Extended log periodic approach in analysing local critical
behaviour–case study for Covid-19 spread in Albania.

Elmira Kushta\textsuperscript{1}, Dode Prenga\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, Faculty of Technical Sciences, University of Vlora
\textsuperscript{2} Department of Physics, Faculty on Natural Sciences, University of Tirana

\textsuperscript{1}dode.prenga@fshn.edu.al, \textsuperscript{2}kushtamira@gmail.com

Abstract. Log-periodic (LP) functions of the general form $y = y_0 + A(t - t_c)^m(1 + B\cos(\omega \log(t - t_c) + \varphi_1))$ have been demonstrated effective in the analysis of the processes characterized by the discrete scale of invariance (DSI) structure and also self-organization behaviour. If other self-organization processes opposing or supporting the principal activity would be present, a multilevel DSI structure is expected to develop and the resulting dynamics would depart from the log-periodic shape. When discussing the processes characterising the daily new positive records for the COVID-19 cases in the country (Albania) we have identified the elements that are responsible to generate modified LP behaviour. The new records that initially represented simply the findings of the state laboratories, are modified each successive days by the pressure for more tests form anxious individuals and other effects which produce a herding behaviour which emergence a LP dynamics. Meanwhile, the reactive behaviour aiming to oppose undesired occurrence would generate additional LP sub-processes that can be trapped by a modified LP function of the form $y = y_0 + A(t - t_c)^m + B(t - t_c)^m \cos (\omega \log(t - t_c) + \varphi_1) + C\cos(\omega - \omega_1) \log(t - t_c) + \varphi_2 + D(t - t_c)^m \cos ((\omega + \omega_1) \log(t - t_c) + \varphi_3)$. During the period of the self-organization behaviour the process is highly nonlinear and therefore the classical models (SIR) based on the mean field assumption and the corresponding ODE equations are not effective to represent the system dynamics. In our case the LP fit to the Covid-19 new cases data series, has challenged the ODE models for the time interval of initial appearance of the positive cases in the country (as of 2 March 2020) up to 3 month later. Also, the LP function has predicated the multiphase waving behaviour and we have forecasted two peak, each of them weeks before recurrence, respectively at 28 April and 10-12 June. Those peaks have been confirmed later within 2 days uncertainty. It resulted that the first regime have been succeeded by another new self-organization regime due to drastic condition changes as result of the socio-economic opening which started in the end of the May 2020. As result, the new regime is juxtaposed over the old one and the LP dynamics remained characteristic and another critical time has appeared. The new critical time has been reproduced with good certainty (1 August) and also the magnitude of the new cases. We observed that our empirical LP function is effective in the describing long term dynamics whereas local techniques involving neural networks approaches have reproduced very well the new cases after the first regime. We concluded that after identification of the LP regimes, we can adopt successfully short time forecasting for new occurrence when working away from the critical time identified by the first method.
1. Introduction

The spread of epidemic diseases is a well-studied phenomenon in the framework of nonlinear dynamics and under the assumptions of the mean field as the SIR or SIER models, see [1], [2] etc. However, in the beginning stage of epidemic outbreak, many assumptions are mathematically exaggerated and therefore application of such models is inappropriate. Related to the COVID-19 pandemic outbreak in Albania, the state agencies have acted energetically by imposing rigid ‘social distancing’ regulation norms that have reduced drastically the human contacts. It is clearly a huge deviance form any classical model assumption. The stringency index has been set soon in a very high level at the values 78-89 for two months [14]. Moreover and like the others, the country is heterogeneous in the sense of the social topology and contact frequency, and also it is an open system that alters epidemiologic and immune system characteristics by time and regions. All those specifics make the models very difficult to apply. Initially, the number of the new cases discovered and declared by the state agencies has been in the range 2-26 for a couple of weeks [3]. Usually the ratio discovered/tested has been at around 1:10 and testing is based on immunological investigation principally [16]. Under those conditions the official data reported are not typically a variable for the system. Consequently, the prediction and forecasting by using mean field models has remain inappropriate and technical groups failed systematically in their epidemic prognosis. Recognising the mathematical difficulties on the modelling the behaviour of the system under investigation, we have considered an alternative physical perceptive of the system: exploring the time dynamics in the sense of the identification of the regimes and possibly the assessment of the time moment when our series would behave as a system variable. In the following, we have noticed significant resemblance with systems described in [4] and consequently we have proposed the modified LP functions for the description of the new positive records for COVID-19 infections in the country. Consequently we have identified some physical parameters for this dynamics.

