New String Theories in Six Dimensions via Branes at Orbifold Singularities

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We present several classes of new 6d string theories which arise via branes at orbifold singularities. They have compact moduli spaces, associated with tensor multiplets, given by Weyl alcoves of non-Abelian groups. We discuss T-duality and Matrix model applications upon compactification.
1. Introduction

It was recently pointed out in [1] that new 6d theories, which include stringy excitations but without gravity, can be obtained in the world-volume of five-branes by taking \( g_s \to 0 \) with \( M_s \) held fixed. Four different classes were obtained in [1]:

(iia) Theories with \( \mathcal{N} = (1,1) \) supersymmetry, which are obtained in type IIB five-branes or, alternatively [2], via type IIA with a \( \mathbb{C}^2/\Gamma_G \) ALE singularity.

(iib) Theories with \( \mathcal{N} = (2,0) \) supersymmetry, which are obtained in the world-volume of type IIA (or M-theory) five branes or, alternatively [2], via type IIB with a \( \mathbb{C}^2/\Gamma_G \) singularity.

(o) Theories with \( \mathcal{N} = (1,0) \) supersymmetry in the world-volume of \( SO(32) \) heterotic small-instantons or type I five-branes.

(e) Theories with \( \mathcal{N} = (1,0) \) supersymmetry in the world-volume of \( E_8 \) small instantons.

The (o) theory has a global \( SO(32) \) symmetry and the (e) theory has a global \( E_8 \times E_8 \) symmetry.

The infrared limit of these theories, with energies small compared to \( M_s \), appear to be local quantum field theories. In the (iib) and (e) cases these are non-trivial, interacting, RG fixed points, while the (iia) and (o) cases are IR free. Despite their different IR behavior, upon compactification to five-dimensions on a circle, \( T \) duality exchanges the (iia) \( \leftrightarrow \) (iib) and (o) \( \leftrightarrow \) (e) theories. Thus the full theories are not local quantum field theories [1].

In this paper, we discuss new 6d \( \mathcal{N} = (1,0) \) theories associated with type II or heterotic five-branes at orbifold singularities in the orthogonal four dimensions. As in [1], we take \( g_s \to 0 \) with \( M_s \) fixed. The fact that new theories could be thus obtained was also mentioned during the course of this work in a footnote in [3]. It was there pointed out that one could have a general, Ricci-flat, non-compact manifold \( \mathcal{M}_4 \) in the remaining four directions, giving theories which, in principle, could depend on the uncountably infinite parameters needed to specify \( \mathcal{M}_4 \). However, as in [4], we expect most of these parameters are irrelevant in the \( g_s \to 0 \) and that only the \( \mathbb{C}^2/\Gamma_G \) singularity type matters. Clearly the singularity itself can not be ignored; indeed, it breaks the supersymmetry of the (iia) or (iib) theories to \( \mathcal{N} = (1,0) \) supersymmetry.

\[^1\] We label \( \Gamma_G \subset SU(2) \) using the well-known correspondence with the simply-laced groups \( G = A_r, D_r, E_{6,7,8} \).
The 6d string theories which we present have compact “Coulomb branches,” associated with expectation values the scalar components of 6d $\mathcal{N} = (1, 0)$ tensor multiplets, which are the “Coxeter boxes” (also referred to as the “Weyl alcove”) of non-Abelian groups. For any group $G$ of rank $r$, the Coxeter box is a compact subspace of $\mathbb{R}^r$ given by all $\vec{\Phi} \in \mathbb{R}^r$ which satisfy

$$\vec{\alpha}_\mu \cdot \vec{\Phi} + M_s^2 \delta_{\mu 0} \geq 0, \quad \mu = 0 \ldots r,$$

where $\vec{\alpha}_\mu$ are the simple roots, including the extended root $\mu = 0$, with $\sum_{\mu=0}^r n_\mu \vec{\alpha}_\mu = 0$ ($n_\mu$ are the Dynkin indices). The $\mu \neq 0$ conditions in (1.1) give the non-compact Weyl chamber $\mathbb{R}^r / W_G$, where $W_G$ is the Weyl-group. Including the $\mu = 0$ condition gives the Coxeter box $\mathbb{R}^r / C_G \cong (S^1)^r / W_G$, where the Coxeter group $C_G$ includes translations in the root lattice of $G$. Compact Coxeter box moduli spaces, of size $R^{-1}$, also arise via Wilson loops upon reducing a $G$ gauge theory on a circle of radius $R$. We have written the size of the Coxeter box (1.1) as $M_s^2$ because here it will be.

