Software of GAMMA-3 system for synthesis and mathematical modelling of UAV control systems

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Abstract. Control system design for complex technical plants includes execution of computationally complicated procedure sets. Many of those demand repeated iterative execution. Iterations are caused by uncertainty of a control task, incompleteness of the information on control plant and conditions of its functioning. Another important reason is related to insufficiently perfect methods of the automatic control theory, because of its ongoing active development. Therefore, the task of developing the computer-aided control system design is as relevant as before. The publication is devoted to the statement of an approach to expansion of GAMMA-3 multipurpose system opportunities. An important feature of GAMMA-3 system includes opportunities for solving the tasks via both traditionally procedural and non-procedural statements. At the first stage, the proposed approach provides the expansion of classes of procedural task statements as the most formalized process. At the following stage, formalization and generalization of saved procedural knowledge in new expanded model of the subject domain takes place. The domain in question allows for non-procedural task statement. Solving non-procedural declared tasks requires using intellectual components of GAMMA-3 system. In this paper, we illustrate the first stage of the proposed approach on the example of creating GAMMA-3 software package for solving the problems on the synthesis of a control system by modeling UAV movement on defined trajectory. One of the main tasks conducted by UAV is surveying remote areas, where data acquisition by habitual ways is complicated or there is a danger to lives and health of the people.

1. Introduction
The relevance of this task proves to be true by numerous publications dedicated to the issues of research and design of UAV control systems [1-8]. It is customary to distinguish vertical, orthophotos and oblique photographs obtained by aerial photography. Vertical photographs are taken straight down [40]. They are mainly used in photogrammetric and image interpretation. Pictures that will be used in photogrammetric are traditionally taken with special large format cameras with calibrated and documented geometric properties. Vertical photographs are often used to create orthophotos, alternatively known as orthophotomaps, photographs which have been geometrically "corrected" so as to be usable as a map. For making orthophotos, the camera is directed vertically downwards, under a right angle to the Earth surface. Resulting photos depict flat surface (an orthogonal projection), reminding an image on geographical maps. This aerial photography type allows defining an interposition of various objects on a plane without taking into account their heights. Basically, such kind of aerial photography is used for creating photographic plans. The similar picture can be obtained by using satellite and traditional aerial photography. Photographs taken at an angle are called oblique
photographs. If they are taken from a low angle relative to the earth's surface, they are called low oblique and photographs taken from a high angle are called high or steep oblique \[39\]. Such kind of aerial photography is impossible in case of satellites and traditional big aircrafts.

Oblique aerial photography yields 3D images (an axonometric projection) of roofs and lateral surfaces (walls). Thus, it is possible to perceive the form of the objects rather than their interposition alone. Besides, oblique aerial photography allows defining heights of the objects relative to each other. Aerial photography happens on the areas and on lines. At air photography on the areas except for longitudinal overlapping pictures still it is necessary to observe and cross-section overlapping. Initial parameters of photography by means of UAV include require picture resolution, aerial camera resolution, sighting angle of the camera objective, degree of single shots’ overlapping. From these data, we can calculate the flight heights and speed of the UAV, along with the frequency of a shutter operation per unit time.

Using UAV as an aerial filming platform has vast prospects at photographing small objects in the area, and at photographing linear objects. Increased requirements to aerial photography quality lead to stricter accuracy demand in terms of UAV following desired trajectory. As a result, much better control systems for UAV are required.

2. Task statement and control law synthesis

Complexity of UAV control tasks has caused the necessity of applying automation devices to problem solving. Despite the presence of various foreign and domestic software packages, creating problem solving opportunities in control system design tasks, efforts on creating new and improving existing packages are on-going. Main objectives of the developers are expansion of task classes to be resolved and increase in automation level.

The most perspective are automation intellectual systems such as the multifunctional GAMMA-3 system \[9\]. Its distinctive feature is the opportunity of solving declarative task statements rather than traditional procedural task statement alone \[10\]. For declaratively tasks statement we observe absence of actions sequence to the task solving. Thus the system of task solution should possess intellectual capabilities providing the opportunities for creating the plans (programs) for solving unknown tasks with a priori unknown task solving plan. It is obvious, that these opportunities are limited by the knowledge available in the system, delivered by the model of formalized task set (MFTS) of the automatic control theory \[10\]. However, MFTS development is a daunting task, solving which requires involving the experts from various scientific schools. That is why GAMMA-3 system implements an approach, presuming two stages for task class expansion of the problems to be solved \[11\]. The first stage includes developing the means for providing automation of new task solving in procedural setting.

