The Riddle of Polarization
in $B \to VV$ Transitions

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Abstract
Measurements of polarization fractions in $B \to VV$ transitions, with $V$ a light vector meson, show that the longitudinal amplitude dominates in $B^0 \to \rho^+\rho^-$, $B^+ \to \rho^+\rho^0$, and $B^+ \to \rho^0K^{*+}$ decays and not in the penguin induced decays $B^0 \to \phi K^{*0}$, $B^+ \to \phi K^{*+}$. We study the effect of rescattering mediated by charmed resonances, finding that in $B \to \phi K^*$ it can be responsible of the suppression of the longitudinal amplitude. For the decay $B \to \rho K^*$ we find that the longitudinal fraction cannot be too large without invoking new effects.


\section{Introduction}

An important result obtained by Belle and BaBar Collaborations is the measurement of the decay widths and of the polarization fractions of several B decays to two light vector mesons \cite{1, 2, 3, 4}. The branching fractions measured by the two Collaborations are collected in Table 1 together with the averages. Together with these data one should collect the upper bound \( B(B^0 \rightarrow \rho^0 \rho^0) \leq 2.1 \times 10^{-6} \) from BaBar \cite{2}. Through the analysis of angular distributions, the polarization fractions of the final states have been measured as reported in Table 2. In the decay modes \( B^0 \rightarrow \rho^+ \rho^- \) and \( B^+ \rightarrow \rho^0 \rho^+ \), \( \rho^0 K^{*+} \) the final states are essentially in longitudinal configuration, with a larger uncertainty for \( B^+ \rightarrow \rho^0 K^{*+} \); on the contrary, in both the observed \( B \rightarrow \phi K^{*} \) transitions the longitudinal amplitude does not dominate, providing nearly 50% of the rate.

\begin{table}[h]
\begin{center}
\begin{tabular}{lcc}
\hline
\textbf{Mode} & \textbf{Belle} \cite{1} & \textbf{BaBar} \cite{2} \\
\hline
\( B^+ \rightarrow \phi K^{**} \) & \((6.7^{+2.4}_{-1.0} \pm 0.7) \times 10^{-6}\) & \((12.7^{+2.2}_{-2.0} \pm 1.1) \times 10^{-6}\) & \((9.5 \pm 1.7) \times 10^{-6}\) \\
\hline
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\begin{center}
\begin{tabular}{lcc}
\hline
\textbf{Mode} & \textbf{Belle} \cite{3} & \textbf{Average} \\
\hline
\( B^+ \rightarrow \rho^0 K^{*+} \) & \((10.6^{+3.6}_{-2.6} \pm 2.4) \times 10^{-6}\) & \(26.2 \pm 6.2) \times 10^{-6}\) \\
\hline
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\begin{table}[h]
\begin{center}
\begin{tabular}{lcc}
\hline
\textbf{Mode} & \textbf{Belle} \cite{3} & \textbf{BaBar} \cite{2, 4} \\
\hline
\( B^+ \rightarrow \rho^0 \rho^+ \) & \((31.7 \pm 7.1^{+3.8}_{-6.7}) \times 10^{-6}\) & \((22.5^{+5.7}_{-5.4} \pm 5.8) \times 10^{-6}\) \\
\hline
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\begin{center}
\begin{tabular}{lcc}
\hline
\textbf{Mode} & \textbf{Belle} \cite{3} & \textbf{Average} \\
\hline
\( B^0 \rightarrow \rho^+ \rho^- \) & \((25^{+5}_{-6} \pm 5) \times 10^{-6}\) & \(25^{+5}_{-6} \pm 5) \times 10^{-6}\) \\
\hline
\end{tabular}
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There are reasons to expect that the light VV final state should be mainly longitudinally polarized, see, e.g., the discussion in \cite{6}. In the following we summarize the arguments, which essentially rely on factorization and on the infinite heavy quark mass limit. Invoking such arguments, the deviation observed in \( B \rightarrow \phi K^{*} \) could be interpreted as a signal of new physics \cite{7}. A more orthodox interpretation \cite{6}, in the framework of QCD improved factorization \cite{8}, relies on the observation that (logarithmically divergent) annihilation diagrams can modify the polarization amplitudes in \( B \rightarrow \phi K^{*} \), producing fractions in agreement with observation.

