PROPERTIES OF $\mathcal{N} = 1$ SUSY YANG-MILLS VACUUMS AND DOMAIN WALLS

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Abstract

It is shown that there is no chirally symmetric vacuum state in the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory. The values of the gluino condensate and the vacuum energy density are found out through a direct instanton calculation. A qualitative picture of domain wall properties is presented, and a new explanation of the phenomenon of strings ending on the wall is proposed.

1. The $\mathcal{N} = 1$ supersymmetric Yang-Mills theory (SYM) partition function is ($Q$ is the topological charge):

$$Z = \sum_k \int dA_\mu d\lambda d\bar{\lambda} \delta(Q - k) \exp \left\{ \frac{i}{4g_0^2} \int dx d\theta \frac{1}{2} W^2_\alpha + \text{h.c.} \right\}. \quad (1)$$

Let us integrate it now over the gluon and gluino fields, but with the chiral superfield $W^2$ being fixed. Proceeding as in \cite{1}, one obtains the partition function for the chiral superfield $\Omega = (W^2/32\pi^2 N_c)$ in the form: \footnote{The notation $\oint$ means that the quantum loop contributions of the superfield $W^2$ have also been integrated out, so that the exact correlators of the superfield $W^2$ are obtained from eq.(2) using the tree diagrams only. See \cite{2} for more detail.}

$$Z = \sum_k \oint d\Omega d\bar{\Omega} \delta(Q - k) \exp\{i \int dxL\},$$
\[
\frac{1}{N_c^2} L = \left\{ \frac{1}{2} \int d\theta \Omega \ln \left( \frac{\Omega}{e\Lambda^3} \right) + h.c. \right\} + \int d\theta d\bar{\theta} M(\Omega, \overline{\Omega}, D^n\Omega, \overline{D^n\Omega}, \ldots). \tag{2}
\]

(where \( D \) and \( \overline{D} \) are the superderivatives). The F-term of the Lagrangian in eq.(2), which accounts for all (super) anomalies, coincides with the F-term of the well known Veneziano-Yankielowicz (VY) \([2]\) effective Lagrangian. The difference is that the meaning of the of the word “effective” was not quite clear for the VY-Lagrangian, as well as its connection with the original fundamental Lagrangian. In our approach (see \([1]\) for detail) its connection with the fundamental YM-Lagrangian and its meaning become clear: it is the exact generating functional of the (one particle irreducible) Green functions of the field \( \Omega \).

The D-term in eq.(2) is nonanomalous and depends both on the field \( \Omega \) and its superderivatives. (For our purposes, we will ignore in what follows all fermionic components of \( \Omega \) and all terms with usual space-time derivatives).

We will show in this section that the chirally symmetric vacuum state obtained by A. Kovner and M. Shifman \([3]\) (KS-vacuum with \( \langle 0 | \lambda \lambda | 0 \rangle = 0 \)) is an artefact of using the total VY-Lagrangian, i.e. with the D-term in eq.(2) chosen in the simplest form:

\[
M = \text{const} \left( \overline{\Omega} \Omega \right)^{1/3}. \tag{3}
\]

In what follows, we prefer to deal with the usual component fields:

\[
\Omega = (\sigma, \theta^2 \chi), \quad \sigma = \frac{\lambda \lambda}{32\pi^2 N_c}, \quad \chi = S + iP = \frac{GG + i\tilde{G}\tilde{G}}{32\pi^2 N_c}, \tag{4}
\]

so that the VY-potential takes the form:

\[
\frac{1}{N_c^2} U = \frac{1}{2} \left\{ (S + iP) \ln \left( \frac{\sigma}{\Lambda^3} \right) + h.c. \right\} - C_0 \frac{S^2 + P^2}{|\sigma|^{4/3}}. \tag{5}
\]

