On the Magnetic-Field Dependence of the Longitudinal Ultrasonic Attenuation in a Type-II Superconductor

Kazue Kudo

Department of Physics, Faculty of Science and Graduate School of Humanities and Sciences, Ochanomizu University, 2-1-1 Otsuka, Bunkyo-ku, Tokyo 112-8610, Japan

Abstract

We propose a simple method by which we can explain the magnetic-field dependence of the longitudinal ultrasonic attenuation in a type-II superconductor. It gives a curve which is in good agreement with experimental data, in particular, near the lower critical field $H_{c1}$. We compare it with conventional methods, which is not in good agreement with the experimental data near $H_{c1}$ but near the upper critical field $H_{c2}$.

Key words: vortex, ultrasonic attenuation, magnetic-field

PACS: 74.25.Ld, 74.60.Ec

1 Introduction

The temperature dependence of the ultrasonic attenuation in a superconductor is well described by the BCS theory [1]. According to the BCS theory, the longitudinal ultrasonic attenuation coefficient in the superconducting state relative to that in the normal state is given by

$$\frac{\alpha_s}{\alpha_n} = \frac{2}{\exp\left[\frac{\Delta(T)}{k_BT}\right] + 1}. \quad (1)$$

The energy gap parameter $\Delta(T)$ depends on the temperature.

Email address: kudo@degway.phys.ocha.ac.jp (Kazue Kudo).
However, the magnetic-field dependence has not been explained very well. Ikushima et al. performed an experiment about the magnetic-field dependence of the longitudinal ultrasonic attenuation in the mixed state of pure niobium [2]. And they explained it theoretically. The spatial average of the energy gap, \( \langle \Delta(H, T) \rangle \), was introduced and assumed to be proportional to the root mean square of the order parameter. Then \( \langle \Delta(H, T) \rangle \) must be proportional to the square root of the magnetization, \( M \), near the upper critical field, \( H_{c2} \). They assumed this relation to be valid throughout the mixed state. And they calculated the attenuation coefficient in the mixed state, \( \alpha_s \), with the measured magnetization. The results of calculation give the curve which is not in good agreement with the experimental data except near \( H_{c2} \).

In this paper, we propose an original simple method to explain the magnetic-field dependence of the longitudinal ultrasonic attenuation, and compare it with conventional methods. First, we study the original method, which is to estimate the ratio of the space occupied with vortices. It gives a curve which is in good agreement with the experimental data, in particular, near \( H_{c1} \). We also study the conventional method, which is to calculate the order parameter by solving the Ginzburg-Landau (GL) equations. It gives a curve which is not in good agreement with experimental data near \( H_{c1} \) but near \( H_{c2} \). In each method, we use eq. (1), in which \( \langle \Delta(H, T) \rangle \) is used instead of \( \langle \Delta(T) \rangle \).

2 To estimate the ratio of the space occupied with vortices

We take \( \eta \) to be the ratio of the space occupied with vortices. For example, if each vortex is assumed to be a cylinder whose radius is the coherence length \( \xi \), \( \eta \) can be written as

\[
\eta = n\pi\xi^2. \tag{2}
\]

Here, \( n \) is the number of vortices per unit area on the plane perpendicular to the magnetic field. We should notice that not only \( n \) but also \( \xi \) varies with the field [3,4]. Let us assume that the energy gap vanishes in each vortex and equals \( \Delta(T) \) in the other area. Then we can introduce the following equation:

\[
\langle \Delta(H, T) \rangle = (1 - \eta)\Delta(T). \tag{3}
\]

Next, let us introduce a simple and phenomenological assumption: the increase in the magnetic-field energy equals that of the free energy when the field increases from \( H_{c1} \) to \( H \):

\[
\frac{1}{8\pi}(H^2 - H_{c1}^2) = \Omega_S(H) - \Omega_S(H_{c1}), \tag{4}
\]
Fig. 1. The longitudinal ultrasonic attenuation coefficient in the mixed state relative to that in the normal state. It is plotted against the applied magnetic field $H$ at 4.2K. The lower end of the solid curve corresponds to $H_{c1}$, and the upper end to $H_{c2}$. The open squares, experimental data, and the dashed curve, theoretical curve, are from ref.[2]. The solid curve is obtained from eqs. (1) and (7).

