Vector dark matter annihilation with internal bremsstrahlung

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Abstract

We consider scenarios in which the annihilation of self-conjugate spin-1 dark matter to a Standard Model fermion-antifermion final state is chirality suppressed, but where this suppression can be lifted by the emission of an additional photon via internal bremsstrahlung. We find that this scenario can only arise if the initial dark matter state is polarized, which can occur in the context of self-interacting dark matter. In particular, this is possible if the dark matter pair forms a bound state that decays to its ground state before the constituents annihilate. We show that the shape of the resulting photon spectrum is the same as for self-conjugate spin-0 and spin-1/2 dark matter, but the normalization is less heavily suppressed in the limit of heavy mediators.
1. INTRODUCTION

It is well understood that dark matter (DM) annihilation or decay to a Standard Model (SM) fermion-antifermion pair \( \bar{f}f \) can be chirality suppressed. Then, the dominant indirect detection process in the current epoch may instead involve the internal bremsstrahlung (IB) of an additional gauge boson (i.e., \( \bar{f}f\gamma, \bar{f}fZ, \bar{f}f g, \bar{f}f'W^\pm \) final states) [1–12]. These processes may not only dominate the annihilation/decay rate, but may also yield a hard boson spectrum which can be more easily distinguished from background. As a result, these internal bremsstrahlung processes have been well studied for the case of spin-0 or spin-1/2 dark matter. In this Letter, we discuss a case that has not been considered so far: chirality suppression of spin-1 dark matter annihilation lifted by internal bremsstrahlung.

The annihilation of spin-1 dark matter particles \( B \) to a fermion-antifermion pair has been studied in several specific models [13–16], and in these cases it is found that there is no chirality suppression, implying that the two-body final state is dominant. We point out that this unsuppressed contribution arises from the \( J = 2 \) s-wave initial state. If the DM initial state is unpolarized, then there is indeed no way to avoid this unsuppressed contribution to the \( 2 \rightarrow 2 \) annihilation cross section. But if the DM initial state is polarized, and the \( J = 2 \) initial state is projected out, then the s-wave \( BB \rightarrow \bar{f}f \) matrix element will be chirality suppressed, and the internal bremsstrahlung process will dominate. This scenario can be realized in a simple model in which the annihilation occurs through the formation of a \( BB \) bound state, which decays to its ground state before the two constituents annihilate. If the ground state is not \( J = 2 \), then the branching fraction for decay to the \( \bar{f}f \) final state will be chirality suppressed, and the primary bound state decay channel will be to a three-body final state.

We focus on the case in which internal bremsstrahlung involves the emission of a photon (\( \bar{f}f\gamma \)). There are general arguments which show that for self-conjugate spin-0 or spin-1/2 dark matter, the photon spectrum adopts a common universal form which depends only on \( r \), the ratio of the mass of the mediating particle \( (m_\Psi) \) to the mass of the dark matter. We will show that this argument generalizes to the case of spin-1 dark matter with one key difference: for spin-0 or spin-1/2 dark matter, the annihilation matrix element necessarily scales as \( m_\Psi^{-4} \) in the \( r \gg 1 \) limit, while for spin-1 dark matter, the matrix element only scales as \( m_\Psi^{-2} \).

The structure of this paper is as follows. In Section 2 we review the general arguments that underly the chirality suppression of dark matter annihilation to the \( \bar{f}f \) final state and apply these arguments to the case of spin-1 dark matter, inferring that IB is only relevant for a polarized initial
state. In section 3, we present the IB photon spectrum for the case of spin-1 dark matter, and demonstrate that its shape is necessarily the same as in the spin-0 and spin-1/2 cases. We also provide a physical realization of internal bremsstrahlung as the dominant annihilation channel in terms of the decay of a dark matter bound state. In section 4, we conclude with a discussion of our results.

2. CHIRALITY SUPPRESSION IN VECTOR DARK MATTER ANNIHILATION

Chirality suppression of the cross section for $s$-wave dark matter annihilation to $\bar{f}f$ arises for spin-0 or spin-1/2 dark matter if the dark matter particle is self-conjugate (i.e., the particle is its own antiparticle) and if minimal flavor violation applies. One can understand this result from general principles; see, for example, Ref. [17]. If the dark matter particle is self-conjugate, then the initial state consists of two identical particles, and must be even under charge conjugation. For an $s$-wave ($L = 0$) initial state, this implies that $S$ is even, which in turn implies $J = 0$. The final state $\bar{f}f$ pair must then have $J_z = 0$, where the $z$-axis is taken to be the direction of motion of the outgoing particles. Since $L_z$ vanishes along the direction of motion, the final state particle and antiparticle must have the same helicity, and thus arise from different Weyl spinors. The matrix element thus violates SM flavor symmetries, and is necessarily chirality suppressed by a factor $(m_f/m_X)^2$ under the assumption of minimal flavor violation, where $m_X$ is the mass of the dark matter.