Coxeter boxes already appear in the theories (ii) and (e) mentioned above. Part of the moduli space of the (ii) theory obtained from $K$ parallel five-branes is the $U(K)$ Coxeter box of size $M_s^2$. The (ii) theory obtained from type IIB string theory on a $\mathbb{C}^2 / \Gamma_G$ ALE singularity has, as part of its moduli space, the Coxeter box of size $M_s^2$ of the corresponding ADE group $G$. The (e) theory obtained from $K$ small $E_8 \times E_8$ instanton five-branes has the Coxeter box, again of size $M_s^2$, for $Sp(K)$ as its Coulomb branch.

We will simply note some basic features of the new 6d string theories, saving a more detailed analysis for further study. In the next section, we discuss theories associated with type IIB NS five-branes at orbifold singularities. The tensor multiplet moduli live on the Coxeter box of the simply laced group $G$ associated with the singularity. In sect. 3 we discuss theories associated with $SO(32)$ heterotic or type I branes at orbifold singularities. In these examples, the tensor multiplet moduli can live in the Coxeter box of a non-simply-laced subgroup of $G$. In sect. 4, we discuss theories associated with $E_8 \times E_8$ branes at orbifold singularities. In sect. 5, we discuss $T$ duality upon compactification. Finally, in sect. 6, we discuss applications of the theories to providing a definition of $M$ theory on $(ALE) \times T^5 \times \mathbb{R}^{1,1}$ and $M$ theory on $(ALE) \times (T^5 / \mathbb{Z}_2) \times \mathbb{R}^{1,1}$. 

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2. Type IIB branes at a $\mathbb{C}^2/\Gamma_G$ orbifold singularity

For our first class of examples, consider $K$ parallel type IIB NS five-branes at a $\mathbb{C}^2/\Gamma_G$ orbifold singularity in the transverse directions. Having five-branes but no ALE singularity would lead to a (iiia) theory of [1]. Having the ALE singularity but no five-branes would lead to a (iib) theory of [1]. Putting the two situations together leads to new $\mathcal{N} = (1,0)$ string theories, whose field theory infra-red limit was discussed in [3].

As discussed in [3], the $\mathcal{N} = (1,0)$ theory has gauge group

$$\prod_{\mu=0}^r U(Kn_\mu),$$

with matter multiplets in the representations $\frac{1}{2} \otimes a_{\mu\nu}(\square, \square)$. In addition, there are $r \equiv \text{rank} \ G$ hyper-multiplets and tensor multiplets (which would give $r \mathcal{N} = (2,0)$ matter multiplets for the theory with no five-branes). $r$ of the $U(1)$ factors in (2.1) have charged matter and are thus anomalous in 6d. As in [3,4], this means that these $U(1)$ factors are spontaneously broken; they pair with the $r$ hyper-multiplets mentioned above to get a mass. The massless, unbroken gauge group is thus

$$U(1) \times \prod_{\mu=0}^r SU(Kn_\mu),$$

with the $U(1)$ factor decoupled, with no charged matter. Although the $U(1)$ factors in (2.1) are massive, their $D$ term equations still constrain the moduli space. Supersymmetry implies that the expectation values of the $r$ hyper-multiplets involved in the $U(1)$ anomaly cancelation appear as Fayet-Iliopoulos terms in these constraints [3]; these are the ALE blowing-up modes, which enter as background parameters in the 6d theory.

