ON THE CAPABILITIES OF SURVEY TELESCOPES OF MODERATE SIZE

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ABSTRACT

To explore the capabilities of moderate-size optical telescopes used in surveys, a set of nine new wide-field designs, having apertures of up to 1 m, was created. The designs were optimized to ensure the widest possible field of view with an image quality better than 3″ across the field. All but one of the systems have an angular field in a range of 3°5–10° and a flat focal surface; the field of the last system is 45° in diameter with an aperture 0.5 m and a spherical focal surface. Relations between the expected limiting magnitude, survey speed, and exposure time allow one to choose the system that is best suited for the objectives of specific observations. In particular, a single wide-field telescope with an aperture of approximately 1 m can detect objects brighter than 22.5′′ over the entire hemisphere within one night. Since the optical layouts are of practical interest, their complete descriptions are given.

Key words: telescopes

1. INTRODUCTION

A number of important astronomical challenges, in particular, the near-Earth asteroid hazard problem, necessitate the continuous observation of all objects in the sky brighter than about 23′′ in the visible waveband. To estimate the required survey speed $S$, as measured in square degrees per second (deg$^2$ s$^{-1}$), we suggest that one needs to cover $10^4$ deg$^2$ of sky within 3 hr. This area is a little smaller than the entire hemisphere visible above the horizon and free of absorption in the Milky Way and Earth’s light pollution at large zenith angles. This gives a survey speed of $S \approx 1 \text{deg}^2 \text{s}^{-1}$, which shows that the problem is non-trivial.

To illustrate its difficulty, note that the field of view for a classical Cassegrain telescope (parabolic primary mirror plus hyperbolic secondary mirror) is only several arcminutes wide. Thus, one would need to acquire approximately $10^6$ images to cover the required area of sky, which is unrealistic even with multiple telescopes. The Ritchey–Chretien telescope, recently considered to be a wide-field instrument, also fails to solve the problem. The typical field of a Ritchey–Chretien telescope does not exceed 20′, which might reduce the number of images mentioned above, but only by an order of magnitude.

To solve the problem, one would naturally turn to the remarkable Bernhardt Schmidt (1930) system, whose modified versions can reach a field of about 10° in diameter (see Wilson 1996). Astronomers had used these systems for over 50 years with photographic plates as light detectors, but one had to bend the plate to match the curved focal surface of the Schmidt camera (its radius of curvature is about equal to the effective focal length). Meanwhile, the majority of modern detectors are flat. One can achieve the flat field either by complicating the optical system or by making the field faceted with small field-flatteners. The last option has been applied to the Kepler telescope, which has the aperture of 95 cm and the field area of 115 deg$^2$. Its detector consists of 21 pairs of the flat 59 mm × 28 mm CCDs covered by sapphire field-flattening lenses. Obviously, this approach is feasible now only for unique projects, so to accommodate flat photon detectors, most of the designs discussed below have a flat focal surface. The exception is an all-spherical design with a 45° field—the particular case of a new system, which was proposed recently by Terebizh (2015, 2016).

The first step toward simplification of the Schmidt camera was made by Schmidt himself. In 1934, he tested a model with the three spherical lenses instead of the aspheric corrector (see Wachmann 1955; Busch et al. 2013). In fact, all the subsequent wide-field catadioptric telescopes—the systems by Richter & Slevoogt (1941), Schmidt–Houghton (Houghton 1942, 1944), Hawkins & Linfoot (1945), Baker (1962), and $\Omega_{2-3}$ (Terebizh 2007)—are the successors of the two generic systems, invented by Schmidt. Modern versions of these systems provide angular fields up to $10^2$ with a flat focal surface and apertures reaching 1 m (Terebizh 2011).

Another approach, employing a lens corrector mounted near the focus of a large aspheric mirror, was introduced by Sampson (1913), Ross (1935), and Wynne (1968). In this way, one cannot achieve a field comparable to that of a mid-size catadioptric system, but the large aperture diameter allows for the detection of faint objects. The modern types of prime-focus correctors have been discussed by Terebizh (2003) and Saunders et al. (2014).

This paper discusses the questions that arise in the development of optical systems for wide-field survey projects. Of primary interest is the question, “What type of optical system is best suited for the specified survey depth and speed?” We compare the efficiency of various catadioptric designs with flat focal surfaces and the angular fields in the range of 3°5–10°, and a 45°-design with a spherical focal surface. The apertures of the catadioptric designs range from 0.4 m up to 1.0 m. For comparison, a 20 cm refractive lens was added with a 15° angular field and a flat focal surface.\footnote{In calculations, we used the Zemax optical program (ZEMAX Development Corporation, U.S.A.).

To ensure a more unbiased comparison of the optical systems being considered, we assume that the linear diameter $B$ of the flat field is the same for all the designs, fixed at 134.5 mm diameter, which coincides with the diagonal length of the frequently used CCD STA 1600 from Semiconductor Technology Associates. For those interested in a detector of another specific size, any of the optical systems in question
may be scaled up or down as necessary. The optical system with the curved focal surface is a separate case that will be discussed in Section 2.2.

Since one should take into account the specifications of light detectors when designing an optical system, we briefly touch on issues related to matching the telescope and detector resolving powers.

