Simulation and Analysis of Centroid Motion Characteristics of Uncontrolled Spinning Projectile on the Plateau

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Abstract. On the plateau, the aerodynamic forces and moments of uncontrolled rotating projectiles in flight have changed greatly compared with those in the plain, which will inevitably affect the trajectory of the projectiles. In this paper, the motion characteristics of projectile moving around the center of mass on Plateau were analysed through the simulation of six-degree-of-freedom rigid body trajectory under Plateau conditions. It is found that the degree of motion around the center of mass increases and the attenuation of rolling motion slows down under Plateau conditions. According to the research results, the paper proposed some suggestions on the use of uncontrolled spinning projectiles under Plateau conditions.

1. Introduction
China's plateau has a vast area, including the Qinghai-Tibet Plateau, Inner Mongolia Plateau, Yunnan-Guizhou Plateau and the Loess Plateau[1]. The area above 1000 meters above sea level is about 5.26 million square kilometers, accounting for 55% of the total land area. The area above 4000 meters above sea level is about 1.85 million square kilometers, accounting for 19% of the total land area. On the plateau, due to the decrease of air density, the aerodynamic forces and moments of the projectile in flight have changed greatly compared with those in the plain, which will inevitably affect the trajectory of the projectile. Although the motion of uncontrolled rotating projectile is directly reflected as the center of mass motion, the motion around the center of mass will affect the movement of the center of mass. Especially in Plateau conditions, the characteristics of the motion around the center of mass have their own characteristics, and will also affect the use of ammunition in Plateau conditions. In this paper, the motion characteristics of projectile moving around the center of mass on the plateau were analyzed through the simulation of six-degree-of-freedom rigid body trajectory under Plateau conditions, and the suggestions for the rational use of the grenade on the plateau were put forward.

2. Six degree of freedom rigid body trajectory model under plateau conditions
According to the classical ballistic theory and method[2][3], the rigid body ballistic model under Plateau condition was established according to the idea of “first establishing coordinate system, then analyzing force and moment, then establishing the relationship between motion parameters and force according to Newton's momentum theorem and theorem of moment of momentum theorem, and then decomposing them into coordinate axes of coordinate system”.

The models include the kinematics model of the center of mass, the dynamic model of the center of mass, the kinematics model of the movement around the center of mass, and the dynamic model of the movement around the center of mass. The models are summarized as follows:
The expressions of forces and moments in the formula are:

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F_{z_2} = F_p \cos \delta_2 \cos \delta_1 - \frac{\rho v}{2} S_a C_D (v - \omega_z) + \frac{\rho S_a}{2} C_{la} \frac{\delta_x}{\sin \delta_r} \left[ \nu_z \cos \delta_2 \cos \delta_1 - v_x (v - \omega_z) \right] \\
+ \frac{\rho v}{2} S_a C_{\gamma_{pa}} \frac{\delta_y}{\sin \delta_r} \left[ (v - \omega_z) \sin \delta_2 + \omega_z \cos \delta_2 \cos \delta_1 \right] - m g'_{\delta z=0} (1 + k_A \sin^2 \Lambda) \left( r_0 / (r_0 + y) \right) \sin \theta_a \cos \psi_2 \\
+ 2 \Omega_x m v (\sin \psi_2 \cos \theta_a \cos \Lambda \cos \alpha_\gamma + \sin \theta_a \sin \psi_2 \sin \Lambda + \cos \psi_2 \cos \Lambda \sin \alpha_\gamma) \\
F_{y_2} = F_p \sin \delta_2 + \frac{\rho v}{2} S_a C_D \omega_z + \frac{\rho S_a}{2} C_{la} \frac{\delta_x}{\sin \delta_r} \left( \nu_z \cos \delta_2 \sin \delta_1 + v_x \omega_z \right) \\
+ \frac{\rho v}{2} S_a C_{\gamma_{pa}} \frac{\delta_y}{\sin \delta_r} \left[ -\omega_z \cos \delta_2 \cos \delta_1 - (v - \omega_z) \sin \delta_2 \sin \delta_1 \right] \\
+ m g'_{\delta x=0} (1 + k_A \sin^2 \Lambda) \left( r_0 / (r_0 + y) \right) \sin \theta_a \sin \psi_2 + 2 \Omega_x m (\sin \lambda \cos \theta_a - \cos \Lambda \sin \theta_a \cos \alpha_\gamma) \\
M_\xi = -\frac{\rho S_a l d}{2} \nu_{r} C_{\text{mw}} \frac{1}{\sin \delta_r} v_{r_z} + \frac{\rho v^2}{2} S_a l \nu_{w_z} \delta_2 \\
M_\eta = -\frac{\rho S_a l d}{2} \nu_{r} C_{\text{mw}} \frac{1}{\sin \delta_r} v_{r_\eta} - \frac{\rho S_a l d}{2} \nu_{r} C_{\text{mw}} \omega_\eta - \frac{\rho S_a l d}{2} \nu_{r} C_{\text{mw}} \frac{1}{\sin \delta_r} v_{w_{\eta}} \\
M_\zeta = -\frac{\rho S_a l d}{2} \nu_{r} C_{\text{mw}} \frac{1}{\sin \delta_r} v_{r_\zeta} - \frac{\rho S_a l d}{2} \nu_{r} C_{\text{mw}} \omega_\zeta - \frac{\rho S_a l d}{2} \nu_{r} C_{\text{mw}} \frac{1}{\sin \delta_r} v_{w_{\zeta}} \\
\]

The parameters of the model see the external ballistics references[2][3]. There is no analytic solution for the above model, and only the numerical solution can be used to solve the trajectory[4].

