Comparative analysis of simulation options for the real geometry of the surfaces of gas turbine engine parts

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Abstract. Geometric deviations of parts surfaces inevitably occur during parts manufacture. It is known that the accumulated defects of parts surfaces and assembly units impact their functional characteristics throughout the entire product life cycle. Often, assembly simulation and dimensional ties analysis of products are performed by using the nominal geometry of parts without taking into account geometric deviations of the shape and location of parts surfaces. The accuracy of dimensional analysis without taking into account geometric deviations of the shape and location of parts surfaces is not sufficient to solve technological problems in digital production. The purpose of this article is the study of the influence of methods and models for constructing the real geometry on the accuracy of its reproduction. The subject of the research is the assembly unit of the rotor of a low-pressure turbine of an aircraft engine. The article considers three methods for constructing the real geometry, which are used to realize assembly virtual simulation. Comparative results of assembly simulation with the use of considered methods of real geometry constructing are presented, as well as their influence on the controlled assembly parameters is estimated.

1. Introduction

Geometric deviations of the shape and location of the surfaces of the part are present throughout the entire production cycle from blanks to the finished product. It is known that the geometric deviations of the surfaces of parts and assembly units affect the operational parameters of the product including resource, reliability, fuel efficiency of engines and other indicators of machines and equipment. To achieve high values of operational characteristics it is necessary to tighten the requirements for dimensional accuracy parameters, such as geometric tolerances of parts or assembly units. In engine manufacturing, the nominal geometry of parts without taking into account geometric deviations is used for modeling and analyzing the assembly of products. In digital production, it is necessary not only to predict assembly parameters but also to control these geometric deviations of parts throughout the product life cycle.

The geometric deviations arising during machining can be divided into two main types: systematic and random [2,3]. This classification is based on extensive production experience. Deviations in the shape and location of the surfaces of the parts are present on each part produced by the machine-shop, but some deviations may be present only on several workpieces. Systematic deviations are deterministic, expected and reproducible [4]. These deviations arise from workpiece clamping errors, incorrect base or kinematic inaccuracy of the machine itself. Random deviations occur as a result of changes in the manufacturing process, such as changes in material properties, tool wear, or environmental fluctuations. Because of these different characteristics, systematic and random
geometric deviations are usually modeled using different methods, even if they can be classified as the same type of geometric deviation (for example, waviness).

The first methods for modeling geometric deviations made it possible to create a 3D object with optimal generated variations based on the use of a parametric solid-state model [1] and the addition of different variations to it [5–7]. The disadvantages of this method can be attributed to parts having several complex geometric deviations. These shortcomings led to the use of continuous displacements [8, 9], which were then extended to extended variational surfaces [10–12] and the tolerance field [13]. Also, various methods for modeling geometric deviations use splines (Bezier curves and NURBS) [14]. Articles [1,15] present approaches based on the construction of tolerance zones based on solid-state modeling. Kinematic formulas such as linear homogeneous matrix transformations [16] or small displacement torsor (SDT) [17], other approaches based on kinematics [18] or vector loops [19,20] are used for modeling geometric deviations (usually within certain tolerance zones) based on a continuous geometric representation [21].

To predict the assembly parameters of the assembly of the gas turbine engine rotor, it is necessary to have reliable data on the dimensional and accuracy parameters of the assembled parts. The universal, widespread and most accurate measurement tool in the engine industry is a coordinate measuring machine. The data obtained from the coordinate measuring machine is used in the construction of real surfaces, as during the measurement there may be emissions that can reach up to 0.1 mm, which can significantly affect the simulated real surface since the monitored assembly parameters are in the range from 0.01 to 0.1 mm, where the accuracy of the construction can significantly affect the forecasting results.

Thus, the urgent task is to develop methods for modeling real surfaces of parts, allowing to solve the problems of dimensional accuracy analysis and synthesis while taking into account defects in the surfaces of parts.

