Constitutive modelling of aluminium alloy sheet at warm forming temperatures

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Abstract. The formability of aluminium alloy sheet can be greatly improved by warm forming. However predicting constitutive behaviour under warm forming conditions is a challenge for aluminium alloys due to strong, coupled temperature- and rate-sensitivity. In this work, uniaxial tensile characterization of 0.5 mm thick fully annealed aluminium alloy brazing sheet, widely used in the fabrication of automotive heat exchanger components, is performed at various temperatures (25 to 250 °C) and strain rates (0.002 and 0.02 s⁻¹). In order to capture the observed rate- and temperature-dependent work hardening behaviour, a phenomenological extended-Nadai model and the physically based (i) Bergström and (ii) Nes models are considered and compared. It is demonstrated that the Nes model is able to accurately describe the flow stress of AA3003 sheet at different temperatures, strain rates and instantaneous strain rate jumps.

1. Introduction
Uniaxial tensile tests show that the work hardening in AA3003 sheet depends on the temperature and strain rate [1]. In numerical modelling of sheet metal forming, a description of material hardening is required. The conventional approach is to fit measured flow stress curves to a convenient empirical equation. The validity of these phenomenological models is often limited to situations that are comparable to the range of experiments on which they are based. Material models based on consideration of the underlying physical processes are expected to have a larger range of applicability than more commonly used empirical models [2-5]. In physically based models, the effect of microstructural-level processes is usually accounted for, perhaps in an average way, at the continuum level. In the current work, a phenomenological extended Nadai [2-4] and two physically based models due to Bergström [6] and Nes [7-9] are considered. The models are fit to measured rate- and temperature-dependent constitutive data for AA3003 brazing sheet. Their ability to capture work hardening and strain rate jump response is assessed.

2. Constitutive model

2.1. Extended Nadai model
In warm forming of aluminium sheet, the strain rate and temperature have a significant, coupled effect on the flow stress. The phenomenological model adopted in this work to capture this behavior is the Nadai hardening model with strain rate dependency [2-4]:
\[
\sigma_{eq} = C\left(\varepsilon_{eq} + \varepsilon_0\right)\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^m
\]  

(1)

Temperature dependence is introduced by allowing \(C\), \(n\) and \(m\) be functions of temperature, \(T\) (in Kelvin). The following functions for the parameters were used in order to fit the tensile tests.

\[
C(T) = C_0 + a_i \left[1 - \exp\left(a_i \frac{T - T_273}{T_m}\right)\right],
\]

\[
n(T) = n_0 + b_i \left[1 - \exp\left(b_i \frac{T - T_273}{T_m}\right)\right]
\]

\[
m(T) = m_0 \exp\left(c_i \frac{T - T_273}{T_m}\right).
\]

(2)

2.2 The Bergström model

In the Bergström physically based hardening model [6], the flow stress is decomposed into strain rate independent stress, \(\sigma_e(T)\) and a work hardening dependent stress, \(\sigma_w(\rho, T)\) due to the evolution of microstructure and is completely described by the total dislocation density \(\rho\). The relation between the dislocation density and \(\sigma_w\) is given by the Taylor equation [2-4]:

\[
\sigma_w = \sigma_0 + \rho \sqrt{\tau} g(T),
\]

where \(\sigma_0\) is a scaling parameter of the order 1, \(G\) the elastic shear modulus and \(b\) the Burgers vector.

The evolution of dislocation density is expressed as the competition between dislocation storage and recovery by remobilization and annihilation; \(U\) represents the storage of mobile dislocations and \(\Omega\) is the dynamic recovery by remobilization and annihilation. The functions \(U\) and especially \(\Omega\) determine the shape of the hardening curve at different temperatures and strain rates.

\[
\frac{d\rho}{d\varepsilon} = U(\rho) - \Omega(\dot{\varepsilon}, T)\rho; \quad \text{with} \quad U(\rho) = U_0 \sqrt{\rho} \quad \text{and} \quad \Omega(\dot{\varepsilon}, T) = \Omega_0 + C \exp\left(-\frac{mQ}{RT}\right)\dot{\varepsilon}^{-m}.
\]

(4)

\(U_0\) is the immobilization rate, \(\Omega_0\) the low temperature, high strain rate limit value of the remobilization probability, \(C\) and \(m\) are constants, \(Q\) the activation energy and \(R\) the gas constant.

