HIGHER ORDER ANGULAR GALAXY CORRELATIONS IN THE SDSS: REDSHIFT AND COLOR DEPENDENCE OF NONLINEAR BIAS

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ABSTRACT

We present estimates of the N-point galaxy, area-averaged, angular correlation functions $\bar{w}_N(\theta)$ for $N = 2, \ldots, 7$ for galaxies from the fifth data release of the Sloan Digital Sky Survey. Our parent sample is selected from galaxies with $18 \leq r < 21$ and is the largest ever used to study higher order correlations. We subdivide this parent sample into two volume-limited samples using photometric redshifts, and these two samples are further subdivided by magnitude, redshift, and color (producing early- and late-type galaxy samples) to determine the dependence of $\bar{w}_N(\theta)$ on luminosity, redshift, and galaxy type. We measure $\bar{w}_N(\theta)$ using oversampling techniques and use them to calculate the projected $s_N$. Using models derived from theoretical power spectra and perturbation theory, we measure the bias parameters $b_1$ and $c_2$, finding that the large differences in both bias parameters ($b_1$ and $c_2$) between early- and late-type galaxies are robust against changes in redshift, luminosity, and $\sigma_8$, and that both terms are consistently smaller for late-type galaxies. By directly comparing their higher order correlation measurements, we find large differences in the clustering of late-type galaxies at redshifts lower than 0.3 and those at redshifts higher than 0.3, both at large scales ($c_2$ is larger by $\sim 0.5$ at $z > 0.3$) and small scales (large amplitudes are measured at small scales only for $z > 0.3$, suggesting much more merger-driven star formation at $z > 0.3$). Finally, our measurements of $c_2$ suggest both that $\sigma_8 < 0.8$ and that $c_2$ is negative.

Subject headings: cosmology: observations — stars: formation

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1. INTRODUCTION

An important subject in cosmology and galaxy formation is galaxy bias. Galaxies trace the underlying dark matter that shapes the universe, but there is no guarantee that they do so faithfully. The galaxy bias defines the relationship between the clustering of dark matter and the clustering of galaxies. Ideally, the relationship between the distribution of galaxies and dark matter is simple—the ratio of the overdensity of galaxies to the overdensity of dark matter (i.e., the bias, hereafter $b$, which is measured relative to clustering dark matter) is a constant factor independent of smoothing scale and the overdensity of dark matter. This ideal case is known as “scale invariant linear bias.” If instead $b$ is a function of the overdensity, it is “nonlinear.” Significantly nonlinear bias would suggest that galaxy formation is dependent on environment, as it would suggest that the efficiency of galaxy formation is dependent on halo mass (i.e., the dark matter overdensity).

Assuming a nonlinear bias, $b$ can formally be expanded into factors $b_N$ via a Taylor expansion, which reduces to linear bias when $b_1 = b$ and $b_N = 0$ for $N > 1$. Several previous studies have found that $b_2$ is nearly zero. Pan & Szapudi (2005) found, by studying the monopole contribution to the 2dFGRS three-point correlation function, that for galaxies with $-21 \leq b_1 \leq -20$, $b_2 = -0.06^{+0.03}_{-0.003}$. Likewise, Hikage et al. (2005) found, using the Sloan Digital Sky Survey (SDSS) spectroscopic galaxy catalog to calculate the bispectrum, that $c_2 (c_N = b_N/b_1)$ is consistent with zero to within 10% for $\sigma_e = 0.9$; however, they measure $c_2$ to be larger for smaller $\sigma_e$. On the other hand, in an earlier study we (Ross et al. 2006, hereafter R06) found $c_2 = -0.30 \pm 0.10$ via determination of the Nth-order angular galaxy correlations for 11 million photometrically selected galaxies from the third data release (DR3) of the SDSS. Gaztañaga et al. (2005) likewise found $c_2 = -0.36^{+0.12}_{-0.09}$ for $b_1 < 19.45$ galaxies in the 2dFGRS by calculating the redshift-space three-point correlation functions, a method that is independent of $\sigma_8$, the rms fluctuation in the matter density averaged in a sphere of radius 8 Mpc.

Determining $\sigma_8$ is necessary because, as yet, there is no clear consensus. When measured from only the third-year WMAP (WMAP3) data, $\sigma_8 = 0.744^{+0.050}_{-0.048}$ (Spergel et al. 2007). Further constraining WMAP3 data via inflationary models, $\sigma_8 = 0.700^{+0.065}_{-0.048}$. When combined with the SDSS power-spectrum data, however, this increases to $0.772^{+0.056}_{-0.048}$. When combined with the 2dFGRS power-spectrum data it decreases to $0.737^{+0.033}_{-0.045}$ (Spergel et al. 2007). Furthermore, the SDSS power spectrum by itself determines $\sigma_8 = 0.842^{+0.063}_{-0.058}$ (Tegmark et al. 2004). Methods that employ clusters of galaxies typically measure smaller values of $\sigma_8$. For example, Voevodkin & Vikhlinin (2004) found $\sigma_8 = 0.72 \pm 0.04$ by using a cluster baryon mass function.

Independent measurements of $b_1$ and $\sigma_8$ are not possible using correlation measurements, as $\sigma_8$ serves as the normalization to the model power spectra one must use to determine $b_1$. The value of $b_2$, however, is not degenerate with $\sigma_8$, as it will change the shape of correlation measurements. A novel technique to provide an independent measure of $\sigma_8$ is to capitalize on the independence of $b_2$. By determining both $b_2$ and $c_2$, $\sigma_8$ can be constrained. Determining the value of $c_2$ alone places loose constraints on the value of $\sigma_8$.

Our most dramatic findings in R06 were that significant nonlinear bias was required to explain the observed clustering of early- and late-type galaxies. The results determined that early-type galaxies’ $c_2$ was larger than that of late-type galaxies by an absolute difference of $1.0 \pm 0.13$. Croton et al. (2006) have since confirmed this measurement—that red galaxies have a higher $c_2$—via their determination of higher order correlations in the 2dFGRS, although they found a more modest difference of $0.36 \pm 0.17$. In another study, Nishimichi et al. (2007) found a difference of closer...
to 0.5 when studying the bispектrum of galaxies selected from the SDSS fourth data release (DR4) spectroscopic survey. It is not surprising that differences are observed between these three studies, as each uses a different color cut to separate the red and blue samples. The existence of nonlinear bias between galaxy types was first found by Gaztañaga (1992) when it was discovered that the higher order clustering of galaxies selected by optical surveys (CFA and Southern Sky Redshift Survey) differs from the clustering of galaxies selected by IRAS (galaxy selection in IRAS was biased toward late-type galaxies). This was confirmed by Fry & Gaztañaga (1993) and interpreted as IRAS (and thus late-type) galaxies having a negative $c_2$ relative to optically selected galaxies.

Clearly, galaxy type affects the nonlinear nature of the bias. The linear bias is also strongly dependent on galaxy type, as red, early-type galaxies have consistently been shown to cluster more strongly (e.g., Willmer et al. 1998; Norberg et al. 2002; Madgwick et al. 2003; Zehavi et al. 2005; R06; Croton et al. 2006) than blue, late-type galaxies. Luminosity has been shown to scale proportionally to $b_1$ (e.g., Madgwick et al. 2003; Zehavi et al. 2005), which logically follows if one expects more luminous objects to generally be more massive. It is also likely that the properties of galaxies, and thus $b$, evolve over cosmic time. Therefore, a true characterization of galaxy bias, both linear and nonlinear, requires quantifying any dependence on galaxy type, luminosity, and redshift.

