Determination of the Critical Current Density in YBa$_2$Cu$_3$O$_{7-\delta}$ Thin Films Measured by the Screening Technique Under Two Criteria

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Abstract—We report the determination of critical current density $J_c$ flowing in YBa$_2$Cu$_3$O$_{7-\delta}$ superconducting thin films by two independent criteria. The estimation of $J_c$ was carried out by an inductive technique, i.e., the so-called screening technique. Both the imaginary part of the fundamental harmonic of susceptibility $\chi_1^\prime$ and the third harmonic of the voltage $V_3$ were used to determine the full penetration field. Our goal is to shed light on the techniques yield similar results in the determination of $J_c$. However, at temperatures relatively far from $T_c$, the $V_3$ criterion showed better agreement with the transport data. Furthermore, using both criteria, we propose an alternative methodology to estimate in situ the coil factor associated with the $V_3$ criterion, avoiding in this way the need for implementing an additional technique.

Index Terms—Alternating current (ac) magnetic susceptibility, coil factor, critical current density, screening technique.

I. INTRODUCTION

The application of high-$T_c$ superconductors in the electronics industry is a promising enterprise, which ranges from superconducting quantum interference devices (SQUIDs) to integrated circuits and superconducting computers. To reach this goal, it is of prime importance to understand the superconducting properties of these materials not only of bulk samples but also of thin films. In recent years, great attention has been devoted to the study of transport properties in superconducting thin films. To measure superconducting properties such as the critical current density $J_c$ of either a bulk or a thin-film superconductor, researchers often use the traditional four-probe technique. Although this technique is quite satisfactory for bulk samples, it is highly intrusive for thin films, and as a consequence, most of the time they can no longer be used in further tests. To overcome this problem, several noninvasive techniques have been proposed. So far, two inductive techniques of this kind are widely used, and each one requires a typical setup. In the first case, which is known as the conventional ac susceptibility technique [1], [2], the sample is embedded in a pick-up coil, which, in turn, is coaxially mounted inside a driving coil. The experimental arrangement is sensitive to the overall magnetic response of the sample when it is subjected to an ac magnetic field of the form $H(t) = H_0 \cos(\omega t)$, where $H_0$ is the amplitude, $\omega = 2\pi f$ is the angular frequency, and $t$ is the time. The induced voltage in the pick-up coil is zero when the sample is in its normal state, but as the superconducting transition occurs, an unbalanced voltage sets in [1]. Due to the nonlinear magnetization of the sample, the harmonics of the voltage will emerge. It has been suggested that the harmonics of the magnetic susceptibility are proportional to the harmonics of the voltage [3], [4]. With the help of this technique and based on the analysis of the imaginary part of the fundamental harmonic of the complex magnetic susceptibility as a function of the temperature $\chi_1^\prime(T)$, Xing et al. [5] proposed a method to determine $J_c$ in superconducting thin films.

The second alternative, i.e., the inductive technique or the screening technique [6], [7], offers a practical approach not only to determine $J_c$ [7]–[10] but also to measure the harmonics of the magnetic susceptibility [3], [4]. In this experimental setup, one or two coils can be used. In the latter case, the film is placed between two small coils forming a sandwich arrangement, and as in the conventional ac susceptibility technique, the induced voltage by the drive coil is measured through the pick-up coil. The reliability of the technique can be guaranteed if the size of the sample is at least twice larger than the outer radius of the coils [6]. Previous experiments [8]–[10] have shown that the amplitude of third-harmonic voltage $V_3$ emerges when the amplitude $I_0$ of the driving current reaches a threshold current $I_{thr}$, which can be related to $J_c$ in the film.

We have described two magnetic criteria to estimate $J_c$. For brevity, we will call them the $\chi_1^\prime(T)$ criterion and the $V_3(I_0)$ criterion. Although both of them are widely used, there is no consensus in their convergence, and there is no fundamental theory to single out one or the other.

In this investigation, we determined $J_c$ under these two criteria only using the setup of the screening technique in
YBa$_2$Cu$_3$O$_{7-\delta}$ thin films. One of the main goals of this paper is to shed light on the selection of the criterion. For this purpose, we compared our inductive results with the transport measurements conducted in microbridges contained in the same films. We found that both inductive criteria yield similar results for temperatures close to $T_c$; however, for temperatures relatively far from $T_c$, the corresponding curves begin to diverge. We take advantage of this inductive technique to suggest an experimental alternative in the determination in situ of the so-called coil factor $k$.

