BAYESIAN ANALYSIS OF EXPONENTIAL LOMAX DISTRIBUTION USING DIFFERENT LOSS FUNCTIONS

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ABSTRACT
The exponential Lomax distribution is an extension of the Lomax distribution proposed by El-Bassiouny et al. (2015). This distribution is very useful and has been found to outperform other extensions of the Lomax distribution such as the exponentiated Lomax, Marshall-Olkin extended-Lomax, beta-Lomax, Kumaraswamy-Lomax, McDonald-Lomax and gamma-Lomax based on some applications to lifetime datasets. In this article, the scale parameter of the exponential Lomax is estimated using the Bayesian method of estimation under two non-informative (Jeffery and Uniform prior) and Informative prior (Gamma prior) distribution and compared to the estimates of Maximum Likelihood using three loss functions (Square error, Quadratic, and Precautionary loss function). The posterior distributions of the said parameter were derived and also the Estimators and risks were also obtained using the priors and loss functions. Furthermore, a simulation study was carried out using R software package to assess the performance of the two methods by means of their MSEs.

Keywords: Exponential Lomax distribution; MLE; Bayesian Method; Uniform prior; Jeffrey's prior; Gamma prior; Square error, quadratic and precautionary loss functions; MSE; Sample sizes.

1. INTRODUCTION
The exponential Lomax distribution is an extension of the Lomax distribution proposed by El-Bassiouny et al. (2015). The Lomax or Pareto type II distribution was proposed by Lomax in (1954). This distribution has found wide applications such as the analysis of business failure life time data, income and wealth inequality, size of cities, actuarial...
Similarly, Aliyu and Yahaya (2016) studied the shape parameter of generalized Rayleigh distribution under non-informative priors with a comparison to the method of maximum likelihood. Besides, a good number of loss functions have been shown to be performing during estimation under Bayesian method in so many studies including Ahmad and Ahmad (2013), Ahmad et al. (2015), Ahmad et al. (2016), Ieren and Oguntunde (2018), Gupta and Singh (2017), Gupta (2017) and Ieren and Chukwu (2018) and many others. Since the approach of estimating a parameter differs from one parameter of a distribution to another, this study aims at estimating the scale parameter of the Exponential Lomax distribution using Bayesian approach and making a comparison between the Bayesian approach and the method of maximum likelihood estimation approach. The rest of this paper is presented in sections and sub-sections as follows:

2. MATERIALS AND METHODS
2.1 PDF and Likelihood function
The pdf of the Exponential Lomax distribution with unknown parameter vector is given as:

\[ f(x) = \frac{\alpha \omega}{\beta} \left( \frac{\beta}{x + \beta} \right)^{\alpha-1} e^{-\omega \left( \frac{\beta}{x + \beta} \right)} - \omega \left( \frac{\beta}{x + \beta} \right)^{-\sigma} \]

\[ x \geq \beta, \alpha, \beta, \omega > 0 \quad (1) \]

Where, \( x \geq \beta, \alpha > 0, \beta > 0, \omega > 0 \) and \( \beta \) are the shape parameters respectively and \( \omega \) is the scale parameter of the exponential distribution.

The total log-likelihood function is obtained from \( f(x) \) as follows:

\[ L(X | \alpha, \beta, \omega) = \left( \frac{\alpha \omega}{\beta} \right)^n \prod_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha+1} e^{-\omega \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)} \quad (2) \]

The likelihood function for the scale parameter, \( \omega \), is given by:

\[ L(X | \omega) \propto \omega^{n} e^{-\omega \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha}} \quad \text{(3)} \]

2.2 Bayesian Analysis under the Assumption of Uniform Prior Using Three Loss Functions
To obtain the posterior distribution \( P(\omega | x) \), the probability distribution of the parameter once the data has been observed, we apply Bayes' Theorem

\[ p(\omega | X) = \frac{p(\omega) L(x | \omega)}{g(x)} \quad (4) \]

Where \( g(x) \) is the marginal distribution of \( X \) and

\[ g(x) = \sum_{\omega} p(\omega) L(x | \omega) \]

\[ \int_{\omega} p(\omega) L(x | \omega) d\omega \]

where \( \sum_{\omega} p(\omega) L(x | \omega) \) when \( \omega \) is discrete and \( \int_{\omega} p(\omega) L(x | \omega) d\omega \) when \( \omega \) is continuous

where \( p(\omega) \) and \( L(x | \omega) \) are the prior distribution and the Likelihood function respectively.

