A Resistive Wideband Space Beam Splitter

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Abstract—We present the design, construction and measurements of the electromagnetic performance of a wideband space beam splitter. The beam splitter is designed to power divide the incident radiation into reflected and transmitted components for interferometer measurement of spectral features in the mean cosmic radio background. Analysis of a 2-element interferometer configuration with a vertical beam splitter between a pair of antennas leads to the requirement that the beam splitter be a resistive sheet with sheet resistance $\eta_0/2$, where $\eta_0$ is the impedance of free space. The transmission and reflection properties of such a sheet are computed for normal and oblique incidences and for orthogonal polarizations of the incident electric field. We have constructed such an electromagnetic beam splitter as a square soldered grid of resistors of value 180 Ohms (approximately $\eta_0/2$) and a grid size of 0.1 m, and present measurements of the reflection and transmission coefficients over a wide frequency range between 50 and 250 MHz in which the wavelength well exceeds the mesh size. Our measurements of the coefficients for voltage transmission and reflection agree to within 5% with physical optics modeling of the wave propagation, which takes into account edge diffraction.

Index Terms—Antenna measurements, radio astronomy, Maxwell equations, physical optics.

I. INTRODUCTION

Events in the cosmological evolution of the gas are predicted to have left imprints in the spectrum of the cosmic radio background. Spectral features caused by events during the epochs of reionization and recombination as well as relatively wider spectral distortions in the cosmic microwave background that arise from energy injection in the early universe—followed by partial or saturated Comptonization—are all orders of magnitude smaller in brightness compared to the brightness of the radio sky; therefore, their detection requires development of purpose-built sensitive spectral radiometers.

A limiting factor in absolute measurements of the spectrum of the sky with a spectral radiometer is unwanted additive contributions generated within the receiver electronics. Radiometers to date rely on switching schemes to cancel these. Most often the input to the receiver is switched between the sensor of the sky electromagnetic radiation and a load that provides a reference spectrum. Another common technique is the use of correlation receivers in which the signal from the sensor is split into two parts, amplified in separate receiver chains and a cross correlation spectrum is computed between the two arms; this nominally eliminates the receiver noise contributions because their noise voltages are statistically independent.

Absolute measurements of the cosmic radio background using a single antenna element as the sensor of the sky radiation will always include emissions from the resistive losses in the antenna and associated balun. Additionally, although switching schemes and correlation receivers do cancel internal sources of receiver noise to a large extent, accuracy in spectral measurements is limited by small magnitude changes in system bandpass, impedance matching in switching schemes and multi-path propagation of internal noise. These manifest as uncalibrated spectral structure in the measurements that often take the form of spectral ripples with a range of periods.

Radio interferometers have the advantage of being unresponsive to additive noise contributions arising from ohmic loss in antennas and noise in amplifiers and receiver chains because these components generated in different arms of an interferometer are are uncorrelated. However, interferometers are also almost completely insensitive to the uniform sky brightness and hence useless for absolute measurement of the cosmic radio background and its spectrum. A way to retain the advantages of interferometers and make absolute measurements of spectral structure in the uniform sky brightness is to construct a space beam splitter that divides and directs the incident sky power into two antennas that form an interferometer pair. The interferometer then responds to and provides a measurement of the common mode power.

We present below this concept, which we call a ‘zero-spacing interferometer’. The sections that follow develop the design of the electromagnetic space beam splitter for this configuration. The system design of the zero-spacing interferometer will be presented in a subsequent manuscript.

II. THE CONCEPT OF A ZERO-SPACING INTERFEROMETER

The configuration consists of a vertical electromagnetic beam splitter placed at the geometric center of the antennas, which are placed at a common height above ground covered with absorber (Fig. 1). Radiation from the sky incident on the beam splitter from either side are partially transmitted to the antenna on the far side and partially reflected to the antenna on the near side. Each antenna, therefore, receives EM radiation transmitted through the beam splitter from the far side as well as reflected off the beam splitter from the near side. The fields sensed by the antennas are amplified and cross-correlated and the spectral distribution of the cross power spectrum is a measurement of the spectral distribution of the absolute sky brightness. The antenna pair forms an interferometer that is potentially sensitive to uniform sky brightness; therefore, we refer to this configuration as a ‘zero-spacing interferometer’. It is of interest to first compute the transmission and reflection properties of the beam splitter that would maximize the sensitivity to this component.

As shown in Fig. 1, $E_1(t)$ and $E_2(t)$ represent the EM waves from the sky that are incident on the two sides of the...
beam splitter and $\Gamma e^{-j\beta}$ and $\tau e^{-j\alpha}$ represent the complex reflection and transmission coefficients. The signals sensed by each of the two antennas, $E_a(t)$ and $E_b(t)$, are vector sums of reflected and transmitted signals:

$$E_a(t) = E_1(t)\Gamma e^{-j\beta} + E_2(t)\tau e^{-j\alpha}, \quad (1)$$

and

$$E_b(t) = E_2(t)\Gamma e^{-j\beta} + E_1(t)\tau e^{-j\alpha}. \quad (2)$$

Cross-correlation of these antenna signals gives the measurement:

$$\langle E_a(t) E_b(t)^* \rangle = (|E_1(t)|^2\Gamma|e^{-j(\beta - \alpha)}| + (|E_2(t)|^2\Gamma|e^{-j(\alpha - \beta)} |)$$

$$+ |E_1(t)||E_2(t)|^2|\tau|^2$$

$$+ |E_1(t)\cdot E_2(t)|^2\tau^2. \quad (3)$$

Since the two incident beams $E_1(t)$ and $E_2(t)$ are from different sky directions they are uncorrelated and hence the last two terms in the above equation average to zero. The product becomes

$$\langle E_a(t) E_b(t)^* \rangle = \Gamma\tau(|E_2(t)|^2 + |E_1(t)|^2)\cos(\beta - \alpha)$$

$$- j\Gamma\tau(|E_2(t)|^2 - |E_1(t)|^2)\sin(\beta - \alpha). \quad (4)$$

Equation (4) is the measurement equation for the zero-spacing interferometer; the real part is proportional to the sum of the powers incident from the two sides of the beam splitter where as the imaginary part is proportional to the difference. For uniform sky, the imaginary component will vanish.

