Wikipedia mining of hidden links between political leaders

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Dated: September 7, 2016

Abstract. We describe a new method of reduced Google matrix which allows to establish direct and hidden links between a subset of nodes of a large directed network. This approach uses parallels with quantum scattering theory, developed for processes in nuclear and mesoscopic physics and quantum chaos. The method is applied to the Wikipedia networks in different language editions analyzing several groups of political leaders of USA, UK, Germany, France, Russia and G20. We demonstrate that this approach allows to recover reliably direct and hidden links among political leaders. We argue that the reduced Google matrix method can form the mathematical basis for studies in social and political sciences analyzing Leader-Members eXchange (LMX).

PACS. 89.75.Fb Structures and organization in complex systems – 89.75.Hc Networks and genealogical trees – 89.20.Hh World Wide Web, Internet

1 Introduction

At present a free online encyclopaedia Wikipedia\textsuperscript{1} becomes the largest open source of knowledge being close to Encyclopaedia Britanica\textsuperscript{2} by an accuracy of scientific entries\textsuperscript{3} and overcoming the later by an enormous amount of available information. A detailed analysis of strong and weak features of Wikipedia is given at\textsuperscript{4,5}.

Since Wikipedia articles make citations to each other they generate a larger directed network with a rather clear meaning of nodes defined by article titles. Due to these reasons it is interesting to apply algorithms developed for search engines of World Wide Web (WWW), those like the PageRank algorithm\textsuperscript{6} (see also\textsuperscript{7}), to analyze the ranking properties and relations between Wikipedia articles. The clear meaning of Wikipedia nodes allows also to use its network as a test bed for machine learning algorithms computing semantic relatedness\textsuperscript{8}.

It is convenient to describe the network of any Wikipedia articles by the Google matrix $G$ constructed from the adjacency matrix $A_{ij}$ with elements 1 if article (node) $j$ points to article (node) $i$ and zero otherwise. Then the matrix elements of the Google matrix take the standard form\textsuperscript{3}

\begin{equation}
G_{ij} = \alpha S_{ij} + (1 - \alpha)/N,
\end{equation}

where $S$ is the matrix of Markov transitions with elements $S_{ij} = A_{ij}/k_{out}(j)$, $k_{out}(j) = \sum_{i=1}^{N} A_{ij} \neq 0$ being the node $j$ out-degree (number of outgoing links) and with $S_{ij} = 1/N$ if $j$ has no outgoing links (dangling node). Here $0 < \alpha < 1$ is the damping factor which for a random surfer determines the probability $(1 - \alpha)$ to jump to any node. The properties of spectrum and eigenstates of $G$ have been discussed in detail for Wikipedia and other directed networks (see e. g.\textsuperscript{9,10}).

The right eigenvectors $\psi_i(j)$ of $G$ are determined by the equation:

\begin{equation}
\sum_{j'} G_{jj'} \psi_i(j') = \lambda_i \psi_i(j).
\end{equation}

The PageRank eigenvector $P(j) = \psi_{i=0}(j)$ corresponds to the largest eigenvalue $\lambda_{i=0} = 1$\textsuperscript{6,7}. It has positive elements which give the probability to find a random surfer on a given node in the stationary long time limit of the Markov process. All nodes can be ordered by a monotonically decreasing probability $P(K_i)$ with the highest probability at $K = 1$. The index $K$ is the PageRank index. Left eigenvectors are biorthogonal to right eigenvectors of different eigenvalues. The left eigenvector for $\lambda = 1$ has identical (unit) entries due to the column sum normalization of $G$. One can show that the damping factor $\alpha$ in\textsuperscript{1} only affects the PageRank vector (or other eigenvectors for $\lambda = 1$ in case of a degeneracy) while other eigenvectors are independent of $\alpha$ due to their orthogonality to the left unit eigenvector for $\lambda = 1$\textsuperscript{7}.

In many real networks the number of nonzero elements in a column of $S$ is significantly smaller then the whole matrix size $N$ that allows to find efficiently the PageRank vector by the PageRank algorithm of power iterations\textsuperscript{7}. Also a certain number of largest eigenvalues (in modulus)
and related eigenvectors can be efficiently computed by the Arnoldi algorithm [10].

For various language editions of Wikipedia it was shown that the PageRank vector produces a reliable ranking of historical figures over 35 centuries of human history [11-15] and Wikipedia ranking of world universities (WRWU) [11,15]. Thus the Wikipedia ranking of historical figures is in a good agreement with the well-known Hart ranking [10], while the WRWU is in a good agreement with the Shanghai Academic ranking of world universities [17]. At the same time Wikipedia ranking produces some new additional insight as compared to these classifications.

In addition to the matrix $G$ it is useful to introduce a Google matrix $G^*$ constructed from the adjacency matrix of the same network but with inverted direction of all links. The statistical properties of the eigenvector $P^*$ of $G^*$ with the largest eigenvalue $\lambda = 1$ have been studied first for the Linux Kernel network [18] showing that there are nontrivial correlations between $P$ and $P^*$ vectors of the network. More detailed studies have been done for Wikipedia and other networks [11-15]. The vector $P^*(K^*)$ is called the CheiRank vector and the index numbering nodes in order of monotonic decrease of probability $P^*$ is noted as CheiRank index $K^*$. Thus, nodes with many ingoing (or outgoing) links have small values of $K$ (or of $K^*$) [10,11,7]. Examples of density distributions (in the $(\ln K, \ln K^*)$ plane) for Wikipedia editions EN, DE, FR, RU from the year 2013 (see network data in [14]) are shown in Fig. 1.

Other eigenvectors of $G$ have $|\lambda| \leq \alpha$ [10]. For Wikipedia is was shown that the eigenvectors with a large modulus of $\lambda$ select some specific communities of Wikipedia network [2,10]. However, a priori it is not possible to know what the meanings of these communities are. Thus other methods are required to determine effective interactions between $N_x$ nodes of a specific subset (group) of the global network of a large size $N$.

Recently, the method of reduced Google matrix has been proposed for analysis of effective interactions between nodes of a selected subset embedded into a large-size network [19]. This approach uses parallels with the quantum scattering theory, developed for processes in nuclear and mesoscopic physics and quantum chaos. In this work we apply this method to subsets (groups) of Wikipedia articles about political leaders (politicians) considering English, French, German and Russian Wikipedia editions and politicians of USA (US), UK, Germany (DE), France (FR) and Russia (RU). The total number of nodes for these Wikipedia networks is $N = 4212493$ (EN), $1532978$ (DE), $1352835$ (FR), $966284$ (RU) [14]. We also analyze interactions between political leaders of the G20 Los Cabos summit in 2012 [20]. The selected subsets have networks of 20 or 40 nodes and are well suited for analysis of direct and hidden links between politicians. In our analysis we use the Wikipedia networks collected in 2013 and described in [14]. The location of selected nodes on the PageRank-CheiRank plane $(\ln K, \ln K^*)$ is shown in Fig. 1. The obtained results allow to determine interesting direct and hidden relations between political leaders of the selected countries.

We should note that the analysis of interactions and relations between political leaders represents a hot topic in social and political sciences [21]. Thus the interactions between leader and group members, known as Leader-Members eXchange (LMX), attracts at present active investigations of researchers in social and political sciences.
However, only very recently the methods of complex networks \[25\] started to be used by in the LMX analysis \[26\]. In this work we argue that the approach to determine the reduced Google matrix \(G_{R}\) represents a useful and efficient tool for the LMX analysis of interactions inside a group of people. Thus for a group of politicians (a group of their articles at Wikipedia) we find that those at the top of PageRank index \(K\) are the dominant leaders being usually country presidents or prime-ministers. It turns out that the obtained \(G_{R}\) matrix, describing the interactions between group members, is composed of three matrix components. These components describe: the direct interactions \(G_{rr}\) between group members, a projector part \(G_{pr}\) which is mainly imposed by the PageRank of group members given by the global \(G\) matrix and a component \(G_{qr}\) from hidden interactions between members which appear due to indirect links via the global network. Thus the reduced matrix \(G_{R} = G_{rr} + G_{pr} + G_{qr}\) allows to obtain precise information about the group environment by taking into account their environment given by the global Wikipedia network. We think that this \(G_{R}\) matrix approach provides mathematical grounds for the LMX studies.

The paper is composed as follows: Section 2 shortly describes the method of reduced Google matrix, Section 3 presents distributions of selected subsets on the PageRank-CheiRank plane of global and reduced networks, Sections 4, 5, 6, 7, 8, and 9 describe the results of the reduced Google matrix analysis for politicians of USA, UK, Germany, France, Russia and G20 respectively. Section 10 provides a particular analysis for the group of French politicians in terms of effective networks of strongest friends or followers using either the matrix \(G_{R}\) or the hidden interactions given by the component \(G_{qr}\). The discussion of the obtained results is given in Section 11. All numerical data of the reduced Google matrix of groups of political leaders considered here are publicly available at the web site \[27\].

2 Reduced Google matrix

The concept of reduced Google matrix \(G_{R}\) was introduced in \[19\] on the basis of the following observation. At present directed networks of real systems can be very large (about 4.2 million articles for the English Wikipedia edition in 2013 \[10\] or 3.5 billion web pages for a publicly accessible web crawl database by the Common Crawl Foundation in 2012 \[25\]). In certain cases one may be interested in the particular interactions among a small reduced subset of \(N_{r}\) nodes with \(N_{r} \ll N\) instead of the interactions in the entire network. However, the interactions between these \(N_{r}\) nodes should be correctly determined taking into account that there are many indirect links between the \(N_{r}\) nodes via all other \(N_{s} = N - N_{r}\) nodes of the network. This leads to the problem of the reduced Google matrix \(G_{R}\) with \(N_{r}\) nodes which describes the interactions of a subset of \(N_{r}\) nodes.

