Black hole formation in the head-on collision of ultrarelativistic charges

Hirotaka Yoshino\textsuperscript{(1,2)} and Robert B. Mann\textsuperscript{(3)}

\textsuperscript{(1)} Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan
\textsuperscript{(2)} Graduate School of Science and Engineering, Waseda University, Shinjuku-ku, Tokyo 169-8555, Japan and
\textsuperscript{(3)} Department of Physics, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1, Canada

(Dated: May 26, 2006)

Abstract

We study black hole formation in the head-on collision of ultrarelativistic charges. The metric of charged particles is obtained by boosting the Reissner-Nordström spacetime to the speed of light. Using the slice at the instant of collision, we study formation of the apparent horizon (AH) and derive a condition indicating that a critical value of the electric charge is necessary for formation to take place. Evaluating this condition for characteristic values at the LHC, we find that the presence of charge decreases the black hole production rate in accelerators. We comment on possible limitations of our approach.

PACS numbers: 04.50.+h, 04.20.Jb, 04.20.Cv, 11.10.Kk
I. INTRODUCTION

Black holes are expected to form in the collisions of elementary particles with energies above the Planck scale. It has been pointed out that the Planck energy could be $O(\text{TeV})$ if our space is a 3-brane situated in a large extra dimensional space and gauge particles and interactions are confined on the brane [1]. If such TeV gravity scenarios are realized, we would be able to directly observe black hole phenomena in planned accelerators such as the Large Hadron Collider (LHC) at CERN. The general scenario is expected to be as follows [2, 3, 4]. First the horizon forms (the black hole production phase), after which the black hole is expected to go to a stationary Kerr black hole by radiating gravitational waves (the balding phase). Then it will evaporate via Hawking radiation (the evaporation phase). In the context of this three-phase scenario, the main problems of interest are the black hole production rate (see [3, 4] and also [5, 6, 8, 9]), the determination of the mass and the angular momentum of the Kerr black hole (see [9, 10, 11] for related issues), and the prediction of Hawking radiation [12]. The interactions between the produced black holes and the brane are discussed in [13]. See also [14] for reviews.

In this paper, we consider an issue related to the black hole production rate. The production rate at the LHC was first calculated in [3, 4]. In proton collisions at the LHC, their constituent partons will collide and form a black hole. The cross section in the parton collision was assumed to be $\sigma_{\text{BH}} \simeq \pi [r_h(2p)]^2$, where $r_h(2p)$ is the gravitational radius corresponding to the system energy $2p$. Integrating this by multiplying the parton distribution functions, the total cross section is derived upon summing all possible parton pairs. Black hole production rate is about 1Hz under this assumption.

A quantitative calculation of (the lower bounds on) $\sigma_{\text{BH}}$ in the framework of general relativity was first done in [5] in the four-dimensional case and was extended to higher-dimensional cases [6] using a system of colliding Aichelburg-Sexl particles [15], obtained by boosting the Schwarzschild black hole to the speed of light with fixed energy $p$. A schematic picture of the spacetime is shown in Fig. 1. The gravitational field of each incoming particle is infinitely Lorentz-contracted and forms a shock wave. Except at the shock waves, the spacetime is flat before the collision (i.e., regions I, II, and III). After the collision, the two shocks nonlinearly interact with each other and the spacetime within the future lightcone of the collision (i.e., region IV) becomes highly curved. No one has succeeded in deriving
the metric in region IV even numerically. However it is possible to investigate the apparent horizon (AH) on the slice $u \leq 0 = v$ and $v \leq 0 = u$ and calculate the cross section for AH formation $\sigma_{\text{AH}} \geq 0$. $\sigma_{\text{AH}}$ provides the lower bound on $\sigma_{\text{BH}}$ because AH formation is a sufficient condition for black hole formation. Recently one of us and Rychkov \cite{8} obtained a somewhat larger value of $\sigma_{\text{AH}}$ by studying the AH on the slice $u \geq 0 = v$ and $v \geq 0 = u$. The result was $\sigma_{\text{AH}} \simeq 3\pi [r_h(2p)]^2$, e.g., for $D = 10$, where $D$ is the total number of spacetime dimensions.

The Aichelburg-Sexl metric describes a high-energy particle with no charge or spin. However these quantities should affect the formation of black holes because the gravitational field of each particle is determined by its energy-momentum tensor. In this paper we consider the effects of electric charge. Although several interesting discussions about phenomena associated with charged black hole formation have appeared in the literature \cite{4, 9, 16}, there has never been a study of black hole formation resulting from the collision of ultrarelativistic charges.

It is difficult to construct a model of colliding charges that takes account of the confinement of the electromagnetic field on the brane. As a first step, we ignore this effect and use higher-dimensional Einstein-Maxwell theory in order to understand the features that do not depend on these details. Furthermore, the Maxwell field would also be higher dimensional in the neighborhood of the particle if the brane is relatively thick. We also ignore the brane tension and the structure of the extra dimensions.

Our approach is as follows. We model the ultrarelativistic charges using the metric obtained by boosting the Reissner-Nortström black hole and taking the lightlike limit. This metric was originally derived by Loustó and Sánchez \cite{17} and was recently rederived in \cite{18}. By combining two charges, we set up the head-on collision of ultrarelativistic charges
(whose structure is similar to Fig. 1) and analyze AH formation on the slice $u \geq 0 = v$ and $v \geq 0 = u$.

Our results indicate that charge has a significant effect, typically preventing black hole formation. We discuss the implications for black hole production in accelerators by choosing parameters appropriate to the LHC. Taking the boosted Reissner-Nordström black hole as a reasonable descriptor of ultrarelativistic charged objects, charge effects will significantly decrease the rate of black hole formation at accelerators. However, simple order-of-magnitude estimates also show that quantum effects of the electromagnetic field could play an important role in such situations. Whether or not they could counteract the effects we obtain remains a subject for future study.

Our paper is organized as follows. In the next section, we introduce the boosted Reissner-Nordström black hole and set up the system of two ultrarelativistic charges. In Sec. III, we derive the AH equation and the boundary conditions. The analytic solution for the AH equation is also represented. In Sec. IV, we provide the results for the condition for the AH formation in the system of two ultrarelativistic charges. Sec. V is devoted to the discussion about the implication of our results for the black hole production in accelerators. We conclude with a discussion of effects that we have neglected in this study.

II. THE SPACETIME OF ULTRARELATIVISTIC CHARGES

In this section, we study the metric of an ultrarelativistic charge that is obtained by boosting the Reissner-Nordström black hole and introduce the system of two ultrarelativistic charges.

