Mathematical modeling of transition processes due to a change in gas consumption at the ends of the inclined section of the gas pipeline

I Khujaev 1, G Shodmonova 2, X Mamadaliyev 1 and X Aminov 1

1 Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, 108 Amir Temur ave., Tashkent, 100200, Uzbekistan
2 Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, 39 Kari Niyazov str., Tashkent, 100000, Uzbekistan

i_k_hujayev@mail.ru

Abstract. The paper presents a solution to the problem of the gas-dynamic state of an elementary section of a gas pipeline with variable input and output mass gas outflows, which is obtained by the method of separation of variables. The quasi-one-dimensional mathematical model of pipeline transport of real gas takes into account factors such as friction, gravity and the local component of the inertia of the gas. By introducing the mass flow rate and averaging the velocity in the quadratic law of resistance, the autonomous equations of the telegraph equation with respect to pressure and mass flow are compiled from a system of equations. Taking into account possible abrupt changes of the unknowns in time and distance, the solution is sought in the form of functional series. The demonstrativeness of this method lies in the fact that the perturbation frequencies relevant to the parabolic and hyperbolic types of equations and the intermediate variant are distinguished. A description is given of a general method for solving the problem when temporary changes in the gas mass flow rate are set at the ends of the section. The exact solutions of mass flow rate and hydrostatic pressure are obtained for the case of a spasmodic change in mass flow rate at the boundaries of the site with a constant slope from the horizon. Numerical results related to the case of instantaneous closure of the output section of a linear section are presented. The results of the work are useful for assessing the energy intensity and reliability of gas pipelines in conditions of large diameters, high working pressures, and also when the gas pipeline is laid along a relief line.

1. Introduction
Pipeline networks designed to transfer the product (water, natural gas, petroleum, petroleum products, etc.) or mechanical energy over a certain distance are designed for a particular steady operation state. The established mode of operation can be stationary, when the indicators of the network and its links remain constant in time, or periodic. In other cases, there are transitions from one operating mode to another, due to a change in the volume of injected and withdrawn gas or operating pressure.

The repeated formation and propagation of shock waves and vacuum leads to the formation of zones of fatigue stresses in the pipeline network. In connection with the presence of waves, it is advisable to solve problems with the use of functional series (for example, Fourier series) in time and distance. Moreover, the formation of aperiodic solutions, due to both the intrinsic frequencies of the
pipeline and the frequencies of external disturbances, can be expected. The eigenfrequencies of the elementary section are determined by a period that is equal to the ratio of the length of the section to the velocity of propagation of small pressure disturbances (sound velocity).

Another reason for addressing to the transition problems in pipelines in general, and in main gas pipelines in particular, is the deviation of the network’s indicators from the intended technological indicators due to the dynamic change in flow. To eliminate these deviations, it is first necessary to study their nature in a theoretical sense, where a quasi-one-dimensional approach is widely used to describe the gas-dynamic state of linear sections and methods for calculating local resistances.

Changes in gas-dynamic parameters of network sections during pipeline transportation are described by quasi-one-dimensional equations of conservation of momentum, mass, energy and the equation of state of the gas [1]. In particular, the temperature of the transported gas changes due to heat transfer from the surrounding pipe environment, the Joule-Thomson effect, phase transformations and other factors [2]. In a track laid over rough terrain, there is a significant influence of the slope of the track from the horizon and, in particular, a “post-pass” mode is formed when a certain part of the potential energy of gravity compensates for the friction forces, and the rest of it leads to a directional increase in hydrostatic pressure [3]. When passing through water bodies, the description of the energy state of the transported gas is complicated by changing the energy state of the environment, for example, glaciation of water [4, 5]. During the underground laying of the gas pipeline, the temperature of the transported gas also changes over distance. There are cases of using soil temperature for domestic purposes [6, 7].

According to the results of [8-13], full or partial blocked pipe sections, as well as gas leakage, increase the energy intensity of the transportation process, loss of the target product, formation environmental disaster and explosive and fire hazard situations. In this regard, various methods are being developed for monitoring the pipeline network [14-17] and gas leak identification [18-22].