As a result, we employ the wealth of information available from the SDSS fifth data release (DR5) to follow up and vastly improve our DR3 measurements on the nature of galaxy bias. Apart from R06, four other studies have used SDSS photometric data to calculate higher order correlations: Gaztañaga (2002a, 2002b) and Szapudi et al. (2002) measured higher order correlations for galaxies in the SDSS Early Data Release (EDR) and found that measurements using SDSS were consistent with previous results and were free of any systematics. Blaizot et al. (2006) measured the first data release (DR1) higher order correlations and found good agreement with simulation. The R06 measurements employed the largest data set ever used to calculate higher order correlations with over 11 million galaxies from DR3. The SDSS DR5 offers an opportunity to significantly improve the DR3 results, as it offers ~70% more data and accurate photometric redshift catalogs complete with rest-frame absolute magnitudes. This wealth of data enables a quantification of the dependence of the $N$th-order correlations of galaxies on luminosity, type, and redshift.

In this paper, we therefore calculate and analyze the area-averaged angular $N$-point correlation functions using SDSS DR5 galaxies, up to seventh order. Our methodology is explained in §3. This present work offers a significant improvement over R06 because we use ~70% more data, extend the range of measurement by an order of magnitude, make measurements using volume-limited samples, and employ improved theoretical modeling. Our main sample of galaxies is used for comparison with the R06 measurements and is split into the same five subsamples as in R06, a process which is described in §2. The measurements made using DR5 are presented in §4, where we illustrate the superiority of DR5 over DR3. We use the SDSS DR5 PhotoZ table to create two volume-limited samples, allowing us to investigate the evolution and luminosity dependence of $N$th-order correlations in §5. Model, area-averaged correlation functions are calculated by integrating over theoretical power spectra and an appropriate redshift distribution, allowing for the calculation of first- and second-order bias parameters. In §6, we present our measurements of the higher order (nonlinear) bias and determine their dependence on galaxy type, luminosity, and redshift.

We adopt the cosmology $(\Omega_m, \Omega_{\Lambda}, h, \Gamma) = (0.28, 0.72, 0.7, 0.15)$, where $\Gamma$ is the shape parameter, based on recent supernovae, large-scale structure, and cosmic microwave background (CMB) measurements (e.g., Riess et al. 2004; Spergel et al. 2007; Cole et al. 2005). This value of the shape parameter arises naturally (see, e.g., eqs. [30] and [31] of Eisenstein & Hu 1998) for a baryon fraction of $\Omega_b/\Omega_m = 0.185$ (e.g., Cole et al. 2005).

2. DATA

The data analyzed herein were taken from the SDSS DR5 (Abazajian et al. 2005). This survey obtains wide-field CCD photometry (Gunn et al. 1998) in five passbands $(u, g, r, i, z)$ and $z_e$ (e.g., Fukugita et al. 1996). The entire DR5 represents 8000 deg$^2$ of observing area. We selected galaxies with positions lying in the northern, contiguous portion of the SDSS from the DR5 PhotoPrimary database and further constrained the sample (using the Schlegel et al. [1998] dust maps) to have reddening-corrected magnitudes in the range $18 \leq r < 21$. Furthermore, significant masking was required to account for bright stars and areas of high reddening and poor seeing (see §3.5). This produced a set of over 18 million galaxies (18,532,911) at a median redshift of about 0.31. This is by far the most galaxies used to conduct this type of measurement and represents over a 70% increase in the number of objects used in our previous DR3 measurements. We split this sample of galaxies into the same five subsamples as in R06 (three samples constrained to the magnitude ranges $18 \leq r < 19, 19 \leq r < 20$, and $20 \leq r < 21$, and two color-selected samples defined by $u - r > 2.2$ for early-type and $u - r \leq 2.2$ for late-type galaxies). These samples are used primarily for comparison with the DR3 measurements; however, due to our improved measurement techniques, they also sample substantially smaller and larger scales than R06.

2.1. Creation of Volume-Limited Samples

Our primary analysis is divided between two volume-limited samples created following the methods outlined in Budavári et al. (2003). Galaxies with $18 < r < 21$ are taken from the DR5 PhotoZ table and matched to galaxies in the DR5 PhotoPrimary table. Using the rest-frame absolute $r$-band magnitudes, $M_r$, for each galaxy in the PhotoZ table, we display the $M_r$-$z$ plane in Figure 1. As can be seen, there is a definite locus that defines the limiting absolute magnitude for a given redshift. To create a volume-limited catalog to a given redshift, $z_i$, we simply select galaxies with $z \leq z_i$ that are also intrinsically brighter than the limiting magnitude defined by the locus of points displayed in Figure 1. We focus our analysis on two volume-limited samples that have roughly similar numbers of galaxies. One sample we take is limited to a redshift of 0.4, requiring that $M_r < -20.5$. This sample, hereafter denoted as z4, contains nearly three and a half million galaxies (3,380,553) after masking (see §3.5). Our other volume-limited sample, hereafter denoted as z3, is limited to a redshift of 0.3, with $M_r < -19.5$, and contains nearly four million objects (3,980,652) after masking.

3. METHODOLOGY

3.1. Angular Correlation Functions

We estimate $N$-point area-averaged angular correlation functions, $\tilde{w}_N(\theta)$, using a counts-in-cells technique (e.g., R06). This basically involves calculating the statistical moments of the overdensities contained in equal-area cells. The overdensity for cell $i$ is defined as

$$\delta_i = \frac{\bar{n} - n_i}{\bar{n}},$$

(1)
where $\bar{n}$ is the average number of galaxies in a cell and $n_i$ is the number of galaxies in cell $i$. The remaining details and equations required to determine $\bar{N} = \frac{1}{C_1 \bar{n}}$ are found in R06.

In a hierarchical model (e.g., Groth & Peebles 1977; Szapudi et al. 1992; Gaztañaga 1994), higher order correlations can be expressed in terms of the two-point correlation function, and the volume-averaged correlations are given by

$$\bar{\xi}_N(R) = S_N \left[ \bar{\xi}_2(R) \right]^{N-1},$$

(2)

where $S_N$ is the hierarchical amplitude. In a similar manner, we can define the analogous relationship for the area-averaged angular correlations,

$$s_N = \frac{\bar{\omega}_N(\theta)}{[\bar{\omega}_2(\theta)]^{N-1}}.$$ (3)

The hierarchical amplitudes of the higher order moments encode much of the pertinent information on the distribution of the data. These amplitudes, therefore, embody the central analysis of this paper.

### 3.2. Redshift Distributions

In order to compare angular measurements to theoretical models, it is necessary to determine $dn/dz$, which we accomplish using the galaxies’ photometric redshifts (see § 2.1). We construct $dn/dz$ by using each published redshift and its error (rejecting any with error greater than 20%) to create a probability density function (pdf). The pdf’s for each redshift are combined to produce the expected number of objects ($n$) in a redshift bin of width 0.001.

This distribution is then normalized and interpolated over in order to estimate $dn/dz$. For SDSS galaxies with $18 \leq r < 21$, the resulting normalized $n(z)$ is plotted in Figure 2. The distribution of $n(z)$ is smooth and roughly Gaussian. For all subsamples (volume-limited and otherwise; see § 2), we likewise use the corresponding DR5 photometric redshifts to obtain estimates of their individual redshift selection functions.

### 3.3. Pixelization

The pixelization schemes we employ are nearly identical to those used in R06. Basically, we reimplemented the SDSSpix pixelization scheme originally developed by Tegmark, Xu, and Scranton,\(^3\) as described in detail in R06. Increased computing resources have allowed efficient usage of SDSSpix at smaller scales, allowing us to make measurements for $\theta > 0.02^\circ$. The “striped” method (see R06) is again employed, but only for scales between 0.02\(^\circ\) and 0.1\(^\circ\) (we note that we are able to probe smaller scales than in R06).

