Efficiency of a feedback process in cavity quantum electrodynamics

H T Fung and P T Leung

Department of Physics and Institute of Theoretical Physics, The Chinese University of
Hong Kong, Shatin, Hong Kong SAR, People’s Republic of China

E-mail: ptleung@phy.cuhk.edu.hk

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Abstract
Utilizing the continuous frequency mode quantization scheme, we study from
first principles the efficiency of a feedback scheme that can generate maximally
entangled states of two atoms in an optical cavity through their interactions with
a single input photon. The spectral function of the photon emitted from the
cavity, which will be used as the input of the next round in the feedback
process, is obtained analytically. We find that the spectral function of the
photon is modified in each round and deviates from the original one. The
efficiency of the feedback scheme consequently deteriorates gradually after
several rounds of operation.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Entanglement has been considered as a fundamental issue to demarcate the border between
quantum and classical physics since the earliest day of quantum theory [1–3], and is presently
an essential ingredient in quantum computation [4, 5]. To achieve the construction of quantum
computers, there has been a surge of interest in the generation of entangled states of atoms and
photons through various implements, including solid-state devices [6–9], nuclear magnetic
resonance in liquid samples [10, 11], ion traps [12–14] and atom–photon interactions in optical
(or microwave) cavities [15–17].

Being a well-studied topic in quantum optics, cavity quantum electrodynamics (CQED)
provides a full arsenal of techniques to generate entangled atom pairs [18–22] and also
entangled photon pairs [23, 24]. In particular, previous studies have shown that cavity loss as
well as environmental noise can help to generate entanglement and increase the success rate
[19, 21, 22, 25]. However, most of these schemes referred above are probabilistic. Therefore,
it presents a challenging task for researchers in this field to work out a deterministic entangling
scheme. With this end in view, an appealing scheme to generate a maximally entangled state of
Two atoms inside a leaky cavity in a ‘quasi-deterministic’ manner has been proposed [21]. Two \( \Lambda \)-type three-level atoms, situated inside a leaky optical cavity and initially prepared in the ground state \( |L\rangle \), interact with a single left-circularly polarized input photon. The polarization of the photon emitted from the cavity after the interaction is then measured. The atoms become maximally entangled if the emitted photon is right-circularly polarized, or otherwise remain intact. The probability of success of the entangling process is shown to approach unity if the input photon is quasi-monochromatic [21]. It is worthwhile to remark that other fundamental processes in quantum computing like quantum state-swapping and controlled phase-flip gates can also be achieved with high success rates via CQED interaction between atoms and such quasi-monochromatic single photons [26, 27].

Needless to say, the feasibility of the scheme proposed in [21] hinges on the availability of quasi-monochromatic single photon sources, which is an issue of current interest in its own right. On the other hand, it is also possible to repeatedly redirect the emitted photon into the cavity till it becomes right-circularly polarized. Hence, the maximally entangled state can be obtained in a quasi-deterministic way with a single photon through this feedback process [28].

The major objective of the present paper is to study the efficiency of such a feedback process, which is crucial to its viability since, due to effects of absorption and leakage, the process will inevitably be terminated after a few rounds. In [28] the master equation approach is adopted. Under the assumption that the probability of generation of entanglement in each round of the whole feedback process is a constant value \( p \) (\( p = 1/2 \) under the optimal condition), the overall success rate after \( N \) rounds is given by \( 1 - (1 - p)^N \) and hence quasi-deterministic generation of entanglement is then achievable in the limit of \( N \to \infty \). However, this crucial assumption has not been thoroughly examined in the paper.

