Direct observation of a dynamical glass transition in a nanomagnetic artificial Hopfield network

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Spin glasses, generally defined as disordered systems with randomized competing interactions, are a widely investigated complex system. Theoretical models describing spin glasses are broadly used in other complex systems, such as those describing brain function, error-correcting codes or stock-market dynamics. This wide interest in spin glasses provides strong motivation to generate an artificial spin glass within the framework of artificial spin ice systems. Here we present the experimental realization of an artificial spin glass consisting of dipolar coupled single-domain Ising-type nanomagnets arranged onto an interaction network that replicates the aspects of a Hopfield neural network. Using cryogenic X-ray photoemission electron microscopy (XEPEM), we performed temperature-dependent imaging of thermally driven moment fluctuations within these networks and observed characteristic features of a two-dimensional Ising spin glass. Specifically, the temperature dependence of the spin glass correlation function follows a power-law trend predicted from theoretical models on two-dimensional spin glasses. Furthermore, we observe clear signatures of the hard-to-observe rugged spin glass free energy in the form of sub-agings, out-of-equilibrium autocorrelations and a transition from stable to unstable dynamics.

Artificial spin ice simulates the geometric frustration of molecular spin ice via patterned magnetic thin films. Each nanomagnet in the pattern acts as a single Ising spin, and the collective behaviour of these magnets leads to phenomena such as emergent interacting magnetic charges, topologically constrained ordered and a variable effective dimension. Despite this long list of success stories with artificial spin ice systems, the realization of an artificial spin glass system has remained elusive. The main challenge has been to design arrays of nanomagnets with a dipolar interaction network that leads to spin glass behaviour. The use of Gaussian-type disorder in arranging Ising-type nanomagnets onto a two-dimensional (2D) plane determined that, despite a balancing competition between ferro- and antiferromagnetic interactions, a spin glass phase is inaccessible at finite temperatures. Employing the concept of effective dimensionality in interacting networks and theoretical predictions that a spin glass phase could only be stabilized at finite temperatures when a critical effective dimension of 2.52 is surpassed, it was shown that tree-like nanomagnetic patterns with elevated effective dimensionality are a successful strategy to increase the effective dimension well above this critical value. However, fabricating extended and quasi-infinite tree-like structures currently remains an unsurpassable challenge.

In this Letter we seek to realize the characteristics of an artificial spin glass by implementing a proof-of-principle Hopfield neural network—a model of associative memory with ties to the Ising model—to guide the disorder of artificial spin systems. Conceptually, associative memory does not require a perfectly identical scenario to identify a memory. Hopfield networks are dynamical systems that evolve towards memories when their inputs are within a neighbourhood of those memories. The memories of these networks correspond to ground states of a spin system and are robust to noise (Fig. 1a and Supplementary Fig. 1). This robustness corresponds to a broad basin of attraction surrounding the spin system’s ground state, allowing the system, in theory, to relax towards the ground state at non-zero temperatures.

We fabricated nanomagnetic Hopfield networks (Methods) consisting of permalloy (Ni80Fe20)—stadium-shaped, Ising-type nanomagnets with length \( L = 300 \text{ nm} \), width \( W = 100 \text{ nm} \) and thickness \( t = 2.7 \text{ nm} \) (Fig. 1c). The dimensions of the nanomagnets were chosen to ensure thermally driven moment reorientations to occur on the timescale of a few seconds at a blocking temperature of \( T_s = 110 \text{ K} \). The sample was kept in vacuum at room temperature for several weeks to allow the Hopfield networks to relax towards equilibrium low-energy states before transfer into the photoemission electron microscope (XEPEM) for magnetic imaging, employing X-ray magnetic circular dichroism (XMCD) at the Fe L\(_3\) edge. For XEPEM, the sample was cooled to 105 K (below the blocking temperature), then imaged to observe the frozen-in low-energy state achieved after thermal annealing (Fig. 2a). The sample was then heated to 120 K to begin our real-time temperature-dependent observations of thermal fluctuations (Supplementary Videos 1 and 2).

