The Value of Information in Selfish Routing

Simon Scherrer, Adrian Perrig, Stefan Schmid

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Network-based path selection

- Suboptimal paths
- No robustness to failures
Source-based path selection

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Suboptimal paths
No robustness to failures
Best path for use case
Fast rerouting on failure
Network-based path selection: Network operator view
Source-based path selection: Network operator view
Goals of our work

Revisit selfish-routing concepts to investigate two issues arising in emerging path-aware Internet architectures:

- **Impact of information**: What network state information should be shared with end-hosts?

- **Impact on network operators**: What is the impact of selfish routing on the cost of network operators?
Price of Anarchy: Three components

\[ \text{PoA} = \frac{C(F_{eq})}{C(F_{opt})} \]

- **C**: Social cost function
- **F_{opt}**: Social optimum
- **F_{eq}**: Equilibrium
Adapted Wardrop model of source-based path selection

\[ d = (d_{1,2}, d_{3,4}) = (1, 1) \]

\[ F = (F_\alpha, F_\gamma\beta, F_\beta, F_{\alpha\gamma}) \]

\[ f = (f_\alpha, f_\beta, f_\gamma) \]

\[ c_\alpha(f_\alpha) = 1 \]

\[ c_\beta(f_\beta) = f_\beta^2 \]

\[ C_\pi(F) = \sum_{\ell \in \pi} c_\ell \]
Total cost functions and social optima

End-host cost function: \[ C^* = \sum_{\text{end-hosts}} \sum_{\text{paths}} \text{flow on path} \cdot \text{path cost} \]

\( (\text{classic}) \)

\[ = \sum_{\pi \in \Pi} F_{\pi} \cdot C_{\pi}(F) = \sum_{\ell \in L} f_\ell \cdot c_\ell(f_\ell) \]

End-host optimum: \[ F^* = \text{argmin}_F C^*(F) \]

Network-operator cost function: \[ C^\# = \sum_{\text{links}} \text{link cost} = \sum_{\ell} c_\ell(f_\ell) \]

Network-operator optimum: \[ F^\# = \text{argmin}_F C^\#(F) \]
Characterizing social optima: Suboptimal path flow pattern

\[ \begin{align*}
\mathbf{d} &= (d_{1,2}) = (1) \\
\mathbf{F} &= (F_\alpha, F_\beta) \\
C(\mathbf{F}) &= C_\alpha(F_\alpha) + C_\beta(F_\beta) \\
\exists \delta. \ |\Delta C_\alpha^+| < |\Delta C_\beta^-| \\
\Rightarrow C \text{ can be reduced}
\end{align*} \]
Characterizing social optima: Optimal path flow pattern

∀ δ. |ΔC_α^+| > |ΔC_β^-|
⇒ C cannot be reduced
⇒ C is optimal

∂C/∂F_α = ∂C/∂F_β
⇒ ∀ δ. |ΔC_α^+| > |ΔC_β^-|
⇒ C is optimal
Socially optimal marginal costs

\[ \frac{\partial C(F')}{\partial F_{\pi}} \] is the marginal cost of path \( \pi \)
given path-flow pattern \( F \)

A path-flow pattern \( F \) is optimal w.r.t. a cost function \( C \in \{C^*, C^\#\} \)

if for every origin-destination pair:

\[ F_{\alpha}, \ldots, F_{\rho} > 0 \quad F_{\sigma}, \ldots, F_{\omega} = 0 \]

\[ \frac{\partial C(F)}{\partial F_{\alpha}} = \ldots = \frac{\partial C(F)}{\partial F_{\rho}} \leq \frac{\partial C(F)}{\partial F_{\sigma}} \leq \ldots \leq \frac{\partial C(F)}{\partial F_{\omega}} \]
Social optimum: Comparison (Example)

\[
F^\# = (F_\alpha, F_\beta, F_\gamma) = \left( \frac{1}{2}, 0, \frac{1}{2} \right)
\]

\[
F^* = (F_\alpha, F_\beta, F_\gamma) = \left( \frac{2}{3}, \frac{1}{3}, 0 \right)
\]

Different optima!