With this form, there are \( N_c \) chirally asymmetric vacuum states:

\[
\bar{\sigma}_n = \langle 0 | \sigma | 0 \rangle_n \sim \langle 0 | \lambda \lambda | 0 \rangle_n \sim \Lambda^3 \exp \left\{ i \frac{2\pi n}{N_c} \right\}, \quad n = 0, \ldots, N_c - 1, \tag{6}
\]

corresponding to the spontaneously broken residual axial symmetry and besides, as emphasized by A. Kovner and M. Shifman \([3]\), there is also the chirally symmetric vacuum solution:

\[
\langle 0 | \lambda \lambda | 0 \rangle_o = 0. \tag{7}
\]
Let us point out first that two solutions, eq.(6) and eq.(7), are not on equal footing. Because we know (from the Witten index) that SUSY is unbroken, we are ensured that $\bar{S} = \langle 0 | S | 0 \rangle \to 0$. So, if $|\sigma| \neq 0$, it is sufficient to use eq.(5) to find out the value of $\sigma$, as higher order terms, like $S(S/|\sigma|^{4/3})^k$, are of no importance in this case. If $|\sigma| \to 0$ however, all higher order terms become of importance and we can not believe, in general, the results obtained from eq.(5). If the VY-potential were exact, the KS-solution will survive. But really, the term $S^2/|\sigma|^{4/3}$ in eq.(5) is only the first term in the expansion in powers of $(S/|\sigma|^{4/3})^k$. So, the KS-solution is not selfconsistent in this respect and we need to know, in particular, the behaviour of the potential at $z = S/|\sigma|^{4/3} \to \infty$. To find it out, let us write first a general form of the potential in eq.(2) ($\sigma = \rho \exp \{i\phi\}$):

$$\frac{1}{N_c^2} U(\sigma, \chi) = S \ln \left( \frac{\rho}{\Lambda^3} \right) - (\phi - \frac{\theta}{N_c}) P + S f_1 \left( \frac{S}{\rho^{4/3}}, \frac{P}{S} \right).$$ (8)

Let us add now to eq.(8) the gluino mass term:

$$\frac{1}{N_c^2} \Delta U = -m_o \rho \cos(\phi),$$ (9)

where $m_o$ is the renormalization group invariant mass parameter. This addition of $\Delta U$ is legitimate as our Lagrangean was obtained integrating out all degrees of freedom, but with all components of the $\Omega$-superfield fixed, and because the gluino mass, $m_o$, can be considered as a source for the field $\lambda \lambda$ (see [1] for more detail). Now, at large $m_o \to \infty$, the heavy gluino will decouple leaving us with the pure YM theory and we know how it decouples, from the renormalization group. In this region (see below): $\bar{S} = O(m_o^{8/11}), \bar{P} = O(m_o^{3/11})$, so that $\bar{S}/\bar{P}^{4/3} = O(m_o^{12/11}) \to \infty$. Therefore, this will allow us to find out the asymptotic behaviour of $f_1$ in eq.(8).

As the gluino contribution to $b_o = 3 = (11/3 - 2/3)$ is $(-2/3)$, it is not difficult to check that the function $f_1$ in eq.(8) has to have the asymptotic behaviour:

$$f_1 \left( \frac{S}{\rho^{4/3}}, \frac{P}{S} \right) \to \frac{1}{4} \ln \left( \frac{S}{\rho^{4/3}} \right) + f_2 \left( \frac{P}{S} \right), \quad \frac{S}{\rho^{4/3}} \to \infty.$$ (10)

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3 Changing the phase of $m_o$ is equivalent to a redefinition of $\theta$ in eq.(8). So, it is convenient to choose $m_o$ in eq.(9) to be real and positive.
In this case, integrating out the $\rho$ and $\phi$ fields, one has:

$$m_o\rho e^{i\phi} = \left(\frac{2}{3}S + iP\right),$$  \hspace{1cm} (11)

and $U(S, P)$ takes the form:

$$\frac{1}{N_c^2} U(S, P) = \frac{11}{12} S \ln \left(\frac{S}{\Lambda_{YM}^4}\right) + \frac{\theta}{N_c} P + S f_3 \left(\frac{P}{S}\right); \Lambda_{YM} = \Lambda^{9/11} m_o^{2/11},$$  \hspace{1cm} (12)

as it should be. \[4\]

Now, we are ready to check the existence of the KS-solution: $\bar{\rho} \to 0$. We can distinguish three cases (we take $\theta = 0$, $\bar{\phi} = \bar{P} = 0$, as they are of no importance for us here).