2. Object of study
The object is the assembly of two parts of a low-pressure turbine: a spacer and a disk. Figure 1 shows a sketch of the assembly unit in question.

![Figure 1. Assembly unit and controlled surfaces, 1 - disk with a deflector, 2 - spacer.](image)

In Figure 1, bases A and B form the axis of rotation (base axis). The surface of the disk D, the radial surface of the disk C. Consider the developed methods for the formation of real models of parts entering the assembly unit under consideration.
3. Methods of forming real models of parts

Figure 2 shows a method of forming a generalization of real models of parts.

![Figure 2. Methodology for the formation of real models.](image)

The real model of the part, reflecting its geometric parameters, is formed based on measurements. The first stage of the method, the measurement of mating and controlled surfaces, is performed using equipment that allows you to obtain data on the coordinates of points in space. In particular, a contact coordinate measuring machine ZEISS MMZ G 20/30/20 with a VAST gold sensor was used.

The first and fifth stages are the same for the compared methods. In the first method, steps two and three are not applied; in the second method, additional processing of the coordinates of the measured points is carried out, but there is no analysis and generalization of the measured data; in the third method, all five stages of the method are present, respectively, in figure 2. Accordingly, the first method is the least time-consuming, and the third is the most.

At the final stage, the points were exported to the NX CAD system, where the construction of real surface models takes place.

To describe the surfaces of parts with geometric shape deviations, we used surfaces formed from bicubic portions (Koons portions [22]). The described surface is a segment corresponding to the parameter values 0 ≤ u ≤ 1, 0 ≤ v ≤ 1. The Koons portion is formed as a result of the conjugation of boundary spline curves and is determined by the expression:

\[ P(u, v) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} u^i v^j, \quad (1) \]

where \( P(u, v) \) – point of a bicubic surface; \( a_{ij} \) – algebraic vector coefficients with components x, y, and z.

Combining Koons portions allows to define a surface of arbitrary shape and size. The surface in (1) is an interpolation tool for setting the surface, i.e. curves and surfaces pass through set points. However, in some tasks, it is required to smooth the data, for example, when processing the measured information. A universal approximation tool is NURBS-splines and surfaces described by parametric functions of the form [22]:

\[ P(u, v) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} h_{ij} \cdot P_{ij} \cdot N_k(u) \cdot N_l(v)}{\sum_{i=1}^{n} \sum_{j=1}^{m} h_{ij} \cdot N_k(u) \cdot N_l(v)}, \quad (2) \]
where \( p_\alpha \) – coordinates \((x_\alpha, y_\alpha, z_\alpha)\) of the defining point in three-dimensional space; \( u, v \) – parameters for calculating the coordinates of the points of the spline surface, varying in the ranges \([t_{k-1}, t_{k+1}]\) and \([s_{j-1}, s_{j+1}]\) respectively; \( h_{ij} \) – homogeneous coordinates of set points; \( N_k(u) , N_j(v) \) – basis functions (or conjugations) of the spline in the parametric directions \( u \) and \( v \) respectively; \( t, s \) – nodal parameter values for which functions \( N_k(u) \) and \( N_j(v) \) are not equal to zero; \( k, l \) – the degree of piecewise spline in the direction of the parameters \( u \) and \( v \) respectively; \( n, m \) – number of points along directions \( u \) and \( v \).

For unstructured data (the first method), the construction of an approximating surface from (2) is used. In the case of processing and analysis of the measured data, and interpolating surface is constructed (1). The stage of loading the coordinates of the processed surface points and constructing the surfaces themselves was automated using a software application in the NX / Open API module in the Visual Basic programming language. Using the constructed surfaces, the nominal CAD model is reconstructed [23]. The resulting model is a real part model that takes into account real geometry deviations.

3.1. Formation of real models without a stage of processing measurement data

The developed “ScanExporter” program developed in C# was used for uploading data on the coordinates of the points of the measured surfaces. The program selects a file or group of files from the directory and extracts all points of the objects. After importing data from Calypso 5.6, it is saved to a *.csv file. The file structure consists of headers, coordinates of points, and components of normal vectors at points.

