Exploring the Energy-dependent Radiation Properties in Dissipative Magnetospheres with Fermi Pulsars

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Abstract

The equatorial current sheets outside the light cylinder (LC) are thought to be promising sites of high-energy emission based on the results of recent numerical simulations. We explore the pulsar light curves and energy spectra by computing the curvature radiation based on the FIDO magnetospheres. The FIDO magnetospheres with a near force-free regime inside the LC and a finite but high conductivity outside the LC are constructed using a spectral algorithm. The pulsar high-energy emission properties are explored by integrating the trajectories of the test particles under the influence of both the accelerating electric field and the curvature radiation losses. As an application, we compare the predicted energy-dependent light curves and energy spectra with those of the Crab and Vela pulsars published in the Fermi 2PC catalog. We find that the observed characteristics of the light curves and energy spectra from the Crab and Vela pulsars can be well reproduced by the FIDO model.

Unified Astronomy Thesaurus concepts: Neutron stars (1108); Rotation powered pulsars (1408); Magnetohydrodynamical simulations (1966)

1. Introduction

Pulsars are rapidly rotating and highly magnetized neutron stars, which were discovered almost half a century ago (Hewish et al. 1968). More than 230 \(\gamma\)-ray pulsars have been detected by Fermi-LAT, in which 117 pulsars are listed in the Second Fermi Pulsar Catalog (2PC; Abdo et al. 2013). The observed typical features of the gamma-ray pulsars are the double-peaked light curves with significant bridge emission and offset emission, and the first peak commonly lags the radio pulse by several rotation periods. The pulsar \(\gamma\)-ray light curves can be used to constrain the location of particle acceleration and radiation mechanisms in the magnetosphere. The high-quality phase-averaged, and phase-resolved spectra provide us with valuable information to study the physical pulsar mechanisms in the magnetosphere. However, the precise locations where particles are accelerated and the mechanisms of how they are accelerated are still unclear.

In the early days of the study of pulsars, the pulsar magnetosphere was usually treated as a vacuum-retarded dipole (VRD; Deutsch 1955). Based on the VRD field geometry, the polar cap (PC; e.g., Ruderman & Sutherland 1975; Harding et al. 1978; Daugherty & Harding 1982, 1996), slot gap (SG; e.g., Arons & Scharlemann 1979; Arons 1983; Dyks & Rudak 2003; Muslimov & Harding 2003, 2004; Dyks et al. 2004; Bai & Spitkovsky 2010a), and outer gap (OG; e.g., Cheng et al. 1986a, 1986b, 2000; Yadigaroglu 1997; Zhang & Cheng 1997, 2000, 2001; Bai & Spitkovsky 2010a) models were proposed for modeling pulsar light curves and energy spectra. The VRD field can produce an accelerating electric field on the surface of the star. Such an accelerating electric field pulls the particles from the neutron star surface to fill the magnetosphere by pair cascades; these particles short out the electric field to form the force-free (FF) magnetosphere (Goldreich & Julian 1969).

Following the work of Goldreich & Julian (1969), Scharlemann & Wagoner (1973) deduced the well-known pulsar equation expressed by the poloidal magnetic flux for an aligned rotator.

Many attempts by several groups were made to solve the pulsar equation (e.g., Michel 1973a, 1973b; Endean 1974). In 1999, Contopoulos et al. (1999, hereafter CKF) first solved the pulsar equation using an iterative algorithm and obtained the CKF solution. The CKF solution was further explored by many groups (e.g., Contopoulos 2005; Gruzinov 2005; Komissarov 2006; McKinney 2006; Timokhin 2006; Yu 2011; Parfrey et al. 2012; Cao et al. 2016a). The CKF solution consists of a closed region extending to the light cylinder (LC), an open region with asymptotically monopolar magnetic field lines, and an equatorial current sheet outside the LC.

