Constraints on the small scale curvature perturbation using Planck-2015 data

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ABSTRACT

The particles emitted from PBHs through the Hawking radiation have interactions with the particles present in the Universe. Due to the interactions, the evolution of the intergalactic medium (IGM) is changed and the changes have imprints on the anisotropies of the cosmic microwave background (CMB). In this paper, we focus on the PBHs with the lifetime in the range of $10^{13}s \lesssim \tau_{PBH} \lesssim 10^{17}s$, corresponding to the mass range of $2.8 \times 10^{13}g \lesssim M_{PBH} \lesssim 2.5 \times 10^{14}g$. We update the constraints on the initial mass fraction of PBHs using the Planck-2015 data. We find that the optimistic upper limits are $4 \times 10^{-29} \lesssim \beta(M_{PBH}) \lesssim 5 \times 10^{-28}$, depending on the mass of PBH. The formation of PBHs is related to the primordial curvature perturbations. Therefore, using the constraints on the initial mass fraction of PBHs, we get the upper limits on the power spectrum of primordial curvature perturbation. For the investigated mass range of PBHs, corresponding to the range of scales $8.9 \times 10^{15} \text{Mpc}^{-1} \lesssim k \lesssim 2.8 \times 10^{16} \text{Mpc}^{-1}$, we find that the upper limits change slightly with a value of $P_R(k) \sim 0.0045$, and the limits are slightly stronger compared with the previous results.

1 INTRODUCTION

It has been predicted by many inflation models that the power spectrum of primordial curvature perturbation, $P_R(k)$, is nearly scale invariant (Lidsey et al. 1997). On large scales, $10^{-4} \text{Mpc}^{-1} \lesssim k \lesssim 1 \text{Mpc}^{-1}$, the constraint is $P_R(k) \sim 10^{-9}$, which is obtained mainly from the CMB, Lyman-α forest and large scale structures (Hlozek et al. 2012; Bird et al. 2011; Tinker et al. 2012). On small scales, 1 Mpc$^{-1} \lesssim k \lesssim 10^{20} \text{Mpc}^{-1}$, the upper limit is $P_R(k) \sim 10^{-2}$, which is obtained from the researching on the PBHs (Josan et al. 2009; Carr et al. 2010). Recently, a new kind of dark matter structures named ultracompact dark matter minihalos (UCMHs) has been suggested, and it is found that researching on UCMHs give much more stringent limits, $P_R(k) \lesssim 10^{-5}$ for the range of scales $5 \text{Mpc}^{-1} \lesssim k \lesssim 10^8 \text{Mpc}^{-1}$ (Josan & Green 2010; Bringmann et al. 2012; Yang et al. 2013; Li et al. 2012; Clark et al. 2015). The limits of IGM, that is, the relevant results from the DM decay obtained Nakama et al. (2014); Jeong et al. (2014) suggested that the energy deposited into the baryonic fluid due to the Silk damping effect can also be used to constrain the primordial curvature perturbation, and they found a boost upper limit $P_R(k) \sim 0.06$ in the range of scales $10^4 \text{Mpc}^{-1} \lesssim k \lesssim 10^5 \text{Mpc}^{-1}$.

PBHs can form in the early Universe via the collapse of large density perturbations. After formation, PBHs can emit photons, electrons, neutrinos, quarks and other particles through the Hawking radiation. The particles emitted from PBHs have interactions with the particles present in the Universe. As a result, the evolution of IGM is changed, and the changes have imprints on the CMB or the other astrophysical processes (Mack & Wesley 2008). Therefore, the data of CMB can be used to constrain the initial mass fraction of PBHs $\beta(M_{PBH})$. On the other hand, the formation of PBHs is related to the primordial curvature perturbations. Using the constraints on the initial mass fraction of PBHs, one can get the limits on the primordial curvature perturbations. It has been pointed that the influences of PBHs on the evolution of IGM, which include heating and ionizing on IGM, are similar to that of DM decay (Carr et al. 2010; Mack & Wesley 2008). Specifically, one can treat the lifetime of a PBH ($\tau_{PBH}$) as the inverse of the DM decay rate ($\Gamma_{DM}^{-1}$), and we follow this method for our calculations. The influences of DM decay on the evolution of IGM have been investigated by previous works (Chen & Kamionkowski 2004; Zhang et al. 2007; Yang 2015). Carr et al. (2010) have shown that the relevant results from the DM decay obtained using the WMAP3 data can be used to constrain the initial mass fraction of PBHs, and the constraints are stronger than that obtained using other observational data. In this paper, we update the constraints on the initial mass fraction of PBHs using the Planck-2015 data, then we use these constraints to get the limits on the power spectrum of primordial curvature perturbation. We find that for the optimistic efficiency of the energy deposited into IGM, $f = 1$, the limits on $\beta(M_{PBH})$ and $P_R(k)$ are about two and one
order of magnitude stronger than that of previous results, respectively (Josan et al. 2009; Carr et al. 2010).