The major concern of the master equation approach used in [28] is the evolution of atoms in the presence of cavity photons inside a leaky cavity. However, the quantum states of the incident and emitted photons are not properly considered. To provide a correct description for the incident and emitted photons, in the present paper we study the same problem using continuous frequency modes [29–32] and describe the quantum states of photons in terms of their spectral functions [21]. In the continuous frequency mode approach [29–32], the incident (emitted) photon is described by a spectral function \( f_{\text{in}}(k) \) (\( f_{\text{out}}(k) \)), with \( k \) being the wave number. We show that if one starts with a cavity photon, which has a spectral function \( f_c(k) \) in the form of a complex Lorentzian with a width given by the leakage rate of the cavity [33], the probability of entanglement is 1/2 under the optimal condition, agreeing with the result obtained in [28]. However, owing to the interaction with the atom, \( f_{\text{in}}(k) \) and \( f_{\text{out}}(k) \) are different. The spectral function of the emitted photon is, therefore, no longer given by \( f_c(k) \) despite the fact that the incident photon is a cavity photon. When the outcome of the measurement is negative, the emitted photon will be redirected into the cavity. However, as the initial state of the photon is now different, the operation cannot be repeated with a constant success rate. Instead, a more elaborated formulation for the overall success rate has to be sought.

Using the continuous frequency mode approach, we show in the present paper that the success rate in each round of the feedback process decreases gradually. Although the probability of entanglement does approach unity in the limit of \( N \to \infty \), the rate at which it approaches unity is much slower than that predicted in [28]. Therefore, the effects of other competing loss mechanisms could become crucial in a prolonged feedback process and great caution has to be exercised in applying the feedback scheme to ensure quasi-deterministic entanglement generation.

The structure of the present paper is as follows. In section 2, we introduce the system considered in the feedback process [21, 28] and expand the interaction Hamiltonian in terms
of the continuous frequency modes [29–32]. In section 3, we show that the dynamics of the two \( \Lambda \)-atoms is reducible to that of a single \( \Lambda \)-atom and in turn obtain the spectral function of the emitted photon. We then analyse the efficiency of the feedback mechanism with our formalism in section 4. In section 5, we present our conclusion and discuss the physical significance of the spectral dependence of CQED processes.

2. Physical system

The experimental setup considered in [28] is schematically sketched in figure 1. Two identical \( \Lambda \)-atoms, \( A \) and \( B \), are placed at the centre of a one-dimensional one-sided leaky optical cavity, which has a length \( l \) and is bounded by two mirrors. The left mirror at \( x = 0 \) perfectly reflects while the right mirror at \( x = l \) is partially transparent.

The normal modes of this leaky cavity are characterized by a continuous wave number \( k \), and are given by [21, 33, 34]

\[
U_k(x) = \begin{cases} 
I(k) \sin kx e^{-ikx} & 0 < x < l, \\
R(k) e^{ikx} & l < x < \infty,
\end{cases}
\]

(2.1)

where

\[
I(k) = -\frac{2it}{1 + r e^{2ikl}}, \\
R(k) = -\frac{r - t + r e^{-2ikl}}{1 + r e^{2ikl}}.
\]

(2.2)

(2.3)

with \( r \) and \( t \) being the reflection and the transmission coefficients of the right mirror, respectively. For a good optical cavity whose transmission coefficient \( |t| \ll 1 \), \( |I(k)| \) sharply peaks at the quasi-mode frequencies [21, 33, 34] and is negligibly small otherwise. Besides, if \( k \) is close to a quasi-mode frequency \( k_c \), \( I(k) \) can be approximated by a single complex Lorentzian such that

\[
|I(k)| \simeq \frac{\kappa}{2\pi |k - k_c + ik/2|},
\]

(2.4)

and the width of the Lorentzian, \( \kappa \), is a measure of the inverse of the lifetime of the quasi-mode. In fact, equation (2.4) is a generic result that holds for any cavities with small leakage rates.
The atomic ground states $|L\rangle$ and $|R\rangle$ are degenerate and separated from the excited state $|e\rangle$ by an excitation energy $\omega_e$. Each atom in its $|L\rangle$ ($|R\rangle$) ground state can be excited to $|e\rangle$ by absorbing a photon in the left-polarized state $|k_L\rangle$ (right-polarized state $|k_R\rangle$), where $k_L$ ($k_R$) signifies the momentum of the photon. As is assumed in [28], the separation between the atoms is comparatively small and therefore the atoms have almost the same coupling strength with the photon field. The Hamiltonian of this system, in units of $\hbar = c = 1$, is given by

