Studies on the $K^+\Sigma$ bound-state via $K^+p \to K^+\phi p$

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(Dated: January 12, 2021)

In the present work, we investigate the hidden-strangeness production process in the $S = +1$ channel via $K^+p \to K^+\phi p$, focusing on the exotic pentaquark molecular $K^+\Sigma$ bound state, assigned by $P_s^+(2071, 3/2^-)$. For this purpose, we employ the effective Lagrangian approach in the tree-level Born approximation. Using the experimental and theoretical inputs for the exotic state and for the ground-state hadron interactions, the numerical results show a small but obvious peak structure from $P_s^+$ with the signal-to-background ratio $\approx 1.7\%$, and it is enhanced in the backward-scattering region of the outgoing $K^+$ in the center-of-mass frame. We also find that the contribution from the $K^+(1680, 1^-)$ meson plays an important role to reproduce the data. The proton-spin polarizations are taken into account to find a way to reduce the background. The effects of the possible 27-plet pentaquark $\Theta^{++}_{27}$ is discussed as well.

PACS numbers: 13.60.Le, 13.40.-f, 14.20.Jn, 14.20.Gk
Keywords: $\phi$-meson production, hidden strangeness, $S = +1$ channel, hadronic molecular, $K^+\Sigma$ bound state, exotic pentaquark, effective Lagrangian method, spin polarization.

I. INTRODUCTION

Hadronic interactions have been one of the most important subjects in the strongly-interacting systems governed by quantum chromodynamics (QCD). From the interactions, various hadronic states can be constructed. The most interesting hadronic states must be exotic hadrons, which are beyond the minimal meson and baryon configurations, i.e., $q\bar{q}$ and $qqq$ in their color singlets: Tetraquarks [1–5], pentaquarks [6–9], hadronic moleculars [10–14], di-baryons [15–18], and so on. However, all of these exotics have not been fully confirmed yet experimentally. For instance, although QCD does not seem to prohibit a $qqq$ configuration, the existence of light pentaquark resonance states, including the famous $\Theta^+$ pentaquark baryon, has been unsettled, while various tetraquark resonance and bound-state mesons reported experimentally as in the above references. In addition, hidden-charm heavy pentaquark molecular bound states were observed in the $J/\psi p$ invariant mass from the heavy baryon decay $\Lambda_b^{+} \to K^-J/\psi p$ at LHCb, $P_s^+(4312, 4440, 4457)$ [10], whereas those states were not measured in the $J/\psi$ photoproduction off the proton target in the GlueX experiment of CLAS at the Jefferson laboratory [19], indicating the possible smaller photon couplings of the states.

We have the following motivations for the present work: As mentioned above, the pentaquark molecular $\bar{D}^{*}\Sigma_c$ bound-state $P_s^+(4457, 3/2^-)$ was found experimentally from the decay with a hidden-charm vector meson [10]: $P_s^+[\bar{D}^{*}\Sigma_c] \to J/\psi[cc]\bar{p}$. Considering an analogous mechanism for the light-flavor sector and assuming that the hidden-flavor channel is the key to observe the pentaquark bound state, there can be a $K^+\Sigma$ bound state appearing in the invariant mass of the hidden-strange vector meson $\phi(1020)$ and proton, being assigned by $P_s^+$ for convenience: $P_s^+[K^+\Sigma] \to \phi[ss]\bar{p}$. Interestingly, from various experimental data [20–22], a bump-like structure is observed at $\sqrt{s} = (2.0 \sim 2.1)$ GeV for the $\phi$-meson photoproduction off the proton target, although the origin of the bump is not fully understood [23–24]. Moreover, from theories, the $K^+\Sigma$ molecular bound state was suggested from the coupled-channel approach with the hidden-local symmetry and turned out to couple to the $\phi p$ channel rather strongly [25]. Hence, we assign this molecular bound state to $P_s^+(2071, 3/2^-)$, in which the mass and spin-parity were estimated theoretically, and it can be viewed as the light-flavor partner of $P_s^+(4457, 3/2^-)$.

