NEUTRON STAR PROPERTIES WITH NUCLEAR
BHF EQUATIONS OF STATE

M. Baldo\textsuperscript{a}, G. F. Burgio\textsuperscript{a}, H. Q. Song\textsuperscript{b} and F. Weber\textsuperscript{c}

\textsuperscript{a} I.N.F.N. Sezione di Catania, c.so Italia 57, I-95129 Catania, Italy
\textsuperscript{b} Institute of Nuclear Research, Academia Sinica, Shanghai 201800, China
\textsuperscript{c} Sektion Physik, Universität München, Am Coulombwall 1, D-85748 Garching, Germany

Abstract

We study the properties of static and rotating neutron stars adopting non-relativistic equations of state (EOS) for asymmetric nuclear matter based on the Brueckner-Hartree-Fock (BHF) scheme. The BHF calculation, with the continuous choice for the single particle potential, appears to be very close to the full EOS, which includes the three-hole line contribution calculated by solving the Bethe-Faddeev equations within the gap choice for the single particle potential. Three-body forces are included in order to reproduce the correct saturation point for nuclear matter. A comparison with fully relativistic many-body calculations of nuclear matter EOS is made.

1 Introduction

The properties of neutron stars such as masses and radii depend on the equation of state (EOS) at densities up to an order of magnitude higher than those observed in ordinary nuclei. Therefore the knowledge of the EOS in the superdense regime is fundamental for the study of astrophysical compact objects \cite{1}. For this purpose we derive an equation of state for asymmetric nuclear matter, using a non-relativistic many-body theory within the framework of the Brueckner-Hartree-Fock (BHF) scheme \cite{2, 3}. In this approach, the basic input is the two-body nucleon-nucleon (NN) interaction. The BHF approximation, with the continuous choice for the single particle potential, reproduces closely the many-body calculations up to three hole-line level \cite{4}. However, as it is well known, the empirical saturation point is not reproduced. Therefore we have included a contribution coming from three-body forces to reproduce the correct saturation point \cite{5}. Those EOS’s are the fundamental input for constructing models of static and rotating neutron stars in the framework of
Einstein’s theory of general relativity by applying a refined version of Hartle’s stellar structure equations \([6, 7]\). We calculate properties of neutron stars like gravitational mass, equatorial and polar radius, for sequences of star models either static or rotating at their respective general relativistic Kepler frequencies. We compare with predictions from fully relativistic microscopic EOS.

## 2 Equation of state

Microscopic calculations of nuclear matter EOS have been performed in the framework of the Brueckner–Hartree–Fock (BHF) scheme \([3]\). The energy per particle \(E/A\) within the BHF scheme is given in terms of the so-called reaction matrix \(G\). The latter is obtained by solving the Brueckner–Bethe–Goldstone (BBG) equation

\[
G(\omega) = V + \frac{Q}{\omega - H_0} G(\omega), \tag{1}
\]

where \(\omega\) is the unperturbed energy of the interacting nucleons, \(V\) is the free nucleon-nucleon (NN) interaction, \(H_0\) is the unperturbed energy of the intermediate scattering states and \(Q\) is the Pauli operator which prevents scattering into occupied states. With the \(G\)-matrix we can calculate the total energy per nucleon

\[
\frac{E}{A} = \frac{3}{5} \frac{h^2 k_F^2}{2m} + U(n), \tag{2}
\]

being \(U(n)\) the contribution of the potential energy to the total energy per particle

\[
U(n) = \frac{1}{2A} \sum_{k,k' \leq k_F} \langle kk' | G(\omega = \epsilon(k) + \epsilon(k')) | kk' \rangle_a \tag{3}
\]

where the subscript \(a\) indicates antisymmetrization of the matrix element. The single-particle energies are denoted by \(\epsilon\). In this scheme, the only input quantity we need is the bare NN interaction \(V\) in the Bethe-Goldstone equation (1).

The Brueckner-Hartree-Fock (BHF) approximation for the EOS in symmetric nuclear matter, within the continuous choice \([3]\), reproduces closely results which include up to three hole-line diagram contributions to the BBG expansion of the energy, calculated within the so called gap choice for the single particle potential \([4]\). In Fig.1 we show the energy per nucleon calculated within this scheme in the case of symmetric and neutron matter using
the Argonne $v_{14}$ model [8] for the two-body nuclear force. The open squares represent the solution of the Bethe-Faddeev equations for the three hole-line in the gap choice. These results extend to higher densities the calculations published in ref.[4] for symmetric nuclear matter. Recently it has been shown that the results up to three hole-lines are independent from the choice of the single particle potential [9], which gives evidence of convergence of the BBG expansion.

For neutron matter the results are preliminary. In any case we found a negligible difference between the BHF results in the continuous and in the gap choice. Correspondingly the three hole-line contribution in neutron matter turns out to be much smaller than in symmetric nuclear matter. All these results together give support to the use of the BHF approximation in the continuous choice in the study of neutron stars.