For larger angular scales an “oversampling” technique (e.g., Szapudi et al. 2002) is applied such that every angular scale uses the same number of data cells. Using the base resolution pixels produced by SDSSpix, we can make cells equivalent to any angular scale $n\theta_b$ at every single base pixel (where $n$ is any integer greater than or equal to 2 and $\theta_b$ is the angular scale at the base resolution). Thus, at a large scale the cells are highly overlapping, allowing more information to be extracted, which allows for more precise calculations at large scales. We therefore perform calculations for $\theta < 20^\circ$. We have verified that the results using the striped method are consistent with using this oversampling implementation at scales between 0.02\(^\circ\) and 0.1\(^\circ\). From here on, we

\(^3\) See http://lahmu.phyast.pitt.edu/~scranton/SDSSPix/.
refer to the method used for small angular scales as the "striped" method and to the method for large angular scales as the "oversampling" method. Oversampling increases the covariance, but this is not a problem since we perform a full covariance analysis for all parameters we attempt to measure.

3.4. Errors and Covariance

We compute errors and covariance matrices using a jackknife method (e.g., Scranton et al. 2002), with inverse-variance weighting for both errors (e.g., Myers et al. 2005, 2006) and covariance (e.g., Myers et al. 2007), nearly identical to the method described in detail in R06. The jackknife method works by creating many subsamples of the entire data set, each with a small part of the total area removed. For the striped method, we utilize the natural geometry of the SDSS. Each of the 29 different stripes in the DR5 forms a natural subset of the overall data. The covariance matrix is calculated using each of the possible subsamples of DR5 that is made up of 28 stripes. For the angular scales that are calculated by the striped method, we find that these 29 subsamples are sufficient to create a stable covariance matrix. For the larger angular scales probed by the oversampling method, we find that 20 jackknife subsamplings are sufficient to create a stable covariance matrix. These 20 subsamples are created by simply eliminating a contiguous grouping of 1/20 of the unmasked pixels in 20 separate areas. To properly constrain fit parameters, we minimize the \( \chi^2 \) using our covariance matrices via

\[
\chi^2 = \sum_{i,j} [\bar{\omega} (\theta_i) - \bar{\omega}_m (\theta_j)] C_{i,j}^{-1} [\bar{\omega} (\theta_j) - \bar{\omega}_m (\theta_j)],
\]

where \( C \) is the covariance matrix and \( i \) and \( j \) refer to the \( i \)th and \( j \)th jackknife subsample.

3.5. Masks

We generally refer to useful observational information (such as seeing and Galactic extinction values) across each pixel in our schema (see § 3.3) as forming a mask of that information. The DR5 area required significant masking, which we performed in the same manner as in R06. Pixels at the base resolution are discarded if they intersect the standard SDSS imaging mask, have a mean reddening \( A_r > 0.2 \), have a mean seeing greater than 1.5\arcsec, or intersect the R06 mask for galaxy M101. Cells at scales above the base resolution have their overdensities corrected for the fractional area of the pixel in the same manner as in R06, thus

\[
\delta_i = \frac{\bar{n} - n_i}{\nbar}.
\]
where $\Delta_i$ is the fractional area of cell $i$. As before, we did not find any systematic variation in correlation measurements as a function of stripe.

4. AREA-AVERAGED CORRELATION FUNCTIONS AND HIERARCHICAL AMPLITUDES: COMPLETE SAMPLE

Figure 3 shows the area-averaged correlation functions for $N \leq 7$ determined for galaxies with $18 \leq r < 21$. Errors for each point were determined by the jackknife method (see R06). Compared to the R06 results, we now sample at more angular scales, and the maximum and minimum angles are extended, resulting in an extra order of magnitude in angular coverage. For each $N$, the correlation function has a shape roughly consistent with a power law to about $0.3^\circ$. At larger scales, there exist obvious features that are not adequately represented by a power law. When looking at $\omega_2$, a power-law representation appears as if it may be appropriate to $1^\circ$. Assuming a form $d\theta^{1-\gamma}$, the $\chi^2$ best fit for $N = 2$ over the angular range $0.02^\circ < \theta < 1^\circ$ is $A = (7.2 \pm 0.1) \times 10^{-3}$ and $\gamma = 1.770 \pm 0.005$. These results are consistent with previous measurements (see, e.g., Gaztañaga 1994; Connolly et al. 2002; Frith et al. 2006; R06). The $\chi^2$ value is 433.6, however, meaning that a power-law form is inappropriate over these scales. In order to produce fits that are not rejected, fits must be performed over ranges that are less than 0.5 dex. We caution, therefore, that our power-law measurement is useful only for comparison purposes.

The hierarchical amplitudes $s_N(\theta)$ for $N \leq 7$ measured in magnitude ranges $18 \leq r < 19, 19 \leq r < 20, 20 \leq r < 21$, and $18 \leq r < 21$ are shown in Figure 4. Compared to the results of R06, the measurements made at scales greater than $1^\circ$ are far more significant, but overall the measurements are quite consistent. We also probe smaller scales, showing that at scales less than $0.02^\circ$ the $s_N$ measured for $18 \leq r < 19$ increase as the angular scales decrease.

As in R06, we employ the simple color criteria determined by Strateva et al. (2001) to separate early-type ($u - r > 2.2$) and late-type ($u - r \leq 2.2$) galaxies from SDSS photometric data. Using this color cut and the magnitude restriction $18 \leq r < 21$, we separate our sample into early- and late-type galaxies and repeat our $N$-point measurements for these two samples. We find that DR5 contains about 25% more late-type galaxies (10,284,575) than early-type galaxies (8,248,336), which is nearly the same proportion as in R06.
Figures 5 and 6 show the results of the $\omega_N$ and $s_N$ measurements for early- and late-type galaxies. The early-type galaxies clearly show stronger clustering at scales $\theta < 5^\circ$, in agreement with previous results (e.g., Willmer et al. 1998; Zehavi et al. 2002; Norberg et al. 2002; Madgwick et al. 2003), but at large scales ($\theta > 5^\circ$), the amplitudes are nearly equal. This suggests that their respective bias might be scale dependent (see § 6.1). In R06 we were not able to probe such large scales; otherwise, our overall results remain consistent. The correlation functions display roughly power-law behavior for all $N$, but there is significant structure that becomes more pronounced as $N$ grows. The $s_N$ of the late-type galaxies show smaller amplitudes for $\theta > 0.1^\circ$, suggesting that higher order bias terms are significant. At the largest scales ($\theta > 2^\circ$), $s_3$ and $s_4$ appear nearly identical for early- and late-type galaxies, yet in this regime, the errors begin to dominate the measurements.

As found in R06, the late-type correlation measurements show extremely interesting behavior. At angular scales between $0.01^\circ$ and $0.1^\circ$, the late-type galaxies exhibit slopes that increase as the scale decreases. Probing smaller scales, this relationship appears to turn over, as evidenced by the $s_N$ measured at $\theta < 0.02^\circ$. For $N = 6$ and 7, there is a dramatic loss of signal at angular scales greater than $0.2^\circ$. A similar loss in signal is shared by the correlation measurements made by using all galaxies, but the measurements made by using early-type galaxies do not show this dramatic loss of signal. This suggests that there is an intrinsic property of late-type galaxy clustering that is so strong it dominates the measurement for $N \geq 6$.

The DR5 PhotoZ table contains estimates of spectral type, given by a number between 0 and 1, where 0 is most red and 1 is most blue. It has been shown (e.g., Budavári et al. 2003) that splitting the sample of galaxies into early- and late-type galaxies at a type value of 0.3 (hereafter the “photo” sample) is roughly equivalent to splitting them using the Strateva et al. (2001) color criteria (hereafter the “color” sample). To test this, we split the entire sample into early- and late-type galaxies by photo and compare the resulting correlation functions to those measured for galaxies split by color. The results for $s_3$ are displayed in Figure 7. While the two cuts produce similar results, there are some significant differences. It appears that the characteristic rise at small scales for late-type galaxies is actually more prevalent in the galaxies split by the photo method. This suggests that type confusion when splitting the galaxies by the color method (which would be expected to be greatest at higher redshift) may dampen the overall signal of late-type galaxies selected this way. To minimize any type confusion, therefore, when we split our galaxy sample by redshift, their types are determined via the photo method, as these
types should be less sensitive to redshift than the color method (see § 5.3 for further justification).