Taking $g_s \to 0$ with $M_s$ fixed, the tensor multiplet moduli space is the Coxeter box (1.1) of the corresponding $ADE$ group $G$. This can be seen starting from the (iib) theory associated with the ALE space and no branes.

Using results found in [3,5] via anomalies, the effective gauge coupling of the $r + 1$ gauge groups on the Coulomb branch can be written as

$$g_\mu^{-2}(\Phi) = \tilde{\alpha}_\mu \cdot \Phi + M_s^2 \delta_{\mu 0}\text{ (2.3)}$$

where, as in (1.1), the $\tilde{\alpha}_\mu$ are the simple and extended roots of the $ADE$ group $G$ associated with the singularity. Using $\tilde{\alpha}_\mu \cdot \tilde{\alpha}_\mu = \tilde{C}_{\mu \nu}$, the extended Cartan matrix of $G$, the couplings
in (2.3) cancel the reducible $\tilde{C}_{\mu\nu}\text{tr}F_{\mu}^{2}\text{tr}F_{\nu}^{2}$ anomaly terms found in [8]. We see that, as required, all $g_{\mu}^{-2} \geq 0$ over the entire Coulomb box (1.1), with the various $g_{\mu}^{-2} = 0$ along the boundaries of the Coulomb box. The “Landau pole” mentioned in [8] has been eliminated by the compactness of the Coulomb branch for finite $M_s$.

There is a Higgs mode of the theory corresponding to moving the $K$ five-branes away from the $X_G \cong \mathbb{C}^2/\Gamma_G$ ALE space. This Higgs branch moduli space is $M_H \cong (X_G)^K/S_K$, as expected, with (2.2) broken to the diagonal $U(K)_D$ away from the origin (or with non-zero Fayet-Iliopoulos parameters). This $U(K)_D$ theory is the (ia) theory of the branes away from the singularity, with gauge coupling $g_D^{-2} = \sum_{\mu=0}^{r} n_{\mu} g_{\mu}^{-2} = M_s^2$ as expected. The low energy theory has an enhanced, accidental $\mathcal{N} = (1,1)$ supersymmetry which is not respected by the massive field theory and stringy modes.

There are also interesting new 6d theories associated with type IIA NS 5-branes at orbifold singularities, which require further understanding. For the case of $K$ branes at a $\mathbb{C}^2/\mathbb{Z}_M$ singularity, the 6d theory could be the same theory as that of $M$ type IIB branes at a $\mathbb{C}^2/\mathbb{Z}_K$ singularity (up to a decoupled tensor multiplet in the former and vector multiplet in the latter).

3. New theories from $SO(32)$ branes at ALE singularities

Our next class of new 6d string theories with $\mathcal{N} = (1,0)$ supersymmetry arise from $SO(32)$ heterotic or type I 5-branes at $\mathbb{C}^2/\Gamma_G$ orbifold singularities. The low energy limit of these theories was discussed in [8] and also, via F-theory, in [10]. The gauge group is

$$\prod_{\mu \in \mathcal{R}} Sp(v_{\mu}) \times \prod_{\mu \in \mathcal{P}} SO(v_{\mu}) \times \prod_{\mu \in \mathcal{C}} U(v_{\mu}),$$

(3.1)

where the nodes of the extended $G$ Dynkin diagram have been grouped into the sets $\mathcal{R}$, $\mathcal{P}$, $\mathcal{C}$, $\overline{\mathcal{C}}$ discussed in detail in [5]. As in the discussion following (2.1), the overall $U(1)$ factor in each $U(v_{\mu})$ is anomalous and thus pairs with a hyper-multiplet to get a mass.