Network operators prefer usage of links with little variable cost (here: \(\gamma\))
Price of Anarchy: Where are we?

\[
\text{PoA} = \frac{C(F_{\text{eq}})}{C(F_{\text{opt}})}
\]

- **C**: Total cost function
- **\(F_{\text{opt}}\)**: Social optimum
- **\(F_{\text{eq}}\)**: Equilibrium
Equilibrium with latency-only information (LI equilibrium)

\[ d = (d_{1,2}) = (1) \]

\[ F = (F_\alpha, F_\beta) = (1, 0) \]

\[ C_\alpha = C_\beta \implies F = (1, 0) \text{ is an LI equilibrium} \]
Characterizing the LI equilibrium

A path flow pattern $F$ is an LI equilibrium if for every origin-destination pair:

$$F_\alpha, \ldots, F_\rho > 0 \quad F_\sigma, \ldots, F_\omega = 0$$

$$C_\alpha(F) = \ldots = C_\rho(F) \leq C_\sigma(F) \leq \ldots \leq C_\omega(F)$$
Equilibrium with perfect information (PI equilibrium)

\[ d_{(1)} = (d_{1,2}) = (1) \]

\[ F_{(1)} = (F_\alpha, F_\beta) \]

\[ \alpha: c_\alpha(f_\alpha) = f_\alpha \]

\[ f_\alpha = F_\alpha + 1 \]

\[ f_\beta = F_\beta + 1 \]

\[ \beta: c_\beta(f_\beta) = 2 \]

Minimize selfish cost

\[ C_{(1)}(F_{(1)}) = F_\alpha \cdot (F_\alpha + 1) + F_\beta \cdot 2 \]

\[ \Rightarrow (F_\alpha, F_\beta) = \left( \frac{2}{3}, \frac{1}{3} \right) \] is a PI equilibrium
Characterizing the PI equilibrium

A path flow pattern $F$ is a PI equilibrium if for every origin-destination pair of any end-host $e$:

$$F_{\alpha}, \ldots, F_{\rho} > 0 \quad F_{\sigma}, \ldots, F_{\omega} = 0$$

$$\frac{\partial C_{(e)}(F)}{\partial F_{\alpha}} = \ldots = \frac{\partial C_{(e)}(F)}{\partial F_{\rho}} \leq \frac{\partial C_{(e)}(F)}{\partial F_{\sigma}} \leq \ldots \leq \frac{\partial C_{(e)}(F)}{\partial F_{\omega}}$$
Capturing the value of information

| Information assumption | Latency-only Information (LI) | Perfect Information (PI) |
|------------------------|-------------------------------|--------------------------|
| Equilibrium            | $F^0$                         | $F^+$                     |
| Price of Anarchy       | $\text{PoA}^0 = \frac{C(F^0)}{C(F^{opt})}$ | $\text{PoA}^+ = \frac{C(F^+)}{C(F^{opt})}$ |

$\Delta = \text{Value of Information (VoI)}$
The benefits of information

$$\text{VoI} > 0$$
The benefits of information: Network of parallel links
(cf. Roughgarden 2003)

\[ \sum_k d_{k,T} = 1 \]

\[ F = (F_{1\alpha}, F_{1\beta}, \ldots, F_{K\alpha}, F_{K\beta}) \]

\[ \alpha \quad c_\alpha(f_\alpha) = 1 \]
\[ \beta \quad c_\beta(f_\beta) = f_\beta \]