$$\hat{H} = \sum_{\alpha=A,B} \omega_e |e_{\alpha}\rangle\langle e_{\alpha}| + \int_0^\infty dk \sum_{\mu=L,R} \hat{a}_{k\mu} \hat{a}_{k\mu}^\dagger + \sum_{\alpha=A,B} \int_0^\infty dk \hat{a}_{k\mu} g_{\mu}(k) |e_{\alpha}\rangle\langle e_{\alpha}| + \text{h.c.}$$

(2.5)

Here the subscripts $A$ and $B$ respectively label relevant quantum states of atoms $A$ and $B$. $\hat{a}_{k\mu}$ and $\hat{a}_{k\mu}^\dagger$ are, respectively, the annihilation and creation operators of a photon with frequency $k$ and polarization $\mu = L, R$. $g_{\mu}(k)$ is the dipole coupling strength, which is frequency dependent and proportional to $I(k)$. Therefore, under the premise that the excitation energy $\omega_e$ is close to a quasi-mode frequency $k_c$, which is assumed in [28], $g_{\mu}(k)$ is approximately given by

$$g_{\mu}(k) = \sqrt{\frac{\lambda_{\mu}}{2\pi}} \frac{k}{k - k_c + i\kappa/2}.$$  

(2.6)

Here,

$$\lambda_{\mu} = \left[ \int_{-\infty}^\infty dk |g_{\mu}(k)|^2 \right]^{1/2}$$  

(2.7)

is a measure of the dipole moment of the relevant atomic transition.

As considered in [28], initially the two $\Lambda$-type atoms are prepared in the ground state $|L\rangle$, while the photon is left-polarized with a normalized spectral function $f(k')$:

$$|\psi\rangle_{in} = \int_{-\infty}^\infty dk' f(k') |LL; k'_L\rangle,$$

(2.8)

where, as usual, we have extended the lower limit of the integration from 0 to $-\infty$ and

$$\int_{-\infty}^\infty dk' |f(k')|^2 = 1.$$  

(2.9)

### 3. Photon–atom interactions

The quantum dynamics of our system can be solved analytically by introducing a new set of bases of the Hilbert space [21]:

$$|E\rangle = \frac{1}{\sqrt{2}}(|e_L; 0\rangle + |e_e; 0\rangle),$$  

(3.1)

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|L R\rangle + |R L\rangle).$$  

(3.2)

Hereafter, $|e_L\rangle$ denotes the product state of $|e_A\rangle$ and $|L_B\rangle$ and analogous notations will be used in our paper. As a result of the initial condition considered in the experiment, an effective Hamiltonian that can fully describe the dynamics of the system is obtained:

$$\hat{H}_{\text{eff}} = \omega_e |E\rangle\langle E| + \int_{-\infty}^\infty dk \sum_{\mu=L,R} k \langle LL; k_L|LL; k_L| + \langle \Phi; k_R|\Phi; k_R\rangle \text{ h.c.}$$

$$+ \int_{-\infty}^\infty dk \sqrt{2} g_{L}(k) |E\rangle\langle LL; k_L| + g_{R}(k) |E\rangle\langle \Phi; k_R\rangle \text{ h.c.}.$$  

