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Formation of clumps and patches in self-aggregation of finite-size particles. (English)

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Strongly coupled reaction-diffusion equations are ubiquitous in physics, chemistry and biomathematics. Starting by a brief, illuminating overview of previous work, the present paper proposes a new set of model equations for the dynamics of self-aggregation of finite-size particles subject to mutual attraction. Distinctive features of the new model are that (a) the mobility of particles depends on the locally averaged particle density and (b) linear diffusion acts on that locally averaged particle density. These facilitate the analytical description of evolution and allow for the complete analysis of stationary states. In the context of strictly positive mobility, starting from smooth initial conditions, the solution for the particle density was found to collapse into a set of delta-functions (clumps). These clumps originate from the nonlinear instability governed by the Riccati equation, which causes the magnitude of any density maximum to grow without bound over finite time. The equations for the evolution of the clumps were computed analytically and the predictions were supported by numerical simulations. If the mobility vanishes for some average density, the evolution leads to the spontaneous formation of jammed patches. Simulations confirm that the final state of the system consists of a combination of patches. The approach proposed in this paper, involving a variational principle, energy and a competition of length scales, is likely to find applicability in many areas of science.

Reviewer: Adrian Neagu (Timisoara)

MSC:
82C21 Dynamic continuum models (systems of particles, etc.) in time-dependent statistical mechanics
35M10 PDEs of mixed type
35K57 Reaction-diffusion equations
92C17 Cell movement (chemotaxis, etc.)

Keywords:
gradient flows; singular solutions; parabolic-elliptic system; chemotaxis; reaction-diffusion models; blow-up

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