MAGNETOSPHERIC ECLIPSES IN THE DOUBLE PULSAR SYSTEM J0737-3039.

Roman R. Rafikov\textsuperscript{1} and Peter Goldreich\textsuperscript{1,2}

ABSTRACT

We argue that eclipses of radio emission from the millisecond pulsar A in the double pulsar system J0737-3039 are due to synchrotron absorption by plasma in the closed field line region of the magnetosphere of its normal pulsar companion B. Based on a plausible geometric model, A’s radio beam only illuminates B’s magnetosphere for about 10 minutes surrounding the time of eclipse. During this time it heats particles at \( r \gtrsim 10^9 \) cm to relativistic energies and enables extra plasma, beyond that needed to maintain the corotation electric field, to be trapped by magnetic mirroring. An enhancement of the plasma density by a factor \( \sim 10^2 \) is required to match the duration and optical depth of the observed eclipses. The extra plasma might be supplied by a source near B through \( B \gamma \) pair creation by energetic photons produced in B’s outer gap. Relativistic pairs cool by synchrotron radiation close to where they are born. Re-excitation of their gyrational motions by cyclotron absorption of A’s radio beam can result in their becoming trapped between conjugate mirror points in B’s magnetosphere. Because the trapping efficiency decreases with increasing optical depth, the plasma density enhancement saturates even under steady state illumination. The result is an eclipse with finite, frequency dependent, optical depth. After illumination by A’s radio beam ceases, the trapped particles cool and are lost. The entire cycle repeats every orbital period. We speculate that the asymmetries between eclipse ingress and egress result in part from the magnetosphere’s evolution toward a steady state when illuminated by A’s radio beam. We predict that A’s linear polarization will vary with both eclipse phase and B’s rotational phase.

Subject headings: pulsars: general — pulsars: individual (J0737-3039A, J0737-3039B) — stars: neutron — radiation mechanisms: non-thermal — plasmas

1. INTRODUCTION.

The recent discovery of the binary pulsar J0737-3039 — a millisecond pulsar (pulsar A with a period \( P_A = 23 \) ms) and a normal pulsar (pulsar B with a period \( P_B = 2.8 \) s) in a tight 2.4 hrs orbit (Burgay et al. 2003) — not only provides us with unprecedented tests of general relativity (Lyne et al. 2004) but also reveals a variety of magnetospheric phenomena. Among the latter are variations of pulsar B’s radio emission correlated with binary orbital phase (Lyne et al. 2004; Ransom et al. 2004) and modulated at the spin frequency of pulsar A (McLaughlin et al. 2004a), and periodic eclipses of pulsar A when it passes behind pulsar B (Lyne et al. 2004; Kaspi et al. 2004; McLaughlin et al. 2004b). It is the latter phenomenon that concerns us in this paper.

Detailed observations by the Green Bank Telescope (Kaspi et al. 2004) established the following frequency-averaged properties of pulsar A’s eclipses: the eclipse duration is about 27 s, which for a relative transverse velocity of 680 km s\(^{-1}\) translates into a size of 18,000 km. Eclipses are significantly asymmetric with ingress taking 3 – 4 times longer than egress; pulsar A’s radio beam is extinguished more strongly post conjugation, consistent with zero flux, than before conjugation when some flux leaks through. Analysis of the same data at higher time resolution by McLaughlin et al. (2004b) uncovered effects of B’s rotational phase on the frequency dependence and shape of the eclipse during ingress; its shape during egress is remarkably independent of either B’s rotational phase or radio frequency.

The spin-down luminosity of pulsar A exceeds that of pulsar B by about 3600, so it is plausible that B’s magnetosphere is compressed by a relativistic wind from A. Calculations by Lyutikov (2004) demonstrate that the ram pressure of A’s wind can be balanced by B’s magnetic field pressure at a stand-off distance \( r_{so} \approx (3.5 - 6) \times 10^9 \) cm which is within B’s light cylinder radius of \( r_{LB} \approx c/\Omega_B \approx 1.3 \times 10^{10} \) cm. A crucial point is that the size of the eclipsing region is considerably smaller than even the compressed size of the B’s magnetosphere. To fully quantify the geometry of the eclipse, the inclination of the system must be accurately known. Measurements of the Shapiro delay established that the inclination is very high, \( i = 87^\circ \pm 3^\circ \) (Lyne et al. 2004). A refined estimate by Colos et al. (2004) based on correlation of interstellar scintillations of both pulsars yields \( i = 90.26^\circ \pm 0.13^\circ \), which corresponds to a minimum distance of the A’s radio beam with respect to B’s position projected on the plane of the sky of only 4000 ± 2000 km. This refinement implies that the extinction during eclipse arises inside pulsar B’s magnetosphere and that the radial extent of the eclipsing region is about 10,000 km.

Our goal is to evaluate the absorption of the radio beam of pulsar A as it passes through the magnetosphere of pulsar B. We show in \( \Box \) that resonant cyclotron absorption in the charged-separated, closed field line region would provide only a small optical depth. However, we demonstrate in \( \bullet \) that absorption of radiation from pulsar A heats charged particles in B’s magnetosphere to relativistic energies. In \( \mathcal{H} \) we describe how this enables the accumulation of additional, charge neutral plasma...
in B’s magnetosphere. As a result, the radio emission of pulsar A can be significantly extinguished by synchrotron absorption. We devote [7] to a discussion of the ramifications of our model.