2. Reason for rushed changes of daily’s new cases

In the beginning of the epidemic spread, we have seen many peoples trying to analyse the number of new cases or their cumulative outcome by simply using standard infection disease models. Clearly they failed but the abrupt changes of the daily new cases become a debating issue. In short here is an explanation for our system that has been typically heterogenous and of a limited size. Consider the number of infected people at the time $t$. The number of new cases identified by the authorities is $n_t = k_tC_t = k_t\gamma_t I_t$ where the coefficient ‘c’ indicates the claim rate or suspicious cases, $k$ is proportionality constant and $\gamma$ indicate the claims over infections ratio. At the next moment ($t+1$) the number of infected peoples has evolved to $I_{t+1}$ whereas $n_{t+1} = k_{t+1}c_{t+1}=k_{t+1}\gamma_{t+1}I_{t+1}$. The mapping equation $n_{t+1} = f(n_t)$ remain unknown as long as all parameters evolve in time. Note that absolute number or ration shows no difference as long as the number of the peoples remains constant (under high stringency index assumption). Let suppose that this mapping approaches to the classical ODE equations for given location or subsystem. For a given homogenous location the probability for a new infection has logistic-like shape or more generally a Richard’s function-form. However each location would have its own origin of time so the SIER solutions look

$$L(t,i) = \frac{KC_0}{\left(c_0^a+(K^a-c_0^a) e^{-\left(K^a-c_0^a\right)^{\frac{rt_{i}}{K}}}}\right)$$

It resulted that if parameters are time-constants, the number of new cases are proportional to the total number $N$ and follows a logistic shape or more generally a Richard’s function-form. However each location would have its own origin of time so the SIER solutions look

$$L(t,i) = \frac{KC_0}{\left(c_0^a+(K^a-c_0^a) e^{-\left(K^a-c_0^a\right)^{\frac{rt_{i}}{K}}}}\right)$$

$$n_{t,1} = k_t\gamma_t L(t, [.])N_1, n_{t,2} = k_t\gamma_t L(t, [.])N_1(N-N_1)$$

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where \( C, K \) and \( r \) are the model parameters in [2] and \( t_c \) is the time calculated form the local first occurrence. So the evolution for the new case after a sufficiently large time becomes

\[
n = n_{1,t} + n_{2,t} = N_1 \left( \frac{KC_{0,1}}{C_{0,1}^a + (K^a - C_{0,1}^a) e^{\frac{rt_1 K^a}{K^a - C_{0,1}^a}}} \right) + N_2 \left( \frac{KC_{0,2}}{C_{0,2}^a + (K^a - C_{0,2}^a) e^{\frac{rt_2 K^a}{K^a - C_{0,2}^a}}} \right)
\]

(3)

It contains two terms with competitive trends that produce changes in the discrete time. After a long period, the curves become smooth. For more subsystems, the feature become characteristic and therefore rushed changes occur in the daily new cases diagnosed as positive. For calculation purposes one can assume later on that the local time origin would follow a given distribution, which is not a goal of this work. Note that each parameter in the SEIR model depends on the subsystems properties making the daily new cases changing abruptly. However, this overlap can cause temporal self-organisation behaviour as long as the time elapsed is not too large. Leaving aside mathematical details we argue that this stage would become a specific regime where temporal self-organisation processes would develop and hence, LP fit can uncover more about the behaviour of the daily new positive cases as key parameter of the system.

