Simulation thermal model of CNC machine tool operating with variable modes

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Abstract. The article presents the approach and the thermal modeling technique of CNC machine tools, implemented at the stage of operation. In the work, as a practical implementation of the fast modeling technology, a simulation method based on the Matlab Simulink application was used. In the work to build a thermal model of the machine tool, we used analytical solutions of the heat conduction equation. This method is applied to the machine tool operating in conditions of variable thermal processes due to varying cutting speeds. The conditions for the coordination of the thermal characteristics of the machine tool at their boundaries, corresponding to the changing spindle rotation speeds, determine the peculiarity of their construction. A feature of the implementation of the mathematical model is the use of experimental modal analysis. In this case, the modal parameters of the approximating functions are determined by solving the optimization problem. To coordinate the adjacent thermal characteristics at their borders, a special condition was adopted. Regardless of the number of temperature modes used in the model, the initial level of the first temperature mode for the next mode of operation of the machine tool was taken to be equal to the achieved level of the thermal characteristic of the machine tool at the last time interval of the current mode of operation.

1. Introduction
Production volume of high precision parts is growing steadily in the general nomenclature of finished products of a modern machine-building enterprise [1]. Analysis of trends in metal-working manufacturing shows that for machine tools of the particular accuracy, machining accuracy is achievable in the range of 1 to 5 µm. Experts predict an increase in accuracy in the range of 0.1 to 1 µm for these machines during the next 10-15 years [2].

High accuracy of machining is associated with increased cutting speeds. During high-speed milling of cast iron, the cutting speed is 50 m/s; when machining steel is 30 m/s; when machining aluminum alloys is 100 m/s [3]. High cutting speeds are accompanied by high rotational speeds of the spindle. Rotational speeds of the spindle in the range from 10,000 to 30,000 rpm are typical for modern machining centers, therefore the questions of the temperature error are actual [4]. The actual temperature error can exceed 60 microns for modern CNC machine tools [4,5]. Achieving the maximum precision of machining is realized due to minimization of the influence of all arising factors during cutting.

According to the estimates of experts in the field of machine tools design, progress in ensuring the accuracy of machining is largely determined by existing methods of modeling [2]. The effectiveness of the modeling methods used to assess the accuracy of the machining is based on the established relationships between design and technological input parameters and output parameters of the accuracy of the machine tool or technological system.
Modeling is carried out at various stages of the life cycle. The stages of design, manufacture and operation are the most important when creating new machine designs. The efficiency of the created structures is largely determined by decisions in the early design stages of the machine tools. The current design stage of optimal constructions is 3D modeling and computer-aided engineering of the machine tools structures [6-8]. At the stages of manufacture and operation for any machine tools, shortcomings are identified. Simultaneously with the implementation of experimental studies, mathematical and computer models are built to eliminate the identified deficiencies. At the stage of machine tools operation, it is most convenient to use not cumbersome finite element models, but models based on thermal characteristics in the form of simple functional dependencies. This allows them to be used both in diagnostic systems and in automated systems of temperature errors compensating for the machine tool. In this case, the use of thermal characteristics, built on the basis of experimental data for the machine tool under study, provides the minimum setting error of the compensation system installed on the machine tool. At this stage, the methods of processing experimental data represented by regression analysis, experimental modal analysis, and artificial intelligence methods (such as Artificial Neural Network, Genetic Algorithms, Ant Colony Optimization, Artificial Immune Systems and Particle Swarm Optimization) are of particular importance [9-20]. In this case, one of the trends in the field of artificial intelligence is the construction of new methods based on combining previously studied methods. For example, researchers combine methods used to analysis of gray systems with methods for constructing artificial neural networks. This allows you to effectively use the advantages of each of the previously known methods and reduce the level of their disadvantages [21-23].