2. RESONANT CYCLOTRON ABSORPTION.

The region of closed field lines in the conventional pulsar magnetosphere model contains a corotating, charge-separated plasma with number density

\[ n_{GJ}(r) = \frac{|\Omega_B \cdot B(r)|}{2\pi ec (1 - v^2/c^2)} \]  \hspace{1cm} (1)

where \( v \) is the corotation velocity (Goldreich & Julian 1969). We are primarily interested in the region of B’s magnetosphere where \( v \ll c \) and the magnetic field is approximately dipolar. For our simplified model it suffices to ignore the angular dependence of the field and set

\[ B(r) \approx B_\star \left( \frac{R_\star}{r} \right)^3, \]
\[ \Omega_B \cdot B(r) \approx \Omega_\star B(r), \]  \hspace{1cm} (2)

where \( R_\star \) is the neutron star radius and \( B_0 \) is its surface magnetic field.

The particle number density \( n \) can be higher than \( n_{GJ} \) because the addition of neutral plasma does not affect the net charge density. We characterize this increase by the parameter \( \lambda \geq 1 \) defined such that

\[ n(r) \approx \lambda(r) \frac{\Omega_B B(r)}{2\pi ec}. \]  \hspace{1cm} (3)

In what follows, we assume that the magnetospheric particles are electrons and positrons.

Resonant cyclotron absorption is the dominant source of extinction for radio waves passing through a conventional pulsar magnetosphere. Cyclotron resonance occurs where

\[ \omega \approx \omega_B \equiv \frac{eB}{m_ee}, \]  \hspace{1cm} (4)

with cross-section (Canuto, Lodenguai & Ruderman 1971; Daugherly & Ventura 1978)

\[ \sigma(\omega) \approx \sigma_T \frac{\omega^2}{(\omega - \omega_B)^2 + \Gamma^2/4}. \]  \hspace{1cm} (5)

Here \( \sigma_T \) is the Thompson cross-section and

\[ \Gamma = \frac{4e^2 \omega_B}{3m_ee^2c^2} \]  \hspace{1cm} (6)

the natural line width. Strictly speaking, this cross section applies to photons in particular modes which differ for electrons and positrons, but we ignore this detail here.

The peak cross section at resonance reaches \( \sigma_{\text{max}} \approx 6\pi(e/\omega_B)^2 \), or roughly the square of the photon wavelength. Nevertheless, the optical depth is modest because an incident photon only resonates with cold electrons in a narrow radius range. From equations (2)- (5) we find that resonance occurs at radial distance from pulsar B of

\[ r_\tau(\omega) \approx R_\star \left( \frac{eB_\star}{m_ee} \omega^{-1} \right)^{1/3} = R_\star \left( \frac{\omega B_\star}{\omega} \right)^{1/3} \approx 1400 R_\star B_{12}^{1/3} \nu_9^{-1/3} \]  \hspace{1cm} (7)

and that the optical depth is given by

\[ \tau_\omega \approx \frac{2\pi}{3} \lambda(r) \frac{\Omega_\star B}{c} \approx 0.2 \lambda(r) B_{12}^{1/3} \nu_9^{-1/3}. \]  \hspace{1cm} (8)

Here \( \nu_0 \) is the radio frequency \( \nu = \omega/(2\pi) \) expressed in GHz and \( \omega_B = 1.8 \times 10^{19} B_{12} \) s\(^{-1} \) is the cyclotron frequency at the surface of pulsar B (i.e. for \( B = B_\star \)) with \( B_{12} \equiv B/(10^{12} \text{G}) \). Throughout the paper we set the neutron star radius to be 10 km and use \( R_\star \) as a unit of distance. Note that for the \( \lambda = 1 \), \( r_\tau \) is approximately the ratio of the resonance radius \( r_\tau \) to the radius of B’s light cylinder, \( c/\Omega_B \), a result obtained previously by Blandford & Scharlemann (1976) and Mikhailovskii et al. (1982).

The determination of the inclination of the binary’s orbit to the plane of the sky based on the scintillation technique (Coles et al. 2004) implies that the radio beam of pulsar A passes pulsar B at an impact parameter \( p \sim 4 \times 10^8 \text{ cm} \). Since \( p \ll r_\tau \), at a first glance cyclotron absorption looks like a viable eclipse mechanism, although a \( \lambda \sim 10 - 100 \) would be required to match the eclipse depth. However, a closer look at cyclotron absorption, as described below, reveals a problem: A’s radio radiation heats the particles in B’s magnetosphere to relativistic energies making synchrotron absorption rather than cyclotron absorption the relevant process.

We denote by \( F_\omega(\omega, r) \) the energy flux per unit frequency from pulsar A at distance \( r \) from pulsar B. A nonrelativistic electron or positron absorbs and emits energy at rates (Rybicki & Lightman 1979)

\[ \dot{E}_+(r) = \frac{1}{2} \int F_\omega(\omega, r) \sigma(\omega) d\omega = 2\pi^2 \frac{e^2}{m_ee} F_\omega(\omega_B(r), r) \]  \hspace{1cm} (9)

and

\[ \dot{E}_-(r) = \frac{4e^4}{9m_ee^2c}(\beta\gamma)^2 B(r)^2. \]  \hspace{1cm} (10)

Here \( \gamma \equiv (1 - \beta^2)^{-1/2} \) is the Lorentz factor of the electrons and positrons which must be close to unity for cyclotron absorption and emission to pertain. Balancing \( \dot{E}_+ \) by \( \dot{E}_- \) yields

\[ \beta \gamma = \frac{3\pi}{2} \left[ \frac{F_\omega(\omega_B(r), r)}{\omega_B^2 m_e} \right]^{1/2}. \]  \hspace{1cm} (11)

