About new forecasting methodology for digital control processes

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Abstract. The article substantiates the need for prompt adjustment of the dynamics of the controlled parameter in the management process and the development of a methodology for prompt adjustment in the proportional-integral-differential (PID) control cycle (in real time). New methods of mixed and optimal digital control on the basis of appropriate mathematical models of digital PID control are proposed, which are distinguished by the formation of the best control action in the tact of preventive evaluation of the control object response.

1. Introduction
The mathematical model of the automatic controller has the form [1-3]:

\[ U(t) = k_p \cdot \Delta x(t) + k_i \cdot \int_{\tau=0}^{\tau=t} \Delta x(\tau) d\tau + k_d \cdot \frac{d\Delta x(t)}{dt}, \]

where \( U(t) \) – control action; \( \Delta x(t) \) – the mismatch (discrepancy) at the current moment \( t \); \( \Delta x(t) = x_0(t) - x(t) \) with setting action \( x_0(t) \) and adjustable parameter \( x(t) \); \( k_p, k_i, k_d \) are the setup variables.

Taking into account discrete time \( t = n \cdot T \) \((n = 0, 1, 2, \ldots)\), discrete transformations of integral (\( I = \int_{\tau=0}^{\tau=nT} \Delta x(\tau) d\tau \)) and differential (\( D = \left. \frac{d\Delta x(t)}{dt} \right|_{\tau=nT} \)) components, (1) is transformed into the expression [4-6]:

\[ U(nT) = k_p \cdot \Delta x(nT) + k_i \cdot \int_{\tau=0}^{\tau=nT} \Delta x(\tau) d\tau + k_d \cdot \left. \frac{d\Delta x(t)}{dt} \right|_{\tau=nT} \]

and the theoretical multiple model of \( \Omega \) controller (hereinafter, the model) will be presented in the form of a record (term) of five components:

\[ \Omega : \quad U(t) = (k_p, k_i, k_d, I, D). \]

Digital control is a consequence of the synthesis process of the analog regulator, which predetermines the dynamics of the transient and set modes by the synthesized structure of the controller.
This is due to the fact that one model is actually implemented (3) without the possibility of adjusting its components in the process of controller’s functioning.

In addition, probably, in any design of the regulator by its models, the real dynamics of regulation will not coincide with the model dynamics of the regulator, that is, with the results obtained in the preliminary modeling.

It can be argued that before sending a control action to actuators, it is necessary to know how the regulatory object reacts to this impact.

To identify the components by means of which the organization of various processes of digital regulation is achieved, we will allocate them with curly braces in the theoretic multiple model. Since the functional $J$ and the operator $D$ are fixed and unchanged in the traditional model (3), it is the change in the set parameters $k_p, k_i, k_D$ in the control method that leads to the organization of various digital control processes:

$$ \Omega : \quad U(nT) = (\{k_p\}, \{k_i\}, \{k_D\}, I, D). $$

When it is possible to form not only this model, but also other variants of control actions, for example, of the type:

$$ \Omega : \quad U(nT) = (\{k_p\}, \{k_i\}, \{k_D\}, \{I_p, I_T, I_S\}, D_p), $$

$$ \Omega : \quad U(nT) = (\{k_p\}, \{I_p, I_T, I_S\}, k_i, k_D, D_p), $$

$$ \Omega : \quad U(nT) = (\{k_p\}, \{k_i\}, \{k_D\}, \{I_p, I_T, I_S\}, D_p), $$

$$ \Omega : \quad U(nT) = (\{k_p\}, \{k_i, k_D\}, \{I_p, I_T, I_S\}, D_p) \quad \text{and so on,} $$

then the efficiency of regulation will be higher, taking into account the results of these different variants in the reaction of the control object.

The current control dynamics is determined by the mismatch at the previous moment of time $\Delta x((n-1)T)$ and the knowledge of how the control object will react at the current time $t = nT$ to the calculated control action made at the previous moment of time. The idea of taking into account the results of “forecasting” on the formation of the control action requires a change in the theoretic-multiple model of the regulatory process by integrating the mismatch operator into it, the time factor ($t$) and the functional $I$ and the operator $D$ within the boundaries of the selected time intervals, for example, in the form:

$$ \Omega : \quad U(t) = (k_p, k_i, k_D, \Delta x, t_B, t_E, \{I\}, \{D\}, T, t), \quad (4) $$

where $[t_B, t_E]$ is an prediction interval.