**III. RESPONSE OF A ZERO-SPACING INTERFEROMETER WITH A LOSSLESS SPACE BEAM SPLITER**

For a beam splitter that has no charge or time varying magnetic field and hence no curl or divergence sources of the electric field, the electric field will be continuous across the surface. This implies that

$$1 + \Gamma e^{-j\beta} = \tau e^{-j\alpha}. \quad (5)$$

The real and imaginary parts of this equation lead separately to the relationships: $1 + \Gamma\cos\beta = \tau\cos\alpha$ and $\Gamma\sin\beta = \tau\sin\alpha$. For a lossless electromagnetic beam splitter, conservation of power requires that $\Gamma^2 + \tau^2 = 1$. These lead to the condition $(\beta - \alpha) = \pi/2$, which implies that for such a lossless beam splitter the measurement $\langle E_a(t) E_b(t)^* \rangle$ has zero response to a uniform sky background.

The characteristics of a space beam splitter may also be derived in terms of scattering parameters. In this approach, the transmission and reflection of the radiation incident on the space beam splitter may be described by a four port network. As shown in Fig. 2 the incoming waves $[V^+]$ and outgoing waves $[V^-]$ are related by the scattering matrix $[S]$ as:

$$[V^-] = [S] \cdot [V^+]. \quad (6)$$

![Fig. 2: A four-port network model for the space beam splitter](image-url)

For a four-port lossless reciprocal network, the relevant terms in the scattering matrix are

$$
\begin{bmatrix}
V_1^- \\
V_2^- \\
V_3^- \\
V_4^- \\
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & S_{13} & S_{14} \\
0 & 0 & S_{23} & S_{24} \\
S_{31} & S_{32} & 0 & 0 \\
S_{41} & S_{42} & 0 & 0 \\
\end{bmatrix}
\cdot
\begin{bmatrix}
V_1^+ \\
V_2^+ \\
V_3^+ \\
V_4^+ \\
\end{bmatrix}.
\quad (7)
$$

The two outputs, $V_3^-$ and $V_4^-$ can be written as:

$$V_3^- = S_{31}V_1^+ + S_{32}V_2^+, \quad (8)$$

and

$$V_4^- = S_{41}V_1^+ + S_{42}V_2^+. \quad (9)$$

The scattering matrix $[S]$ for any lossless reciprocal network is symmetric and unitary[5], which implies that

$$S_{13} = S_{31}, \quad S_{23} = S_{32}, \quad S_{14} = S_{41}, S_{24} = S_{42}, \quad (10)$$

$$\sum_{k=1}^{4} S_{ki}S_{ki}^* = 1 \quad \forall \ i, \quad (11)$$
and
\[ \sum_{k=1}^{4} S_{k}S_{k}^{*} = 0 \quad \forall \; i \neq j. \]  
(12)

From the above conditions we infer that
\[ S_{31}S_{41}^{*} = -S_{32}S_{42}^{*}, \]  
(13)

which may be written in terms of the transmission and reflection coefficients as
\[ \Gamma e^{-j\beta}e^{j\alpha} = -\tau e^{-j\alpha}\Gamma e^{j\beta}. \]  
(14)

This leads to the result that \((\alpha - \beta) = \pi/2\) or that for a lossless beam splitter the measurement \((E_{a}(t)) \cdot \bar{E}_{b}(t)\) yields null value for a uniform sky.

A lossless space beam splitter will manifest a phase difference of \(\pi/2\) between the transmitted and reflected components as is the case for a partially silvered mirror. The lack of response to uniform sky is a consequence of this property. Which leads to the conclusion that the beam splitter for a zero-spacing interferometer requires to be lossy and not have a \(\pi/2\) phase shift between transmission and reflection coefficients if the zero-spacing interferometer should respond to uniform sky.

IV. EM PROPAGATION THROUGH A BEAM SPLITTER WITH FINITE SHEET CONDUCTIVITY

The space beam splitter is taken to be a sheet of conductance \(S = \sigma \cdot \delta x\) (Siemen-square), where \(\sigma\) is the material conductivity and \(\delta x\) is the sheet thickness. When an electromagnetic wave traveling in one medium is incident on a second medium with a different intrinsic impedance (\(\eta\)) the wave, in general, will be partially reflected and partially transmitted. Since the second medium considered here has a definite value of conductivity a part of the incident energy will also be absorbed at the sheet.

A. The case of normal incidence

We adopt right handed Cartesian coordinates with the sheet placed in the \(yz\)-plane. The case of a plane wave traveling in the positive \(x\) direction normal to the surface of the sheet is considered. Let \(E_{i}\) be the electric field intensity of the incident wave at the sheet, \(E_{r}\) be the electric field intensity of the reflected wave and \(E_{t}\) be the electric field intensity of the transmitted wave on the far side of the sheet. Similar subscripts are used to denote the magnetic field intensities of the corresponding waves. The \(E\) and \(H\) vector fields are assumed to be along the \(z\) and \(y\) directions respectively; their components are written with a negative sign if their corresponding vector directions are opposite to that of the Cartesian axes. The incident, reflected and transmitted waves travel through the same medium, free space, whose intrinsic impedance \(\eta_{o}\) is 377 \(\Omega\). These are illustrated in Fig. 3.