3. Log-periodic dynamics

Log-periodic functions (LP) have been used in analysing the self-organization processes and DSI structures, mostly in the study of the financial indexes dynamics; see [4], [5], [6], [7] and references therein. According to [4] the DSI is a specific scale invariance where the scaling parameter is discrete that is in the equation \( \Theta(x) = \mu(x) \), parameter \( \alpha \) is discrete ay \( \alpha \rightarrow [\alpha_1, \alpha_2, \ldots] \) and \( \alpha_0 = \alpha^n \) It give rise to a specific scaling where the transformation is realised in logarithmic scale. General conditions of DSI regimes have been meet in other systems as avalanches, earthquake energy release etc., [4], [6]. There is a large literature about LP specifics and their application, see [6], [8]. To describe the outcome that emerge as result of the herding behaviour Sornette in [5], [4] etc., has used the log-periodic function

\[
y = y_0 + A(t - t_c)^{\alpha n} (1 + Bcos(\omega(t - t_c) + \varphi))
\]

(4)

where \( t_c \in \) is the critical time, \( y \) is the logarithm of the observable, \( \omega \) is the cyclic frequency related to the DSI parameters and \( A, B \) are constants. The critical point is interpreted as the time moment when the regime is most likely to change. Alternative and modified forms of (4) have been used successfully to describe the self-organisation behaviour in many processes evolving criticality.

3.1. Complementary view on LP: frustrated DSI and the multimode LP functions

For specific DSI structures EURO-ALL exchange rate in Albania we have employed an extended q-version of the (1). So the ad hoc q-LP form

\[
y = y_0 + A(t - t_c)^{\alpha n} (1 + Bcos(\alpha(t - t_c) + \varphi_1) + Ccos(2\omega(t - t_c) + \varphi_2))
\]

(5)

is used to embodied the effects of many factors that makes the DSI feature not strongly characteristic for the system [9]. The q-logarithm has been used in the framework of q-functions application in the
physical contexts of q-Entropy application in [10] and has been discussed in details in [11]. In [9] the logarithmic part of the function is related directly with DSI structure and henceforth if this property is weakly characteristic, the logarithm is supposed to be transformed in its approximate q-form. In another case we have considered the effect of the activities that try to reduce the dangerous amplitudes near the critical point [12]. Therein we have considered the modified log-periodic forms which we will call in the following the multi-mode LP function
\[
y = y_0 + A(t - t_c)^m \cos(\omega \log(t - t_c) + \varphi_1) + B \cos((\omega - \omega_1) \log(t - t_c) + \varphi_2) + C(t - t_c)^m \cos(\omega + \omega_1) \log(t - t_c) + \varphi_3)
\]

Similarly, in the COVID-19 related processes in Albania, we have identified elements of the herding behaviour in some scale and also similar activities discussed in [12]. So, the COVID tests in the early stage have been conditioned by the decision of sanitary authorities whom to test and how many tests have to be carried out. Next, under the pressure of the increasing suspicions cases and general herding claims, the test supposedly are performed more generally following a convergent route to the rate of the daily new infections. So, the state agencies have started to provide proportionally more tests and are expected to try anticipating the behaviour in the framework of their routine works to prevent the outbreak for hospitalization. Under those condition, similarly to the processes discussed in [12] we argued that the system would follow a near to DSI behaviour amended by two additional terms in the form (3) as follows. Suppose for a given time moment the quantity would have the value given by (1). It is expected to culminate in the time moment \(t_c\) which is not a pleasant outcome for the authorities. They undertake countermeasures to oppose it so an added phase-shifted term would appear in the overall dynamics. This term would try to add appropriate precedent or incoming value as to eliminate the trend that has started before, so the new LP function would contain a progressive and a retarded mode. Formally this process is expected to generate two terms similar with Stokes and anti-Stokes line in Raman lines. By assuming that such effects could not modify the critical time an acceptable idea is to consider a cyclic frequency shift and therefore the terms of the form \((t - t_c)^m \cos(\omega \pm \omega_1 \log(t - t_c) + \varphi)\) have been added in (2). For the limit case where \(\omega = \omega_1\) we regain the three-term log-periodic form
\[
y = y_0 + A(t - t_c)^m(1 + B \cos(\omega \log(t - t_c) + \varphi) + B \cos(2 \omega \log(t - t_c) + \varphi_2))
\]
which is found appropriate in the fitting of the rare data point series with underground DSI dynamics. From the form (2) we deduced the q-LP form (3) which worked well in the case where DSI mechanism is present but not dominant, similarly with our systems considered in [9]. The fitting of the forms (1), (3) and their q-form could be realised by the step by step procedure described in [7] or [8] and we are not repeating it herein. Note that the ad hoc genetic algorithm we constructed can be easy executed on a standard PC machine and converge quite soon, from some minutes to a few hours.