The time-dependent simulation of FF pulsar magnetospheres for an oblique rotator was first performed using the finite-difference time-domain approach by Spitkovsky (2006). The algorithm of Spitkovsky (2006) was then improved on by Kalapotharakos & Contopoulos (2009) by implementing nonreflecting absorbing boundaries so that a final steady-state solution can be reached when evolving many stellar periods. The pseudo-spectral method was also developed to simulate the 3D FF magnetosphere (Parfrey et al. 2012; Pétri 2012; Cao et al. 2016a). All of these simulations converge to a similar CKF magnetosphere with an equatorial current sheet outside the LC. The FF approximation is also extended to the full magnetohydrodynamic regime where the plasma inertia and pressure are taken into account (Tchekhovskoy et al. 2013) and to the general-relativistic regime that takes into account the time-space curvature and frame-dragging effects (Pétri 2016; Carrasco et al. 2018). The FF solution provides a different field structure compared to the vacuum one. The pulsar light curves are also explored by assuming the location of the zone of acceleration based on the FF field (e.g., Bai & Spitkovsky 2010b; Contopoulos & Kalapotharakos 2010; Harding & Kalapotharakos 2015; Chen et al. 2020).

The FF solution is nondissipative and thus precludes the production of pulsed emission in the magnetosphere. A more realistic magnetosphere should allow the local dissipation to accommodate the production of radiation in some regions. Therefore, a resistive magnetosphere was proposed to model...
the pulsar magnetosphere by involving a macroscopic conductivity parameter (Kalapotharakos et al. 2012a; Li et al.
2012; Cao et al. 2016b). The resistive solution ranges from the VRD to FF magnetospheres with increasing conductivity, and the dissipative region appears in the equatorial current sheet outside the LC at high conductivities (e.g., Cao et al. 2016b).

The resistive magnetosphere was also used to study the high-energy phenomena of pulsars (Kalapotharakos et al. 2012b, 2014; Brambilla et al. 2015; Cao & Yang 2019). Recently, the particle-in-cell (PIC) method was used to model the pulsar magnetosphere by self-consistently treating the particle motions and the electromagnetic fields (e.g., Chen & Beloborodov 2014; Philippov & Spitkovsky 2014; Belyaev 2014; Philippov & Spitkovsky 2014; Kalapotharakos et al. 2018). The pulsar light curves are explored by including the radiation reaction in the PIC code (e.g., Cerutti et al. 2016; Kalapotharakos et al. 2018; Philippov & Spitkovsky 2018).

In the previous works, the γ-ray light curve in the dissipative magnetosphere—the FIDO magnetosphere with a near-FF regime inside the LC and finite conductivity outside the LC—is produced by collecting the bolometric luminosity from all emitting particles (Kalapotharakos et al. 2014; Cao & Yang 2019). However, these studies did not compute the curvature spectrum from the individual particles, and the light curves are only produced by collecting the bolometric luminosity from all the emitting particles. Brambilla et al. (2015) explored the impact of the conductivity parameter on the light curves and spectra in the FIDO magnetospheres by using an approximate expression for the accelerating electric field. Recently, Kalapotharakos et al. (2017) further refined the FIDO models and calculated the curvature radiation spectra using the realistic accelerating fields given by the models.

In this paper, we further explore the γ-ray-energy-dependent radiation patterns by extending the study of Cao & Yang (2019) and computing the curvature spectra from the emitting particles. In Section 2, we describe the dissipative magnetosphere model. In Section 3, we describe how the test particles are injected and tracked. In Sections 4 and 5, we elaborate on the modeling of the curvature spectra, and how the sky maps and light curves are produced. In Section 6, the FIDO magnetospheres are applied to the Crab and Vela pulsars, their sky maps, light curves, and spectra are produced. Finally, the conclusions and discussions are listed in Section 7.

