Learning to Broadcast for Ultra-Reliable Communication with Differential Quality of Service via the Conditional Value at Risk

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Abstract

Broadcast/multicast communication systems are typically designed to optimize the outage rate criterion, which neglects the performance of the fraction of clients with the worst channel conditions. Targeting ultra-reliable communication scenarios, this paper takes a complementary approach by introducing the conditional value-at-risk (CVaR) rate as the expected rate of a worst-case fraction of clients. To support differential quality-of-service (QoS) levels in this class of clients, layered division multiplexing (LDM) is applied, which enables decoding at different rates. Focusing on a practical scenario in which the transmitter does not know the fading distribution, layer allocation is optimized based on a dataset sampled during deployment. The optimality gap caused by the availability of limited data is bounded via a generalization analysis, and the sample complexity is shown to increase as the designated fraction of worst-case clients decreases. Considering this theoretical result, meta-learning is introduced as a means to reduce sample complexity by leveraging data from previous deployments. Numerical experiments demonstrate that LDM improves spectral efficiency even for small datasets; that, for sufficiently large datasets, the proposed mirror-descent-based layer optimization scheme achieves a CVaR rate close to that achieved when the transmitter knows the fading distribution; and that meta-learning can significantly reduce data requirements.

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I. INTRODUCTION

Layered division multiplexing (LDM) has been introduced in several standards as an effective means to support differential quality-of-service (QoS) in broadcast and multicast services. With LDM, multiple independent sub-messages, or layers, are superimposed, enabling the decoding of a different number of messages depending on the channel conditions, thus supporting communication at a variable rate [1]–[4]. The most common use of LDM is for multimedia broadcast, as adopted by the Advanced Television Systems Committee (ATSC 3.0) [4], [5], in which LDM supports a robust configuration for mobile receivers and a high-capacity connection for fixed receivers. Other applications include Machine-Type Communication (MTC) and Industry 4.0, in which LDM is considered as a tool to deliver critical control services and best-effort monitoring services [6]–[8]; as well as vehicle-to-everything (V2X) communications [9]. In V2X systems, traffic light status and pedestrian detection information in intelligent intersections [10] can be broadcast together with high-definition map transmission for enhanced autonomous driving [11].

In many of the use cases of broadcast and multicast services, it is essential to ensure that a large fraction of clients receives at least some of the information being transmitted, such as basic safety messages (BSMs) in V2X [12]. The corresponding standard design objective is the transmission of a single message at the outage rate [13]–[16]. For outage probability $\beta \in [0, 1]$, the $\beta$-outage rate is the largest rate that can be guaranteed with probability $1 - \beta$, i.e., that can be achieved by a fraction $1 - \beta$ of all possible clients (see Fig. 1). Transmitting at the $\beta$-outage rate implies that the fraction $\beta$ of clients with the worst channel conditions cannot decode the (single) message being broadcast.

Targeting the design of LDM broadcasting/multicasting in ultra-reliable communication systems, this work takes the complementary approach of focusing on the differential QoS performance of the $\beta$-fraction of clients with the worst instantaneous channel conditions (see Fig. 1). To this end, we introduce the $\beta$-conditional value-at-risk (CVaR) rate as the expected rate achieved by the $\beta$-fraction of clients with the worst fading channels via LDM. Unlike the $\beta$-outage rate criterion, the $\beta$-CVaR rate is concerned with the performance of the worst-case $\beta$-fraction of clients, enabling clients in this class to decode at different rates.

Maximizing the $\beta$-CVaR rate requires adjusting the layers’ rates and power levels as a function of the channel distribution [17]. However, in practice, this distribution is unknown.
Fig. 1. Illustration of the standard $\beta$-outage rate and of the proposed $\beta$-CVaR rate.

Accordingly, in this paper, we assume the transmitter has access to a dataset sampled during deployment, from which the rate and power allocation for each layer are optimized. We explore theoretic and algorithmic aspects of this design problem, including also extensions to learning to learn, or meta-learning [18].

**Related Work:** LDM, also known as the broadcast approach, has been extensively studied as means to improve spectral efficiency in various scenarios. A comprehensive survey of the state-of-the-art is available in [1], and we mention here some representative examples. The broadcast approach for slowly fading single-user channels was investigated in [17], where it was shown that transmitting multiple layers can increase the expected achievable rate. The gain of the broadcast approach was also demonstrated in [19] for finite number of layers. Specifically, for quasi-static Rayleigh fading channel, two layers were shown to achieve most of the throughput gain. Importantly, unlike our work, both references [17] and [19] assume that the transmitter knows the fading distribution.

With respect to ultra-reliable communication, multicast beamforming was studied in [20] with the goal of minimizing the outage probability, and an approximate solution was obtained for a Gaussian mixture channel with up to three Gaussian kernels. For unknown fading distribution, several gradient-based algorithms were proposed in [21] to optimize beamforming based on a dataset of channel samples. Similarly, an alternating gradient descent algorithm was recently proposed in [22] for the joint optimization of the precoding weights and the reconfigurable intelligent surface (RIS) reflection pattern in RIS-aided communication system.

Optimization of the CVaR statistic was first introduced in [23] for financial applications. Since then, it has been considered in a variety of fields [24]. For communication systems, the CVaR statistic was applied in [25] for risk-sensitive resource allocation scheme targeting
low-latency traffic; in [26] for statistical QoS estimation in a shared spectrum; and in [27] for robust computation offloading from mobile devices to infrastructure nodes.

A review of meta-learning with emphasis on applications to communication systems is available in [28]. Representative examples include meta-learning for learning to demodulate [29], [30] or decode [31]; for end-to-end learning of encoder and decoder [32]; for beam-forming adaptation [33], [34]; for proactive resource allocation [35], [36]; and for channel estimation [37].

**Main Contributions:** This work introduces and studies the concept of $\beta$-CVaR rate for the problem of LDM-based broadcasting/multicasting in systems with a single-antenna base station (BS) serving single-antenna clients. The channel coefficients and the fading distribution are assumed to be unknown to the BS, which optimizes layer allocation based on a dataset sampled during deployment. We address both information-theoretic and algorithmic aspects, with the specific contributions being as follows.

- We introduce the concept of $\beta$-CVaR rate as the average rate obtained for the $\beta$-fraction of clients with the worst channel conditions (see Fig. 1). This novel criterion targets ultra-reliable broadcasting/multicasting applications, while allowing for differential worst-case QoS guarantees.
- As a special case of the problem of maximizing the $\beta$-CVaR metric, we review the optimization of power and rate allocation for the expected achievable rate when the fading distribution is known and infinite layers are applied. For this scenario, we bound the optimality gap caused by the availability of limited data via a generalization analysis [38].
- Moving beyond the expected rate metric, we address the problem of maximizing the $\beta$-CVaR rate. At a theoretical level, we characterize the number of samples required to maintain a desired optimality gap, showing that the sample complexity increases as the fraction $\beta$ decreases. At an algorithmic level, we introduce a mirror-descent based scheme [39] to maximize an empirical estimate of the $\beta$-CVaR rate.
- In light of our theoretical result that the sample complexity increases as $\beta^{-1}$ as $\beta$ decreases, we address the problem of reducing sample complexity via meta-learning. By leveraging data from multiple previous deployments, each with different fading distributions, meta-learning aims at decreasing data requirements on new deployments.
- Numerical results demonstrate that broadcasting multiple layers improves spectral efficiency even for small datasets, and that, for sufficiently large dataset, the expected rate and $\beta$-CVaR rate are close to that achieved when the BS knows the fading distribution, confirming the sample complexity analysis. In addition, meta-learning is shown to be effective in
decreasing the sample complexity for low outage probabilities.

Organization: The rest of the paper is organized as follows. In Section II, we present an information-theoretic model for a multi-layer broadcast channel with no channel state information (CSI). Maximization of the expected achievable rate is studied in Section III. In Section IV, we define the $\beta$-CVaR rate performance measure and characterize the sample complexity. In Section V, we describe a mirror-descent-based algorithm for empirical $\beta$-CVaR rate maximization. Meta-learning is introduced in Section VI as a means to reduce sample complexity. In Section VII, we present numerical results in order to evaluate the expected rate and $\beta$-CVaR rate for layer allocation, and to assess the impact of meta-learning on performance. Finally, in Section VIII, we conclude the paper and highlight some open problems.

Notation: Random variables and vectors are denoted by lowercase and boldface lowercase Roman-font letters, respectively. Realizations of random variables and vectors are denoted by lowercase and boldface lowercase italic-font letters, respectively. For example, $x$ is a realization of random variable $X$ and $\mathbf{x}$ is a realization of random vector $\mathbf{x}$. For any positive integer $K$, we define the set $[K] \triangleq \{1, 2, \ldots, K\}$. The cardinality and convex hull of a set $\mathcal{L}$ are denoted by $|\mathcal{L}|$ and $\text{conv}(\mathcal{L})$, respectively. The $\ell^1$-norm and $\ell^2$-norm of a vector $s$ are denoted by $\|s\|_1$ and $\|s\|_2$, respectively. For two scalars $a$ and $b$, the indicator of the event $a \geq b$ is denoted by $\mathbf{1}_{a \geq b}$. That is, $\mathbf{1}_{a \geq b}$ equals one if $a \geq b$ and zero otherwise. The maximum between real scalar $r$ and zero is denoted by $(r)^+$. The set of non-negative real numbers is denoted by $\mathbb{R}_+$. The nearest positive integer to scalar $x$ is denoted by $\lceil x \rceil$. $\text{diag}(\mathbf{u})$ represents a diagonal matrix with diagonal given by the vector $\mathbf{u}$.

II. System Model and Problem Definition

We consider the system depicted in Fig. 2 in which a single-antenna BS broadcasts a common message to single-antenna clients over a fading broadcast channel. The fading coefficient for each client is drawn from a common fading distribution $p_h(h)$, and is assumed to remain constant for the duration of a coding block consisting of $n$ symbols. The common fading distribution $p_h(h)$ may take the form of a mixture model, as in [20], in order to account for heterogeneous long-term effects such as path loss and shadowing.

The signal received by a client at time $t \in [n]$, denoted by $y(t)$, can be expressed as

$$y(t) = \sqrt{P}h x(t) + z(t),$$

(1)
Fig. 2. Illustration of the broadcast setting under study. A single-antenna base station (BS) broadcasts a common message to single-antenna clients. The signal to each client undergoes a fading channel in which the fading coefficient is drawn from a common fading distribution \( p_h(h) \). This paper is concerned with the average rate of the \( \beta \)-fraction of users with the worst instantaneous channel conditions (see Fig. 1).

where \( P > 0 \) denotes the BS transmission power; \( x(t) \in \mathbb{C} \) denotes the signal transmitted at time \( t \), which is subject to the average power constraint

\[
\mathbb{E} \left[ |x(t)|^2 \right] \leq 1; \tag{2}
\]

channel coefficient \( h \sim p_h(h) \) denotes the quasi-static fading coefficient; and \( z(t) \sim \mathcal{CN}(0, 1) \) denotes the additive white Gaussian noise (AWGN).