In this note we wish to address another effect that potentially changes the result in the penguin induced \( B \rightarrow \phi K^{*} \) decay without affecting the observed \( B \rightarrow \rho \rho \) transition: rescattering of intermediate charm states. Such effects, studied long ago in \( B \rightarrow K \pi \) transitions \cite{9} and investigated recently in other \( B \rightarrow P P \) and \( VP \) decays \cite{10} as well...
as in factorization forbidden B transitions to charmonium final states \[11\], can invalidate the arguments on the basis of which the dominance of the longitudinal configuration is argued.

We discuss factorization and its consequences in Section 2 and the analysis of rescattering effects for \(B^0 \rightarrow \phi K^{*0}\) in Section 3. At the end we discuss a few consequences.

\[\begin{array}{|c|c|c|c|c|}
\hline
\text{Mode} & \text{Pol. fraction} & \text{Belle [1]} & \text{BaBar [2]} & \text{Average} \\
\hline
B^+ \rightarrow \phi K^{*+} & \Gamma_L/\Gamma & 0.46 \pm 0.12 \pm 0.03 & & \\
\hline
B^0 \rightarrow \phi K^{*0} & \Gamma_L/\Gamma & 0.43 \pm 0.09 \pm 0.04 & 0.65 \pm 0.07 \pm 0.02 & 0.58 \pm 0.06 \ (0.52 \pm 0.07 \pm 0.02) \\
\hline
B^0 \rightarrow \phi K^{*0} & \Gamma_L/\Gamma & 0.41 \pm 0.10 \pm 0.02 & (0.27 \pm 0.07 \pm 0.02) & \\
\hline
\end{array}\]

Table 2: Polarization fractions in \(B \rightarrow VV\) transitions. The BaBar results reported in brakets are preliminary data quoted in ref. [5].

2 Polarization in factorization-based approaches

The decay \(B^0 \rightarrow \phi K^{*0}\) is described by the amplitude

\[
\mathcal{A}(B^0(p) \rightarrow \phi(q, \epsilon)K^{*0}(p', \eta)) = A_0 \epsilon^* \cdot \eta^* + A_2 (\epsilon^* \cdot p)(\eta^* \cdot q) + i A_1 \epsilon^\alpha \beta \gamma \delta \epsilon^*_\alpha \eta^*_\beta p_\gamma p'_\delta \ 
\]

with \(\epsilon(q, \lambda)\) and \(\eta(p', \lambda)\) the \(\phi\) and \(K^*\) polarization vectors, respectively, with \(\lambda = 0, \pm 1\) the three helicities. Since the decaying \(B\) meson is spinless, the final vector mesons share the same helicity. \(A_0\) and \(A_2\) are associated to the S- and D-wave decay, respectively, and \(A_1\) to the P-wave transition.

The three helicity amplitudes \(A_L\) and \(A_\pm\) can be written in terms of \(A_{0,1,2}\):

\[
A_L = -\frac{1}{M_\phi M_{K^*}}[(p \cdot p' - M_{K^*}^2)A_0 + M_B^2 |\vec{p}'|^2 A_2] \\
A_\pm = -A_0 \mp M_B |\vec{p}'| A_1
\]
in the transversity basis, the transverse amplitudes
\[ A_{\parallel} = \frac{A_+ + A_-}{\sqrt{2}} = -\sqrt{2}A_0 \]
\[ A_\perp = \frac{A_+ - A_-}{\sqrt{2}} = -\sqrt{2}M_B|\vec{p}'|A_1 \]
can also be defined, with \(|\vec{p}'| = \lambda^\frac{1}{2}(M_B^2, M_{K^*}^2, M_\phi^2)/2M_B \) (\(\lambda\) the triangular function) the common \(\phi\) and \(K^*\) three-momentum in the rest frame of the decaying B-meson. In terms of such amplitudes the expression of the decay rate is simply:
\[ \Gamma = \frac{|\vec{p}'|}{8\pi M_B^2} (|A_L|^2 + |A_{\parallel}|^2 + |A_\perp|^2) \]
while the three polarization fractions are given by
\[ f_L = \frac{\Gamma_L}{\Gamma} = \frac{|A_L|^2}{|A_L|^2 + |A_{\parallel}|^2 + |A_\perp|^2} \]
\[ f_{\parallel} = \frac{\Gamma_{\parallel}}{\Gamma} = \frac{|A_{\parallel}|^2}{|A_L|^2 + |A_{\parallel}|^2 + |A_\perp|^2} \]
\[ f_\perp = \frac{\Gamma_\perp}{\Gamma} = \frac{|A_\perp|^2}{|A_L|^2 + |A_{\parallel}|^2 + |A_\perp|^2} \]

