A Reinforcement Learning Perspective on the Optimal Control of Mutation Probabilities for the (1+1) Evolutionary Algorithm: First Results on the OneMax Problem

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Abstract

We study how Reinforcement Learning can be employed to optimally control parameters in evolutionary algorithms. We control the mutation probability of a (1+1) evolutionary algorithm on the OneMax function. This problem is modeled as a Markov Decision Process and solved with Value Iteration via the known transition probabilities. It is then solved via Q-Learning, a Reinforcement Learning algorithm, where the exact transition probabilities are not needed. This approach also allows previous expert or empirical knowledge to be included into learning. It opens new perspectives, both formally and computationally, for the problem of parameter control in optimization.

1 Problem statement

We maximize the OneMax function: $OM(x) = \sum_{i=1}^{n} x_i, \forall x_i \in \{0, 1\}$ via the (1+1) Evolutionary Algorithm (EA) by which, given a random initialization of $x \in \{0, 1\}^n$, at every iteration, each of the bits is flipped (mutated) with probability $\theta$, yielding a solution candidate $x'$. If $OM(x') > OM(x)$, $x'$ is kept. We proceed until the terminal condition $OM(x) = n$ is met. We regard the evolution of $x$ as a stochastic process, conditioned at each step by $\theta$. This yields a Markov Decision Process (MDP) [Puterman 2014], whose optimal control policy can be found via Dynamic Programming when the transition probabilities are known and Reinforcement Learning (RL) when only experience data is available.\[1\]

1An online compendium with proofs and code to replicate our results is available at https://github.com/**********

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2 Related Work

Recent results [Karafotias et al. 2015] have proposed new mechanisms to dynamically control parameters in evolutionary algorithms, in opposition to just tuning and fixing them prior to optimization. Some theoretical results [Bottcher et al. 2010; Doerr and Wagner 2018; Doerr and Doerr 2015; Giessen and Witt 2015] have demonstrated the intuition (e.g. 1/5th rule) that adaptive parameters can perform substantially better than static tuning, producing also optimal behaviours in some cases. When exact analyses are not possible, we propose to use RL [Sutton et al. 1998] to estimate such optimal behaviours. Indeed, promising results [Karafotias et al. 2014; Buzdalova et al. 2014] have hinted the potential of the generic use of RL in EA.

3 Markov Decision Process

During the execution of the EA, we want to sequentially change $\theta$ to minimize the expected termination time. This problem can be formulated as an MDP, with states $S = \{0, 1, 2, \ldots, n\}$, $s = OM(x)$ and actions $A = \{\theta_1, \theta_2, \ldots\} = \{0.01, 0.02, \ldots, 0.99, 1\}$ (a discretization of the mutation probability $\theta \in [0, 1]$).

At each step, a reward $r(s, \theta)$ is obtained, where $r(s, \theta) = 0$ if the terminal state $s = n$ is reached, and $r(s, \theta) = -1$ otherwise. An optimal parameter control policy $\pi(s) = \theta$ maximizes $E(\sum_{i=0}^{\infty} r_i)$ for any initial state $s$.

3.1 Transition Probabilities

The transition matrix $P = [P(s' | s, \theta)]_{(s, \theta) \in (S, A)}$, describes the probability of transitioning to a state $s' = OM(x')$ from any state $s = OM(x)$ given any action $\theta \in A$. At any iteration $t$, $x_t \in \{0, 1\}^n$ has $OM(x_t) = n_1$ ones and $n_0 = n - n_1$ zeros. Let $W \sim Bin(n_0, \theta)^n$ be the random variable (r.v.) describing the ones gained at the end of an iteration and $L \sim Bin(n_1, \theta)$ be the r.v. for the ones lost. $Z' = W - L \in [-n_1, -n_1 + 1, \ldots, n_0 - 1, n_0]$ is the r.v. for the net gain after a mutation. Note that $Z' = W - L$ is the difference of independent binomial distributions. By convolution, it follows that the probability mass function of $Z'$ is:

$$p_{Z'}(z) = \begin{cases} \sum_{i=0}^{n_0} p_W(i + z; n_0, \theta) \times p_L(i; n_1, \theta) : z \geq 0 \\ \sum_{i=0}^{n_0} p_W(i; n_0, \theta) \times p_L(i + z; n_1, \theta) : z < 0. \end{cases}$$

Under the (1+1)EA, if $Z' \leq 0$, the solution candidate $x'$ is rejected as no negative values are admissible. The r.v. $Z'_k$ for the state $s'$ of our EA process has thus values $Z'_k \in [0, 1, 2, \ldots, n_0 - 1, n_0]$, where $P(Z'_k = 0) = P(Z' < 0) + P(Z' = 0)$ and $P(Z'_k = k) = P(Z' = k) \forall k > 0$.