In a certain sense we can trace parallels with the problem of quantum scattering appearing in nuclear and mesoscopic physics \[29\] and quantum chaotic scattering \[31\]. Indeed, in the scattering problem there are effective interactions between open channels to localized basis states in a well confined scattering domain where a particle can spend a certain time before it escapes by open channels. Having this analogy in mind we construct the reduced Google matrix \(G_{R}\) which describes interactions between selected \(N_{r}\) nodes and satisfies the standard requirements of the Google matrix.

Let \(G\) be a typical Google matrix of Perron-Frobenius type for a network with \(N\) nodes such that \(G_{ij} \geq 0\) and the column sum normalization \(\sum_{i=1}^{N} G_{ij} = 1\) are verified. We consider a sub-network with \(N_{r} < N\) nodes, called “reduced network”. In this case we can write \(G\) in a block form:

\[
G = \begin{pmatrix}
G_{rr} & G_{rs} \\
G_{sr} & G_{ss}
\end{pmatrix}
\]  

where the index “\(r\)” refers to the nodes of the reduced network and “\(s\)” to the other \(N_{s} = N - N_{r}\) nodes which form a complementary network which we will call “scattering network”.

We denote the PageRank vector of the full network as

\[
P = \begin{pmatrix}
P_r \\
P_s
\end{pmatrix}
\]

which satisfies the equation \(GP = P\) or in other words \(P\) is the right eigenvector of \(G\) for the unit eigenvalue. This eigenvalue equation reads in block notations:

\[
(1 - G_{rr}) P_r - G_{rs} P_s = 0,
\]

\[
G_{sr} P_r + (1 - G_{ss}) P_s = 0.
\]

Here \(1\) is the unit matrix of corresponding size \(N_{r}\) or \(N_{s}\). Assuming that the matrix \(1 - G_{ss}\) is not singular, i.e. all eigenvalues \(G_{ss}\) are strictly smaller than unity (in modulus), we obtain from \(6\) that

\[
P_s = (1 - G_{ss})^{-1} G_{sr} P_r
\]

which gives together with \(5\):

\[
G_{R} P_r = P_r, \quad G_{R} = G_{rr} + G_{rs} (1 - G_{ss})^{-1} G_{sr}
\]

where the matrix \(G_{R}\) of size \(N_{r} \times N_{r}\), defined for the reduced network, can be viewed as an effective reduced Google matrix. Here the contribution of \(G_{rr}\) accounts for direct links in the reduced network and the second term with the matrix inverse corresponds to all contributions of indirect links of arbitrary order. We note that in mesoscopic scattering problems one typically uses an expression of the scattering matrix which has a similar structure where the scattering channels correspond to the reduced network and the states inside the scattering domain to the scattering network \[31\].

The matrix elements of \(G_{R}\) are non-negative since the matrix inverse in \(8\) can be expanded as:

\[
(1 - G_{ss})^{-1} = \sum_{l=0}^{\infty} G_{ss}^{l}.
\]

In \(8\) the integer \(l\) represents the order of indirect links, i.e. the number of indirect links which are used to connect
indirectly two nodes of the reduced network. The matrix inverse corresponds to an exact resummation of all orders of indirect links. According to (9) the matrix \((1 - G_{ss})^{-1}\) and therefore also \(G_R\) have non-negative matrix elements. It remains to show that \(G_R\) also fulfills the condition of column sum normalization being unity. For this let us denote by \(E^T = (1, \ldots, 1)\) the line vector of size \(N\) with unit entries and by \(E^T_r\) (or \(E^T_s\)) the corresponding vectors for the reduced (or scattering) network with \(N_r\) (or \(N_s\)) unit entries such that \(E^T = (E^T_r, E^T_s)\). The column sum normalization for the full Google matrix \(G\) implies that \(E^T G = E^T\), i.e. \(E^T\) is the left eigenvector of \(G\) with eigenvalue 1. This equation becomes in block notation:

\[
E^T_r (1 - G_{rr}) - E^T_s G_{ss} = 0, \quad (10)
\]

\[-E^T_r G_{rs} + E^T_s (1 - G_{ss}) = 0. \quad (11)
\]

From (11) we find that \(E^T = E^T_r G_{rr}(1 - G_{ss})^{-1}\) which implies together with (10) that \(E^T G_R = E^T\) using \(G_R\) as in (9). This shows that the column sum normalization condition is indeed verified for \(G_R\) justifying that this matrix is indeed an effective Google matrix for the reduced network.

We can question how to evaluate practically the expression \(G_R\) for a particular sparse and quite large network with a typical situation when \(N_r \sim 10^2\cdot 10^3\) is small compared to \(N\) and \(N_s \approx N \gg N_r.\) If \(N_s\) is too large (e. g. \(N_s > 10^3\)) a direct naive evaluation of the matrix inverse \((1 - G_{ss})^{-1}\) in (9) by Gauss algorithm is not possible. In this case we can try the expansion (8) provided it converges sufficiently fast with a modest number of terms. However, this is most likely not the case for typical applications since \(G_{ss}\) is likely to have at least one eigenvalue very close to unity.

Therefore, we consider the situation where the full Google matrix has a well defined gap between the leading unit eigenvalue and the second largest eigenvalue (in modulus). For example if \(G\) is defined using a damping factor \(\alpha\) in the standard way, as in (1), the gap is at least \(1 - \alpha\) which is 0.15 for the standard choice \(\alpha = 0.85\) [4]. For such a situation we expect that the matrix \(G_{ss}\) has a leading real eigenvalue close to unity (but still different from unity so that \(1 - \lambda_{c}\) is singular) while the other eigenvalues are clearly below this leading eigenvalue with a gap comparable to the gap of the full Google matrix \(G\).

In order to evaluate the expansion (8) efficiently, we need to take out analytically the contribution of the leading eigenvalue close to unity which is responsible for the slow convergence.

Below we denote by \(\lambda_c\) this leading eigenvalue and by \(\psi^L_R\) the corresponding right (left) eigenvector such that \(G_{ss} \psi^L_R = \lambda_c \psi^L_R\) (or \(\psi^L_s G_{ss} = \lambda_c \psi^L_s\)). Both left and right eigenvectors as well as \(\lambda_c\) can be efficiently computed by the power iteration method in a similar way as the standard PageRank method. We note that one can easily show that \(\lambda_c\) must be real and that both left/right eigenvectors can be chosen with positive elements. Concerning the normalization for \(\psi_R\) we choose \(E^T_s \psi_R = 1\) and for \(\psi_L\) we choose \(E^T_r \psi_R = 1\). It is well known (and easy to show) that \(\psi^L_s\) is orthogonal to all other right eigenvectors (and \(\psi_R\) is orthogonal to all other left eigenvectors) of \(G_{ss}\) with eigenvalues different from \(\lambda_c\). We introduce the operator \(P_c = \psi_R \psi^T_L\) which is the projector onto the eigenspace of \(\lambda_c\) and we denote by \(Q_c = 1 - P_c\) the complementary projector. One verifies directly that both projectors commute with the matrix \(G_{ss}\) and in particular \(P_c G_{ss} G_{ss} P_c = \lambda_c P_c\). Therefore we can write:

\[
(1 - G_{ss})^{-1} = (P_c + Q_c)(1 - G_{ss})^{-1}(P_c + Q_c) \quad (12)
\]

\[
= P_c \frac{1}{1 - \lambda_c} + Q_c (1 - G_{ss})^{-1} Q_c \quad (13)
\]

\[
= P_c \frac{1}{1 - \lambda_c} + (1 - G_{ss})^{-1} Q_c \quad (14)
\]

\[
= P_c \frac{1}{1 - \lambda_c} + Q_c \sum_{l=0}^{\infty} G_{ss}^l \quad (15)
\]

with \(G_{ss} = Q_c G_{ss} Q_c\) and using the standard identity \(P_c Q_c = 0\) for complementary projectors. The expansion in (15) has the advantage that it converges rapidly since \(G_{ss}^l \sim |\lambda_{c,2}|^l\) with \(\lambda_{c,2}\) being the second largest eigenvalue which is significantly lower than unity (e. g. \(|\lambda_{c,2}| \approx 0.85\) for the case with a damping factor). The first contribution due to the leading eigenvalue \(\lambda_c\) close to unity is taken out analytically once the left and right eigenvectors, and therefore also the projector \(P_c\), are known.

The combination of (8) and (15) provides an explicit algorithm feasible for a numerical implementation for the case of modest values of \(N_r\), large values of \(N_s\) and of course for sparse matrices \(G, G_{ss}\), etc. The described method can also be modified to take out analytically the contributions of several leading eigenvalues close to unity as described in [19]. We note that the numerical methods described in [35] allow to determine the eigenvalues \(\lambda_c\) (and corresponding eigenvectors) which are exponentially close to unity (e. g. \(1 - \lambda_c \sim 10^{-16}\)) so that the expression (12) can be efficiently computed numerically.