5. HIGHER ORDER CORRELATIONS IN VOLUME-LIMITED SAMPLES

5.1. z < 0.3

The volume-limited sample z3 contains roughly four million galaxies with $M_r < -19.5$. We split this sample into 15 different subsamples (although each is not mutually exclusive). The sample is split by magnitude into two groups: $-20.5 < M_r < -19.5$ and $-21.5 < M_r < -20.5$; and by redshift into two groups: $0 < z < 0.2$ and $0.2 < z < 0.3$. Each group itself contains three groups: early-type, late-type, and all types, where the split is done by using the photo method.

Figure 8 displays the $s_N$ measurements made using all, late-type, and early-type galaxies in the z3 volume-limited sample. Immediately, one notices that the late-type galaxies do not resemble the late-type galaxy measurements made on the full sample, especially at smaller scales. The late-type galaxies also display larger error bars than the early-type galaxies, despite the fact that there are nearly the same number of late-type galaxies (1,984,021) and early-type galaxies (1,996,631) in this sample. The early-type galaxies show significantly higher amplitudes than the late-type galaxies, and the difference is largest at $\theta > 0.1^\circ$, just as in the full sample of galaxies.

For each subset of galaxies (all, late, and early) the amplitudes for the full sample are slightly larger than those of z3. This suggests that the bias is slightly larger in the z3 sample than in the full sample, as $s_N \propto b^2 - N$. This is not surprising, as the full sample contains less luminous galaxies than z3, and bias is known to increase with luminosity (e.g., Madgwick et al. 2003; Zehavi et al. 2005). The bias measurements for each sample are calculated in § 6.

Figure 9 displays the $s_3$ and $s_4$ measurements made by separating the z3 galaxies by luminosity into two groups, $-21.5 < M_r < -20.5$ and $-20.5 < M_r < -19.5$. These measurements for all, late-type, and early-type galaxies are shown in the bottom, middle, and top panels, respectively. For each galaxy type, the shapes of $s_N$ for brighter galaxies are quite similar to those for the fainter galaxies. The largest difference is that the amplitudes are higher for the fainter galaxies, which is consistent with the first-order bias.
increasing with luminosity. Based solely on Figure 9, it does not appear that nonlinear clustering (i.e., \(c_2, c_3\)) is dependent on luminosity, as the differences in the amplitudes appear consistent with a linear bias model (see § 6, Table 1).

In order to test for evolution, \(z_3\) is split by redshift into \(0 < z < 0.2\) and \(0.2 < z < 0.3\) samples. Figure 10 shows this split for all, late-type, and early-type galaxies in the bottom, middle, and top panels, respectively. After accounting for the fact that the galaxies with \(0.2 < z < 0.3\) probe scales that are about 1.6 times larger (based on median redshifts of 0.26 and 0.15) than the \(0 < z < 0.2\) galaxies, the shapes of the \(s_N\) are quite similar to a physical scale of about 5 \(h^{-1}\) Mpc (\(~0.06^\circ\) for \(0 < z < 0.2\), \(~0.04^\circ\) for \(z < 0.3\)). Despite the visual differences, there does not appear to be significant evolution in the bias, as the differences in the measurements can be explained by the differences in the physical scales.

One concern with the volume-limited samples we create is the possible effects of photometric redshift errors on the creation of our samples. To quantify any potential bias, we created 10 separate samples for which instead of taking the stated photometric redshift of each galaxy, we sampled its probability density function (pdf) given by its 1 \(\sigma\) error, thereby assigning a new redshift for each galaxy. This allowed us to create 10 separate samples with the same magnitude and redshift limits as \(z_3\). Figure 11 displays the average measured \(s_3\) of these samples, with errors (thick black error bars) calculated by finding the standard deviation of the 10 measurements. Underneath these points, the jackknife errors of the \(z_3\) sample are plotted with thin gray error bars. It is clear that the normal jackknife errors dominate the error budget, especially at larger scales where the errors are most important for fitting bias values. Therefore, we conclude that photometric redshift errors should not make any significant difference in our results.

5.2. \(z < 0.4\)

The volume-limited sample \(z_4\) contains just under three and a half million galaxies with \(M_r < -20.5\) and \(z < 0.4\). Figure 12 shows the \(s_N\) measurements for all, late-type, and early-type galaxies in \(z_4\), split using the photo method. Their shapes are quite similar to the measurements made on the full sample. The \(z_4\) sample, however, shows a strong rise at small scales in \(s_N\) for late-type galaxies, unlike the \(z_3\) sample. The error bars on the late-type galaxies are again larger than those of the early-type galaxies, but in this case it can be explained by the fact that there are significantly fewer late-type galaxies (1,325,488 late-type vs. 2,055,065 early-type).

Splitting \(z_4\) into two redshift bins, \(z < 0.3\) (1,302,750 galaxies) and \(0.3 < z < 0.4\) (2,077,803 galaxies), produces extremely
interesting results. Figure 13 shows the $s_N$ measured in these redshift bins for all, late-type, and early-type galaxies in the bottom, middle, and top panels, respectively. For each galaxy type, the $s_N$ are much closer to constant at the lower redshift. For the late-type galaxies, the errors are much larger at small scales for the low-redshift bin. This is because there are nearly 75\% more high-redshift (843,527) galaxies than low-redshift (481,961). Low-redshift, late-type galaxies have a weak signal at small scales, while the high-redshift galaxies have a relatively strong signal, meaning that in the full sample the high-redshift signal dominates. This explains the small-scale features seen in the full sample of late-type galaxies.

The late-type galaxies can be further split by their type value, $t$. Galaxies with $0.3 < t < 0.65$ are put into a sample we denote as $L1$. These galaxies correspond roughly to late-type spirals (Budavári et al. 2003). Galaxies with $0.65 < t$ are put into a sample we denote as $L2$ and correspond roughly to irregular galaxies (Budavári et al. 2003). Figure 14 shows the measured $S_3$ and $S_4$ for $L1$ (filled symbols) and $L2$ (open symbols) for $0.3 < z < 0.4$. The rise in the amplitudes at small scales is much stronger for the $L2$ galaxies, which we interpret as evidence that the small-scale rise correlates with star formation in dense environments, which we discuss further in §7.3.

5.3. Comparison of Full Sample to Volume-Limited Sample

By comparing the $s_N$ measurements made on the full sample to those made on the volume-limited samples, features in the full sample of galaxies are isolated in the redshift/luminosity plane. The most obvious conclusion is that the measurements of late-type galaxies are dominated by galaxies with $z > 0.3$, especially on angular scales smaller than $0.5^\circ$, as the $s_N$ signal is much stronger at $z > 0.3$ for late-type galaxies. For early-type galaxies, the shapes of $s_N$ measured in all subsamples are consistent both with each other and with the measurement of early-type galaxies drawn from the full sample.

For many of the subsamples, there appears to be a feature at $\sim 2^\circ$. This is especially true for all galaxy types in the main sample of galaxies and for early-type galaxies with $0.3 < z < 0.4$ and $z < 0.2$. It is quite prevalent for all galaxy types with $z < 0.4$ and is strangely not present for galaxies with $0.2 < z < 0.3$. A close inspection of the feature reveals that it actually occurs at slightly different angular scales in the different redshift shells. Figure 15 shows $S_3$ measured for early-type galaxies in the $z < 0.2$ redshift bin of $\sim 3^\circ$ (gray circles) and the $0.3 < z < 0.4$ redshift bin of $\sim 4^\circ$ (black triangles). These are the ideal samples to compare, as they represent the largest difference in physical scale. The left panel...
shows the two measurements plotted on an angular scale, while the right panel plots \( s_3 \) against the equivalent physical scale (based on their median redshifts of 0.154 and 0.355). When the two measurements are plotted against their equivalent scale, the match is much better. This suggests that the feature is physical in nature and is characterized by a minima at 10 units.

