Analysis of effect of magneto-hydrodynamic, couple-stress and roughness on conical bearing

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Abstract: The current paper explains the effect of couple stress fluid between the conical bearing when magnetic field and surface roughness is applied jointly. Modified Reynolds equation is acquired for various roughness patterns. Various parameters are tested on both these roughness pattern and the results are presented in the current paper. It is observed that non dimensional pressure, load carrying capacity and squeeze film time increases for magnetic field and couple stress increases. We also noticed that all these non-dimensional parameters increases for higher values of Azimuthal pattern, whereas in Radial roughness pattern these parameters decreases.

1. INTRODUCTION
Magnetohydrodynamics (MHD) is the study that explains the phenomenon of magnetic properties on electrically conducting fluids. The effect of MHD was widely studied in industrial processes ranging from metallurgy to production of pure crystals [1]. The MHD effect has been practiced in MHD generators, a large number of aerodynamics devices and small machines. Stokes[2] explains the theory of fluids that permits the polar effect namely the existence of non-symmetric stress tensor and couple stress known as the theory of fluids with couple stress. Ramanian [3] studied the effect of squeeze film between finite plates lubricated with couple stress fluid and observed that squeeze film time increases if couple stress is used as lubricant. Burjurke and Naduvinamani [4] proved that the result of couple stress on cavitations is quit important when compared to viscous fluid with same viscosity. Wu[5], Verma [6], Bhat and Deheri[7] researched on the squeeze film behavior between rotating porous annular disk and prove that magnetic field gives a better results on the performance of squeeze film. These studies were assumed for only smooth bearing surfaces. But we know that surfaces with bearing after a prolong use generates roughness. Various studies[8-10] in the field of surface roughness has helped to deploy the stochastic model. Syeda Tasneem Fathima et al., [11] studied the impact of magnetic field and surface roughness on the circular plates lubricated with couple stress fluid. It was concluded that the effect of surface roughness are more prominent in case of couple stress fluids compared to Newtonian fluids in the existence of magnetic field. Naduvinamani [12] studied the response of surface roughness on the behavior of curved pivoted slider to find a general type of surface roughness modeled by a random variable with non-zero mean, variance and skewness. The impact of magnetic field and couple stress fluid on rough conical bearing is not been studied. In this paper we
analyze on the mixed effect of couple stress and surface roughness on MHD squeeze film characteristic between conical bearing.

2. MATHEMATICAL FORMULATION:
We assume a squeeze film in between the plates of conical bearing that approaches each other with normal velocity, with rough surface (at $z=0$) as given in figure:1. A magnetic field $B_0$ is enforced at the bottom of the bearing. Also we consider couple stress fluid as a lubricant in the film region. With the assumption of MHD and couple stress theory, the basic equations are:

![Figure 1: Conical plates lubricated with Couple Stress fluid bearing rough surface](image)

\[
\begin{align*}
\frac{\partial^2 u}{\partial z^2} - \frac{\eta}{\mu} \frac{\partial^4 u}{\partial z^4} + \frac{M_0^2}{h_0^2} u &= \frac{1}{\mu} \frac{\partial p}{\partial r} \\
\frac{\partial p}{\partial z} &= 0 \\
\frac{1}{r} \frac{\partial}{\partial r} (ru) + \frac{\partial w}{\partial z} &= 0
\end{align*}
\]

With Boundary conditions on velocity components:

i) At the upper surface $z = h \sin \theta$
\[ u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0 \quad (4) \]

\[ w = \sin \theta \frac{dh}{dt} \quad (5) \]

ii) At the lower surface, \( z = 0 \)

\[ u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0 \quad (6) \]

\[ w = 0 \quad (7) \]

Solving equation (1) using the boundary condition (4) and (6), we get

\[ u = [(g_1 - g_2) - 1] \frac{h_0^2}{\mu M_0^2} \frac{\partial \rho}{\partial r} \quad (8) \]

\( l = (\eta/\mu)^{1/2} \) is the couple stress parameter, \( M_0 = B_0 h_0 (\eta/\mu)^{1/2} \) is the Hartmann number.