**EH Opt:** \( F^* \) s.t. \( f_\beta = 1/2 \)

**LI Eq:** \( F^0 \) s.t. \( f_\beta = 1 \)

**NO Opt:** \( F^\# \) s.t. \( f_\beta = 0 \)

**PI Eq:** \( F^+ \) s.t. \( f_\beta = K/(K+1) \)
## The benefits of information: Network of parallel links

| Perspective                  | LI equilibrium | PI equilibrium |
|------------------------------|----------------|----------------|
| **End-host perspective**     | $\text{PoA}_{0} = \frac{4}{3}$ | $\text{PoA}_{0} = \frac{4}{3}$ | $\text{PoA}_{+} = \frac{(K^2 + K + 1)}{(K^2 + 2K + 1)} \cdot \frac{4}{3}$ \leq \text{PoA}_{0}$ |
| **Network-operator perspective** | $\text{PoA}_{0} = 2$ | $\text{PoA}_{+} = 1 + \frac{K}{K + 1}$ \leq 2 = \text{PoA}_{0}$ |
**The benefits of information: Network of parallel links**

| LI equilibrium | PI equilibrium |
|----------------|----------------|
| End-host perspective | PI equilibrium cheaper than LI equilibrium |
| Network operator perspective | $\text{VoI} > 0$ |

For the PI equilibrium:

- $\text{PoA}^{*+} = \frac{(K^2 + K + 1)}{(K^2 + 2K + 1)} \cdot \frac{4}{3} \leq \text{PoA}^{*0}$

For the LI equilibrium:

- $\text{PoA}^{#+} = \frac{1 + K/(K + 1)}{2} \leq 2 = \text{PoA}^{#0}$
The drawbacks of information

\[ \text{VoI} < 0 \]
The drawbacks of information: Ladder network

\[ \mathbf{d} = (d_{11,12}, d_{21,22}) = (1, 1) \]

\[ \mathbf{F}^\rightarrow = (1, 0, 1, 0) \]

Direct-only \( \mathbf{F}^\rightarrow \) is universally optimal:

\[ \mathbf{F}^\rightarrow = \mathbf{F}^* = \mathbf{F}^# \]
The drawbacks of information: Ladder network

$$F^- = (1, 0, 1, 0)$$

$$c_{h_1}(f_{h_1}) = f_{h_1}^2$$

$$c_{v_1}(f_{v_1}) = f_{v_1}$$

$$c_{v_2}(f_{v_2}) = f_{v_2}^2$$

$$C_{1H}(F^-) = 1 = C_{1V}(F^-) \Rightarrow F^- = F^0$$

(LI equilibrium is optimal)
The drawbacks of information: Ladder network

\[ F^\rightarrow = (1, 0, 1, 0) \]
\[ F^\sim = (0.9, 0.1, 1, 0) \]

\[ C_{(1)}(F^\rightarrow) = 1 > C_{(1)}(F^\sim) = 0.87 \Rightarrow F^\rightarrow \neq F^+ \]

(PI equilibrium is suboptimal)
The drawbacks of information: Ladder network

\[ F^\rightarrow = (1, 0, 1, 0) \]

**PI equilibrium**

more costly than

**LI equilibrium**

\[ C^{(1)}(F^\rightarrow) = 1 > C^{(1)}(F^\sim) = 0.87 \implies F^\rightarrow \neq F^+ \]

(PI equilibrium is suboptimal)

\[ h_1 c_{h_1}(f_{h_1}) = f_{h_1}^2 \]

\[ h_2 c_{h_2}(f_{h_2}) = f_{h_2}^2 \]

\[ h_2 c_{h_2}(f_{h_2}) = f_{h_2}^2 \]

\[ f_{v_1} = f_{v_1} \]

\[ f_{v_1} = f_{v_1} \]

\[ f_{v_2} = f_{v_2} \]

\[ f_{v_2} = f_{v_2} \]

\[ F^\sim = (0.9, 0.1, 1, 0) \]

\[\text{VoI} < 0\]
The drawbacks of information: Generalized ladder network

Upper bound on PoA for network operators:

\[ \text{PoA}^+ \leq 1 + \frac{2(H-1)}{3H} p \]

\[ \leq 1 + \frac{2}{3} p \]
Drawback of information: Abilene Topology Case Study
Thank you for your attention!

Happy to answer questions in the chat forum!

Or by email: Simon Scherrer
simon.scherrer@inf.ethz.ch