In uplink sensing, multiple user equipments (UEs) send uplink signals to one base station (BS) for data transmission [5]. When the number of UEs is large enough, the targets around the BS can be completely covered and the BS can perform simultaneous data transmission and target detection. In [6], the authors designed an uplink channel estimation and sensing scheme based on deep learning. The authors in [7] analyzed the Cramér-Rao bound for the uplink ISAC and concluded that the uplink multi-path environment is beneficial for improving the radar sensing accuracy. Besides estimating the element-wise channel, parameter extractions, which only extract the parameters of interest from the overall channel, can also be adopted for obtaining radar channels [15], [16], [17]. Some papers discussed how to extract the parameters in the ISAC channel environment. In [18], the authors used a low-rank tensor metric to extract three parameters including delay, angle, and Doppler of targets. In [19], the authors proposed a range-and-Doppler estimation scheme based on multiple-signal classification (MUSIC) estimators. These papers assumed perfect synchronization between transceivers. The synchronization is not easy to realize between BS and multiple UEs since this process can be time-consuming [20]. When the transmitter is asynchronous with the BS, there exist timing offset (TO) and carrier-frequency offset (CFO) in the channel [21], [22], [23], [24]. As detailed in [25], such offsets cause sensing ambiguity for the delay and Doppler-frequency estimation, and they also cause unknown and time-varying phase shifts across CSI measurements, preventing coherent processing of multiple CSI measurements. Conventional direct sensing techniques are not effective in handling such offsets.

Recently, some WiFi-sensing papers have dealt with asynchronous transceiver setups and obtained key parameters including delay, angle-of-arrival (AoA), and Doppler frequency.
These techniques can be classified into cross-antenna cross-correlation (CACC) and CSI-ratio classes. Both techniques exploit common clock offsets across multiple antennas at the receiver. They can directly remove all the offsets from the signal models, and enable coherent processing of CSI and the estimation of dynamic CSI only. But new challenges also exist, as introduced in [25]. In [21], CACC was applied to obtain the AoA with commodity WiFi devices. In [22], CACC was used to resolve the ranging estimation problem for passive human tracking using a single WiFi link. In [23], the authors also applied CACC to cancel the offsets and used the modulus of received signals to obtain the parameter of a human target. The CACC operation results in mirrored parameters in the output. The authors in [22] used the average signal to suppress the mirrored side product. In [24], the authors considered asynchronous PMNs and estimated the mirrored unknown parameters using a mirrored MUSIC. All of these works are based on CACC operations, and they would require a static and dominating line-of-sight (LOS) path and other assumptions for system setups [25]. Another way to combat the offsets is to use division/ratio, rather than the cross-correlation, between the signals (CSI) obtained on different antennas [26], [27], [28]. But the CSI ratio methods lead to nonlinear signal models, which makes it challenging to apply conventional spectrum analysis techniques and to sense multi-target channels. These techniques have worked reasonably well in indoor sensing using WiFi signals. However, they become less efficient for PMNs in outdoor environments where a dominating LOS path is often absent and multiple dynamic targets are present. These issues reduce the sensing resolution and substantially raise both false alarm and miss rate [29]. In the application where multiple dynamic/moving objects are needed to be detected in the PMNs, most of the previous papers cannot be used either as they can detect only one moving target.

In this paper, we develop an uplink sensing scheme for estimating key sensing parameters, including Doppler frequency, AoA, and propagation delay, of all moving targets. Under an uplink channel of PMN, we perform the uplink sensing based on the unprecedentedly employed Taylor series of the CSI ratio. Compared with CACC, the CSI ratio has no requirement for a LOS path and can extract the specific targets in movements. This work can also be used in other applications, such as WiFi sensing and indoor tracking. The main contributions of this paper are

- We use the complex Taylor series to convert the CSI ratio from a nonlinear function into a linear function, which enables us to sense multiple moving targets in the presence of asynchronous offsets, without requiring a LOS path. We also analyze the convergence of the Taylor series of the CSI ratio.
- We extract key parameters exclusively for dynamic paths of the ISAC channel with a small number of observations. For Doppler frequency, we reconstruct the signal variation in the temporal domain. The zero frequency is suppressed and the non-zero dynamic Doppler frequencies can be extracted in the proposed Doppler estimator.

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**II. SYSTEM AND CHANNEL MODELS**

We consider the uplink communication and sensing in a PMN, as shown in Fig. 1. Multiple static UEs communicate with one static BS that uses received uplink signals for both communication and sensing. Each UE has one antenna. The BS uses a uniform linear array (ULA) of \( N \) antennas. The uplink channel between the BS receiver and the UE’s transmitter has multiple paths including both static and dynamic ones. The static paths refer to the LOS path, the paths reflected by static objects, and the ones that have negligible moving speed. The Doppler frequencies of static paths are assumed to be zeros. The dynamic paths are reflected by moving objects, such as vehicles. The Doppler frequencies of dynamic paths are non-zeros and cause temporal phase variations in CSI. Since all UEs are assumed to be static, the uplink channel mainly consists of static paths and probably has several dynamic paths. In this paper, we treat these two types of paths differently and focus on estimating the parameters of dynamic paths solely.

Although logical channels are used in mobile networks and signals are transmitted in well-defined timeslots, we adopt a simplified packet structure to generate transmitted signals. In
each packet, training symbols, denoted as preambles, are followed by a sequence of data symbols. Orthogonal frequency-division multiplexing (OFDM) modulation is applied across the whole packet. The data symbols can be empty if the packet is a demodulation reference signal (DMRS). For both preamble and data symbols, each of them has G subcarriers with a subcarrier interval of $1/T$, where $T$ denotes the length of an OFDM symbol. Each of the OFDM symbols is prepended by a cyclic prefix (CP) of period $T_C$. The $m$th transmitted packet at the UE’s baseband can be expressed as [16], [30]

$$s(t, m) = \sum_{g=0}^{G-1} e^{j2\pi(f_c + f_g)T} \text{rect} \left( \frac{t}{T + T_C} \right) x[m, g],$$

$m \in \{0, \ldots, M-1\}, g \in \{0, \ldots, G-1\},$

where $f_c$ is the carrier frequency, $\text{rect}(\frac{t}{T + T_C})$ denotes a rectangular window of length $T + T_C$, $x[m, g]$ is a preamble transmitted on the $g$th subcarrier of the $m$th OFDM packet, and $T_A$ is the interval between packets. For notational simplicity, we let one specific UE occupy the whole frequency band of $G/T$ and omit the index related to different UEs, but the proposed scheme can be readily applied to the case of multiple UEs by using other subcarrier assignments [31]. This packet structure is generalized and can be used to represent signals in many wireless systems, such as WiFi and Bluetooth, in addition to mobile networks. Therefore, the scheme presented in this paper can also be applied to all these systems.

In this paper, we assume there are $L_S$ static paths and $L$ dynamic paths. Without loss of generality, we let the first $L$ paths, $1 \leq l \leq L$, be dynamic ones, and the rest $L_S$ paths, $1 \leq l \leq L + L_S$, be static ones. Let $\alpha_l$, $f_{d,l}$, $\tau_l$ and $\theta_l$ denote the complex channel gain, Doppler frequency, delay (propagation delay), and AoA of the $l$th path, $(1 \leq l \leq L + L_S)$, respectively. Since there is typically no synchronization at the clock level between BS and UEs, the received signal has an unknown time-varying TO, denoted as $\tau_0[m]$, associated with the delay, even if the packet level synchronization is achieved. Hence, the total time delay during the signal propagation as seen by BS equals $\tau_l + \tau_0[m]$. There also exists an unknown time-varying CFO, denoted as $f_{0}[m]$, due to the asynchronous carrier frequency. We let the CFO absorb the random phase shift term, which is also present in the received signal and explicitly considered in [22], [26]. The channel model is given by

$$h(t, m) = \sum_{l=1}^{L+L_S} \alpha_l \times \delta(t - \tau_l - \tau_0[m]) - mT_A + (f_{d,l} + f_{0}[m])t/f_c \cdot \mathbf{a}(\Omega_l),$$

where $\delta(t)$ is an impulse signal and $\mathbf{a}(\Omega_l) = \exp(j\Omega_l(0, 1, \ldots, N-1)^T)$ is the array response vector of size $N \times 1$, with $\Omega_l = \frac{2\pi d}{\lambda} \sin \theta_l$, $d$ denoting the antenna interval, $\lambda$ denoting the wavelength, and $\theta_l$ denoting the AoA from the $l$th path.

