Multiplicity fluctuation and correlation of identified baryons in quark combination model

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The dynamical fluctuation and correlation of multiplicity distributions of identified baryons and antibaryons produced by the hadronization of the bulk quark system are systematically studied in quark combination model. Starting from the most basic dynamics of quark combination which are necessary for multiplicity study, we analyze moments (variance, skewness and kurtosis) of inclusive multiplicity distribution of identified baryons, two-baryon multiplicity correlations, and baryon-antibaryon multiplicity correlations after the hadronization of quark system with given quark number and antiquark number. We obtain a series of interesting findings, e.g., binomial behavior of multiplicity moments, coincide flavor dependent two-baryon correlation and universal baryon-antibaryon correlation, which can be regarded as general features of quark combination. We further take into account correlations and fluctuations of quark numbers before hadronization to study their influence on multiple production of baryons and antibaryons. We find that quark number fluctuation and flavor conservation lead to a series of important results such as the negative $p\Omega^-$ multiplicity correlation and universal two-baryon correlations. We also study the influence of resonance decay in order to compare our results with future experimental data in ultra-relativistic heavy ion collisions at LHC.

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I. INTRODUCTION

In ultra-relativistic heavy ion collisions, a new state of the matter — Quark Gluon Plasma (QGP) is created at the early stage of collisions. The produced QGP expands, cools and changes into a hadron system at a critical energy density [1]. Because of the non-perturbative difficulty of quantum chromodynamics, the transition from QGP to hadrons (i.e. hadronization) can only be described currently by phenomenological models such as statistical hadronization models [2, 3] and quark (re-)combination/coalescence models [4–10]. These models have been tested against the available experimental data of hadronic yields, momentum spectra and flows.

Dynamical correlation and fluctuation of multi-hadron production carry more sophisticated hadronization dynamics. They are quantified by various covariances and moments in multiplicities or momenta of identified hadrons, and are measured in experiments via event-by-event method. Study of them can further test those existing phenomenological models of hadron production at hadronization and gain deep insights on dynamics of realistic hadronization process. We can also obtain the information of the correlations and fluctuations of quarks and antiquarks just before hadronization by studying their projection on hadronic observables. On the other hand, study of identified hadrons is also helpful for the investigation of correlation and fluctuation of conservative charges which is a recently hot topic both in experimental and theoretical studies [11–15]. There one should know how conservative charges populate in various identified hadrons, which depends on their coherent abundances and thus is directly related their multiple production dynamics at hadronization.

In the past few years, only the experimental data of pion, kaon and proton are reported [16–18] and available theoretical studies/predictions are mainly of them usually based on statistical model [19–24]. With the improvement of statistics and experimental measurement precision, observation of more hadron species such as $\Lambda$, $\Xi^-$ and $\Omega^-$ can be expected in the near future. Therefore, the corresponding theoretical predictions by different hadron production models are necessary, which are used to guide the experimental data analysis, reveal the underlying dynamics of the observation and test these models.

In this paper, we study the multiplicity fluctuations and correlations of various identified baryons and antibaryons produced directly by the hadronization. We focus on the $J^p = \frac{1}{2}^+$ and $\frac{3}{2}^-$ baryons in flavor SU(3) ground state with particular emphasis on various strange baryons. There are obvious advantages in measuring these baryons: (1) baryon is a sensitive probe of hadron production mechanism at hadronization. (2) rapidity shift in baryon production and in baryon resonance decays is small, which is suitable to experimental observation at finite rapidity window size.

We use the quark combination mechanism (QCM) to describe the production of hadrons at quark system hadronization. QCM has been used to reproduce lots of low and intermediate $p_T$ data at RHIC and LHC experiments, in particular the data of yields and rapidity distributions [10, 25–27]. The related entropy and pion production issues have been extensive addressed in literatures [28–31]. Explaining the fluctuations and/or correlations of hadron production is very intuitive in QCM. When a quark hadronizes, it can either come into a baryon or into a meson, which leads to the fluctuations of global baryon multiplicity; it can either come into a specific baryon (e.g. into a proton for a $u$ quark hadronization) or into another specific baryon (e.g. a $\Delta^+$), which leads to the fluctuations of proton and $\Delta^+$ multiplicity and also an anti-correlation between two baryons. In addition, correlations and fluctuations of quarks and antiquarks will also pass to hadrons.

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after hadronization.

Concretely, we calculate various moments of inclusive multiplicity distributions of baryons, e.g. variance, skewness and kurtosis, the correlations between two baryons and correlations between baryons and antibaryons. We analyze the dominant dynamics among these correlations and fluctuations and give predictions of QCM that can be tested by the future experimental data. This paper mainly discuss baryon production at zero baryon number density at LHC, and the extension to RHIC energies and meson sector is the goal of future work.

The paper is organized as follows. In section II, we introduce a working model which includes the necessary dynamics of QCM for multiplicity study and discuss the dynamical sources of the multiplicity correlation and fluctuations in multiple baryon production. In Section III, we study multiplicity fluctuations and correlation of baryons and antibaryons which are produced from the quark system with the given numbers of quarks and antiquarks. In Section IV, we take into account the fluctuations and correlations of quark number before hadronization to study their influence on baryon and antibaryon production. In Section V, we further take into account the effects of resonance decay. Summary and discussion are given in Sec VI.

II. A WORKING MODEL

Due to the difficulty of non-perturbative QCD, there isn’t a widely-accepted theoretical framework of QCM established so far which can self-consistently describe the whole picture of hadronization dynamics. In this paper, we need a working model which includes the necessary dynamics of QCM for multiplicity study and obtain the correlations and fluctuations of the produced hadrons. Meanwhile we should induce as least specific/particular assumptions as possible which effects are addressed clearly whenever it is imposed. Because there is no relevant works in previous literatures, the purpose of this paper is to focus on the results of most basic QCM dynamics, which will serve as a preliminary test of the model using the future experimental data and a baseline for the sophisticated hadronization dynamics.

We consider a system consisting of various quarks and antiquarks with constituent masses, corresponding to the “dressed” quarks and antiquark in non-perturbative QCD regime. We denote the number of quarks of flavor $q_i$ in the system by $N_{q_i}$ and similarly antiquarks by $N_{ar{q}_i}$. Three flavors, up, down and strange, are considered in this paper. As the system hadronizes, these quarks and antiquark combine with each other to form the color singlet hadrons. Finally, the system produces, in an event, various hadrons with numbers $\{N_{h}\}$ where $i = \pi, K, \rho, K^*$, ..., $p, \Lambda, \Xi, \Omega^*$ up to all included hadron species. Here, we consider only the ground state $J^P = 0^-$ and $1^-$ mesons and $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ baryons in flavor SU(3) group. The numbers of these hadron are varied event-by-event around their averaged values and follow a certain distribution $P(\{N_{h}\}; \{N_{q_i}, N_{\bar{q}_i}\})$ which is governed by hadronization dynamics.

The precise form of $P(\{N_{h}\}; \{N_{q_i}, N_{\bar{q}_i}\})$ depends on the full knowledge of combination hadronization dynamics. On the all “on market” QCM-models, few ones can give their specific solution of $P$. In addition, high dimensionality feature of $P$ makes the analytic solution quite difficult to get. In this paper, we generalize the quark combination simulation in SDQCM [10] to focus only on the multiplicity properties of the produced hadrons and model the $P(\{N_{h}\}; \{N_{q_i}, N_{\bar{q}_i}\})$, considering that this model has reproduced lots of experimental data of multiplicity of various hadrons in relativistic heavy ion collisions at different energies.

The main idea of quark combination simulation in SDQCM is as follows: (1) Assign all quarks and antiquarks in system into a abstract one-dimensional sequence. The relative distance between any two quarks and/or antiquark in sequence represent their map in realistic phase space. (2) combine these quarks and antiquarks in the sequence into hadrons according a quark combination rule(QCR). A schematic example is as follows.

$$q_1q_2q_3q_4\bar{q}_5q_6q_7q_8q_9q_{10}q_{11}q_{12}q_{13}q_{14}q_{15}\bar{q}_{16}q_{17}\bar{q}_{18}q_{19}\bar{q}_{20}$$

$$\text{QCR} \rightarrow M(q_1q_2)B(q_3q_4q_5)M(q_6q_7)M(q_8q_9)B(q_{10}q_{11})$$

$$M(q_{12}q_{13})B(q_{14}q_{15}q_{16}q_{17})B(q_{18}q_{19}q_{20})$$

(1)

QCR depends on the combination dynamics. As shown directly by above example, QCR should firstly satisfy two basic dynamics: (1) baryon formation is by the combination of three quarks which are close with each other in phase space and meson by a quark and an antiquark. Therefore, neighboring or next-neighboring quark combination in sequence is needed. (2) after hadronization, there is no free quarks and antiquarks left.

Considering the fact that the produced baryon is much smaller than mesons after hadronization, the key content of QCR is how to describe the production of baryons relative to that of mesons for a given quark configuration. We adopt the following procedure. For the local quark populations such as $qq$ and $qqq$, we can assign $qq \rightarrow M$ and $qqq \rightarrow M + q$ with relative probability $1$. When the case of possible baryon production $qqq$ is occurred, we give a probability or conditional criteria. If the nearest neighbor of $qqq$ is still a quark, opportunity for baryon formation should be significantly increased, and we can assign $qqq \rightarrow B + q$ with relative probability $1$. One the contrary, if the nearest neighbor of $qqq$ is an antiquark $\bar{q}$, then this $\bar{q}$ can have the chance of capturing one $q$ to form the meson and two quarks are left to combine with other quarks and antiquarks. We denote the probability of this channel by $P_{qqq \rightarrow M+q}$. The baryon formation probability in $qqq$ configuration is then $P_1 \equiv P_{qqq \rightarrow B+q} = 1 - P_0$.

A naive analysis gives $P_0/P_1 \sim (3 \times \frac{1}{3})/(1 \times \frac{1}{3}) = 9$ where the factor 3 is the number of the possible combinations for meson formation in $qqq$ configuration and factor 1 for baryon formation. Factor $\frac{1}{3}$ and $\frac{1}{3}$ are the color weight of forming color singlet meson and baryon in the stochastically colored quark combination, respectively. Therefore, baryon formation probability $P_1$ in $qqq$ case should be a small value ~ 0.1. In practice, a value of about 0.04 for $P_1$ can well explain the observed baryon yields in relativistic heavy ion collisions.
Above considerations in baryon formation is one kind of non-isolation approximation for the quark combination process, i.e., baryon formation is non-trivially influenced by the environment (the surrounding quarks and antiquarks). It is different from those (re)combination/coalescence models that are popular at early RHIC experiments [5–9]. They apply the sudden hadronization (i.e., isolation) approximation for the combination probability (by the overlap between quark wave function and hadron one they formed).

