Grain Propellant Optimization Using Real Code Genetic Algorithm (RCGA)

Muhammad Farraz Al Farizi\textsuperscript{1}, Romie Oktovianus Bura\textsuperscript{3}, Soleh Fajar Junjunan\textsuperscript{3} and Bagus H. Jihad\textsuperscript{4}

1,2) Flight Physics Research Group, Faculty of Mechanical and Aerospace Engineering, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia
3,4) Rocket Technology Center, National Institute of Aeronautics and Space (LAPAN), Indonesia

*Muhammad.farraz93@gmail.com

Abstract. Grain propellant design is important in rocket motor design. The total impulse and ISP of the rocket motor is influenced by the grain propellant design. One way to get a grain propellant shape that generates the maximum total impulse value is to use the Real Code Genetic Algorithm (RCGA) method. In this paper RCGA is applied to star grain Rx-450. To find burn area of propellant used analytical method. While the combustion chamber pressures are sought with zero-dimensional equations. The optimization result can reach the desired target and increase the total impulse value by 3.3% from the initial design of Rx-450.

1. Introduction

One of the parameters of the rocket motor is the total impulse. To get the maximum impulse required a good grain propellant design that is grain propellant has a little sliver, high volumetric loading, and high combustion chamber pressure. To get it is required iteration process. The iteration process is iterate the variable independent of propellant (star grain has 7 independent variables, there are r1, r2, \( \omega \), \( \eta \), \( \xi \), N, \( R_0 \)). Currently, Lapan rocket motor process design still use trial and error method (change the value of 7 independent variables manually), so it takes a long time in the design process and not sure to get optimal results. One method that can be used to speed up the design process is genetic algorithm. Moreover, the use of this method will produce an optimal design. Genetic algorithm (GA) method is widely used because Genetic algorithm (GA) can be applied to functions that have discontinuity (Robust), can capture global optimum and does not require initial design to do optimization process. In addition GA has a very high level of code simplicity so it is very easy to make the code. Raza and Liang (2010) compared Genetic algorithms (GA), Simulated Annealing (SA), and Combined GA and SA for dual thrust propellant optimization. From the results, they found that Genetic algorithm (GA) produces the largest results, while the method of hybrid GA and SA can reduce the computation time. Then Kamran et al (2010) using Genetic algorithm (GA) to get optimal star grain results. Kamran use a drafting method to calculate changes in the area of burning by connecting software optimization with CAD software. For this research the change of the area of combustion is solved by analytic method, the advantage of analytical method to find the area of burning is the process is faster than drafting method, and disadvantage is not flexible (different type of grain, different equation).

Genetic algorithm (GA) is divided into two method, binary code (Binary Code Genetic Algorithm) and real code (Real Code Genetic Algorithm). Real Code Genetic Algorithm (RCGA) is widely used...
because it has the advantage of computation time because there is no encoding and decoding process.

Besides it, Real Code Genetic Algorithm can be applied to large domain when compared with Binary code Genetic algorithm (BCGA) because the use of domain Large on the Binary code Genetic algorithm will reduce precision if the chromosome length is limited [3]. One problems of this method is difficult to obtain integers during the mutation and crossover process. Fortunately, from the previous study [12] was obtained for the case of star grain value of $N = 5,6,7$ can produce total maximal impulse. So the difficulty of obtaining integers can be resolved by optimizing the value of $N$ at outside optimization.

