The Characteristics and Optimal Implementation of Mating Disruption in Managing Pest Insect Population

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Abstract. Conventional pesticide application is the most common method in dealing with pests. However, the application of conventional pesticide has a considerable drawback to the environment and the agricultural sector in the long run. Therefore, a more environmentally friendly pest control method is needed. Mating disruption is one of the alternative methods available. This paper discusses the characteristics and the optimal implementation of a mating disruption using optimal control approach. A generic insect life cycle model is examined and modified to add mating disruption as a dynamic control method. 4th order Runge-Kutta method and forward-backwards sweep method was used to numerically solve the control problem. An illustrative comparison between the implementation of constant control and dynamic control was also given for a variety of scenarios. From the simulations, it was found that mating disruption generates a large amount of benefit when it is fully effective. When it has imperfect effectiveness in disrupting the mating process, mating disruption will generate significantly less benefit. The optimal control approach reduces the application cost significantly while still preserving the benefit.

1. Introduction

The development of pest insect management methods is rising along with the increasing human population and increasing demand for agricultural products. The use of conventional pesticides is the most common method in controlling the pest insect population. Even though the increase of productivity in the agricultural sector is enormous, the environmental damage caused by the application of conventional pesticide is also great such as the contamination of soil, water surface, and groundwater that give threat to the consumer of agricultural products and also the workers [15]. The reduction of organism diversity including the reduction of pest natural predator in the environment and also the possibility of resistance are also threats to the environment and the agricultural sector in the long run. Thus, the development of more eco-friendly pest control methods is needed to reduce the possible environmental damage. The measures to manage the crop pests with the environmentally friendly method are discussed within the scope of integrated pest management (IPM) [3].

There are many eco-friendly methods developed as alternatives to the use of conventional pesticide. One of the examples that are studied and used is mating disruption. Mating disruption is commonly used as the control of many Lepidoptera population [6], such as Tuta absoluta [7], Lobesia botrana [13], Cydia nigricana [4], and many more examples. Failures are also found in implementing mating disruption. This was the case for Leucoperta coffeela [1]. Many hypothetical factors may cause
the failure of mating disruption implementation such as the concentration of the substance used, the
area of implementation, and the size of the insect population.

Mating disruption is a control method that uses synthetic sex pheromone to prevent the mating
process of the insect [10]. This method considers the natural behaviour of the insect in the mating
process. Female insect secretes its sex pheromone to the air and fill aerial trail to attract male insect
[17]. The implementation of mating disruption can be conducted in a variety of ways. One of the most
common ways to implement mating disruption is to combine it with a trapping method [9]. Aerial
dispersal is also one of the ways to apply mating disruption [5].

In this paper, we compared the implementation of mating disruption between constant control
implementation and dynamic control implementation for several possible effectiveness values. This
was conducted to assess how effective is the control method in reducing the insect population, then we
conducted optimal control analysis to find the optimal strategy in implementing the control method.

2. Mathematical model

2.1. Model

In this paper, we used the generic model to illustrate the life cycle of insect proposed in [2]. There are
four compartments used in this model namely larva (I), male insect (M), the unfertilized female insect
(Y), and fertilized female (F). The compartment model is provided in Figure 1. The larva is produced
by fertilized female population and grows logistically with the growth rate of \( b \left( 1 - \frac{I}{k} \right) \) for \( b \) is the
intrinsic egg-laying rate and \( k \) is the carrying capacity. The larva grows to be male insect and female
insects at the rate of \( v_I \) with the \( r \) proportion of the population will be female insect and \( (1 - r) \) the
proportion of the population will be male insects. The unfertilized female insect will be fertilized
female when it is mated by the male population with the rate of \( v_Y \) and the possibility of a single
female to be mated by a single male is \( \min \left( 1, \frac{yM}{Y} \right) \) for \( y \) is the number of females that can be mated
by a single male in a day. The fertilized female can be unfertilized female again at the rate of \( \delta \). All of
the subpopulations have a constant mortality rate \( \mu_i \) for \( i \in \{I, Y, F, M\} \).