On the basis of the above equations (8)-(12) the reduced Google matrix can be presented as a sum of three

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**Table 1.** Values of \(1 - \lambda_c\) and of \(\Sigma_P\) for the five groups of politicians and four cases of G20 subnetworks with \(\lambda_c\) being the leading eigenvalue of the matrix \(G_{ss}\) and \(\Sigma_P = \|P_c\|\) being the 1-norm of the projected PageRank representing the relative PageRank weight of the subset in the full network.

| Network | Group      | \(1 - \lambda_c\) | \(\Sigma_P\) |
|---------|------------|-------------------|--------------|
| Enwiki  | Politicians US | 0.0003680        | 0.0003757    |
| Enwiki  | Politicians UK | 0.0001041        | 0.0001068    |
| dewiki  | Politicians DE | 0.0002705        | 0.0002802    |
| Frwiki  | Politicians FR | 0.0003810        | 0.0004005    |
| Ruwiki  | Politicians RU | 0.0003136        | 0.0003247    |
| Enwiki  | G20 EN      | 0.0002465        | 0.0002508    |
| Dewiki  | G20 DE      | 0.0001492        | 0.0001513    |
| Frwiki  | G20 FR      | 0.0002131        | 0.0002159    |
| Ruwiki  | G20 RU      | 0.0002707        | 0.0002734    |
components

\[ G_R = G_{rr} + G_{pr} + G_{qr}, \] (16)

with the first component \( G_{rr} \) given by direct matrix elements of \( G \) among the selected \( N_r \) nodes. The second projector component \( G_{pr} \) is given by

\[ G_{pr} = G_{rs} \mathcal{P}_c \mathbf{G}_{sr}/(1 - \lambda_c), \quad \mathcal{P}_c = \psi_R \psi_L^T. \] (17)

We mention that this contribution is of the form \( G_{pr} = \tilde{\psi}_R \tilde{\psi}_L^T/(1 - \lambda_c) \) with \( \psi_R \), \( \psi_L \) and \( \tilde{\psi}_R = \tilde{\psi}_L = \psi_L^T \mathbf{G}_{sr} \) being two small vectors defined on the reduced space of dimension \( N_r \). Therefore \( G_{pr} \) is indeed a (small) matrix of rank one which is also confirmed by a numerical diagonalization of this matrix. The third component \( G_{qr} \) of indirect or hidden links is given by

\[ G_{qr} = G_{rs} \{ Q_c \sum_{i=0}^{\infty} G_{ss}^i \} \mathbf{G}_{sr} - \mathcal{P}_c, \quad \tilde{G}_{ss} = Q_c G_{ss} Q_c. \] (18)

Even though the decomposition (16) is at first motivated by the numerical efficiency to evaluate the matrix inverse, it is equally important concerning the interpretation of the different terms and especially the last contribution (15) which is typically rather small as compared to (17) but plays in an important role as we will see below.

Concerning the numerical algorithm to evaluate all contributions in (16), we mention that we first determine by the power iteration method the leading left \( \psi_L \) and right eigenvector \( \psi_R \) of the matrix \( G_{ss} \) which also provides an accurate value of the corresponding eigenvalue \( \lambda_c \) or better of \( 1 - \lambda_c \) (by taking the norm of the projection of \( G \psi_R \) on the reduced space which is highly accurate even for \( \lambda_c \) close to 1). These two vectors provide directly \( G_{pr} \) by (17) and allow to numerically apply the projector \( Q_c \) to an arbitrary vector (with \( N \) operations). The most expensive part is the evaluation of the last contribution according to (15). For this we apply successively \( G_{ss} = Q_c G_{ss} Q_c \) to an arbitrary column of \( G_{sr} \) which can be done by a sparse matrix vector multiplication or the efficient algorithm of the projector.

Therefore, we can calculate in parallel, for each column \( j \) of \( G_{qr} \), the following product \( Q_c \sum_{i=0}^{\infty} G_{ss}^i G_{sj} \). This computation can be performed using the projection algorithm of PageRank which converges after about \( \sim 200 - 250 \) terms. Indeed, the contribution of the leading eigenvalue (of \( G_{ss} \)) has been taken out and the eigenvalues of \( G_{ss} \) are roughly below the damping factor \( \alpha = 0.85 \). In the end the resulting vector is multiplied with the matrix \( G_{ss} \) which provides one column of \( G_{qr} \). This procedure has to be repeated for each of the \( N_r \) columns but the number \( N_r \) is typically very modest (20 or 40 in this work) and the computation of the different columns can actually be done in parallel on typical multicore machines.

Concerning the choice of the reduced space we use 5 groups of 20 or 40 political leaders of 5 countries (US, UK, DE, FR, RU) for 4 Wikipedia editions (EN, DE, FR, RU with EN-Wikipedia for both US and UK politicians). We also consider the group of G20 state leaders for which we use all 4 of these Wikipedia editions even though here we concentrate on the G20 data obtained for EN-Wikipedia. A detailed description of these subsets is given in Section 3. For the data sets of politicians considered in this work we find that typically \( 1 - \lambda_c \approx 10^{-4} \) and the right eigenvector \( \psi_R \) of \( G_{ss} \) is rather close to the full PageRank of \( G \) (for the leading nodes in the full PageRank not belonging to the reduced space). Furthermore, we find that an approximate relation holds: \( 1 - \lambda_c \approx \Sigma P = \| P_r \| \) where \( \Sigma P \) is the PageRank probability of the global network concentrated on the subset of \( N_r \) selected nodes. The data of Table 1 show that this relation works with an accuracy of a couple of percent. To understand this result mathematically, we replace in (7) the matrix inverse by the first term of (15) which gives \( P_r \approx \psi_R \psi_L^T G_{sr} P_r/(1 - \lambda_c) \) (for \( N_s \gg N_r \) and \( 1 - \lambda_c \ll 1 \)). Furthermore we note that \( \tilde{\psi}_L \approx E_s \) (for most nodes) and \( E_s^T G_{sr} P_r \approx E_s^T P_r = \| P_r \| \) such \( P_r \approx \psi_R \| P_r \|/(1 - \lambda_c) \). Since \( \| P_r \| \approx \| \psi_R \| \approx 1 \) we find indeed that \( \| P_r \| \approx 1 - \lambda_c \).

The numerical computations show that the vectors \( \psi_R \) and \( \tilde{\psi}_L \) introduced below equation (17) are approximately given by \( \tilde{\psi}_R = P_r \) and \( \tilde{\psi}_L = E_r^T \) such that \( G_{pr} \approx P_r E_r^T/(1 - \lambda_c) \) is rather close to a rank one matrix (since \( \| P_r \| \approx 1 - \lambda_c \) and with identical columns given by the normalized vector \( P_r/(1 - \lambda_c) \)). More precisely, we will indeed see in Sections 4-9, that the overall column sums of \( G_{pr} \) account for \( \sim 95-97\% \) of the total column sum of \( G_R \). In other words, in terms of probability the contribution of \( G_{pr} \) is dominant in \( G_R \) but it is also kind of trivial with nearly identical columns. Therefore the two small contributions of \( G_{rr} \) and \( G_{qr} \) are indeed very important for the interpretation even though they only contribute weakly to the overall column sum normalization.

The meaning of \( G_{rr} \) is rather clear since it gives direct links between the selected nodes. In contrast, the meaning of \( G_{qr} \) is significantly more interesting since it generates indirect links between the \( N_r \) nodes due to their interactions with the global network environment. We note that \( G_{sr} \) is composed of two parts \( G_{qr} = G_{qrd} + G_{qrn} \) where the first diagonal term \( G_{qrd} \) represents a probability to stay on the same node during multiple iterations of \( G_{ss} \) in (15) while the second nondiagonal term \( G_{qrn} \) represents indirect (hidden) links between the \( N_r \) nodes appearing via the global network. We note that in principle certain matrix elements of \( G_{qr} \) can be negative, which is possible due to negative terms in \( Q_c = 1 - \mathcal{P}_c \) appearing in (15). However, for all subsets considered in this work the total weight of negative elements was negligibly small (about \( 10^{-10} \) for the data of UK politicians, 0 for data of politicians of other countries, and \( 10^{-5} \) for the G20 state leader data, of the total weight 1 for \( G_R \)).

It is convenient to characterize the strength of 3 components in (16) by their respective weights \( W_{rr} \), \( W_{pr} \), \( W_{qr} \) given respectively by the sum of all matrix elements of \( G_{rr} \), \( G_{pr} \), \( G_{qr} \) divided by \( N_r \). By definition we have \( W_{rr} + W_{pr} + W_{qr} = 1 \).

In the following sections we will see that all three components of (16) play important roles. We present here only results for \( G_R \) obtained from \( G \). The results for the net-
Fig. 2. Position of nodes in the local \((K, K^*)\) plane of the reduced network for 20 US politicians in the Enwiki network. The names are shown on the same lines of the corresponding data points.