Rx-450 is one of Lapan’s sonda rocket. This rocket use star grain propellant with a length of 3700 mm. The star propellant is chosen because LAPAN is experienced in making it. In addition, star-shaped propellants can produce large areas of large initial fuel without compromising volumetric efficiency [10]. The large volumetric loading fraction that is owned by star-shaped propellants is 0.75-0.9 and the star propellant has a sliver of 5-10% [1]. Assuming volumetric loading fraction 0.8, sliver 2%, and not optimal throat nozzle affecting 2%, should propellant with existing propellant can produce total impulse equal to 1530 kNs.

\[
I_{Toptimal} = (254 \times 0.96 \times 0.92) \times (0.00167 \frac{kg}{cm^3}) \times 3.14 \times (21.6 cm)^2 \times 370 cm \times 0.8 \times 0.98 \times (9.81 \frac{m}{s^2})^{0.98}
\]

2. Method
2.1. Optimization Method

![Flow Chart Optimization](image-url)

**Figure 1** flow chart optimization
Table 1 Parameter Optimization

| Parameter                  | Value |
|----------------------------|-------|
| Max Fitness evaluation     | 10000 |
| Population                 | 100   |
| Crossover probability      | 1     |
| Mutation probability       | 1     |
| Variable                   | 5     |

The method used optimization is Real code genetic algorithm. But at research the crossover probability and Mutation probability equal one. The best chromosome always do crossover with other chromosome.

For the case of grid propellant optimization BLX- α is chosen because this method produces a new population around chromosomes 1 and 2. While the mutation method used is Non-Uniform Mutation. This method is simple and has mutation characteristics occurring at the beginning of a temporary iteration when approaching the maximum limit of individual mutation iteration of mutation results is not large. The following is a crossover process using the BLX- α and Non uniform mutation methods [3].

Crossover

Crossover is an operator to find the optimum value (convergence operator), so crossover probability is set high (crossover probability> 0.95). In this research, crossover probability is not set, but the best population always crossover with other populations, in hopes to get better results from the best population.

Assume \( C_1 = (c_{11}, \ldots, c_{1n}) \) and \( C_2 = (c_{21}, \ldots, c_{2n}) \) is chromosome selected to crossover. An offspring is generated: \( H = (h_1, \ldots, h_i, \ldots, h_n) \). Where \( h_i \) is a randomly (uniformly) chosen number of interval \([c_{\text{min}} - \alpha, c_{\text{max}} + \alpha]\). With \( c_{\text{max}} = \max(c_{11}, c_{21}) \) and \( c_{\text{min}} = \min(c_{11}, c_{21}) \) [3].

\[
\begin{align*}
C'_i &= c_i + \Delta(t, b_i - c_i) \quad \text{if } \tau = 0 \\
C'_i &= c_i - \Delta(t, c_i - a_i) \quad \text{if } \tau = 1
\end{align*}
\]

\( c_i \in [a_i, b_i], \tau \) is random number 0 or 1

\[
\Delta(t, y) = y \left(1 - r^{(1 - t/g_{\text{max}})^b}\right),
\]

\( t \) is generation number, \( g_{\text{max}} \) is max number generation

Mutation

Usually mutation probability is set low because the function of this mutation operator is to keep the population diversity (divergence operator). In this research, there is no mutation probability but mutation always happen at 10 worst population. \( C'_i \) is the result of mutation [3]

2.2 Grain Burn Back Analysis

The method used for the analysis of burning area of propellant is analytical method. The analytic method has advantages in fast computing time and simple code. One of the things that must be considered when using the analytical method for optimization propellant is the order of phase combustion. Star grain burning can be divided into 4 Phases. This is the analytic equations for star grain for each phase and the order of phase combustion:
Variabel star grain = \((R_0, N, r1, r2, \omega, \eta, \xi)\)

\[ R_p = R_0 - r_1 - \omega \]

Sudut = \(360 - ((180 - 180/N) + \eta + 90) - (180/N) \)

\[ y_{max} = ( (R_0 - R_p \cos(\xi))^2 + (R_p \sin(\xi))^2 )^{0.5} - (r_1 + r_2) \]

Figure 2. Star Grain Propellant [7]

Phase 1 (when \(r_2 > 0\))