The remaining quarks and antiquarks in \( q\bar{q}q \to M + q \), \( q\bar{q}q \to M + q\bar{q} \) and \( q\bar{q}q \to B + q \) processes will subsequently combine with following quarks and antiquarks in sequence to form hadrons until at last all quarks and antiquarks are combined into hadrons. This procedure reflects, to a certain extent, the spread of the hadronization in space-time.

Another point of QCR is the order of the combination. As long as the quark number is large, different orders such as from left to right, from right to left, and from middle to the sides are equivalent and give the same result.

Based on above discussions, we give the following combination algorithm for the hadronization of quark system:

(i) start from the first parton (\( q \) or \( \bar{q} \)) in the sequence.

(ii) if the first and second partons are either \( \bar{q}q \) or \( q\bar{q} \), they combine into a meson and are removed from the sequence, then go back to (i); if first two are \( qq \) or \( \bar{q}\bar{q} \), then go to the next.

(iii) look at the third parton, if three partons are \( q\bar{q}q \) or \( \bar{q}\bar{q}q \), then the first and third partons combine into a meson and are remove from the sequence, then go back to (i); if three partons are \( qq\bar{q} \) or \( \bar{q}q\bar{q} \) then go to the next.

(iv) look at the fourth parton, if four partons are \( qqq\bar{q} \) or \( \bar{q}qq\bar{q} \), then first three partons combine into a baryon or antibaryon and are removed from the sequence, then go back to (i); If four partons are \( q\bar{q}q\bar{q} \) or \( \bar{q}q\bar{q}q \), there are two choices: (a) the first and fourth partons combine into a meson with probability \( P_0 \) and are remove from the sequence, then go back to (ii); (b) the first three partons combine into a baryon or an antibaryon with probability \( P_1 \) and are removed from the sequence, then go back to (i).

Above algorithm does not differentiate quark flavors in consideration of the flavor independence of strong interactions. Compared with the combination rule in Ref.[10], this algorithm address more explicitly the baryon production by the addition of step (iv) to fine tune baryon meson production competition. In essence, it can be regarded as an generalization of the combination rule in Ref.[10] in multiplicity description of the produced baryons.

For a given \( q_1\bar{q}_2q_3 \) that is known to form a meson by above combination algorithm, it can form either a \( J^P = 1^− \) vector (V) meson or a \( J^P = 0^− \) pseudo scalar (PS) meson. Similarly, a \( q_1q_2\bar{q}_3 \) (except for three identical \( q\bar{q}q \) case) can form either a \( J^P = \frac{1}{3}^− \) baryon or a \( J^P = (\frac{1}{3})^+ \) baryon. Following previous works [10, 26], we use the parameter \( R_{VP} \) to denote the relative production ratios of vector mesons to pseudoscalar mesons and \( R_{OD} \) the ratio of octet baryons to decuplet baryons. Then we get the branch ratios of each hadronization channel for a \( q_1\bar{q}_2q_3 \) combination

\[
C_{M_i} = \begin{cases} 
1/(1 + R_{VP}) & \text{for } J^P = 0^- \text{ mesons}, \\
R_{VP}/(1 + R_{VP}) & \text{for } J^P = 1^- \text{ mesons}, 
\end{cases}
\]

and for a \( q_1q_2q_3 \) combination

\[
C_{B_i} = \begin{cases} 
R_{OD}/(1 + R_{OD}) & \text{for } J^P = (1/2)^+ \text{ baryons}, \\
1/(1 + R_{OD}) & \text{for } J^P = (3/2)^+ \text{ baryons}. 
\end{cases}
\]

As did in previous works, we can apply above combination algorithm to the relativistic heavy ion collision by considering some properties of the produced quark system. It is observed that (1) the longitudinal expansion is predominate both in momentum space and in spatial space; (2) the longitudinal velocity/rapidity of quarks is closely correlated to their spatial position; (3) the rapidity density of quark numbers is very large and is relatively slowly varied. Therefore, we can sort all quark and antiquark according to their rapidity into an one-dimensional sequence, and then combine neighboring quarks and antiquarks into hadrons. In the transverse direction, one can not combine directly the neighboring quarks because their relative transverse momentum intervals exponential increase with the transverse momenta of quarks/antiquarks (transverse momentum distribution of quarks is exponential decreased). So we use the statistical combination approach, i.e., the \( p_T \) distribution of hadron is the convolution of quark \( p_T \) distribution and combination kernel, where the combination kernel is mainly dependent on the relative transverse momenta between two quarks/antiquarks. It is thus similar to those inclusive recombination/coalescence approach using the hadron wave function [5–9]. But our model is different from those inclusive methods in the proper treatment of unitarity in hadronization and the ability of well explanation of hadronic yield and longitudinal rapidity distributions observed in relativistic heavy ion collisions [10, 25–27].

Let us to summarize the origin of correlations and fluctuation of the produced baryons and antibaryons. First, local \( q\bar{q}q \) aggregation in phase space is stochastic for the system consisting of free quarks and antiquarks. Second, the \( q\bar{q} \to B \) process is probabilistic under the noise surrounding (i.e. stochastic populated quarks and antiquarks in neighbourhood). Together with the branch ratio of a given \( q_1q_2q_3 \) to a specific hadron state, they lead to the multiplicity fluctuations of the produced identified baryons. The conservation of baryon number in quark combination process constrains the global production of baryons and antibaryons and also the production of identified baryons and their anti-particles. The production correlation between two baryons mainly comes from a so-called “exclusion” effect, i.e., once a quark enters into a \( B_i \) at hadronization it is consumed and therefore can not re-act into \( B_j \). These effects lead to a nontrivial and complex multi-particle multiplicity distribution \( \mathcal{P}(dN_q; \{N_{q_i}, N_{\bar{q}_j}\}) \).
III. BARYON PRODUCTION FROM A GIVEN QUARK SYSTEM

In this section, we study fluctuations and correlations of baryons and antibaryons which are produced from the quark system with the given number of quarks and antiquarks. This enables us to learn more clearly the properties of baryon production coming from the quark combination process itself. Analytical results of various moments (mean, variance, skewness, kurtosis) of the inclusive multiplicity distribution of identified baryons are given firstly, according to the basic dynamics of quark combination discussed in previous section. Then two-baryon multiplicity correlation, baryon-antibaryon correlation and multi-baryon multiplicity correlation are studied systematically.

A. moments of multiplicity distribution of baryons

Firstly, we discuss the properties of inclusive multiplicity distribution of various identified baryons calculated from above combination algorithm. As a demonstration, Fig. 1 shows multiplicity distributions of total baryons and those of identified \( p \), \( \Lambda \), \( \Xi^0 \) and \( \Omega^- \), as the quark system with \( N_q = N_{\bar{q}} = 500 \) hadronizes. Here, the relative ratios of different quark flavors are set to be \( N_u : N_d : N_s = 1 : 1 : 0.43 \). We see that distribution of total baryon is close to the Gaussian shape while those of identified baryons exhibit more obvious deviation from it with the decrease of multiplicity. We see clearly a finite and nontrivial (species dependent) spectrum widths for their multiplicity distributions, which are determined by the dynamics of quark combination.

![Diagram](https://via.placeholder.com/150)

**FIG. 1:** (Color online) Normalized multiplicity distribution of total baryons (a) and those of identified protons \( p \) (b), \( \Lambda \) (c), \( \Xi^0 \) (d) produced by hadronization of a quark system with \( N_q = N_{\bar{q}} = 500 \). Here, the relative ratios of different quark flavors are set to be \( N_u : N_d : N_s = 1 : 1 : 0.43 \).

For the averaged multiplicity of identified baryons,

\[
\overline{N}_{B_j} = \sum_{|N_q|} N_{B_j} \mathcal{P}(|N_q|; |N_q|, N_{\bar{q}}),
\]

we have obtained the empirical solution in previous studies [10, 26]

\[
\overline{N}_{B_j} = P_{B_j} \overline{N}_B
\]

where \( \overline{N}_B = \sum |N_{B_j}| \) is the averaged total baryons and \( P_{B_j} \) denotes the production weight of \( B_j \) in all baryons. \( P_{B_j} \) can be decomposed as \( C_{B_j} P_{q_1 q_2 q_3 B} \) where \( P_{q_1 q_2 q_3 B} \) is the probability that, as a baryon is known to be produced, the flavor content of this baryon is \( q_1 q_2 q_3 \). Considering that every \( q_1 \), \( q_2 \) and \( q_3 \) quark in the system can have the chance of entering into \( B_j \) at hadronization, we get \( P_{q_1 q_2 q_3 B} = N_{B_j}^{q_1 q_2 q_3} / N_{qqq} \). \( N_{qqq} = N_q(N_q - 1)(N_q - 2) \) is total possible three quark combination number where \( N_q = \sum f N_j \) is total quarks in system. \( N_{B_j}^{q_1 q_2 q_3} = N_{iter} \prod f \prod_{i=1}^{N_j} (N_q - i + 1) \) is the possible number of \( q_1 q_2 q_3 \) combinations where \( n_{f,B} \) is the number of valance quark \( f \) contained in hadron \( B_j \). Here index \( f \) runs over all quark flavors. \( N_{iter} \) is the iteration factor taking to be 1, 3, and 6 for the case of three identical flavor, two different flavor and three different flavor contained in a baryon, respectively.

We have used Eq. (3) to reproduce the experimental data of yields and yield ratios of various identified baryons in relativistic heavy ion collisions at different collision energies [10, 25–27]. For the detailed discussions of the average yield formulas of identified baryons as well as those of antibaryons we refer the readers Refs. [25–27]. We argue that just based on these well performance of such kind of combination algorithm on the event averaged yields, we in this paper make further test in fluctuations and correlations.