Considering the motivations given above, we would like to study the hidden-strangeness production process with the $S = +1$ kaon beam off the proton target, i.e., $K^+p \to (K^+P_s^+) \to K^+\phi p$. For this purpose, we employ the effective Lagrangian method at the tree-level Born approximation. As for the interaction structures, we basically make use of the pseudo-scalar(P5)-meson–baryon Yukawa interactions for the baryon-intermediate s- and u-channel diagrams, in addition to the $\phi$-meson exchange t-channel diagrams. As shown in the LHCb experiment for $B^+ \to J/\psi\phi K^+$ [19],

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FIG. 1: Relevant tree-level Feynman diagrams for $K^+p \to K^+\phi p$. The dashed, solid, and wavy lines indicate the meson, baryon, and $\phi(1020)$ meson, respectively. The four momenta of the initial state ($K^+, p$) and final state ($K^+, \phi, p$) are defined by $(k_1, k_2)$ and $(k_3, k_4, k_5)$, respectively. For convenience, we separate the diagrams into three categories: The $\phi$-meson exchange diagrams in the $t$ channel ($t_{1-5}$), the $\Theta^{++}_{27}$-pole diagram in the $s$ channel ($s_1$), and the $\Lambda^{(*)}$-intermediate diagrams in the $u$ channel ($u_{1-5}$). The off-shell strange mesons can be $K(495, 0^-)$ for the $(t, u, s)$ and $K^*(1680, 1^-)$ for the $t_{1,2}$ channel, whereas the off-shell baryons for the $t_{1-4}$ and $u_{1-5}$ channels are $(p, P^+_s)$ and $(p, \Lambda^{(*)})$.

the contribution from $K^*(1680, 1^-)$, which decays into $K\phi$, is also taken into account. A possible 27-plet pentaquark state $\Theta^{++}_{27}$ is included as well to verify the effects of its existence to physical observables. The relevant interaction strengths are taken from the well-known Nijmegen potential model [20] and the coupled-channel method with the hidden local symmetry [25]. Phenomenological form factors are also taken into account, and the cutoff masses for the form factors determined to reproduce the presently available experimental data for $K^+p \to K^+\phi p$ [27–29].

From the numerical results, we observe the strong enhancements of the total cross section with the $K^*(1680)$ contribution as the center-of-mass (cm) energy increases, due to the higher momentum-dependent Lorentz structure of the $KK^*\phi$ interaction vertex. In the Dalitz plot analyses, we find the band structure with $\Gamma = 14$ MeV for $P^+_s$, and the $P^+_s$ contribution interferes with the background constructively. As the energy increases larger than $\sqrt{s} \approx 2.70$ GeV, the $K^*(1680)$ contribution dominates the cross section. In the $\phi p$-invariant-mass plots, we find an obvious peak structure from $P^+_s$ with the signal-to-background ratio $\approx 1.7\%$ at $\sqrt{s} = 2.65$ GeV. There is the considerable cross-section enhancement from the $K^*(1680)$ contribution for $M(\phi p) \lesssim 2.05$ GeV in the $\phi p$-invariant-mass plots as expected from the total cross section. There appears a broad $K^*(1680)$ peak for the higher cm energy in the $K^+\phi$-invariant-mass plots. In addition to the $P^+_s$ and $K^*(1680)$ contributions, the backgrounds with the $\Lambda(1115)$-hyperon intermediate states with the $\phi$ meson emitted from the kaon beam turns out out be the most important and largest contribution to reproduce the cross section data.

A method to reduce the background is suggested for the better signals of $K^*(1680)$ and $P^+_s$, using the different initial- and final-state proton-spin combinations. When the two spins are opposite to each other, the background is largely suppressed, since the spin non-flip process only survives in the scattering amplitude. As for the opposite case, the $K^*(1680)$ peak manifests itself in the $K^+\phi$-invariant-mass plots. In contrast, the $P^+_s$ peak is rather dubious for the opposite case in the $\phi p$-invariant-mass plots, since $P^+_s$ strongly interferes with the background constructively, not with $K^*(1680)$. The angular dependence shows mild backward-scattering enhancements as functions of the outgoing $K^+$ angle in the cm frame, and this behavior was originated from the $u$-channel propagators in the scattering amplitudes for the $K^*(1680)$ and background contributions. Therefore, the signal of $P^+_s$ is also amplified in the backward-scattering region. Finally, we investigate the possible contribution from the 27-plet pentaquark $\Theta^{++}_{27}$. Tuning the coupling strength of the pentaquark, we observe a diagonal band structure from the $\Theta^{++}_{27}$ in the Dalitz plot, and it starts to interfere strongly with $K^*(1680)$ for $M(\phi p) \gtrsim 2.05$ GeV.