We assume that the BS does not know the fading realizations nor the common fading distribution \( p_h(h) \), while each client knows its own channel \( h \). Due to the lack of CSI, the BS applies layered division multiplexing (LDM) [17] with \( \rho \) layers, or sub-messages, in order to enable differential quality of service at the clients. The transmitted signal \( x(t) \) in (1) is accordingly given as

\[
x(t) = \sum_{m=1}^{M} x_m(t), \tag{3}
\]

where \( x_m(t) \sim \mathcal{CN}(0, \lambda_m) \), with \( m \in [M] \), denotes a symbol from a Gaussian random codebook with average power \( \lambda_m \) that is used to encode sub-message \( w_m \in [2^{\rho_m}] \) of rate \( \rho_m \geq 0 \). To satisfy the normalized power constraint in (2), the power-allocation vector \( \lambda \triangleq (\lambda_1, \ldots, \lambda_M) \) must thus lie in the simplex

\[
\Delta_c^M \triangleq \left\{ \lambda \in \mathbb{R}_+^M : \sum_{m=1}^{M} \lambda_m \leq 1 \right\}. \tag{4}
\]

We refer to message \( w_m \) and corresponding encoded signal \( x_m(t) \) as the \( m \)th layer.

Each client decodes sub-messages by applying successive cancellation decoding (SCD) with the order \( w_1, \ldots, w_M \). When decoding layer \( m \in [M] \), all subsequent layers are treated as AWGN. Each client can hence decode only a subset of layers depending on its channel gain
g \triangleq |h|^2. We denote by \( I_m \triangleq \sum_{i=m+1}^{M} \lambda_i \) the normalized power level of the inter-layer interference affecting the decoding of layer \( m \), and as \( p_g(g) \) the distribution of the channel gain \( g \).

We parametrize the rate \( \rho_m \) of layer \( m \) as [17]

\[
\rho_m(s^m, \lambda) \triangleq \log_2 \left( 1 + \frac{\|s^m\|_1 \lambda_m P}{1 + \|s^m\|_1 I_m P} \right),
\]

where \( s \triangleq (s_1, \ldots, s_M) \in \mathbb{R}_+^M \) is a non-negative vector set by the BS, and vector \( s^m \triangleq (s_1, \ldots, s_m) \in \mathbb{R}_+^m \) consists of the first \( m \) elements of \( s \). Assuming that all previous layers are correctly decoded, the rate achievable for layer \( m \) by a client with channel gain \( g \) is \( \log_2 (1 + g \lambda_m P / (1 + g I_m P)) \). Therefore, the client can decode all layers up to layer \( m \) if and only if its channel gain satisfies the inequality \( g \geq \|s^m\|_1 \). Accordingly, given the power and rate allocation vectors \( \lambda \) and \( s \), the total rate that can be decoded by a client with channel gain \( g \) is given as

\[
R(s, \lambda, g) \triangleq \sum_{m=1}^{M} \rho_m(s^m, \lambda) 1_{g \geq \|s^m\|_1}.
\]

We study the optimization of the rate and power allocation vectors \((s, \lambda)\) under two performance metrics. Specifically, we first consider the optimization of the expected achievable rate

\[
\bar{R}(s, \lambda) \triangleq \mathbb{E}_{g}[R(s, \lambda, g)],
\]

where the expectation is over the fading distribution \( p_g(g) \), in Section III. Then, in Section IV, we investigate a more general metric, the CVaR, which can naturally account for the performance of ultra-reliable communication.

III. EXPECTED ACHIEVABLE RATE

In this section, we study the maximization of the expected achievable rate \( \bar{R}(s, \lambda) \) in (7) over the power and rate allocation vectors \( \lambda \) and \( s \). That is, we consider the optimization problem

\[
(s^*, \lambda^*) \in \arg \max_{(s, \lambda) \in \mathbb{R}_+^M \times \Delta_c^M} \bar{R}(s, \lambda).
\]

The optimization problem (8) depends on the unknown fading distribution \( p_g(g) \). In this section, we first review known optimality results for the case in which \( p_g(g) \) is known, and then we address the setting of interest in which the distribution \( p_g(g) \) is unavailable.
A. Known Channel Distribution

In [17], problem (8) was studied under the assumption that the BS knows the fading distribution $p_g(g)$, and the optimal power allocation density was derived for an infinite number of layers $M \to \infty$. For reference, we report the main result in the following proposition.

Proposition 1 ([17, Sec. II.B]): The expected rate achieved when the BS knows the fading distribution $p_g(g)$ and LDM is applied with an infinite number of layers ($M \to \infty$) is given as

$$\bar{R}_\infty = \int_0^\infty \Pr[g \geq u] \frac{u\rho(u)}{1 + uI(u)} du,$$

with power allocation density $\rho(u) = -\frac{dI(u)}{du}$ and accumulated interference

$$I(u) = \begin{cases} \frac{\Pr[g \geq u] - u p_g(u)}{u e^{-\frac{u}{P}}}, & \text{for } u_0 \leq u \leq u_1, \\ 0, & \text{otherwise}, \end{cases}$$

where $u_0$ is determined by the equality $I(u_0) = P$, and $u_1$ is determined by the equality $I(u_1) = 0$.

Note that the result (10) holds only for continuous fading distributions $p_g(g)$. For the special case of Rayleigh fading, this result can be specialized as follows.

Corollary 1 ([17, Sec. II.C]): For Rayleigh fading, i.e., for $h \sim \mathcal{CN}(0, 1)$, the expected rate achieved when the BS knows the fading distribution $p_g(g)$ and LDM is applied with an infinite number of layers ($M \to \infty$) can be expressed as

$$\bar{R}_\infty = 2 \int_{u_0}^\infty \frac{\exp(-u)}{u} du - 2 \int_1^\infty \frac{\exp(-u)}{u} du - (\exp(-u_0) - \exp(-1)),$$

where

$$u_0 = \frac{2}{1 + \sqrt{1 + 4P}}.$$

To the best of our knowledge, finding an explicit solution to problem (8) for a finite number of layers is an open problem even when the fading distribution $p_g(g)$ is known. That said, the expected achievable rate in Proposition 1 can be viewed as an upper bound on the expected rate achieved for a finite number of layers and for unknown fading distribution.

B. Unknown Channel Distribution: Empirical Average Rate Maximization

In this paper, we assume that the BS does not know the fading distribution $p_g(g)$, and hence it cannot directly optimize the expected achievable rate $\bar{R}(s, A)$. Instead, we assume that the BS has access to a dataset

$$\mathcal{G} = \{g_1, \ldots, g_N\}$$
consisting of $N$ fading realizations sampled in an independent and identically distributed (i.i.d.) manner from distribution $p_g(g)$. Based on dataset $\mathcal{G}$, the BS approximates the expected achievable rate with the empirical average

$$
\bar{R}^G(s, \lambda) = \frac{1}{N} \sum_{i=1}^{N} R(s, \lambda, g_i). \tag{14}
$$

The maximization of the average rate (14) over power and rate allocation vectors $\lambda$ and $s$ can be expressed as the optimization problem

$$(s^G, \lambda^G) \in \arg \max_{(s,\lambda)\in \mathbb{R}^M \times \Delta_c^M} \bar{R}^G(s, \lambda). \tag{15}$$

A solution to problem (15) can be practically obtained via an iterative optimization scheme as detailed in Section V.

We emphasize that optimizing the average rate $\bar{R}^G(s, \lambda)$ via problem (15) is useful not only when the fading distribution $p_g(g)$ is unknown, but also when the direct optimizations in (8) based on knowledge of the distribution $p_g(g)$ is not tractable. In this latter case, one can potentially generate the dataset $\mathcal{G}$ with an arbitrary number of fading realizations $N$.

### C. Optimality Gap and Sample Complexity

An important theoretical question is whether the expected achievable rate obtained under the power and rate allocation vectors (15) approaches the ground-truth maximum expected achievable rate obtained with vectors (8) as the size of the dataset increases. If so, it would also be interesting to quantify how many samples $N$ are required to achieve a desired level of approximation. This is the subject of this subsection.

To proceed, we define the optimality gap

$$e^G \triangleq \bar{R}(s^*, \lambda^*) - \bar{R}(s^G, \lambda^G) \tag{16}$$

as the difference between the expected rate achieved with optimal power and rate allocation vectors (8) and the expected rate achieved by the empirical rate maximization (15). The optimality gap is random due to the stochastic nature of the dataset $\mathcal{G}$.

To bound the optimality gap, we assume that the norms of the optimal vectors $s^*$ and $s^G$ in (8) and (15), respectively, can be bounded as $\max\{\|s^*\|_1, \|s^G\|_1\} \leq S$ for some known constant $S > 0$. Note that this assumption is not restrictive since, in practice, $S$ represents the largest fading gain $g$ that a client is expected to experience. The following proposition bounds the optimality gap under this assumption.
Proposition 2: Let \( \mathcal{G} = \{g_1, \ldots, g_N\} \) be a dataset of \( N \) fading realizations drawn independently from the fading distribution \( p_g(g) \), and let \( \delta \in (0, 1] \). With probability at least \( 1 - \delta \), the optimality gap (16) is bounded, for rate allocation vectors with bounded norms \( \max\{\|s^+\|_1, \|s^-\|_1\} \leq S \), as
\[
e^\mathcal{G} \leq \left( 4\sqrt{(2N + 1) \ln(N + 1)} + \sqrt{\frac{2 \ln(2/\delta)}{N}} \right) 2 \log_2 (1 + SP).
\]

Proof: See Appendix A. 

This result shows that the optimality gap scales with number of data points, \( N \), as \( \mathcal{O}(\sqrt{\ln(N)/N}) \), implying that any level of accuracy can be attained as the dataset grows larger, i.e., as \( N \to \infty \). Furthermore, for a given desired optimality gap \( e^\mathcal{G} \leq \epsilon \), the required number of data points \( N \), i.e., the sample complexity, satisfies the approximate inequality
\[
\frac{N}{\ln(N)} \gtrapprox \left( \frac{\log_2(SP)}{\epsilon} \right)^2
\]
for large \( N \). Intuitively, the sample complexity increases with the signal-to-noise ratio (SNR) metric \( SP \) since, as the achievable rate increases, a better approximation is required to achieve the same subtractive optimality gap.

IV. CONDITIONAL VALUE AT RISK

In this section, we move beyond the expected rate metric with the goal of investigating a performance measure that is closer to the requirements of ultra-reliable systems, namely the conditional value-at-risk (CVaR) rate.

A. Outage Rate and CVaR Rate

The standard performance measure used for ultra-reliable systems is the outage rate [13]–[16]. For outage probability \( \beta \in [0, 1] \) and vectors \( s \) and \( \lambda \), the \( \beta \)-outage rate \( r_\beta(s, \lambda) \) is the largest total rate in (6) that can be guaranteed with probability \( 1 - \beta \). Mathematically, it is defined as
\[
r_\beta(s, \lambda) \triangleq \max \{r \in \mathbb{R}_+ : \Pr[R(s, \lambda, g) \geq r] \geq 1 - \beta\}.
\]

For the broadcast setting under study, a \( \beta \)-outage rate \( r_\beta(s, \lambda) = r \) indicates that a fraction \( 1 - \beta \) of all possible clients is guaranteed to attain a rate at least equal to \( r \); or, conversely, that a fraction \( \beta \) of all possible clients cannot decode at rate \( r \), as illustrated in Fig. 1.