In thermal modeling of machine tools, it is necessary to solve the problems from various areas of mathematics and physics. The methodology of computer modeling, which combines methods from different fields of knowledge, characterized by a highest degree of formalization is relevant to the production in the conditions of implementation of the digital economy. The practice of equipment operation shows that at this stage, fast modeling technologies, which allow one to quickly build models, and on the basis of the obtained results, introduce corrective actions into the operation of various automated systems providing the thermal stability of the machine tool are the most demanded.

Simulation modelling based on the application of Simulink Matlab are beginning to gain more popularity among researchers in the field of thermal modeling as fast modeling technology [24-26].

2. The Modeling Method

2.1 Theoretical principles

An analysis of the possibilities of the Simulink interactive environment has shown that fast modeling technology can be implemented in three ways: using analytical solutions of the heat conduction equation; solving the system of differential equations by the built-in methods of Simulink; using the transfer functions apparatus.

Each of these methods leads to the need to develop independent procedures for practical implementation. In this paper, the procedure for constructing the thermal model of temperature error in machine tool based on analytical solutions to the heat conduction equation is considered.

Fast modeling technology with building thermal characteristics of the machine tool working in conditions of variable thermal processes due to the variable cutting speeds is of particular relevance. The matching conditions at the boundaries of the thermal characteristics corresponding to the changing spindle rotational speeds of the machine tool are a feature of the construction of such thermal characteristics. The complexity of matching is due to the fact that the solutions of two parametrically different thermal processes converge at one point (in time).

We have considered the construction of the temperature characteristic for a machine tool working according to the scheme: at a rotation speed of 250 rpm, the machine tool runs for 10 min; at a rotation speed of 1000 rpm – 40 min; at a rotation speed of 3000 rpm – 40 min; at a rotation speed of 5000 rpm – 40 min; return to the rotation speed of 3000 rpm – for 40 min; completion of the machine tool at a rotation speed of 1000 rpm within 120 min.
The implementation of this modeling way is based on the use of experimental modal analysis [13,25], when the thermal characteristic can be represented as:

\[ Y(t) = \sum_{i=1}^{m} \left( x_{i,j} \cdot (1 - e^{-t/x_{2i,j}}) + x_{j,i} \cdot e^{-t/x_{2i,j}} \right) \]  

(1)

where \( x_{i,j} \) is the modal amplitude, \( x_{2i,j} \) is the modal thermal time constant, \( x_{j,i} \) is the initial level of the temperature mode, \( m \) is the number of temperature modes, and \( t \) is the time.

Equation (1) is applicable for constructing the temperature field of a machine tool and describing the behavior of its thermal deformations. This is possible due to the high degree of correlation between temperatures and temperature displacements. Therefore, the dimensions of the modal amplitude and the initial level of the temperature mode are determined by the chosen purpose of equation (1) such as either temperatures or temperature displacements.

Based on the analysis of processes of heating and cooling, equation (1) has the following interpretation. The first of the \( m \) terms determine heating from the zero level to the amplitude, and the following \( m \) terms describe cooling from the initial level to zero.

At the boundaries of adjacent thermal characteristics, the matching condition is taken as: the initial level of the sought-for function (temperatures or temperature displacements) for a subsequent operating mode of the machine tool is equal to the level of thermal features of the machine tool achieved in the last moment of the time interval for the current operating mode. The number of temperature modes in the model can be any. This creates the complexity of the practical implementation of equation (1). From the experiment, we can determine the value of the sought-for function at a concrete point in time, but it is impossible to determine individual parameters \( x_{j,i} \). This means that on the boundary of adjacent thermal characteristics of the initial level a function of temperatures or thermal displacements is specified. In equation (1) the components associated with the implementation of the heating process are always known. The uncertainty introduced by the cooling process leads to additional modeling errors. In practice, equation (1) can be implemented in three ways. Below three versions of equation (1) are given for the second time interval of the operating cycle of machine tool (in the future we will use only the term “operating cycle”):