The boundary conditions for a conductive sheet are:
- the discontinuity in the normal component of \(E\) is proportional to the surface charge density,
- the normal component of \(B\) is continuous,
- the tangential component of \(E\) is continuous, and
- the discontinuity in the tangential component of \(H\) equals the surface current density.

For waves whose directions of incidence, reflection and transmission are normal to the conductive sheet, the normal components of \(E\) and \(B\) are zero. The boundary conditions for the tangential components across the sheet of thickness \(\delta x\) may be expressed as follows:
\[ \frac{\delta E_{z}}{\delta x} = 0, \]

or
\[ E_{i} + E_{r} - E_{t} = 0. \]  
(15)

\[ \frac{\delta H_{y}}{\delta x} = J_{z}, \]

where \(J_{z}\) is the \(z\)-component of the current density in the sheet. This may be expanded to
\[ H_{i} - H_{r} - H_{t} = \sigma \cdot E_{c} \cdot \delta x, \]

where the electric field at the sheet is denoted by \(E_{c}\). It follows that
\[ H_{i} - H_{r} - H_{t} = S \cdot E_{c}. \]  
(16)

The negative sign for \(H_{i}\) is because the propagation of the reflected wave is in the negative \(x\) direction.

The relation between electric and magnetic field intensities is \(H = E/\eta\). Substituting this relation in equation
\[ \frac{1}{\eta_{o}}(E_{i} - E_{r} - (E_{i} + E_{r})) = S \cdot E_{c}, \]

or
\[ E_{r} = -S\frac{\eta_{o}}{2} E_{c}. \]  
(17)

The net electric field \(E_{c}\) that drives the current density in the sheet is the sum of the incident electric field and the reflected
electric field, \((E_i + E_r)\), which is also the transmitted electric field \(E_t\):

\[ E_t = E_i - S \eta_o \frac{E_i}{2 (1 + S \eta_o/2)}. \]

The reflection coefficient for the electric field, which is defined as the ratio of the reflected to the incident electric fields, can hence be written as:

\[ \Gamma = \frac{S \eta_o}{(1 + S \eta_o/2)} \angle 180^\circ. \tag{18} \]

The transmitted electric field may be written in terms of the incident field by substituting the expression derived above for \(E_t\) in equation (15):

\[ E_t = E_i \left(1 - \frac{S \eta_o}{1 + S \eta_o/2}\right). \]

This leads to an expression for the transmission coefficient for the electric field, which is the ratio of the transmitted to the incident electric fields:

\[ \tau = \frac{1}{1 + S \eta_o/2} \angle 0^\circ. \tag{19} \]

### B. The desired impedance of a space beam splitter in a zero-spacing interferometer

The selection of the sheet impedance of the electromagnetic beam splitter has been carried out with the goal of maximizing the response of the zero-spacing interferometer or correlation response between the two arms of the interferometer to uniform sky. Since the measurement equation \(\text{Equation 4}\) gives the real component to be the sum of the sky radiations on the two sides of the beam splitter, maximizing the response is just maximizing this real term neglecting the imaginary component.

The reflection and transmission coefficients computed above \(\text{Equations 18 and 19}\) have a phase difference of \(180^\circ\). The response to uniform sky is a maximum when \(\cos(\beta - \alpha)\) is unity, which requires that the conductance of the space beam splitter should be a real value, \(i.e.,\) the sheet is purely resistive.

The value of \(S\) should also be such that it maximizes the product:

\[ \Gamma \cdot \tau = \frac{S \eta_o}{(1 + S \eta_o/2)^2}, \tag{20} \]

which is a maximum when \(S = 2/\eta_o\). At this point of inflection the magnitudes of the reflection and transmission coefficients become equal and equal to \(1/2\). Therefore, maximizing the product and thereby maximizing the real term in \(\text{Equation 4}\) leads to the result that the sheet conductance \(S\) needs to be real and equal to \(2/\eta_o\) Siemen-square.

For such a resistive sheet, the power reflected, transmitted and absorbed per unit area may be expressed in terms of the incident power per unit area as:

\[ \text{Incident power} = |E_i \times H_i| = \frac{E_i^2}{\eta_o}. \tag{21} \]

Reflected power \(\text{Equation 22}\):

\[ \text{Reflected power} = \frac{E^2}{\eta_o} = \frac{1}{\eta_o} \left(\frac{S \eta_o}{2}\right)^2 \left(\frac{E_i}{1 + S \eta_o/2}\right)^2 \]

\[ = 1 + \frac{E^2}{4 \eta_o}. \tag{22} \]

Transmitted power \(\text{Equation 23}\):

\[ \text{Transmitted power} = \frac{E^2}{\eta_o} = \frac{1}{\eta_o} \left(\frac{1}{1 + S \eta_o/2}\right)^2 = \frac{1}{4} \frac{E_i^2}{\eta_o}. \tag{23} \]

The power absorbed by the resistive sheet per unit area is calculated as the ohmic dissipation by the current density, which is given by:

\[ \text{Power absorbed} = \frac{(J \cdot \delta x)^2}{S} = E^2 C. \tag{24} \]

Thus, the power absorbed by the resistive sheet is half of the incident power, with a quarter reflected and the remaining quarter transmitted.