2. The Dissipative Magnetosphere

A dissipative magnetosphere can be obtained by solving the time-dependent Maxwell equations:

\[
\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E},
\]

\[
\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \frac{4\pi}{c} \mathbf{J},
\]

\[
\nabla \cdot \mathbf{B} = 0,
\]

\[
\nabla \cdot \mathbf{E} = 4\pi \rho,
\]

where \( \mathbf{B} \) is the magnetic field, \( \mathbf{E} \) is the electric field, \( \rho \) is the charge density, and \( \mathbf{J} \) is the current density. In the dissipative magnetosphere, the current density \( \mathbf{J} \) ensuring the closure of the system is defined as a form of Ohm’s law using the local electromagnetic fields with a conductivity parameter (Kalapotharakos et al. 2014; Cao et al. 2016b):

\[
J = c\rho \frac{\mathbf{E} \times \mathbf{B}}{B^2 + E_0^2} + \sigma \mathbf{E} |, \]

where the first term in Equation (5) is the drift velocity perpendicular to the magnetic fields, the term \( E_0 \) in the denominator ensures the drift velocity is subluminal and satisfies the following conditions: \( B_0^2 - E_0^2 = B^2 - E^2 \), \( E_0B_0 = E \cdot B \), \( E_0 \geq 0 \). The second term, \( \mathbf{E} \cdot \mathbf{B}/B \), is the accelerating electric field parallel to the magnetic field, which is consistently controlled by the conductivity parameter \( \sigma \).

In this paper, the FIDO magnetospheres with the near-FF regime (the conductivity fixed as 60Ω) inside the LC and a dissipative regime of finite conductivity \( \sigma \) outside the LC are constructed by the spectral method, where any \( E_\parallel \) component within the LC is discarded (see also Kalapotharakos et al. 2014; Cao & Yang 2019). The neutron star is treated as a perfect conductor with a magnetic moment in the center of the star. The boundary condition on the star surface is enforced by a corotating electric field \( \mathbf{E} = -(\Omega \times \mathbf{r}) \times \mathbf{B}/c \). We evolve the Maxwell equations for a series of magnetic inclination angles \( \alpha \) ranging from 15° to 90° with an interval of 5°. The conductivities \( \sigma \) outside the LC are set to 0.3Ω, 3Ω, 10Ω, and 30Ω. The computational domain extends from the star surface \( r_s = 0.2R_{1.5} \) up to \( r_{max} = 3R_{1.5} \). Better accuracy can be obtained with the resolution of \( N_r \times N_\theta \times N_\phi = 128 \times 64 \times 128 \). Moreover, the magnetospheres are obtained after several pulsar spin periods to reach the final stable state. The \( E_\parallel \) values outside the LC provided by the FIDO solutions are used to compute the curvature radiation. This is very different from the ones used in Kalapotharakos et al. (2014) and Brambilla et al. (2015), in which they used an approximate expression to produce the accelerating electric field based on the corresponding FF solutions.

3. Particles Injection and Acceleration

The realistic pulsar magnetospheres are filled with abundant electron/positron pairs, which are accelerated by the parallel electric fields to relativistic velocities and radiate γ-ray photons. In order to imitate the behavior of the electron/positron in the dissipative magnetospheres, a set of \( \sim 1.5 \times 10^5 \) electron/positron pairs with small initial Lorentz factors \( (\gamma_{ini} \lesssim 100) \) are randomly ejected from the PCs. We track the test particles from the neutron surfaces to 2.5\( R_{1.5} \) to produce the pulsar light curves and spectrum by including both the accelerating electric field and curvature radiation losses. The trajectory of the particles (Kalapotharakos et al. 2014; Cao & Yang 2019) in the inertial observe frame (IOF) is given by the local electromagnetic fields,

\[
v \equiv \frac{dx}{dt} = \left( \frac{\mathbf{E} \times \mathbf{B}}{B^2 + E_0^2} + f \frac{\mathbf{B}}{B} \right) c,
\]

where the first term in Equation (6) is the drift velocity, while the second term is the velocity component parallel to the magnetic fields. The sign and the value of the scalar factor \( f \) are determined by setting \( v \approx c \) and ensuring that the particle motions are always outward.
4. Modeling of the Curvature Radiation

