Decoherence in quantum cosmology and the cosmological constant

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We discuss a spacetime having the topology of $S^3 \times \mathbb{R}$ but with a different smoothness structure. The initial state of the cosmos in our model is identified with a wildly embedded 3-sphere (or a fractal space). In previous work we showed that a wild embedding is obtained by a quantization of a usual (or tame) embedding. Then a wild embedding can be identified with a (geometrical) quantum state. During a decoherence process this wild 3-sphere is changed to a homology 3-sphere. We are able to calculate the decoherence time for this process. After the formation of the homology 3-sphere, we obtain a spacetime with an accelerated expansion enforced by a cosmological constant. The calculation of this cosmological constant gives a qualitative agreement with the current measured value.

Keywords: decoherence in cosmology; homology 3-spheres; cosmological constant.

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1. Introduction

General relativity (GR) has changed our understanding of spacetime. In parallel, the appearance of quantum field theory (QFT) has modified our view of particles, fields and the measurement process. The usual approach for the unification of QFT and GR, to a quantum gravity, starts with a proposal to quantize GR and its underlying structure, spacetime. There is a unique opinion in the community about the relation between geometry and quantum theory: The geometry as used in GR is classical and should emerge from a quantum gravity in the limit (Planck’s constant tends to zero). Most theories went a step further and try to get a spacetime from quantum theory. Then, the model of a smooth manifold is not suitable to describe quantum gravity. But, there is no sign for a discrete spacetime structure or higher dimensions in current experiments. Hence, quantum gravity based on the concept of a smooth manifold should also able to explain the current problems in the standard cosmological model ($\Lambda$CDM) like the appearance of dark energy/matter, or
the correct form of inflation etc. But before we are going in this direction we will motivate the usage of the 'good old' smooth manifold as our basic concept.

When Einstein developed GR, his opinion about the importance of general covariance changed over the years. In 1914, he wrote a joint paper with Grossmann. There, he rejected general covariance by the now famous hole argument. But after a painful year, he again considered general covariance now with the insight that there is no meaning in referring to the *spacetime point A* or the *event A*, without further specifications. Therefore the measurement of a point without a detailed specification of the whole measurement process is meaningless in GR. The reason is simply the diffeomorphism-invariance of GR which has tremendous consequences. Furthermore, GR do not depend on the concrete topology of spacetime. All restrictions on the topology of the spacetime were formulated using additional physical conditions like causality (see [1]). This ambiguity increases in the 80's when the first examples of exotic smoothness structures in dimension 4 were found. The (smooth) atlas of a smooth 4-manifold $M$ is called the smoothness structure (unique up to diffeomorphisms). One would expect that there is only one smooth atlas for $M$, all other possibilities can be transformed into each other by a diffeomorphism. But in contrast, the deep results of Freedman [2] on the topology of 4-manifolds combined with Donaldson's work [3] gave the first examples of non-diffeomorphic smoothness structures on 4-manifolds including the well-known $\mathbb{R}^4$. Much of the motivation can be found in the FQXI essay [4]. Here we will discuss another property of the exotic smoothness structure: its quantum geometry.

2. Exotic $S^3 \times \mathbb{R}$

Let us consider the spacetime with topology $S^3 \times \mathbb{R}$ where the 3-sphere has growing radius. This spacetime can admit uncountable many, different (=non-diffeomorphic) smoothness structures, denoted by $S^3 \times_\theta \mathbb{R}$ (a first example was constructed in [5]). For the construction of $S^3 \times_\theta \mathbb{R}$, one needs a homology 3-sphere $\Sigma$, i.e. a compact 3-manifold with the same homology like the 3-sphere. The Poincare sphere (or the dodecahedral space used in cosmology, see [6]) is an example. Usually, every homology 3-sphere is the boundary of a contractable 4-manifold but not every homology 3-sphere is the boundary of a SMOOTH, contractable 4-manifold. We used this fact to formulate restrictions on possible smooth spacetimes in cosmology [7]. For the construction of an exotic $S^3 \times_\theta \mathbb{R}$ one needs the following pieces:

1. $W_1$ as cobordism between $\Sigma$ and its one-point complement $\Sigma \backslash pt.$ and
2. $W_2$ as cobordism between $\Sigma \backslash pt.$ and $\Sigma \backslash pt.$

Then the non-compact 4-manifold

$$W = \ldots \cup -W_2 \cup -W_2 \cup (-W_1 \cup W_1) \cup W_2 \cup W_2 \cup \ldots$$

(see [5], $-W_i$ has reversed orientation) is homeomorphic to $S^3 \times \mathbb{R}$ but not diffeomorphic to it, i.e. $W = S^3 \times_\theta \mathbb{R}$. But $S^3 \times_\theta \mathbb{R}$ has the topology of $S^3 \times \mathbb{R}$, i.e. for
every $t \in \mathbb{R}$ there is a 3-sphere $S^3 \times \{t\}$ topologically but not smoothly embedded into $S^3 \times_\theta \mathbb{R}$. Or,

The 3-sphere $S^3 \hookrightarrow S^3 \times_\theta \mathbb{R}$ is wildly embedded, i.e. it is only represented (better triangulated) by infinite many polyhedrons. The wild 3-sphere is a fractal space.

In [8, 9] we discussed wild embeddings and its relation to quantum geometry. We were able to show that the (deformation) quantization of a tame embedding (see the appendix for a definition) is a wild embedding. The idea of the proof can be simply expressed in the formalism of GR. First we consider a tame embedding $S^3 \hookrightarrow S^3 \times \mathbb{R}$ for a non-exotic spacetime. The 3-sphere can be triangulated and we consider the 1-skeleton, i.e. a finite graph. The holonomies along this graph with respect to a suitable connection representing the geometry. The geometry of $S^3$ can be at least locally approximated by a homogenous ($SO(3)$ invariant) metric (in the Cartan geometry formalism, see [10, 11]). The observables in this theory (as the functions over the space of holonomies) form a Poisson algebra [12]. The (deformation) quantization of this Poisson algebra [13] transformed the graph into a knotted, infinite graph. As shown by us, this knotted graph is a wild embedding which can be interpreted as the quantum state. This quantum state contains an infinite number of homogenous metrics, i.e. metrics with different values of the curvature. Then the transition of a wild embedding to a tame embedding is the transition from the quantum state to a classical state which can be interpreted as decoherence.

3. Decoherence as Topology-change

Now we will interpret the wild embedded 3-sphere in cosmology. In our model $W$ of the exotic $S^3 \times_\theta \mathbb{R}$ we made a rescaling so that the 3-sphere at $t = -\infty$ is the initial state of the cosmos (at the big bang), i.e. we assume that the cosmos starts as a small 3-sphere (of Planck radius). As we claimed above, this 3-sphere is a wildly embedded 3-sphere $S^3_\theta$. In the model [11] of $W$ above we have the part $-W_1 \cup W_1$ as a cobordism from $\Sigma \backslash pt.$ to $\Sigma$ and back. As Freedman [5] showed this cobordism is equivalent to a cobordism from $S^3 \backslash pt.$ to $\Sigma$ and back. But now we can identify the wild $S^3_\theta$ (partly) with $S^3 \backslash pt.$ in this cobordism. But then we obtain a transition from the wild $S^3_\theta$ (the quantum state) to the homology 3-sphere $\Sigma$, which is smoothly embedded $\Sigma \hookrightarrow W = S^3 \times_\theta \mathbb{R}$. Or,

$$\text{quantum state } S^3_\theta \xrightarrow{\text{decoherence}} \text{ classical state } \Sigma$$

and we studied this process in [14] more carefully. In case of a hyperbolic homology 3-sphere, we showed that there is a single parameter, a topological invariant