### C. Analysis of an impedance sheet as a shunt impedance in free space

The transmission and reflection coefficients for an EM wave encountering an impedance sheet may also be evaluated by modeling the sheet as a shunt in free space using a transmission line analogy. As shown in Fig. 4 a sheet of effective impedance \(Z_s\) may be modeled as forming a shunt in a transmission line that has line impedance equal to free space impedance \(\eta_o\) on either side. The incident EM wave is considered as arriving from the left in the figure, and the infinite transmission line to the right of the sheet is modeled as a termination of the transmission line with impedance equal to the free space impedance \(\eta_o\).

![Fig. 4: A transmission line analog for an impedance sheet in free space.](image)

The voltage reflection and transmission coefficients at the impedance sheet are given by:

\[ \Gamma = \frac{-\eta_o/2}{\eta_o/2 + Z_s}. \tag{25} \]

and

\[ \tau = \Gamma + 1 = \frac{Z_s}{\eta_o/2 + Z_s}. \tag{26} \]

Using these values of \(\Gamma\) or \(\tau\), the fractional power dissipated per unit area of the impedance sheet is:

\[ P_{abs} = 1 - \Gamma^2 - \tau^2 = \frac{Z_s \eta_o}{(\eta_o/2 + Z_s)^2}. \tag{27} \]
If the sheet has impedance $Z_o = \eta_o/2$, $\Gamma = -1/2$, $\tau = 1/2$ and $P_{\text{abs}} = 1/2$, which are same as the results derived earlier in that half the incident power is dissipated in the resistive sheet and the reflected and transmitted waves have half the amplitude of the incident EM wave.

D. The case of oblique incidence

We next compute the transmission and reflection coefficients for oblique incidence of the incoming radiation on the sheet. The Poynting vector for the incident wave is assumed to be in the $xz$-plane and at an angle $\theta$ (the angle of incidence) to the sheet normal. Snell’s law applies here and the angle of reflection is equal to that of incidence. The angle of transmission will also remain the same as incidence because the wave travels into the same medium (air) following propagation through the sheet. For oblique incidence it is necessary to consider a pair of polarizations for the incident electric field: (1) an E-plane incidence, where the E-field is in the plane of incidence and (2) an H-plane incidence, where the incident E-field is perpendicular to the plane of incidence.

We first consider the case of H-plane incidence. The incident, reflected and transmitted waves and the orientations of their respective components $E$ and $H$ are shown in Fig. 5. $H_i$ makes the same angle $\theta$ to the $z$-axis, and this field may be resolved into two perpendicular components, $H_{zi}$ and $-H_{zr}$. $E_i$, $E_r$ and $E_t$ are all along $-y$-axis.

![Diagram of EM wave orientations]

Fig. 5: The orientations of E and H fields of the incident, reflected and transmitted EM waves for the case where the E-field of the incident radiation is polarized perpendicular to the plane of incidence, which is the $xz$-plane. The angle of incidence is $\theta$.

There are no static charges on the sheet and currents driven by the electric field are along $y$-axis; we define $\mathbf{J}$ to represent the vector current along positive $y$ axis. Integral forms of Maxwell’s equations lead to the relations:

\[ E_i + E_r - E_t = 0, \]  \hspace{1cm} (28)

\[ H_{zi} - H_{zx} - H_{zt} = -J_y \delta x \]  \hspace{1cm} (29)

and

\[ H_{xi} + H_{xr} - H_{xt} = 0. \]  \hspace{1cm} (30)

Writing Equation (29) in terms of $E$ fields and substituting $\delta x \cdot S$ for $-J_y \cdot \delta x$,

\[ \frac{1}{\eta_o} (E_i \cos \theta - E_r \cos \theta - E_t \cos \theta) = SE_c. \]

Using Equation (28) we may rewrite this expression in the form

\[ E_r = -\frac{\eta_o S E_c}{2 \cos \theta}. \]  \hspace{1cm} (31)

The net electric field $E_c$ along the sheet is given by the sum of $E_i$ and $E_r$:

\[ E_c = E_i - \frac{\eta_o S E_c}{2 \cos \theta}. \]

We may now substitute the above expression for $E_c$ in Equation (31) to derive the reflection coefficient:

\[ \Gamma = \frac{\eta_o S}{2 \cos \theta + \eta_o S} \angle 180^\circ. \]  \hspace{1cm} (32)

The transmission coefficient may be evaluated from the relation $1 + \Gamma = \tau$, which follows from Equation (28). We derive that

\[ \tau = \frac{\cos \theta}{\cos \theta + \eta_o S/2} \angle 0^\circ. \]

If the conductance of the sheet is set to be $S = 2/\eta_o$, $\Gamma$ and $\tau$ reduce to the forms

\[ \Gamma = \frac{1}{\cos \theta + 1} \angle 180^\circ, \]  \hspace{1cm} (33)

\[ \tau = \frac{\cos \theta}{\cos \theta + 1} \angle 0^\circ. \]  \hspace{1cm} (34)

We next consider the case of E-plane incidence. For an incoming wave with electric field polarized to be in the plane of incidence, the electric field will make an angle $\theta$, equal to the angle of incidence, to the $z$-axis. The $E$ component of the incident, reflected and transmitted waves may be resolved into $E_x$ and $E_z$ components as shown in Fig. 6.
Fig. 6: The orientations of E and H components of the incident, reflected and transmitted EM waves when the incident wave is polarized to have the electric field in the plane of incidence.