\[ g_1 = g_{11}, \quad g_2 = g_{12}, \text{ for } 4 M_0 j^2 / h_0^2 < 1 \quad (9) \]

\[ g_1 = g_{21}, g_2 = g_{22}, 4 M_0 j^2 / h_0^2 = 1 \quad (10) \]

\[ g_1 = g_{31}, g_2 = g_{32}, 4 M_0 j^2 / h_0^2 > 1 \quad (11) \]

Appendix A has the related information on equation (9),(10) and (11)

We get a non-linear modified Reynolds equation on integrating the continuity equation (3) with the associated boundary conditions

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r f(h, \theta, l, M_0) \frac{\partial \rho}{\partial r} \right) = -\mu V \sin \theta \quad (12) \]

Where

\[ f(h, \theta, l, M_0) = \begin{cases} \frac{h_0^3}{M_0^2} \frac{2l}{A^2 - B^2} \left( \frac{B^2}{A} \tanh \frac{A h_0}{2} - A^2 \tanh \frac{B h_0}{2} \right) + h \sin \theta & \text{for } 4 M_0 j^2 / h_0^2 < 1, \\ \frac{h_0^3}{M_0^2} \left( \frac{h_0 \sin \theta}{2} \sec h \right) - 3 \sqrt{2l} \tanh \left( \frac{h \sin \theta}{2 \sqrt{2l}} \right) + h \sin \theta & \text{for } 4 M_0 j^2 / h_0^2 = 1, \\ (A \cos \theta - B_0 \sin \theta) \left( \frac{(A \cos \theta - B_0 \sin \theta) \cosh (A \sin \theta) + \cosh (B \sin \theta)}{A \cos \theta + B_0 \sin \theta} \right) + h \sin \theta & \text{for } 4 M_0 j^2 / h_0^2 > 1, \end{cases} \]
For mathematical modeling of surface roughness, known as the Stochastic film thickness $H$ which consist of two parts represented as

$$H = h + hs(r, \Theta, \xi)$$  \hspace{1cm} (13)

The probability density function given by $f(h_s)$ where $h_s$ is the stochastic film thickness. We obtain the stochastic modified Reynolds equation on taking the average of (12) with respect to $f(h_s)$

$$\frac{1}{r} \frac{\partial}{\partial r} \left( E(f(h, \Theta, l, M_0)) r \frac{\partial E(p)}{\partial r} \right) = -\mu V \sin \Theta$$  \hspace{1cm} (14)

Where

$$E(*) = \int_{-\infty}^{\infty} (*) f(h_s) dh_s$$  \hspace{1cm} (15)

Christensen [9] assumes the roughness distribution function as

$$f(h_s) = \begin{cases} \frac{35}{32c^3} (c^2 - hs^2)^3, & -c < hs < c \\ 0, & \text{elsewhere} \end{cases}$$

Where $\sigma = c / 3$ is the standard deviation

Here we assume two roughness patterns i.e, the one-dimensional Radial roughness pattern and the Azimuthal roughness pattern.

**Radial Roughness Pattern.**

The structure for the one-dimensional radial roughness pattern are straight ridges and valley passing through $z=0$, $r=0$. For the present study the film thickness takes the form

$$H = h(t) + h_r(\Theta, \xi)$$  \hspace{1cm} (16)

The stochastic modified Reynolds equation becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial E(p)}{\partial r} E(f(h, \Theta, l, M_0)) \right) = -\mu V \sin \Theta$$  \hspace{1cm} (17)

**Azimuthal Roughness Pattern.**

The structure for the one-dimensional azimuthal roughness pattern on the bearing surface are ridges circular and valley on the flat plates that are concentric on $z=0$, $r=0$. In this case the film thickness takes the form

$$H = h(t) + h_r(r, \xi)$$  \hspace{1cm} (18)

The stochastic modified Reynolds equation takes the form
\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial E(P)}{\partial r} \right] = -\mu V \sin \theta
\]  

(19)

Equation (17) and (19) together can be expressed as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial E(p)}{\partial r} \right] G(h, \theta, l, M_0, c) = -\mu V \sin \theta
\]  

(20)