$$\vartheta = \frac{3 \cdot vol(\Sigma)}{2 \cdot CS(\Sigma)}$$

of $\Sigma$, which characterizes all properties of this process [2]. Hyperbolic $n$–manifolds for $n > 2$ show Mostow rigidity, i.e. every diffeomorphism or conformal transformation is an isometry. Therefore the (unit) volume $vol(\Sigma)$ and the Chern-Simons
functional (as integral over the scalar curvature of $\Sigma$) are topological invariants. In [14], we interpreted the process (2) as inflation and calculated the exponential increase to be

$$\exp\left(\frac{3 \cdot \text{vol}(\Sigma)}{2 \cdot CS(\Sigma)}\right) = \exp(\vartheta)$$  (3)

Now we further interpret the process as decoherence of a quantum state (the wild 3-sphere) to the classical state (the tame hyperbolic homology 3-sphere). But then the decoherence time can be calculated from [14]. Assume the Planck time $T_{\text{Planck}}$ as time scale for the quantum state then we obtain for the decoherence time $T_{\text{decoherence}}$

$$T_{\text{decoherence}} = T_{\text{Planck}} \cdot \sum_{n=0}^{5} \frac{\vartheta^n}{n!}$$

(see section 4.3 in [14]). Consider as an example the hyperbolic homology 3-sphere $\Sigma(8_{10})$ obtained by Dehn surgery along the knot $8_{10}$ (in Rolfsen notation), see Fig. 1. Then we obtain the values (using SnapPea of J. Weeks)

$$\text{vol}(\Sigma(8_{10})) = 8.65115...$$
$$CS(\Sigma(8_{10})) = 0.15616...$$

or

$$\vartheta = \frac{3 \cdot \text{vol}(\Sigma)}{2 \cdot CS(\Sigma)} \approx 83.131....$$

and the decoherence time

$$T_{\text{decoherence}} = T_{\text{Planck}} \cdot \sum_{n=0}^{5} \frac{\vartheta^n}{n!} \approx 3 \cdot 10^{-36} \text{s}.$$  

According to this model, the cosmos starts in a quantum state (represented by a wild 3-sphere of Planck size) which undergoes a transition to a homology 3-sphere (classical state) by a decoherence process with time $\approx 3 \cdot 10^{-36} \text{s}$.
4. The Cosmological Constant

But what does the model predict for the future evolution? After the formation of the (hyperbolic) homology 3-sphere, the whole spacetime admits a hyperbolic structure (metric of negative scalar curvature). At the same time, the hyperbolic homology 3-sphere changes back to a 3-sphere. Therefore the spatial curvature is positive whereas the curvature of the spacetime is negative, i.e. one has a negative curvature along the time-like component of the curvature. We will show this fact now. Let

\[ \tilde{W} = W_1 \cup W_2 \cup W_3 \cup \ldots \]

be the spacetime for this phase. Then using the Mostow rigidity (for the 4-manifold \( \tilde{W} \)), we have a constant negative scalar curvature

\[ R_{\tilde{W}} = -\Lambda < 0 \quad (4) \]

for the 4-manifold \( \tilde{W} \). The spatial component (the 3-sphere) is assumed to carry a metric of constant positive curvature

\[ R = \frac{1}{r^2} \quad (5) \]

with radius \( r \). For the whole 4-manifold \( \tilde{W} \) we have the topology of \( S^3 \times [0, 1) \) and we choose a product metric of the form

\[ ds^2 = dt^2 - R^2 \cdot h_{ik} dx^i dx^k \]

with the scaling factor \( R = R(t) \) and the homogeneous metric \( h_{ik} \) of the 3-sphere (of unit radius). Then one obtains for the Ricci tensor

\[ R_{00} = -3 \frac{\dddot{R}}{\dot{R}} \]

\[ R_{ii} = \frac{\dddot{R}}{R} + 2 \frac{\ddot{R}^2}{R^2} + 2 \frac{\dot{R}^2}{R^2} \quad i = 1, 2, 3 \]

