Research on Similarity Correlation and Compactness of Divided Regions Based on Gray Scale

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Abstract-In the indicators of the conventional gradation division area, it is usually only considering the fluctuation of the regional gradation variance or the rationality of the division is rarely considered, and the effect is determined only by qualitative observation. The purpose of this paper is to give a quantitative representation of the similarity and compactness of the gray areas, which is used to measure the pros and cons of the divided areas and determine whether it needs to be re-divided. The average similarity of the pixels in the region and the variance between the gray values of the pixel points in the region can be used as parameters to jointly measure the similarity and compactness of the divided regions based on the gray scale, and are represented by the parameter S. Experiments show that the larger the s, the smaller the similarity and compactness of the divided regions, and vice versa. It has a certain help for image processing and region division.

Keywords-region division; similarity; gray mean; gray variance

I. INTRODUCTION

Image processing often requires grayscale processing and area division. Grayscale region division plays an important role in subsequent graphics image processing. Generally speaking, the quality of grayscale region directly affects the advantages and disadvantages of subsequent image processing. In the indicators of the conventional gradation division area, it is usually only considering the fluctuation of the regional gradation variance or the rationality of the division is rarely considered, and the effect is determined only by qualitative observation. The quantitative representation of the similarity and compactness of a divided gray area is incalculable in image processing, which helps to quantitatively determine the quality of a gray area, cycle evaluation and modification until the requirements are met. The method proposed in this paper is that the similarity and compactness of the divided gray regions can be quantitatively determined by the similarity between the pixels in the determined region and the joint relationship of the gray value variances between the pixels.

II. DETAILED EXPLANATION OF THE ALGORITHM STEPS

A. Mean Filtering

Firstly, while retaining the gray information of the original image area, a smaller neighborhood can be selected for mean filtering, which not only ensures the elimination of sharp noise, but also guarantees a certain degree of smoothness, and ensures that it is not excessively smooth and loses too much detail information. In the formula, \( \bar{x} \) represents the filtered gray value of the i-th pixel, and the parameter \( \delta \) functions as a balance experiment in the formula. (citing literature [7])

\[
X_i = \frac{1}{1+\delta} \left[ x_i + \frac{\delta}{M} \sum_{r \in N_r} X_r \right]
\]

(1)

B. Similarity between Two Pixels in the Region

The similarity between two pixels in a region describes the correlation between two pixels in the region, which can reflect the intrinsic relationship of the region and the degree of aggregation compactness. The method for solving the similarity of any two pixels in a region is as follows: (citing literature [6]).

Substitute the pixel value formula in the first step of the mild average filter, so that \( x_i \) represents the gray value of the i-th pixel in the image, and \( (p_i, q_i) \) represents the coordinate of \( x_i \). Substituting equation (1) into a formula that gives the degree of similarity of two pixels.

\[
S_{ij} = \exp \left\{ -\max(\frac{\|p_i-p_j\|}{\lambda_g}, \|q_i-q_j\|) \left( \frac{\|x_i-x_j\|}{\lambda_s} \right)^{\delta} \right\}_{i \neq j}
\]

(2)

Here, \( \lambda_s \) and \( \lambda_g \) are scale parameters for controlling distance difference and gray scale difference. \( \lambda_s \) usually takes 3, \( \lambda_g \) is usually selected between 0.5 and 6. \( \rho \) is a value that reflects the flatness of the local area. The smaller the formula value, the similarity between the two pixels is higher.

C. Average Pixel Similarity Value of the Region

The similarity in the above equation can only reflect the relationship between two pixels in the region, but to map to the entire plane, this needs to solve the similarity value between any two pixels, and then find the sum and then average, get the formula for the average pixel similarity value of the region. The steps are as follows: Substituting (2) into (3), the mean of the similarity between any two pixels in the region is obtained, and the correlation similarity of the entire divided region is reflected. \( \bar{C}_r \) represents the mean of the similarity of the nth region, \( C_r \) represents the number of all cases, and M represents the total number of pixels of the nth region.
\[ b_n = \frac{\sum S_i}{C_M} \]  

(3)