and the increase of the free energy should be related to $\eta$. Thus we have

$$\eta = A(H^2 - H_{c1}^2). \tag{5}$$

Here, $A$ is a constant. Taking into account that $\langle \Delta(H,T) \rangle$ vanishes at $H = H_{c2}$, from eqs. (3) and (5) we have

$$A = \frac{1}{H_{c2}^2 - H_{c1}^2}. \tag{6}$$

Combining eqs. (3), (5) and (6), we obtain

$$\langle \Delta(H,T) \rangle = \left(1 - \frac{H^2 - H_{c1}^2}{H_{c2}^2 - H_{c1}^2}\right) \Delta(T). \tag{7}$$

The solid curve in Fig. 1 is obtained from eqs. (1) and (7). It is in good agreement with the experimental data, particularly near $H_{c1}$. 
3 To calculate the order parameter by solving the GL equations

Now, we calculate $\langle |\Psi|^2 \rangle$. Here, $\Psi$ is the order parameter: $\Psi = \Psi_o f e^{i\gamma}$; $\Psi_o$ is the order parameter in the absence of field; $f$ and $\gamma$ are the normalized magnitude and phase of the order parameter. Hao et al. [5] solved the Ginzburg-Landau equations with a trial function,

$$f = \frac{r}{(r^2 + \xi_v^2)^{1/2}} f_\infty.$$  \hspace{1cm} (8)

Here, $\xi_v$ and $f_\infty$ are two variational parameters; $r$ is distance from a vortex axis. And, especially for $\kappa \simeq 5$, the parameters were approximated by the following formulas:

$$f_\infty^2 = 1 - \left(\frac{B}{\kappa}\right)^4,$$ \hspace{1cm} (9)

$$\left(\frac{\xi_v}{\xi_{vo}}\right)^2 = 1 + \left(\frac{B}{\kappa}\right)^4,$$ \hspace{1cm} (10)

where $B = 2\pi/(\kappa A_{cell})$ is the averaged magnetic flux density; $A_{cell}$ is the unit cell area of the vortex lattice; $\xi_{vo}$ is the value of $\xi_v$ at $B = 0$. Using eqs. (8), and (9) and (10), the order parameter should be given by

$$\frac{\langle |\Psi|^2 \rangle}{\Psi_o^2} = \frac{1}{A_{cell}} \iint f^2 dS = \frac{1}{A_{cell}} \iint \frac{f_\infty^2 r^2}{r^2 + \xi_v^2} dS.$$ \hspace{1cm} (11)

Here, the integral is taken over one lattice cell, which is approximated by a circle centered at a vortex axis and has the same cell area. By calculating eq. (11), we find

$$\frac{\langle |\Psi|^2 \rangle}{\Psi_o^2} = f_\infty^2 \left[ 1 - \frac{\xi_v^2}{R^2} \log \left( \frac{R^2}{\xi_v^2} + 1 \right) \right],$$ \hspace{1cm} (12)

where $R$ is the radius of the circle,

$$R^2 = \frac{A_{cell}}{\pi} = \frac{2}{B\kappa}.$$ \hspace{1cm} (13)

And, in order to write $\langle |\Psi|^2 \rangle$ as a function of $H$, we assume that $B$ is related to $H$ by the following equation [5],

$$H = \frac{\kappa f_\infty^2 \xi_v^2}{2} \left[ \frac{1 - f_\infty^2}{2} \ln \left( \frac{2}{B\kappa \xi_v^2} + 1 \right) - \frac{1 - f_\infty^2}{2 + B\kappa \xi_v^2} + \frac{f_\infty^2}{(2 + B\kappa \xi_v^2)^2} \right]$$.
Fig. 2. The longitudinal ultrasonic attenuation coefficient in the mixed state relative to that in the normal state. The solid curve is obtained from eqs. (1) and (9)-(15). The dashed curve is obtained from eqs. (1) and (7). The open squares are experimental data from ref. [2].

