Holographic energy loss in an anisotropic strong coupled plasma

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(Dated: November 29, 2022)

We study the energy loss of a quark moving in the strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) plasma under the influence of spatial anisotropy. The heavy quark drag force, the diffusion coefficient and the jet quenching parameter are calculated within the Einstein-Maxwell-dilaton model, in which anisotropic background is specified by an arbitrary dynamical exponent $A$. It is shown that with anisotropic factor $A$ increasing, the drag force and the jet quenching parameter go up, while the diffusion coefficient goes down. We find that the energy loss becomes larger when the quark moving perpendicular to anisotropy direction in transverse plane. The enhancement of drag forces for a fast moving heavy quark as well as jet quenching parameters near critical temperature $T_\text{c}$ is observed, which presents one of typical features of QCD phase transition.

PACS numbers:

I. INTRODUCTION

The heavy ion collisions (HICs) at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) created a strongly-coupled Quark Gluon Plasma (QGP) [1–3]. This provides a novel window to study the physics of strongly coupled Quantum Chromodynamics (QCD). Since the properties of a strongly coupled system cannot be reliably calculated directly by perturbative techniques, we have to resort to some nonperturbative approaches such as lattice QCD [4], AdS/CFT duality [5–8] to conquer the challenges.

The AdS/CFT correspondence, initially proposed by Maldacena in 1997 [5], provides a conjecture between a strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory and a classical supergravity in the asymptotic AdS$_5$ background in the limit of large t’Hooft coupling $\lambda \equiv g^2 N_c$. Following efforts of pioneers, the duality is introduced to handle problems in the strongly coupled QCD scenario [6–8]. Although the precise gravity dual of QCD is unknown, the SYM and QCD may share same qualitative features in the strongly coupled regime at finite temperature, which means one could capture physics of strong coupled QCD by deformed AdS$_5$ [9–11]. One of the significant achievements is the calculation of the ratio of shear viscosity over to entropy density of the QGP, which has been shown to be $1/4\pi$, a simple universal value in gravity side [12]. In weak coupling gravity side, plenty of real time dynamical quantities were computed within top-down and bottom up holographic QCD models, such as hydrodynamic transport coefficients [13–19], energy loss of energetic parton quenching through the QGP [20–28], the thermal photon and di-lepton production rates [29–31], elliptic flow [32–36] and so on [37, 38].

With the gauge theory and gravity duality, the anisotropic geometries have been investigated to understand the properties of the QGP for a long time [39–48]. It should be noticed that only the holographic QCD models with anisotropy succeeded in attempting to reproduce energy dependence of the total multiplicity of experiments in HICs [49–52]. Theoretically, it is also very interesting to study spatial anisotropic systems in deformed $\mathcal{N} = 4$ SYM theory. The neutral anisotropic black brane solution at zero temperature was found originally [40]. Soon nonzero temperature was constructed from type-IIB supergravity by Mateos and Trancane [41, 42]. The gravity solution they constructed is firmly embedded in type IIB superstring theory and dual to a topologically deformed SYM theory, where the topological deformation injects anisotropy into the theory. Although the sources of anisotropy in such models may be different from the hot and dense QCD matter created in the RHIC and the LHC, we expect this kind of models to capture some intrinsic features of the QGP [53–58].

Most of the earlier holographic efforts on anisotropic systems focus on top-down scale-invariant systems and non-conformal charged systems where the anisotropy is introduced by a magnetic field. Recently, the authors of [59, 60] proposed a new bottom-up Einstein-Maxwell-dilaton (EMD) model, where the isotropy is broken by introducing a source at a spatial direction in metric. It is illuminating to take an investigation on the energy loss of an energetic parton within this neutral anisotropic bottom-up system. Besides, since the EMD model is designed to mimic the QCD deconfinement phase transition, it is also of great interest to utilize this anisotropic EMD model to study the propagation of a quark around critical temperature $T_\text{c}$. Much attention having been attracted by recent BES program in HICs, we hope our work could shed light on studying the real time dynamical properties around critical point.