(3.3)
The effective Hamiltonian $\hat{H}_{\text{eff}}$ can be further simplified by introducing the states \[ |\psi_1(k)\rangle = \frac{1}{V(k)} (\sqrt{2} g_L^*(k) |LL; k_L\rangle + g_R^*(k) |\Phi_1\rangle), \tag{3.4} \]
\[ |\psi_2(k)\rangle = \frac{1}{V(k)} (g_R(k) |LL; k_L\rangle - \sqrt{2} g_L(k) |\Phi_1\rangle), \tag{3.5} \]
where
\[ V(k) = \sqrt{2|g_L(k)|^2 + |g_R(k)|^2}. \tag{3.6} \]
In terms of these states, $\hat{H}_{\text{eff}}$ can thus be written as \[ \hat{H}_{\text{eff}} = \hat{H}_0 + \hat{V} + \hat{H}_{\text{dark}}, \tag{3.7} \]
where
\[ \hat{H}_0 = \omega |E\rangle\langle E| + \int_{-\infty}^{\infty} dk |\psi_1(k)\rangle\langle \psi_1(k)|, \tag{3.8} \]
\[ \hat{V} = \int_{-\infty}^{\infty} dk \ V(k) |\psi_1(k)\rangle\langle \psi_1(k)| + \text{h.c.}, \tag{3.9} \]
\[ \hat{H}_{\text{dark}} = \int_{-\infty}^{\infty} dk |\psi_2(k)\rangle\langle \psi_2(k)|, \tag{3.10} \]
with $\hat{H}_{\text{dark}}$ being the free Hamiltonian of the dark states $|\psi_2(k)\rangle$ that does not take part in the interaction. Therefore, the system is now analogous to a two-level atom and we will omit the term $\hat{H}_{\text{dark}}$ in the following discussion.

The evolution of the system at a later time $t > 0$ can be obtained once the retarded Green’s function, $\theta(t) \exp[-i(\hat{H}_0 + \hat{V})t]$, is known. To this end, we consider the resolvent of the Hamiltonian $\hat{H}_0 + \hat{V}$, which is defined by \[ \hat{G}(\omega) = \frac{1}{\omega - \hat{H}_0 - \hat{V}}, \tag{3.11} \]
with $\omega$ being a complex variable. It yields the retarded Green’s function, $\theta(t) \exp[-i(\hat{H}_0 + \hat{V})t]$, through an integral transformation:
\[ \theta(t) \exp[-i(\hat{H}_0 + \hat{V})t] = \lim_{\epsilon\to0^+} \frac{i}{2\pi} \int_{-\infty+\text{i}\epsilon}^{\infty+\text{i}\epsilon} \hat{G}(\omega) \ e^{-\text{i}\omega t} \ d\omega. \tag{3.12} \]
The exact form of the resolvent $\hat{G}(\omega)$ has been worked out in \[21\]. In the following, we shall make use of it to evaluate the final state of our system subject to two different initial conditions.

As considered in \[28\], the two atoms first interact with a photon existing in the cavity. Such a photon is termed as a cavity photon (or a quasi-mode photon) \[36\], which is characterized by a spectral function $f(k') = f_c(k')$ given by
\[ f_c(k') \equiv \frac{1}{\kappa} s^*_n(k') = \frac{1}{\sqrt{2\pi}} \frac{1}{k' - k_e + \text{i} \kappa/2}. \tag{3.13} \]
At $t = 0$ the cavity photon is confined in the cavity and leaks out of the cavity at a rate $\kappa$ for $t > 0$. If a left-polarized cavity photon is prepared in the cavity at $t = 0$, the initial state can be represented as
\[ |\psi\rangle_m = \int_{-\infty}^{\infty} dk' \frac{f_c(k')}{V(k')} (\sqrt{2} g_L(k') |\psi_1(k')\rangle + g_R^*(k') |\psi_2(k')\rangle). \tag{3.14} \]
By using the resolvent method developed in [21], we obtain the output state of the first round:

$$|\psi\rangle_{\text{out},1} = |LL\rangle \otimes \int_{-\infty}^{\infty} dk \, f_c(k) D_L(k) e^{-ikt}|k_L\rangle + |\Phi\rangle \otimes \int_{-\infty}^{\infty} dk \, f_c(k) D_R(k) e^{-ikt}|k_R\rangle,$$

(3.15)

where

$$D_L(k) = \frac{(\Delta k - \delta_e)(\Delta k + i\kappa/2) - \lambda_R^2}{(\Delta k - \omega_+)(\Delta k - \omega_-)},$$

(3.16)

$$D_R(k) = \frac{\sqrt{2}\lambda_L\lambda_R}{(\Delta k - \omega_+)(\Delta k - \omega_-)},$$