a) Let $\bar{z} = (\bar{S}/\bar{\rho}^{4/3}) \to 0$, so that $f_1(z = S/\rho^{4/3}) \sim z$. As was pointed out above, this variant is selfcontradictory at $\bar{\rho} \to 0$, as $\partial U/\partial S = 0$ leads to: $\bar{z} \sim \ln(\Lambda^{3}/\bar{\rho}) \to \infty$.

b) Let $\bar{z} \to z_o = const \neq 0$. Then (barring pathological singularities) the saddle point equations are: $z_o f_1'(z_o) = 3/4$; $\ln(\Lambda^{3}/\bar{\rho}) = f_1(z_o) + z_o f_1'(z_o)$. The first equation shows that $f_1'(z)$ (and so $f_1(z)$) is nonsingular at $z = z_o$, but we are in trouble then with the second equation at $\bar{\rho} \to 0$.

c) Finally, let $\bar{z} \to \infty$, so that $f_1(z) \to (1/4)\ln z$. This case is also in trouble, as $\partial U/\partial S = 0$ leads to $(\bar{S}/\bar{\rho}) = O(1/\bar{\rho}^{11/3}) \to \infty$ at $\bar{\rho} \to 0$, while $\partial U/\partial \rho = 0$ leads to $(\bar{S}/\bar{\rho}) \to 0$.

On the whole, we conclude that there is no chirally symmetric vacuum state in $\mathcal{N} = 1$ SYM, so that the residual axial symmetry is spontaneously broken in all vacuum states.

2. We will show now that a spontaneous breaking of the residual axial symmetry and the value of the gluino condensate can be obtained in a quite different way, through a direct calculation of the instanton contributions into the partition function. With this purpose, let us return to the original partition function, eq.(1), add the gluino mass term with a small but finite mass $m_o$ to the action, and consider the instanton contributions.

It has been shown in \[4\] that, under a special choice of the collective coordinates, the $n$-instanton contribution splits up into $nN_c$ "instantonic

\[4\] Another way to check eq.(10) is to recall that eq.(11) can be obtained through a direct calculation of the heavy gluino loop in the gluon background, and is directly related to the trace and axial anomalies.
In our case of $\mathcal{N} = 1$ SYM, the result is especially simple. Because all nonzero mode contributions cancel exactly between the gluon and gluino contributions, there remains no residual interaction between these instantonic quarks. For instance, the $n=1$ instanton contribution takes the form ($b_o = 3, m_\theta = m_o \exp (i \theta / N_c)$):

$$Z_1 = \int dx_1...dx_{N_c} \frac{1}{N_c!} \left[ N_c^2 \frac{m_\theta}{2} \Lambda^{b_o} \right]^{N_c}, \quad (13)$$

where: $x_i$ are the collective coordinates (the positions of the instantonic quarks), and $m_\theta$ is due to the gluino zero modes. The $n$-instanton contribution is exactly of the same form, so that summing up over $n$ (and adding antiinstantons) one obtains the partition function in the form:

$$Z_{tot} = Z \ Z^*, \quad Z = \frac{1}{N_c} \sum_{k=0}^{N_c-1} e^{I(k)},$$

$$I(k) = \int dx \ N_c^2 \left\{ \frac{m_\theta}{2} \Lambda^3 \left[ 1 + O(|m_\theta|^2) \right] \exp \left( i \frac{2\pi k}{N_c} \right) + O(|m_\theta|^2) \right\}. \quad (14)$$

Here, the factor $Z_{N_c}(k) = \exp \{i 2\pi k / N_c\}$ appeared because we have extracted the $N_c$-th power root from unity, when going from eq.(13) to eq.(14). It plays the role of the "neutralizator", i.e. when $\exp \{I(k)\}$ in eq.(14) is expanded back into a power series, it ensures that instantonic quarks appear in the $N_c$-fold clusters only (i.e. in the form of instantons). Besides, it ensures the periodicity: $Z(\theta) = Z(\theta + 2\pi l)$, which was explicit before summation over $n$.