Thus, in order to avoid undesirable temporal deviations of the regulated parameter and to be able to correct the dynamics in the course of operation, it is expedient to develop a methodological direction of the organization of digital control for models of the type (4) to formation of the best control action in the moment of preventive assessment of reaction of the control object and create methods for operative correction and control dynamics in the control cycle (in real time).

As in control theory there are no methods of formation of control actions of the PID controller with the involvement of prediction procedures for preventive assessment of the response of control object and methods of constructing an adequate assessment formula regulation, the issues of developing methodological directions of the organization of digital control system for models such as (4) constitute the basic problems of digital control with forecasting.

2. **Control with forecasting of the situation on a step forward**
The simplest version of model (4) can be the model using the formula of rectangles \((I_R)\), trapezoidal formula \((I_T)\) and for Simpson’s rule \((I_S)\) to represent an integral component [5-7]:

\[
\Omega : \quad U(t) = (k_p, k_j, k_d, \Delta x, nT, (n+1)T, \{I_R, I_T, I_S\}, D, T, t).
\]

with a prediction interval:

\[
[t_R, t_E] = [nT, (n+1)T]
\]

and a prediction function in the form of an estimate of the mismatch at the predicted (following from the current \(t = nT\) ) step \(t = (n+1)T\), followed by the transfer to the actuators of that control action, which will lead to a minimum mismatch in the next step \(t = (n+1)T\).

Here we distinguish the methods of mixed and “optimal” digital control.

2.1. Mixed digital control

The idea of mixed digital control is to select several mathematical models of the controller, calculate the control actions corresponding to the controller models, to search for the minimum mismatch \(\Delta x_{n, M} ((n+1)T)\):

\[
\Delta x_{n, M} ((n+1)T) = \min \{ |\Delta x_1 ((n+1)T, U_1 (nT))|, |\Delta x_2 ((n+1)T, U_2 (nT))|, \ldots, |\Delta x_k ((n+1)T, U_k (nT))| \}
\]

and choosing the best control action \(U_M (nT)\) corresponding to the deviation:

\[
U_M (nT) = U_j (nT) \quad \text{for} \quad \Delta x_{n, M} ((n+1)T) = \Delta x_j ((n+1)T) \quad (i = 1, 2, \ldots, k).
\]

In this case, when calculating the control actions \(U_j (nT)\) at the current step \(t = nT\) is calculated taking into account the previous values \(U_M ((n+1)T)\) and \(\Delta x_M = \Delta x_{n, M} ((n+1)T)\).

The mixed digital control method can be described by the following sequence of actions:

- a set of discrete mathematical models for forming a control action with the corresponding values of the set parameters \(k_p, k_j\) and \(k_d\) is selected:
  
  \[
  M_1, M_2, \ldots, M_{k-1}, M_k ;
  \]

- a criterion for selecting models is given in case of mismatch coincidence of, for example, the residual coincide:
  
  \[
  |\Delta x_1 ((n+1)\cdot T)| = |\Delta x_2 ((n+1)\cdot T)| = \ldots = |\Delta x_{k-1} ((n+1)\cdot T)| = |\Delta x_k ((n+1)\cdot T)|
  \]

  for calculating control actions, the model is selected \(M_j (j = 1, 2, \ldots, k)\);

- setting is given \(x_0 (t)\);

