THE WARDLE INSTABILITY IN INTERSTELLAR SHOCKS. II. GAS TEMPERATURE AND LINE EMISSION

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ABSTRACT

We have modeled the gas temperature structure in unstable C-type shocks and obtained predictions for the resultant CO and H$_2$ rotational line emissions, using numerical simulations of the Wardle instability presented in Paper I. Our model for the thermal balance of the gas includes ion-neutral frictional heating; compressional heating; radiative cooling due to rotational and ro-vibrational transitions of the molecules CO, H$_2$O, and H$_2$; and gas-grain collisional cooling. We obtained results for the gas temperature distribution in—and H$_2$ and CO line emission from—shocks of neutral Alfvénic Mach number 10 and velocity 20 or 40 km s$^{-1}$ in which the Wardle instability has saturated. Both two- and three-dimensional simulations were carried out for shocks in which the preshock magnetic field is perpendicular to the shock propagation direction, and a two-dimensional simulation was carried out for the case in which the magnetic field is obliquely oriented with respect to the shock propagation direction. Although the Wardle instability profoundly affects the density structure behind C-type shocks, most of the shock-excited molecular line emission is generated upstream of the region where the strongest effects of the instability are felt. Thus the Wardle instability has a relatively small effect on the overall gas temperature distribution in—and the emission-line spectrum from—C-type shocks, at least for the cases that we have considered. In none of the cases that we have considered thus far did any of the predicted emission-line luminosities change by more than a factor of 2.5, and in most cases the effects of instability were significantly smaller than that. Slightly larger changes in the line luminosities seem likely for three-dimensional simulations of oblique shocks, although such simulations have yet to be carried out and lie beyond the scope of this study. Given the typical uncertainties that are always present when model predictions are compared with real astronomical data, we conclude that Wardle instability does not imprint any clear observational signature on the shock-excited CO and H$_2$ line strengths. This result justifies the use of one-dimensional steady shock models in the interpretation of observations of shock-excited line emission in regions of star formation. Our three-dimensional simulations of perpendicular shocks revealed the presence of warm filamentary structures that are aligned along the magnetic field, a result that is of possible relevance to models of water maser emission from C-type shocks.

Subject headings: infrared: ISM: lines and bands — instabilities — ISM: molecules — masers — molecular processes — shock waves

1. INTRODUCTION

Shock waves have been recognized as a widespread phenomenon in regions of active star formation, where supersonic protostellar outflows interact with surrounding interstellar gas (see Draine & McKee 1993, and references therein). Shock waves in star-forming regions typically give rise to hot molecular gas at temperatures of several hundred to several thousand kelvins, which has been observed by means of the radiation it emits in high-lying rotational lines of CO (e.g., Watson et al. 1985; Genzel, Poglitsch, & Stacey 1988), in ro-vibrational transitions of H$_2$ (e.g., Brand et al. 1988), as well as in pure rotational H$_2$ transitions (e.g., Wright et al. 1996).

Thus far, the most successful models (e.g., Draine & Roberge 1982; Chernoff, Hollenbach, & McKee 1982; Draine, Roberge, & Dalgarno 1983; Smith 1991; Kaufman & Neufeld 1996a, hereafter KN96) for these CO and H$_2$ emissions have invoked the presence of “continuous” or C-type shocks (Draine 1980) in which the ionized and neutral species drift relative to each other, but each shows a continuous variation in flow velocity (in contrast to “jump” or J-type shocks).

Given the presence of magnetic fields of strengths that are typically observed in the interstellar medium (Heiles et al. 1993), C-type shocks are expected to arise (Mullan 1971; Draine 1980) when shocks propagate at speeds of less than ~40–50 km s$^{-1}$ in molecular media of low fractional ionization. Models that invoke C-type shocks of speeds of ~35–40 km s$^{-1}$ or (better) an admixture of shock speeds of up to ~40 km s$^{-1}$ (Smith & Brand 1990; Wright et al. 1996) have been successful in quantitatively accounting for the H$_2$ ro-vibrational line fluxes observed from the Orion KL and Cepheus A (W) outflow regions and the CO line fluxes observed from Orion KL. Models for C-type shocks in very dense molecular gas (Kaufman & Neufeld 1996b) have also been successfully invoked in explaining the radio and submillimeter wavelength water maser lines that have been observed in star-forming regions.

