The Van Hiele geometry thinking levels of mild mental retardation students

Z A Shomad, T A Kusmayadi and Riyadi
Universitas Sebelas Maret, Jl.Ir. Sutami No. 36A, Surakarta 57126, Indonesia

E-mail: zahidshomad@gmail.com

Abstract. This research is to investigate the level of mild mental retardation geometry students thinking. This research focuses on the geometry thinking level based on Van Hiele theory. This study uses qualitative methods with case study strategy. Data obtained from observation and tests result. The subjects are 12 mental retardation students. The result show that ability of mild mental retardation students with each other is different but have same level of level thinking geometry. The geometry thinking level of mental retardation students was identified in level 1 of the Van Hiele theory. Based on the level thinking geometry of mental retardation students simplify geometry thinking teachers in selecting appropriate learning methods, choose the materials in accordance with ability, and can modify the material following the geometry thinking level of mental retardation students.

1. Introduction
One of the most important topics in mathematics is geometry [1-8] who learn about points, lines, figures, the relationships between the line, length, area, volume, and others [9]. Geometry teach the students in the abstract and visually about the size and structure of geometry, such as the characteristics and relationships between one figure with another figures.

The aims of learning geometry is to provide an opportunity for students to have and develop the critical thinking ability, problem-solving ability, and better understanding ability of mathematic by gaining high level of geometry thinking ability [10]. To reach these goals, students need to understand the terms of geometry, spatial problems, and have cause and affect relationship between the features of geometric figures instead of memorizing definitions and theorems [11]. It would be very appropriate to divide the teaching in accordance with the geometry thinking level. However, the increase in the geometry thinking level does not occur naturally, need for learning which stimulates progress level thinking geometry. Geometric thinking levels facilitate students in learning geometry gradually.

One of the important researches about the geometry thinking level is the Van Hiele Geometry Thinking Level. The theory of geometry thinking Levels from the Van Hiele advanced by mathematician Dina Van Hiele and his wife Marie Van Hiele Pieree in 1957. Geometry thinking levels of Van Hiele consists of five levels, namely: 1) Level 0 (Visualization), 2) Level 1 (analysis), 3) Level 2 (Informal Deduction), Level 3 (Deduction), and Level 4 (Rigor) [12, 13]. Geometry thinking levels compiled by Van Hiele have hierarchical stages, making it easier for students in learning high level geometry.

At Level 0, the student identify, resolving names, and operate on geometric figures (e.g., angles, triangles intersecting or parallel lines) according to their appearance [14]. Students can mention the name based on
appearance in general. They can mention triangle, square, cube, etc., but they do not explicitly identify the properties of the figures [15], they can’t analyze. At Level 1, the student analyzes figures in terms of their components and relationships among components and discovers properties/rules of a class of figures empirically (e.g., by folding, measuring, and using a grid or diagram) [14]. Students analyze the existing components and the relationships between components, and then compile the tools from the figures of empirically. The forms are recognized by their components. They not only look at the visualization, but give attention to the properties. If someone describes a figure with four corners, it is a quadrilateral, though depicted with ugly. However, at this level, properties are not yet an ordered, such as a square is not necessarily identified being a rectangle [16, 17]. At Level 2, the student logically interrelates previously discovered properties/rules by giving or following informal arguments [14]. At this level, students can distinguish the features of figure. Students will understand why each square is a square, but they can’t organize necessary lists in order to proof this observation [18, 19]. At Level 3, the student proves theorems deductively and establishes interrelationships among system of theorems [14]. At this level, students are able to explain the relationships between figures using theorems, definitions, and axioms. Students can use their critical thinking ability and problem solving ability. At Level 4, the student establishes theorems in different postulation systems and analyzes/album these systems [14]. They understand that the system is a work area.

Average high students are on level 2-3 [19] and elementary school students are at a level 1-2 [20]. Lately, mathematics and geometry are not only developed for regular students, but also for students with special need. Individuals with Disabilities Education Act (IDEA) stated that it is important to provide access to students with special need in order to get the same curriculum with students normal and expected both get the same instruction on the concepts that are important [21]. One of them is mental retardation. Mental retardation/intellectual disability is a condition of incomplete development of the mind, characterized by impairment of skills manifested during the developmental period, i.e. cognitive, language, motor and social abilities [22].