In order to compute the amplitude eq.(1), we consider the effective weak Hamiltonian inducing the \(\bar{b} \to \bar{s}s\bar{s}\) transitions, which can be written as
\[ H_W = \frac{G_F}{\sqrt{2}}(-V_{tb}^* V_{ts})(\sum_{i=3}^{10} c_i O_i + c_{7\gamma} O_{7\gamma} + c_{8g} O_{8g}) \]
with the operators
\[ O_3 = (\bar{b}_\alpha s_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q_\beta)_{V-A} \]
\[ O_4 = (\bar{b}_\beta s_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q_\beta)_{V-A} \]
\[ O_5 = (\bar{b}_\alpha s_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q_\beta)_{V+A} \]
\[ O_6 = (\bar{b}_\beta s_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q_\beta)_{V+A} \]
\[ O_7 = \frac{3}{2}(\bar{b}_\alpha s_\alpha)_{V-A} \sum_{q'} c_{q'}(\bar{q}'_\beta q_\beta)_{V+A} \]
\[ O_8 = \frac{3}{2}(\bar{b}_\beta s_\alpha)_{V-A} \sum_{q'} c_{q'}(\bar{q}'_\alpha q_\beta)_{V+A} \]
\[ O_9 = \frac{3}{2} \langle \bar{b} \sigma \gamma_5 q \rangle_{V-A} \sum_{q'} e_{q'} \langle \bar{q}' q' \rangle_{V-A} \]
\[ O_{10} = \frac{3}{2} \langle \bar{b} \sigma \gamma_5 q \rangle_{V-A} \sum_{q'} e_{q'} \langle \bar{q}' q' \rangle_{V-A} \]

(\alpha, \beta \text{ are colour indices and } \langle \bar{q} q \rangle_{V+A} = \bar{q} \gamma^\mu (1 \mp \gamma_5) q). \ O_{3-6} \text{ are gluon penguin operators, } O_{7-10} \text{ electro-weak penguin operators, } O_{7,8} = \frac{\alpha}{s m_b} (1 + \gamma_5) T^a s G_{\mu\nu}^a, \text{ with } F_{\mu\nu} \text{ and } G_{\mu\nu}^a \text{ the electromagnetic and the gluon field strength, respectively; } c_i\gamma_{1,7,8,9}(p) \text{ are the Wilson coefficients.}

The amplitude \( A(B^0 \to \phi K^{*0}) \) obtained from (6) admits a factorized form

\[ A_{\text{fact}}(B^0 \to \phi K^{*0}) = \frac{G_F}{\sqrt{2}} (\bar{V}_{tb} V_{ts} a_W) \langle K^{*0}(p', \eta) \rangle \langle \bar{b} s \rangle_{V-A} \langle B^0(p) \rangle \langle \phi(q, \epsilon) | \langle \bar{s} s \rangle | 0 \rangle \]

with \( a_W = a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \), \( a_i = c_i + \frac{c_{i+1}}{N_c} \) for \( i = 3, 5, 7, 9 \) and \( a_i = c_i + \frac{c_{i-1}}{N_c} \) for \( i = 4, 10 \) \((N_c \text{ is the number of colours})\). This formula presents the drawbacks of naive factorization, namely there is not a compensation of the scale dependence between Wilson coefficients and operator matrix elements. However, it allows us to immediately write down the polarization fractions, once the \( B \to K^* \) matrix element has been expressed in terms of form factors \(^1\), and the \( \phi \) meson leptonic constant has been introduced:

\[ \langle \phi(q, \epsilon) | \langle \bar{s} s \rangle | 0 \rangle = f_\phi M_\phi \epsilon^\mu \]

\[ \langle K^*(p', \eta) \rangle \langle \bar{b} \gamma_\mu (1 - \gamma_5) s \rangle B(p) \rangle = -i \epsilon_{\mu\nu\rho\sigma} \eta^{\nu\rho} p^{\mu} p^{\sigma} \frac{2V}{M_B + M_{K^*}} - [(M_B + M_{K^*}) A_1 \eta^{\mu}_e \]

\[ = - \frac{A_2}{M_B + M_{K^*}} (\eta^\mu \cdot p) (p + p')_\mu - 2M_{K^*} (A_3 - A_0) \frac{q^2}{q^2} (\eta^\mu \cdot p) q_\mu \],

with the form factors \( V, A_1, A_2, A_3 \) and \( A_0 \) functions of \( q^2 \). From (8-10) it is easy to write down the polarization amplitudes and check that, for large values of \( M_B \),

\[ A_L \propto M_B^2 [(A_1(M_\phi^2) - A_2(M_\phi^2)) + \frac{M_{K^*}}{M_B} (A_1(M_\phi^2) + A_2(M_\phi^2))] \]
\[ A_{||} \propto M_B A_1(M_\phi^2) \]
\[ A_{\perp} \propto M_B V(M_\phi^2) \]

expressions which determine the behaviour of the three amplitudes once the parametric dependence on the heavy quark mass of the form factors close to the maximum recoil

\(^1\text{For the } B \to K^* \text{ and } B \to D^* \text{ matrix elements eqs.}(10) \text{ and } (18) \text{ we use the same phase convention.}\)
point has been established. In the limit \( M_B \to \infty \) and for \( q^2 = 0 \) such a dependence has been investigated [12] with the result that the three form factors \( V, A_1 \) and \( A_2 \) should be equal: \( A_2/A_1 = V/A_1 = 1 \). One therefore expects:

\[
\frac{\Gamma_L}{\Gamma} \simeq 1 + \mathcal{O}(\frac{1}{M_B}) \quad , \quad \frac{\Gamma_{\parallel}}{\Gamma_{\perp}} \simeq 1
\]  

regardless, in this scheme, of the Wilson and CKM coefficients. Assuming generalized factorization, with the substitution of the Wilson coefficients \( a_i \) with effective parameters \( a_{i}^{eff} \), it is eventually possible to reconcile the branching ratio with the experimental measurement, but not to modify the polarization fractions, since the dependence on the \( a_i \) cancels out in the ratios. Therefore, in order to explain the small ratio \( \Gamma_L/\Gamma \) within the Standard Model one has to look either at the finite mass corrections, or at effects beyond factorization.

For finite heavy quark mass, one can compare the experimental result for the polarization fractions in \( B^0 \to \phi K^{*0} \) decays (Table 2) with the predictions of various form factor models [13, 14, 15, 16]. As shown in fig. 1, in many models the ratios \( A_2/A_1 \) and \( V/A_1 \) deviate from the asymptotic prediction, suggesting that the regime of finite \( M_B \) does not fully coincide with the asymptotic regime. In one case there is a marginal agreement between the form factor model and data. However, the indication of effects beyond naive and generalized factorization is clear.

