Candidate Software Process Flaws for the Boeing 737 Max
MCAS Algorithm and Risks for a Proposed Upgrade

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Abstract
By reasoning about the claims and speculations promised as part of the public discourse, we
analyze the hypothesis that flaws in software engineering played a critical role in the Boeing 737
MCAS incidents. We use promise-based reasoning to discuss how, from an outsider’s perspective, one
may assemble clues about what went wrong. Rather than looking for a Rational Alternative Design
(RAD), as suggested by Wendel, we look for candidate flaws in the software process. We describe four
such potential flaws. Recently, Boeing has circulated information on its envisaged MCAS algorithm
upgrade. We cast this as a promise to resolve the flaws, i.e. to provide a RAD for the B737 Max. We
offer an assessment of B-Max-New based on the public discourse.

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1 Introduction

This paper is a sequel to our introductory paper [1], which described the unfolding of the Boeing-MCAS affair in the wake of the tragic crashes of two Boeing 737 Max aircraft. Below we will abbreviate Boeing 737 to B737. Our tool for the previous analysis was Promise Theory, and our source information came from the viewpoint of the resulting public debate. Unavoidably, as outsiders to Boeing’s software engineering process, we have—at best—only a rudimentary technical insight into the specifications for the software component MCAS, and its original requirements. This limits the kind of reasoning available to us, as outsiders. Nevertheless, there are still matters which are amenable to analysis, and may contribute to the public debate.

In this follow-up paper, we have two objectives: to investigate the extent to which software (or software engineering) flaws can be said to have played a role in the incidents, and to comment on the software-related aspects of plans for a redesign. We make use of information concerning the B737 Max MCAS algorithm affair, made public soon after the occurrence of the second disaster, and we consider two questions: (i) to what extent does the discourse around the incidents point to possible software design flaws?, and (ii) given that there are candidates for such flaws, can we narrow the scope of possibility to be more specific about them? These considerations lead us to the notion of a software process flaw, as discussed in [2]. We refer to such indications as ‘candidate software process flaws’.

Without an investigation based on inside knowledge, it’s impossible to accurately determine whether or not these candidate software process flaws correspond to verifiable problems with the software process that took place. Thus—as is the case with all public discourse—the discourse is fundamentally speculative, and conclusions are limited. Nevertheless, we can sharpen the discussion somewhat. Johnston & Harris [3] focus on the question of whether software engineering can (or must) be considered the cause of the problem. Their paper provides useful information on the intended working of MCAS. They conclude that the complexity of the entire problem renders it too much of a simplification to speak of a software engineering problem. This is an easy position to take, as being overwhelmed by complexity is the default state; indeed, the willingness to be satisfied with such a position might be a result of the popularity of so-called ‘blame free’ postmortem analysis, and its preference for pointing away from human factors. We are unconvinced by that conclusion, however.
Promise Theory helps to clarify the roles of intent in a causative network. What Johnston & Harris indicate in detail is that there was a high risk of feature interaction,\textsuperscript{3} a suggestion which we have detailed in [1]. Feature interaction, however, is a classic phenomenon in software engineering and we consider it axiomatic that software engineers must always be on the lookout for such issues, a task which bestows on them an overarching responsibility for due diligence.\textsuperscript{4} Yoshida [4] claims similarly that automation is to blame for the disasters, though the paper does not clearly point out what went wrong in terms of software design or development. Automation is merely a proxy for human intent, so this suggests instead that there may have been problems in the intent or implementation of the automation by software engineers, mixed with many other issues arising from the intrinsic complexity of Just In Time design of a B737 NG follow up model. In a Promise Theory analysis, the role of trust plays a central role: which agents in a process trusted which others, leading to acceptance of promises being kept in good faith, and which of those were most vulnerable to being broken?\textsuperscript{5}

Choi [5] assumes that the software engineering for MCAS, when considered in hindsight, was indeed flawed or in error\textsuperscript{6}—a viewpoint which we consider not to be self-evident, and insufficiently supported by the remarks made by Choi in his paper. He then takes the case as an illustration for the claim that writing safety-critical software ought to be considered (legally) as a profession, which currently is not the case in the US. Choi’s paper is obviously relevant for our discussion, and we advise any readers who might seek to impute a conclusion that any part of our work below is a claim that Boeing’s software engineers may have failed in their due diligence, to first to read Choi [5] in full.\textsuperscript{7}

The analysis and discussion concerning software flaws depends on various propositions, which might be accorded probabilistic interpretations. This is a common technique in high risk industries such as aviation and nuclear power. Fault Tree Analysis and other forms of forensic logic have been in use for decades.\textsuperscript{7} In keeping with Promise Theory, and the necessary matching of propositional (+) and interpretive (-) promises, for every interaction and observation, we take it as given that a lack of information confounds the attempt to assign numerical quantifiers, even of subjective beliefs, within propositions. All uncertainties must be taken into account when determining one’s subjective belief in a specific statement. We refrain, however, from asserting any precise probabilities, and use probabilities only as typical scales about which one may reason by relative orders of magnitude. We take these from a semantic set, without specific quantitative representation: false, very implausible, implausible, fifty/fifty, plausible, very plausible, and true (verified). This will be sufficient for our limited purpose.

In the wake of the accidents, Boeing has issued various announcements\textsuperscript{8} concerning the redesign of the B737 planes, which it proposes to apply. We have combined these snippets of information into a de facto promise named B-Max-New which we ascribe to Boeing so to say.\textsuperscript{9} We suggest the idea that the combination of proposals in B-Max-New constitutes novel subsequent information which, in a sense of Bayesian statistics can be used as an incentive to change one’s current probability function on the space of propositions.

1.1 An algorithm upgrade acquires prominence

Boeing has announced that—in order to obtain recertification of the B737 Max—an upgrade of MCAS has been developed which will guarantee that similar accidents cannot take place in the future anymore.\textsuperscript{10} Moreover, no hardware modifications will be made to the B737 Max design beyond providing a standardized warning for pilots should the two AOA sensors disagree. This warning was an optional feature previously. We infer, from these promises, that Boeing has assessed the problem with the B737 Max to reside solely in the design of the MCAS algorithm, but not so much in the actual implementation thereof.\textsuperscript{11} From these statements, one cannot infer that the the MCAS software component contains a fault or defect (IEEE terminology, see [6]), because that would imply that MCAS had made a promise that it did not keep. Nor does it follow that the software process, by which the MCAS software component was developed was flawed in some way, since it may have implemented the design promises with perfect fidelity. If the MCAS software component works according to its promises or requirements, the requirements and promises themselves may have been at fault. This is a question of whether the software was a faithful proxy for appropriately represented design intentions. Following Wendel [7] we use the phrase rational
alternative design (RAD) for a proposed modification of the B737 Max which would have overcome the difficulties as experienced. A RAD is a promise, issued by a designer, that upon incorporating the proposed improvements would result in an alternative design which would not have suffered from the assumed deficiencies of the design involved in the incidents. The problem with third party RADs is that the flight systems are so complex that one can hardly trust any specific RAD proposal without actual testing, as the mutual dependencies of system components are many and intricate. Thus, as public observers, we may assess a lack of trust in the ability of exterior promisers to come up with a RAD for the B737 Max—but we cannot distrust Boeing as much, by the same argument, since Boeing’s engineers have maximal inside knowledge of the systems. For Boeing, our trust can only be in their presumed diligence.

We interpret Boeing’s recent proposed improvements as a promise (named B-Max-New in section 3.1). It was made by Boeing, and with the public in scope. This promise brings new information, which we may use to update our subjective probabilities, and which we subsequently use as basis for making two promises about our expectations for the outcome of the B737 Max improvements:

**Promise 1.1 (A 3th AOA sensor makes trim wheel use less likely)** *From Authors (promiser) to Public (promisee):*

**Promise Body:**

Let \( n \) be a positive integer \( 1, 2, 3, \ldots \). If Boeing provides an upgrade of the B737 Max, as suggested in B-Max-New, then the new model would be operationally closer to the B737 NG if MCAS were based on the inputs of \( 3 \) (or greater odd number \( 2n + 1 \) of) AOA sensors, rather than on \( 2 \).

The use of majority voting for positive sensor outcomes with \( 2n + 1 \) AOA sensors would, in particular, reduce the probability that the trim wheel needs to be used (after a single AOA sensor failure has led to an AOA sensor disagreement signal to MCAS), instead of the pilot controlled stabilizer control by wire.

**END PROMISE**

**Promise 1.2 (Using 3 AOA sensors is “better”)** *From Authors (promiser) to Public (promisee):*

**Promise Body:**

If Boeing redesigns the B737 Max, according to what it has promised (in our assessment) in promise B-Max-New, then there is a strong case that the resulting aircraft warrants pilots to take on systematic simulator training, preparing them for the situation that MCAS (as upgraded) has been switched off for the rest of the flight. Without MCAS active, the behaviour of the airplane does not promise to be within the constraints imposed by US regulations on avionics: 14CFR §25.203(a) “Stall characteristics”.