As a first characterization step, we extracted the temperature-dependent dimensionless magnetic susceptibility \( \chi(T) \) (Methods), where \( \chi \) is the magnetic susceptibility, \( k_B \) is the Boltzmann constant, and \( m \) is the magnetic moment of a single nanomagnet, and plotted its inverse as a function of temperature (Fig. 3a). Extracting this directly from the spin orientations allows us to filter out the bulk properties of the constituent permalloy and directly analyse the emergent properties of the nanomagnet network. Fitting this temperature dependence to a Curie–Weiss law, \( \frac{1}{\chi(T)} = \frac{T - T_C}{A} \), we found that the effective dimension \( D_{\text{eff}} \) of our artificial spin glass was comparable to that of a random system of Ising spins, indicating the presence of a dynamical glass transition.
where $A$ is the Curie constant,(green dashed line, Fig. 3a), revealed a system-related Curie temperature $T_{C_{\text{Hopfield}}} = 27.6 \pm 15.7$ K, which is far below the blocking temperature $T_g = 110$ K of the patterned nanomagnets and their material-based Curie temperature $T_C$ (Methods and Supplementary Fig. 4). This confirms that competing interactions are well randomized in these Hopfield networks. Despite the large error, a non-zero transition temperature suggests the effective dimension may be above two owing to the long-range interactions in the system, making the interaction graph non-planar.

In bulk experimental spin glass, a comparison between field-cooled and zero field-cooled systems typically shows signatures of a spin glass. Here we provide a more direct characterization of the thermodynamics to explore spin glass behaviour in these artificial Hopfield networks. We extracted both the standard spin correlation function $[C^*(r)]_{\text{nuc}}$ and the unbiased spin glass correlation function $[\Sigma^*(r)]_{\text{nuc}}$ (Methods) and plotted them as a function of distance (Fig. 3b). We then fit these correlation functions with a spatial decay function in the form of $e^{-r/\nu}$ and $e^{-r/v}$, with $L(T)$ and $L_{SG}(T)$ being the temperature-dependent standard (blue squares, Fig. 3b) and spin glass correlation lengths (red asterisks, Fig. 3b), respectively. We fit the temperature dependence of these correlation lengths to a power law of the form $f(T) = B (T - T_c)^{-\nu}$, where $T_c$ is the Curie temperature and $B$ is the scaling constant, by performing a linear least-squares fit of the linearized form of the equation, $\log_a f(T) = -\nu \log_a(T - T_c) + \log_a B$ (blue and red dashed lines, Fig. 3b), and calculated a standard critical exponent $\nu = 0.143 \pm 0.482$ and a spin glass critical exponent $\nu_{SG} = 3.72 \pm 1.04$, where the errors are calculated through the standard error of the regression. The latter value comes close to the critical exponent $\nu_{SG} = 3.559 \pm 0.025$ predicted for a 2D Ising spin glass, indicating that our artificial Hopfield networks are ordering towards a spin glass transition.

The dynamics of spin glasses vary substantially when two factors are changed, whether the system is in or out of equilibrium and whether it is above or below the glass transition. To complicate matters further, evidence suggests that there is not simply a single, fixed glass transition. Often, there is a second dynamical transition temperature. This usually exceeds the ‘static’ critical temperature and is characterized by shifting peaks of a.c. susceptibility in experiments, which occurs in part because of an increasingly prominent ‘memory’ of previous states resulting from the slow exploration of phase space. Computational studies observe this transition through how different initial states maintain a finite overlap with one another over time, settling into distinct regions in phase space. Others characterize this as a transition from high-temperature chaotic dynamics to low-temperature stable dynamics. Here we employ an analysis of the system's autocorrelation function, its imperfect power law decay and the Lyapunov exponent (Methods), all as a function of temperature.

Signatures of the system's state may be found directly from the two-point autocorrelation function (Methods). Both the general shape of the function and the critical exponent resulting from a power-law fit can help categorize the system. The log–log plots of the autocorrelation function (Fig. 4a) all decrease in slope over time, indicating a variable critical exponent and, by extension, that the system has not yet relaxed to equilibrium. The critical exponent itself (Fig. 4b) reinforces this conclusion, as it is notably lower than the minimum values predicted in equilibrium, $\nu(T) = 0.395$ (ref. 11) or $\nu(T) = 0.5$ (ref. 12). Notably, non-equilibrium autocorrelations are often flatter with less time elapsed.

To extract more information about the chaoticity of the system, we studied the Lyapunov exponent from the spin dynamics. Transitions from stable to chaotic behaviour begin when similar trajectories through phase space diverge exponentially and continue to diverge despite the phase space being bound. The time rate of the exponential behaviour, the Lyapunov exponent, is positive when the system is unstable, potentially chaotic, and negative when the system is stable. Using a data-driven method (Methods), we find similar initial paths and use their average distance over time to extract the Lyapunov exponent for each temperature. The exponents transition from negative values at low temperatures to positive values at high temperatures (Fig. 4c), which is consistent with a dynamical transition.