Using once again the integral forms of Maxwell’s equations to derive the change in fields across the sheet, and assuming that we have no net charges in the resistive sheet and that the net current density is in the sheet and in the plane of incidence where the electric fields are oriented:

\[ E_{zi} + E_{zr} - E_{zt} = 0, \]  
\[ (35) \]

\[ H_i - H_r - H_t = J_z \delta x, \]  
\[ (36) \]

and

\[ (-E_{xi} + E_{xr} + E_{xt}) = 0. \]  
\[ (37) \]

We have taken the current density vector \( J \) in this case study to be along positive \( z \)-axis.

Equation (36) may be written in terms of the electric field components as follows:

\[ \frac{1}{\eta_0}(E_i - E_r - E_t) = S.E_c, \]

where the vector electric field in the sheet \( E_c \) in this case is also along positive \( z \)-axis. Since the angles of the incident, reflected and transmitted waves are equal, we may solve the above equation together with the relationship between the \( z \)-components of the electric fields (Equation 35) to get:

\[ E_r = -\frac{\eta_0 S E_c}{2}. \]

\[ (38) \]

The net electric field along the sheet is due to the \( z \)-components of the electric fields of the incident and reflected waves:

\[ E_c = E_{zi} - \frac{\eta_0 S E_c \cos \theta}{2}. \]

The electric field of the reflected wave (Equation 38) expressed in terms of the above \( E_c \) gives the reflection coefficient to be:

\[ \Gamma = \frac{\eta_0 S \cos \theta}{1 + \frac{\eta_0 S \cos \theta}{2}} \angle 180^\circ. \]

\[ (39) \]

The transmission coefficient is calculated to be:

\[ \tau = \frac{1}{1 + \frac{\eta_0 S \cos \theta}{2}} \angle 0^\circ. \]

Substituting the value of sheet conductivity \( S = 2/\eta_0 \) reduces the expressions for \( \Gamma \) and \( \tau \) to the forms:

\[ \Gamma = \frac{\cos \theta}{1 + \cos \theta} \angle 180^\circ, \]

\[ (41) \]

and

\[ \tau = \frac{1}{1 + \cos \theta} \angle 0^\circ. \]

\[ (42) \]

The expressions for transmission and reflection coefficients for oblique incidence of the EM waves show a dependence on the incidence angle. Their values approach 1/2 as \( \theta \) tends to 0\(^\circ\), consistent with the results in the previous section. The transmission and reflection coefficients are real with zero phase. Hence when such a resistive sheet is deployed as the space beam splitter, the loss of response to the uniform sky brightness would depend on incidence angle via the \( \Gamma \tau \) product term in Equation 4. In Fig. 7 we plot the ratio of this loss of response normalized to the maximum value of 0.25 that this product might take for normal incidence and for different values for the sheet conductivity.

Fig. 7: Response of the zero-spacing interferometer for different angles of incidence. The response is computed as \( \Gamma \tau \) product normalized to the maximum value of 0.25. Different traces are for different values of sheet conductance \( S \): shown as a continuous line is the curve corresponding to \( S = 2/\eta_0 \) or sheet resistance of 188.5 \( \Omega \) square\(^{-1}\), the three lines above the continuous line are, progressively for sheet resistance \((1/S)\) of 225 to 275 \( \Omega \) square\(^{-1}\) in increments of 25 and the six lines below the continuous line are for decreasing values of sheet resistance from 175 to 50 \( \Omega \) square\(^{-1}\) again in intervals of 25.

As seen in the figure, for normal incidence the response is a maximum for \( S = 2/\eta_0 \) or sheet resistance of 188.5 \( \Omega \) square\(^{-1}\). For larger values of sheet resistance the response is maximum at progressively larger angles of incidence. It is unsurprising that the change in transmission and
reflection with angle of incidence \( \theta \) is exactly what would be expected if the effective conductance of the sheet is assumed to be \( S/\cos \theta \), which corresponds to the conductance of a sheet with thickness equal to the line-of-sight thickness of an equivalent resistive slab when the angle of incidence is \( \theta \). As seen in Fig. 7 as the sheet resistance increases above \( \eta_o/2 \) \( \Omega \) square\(^{-1} \) the response is a maximum along the line of sight at which \( S/\cos \theta \) equals \( 2/\eta_o \). A value somewhat exceeding \( \eta_o/2 \) \( \Omega \) square\(^{-1} \) would provide a good response over a wide range in incidence angle - the appropriate value would depend on the beam patterns of the antennas and the geometry of the zero-spacing interferometer.

E. Considerations arising from skin depth of the sheet material

At frequency \( f \), the skin depth \( \delta_s \) is given by:

\[
\delta_s = 1/\sqrt{\pi \sigma_f \mu_o \mu_r}.
\]

where \( f \) is the frequency of the incident EM wave and \( \mu_o \) and \( \mu_r \) are, respectively, the permeability of free space and the relative magnetic permeability of the sheet material. At frequencies where the skin depth \( \delta_s \) becomes comparable to or less than the sheet thickness \( \delta x \), the sheet conductance falls below the d.c. conductance \( \sigma \cdot \delta x \) and is given by

\[
S = \sigma \cdot \delta x \{1 - e^{-\delta x/\delta_s}\}.
\]

In general, it is this a.c. conductance that is required to be \( \eta_o/2 \), which implies that

\[
\sigma \cdot \delta x \{1 - e^{-\delta x/\delta_s}\} = \eta_o/2
\]

for operation as a beam splitter.

