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Finite element analysis of the biaxial cyclic tensile loading of the elastoplastic plate with the central hole: asymptotic regimes

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Abstract. For elastoplastic structure elements under cyclic loading three types of asymptotic behavior are well known: shakedown, cyclic plasticity or ratcheting. In structure elements operating in real conditions ratcheting must always be excluded since it caused the incremental fracture of structure by means of the accumulation of plastic strains.

In the present study results of finite-element (FEM) calculations of the asymptotical behavior of an elastoplastic plate with the central circular and elliptic holes under the biaxial cyclic loading for three different materials are presented.

Incremental cyclic loading of the sample with stress concentrator (the central hole) is performed in the multifunctional finite-element package SIMULIA Abaqus. The ranges of loads found for shakedown, cyclic plasticity and ratcheting are presented. The results obtained are generalized and analyzed. Convenient normalization is suggested. The chosen normalization allows us to present all computed results, corresponding to separate materials, within one common curve with minimum scattering of the points. Convenience of the generalized diagram consists in a possibility to find an asymptotical behavior of an inelastic structure for materials for which computer calculations were not made.

1. Introduction
Asymptotic states of inelastic structures subjected to cyclic loading cause the particular interest in Solid Mechanics. Response of the inelastic structure under periodic mechanical loading is complicated and may contain inelastic strains. The reason of these difficulties is the need of calculations, which include the whole history of structure loading.

Elements of structures are often subjected to variable temperatures and loadings. If the body is elastic, then the durability is defined by the fatigue material data and fracture comes after large number of cycles. If the body is elastoplastic, the dangerous state can be reached after rather small number of cycles. It is necessary to distinguish two cases. The first one is when fracture comes owing to alternation of the plastic strains sign (e.g. after plastic stretching – plastic compression etc.). It is alternate plasticity (plastic or low-cyclic fatigue). The second one is when plastic strains increase with every cycle, which leads to its accumulation and then failure (progressing plastic deformation - ratcheting) [1]–[24].

In structural design shakedown is considered to be safe, ratcheting should be avoided. In this regard, requirement in knowledge on the early steps of configuration design of the asymptotic behavior of the inelastic structure after a large number of loading cycles becomes actual [2], [3].
Two classes of methods are developed in order to determine the asymptotic state: direct methods [2]–[7] and incremental methods [8]–[12].

Some works contain both analytical and incremental approaches, as in [8], where cyclic plasticity theory is used to study the three dimensional effects at the tip of rounded notches in plates of finite thickness to determine the actual stress and strain state arising. Different notch geometries and loading conditions are investigated.

As the problems of determining the asymptotic states of inelastic bodies cause considerable interest recently, in literature are proposed plenty solutions of applied durability problems (cyclic loading of rails, tools and machine parts) [9]–[25].

In recent work incremental step-by-step biaxial loading of the elastoplastic plate with the central hole is carried out. The applied load is cyclic. Three types of asymptotic behavior of the plate are revealed during the analysis of FEM calculations. Domains of loadings for three asymptotic regimes of an inelastic structure are found. Calculations are carried out for three different materials. The obtained ranges of shakedown, alternate plasticity and ratcheting were analyzed and general patterns of different regimes were revealed.

2. Computing experiments
Unfortunately, incremental methods are time-consuming and demand a considerable number of numerical experiments [1-9]. The aim of direct numerical methods is to overcome shortcomings of incremental analysis, however the developed theory is far from completion and practical use [2, 4, 9, 11, 13]. Owing to the specified reasons, in the recent work incremental loading of elastoplastic plate with central hole was carried out in order to identify different types of asymptotic behavior of inelastic structure and determine loads, which lead to regime changings.

Implementation and research of the three asymptotic regimes have been held out on the example of a square plate with central hole (elliptic and then circular). The geometry of the plate is shown in figure 1.

The length of a side of the plate $L$ is $16dm$. The plate’s side length to its thickness ratio is 0.02. The semi major axis of the elliptical hole is $a = 3dm$, and the semi minor axis is $b = 2dm$. For the circular hole radius $r = 2dm$.