Lyne et al. (2004) measure a time-averaged flux density \( \approx 1.6 \text{ mJy at } \nu_0 = \omega_0/(2\pi) = 1.4 \text{ GHz from pulsar A. Taking } 600 \text{ pc } \text{ for the distance to J0737-3039 one obtains } F_0 = F_\omega(\omega_0) \approx 10^{-6} \text{ ergs cm}^{-2} \text{ at the position of B which is separated from A by about } 9 \times 10^{10} \text{ cm}. Using equation (11), we find that at \( r_\tau \approx 1250 R_\star B_{12} \) where 1.4 GHz photons get absorbed the electrons and positrons have \( \beta \gamma \approx 25 \). This means that the electrons and positrons are relativistic and that the cyclotron approximation is inapplicable. The timescale for particles to become mildly relativistic (\( E \sim m_ee^2 \)) due to cyclotron absorption is

\[ t_{\text{heat}}(r) \approx \frac{m_ee^2}{E_+(r)} = \frac{m_ee^2 c^2 / e^2}{2\pi^2 F_\omega(\omega_B(r), r)}, \]  \hspace{1cm} (12)

which is about 5 s at the position where 1.4 GHz photons are resonantly absorbed.
A related example of the heating of particles to relativistic energies through resonant cyclotron absorption of radio waves is given in Lyubarskii & Petrova (1998). In their example the pulsar’s radio waves heat particles streaming along its open field lines.

3. SYNCHROTRON ABSORPTION.

The mean cross-section for synchrotron absorption by an isotropic distribution of particles with energy $\gamma m_e c^2$ is (Rybicki & Lightman 1979)

$$\sigma_s(\omega) \approx \frac{8\pi^2}{3^2/3 \Gamma(1/3)} B \left(\frac{\omega_B}{\gamma \omega}\right)^{5/3},$$

(13)

provided $\omega_B / \gamma \lesssim \omega \lesssim \gamma^2 \omega_B$. Here $\Gamma(x)$ is a complete gamma function.

In the absence of published measurements of pulsar A’s radio spectrum, we assume that it is a power law with most of the energy concentrated at low frequencies (i.e. $F_\nu \propto \omega^{-\delta}$ with $\delta > 1$) as is typical for millisecond pulsars (Kuzmin & Losovsky 2001). The position of the low frequency cutoff of the spectrum is not very important for our problem (although see [4]).

For the cross-section [13] and an incident spectrum with $\delta > 1$, particle heating is dominated by the lowest frequencies. This also implies that A’s radio beam suffers a low frequency cutoff which progresses toward higher frequencies with increasing depth in B’s magnetosphere. Consequently, the absorbed spectrum of A’s radio beam at distance $r$ from pulsar B takes the form

$$F_\nu(\omega, r) = F_0 \left(\frac{\omega_0}{\omega}\right)^{\delta} \exp[-\tau_a(\omega, r)],$$

(14)

where $\tau_a(\omega, r)$ is the frequency-dependent optical depth at $r$. Using equations [15] & [16] one finds

$$\tau_a(\omega, r) = \int r \sigma_s(\omega) n(\omega) d\omega \approx \frac{4\pi}{3^2/3 \Gamma(1/3)} \lambda(r) \frac{\Omega_B r}{c} \left(\frac{\omega_B}{\gamma \omega}\right)^{5/3},$$

(15)

where in arriving at the last expression we have assumed that $\lambda(r)(\omega_B / \gamma)^{5/3}$ is a steeply decreasing function of $r$ and

$$\lambda_1 = 4 + \frac{5d \ln \gamma}{3d \ln r} - \frac{d \ln \lambda}{d \ln r},$$

(16)

Expression [15] is valid provided $\omega \gtrsim \omega_B / \gamma$; below this frequency the relativistic plasma becomes transparent. Since extinction at observed frequencies occurs at $r \sim 10^9$ cm $\ll t_L = c/\Omega_B \approx 10^{10}$ cm, a large $\lambda(r)$ is required to account for $\tau_a \gtrsim 1$.

Next we give a simplified evaluation of the energies to which magnetospheric particles are heated assuming $\delta = 2$ and $\lambda$ independent of $r$. We define a local cutoff frequency $\omega_1(r)$ such that $\tau_a(\omega_1(r), r) = 1$. Using equation [15] and dropping constant coefficients we find

$$\omega_1(r) \approx \frac{\omega_B(r)}{\gamma(r)} \left[\lambda(r) \frac{\Omega_B r}{c}\right]^{3/5}.$$  

(17)

Clearly $\omega_1(r)$ increases as $r$ decreases. Figure 1 illustrates how the low frequency part of $F_\nu(\omega, r)$ erodes with increasing depth in B’s magnetosphere. The synchrotron heating rate, $\dot{E}_s^r$, is given by

$$\dot{E}_s^r(r) \approx \sigma(\omega_1(r)) \omega_1(r) F_\nu(\omega_1(r), r)$$

$$\approx F_0 \left(\frac{\omega_0}{\omega_B}\right)^2 \exp\left[-\frac{\lambda(r) \Omega_B r}{c}\right] \approx\frac{\omega_0}{\omega_B} \left(\frac{r}{R_*}\right)^{5/3},$$

(18)

which reflects the dominance of radiation with $\omega \sim \omega_1(r)$. Balancing the synchrotron heating rate by the synchrotron cooling rate $\dot{E}_s$, we obtain

$$\gamma(r) \approx 5 \times 10^2 \frac{F_0}{m_e \omega_B} \left(\frac{\omega_0}{\omega_B}\right)^2$$

$$\times \left[\frac{\lambda \Omega_B R_*}{c}\right]^{8/5} \left(\frac{r}{R_*}\right)^{52/3}$$

$$\approx 2.5 \times 10^4 \lambda^{-8/5} \left(r/R_\star\right)^{52/3} B_{12}^{-4}.$$  

(19)

The calculation of $\gamma(r)$ tacitly assumes that synchrotron absorption of A’s radio emission is the only source for heating particles in B’s magnetosphere [cf. Lyutikov & Thompson (2005)].