\[ + \frac{f_\infty^2(2 + 3B\kappa\xi_v^2)}{2\kappa(2 + B\kappa\xi_v^2)} + B + \frac{f_\infty}{2\kappa\xi_vK_1(f_\infty\xi_v)} \]
\[ \times \left[ K_0 \left( \xi_v(f_\infty^2 + 2B\kappa)^{1/2} \right) - \frac{B\kappa\xi_vK_1 \left( \xi_v(f_\infty^2 + 2B\kappa)^{1/2} \right)}{(f_\infty^2 + 2B\kappa)^{1/2}} \right] \]

Then, if we use the relation,

\[ \frac{\langle \Delta(H, T) \rangle}{\Delta(T)} = \sqrt{\frac{\langle |\Psi|^2 \rangle}{\Psi_0^2}}. \quad (15) \]

The solid curve in Fig. 2 is obtained from eqs. (1) and (9)-(15). It is not in good agreement with the experimental data near \( H_{c1} \) but near \( H_{c2} \).

4 Discussion

In this section, we discuss the validity for each of the approximations which are adopted to calculate the three theoretical curves.

The theoretical curve calculated by Ikushima \textit{et al.} [2] was proposed to explain the experimental data particularly near \( H_{c2} \). They used the approximation
that $\langle \Delta(H, T) \rangle$ should be proportional to the square root of the magnetization $M$. It is valid near $H_{c2}$. However, it is not necessarily valid near $H_{c1}$.

In Section 2, we introduced simple and phenomenological assumptions and approximations. Equations (4) and (5) should be effective near $H_{c1}$, where the number of the vortices is small. However, they may be less effective near $H_{c2}$. Because the relation between the free energy and $\eta$, the ratio of the space occupied with vortices, may be more complex when the number of the vortices is large.

The approximation leading to eqs. (9) and (10) in Section 3 is not necessarily effective in explaining $\langle \Delta(H, T) \rangle$, on which the ultrasonic attenuation depends. It was originally adopted to explain the magnetization curves in ref. [5]. The magnetization should be strongly dependent on the penetration depth $\lambda$. Also, the order parameter $\Psi$ in Section 3 should be dependent on $\lambda$. On the other hand, $\langle \Delta(H, T) \rangle$ should be dependent on the coherence length $\xi$. In eq. (15), we assumed the behavior of $\langle \Delta(H, T) \rangle$ to be equivalent to that of $\langle |\Psi| \rangle$. In other words, we assumed the magnetic-field dependence of $\lambda$ to be equivalent to that of $\xi$. However, the magnetic-field dependence of $\lambda$ is generally different from that of $\xi$ [3]. Therefore the magnetic-field dependence of magnetization should be generally different from that of $\langle \Delta(H, T) \rangle$. Equations (9) and (10) may be effective in explaining $\langle \Delta(H, T) \rangle$ not through the whole mixed-state region but only near $H_{c2}$.

5 Conclusion

The original method discussed in this paper gives the theoretical curve which is in good agreement with experimental data particularly near $H_{c1}$. On the other hand, the theoretical curves given by the two conventional methods are not in good agreement with experimental data near $H_{c1}$ but $H_{c2}$. Though the original method is simple and phenomenological, it may give some information on the conformation of the vortices in type-II superconductors particularly near $H_{c1}$.

Acknowledgement.

The author wishes to thank Dr. T. Deguchi of Ochanomizu University for his advice.
References

[1] J. Bardeen, L. N. Cooper and J. R. Schrieffer: Phys. Rev. 108 (1956) 1175.

[2] A. Ikushima, T. Suzuki, N. Tanaka and S. Nakajima: J. Phys. Soc. Jpn. 19 (1964) 2235.

[3] J. E. Sonier et al.: Phys. Rev. Lett. 79 (1997) 1742.

[4] A. A. Golubov and U. Hartmann: Phys. Rev. Lett. 72 (1994) 3602.

[5] Z. Hao et al.: Phys. Rev. B 43 (1991) 2844.