The tensor multiplet structure is related to the Coxeter box of the corresponding simply-laced group $G$, but modded out by a $\mathbb{Z}_2$ action $*$ which takes $\mathcal{C} \leftrightarrow \overline{\mathcal{C}}$. From the analysis in [5], the result is that the tensor multiplets for a $\mathbb{C}^2/\Gamma_G$ singularity live in the Coxeter box of $H \subset G$ with $G \rightarrow H$ as

$$\begin{align*}
SU(2P) & \rightarrow Sp(P) \\
SO(4P + 2) & \rightarrow SO(4P + 1) \\
SO(4P) & \rightarrow SO(4P) \\
E_6 & \rightarrow F_4 \\
E_7 & \rightarrow E_7 \\
E_8 & \rightarrow E_8.
\end{align*}$$

(3.2)
The operation in (3.2) is the same modding out which appeared in the description of \[12,13\] for obtaining composite gauge invariance with non-simply-laced gauge groups. Although it is outside of the focus of this work, we note that the hyper-Kahler quotient construction of \[8,5\] for the moduli space of \(SO(N)\) instantons on ALE spaces suggests an interesting analog of the results of Nakajima. Briefly put, Nakajima [14] showed that \(\hat{G}_N\) affine Lie algebras arise in analyzing the moduli space of \(U(N)\) instantons on \(\mathbb{C}^2/\Gamma_G\). Similarly, we expect \(\hat{H}_N\) affine Lie algebras to arise in analyzing the moduli space of \(SO(N)\) instantons on \(\mathbb{C}^2/\Gamma_G\), with \(G \rightarrow H\) as in (3.2). The results of [14] find physical application, for example in [15], in showing that simply-laced composite gauge invariance is properly represented on massive modes. The conjectured appearance of \(\hat{H}_N\) affine Lie algebras could find similar application in compactifications with non-simply-laced composite gauge invariance.

Other 6d theories can be obtained by making use of the fact, as in [7], that the gauge group of the heterotic or type I theory is actually \(Spin(32)/\mathbb{Z}_2\). The low-energy limit of these string theories in the case of \(\mathbb{C}^2/\mathbb{Z}_2\) singularities was discussed in [8], where it was (sloppily) referred to as the case without vector structure. The result is a theory based on the “type I5 quiver diagrams” of [6], with gauge group

\[
\prod_{i=1}^{P} SU(v_{i})
\]  

and tensor multiplets which live in the Coxeter box, of size \(M^2_s\), of \(Sp(P - 1)\). For the simplest example, \(\mathbb{C}^2/\mathbb{Z}_2\), the low energy theory is \(SU(2K)\) with two matter fields in the \(\square\) and sixteen in the \(\blacksquare\) and no tensor multiplet.

4. **New theories from \(E_8 \times E_8\) branes at orbifold singularities**

Our next class of new 6d string theories with \(\mathcal{N} = (1, 0)\) supersymmetry arise via \(E_8 \times E_8\) 5-branes at orbifold singularities in the \(g_s \rightarrow 0\) with \(M_s\) fixed limit. The gauge group and number of tensor multiplets associated with point-like \(E_8\) instantons at ADE orbifold singularities was obtained via F-theory in [11]. We take this opportunity to briefly spell out the massless matter content of these theories, which we determine from the results of [11] combined with anomaly considerations, as it was not presented in [11]. First, the irreducible \(\text{tr}F^4\) gauge anomalies must vanish; remaining reducible anomalies must then be canceled by coupling to the tensor multiplets. In addition, as discussed in [10], a \(\pi_6\)
anomaly restricts $SU(2)$ to have $n_2 = 4 \text{ mod } 6$, $SU(3)$ to have $n_3 = 0 \text{ mod } 6$, and $G_2$ to have $n_7 = 1 \text{ mod } 3$. A further general condition is

\[ n_H - n_V + 29n_T = 30K + r, \tag{4.1} \]

where $n_H$ is the total number of hyper-multiplets, $n_V$ is the total number of vector multiplets, $n_T$ is the number of tensor multiplets, $K$ is the number of small instantons or five-branes, and $r \equiv \text{rank} G$ is the number of ALE blowing-up modes. The condition (4.1) is a 6d analog of a ’t Hooft anomaly matching condition for the gravitational anomaly.

