Application of shooting method on MHD thermally stratified mixed convection flow of non-Newtonian fluid over an inclined stretching cylinder

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Abstract: An analysis is made to examine the magnetohydrodynamic mixed convection boundary layer flow of Eyring-Powell fluid brought by an inclined stretching cylinder. Flow field analysis is accounted by thermal stratification phenomena. The temperature is assumed to be higher across the surface of cylinder as compared to ambient fluid. The arising mathematical model regarding Eyring-Powell fluid is governed by interesting physical parameters which includes mixed convection parameter, thermal stratification parameter, heat generation/absorption parameter, curvature parameter, fluid parameters, magnetic field parameter and Prandtl number. The numerical solutions are computed through the application of shooting technique conjunction with fifth order Runge-Kutta algorithm. In addition, numeric values for two unlike geometries namely, plate and cylinder for skin friction coefficient and Nusselt number are presented with the aid graphs and some particular cases are discussed. The present study is validated by establishing comparison with previously published works, which sets a benchmark of quality of shooting method.

Keywords: Shooting Method; MHD; Thermal Stratification; Mixed Convection; Eyring-Powell Model; Heat Generation/Absorption, An Inclined Stretching Cylinder.

1. Introduction
The mutual interaction of fluid flow and magnetic field relates MHD. Such type of analogy comprises those fluids which are electrically conducting and nonmagnetic comprise liquid metals, strong electrolytes and hot ionized gases. Magnetohydrodynamic fluid flows have various applications in the area of polymer and metallurgical industry. The cooling of filaments or continuous strips confirms the metallurgical use, for example process involves drawing of and thinning of copper wires. The characteristics of final products highly depends on cooling rate. Drawing such strips under magnetic field, we can control stretching and rate of cooling. In a same way many other properties of such materials can be adjusted by iterating magnetic field. Pavlov [1] studied the magnetohydrodynamic fluid flow past over a stretching surface and provided an exact solution of momentum equation. Kumari and Nath [2] examined the magnetohydrodynamic effects on a non-Newtonian fluid flow over a continuously moving surface. The researchers are still showing interest to study magnetohydrodynamic flows over a plane geometry, as far as cylindrical geometry is concern, Ishak et al. [3] explored the magnetohydrodynamic heat mass transfer flow induced by stretching cylinder. Mukhopadhyay et al. [4] identified the characteristics of magnetohydrodynamic boundary layer...
flow by way of stretching cylinder. Malik et al. [5] reported the magnetohydrodynamic flow of tangent hyperbolic fluid past over a stretching cylinder.

Convective heat transfer involves both heat diffusion known as conduction and by bulk heat transfer of fluid known as advection. Convection heat transfer may be natural, forced or mixed. Convection phenomena play a key role in practical life. The chemical transportation in packs bed reactors, drying of porous solid and solar power collectors to mention just a few are the practical applications of mixed convection phenomena. So that, the mixed convection was identified by many scientists as Heckel et al. [6] studied mixed convection flow along slender vertical cylinder by considering variable surface temperature. The characteristics of heat transfer along mixed convection flow of non-Newtonian fluid past a vertical plate was taken by Wang [7]. Chen [8] examined the laminar mixed convection flow over a vertical continuously stretching sheet. Seddeek et al. [9] reported mixed convection flow past a continuously stretching vertical cylinder by considering variable viscosity and thermal diffusivity. The properties of mixed convection flow past a permeable vertical cylinder along surface heat flux was provided by Bachok and Ishak [10]. Hsiao [11] examined the magnetohydrodynamic mixed convection flow of viscoelastic fluid over a porous wedge. Mukhopadhyay and Ishak [12] explored the mixed convection flow past a stretching cylinder by means of thermal stratification. The mixed convection flow of viscoelastic fluid over a stretching cylinder was discussed by Hayat et al. [13]. The mixed convection nanofluid flow over an inclined stretching cylinder with thermal slip effects by way of Buongiorno’s model was considered by Dhanai et al. [14].