\( t \in [t_1,t_2] \)

\[ \begin{align*}
(2.a) \quad Y(t) &= \sum_{i=1}^{m} \left( x_{i,H} \cdot (1 - e^{-t/x_{2i,H}}) \right) + Y(t_1) \cdot e^{-t/x_{2i,H}} \quad (2.a) \\
(2.b) \quad Y(t) &= \sum_{i=1}^{m} \left( x_{i,H} \cdot (1 - e^{-t/x_{2i,H}}) \right) + \tilde{Y}(t_1) \cdot e^{-t/x_{2i,H}} \quad (2.b) \\
(2.c) \quad Y(t) &= \sum_{i=1}^{m} \left( x_{i,H} \cdot (1 - e^{-t/x_{2i,H}}) \right) + \tilde{Y}(t_1) \cdot e^{-t/x_{2i,H}} \quad (2.c)
\end{align*} \]

where \( \tilde{Y}(t_1) \) is the experimental value of the initial level of the sought-for function on the second section of the operating cycle; \( x_{2i,H} \) is the modal thermal time constant of the first temperature mode in the second section of the operating cycle; \( x_{22,H} \) is the modal thermal time constant of the second temperature mode in the second section of the operating cycle; \( \tilde{Y}(t_1) \) is estimate of the initial level of the corresponding temperature mode.

In equation (2) subscripts of the Roman numerals correspond to the numbers of intervals of operation of the machine tool with the different spindle speeds. Thus, equations (2.a) and (2.b) describe the thermal characteristics in which the cooling process described by the first and second temperature modes is realized in each section of the operating cycle, respectively. Equation (2.c) is one possible representation of the thermal characteristic in which the cooling in each section of the operating cycle is carried out simultaneously for all modes, or the process of multimodal cooling is realized. For building thermal characteristics according to equation (2.c) you must perform additional calculations:
\[ A_j = \sum_{i=1}^{m} (x_{ij}), b_{ij} = \frac{x_{ij}}{A_j}, \ddot{Y}(t_j) = b_{ij}, Y(t_j), i = 1, ..., m, j = 1, ..., N, \]

where \( A_j \) is a value asymptote \( Y(t) \) of the function achieved in the corresponding section \( j \) of the operating cycle, \( b_{ij} \) is a coefficient of distribution of the initial level between temperature modes, \( N \) is the number of time intervals.

When creating a model in Simulink, the following set of basic components is used to describe the complex operating mode of the machine: source blocks of the modeling; blocks of functions defined by the user for equations (1) and (2); blocks of mathematical and logical operations, subsystems and switches; blocks of presentation of modeling results.

The model in the Simulink environment uses Clock as the source of the time signal. The thermal characteristics are represented by the blocks for specifying the function Fcn, in which equations (1) and (2) are realized. Modal parameters \( x_{ij}, x_{2j}, \) and \( x_{3j} \) are determined by the methods of experimental modal analysis, which is based on the solution of the optimization problem for the objective function as:

\[ J(n, t, x_{1j}, x_{2j}, x_{3j}, ...) = \sqrt{\sum_{k=1}^{n} (Y^2(t_k) - Y(t_k))^2}, \]

where \( Y^2(t_k), Y(t_k) \) are the experimental (measured) and calculated values of the function \( Y \) at fixed instants of time \( t_k \).

The optimization problem is formulated as follows:

\[ \min J(n, t, x_{1i}, x_{2i}, x_{3i}, ..., x_{1m}, x_{2m}, x_{3m}) \rightarrow 0 \]

(5)

Parametric constraints for the modal parameters are given as:

\[ x_{1i,\text{min}} \leq x_{1i} \leq x_{1i,\text{max}}; \]
\[ x_{2i,\text{min}} \leq x_{2i} \leq x_{2i,\text{max}}; \]
\[ x_{3i,\text{min}} \leq x_{3i} \leq x_{3i,\text{max}}; \]

(6)

where \( x_{1i,\text{min}}, x_{2i,\text{min}}, x_{3i,\text{min}}, x_{1i,\text{max}}, x_{2i,\text{max}}, x_{3i,\text{max}} \) are the limit boundaries of the corresponding modal parameters.