The data from this file is loaded into the MATLAB environment program and only the surface points of interest from the total measured data array are saved in the *.txt or *.xlsx format. This is followed by stage 4 of the general methodology in Figure 2, and NURBS (2) is used to describe the surfaces.

3.2. The second method, including pre-processing of the measured data

In the second method, the processing of the coordinates of the points is carried out, which consists of filtering and smoothing the values in the measurements and ordering the structure of points on surfaces having sufficiently high shape deviations. Let us take a closer look at these two processing subtasks. The data after the measurements are loaded into a mathematical package, in our case, the MATLAB application package.

The coordinates after the measurement may have “outliers” due to the presence of contaminants on the surface, improper operation of the sensor, etc. On a certified measuring device such values are rare, for example, when measuring 160 points, ten such values can be accumulated, the value of which can be on average from 5 to 30 microns and higher. To filter such values on cylindrical and flat surfaces, as well as to calculate location deviations, it is necessary first enter the replacement elements in the array of measured points: respectively, “cylinder” and “plane”, using the least squares method.

The moving average method was used for filtering. The values after filtration are numerically equal to the arithmetic average of the values of the original function for the specified period and are calculated by the formula [24]:

\[
SMA_t = \frac{1}{n} \sum_{i=0}^{n-1} p_{t-i},
\]

where \( SMA_t \) – moving average value at point \( t \); \( n \) – the number of values of the original function for calculating the moving average (smoothing interval); \( p_{t-i} \) – value of the original function at the point \( t-i \).

In the case of cylindrical faces, a filter is made for the value of polar radii, and in the case of end faces, a filter is made for the coordinates of points along an axis perpendicular to the end face. Points with outliers above 5 microns were simply removed from the array.
When using filters to reduce the effect or even eliminate emissions, additional errors occur. These errors are due to the fact that the filtered sections of the profile can be distorted due to the influence of emissions located in some proximity to these sections. The estimation of errors introduced by filters is a separate complex task and can be performed using the simulation method [25]. In the course of the simulation method, it is necessary to form an initial profile that is accepted as real, add random outliers to it, filter it using this filter, and compare the resulting profile after filtering with the profile that was laid out as the real one. The filtering error depends on the magnitude of the emissions and the discreteness of the measured points. The size of the errors during filtration is up to 50% of the value of the random error, that is, they reach 1.5 microns.

The ordering of the point structure consists in calculating the uniformly distributed coordinates of the points and calculating the coordinates of the points that are not enough to construct the data structure so that they form a grid of curves to define the surface (1). The calculation of the points coordinates consists of the formation of sections of the surfaces of the parts by approximating or interpolating the measured arrays of the surface points coordinates by means of splines.

3.3. The third method
As described previously, in the third method, after the stage of preliminary processing of the measured data, there is a stage of analysis and generalization. To describe the shape deviation, a harmonic series is selected, which can be used to describe the discrete series of shape deviation data obtained in the previous steps with sufficient accuracy. The expression of the harmonic series used has the following form:

$$\delta_p(x) = \sum_{k=1}^{\infty} A_k \sin(k \cdot (2 \cdot \pi \cdot x / \tau + \theta_k) + \left(1 + (-1)^k\right) \cdot \pi / 2),$$

where $A_k$ – set of amplitudes of members of the harmonic series; $\theta_k$ – phases of the members of the harmonic series; $k$ – frequencies of members of the harmonic series; $\tau$ – function period (curve length); $x$ – the current length of the curve bounded by the starting and current points (angle).

To obtain the parameters of the series, the discrete Fourier transform was used. In addition to the analysis of shape deviations, the following parameters are calculated: deviations of the radii of cylindrical surfaces; location deviation parameters (inclination angles to coordinate axes, surface displacement). The location deviation parameters are calculated based on the parameters of the replacement elements “cylinder” and “plane”, calculated by the least squares method (LSM) from the coordinates of the measured points.