The theory (e) for $K E_8 \times E_8$ five-branes and no singularity has a Coulomb branch with $n_T = K$ tensor multiplets and no vector-multiplet gauge group. Putting the $K$ 5-branes at a $\mathbb{C}^2/\mathbb{Z}_M$ singularity, with $K \geq 2M$, the result of [11] is that there is a Coulomb branch, again with $n_T = K$ tensor multiplets, but with new gauge fields, with gauge group

\[ SU(2) \otimes SU(3) \otimes \cdots \otimes SU(M - 1) \otimes SU(M)^{\otimes (K - 2M + 1)} \otimes SU(M - 1) \otimes \cdots \otimes SU(2). \tag{4.2} \]

The massless matter content consists of bi-fundamentals charged under each neighboring pair of gauge groups in (4.2) as well as an extra fundamental flavor for each of the two $SU(2)$s at the ends and for each of the two $SU(M)$s at the end of the string of $SU(M)$s. As remarked in [11], the gauge group in (4.2) agrees (up to replacing the $SU(n)$ with $U(n)$) with that of [17,18] which is mirror dual in three dimensions to $U(M)$ gauge theory with $K$ flavors; the above hyper-multiplet content also agrees with that of [17,18]. The theory with this gauge group and matter content is properly free of gauge anomalies (making use of couplings to $K - 3$ of the tensor multiplets to cancel the reducible gauge anomalies).

The theory with the above gauge group and matter content properly has a $K + M - 1$ dimensional Higgs branch, with the gauge group generically completely broken. $M - 1$ of the Higgs-branch moduli correspond to the blowing-up modes of the $\mathbb{C}^2/\mathbb{Z}_M$ orbifold. The remaining $K$ dimensions is the $K$-fold symmetric product of the ALE space with those $M - 1$ moduli, corresponding to the locations of the $K$ identical, point-like instantons on the ALE space. For generic values of these moduli, the 5-branes are away from any singularity and there are no vector-multiplets; the gauge symmetry (4.2) is unHiggsed when the moduli are tuned, corresponding to putting the 5-branes on the singularity.

For $K = 6$ five-branes at a $G = D_4$ singularity, the result of [11] is that the gauge group is $SU(2) \times G_2 \times SU(2)$ with $n_T = 6$ tensor multiplets. The matter content is determined by anomaly considerations to be $\frac{1}{2}(2, 1, 1) \oplus \frac{1}{2}(2, 7, 1) \oplus \frac{1}{2}(1, 7, 2) \oplus \frac{1}{2}(1, 1, 2) \oplus 2(1, 7, 1)$. This
theory has a 10 dimensional Higgs branch, with the gauge group generically completely broken, corresponding to the location of the six point-like instantons on the ALE space and its four blowing-up modes. Giving an expectation value to a matter fields in the \((1, 7, 1)\) corresponds to smoothing the \(D_4\) singularity to an \(A_2\) singularity.

For \(K \geq 7\) five-branes at a \(G = D_4\) singularity, the result of \([1]\) is gauge group 
\[SU(2) \times G_2 \times SO(8)^{K-7} \otimes G_2 \otimes SU(2)\] 
with \(n_T = 2K - 7\). The matter content is determined by anomaly considerations to be \(\frac{1}{2}(2, 1) \oplus \frac{1}{2}(2, 7)\) for each \(SU(2) \times G_2\) pair and no other matter fields.

For \(K > 7\) is no other matter.

For \(E_6\), the result of \([1]\) is \(n_T = 4K - 22\), with gauge group 
\[SU(2) \times G_2 \times F_4 \times G_2 \times SU(2)\] 
for \(K = 8\) and gauge group 
\[SU(2) \times G_2 \times F_4 \times SU(3) \times (E_6 \times SU(3))^{K-9} \times F_4 \times G_2 \times SU(2)\] 
for \(K > 8\). The matter content is determined by anomaly considerations to consist, as above, of the minimal \(SU(2) \times G_2\) matter \(\frac{1}{2}(2, 1) \oplus \frac{1}{2}(2, 7)\) in each pair of \(SU(2) \times G_2\).