This paper is organized as follows: In Sec. II, taking into account the influences of PBHs on the evolution of IGM, we get the constraints on the initial mass fraction of PBHs using the Planck-2015 data. In Sec. III, the limits on the power spectrum of primordial curvature perturbation are obtained using the relevant results given in Sec.II. The conclusions are given in Sec. IV.

2 THE INFLUENCES OF PBHS ON THE EVOLUTION OF IGM AND CONSTRAINS ON THE PBHS INITIAL MASS FRACTION

Through the Hawking radiation, PBHs can emit different particles depending on their masses (Page 1976a,b, 1977). The temperature and lifetime of a PBH are (Carr et al. 2016)

\[ T_{\text{PBH}} \approx 106 \left( \frac{M_{\text{PBH}}}{10^{15} \text{g}} \right)^{-1} \text{MeV} \]  
\[ \tau_{\text{PBH}} \approx 2.7 \times 10^{14} \left( \frac{M_{\text{PBH}}}{10^{15} \text{g}} \right)^{3} \frac{1}{f(M_{\text{PBH}})} \text{s}, \]  

where \( f(M_{\text{PBH}}) \) is a function of PBH mass (Carr et al. 2016, 2010; Josan et al. 2009). For the mass range \( M_{\text{PBH}} = 10^{15} \text{g} - 10^{17} \text{g} \) \( (T_{\text{PBH}} = 0.106 \text{MeV} - 10.6 \text{MeV}) \), electrons can be emitted, and muons can be emitted for smaller masses \( M_{\text{PBH}} = 10^{14} \text{g} - 10^{15} \text{g} \) \( (T_{\text{PBH}} = 10.6 \text{MeV} - 106 \text{MeV}) \). Pions can be emitted for PBHs in the mass range \( M_{\text{PBH}} \lesssim 5 \times 10^{14} \text{g} \), where PBHs have completed their evaporation at the present epoch. For the temperature of PBHs exceeding the QCD confinement scales, \( \Lambda_{\text{QCD}} = 250 - 300 \text{MeV} \) \( (M_{\text{PBH}} = 3.5 \times 10^{15} \text{g} - 4.2 \times 10^{15} \text{g}) \), other fundamental particles such as quarks and gluons can be emitted (Carr et al. 2010, 2016). In this paper, we focus on the PBH with a lifetime in the range of \( 10^{13} \text{s} \lesssim \tau_{\text{PBH}} \lesssim 10^{17} \text{s} \), which corresponds to the mass range of \( 10^{13} \text{g} \lesssim M_{\text{PBH}} \lesssim 10^{14} \text{g} \). For this mass range, PBHs complete their evaporation between the epoch of recombination \( (z \sim 1100) \) and reionization \( (z \sim 6) \).

The particles emitted by PBHs have interactions with particles present in the Universe. The evolution of IGM can be changed due to the interactions. The main influences on IGM are ionization and heating, which have imprints on the CMB (Shull & van Steenberg 1985; Chen & Kamionkowski 2004; Mack & Wesley 2008). Although the mechanism of Hawking radiation is not the same as that of DM decay, the influences of PBHs on the evolution of IGM are similar to the DM decay case (Mack & Wesley 2008). 1 Specifically, for the purposes of our calculations, the lifetime of PBH with a fixed mass, \( \tau_{\text{PBH}}(M_{\text{PBH}}) \), can be equivalent to the inverse of the DM decay rate \( \Gamma_{\text{DM}}^{-1} \). Following the methods given by Chen & Kamionkowski (2004); Zhang et al. (2007), taking into account the Hawking radiation of PBHs, the evolution of the ionization degree \( (x_e) \) and the temperature of IGM \( (T_k) \) can be written as