3 Rescattering effects

If one considers the possibility of rescattering effects, there are other terms in the effective weak hamiltonian that can induce the transition \( B^0 \to \phi K^{*0} \). Processes that should be the most relevant ones are \( \bar{b} \to c\bar{c}s \to s\bar{s}s \). Such processes can give sizeable contribution to the penguin amplitudes obtained from (6) since they involve Wilson coefficients of \( \mathcal{O}(1) \) (that multiply current-current quark operators), while the Wilson coefficients in penguin \( \bar{b} \to s\bar{s}s \) operators are smaller (\( \mathcal{O}(10^{-2}) \)). On the other hand, there is not a CKM suppression in such processes, since \( |V_{tb}^*V_{ts}| \) and \( |V_{cb}^*V_{cs}| \) are nearly equal. An example of processes of this type is depicted in fig. 2, where a sample of intermediate charm mesons is shown. As far as the polarization of the final state is concerned, one has to notice that different intermediate states in fig. 2 contribute to different polarization amplitudes, so that the longitudinal as well as the transverse amplitudes can be modified. For example, considering only intermediate pseudoscalar and vector charmed mesons coming from the \( B \) meson vertex, there are eight diagrams of the kind depicted in fig. (2). Intermediate states
Figure 1: Ratios of $B \to K^*$ form factors: $V(M_{\phi}^2)/A_1(M_{\phi}^2)$ versus $A_2(M_{\phi}^2)/A_1(M_{\phi}^2)$. The continuous lines correspond to the (one, two and three-$\sigma$) regions of the Belle data in Table 2 (a) and of the average of Belle and BaBar data (b) for $\Gamma_L/\Gamma$ and $\Gamma_\perp/\Gamma$ in $B^0 \to \phi K^{*0}$. The points correspond to different form factor models: QCDSR [13] (dot), LCSR [14] (triangle), MS [15] (square), BSW [16] (diamond).

comprising one vector and one pseudoscalar meson (four diagrams) only contribute to the $P-$wave transition and therefore to the amplitude $A_{\perp}$. On the other hand, intermediate states comprising two pseudoscalar mesons (two diagrams) only contribute to $A_L$ and $A_\parallel$, while intermediate states with two vector mesons (2 diagrams) contribute to the three polarization amplitudes $A_L$, $A_{\perp}$ and $A_\parallel$.

In order to estimate the contribution of diagrams of the type in fig.2 we can use a formalism that accounts for the heavy quark spin-flavour symmetries in hadrons containing a single heavy quark [17] and for the so called hidden gauge symmetry to describe their interaction with light vector mesons [18]. As well known, in the heavy quark limit, due to the decoupling of the heavy quark spin $|\vec{s}_Q|$ from the light degrees of freedom total angular momentum $|\vec{s}_t|$, it is possible to classify hadrons with a single heavy quark $Q$ in terms of
Figure 2: Rescattering diagrams contributing to $B \to \phi K^*$. The box represents a weak vertex, the dots strong couplings.

$s_\ell$. Mesons can be collected in doublets the members of which only differ for the relative orientation of $\vec{s}_Q$ and $\vec{s}_\ell$ [17]. The doublets with $J^P = (0^-, 1^-)$ corresponding to $s_\ell^P = \frac{1}{2}^-$ can be described by the effective fields

$$H_a = \frac{(1 + v)}{2}[P^*_a \gamma^\mu - P_a \gamma_5]$$

(13)

where $v$ is the meson four-velocity and $a$ is a light quark flavour index. The field $\overline{H}_a$ is defined as $\overline{H}_a = \gamma^0 H^1_a \gamma^0$; all the heavy field operators contain a factor $\sqrt{M_H}$ and have dimension $3/2$.

It is possible to formulate an effective Lagrange density for the low energy interactions of heavy mesons with light vector mesons [18]. The interaction term of such a Lagrangian reads as

$$\mathcal{L}_{HHV} = -i \beta \text{Tr}\{H_b (v^\mu \rho_\mu)_b \overline{H}_a\} + i \lambda \text{Tr}\{H_b (\sigma^{\mu\nu} F_{\mu\nu})_b \overline{H}_a\}.$$  

(14)

Light vector mesons are included in (14) through the fields $\rho = i\frac{g_{\rho}}{2} \hat{\rho}$ representing the low-lying vector octet:

$$\hat{\rho} = \begin{pmatrix}
\sqrt{\frac{1}{2}} \rho^0 + \sqrt{\frac{1}{6}} \omega_8 \\
\rho^- \\
K^{*-} \\
\sqrt{\frac{1}{2}} \rho^0 + \sqrt{\frac{1}{6}} \omega_8 \\
K^{*0} \\
-\sqrt{\frac{2}{3}} \omega_8
\end{pmatrix}$$

(15)

with $F_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + [\rho_\mu, \rho_\nu]$. Invoking the mixing $\omega_8 - \omega_0$ one gets the interaction term involving $\phi$. The interactions of heavy mesons with the light vector mesons are thus governed, in the heavy quark limit, by two couplings $\beta$ and $\lambda$. From light cone QCD sum
rules [19] as well as from vector mesons dominance arguments [10] one estimates $\beta \simeq 0.9$ and $\lambda \simeq 0.56 \text{ GeV}^{-1}$, while $g_{\nu}$ is fixed to $g_{V} = 5.6$ by the KSRF relation [20].