The software of the design operations necessary for the solving of a new task is developing. The sequence of their execution is set by the procedure performed in specialized language with using habitual forms of mathematical expressions. The second stage assumes generalization and formalization of new methods in MFTS concept via the probable inclusion into existing knowledge model of new kinds and types of mathematical models, characteristics, properties and attributes of the formalized descriptions, belonging to control system components, new operations with them, etc. As such, the process is so far insufficiently formalized. Hence, building new formalized model of knowledge appears rather complex and time-consuming. Nevertheless, the results of the first-stage implementation of proposed approach allow giving an opportunity to solve new tasks in procedural setting to users-designers of control systems.

The goal of this publication is the illustration of tasks classes expansion of multifunctional GAMMA-3 system. For this purpose, we use an example of developing the software package ensuring an opportunity of control law synthesis tasks, along with observer creation and mathematical modelling of automatic control systems by UAV movement on the given trajectory. The problem of UAV trajectory movements can be divided into predictive control task and stabilization task. Accuracy of tracking given trajectory is a necessary condition of obtaining trustworthy information. Process of
UAV predictive control realization demands the presence of exact mathematical model of the flying device used for control law synthesis and choice of control action formation algorithms. The task of stabilization is considered relatively to the given predictive movement. Stabilization is process of liquidation of angular deviations of the connected system of coordinates of the flying device appearing in flight from basic system of coordinates. Such approach allows the flying device returning to required position broken by internal or external disturbance. Because tasks of control and observation have dual nature, we will separately solve a problem of control and a problem of observation.

For fulfilling this objective, it is necessary to execute the following:
- Description of a UAV movement mathematical model;
- Development of the control law providing necessary quality of regulation;
- Modelling of an automatic control system.

2.1. Mathematical model of the UAV

As an example, the UAV model in the form of a mass point in the normal system of coordinates is considered:

\[
\begin{align*}
\ddot{x} &= u_x + f_x, \\
\ddot{y} &= u_y + f_y, \\
\ddot{z} &= u_z - mg,
\end{align*}
\]

where \(x, y, z\) is UAV coordinates, \(u_x, u_y, u_z\) is controls on corresponding axes, \(f_x, f_y, f_z\) is perturbing influences on corresponding axes, \(m\) is UAV mass, \(g\) is free fall acceleration.

Let us make the following designations:

\[\begin{align*}
x_1 &= x, x_2 = \dot{x}, x_3 = y, x_4 = \dot{y}, x_5 = z, x_6 = \dot{z}.
\end{align*}\]

The model of a control plant in the state space form of Cauchy equation:

\[
\begin{align*}
\dot{x} &= A_o x + B_o u + M_o f, \\
y &= C_o x,
\end{align*}
\]

where \(x = [x_1, x_2, x_3, x_4, x_5, x_6]^T\), \(x \in \mathbb{R}^{n_o}\) is \(n_o\)-dimensional vector of states; \(y \in \mathbb{R}^r\) is \(r\)-dimensional vector of output variables; \(u \in \mathbb{R}^m\) is \(m\)-dimensional vector of controls; \(f \in \mathbb{R}^\mu\) is \(\mu\)-dimensional vector of external perturbations; \(A_o, B_o, M_o, C_o\) is numerical matrices of corresponding sizes

\[
A_o = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
B_o = \begin{bmatrix}
0 & 0 & 0 \\
\frac{1}{m} & 0 & 0 \\
0 & 0 & 0 \\
0 & \frac{1}{m} & 0 \\
0 & 0 & 0 \\
0 & 0 & \frac{1}{m} \\
\end{bmatrix},
M_o = \begin{bmatrix}
0 & 0 & 0 \\
\frac{1}{m} & 0 & 0 \\
0 & 0 & 0 \\
0 & \frac{1}{m} & 0 \\
0 & 0 & 0 \\
0 & 0 & \frac{1}{m} \\
\end{bmatrix},
C_o = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}.
\]

2.2. Synthesis of control law

Special attention is paid to the development of automatic control system of unmanned aerial vehicle. In particular, it belongs to software and algorithm provisioning. With its help, it will be possible to provide fault tolerance, accuracy and required quality of control processes. Modern UAV automatic control system represents control information complex capable of processing quickly input measurement information, providing automatic flight according to downloaded program, and many other things. Plant is affected at the same time by the control and perturbing influences (\(f\) is perturbing influence, \(u\) is control influence).
Because only the vector of measurements $y$ is available to measurement, for recovery of a state vector $x$ it is necessary to construct the observer. The structure of control system is presented at Figure 1. Due to duality of control and observation tasks, we carry out synthesis of the control law and the observer separately.