One can also see why the annihilation of unpolarized spin-1 dark matter does not exhibit chirality suppression. If the initial state consists of two identical real spin-1 particles, then it can also be in an $L = 0$, $S = 2$, $J = 2$ state. In this case, the final state need not mix different Weyl spinors, and thus the matrix element need not be chirality suppressed.

These results also follow from an analysis of the 4-point effective contact operators that can mediate dark matter annihilation to a fermion-antifermion pair [17, 18]. In particular, for spin-0 or spin-1/2 dark matter, one finds that there exists no operator of dimension $\leq 6$ which has a nontrivial matrix element with an $s$-wave initial state of identical dark matter particles, and which also does not mix SM Weyl spinors. But for spin-1 dark matter, there are two such dimension-6 operators:

$$
\mathcal{O} = \frac{1}{2\Lambda^2} B_{\mu B} \left( \bar{f} \gamma^{\{\mu} \partial^{\nu\}} f - \partial^{\{\nu} \bar{f} \gamma^\mu \} f \right),
$$
$$
\mathcal{O}' = \frac{1}{2\Lambda^2} B_{\mu B} \left( \bar{f} \gamma^{\{\mu \gamma^5} \partial^{\nu\}} f - \partial^{\{\nu} \bar{f} \gamma^\mu \} \gamma^5 f \right). \tag{1}
$$
These operators (which were not discussed in Refs. [17, 18]) yield a nontrivial matrix element between an s-wave $J = 2$ dark matter initial state and an $\bar{f}f$ final state with no Weyl spinor mixing. As these operators are both $CP$-even and respect SM flavor symmetries, they cannot be projected out.

We can verify this result with explicit calculation. Henceforth, we assume $m_f = 0$. We consider a model in which the spin-1 dark particle $B$ couples to a SM fermion $f$ through exchange of a heavy charged fermion $\Psi$ via the interaction Lagrangian,

$$
\mathcal{L} = \lambda_L \bar{\Psi} \gamma^\mu P_L f B_\mu + \lambda_L^* \bar{f} \gamma^\mu P_L \Psi B_\mu, \tag{2}
$$

where $\lambda_L$ is a dimensionless coupling and we have assumed that the dark sector only couples to left-handed SM fermions. The unpolarized annihilation cross section for the $t$-channel annihilation process $B(p_1) B(p_2) \rightarrow f(k_1) \bar{f}(k_2)$ has been computed in Refs. [13, 15, 16], and indeed it is nonvanishing in the nonrelativistic limit.

The $L = 0, S = 0, J = 0$ initial state can be written in the individual spin basis as

$$
|J = 0, J_z = 0\rangle = \frac{1}{\sqrt{3}}|S_z^1 = +1, S_z^2 = -1\rangle - \frac{1}{\sqrt{3}}|S_z^1 = 0, S_z^2 = 0\rangle + \frac{1}{\sqrt{3}}|S_z^1 = -1, S_z^2 = +1\rangle, \tag{3}
$$

and one can verify that the matrix element for the annihilation of this state to $\bar{f}f$ vanishes in the nonrelativistic limit. Similarly the matrix element for annihilation of the $J = 1$ initial state also vanishes in the nonrelativistic limit; the two diagrams cancel, indicative of the fact that two identical particles cannot be in an $L = 0, S = 1, J = 1$ initial state. But the matrix element for annihilation of the $J = 2$ initial state is nonvanishing.

3. INTERNAL BREMSSTRAHLUNG

We now consider the matrix element for the annihilation of the $J = 0$ initial state to the $\bar{f}f\gamma$ final state.

The squared amplitude for annihilation of the $J = 0$ initial state (summed over final state spins) is

$$
\sum_{\text{spins}} |\mathcal{M}_{J=0}|^2 = \frac{32\pi\alpha\lambda_L^2}{3m_B^2} \frac{(2 + r^2)^2}{4(1 - x)(2 + 2y^2 + 2y(x - 2) - 2x + x^2)} \frac{4(1 - x)(2 + 2y^2 + 2y(x - 2) - 2x + x^2)}{(1 - r^2 - 2y)^2(3 + r^2 - 2x - 2y)^2}, \tag{4}
$$

where $r \equiv m_\Psi/m_B$. Here $x \equiv 2E_\gamma/\sqrt{s}$, $y \equiv 2E_f/\sqrt{s}$, and $\sqrt{s}$ is the center-of-mass energy. The reduced energy parameters $x$ and $y$ are subject to the kinematic constraints $0 \leq x, y, \leq 1$, $x + y \geq 1$. The
3.1. Relation to the spin-0 and spin-1/2 cases

