Integrable impurity spin ladder systems

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Abstract

Two integrable spin ladder systems with different types of impurities are proposed. The impurities are introduced in such a way that the integrability of the models is not violated. The models are solved exactly and the Bethe ansatz equations as well as the energy eigenvalues are obtained. We show for both models that a phase transition between gapped and gapless spin excitations occurs at a critical value of the rung coupling $J$. In addition, the dependence of the impurities on this phase transition is determined explicitly. Remarkably, in one of the models a decreasing of the spin gap with increasing impurity strength is found.

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The study of spin ladder systems continues to generate activity in order to understand the crossover from one-dimension to two-dimensions in condensed matter physics. In fact, with the rapid progress presently being made in nano-engineering, several compounds have been realized experimentally with a ladder structure (see for example [1,2]). In addition, experiments using different techniques, such as magnetic susceptibility measurements [3] or nuclear magnetic ressonance [4], report on the existence of a spin gap in the spectrum of elementary excitations for even leg ladders. The existence of such a spin gap is crucial for superconductivity to occur under hole doping, as verified experimentally in some of these compounds and also predicted theoretically.

Initially, most of the theoretical results concerning ladder systems were obtained from studies of the standard Heisenberg ladder. However, in contrast to its one-dimensional analogue, this model can not be solved exactly. Subsequently, in order to gain insight into the theory of spin ladder systems, other models with generalized interaction terms which guarantee integrability have been proposed [5–16]. Remarkably, such generalized models exhibit realistic physical properties such as the existence of a spin gap [5] and the prediction of magnetization plateaus at fractional values of the total magnetization [6]. However, although there is a considerable amount of work on integrable spin ladder systems in the literature, very few are concerned with the presence and influence of impurities. (One exception however is [15].)

The role of impurities in the context of spin chains and strongly correlated electron systems is established to be an important aspect, particularly in low-dimensional cases. When undertaking studies appealing to exact solutions afforded by integrable systems, it is possible to incorporate impurities into the system without violating integrability. This can be achieved via two methods. In the first case, the impurities are given by changing the representation of the underlying Lie algebraic structure at some lattice sites from the fundamental representation to some other representation. In this context, several versions of the Heisenberg and $t-J$ models, for example, have been investigated [17–20]. Another possibility is to introduce the impurities by way of inhomogeneities in the transfer matrix of the system. This was explored, for instance in [21–24].

Here we wish to adapt these two known methods in one-dimension to incorporate impurities to the quasi-bidimensional case for the purpose of studying impurities in spin ladder systems. In particular, we will construct two integrable spin ladder models based on the $su(4)$ algebra (Wang’s model [5]) with impurities. The effect of these impurities on the phase transition between the gapped and gapless spin excitations of both models will be investigated. We will show that in one of the cases the gap does not depend explicitly on the impurity, while it does in the other case. Moreover, it turns out in the latter instance that the spin gap decreases by increasing the impurity.

Now, we begin by introducing the first model whose Hamiltonian reads

$$H = \sum_{i=1}^{N} h_{i,i+1} + \frac{6}{\Lambda (\Lambda - 8)} Q_{i,i} - \frac{2}{\Lambda - 8} Q_{i,i} h_{i,i+1} + \frac{J}{2} \sum_{i=1}^{N} (\vec{\sigma}_i \cdot \vec{\tau}_i - 1) - \frac{J}{2} (\vec{\sigma}_i \cdot \vec{\tau}_i - 1),$$

where $h_{i,i+1} = -\frac{1}{2} P_{i,i+1}$ with $P_{i,i+1}$ and $Q_{i,i}$ given by

$$h_{i,i+1} = \frac{1}{4} (1 + \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}) (1 + \vec{\tau}_i \cdot \vec{\tau}_{i+1})$$

(1)
Above $\vec{\sigma}_i$ and $\vec{\tau}_i$ are Pauli matrices acting on the site $i$ (or the impurity site $\bar{i}$) of the upper and lower legs of the ladder, respectively, $J$ is the coupling constant across the rungs (including the impurity rung) and $\Lambda$ is an arbitrary parameter. Throughout, $N$ is the number of rungs (equivalently, the length of the ladder) and periodic boundary conditions are imposed. The picture below shows the 2-leg spin $1/2$ ladder in detail.