Where

\[
G(h, \theta, l, M_0, c) = \begin{cases} 
E(f(h, \theta, l, M_0)), & \text{for radial} \\
E \left( \frac{1}{f(h, \theta, l, M_0)} \right)^{-1}, & \text{for azimuthal} 
\end{cases}
\]  

(21)

Where, \(E(f(h, \theta, l, M_0)) = \frac{35}{32c^2} \int_{-c}^{c} f(h, \theta, l, M_0)(c^2 - h^2)^3 dh\)

\[
E \left( \frac{1}{f(h, \theta, l, M_0)} \right) = \frac{35}{32c^2} \int_{-c}^{c} (c^2 - h^2)^3 dh
\]

Introducing the dimensionless parameters as below:

\[
\begin{align*}
\rho^* &= \frac{r}{a \cos \theta}, l^* = \frac{2l}{h_0}, \mu^* = \frac{h}{h_0}, p^* = \frac{E(P)h_0^3}{\mu a^2 (dh dt) \cos \theta}, C = \frac{c}{h_0}
\end{align*}
\]

Equation (20) reduces to

\[
\frac{1}{r^*} \frac{\partial}{\partial r^*} \left[ r^* \frac{\partial p^*}{\partial r^*} \right] G(h^*, \theta^*, l^*, M_0, C) = -1
\]  

(22)

\[
\frac{\partial p^*}{\partial r^*} = 0 \text{ at } r^* = 0 \text{ and } p^* = 0 \text{ at } r^* = 1
\]  

(23)

On Integrating (22) using the condition (23) we obtain film pressure \(p^*\) as
\[ p^* = \frac{1 - r^2}{4G(h^*, \theta, l^*, M_0, c)} \]  

(24)

On integrating the film pressure we get the load carrying capacity

\[ E(W) = \int_0^{a \cos \theta} 2\pi r E(p)dr \]

We obtain the dimensionless load carrying capacity as.

\[ W^* = \frac{E(W)h_0^3}{\mu a^3 \pi (-dh/dt) \cos \theta} = \frac{1}{8G(h^*, \theta, l^*, M_0, c)} \]  

(25)

Integrating equation (25) using the initial condition for the film height

\[ h^*(t^* = 0) = 1 \]

We get the dimensionless time for squeeze-film cone plates as.

\[ T^* = \frac{E(W)h_0^2}{\mu a^2 \pi \cos \theta} = \int_{h_1}^{h} \left\{ \frac{1}{8G(h^*, \theta, l^*, M_0, c)} \right\} dh^* \]  

(26)

3. RESULTS AND DISCUSSION:

In order to study the overall effect of applied magnetic field, couple stress, half-cone angle and surface roughness on conical bearing various analysis were carried out on non-dimensional parameters like roughness parameter \( C \), Hartmann number \( M_0 \), couple stress parameter \( l^* \) and half cone angle \( \theta \) using Stokes couple stress theory and Christensen's stochastic theory for rough surface. For the detailed numerical discussion on these parameters we have chosen the following ranges: \( C = 0, 0.3, 0.6 \), \( M_0 = 0, 2, 4 \), \( l^* = 0, 0.2, 0.4 \) and \( \theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \).

3.1 Pressure.
Figure 2: Variation of $P^*$ with $r^*$ for different values of $C$ with $M_0=4$, $l^*=0.3$, $\theta_0=0.3$

Figure 3: Variation of $P^*$ with $r^*$ for different values of $M_0$ with $l^*=0.3$, $C=0.3$
Figure 4: Variation of $P^*$ with $r^*$ for different values of $l^*$ with $M_0 = 4, C = 0.3$

Figure 5: Variation of $P^*$ with $r^*$ for different values of $\theta$ with $M_0 = 4, C = 0.3$