3. Motion characteristics of uncontrolled spinning projectile around the center of mass on the plateau
The motion characteristics around the center of mass mainly refer to the change of the direction of the projectile axis relative to the ground reference coordinate system and the change of the angle of attack of the projectile. The change of the projectile axis direction relative to the ground reference coordinate system includes the variation of elevation angle and azimuth angle. The angle of attack can also be decomposed into elevation attack angle and azimuth attack angle. Both of them mainly describe the pitching motion characteristics of projectiles. In addition, rolling motion of projectiles is also an aspect of moving around the center of mass.

3.1. Variation of projectile axis direction with elevation of position

Taking the full-scale charge of a certain grenade as an example, the trajectory simulation are carried out with the firing angles of 30°, 45° and 60°, and the variation laws of the azimuth angle and the elevation angle of the projectile axis with elevation of position were analyzed. The variation rules of the elevation angle and azimuth angle of the projectile axis with elevation of position variation are shown in figures 1 to 6.

3.2. Variation of attack angle with elevation of position

The results show that the azimuth angle of the projectile axis increases with the elevation of position increasing. Taking full-scale charge as an example, the altitude increases from 0 m to 5000m, and the range of the maximum azimuth angle of the projectile axis varies from 2.05° to 2.48° when the firing angle is 30°. When the firing angle is 45°, the range of maximum azimuth angle of the projectile axis is 3.23° to 5.01°, and when the firing angle is 60°, the range of maximum azimuth angle of the projectile axis is 8.13° to 22.03°.

With the increase of position altitude, the change trend of elevation angle of the projectile axis becomes slower, and the variation law of the ballistic inclination angle is similar to this.

3.2. Variation of attack angle with elevation of position
Attack angle is an important parameter in projectile flight attitude, and is also an important index to investigate the stability of projectile flight. The variation of attack angle with elevation of position is shown in figures 7 to 9.

As the height increases, the angle of attack increases. Taking full-scale charge as an example, the altitude increases from 0 m to 5000 m, and the range of the maximum angle of attack varies from 0.47° to 0.71° when the firing angle is 30°. When the firing angle is 45°, the range of maximum angle of attack is 1.23° to 2.70°, and when the firing angle is 60°, the range of maximum angle of attack is 6.32° to 17.64°. In addition, when the firing angle is 60°, the angle of attack oscillates at trajectory descent stage, and the oscillation phenomenon becomes more obvious with the increase of altitude. This shows that the flight stability of projectiles at high altitude becomes worse and the increase of altitude will aggravate the deterioration of stability.

3.3. Variation of rotational speed with elevation of position

For a spinning stabilized projectile, the speed of the projectile decreases gradually after the exit. According to the calculation results of 6D rigid body ballistic model, the variation of the rotational speed of a certain grenade with the increase of altitude is shown in figures 10 to 12.

It can be seen that the attenuation trend of speed decreases slowly due to the decrease of air density at high altitude. The simulation results also show that there are two inflection points in the attenuation curve with the increase of the firing angle, which indicates that the rotational speed attenuates faster at the beginning, slows down at the middle of the ballistic trajectory and increases at the end. As shown in figure 12, the performance is particularly evident at the 60° angle. This trend of change requires attention in the design and use of projectiles that need to be opened or guided at the end of the trajectory.
4. Conclusion
The simulation results show that the elevation angle and azimuth angle of projectile axis of the uncontrolled rotating projectile increase under Plateau conditions, indicating that the angular motion increases and the rolling motion attenuates slowly. Under the same firing conditions, the change of the direction swing of projectile axis is greater than that of the vertical swing under Plateau condition. Especially when the firing angle is large, the swing of the shafts appears obvious oscillation in the arc falling section. This obviously increases the overload of the internal parts of the projectile, and is not conducive to ensuring the reliability of fuse and other components. At the same time, the projectile axis oscillation is not conducive to the stability and firing accuracy of the two trajectory of the scattered ammunition. Under the condition of large firing angle in Plateau area, with the increase of altitude, the increase of attack angle is especially obvious, which is not conducive to the stability of flight trajectory. Under the condition of large firing angle in Plateau area, with the increase of altitude, the change of projectile rotational speed shows that the attenuation speed in middle trajectory decreases slowly and the attenuation speed increases in the end trajectory. This trend of change requires attention in the design and use of projectiles that need to be opened or guided at the end of the trajectory.

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