Notice here that besides the usual Heisenberg interactions along the legs and rungs we also have Heisenberg type interactions between the lattice site $i$ and the impurity site $\bar{i}$, as well as biquadratic interactions (also involving the impurity site $\bar{i}$). In addition, there are three site interactions, involving the sites $i$ and $i+1$ as well as the impurity site $\bar{i}$. Finally, we mention that the Hamiltonian for the ladder model based on the $su(4)$ symmetry [3] can be recovered from eq. (1) by taking the limit $\Lambda \to \infty$.

The energy eigenvalues of the Hamiltonian (1) are given by

$$E = 2J + (1 - 2J)N - 4 \sum_{i=1}^{M_1} \left( \frac{1}{\lambda_i^2 + 1} - \frac{J}{2} \right)$$

where $\lambda_i$ are solutions to the Bethe ansatz equations (BAE) below. The BAE arise from the exact solution of the model through the nested algebraic Bethe ansatz method and read

$$\left( \frac{\lambda_l - i}{\lambda_l + i} \right)^N = \prod_{i \neq l}^{M_1} \frac{\lambda_l - \lambda_i - 2i}{\lambda_l - \lambda_i + 2i} \prod_{j=1}^{M_2} \frac{\lambda_l - \mu_j + i}{\lambda_l - \mu_j - i}$$

$$\prod_{j \neq l}^{M_2} \frac{\mu_j - \mu_j - 2i}{\mu_j - \mu_j + 2i} = \prod_{i=1}^{M_1} \frac{\mu_l - \lambda_i - i}{\mu_l - \lambda_i + i} \prod_{k=1}^{M_3} \frac{\mu_l - \nu_k + i}{\mu_l - \nu_k - i}$$

$$\left( \frac{\nu_l - \Lambda + i}{\nu_l - \Lambda - i} \right)^{M_3} \prod_{k \neq l}^{M_3} \frac{\nu_l - \nu_k - 2i}{\nu_l - \nu_k + 2i} = \prod_{j=1}^{M_2} \frac{\nu_l - \mu_j - i}{\nu_l - \mu_j + i}$$

The presence of the impurity can be detected explicitly through the presence of the parameter $\Lambda$ in the first term of the last equation above.

Here the ground state is given by a product of rung singlets when $J > 2$ and the energy is $E_0 = 2J + (1 - 2J)N$. This is in fact the reference state used in the Bethe
ansatz calculations and corresponds to the case $M_1 = M_2 = M_3 = 0$ of the BAE (3). To describe an elementary excitation, we choose $M_1 = 1$ and $M_2 = M_3 = 0$ in the BAE (3) which gives the minimal excited state energy, $E_1 = -4 + 4J + (1 - 2J)N$.

The energy gap can easily be calculated and is found to be

$$
\Delta = 2\left(J - 2\right).
$$

By solving $\Delta = 0$ for $J$ we find the critical value $J_c = 2$, indicating the critical point at which the quantum phase transition from the dimerized phase to the gapless phase occurs. Notice that there is no effect of the impurity $\Lambda$ on the gap.