Assessing the system's time-independent measures holistically, the dominance of the spin glass correlation length over the standard correlation length and its temperature dependence are hallmarks of a system with a glass ground state. Despite the system ordering as indicated by the increasing magnetic susceptibility with decreasing temperature (Fig. 3a), the standard correlation lengths (Fig. 3b, blue squares) are essentially noise. The power-law fit determines that $\nu = 0.143 \pm 0.482$, confirming that the standard correlation function can no longer determine the order parameter. On the other hand, the spin glass correlation length grows rapidly as the system is cooled (Fig. 3b, red asterisks). Its power-law fit produces a critical exponent of $\nu = 3.72 \pm 1.04$, which, within the uncertainty range, matches the theoretically known value for a 2D spin glass, $\nu_{SG} = 3.559 \pm 0.025$ (ref. 13).
A dynamical analysis indicates a non-equilibrium temporal correlation and a dynamical transition, which supports the hypothesis of a rough free-energy landscape. The exact temperature dependence is non-universal, but the autocorrelation function of many spin glasses decays with a power law with an exponent \( \nu(T) = 0.5 \) at the Alameda–Thouless line in equilibrium and \( \nu(T) = 0.395 \) for the Edwards–Anderson model. Experimental results and out-of-equilibrium simulations find that the critical exponent varies over long periods of spin glass aging. It is common for \( \nu(T) \) to start small as the system initially explores the phase space (sub-aging) and then increases as a path towards lower energy states is found (aging). The exponent \( \nu(T) \) varies earlier when the temperature is higher. Combining this observation with the fact that all \( \nu(T) \) values (Fig. 4b) are far below anything predicted by equilibrium theory suggests that our system is out of equilibrium at all temperatures and relaxing in the sub-aging regime. Furthermore, the faster relaxation of the higher-temperature systems allows the systems to leave the sub-aging regime more quickly, resulting in more variable slopes and increasing the \( \nu \) determined by the fit as it increases over time. Aside from this continuous evolution towards faster relaxation...
from sub-aging, there is another prominent trend in the autocorrelation functions. At 157 K and above, the values of the autocorrelation function remain relatively similar despite a decreasing slope. However, as the temperature drops between 157 K and 147 K, systems more rapidly diminish in their average autocorrelation, then slowly increase in their average autocorrelation after this initial decrease in temperature. The secondary increase in autocorrelation is probably due to lower fluctuation rates of the magnetic moments, but the initial dramatic decrease between 157 K and 147 K seems to arise from a dynamic transition. The Lyapunov exponents and the rough free-energy landscape of spin glasses further solidify this conclusion.

The Lyapunov exponents increase with increasing temperature (Fig. 4c), showing a tendency for similar initial states to diverge as the system heats up. The system transitions from stable dynamics ($\lambda < 0$) to unstable dynamics ($\lambda > 0$) around 157 K, the same temperature where average autocorrelation jumps dramatically. This is consistent with the system settling into deeper free-energy minima, after losing enough energy to no longer traverse a broader section of phase space (Supplementary Video 3), increasing the rate of relaxation and grouping together similar trajectories in the same basin. This convergence of trajectories explains the energetic origins of both dynamic transition and memory in spin glass, especially when considering that the basin is probably centred around a state encoded into the underlying Hopfield network. Overall, the varying relaxation over time and a rough free-energy landscape are consistent with the behaviour of spin glasses.

As annealing for these systems is further improved, direct real-space studies and investigation of the spin glass ground state will be accessible and assist in our understanding of equivalent non-deterministic polynomial-time hard problems (NP-hard) and brain science models. Further experiments on artificial spin glasses and a more faithful representation of annealing-based Ising model problems, such as determining graph isomorphism and optimal binary linear programming, have already been shown that nanomagnetic systems may potentially approach the Landauer limit at room temperature and make excellent candidates for low-energy computing.

### Online content

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41567-022-01538-7.

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**Fig. 4 | Dynamical behaviour of a nanomagnetic Hopfield network.**

**a.** Two-point autocorrelation function plotted on a log–log plot for all lattice temperatures, with error bars generated from the standard deviation of the mean. If the anticipated power-law decay is observed, the plots will be linear.

**b.** Decay power $\nu(T)$ fit from the autocorrelation function of the form $C(\tau) = C_0|\tau|^{-\nu(T)}$, with the standard error of fit represented by error bars.