The received time-domain signal corresponding to (1) and (2) can be represented as [22]

$$y(t, m) = \sum_{l=1}^{L+L_S} \alpha_l \times e^{j2\pi mT_A(f_{d,l} + f_{0}[m])},$$

where $w(t, m)$ is a complex additive-white-Gaussian-noise (AWGN) vector with zero mean and variance of $\sigma^2$.

Recall that we only use the preambles, $x[m, g]$, for sensing. Hence, $x[m, g]$ is available at the BS and can be easily removed by multiplying $(x[m, g])^{-1}$. After removing CP and $x[m, g]$, we transform the time-domain signal into the frequency domain via a fast-Fourier-transform (FFT)’s. Referring to (3) and neglecting the noise, the received frequency-domain signal at the $n$th antenna is

$$y_n[m, g] = \mathcal{F}\{h(t, m) \ast s(t, m)\}(x[m, g])^{-1} = \sum_{l=1}^{L+L_S} \alpha_l e^{jn\Omega_l} e^{j2\pi mT_A(f_{d,l} + f_{0}[m])} e^{-j2\pi \frac{f_c}{T}(\tau_l + \tau_0[m])} \sum_{l=1}^{L+L_S} \alpha_l e^{jn\Omega_l} e^{j2\pi mT_A(f_{d,l} + f_{0}[m])} e^{-j2\pi \frac{f_c}{T}(\tau_l + \tau_0[m])} D_n[m, g] + S_n[g] e^{j2\pi mT_A f_{0}[m]} e^{-j2\pi \frac{f_c}{T} \tau_0[m]},$$

$\Omega_n = \{0, \ldots, N-1\}$. From (4), it is seen that the CSI of interest is mixed with the offsets. Such offsets need to be removed to enable coherent processing of the CSI across time and deliverable of absolute estimates for the delay and Doppler frequencies. The CACC and CSI ratio methods have been proposed to resolve this issue, with respective advantages and disadvantages as detailed in [21], [22], [23], [24], [25], [26], [27], [28]. Our scheme aims to solve the major nonlinearity problem of the CSI ratio method, based on the Taylor series representation of the CSI ratio, as will be detailed next.

### III. Taylor Series of CSI Ratio

In typical cases where there are fewer dynamic paths than static paths, it will be attractive if we can extract the dynamic parameters without knowing the static paths, as fewer observations are required. The CSI ratio method can meet such an objective.

Referring to (4) and neglecting the noise term, the CSI ratio between the $n$th antenna and the $(n - q)$th antenna is given by
\[
\xi_{n,n-q}[m,g] = \frac{y_n[m,g]}{y_{n-q}[m,g]} = \frac{(S_n[g] + D_n[m,g]) e^{j2\pi m T_{f_0}[m]} e^{-j2\pi \frac{\pi}{L} \tau_0[m]}}{(S_{n-q}[g] + D_{n-q}[m,g]) e^{j2\pi m T_{f_0}[m]} e^{-j2\pi \frac{\pi}{L} \tau_0[m]}} \]

\[
\approx \frac{S_n[g] + \sum_{l=1}^{L} \alpha_l e^{j\Omega_l L} e^{j2\pi m T_{f_0}[m]} e^{-j2\pi \frac{\pi}{L} \tau_0}}{S_{n-q}[g] + \sum_{l=1}^{L} \alpha_l e^{j(n-1)\Omega_l L} e^{j2\pi m T_{f_0}[m]} e^{-j2\pi \frac{\pi}{L} \tau_0}} \]

= \frac{f(z_m)}{f(z_{n-q})}, \quad q \in \{n - N + 1, \ldots, n\},

(5)

where \( z_{n,q}[m,g] = \alpha_l e^{j\Omega_l L} e^{j2\pi m T_{f_0}[m]} e^{-j2\pi \frac{\pi}{L} \tau_0} \) and \( z_m = [z_{n,1}[m,g], \ldots, z_{n,L}[m,g]]^T \). Note that different from existing works [26], [27], we consider a general and more complicated case where multiple dynamic and static paths are present. Also note \( f(z_m) \) is related to \( g, q, n \) as well, while we omit these subscripts, \( g, q, n \), and \( L \), for the notational simplicity of the following derivations. From the outcome of (5), both TOs and CFOs are fully canceled. It shall be highlighted that the offsets can only be removed by dividing the signals across the spatial domain. The ratio will still contain the offsets if the division is across other domains, i.e., \( g \) or \( m \).

Since there are \( L \) dynamic components, the CSI ratio, \( f(z_m) \), is a multi-element function with respect to \( L \) \( z_{n,l}[m,g] \)'s in the temporal domain. We write \( z_{n,l}[m,g] \) as \( z_l \) for notational simplicity. These \( L \) variables are stacked into the vector, \( z_m = [z_1, \ldots, z_L]^T \). Let \( z \) denote an arbitrary complex vector of dimension \( L \times 1 \). By using the multi-element complex Taylor series w.r.t \( z_m \), the CSI ratio can be represented as

\[
f(z) = f(z_m) + \nabla f(z_m)^T (z - z_m) + \frac{1}{2!} (z - z_m)^T H(z_m) (z - z_m) + O^3(z),
\]

where \( \nabla f(z_m) = \left[ \frac{\partial f}{\partial z_1}, \ldots, \frac{\partial f}{\partial z_L} \right]^T \) and \( H(z_m) \) is

\[
H(z_m) = \begin{bmatrix}
\frac{\partial^2 f}{\partial z_1^2} & \cdots & \frac{\partial^2 f}{\partial z_1 \partial z_L} \\
\vdots & \ddots & \vdots \\
\frac{\partial^2 f}{\partial z_L \partial z_1} & \cdots & \frac{\partial^2 f}{\partial z_L^2}
\end{bmatrix}.
\]

(7)

Then, we denote \( f^{(1)}(z_l) \) and \( f^{(2)}(z_{l_1}, z_{l_2}) \) as the \( l \)th entry of \( \nabla f(z_m) \) and the \( (l_1, l_2) \)-th entry of \( H(z_m) \), respectively, where \( l, l_1, l_2 \in \{1, \ldots, L\} \). Note that we use the truncated series up to the second order in this paper. It is also possible to use higher orders to achieve a lower truncation error, at the cost of higher complexity.

Referring to (5), the 0th-order, the 1st-order, and the 2nd-order derivatives are given by

\[
f(z_m) = f(z_m) = \xi_{n,n-q}[m,g],
\]

\[
f^{(1)}(z_l) = \frac{\partial f}{\partial z_l},
\]

\[
f^{(2)}(z_{l_1}, z_{l_2}) = \frac{\partial^2 f}{\partial z_{l_1} \partial z_{l_2}},
\]

(8)

\[
= \frac{y_{n-q}[m,g] - e^{-j\xi_{n,n-q}[m,g]}}{y_{n-q}[m,g]^2} e^{j2\pi m T_{f_0}[m]} e^{-j2\pi \frac{\pi}{L} \tau_0[m]} \]

\[
\Delta z_l \Delta z_{l_2}, p \in \{1, \ldots, P\},
\]

(13)
where \( \Delta z_i = z_{n,l}[m+1, g] - z_{n,l}[m, g] \). Letting \( \hat{z}_i \) be \( z_{n,l}[m+1, g] - j e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)} \), we have

\[
f(z_{m+1}) = f(z_m) + \sum_{l=1}^{L} h_{n,q}(\Omega_1) \hat{z}_i \cdot (e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)}) - 1 \]

\[
+ \frac{1}{2} \sum_{l_1=1}^{L} \sum_{l_2=1}^{L} e^{2 \pi p T A f_0} \hat{z}_1 \hat{z}_2 \]

\[
\times (e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)}) = 1 \), \( (e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)}) - 1 \). \] \( (14) \)