Fig. 3. Position of nodes in the local \((K, K^*)\) plane of the reduced network for 20 UK politicians in the Enwiki network. The names are shown on the same lines of the corresponding data points.

| Names (US)             | \(K\) | \(K^*\) | \(K_G\) |
|------------------------|-------|---------|---------|
| Barack Obama           | 1     | 4       | 2       |
| George W. Bush         | 2     | 2       | 1       |
| Hillary Rodham Clinton | 3     | 6       | 6       |
| John Kerry             | 4     | 7       | 4       |
| John McCain            | 5     | 1       | 3       |
| Joe Biden              | 6     | 5       | 7       |
| Dick Cheney            | 7     | 14      | 5       |
| Michael Bloomberg      | 8     | 3       | 14      |
| Condoleezza Rice       | 9     | 13      | 10      |
| Colin Powell           | 10    | 9       | 12      |
| Nancy Pelosi           | 11    | 15      | 8       |
| Sarah Palin            | 12    | 12      | 9       |
| Donald Rumsfeld        | 13    | 10      | 11      |
| Donald Trump           | 14    | 11      | 20      |
| Michele Bachmann       | 15    | 8       | 16      |
| Bernie Sanders         | 16    | 16      | 15      |
| Robert Mueller         | 17    | 18      | 13      |
| Ashton Carter          | 18    | 19      | 19      |
| Jack Lew               | 19    | 17      | 18      |
| James B. Comey         | 20    | 20      | 17      |

Table 2. List of names of 20 selected US politicians and the corresponding PageRank index values \(K\), \(K^*\) and \(K_G\) of the reduced network matrices \(G_R\), \(G^*_R\) and \(G_{r+qr}\) given as the sum of \(G_{rr}\) and \(G_{qr}\) (without diagonal elements). All matrices were computed for the English Wikipedia edition of 2013.

For our studies we choose 6 independent groups of articles of 20 US and 20 UK politicians from Enwiki, 40 German and 40 French politicians from Dewiki and Frwiki respectively, 20 Russian politicians from Ruwiki and the 20 G20 state leaders from Enwiki. The information about number of nodes and links for each Wikipedia edition is available at [14]. In the selection of names of political leaders of each country we used the names appearing at the top of Google search on e. g. “politicians of Russia”, in addition we take only those politician who were active in the period not more than 10 - 20 years before the collection date of our Wikipedia editions of 2013. A few names are used to have a group of 20 or 40. We do not pretend that we selected all important politicians of a given country but we suppose that the main part of them is present in our selection.

For each group (or subset of \(N_r\) nodes) we order politicians by their PageRank probability in the corresponding global Wikipedia network. After such ordering we obtain local rank PageRank index \(K\) changing from 1 to 20 (or 40). The best known politicians are found to be at the top values \(K = 1, 2, ...\). In addition we determine the local CheiRank index \(K^*\) of the selected names using the work with inverted direction of links, corresponding to CheiRank of \(G^*_R\) of \(G^*\) are given at [24].
Table 3. Same as Table 2 for 20 selected UK politicians and the English Wikipedia edition of 2013.

| Names (UK)     | $K$ | $K^*$ | $K_G$ |
|----------------|-----|-------|-------|
| Tony Blair     | 1   | 1     | 2     |
| David Cameron  | 2   | 4     | 3     |
| Gordon Brown   | 3   | 2     | 1     |
| Boris Johnson  | 4   | 3     | 7     |
| Alex Salmond   | 5   | 6     | 5     |
| George Osborne | 6   | 9     | 6     |
| Ed Miliband    | 7   | 5     | 4     |
| Peter Robinson | 8   | 10    | 12    |
| Carwyn Jones   | 9   | 19    | 13    |
| Nicola Sturgeon| 10  | 8     | 9     |
| Rhodri Morgan  | 11  | 16    | 15    |
| Theresa May    | 12  | 7     | 8     |
| Angela Eagle   | 13  | 13    | 15    |
| Nigel Farage   | 14  | 11    | 10    |
| Arlene Foster  | 15  | 18    | 14    |
| Sadiq Khan     | 16  | 14    | 11    |
| Jeremy Corbyn  | 17  | 15    | 17    |
| Leanne Wood    | 18  | 12    | 18    |
| Andrea Leadsom | 19  | 17    | 19    |
| Mohammed Shabq | 20  | 20    | 20    |

CheiRank vector of the global network. At the top of $K^*$ we have most communicative articles of politicians. Then we present the distribution of politicians or G20 state leaders on the PageRank-CheiRank plane of local indexes $(K, K^*)$ in Figs. 2, 3, 4, 5, 6 and 7 for US, UK, DE, FR, RU and G20 respectively. The full names of political leaders are given respectively in Tables 2, 3, 4, 5, 6 and 7 with corresponding values of local $K, K^*$ indexes (we discuss the meaning of the additional index $K_G$ later).

For the US case shown in Fig. 2 and Table 2 we find that Obama, Bush and Clinton take the top three $K$ positions which appears to be rather natural. However, the most communicative politicians being at the top of CheiRank with $K^* = 1, 2, 3$ are McCain, Bush and Bloomberg.

In Fig. 3 and Table 3 the names and distribution of 20 UK politicians are shown. The top 3 positions of PageRank are taken by UK prime ministers Blair, Cameron, Brown with $K = 1, 2, 3$. The distribution in the $(K, K^*)$ plane is more centered in a diagonal vicinity as compared to the US case and other countries discussed below. For $K^* = 1, 2, 3$ we have Blair, Brown and Johnson. The present prime minister May is rather far in rank indexes.

The PageRank-CheiRank distribution of German politicians is shown in Fig. 4 with the full names and ranks given in Table 4. The top PageRank values $K = 1, 2, 3$ are taken by the chancellors Kohl, Schmidt and Merkel with Schröder at $K = 4$. However, the most communicative politicians are Gauck (DE president, $K^* = 1$), Merkel ($K^* = 2$) and Lafontaine ($K^* = 3$).

The French politicians are presented in Fig. 5 and Table 5. Here we choose names of those who are really active
Section.

and links between different parties is discussed in the last

membership in political parties. The effect of interactions

have rather high values of

perspective on the

Marie) Le Pen at the third position. We note a large dis-

are taken by two presidents Sarkozy, Hollande with (Jean-

Table 4.

the German Wikipedia edition of 2013.

in the period 2007 - 2013, thus e. g. Jacques Chirac is

not included in the list. The top 3 positions in $K$ and $K^*$

are taken by two presidents Sarkozy, Hollande with (Jean-

Marie) Le Pen at the third position. We note a large dis-

position of the $(K, K^*)$ plane for the main part

of politicians. Thus Royal ($K = 4$) and Raffarin ($K = 5$

have rather high values of $K^* = 13$ or 32 respectively.

For French politicians in Table 6, we mark by color their

membership in political parties. The effect of interactions

and links between different parties is discussed in the last

Section.

The names of 20 Russian politicians and their distribution on the $(K, K^*)$ plane are presented in Fig. 6 and Table 5. Similar to the case of France the two top PageR-

ank positions are taken by presidents Putin, Medvedev with

Zhirinovsky at the third position with

and $K^* = 32$ respectively.

plane are presented in Fig. 6 and

showing low com-

$K^* = 12$ showing low com-

munication properties of his Wikipedia article.

All the groups of politicians have been considered above

in the frame of Wikipedia editions corresponding to their

Table 4. Same as Table 2 for 40 selected DE politicians and the German Wikipedia edition of 2013.

| Names (DE)       | $K$ | $K^*$ | $K_G$ |
|------------------|-----|-------|-------|
| Helmut Kohl      | 1   | 7     | 3     |
| Helmut Schmidt   | 2   | 6     | 6     |
| Angela Merkel    | 3   | 2     | 1     |
| Gerhard Schröder | 4   | 5     | 2     |
| Wolfgang Schäuble| 5   | 13    | 9     |
| Joschka Fischer  | 6   | 11    | 7     |
| Oskar Lafontaine | 7   | 3     | 4     |
| Horst Seehofer   | 8   | 20    | 20    |
| Joachim Gauck    | 9   | 1     | 24    |
| Guido Westerwelle| 10  | 16    | 14    |
| Karl-Theodor zu Guttenberg | 11 | 8 | 13 |
| Thomas de Maizière | 12 | 23 | 16 |
| Franz Müntefering | 13 | 26 | 5 |
| Frank-Walter Steinmeier | 14 | 17 | 10 |
| Ursula von der Leyen | 15 | 14 | 25 |
| Peer Steinbrück  | 16  | 12    | 8     |
| Gregor Gysi      | 17  | 22    | 17    |
| Norbert Lammert  | 18  | 28    | 22    |
| Sigmar Gabriel   | 19  | 24    | 12    |
| Winfried Kretschmann | 20 | 21 | 35 |
| Peter Altmaier   | 21  | 34    | 29    |
| Jürgen Trittin   | 22  | 9     | 32    |
| Volker Beck      | 23  | 15    | 23    |
| Renate Künast    | 24  | 4     | 11    |
| Hans-Christian Ströbele | 25 | 36 | 19 |
| Ronald Pofalla   | 26  | 27    | 21    |
| Claudia Roth     | 27  | 25    | 28    |
| Sahra Wagenknecht| 28  | 25    | 28    |
| Volker Kauder    | 29  | 18    | 34    |
| Andrea Nahles    | 30  | 32    | 18    |
| Katrin Göring-Eckardt | 31 | 30 | 26 |
| Heiko Maas       | 32  | 29    | 30    |
| Cem Ozdemir      | 33  | 19    | 33    |
| Hermann Gröhe    | 34  | 37    | 31    |
| Ralf Brauksiepe  | 35  | 40    | 38    |
| Barbara Hendricks| 36  | 35    | 36    |
| Wolfgang Bosbach  | 37  | 31    | 27    |
| Philipp Mählender| 38  | 33    | 37    |
| Rainer Arnold    | 39  | 39    | 39    |
| Florian Hahn     | 40  | 38    | 40    |

Table 5. Same as Table 2 for 40 selected FR politicians and the French Wikipedia edition of 2013. Color marks membership in political parties: red CR (far-left parties), magenta CM (socialist party PS), green CG (green parties), blue CB (right parties UMP, UDI), CV violet (far-right and FN).