**END PROMISE**

The rationale for MCAS is rendered less convincing if extensive simulator training is needed regarding aircraft control (of the new B737 Max) in circumstances which have no counterpart in the behaviour of the B737 NG, that is without any forthcoming MCAS intervention (though with the B737 Max aerodynamics). Given the need for such training, in any case, one may contemplate additional training with a B737 Max configuration but without MCAS (denoted as B737 Max-min below). Making use of quantitative AOA information might also provide be attractive to pilots. We combine these promises into an implicit one, encompassing the points:

**Promise 1.3 (Implausible requirements on simulator training)** *From Authors (promiser) to Public (promisee):*

**Promise Body:**
If Boeing redesigns the B737 Max, as promised in promise B-Max-New, then the resulting aircraft is such that either (i) additional simulator training with MCAS (i.e. its successor) in disabled mode is essential (for pilots with a B737 NG type rating), or (ii) a further redesign is required which involves the installation of one or more additional AOA sensors, in which case simulator training with MCAS disabled may even become irrelevant, and will become less critical in any case.

END PROMISE

1.2 Some propositions

There are various propositions (hypotheses, assertions, assumptions, beliefs, and logical variables) which will play a role in evaluating these promises. It’s helpful to introduce some definitions:

Definition 1.1 (Positive input to MCAS) By this we denote an MCAS event, in which sampled input from one or both AOA sensors is considered to be out of range, so that MCAS intervention is now required, unless its data set for recent interventions suggests otherwise (while the Mach number as estimated for the moment is used to determine the strength of the intervention).

Definition 1.2 (The B737 Max-min) This denotes the hypothetical aircraft type which is obtained upon disabling MCAS interventions in the B737 Max.

Now consider a number of propositions:

Proposition 1.1 (C-B737 Max-min certification) This denotes the assertion that a B737 Max-min class of aircraft would obtain FAA certification with the same type rating as the B737 NG.

Proposition 1.2 (Cn-B737 Max-min) This denotes the assertion that the B737 Max-min class of aircraft would obtain FAA certification, possibly with a new type rating.

Proposition 1.3 (MCAS not anti-stall) The assertion that the B737 Max class of aircraft’s MCAS software component does not intend to serve as anti-stall protection in any way. In other words, switching off MCAS would not increase the probability of a stall, compared to the B737 Max, which would stand in the way of certification.

Taking our own position on trust, we shall assume that Boeing engineers have considered and made up their minds about each of these propositions—perhaps formulated differently—even though such information is unknown to us. The information about the B737, collected by Chris Brady on a website, provides the following quote about MCAS:

The LEAP engine nacelles are larger and had to be mounted slightly higher and further forward from the previous NG CFM56-7 engines to give the necessary ground clearance. This new location and larger size of nacelle cause the vortex flow off the nacelle body to produce lift at high AoA. As the nacelle is ahead of the C of G, this lift causes a slight pitch-up effect (i.e. a reducing stick force) which could lead the pilot to inadvertently pull the yoke further aft than intended bringing the aircraft closer towards the stall. This abnormal nose-up pitching is not allowable under 14CFR §25.203(a) “Stall characteristics”. Several aerodynamic solutions were introduced such as revising the leading edge stall strip and modifying the leading edge vortilons but they were insufficient to pass regulation. MCAS was therefore introduced to give an automatic nose down stabilizer input during elevated AoA when flaps are up.
From this, we may conclude that its author would consider Proposition 1.1 (C-B737 Max-min) and Proposition 1.2 (Cn-B737 Max-min) to be both false, because the design could not comply with the regulations, without some form of MCAS. Avoiding non-compliance with regulations appears to be sufficient rationale for introducing MCAS and—whether or not this particular form of non-compliance would without MCAS, contribute to a risk of stall—does not really matter for giving a rationale for the presence of MCAS. We expect, then, that the author of the quote would consider Proposition 1.3 (MCAS not anti-stall) to be true, the argument being that without MCAS certification will not be obtained, but not because of the increased risk of a stall, but because of the invalidity of Proposition 1.2.

We maintain that Proposition 1.1 implies Proposition 1.2, and Proposition 1.2 implies Proposition 1.3. We assign the following ad hoc probabilities to these Propositions:

- Proposition 1.1 (C-B737 Max-min) is very implausible (if it were true the rationale for MCAS would have been too weak),
- Proposition 1.2 (Cn-B737 Max-min) is implausible (mainly based on the quote taken from Brady’s website), and
- Proposition 1.3 (MCAS not anti-stall) is very plausible (mainly based on repeated information on the matter from Boeing).

It should be mentioned, however, that many authors have a different view on the rationale for MCAS, to mention Johnston & Harris [3] and Wendel [7] who both state that the need for MCAS is rooted in anti-stall protection. We remark that—should Boeing latterly announce that it did indeed understand, from the outset, that MCAS was meant to serve as an anti-stall system, then—our trust in Boeing would be decreased, and, as a result, our odds for Proposition 3.1 (B-Max-New) would drop significantly.

1.3 AOA sensor system: false positives versus false negatives

Let’s assume that the AOA sensors, when sampled and the results combined by some algorithm, effectively promise one of the following three possible outcomes to MCAS: “positive” (AOA too high’), “negative” (AOA not too high), “neutral” (AOA sensor system out of order). Promise theoretically, these correspond to a promise for determining the condition for intervention to be either: kept, not kept, or indeterminate. We speak of a false negative if the AOA sensor system (which includes both the sensor output (+) and its interpretation (-), in promise terms [8]) produces a negative signal when it should have been positive. We speak of a false neutral signal if the AOA sensor system produces neutral value while it should have been positive.

The idea behind MCAS is principally that, if it receives positive input (AOA too high), it may impose interventions which overrule pilot commands, at least initially. Now let us assume that the MCAS algorithm only intervenes upon a positive input from the AOA sensor system. In practice neutral and negative are both negative. We notice that, if MCAS is meant as an anti-stall system (and thus Proposition 1.3 fails), then (i) MCAS interventions are sometimes needed because otherwise no certification can be obtained, and therefore (ii) the occurrence of false negatives and of false neutrals are mission critical problems.

By minimizing these false negative and neutral assessments, however, the complementary risk of false positives may increase. Therefore it must be promised, at the design level, that MCAS driven interventions, triggered by false positives will not constitute a significant safety risk. This cannot be done by means of an intelligent processing of AOA sensor information alone; a completely different safety system would be needed—for instance, by pilots enacting a transition to manual stabilizer control, using the trim wheel, thereby deactivating MCAS interventions.

If MCAS does not promise to function as an anti-stall system, false negatives are unfortunate but manageable, while MCAS interventions triggered by false positives might constitute a serious risk. That risk can be dealt with in two ways: reducing the probability of false positives, by requiring that several AOA sensors are in agreement, or handing over to the pilot just as in the preceding case, and making sure, by other design criteria, that MCAS interventions cannot have adverse safety consequences.
1.4 How to use two AOA sensors

If MCAS is meant as an anti-stall system, then it would be natural for MCAS to intervene if either one of the two sensors notified an “excessive AOA”, whereas if MCAS is not meant as an anti-stall system, and false negatives were not so much of a concern, the prevention of false positives becomes relatively more important. Then one might then promise that MCAS intervene only if both AOA sensors agreed about an excessive value. Thus, there is no unambiguous meaning to “making use of both sensors in order to avoid a single point of failure” if it has not been determined which failure must be prevented in the first place.

However, if the method for preventing safety risks from an MCAS intervention under false positive information is dealt with sufficiently well (as it must be when it serves as an anti-stall system), then using the value of a single AOA sensor could only be defended, if one could disregard the fact that the single point of failure renders an aircraft system formally ‘out of order’. Speculating for a moment that Boeing engineers did not, in fact, know whether MCAS would be needed for its anti-stall functionality, and assigned a subjective probability of ‘fifty/fifty’ to Proposition 1.2, then it could further be argued that the alternating use of a single AOA sensor makes a kind of naive ‘Monte Carlo method’ or mixed strategy sense (e.g. as a Game Theory strategy), because having too many false positives would suggest a need for additional pilot training on how to use the trim wheel, which constitutes an option of last resort. This view is hard to defend, however, as the presence of systems that intentionally limit their own certainty regarding mission actual information is not the basis for any kind of engineering standard. Either one assures dynamic stability without correction, or the envelope for correction should be dynamically stable. By not using the disjunction of the “excessive AOA” judgements from both AOA sensors, they made a design decision consistent with not using MCAS as an anti-stall system, while they did not fully “vote against” that idea either.

1.5 White-spots in outsider knowledge

Regrettably, no perspectives from Boeing software engineers have percolated to the media, so we lack crucial expert information. We do have test pilot perspectives, from scattered emails and quotes that have found their way to the media as tabloid gossip. Much attention was paid to the testimony of a Boeing factory ‘whistleblower’, who claims to have identified problematic working practices and much attention was given to emails and communications from several Boeing test pilots at the time of MCAS introduction and tuning.

