Calculation of Thermal Stress in Elastohydrodynamic Lubrication of Line Contact

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Abstract—A numerical method is proposed to calculate the surface thermal stresses of solids in thermal elastohydrodynamic lubrication of line contact. According to the method, the material line thermal expansion coefficients of lubricated contact surface are calculated after obtaining the pressure and temperature that meet the convergence conditions, then the thermal stresses are calculated. The results show that the values of the circumferential thermal stress and the axial thermal stress are negative. Both the circumferential thermal stress and the axial thermal stress are compressive stresses. When the entrainment speed increases, the absolute values of the circumferential and axial thermal stresses become larger.

1. INTRODUCTION

Researchers pointed the line EHL contact occurs in, for example, rolling bearings, cam-tappet systems, gears, and flexible seals [1]. The theory of elastohydrodynamic lubrication (EHL) of line contacts has been widely studied by researchers at home and abroad. A method was presented to solve isothermal elastohydrodynamic lubrication of line contact problems [2]. An efficient numerical analysis was studied for solving contact zone temperatures [3]. A model of rolling line contact was built to calculated lubrication characteristics [4]. The adjoint error estimation techniques was used to estimate line contact film pressure and thickness [5]. Some surface roughness models were adopted for the analysis of the effects of roughness on film pressure and thickness in a steady state EHL line contact without considering the influences of oil film temperature rises [6-7]. The effects of slide-roll ratios on the performance of thermal EHL in line contact were studied by using the multilevel method [8]. A full numerical analysis was presented to calculate the thermal EHL of steel–steel line contact characteristics under reciprocating motion [9]. Further more, the thermal effect and entrainment speeds effect have been widely investigated [10]. The material parameter and the inlet temperature have been considered in the study of TEHL [11]. However, many researchers ignored the problem that temperature changes can produce thermal stresses. In this paper, the surface thermal stresses of solids were studied and the effect of entrainment speed on thermal stress was further analysed. The results can be used as a reference for other research on elastohydrodynamic lubrication.
2. EQUATIONS

2.1. Reynolds Equation
Considering the thermal effect, the generalized Reynolds equation for a Newtonian steady-state line contact can be written as [12]:

\[
\frac{\partial}{\partial x} \left[ \left( \rho K e \right) \frac{\partial p}{\partial x} \right] = 12u_e \frac{\partial}{\partial x} \left( \rho^* h \right)
\]  (1)

The viscosity equation and the density equation proposed by Roe lands and Dowson-Higginson are employed to calculate the values of film viscosity and density in the domain of the lubrication contact [13-14]. The load balance equation and the film energy equation are used to calculate the oil film pressure and temperature rise respectively [10].

2.2. Film Thickness Equation
Based on the previous oil film thickness equation, the effect of the thermal elastic deformation caused by film temperature rise is further considered, the film thickness equation for the line contact can be expressed by [15]:

\[
h(x) = h_0 + \frac{x^2}{2R} - \frac{2}{\pi E} \int_{-\infty}^{\infty} p(x') \ln(x-x') \, dx' - s(x)
\]  (2)

where, \( s \) is thermal elastic deformation, \( R = R_1R_2/(R_1 + R_2) \).

2.3. Thermal Stress Equation
According to the theory of thermoelasticity, using stress function method to analyze the thermodynamic characteristics of a long cylinder [15], thermal stresses in the cross section can be expressed as:

\[
\sigma_{ri} = 2\mu \frac{1 + \nu}{1 - \nu} \alpha_i \Delta T_i \sum_{s=1}^{E} \frac{R_i}{R_s} \frac{2g^2}{(g^2 + \beta_i^2)^2} \beta_i^{2s} J_1((\beta_i p R_i)^{1/2}) - J_1((\beta_i R_i))
\]  (3)

\[
\sigma_{ri} = 2\mu \frac{1 + \nu}{1 - \nu} \alpha_i \Delta T_i \sum_{s=1}^{E} \frac{R_i}{R_s} \frac{2g^2}{(g^2 + \beta_i^2)^2} \beta_i^{2s} J_1((\beta_i p R_i)^{1/2}) - J_1((\beta_i R_i))
\]  (4)

\[
\sigma_{ri} = 2\mu \frac{1 + \nu}{1 - \nu} \alpha_i \Delta T_i \sum_{s=1}^{E} \frac{R_i}{R_s} \frac{2g^2}{(g^2 + \beta_i^2)^2} \beta_i^{2s} J_1((\beta_i p R_i)^{1/2}) - J_1((\beta_i R_i))
\]  (5)

where, the material line thermal expansion coefficient is \( \alpha_i = p(1-2v)/(E_i \cdot \Delta T_i) \), \( i = 1 \) or 2. \( \sigma_{ri} \), \( \sigma_{r} \) and \( \sigma_{z} \) are radial, circumferential and axial thermal stress respectively. Other parameters mean the same as those in the reference [15]. The interaction between points is ignored in the calculation, and the thermal stresses are calculated point by point.