This paper is organized as follows. In Sec. II, we briefly
introduce the EMD model with the spatial anisotropic background [60]. In Sec. III we derive the drag force of heavy quark energy loss when passing through the QGP with the classic trailing string model. In Sec. IV we compute non-relativistic diffusion parameters by using Einstein relation together with the results of Sec. III. And the numerical results of jet quenching parameters are discussed in Sec. V. In the end we present a brief summary in Sec. VI.

II. THE EMD MODEL

The EMD system with spatial anisotropy has been studied by the author of [59, 60]. In this section, we briefly review this spatial anisotropic holographic model starting from the Einstein-dilaton-two-Maxwell action,

\[ S = \int \frac{d^5 x}{16\pi G_5} \sqrt{-g} \left[ R - \frac{f_1(\phi)}{4} F_{(1)}^2 - \frac{f_2(\phi)}{4} F_{(2)}^2 - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \right] \]

(1)

where \( F_{(1)} \) and \( F_{(2)} \) are the field strength tensors of the two U(1) gauge fields, \( \phi \) is the dilaton field and \( V(\phi) \) denotes the dilaton potential. \( f_1(\phi), f_2(\phi) \) are the gauge kinetic functions, representing the coupling with the two U(1) gauge fields respectively.

For holographic description of the hot and dense anisotropic QGP, the metric ansatz is given by

\[ ds^2 = \frac{L^2 b(z)}{z^2} \left[ -g(z) dt^2 + dx^2 + z^2 \left( dy_1^2 + dy_2^2 \right) + \frac{dz^2}{g(z)} \right] \]

(2)

where \( L \) gives the AdS-radius, \( b(z) \) denotes the warp factor, \( g(z) \) stands for the blackening function. \( A \) is the parameter of anisotropy and a slight anisotropy \( A = (1.01 - 1.04) \) is taken in our calculation [60, 61]. Since there is rotational invariance in the \( y_1y_2 \)-direction, the anisotropic direction is considered to be \( x \)-direction [62]. The warp-factor is chosen from light quark system and take the form \( b(z) = e^{-2a\log(bz^2+1)+\sqrt{2}\phi(z)} \) in the string frame. In the following calculations we set the AdS radius \( L \) to be one for convenience.

The solution for the blackening function may be obtained in

\[ g(z) = 1 - \frac{\int_0^z (1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}} d\xi}{\frac{2\mu^2 c}{L^2 (1 - e^{c\xi^2})^2} \int_0^z e^{c\xi^2} (1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}} d\xi} \times \left[ 1 - \frac{\int_0^z (1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}} d\xi}{\int_0^{z_h} (1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}} d\xi} \right] \]

Then the temperature is

\[ T = \frac{|g'(z)|}{4\pi} |_{z=z_h} \]

\[ = \frac{1}{4\pi} \left[ \frac{(1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}}}{\int_0^{z_h} (1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}} d\xi} \right] \left[ 1 - \frac{2\mu^2 c e^{2cz_h}}{L^2 (1 - e^{cz_h})^2} \right] \]

\[ \times \left[ 1 - e^{-cz_h} \frac{\int_0^{z_h} e^{c\xi^2} (1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}} d\xi}{\int_0^{z_h} (1 + b\xi^2)^{3a} \xi^{1+\frac{\phi}{2}} d\xi} \right] \]

(4)

where \( z_h \) denotes the location of horizon.

And the dilaton field \( \phi(z) \) reads

\[ \phi(z) = \int_0^z d\xi \times 2\sqrt{A - 1 + [2(A - 1) + 9aA^2] b\xi^2 + K} \]

(5)

\[ \frac{A - 1 + 3a(1 + 2a)A^2}{(1 + b\xi^2)A\xi} \]

where \( K = \frac{A - 1 + 3a(1 + 2a)A^2}{(1 + b\xi^2)A\xi} \).

There are no divergences in the isotropic case \( A = 1 \) for the dilaton field, but in the anisotropic case the dilaton field has a logarithmic divergence with \( \phi(z) \approx \frac{2\sqrt{A - 1}}{A} \log \left( \frac{\xi}{\xi_0} \right) \). It is proposed in [60, 61] that a sufficiently small boundary condition point \( z_0 \) should reproduce the proper behavior of the scalar field. In this paper we take \( a = 4.046, b = 0.01613, c = 0.227 \) to be compatible with results in the isotropic case [63], where a first-order Hawking-Page-like phase transition happen at critical temperature \( T_c = 157.8 \text{ MeV} \).

