A Many-Objective Joint Parallel Simulation Method for Acoustic Optimization Design of Sound-Absorbing Structures

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Abstract: This paper establishes a many-objective MATLAB with a COMSOL joint parallel simulation optimization method in order to solve the current situation of low efficiency, single objective, and poor effect in acoustic optimization design research for a sound-absorbing structure. Our proposed method combines the means for population partitioning, monitoring, and adaptive normalization, within the framework of the NSGA-III algorithm, which takes the hyperplane deployment scheme into account in its entirety. Compared to the traditional genetic algorithm toolbox of the joint COMSOL optimization scheme, it is shown that the joint parallel simulation optimization method that is constructed in this paper achieves a higher optimization efficiency and a better experimental performance, thereby aiding in the identification of the optimal solution to multiple objectives. The optimization efficiency can increase linearly as the number of available cores on the computer increases. This method is then used to construct a parallel, low-frequency, broadband, highly-sound-absorbing structure. Without any constraints on the optimization objective, the diversity of the optimization results is evident within the parameter optimization range of this paper. The optimization results are stable and substantial, with constrained optimization objectives that have some reference value. In addition, the proposed method can solve acoustic vibration optimization problems and can be applied to other finite element optimization problems.

Keywords: NSGAIII; many-objective; parallel; acoustic optimization; sound-absorbing structure

1. Introduction

Currently, with the advancement of materials research and manufacturing technology, sound-absorbing structures that can modulate the acoustic properties of the environment are utilized in a wide variety of applications [1]. Given the high operability and flexibility of acoustic simulations, they appear in numerous acoustics-related studies [2,3]. In this regard, optimizing the initial sound absorption structure for further investigation has become the mainstream trend.

Using the two commonly used optimization methods as examples, the literature on optimization by parametric scanning examines these two techniques. Li et al. [4] have developed a new light and thin sound absorption structure that is composed of melamine foam and hollow porous spheres, which significantly enhances the low-frequency sound absorption performance. Jimenez et al. [5] have investigated the impact of the incident angle on locally-reactive acoustic metamaterials and the relationship between the ideal absorption peak, the incident angle, and the frequency. Using parameters, such as the total stiffness and damping, as independent variables, and the average sound absorption coefficient in a specific frequency band as the optimization objective, Li et al. [6] have created a type of loudspeaker with a good sound absorption performance. Han et al. [7] have constructed a labyrinthine Helmholtz phonon crystal and have concluded that increasing the tube length and preserving a small lattice constant could significantly enhance the structure’s low-frequency sound insulation capacity. Chen et al. [8] have constructed a...
resonant metamaterial plate with a front radial film and have concluded that increasing the plate thickness and decreasing the film thickness could effectively improve the broadband and low-frequency properties.

In the literature concerning the application of genetic algorithms (GA, NSGA-II), Zhao et al. [9] have proposed a tunable low-frequency absorber, achieving 84% absorption of 500–800 Hz sound waves with an 82 mm thickness structure. By combining the gradient-based finite element difference method with the original method, Auriemma et al. [10] have maximized the acoustic transmission loss of suspended glass panels. Xin et al. [11] have modified the structural geometrical parameters, obtaining the optimal sound absorption performance of the structure when the thickness and the boundary are fixed. Silva et al. [12] have designed compact high-performance silencers that can effectively reduce pipeline noise. Jing et al. [13] have achieved the desired sound absorption performance by optimizing the supercell area, as well as the number and shape of the cellular structures. Bhattacharya et al. [14] have utilized finite elements and hybrid regression in order to optimize the geometry to attenuate the broadband acoustic signals. Redondo et al. [15] have extended the absorption bandwidth in order to encompass the entire low-frequency spectrum by evaluating the various cost functions. Torre et al. [16] have designed an efficient method for calculating the acoustic transmission losses by combining the analytical and numerical methods. Broyles et al. [17] have combined hypercubic sampling with constrained optimization in order to enhance the implied energy and sound transmission levels of the hybrid slab. Sanders et al. [18] have applied a gradient-based partial differential equation constrained optimization method that has realized the optimal continuous radial distribution of solid elastic materials, thereby constructing a gradual acoustic cloak with effective low-frequency sound absorption.

However, parametric scanning and genetic algorithms both have limitations when they are applied to the optimization of acoustic structures. Parametric scanning is no longer effective in solving the hyperparameter problem, since the efficiency of parametric scanning by exhaustive traversal is severely affected by the proliferation of the parameters. In addition, the resulting law has limited applicability when a single parameter is investigated using the control variables method. It is likely to lead to producing contradictory results, due to variations in another parameter.