For \(K = 8\) the \(F_4\) has a single matter field in the \(26\) (giving it an expectation value breaks 
\(F_4 \to SO(9) \to SO(8)\), corresponding to smoothing the singularity from \(E_6 \to D_5 \to D_4\). For \(K > 8\) each \(SU(2) \times G_2\) pair has the same minimal matter content as above, and there is no other matter.

For \(K \geq 10\) five-branes at a \(E_7\) singularity, the result of \([1]\) is 
\[(SU(2) \times G_2)^4 \times F_4^2 \times E_7 \times (SU(2) \times SO(7) \times SU(2) \times E_7)^{K-10}\] 
with \(n_T = 6K - 40\). Each \(SU(2) \times G_2\) factor has the minimal matter appearing above. Each \(SU(2) \times SO(7) \times SU(2) \times E_7\) factor has matter \(\frac{1}{2}(2, 8, 1, 1) \oplus \frac{1}{2}(1, 8, 2, 1)\). There is no other matter.

For \(K \geq 10\) five-branes at a \(E_8\) singularity, the result of \([1]\) is gauge group 
\[E_8^{(K-9)} \times F_4^{(K-8)} \times (SU(2) \times G_2)^{2K-16}\] 
with \(n_T = 12K - 96\). Each \(SU(2) \times G_2\) factor has the minimal matter content appearing above and there is no other matter.

The result of \([1]\) for \(K = 2m + 6\) five-branes at a \(D_{m+4}\) singularity is \(n_T = 2K - 6\) and gauge group 
\[SU(2) \times G_2 \times SO(9) \times SO(3) \times SO(11) \times SO(5) \times \cdots \times SO(2m + 5) \times SO(2m - 1) \times SO(2m + 7) \times SO(2m - 1) \times \cdots \times SO(9) \times G_2 \times SU(2)\]. For \(K > 2m + 8\) five-branes at a \(D_{m+1}\) singularity, \([1]\) again find \(n_T = 2K - 6\) and, in addition to the gauge group factors for \(K = 2m + 6\), \(Sp(m) \times (SO(2m + 8) \times Sp(m))^{(K-2m-8)} \times SO(2m + 7)\).

For \(m > 1\), we were not able to find a solution for matter content which is compatible with anomaly considerations and these gauge groups, though perhaps one does exist.

\[2\] Note added (in revised version, 9/3/97): There is a slight modification of the above gauge groups for which there is a matter content which is nicely compatible with all of the anomaly considerations. For \(K = 2m + 6\) five-branes at a \(D_{m+4}\) singularity, with \(n_T = 2K - 6\) as in \([1]\),
5. Compactification and $T$ duality

It is natural to expect that, upon compactification on a circle, the new theories associated with five-branes at singularities are related by $T$ duality, generalizing that of [1] between $(\text{iia}) \leftrightarrow (\text{iib})$ and $(\sigma) \leftrightarrow (\varepsilon)$. As in [1], this can be put to a simple test.

Upon compactifying on a circle, both the Cartan of the 6d gauge group and the 6d tensor multiplets lead to 5d $U(1)$ gauge fields with scalar moduli. The number of 5d scalar moduli is thus $r_V + n_T$, where $r_V$ is the rank of the 6d vector multiplet gauge group and $n_T$ is the number of 6d tensor multiplets. Two 6d theories related by $T$ duality must thus have $r_V + n_T = \tilde{r}_V + \tilde{n}_T$. More precisely, tensor multiplets in 6d have a compact “Coulomb branch,” with the scalar moduli living on a box of size $M_s^2$. Upon reducing to 5d and rescaling the modulus to have dimension one, it lives on a box of size $M_s^2 R$. On the other hand, reducing a 6d vector multiplet to 5d leads to a scalar modulus which lives on a box of size $R^{-1}$. Because $T$ duality relates a theory compactified on a circle of radius $R$ to another theory compactified on a circle of radius $\tilde{R} \equiv (M_s^2 R)^{-1}$, it exchanges 5d moduli associated with 6d tensor multiplets with those associated with 6d vector multiplets. Thus $T$ dual theories must satisfy the stronger conditions $\tilde{r}_V = n_T$ and $\tilde{n}_T = r_V$.