A. Metric of an ultrarelativistic charge

We begin by reviewing the ultrarelativistic boost of the Reissner-Nordström spacetime metric in $D$ dimensions originally studied in [17]. The metric of the Reissner-Nordström spacetime [19] is given as

$$ds^2 = -g(R)dt^2 + g(R)^{-1}dR^2 + R^2d\Omega_{D-2}^2,$$

$$g(R) = 1 - \frac{2M}{R^{D-3}} + \frac{Q^2}{R^{2(D-3)}},$$

where $M$ and $Q$ are the mass and charge of the black hole, respectively.
where $Q$ and $M$ are related to charge $q$ and mass $m$ as follows:

$$Q^2 = \frac{8\pi G_D q^2}{(D-2)(D-3)},$$

$$M = \frac{8\pi G_D m}{(D-2)\Omega_{D-2}}.$$  \hfill (3)

Here, $G_D$ is the gravitational constant and $\Omega_{D-2}$ is the $(D-2)$-dimensional area of a unit sphere. The electromagnetic field strength tensor $F_{\mu\nu}$ is given as

$$F = \frac{1}{2} E_0dT \wedge dR, \quad E_0 = \frac{q}{R^{D-2}}.$$  \hfill (5)

Introducing the isotropic coordinates $(\bar{T}, \bar{Z}, \bar{r}, \bar{\phi}_1, ..., \bar{\phi}_{D-3})$, the metric becomes

$$ds^2 = -\frac{\left[\bar{R}^{2(D-3)} - (M^2 - Q^2)/4\right]^2}{\left[\bar{R}^{2(D-3)} + M\bar{R}^{D-3} + (M^2 - Q^2)/4\right]^2 d\bar{T}^2} + \left(1 + \frac{M}{\bar{R}^{D-3}} + \frac{M^2 - Q^2}{4\bar{R}^{2(D-3)}}\right)^{2/(D-3)} (d\bar{Z}^2 + d\bar{r}^2 + \bar{r}^2d\bar{\Omega}_{D-3}),$$

where $\bar{R} = \sqrt{\bar{Z}^2 + \bar{r}^2}$ and $d\bar{\Omega}_{D-3}$ is the metric of a $(D-3)$-dimensional unit sphere spanned by $\bar{\phi}_i$. We apply a boost in the $\bar{Z}$ direction

$$\bar{T} = \gamma(\bar{t} - v\bar{z}),$$

$$\bar{Z} = \gamma(-v\bar{t} + \bar{z}),$$

where $\gamma$ is the Lorentz factor $\gamma = 1/\sqrt{1 - v^2}$. We fix both the energy

$$p = m\gamma,$$

and the following quantity

$$p^2_e = q^2\gamma,$$

and take the lightlike limit $\gamma \to \infty$. This yields a finite result that is the charged version of the Aichelburg-Sexl metric [17]:

$$ds^2 = -d\bar{u}d\bar{v} + d\bar{r}^2 + \bar{r}^2d\bar{\Omega}_{D-3} + \Phi(\bar{r})\delta(\bar{u})d\bar{u}^2,$$

$$\Phi(\bar{r}) = \begin{cases} 
-8G_4p \ln \bar{r} - \frac{2a}{\bar{r}} & (D = 4), \\
\frac{16\pi G_{DP}}{(D-4)\Omega_{D-3}\bar{r}^{D-4}} - \frac{2a}{(2D-7)\bar{r}^{2D-7}} & (D \geq 5), 
\end{cases}$$

(D = 4),
where
\[ a = \frac{2\pi(4\pi G_D p_e^2)}{(D-3)} \left( \frac{2D-5}{2D-4} \right)!! \quad (D \geq 4) \] (13)
and our normalizations of \( p \) and \( p_e \) differ from those of Ref. [18]. The metric (11) reduces to the Aichelburg-Sexl metric in the limit \( p_e \to 0 \). The stress-energy tensor has the form
\[ T_{\mu\nu} = T_{\mu\nu}^{(0)} + T_{\mu\nu}^{(em)} \]
where \( T_{\mu\nu}^{(0)} \) and \( T_{\mu\nu}^{(em)} \) are proportional to \( p\delta(\bar{u})\delta^{D-2}(\bar{r}) \) and \( p_e^2\delta(\bar{u})/\bar{r}^{2D-5} \), respectively. Note that although the value of \( q^2 \) goes to zero in the infinite boost limit, the electromagnetic energy-momentum tensor \( T_{\mu\nu}^{(em)} \) has a nonzero distributional value. For convenience we adopt the quantity
\[ r_0 = \left( \frac{8\pi G_D p}{\Omega_{D-3}} \right)^{1/(D-3)} \] (14)
as the unit of the length in the following (i.e., \( r_0 = 1 \)).

We pause here to comment on the validity and limitations of the metric (11). First note that when the the Reissner-Nordström metric is boosted in a usual way, the rest mass \( m \) and charge \( q \) are fixed and boosted to a finite value of \( \gamma \). In this case, the leading order contributions of the mass and the charge to the metric are \( O(\gamma m) \) and \( O(\gamma q^2) \), respectively. Setting \( p = \gamma m \) and \( p_e^2 = \gamma q^2 \), the boosted metric approximately coincides with the metric (11)–(13), because the difference is subleading order \( O(m) = O(p/\gamma) \) and \( O(q^2) = O(p_e^2/\gamma) \). Furthermore, in the ultrarelativistic limit both of these terms diverge unless we take both \( m \) and \( q \) to vanish in this limit [17]. Since we expect that there will be energy-momentum due both to mass and to the electromagnetic field of the particle, it is reasonable to put \( p = \gamma m \) and \( p_e^2 = \gamma q^2 \). Indeed, as noted above the electromagnetic contribution to the stress-energy of the charged particle has a nonzero (distributional) contribution [17]. Consequently we regard the charged version of the Aichelberg-Sexl metric (11) as a good approximation to an ultrarelativistic massive charged body with finite \( \gamma \).

Second, there is a restriction on the reliability of the metric (11) that comes from the charge’s electrostatic energy\(^1\). Since the electrostatic energy of a point charge diverges, there is some radius \( r_c \) at which the outside electrostatic energy is equal to the rest mass of the point charge:\(^2\)
\[ r_c = \left[ \frac{\Omega_{D-3}q^2}{2(D-3)m} \right]^{1/(D-3)}. \] (15)

---

\(^1\) We thank an anonymous referee for this point.