The present paper is organized as follows: The theoretical framework is briefly introduced in Section II. The Section III is devoted to the numerical results with detailed discussions. The summary is given in the final Section.

II. THEORETICAL FRAMEWORK

In this Section, we provide the theoretical framework to investigate the $K^+p \to K^+\phi p$ reaction process. In Fig. 1, we depict the relevant tree-level Feynman diagrams for it. The dashed, solid, and wavy lines indicate the meson, baryon, and $\phi(1020)$ meson, respectively. The four momenta of the initial state ($K^+, p$) and final state ($K^+, \phi, p$) are defined by $(k_1, k_2)$ and $(k_3, k_4, k_5)$, respectively. For convenience, we separate the diagrams into three categories: The $\phi$-meson exchange diagrams in the $t$ channel ($t_{1-5}$), the $\Theta^{++}_{27}$-pole diagram in the $s$ channel ($s_1$), and the $\Lambda^{(*)}$-
intermediate diagrams in the u channel (u_{1~5}). The off-shell strange mesons can be K(495, 0^-) for the (t, u, s) and K*(1680, 1^-) for the t_{1,2} channel, whereas the off-shell baryons for the t_{1~4} and u_{1~5} channels are (p, P^+_t) and (p, Λ^{1*}).

As for $P^+_t$ as the pentaquark molecular $K^*Σ$ bound state, we employ the theoretical results from Ref. [25], in which the hidden-local symmetry was taken into account with the coupled-channel Bethe-Salpeter equation, resulting in $P^+_s(2071, 3/2^-)$ as the isospin 1/2 pole, with its full decay width $Γ_{P^+_s}$ = 14 MeV. Note that in Ref. [30], the s-wave exotic bound state was estimated with $J^P = 3/2^-$ nucleon resonance with its mass $≈ 2064$ GeV via the quark de-localization color screening model (QDCSM). We consider the ground state $Λ(1115, 1/2^+)$, $Λ(1405, 1/2^-)$, and $Λ(1520, 3/2^-)$ for the $Λ$-hyperon contributions. We, however, take only the ground-state one for the diagram ($u_5$) into account, since little information is available for the $ϕΛ^*Λ^*$ vertex. Similarly, we do not include the proton resonances for brevity.

In the Review of Particle Physics [31], there are several strange mesons, decaying into $K^+ϕ$, such as $K_1(1650, 1^+)$ and $K^*(1680, 1^-)$, although their experimental confirmations are still poor. Nonetheless, as shown in the LHCb experiment for $B^+ → J/ψφK^+$ [22], among the various strange mesons, in the vicinity of the invariant mass $M(K^+ϕ) = 1.7$ GeV, the $K^*(1680, 1^-)$ contribution is the most dominant one, in addition to the non-resonant background, whereas the $K_1(1650, 1^+)$ contribution seems appearing in the higher-mass region $M(K^+ϕ) ≈ 1.8$ GeV. Note that our purpose of the present work is to study the $P^+_s$ bound state, which appears in the vicinity of $M(ϕp) = 2$ GeV, and this mass region corresponds to $M(K^+ϕ) = (1.6 ∼ 1.7)$ GeV. Hence, we exclude the $K_1(1650, 1^+)$ contribution rather safely for the numerical calculations and to reduce theoretical uncertainties.

As shown in the diagram ($s_1$) of Fig. 1, there can be a contribution from the baryonic exotic state, such as the 27-plet pentaquark baryon $Ω_{27}^{1+}$. Its physical property was studied in Ref. [23], and its mass, spin-parity, and full width turn out to be 1.60 GeV, 3/2^-, and $≤ 43$ MeV, respectively, via the chiral soliton model (CHSM). From the decay width we obtain $g_{KNππππ}^{ωπππ} ≤ 2.06$.