The feature of solving the optimization problem (4)-(6) is both the choice of the initial optimization point by specifying some fixed values of the modal parameters, and the formation of limit values in the constraint system (6). Specific values are determined from the accumulated experience of experimental thermal studies for a fixed-size machine tool. This is due to the presence of a strong correlation of the modal parameters with the heat capacity of the machine tool.

Complex mode of operation of the machine tool is specified by blocks Switch Case and Switch Case Action (Matlab system blocks). In the block Switch Case is formed sequence in time as follows:

\[ < t_0, t_j, t_j, ..., t_{n-1}, t_n > \]

(7)

where \( t_j, t_n \) are interval values of time and the time of completion of the machine tool; in the example considered in the article the shutdown time of machine tool is 290 min.

Figure 1 shows the final thermal model of the machine tool, and during the simulation a thermal characteristic is constructed.
Figure 1. The model of formation of the thermal characteristic of the machine tool.

The model incorporates a similar notation for writing blocks Fcn, for example, Fcn_3000 and Fcn_3000_2. In the first case, this means the formation of the approximating function $Y$ when the machine is operating with a spindle speed of 3000 rpm, and the previous spindle speed was 1000 rpm. In the block Fcn_3000_2 an approximating function is formed for the thermal characteristics of the machine operating at the same spindle speed of 3000 rpm, and the previous spindle speed was 5000 rpm. Blocks Switch Case Action are denoted as a combination of letters and numbers of abbreviation SCA and the numbers of the corresponding blocks.

2.2 Experiment and discussion of the simulation results

The full-scale experiment was carried out on a vertical CNC machine tool of 400V (Russia). Thermal tests were conducted with idling at spindle for four rotation speeds of 250, 1000, 3000 and 5000 rpm. As shown by the results of experiments at the first spindle speed, the temperature displacements of the spindle head did not occur, as it was very small. The purpose of this speed is explained by the operational need. During this time, initial heating of the spindle bearings is carried out. Figure 2 shows a photography of the machine tool with the installed temperature sensors and indicator heads for measuring the temperature displacements of the spindle head. Indicator heads for measuring displacements along each axis were installed in magnetic holders, which were mounted on the table of the machine tool. The leg of the corresponding indicator head was touching the immovable flange of the spindle head. Temperature measurements were carried out using a digital multichannel (12 channels) MIT-12TP-11 temperature meter equipped with an RS-232 interface.
This instrument is designed for high-precision measurements of temperature with a scale division equal to 0.1 °C. Twelve temperature sensors with magnetic fastening were installed on the surfaces of the machine tool, available for measurement. The information received for each channel was displaying on the digital display of the instrument and transmitted to a computer as a data table. It was possible to observe the change of the instrument readings on the monitor.

Figure 2. Configuration of the temperature sensors and the indicator heads for measuring the thermal displacements along the X, Y, and Z axes.

Approximated thermal characteristics, formed from the averaged experimental values for each of the four spindle speed, are shown in Figure 3.

Figure 3. Approximated experimental curves of the temperature T (1–4) as a function of the time, at spindle speeds of 250 (1), 1000 (2), 3000 (3) and 5000 (4) rpm.

Figure 4 shows four curves. Curve 1 is an experimental thermal characteristic obtained for the excess temperature measured by the temperature sensor installed near front bearing of spindle assembly. The use of the readings of this sensor is explained by the strong correlation between the temperature values fixed by it and experimental values of thermal displacements. Curves 2, 3, and 4 are thermal characteristics constructed according to the mathematical models (2.a), (2.b), (2.c) and the structural scheme of the simulation model presented in Figure 1.