3.2. LP analysis for the behaviour of the new cases of COVID-19 in Albania

By fitting the first weeks data to the ad hoc LP function (6) we observed an early stage of self-organisations behaviour that drive the system toward a local peak, Figure 2. So, using the data series for 28 days of the epidemics presence, the critical time has been estimated in nearly two weeks in advance and pointed at 25-26 April. It happened really at 28 of April. In our calculation the expected number of new case in this moment is estimated to be 90-110 that does have not matched the real occurrences but remember the nature of the new cases diagnosed: they are mostly mechanical. Note that any prediction with other model gives wrong results. In fact, it resulted that at this date, the number of new diagnosed positive has raised rapidly to 35 from 12 individuals diagnosed a day before. Such a sharp change corresponds to the critical behaviour and is typical for LP dynamics.
Figure 1: Early stage log periodic behaviour

Figure 2: Prolonged regime
Next, the outcomes of the tactical opening ruled by the authorities are expected to activate other DSI mechanisms and therefore other log-periodic sub processes would possibly emerge. However, initial self-organization persists and we argue that a long term LP would continue to characterise that behaviour of the data series. We observe that the critical time of a longer pre-assumed LP process covering 85 data points would be around 12-14 June and the process would enter in another frustrated mode. Again, this peak has occurred at the date 14, and the other one has been predicted to occur at the end of the July, Figure 3. It happened also within 2 days uncertainty and the peak values have been 126 against 160 predicted by the model. In the next step we adopt the form (7) to estimate the significance of the LP shapes. We observe that longer q-LP fits have higher q-values in the forms (7) which indicate that the all process has started to lose its initial LP behaviour the best q-value is found at 1.07, so the departure form LP-classical form is not negligible. Therefore the period of the time that started at a couple of days after first occurrence and the finished at the third critical time (around 4 August) is described as a regime in the dynamical process analysed. This regime is not classified as an epidemic phase, because other arguments are needed and some of them go beyond the scope of this study. However, it is possible that after this date the new cases diagnosed would represent better the infections rates and classical models become to be more effective.

4. Other evidence by empirical analysis for the data

By LP-analysis we have concluded that the time evolution of the new positive is characterised by two short term self-organization behaviour and a long term ones. This last covers the period from (around) 8-10 March to around 2-4 August 2020. Although it is not qualified herein as an epidemic phase. It merely represent the stage of the transformation of new cases diagnosed to the new infection occurrences. Firstly we considered the evidencing of the epidemic steady phase where the new cases would follow a specific distribution. Similarly as in [12], we have analysed it by checking the fit of the q-Gaussian or q-exponential to the empirical distribution of the data. Note that those functions are very useful to describe the non-stationary distributions as detailed in [10], [11]. We observe that the q-values obtained in the q-Gaussian fitted is greater than 3 which indicates that the series of new cases were not drown form a distribution. Next by excluding low values that correspond to the lockdown time the q-Gaussian fitted has q~2.8 but the fitness estimators weren’t satisfactory. So, the system consisting on the series of new cases for the period 2 March, 4 August remain in a highly dynamical stage.

