Detectability of Gravitational Waves from the Coalescence of Massive Primordial Black Holes with Initial Clustering

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We show that the effect of initial non-Gaussian clustering can significantly enhance the event rate for primordial black hole (PBH) coalescence. The impact of such clustering is studied in a specific scenario of multi-stream inflation. Initial clustering enables the possibility of detecting massive PBH coalescence by space-based gravitational wave interferometers such as LISA and DECIGO/BBO. The parameter regime for the ground-based detectors to detect PBH coalescence is also extended.

Black holes of a variety of masses could have formed in the early Universe. They are known as primordial black holes (PBHs) [1–3]. PBHs with initial mass less than $10^{15}$g have already evaporated by now through Hawking radiation [4]. More massive PBHs, once formed, are still present in the Universe today, constituting part of the dark matter. See [5–7] for production mechanisms and observational constraints on PBHs of different masses.

Recently, stellar mass PBHs have received renewed interest [8–10] since the first detection of gravitational waves from the coalescence of a black hole binary [11]. Stellar-mass PBHs could have formed sufficiently frequently in the early Universe so that mergers of PBHs may be observable by ongoing and/or planned ground-based gravitational-wave interferometers, such as LIGO [12], Virgo [13] and KAGRA [14].

Targeting much lower frequencies, future space-based gravitational-wave interferometer experiments are planned, such as LISA [15], DECIGO [16], Taiji [17] and Tianqin [18]. It thus becomes interesting to consider the possibility of observing mergers of PBHs with heavier masses.

In the simplest PBH formation scenarios, mergers of massive PBHs in the sensitive frequency ranges of space-based interferometers are unlikely to be detectable, due to tight constraints on the PBH abundance from various astrophysical observations. This can be seen by noting that, in [10], assuming a uniform distribution of PBHs and a monochromatic PBH mass function, the probability that the coalescence occurs in the time interval $(t, t+dt)$ can be estimated as $dP_t \propto T^{-3/7} dt$ for $t < t_c$ and $dP_t \propto T^{-3/8} t_c^{29/56} dt$ for $t > t_c$, with $t_c, T \propto M^{-5/3}$. Hence, the event rate is $P_t \propto M^{-32/37}$ or $P_t \propto M^{-26/21}$. This shows that, for more massive PBH binaries, to get a reasonable event rate (say, $1Gpc^{-3}\text{yr}^{-1}$), we will need a greater PBH fraction $f = \Omega_{PBH}/\Omega_{DM}$. However, several constraints have been put on large $f$ by Eridanus II [26], Planck [27], wide-binary disruption [28] and millilensing of quasars [29]. Hence, massive PBH mergers in such a simple setup are unlikely to be observed by future space-based interferometers.

The inclusion of initial spatial clustering of PBHs changes the story. The possibilities of clustered PBHs are discussed in [31–34] and references therein. We will show that the initial clustering can significantly enhance the detectability of gravitational waves from massive PBHs. This is because of an increased formation rate of PBH binaries inside the clusters.

To construct a simple model of clustering, we consider the scenario of multi-stream inflation [35–38] (see also [39] for a similar model). As illustrated in Fig. 1, the inflationary trajectory bifurcates at an encounter of a potential barrier in field space. Most of the observable universe (region A) follows one trajectory, while as rare events, a different trajectory with PBH formation leads to the generation of small bubbles (patch B) in the observable universe.

We expect that our analysis should qualitatively apply for general models with significant initial clustering. However, the continuous variation of PBH densities in those models makes the analysis complicated. Thus, in this Letter, we focus on the clustering from multi-stream inflation, where there is no PBH in region A, and the PBH density is approximately constant in B patches. We

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1 Even for PBHs which are formed from Gaussian primordial fluctuations, strictly speaking, this assumption is over-simplified, see [19–23] for the effects of clustering for Gaussian cases. Also, [24, 25] thoroughly revisited estimates of the PBH merger rate.