\[ (1 + z) \frac{dx_e}{dz} = \frac{1}{H(z)} \left[ R_e(z) - I_e(z) - I_{\text{PBH}}(z) \right], \]
\[ (1 + z) \frac{dT_k}{dz} = \frac{8 \pi T^4_{\text{CMB}}}{3 m_e c H(z)} \left( 1 + f_{\text{He}} + x_e \right) \left( T_k - T_{\text{CMB}} \right) \]
\[ \frac{2}{3 k_B H(z)} \left( 1 + f_{\text{He}} + x_e \right) + T_k, \]

where \( R_e(z) \) and \( I_e(z) \) are the standard recombination rate and ionization rate, respectively. \( I_{\text{PBH}} \) and \( K_{\text{PBH}} \) are the ionization rate and heating rate caused by PBHs, which can be written as

\[ I_{\text{PBH}} = \chi_i f \left( f'(\Omega_{\text{PBH}}/\Omega_0) (m_e c^2 / E_b) \tau_{\text{PBH}}^{-1} e^{-t/t_{\text{PBH}}} \right), \]
\[ K_{\text{PBH}} = \chi_h f \left( f'(\Omega_{\text{PBH}}/\Omega_0) (m_e c^2 / E_b) \tau_{\text{PBH}}^{-1} e^{-t/t_{\text{PBH}}} \right), \]

where \( f \) is the fraction of the energies deposited in the IGM and it is generally a function of redshift (Madhavacheril et al. 2014). In this paper, we treat \( f \) as a free parameter and the relevant discussions are given in following sections. \( f' \) is the fraction of electrons and positrons among the particles emitted by PBHs. As mentioned above, PBHs can emit different particles depending on their masses. Moreover, the mass of PBH can be changed with the particle emission. For the masses of PBHs considered in this paper, the most influences of particles on the IGM are caused by electrons, positrons and photons (Chen & Kamionkowski 2004). For the photons, due to the processes of pair production, the energy deposited in the IGM can be treated as that of the electrons and positrons (Mack & Wesley 2008). Therefore, for the masses of PBHs considered in this paper, following Carr et al. (2010), we adopt \( f' = 0.1 \). \( \Omega_{\text{PBH}} \) and \( \Omega_0 \) are the density parameters of PBH and baryon, respectively. \( E_b = 13.6 \text{ eV} \) is the ionization energy. \( \chi_i \) and \( \chi_h \) are the fractions of deposited energy for the ionization and heating of IGM, and which have been computed in detail by Shull & van Steenberg (1985). In this paper, we use the forms suggested by Chen & Kamionkowski (2004), \( \chi_i = (1 - x_e) / 3 \), \( \chi_h = (1 + 2 x_e) / 3 \). The more accurate calculations and discussions about \( \chi_i,h \) have been done by Galli et al. (2013); Chluba (2010). According to the discussions given by Galli et al. (2013), the forms of \( \chi_i,h \) used in this paper are enough for our calculations and it is excepted that the much more accurate ones can effect our final results slightly. 2

In order to investigate the evolution of IGM described by Eqs. (5) and (6), we have modified the public code RECFAST 3 to account for the contributions of PBHs. For parameter fitting, we have used the Markov Chain Monte Carlo (MCMC) techniques. We modify the public MCMC code CosmoMC 4 in order to vary the new parameters along with

1 Here we do not consider the accretion of gas onto PBHs. The energy released from the accretion process can deposit into IGM, and the evolution of IGM is also changed. For this case, the PBHs is not similar to the DM decay case.