III. DRAG FORCE

In small momentum transfer limit, the multiple scattering of heavy quarks with thermal partons in the QGP can be treated as Brownian motion [64–66], which can be described by the Langevin equation as,

\[ \frac{dp}{dt} = -\eta dp + f_{\text{drive}}. \]

(7)

When the heavy quark moving with a constant velocity \( v \), the driving force \( f_{\text{drive}} \) is equal to the drag force \( f_{\text{drag}} = \eta dp \).

In gauge theory side, the heavy quark suffers a drag force and consequently loses its energy while traversing through the strongly coupled plasma. In gravity side, this process could be modeled by a trailing string [22, 23], and the drag force \( f_{\text{SYM}} \) in isotropic SYM plasma with zero chemical potential is then given by

\[ f_{\text{SYM}} = -\frac{\pi T^2}{\sqrt{1 - v^2}} \left( \frac{v}{2} - \sqrt{1 - v^2} \right), \]

(8)

where \( \sqrt{\lambda} = \frac{L^2}{\pi} = \sqrt{g_{YM} N_c} \). The energy loss of the heavy quark can be understood as the energy flow from
the endpoint along the string towards the horizon of the world-sheet.

We follow the argument in [22, 23] to analyze the energy loss of a heavy quark in the anisotropic background. The drag forces are calculated near the critical temperature $T_c$, and the string dynamics is captured by the Nambu-Goto string world-sheet action

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det g_{\alpha\beta}},$$

(9)

$$g_{\alpha\beta} = \frac{\partial X^\mu}{\partial \sigma^\alpha} \frac{\partial X^\nu}{\partial \sigma^\beta},$$

(10)

where $g_{\alpha\beta}$ is induced metric, and $g_{\mu\nu}$ and $X_\mu$ are the brane metric and target space coordinates.

The trailing string corresponding to a quark moving on the boundary along the chosen direction $x_p(x_p = x, y_1, y_2)$ with a constant velocity $v$ has the usual parametrization

$$t = \tau, x_p = vt + \xi(z), z = \tau.$$  

(11)

Plugging static gauge Eq. (11) into the metric Eq. (2), we have

$$ds^2 = g_{tt} dt^2 + g_{xx} dx^2 + g_{zz} dz^2,$$

(12)

$$g_{tt} = -L^2 b(z) g(z),$$

(13)

$$g_{xx} = \frac{L^2 b(z)}{z^2 g(z)} (x_p = x),$$

(14)

$$g_{xx} = \frac{L^2 b(z)}{z^2 g(z)},$$

(15)

$$g_{zz} = \frac{L^2 b(z)}{z^2 g(z)}.$$  

(16)

The Lagrangian density can be obtained from the Nambu-Goto action as

$$\mathcal{L} = \sqrt{-g_{tt} g_{zz} - g_{zz} g_{xx} v^2 - g_{tt} g_{xx} v^2}.$$  

(17)

The Lagrangian density does not depend on $\xi$ from Eq. (17), which implies that the canonical momentum is conserved,

$$\Pi_\xi = \frac{\partial \mathcal{L}}{\partial \xi'} = \frac{-g_{tt} g_{xx} \xi'}{\sqrt{-g_{tt} g_{zz} - g_{zz} g_{xx} v^2 - g_{tt} g_{xx} v^2}}.$$  

(18)

Then one can get

$$\xi^2 = \frac{-g_{zz}(g_{tt} + g_{xx} v^2) \Pi_\xi^2}{g_{tt} g_{xx}(g_{tt} g_{xx} + \Pi_\xi^2)}.$$  

(19)

Both the numerator and the denominator must change sign at the same location $z_c$ from Eq. (19). The critical point $z_c$ can be written as

$$g_{tt}(z_c) = -g_{xx}(z_c) v^2,$$  

(20)

and

$$\Pi_\xi^2 = -g_{tt}(z_c) g_{xx}(z_c).$$  

(21)

Finally, we obtain the drag force

$$f = -\frac{1}{2\pi\alpha'} \Pi_\xi = -\frac{1}{2\pi\alpha'} g_{xx}(z_c) v.$$  

(22)