Stratification has an abundant applications in real world. For example environmental heat rejection such as seas, lakes and rivers, thermal energy storage systems similar to solar ponds etc. So, due to huge implementations in fluid mechanics many researchers probed stratification phenomena. Whereas geology, biology, oceanography, agriculture, astrophysics and many chemical processes also enclosed double diffusion occurrence. In addition, closed containers, ecological heated walls chambers are supported by convectional environments with thermal and solutal stratification. Such a stratification of the medium is due to temperature and concentration differences, which results density variation in the medium. In short, stratification contribute a vital role in many industrial and natural phenomena. The double stratification flow of nanofluid over a vertical plate was taken by Ibrahim and Makinde [15]. The radiative Jeffrey fluid flow near a stretching sheet with double stratification effects was examined by Hayat et al. [16]. The influence of magnetohydrodynamic flow of micropolar fluid was presented by Srinivasacharya and Upendra [17].

In 1944, Eyring and Powell introduced a fluid model known as Eyring-Powell fluid model [18]. Even though it is complex model but it has certain advantages over a non-Newtonian models in this sense, it is derived from kinetic theory of liquid rather than the experimental relation and turn into Newtonian mode of behaviour for both low and high shear rates. The Eyring-Powell model with heat transfer play a important role in different industrial, natural and geophysical processes. Such a processes contains dispersion and formulation of fog, groves of fruit trees, crops damaging period due to freezing, thermal insulation and underground energy transport etc. A countable attempts has been made to study Eyring-Powell fluid flow over a stretching cylinder. To mention just a few, Malik et al. [19] examined the Eyring-Powell fluid past a stretching cylinder along variable viscosity effect. Recently, Hayat et al. [20] studied the Eyring-Powell magnetohydrodynamic nanofluid flow due to stretching cylinder by considering thermal radiation effect. More recently, Rehman et al. [21] identified stagnation point Eyring-Powell fluid flow over a vertical cylinder.

The above mentioned literature assessment indicates that the boundary layer flow of non-Newtonian fluid (Eyring-Powell) over an inclined stretching cylinder is not investigated widely, as yet. Therefore, the main focus of present study is to analyse magnetohydrodynamic mixed convectional thermally stratified flow of Eyring-Powell fluid induced by an inclined stretching cylinder by way of heat generation/absorption effect. The flow directing differential equations are converted into a system of non-linear ordinary differential equations by using suitable transformations. The numerical solution is obtained by shooting
method with the aid of fifth order Runge-Kutta algorithm. The effect logs regarding achieved results for velocity and temperature profiles are presented graphically. The numeric values for local skin friction coefficient and Nusselt number against two different geometries namely, plate and cylinder are reported with the aid of graphs, which is very essential for industrial application point of view. It is also trusted here, that the results found will not only offer convenient information for applications, but also serve as a complement to the preceding studies.

Figure 1. The physical model and coordinate’s system.

2. Flow Field Analysis
In this study, we have consider steady two dimensional, an incompressible boundary layer flow of Eyring-Powell fluid induced by an inclined stretching cylinder. Flow field analysis is taken with thermal stratification phenomena. The temperature is supposed to be higher near the cylinder surface as compare to ambient fluid. For the Eyring-Powell fluid model, the rheological equation of state is given as:

\[ \Gamma = \mu \frac{1}{\beta \gamma^1} \sinh^{-1} \left( \frac{1}{c} \gamma^1 \right) A, \]  

\[ \gamma^1 = \sqrt{\frac{1}{2} tr(A)} \]  

The second order approximation for \( \sinh^{-1}(\cdot) \) function is estimated as:

\[ \sinh^{-1} \left( \frac{1}{c} \gamma^1 \right) = \frac{\gamma^1}{c} - \frac{\gamma^1}{6c^3}, \quad \text{where} \quad \left| \frac{\gamma^1}{c} \right| << 1. \]