4.1. Evidence of the trend and traces of phase by EMD approach

Aiming in the analysis of the trend of the new cases diagnosed daily which would testify the existence of an evolution phase for the dynamics of the infections, we used empirical mode decomposition technique (EMD) which is known as a suitable technique in the study of nonlinear systems, see [13] and [14]. In this technique the series is decomposed in constitutive modes (IMF) similar to the Fourier analysis by an ad-hoc sifting process, see [10] etc. Note that in many cases the IMF are not related to the physical processes underlying the dynamics of the system, whereas the last IMF represents the trend of the dominant process. In this case, aiming in the identification of a possible regime, we have explored some daughter series by choosing different starting and ending time moment. We observe that for such series the EMD analysis produced usually 7-12 intrinsic modes (IMF). By selecting the starting date of the series at around 18-20 March that is around two weeks after first appearance of the infection, we observe that magnitude of the last mode fall below 1. We acknowledge this as the sign that the data series has started to represent an underlining system dynamics which coincide with the presence of a dominant regime. Herein, we considered that a logical tolerable error or deviances in the slowest mode (the last IMF) could be one unit (one person) given the relatively small number of occurrences. After evidences for the trend of the data representing new cases, we see that the developing time for the LP regime identified above coincide with less than half of the inner period for last IMF mode. Therefore the epidemic phase would be expected to last at least twice of the LP regime that is e more than six month phase. It might be another LP regime or another smother evolution of the key variable considered in this paper. To complete the picture we checked for fractal and multifractality properties of the series.
based on the analysis given in [15]. By fixing the end time and changing the starting time we observed that multifractal asymmetry and q-entropy production change abruptly, which indicates high change rate of the multifractal properties of the system themselves. We consider the variation of the multifractal asymmetry from series starting in successive time as an indicator of a non-dominant regime for the system. A detailed study on this framework is scheduled for a successive work when more data would have been gathered from the system.

![Figure 3: IMF preliminarily analysis](image)

**Figure 3: IMF preliminarily analysis**

4.2. *Aftermath short term forecasting by NN Modeling*

In the final step we propose some practical use of the above analysis in the framework of prediction and forecasting for new occurrences. It consisted in the prior identification of the critical behaviour and applying afterwards the network training tool for series populating a regime but by avoiding the critical regions. So, from the phenomenological point of view, the number of positive cases trapped by the sanitary system turn out to be monotonically correlated with the number of infected peoples as the times goes on. Therefore, hidden interrelationship will become more robust and we can use the autoregressive mapping $n_t = f(n_{t-\tau}, \ldots, n_{t-1})$ to produce new values instead of modelling by mean field approaches which are not suitable based on the above discussion. In this case, we do not need other system variables and records. Note that the forecasting and predication by using NNM would be more reliable and physical if the process has been stabilised. In this view, the LP analysis is very helpful once again: it announce the critical time around which the behaviour is highly dynamical. Herein we discretise the time interval in periods $[t_i, t_{i+14}]$ taking into account the incubation time which is largely known as 14 days. We obtained acceptable forecasted results for 5 days. The simulation give good averaged new cases for 5 days after the edge of the series, as seen from the comparison of the values with real data.
added in next 5 days, Figure 4. Note that in the case of Figure 4 we have considered the edge of the LP regime that affected the quality of the forecasting.

5. Conclusions
The evolution of the recorded data for COVID-SARS-2 in Albania during the first months of its appearance has been expressed well by a modified log-periodic function and from the other side, significant departure from infectious diseases propagation. This behaviour is believed to be a consequence of the mechanical nature of the recorded data in the first period, but the restraining protocol imposed by the state agencies can be listed as another factor as well. We observed that after two weeks of the first declared occurrence, the system has been organized in a specific LP regime with critical time in the beginning of the August. By this analysis we have concluded that this regime coincides with the first stage of the epidemic outbreak which is expected to last more than twice of the LP regime period of 20 March-04 August. By using those findings and therefore excluding a narrow period around the critical points, we have performed the prediction and the forecast for daily new occurrences by using neural network models. The uncertainty in this case is obtained around 2-5 points (individuals) for around 100 new cases. In practical terms this is a good prediction result with usable capability if we consider the unpredictable behaviour of this new epidemic disease. This analysis would be valuable for the period when the epidemic is not transformed in theoretical pandemic and therefore the routine or standard models are applicable. Adding to that, it can be performed for other cases with similar paradigmatic conditions or in the analysis of the moderate term dynamics of this disease spreading process.
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