From this model, we can see the generic life stages of the insect and put a specific control to
manage the whole insect population. We modify the interpretation of mating disruption in [2]. The
first modification is that instead of considering it as a measurement of the additional female insect, we
interpreted mating disruption control as the measurement of female insects that are hidden by the
synthetic sex pheromone. The second modification is that instead of considering mating disruption as
constant control, we considered a dynamic form of control. The control is denoted as a function of
time \( u(t) \) for \( 0 \leq u(t) \leq 1 \) and it represents the portion of the area in which mating disruption is
being applied to and \( \varepsilon \) is the effectiveness of control that depends on the method of application, the
substance used, the concentration of the substance, and other factors.

![Figure 1: Compartment model of insect life cycle](image)
The compartment model presented in Figure 1 can also be presented in a system of differential equations in equations (1)-(4).

\[
\frac{dI(t)}{dt} = b \left(1 - \frac{I(t)}{K}\right) F - (v_I + \mu_I)I(t)
\]

\[
\frac{dM(t)}{dt} = (1 - r)v_I(t) - \mu_M M(t)
\]

\[
\frac{dY(t)}{dt} = rv_I(t) + \delta F - \left(1 - \epsilon u(t)\right)v_Y \min \left\{1, \frac{\gamma M(t)}{Y(t)}\right\} Y(t) - \mu_Y Y(t)
\]

\[
\frac{dF(t)}{dy} = \left(1 - \epsilon u(t)\right)v_Y \min \left\{1, \frac{\gamma M(t)}{Y(t)}\right\} Y(t) - (\delta + \mu_F)F(t)
\]

It is important to note that the system is piecewise because of the expression \(\min \left\{1, \frac{\gamma M(t)}{Y(t)}\right\}\). The system is divided into two cases. First is the male abundant case where the population of male insect is large enough to fertilize all unfertilized female available and it happens if \(\gamma M(t) - Y(t) \geq 0\). The second case is male scarce where the population of male insect is not large enough to fertilize all unfertilized female available and it happens if \(\gamma M(t) - Y(t) < 0\). When \(u(t) = 0\), then we refer to the model without control in [2] where the stability analysis is already done.

2.2. Control Problem

To find the optimal strategy in implementing mating disruption with an optimal control approach, we constructed a performance index and the objective functional. The objective of the optimization process is to minimize the total insect population and the cost of control application. The objective functional is presented in equation (5).

\[
\min J = \int_0^T C_0 N(t) + \frac{C_1}{2} u^2(t) dt
\]

for \(C_j\) for \(j = 0, 1\) are the balancing cost weights and \(N(t) = I(t) + M(t) + Y(t) + F(t)\). The minimization process is subject to the system (1)-(4). The dynamics of each function in the system (1)-(4) are the response to the implementation of the control \(u(t)\). Optimal control is an approach to find the best admissible control \(u(t)\) to optimize the objective functional (5). In this context, we also need to note that \(0 \leq u(t) \leq 1\) for all \(t \in [0,T]\). The control \(u(t)\) is the admissible control and a part of a set of all admissible controls \(\mathcal{U} = \{u| u(t)\) is a Lebesgue measurable for \(t \in [0,T]\}\).

