PRISTINE DETERMINATION OF THE UNITARITY TRIANGLE USING 
B → K D⁰ PROCESSES

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There is no good reason to think that BSM-CP-odd phases(s) will necessarily cause large deviations in B-physics from predictions of the SM. Therefore, residual theory error in extraction of the unitarity triangle can undermine experimental efforts to search for BSM phase(s). We stress that final states containing D⁰ or ¯D⁰ in decays of charged and neutral B’s can yield all the angles of the unitarity triangle with negligible theory error (i.e. O(0.1%)).

I Introduction & Motivation: why must we target unitarity triangle with “zero” theory error

The two B-factories have made considerable progress in determining β by measurement of the time dependent CP asymmetry in B⁰ (B̄⁰) → ψKs and related modes. These machines have also been performing remarkably well. Very soon each of them should have ~ 10⁸ B- ¯B pairs and with improvements in luminosity to ≥ 10³⁴ cm⁻² s⁻¹ that are anticipated, 10⁹ B- ¯B pairs should become accessible in the next few years. In addition, Tevatron experiments CDF/D∅, and hopefully in the not too distant future BTeV and LHCB should also enable much larger data samples. Furthermore, encouraged by the success of the two B-factories, there is also considerable interest in very high luminosities (~ 10³⁶ cm⁻² sec⁻¹) facilities, Super-BELLE and Super-BABAR. With these anticipated experimental developments it is not enough that we can determine β with essentially negligible theory error (actually < 1%) we must target α and γ extraction also with zero theory error. Indeed extremely accurate determinations of all 3 unitarity angles is not just desirable but may well be an essential prerequisite for a successful search of the effect of any CP-odd phase in B-decays due to physics beyond the Standard Model.

In this regard it is important to realize that although the CP-odd phase in the CKM picture is of O(1) and not small and it leads to large asymmetries in B-physics, it yields very small CP-asymmetries in K decays; recall εK ~ 10⁻³ and ε' ~ 10⁻⁶ even though aCP(B → ψKs) ~ 75%. There are two important repercussions of this realization.

1. Failure of the (b → d) unitarity triangle [UT] due to effects of new physics may well be small and subtle. Therefore, residual theory error in the determination of the angles of the UT may mask the effect of new physics and thwart experimental attempts to find them.
Table 1: Comparison of some fits.

| Input Quantity                     | Atwood & Soni | Ciuchini et al | Hocker et al |
|------------------------------------|---------------|----------------|--------------|
| $R_{uc} \equiv |V_{ub}/V_{cb}|$               | .085 ± .017   | .089 ± .009   | .087 ± .006 ± .014 |
| $F_{B_d} \sqrt{|B_{B_d}|}\text{ MeV}$ | 230 ± 50      | 230 ± 25 ± 20  | 230 ± 28 ± 28 |
| $\xi$                              | 1.16 ± .08    | 1.14 ± .04 ± .05 | 1.16 ± .03 ± .05 |
| $\hat{B}_K$                        | .86 ± .15     | .87 ± .06 ± .13 | .87 ± .06 ± .13 |

**Output Quantity**

| sin 2$\beta$                      | .70 ± .10     | .695 ± .065   | .68 ± .18    |
| sin 2$\alpha$                     | -.50 ± .32    | -.425 ± .220  |             |
| $\gamma$                          | 46.2° ± 9.1°  | 54.85 ± 6.0  | 56 ± 19     |
| $\hat{\eta}$                      | .30 ± .05     | .316 ± .040  | .34 ± .12   |
| $\hat{\rho}$                      | .25 ± .07     | .22 ± .038   | .22 ± .14   |
| $|V_{td}/V_{ts}|$                  | .185 ± .015   | .19 ± .04    |             |
| $\Delta m_{B_s}(ps^{-1})$         | 19.8 ± 3.5    | 17.3$^{+1.5}_{-0.7}$ | 24.6 ± 9.1  |
| $J_{CP}$                           | (2.55 ± .35) × 10^{-5} | (2.8 ± .8) × 10^{-5} |
| $BR(K^+ \rightarrow \pi^+\nu\bar{\nu})$ | (0.67 ± 0.10) × 10^{-10} | (.74 ± .23) × 10^{-10} |
| $BR(K_L \rightarrow \pi^0\nu\bar{\nu})$ | (0.225 ± 0.065) × 10^{-10} | (.27 ± .14) × 10^{-10} |