Using (14) it is easy to work out the matrix elements $D_{s}^{(s)}D^{(*)}K^{*}$ appearing in one of the vertices in fig.2:

\[
\langle D_{s}^{(*)} (p_{D} - p') K^{*0}(p') | D^{-}(p_{D} = M_{D}v_{D}) \rangle = \bar{\beta} \sqrt{M_{D}M_{D_{s}}} \langle v_{D} \cdot \eta \rangle
\]

\[
\langle D_{s}^{-} (p_{D} - p', \epsilon_{1}) K^{*0}(p', \eta) | D^{-}(p_{D}, \eta_{1}) \rangle = i\lambda \sqrt{M_{D}M_{D_{s}}} \epsilon^{\alpha\mu\beta} v_{D_{s}} \eta_{1} \epsilon_{1 \beta} \epsilon_{\alpha
\]

\[
\langle D_{s}^{-} (p_{D} - p', \epsilon_{1}) K^{*0}(p', \eta) | D^{-}(p_{D}, \eta_{1}) \rangle = -i\lambda \sqrt{M_{D}M_{D_{s}}} \epsilon^{\alpha\mu\beta} v_{D_{s}} \eta_{1} \epsilon_{1 \beta} \epsilon_{\alpha
\]

\[
+\bar{\lambda} \sqrt{M_{D}M_{D_{s}}} [(\eta_{1} \cdot \eta_{*})(\epsilon_{1} \cdot p') - (\eta_{1} \cdot p') (\epsilon_{1} \cdot \eta_{*})] \]

(16)

where $\bar{\beta} = \frac{2\beta g_{\nu}}{\sqrt{2}}$ and $\bar{\lambda} = \frac{4\lambda g_{\nu}}{\sqrt{2}}$. Matrix elements involving $\phi$ in the other vertex in fig.2 are obtained analogously.

As for the weak amplitude $B^{0} \rightarrow D_{s}^{(*)}+D^{(*)}-$, since there is empirical evidence that factorization reproduces the main experimental findings [21], we write it as

\[
\langle D_{s}^{(*)}+D^{(*)}- | H_{W} | B^{-} \rangle = \frac{G_{F}}{\sqrt{2}} V_{cb} V_{cs}^{*} a_{1} \langle D^{(*)}- | (V - A)^{\mu} | B^{0} \rangle \langle D_{s}^{(*)}+ | (V - A)^{\mu} | 0 \rangle
\]

(17)

with $a_{1} \simeq 1$. In the heavy quark limit the matrix elements in (17) involve the Isgur-Wise function [17]:

\[
\langle D^{-}(v') | V^{\mu} | B_{0}^{0}(v) \rangle = \sqrt{M_{B}M_{D}} \xi(v \cdot v')(v + v')^{\mu}
\]

\[
\langle D^{*-}(v', \epsilon) | V^{\mu} | B_{0}^{0}(v) \rangle = -i\sqrt{M_{B}M_{D^{*}}} \xi(v \cdot v') \epsilon_{\beta}^{*} \epsilon^{\alpha\beta\gamma\mu} v_{\alpha} v_{\gamma}
\]

(18)

\[
\langle D^{*-}(v', \epsilon) | A^{\mu} | B_{0}^{0}(v) \rangle = \sqrt{M_{B}M_{D^{*}}} \xi(v \cdot v') \epsilon_{\beta}^{*}[(1 + v \cdot v')g^{\beta\mu} - v^{\beta} v^{\mu}] ,
\]