The integrability of this model (for $J = 0$) can be shown by the fact that there are matrices $R$ and $R^*$ given by

$$
R_{12} = \begin{pmatrix}
    a & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & b & 0 & 0 & | & c & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & b & 0 & | & 0 & 0 & 0 & 0 & | & c & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & b & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & c & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    & & & & | & & & & & | & & & & & & & & & | & & & & & | & & & & & | & & & & & |
    0 & c & 0 & 0 & | & b & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & | & 0 & a & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & | & 0 & 0 & b & 0 & | & 0 & c & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & | & 0 & 0 & 0 & b & | & 0 & 0 & 0 & 0 & | & c & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    & & & & | & & & & & | & & & & & & & & & | & & & & & | & & & & & | & & & & & |
    0 & 0 & c & 0 & | & 0 & 0 & 0 & 0 & | & b & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & | & 0 & 0 & c & 0 & | & 0 & b & 0 & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & | & 0 & 0 & 0 & c & | & 0 & 0 & 0 & 0 & | & 0 & b & 0 & 0 & | & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & | & c & 0 & b & 0 & | & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & a \\
    & & & & | & & & & & | & & & & & & & & & | & & & & & | & & & & & | & & & & & |
\end{pmatrix},
$$

with

$$
a = -x/2 + 1, \quad b = -x/2, \quad c = 1,
$$
\( R_{12} = \begin{pmatrix}
    c & 0 & 0 & 0 & 0 & d & 0 & 0 & 0 & 0 & d & 0 & 0 & 0 & 0 & d \\
    0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 & 0 \\
    d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0 & 0 \\
    d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 & 0 \\
    d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c & 0 & 0 \\
    d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e & 0
\end{pmatrix} \)

with

\[ c = 1, \quad d = 2/x, \quad e = 1 + \frac{2}{x}, \]

obeying the Yang-Baxter algebra

\[ R_{12}(x - y)R_{13}(x)R_{23}(y) = R_{23}(y)R_{13}(x)R_{12}(x - y). \]  
\[ R_{12}(x - y)R_{13}^*(x)R_{23}^*(y) = R_{23}^*(y)R_{13}^*(x)R_{12}(x - y). \]

Above \( R(x) \) is the fundamental \( R \)-matrix associated to the \( su(4) \) algebra while \( R^*(x) \) is the solution which acts in the tensor product of the fundamental representation and its dual. By applying the standard procedure of the Quantum Inverse Scattering Method (QISM), the global Hamiltonian is obtained from

\[ H = \frac{d}{dx} \ln(\tau(x, \Lambda))|_{x=0} \]

where \( \tau(x, \Lambda) \) is the transfer matrix

\[ \tau(x, \Lambda) = \text{tr}_0(R_{0,1}(x) \ldots R_{0,i}(x)R_{0,i+1}^*(x - \Lambda)R_{0,i+1}(x)\ldots R_{0,N}(x)) \]

and \( \text{tr}_0 \) denotes the trace over the auxiliary space, labelled by 0.

Notice here that the impurity is incorporated into the system through the inclusion of the operator \( R^*(x - \Lambda) \) in the transfer matrix and its effect on the spectrum can be detected by the presence of the extra parameter \( \Lambda \).
In this way, the Hamiltonian (1) can be mapped to the following Hamiltonian, which can be derived from the $R$ and $R^*$ matrix (eqs. (3,4)) obeying the Yang-Baxter algebra for $J = 0$, while for $J \neq 0$ the rung interaction takes the form of a chemical potential term, commuting with the Hamiltonian.

\[ \bar{H} = \sum_{i=1}^{N} \bar{h}_{i,i+1} + \frac{6}{\Lambda(\Lambda - 8)} \bar{Q}_{i,i} + \frac{2}{\Lambda - 8} \bar{h}_{i,i+1} \bar{Q}_{i,i} - \frac{2}{\Lambda} \bar{Q}_{i,i} \bar{h}_{i,i+1} - 2J \sum_{i=1}^{N} X^0_{i} + 2JX^0_{i}, \] (11)

where

\[ \bar{h}_{i,i+1} = \sum_{\alpha,\beta=0}^{3} X^\alpha_i X^{\beta}_i \]
\[ \bar{Q}_{i,i} = \sum_{\alpha,\beta=0}^{3} X^\alpha_i X^{\beta}_i. \]

In the above, $X^{\alpha}_i = |\alpha_i\rangle\langle\beta_i|$ are the Hubbard operators with $|\alpha_i\rangle$ being the orthogonalised eigenstates of the local operator $(\vec{\sigma}_i, \vec{\tau}_i)$.