2 For more detailed discussions, one can refer to the Sec.V given by Galli et al. (2013).

3 http://camb.info

4 http://cosmologist.info/cosmomc
the cosmological parameters. As shown above, for our purposes, we should consider the set of six cosmological parameters, \( \{ \Omega_m h^2, \Omega_b h^2, \theta, \tau, n_s, A_s \} \), and two new parameters, \( \tau_{\text{PBH}} \) and \( \zeta \equiv f_{\text{PBH}} \), \( \Omega_b h^2 \) and \( \Omega_m h^2 \) are the density parameters of baryon and dark matter, \( \theta \) is the ratio of the sound horizon at recombination to its angular diameter distance multiplied by 100, \( \tau \) is the optical depth, \( n_s \) and \( A_s \) are the spectral index and amplitude of the primordial density perturbation power spectrum. For the PBH with a lifetime larger than the age of the Universe, \( \tau_{\text{PBH}} \gtrsim 10^{17} \) s, the factor \( e^{-t_{\text{PBH}}} \) in Eqs. (5) and (6) is close to 1. Therefore, only one new combined parameter \( \tau_{\text{PBH}}^{-1} \zeta \equiv f_{\text{PBH}} \) is needed to be fitted (Zhang et al. 2007; Chen & Kamionkowski 2004; Mack & Wesley 2008; Yang 2015). However, for the PBH with a short lifetime as being considered in this paper, one must fit two new parameters \( \tau_{\text{PBH}}^{-1} \) and \( \zeta \equiv f_{\text{PBH}} \) simultaneously (Zhang et al. 2007; Chen & Kamionkowski 2004).

For our purposes, the final constraints on the power spectrum of curvature perturbation, we are most interested in the two new parameters \( \tau_{\text{PBH}}^{-1} \) and \( \zeta \). After running the MCMC code CosmoMC with the Planck-2015 data, the constraints on the parameters are obtained. For our purpose, in Fig. 1, we plot the constraints on the PBH parameters in the two-dimensional parameter space \([ \log_{10}(\tau_{\text{PBH}}^{-1}), \log_{10}(\zeta)/10 ]\), which are obtained after marginalization over the other parameters (6 cosmological parameters), and the constraints on the other parameters are not shown. As shown in Fig. 1, the region above the black line is excluded by the Planck-2015 data and the allowed region is under the black line. The boundary line can be written in a simple form and the allowed parameters space is given by

\[
\frac{1}{10} \log_{10}(\zeta) < p_1 + p_2 x + p_3 x^2 + p_4 x^3 + p_5 x^4 + p_6 x^5, \tag{7}
\]

where \( p_1 = -555.461, p_2 = -1836.23, p_3 = -2418.65, p_4 = -1583.48, p_5 = -515.301, p_6 = -66.6964 \) and \( x = \frac{1}{10} \log_{10}(\tau_{\text{PBH}}^{-1}) \).

The formation of PBHs is related to the large density perturbations existed in the early Universe. One of the most important issues is the initial mass fraction of PBHs, which is defined as \( \beta(M_{\text{PBH}}) \equiv \rho_{\text{PBH}}/\rho_c \) (Carr et al. 2010), and it stands for the fraction of the horizon mass which collapses into the formation of PBHs. \( \rho_{\text{crit}} \) is the critical energy density at the formation time of PBHs. The initial mass fraction has a relation to the parameter \( \zeta \) as (Josan et al. 2009; Carr et al. 2010)

\[
\beta(M_{\text{PBH}}) = 1.5 \times 10^{-18} \, \zeta \left( \frac{M_{\text{PBH}}}{5 \times 10^{14} \text{g}} \right)^{1/2}. \tag{8}
\]

Using Eqs. (7) and (8), one can obtain the upper limits on the initial mass fraction of PBH for different masses, the results are shown in Fig. 2. For comparison, the upper limits on \( \beta(M_{\text{PBH}}) \) from WMAP3 and EGB are also shown (Zhang et al. 2007; Carr et al. 2010). For the mass \( M_{\text{PBH}} \sim 10^{14} \) g, the initial mass fraction is \( \beta(M_{\text{PBH}}) \sim 10^{-28} \) and it is about 2 orders of magnitude stronger compared with that obtained from WMAP3 data (Carr et al. 2010). There are many other observations which have been used to constrain \( \beta(M_{\text{PBH}}) \) for different masses of PBHs. For the mass range considered by us, the limits on \( \beta(M_{\text{PBH}}) \) are mainly from the extragalactic antiprotons, extragalactic neutrinos and the extragalactic photon background (EGB) (Weidenspointner et al. 2000; Strong et al. 2004; Abdo & Ackermann 2010; Carr et al. 2010). Among them, the most stringent constraints are from the EGB. With the non-observation of the excess in EGB, Carr et al. (2010) found a upper limit \( \beta(M_{\text{PBH}}) \sim 10^{-25} \) for \( M_{\text{PBH}} \sim 10^{14} \) g. On the other hand, utilizing the relevant results obtained from the WMAP3 data (Zhang et al. 2007), Carr et al. (2010) found a upper limit \( \beta(M_{\text{PBH}}) \sim 10^{-26} \) for \( M_{\text{PBH}} \sim 10^{15} \) g. The future 21cm surveys, such as the Square Kilometer Array, could give a much better upper limit, e.g. \( \beta(M_{\text{PBH}}) \sim 10^{-29} \) for \( M_{\text{PBH}} \sim 10^{15} \) g, if the foreground can be removed totally (Mack & Wesley 2008).