3. Optimality conditions

At this point, we have bounded Lebesgue measurable controls and non-negative initial values of the state system, then the non-negative solution to the system (1)-(4) exists. In addition, the objective functional is concave with respect to \(u\) and closed admissible control in \(\mathcal{U}\), the existence of optimal control \(u^*\) is guaranteed [11]. The further optimality conditions are derived to find the optimal control is provided by Pontryagin’s maximum principle [16]. Pontryagin’s maximum principle represents the control problem as the optimization of a Hamiltonian function, then derives the optimal control, the state system, and the adjoin system. We constructed a Hamiltonian function presented in equation (6).

\[
H = C_0 N(t) + \frac{C_1}{2} u^2(t) + \sum_{i=1}^{4} p_i(t)R_i(t)
\]

for \(p_i(t); i = 1, 2, 3, 4\) are the adjoint functions corresponding to equations (1)-(4) while the \(R_i\) are the right-hand side of the system (1)-(4). It is important to note that the Hamiltonian function is also piecewise. Pontryagin’s maximum principle provides the set requirements presented by equation (7)-(9).

\[
\frac{\partial H}{\partial u} = 0
\]
\[
\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i}, \quad (8)
\]
\[
\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i} \quad (9)
\]
for \( i = 1, 2, 3, 4 \) and \( x_i \in \{I, M, Y, F\} \). Requirement (7) provides the optimal control, requirement (8) provides the state system, and requirement (9) provides the adjoin system.

Using condition (7), we derived the optimal control solution for the Hamiltonian function (6). The derivation of the optimal control solution was also considering the lower and the upper bounds of the control. Since control \( u(t) \) is put in the piecewise part of the system, then the solution for the optimal control is also piecewise. If \( \gamma M(t) - Y(t) \geq 0 \), the system enters male abundant case and the optimal for the system is presented in equation (10) while also considering the bounds for controls.

\[
u(t) = \min \left\{ 1, \max \left\{ 0, \frac{(p_3(t) - p_4(t))v_y Y(t)}{C_1} \right\} \right\}, \quad (10)\]

and if \( \gamma M(t) - Y(t) < 0 \), the system enters male scarce case and the optimal control solution for the system is presented in equation (11).

\[
u(t) = \min \left\{ 1, \max \left\{ 0, \frac{(p_3(t) - p_4(t))v_y \gamma M(t)}{C_1} \right\} \right\}, \quad (11)\]

From equations (10)-(11), we can intuitively conclude that the control effort depends on the mating rate and the size of the possible mated population. In (10), the possible mated population is \( v_Y Y(t) \) while in (11), the possible mated population is \( v_Y \gamma M(t) \). The other factor is the corresponding adjoin functions which are in this case \( p_3(t) \) and \( p_4(t) \). The associated cost impacts negatively to the control effort where the more expensive the control cost, the lesser the control effort.

Requirement (8) provides the state system which is the system (1)-(4) while requirement (9) provides the adjoin system. The adjoin system is piecewise since the Hamiltonian function is piecewise. The first part of the adjoin system is corresponding to the state function \( I(t) \) and is presented in equation (12).

\[
\frac{dp_1(t)}{dt} = -C_0 - p_1(t) \left[ -\frac{bF(t)}{K} - (v_Y + \mu_Y) \right] - p_2(t) [(1 - r) v_Y] - p_3(t) [r v_Y]. \quad (12)
\]

The second and the third equations of the adjoin system are piecewise. If \( \gamma M(t) - Y(t) \geq 0 \), the system enters male abundant case and the second and the third part of the adjoin system are presented in equations (13)-(14).

\[
\frac{dp_2(t)}{dt} = -C_0 - p_2(t) [-\mu_M], \quad (13)
\]
\[
\frac{dp_3(t)}{dt} = -C_0 - p_3(t) [-(1 - \epsilon u(t)) v_Y + \mu_Y] - p_4(t) [(1 - \epsilon u(t)) v_Y]. \quad (14)
\]

If \( \gamma M(t) - Y(t) < 0 \), the system enters the male scarce case and the second and the third part of the adjoin system are presented in equations (15)-(16).

\[
\frac{dp_2(t)}{dt} = -C_0 - p_2(t) [-\mu_M] \quad (15)
\]
\[
\frac{dp_3(t)}{dt} = -C_0 - p_3(t) [-(1 - \epsilon u(t)) v_Y] - p_4(t) [(1 - \epsilon u(t)) v_Y]. \quad (16)
\]