(3.17)

and the complex Rabi frequencies $\omega_{\pm}$ are defined by

$$\omega_{\pm} = \frac{\delta_e - \kappa/2}{2} \pm \sqrt{\left(\frac{\delta_e + \kappa/2}{2}\right)^2 + 2\lambda_L^2 + \lambda_R^2},$$

(3.18)

with the detuning $\delta_e = \omega_e - k_c$. Since the output state $|\psi\rangle_{\text{out}}$ is also normalized, it is obvious that

$$\int_{-\infty}^{\infty} dk |f_c(k)D_L(k)|^2 + \int_{-\infty}^{\infty} dk |f_c(k)D_R(k)|^2 = 1.$$  

(3.19)

After the first encounter with the atoms, the photon is emitted from the cavity and its polarization is detected. If the emitted photon is right-polarized, then the two atoms will remain in the state $|LL\rangle$ and the emitted photon will be sent back to the cavity. In this case, the two atoms have to interact with a photon injected from the exterior of the cavity. Such an initial photon satisfies the scattering condition [21], and has a modified normalized spectral function $f_n(k)$ after $n$ rounds of interactions. As a result, the final state of our system after the $(n+1)$th interaction is then given by [21]

$$|\psi\rangle_{\text{out},n+1} = |LL\rangle \otimes \int_{-\infty}^{\infty} dk \, f_n(k) C_L(k) e^{-ikt}|k_L\rangle + |\Phi\rangle \otimes \int_{-\infty}^{\infty} dk \, f_n(k) C_R(k) e^{-ikt}|k_R\rangle,$$

(3.20)

where

$$C_L(k) = \frac{(\Delta k - \delta_e)(\Delta k^2 + \kappa^2/4) - \Delta k(\lambda_R^2 + 2\lambda_L^2) + i\kappa(\lambda_R^2 - 2\lambda_L^2)/2}{(\Delta k - i\kappa/2)(\Delta k - \omega_+)(\Delta k - \omega_-)},$$

(3.21)

$$C_R(k) = \frac{\sqrt{2}i\kappa\lambda_L\lambda_R}{(\Delta k - i\kappa/2)(\Delta k - \omega_+)(\Delta k - \omega_-)}.$$  

(3.22)

It is interesting to note that $C_R(k)$ and $C_L(k)$ themselves satisfy the unitarity condition:

$$|C_L(k)|^2 + |C_R(k)|^2 = 1,$$

(3.23)

reflecting the fact that waves with different frequencies are not mixed upon such scattering.

4. Feedback mechanism

From the functions $D_L(k)$, $D_R(k)$, $C_L(k)$ and $C_R(k)$ obtained in the previous section, one can analyse the feedback mechanism initiated by an initial cavity photon [28]. After the first
round, the output state can be obtained from equation (3.15). The probability of getting a
right-polarized photon and hence a maximally entangled state in the first round is given by

\[ P_R^1 = \int_{-\infty}^{\infty} dk |D_R(k) f_c(k)|^2. \]  

(4.1)

Under the optimal condition where \( \lambda_R = \sqrt{2} \lambda_L \) and in the strong coupling limit, \( P_R^1 \) is equal
to \( 1/2 \) [21]. This is in perfect agreement with the numerical result obtained in [28].

However, if the emitted photon is left-polarized, which has a normalized spectral function:

\[ f_1(k) = D_L(k) f_c(k) \]

(4.2)
it will be reinjected into the cavity as the initial state of the second round. As \( f_1(k) \) is in
general different from \( f_c(k) \), its interaction with the two atoms will not be the same as that of
the initial cavity photon. Therefore, the efficiency of the feedback process has to be examined
with extra care.