2 For the PBH generation mechanisms, see, for example, [40–43]. Our discussion does not depend on the details of these PBH formation mechanisms, though for simplicity we assume the mass function is monochromatic. In addition to PBHs, ultracompact minihalos can also be formed in B patches after matter-radiation equality [44]. However, they would not significantly affect the subsequent discussions, though they themselves would have interesting observational implications (see [44] and references therein.)
leave the detailed analysis of general clustering and continuously varying PBH density to future work.

\[ k_2 \approx 7.5 \times 10^5 \text{Mpc}^{-1} \left( \frac{M}{30M_{\odot}} \right)^{-1/2}. \] (3)

Hence, the above inequality can be regarded as providing an upper limit on \( k_1 \), for each \((\beta_1, M, f)\).

We also assume that gravitational wave experiments can cover a large number \( N_B \) of B patches. The observable volume depends on the sensitivity of experiments. For simplicity, we fix the observable distance to be \( k_{\text{obs}}^{-1} = 1 \text{Gpc} \). We express the above condition as

\[ N_B = (k_1/k_{\text{obs}})^3 \beta_1 > 2^3. \] (4)

In addition, we also focus on \( \beta_2 \ll 1 \), so that gravitationally-bound clusters of PBHs (PBH clusters) are not formed during radiation domination. This is to ensure the validity of the calculations of [10], which we will use later. This condition can be quantified as follows.

Let us denote the formation redshift of PBH clusters by \( z_c \), which would be determined by \( k_2 \) and \( \beta_2 \). The calculations of [10] are only applicable before the formation of such PBH clusters. The formation redshift \( z_c \) can be estimated as follows. PBH clusters would be formed approximately when B patches become locally-matter dominant during radiation domination, since the dynamical timescale of the would-be cluster then becomes comparable to the Hubble timescale. Noting the ratio between the energy density of PBHs and that of radiation grows in proportion to the scale factor in B patches, we find \( \beta_2(z_+/z_c) = 1 \), or \( z_c = \beta_2 z_* \), where \( z_* \) is the redshift when PBHs are formed. We focus on the parameter region where \( z_c < z_{\text{eq}} \) is satisfied, so that the analysis of [10] can be used.

Noting \( \beta_2 = \beta_1^{-1} \beta \approx \beta_1^{-1} f(a_*/a_{\text{eq}}) \), the condition \( z_c < z_{\text{eq}} \) can also be rewritten as

\[ f < \beta_1. \] (5)

This condition is roughly equivalent to the condition that PBHs comprise only a subdominant component of the dark matter in B patches.

With these constraints, let us now calculate the enhanced merger rate of PBHs with initial clustering.

In [10], the merger probability and event rate of PBH binaries were calculated for the standard cases with small initial clustering, i.e. \( \beta_1 = 1 \). The merger probability crucially depends on \( f = \Omega_{\text{PBH}}/\Omega_{\text{DM}} \). For more initial clustering cases with \( \beta_1 < 1 \), the merger probability is determined by the fraction \( f_1 \) of PBHs to the dark matter inside B patches, that is, \( f = \beta_1 f_1 \). For the same \( f \), \( f_1 \) is larger when \( \beta_1 < 1 \), so the merger probability is enhanced relative to the standard cases with \( \beta_1 = 1 \). Note that when the number density of black holes \( n_{\text{BH}} \) is multiplied by the merger probability per time \( dP_c/dt \) calculated in [10], in order to obtain the merger rate at
some time \( t \), the corresponding number density should be the average number density of black holes in the observed volume, instead of the local number density in B patches. And if the condition (4) is satisfied, then \( f \) (instead of \( f_1 \)) should be used for the calculation of \( n_{\text{BH}} \). That is, with clustering we have

\[
\text{(event rate)} = \frac{f\Omega_{\text{DM}}\rho_c(t)}{M_{\text{BH}}} \frac{dP_c}{dt} \bigg|_{f=f_1},
\]

where \( \rho_c \) denotes the critical density.