The ac conductance of the sheet tends to \( \sigma \cdot \delta x \) for large sheet thickness and to \( \sigma \cdot \delta x \) for the case of sheet thickness much smaller than the skin depth. Frequency independent performance as a beam splitter argues for adoption of thin screens, which implies that the product of bulk conductivity and the square of sheet thickness must satisfy the relation

\[
\sigma \cdot (\delta x)^2 \ll 1/(\pi f \mu_o \mu_r)
\]

over the entire frequency range of operation. If we assume that the relative magnetic permeability of the sheet material is unity, this leads to the result that the sheet thickness needs to be

\[
\delta x \ll (6.913/f_{100 \text{MHz}}) \text{ cm},
\]

where \( f_{100 \text{MHz}} \) is the frequency in units of 100 MHz: \( f_{100 \text{MHz}} = f/(100 \text{ MHz}) \).

F. Mutual coherence in the emissivity on opposite sides of a resistive sheet: an additive response in the zero-spacing interferometer

There will be thermal emission from a resistive sheet that emerges on the two sides and will have a partial mutual coherence. The emissivity will depend on the physical temperature of the resistive sheet and its opacity. The fractional coherence between the emission on the opposite sides will depend on the opacity. An opaque screen will have maximum emissivity and zero mutual coherence and as the opacity reduces the emissivity reduces and the fractional mutual coherence increases. The emission on opposite sides of a resistive sheet that forms the beam splitter in a zero-spacing interferometer would add to the system temperature of the receivers on the two sides and also appear as an additive response owing to the mutual coherence. This additive response would depend on the geometry and antenna patterns and its computation is deferred to when the system level design of the zero-spacing interferometer is presented in a subsequent manuscript.

V. A resistive wire mesh or square grid of resistors functioning as a resistive sheet

A resistive sheet may be approximated by a square wire grid made of resistive wire in which the grid size is considerably smaller than the wavelength of the EM waves. Such a realization has been used previously for the construction of an absorber screen[6]. Frequency independent operation as a beam splitter requires that the wire radius be much smaller than the skin depth over the frequency range of operation, so that the a.c. conductance of the wires is the same as the d.c. conductance. The sheet resistance of a wire grid that has square grids is equal to the resistance value of a single wire segment; therefore, every wire segment of the grid is to be made equal to \( (\eta_o/2) \) \( \Omega \) for operation as a beam splitter.

Alternatively, the resistive sheet may be constructed as a square grid of resistors. The sheet resistance of a square grid of resistors is equal to the value of the individual resistors, which implies that our beam splitter resistive sheet may be constructed as a grid of resistors in which every resistor has a value of \( (\eta_o/2) \) \( \Omega \). It is assumed here that the resistors have a.c. resistance equal to this value.

A. Performance dependence on the size of the resistor grid

The EM space beam splitter constructed from a grid of resistive wire or resistors will be frequency dependent since the grid size sets the upper limit of the frequency up to which the grid may be usefully approximated to be a continuous sheet for the incoming radiation. The reflection and transmission coefficients for a soldered resistive wire grid has been computed by Astrakhan[7]. Using their relations, we plot in Fig. 8 the gain loss of the zero-spacing interferometer arising from the use of a coarse grid for the resistive sheet. It is seen that provided the wavelength of operation exceeds eight grid units, the loss in response is less than 10%.

B. Performance dependence on the polarization of the incident EM wave

The values of the reflection and transmission coefficients of a wire grid were computed for different incidence angles (Fig. 9). The dependence of \( \Gamma \tau \) product on the angle of incidence is same for both polarizations—the gain loss versus angle of incidence is independent of the plane of polarization—and for both polarizations the gain loss of the zero-spacing interferometer follows the \( \Gamma \tau \) product given in Fig. 7.
C. Fabrication of a resistive sheet as a resistor grid

We have constructed a resistive sheet beam splitter as a square grid of resistors as a proof of concept. Every segment of the soldered wire grid is made of conductive copper wire with a carbon resistor of value $180 \, \Omega$ at the center; this was the closest value to $\eta_o/2 = 188.5 \, \Omega$ that is readily available commercially.

The resistor grid was built across a $4 \, \text{m} \times 3 \, \text{m}$ wooden frame that was constructed with using any conductive metal - no nails or metal clips were used at joints. The resistor network is supported by strapping tapes: a square grid of strapping tape was fastened on the wooden frame to serve as a base to support the resistor grid. The advantage of using a grid of tapes rather than a continuous sheet for support is the reduction of wind loading on the structure when deployed in the field. Strapping tape is commercially available as polypropylene (PP) and polyester (PET); we selected PP strapping tape since the dielectric constant is 1.5–2.5 and closer to unity compared to the 2.8–4.8 of PET. The resistor grid network is formed by first making a square grid of copper wires over the PP strapping tapes, then soldering the wires at the junctions, then soldering a resistor parallel to the wires and at the center of each wire segment, then cutting away the shorting lengths of copper wire across each resistor so as to leave a resistor grid. The resistors and soldered junctions are glued to the PP grid, which serves as a base that supports the resistor grid. To enable the grid to have frequency independent characteristics up to about 350 MHz the grid spacing is chosen to be 10 cm, which corresponds to $\lambda/8$ at 375 MHz. Photographs of the prototype resistor-grid type EM beam splitter is shown in Fig. 10 and a close-up of the grid is shown in Fig. 11.

D. Measurements of transmission and reflection coefficients: comparison with theory

The beam splitter we have constructed as a resistor grid was evaluated for its transmission and reflection properties at a number of discrete frequencies to compare with expectations: we made measurements over the range 50 to 250 MHz in which the wavelength is greater than the grid cell size but also not much larger than the total dimensions of the resistor grid. Transmission and reflection coefficients were measured using an Agilent FieldFox Network analyzer (N9915A) in network analyzer mode and the coefficients were measured as $S_{11}$ and $S_{21}$ scattering parameters. Antenna elements from the ETS-Lindgren Model 3121D dipole set were used.