It should be noticed that for the results shown in Fig. 2 we have set the parameter \( f = 1 \), which means that all of the energies of electrons and positrons emitted by PBHs deposit into the IGM. In general, \( f \) is a function of redshift, and it is also different for different particles. For more detailed discussions, one can refer to e.g. Madhavacheril et al. (2014).

### 3 CONSTRAINTS ON THE PRIMORDIAL CURVATURE PERTURBATION

After obtaining the limits on \( \beta(M_{\text{PBH}}) \), in this section we briefly review how the initial mass fraction of PBHs relates to the primordial curvature perturbation, and we get the limits on the power spectrum of the primordial curvature perturbation.

At the end of inflation, PBHs can be formed at the scales which have left the horizon. The scales which never leave the horizon during inflation can also form

![Figure 1. The constraints (95% C.L.) on the PBH parameters in the two-dimensional parameter space \([ \log_{10}(\tau_{\text{PBH}}^{-1}), \log_{10}(\zeta)/10 ]\), which are obtained after marginalization over the other parameters (6 cosmological parameters). The region above the black line is excluded by the Planck-2015 data. In this plot, we have set the lifetime range of PBH as \( 10^{13} \) s \( \leq \tau_{\text{PBH}} \leq 10^{17} \) s.](image-url)
PBHs (Zaballa et al. 2007). For more detailed discussions about the formation of PBHs one can refer to e.g. Carr (2005). The primordial density perturbation could be gaussian or non-gaussian (Hidalgo 2007). In this work, we considered the gaussian perturbations. According to the Press-Schechter theory (W.H.Press & Schechter 1974), for the gaussian perturbations, the initial mass fraction of PBHs can be written as

$$\beta(M_{PBH}) = 2 \frac{M_{PBH}}{M_H} \int_{0.3}^{1} P(\delta_H(R)) d\delta_H(R),$$

where $M_{PBH} = f_M M_H$, $M_H$ is the horizon mass, $f_M = (1/3)^{1.5}$ is the fraction of the horizon mass which can form PBHs (Josan et al. 2009). $\delta_H(R)$ is the smoothed density contrast at horizon crossing, where $R = (aH)^{-1}$. $P(\delta_H(R))$ is the probability distribution of the smoothed density contrast with the gaussian perturbations at horizon crossing,

$$P(\delta_H(R)) = \frac{1}{\sqrt{2\pi \sigma_H(R)}} \exp\left(-\frac{\delta_H^2(R)}{2\sigma_H^2(R)}\right),$$

where $\sigma(R)$ is the mass variance. Then the initial mass fraction of PBHs can be written as

$$\beta(M_{PBH}) = \frac{2 f_M}{\sqrt{2\pi \sigma_H(R)}} \times \int_{0.3}^{1} \exp\left(-\frac{\delta_H^2(R)}{2\sigma_H^2(R)}\right) d\delta_H(R),$$

The mass variance $\sigma(R)$ is related to the power spectrum of density perturbations, $P_\delta(k, t)$, as following form,

$$\sigma^2(R) = \int_0^\infty W^2(kR)P_\delta(k)\frac{dk}{k},$$

where $W(kR)$ is the Fourier transform of the window function used to smooth the density contrast. The power spectrum of primordial curvature perturbation, $P_\mathcal{R}(k)$, is related to the power spectrum of primordial density perturbation as (Josan et al. 2009)

$$P_\mathcal{R}(k) = \frac{16}{3} \left(\frac{k}{aH}\right)^2 j_0^2(k/\sqrt{3}aH)P_\delta(k),$$