(the dot is the time derivative) leading to a negative scalar curvature

\[ R = -\frac{6}{R^2} \left( R \dddot{R} + \ddot{R}^2 + 1 \right) \]

(in agreement with our setting above). The spatial components of the Ricci tensor are positive for \( \dddot{R} \geq 0 \) and negative along the time-like component. Secondly, the spatial scalar curvature \( (3) \) is also positive. Then according to \( (4) \) we obtain

\[ R \dddot{R} + \ddot{R}^2 + 1 = \frac{\Lambda}{6} R^2 \]

with an accelerated expansion. Furthermore we also shown the claim above. We call \( \Lambda \) the cosmological constant. This constant can be determined by the expansion rate \( (3) \) of the inflation. We assumed a Planck-size cosmos at the big bang which grows to a cosmos of size

\[ L = L_P \cdot \exp (\vartheta) \]
and curvature
\[ \frac{1}{L_P^2} \cdot \exp(2 \cdot \vartheta). \]
Then the Mostow rigidity of the homology 3-sphere Σ implies a constant curvature which determines by the same argument (now for the 4-manifold) the curvature \( R_{\tilde{W}} \) to
\[ R_{\tilde{W}} = -\Lambda = -\frac{1}{L_P^2} \cdot \exp(-2 \cdot \vartheta). \]
Thus we obtain an exponential small expression for the cosmological constant! For the homology 3-sphere \( \Sigma(8_{10}) \) above, we obtain
\[ \Lambda \cdot L_P^2 = \exp \left( \frac{3 \cdot \text{vol}(\Sigma)}{\text{CS}(\Sigma)} \right) \approx \exp(-166.262..) \approx 6.2 \cdot 10^{-73} \]
which is not small enough to explain the current value of the cosmological constant. But there is a lot of freedom to construct a hyperbolic homology 3-sphere from a knot. In particular, the closing of the knot complement by using a so-called cusp is very important. Therefore, for a +3 Dehn-surgery with a special cusp (generating a geodesic of minimal length 0.5054), we obtain a homology 3-sphere \( \tilde{\Sigma}(8_{10}) \) with
\[ \text{vol}(\tilde{\Sigma}(8_{10})) = 4.67277013... \]
\[ \text{CS}(\tilde{\Sigma}(8_{10})) = 0.05095345... \]
so that
\[ 2 \cdot \vartheta \approx 275.12021... \]
and
\[ \Lambda \cdot L_P^2 \approx 3.3 \cdot 10^{-120}. \]
In cosmology, one usually relate the cosmological constant to the Hubble constant \( H_0 \) and the critical density leading to the length scale
\[ L_c = \frac{c^2}{3H_0^2}. \]
The corresponding variable is denoted by \( \Omega_\Lambda \) and we obtain
\[ \Omega_\Lambda = \frac{c^5}{3hG H_0^2} \cdot \exp \left( \frac{-3 \cdot \text{vol}(\Sigma(8_{10}))}{\text{CS}(\Sigma(8_{10}))} \right) \]
in units of the critical density and using the Planck length \( L_P = \sqrt{\frac{\hbar G}{c^3}} \). By using the measured value for the Hubble constant (Planck satellite)
\[ H_0 = 68 \frac{km}{s \cdot Mpc} \]
we are able to calculate the dark energy density (as expression for the cosmological constant)

\[ \Omega_\Lambda = 0.513 \]

and we obtain only the rough order of the constant (in contrast to the current value \((\Omega_\Lambda)_{\text{measure}} = 0.69\) measured by the Planck satellite).

5. Conclusion

The paper discussed a simple model of a cosmic evolution starting with a quantum state (represented by a wildly embedded 3-sphere) which changed to a classical state (tame embedded homology 3-sphere \(\Sigma\)). In the model we are able to calculate the decoherence time of the quantum state. Furthermore, we obtain a small cosmological constant with a value which is in qualitative agreement with the current measured value. The constant value of the cosmological constant can be geometrically understood by Mostow rigidity. We speculate that the correct value of the cosmological should be obtained by a more realistic model including matter coupling.