Note that \( (14) \) can be transformed into \( f(z_{m+1}) - f(z_m) \) that denotes the difference of CSI-ratio (D-CSIR), denoted as

\[
\psi_{n,q}[m, p, g] = f(z_{m+1}) - f(z_m) = \xi_{n,m,q}[m, p, g] - \xi_{n,m,q}[m, g] \]

\[
n \in \{0, \cdots, N - 1\}, q \in \{n - N + 1, \cdots, n\}. \] \( (15) \)

Regarding the samples of D-CSIR, \( f(z_{m+1}) - f(z_m) \), the Doppler frequencies of dynamic paths are clearly shown on the right-hand side of \( (14) \) and can be retrieved by analyzing the phase variance of the D-CSIR in the temporal domain. Unfortunately, the delays make both dynamic paths and static paths vary in the frequency domain, and hence, it is invalid to use the Taylor series w.r.t. \( g \). For AoAs, the offsets are not present in the formulated estimation problem. Therefore, traditional AoA estimation methods could be used but face the estimation of all paths including both dynamic and static paths.

IV. DYNAMIC PARAMETER ESTIMATION

In this section, we propose a novel estimation scheme for obtaining Doppler frequencies, AoAs, and delays of dynamic paths only. Via using the CSI ratio, the scheme can exclusively extract the dynamic parameters. For the Doppler frequency, the proposed estimator is presented in Section IV-A. The proposed AoA estimator is presented in Section IV-B. We further present a delay estimator in Section IV-C for the case of \( L = 1 \). The general delay estimator is finally presented in Section IV-D.

A. Doppler Frequency Estimator

Intuitively, the Doppler frequencies can be obtained by analyzing the phase variance of the D-CSIR from \( p = 1 \) to \( p = P \). By assembling \( p \) from 1 to \( P \), we obtain a D-CSIR vector, denoted as \( \psi_{n,q}[m, g] = [\psi_{n,q}[m, 1, g], \cdots, \psi_{n,q}[m, P, g]] \). The selection of \( P \) is limited. On the one hand, a larger \( P \) is preferred as we can use more samples of D-CSIR. On the other hand, \( P \) shall not be too large, such that the samples are from a coherent processing interval (CPI) and the coefficients of the Taylor series can be regarded as unchanged. Referring to \( (14) \), the MUSIC method can be used to estimate the Doppler frequencies of the dynamic paths. Given the first two orders of the Taylor series, the non-linear CSI ratio becomes a linear function w.r.t. \( (e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)}) - 1 \).

The MUSIC-type estimators require the basis vectors of \( \psi_{n,q}[m, g] \). Since the CSI ratio has been transformed into linear expressions via the Taylor series, the first-, the second-, and/or the higher-order Taylor series can be used to represent the basis vectors of \( \psi_{n,q}[m, g] \). Referring to \( (14) \) and supposing that only the first two-order Taylor series are used to linearly express \( \psi_{n,q}[m, g] \), the basis vectors of \( \psi_{n,q}[m, g] \) are given by

\[
b_1(f_{D,l}) = (e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)}) - 1 , \]

\[
l \in \{1, \cdots, L\} \] \( (16) \)

and

\[
b_2(f_{D,l}, f_{D,l}) = (e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)}) - 1 \times (e^{2 \pi p T A f_0} e^{j 2 \pi f (\tau_0 + 1)}) - 1 \]

\[
, l_1, l_2 \in \{1, \cdots, L\}. \] \( (17) \)

Note that there are \( L(L + 1)/2 \) independent basis vectors in \( (17) \), and both \( f_{D,l_1} \) and \( f_{D,l_2} \) have independent indices. Their indices, \( l_1 \) and \( l_2 \), correspond to those entries in the second order of the Taylor series. With given \( L \) dynamic paths, there are \( L \) 1st-order and \( L(L + 1)/2 \) 2nd-order basis vectors. Note that the number of basis vectors of \( \psi_{n,q}[m, g] \) is no larger than \( P \).

We note that the basis vectors for \( \psi_{n,q}[m, g] \) are not related to \( m, g, n, \) and \( q \), which means that these indices can be arbitrary integers in the range. Stacking all samples increases computational complexity and is not necessary. Alternatively, we can select the indices of \( m, g, n, \) and \( q \), such that the following MUSIC function has the largest peak-to-average-power ratio (PAPR). A globally optimal solution is exhaustively searching all possibilities of \( \psi_{n,q}[m, g] \) but this will result in high complexity too. The selection of \( m, g, n, \) and \( q \) only needs to guarantee that the received signal power, \( |\psi_{n,q}[m, g]|^2 \), is larger than the noise level, such that the CSI ratio would not enhance the noise. Therefore, we propose to select some candidates of \( m, g, n, \) and \( q \), such that the received signal power is sufficiently large among all \( m, g, n, \) and \( q \), and then we select the best candidate of which MUSIC function has the highest PAPR. Since the signal variation across \( m \) is the least, we fix \( n = n_0, q = q_0, g = g_0, \) and stack the vectors, \( \psi_{n,q}[m, g] \), from \( m = 0 \) to \( m = M - P - 1 \). The corresponding stacked matrix is \( P \).

To obtain the Doppler frequency, we substitute a test value, \( f \), into \( f_{D,l_1}, f_{D,l_2}, f_{D,l_2} \), and \( f_{D,l_2} \). Additionally, note that Appendix A has given the expression for all Taylor series. Hence, we can also construct the third-order basis vectors, denoted as \( b_3(f) \), or even higher-order basis vectors. After dismissing the harmonic vectors and omitting the \( i \)th orders of the Taylor series, \( \forall i \geq J + 1 \), the required number of basis vectors equals \( J \). For simplicity of exposition, we assume \( J = 2 \). The basis vectors are then normalized, e.g., \( b_1(f_{D,l}) = b_1(f_{D,l})/\|b_1(f_{D,l})\|_p \). Using the MUSIC method, the dynamic Doppler frequencies can be obtained by solving

\[
f_{D,l} = \text{Peak}_l \left( \frac{1}{\|b_1(f), b_2(f, f)\|_p N_p} \right), \] \( (18) \)

where \( N_p \) is the null-space of \( P \) that is obtained from the left singular matrix of \( P \) and \( \text{Peak}_l(\cdot) \) is a function that obtains \( f \).
such that the objective function reaches the $l$th highest peak. It is noted that there is a trivial solution, $f = 0$, to the problem of (18), because $b_1(0) = b_2(0) = 0$. The normalization of $b_1(f)$ and $b_2(f, f)$ can greatly suppress the peak generated by the trivial solution. Hence, the value of $f$ can be an arbitrary value except 0.

Given a signal block $P$, we let $P_s$ be the desired signal block consisting of basis vectors, and $\Psi$ be the perturbations of the signal block, such that $P = P_s + \Psi$. Based on the analysis performed in our previous work [24], the theoretical error between the MUSIC estimate and the practical value is

$$\Delta f = \text{Re} \left[ \frac{p_0(f) H \Delta \Psi^H P_{\text{e}}(f) P_{\text{e}}(f)}{p_0(f) H \Delta \Psi^H P_{\text{e}}(f) P_{\text{e}}(f)} \right], \quad (19)$$

where $p_0(f)$ denotes the basis vector with respect to $f$, $P_{\text{e}}(f)$ is the first order derivative of $p_0(f)$, $\Delta \Psi$ is the null-space that corresponds to $P_s$ and $\Delta \Psi$ is the related perturbations.