We notice that the BHF fails to reproduce the empirical saturation point of nuclear matter. This well known deficiency, which does not depend on the choice of the two-body force, is commonly corrected introducing three-body forces (TBF). We adopted the Urbana three-nucleon model [10], which consists of an attractive term due to two–pion exchange with excitation of an intermediate $\Delta$-resonance, and a repulsive phenomenological central term. Several details are given in ref.[5].

The corresponding EOS obtained using the $v_{14}$ potential is depicted in Fig.2a) for symmetric and neutron matter (solid line). This EOS saturates at $n_0 = 0.178 \, fm^{-3}$, $E_0/A = -16.46 \, MeV$ and it is characterized by an incompressibility $K_\infty = 253 \, MeV$. In Fig.2a) we plot also the EOS from a recent Dirac-Brueckner calculation (DBHF) [14] with the Bonn–A two–body force (dashed line). In the low density region the BHF equation of state with TBF and DBHF equation of state are very similar, whereas at higher density the DBHF is stiffer. The discrepancy between the non-relativistic and relativistic calculation of the EOS can be easily understood by noticing that the DBHF treatment is equivalent [12] to introduce in the non-relativistic BHF the three-body force corresponding to the excitation of a nucleon-antinucleon pair, the so-called Z-diagram which is repulsive at all densities. In BHF treatment, on the contrary, both attractive and repulsive three-body forces are introduced, and therefore a softer EOS is expected.

The properties of neutron stars (NS) depend on the knowledge of the EOS over a wide range of densities, i.e. from the density of iron at the star’s surface up to several times the density of normal nuclear matter encountered in the core [1]. It is commonly accepted that the interior part of a neutron star is made mainly by nuclear matter (eventually superfluid) with a certain
lepton fraction, although the high-density core might suffer a transition to other hadronic components. Here we assume that a neutron star is composed only by nucleons and leptons, \textit{i.e.} an uncharged mixture of neutrons, protons, electrons and muons in $\beta$-equilibrium. The presence of leptons softens the EOS with respect to the pure neutron matter case. In Fig. 2b) the EOS for $\beta$-stable matter is shown for the cases previously discussed. It has to be stressed that in $\beta$-stable matter the density dependence of the nuclear symmetry energy affects the proton concentration \cite{13}. As it has been recently pointed out by Lattimer et al. \cite{14}, the value of the proton fraction in the core of NS is crucial for the onset of direct Urca processes, whose occurrence enhances neutron star cooling rates.

From the energy per baryon of asymmetric nuclear matter in $\beta$-equilibrium, we calculate the nuclear contribution $P_{\text{nucl}}$ to the total pressure of stellar matter as well as the mass density $\rho_{\text{nucl}}$. Then the total pressure and total mass density can be easily calculated by adding the leptonic contributions. For more details, see ref.\cite{5}.

\section{Neutron star structure}

Neutron stars are objects of highly compressed matter so that the geometry of space-time is changed considerably from flat space. Thus Einstein’s general theory of relativity must be applied. Therein the Einstein curvature tensor $G_{\mu\nu}$ is coupled to the energy-momentum density tensor $T_{\mu\nu}$ of matter (G denotes the gravitational constant):

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}(\rho, P(\rho))$$

The knowledge of the EOS, \textit{i.e.} pressure $P$ as function of the energy density $\rho$ is therefore required in order to solve eq. (4). For a spherically symmetric and static star, Einstein’s equations reduce to the familiar Tolman–Oppenheimer–Volkoff (TOV)\cite{15} equations

\begin{align*}
\frac{dP(r)}{dr} &= -\frac{G m(r) \rho(r)}{r^2} \left(1 + \frac{P(r)}{c^2 \rho(r)} \right) \left(1 + \frac{4\pi r^3 P(r)}{c^2 m(r)} \right) \\
\frac{dm(r)}{dr} &= 4\pi r^2 \rho(r)
\end{align*}

For a given EOS \textit{i.e.} $P(\rho)$, one can solve the TOV equations by integrating them for a given central energy density $\rho$, from the star’s center to the
star’s radius defined by $P(R_s) = 0$. This gives the stellar radius $R$ and the gravitational mass is then

$$M_G \equiv m(R) = 4\pi \int_0^R dr \ r^2 \rho(r). \quad (7)$$

The case of rotating stars is more complicated, since changes occur in the pressure, energy density and baryon number density because of the rotation. In this work we adopt the method developed in ref.[6], which is a redefined version of Hartle’s perturbative method for the investigation of the general relativistic Kepler frequency of a rotating neutron star. We recall that for rotation at frequencies beyond the Kepler value $\Omega_K$, mass shedding at the equator sets in which makes the star unstable. Therefore $\Omega_K$ sets an absolute upper bound on the rotational frequency. For every nuclear EOS there are uniquely determined values of $\Omega_K$ for each star in the sequence up to the limiting mass value. In Fig.3 we show the results obtained with our BHF plus TBF equation of state for asymmetric nuclear matter and with the Dirac-Brueckner one. We display the gravitational mass $M_G$, in units of the solar mass $M_\odot$ ($M_\odot = 1.99 \times 10^{33}$ g), as a function of the radius $R$ and the central energy density $\rho$ (in units of $\rho_0 = 140 \text{ MeV/fm}^3$). The upper lying curves show the increase of mass due to rotation at the (absolute limiting) Kepler frequency, i.e. $\Omega = \Omega_K$. The solid (dashed) line indicates a sequence of star models obtained with the BHF+TBF (DBHF) equation of state. We observe larger gravitational masses, i.e. up to 14% mass increase for BHF+TBF and 15% for DBHF, relative to the spherical (non-rotating) Oppenheimer-Volkoff star model of the same $\rho$ value. The large mass increase obtained is accompanied by relatively large (equatorial) radius value. Moreover one sees that, for a fixed value of the gravitational mass, the central star density $\rho$ decreases for increasing values of the rotational frequency. This is because of the centrifugal force acting on the star’s matter together with the nuclear force. Their intensity must be counterbalanced by the attractive gravitational forces. More details on rotating stars will be given in a forthcoming paper [16].