The uniform prior is defined as:

\[ p(\alpha) \propto 1; \quad 0 < \alpha < \infty \]

The posterior distribution of the scale parameter \( \omega \) under uniform prior is obtained from equation (4) using integration by substitution method as
The Bayes estimators and posterior risks under uniform prior using $SELF$, $QLF$ and $PLF$ are given respectively as follows:

\[ \Omega_{SELF} = E(\Omega | \mathbf{X}) = \frac{n+1}{\sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha}} \] (6)

\[ P(\omega_{SELF}) = \frac{n+1}{\sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha}} \] (7)

\[ \Omega_{QLF} = \frac{n-1}{\sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha}} \] (8)

\[ P(\omega_{QLF}) = \frac{1}{n} \] (9)

\[ \Omega_{PLF} = \left\{ E(\Omega^2 | \mathbf{X}) \right\}^{\frac{1}{2}} = \frac{(n+2)(n+1)^{\frac{1}{2}}}{\sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha}} \] (10)

\[ P(\omega_{PLF}) = 2 \left\{ \frac{(n+2)(n+1)^{\frac{1}{2}}}{\sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha}} - (n+1) \right\} \] (11)

2.3 Bayesian Analysis under the Assumption of Jeffrey’s Prior Using Three Loss Functions

Also, the Jeffrey’s prior is defined as:

\[ p(\alpha) \propto \frac{1}{\alpha}; 0 < \alpha < \infty \] (12)

The posterior distribution of the scale parameter $\omega$ for a given data under Jeffrey prior is obtained from equation (4) using integration by substitution method as

\[ p(\omega | \mathbf{X}) = \frac{\omega^{n-1} \left( \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha} \right)^{\alpha} \times e^{-\omega \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-\alpha}}}{\Gamma(n)} \] (13)

The Bayes estimators and posterior risks under Jeffrey’s prior using $SELF$, $QLF$ and $PLF$ are given respectively as follows:
2.4 Bayesian Analysis under the Assumption of Gamma Prior Using Three Loss Functions

Also, the gamma prior is defined as:

\[
P(\omega) = \frac{a^b}{\Gamma(b)} \omega^{b-1} e^{-a\omega}; a, b, \omega > 0 \quad (20)
\]

The posterior distribution of the scale parameter \( \omega \) for a given data under gamma prior is obtained from equation (4) using integration by substitution method as

\[
P(\omega | X) = \frac{\left( a + \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right) \right) ^ {n+b} \omega ^ {a - \left( \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right) \right) ^ a} \omega ^ {b-1} e ^ {-a \left( \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right) \right) ^ a}} {\Gamma(n+b)}
\]

\[
(21)
\]

The Bayes estimators and posterior risks under gamma prior using SELF, QLF and PLF are given respectively as follows:

\[
\omega_{SELF} = E(\omega | X) = \left( \frac{n+b}{a + \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right) ^ {a-\alpha}} \right) \quad (22)
\]

\[
P(\omega_{SELF}) = \left( \frac{n+b}{a + \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right) ^ {a-\alpha}} \right) ^ {\frac{1}{2}} \quad (23)
\]
\( \omega_{QLF} = \frac{(n+b-2)}{a + \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a}} \) (24)

\[ P(\omega_{QLF}) = \frac{1}{(n+b-1)} \] (25)

\[ \omega_{PLF} = \left\{ E \left( \omega^2 | X \right) \right\}^{\frac{1}{2}} = \frac{[(n+b+1)(n+b)]^{\frac{1}{2}}}{a + \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a}} \] (26)

\[ P(\omega_{PLF}) = 2 \left\{ \frac{[(n+b+1)(n+b)]^{\frac{1}{2}} - (n+b)}{a + \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a}} \right\} \] (27)

### 2.4 Maximum Likelihood Estimation

Let \( x_1, x_2, \ldots, x_n \) be a random sample from a population \( X \) with probability density function \( f(x) \).