For a fixed event rate, the fraction \( f \) as a function of \( M \) is plotted in Fig. 2. With clustering, smaller values of \( f \) are needed to achieve the given event rate, as a result of the enhanced merger probability, controlled by the parameter \( \beta_1 < 1 \).

![Graph showing the relationship between f, M, and the event rate](image)

**FIG. 2.** The fraction of PBHs \( f = \Omega_{\text{PBH}}/\Omega_{\text{DM}} \) as a function of PBH mass \( M \), needed to realize an event rate of 1Gpc\(^{-3}\)yr\(^{-1}\). We compare the cases of no clustering (\( \beta_1 = 1 \)) with those of enhanced initial clustering (\( \beta_1 = 10^{-2}, 10^{-4} \)). We have taken \( \Omega_{\text{DM}} = 0.27, H_0 = 70\text{kmMpc}^{-1}\text{s}^{-1}, z_{\text{eq}} = 3000 \) and \( t_0 = 13.7\text{Gyr} \). The space-based experiments are sensitive to the frequency range \( 10^{-3} \sim 1\text{Hz} \). Limits from Eridanus II [26], Plank [27], wide-binary disruption [28], and millilensing of quasars [29] are also plotted. Note that these limits are calculated assuming no initial clustering (\( \beta_1 = 1 \)) and hence cannot be directly applied to cases with initial clustering (\( \beta_1 < 1 \)). We also show the frequency of gravitational waves for each mass of merging PBHs along the upper horizontal axis, for which we use the frequency at the Innermost Stable Circular Orbit [30].

In Fig. 2, we have also included limits on massive PBHs [6]. It is important to note that these limits only apply for the case without initial clustering (\( \beta_1 = 1 \)), in which case the parameter space for detecting PBH merger event from space-based interferometers is almost vanishing. However, they should not be interpreted as constraints for the cases of \( \beta_1 \ll 1 \).

Three types of limits have been considered in Fig. 2. We briefly discuss the implication of clustering on these limits here: (1) Limits from the local density of PBHs within the local group, including Eridanus II [26] and wide-binary disruption [28]. When \( \beta_1 \ll 1 \), the MWG halo and the Local Group are very unlikely to be in a PBH-rich region (patch B). So these limits do not constrain the case with strong initial clustering at all. (2) Millilensing of quasars [29]. The validity of this limit will depend on the number and size of PBH-rich patches. If there are unlikely to be PBH-rich regions along all lines-of-sight between us and the quasars, the limit does not apply. Otherwise the quasar bound can apply. (3) CMB limits [27](see also [46, 47]). The effect of PBH accretion (which is an indirect limit depending on astrophysical assumptions) mainly affects the reionization history and thus modifies the temperature and polarization power spectra at low \( \ell \). In addition, for larger mass PBHs (which is the focus of this Letter), the ionization effects shift to lower redshifts and thus affect even lower \( \ell \). However, the initial clustering of PBHs is a high \( \ell \) effect when \( \beta_1 \ll 1 \). Due to spatial inhomogeneity, the affected \( \ell \) should be determined by the size of PBH-rich regions and the size of reionization bubbles, which correspond to smaller \( \ell \). It remains interesting to work out the precise CMB limits considering initial clustering. We provide additional remarks on this issue later.

We have assumed that the merger probability of PBHs up to the present moment \( P_c(t_0) \), which can be calculated from the formulae in [10], is sufficiently small so that the evolution of PBH population is negligible. This leads to an additional constraint on the model parameters. Let us require

\[
P_c(t_0) < 0.1.
\]

Let us check whether the model parameters in Fig. 2 satisfy the conditions on the parameters. First let us consider the conditions which do not depend on \( k_1 \), that is, Eqs. (5) and (7). Each solid line in Fig. 2 is terminated at some large \( M \) so that these two conditions are satisfied along the lines. It turns out that Eq. (5) is more restrictive than (7).