- for the moment of time \(t = nT\) \(n = 0, 1, \ldots\), control actions are calculated:
  
  \[
  U_1 (nT, U_M ((n-1)\cdot T), \Delta x_M (nT), \Delta x_M ((n-1)\cdot T), \Delta x_M ((n-2)\cdot T), \ldots),
  \]

  \[
  U_2 (nT, U_M ((n-1)\cdot T), \Delta x_M (nT), \Delta x_M ((n-1)\cdot T), \Delta x_M ((n-2)\cdot T), \ldots), \ldots,
  \]

  \[
  U_{k-1} (nT, U_M ((n-1)\cdot T), \Delta x_M (nT), \Delta x_M ((n-1)\cdot T), \Delta x_M ((n-2)\cdot T), \ldots),
  \]

  \[
  U_k (nT, U_M ((n-1)\cdot T), \Delta x_M (nT), \Delta x_M ((n-1)\cdot T), \Delta x_M ((n-2)\cdot T), \ldots);
  \]
• for the moment of time \( t = (n + 1) \cdot T \), the reactions of the control object are modeled \( x_1((n + 1) \cdot T), x_2((n + 1) \cdot T), \ldots, x_{k-1}((n + 1) \cdot T), x_k((n + 1) \cdot T) \):

\[
x_1((n + 1) \cdot T, U_1(nT), x_M(nT), \ldots), x_2((n + 1) \cdot T, U_2(nT), x_M(nT), \ldots), \ldots,
\]

\[
x_{k-1}((n + 1) \cdot T, U_{k-1}(nT), x_M(nT), \ldots), x_k((n + 1) \cdot T, U_k(nT), x_M(nT), \ldots);
\]

• for the moment of time \( t = (n + 1) \cdot T \), mismatches are calculated \( \Delta x_1((n + 1) \cdot T), \Delta x_2((n + 1) \cdot T), \ldots, \Delta x_{k-1}((n + 1) \cdot T), \Delta x_k((n + 1) \cdot T) \):

\[
\Delta x_1((n + 1) \cdot T, x_1((n + 1) \cdot T), x_0), \Delta x_2((n + 1) \cdot T, x_2((n + 1) \cdot T), x_0), \ldots,
\]

\[
\Delta x_{k-1}((n + 1) \cdot T, x_{k-1}((n + 1) \cdot T), x_0), \Delta x_k((n + 1) \cdot T, x_k((n + 1) \cdot T), x_0);
\]

• the minimum absolute value of the mismatch at the time \( t = (n + 1) \cdot T \) is calculated taking into account the criterion for selecting point 2:

\[
\Delta x_{\text{min}, M}((n + 1) \cdot T) =
\]

\[
= \min \left\{ |\Delta x_1((n + 1) \cdot T)|, |\Delta x_2((n + 1) \cdot T)|, \ldots, |\Delta x_{k-1}((n + 1) \cdot T)|, |\Delta x_k((n + 1) \cdot T)| \right\};
\]

• the best control action is chosen \( U_M(nT) \):

\[
U_M(nT) = U_i(nT) \quad \text{when} \quad \Delta x_{\text{min}, M}((n + 1)T) = \Delta x_i((n + 1)T) \quad (i = 1, 2, \ldots, k).
\]

2.2. “Optimal” digital control

“Optimal” digital control is aimed at implementing the same idea, which is embodied in mixed digital control. Several mathematical models \( M_1, M_2, \ldots, M_{k-1}, M_k \) of the controller are being selected, calculation of control actions \( U_1(nT), U_2(nT), \ldots, U_{k-1}(nT), U_k(nT) \) corresponding to controller models, searching for a minimum mismatch \( \Delta x_{\text{min}, O}((n + 1)T) \):

\[
\Delta x_{\text{min}, O}((n + 1)T) = \min \left\{ |\Delta x_1((n + 1)T, U_1(nT))|, |\Delta x_2((n + 1)T, U_2(nT))|, \ldots,
\]

\[
|\Delta x_{k}(((n + 1)T, U_k(nT))| \right\}
\]

and selecting the best control action \( U_O(nT) \), corresponding to a deviation \( \Delta x_{\text{min}, O}((n + 1)T) \):

\[
U_O(nT) = U_i(nT) \quad \text{at} \quad \Delta x_{\text{min}, O}((n + 1)T) = \Delta x_i((n + 1)T) \quad (i = 1, 2, \ldots, k).
\]

However, in this case, when calculating control actions \( U_i(nT) \) the current step \( t = nT \) is carried out not taking into account the previous values \( U_O((n - 1)T) \) and \( \Delta x_O = \Delta x_{\text{min}, O}((n - 1)T) \), but based on the values:

\[
U_i((n - 1)T) \quad \text{and} \quad \Delta x_O = \Delta x_i((n - 1)T).
\]