The last part of the adjoin system is presented in equation (17).

\[
\frac{dp_4(t)}{dt} = -C_0 - p_4(t) \left[ -b \left( 1 - \frac{l(t)}{K} \right) \right] - p_3(t) [\delta] - p_4(t) [-\delta + \mu_F)]. \quad (17)
\]

The additional requirement that is needed to be satisfied is transversality condition. The transversality condition is generally written as presented in equation (18).

\[
(S_x - p) \delta x|_T + (H + S_I) \delta t|_T = 0. \quad (18)
\]
We assumed that the terminal values for the state system are free and the terminal time value is fixed. From the objective functional, \( S = 0 \). Thus, the transversality condition (18) can be rewritten as presented in equation (19).

\[
p_i(t) = 0; i = 1,2,3,4.
\]  

(19)

To solve the control problem, we need to solve the state system and the adjoin system simultaneously, then calculate the solution for the optimal control for the system. In the next section, we provided numerical solutions for the control problem.

4. Numerical solutions

In this section, we provided numerical solutions for the system. We exploited the 4th order Runge-Kutta method [14] to find the solution for the system without control and with constant control. Then we use the forward-backwards sweep method [12] that is combined with 4th order Runge-Kutta to find the dynamic optimal implementation of the control. The parameter values used are collected from [2]. We set the terminal time value \( T = 80 \) and the balancing cost weights \( C_0 = 1 \) and \( C_1 = 100 \). We also set the initial values of the state functions \( I(0) = 100, M(0) = 20, Y(0) = 400, \) and \( F(0) = 120 \). For the constant control, we set that \( u(t) = 1 \) which is the maximum control implementation capacity. We examined four values of effectiveness which are \( \varepsilon = 1 \) which means that the control has 100% effectiveness, \( \varepsilon = 0.9 \) which means the control has 90% effectiveness, \( \varepsilon = 0.8 \) which means the control has 80% effectiveness, and \( \varepsilon = 0.7 \) which means the control has 70% effectiveness.

![Figure 2 Constant control for different effectiveness levels](image)

Figure 2 provides the dynamics of the insect population with constant control for different effectiveness levels. It is found that the effect of mating disruption is prevented mating process. This is seen in the dynamics of the unfertilized female population and fertilized female population. Without control, the unfertilized female population decreased immediately and the population was transferred to be the fertilized female population. It is seen that without control, the fertilized female population
increased significantly indicating a high rate of mating success. This then caused an increased number of larva. With control, the number of unfertilized females increased because there is less mating process. With 100% effectiveness, mating disruption prevented all mating possibility and resulted in an increased number of unfertilized female. The unfertilized female population decreased because of natural causes while there is no recruitment to the unfertilized female population since no new larva generation is produced. This then resulted in a significant decrease in all four subpopulations.

However, when the effectiveness of control was decreased, the unfertilized female population increased indicating the application of control, but then the population kept increasing because of constant recruitment. There were still some females that were successfully mated by male and new generations of larva were still produced. This shows that mating disruption with imperfect effectiveness should not solely be implemented. The implementation of mating disruption should be complemented with other control methods. This is consistent with the explanation in [7] and [8]. Knowing the lack of efficacy, then implementing mating disruption recklessly should be avoided, therefore, finding an optimal implementation of mating disruption is important. The effectiveness of the control effort using mating disruption also depends on the species and the dose of the active substance of the synthetic sex pheromone. This is particularly indicated in [2] where there is a threshold for the dose of synthetic pheromone used in the system with the unit of female individual.

Figure 3 provides the optimal control profile for the system. It is clearly shown that to minimize the objective functional, we should not implement mating disruption at full capacity for the whole control period. When the effectiveness of control was 100%, the implementation of control was excessive for almost the whole control period. When the effectiveness was decreased to 90%, 80%, and 70%, the implementations of control were also decreased to conserve capital because implementing more control would not give a significant effect in decreasing the total insect population. The less effective is the control, the less control is implemented.