We would like to emphasize that the above expression for the action in eq. (14) is exact, within the indicated accuracy. Indeed:

a) The perturbation theory (i.e. the $Q = 0$ sector of the partition function) contribution is exactly zero at $m_o = 0$ due to SUSY, and the corrections from this sector start with $O(|m_\theta|^2)$. This is because the replacement $m_o \rightarrow -m_o$ is equivalent to changing the $\theta$- angle, and the $Q = 0$-sector is $\theta$-independent.

b) Because the one-loop $Z_Q$-contribution contains already the factor $(m_\theta)^{QN_c}$, all higher loop corrections to it can be calculated with $m_o = 0$, and they all cancel due to SUSY. For the same chirality reasons, the (relative) corrections in this sector also start with $O(|m_\theta|^2)$, including those which originate from disturbing the exact cancelation between the gluon and gluino nonzero
modes at $m_o = 0$.

c) As for the instanton-antiinstanton interaction contributions, they should not be considered as independent ones, but rather as belonging to perturbation theory (its asymptotic tail), in the sector with fixed $Q$. So, they are also zero in the sense that they are accounted for already in the points "a" and "b" above.

Eq. (14) shows clearly that the residual axial symmetry is broken spontaneously in our system (in the infinite volume limit). Indeed, before summation over $n$ each $n$-instanton contribution was invariant by itself under $\theta \to \theta + 2\pi l$, as a result of the residual axial symmetry. But after summation, the instantonic quarks have released and the above symmetry acts nontrivially now, interchanging $N_c$ branches between themselves. As a result, because the small perturbation ($m_o \neq 0$) was introduced, one definite branch dominates the whole partition function, - those one which minimizes the energy (at given $\theta$). So, we obtain for the vacuum energy density:

$$E_{\text{vac}} = -N_c^2 \Lambda^3 \frac{1}{2} [m_\theta + m_\theta]_{2\pi} + O(|m_\theta|^2),$$

where the notation $[f(\theta)]_{2\pi}$ means that this function is $f(\theta)$ at $-\pi \leq \theta \leq \pi$, and is glued then to be periodic in $\theta \to \theta + 2k\pi$, i.e.: $[f(\theta)]_{2\pi} = \min_k f(\theta + 2\pi k)$.

Further, because the $O(m_o)$ term appeared in the energy, this shows that the order parameter (the gluino condensate) is nonzero. Indeed, let us consider $\langle \theta, k | \lambda \lambda | \theta, k \rangle$:

$$\langle \theta, k | \lambda \lambda | \theta, k \rangle = N_c \Lambda^3 \exp \left\{ i \frac{\theta + 2\pi k}{N_c} \right\}. \quad (16)$$

Thus, it is clear from eqs. (14), (16) that (at $m_o \to 0$), there are $N_c$ degenerate vacuum states differing by the phase of the gluino condensate: $\langle \theta, k | \lambda \lambda | \theta, k \rangle = N_c \Lambda^3 \exp \left\{ i \frac{\theta + 2\pi k}{N_c} \right\}$.

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5 We can keep $m_o$ infinitesimal but finite, and $V \to \infty$, and to separate out one term from the sum over "k" in eq. (16).
Moreover, it is possible to replace \( m_\theta \) by the local function \( m_\theta(x) \) in eqs. (13) and (14). Indeed, in eq. (13) the gluino zero mode contributions will take the form:

\[
I_o = \int dx_1 \ldots dx_{N_c} \int dy_1 m_\theta(y_1) \ldots dy_{N_c} m_\theta(y_{N_c}) \Pi,
\]

\[
\Pi = |\psi_o^{(1)}(y_1 - x_k)|^2 \ldots |\psi_o^{(N_c)}(y_{N_c} - x_k)|^2,
\]

where \( \psi_o^{(i)}(y_i - x_k) \) means \( \psi_o^{(i)}(y_1 - x_1, y_i - x_2, \ldots, y_i - x_{N_c}) \), and \( \int dy|\psi_o^{(i)}(y - x_k)|^2 = 1 \). It is not difficult to see that:

\[
I_1 = \int dx_1 \ldots dx_{N_c} \Pi = 1.
\]