The primary issue with the genetic algorithms (GA, NSGA-II) currently in widespread use is the absence of optimization objectives. The former can only optimize single-objective problems and the latter can optimize no more than two or three multi-objective problems. In addition, when the genetic algorithm interacts with the finite element software, the optimization of the two-dimensional model is limited in scope and can only optimize some of the completely axisymmetric structures; the optimization of the three-dimensional model is so inefficient that it may take more than ten days or even tens of days to optimize a better structure.

In order to address the issues of low efficiency, a small number of optimization targets and the poor performance of the existing optimization are required. A many-objective MATLAB with a COMSOL joint parallel simulation optimization method is presented in this paper. We have improved the non-open-source NSGA-III [19] framework. The experimental results demonstrate that our method is applicable to acoustic vibration optimization problems and can be extended to other finite element optimization problems.

2. The Applied Model

This section presents the adopted optimization model, which will be used for the subsequent verification of the optimization effect and the design of low-frequency, broadband, high sound-absorbing structures.

2.1. Design of the Sound-Absorbing Structure

According to a study by Du [20], cavities with circular cross-sections are widely utilized in underwater sound-absorption structures and have excellent sound absorption
properties. In this paper, the interpolated cavity structure—an asymptotic cavity structure as depicted in Figure 1—has been selected.

![Figure 1. Sound-absorbing structure model, (a) Elevation view; (b) Vertical view.](image)

The thickness of the steel plate and the anechoic coating are 10 mm and 40 mm, respectively. The length and width of the whole model are both 40 mm. $h_1$ represents the distance from the first interpolation point of the cavity to the steel plate, $r_1$ corresponds to the radius of the first interpolation point to the central axis; $h_2$ has been chosen to be the vertical distance from the second interpolation point of the cavity to the first interpolation point, $r_2$ denotes the radius of the second interpolation point to the central axis; $h_3$ represents the vertical distance from the third interpolation point of the cavity to the second interpolation point, $r_3$ has been set as the radius of the third interpolation point to the central axis. Figure 1 only depicts one instance of this structure type. Changing the parameters enables the formation of three additional structures, as shown in Figure 2.

![Figure 2. Schematic diagram of the structures, (a) Structure 1; (b) Structure 2; (c) Structure 3.](image)

2.2. Mathematical Models

This paper studies the sound absorption characteristics of the structure, with which the absorption characteristics can be easily calculated by using the periodic boundary conditions. In order to obtain the sound absorption characteristics of the structure, the finite element method can reduce the complex structures to unit-cell models by imposing constraints and absorbing boundaries when analyzing the periodic structures. In this paper, a finite unit-cell model serves as the fundamental research object, and Floquet periodic boundary conditions are used to simulate infinitely large periodic structures and waters by setting them at the boundary, which is also a common method for studying sound-absorption structures [21,22].

In this instance, the steel-backed periodic cell with a cavity is used in order to simulate the structure of an infinite sample. As depicted in Figure 3, the incident wave strikes the sound-absorbing structure from the waterside, and the domain probe point receives the scattered, reflected wave. In order to simulate infinitely far waters, a perfectly matched
layer (PML) is placed at the water end [23]. The complete absorption of the acoustic wave incident on the PML is achieved by setting a fluctuation equation containing an attenuation factor in the layer.

![Figure 3. Cavity element model.](image)

Among them, the material parameters of the substrate, the steel plate, and the water are shown in Table 1.

**Table 1. Material parameters.**

| Materials   | Young’s Modulus (Pa) | Poisson’s Ratio | Loss Factor | Density (kg/m$^3$) |
|-------------|----------------------|----------------|-------------|-------------------|
| Substrate   | $1.4 \times 10^8$    | 0.49           | 0.48        | 1100              |
| Steel       | $2.1 \times 10^{11}$ | 0.3            | 0           | 7800              |
| Water       | -                    | -              | -           | 1000              |

2.2.1. Control Equations

This section utilizes the COMSOL finite element simulation platform to extract the acoustic coefficients of the model. The calculation utilizes the pressure acoustic-frequency domain module. The controlling equation is [24]:

$$\nabla \cdot \left[ -\frac{1}{\rho_c} (\nabla p_t - q_d) \right] - k^2_{eq} \frac{p_t}{\rho_c} = Q_m$$  \hspace{1cm} (1)

$$p_t = p + p_b$$  \hspace{1cm} (2)

$$k^2_{eq} = \left( \frac{\omega}{c} \right)^2$$  \hspace{1cm} (3)

where

$\rho_c$—material density, kg/m$^3$;

$p_t$—total pressure, Pa;

$q_d$—dipole source, N/m$^3$;

$Q_m$—monopole source, 1/s$^2$;

$p$—pressure, Pa;

$p_b$—background pressure, Pa;

$\omega$—angular frequency;

$c$—speed of sound in water, m/s.