The likelihood function, \( L(X | \alpha, \beta, \omega) \), is defined to be the joint density of the random variables \( x_1, x_2, \ldots, x_n \). The pdf of the ELD is given as

\[ f(x) = \frac{\alpha \omega}{\beta} \left( \frac{\beta}{x + \beta} \right)^{-a} e^{-\omega \left( \frac{\beta}{x + \beta} \right)^{-a}} \]

\[ L(X | \alpha, \beta, \omega) = \left( \frac{\alpha \omega}{\beta} \right)^n \prod_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a} e^{-\omega \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a}} \] (28)

The likelihood function for the scale parameter, \( \omega \), is given by;

\[ L(X | \omega) \propto \omega^n e^{-\frac{\omega \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)}{a}} \] (29)

Let the log-likelihood function, \( l = \log L(X | \omega) \), therefore

\[ l = n \log \omega - \omega \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a} \] (30)

Differentiating \( l \) partially with respect to \( \omega \), the scale parameter and solving for \( \hat{\omega} \) gives;

\[ \frac{\partial l}{\partial \omega} = \frac{n}{\omega} - \sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a} = 0 \Rightarrow \hat{\omega} = \frac{n}{\sum_{i=1}^{n} \left( \frac{\beta}{x_i + \beta} \right)^{-a}} \] (31)

Which is the MLE of the scale parameter, \( \omega \).
3. RESULTS AND DISCUSSIONS

3.1 Simulation and Comparison

In this section, a package in R software is considered to generate random samples of sizes \( n = (10, 15, 20, 27, 35, 55, 95, 125) \) from Exponential Lomax distribution under the following combination of parameter values:

- \( \alpha = 1, \beta = 1 \) and \( \omega = 0.5 \),
- \( \alpha = 1, \beta = 3 \) and \( \omega = 0.5 \),
- \( \alpha = 3, \beta = 1 \) and \( \omega = 0.5, \omega = 0.5 \),
- \( \alpha = 1, \beta = 1 \) and \( \omega = 0.5, \omega = 0.5 \).

The following tables present the results of our simulation study by listing the estimates of the scale parameter with their respective Mean Squared Errors (MSEs) under the appropriate estimation methods such as the Maximum Likelihood Estimation (MLE), Squared Error Loss Function (SELF), Quadratic Loss Function (QLF), and Precautionary Loss Function (PLF) under Uniform Jeffrey and gamma priors respectively.

**Table 1**: Estimators their Estimates and Mean Squared Errors based on the replications and sample sizes where \( \alpha = 1, \beta = 1 \) and \( \omega = 0.5 \)