Disregarding the difference in the spectral functions of the incident photons, the dynamics
of the second round is exactly the same as the first one and the output state is given by
equation (3.20). The probability of getting a left-polarized photon in the first round and a
right-polarized photon in the second round is

\[ P_R^2 = \int_{-\infty}^{\infty} dk |C_R(k) f_1(k)|^2 \times \int_{-\infty}^{\infty} dk |D_L(k) f_c(k)|^2 \]
\[ = \int_{-\infty}^{\infty} dk |C_R(k) D_L(k) f_c(k)|^2. \]  

(4.3)

The photon leaks out from the cavity after its second encounter with the atom, and its
polarization is measured. If once again, a left-polarized photon is detected, the normalized
spectral function of this photon is given by

\[ f_2(k) = \frac{C_L(k) f_1(k)}{\left[ \int_{-\infty}^{\infty} dk |C_L(k) f_1(k)|^2 \right]^{1/2}} \]
\[ = \frac{C_L(k) f_1(k)}{\left[ \int_{-\infty}^{\infty} dk |C_L(k) D_L(k) f_c(k)|^2 \right]^{1/2}}. \]  

(4.4)

This feedback process is carried on iteratively until a right-polarized photon is detected outside
the cavity. After \( n \) rounds of interactions, the normalized spectral function of the left-polarized
photon can be found by using equations (3.15) and (3.20), yielding

\[ f_n(k) = \frac{C_L(k) f_{n-1}(k)}{\left[ \int_{-\infty}^{\infty} dk |C_L(k) f_{n-1}(k)|^2 \right]^{1/2}} \]
\[ = \frac{C_L(k) f_{n-1}(k)}{\left[ \int_{-\infty}^{\infty} dk ||C_L(k) D_L(k) f_c(k)||^2 \right]^{1/2}}. \]  

(4.5)

Following the argument developed above and using mathematical induction, one can
readily show that the probability of getting a left-polarized photon in the first \( (n-1) \) rounds
and a right-polarized one in the \( n \)th round \( (n \geq 2) \) is given by

\[ P_n^R = P_{n-1}^L \int_{-\infty}^{\infty} dk |C_R(k) f_{n-1}(k)|^2 \]
\[ = \int_{-\infty}^{\infty} dk |C_R(k) C_L(k) D_L(k) f_c(k)|^2, \]  

(4.6)
where $p_{n-1}^L$ is the probability of detecting a left-polarized photon in the first $n - 1$ round, and is given by

$$p_{n-1}^L = \int_{-\infty}^{\infty} dk |C_L(k)|^{n-2} D_L(k) f_c(k)^2.$$  \hfill (4.7)

The cumulative probability of generating a pair of maximally entangled atoms after the first $N$ rounds of interaction is therefore

$$P_R^N = \sum_{n=1}^{N} p_n^R$$  \hfill (4.8)

$$= 1 - \int_{-\infty}^{\infty} dk |C_L(k)|^{N-1} D_L(k) f_c(k)^2,$$  \hfill (4.9)

where the unitarity condition (3.23) has been applied to simplify the expression. Since the magnitude of $C_L(k)$ is always less than unity, it is obvious that

$$\lim_{N \to \infty} P_R^N = 1,$$  \hfill (4.10)

leading to the conclusion that after a sufficiently large number of rounds the two atoms will surely become maximally entangled. However, the limit $N \to \infty$ in equation (4.10) can be taken only if other sources of energy leakage can be safely ignored. To eliminate (or at least minimize) the effects arising from photon loss and atomic spontaneous decay, it is advantageous to get a high enough success probability in a few rounds. Otherwise, the plausibility of the feedback mechanism to ensure quasi-deterministic entanglement still remains precarious. Therefore, a study of rate at which $P_R^N$ approaches unity is called for.

In figure 2 we show the absolute values of $C_L(k)$ and $D_L(k)$ for an optimal case with $\delta = 0, \kappa = 1$ and $\lambda_L = \lambda_R / \sqrt{2} = 2.5 \kappa$ [28, 21]. Figure 3 clearly demonstrates the variation in the spectral function of the photon during the feedback process. In contrast to the behaviour of $| f_1(k) |$, which have three maxima at $\Delta k \equiv k - k_c = 0, \omega_0, |C_L(k)|$ vanishes at $\Delta k = 0, \omega_0$ and is quite small around there. $p_n^R$ consequently decreases significantly as $n$ increases. As shown in figure 4, the total probability of entanglement after $N$ rounds of operation is almost saturated after about 10 rounds and is still away from unity even after 100 rounds. This differs markedly from the result predicted in [28], which converges to unity rapidly. We also note that the saturation phenomenon persists for another case with $\lambda_L = \lambda_R / \sqrt{2} = 25 \kappa$. Therefore, the efficiency of the feedback scheme remains low even for a much larger interaction strength.