| Names (FR)       | $K$ | $K^*$ | $K_G$ |
|------------------|-----|-------|-------|
| Nicolas Sarkozy CB | 1  | 1    | 1     |
| François Hollande CM | 2  | 2    | 2     |
| Jean-Marie Le Pen CV | 3  | 3    | 11    |
| Ségolène Royal CM  | 4  | 13   | 3     |
| Jean-Pierre Raffarin CB | 5  | 32   | 10    |
| Dominique de Villepin CB | 6  | 18   | 12    |
| François Fillon CB  | 7  | 8    | 4     |
| François Bayrou CB  | 8  | 5    | 6     |
| Laurent Fabius CM   | 9  | 10   | 8     |
| Dominique Strauss-Kahn CM | 10 | 14   | 7     |
| Jack Lang CM        | 11 | 19   | 13    |
| Alain Juppé CB      | 12 | 6    | 5     |
| Jean-Louis Borloo CB | 13 | 12   | 19    |
| Bertrand Delanoë CM | 14 | 9    | 18    |
| Jean-Luc Mélenchon CM | 15 | 4    | 14    |
| Marine Le Pen CV    | 16 | 15   | 17    |
| Christine Lagarde CB | 17 | 28   | 25    |
| Martine Aubry CM    | 18 | 16   | 9     |
| Daniel Colin-Bendit CG | 19 | 23   | 32    |
| Valérie Pécresse CB | 20 | 27   | 29    |
| Jean-François Copé CB | 21 | 22   | 15    |
| Nathalie Kosciusko-Morizet CB | 22 | 7   | 26    |
| Arnaud Montebourg CM | 23 | 11   | 16    |
| Claude Bartolone CM | 24 | 37   | 28    |
| Rachida Dati CB     | 25 | 17   | 35    |
| Olivier Besancenot CR | 26 | 35   | 37    |
| Nicolas Dupont-Aignan CV | 27 | 33   | 30    |
| Eva Joly CG          | 28 | 20   | 21    |
| Christiane Taubira CM | 29 | 30   | 38    |
| Elisabeth Guigou CM | 30 | 26   | 23    |
| Brice Hortelano CB  | 31 | 24   | 27    |
| Rama Yade CB        | 32 | 31   | 39    |
| Pierre Moscovici CM | 33 | 21   | 20    |
| Manuel Valls CM     | 34 | 29   | 22    |
| Claude Guéant CB    | 35 | 39   | 34    |
| Hervé Morin CB      | 36 | 34   | 24    |
| Cécile Duflot CG    | 37 | 35   | 26    |
| Michel Sapin CM     | 38 | 36   | 31    |
| Henri Guaino CB     | 39 | 38   | 33    |
| Florian Philippot CV | 40 | 40   | 40    |
Fig. 6. Position of nodes in the local \((K, K^*)\) plane of the reduced network for 20 RU politicians in the Ruwiki network. The names are shown on the same lines of the corresponding data points.

Table 6. Same as Table 2 for 20 selected RU politicians and the Russian Wikipedia edition of 2013.

| Names (RU)                     | \(K\) | \(K^*\) | \(K_G\) |
|-------------------------------|-------|---------|---------|
| Putin, Vladimir Vladimirovich | 1     | 1       | 1       |
| Medvedev, Dmitry Anatolyevich| 2     | 12      | 2       |
| Zhirinovsky, Vladimir Wollovich| 3 | 10      | 15      |
| Matviенко, Valentina Ivanovna | 4     | 2       | 11      |
| Zyuganov, Gennady Andreyevich | 5     | 7       | 13      |
| Mironov, Sergey Mikhailovich  | 6     | 6       | 8       |
| Chubais, Anatoly Borisovich   | 7     | 4       | 6       |
| Sobyanin, Sergey Semenovich   | 8     | 16      | 9       |
| Nemtsov, Boris Yelmovich      | 9     | 8       | 10      |
| Fradkov, Mikhail Yelmovich    | 10    | 13      | 3       |
| Yavlinsky, Grigory            | 11    | 3       | 14      |
| Kudrin, Alexei Leonidovich    | 12    | 14      | 7       |
| Ivanov, Sergey Borisovich     | 13    | 18      | 4       |
| Lavrov, Sergey Viktorovich    | 14    | 15      | 20      |
| Prokhorov, Mikhail Dmitrievich| 15    | 5       | 18      |
| Gref, Herman Oskarovich       | 16    | 17      | 5       |
| Shoygu, Sergei                | 17    | 9       | 19      |
| Hakamada, Irina Matsuonova    | 18    | 11      | 17      |
| Silanov, Anton Germanovich    | 19    | 20      | 16      |
| Nabiullina, Elvira            | 20    | 19      | 12      |

Table 7. List of the 20 state leaders of the G20 states and the corresponding PageRank index values \(K\) of the reduced network matrices \(G_R\) for the English (EN), German (DE), French (FR) and Russian (RU) Wikipedia editions of 2013. For each state leader the country name is given in ISO 3166-1 alpha-2.

| G20 / Wikipedia edition | EN | DE | FR | RU |
|--------------------------|----|----|----|----|
| Names                    | \(K\) | \(K\) | \(K\) | \(K\) |
| Barack Obama             | 1  | 1  | 1  | 2  |
| Vladimir Putin           | 2  | 3  | 2  | 1  |
| David Cameron            | 3  | 4  | 6  | 4  |
| Stephen Harper           | 4  | 10 | 7  | 16 |
| Angela Merkel            | 5  | 2  | 4  | 3  |
| Manmohan Singh           | 6  | 13 | 13 | 10 |
| Hu Jintao                | 7  | 9  | 5  | 5  |
| François Hollande         | 8  | 5  | 3  | 6  |
| Julia Gillard            | 9  | 14 | 16 | 11 |
| José Manuel Barroso      | 10 | 6  | 8  | 14 |
| Jacob Zuma               | 11 | 18 | 18 | 18 |
| Mario Monti              | 12 | 8  | 9  | 9  |
| Cristina Fernández de Kirchner | 13 | 12 | 10 | 8  |
| Recep Tayyip Erdoğan     | 14 | 7  | 14 | 12 |
| Felipe Calderón          | 15 | 11 | 12 | 7  |
| Abdullah of Saudi Arabia  | 16 | 15 | 15 | 13 |
| Susilo Bambang Yudhoyono | 17 | 19 | 20 | 19 |
| Dilma Rousseff           | 18 | 17 | 11 | 15 |
| Lee Myung-bak            | 19 | 16 | 17 | 17 |
| Yoshihiko Noda           | 20 | 20 | 19 | 20 |
narrow language. Indeed, we find that in other language editions the articles about many politicians are rather short or sometimes they are even absent (e. g. for 40 French politicians in Enwiki or Ruwiki). However, the political leaders of countries are usually well present in the editions discussed here. Therefore, we take for our analysis 20 world political leaders that have participated in the G20 meeting at Los Cabos summit in 2012. Their names and local PageRank indexes according to Enwiki, Dewiki, Fwiki, Ruwiki are given in Table 7. The distribution of politicians on PageRank-CheiRank plane is shown in Fig. 7 for Enwiki. We take the name of country leader Abdullah of Saudi Arabia (Saudi Arabia) since the name of the minister of finance, who was representing Saudi Arabia, is not listed in Dewiki, Fwiki, Ruwiki; José Manuel Barroso is taken as EU representative.

Among G20 leaders the top 2 PageRank positions are taken by Obama and Putin (see Table 7 in Enwiki, Fwiki; Putin and Obama in Ruwiki and Obama and Merkel in Dewiki. So there is a definite trend for leaders being promoted in their native editions. The language preference is probably the reason to have Singh (India) ahead of Jintao (China) in Enwiki while in other editions Jintao is well ahead of Singh. At the top CheiRank positions K∗ of Enwiki we have Putin, Erdoğan, Obama (see Fig. 7) showing very different communicative strengths of political leaders.

In the next sections we consider interactions between selected political leaders using the reduced Google matrix approach.

4 Direct and hidden links of US politicians

The reduced Google matrix $G_R$ of 20 US politicians, listed in Fig. 2 and Table 2 is shown in Fig. 8 with its three matrix components $G_{pr}$, $G_{rr}$, $G_{qr}$ from (16). The amplitudes of matrix elements are shown by color with maximum for red and minimum for blue. We mention that for the data of US Politicians in this section there are no negative matrix elements of $G_{qr}$. The same holds for the cases of DE and FR politicians (see sections below). However for UK politicians (G20 data, also sections below for both cases) there are few very (rather) small negative elements of $G_{qr}$ of order $\sim 10^{-8}$ (or $\sim 10^{-5}$) with no (or very small) effects on colors. The data of US politicians in Fig. 8 clearly show that the main contribution to $G_R$ is given by the projector component $G_{pr}$ with a weight of approximately 96%. The remaining weight is distributed between the component of direct links $G_{rr}$ (1.9%) and the one of indirect links $G_{qr}$ (1.6%) (see Fig. 8). Of course, the total weight of three components is equal to unity by construction.

Thus the main component $G_{pr}$ imposes to $G_R$ a large contribution proportional to the PageRank probability $P(K)$ which is mainly produced by the environment of the huge remaining part $G_{ss}$ of the global network with $N - N_r \gg N_r$ nodes. Due to this the structure of $G_R$ is close to a solution (discussed above) when each column of $G_R$ is roughly given by the same PageRank vector of $N_r$ nodes which up to a constant factor coincides with the PageRank probabilities of the selected $N_r$ nodes in the global network of $N$ nodes. Due to the simple rank-one structure of $G_{pr}$ the smaller contributions of the other two components $G_{rr}$ of direct and $G_{qr}$ of indirect links (friends) play an important role even if their weight is significantly smaller as compared to the weight of $G_{pr}$.