Burn Area (\(A_B\))

\[ S1 = (R_p \frac{\sin(\xi)}{\sin(\eta)}) - (y + r_1 + r_2) \times (\cot(\eta)) \]

\[ S2 = (\Delta y + r_1) \times (\frac{\pi}{2} - (\eta) + \xi) \]

\[ S3 = (R_p + y + r_1) \times (\frac{\pi}{N} - \xi) \]

\[ S4 = 2\pi r_2 \left(\frac{\text{sudut}}{360}\right) \]

\[ A_B = 2N \times (S1 + S2 + S3 + S4) \times L \]

Port Area (\(A_P\))

\[ A_{P1} = \left(0.5R_p \sin(\xi) \times \left(R_p \cos(\xi) + R_p \sin(\xi) \tan(\eta)\right)\right) \]

\[ A_{P2} = (-1) \times 0.5 \times \left(\left(R_p \times \frac{\sin(\xi)}{\sin(\eta)} - (y + r_1 + r_2) \times \cot(\eta)\right)^2 \times \tan(\eta)\right) \]

\[ A_{P3} = (0.5)(y + r_1)^2 \times (\frac{\pi}{2} - (\eta) + \xi) \]

\[ A_{P4} = (0.5)(R_p + y + r_1)^2 \times (\frac{\pi}{N} - \xi) \]

\[ A_{P5} = (S1 \times r_2) + (\pi \times (r_2)^2 \times (\text{sudut}/360)) \]

\[ A_P = 2N \times (A_{P1} + A_{P2} + A_{P3} + A_{P4} + A_{P5}) \]

Furthermore there are two possible processes of burning propellant.

1. \(S1\) finished, after that \(S3\) finished, after that \(S2\) finished

Phase 2 (when \(r_2 = 0\), \(S1 > 0, S3 > 0, S2 > 0\))

Burn Area

\[ S1 = (R_p \frac{\sin(\xi)}{\sin(\eta)}) - (y + r_1) \times (\cot(\eta)) \]

\[ S2 = (y + r_1) \times \left(\frac{\pi}{2} - (\eta) + \xi\right) \]

\[ S3 = (R_p + y + r_1) \times (\frac{\pi}{N} - \xi) \]

\[ S4 = 0 \]

\[ A_B = 2N \times (S1 + S2 + S3 + S4) \times L \]

Port Area

\[ A_{P1} = \left(0.5R_p \sin(\xi) \times \left(R_p \cos(\xi) + R_p \sin(\xi) \tan(\eta)\right)\right) \]
Ap2 = \((-1) \times 0.5 \times \left(\left(\frac{R_p \times \sin(\xi)}{\sin(\eta)}\right) - (y + r_1) \times \cot(\eta)\right)^2 \times \tan(\eta)\)

AP3 = \((0.5)(y + r_1)^2 \times \left(\left(\frac{\pi}{2}\right) - (\eta) + \xi\right)\)

AP4 = \((0.5)(R_p + y + r_1)^2 \times \left(\left(\frac{\pi}{N}\right) - \xi\right)\)

AP5 = 0

A_p = (2N \times (AP1 + AP2 + AP3 + AP4 + AP5))

Phase 3 (when \(r_2 = 0, S1 = 0, (y) < (\omega) \) \([S3 > 0], S2 > 0\))

Burn Area

S1 = 0

S2 = \((y + r_1) \times \left(\left(\frac{\pi}{2}\right) - (\eta) + \xi - \tan^{-1}\left(\frac{\sqrt{(y + r_1)^2 - (R_p \sin(\xi))^2}}{R_p \sin(\xi)}\right)\right)\)

S3 = \((R_p + y + r_1) \times \left(\left(\frac{\pi}{N}\right) - \xi\right)\)

S4 = 0

A_p = (2N \times (S1 + S2 + S3 + S4)) \times L

Port Area

Ap1 = \(0.5R_p \sin(\xi) \times \left(R_p \cos(\xi) + \sqrt{(y + r_1)^2 - (R_p \sin(\xi))^2}\right)\)