Let's consider cases when the copper plate has different mechanical properties:

- a) density is $\rho = 5400kg/m^3$, Young's modulus is $E = 4.3 \times 10^8 kg/m^2$, Poisson's ratio is $\nu = 0.28$ [26];
- b) density is $\rho = 8920kg/m^3$, Young's modulus is $E = 13 \times 10^9 kg/m^2$, Poisson's ratio is $\nu = 0.28$ [27];
- c) density is $\rho = 8920kg/m^3$, Young's modulus is $E = 1 \times 10^{10} kg/m^2$, Poisson's ratio is $\nu = 0.3$ [27].

The plate is subjected to the biaxial loading $P_1$ and $P_2$ (figure 1). The applied load $P_2(t)$ is cyclic. The applied loading is schematically shown in figure 1.
2.1. Elasto-plastic analysis

In this section the finite-element method solution of the cyclic loaded elasto-plastic plate with central hole problem is given. The purpose of the present finite element analysis is to define load amplitudes (boundaries of shakedown, cyclic plasticity and ratcheting domains) when these regimes are realized.

Let’s consider a body, occupying volume $V$ (the plate with central hole, the quarter of which is given in figure 1). The boundary conditions are:

- When $\bar{x} \in \Gamma_2$, $P_2(t) = k_2 \sigma_T \sin(2\pi t/T)$, see figure 2. When $\bar{x} \in \Gamma_1$, $P_1 = k_1 \sigma_T$, where $\sigma_T$ is material yielding strength; $k_1, k_2$ are constants of proportionalities, which are used to vary the amplitude of the applied load.

![Figure 1. Geometry of the model: a quarter of a loaded plate.](image1)

![Figure 2. Periodic law for the load.](image2)

It is supposed that the plate is made of elasto-plastic material. The total strains consist of elastic and plastic components: $\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p$, where $\varepsilon_{ij}^e$ is elastic strain and $\varepsilon_{ij}^p$ is plastic strain. For the elastic part Hooke’s law is valid: $\varepsilon_{ij}^e = C_{ijkl}\sigma_{kl}$. As physical ratios for plastic deformation the equations of the flow plasticity theory are applied. For the plastic environment in space of tension the flow surface $f(\sigma_{ij}) = s_{ij} s_{ij} - 2\sigma_T^2 / 3$ is introduced. Plastic strains are defined by the associate flow rule:

$$d\varepsilon_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}}.$$  

It is necessary to determine the asymptotic state of the structure after a large number of loading cycles and define the possible regimes of the body behavior under the applied cyclic and static loadings. The mathematical problem is reduced to the system of equations:
equilibrium equation: $\sigma_{ij,j} = 0$ in $V$,

the constitutive law:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^p = \frac{1+\nu}{E} \ddot{\sigma}_{ij} - \frac{\nu}{E} \ddot{\sigma}_{kk} \delta_{ij} + \lambda \frac{\ddot{f}}{\ddot{\sigma}_{ij}} \text{ in } V,$$

Cauchy’s relations: $\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2$ in $V$.

The boundary conditions are: periodic load $\sigma_{22} = P_2(t)$ on $\bar{x} \in \Gamma_2$ and static load $\sigma_{11} = P_1$ on $\bar{x} \in \Gamma_1$.

Boundary conditions on the symmetry planes of the quarter of a plate are shown in the figure 1.

### 2.2. Finite element analysis

The series of calculations was carried out in multifunctional finite-element package SIMULIA Abaqus [26].

Plastic material properties of the model are set by the table 1 (a, b, c). The first value in the first column is the material yielding strength, and the last one in the first column is the ultimate strength of material [26]–[27].

**Table 1. Stress – plastic strain dependence for copper.**

|       | (a)        | (b)        | (c)        |
|-------|------------|------------|------------|
| $\sigma$, $\varepsilon^{pl}$, $\rho$, $\sigma$, $\varepsilon^{pl}$, $\sigma$, $\varepsilon^{pl}$ | $\sigma$, $\varepsilon^{pl}$, $\rho$, $\sigma$, $\varepsilon^{pl}$, $\sigma$, $\varepsilon^{pl}$ | $\sigma$, $\varepsilon^{pl}$, $\rho$, $\sigma$, $\varepsilon^{pl}$, $\sigma$, $\varepsilon^{pl}$ |
| kg / m$^2$ | kg / m$^2$ | kg / m$^2$ |
| 54×10$^4$ | 210×10$^6$ | 60×10$^6$  |
| 58×10$^4$ | 240×10$^6$ | 100×10$^6$ |
| 63×10$^4$ | 280×10$^6$ | 140×10$^6$ |
| 69×10$^4$ | 300×10$^6$ | 170×10$^6$ |
| 74×10$^4$ | 320×10$^6$ | 200×10$^6$ |
| 78×10$^4$ | 360×10$^6$ | 220×10$^6$ |