The \( \beta \)-outage rate does not provide any information about the rates achieved by the \( \beta \)-fraction of users with the worst channel gain. To obtain a more refined analysis of the
\(\beta\)-fraction of the least performing users, in this paper, as illustrated in Fig. 1, we introduce the \(\beta\)-CVaR rate \(R_\beta(s, \lambda)\) as the expected rate achieved by the \(\beta\)-fraction of clients with the worst channels. Formally, the \(\beta\)-CVaR rate is defined as

\[
R_\beta(s, \lambda) \triangleq \mathbb{E}_g \left[ R(s, \lambda, g) \big| R(s, \lambda, g) \leq r_\beta(s, \lambda) \right], \tag{20}
\]

where the expectation is conditioned on the event that the rate is lower than the \(\beta\)-outage rate.

For \(\beta = 0\), we have \(r_0(s, \lambda) = R_0(s, \lambda) = 0\) for all \(s \in \mathbb{R}_+^M\) and \(\lambda \in \Delta^M\), so, we limit the range of \(\beta\) to \(\beta \in (0, 1]\). Furthermore, at the other extreme, for \(\beta = 1\), the 1-CVaR rate can be seen to coincide with the expected achievable rate, i.e., \(R_1(s, \lambda) = \bar{R}(s, \lambda)\). In practice, for ultra-reliable applications, smaller values of \(\beta\) are typically of interest.

For any \(\beta \in (0, 1)\), the \(\beta\)-CVaR rate can be expressed via the \textit{variational representation} \cite[Theorem 1]{23}

\[
R_\beta(s, \lambda) = \max_{r \in \mathbb{R}_+} f_\beta(s, \lambda, r), \tag{21}
\]

where the function \(f_\beta(s, \lambda, r)\) is defined as

\[
f_\beta(s, \lambda, r) \triangleq r - \beta^{-1} \mathbb{E}_g \left[ (r - R(s, \lambda, g))^+ \right]. \tag{22}
\]

Furthermore, the maximum in (21) is attained at the \(\beta\)-outage rate \(r = r_\beta(s, \lambda)\), i.e.,

\[
R_\beta(s, \lambda) = f_\beta(s, \lambda, r_\beta(s, \lambda)). \tag{23}
\]

Finally, the maximization of the \(\beta\)-CVaR rate over the power and rate allocation vectors \(\lambda\) and \(s\) can be expressed as the joint optimization problem

\[
(s_\beta^*, \lambda_\beta^*) \in \arg \max_{(s, \lambda) \in \mathbb{R}_+^M \times \Delta^M} R_\beta(s, \lambda) = \arg \max_{(s, \lambda) \in \mathbb{R}_+^M \times \Delta^M} f_\beta(s, \lambda, r_\beta(s, \lambda)). \tag{24}
\]

To the best of our knowledge, finding an explicit solution to the maximization (24) for any \(\beta \in (0, 1)\) is an open problem even when the fading distribution \(p_\beta(g)\) is known.

\textbf{B. Empirical CVaR Maximization}

Since the BS does not know the fading distribution \(p_\beta(g)\), it cannot directly optimize the \(\beta\)-CVaR rate. Instead, similar to the expected rate maximization problem studied in Section III, the BS approximates the \(\beta\)-CVaR rate with an empirical average over the dataset \(\mathcal{G}\) in (13).

Specifically, following the variational representation (21), the \textit{empirical \(\beta\)-CVaR rate}, for any \(\beta \in (0, 1)\), is defined as

\[
R_\beta^G(s, \lambda) \triangleq \max_{r \in \mathbb{R}_+} f_\beta^G(s, \lambda, r), \tag{25}
\]
with \( f^\beta_G(s, \lambda, r) \) being the empirical approximation of function \( f_\beta(s, \lambda, r) \) in (22), i.e.,

\[
f^\beta_G(s, \lambda, r) = r - \frac{1}{N \beta} \sum_{i=1}^{N} (r - R(s, \lambda, g_i))^+ .
\]

(26)

Furthermore, we define the empirical \( \beta \)-outage rate \( r^\beta_G(s, \lambda) \) as the optimal \( r \) for problem (25). Hence, we have

\[
R^\beta_G(s, \lambda) = f^\beta_G(s, \lambda, r^\beta_G(s, \lambda)).
\]

(27)

Overall, the maximization of the empirical \( \beta \)-CVaR rate over power and rate allocation vectors \( \lambda \) and \( s \) can hence be expressed as the optimization problem

\[
(s^\beta_G, \lambda^\beta_G) \in \arg \max_{(s, \lambda) \in \mathbb{R}^M \times \Delta^M} R^\beta_G(s, \lambda) = \arg \max_{(s, \lambda) \in \mathbb{R}^M \times \Delta^M} f^\beta_G(s, \lambda, r^\beta_G(s, \lambda)).
\]

(28)

In closing this section, we observe that, unlike the \( \beta \)-outage rate (19), the empirical \( \beta \)-outage rate \( r^\beta_G(s, \lambda) \) has a closed-form expression. Defining as \( g_i \) the \( i \)th smallest channel gain in dataset \( \mathcal{G} \), i.e., \( g_{[1]} \leq g_{[2]} \leq \cdots \leq g_{[N]} \), we have the following proposition.

**Proposition 3:** For any power and rate allocation vectors \( \lambda \in \Delta^M \) and \( s \in \mathbb{R}^M \), the empirical \( \beta \)-outage rate is given as \( r^\beta_G(s, \lambda) = R(s, \lambda, g_{[N \beta]}) \).

**Proof:** See Appendix C.

By Proposition 3, the objective in (28) can be expressed as

\[
R^\beta_G(s, \lambda) = \frac{[N \beta]}{N \beta} R^{\beta^*}(s, \lambda) + \left( 1 - \frac{[N \beta]}{N \beta} \right) R(s, \lambda, g_{[N \beta]}),
\]

(29)

where we have defined the subset

\[
\mathcal{G}_\beta \triangleq \{ g_{[1]}, g_{[2]}, \ldots, g_{[N \beta]} \} \subseteq \mathcal{G},
\]

(30)

which consists of the lowest \( \beta \)-fraction of channel gains. Therefore, the \( \beta \)-CVaR rate can be obtained from the expected achievable rate calculated for the subset \( \mathcal{G}_\beta \), with a minor correction (the second term in (29)) if the product \( N \beta \) is not an integer.

**C. Optimality Gap and Sample Complexity**

In this section, we study the sample complexity for the \( \beta \)-CVaR rate metric. To this end, for any \( \beta \in (0, 1) \), we define the optimality gap

\[
\epsilon^\mathcal{G}_\beta \triangleq R_\beta(s^*_\beta, \lambda^*_\beta) - R_\beta(s^\mathcal{G}_\beta, \lambda^\mathcal{G}_\beta)
\]

(31)

as the difference between the \( \beta \)-CVaR rate achieved with optimal power and rate allocation vectors (24) and the \( \beta \)-CVaR rate achieved by the empirical maximization (28).
Similar to the case of expected rate maximization studied in Section III-C, we assume rate allocation vectors with bounded norms, i.e., \( \max\{\|s^*_\beta\|_1, \|s^0\beta\|_1\} \leq S \). The following proposition bounds optimality gap (31) under this assumption.

**Proposition 4:** Let \( \mathcal{G} = \{g_1, \ldots, g_N\} \) be a dataset of \( N \) fading realizations drawn independently from the fading distribution \( p_g(g) \), and let \( \delta \in (0, 1] \). With probability at least \( 1 - \delta \), the optimality gap (31) is bounded, for \( \beta \in (0, 1) \) and rate allocation vectors with bounded norms \( \max\{\|s^*_\beta\|_1, \|s^0\beta\|_1\} \leq S \), as

\[
e^\mathcal{G}_\beta \leq \beta^{-1} \left( 4 \sqrt{\frac{(2N + 1) \ln(N + 1)}{3N(N + 1)}} + \sqrt{\frac{2 \ln(2/\delta)}{N}} \right) 2 \log_2(1 + SP).
\]

*Proof:* See Appendix D.

For any fixed \( \beta \in (0, 1) \), Proposition 4 demonstrates that, in a manner similar to Proposition 2, any level of accuracy can be attained as \( N \to \infty \). Furthermore, the scaling of the optimality gap in terms of number of data points, \( N \), and SNR metric, \( SP \), is equivalent to that described for expected rate maximization in Proposition 2. However, Proposition 4 also shows that the number of samples required to maintain a desired optimality gap \( \epsilon \) increases as \( \beta \) decreases according to the approximate inequality

\[
\frac{N}{\ln(N)} \gtrsim \left( \frac{\log_2(SP)}{\beta \epsilon} \right)^2
\]

for sufficiently large \( N \). Intuitively, this is because, as \( \beta \) becomes smaller, the \( \beta \)-CVaR rate is calculated with respect to increasingly rarer outage events, which require more data to be observed in sufficient numbers.

**V. Mirror Gradient Descent for \( \beta \)-CVaR Rate Maximization**

In this section, we introduce a gradient-based iterative optimization procedure to tackle the empirical \( \beta \)-CVaR rate maximization problem (28). The approach is based on the introduction of a surrogate smooth objective and on mirror descent, as described in the rest of this section and summarized in Algorithm 1.

**A. Smooth Surrogate Objective**

A first challenge in developing iterative solutions to problem (28), is that the partial derivative of the indicator in the achievable rate expression (6) with respect to vector \( s \)
Algorithm 1: Empirical CVaR maximization

**Input:** Dataset \( G \), fraction \( \beta \in (0, 1] \)

**Initialization:** Initialize \( u \in \mathbb{R}^M \) and \( \lambda \in \Delta_c^M \)

1. set \( i = 0 \)
2. set \( u^{(i)} = u \) and \( \lambda^{(i)} = \lambda \)
3. while not converged do
   4. set \( i \leftarrow i + 1 \)
   5. set \( u^{(i)} \leftarrow \text{GD}_\beta(u^{(i-1)}; G, \lambda^{(i-1)}) \) (defined in (39))
   6. set \( \lambda^{(i)} \leftarrow \text{EG}_\beta(\lambda^{(i-1)}; G, u^{(i-1)}) \) (defined in (40))
4. return \( (s^{(i)} = \exp(u^{(i)}), \lambda^{(i)}) \)

equals zero almost everywhere. Therefore, in order to facilitate the application of a gradient-based optimization procedure, we replace the rate \( R(s, \lambda, g) \) in (6) with the smooth surrogate objective

\[
R_\sigma(s, \lambda, g) \triangleq \sum_{m=1}^{M} \rho_m(s^m, \lambda) \sigma(c(g - \|s^m\|_1)),
\]

where \( \sigma(x) \triangleq 1/(1 + \exp(-x)) \) is the sigmoid function, and the parameter \( c > 0 \) determines the trade-off between smoothness and accuracy of the surrogate approximation. As \( c \rightarrow \infty \), the surrogate (34) tends uniformly to the original rate (6), while smaller values of \( c \) yield non-zero partial derivatives with respect to \( s \).