| n  | Measures | MLE | Uniform Prior | Jeffrey's Prior | Gamma Prior |
|----|----------|-----|---------------|-----------------|-------------|
|    |          | SELF| QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF |
| 10 | Estimate | 0.556 | 0.6116 | 0.5004 | 0.6388 | 0.556 | 0.4448 | 0.5831 | 0.5548 | 0.4694 | 0.5757 |
|    | MSE      | 0.0411 | 0.0584 | 0.0308 | 0.0694 | 0.0411 | 0.0274 | 0.0487 | 0.0234 | 0.0155 | 0.0277 |
| 15 | Estimate | 0.5365 | 0.5723 | 0.5008 | 0.5899 | 0.5365 | 0.465 | 0.5541 | 0.5365 | 0.465 | 0.5541 |
|    | MSE      | 0.0228 | 0.0296 | 0.0187 | 0.034 | 0.0228 | 0.0173 | 0.0258 | 0.0165 | 0.0121 | 0.0188 |
| 20 | Estimate | 0.5257 | 0.552 | 0.4994 | 0.565 | 0.5257 | 0.4731 | 0.5387 | 0.5314 | 0.4852 | 0.5428 |
|    | MSE      | 0.016 | 0.0196 | 0.0138 | 0.0219 | 0.016 | 0.0131 | 0.0176 | 0.0127 | 0.01 | 0.0141 |
| 27 | Estimate | 0.5194 | 0.5386 | 0.5002 | 0.5482 | 0.5194 | 0.4809 | 0.5289 | 0.5248 | 0.4898 | 0.5335 |
|    | MSE      | 0.0113 | 0.0133 | 0.0102 | 0.0145 | 0.0113 | 0.0098 | 0.0122 | 0.0097 | 0.0081 | 0.0105 |
| 35 | Estimate | 0.5149 | 0.5296 | 0.5002 | 0.537 | 0.5149 | 0.4855 | 0.5222 | 0.5198 | 0.4924 | 0.5266 |
|    | MSE      | 0.0082 | 0.0093 | 0.0075 | 0.01 | 0.0082 | 0.0073 | 0.0087 | 0.0073 | 0.0063 | 0.0078 |
| 55 | Estimate | 0.5098 | 0.5191 | 0.5005 | 0.5237 | 0.5098 | 0.4913 | 0.5144 | 0.5134 | 0.4957 | 0.5178 |
|    | MSE      | 0.005 | 0.0054 | 0.0047 | 0.0057 | 0.005 | 0.0046 | 0.0052 | 0.0047 | 0.0042 | 0.0049 |
| 95 | Estimate | 0.5047 | 0.51 | 0.4994 | 0.5126 | 0.5047 | 0.494 | 0.5073 | 0.507 | 0.4967 | 0.5096 |
|    | MSE      | 0.0027 | 0.0029 | 0.0027 | 0.003 | 0.0027 | 0.0026 | 0.0028 | 0.0026 | 0.0025 | 0.0027 |
| 12 | Estimate | 0.5042 | 0.5082 | 0.5002 | 0.5102 | 0.5042 | 0.4961 | 0.5062 | 0.506 | 0.4981 | 0.508 |
|    | MSE      | 0.0021 | 0.0022 | 0.002 | 0.0022 | 0.0021 | 0.002 | 0.0021 | 0.002 | 0.0019 | 0.0021 |