To evaluate the size of the eclipsing region $r_e(\omega)$, defined as a distance from pulsar B at which $\tau_a(\lambda, \omega) = 1$, we combine equations [17] and [19] to arrive at

$$r_e(\omega) = 0.54 \frac{R_\star}{\omega_0} \left(\frac{\omega_0}{\omega_B}\right)^{5/64}$$

$$\times \left[\frac{F_0}{m_e \omega_B^2} \left(\frac{\omega_0}{\omega_B}\right)^2 \frac{\lambda \Omega_B R_*}{c}\right]^{11/64} \approx 380 R_\star \lambda^{11/64} \nu_9^{-5/64} B_{12}^{-25/64}.$$  

(20)

The weak frequency dependence of $r_e$ is an artifact of the assumed constancy of $\lambda$.

The observed eclipse duration corresponds to $r_e \approx 10^9$ cm at $\nu = 1$ GHz. From equation [20] we find that this implies $\lambda \approx 270$. For this value of $\lambda$, equation [19] yields $\gamma \approx 3.2 B_{12}^{-4}$ at $r_e = 10^9$ cm so our assumption of synchrotron absorption is marginally valid. The optical depth $\tau_a$ cannot be arbitrarily high because radio photons of frequency $\omega$ are not absorbed within the radius at which $\omega \approx \omega_B / \gamma = \omega$. This “saturation” of $\tau_a$ can be used to probe the dependence of $\lambda$ on $r$ (see equation [22] for a particular example).

4. ENHANCEMENT OF PARTICLE NUMBER DENSITY IN THE MAGNETOSPHERE OF PULSAR B.

We hypothesize that there is a continuous supply of energetic particles within the closed field line region of B’s magnetosphere from a source located close to the star where the magnetic field is strong. Radiation damps the gyrational motions of particles born in this region before they can slide out along magnetic field lines. Under normal circumstances each particle would loop along the closed field line on which it was born and strike the star’s surface in the opposite magnetic hemisphere. However,
illumination by A’s radio beam can re-excite the gyra-
tional motions of particles that stream far away from B
and then keep them suspended between conjugate mirror
points (see 11.12). Thus, both particle heating, making
synchrotron absorption possible, and particle trapping,
leading to a higher optical depth eclipse, are mediated
by A’s radio beam.

4.1. Lifetimes of Energetic Particles.

After illumination by A’s radio beam ceases, particles
trapped by magnetic mirroring in B’s magnetosphere
cool and are lost. Synchrotron emission reduces their
energies to \( m_e c^2 \) on a timescale, \( t_{cool} \), independent of
their initial \( \gamma \). From equation (22), we estimate
\[
t_{cool} \sim \frac{m_e^3 c^5}{e^2 B^2} \approx 350 \text{ s} \left( \frac{r/R_s}{10^3} \right)^6 B_{12}^{-2}
\]
where the last line is evaluated for \( \delta = 2 \). Use of the
cyclotron absorption formula [9] is appropriate because
particles streaming past \( r_c \) from a source interior to \( r_{cool} \)
are at most mildly relativistic (see 11.3).

Comparing \( \Delta E_{min} \) with \( \Delta E_+ \) we find that trapping via
cyclotron absorption and subsequent mirroring is possible
for particles with
\[
\beta_{||}^{12/5} \gamma \lesssim 0.12 \left( \frac{r_c/R_s}{10^3} \right)^{10}.
\]
Thus particles reaching \( r_c \approx 10^9 \text{ cm} \) with \( \beta_{||} \lesssim 0.4 \) can be
trapped. This threshold is not very restrictive (see 11.3),
so the rate at which \( \lambda \) can grow is primarily
determined by the rate at which particles are injected by
the source near B’s surface.6

4.3. Source of Particles.

We can only speculate about possible sources of parti-
cles in the corotating magnetosphere of pulsar B. The
requirement that B’s magnetosphere fill with an appro-
priate density of plasma while it is illuminated by A’s radio
beam is not demanding. The total number of particles
needed to provide the corotating charge density in B’s
magnetosphere is \( N_{min} \approx (2B_r R_s^2 / \epsilon c) \ln (r_{cool}/R_s) \sim 10^{30} \).
Boosting this number by a factor \( \sim 10^2 \) in \( 10^3 \text{ s} \)
implies a trapping rate \( \dot{N} \sim 10^{29} \text{ s}^{-1} \). If every
trapped particle were born with the energy \( f_1 m_e c^2 \) (sub-
sequently lost as synchrotron radiation) and only a frac-
tion \( f_2 < 1 \) of them were trapped, the source power would
be \( P \sim 10^{21} f_1 f_2 \text{ ergs s}^{-1} \). For \( f_1 = 10 \) and \( f_2 = 0.1 \), this
amounts to \( \sim 10^{20} \text{ ergs s}^{-1} \), much smaller than the
spin-down luminosity, \( \sim 10^{30} \text{ ergs s}^{-1} \), estimated for pul-
sar B by Lyne et al. (2004).