\(^2\) For an electron in four dimensions, \( 2r_c \simeq 2.8 \text{fm} \) is called the classical electron radius [20].
Because the classical electromagnetism has the contradiction inside of \( r_c \), quantum electromagnetic (QED) effects become important there. Hence the necessary condition for the reliability of the metric (11)–(13) is \( \bar{r} \gtrsim r_c \). Since our analysis of the formation of an AH will be done around \( \bar{r} = r_0/2^{1/(D-3)} \), the necessary condition for reliability of our results is \( r_c^{D-3} \lesssim r_0^{D-3}/2 \). Using Eqs. (9), (10), (13) and (14), this condition is rewritten as

\[
\frac{a}{r_0^{2(D-3)}} \lesssim \frac{\pi \Omega_{D-3}(2D - 5)!!}{\Omega_{D-2}(2D - 4)!!}.
\] (16)

The value of the right hand side ranges from 0.58 to 0.68 for \( 4 \leq D \leq 11 \).

**B. Geodesic coordinates**

The delta function in Eq. (11) indicates that the \((\bar{u}, \bar{v}, \bar{r})\) coordinate is discontinuous at \( \bar{u} = 0 \). Seeking new coordinates that are continuous and smooth across the shock, we introduce \((u, v, r, \phi_i)\) by the coordinate transformation

\[
\bar{u} = u, 
\] (17)

\[
\bar{v} = v + F(u, r),
\] (18)

\[
\bar{r} = G(u, r),
\] (19)

\[
\bar{\phi}_i = \phi_i,
\] (20)

where \( F(u, r) = 0 \) and \( G(u, r) = r \) for \( u < 0 \). In the new coordinate system we require that \( v, r, \phi_i = \text{const.} \) is a null geodesic with \( u \) its affine parameter. By directly calculating the geodesic equation, we find that the requirement is satisfied if and only if

\[
F_{,u} = G_{,u}^2 + \Phi(G)\delta(u),
\] (21)

\[
F_{,r} = 2G_{,u}G_{,r}.
\] (22)

are satisfied. The solution for \( F \) and \( G \) is

\[
F(u, r) = \theta(u) \left[ \Phi(r) + \frac{u}{4} (\Phi'(r))^2 \right]
\] (23)

\[
G(u, r) = r + \frac{u\theta(u)}{2} \Phi'(r),
\] (24)
The trajectories of null rays $v, r, \phi_i = \text{const.}$ in the coordinate $(\bar{u}, \bar{r})$ in the $D = 4$ case. The value of $a$ is 0.75. $\bar{\phi}_i$ and $\bar{v}$ are suppressed. Because the gravitational field of the shock is repulsive around the center, the null rays exhibit a crossing singularity (the gray dashed line). For large $r$, the gravitational field is attractive and the light rays will cross at a focusing singularity (the thick gray line).

where $\theta(u)$ is the unit step function. The metric in the coordinate $(u, v, r, \phi_i)$ becomes

$$ds^2 = -dudv + G^2_{r} dr^2 + G^2 d\Omega^2_{D-3},$$

where $G$ and $G_{r}$ are explicitly given by

$$G = r + \frac{u\theta(u)}{r^{D-3}} \left(1 - \frac{a}{r^{D-3}} \right),$$

$$G_{r} = 1 + (D - 3) \frac{u\theta(u)}{r^{D-2}} \left(1 - \frac{2a}{r^{D-3}} \right).$$

In the coordinate $(u, v, r, \phi_i)$, the metric coefficients $G^2$ and $G^2_{r}$ are continuous across the shock. On the other hand, two coordinate singularities appear in the region $u > 0$. The first one is

$$u = \frac{-r^{2D-5}}{(D-3)(r^{D-3} - 2a)}$$

at which $G_{r} = 0$ and the other is

$$u = \frac{r^{2D-5}}{r^{D-3} - a}$$

at which $G = 0$. The two singularities cross each other at $r = r_c \equiv [(2D - 5)a/(D - 2)]^{1/(D-3)}$. The light ray $v, r, \phi_i = \text{const.}$ with $r < r_c$ will reach the first singularity $G_{r} = 0$ and the one with $r > r_c$ will plunge into the second singularity $G = 0$.

To understand the physical meaning of this it is useful to revert to the coordinates $(\bar{u}, \bar{v}, \bar{r}, \bar{\phi}_i)$. Figure 2 shows the trajectories of null rays $v, r, \phi_i = \text{const.}$ in the $(\bar{u}, \bar{r})$-plane.
In the neighborhood of \( r = 0 \), the gravitational field of the shock is strongly repulsive and the light rays expand. Because of this effect, the two neighboring light rays cross each other. Since the crossing point corresponds to the point \( G_r = 0 \), we call it the crossing singularity. It does not appear in the case of a neutral particle. For sufficiently large \( r \), the gravitational field is attractive and light rays with the same value of \( r \) will be focused on the axis. This is the point \( G = 0 \) and we call it the focusing singularity. In the coordinate system \((u, v, r, \phi)\), we can only consider the region prior to the two singularities.

C. The spacetime with two high-energy charges

Since we have obtained smooth coordinates for an ultrarelativistic charge, we can set up a system of two ultrarelativistic charges as follows. We assume without loss of generality that the two particles have the same energy \( p \) and different charge parameters \( p_e^{(1)} \) and \( p_e^{(2)} \). Using (13) this implies the two particles have different values of \( a \) denoted by \( a_1 \) and \( a_2 \). Because there is no interaction between two particles before the collision, we simply combine the metric of each particle in order to obtain the metric of the region outside the future light cone of the shock collision:

\[
d s^2 = \begin{cases} 
-\,dudv + dr^2 + r^2 d\Omega^2_{D-3}, & (u \leq 0, v \leq 0), \\
-\,dudv + (G_r^{(1)}(u, r))^2 \, dr^2 + (G^{(1)}(u, r))^2 \, d\Omega^2_{D-3}, & (u \geq 0, v \leq 0), \\
-\,dudv + (G_r^{(2)}(v, r))^2 \, dr^2 + (G^{(2)}(v, r))^2 \, d\Omega^2_{D-3}, & (u \leq 0, v \geq 0),
\end{cases}
\]

where \( G^{(1)} \) and \( G^{(2)} \) are the functions obtained by substituting \( a = a_1 \) and \( a_2 \) for Eq. (25), respectively. Nonlinearities in the field equations obstruct us from obtaining the metric in the region \( u > 0, v > 0 \).