Using the approximation (34), we define the surrogater empirical CVaR maximization problem as

\[
(\tilde{s}_\beta^g, \tilde{\lambda}_\beta^g) = \arg \max_{(s, \lambda) \in \mathbb{R}^M \times \Delta_c^M} \tilde{R}_\beta^g(s, \lambda),
\]

where, based on (29), we have defined

\[
\tilde{R}_\beta^g(s, \lambda) \triangleq \frac{[N\beta]}{N\beta} \tilde{R}_{\beta}^g(s, \lambda) + \left(1 - \frac{[N\beta]}{N\beta}\right) R_\sigma(s, \lambda, g_{[\lfloor N\beta \rfloor]}),
\]

with the surrogate average rate

\[
\tilde{R}_{\beta}^g(s, \lambda) \triangleq \frac{1}{[N\beta]} \sum_{i=1}^{[N\beta]} R_\sigma(s, \lambda, g_{[i]}).
\]

B. Mirror Descent

Although the objective in (35) is smooth, plain-vanilla gradient descent cannot be applied to address the optimization (35) due to the domain constraints on the optimization variables
\((s, \lambda) \in \mathbb{R}^M_+ \times \Delta^M_\epsilon\). To tackle the constraint \(s \in \mathbb{R}^M_+\), we parametrize the rate-allocation vector \(s\) with a vector \(u \in \mathbb{R}^M\) as
\[
s = \exp(u) \triangleq (\exp(u_1), \ldots, \exp(u_M)). \tag{38}
\]
Furthermore, to satisfy the constraint \(\lambda \in \Delta^M_\epsilon\), we consider a mirror-decent based scheme which adapts the updates to the geometry of the simplex \(\Delta^M_\epsilon\) via the exponentiated gradient [40]. Overall, this leads to the updates
\[
u \leftarrow u + \eta \text{diag} \left( \exp(u) \right) \left. \nabla_s \tilde{R}^S_{\beta}(s, \lambda) \right|_{s = \exp(u)} \triangleq \text{GD}_{\beta}(u; \mathcal{S}, \lambda) \tag{39}
\]
and
\[
\lambda_m \leftarrow \frac{\lambda_m \exp \left( \gamma \left[ \nabla_A \tilde{R}^S_{\beta}(\exp(u), \lambda) \right]_{m'} \right)}{\sum_{m'=1}^M \lambda_{m'} \exp \left( \gamma \left[ \nabla_A \tilde{R}^S_{\beta}(\exp(u), \lambda) \right]_{m'} \right)} \triangleq \text{EG}_{\beta}(\lambda; \mathcal{S}, u), \quad \forall m \in [M]. \tag{40}
\]
The resulting procedure to optimize the empirical \(\beta\)-CVaR rate is summarized in Algorithm 1. By (36), the algorithm can be also applied to maximize the average rate by setting \(\beta = 1\).

VI. REDUCING SAMPLE COMPLEXITY VIA META-LEARNING

In Section IV, we have shown that, given the focus of the \(\beta\)-CVaR metric on the performance of a small fraction \(\beta\) of the clients with the worst channel gains (see Fig. 1), the number of samples required to maintain a desired optimality gap increases as \(\beta\) decreases (see (33)). In this section, we propose meta-learning as a means to reduce sample complexity by leveraging historical data from other deployments, each generally characterized by different fading distributions, as detailed next.

A. Setting

Let \(y_\tau(t)\) be the signal received by a client in a previous deployment described by variable \(\tau\). Similar to the model (1), the received signal can be expressed as
\[
y_\tau(t) = \sqrt{p}_\tau x_\tau(t) + z_\tau(t), \tag{41}
\]
where \(x_\tau(t)\) denotes the signal transmitted by BS \(\tau\) at time \(t \in [n]\); \(z_\tau(t) \sim \mathcal{CN}(0, 1)\) denotes the AWGN; and \(h_\tau \sim p_{h_\tau}\) denotes the quasi-static fading coefficient. The dependence of the fading distribution on the deployment variable \(\tau\) indicates that different deployments may have distinct channel statistics. As an example, each deployment may be characterized by Rician fading coefficients \(h_\tau \sim \mathcal{CN}(\mu, \sigma^2)\) with parameters \(\tau = (\mu, \sigma^2)\).
To support meta-learning, we assume that we have access to data from a subset \( \mathcal{T} = \{\tau_1, \ldots, \tau_D\} \) of \( D \) deployments. For each deployment \( \tau \in \mathcal{T} \), we specifically have a dataset \( \mathcal{G}_\tau \triangleq \{g_{\tau,1}, \ldots, g_{\tau,N}\} \) of \( N \) fading realizations sampled in an i.i.d. manner from distribution \( p_{g_\tau}(g_\tau) \), where \( g_\tau \triangleq |h_\tau|^2 \) is the gain coefficient. We refer to the collection of datasets \( \mathcal{G}^\mathcal{T} \triangleq \{\mathcal{G}_\tau\}_{\tau \in \mathcal{T}} \) as the \emph{meta-training data}. The goal of meta-learning is to use this meta-training data prior to deploying a new system in order to reduce the amount of data needed to optimize effectively the power and rate allocation vectors for the latter.

### B. MAML

To demonstrate the gain of meta-learning in reducing the sample complexity, we focus on the model-agnostic meta-learning (MAML) approach introduced in [41]. In this approach, the meta-training data \( \mathcal{G}^\mathcal{T} \) is used to optimize an initialization \( (s, \Lambda) \) for a gradient-based scheme that addresses the \( \beta \)-CVaR optimization (35). Specifically, we adapt MAML for the gradient-based scheme described in the previous section.

First, given a random initialization \( (s, \Lambda) \), with \( s = \exp(u) \) for some \( u \in \mathbb{R}^M \), the model parameters are individually adapted to each deployment by applying a single update of the gradient-based scheme in Algorithm 1. That is, the updated model parameters \( (u_\tau, \Lambda_\tau) \), for deployment \( \tau \in \mathcal{T} \), are given as

\[
    u_\tau(u, \Lambda) = GD_\beta(u; \mathcal{G}^\mathcal{T}, \Lambda), 
\]

and

\[
    \Lambda_\tau(u, \Lambda) = EG_\beta(\Lambda; \mathcal{G}^\mathcal{T}, u), 
\]

where functions \( GD_\beta(\cdot; \cdot, \cdot) \) and \( EG_\beta(\cdot; \cdot, \cdot) \) are defined in (39) and (40), respectively. Then, the initialization \( (u, \Lambda) \) is optimized by maximizing the surrogate average empirical \( \beta \)-CVaR across all deployments as

\[
    \max_{(u, \Lambda) \in \mathbb{R}^M \times \Delta^M} \phi(u, \Lambda), 
\]

where we have defined the function

\[
    \phi(u, \Lambda) \triangleq \sum_{\tau \in \mathcal{T}} R_{\beta}^{\mathcal{G}_\tau}(\exp(u_\tau(u, \Lambda)), \Lambda_\tau(u, \Lambda)). 
\]

To address the optimization (44), a gradient-based scheme is applied in which, similar to the scheme described in the previous section, the vector \( u \) is updated via gradient descent,
whereas the power-allocation vector $\lambda$ is updated via mirror descent. That is, the meta-updates are given as

$$u \leftarrow u + \eta \nabla u \phi(u, \lambda)$$  \hspace{1cm} (46)

and

$$\lambda_m \leftarrow \frac{\lambda_m \exp (\gamma [\nabla_{\lambda} \phi(u, \lambda)]_m)}{\sum_{m'=1}^{M} \lambda_{m'} \exp (\gamma [\nabla_{\lambda} \phi(u, \lambda)]_{m'})}, \quad \forall m \in [M].$$  \hspace{1cm} (47)

In practice, to evaluate the gradients $\nabla_u \phi(u, \lambda)$ and $\nabla_{\lambda} \phi(u, \lambda)$ of the objective $\phi(u, \lambda)$ in (44), we apply the chain rule of differentiation. Specifically, the meta-update of initialization $u$ in (46) can be expressed as

$$u \leftarrow u + \eta \sum_{\tau \in \mathcal{T}} \mathbf{J}_{u, \tau}^T(u) \nabla_u R^\beta_\tau (\exp(u_\tau(u, \lambda)), \lambda_\tau(u, \lambda))$$

$$+ \sum_{\tau \in \mathcal{T}} \mathbf{J}_{\lambda, \tau}^T(u) \nabla_{\lambda} R^\beta_\tau (\exp(u_\tau(u, \lambda)), \lambda_\tau(u, \lambda))$$

$$\triangleq \text{MGD}_\beta(u; \mathcal{G}^\tau, \{u_\tau\}_{\tau \in \mathcal{T}}, \{\lambda_\tau\}_{\tau \in \mathcal{T}}),$$  \hspace{1cm} (48)

where $\mathbf{J}_{f}(v)$ denotes the transposed Jacobian matrix of vector function $f(v)$, i.e.,

$$\mathbf{J}_{f}^T(v) \triangleq \left( \begin{array}{ccc} \frac{\delta f_1(v)}{\delta v_1} & \cdots & \frac{\delta f_M(v)}{\delta v_1} \\ \vdots & \ddots & \vdots \\ \frac{\delta f_1(v)}{\delta v_M} & \cdots & \frac{\delta f_M(v)}{\delta v_M} \end{array} \right).$$  \hspace{1cm} (49)

Similarly, for $m \in [M]$, the meta-update of initialization $\lambda_m$ in (47) can be expressed as

$$\tilde{\lambda}_m \leftarrow \lambda_m \exp \left( \gamma \left[ \sum_{\tau \in \mathcal{T}} \mathbf{J}_{u, \tau}^T(\lambda) \nabla_u R^\beta_\tau (\exp(u_\tau(u, \lambda)), \lambda_\tau(u, \lambda)) \right] \right)$$

$$\times \exp \left( \gamma \left[ \sum_{\tau \in \mathcal{T}} \mathbf{J}_{\lambda, \tau}^T(\lambda) \nabla_{\lambda} R^\beta_\tau (\exp(u_\tau(u, \lambda)), \lambda_\tau(u, \lambda)) \right] \right)$$

$$\triangleq \text{MEG}_\beta(\lambda; \mathcal{G}^\tau, \{u_\tau\}_{\tau \in \mathcal{T}}, \{\lambda_\tau\}_{\tau \in \mathcal{T}})$$  \hspace{1cm} (50)

with $\lambda_m = \tilde{\lambda}_m / \|\tilde{\lambda}\|_1$. In addition, the gradient $\nabla_u R^\beta_\tau (\exp(u_\tau(u, \lambda)), \lambda_\tau(u, \lambda))$ in (48) and (50) can be further expressed as

$$\nabla_u R^\beta_\tau (\exp(u_\tau(u, \lambda)), \lambda_\tau(u, \lambda)) = \text{diag}(\exp(u_\tau(u, \lambda))) \nabla_s R^\beta_\tau(s, \lambda_\tau) \bigg|_{s = \exp(u_\tau(u, \lambda))}. \hspace{1cm} (51)$$