The global structure of $G_{rr}$ of 20 US politicians is shown in Fig. 8 (bottom left panel) and in more detail in Fig. 9 where the lines and columns are marked by short names of politicians (up to 7 letters). We can say that large matrix elements in a column of a given politician can be considered as direct friends to whom he/she points in his/her Wikipedia article. However, we should note that by construction of $G$ all elements inside a given column have the same amplitudes given by a fraction of total number of outgoing links of a given article which point inside $N_r$ nodes of selected subset. Thus on the basis of the $G_{rr}$ component it is not possible to say that some direct friend (link) of a given politician is more preferable than another one: all of them have the same amplitude. Of course, when we are speaking about a friend we simply mean that one politician points to another but at present we cannot say if this link has a positive or negative content. Such a classification would required further extension of our $G_R$ analysis. However, since the PageRank probability is on average
Due to the above reasons the most interesting matrix component $G_{qr}$ is the one of indirect links shown in Fig. 8 (bottom right panel) and in Fig. 10 where diagonal elements have been removed. The weight $W_{qrd} = 0.00487$ of diagonal components of $G_{qr}$ is approximately twice as small than the weight of the nondiagonal part with $W_{qrnd} = 0.01143$ : nondiagonal elements play a more significant role. One can understand that indirect links produce considerable diagonal contributions. However, for the analysis of indirect links between different nodes there are not of interest.

As discussed above the largest matrix elements of $G_{qr}$ in a column of a given politician give his/her strongest indirect or hidden friends while those in a line give his/her strongest hidden followers. The names of top friends and top followers of top 5 PageRank US politicians are given in Table 8. Surprisingly we find that the strongest hidden
followers of Obama are not his colleagues from the democratic but Lew (actual secretary of treasury), Comey (actual FBI director), Carter (actual secretary of defense).

Interestingly, Clinton has as main followers Lew, Sanders, Pelosi so that already in 2013 there existed the strong hidden links between Sanders and Clinton. At the same time the top 3 hidden friends (followers) of Trump are Obama, Bush, Clinton (Palin, Bloomberg, Clinton) highlighting hidden links between these two political leaders who are now fighting for the US presidency (see data at [27]).

It is possible to try not to take into account the projector component and construct a modified reduced Google matrix \( \tilde{G_R} \) obtained from \( G_{rr} + G_{qrnd} \) by renormalization of each column to unity. Then the PageRank vector of \( \tilde{G_R} \) gives the new ranking of the selected group with index \( K_G \) given in Table 2. We then see that the top 3 positions are taken by Bush, Obama, McCain. This gives a rearrangement of the ranking which stresses in a stronger way previous presidential teams. We will see that for other countries the effects can work in a different direction (e. g. for DE).

5 Direct and hidden links of UK politicians

The reduced Google matrix analysis of 20 UK politicians from Fig. 3 and Table 3 is presented in Figs. 11, 12, 13. As for the US case we find that \( G_R \) has the dominant contribution from the projector component \( G_{pr} \) which has a similar weight of 96%. The weights of other two components of direct \( G_{rr} \) (1.7%) and hidden \( G_{qr} \) (1.5%) links is also similar to those of US. However, on average the direct links are distributed in a more homogeneous manner for UK then for US (see Figs. 12, 9). The strongest direct link is from Leadsom to Osborne.

The distribution of hidden links in Fig. 13 is much more broad as compared to the direct ones in Fig. 12. The top 3 hidden friends and followers of top 5 PageRank politicians are given in Table 9. There are strong links between top 3 leaders Blair, Brown and Cameron. More surprisingly we find that already in 2013, May was the strongest follower of Cameron (there is only a moderate direct link between them) and Johnson (there is no direct link). The strongest amplitudes of links are from Sturgeon (actual first minister of Scotland) to Salmond (previous first minister of Scotland) and from Foster to Robinson (first ministers of Northern Ireland). Even if the direct links are present in \( G_{rr} \) they are definitely not so pronounced as in the indirect part \( G_{qr} \). Also we find that Khan (actual mayor of London) is a second by strength indirect follower of Cameron even if there are no direct links between them.

The ranking index \( K_G \) of UK politicians from \( G_{rr} + G_{qrnd} \) is given in the last column of Table 3. It places on top positions Brown, Blair, Cameron followed by Miliband and Salmond. Such a ranking looks to be less natural as compared to the global rank index \( K \). It stresses that that the projector component \( G_{pr} \) still plays an important role.

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![Fig. 11. Same as Fig. 8 for 20 UK politicians in the Enwiki network. The weights of the three matrix components of \( G_R \) are \( W_{pr} = 0.9668 \) for \( G_{pr} \), \( W_{rr} = 0.01729 \) for \( G_{rr} \), \( W_{qr} = 0.01588 \) for \( G_{qr} \).](image)

![Fig. 12. Density plot of the matrix \( G_{rr} \) for the reduced network of 20 UK politicians in the Enwiki network with short names at both axes.](image)
6 Direct and hidden links of DE politicians

For 40 German politicians the matrix $G_R$ and its three components are shown in Figs. 14, 15 and 16. The component weights are similar to the cases of US, UK but the percentage of $G_{rr}$ is now slightly higher.

The strongest direct links are from Maas to Lafontaine (Maas was supported by Lafontaine, chairman of the Social Democratic Party - SPD) and Arnold to Ströbele. For the indirect links component $G_{qr}$ the strongest links remains the one of Maas to Lafontaine influenced by their direct link. However, in global the number of indirect links is significantly larger compared to direct links. The top 5 PageRank politicians have strongest direct friend links mainly between their own group as it is seen in Table 10. However, the list of followers is rather different. Thus Merkel is the first follower of Kohl who strongly supported her. Kohl is the strongest follower of Schmidt, probably because many Wikipedia articles refer to the change of power between them in 1982 but also because Kohl was the opposition leader during the Schmidt government of 1974-1982. Furthermore, Hendricks is the first follower of Merkel. She is a recent member of Merkel’s government (since end 2013 and at least up to 2016) despite being member of the socialist party. The strongest follower of Schöder is Maas. Despite the fact that both belong to the socialist party it is difficult to establish a direct political link due to a considerable time difference of more than 10 years between their time periods in government. However, their names appear both (and together with other names) in certain news articles discussing general marital prob-
problems of certain German politicians which may indeed produce indirect links on a different than professional level.

The ranking \( K_E \) from the matrix \( G_{rr} + G_{qrr} \) is given for 40 German politicians in the last column of Table 4, placing on the top positions Merkel, Schröder, Kohl.

### Table 10. Same as Table 8 for leading DE politicians and the German Wikipedia edition of 2013.

Table 10. Same as Table 8 for leading DE politicians and the German Wikipedia edition of 2013.

| Politicians | DE Name | Dewiki Name |
|-------------|---------|-------------|
| Kohl        | Schmidt | Merkel      |
| Schmidt     | Merkel  | Westerwelle |
| Merkel      | Schröder| Lafontaine  |
| Schröder    | Kohl    | Steinbrück |
| Schäuble    | Kohl    | Steinmeier  |
|            | Schmidt | Hendricks   |
|            |         | Maas        |
|            |         | Steinbrück  |
|            |         | Lammert     |

### 7 Direct and hidden links of FR politicians

The reduced Google matrix \( G_R \) and its three components for 40 French politicians (see Fig. 5 and Table 5) are shown in Figs. 17, 18, 19. Here the weight of \( W_{rr} \) is the largest among all groups of politicians considered in this work. Also the distribution of direct links (see Fig. 18) is very broad compared to the case of 40 DE politicians in Fig. 15. The two strongest direct links go from Philippot to Jean-Marie Le Pen and Marine Le Pen, respectively. They all belong to the far-right FN party.

The hidden links between 40 FR politicians without diagonal terms are shown in Fig. 19. The 3 strongest friends and followers are given in Table 11. Friends of current or former presidents Hollande and Sarkozy are main election opponents or close collaborators (Fillon, Fabius). Raffarin, a former prime minister and leader of UMP party, has only same-party hidden links. This could be explained by the fact that he never participated in the last round of any presidential election. Main relationships of Table 11 are reasonable and can be explained. The most surprising connection is the hidden link from Taubira to J.M. Le Pen (there is no direct link). Taubira issued an important law allowing same-sex marriage in 2013, creating virulent opposition from far-right parties that are documented in Wikipedia.

As pinpointed earlier and in previous works [13], using \( G_{rr} \) for extracting knowledge on followers is not meaningful due to column normalization. A good example can
Fig. 17. Same as Fig. 8 for 40 FR politicians in the Frwiki network. The weights of the three matrix components of $G_R$ are $W_{pr} = 0.9481$ for $G_{pr}$, $W_{rr} = 0.03248$ for $G_{rr}$, $W_{qr} = 0.01941$ for $G_{qr}$.

Fig. 18. Density plot of the matrix $G_{rr}$ for the reduced network of 40 FR politicians in the Frwiki network with family names at both axes.