We further study the variance, skewness and kurtosis of multiplicity distribution for various identified baryons. Their definitions are

\[
\overline{\sigma}^2_{B_j} = \frac{\delta N_{B_j}^2}{\overline{N}_{B_j}} = \frac{(N_{B_j} - \overline{N}_{B_j})^2}{\overline{N}_{B_j}} = \sum_{|N_q|} (N_{B_j} - \overline{N}_{B_j})^2 \mathcal{P}(|N_q|; |N_q|, N_{\bar{q}}),
\]

and similarly

\[
\overline{S}_{B_j} = \frac{\delta N_{B_j}^3}{\overline{N}_{B_j}^{1/2}} \quad \overline{K}_{B_j} = \frac{\delta N_{B_j}^4}{\overline{N}_{B_j}^{3/2}} - 3.
\]

Note that we always use the superscript \( \overline{\cdot} \) to denote the averaged hadronic quantities for a give quark system hadronization.

To analyze their properties, we have to consider joint production of multi-baryons. Taking variance for example, two-\( B_j \) pair production is given by

\[
N_{B_j}(N_{B_j} - 1) = P_{2B_j} \overline{N}_B(N_{B_j} - 1),
\]

where the production probability of two-\( B_j \) pair can be evaluated by \( P_{2B_j} = N_{2B_j}^{q_1 q_2 q_3 q_4} / N_{qqq} \) with the number of six
FIG. 2: (Color online) The square root of variance, skewness and kurtosis of the multiplicity distribution of various identified baryons with respect to their production weights $P_i = \bar{N}_B / N_B$. The size of quark system before hadronization is chosen to be $N_s = N_q = 500$ and the relative ratios of different quark flavors are set to be $N_s : N_q : N_{\bar{q}} = 1 : 1 : 0.43$. Symbols are full results and lines are leading terms of full results which have the form of binomial distribution with parameter $(N_B, P_i)$.

where the first term in the right hand of the equation is the dominant part. Similarly, we have

\[
\overline{\sigma^2}_{B_j} = \frac{1}{\overline{\sigma^2}_{B_j}}\left[\bar{N}_B P_{B_j} (1-P_{B_j})(1-2P_{B_j}) + 3P_{B_j}^2 \left[(1-A_1)\overline{\sigma^2}_{B_j} - A_1\bar{N}_B(\bar{N}_B-1)\right]
+ P_{B_j}^3 \left[(1-A_2)\overline{\sigma^2}_{B_j} + 3(A_1-A_2)\bar{N}_B\overline{\sigma^2}_{B_j} - 3(1-A_2)\overline{\sigma^2}_{B_j} + \bar{N}_B(\bar{N}_B-1)((3A_1-A_2)\bar{N}_B + 2A_2)\right]\right],
\]

and

\[
K_{B_j} = 3 + \frac{1}{\overline{\sigma^2}_{B_j}}\left[\bar{N}_B P_{B_j} (1-P_{B_j})\left[1 - 6P_{B_j}(1-P_{B_j}) + 3\bar{N}_B P_{B_j}(1-P_{B_j})\right] + 7P_{B_j}^2 \left[(1-A_1)\overline{\sigma^2}_{B_j} - A_1\bar{N}_B(\bar{N}_B-1)\right]
+ 6P_{B_j}^3 \left[(1-A_2)\overline{\sigma^2}_{B_j} + (1-3A_2+2A_1)\bar{N}_B - 3(1-A_2)\overline{\sigma^2}_{B_j} + \bar{N}_B(\bar{N}_B-1)((2A_1-2A_2)\bar{N}_B + 2A_2)\right]
+ P_{B_j}^4 \left[(1-A_3)(K_B + 3)\overline{\sigma^2}_{B_j} + [4(A_2-A_3)\bar{N}_B - 6(1-A_3)]\overline{\sigma^2}_{B_j} + (12A_2 - 6A_3 - 6A_1)\bar{N}_B^2 + (18A_3 - 12A_2 - 6)\bar{N}_B + 11(1-A_3)\overline{\sigma^2}_{B_j} + (4A_2 - 6A_1 - A_3)\bar{N}_B^3 + (6A_1 + 6A_3 - 12A_2)\bar{N}_B^2 + (8A_2 - 11A_3)\bar{N}_B^3 + 6A_3\bar{N}_B\right]\right],
\]

where three coefficients $A_1, A_2$ and $A_3$ are

\[
A_L = 1 - \prod_{k=1}^{L} \left(1 - \frac{1}{\overline{\sigma^2}_{B_j}}\left[\bar{N}_B P_{B_j} (1-P_{B_j}) + \frac{3}{\bar{N}_B} \left[(1-A_1)\overline{\sigma^2}_{B_j} - A_1\bar{N}_B(\bar{N}_B-1)\right]\right] \right) \prod_{m=1}^{3} \left(1 - \frac{3}{\bar{N}_B} \left[(1-A_2)\overline{\sigma^2}_{B_j} + 3(A_1-A_2)\bar{N}_B\overline{\sigma^2}_{B_j} - 3(1-A_2)\overline{\sigma^2}_{B_j} + \bar{N}_B(\bar{N}_B-1)((3A_1-A_2)\bar{N}_B + 2A_2)\right]\right),
\]

with $L = 1, 2, 3$.

In above formulas of variance, skewness and kurtosis, the first term in right hand side of equation is always the dominant part and we find that there are just the results of binomial distribution with parameters $(\bar{N}_B, P_i)$. In Fig. 2, we plot $\overline{\sigma^2}_{B_j}$,
\( \bar{S}_{B_i} \) and \( \bar{K}_{B_j} \) of various identified baryons as the function of their production weights \( P_{B_i} \). The symbols are full results and the lines binomial distribution as their leading approximation. The size of quark system here is choose to be \( N_q = N_s = 500 \) and the relative ratios of different quark flavors are set to be \( N_u : N_d : N_s = 1 : 1 : 0.43 \).

Multiplicity distribution of total baryons shows some slightly different properties from those of identified baryons. Variances of total baryon multiplicity is proportional to system size via \( \bar{\sigma}^2_{B_i}/N_B \approx 0.35 \) at current baryon-meson competition and skewness inversely proportional to system size via \( \bar{\Sigma}^3_{B_i}/N_B^{3/2} \approx 0.37 \). These properties are in general expectations of stochastic combination process. But the proportional coefficients is no longer to be able to explained in terms of binomial distribution. This is easily understood. The number of quarks consumed by total baryon formation is about 20\% of total quark number in the system. This feature causes the deviation from the independent and stochastic feature of the binomial distribution/trial.

### B. two-baryon correlations

Production of two different kinds of baryons are usually anti-associated in the hadronization of quark system with fixed quark numbers, characterising by the negative covariances of their multiplicities. The multiplicity covariance is defined as

\[
\bar{C}_{B_iB_j} = \delta N_{B_i}\delta N_{B_j} = N_{B_i}N_{B_j} - N_{B_i}N_{B_j},
\]

where the joint production probability of \( B_iB_j \) pair can be evaluated by

\[
P_{B_iB_j} = N^\text{th}_{B_iB_j}/N_q \quad \text{with the number of six quark cluster possible for } B_iB_j \text{ pair production } N^\text{th}_{B_iB_j} = N^\text{th}_{i \text{ter}}N^\text{th}_{j \text{ter}} \prod_j \sum_i n_i \langle n_i / n_{q \text{ iter}} \rangle (N_f - i + 1) \text{ and that for all two-baryon pair production } N_q = \prod_{i=1}^{N_{B_i}} (N_q - i + 1).
\]

Substituting Eqs. (2) and (12) into the covariance of two baryons

\[
\bar{C}_{B_iB_j} = \frac{P_{B_iB_j}N_{B_i}(N_{B_i} - 1)}{P_{B_i}P_{B_j}} - 1, \tag{13}
\]

in which

\[
P_{B_iB_j} = \frac{\prod_{f} \sum_{i=1}^{n_i} \langle 1 - \frac{n_i}{N_q} \rangle (N_f - i + 1)}{\prod_{m=1}^{3} \sum_{m=1}^{N_q - m + 1} \langle 1 - \frac{3}{N_q} \rangle (N_q - m + 1)} \tag{14}
\]

\[
= 1 - \sum_{f} n_{f,B_i} n_{f,B_j} \frac{1}{N_f} + \frac{9}{N_q} + O(N^{-2}),
\]

where the products and sumation with index \( f \) run over all quark flavors and \( n_{f,B_i} \) is the number of valance quark \( f \) contained in hadron \( B_i \). Finally, we have

\[
\bar{C}_{B_iB_j} = \frac{\bar{\Sigma}_{B_iB_j}}{N_{B_i}N_{B_j}} = - \sum_{f} \frac{n_{f,B_i} n_{f,B_j}}{N_f} - \frac{1}{N_B} \frac{\bar{\sigma}_{B_i}^2}{N_B} + \frac{9}{N_q} + O(N^{-2}). \tag{15}
\]

The first part in the right hand side of the equation is the leading order contribution. It essentially originates from the fact that at hadronization once a quark enters into a \( B_i \) it is consumed and therefore can not recombine into \( B_j \). This part is inversely proportional to the quark number of the coincide flavors in two baryons, so the relative anti-correlation among strange baryons is usually greater than those of light flavor baryons. The part in bracket is the next-leading order contribution, which is usually a few percentages of the first part. It is negligible in correlations for most baryon pairs with the coincide valance quark content but becomes important for correlations between baryon pairs with totally different quark flavors, such as \( C_{p \Omega^-, \Lambda^+ \Delta^-}, \) etc.

![FIG. 3: (Color online) Multiplicity covariance between two identified baryons. Different baryon-baryon pairs are distinguished in the horizontal axis by their multiplicity products. The size of quark system before hadronization is chosen to be \( N_q = N_s = 500 \) and the relative ratios of different quark flavors are set to be \( N_u : N_d : N_s = 1 : 1 : 0.43 \). Symbols are numerical results of QCM algorithm in Sec. II and the short solid line is the analytic result in Eq. (15).](image-url)
in combination is suppressed by the baryon number conservation.