The series of calculations of the structure asymptotic conditions have been carried out, where constants of proportionality $(k_1, k_2)$ varied from 0 to 1.3. All the results in this section are given for the plate with elliptic hole. For the circular hole they are just similar.

Periodic nature of loading is seen in the figure of changes of equivalent stress during time for different types of structure behavior (figure 3).
Figure 3. Cyclic character of the equivalent stress (blue curve corresponds to shakedown; green curve corresponds to cyclic plasticity, red curve shows ratcheting).

It is obvious that for different asymptotic regimes of the structure stresses always behave cyclically. This feature is the basis for the most part of direct methods which are used to determine the asymptotic state [2, 4, 9, 11, 13].

For the analyzed elasto-plastic plate all three regimes were obtained by changing the magnitude of the load (figures 4–6) and loadings that lead to the change of the regime were defined.

One can determine the asymptotic type of the structure after a number of loading cycles by the use of the plastic deformations character.

In the figure 4 plastic strain components of the copper plate (material b) for shakedown are shown:
1) $\varepsilon_{11}^p$ (blue curve) and $\varepsilon_{22}^p$ (green curve) for $P_1 = P_{2\text{max}} = 100kN / m^2$;
2) $\varepsilon_{11}^p$ (blue curve) and $\varepsilon_{22}^p$ (green curve) for $P_1 = 100kN / m^2$; $P_{2\text{max}} = 250kN / m^2$.

Figure 4. Plastic strains in the model for shakedown (case 1(left), 2(right)).

In the figure 5 plastic strain components of the copper plate (material a) for cyclic plasticity are shown:
1) $\varepsilon_{11}^p$ (blue curve) and $\varepsilon_{22}^p$ (green curve) for $P_1 = P_{2\text{max}} = 380kN / m^2$;
2) \( \varepsilon_{11}^p \) (blue curve) and \( \varepsilon_{22}^p \) (green curve) for \( P_1 = P_{2\text{max}} = 400\text{kN} / \text{m}^2 \).

In the figure 6 plastic strain components of the copper plate (material b) for ratcheting are shown:
1) \( \varepsilon_{11}^p \) (blue curve) and \( \varepsilon_{22}^p \) (green curve) for \( P_1 = 300\text{kN} / \text{m}^2 ; P_{2\text{max}} = 50\text{kN} / \text{m}^2 \);  
2) \( \varepsilon_{11}^p \) (blue curve) and \( \varepsilon_{22}^p \) (green curve) for \( P_1 = 100\text{kN} / \text{m}^2 ; P_{2\text{max}} = 220\text{kN} / \text{m}^2 \).

Stress – strain curve also can brightly characterize the type of structure asymptotic behavior. In figure 7 dependence of \( \varepsilon_{11} \) and \( \sigma_{11} \) for shakedown is shown. Mathematically this relation for every moment \( \tau = t / T \) (where \( t \) is the specific moment of time, \( T \) is the period of loading) inside the stabilized asymptotic cycle can be presented as: \( \dot{\varepsilon}_{ij}^{\text{pl,cs}} = \lim_{n \to \infty} \dot{\varepsilon}_{ij}^{\text{pl}}(\tau) = 0 \), where \( \dot{\varepsilon}_{ij}^{\text{pl,cs}} \) is the plastic strain rate in the stabilized cycle.