The solid–liquid interface is determined by the coupling equation [25] as follows:

$$-n \cdot \left[ -\frac{1}{\rho_c} (\nabla p_t - q_d) \right] = -n \cdot u_{it}$$  \hspace{1cm} (4)

$$F_A = p_i n$$  \hspace{1cm} (5)

where $n$ represents the surface normal, $u_{it}$ denotes the structural displacement, and $F_A$ represents the load that is acting on the structure.
2.2.2. Boundary Conditions

Floquet periodic conditions (6) and (7) are set on the surface of the structure and the water, respectively, in order to simulate the infinite structure and the water [26].

\begin{equation}
pt,\text{dist} = pt,\text{src}e^{-ik_F(r_{\text{dist}}-r_{\text{src}})}
\end{equation}

\begin{equation}
-n_{\text{dist}}\cdot\left(-\frac{1}{\rho_c}\left(\nabla p_t - q_d\right)\right)_{\text{dist}} = -n_{\text{src}}\cdot\left(-\frac{1}{\rho_c}\left(\nabla p_t - q_d\right)\right)_{\text{src}} e^{-ik_F(r_{\text{dist}}-r_{\text{src}})}
\end{equation}

where \( \text{dist}, \text{src} \) represent the two opposite sides of the given periodic boundary conditions, and \( K_F \) denotes the wave number.

In addition, the back of the steel plate can be approximated as a free boundary because the medium on the transmission side is air and the amplitude of the transmission coefficient is very small.

2.2.3. Mesh

When performing the finite element calculations for the acoustic problems, the setting of both the structural mesh and the medium mesh requires extra attention, therefore, the mesh sensitivity is thoroughly analyzed before writing the paper. In general, the maximum grid size should be set to one-sixth of the wavelength [27], with the grid size at the corresponding frequency being directly calculated for the water and air domains based on their respective sound velocities. In accordance with the substrate and the steel plate’s transverse wave velocities, the grid size at the corresponding frequency is calculated.

In this paper, the calculation frequency range is between 100 Hz and 3000 Hz, and the number of grids will not vary significantly with the frequency change. In the simulation process, using a small number of grids will save a significant amount of regridding time, based on the assumption that the grid does not change with the frequency. Consequently, after combining the sensitivity analyses, the maximum grid size of the selected structure is set to 10 mm, while the maximum grid size of the watershed is set to 80 mm. This ensures the accuracy of the model at all of the frequencies and efficiencies.

As depicted in Figure 4, the acoustic structure of a particular configuration was meshed using the meshing principles mentioned above; the number of meshes in this model was 9466 and the number of nodes was 1987.

![Figure 4. Mesh model.](image)

2.2.4. Evaluation Indicators

In order to grasp the sound absorption effect of the structure, this paper adopts the sound absorption coefficient \( \alpha \) [28] as the evaluation index of the structure, which is given as follows:

\begin{equation}
\alpha = 1 - T^2 - R^2
\end{equation}

where \( T \) denotes the transmission coefficient that is extracted by finite simulation and \( R \) is the reflection coefficient. Since \( T \ll 0 \), Equation (8) can be directly expressed as follows:

\begin{equation}
\alpha = 1 - R^2
\end{equation}

In the COMSOL finite element model, the domain point probe is used to extract the reflection coefficient. A background sound pressure field with an amplitude of one Pascal
is used as the sound source in this paper, so the sound pressure of the reflected sound field can be extracted directly as the reflection coefficient by using the domain point probe [29].

2.2.5. Validation

The validity of the model that is presented in this paper has been established through a review of the relevant literature [30]. Figure 5a depicts a model from the literature that employs a cavity structure that is similar to that of this paper, and the medium environment is essentially identical to that of this paper; therefore, the simulation model that is depicted in Figure 5b was constructed for comparison. Comparing the results in Figure 6 demonstrates that the finite element model that is constructed in this paper is in complete agreement with the results that were obtained from models in the literature, thereby demonstrating the efficacy of the method that is presented in this paper.

![Figure 5](image)

**Figure 5.** (a) Models in the literature, Reprinted with permission from Ref. [30] 2019, Zhong, J.; (b) The simulation model in the original manuscript.

![Figure 6](image)

**Figure 6.** Comparison of the simulation results of this paper with the literature results.

3. Many-Objective Joint Parallel Simulation Method

3.1. Multi-Objective Optimization Algorithm

The essence of the multi-objective optimization problem is to find the optimal solution to multiple objectives under the constraints of independent variables, as described by Equation (10) as follows:

$$\min \left[ f_1(x), f_2(x), f_3(x), \ldots, f_n(x) \right]$$  \hspace{1cm} (10)

subject to:

$$lb \leq x \leq ub$$  \hspace{1cm} (11)

$$A_{eq} \cdot x = beq$$

$$A \cdot x \leq b$$

where $f_1(x)$ is the objective function to be optimized, $x$ represents the independent variable to be optimized, and Equation (11) is the constraint on $x$, including the range constraint, linear constraint, and inequality.
Multi-objective optimization differs fundamentally from single-objective optimization in that the optimization of one objective may degrade the performance of the other objectives. It is nearly impossible to simultaneously optimize all of the objectives; the trade-offs, weighing, and judging must be used to make all objectives as optimal as possible. In addition, the optimal solution to a multi-objective optimization problem is often not a single solution, but a set of optimal solutions.