For an assessment of the flight system design promises, it seems more relevant to know what Boeing engineers thought of Proposition 1.1, Proposition 1.2, and Proposition 1.3, and how they made up their mind on what use to make of the presence of two AOA sensors for providing input data to MCAS. As outsiders, we have no clue, and it’s conceivable the engineers came to a fifty/fifty assessment of Proposition 1.2, and that the use of a single AOA sensor for each flight emerged as a convincing compromise.

In Appendix 5.4, we have made some simplified estimates concerning the use of one and 2 sensors which illustrate some of the observations made in the text. We summarize these here.

**Proposition 1.4 (Deliberate choice for single AOA sensor input)** The design decision to take input from a single AOA sensor, exclusively, was made. This occurred during the years 2012-2014, in a context of deliberation, involving software engineers, airframe engineers, and perhaps test pilots. Various alternatives, including conjunctive and disjunctive reading of sensor warnings, as well as disagreements were balanced as options.

We choose to assign the subjective probability “very plausible” to Proposition 1.4, based solely on Boeing’s history of technical standing.

**Proposition 1.5 (Single AOA sensor input as a compromise)** The option to take “AOA too high” input from a single AOA sensor only—and not to require agreement between AOA sensors as a precondition—was chosen, because otherwise the probability of having to disable MCAS would become too high.
We assign subjective probability “plausible” to this Proposition 1.5. It expresses only the fact that we have no evidence in either direction. The main virtue of 1.5 is that it summarizes the best argument we can think of for the decision to use positive input generated by a single AOA sensor, assuming the truth of Proposition 1.3 (MCAS non anti-stall). We choose to disregard the worst case notion that it was a result of negligent oversight.

2 The MCAS affair as a software engineering issue

Boeing announced an adaptation of the B737 Max design, which essentially consists of an upgrade for the MCAS algorithm. From this announcement, we can draw a number of conclusions about Boeing’s beliefs:

1. Without an upgrade of the MCAS algorithm, no solution to the MCAS affair is in sight. Stated differently, without such an upgrade recertification of the currently grounded B737 Max 800 aircraft is not conceivable.

2. Both accidents may be causally connected to the collective software engineering decisions made for MCAS under each set of circumstances.

3. For an outsider to the MCAS software process, there is no current evidence that with a different algorithm, or with the requirement of supplementary MCAS-aware pilot training, that there would have been a different outcome—to the extent that either or both accidents would not have occurred.

4. It’s relatively easy to imagine a modification of MCAS, say MCASp (meaning “patched MCAS”) which has the property that aircrafts of the form B737 Max-min[MCASp] would not have become victim of the same accidents. In fact, this holds for B737 Max-min. It follows that merely proposing a modification of MCAS, which would have blocked both scenarios, comes nowhere near providing a relevant RAD (Rational Alternative Design). The question of which role MCAS ought to play in the first place, has to be taken into account too, before stripping it of features that may be considered causes of incidents in the recent past.

5. It’s unclear to what extent software engineering could indicate a level of training needed for pilots, without user experience. Software engineering does not promise such assessments. If training methods and efforts, for B737 Max pilots, were considered defective, in relation to MCAS, then it would not be natural to consider such a deficiency a software engineering procedural failure.

For the MCAS software component at hand (as in a B737 Max), it is entirely conceivable to promise that pilots need not know about its existence in order to fly the aircraft safely. From the software engineering perspective, however, it is the system behaviour (as perceived by the pilots) which matters, not the view of engineers. This is a matter of cognitive psychology to determine whether or not some form of knowledge of MCAS is helpful or needed for pilots whose entire focus must be on quick and reliable determination of which of the available options for action need to be taken, and on then acting accordingly and decisively (see for instance [9]).

6. It is probably the case that fully informed pilot action would have increased the chance of survival in the Lion Air case, while in the second case a detailed knowledge of MCAS working and timing might perhaps have enabled the pilots to recover from what they understood as a stabilizer runaway, by making use of the control-by-wire of the motor driving the jackscrew, timed so as to switch off that motor (and thereby disabling MCAS intervention) just before the intervention took place (it seems that they tried to do this at some stage), but this “method” would be a far cry of what the MCAS designers had in mind for the resolution of false positive sensor readings, and its successful application would have been a highly laudable achievement rather than a reasonably predictable professional pilot action.
7. Assuming the software engineers were aware that the use of a single AOA sensor gave relatively many false positives, implying a high probability that pilots would have to fall back on use of the stabilizer wheel, then they may yet have had no way of knowing that the stabilizer wheel does not so easily serve as a way out of last resort in the absence of pilot input. It’s unclear which parts of the inherited B737 NG design the software engineers could have expected flawless and unproblematic operation, based on their own understanding. This is a ever-present risk for engineering about scenarios for which one has no hands-on knowledge. By the downstream principle of responsibility in Promise Theory, the aircraft integrators would have the responsibility to promise this (see [8]).

8. Assuming that the software engineers were aware that MCAS might be needed, in the capacity of a safety critical anti-stall system, then they had to keep open that option. In particular they would know that false negatives must be prevented (otherwise the system might fail to counteract when the risk of stall mounted), which may in turn come with an increased potential for false positives, and by consequence they would need to rely on the assumption that alternative stabilizer control could help with a stabilizer runaway. Under these circumstances, it is understandable that they did not enforce the nearly full elimination of false positives by means of the conjunctive reading of a number of AOA sensor outputs (thereby designing as if false negatives don’t really matter).

9. After the Lion Air accident, it seemed to be the case that elementary schooling of pilots would be of great and sufficient help. It was only after the second accident that it became clear that the presence of MCAS (given the particular features of the timing of its interventions) might have somehow undermined the capability for the trim wheel to serve as the solution of last resort in case of a stabilizer runaway-like problem. It’s normal to seek to attribute the ‘blame’ of accidents to human error, in first instance, since there is always a human at the end of every intentional casual chain, and humans are assumed to be less predictable components than technological ones. Therefore pointing at pilot error after the first accident was a plausible idea.

The situation is even more complicated for the Lion Air pilots, however. It has been claimed that they failed to properly use the manual trim wheel, having first switched off both CUTOUT switches, in order to deal with a problem for which it would have been the appropriate and necessary first step. That observation still leaves the question unanswered if—in the Lion Air case—the pilots would have been more successful than the pilots were, had they ‘performed better’ according to the designers intended script\textsuperscript{20}. A pilot error may have occurred, even if avoiding that very error would not have saved the plane. The first accident brought to light the defectiveness of B737 Max pilot training, and preparation in certain circumstances, but it did not clearly point to a systemic problem in the design of the flight control system.

2.1 Hardware versus software

We assume that, for many objectives, hardware solutions as well as software solutions can be developed. Moreover, we assume that determination of precisely which issues are delegated to software takes place at least in part during the algorithm design phase. The principle of separation of scales ought to play a role in this, as well as the assessment of the intrinsic stability (inevitability) of the processes (algorithms) used in the implementation (see [2, 10]). Consider the following scenarios and design possibilities concerning the scope of software control relevant to the discussion:

- It might have been natural—in a software design process for MCAS—to insist on a hardware addition, in the form of a third AOA sensor to simplify reasoning. This would be in keeping with ‘standard lore’ for quorum systems, where $2n + 1$ agents are expected to reach a majority outcome, e.g. see [2].

- One may imagine that, instead of producing repeated interventions, MCAS would switch off itself after two interventions and then instruct pilots to engage in full manual flight with focus on keeping the AOA within safe limits.
• One may also imagine that frequent inspection of both AOA sensors could be integrated with a machine learning system, allowing software to diagnose sudden anomalous changes in one of the two sensors, e.g. by simple machine learning, which—in the absence of pilot actions—may be taken for an indication that the sudden change was anomalous, and indicating that the other sensor should be consulted.

• One may imagine an MCAS algorithm in which manual use of the trim wheel deactivates MCAS, thereby reducing the range of conditions which are to be handled by means of the MCAS algorithm. This approach is used in car cruise control systems, for instance.

We don’t claim that any of these suggestions would have been helpful in avoiding an accident, but we find it plausible that the the precise conditions under which software has dominion are determined, in part, during software design. This points to the need for a robust iterative dialogue between hardware and software throughout the design of any specialized system—not least one that is mission critical.

### 2.2 Criteria for the presence of a software issue

We now focus on which criteria might qualify the MCAS affair to be classified as a software or software engineering related incident. Software must play a role if and only if the following conditions are met in combination:

1. The B737 Max cannot be recertified as is under the B737 NG type rating (there is a problem, with software or otherwise).  
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2. The B737 Max-min cannot be certified, not even with a new type rating (MCAS serves a necessary purpose, although its current form does not solve all problems).  
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3. Assuming that MCAS development took place, within the bounds of a software process model $M_{sp}$, adopted by Boeing for the design and production of avionics control software. Then either (a), (b), or (c):

   (a) There was an error in executing the software process—a convincing indication that the actual software process, as it was followed, for the design and implementation of the MCAS algorithm, deviated from the prescriptions of $M_{sp}$ to such an extent that the following holds: Had the prescriptions of $M_{sp}$ been followed, then either the MCAS algorithm would have become different (due to deviating requirements) or else the MCAS software component would have been different (as the result of more rigorous testing or as a consequence of attempts to deliver a formal verification).