The steady two dimensional equations under boundary layer approximation for Eyring-Powell fluid flow, in usual notation are given as follows:

\[ \frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0, \]
The axial line of cylinder is considered as \( x \)-axis and radial direction perpendicular to axial line is taken as \( r \)-axis. So that, \( u, v, \alpha, \beta, \gamma, c, \rho, Q_0 \), and \( \alpha' \), denotes velocity components in the \( x \) and \( r \) direction, kinematic viscosity, gravity, coefficient of thermal expansion, inclination, specific heat at constant pressure, fluid density, heat generation/absorption coefficient and thermal diffusivity respectively. Further, \( A_1, tr, c, \beta \) and \( \mu \) denotes first Rivlin-Ericksen tensor, trace, fluid parameters and dynamic viscosity respectively.

The endpoint conditions are given as:

\[
\begin{align*}
0 \quad & \text{at } r = R, \\
u(x, r) & \to 0 \quad \text{at } r \to \infty, \\
T(x, r) & = T_0(x) = T_0 + \frac{b x}{L}, \quad \text{at } r = R, \\
T(x, r) & \to T_\infty(x) = T_0 + \frac{c x}{L}, \quad \text{as } r \to \infty.
\end{align*}
\]

To trace out the solution of Eqs. (5)-(6) under end point conditions Eq. (7), we have considered following transformations:

\[
\begin{align*}
u & = \frac{U_0 x}{L} F'(\eta), \\
\psi & = \left( \frac{U_0 \nu x^2}{L} \right)^{\frac{1}{2}} R F(\eta), \\
T(\eta) & = \frac{T - T_\infty}{T_w - T_0} = \frac{T - T_0}{T_\infty - T_0},
\end{align*}
\]

where \( T_\infty(x), T_0(x), T_0, U_0, L, F(\eta), F'(\eta) \) denotes prescribed surface temperature, variable ambient temperature, reference temperature, free stream velocity, reference length, dimensionless variable, velocity of fluid over an inclined stretching cylinder because prime denotes differentiation with respect to \( \eta \) (similarity variable) and \( b, c \) are positive constants, \( \psi \) is the stream function, which identically satisfies the continuity Eq. (4) and is defined as:

\[
\begin{align*}
u & = \frac{1}{r} \left( \frac{\partial \psi}{\partial r} \right), \\
\psi & = -\frac{1}{r} \left( \frac{\partial \psi}{\partial x} \right),
\end{align*}
\]

by substituting Eqs. (8)-(9) in Eqs. (5)-(6), we get:
the transformed endpoint conditions are:

\[ F^\prime = 1, \quad F = 0, \quad T = 1 - \delta, \quad \text{at} \quad \eta = 0, \]
\[ F^\prime \to 0, \quad T \to 0 \quad \text{as} \quad \eta \to \infty, \]

where \( \gamma, K, M, \lambda, \lambda_r, \text{Pr}, \delta, \delta \), denotes magnetic field parameter, curvature parameter, fluid parameters, mixed convection parameter, Prandtl number, thermal stratification parameter and heat generation/absorption parameter respectively and defined as follows:

\[ \gamma = \sqrt{\frac{\sigma B_0^2}{\rho a}}, K = \frac{1}{R} \sqrt{\frac{\nu}{a}}, \quad M = \frac{1}{\mu \beta c}, \quad \lambda = \frac{a^3 x^2}{2 c^2 v}, \]
\[ \lambda_r = \frac{Gr}{Re_x^2}, \quad \text{Pr} = \frac{v}{\alpha}, \quad \delta = \frac{c}{b}, \quad \text{and} \quad \delta = \frac{LQ_o}{U_0 \rho c_p}. \]

Furthermore, \( Gr \) denotes thermal Grashof number which is defined as:

\[ Gr = \frac{g \beta T (T_w - T_0)}{\nu^2}. \]