There are two different drag forces, $f^{v||A}$ and $f^{v\perp A}$, for the anisotropy in background metric in Eq. (2). To be specific, $f^{v||A}$ stands for the drag force in parallel with anisotropy direction, when the jet parton moving along anisotropy direction. And $f^{v\perp A}$ denotes the drag force in parallel with its motion direction, when the jet parton moving perpendicular to anisotropy direction. Plugging Eq. (12) into Eq. (22), we have

$$f^{v||A} = -\frac{1}{2\pi\alpha'} g_{xx}(z_c) v |_{x_p = x}$$  

(23)

$$= -\frac{v}{2\pi\alpha'} b(z_c) z_c^{-\frac{2}{3}},$$  

(24)

and

$$f^{v\perp A} = -\frac{1}{2\pi\alpha'} g_{xx}(z_c) v |_{x_p = y_1}$$  

(25)

$$= -\frac{v}{2\pi\alpha'} b(z_c) z_c^{-\frac{2}{3}}.$$  

(26)

![FIG. 1: Perpendicular (dashed line) and parallel (solid line) drag force at lower speed ($v = 0.6$) normalized by conformal limit as a function of the temperature for different values of the anisotropy factor $A$.](image)

The influence of spatial anisotropy on drag forces are illustrated in Fig. 1 and Fig. 2, where the drag forces in anisotropic plasma are rescaled by the isotropic SYM result at zero chemical potential given in Eq. (8). Fig. 1 shows, at lower speed ($v = 0.6$) the drag force $f^{v||A}$ always becomes larger with increasing anisotropic factor $A$. Similar trend is also observed for drag forces perpendicular to anisotropy direction $f^{v\perp A}$. It is seen that the perpendicular direction drag force $f^{v\perp A}$ is larger than parallel direction drag force $f^{v||A}$ with the same anisotropic factor around critical temperature $T_c$. 
At higher speed $v = 0.96$, corresponding to faster charm quark, the situation becomes more complicated as presented in Fig. 2. Plots (a), (b), (c) and (d) in Fig. 2 present drag forces at different anisotropy $A = 1$, $A = 1.01$, $A = 1.02$ and $A = 1.03$ respectively. We find the drag force $f^{v \parallel A}$ always goes up with increasing anisotropic factor $A$. The perpendicular direction drag force $f^{v \perp A}$ is larger than the parallel direction drag force $f^{v \parallel A}$ around $T_c$. Furthermore, there is a peak near critical temperature $T/T_c = 1.1$ when the velocity of quark ($v = 0.96$) is approaching the speed of light. The enhancement of energy loss around critical temperature $T_c$ is one of typical features of QCD phase transition.

From Fig. 1 and Fig. 2, we also find that charm quark (with faster velocity and lighter mass) is more sensitive to properties of the anisotropy QGP than bottom quark (with slower velocity and heavier mass), when they pass through the anisotropic plasma with a fixed initial energy $E_i$.

IV. DIFFUSION COEFFICIENTS

The diffusion coefficient, another important transport parameter of plasma, has been studied extensively at the RHIC and the LHC. It is of a general practice to utilize Einstein-Maxwell system to study this transverse momentum broadening when heavy quark propagation in plasma [68, 69]. The heavy quark transverse momentum diffusion constant $D$ in the strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills theory was first computed in [70, 71], and then it was generalized to non-conformal theories in [72]. The Langevin dynamics of non-relativistic heavy quarks is completely determined by the momentum broadening $D$. The Einstein relation together with the expression of $\eta_D$ allow us to infer the value of $D$ for this strongly coupled anisotropic plasma. The diffusion coefficient in the isotropic SYM theory [71] is

$$D_{SYM} = \frac{T}{mt_{SYM}} = \frac{2}{\pi T \sqrt{\lambda}},$$  \hspace{1cm} (27)

where $t_{SYM} = \frac{1}{\eta_D}$ is the diffusion time.