In works [23-26] problems of modeling transition processes in gas supply systems are considered and various systems of equations and methods of their solution are presented. The options are analyzed when the pressure loss to overcome the friction force is large or small.

With regard to the power factors taken into account and the methods of solving problems, this work is close to [27-29]. Introduction of gas mass flow and linearization of I.E. Charny allows us to make separate equations for hydrostatic pressure and mass flow, which are the equations of telegraph type. The problems are solved by the Fourier method involving functional series. The advantages of these solutions are that in them the selection of frequencies corresponding to the approaches of the “long” and “short” pipelines are performed automatically.

A similar approach was used in [30], where the problem was solved for given periodic boundary conditions of mass flow and gas pressure.

In the framework of this work, we set limits for ourselves by only considering cases when the values of one or another indicator are set at the borders. At the same time, if the mass flow values are set at the boundaries, then the problem with respect to pressure has boundary conditions of the third kind. Accordingly, if pressure values are set at the boundaries, then, when solving the problem with respect to mass flow, we have another kind of boundary conditions. In this regard, when setting a fixed indicator at the border, to determine another indicator, we use the already known solution of the indicator, the boundary conditions of which are set.

The expected solution summarizes the results of a number of problems on the gas-dynamic state of the elementary section of the gas pipeline during transient processes, including those from [28-30].

2. Problem Statement
For the description of transients occurring in the area with a constant slope $\sin \alpha$ and constant diameter $D$, we use the equations
Hereinafter, the variables are pressure \( p \), density \( \rho \) and velocity \( w \), the values of which are averaged over the cross-sectional area \( f \pi D^2 / 4 \) and depend on the coordinate \( x \) and time \( t \). The acceleration of gravity \( g \), the velocity of propagation of small perturbations of pressure (sound) in the gas-pipe system \( c \), the gas super-compressibility coefficient \( Z \), the reduced gas constant \( R \) and the temperature \( T \) of the transported gas have constant or averaged values.

In three steps, we simplify the equations of system (1). In the first step in the first two equations, the density \( \rho \) is replaced by the expression \( p \) found from the third equation. In the second step, we introduce the linearization of a member of the Darcy-Weisbach law by replacing \( w^2 \approx w_c w \), where \( w_c = \text{const} \) is the characteristic speed of the process or the linearization parameter. In the third step, we introduce the mass flow.

\[
M = \rho w f .
\]

As a result of these steps, the system of equations becomes linear:

\[
\begin{align*}
\frac{\partial p}{\partial x} & = \frac{b}{f} M + \frac{a}{c^2} \rho + \frac{1}{f} \frac{\partial M}{\partial t}, \\
\frac{\partial p}{\partial t} & = \frac{c^2}{f} \frac{\partial M}{\partial x}.
\end{align*}
\]  

(2)

In the first equation, the term with the coefficient \( a = \frac{g \sin \alpha}{ZRT} c^2 \) reflects the force of gravity, the term with the coefficient \( b = \frac{\lambda w_c}{2D} \) represents the friction force, and the third term to the right of the equal sign indicates the fraction of the local component of the gas inertia force in the pressure drop. According to the physics of the problem \( b \) has a non-negative value, and \( a \) can have positive, zero or negative values.

With the exclusion of pressure \( p \) from system (2), we obtain an equation for mass flow \( M \) :

\[
\frac{\partial^2 M}{\partial t^2} + b \frac{\partial M}{\partial t} = c^2 \frac{\partial^2 M}{\partial x^2} + a \frac{\partial p}{\partial x},
\]

(3)

and with the exception of mass flow \( M \) from (2) we obtain the equation for hydrostatic pressure \( p \) :

\[
\frac{\partial^2 p}{\partial t^2} + b \frac{\partial p}{\partial t} = c^2 \frac{\partial^2 p}{\partial x^2} + a \frac{\partial p}{\partial x}.
\]

(4)