This can be thought of as a reason why, as we have seen, the Coulomb branch of 6d tensor multiplets is the Coxeter box of a non-Abelian group. Compactifying on a circle, there should be a $T$ dual theory where these moduli do arise from a gauge theory with that gauge group.

For example, the vector multiplets of the (iia) theory compactified on a circle of radius $R$ and the tensor multiplets of the (iib) theory compactified on a circle of radius

the modified gauge group is $SU(2) \times G_2 \times SO(9) \times Sp(1) \times SO(11) \times Sp(2) \times \cdots \times SO(2m + 5) \times Sp(m - 1) \times SO(2m + 7) \times Sp(m - 1) \times \cdots \times SO(9) \times G_2 \times SU(2)$. The matter content which satisfies all of the anomaly equations is given by the minimal $1\over 2)((2, 1) \oplus (2, 7))$ in each $SU(2) \times G_2$ factor and a half-hypermultiplet bi-fundamental charged under each neighboring $SO$ and $Sp$, i.e. a $1\over 2(2k + 7, 2k)$ under each neighboring $SO(2k + 7) \times Sp(k)$ and a $1\over 2(2k, 2k + 9)$ under each neighboring $Sp(k) \times SO(2k + 9)$. In addition, the middle $SO(2m + 7)$ gauge group has a hypermultiplet in the $2m + 7$ which is uncharged under the other gauge groups. For the cases $m = 2, 3$, where the gauge group agrees with that of [1] (as $Sp(1) \cong SO(3)$ and $Sp(2) \cong SO(5)$), this matter content was first worked out by G. Rajesh. I am very grateful for his correspondence on the $m = 2, 3$ cases, which helped to inspire the above modified gauge groups and matter content for $m > 3$. I also thank P.S. Aspinwall and D.R. Morrison for helpful correspondence on these issues. A similar modification of the gauge group and matter content applies for $K > 2m + 8$. 

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\[ \tilde{R} \equiv (M_s^2 R)^{-1} \] both lead to moduli living on a Coxeter box of size \( R^{-1} \), compatible with their equivalence \([1]\). Similarly, both the \( SO(32) \) theory \((o)\), compactified on a circle of radius \( R \), with Wilson lines which break it to \( SO(16) \times SO(16) \), and the theory \((e)\) on a circle of radius \( \tilde{R} \equiv (M_s^2 R)^{-1} \) lead to a 5d moduli space which is the Coxeter box of \( Sp(K) \), of size \( R^{-1} \).

The theories associated with \( SO(32) \) and \( E_8 \times E_8 \) branes at \( C^2/\Gamma_G \) singularities do satisfy the condition \( r_V + n_T = \tilde{r}_V + \tilde{n}_T \). Indeed, as also noted in \([19]\), in both cases, \( r_V + n_T = C_2(G)K - |G| \), where \( C_2(G) \) is the dual Coxeter number of the \( ADE \) group \( G \) and \( |G| \) is its dimension.\(^3\) On the other hand, the two theories do not satisfy the stronger conditions \( \tilde{r}_V = n_T \) and \( \tilde{n}_T = r_V \). It is not presently known how this failure should be interpreted or resolved.

6. Matrix Model Applications of the Theories.

Following \([20]\), it was suggested in \([21]\) that a M(atrix) description of \( M \) theory on \( X_G \times \mathbb{R}^6,1 \), where \( X_G \) is an ALE space asymptotic to \( C^2/\Gamma_G \), is given by quantum mechanics with 8 supersymmetries and gauge group \( \prod_{\mu=0}^r U(v_{\mu}) \) with matter \( \frac{1}{2} \bigoplus_{\mu=0}^r a_{\mu\nu}(\square_\mu, \square_\nu) \). The (classical) moduli space of vacua of this theory for \( v_{\mu} = K n_{\mu} \) is \( (X_G \times \mathbb{R}^5)^K/S_K \), corresponding to the location of \( K \) identical zero branes in the light-cone \( X_G \times \mathbb{R}^5 \). We propose a slight variant of this conjecture.