2. OPTICAL LAYOUTS

2.1. Flat Focal Surface

A set of catadioptric systems discussed below includes the eight flat-field designs and one design with a spherical focal surface (Figure 1). Most of these optical layouts have long been known, and almost all of them have been implemented. For this paper, new versions were designed to provide the widest possible angular field of view given an image quality better than 3″ across the field. All of these optical designs are of independent practical interest. A purely lens objective (No. 9 in Table 1) has been added only to compare its efficiency with that of the catadioptric systems.

The term “sky survey rate” and the corresponding “Herschel” unit of measure, are explained in Section 5. All systems have been optimized for the wavelength boundaries as specified in Table 1, but they can be used in a wider spectral range. The common linear obscuration coefficient is given as \( \eta = D_{\text{obs}}/D = \sqrt{1 - U} \), where both \( \eta \) and \( U \) are dependent on the field angle. The effective aperture diameter is given as \( D_e = D \sqrt{U} \). Finally, the image diameter, \( D_{80} \), corresponds to the integral wavelengths within the range as specified in Table 1. Certainly, one can improve image quality by using a narrow-band filter.

In this paper, we have not considered some attractive systems capable of providing good image quality within the field up to 3″5, in particular, the three-mirror anastigmat by Korsch (1972, 1977) and the Mersenne-Schmidt telescope by Paul (1935); see also Willstrop (1984). The reason is that for apertures less than 1 m, one can attain the same image quality and field size with more simple optics and with lower obscuration than is possible with the three-mirror designs. However, for large telescopes, where full-aperture field correctors cannot be made, the systems mentioned above provide the maximum survey depth. Thus, the layouts by Korsch and Paul were selected, respectively, for the space telescope SNAP with an aperture of 2 m and a field of 1″5 diameter, and for the 8.4 m ground-based telescope LSST with a field of 3″5.

Figure 1. Optical layouts of survey telescopes. Top row, from left to right (numbering corresponds to Table 1): (1) Prime-focus corrector, (2) corrected Cassegrain, and (3) Schmidt camera with a three-lens corrector-flattener. Middle row: (4) all-spherical Schmidt–Houghton, (5) all-spherical modified Richter–Slevogt, and (6) all-spherical Richter–Slevogt with a Mangin primary (Amon et al. 1971). Bottom row: (7), (8) All-spherical \( \Omega_2 \) and \( \Omega_3 \) systems (Terebizh 2007), and (10) all-spherical system with a four-lens corrector (Terebizh 2015).
A complete description of all the systems discussed in this paper is given in Tables 3–11 and the Appendix.

Figure 2 gives an idea of how the angular field of view, $2w$, varies with the telescope aperture, $D$, for the flat-field systems listed in Table 1. Similar to the extensive set of existing telescopes discussed by Terebizh (2011), the field of view is, to a first approximation, inversely proportional to the aperture diameter: $2w^2 \approx 3.51/D_m$. This dependence arises from a simple relation

$$B \approx D_\phi \cdot 2w^2/57.3,$$

given that the linear size of the detector $B$ and focal ratio $\phi \equiv F/D$ are not changing significantly. In our case, this condition is satisfied, because the detector diagonal is fixed and values of $\phi$ are close to the mean value ($\phi \approx 2.20$).

Although a corrected Cassegrain system is, in some essential respects, inferior to a system with the prime-focus corrector, its compactness may play the decisive role, if making a number of identical instruments is assumed.

In this regard, it is worth adding that the image quality in corrected Cassegrain systems depends weakly on the shape of the secondary mirror. In cases where the squared eccentricity of this mirror reaches a value on the order of 20 or even higher, the gain in the image quality is lost due to severe tolerances for manufacturing, assembly, and alignment. On the contrary, tolerances are much more loose for a corrected Cassegrain system with a spherical secondary mirror; this significantly increases the productivity of observations (a good example is GEDSS; Jeas 1981). The fact that the spherical secondary mirror contributes to the achieve of a wide field of view in corrected Cassegrain telescopes was discovered by Harmer & Wynne (1976) a long time ago. The reason is likely that the secondary mirror should be equally optimal for the light beams falling on it at very different angles, and this is possible only when this mirror is close to a sphere.

2.2. Spherical Focal Surface

A general description of the all-spherical telescope with an extremely wide field of view, along with a few examples, was given by Terebizh (2015, 2016). We present here one more sample of this system with an aperture of 500 mm and a 45° angular field of view. It is included in Table 1 as system No. 10; see also Figure 1 Table 11 and the Appendix for the optical prescription. Since the fraction of unvignetted rays $U$ and the survey rate $\Gamma$ depend on the shape of the detector, those values for this system are left blank in Table 1. Generally speaking, the lenses can be made of arbitrary types of glass; for this example, we choose fused silica for all of the lenses.

As shown in Figure 3, the design provides a very wide field with good image quality. The $D_{80}$ image diameter in the polychromatic waveband varies in the range of $1''2–1''4$ across the field, whereas the rms spot diameter is close to the Airy disk diameter of the corresponding diffraction pattern.

Evidently, the drastic extension of the angular field of high image quality results from the transition to a design with purely spherical optics. This feature provides the true point symmetry of an optical system about the center of curvature of the mirror that is limited only by the inevitable vignetting on the aperture stop. The second feature of the system, which provides almost complete absence of chromaticity, is the afocality of the four-lens corrector: it operates here at a focal ratio of $f/87$. Finally,
the third property necessary to ensure a wide field is the close proximity of the entrance pupil to the aperture stop; their separation is only 31 mm in this design.