2. Search for effect of any BSM phase(s) may well require very large data samples. A model independent estimate is very difficult to make. If the asymmetry due to new physics is of order $\epsilon_K \sim 10^{-3}$, then even with a branching ratio of $\sim 10^{-3}$ (there are very few relevant $B$-decay modes that have branching ratios this big) we may need $\sim 10^{10} B'$s to find such an effect. Efforts at developing the capabilities for large data samples of $B'$s in clean environments and/or specialized $B$-detectors are therefore very worthwhile.

These considerations lead us to suggest that while attempts at using proposed methods for $\alpha$ and $\gamma$ via $\pi\pi$, $\rho\pi$, $K\pi$ etc. should continue, we should realize that the presence of penguin, especially electroweak penguins (EWP), contributions along with the use of flavor symmetries could easily lead to theory errors of 10% or more due to any model dependence and theoretical assumptions that need be invoked. Clearly on both the theory as well as the experimental front methods for determining $\alpha$, $\beta$, $\gamma$ with zero theory error should be vigorously pursued.

Recall that the current measurements of sin $2\beta$ agree very well with theoretical expectations; the experimental world average sin $2\beta^{WA}$ = 0.78±0.14 is completely consistent with theoretical expectations, (sin $2\beta^{SM}$ ≈ 0.70 ± 0.14). See Table 1 for a comparison of some theoretical fits. It is important though to realize that this test of the SM has serious limitations. A lot of theoretical input is used in making these fits as they rely heavily on theoretical evaluations of several of the hadronic matrix elements. Improving the accuracy in these calculations is extremely difficult and painfully slow. To underscore this we mention two problems with the theoretical input being used in these fits.

First, even for the highly matured calculation of $\hat{B}_K$ there are some reasons to believe that the JLQCD (quenched) result, $\hat{B}_K^{\text{Staggered}} = 0.860 ± 0.054$, which has been widely used in the past many years, may well be 10–15% higher than the true value. This expectation is based on results obtained by using the newer discretization, domain wall quarks (DWQ) which has much better chiral-flavor symmetry properties. With DWQ both CP-PACS and RBC get smaller values; averaging their numbers one gets $\hat{B}_K^{\text{DFW}} = 0.758 ± 0.033$; (again this is quenched). This is about ± 13% below the older JLQCD result.

Second problem is with the SU(3) breaking ratio, $\xi$, which monitors $B_s$ versus $B_d$ oscillations. The concern with regard to $\xi$ that we have been voicing in the past couple of years is that the widely used central value ($\sim 1.15$) provided by some lattice calculations, is quite likely an
The worry is based on the observation that most lattice calculations of $\xi$ use the “indirect” method in which the matrix element of the 4-quark operator is parametrized in terms of a B-parameter and the pseudoscalar decay constant. In actual numerical calculations of the decay constant, the usual practice is to fit linearly to the light quark mass ($m_q$) dependence. Since the original matrix element of interest depends quadratically on the decay constant, this procedure is unlikely to get the right coefficient, for example, of $m_q^2$. Instead, one may equally well directly calculate (on the lattice) the mixing matrix element without introducing B-parameters in calculating the $\Delta F = 2$ mixing matrix elements is purely a historical accident. Recall that originally B-parameters were introduced in such calculations for dealing with the analogous $K - \bar{K}$ mixing matrix element. There $f_K$ was known experimentally and in the phenomenological literature $B_K$ was introduced as a measure of deviation of the matrix element for the idealised case of vacuum saturation. For B-mesons, the decay constant is not known from experiment and determining that from theory becomes the central issue. One can equally well directly calculate (on the lattice) the mixing matrix element without introducing decay constant or B-parameter. The direct method seems to give largish central value but within (rather large) errors is consistent with the indirect method. To be on the safe side one ought to use both methods with very good control over errors in each case and then an average of the two methods should be used for $\xi$.