$v$ and $v'$ being $B$ and $D^{(*)}$ four-velocities, $\epsilon$ the $D^{*}$ polarization vector and $\xi(v \cdot v')$ the Isgur-Wise form factor. As for the $D^{(*)}$ current-vacuum matrix elements defined as

\[
\langle 0 | \bar{q}_{a} \gamma^{\mu} \gamma_{5} c | D_{a}(v) \rangle = f_{D_{a}} M_{D_{a}} v^{\mu}
\]

\[
\langle 0 | \bar{q}_{a} \gamma^{\mu} c | D_{a}^{*}(v, \epsilon) \rangle = f_{D_{a}^{*}} M_{D_{a}^{*}} \epsilon^{\mu} ,
\]

(19)

they can be parameterized in the heavy quark limit in terms of a single quantity $f_{D_{a}} = f_{D_{a}^{*}}$.

Now, the estimate of the absorptive part of the rescattering diagrams in fig. 2

\[
\text{Im} A_{\text{resc}} = \frac{\lambda^{\frac{1}{2}}(M_{B}^{2}, M_{D_{s}^{(*)}}^{2}, M_{D_{s}^{(*)}}^{2})}{32\pi M_{B}^{2}} \int_{-1}^{1} dz A(B^{0} \rightarrow D_{s}^{(*)}+D^{(*)}-) A(D_{s}^{(*)}+D^{(*)}- \rightarrow \phi K^{*0})
\]

(20)
can be carried out. The integration variable \( z = \cos \theta \) is related to the angle between the three-momenta of \( \phi \) and of the emitted \( D_s^{(*)} \) from \( B \) vertex in fig.2. We use \( |V_{cb}| = 0.042, |V_{cs}| = 0.974 \) (the central values reported by the Particle Data Group [22]), \( f_{D_s} = f_{D_s} = 240 \text{ MeV} \) [23] and \( \xi(y) = \left( \frac{2}{1+y} \right)^2 \).

The couplings in (16) do not account for the off-shellness of the exchanged \( D_s^{(*)} \) mesons in fig.2. One can introduce form factors:

\[
g_i(t) = g_{i0} F(t)
\]

(21)

to account for the \( t \)-dependence of the couplings (the vertices in rescattering diagrams cannot be considered point-like since they do not involve elementary particles), \( g_{i0} \) being the on-shell couplings. However, the form factors are unknown. We use

\[
F(t) = \frac{\Lambda^2 - M_{D_s}^2}{\Lambda^2 - t}
\]

(22)

to satisfy QCD counting rules. We could vary the value of \( \Lambda \), considering the uncertainty from the form factor \( F(t) \) in the final numerical result. Instead, since the relative sign of rescattering and factorized amplitude is also unknown, as well as the role of diagrams involving excitations and the continuum, we fix \( \Lambda = 2.3 \text{ GeV} \) and analyze the sum

\[
A = A_{\text{fact}} + r A_{\text{resc}}
\]

(23)

varying the parameter \( r \) and approximating the long distance amplitude with eq.(20).

We compute the short-distance factorized amplitude using the \( B \to K^* \) form factors appearing in two extreme cases in fig.1, the model [13] and the model [14], with Wilson coefficients \( a_3 = 48 \times 10^{-4}, a_4 = (-439 - 77i) \times 10^{-4}, a_5 = -45 \times 10^{-4}, a_7 = (-0.5 - 1.3i) \times 10^{-4}, a_9 = (-94 - 1.3i)48 \times 10^{-4} \) and \( a_{10} = (-14 - 0.4i) \times 10^{-4} \), as computed in [24] for \( N_c = 3 \).

The result is depicted in fig.3. For the model [13], a contribution of the rescattering amplitude is in order to obtain the measured \( B \to \phi K^* \) branching fraction. Of the two possible values of the parameter \( r \) which reproduce the experimental rate, \( r \approx 0.08 \) and \( r \approx -0.3 \), the former allows us to simultaneously obtain a small longitudinal polarization fraction: \( \Gamma_L/\Gamma \approx 0.55 \), compatible with the measurements. The transverse polarization fractions turn out \( \Gamma_\parallel/\Gamma \approx 0.30 \) and \( \Gamma_\perp/\Gamma \approx 0.15 \). They are both consistent with measurement, but with the hierarchy \( \Gamma_\parallel/\Gamma > \Gamma_\perp/\Gamma \).