Thus, the “optimal” digital control method is the following:

• determining mathematical models of digital control:

\[
M_1, M_2, \ldots, M_{k-1}, M_k;
\]

• setting \( x_0(t) \);

• determining the criteria for selecting a model is analogous to a mixed regulation;
• for the moment of time \( t = nT \) (\( n = 0, 1, \ldots \)) control actions \( U_1(nT), U_2(nT), \ldots, U_{k-1}(nT), U_k(nT) \) are calculated according to models \( M_1, M_2, \ldots, M_{k-1}, M_k \):

\[
U_1(nT, U_1((n-1)\cdot T), \Delta x_1(nT), \Delta x_1((n-1)\cdot T), \Delta x_1((n-2)\cdot T)),
\]

\[
U_2(nT, U_2((n-1)\cdot T), \Delta x_2(nT), \Delta x_2((n-1)\cdot T), \Delta x_2((n-2)\cdot T)), \ldots,
\]

\[
U_{k-1}(nT, U_{k-1}((n-1)\cdot T), \Delta x_{k-1}(nT), \Delta x_{k-1}((n-1)\cdot T), \Delta x_{k-1}((n-2)\cdot T)),
\]

\[
U_k(nT, U_k((n-1)\cdot T), \Delta x_k(nT), \Delta x_k((n-1)\cdot T), \Delta x_k((n-2)\cdot T));
\]

• the simulation of the response of the control object \( x_1((n+1)\cdot T), x_2((n+1)\cdot T), \ldots, x_{k-1}((n+1)\cdot T), x_k((n+1)\cdot T) \) at the time \( t = (n+1)\cdot T \) is given:

\[
x_1((n+1)\cdot T, U_1(nT), x_0(nT), \ldots), x_2((n+1)\cdot T, U_2(nT), x_0(nT), \ldots), \ldots,
\]

\[
x_{k-2}((n+1)\cdot T, U_{k-1}(nT), x_0(nT), \ldots), x_{k-1}((n+1)\cdot T, U_k(nT), x_0(nT), \ldots);
\]

• calculation of mismatches \( \Delta x_1((n+1)\cdot T), \Delta x_2((n+1)\cdot T), \ldots,
\]

\[
\Delta x_{k-1}((n+1)\cdot T), \Delta x_k((n+1)\cdot T), \ldots,
\]

\[
\begin{align*}
\Delta x_1((n+1)\cdot T, x_1((n+1)\cdot T), x_0), & \quad \Delta x_2((n+1)\cdot T, x_2((n+1)\cdot T), x_0), \ldots, \\
\Delta x_{k-2}((n+1)\cdot T, x_{k-2}((n+1)\cdot T), x_0), & \quad \Delta x_{k-1}((n+1)\cdot T, x_{k-1}((n+1)\cdot T), x_0). 
\end{align*}
\]

• a criterion for selecting models is given in case of mismatch coincidence, for example, the residual coincide:

\[
|\Delta x_1((n+1)\cdot T)| = |\Delta x_2((n+1)\cdot T)| = \ldots = |\Delta x_{k-1}((n+1)\cdot T)| = |\Delta x_k((n+1)\cdot T)|
\]

for calculating the control actions, the model \( M_j (j = 1, 2, \ldots, k) \) is selected;

• the final control action \( U_0(nT) \) is calculated.

3. Conclusion
Experimentation with new models and methods of digital regulation with forecasting a step forward [8] showed a better dynamics of the regulated parameter in comparison with traditional mathematical models.

The paper proposed:

• the theoretic multiple models of the digital PID controller based on the discretization of the continuum model, differing in the component composition that significantly influences the dynamics of regulation;

• new methods of mixed and optimal digital control based on known mathematical models of digital PID control, which are distinguished by the formation of the best control action in the tact of preventive evaluation of the control object response.

The main result of the study can be considered. The proposed new methods of mixed and optimal digital control can become the basis of the methodological direction of the organization of digital control on the formation of control actions at the time of preventive evaluation of the reaction of the control object.
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