The skin friction coefficient at the surface of cylinder is considered as:

\[ C_f = \frac{\tau_w}{\rho u_T^2}, \quad \tau_w = \left[ \mu \left( \frac{\partial u}{\partial r} \right) + \frac{1}{\beta \varepsilon} \frac{\partial u}{\partial r} - \frac{1}{6 \beta c^3} \left( \frac{\partial u}{\partial r} \right)^3 \right]_{r=R}, \]

where \( \tau_w \) and \( \mu \) denotes shear stress and viscosity of fluid respectively. The dimensionless form of skin friction coefficient is prearranged as:

\[ \frac{1}{2} C_f Re_x^{1/2} = (1 + M) F^* (0) - \frac{M \lambda}{3} \left( F^* (0) \right)^3, \]

with \( \text{Re}_x = \frac{U_0 x^2}{\nu L} \) as a local Reynolds number.

The local Nusselt number is defined as:

\[ Nu_x = \frac{\alpha q_w}{k(T_w - T_0)}, \quad q_w = -k \left( \frac{\partial T}{\partial r} \right)_{r=R}, \]

in dimensionless form it can be written as:

\[ Nu_x Re_x^{1/2} = -T^* (0). \]

3. Special Cases

3.1. Case-A

A mathematical formulation for steady two dimensional flow of Eyring-Powell fluid brought by an inclined stretching cylinder is given by Eq. (10). In the absence of magnetic field parameter and curvature...
parameter, we obtained a steady, two-dimensional incompressible mixed convection flow of Eyring-Powell fluid past over an inclined stretching plate. So that, the comparative study of cylindrical and plane geometry is examined and plotted through figures 11-12 for skin friction coefficient and heat transfer rate respectively.

Mathematically, by using $\gamma = 0$ and $K = 0$, Eq. (10) takes the form:

$$F'' - (F')^2 + (1 + M)F'' + M \lambda (F^n)'^2 F'' + \lambda_c T(\eta) \cos \alpha = 0,$$

$$T'' + \text{Pr}(FT' - F'T - F\delta_t + \delta T) = 0,$$

with boundary conditions:

$$F' = 1, \quad F = 0, \quad T = 1 - \delta_t \quad \text{at} \quad \eta = 0,$$

$$F' \to 0, \quad T \to 0 \quad \text{as} \quad \eta \to \infty.$$ (19a)

3.2. Case-B

In addition, by incorporating inclination $\alpha = 0$ with mixed convection parameter $\lambda_c = 0$, we achieved two-dimensional steady, an incompressible flow of Eyring-Powell fluid past over a stretching plate. Mathematics for such a physical problem is given by:

$$F'' - (F')^2 + (1 + M)F'' + M \lambda (F^n)'^2 F''$$

with boundary conditions

$$F' = 1, \quad F = 0, \quad \text{at} \quad \eta = 0,$$

$$F' \to 0, \quad \text{as} \quad \eta \to \infty.$$ (20)

Which was discussed by Javed et al. [24]

4. Shooting Method

The non-linear boundary value problem (BVP) given in Eq. (10) and (11) subjected to endpoint conditions Eq. (12) has been computed numerically by employing shooting method with the aid of fifth order Runge-Kutta algorithm. We reduce Eq. (10) and (11) into a system of five first order simultaneous equations by appropriate letting i-e

$$x_2 = F',$$

$$x_3 = x_2' = F''',$$

$$x_5 = T',$$

the equivalent form of Eqs. (10) and (11) under newly signed variables is given by:
\[ \begin{align*}
x'_1 &= x_2 \\
x'_2 &= x_3 \\
x'_3 &= \frac{(x_2)^2 - x_1 x_3 - (2K)(1+M)x_3 + \frac{4}{3} \lambda MK (1 + 2K \eta) x_3^3 + \gamma^2 x_2 - \lambda_\gamma x_4 \cos \alpha}{(1 + 2K \eta)(1+M) - M \lambda (1 + 2K \eta)^2 x_3^2} \\
x'_4 &= x_4 \\
x'_5 &= \frac{\Pr(x_3 x_4 + \delta x_2 - x_1 x_5 - \delta x_4) - 2K x_5}{1 + 2K \eta} \tag{21}
\end{align*} \]