Substituting Eq.(13) into Eq.(12) and setting $R = (aH)^{-1}$, the mass variance is written as

$$\sigma_H^2(R) = \frac{16}{3} \int_0^\infty (kR)^2 j_0^2(kR/\sqrt{3})\exp(-k^2R^2)P_\mathcal{R}(k)\frac{dk}{k},$$

The integral result of Eq. (14) is dominated in the scales $k \sim 1/R$. Following Josan et al. (2009) we use the form of $P_\mathcal{R}(k)$ which is valid for general slow-roll inflation models as (Kohri et al. 2008; Leach et al. 2000)

$$P_\mathcal{R}(k) = P_\mathcal{R}(k_0) \left(\frac{k}{k_0}\right)^{n(k_0)-1},$$

where $n(k_0) = 1$, and the changes of $n(k_0)$ in the ranges being allowed by the present observations would effect our final results slightly (Josan & Green 2010; Josan et al. 2009).

Using Eq. (11), the constraints on the initial mass fraction of PBHs, $\beta(M_{PBH})$, can be used to get the constraints on the mass variance, $\sigma(R)$. Then using Eq. (14) one can obtain the constraints on the power spectrum of primordial curvature perturbation, $P_\mathcal{R}(k)$. The final results are shown in Fig. 3. For the scales corresponding to the lifetime (or mass) range considered in this paper, 8.9 $\times$ 10$^{15}$ Mpc$^{-1}$ $\lesssim k \lesssim 2.8 \times 10^{16}$ Mpc$^{-1}$, the limits do not change nearly with a value of $P_\mathcal{R}(k) \sim 0.0045$. $P_\mathcal{R}(k)$ can also be constrained by many other observations (Josan et al. 2009). In Fig. 3, the upper limits on $P_\mathcal{R}(k)$ from WMAP3 and EGB are also shown, which are obtained by converting the upper limits on $\beta(M_{PBH})$ given in Fig. 2. From Fig. 3, it can be seen that our limits are slightly improved compared with that obtained from WMAP3 and EGB. One should be noticed that for the constraints on $P_\mathcal{R}(k)$, we have adopted $f = 1$ as done in Sec. II. Be similar to the constraints on $\beta(M_{PBH})$, the limits on $P_\mathcal{R}(k)$ are weaker for the other values of $f$ ($f < 1$).

4 CONCLUSIONS

Constraints on the power spectrum of primordial curvature perturbation are very important for the cosmological researches. On large scales, the constraints are mainly from the observations and researches on the CMB, Lyman-$\alpha$ and large scale structures. On small scales, the constraints are mainly from the researches on PBHs but these constraints are fairly weak. Be similar to the DM decay, PBHs have influences on the evolution of the IGM through the Hawking radiation. One of the results of the influences is that the anisotropies of the cosmic microwave background are changed. In this paper, taking into account the influences of PBHs on the evolution of the IGM, we used

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Figure2.png}
\caption{The upper limits (95% C.L.) on the initial mass fraction of PBH in the mass range of $2.8 \times 10^{13} \lesssim M_{PBH} \lesssim 2.5 \times 10^{14}$ (red solid line). Here, we have set the free parameter $f$ in Eqs. (5) and (6) as $f = 1$. The line is truncated due to the lifetime range of PBH considered in this paper. For comparison, the upper limits from WMAP3 (black dot-dashed line) and EGB (green dotted line) are also shown (Zhang et al. 2007; Carr et al. 2010).}
\end{figure}
Figure 3. The upper limits (95% C.L.) on the power spectrum of primordial curvature perturbation $P_R(k)$ for the scale range of $8.9 \times 10^{15}$ Mpc$^{-1} \lesssim k \lesssim 2.8 \times 10^{16}$ Mpc$^{-1}$ (red solid line). Corresponding to the Fig. 2, the line is truncated due to the lifetime range of PBH considered in this paper. For comparison, the upper limits from WMAP3 (black dotted-dashed line) and EGB (green dotted line) are also shown (Zhang et al. 2007; Carr et al. 2010), which are obtained by converting the upper limits shown in Fig. 2.

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