![Figure 2](image-url)
Figure 3. A plot of (a) $|f_c(k)|^2$, (b) $|f_1(k)|^2$, (c) $|f_2(k)|^2$ and (d) $|f_{10}(k)|^2$ for the optimal case ($\lambda_R = \sqrt{2} \lambda_L$) with $\lambda_L = 2.5 \kappa$.

Figure 4. Probability of entanglement generation within $N$ rounds of operation for the optimal case ($\lambda_R = \sqrt{2} \lambda_L$). The triangles ($\lambda_L = 25 \kappa$) and the squares ($\lambda_L = 2.5 \kappa$) show the results obtained from our theory, while the circles are data obtained from the constant success rate assumption with $p = 1/2$.

5. Conclusion

Adopting the continuous frequency mode quantization scheme, we studied the evolution of a photon under the feedback mechanism and showed that the spectral function of the photon is in general modified in each round of the feedback process. Thus, the feedback scheme is not as effective as what [28] claimed. Whether the above-mentioned scheme can ensure quasi-deterministic entanglement after a large number of iterations depends crucially on the possibility of elimination of other competing processes, say, photon losses in the cavity and in the optical fibres. Nevertheless, the feedback scheme could still be a useful method to improve the probability of success until other mechanisms interfere with its operation.

The aim of the feedback process proposed in [28] is to ensure quasi-deterministic generation of entangled states with only one single input photon. If, instead, multiple independent cavity photons are used, the probability of success in each interaction is at best 0.5 [28]. Besides, as argued by Hong and Lee in [28], for an experiment using multiple photons the finite detection efficiency of the detector $D_2$ that measures right-polarized output photons may give rise to ambiguity stated as follows. When no photon is detected by the
detector $D_2$, there are two possible cases: (1) the entanglement process has not yet succeeded or (2) the detector $D_2$ failed to detect the output photon. The introduction of the feedback process, which has a success rate close to unity, in effect makes the detector $D_2$ redundant and eliminates the ambiguity mentioned above.

On the other hand, it is worthwhile to note the accumulative effect resulting from the difference between the spectra of the input and output photons. As demonstrated in the present paper (see figure 3), the spectrum of an input photon is in general different from that of the output photon. This point is often overlooked in the conventional master equation approach to leaky cavities. If the output photon is then used as the input of a new process, such difference in the spectra might give rise to accumulative effect and harm the overall performance. Needless to say, photons are ideal mediators of quantum information and can generate entanglement between well-separated atoms. However, one must be conscious of the change in the spectra of a photon before and after interacting with an atom as such change might lead to a marked difference in the long run.

Finally, we would also like to remark that the probability of success in the process examined here depends sensitively on the spectrum of the input photon. For example, instead of using cavity photons with a spectral width equal to that of the cavity, one can also consider an input photon with an arbitrary width $\kappa_{\text{in}}$, i.e.

$$f(k') = \frac{\kappa_{\text{in}}}{2\pi} \frac{1}{k' - k_c + i\kappa_{\text{in}}/2}.$$  

As shown in figure 5, for a ‘quasi-monochromatic’ photon which has a sufficiently narrow spectral width, the probability of success in one single trial can be close to unity irrespective of the value of $\lambda_L/\kappa$ [21]. Similarly, Chen et al [26, 27] suggest that quantum state-swapping and controlled phase-flip gates are achievable with high success rates with quasi-monochromatic single photons. Therefore, the quantum state of single photons is likely to play a crucial role in quantum computing.

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