Finally, the MAML algorithm is summarized in Algorithm 2.
Algorithm 2: MAML

**Input:** Meta-training data $\mathcal{G}$, fraction $\beta \in (0, 1]$

**Initialization:** Randomly initialize $u \in \mathbb{R}^M$ and $\lambda \in \Delta_c^M$

```plaintext
while not converged do
  for each deployment $\tau \in \mathcal{T}$ do
    set $u_\tau \leftarrow \text{GD}_\beta(u; \mathcal{G}, \lambda)$ (defined in (39))
    set $\lambda_\tau \leftarrow \text{EG}_\beta(\lambda; \mathcal{G}, u)$ (defined in (40))
    set $u \leftarrow \text{MGD}_\beta(u; \mathcal{G}, \{u_\tau\}_{\tau \in \mathcal{T}}, \{\lambda_\tau\}_{\tau \in \mathcal{T}})$ (defined in (48))
    set $\tilde{\lambda}_m \leftarrow \text{MEG}_\beta(\lambda; \mathcal{G}, \{u_\tau\}_{\tau \in \mathcal{T}}, \{\lambda_\tau\}_{\tau \in \mathcal{T}})$, $\forall m \in [M]$ (defined in (50))
  set $\lambda = \tilde{\lambda}/\|\tilde{\lambda}\|_1$
return $(u, \lambda)$
```

VII. NUMERICAL RESULTS

In this section, we evaluate the expected rate $\bar{R}(\hat{s}_i^g, \hat{\lambda}_i^g)$ and $\beta$-CVaR rate $R_\beta(\hat{s}_i^g, \hat{\lambda}_i^g)$ for parameters $(\hat{s}_i^g, \hat{\lambda}_i^g)$ and $(\bar{s}_i^g, \bar{\lambda}_i^g)$ obtained via Algorithm 1 with learning rates $\eta = \gamma = 0.01$ and sigmoid smoothness parameter $c = 10$. The expected rate and $\beta$-CVaR rate are averaged over 1000 datasets $\mathcal{G}$, which we denote as $\mathbb{E}_\mathcal{G}[\bar{R}(\hat{s}_i^g, \hat{\lambda}_i^g)]$, and $\mathbb{E}_\mathcal{G}[R_\beta(\hat{s}_i^g, \hat{\lambda}_i^g)]$, respectively. Furthermore, we demonstrate the gain of MAML as means to reduce sample complexity by assessing the $\beta$-CVaR rate achieved when an initialization $(s, \lambda)$ is optimized based on previous deployments via Algorithm 2 with learning rates $\bar{\eta} = \bar{\gamma} = 0.01/D$.

A. Expected Achievable Rate

In Fig. 3, we plot the expected achievable rate as a function of the number of layers $M$ with power $P = 20$dB, Rayleigh fading distribution, and dataset of size $N = 10, 100,$ and 1000. For this special case, the ideal optimal solution in Corollary 1 obtained by using infinite layers and assuming that the fading distribution is known is used as an upper bound. Furthermore, we plot for reference the expected rate achieved with finite number of layers when the BS knows the fading distribution, which is obtained by replacing the surrogate empirical average rate $\bar{R}_1^g(s, \lambda)$ with the expected rate $\bar{R}(s, \lambda)$ in the gradient-based updates (39)–(40). First, confirming the sample complexity analysis in Section III-C, for sufficiently large datasets, the expected rate is close to that achieved when the BS knows the fading distribution. Furthermore, using multiple layers provides notable gain over a single layer,
Fig. 3. The expected achievable rate as a function of $M$ with $P = 20$ dB and $N = 10, 100, \text{and } 1000$.

even for small datasets. Finally, the expected rate achieved with $M = 6$ layers and sufficiently large dataset is seen to be close to the upper bound.

In Fig. 4, we plot the ratio of the expected rate achieved via LDM with $M$ layers to the expected rate achieved with a single layer as a function of the power $P$ with Rayleigh fading distribution and dataset of size 1000. It is observed that the gain of LDM increases with power $P$. Intuitively, this is because, for sufficiently high power, splitting the last layer, while keeping the same norm $\|s\|_1$, has a negligible impact on the rate $\rho_M(s, \lambda)$ but adds another layer that is much more likely to be decoded (see eqs. (5)–(7)).

B. Conditional Value at Risk

To demonstrate the importance of optimizing the $\beta$-CVaR rate for applications that focus on the performance of a small fraction $\beta$ of the clients with the worst channel gains, in Fig. 5, we plot the expected $\beta$-CVaR rate as a function of the fraction $\beta$ with power $P = 20$ dB, Rician fading distribution $h \sim CN(2,1)$, $M = 6$ layers, and datasets of size $N = 10^4$, and for rate- and power-allocation vectors $s$ and $\lambda$ optimized based on different metrics: (i) the surrogate empirical $\beta$-CVaR rate $\tilde{R}_\beta(s, \lambda)$ defined in (36), (ii) the surrogate average rate $\tilde{R}_1(s, \lambda)$ defined in (37), and (iii) the surrogate empirical outage rate $\tilde{R}_{\beta}(s, \lambda) = \rho_M(s, \lambda, g(\lfloor N\beta \rfloor))$. It is observed that, for most values of $\beta \in (0,1]$, the average achievable rate for the $\beta$-fraction of clients with the worst channel gains increases significantly if the rate- and power-allocation
Fig. 4. The expected achievable rate gain as a function of $\sigma^2$ with $h \sim \mathcal{CN}(0, \sigma^2)$, $P = 20$ dB, and $N = 10^4$.

Fig. 5. The expected $\beta$-CVaR rate as a function of $\beta$ with $P = 20$ dB, $M = 6$, and $N = 10^4$.

vectors are optimized to maximize the surrogate empirical $\beta$-CVaR rate. In contrast, for sufficiently high fraction $\beta$, the surrogate average rate can also be used as the optimization objective. This is because the surrogate average rate $\bar{R}^G_{\beta}(s, \lambda)$ is a special case of the surrogate empirical $\beta$-CVaR rate $\bar{R}^G_{\beta}(s, \lambda)$ for $\beta = 1$. Similarly, for sufficiently low fraction $\beta$, the surrogate empirical outage rate can be used as the optimization objective. This is due to the limit $\lim_{\beta \to 0} \bar{R}^G_{\beta}(s, \lambda) = \lim_{\beta \to 0} \bar{e}^G_{\beta}(s, \lambda) = R_{\sigma}(s, \lambda, g_{[1]})$ (see eq. (36)).
In Fig. 6, we plot the expected $\beta$-CVaR rate, $E_\beta[R_\beta(\hat{s}^\beta_\beta, \hat{\lambda}^\beta_\beta)]$, as a function of the dataset size $N$ with power $P = 20$dB, Rician fading distribution $h \sim CN(\sqrt{20}, 16)$, $M = 1, 2,$ and 6 layers, and for $\beta = 1, 0.1,$ and 0.01. It is observed that, as $\beta$ decreases, larger dataset is required to obtain an accurate estimate of the achievable $\beta$-CVaR rate. This is because, as discussed in Section IV-C, the number of samples required to maintain a desired optimality gap increases as $\beta \to 0$. In addition, using multiple layers is shown to be advantageous even for very small datasets ($N \geq 4$).

C. Meta-Learning

To evaluate the gain of meta-learning in reducing the sample complexity, we compare the expected $\beta$-CVaR rate achieved with random initialization to that achieved by optimizing an initialization $(s, \lambda)$ based on the data from previous deployments via Algorithm 2. To this end, in Fig. 7, we plot the expected $\beta$-CVaR rate as a function of the dataset size $N$ with power $P = 20$dB, $M = 6$ layers, $\beta = 0.1,$ and $D = 10$ previous deployments. The channel for the new deployment is taken to be $h \sim CN(\sqrt{10}, 5)$, whereas the channel for each previous deployment is given as $h_\tau \sim CN(\sqrt{10} + \mu_\tau, 5)$ with random deviation $\mu_\tau \sim CN(0, 2)$. For small datasets, it is observed that MAML provides significant performance gains over the $\beta$-CVaR rate achieved with random initialization, i.e., when the power and rate allocation vectors are optimized without taking into account data from previous deployments. In contrast, for sufficiently large datasets, local optimization for each new deployment is sufficient for achieving a desired optimality gap.

In Fig. 8, we plot the expected $\beta$-CVaR rate as a function of the number of previous deployments $D$ with power $P = 20$dB, $M = 6$ layers, $\beta = 0.1$, datasets of size $N = 10$, and channel coefficients $h$ and $\{h_\tau\}_\tau \in \mathcal{T}$ distributed as in the previous numerical experiment. While the expected $\beta$-CVaR rate is monotonically increasing with the number of previous deployments, Fig. 8 demonstrates that the sample complexity can be reduced substantially even if MAML is applied with only a few deployments.

VIII. Conclusion

In this work, we have studied LDM as an enabler of differential QoS for ultra-reliable broadcast/multicast communication systems. To this end, we have introduced the $\beta$-CVaR rate as the average rate of the $\beta$-fraction of clients with the worst instantaneous channels. We have focused on a practical model in which the fading distribution is unknown, and the transmitter optimizes rate and power allocation for each layer based on a dataset sampled...
during deployment. The optimality gap caused by the availability of limited data was bounded via a generalization analysis, and the sample complexity was shown to increase as the fraction $\beta$ decreases. To optimize the rate and power allocation parameters, a mirror-descent based scheme was introduced, which, for sufficiently large dataset, was demonstrated via numerical experiments to achieve a $\beta$-CVaR rate close to that achieved when the BS known the fading distribution. Furthermore, meta-learning was shown to be instrumental in decreasing the sample complexity for ultra-reliable communication with low outage probabilities. Among related problems left open by this study, we mention the extension to multiple transmit
antennas [21], to channels with multiple uncoordinated transmitters [42], [43], and the analysis of LDM with an infinite number of layers [17].