To date, all models for the emission properties of C-type shocks have made use of one-dimensional time-independent solutions of the shock evolution equations, even though the semianalytic work of Wardle (1990, 1991a, 1991b) has established clearly that such solutions are subject to a rapidly growing instability when the neutral Alfvénic Mach
number exceeds ~5 (as it does typically in shock models for regions of active star formation). Numerical calculations by Tóth (1994, 1995) of the nonlinear evolution of the Wardle instability have suggested that it leads to large-amplitude compressions of the gas into structures that are elongated with the flow. In results described in a companion paper, Stone (1997, hereafter S97) has recently carried out two- and three-dimensional simulations that allow the nonlinear evolution of the Wardle instability to be followed to point at which the instability saturates. These simulations allow us to address the first time the question of how the Wardle instability affects the shock-excited line emission from C-type shocks. The question is important because if the effects are large, previous models for the line emission observed from such shocks will need to be revised.

In this paper, we present a study of the molecular line emission that is expected to result from unstable C-type shocks, given the ion and neutral velocity fields computed in the dynamical calculations of S97. In § 2, we describe the details of our calculation, and in § 3 we present and discuss the results obtained for three different geometries and two different shock velocities. The implications of our results are discussed in § 4, and a brief summary is given in § 5.

2. CALCULATIONS

In this study, we have computed the gas temperature and the molecular line emission that is expected to result from unstable C-type shocks. Since thermal pressure does not significantly affect the dynamics in the magnetically dominated C-type shocks, we have considered (see the dynamic calculations carried out recently by Here the ionized and neutral drag terms so as to ensure their unconditional stability. Similar methods are being implemented independently by Mac Low & Smith (1997). Two-dimensional simulations of C-type shocks were carried out for a variety of shock parameters typical of the dense interstellar medium, viz., Mach number $M = 40, 100, 200$; neutral Alfvenic Mach number $A = 10$ or 20; angle between pre-shock magnetic field and flow direction $\theta = \pi/6, \pi/4, \pi/2$. S97 also carried out a three-dimensional simulation for the single case $M = 100, A = 10, \theta = \pi/2$. In each case, the initial condition was the steady state solution of the one-dimensional evolution equations.

While the shock parameters and results obtained in the dynamical calculation could be expressed in terms of dimensionless quantities, our calculation of the resultant gas temperatures and line emission requires the flow velocities and spatial coordinates to be expressed in dimensional units. In particular, the characteristic size scale $L_\star = v_\star^SN/\langle w_\star \rangle$ must be specified: given a preshock H$_2$ density of $10^5$ cm$^{-3}$, the treatment of ion-neutral coupling presented by KN96 implies an effective value of $2.68 \times 10^{15}$ cm for $L_\star$. Our selection of the shock velocity and preshock magnetic field strength is discussed in § 3 below.

2.2. Gas Temperature

Once the ion and neutral velocities in the shocked region have been computed, the gas temperature, $T$, may be determined by solving the energy equation for the neutral fluid:

$$\Lambda(T) = G - n_k kTV \cdot v_\star + \nabla \cdot (\rho_\star \langle \omega \rangle) - \partial u/\partial t, \quad \text{(2)}$$