Figure 7 suggests that separating early- and late-type galaxies via the color method may break down at higher redshifts. Figure 16 displays the same information as Figure 13, with galaxy type determined via the color method instead of the photo method. The measurements for \( z < 0.3 \) are nearly identical to those measured splitting by the photo method, but for \( 0.3 < z < 0.4 \) the results are quite different, for both the early- and late-type galaxies. This suggests that for \( z > 0.3 \), the color method cannot adequately distinguish between early- and late-type galaxies, and it justifies our preference for separating galaxy type based on the photo method.

### 6. BIASES MEASUREMENTS

There is no guarantee that galaxies cluster with the same amplitude as the dark matter they trace. Indeed, it has often been shown that galaxies display different clustering amplitudes when separated by luminosity or type (e.g., Fry & Gaztañaga 1993; Willmer et al. 1998; Norberg et al. 2002; Madgwick et al. 2003; Zehavi et al. 2005; R06; Croton et al. 2006). The simplest model for this difference is the linear bias model. In this model, the overdensity of galaxies is a linear function of the overdensity of dark matter:

\[
\delta_g = b \delta_{DM},
\]

where \( g \) denotes galaxy, \( DM \) denotes dark matter, and \( b \) is the linear bias factor. In this approximation, one finds the simple relationship that

\[
\tilde{\omega}_{2,g} = b^2 \tilde{\omega}_{2,DM}.
\]

It has been shown, however (e.g., Fry & Gaztañaga 1993; Gaztañaga et al. 2005; R06; Croton et al. 2006), that linear bias may not be a good approximation. One can represent the relationship between overdensities more generally, such that the measured overdensity is some function of the dark matter overdensity. As such, the relationship can be expanded into a Taylor series (Fry & Gaztañaga 1993):

\[
\delta_g = \sum_{N=0}^{\infty} \frac{b_N}{N!} \delta_{DM}^N,
\]

where \( b_N \) is the Nth-order bias term. Thus, it follows that if nonlinear bias is important, a measurement of the linear bias will increase as the overdensities increase (i.e., as the scale gets smaller) and the nonlinear terms (i.e., any \( b_N \) for \( N > 1 \)) grow in importance. When \( \delta \ll 1 \), \( b_1 \) can be determined via equation (7), as higher order terms will be negligible.

#### 6.1. First-Order Bias Measurements

In order to find \( b_1 \), one must first determine the scales at which higher order terms make a negligible contribution to \( \tilde{\omega}_2 \). This can be done by calculating \( \omega_{2,0} \), incorporating a second-order bias term, and comparing it to the \( \omega_{2,2} \) calculated normally. To second order, the dark matter overdensity can be expressed (via trivial manipulation of eq. [8]) as

\[
\delta_{DM} = b_2^{-1} \left( -b_1 \pm \sqrt{b_1^2 + 2b_2 \delta_g} \right).
\]

Thus, when calculating \( \omega_{2,2} \), if equation (9) is used to correct the measured overdensities in each cell, a measurement is returned that assumes a first and second-order bias. If the first-order bias is set to 1 and a reasonable second-order bias is applied, the resulting measurement will begin to deviate significantly from the standard measurement when the second-order effects become important. Based on previous measurements (Gaztañaga et al. 2005; R06), we select \(-0.3\) as a reasonable value to use. Figure 17 shows the ratio of \( \omega_{2,2} \) calculated with this value to \( \omega_{2,2} \) calculated in the standard way, using the sample of galaxies from \( z_3 \) with \(-20.5 < M_r < -19.5 \). The ratio grows significantly greater than 1 for \( \theta < 0.66 \), corresponding to a physical scale of about \( 8 \) Mpc. This is right where one would expect nonlinear effects to become important, as it marks the generally accepted transition to the weakly linear regime. As a result, we calculate the first-order bias for scales where the corresponding physical scale (\( \tilde{\theta} \)) is greater than \( 8 \) Mpc.

Calculating the bias requires knowing \( \omega_{2,2,DM} \). Of course this is not directly measurable, so we must resort to using a theoretical model. Smith et al. (2003) have derived fitting formulae based on N-body simulations, which produce matter power spectra when given specific input cosmological parameters. By using
a modified version of Limber’s equation (Peebles 1980), one can use the appropriate redshift distribution to invert the $P(k)$ to obtain $\bar{\Omega}^2(C^{18})$:

\[
\bar{\Omega}^2(C^{18}) = \frac{1}{C^{25}} \int \frac{dn}{dz} \int \frac{dZ}{d\chi} F(\chi) \int P(k) W_{2D}(D\theta k) dk,
\]

where $W_{2D} = 2J_1(x)/x$ is the top-hat two-dimensional window function, $D$ is the survey depth (determined by the median redshift), $P(k)$ is the matter power spectra, $k$ is the spectral index, and $J_1$ is the first-order Bessel function of the first kind. In a flat universe, $F(\chi) = 1$, and $dz/d\chi = H(z)/c = H_0[\Omega_m(1 + z)^3 + \Omega_{\Lambda}/c]^{1/2}$, simplifying equation (10) to

\[
\bar{\omega}_2(\theta) = \frac{H_0 \pi}{c} \left( \frac{dz}{d\chi} \right)^2 \sqrt{\Omega_m(1 + z)^3 + \Omega_{\Lambda}} \int P(k) W_{2D}(D\theta k) dk.
\]

This integral was numerically determined with our assumed cosmology and fixing the value of $\sigma_8$ for the matter $P(k)$ to 0.8. Following this procedure, model $\bar{\omega}_2$ were produced for each subsample studied, using the appropriate redshift distribution. Finally, the first-order bias was calculated by using the covariance matrix at scales $\theta > 8 h^{-1}$ Mpc. We note that this approach is much more accurate than the methods employed in R06, which inverted the measurements to real space, required $\bar{\Omega}^2$ be a power law, calculated only the relative bias, and were insensitive to any changes in the bias as a function of scale.

Tables 1 and 2 display our calculated $b_1$ values for all of the measurements made using the volume-limited samples $z < 0.4$ and $M_r < -21.5$. These measurements look quite similar to those made on the full sample. [See the electronic edition of the Journal for a color version of this figure.]

The most important information can be summarized as follows: (1) for $z < 0$, there is no significant evolution in $b_1$ as a function of redshift for any galaxy type, but there is significant evolution seen in $z > 0.4$ and it is most dramatic for the late-type galaxies; (2) $b_1$ grows larger with luminosity, independent of galaxy type; and (3) $b_1$ is consistently larger for early-type galaxies, although the ratio of $b_{1,\text{early}}$ to $b_{1,\text{late}}$ is larger for $z < 0.3$. The determined luminosity and type dependences of $b_1$ are consistent with the general results of previous findings (Fry & Gaztañaga 1993; Willmer et al. 1998;
Norberg et al. 2002; Madgwick et al. 2003; Zehavi et al. 2005; R06; Croton et al. 2006). All of the determined values are dependent on the true value of $\sigma_8$. Since all of the models use $\sigma_8 = 0.8$, the true values of the $b_1$ are $0.8b_{1,m}/\sigma_8$, where $b_{1,m}$ are the measured values reported in Tables 1 and 2.

Most of the fits to $b_1$ using the $z_3$ sample minimize $\chi^2$ such that it is approximately one or smaller per degree of freedom (dof), meaning most fits favor a scale-invariant $b_1$ over the fitted range. The notable exception is the measurement for late-type galaxies with $0.2 < z < 0.3$, with a $\chi^2$/dof value of nearly two. The fits to $b_1$ for $z_4$ are in general worse than those of $z_3$. All but two have fits with $\chi^2$/dof greater than one. It is thus unlikely that $b_1$ is scale invariant at higher redshifts, a fact which must be taken into account when calculating higher order bias terms.