The corresponding endpoint conditions regarding new variables are given as follows:
\[ \begin{align*}
x_1(0) &= 0, \\
x_2(0) &= 1, \\
x_3(0) &= unknown, \\
x_4(0) &= 1 - \delta_1, \\
x_5(0) &= unknown. \tag{22}
\end{align*} \]

For the integration of first order system i.e. Eq. (21) as a IVP with the aid of fifth order Runge-Kutta scheme, we required values for \( x_3(0) \) i.e. \( F''(0) \), and \( x_5(0) \) i.e. \( T'(0) \). Newton’s method is then used to estimate the values of \( F''(0) \) and \( T'(0) \) till the solution approaches zero with the desired efficiency of \( 10^{-4} \) along step size \( \Delta \eta = 0.025 \) by way of additional conditions given as:
\[ \begin{align*}
x_2(\infty) &= 0, \\
x_4(\infty) &= 0. \tag{23}
\end{align*} \]

5. Results and Discussion

5.1. Velocity profiles

Table 1 includes the assessment sample of our results for heat transfer rate corresponding to different values of Prandtl number \( Pr \) with results obtained in [20], [22]-[23]. An excellent match has been found among existing work, which yields a conformity of present results. The influence of magnetic field parameter on velocity profile is presented in figure 2. It is seen that the velocity profile decreases for higher values of magnetic field parameter \( \gamma \). Physically, when we increase magnetic field parameter \( \gamma \) the Lorentz force increases, which is resistive force. As a result fluid velocity decreases. In figure 3, velocity profile is evaluated at different values of thermal stratification parameter \( \delta_1 \). It is evident that the velocity profile decreases for higher values of thermal stratification parameter \( \delta_1 \). Essentially, this effect is due to decline in convective potential between surface of cylinder and ambient temperature. Figure 4 includes the effect of mixed convection parameter \( \lambda_\gamma \) on velocity profile. It is noticed that for higher iterations of mixed convection parameter \( \lambda_\gamma \), velocity of the fluid increases. Physically, it is due to inciting of thermal buoyancy force. The impact of fluid parameter \( M \) is described through figure 5. It is found that the velocity
of fluid increases for increasing values of fluid parameter $M$, because fluid parameter $M$ has inverse relation towards viscosity of fluid, so increase in fluid parameter $M$ turns fluid to be less viscous and hence deformation rate increases which yields increase in velocity. Figure 6 presents the influence of curvature parameter $K$ on velocity profile. It is observed that the higher values of curvature parameter $K$ is the cause of increase in velocity profile. Curvature parameter $K$ is inversely proportional to radius of curvature. So when we increase curvature parameter, the radius of cylinder decreases and hence contact surface area of cylinder with fluid reduces which offers less resistance to fluid flow. So increase in curvature parameter $K$ causes increase in velocity profile.

Figure 2. Influence of $\gamma$ on velocity profile.

Figure 3. Influence of $\delta_1$ on velocity profile.

Figure 4. Influence of $\lambda_\gamma$ on velocity profile.

Figure 5. Influence of $M$ on velocity profile.
Figure 6. Influence of $K$ on velocity profile.

Figure 7. Influence of $\lambda_T$ on temperature profile.

Figure 8. Influence of $\delta_1$ on temperature profile.

Figure 9. Influence of $K$ on temperature profile.

Figure 10. Influence of $\delta$ on temperature profile.

Figure 11. Influence of $\lambda_T$ and $\delta$ on skin friction coefficient.
Figure 12. Effect of $\alpha$ and $\delta$ on local Nusselt number.