   In either case a software process flaw has been found (which may but need not explain the accidents). What matters is: (i) the resulting MCAS software component was deemed adequate, but only on insufficient grounds, and, (ii) that fact could have been noticed had $M_{sp}$ been properly applied. In this case, a promise being made by a low fidelity agent [2].

   (b) There was a flawless application of software process $M_{sp}$, which lead to say MCAS, yet it is concluded in hindsight that the MCAS algorithm or the MCAS software component contains an error, which could be repaired once detected, by making a few relatively minor modifications to the algorithm and to its implementation.

   In this case it must be analyzed why proper application of $M_{sp}$ failed to detect the error, and why the error was not noticed during certification either.

   (c) Alternatively, there was a flawless application of software process $M_{sp}$, which lead to say MCAS and in hindsight it is concluded that MCAS does not solve the engineering problems so that in any case MCAS must be redesigned, though without the guarantee that such a redesign can or will be successful.
In this case, a rather fundamental flaw in the software process model has been discovered and a modified software process $M_{sp}$ must be promised first. Moreover it is not clear whether or not an upgrade of the software component MCAS can be found at all (because, unfortunately, it has come about that a "correct" process for the development of such a solution within $M_{sp}$ does not conclusively prove the existence of an outcome that can keep all promises.)

We have no reason to assume (b), and the presence of a straightforward implementation error (in a context with adequate specifications for MCAS) would certainly have reached the media by now. Moreover, we have no means to establish (c) even if that were the case, and we limit attention to options for software process flaws as classified under (a).

2.3 Candidate software process flaws

From our vantage point, it is impossible to determine whether or not the software process for MCAS contained flaws, as referred to under 3 (i) above. What we can do, however, is to suggest some candidate flaws. In this section we propose four candidate software process flaws.

2.3.1 Questionable safety level classification

This section proposes, by a hypothetical argument, that the acceptance of a safety certification assigned to MCAS was itself a design flaw that may have warranted further attention.

As reported, in various documents, the MCAS software component has been assigned a DO 178c safety level B (deemed ‘hazardous’). It may be the case that no convincing scenarios have been provided which support this classification. In that case, one candidate process flaw consists of insufficiently matching high level requirements (e.g. anti-stall) and low level requirements (e.g. increase counter when event received) on the MCAS algorithm. A justification for this could go as follows. If MCAS were meant to serve as an anti-stall system, and if an MCAS failure would somehow prevent it from playing that role, a stall might occur which may be catastrophic (hull loss) rather than hazardous (risking the lives of some passengers or crew members at worst).

It seems to follow that the level B assignment corresponds to not taking MCAS for an anti-stall system, so the question arises which other scenario could justify classification B. Now assuming that the main virtue for MCAS is supposed to be to make the B737 MCAS maneuvering characteristics similar to those of a B737 NG, it is hard to imagine that failing to play that role could cause a hazardous outcome (for instance destabilized by sudden turbulence, or side wind, which risks lives of passengers and crew not wearing seat belts).

The safety classification ‘hazardous’ (assuming it had been justified in a systematic manner) thus relates apparently to the handling of MCAS interventions, which may have been based on false positive information. Adopting that assumption, however, means nothing less than that it was known, at design time, that pilots handling one or more MCAS interventions were confronted with a level B safety risk. Now, the rationale of the MCAS system becomes doubtful: the cure is worse (in terms of its safety risk) than the problem it is supposed to solve (again as measured in terms of a safety risk).

The above argument does not change much if MCAS were given safety level A (potentially catastrophic). It seems that the only reasonable safety certification one could accept of a properly functioning MCAS component would have been C (major problems) or below.

2.3.2 Implicit awareness of the state of the art

The question of whether or not MCAS is supposed to serve as an an anti-stall system may have been on the table, even though undetermined. In any case, the experience and knowledge associated with its Pegasus predecessor may have been insufficiently acknowledged. In this case, it’s arguable that not enough use was made of the existing experience with the MCAS high level requirements specification, inherited from the Pegasus project. This specification probably has been imported in the B737 Max software process.
2.3.3 Unfamiliar requirements

Assuming that MCAS is supposed to make sure that say a B737 NG is “pilot experience equivalent” to B737 Max, that problem amounts to solving the informal promise equation:

\[ \text{B737 NG} \equiv_{\text{pe}} \text{B737 Max-min} [X] \]

where \( \equiv_{\text{pe}} \) signifies equivalence, or the indistinguishability of promises assessed by an agent for the left and right hand sides. Here X is a possibly multi-variate variational parameter, for a software component, for which MCAS is claimed to constitute a solution, and X implements a thread (i.e. is the mathematical semantics of an instruction sequence), which is combined in parallel with the threads of B737 Max-min by way of strategic interleaving. Now, in this case, the candidate flaw is that assessing high level requirements depicted in this this equation is not currently possible—it can be considered too remote from what computer science has on offer, in terms of languages and concepts capturing requirements. Formally defining \( \equiv_{\text{pe}} \) is very difficult. Aside from Promise Theory, which can address a scaling of issues and assess their gaps and flaws, and yet is still insufficient, we are not aware of papers on this or similar issues. Then there is the question of assessment, which may be performed differently by each assessor. Thus, it’s unclear how compliance with such high level issues could be demonstrated at all. This is a major unsolved issue in computer science.

To put it another way, if a set of promises (perhaps arising from a requirements specification) were the starting point of MCAS development, then there is no evidence that merely applying a traditional software engineering process (such as DO 178c) could produce such an X. Software engineers should not have accepted the assignment in this case, as it would be clear that the promises could not be kept.

2.3.4 Intervention overshoot

The MCAS algorithm commands highly disruptive interventions, in terms of the size and the duration of the movement of the stabilizer. The candidate flaw is that not enough research and development were performed to justify the quantitative aspects of MCAS interventions. This candidate flaw is connected with the assessment of the safety level of MCAS. The stronger and longer its interventions the higher the risks connected with interventions triggered by false positive sensor inputs.

2.4 Algorithms: a definition

The concept of an algorithm is becoming more important day by day, but there is no consensus on what it means. Algorithms are now the subjects for public debates, e.g. in facial recognition, game playing, and of course in flight systems. The rules of engagement for such public debates cannot simply be inferred from a scientific tradition in computer science, or from a practical tradition in software engineering.

A difficulty faced by a public debate about any specific algorithm comes from the fact that none of the participants are likely to be aware of the interior details of the design of the algorithm, nor with the specifics of implementations of it. Both in computer science and in software engineering, this removal between the actors in a discourse from the technicalities of its subject matter makes discussions rest on speculative and principled arguments rather than facts. It seems to have gone unnoticed by the computer science community that the word ‘algorithm’ serves to decouple software technicalities from their underlying ideas, so as to make implementations amenable to public debate. This aspect constitutes part of our definition of an algorithm for the purposes of this paper:

**Definition 2.1 (Algorithm)** An algorithm is a method for solving a certain problem (i.e. for achieving a promised outcome) in a stepwise manner. An algorithmic method is documentable by a finite sequence of instructions. An execution of the method implies performing the instructions, possibly more than once, so that no a priori bound on the number of steps can necessarily be given in terms of the number of instructions. Algorithm refers collectively to this method as a conceptual black box or semantic boundary, which makes certain exterior promises, without having all interior details at hand.
latter understanding of algorithm renders it amenable for debate among software non-specialists, on the assumption that it keeps its promises.

We assume that the concept of a public debate about an algorithm and its implementation is reasonable, and that its implementation as a software component makes sense, as these represent intentions, which may have societal and even legal ramifications. Our discussion, as part of such a debate, is motivated by the question of whether the design and implementation of the MCAS algorithm may have contributed causally to the fatal accidents, and we proceed exclusively with the handicap of information pertaining only to the public debate on the matter. Our ruminations are therefore likely to be of particular interest to lawyers and public officials.

3 A RAD for MCAS and a potential candidate software process flaw

According to Wendel [7], if a litigation lawyer plans to make a case concerning a suspicion of a design problem in a product, it’s a wise idea to provide a so-called RAD, or a Rational Alternative Design. The RAD indicates how the design problem could have been avoided with the implication that it should have been avoided. Indeed many authors have made suggestions on how the MCAS system might alternatively have been designed. We take the following position, however: the complexity of flight control software is so high that it is not plausible for an outsider to suggest an RAD because too many factors must be taken into account. However, a RAD for the B737 Max which has been proposed by Boeing cannot reasonably be dismissed as potentially not taking the complexity of the problem into account—since they are the insiders we seek. Therefore, it’s reasonable to look into such an RAD (if available) in some detail.

3.1 An RAD originating from Boeing

By RAD-B, for the B737 Max, we denote the following outline:

- Improvement of various software components (different from MCAS), as well as further development of the computing architecture within the aircraft in such a manner that in the future both main flight control computers will permanently work in parallel, rather than that in an alternating fashion one of these is in the lead during a flight, as was the case until now in the B737 NG 800 and the B737 Max 800.