where $\Lambda$ is the gas cooling rate per unit volume, $v_\star$ is the neutral velocity, $G = \rho_\star \rho_i \langle \omega \rangle^2$ is the frictional heating rate per unit volume, $n_\star = \rho_\star/\mu$ is the neutral particle density, $\mu$ is the mean particle mass, and $u$ is the internal energy density of the gas. The four terms on the right represent frictional heating, compressional heating, the advection of thermal energy, and the effect of time dependence on the internal energy. A considerable simplification is afforded by the fact that the cooling timescale within a C-type shock, $u/\Lambda(T)$, is necessarily short, compared to the flow timescale. This allows the advection and time-dependent terms to be neglected, and the energy equation becomes an algebraic rather than a partial differential equation for $T$, the quantities $G, n_\star, v_\star$ having already been determined in the numerical simulations of S97.

In computing the gas cooling function $\Lambda(T)$, we included cooling due to rotational and vibrational emissions from the molecules H$_2$, H$_2$O, and CO, adopting the radiative cooling functions of Neufeld & Kaufman (1993, hereafter NK93) and treating optical depth effects by an escape probability method described in the Appendix. Previous studies of gas-phase chemistry in C-type shocks (Draine et al. 1983; KN96) have indicated that for shock velocities greater than $\sim 15$ km s$^{-1}$, atomic oxygen in the preshock gas is rapidly converted into water once the temperature reaches $\sim 400$ K, while the preshock CO abundance is unaffected by the passage of the shock. We have therefore assumed constant H$_2$O and CO abundances throughout the shocked region, adopting values relative to H$_2$ of $8.5 \times 10^{-4}$ and $2.4 \times 10^{-5}$, respectively (see KN96). The cooling function $\Lambda(T)$ also includes the collisional energy transfer that results
from inelastic collisions between gas molecules and grains, given an assumed grain temperature of 50 K; for this process we adopted the gas-grain cooling rate of Hollenbach & McKee (1989).

### 2.3. Line Emission

After determining the temperature structure of the shocked region, we computed the emission-line spectrum. We solved the equations of statistical equilibrium for CO and H$_2$ for 36,000 different values of $T$, $n_n$, and an optical depth parameter $N$ (see Appendix), and then interpolated the solutions thereby obtained to determine the H$_2$ level populations and CO line emissivities at every point in the shock simulation. Integrating over the entire volume in which the gas temperature exceeds the dust temperature, we obtained predictions for the CO line luminosity from the warm, shock-heated gas and for the column densities in various rotational states of H$_2$.

In solving the equations of statistical equilibrium for the CO level populations, we treated the effects of radiative trapping using the same escape probability approximation that was used to compute the total radiative cooling rates (see Appendix). We adopted the same molecular data for CO and H$_2$ that were used by KN96.

### 3. RESULTS

In the present paper, we have confined our attention to the fiducial shock parameters considered by S97 (i.e., to shocks with Mach number $M = 100$ and neutral Alfvénic Mach number $A = 10$) and to the case in which the preshock H$_2$ density is $10^4$ cm$^{-3}$. All our results apply to shocks that are initially plane-parallel and that propagate in an initially homogeneous medium.

We obtained results for three different geometries: two-dimensional simulations of perpendicular shocks ($\theta = \pi/2$), two-dimensional simulations of oblique shocks ($\theta = \pi/6$), and three-dimensional simulations of perpendicular shocks. We also considered two different shock velocities: $v_s = 20$ km s$^{-1}$ and $v_s = 40$ km s$^{-1}$. Note, however, that the assumed neutral Alfvénic Mach number was 10 for both shock velocities, so the assumed preshock magnetic fields were different in the two cases: the assumed field strengths were 0.45 and 0.89 mG, respectively, for the 20 and the 40 km s$^{-1}$ shocks.

For each of the six resultant cases, we have computed the gas temperatures in the shocked region and the predicted CO and H$_2$ line emissions for two snapshots: (1) the initial state, in which the flow solution is the steady state solution of the one-dimensional dynamical equations, and (2) after the Wardle instability has reached saturation. A crucial feature of the results presented here is that an identical method (with identical microphysical assumptions and approximations) was used for each snapshot; thus, any differences in the predicted gas temperatures and line emissions for the second snapshot are directly attributable to the effects of the Wardle instability.