Meantime, in light of this observation, we adopt a conservative attitude and had used $\xi = 1.16 \pm 0.08$ with an error that is considerably bigger compared to Hocker et al. and Ciuchini et al. and we have been stressing for quite sometime that their errors are an underestimate. We also examined the effect on $\sin 2\beta$ of an increase in $\xi$ (including a larger error) along with a decrease in $B_K$, following indications from DWQ. (See Table 2 from [3].) With the size of uncertainties currently present, $\sin 2\beta$ is hardly effected with $(\sin 2\beta)^{SM} = 0.72 \pm 0.10$ giving us additional confidence in the comparison of the experimental measurements to the predictions of the SM. However, the point still remains that use of theory input has its limitations and we must try hard to develop methods that can yield angles of the unitarity triangle very “cleanly”, i.e. with zero theory error and with no theoretical assumptions.

We emphasize here that final states containing $D^0$, $\bar{D}^0$ in decays of charged or neutral B’s

| Input Quantity | Atwood & Soni |
|----------------|--------------|
| $R_{ac} \equiv |V_{ub}/V_{cb}|$ | 0.85 ± 0.17 |
| $F_{B_d} \sqrt{B_{B_d}}$ | 230 ± 50 MeV |
| $\xi$ | 1.16 ± 0.08 |
| $B_K$ | 0.86 ± 0.15 |

| Output Quantity | Atwood & Soni |
|----------------|--------------|
| $\sin 2\beta$ | 0.70 ± 0.10 |
| $\sin 2\alpha$ | 0.73 ± 0.10 |
| $\gamma$ | 0.72 ± 0.10 |
| $\eta$ | 46.2° ± 9.1° |
| $\tilde{\rho}$ | 48.7 ± 8.5 |
| $|V_{td}/V_{ts}|$ | 0.30 ± 0.05 |
| $\Delta m_{B_s}(ps^{-1})$ | 0.32 ± 0.05 |
| $J_{CP}$ | 52.3 ± 12.1 |
| $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ | 25 ± 0.07 |
| $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ | 0.185 ± 0.015 |
| $J_{CP}$ | 19.8 ± 3.5 |
| $J_{CP}$ | (2.55 ± 0.35) \times 10^{-5} |
| $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ | (0.67 ± 0.10) \times 10^{-10} |
| $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ | (0.225 ± 0.065) \times 10^{-10} |
can be used very effectively to determine all three angles with essentially zero theory error, i.e. $\lesssim 0.1\%$.

$B^\pm$ and $B^0$ decays to (e.g.) $K^\pm D^0(\bar{D}^0)$ and $K^0/\bar{K}^0 D^0(\bar{D}^0)$ respectively involve only decays via two tree graphs ($b \rightarrow c$ and $b \rightarrow u$), no penguin strong or electroweak are involved. Methods based on these using direct CP and time dependent CP respectively can give all three angles with no theory error or theory assumptions. In the case of $B^\pm \rightarrow K^\pm D^0(\bar{D}^0)$, the $D^0(\bar{D}^0)$ decays must proceed to CP-non-eigenstates. Using only $D^0, \bar{D}^0$ decays to CP eigenstates as advocated in [16] has the difficulty that the suppressed branching ratio is not accessible to experiment as $D^0, \bar{D}^0$ flavor is difficult to tag in these B-decays [17]. Estimating this branching ratio with the use of theory input or assumptions defeats our original goal of determination of the UT with zero theory error. A second undesirable feature of using only CP-eigenstates of $D^0$ is that the resultant CP asymmetry is small $\sim 0$ (a few %), with CPNES method [19] the asymmetries are large.

I.1 $\gamma$ with zero theory error

This is a uniquely clean method with no theoretical assumption and involving no penguin contribution, QCD or EW. Interference between two tree graphs, $b \rightarrow u$ and $b \rightarrow c$ is exploited. Consequently, the limiting theory error is completely negligible. Furthermore, the interference between the amplitudes contributing to common final states of $D^0, \bar{D}^0$ that are not CP-eigenstates is large resulting in large direct CP asymmetry $\sim$ tens of percents which can be studied, in principle, at any $B$-facility [18]. Furthermore, while only two modes are essential for the analysis, many modes are available. Thus discrete ambiguity in determination of $\gamma$ can be removed by use of several modes.