Indeed, let us take temporarily \( m_\theta = 1 \) and put our fields into a large 4-dimensional Euclidean volume \( V \), of an arbitrary form. Because \( \int dy_1 \ldots dy_{N_c} \Pi = 1 \), \( I_o(m = 1) = V^{N_c} \). Now, if \( I_1 \) were a nontrivial function of the ratios like \( (y_1 - y_2)^2/(y_2 - y_3)^2 \) etc., then \( I_o(m = 1) \) will be of the form: \( I_o(m = 1) = V^{N_c} f_{\text{geom}} \), where the function \( f_{\text{geom}} \) will depend on the geometry of our volume, and this will be a wrong answer. Therefore, \( I_o = \prod_{i=1}^{N_c} \int dy_i m_\theta(y_i) \), and \( m_\theta \) can be replaced by \( m_\theta(x) \) in eqs. (14) and (15). While the corrections \( O(|m_\theta|^2) \) in eq. (14) remain uncontrollable, it is important that there are no uncontrollable pure chiral corrections of the type \( O(m_\theta^l) \), \( l \geq 2 \). As a result, taking derivatives \( \sim \delta/\delta m_\theta(x) \) we can obtain even local pure chiral Green functions, like: \( \langle k|\lambda\lambda(x_1) \ldots \lambda\lambda(x_l)|k \rangle \), and all of them will be pure constants.

There is nothing mysterious in this behaviour and it does not imply that the theory is trivial. For instance, let us consider \( \langle k|\lambda\lambda(x) \lambda\lambda(0)|k \rangle \), and let us denote:

\[
\lambda\lambda(x) = \exp(i2\pi k/N_c) \rho(x) \exp(i\phi_k(x)),
\]

so that:

\[
\langle k|\rho(x)|k \rangle \sim N_c \Lambda^3
\]

and \( \langle k|\phi_k(x)|k \rangle = 0 \). Then \( \rho \exp(i\phi_k) = \sigma_k + i\pi_k \):

\[
\langle k|\lambda\lambda(x) \lambda\lambda(0)|k \rangle = \langle k|\lambda\lambda(x)|k \rangle \langle k|\lambda\lambda(0)|k \rangle +
\]

\[
+ \exp \left\{ \frac{i4\pi k}{N_c} \right\} \{ \langle k|\sigma_k(x)\sigma_k(0)|k \rangle_{\text{con}} - \langle k|\pi_k(x)\pi_k(0)|k \rangle_{\text{con}} \} = \langle k|\lambda\lambda(0)|k \rangle^2,
\]

as the nontrivial connected correlators cancel each other due to SUSY.

As was pointed out above, supposing only that the gluino condensate is really nonzero, it becomes legitimate to use the VY-Lagrangian to investigate the vacuum properties, i.e. to find out the gluino condensate, eq.(6) \[4\], and the vacuum energy density, eq.(15) \[5\]. In other words, it is not an approximation in this case as higher order terms are of no importance. In
contrast, if we want to deal with some excitations, say domain walls, the VY-Lagrangean is insufficient.

3. Because there is a spontaneous breaking of the residual axial symmetry, there are the domain wall excitations interpolating between the above $N_c$ chirally asymmetric vacuums. The purpose of this section is to give a new qualitative description and interpretation of the domain wall properties and, in particular, their ability to screen the quark charge.

Let us recall in short the interpretation of the vacuum energy density behaviour in the pure gluodynamics which was proposed in [1]. The vacuum state at $\theta = 0$ is supposed to be the condensate of pure magnetic monopoles, i.e. the dyons with the magnetic and electric charges $d^\theta_{1=0} = (1, 0)$ (and of all $N_c - 1$ types, as there are $N_c - 1$ types of monopoles due to $SU(N_c) \to U(1)^{N_c-1}$).