Appendix A. Wild and Tame embeddings

We call a map \(f: N \to M\) between two topological manifolds an embedding if \(N\) and \(f(N) \subset M\) are homeomorphic to each other. From the differential-topological point of view, an embedding is a map \(f: N \to M\) with injective differential on each point (an immersion) and \(N\) is diffeomorphic to \(f(N) \subset M\). An embedding \(i: N \to M\) is tame if \(i(N)\) is represented by a finite polyhedron homeomorphic to \(N\). Otherwise we call the embedding wild. There are famous wild embeddings like Alexanders horned sphere \([15]\) or Antoine’s necklace. In physics one uses mostly tame embeddings but as Cannon mentioned in his overview \([16]\) one needs wild embeddings to understand the tame one. As shown by us \([17]\), wild embeddings are needed to understand exotic smoothness.

References

References

1. S.W Hawking and G.F.R. Ellis. The Large Scale Structure of Space-Time. Cambridge University Press, 1994.
2. M.H. Freedman. The topology of four-dimensional manifolds. J. Diff. Geom., 17:357–454, 1982.
3. S. Donaldson. An application of gauge theory to the topology of 4-manifolds. J. Diff. Geom., 18:269–316, 1983.
4. T. Asselmeyer-Maluga. A chicken-and-egg problem: Which came first, the quantum state or spacetime? see http://fqxi.org/community/forum/topic/1424, 2012. Fourth Prize of the FQXi Essay contest ”Questioning the Foundations” (see http://fqxi.org/community/essay/winners/2012.1).
5. M.H. Freedman. A fake \(S^3 \times R\). Ann. of Math., 110:177–201, 1979.
6. J.-P. Luminet, J.R. Weeks, A. Riazuelo, R. Lehoucq, and J.-P. Uzan. Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background. *Nature*, \textbf{425}:593–595, 2003.

7. T. Asselmeyer and J. Król. On topological restrictions of the spacetime in cosmology. *Mod. Phys. Lett. A*, 27 No. 24:1250135, 2012. [arXiv:1206.4796]

8. T. Asselmeyer-Maluga and J. Krol. Quantum D-branes and exotic smooth $\mathbb{R}^4$. *Int. J. Geom. Methods in Modern Physics*, \textbf{9}:1250022, 2012. [arXiv:1102.3274]

9. T. Asselmeyer-Maluga and J. Król. Quantum geometry and wild embeddings as quantum states. *Int. J. of Geometric Methods in Modern Physics*, \textbf{10}(10), 2013. will be published in Nov. 2013, [arXiv:1211.3012]

10. D.K. Wise. Symmetric space Cartan connections and gravity in three and four dimensions. *SIGMA*, \textbf{5}:080, 2009. [arXiv:0904.1738]

11. D.K. Wise. Macdowell-Mansouri gravity and Cartan geometry. *Class. Quantum Grav.*, \textbf{27}:155010, 2010. [arXiv:gr-qc/0611154]

12. W.M. Goldman. The symplectic nature of the fundamental groups of surfaces. *Adv. Math.*, 54:200–225, 1984.

13. V.G. Turaev. Skein quantization of poisson algebras of loops on surfaces. *Ann. Sci. de l’ENS*, 24:635–704, 1991.

14. T. Asselmeyer-Maluga and J. Król. On the origin of inflation by using exotic smoothness. [arXiv:1301.3628] 2013.

15. J.W. Alexander. An example of a simple-connected surface bounding a region which is not simply connected. *Proceedings of the National Academy of Sciences of the United States*, 10:8 – 10, 1924.

16. J.W. Cannon. The recognition problem: What is a topological manifold? *BAMS*, 84:832 – 866, 1978.

17. T. Asselmeyer-Maluga and J. Król. Abelian gerbes, generalized geometries and exotic $R^4$. arXiv: 0904.1276, subm. to J. Math. Phys., 2009.
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