Like the classical system of Schmidt, the system under consideration meets the difficulties caused by the curvature of the focal surface. The following methods seem to be preferred now in this regard:

1. the use of large detectors with curved surfaces;
2. the application of the long-known technology based on the plurality of delicate waveguides with a curve in the aggregate input surface (figured fiber-optic plates); and
3. the use of relatively small flat detectors equipped with local field-flattening optics.

The principal issues and examples of curved detectors were discussed by Iwert & Delabre (2010) and Iwert et al. (2012); the first of these articles includes a photograph of a curved detector a size of 60 mm × 60 mm and a curvature radius of 500 mm. There are also working examples of curved detectors of this type. In particular, a mosaic of curved detectors has been implemented in the DARPA-developed 3.5 m Space Surveillance Telescope (Blake et al. 2013).

The second option, being considered in a modern context, involves a number of technological problems. One may expect that these problems will be solved within the framework of the 2013 program announced by the European Space Agency, which provides a solution for mapping a curved image field onto a flat imaging detector array.²

² Details can be found in a note: “Imaging optics and optical device for mapping a curved image field” at http://www.esa.int/ESA on 2013 March 19.

In Section 1, we mentioned the third option in connection with the Kepler space telescope. For the VT-119j design, the radius of the curvature of the focal surface is rather large, 1329 mm, so additional flattening optics can be made with a single lens for each detector. Let us consider, for example, a 30 mm × 30 mm flat detector with a diagonal length of 42.4 mm. If we attempt to keep the original image quality at the center of each detector, then their corner size will be about 50 μm. Alternatively, we can achieve uniform image quality across the entire detector by shifting the detector 0.1 mm and allowing the center spots to blur slightly, resulting in a spot diameter across the field of less than 23 μm. Finally, the image quality is restored completely, when a weak lens made of fused silica is installed in front of the detector. Since the lens radii of curvature are of about 300 and 200 mm, one can simply use it as the detector window. Thus, in the case under consideration, the third option—faceting of the spherical focal surface—is the most straightforward.

As in all Schmidt-like telescopes, obscuration of the useful light occurs in this design, so one needs to find a reasonable compromise between the field size and limiting magnitude. A corresponding example will be discussed in Section 4.

It is important to note that individual detectors of any form can be placed, both continuously and discretely, on the curved focal surface to match the shape of the area of the sky being studied.

The high optical performance of design No. 10 will ultimately have to be weighed against the additional cost of the large primary mirror and telescope. At the same time, the extreme simplicity of the optical surfaces and glass types...
should be considered, as well as the softness of tolerances both in manufacturing and in operation.

2.3. Notes Concerning Manufacturing

Since we are primarily interested in the performance of various optical systems, we touch only in passing upon issues relating to their manufacturing. For many reasons, the technological challenges increase rapidly with higher speed optical systems, say, when the focal ratio \( f / D \) becomes less than 1.5. As seen in Table 1, these concerns do not apply to the sample of designs under consideration here.

The primary mirrors of the telescopes No. 1 and No. 2 are aspheres. The main problem that arises in polishing of aspheric surfaces is not as much concerned with the maximum deviation of the surface from a nearest sphere but with the asphericity gradient \( G \) (\( \mu m \ mm^{-1} \)), i.e., the rate at which the deviation changes along the radial coordinate. An approximate expression for the maximum asphericity gradient \( G_{max} \) of a conic section as a function of its diameter \( D \), the radius of curvature at the vertex \( R_0 \), and the eccentricity \( e \) is

\[
G_{max} \simeq 31.25 \varepsilon^2 (D / |R_0|)^3, \quad \mu m \ mm^{-1}
\] (Terebizh 2011). Usually, the asphericity gradient of the primary mirror does not exceed 0.6 \( \mu m \ mm^{-1} \). For the fastest existing wide-field telescopes, this value reaches 1.5 \( \mu m \ mm^{-1} \) (see Figure 4 in the paper mentioned above). For our designs, the values of \( G_{max} \) are 0.45 \( \mu m \ mm^{-1} \) and 0.58 \( \mu m \ mm^{-1} \) for the systems No. 1 and No. 2, respectively, i.e., their aspheric gradients are relatively small.

It is also worth adding that each of the designs Nos. 1–3 contains only one aspheric surface. Thus, the tolerances are quite reasonable for these systems, while for the all-spherical designs Nos. 4–10 the tolerances are as mild as possible, given the system’s speed.

3. SAMPLING FACTOR

As one can see, the value of \( D_{80} \) characterizes the image quality provided by a telescope alone; to distinguish \( D_{80} \) from similar quantities, we designate it hereafter as \( \beta_{tel} \). For the telescopes considered in this paper, it varies from 0.96 up to 2.9, i.e., has the same order of magnitude as typical atmospheric blurring \( \beta_{atm} \). For our purposes, it is enough to accept that the angular diameter of a star image due to these two factors is

\[
\beta = \sqrt{\beta_{atm}^2 + \beta_{tel}^2}.
\] (3)

Accepting \( \beta_{tel} \) according to Table 1 and fixed \( \beta_{atm} = 1''/5 \), we obtain the resulting values of the image quality \( \beta \) shown in the third column of Table 2. We also assume the detectors pixel pitch to be 9 \( \mu m \), as found on the CCD STA 1600. The corresponding angular sizes of pixels are given in the forth column of Table 2.