As has been shown by E. Witten long ago [6], as $\theta$ becomes nonzero the monopoles turn into the dyons with the charges: $d^\theta_{1=\pi} = (1, 1/2)$. For this reason, the vacuum energy density, $E_{\text{vac}}(\theta)$, increases. This continues up to $\theta \to \pi$ where the above dyons look as: $d^\theta_{1=\pi} = (1, 1/2)$. The above vacuum becomes unstable in the infinitesimal vicinity of $\theta = \pi$ because there is another state, the condensate of $d^\theta_{2=\pi} = (1, -1/2)$ - dyons, degenerate in energy with the first one. Thus, there occurs rearrangement of the electrically charged degrees of freedom to recharge the $d_1$ - dyons into the $d_2$ - ones. For instance, a copious "production" of the charged gluons, $\bar{g} = (0, -1)$ takes place, so that: $(\bar{g}) + d_1 \to d_2$. This recharging allows the system to have a lower energy at $\theta > \pi$. Indeed, there are now only the $d^\theta_{2} = (1, -1 + \theta / 2\pi)$ - dyons in the condensate at $\theta > \pi$, their electric charge decreases with increasing $\theta$ and the vacuum energy density decreases with it. As $\theta \to 2\pi$, the $d^\theta_{2}$ - dyons become the pure monopoles, and the vacuum state becomes exactly as it was at $\theta = 0$. On the whole, the vacuum energy density, $E_{\text{vac}}(\theta)$, increases in some way at $0 \leq \theta \leq \pi$; there is a cusp due to the above described recharging at $\theta = \pi$, and it decreases then (in a symmetric way) reaching its minimal value at $\theta = 2\pi$.

Now, let us return to SYM and let us suppose that we have integrated out all, but the $\rho$ and $\phi$ ($\lambda \lambda \sim \rho \exp\{i\phi\}$) fields (really, we expect the field $\rho$ is unimportant for a qualitative picture discussed below and we will ignore it, supposing simply that it takes its vacuum value $\bar{\rho} \sim \Lambda^3$).

As the field $N_c \phi$ in SYM is the exact analog of $\theta$ in YM, the above described interpretation of the behaviour of $E_{\text{vac}}(\theta)$ in YM can be transferred
to SYM, with only some evident changes: a) $E_{vac}(\theta) \to U(N_c \phi)$, and it is not the vacuum energy density now but rather the potential of the field $\phi$; b) if we start with the condensate of pure monopoles at $\phi = 0$, the recharge $d_1^\phi = (1, N_c \phi/2\pi) \to d_2^\phi = (1, -1 + N_c \phi/2\pi)$ and the cusp in $U(N_c \phi)$ will occur now at $\phi = \pi/N_c$, so that at $\phi = 2\pi/N_c$ we will arrive at the next vacuum with the same pure monopole condensate but with the shifted phase of the gluino condensate.

Let us consider now the domain wall exitation, $\phi_{dw}(z)$, interpolating along the "z" axis between, say, two nearest vacuums: $\phi(z \to -\infty) \to 0$ and $\phi(z \to \infty) \to 2\pi/N_c$. There is a crucial difference between this case and those just described above where the field $\phi$ was considered as being space-time independent, i.e. $\phi(z) = \text{const}$. The matter is that the system can not behave now in a way described above (which allowed it to have a lowest energy at each given value of $\phi(z) = \phi = \text{const}$): i.e. to be the pure condensate of $d_1^\phi$ - dyons at $0 \leq \phi < \pi/N_c$, the pure condensate of $d_2^\phi$ - dyons at $\pi/N_c < \phi \leq 2\pi/N_c$, and to recharge suddenly at $\phi = \pi/N_c)$. The reason is that the fields corresponding to electrically charged degrees of freedom also become functions of "z" at $q = \int dz [d\phi_{dw}(z)/dz] \neq 0$. So, they can not change abruptly now at some $z = z_o$ where $\phi_{dw}(z)$ goes through $\pi/N_c$, because their kinetic energy will become infinitely large in this case. Thus, the transition will be smeared necessarily.

The properties of the domain wall in the pure gluodynamics with $\theta = \pi$ were described in [1]. The properties of the domain wall under consideration here will be similar to those described in [1]. The main difference is that $\theta$ was fixed at $\pi$ in [1], while $N_c \phi_{dw}(z)$ varies here smoothly between its limiting values, and the electric charges of dyons follow it.