### 6.2. Second-Order Bias Measurements

In order to measure the second-order bias, we use the equation (Fry & Gaztanaga 1993)

$$S_{3,T} = b_{1,T}^{-1}(S_{3,DM} + 3c_{2,T}),$$  \hspace{1cm} \text{(12)}

where $c_2 = b_2/b_1$. This equation was derived for real-space hierarchical amplitudes, but given a theoretical $s_{3,DM}$, $c_2$ can be determined in the same way from $s_{3,g}$. Using perturbation theory (PT), an expression valid in the weakly nonlinear regime can be derived for $\tilde{\omega}_{3,DM}$ (Bernardeau 1995):

$$\tilde{\omega}_{3,DM} = 6 \left( \frac{H_0 \pi}{c} \right)^2 \int \left( \frac{dn}{dz} \right)^3 \left[ \Omega_m (1+z)^3 + \Omega_\Lambda \right] dz \times \left\{ \frac{6}{7} \left\{ \int kP(k) \left[ W_{2D}^2(D\theta k) \right]^2 dk \right\} + \int kP(k) \left[ W^2_{2D}(D\theta k) \right] dk \times \int k^2 D\theta P(k) W_{2D}(D\theta k) W_{2D}^T(D\theta k) dk \right\}. \hspace{1cm} \text{(13)}$$

From equation (3), it is clear that $s_{3,DM}$ can be obtained by dividing this $\tilde{\omega}_{3,DM}$ by $\tilde{\omega}_{2,DM}^2$. Equation (13) uses PT, which means that a linear power spectrum must be used when calculating $s_{3,DM}$, and it is only valid at scales with $r \gtrsim 8 h^{-1}$ Mpc. Using the model $s_{3,DM}$ and $b_1$ calculated at each scale (allowing for any scale dependence in $b_1$), one can calculate $c_2$ at each scale and thus construct its covariance matrix and determine the $\chi^2$ best-fit average values (see § 3.4) of $c_2$ for each subsample. Once again, we note that this method is superior to that employed by R06, as it has been
shown (e.g., Gaztañaga & Bernard 1998; Bernard et al. 2002) that this method is a better match to simulations than assuming a hierarchy and inverting the bias.

The values of $c_2$ for the $z_3$ and $z_4$ samples are presented in column (7) of Tables 1 and 2, respectively. The important aspects are as follows: (1) as first seen in R06, the $c_2$ values for late-type galaxies are significantly lower than those of early-type galaxies; (2) early-type galaxies have a value of $c_2$ that is independent of both luminosity and redshift; and (3) $c_{2, \text{late}}$ varies slightly with luminosity and evolves significantly between redshifts of 0.3 and 0.4, which we explore in § 7.3. The $\chi^2$/dof for all but three measurements are less than 1.24. The three exceptions are for measurements drawn from samples of early-type galaxies, and the poor fits are largely due to the strength in the feature at 10 h$^{-1}$ Mpc.

To the limit on our data, one would expect most measurements can be fit by a constant $c_2$, as the errors on the $s_3$ measurements over the fit ranges are typically large.

7. DISCUSSION

7.1. $c_2$ versus $\sigma_8$

The relationship between $c_2$ and $\sigma_8$ is more complicated than for the first-order bias. Given that our measured $c_{2,m}$ are calculated using $\sigma_8 = 0.8$ for the matter $P(k)$, it can be shown that

$$c_2 = \frac{\sigma_8}{0.8}c_{2,m} + \frac{s_3}{3} \left(1 - \frac{\sigma_8}{0.8} \right),$$

where $c_2$ is the true $c_2$. Since $s_3$ is not constant, the difference between $c_2$ and $c_{2,m}$ is not constant either. Thus, it is possible to perform a two-parameter fit on the values of $c_2$ and $\sigma_8$, thereby placing loose constraints on $\sigma_8$. Figure 18 shows the 1σ allowed regions of parameter space for $c_2$ and $\sigma_8$ for all (solid lines), early-type (dotted lines), and late-type (dashed lines) galaxies in the $z_3$ volume-limited sample. Clearly, there is a large spread in the allowed values of $c_2$ even if one constrains $0.7 < \sigma_8 < 0.9$ (changing $\sigma_8$ from 0.7 to 0.9 decreases $c_2$ by about 0.6 for all galaxy types). It is clear that a significant difference in the $c_2$ of early- and late-type galaxies is robust against changes in $\sigma_8$, as increasing $\sigma_8$ has only a slight effect (the difference is 1.0 for $\sigma_8 = 0.7$ and is 1.1 for $\sigma_8 = 0.9$). Thus, irrespective of $\sigma_8$, there is clear evidence for nonlinear bias differences between early- and late-type galaxies.

Focusing on the $z_3$ sample of all galaxy types, if we set $c_2 = 0$ we constrain the value of $\sigma_8$ to be in the range $0.64_{-0.03}^{+0.04}$. This is inconsistent at the 1σ level with the WMAP3 best-fit parameters ($\sigma_8 = 0.74_{-0.06}^{+0.05}$) but consistent with the 1σ lower bound on WMAP3 as constrained by inflationary models (e.g., $\sigma_8 = 0.70_{-0.06}^{+0.05}$). On the other hand, it is highly at odds with the typical effect of combining WMAP3 and large-scale structure constraints, which tend to higher values of $\sigma_8$. We would contend that these higher values of $\sigma_8$ may be inconsistent with linear theory, as our data suggest that $c_2$, for all galaxies, deviates by more than 2σ from zero for $\sigma_8 > 0.71$. If we instead set $c_2 = -0.36$, as found by Gaztañaga et al. (2005), then $\sigma_8 = 0.77_{-0.04}^{+0.04}$ nearly identical to the WMAP3-SDSS combined measurement. Thus, considering that it is both unlikely that $c_2$ is greatly negative and unlikely that $\sigma_8$ is less than 0.68, we contend that the best interpretation of our results is that $c_2$ is at least slightly negative and that $\sigma_8 < 0.8$.

With the methods employed in this paper, it is not possible to break the degeneracy between the bias parameters and $\sigma_8$. It may be possible, however, using measurements similar to those presented herein. When we tested the effects of $b_2$ on the calculation of $\tilde{\omega}_2$, we created a way in which one could compare a $\tilde{\omega}_2$ measurement “corrected” for $b_2$ to a model $\tilde{\omega}_2$ and thus test whether...
or not a particular value of $b_2$ was a good fit. In principle, one could calculate $\bar{\omega}_2$, varying the input value of $b_2$, to find a $\chi^2$ best-fit value of $b_2$. This best-fit value of $b_2$ would be independent of $\sigma_8$, as $b_2$ affects the shape of $\bar{\omega}_2$, and $\sigma_8$ affects only the normalization. Such an effort would require significant computational resources and would certainly be prohibitive on the 24 separate subsamples we studied. Endeavoring to such a task using a sample that probes large volumes with minimal shot noise and precise photometric redshift estimations, such as the luminous red galaxies (LRGs), is more feasible (A. Ross et al. 2007, in preparation).

7.2. $c_2$ versus $b_1$

Recent studies have found a relationship between $c_2$ and $b_1$. Gaztañaga et al. (2005) suggested the relationship $c_2 = b_1 - 1.2$. Nishimichi et al. (2007) measured results consistent with this relationship, and they showed that $c_2$’s dependence on $b_1$ is physically motivated. The major difference between these studies and ours is that we use galaxies over a much larger redshift range. Thus, we must determine whether the theoretical relationship between $c_2$ and $b_1$ is robust against changes in redshift.