### B. AoA Estimator

For estimating AoAs of dynamic paths, similar to the proposed estimator above, we exploit the D-CSIR, such that the dynamic AoAs can be extracted solely. From (15), we can observe that the AoAs are dependent on both $n$ and $g$. By assembling the samples in the spatial domain, $n$ and $g$, we also use the MUSIC-type estimators to obtain AoAs. Note that the basis vectors in the spatial domain are related to $m$ and $g$ too, which is observed from the expressions of $h_{n,q}^{m,g}(\Omega_1)$ and $H_{n,q}^{m,g}(\Omega_1, \Omega_2)$. This indicates that $m$ and $g$ are coupled with the spatial domain and need to be fixed during the AoA estimation. Fortunately, the index, $p$, is not coupled with the spatial-domain samples. Therefore, for estimating AoAs, only measurements with different $p$ can be stacked.

We form a spatial-domain matrix with the $(n + 1, n - q + 1)$th entry being the D-CSIR, $\psi_{n,q}[m, p, g]$, $n \in \{0, \cdots, N - 1\}$, $q \in \{n - N + 1, \cdots, n\}$, given by

$$A[m, p, g] = \begin{bmatrix}
\psi_{0,0}[m, p, g] & \cdots & \psi_{0,-N+1}[m, p, g] \\
\psi_{1,0}[m, p, g] & \psi_{1,-1}[m, p, g] & \cdots & \psi_{1,-N+2}[m, p, g] \\
& \ddots & \ddots & \ddots \\
\psi_{N-1,N-1}[m, p, g] & \cdots & \psi_{N-1,0}[m, p, g]
\end{bmatrix}. \quad (20)$$

Note that the diagonal entries of $A[m, p, g]$ are 0’s since $\psi_{n,0}[m, g] = \xi_{n,0}[m + p, g]$, $\psi_{n,n}[m, g] = 0$. The number of effective entries in $A[m, p, g]$ is $N^2 - N$. With fixed $m$, $p$, and $g$, the matrix becomes a manifold influenced by AoAs only.

Referring to (14), we can obtain the basis vectors for each column of $A[m, p, g]$. The basis vectors for the $(n' + 1)$th column of $A[m, p, g]$ are written as

$$d_1[\Omega_1, n'] = \left[ h_{0,-n'}^{m,g}(\Omega_1) e^{j0\Omega_1}, \cdots, h_{N-1,-n'+N-1}^{m,g}(\Omega_1) e^{j(N-1)\Omega_1} \right]^T,$$

$$n' \in \{0, \cdots, N - 1\}, \quad (21)$$

and

$$d_2[\Omega_1, \Omega_2, n'] = \left[ H_{0,-n'}^{m,g}(\Omega_1, \Omega_2) e^{j0\Omega_1}, H_{1,-n'+1}^{m,g}(\Omega_1, \Omega_2) e^{j2\Omega_1+\Omega_2}, \cdots, H_{N-1,-n'+N-1}^{m,g}(\Omega_1, \Omega_2) e^{j2(N-1)\Omega_1+\Omega_2} \right]^T. \quad (22)$$

To use all columns effectively, we vectorize the manifold into an $N^2 \times 1$ vector. Since the number of effective entries in $A[m, p, g]$ is $N(N - 1)$, the maximum number of dynamic AoAs that can be distinguished is $N(N - 1)$. The vectorized basis vector is expressed as

$$d_1(\Omega_1) = [d_1^T[\Omega_1, 0], \cdots, d_1^T[\Omega_1, N - 1]]^T, \quad (23)$$

and then it is normalized to one. Likewise, $d_2(\Omega_1, \Omega_2)$, is similarly obtained.

To remove the harmonic components, we dismiss the terms with $\Omega_1 \neq \Omega_2$. The basis vectors of AoAs are dependent on $m$ and $g$ as well. To reduce the computational complexity, we let $m = m_0$, $g = g_0$, and stack the vectors from $p = 1$ to $p = P$ into a matrix, given by

$$\bar{A} = [\text{vec}(A[m_0, 1, g_0]), \cdots, \text{vec}(A[m_0, P, g_0])]. \quad (24)$$

Based on the MUSIC estimators, the AoAs can be estimated by

$$\hat{\Omega} = \text{Peak} \left( \frac{1}{\|d_1(\Omega), d_2(\Omega), \bar{A}\|^2 F} \right), \quad (25)$$

where $\Omega$ is a test value of AoA and $N_3$ is the null-space of $\bar{A}$. Note that we can also use $d_1(\Omega)$ only when the basis vectors have good orthogonality with the null space.

There is a trivial solution to the problem of (25) under a specific condition, as stated in Proposition 2.

### Proposition 2: $\Omega = 0$ is the trivial solution to (25) when $S_{n_1}[g] = S_{n_2}[g], \forall n_1, \forall n_2 \in \{0, \cdots, N - 1\}$ and $L = 1$. See the proof in Appendix B. According to Proposition 2, the trivial solution exists when there is only one dynamic path and the static components, $S_{n}[g]$, are the same for all antennas. Such a case rarely happens in PMNs because the number of antennas for a BS is large enough to assure $S_{n}[g]$ to be different from one another. When there are only a small number of antennas, the trivial solution may exist and influence the accuracy of the AoA estimator.

### C. Delay Estimator for One Dynamic Path

Referring to (14), the delay is not explicitly expressed and only exists in $\tilde{\tau}_i$. From the expression of $\tilde{\tau}_i$ above (14), we see that the delay is mixed with TO.

### Proposition 3: For a single dynamic path, the dynamic delay can be obtained from the CSI ratio if and only if $S_n[g]$ is known.

**Proof:** For a single dynamic path, the CSI ratio in (5) can be rewritten as

$$\xi_{n,n-q}[m, g] = \frac{S_{n}[g] e^{j2\pi x} + 2 \tau_1 e^{j2\pi x} m T_{\text{d}} e^{j2\pi x} (x + \tau_1)}{S_{n-q}[g] e^{j2\pi x} + 2 \tau_1 e^{j2\pi x} m T_{\text{d}} e^{j2\pi x} (x + \tau_1)}, \quad (26)$$

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where \( x \) is an arbitrary value. Without knowing \( S_n[g] \), it is impossible to differ \( S_n[g] \) from \( S_n[g]e^{j2\pi \frac{2}{\pi} x} \) because both \( S_n[g] \) and \( S_n[g]e^{j2\pi \frac{2}{\pi} x} \) can be generated using different values of static delays. Then, \( \tau_1 \) shows no difference with \( \tau_1 - x \). This means that no matter what \( \tau_1 \) becomes, the CSI ratio is unchanged. Therefore, if the CSI ratio is used for obtaining \( \tau_1 \) in the dynamic path, \( S_n[g] \) must be known first.

According to Proposition 3, without knowing the information of static components, the dynamic delay cannot be obtained from the CSI ratio. If using the ratio between adjacent \( g \), the delay is entangled with \( \Omega \) and hence cannot be obtained either.

The ratio between adjacent packets is small. Generally, the interval should be larger than \( M \).

Let us compute the following ratio

\[
\frac{S_n'[g]}{S_n'[g] + e^{-j\Omega_n e^{j2\pi T\tau_1}} \frac{\tau_1 - \tau_1^0}{\alpha_1}} = \frac{D'[m]}{S_n'[g] + e^{-j\Omega_n e^{j2\pi T\tau_1}} \frac{\tau_1 - \tau_1^0}{\alpha_1}}.
\]

where \( D'[m] = e^{j2\pi T\tau_1} \) and \( S_n'[g] = (S_n[g]/\alpha_1) \times e^{-j\Omega_n e^{j2\pi T\tau_1}} \). where \( D'[m] = e^{j2\pi T\tau_1} \) is known value and \( S_n'[g] = (S_n[g]/\alpha_1) e^{-j\Omega_n e^{j2\pi T\tau_1}} \) is to be obtained.