The transmission measurements were made using the linearly polarized antennas, one connected to the output terminal of the network analyzer on one side of the resistor grid and the second placed on the other side and connected via a coaxial transmission line to the input terminal of the analyzer. The resistor grid is mounted vertically between the two antennas. The antennas were adjusted to be resonant at the measurement frequency. The magnitude and phase of $S_{21}$ were recorded in a narrow bandwidth about the resonant frequency. The $S_{21}$ measurement requires to be corrected for the return loss characteristics of the antennas used and also the space loss in order to derive the transmission coefficient of the resistor.
Fig. 10: Photograph of the resistor grid, which was constructed using a wooden frame, polypropylene strapping tape and 180 Ω resistors soldered at the centers of a 10-cm square conductive copper grid.

Fig. 11: A close up view of the resistor grid.

grid alone. Therefore, additional calibration measurements were made: the measurement of $S_{21}$ is carried out for two cases, one with the resistor grid between the transmitter and receiver antennas and a second calibration measurement with the resistor grid removed but with everything else in the setup and environment unchanged. Calibration involves making two measurements, which we refer to as $S_{21}^{on}$ and $S_{21}^{off}$ respectively for the measurements made with and without the resistor grid between the antennas. In terms of the corresponding magnitudes $V_{S_{21}^{on}}$ and $V_{S_{21}^{off}}$ and phases $\phi_{S_{21}^{on}}$ and $\phi_{S_{21}^{off}}$, the measured transmission coefficient is computed as the ratio

$$
\tau_m = \frac{V_{S_{21}^{on}} e^{-j\phi_{S_{21}^{on}}}}{V_{S_{21}^{off}} e^{-j\phi_{S_{21}^{off}}}}.
$$

(48)

The ratio of the magnitudes of the pair of $S_{21}$ measurements yields the magnitude of $\tau_m$ and the difference in the phases yields the phase of the transmission coefficient. Such a referenced measurement takes into account the gains of the antennas used, corrects for space propagation, and also cancels response arising from unwanted propagation paths via ground reflections and reflections off other structures in the surroundings.

For the measurement of $\tau_m$ at normal incidence, both the linearly polarized antennas are kept vertical with respect to ground and with their phase centers at the same height of 2.0 m above ground, which is half the total height of the $4 \times 3$ m$^2$ resistor grid. Measurements of $\tau_m$ were also made for oblique incidence of $30^\circ$ by raising the transmitter antenna and lowering the receiving antenna so that the straight ray propagation path intersects the resistor grid close to its center and with this incidence angle of $30^\circ$. For oblique incidence, measurements of $\tau_m$ were made separately for $H$-plane incidence, in which the transmitter and receiver antennas were mounted horizontally, and for $E$-plane incidence, in which the antennas were mounted in the plane of incidence and inclined at $30^\circ$ to the vertical. At each frequency, the distance between the antennas and resistor grid were adjusted to be four times the Fresnel number $d^2/\lambda$, where $d$ is the dimension of the resonant antenna and $\lambda$ is the wavelength of the measurement.

Fig. 12 shows the measured transmission coefficient $\tau_m$ versus frequency for normal incidence. Plots of $\tau$ versus frequency for oblique incidence at $30^\circ$ are shown in Figs. 13 and 14 for the incident electric field in the $H$ and $E$ planes respectively. Also shown, for comparison, are the expectations for $\tau$ based on Astrahan[7].

Measurements of reflection coefficients were also carried out for three cases: normal incidence, $H$ plane with $30^\circ$ incidence and $E$ plane with $30^\circ$ incidence. For normal incidence, a single antenna was used as a trans-receiver and connected to the output terminal of the network analyzer; the trans-receiver was placed on one side of the resistor grid. In this particular case, the $S_{11}$ scattering parameter enabled calculation of the reflected power, the antenna was kept vertical above the ground at a constant height of 2.0 m and at each measurement frequency the distance between the antenna and resistor grid was adjusted to four times the Fresnel number. The oblique
Fig. 12: Transmission coefficient $\tau$ versus frequency for the resistor grid type space beam splitter for normal incidence. The measured $\tau_m$ are shown using symbols. Expectation of $\tau$ for an infinite resistor grid is shown as a continuous black line. Also shown for comparison using a red dashed line is expectation based on physical optics modeling of the transmission done specifically for a finite size resistor grid of $4 \times 3$ m$^2$, same as the one on which the measurements were made.

Fig. 13: $\tau$ versus frequency for H plane incidence at $30^\circ$. As in the previous figure, comparisons are provided for corresponding expectations for finite size and infinite resistor grids.

incidence measurements were made as $S_{21}$ scattering matrix element and for these reflection coefficient measurements both antennas were placed on the same side of the resistor grid. As in the transmission case, the linearly polarized antennas are kept horizontal for H-plane measurements and adjusted in relative heights so that the reflected ray between the antennas makes $30^\circ$ incidence angle and is reflected from the center of the screen. The antennas were placed in the plane of incidence for E-plane measurements, with one tilted to $30^\circ$ and the other to $270^\circ$ with respect to the vertical.

