The conditions of similarity and generalized dependences for calculating convective heat transfer in supercritical pressure coolants

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Abstract. In this report the assessment of the results of recent experimental investigations of heat transfer in turbulent flow of supercritical water and modeling fluids (carbon dioxide, Freon) in vertical channels of different geometry (tubes, annular gaps and rod bundles) is presented. The conditions of similarity and the system of criteria, which determine the intensity of heat exchange in the fluids near the critical point, are considered. Due to the small hydraulic diameter of the heat exchange channels in the core of nuclear reactors it is possible to neglect the gravitational forces compared to the acceleration caused by the thermal inertia effects and the forces of viscosity. Based on these ideas two comprehensive criteria were proposed. Their application in the basic equation of heat transfer suggested by the authors earlier for the normal regimes satisfactorily (with an error of 20–25%) describes the features of change of heat transfer coefficient in the deteriorated and mixed regimes of heat transfer. The system of equations suitable for engineering calculation of heat transfer in channels of nuclear reactors cooled with supercritical pressure water was developed.

1. Introduction
The future development of nuclear power engineering is connected with the transition to supercritical parameters of a working steam. The supercritical pressure light water-cooled reactor (SCWR) has been chosen as a generation IV reactor for innovative nuclear power plants. The application of this reactor type makes it possible to significantly increase the efficiency of the thermodynamic cycle of energy conversion and reduce capital and operational costs [1]. To realize this concept it is necessary to solve a number of problems concerning reactor core design. Among them the thermal-hydraulic analysis of SCWR fuel assemblies represents an important issue for the design of new nuclear reactors, making use of supercritical pressure water as primary coolant. The prediction of cladding temperature of fuel elements is required to ensure their safe and reliable operation. In the prediction of the complex phenomena occurring owing to the changes of fluid properties exhibited in the vicinity of the pseudo-critical temperature the special models and engineering correlations for the analysis of heat transfer and fluid dynamics must be worked out. In particular, phenomena of heat transfer deterioration must be carefully considered. In the open literature there exist a lot of empirical correlations which were derived on the base of the data for round tubes. As to heated rod bundles cooled by supercritical...
pressure water the data for the present are scanty therefore the analysis of experimental results obtained both for water and modeling fluids is very useful.

This paper presents a new method for the prediction of normal, deteriorated and mixed heat transfer in supercritical pressure fluids. In order to derive the correlations experimental data for various modes of turbulent heat transfer in water, carbon dioxide and Freon-12, Freon-134a flowing in tubes, annular channels and rod bundles were generalized over a wide range of parameters. Emphasis was put on the simplicity and reliability of calculation dependencies.

2. Analysis of heat transfer modes
As a consequence of the strong variation of fluid properties under such conditions the heat transfer has very complex character. Due to the combined influence of buoyancy and thermally-induced bulk flow acceleration the effectiveness of heat transfer can be reduced. This mode of heat transfer has been referred to impaired or deteriorated regime of forced convection. In the absence of the significant effects of buoyancy and acceleration the heat transfer is considered as normal or improved. Modes of normal and improved heat transfer meet the requirements of reliability and safety of power equipment to the utmost extent; therefore, they should be used as the basis for operational regimes of nuclear reactors with supercritical pressure water. At the same time, special attention should be paid to the regimes of deteriorated heat transfer, since the transition to them can lead to unacceptable overheating of the fuel elements of the reactor and, as a result, cause their destruction.

In the case of reactor fuel assemblies the diametrical size of coolant channels is sufficiently small, therefore, one would expect the influence of buoyancy forces to be negligible. Thus, it is supposed further that deteriorated heat transfer regimes occur due to turbulence production in the flow being reduced as a result of thermally-induced bulk flow acceleration.

2.1. Base relation
As a base for generalized dependencies the equation suggested before in paper [2] for normal heat transfer regimes was used

\[ \frac{\nu_{b,N}}{\nu_0} = \left( \frac{\rho_w}{\rho_b} \right)^{0.25} \left( \frac{c_p}{c_{p,b}} \right)^n. \]  