Our favored source is the creation of \( e^\pm \) pairs on closed
field lines by gamma rays with energies \( \sim 100 \text{ MeV} \)
emitted by particles accelerated in the outer gap of pul-
sar B (Cheng, Ho, & Ruderman 1986a,b; Wang et al.
1998).7 Photons from the outer gap can enter the coro-
tating magnetosphere and propagate at significant angles
to magnetic field lines. This facilitates \( B \gamma \) pair
creation close to the neutron star’s surface.

Pairs born relativistically initially lose energy through syn-
chrotron radiation on a timescale \( \Gamma^{-1} \sim 10^{-12} s B_{\perp}^{-2} \)
(see 9). As a consequence of relativistic beaming, their
pitch angles remain constant until they become transrela-
tivistic. Subsequent cooling by gyrosynchrotron radi-
aton completely damps gyrational motions but preserves
the component of velocity parallel to the magnetic field.
Thus particles streaming away from the neutron star
have \( \beta_{||} \lesssim 1 \). As we discussed in 11.2, this allows for
their efficient trapping when they are illuminated by A’s
radio beam.

5. Steady State Magnetosphere.

Under constant illumination by A’s radio beam, B’s
magnetosphere would achieve a steady state. We have
already described how particle energies would be set by

\* The expression for the residence time, \( \sim r / (\beta_c c) \), and that for
\( r_{cool} \) given by equation 22 hold in both relativistic (\( \beta \approx 1 \)) and
non-relativistic (\( \beta \ll 1 \)) regimes.

\* The two-stream instability might in principle assist in the trap-
ping of particles, but an unrealistically high number density is re-
quired for its growth time to be comparable to the particle residence
time in the magnetosphere.

\* We are grateful to Jonathan Arons for drawing our attention
to this possibility.
a balance between the absorption and emission of radiation. Here we concentrate on how the particle enhancement factor $\Lambda$ would be fixed in the context of the discussion in (4)

The efficiency of particle trapping drops sharply as the optical depth at the local cyclotron frequency, $\tau_{\omega}(\omega_B(r), r)$, increases above unity. The energy that fresh particles acquire by cyclotron absorption in passing through the magnetosphere is reduced by a factor $\exp[-\tau_{\omega}(\omega_B(r), r)]$ compared to $\Delta E_{\perp}$ (see equation (25)). Thus increasing $\tau_{\omega}(\omega_B(r), r)$ lowers the maximum $\beta^2$ at which particle trapping can occur (see (14)). The exponential dependence of trapping on optical depth suggests that $\tau_{\omega}(\omega_B, r)$ does not greatly exceed unity and also that it depends at most logarithmically on $r$. We set

$$\tau_{\omega}(\omega_B, r) = \chi,$$

where $\chi$ is a parameter we treat as independent of $r$ and whose precise value depends upon the rate at which trapped particles are lost by unspecified relaxation processes.

For $\omega > \omega_B/\gamma$ we find using equation (15) that $\tau_{\omega}(\omega, r) = \tau_{\omega}(\omega_B, r)(\omega_B/\omega)^{5/3}$. Thus, in the steady state magnetosphere photons of frequency $\omega$ are absorbed at distance $r_{e}(\omega)$ such that $\omega_B(r_{e}) = \omega\chi^{-3/5}$. This gives

$$r_{e}(\omega) = \chi^{1/5}r_{e}(\omega) = R_\star\chi^{1/5}\left(\frac{\omega_B}{\omega}\right)^{1/3} \approx 1400 R_\star\chi^{1/5}B_{12}^{-1/3}$$

where $r_{e}(\omega)$ is defined in equation (4). In steady state the eclipse duration would scale as $r^{-1/3}$. The structure of the steady state magnetosphere for an arbitrary power law spectrum of incident radio radiation is described by equations (11) and (12) in Appendix B. With typical parameters for J0737-3039 and $\delta = 2$ we obtain

$$\gamma(r) \approx 1.4\chi^{-24/55} \left(\frac{r/R_\star}{10^{13}}\right)^{36/11} B_{12}^{-12/11},$$

$$\lambda(r) \approx 10^2\chi^{3/11} \left(\frac{r/R_\star}{10^{13}}\right)^{49/11} B_{12}^{-20/11}.$$ 

Absorption by the extra plasma of A’s radio radiation reduces the efficiency of particle heating thus lowering the value of $\gamma$. The dispersion measure variation during the eclipse caused by extra plasma within the steady state magnetosphere is at least 2 orders of magnitude below the current upper bound of 0.016 pc cm$^{-3}$ (Kaspi et al. 2004).

The important distinction of the steady state model is that it predicts $\lambda$. This determines the lower and upper frequencies between which synchrotron absorption is effective at a given $r$:

$$\frac{\omega B}{2\pi} \gamma^{-1} \approx 2 \text{ GHz} \chi^{24/55} \left(\frac{r/R_\star}{10^{13}}\right)^{-69/11} B_{12}^{23/11},$$

$$\frac{\omega B}{2\pi} \gamma^{-2} \approx 5.6 \text{ GHz} \chi^{-48/55} \left(\frac{r/R_\star}{10^{13}}\right)^{-39/11} B_{12}^{-13/11}.$$ 

It follows that the maximum optical depth at frequency $\nu$, an observable quantity, is given by

$$\tau_{\max}(\nu) \approx 3.2\chi^{15/23} \nu_9^{-20/23}.$$ (32)

Figure 1 depicts the evolution of the spectrum of A’s radio beam with depth in B’s magnetosphere. Power at low frequencies is gradually eaten out by synchrotron absorption as the beam propagates deeper into B’s magnetosphere.