Let us now introduce another spin ladder model with an impurity, whose Hamiltonian reads

\[ H = \sum_{k=1}^{i-1} h_{k,k+1} + \sum_{k=i+1}^{N} h_{k,k+1} + H_{imp} + \frac{J}{2} \sum_{k=1}^{N} (\vec{\sigma}_k \cdot \vec{\tau}_k - 1) + \frac{J}{2} (\vec{\sigma}_i \cdot \vec{\tau}_i - 1) \] (12)

where $H_{imp}$ represents the interaction of the chain with an impurity localized in the $\bar{i}$ position and is given by

\[ H_{imp} = \frac{4}{4 - \Lambda^2} \left\{ P_{i,i} + P_{i,i+1} - \frac{\Lambda^2}{4} P_{i,i+1} \right\} \]
\[ + \frac{2\Lambda}{4 - \Lambda^2} \left\{ P_{i,i+1}P_{i,i} - P_{i,i}P_{i,i+1} \right\} - \frac{2\Lambda}{4 - \Lambda^2} I_{i,i} \]

(13)

Above, $h_{k,k+1} = P_{k,k+1}$ represents the permutation operator between the lattice sites $k$ and $k + 1$ and has the form

\[ P_{k,k+1} = \frac{1}{4} \left( 1 + \vec{\sigma}_k \cdot \vec{\sigma}_{k+1} \right) \left( 1 + \vec{\tau}_k \cdot \vec{\tau}_{k+1} \right) \]

This term contains Heisenberg interactions along the legs and rung, as well as biquadratic interactions, whose physical importance has been pointed out in [23]. Here we also have three site interactions involving the lattice sites $i, i + 1$ as well as the impurity site $\bar{i}$.

Finally, we mention that Wang’s ladder system [3] with length $N + 1$ can be recovered by taking the limit $\Lambda = 0$. 

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The energy eigenvalues of the Hamiltonian (12) are given by

$$E = -2J + (1 - 2J)N + \frac{2}{2 + \Lambda} - 4 \sum_{i=1}^{M_1} \left( \frac{1}{\lambda_i^2 + 1} - \frac{J}{2} \right)$$  \hspace{1cm} (14)$$

where $\lambda_i$ are solutions to the Bethe ansatz equations below

$$\left( \frac{\lambda - i}{\lambda + i} \right)^N \left( \frac{\lambda - i(1 - \Lambda)}{\lambda + i(1 + \Lambda)} \right) = \prod_{i \neq i}^{M_1} \frac{\lambda - \lambda_i - 2i}{\lambda - \lambda_i + 2i} \prod_{j=1}^{M_2} \frac{\lambda_i - \mu_j + i}{\lambda_i - \mu_j - i}$$

$$\prod_{j \neq l}^{M_2} \frac{\mu_i - \mu_j - 2i}{\mu_i - \mu_j + 2i} = \prod_{i=1}^{M_1} \frac{\mu_i - \lambda_i - i}{\mu_i - \lambda_i + i} \prod_{k=1}^{M_3} \frac{\mu_i - \nu_k - i}{\mu_i - \nu_k + i}$$

$$\prod_{k \neq l}^{M_3} \frac{\nu_l - \nu_k - 2i}{\nu_l - \nu_k + 2i} = \prod_{j=1}^{M_2} \frac{\nu_l - \mu_j - i}{\nu_l - \mu_j + i}$$  \hspace{1cm} (15)$$