4 Conclusions

In conclusion, we computed some properties of NS’s on the basis of a microscopic EOS obtained in the framework of BHF many–body theory with two plus three–body nuclear interactions. Our EOS with three-body forces is able to reproduce the correct saturation point of nuclear matter. The comparison with the DBHF method shows that the relativistic effects represent a particular repulsive three-body force and gives rise to a stiffer EOS. Therefore the
predicted values for the limiting mass and radius of neutron stars are higher in the DBHF case than in the non-relativistic BHF calculation. The inclusion of rotation substantially changes the limiting values of the gravitational mass by about 14-15 % for configurations rotating at their respective general relativistic Kepler frequency. A more detailed comparison between the predictions of non-relativistic BHF microscopic EOS and observational data on pulsars, i.e. fast rotation and large enough neutron star masses, is currently in progress.

References

[1] S. Shapiro and S. Teukolsky, ”Black Holes, White Dwarfs and Neutron Stars”, (John Wiley & Sons 1983) USA

[2] H. A. Bethe, Ann. Rev. Nucl. Sci. 21 (1971) 93.

[3] M. Baldo, I. Bombaci, L.S. Ferreira, G. Giansiracusa and U. Lombardo, Phys. Rev. C43 (1991) 2605 and references therein.

[4] H.Q. Song, M. Baldo, G. Giansiracusa and U. Lombardo, Phys. Lett.B411 (1997) 237; B.D. Day and R.B. Wiringa, Phys. Rev. C32 (1985) 1057.

[5] M. Baldo, I. Bombaci, and G.F. Burgio, Astron. and Astrophys. 328 (1997) 274.

[6] F. Weber and N.K. Glendenning, Phys. Lett. B265 (1991) 1; F. Weber and N.K. Glendenning, Astrophys. Journ. 390 (1992) 541.

[7] J.B. Hartle, Astrophys. Journ. 150 (1967) 1005; J.B. Hartle and K.S. Thorne, Astrophys. Journ. 153 (1968) 807.

[8] R.B. Wiringa, R.A. Smith and T.L. Ainsworth, Phys. Rev. C29 (1984) 1207.

[9] H.Q. Song, M. Baldo, G. Giansiracusa and U. Lombardo, Phys. Rev. Lett. (1998) submitted.

[10] J. Carlson, V.R. Pandharipande and R.B. Wiringa, Nucl. Phys. A401 (1983) 59.

[11] G.Q. Li, R. Machleidt and R. Brockmann, Phys. Rev. C45 (1992) 2782.

[12] M. Baldo, G. Giansiracusa, U. Lombardo, I. Bombaci and L.S. Ferreira, Nucl. Phys. A583 (1995) 599.
[13] I. Bombaci and U. Lombardo, Phys. Rev. C44 (1991) 1892.

[14] J. Lattimer, C. Pethick, M. Prakash and P. Haensel, Phys. Rev. Lett. 66 (1991) 2701.

[15] R.C. Tolman, 1934, Proc. Nat. Acad. Sci. USA 20 (1934) 3; J. Oppenheimer and G. Volkoff, Phys. Rev. 55 (1939) 374.

[16] M.Baldo et al., in preparation.
Figure captions

Fig.1: The energy per baryon $E/A$ is plotted vs. the number density $n$ for symmetric matter (lower curve) and for neutron matter (upper curve). The solid line represents a Brueckner calculation with Av14 potential with the continuous choice. The squares are the solutions of the Bethe-Fadeev equations with the gap choice.

Fig.2: The energy per baryon $E/A$ is plotted vs. the number density $n$ in panel (a) for symmetric matter (lower curves) and for neutron matter (upper curves). The solid line represents non-relativistic BHF calculations with three-body forces and the dashed line a relativistic Dirac-Brueckner one. Panel (b): as in panel (a) but for $\beta$-stable nuclear matter.

Fig.3: The gravitational mass $M_G$, expressed in units of the solar mass $M_\odot$, is displayed vs. radius $R$ (panel (a)) and the central energy density $\rho$ (in units of $\rho_0 = 140$ MeV/fm$^3$) (panel (b)) for sequences of star models constructed for BHF+TBF (solid line) and DBHF (dashed line) equations of state. The two (lower) upper lying curves refer to (non-) rotating star models.