Metacognitive Knowledge of a Student in Planning the Solution of Limit Problems

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Abstract. This study aims to describe metacognitive knowledge of a student in choosing an approach or strategy in solving the limit problems. The type of this research is explorative with qualitative approach, that is from written works and depth interviews. These collected data will be analysed in depth how the subject used metacognitive knowledge. The results show that the student in choosing approach or strategy in solving the limit problems uses metacognitive knowledge, either declarative, procedural, or conditional ones.

1. Introduction

As a basic skill of calculus, an understanding of limit is needed in order to build an understanding of the basic concepts of calculus. This suggests that a good students’ understanding of the limit is needed so that they understand the concepts that are based on the concept of limit. Metacognition is part of the ability of self–knowledge-monitoring. Furthermore, Solso explains that metacognition has an impact on the supervision and control of information retrieval processes and inference processes that occur in the memory system. Monitoring refers to the way we evaluate what we know or what we do not know. Metacognition as an advanced thought involves active control over cognitive processes [1, 2].

Metacognition is widely used, among others related to efforts in optimizing the ability of students in solving problems [3-6], optimizing learning outcomes that can be achieved by students [5,7,8], understanding the contents of a reading topic [9, 11], or improving someone’s abilities to become a successful learner [12,13]. Therefore, the efforts that involve metacognition in various learning activities are expected to provide benefits to improve the quality of learning being implemented.

Metacognition as a thought is about thinking itself which is the interaction between three important aspects they are: 1). knowledge of self-thinking process, 2) control or self-regulation, and 3). beliefs and intuition. Knowledge of thinking processes concerns how accurate a person is in expressing their thinking processes, self-awareness or self-regulation, the accuracy of a person in maintaining and managing what they must do when it comes to solving mathematical problems, and how accurately someone in using input from his/her observations to direct activities to solve problems, whereas beliefs and intuitions concern any mathematical ideas that are prepared to solve mathematical problems and how those ideas form ways to solve math problems [3,6,14].

This interaction is very important because our knowledge of our cognitive process can help in regulating the things around us and selecting strategies to improve our cognitive abilities in the future.
The metacognitive process includes the ability to ask and answer questions about (1) What do I know about this subject, topic and subject matter? (2) Do I know what I should know? (3) Do I know where I can get some knowledge information? (4) How long will it take me to learn this? (5) What strategies and tactics can I use to learn this? (6) Do I understand what I hear, read or see? (7) How do I know if I am studying at the appropriate level? (8) How can I see if I make a mistake? (9) How should I revise my plan if it does not fit my expectations and satisfaction? [6, 14]

Metacognitive knowledge is part of someone’s knowledge that acquired through interaction with others as a cognitive being and with different tasks, goals, actions and experiences of cognition. Metacognitive knowledge consists primarily of knowledge or belief about what factors or variables play a role and interact and influence the direction and outcome of cognitive activity. This kind of knowledge is also known as awareness of cognition. There are three main factors or variables in metacognitive knowledge which are: (1) individual, (2) task, and (3) strategy. The "individual" category includes everything someone believes about their nature and others as a cognitive processor [12, 15, 16]. It relates to the kind of knowledge gained and the beliefs about human beings as cognitive beings. The "task" category relates to information available to a person during cognitive activity. The "strategy" category deals with an important issue of knowledge gained through a strategy that is likely to be effective for achieving goals in a cognitive endeavour [11].

Metacognitive knowledge is distinguished by three types of metacognition, that are: (1) person metacognition that is knowledge of the ability of memory and limitations of a person, including the limitations of self, and ability to monitor the experience of his/her memory directly, (2) task metacognition is knowledge of how difficult a memory problem related to task specificity, and (3) strategy metacognition is the knowledge of something that can be done to influence the effectiveness of memory [5, 17].

Brown classifies metacognitive knowledge based on someone’s consciousness of their metacognitive knowledge which are: declarative, procedural and conditional [16,18,19]. And then Pierce suggests that: “To increase their metacognitive abilities, students need to possess and be aware of three kinds of content knowledge: declarative, procedural, and conditional. Declarative knowledge is the factual information that one knows; it can be declared—spoken or written. Procedural knowledge is knowledge of how to do something, of how to perform the steps in a process. Conditional knowledge is knowledge about when to use a procedure, skill, or strategy and when not to use it [11, 20].