For $J > \frac{2}{\Lambda^2 + 1}$ (see eqs. (16, 17) below) the ground state is given by the product of rung singlets and the energy is $-2J + \frac{2}{2 + \Lambda} + (1 - 2J)N$. This is again the reference state used in the Bethe ansatz calculation and corresponds to the case $M_1 = M_2 = M_3 = 0$ for the BAE (15). To describe an elementary excitation, we take $M_1 = 1$ and $M_2 = M_3 = 0$ in the BAE (15), which leads to the following solution for the variable $\lambda^1$

$$\lambda = \frac{\Lambda}{4} - \frac{1}{24} \left\{ (1 + i\sqrt{3})K^{1/3} - \frac{1}{8}(1 - i\sqrt{3})(3\Lambda^2 + 16)K^{-1/3} \right\}$$  \hspace{1cm} (16)$$

with $K$ given by

$$K = 27\Lambda^3 + 12i\sqrt{81\Lambda^4 + 432\Lambda^2 + 768}.$$  

Notice that the solution of the BAE $\lambda$ depends explicitly on the impurity $\Lambda$. This relation, although complicated, is illustrated in Fig. 1 below.

The energy gap can be calculated using the Bethe ansatz solution and has the form

$$\Delta = 2\left( J - \frac{2}{\Lambda^2 + 1} \right).$$  \hspace{1cm} (17)$$

Here, in contrast to the previous case, the impurity does affect the gap. By solving $\Delta = 0$ for $J$ we find the critical value $J_c = \frac{2}{\Lambda^2 + 1}$, indicating the critical line at which the quantum phase transition from the dimerized phase to the gapless phase occurs. The phase transition line is shown in Fig. 2.

Notice that by increasing the impurity $\Lambda$, the critical value $J_c$ also increases. A further analysis of both graphs together with the gap expression (17) reveals that there is a reduction of the gap by increasing $\Lambda$. This result can be easily confirmed by inspection of Fig. 3.

The exact solvability of this model, as for the previous case, can be shown by the fact that for $J = 0$ there is a $R$-matrix given by equation (18) obeying Yang-Baxter algebra

$$R_{12}(x - y)R_{13}(x)R_{23}(y) = R_{23}(y)R_{13}(x)R_{12}(x - y).$$  \hspace{1cm} (18)$$

\text{strictly, the lattice length is assumed odd, such that in the limit where there is no impurity ($\Lambda = 0$) the correct gap expression discussed in [5] can be recovered}
Figure 1: This graph shows how the solution of the BAE ($\lambda^2$) depends on the impurity $\Lambda$.

Figure 2: Rung coupling $J$ versus impurity $\Lambda$. This graph represents the phase diagram. The curve ($J^c = 2/($$\lambda^2 + 1$)) divides the gapped and gapless phases.
Figure 3: This graph shows how the spin gap $\Delta$ depends on the impurity $\Lambda$ for different values of the rung coupling $J$.

By the standard procedure the global Hamiltonian reads

$$H = \frac{d}{dx} \ln(\tau(x, \Lambda))|_{x=0}$$

(19)

where $\tau(x, \Lambda)$ is the transfer matrix

$$\tau(x, \Lambda) = \text{tr}_0(R_{0,1}(x) \ldots R_{0,i}(x)R_{0,i}(x - \Lambda)R_{0,i+1}(x) \ldots R_{0,N}(x))$$

(20)

The shift that appears in $R_{0,i}(x - \Lambda)$ represents the impurity.

The Hamiltonian (12) can be mapped, as in the previous case to the Hamiltonian that is obtained by the standard procedure from $R$-matrix (5) that obeys the Yang-Baxter algebra for $J = 0$, while for $J \neq 0$ the rung interaction take the form of a chemical-potencial term.

To summarize, we have introduced two spin ladder models with an impurity $\Lambda$. In the limit of vanishing impurity both models reduce to that introduced by Wang $[$5$]$ based on the $su(4)$ symmetry. The Bethe ansatz equations as well as the energy expression for the models were presented. We have shown that in one of the cases the impurity affects the gap of the system non-trivially, while in the other case there is no dependence.
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