To compute the invariant amplitudes for the diagrams in Fig. 1, the effective Lagrangians for the relevant interaction vertices are defined as follows:

$$\mathcal{L}_{VP} = \frac{ig_{VP}}{\sqrt{2}} V_{\mu} \mathcal{A}_\mu - (\partial_\mu P) P^\mu,$$

$$\mathcal{L}_{VPP} = \frac{g_{VPP}}{\sqrt{2}} \mathcal{A}_\mu \mathcal{A}_\mu,$$

$$\mathcal{L}_{PNB_{1±}} = \frac{g_{PNB_{1±}}}{\sqrt{2}} N P_{T±} B_{1±} + h.c.,$$

$$\mathcal{L}_{PNB_{2±}} = \frac{g_{PNB_{2±}}}{\sqrt{2}} N (\partial_\mu P) \gamma_5 \gamma_\mu B_{2±} + h.c.,$$

$$\mathcal{L}_{VPN1±} = \frac{g_{VPN1±}}{\sqrt{2}} N \gamma_\mu \gamma_\nu V_{\mu \nu} B_{3±} + h.c.,$$

$$\mathcal{L}_{VPN2±} = \frac{g_{VPN2±}}{\sqrt{2}} N \gamma_\mu \gamma_\nu V_{\mu \nu} B_{3±} + h.c.,$$

where $P$ and $V$ designate the fields for the pseudo-scalar and vector mesons, whereas $N$ and $B_{1±}$ indicate those for the nucleon and baryon with its spin-parity $1, 3/2^+/2$, respectively. $F_{\mu \nu}^{\pm}$ stands for the anti-symmetric field strength tensor for the massive vector meson. Here, we define the notation $\Gamma^{±} = (\gamma_5, 1 \times 4)$ corresponding to the parities. The invariant amplitudes can be evaluated straightforwardly, using the effective Lagrangians given above.

Here, we have an issue that the phase factors between the scattering amplitudes are not uniquely determined. The total scattering amplitude can be written with the phase factors in general as follows:

$$M_{\text{total}} = \epsilon^{q1} M_{q1}, K^*(1680) + \epsilon^{q4} M_{q4, p^+_3} + \epsilon^{q1} M_{q1, BKG} + \epsilon^{q,u, BKG} M_{q,u, BKG},$$

where the first and second terms in the r.h.s. of Eq. (2) provide peak structures from $K^*(1680)$ and $P^+_s$, whereas the third and fourth ones are for the $t$- and $u$-channel backgrounds (BKG), respectively. The corresponding phase angles are given by $\psi$ here. Although we do not show all the possible combinations of the different phase angles, in order to reduce theoretical uncertainties in the present work, we choose $(\psi_{q1, K^*(1680)}, \psi_{q4, P^+_3}, \psi_{q1, BKG}, \psi_{q,u, BKG}) = (0, 0, 0, \pi)$, which provide the most appropriate interference patterns, indicating clear peak structures and reproducing properly the experimental data for the total cross section, as will be shown in the next Section.

In order to take the spatial extension of the hadrons into account, the following phenomenological form factors are employed as follows:

$$F(q^2, M) = \left[ \frac{A^4}{A^4 + (q^2 - M^2)^2} \right]^{1/2}.$$

As shown in Eq. (3), because the form factors are functions of the off-shell momenta of the intermediate hadrons, two form factors are multiplied to a bare scattering amplitude in general: $iM_{q1}^K → iM_{q1}^K F(q_{-3}^2, M_K) F(q_{-3}^2, M_φ), \text{ where}$
FIG. 2: (Color online) Total cross section for $K^+p \rightarrow K^+\phi p$. The experimental data are taken from Refs [27–29]. The solid and dashed lines denote those with and without $K^*(1680)$, respectively.