Metacognitive components in solving mathematical problems in learning are: (a) metacognitive knowledge, (b) metacognitive skills. Metacognitive knowledge of problem solving refers to declarative knowledge, procedural knowledge, and a person’s conditional knowledge on problem solving. While metacognitive skills are the controls of their own thinking processes that are differentiated in prediction skills, such as "how difficult is the task", planning skills, such as "what will I do to execute this task?" , monitoring skills, such as "What do I not know I’ve been getting to achieve goals?" and evaluation skills, such as "are all the tables I created are complete for problem solving"? [13, 21- 23].

Brown further argues that metacognitive knowledge is related to resolute or state, the ability to reflect on complex cognitive processes and discuss them with others (stateable), the ability to know certain facts about cognition that are not true (fallible), and its development is affected by age [7, 16]. This Brown’s view indicates that one’s metacognitive knowledge will develop as the age and learning experience progresses.

Considering the above opinion, it can be said that metacognitive knowledge is a knowledge that is possessed by a person about his/her cognitive ability, tasks to be performed and cognitive strategies that must be selected. This metacognitive knowledge is the knowledge of a person acquired through interaction with others, through the execution of a task, or through an attempt to achieve a goal. Metacognitive knowledge consists of declarative knowledge, procedural knowledge, and conditional knowledge.

Problem solving is one of the proper methods for learning and teaching mathematics. A person with problem-solving skills has several advantages, including developing critical thinking skills and strengthening math skills. There are four categories in teaching problem solving, namely: solving the
problem develops cognitive skills in general, problem solving encourages creativity, problem solving is part of the mathematical application process, and problem solving motivates learners to learn math [6, 22]. With the ability to solve problems obtained from mathematics lessons, students are expected to use to solve problems in everyday life, as revealed by Cooney that is, teaching learners to solve problems, allows learners to be more analytical in making decisions in his/her life [24, 25].

2. Method
This research is explorative type with qualitative approach. The subject is one-student from Mathematics Education department, Faculty of Teacher Training and Education, Tadulako University. He is the student who have followed the calculus course, we assumed that he has adequate knowledge about the concept of limit of a function in one variable.

The data collection includes written assignment and in-depth, unstructured interview for assuring credible data we utilized a time triangulation. To analyse the credible data, we followed the Miles and Huberman model with stages: 1) data reduction, 2) data display, 3) conclusion drawing [10].

3. Results and Discussion
In the following we label the subject of the research with S and the researcher with P. The attached number string indicates the sequence of the interview stage (e.g. (005)S means that the subject respond is in fifth stage in the interview sequence).

The second stage in solving the problems proposed by Polya is the stage of preparing a settlement plan, that is choosing an approach or problem-solving strategy. After understanding the problem, S is to arrange a plan (devising a plan), that is choosing an approach or problem-solving strategy [26]. At this stage S is convinced to draw up a plan of completing the task using a certain strategy to achieve his cognitive goals ((035)S, (036)S). Therefore, S predicts that he is able and sure to retrieve information that has been obtained and contained in long-term memory. Explanation given by S about it can be known from the following interview transcripts:

(035)P : Because you already understand the problem, then what will you do?
(035)S : Prepare a settlement plan, Sir.
(036)P : Are you sure you can make that plan?
(036)S : Yes, I am, Sir.
(037)P : Why are you so convinced?
(037)S : I’ve been taught about this kind of thing, and the limit material has also been studied in Calculus I, Sir.

S predicts the time to be used in developing an approach or problem-solving strategy that is about nine minutes. S can devise a problem-solving plan because he has already worked on the problem of function limit ((038)S, (039)S). In solving the problem S plans to use the definitions of left and right limits, as well as limit theorems ((040)S). The cognitive strategy was chosen to make it easier in solving the problem ((041) S). The explanation given by S about it can be known through the following interview transcripts.