3. NUMERICAL METHOD
The main equations need to be written in dimensionless forms. Dimensionless quantities are used to be defined as: \( X = x / h_i \), \( Z = z / h_0 \), \( S = sR / h_i \), \( H = hR / h_i \), \( P = p / p_i \). The numerical solution is achieved by pressure-temperature-deformation iteration between the equations with their boundary conditions. The computational domain is \( x_m = -4h_i \) and \( x_{out} = 1.4h_i \). The numbers of nodes on the finest level are 129 in the x-direction. Five levels of grids are used to calculate the film pressure and thickness. Note that temperature field is calculated only on the finest level of grids. The calculation process of thermal stress is carried out in the iterative process of solving the thermal elastic deformation after obtaining the numerical solutions that satisfy both the pressure convergence and the temperature convergence. The thermal stress is used to calculate the thermal elastic deformation that will return and correct the film thickness in the computational domain. It should be emphasized that because the solid surfaces are
constrained by the non-uniform oil film pressure and temperature field, the line thermal expansion coefficient in the thermal stresses and thermoelastic deformation formulas cannot be calculated using the average line thermal expansion coefficient during free thermal expansion conditions, which requires the help of micromolecular mechanics theory. During the calculations, the line thermal expansion coefficient needs to be corrected to avoid large thermal deformations in the calculation area of low temperature and low pressure. The correction method is as follows: if the calculated line thermal expansion coefficient is larger than the average line thermal expansion coefficient, the calculated line thermal expansion coefficient is made to be equal to the average thermal expansion coefficient; in a region where the temperature change does not exceed 1K, the line thermal expansion coefficient is regarded as zero to calculate the thermal stresses and deformation.

4. RESULTS AND DISCUSSIONS

The values of parameters used in the calculation are: \( R_1=0.01 \text{ m}, \ R_2=0.1 \text{ m}, \ u_0=1.5 \text{ m/s}, \ k_0=0.14 \text{ W/(m·K)}, \ T_0=303 \text{ K}, \ \rho_{1,2}=7850 \text{ kg/m}^3, \ E=2.16\times10^{11} \text{ Pa}, \ c=2000 \text{ J/(kg·K)}, \ \eta_0=0.05 \text{ Pa·s}, \ c_{1,2}=470 \text{ J/(kg·K)}, \ \rho_0=970 \text{ kg/m}^3, \ w=1.8\times10^5 \text{ N/m}.

4.1. Results

Figure 1 shows the calculated results obtained by using the numerical method. The dimensionless film pressure shows a secondary peak at the outlet where a necking of the dimensionless oil film thickness appears. When the film thickness is minimal, the temperature rise (\( \Delta T_1 \) and \( \Delta T_2 \)) on both solid surfaces are the largest. The values of \( \Delta T_1 \) and \( \Delta T_2 \) are very similar, and the maximum values are about 22 K. The material line thermal expansion coefficients change obviously and the maximum values of \( \alpha_1 \) and \( \alpha_2 \) are \( 4.17\times10^{-7} \text{ K}^{-1} \) and \( 4.28\times10^{-7} \text{ K}^{-1} \). It can be seen in the figure that the change curves of the thermal expansion coefficients are similar to the change curves of the temperature rises. The circumferential thermal stress and axial thermal stress have similar trends and they are negative. That is to say, both the circumferential thermal stress and the axial thermal stress are compressive stresses under the constraints of the external oil film pressure. It coincides with other common sense of mechanics. Whether it is circumferential or axial stress, the absolute value of the surface thermal stress of solid 2 is much larger than that of solid 1. The minimum values of \( \sigma_{1c} \) and \( \sigma_{2c} \) are \( -2.36\times10^{-2} \text{ MPa} \) and \( -1.97\times10^{-1} \text{ MPa} \). The minimum value of circumferential thermal stress \( \sigma_{2c} \) is 8 times more than that of \( \sigma_{1c} \). The minimum values of \( \sigma_{1z} \) and \( \sigma_{2z} \) are \( -2.38\times10^{-2} \text{ MPa} \) and \( -2.22\times10^{-1} \text{ MPa} \). The minimum value of axial thermal stress \( \sigma_{2z} \) is nearly 9 times more than that of \( \sigma_{1z} \). It is clear that solids with a larger radius are subject to relatively larger thermal stresses, and the axial thermal stress of any solid will have a greater effect on the solids.
(b) Film thickness

Temperature rise, $\Delta T$ (K)

(c) Film temperature rise

Line thermal expansion coefficients, $\alpha$ (1/K)

(d) Line thermal expansion coefficient
4.2. Effect of Entrainment Speed
As can be seen from Figure 2, the circumferential thermal stress on the surface of solid 1 and the axial thermal stress on the surface of solid 2 change significantly at different entrainment speeds. The variation trend of axial thermal stress is the same as that of the circumferential thermal stress. The minimum values of two kind thermal stress appear at the same position of the curves. When the entrainment speed increases, the absolute values of the circumferential and axial thermal stresses become larger. The reason is that the temperature rise of oil film decreases with the increase of speed, but the pressure of oil film almost does not change, which leads to the increase of line thermal expansion coefficient and makes the thermal stress become larger. Because the thermal stresses are affected by many factors, such as temperature, expansion coefficient, pressure and so on, the nonlinearity of thermal stresses become more obvious. However, in previous research conclusions, increasing speed is beneficial for lubrication, but now it seems that increasing speed is also harmful to solids in lubrication contact.
5. CONCLUSIONS
Based on thermal elastohydrodynamic lubrication theory of line contact, a method is proposed to calculate the thermal stresses on the surface of solids. By some examples, the convergence results of lubrication characteristics and thermal stresses are obtained. The circumferential thermal stress and the axial thermal stress have the same variation tendency in the computational domain. Both the circumferential thermal stress and the axial thermal stress are compressive stresses. The values of axial thermal stress at each point are greater than those of circumferential thermal stress on the same solid. Increasing the entrainment speed can make the absolute values of the circumferential and axial thermal stresses become larger.

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