Here, we assume AoA is obtained and only estimate the delay using the least-square (LS) method, i.e.,

\[
\begin{bmatrix}
S_n'[g] \\
S_n'[g]
\end{bmatrix} = \begin{bmatrix}
1, -\xi_{m,n,q}[m_0, g] \\
1, -\xi_{m,n,q}[m_1, g]
\end{bmatrix}^{-1}
\begin{bmatrix}
1, -\xi_{m,n,q}[m_0, g] \\
1, -\xi_{m,n,q}[m_1, g]
\end{bmatrix} \begin{bmatrix}
S_n'[g] \\
S_n'[g]
\end{bmatrix}.
\]

where \( m_0 \) and \( m_1 \) are the indices of two selected packets. It is noted that the interval between \( m_0 \) and \( m_1 \) should be selected sufficiently large to make the equations linearly independent. Generally, the interval should be larger than \( P \).

Note that \( S_n[g] \) denotes the static component of a received signal without offsets and it is assumed to be unchanged over different CPIs. The obtained \( S_n'[g] = S_n[g] e^{-j\Omega_n e^{j2\pi T\tau_1}} \) varies slowly over different CPIs. Hence, in the \( k \)th CPI (one CPI has \( M \) packets), the obtained \( S_n'[g] \) is rewritten as \( S_n'[g](k) = S_n'[g] e^{-j\Omega_n(k) e^{j2\pi T\tau_1(k)}} / \alpha_1 \). Let \( k = 0 \) be a reference CPI that starts at any time. Using \( \tau_1(0) \) as the reference, we compute the following ratio

\[
\frac{S_n'[g](k)}{S_n'[g](0)} = e^{j(\Omega_1(k) - \Omega_1(0)) e^{j2\pi T\tau_1(0)} / \alpha_1}.
\]

Staking the ratios for \( G \) subcarriers into a vector and computing its FFT, we can obtain

\[
\mathbf{F} \left[ \begin{array}{c}
S_n'[0](k) \\
\vdots \\
S_n'[G-1](k)
\end{array} \right] = e^{jT(\tau_1(0) - \tau_1(k)) / \alpha_1}.
\]

where \( \mathbf{F} \) is the FFT matrix of dimension \( G \times G \). Let \( g^* \) be the index of the entries in (30) with the maximal absolute value.

The relative delay of each individual CPI is obtained as \( \tau_1(0) - \tau_1(k) = g^* T / G \).

When estimates for absolute delays are needed, the reference value \( \tau_1(0) \) can be obtained in various ways, e.g., system calibration or exploiting prior knowledge of delays. For example, when the range of delay is known as \( [T_a, T_b] \), the range of the estimated relative delay is \( [T_a - \tau_1(0), T_b - \tau_1(0)] \). By observing the range of the estimates, we can obtain the value of \( \tau_1(0) \). Then, the delay \( \tau_1 \), i.e., \( \tau_1(k) \), is obtained after knowing the reference value \( \tau_1(0) \). With the obtained estimates of \( \tau_1 \) and \( \Omega_1 \), the static component, scaled by an unknown complex value, \( 1/\alpha_1 \), can be obtained as \( S_n[g]/\alpha_1 = S_n[g] e^{-j\Omega_n(0) e^{j2\pi T\tau_1}} \).

It should be highlighted that the AoA of one dynamic path can also use the method in this subsection as long as the Doppler is estimated and the AoA has a limited range in \((-\pi, \pi)\). The estimation of AoA and delay are independent since the term, \( S_n[g] e^{-j\Omega_n(0) e^{j2\pi T\tau_1}} \), can be obtained. Estimating the AoA by using the method in this subsection can effectively address the issue of trivial solution pointed out in the end of Section IV-B.

D. Delay Estimator for Multiple Dynamic Paths

We can extend the method in Section IV-C to estimate the delays in a general way.

When there are multiple dynamic paths, we need to first match the estimates of Doppler frequencies and AoAs obtained from Sections IV-A and IV-B.

For each row of \( \mathbf{A} \) in (24), the indices, \( n, q, m, g \) are fixed and only \( p \) varies. Observing (14) and (15), it is clear that, the rows of \( \mathbf{A} \), \( \psi_{n,q}[m, 1, g], \ldots, \psi_{n,q}[m, P, g] \), have the basis vectors of \( b_1(f_{D,l,i}) \). As each column of \( \mathbf{A} \), it is clear that the columns of \( \mathbf{A} \) have the basis vectors of \( d_1(\Omega_l) \) since \( \mathbf{A} \) itself is used for AoA estimation in Section IV-B. Hence, \( \mathbf{A} \) can be used for pair matching the estimated AoAs and Doppler frequencies. The matching of the estimated AoAs and Doppler frequencies can be realized by

\[
(f_{D,l,i}, \Omega_l) = \max_{l,i} |\mathbf{d}_l^H(\Omega_l) \mathbf{A} b_1(f_{D,l,i})|, l,i \in \{1, \ldots, L\},
\]

where \( \max_{l,i} \) denotes to find \( L \) the highest peaks in different rows and different columns of the map. Note that both Doppler frequencies and AoAs are estimated using the Peak function. Hence, the number of dynamic paths can be estimated from the number of peaks generated by the Peak function, as will be shown in Fig. 9.

We can obtain multiple delays from

\[
\xi_{n,n-q}[m,g] = \frac{y_{n,q}[m,g]}{y_{n-q}[m,g]} = \frac{S_n[g] + D_n[m,g]}{S_{n-q}[g] + D_n-q[m,g]} = \frac{S_n'[g] + D_n'[m]}{S_{n-q}[g] + D_n-q'[m]} e^{-j\Omega_n e^{j2\pi T\tau_1}}
\]

where \( l = 1, \ldots, L \), the range of delay is\( [T_a - \tau_1(0), T_b - \tau_1(0)] \). By observing the range of the estimates, we can obtain the value of \( \tau_1(0) \). Then, the delay \( \tau_1 \), i.e., \( \tau_1(k) \), is obtained after knowing the reference value \( \tau_1(0) \). With the obtained estimates of \( \tau_1 \) and \( \Omega_1 \), the static component, scaled by an unknown complex value, \( 1/\alpha_1 \), can be obtained as \( S_n[g]/\alpha_1 = S_n[g] e^{-j\Omega_n(0) e^{j2\pi T\tau_1}} \).
where \( D'_n, l[m] = e^{j2\pi m T_A f_{c, l} + j \Omega_l} \) and \( S'_n[g] = (S_n[g]) / \alpha_1 \). Note that \( D'_n, l[m] \) is known. Also using the LS method, we can obtain multiple dynamic delays from
\[
\begin{bmatrix}
[S'_n[g], S'_{n-q}[g], \alpha_2 e^{-j2\pi \frac{2}{\pi}(\tau_2 - \tau_1)}, \ldots, \\
\alpha L e^{-j2\pi \frac{2}{\pi}(\tau_L - \tau_1)}
\end{bmatrix}^T
\]
\[
= \begin{bmatrix}
1^T \\
D'_{n, 2}[m^T] - \xi_{n, n-q}[m^T, g] \odot D'_{n-q, 2}[m^T] \\
\vdots \\
D'_{n, L}[m^T] - \xi_{n, n-q}[m^T, g] \odot D'_{n-q, L}[m^T] \\
\times \xi_{n, n-q}[m, g] \odot D'_{n-1, q}[m] - D'_{n-q, 1}[m]
\end{bmatrix},
\]
(33)
where \( m = [m_0, \ldots, m_L]^T \) are the indices of \((L + 1)\) packets ranging from 0 to \( M - 1\). As long as \( \tau_1 \) can be obtained, other delays are easy to be obtained. We note that \( \tau_1 \) can be obtained from the first row of the LS output, which is realized in the same way as in Section IV-C. Denote the \((l + 1)\)th row of the LS output as \( O^L_{n+1,q}[g] \). The unknown \( \tau_1, l \geq 2 \), is obtained by finding the peak from the FFT of the LS outputs, given by
\[
(\hat{\tau}_l - \tau_1) = \frac{T}{G} \text{Peak} \begin{bmatrix} G & [O^L_{n+1,0}, \ldots, O^L_{n+1,G-1}]^T \end{bmatrix},
\]
l \geq 2,
(34)
where the Peak function obtains the peak value (the maximal absolute value) of a complex vector from \( g = 0 \) to \( g = G - 1 \).