Once the particle trajectories are determined, the Lorentz factors $\gamma$ of the radiating particle along each trajectory can be calculated by integrating the following expression:

$$\frac{d\gamma}{dt} = f q_e E_{||} - \frac{2q_e^2 \gamma^4}{3R_{CR}^2 m_e c^2},$$

(7)

where the first term in Equation (7) is the energy gain rate of the particles due to the accelerating electric field, and the second term is the energy loss rate due to curvature radiation (CR) reaction. $q_e$ and $m_e$ are the electron charge and rest mass, and $E_{||}$ is the component of the accelerating electric fields provided by the solutions themselves. $R_{CR}$ is the local curvature radius at each point of the trajectory, which is calculated in the IOF using

$$R_{CR} = \frac{dl}{d\theta},$$

(8)

where $dl$ is the segment length along the particle trajectory, while $d\theta$ is the angle between two adjacent velocities. The expression of the curvature radius used in the paper is essentially the same as the one given by Harding & Kalapotharakos (2015), but in different forms. Recently, a similar approach to determine the curvature radius was also used by Viganò & Torres (2020), introducing the differential geometry Frenet–Serret equations to express the trajectories of the particles. The equilibrium $\gamma_L$ values balanced by the acceleration of $E_{||}$ and the CR loss can be obtained from the equilibrium of the radiative reaction region ($d\gamma/dt = 0$) from

$$\gamma_L^4 = \frac{3|E_{||}| R_{CR}^2}{2q_e^2},$$

(9)

we also note that the $\gamma_L$ values are weakly affected by $E_{||}$ and $R_{CR}$ when the equilibrium states are reached.

The energy spectrum of the curvature radiation from a single particle at each radiating location $r$ with the Lorentz factor $\gamma$ is calculated by integrating the expression (Tang et al. 2008)

$$F(E_{\gamma}, r) = \frac{\sqrt{3} e^2 \gamma^\alpha}{2\pi \hbar R_{CR} E_{\gamma}} F(x),$$

(10)

where $x = E_{\gamma}/E_{\text{cur}}$, $E_{\gamma}$ is the energy of the emitting photon, $E_{\text{cur}} = \frac{3}{2} c R_{CR}^2$ is the characteristic energy of the curvature radiation photon, and the function $F(x)$ is defined as

$$F(x) = x \int_x^\infty K_{5/3}(\xi) \, d\xi,$$

(11)

where $K_{5/3}$ is the modified Bessel function of order 5/3. The function $F(x)$ is calculated by using the approximate expression given by Aharonyan et al. (2010). In fact, we only sample $\sim 1.5 \times 10^9$ particles from the stellar surface, which do not reflect real particle numbers in the pulsar magnetospheres. Therefore, we weight the individual flux of the curvature radiation by the surface charge density $\rho_s$.

In Figure 1, we plot the maximum Lorentz factor $\gamma_{\text{max}}$ values of the particles along the trajectories onto the PC for $\alpha = 45^\circ$ and $\sigma = 1 \Omega$, we find that larger Lorentz factors ($\gtrsim 10^7$) come from the trajectories both on the edge and within the interior of the PC, and that the maximum Lorentz factors reaching up to ($\gtrsim 10^8$) are almost coming from the particles originating from the edge of the PC. This is due to the fact that, for much lower conductivity values, especially for $\alpha < 45^\circ$ and $\sigma < 1 \Omega$, the FIDO magnetospheres will significantly deviate from the FF geometry; the equatorial current sheet and the high accelerating electric field will also be destroyed. The high-acceleration electric field will be distributed in a wider space outside the LC. Similar phenomena were also noticed by other groups (Kalapotharakos et al. 2014; Brambilla et al. 2015; Cao & Yang 2019). Therefore, the particles both originating from the edge and the interior of the PC will encounter the high-acceleration electric fields and be accelerated to larger Lorentz factors to radiate GeV photons.