**c.** Lyapunov exponent of moment dynamics plotted versus temperature. Error bars result from the standard error of fitting the exponents.

\[
\nu(T) = \frac{1}{T} \ln \left( \frac{1}{t_{\text{max}} - t} \right)
\]
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Methods

Designing a nanomagnetic Hopfield network. Both Hopfield networks and Ising spin systems evolve as governed by their ‘interaction’ networks. Here we describe how these networks are defined and how they may be modified computationally to match one another before fabrication.

A Hopfield neural network of size $N$ is represented by a vector $S^m$ of binary states ($-1$ and $1$) at iteration $m$. A connectivity matrix $w_{ij}$ governs its dynamics via the rule

$$S^{m+1}_i = \sum_{j=1}^{N} w_{ij} S^{m}_j.$$  

To maintain a binary range, the activation function is defined as $f(x) = \frac{1}{1 + e^{-x}}$, the ‘sign’ function. The connectivity matrix is created from a set of $n$ patterns, $\xi^p$, each labelled by $\gamma$, so that $S^m$ intends to ‘recall’, or grow closer to, over several iterations. The storage is encoded in the connection or weight matrix by the Hebbian learning rule:

$$w_{ij} = \frac{1}{2} \sum_{\gamma=1}^{n} \xi^p_i \xi^p_j.$$  

Here we consider $\xi^p_i$ to be a random vector whose entries are independently drawn from the probability distribution $p(\xi^p_i = 1) = p(\xi^p_i = -1) = 0.5$. Practically speaking, patterns of interest may not take on this form, but a mapping of all bits from a set of patterns $\xi^p$ onto $\xi^q$ is possible if $n < N$. The attractors of this dynamical system are $\xi^p$, making them the ‘memorized’ patterns of the system.

Hopfield showed that the iterative evolution described in equation (1) always decreases an effective Hamiltonian:

$$H_{eff} = -\frac{1}{2} \sum_{\gamma} w_{ij} S^m_i S^m_j.$$  

The Ising Hamiltonian in zero field has a similar form:

$$H_0 = -\frac{1}{2} \sum_{\gamma} \beta S^m_i S^m_j.$$  

$J_{ij}$ here determined by magnetic interactions and is analogous to the connectivity matrix $w_{ij}. How the distributions of these two matrices differ can widely influence what states minimize the system’s Hamiltonian. In artificial nanomagnets, dipolar interaction strength is determined by the distribution of magnetization and the positions and orientations of the nanomagnets for patterned nanomagnetic systems and $\theta^i$ is the binary Ising variable indicating the orientation of the magnetization. To model the exact interaction strength for a collection of nanomagnets with positions $r_i$ and orientations $\theta_i$, we implement the compass needle model:

$$J_{ij} = -\left(\frac{1}{|r_{ij} - r_{ij}|} - \frac{1}{|r_{ij} - r_{ij}|} + \frac{1}{|r_{ij} - r_{ij}|}\right).$$  

Because this model assumes interactions occur between magnetic charges at the ends of the nanomagnets, $r_i$ and $r_j$ are the positions of the positive and negative charge belonging to spin $i$ as determined by the lengths, positions and orientations of the magnets.

To fabricate an Ising system equivalent to a Hopfield network, one must first reduce the difference between $w_{ij}$ and $J_{ij}$ as much as possible (Fig. 1a,b). The scale of each is irrelevant, so they are both normalized by the average absolute interaction strength per neuron or spin. Specifically, $w_{ij}' = w_{ij} N \sum_i |w_{ij}|$ and $J_{ij}' = J_{ij} N \sum_i |J_{ij}|$. We then use machine learning methods to change $\xi^p$ and the positions and angles of a nanomagnetic design to minimize a cost function $C = \sum_{ij} |w_{ij}' - J_{ij}'|$. We determine this through gradient descent of the continuous variables, $r_i$ and $\theta_i$, and relatively quickly reach local minima. The cost function may be further reduced through modification of the discrete pattern states, $\xi^p$, as any states of interest may be mapped onto arbitrary stored patterns. A Monte Carlo Metropolis annealing of ‘energy’ $C$ with ‘spins’ $\xi^p$ is an appropriate method of further reducing the cost function. This is carried out with parallel tempering at 100 separate temperatures, the lowest temperature of which is used for the new $\xi^p$. The overall process of matching the systems progresses by alternating gradient descent and annealing until C converges. The positions and orientations are then used to fabricate our nanomagnetic system (Fig. 1c).