Note that the \( x \) and \( y \) dependence of Eq. (4) is identical to that for the cases of spin-0 and spin-1/2 dark matter [4, 8]. An explanation for the identicalness of the internal bremsstrahlung spectra for spin-0 and spin-1/2 dark matter that relies on effective operators was put forward in Ref. [8], and extended in Ref. [11] using the operator classification of Ref. [10]. It has been shown that in the heavy mediator limit \((r \gg 1)\), the effective 5-point contact operators that lead to internal bremsstrahlung must be of dimension \( \geq 8 \) [8, 10]. The dominant operators are thus dimension 8: there are 5 such operators for Majorana fermion dark matter, and 7 such operators for real scalar dark matter [10]. But it turns out that all of these operators produce identical photon spectra; although any particular model will be realized as one particular linear combination of these operators, the resulting photon spectrum is necessarily universal. If one does not take the limit \( r \gg 1 \), the only change to the form of the amplitude arises from the denominators of the propagators of the heavy mediators [11]. But as the annihilation process is \( t \)-channel for both spin-0 and spin-1/2 dark matter, the denominators of the propagators in these two cases are the same, implying that the spectrum remains universal even outside of the contact operator limit.\(^1\)

This argument easily generalizes to the case of spin-1 dark matter. Each of the contact operators can be written as a dark matter bilinear with some Lorentz tensor structure, contracted with a SM trilinear with the same Lorentz structure, provided that the SM factor can produce an \( \bar{f}f\gamma \) final state and that the DM factor has a nontrivial matrix element with an \( s \)-wave initial state of identical particles. The shape of the photon spectrum is determined only by the SM trilinear factor because the DM bilinear contributes a constant factor to the matrix element which is independent of \( x \) and \( y \).

For the spin-0 case, the DM bilinear is necessarily either a 0-index or 2-index tensor, while in the spin-1/2 case the DM bilinear is a 1-index tensor [10]. For the case of spin-1 dark matter, the only bilinears that have a nontrivial matrix element with the \( J = 0 \) \( s \)-wave initial state are \( B_\mu B^\mu \) and \( B_\mu B_\nu \). Since the initial state has \( J = 0 \), only a rotationally invariant piece of the tensor can contribute; for the bilinear \( B_\mu B_\nu \), this piece scales as \( \delta_{\nu\nu} \) in the nonrelativistic limit. The contact operators for internal bremsstrahlung of spin-1 dark matter can therefore be written in terms of the operators for spin-0 dark matter, found in Ref. [10], by the replacements \( \phi^2 \rightarrow B_\mu B^\mu \), \( \partial_\mu \phi \partial_\nu \phi \rightarrow B_\mu B_\nu \). Because the photon spectrum is determined by the SM factor, these replacements

\(^1\) This generalization fails if internal bremsstrahlung can occur from an \( s \)-channel diagram, as is the case with internal Higgsstrahlung [19].
do not alter the spectrum, which is thus identical for the case of spin-1 and spin-0 dark matter.

There is one subtlety to this argument. For the case of spin-0 dark matter, the bilinear $\partial_\mu \phi \partial_\nu \phi$ has a nontrivial matrix element with the $J = 0$ initial state only if $\mu = \nu = 0$, so only the corresponding terms in the SM factor contribute to the photon spectrum. But with the replacement $\partial_\mu \phi \partial_\nu \phi \rightarrow B_\mu B_\nu$, only the terms in the DM bilinear with $\mu = \nu = i$ are nontrivial, implying that a different set of terms in the SM trilinear contribute to the photon spectrum for the spin-1 case. The SM trilinear with which the 2-index DM bilinear is contracted is $\bar{f} \gamma^\mu \not{D}^\nu P_L f - \not{f} \gamma^\mu \not{D}^\nu P_L f$ [10], where $D$ is the SM covariant derivative. Since the matrix element for this factor vanishes if contracted with $g_{\mu\nu}$ [10], the contributions from the SM trilinear corresponding to $\mu = \nu = i$ and $\mu = \nu = 0$ are identical, thereby implying that the contribution to the matrix element for the spin-0 case is the same as for the spin-1 case.

There is one last interesting feature to note from this construction. The replacement $\partial_\mu \phi \partial_\nu \phi \rightarrow B_\mu B_\nu$, gives the effective operator $B_\mu B_\nu (\bar{f} \gamma^\mu \not{D}^\nu P_L f - \not{f} \gamma^\mu \not{D}^\nu P_L f)$, which is dimension-6. So, this operator can provide a contribution to the squared matrix element which scales as $r^{-4}$, instead of the $r^{-8}$ scaling which necessarily appears for spin-0 or spin-1/2 dark matter. Indeed, we see this scaling in Eq. (4). This implies that in the heavy mediator limit, there is less suppression of the internal bremsstrahlung cross section for the case of spin-1 dark matter.