Currently, there are many methods that are widely used to solve multi-objective optimization problems, mainly genetic algorithms, particle swarm algorithms, simulated annealing algorithms, ant colony algorithms, etc. Based on the single-objective genetic algorithm Srinivas [31], the non-dominated genetic algorithm (NSGA) was proposed in 1995. NSGA, which is based on the concept of Pareto optimality, can be used to solve multi-objective problems, despite its high computational complexity and lack of an elite strategy. In 2002, in response to this motivation, Deb developed the most popularly used fast non-dominated ranking genetic algorithm with an elite strategy based on NSGA [32] (NSGA-II).

NSGA-II is unquestionably one of the most popular and effective algorithms for solving two- or three-objective multi-objective optimization problems. However, the number of optimization targets in acoustic optimization problems is often huge. In the process of optimizing an anechoic coating, for instance, it is not the characteristics of a specific frequency that must be satisfied, but rather the optimization of the entire frequency band to the greatest extent. This indicates that the number of optimization targets could reach tens or even dozens. The NSGA-II is slightly insufficient for such high-dimensional objective problems. The reasons for this are as follows: First, the proportion of non-dominated solutions in the population rises dramatically as the optimization objective is increased. In addition, the search process is slow and the calculation of the index for maintaining the population diversity is overly complex for the high-dimensional objective. Moreover, the recombination operator’s search capability is inefficient.

Therefore, using NSGA-II to solve an optimization problem with a large number of objectives remains extremely difficult. Hence, based on NSGA-II, Deb [19] proposed the non-dominated genetic algorithm with reference points (NSGA-III) in 2014. NSGA-III inherited most of the NSGA-II framework but made a significant change in the selection mechanism, abandoning the ranking method by crowding and adopting the extensive introduction of distribution reference points to preserve the population diversity in the high-dimensional space. The primary procedure is depicted in Figure 7.

![Figure 7. NSGA-III algorithm main flowchart.](image-url)
The NSGA-III algorithm resembles the NSGA-II algorithm, as depicted in Figure 4. However, the NSGA-III algorithm modifies the selection process in the following three main ways:

1. Determine reference points for hyperplanes

   The diversity of non-dominated solutions is maintained by pre-defined reference points, which can be defined in relation to the structured method or set by one’s self;

2. Target space normalization

   The normalization procedure aims to transform the objective function into an adaptively normalized objective function value by generating ideal points, so that the reference points are uniformly distributed in the high-dimensional objective space;

3. Associated individuals and reference points

   This step identifies the reference line between each individual at the nearest reference point.

   Based on the results of Deb et al., NSGA-III is diverse and convergent in its ability to solve high-latitude target problems. However, the team has not disclosed the code related to NSGA-III at present, so this paper focuses on the NSGA-III algorithm created by Ye et al. [33] in the platEMO platform and develops a many-objective MATLAB with a COMSOL joint parallel simulation optimization method for acoustic optimization design of sound-absorbing structures based on it.

3.2. Joint Parallel Simulation Method

   The algorithm for many-objective optimization focuses on the fitness value. It iteratively evolves the optimal individuals, whereas the traditional genetic algorithm toolbox (including GA and NSGA-II) utilizes a sequential calculation method, i.e., the initial population and the evolved population are calculated sequentially. If the fitness function runs quickly, the evolution process as a whole will be rapid; however, if the fitness function runs slowly, the evolution process as a whole will be extremely lengthy.

   Using MATLAB with a COMSOL joint optimization simulation as an example, the fitness function modifies the simulation model in COMSOL with the optimization algorithm’s parameter variables. After running the new model in COMSOL, it returns the result of the fitness function, forming a closed-loop, and causing the optimization algorithm to evolve iteratively. Assuming a run time of 10 min, a population size of 100 would require at least 1000 min to complete one evolution, and more than 500 h to complete 30 evolutions, which is an insufficient optimization efficiency. Therefore, it is urgent to improve the efficiency of optimization. At the initial and evolutionary stages of the multi-objective optimization algorithm, populations are generated in batches; consequently, it is essential to calculate the fitness values of a batch of individuals simultaneously, rather than sequentially. Obviously, this also necessitates that the size of a single operation is negligible and that the current computer conditions can accommodate the simultaneous operation of multiple individuals.