Among the set of the most important parameters that define the utility of a telescope for a specific observing problem is the sampling factor \( \chi \), which is the ratio of the diameter of the star image to the pixel size:

\[
\chi = \beta / p,
\] (4)

where both \( \beta \) and \( p \) are either in angular or linear units. According to the well-known sampling theorem by V. Kotelnikov and C. Shannon (see, e.g., Press et al. 1992), the

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**Table 2**

Sampling Factor \( \chi \) at the Atmospheric Blurring 1''/5 and Linear Pixel Size 9 \( \mu m \)

| System No. | Scale (\( \mu m \)) | \( \beta \) | \( p \) | \( \chi \) |
|-----------|---------------------|-----------|-------|--------|
| 1         | 10.6                | 1.62      | 0.85  | 1.9    |
| 2         | 10.6                | 1.83      | 0.85  | 2.2    |
| 3         | 4.97                | 2.42      | 1.81  | 1.3    |
| 4         | 3.71                | 3.05      | 2.43  | 1.3    |
| 5         | 7.46                | 2.16      | 1.21  | 1.8    |
| 6         | 5.30                | 2.27      | 1.70  | 1.3    |
| 7         | 5.30                | 2.16      | 1.70  | 1.3    |
| 8         | 5.29                | 2.34      | 1.70  | 1.4    |
| 10        | 6.58                | 1.98      | 1.37  | 1.4    |

**Table 3**

VT-56y Design with 1.0 m Aperture and 3''/5 Field

| Surf. No. | Comments | \( R_0 \) (mm) | \( T \) (mm) | Glass | \( D \) (mm) |
|-----------|----------|----------------|-------------|-------|-------------|
| 1         | Stop     | \( \infty \)   |             |       | 1000.0      |
| 2         | Pri      | –4708.58       | –1645.73    | Mirror| 1000.0      |
| 3         | L_1      | –419.235       | –45.0       | N-BK7 | 391.6       |
| 4         | L_2      | –1199.06       | –101,877    |       | 384.5       |
| 5         | L_3      | –504.582       | –24.40      | FS    | 304.5       |
| 6         | L_4      | –244.457       | –61.976     |       | 274.4       |
| 7         | L_5      | 1854.23        | –34.50      | N-LAK8| 267.9       |
| 8         | L_6      | –344.778       | –23.148     |       | 255.9       |
| 9         | L_7      | –2822.80       | –20.0       | S-PHM52| 256.2      |
| 10        | L_8      | 757.208        | –187.357    |       | 256.9       |
| 11        | L_9      | –406.165       | –42.40      | S-FPL53| 252.7      |
| 12        | L_10     | 496.992        | –148.232    |       | 251.4       |
| 13        | F        | \( \infty \)   | –7.0        | N-BK7 | 157.6       |
| 14        | \( \infty \) | –18.0    |             |       | 155.1       |
| 15        | \( \infty \) | –6.0     |             | FS    | 144.8       |
| 16        | \( \infty \) | –14.0    |             |       | 142.5       |
| 17        | Ima      | \( \infty \)   |             |       | 134.5       |

Note. Effective focal length is 2183 mm. The design waveband is 0.40–0.85 \( \mu m \). Conic constant of the primary mirror is \( –1.506709 \), all other surfaces are spheres. Light obscuration corresponds to the round screen of diameter 392 mm.

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**Figure 4.** Limiting magnitude as a function of the exposure time for the flat-field systems listed in Table 1.
Table 4
VT-112 Design with 1.0 m Aperture and 3:5 Field

| Surf. No. | Comments | R₀ (mm) | T (mm) | Glass | D (mm) |
|-----------|----------|---------|--------|-------|--------|
| 1         | Stop     | ∞       | 28.944 |       | 1000.0 |
| 2         | Pri      | −4311.49| −1139.21| Mirror | 1000.0 |
| 3         | Sec      | −11457.9| 601.547| Mirror | 549.5  |
| 4         | L₁       | 342.969 | 32.0   | N-BK7 | 350.0  |
| 5         |          | 239.293 | 44.296 |       | 322.7  |
| 6         | L₂       | 622.257 | 32.0   | N-LAK9| 322.6  |
| 7         |          | 20122.9 | 130.451|       | 320.0  |
| 8         | L₃       | 766.109 | 26.0   | N-LAK10| 265.7  |
| 9         |          | 249.604 | 102.599|       | 248.2  |
| 10        | L₄       | 363.158 | 70.0   | S-FPL53| 263.8  |
| 11        |          | −299.997| 94.073 |       | 262.1  |
| 12        | L₅       | −595.004| 17.360| N-LAK14| 178.9  |
| 13        |          | −11372.6| 30.005|       | 172.6  |
| 14        | F        | ∞       | 5.0    | N-BK7 | 158.6  |
| 15        |          | ∞       | 25.0   |       | 157.1  |
| 16        | W        | ∞       | 4.0    | FS    | 145.5  |
| 17        |          | ∞       | 21.0   |       | 144.2  |
| 18        | Ima      | ∞       | ...    |       | 134.5  |

Note. Effective focal length is 2189 mm. The design waveband is 0.45–0.85 μm. Conic constant of the primary mirror is −1.495564. all other surfaces are spheres. Light obscuration corresponds to the round screen of diameter 560 mm.