APPENDIX

A. Proof of Proposition 2

The optimality gap $e^G$ (16) can be upper bounded as

$$e^G = \bar{R}(s^*, \lambda^*) - \bar{R}^G(s^*, \lambda^*) + \bar{R}^G(s^*, \lambda^G) - \bar{R}^G(s^G, \lambda^G) - \bar{R}(s^G, \lambda^G) \leq \left( \bar{R}(s^*, \lambda^*) - \bar{R}^G(s^*, \lambda^*) \right) + \left( \bar{R}^G(s^G, \lambda^G) - \bar{R}(s^G, \lambda^G) \right),$$

where the inequality holds since $(s^G, \lambda^G)$ maximize the average rate $\bar{R}^G(s, \lambda)$. Next, to further bound the optimality gap, we bound, uniformly, the difference $|\bar{R}(s, \lambda) - \bar{R}^G(s, \lambda)|$ for all $\lambda \in \Delta_c^M$ and $s \in \mathbb{R}_+^M$ with $\|s\|_1 \leq S$. Note that the expected achievable rate (7) can be expressed as

$$\bar{R}(s, \lambda) = \mathbb{E}_g \left[ R(s, \lambda, g) \right] = \sum_{m=1}^M \rho_m(s^m, \lambda) \tilde{F}_g(\|s^m\|_1),$$

where $\tilde{F}_g(\|s^m\|_1)$ denotes the complementary cumulative distribution function (CCDF)

$$\tilde{F}_g(\|s^m\|_1) \triangleq \operatorname{Pr}[g \geq \|s^m\|_1].$$

Fig. 8. The expected $\beta$-CVaR rate as a function of $D$ with $P = 20$dB, $M = 6$, $\beta = 0.1$, and $N = 10$. 


Similarly, the average rate (14) can be expressed as
\[
\bar{R}^g(s, \lambda) = \frac{1}{N} \sum_{i=1}^{N} R(s, \lambda, g_i) = \frac{1}{N} \sum_{i=1}^{N} \sum_{m=1}^{M} \rho_m(s^m, \lambda) 1_{g_i \geq \|s^m\|_1},
\]
\[
= \frac{1}{N} \sum_{m=1}^{M} \rho_m(s^m, \lambda) \sum_{i=1}^{N} 1_{g_i \geq \|s^m\|_1},
\]
\[
= \sum_{m=1}^{M} \rho_m(s^m, \lambda) \bar{F}^g_{g} (\|s^m\|_1), \tag{55}
\]
where \( \bar{F}^g_{g} (\|s^m\|_1) \) denotes the empirical CCDF
\[
\bar{F}^g_{g} (\|s^m\|_1) \triangleq \frac{1}{N} \sum_{i=1}^{N} 1_{g_i \geq \|s^m\|_1}. \tag{56}
\]
Therefore, to uniformly bound the difference \(|\bar{R}(s, \lambda) - \bar{R}^g(s, \lambda)|\), we first uniformly bound \(|\bar{F}_g(s) - \bar{F}^g_{g} (s)|\) using the following proposition.

**Proposition 5:** Let \( \mathcal{G} = \{g_1, \ldots, g_N\} \) be a dataset of \( N \) fading realizations drawn independently from the fading distribution \( p_{g}(g) \), and let \( \delta \in (0, 1] \). With probability at least \( 1 - \delta \), uniformly over all \( s \in \mathbb{R}_+ \), we have
\[
\left| \bar{F}_g(s) - \bar{F}^g_{g} (s) \right| \leq 4 \sqrt{\frac{(2N + 1) \ln(N + 1)}{3N(N + 1)}} + \sqrt{\frac{2 \ln(2/\delta)}{N}}. \tag{57}
\]

**Proof:** See Appendix B.

Proposition 5 implies that, with probability at least \( 1 - \delta \), we can bound the difference \(|\bar{R}(s, \lambda) - \bar{R}^g(s, \lambda)|\), uniformly over all \( \lambda \in \Delta_c^M \) and \( s \in \mathbb{R}_+^M \) with \( \|s\|_1 \leq S \), as
\[
\left| \bar{R}(s, \lambda) - \bar{R}^g(s, \lambda) \right| \overset{(a)}{=} \left| \sum_{m=1}^{M} \rho_m(s^m, \lambda) \left[ \bar{F}_g(\|s^m\|_1) - \bar{F}^g_{g} (\|s^m\|_1) \right] \right|
\]
\[
\overset{(b)}{\leq} \sum_{m=1}^{M} \rho_m(s^m, \lambda) \left| \bar{F}_g(\|s^m\|_1) - \bar{F}^g_{g} (\|s^m\|_1) \right|
\]
\[
\overset{(c)}{\leq} 4 \sqrt{\frac{(2N + 1) \ln(N + 1)}{3N(N + 1)}} + \sqrt{\frac{2 \ln(2/\delta)}{N}} \sum_{m=1}^{M} \rho_m(s^m, \lambda)
\]
\[
\overset{(d)}{\leq} 4 \sqrt{\frac{(2N + 1) \ln(N + 1)}{3N(N + 1)}} + \sqrt{\frac{2 \ln(2/\delta)}{N}} \log_2(1 + SP), \tag{58}
\]
where (a) follows from (53) and (55); (b) follows from the triangle inequality and since the rate of each layer is non-negative; (c) follows from Proposition 5; and (d) holds since \( S \geq \|s\|_1 \). Finally, based on inequalities (52) and (58), we can upper bound the optimality gap as (17).
B. Proof of Proposition 5

Let function \( \ell : \mathbb{R} \times \mathbb{C} \mapsto \{0, 1\} \) be defined as

\[
\ell(s, g) \triangleq 1_{g \geq s}.
\]

The true and empirical CCDF can hence be expressed as

\[
\bar{F}_g(s) = \mathbb{E}_g[\ell(s, g)]
\]

and

\[
\bar{F}^\mathcal{G}_g(s) = \frac{1}{N} \sum_{i=1}^N \ell(s, g_i),
\]

respectively, where \( g_1, \ldots, g_N \in \mathcal{G} \) are \( N \) fading realizations. In addition, let \( \mathcal{L}(g_1, \ldots, g_N) \subset \{0, 1\}^N \) be the set

\[
\mathcal{L}(g_1, \ldots, g_N) \triangleq \{(\ell(s, g_1), \ldots, \ell(s, g_N)) : s \in \mathbb{R}\}.
\]

Furthermore, denote by \( \text{Rad}(\mathcal{L}(g_1, \ldots, g_N)) \) the Rademacher complexity of set \( \mathcal{L}(g_1, \ldots, g_N) \), i.e.,

\[
\text{Rad}(\mathcal{L}(g_1, \ldots, g_N)) \triangleq \frac{1}{N} \mathbb{E}_b \left[ \sup_{\ell \in \mathcal{L}(g_1, \ldots, g_N)} \left| \frac{1}{N} \sum_{i=1}^N b_i \ell_i \right| \right],
\]

where the elements of random vector \( b = (b_1, \ldots, b_N) \in \{\pm 1\}^N \) are i.i.d. with \( \mathbb{P}[b_i = 1] = \mathbb{P}[b_i = -1] = 1/2 \). Since \( |\ell(s, g)| \leq 1 \) for all \( g \in \mathbb{R}_+ \) and \( s \in \mathbb{R} \), by [44, Thm 26.5] and [45, Prop. 8], for random variables \( g_1, \ldots, g_N \) that are i.i.d. according to \( p_g(g) \), we have

\[
\mathbb{P} \left[ \bigcap_{s \in \mathbb{R}} \left( \left| \bar{F}_g(s) - \bar{F}^\mathcal{G}_g(s) \right| \leq 4 \mathbb{E} [\text{Rad}(\mathcal{L}(g_1, \ldots, g_N))] + \sqrt{\frac{2 \ln(2/\delta)}{N}} \right) \right] \geq 1 - \delta.
\]

Next, we bound the expected Rademacher complexity \( \mathbb{E} [\text{Rad}(\mathcal{L}(g_1, \ldots, g_N))] \) in (64). We assume, without loss of generality (w.l.o.g.), that the channel realizations \( g_1, \ldots, g_N \in \mathcal{G} \) are ordered such that \( g_i \geq g_j \) for all \( j \in [i] \). Note that, if \( \ell(s, g_j) = 1 \) for some \( s \in \mathbb{R} \) then \( \ell(s, g_i) = 1 \) for all \( j \leq i \leq N \). Therefore, we have

\[
|\mathcal{L}(g_1, \ldots, g_N)| = N + 1.
\]

Denote by

\[
\bar{\ell} \triangleq \frac{1}{N+1} \sum_{\ell \in \mathcal{L}(g_1, \ldots, g_N)} \ell = \frac{1}{N+1} (1, 2, \ldots, N)
\]

the average vector in \( \mathcal{L}(g_1, \ldots, g_N) \). Note that

\[
\max_{\ell \in \mathcal{L}(g_1, \ldots, g_N)} \|\ell - \bar{\ell}\|_2 = \|ar{\ell}\|_2 = \sqrt{\frac{N(2N + 1)}{6(N + 1)}}.
\]
Hence, by Massart Lemma [44, Lemma 26.8], we have
\[
\text{Rad}(\mathcal{L}(g_1, \ldots, g_N)) \leq \sqrt{\frac{N(2N+1)}{6(N+1)}} \cdot \sqrt{\frac{2\ln(N+1)}{N}} = \sqrt{\frac{(2N+1)\ln(N+1)}{3N(N+1)}} \tag{68}
\]
for any channel realizations \(g_1, \ldots, g_N \in \mathbb{R}_+\). This implies that the upper bound in (68) bounds the expected Rademacher complexity \(\mathbb{E}[\text{Rad}(\mathcal{L}(g_1, \ldots, g_N))]\) as well. By substituting (68) in (64) we get (57).