According to the American Association on Mental Deficiency (AAMD), there are several types of mental retardation which is measured by IQ, are: 1) Mild mental retardation with IQ score 50-70, 2) Moderate mental retardation with IQ score 36-50, Severe mental retardation 3) with IQ score 20-40, Profound mental retardation 4) with an IQ score below 20, and 5) Unspecified [23]. Elliot further explain some of the difficulties experienced by students studying mental retardation, are: 1) attention, 2) cognitive processes, especially with regard to organizing, classification, and strategy setting, 3) Power Remember, which tends to be bad for short-term memory [24]. The mental age of a student's mental retardation the equivalent age of regular students at the elementary school level, [21]. It implies that although the conditions of the students are weak in the cognitive aspects of mental retardation, but students also have mental retardation geometry thinking level.

Based on the above explanation, geometry thinking level is an important component in learning geometry. Although there has been no research on the level of thinking of students in need special, but there is some literature that describes the geometry for students in need of special students, particularly mental retardation. Therefore, this research aims to analyze the level of geometric thinking about mental retardation students.

2. Method
This study uses qualitative methods with strategy case studies. The qualitative method is a method that aims to find out the events or phenomena experienced by the subject with the description with words and sentences [25]. Case study is an empirical inquiry that purposed to investigates a contemporary phenomenon within its real-life context [26]. Qualitative methods of research with strategy case study is a research aims to know the contextual problem by administering treatment. The problem in this research is a level thought geometry students mild mental retardation.

The research was held at the High School in Sukoharjo in March– May 2017. The subject of this research is the mild mental retardation students chosen by purposive sampling. The subject is used as many as 12 students with mild mental retardation the same capabilities.

Data taken with the awarding of the test on the whole subject,. Data were analyzed with Miles and Huberman steps of analyzing involve data reduction, data display, and verification [27]. The data in the form of the results of the students and then analyzed to find out the level of geometric thinking students.
3. Result and Discussion

Table 1 describes the results of level thinking geometry students with mental retardation subject as many as 12 students at a high school in Sukoharjo, Indonesia. In Table 1, there is a code of the student and the geometry thinking level of mild mental retardation student. In the grouping of students in levels is done by analyzing the test results of geometric thinking level students.

Table 1. The task of level thinking geometry students mild mental retardation

| Subject | TT01 | TT02 | TT03 | TT04 | TT05 | TT06 | TT07 | TR08 | TR09 | TR10 | TR11 | TR12 |
|---------|------|------|------|------|------|------|------|------|------|------|------|------|
| Level 0 | 1    | 1    | 1    | 1    | 1    | 1    | 1    | 0    | 0    | 0    | 0    | 0    |
| Level 1 | T    | F    | T    | F    | T    | F    | T    | F    | F    | F    | F    | F    |
| Level 2 | T    | T    | T    | T    | T    | T    | T    | T    | T    | T    | T    | T    |

This level think geometry is based on geometric thinking levels test students who have been validated by experts. The question of the test consists of 10 numbers. Number 1-4 measures the geometric thinking level 0, number 5-7 measures the geometric thinking level 1, and number 8-10 measures the geometric thinking level 2. Subject categorized in level thinking 0 if at least 70% correct can work out between the reserved number 1-4. Students are categorized in level 1 can work out if at least 70% correctly on the numbers 5-7, and students are categorized in level 2 may be exercised if at least 70% correctly on the number 8-10.

Table 2. The result of subject geometry thinking test level

| Subject | TT01 | TT02 | TT03 | TT04 | TT05 | TT06 | TT07 | TR08 | TR09 | TR10 | TR11 | TR12 |
|---------|------|------|------|------|------|------|------|------|------|------|------|------|
| Level 0 | F    | T    | T    | T    | T    | T    | T    | F    | T    | F    | T    | F    |
| Level 1 | T    | F    | T    | F    | T    | F    | T    | F    | F    | F    | F    | F    |
| Level 2 | T    | T    | T    | T    | T    | T    | T    | T    | T    | T    | T    | T    |