- The MCAS algorithm is modified, and accordingly the MCAS software component is upgraded, resulting in MCASu in our terminology.

- The upgrade of MCAS to MCASu is expected to enjoy the following properties:
  - Upon discovery of an AOA sensor disagreement, the MCASu component will be terminated and act as if no “AOA too high” triggers are being received until reactivated. In addition the pilot will be expected to do without autopilot, without autothrottle, and without ‘control by wire’ of stabilizer trimming,
  - Interventions by MCASu will be more moderate (changing the trim less) than before,
  - MCASu interventions won’t be repeated (within the same intervention episode), i.e. unless in between an episode with adequate AOA and normal flight parameters has occurred (i.e. has been observed),
  - MCASu will allow overriding (or rather, successful counteraction) by ordinary pilot control via the yoke.

- Each cockpit will routinely be equipped with the same visual and acoustic signal for warning in case of an AOA disagreement, (this upgrade constitutes the only hardware modification, and was already available as an optional feature).
Now Boeing has issued a promise, B-Max-new, as follows:

**Promise 3.1 (B-Max-new)**  *From promiser: Boeing to promisee: Public.*

**Promise Body:**

- A redesign of the B737 Max has been realized (we, not Boeing, call it B737 Max-min[MCASu], the B737 Max with MCAS upgraded to MCASu).
- Many test flights have been made with the available B737 Max-min[MCASu] prototypes, and simulator sessions were carried out with a novel dedicated simulator for the B737 Max-min[MCASu],
- The B737 Max-min[MCASu] will obtain within a reasonable time the FAA (re)certification for the same type rating as the B737 Max had been certified with (and the same as the B737 NG).
- Preparing existing aircrafts to the mentioned upgrade is likely to involve not more than 150 expert working hours per aircraft.

**END PROMISE**

From B-Max-New the following conclusions can be drawn:

- Boeing engineers assume the truth of Proposition 1.3 (MCAS not anti-stall). A reasonable consequence of taking notice of the announcement of B-Max-new is to raise the subjective probability of Proposition 1.3 (MCAS not anti-stall) to “true”. If one of both AOA sensors is damaged during the start, say by way of collision with a bird, the MCASu system will be disabled so that MCAS cannot provide any protection against stall. As the risk of a single sensor failure is relatively high, the importance of MCASu for anti-stall protection must have been considered minimal or absent by Boeing engineers (and so may we).
- The MCASu feature is considered useful, although it will be less intrusive than its predecessor MCAS. This makes one wonder why the design decisions for the MCAS algorithm were as they were. The question arises whether the functionality of MCASu could be specified in simple terms, for a large audience, now that the suggestion that its claim of anti-stall functionality is conclusively outdated.

### 3.2 Assessing RAD-B and B-Max-New

Our assessment of Promise 3.1 (B-Max-new) is:

**Assessment 3.1 (B-Max-new)**  *Boeing may not be able or willing to keep promise (B-Max-new) and therefore we qualify our assessment of Promise 3.1 (B-Max-new) as implausible.*

We shall argue in a number of steps, while assuming significant trust in Boeing to begin with.

**Step 1.** We remark first that without trust in Boeing there no reason to believe that the promise of (re)certification of the B737 Max-min[MCASu] will be kept at all. The challenge is then to justify a low probability assessment of promise 3.1, while maintaining significant trust in Boeing.

**Step 2.** Given the assumption of significant trust in Boeing, our assessment of promise 3.3 (non-anti-stall promise) in [1] is that it will be kept. If follows that MCAS is not helping out in life threatening situations but only in cases where a mismatch (in terms of expected pitch stability according to Boeing) with pilot experience with a Boeing 737 NG can result.
Step 3. If Proposition 1.1 (C-B737 Max-min) were the case, the introduction of MCAS can hardly be justified, the only remaining relevance of it being that it makes the B737 Max pilot experience so close to the B737 NG pilot experience that not only the same type rating as for the B737 NG can be used but also, and more importantly, no costly training is needed for B737 NG pilots who plan to pilot a B737 Max. So we find that C-B737 Max-min is probably false.

Step 4. Further from promise 4.1 of [1], as well as our trust in the promiser (Boeing) we infer that Boeing expects the B737 Max-min[MCASu] to have a single minor hardware improvement only (except for the modernization of its computing platform with both computers permanently active and working in parallel, an overdue innovation which is unrelated to MCAS): the pilots will receive a uniform standard warning of AOA disagreement of such a disagreement is noticed, upon which MCASu will disengage and pilots are supposed to perform manual (non autopilot flight).

Step 5. With two AOA sensors on board the likelihood of an AOA sensor disagreement is maximal. Hundreds of such events have been observed in recent years. It takes only a bird to collide with one of both sensors and the subsequent presence of an AOA sensor disagreement is plausible.

Step 6. Upon having received an AOA disagreement warning, a B737 Max-min[MCASu] behaves just as a B737 Max-min until the end of the flight.

Step 7. Now Boeing will argue that for piloting a B737 Max-min[MCASu], just as they did before for the B737 Max, that no substantial further training (either in theory or in the simulator) is required. From this it follows that, without further training B737 NG pilots can fly a B737 Max-min, for almost a full cycle (a collision with a bird may well take place just after lift off.)

Step 8. We conclude that the circumstance in which pilots (or prospective pilots) of a B737 Max-min[MCASu] have received an AOA warning constitutes an emergency—a high stress cognitive burden. Prospective B737 Max-min[MCASu] pilots will require a meaningful training for that particular course of events, and moreover they will require additional proactive simulator training on how to fly the B737 Max-min (i.e. how to fly the B737 Max-min[MCASu] by hand). They will likely find it hard to accept that MCAS (MCASu) is needed for their well-being, as pilots, while its absence creates no difficulty worth thinking through in detail or worth being tried out in a simulator. These paradoxical positions cannot easily be explained to the public: after MCAS has been upgraded (to MCASu), pilots must be specifically trained in how to work without MCASu.

A further modification of the B737 Max-min [MCASu] which simplifies these matters is in order.

Step 9. Having three AOA sensors, instead of 2, with majority voting (the A320 architecture for that matter) constitutes a design which we will denote as B737 Max-min[MCASu/3mv], or even five AOA sensors (B737 Max-min[MCASu/5mv]) will drastically alter the picture.

In a B737 Max-min[MCASu/5mv] plane, the probability of an AOA mismatch which disables MCASu, is very low (three AOA sensors must malfunction simultaneously), and for that reason piloting that plane comes much closer to handling a B737 NG than is the case with a B737 Max-min[MCASu/2mv] i.e. a B737 Max-min[MCASu] (at least under the assumption that MCASu works well in the absence of false positives while not being hit with false negatives too much).

4 Why does it all matter, and when?

It’s a challenge to express in a reliable manner when, how, why, and for whom the observations made in preceding sections may be of relevance. As authors, we must define our own role. We view our role as that of limited ‘experts’, in a particular field of automated systems and reasoning, who provide
knowledge about a case which may be of use to all persons and organizations who take an interest in the affairs concerning the B737 Max. Our combined expertise comprises: (i) practice and theory of software engineering, (ii) Promise Theory, as used as a tool for the description of complex multi-agent scenes, (iii) Logic, including theoretical aspects of forensic reasoning. Our expertise, however, does not include the following areas: (i) aircraft engineering, (ii) the actual writing of safety critical software, (iii) working within the DO 178c software process framework, (iv) piloting, (v) airline management. Readers may derive an initial level or trust in us—the authors—from this brief declaration of claimed expertise. Indeed, readers are free to assign ad hoc probabilities, as we have, if they so desire.

4.1 Potential relevance of both forecasting promises

The potential relevance of the forecasting promises, Promise 1.2 (using 3 AOA sensors is better) and Promise 1.3 (implausible requirements on simulator training) is now as follows: these may be taken for our promises on how the acceptance may work out of an aircraft model B737 Max-new, engineered along the lines of Promise 3.1 (B-Max-new). In particular, pilots (and airlines) may insist on rigorous simulator training on the basis of a positive assessment of Promise 1.3 and they may insist that simulators represent the various scenarios for using the manual trim wheel very well, and may ask for evidence for the latter.

A positive assessment of Promise 1.2 (using 3 AOA sensors is better) and the arguments given for it, may produce an incentive for airlines as well as for some members of the public to insist that more than 2 AOA sensors will be installed in a renewed B737 Max model, and to insist that Boeing won’t keep Promise 3.1 (B-new-max), and instead will withdraw that promise and issue an upgraded Promise including one or more additional AOA sensors.

4.2 Potential relevance of the candidate software process flaws

Adequately analyzing the potential merits of pointing out candidate software process flaws, for the software engineering process which led up to the MCAS algorithm, is significantly more difficult than the argument just given, in cases of the Promises 1.2 (using 3 AOA sensors is better) and Promise 1.3 (implausible requirements on simulator training). We shall distinguish five different lines of discourse, for the appreciation of the potential relevance of this work, each of which may be relevant for some readers, and may be irrelevant for other readers.