C. baryon-antibaryon correlations

It is generally expected that baryons and antibaryons are associated in their productions, characterized by the positive covariance \( \overline{C}_{B,\bar{B}} = \delta N_B \delta N_{\bar{B}} = N_B N_{\bar{B}} - \overline{N}_B \overline{N}_{\bar{B}} \) of their multiplicities. One main reason of this association comes from the global baryon number conservation in hadronization which is denoted by the quark number conservation \( N_B - N_{\bar{B}} = \frac{1}{2}(N_q - N_{\bar{q}}) \) in the combination process. This causes the following correlation between baryon and antibaryon

\[
\overline{C}_{B,\bar{B}} = p_B p_{\bar{B}} \delta N_B \delta N_{\bar{B}} = p_B p_{\bar{B}} \overline{\sigma}_B^2,
\]

where we have used \( \delta N_B \delta N_{\bar{B}} = \overline{\sigma}_B^2 \) at fixed quark numbers. Inserting posterior production weight \( p_B = \langle N_B \rangle / \langle N_B \rangle \), we get a scaling property

\[
\overline{C}_{B,\bar{B}} = \frac{\overline{\sigma}_B^2}{\overline{N}_B \overline{N}_{\bar{B}}},
\]

for baryon-antibaryon multiplicity correlations.

![Covariance between identified baryons and antibaryons](image)

**FIG. 4:** Covariance between identified baryons and antibaryons. Different baryon-antibaryon pairs are distinguished in the horizontal axis by their multiplicity products. The size of quark system before hadronization is chosen to be \( N_q = N_{\bar{q}} = 500 \) and the relative ratios of different quark flavors are set to be \( N_u : N_d : N_s = 1 : 1 : 0.43 \). Symbols are numerical results of QCM algorithm in Sec. II and the solid line is the scaling given by Eq. (17).

In Fig. 4, we compare above formula with numerical results obtained from the algorithm in Sec. II with quark system \( N_q = N_{\bar{q}} = 500 \) in which the relative ratios of different quark flavors are set to be \( N_u : N_d : N_s = 1 : 1 : 0.43 \). The well agreement suggests that global baryon number conservation is the dominated reason for the production correlation between identified baryons and antibaryons. One interesting result is that both \( \overline{C}_{B,\bar{B}} \) and \( \overline{C}_{B,\bar{B}} \) (\( i \neq j \)) follow the same scaling line, which indicates that the production of baryon-antibaryon pair does not suffer more important constrain than that of two different baryons. This is reasonable in the case of free combination of quarks and antiquarks. Using the \( \overline{N}_B / \overline{N}_{\bar{B}} \approx 1/12 \) and \( N_i / N_u \approx 0.43 \) reproducing yield data in relativistic heavy ion collisions, we can estimate \( \overline{N}_B \approx \frac{1}{2} N_u \approx \frac{1}{2} N_s \), which means the baryon number conservation is strongest constraint in baryon antibaryon joint production. The production of identified baryons \( i \) and antibaryons \( j \) consume only small fraction of total quarks and antiquarks and thus has not reach the conservation threshold of identified quark flavors.

D. multi-body correlations

Following the similar procedure, we also get multi-baryon correlations due to the exclusion effect of successive baryon production discussed in Sec. III B and baryon number conservation in baryon antibaryon production in Sec. III C. The three-baryon correlation is

\[
\overline{C}_{\alpha\beta\gamma} = \delta N_{\alpha} \delta N_{\beta} \delta N_{\gamma} = \overline{N}_{\alpha} \overline{N}_{\beta} \overline{N}_{\gamma} - \overline{N}_{\alpha} \overline{C}_{\beta\gamma} - \overline{N}_{\beta} \overline{C}_{\alpha\gamma} - \overline{C}_{\alpha\beta\gamma},
\]

and four-baryon correlation is

\[
\overline{C}_{\alpha\beta\gamma\delta} = \delta N_{\alpha} \delta N_{\beta} \delta N_{\gamma} \delta N_{\delta} = \overline{N}_{\alpha} \overline{N}_{\beta} \overline{N}_{\gamma} \overline{N}_{\delta} - \overline{N}_{\alpha} \overline{N}_{\beta} \overline{C}_{\gamma\delta} - \overline{N}_{\alpha} \overline{N}_{\gamma} \overline{C}_{\beta\delta} - \overline{N}_{\alpha} \overline{N}_{\delta} \overline{C}_{\beta\gamma} - \overline{N}_{\beta} \overline{N}_{\gamma} \overline{N}_{\delta} \overline{C}_{\alpha\delta} - \overline{N}_{\beta} \overline{N}_{\delta} \overline{C}_{\alpha\gamma} - \overline{N}_{\gamma} \overline{N}_{\delta} \overline{C}_{\beta\alpha} - \overline{N}_{\gamma} \overline{N}_{\delta} \overline{C}_{\alpha\beta} - \overline{C}_{\alpha\beta\gamma\delta}.
\]

The averaged multiplicity product of three baryons (\( \alpha\beta\gamma \in \text{baryon} \)) can be written as

\[
\overline{N}_\alpha \overline{N}_\beta \overline{N}_\gamma = (1 - A_{\alpha\beta\gamma}) \overline{N}_\alpha \overline{N}_\beta \overline{N}_\gamma \times \frac{\delta^3 N_B + 3 \overline{\sigma}_B^3 (N_B - 1) + N_B \overline{N}_B - 2}{\overline{N}_B} + \delta_{\alpha\beta\gamma} (1 - \delta_{\alpha\beta}) (\overline{C}_{\alpha\beta\gamma} + \overline{N}_\alpha \overline{N}_\gamma) + (\delta_{\alpha\beta} + \delta_{\alpha\gamma}) (1 - \delta_{\alpha\beta})(\overline{C}_{\alpha\beta\gamma} + \overline{N}_\alpha \overline{N}_\beta) + \overline{N}_\alpha \overline{N}_\beta \overline{N}_\gamma [3 \overline{\sigma}_B + 3 \overline{N}_B (N_B - 1) + \overline{N}_B].
\]

where \( \delta_{\alpha\beta\gamma} \) is Kronecker delta function and \( \overline{\delta}_3 N_B \equiv \overline{S}_{B} \overline{\sigma}_B^3 \) is the third moment of total baryons. For that of two baryons and one anti-baryon, e.g., \( \alpha\beta \in \text{baryon} \), \( \gamma \in \text{anti-baryon} \), we have
Here, \( c = \bar{N}_B - \bar{N}_\bar{B} \) is the number of net baryons and taken to be zero at LHC.

The averaged multiplicity product of four baryons (\( \alpha \beta \gamma \epsilon \) e baryon) can be written as

\[
\bar{N}_a\bar{N}_\beta\bar{N}_\gamma\bar{N}_\epsilon = (1 - A_{\alpha\beta\gamma\epsilon}) \bar{N}_a\bar{N}_\beta\bar{N}_\gamma\bar{N}_\epsilon \frac{1}{\bar{N}_B} \times \left[ \frac{\delta N_B^2}{\bar{N}_B} + (4\bar{N}_B - 6)\delta N_B^3 + \sigma_B^2(6\bar{N}_B^2 - 18\bar{N}_B + 11) \right]
\]

\[
+ \bar{N}_B(\bar{N}_B - 1)(\bar{N}_B - 2)(\bar{N}_B - 3) + \delta_{\alpha\beta\gamma\epsilon}\alpha\beta\gamma\epsilon(3N_B - 11N_B^2 + 6N_a) + \delta_{\alpha\beta\gamma\epsilon}(1 - \delta_{\alpha\beta\gamma\epsilon})(3N_B^2 - 18N_B + 1) \tag{22}
\]

and for that of two baryons and two antibaryons, e.g. \( \alpha \beta \) in baryon, \( \bar{\gamma} \bar{\epsilon} \) in anti-baryon, we have

\[
\bar{N}_a\bar{N}_\beta\bar{N}_\gamma\bar{N}_\epsilon = (1 - A_{\alpha\beta\gamma\epsilon})\bar{N}_a\bar{N}_\beta\bar{N}_\gamma\bar{N}_\epsilon \frac{1}{\bar{N}_B^2} \times \left[ \frac{\delta N_B^2}{\bar{N}_B^2} + (4\bar{N}_B - 3)\delta N_B^3 + \sigma_B^2(6\bar{N}_B^2 + 9(c - 1)\bar{N}_B + 3c^2 - 6c + 2) \right]
\]

\[
+ \bar{N}_B(\bar{N}_B - 1)(\bar{N}_B - 2)\bar{N}_B + \delta_{\alpha\beta\gamma\epsilon}\alpha\beta\gamma\epsilon(3N_B - 11N_B^2 + 6N_a) + \delta_{\alpha\beta\gamma\epsilon}(1 - \delta_{\alpha\beta\gamma\epsilon})\bar{N}_a\bar{N}_\beta\bar{N}_\gamma\bar{N}_\epsilon + (\delta_{\beta\gamma} + \delta_{\beta\epsilon})(1 - \delta_{\alpha\beta})(\bar{N}_a\bar{N}_\beta\bar{N}_\gamma\bar{N}_\epsilon) \tag{23}
\]

\[
+ \bar{N}_B(\bar{N}_B - 1)(\bar{N}_B - 2)\bar{N}_B + \delta_{\alpha\beta\gamma\epsilon}\alpha\beta\gamma\epsilon(3N_B - 11N_B^2 + 6N_a) + \delta_{\alpha\beta\gamma\epsilon}(1 - \delta_{\alpha\beta})(\alpha\beta\gamma\epsilon)\bar{N}_a\bar{N}_\beta\bar{N}_\gamma\bar{N}_\epsilon \tag{24}
\]

where \( \delta N_B^2 \equiv \langle K_B + 3\rangle \sigma_B^2 \) is the fourth moment of total baryons. The average multiplicity product of three or two baryons read from Eqs. (20) and (12).

\section*{Coefficients \( A_{\alpha\beta\gamma\epsilon} \), \( A_{\alpha\beta\gamma\epsilon} \), and \( A_{\alpha\beta\gamma\epsilon} \) are extension of Eq. (10),}

\[
A_{\alpha\beta\gamma\epsilon} = 1 - \prod_{k=1}^{n_b} \prod_{s=1}^{n_f} \left( 1 - \frac{\sum_{k=1}^{n_b} n_{\alpha\beta\gamma\epsilon} k}{N_{\alpha\beta\gamma\epsilon}} \right).
\]

\section*{where \( n_b \) in the denominator denotes the number of involved baryons, i.e., \( n_b = 4 \) for \( \alpha \beta \gamma \epsilon \) and \( 3 \) for \( \alpha \beta \). \( n_{\alpha\beta\gamma\epsilon} \) is the number of valance quark of flavor \( f \) contained in hadron \( h \). \( h - 1 \) in numerator denotes the hadron before \( h \) in combination \( \alpha\beta\gamma\epsilon \).