We briefly summarize the computational complexity of the three estimators that are proposed in Sections IV-A, IV-B, and IV-D. The complexity of obtaining CSIR samples is \( O(MGN(N - 1)/2) \). After obtaining all CSIR samples, the highest complexities of the Doppler and AoA estimators are \( O(P_L(M - P - 1)) \) and \( O((N - 1)^4P) \), respectively, which are resulted from obtaining the null-space signal matrix in the MUSIC algorithms. As for the delay estimator, its complexity is relatively low compared with calculating the CSIR samples.

V. EXPERIMENTAL AND NUMERICAL RESULTS

In this section, we provide both experimental and numerical results to validate the proposed parameter estimators. We use both practical data collected by a 3-antenna COTS WiFi device and the simulated data generated by MATLAB.

In experimental results, the detailed setup is the same as that in [23] as we implement our estimator by using their raw data. In the setup of a COTS WiFi system, the receiver has three antennas and the transmitter has one antenna, as shown in Fig. 2.

In the figure, \( d_2 = 2.251 \text{ cm}, d_1 = 2.682 \text{ cm}, \) and \( d_0 = 270 \text{ cm} \). The carrier frequency is 5.32 GHz with the subcarrier interval being 312.5 kHz. The number of subcarriers, \( G \), is 30. The sampling frequency, \( 1/T \), is 1 kHz and \( T_A \) is 0.1 s. The fixed phase shifts across antennas are removed from the dataset with system calibration. A low pass filter with a cutoff frequency of 60 Hz is also applied to smooth the raw CSI measurements, and the filtered outputs are used as inputs to our proposed scheme. For the proposed sensing parameters, we let \( P \) equal 30.

A. Experimental Results

In Fig. 3, we illustrate the Doppler frequency, AoA, delay, and trajectory obtained by processing the WiFi received signals. In the experiments, there is one moving human target in the indoor environment. It is noted that when the delay is too small with respect to \( T \), its estimation accuracy is low. Hence, a Kalman filter is typically used to smooth the delay estimates and the trajectory. The detailed setup of the Kalman filter used here can be referred to [23]. As can be seen from Fig. 3(a)–3(c), our method shows a similar trend to the WiDFS method. In Fig. 3(b), the AoAs of the WiDFS method have mutated points around 28 s and 58 s. The original delay estimates without the Kalman filter are shown in Fig. 3(c) for comparison. We note that the delay estimates are relative to the initial value, which corresponds to the initial location of the target (in the origin of the coordinate). Fig. 3(d) plots the trajectory calculated by using the AoA and the smoothed delay estimates from 20 s to 50 s. The coordinates of the trajectory are expressed as
\[
x_{\text{traj}} = d_l \sin(\theta) - d_0/2, y_{\text{traj}} = d_l \cos(\theta),
\]
(35)
where \( \theta \) is the AoA and \( d_l \) is the distance between the receiving antenna \( n = 0 \) and the human target. The ground truth is obtained by the millimeter-wave (mmWave) radar device that is located beside the commodity WiFi. We can see that both WiDFS and our methods exhibit some mutated trajectories. Our obtained AoA has more fluctuation, while the WiDFS method has mutated AoA. It is noted that our methods do not require the existence of the LOS path, and more importantly, our method can handle multiple dynamic paths, while the WiDFS method cannot. The performance of our method in multidynamic path scenarios will be shown in Fig. 9.

B. Numerical Results

In numerical results, the carrier frequency is 3 GHz, the number of subcarriers is \( G = 64 \), and the frequency bandwidth is 64 MHz. Hence, the OFDM symbol period \( T \) is 1 \( \mu \text{s} \). The propagation delay is randomly distributed over \([0.2, 0.4] \mu \text{s} \). The CP period \( T_C \) is 0.4 \( \mu \text{s} \). The approximate interval between two packets, \( T_A \), is 1 ms. We use \( M = 128 \) packets for the parameter estimation. The velocity of targets ranges from -30 meter-per-second (m/s) to 30 mps, and the Doppler frequency is randomly distributed over \([-0.3, 0.3] \text{kHz} \). The AoAs of targets are random values uniformly distributed from \(-\pi/2\) to \(\pi/2\).
to $\pi/2$. All the targets are modeled as point sources, and the radar cross-sections are assumed to be 1. The BS employs a ULA with $N = 8$ antenna elements. There is one dynamic path and $L_S = 5$ static paths unless stated otherwise. For simplicity, we use a complex Gaussian distribution to generate the path gain, i.e., $\alpha_l \sim \mathcal{CN}(0, 1)$. This model can cover a large range of path gains generated by a detailed path-loss model, such as the Ray-Tracing model. We assume there does not exist a dominant LOS path. For estimating Doppler frequency, we fix $n_0 = 1, q_0 = 1, g_0 = 0$ to obtain $P$. For estimating AoA, we fix $m_0 = 0$ and $g_0 = 0$ to obtain $\bar{A}$. The gap between $m_0$ and $m_1$ in our delay estimator is 30. The SNR is defined as $P_o/\sigma^2$, where $P_o = 1$ W is the power of $x[m, g]$ in (1) and $\sigma^2$ is the variance of noise that is defined below (3).

Fig. 4 shows the convergence probability and truncation error of the Taylor series versus $L$ and $L_S$. The truncation error is defined as the ratio of the energy of $O_3(z)$ to that of CSIR.

$$e_{\text{Tay}}(p) = \sum_{g=0}^{G-1} \sum_{m=0}^{M-1} \sum_{n=1}^{N} \sum_{l_1 = 1}^{L} \sum_{l_2 = 1}^{L} MGN|\xi_{n, g}[m + p, g]|^2. \quad (36)$$

Here, we let $p = 2$. Note that we regard the harmonic of the second order of the Taylor series, i.e., $f^{(2)}(z_{l_1}, z_{l_2}), l_1 \neq l_2$, as an error too. We only count in the truncation error in (36) when the Taylor series is convergent. The convergence probability gives the probability that the Taylor series is actually convergent. From Fig. 4(a) and 4(b), we observe that the truncation error decreases and the convergence probability increases when $L_S$ increases and $L$ decreases. This means that a higher power ratio between static and dynamic paths leads to a better Taylor series approximation to the CSIR. In general, the number of static paths is far more than that of dynamic paths. When there is a LOS path, the power of static paths can be 10 times higher than that of the dynamic path, and hence, the expected truncation error should be around 5% and the expected convergence probability is about 0.95. In the case of $L = L_S$, which rarely happens in practice, the convergence probability is around 0.5. According to our simulation results in Fig. 9, our estimators work well when $L_S/L$ is as low as 2.5, corresponding to a truncation error of about 18%.

Fig. 5 unfolds how the SNR influences the projection distance that is in the denominator of the MUSIC objective function. The projection distance has a close relationship with the sensing performance. For the Doppler estimator, the projection distance is given by $\|b_1(f_{D,l}, b_2(f_{D,l}, f_{D,l}))^HTP\|_F$. For the AoA estimator, the projection distance is given by $\|d_1(\Omega_l), d_2(\Omega_l, \Omega_l))^HTA\|_F$. Our delay estimator is not based on the Taylor series of CSIR and is excluded from the figure. Here, $L = 1$ and $L_S = 5$. The largest projection distance is normalized to 1. From the figure, we note that the projection distance is lower than 0.4 at low SNRs and drops to around 0.1 at high SNRs. Since the MUSIC estimator is a non-linear approach, we can see that, even at 0 dB, the projection distance...
is still lower than 0.5, which means that the space spanned by the adopted Taylor series and the signal space of D-CSIR has an angle less than $\frac{\pi}{6}$.