As a specific example one may consider $B^- \rightarrow K^- D^0(\bar{D}^0)$ with $D^0(\bar{D}^0) \rightarrow K^+ \pi^-$ so the overall reaction being studied is just $B^- \rightarrow K^- K^+ \pi^-$. It is important to understand that the suppressed branching ratio $B^- \rightarrow K^- D^0$ is not needed and in fact is an output, i.e. is determined in the analysis along with $\gamma$. The FS (e.g. $K^- K^+ \pi^-$) results from interference between two amplitudes one of which is color allowed ($B^- \rightarrow K^- D^0$) but doubly-Cabibbo-suppressed ($D^0 \rightarrow K^+ \pi^-$) whereas the other is color-suppressed ($B^- \rightarrow K^- D^0$) but Cabibbo allowed ($\bar{D}^0 \rightarrow K^+ \pi^-$).

$B^-$ and $B^+$ decay amplitudes to two such final states (say, e.g. $B^\pm \rightarrow K^\pm D^0, \bar{D}^0$ with $D^0, \bar{D}^0 \rightarrow K^\pm \pi^\mp$ and $K^+ K^- \pi^0$) result in 4 equations and 4 unknowns [18]. The four unknowns are the 2 strong phases (one for each final state), the CP-odd weak phase $\gamma$ that we are after and the branching ratio (denoted by $b$) of $B^- \rightarrow K^- D^0$ which is extremely difficult to measure experimentally due to severe backgrounds.

One of the advantages of the method is that it can be applied to many modes e.g. $B^- \rightarrow K^-, K^- D^0, (\bar{D}^0)$ with $D^0, \bar{D}^0 \rightarrow K^+ \pi^-, K^+ \rho^-, K^{*+} \pi^-, K^+ a_1^-, K^{+\pi^+ \pi^-}$ etc. Another important point is that the method allows to include $D^0, \bar{D}^0$ decays to CP-eigenstates [13] so long as one CP-non-eigenstate is also included. So as a specific example one could use $D^0, \bar{D}^0 \rightarrow K^+_\pi^0$ (i.e. a CPES) with $D^0, \bar{D}^0 \rightarrow K^+ \pi^-$ (a CPNES). The point is that once one CPNES is included sufficient number of observables become available to solve for the branching ratio $B^- \rightarrow K^- D^0$ as an output along with $\gamma$.

Fig. 1 illustrates use of only two modes, one CPNES ($K^+ \pi^-$) and one CPES ($K^0_\pi \pi^0$) of $D^0, \bar{D}^0$ assuming $\bar{N}_B = (#\text{ of } B\bar{B}\text{ pairs}) \times \text{acceptance} = 10^8$. Solutions to the equations for the two modes intersect in four places in the $b$-$\gamma$ plane. Regions with 68%, 90% and 99% CL are shown. Multiple solutions are clearly a limitation. Fig. 2 shows the result when several more modes of $D^0, \bar{D}^0$ are also combined. Now the improvement over Fig. 1 is significant and $\gamma$ with a (1-sigma) accuracy of about $7^\circ$ is obtained; in this calculation true value of $\gamma$ is assumed to be 60 degrees.
Figure 1: The likelihood distribution is shown as a function of $\gamma$ and $b(K^*)$ (which is branching ratio of $B^- \to K^*-\bar{D}^0$) assuming that $N_B = 10^8$ and assuming only the $K^+\pi^-$ and $K_s\pi^0$ modes of $D^0$ are measured. The outer edge of the shaded regions correspond to 90% confidence while the inner edge corresponds to 68% confidence. The solid lines show the locus of points which give the $K^+\pi^-$ results while the short dashed curve shows the points which give the $K_s\pi^0$ results.

Figure 2: The likelihood distribution as in Fig.1 but now using 6 decay modes of $D^0, \bar{D}^0$. The solution for the $K^+\pi^-$ data is shown with the solid curve; that for the $K_s\pi^0$ is shown with the short dashed curve; $K^+\rho^-$ is shown with the long dashed curve; $K^+a_1^-$ is shown with the dash-dot curve; $K_s\rho^0$ data is shown with the dash-dot-dot curve and the solution for the $K^*+\pi^-$ data is shown with the dash-dash-dot curve.