The parameters obtained as a result are used to summarize the data, tasks modeled by using the calculated parameters of the point deviations:

$$\bar{p}_p = \left(\bar{p}_n + \bar{n} \cdot \delta_{\phi}\right) \cdot R + \bar{t},$$

where $p_p$, $\bar{p}_n$ – point coordinate vector $(x, y, z)$ respectively measured (simulated) and nominal (CAD) surfaces; $\bar{n}$ – normal vector at the point $\bar{p}_n$; $\delta_{\phi}$ – shape deviation at a point $\bar{p}_n$; $R$, $\bar{t}$ – rotation matrix and transposition vector of point coordinates $\bar{p}_n$ characterizing the deviation of the location.

Thus, in the third method, the coordinates of the measured points are first “decomposed” into components by which they can be reproduced artificially. The end and flat faces of the parts considered in the study at stage four in the third method are sweeping [26], they can be obtained by moving one curve along a guide curve in space.

4. CAE - calculation of assembly parts
Real models are used for virtual verification of assembly capabilities and quality indicators. Virtual assembly is performed using the finished elements method. The finite element model (FEM) was
created and the boundary conditions were set in the ANSYS Workbench software package in the “Static Structural” module.

The module used does not allow modeling with a gap between parts. The pre-set preload in 3D models leads to a significant deterioration in the results; the following actions were performed to eliminate this problem:
- A FEM grid with a given structure was generated;
- The size of the gap between the end mating surfaces of 3D models was determined;
- The part was moved in the axial direction by a previously determined value.

All contacts between the mating surfaces of the FEM were used with “Frictional” friction, with a value equal to 0,15. The gap between the end surfaces of the part and the nut (bolt head) was compensated by the “Adjust to Touch” function.

The remaining set parameters in contact pairs are general and necessary to increase the convergence of the problem and the accuracy of the calculation. The algorithm for determining contact points on the nodes of the elements of the target surface was set to “Nodal-Normal To Target”, which allows you to more accurately determine the interaction of parts with real faces.

The next step is to set limits and apply a power load. To simulate the application of force on the parts under consideration, a simplified 3D model of a bolted connection was used. The axial force in the bolt joint FEM was modeled with the “Bolt Pretension” function with the specified loading stages. The spacers were fixed on the base surfaces A and B using the “Fixed Support” function, which eliminates movements and rotations along all three axes.

For further analysis, the deformed models formed for each mutual angular position are generated by the “Total Deformation” function and saved in the *.stl format.

5. Results of work
The assembly experiment was carried out in the following sequence:
1. Measurement of contacting and control surfaces of parts;
2. Assembling parts at zero marks, on special equipment that simulates the connection of a spacer to a disk of 3 steps, a shaft and a disk simulator of 4 steps;
3. The measurement of the points of the control surfaces C and D (Figure 1) relative to the bases of the shaft simulator (special equipment) and the calculation of end and radial beats;
4. The disk is removed and rotated 120 degrees relative to zero marks and the measurement of assembly parameters is repeated.

Stages 3 and 4 are performed for three angular positions.
5. Data is exported in the form of coordinates of the measured points in the *.txt format.

In MATLAB, steps 1–3 of the general procedure in Figure 1 were carried out, real models were created in the NX system (steps 4–5 of the method in Figure 1), and the virtual assembly parameters were calculated in the ANSYS program. Table 1 shows the results of predicted assembly parameters using three different methods for generating real models.

| Mutual angular laid, deg. | Assembly parameter, surface | Experiment, mm | Methodology for the formation of real models, mm |
|--------------------------|---------------------------|----------------|-----------------------------------------------|
|                          | Radial runout, C          | 0.159          | 0.135 0.100 0.167 |
|                          | End runout, D             | 0.098          | 0.127 0.127 0.124 |
| 0                        |                           |                | Î 1 Î 2 Î 3              |
| 120                      | Radial runout, C          | 0.134          | 0.092 0.147 0.097       |
|                          | End runout, D             | 0.079          | 0.096 0.098 0.063       |
| 240                      | Radial runout, C          | 0.133          | 0.144 0.151 0.109       |
|                          | End runout, D             | 0.084          | 0.104 0.103 0.085       |