\footnote{For readers used to MDP notations: this total reward criterion (no discount factor $\gamma$) is well defined for Stochastic Shortest Path problems such as the one considered here.}

\footnote{binomial distribution of parameters $(n_0, \theta)$.}
4 Optimal Parameter Control

We briefly introduce the two main methods used to compute the optimal policy: one based on Dynamic Programming [Bellman, 1957], the other relying on Q-Learning [Watkins, 1989].

4.1 Dynamic Programming

The function $V_\pi: s \mapsto E(\sum_{t=0}^{\infty} r_t | s_0 = s)$ (called $\pi$’s value function) maps state $s$ to (minus) their expected time-to-termination. The optimal policy’s value function $V^*$ is defined recursively by Equation 1. Value Iteration is the Dynamic Programming algorithm that repeatedly applies Equation 1 until convergence to $V^*$.

$$V_{n+1}(s) = \max_{\theta} \left[ r(s, \theta) + \sum_{s'} P(s'|s, \theta)V_n(s') \right]$$ (1)

Figure 1 reports the computed value functions for the following three policies (plotted in Figure 2):

- the constant $\theta = \frac{1}{n}$ commonly used in (1+1)EA,
- the $\pi(s) = \frac{1}{1+s}$ policy from [Bottcher et al., 2010] (originally designed for the LeadingOnes function),
- the optimal policy found via Value Iteration.

In Figure 1, the marks corresponds to the empirical average $T$, for 2000 runs initialized respectively at $S_{\text{init}} = \{5, 10, 22, 45\}$. In Table 1, one can find the average $T$ for a random starting state.
Table 1: Empirical time-to-termination, 2000 runs

|                | Policy | Constant | 1/(s+1) | MDP |
|----------------|--------|----------|---------|-----|
| Average        |        | 442      | 430     | 412 |
| Standard Deviation | 163 | 165      | 164     |     |

4.2 Q-Learning

Although one can explicitly compute the transition probabilities for the parameter control problem based on the OneMax function, such probabilities are generally not available. Learning mechanisms such as Q-Learning, allow to obtain $\pi^*$, using sampled transitions, without explicitly requiring $P$.

$Q(s, \theta) \leftarrow Q(s, \theta) + \alpha \left[ r + \max_{\theta'} Q(s', \theta') \right]$  

(2)

To that end, it learns the optimal state-action value function $Q^*(s, \theta) = r(s, \theta) + \mathbb{E}_{s'} (V^*(s'))$. Q-learning is a stochastic approximation process: it repeats the operation of Equation 2 in all states and actions until convergence to $Q^*$ (which boils down to solving Equation 1). The optimal policy is then the greedy policy $\pi^*(s) = \arg \max_{\theta} Q^*(s, \theta)$.

5 Discussion

The approach presented above generalizes straightforwardly to other problems and algorithms. Our goal in this contribution was to illustrate how a RL perspective on (optimal) Parameter Control can help bring new contributions to the Optimization field. Extending this contribution to a larger class of problems opens new challenges:
• Continuous actions (parameters) are a common limitation in RL, generally overcome using Policy Gradient methods.

• The state of an optimization process is problem and algorithm specific and might not always define a Markov process, thus leading to partial observability and/or approximations.

• The curse of dimensionality is a crucial issue in RL and introducing expert knowledge in the learning process can greatly help the convergence.

• Convergence to an optimal parameter control policy can take advantage of sampling the optimization process at will.

• Minimizing the expected termination time is not the only relevant criterion. For instance, a natural alternative would be to maximize the time-discounted value function improvements (an approach close to the idea of regret minimization).

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