Following the methods of Nishimichi et al. (2007) it is possible to calculate the first- and second-order bias parameters using a simple halo occupation distribution (HOD) model in which the mean number of galaxies in a halo of mass $M$ is given by

$$\langle N(M) \rangle = \begin{cases} 1 + \left( \frac{M}{M_1} \right)^\alpha, & M > M_{\text{min}}, \\ 0, & M \leq M_{\text{min}}, \end{cases}$$

where $\alpha$, $M_1$, and $M_{\text{min}}$ are free parameters and in general are fit such that this HOD model reproduces the measured clustering. Given this HOD model, the bias parameter $b_N$ is given by

$$b_N = \frac{\int_{M_{\text{min}}}^{M_{\text{max}}} dM \ n_{\text{halo}}(M,z) \langle N(M) \rangle B_n(M,z)}{\int_{M_{\text{min}}}^{M_{\text{max}}} dM \ n_{\text{halo}}(M,z) \langle N(M) \rangle},$$

where $n_{\text{halo}}$ is the mass function of halos with mass $m$ at redshift $z$ determined via an ellipsoidal collapse model (e.g., Sheth et al. 2001) and $B_n(m,z)$ is the $N$th-order bias coefficient of halos. We calculate both factors following the methods described in detail in Nishimichi et al. (2007) but with $\sigma_8 = 0.8$ so that we can compare the calculated bias with our bias measurements (made assuming $\sigma_8 = 0.8$). We adopt the fit parameters of Zehavi et al. (2005) determined for $M_r < -19.5$ (log $M_{\text{min}} = 11.76$, log $M_1 = 13.15$, and $\alpha = 1.13$) and $M_r < -20.5$ (log $M_{\text{min}} = 12.30$, log $M_1 = 13.67$, and $\alpha = 1.21$) for our HOD models, which we
use to calculate $b_1$ and $c_2$ at the median redshifts of the different subsamples.

In Figure 19 we display our calculated and measured $c_2$ and $b_1$, where the gray open circles are the calculated values, solid (all), dotted (early-type), and dashed (late-type) error bars denote all of the measured values, and the three lines represent $c_2 = b_1 - 1.4$, $c_2 = b_1 - 1.6$, and $c_2 = b_1 - 1.8$. Using the HOD fits of Zehavi et al. (2005) does not predict a strong relationship between $c_2$ and $b_1$. Changing the redshift weakly affects the calculated bias parameters, as the results show a significant change only when the HOD parameters are changed (the three calculations for $M_r < -19.5$ are hardly distinguishable, as are the three calculations for $M_r < -20.5$). When these results are compared to each of our measured $c_2$ values for all galaxy types, the calculated values are consistent, although they are all greater than the measured values. (The agreement is stronger if one considers that changing $\sigma_8$ will shift the values around.) Both the late-type and all galaxies appear consistent with $c_2$ being linearly dependent on $b_1$, but the early-type galaxies’ $c_2$ measurements show no clear dependence on $b_1$. Our basic modeling suggests that there is nothing unusual about the relationship between $c_2$ and $b_1$ for early-type galaxies. It further suggests that for each galaxy type there is redshift evolution of the HOD for a given luminosity.

### 7.3. Late-Type Galaxies

The measurements made for late-type galaxies are significantly different for $z < 0.3$ than for $z > 0.3$ (e.g., Fig. 13). The rise in correlation amplitudes at small scales, first reported by R06, happens only for $z > 0.3$. This suggests that late-type galaxies become much more likely to exist in close groupings at redshifts greater than 0.3. We are essentially measuring a preponderance of star-forming galaxies in tight configurations as the redshift grows larger than 0.3. This in turn suggests that merger-driven star formation becomes common at $z > 0.3$. This hypothesis is further supported by the fact that the bluest galaxies ($L_2$) at $z > 0.3$ display the largest amplitudes at small scales (see Fig. 14), meaning that the galaxies with the most star formation are most likely to be found in tight groupings.

This picture is broadly consistent with the concept of downsizing (e.g., Cowie et al. 1996), which essentially states that higher mass galaxies form stars earlier and more quickly than lower mass galaxies. The galaxies included in the volume-limited samples are relatively bright ($M_r < -19.5$), and thus we find evidence of a

### Table 1

| No. | $M_r$ Range | $z$ Range | $h^{-1}$ Mpc Range | $b_1$ | $\chi^2$/dof | $c_2$ | $\chi^2$/dof |
|-----|-------------|-----------|-------------------|-------|-------------|-------|-------------|
| All |
| 1.   | $<-19.5$    | $<0.3$    | 8–36              | 1.06 ± 0.01 | 1.10 | $-0.45 ± 0.13$ | 0.44 |
| 2.   | $<-19.5$    | $<0.3$    | 8–36              | 0.97 ± 0.01 | 0.36 | $-0.58 ± 0.20$ | 0.29 |
| 3.   | $<-19.5$    | $<0.3$    | 8–36              | 1.18 ± 0.01 | 0.76 | $-0.38 ± 0.14$ | 0.16 |
| 4.   | $<-19.5$    | $<0.3$    | 8–36              | 1.03 ± 0.01 | 0.24 | $-0.36 ± 0.19$ | 0.52 |
| 5.   | $<-19.5$    | 0.2–0.3   | 8.5–38            | 1.03 ± 0.02 | 1.13 | $-0.32 ± 0.15$ | 0.29 |
| Early |
| 6.   | $<-19.5$    | $<0.3$    | 8–36              | 1.33 ± 0.03 | 0.31 | $0.06 ± 0.12$ | 2.40 |
| 7.   | $<-19.5$    | $<0.3$    | 8–36              | 1.21 ± 0.02 | 0.51 | $0.15 ± 0.15$ | 3.94 |
| 8.   | $<-19.5$    | $<0.3$    | 8–24              | 1.50 ± 0.03 | 0.17 | $0.08 ± 0.13$ | 0.34 |
| 9.   | $<-19.5$    | $<0.2$    | 8–24              | 1.27 ± 0.03 | 0.43 | $0.10 ± 0.08$ | 1.23 |
| 10.  | $<-19.5$    | 0.2–0.3   | 8.5–38            | 1.27 ± 0.03 | 0.18 | $0.05 ± 0.12$ | 1.21 |
| Late |
| 11.  | $<-19.5$    | $<0.3$    | 8–36              | 0.87 ± 0.01 | 0.98 | $-0.93 ± 0.34$ | 0.80 |
| 12.  | $<-19.5$    | $<0.3$    | 8–36              | 0.84 ± 0.01 | 0.88 | $-0.98 ± 0.27$ | 0.46 |
| 13.  | $<-19.5$    | $<0.3$    | 8–36              | 1.04 ± 0.02 | 1.14 | $-0.84 ± 0.25$ | 0.04 |
| 14.  | $<-19.5$    | $<0.2$    | 8–24              | 0.79 ± 0.02 | 0.10 | $-1.03 ± 0.35$ | 0.27 |
| 15.  | $<-19.5$    | 0.2–0.3   | 8.5–38            | 0.84 ± 0.01 | 1.90 | $-1.03 ± 0.35$ | 0.20 |