Figure 2 indicates the variation of non-dimensionless $P^*$ with respect to $r^*$ for various values of $C$ with $M_0 = 4, l^* = 0.3, \theta = \pi/3$. Here we notice that with increasing roughness parameter $C$ pressure $P^*$ increases (decreases) in case of Azimuthal (Radial) roughness pattern. Figure 3 displays change in $P^*$ against the values of $r^*$ for various values of Hartmann number $M_0$ with $l^* = 0.3, C = 0.3$ and half-cone angle $\theta = \pi/3$. It is noticed that $P^*$ increases for both radial and azimuthal roughness pattern with increasing values of Hartmann number $M_0$. The impact of dimensionless parameter $l^*$ with $C = 0.3, M_0 = 4, \theta = \pi/3$ on the variation
of non-dimensional pressure \( P^* \) is shown in Figure 4. We see that with increasing value of \( r^* \) and couple stress parameter \( l^* \), pressure \( P^* \) increases for both the roughness pattern when compared with Newtonian case \( (l^*=0) \). The change in non-dimensional pressure \( P^* \) with \( r^* \) for distinct values of half-cone angle \( \theta \) \( C=0.3, M_0=4, l^*=0.3 \) is shown in figure 5. Here we note that pressure reduces as half-cone angle \( \theta \) increases for radial and azimuthal roughness pattern.

3.2 Load Carrying Capacity.
The difference in the load carrying capacity \( W^* \) against \( h^* \) for various values of \( C \) with \( M_0=4, l^*=0.3, \theta=\pi/3 \) is shown in Figure 6. We note that load carrying capacity \( W^* \) for increasing values of \( C \) for Azimuthal roughness pattern when compared to Radial roughness pattern. Figure 7 shows that for higher values of Hartmann number \( M_0 \), the non-dimensional load carrying capacity \( W^* \) increases in case of radial and azimuthal roughness pattern with \( l^*=0.3, C=0.3 \) and half-cone angle \( \theta=\pi/3 \). The effect of dimensionless parameter \( l^* \) with \( C=0.3, M_0=4, \theta=\pi/3 \) on \( W^* \) is depicted in Figure 8. We notice that for both the roughness pattern the couple stress parameter increases the load carrying capacity \( W^* \) compared to Newtonian case \( (l^*=0) \). Figure 9 shows the change in \( W^* \) against \( h^* \) for various values of half-cone angle \( \theta \) having \( C=0.3, M_0=4, l^*=0.3 \). We note that load carrying capacity reduces as half-cone angle \( \theta \) increases for radial and azimuthal roughness pattern.

![Figure 6: Variation of \( W^* \) with \( h^* \) for different values of \( C \) with \( M_0=4, l^*=0.3, \theta=\pi/3 \)](image-url)
Figure 7: Variation of $W^*$ with $r^*$ for different values of $M_0$ with $l^*=0.3, C=0.3$ and $\theta = \pi/3$.

Figure 8: Variation of $W^*$ with $r^*$ for different values of $l^*$ with $M=4, C=0.3$ and $\theta = \pi/3$. 

Radial Azimuthal
Roughness Roughness
$M_0=0$ $M_0=0$
$M_0=2$ $M_0=2$
$M_0=4$ $M_0=4$

Radial Azimuthal
Roughness Roughness
$l^*=0$ $l^*=0$
$l^*=0.2$ $l^*=0.2$
$l^*=0.4$ $l^*=0.4$
Figure 9: Variation of $W^*$ with $r^*$ for different of $\theta$ with $M_0=4, C=0.3$ and $l^*=0.3$

3.3 Squeeze film time.

Figure 10 indicates the change in the values of $T^*$ with respect to $h_1^*$ for various values of C having $M_0=4$, $l^*=0.3$, $\theta=\pi/3$. Here we notice that as the roughness parameter C increases the non-dimensional squeeze film time $T^*$ increases for Azimuthal roughness where as it decreases for Radial pattern.
Figure 11: Variation of $T^*$ with $h_1^*$ for different values of $M_0$ with $l^*=0.3, C=0.2, \theta = \pi/3$

Figure 12: Variation of $T^*$ with $h_1^*$ for different values of $l^*$ with $M_0=4, C=0.3$ and $\theta = \pi/3$
The change in squeeze film time $T^*$ against $h_{1^*}$ for various values of Hartmann number having $l^*=0.2$ and $\theta=\pi/3$ is described in Figure 11. We notice that as compared to non-magnetic field ($M_0=0$), the impact of magnetic field was to increase the squeeze film time. Figure 12 exhibits the change in squeeze film time $T^*$ against $h_{1^*}$ for various values of $l^*$ having $M_0=1$, $C=0.001$, $\theta=\pi/3$. Here we note that the couple stress increases the squeeze film time as compared with the Newtonian case ($l^*=0$). Figure 13 depicts the change in the non-dimensional squeeze film time $T^*$ against $h_{1^*}$ for various values of $l^*$ having $M_0=1$, $C=0.001$, $l^*=0.3$. We found that squeeze film time reduces with increasing $h_{1^*}$ and $l^*$ values.