Moreover, we note that, as the conductivity increases, the distribution of the accelerating electric fields will gradually contract and concentrate mainly near the equatorial current sheet outside the LC, and the particles contributing to the GeV emission will almost be the ones starting from the edges of the PCs. We also note that the lower (larger) the conductivity values outside the LC, the larger (lower) the Lorentz factors. However, for much lower conductivity values ($\ll 30 \Omega$), the Lorentz factors will be expected to scarcely exceed $10^6$, and this will lead to MeV emission. These results are due to the fact that the larger conductivity values always restrain the extension and strength of the accelerating electric fields in the equatorial current sheet. Therefore, the appropriate values of conductivity used to generate the light curves and GeV photons consistent with the ones observed are important in the FIDO models. Similar conclusions are also reached by Kalapotharakos et al. (2014) and Cao & Yang (2019).
edges, which are the origin of the dissipative regions outside the LC, where almost all of the effective radiation particles are coming from. Because particles originating from the PC edges will reach the equatorial current sheet around the rotating equator, they will encounter the high $E_{||}$ values and be quickly accelerated to the $\gamma_{\text{max}}$ values. We also see that the distributions of the $\gamma_{\text{max}}$ values are asymmetric around the PC edges and all of the $\gamma_{\text{max}}$ values are almost concentrated on the leading side of the PC edges. The asymmetries of the high $\gamma_{\text{max}}$ values around the PC also indicate that the effects of the equatorial current sheet on the particles are not symmetrical. Moreover, we also note that distributions of the high $\gamma_{\text{max}}$ values shrink with increasing $\alpha$ for a fixed $\sigma$ due to the decreasing dissipative region in the current sheets (Kalapotharakos et al. 2014; Cao & Yang 2020).

In the bottom row of Figure 2, we plot the 3D volume rendering of the equatorial current sheet outside the LC and the projections of the Lorentz factor along their trajectories, in the corotating frame, for a sample of 300 particles. We see that the larger Lorentz factors are produced by the trajectories originating from the PC edges and reaching around the equatorial current sheet. Moreover, the standard pulsar parameters $P = 0.1$ s and $B = 10^{12}$ G are used.

5. The Energy-dependent Sky Maps and Light Curves

The sky maps and light curves are produced by collecting the curvature photon spectrum. We construct the sky maps and light curves by collecting the curvature photons from all emitting particles. To construct the pulsar light curves, we need to determine the emission direction of the photon in the IOF. The emission direction of the photon $\eta_{\text{em}}$ is assumed to be along the direction of particle motion $\beta = v/c$ in the IOF, and the viewing angles and observing (azimuthal) phase are defined by $\zeta$ and $\phi$, respectively, relative to the pulsar rotation axis

$$\zeta = \cos(\beta_z)$$

and

$$\phi = \phi_{\text{rot}} - \phi_{\text{em}} - \eta_{\text{em}} / R_{1C},$$

where the effects of the field rotation and time delay are taken into account. $\phi_{\text{rot}} = \Omega t$ is the rotation phase, $\phi_{\text{em}} = \arctan(\beta_z / \beta_\parallel)$ is the phase of the emitting photon, and the third term is the phase from the time-delay correction.

For a given energy region (say 0.1–50 GeV), the curvature photons ($N_{\text{ph}}$) from all emitting particles at the distance $r$ along their trajectories are collected by integrating the curvature spectrum in the energy interval $(E_\gamma, E_\gamma' )$,

$$N_{\text{ph}}(r) = \int_{E_\gamma}^{E_\gamma'} F(E_\gamma, r) dE_\gamma.$$  

In our calculations, the observed phase $\phi$ is uniformly divided into 100 bins (between 0° and 360°), 180 bins for $\zeta$ (between 0°
and 180°), and 220 bins for the energy (between 0.1 and 50 GeV). Here, a Gaussian profile is used to smooth the collected curvature photons with \( \Delta \zeta = 4° \) at viewing angle \( \zeta^0 \) by

\[
N_{ph} \propto \exp \left( -\frac{(\zeta - \zeta^0)^2}{2\Delta \zeta^2} \right). \tag{15}
\]

The sky maps are produced by collecting all of the curvature photons from each emitting particle, the light curves are then obtained by cutting the sky maps in a fixed viewing angle \( \zeta^0 \). We find that the sky maps and light curves are very sensitive to the \( \sigma \) values. At lower \( \sigma \) values, the light curve is broad in shape and has only one peak, which does not match most of the pulsars published in Fermi 2PC (also see Kalapotharakos et al. 2014; Cao & Yang 2019). As the \( \sigma \) values increase, however, the peaks become narrower and more double-peaked light curves will appear.