Ap2 = 0

AP3 = \((0.5)(y + r_1)^2 \times \left(\left(\frac{\pi}{2}\right) + \xi - \tan^{-1}\left(\frac{\sqrt{(y + r_1)^2 - (R_p \sin(\xi))^2}}{R_p \sin(\xi)}\right)\right)\)

AP4 = \((0.5)(R_p + y + r_1)^2 \times \left(\left(\frac{\pi}{N}\right) - \xi\right)\)

AP5 = 0

A_p = (2N \times (AP1 + AP2 + AP3 + AP4 + AP5))

Phase 4 when \(r_2 = 0, S1 = 0, (y) \geq (\omega) \) \([S3 = 0], y < y_{max} \) \([S2 > 0 \) (sliver)]

\[\beta = \left(\left(\frac{\pi}{2}\right) - (\eta) + \xi\right)\]

\[\gamma = \sqrt{(y + r_1)^2 - (R_p \sin(\xi))^2} - \eta\]

\[\varepsilon = \pi - \cos^{-1}\left((-1) \times \left(\frac{R_0^2 - R_p^2 - (y + r_1)^2}{2R_p(y + r_1)}\right)\right)\]

\[\mu = \cos^{-1}\left(\frac{(y + r_1)}{R_0} \sin(\pi - \varepsilon)\right)\]
Burn Area
S1 = 0
S2 = (y + r_1)(\beta - \gamma - \epsilon)
S3 = 0
S4 = 0
A_b = (2N \times (S1 + S2 + S3 + S4)) \times L

Port Area
AP1 = (0.5)(R_0)^2 \times \left(\frac{\pi}{N} - \xi + \mu\right)
AP2 = 0.5 (y + r_1)^2(\beta - \gamma - \epsilon)
AP3 = 0.5 \left(2 \left(\frac{R_p \sin(\xi)}{\sin(\eta)}\right) + (R_p \sin(\xi))(\frac{\pi}{N} - \xi)\right)^{0.5}
AP4 = -0.5 \left(2 \left(\frac{R_p \sin(\mu)}{\sin(\xi)}\right) + (R_p \sin(\mu))(\frac{\pi}{N} - \xi)\right)^{0.5}
A_p = (2N \times (AP1 + AP2 + AP3 + AP4))

2. S3 finished, after that S1 finished, after that S2 finished

Phase 2 (when \(r_2 = 0\), S1 > 0, S3 > 0, S2 > 0)
Burn Area
S1 = \left(R_p \sin(\xi)\right) - (y + r_1) \times (\cot(\eta))
S2 = (y + r_1) \times \left(\frac{\pi}{2} - (\eta) + \xi\right)
S3 = (R_p + y + r_1) \times \left(\frac{\pi}{N} - \xi\right)
S4 = 0
A_b = (2N \times (S1 + S2 + S3 + S4)) \times L

Port Area
AP1 = \left(0.5R_p \sin(\xi) \times \left(R_p \cos(\xi) + R_p \sin(\xi) \tan(\eta)\right)\right)
AP2 = (-1) \times 0.5 \times \left((\frac{R_p \times \sin(\xi)}{\sin(\eta)}) - (y + r_1) \times \cot(\eta)\right)^2 \times \tan(\eta)
AP3 = (0.5)(y + r_1)^2 \times \left(\frac{\pi}{2} - (\eta) + \xi\right)
AP4 = (0.5)(R_p + y + r_1)^2 \times \left(\frac{\pi}{N} - \xi\right)
AP5 = 0
A_p = (2N \times (AP1 + AP2 + AP3 + AP4 + AP5))