If we use the form factors in [14], for \( r = 0 \) the predicted rate exceeds the experimental datum, so that the rescattering contribution should be weighted by a negative \( r \) to
Figure 3: Dependence of the branching ratio and polarization fractions of $B^0 \to \phi K^{*0}$ on the long distance contribution. $B \to K^*$ form factors computed in [13] (left) and in [14] (right) are used in the short-distance amplitude. $r = 0$ corresponds to the absence of rescattering. The three curves in (b) correspond to $\Gamma_L/\Gamma$ (continuous curve), $\Gamma_\perp/\Gamma$ (dashed) and $\Gamma_\parallel/\Gamma$ (dot-dashed). The horizontal lines represent the experimental result in Table 1 for the branching ratio (a) and for $\Gamma_L/\Gamma$ (b).

reconcile the branching fraction; as depicted in fig.3, in such a region ($r \simeq -0.05$) the longitudinal fraction increases. However, this conclusion crucially depends on the value of the Wilson coefficients $a_3 - a_{10}$ used as an input in the evaluation of the short-distance amplitude. As shown in [24], for example, $a_4$ varies from $-402 - 72i$ to $-511 - 87i$ changing $N_c$ from 2 to $\infty$. For a smaller value of the sum of Wilson coefficients, both the sets of form factors would require a similar long-distance contribution, with the effect of reducing the longitudinal fraction.

A feature of both the sets of data is that, in the region of $r$ where the experimental rate is reproduced, $\Gamma_\parallel$ is larger or similar to $\Gamma_\perp$. The ratio $\frac{\Gamma_\parallel}{\Gamma_\perp}$ is sensitive to operators of different chirality which would appear in the effective Hamiltonian in extensions of the Standard Model [6].
4 Discussion

The conclusion of this analysis is that FSI effects can modify the helicity amplitudes in penguin dominated processes. The numerical result depends on the interplay between Wilson coefficients, form factors and rescattering amplitude, and we have shown that the experimental observation can be reproduced. At the same time, the rescattering effects we have considered are too small to affect the observed $B \to \rho \rho$ decays. As a matter of fact, while the CKM factors in the tree diagram in $B^0 \to \rho^+ \rho^-$ transition ($V^*_{ub}V_{ud}$) have similar size to the CKM factor in the FSI diagram in fig.2 ($V^*_{cb}V_{cd}$), the Wilson coefficient in current-current transition is $\mathcal{O}(1)$. We can expect to observe FSI effects in colour-suppressed and other penguin induced $B \to VV$ decays, such as $B^0 \to \rho^0 K^{*0}$, $B^0 \to \omega K^{*0}$, and $B^0 \to \rho^0 \rho^0$, $B^0 \to \rho^0 \omega$, $B^- \to \rho^- K^{*0}$, $B^- \to K^{*-} K^{*0}$.

Let us consider $B^+ \to \rho^0 K^{*+}$. On the basis of general arguments, we cannot assess the role of FSI without an explicit calculation, due to the CKM suppression of the factorized amplitude. The determination of the rescattering amplitude, similar to that in fig.2, can be done following the same method discussed above, obtaining $\Gamma_L/\Gamma \simeq 0.7$, i.e. smaller (even though compatible within 2-σ) than the measurement in Table 2. Therefore, in our approach we can accommodate a small $\Gamma_L$ for $B \to \phi K^*$ at the prize of having a smaller value of $\Gamma_L$ for $B \to \rho K^*$, which is not currently excluded due to the uncertainty in the data for this mode. It is interesting to notice that an analogous prediction is done in QCD improved factorization [6], where one gets $\frac{\Gamma_L}{\Gamma}(B \to \rho K^*) < \frac{\Gamma_L}{\Gamma}(B \to \phi K^*)$. More precise measurements are in order to suggest a solution to this polarization riddle. If further measurements of polarization fractions will confirm the present situation of a small longitudinal fraction in $\phi K^*$ and a large longitudinal fraction in $\rho K^*$, in that case we cannot identify uniquely the rescattering mechanism for explaining the data, envisaging the exciting necessity of new effects.

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