II. THEORY

The experimental geometry used in this paper is shown in Fig. 1. An ac magnetic field $H = H_0 \cos(\omega t)$ is generated just above the film by a current $I = I_0 \cos(\omega t)$ passing through the drive coil. The superconducting response is registered through the pick-up coil placed on the rear of the film. The induced voltage depends on the coupling between the coils and is strongly influenced by the superconducting properties of the film.

A. $\chi_1''(T)$ Criterion

Xing et al. [5] used the imaginary part of the fundamental harmonic of susceptibility $\chi_1''(T)$ to estimate $J_c$. They found that the magnetic moment or the shielding current in the hysteresis cycle saturates when amplitude $H_0$ reaches penetration field $H^*$. Based on the Sun model [11], the imaginary part of the fundamental harmonic for a thin film is found to be

$$\chi_1'' \propto \frac{e^{-h}}{h} [h \cosh (h) - \sinh (h)]$$  \hspace{1cm} (1)

where $h = H_0/H^*$ is the reduced field. A plot of $\chi_1''$ versus $h$ shows a peak at $h = 1.344$ [5]. If $\chi_1''$ is plotted as a function of the temperature, a peak appears at a temperature $T_p$, where the aforementioned relation is satisfied, i.e., $H_0 = 1.344H^*(T_p)$. Following the calculations of Brandt [12], the position of the peak of $\chi_1''(h)$ is related to the current density by the following equation:

$$H^* = \alpha d J_c$$  \hspace{1cm} (2)

where $d$ is the film thickness, and $\alpha$ is a constant between 0.8 and 0.9, slightly depending on the geometry (disk, stripe, and square). Using $\alpha = 0.9$ and substituting in the last relation of $H_0$, the following formula is obtained:

$$J_c(T_p) = 1.17\frac{H_0}{d}.$$  \hspace{1cm} (3)

Here, $d$ is given in meters, and $H_0$ is the RMS value in amperes per meter. Equation (3) assumes a homogenous magnetic field. To verify this, we calculated the possible inhomogeneity in the region inside the coil. Using finite-element calculations, we found that there is a variation of 5% with respect to the average magnetic field. Therefore, for the region of interest, the magnetic field can be assumed homogeneous. Since $V_3''(T) \propto \omega H_0 \chi_1''$ [3], one can obtain $J_c$ by plotting $V_3''(T)$ as a function of the temperature for distinct magnetic field amplitudes. For each curve, a peak appears at a temperature $T_p$, and using relation (3), we can obtain the temperature dependence of $J_c$.

B. $V_3(I_0)$ Criterion

Consider that the sample is held at a constant temperature $T$ ($T < T_c$) and that a small field $H_0$ has been applied to the film. If the value of $I_0$ is smaller than a certain threshold value $I_{th}$ ($I_0 < I_{th}$), the magnetic field under the film is screened by a superficial superconducting current $K_1$ (the sheet current) that flows in the film. The magnetic field amplitudes at the upper and lower surfaces of the film are $H_1 = 2H_0$ and $H_2 = 0$, respectively [6], [7]. In this case, the film is regarded as an image coil reflected through the upper surface of the film, carrying the same current but in the opposite direction. The magnetic response of the film is linear, and no harmonics of the voltage are induced in the pick-up coil. When $I_0 = I_{th}$, the magnetic field achieves the full penetration, and the film response is no longer linear. As a result, the third-harmonic voltage $V_3$ in the pick-up coil starts to emerge. At this point, the amplitude of the ac magnetic field near the surface of the film is $H_1 = 2H_0 = J_c d$ [6], [7]. Then, $J_c$ can be expressed as

$$J_c = \frac{k}{d} I_{th}$$  \hspace{1cm} (4)

where $k$ is a coil factor, and $J_c$ can be determined by the occurrence of $V_3$. However, since $V_3$ is proportional to $I_{th}$, a constant resistance criterion must be taken to accurately determine $I_{th}$ [9]. Accordingly, $I_{th}$ must be determined from the $(V_3/I_0)$ versus $I_0$ curve.