$q_{i\pm j} \equiv k_i \pm k_j$, for instance. The cutoff mass is determined to reproduce relevant experimental data, and will be discussed in the next Section in detail. Note that $g_{KK\phi}$, $g_{\phi NN}$, and $g_{KN\Lambda}$ come from the Nijmegen soft-core potential model [26], while $g_{\phi P_s^+P_s}$ and $g_{K_{NN}\Theta^{++}}$ are estimated theoretically in Refs. [25] and [33], respectively. However, $g_{K_{NN}\Theta^{++}}$ will be treated as a free parameter in the actual calculations, since the maximum value 2.06 is too large to describe the data.

$g_{KN\Lambda(1405)}$ and $g_{KN\Lambda(1520)}$ are determined using the interaction Lagrangians in Eq. (1) and experimental data [31]. Finally, $g_{KK^*\phi}$ will be determined by fitting with the experimental data, since we do not know its branching ratio to $K^+\phi$ exactly, although its full decay width reads 322 MeV from the Review of Particle Physics [31]. Relevant coupling constants and full decay widths for the present numerical calculations are listed in Table I.

TABLE I: Relevant coupling constants and full decay widths.
structure of the plot is uncomplicated, i.e., the sum of the background and $P_s^+$ contributions. As the energy increases, the $K^*(1680)$ contribution comes into play and the interference between $P_s^+$ and $K^*(1680)$ takes place. As expected from the total cross section in Fig. 2, as the energy gets higher than $\sqrt{s} = 2.70$ GeV, the $K^*(1680)$ contribution plays a dominant role to reproduce the strength, and the interference becomes enhanced more.

In order to understand the details of the $P_s^+$ band structure and the effects of $K^*(1680)$, we show the $d\sigma/dM(K^+\phi)$ (left) and $d\sigma/dM(\phi p)$ (right) invariant-mass plots for several $\sqrt{s}$ values in Fig. 4. The shaded areas indicate the full results, where the solid lines indicate those without $P_s^+$ and the dotted ones without $P_s^+$ and $K^*(1680)$. In the left panel for $d\sigma/dM(K^+\phi)$, we observe that the $K^*(1680)$ contribution becomes larger with respect to the energy, and almost 50% of the strength of the differential cross section is made from the $K^*(1680)$ one as seen in the case for $\sqrt{s} = 2.70$ GeV for instance. Note that the broad peak of $K^*(1680)$ starts to appear as the maximum value of $M(K^+\phi)$ gets larger than its mass 1718 MeV, which is denoted by the vertical solid line [31]. As expected, the $P_s^+$ contribution does not make significant contributions here. In the right panel of Fig. 4, we show the numerical results for $d\sigma/dM(\phi p)$ in the same manner with that of the left one. The vertical solid line denotes the $K^*\Sigma$ threshold. Similarly, we observe stronger $K^*(1680)$ contribution for the higher energies, and it increases the differential cross section in the lower $M(\phi p)$ region, as understood from the Dalitz plots in Fig. 3. The peak structure of $P_s^+$ is clearly observed below the $K^*\Sigma$ threshold, and it gives a signal-to-background ratio $\sim 1.7\%$ for $\sqrt{s} = 2.65$ GeV for instance.

In Fig. 5, we draw each contribution separately for $d\sigma/dM(K^+\phi)$ for $\sqrt{s} = 2.70$ GeV (left) and $d\sigma/dM(\phi p)$ for $\sqrt{s} = 2.65$ GeV (right). From the left panel of the figure, we note that the diagram ($t_2$) with $K^*(1690)$ of Figure 4 (dotted) provides the dominant contribution to the differential cross sections, showing the broad peak around the $K^*(1690)$ mass, indicated by the vertical solid line. The deviation of the peak-position from its mass can be understood.
by the interference and boundary effects. As a pure background contribution only with the ground-state hadrons, the diagram \((u_1)\) with \(\Lambda(1115)\) (dot-dashed) provides the larger contribution than others. Note that other diagrams are contributing only small or negligible portion of the total differential cross section. Note that the \(P_2^+\) contribution is almost unseen. In the right panel of Fig. 5, we plot \(d\sigma/dM(\phi p)\) in the same manner. Again, the shape of the total curve is almost made from the diagrams \((t_2)\) and \((u_1)\) being similar to \(d\sigma/dM(K^+\phi)\), while the diagram \((t_4)\) gives the \(P_2^+\) peak, whose strength is enhanced by the constructive interference with other contributions.