4.2.1 Curiosity

A general and impartial curiosity drives interest in possible explanations for the tragic incidents. Our work may contribute to that perspective by providing hypothetical candidate explanations in some detail.

4.2.2 Litigation support: top level division of responsibilities

Victims and stakeholders in the crashes may intend to sue some combination of an airline, the airframer, suppliers of the airframer, one or more certification authorities. Supporting these parties is another line of interest. A fuller picture may enable the plaintiff or sued parties to an agreement, to share responsibility for the problem each of them will accept, or reach a corresponding division of claims. It may be useful for all sides involved to have a common language for talking about what might have gone wrong with the software engineering, even if parties agree not to investigate in detail what actually went wrong.

4.2.3 Litigation support: if details on software engineering matter

Several different scenarios may involve a setting where the details of software engineering do matter. If Boeing, airlines, certification authorities, and perhaps other parties cannot arrive at an agreement about a division of responsibilities, as mentioned in Paragraph 4.2.2, then they may want to settle the matter on the basis of a detailed investigation of facts so that many more details of organization and production come into play.
In a different scenario, a part of the software process has been outsourced to one or more suppliers and a division of responsibilities must be found for the parties involved in the relevant software process. Then it may matter for all sides to have a survey of candidate software process flaws at hand.

- By using the language of software process flaws, rooted in established Promise Theory, we provide a way to get around the straightjacket of having to suggest a RAD, which may then be easily judged technically naive by the defendant’s lawyers.

- If the framework and the logic of software process flaws has been accepted by various sides in principle, then the candidate flaws point at options for the prosecution or for the claiming side in a civil case.

By accepting said framework, including a survey of candidate software process flaws, a court (providing it uses Bayesian reasoning with a subjective understanding of any assigned probability) also promises to adopt (and keep fixed for the time being) non-zero (undisclosed) prior odds for each of the listed candidate software flaws. The choice of such prior odds is entirely left to the discretion of the court. By adopting non-zero prior odds the court opens a potential path forward to proof of existence in case no decisive and confirmatory testimony from software engineers who participated in the process can be obtained.

- As it stands the four candidate software process flaws constitute mere speculation and carry no weight of proof. Evidence may be gathered, however, first of all by means of interviewing relevant software engineers (if they are willing to cooperate).

- In lieu of confirmation, for the mentioned software process flaws, the claimant may need to use indirect proof. In an indirect proof, say for an occurrence of the second software process flaw (wrong assignment of safety level to a software component) a certain type of evidence $E$ is brought forward (by the claimant or by the prosecution, the information resulting from forensic investigation) and the validity of $E$ is confirmed by witness testimony. Moreover, in advance of the testimony and independently of that testimony a software process expert (not one of the authors, however, as we both don’t possess the required expertise in software metrics) have been able to determine a significant likelihood ratio from which it follows that the evidence $E$ was much more likely to hold true in the presence of the second software process flaw than in its absence.

As stated, the court may adopt non-zero prior odds for each of the listed candidate flaws, including the third one. Now the mentioned software process experts (in general, not specific for Boeing software engineering) may communicate the likelihood ratio which they have determined to the court from which it may derive (compute) its posterior odds, on which a final decision may be grounded.

4.2.4 Software process improvement

Engineers may seek to improve the execution of the software process, or in case no single software process flaw can be identified retrospectively, they may pursue the more far reaching objective to improve the software process at hand.

4.2.5 Towards responsible software engineering

A requirement, which might be imposed in the long run for MCASu, is that it will be the outcome of so-called responsible software engineering (RSE). A criterion for RSE as mentioned in Schieferdecker [11] is explicability, which goes beyond the narrow circle of software technicians involved in a project. We expect that thinking in terms of the removal of software process flaws may be helpful for obtaining explainable algorithms.
5 Concluding remarks

In this paper we have obtained three results: (i) criteria under which the B737 Max MCAS algorithm affair may be considered a software issue, (ii) a list of four so-called candidate software process flaws each of which might explain in part what went wrong in the B737 Max MCAS algorithm affair (questionable safety level classification, implicit awareness of the state of the art, unfamiliar requirements, and intervention overshoot), and (iii) an argument why the recent promise by Boeing about the alternative design of the B737 Max may require a further revision.

5.1 MCAS design as a case of DSR (Design Science Research)

Concerning the design of MCAS—and now of MCASu—one may notice that, from the outset, the very existence of a software component X which solves the equation mentioned in Paragraph 2.3.3 is an open question. It follows that there may be none or perhaps many solutions to this problem. It also follows that we are faced with a research problem to better understand what we might call Design Science Research (DSR), i.e. “how to solve a problem by implementation of a process leading to an innovative artefact” for which each of the 7 guidelines listed in Hevner et. al. [12] are relevant. The (re)engineering of MCASu might conceivably be cast in the framework of DSR. Doing so would focus attention on guideline 6 (design as a search process) which calls for a thorough understanding of the relevant search space and for the application of a systematic method for selecting a solution within the given search space.

Constraints on the search space just mentioned may be split into intrinsic constraints and extrinsic constraints. Intrinsic constraints guarantee safety and compliance with strict rules of airframe behaviour (if any such rules outside the realm of safety are being imposed), while extrinsic constraints are introduced in order to optimize physical pilot experience, amenability to simulator based training, compatibility with the minimization of memory items, passenger experience, and explicability to prospective customers. Expanding on external requirements, one may imagine that an MCAS successor system includes assistance for the pilots when its interventions take place and pilots may feel the need to intervene nevertheless with their own interventions. The terminology of intrinsic requirements and extrinsic requirements has been elaborated in detail in the context of care robots, see e.g. Poulsen, et. al. [13].

Finally, MCASu may be considered a software robot which can therefore be thought of in terms of robotics and AI. The ethical issues arising from this are a separate issue, and Poulsen et. al. [13] suggest that a “school” must be chosen concerning machine ethics. There are some options: (i) moral (software) robotics in which decision-taking is done by the robot which is given ethical principles from which it must compute adequate decisions, (ii) good robotics, an approach in which the designer views the (software) robot as an ordinary program which must be ethically adequate by design, (iii) a combination of the two where the system implements moral robotics, while performing as if it were designed as an artefact for good robotics. The combined approach appears in a different terminology in the context of autonomous weapons, e.g. in [14].

5.2 From secrecy about the presence of a software component to secrecy about its internals

It has been stated, in many blogs and papers on this affair, that Boeing was negligent in not disclosing the existence of MCAS algorithm and its implementation in a software component MCAS to pilots. For instance Hutton & Rutkowski [15] qualify the secrecy about the presence of MCAS as implausible. Obviously, when announcing the existence of the MCAS software component to customers, pilots, and perhaps the public, some abstraction of its algorithm must be communicated, as the whole algorithm may comprise too many instructions for a non-(software)technical audience to swallow. In this case secrecy is a matter of degree and is plausible that some parts of the MCASu algorithm will remain unexplained to pilots and customers.\footnote{If there is any principle which applies to the amount of information conveyed to pilots, it appears only to be that such information be minimized rather than maximized. It seems still to be a useful design.
requirement on MCASu that its very existence need not be explained or announced to pilots, if only as an ideal which for other reasons won’t be achieved in this particular case. Forgetting for a moment that it has become impossible to hide existence of MCASu after the existence of MCAS has been made public, it seems to be the case that accepting the need to explain the working and the existence of MCAS (MCASu) to pilots goes hand in hand with acknowledging its potential deficiencies.\textsuperscript{34}

MCASu might become more complex in the future as its presence turns the plane in part into an “Autonomous Thing (AuT)”, making its own decisions to intervene during a flight. It may in due time be equipped with learning capabilities, and be made adaptive to the operational style of a certain pilot. Following Linz \textsuperscript{[16]} an MCASu with learning capability must not be safety critical, however, at least not for the foreseeable future, and from this it follows that the degrees of freedom for its design are limited by the safety level classification which is assigned to it.

5.3 Addendum: Promise Theory

For expositions on Promise Theory and applications thereof we refer to Burgess \textsuperscript{[17]} and Bergstra & Burgess \textsuperscript{[8, 18]}. In Bergstra & Burgess \textsuperscript{[1]} an extensive introduction to the connection between Promise Theory and the rather singular technical MCAS affair is provided.

5.4 Disclaimer

None of our remarks are based on new evidence. We work solely with publicly available information only as a principle of methodology. The validity of the information we have used cannot be taken for granted. We combine available information into patterns and, in that manner, try to extract meaning out of disparate snippets of promised information. We believe that reasoning about technology on the basis of incomplete, and sometimes invalid, non-technical and publicly available information, has become important, and requires the development of its own methodologies. Our “promise” that Promise Theory is a helpful tool for such methodologies is implicit in this. The B737 Max MCAS affair provides a compelling and significant case study to that end. The B737 Max MCAS affair merits significant further attention from researchers working with or without the use of Promise Theory.

Notes

\textsuperscript{1}See e.g. the informative YouTube item about the Ethiopian Airlines flight 302 crash on March 10, 2019, in \url{https://www.youtube.com/watch?v=_T5xHzZjPQ}.

