Pairing properties of hot nuclei within the finite temperature
Hartree Fock+BCS approximation

E Yüksel¹, E Khan² and K Bozkurt¹

¹Physics Department, Yıldız Technical University, 34210 Esenler, Istanbul, Turkey ²Institut de Physique Nucléaire, Universite Paris-Sud, IN2P3-CNRS, F-91406 Orsay Cedex, France

E-mail: eyuksel@yildiz.edu.tr

Abstract. In this work, finite temperature Hartree Fock BCS (FT+HFBCS) calculations are performed in Sn isotopes with the zero-range Skyrme type interactions. Vanishing of pairing correlations due to the increase in temperature are investigated with various types of Skyrme interactions. In addition, dependence of the critical temperature with respect to the several properties of these energy density functionals are analysed.

1. Introduction

Pairing correlations constitutes short-range part of the nucleon-nucleon interaction and have been investigated over the years within the mean field models. Pairing correlations are included either with BCS or Bogoliubov approximations at zero temperature [1].

One of the most remarkable results in finite nuclei was obtained with the disappearance of pairing properties at critical temperatures as in the metal superconductors. Experimental signatures for the phase transition in hot nuclei were firstly investigated with the formation of S-shape in the heat capacities of ¹⁶¹,¹⁶² Dy ¹⁷¹,¹⁷² Yb [2] and ¹⁶⁶,¹⁶⁷ Er [3] nuclei using level density measurements. The formation of the S-shape in the heat capacities is related to the phase transition of nuclei from superfluid state to the normal state. Also, the critical temperature for quenching of pairing correlations was found at around 0.5 MeV.

Theoretically, investigations about the effect of temperature on finite nuclei have also been carried out within mean-field theories. First studies in hot nuclei were performed with Hartree Fock-BCS model [4, 5]. It has been shown that pairing correlations disappears and nuclei go through a sharp transition from superfluid state to the normal state at a critical temperature. In addition, critical temperature is related to the pairing gap value at zero temperature and follows the \( T_c = 0.567\Delta(T = 0) \) equation according to the BCS model of nuclei. Thereafter, finite temperature HFB model has been developed which gave same results as in the BCS model [6-12].

The aim of this proceeding is to investigate the relation between pairing and temperature with various Skyrme energy density functionals. In addition, we investigate the relation between effective nucleon mass and \( K_{\text{inf}} \) (incompressibility) Skyrme parameters and critical temperature. In Sec 2. We briefly give the main ingredients of the finite temperature HFBCS model (FT+HFBCS). In Sec. 3 the results are presented for Tin isotopic chain and interpreted. The main outcome is summarized in Sec.4.
2. Finite temperature Hartree Fock BCS model

In this work, we used finite temperature HFBCS approach for the study of temperature and pairing correlations in Sn isotopes. BCS approach of nuclei is well known [1]. For the finite temperature extension of BCS model, we used temperature dependent Fermi-Dirac distribution function which is given by,

\[ f_k = \left[ 1 + \exp\left(\frac{E_k}{k_B T}\right) \right]^{-1} \tag{1} \]

where \( f_k \) is the Fermi function, \( E_k \) is the quasiparticle energy, \( k_B \) is the Boltzmann constant and \( T \) is the temperature. For the FT+HFBCS model, all equations keep the same structure except for particle number, gap equation, normal and abnormal densities [7]. Thermal averaged densities: particle density, spin density and kinetic energy density are given as [7, 14]:

\[ \sum \left\{ \frac{1}{2} \left[ v_k^2 \right] \right\} \tag{2} \]

\[ \sum \left\{ \left( \frac{u_k}{v_k} \right)^2 \right\} \tag{3} \]

\[ \sum \left\{ \left( \frac{\partial u_k}{\partial r} - \frac{u_k}{r} \right)^2 \right\} \tag{4} \]

The thermal averaged pairing field is given by:

\[ \Delta_T(r) = \frac{\nu_p}{2} \left( 1 - \eta \left( \frac{\rho(r)}{\rho_0} \right) \right) \frac{1}{4\pi} \sum_k (2j_k + 1) u_k^2(r) v_k(r)(1 - 2f_k) \tag{5} \]

We used zero-range Skyrme interactions in the calculations. Calculations are performed in a 20 fm box with 0.1 fm mesh point. In the HFBCS framework, the pairing correlations are taken into account with,

\[ V_{pair}(r_1, r_2) = V_0 \left[ 1 - \eta \left( \frac{\rho(r)}{\rho_0} \right) \right] \delta(r_1 - r_2) \tag{6} \]

where \( \rho(r) \) is the particle density, \( \rho_0 = 0.16 \text{ fm}^3 \) is the nuclear saturation density and \( \eta = 1 \) for the surface pairing interaction. The pairing strength \( V_0 \) is fixed for each nuclei according to \( 12/\sqrt{A} \).