C. Proof of Proposition 3

First, we prove that \(f^G_\beta(s, \lambda, r)\) is concave with respect to \(r \geq 0\). To this end, let \(0 \leq r_1 \leq r_2, \delta \in [0, 1]\), and assume that, w.l.o.g., dataset \(G\) is sorted such that \(g_1 \leq g_2 \leq \cdots \leq g_N\). Therefore, there exist \(0 \leq N_1 \leq N_2\) such that we can express \(f^G_\beta(s, \lambda, r_1)\) and \(f^G_\beta(s, \lambda, r_2)\) as
\[
f^G_\beta(s, \lambda, r_1) = r_1 - \frac{1}{N\beta} \sum_{i=1}^{N_1} (r_1 - R(s, \lambda, g_i)) \tag{69}
\]
and
\[
f^G_\beta(s, \lambda, r_2) = r_2 - \frac{1}{N\beta} \sum_{i=1}^{N_2} (r_2 - R(s, \lambda, g_i)), \tag{70}
\]
respectively. Furthermore, there exists \(N_\delta\), where \(N_1 \leq N_\delta \leq N_2\), such that we have
\[
f^G_\beta(s, \lambda, \delta r_1 + (1 - \delta)r_2) = \delta r_1 + (1 - \delta)r_2 - \frac{1}{N\beta} \sum_{i=1}^{N_\delta} (\delta r_1 + (1 - \delta)r_2 - R(s, \lambda, g_i)) \tag{71}
\]
Note that we can lower bound \(f^G_\beta(s, \lambda, \delta r_1 + (1 - \delta)r_2)\) as
\[
f^G_\beta(s, \lambda, \delta r_1 + (1 - \delta)r_2) = \delta \left[ r_1 - \frac{1}{N\beta} \sum_{i=1}^{N_\delta} (r_1 - R(s, \lambda, g_i)) \right]
+ (1 - \delta) \left[ r_2 - \frac{1}{N\beta} \sum_{i=1}^{N_\delta} (r_2 - R(s, \lambda, g_i)) \right]
= \delta \left[ f^G_\beta(s, \lambda, r_1) - \frac{1}{N\beta} \sum_{i=N_1+1}^{N_\delta} (r_1 - R(s, \lambda, g_i)) \right]
+ (1 - \delta) \left[ f^G_\beta(s, \lambda, r_2) + \frac{1}{N\beta} \sum_{i=N_\delta+1}^{N_2} (r_2 - R(s, \lambda, g_i)) \right]
\geq \delta f^G_\beta(s, \lambda, r_1) + (1 - \delta) f^G_\beta(s, \lambda, r_2), \tag{72}
\]
where the inequality holds since \(r_1 < R(s, \lambda, g_i)\) for \(i > N_1\), and \(r_2 \geq R(s, \lambda, g_i)\) for \(i \leq N_2\). Therefore, \(f^G_\beta(s, \lambda, r)\) is concave with respect to \(r \geq 0\).
Similarly, for $r \geq 0$, let $N_r \geq 0$ such that
\[ f^G(\mathbf{s}, \mathbf{\lambda}, r) = r - \frac{1}{N\beta} \sum_{i=1}^{N_r} (r - R(\mathbf{s}, \mathbf{\lambda}, g_i)) \cdot \tag{73} \]
The derivative of $f^G(\mathbf{s}, \mathbf{\lambda}, r)$ with respect to $r$ is hence given as
\[ \frac{\partial}{\partial r} f^G(\mathbf{s}, \mathbf{\lambda}, r) = 1 - \frac{N_r}{N\beta}. \tag{74} \]
We find $r^G(\mathbf{s}, \mathbf{\lambda})$ by analyzing the case of $N\beta > 1$ and the case of $N\beta \leq 1$ separately.

1) $N\beta > 1$: If $N\beta$ is an integer, the derivative in (74) is zero iff $N_r = N\beta$. Therefore, the optimal $r$ is given as $r^G(\mathbf{s}, \mathbf{\lambda}) = R(\mathbf{s}, \mathbf{\lambda}, g_{N\beta})$. If $N\beta$ is not an integer, the optimal $r$ is that which induces $N_r$ closest to $N\beta$, i.e., $r^G(\mathbf{s}, \mathbf{\lambda}) = R(\mathbf{s}, \mathbf{\lambda}, g_{\lceil N\beta \rceil})$.

2) $N\beta \leq 1$: In this case, the derivative in (74) cannot be zero and is positive iff $N_r = 0$. Therefore, the optimal $r$ is the largest $r$ for which $f^G(\mathbf{s}, \mathbf{\lambda}, r) = r$, i.e., $r^G(\mathbf{s}, \mathbf{\lambda}) = R(\mathbf{s}, \mathbf{\lambda}, g_1)$.

Note that, for $N\beta \leq 1$, we have $\lceil N\beta \rceil = 1$ since we defined $\lfloor x \rfloor$ to be the closest positive integer to scalar $x$.

D. Proof of Proposition 4

As detailed in Section IV, the BS optimizes the $\beta$-CVaR rate $R_\beta(\mathbf{s}, \mathbf{\lambda})$ by optimizing the function $f_\beta(\mathbf{s}, \mathbf{\lambda}, r)$ in (22). Therefore, the optimality gap $e_\beta^G$ (31) can be restated as
\[ e_\beta^G = f_\beta(s^*_\beta, \mathbf{\lambda}^*_\beta, r_\beta(s^*_\beta, \mathbf{\lambda}^*_\beta)) - f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r^G_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)). \tag{75} \]
Next, we bound the optimality gap (75) as
\[ e_\beta^G = f_\beta(s^*_\beta, \mathbf{\lambda}^*_\beta, r_\beta(s^*_\beta, \mathbf{\lambda}^*_\beta)) - f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r^G_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) \]
\[ + f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) - f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r^G_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) \]
\[ + f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r^G_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) - f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) \]
\[ \leq \left( f_\beta(s^*_\beta, \mathbf{\lambda}^*_\beta, r_\beta(s^*_\beta, \mathbf{\lambda}^*_\beta)) - f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r^G_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) \right) \]
\[ + \left( f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r^G_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) - f_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta, r_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta)) \right), \tag{76} \]

where the inequality holds since $(s^G_\beta, \mathbf{\lambda}^G_\beta, r^G_\beta(s^G_\beta, \mathbf{\lambda}^G_\beta))$ maximize the empirical approximation $f_\beta^G(\mathbf{s}, \mathbf{\lambda}, r)$ (26). In addition, note that, for rate allocation vectors with bounded norms $\|s\|_1 \leq S$, we have $r_\beta(\mathbf{s}, \mathbf{\lambda}), r^G_\beta(\mathbf{s}, \mathbf{\lambda}) \leq \log_2(1 + SP) \triangleq \tilde{R}$.

Similar to the proof of Proposition 2, to further bound the optimality gap, we bound, uniformly, the difference $|f_\beta(\mathbf{s}, \mathbf{\lambda}, r) - f_\beta^G(\mathbf{s}, \mathbf{\lambda}, r)|$ for all $\mathbf{\lambda} \in \Delta^M_\mathbf{\lambda}$, $0 \leq r \leq \tilde{R}$ and $\mathbf{s} \in \mathbb{R}^M_+$. Note that the difference $|f_\beta(\mathbf{s}, \mathbf{\lambda}, r) - f_\beta^G(\mathbf{s}, \mathbf{\lambda}, r)|$ can be expressed as
\[ \left| f_\beta(\mathbf{s}, \mathbf{\lambda}, r) - f_\beta^G(\mathbf{s}, \mathbf{\lambda}, r) \right| = \beta^{-1} \left| \frac{1}{N} \sum_{i=1}^{N} \delta^+(s, \mathbf{\lambda}, g_i, r) - E_g[\delta^+(s, \mathbf{\lambda}, g, r)] \right|, \tag{77} \]
where we have defined function \( \delta^+(s, \lambda, g, r) \triangleq (r - R(s, \lambda, g))^+ \). Let \( D^+(g_1, \ldots, g_N) \subset \mathbb{R}_+^N \) be the set

\[
D^+(g_1, \ldots, g_N) \triangleq \{ (\delta^+(s, \lambda, g_1, r), \ldots, \delta^+(s, \lambda, g_N, r)) : \\
 s \in \mathbb{R}_+^M, \ |s|_1 \leq S, \ \lambda \in \Delta_c^M, \ 0 \leq r \leq \tilde{R} \} .
\] (78)

Since \(|\delta^+(s, \lambda, g, r)| \leq \tilde{R}\) for all \( s \in \mathbb{R}_+^M, \ \lambda \in \Delta_c^M, \ g \in \mathbb{R}_+ \), and \( 0 \leq r \leq \tilde{R} \), by [44, Thm 26.5] and [45, Prop. 8], for random variables \( g_1, \ldots, g_N \) that are i.i.d. according to \( p_g(g) \), we have, with probability at least \( 1 - \delta \),

\[
|f_\beta(s, \lambda, r) - f_\beta^*(s, \lambda, r)| \leq \beta^{-1} \left( 4E \[ \text{Rad}(D^+(g_1, \ldots, g_N)) \] + \tilde{R} \sqrt{\frac{2 \ln(2/\delta)}{N}} \right) .
\] (79)

Next, we bound the Rademacher complexity \( \text{Rad}(D^+(g_1, \ldots, g_N)) \) in (79). Let \( D(g_1, \ldots, g_N) \) be the set

\[
D(g_1, \ldots, g_N) \triangleq \{ (\delta(s, \lambda, g_1, r), \ldots, \delta(s, \lambda, g_N, r)) : \\
 s \in \mathbb{R}_+^M, \ |s|_1 \leq S, \ \lambda \in \Delta_c^M, \ 0 \leq r \leq \tilde{R} \},
\] (80)

where we have defined function

\[
\delta(s, \lambda, g, r) \triangleq r - R(s, \lambda, g).
\] (81)

Note that the set \( D^+(g_1, \ldots, g_N) \) can be expressed as

\[
D^+(g_1, \ldots, g_N) = \{ (q(\delta(s, \lambda, g_1, r)), \ldots, q(\delta(s, \lambda, g_N, r))) : \\
 s \in \mathbb{R}_+^M, \ |s|_1 \leq S, \ \lambda \in \Delta_c^M, \ 0 \leq r \leq \tilde{R} \},
\] (82)

where function \( q : \mathbb{R} \to \mathbb{R}_+ \) is defined as \( q(x) \triangleq (x)^+ \). Since \( q(\cdot) \) is a 1-Lipschitz function, i.e., for all \( x, y \in \mathbb{R} \), we have \( |q(x) - q(y)| \leq |x - y| \), then, by the Contraction Lemma [44, Lemma 26.9], we have

\[
\text{Rad}(D^+(g_1, \ldots, g_N)) \leq \text{Rad}(D(g_1, \ldots, g_N)).
\] (83)

In addition, let \( \mathcal{R}(g_1, \ldots, g_N) \subset \mathbb{R}_+^N \) be the set

\[
\mathcal{R}(g_1, \ldots, g_N) \triangleq \{ (R(s, \lambda, g_1), \ldots, R(s, \lambda, g_N)) : s \in \mathbb{R}_+^M, \ |s|_1 \leq S, \ \lambda \in \Delta_c^M \} .
\] (84)

It follows from the definition of \( \delta(s, \lambda, g, r) \) in (81) and from [44, Lemma 26.6] that

\[
\text{Rad}(D(g_1, \ldots, g_N)) \leq \text{Rad}(\mathcal{R}(g_1, \ldots, g_N)).
\] (85)