In Figure 3, we give the sky maps and light curves in the energy band (0.1, 50 GeV) for the FIDO magnetosphere with \( \sigma = 30 \) outside the LC for the magnetic inclination angles \( \alpha = 45°, 60°, 75°, 90° \) and viewing angles \( \zeta = 30°, 45°, 60°, 75°, 90° \). The standard pulsar parameters \( P = 0.1 \) s and \( B_\ast = 10^{12} \) G are adopted. We find that the light curves tend to the double-peaked profiles toward increasing \( \alpha \) and/or \( \zeta \) direction, some of which are similar to the observed ones published in the Fermi 2PC. The light curves are similar to the ones obtained by collecting the bolometric luminosity in larger viewing angles, while they are different from those in lower viewing angles (Cao & Yang 2019) for the spectral difference of the phase. Similar conclusions are also found in Kalapotharakos et al. (2014) and Brambilla et al. (2015).

Figure 3. The sky maps and light curves for FIDO magnetospheres with \( \sigma = 30 \) \( \Omega \) outside the LC for a series of magnetic inclinations \( \alpha = 45°, 60°, 75°, \) and 90° (from top to bottom). The light curves in each row are obtained by cutting \( \zeta = 30°, 45°, 60°, 75°, \) and 90° (from left to right). The pulsars parameters \( P = 0.1 \) s and \( B_\ast = 10^{12} \) G are used.
6. The Energy-dependent LCs and the Energy Spectra for the Crab and Vela Pulsars

In order to further constrain the model parameters, the emission characteristics of the Crab and Vela pulsars are studied using our FIDO model. Their energy-dependent sky maps and light curves, phase lag $\delta$\,, peak width $\Delta$, phase-averaged spectra, luminosity $L_\gamma$, cutoff energy $E_{\text{cut}}$, the spectral index $\Gamma$ are produced and compared with the ones observed by Fermi.

The model luminosity of the pulsar is determined by the following expression,

$$L_\gamma = 4\pi d^2 f_\Omega \int_{0.1}^{50} E_\gamma F(E_\gamma) dE_\gamma,$$

(16)

where $d$ is the real distance from the pulsars from 2PC, $f_\Omega$ is the beam correction factor, which we choose to be 1 because the outer magnetosphere fan-like beam sweeping the entire sky gives $f_\Omega \approx 1$ (Abdo et al. 2013). $F(E_\gamma)$ is the total energy flux in the dissipative region determined by

$$F(E_\gamma) = \frac{1}{\Delta\Omega d^2} \sum_r F(E_\gamma, r),$$

(17)

where $\Delta\Omega$ is the solid angle, which we choose to be $4\pi$ sr. The model cutoff energy $E_{\text{cut}}$ and spectral index $\Gamma$ from the phase-averaged spectrum are obtained by fitting the model differential spectrum with an exponential cutoff power law (Abdo et al. 2013):

$$\frac{dN}{dE} = K\left(\frac{E}{E_0}\right)^{-\Gamma} \exp\left(-\frac{E}{E_{\text{cut}}}\right)^b,$$

(18)

where $K$ is the normalization factor, $\Gamma$ the photon index, $E_{\text{cut}}$ the cutoff energy, $b$ the sharpness of the cutoff which is fixed to 1 in the paper, and the energy $E_0$ at which $K$ is defined is arbitrary.