   To calculate the batch of individuals within a population, we must first divide it. Taking a population of size \( n \) as an example, the population is divided into \( m \) subpopulations, each containing \( x \) individuals. The fitness function is calculated for each of the \( m \) subpopulations and the fitness values of each individual are aggregated into the larger population after calculating the subpopulations. The simultaneous computation of each small population necessitates the simultaneous call of \( m \) COMSOL for the simulation. For example, using a 32-core workstation, one COMSOL for every two cores can ensure that 16 COMSOL are running simultaneously while maintaining the computational speed. Therefore, the key to the solution is to make batch calls to COMSOL for the multi-objective genetic algorithm. Figure 8 illustrates the details of our proposed method.
First, the algorithm’s core is still based on the NSGA-III algorithm, with the following specific steps:

1. The initial phase generates consistent reference points based on the number of populations specified by the user. The original authors used the structured reference point method by Das et al. [34]. This method has superior performance when the objectives are small and can be optimized by generating fewer reference points; however, when the objective function is significant, the number of reference points dramatically increases, resulting in a decrease in the overall optimization speed. With the aid of two hyperplanes, we employ the deployment scheme of Deb and Jain et al. [35] to ensure a small number and wide distribution of reference points as follows:

   a) Generate reference points on the hyperplane space of \( (M - 1) \) latitude, and the number of reference points is as follows:
   \[
   X = C^H_{M+H-1}
   \]  
   where \( H \) represents the number of copies of each individual target divided.

   b) For every \( x_{ij} \in X \) (the \( ij \) subscript denotes the \( i \)-th element of the \( i \)-th combination in \( X \) ), the following exists:
   \[
   x_{ij} = x_{ij} - \frac{j - 1}{H}
   \]

   c) Define \( S_1 \) as the set of reference points, for each \( S_{ij} \in S_1 \) and \( x_{ij} \in X \), and the following exists:
   \[
   \begin{align*}
   s_{ij} &= x_{ij}, j = 1 \\
   s_{ij} &= x_{ij} - x_{i(j-1)}, 1 < j < M \\
   s_{ij} &= 1 - x_{i(j-1)}, j = M
   \end{align*}
   \]
(d) Taking $S_1$ as the set of points on the boundary layer and defining $S_2$ as the set of points on the inner layer, for each $S_{ij}' \in S_2$ and $S_{ij} \in S_1$, the following exists:

$$S_{ij}' = \frac{1}{2} S_{ij} + \frac{1}{2M}$$  \hspace{1cm} (15)

(e) Define the set of reference points as follows:

$$S = S_1 \cup S_2$$  \hspace{1cm} (16)

2. Create a batch of $n$ files based on the actual number of cores available on the computer. Each file contains an .m file that waits for parameter input and an .mph file that waits for the command execution. In addition, two “sentry” files are created, each of which also includes an .m file and an .mph file; this file is used to monitor the status of the .mph file, and if an execution exception is detected, it will replace the .mph file and the optimization process will be detected. “Sentry” operation process is shown in Figure 9.

3. Population initialization. The primary function generates the first batch of population variables and stores them in a .txt file; the n MATLAB, with COMSOL running in the background, reads the .txt file and calls the corresponding COMSOL to start running, and then stores the result values in the corresponding .txt file after running; the fitness function in NSGA-III reads the result values in the .txt file, and returns to the primary function when all of the result values in the population have been read.

4. Determining constraint relationships and creating ideal points According to the non-linear inequality relationship used to solve the constraint relationship, and to extract the minimum value of population fitness, an ideal point is constructed, through which the objective function is transformed into an adaptive normalization function, with the following specific operation:

(a) The ideal point of the population is $S_t$, which is defined as the minimum of $\cup_{t=0}^T \{z_{min}^i, i = 1, 2, \ldots, M\}$, and an ideal point $z = (z_{min}^1, z_{min}^2, \ldots, z_{min}^M)$ is constructed. The objective function is transformed according to Equation (17) [19] as follows:

$$f_i'(x) = f_i(x) - z_{min}^i$$  \hspace{1cm} (17)

(b) Find the additional points of each axis as follows:

$$ASF(x, w) = \frac{\max_{i=1}^M f_i'(x)}{w}, x \in S_t, w = (\tau, \ldots, \tau), \tau = 10^{-6}$$  \hspace{1cm} (18)

(c) Construct the hyperplane using the extra points and find the intercept of the plane with the coordinate axis $a_i$;

(d) Normalization of the objective function is as follows:

$$f_i^n(x) = \frac{f_i'(x) - z_{min}^i}{a_i - z_{min}^i}, i = 1, 2, \ldots, M$$  \hspace{1cm} (19)

5. Following the termination determination of the population selection, crossover, and mutation steps were implemented to initiate population evolution.