I.2 Extracting $\beta-\alpha$ with zero theory error

The angle $\delta \equiv \beta - \alpha + \pi = 2\beta + \gamma$, can be obtained by time dependent $CP$ asymmetry measurements in $B^0, \bar{B}^0 \to K^0D^0(\bar{D}^0)$. As in the case of $B^\pm$ decays to $K^\pm D^0(\bar{D}^0)$ only tree graph decays are involved and no penguin, strong or electroweak, enters. Also once again it is simply a matter of writing down a bunch of equations and providing enough experimental information via observables to render the system completely soluble yielding the unitarity angles that we are after. Indeed there is so much redundancy that not only $\alpha$ but also $\beta$ can be determined in this way providing a valuable comparison against the $\beta$ determined from $B \to \psi K_s^0$.

Some aspects of this method have been previously studied by quite a few authors. In principle, time-dependent CP asymmetry (TDCPA) measurements in $B^0(\bar{B}^0) \to K_s^0D^0, K_s^0\bar{D}^0$ is all that is needed to extract $\delta$. However, as in the case of $B^\pm \to K^\pm D^0(\bar{D}^0)$, the $D^0, \bar{D}^0$ flavor tagging is problematic. The resolution of this problem using CP-non-eigenstates of $D^0, \bar{D}^0$ (just as in the case of $B^\pm$ decays) has been suggested to give information on $\delta$ and also possibly on $\beta$. Actually in this case $D^0(\bar{D}^0)$ decays to CP eigenstates can also be used. However, if only an exclusive CPES (e.g. $K_s\pi^0$) is used then the number of observables is 3 and the number of unknowns, including $\delta$, is 4 so not information is available for a separate solution.

Combining the CPES and CPNES methods seems very effective as the number of unknowns involved for the CPES case are just a subset of those for the CPNES case. If one exclusive CPNES mode (such as $K^+\pi^-$) and a CPES mode (e.g. $K_s\pi^0$) are used then we get 9 observables for 5 unknowns; if $\beta$ is treated as an unknown even then the system of equations is solvable.

Of course several CPNES modes can be included. For each CPNES that is included we have 6 new observables at the expense of only one additional unknown. Many exclusive modes are available e.g. $K^-\pi^+(Br \sim 3.8\%), K^-\rho^+(10.8\%), K^{*-}\pi^+(5.0\%), K^*0\pi^0(3.1\%), K^+\rho^- (6.1\%), \text{and } K^{*-}a_1^+(7.1\%)$, for a total of 36%. A nice 4-body mode with all charged track is
Figure 3: The $\chi^2_{\min}$ vs. $\delta$ for the toy model calculation given $\hat{N}_B = 10^9$. The thin solid line is the result for $D^0 \to K^-\pi^+$ alone. The dashed line the result for CPES containing $K_S$ together with related CPES containing $K_L$. The dotted lines the result obtained combining $K^-\pi^+$ with CPES containing $K_S$. The dashed-dotted line gives the result for $K^- + X$ alone and the thick solid line combines $K^- + X$ with CPES containing $K_S$. (Note $\delta = 110^\circ$ is assumed here)

Figure 4: The $\chi^2_{\min}$ vs. $\beta$ for the toy model calculation given $\hat{N}_B = 10^9$ using $K^- + X$ with CPES containing $K_S$. (Note $\beta = 25^\circ$ is assumed.)

$K^-\pi^+\pi^+\pi^-$ (BR > .6%).

In fact a very nice way to solve for $\delta$ (and $\beta$ simultaneously) is to generalize the above exclusive (CPNES) case to inclusive CPNES via $D^0 \to K^- + X$. Then the Br$\sim$ 53%; one has 6 observables and 6 unknowns. So it is a solvable system but with discrete ambiguities. A very promising way to overcome the ambiguities is to combine this inclusive CPNES case with a CPES mode.

Fig. 3 illustrates the results of our study of extracting $\delta$ via this method. Combining the inclusive CPNES ($D^0 \to K^-+X$) with exclusive CPES seems to do a very good job of eliminating the ambiguities and give $\delta$ with an error of $\pm 2.5^\circ$ (the true value of $\delta$ in this case study is $110^\circ$). (See also Table 3.) In this example we have used $\hat{N}_B = (\text{number of } B-\bar{B} \text{ mesons})$ (acceptance) $= 10^9$.