### 3.2. Realization of internal bremsstrahlung as the dominant annihilation channel

As we have seen, the internal bremsstrahlung annihilation process can only be significant if annihilation from the $J = 2$ initial dark matter state is suppressed. So internal bremsstrahlung is only relevant if the initial state is polarized. A simple scenario in which this can happen is if the dark matter particles predominantly annihilate by first forming a nonrelativistic $BB$ bound state, which then decays to its ground state, before the constituents finally annihilate. If the ground state is $J = 2$, then s-wave annihilation to a two-body final state will dominate, while if the ground state is $J = 0$, then s-wave annihilation to a three-body $\bar{f} f \gamma$ final state will dominate. If the ground state is $J = 1$, then p-wave annihilation to a two-body final state will dominate.

There is a large body of work on self-interacting dark matter, including models in which dark sector particles form composite bound states; see, for example, Refs. [20–22]. A detailed formulation of the confining potential, and the resulting spectroscopy, is beyond the scope of this work. We note, however, that the dynamics which generate the confining potential are independent of the field $\Psi$ that mediates the interaction between the dark matter and the SM. Thus, $m_\Psi$ can be much
larger than the scale of the confining potential, and the lifetime of the bound state can easily be 
large compared to the timescale on which it deexcites to the ground state.

There are two distinct classes of models within this scenario: dark matter may largely consist 
of such bound states, or the dark matter particles may largely be unbound, with the formation 
of bound states followed relatively quickly by annihilation of the constituents. In the former case, 
one may just as well treat the $J = 0$ bound state as a composite spin-0 dark matter particle, whose 
two-body decays to SM fermions are chirality suppressed [4]. In the latter case, the spin-1 particles 
$B$ in fact constitute the dark matter. This distinction can be significant for the purposes of direct 
detection. But in either case, the doubly-differential decay rate of the bound state may be written 
as

$$\frac{d^2 \Gamma}{dx\,dy} = \left| \phi(0) \right|^2 \frac{1}{128\pi^3} \left( \sum_{\text{spins}} |M_{J=0}|^2 \right), \quad (5)$$

where $\phi(0)$ is the wavefunction of the bound state evaluated at the origin.

4. CONCLUSIONS

We considered the annihilation of self-conjugate spin-1 dark matter to Standard Model fermions, 
assuming that flavor violation is minimal. We have shown that the $BB \rightarrow \bar{f}f$ annihilation cross 
section can be chirality suppressed, but only if the initial state is $J = 0$; the $J = 2$ state has an 
unsuppressed $2 \rightarrow 2$ annihilation cross section. For the $J = 0$ state, the dominant $s$-wave annihila-
tion process yields a three-body final state $\bar{f}f\gamma$ via internal bremsstrahlung, with a spectrum that 
is identical to the case of self-conjugate spin-0 or spin-1/2 dark matter.

The typical suppression of the internal bremsstrahlung cross section by a factor of the fine 
structure constant implies that this process is unimportant for the case in which the initial DM 
state is unpolarized. But there are scenarios for which the DM state is polarized, as in the context 
of self-interacting dark matter. In particular, if dark matter forms a nonrelativistic $BB$ bound 
state, which decays to a $J = 0$ ground state before the constituents annihilate, then internal 
bremsstrahlung could be the dominant annihilation process.

We have shown that for spin-1 dark matter, the shape of the photon spectrum arising from 
internal bremsstrahlung is necessarily the same as for spin-0 and spin-1/2 dark matter, generalizing 
previous arguments regarding the universality of the photon spectrum. But unlike the spin-0 or 
spin-1/2 cases, in which the annihilation matrix element is suppressed by $m_{\Psi}^{-4}$ in the heavy mediator 
limit, in the spin-1 case the matrix element is only suppressed by $m_{\Psi}^{-2}$. This is particularly
interesting because of its impact on complementary searches at the LHC. In order for internal bremsstrahlung to be possible, the mediator $\Psi$ must be charged, and there are tight constraints on new charged particles from collider experiments. If $m_\Psi$ is increased in order to evade those constraints, then the internal bremsstrahlung cross section becomes heavily suppressed. As a result, many studies of the IB photon spectrum have necessarily focused on the regime where the dark matter and the mediator are nearly degenerate; in this region of parameter space, the internal bremsstrahlung cross section is not heavily suppressed, and the mediator escapes collider searches because the jets/leptons produced by its decay are soft. For the case of spin-1 dark matter, however, $r$ can be made much larger without heavily suppressing the annihilation cross section. This opens a new window in parameter space in which one can search for dark matter annihilation via internal bremsstrahlung.

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