The results in table 1 show that the estimator of the scale parameter using QLF under Gamma is better than the other estimators (uniform and Jeffrey prior and MLE) with small MSE irrespective of the variation in the samples. This behavior of minimum MSE for Bayesian estimation (using QLF under Uniform, Jeffrey and gamma priors) is an indication that the method for this parameter is better than the Method of Maximum Likelihood estimation (MLE) for the chosen parameter values irrespective of small, medium or large sample sizes.
| Table 2: Estimators their Estimates and Mean Squared Errors based on the replications and sample sizes where \( \alpha = 1, \beta = 3 \) and \( \omega = 0.5 \). |
|---|---|---|---|---|---|---|---|
| \( n \) | Measures | Uniform Prior | Jeffrey's Prior | Gamma Prior |
| | | SELF | QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF |
| 10 | Estimate | 0.5508 | 0.6058 | 0.4957 | 0.6328 | 0.5508 | 0.4406 | 0.5776 | 0.5507 | 0.466 |
| | MSE | 0.04 | 0.0565 | 0.0304 | 0.0671 | 0.04 | 0.0275 | 0.0472 | 0.0227 | 0.0155 |
| 15 | Estimate | 0.5351 | 0.5707 | 0.4994 | 0.5883 | 0.5351 | 0.4637 | 0.5526 | 0.5399 | 0.4799 |
| | MSE | 0.0227 | 0.0294 | 0.0187 | 0.0337 | 0.0227 | 0.0174 | 0.0257 | 0.0165 | 0.0122 |
| 20 | Estimate | 0.524 | 0.5502 | 0.4978 | 0.5632 | 0.524 | 0.4716 | 0.537 | 0.5299 | 0.4839 |
| | MSE | 0.0154 | 0.0189 | 0.0134 | 0.0211 | 0.0154 | 0.0128 | 0.017 | 0.0124 | 0.0098 |
| 27 | Estimate | 0.52 | 0.5392 | 0.5007 | 0.5488 | 0.52 | 0.4815 | 0.5295 | 0.5253 | 0.4903 |
| | MSE | 0.0116 | 0.0136 | 0.0104 | 0.0149 | 0.0116 | 0.01 | 0.0125 | 0.01 | 0.0082 |
| 35 | Estimate | 0.5137 | 0.5284 | 0.499 | 0.5357 | 0.5137 | 0.4844 | 0.521 | 0.5186 | 0.4913 |
| | MSE | 0.0084 | 0.0095 | 0.0077 | 0.0102 | 0.0084 | 0.0075 | 0.0089 | 0.0075 | 0.0065 |
| 55 | Estimate | 0.5089 | 0.5181 | 0.4996 | 0.5227 | 0.5089 | 0.4904 | 0.5135 | 0.5125 | 0.4948 |
| | MSE | 0.005 | 0.0054 | 0.0047 | 0.0057 | 0.005 | 0.0047 | 0.0052 | 0.0047 | 0.0042 |
| 95 | Estimate | 0.5051 | 0.5104 | 0.4998 | 0.5131 | 0.5051 | 0.4945 | 0.5078 | 0.5074 | 0.4971 |
| | MSE | 0.0028 | 0.003 | 0.0027 | 0.0031 | 0.0028 | 0.0027 | 0.0029 | 0.0027 | 0.0026 |
| 12 | Estimate | 0.504 | 0.5081 | 0.5 | 0.5101 | 0.504 | 0.496 | 0.506 | 0.5058 | 0.4979 |
| | MSE | 0.002 | 0.0021 | 0.002 | 0.0022 | 0.002 | 0.0021 | 0.002 | 0.0019 | 0.002 |

Table 2 also gives a similar pattern of the result found in table 1 with lower values of MSEs for the estimators using PLF under Uniform, Jeffrey and gamma priors. This result indicates that QLF under gamma prior produces the best estimator more than the QLF under Uniform and Jeffrey priors and these effects are found to be continuous despite the different sample sizes used.