#### 3.1. Gas Temperatures

In Figure 1a (Plate 2), we present temperature and density maps for neutral species (i.e., molecular hydrogen) in two-dimensional simulations of a perpendicular shock of velocity $v_s = 20$ km s$^{-1}$. The upper panel shows the results for a steady shock (initial condition; elapsed time $t = 0$), and the lower panel shows the results after the Wardle instability has saturated ($t = 7t_{flow} = 3000$ yr). The left side of each panel shows the H$_2$ density and the right side shows the H$_2$ temperature. In each case, the preshock gas is located at the top of the panel and flows downward, relative to the shock.

In the initial state, the temperature structure agrees well with the earlier one-dimensional steady shock calculations of KN96. The agreement is not exact, however, because in the present study we have slightly simplified the detailed treatment of the microphysics undertaken by KN96 (see § 2.2). In particular, we have (1) neglected the advection term in the energy equation, (2) assumed a constant water abundance throughout the computational volume, and (3) neglected the dependence of the collisional coupling constant $x$ upon the ion-neutral drift velocity. Despite these approximations, the differences between the temperature structure in the initial state and that predicted by KN96 are small: the peak temperatures in the shocked region are, respectively, 1160 and 1150 K, and the predicted fluxes for far-infrared CO lines ($J = 14–13$ and above) differ at most by 30% (see § 3.2 below).

As discussed by S97, the Wardle instability leads to the formation of dense sheets in which the neutral particle density is very much larger than the postshock density in a one-dimensional steady shock. These dense sheets are readily apparent on the left side of the lower panel. The right side of the panel, however, shows that the dense sheets are very cold; thus, they are not expected to contribute to the high-temperature line emission that is characteristic of interstellar shocks. Outside the dense sheets, the effect of the instability upon the temperature structure is modest. Hot spots form where the incoming gas hits the edges of the dense sheets, and the peak temperature increases from 1160 K in the initial state to 1420 K after the Wardle instability has saturated, but the volume occupied by the hot spots is rather small. These results are discussed further in § 4 below.

Analogous results are presented for a two-dimensional simulation of an oblique shock in Figure 1b (Plate 3), and for a three-dimensional simulation of a perpendicular shock in Figure 2 (Plate 4). As in the two-dimensional simulation of a perpendicular shock, the effects upon the high-temperature part of the shock are modest. In the three-dimensional simulations, however, additional instability modes are apparent. In particular, the Wardle instability results in filaments oriented along the magnetic field and perpendicular to the cold dense sheets in which the heating rate (see S97) and the temperature are enhanced. The possible implications of this result for the generation of water maser emission in C-type shocks is discussed in § 4.

We have found an alternative representation of the temperature structure to be a very useful supplement to Figures 1a, 1b, and 2. In Figures 3a and 3b we have characterized the temperature distribution in the shocked gas by plotting the column density of H$_2$ that is hotter than a given temperature, $T$, as a function of that temperature. The column density is an average along the shock propagation direction. The results shown in Figure 3a apply to perpendicular shocks of velocity 20 and 40 km s$^{-1}$, while those in Figure 3b apply to oblique shocks. Dotted lines show the temperature distributions for the initial state, corresponding to the steady state solution for a one-dimensional shock. Solid lines show the results of two-dimensional simulations after the Wardle instability has saturated, and the dashed line
Fig. 3.—(a) Temperature distribution in the neutral fluid. The column density of shock-heated H$_2$ warmer than gas temperature $T$ is plotted as a function of $T$ for a perpendicular shocks ($\theta = \pi/2$) with Alfvén Mach number $A = 10$, preshock H$_2$ density $10^5$ cm$^{-3}$, and shock velocity $v_s = 20$ and 40 km s$^{-1}$. Dotted lines show the results for a steady shock (initial condition; elapsed time $t = 0$), and solid lines show the results of two-dimensional simulations in which the Wardle instability has saturated ($t = 7t_{\text{flow}} = 3000$ yr). Dashed lines apply to three-dimensional simulations in which the Wardle instability has saturated ($t = 9.6t_{\text{flow}} = 4100$ yr). (b) Same as for $a$ but for an oblique shock (two-dimensional simulations only).