Figs. 6–8 plot the normalized mean-squared-error (NMSE) versus signal-to-noise ratio (SNR) of Doppler frequency, AoA, and delay, respectively. Other uplink sensing benchmarks, which addressed the TO and CFO in asynchronous systems, are included in our simulations. For Doppler and delay estimators, the benchmarks are the AMS method [21], CACC with MUSIC, and CACC with mirrored MUSIC [24]. For the AoA estimator, we compare our AoA estimator with the AMS, H-MUSIC [32], and the average NMSE of quantization grids, i.e., $1/N_f^2 \approx 1.56 \times 10^{-2}$. We also plot the Cramér-Rao bound (CRB) of synchronized signals with known offsets, where the CRB is derived in [33]. The CRB will be lower than the actual CRB in the presence of offsets, but the actual CRB is very challenging to derive, if not impossible. We consider the case of $L = 1$ and $L_S = 5$ with/without a LOS path. When a LOS path is present, its power is 10 dB higher than that of the NLOS path. The parameter of our proposed estimators, $P$, equals 30. Other system setups are the same as those given at the beginning of this subsection. The figures demonstrate that our proposed estimator outperforms the AMS method for all three parameters.

For NMSE of Doppler frequency, our estimator is nearly the same as CACC-MUSIC in the LOS scenario. Since the received signals involve a dominant LOS path, CACC-mirrored-MUSIC has higher accuracy than our proposed estimator, however, CACC methods only work with the existence of a LOS scenario. Without the LOS path, it is seen that the NMSEs of CACC methods rise dramatically. Without the LOS path, our proposed estimator is better than all the benchmarks.

For NMSE of AoA, our method can achieve nearly the same NMSE as H-MUSIC at high SNRs. In low SNRs, the performance of our proposed AoA estimator degrades. This is because the Taylor series is convergent when the dynamic component is little. Since the noise can be treated as the dynamic component, the Taylor series will be divergent with a high noise...
Despite the defect, our method obtains the dynamic path separately while both H-MUSIC and CACC need to estimate all paths’ parameters.

For NMSE of delay, our method can achieve the best performances without the LOS path. With the existence of the LOS path, our delay estimator still achieves the lowest NMSE at a high SNR. Besides, CACC methods can only obtain relative delays, which means that the delay of the LOS path is necessary to be known at the BS. Our delay estimator can obtain the absolute delays as long as $S_n[g]$ is known. Otherwise, it obtains relative delays.

Fig. 9 illustrates the shapes of the ‘Peak’ functions with considering multiple dynamic paths in a noiseless environment. Here, $L = 2$, $L_S = 5$. Other system parameters are the same as Fig. 6. For the Doppler frequency, we can observe that two peaks tightly match the practical Doppler frequencies. We compare its shape with the traditional MUSIC which has a large peak near 0 Hz due to the existence of static components. In traditional MUSIC, the estimated Doppler frequencies have unknown CFOs mixed with practical values. Even worse, it is noted that some dynamic paths are missing in the peaks of traditional MUSIC. For the AoA estimator, the peaks have a sharp shape and match with the practical AoAs tightly. Traditional MUSIC cannot separate dynamic AoAs from static ones and hence would generate 7 peaks, which is not shown in this figure. For the delay estimator, we see that each peak occupies the entire range of $[0, T]$ individually because the delays are obtained from different rows of the LS outputs in (33). This means that the delays do not have ambiguity problems when multiple delays are close to each other.

Our scheme has been validated via simulation results for the cases when the variation of path gains is up to about 15 dB. Thus it is particularly effective for sensing targets near the BS. For paths with significant power variation, e.g., when sensing targets with very large range separation, weaker dynamic paths may be buried in stronger ones and our scheme may fail to sense the weaker paths.

VI. CONCLUSION

This paper has proposed a Doppler-AoA-delay estimation scheme under the uplink ISAC systems, where only the parameters of moving targets are estimated. The system does not require clock synchronization between transceivers thanks to the CSI ratio. This enables effective sensing in a bi-static ISAC setup, including uplink sensing in PMNs. Our novel scheme is based on the truncated complex Taylor series of the CSI ratio, which is demonstrated to have a good convergence property. The Taylor series transform the non-linear CSI ratio into linear forms. The simulation results show that our scheme works effectively for target sensing without a dominating LOS path, and its performance improves when the power ratio between static and dynamic paths increases. The proposed AoA and delay estimators outperform the benchmark at high SNRs. The Doppler estimator outperforms the benchmarks in the NLOS scenario. Our work can be effectively applied in the PMN and WiFi sensing networks. It will become more attractive if its effectiveness in non-static-path dominating channels can be improved. Such improvements may be achieved via an iterative
approach after obtaining an initial estimation of the sensing parameters.

APPENDIX A

CONVERGENCE OF TAYLOR SERIES OF CSI RATIO

From (9) and (10), we can generalize the $k$th derivative of $f(z_n)$ as (37), where $y_n$ is short for $y_n[m, g]$, $y_{n-q}$ is short for $y_{n-q}[m, g]$, $e^{jk_0y_n} = e^{jk_TX_k,f_0[m]}$, and $e^{-jk_0y_{n-q}} = e^{-jk_TX_k,f_0[m]}$. Both derivatives w.r.t. $k = 1$ and $k = 2$ satisfy (37). The higher-order Taylor series can be proved by using the induction method, while omitted due to the page limits. Note that the upper bound approaches to zero with increasing. Note that (21) becomes

$$d_2(0, n') = [H_{0_n}^{m,g}(0, 0), \cdots, H_{0_n}^{m,g}(0, 0)]^T$$

and $d_2(0, n')$ becomes

$$d_2(0, n') = [D_0-n(0), D_{n-1}(0)]^T$$

where $n' = n' \in \mathbb{N}^+$. Likewise, the higher-order vectors of $d_1(0, n')$, $d_2(0, n')$, and $n'$ column of $A[m, p, g]$ is linearly dependent. Hence, the rank of the matrix, $d_1(0, n')$, $d_2(0, n')$, $v_n(A[m, p, g])$, is 1.

Note that, for an arbitrary $p$, the rank of the matrix, $d_1(0, n')$, $d_2(0, n')$, $v_n(A[m, p, g])$, is 1. With $n'$ fixed, we have

$$\text{rank}([d_1(0, n'), d_2(0, n'), A_n[m, g]]) = 1$$

where $A_n[m, g] = [v_n(A[m, 1, g]), \cdots, v_n(A[m, P, g])]$. Also noting that

$$\hat{A} = [A_0^T[m,g], \cdots, A_{N-1}^T[m,g]]^T$$

and $d_2(0, 0) = [d_2^T(0, 0), \cdots, d_2^T(0, N-1)]^T$, and $d_2(0, 0) = [d_2^T(0, 0), \cdots, d_2^T(0, N-1)]^T$, we have

$$\text{rank}(\hat{A}) = \text{rank}([\hat{A}, d_1(0, d_2(0, 0))]) \leq N.$$
REFERENCES

[1] M. L. Rahman, J. A. Zhang, X. Huang, Y. J. Guo, and R. W. Heath, “Framework for a pervasive mobile network using joint communication and radar sensing,” IEEE Trans. Aerosp. Electron. Syst., vol. 56, no. 3, pp. 1926–1941, Jun. 2020.

[2] J. A. Zhang, M. L. Rahman, K. Wu, X. Huang, Y. J. Guo, S. Chen, and J. Yuan, “Enabling joint communication and radar sensing in mobile networks—A survey,” IEEE Commun. Surveys Tuts., vol. 24, no. 1, pp. 306–345, 2022.

[3] P. Kumar, J. Choi, N. González-Prelinc, and R. W. Heath, “IEEE 802.11ad-based radar: An approach to joint vehicular communication-radar system,” IEEE Trans. Veh. Technol., vol. 67, no. 4, pp. 3012–3027, Apr. 2018.

[4] C. Sturm and W. Wiesbeck, “Waveform design and signal processing aspects for fusion of wireless communications and radar sensing,” Proc. IEEE, vol. 99, no. 7, pp. 1236–1259, Jul. 2011.