Taking the charge conjugation operation, we get coefficients of antibaryons.

We can check that the following normalization is satisfied, i.e.,

\[
\sum_{\alpha\beta\gamma\epsilon} \overline{C}_{\alpha\beta\gamma\epsilon} = \delta N_B^3,
\]

\[
\sum_{\alpha\beta\gamma\epsilon} \overline{C}^2_{\alpha\beta\gamma\epsilon} = \delta N_B^2.
\]

\section*{Baryon Production from the Quark System with Variational Quark Numbers}

In this section, we take into account the effects of fluctuations and correlations of quark numbers in system before hadronization on multiple production of baryons and antibaryons. We firstly give the general procedure of including quark number fluctuations and correlations in hadronic observables and then show the specific formulas for moments and two body correlations of baryons and antibaryons. Then we discuss the properties of quark number fluctuations and
correlations in the context of ultra-relativistic heavy ion collisions and we show the numerical results of baryon moments, two baryon correlations and baryon antibaryon correlations.

A. general formulas of including quark number correlations

The produced quark system in heavy ion collisions at a specific collision energy is always varied in size event-by-event and the number of quarks and that of antiquarks in system at hadronization should follow a certain distribution \( \mathcal{P}(N_q, \bar{N}_\bar{q}; \langle N_q \rangle, \langle \bar{N}_\bar{q} \rangle) \) around the event-averaged quark number \( \langle N_q \rangle \) and antiquark numbers \( \langle \bar{N}_\bar{q} \rangle \), where \( q_i = u, d, s \) considered in this paper. In QCM, the distribution includes also the possible contribution of small-amount dynamical production of newborn quarks and antiquarks during hadronization process due to the requirement of exact energy conservation and entropy increase \([31]\). The event average of a hadronic physical quantity \( A_h \) is

\[
\langle A_h \rangle = \sum_{\{N_q, \bar{N}_\bar{q}\}} A_h \mathcal{P}(\{N_q\}; \{N_q\}, \{\bar{N}_\bar{q}\}; \{\langle N_q \rangle, \langle \bar{N}_\bar{q} \rangle\}) = \sum_{\{N_q, \bar{N}_\bar{q}\}} \bar{A}_h \mathcal{P}(\{N_q\}; \{\langle N_q \rangle, \langle \bar{N}_\bar{q} \rangle\}).
\] (28)

If \( \bar{A}_h \) is known already, we can expand it as the Taylor series at the event average of quark numbers \( \langle N_q, \bar{N}_\bar{q} \rangle \)

\[
\bar{A}_h = \bar{A}_h(\{N_q\}) + \sum_{f_1} \frac{\partial^2 \bar{A}_h}{\partial N_{f_1} \partial N_{f_2}} \delta N_{f_1} \delta N_{f_2} + \frac{1}{2} \sum_{f_1, f_2} \frac{\partial^2 \bar{A}_h}{\partial N_{f_1} \partial N_{f_2}} \delta N_{f_1} \delta N_{f_2} + \frac{1}{3!} \sum_{f_1, f_2, f_3} \frac{\partial^3 \bar{A}_h}{\partial N_{f_1} \partial N_{f_2} \partial N_{f_3}} \delta N_{f_1} \delta N_{f_2} \delta N_{f_3} + O(\delta^4),
\] (29)

where indexes \( f_1, f_2, f_3 \) run over all quark and antiquark flavors and \( \delta N_{f_1} = N_{f_1} - \langle N_{f_1} \rangle \). The subscript \( \langle N_q, \bar{N}_\bar{q} \rangle \) denote the evaluations at event average point. Substituting it into Eq. (28), we get

\[
\langle A_h \rangle = \bar{A}_h(\{N_q\}) + \sum_{f_1, f_2} \frac{\partial^2 \bar{A}_h}{\partial N_{f_1} \partial N_{f_2}} \delta N_{f_1} \delta N_{f_2} + \frac{1}{2} \sum_{f_1, f_2, f_3} \frac{\partial^2 \bar{A}_h}{\partial N_{f_1} \partial N_{f_2} \partial N_{f_3}} \delta N_{f_1} \delta N_{f_2} \delta N_{f_3} + O(\delta^4),
\] (30)

where \( C_{f_1, f_2} = \langle \delta N_{f_1} \delta N_{f_2} \rangle \) and \( C_{f_1, f_2, f_3} = \langle \delta N_{f_1} \delta N_{f_2} \delta N_{f_3} \rangle \) are two-body and three-body correlation functions of quarks and antiquarks, respectively. Then we can calculate the influence of quark number fluctuations and correlations on hadronic quantities order by order. In the following equations we drop the subscript \( \langle N_q, \bar{N}_\bar{q} \rangle \) for convenience.

B. formula of identified baryons

Using Eq. (30), we first get the event average of baryon multiplicity

\[
\langle N_B \rangle = N_B + \frac{1}{2} \sum_{f_1, f_2} \frac{\partial^2 N_B}{\partial N_{f_1} \partial N_{f_2}} C_{f_1, f_2} + O(N_f^{-2}).
\] (31)

The effect of two-quark correlations to baryon multiplicity is the order of magnitude of \( 1/\langle N_f \rangle \), which is only a few percentages of the leading term due to the large quark number (i.e., thousand of quarks and antiquarks per unity rapidity at RHIC and LHC energies). The influence of three-body and four-body correlations of quarks and antiquarks is suppressed further by \( 1/N_f^2 \). Therefore, effects of quark number correlations and fluctuations can be safely neglected in studies of inclusive multiplicity of identified hadrons in relativistic heavy ion collision, as we did in previous works.

For moments of multiplicity distribution of identified hadrons, we have

\[
\sigma_B^2 = \bar{\sigma}_B^2 + \sum_{f_1, f_2} \left( \frac{\partial \bar{N}_B}{\partial N_{f_1}} \frac{\partial \bar{N}_B}{\partial N_{f_2}} + \frac{1}{2} \frac{\partial^2 \bar{N}_B}{\partial N_{f_1} \partial N_{f_2}} \right) C_{f_1, f_2} + O(N_f^{-2}),
\] (32)

\[
S_B = \bar{S}_B + \sum_{f_1, f_2} \left( \frac{\partial \bar{N}_B}{\partial N_{f_1}} \frac{\partial \bar{N}_B}{\partial N_{f_2}} + \frac{3}{2} \frac{\partial^2 \bar{N}_B}{\partial N_{f_1} \partial N_{f_2}} \right) C_{f_1, f_2} + O(N_f^{-2}),
\] (33)

\[
K_B = \bar{K}_B + (\bar{K}_B + 3) \left\{ \sum_{f_1, f_2} \left( \frac{\partial \bar{N}_B}{\partial N_{f_1}} \frac{\partial \bar{N}_B}{\partial N_{f_2}} + \frac{8}{3} \frac{\partial^2 \bar{N}_B}{\partial N_{f_1} \partial N_{f_2}} \right) C_{f_1, f_2} + O(N_f^{-2}) \right\}.
\] (34)
Here, we have used $\partial_{1} \equiv \frac{\partial}{\partial N_{g_{1}}} \partial_{12} \equiv \frac{\partial}{\partial N_{g_{1}} N_{g_{2}}}$ for abbreviation. Because higher order contributions of quark correlations and fluctuations are usually suppressed by the factor $1/(\langle N_{f} \rangle)$ order by order, here we show only the effects of the second order correlations and fluctuations of quark numbers on the directly produced baryons.

For two baryons/antibaryons correlations, we have

$$C_{\text{of}} = \overline{C}_{\text{of}} + \frac{1}{2} \sum_{f_{1}, f_{2}} \left[ 2 \partial_{1} \overline{C}_{\text{of}} \partial_{2} \overline{C}_{\text{of}} + \partial_{12} \overline{C}_{\text{of}} \right] C_{f_{1}, f_{2}} + O(N_{f}^{-2}). \ (35)$$

Here, the contribution of second order quark correlations is the same order as $\overline{C}_{\text{of}}$, and they might cancel with each other significantly. The influence of the higher order contributions of quark correlations is about few percentages at LHC and is neglected here. As $\alpha = \beta$, we obtain Eq. (32) which is also hardly influenced by higher order quark correlations.

In appendix, we give the derivation of the full expression of Eqs. (32)-(34) up to the four-body quark correlations for reader’s convenience and the decay calculations in the next section.

### C. quark number correlations and fluctuations

We firstly determine the size of quark system before hadronization which is consistent with that produced in relativistic heavy ion collisions at LHC energy. By fitting the rapidity density of hadronic yield in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, we obtain $\langle N_{u} \rangle = \langle N_{d} \rangle = 1710$ and the strangeness content $\langle N_{s} \rangle/(N_{u}) = (N_{d})/(N_{d}) = 0.43$ for quark system in unit rapidity window size in central rapidity region. We note that the obtained strangeness suppression factor $\lambda_{s} \equiv \langle N_{s} \rangle/(N_{u}) = (N_{d})/(N_{d}) = 0.43$ is in agreement with the Wroblewski parameter calculated by Lattice QCD [32, 33]. In the following text, we use it as the default size of quark system. If different rapidity window size is selected, the quark system is obtained linearly.

For two-body correlation $C_{f_{1}, f_{2}}$ of quarks and antiquark, using the charge conjugation symmetry and isospin symmetry between $u$ and $d$ quarks for the quark system produced at LHC, there are only 8 relevant quark correlations, i.e.,

- two variances $C_{uu} \equiv \sigma_{u}^{2}$ and $C_{ss} \equiv \sigma_{s}^{2}$,
- two pair correlations $C_{ud}$ and $C_{sd}$,
- four off-diagonal correlations $C_{ud}$, $C_{us}$, $C_{sd}$ and $C_{us}$.