| Table 3: Estimators their Estimates and Mean Squared Error and based on the replications and sample sizes where \( \alpha = 3, \beta = 1 \) and \( \omega = 0.5 \). |
|---|---|---|---|---|---|---|---|
| \( n \) | Measures | Uniform Prior | Jeffrey's Prior | Gamma Prior |
| | | SELF | QLF | PLF | SELF | QLF | PLF | SELF | QLF | PLF |
| 10 | Estimate | 0.554 | 0.6094 | 0.4986 | 0.6365 | 0.554 | 0.4432 | 0.581 | 0.5621 | 0.4684 | 0.585 |
| | MSE | 0.0405 | 0.0575 | 0.0305 | 0.0683 | 0.0405 | 0.0273 | 0.0479 | 0.0298 | 0.019 | 0.0353 |
| 15 | Estimate | 0.5357 | 0.5714 | 0.5 | 0.589 | 0.5357 | 0.4643 | 0.5533 | 0.5448 | 0.4807 | 0.5606 |
| | MSE | 0.023 | 0.0298 | 0.0189 | 0.0342 | 0.023 | 0.0176 | 0.026 | 0.0196 | 0.0141 | 0.0223 |
| 20 | Estimate | 0.5275 | 0.5538 | 0.5011 | 0.5669 | 0.5275 | 0.4747 | 0.5405 | 0.5356 | 0.4869 | 0.5477 |
| | MSE | 0.0165 | 0.0203 | 0.0142 | 0.0227 | 0.0165 | 0.0134 | 0.0182 | 0.0149 | 0.0114 | 0.0165 |
| 27 | Estimate | 0.5199 | 0.5391 | 0.5006 | 0.5487 | 0.5199 | 0.4813 | 0.5294 | 0.5268 | 0.4904 | 0.538 |
| | MSE | 0.0114 | 0.0133 | 0.0102 | 0.0146 | 0.0114 | 0.0098 | 0.0122 | 0.0106 | 0.0087 | 0.0115 |
| 35 | Estimate | 0.5157 | 0.5304 | 0.501 | 0.5377 | 0.5157 | 0.4862 | 0.523 | 0.5214 | 0.4933 | 0.5284 |
| | MSE | 0.0082 | 0.0094 | 0.0075 | 0.0101 | 0.0082 | 0.0073 | 0.0088 | 0.0079 | 0.0067 | 0.0085 |
| 55 | Estimate | 0.51 | 0.5192 | 0.5007 | 0.5239 | 0.51 | 0.4914 | 0.5146 | 0.514 | 0.4959 | 0.5184 |
| | MSE | 0.0051 | 0.0056 | 0.0048 | 0.0058 | 0.0051 | 0.0047 | 0.0053 | 0.005 | 0.0045 | 0.0052 |
| 95 | Estimate | 0.505 | 0.5103 | 0.4997 | 0.513 | 0.505 | 0.4944 | 0.5076 | 0.5074 | 0.497 | 0.51 |
| | MSE | 0.0028 | 0.0029 | 0.0027 | 0.003 | 0.0028 | 0.0027 | 0.0029 | 0.0028 | 0.0026 | 0.0028 |
| 12 | Estimate | 0.5039 | 0.5079 | 0.4998 | 0.5099 | 0.5039 | 0.4958 | 0.5059 | 0.5058 | 0.4978 | 0.5078 |
| | MSE | 0.0021 | 0.0022 | 0.0021 | 0.0022 | 0.0021 | 0.0021 | 0.0021 | 0.002 | 0.0021 |

Again from table 3, it is confirmed that QLF under gamma priors gave the best estimators for the scale parameter irrespective of the changes in the allocation of sample sizes. This efficiency is again followed by the same QLF under Uniform and Jeffrey priors.
The above table, table 4 also reveals finally that gamma prior with QLF is the most efficient for the scale parameter, and looking at all the results presented in the tables, we can conclude that Bayes estimates using Quadratic loss function (QLF) are associated with minimum MSE when compared to those obtained using MLE, SELF and PLF irrespective of the parameter values as well as the allocated sample sizes of n=10, 15, 20, 27, 35, 55, 95 and 125.

4. CONCLUSIONS

In this article, we obtain Bayesian estimators of the scale parameter of Exponential Lomax distribution. The posterior distributions of this parameter are derived by using Uniform, Jeffrey and gamma priors. Bayes estimators and their risks have been derived by using three loss functions under the three prior distributions. The three loss functions are Squared Error Loss Function (SELF), Quadratic Loss Function (QLF) and Precautionary Loss Function (PLF). The performance of these estimators is assessed on the basis of their relative Biases and mean square errors. Monte Carlo Simulations are used to compare the performance of the estimators. It is discovered that using the QLF produces very low minimum measures of MSE under all the priors (gamma, Jeffreys and uniform) and most especially under gamma prior, then the SELF, MLE and lastly the PLF irrespective of the parameter values and difference in sample size. Most importantly, we found that Bayesian Method using Quadratic Loss Function (QLF) under gamma prior produces the best estimators of the scale parameter compared to estimators using Maximum Likelihood method, Squared Error Loss Function (SELF) and Precautionary Loss Function (PLF) under Uniform and Jeffrey priors irrespective of the values of the parameters and the different sample sizes. It is also discovered that the other parameters have no effect on the estimates of the scale parameter.

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