Table 11. Same as Table 8 for leading FR politicians and the French Wikipedia edition of 2013.
be highlighted here as in $G_{rr}$, top three followers of J.-M. Le Pen (FN party founder) are F. Philippot, N. Dupont-Aignan and A. Montebourg (socialist party), while the leader of FN, Marine Le Pen, only ranks as its 27th follower (due to her increased number of outgoing links compared to others). Looking now at the hidden links in Fig. 19 and at the 3 strongest followers of J.M. Le Pen, Marine Le Pen ranks two, just after F. Philippot, the other important figure of that party.

Several hidden links appear in matrix $G_{qr}$ as well in this network: Philippot to Sarkozy is strong in Fig. 19 and non-existing in Fig. 18; same with Pécresse to Hollande or Strauss-Kahn to Sarkozy. An in-depth analysis of the network of FR politicians is given in Section 10.

8 Direct and hidden links of RU politicians

The reduced Google matrix $G_R$ for 20 Russian politicians from Fig. 6 and Table 6 are shown in Figs. 20, 21, 22. The weight of the component of direct links is a bit larger than in the case of US and UK but comparable to DE. The strongest direct links are from Siluanov (minister of finance) to Putin and Medvedev and Kudrin (previous minister of finance).

The matrix of hidden links $G_{qr}$ without diagonal is shown in Fig. 22 with the list of 2 top friends and followers for top 5 RU politicians in Table 12. Among hidden friends of Putin we find naturally Medvedev, Chubais, Ivanov who are closely linked with him during his political career. More surprising is that his strongest followers are

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**Fig. 20.** Same as Fig. 8 for 20 RU politicians in the Ruwiki network. The weights of the three matrix components of $G_R$ are $W_{pr} = 0.9644$ for $G_{pr}$, $W_{rr} = 0.02408$ for $G_{rr}$, $W_{qr} = 0.011148$ for $G_{qr}$.

**Fig. 21.** Density plot of the matrix $G_{rr}$ for the reduced network of 20 RU politicians in the Ruwiki network with short names at both axes.

**Fig. 22.** Density plot of the matrix $G_{qr}$ without diagonal elements for the reduced network of 20 RU politicians in the Ruwiki network with short names at both axes. The weight of this matrix component without diagonal is $W_{qrnd} = 0.007912$ (see Fig. 20 for $W_{qr}$ weight with diagonal).
Table 12. Same as Table 8 for leading RU politicians and the Russian Wikipedia edition of 2013.

| Politicians | RU | Ruwiki |
|-------------|----|--------|
| Name        | Friends | Followers |
| Putin       | Medvedev | Hakamada |
|             | Chubais  | Fradkov |
|             | Ivanov   | Nabiullina |
| Medvedev    | Putin    | Putin |
|             | Chubais  | Mironov |
|             | Ivanov   | Gref |
| Zhirinovsky | Putin    | Zhiruganov |
|             | Medvedev | Hakamada |
|             | Zyuganov | Yavlinsky |
| Matvienko   | Putin    | Mironov |
|             | Medvedev | Kudrin |
|             | Mironov | Nemtsov |
| Zyuganov    | Putin    | Hakamada |
|             | Medvedev | Yavlinsky |
|             | Zhirinovsky | Zhirinovsky |

Hakamada, Fradkov, Nabiullina. Among friend relations it is somewhat unexpected that Zhirinovsky and Zyuganov are friends of each other in a third position. It is also interesting that Russian democratically oriented politicians (Hakamada, Yavlinsky) are among top followers of Zhirinovsky and Zyuganov. More naturally, top 2 to 5 PageRank politicians (Medvedev, Zhirikovsly, Matvienko, Zyuganov) have Putin as the first friend (see data at [27]). In fact for all 19 politicians the first friend from the matrix of indirect links $G_{qr}$ is Putin confirming well the vertical power of politics in Russia. On a level of direct friends from $G_{rr}$ only Chubais and Hakamada do not point directly to Putin but on a level of indirect links of $G_{qr}$ all 19 politicians point first to Putin. For RU democrats like we have among top 3 friends (followers) for Yavlinsky: Putin, Zyuganov, Chubais (Hakamada, Nemtsov, Zyuganov); for Nemtsov: Putin, Chubais, Hakamada (Hakamada, Chubais, Yavlinsky); for Hakamada: Putin, Nemtsov, Zyuganov (Nemtsov, Zyuganov, Yavlinsky).

This shows relatively strong links between democrats even contacts with Zyuganov (leader of communist party) are surprisingly well present.

The rank index $K_G$ from the matrix $G_{rr} + G_{qr}$ and $G_{qr}$ is given in last column of Table 8 with the top leaders being Putin, Medvedev, Fradkov that seems to overestimate the importance of Fradkov, even if he is followed by Ivanov and Gref that looks to be more reasonable. So the above analysis of politicians of US, UK, DE, FR, RU suggests that the projector component should be taken into account even if we want to analyze the relations inside the selected group of politicians since the environment links of the global matrix $G$ still play an important role.

9 Direct and hidden links of G20 state leaders

Above we considered interactions between political leaders of the same country from the view point of the Wikipedia edition of their main language. It is interesting to see the results of the reduced Google matrix analysis for interactions of state leaders of the G20 summit of 2012 [20]. We analyze these interactions from the view point of 4 Wikipedia editions EN, DE, FR, RU.

The list of names of politicians of G20 and their distribution over PageRank-CheiRank plane are given in Fig. 7 and Table 7. In our presentation below we keep the names in the PageRank order of Enwiki of Table 7. For EN Wikipedia the reduced Google matrix $G_R$ and its three components are shown in Figs. 23, 24, 25. We see that, compared to previous cases inside one country, the weights $W_{rr}$ and $W_{qr}$ are reduced approximately by a factor 2. Indeed, there are significantly less direct links between leaders of different states (see data at [27]). For example, the only direct links for RU Wikipedia are between Merkel and Putin, Erdoğan, Putin and Jintao.

Thus the importance of indirect links from $G_{qr}$ becomes more significant even if the weight of nondiagonal matrix elements is reduced by a factor 4-5 compared to the case of politicians in the same country. In Fig. 25 for Enwiki we find the strongest indirect links between Monti and Barroso (EU), Abdullah (SA) and Obama (even if direct links also exist), Hollande and Merkel, Noda (JP) and Obama (without direct links). The list of top 3 friends...
The reduced Google matrices $G_{rr}$ from 4 Wikipedia editions are shown in Fig. 26. It is clear that each culture (which, in first approximation, can be associated with the language) has its own view on relations between state leaders of G20. Indeed, even top 3 PageRank leaders are different for these cultures which creates different structures of matrix elements.

The reduced Google matrices $G_{qr}$ are shown in Fig. 27 for Dewiki, Fig. 28 for Frwiki and Fig. 29 for Ruwiki. For Dewiki the strongest indirect links are from Gillard (AU) and Barroso to Merkel, Abdullah to Erdogan. For Frwiki the strongest link is from Yudhoyono (ID) to Putin, Monti and Merkel to Barroso. In contrast for Ruwiki the strongest links are from Barroso to Obama, Myung-bak (KR) to Putin. Being at the top of PageRank in Ruwiki, Putin accumulates the largest number of followers. This demonstrates a large variety of cultural views on interactions between state leaders.

The English version provides the richest information thanks to its volume and the variety of its contributions. However, main trends in other countries are not necessar-
Fig. 26. Density plots of the matrix $G_R$ for the reduced networks of 20 state leaders of G20 states in the Enwiki (top left), Dewiki (top right), Frwiki (bottom left) and Ruwiki (bottom right) networks. The order of the 20 state leaders is in all cases given by the PageRank order for the Enwiki network and corresponds to the same order of Figs. 23-25.

ily pictured in the English version due to cultural bias, and vice versa. For instance, the very strong link from Hollande to Merkel in Enwiki is really thin in the French edition, while it is clearly visible in the German one.

10 Network of political leaders of France

To have a more direct pictorial representation of interactions in the framework of the reduced matrix $G_R$ we choose the case of 40 politicians of France discussed above (see Fig. 5 and Table 5). Here all politicians are attributed to the main political parties marked by corresponding 5 colors in Table 5. For each color we take the top PageRank politicians: Sarkozy (blue, right parties, UMP-UDI), Hollande (magenta, left, PS), Jean-Marie Le Pen (violet, ultra-right, FN), Mélanchon (red, ultra-left), Cohn-Bendit (green, green party). These 5 “leaders”1 are positioned on a circle of fixed radius forming the first level. Then for the directed network of $G_R$ of these 5 leaders we show for each of them 4 strongest links from them to other politicians (considered as their direct friends). These links are placed on second level circles around the (primary) politicians of the first level to which they correspond (the preference is given to the primary politicians with the same color if there are several corresponding primary politicians). For the politicians of the second level new red links with arrows are drawn for each of them with top 4 strongest links forming the third level circles if any. After these two iterations of 4 strongest friends we obtain the network of friends of $G_R$ with only 7 politicians. The politicians are marked by their PageRank numbers $K$ from Table 5. At blue color we find Sarkozy with Raffarin, at magenta we have Hollande with Royal, and for other colors we have only one politician of the first circle. Thus we find that the circle of close friends is very narrow. We also mention that for this case the subnetwork actually saturates completely at level 3 (i.e. including the group of tertiary politicians visible in Fig. 30) with only these 7 politicians and does not increase even if we try to include higher level circles/politicians. The reason of the saturation on a small sub network is that $G_R$ is dominated by the rank one contribution $G_{pr}$ which selects essentially top PageRank nodes. Therefore we simply find the top 5 PageRank nodes plus the two late PageRank position nodes of Mélanchon and Cohn-Bendit that have been selected to belong to the set of primary nodes.