Variance of quark number is usually approximated to follow Poisson statistics $\sigma_{u}^{2} \approx \langle N_{u} \rangle$ and $\sigma_{s}^{2} \approx \langle N_{s} \rangle$ for a thermalized quark system with grand canonical ensemble. Lattice QCD calculations at vanishing chemical potential provides important constraint on above quark correlations [34], which show the weak off-diagonal flavor susceptibilities of quark numbers $\chi_{us}/\chi_{ss} \approx -0.05$ and $\chi_{ud}/\chi_{uu} \approx -0.05$ as temperature closes to the phase boundary. Here, $\chi_{uu} \equiv C_{uu} + C_{ud} - C_{us} + C_{sd} = 2(C_{us} - C_{ud})$ and others are similarly defined. Because of the lack of further theoretical constraints on those quark number correlations at present, we have to adopt some symmetry approximations on quark correlations, i.e., $C_{uu}/C_{uu} = C_{ud}/C_{ud} = \lambda_{1}$ and $C_{uu}/\sigma_{u}^{2} = C_{ss}/\sigma_{s}^{2} = \lambda_{2}$ where $\lambda_{1}$ and $\lambda_{2}$ are treated as parameters of this work. The value of $\lambda_{2}$ is smaller than one if we consider a slice of quark system, e.g., mid-rapidity region, produced in heavy ion collisions. The off-diagonal flavor correlations are usually expected to be much smaller than the variance of quark numbers. Inspired by the weak off-diagonal flavor susceptibilities in Lattice QCD calculations, we assume $\lambda_{ud}/\lambda_{uu} \approx 0.05$ (correspondingly $\lambda_{1} \approx 2.0$) with some arbitrariness in this work to study the effects of the weak flavor off-diagonal quark correlations on baryon and anti-baryon production.

Since this work mainly focuses on the baryon sector, we introduce the total baryon number balance coefficient $\rho_{B}^{(q)}$ as one physical characteristic of the quark system,

$$\rho_{B}^{(q)} \equiv \frac{\sum_{f_{1}, f_{2}} \frac{1}{2} C_{f_{1}, f_{2}}}{N_{B}^{(q)}} = \lambda_{2} - 0.1 \lambda_{1} \frac{1 - \lambda_{2}}{1 - \lambda_{1}} \frac{1 + 2 \lambda_{2}}{2 + \lambda_{2}}, \ \ (36)$$

where indexes $f_{1}, f_{2}$ run over all flavors of quarks and $N_{B}^{(q)} = \frac{1}{3}(\langle N_{u} \rangle + \langle N_{d} \rangle + \langle N_{s} \rangle)$. Note that the factor $1/3$ before $C_{f_{1}, f_{2}}$ denotes the balanced baryon number if $f_{1}$ and $f_{2}$ are correlated. The second equal uses above approximated quark correlations. We also introduce the electric charge balance coefficient of quark system, which is defined as

$$\rho_{C}^{(q)} = \frac{1}{N_{C}^{(q)}} \sum_{f_{1}, f_{2}} \min(Q_{f_{1}}, Q_{f_{2}}) C_{f_{1}, f_{2}}, \ \ (37)$$

where indexes $f_{1}, f_{2} = u, d, s \bar{s}$ run over all positively charged quarks with electric charge $Q_{f_{1}}$ and $Q_{f_{2}}$, respectively. $N_{C}^{(q)} = \frac{1}{3}(2\langle N_{u} \rangle + \langle N_{d} \rangle + \langle N_{s} \rangle)$. The balanced charge for $f_{1}f_{2}$ pair is the minimum of their individual electric charges. Above approximated quark two-body correlations guarantee correct boundary behavior of conserved charge for quark system, i.e., as $\lambda_{2}$ goes to one both $\rho_{B}^{(q)}$ and $\rho_{C}^{(q)}$ go to one. Using the measured charge balance function of thermal particles in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [35], we can roughly constrain the $\rho_{C}^{(q)}$ of quark system

$$\rho_{C}^{(q)}(y_{w}) \approx \int^{y_{w}} B(\delta \eta) d\delta \eta, \ \ (38)$$

if we expect the small change of charge balance property of system during hadronization [36]. Here, $y_{w}$ is the rapidity window size related to the size of quark system.

Three-body and four-body correlations of quarks and antiquarks influence relatively less on the physical quantities of initial baryons in previous subsection than the two-body correlations of quark numbers. But they will influence those of final baryons through resonance decay (as shown in next section), so we need them also. Because there are no theoretical calculations at present that we can borrow, we take the following approximation for three-body quark correlations, i.e., $C_{fff} \equiv \langle \delta N_{f} \rangle$ and off-diagonal correlations $C_{f_{1}f_{2}f_{3}} = 0$ where $f_{1}, f_{2}, f_{3}$ are different flavors. For four-body off-diagonal correlations, we approximate them using
shows moments of various identified baryons after considering quark number fluctuations (QNF) and quark number flavor conservation (QFC) with parameter $\lambda_2$. The $\lambda_2 = 0.91$ is chosen to be consistent with the observed charge balance function of thermal particles in unit pseudo-rapidity window size observed in Pb+Pb collisions at 2.76 TeV [35]. With $\lambda_2 = 0.18$, we get almost unitary baryon moments similar to Poisson distribution.

By this approximation, the kurtosis of net baryons has the property $K_{\text{net b}} \sigma_{\text{net b}}^2 = 1$ which is suggested in ultra-relativistic heavy ion collisions [11].

D. numerical results of multiplicity moments of identified baryons

Fig. 5 shows moments of various identified baryons after taking into account the effects of quark correlations and fluctuations. In order to clearly present effects of quark correlations and fluctuations, the variance $\sigma_i^2$, skewness $S_i$, and kurtosis $K_i$ of identified baryons are multiplied by factors $1/\langle N_i \rangle$, $\sqrt{\langle N_i \rangle}$ and $\langle N_i \rangle$, respectively, to make them all in the order of one. Here, the usage of $\langle N_i \rangle$ as the scaling factor is due to its insensitive to the correlations and fluctuations of quark numbers.

Solid circles in Fig. 5(a) are variance of initial baryons directly produced by hadronization. As discussed previously, $\sigma_i^2/\langle N_i \rangle$ of identified baryons, roughly following binomial distribution, is always smaller than one and usually decreases with the increasing of the baryon multiplicity or production weight. $\Omega^-$ is only 2% smaller than one while proton about 10%. But there are several exceptions for such a decreasing trend. For example, variance of $\Delta^{++}$ is smaller than its iso-spin partner $\Delta^+$ although their multiplicity are nearly same. This is due to the effect of identical quark flavor in baryon production encoded via coefficient $\Lambda_L$ in their variance formula in Eq. (7). Others exceptions including those between $\Xi$ and $\Sigma$ and those between $\Sigma^*$ and $\Lambda$ are due to the same reasons either in strange or light flavor sector. These properties are also observed in baryon’s skewness Fig. 5(b) and kurtosis Fig. 5(c) with larger amplitudes.

Open circles show the baryon moments after considering only the effects of quark number fluctuations. We can see that fluctuations of quark numbers obviously increase the baryon’s multiplicity fluctuations. $\sigma_i^2/\langle N_i \rangle$ of various baryons exceed one. Proton is about 3% greater than one while $\Omega^-$ slightly exceeds one. Skewness and kurtosis of baryons are also greater than one and they are more sensitive to quark number fluctuations, e.g. proton skewness increase about 5% and kurtosis about 10%, respectively.

Solid up-triangles show baryon moments after considering further the effects of quark flavor conservations with parameter value $\lambda_2 = 0.91$, besides of quark number fluctuations. Here the value of parameter $\lambda_2$ is chosen so that the electric charge balance coefficient $\mu_2^{\text{eq}}$ of quark system, according to Eq. (38), is consistent with the measure charge balance in unit rapidity window size $y_w = 1$ in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ GeV [35]. Because moments of baryons are insensitive to parameter $\lambda_1$, so we do not show its effects on the Fig. 5. Considering the pair association of quark and antiquark will facilitate meson production and suppress baryon production. Comparing to open circles, we therefore observe an significant decrease of proton variance, skewness and kurtosis. For baryons with small multiplicity such as $\Omega^-$ and $\Xi^*$, they are weakly influenced by flavor conservation of quark numbers and their moments are always almost one. If we choose smaller flavor conservation parameter $\lambda_2 = 0.18$ which corresponds to the observed charge balance in small rapidity window size $y_w = 0.15$, we can observe almost unitary baryon’s moments, shown as star symbols, which is similar to Poisson
E. numerical results of two baryon correlations

Fig. 6 shows two-baryon multiplicity correlations after considering effects of quark number fluctuations and correlations. Solid circles show initial two-baryon correlations due to the hadronization of the quark system with given quark numbers and antiquark numbers. They exhibit a sensitive dependence on baryon species, see detailed discussions in Sec. III(B). After taking into account effects of quark number fluctuations, all two-baryon correlations, open circles, flip their sign and become a positive and almost universal value. The positive value means the production of two baryons are associated, which is because that both baryons parallelly respond to the change of quark numbers or that of antiquark numbers. This association is suppressed and/or canceled by further taking into account the flavor conservation of quark numbers. With small flavor conservation parameter value $\lambda_2 = 0.18$, all two-baryon correlations, open up-triangles, tend to be zero. As increases the $\lambda_2$ to 0.91, open squares, we see a strong anti-association between two baryon production, and interestingly we see a universal value for all two-baryon correlations. This is a striking characteristic of two-baryon production in QCM.

![FIG. 6: (Color online) Two-baryon multiplicity correlations after considering the effects of quark number fluctuations (QNF) and quark flavor conservation (QFC) just before hadronization. The labels for solid circles denoting which two-baryon correlation are the same as those in Fig. 3.](image)

We also study in Fig. 7 the parameter dependence of two-baryon correlations. In order to be closely related to the experimental measurement at finite rapidity window, we constrain, according to Eq. (38), the charge balance coefficient of quark system $\rho^{(q)}_C$ by the measured charge balance function of thermal particles in central Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV [35]. Because $\rho^{(q)}_C$ is usually weakly dependent on parameter $\lambda_1$, we determine the flavor conservation parameter $\lambda_2$ by $\rho^{(q)}_C$ at $\lambda_1 = 2.0$. Fig. 7 shows two-body correlations of stable baryons $p, \Lambda, \Xi^-$, $\Omega^-$ at different rapidity window size (correspondingly at different flavor conservation $\lambda_2$). Note that, results in Fig. 7 are different from results in Fig. 6 where average quark numbers of quark system are constant respect to the change of $\lambda_2$. Here we take the average quark numbers of quark system to be linearly proportional to rapidity window size and then we fix $\lambda_2$ by $\rho^{(q)}_C(y_N)$. We see a non-monotonic behavior of two baryon correlations with respect to window size, which is due to the competition of changed flavor conservation and the changed quark numbers of system. As window size increases from 0.3 to 0.6, the flavor conservation coefficient $\lambda_2$ increases rapidly up to about 0.7 and this leads to the increased anti-association between two baryons. However, as the window size continues to enlarge, effects of increased flavor conservation is overwhelmed by that of increased quark numbers and we see a decreased anti-association between two baryons. We also see that with the increased window size the difference between different two-baryon correlations decreases and we have an almost universal correlation magnitude for all two-baryon correlations, as shown in Fig. 6.