Fig. 17.—Ratio of $\omega_{c2}$ to $\omega_{c6}$, where $\omega_{c6}$ is calculated by correcting overdensities for a second-order bias term ($b_2$) equal to $-0.3$ and $c_0$ is calculated in the standard way. Both are calculated using the $z_3$ sample of galaxies with $-20.5 < M_r < -19.5$. The ratio begins to grow significantly greater than 1 for $\theta < 0.66^\circ$, corresponding to a physical scale of approximately $8 h^{-1}$ Mpc. This corresponds to the lower bound of the weakly nonlinear regime. All bias values are thus calculated for $r > 8 h^{-1}$ Mpc.
The Measured Values of Bias Parameters $b_1$ and $c_2$ for All Measurements Made Using the Nine $24 \times 24$ Volume-Limited Samples

| No. | $M_r$ Range | $z$ Range | $h^{-1}$ Mpc Range | $b_1$ | $\chi^2$/dof | $c_2$ | $\chi^2$/dof |
|-----|-------------|-----------|---------------------|-------|-------------|-------|-------------|
| (1) | (2)         | (3)       | (4)                 | (5)   | (6)         | (7)   | (8)         |
| All |             |           |                     |       |             |       |             |
| 1.  | $<-20.5$    | $<0.4$    | 10–45               | 1.35 ± 0.01 | 1.73 | $-0.29 \pm 0.17$ | 0.89 |
| 2.  | $<-20.5$    | $<0.3$    | 8–36                | 1.22 ± 0.03 | 1.36 | $-0.34 \pm 0.21$ | 0.20 |
| 3.  | $<-20.5$    | 0.3–0.4   | 8–51                | 1.39 ± 0.01 | 2.81 | $-0.17 \pm 0.10$ | 0.46 |
| Early |             |           |                     |       |             |       |             |
| 4.  | $<-20.5$    | $<0.4$    | 10–45               | 1.64 ± 0.03 | 0.31 | 0.01 ± 0.14 | 1.77 |
| 5.  | $<-20.5$    | $<0.3$    | 8–36                | 1.53 ± 0.03 | 0.16 | 0.08 ± 0.12 | 0.70 |
| 6.  | $<-20.5$    | 0.3–0.4   | 8–51                | 1.66 ± 0.02 | 1.12 | 0.01 ± 0.08 | 0.88 |
| Late |             |           |                     |       |             |       |             |
| 7.  | $<-20.5$    | $<0.4$    | 10–45               | 1.23 ± 0.02 | 2.20 | $-0.52 \pm 0.27$ | 0.44 |
| 8.  | $<-20.5$    | $<0.3$    | 8–36                | 1.04 ± 0.02 | 1.7 | $-0.83 \pm 0.21$ | 0.03 |
| 9.  | $<-20.5$    | 0.3–0.4   | 8–51                | 1.25 ± 0.02 | 3.66 | $-0.38 \pm 0.30$ | 0.37 |

The preponderance of merger-driven star formation only for $z > 0.3$. Our interpretation is supported by the results of Heavens et al. (2004), who found that star formation peaked between a redshift of 0.3 and 0.8, using the fossil records of local SDSS galaxies. Furthermore, they found the star formation rate to be a strong function of galaxy mass, implying that for $L > L^*$ there is little star formation at low redshift (this is supported by the dearth of late-type galaxies in the volume-limited samples at $z < 0.2$ in our current analysis). This interpretation is further supported by recent results from the Cosmic Evolution Survey (Scoville et al. 2007), which found dramatically smaller star formation rates for galaxies $0.20 < z < 0.43$ than for galaxies $0.43 < z < 0.65$.

The implications of $c_{2,\text{late}}$ being significantly smaller than that of early-type galaxies can be explained physically (see Tables 1 and 2). As the overdensity of dark matter increases, the overdensity of late-type galaxies becomes a smaller percentage of the dark matter overdensity. This is not a surprise, as the well-known morphology-density relationship (Dressler 1980) tells us that the centers of clusters (i.e., the most overdense regions) are filled with a smaller percentage of late-type galaxies than the outskirts of the clusters. It naturally follows that late-type galaxies will have a smaller value of $c_2$ than early-type galaxies. In terms of a halo occupation distribution (HOD), one would expect the fraction of late-type galaxies to decrease with the mass of the host dark matter halo, which is the general trend recently determined by Zehavi et al. (2005).
For $z > 0.3$, $c_{2, \text{late}}$ is much closer to $c_{2, \text{early}}$ than any of the measurements made at lower redshift (see Tables 1 and 2). This suggests that at higher redshift, the fraction of red galaxies as a function of density should display a shallower slope than at low redshift. This is observed by Yee et al. (2005); as for galaxies with $M_r < -19.5$, the slope in this relationship is on average smaller for galaxies with $0.4 < z < 0.6$ than for galaxies with $0.2 < z < 0.4$. While these redshift ranges are different from those we employ, it confirms that the fraction of red galaxies versus density relationship shows a decrease in slope as the redshift increases (for $M_r < -19.5$). This is in line with the results of Dressler et al. (1997), who found that at redshifts $\sim 0.5$ the fraction of spiral galaxies in clusters is $2-3$ times larger than in local clusters and that the spirals at higher redshift essentially replace the SO fraction. It is thus likely that the increase in $c_{2, \text{late}}$ at $z > 0.3$ is due to cluster spirals that have yet to evolve into SO galaxies. At smaller redshifts, the spirals have likely evolved into S0 galaxies. Galaxies that we classify as late-type are therefore unlikely to be found in dense environments at low redshift, and thus $c_{2, \text{late}}$ is significantly smaller for $z < 0.3$ than for $z > 0.3$.

8. CONCLUSIONS

The results presented in this paper represent the most complete and accurate determination of the Nth-order correlations of photometrically selected galaxies. The measurements and the theoretical modeling used to interpret the measurements represent a significant improvement over the R06 measurements. Taking all SDSS galaxies with $18 < r < 21$ and measuring $\omega_N$ produces extremely interesting results, but it is only through volume-limiting the sample and splitting by type, redshift, and luminosity that we are able to analyze the subtle effects that produce the measurements displayed for all galaxies. In doing so, we are able to quantify the nature of linear and nonlinear clustering and its dependence on type, redshift, and luminosity.

We find that the linear bias parameter $b_1$ is smaller for late-type galaxies than for early-type galaxies, a result that is robust against changes in redshift and luminosity, but the ratio of $b_1, \text{early}$ to $b_1, \text{late}$ does vary between 1.2 and 1.5 depending on the specific redshift/luminosity bin. We confirm that $b_1$ increases proportional to luminosity, as found in many previous studies. Significant evolution appears to occur in galaxies between a redshift of 0.3 and 0.4 as there is a large increase in $b_1$ going between galaxies with $z < 0.3$ and $0.3 < z < 0.4$ but no significant change in $b_1$ between $z = 0.2$ and 0.3.

The second-order bias parameter, characterized by $c_2$, is significantly smaller for late- than for early-type galaxies, and this is robust against any changes in the luminosity, redshift, or $\sigma_8$. This relationship between the nonlinear bias of early- and late-type galaxies can be seen as a rigorous statistical restatement of the density-morphology relationship and agrees with the results of the HOD analysis by Zehavi et al. (2005). By applying a basic HOD model, we find our measured results are in fair agreement with the HOD parameters determined by Zehavi et al. (2005).

There are large differences in the correlation measurements of late-type galaxies at redshifts greater and less than 0.3. This is broadly consistent with cosmic downsizing (Cowie et al. 1996). These differences predict a great amount of merger-driven star formation at $z > 0.3$ and are consistent with the observed evolution in the density/morphology relationship with redshift. Our results suggest that a detailed study of the density/morphology relationship as a function of redshift would find significant evolution at $z \sim 0.3$.

If we require that bias be linear and set $c_{2, \text{all}} = 0$, we find $\sigma_8 = 0.64^{+0.04}_{-0.03}$ consistent with the lower limit on WMAP3 measurements constrained by inflationary models. If instead we set $c_{2, \text{all}}$ equal to the $\sigma_8$ independent value found by Gaztañaga et al. (2005), we find that $\sigma_8$ is a great match to the WMAP3-SDSS combined measurement. Considering all of the results, the most likely conclusion is that $c_2$ is at least slightly negative and that $\sigma_8 < 0.8$.

As usual, the results of our study demand more investigation. To this end, we are currently working to extend the analyses presented herein in two complementary directions. First, as was discussed in § 7.1, the value of $c_2$ can be constrained further by correcting the $\omega_N$ measurements for the assumed $c_2$ and measured $b_1/\sigma_8$. Thus, by performing this analysis on the large, homogeneous, photometric Luminous Red Galaxy sample from the SDSS and applying the results to our current analysis, we are thereby improving our measurements of higher order bias terms and their dependence on galaxy type, redshift, and luminosity. Second, we are also improving our theoretical interpretation of these results by performing a more rigorous halo occupation distribution model analysis of our higher order correlation function measurements.

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