2.1. Statement of a general problem in terms of mass flow

We suppose that before the start of changes at the ends of the section, the distribution of the mass flow rate and its time derivative at the site are known:

\[
M(x,0) = \varphi(x), \quad \frac{\partial M(x,0)}{\partial t} = \psi(x) \quad \text{when} \quad 0 \leq x \leq 1, \quad t < 0.
\]  

(5)

Starting from the moment of time \( t = 0 \) at the beginning \( (x = 0) \) and end \( (x = l) \) of the section, certain laws of changing the mass flow rate of gas will be established:

\[
M(0,t) = M_0(t), \quad M(l,t) = M_l(t) \quad \text{when} \quad t \geq 0.
\]  

(6)
3. Solution Method

3.1. The solution of the problem (3), (5) - (6)
To solve the problem, we use the method of separation of variables [28-30].

In order to facilitate the process of solving equation (3), a new unknown \( u(x,t) \) is introduced according to

\[
M(x,t) = e^{\frac{-ht}{2c^2}} u(x,t).
\]

\[
u(x,t) = v(x,t) + \frac{x}{l} e^{\frac{al}{2c^2}} \left[ e^{2c^2M_l(t)} - M_0(t) \right]
\]

boundary conditions become uniform

\[
v(0,t) = v(l,t) = 0
\]

and the task is reduced to a form that allows the use of the method of separation of variables.

The reverse transition to mass consumption in (7) gives the final form of the solution to problem (3), (5) - (6) with respect to mass consumption:

\[
M(x,t) = e^{\frac{ax}{2c^2}} M_0(t) + \frac{x}{l} e^{\frac{ax}{2c^2}} \left[ e^{2c^2M_l(t)} - M_0(t) \right] + \sum_{n=1}^{\infty} \left[ C_n \sin \sqrt{D_n} t + D_n \cos \sqrt{D_n} t \right] \sin \frac{\pi nx}{l}.
\]

Here, \( L_n(t) \) is a particular solution to the following equation

\[
L_n''(t) - D_n L_n(t) = \frac{2}{\pi n} \left[ 1 - (-1)^n \right] F(t) - (-1)^n \frac{2l}{\pi n} Q(t);
\]

\[
D_n = \frac{b^2 c^2 - a^2 - c^2 \lambda_n^2}{4c^2}, \quad \lambda_n = \frac{\pi n l}{l}, \quad I_n = \int_0^l e^{2c^2 \varphi(\xi)} \sin \frac{\pi n \xi}{l} d\xi,
\]

\[
J_n = \int_0^l e^{2c^2 \varphi(\xi)} \frac{\pi n \xi}{l} d\xi, \quad \gamma_n = \begin{cases} \sqrt{D_n} & \text{when } D_n > 0 \\ 1 & \text{when } D_n = 0 \\ \sqrt{D_n} & \text{when } D_n < 0 \end{cases},
\]

\[
C_n = -M_0(0) q_n - \frac{1}{l} \left[ e^{2c^2M_l(0)} - M_0(0) \right] g_n + L_n(0) + \frac{2}{l} I_n,
\]

\[
D_n = \frac{1}{\gamma_n} \left[ -L_n'(0) + \frac{b}{2} L_n(0) + \frac{b}{2} C_n - M_0'(0) q_n - \frac{1}{l} \left[ e^{2c^2M_l'(0)} - M_0'(0) \right] g_n + J_n \right].
\]

3.2. The solution of the problem of pressure using (7)
The second equation of system (2) is written in the form of

\[
\frac{\partial p}{\partial t} = -\frac{c^2}{f} \frac{\partial M}{\partial x}
\]

and we integrate both sides of the equation in time from zero to \( t \):

...
\[ p(x,t) = p(x,0) - \frac{c^2}{f} \int_0^t \frac{\partial M(x,\xi)}{\partial x} d\xi. \]  \tag{8}

The expression of the initial pressure distribution \( p(x,0) \) is found from the first equation of the system (2). The values \( M(x,0) \) and \( \frac{\partial M(x,0)}{\partial t} \) are known for \( 0 \leq x \leq l \) and \( t > 0 \). In this regard, when \( t \to 0 \) the first equation of system (2) can be written in the form:

\[ \frac{\partial p}{\partial x} + \frac{a}{c^2} p = -\frac{b\varphi(x) + \psi(x)}{f}. \]