Phase 3 (when \(r_2 = 0\), S1 > 0, S3 = 0, S2 > 0) (Sliver)
\[
\beta = \left(\frac{\pi}{2} - \eta + \xi\right)
\]
\[
\gamma = 0
\]
\[
\epsilon = \pi - \cos^{-1}\left((-1) \times \frac{R_0^2 - R_p^2 - (y + r_1)^2}{2R_p(y + r_1)}\right)
\]
\[
\mu = \cos^{-1}\left(\frac{(y + r_1)}{R_0} \sin(\pi - \epsilon)\right)
\]
Burn Area
S1 = \left( R_p \frac{\sin(\xi)}{\sin(\eta)} - (y + r_1) \times \cot(\eta) \right)
S2 = (y + r_1)(\beta - 0 - \epsilon)
S3 = 0
S4 = 0
A_p = (2N \times (S1 + S2 + S3 + S4)) \times L
Port Area
Ap1 = 0.5 \left( R_p \sin(\xi) \times \left( R_p \cos(\xi) + R_p \sin(\xi) \tan(\eta) \right) \right)
Ap2 = (-1) \times 0.5 \times \left( \left( \frac{R_p \times \sin(\xi)}{\sin(\eta)} \times (y + r_1) \times \cot(\eta) \right)^2 \times \tan(\eta) \right)
AP3 = (0.5)(R_0)^2 \times \left( \frac{\pi}{N} \right) - \xi
AP4 = 0.5 \left( R_p \sin(\mu) \right) \left( R_p \cos(\mu) \right) - 0.5 \times \left( R_p \sin(\mu) \right) \times \left( y + r_1 \right)^2 - \left( R_p \sin(\mu) \right)^2 + 0.5 \left( y + r_1 \right)^2(\beta - \epsilon)
AP5 = 0
A_p = (2N \times (AP1 + AP2 + AP3 + AP4 + AP5))

Phase 4 when \( r_2 = 0, S1 = 0, (y) \geq (\omega) [S3 = 0], y < y_{max} \) [S2>0 (sliver)]
Same with case 1
\[ \beta = \left( \frac{\pi}{2} - (\eta) + \xi \right) \]
\[ \gamma = \left( \frac{(y + r_1)^2 - (R_p \sin(\xi))^2}{R_p \sin(\xi)} \right) - \eta \]
\[ \epsilon = \pi - \cos^{-1} \left( (-1) \times \left( \frac{R_0^2 - R_p^2 - (y + r_1)^2}{2R_p(y + r_1)} \right) \right) \]
\[ \mu = \cos^{-1} \left( \frac{(y + r_1)}{R_0 \sin(\pi - \epsilon)} \right) \]

Burn Area
S1 = 0
S2 = (y + r_1)(\beta - \gamma - \epsilon)
S3 = 0
S4 = 0
A_p = (2N \times (S1 + S2 + S3 + S4)) \times L
Port Area
Ap1 = (0.5)(R_0)^2 \times \left( \frac{\pi}{N} \right) - \xi + \mu
Ap2 = 0.5 \left( y + r_1 \right)^2(\beta - \gamma - \epsilon)
AP3 = 0.5 \left( R_p \sin(\xi) \right) \left( R_p \cos(\xi) \right) + \left( R_p \sin(\xi) \right) \left( y + r_1 \right)^2 - \left( R_p \sin(\xi) \right)^2 \right)^{0.5}
AP4 = -0.5 \left( R_p \sin(\mu) \right) \left( R_p \cos(\mu) \right) + \left( R_p \sin(\mu) \right) \left( y + r_1 \right)^2 - \left( R_p \sin(\mu) \right)^2 \right)^{0.5}
A_p = (2N \times (AP1 + AP2 + AP3 + AP4))
2.3. Fitness Function

The purpose of this optimization is to obtain the maximum total impulse value with the weight limit of propellant should not exceed 815 kg, \( \frac{A_p}{A_t} \leq 0.65 \), combustion chamber pressure \( \leq 70 \) bar, \( \frac{A_b}{A_p} \leq 190 \) by optimizing independent variables of grain propellant (Figure2). With the upper and lower limits of the geometry variable are as follows:

Diameter = 432 mm

\[
\begin{align*}
N &= 6 \\
40 &\leq w \leq 200 \\
0.5 &\leq r_2 \leq 20 \\
0.5 &\leq r_f \leq 15 \\
10^o &\leq \zeta \leq 54^o \\
10^o &\leq \eta \leq 60^o
\end{align*}
\]

The diameter of propellant and N becomes input optimization so that the variable to be optimized is

\[ X = (r_1, r_2, \omega, \eta, \xi) \]

From the optimization objective, mathematically objective function and fitness function can be written

\[ F(x) = \max(I_T) \]

With constraint:

\[ \text{Weight} = G_1(x) \leq 815 \text{ kg} \]

\[ \text{Chamber pressure} = G_2(x) \leq 70 \text{ bar} \]

\[ \frac{A_p}{A_t} G_3(x) \leq 0.65 \]

\[ \frac{A_b}{A_p} = G_4(x) \leq 190 \]

Where \( A_p \): Port Area, \( A_p \): Burn Area, \( A_t \): Throat Area

Due to Rao’s book, the optimization methods are used to find the minimum value, to be similar to the book, the objective function and the fitness function to be

\[ F(x) = \max(I_T) = \min(-I_T) \]

From the optimization goal, the grain propellant optimization is a non-linear optimization with constraint. To simplify the case and code generation, this case will be forced into non-linear optimization without constraint by using the concept of punishment/penalty, so it is expected that when a chromosome selection process that results in value does not meet the constraint it will be eliminated. The penalty/punishment is:

\[
\text{If (} P_c > 70 \text{ bar } \cup \text{Berat} > 815 \text{ kg } \cup \frac{A_p}{A_t} > 0.65 \cup \frac{A_b}{A_p} > 190 \text{,} I_T = -1 \times I_T \text{)}
\]
The punishment will cause the chromosome that does not meet the constraint has a fitness function value worse than the chromosome that meets the constraint. The value of the total impulse can be searched by solving the following equations:

\[ F(x) = \min(-I_T) = \min(-\int_0^{t_b} F \, dt) = \min(-\int_0^{t_b} P_c A_t C_{f\text{actual}} \, dt) \]

With \( P_c A_t C_{f\text{actual}} \) from equation:

\[ C_{f\text{actual}} = \lambda \eta_f C_{f\text{ideal}} \]

\[ C_{f\text{ideal}} = \sqrt{\frac{2\gamma^2}{\gamma - 1}\left(\frac{\gamma + 1}{\gamma - 1}\right)^{\gamma - 1}(1 - \frac{P_e}{P_c})^{\gamma - 1} + \left(\frac{P_e - P_{\text{amb}}}{P_c}\right) \varepsilon} \]

\[ \frac{dP_c}{dt} = \frac{1}{v(t)} \left[ RT_c \left( \rho_f A_0(t) a P_c^n - \frac{P_{\text{amb}}}{c^*}\right) - \frac{P_e dv}{dt} \right] \]

\[ aP_c^n = r = \frac{dy}{dt} \]

With the value of the variable from the above equation with the value as follows:

\[ a = 0.03467 \text{ dan } n = 0.3180 \text{ (from strand burner)}, A_t = 176.25 \text{ cm}^2, \lambda = 0.9872, \varepsilon = 6.55, \gamma = 1.19, P_{\text{amb}} = 1 \text{ atm}, \eta_f = 0.92, c^*_{\text{actual}} = 1490 \frac{m}{s} \]

\( P_c \): Chamber Pressure, \( P_e \): Exit Pressure, \( P_{\text{amb}} \): Ambient Pressure, \( C_f \): Coefficient thrust, \( t_b \): Burning time, \( \lambda \eta_f \): Nozzle efficiency, \( \eta_c \): Combustion efficiency, \( r \): Burn rate, \( a \): Coefficient burn rate (Saint Robert’s burn rate law), \( n \): Coefficient burn rate (Saint Robert’s burn rate law), \( c^* \): Characteristic velocity, \( \gamma \): Specific heat ratio, \( \varepsilon \): Expansion heat ratio