Table 1: Presenting the numerical comparison of the squeeze film time $W^*$ and $T^*$ between Hanumagowda et al.[13] and the present analysis with $\theta=(\pi/3)$.

| Hanumagowda et. Al.[13] Analysis | Present Analysis |
|----------------------------------|-----------------|
| $M_0$ | C=0 | C=0.2 |
| $l^*=0.2$ | $l^*=0.4$ | $l^*=0.2$ | $l^*=0.4$ |
| Radial | Azimuthal 1 | Radial | Azimuthal 1 |
| $M_0=0$ | $M_0=1$ |
| 14.752 | 26.650 | 14.752 | 26.650 | 14.041 | 24.757 | 2 | 30.9264 |
| 5 | 5 | 5 | 5 | 3 | 16.4715 | 2 | 30.9264 |
| $W^*$ | $M_0=2$ | $M_0=1$ |
| 15.929 | 27.832 | 15.929 | 27.832 | 15.297 | 26.065 | 1 | 31.1231 |
| 2 | 2 | 2 | 2 | 1 | 17.6637 | 9 | 31.1231 |
| 19.456 | 31.375 | 19.456 | 31.375 | 19.001 | 29.929 | 4 | 35.712 |
| 4 | 3 | 9 | 3 | 9 | 21.2357 | 8 | 35.712 |
4. CONCLUSION:
The current study explains the squeeze film lubrication on conical bearing with a joined effect of surface roughness, couple stress fluid and applied magnetic field. The results and discussions stated above draws the following conclusion:

1. The non-dimensional parameters like pressure \( P^* \), load carrying capacity \( W^* \) and squeeze film time \( T^* \) rises for large values of roughness parameter \( C \) in Azimuthal pattern, whereas in Radial roughness pattern these parameters decreases

2. As Hartmann number increases pressure \( P^* \), load carrying capacity \( W^* \) and squeeze film time \( T^* \) increases compared to non-magnetic case \( (M_0=0) \) for both the roughness pattern

3. As couple stress parameter increases pressure \( P^* \), load carrying capacity \( W^* \) and squeeze film time \( T^* \) increases compared to Newtonian case \( (l=0) \) in case of both the roughness pattern

4. In both the roughness pattern as the half-cone angle increases the pressure, load carrying capacity and squeeze film time decrease.

Nomenclature:

\( B_0 \): Applied magnetic field

\( C \): Dimensionless roughness parameter \((c/h_0)\)

\( h \): Film thickness

\( h_0 \): Initial film thickness

\( h^* \): Non-dimensional film thickness \((h^* = \frac{h}{h_0})\)

\( h_t \): Stochastic film thickness

\( l \): Couple stress parameter \((\frac{h}{\mu})^{1/2}\)

\( l^* \): Dimensionless couple stress parameter \(2l/h_0\)

\( M_0 \): Hartmann number \(B_0h_0(\frac{\sigma}{\mu})^{1/2}\)

\( E(P) \): Pressure in the film region

\( p^* \): Non-dimensional pressure \(p^* = \frac{E(P)h_0^3}{\mu a^2(dh/dt)Co sec \theta}\)
\( W^* \): Non-dimensional carrying capacity 
\[
W^* = \frac{E(W)h_0^3}{\mu a^4 \pi (-dh/dt) \sec^2 \theta}
\]

\( T^* \): Non-dimensional response time 
\[
T^* = \frac{E(W)h_0^2}{\mu a^4 \pi \cos \sec^2 \theta}
\]