A very important feature of this method is that you can also use it to solve for $\beta$ very cleanly, i.e. no theory assumptions are involved. In fact this method is even cleaner than the $B \to \psi K^0_s$ method as the latter does receive some (although very small) penguin contributions whereas the former has none. However, it is not as efficient as the $\psi K^0_s$ method, i.e. more number of B mesons are needed to get similar quantitative accuracy on $\beta$. Fig.4 illustrates how well the method works for determining $\beta$. Here the input used is the one given in the 5th row of Table 3. With $\hat{N}_B = 10^9$ the one sigma error on $\beta$ is around 2 degrees.

Table 4 shows a brief summary for determination of the three angles of the unitarity triangle. It also highlights the limiting theory error of each method. With the large data samples that should become available, it is reasonable to expect that with these $B \to \bar{K}(K^*)D^0$ methods all three angles could be determined precisely providing an extremely important test of the CKM paradigm. A notable feature of these methods is that in charged or neutral B-decays final states relevant to extracting all three angles cleanly all contain $D^0$ or $\bar{D}^0$; this should help in improving the experimental efficiency.

Recall that two prominent methods for $\alpha$, both using time dependent CP, have been studied for quite some time. The first method requires CP asymmetry measurements in $B^0(\bar{B}^0) \to \pi^+\pi^-$ as well as BR for $B^0, \bar{B}^0 \to \pi^0\pi^0$ and $B^\pm \to \pi^\mp\pi^0\pi^0$. Although these measurements should enable one to perform an isospin analysis and remove the penguin contribution the value of $\alpha$ thus deduced suffers from contamination from electroweak penguin contribution as EWP evade the isospin analysis. To that extent this method for $\alpha$ determination has some residual theory
Table 3: Attainable one sigma accuracy with various data sets given $N_B = 10^9$; note the 2nd and 5th cases are omitted from Fig 3 for clarity.

| Case | Accuracy |
|------|----------|
| CPES with $K_S$ and with $K_L$ | $\pm 8.5^\circ$ |
| CPNES $K^- \pi^+$ with $K_S$ and with $K_L$ | $\pm 5^\circ$ |
| The CPNES $K^- \pi^+$ together with CPES, both with $K_S$ only | $\pm 9.0^\circ$ |
| $K^- + X$ together with $K_S$ CPES | $\pm 2.5^\circ$ |
| $K^- + X$ together with $K_S$ as well as $K_L$ CPES | $\pm 2.4^\circ$ |

error and has to invoke model dependent estimates in the evaluation of the EWP contribution.

The key experimental difficulty in this set of measurements is the $2\pi^0$ mode due to the small branching ratio ($\sim 2 \times 10^{-7}$) that is expected, made harder by the relative low detection efficiency, perhaps also compounded by the fact that the $\pi^0$’s are very energetic giving rise to a small opening angle between the $\gamma$-pairs. Presumably, these experimental difficulties will be surmounted as luminosities improve. Already the two groups have made attempts to measure TDCPA in $B^0 \to \pi^+ \pi^-$ which should become quite accurate relatively shortly. However, the interpretation of this in terms of the angle $\alpha$ requires important input from theory.

Another important method wherein isospin analysis, can be used is $B \to \rho \pi$. In this approach one can make use of resonance effects in the Dalitz plot; however, some model dependence is likely to occur in handling the continuum of $B \to 3\pi$. Once again, EWP are assumed to be negligible and this is also an important source of the limiting theory error in the $\rho \pi$ analysis.

Recently we have also proposed two other methods for extracting $\alpha$ and $\gamma$ that use penguin and tree interference effects and therefore are also not completely clean. Table 5 shows a sample of these methods that uses penguin-tree interference along with the very clean methods of Table 4. It is also important to emphasize that $B \to KD^0$ methods require roughly the same number of B’s as the $\rho \pi$ or $\pi \pi$ methods.