Now we are in a position to discuss a method to reduce the background contributions to enhance those particle signals that we are interested in. Here, to make the discussion simple, we consider the most important contributions, i.e., diagrams \((t_2,4)\) for \(K^+(1680)\) , \((t_4)\) for \(P_2^+\), and \((u_1)\) for a pure background as understood in Fig. 5. For this purpose, we introduce different target initial-state \((i)\) and final-state \((f)\) proton-spin combinations, i.e., parallel or opposite to each other:

\[
\sigma_{\text{parallel}} \equiv \sigma(\uparrow\uparrow) + \sigma(\downarrow\downarrow), \quad \sigma_{\text{opposite}} \equiv \sigma(\uparrow\downarrow) + \sigma(\downarrow\uparrow),
\]

where we define these cross sections by \(\sigma(S_i S_f)\). By a simple Clebsch-Gordan coefficient analyses, as for the the diagrams \((t_2,4)\), \(\sigma_{\text{parallel}}\) and \(\sigma_{\text{opposite}}\) are both finite, whereas \(\sigma_{\text{parallel}}^{u_1}\) is finite but \(\sigma_{\text{opposite}}^{u_1} \sim 0\), since the spin-0 PS mesons couple to the spin-1/2 ground-state baryons in the diagram \((u_1)\). Hence, by making the the proton spins opposite to each other, the largest background contribution from the diagram \((u_1)\) is suppressed considerably. In Fig. 5, we show the numerical results for the differential cross sections of Eq. \((4)\) in the same manner with that of Fig. 5. As shown in the left panel of the figure for \(d\sigma/dM(K^+\phi)\), the background nearly disappears for the opposite case as expected, and the \(K^+(1680)\) contribution becomes obvious at its mass. In the right panel for \(d\sigma/dM(\phi p)\), the situation is similar but we observe the \(P_2^+\) peak as well for the opposite case. Thus, we can conclude that the \(K^+(1680)\) contribution dominates the cross section for the opposite case, and it is rather useful to investigate that vector kaon properties via experiments, although the peak of \(P_2^+\) is rather dubious, since \(P_2^+\) is strongly interfered with the background contribution constructively, not with the \(K^+(1680)\) one.

The numerical results for the angular distribution of the cross section \(d\sigma_{K^+ p \rightarrow K^+ \phi p}/d\cos \theta\) for \(\sqrt{s} = 2.7\) GeV is given in the left panel of Fig. 7. The angle \(\theta\) is defined by that of the outgoing kaon in the cm frame with respect to the \(+z\) beam direction. We observe the mild enhancement in the backward scattering region for the cases with (solid) and without (dotted) \(K^+(1680)\). This behavior can be easily understood from the amplitude for the diagram \((u_1)\), which is the largest contribution of the present reaction process. From the \(u\)-channel propagator in the amplitude in the cm frame

\[
i\mathcal{M}_{u_1} \propto \left[ (M_{K^+}^2 - M_N^2 - M_K^2) + 2E_p E_{K^+} - 2 \left( \vec{k}_{p_i} \cdot \vec{k}_{K_f} \right) \right]^{-1},
\]

we find that, when the three momenta of the target proton \(\vec{k}_{p_i}\), which is in the \(-z\) direction, and the outgoing kaon \(\vec{k}_{K_f}\) are parallel in the cm frame, the amplitude becomes maximized. The contribution from the \(K^+(1680)\) also gives

\[
\sqrt{s} = 2.70 \text{ GeV (left)} \text{ and } d\sigma/dM(\phi p) \text{ for } \sqrt{s} = 2.65 \text{ GeV (right). Each contribution is given in different line styles as indicated.}
\]
FIG. 6: (Color online) Polarized invariant-mass plots $d\sigma/dM(K^+\phi)$ for $\sqrt{s} = 2.70$ GeV (left) and $d\sigma/dM(\phi p)$ for $\sqrt{s} = 2.65$ GeV (right). The solid and dotted lines denote the proton-spin parallel and opposite combinations, respectively, as defined in Eq. (4).