\( \mu \): Viscosity coefficient

\( \eta \): Material constant characterizing couple stress

\( \sigma \): Electrical conductivity

\( \xi \): Random Variable

**Appendix A**

\[
g_{11} = \frac{A^2}{(A^2 - B^2)} \frac{\cosh(B(2z - h \sin \theta)/2l)}{\cosh(Bh \sin \theta/2l)}, \quad g_{12} = \frac{B^2}{(A^2 - B^2)} \frac{\cosh(A(2z - h \sin \theta)/2l)}{\cosh(Ah \sin \theta/2l)}, \quad A = \left[ 1 + (1 - 4M_0^2 z^2 / h^2)^{1/2} \right]^{1/2}
\]

\[
B = \left[ \frac{1 - (1 - 4M_0^2 z^2 / h^2)^{1/2}}{2} \right]^{1/2}, \quad g_{21} = \frac{2 \cosh((z - h \sin \theta)/\sqrt{2l}) + 2 \cosh(z/\sqrt{2l})}{2(\cosh(h \sin \theta/\sqrt{2l}) + 1)}
\]

\[
g_{22} = \frac{(z/\sqrt{2l}) \sinh((z - h \sin \theta)/\sqrt{2l}) + (z - h \sin \theta)/\sqrt{2l} \sinh(z/\sqrt{2l})}{2(\cosh(h \sin \theta/\sqrt{2l}) + 1)}
\]

\[
g_{31} = \frac{\cosh(B_2 z \cosh A_2 (z - h \sin \theta) + \cosh A_2 z \cos B_2 (z - h \sin \theta))}{\cosh(A_2 h \sin \theta) + \cos(B_2 h \sin \theta)}
\]

\[
g_{32} = \frac{\cot \theta(\sin B_2 z \sin B_2 (z - h \sin \theta) + \sin B_2 z \sin A_2 (z - h \sin \theta))}{\cosh(A_2 h \sin \theta) + \cos(B_2 h \sin \theta)}
\]

\[
A_2 = \sqrt{M_0/l h_2 \cos(\theta/2)}, \quad B_2 = \sqrt{M_0/l h_2 \sin(\theta/2)}, \quad \theta = \tan^{-1}(\sqrt{4l^2 M_0^2 / h^2} - 1)
\]

**REFERENCES :**

[1] Moreau R 1990 Magnetohydrodynamics *Kluwer Academic Publishers* Amsterdam

[2] Stokes V K 1966 Couple Stress in Fluids *Phys.Fluids* 19709-1715

[3] Ramaniah G 1979 Squeeze films between Finite Plates lubricated by Fluids with Couple Stress, *Wear* 54315-320

[4] Bujurke N M and Naduvinamani N B 1990 The lubrication of lightly cylinders in combined rolling, sliding and normal motion with couple stress fluid *Int. J. mech.Sci* 32969-979

[5] Wu. H1971 The Squeeze film between rotating porous annular disk *Wear* 18461-470

[6] Verma PDS 1986 Magnetic fluid-based squeeze films *International Journal of Engineering Science* 24395-401

[7] Bhat M V and Deheri G M 1993 Magnetic fluid based squeeze film between porous circular disk *Journal of Indian Academy of Mathematics* 15145-147

[8] Christen H 1969-70 Stochastic models for hydrodynamic lubrication of rough surfaces *Proceeding of the Institution of Mechanical Engineers Part* 184(55)101-103,
[9] Hsu CH, Lai C, LU RF and Lin JR 2009 Combined effect of surface roughness and rotating inertia on squeeze film characteristic of parallel circular disks Journal of Marine Science and Technology1760-66

[10] Syeda Tasneem Fathima ,Naduvinamani J B, Santhosh Kumar J and Hanumagowda B N 2015 Effect of surface roughness on the squeeze film characteristics of circular plates in the presence of conducting couplestress fluid and transverse magnetic field Advances in Tribology 20151-7

[11] Naduvinamani N B and Biradar K 2006 Surface roughness effect on curved pivoted slider bearing with couple stress fluid Lubrication Science 18293-307

[12] Hanumagowda B N ,Swapna Nair and Vishu Kumar M 2017 Effect of MHD and Couple Stress on Conical Bearing International Journal of Pure and Applied Mathematical 113(6)316-324.