While our emphasis here has been on $B^+, B^-, B^0, \bar{B}^0$ mesons, if time dependent oscillation studies in $B_s, \bar{B}_s$ become feasible, then a clean way to get $\gamma$ may also be accessible via final states of $D_s^+(D_s^-)K^- (K^+)$ or their vector counterparts. Experimental feasibility especially in a hadronic environment of some of these is studied in [3].

Let us briefly remark that there is also a very clean way to get the magnitude of the CP-odd CKM phase directly from $K_L \to \pi^0 \nu \bar{\nu}$.

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3. The phrase zero theory error will be used here loosely to emphasize negligible theory error i.e. $\lesssim 0.1\%$. 
Table 4: Pristine methods for extracting the UT with negligible theory error. Lower range of # of B’s is the estimate for initial determination and the upper for precise measurements.

| Angle | Mode(s) | Ref. | Type of CP | Pollution | Limiting Theory Error | # of B’s Needed /10^8 |
|-------|---------|------|------------|-----------|-----------------------|-----------------------|
| $\beta$ ($\phi_1$) | $B \rightarrow \psi K^0$ | Bigi + Sanda | time dep. | $\sim 1\%$ | $\lesssim 1\%$ | $\sim 1\%$ | 0.5–5 |
| $\gamma$ ($\phi_3$) | $K^\pm D^0_0(\bar{D}^0_0)$ $\rightarrow K^+\pi^-$ | Atwood Dunietz, Soni | Direct | 0 | 0 | $\sim 0$ | 5–50 |
| $\alpha(\phi_2)$ and $\beta(\phi_1)$ | $K^0 D^0_0(\bar{D}^0_0)$ ↓ CPES,CPNES, Inclusive | Atwood +AS | Direct | 0 | 0 | $\sim 0$ | 5–50 |

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23. With regard to $K_L^*$ in this Table, note that their detection is not necessary for this method to work. However, they can aid the analysis considerably as they add to the number of
Table 5: Illustrative sample of methods for extracting the UT. Lower range of # of B’s is the estimate for initial determination and the upper for precise measurements.

| Angle  | Mode(s) | Ref. | Type of Theory Needed | Pollution | Limiting Error | # of B’s Needed /10^8 |
|--------|---------|------|-----------------------|-----------|----------------|----------------------|
| $\beta$ ($\phi_1$) | $B \rightarrow \psi K^0$ | Bigi + Sanda | time dep. | $\sim 1\%$ | $\lesssim 1\%$ | $\sim 1\%$ | 0.5–5 |
| $\gamma$ ($\phi_3$) | $K^\pm D_0^0(\bar{D}_0^0) / K^+\pi^-$ | Atwood Dunietz, Soni | Direct | 0 | 0 | $\sim 0$ | 5–50 |
| $\alpha(\phi_2)$ and $\beta(\phi_1)$ | $K^0 D_0^0(\bar{D}_0^0) \downarrow$ CPES,CPNES, Inclusive | Atwood +AS | time dep. | 0 | 0 | $\sim 0$ | 5–50 |
| $\gamma(\phi_3)$ | $\pi\pi$ | Gronau + London | time dep. | $\approx 30\%$ | few% | $\sim 5$–10% | 10–50 |
| $\alpha$ ($\phi_2$) | $\rho\pi$ | Quinn et al | " | $\approx 30\%$ | " | $\sim 5$–10% | 5–50 |
| | $\rho(\omega)P$ $\left( P = \pi, \eta, a_0 \ldots \right)$ | Atwood + Soni | " | $\approx 30\%$ | " | $\sim 1$–2% | 5–50 |
| | $\rho\pi + \rho(\omega)P$ | Comb. of above 2 | " | " | " | $\sim 1\%$ | 5–50 |
| $\alpha(\phi_2)$ and $\beta(\phi_1)$ | $B_{\pm}, B_{\mp}^0(B_{\pm}^0)$ $\rho\omega, \bar{K}^{*0}\rho^+$ | Atwood +AS | Direct | $\approx 20\%$ | $\approx 5\%$ | $\lesssim 5\%$ | 5–50 |
| $\gamma(\phi_3)$ | $B \rightarrow K^{*}\rho(\omega)$ | " | " | " | " | " | 5–50 |
observables; see Ref. For numerical results in this Table we assumed $K_L$ detection efficiency to be half of that for $K_S$.

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