(Fig. 3a only) shows the result of a three-dimensional simulation.

Figures 3a and 3b show that although the Wardle instability increases the maximum temperature in the postshock gas, only a small H$_2$ column density ($\sim 10^{19}$ cm$^{-2}$) is present at temperatures higher than those achieved in an steady state one-dimensional shock. In fact, the temperature of the warmest $\sim 10^{20}$ cm$^{-2}$ of H$_2$ is typically somewhat

Fig. 4.—(a) CO rotational emission from the shock-heated gas. The CO line fluxes are plotted as a function of the quantum number of the upper state, $J_u$. Results are for perpendicular shocks ($\theta = \pi/2$) with Alfvén Mach number $A = 10$, preshock H$_2$ density $10^5$ cm$^{-3}$, and shock velocity $v_s = 20$ and 40 km s$^{-1}$. Dotted lines show the results for a steady shock (initial condition; elapsed time $t = 0$), and solid lines show the results of two-dimensional simulations in which the Wardle instability has saturated ($t = 7t_{\text{flow}} = 3000$ yr). Dashed lines refer to three-dimensional simulations in which the Wardle instability has saturated ($t = 9.6t_{\text{flow}} = 4100$ yr). (b) Same as for $a$ but for an oblique shock (two-dimensional simulations only).
factor 2. Density of gas warmer than 300 K is increased by up to a small factor of 2.5, and in most cases the effect of the Wardle instability is considerably smaller than that. Since the effects of the Wardle instability upon the line luminosities were found to be (1) somewhat larger in oblique shocks than in perpendicular shocks and (2) slightly larger in three-dimensional simulations of perpendicular shocks than in two-dimensional simulations, we speculate that effects slightly larger than a factor of 2.5 are likely for three-dimensional simulations of oblique shocks. Future numerical simulations of three-dimensional oblique shocks will be needed to test this speculation; unfortunately, such computations require significantly more computational resources than any of the calculations carried out thus far by S97.

4. DISCUSSION

The primary result of this study is a negative one. At least for the range of parameters and geometries that we have considered here, we conclude that the Wardle instability does not imprint any clear observational signature upon the shock-excited CO and H$_2$ line spectra, given the comparable or larger effects that may result from (1) uncertainties in the microphysics of ion-neutral coupling and molecular excitation, (2) observational errors in the measurement of line fluxes, (3) the superposition of shocks of different velocities in the telescope beam, (4) inhomogeneities in the preshock conditions, and (5) the effects of the Wardle instability upon the shock-excited emission-line fluxes are quite modest and follow the behavior that would be expected from the temperature distributions plotted in Figures 3a and 3b.

3.2. Line Emission

Figures 4a and 4b show the expected CO emission for perpendicular and oblique shocks. Here the fluxes (i.e., the luminosities per unit surface area of the shock) for pure rotational lines of CO are plotted as a function of the rotational quantum number of the upper state of the transition, J$_u$. As before, the dotted lines show results for the initial snapshot, corresponding to the steady state solution for a one-dimensional shock. Solid lines show the results of two-dimensional simulations after the Wardle instability has saturated, and the dashed line (Fig. 4a only) shows the result of a three-dimensional simulation. Analogous results for H$_2$ rotational states are shown in Figures 5a and 5b, in which the column densities of H$_2$ in different states of rotational excitation, N$_J$, have been divided by the statistical weight, g$_J$, and plotted as a function of the energy of the rotational state, E$_J$ (in temperature units E$_J$/k). The meaning of the different line types is identical to that adopted in Figures 3a, 3b, 4a, and 4b. Since the quadrupole-allowed rotational transitions of H$_2$ are all optically thin, the results shown in Figures 5a and 5b specify the predicted H$_2$ rotational line strengths unambiguously.