In Figures 4 and 5, we compare the predicted energy-dependent light curves with those published in Fermi 2PC for

Figure 4. The evolution of the energy-dependent sky maps and the light curves of the Crab pulsar for $\alpha = 75^\circ$ and $\sigma = 10\Omega$ outside the LC. Top panels are the sky maps for energy bands of (0.1, 50 GeV), (3, 50 GeV), (1, 3 GeV), (0.3, 1 GeV), and (0.1, 0.3 GeV), from left to right, respectively. Bottom panels: corresponding model light curves (magenta) are obtained by cutting the sky maps in $\zeta = 50^\circ$ and the observed light curves (red) taken from 2PC (Abdo et al. 2013).

Figure 5. Same as the panels in Figure 4, but for the Vela pulsar $\alpha = 60^\circ$, $\sigma = 30\Omega$, and $\zeta = 42^\circ$. 
the Crab and Vela pulsars, respectively. The evolutionary patterns of the energy-dependent observed light curves with the energy bands can be well explained by the FIDO models for both Crab and Vela. The dependence of the relative ratio between the two peaks on the photon energy are well reproduced by our FIDO model. We also find that the sky maps in different energy bands are not changed predominantly; similar results are also found by Pétri (2020) in the VRD magnetic fields.

Further, for the Vela pulsar, we note that the ratio of the intensity of the first peak to that of the second peak decreases as the photon energy increases. This may due to the slightly different curvature radius along the trajectories of the particles contributing to the two peaks, separately. We find that the particles contributing to the second peak will collectively possess larger $R_{CR}$ than that to the first peak in the equatorial current sheet. In the CR reaction limit, the CR cutoff energy $E_c$ is related by $E_{||}$ and the curvature radius $R_{CR}$

$$E_c \propto E_{||}^{3/4} R_{CR}^{1/2}.$$  

(19)

Even though this limit is not reached, the $E_c$ values are also expected to be larger at the second peak of the light curve than at the first peak, which will give a larger cutoff energy for the second peak than the first peak, consistent with the observed data (DeCesar et al. 2011). Similar results are also found by Brambilla et al. (2015) in similar FIDO models and by Barnard et al. (2018) in the 3D FF magnetospheres using the SG model. Moreover, Kalapotharakos et al. (2017) even explored the dependence of $E_c$ on the spin-down rate, which put more constraints on the FIDO models.

In Figure 6, we plot the phase-averaged spectra, and the corresponding model differential spectra are fitted to obtain the photon index and the cutoff energy for the Crab (top row) and
Vela (bottom row) pulsars, respectively. We see that the model CR phase-averaged spectra are well consistent with the Fermi data. Moreover, the shapes of the model spectra are very similar to the ones obtained by Harding & Kalapotharakos (2015) and Pétri (2020) in the FF and VRD magnetic fields, separately. The parameters adopted to produce the modeling results for the FIDO models and the ones observed by Fermi are listed in the Appendix. From which we can see that our FIDO models can better produce the results that are consistent with the ones observed. We note that the model results, i.e., the evolutionary patterns of the light curves with energies, the cutoff energy, and the spectral index for the Crab and Vela pulsars, are consistent with those obtained by Brambilla et al. (2015).

7. Conclusions and Discussions

In this paper, we study the pulsar γ-ray light curves and spectra based on the FIDO magnetospheres. The FIDO magnetospheres with a near-FF regime inside the LC and finite but high conductivity outside the LC are constructed by a spectral algorithm. We expand the study of Cao & Yang (2019) by computing the curvature radiation spectrum. A realistic particle trajectory is defined by using the FIDO field structures. The particle Lorentz factors along each trajectory are computed under the effects of the accelerating electric field and curvature radiation losses. The γ-ray sky maps and light curves are then produced by collecting the curvature photons from all emitting particles. Our results show that the distributions of $\gamma_{\text{max}}$ are asymmetric around the PC edges, and all $\gamma_{\text{max}}$ values are almost concentrated on the leading side of the PC edges, and that the higher Lorentz factors are coming from particles whose trajectories reach close to the equatorial current sheet outside the LC. The asymmetries of high $\gamma_{\text{max}}$ values around the PC also indicate that the effects of the equatorial current sheet on the particles are not symmetrical. As an application, we compare the predicted light curves and energy spectra with those of the Crab and Vela pulsars observed by Fermi. We find that the light curves and energy spectra from the Crab and Vela pulsars can be well reproduced by the FIDO model. For the Vela pulsar, the relative ratio of the first peak to the second peak decreases with increasing energy. This may due to the geometrical differences of the trajectories of the particles contributing to the two peaks, respectively.