\textsuperscript{2}In any complex system, like flight operations, there is a network of causation that might be ambiguous and tricky to decompose. Nonetheless, certain factors present themselves at heart of such a network, and play a decisive role. Software is clearly such a issue, as basic and commonly held software engineering principles were violated.

\textsuperscript{3}Feature interaction was noticed as a complicating factor in the area of telecom software, see for instance \textsuperscript{[19]}. Original work on feature interaction had a focus on unforeseen and unintended phenomena, oversights of designers, whereas more recent work on feature interaction has a focus on the handling of conflicting actuator inputs (see \textsuperscript{[20]}). In the B737 case conflicting actuator inputs were in fact present if, during an MCAS intervention, the pilots tried to use the handle on the yoke to control the stabilizer pitch. Following the interpretation of feature interaction of \textsuperscript{[20]} the B737 MCAS algorithm affair is in part about a potentially unsatisfactory solution that was chosen for a specific feature interaction problem.

\textsuperscript{4}We feel that it is not necessarily a service to the discipline of software engineering not to be fully open about the possibility that software engineering was at fault, just as much as it is not a service to piloting not to be open to the possibility that pilot errors were involved. There is a double bind involved, however: what may be healthy for the field at large may not be good news for some of its specialists. In other words for an author loyalty to software engineering as a discipline and loyalty to its specialists may diverge. We start out with a loyalty to software engineering, and a loyalty by default to software engineers, which gives an incentive only to accept that software engineering was at fault if there are convincing arguments for that hypothesis. In this paper we cannot confirm or disconfirm any hypothesis to that extent, but we make some progress towards understanding how that might eventually be done.

\textsuperscript{5}Again, we use the definitions of error, flaw, and fault from \textsuperscript{[2]}.

\textsuperscript{6}The conclusions drawn by Choi in \textsuperscript{[5]} may be counter-intuitive for a computer scientist: it is the established failure of computer science to provide an empirical or scientific foundation for developing safety critical software which apparently justifies the transition to a legal regime as it is used for doctors and lawyers. Every bug in the software is trivial in hindsight, but the fact of the matter is that no software process guidelines have been put forward which guarantee the writing of perfect software, just
as no doctor is able to avoid making mistakes, or to cure all patients. Choi emphasizes that writing safety critical software is unlike many other forms of engineering where scientific progress increasingly moves what used to be professional activity out of the legal regime of judging professional malpractice by professional peers, and that for writing safety critical software the methodological scientific progress has demonstrated that moving in the opposite direction would be appropriate.

The reasoning of Choi [5] might, however, also be applied to the results in our paper. This works as follows: assuming that several convincing candidate software process flaws have been spotted, and assuming that further (forensic) investigation would confirm that one or more of these software process flaws have actually occurred during the construction of the MCAS algorithm and the corresponding implementing software component. Now following Choi: a survey of industrial practice would plausibly lead to the conclusion that, in most software processes, flaws frequently occur; that principled methods for avoiding such flaws have not been successful in the past and have not guaranteed the construction of high quality software, however defined. Thus, it is only after the fact that these software process flaws have been spotted, and the suggestion that successfully avoiding software process flaws is what competent programmers can and should do is unrealistic. Software engineers deserve to be protected from accusations based on such illusions, and that protection is what a legal professional status may bring them. The downstream principle for responsibility, in Promise Theory, makes it clear that no upstream promise should be taken as gospel under any circumstances.

For the a discussion of the downstream principle for responsibility in Promise Theory we refer to Paragraph 13.6.3 in [18].

7For references to Fault Tree Analysis, see [10, 21]. An extensive discussion of propositions in forensics can be found in Bergstra [22].

8See http://www.boeing.com/737-max-updates/en-ca/ and the FAQ list on that page. From other sources we infer that the system architecture will be redesigned in order to have the two on board computers working permanently parallel, a modification which as far as we understand is unrelated to the MCAS affair, although the original alternating scheduling of operation also determines which of the two AOA sensors is regularly being inspected by MCAS, a mechanism which would then change.

9Clearly Boeing did not express any of this in the format of promise theory. In fact we express a meta-promise that we consider Boeing to have made the promise B-Max-New.

10This wording is typical in informal usage. In Promise Theory, this is only a promise, as there is no such thing as a guarantee in practice.

11After ignoring pilot errors (including training and pilot certification deficiencies), and also maintenance problems (each of which may or may not have causally contributed to either of the accidents, what remains is the algorithm’s intent.

12This is standard software practice for quorum determination in redundant algorithms.

13CFR (Code of Federal Regulation), title 14 (Aeronautics and Space), part 25 (Airworthiness Standards For Airplanes In The Transport Category, including the B737).

14This option is vacuous, however, if MCAS has an essential role to play in obtaining certification as mentioned in Promise 1.2. Wendel [7] discusses this issue in some detail and claims that use of AOA values from AOA sensors is common place in military aviation but not in commercial jetliners.

15The Mach number expresses speed w.r.t. air rather than ground, relative to the speed of sound.

16The B737 Max has no switch which allows pilots to disable MCAS, which is understandable from the perspective that pilots are not even supposed to know of the existence of MCAS.

17Website about the B737 maintained by Chris Brady: www.b737.org.uk; the information on the site is also available in book format.

18Although the engineers knew that the background for MCAS was based on anti-stall, from the Pegasus design.

19On 4 October 1992 a Boeing 747-200F freighter aircraft (El Al flight 1862) came down in Amsterdam. Years later it was concluded on the basis of extensive research that with unconventional methods, there are indications that the plane and its crew, not to mention the civilian victims, might have been saved by its pilots, Smaili et. al. [23]. This remarkable result does not imply that the pilots should or could have discovered a recovery strategy in real time. The work indicates, however, that determination of the flight envelope of a damaged aircraft is a highly non-trivial matter, which may produce surprising results when pursued systematically.

20A conceptual difficulty in this matter is whether or not repeated MCAS intervention based on false positive AOA sensor input may or can be classified as a stabilizer runaway for which classic B737 problem handling procedures apply.

21Perhaps the requirements on non-certifiability can or should be relaxed by dropping the requirement that it is done within the B737 NG type rating. That is a rather strict requirement, and it is not self-evident that software engineers would have the task to achieve that sort of objective.

22It may be an open question whether or not an update MCASu of MCAS can be found at all (i.e. in principle) such that B737 Max-min[MCASu] (with standard AOA disagreement warning to pilots) could be certified by the FAA under the same type rating as the B737 NG, where certification is performed in compliance with the highest currently available standards of certification methodology. The quality criterion on certification may involve improvements on the certification process which has been in place for the B737 Max, but which does not require the introduction of a completely novel certification methodology, a path which has been suggested in [24]. It would be unreasonable to ask for a (mathematical) proof that no adequate update MCASu of of MCAS can be found, even if that were the case. Indeed there are no practical proof systems for the non-existence of algorithms or of implementations algorithms.

23Some software process flaws, candidate or actual, clearly don’t matter for the case at hand, for instance if the testing coverage of a software component taking care of unfolding the landing gear is considered insufficient, there is a process flaw, even if no
defect has been missed which might have been detected by means of a check against a proper test suite, but such a flaw cannot possibly have any relation to the two accidents from which the MCAS affair has arisen.  

24 We refer to [25] and [26] for multithreading with strategic interleaving.  

25 Not all problems may necessarily be amenable to this mode of solution, but we shall assume they are in this case.  

26 For the notion of an instruction sequence and the consequences of requiring finiteness thereof we refer to [27]. Our definition of an algorithm is somewhat more specific than most definitions conventionally used in computer science, e.g. in Corman et. al. [28].  

27 This is what is known as the separation of interior and exterior promises in Promise Theory.  

28 It would be unreasonable to hold the existence of an RAD emerging from Boeing against Boeing, because it can hardly be assessed by an outsider how difficult it has been to come up with and to validate its practical feasibility.  

29 Wendel [7] provides convincing arguments why the feature has been offered by Boeing as an optional one.  

30 A growing base of documented experience is emerging from DNA related cases where likelihood transfer mediated reasoning is used quite often (see also [22]).  

31 The actual values of prior odds and of posterior odds as used and computed in a case, as well as height of the threshold for the posterior odds which is used by the court to arrive at a discrete decision (yes or no), is all private to the court and will not be communicated to any other participant in the process. The likelihood ratio, however, as communicated by the experts to the court is included in the formal proceedings of the case.  

32 In [11] software is to be defined by so-called meta-algorithms, a role which we ascribe to algorithms proper. We mention that (in [11]) following ISO terminology software is considered an intellectual creation which is independent of the medium on which it is represented. Following our definition of an algorithm (see Paragraph 2.4), the algorithm is an intellectual creation which is even more abstract than software components (a part of software), and more abstract than the documentation of software components (another constituent of software) it is independent from the precise form of its various documentations, and for that reason and by definition not a part of the software.  

33 In Bergstra [29] the number of instructions of an instruction sequence $X$, denoted $LLOC(X)$ for “logical lines of code of $X$” is used as a metric for its size.  