F. numerical results of baryon-antibaryon correlation

Fig. 8 shows various baryon-antibaryon multiplicity correlations after considering effects of quark number fluctuations and correlations. Solid circles shows baryon-antibaryon correlations for the hadronization of the quark system with given quark numbers and antiquark numbers. They exhibit a uni-
After considering quark number flavor correlations (QFC) with parameter value \( \lambda_2 = 0.91 \), note that these results are not sensitive to parameter value \( \lambda_1 \) for large \( \lambda_2 \) value and thus we shown only results at \( \lambda_1 = 2.0 \). Most of baryon-antibaryon correlations returns to the positive case which means their production is associated. In particular, hyperon-antihyperon correlations, e.g. \( \Omega^- \bar{\Omega}^+ \), \( \Omega^- \bar{\Xi}^0 \), are much larger than \( p \bar{p} \) correlation. This suggests that the strangeness conservation plays an important role in hyperon-antihyperon joint production. Surprisingly, see Fig. 8 panel (b), some baryon-antibaryon pairs, e.g. \( p \bar{\Xi}^+ \), \( p \bar{\Omega}^- \), have negative values. This is because their production do not or less involve the matched \( u \bar{u}, d \bar{d}, s \bar{s} \) pairs and thus flavor conservation less directly constrain their joint production and therefore the effect of quark number fluctuations dominates their production. With small flavor conservation parameter value \( \lambda_2 = 0.18 \), all baryon-antibaryon correlations tend to zero (with maximum deviation about 0.002) and we do not show them in Fig. 8 for clarity.

In Fig. 9, we show the rapidity window size \( y_w \) dependence of some baryon-antibaryon correlations. The relationship between \( \lambda_2 \) and \( y_w \) is the same as that in above subsection. We observe from panel (a) that \( C_{\bar{p}\bar{p}} \) is always negative at different \( y_w \) (\( \lambda_2 \)) and \( C_{p\bar{p}} \) is negative at small \( y_w \) and tends to zero with increasing \( y_w \) due to the increased effect of flavor conservation \( \lambda_2 \). For \( p \bar{p} \), \( \Lambda \bar{\Lambda} \) correlations in Fig. 9(a), \( \Xi^- \bar{\Xi}^+ \), \( \Omega^- \bar{\Omega}^+ \) and other hyperon-antihyperon correlations in panel (b) that
largely involve the matched $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ pairs, they are all positive under the influence of quark flavor conservation. We also observe that as $y_w \geq 0.6$, $p\bar{p}$, $\Lambda\bar{\Lambda}$, $\Xi^-\Xi^+$, $\Omega^-\Omega^+$ correlations decrease with the increasing $y_w$, which is because of the increased quark numbers (or system size).

V. DECAY EFFECTS

Multiplicity of final baryons observed in experiments usually contains the decay contribution of unstable resonances. In this section, we study the $e^+$-dependence of final baryons. We firstly derive the formulas of decay influence on stable baryons and then show the numerical results of stable baryons $p$, $\Lambda$, $\Xi^-$ and $\Omega^-$. 

\begin{equation}
F(N_a, N_b, N_c, \ldots) = \sum_{N_j} \mathcal{P}((N_j); \langle N_q \rangle) \prod_i \left( \sum \mathcal{f}(N_{a_i}^j, N_{b_i}^j, \ldots, N_i, \{D_{ia}, D_{ib}, D_{ic}, \ldots\}) \right) \prod_k \delta_{N_k, N_i^k}, \tag{42}
\end{equation}

where index $i$ runs over all kinds of directly produced baryons and $k$ runs over all stable hadrons we studied.

The inclusive yield of final-state identified baryons receives the linear superposition of resonance decays,

\begin{align}
\langle N_a \rangle &= \sum_{N_{a_0}, \ldots} N_{a_0} F(N_a, N_b, N_c, \ldots) \\
&= \sum_{N_j} \mathcal{P}((N_j); \langle N_q \rangle) \prod_i \left( \sum \mathcal{f}(N_{a_i}^j, N_i, D_{ia}) \right) \sum_{N_{a_i}^k} N_{a_i}^k \\
&= \sum_k \left( \sum_{N_j} \mathcal{P}((N_j); \langle N_q \rangle) N_k D_{ka} \right) \\
&= \sum_k \langle N_k \rangle D_{ka}.
\end{align}

Note that we have used the abbreviation $\mathcal{P}((N_j); \langle N_q \rangle) \equiv \mathcal{P}((N_j); \langle N_q \rangle)$ for the joint distribution of directly produced baryons and written $D_{bk} = 1$ to obtain the compact formulation. Similarly, we can calculate the various moments of multiplicity of stable baryons as

\begin{align}
\langle N_a^m \rangle &= \sum_{N_{a_0}, \ldots} N_{a_0}^m F(N_a, N_b, N_c, \ldots) \\
&= \sum_{N_j} \mathcal{P}((N_j)) \prod_i \left( \sum \mathcal{f}(N_{a_i}^j, N_i, D_{ia}) \right) \left( \sum_k N_k^b \right)^m,
\end{align}

and finally have

\begin{align}
\sigma_a^2 &= \sum_{m,n} C_{mn} D_{ma} D_{na} + \sum_m \langle N_m \rangle D_{ma} (1 - D_{ma}), \tag{44}
\end{align}

A. formulas of decay effects

For baryon resonance $i$, its stable daughter baryons are denoted as $a, b, c, \ldots$ with decay branch ratios $D_{ia}, D_{ib}, D_{ic}, \ldots$, respectively. $D_{ij}$ is taken from PDG [37]. The joint multiplicity distribution of daughter baryons from the parent baryon $i$ of number $N_i$ are taken to be the multinomial distribution $f((N_{i_a}^i, N_{i_b}^i, N_{i_c}^i, \ldots), N_i, \{D_{ia}, D_{ib}, D_{ic}, \ldots\})$, where $N_{i_a}^i, N_{i_b}^i, N_{i_c}^i, \ldots$ denote the numbers of decayed baryons $a, b, c, \ldots$ respectively. Recalling the joint distribution of directly produced baryons in Sec.IV, we write the joint multiplicity distribution of stable baryons

\begin{align}
S_a &= \frac{1}{\sigma_a^2} \left( \sum_{k,m,n} C_{kmn} D_{ka} D_{ma} D_{na} + 3 \sum_{m,n} C_{mn} D_{ma} (1 - D_{ma}) D_{na} + \sum_m \langle N_m \rangle D_{ma} (1 - D_{ma}) (1 - 2D_{ma}) \right), \tag{45}
\end{align}

\begin{align}
K_a + 3 &= \frac{1}{\sigma_a^4} \left( \sum_{m,n,k,l} C_{mnl} D_{ma} D_{ma} D_{ka} D_{la} + 6 \sum_{m,n} C_{mn} D_{ma} (1 - D_{ma}) D_{ma} D_{ka} \\
&+ 4 \sum_{m,n} C_{mn} D_{ma} (1 - D_{ma}) (1 - 2D_{ma}) D_{na} + 3 \sum_{m,n} C_{mn} \langle N_m \rangle D_{ma} (1 - D_{ma}) D_{ma} (1 - D_{ma}) \right) \sum_m \langle N_m \rangle D_{ma} (1 - D_{ma}) (1 - D_{ma}) \right), \tag{46}
\end{align}
The average of the multiplicity product of two stable baryons are evaluated by

\[
\langle N_a N_b \rangle = \sum_{\langle N_a, N_b, \ldots \rangle} N_a N_b f(N_a, N_b, N_c, \ldots) = \sum_{\langle N_i \rangle} \mathcal{P}(\langle N_i \rangle) \prod_i \left( \sum_{\langle N \rangle} f(N_i^a, N_i^b, D_{ia}, D_{ib}) \right) \left( \sum_m N^m_a N^m_b \right)
\]

\[
= \sum_{\langle N_i \rangle} \mathcal{P}(\langle N_i \rangle) \prod_i \left( \sum_{\langle N \rangle} f(N_i^a, N_i^b, D_{ia}, D_{ib}) \right) \left( \sum_{m \neq n} N^m_a N^n_b + \sum_{m = n} N^m_b N^m_n \right)
\]

(47)

Substituting it into the definition of two body correlation we get

\[
C_{ab} = \sum_{m,n} \left[ C_{mn} - \delta_{mn} \langle N_m \rangle \right] D_{ma} D_{nb},
\]

(48)

which receives the coherent superposition of two resonance decays as well as the anti-association due to the possible same parent baryon resonance.