Illustration of B’s magnetosphere by A’s radio beam probably starts only short time prior to eclipse. So we may be witnessing radio beam attenuation by dynamically evolving plasma in B’s magnetosphere. The timescale for cold particles to become trans-relativistic via cyclotron absorption evaluated from equation (14) is rather short, typically $\lesssim 10$ s. Energies of relativistic particles rise exponentially via synchrotron absorption on timescale

$$t_{\text{heat}} \approx 0.78 \left[\frac{e^2}{mc^2} F_\omega(\omega_B)\right]^{-1} \approx 320 \left(\frac{r/R_\star}{10^{13}}\right)^{-6}.$$ (33)

so long as the radio spectrum of pulsar A remains unattenuated. Heating has an exponential character for $\delta = 2$ because as $\gamma$ grows particles can absorb the incoming radio photons at lower frequencies because $\omega_B/\gamma$ decreases making more energy from the unabsorbed spectrum available to heat them. This partly compensates for the decrease of the absorption cross section with increase of $\gamma$ (see eq. (13)). Once $\gamma$ grows so large that $\tau_{\omega}(\omega_B, \gamma, r)$ becomes comparable to unity, heating is less efficient and $t_{\text{heat}} \propto \gamma^{8/3}$. We estimate that particles reach $\gamma \approx 10^2$ at $r \approx 10^9$ cm on a timescale of $\sim 5$ min. This is comparable to estimates of several tens of minutes for the duration of the illumination period judged from the orientation of pulsar A’s spin and dipole axes suggested by Demorest et al. (2004). Not much can be
said about the timescale needed for the particle density to reach its steady state value. Presumably this is largely controlled by the rate at which the source near pulsar B is able to supply fresh trans-relativistic particles since these are readily trapped.

6. Reprocessed Radiation.

Energy absorbed by particles in B’s magnetosphere from A’s radio beam is reemitted as synchrotron radiation, albeit with some time delay. Absorption takes place at \( \omega \sim \omega_1 \), see equation (11), close to the minimum frequency possible, \( \omega_B / \gamma \), and is reemitted at the considerably higher frequency \( \omega \sim \gamma^2 \omega_B \). This reemitted radiation, although unimportant locally, may dominate the primary radiation from pulsar A deeper in pulsar B’s magnetosphere.

Here we attempt to calculate the properties of the reprocessed radiation for the steady state magnetosphere by applying results from \( \S \) 4. This requires choosing a low frequency cutoff, \( \nu_{\text{min}} \), for the assumed power law spectrum of A’s radio emission. Because there is little evidence for low-frequency cutoffs in the spectra of millisecond pulsars above 100 MHz (Kuzmin & Losovsky 2001), we normalize \( \nu_{\text{min}} \) to this frequency. From equation (28) we deduce that the energy carried by photons with \( \nu \sim \nu_{\text{min}} \) is absorbed at a distance \( r_{\text{max}} \approx 3500R_\star \nu_{\text{min},8}^{-1/3} \) from pulsar B and reemitted at frequency

\[
\nu_{\text{max}} \approx \gamma^3 \left( r_{\text{max}} \omega_B (r_{\text{max}}) / 2\pi \right) \approx 360 \nu_{\text{min},8}^{-13/11} \text{GHz.}
\]

The latter represents the upper cutoff frequency of the reprocessed radiation because \( \gamma^2 \omega_B \) increases with increasing \( r \) (see \( \S \) 29). A detailed calculation shows that radiation reemitted by all the relativistic particles within \( r_{\text{max}} \) has a flat power law spectrum \( F_\nu \) with index close to zero. The local flux of reprocessed photons at \( \nu_{\text{max}} \) can be estimated from

\[
F_\nu (\nu_{\text{max}}) \nu_{\text{max}} \approx F_\nu (\nu_{\text{min}}) \nu_{\text{min}} \nu_{\text{min}} \nu_{\text{max}} \approx 2 \times 10^{-9} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}
\]

for a primary spectrum with \( \delta = 2 \) and time-averaged intensity (at B’s location) \( F_\nu (1.4 \text{ GHz}) = 6 \times 10^{-6} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \). Reprocessed spectrum intersects the primary radio spectrum of pulsar A at \( \sim 50 \text{ GHz} \) and at frequencies higher than that one has to solve the radiation transfer problem to determine particle heating. Luckily, this is far enough from 1 GHz region of spectrum where eclipse observations are usually taken so that we do not need to worry about such complications. Unfortunately, the reprocessed radiation can hardly be detected on Earth because of very small covering fraction of B’s magnetosphere as seen from A.

7. Discussion.

A simple dynamical picture of pulsar A’s eclipse by pulsar B’s magnetosphere emerges from our considerations. Prior to the onset of illumination by A’s radio beam, the number density in the corotating part of B’s magnetosphere is equal to \( n_{\text{GCJ}} \). When A’s radio beam strikes the magnetosphere of pulsar B, perhaps \( \sim 10 \) min prior to B’s inferior conjunction, particles initially present in the magnetosphere rapidly (in \( \lesssim 10 \) s) become relativistic. Their energies continue to rise until either synchrotron absorption is balanced by synchrotron emission or illumination by A’s radio beam ceases. At the same time neutral plasma accumulates in the magnetosphere as the result of the trapping of particles that stream out in cold beams from a source near B. It is unclear whether the magnetosphere reaches steady state prior to the eclipse or whether is still evolving. In either case a density enhancement \( \lambda \sim 200 - 300 \) would be required to match the observed depth and duration of the eclipse. Some time, perhaps several tens of minutes, after the eclipse illumination of B’s magnetosphere by A’s radio beam ceases and the particles cool on a typical timescale \( t_{\text{cool}} \sim 10^{-2} - 10^{-3} \text{ s} \) given by equation (21).