III. FINDING APPARENT HORIZONS

In this section, we study the AH on the slice \( u > 0 = v \) and \( v > 0 = u \). Figure shows the schematic shape of the AH in the slice. Because the system is axi-symmetric, the location of the AH surface in each side is given by a function of \( r \). We assume that the AH is given by the union of two surfaces \( S_1 \) and \( S_2 \) where

\[
S_1 : u = h^{(1)}(r) \quad (r_{\text{min}} \leq r \leq r^{(1)}_{\text{max}}) \quad \text{on} \quad u > 0 = v,
\]

(31)
FIG. 3: The schematic shape of the AH ($S_1$ and $S_2$ shown by thick black lines) on the slice $u > 0 = v$ and $v > 0 = u$. The crossing singularities (CS) and the focusing singularities (FS) are shown by the dark and light gray lines, respectively. $S_1$ and $S_2$ cross $u = v = 0$ at $r = r_{\text{min}}$ and the focusing singularities.

\[
S_2 : v = h^{(2)}(r) \quad (r_{\text{min}} \leq r \leq r_{\text{max}}^{(2)}) \quad \text{on} \quad v > 0 = u, \tag{32}
\]

where $h^{(1)}$ and $h^{(2)}$ are monotonically increasing functions of $r$. Continuity of the metric at the AH yields the constraint

\[
h^{(1)}(r_{\text{min}}) = h^{(2)}(r_{\text{min}}) = 0 \tag{33}
\]

so that $S_1$ and $S_2$ coincide with each other at $u = v = 0$. At $r = r_{\text{max}}^{(n)}$ (where $n = 1$ or 2), we require that $h^{(n)}(r)$ cross the coordinate singularity, i.e.,

\[
G^{(1)}(h^{(1)}(r_{\text{max}}^{(1)}), r_{\text{max}}^{(1)}) = G^{(2)}(h^{(2)}(r_{\text{max}}^{(2)}), r_{\text{max}}^{(2)}) = 0. \tag{34}
\]

Because proper circumference at the focusing singularity is zero, the surface becomes a closed surface by the above requirements.

\section{AH equation}

Next we derive the equation for $h^{(1)}(r)$ and $h^{(2)}(r)$. Because the equation for $h^{(2)}(r)$ is obtained by just changing the index of the equation for $h^{(1)}$, we only have to consider $S_1$. We put $h(r) = h^{(1)}(r)$ and $G(u, r) = G^{(1)}(u, r)$. 
The AH equation is derived by calculating the expansion $\theta_+$ of the null geodesic congruence from the surface and demanding it to vanish. In order to find the tangent vector $k^\mu$ of the congruence, we consider the lightcone at each point on the surface and adopt the outermost one. It is given by

$$
k^u = \frac{(h, r(r))^2}{2 (G, h(r), r)^2}, \tag{35}
$$

$$
k^v = 2, \tag{36}
$$

$$
k^r = \frac{h, r(r)}{(G, h(r), r)^2}, \tag{37}
$$

$$
k^{\phi_i} = 0. \tag{38}
$$

By calculating the evolution of the area along the congruence, we find that the expansion $\theta_+$ is

$$
\theta_+ = \partial_r k^r + (D - 3) \frac{G, u k^u + G, r k^r}{G} + \frac{G, r u k^u + G, r r k^r}{G, r}.
$$

(39)

Substituting Eqs. (35) and (37) and imposing $\theta_+ = 0$, we find

$$
h, r + h, r \left[ (D - 3) \frac{1/2 G, u h, r + G, r}{G} - \frac{3/2 G, r u h, r + G, r r}{G, r} \right] = 0. \tag{40}
$$

This is the AH equation.

### B. Boundary conditions

The continuity of the tangent vector $k^\mu$ of the congruence should be imposed at $r = r_{\text{max}}$ and $r = r_{\text{min}}$. Otherwise, the surface would have a delta function expansion and would not satisfy the AH condition (40).

At $r = r_{\text{max}}$, we return to the coordinates ($\bar{u}, \bar{v}, \bar{r}, \bar{\phi}_i$) and impose $k^\rho = 0$. This is equivalent to

$$
h, r(r_{\text{max}}) = \frac{-2 G, r(h(r_{\text{max}}), r_{\text{max}})}{G, u(h(r_{\text{max}}), r_{\text{max}})}.
$$

(41)

From the AH Eq. (40), we see that this is equivalent to the regularity condition at the focusing singularity $G = 0$. Hence, if we find a regular solution of Eq. (40) that crosses the focusing singularity, it will automatically satisfy the boundary condition at $r = r_{\text{max}}$.

In order to find the boundary condition at $r = r_{\text{min}}$, we consider both $S_1$ and $S_2$. The tangent vectors $k_1^\mu$ and $k_2^\mu$ of the congruence of surfaces $S_1$ and $S_2$ are given by

$$
k_1^v = 2, \quad k_1^u = (h_{,r}^{(1)})^2/2, \tag{42}
$$
respectively, at \( r = r_{\text{min}} \). Because \( k_1^\mu \) and \( k_2^\mu \) point in the same direction, \( k_1^\mu k_2^\mu = k_1^\nu k_2^\nu \) holds. This is equivalent to

\[
h_{r'}(r_{\text{min}})h_{r'}(r_{\text{min}}) = 4. \tag{44}
\]

which is the boundary condition that must be imposed at \( r = r_{\text{min}} \). Note that both \( h_{r'}(r_{\text{min}}) \) and \( h_{r'}(r_{\text{min}}) \) are positive.

### C. Solutions

The AH Eq. (40) can be solved exactly, yielding a one parameter family of regular solutions given by

\[
h(r) = \frac{2r^2}{(1-a/r)^2} \left[ \ln \left( \frac{r}{r_{\text{min}}} \right) + a \left( \frac{1}{r} - \frac{1}{r_{\text{min}}} \right) \right], \tag{45}
\]

for \( D = 4 \) and

\[
h(r) = \frac{2}{(D-4)} \frac{r^{D-2}}{(1-a/r^{D-3})^2} \left[ \left( 1 - \frac{D-4}{2D-7} \frac{a}{r_{\text{min}}^{D-3}} \right) \left( \frac{r}{r_{\text{min}}} \right)^{D-4} - 1 + \frac{D-4}{2D-7} \frac{a}{r_{\text{min}}^{D-3}} \right], \tag{46}
\]

for \( D \geq 5 \). These solutions satisfy the boundary condition at \( r = r_{\text{max}} \) and \( h(r_{\text{min}}) = 0 \). The quantity \( h_{r'}(r_{\text{min}}) \) becomes

\[
h_{r'}(r_{\text{min}}) = \frac{2x^2}{x-a}, \tag{47}
\]

where

\[
x \equiv r_{\text{min}}^{D-3}. \tag{48}
\]

Then, the boundary condition (44) becomes

\[
x^4 = (x-a_1)(x-a_2). \tag{49}
\]

This equation determines the value of \( r_{\text{min}} \); indeed the AH exists if and only if there is a solution to Eq. (49). Note that \( x \) must be larger than \( a_1 \) and \( a_2 \) because \( h_{r'}(r_{\text{min}}) \) is positive.