The coil factor depends on the shape and location of the coils [10]. Although it can be estimated theoretically [13], in practice, due to the small dimensions of the coils, it is difficult to know the actual radius of the windings or to exactly keep the same spacing of the coils and the film between tests. Some authors have determined the $k$ coil factor using a reference film that is used in combination with inductive and transport measurements or using other magnetic techniques, such as dc magnetization [8]–[10] or magnetization relaxation [14]. However, regardless of the approach, (4) establishes that the thickness of the reference film must be equal to the test film, which is a practical restriction. Therefore, an independent method capable of estimating in situ the coil factor without the requirement of
III. EXPERIMENTAL SETUP

Three films were deposited on CeO$_2$-buffered R-cut Al$_2$O$_3$ substrates by the so-called thermal coevaporation method and were coated in situ with a layer of Au (Theva Company). Film thicknesses were 100-nm Au/200-nm YBCO for the film labeled M1 and 300-nm Au/330-nm YBCO for the films labeled M2 and M3, respectively. Only films M1 and M2 had regions patterned for transport measurements. The growth and patterned details were reported elsewhere [15]. Fig. 2 shows a schematic representation of M1. This 15 mm × 15 mm film has a region for inductive measurements (15 mm × 10 mm) in the upper zone and a region for transport measurements with four microbridges in the lower zone. Films M2 and M3 were smaller than M1 with dimensions of 10 mm × 10 mm. Similar to M1, M2 had an inductive region (10 mm × 7 mm) with only two microbridges in the lower zone. M3 had a continuous surface without microbridges. In the inductive region of M1 and M2, the pair of coils was placed at the center, as sketched in Fig. 1. Each coil had 360 turns, an outer diameter of 4 mm, an inner diameter of 1 mm, and a height of 1 mm. The block diagram of the experimental setup for inductive measurements is shown in Fig. 3. A sinusoidal voltage was supplied with a low-distortion generator (SRS-DS360) to the drive coil. For the $V_3$ criterion, the current was determined measuring the drop voltage in resistor $R_s$ in series with the drive coil by means of a data acquisition board (NI-USB6216). For the case of the $\chi''_1(T)$ criterion, we wish to maintain a constant magnetic field as the temperature is changed; thus, the current in $R_s$ was controlled by means of a proportional–integral–derivative control implemented in a computer program. For both criteria, we applied a frequency of 1 kHz, and a phase-sensitive lock-in amplifier (SRS-SR830) was used to analyze the signal coming from the pick-up coil. The superconducting sample was cooled down by means of a He cold finger coupled to a cryostat (Cryotech-ST15) compressor, and the temperature was controlled via a temperature controller (SI-9600) and monitored by a diode (SI-410A) placed near the sample.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

We first examined the homogeneity of M1 and M2. Fig. 4 shows the temperature dependence of the $J_c$ of the four microbridges of M1 and the two microbridges of M2. For M1, the temperature was varied from 88 to 72 K. For M2, both
microbridges were explored in a smaller range, i.e., from 87.7 to 81.5 K. The transport measurements were carried out with the conventional four-probe method, and a criterion of electric field \( E = 10^{-3} \) V/m to determine \( J_c \) was considered. This criterion was defined when a drop voltage reached 1 \( \mu \)V through the microbridges of 1-mm length. At this voltage value, we clearly observed a sudden increase in the voltage of the \( I-V \) curve. The average \( T_c \) measured was 88.2 and 87.7 K for M1 and M2, respectively. In both cases, the \( T_c \) difference between the microbridges was less than 0.1%.

The next step was to estimate \( J_c \) by the screening technique according to the two criteria outlined previously. Since the same measurement procedure was performed for all films, we will present, for the sake of illustration, the procedure for M1 only. In Fig. 5, the imaginary part of the fundamental harmonic \( V_1'' \) as a function of the temperature is shown. These curves were performed for magnetic field amplitudes \( H_0 \) ranging from 58 to 901 A/m at a slow cooling rate of 2 K/min. We can see that, as the amplitude increases, the peak shifts toward lower temperatures, and at the same time, the width of the curve increases. It is well known that these effects are due to the field and temperature dependence of \( J_c \) [4], [5], [11].