Following the spirit of Eq. (47), we obtain the three body correlation with different species \(C_{abc}\)

\[
C_{abc} = \sum_{m,n,k} \left( C_{mnk} + (\delta_{mk} + \delta_{nk})C_{mn} - \delta_{mn} C_{mk} \right)
\]

\[
+ 2 \delta_{mn} \delta_{nk} \langle N_m \rangle D_{ma} D_{nb} D_{kc},
\]

(49)

and \(C_{aac}\) with one identical pair can be obtained by \(C_{aac} = (C_{abc})_{a \neq b} + C_{ac},\) and four-body correlations \(C_{abcd}\) with four different species,

\[
C_{abcd} = \sum_{m,n,k,l} \left( C_{mnkl} D_{ma} D_{nb} D_{kc} D_{ld} - \sum_{m,n,k} \left( C_{mnk} + C_{kl} \langle N_m \rangle \right) D_{mkl}^{(211)}(a, b, c, d) \right.
\]

\[
+ \sum_{m,l} \left( C_{ml} + \langle N_m \rangle \right) D_{ml}^{(22)}(a, b, c, d) \right]
\]

\[
+ 2 \sum_{m,l} C_{ml} D_{ml}^{(3)}(a, b, c, d) \right]
\]

\[
- 6 \sum_{m,n} \langle N_m \rangle D_{ma} D_{nb} D_{mc} D_{ld} D_{kd}.
\]

Here, \(D_{mkl}^{(211)}(a, b, c, d) = D_{ma} D_{mb} D_{kc} D_{ld} + D_{ma} D_{mc} D_{kb} D_{ld} + D_{ma} D_{mb} D_{kc} D_{ld} + D_{ma} D_{mb} D_{kc} D_{ld} + D_{ma} D_{mb} D_{kc} D_{ld} + D_{ma} D_{mb} D_{kc} D_{ld} \) denote the summation over all possible joint-decay probabilities for three resonances \(mkl\) into four stable baryons where the superscript \((211)\) denotes that one of the parent resonances has two decay channels to two different stable baryons, respectively. Similarly, we have \(D_{ml}^{(22)}(a, b, c, d) = D_{ma} D_{mb} D_{mcl} D_{ld} + D_{ma} D_{mc} D_{ld} D_{km} + D_{ma} D_{mc} D_{ld} D_{km} + D_{ma} D_{mc} D_{ld} D_{km} \) and \(D_{ml}^{(3)}(a, b, c, d) = D_{ma} D_{mb} D_{mcl} D_{ld} + D_{ma} D_{mb} D_{mcl} D_{ld} + D_{ma} D_{mb} D_{mcl} D_{ld} + D_{ma} D_{mb} D_{mcl} D_{ld} \). Other four-body correlations of stable baryons with one identical pair, two identical pairs, and three identical species can be obtained as follows

\[
C_{aacd} = (C_{abcd})_{a \neq c} + C_{abcd} + \langle N_a \rangle C_{bd},
\]

(51)

\[
C_{aacb} = (C_{abcd})_{a = c = b} + 3C_{aac} + (3\langle N_a \rangle - 2) C_{ab},
\]

(52)

\[
C_{aabb} = (C_{abcd})_{a = b} + C_{a = b} + C_{ab} + \langle N_a \rangle \sigma^2_a - C_{a = b} - \langle N_a \rangle \langle N_b \rangle.
\]

(53)

B. numerical results of stable baryons

Fig. 10 shows multiplicity moments of final proton, \(\Lambda\) and \(\Xi^-\) at different rapidity window size (correspondingly at different flavor conservation parameter \(\alpha_2\)). Lines show moments of baryons without including resonance decays. Open symbols show results including weak decays, strong decays and electromagnetic decays. Solid symbols show results including only strong and electromagnetic decays. We can see that due to the large decay contribution to final proton and \(\Lambda\) multiplicity, moments of final proton and \(\Lambda\) and \(\Xi^-\) at different rapidity window size. Open squares show results including only strong and electromagnetic (S&EM) decays. Comparing to initial baryon-antibaryon correlations without resonance decays (dashed lines), we can see that all correlations except \(p\bar{p}\) are almost
unaffected by S&EM decays. However, for baryon-antibaryon correlations except Ξ−Ω+ and Ξ−Ω+, after further including weak decays, they (open circles) are significantly changed. In addition, we observe that final pɔ, pΛ, ɔΞ+ and pΩ+ with full decay contributions, open circles, have almost the same correlations. This is because that they all reflect such a baryon-antibaryon production association, i.e., when an antibaryon either ɔ, Λ or ɔΞ+ is produced, a baryon of any species (via final proton) should be produced with a certain associated probability to balance the baryon quantum number.

There are some striking properties in above decay calculations which are suitable for the future experimental measurement. First, Ξ−Ω+ and Ξ−Ω+ correlations are almost unaffected by resonance decays. Second, pΩ+ correlations with only S&EM decays are negative while including weak decays is positive for moderate and large rapidity window size. Third, final pΛ correlations change the sign around moderate window size ɔ. Fourth final pΞ+ with full decay contribution are positive while including only S&EM decays it tends to zero at moderate and large ɔ.

VI. SUMMARY AND DISCUSSION

We have studied the dynamical multiplicity fluctuation and correlation of identified baryons and antibaryons produced by the hadronization of bulk quark system in quark combination model. We firstly develop a working model to discuss the most basic dynamics of quark combination which is necessary to multiplicity study. Then, for the quark system with given quark numbers and antiquark numbers, we derive analytically moments (variance, skewness and kurtosis) of multiplicity distributions of produced baryons, two-baryon multiplicity correlation, and baryon-antibaryon multiplicity correlations. We obtain a series of interesting findings in baryon multiplicity

(1) Multiplicity moments of identified baryons nearly exhibit the behavior of binomial distribution.

(2) Anti-association of two-baryon production are mainly determined by the coincide flavors of two baryon.
(3) All baryon-antibaryons correlation show a positive and universal magnitude, which means that the joint production of baryon and antibaryon is mainly constrained by baryon quantum number conservation in combination.

These properties come from the basic dynamics of quark combination and, therefore, can be regarded as the general features of quark combination mechanism.

We also take into account the correlations and fluctuations of quark numbers and antiquark numbers before hadronization to study their effects on multiple production of baryons and antibaryons. Suppose the weak off-diagonal flavor correlations of quarks and antiquarks, we focus on the effects of quark number fluctuations via Poisson distribution and flavor conservation via the parameter \( \lambda_2 \equiv \frac{C_{ff}}{\sigma_f^2} \). In order to relate the experimental measurement at finite rapidity window size \( y_w \), we use the charge balance function of thermal particles measured in central Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV to constrain the parameter \( \lambda_2 \) at different rapidity window size. We calculate the moments of inclusive baryon multiplicity, two-baryon multiplicity correlations and baryon-antibaryon correlations at mid-rapidity with unit window size and these at different rapidity window size. Different from those results directly from quark combination, after including quark number fluctuations and correlations we find

1. Multiplicity moments of baryons deviate from binomial distribution, and at small flavor conservation parameter value we can observe the Poisson statistics.
2. All two-baryon correlations within unit rapidity window size tend to be a negative and universal value.
3. Baryon-antibaryon correlations exhibit large species difference. In particular, \( C_{p\Omega^+} \) is negative showing the anti-association between \( p \) and \( \Omega^+ \) production. At moderate window size we observe the negative sign of \( p\Xi^+ \).
correlation but at large window size we observe the vanishing $p\Xi^+$ correlation. We also observe the sign change of $p\Lambda$ correlation at moderate window size.

We also study the influence of resonance decays on multiplicity fluctuations and correlations of baryons and antibaryons. We separately calculate above quantities including strong and electromagnetic (S&EM) decays and those further including weak decays. Our final results of stable baryons $p$, $\Lambda$, $\Xi^-$ and $\Omega^-$ show several interesting properties

1. The moments of final proton and $\Lambda$ are obviously smaller than those of directly produced baryons. However, the scaled moments of final $\Xi^-$ is weakly influenced by resonance decay and is close to Poisson distribution.

2. Two-baryon correlations are hardly influenced by either S&EM decays or weak decays. In addition, they are dependent on rapidity window size in a non-monotonic way.

3. Effects of resonance decays on baryon-antibaryon correlation are sophisticated. $\Xi^-\Xi^+$ and $\Xi^-\Omega^-$ correlations are almost unaffected by S&EM and weak decays. $p\Omega^+$ correlations with only S&EM decays are negative while including weak decays is positive for moderate and large window size. $p\Lambda$ correlations change the sign around moderate window size $y_w$.

They are striking phenomena which are suitable for the future experimental measurement.

Some discussions related to experimental observation at finite window size are in order. In Sec. III and IV, we choose a quark system of specific size which corresponds to a specific slice of the quark system produced in relativistic heavy ion collisions. Here we do not consider the possible rapidity shift between (anti-)quarks and the formed (anti-)baryon, which may lead to the produced baryons fly off the studied window and baryons produced in other region fly in this window. However, the effect of rapidity shift in combination is quite small because of the following two reasons. First, there is small discrepancy between the total mass of three quarks and the mass of the formed baryon. Note that we usually use the constituent quark mass in QCM, i.e. $m_u \sim 330$ MeV and $m_s \sim 500$ MeV. Therefore, there is no large rapidity shift occurs in combination due to the large energy mismatch between three quarks neighboring in phase space and the baryon they formed. Second, we apply the quark combination rule as explain in Sec. II to longitudinal rapidity direction to solve the unitary issue which is necessary for multiplicity study. This approach has reproduced experimental data of rapidity distribution of identified hadrons in relativistic heavy ion collisions at different collisional energies. The rapidity interval between neighboring quarks is only the order of $10^{-3}$ due to the high quark number density $dN/dy \sim 10^3$ in ultra-relativistic heavy ion collisions. Therefore, rapidity shift in baryon production is quite small and it hardly influences results in this work. In Sec. V, we also neglect the rapidity shift in resonance decay. Because the rapidity shift in baryon decay is small ($\lesssim 0.1$), its influence is also expected to be small.

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Appendix: derivation of Eqs. (32-34)

Applying the Eq. (30) and substituting the following expansion

$$\langle N_a^m N_b^n \rangle = \frac{1}{2} \sum_{f_1 f_2} \delta_{12} N_a^{m_1} N_b^{m_2} C_{f_1 f_2}$$

into the definition of multiplicity moments

$$\sigma_a^2 = \langle N_a^3 \rangle - \langle N_a \rangle^2,$$

$$\langle \delta N_a^3 \rangle = \langle N_a^3 \rangle - 3\langle N_a \rangle\sigma_a^2 - \langle N_a \rangle^3,$$

$$\langle \delta N_a^4 \rangle = \langle N_a^4 \rangle - 4(\delta^2 N_a)^2 - 6(\delta N_a)^2 \sigma_a^2 - \langle N_a \rangle^4,$$

and two-body multiplicity correlation

$$C_{ab} = \langle N_a N_b \rangle - \langle N_a \rangle \langle N_b \rangle,$$

we can get the expression of Eqs. (32-34) up to two body quark correlations. The complete expansion of Eqs. (A.2) and (A.3) up to four body quark correlations are too lengthy and are not shown. In addition, direct calculations according to Eqs. (A.2) and (A.3) is numerically convenient.

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