Multiplying both sides of the equation by \( e^{-ax} \), its left-hand side can be represented as a monomial:

\[ \frac{\partial}{\partial x} (e^{-ax} p) = -e^{-ax} \frac{b\varphi(x) + \psi(x)}{f}. \]

Now, supposing \( p(0,0) = p_{00} \) we integrate the equation from zero to \( x \):

\[ e^{-ax} p(x,0) - p_{00} = \Phi(x), \]

where

\[ \Phi(x) = -\frac{b}{f} \int_0^x e^{a\eta} \varphi(\eta) d\eta - \frac{1}{f} \int_0^x e^{a\eta} \psi(\eta) d\eta. \]

Hence we find that, according to the initial conditions (5), the initial distribution of pressure over the section can be described by the formula

\[ p(x,0) = e^{-ax} \left[ p_{00} + \Phi(x) \right]. \]

The integral is involved in (8). First we find the integrand function:

\[
\frac{\partial M(x,\xi)}{\partial x} = -\frac{a}{2c^2} e^{ax} M_0(\xi) + \\
+\frac{1}{l} e^{ax} \left( 1 - \frac{a}{2c^2} x \right) \left( e^{ax} M_1(\xi) - M_0(\xi) \right) + \\
+e^{\frac{ax}{2}} \sum_{n=1}^{\infty} \left( Y_n(\xi) + L_n(\xi) \right) \left( \frac{\pi n}{l} \cos \frac{\pi n x}{l} - \frac{a}{2c^2} \sin \frac{\pi n x}{l} \right).
\]

The integration of this expression concerns parts that depend on time, and the remaining factors will appear as coefficients of integration.

Then we select the parts that depend on time, and integrate them:

\[ \mathcal{M}_0(t) = \int_0^t M_0(\xi) d\xi, \quad \mathcal{M}_1(t) = \int_0^t M_1(\xi) d\xi, \]

\[ \mathcal{L}_n(t) = \int_0^t e^{\frac{ax}{2}} L_n(\xi) d\xi, \quad \mathcal{Y}_n(t) = \int_0^t e^{\frac{ax}{2}} Y_n(\xi) d\xi. \]

The first three integrals depend on the functions \( M_0(t) \) and \( M_1(t) \). They can be calculated analytically or numerically. The fourth integral is obviously independent of the boundary conditions, but is defined as a conditional operator depending on the value of the expression \( D_n \). The value of the integral is taken from [4].
When \( D_n > 0 \) only time dependent part has integral

\[
\bar{Y}_n^{(1)}(t) = \int_0^t e^{-\frac{b^2}{2}} \left( C_n \text{ch} \sqrt{|D_n|} \xi + D_n \text{sh} \sqrt{|D_n|} \xi \right) d\xi =
\]

\[
= \frac{1}{a^2 + e^2 \lambda^2_n} \left[ \left( -\frac{b}{2} C_n - \sqrt{|D_n|} D_n \right) e^{-\frac{b^2}{2} \sqrt{|D_n|} t - 1} + \right.
\]

\[
\left. + \left( \sqrt{|D_n|} C_n - \frac{b}{2} D_n \right) e^{-\frac{b^2}{2} \sqrt{|D_n|} t} \right],
\]

when \( D_n = 0 \) –

\[
\bar{Y}_n^{(2)}(t) = \int_0^t e^{-\frac{b^2}{2}} \left( C_n + D_n t \right) d\xi =
\]

\[
= -\frac{2}{b} \left( e^{-\frac{b^2}{2} - 1} C_n + \frac{2}{b} e^{-\frac{b^2}{2} - 1} - \frac{4}{b^2} \left( e^{-\frac{b^2}{2} - 1} \right) \right) D_n,
\]

when \( D_n < 0 \) –

\[
\bar{Y}_n^{(3)}(t) = \int_0^t e^{-\frac{b^2}{2}} \left( C_n \cos \sqrt{|D_n|} \xi + D_n \sin \sqrt{|D_n|} \xi \right) d\xi =
\]

\[
= \frac{1}{a^2 + e^2 \lambda^2_n} \left[ \left( -\frac{b}{2} C_n - \sqrt{|D_n|} D_n \right) e^{-\frac{b^2}{2} \sqrt{|D_n|} t - 1} + \right.
\]

\[
\left. + \left( \sqrt{|D_n|} C_n - \frac{b}{2} D_n \right) e^{-\frac{b^2}{2} \sqrt{|D_n|} t} \right].
\]