In fact, the origin of the conductivity parameter $\sigma$ is not yet completely understood. It is usually assumed that $\sigma$ is constant in some region of the magnetosphere, which may be a strong constraint. A possible cause is that $\sigma$ is a function of the distance from the neutron star (Kato 2017). Therefore, it is worth studying the feature of the conductivity parameter in the future. We also note that there are no back-reaction of photons onto radiative particles in the resistive model. A better approximation is the radiation reaction limit (called Aristotelian electrodynamics), where particle acceleration is fully balanced by radiation. Dissipative pulsar magnetospheres with a radiation reaction limit have been presented in our other paper, Cao & Yang (2020). The accelerating electric field is found to be restricted to the current sheet, and the acceleration region is self-consistently controlled by the pair multiplicity. In the next step, we will use the dissipative magnetosphere with the radiation reaction limit to explore the influence of pair multiplicity on the pulsar γ-ray emission.

It is also noted that the current numerical simulation still cannot resolve realistic ratios of the stellar to LC radius. A large ratio with $R_* / R_{\text{LC}} = 0.2$ corresponding to a 1 ms pulsar is used to model the light curves and energy spectra for all pulsars (Kalapotharakos et al. 2014; Brambilla et al. 2015; Cao & Yang 2019). The spectral algorithm can allow us to look deeply into the magnetosphere with physically realistic ratios of the stellar to LC radius, and the effect of the ratios of the stellar to LC radius on the pulsar magnetosphere will be explored with higher-resolution simulation using the spectral algorithm in our future work.

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Appendix

Below, we list the comparison between the Fermi-observed parameters and those given by our FIDO model for the Crab and Vela pulsars. The units for the period $P$, magnetic field $B$, distance $d$, luminosity $L_{\gamma}$, and the cutoff energy $E_{\text{cut}}$ are second (s), Gauss (G), kpc, erg cm$^{-2}$ s$^{-1}$, and GeV, respectively.

Appendix A

Crab Pulsar

(I) The Fermi-observed parameters: $P = 0.033, B = 3.8 \times 10^{12}$, $d = 2.0, E_{\text{cut}} = 4.2, \Gamma = 1.9, L_{\gamma} = 6.2 \times 10^{35}$, $\delta = 0.12$, and $\Delta = 0.40$.

(II) The model-adopted parameters: $E_{\text{cut}} = 4.2, \Gamma = 1.5, L_{\gamma} = 1.31 \times 10^{35}$, $\delta = 0.12$, and $\Delta = 0.45$.

Furthermore, the geometric parameters used in the models are $P = 0.033, B = 4 \times 10^{12}$, $d = 2.0$, magnetic inclination angle $\alpha = 75^\circ$, $\sigma = 10$, and viewing angle $\zeta = 50^\circ$.

Appendix B

Vela Pulsar

(I) The Fermi-observed parameters: $P = 0.089, B = 4 \times 10^{12}$, $d = 0.29, E_{\text{cut}} = 3.0, \Gamma = 1.5, L_{\gamma} = 8.9 \times 10^{34}$, $\delta = 0.14$, and $\Delta = 0.43$.

(II) The model-adopted parameters: $E_{\text{cut}} = 4.3, \Gamma = 1.5, L_{\gamma} = 6.9 \times 10^{34}$, $\delta = 0.15$, and $\Delta = 0.40$.

Furthermore, the geometric parameters used in the models are $P = 0.1, B = 4 \times 10^{12}$, $d = 0.29$, magnetic inclination angle $\alpha = 60^\circ$, $\sigma = 30$ and viewing angle $\zeta = 42^\circ$.

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