Now consider \( M \) theory on \( X_G \times T^5 \times R^{1,1} \). Following \([1]\), it is expected\(^2\) that a definition of this theory is given by compactifying the new 6d theory of sect. 2 on a \( \hat{T}^5 \). As

\(^3\) As also noted in \([19]\), this agrees with the dimension (in hyper-multiplet units) of the moduli space of \( K G \) instantons on \( K3 \). Duality between the heterotic theory on \( T^3 \) and \( M \) theory on \( K3 \) suggests that the quantum-corrected Coulomb branch for the theory compactified to 3d on a \( T^3 \) actually is the moduli space of \( K G \) instantons on \( K3 \). Similarly, compactifying the theory of sect. 2 associated with type II branes at orbifold singularities, the dimension of the Coulomb branch is \( C_2(G)K \). Duality between type II on a \( T^3 \) and \( M \) theory on \( T^4 \) suggests that the quantum-corrected Coulomb branch for the theory compactified to 3d on a \( T^3 \) is the moduli space of \( K G \) instantons on \( T^4 \).

\(^4\) This is the moduli space for generic Higgs expectation values. There is a larger Coulomb branch, of dimension \( 5K C_2(G) \), at the origin.

\(^5\) I thank N. Seiberg for suggesting this.
in [1], there are 25 compactification parameters living in $SO(5,5,\mathbb{Z})/SO(5,5)/(SO(5) \times SO(5))$. Taking a rectangular torus with no $B$ field, $\hat{T}^5$ is related to $T^5$ as in [1], by:

$$\hat{L}_i = \frac{l_p^3}{RL_i},$$
$$M_s^2 = \frac{R^2L_1L_2L_3L_4L_5}{l_p^9},$$

(6.1)

where $R$ is the radius of the longitudinal direction and $l_p$ is the eleven-dimensional Planck-length. Indeed, this gives the correct light-cone $X_G \times T^5$ space-time from the moduli space of vacua (subject to the same discussion about the situation at the quantum level as in [22,1]).

In the limit of large $T^5$, this reduces to a slight variant of the suggestion of [21] outlined above. The massless gauge group of the 6d theory is given by (2.2) rather than $\prod_{\mu=0}^r U(v_\mu)$; in addition, there are the $n_T = r$ tensor multiplets. Upon compactification, the tensor multiplets yield $U(1)^r$ gauge fields, the same number which became massive because of the anomaly. It is thus tempting to conclude that, upon compactification, the tensor multiplets simply give back the same $U(1)$ factors which became massive in 6d because of the anomaly, giving back the original $\prod_\mu U(K_n_\mu)$ theory in lower dimensions. However, this does not seem to be the case. The difference is that the matter fields $\frac{1}{2} \otimes_{\mu\nu=0}^r a_{\mu\nu}(\square_\mu, \square_\nu)$ were charged under the $U(1)^r$ which became massive because of the 6d anomaly. On the other hand, these matter fields are neutral under the $U(1)^r$ which the tensor multiplets give back upon compactification; the new $U(1)^r$ has no charged matter. Taking the limit of large $T^5$ in (6.1) thus yields a slight variant of the gauge theory of [21].

Following [1], we similarly expect that the 6d string theory from $SO(32)$ or $E_8 \times E_8$ heterotic five-branes at a $X_G$ singularity, when compactified on $\hat{T}^5$ (which depends on the 105 parameters in $SO(21,5,\mathbb{Z})/SO(21,5)/(SO(21) \times SO(5))$), gives a definition of $M$ theory on $X_G \times (T^5/\mathbb{Z}_2) \times \mathbb{R}^{1,1}$.

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