From Eq. (22) and Eq. (27), we obtain diffusion coefficient in anisotropic plasma normalized by isotropic SYM results as,

$$D = D_{SYM} \frac{\pi^2 T^2}{g_{xx}(z_c) \sqrt{1 - v^2}|(x_p=x_1,y_2)|},$$  \hspace{1cm} (28)

Now there are also two different diffusion coefficients, $D^{v \parallel A}$ and $D^{v \perp A}$, for the anisotropy in background metric Eq. (2). $D^{v \parallel A}$ gives the diffusion coefficient when jet partons moving along anisotropy direction, while $D^{v \perp A}$ gives the one when the jet parton moves perpendicular to anisotropy direction. Plugging Eq. (12) into Eq. (28),

![FIG. 2: Perpendicular (dashed line) and parallel (solid line) drag force at higher speed ($v = 0.96$) normalized by conformal limit as a function of the temperature for different values of the anisotropy factor $A$.](image-url)
we have

\[
\frac{D_{\parallel}^A}{D_{\text{SYM}}} = \frac{\pi^2 T^2}{g_{xx}(z_c)\sqrt{1-v^2}} |_{x_p=x} (29)
\]

and

\[
\frac{D_{\perp}^A}{D_{\text{SYM}}} = \frac{\pi^2 T^2}{g_{xx}(z_c)\sqrt{1-v^2}} |_{x_p=y_1} (30)
\]

\[
\frac{D_{\parallel}^A}{D_{\text{SYM}}} = \frac{\pi^2 T^2}{b(z_c)z_c^2\sqrt{1-v^2}} (31)
\]

\[
\frac{D_{\perp}^A}{D_{\text{SYM}}} = \frac{\pi^2 T^2}{b(z_c)z_c^2\sqrt{1-v^2}} (32)
\]

The numerical results of the influences on diffusion constants \(D\) from anisotropy factor are displayed in Fig. 3 and Fig. 4, normalized by the isotropic SYM result at zero baryon density given in Eq. (27). It is seen from Fig. 3 that, at lower speed \((v = 0.6)\) both \(D_{\parallel}^A\) and \(D_{\perp}^A\) suffer stronger suppression with increasing anisotropic factor \(A\). In addition, the perpendicular direction diffusion constant \(D_{\perp}^A\) has stronger suppression than parallel direction diffusion constant \(D_{\parallel}^A\) around critical temperature \(T_c\).

Fig. 4 gives the results at higher speed \((v = 0.96)\), and plots (a), (b), (c) and (d) in Fig. 4 present diffusion constants at different anisotropy \(A = 1, A = 1.01, A = 1.02\) and \(A = 1.03\) respectively. We find diffusion constant \(D_{\parallel}^A\) goes down with increasing anisotropic factor \(A\). It is also seen that the perpendicular direction diffusion constant \(D_{\perp}^A\) has stronger suppression than parallel direction diffusion constant \(D_{\parallel}^A\) around critical temperature \(T_c\). One may observe the strongest suppression near critical temperature \(T/T_c = 1.1\) when quark moves almost with the speed of light \((v = 0.96)\). All numerical results show the same trend that the energy loss in perpendicular direction is larger than the one in parallel direction.
V. JET TRANSPORT COEFFICIENT

In dual gravity theory, a non-perturbative definition of jet transport coefficient \( \hat{q} \) has been provided \[21\], based on the computation of light-like adjoined Wilson loops for \( \mathcal{N} = 4 \) SYM plasma. It has been shown that the jet quenching parameter at an isotropic SYM plasma with zero chemical potential is

\[
\hat{q}_{\text{SYM}} = \frac{\pi^2 \sqrt{T_{33}^\tau}}{\Gamma(\frac{1}{3})}. \tag{33}
\]

Later, this work has been extended to various cases, for instance, the impact of chemical potential on \( \hat{q} \) \[73, 74\]. In this section we discuss the jet quenching parameter \( \hat{q} \) in the anisotropic background. We follow the argument in \[62\] to study the jet quenching parameter of a light quark system in anisotropic medium, in which the jet quenching parameter \( \hat{q} \) is directly related to light-like adjoined Wilson loop \[21\] as

\[
\langle W^A[C]\rangle \approx \exp[-\frac{1}{4\sqrt{2}}\hat{q}L^2L^-], \tag{34}
\]

where \( C \) is a null-like rectangular Wilson loop formed by a quark-antiquark pair, \( L \) gives the separated distance and \( L^- \) the traveling distance along light-cone time duration.