As a result particles no longer mirror and all plasma beyond that needed to maintain the corotation electric field is lost. This entire cycle repeats each orbital period.

We speculate that part of the asymmetry between eclipse ingress and egress reflects a rise in plasma density in B’s magnetosphere during the \( \sim 30 \) s the eclipse lasts. The optical depth, \( \tau_\omega \), is very sensitive to increasing \( \lambda \). Not only is \( \tau_\omega \) directly proportional to the total column density of absorbing particles, it is also proportional to the absorption cross section per particle which varies as \( \gamma^{-5/3} \) and \( \gamma \) decreases with increasing \( \lambda \). Thus if \( \lambda \) were growing on a timescale of minutes during eclipse, egress would be deeper than ingress and the eclipse centroid would occur slightly after B reached inferior conjunction since \( r_e \) grows as \( \lambda \) increases (see eq. 29). In this picture the smooth ingress may partly be caused by the time-variable optical depth at a fixed location rather than the \( \tau_\omega \) variation as the radio beam samples smaller values of \( r \). On the contrary, during egress the optical depth is higher and the abrupt termination of eclipse may signal the emergence of the radio beam from behind an almost opaque screen. This explanation requires proper timing.

Our model predicts a variation of the polarization of A’s radio emission during the course of the eclipse. The polarization signal should be strongly correlated with B’s rotational phase in a manner similar to the eclipse lightcurve variations found by McLaughlin et al. (2004b). This prediction stems from the fact that synchrotron absorption of photons in different polarization states is sensitive to the angle between the photon \( k \) vector and the direction of the magnetic field. Modeling the polarization signature would be facilitated because B’s magnetic field should be nearly dipolar at \( r_e \) since \( R_\star \ll r_e \ll r_{\text{so}} \).

Effects of general relativity are very important in J0737-3039. Lai & Rafikov (2004) have demonstrated that gravitational light bending can significantly (by \( \sim 30\% \)) change the minimum impact parameter at which A’s radio beam passes B. Thus gravitational lensing must play a significant role in shaping the eclipse profile. Moreover, because of the binary’s finite orbital eccentricity, \( e \approx 0.088 \), periastron precession driven mainly by the effects of general relativity forces a 21 yr periodic variation, from \( a|\cos i|(1-\epsilon) \) to \( a|\cos i|(1-\epsilon) \) neglecting lensing effects, of the pulsars’ minimum projected separation on the plane of the sky (Burgay et al. 2003). By changing the minimum impact parameter of A’s radio beam with respect to B, this should produce observable eclipse profile variations.

Lyutikov (2004) and Arons et al. (2004) have suggested that the eclipse of A’s radio beam is caused by synchrotron absorption in the magnetosheath that forms when A’s relativistic wind impacts B’s magnetosphere. Detailed polarization observations offer a means to distinguish this viable alternative from our model.
The model we have presented is necessarily rather simplistic — it is one-dimensional, it assumes that particle distribution functions are isotropic, and so on. Where possible, future work should relax these constraints. A quantitative description of the particle source giving rise to the plasma density enhancement might also be pursued. Further observations have the opportunity to reveal additional clues to properties in the eclipse region. Knowledge of A’s radio spectrum is crucial for calculating particle energies, trapping efficiencies, and the eclipse duration. Time-resolved polarization of A’s radio emission during eclipse would constrain the magnetic field geometry and particle distribution anisotropy.

Clarifying details of this remarkable example of nonlinear coupling of relativistic plasma to external radiation may provide clues for understanding other puzzling phenomena such as the modulation of B’s radio emission by radiation from A (McLaughlin et al. 2004a) and the periodic variations of the pulsar B’s brightness that correlate with its orbital phase (Ransom et al. 2004).

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APPENDIX

PARTICLE ENERGY.

To properly compute the heating of particles by synchrotron absorption we evaluate

$$\dot{E}_s^+(r) = \frac{1}{2} \int_{\omega_B/\gamma}^{\infty} F_\omega(\omega, r) \sigma_s(\omega) d\omega \approx \frac{4\pi^2}{3^{4/3} \Gamma(1/3)} \frac{F_0 e}{B} \int_{0}^{\infty} \left( \frac{\omega_B}{\gamma \omega} \right)^{5/3} \frac{\omega_0}{\omega} e^{-\tau_\omega(\omega, r)} d\omega, \quad (A1)$$

where the factor 1/2 roughly accounts for shadowing as B rotates. The lower limit of integration is extended to zero (instead of $\omega_B/\gamma$) because we are assuming that $\tau_\omega(\omega_B/\gamma, r) \gg 1$. The largest contribution to the local heating comes from $\omega$ such that $\tau_\omega(\omega, r) \sim 1$ which, coupled to the condition that $\omega_B/\gamma < \omega$ and equation (15), assures that $\tau_\omega(\omega_B/\gamma, r) \gg 1$.