D. The Mean Value of the Pixel Gray Value in the Area

In addition to the similarity between the pixels in the region, the variance of the gray value of the pixel in the region is also an important indicator reflecting the similarity and compactness of the divided regions. In order to obtain the variance in the region, it is necessary to first substitute the gray value of the equation (1) into the equation (4) to obtain a formula representation of the gray value mean \( \mu_n \) in the region.

\[ \mu_n = \frac{\sum x_i}{M} \]  

(4)

E. The Variance of the Pixel Gray Value in the Area

In the case where the mean value of the region gray value is obtained, the formula for obtaining the variance of the gray value in the nth region is expressed. The \( \delta_n \) represents the variance, \( x_i \) represents the gray value of the pixel in the nth region, and \( \mu_n \) is the mean value of the gray value. M is the total number of pixels in the area.

\[ \delta_n^2 = \frac{\sum (x_i - \mu_n)^2}{M} \]  

(5)

F. An Indicator of the Pros and Cons of a Regional Division Performance--s

Considering the two factors of the average pixel similarity of the region and the variance of the pixel gray value in the region, the two equations (3) and (5) are substituted to obtain the index \( s \) of the superiority and inferiority of the region partitioning performance. \( s \) represents the similarity correlation of pixels in the region, \( \tau \) is the similarity mean parameter of the nth region, and \( \omega \) is the variance parameter of the gray value in the nth region. (To ensure the monotonic increment of its \( s \) function, take the reciprocal of the average pixel similarity value \( b_n \) of the region and record it as the average pixel similarity value of the region \( b_n^1 \))

\[ s = \frac{\tau}{b_n} + \omega \delta_n^2 \]  

(6)

For the study of the similarity and compactness of the regional division, it can be seen that the smaller the \( s \), the better the similarity of the regional division.

III. EXPERIMENTAL RESULTS AND ANALYSIS

To verify the conclusion, three matrixes of gray-scale pixel regions are assumed here, which are respectively denoted as region a, region b, and region c, and the matrix of the gray-scale region after the gentle mean filtering is as follows:

\[
\begin{array}{cccc}
20 & 20 & 20 & 20 \\
20 & 20 & 20 & 20 \\
20 & 20 & 20 & 20 \\
20 & 20 & 20 & 20 \\
\end{array}
\]

\[
\begin{array}{cccc}
54 & 57 & 52 & 40 \\
53 & 55 & 50 & 53 \\
54 & 47 & 52 & 60 \\
56 & 57 & 44 & 51 \\
\end{array}
\]

\[
\begin{array}{cccc}
64 & 57 & 52 & 60 \\
57 & 51 & 54 & 43 \\
54 & 57 & 52 & 60 \\
54 & 57 & 51 & 40 \\
\end{array}
\]

The average values of the gray areas of the region a, the region b, and the region c are calculated by (4) to be 20.52.3, and 32.5, respectively. Then, by using equation (5), the gray-scale variance values of region a, region b, and region c are 0, 26.8, 341.5. (all calculated data retains only one decimal place), and then their variance graphs are obtained. (The number 1 on the horizontal axis represents the gray matrix a, the number 2 on the horizontal axis represents the gray matrix b, and the number 3 on the horizontal axis represents the gray matrix c. Vertical axis represents variance).
Similarly, the $b_1$ value of the region a, the region b, and the region c can be calculated by (2) and (3) (all calculated data retains only one decimal place). The values are 6.0, 47.1, 72.4. The diagrams are as follows: (where the number 1 on the horizontal axis represents the gray matrix a, the number 2 on the horizontal axis represents the gray matrix b, and the number 3 on the horizontal axis represents the gray matrix c. Vertical axis indicates similarity).

Finally, the index $s$ values of the area division performance of the area a, the area b, and the area c can be obtained by (6) and the above two figures, and the values are 2, 32.9, and 260.6, respectively. That is, the smaller the $s$ value, the greater the degree of regional aggregation and similarity. (All calculations only retain one digit after the decimal point) and then get a graphical representation of $s$. (where the number 1 on the horizontal axis represents the gray matrix a, the number 2 on the horizontal axis represents the gray matrix b, and the number 3 on the horizontal axis represents the gray matrix c.).

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