### IV. RESULTS

In the study in the previous section, the problem of finding the AH was reduced to solving the quartic equation (49). Now we study the condition for the AH existence.
FIG. 4: The shape of the AH (the black lines) that is formed in the collision of two charges with the same $a$ for $D = 4$. The values of $a$ are 0, 0.2, 0.25. The inner boundary of the trapped region is also shown by dashed lines. The gray lines indicate the coordinate singularities. For $a > 0.25$, the trapped region disappears.

A. Collision of charges with the same $a$

As a concrete example, we first consider the situation where both charges are the same, i.e., $a_1 = a_2 = a$. In this case, $x$ is solved as

$$x = \frac{1 \pm \sqrt{1 - 4a}}{2}. \quad (50)$$

The AH exists only when $a \leq 1/4$.

In Fig. 4, we show the examples of the solutions in the $D = 4$ case for $a = 0, 1/5, 1/4$. For $a > 0$, there are two solutions that correspond to the inner and outer boundaries of the trapped region. If we increase $a$, the trapped region shrinks and the two solutions become degenerate at $a = 1/4$. There is no AH for $a > 1/4$.

B. Collision of a charged and a neutral particle

Next, we consider the case where one particle has $a_1 = a$ and the other is neutral, i.e., $a_2 = 0$. In this case, Eq. (49) becomes

$$x^3 - x + a = 0. \quad (51)$$

Solutions exist only for $a \leq 2/(3\sqrt{3})$. 

13
FIG. 5: The shape of the AH (the black line) that is formed in the collision of a charge with parameter $a$ and a neutral particle in the $D = 4$ case. $S_1$ is shown in the upper side and $S_2$ is shown in the lower side. The values of $a$ are 0, 0.3, 0.3849.

FIG. 6: The region of the AH formation in the $(a_1, a_2)$-plane.

Figure 5 shows the shape of the AH in the $D = 4$ case. Similarly to the same charge case, two solutions appear and they degenerate at $a = \frac{2}{3\sqrt{3}} \approx 0.3849$. For $a > \frac{2}{3\sqrt{3}}$, there is no AH.
C. General charge parameters

Now we consider the condition for AH formation for general $a_1$ and $a_2$. In general, Eq. (49) has four solutions. Under the condition $a_1 > 0$ and $a_2 > 0$, there is always one negative solution and one positive solution smaller than $\min[a_1, a_2]$. However, these two solutions do not correspond to an AH because $x$ must be greater than $a_1$ and $a_2$.

We therefore investigate the existence of the other two solutions. Setting

$$f(x) = x^4 - (x - a_1)(x - a_2), \quad (52)$$

we find that an AH exists if and only if the local minimum of $f(x)$ in the $x > 0$ region is less than or equal to zero. The location of the local minimum is given by the following equation:

$$\frac{df}{dx} = 4x^3 - 2x + (a_1 + a_2) = 0, \quad (53)$$

whose positive solution is

$$x = \frac{1}{3} \left[ -(a_1 + a_2) + \sqrt{(a_1 + a_2)^2 - 8/27} \right]^{-1/3} + \frac{1}{2} \left[ -(a_1 + a_2) + \sqrt{(a_1 + a_2)^2 - 8/27} \right]^{-1/3}. \quad (54)$$

Substituting this value into $f(x)$ and drawing the contour for $f(x) = 0$, we find the region for the AH formation on the $(a_1, a_2)$-plane, shown in Fig. 6. Both $a_1$ and $a_2$ must be sufficiently small for AH formation.

D. Physical interpretation

Since $a_1$ and $a_2$ are proportional to $(p^{(1)}_e)^2$ and $(p^{(2)}_e)^2$, the condition derived above does not depend on the sign of the charge of either particle. This is because the gravitational field due to each charge is generated by an electromagnetic energy-momentum tensor $T^{(em)}_{\mu\nu}$ that depends on the squared charge. As pointed out in Sec. II, the gravitational field induced by $T^{(em)}_{\mu\nu}$ of the incoming particles is repulsive, and its effect becomes dominant around the center. As the value of $a$ increases, the repulsive region becomes large, preventing formation of the AH.

This effect is reminiscent of that found in the original Reissner-Nordström black hole. If we increase the charge $|q|$, the inner and outer horizons become closer, coalescing at $|q| = \frac{m}{\Omega_{D-2}} \sqrt{\frac{8\pi (D-3)G_D}{(D-2)}}$. This example also makes clear that the gravitational field generated
by a Coulomb field is repulsive regardless of the sign of the charge and tends to obstruct black hole formation. However the analogy with the Reissner-Nordström black hole has its limitations. One might naturally expect that a black hole forms from the collision of two charges \( q_1 \) and \( q_2 \) when and only when the total charge \( |q_1 + q_2| \) of the system is less than \( \frac{(2p)}{\Omega_{D-2}} \sqrt{\frac{8\pi (D-3) G_D}{(D-2)}} \) where \( 2p \) is the total system energy. However our analysis shows that an AH does not necessarily form even in the case \( |q_1 + q_2| \ll \frac{(2p)}{\Omega_{D-2}} \sqrt{\frac{8\pi (D-3) G_D}{(D-2)}} \), because the repulsive gravitational effect due to the electric field is enhanced by a factor of \( \gamma \). The critical value of \( a \) for AH formation is where this enhanced repulsive force becomes equivalent to the self-attractive force due to the energy of the system. We will return to this point in the next section when we apply our results to the LHC phenomena.

Note that there remains a possibility that a black hole will form upon collision even if there is no AH on the slice we have studied, because AH formation is only a sufficient condition for black hole formation. While a study of the temporal evolution after collision is beyond the scope of this paper, let us briefly discuss how the condition for black hole formation is expected to be modified from relative to the condition for AH formation.

In the collision between a charged and a neutral particle, the gravitational interaction begins after the collision and there is no electromagnetic interaction between the two particles. Because the gravitational field of a charged particle remains repulsive, the condition for the black hole formation is also given by \( a \ll \frac{2}{3\sqrt{3}} \) up to a factor close to unity. In the case of a collision between particles with equal charge, electromagnetic interactions will begin after the collision. Both the gravitational and electromagnetic forces acting between the two particles are repulsive; there is no reason to expect that the electromagnetic interaction enhances black hole formation. Modifications from the condition for AH formation are again expected to be small and the condition for the black hole formation should also be \( a \lesssim 1/4 \).