Sample fabrication. Liftoff assisted electron beam lithography was used to generate nanomagnetic Hopfield networks. A 1 X 1 cm$^2$ silicon (100) substrate was first spin-coated with a 70-nm-thick layer of poly(methylpentacrylate) (PMMA) resist. A VISTEC VB300 e-beam writer was then used to define Hopfield patterns with stadium-shaped nanomagnets onto the substrate. Following the development of the exposed resist layer, a 2.7 nm thin permalloy ($NiFe_{81}P_{19}$) film was deposited on the substrate at a base pressure of 1 x 10$^{-7}$ torr, together with an aluminum capping layer of 2 mm to avoid fast oxidation. The substrate was next placed into acetone for liftoff. The resulting nanomagnetic artificial Hopfield networks consisted of nanomagnets with lengths $L = 300$ nm and widths $W = 100$ nm. Considering the strong dependence of the magnetization on the film thickness and temperature, we performed thickness-dependent superconducting quantum interference device (SQUID) magnetometry (Supplementary Figs. 3 and 4) to extract relevant properties such as the saturation magnetization $M_s$ and blocking temperatures for various permalloy thicknesses. For accurate determination of the deposited film thicknesses, a continuous film was deposited in parallel and analysed using X-ray reflectometry (Supplementary Fig. 5).

PEEM. Magnetic imaging was performed at the PEEM endstation of the SIM beamline at the Swiss Light Source (SLS)36, employing XMCD at the Fe L$_3$ edge37. An XMCD image is a result of the pixelwise division of images obtained with circularly left- and circularly right-polarized light. The typical dark and bright contrast is a direct measure of the orientation of a magnetic moment with respect to the incoming X-ray propagation vector. Moments with a non-zero component towards the incoming X-rays will appear dark, while moments pointing in the opposite direction will appear bright (Fig. 2a). Seventy XMCD images were recorded every 14 s at 120, 130, 147, 157, 168, 181 and 196. System-wide time evolution occurred on the order of seconds, as indicated by Fig. 2b.

Spin–spin correlations and magnetic susceptibility. Temperature–dependent spatial spin correlations were extracted using our previously employed method34. The spatial correlation function was calculated according to

$$C(r = \langle S_i S_j \rangle_T,$$  

where $\gamma = \pm 1$ to represent the Ising state of spin $i$, $r_{ij}$ is the distance between spins $r_i$ and $r_j$, and $(\cdots)_T$ denotes a thermal average. The absolute value of this, $C(r) = |C(r)|$, was used for correlation function calculations. All correlation functions corresponding to $\gamma = \pm 1$ and $r_{ij} < \Delta$, where $\Delta$ is the distance between consecutive $r_i$ were averaged to a single value:

$$C'(r) = \frac{1}{\alpha} \sum\alpha C'(r_{ij}).$$  

The decay of the correlation function is expected to follow an exponential function

$$\langle C' (r) \rangle_T = e^{-r^2/\Delta^2},$$  

where $L(T)$ is the standard correlation length, which can also be plotted as function of temperature.

The dimensionless magnetic susceptibility $\chi$ was calculated from this correlation using the fluctuation dissipation theorem. This susceptibility $\chi$ was returned to appropriate dimensions by an additional factor, $m$, the magnetic moment of a single Ising macrospin:

$$\chi(T) = \frac{2m^2}{h} \sum\gamma C(r_{ij}).$$  

For the nanomagnets discussed here, the magnetic moment $m$ is measured using SQUID–vibrating sample magnetometer (VSM) measurements of permalloy structures, finding a magnetization at saturation, $M_s = 800 \pm 21$ kA m$^{-1}$ (Supplementary Fig. 4), and its magnetic moment $m = 7.3 \pm 1.8 \times 10^{-10}$ A m$^2$.