3. Result

![Figure 5 Iteration Process](image)

**Figure 5** Iteration Process

**Figure 5** Show that the iteration process leads to a convergent value. From optimization is obtained graph of thrust as shown at **Figure 6**.
Figure 6 Comparison Initial design and optimized design (Pressure and Thrust)

Table 2 Comparison Initial design and optimized design

|               | initial | optimization | Increment |
|---------------|---------|--------------|-----------|
| weight (kg)   | 735.2   | 736.07       | 0.11 %    |
| Sliver weight (kg) | 12.85   | 11.73       | -8.7%     |
| Total Impulse (kNs) | 1480   | 1530        | 3.3%      |
| $I_{sp}$ (s)  | 205.2   | 211.8        | 3.3%      |

The optimization process takes about 5 min. Optimization results with the Real Code Genetic Algorithm increase the total impulse by 3.3%, this is because the sliver weight of the optimization result is less than the initial design, the rocket carrying more propellant mass and the combustion chamber optimization is greater than the initial design. Optimized combustion chamber pressures greater than the initial design due to the burn area of optimization results is greater than the initial design.
4. Conclusion

Real Code Genetic algorithm method can be applied in rocket motor design process. The use of RCGA optimization method can increase the total impulse by 3.3% and takes about 5 min.

Future research will investigate new strategies for improving the speed. To accelerate the process, we must research about influence of GA parameter (population, crossover rate, and mutation rate) on grain propellant optimization case. Another future research, we will change the objective function from max total impulse to profile thrust which are desired (input from user).

5. Acknowledgement

The authors would like to thank director of Rocket Technology Center (Pustekroket) LAPAN for providing research support facilities.

References

[1] Davenas, Alain. Solid rocket propulsion technology. Newnes, 2012.
[2] Hartfield, Roy, et al. "A review of analytical methods for solid rocket motor grain analysis." 39th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exhibit. 2003.
[3] Herrera, Francisco, Manuel Lozano, and Jose L. Verdegay. "Tackling real-coded genetic algorithms: Operators and tools for behavioural analysis." Artificial intelligence review 12.4 (1998): 265-319.
[4] Junjunan, S. F. Design And Performance Prediction of Solid Rocket Motor For RX450 Ballistic Rocket.
[5] Kamran, A., Guozhu, L., Rafique, A. F., Naz, S., & Zeeshan, Q. "Star grain optimization using genetic algorithm." Proceedings of the 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Orlando, FL, 12-15 April, AIAA 2010. Vol. 3084. 2010.
[6] Design, Solid Propellant Grain, and Internal Ballistics. "NASA/SP-8076." NASA (March 1972) (1972).
[7] Rao, Singiresu S., and Singiresu S. Rao. Engineering optimization: theory and practice. *John Wiley & Sons*, 2009.

[8] Raza, Muhammad Aamir, and Wang Liang. "Design and Optimization of 3D Wagon Wheel Grain for Dual Thrust Solid Rocket Motors." *Propellants, Explosives, Pyrotechnics* 38.1 (2013): 67-74.

[9] Stein, S. D. (2007). Benefits of the Star grain Configuration for a Sounding Rocket. From [http://spacegrant.colorado.edu/COSGC_Projects/symposium_archive/2007/papers/S07_14_Star_Grain_Configuration_for_Sounding_Rocket.pdf](http://spacegrant.colorado.edu/COSGC_Projects/symposium_archive/2007/papers/S07_14_Star_Grain_Configuration_for_Sounding_Rocket.pdf)

[10] Sutton, George P., and Oscar Biblarz. Rocket propulsion elements. *John Wiley & Sons*, 2001.

[11] Al Farizi, M.F, Bura,R.O., Junjunan,S.F. Jihad,B.H. Pemilihan jumlah –n pada star grain propellant dengan metode trial and error. *SIPTEGAN.2017*