In a collision between two particles with charges of opposite sign these results might change. The electromagnetic force acting between two particles after the collision is attractive in this case. The Coulomb fields of the two particles will tend to cancel each other, suppressing the repulsive force of the contributions to the gravitational field due to each charge. If this effect is significant, the electromagnetic interaction will enhance black hole formation and it is conceivable that the condition for the black hole formation might significantly differ from \( a < 1/4 \). Since we cannot know how effective the electromagnetic interaction is in enhancing black hole formation without directly computing the subsequent
temporal evolution, we leave the derivation of the correct condition for future research.

The electromagnetic radiation emitted in the collision process crucially depends on whether a black hole forms or not. If a black hole forms, it will shed hair by radiating gravitational and electromagnetic waves. An investigation of the ratio of these two kinds of radiated energy in a slightly different setup found that the electromagnetic energy is suppressed relative to the gravitational energy for very high energies \[ 21 \]. We expect that similar results hold for our system, because most of the bremsstrahlung radiation due to the collision will be hidden inside of the horizon. On the other hand, if a black hole does not form, the scattering of two particles will occur. In this case, strong bremsstrahlung radiation is expected because the electromagnetic interaction is not hidden inside of the horizon.

We now discuss the reliability of our results on the condition for AH formation. In Sec. IIA, we introduced the radius \( r = r_c \) at which the exterior electrostatic energy becomes equal to the rest mass. The necessary condition for the reliability of our model is given by Eq. \( 16 \), which is approximately \( a \lesssim 0.6 \). Hence the range of \( a \) where the AH is prohibited is \( 1/4 \leq a \lesssim 0.6 \) in the case of the collision of two particles with equal charge. However \( r < r_c \) is a sufficient condition for the importance of QED effects, but is not a necessary condition. It is possible that QED effects are important also in a neighborhood of \( r \sim r_c \).

An exact description of ultrarelativistic collisions of charged bodies thus entails inclusion of QED effects. However the discussion above indicates that we cannot be sure if QED effects suppress or enhance the repulsive effect we have obtained.

In the next section, we discuss what phenomena will result at the LHC assuming QED effects are small and thus our results are applicable.

V. DISCUSSION

We discuss here possible implications of our results in the context of the TeV gravity scenarios by evaluating the characteristic value of \( a \) in future accelerators, assuming that boosted Reissner-Nordström black holes represent the gravitational field of elementary particles with electric charge moving at high speed. We also discuss the reliability of our results by discussing other possible effects that are not included in our current analysis.

To simplify the discussion, let us consider the case of a head-on collision of two particles with equal charge and mass. In this case, the condition for the AH formation in the head-on
collision is given by $a/r_0^{2(D-3)} \leq 1/4$, where we have restored the length unit $r_0$. This is equivalent to

$$\frac{p_e^2}{G_D p^2} \leq \frac{2(D - 3)(2D - 4)!!}{\Omega_{D-3}^2(2D - 5)!!}$$

(55)

Because both $p_e^2$ and $p$ are proportional to the Lorentz factor $\gamma$, the left hand side goes to zero and charge effects are not significant at high energies. But is the energy sufficiently high at the LHC?

In order to evaluate the value of $p_e^2$, we use the original definition $p_e^2 = \gamma q^2$, where $q^2$ is the squared charge in the higher-dimensional Maxwell theory. We first establish the relationship between the higher-dimensional charge $q^2$ and the four-dimensional one $q_4^2$. For this purpose, we consider the two particles with the same charge $q$ at rest. The force acting between two particles is given by

$$F = \frac{q^2}{r^{D-2}}.$$  

(56)

If we assume that the gauge field is confined on the brane, the unique characteristic length scale is the width of the brane, which should be of the order of the Planck length ($C_{\text{brane}}/M_p$), where $M_p$ is the Planck mass and $C_{\text{brane}}$ is a dimensionless quantity of order one. Hence, for sufficiently large $r$, $F$ becomes

$$F \rightarrow \frac{q^2}{r^2} \left( \frac{M_p}{C_{\text{brane}}} \right)^{D-4} = \frac{q_4^2}{r^2},$$

(57)

and we find

$$q^2 = q_4^2 \left( \frac{C_{\text{brane}}}{M_p} \right)^{D-4}.$$  

(58)

For the characteristic value of $q_4^2$, we adopt

$$q_4^2 = C_q^2 \alpha,$$

(59)

where $\alpha$ is the fine structure constant and $C_q$ is the charge in units of the elementary charge $e$. In the quark case, $C_q$ is $1/3$ or $2/3$. We also rewrite the gravitational constant $G_D$ in terms of the Planck mass $M_p$. Notwithstanding several definitions of the Planck mass (summarized in [22]), we adopt the definition

$$G_D^{-1} = \frac{4\pi}{(2\pi)^{D-4}M_p^{D-2}}.$$  

(60)

Then the condition is rewritten as

$$C_q^2 \alpha \left( \frac{M_p}{m} \right) \left( \frac{M_p}{p} \right) \lesssim \frac{(D - 3)(2D - 4)!!}{2\pi \Omega_{D-3}^2(2D - 5)!!} \left( \frac{2\pi}{C_{\text{brane}}} \right)^{D-4},$$

(61)
TABLE I: The values of the right hand side of Eq. (61) for two cases $C_{\text{brane}} = 1$ and $2\pi$.

| $D$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|-----|---|---|---|---|---|---|----|----|
| $C_{\text{brane}} = 1$ | 0.01 | 0.04 | 0.2 | 1 | 6 | 40 | 300 | 3000 |
| $C_{\text{brane}} = 2\pi$ | 0.01 | 0.006 | 0.004 | 0.004 | 0.004 | 0.004 | 0.005 | 0.008 |

TABLE II: The values of the right hand side of Eq. (62) for two cases: $C_{\text{brane}} = 1$ and $2\pi$.

| $D$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|-----|---|---|---|---|---|---|----|----|
| $C_{\text{brane}} = 1$ | 0.006 | 0.02 | 0.09 | 0.4 | 2 | 20 | 100 | 1000 |
| $C_{\text{brane}} = 2\pi$ | 0.006 | 0.003 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.003 |

where $m$ denotes the rest mass of each incoming particle.

The values of the right hand side are summarized in the cases $C_{\text{brane}} = 1$ and $2\pi$ in Table I. In the case $C_{\text{brane}} = 1$, it is less than 1 for $D \leq 7$ and becomes larger as $D$ increases. In the case $C_{\text{brane}} = 2\pi$, it is less than 0.01 for all $5 \leq D \leq 11$. On the other hand, the natural values for the factors in the left hand side at the LHC would be $\alpha \simeq 1/137$ \(^3\), $(M_p/m) \sim 1\text{TeV}/5\text{MeV} = 200000$ (for a up or down quark) and $(M_p/p) = 1/\text{few}$. In quark collisions at the LHC, AH formation will not occur at the instant of collision, if the brane is somewhat thick or if the dimensionality $D$ is not too large.