Note that the achievable rate \( R(s, \lambda, g) \) can be expressed as

\[
R(s, \lambda, g) = \log_2 (1 + SP) \sum_{m=1}^M \alpha_m(s^m, \lambda) \ell(\|s^m\|_1, g),
\] (86)
where we have defined functions
\[ \alpha_m(s^m, \lambda) \triangleq \frac{\rho_m(s^m, \lambda)}{\log_2(1 + SP)}, \quad m \in [M], \] (87)
and function \( \ell(\cdot, \cdot) \) is defined in (59). Now, let \( \mathcal{R}(g_1, \ldots, g_N) \subset \mathbb{R}_+^N \) be the set
\[ \mathcal{R}(g_1, \ldots, g_N) \triangleq \left\{ \sum_{m=1}^{M} \alpha_m(s^m, \lambda) (\ell(\|s^m\|_1, g_1), \ldots, \ell(\|s^m\|_1, g_N)) : \right. \\
\left. \quad s \in \mathbb{R}_+^M, \quad \|s\|_1 \leq S, \quad \lambda \in \Delta_c^M \right\}. \] (88)
It follows from [44, Lemma 26.6] that
\[ \text{Rad}(\mathcal{R}(g_1, \ldots, g_N)) \leq \log_2(1 + SP)\text{Rad}(\mathcal{R}(g_1, \ldots, g_N)). \] (89)
Furthermore, since \( \alpha_m(s^m, \lambda) \geq 0 \) and \( \sum_{m=1}^{M} \alpha_m(s^m, \lambda) \leq 1 \), then \( \text{Rad}(\mathcal{R}(g_1, \ldots, g_N)) \subseteq \text{conv}(\mathcal{L}(g_1, \ldots, g_N)) \), where set \( \mathcal{L}(g_1, \ldots, g_N) \) is defined in (62). Hence, we have
\[ \text{Rad}(\mathcal{R}(g_1, \ldots, g_N)) \leq \text{Rad}(\text{conv}(\mathcal{L}(g_1, \ldots, g_N))) \]
\[ \quad \overset{(a)}{=} \text{Rad}(\mathcal{L}(g_1, \ldots, g_N)) \]
\[ \overset{(b)}{\leq} \sqrt{\frac{2(N + 1) \ln(N + 1)}{3N(N + 1)}}, \] (90)
where (a) follows from [44, Lemma 26.7] and (b) follows from (68). Overall, we have
\[ \text{Rad}(\mathcal{D}^+(g_1, \ldots, g_N)) \leq \log_2(1 + SP) \sqrt{\frac{2(N + 1) \ln(N + 1)}{3N(N + 1)}}. \] (91)
Substituting (91) in (79) yields
\[ \left| f_{\beta}(s, \lambda, r) - f_{\beta}^g(s, \lambda, r) \right| \leq \beta^{-1} \left( 4\sqrt{\frac{2(N + 1) \ln(N + 1)}{3N(N + 1)}} + \sqrt{\frac{2 \ln(2/\delta)}{N}} \right) \log_2(1 + SP). \] (92)
Finally, based on inequalities (76) and (92), we can upper bound the optimality gap as (32).

REFERENCES

[1] A. Tajer, A. Steiner, and S. Shamai (Shitz), “The broadcast approach in communication networks,” *Entropy*, vol. 23, no. 1, p. 120, 2021.
[2] D. Gómez-Barquero and O. Simeone, “LDM versus FDM/TDM for unequal error protection in terrestrial broadcasting systems: An information-theoretic view,” *IEEE Trans. Broadcast.*, vol. 61, no. 4, pp. 571–579, 2015.
[3] S. Verdu and S. Shamai (Shitz), “Variable-rate channel capacity,” *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2651–2667, 2010.
[4] L. Zhang et al., “Layered-division-multiplexing: Theory and practice,” *IEEE Trans. Broadcast.*, vol. 62, no. 1, pp. 216–232, 2016.
[5] S. I. Park et al., “Low complexity layered division multiplexing for ATSC 3.0,” *IEEE Trans. Broadcast.*, vol. 62, no. 1, pp. 233–243, 2016.
[6] E. Arruti, M. Mendicute, and M. Barrenechea, “QoS in industrial wireless networks using LDM,” in Proc. IEEE Int. Workshop Elect., Cont., Meas., Sig. App. Mecha. (ECMSM), 2017, pp. 1–6.
[7] E. Arruti, M. Mendicute, and M. Barrenechea, “Unequal error protection with ldm in inside carriage wireless communications,” in Proc. Int. Conf. ITS Telecommunications (ITST), 2017, pp. 1–5.
[8] J. Montalban et al., “NOMA-based 802.11n for industrial automation,” IEEE Access, vol. 8, pp. 168 546–168 557, 2020.
[9] M. Boban et al., “Connected roads of the future: Use cases, requirements, and design considerations for vehicle-to-everything communications,” IEEE Veh. Technol. Mag., vol. 13, no. 3, pp. 110–123, 2018.
[10] A. Kostopoulos et al., “Use cases and standardisation activities for eMBB and V2X scenarios,” in Proc. IEEE Int. Conf. Communications Workshops (ICC Workshops), 2020, pp. 1–7.
[11] F. Wang, D. Guan, L. Zhao, and K. Zheng, “Cooperative V2X for high definition map transmission based on vehicle mobility,” in Proc. IEEE Vehic. Tech. Conf. (VTC2019-Spring), 2019, pp. 1–5.
[12] J. Liu and A. J. Khattak, “Delivering improved alerts, warnings, and control assistance using basic safety messages transmitted between connected vehicles,” Transpor. res. part C: emerg. tech., vol. 68, pp. 83–100, 2016.
[13] M. Angjelichinoski, K. F. Trillingsgaard, and P. Popovski, “A statistical learning approach to ultra-reliable low latency communication,” IEEE Trans. Commun., vol. 67, no. 7, pp. 5153–5166, 2019.
[14] R. Jurdi, S. R. Khosravirad, and H. Viswanathan, “Variable-rate ultra-reliable and low-latency communication for industrial automation,” in Proc. Information Science and Systems (CISS), 2018, pp. 1–6.
[15] P. Popovski et al., “Wireless access in ultra-reliable low-latency communication (URLLC),” IEEE Trans. Commun., vol. 67, no. 8, pp. 5783–5801, 2019.
[16] G. Hampel, C. Li, and J. Li, “5g ultra-reliable low-latency communications in factory automation leveraging licensed and unlicensed bands,” IEEE Commun. Mag., vol. 57, no. 5, pp. 117–123, 2019.
[17] S. Shamai and A. Steiner, “A broadcast approach for a single-user slowly fading MIMO channel,” IEEE Trans. Inf. Theory, vol. 49, no. 10, pp. 2617–2635, 2003.
[18] S. Thrun and L. Pratt, Learning to learn. Springer Science & Business Media, 2012.
[19] Y. Liu, K. Lau, O. Takeshita, and M. Fitz, “Optimal rate allocation for superposition coding in quasi-static fading channels,” in Proc. IEEE Int. Symp. Inform. Theory (ISIT), 2002, pp. 111–111.
[20] V. Ntranos, N. D. Sidiropoulos, and L. Tassiulas, “On multicast beamforming for minimum outage,” IEEE Trans. Wireless Commun., vol. 8, no. 6, pp. 3172–3181, 2009.
[21] Y. Shi et al., “Learning to beamform for minimum outage,” IEEE Trans. Signal Process., vol. 66, no. 19, pp. 5180–5193, 2018.
[22] W. Fang, M. Fu, Y. Shi, and Y. Zhou, “Outage minimization for intelligent reflecting surface aided MISO communication systems via stochastic beamforming,” in Proc. IEEE Sens. Arr. Multichann. Sig. Process. Workshop (SAM), 2020.
[23] R. T. Rockafellar and S. Uryasev, “Optimization of conditional value-at-risk.” Journal of risk, vol. 2, pp. 21–42, 2000.
[24] C. Filippi, G. Guastaroba, and M. G. Speranza, “Conditional value-at-risk beyond finance: a survey,” International Transactions in Operational Research, vol. 27, no. 3, pp. 1277–1319, 2020.
[25] M. Alsenwi et al., “eMBB-URLLC resource slicing: A risk-sensitive approach,” IEEE Commun. Lett., vol. 23, no. 4, pp. 740–743, 2019.
[26] X. Li et al., “Statistical QoS provisioning over uncertain shared spectrum in cognitive iot networks: A distributionally robust data-driven approach,” IEEE Trans. Veh. Technol., vol. 68, no. 12, pp. 12 286–12 300, 2019.
[27] Z. Wu et al., “Energy-efficient robust computation offloading for fog-iot systems,” IEEE Trans. Veh. Technol., vol. 69, no. 4, pp. 4417–4425, 2020.
[28] O. Simeone, S. Park, and J. Kang, “From learning to meta-learning: Reduced training overhead and complexity for communication systems,” in 2020 2nd 6G Wireless Summit (6G SUMMIT), 2020, pp. 1–5.

[29] S. Park, H. Jang, O. Simeone, and J. Kang, “Learning to demodulate from few pilots via offline and online meta-learning,” IEEE Trans. Signal Process., vol. 69, pp. 226–239, 2021.

[30] K. M. Cohen, S. Park, O. Simeone, and S. Shamai, “Learning to learn to demodulate with uncertainty quantification via bayesian meta-learning,” arXiv preprint arXiv:2108.00785, 2021.

[31] Y. Jiang, H. Kim, H. Asnani, and S. Kannan, “MIND: Model independent neural decoder,” in Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC), 2019, pp. 1–5.

[32] S. Park, O. Simeone, and J. Kang, “Meta-learning to communicate: Fast end-to-end training for fading channels,” in Proc. IEEE Int. Conf. Acoust., Speech and Sig. Process. (ICASSP), 2020, pp. 5075–5079.

[33] Y. Yuan et al., “Transfer learning and meta learning-based fast downlink beamforming adaptation,” IEEE Trans. Wireless Commun., vol. 20, no. 3, pp. 1742–1755, 2021.

[34] J. Zhang et al., “Embedding model based fast meta learning for downlink beamforming adaptation,” IEEE Trans. Wireless Commun., 2021.

[35] Y. Yuan, G. Zheng, K.-K. Wong, and K. B. Letaief, “Meta-reinforcement learning based resource allocation for dynamic V2X communications,” IEEE Trans. Veh. Technol., vol. 70, no. 9, pp. 8964–8977, 2021.

[36] S. Park and O. Simeone, “Predicting flat-fading channels via meta-learned closed-form linear filters and equilibrium propagation,” arXiv preprint arXiv:2110.00414, 2021.

[37] H. Mao, H. Lu, Y. Lu, and D. Zhu, “Roemnet: Robust meta learning based channel estimation in OFDM systems,” in Proc. IEEE Int. Conf. Communications (ICC), 2019, pp. 1–6.

[38] J. Lee, S. Park, and J. Shin, “Learning bounds for risk-sensitive learning,” arXiv preprint arXiv:2006.08138, 2020.

[39] A. Beck and M. Teboulle, “Mirror descent and nonlinear projected subgradient methods for convex optimization,” Operations Research Letters, vol. 31, no. 3, pp. 167–175, 2003.

[40] J. Kivinen and M. K. Warmuth, “Exponentiated gradient versus gradient descent for linear predictors,” information and computation, vol. 132, no. 1, pp. 1–63, 1997.

[41] C. Finn, P. Abbeel, and S. Levine, “Model-agnostic meta-learning for fast adaptation of deep networks,” in International Conference on Machine Learning. PMLR, 2017, pp. 1126–1135.

[42] M. Zohdy, A. Tajer, and S. Shamai (Shitz), “Broadcast approach to multiple access with local CSIT,” IEEE Trans. Commun., vol. 67, no. 11, pp. 7483–7498, 2019.

[43] M. Zohdy, A. Tajer, and S. Shamai, “Distributed interference management: A broadcast approach,” IEEE Trans. Commun., vol. 69, no. 1, pp. 149–163, 2021.

[44] S. Shalev-Shwartz and S. Ben-David, Understanding machine learning: From theory to algorithms. Cambridge university press, 2014.

[45] N. Weinberger, “Generalization bounds and algorithms for learning to communicate over additive noise channels,” IEEE Trans. Inf. Theory, 2021.