Substituting $\tau_\omega$ from equation (16) into equation (A1) and calculating the integral over $d\omega$ we find

$$\dot{E}_s^+(r) = \frac{3\pi \zeta_1}{5} \frac{2 + 3\delta}{5} \left[ \frac{4\pi}{3^{4/3} \Gamma(1/3)} \right]^{3(1-\delta)/5} \frac{\epsilon_{\omega B} F_0}{B} \left( \frac{\omega_0}{\omega_B} \right)^{\delta} \left[ \lambda(r) \frac{\Omega_B r}{c} \right]^{-(2+3\delta)/5} \gamma^{\delta-1}. \quad (A2)$$

Balancing this heating rate by the cooling rate (10) yields

$$\gamma(r) = \left[ \frac{27\pi \zeta_1}{20} \frac{2 + 3\delta}{5} \left( \frac{4\pi}{3^{4/3} \Gamma(1/3)} \right)^{3(1-\delta)/5} \left[ \frac{F_0}{m_e \omega_B^2} \left( \frac{\omega_0}{\omega_B} \right)^{\delta} \left[ \lambda(r) \frac{\Omega_B r}{c} \right]^{-(2+3\delta)/5} \right]^{3/2} \right]. \quad (A3)$$

Substituting equation (A3) into equation (15), assuming $\tau_\omega(\omega) = 1$, and solving for $r_e$, we obtain the size of eclipsing region at frequency $\omega$:

$$r_e(\omega) = R_s \left[ \frac{4\pi}{3^{4/3} \Gamma(1/3) \zeta_1} \right]^{3/32} \left[ \frac{20}{27\pi \zeta_1 (2 + 3\delta)} \right]^{5/64} \left( \frac{\omega_B \omega}{\omega} \right)^{\delta} \left( \frac{5(3-\delta)}{64} \right)^{\delta} \left[ \frac{F_0}{m_e \omega_B^2} \left( \frac{\omega_0}{\omega_B} \right)^{\delta} \left[ \lambda(r) \frac{\Omega_B r}{c} \right]^{-(2+3\delta)/5} \right]^{11/64}. \quad (A4)$$

The constant coefficients in these expressions depend upon the values of the parameters $\delta$ and $\zeta_1$. For the radio spectrum of pulsar A with $\delta = 2$ and $\lambda$ independent of $r$, it follows that $\zeta_1 = 64/3$. Then the coefficient in equation (A4) is $\approx 481$.

STEADILY ILLUMINATED MAGNETOSPHERE.

Combining equations (15), (27), and (A3), $\gamma$ is self-consistently determined to be

$$\gamma(r) = \left[ \frac{9\pi^2 \Gamma(2 + 3\delta)}{3^{4/3} \Gamma(1/3)} \right] \left[ \frac{F_0}{m_e \omega_B^2} \left( \frac{\omega_0}{\omega_B} \right)^{\delta} \right]^{3/11}. \quad (B1)$$

From equations (15), (27), and (A3) we find that

$$\lambda(r) = \frac{3^{4/3} \Gamma(1/3)}{4\pi} \left[ \frac{9\pi^2 \Gamma(2 + 3\delta)}{3^{4/3} \Gamma(1/3)} \right]^{5/11} \left[ \frac{F_0}{m_e \omega_B^2} \left( \frac{\omega_0}{\omega_B} \right)^{\delta} \right]^{5/11}. \quad (B2)$$

Equations (B1) and (B2) imply that $\zeta_1 = 5$, see equation (16). These formulae determine the structure of the steady state magnetosphere for arbitrary $\delta > 1$.

REFERENCES

Arons, J., Backer, D. C., Spitkovsky, A., & Kaspi, V. 2004, in Binary Radio Pulsars, ASP Conf. Series; eds. F. A. Rasio & I. H. Stairs

Blandford, R. D. & Scharlemann, E. T. 1976, MNRAS, 174, 59
Burgay, M., D’Amico, N., Possenti, A. et al. 2003, Nature, 426, 531
Canuto, V, Lodenquai, J., & Ruderman, M. 1971, Phys. Rev. D, 3, 2303
Cheng, K. S., Ho, C., & Ruderman, M. 1986a, 300, 500
Cheng, K. S., Ho, C., & Ruderman, M. 1986b, 300, 522
Coles, W. A., McLaughlin, M. A., Rickett, B. J. et al. 2004, astro-ph/0409204
Daugherty, J. K. & Ventura, J. 1978, Phys. Rev. D, 3, 2303
Demorest, P., Ramachandran, R., Backer, D. C., et al. 2004, astro-ph/0402025
Goldreich, P. & Julian, W. H. 1969, ApJ, 157, 869
Kaspi, V., Ransom, S., Backer, D. C. et al. 2004, ApJL, 613, 137
Lai, D. & Rafikov, R. R. 2004, astro-ph/0411726
Lyne, A. G., Burgay, M., Kramer, M., et al. 2004a, ApJL, 613, 57
McLaughlin, M. A., Lyne, A. G., Lorimer, D. R., et al. 2004b, astro-ph/0408297
Mikhailovskii, A. B., Onishchenko, O. G., Suramlishvili, G. I., & Sharapov, S. E. 1982, Sov. Astron. Lett., 8(6), 369
Ransom, S., Demorest, P., Kaspi, V., Ramachandran, R., & Backer, D. C. 2004, in *Binary Radio Pulsars*, ASP Conf. Series; eds. F. A. Rasio & I. H. Stairs
Wang, F. Y.-H., Ruderman, M., Halpern, J. P., & Zhu, T. 1998, ApJ, 498, 373
Lyutikov, M. 2004, MNRAS, 353, 1095
Lyutikov, M. & Thompson, C. 2005, in preparation
McLaughlin, M. A., Kramer, M., Lyne, A. G., et al. 2004a, ApJL, 613, 57
Lyubeznikii, Y. E. & Petrova, S. A. 1998, A&A, 337, 433