Until now charge effects were presumed to be small \(^4\), because they were expected to be proportional to the fine structure constant $\alpha \simeq 1/137$. Our analysis indicates that charge effects can be quite large, because the electromagnetic energy-momentum tensor $T_{\mu \nu}^{(\text{em})}$ is proportional to $p_e^2 \sim \gamma \alpha$ and the Lorentz factor $\gamma$ is much larger than $1/\alpha$ for ultrarelativistic charges and so our results are quite nontrivial. Actually charged black holes were expected to form in collisions between ultrarelativistic charges at the LHC \(^4,9,16\), because a black hole with mass few TeV and elementary charge $e$ is able to exist, as shown in the following. The condition for the existence of an horizon in the Reissner-Nordstr"om spacetime is $|q| \leq \frac{m}{\Omega_{D-2}} \sqrt{\frac{8(D-3)\pi G_P}{(D-2)}}$. Using the definition of the Planck mass \(^6\) and Eq.

\(^3\) If we take the running of the coupling constant into account, a somewhat larger value $\alpha \simeq 1/120$ might be better. Adopting this value, the condition of the black hole formation will become a bit stricter.
we find that this condition becomes

\[ q_4^2 \left( \frac{M_p}{m} \right)^2 \leq \frac{2(D-3)}{(D-2) \Omega_{D-2}} \left( \frac{2\pi}{C_{\text{brane}}} \right)^{D-4}. \]

(62)

Setting \( q_4^2 = \alpha = 1/137 \) and \( m = \text{few} \times M_p \), the typical value of the left hand side becomes \( \sim 0.001 \). The values of the right hand side are given in Table II. The inequality (62) is satisfied and there is indeed a black hole\(^4\), although it is in a near extremal state in the thick brane case. So the important point of our results is that the collision of two charges \( q_1 \) and \( q_2 \) with the center-of-mass energy \( 2p \) does not necessarily lead to black hole formation even if there exists a black hole of mass \( 2p \) and charge \( |q_1 + q_2| \), because the charge effect is enhanced by a factor of \( \gamma \).

Keep in mind that the condition for the black hole formation is different from the condition for AH formation. As discussed in Sec. IVD, however, two conditions become similar in the case of a collision of particles whose charges have the same sign or a collision between a charged and a neutral particle. There is a possibility that the condition for black hole formation is significantly modified from the condition for AH formation in the case of a collision between two equal but opposite-signed charged particles, because the electromagnetic interaction can enhance gravitationally attractive effects contributing to black hole formation. However even if we assume that a black hole forms in this case, black hole production occurs only when a quark and its antiquark (or two gluons\(^5\)) collide at the LHC. Then black hole formation would become a rare process – just scattering with the associated bremsstrahlung radiation would occur in most cases. As a result, the black hole production rate could significantly decrease relative to previous expectations, and detecting black hole signals will be much more difficult than originally expected. In order to specify more detailed criteria for black hole formation it will be necessary to study temporal evolution of the spacetime after collision. Inclusion of brane effects on gauge field confinement may also be necessary.

We note also that additional effects such as inclusion of the spin of incoming particles

\(^4\) Whether a charged black hole can exist in the ADD scenario was first discussed in [23] using the Reissner-Nordström metric. There it was stated that an elementary-charged object a few TeV in mass should be a naked singularity because it violates the condition admitting the existence of an horizon. In this discussion, however, the effect of gauge-field confinement on the brane seems to have been ignored.

\(^5\) Although the gluons do not have electric charge, they have color charge. Hence we should note that if color charge has an effect analogous to that which we have found in this paper, black holes would not be produced also in gluon collisions.
are also required. If such effects also weaken the gravitational field of incoming particles in a manner similar to that of electric charge, the black hole production rate at the LHC might further decrease. Inclusion of spin might be carried out via the interesting “gyraton” model [24] proposed recently. This is the spacetime of a source of finite width propagating at the speed of light. It has internal angular momentum (spin) and its gravitational field produces a frame-dragging effect. Since its gravitational field is also repulsive around its center we expect similar inhibition of AH formation to be observed in the gyraton collisions.

Finally, we revisit the issue of reliability of the charged particle model that we used. As we pointed out in Sec. II A and IVD, QED effects become important within the radius \( r_c \) where the exterior electrostatic energy is equal to the rest mass energy. Substituting the relation (58) into Eq. (15), \( r_c \) is given by

\[
\frac{r_c}{C_{\text{brane}}^{(D-4)/(D-3)}} \left( \frac{q_4^2}{m M_p^{D-4}} \right)^{1/(D-3)}.
\]  

(63)

For a quark, \( r_c \) ranges from \( 10^{-4} \)–\( 10^{-3} \) fm and is larger than \( r_h(2p) \sim 10^{-4} \) fm. Consequently our AH analysis was carried out in a regime where the quantum electrodynamic effects may be important. In order to study QED effects, it will be necessary to derive the gravitational field of a charge produced by the expectation value of the energy-momentum tensor \( \langle T^{(em)}_{\mu\nu} \rangle \). One realization of QED effects is the existence of a nonzero trace anomaly \( \langle T^{(em)}_{\mu\mu} \rangle \). Such QED effects should affect the condition for the AH formation in ultrarelativistic charged collisions, an interesting subject for future study. While we cannot definitely conclude that charge inhibits (or even prevents) black hole formation at the LHC, it is not obvious whether the QED effects weaken or further strengthen the repulsive effect we obtained.

To summarize, we studied AH formation in the head-on collision of the ultrarelativistic charges, modeled by boosted Reissner-Nordström black holes. We find that charge inhibits the formation of an AH. Our results suggest a decrease of the black hole production rate at the LHC in TeV gravity scenarios, although further studies on the evolution after the collision and QED effects are required.