Equation (4) can be integrated analytically to obtain the total dislocation density and is expressed in terms of an incremental algorithm that can easily be implemented in FE code (Kurukuri et al., 2010):

\[
\rho_{i+1} = \frac{U_0}{\Omega} \left[\exp\left(\frac{1}{2} \Omega\Delta \varepsilon - 1\right) + \sqrt{\rho_i}\right] \exp(-\Omega\Delta \varepsilon).
\]

(5)

The final equation for the flow stress is:

\[
\sigma_{eq} = g(T)(\sigma_0 + \rho \sqrt{\tau} G), \quad \text{and} \quad g(T) = 1 - C_{\rho} \exp\left(-\frac{T}{T'}\right).
\]

(6)

where \(C_{\rho}\) and \(T'\) are fitting parameters, \(g(T)\) is the shear modulus, \(G\), divided by the reference value \(G_{ref}\). Some of the parameters in the Bergström model can be selected beforehand. The rest are determined by a least squares approximation of experimental results. The initial dislocation density \(\rho_0\) was chosen to be \(10^{16} m^{-2}\), which seems to be a reasonable value for annealed aluminium [2]. The magnitude of the Burgers vector \(b\) and the shear modulus at room temperature \(G_{ref}\) were taken from the literature.

2.3 The Nes model

The Nes model approach relies on a multi-parameter description for the microstructure evolution. At small strains the stored dislocations are arranged in a cell structure with dislocation density within the cells \(\rho_c\) and cell size \(\delta\). The cells have finite walls of higher dislocation density. At larger strains the dynamic recovery of dislocations becomes important and the cell walls collapse into sub-boundaries of well-defined misorientation \(\varphi\). Extensive presentations of the model are given in [7-9]. The microstructure evolution is obtained by solving a set of differential equations describing the evolution of these parameters. Leaving out the details, these can be written as:
\[
\frac{d\rho}{d\gamma} = \frac{l}{1+f(q^2_b-1)bL_{eff}} - \frac{2}{\gamma} \rho \frac{\gamma}{\rho}, \quad \frac{d\delta}{d\gamma} = -\frac{2\delta \rho \frac{SL_{eff}}{\gamma} + \frac{bv_i}{\gamma}}, \quad \text{and} \quad \frac{d\varphi}{d\gamma} = g(\rho, \delta, \varphi). \quad (7)
\]

The first term in these equations represents storage of dislocations (which increases work hardening), the second term represents dynamic recovery by annihilation of dislocations (which decreases work hardening). It describes the flow stress as a function of microstructure evolution parameters:

\[
\tau = \tau_{\gamma} + \alpha_G b \left[ \Gamma_1 \left( \frac{q}{\delta \sqrt{\rho_i}} \right) \sqrt{\rho_i} + \Gamma_2 \left( \frac{q}{\delta \sqrt{\rho_i}} \right) \frac{q}{\delta} \right] + \hat{\alpha}_G b \left[ \Gamma_1(0) \frac{l}{\delta} + \frac{l}{D} \right]. \quad (8)
\]

Here, \( \tau_{\gamma} \) is thermal stress, due to short range interactions between mobile dislocations and intersecting stored ones, dragging of jogs, and elements in solid solution. The \( \alpha_1 \)-term is the contribution from stored dislocations and the \( \hat{\alpha}_2 \)-term is the contribution from sub-grains and grain boundaries. \( G \) is the shear modulus, \( b \) is the Burgers vector. In applications of the model, the stress tensor at a macroscopic continuum scale is required representing contributions from many grains of various crystallographic orientation and microstructure. Hence, the flow stress and strain at continuum scale can be represented as, \( \sigma_{\alpha} = M \tau_{\alpha} \) and \( \epsilon_{\alpha} = \gamma/M \), here \( M \) is the Taylor factor. The Nes model involves a large number of parameters (nearly 40), however, most of these are pre-determined based on material constants, while some are fit through manual methods as described in [5].

3. Results

The various parameters of the models described above are obtained by fitting simultaneously with the experimental stress-strain data at temperatures of 25, 150 and 250 °C and strain rates of 0.002 and 0.02 s\(^{-1}\) using MATLAB simplex optimization algorithm. In figure 1, the simulated flow curves for the extended Nadai, Bergström and Nes models are plotted together with the experimental data. It can be seen that the Bergström and extended Nadai models are capable of describing the overall trends in material behaviour over the range of temperatures and strain rates with a reasonable accuracy. The Nes model, on the other hand, predicts the behaviour with exceptional accuracy at all strain rates and temperatures due to its strong physically motivated nature.

![Figure 1](image-url)

**Figure 1.** True stress-plastic strain curves—experiments and models.

Even though all three models considered yield reasonable results for constant strain rate conditions, the predictions are quite different if a jump in the strain rate is considered. In figure 2, the simulated stress-strain curves using three models are compared with the experiments are plotted for deformation at 150 °C and 250 °C and for strain rates of 0.002 and 0.02 s\(^{-1}\). If strain rate changes from 0.002 to 0.02 s\(^{-1}\) or from 0.02 to 0.002 s\(^{-1}\) are applied after a strain of 15%, the extended Nadai model immediately follows the curve corresponding to a constant strain rate. With the Bergström model the
constant strain rate curve is only slowly approached after continuous straining. With the Nes model, an initial jump in the flow stress is observed, followed by slowly approaching the constant strain rate curve, which is a better representation of the experiments.

Figure 2. Flow curves with and without strain rate jumps—experiments and models.

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