Next, let us consider the other conditions which depend on \( k_1 \), which are (2) and (4). Of these, (2) gives the upper bound on \( k_1 \), whereas (4) gives the lower bound on \( k_1 \), for fixed \( f, M \) and \( \beta_1 \). Let us denote the upper bound on \( k_1 \), determined by (2), as \( k_{M} \), and the lower bound determined from (4) by \( k_m \). If \( k_m < k_M \), there exist values of \( k_1 \), which satisfy both conditions for fixed \( f, M \) and \( \beta_1 \). It turned out that these conditions are much less restrictive than the condition given by Eq. (5).

To summarize, we have studied the detectability of PBH coalescence with initial clustering, with an emphasis on the more massive PBHs to be detected in the future by LISA-like experiments.

In this work, we have adopted a simple model of initial clustering of PBHs, where the PBH-rich regions have sharp boundaries. But one may also generalize our
discussions to other situations, such as other kinds of primordial non-Gaussianity causing initial clustering of PBHs [48, 49]. Non-Gaussianity can also alleviate the fine-tuning problem associated with PBH formation [50]. CMB distortion limits on massive PBHs [51, 52] would also be avoided by clustering [39], since it is suppressed by $\beta_1$ in our model. However, if $\beta_1$ is not so small, our scenario may also lead to CMB $\mu$ distortions observable by future high-sensitivity experiments for massive PBHs [52]. Our scenario may be tested by investigating spatial distributions of gravitational-wave events in details in future. Though we have restricted attention to the coalescence of PBH binaries at very low redshifts, following [10], it would also be interesting to discuss high-redshift PBH mergers. It would also be worthwhile to investigate stochastic gravitational-wave backgrounds for PBHs of different masses with initial clustering. Depending on the model parameters, our scenario would cause inhomogeneous big bang nucleosynthesis, which might lead to interesting observational traces [53].

We have focused our attention on the parameter region $f < \beta_1$. This intersects a large part of the parameter regions of interest. However, we should note that this constraint is for theoretical simplicity instead of an observational bound. For $f > \beta_1$, in B patches PBHs dominate over other dark matter in energy density. Thus, the previous calculation of the coalescence time needs to be reconsidered. We will leave the detailed calculation to future work, but add a few more remarks:

**Disruption of PBH binaries:** As shown in theoretical models and simulations in [25], when PBHs are the dominant component of dark matter, it is highly probable that PBH binaries are disrupted by other PBHs. And the disruption usually results in swapping of a PBH in the binary. The swapping typically reduces the eccentricity of the PBH pairs, and thus will lead to a much longer coalescence time. Considering that PBH binaries (without disruption) typically have a coalescence time much shorter than $t_0$, the effect of disruption may increase the event rate of coalescence around $t_0$.

**Further clustering in the radiation-dominated era for $f > \beta_1$:** PBHs may further evolve and cluster in the early universe. Inside such PBH clusters, binary formation may be understood similarly to that in globular clusters [54–57]. Runaway tidal capture may also take place [58]. GWs from hyperbolic encounters of PBHs can also yield interesting observational implications [59]. PBH clusters may also eventually form supermassive black holes, observed as high-redshift quasars (see [39] and references therein), whose origin still remains to be understood.

**Ultracompact minihalos and locally enhanced structure formation:** PBH-rich patches will be dark-matter overdense regions, which collapse much earlier than standard structure formation, which may also be called ultracompact minihalos [60]. In addition, Poisson fluctuations in the number density of B patches would also cause enhancement of structure formation, similarly to limits on massive PBHs due to the Poisson fluctuations in their number density [5]. These effects would be constrained by observations of the Lyman-$\alpha$ forest. If these constraints are too severe, one may consider some underdense regions to compensate the overdensities (which can also be realized in multi-stream inflation). Also, binary formation may be efficient in ultracompact minihalos. Moreover, in ultracompact minihalos, PBHs are likely to have relatively high velocities; so accretion effects may be suppressed, which would further weaken the CMB limits. Note also that small-scale structure formation may also be enhanced inside B patches due to Poisson fluctuations of PBHs, possibly leading to the formation of smaller ultracompact minihalos, before B patches themselves collapse.

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