In addition to the C-type shocks from which we have computed the line emission, S97 found that the Wardle instability may also give rise to J-type shocks (in a region that has been advected out of the computational volume used here). Although interesting in their own right, these J-type shocks are expected to make a negligible contribution to the total line emission because the shock velocity is small (\( \sim 1 \) km s$^{-1}$) and the energy dissipated within the J-type shocks amounts to less than 1% of that dissipated in the C-type shocks modeled here.

Figures 4a, 4b, 5a, and 5b show that the effects of the Wardle instability upon the shock-excited emission-line fluxes are quite modest and follow the behavior that would be expected from the temperature distributions plotted in Figures 3a and 3b.

At least for the cases that we have considered thus far, the H$_2$ and high-J CO line strengths are changed at most by a factor of 2.5, and in most cases the effect of the Wardle instability is considerably smaller than that.
density, and so forth. This result provides a justification for the use of one-dimensional steady shock models in the interpretation of observations of shock-excited line emission.

The rather surprising negative result that we have obtained requires some explanation, particularly in light of the profound effects upon postshock density structure that result from the Wardle instability. Figure 6 provides a graphical explanation of why the effects of the Wardle instability upon the shocked-excited line emission are relatively small. Here we plot the ion-neutral drift speed, \( v_d = |v_i - v_n| \), and the compression of the ionized fluid, \( C_i = \rho_i/\rho_0 \), as a function of displacement along the shock propagation direction. The heating rate per neutral particle is proportional to \( \rho_0^2 C_i \). Results are shown for a one-dimensional steady state shock (dotted line), and for a trajectory that intersects the tip of a dense sheet within a two-dimensional perpendicular shock in which the Wardle instability has saturated. In each case, the displacement is expressed relative to the location at which the temperature peaks, negative values referring to the preshock side. As Figure 6 shows, the final compression of the ions is increased enormously by the Wardle instability. However, by the time that the neutral particles reach the region of enhanced ion density, the drift velocity has dropped significantly. Most of the shock-excited line emission arises upstream of the region where the effects of the Wardle instability are felt. Indeed, the maximum heating rate per neutral particle is enhanced only by a relatively small factor even along trajectories that hit the dense sheets edge-on.

Despite the lack of an obvious signature in the spectrum of shock-excited line emission, our three-dimensional simulation of the Wardle instability in a perpendicular shock does predict a distinctive phenomenon: the formation of hot filaments that are oriented along the magnetic field and perpendicularly to the cold dense sheets. At least in the perpendicular shock case that we have considered thus far, these filaments are aligned perpendicularly to the direction of the maximum velocity gradient, yielding a long, coherent path length for the amplification of maser radiation. This result is potentially important because C-type shocks in very dense molecular gas have been proposed (Neufeld & Melnick 1990; KN96) as a likely source for the water maser emission observed in regions of active star formation. Further study is needed to determine what maser emission properties would be expected from these filaments and whether the filaments are apparent in three-dimensional simulations of oblique shocks.

5. SUMMARY

1. We have modeled the neutral gas temperature and the CO and \( \mathrm{H}_2 \) rotational line emissivities in unstable C-type shocks using numerical simulations of the Wardle instability that were carried out by Stone (1997) and described in a companion paper. The results apply to shocks that are initially plane-parallel and that propagate in an initially homogeneous medium.