3. Numerical results for Tin nuclei

In this work, we employed FT+HFBCS approximation for the selected Sn isotopes. Since we deal with magic proton number \( Z=50 \), calculations are performed with average neutron pairing gap value which is defined as [11],

\[ < \Delta_n >_k = \frac{\int d\mathbf{r} \Delta_n(r) \kappa(r)}{\int d\mathbf{r} \kappa(r)} \tag{7} \]

In here, \( \Delta(r) \) and \( \kappa(r) \) are the pairing field and pairing tensor which are obtained from the solution of the FT+HFBCS model.

In Figure 1, we present the change of average neutron pairing gaps as a function of temperature with different Skyrme energy density functionals: SLy5, SGII, SIII and SkM*. According to the BCS theory, critical temperature is related with the pairing gap as; \( T_c \approx 0.57 \Delta_n(T = 0) \) [4, 5]. In our calculations, disappearance of pairing correlations is obtained around 0.65 MeV for all Skyrme interactions. Phase transition is obtained from superfluid to normal state and critical temperature follows the generally admitted rule: \( T_c \approx 0.57 \Delta_n(T = 0) \) for all Sn isotopes and Skyrme interactions.
Because of the high temperature, cooper pairs around Fermi level are broken and phase transition from superfluid state to normal state occurred [6].

Figure 1. Mean value of the neutron pairing gap as a function of temperature in $^{106,110,114,118}$Sn nuclei, calculated with different Skyrme interactions.

Figure 2. Change of critical temperature with the effective nucleon mass values of the used Skyrme interactions.

To see the effect of various Skyrme energy density functionals on the critical temperature, the relation between the effective mass and $K_{inf}$ parameters of Skyrme interactions are investigated as a function of critical temperatures of $^{120}$Sn and $^{160}$Sn nuclei. The aim is to check whether the nuclear incompressibility and/or the effective mass of a given interaction has any impact on the critical temperature of nuclei. Since we want to investigate the effect of effective nucleon mass and $K_{inf}$ parameters, we used constant pairing strength which is $\Gamma_0 = 680 \text{ MeV f m}^3$. In Figure 2, we display the results in terms of the effective nucleon mass and critical temperature of nuclei with 14 different Skyrme interactions. Critical temperature values are obtained between 0.5 MeV and 0.9 MeV for all...
Skyrme interactions. A weak relationship is obtained between the effective mass and critical temperature. The effective mass is known to impact the level density which in turn may have an effect on the critical temperature. In Figure 3, we present the results for $K_{\text{inf}}$ and critical temperature values of $^{120}\text{Sn}$ and $^{160}\text{Sn}$ nuclei. However no relation is found between $K_{\text{inf}}$ and critical temperatures.

4. Conclusions
Quenching of pairing properties of Tin isotopes with temperature are investigated within the framework of the finite temperature HFBCS approximation by using zero-range Skyrme interactions. All calculations are carried out with surface pairing interaction. The relation between critical temperature and average neutron pairing gap obeys the generally admitted rule in the BCS theory and found around $T_c \approx 0.57\Delta(T = 0)$ with all Skyrme interactions. To see the effect of different Skyrme interactions on the critical temperature values of $^{120}\text{Sn}$ and $^{160}\text{Sn}$, we investigated the effective nucleon mass and $K_{\text{inf}}$ parameters of Skyrme interactions as a function of critical temperature. Although, no meaningful relation is obtained between incompressibility and critical temperature, we obtained a weak correlation in the case of the effective mass.

References
[1] Ring P, Schuck P 1980 The Nuclear Many-Body Problem (Verlag, Heidelberg: Springer).
[2] Schiller A, Bjerve A, Guttormsen M, Hjorth-Jensen M, Ingebritsen F, Melby E, Messelt S, Rekstad J, Siem S, and Ødegård S W 2001 Phys. Rev. C 63 021306
[3] Melby E, Guttormsen M, Rekstad J, Schiller A, Siem S, and Voinov A 2001 Phys. Rev. C 63 044309
[4] Goodman A L 1981, Nucl. Phys. A 352 30
[5] Goodman A L 1986 Phys. Rev. C 34 1942
[6] Egido J L, Robledo L M and Martin V 2000 Phys. Rev. Lett. 85 26
[7] Martin V, Egido J L and Robledo L M 2003 Phys. Rev. C 68 034327
[8] Khan E, Van Giai N, Sandulescu N 2007 Nucl. Phys. A 789 94
[9] Khan E, Giai N V, Grasso M 2004 Nucl. Phys. A 731 311