From Figure 4, it is clear that the effect of decreasing the control capacity due to cost made the difference between the dynamics without control and with control not as significant as the dynamics of the population with constant full control implementation. However, it is done to conserve resources relative to the benefit that the control implementation could give. When the effectiveness of control was 100%, the insect population decreased significantly because for almost 80 days, there was no new larva generation and the population decreased because of natural causes. When the effectiveness of control was decreased to 90%, 80%, and 70%, the difference between the population without control and with control became less significant because there were still new generations of larva produced.

From Figure 5, it is found that the difference between the dynamics of the total insect population with constant full control implementation and the optimal dynamic control implementation is not significant. However, the control used in the optimal dynamic control profile was considerably lesser
than the constant control. This means less capital that needs to be spent using the optimal implementation of control while not losing significant benefit.

Figure 4 Dynamics of insect population with dynamic control

Figure 5 Dynamics of total insect population (a) with constant control and (b) with dynamic control

5. Conclusion
In this paper, we have presented a generic model of the insect life cycle and the modification made to add mating disruption as a control function. We have examined the comparison between the implementation of constant control and the implementation of dynamic control for a variety of
effectiveness values which were 100%, 90%, 80%, and 70%. It is found that mating disruption will give a significant benefit if it has perfect effectiveness of 100%. However, if the effectiveness to disrupt mating is imperfect, its effectiveness to decrease the whole insect population will also decrease significantly. Using the optimal control, the capital that needs to be spent can be reduced significantly while not losing too much benefit. With the knowledge that imperfect effectiveness of control application can reduce its effectiveness to reduce the total insect population, then it is necessary to apply other complementary control methods.

References

[1] Ambrogi BG, Lima ER and Sousa-Souto L 2006 BioAssay 1(8) 1-5.
[2] Anguelov R, Dufourd C and Dumont Y 2017 Appl. Math. Model. 52 437-457.
[3] Apple JL and Smith RFE 1976 Integrated Pest Management (New York: Plenum Press).
[4] Bengtsson M, Karg G and Kirsch PA 1994 J. Chem. Ecol. 20(4) 871-887.
[5] Brockerhoff EG, Suckling DM, Kimberly M, Richardson B, Coker G, Gous S, Kerr JL, Cowan DM, Lance DR, Strand T and Zhang A 2012 PLoS ONE. 7(8) e43767.
[6] Carde RT and Minks AK 1995 Annu. Rev. Entomol. 40 559-585.
[7] Cocco A, Deliperi S and Delrio G 2013 J. Appl. Entomol. 137(2013) 16-28.
[8] Cocco A, Lentini A and Serra G 2014 J. Insect Sci. 14(144) 1-8.
[9] Evenden ML, Judd GJR and Borden JH 1998 Entomol. Exp. Appl. 90 27-47.
[10] Foster SP and Harris MO 1997 Annu. Rev. Entomol. 42 123-146.
[11] Flemming WH 1975 Deterministic and Stochastic Optimal Control (New York: Springer-Verlag)
[12] Lenhart S and Workman JT 2007 Optimal Control Applied to Biological Models (Florida: CRC Press)
[13] Lousi F and Schirra KJ 2001 IOBC wprs Bull. 24(2) 75-79.
[14] Mathews JH and Fink KD 20004 Numerical Method using MATLAB 4th Edition (New Jersey: Prentice Hall)
[15] Mourato S, Ozdemiroglu E and Foster V 2000 Environ. Sci. Tech 34 1456-61.
[16] Tu PNV 1984 Introductory Optimization Dynamics Optimal Control with Economics and Management Science Applications (New York: Springer).
[17] Welter SC, Pickel C and Miller JG 2005 California Agric. 59(1) 16-22.