FIG. 7: (Color online) Left: Differential cross section $d\sigma_{K^+p \to K^+\phi p}/d\cos\theta$ for the outgoing-$K^+$ angle in the cm frame at $\sqrt{s} = 2.7$ GeV. Right: Double-differential cross section $d^2\sigma_{K^+p \to K^+\phi p}/dM(\phi p)\,d\cos\theta$ at $\sqrt{s} = 2.7$ GeV.

the backward-scattering enhancement as shown in the left panel of the figure, due to the same reason with that of the diagram $(u_1)$. In the right panel of Fig. 7, we show the double differential cross section $d^2\sigma_{K^+p \to K^+\phi p}/dM(\phi p)\,d\cos\theta$ at $\sqrt{s} = 2.7$ GeV. There, we clearly see that the $P_s^+$ peak manifests itself in the backward-scattering region $\cos\theta \gtrsim 0.5$.

Finally, we explore the effect from the possible 27-plet pentaquark $\Theta_{27}^{++}$, appearing in the diagram $(s_1)$ of Fig. 1. Although its physical properties estimated in Ref. [33], we treat the coupling constant $g_{KN\Theta_{27}^{++}}$ as a free parameter, while the theoretical estimations for its mass 1600 MeV and full width 43 MeV are taken as they are, since the theory value for the coupling constant gives too large cross section. We note that, in the numerical calculations, we choose 5% of the theory value, providing a similar strength to other contributions. In the left panel of Fig. 8, we depict the Dalitz plot for the process at $\sqrt{s} = 2.7$ GeV with the pentaquark. There appears a diagonal band for the pentaquark and it gives enhancements by interfering with the $K'(1680)$ contribution. Also, the peak of $P_s^+$ is slightly affected by the inclusion of the pentaquark constructively. In the right panel of figure, the $\phi p$-invariant-mass plots are given with (solid) and without (shaded) the $\Theta_{27}^{++}$ contribution. By seeing that, we can expect that the cross section below $M(\phi p) \lesssim 2.05$ will be enhanced more as $g_{KN\Theta_{27}^{++}}$ increases.
FIG. 8: (Color online) Left: Dalitz plots for $K^+p \rightarrow K^+\phi p$ for $\sqrt{s} = 2.70$ GeV with the $\Theta^{++}_{27}$ contribution. Right: Differential cross section $d\sigma/dM(\phi p)$ for $\sqrt{s} = 2.75$ GeV with (solid) and without (shaded) the pentaquark contribution.

IV. SUMMARY

In the present work, we investigated the hidden-strangeness production process via $K^+p \rightarrow K^+\phi p$. Here, assuming that the hidden-flavor pentaquark molecular bound state is easier to be formed than open-flavor ones, since $P^+_s(4457)$ is the only exotic pentaquark baryon ever observed firmly in $P^+_s[uudc\bar{c}] \rightarrow J/\psi[cc]\bar{p}$, we considered that $P^+_s[uuds\bar{s}] \rightarrow \phi[s\bar{s}] p$, in which $P^+_s$ is an exotic molecular bound state of $K^*$ and $\Sigma$, can be observed in this specific production process with higher possibility than other light-flavor baryonic exotics. We employed a simple phenomenological model based on the effective Lagrangian approach and provided the relevant numerical results by making use of presently available experimental and theoretical information for the purpose. We used the theory estimations for the $P^+_s(2071, 3/2^+)$ from the hidden-local symmetry arguments. In addition to the background and $P^+_s$ contributions, we also took into account $K^*(1680)$, which decays into $K\phi$, as indicated in the LHCb experiments. Below, we list up the relevant observations in the present work:

- First we determined the cutoff mass, which is one of the most important model parameters, by fitting the experimental data for the total cross section of $K^+p \rightarrow K^+\phi p$ for $\sqrt{s} \gtrsim 2.7$ GeV. We observed strong enhancements of the cross section with the $K^*(1680)$ contribution as the cm energy increases, due to the higher momentum-dependent nature of the $KK^*\phi$ interaction vertex. From the Dalitz plot analyses, we observed the narrow ($\Gamma = 14$ MeV) band structure for $P^+_s$ with the increasing background contributions as the cm energy does, and the $P^+_s$ contribution is constructively interfered with the background. As the energy becomes larger than $\sqrt{s} \approx 2.65$ GeV, the $K^*(1680)$ contribution comes into play and starts to dominate the cross section of the process.