2. In modeling the gas temperature within the region of shocked gas, we included ion-neutral frictional heating; compressional heating; radiative cooling due to rotational and ro-vibrational transitions of the molecules CO, \( \mathrm{H}_2\mathrm{O} \), and \( \mathrm{H}_2 \); and gas-grain collisional cooling. In modeling the line emission from CO and \( \mathrm{H}_2 \), we solved the equations of statistical equilibrium for the populations in excited rotational states using an escape probability method to treat the effects of radiative trapping in the CO transitions.

3. We obtained results for the gas temperature distribution in—and \( \mathrm{H}_2 \) and CO line emission from—shocks of neutral Alfvénic Mach number 10 and velocity 20 or 40 km s\(^{-1}\), in which the Wardle instability has saturated. Both two- and three-dimensional simulations were carried out for shocks in which the preshock magnetic field is perpendicular to the shock propagation direction; and a two-dimensional simulation was carried out for the case in which the magnetic field is oriented obliquely with respect to the shock propagation direction.

4. Although the Wardle instability profoundly affects the density structure behind C-type shocks, most of the shock-excited molecular line emission is generated upstream of the region where the strongest effects of the instability are felt. Thus, the Wardle instability has a relatively small effect upon the overall gas temperature distribution in, and the emission line spectrum from, C-type shocks, at least for the cases that we have considered. In none of these cases did any of the predicted emission-line luminosities change by more than a factor of 2.5, and in most cases the effects of the instability were significantly smaller than that.

Slightly larger changes in the line luminosities seem likely for three-dimensional simulations of oblique shocks (see § 3.2 above), although such simulations have yet to be carried out and lie beyond the scope of this paper.

Given the comparable or larger uncertainties in the microphysics, observational errors in the measurement of line fluxes, and uncertainties in the physical parameters for real interstellar regions where shocks are present, we conclude that Wardle instability does not imprint any clear

![Figure 6](image-url)

**Figure 6.** Ion-neutral drift speed, \( v_d = |v_i - v_n| \), and the compression of the ionized fluid, \( C_i = \rho_i/\rho_0 \), as a function of displacement along the shock propagation direction. The heating rate per neutral particle is proportional to \( \rho_0^2 C_i \). Results are shown for a one-dimensional steady state shock (dotted line), and for a trajectory that intersects the tip of a dense sheet within a two-dimensional perpendicular shock in which the Wardle instability has saturated. In each case, the displacement is expressed relative to the location at which the temperature peaks, negative values referring to the preshock side.
observational signature on the shock-excited CO and H$_2$ line strengths. This result provides a justification for the use of one-dimensional steady-shock models in the interpretation of observations of shock-excited line emission.

5. Our three-dimensional simulations of perpendicular shocks revealed the presence of warm filamentary structures that are aligned along the magnetic field, a result of possible relevance to models of water maser emission from C-type shocks.

6. The results presented in this paper apply only to a rather limited region of parameter space, and the conclusions of this study may not be generally applicable throughout the parameter space of astrophysical interest. In particular, future work will be needed to investigate the effects of the Wardle instability in three-dimensional simulations of oblique shocks, in bow shocks, and in shocks that propagate in inhomogeneous or in weakly magnetized media.

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APPENDIX

TREATMENT OF OPTICAL DEPTH EFFECTS

The volume cooling rate $\Lambda$ (§2.2) depends not only on the temperature but also on the gas density and the optical depth in CO and H$_2$O line transitions that contribute to the cooling. NK93 used an escape probability method to treat optical depth effects, making use of the Sobolev approximation to model the escape of radiation from a one-dimensional fluid flow in which the velocity gradient is large. They introduced an optical depth parameter $\tilde{N} \equiv n_v/|dv/\nu ds|$ to characterize the importance of optical depth effects. In the two- and three-dimensional velocity fields considered in the present paper, this expression for $\tilde{N}$ must be generalized by the replacement of $|dv/\nu ds|$ with an appropriate expression involving the velocity gradient tensor, $\partial v/\nu \partial x_j$.