We now perform the same procedure for the followers in $G_R$, using the strongest 4 incoming links on each level, instead of friends, as it is shown in the bottom panel of

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1 This definition of leader does not necessarily correspond to the official “leader” of a political party or group since in the Wikipedia PageRank index also older, more historical aspects, e. g. “former leadership”, are taken into account.
There is also Taubira (compact having all its 4 members being grouped together. Mélanchon. The group of the violet party becomes more is less pronounced with a smaller number of followers for work the separation of the magenta PS party into two groups centered around Hollande and another centered around Mélanchon. This clearly indicates fundamental structural it is the case for G_R, the subnetwork would saturate at all 40 politicians after 5 iterations (the corresponding higher level links are not shown in Fig. 30 in order to keep the presentation simpler).

We also used the same approach to construct the network of hidden friends or hidden followers using the matrix G_qr (instead of G_R) with the result shown in both panels of Fig. 31. Now, the network of hidden friends contains a significantly larger number of politicians after two iterations (15 instead of 7 for G_R but also with saturation at level 3). At the same time the network of followers has a similar number of nodes (34 instead of 32 for G_R with saturation at 35 after level 4). For the hidden follower network the separation of the magenta PS party in two groups is less pronounced with a smaller number of followers for Mélanchon. The group of the violet party becomes more compact having all its 4 members being grouped together. There is also Taubira (K = 29) who is closely linked to this violet group for similar reasons as the ones presented earlier.

Several follower interactions seem more reasonable with G_qr. For instance, Duflot (K = 37, Green party) isn’t linked anymore to PS but mainly to its own party members. Similarly, ultra-left members are connected together at the first level, and not only at the secondary level as it is the case for G_R. Interesting to notice is that in G_qr Philippot (K = 40, ultra-right FN party) clearly follows four of the main party leaders (J.-M. Le Pen, Sarkozy, Hollande and Cohn-Bendit), while this is not the case in G_R. This is clearly related to his political acquaintances of before 2009 where he backed up Chevènement (socialist party) and Mélanchon, among others.

11 Discussion

In conclusion, we have presented a new mathematical method which establishes an effective directed network for a selected subset of nodes belonging to a significantly larger network. This approach was tested on examples of several groups of political leaders of 5 countries and world state leaders of G20 analyzed in the frame of several Wikipedia networks. Our results show that the proposed method allows in a reliable way to determine direct and hidden links between political leaders. We think that this approach can provide firm mathematical grounds for the LMX studies.
Fig. 30. Network structure of friends (followers) in top (bottom) panel induced by the largest four nondiagonal elements per column (row) of the matrix $G_R$ obtained for the group of 40 French politicians in the French Wikipedia edition of 2013. The five colors of the nodes represent the five main political movements and the numbers of the nodes represent the local PageRank index in Table 5. For each movement the leading node in PageRank ordering is drawn on the largest primary circle with name. The thick black arrows represent links from (to) each primary node to (from) its four most relevant nodes and the thin red arrows represent links from (to) each secondary node to (from) its four most relevant nodes. New secondary (tertiary) nodes are drawn on the secondary (tertiary) circles. Most of these circles contain less than four nodes due to possible multiple links with center nodes on other circles of the same level. When possible, a secondary (tertiary) node was associated to a primary (secondary) node of the same color.

Fig. 31. Same as Fig. 30 for the matrix $G_{qr}$ which recovers hidden links between politicians.

Our results show that the Wikipedia network can be used in an efficient way to determine direct and hidden relations between different subjects appearing in Wikipedia. We also point that the reduced Google matrix approach can be applied to a variety of directed networks where the relations between selected subgroup of nodes are not straightforward to identify.

We thank Leonardo Ermann for stimulating discussions of properties of the reduced Google matrix for various directed networks. This research is supported in part
References

1. http://www.wikipedia.org (accessed August 2016)
2. Encyclopaedia Britannica http://www.britannica.com/ (accessed August 2016).
3. J.Giles, Internet encyclopaedias go head to head, Nature 438, 900 (2005)
4. J.M. Reagle Jr. Good Faith Collaboration: The Culture of Wikipedia, MIT Press, Cambridge MA (2010)
5. F.A. Nielsen, Wikipedia research and tools: review and comments, (2012), available at SSRN: dx.doi.org/10.2139/ssrn.2129874
6. S. Brin and L. Page, Computer Networks and ISDN Systems 30, 107 (1998)
7. A.M. Langville and C.D. Meyer, Google’s PageRank and beyond: the science of search engine rankings, Princeton University Press, Princeton (2006)
8. E. Gabrilovich and S. Markovitch, Computing Semantic Relatedness using Wikipedia-based Explicit Semantic Analysis, Proc. 20th Int. Joint Conf. on Artificial Intelligence 7, p.1606 (2007)
9. L. Ermann, K.M. Frahm and D.L. Shepelyansky, Spectral properties of Google matrix of Wikipedia and other networks, Eur. Phys. J. B 86, 193 (2013)
10. L. Ermann, K.M. Frahm and D.L. Shepelyansky, Google matrix analysis of directed networks, Rev. Mod. Phys. 87, 1261 (2015)
11. A.O. Zhirov, O.V. Zhirov and D.L. Shepelyansky, Two-dimensional ranking of Wikipedia articles, Eur. Phys. J. B 77, 523 (2010)
12. Y.-H. Eom, K.M. Frahm, A. Benczur and D.L. Shepelyansky, Time evolution of Wikipedia network ranking, Eur. Phys. J. B 86, 492 (2013)
13. Y.-H. Eom and D.L. Shepelyansky, Highlighting entanglement of cultures via ranking of multilingual Wikipedia articles, PLoS ONE 8(10), e74554 (2013)
14. Y.-H. Eom, P. Aragon, D.Laniado, A.Kaltenbrunner, S.Vigna and D.L. Shepelyansky, Interactions of cultures and top people of Wikipedia from ranking of 24 language editions, PLoS ONE 10(3), e0114825 (2015)
15. J.Lages, A.Patt and D.L. Shepelyansky, Wikipedia ranking of world universities, Eur. Phys. J. B 89, 69 (2016)
16. M.H. Hart, The 100: ranking of the most influential persons in history, Citadel Press, N.Y. (1992)
17. Academic Ranking of World Universities, http://www.shanghairanking.com/ (accessed Aug. 2016)
18. A.D. Chepelianskii, Towards physical laws for software architecture, arXiv:1003.5455 [cs.SE] (2010)
19. K.M. Frahm and D.L. Shepelyansky, Reduced Google matrix, arXiv:1602.02394[physics.soc] (2016)
20. 2012 G20 Los Cabos summit, https://en.wikipedia.org/wiki/2012_G20_Los_Cabos_summit (accessed Aug. 2016)
21. B.M. Bass, Bass & Stogdill’s Handbook of Leadership: Theory, Research and Managerial Applications, Free Press, N.Y. 3rd Edition (1990)
22. G.B. Graen and M. Ulh-Bien, Relationship-based approach to leadership: development of leader-member exchange (LMX) theory of leadership over 25 years: applying a multi-level multi-domain perspective, The Leadership Quarterly 6, 219 (1995)
23. M. Ulh-Bien and R. Marion (Eds.), Complexity leadership, Inf. Age Publ. Charlotte North Carolina, USA (2008)
24. T.N. Bauer and B. Erdogon (Eds.), The Oxford Handbook of Leader-Member Exchange, Oxford University Press, Oxford (2016)
25. S. Dorogovtsev, Lectures on complex networks, Oxford University Press, Oxford (2010).
26. B. Erdogon, T.N. Bauer and J. Walter, Deeds that help and words that hurt: helping and gossip as moderators of the relationship between leader-member exchange and advice network centrality, Personal Psychology 68, 185 (2015)
27. http://www.quantware.ups-tlse.fr/QWLIB/wikipolitnet/ (accessed Sept. 2016)
28. R. Meusel, S. Vigna, O. Lehemberg and C. Bizer, The graph structure in the web - analyzed on different aggregation levels, J. Web Sci. 1, 33 (2015)
29. V.V. Sokolov and V.G. Zelevinsky, Dynamics and statistics of unstable quantum states, Nucl. Phys. A 504, 562 (1989)
30. V.V. Sokolov and V.G. Zelevinsky, Collective dynamics of unstable quantum state, Annals of Physics 216, 323 (1992)
31. C.W.J. Beenakker, Random-matrix theory of quantum transport, Rev. Mod. Phys. 69, 731 (1997)
32. T. Guhr, A. Müller-Groeling and H.A. Weidenmüller, Random Matrix Theories in Quantum Physics: Common Concepts, Phys. Rep. 299, 189 (1998)
33. R.A. Jalabert, Mesoscopic transport and quantum chaos, Scholarpedia 11(1), 30946 (2016)
34. P. Gaspard, Quantum chaotic scattering, Scholarpedia 9(6), 9806 (2014)
35. K.M. Frahm, B. Georgeot and D.L. Shepelyansky, Universal emergence of PageRank, J. Phys. A: Math. Theor. 44, 465101 (2011)
36. P. Aragón, D. Laniado, A. Kaltenbrunner and Y. Volkovich, Biographical social networks on Wikipedia: a cross-cultural study of links that made history, Proc. of the 8th Intl. Symposium on Wikis and Open Collaboration (WikiSym 2012), ACM, New York No 19 (2012)