Table 5
VT-110f Design with 500 mm Aperture and 7:5 Field

| Surf. No. | Comments | R₀ (mm) | T (mm) | Glass | D (mm) |
|-----------|----------|---------|--------|-------|--------|
| 1         | L₁       | ∞       | 25.142 | FS    | 502.2  |
| 2         | Stop     | ∞       | 1472.789|       | 500.0  |
| 3         | Pri      | −2336.261| −979.429| Mirror | 680.2  |
| 4         | L₁       | −376.877| −24.832| SF10 | 225.0  |
| 5         |          | 9297.360| −2.035 |       | 221.3  |
| 6         | L₂       | 2346.990| −18.356| N-LAF7| 221.2  |
| 7         |          | −210.890| −3.828 |       | 198.8  |
| 8         | L₃       | −196.733| −45.723| S-FPL53| 198.2  |
| 9         |          | 946.597 | −40.557|       | 192.6  |
| 10        | F        | ∞       | −4.0   | N-BK7 | 160.6  |
| 11        |          | ∞       | −16.0  |       | 158.8  |
| 12        | W        | ∞       | −4.0   | FS    | 147.6  |
| 13        |          | ∞       | −16.0  |       | 145.7  |
| 14        | Ima      | ∞       | ...    |       | 134.5  |

Note. Effective focal length is 1026 mm. The design waveband is 0.42–0.82 μm. Obscuration on surface #6 of diameter 250.0 mm. Surface #2 is even asphere with 0.82 μm. Obscuration on surface #6 of diameter 250.0 mm. Surface #2 is even asphere with 0.82 μm. Obscuration on surface #6 of diameter 250.0 mm. All others are aspheres.

where detecting faint objects is of primary importance, the sampling factor is reduced to one to two.

The χ values corresponding to the conditions we have adopted are given in the last column of Table 2. We can see that the telescopes under discussion are well suited to the work of a search or exploratory nature.

4. LIMITING MAGNITUDE AND SURVEY SPEED

Let us now consider the characteristics of wide-field telescopes that are of special interest within the scope of this paper. Namely, these are the limiting magnitude (m<sub>lim</sub>) and survey speed (S, degree²s⁻¹) determined by the telescope+detector system and the observational conditions. In calculations, we took into account the entrance pupil
Table 8
VT-98v Design with 500 mm Aperture and 7/10 Field

| Surf. No. | Comments | $R_0$ (mm) | $T$ (mm) | Glass | $D$ (mm) |
|-----------|----------|------------|----------|--------|----------|
| 1         |          | 1961.024   | 65.083   | FS     | 582.3    |
| 2         |          | -1654.074  | 60.033   |        | 579.1    |
| 3         |          | -1858.116  | 39.028   | FS     | 550.2    |
| 4         |          | 79341.23   | 405.393  |        | 538.4    |
| 5         | Stop     | ∞           | 22.105   |        | 421.8    |
| 6         |          | -1086.270  | 45.923   | FS     | 421.8    |
| 7         |          | -2128.737  | -45.923  | Mirror | 429.1    |
| 8         |          | -1086.270  | -427.499 |        | 414.0    |
| 9         |          | 79341.23   | 515.142  | Mirror | 323.1    |
| 10        |         | 305.528    | 23.460   | FK3    | 200.0    |
| 11        |          | -3053.207  | 30.473   |        | 198.2    |
| 12        |          | -661.789   | 16.418   | S-TIM22| 182.7    |
| 13        |          | -768.247   | 28.810   |        | 178.7    |
| 14        |          | ∞           | 7.0      | N-BK7  | 159.5    |
| 15        |          | ∞           | 23.0     |        | 156.9    |
| 16        |          | ∞           | 10.0     | FS     | 143.9    |
| 17        |          | ∞           | 10.0     |        | 140.2    |
| 18        |          | ∞           |          |        | 134.5    |

Note. Effective focal length is 1093 mm. The design waveband is 0.45–0.85 $\mu$m. Obscuration on surface #4 of diameter 270.0 mm. All surfaces are spheres.

Table 9
VT-102j Design with 525 mm Aperture and 7/10 Field

| Surf. No. | Comments | $R_0$ (mm) | $T$ (mm) | Glass | $D$ (mm) |
|-----------|----------|------------|----------|--------|----------|
| 1         | Stop     | ∞           | 150.0    |        | 525.0    |
| 2         | Obsc     | ∞           | 64.192   |        | 240.0    |
| 3         | L₁      | 4516.956   | 61.791   | FS     | 552.3    |
| 4         |          | -4110.190  | 828.460  |        | 554.3    |
| 5         | L₂      | -1249.675  | 53.0     | FS     | 558.6    |
| 6         | Pri      | -2262.803  | -53.0    | Mirror | 568.4    |
| 7         | L₃      | -1249.675  | -828.460 |        | 544.8    |
| 8         | L₄      | -4110.190  | -61.791  | FS     | 285.2    |
| 9         |          | 4516.956   | -64.192  |        | 271.5    |
| 10        | L₁      | -419.927   | -50.0    | S-PHM52| 239.4    |
| 11        |          | 1169.041   | -0.20    |        | 226.8    |
| 12        | L₄      | 1167.046   | -44.897  | S-LAH55| 226.6    |
| 13        |          | -16723.57  | -63.966  |        | 207.4    |
| 14        | F       | ∞           | -5.0     | S-BSL7 | 161.4    |
| 15        |          | ∞           | -15.0    |        | 159.1    |
| 16        | W       | ∞           | -3.0     | FS     | 148.3    |
| 17        |          | ∞           | -17.0    |        | 146.8    |
| 18        | Ima     | ∞           |          |        | 134.5    |