Substituting the obtained integrals into equation (8) leads to the solution of the problem with respect to pressure:

\[
p(x,t) = e^{-\frac{ax}{c^2}} \left[ p_{00} + \Phi(x) \right] -
\]

\[
- \frac{e^2}{f} e^{-\frac{ax}{c^2}} \left\{ \frac{a}{2c^2} \tilde{M}_0(t) + \frac{1}{l} \left( 1 - \frac{a}{2c^2} x \right) \left[ \frac{al}{e^{2c^2}} \tilde{M}_1(t) - \tilde{M}_0(t) \right] + \right.
\]

\[
\left. + \sum_{n=1}^\infty \left[ \bar{Y}_n(t) + \bar{I}_n(t) \right] \left( \frac{\pi n}{l} \cos \frac{\pi nx}{l} - \frac{a}{2c^2} \frac{\pi nx}{l} \right) \right\}.
\]

4. Results

A program has been compiled for the case when boundary conditions are specified with respect to mass flow in the form of constant values. The initial data are taken as \( l = 1 \text{km}, \ w_0 = 13.24 \text{ m/s}, \ w_s = 6.62 \text{ m/s}, \ p_{00} = 5 \text{ MPa}, \ \sin \alpha = 0.1, \ c = 200 \text{ m/s}, \ \lambda = 0.01, \ M_0 = M_H = 400 \text{ kg/s}, \ M_K = 0 \text{ kg/s}. \)
Figure 1. Graphs of mass flow at the exit from the site for various moments of dimensionless time. 

\[ l = 1 \text{ km}, \quad w_0 = 13.24 \text{ m/s}, \quad w_* = 6.62 \text{ m/s}, \quad p_{00} = 5 \text{ MPa}, \quad \sin \alpha = 0.1, \quad c = 200 \text{ m/s}, \quad \lambda = 0.01, \]

\[ M_0 = M_H = 400 \text{ kg/s}, \quad M_K = 0 \text{ kg/s}. \]

Figure 2. The change in the average flow rate in the site at different points in time. Data see fig. one.

Graphs of mass flow, hydrostatic pressure, and gas velocity for dimensionless times 0. 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1.0, 1.5, 2.0 and 2.5 correspond to the case are presented in figures 1 and 2.

Comparisons of the calculation results using the above formulas with the results of [30] showed their identity. This proves the reliability of the proposed solution for the first problem. Works on the implementation of the solution method for the cases when the boundary conditions are specified in the form of Fourier series are carried out.

5. Conclusion

Taking into account the relevance of studying the characteristics of transition processes in gas pipelines, we formulated and analytically solved the problems for the cases of given boundary conditions with respect to mass flow of gas. With the account of the possibility of the presence or formation of gaps in the boundary conditions, solutions were sought in the form of functional series. For cases of transition from a stationary or periodic mode of operation of the considered elementary segment to another stationary or periodic mode, the integrals given in the text are calculated in quadratures.
In the solutions obtained (7) and (8), the mechanisms of perturbation suppression due to the friction force are embedded. At the same time, the frequencies of disturbances, which are stored longer \( (D_n < 0) \), are highlighted, have a quasi-resonant nature \( (D_n = 0) \) or are quenched intensively \( (D_n > 0) \).

Taking into account the influence of gravity and the local component of inertia, as well as variable boundary conditions in a mathematical model of transition processes helps to ensure the adequacy of the solutions obtained for problems of pipeline gas transportation. And this corresponds to the trend of further development of the network of gas pipelines with the transition to large diameters and high working pressures.

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