- From the $\phi p$-invariant-mass plots, we found an obvious peak structure from $P^+_s$ with the signal-to-background ratio $\approx 1.7\%$ at $\sqrt{s} = 2.65$ GeV. There were considerable cross section enhancements from the $K^*(1680)$ contribution for $M(\phi p) \lesssim 2.05$ GeV. We could observe a broad $K^*(1680)$ peak as the cm energy increases from the $K^+\phi$-invariant-mass plots. Beside the $P^+_s$ and $K^*(1680)$ contributions, the scattering amplitude $(u_1)$ with the $\Lambda(1115)$-hyperon intermediate states with the $\phi$ meson emitted from the kaon beam were the most largest source to produce the cross section as background.

- We tried to find a way to reduce the background to enhance the signals of the $K^*(1680)$ and $P^+_s$ by different initial- and final-state proton-spin combinations. When the spins are opposite to each other, the backgrounds are greatly suppressed, since the spin non-flip process only survives in the scattering amplitude. As for the proton-spin opposite case, the $K^*(1680)$ peak was seen with clarity for $\sqrt{s} \gtrsim 2.7$ GeV in the $K^+\phi$-invariant-mass plots. On the contrary, the $P^+_s$ peak was dubious for the proton-spin opposite case in the $\phi p$-invariant-mass plots, since $P^+_s$ is strongly interfered with the background contributions, not with the $K^*(1680)$ one, constructively.

- The angular dependences of of the cross sections showed mild backward-scattering enhancements with respect
to the outgoing $K^+$ angle in the cm frame, and this behavior was originated from the nature of the $u$-channel propagators in the $K^+(1680)$ and background amplitude, mentioned above, i.e., the scattering amplitude is maximized when three momenta of the incident and scattered particles are opposite to each other in the cm frame. Hence, the signal of $P_s^+$ was also amplified in the backward-scattering region as shown in the numerical results of the double differential cross sections $d^2\sigma_{K+p\rightarrow K+\phi}/dM(\phi)p\ d\cos\theta$.

- Finally, we examined the possible contribution from the 27-plet pentaquark $\Theta_{27}^{++}$, being based on the theoretical estimations for it from the chiral soliton model. Tuning the coupling strength of the pentaquark to match with other contributions, we observed a band structure from the $\Theta_{27}^{++}$ in the Dalitz plot and it starts to interfere strongly with $K^+(1680)$ for $M(\phi)p \gtrsim 2.05$ GeV.

The numerical results of the present work can be a useful guide to measure the exotic pentaquark molecular bound state $P_s^+$ as well as to extract the information of $K^+(1680)$ possibly in the J-PARC experiments with the high-momentum kaon beam in the future. It is interesting to note that Ref. [25] also suggested another possible $K^*\Sigma$ bound state with its pole mass (1977 − i22) MeV and spin-parity $1/2^-$ in addition to $P_s^+(2071,3/2^-)$, although we did not consider this lower-mass state in the present work, since it exists near the Dalitz-plot boundary $M(\phi)p \approx 1900$ MeV and its wider width. The situation is similar to that there are two bound states $P_s^+(4440)$ and $P_s^+(4457)$ below the $D^*\Sigma$ threshold. Nonetheless for the mass differences are significantly different $\Delta M_s \approx 100$ MeV and $\Delta M_s \approx 20$ MeV, the similarities between the light- and heavy-flavor sectors are very peculiar. More detailed works together with more exotics such as $P_s^+(1977)$ in a specific reaction process and theoretical studies for the structures of the exotics are in progress, and appear elsewhere.

Acknowledgment

The author is grateful to fruitful discussions with K. P. Khemchandani (Sao Paulo Univ.), J. K. Ahn (Korea Univ.), and A. Hosaka (RCNP). This work was supported by a Research Grant of Pukyong National University (2019). All the Feynman diagrams were generated via https://feynman.aivazis.com/.

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