So, at far left there will be a large coherent condensate of $d_1^\phi = (1, N_c \phi/2\pi)$-dyons and a small (incoherent) density of $d_2^\phi = (1, -1 + N_c \phi/2\pi)$-dyons. The $d_2^\phi$-dyons can not move freely in this region as they are on the confinement and appear as a rare and tightly connected pairs, $\overline{d_2^\phi} d_2^\phi$, only. Therefore, their presence does not result in the screening of the corresponding charge. As we move to the right, the density of $d_1^\phi$-dyons decreases while those of $d_2^\phi$ - increases. These last move more and more freely, but are still on the confinement. Finally, their density reaches a critical value so that a "percolation"

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\[6\] Other possible dyons play no role in the transition we consider, and we will ignore them.
takes place, and the $d_2^\phi$-dyons form a continuous coherent network within which individual $d_2^\phi$-dyons can move freely to any distance. At the same time, there still survives a sufficiently large coherent condensate of $d_1^\phi$-dyons, which still can freely move individually within their own network.

At the symmetrical place to the right of the domain wall centre the ”inverse percolation” takes place, so that the network of $d_1^\phi$-dyons decays into separate independently fluctuating droplets, whose density (and size) decreases with further increasing $z$. At large $z$ we arrive at the vacuum state with a large coherent condensate of monopoles (former $(1, -1)$-dyons at large negative $z$).

The above described system has some features in common with the mixed state of the type-II superconductor in the external magnetic field. The crucial difference is that the magnetic flux tubes are sourceless inside the superconductor, while in our case there is a finite density of freely moving real charges (and anticharges) within each network.

Each time when there will coexist the condensates of two mutually non-local fields, they will try to keep each other on the confinement, and will resemble the above described case.

For instance, in SUSY SU(2)-YM with one matter flavour, there will be three phases, depending on the value of $m$, - the mass parameter of the (s)quark. At small $m < m_1 = C_1 \Lambda$, there will be the usual electric Higgs phase, with the magnetic charges being on the confinement, and with the monopoles appearing as independently fluctuating neutral droplets only. At $m = m_1$ the ”percolation” of the monopole droplets takes place, so that at $C_1 < (m/\Lambda) < C_2$ there will be ”the double Higgs phase” with two coherent networks of monopoles and electric Higgs particles, with their averaged densities being constant over the space, and following only the value of $m$.

There will be screening rather than confinement (although the difference between these becomes somewhat elusory here) of any test charge in this interval of $m$. Finally, at $m = m_2 = C_2 \Lambda$ the ”inverse percolation” of the electric Higgs condensate takes place, so that there will be only independently fluctuating neutral droplets of the electric Higgs particles at $m > m_2$, and the usual confinement of the electric charge.

Now, let us return to our original theory and consider what happens when

7 The kinetic terms of magnetically charged fields will have peculiarities at $m = m_1$, while the point $m = m_2$ will be special for the kinetic terms of electrically charged fields.
a heavy quark is put inside the bulk of the domain wall. The crucial point
is that there is a mixture of all four dyon and antidyon species (of all \( N_c - 1 \)
types): 
\[
d_1^\phi = (1, N_c \phi/2\pi), \quad \bar{d}_1^\phi = (-1, -N_c \phi/2\pi), \quad d_2^\phi = (1, -1 + N_c \phi/2\pi) \quad \text{and} \quad \bar{d}_2^\phi = (-1, 1 - N_c \phi/2\pi)
\]
in this "percolated region", with each dyon moving freely inside its coherent network. So, this region has the properties of "the double Higgs phase", as here both the \( d_1^\phi \) and \( d_2^\phi \)-dyons are capable to screen corresponding charges. And because the charges of \( d_1^\phi \) and \( d_2^\phi \)-dyons are linearly independent, polarizing itself appropriately this mixture of dyons will screen any test charge put inside, the heavy quark one in particular.

If the test quark is put at far left (right) of the wall, the string will originate from this point making its way toward a wall, and will disappear inside the bulk (i.e. the region of the double Higgs phase) of the wall. The above described explanation differs from both, those described by E. Witten in \[7\] and those proposed by I. Kogan, A. Kovner and M. Shifman in \[8\].

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