Using the equations

\[
\langle W^A[C]\rangle \approx \langle W^F[C]\rangle^2 \tag{35}
\]

and

\[
\langle W^F[C]\rangle \approx \exp[-S_I], \tag{36}
\]

we obtain a general relation of jet quenching parameter

\[
\hat{q} = 8\sqrt{2} \frac{S_I}{L^-L^2}. \tag{37}
\]

To calculate the Wilson loop, we take advantage of the light-cone coordinates

\[
x_+ = \frac{t + x_p}{\sqrt{2}}, \quad x_- = \frac{t - x_p}{\sqrt{2}}, \tag{38}
\]

where \( x_p \) is chosen to be the direction of motion.

The metric Eq. (2) is then given by

\[
ds^2 = G_{--}(dx_*^2 + dx_+^2) + G_{++} dx_+ dx_-, \tag{39}
\]

\[
+G_{zt} dx_+ dz^2 + G_{zz} d^2 z, \tag{40}
\]

\[
G_{--} = \frac{g_{tt} + g_{pp}}{2}, \tag{41}
\]

\[
G_{++} = \frac{g_{tt} - g_{pp}}{2}, \tag{42}
\]

\[
G_{ii} = g_{ii}(i=x_1, y_1), \tag{43}
\]

\[
G_{zz} = g_{zz}. \tag{44}
\]

Given the Wilson loop extending along the \( x_k \) direction, we choose static gauge coordinates

\[
x_- = \tau, x_k = \sigma, u = u(\sigma). \tag{45}
\]

The Nambu-Goto action Eq. (9) can be given as

\[
S = \frac{L}{\pi\alpha'} \int_0^{\frac{L^-}{2}} d\sigma \sqrt{G_{zz}G_{--}(G_{zz}z^2 + G_{kk})}. \tag{46}
\]

As action Eq. (46) does not depend explicitly on \( \sigma \) explicitly, we could have a conserved quantity

\[
E = \int_0^{\frac{L^-}{2}} \frac{\partial L}{\partial \dot{z}} \dot{z} - L = E, \tag{47}
\]

resulting in

\[
z^2 = \frac{(G_{kk}G_{--} - c^2)}{c^2 G_{zz}}. \tag{48}
\]

Combining Eq. (48) and Eq. (46), we get

\[
S_0 = \frac{L}{\pi\alpha'} \int_0^{\frac{L^-}{2}} dz \sqrt{G_{iiu}G_{--}}. \tag{49}
\]

The total action is divergent and should be subtracted by the self-energy of the two free quarks part

\[
S - S_0 = \frac{L}{\pi\alpha'} \int_0^{\frac{L^-}{2}} dz \sqrt{G_{zz}} \sqrt{\left(\frac{G_{zz} - G_{kk}}{G_{zz}G_{kk} - E^2} - 1\right)} \tag{50}
\]

In our calculation, indices \( p \) and \( k \) here denote a chosen direction. Substituting Eq. (50) into Eq. (37), we show

\[
\hat{q}_{(p,k)} = \frac{\sqrt{2}}{\pi\alpha'} \left( \int_0^{\frac{L^-}{2}} dz \, \frac{1}{g_{kk}} \sqrt{\frac{2 g_{zz}}{g_{tt} g_{zz}}} \right)^{-1} \tag{51}
\]

Now we see there are three types of jet quenching parameters, \( \hat{q}_{(||, \perp)} \), \( \hat{q}_{(\perp, ||)} \) and \( \hat{q}_{(||, \perp)} \) for anisotropic background given in Eq. (2). Here \( \hat{q}_{(||, \perp)} \) denotes the jet transport coefficient when energetic partons moving along anisotropy, and the momentum broaden perpendicular to anisotropy direction; \( \hat{q}_{(\perp, ||)} \) stands for the jet quenching parameter when energetic partons moving perpendicular to anisotropy direction, and the momentum broadening along anisotropy; \( \hat{q}_{(||, \perp)} \) gives the coefficient with fast parton moving perpendicular to anisotropy direction, and the momentum broadening perpendicular to anisotropy direction.