[5] Z. Abu-Shaban, X. Zhou, T. Abhayapala, G. Seco-Granados, and H. Wymeersch, “Performance of location and orientation estimation in 5G mmwave systems: Uplink vs downlink,” in Proc. IEEE Wireless Commun. Netw. Conf. (WCNC). Piscataway, NJ, USA: IEEE, 2018, pp. 1–6.

[6] S. Huang, M. Zhang, Y. Gao, and Z. Feng, “MIMO radar aided mmWave time-varying channel estimation in MU-MIMO V2X communications,” IEEE Trans. Wireless Commun., vol. 20, no. 11, pp. 7581–7594, Nov. 2021.

[7] C. Li, N. Raymond, B. Xia, and A. Sahharwal, “Outer bounds for a joint communicating radar (Comm-Radar): The uplink case,” IEEE Trans. Commun., vol. 70, no. 2, pp. 1197–1213, Feb. 2021.

[8] L. Zheng and X. Wang, “Super-resolution delay-Doppler estimation for OFDM passive radar,” IEEE Trans. Signal Process., vol. 65, no. 9, pp. 2197–2210, May 2017.

[9] C. R. Berger, B. Demissie, J. Heckenbach, P. Willett, and S. Zhou, “Signature processing for passive radar using OFDM waveforms,” IEEE J. Sel. Topics Signal Process., vol. 4, no. 1, pp. 226–238, Feb. 2010.

[10] A. Ali, N. Gonzalez-Prelinc, R. W. Heath, and A. Ghosh, “Leveraging sensing at the infrastructure for mmWave communication,” IEEE Commun. Mag., vol. 58, no. 7, pp. 84–89, Jul. 2020.

[11] N. García, H. Wymeersch, E. G. Ström, and D. Stock, “Location-aided mm-wave channel estimation for vehicular communication,” in Proc. IEEE 17th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC). Piscataway, NJ, USA: IEEE, 2016, pp. 1–5.

[12] B. Friedlander, “Waveform design for MIMO radars,” IEEE Trans. Aerosp. Electron. Syst., vol. 43, no. 3, pp. 1227–1238, Jul. 2007.

[13] D. Cataldo, L. Gentile, S. Ghio, E. Giusti, S. Tomei, and M. Martorella, “Multibistatic radar for space surveillance and tracking,” IEEE Trans. Aerosp. Electron. Syst., vol. 35, no. 8, pp. 14–30, Aug. 2020.

[14] N. Cao, Y. Chen, X. Gu, and W. Feng, “Joint bi-static radar and communications designs for intelligent transportation,” IEEE Trans. Veh. Technol., vol. 69, no. 11, pp. 13060–13071, Nov. 2020.

[15] X. Zhang, L. Xu, L. Xu, and D. Xu, “Direction of departure (DOD) and direction of arrival (DOA) estimation in MIMO radar with reduced-dimension MUSIC,” IEEE Commun. Lett., vol. 14, no. 12, pp. 1161–1163, Dec. 2010.

[16] J. Gu, J. Moghaddasi, and K. Wu, “Delay and Doppler shift estimation for OFDM-based radar-radio (RadCom) system,” in Proc. 2015 IEEE Int. Wireless Symp. (IWS 2015), Mar. 2015, pp. 1–4.

[17] O. Mihan and N. D. Sidiroopoulos, “Maximum likelihood passive and active sensing of wideband power spectra from few bits,” IEEE Trans. Signal Process., vol. 63, no. 6, pp. 1391–1403, Mar. 2015.

[18] B. Kong, Y. Wang, X. Deng, and D. Qin, “Joint range-Doppler-angle estimation for OFDM-based RadCom system via tensor decomposition,” Wireless Commun. Mobile Comput., vol. 2018, pp. 1–12, 2018.

[19] J. B. Sanson, P. M. Tomé, D. Castanheira, A. Gameiro, and P. P. Montezino, “High-resolution delay-Doppler estimation using received communication signals for OFDM radar-communication system,” IEEE Trans. Veh. Technol., vol. 69, no. 11, pp. 13112–13123, Nov. 2020.

[20] M. Hyder and K. Mahata, “Zadoff–Chu sequence design for random access initial uplink synchronization in LTE-like systems,” IEEE Trans. Wireless Commun., vol. 16, no. 1, pp. 503–511, Jan. 2017.

[21] X. Li, D. Zhang, Q. Li, J. Xiong, S. Li, Y. Zhang, and H. Mei, “IndoTrack: Device-free indoor human tracking with commodity Wi-Fi,” in Proc. ACM Interact. Mob. Wearable Ubiquitous Technol., vol. 1, no. 3, pp. 1–22, Sep. 2017, doi: 10.1145/3130940.

[22] K. Qian, C. Wu, Y. Zhang, G. Zhang, Z. Yang, and Y. Liu, “Widar2.0: Passive human tracking with a single Wi-Fi link,” in Proc. 16th Ann. Int. Conf. Mobile Syst. Appl. Serv. New York, NY, USA: ACM, 2018, pp. 350–361, doi: 10.1145/3210240.3210314.

[23] Z. Wang, J. A. Zhang, M. Xu, and J. Guo, “Single-target real-time passive WiFi tracking,” IEEE Trans. Mobile Comput., vol. 22, no. 6, pp. 3724–3742, Jun. 2023.

[24] Z. Ni, J. A. Zhang, X. Huang, K. Yang, and J. Yuan, “Uplink sensing in perceptive mobile networks with asynchronous transceivers,” IEEE Trans. Signal Process., vol. 69, pp. 1287–1300, Feb. 2021.

[25] J. A. Zhang, K. Wu, X. Huang, Y. J. Guo, D. Zhang, and R. W. Heath, “Integration of radar sensing into communications with asynchronous transceivers,” IEEE Commun. Mag., vol. 60, no. 11, pp. 106–112, Nov. 2022.

[26] Y. Zeng, D. Wu, J. Xiong, E. Yi, R. Gao, and D. Zhang, “FarSense: Pushing the range limit of WiFi-based sensing with CSI ratio of two antennas,” Proc. ACM Interact. Mob. Wearable Ubiquitous Technol., vol. 3, no. 3, pp. 1–26, 2019.

[27] Y. Zeng, D. Wu, J. Xiong, J. Liu, Z. Liu, and D. Zhang, “Multisense: Enabling multi-person resonance sensing with commodity WiFi,” Proc. ACM Interactive Mobile Wearable Ubiquitous Technol., vol. 4, no. 3, pp. 1–29, 2020.

[28] X. Li, J. A. Zhang, K. Wu, Y. Cui, and X. Jing, “CSI-ratio-based Doppler frequency estimation in integrated sensing and communications,” IEEE Sens. J., vol. 22, no. 21, pp. 20886–20895, Nov. 2022.

[29] C. Lasoudias, A. Moreira, S. Kim, S. Lee, L. Wirola, and C. Fischione, “A survey of enabling technologies for network localization, tracking, and navigation,” IEEE Commun. Surveys Tuts., vol. 20, no. 4, pp. 3607–3644, Jul. 2018.

[30] C. Sturm, Y. L. Sit, M. Braun, and T. Zwick, “Spectrally interleaved multi-carrier signals for radar network applications and multi-input multi-output radar,” IET Radar Sonar Navigat., vol. 7, no. 3, pp. 261–269, Mar. 2013.

[31] Y. L. Sit, B. Nuss, and T. Zwick, “On mutual interference cancellation in a MIMO OFDM multiuser radar-communication network,” IEEE Trans. Veh. Technol., vol. 67, no. 4, pp. 3339–3348, Apr. 2018.

[32] S. Chuang, W. Wu, and Y. Liu, “High-resolution AoA estimation for hybrid antenna arrays,” IEEE Trans. Antennas Propag., vol. 63, no. 7, pp. 2955–2968, Jul. 2015.

[33] M. D. Larsen, G. Seco-Granados, and A. L. Swindlehurst, “Pilot optimization for time-delay and channel estimation in OFDM systems,” in Proc. 2011 IEEE Int. Conf. Acoust. Speech Signal Process. (ICASSP). Piscataway, NJ, USA: IEEE, 2011, pp. 3564–3567.

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