In the figure 8 the strain \( \varepsilon_{11} \) and stress \( \sigma_{11} \) curve for cyclic plasticity is shown:
\[ \dot{\varepsilon}_{ij}^{\text{pl,cs}} = \lim_{n \to \infty} \dot{\varepsilon}_{ij}^{\text{pl}}(\tau) \neq 0, \quad \Delta \varepsilon_{ij}^{\text{pl,cs}} = \int \dot{\varepsilon}_{ij}^{\text{pl}}(\tau) d\tau = 0. \]

In the figure 9 stress – strain curve for ratcheting is shown [13]:

![Figure 5](image5.png)

**Figure 5.** Plastic strains in the model for cyclic plasticity (case 1(left), 2(right)).

![Figure 6](image6.png)

**Figure 6.** Plastic strains in the model for ratcheting (case 1(left), 2(right)).
\[ \hat{\varepsilon}_{ij}^{pl,cs} = \lim_{n \to \infty} \hat{\varepsilon}_{ij}^{pl}(\tau) \neq 0, \quad \Delta \varepsilon_{ij}^{pl,cs} = \int \hat{\varepsilon}_{ij}^{pl}(\tau)d\tau \neq 0. \]

**Figure 7.** Stress – strain curve (30 loading cycles) for the model: shakedown.

**Figure 8.** Stress – strain curve (30 loading cycles) for the model: cyclic loading.

**Figure 9.** Stress – strain curve (30 loading cycles) for the model: ratcheting.

When modeling this regime one can definitely see the birth and progress of plastic flow in the plate (figure 10).

(a) 

(b)
Figure 10. Plastic strains in the model.

The obtained data were generalized and represented in the form of the diagram for both of the plates (figure 11) in which the domains of loadings corresponding to each type of an asymptotical behavior of a plate are defined. The type of structure behavior was defined for the most deformed finite element of the plate. The location and number of this element varied according to the exact loading case.

Figure 11. Results of numerical analysis: the diagram of loading domains for different asymptotic regimes of the plate: left – with elliptic hole, right – with circular hole.
Boundaries of the loading limits $P_1$ and $P_{2\text{max}}$ for ratcheting are marked with red color, for cyclic plasticity – with blue, for shakedown – with green. From the diagram it is obvious that without cyclic loading ratcheting comes at the magnitude of the load which is equal to the material ultimate strength.

This diagram allows us to define safe magnitudes of loads for the structure and avoid dangerous (unsafe) regimes.

From figure 11 it is seen that for different materials limits of shakedown, cyclic plasticity and progressing plastic flow differ, but qualitative distribution of domains remains similar for all the materials.

Using this hypothesis, based on the supervision, we can introduce dimensionless parameters: $\pi_1 = P_1 / \sigma_B$, $\pi_2 = P_2 / \sigma_B$ and construct diagram $P_1$ and $P_{2\text{max}}$ on the plane $\pi_1, \pi_2$ for all the three materials. Results are shown in the figure 12.

**Figure 12.** Results of numerical analysis: the generalized diagram of loading domains for different asymptotic regimes of the plate: left – with elliptic hole, right – with circular hole.

From figure 12 it is seen that all curves defining domain boundaries match and lay down on the uniform curve. On the plane $\pi_1, \pi_2$ all normalized calculated points from the numerical experiment lie on the uniform curve. The variables $\pi_1, \pi_2$ can be interpreted as similarity variables.

Thus, numerical analysis showed independence of characteristic domains from mechanical properties of materials. Therefore there is no need to calculate domains for every distinct material.

The chosen normalization allows us to present all computed charts for certain materials within one curve with the minimum point dispersion. The convenience of the generalized diagram is in
opportunity to know the asymptotic behavior of an inelastic structure for different materials without computing.

3. Conclusion
By means of varying $k_1$ and $k_2$ we can study the whole range of static and cyclic loads, applied to the structure. In the present paper large series of numerical computer modeling experiments for plate with central hole under cyclic loading (in the whole range values of coefficients $k_1$ and $k_2$) have carried out and interpreted. On the basis of the computer experiment ranges of loads for shakedown, cyclic plasticity and ratcheting have been defined.

Types of asymptotic behaviors of non-elastic structures are discovered. The typical diagrams shown in figure 12 are constructed; their main and intrinsic features are defined. Borders of asymptotic behavior types of a certain structure subjected to identical structure loading, for different materials match and lay down on one curve.

For three materials with different mechanical features domains are similar. Thus a convenient normalization of computing results is offered. Experimental data are presented in a generalized diagram of structure’s asymptotic behavior.

The obtained incremental analysis results may be useful for checking the data received with the direct methods. Direct methods give us an opportunity to decrease computing expenses while determining ranges of safe working regimes of test samples. This will allow us to organize design of structures in a new way.

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