(038)P : How much time do you need to create a settlement plan in this question?
(038)S : Approximately nine minutes, Sir.
(039)P : Roughly, do you able to do with that time?
(039)S : (S bows the head) Yes I am, Sir, because I’ve learned limit of a function.
(040)P : What your plan is like?/What is your plan?
(040)S : My plan is, I will solve it by using the definition and limit theorem, Sir.
(041)P : Why do you use that?
(041)S : To make it easier and normally, a question like that requires the definition and limit theorem.
Based on the result of the data presentation above, then it is concluded that S used a metacognitive knowledge at the stage of preparing problem-solving plan. To plan, S involves his metacognitive activity with planning the preparation of problem-solving strategy consciously using certain strategy to reach his cognitive goals and by considering the initial knowledge as his declarative knowledge. At this moment S can mention the symbols related to function limit, write the limit definition, and this knowledge helps to prepare a settlement plan. S predicts the time to be used in preparing a problem-solving plan by considering his initial knowledge. This is in line with some expert’s opinion that declarative knowledge is factual information or knowledge of known facts [21, 22, 27].

In order to achieve his cognitive goals, firstly S planned to determine the value of the left and right limits ((045)S), so that, from the result he will be able to determine the value of the limit ((046)S). S realized that the limit value of a function at a point does not always exist and the value of a limit of a function at a point can either exist or not. So it depends on the value of the left and right limits ((047)S, (048)S, (049)S, (050)S). Explanation given by S about it can be known from his work and the following interview transcripts:

(042)S : (handwriting S write down the theorem to be used)

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(045)P : Continue.
(045)S : Firstly, determine left and right limits.
(046)P : Continue.
(046)S : From that results I will determine the value of the limit, Sir.
(047)P : What do you mean?
(047)S : (S bows the head)

I mean, the value of the function limit at a point does not always exist, Sir.
(048)P : And then?
(048)S : The limit value of a function at a point can exist and can’t exist.
(049)P : How to make it sure?
(049)S : Pardon, Sir?
(050)P : The value of the limit that you said before.
(050)S : The value of the limit can be known after calculating the left and right limits.

Based on the results of the data presentation above, it can be concluded that the used his metacognitive knowledge at the stage of preparing a problem-solving plan. To be able to plan, S involves his metacognitive activity, that is knowing the procedure that will be used in solving the problem so that the troubleshooting plan can be achieved. Accordingly, this is in line with the notion of procedural knowledge, that is knowledge of how to do something, how to do every step in the process [8, 21, 22, 27, 28]. In this case, S shows its awareness of its procedural knowledge, where S can perform a procedure or stages by using the limit theorem, so the value of the limit can be obtained.

In evaluating the solution plan that has been settled, S is rechecking whether the plans that have been made are in accordance with the desired results and evaluating the appropriateness of the procedures that had been selected as a basis for concluding that the purpose of the planned cognition has been reached ((052) S (053) F). So that, S can state that the arranged plan can be used to solve the problem and suggest that there may be other ways, but that is all that he thought of ((054) S, (055) S and (056) S). The explanation by S about it can be known from the following interview transcripts:

(052)P: You said that you will make a plan. Is that enough?
(052)S : (S is silent)

I think that is enough, Sir.
(053)P : Do you think this plan can be used to solve the problem?
(053)S : (S looks back at the theorem that he wrote)

Yes, Sir.

(054)P : Are you sure?
(054)S : Sure, Sir.

(055)P : Do you have any other plans that can be used to solve that problem?
(055)S : No, Sir.

(056)P : Why?
(056)S : Maybe there is, but that's just what I think, Sir.

Based on the results of the data presentation above, it can be concluded that he used his metacognitive knowledge S at the stage of preparing a problem-solving plan. To be able to plan, S involves his metacognitive activity, that is by monitoring the implementation of a problem-solving plan with his consciousness planning to use the limit theorem for solving the problems by considering an easy completion. In this case, his awareness of utilizing conditional knowledge, can be seen when setting out to use the limit theorem in solving the problem. S consciously evaluates the steps of the solution plan for the problem in how to re-examine the steps that have been made, whether these have led to the achievement of goals or not, and checks the adequacy of information needed to solve the problem. So, S does not need to plan another way to develop a problem-solving plan [7, 20, 27].

4. Conclusion
Based on a description of metacognitive activity, when choosing the strategy in solving the limit problem he has shown the metacognitive activities, that is by utilizing his metacognitive knowledge, whether in the form of declarative knowledge, procedural knowledge, or conditional knowledge.

Involving metacognitive knowledge is very useful in building the subject's awareness of existing knowledge when solving problems, in the proper context. Therefore, it is suggested to the researcher having a relevant problem to examine more deeply the involvement of metacognitive knowledge in solving the function limit problems in particular and for other problems in everyday life.

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