Note. Effective focal length is 1094 mm. The design waveband is 0.43–0.85 $\mu$m. Obscuration on surface #2 of diameter 240.0 mm. All surfaces are spheres.

diameter, effective focal length, angular field of view, telescope transmission efficiency, fraction of unvignetted rays, spectral bandwidth, and value of $\beta_{tot}$. The detector parameters are the same for all telescopes; the quantum efficiency is 0.85 events/photon and the pixel size is 9 $\mu$m (CCD STA 1600). It was assumed that noise obeys the Poisson distribution. For observational conditions, we have assumed that $\beta_{tot} = 1.5$, sky background is 20.0$^\circ$arcsec$^{-2}$, optical thickness of the atmosphere in zenith is 0.30, object zenith angle is 40$^\circ$, the dead time is 5 s, and the threshold signal-to-noise ratio $S/N = 8$. The parameter we call “dead time” is the sum of the time required for image read-out and telescope slewing. The $S/N$ corresponds to the total number of pixels in the detector. The variable value in our calculations is the exposure time $T$. We tried to adopt the above parameters as close as possible to their typical values. Of course, variations of initial values change estimates, but not radically.

Figures 4 and 5 present the resulting values for limiting magnitude and survey speed, respectively. The values of $m_{lim}$ are in good agreement with the estimates according to the SIGNAL package created by the team of the Isaac Newton Group of Telescopes (http://cutserver.ing.iac.es/signal/). However, we do not require the calculations to match the real
data exactly because our primary goal is to evaluate the comparative characteristics for the various types of optical systems.

As one can see, the flat-field optical systems are clearly divided into three groups: (1) the systems No. 1 and No. 2; (2) the systems Nos. 3–8; and (3) the lens objective No. 9. This division follows from the initial grouping according to the $D$ and $2w$ values (see Figure 2), because limiting magnitude depends primarily on the aperture diameter and does not depend on the field size, while the survey speed is proportional to the field area.

Since $m_{\text{lim}}$ grows and $S$ decreases with increasing of $T$, we can expect that some combination of these parameters will be independent, to a first approximation, on the exposure time. Indeed, the limiting magnitude $m_{\text{lim}} = 2.5 \log(DT^{4/2}) + A$, where $A$ incorporates all other parameters. Taking into account that, for a relatively short dead time, the survey speed $S \simeq (2w)^2/T$, we obtain

$$Q \equiv m_{\text{lim}} + 1.25 \log S \simeq \text{const.} \quad (5)$$

In our case, for $T$ in the range from 10 to 90 s, the rms variation of $Q$ for any particular design is only 0.04. The individual values of $Q$ vary from 19.5 for the 200 mm refractor to 20.7 for the 1 m system with a prime-focus corrector. Obviously, the $Q$ value depends not only on the telescope + detector system, but also on the observational conditions, so, the greater $Q$ we reach, the more effective an observational system we have. An adequate criterion of effectiveness of a telescope alone is discussed below in Section 5.

When planning a program of observations, unequal priorities are usually assigned to the stellar magnitude and survey speed. Therefore, the main problem is choosing an appropriate exposure time $T$ required to obtain the desired $m_{\text{lim}}$ and $S$ values. In turn, the triplet $(m_{\text{lim}}, S, T)$ adopted by an observer determines the proper choice of telescope, namely, its optical system and the set of specifications listed in Table 1.

For example, suppose that we are going to provide $S \sim 1 \text{deg}^2 \text{s}^{-1}$, as mentioned in the Introduction, with a limiting magnitude $m_{\text{lim}} \sim 20.5$. Figure 4 shows that telescopes No. 1 and No. 2 are too big for this purpose, because the exposure time required is short in comparison to the dead time. The desired limiting magnitude can be achieved by using system No. 3 or system Nos. 5–8 with a more appropriate exposure time of 25–50 s. However, further consideration of Figure 5 excludes system No. 5 (a modified Richter–Slevogt), because it does not provide a large enough field. The system No. 3 (Schmidt camera) contains an aspheric corrector plate; besides, to achieve a good image quality one should use an expensive and not simple glass for additional correcting lenses. These considerations suggest that we opt for one of the all-spherical telescopes containing only simple types of glass, namely, No. 6 (Amon et al. 1971), No. 7 ($\Omega_2$), or No. 8 ($\Omega_3$). The final choice requires a detailed discussion of more subtle properties of these systems.

To show the essential conditionality of choosing a proper optical layout, we note that system No. 4 (an all-spherical Schmidt–Houghton) is clearly preferable at necessity to achieve the same survey speed of $S \sim 1 \text{deg}^2 \text{s}^{-1}$ but $m_{\text{lim}} \sim 20.0$ for an ~40 s exposure. A number of telescopes based on this design were made within the last decade (Terebizh 2011).

Naturally, only the 1 m telescopes, designs No. 1 and No. 2, allow for the detection of faint objects in the range of $21.5^m \sim 22.5^m$. The difference in the limiting magnitude between these two systems is $\sim 0.21^m$; it equally results from the lower obscuration and better image quality provided by the prime-focus corrector approach. Both of these factors are typical for the systems compared.