Calibration of the reflection measurements to derive the reflection coefficients of the resistor grid required measurements of the scattering matrix element with the grid in place, without the grid and, additionally, a third measurement with a reflector plate (aluminum sheet) replacing the resistor grid. The amplitude and phase in this third measurement are denoted by $V_{S_{ij}}^o$ and $\phi_{S_{ij}}^o$, where $ij$ is either 11 or 21. The measured complex scattering matrix element without the resistor grid is subtracted from each of the measurements done with the resistor grid and the reflecting sheet to obtain the value of $\Gamma$ as follows:

$$\Gamma_m = \frac{V_{S_{ij}} e^{-j \phi_{S_{ij}}}}{V_{S_{ij}} e^{-j \phi_{S_{ij}}}} - \frac{V_{S_{ij}}^o e^{-j \phi_{S_{ij}}}}{V_{S_{ij}}^o e^{-j \phi_{S_{ij}}}}.$$  \hspace{1cm} (49)

The subtraction of the complex $S_{11}^o$ and $S_{21}^o$ measurements that were carried out without the resistor grid, from the values obtained with the resistor grid and with aluminum reflector, is aimed at cancellation of space propagation terms and invariant extraneous reflections from the environment.

Fig. 14: $\tau$ versus frequency for E plane incidence at $30^\circ$. As in the previous two figures, comparisons are provided for corresponding expectations for finite size and infinite resistor grids.

Fig. 15 shows reflection coefficient $\Gamma$ versus frequency for normal incidence. Plots of $\Gamma$ versus frequency for oblique incidence at $30^\circ$ are shown in Figs. 16 and 17 for cases where the incident electric fields are, respectively, in the H and E planes.

As expected, the modeling of the reflection and transmission that takes into account the finite size of the resistor grid provides a better match to the measurements. These refined computations of the expected $\tau$ and $\Gamma$ are based on physical optics modeling of the diffraction along the edges. The electric field over a finely spaced square array of points on the plane of the resistor grid, and extending over areas up to ten times larger in extent on three adjacent sides of the resistor grid (the fourth side was omitted since the ground blocks paths in that...
Fig. 15: Reflection coefficient $\Gamma$ versus frequency for the resistor grid type space beam splitter for normal incidence. As in the previous figures, measurements are compared with corresponding expectations for finite size and infinite resistor grids.

Fig. 16: $\Gamma$ versus frequency for H plane incidence at $30^\circ$. As in the previous figures, measurements are compared with corresponding expectations for finite size and infinite resistor grids.

Fig. 17: $\Gamma$ versus frequency for E plane incidence at $30^\circ$. As in the previous figures, measurements are compared with corresponding expectations for finite size and infinite resistor grids.

In the case of transmission, the computation of the net received field is repeated for the case where no resistor grid is placed between the antennas, and the pair of computations made for the cases with and without the resistor grid are used as in Equation 48 to derive the expectation for $\tau$. The computation of the net received field for reflection is repeated for the case where the resistor grid is replaced with a perfect reflector with $\Gamma = -1$, and the pair of computations used as in Equation 49 to derive the expectation for $\Gamma$. The results of this physical optics based expectations for the measurements of $\tau$ and $\Gamma$ are also shown in the figures along with expectations corresponding to an infinite size resistor grid. The physical optics modeling demonstrates good agreement between measurements and modeling for the resistor grid based space beam splitter: the discordance between modeling and measurements has standard deviations of 5% in $\tau$ and $\Gamma$.

VI. SUMMARY

Events in the early history of the Universe are believed to have left imprints in the spectrum of the cosmic radio background whose detection requires precision receivers. A major problem in their detection is confusion from internal noise of spectral radiometers, which includes self-generated noise in the low-noise amplifiers as well as noise added by ohmic losses in the antenna and passive interconnects. Radiometers operating as interferometers avoid this problem since ohmic losses and amplifier noise are uncorrelated between elements in different arms; however, detection of uniform sky brightness with an interferometer requires a space beam splitter operating at radio wavelengths.

Towards building such a zero-spacing interferometer for absolute measurements of spectral signatures in the radio sky, we have developed an electromagnetic space beam splitter. It has been shown that the sensitivity to mean sky brightness is...
a maximum when the beam splitter is resistive and is a semi-transparent resistive sheet of sheet resistance $(\eta_o/2)$, where $\eta_o$ is the impedance of free space. A two-element interferometer with antennas on either side of a vertical resistive sheet with this sheet resistance forms a zero-spacing interferometer. The antenna beam solid angle is expected to be directed at the vertical resistive sheet so as to dominantly respond to sky brightness that is either reflected or transmitted from the sheet on the two sides. In this configuration, half the incident sky power is absorbed in the resistive sheet, a quarter is transmitted and a quarter reflected. Resistive sheets are frequency independent and hence their reflection ($\Gamma$) and transmission ($\tau$) properties are independent of frequency, thus providing smooth transfer function for the sky spectrum without altering the cosmological signatures embedded therein.

We show, further, that a resistive sheet may be realized as a square grid of resistive wire or a square grid of resistors in which each resistance has value $(\eta_o/2)$ Ω. Such a realization would, admittedly, have frequency independent characteristics only at wavelengths substantially longer than the grid size. We have constructed such a square resistor grid with a size $4 \times 3$ m$^2$ and grid spacing of 0.1 m using resistors of 180 Ω and measured its performance at a range of wavelengths longer than 1 m. Reflection and transmission coefficients were measured, both for normal as well as oblique incidence, as well as for cases where the incident electric field is in the plane of incidence and perpendicular to this plane. The measurements were done as $S_{11}$ and $S_{21}$ scattering matrix parameters using a network analyzer and a pair of linearly polarized antennas resonant at the measurement frequency. Calibration measurements were made by removing the resistor grid and also replacing it with an aluminum metal sheet. The measured $\Gamma$ and $\tau$ were compared with expectations that used physical optics modeling to account for edge diffraction; the measurements agree with expectations within 5%.

The development continues with designing antennas appropriate for the zero-spacing interferometer configuration leading on to a system design.

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