6. Generating subpopulations. To avoid confusion between subpopulation and parent population parameters, the command to clear the parameter file and the result file in real-time is added to the subpopulation execution process. Subpopulations are then subjected to constraint determination and ideal point finding operations.

7. The parent and child populations are merged, and sequential operations, including non-dominated sorting, target space normalization, a calculation of reference lines,
an association of individuals with reference points, and a selection of dominant populations are carried out.

(8) If the termination condition is met, the optimal Pareto solution is output; otherwise, steps (5) through (8) are repeated.

![Figure 9. “Sentry” operation process.](image)

4. Advantage Verification

This section employs the control variables method for comparative analysis, ensuring that the three genetic algorithms stop evolving after executing the same number of models, allowing for a clearer comparison of the efficiency of the three algorithms, as well as their optimization effects.

In order to improve the sound absorption performance of the structure at 1000–3000 Hz and to achieve the best overall sound absorption performance, a local objective function is set at 300 Hz intervals, and the following optimization model is employed:

\[
\begin{align*}
\text{min } f_1 &= -\frac{1}{2}(\alpha_{1000\text{Hz}} + \ldots + \alpha_{1200\text{Hz}}) \\
\text{min } f_2 &= -\frac{1}{2}(\alpha_{2800\text{Hz}} + \ldots + \alpha_{3000\text{Hz}}) \\
2 \text{ mm } &\leq r_1 \leq 19 \text{ mm} \\
2 \text{ mm } &\leq r_2 \leq 19 \text{ mm} \\
2 \text{ mm } &\leq r_3 \leq 19 \text{ mm} \\
1 \text{ mm } &\leq h_1 \leq 15 \text{ mm} \\
1 \text{ mm } &\leq h_2 \leq 15 \text{ mm} \\
1 \text{ mm } &\leq h_3 \leq 15 \text{ mm} \\
h_1 + h_2 + h_3 &\leq 39 \text{ mm}
\end{align*}
\] (20)

Scheme one is the genetic algorithm toolbox (GA) combined with MATLAB with COMSOL. Since the GA algorithm can only handle single-objective problems, the objective function is simplified to \( \text{min } = -1/21(\alpha_{1000\text{Hz}} + \ldots + \alpha_{3000\text{Hz}}) \). Scheme two is the genetic algorithm toolbox (NSGA-II) combined with MATLAB with COMSOL. Scheme three is the joint parallel simulation optimization method of MATLAB with COMSOL that is proposed in this paper, and seven COMSOL are optimized in parallel at the same time. The main parameters of the three algorithms are set in the same way, as shown in Table 2.

| Parameter Name | Generations | Population Size | Stall Genlimit | Crossover Fraction | Migration Fraction |
|----------------|-------------|-----------------|---------------|-------------------|-------------------|
| Parameter Value| 20          | 30              | 20            | 0.8               | 0.2               |

Table 3 displays the time spent in the three schemes, and it can be seen that the efficiency of the joint parallel simulation method that has been developed in this paper is 6.36 times that of Scheme one and 6.28 times that of Scheme two in the case of seven
COMSOL running in parallel. However, these are not the only benefits of the joint simulation approach that are described in this paper, the optimization efficiency increases exponentially with the number of parallel COMSOL, i.e., the more actual cores that are available on the computer, the greater the improvement in the optimization efficiency.

Table 3. Time taken for each scheme.

| Scheme     | Scheme 1 | Scheme 2 | Scheme 3 |
|------------|----------|----------|----------|
| Time spent(s) | 46,224 | 45,572 | 7261 |

In addition, some optimized parameters are listed in Table 4, and Figure 10 depicts the optimized results. Scheme one gives only one optimal solution as a single-objective optimization scheme, while Schemes two and three with Pareto optimal solutions give multiple optimal solutions. As can be seen, all three of the schemes can be optimized in order to obtain better results, the peak absorption coefficient is close to one, and the average absorption coefficient of the whole frequency band is around 0.78. However, the results that were obtained from Scheme two and Scheme three have significant advantages over Scheme one in terms of diversity. In addition, Scheme three is better than Scheme two, because Scheme three can guarantee the overall broadband absorption performance while focusing on the seven frequency bands. At the same time, Scheme two can also guarantee the overall broadband absorption performance, but due to the shortcomings of the algorithm itself, it can only focus on the first three frequency bands and cannot completely take into account the last four frequency bands. With combined efficiency and optimization effect, Scheme three has significant advantages.