\[
\hat{q}_{(||, \perp)} = \frac{\sqrt{2}}{\pi\alpha'} \left( \int_0^{\frac{L^-}{2}} dz \, \frac{1}{g_{zz}} \sqrt{\frac{2 g_{zz}}{g_{tt} (f(z) + 1)}} \right)^{-1} \tag{52}
\]

\[
\hat{q}_{(\perp, ||)} = \frac{\sqrt{2}}{\pi\alpha'} \left( \int_0^{\frac{L^-}{2}} dz \, \frac{1}{g_{zz}} \sqrt{\frac{2 g_{zz}}{g_{tt} g_{yy}}} \right)^{-1} \tag{53}
\]
\[ \hat{q}_{(\perp,\perp)} = \frac{\sqrt{2}}{\pi \alpha'} \left( \int_0^{z_h} dz \frac{1}{g_{y_1 y_1}} \sqrt{\frac{1}{g_{tt} + g_{y_1 y_1}}} \right)^{-1} \]

\[ = \frac{\sqrt{2}}{\pi \alpha'} \left[ \int_0^{z_h} dz \left( b(z) z^2 \sqrt{\frac{2}{g(z)(-f(z) + z^2 - \frac{4f}{z})}} \right)^{-1} \right] \]

Fig. 5 demonstrates the impact of spatial anisotropy on jet quenching parameters, normalized by the isotropic SYM result at zero baryon density given in Eq. (33). Plots (a), (b), (c) and (d) in Fig. 5 show jet quenching parameters at different anisotropy \( A = 1, A = 1.01, A = 1.02 \) and \( A = 1.03 \) respectively. Figure (a) with \( A = 1 \) corresponds to the isotropic case. One see that all three jet quenching parameters \( \hat{q}_{(||,\perp)} \), \( \hat{q}_{(\perp,||)} \) and \( \hat{q}_{(\perp,\perp)} \) increase with anisotropic factor \( A \). And we observe the small peak around critical temperature \( T/T_c = 1.4 \), which is one of typical features of QCD phase transition. It is noted that our results are consistent with recent lattice simulations in [75]. One reads from different anisotropic cases in Figure (b), (c) and (d) of Fig. 5, that in general \( \hat{q}_{(||,\perp)} \geq \hat{q}_{(\perp,\perp)} \geq \hat{q}_{(\perp,\perp)} \), which indicates that the energy loss is larger in the transverse plane than along the anisotropic direction. It also shows that the momentum broadening of an energetic parton in anisotropic medium depends more on the direction of motion rather than the direction of momentum broadening.

VI. CONCLUSION

The study of jet quenching properties as functions of parameters such as the temperature, chemical potential, and anisotropy factor is of great relevance for the understanding the anisotropic QGP. In present work, we have taken an investigation on energy loss of a fast parton passing through a strongly coupled plasma from an EMD model with anisotropy.

We focus on the influences of spatial anisotropy on several important quantities related to parton energy loss. It is demonstrated that with increasing anisotropic factor \( A \), the drag force and jet quenching parameter go up, while the diffusion constant goes down. The comparison of drag forces in different directions shows that energy loss is larger when moving perpendicular to anisotropy direction than paralleled to anisotropy direction. The jet quenching parameter and diffusion constant also give the same conclusion that energy loss is stronger when the jet parton moves perpendicular to anisotropy direction.

We also observe a peak near critical temperature \( T_c \) both on the drag force and the jet quenching parameter when energetic partons move nearly with the speed of light. The small enhancement of the drag forces \( (f^{||,A} + f^{\perp,\perp}) \) is around \( T_c/T = 1.1 \), while the enhancement of the jet quenching parameters \( \hat{q}_{(||,\perp)}, \hat{q}_{(\perp,\perp)}, \) and \( \hat{q}_{(\perp,\perp)} \) is around \( T_c/T = 1.4 \). It is noticed that our results are consistent with recent lattice calculations given
in [75]. However, when parton moving at lower speed the peak of the drag force near \( T_c \) may disappear. Comparing numerical results of the drag force at different speeds, we see charm quark is more sensitive to the properties of the plasma than bottom quark when the initial jet energy is fixed.

ACKNOWLEDGMENTS

We thank Zhou-Run Zhu for his enlightening advice and very useful discussions. This research is supported by the Guangdong Major Project of Basic and Applied Basic Research No. 2020B0301030008, and Natural Science Foundation of China with Project Nos. 11935007.

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