In general, the optical depth parameter is inversely proportional to $\langle |dv/\nu ds/\rangle$, where $|dv/\nu ds|$ is the line-of-sight neutral velocity gradient along a given ray, and angle brackets denote an angle average. For a one-dimensional flow, $\langle |dv/\nu ds/\rangle = |dv/\nu ds/\nu ds|/3$. For a three-dimensional flow, we may write $\langle |dv/\nu ds/\rangle = (|e_1| + |e_2| + |e_3|)/3$, where $e_1$, $e_2$, and $e_3$ are the eigenvalues of the symmetrized velocity gradient tensor, $(\partial v/\nu \partial x_j + \partial v/\nu \partial x_i)/2$, in descending order of magnitude, and $f$ is a number of order unity that depends on $e_2/e_1$ and $e_3/e_1$. Thus the expression for the optical depth parameter becomes $\tilde{N} \equiv n_v/f(|e_1| + |e_2| + |e_3|)$. By direct integration of $dv/\nu ds$ over all angles, we have evaluated $f$ in the limit where $e_2/e_1 \leq 1$ (a limit that applies exactly in two-dimensional flows and to a good approximation in the three-dimensional flows of present interest). We find that $f$ always lies in the range $2/\pi$ to 1 and is given by

$$f = 1 - \frac{e_2/e_1 \geq 0}{1 + R} ,$$

$$f = 1 - \frac{R - 1}{R} + \frac{4}{\pi} \left( \frac{R}{1 + R^2} - \frac{1 - R}{1 + R} \tan^{-1} R \right) \left( \frac{e_2}{e_1} \leq 0 \right) ,$$

where $R = -e_2/e_1$.

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Fig. 1.—(a) Temperature and density maps for molecular hydrogen in two-dimensional simulations of a perpendicular shock ($\theta = \pi/2$) of velocity $v_n = 20$ km s$^{-1}$, Alfvén Mach number $A = 10$, and preshock $H_2$ density $10^5$ cm$^{-3}$. The upper panel shows the results for a steady shock (initial condition; elapsed time $t = 0$), and the lower panel shows the results after the Wardle instability has saturated ($t = 7t_{\text{flow}} = 3000$ yr). The left side of each panel shows the $H_2$ density, and the right side shows the $H_2$ temperature. In each case the preshock gas is located at the top of the panel and flows downwards relative to the shock. (b) Same as for 1a but for an oblique shock ($t = 3.4t_{\text{flow}} = 1640$ yr in lower panel).

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PLATE 2
t = 0  (Steady Shock Solution)

```
| Distance / L_s |
|----------------|
| 3.0            |
| 2.0            |
| 1.0            |
| 0.0            |
| -1.0           |
| -2.0           |
| -3.0           |
```

Neutral Density

```
| log n(H_2) [cm^{-3}] |
|-----------------------|
| 7.1                   |
| 4.9                   |
```

Temperature

```
| Temperature [K] |
|-----------------|
| 1913            |
| 23.8            |
```

```
| Distance / L_s |
|----------------|
| 3.0            |
| 2.0            |
| 1.0            |
| 0.0            |
| -1.0           |
| -2.0           |
| -3.0           |
```

Neutral Density

```
| log n(H_2) [cm^{-3}] |
|-----------------------|
| 7.1                   |
| 4.9                   |
```

Temperature

```
| Temperature [K] |
|-----------------|
| 1913            |
| 23.8            |
```

Fig. 1b

Neufeld & Stone (see 487, 285)

Plate 3
Fig. 2.—Temperature map for molecular hydrogen in three-dimensional simulations of a perpendicular shock ($\theta = \pi/2$) of velocity $v = 20$ km s$^{-1}$, Alfvén Mach number $M = 10$, and preshock H$_2$ density $10^5$ cm$^{-3}$. The figure shows the temperature structure after the Wardle instability has saturated ($t = 9.6t_{\text{turn}} = 4100$ yr).

Neufeld & Stone (see 487, 285)