In our analysis, we have not included an effect of the stray light or direct exposure of the detector by the sky background, because these factors are highly dependent on both the optical scheme of the telescope and on its structure including baffling. Evidently, in this respect the telescope with a prime-focus corrector would clearly be superior to systems such as a corrected Cassegrain.

With regards the design No. 10, with a field of $45^\circ$ in diameter, the corresponding limiting magnitude and survey speed depend on the relative area of detectors placed in the focal surface. Let us suppose, for example, that a strip of sizes $45^\circ \times 4^\circ (\sim 1000 \text{ mm} \times 95 \text{ mm})$ is occupied by detectors. Then, the fraction of unvignetted rays $U \simeq 0.89$, which gives the equivalent aperture diameter $D_e \simeq 472 \text{ mm}$. Thus, the limiting magnitude for system No. 10, in this case, would be approximately equal to that for system No. 7 (see Figure 4).

Note that the strip’s area, $180 \text{ deg}^2$, corresponds to diameter $2 w_e \simeq 15^\circ 1$ of the equivalent circular field of view, so the expected survey speed is very high. In particular, with an exposure time of 20 s and the assumed dead time 5 s, we obtain $S \simeq 7 \text{deg}^2 \text{s}^{-1}$; at such a speed, the sky region with an area of $10^4 \text{deg}^2$ will be covered in less than half an hour.

5. SKY SURVEY RATE

Along with a number of standard parameters of telescopes, it is useful to have a parameter to give an idea of the efficiency of the telescope for use as a survey tool. To date, a widely used parameter is the etendue $E \equiv \pi w^2 \cdot \pi D_e^2 / 4$, a product of the observed sky area ($\text{deg}^2$) and the effective area of the telescope aperture ($\text{m}^2$). The inadequacy of this measure is clear from the fact that $E$ does not take into account the quality of images provided by a survey telescope. Meanwhile, there is no doubt that with better angular resolution, higher survey efficiency can be achieved.
An adequate measure of the survey efficiency, the sky survey rate $\Gamma$, was proposed by V.V. Biryukov and the author (Terebizh 2011). By definition, the sky survey rate is proportional to the ratio of the observed sky area $\pi w^2$ to the exposure time $T$, needed to achieve the required S/N value. It is not difficult to show that such a choice leads to the parameter

$$\Gamma \equiv \frac{\pi w^2 \cdot \pi D_e^2/4}{\Delta^2} = E/\Delta^2,$$

(6)

where $\Delta$ is the so-called delivered image quality, measured in angular units. For our purpose, it is sufficient to accept

$$\Delta = \sqrt{\beta_{aim}^2 + \beta_{tel}^2 + (p')^2}. \quad (7)$$

In essence, $\Gamma$ is the product of the number of resolution elements in the observed region of the sky and the effective area of the telescope’s aperture. For practical purposes, a convenient unit of measurement for $\Gamma$ is

Herschel $\equiv 1 \text{m}^2 \text{deg}^{-2} \text{arcsec}^{-2}$,

(8)

named after William Herschel (1738–1822). Hereafter, we use the abbreviation $H$.

Values of $\Gamma$ for the systems considered here are given in Table 1. Note the significant superiority of the telescope No. 1 when compared to system No. 2. This shows, once again, merits of the location of the lens corrector in a prime focus.

Next, one might assume that the system No. 4 is significantly more effective than system No. 5 because their apertures do not differ very much; however, the field of view of the system No. 4 is twice as larger. In reality, the two systems have the same value of $\Gamma$, which stresses the importance of good image quality for survey telescopes.

As for the lens objective No. 9, its small diameter and a comparatively lower quality of images do not provide a large value of $\Gamma$. Nevertheless, very high survey speed makes it quite suitable for observations of rapidly varying objects. For example, with an exposure time of 20 s the survey speed reaches 4.5 deg$^2$ s$^{-1}$, which allows for the registering of objects brighter than 18.6$^m$ across a 10$^9$ deg$^2$ area of sky in just 40 minutes. If necessary, the image quality of the lens can be improved by a slight aspherization of certain surfaces or by using more expensive glass.

The high efficiency of the $\Omega_{2-3}$-designs No. 7 and No. 8 deserves special attention. These all-spherical systems offer good images over a large field of view; the secondary passage of light through the input optics enables a significant reduction of both aberrations and light obscuration, so the effective aperture diameter is close to the entrance pupil diameter. Further, the use of spherical optics significantly softens the tolerances in manufacture and operation of a telescopes; ultimately this strongly affects the total cost of a sky survey system. It can be assumed that both these systems will be widely used in the future surveys.

As one might expect, the design No. 10 provides the highest survey rate. Continuing the discussion from the preceding section, we have a field area $180$ deg$^2$, an equivalent aperture diameter $D_e = 0.472$ m and a delivered image quality $\Delta = 2'41$. According to Equation (6), the resulting survey rate $\Gamma \approx 5.4 H$, which is almost three times greater than the maximum value for any other system listed in Table 1.

### 6. CONCLUSIONS

We have seen that even a single wide-field telescope with an aperture less than $\sim1$ m and a flat detector, designed and manufactured properly, ensures registration of the objects brighter than 21.5$^m$–22.5$^m$ over the entire visible hemisphere of the sky within one night. The system with a spherical focal surface provides for a much faster but not so deep a survey.