![Figure 1](image_url)

Figure 1. An automatic control system with static controller.

Let us compile a system (4) of equations describing this automatic control system.

$$
\begin{align*}
\dot{x}_o &= A_o x_o + B_o u + M_o f, \\
y &= C_o x_o + D_o u + G_o f, \\
\dot{\hat{x}} &= A_o \hat{x} + B_o u + L(y - \hat{y}), \\
\hat{y} &= C_o \hat{x}, \\
\dot{x}_g &= A_g x_g + B_g g, \\
u &= K(x_g - \hat{x}).
\end{align*}
$$

(4)

The first two equations designate the plant, while the following two describe the observer. Further on, consequent desirable behaviour of the plant is described, and the last equation denotes the control law. We conduct the synthesis of the control law by LQR-method (linearly - quadratic optimization) for optimization criteria in the form:

$$
J = \int_0^\infty (x'Qx + u'Ru)dt
$$

(5)

We define numerical values of functional matrices’ elements according to required accuracy and quality of a control process: steady state errors on each channel should not exceed 0.3 m.

$$
Q = \text{diag}[1000001 \ 50000001 \ 1000001], \ R = \text{diag}[1 \ 1 \ 1].
$$

(6)

We solve the problem using the software package of the multifunction GAMMA-3 system synthesis [9] supporting a procedural paradigm of tasks definition.

The matrix of the control law appears as:

$$
K = \begin{bmatrix}
316.229 & 795.273 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
2236068 & 21147428 & 0 & 0 & 0 & 1000 \\
4427.189 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

(7)

2.3. Creation of the observer

Because not all components of a state vector are available to direct measurement, creation of the observer is necessary. Let us use in this case a Kalman filter as the observer.

For a plant model in a look:

$$
\begin{align*}
\dot{x} &= Ax + Bu + w, \\
y &= Cx + v,
\end{align*}
$$

(8)

the equation of Kalman filter has an appearance:
\[ \dot{x} = A\hat{x} + Bu + L(C\hat{x} - y) , \]  
\[ (9) \]

where \( \hat{x} \) is vector of estimates of state variables of initial system, \( A, B, C \) is matrices of plant, \( y \) is vector of output signal, \( L \) is matrix of transfer coefficients of the observer, \( w \in R^n \) is random external disturbances, \( v \in R^r \) is random noises of a measuring device.

The optimization criterion has an appearance:

\[ J = \int_{0}^{\infty} (x'Q_n x + y'R_n y) dt . \]  
\[ (10) \]

where:

\[ Q_n = \begin{bmatrix} 1000000 & 0 & 0 \\ 0 & 1000000 & 0 \\ 0 & 0 & 1000000 \end{bmatrix} , R_n = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \]  
\[ (11) \]

Also the observer parameters are determined by tools of the GAMMA-3 system for solution of observation tasks:

\[ L = \begin{bmatrix} 1.414 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1.414 & 0 \\ 0 & 0.999 & 0 \\ 0 & 0 & 0.451 \\ 0 & 0 & 0.102 \end{bmatrix} . \]  
\[ (12) \]

3. **UAV movement modelling with synthesized control system on the set trajectory**

For modelling of the UAV movement under control of the synthesized control system on the set trajectory, we will construct a mathematical model of the closed system in the state space:

\[ \dot{x} = Ax + Bu , \]
\[ y = Cx + Du , \]  
\[ (13) \]

where \( x \in R^n \) is \( n \)-dimensional vector of states; \( y \in R^r \) is \( r \)-dimensional vector of output variables \( y \); \( u \in R^m \) is \( m \)-dimensional vector of control.