Table 4. Optimized parameters.

| Optimized Structural Parameters | $r_1$ (mm) | $r_2$ (mm) | $r_3$ (mm) | $h_1$ (mm) | $h_2$ (mm) | $h_3$ (mm) |
|--------------------------------|------------|------------|------------|------------|------------|------------|
| Scheme 1                        | 7.14       | 17.96      | 8.31       | 14.85      | 4.64       | 2.39       |
| Scheme 2                        | 5.12       | 16.02      | 2.50       | 1.78       | 2.17       | 10.36      |
|                                 | 10.59      | 18.91      | 4.08       | 6.59       | 5.64       | 11.35      |
|                                 | 6.03       | 18.60      | 2.92       | 2.16       | 3.20       | 9.76       |
| Scheme 3                        | 6.13       | 11.98      | 17.78      | 14.02      | 13.20      | 3.26       |
|                                 | 16.17      | 11.93      | 16.64      | 11.71      | 7.04       | 9.51       |
|                                 | 12.88      | 13.52      | 11.21      | 13.33      | 1.00       | 7.50       |
|                                 | 5.76       | 18.52      | 11.36      | 14.40      | 13.23      | 7.54       |
|                                 | 11.32      | 13.15      | 11.36      | 8.46       | 12.35      | 4.04       |

Figure 10. Optimized results, (a) Scheme 1; (b) Scheme 2; (c) Scheme 3.
5. Design of the Low-Frequency Broadband High Sound-Absorbing Structure

Based on the verification effect, it is evident that the scheme that has been constructed in this paper has significant efficiency and diversity advantages. The optimized results have a wide range of peak absorption coefficients, from 1100 Hz to 2700 Hz for structures that have been similarly optimized. As shown in Figure 11, we have designed a low-frequency broadband structure with high sound absorption to elaborate on our proposed method’s benefits. In this structure, the unit-cell models are connected in parallel in order to combine the benefits of each unit-cell.

![Schematic diagram of the parallel model.](image)

**Figure 11.** Schematic diagram of the parallel model.

5.1. Optimization Objectives Are Unconstrained

The optimized frequency band is 100–3000 Hz, a local objective function is set for every 300 Hz, and the following optimization model is used:

\[
\begin{align*}
\min f_1 &= -\frac{1}{3}[\alpha_{100\text{Hz}} + \ldots + \alpha_{300\text{Hz}}] \\
\min f_{10} &= -\frac{1}{3}[\alpha_{2800\text{Hz}} + \ldots + \alpha_{3000\text{Hz}}] \\
2 \text{ mm} &\leq r_1(i) \leq 19 \text{ mm} \\
2 \text{ mm} &\leq r_2(i) \leq 19 \text{ mm} \\
2 \text{ mm} &\leq r_3(i) \leq 19 \text{ mm} \\
1 \text{ mm} &\leq h_1(i) \leq 15 \text{ mm} \\
1 \text{ mm} &\leq h_2(i) \leq 15 \text{ mm} \\
1 \text{ mm} &\leq h_3(i) \leq 15 \text{ mm} \\
h_1(i) + h_2(i) + h_3(i) &\leq 39 \text{ mm} \\
i &= 1, 2, 3, 4
\end{align*}
\] (21)

After conducting a convergence analysis, the primary parameters of the NSGA-III algorithm are set as follows: an evolution number of 30, a population size of 60, a maximum stopping generation of 30, a crossover probability of 0.8, a variation probability of 0.2, and 11 COMSOL are optimized simultaneously in parallel. The optimization took 30 h, and the results are displayed in Figure 12. From Figure 11, it is clear that the optimization objective without constraint falls within the parameter optimization range of this paper, and that the diversity characteristics of the optimization results are evident.
parameters are set in the same manner as they are in the preceding section, and the present a variety of types. However, fixed indicators are frequently employed in the actual

![Figure 12. Optimized results (optimization goals are unconstrained).](image1)

5.2. Optimization Objectives Are Constrained

When the optimization objective is not constrained, the optimization direction is diverse, as demonstrated in the previous section. The final Pareto optimal solution will present a variety of types. However, fixed indicators are frequently employed in the actual design of acoustic structures. Consequently, based on the optimization model that is presented in the preceding subsection, the following objective constraints are added in this subsection:

\[
\begin{align*}
\min (f_3 + f_4 + f_5) & \geq 0.6 \\
\min (f_6 + \ldots + f_{10}) & \geq 0.9
\end{align*}
\]

(22)

The meaning of Formula (22) is that the average absorption coefficient for the frequency range of 700–1500 Hz is greater than 0.6, and the average absorption coefficient for the frequency range of 1600–3000 Hz is greater than 0.9. The NSGA-III algorithm’s primary parameters are set in the same manner as they are in the preceding section, and the optimization outcomes are depicted in Figure 13. With a constrained optimization objective, the optimization outcomes are extremely stable, and all of the results satisfy the objective constraints. In addition, the algorithm performs diversity attempts after reaching the predetermined constraint target, and the average absorption coefficient from 700 Hz to 1500 Hz varies between 0.6 and 0.65. The average absorption coefficient between 1600 Hz and 3000 Hz varies between 0.94 and 0.9.