Table 1. Comparison value of $-T'(0)$ against $Pr$.

| $Pr$ | Hayat et al. [20] | Ishak and Nazar [22] | Vajravelu et al. [23] | Current results |
|------|------------------|----------------------|-----------------------|-----------------|
| 1    | 1.00000          | 1.0000               | 1.000002              | 1.0021          |
| 1.1  | -                | -                    | -                     | 1.0924          |
| 1.2  | 1.120662         | -                    | -                     | 1.1589          |
| 1.4  | 1.2135           | -                    | -                     | 1.2235          |

5.2. Temperature profiles

In figure 7, the effect of mixed convection parameter $\lambda_r$ on temperature profile is presented. It is found that increasing values of mixed convection parameter $\lambda_r$ corresponds decrease in temperature profile. When mixed convection parameter $\lambda_r$ increases, thermal buoyancy forces enhanced which results inciting in heat transfer rate and hence temperature profile decreases. The influence of thermal stratification parameter $\delta_\gamma$ is depicted in figure 8, it is observed that the temperature profile decreases for increase in thermal stratification parameter $\delta_\gamma$. This is due to drop of temperature difference between surface of cylinder and ambient fluid therefore, temperature profile decreases. Figure 9 paints that the temperature profile increases for increase in curvature parameter $K$. As Kelvin temperature is defined as an average kinetic energy so when we increase curvature of cylinder, velocity of the fluid increases, resultantly kinetic energy increases and due to this temperature increases. Note that temperature of fluid decreases near the surface of cylinder and increases away from the surface. The effects of heat generation/absorption parameter $\delta$ on temperature profile can be seen through figure 10. It is clearly seen that for altered values heat generation/absorption parameter $\delta$ temperature of fluid increases. During heat generation process energy is produced which brings enhancement in temperature distribution. In addition, for higher values of heat generation/absorption parameter $\delta$, over shoot in temperature profile is observed. Such type of overshoot can be controlled by introducing heat sink which helps to reduce temperature of fluid. Figure 11 includes the plots of skin friction coefficient verses mixed convection parameter and heat generation/absorption.
parameter (\(\delta =0.2,0.3,0.4\)). It is worth pointing here the skin friction decreases (in absolute sense) for larger values of mixed convection parameter \(\lambda_r\) and shows opposite attitude towards heat generation/absorption parameter \(\delta\). It is also observed that the magnitude in case of cylinder is higher than the plate. Figure 12 witnessed that the heat transfer rate decreases (absolute sense) for higher values of inclination (i-e \(\alpha=30^0,45^0,60^0\)) and heat generation/absorption parameter \(\delta\). The significant enhancement of local Nusselt number is observed in case of cylinder as compare to plate.

6. Concluding Remarks
The shooting method conjunction with fifth order Runge-Kutta algorithm has been successfully applied to perform a comparative study of mixed convectional thermally stratified boundary layer flow of Eyring-Powell fluid induced by an inclined stretching cylinder by means of heat generation/absorption effect. The behaviour of dimensionless velocity and temperature profiles are identified under different physical parameters. The key results of this study are summarized and itemized as follows:

- The velocity profile increases for higher values of mixed convection parameter \(\lambda_r\), curvature parameter \(K\) and fluid parameter \(M\), while it shows opposite behaviour towards magnetic field parameter \(\gamma\) and thermal stratification parameter \(\delta_1\).
- The temperature profile increases significantly for larger values of heat generation/absorption parameter \(\delta\) and curvature parameter \(K\), whereas it shows decline via mixed convection parameter \(\lambda_r\) and thermal stratification parameter \(\delta_1\).
- The skin friction coefficient shows decreasing behaviour towards mixed convection parameter \(\lambda_r\) and increasing via heat generation/absorption parameter \(\delta\).
- The local Nusselt number is decreasing function of mixed convection parameter \(\lambda_r\) and heat generation/absorption parameter \(\delta\).
- It is seemed that magnitude of skin friction as well as heat transfer rate is remarkably large for cylinder as compare to plane geometry (i-e plate).
- The computed results are validated for heat transfer rate by iterating Prandtl number \(Pr\), which leads to surety of the present work.

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