Let us enter designations:

\[ \bar{x} = [x_o \ \hat{x} \ x_g]^T , \ u = [f \ g]^T . \]  
\[ (14) \]

Then:

\[ A = \begin{bmatrix} A_o & -B_o K \\ LC_o & (A_o - LC_o) - (B_o + LD_o)K \end{bmatrix} , \quad B = \begin{bmatrix} M_o \\ 0 \end{bmatrix} , \quad C = \begin{bmatrix} C_o \\ 0 \end{bmatrix} , \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 \end{bmatrix} . \]

Now we investigate functioning of the synthesized control system, setting by means of vector \( g = [g_x \ g_y \ g_z] \) - desirable behavior of an plant at external perturbation \( f = [f_x \ f_y \ f_z] \) on the corresponding axes \( x, y, z \).

Let mass of the UAV \( m = 1000 \) kg.

Let us consider several modes of flight:

1) take-off on a straight line, flight and smooth landing on a straight line (Figures 2-4);
Figure 2. The plant $y[1]$ is X-flight trajectory on $x$-axis; $y[4]$ is gx-wished a trajectory on $x$-axis.

Figure 3. The plant $y[2]$ is Y-flight trajectory on $y$-axis; $y[5]$ is gy-wished a trajectory on $y$-axis.

Figure 4. The plant $y[3]$ is Z-flight trajectory on $z$-axis; $y[6]$ is gz-wished a trajectory on $z$-axis.

2) smooth take-off on a straight line, flight on a straight line, flight of the area on circles and return to a starting point (Figures 5-7);
Figure 5. Control system research in the mode 2: the plant $y[1]$ is X-flight trajectory on an axis $x$; $y[4]$ is $gx$-wished a trajectory on an axis $x$.

Figure 6. Control system research in the mode 2: the plant $y[2]$ is Y-flight trajectory on $y$-axis; $y[2]$ is $gy$-wished a trajectory on $y$-axis.

Figure 7. Control system research in the mode 2: the plant $y[3]$ is Z-flight trajectory on $z$-axis; $y[6]$ is $gz$-wished a trajectory on $z$-axis.
3) take-off on spirals, flight on a straight line, flight of the area on circles and return to a required point (Figures 8-10).

**Figure 8.** Control system research in the mode 3: the plant $y[1]$ is X-flight trajectory on an $x$ axis; $y[4]$ is gx-wished a trajectory on $x$ axis.

**Figure 9.** Control system research in the mode 3: the plant $y[2]$ is Y-flight trajectory on $y$ axis; $y[5]$ is gy-wished a trajectory on $y$ axis.

**Figure 10.** Control system research in the mode 3: the plant $y[3]$ is Z-flight trajectory on $z$ axis; $y[6]$ is gz-wished a trajectory on $z$ axis.
The made computing experiments allowed defining the accuracy figures and qualities of traffic control of the UAV on the set trajectory provided in table 1.

| Criterion                                                                 | Value |
|---------------------------------------------------------------------------|-------|
| Error over control at the intermittent setting influence on channels     | X: 20%, Y: 15%, Z: 30% |
| Regulation error at the intermittent setting influence on channels        | X: 0.2, Y: 0.15, Z: 0.3 |
| Regulation time at the intermittent setting influence                     | 15c.  |
| Error over control at linearly - the accruing setting influence on channels | X: 1.25%, Y: 0%, Z: 5% |
| Regulation error at linearly - the accruing setting influence             | X: 1, Y: 0, Z: 8 |
| Regulation time at linearly - the accruing setting influence              | 2c.   |

### 4. Conclusion
The conducted researches confirm operability of the selected scheme of an automatic control system.

Using of duality of a task of synthesis of the control law and a task of creation of the observer allowed successfully a problem of creation of control systems. The control law in combination with the observer is not too difficult. Therefore, its implementation is possible with use of rather inexpensive computer system. Nevertheless, as showed results of modelling, accuracy figures and qualities of control process are different at different stages of work. At the same time at a stage of rectilinear flight (operating duty) requirements to following accuracy on the set trajectory are fulfilled. At stages of take-off and decrease the deviation from a desirable trajectory exceeds the set requirements. However these stages are auxiliary. On them the area research is not conducted. Thus, in this work the operability of implementation as software tools of the multifunction GAMMA-3 system for procedures of control law synthesis, creation of the Kalman-observer and mathematical modelling of UAV movement in the different modes of flight with control systems and observer is shown.

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