![Figure 13. Optimized results (optimization goals are constrained).](image2)
In addition, a subset of the optimization results is analyzed using the parameters shown in Table 5, and the model depicted in Figure 14.

Table 5. Optimized parameters (one of them).

| Parameter Name | Structure 1 | Structure 2 | Structure 3 | Structure 4 |
|----------------|-------------|-------------|-------------|-------------|
| $r_1$ (mm)     | 9.39        | 7.69        | 16.83       | 16.12       |
| $r_2$ (mm)     | 15.33       | 12.67       | 8.46        | 5.10        |
| $r_3$ (mm)     | 16.46       | 12.82       | 8.95        | 10.08       |
| $h_1$ (mm)     | 11.49       | 7.06        | 1.00        | 1.38        |
| $h_2$ (mm)     | 12.89       | 11.93       | 4.79        | 3.50        |
| $h_3$ (mm)     | 10.74       | 12.05       | 4.67        | 3.95        |

![Figure 14. Optimized structure (one of them).](image)

According to Lin et al. [36], this cavity absorption mechanism is primarily composed of resonance energy loss, scattering, and waveform conversion. The low-frequency absorption is primarily determined by resonance energy loss from the three concepts. The absorption peak of Unit-cell model one is 1 kHz and the absorption coefficient is 0.993, whereas the absorption peak of Model two is 1.9 kHz, and the absorption coefficient peaks at 0.969. Figure 15 illustrates the vibrational velocity of the model [25], and the vibrational velocity $v_H$ (23) is derived from the structural displacement $u_H$ in Equation (4).

$$v_H = \frac{du_H}{dt}$$

![Figure 15. Vibrational velocity diagram at the maximum absorption coefficient, (a) Unit-cell model 1, 1000 Hz; (b) Unit-cell model 2, 1900 Hz.](image)
In comparing the vibrational velocities of Unit-cell models one and two at the maximum absorption coefficient, Model two has resonance energy loss, waveform conversion, and scattering, in addition to resonance energy loss compared to Model one. Shear waves are more likely to dissipate in solid media than in longitudinal waves. The cavity in Model two of the unit-cell transforms the incident longitudinal waves into shear waves and increases the propagation distance, resulting in a wider absorption peak width in Model two.

Furthermore, due to the larger cavity volume of Model one’s unit-cell, the absorption peak frequency is lower than that of Models two, three, and four, which is consistent with the literature’s conclusions. After connecting the unit-cell models in parallel, as shown in Figure 16, the resonance peak is shifted due to cavity coupling, and the absorption peak is between Models one and two. The introduction of Unit-cell models three and four simultaneously widened the absorption peak width. Model one, with an absorption coefficient greater than 0.86, has an absorption peak width of 340 Hz. Model two has an absorption peak width of 860 Hz, while the parallel model reaches 1760 Hz.

![Comparison diagram of the parallel model and unit-cell model.](image)

**Figure 16.** Comparison diagram of the parallel model and unit-cell model.

6. Conclusions

This paper develops a many-objective MATLAB with a COMSOL joint parallel simulation optimization method, based on the NSGA-III algorithm framework, for acoustic optimization design of sound-absorbing structures, and builds on this scheme in order to build a parallel low-frequency broadband high sound-absorbing structure.

On the one hand, a MATLAB with a COMSOL parallel simulation optimization platform was developed by using the platEMO platform and the framework of the NSGA-III algorithm, which takes into account the hyperplane deployment schemes, the monitoring means, and adaptive normalization. It has been demonstrated that the parallel simulation optimization method that has been developed in this paper outperforms the conventional genetic algorithm toolbox jointly with COMSOL. The benefits include high efficiency, multiple optimization objectives, and excellent optimization results, among others. In addition, the optimization efficiency increases linearly with the actual number of computer cores.

This paper has obtained a low-frequency broadband structure with high sound absorption by combining the above method with the characteristics of a structure with a tapering cavity and employing parallel means. In the case of unconstrained optimization objectives, and within the parameter optimization range, the diversity characteristics of the optimization results are readily apparent. With an average absorption coefficient above 0.6 for 700–1500 Hz and above 0.9 for 1600–3000 Hz, the constrained optimization objective results are substantial and stable.
In addition, the joint parallel simulation method that has been developed in this paper can solve the more challenging 3D finite element model optimization problems, such as the optimization of acoustic structures with a complex internal filling or a mixture of multiple single-cell element models. The enhancement of optimization efficiency drastically reduces the optimization time of such problems, allowing optimizations that previously required months to complete to be completed in a matter of days.

In conclusion, the many-objective parallel simulation optimization method that has been presented in this paper has made significant strides in solving acoustic vibration optimization problems. It can be applied to additional finite element optimization issues.

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