Unbiased spin glass spin–spin correlation and correlation length. The measurements determine $C_{\gamma}$, an estimate for the spin–spin correlation $\langle S_i S_j \rangle$, between all pairs of spins $i$ and $j$. Naturally, there is an uncertainty in the experimental results. Let us write

$$C_{\gamma} = \langle S_i S_j \rangle + e_{\gamma},$$  

Assuming that the system is equilibrated, the error $e_{\gamma}$ is a random variable with zero mean. We say that $C_{\gamma}$ is an unbiased estimator for $\langle S_i S_j \rangle$. Unbiased means that if one repeats the set of measurements many times, then the average gets arbitrarily close to the exact answer. However, for spin glass correlations we need the square of the correlation function (equation (1) of ref. 3). In this case, $C_{\gamma}$ is a biased estimator for the spin glass correlation function$^{36} \langle S_i S_j \rangle^2$ because

$$C_{\gamma} = \langle S_i S_j \rangle^2 + 2 \langle S_i S_j \rangle e_{\gamma} + e_{\gamma}^2$$  

and the term $e_{\gamma}^2$ has a non-zero mean. As a simple example, suppose that $C_{\gamma}$ is obtained from just one spin configuration. Then $C_{\gamma}^2 = \langle S_i S_j \rangle^2 = 1$, for all pairs. Hence summing over all pairs to give the spin glass susceptibility gives a completely wrong result. However, even if $C_{\gamma}$ is obtained from just one spin configuration, summing $C_{\gamma}$ over all pairs to get the ferromagnetic susceptibility gives a result that, although having quite large error bars, is nonetheless unbiased. From the spin configurations, we can calculate

$$\langle S_i S_j \rangle \text{ estimated from } C_{\gamma} = \frac{1}{N} \sum_{r=1}^{N} \langle S_i S_j \rangle (r_t).$$  

These exponents were calculated at every temperature and are plotted in Fig. 4c. The exponents increase with temperature, transitioning from negative to positive values between 157 K and 168 K. This indicates a leap into chaotic or unstable behaviour.

Magnetometry measurements. To obtain quantitative information of the magnetic moments of nanomagnets, we patterned five large-area samples (4 × 4 mm²) with thicknesses of 1.9, 2.0, 2.5, 3.3 and 13 nm (measured by X-ray reflectivity measurements; Supplementary Fig. 5). We then measured hysteresis loops, extracting quantitative information about the magnetic moments as a function of temperature for the various thicknesses. The blocking temperature was extracted from temperature-dependent d.c. magnetization curves using a SQUID-VSM. Each sample was cooled to 30 K and the magnetic moment \(M(T,H)\) measured using a sweeping field \(H\) from −750 Oe to 750 Oe at a rate of 500 Oe/s, applied along the long axis of the nanomagnets. This was repeated at an interval step size of 10 K up to 400 K (Supplementary Fig. 3 presents a few typical loops), and from each hysteresis loop we extracted the magnetization at saturation \(M_s(T)\) and coercivity \(H_c(T)\), both as a function of temperature. These measurement series were repeated for all thicknesses of the patterned samples, as well as for a thin-film equivalent of the 1.9-nm patterned film, indicating that patterning does not alter the magnetic properties (Supplementary Figs. 1a and 3b, respectively). The blocking temperature \(T_B\) of each patterned sample can be estimated using two methods. Either the coercivity measurements can be fitted using

\[
H_c = H_0 \left[ 1 - \left( \frac{T}{T_B} \right)^{\frac{1}{3}} \right],
\]

to obtain an estimate of the blocking temperature \(T_B\), or we take the blocking temperature of the nanomagnets when \(H_c(T = T_B) = 0\) (Supplementary Fig. 4b and c, respectively). The blocking temperature is dependent on the measurement technique and its characteristic timescale, and the PEEM blocking temperature is within the range coming from the two methods, usually close to the one obtained by fitting \(H_c(T)\). An assessment of deposited permalloy film thickness via X-ray reflectivity measurements, together with thorough thickness-dependent magnetometry, provides an accurate determination of \(M_s(T)\) and the coercivity \(H_c\) as a function of thickness. It should be noted that the saturation magnetization \(M_s\) reported in our current study differs substantially from previously reported values \(^{3b}\) and can therefore be seen as a parameter that can differ depending on which deposition facilities and fabrication processes are employed.

Data availability

The data that support the findings in this study are available in the public repository at https://zenodo.org/record/3879444. Source data are provided with this paper.

Code availability

The code used in this study is available from the authors upon request.

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Author contributions

M.S. and A.F. conceived the project. A.F. designed and performed the experiments with support from K.H., S.P. and A.K. S.D. fabricated the samples. K.H. fabricated the samples...
for SQUID magnetometry and Y.A.B. performed the X-ray reflectivity measurements and fitting. M.S. analysed and interpreted the data with support from F.C. and A.F. M.S. and A.F. wrote the manuscript with input from all other authors. S.v.D., C.N. and A.F. supervised the project.

**Competing interests**
The authors declare no competing interests.

**Additional information**

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