Of course, more extensive and reliable data should come from an observational program using a hierarchic set of different telescopes with optimally chosen specifications. Some such systems using previously built telescopes are at work today effectively, in particular, the Palomar Transient Factory (Lav et al. 2009) and the Catalina Real-time Transient Survey (Djorgovski et al. 2011). The ATLAS (Asteroid Terrestrial-impact Last Alert System), using newly built instruments, is close to completion (Tonry 2010).

As far as the choice of telescope optical design depends on the specific goals of any particular wide-field observations program, there are no preferred options suitable for all occasions. Besides, as is well known, the choice of an optical layout depends on the characteristics of the selected photon detector. In addition to the two mentioned factors, the observation project goals, and the properties of the detector, one also needs to include consideration of the project’s cost. Together, these three factors almost uniquely define the required optical scheme.

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### APPENDIX

EXPLANATORY NOTES TO THE OPTICAL LAYOUTS

The following is a brief explanation to the designs, which are included in Table 1 at No. 1–No. 8 and No. 10; the corresponding optical layouts are shown in Figure 1.

The numbering of surfaces in the tables corresponds to the optical path of light. The terms “aperture” and “entrance pupil” are considered to be equivalent. All distances are given in millimeters. The radius of curvature and the inter-element distances are presented with the precision required by optical design soft packages. We do not round the thicknesses of the lenses, because variations of the optical constants of each glass type at manufacturing will inevitably require a minor adjustment of the thicknesses.

The types of optical glass correspond to those in the catalogs of Schott (N…N) and Ohara (S…S). In both catalogs, the selection of glass was limited to those with the maximum melt frequency. Glasses S-BSL7 and N-BK7 are equivalent. The optical parameter data for the fused silica came from “The Infrared and Electro-Optical Systems Handbook,” Vol. III, Ch. 1.
Since the filter and the detector window have zero optical power, their position and thickness can be changed with slight correction of the inter-element distances.

As usual, optical designs can be scaled up or down. Since image quality is not far from the diffraction limit, the scaling up should be performed along with a slight optimization.

The brief designations are the following: $R_0$—the paraxial radius of curvature, $T$—the distance to the next surface, $D$—the light diameter, FS—the fused silica, Stop—the aperture stop, SP—the stop position, $L_k$—the $k$th lens, Pri—the primary mirror, Sec—the secondary mirror, Obsc—obscuration, F—filter, W—window of the detector, Ima—image on the focal surface.

REFERENCES

Amon, M., Rosin, S., & Jackson, B. 1971, ApOpt, 10, 490
Baker, J. G. 1962, U.S. Patent 3,022,708
Blake, T., Pearce, E., Gregory, J. A., et al. 2013, in AMOS Technical Conf., 57
Busch, W., Ceragioli, R. C., & Stephani, W. 2013, JAHH, 16, 107
Djorgovski, S. G., Drake, A. J., Mahabal, A. A., et al. 2011, arXiv:1102.5004v1 [astro-ph.IM]
Harmer, C. F. W., & Wynne, C. G. 1976, MNRAS, 177, 25
Hawkins, D. G., & Linfoot, E. H. 1945, MNRAS, 105, 334
Houghton, J. L. 1942, Brit. Patent 546,307
Houghton, J. L. 1944, U.S. Patent 2,350,112
Iwert, O., & Delabre, B. 2010, Prog. SPIE, 7742, 27
Iwert, O., Ouellette, D., Lesser, M., & Delabre, B. 2012, Prog. SPIE, 8453, 1
Jeas, W. C. 1981, MIEIC
Korsch, D. 1972, ApOpt, 11, 2986
Korsch, D. 1977, ApOpt, 16, 2074
Law, N. M., Kulkarni, S. R., Dekany, R. G., et al. 2009, PASP, 121, 1395
Paul, M. 1935, Rev. Opt. Theor. Instrum, 14, 169
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in C (Cambridge: Cambridge Univ. Press)
Richter, R., & Slevogt, H. 1941, Deutsche Patentanmeldung (Zeiss) Z, 26, 592
Ross, F. E. 1935, ApJ, 81, 156
Sampson, R. A. 1913, RSPTA, 213, 27
Saunders, W., Gillingham, P., Smith, G., Kent, S., & Doel, P. 2014, Prog. SPIE, 9151, 91511M
Schmidt, B. 1930, Mitteilungen Hamburger Sternwarte in Bergedorf 7 (36), 15, Reprinted: Selected Papers on Astronomical Optics, SPIE Milestone Series 73, ed. D. J. Schroeder, 165, 1993
Terebizh, V. Y. 2003, AURA-CTIO Rep. No. C10430A; Astronomy Letters, 30, 200 2004
Terebizh, V. Y. 2007, arXiv:0710.2165v1 [astro-ph]
Terebizh, V. Y. 2011, AN, 332, 714
Terebizh, V. Y. 2015, arXiv:1507.07110v1 [astro-ph.IM]
Terebizh, V. Y. 2016, AN, 337, 571
Tonry, J. 2010, arXiv:1011.1028v1 [astro-ph.IM]
Wachmann, A. A. 1955, S&T, 15, 4
Willstrop, R. V. 1984, MNRAS, 210, 597
Wilson, R. N. 1996, Reflective Telescope Optics, Vol. I (Berlin: Springer)
Wynne, C. G. 1968, ApJ, 152, 675