Based on indicators of the level of thinking of geometry of Van Hiele [28], subject TT01, TT02, TT03, TT04, TT05, TT06, TT07 categorized in level 1 and subject TR08, TR09, TR10, TR11, TR12 categorized in level 0. Subject level 0 able to describe triangle, rectangle, and the rectangle figures appropriately. The subject is not capable of analyzing the characteristics a triangle, square, and
rectangular figure, so that the subject is incorrect in choosing a wake which is included in the triangle, square, and rectangular. Based on the analysis, the subject TR08, TR09, TR10, TR11, TR12, and categorized in level 0. While the subject is categorized in level 1 because the subject is able to describe exactly a triangle, square, and rectangular figures. The subject is also capable of analyzing the characteristics of wake up, so you can sort and select the shapes included in the triangle, square, and rectangular figures. The subject of level 1 is unable to associate the relationship figure one with another figure, for example a rectangular including square has four sides of equal length.

3.1 Mild mental retardation student of geometry thinking level 1

![Figure 1. The test result of subject with geometry thinking level 1](image1)

Based on Figure 1, the subject can show the image rectangle. The subject of understanding about the characteristics and components of the rectangle. The subject knows the sides that parallel and have the same length. The subject also knows that figure has four right angles, so that students can show the image rectangle. Subject with level 1 can also compare a figure to another figure and describe with precision. However, the subject with level 1 cannot show that a square is a rectangle with a length of the same side.

![Figure 2. The test result of subject with geometry thinking level 1](image2)
Based on Figure 2, the subject of geometric thinking with level 1 can not analyze the relationships between the components of a square and a rectangle. The subjects still think that square is not part of the rectangle [29].

Based on research of Elliot, mental retardation student possible having difficulties with an unusual task they do [24]. Tests and activities from the level of thinking is the new test geometry for mild mental retardation students . However, when compared to the condition of the subject that are experiencing a shortage in the cognitive aspects, then the achievement level thinking geometry students mental retardation level 1 is a good thing. The Van Hiele levels of geometric thinking is a step that the hierarchy, so it is not possible to have the subject thinks the geometry of 3 and 4.

3.2 Mild mental retardation student of geometry thinking level 0

![Figure 3. The test result of subject with geometry thinking level 0](image)

Based on the results of the analysis, the subject of geometric thinking with level 0 are having difficulty in naming or identifying figure. The subject was wrong in choosing to figure which is a square. The subject is the difficulty in understanding the characteristics of the figure, so wrong in naming the figure. In Figure 3 above, it is possible to interpret the wrong subject that figure has chosen a right angle so the frontage. Koberlein defines that a square is a quadrilateral wake has a right angle and have a contiguous congruent sides [30, 31]. The subject confuse in comparing the characteristics of figure so that the subject made a mistake in identifying the square. The subject is also unable to identify the characteristics of another figure. This is possible because of the condition of the subject which is a mild mental retardation that students have a less in cognitive aspects.

Based on the above explanation, there is a difference of level thinking geometry students mild mental retardation. It is in accordance with the research conducted by Kemis and Rosnawati, barriers and conditions experienced by each student's mental retardation can be varied, although it has the same type of disability [32]. Although students have mild mental retardation barriers to smarts, but students can still be taught mental retardation. So the thinking level geometry can also be one of the skills that can be developed for students ’ mental retardation. Some of the skills that must be mastered by the student's mental retardation are the ability of calculation and mathematics reasoning [33, 34]. Because the stages in thinking level geometry are hierarchical stages, it can help students develop the ability of reasoning in mental retardation.
4. Conclusion
The result of this research shows those students who have mild mental retardation level of geometric thinking 0 experience difficulties in identifying and comparing the figure. While students with mental retardation level of geometric thinking 1 has difficulty in connecting the components of a figure. It shows that any individual included in the mild mental retardation may have different geometric thinking levels, although equally included in the category of mental retardation. However, it is important for teachers to know the level of mild mental retardation students thinking for simplicity in explaining the material geometry. If the teacher knows the condition of the student's mental retardation, then handling for student's mental retardation when learning will be easier. The selection of methods, models, strategies, and media in learning geometry are expected to raise the level of mild mental retardation geometry students thinking. One of them with the learning phase geometry developed by Van Hiele.

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