34 A definite need to be open about the existence of MCASu would arise if the idea is that pilots would apply some form of reinforcement learning in order to minimize the number of MCASu interventions on the long run.

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A False positives and false negatives

We assume that AOA sensors produce a rational number in the range \([a_{\text{low}}, a_{\text{high}}]\), for which after normalization we will use the range \([0, 1]\). Further we assume that there are sensors AOA\(_L\) (left AOA sensor), and AOA\(_R\) (right AOA sensor), and there may be an additional sensor AOA\(_3\) (third AOA sensor), or even more AOA sensors (AOA\(_4\) etc.).

With AOA we denote at any time the actual angle of attack, while AOA\(_s\) denotes the value as measured by sensor \(s\). We assume that \(a \in (0, 1)\) is the threshold value beyond which the angle of attack is considered too high. The basic idea is that if sensor \(s\) is used and AOA\(_s\) > \(a\) the input is given to MCAS (or any variation of it) that it must come into action and change the pitch of the trim for some duration and angle.

A.1 Working with a single sensor \(s\)

In the B737 Max in an alternating fashion per flight a different sensor is used. During a given flight that may, say AOA\(_L\). We say that reading AOA\(_s\) > \(a\) constitutes a positive AOA result on sensor \(s\). If sensor \(s\) is flawless then we write AOA\(_s^+\) for the sensor (the superscript indicates that the sensor is perfect). We find AOA\(_s^+\) > \(a\) \iff AOA > \(a\), and the probability of false positives (AOA\(_s^+\) > \(a\) while AOA \(\le\) \(a\)) is 0 just as the probability of false negatives (AOA\(_s^+\) \(\le\) \(a\) while AOA > \(a\)) is 0.

Now there may be a probability \(f\) that a sensor is defective. With Defect\(_s\) we denote that sensor \(s\) is defective. We will assume that the sensor output for a defective sensor is randomly distributed over the interval \([0, 1]\) in such a manner that for \(p < q \in [0, 1]\): \(P(\text{AOA}_s \in [p, q] | \text{Defect}_s) = q - p\). (Here \(P(A|B)\) denotes the conditional probability of \(A\) relative to \(B\).) We further assume, by way of a simplification, that if a sensor is not defect it produces the right value, and therefore two non-defect sensors will produce the same value.

The probabilities for a false positive reading of \(s\) and for a false negative reading of \(s\) now both are non-zero:

\[
P(\text{AOA}_s > a | \text{AOA} \leq a) =
\]

\[
P(\text{Defect}_s) \cdot P(\text{AOA}_s > a | \text{Defect}_s) + P(\neg \text{Defect}_s) \cdot P(\text{AOA}_s^+ > a | \text{AOA} \leq a) = f \cdot (1 - a),
\]

and

\[
P(\text{AOA}_s \leq a | \text{AOA} > a) =
\]

\[
P(\text{Defect}_s) \cdot P(\text{AOA}_s \leq a | \text{Defect}_s) + P(\neg \text{Defect}_s) \cdot P(\text{AOA}_s^+ \leq a | \text{AOA} > a) = f \cdot a.
\]

Assuming for a moment that MCAS is an anti-stall device, and that stalling is considered a significant problem, then false negatives constitute a fundamental problem, because in those cases MCAS intervention might prevent a stall, and might prevent an accident, and it appears that a significant risk of false negatives arises. Under the assumption that MCAS interventions are merely a help for the pilot but inessential in terms of flight safety, primary attention must be paid to false positives, for which there is a non-trivial risk just as well.

A.2 Working with two sensors

Having two sensors available several scenario’s exits for combining two readings into adequate input for MCAS.

(i) conjunctive reading: one may require that both sensors show a positive reading and only then communicate a positive reading to MCAS,

(ii) disjunctive reading: one may require that at least one of the sensors shows a positive reading,

(iii) guarded reading: one may require that the readings of both sensors don’t disagree too much, for instance it may be required that \(|\text{AOA}_L - \text{AOA}_R| < d\) for a chosen threshold \(d \in (0, 1)\), and if the disagreement is larger to infer that at least one of the sensors is defective and for that reason to disable MCAS for the rest of the flight, while if the disagreement is below the chosen threshold to use in an alternating manner either of both sensors (with two sensors it is difficult to guess which one is defective,
though some informative guesswork is conceivable if one assumes a probability distribution for the actual value of AOA which may be computed from recent flight data).

Because we have assumed that a non-defect sensor yields the correct rational output will full precision, the criterion for a sensor being defect simplifies to $\text{AOA}_L \neq \text{AOA}_R$.

The probabilities for false positives and negatives are different each of these cases.

(i) We write $\text{AOA}^{2c}$ for the conjunctive reading of both sensors. In this case we find that a false positive can only arise if both sensors fail:

$$P(\text{Defect}_R | a | \text{AOA} \leq a) = P(\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot (1 - (1 - P(\text{AOA}_L \leq a | \text{Defect}_L)) \cdot (1 - P(\text{AOA}_R \leq a | \text{Defect}_R))) +$$

$$P(\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot P(\text{AOA}_L \leq a | \text{Defect}_L) \cdot P(\text{OA}_R \leq a | \text{Defect}_R) =$$

$$f^2 \cdot (1 - (1 - a)^2) + 2 \cdot f \cdot (1 - f) \cdot a =$$

$$f \cdot a \cdot (4 - a - 2 \cdot f) > f \cdot a$$

This value exceeds $f \cdot a$ as $a < 1$. We find that a conjunctive reading increases the probability of a false negative outcome.

(ii) We write $\text{AOA}^{2d}$ for the disjunctive reading of both sensors. In this case we find that a false positive can occur if at least one of the sensors fails:

$$P(\text{AOA}^{2d} \supset a | \text{AOA} > a) =$$

$$P(\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot (1 - (1 - P(\text{AOA}_L > a | \text{Defect}_L)) \cdot (1 - P(\text{AOA}_R > a | \text{Defect}_R)) +$$

$$P(\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot P(\text{AOA}_L > a | \text{Defect}_L) +$$

$$P(-\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot P(\text{AOA}_R > a | \text{Defect}_R) =$$

$$f^2 \cdot (1 - a^2) + 2 \cdot f \cdot (1 - f) \cdot (1 - a) =$$

$$f \cdot (1 - a) \cdot (1 + a) + 2 \cdot f \cdot (1 - f) \cdot (1 - a) =$$

$$f \cdot (1 - a) \cdot (f \cdot (1 + a) + 2 \cdot (1 - f)) > f \cdot (1 - a)$$

The probability of a false positive has increased w.r.t. the case of reading a single AOA sensor only (assuming $f > 0$).

For false negatives we find that the probability has increased:

$$P(\text{AOA}^{2d} \supset a | \text{AOA} > a) =$$

$$P(\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot (1 - (1 - P(\text{AOA}_L \leq a | \text{Defect}_L)) \cdot (1 - P(\text{AOA}_R \leq a | \text{Defect}_R))) =$$

$$f^2 \cdot (1 - (1 - a)^2) = f^2 \cdot (2 - a - a^2) = f^2 \cdot a \cdot (1 - a) < f \cdot a.$$

(iii) In the case that it is checked first that both sensors disagree too much, we may say that such a finding potentially contributes to the false negatives. We write $\text{AOA}^{2Leq}$ for the reading protocol which first checks that outputs are equal, and if not returns the result “negative” to MCAS and otherwise reads the sensor $\text{AOA}_L$ compares the result with $a$ and sends that result to MCAS. Now we find for false negatives:

$$P(\text{AOA}^{2Leq} \supset a | \text{AOA} > a) =$$

$$P(\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot P(\text{AOA}_L \leq a | \text{Defect}_L) +$$

$$P(\text{Defect}_R) \cdot P(-\text{Defect}_L) \cdot P(\text{AOA}_L \leq a | \text{AOA} > a) +$$

$$P(-\text{Defect}_R) \cdot P(\text{Defect}_L) \cdot P(\text{AOA}_L \leq a | \text{AOA} > a) +$$

$$P(-\text{Defect}_R) \cdot P(-\text{Defect}_L) \cdot P(\text{AOA}_L \leq a | \text{AOA} > a) =$$

$$f^2 \cdot a + (1 - f) \cdot f \cdot a = f \cdot a$$

It now appears that until a disagreement has been found the probability for a false negative is the same as with the protocol that does not check for an agreement between both sensors in advance of returning a positive result.

A reason for not making this choice, however lies in the probability $P^\neq$ that sensor disagreement will switch off MCAS: $P^\neq = 1 - (1 - f) \cdot (1 - f) = f \cdot (2 - f)$ which is quite a significant probability that MCAS will be out of use.

It is the latter probability which one may reduce by making use of majority voting with three sensors, in
which case the probability of not finding two equal sensor outputs is \(3 \cdot (1 - f) \cdot f^2 + f^3 = f^2 \cdot (3 - 2 \cdot f)\) which is much less for say \(f = 10^{-6}\).