The maximum error in the simulation did not exceed 5 %. In this case, the experimental thermal characteristic is constructed from the averaged values with a relatively small spread of the ambient temperature in the range 2 °C.
2.3 The modeling technique

For practical realization of the mathematical model and the structural scheme of the thermal model, a thermal modeling technique was developed, including nine stages represented by nine blocks (Figure 5). The purpose of the technique is to create a formalized sequence of stages of development of a thermal model that is invariant to the machine layout.

Figure 4. One experimental and three calculated thermal characteristics.

At the first stage, the simulation task is formulated, which consists in the formation of a test operating cycle (block 1). In general, it is possible to generate several operating cycles. At this stage, not only the operating cycles are described in the form of a sequence (7), but a full-scale experiment is also conducted, during which the experimental thermal characteristic is determined.

At the second stage (block 2), experimental thermal characteristics are formed for fixed spindle speeds. For intermediate rotational speeds, thermal characteristics can be obtained using interpolating functions.

In the third stage (block 3), construction of approximating functions of the form (1) and (2) is performed using the solution of the optimization problem in the formulation (4) - (6). As an optimization method for solving this class of problems, the most efficient method was shown by the method of sequential quadratic programming.

At the fourth stage, a structural diagram of the thermal model is developed (Figure 1). Formalization of the operating cycle in the Switch Case component is a practical feature of the implementation of the scheme. The operating cycle of the machine tool determines the number of Fcn blocks and their sequence.

For the implementation of the fifth stage (block 5) blocks Display are used. The data displayed on them correspond to the boundary excess values of the sought-for function \( Y(t) \) for each switching of the speed. The use of these data allows to reconcile the values of the function \( Y(t) \) when switching speed according to equation (2).

In the sixth block, the condition for completing the formation of the thermal model of the machine tool is checked. This is explained by the iterative and consistent character of the formation of the complete thermal model for the operating cycle. For instance, at the first iteration of the simulation of the thermal model, the value of the sought-for function \( Y(t) \), displayed in the Display1 block (Figure 1), is formed; at the second iteration, the generated value is displayed in the Display2 block and so on.

At the seventh stage (block 7), a verification calculation is carried out and a model thermal characteristic of type 2 is constructed (Figure 2).

To make a decision on the completion of the model adjustment, a transition is made to block 8, in which the simulation errors are estimated \( \epsilon \) as:
It is possible to define quality of thermal model if to accept some the preassigned value $\varepsilon$. At non-compliance with a condition (8) transition to the beginning of the scheme, to the block 2 is carried out. It means that the experimental thermal characteristics received at the second stage have been created at significantly differing basic data, for instance, at ambient temperature.

Having achieved the required accuracy of modeling, it is possible to form the final model for equations (1) and (2) and go to the stage of its operation described by block 9. Thus, in this block a thermal model of the machine tool is formed, which allows to build its thermal characteristic for a complex operating mode.

![Figure 5. Scheme of the thermal modeling technique.](image)

### 3. Conclusion

The paper presents one of the possible ways to build a simulation thermal model of the machine tool at the operation stage. The use of analytical solutions of the heat conduction equation allowed us to apply an interactive system for analyzing the dynamic systems of Simulink. Its use allows you to effectively conduct a study of the thermal characteristics of the machine tool working in complex operating modes. The use of experimental modal analysis and the accepted condition of the matching of adjacent thermal characteristics at their boundaries determine the feature of implementation of the mathematical model. Three variants of constructing thermal characteristics are considered in the article. Based on the comparison of calculated and experimental values, the choice of the type of function used to approximate the experimental thermal characteristics was justified. In this case, a small error of simulation is provided (less than 5%). For practical implementation of the proposed approach in the thermal modeling of machine tools, a new technique of thermal modeling was presented.
The thermal characteristic obtained for the complex operating mode of the machine tool can be used not only as an estimate of the possible temperature error, but also as the calculated compensating influences for the working parts of the machine tool.

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