The convergence of the simulation results with the actual parameters obtained during the assembly was evaluated by calculating the absolute deviations:
$$\delta_a = P_s - P_{mes},$$

and relative deviations:

$$\delta_{rel} = \delta_a / P_{mes} \cdot 100\%,$$

where $P_s$ - simulation parameter; $P_{mes}$ - measured parameter.

Analysis of methods for constructing surfaces with real geometry is presented in table 2.

| Mutual angular laid. details, deg. | Methodology for the formation of real models | Assembly parameter, surface | Deviation |
|-----------------------------------|---------------------------------------------|-----------------------------|-----------|
|                                   | № 1                                         | № 2                                         | № 3       |
|                                   |                                              | $\delta_a$, mm  | $\delta_{rel}$, % | $\delta_a$, mm  | $\delta_{rel}$, % | $\delta_a$, mm  | $\delta_{rel}$, % |
| 0                                 | Radial runout, C                            | -0.024                     | -15.15     | -0.060                   | -37.56             | 0.007                      | 4.64                     |
|                                   | End runout, D                              | 0.029                      | 29.14      | 0.029                    | 29.46              | 0.026                      | 26.56                     |
| 120                               | Radial runout, C                           | -0.043                     | -31.72     | 0.013                    | 9.62               | -0.037                     | -27.46                    |
|                                   | End runout, D                              | 0.017                      | 21.53      | 0.019                    | 24.00              | -0.016                     | -20.57                    |
| 240                               | Radial runout, C                           | 0.011                      | 8.00       | 0.018                    | 13.28              | -0.024                     | -17.91                    |
|                                   | End runout, D                              | 0.020                      | 24.09      | 0.019                    | 22.48              | 0.001                      | 1.37                      |

Based on the results shown in tables 1 and 2, the minimum deviations from the experimental results are obtained using 3rd method, higher when using 1st and 2nd methods. The small discrepancy in predicting the end runout when using methods 1 and 2 is explained by the fact that measurements of surface points were carried out practically in laboratory conditions, and the stage of processing the measured data (the method in Figure 2) has little effect on the accuracy of geometry reproduction. The maximum deviation when using 3rd method was 27.5%.

![Figure 3. Prediction results of radial runout of surface C.](image-url)

Deviations between the predicted and measured values of the radial runout are largely due to the fact that the angular position of the spacers in the assembly unit is ensured by bolted joints. The positioning of the bolt hole is 0.02 mm, which significantly affected the assembly parameters in the full-scale experiment. When simulating the assembly process, the position of the holes remained nominal, which led to high discrepancies between theoretical and experimental data. Figures 3 and 4 show comparative results of experimental and predicted results of assembly parameters.
6. Conclusion
The article presents the results of the research on the impact of methods and models for representing real geometry on the accuracy of the prediction of assembly parameters. The considered methods of modeling real surfaces of parts allow solving the problems of dimensional accuracy analysis and synthesis taking into account defects in the surfaces of parts. The obtained results of predicting the assembly parameters of the assembly of parts of the turbine rotor allow us to talk about the application of the considered methods in real manufacturing.

In the distant future, more experiments on assembly and measurement are planned. A promising approach to constructing real models for solving the problem of digital forecasting of product quality is the third method, which allows us to generalize the errors of a batch of parts and thereby simulate a much larger sample of surfaces, reaching up to 1000 or more cases. Besides, a limited set of parameters characterizing the real geometry obtained in the implementation of the third method allows us to use other methods for calculating assembly parameters (for example, machine learning) based on the considered assembly simulation to reduce the complexity.

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