From these curves, we obtained the value of the temperature at the maximum \( (T_p) \) of each curve, and using (3), \( J_c(T_p) \) was estimated for a temperature range between 85 and 88 K. Finally, we estimated \( J_c \) by the \( V_3 \) criterion. The curves for third-harmonic resistance \( V_3/I_0 \) as a function of the driving current amplitude are shown in Fig. 6. Note that the curves at lower temperatures rise smoother than those at temperatures close to \( T_c \). The \( I_{th}(T) \) curve was determined considering a third-harmonic resistance criterion of 0.2 m\( \Omega \) [9]. However, for the \( V_3 \) criterion, the \( k \) coil factor is required [see (4)] to obtain the \( J_c(T) \) curve. In order to obtain the \( k \) coil factor, we performed the transport measurements of microbridge #1 simultaneously. Then, the \( I_{th}(T) \) curve was fitted to the \( J_c \) that was obtained via the transport measurement. The coil factor was found to be \( k = 217 \pm 23 \) cm\(^{-1}\). If \( k \) is obtained from the curve of \( J_c \) based on the \( \chi_1'' \) criterion, we will get \( k = 230 \pm 33 \) cm\(^{-1}\), which, within the experimental errors, is in agreement with the value found previously. The temperature dependence of \( J_c \) by transport and inductive measurements are shown in Fig. 7. We can observe that there is overall agreement among different approaches. In Fig. 8, the \( J_c \) for M2 is shown. We note, in general, that there is good agreement with the inductive and transport measurements. For the \( V_3 \) criterion, a coil factor \( k = 379 \pm 54 \) cm\(^{-1}\) was obtained from the transport measurements of microbridge #1 carried out simultaneously. Moreover, if \( k \) is determined using the \( J_c \) from the \( \chi_1'' \) criterion, then we will
get $k = 346 \pm 53 \text{ cm}^{-1}$, which is the same $k$ value obtained via the transport measurements. The temperature dependence of $J_c$ for M3 is shown in Fig. 9. For the $V_3$ criterion, the same $k$ value of M2 was considered. Note that, as in the previous cases, there is good agreement with both inductive criteria.

Unlike the transport measurement where a constant electric field was considered the criterion for determining $J_c$, in the inductive methods, $H^*$ and $I_{th}$ were considered criteria. To have an idea of the electric field induced in the film with the inductive measurement, we can use the well-known relation $E \approx 2.04 \mu_0 f d^2 J_c$ [16]. For M2, it is found that $E \approx 5 \mu \text{V/m at } 77.3 \text{ K}$, and for M1, we found that $E \approx 1 \mu \text{V/m at } 84 \text{ K}$. These values are about two orders of magnitude smaller than the value from the transport measurements, which is in agreement with those previously reported in literature [8], [14]. If a power-law current–voltage characteristic $E = \alpha J^n$ (with $\alpha$ as a constant) is assumed, the difference caused by the 100-times different $E$ is calculated to be $26\%$ for $n = 20$, $17\%$ for $n = 30$, and $12\%$ for $n = 40$. According to flux creep theories [4], $n = U_c(J)/k_B T$, where $U_c$ is the activation energy, and $k_B$ is the Boltzmann constant. Thus, at lower temperatures, the $I - V$ curves for the films should exhibit a sharp increase, indicating a high $n$ value. Therefore, at lower temperatures, one would expect that the discrepancy in $J_c$ between $V_3$ and the transport measurements diminishes. Note that, to estimate $E$ for the $V_3$ criterion, $J_c$ is required, i.e., equation $E \approx 2.04 \mu_0 f d^2 J_c$ implies that $J_c$ is known by means of some other method. Therefore, one could use the $J_c$ obtained via the $\chi''_3$ criterion to estimate $E$ and then select a suitable frequency $f$ such that $E$ approaches the value of the transport measurements.

V. SUMMARY

In this paper, the $V_3$ and $\chi''_3$ inductive approaches have been confronted with the conventional transport method. We found that both criteria yield similar results for temperatures close to $T_c$ and agree with the transport measurements. Our investigation also provided an alternative methodology to estimate \textit{in situ} the coil factor $k$ that is associated with the $V_3$ criterion. This is achieved by using the $\chi''_3$ criterion with the setup of the screening technique.

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