**Acknowledgments**

H.Y. thanks Tetsuya Shiromizu, Antonino Flachi, Hideo Kodama and Masayuki Asakawa for helpful comments. R.B.M. is grateful for the hospitality of TiTech, where this work was
initiated. The work of H.Y. was partially supported by a Grant for The 21st Century COE Program (Holistic Research and Education Center for Physics Self-Organization Systems) at Waseda University. R.B.M. was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) [arXiv:hep-ph/9803315].
   I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *ibid.* 436, 257 (1998) [arXiv:hep-ph/9804398].
   L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].
[2] T. Banks and W. Fischler, [arXiv:hep-th/9906038].
[3] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. 87, 161602 (2001) [arXiv:hep-ph/0106295].
[4] S. B. Giddings and S. Thomas, Phys. Rev. D 65, 056010 (2002) [arXiv:hep-ph/0106219].
[5] D. M. Eardley and S. B. Giddings, Phys. Rev. D 66, 044011 (2002), [arXiv:gr-qc/0201034].
[6] H. Yoshino and Y. Nambu, Phys. Rev. D 67, 024009 (2003) [arXiv:gr-qc/0209003].
[7] S. B. Giddings and V. S. Rychkov, Phys. Rev. D 70, 104026 (2004) [arXiv:hep-th/0409131].
[8] H. Yoshino and V. S. Rychkov, Phys. Rev. D 71, 104028 (2005) [arXiv:hep-th/0503171].
[9] Cardoso, E. Berti and M. Cavaglia, Class. Quant. Grav. 22, L61 (2005) [arXiv:hep-ph/0505125].
[10] P. D. D’Eath and P. N. Payne, Phys. Rev. D 46, 658 (1992); 46, 675 (1992); 46, 694 (1992).
[11] V. Cardoso and J. P. S. Lemos, Phys. Lett. B 538, 1 (2002) [arXiv:gr-qc/0202019].
   E. Berti, M. Cavaglia and L. Gualtieri, Phys. Rev. D 69, 124011 (2004) [arXiv:hep-th/0309203].
   H. Yoshino, T. Shiromizu and M. Shibata, Phys. Rev. D 72, 084020 (2005) [arXiv:gr-qc/0508063].
[12] R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85, 499 (2000) [arXiv:hep-th/0003118].
   P. Kanti and J. March-Russell, Phys. Rev. D 66, 024023 (2002) [arXiv:hep-ph/0203223].
   P. Kanti and J. March-Russell, Phys. Rev. D 67, 104019 (2003) [arXiv:hep-ph/0212199].
   V. P. Frolov and D. Stojkovic, Phys. Rev. D 67, 084004 (2003) [arXiv:gr-qc/0211055].
D. Ida, K. y. Oda and S. C. Park, Phys. Rev. D 67, 064025 (2003) [Erratum-ibid. D 69, 049901 (2004)] arXiv:hep-th/0212108;
M. Cavaglia, Phys. Lett. B 569, 7 (2003) arXiv:hep-ph/0305256;
D. Ida, K. y. Oda and S. C. Park, Phys. Rev. D 71, 124039 (2005) arXiv:hep-th/0503052;
C. M. Harris and P. Kanti, Phys. Lett. B 633, 106 (2006) arXiv:hep-th/0503010;
A. S. Cornell, W. Naylor and M. Sasaki, JHEP 0602, 012 (2006) arXiv:hep-th/0510009;
V. Cardoso, M. Cavaglia and L. Gualtieri, Phys. Rev. Lett. 96, 071301 (2006) arXiv:hep-th/0512002;
V. Cardoso, M. Cavaglia and L. Gualtieri, JHEP 0602, 021 (2006) arXiv:hep-th/0512116;
D. Ida, K. y. Oda and S. C. Park. arXiv:hep-th/0602188.

[13] V. P. Frolov and D. Stojkovic, Phys. Rev. D 66, 084002 (2002) arXiv:hep-th/0206046;
V. P. Frolov and D. Stojkovic, Phys. Rev. Lett. 89, 151302 (2002) arXiv:hep-th/0208102;
V. P. Frolov, D. V. Fursaev and D. Stojkovic, JHEP 0406, 057 (2004) arXiv:gr-qc/0403002;
V. P. Frolov, D. V. Fursaev and D. Stojkovic, Class. Quant. Grav. 21, 3483 (2004) arXiv:gr-qc/0403054;
D. Stojkovic, Phys. Rev. Lett. 94, 011603 (2005) arXiv:hep-ph/0409124;
A. Flachi and T. Tanaka, Phys. Rev. Lett. 95, 161302 (2005) arXiv:hep-th/0506145;
A. Flachi, O. Pujolas, M. Sasaki and T. Tanaka, arXiv:hep-th/0601174
A. Flachi, O. Pujolas, M. Sasaki and T. Tanaka, arXiv:hep-th/0604139

[14] G. Landsberg, arXiv:hep-ph/0211043
M. Cavaglia, Int. J. Mod. Phys. A 18, 1843 (2003) arXiv:hep-ph/0210296;
P. Kanti, Int. J. Mod. Phys. A 19, 4899 (2004) arXiv:hep-ph/0402168;
S. Hossenfelder, arXiv:hep-ph/0412265

[15] P. C. Aichelburg and R. U. Sexl, Gen. Rel. Grav. 2, 303 (1971).
[16] S. Hossenfelder, M. Bleicher, S. Hofmann, H. Stoecker and A. V. Kotwal, Phys. Lett. B 566, 233 (2003) arXiv:hep-ph/0302247;
S. Hossenfelder, B. Koch and M. Bleicher, arXiv:hep-ph/0507140

[17] C. O. Lousto and N. Sanchez, Int. J. Mod. Phys. A 5, 915 (1990).
[18] M. Ortaggio, arXiv:gr-qc/0601093
[19] R. C. Myers and M. J. Perry, Annals Phys. 172, 304 (1986).
[20] J. D. Jackson, Classical Electrodynamics (John Wiley, New York, 1975).
[21] V. Cardoso, J. P. S. Lemos and S. Yoshida, Phys. Rev. D 68, 084011 (2003) arXiv:gr-qc/0307104.
[22] S. B. Giddings, in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. N. Graf, eConf C010630, P328 (2001) arXiv:hep-ph/0110127.
[23] R. Casadio and B. Harms, Int. J. Mod. Phys. A 17, 4635 (2002) arXiv:hep-th/0110255.
[24] V. P. Frolov and D. V. Fursaev, Phys. Rev. D 71, 104034 (2005) arXiv:hep-th/0504027; V. P. Frolov, W. Israel and A. Zelnikov, Phys. Rev. D 72, 084031 (2005) arXiv:hep-th/0506001; V. P. Frolov and A. Zelnikov, Class. Quant. Grav. 23, 2119 (2006) arXiv:gr-qc/0512124.
[25] R. J. Crewther, Phys. Rev. Lett. 28, 1421 (1972); M. S. Chanowitz and J. R. Ellis, Phys. Lett. B 40, 397 (1972); S. L. Adler, J. C. Collins and A. Duncan, Phys. Rev. D 15, 1712 (1977); J. C. Collins, A. Duncan and S. D. Joglekar, Phys. Rev. D 16, 438 (1977).