In Equation (1) the Nusselt number \( \nu_0 \) is determined from the well-known Dittus–Boelter correlation

\[ \nu_0 = 0.023 \Re^{0.8} \Pr^{0.4} \]  

for turbulent forced convection heat transfer in tubes in the absence of significant non-uniformity of fluid properties. The property variations over the channel cross-section are accounted by two empirical corrections: \( \rho_w/\rho_b \) is the ratio of fluid densities at the wall temperature \( t_w \) and bulk temperatures \( t_b \) correspondingly, \( c_p/c_{p,b} \) is the ratio of the average integral heat capacity \( c_p \) and the heat capacity of the fluid \( c_{p,b} \) at bulk temperature. The value of \( c_p \) is expressed, as usual, by the ratio of the corresponding differences of fluid enthalpies \( h \) and temperatures \( t \) and it is found as \( c_p = (h_w - h_b)/(t_w - t_b) \). The index \( n \) depends on the value of \( c_p/c_{p,b} \); \( n = 0.6 \) at \( c_p/c_{p,b} < 1 \) and \( n = 0.4 \) at \( c_p/c_{p,b} \geq 1 \). It has been shown [2] that Equation (1) with an error of 20–25% describes a large array of experimental data on the normal heat transfer to the flow of water of supercritical pressure in vertical tubes.
2.2. Method for determining the heat exchange regime

When analyzing the features of the heat exchange modes in supercritical pressure fluids, the data obtained in [3–24] (Table 1) were used. By comparing the calculation data from Equation (1) with the results of heat transfer measurements in the works mentioned, the types of heat exchange regime were determined. A quantitative measure in the diagnosis of regimes was the ratio of the Nusselt number $Nu_{b,Exp}$ calculated from the experimental data to the $Nu_{b,N}$ found from Equation (1).

The case $Nu_{b,Exp}/Nu_{b,N} ≈ 1$ was related to the normal regime, the cases $Nu_{b,Exp}/Nu_{b,N} > 1.15$ and $Nu_{b,Exp}/Nu_{b,N} < 0.85$ – to the modes of improved and deteriorated heat exchange respectively.

Table 1. Characteristics of the experimental data used in generalization

| Authors            | Test section | $p_c$, MPa | $G$, kg/(m$^2$·s) | $q_t$, kW/m$^2$ | $h_{b}$, kJ/kg |
|--------------------|--------------|------------|-------------------|-----------------|---------------|
| **Water** $p_c = 22.06$ MPa, $t_c = 374.0^{\circ}$C |
| Shitsman [3]       | Tube $d = 8$ mm | 22.6       | 323, 430, 1500    | 278–1083        | 1219–2410     |
| Ishigai et al. [4] | Tube $d = 3.92$ mm | 24.5       | 1000             | 1116, 1372      | 1014–3269     |
| Kirillov et al. [5]| Tubes $d = 10$ mm, $L = 1$&$4$ m | 24.6–25 | 198–1494         | 88–1029         | 1336–2459     |
| Gang et al. [6]    | Annular channel $d_{h} = 8$&$12$ mm | 25       | 400, 980, 1000    | 200–1000        | 993–3144      |
| Li Y. et al. [7]   | Tube $d = 6$ mm | 23, 24, 25 | 649–1207         | 600–860         | 1225–2765     |
| Li H. et al. [8]   | Tube $d = 7.6$ mm | 23, 25, 26 | 448–799         | 189–665         | 1356–2450     |
| Wang et al. [9, 10]| Bundle $2\times2$ $d_{h} = 4.32$ mm | 23, 25, 28 | 350–1000       | 200–1000        | 848–3034      |
| Gu et al. [11]     | Bundle $2\times2$ $d_{h} = 5.4$ mm | 23, 25, 26 | 400, 800, 1200   | 300–1000        | 1034–2773     |
| Razumovskiy et al. [12]| Annular channel $d_{h} = 2.67$ mm | 22.6 | 2000             | 1543–2547       | 939–1284      |
| Hu and Gu [13]     | Bundle $2\times2$ $d_{h} = 4.79$ mm | 23–26    | 450–1200        | 200–1000        | 1035–2770     |

| **Carbon dioxide** $p_c = 7.38$ MPa, $t_c = 31.0^{\circ}$C |
| Kim, Bae et al. [14–18]| Tubes $d = 4.4$, 4.5, 6.3, 9 mm | 7.75–8.85 | 400–1200 | 30–110 | 220–510 |
| Bae [18]           | Annular channel $d_{h} = 2$ mm | 8.12      | 400, 1200    | 50, 90  | 210–560  |
| Gupta et al. [19]  | Tube $d = 8$ mm | 8.4       | 910, 1610    | 90, 240 | 260–520  |
| Eter et al. [20]   | Bundle of 3 rods $d_{h} = 4.08$ mm | 7.7, 8.3 | 330–1170     | 60–130  | 220–410  |
| Eter et al. [21]   | Tube $d = 8$ mm | 7.67–8.41 | 194–314     | 41–88  | 236–490  |
| Liu et al. [22]    | Tube $d = 10$ mm | 7.45, 7.61 | 304, 902     | 122–205 | 237–400  |

| **Freon-12** $p_c = 4.14$ MPa, $t_c = 112.0^{\circ}$C |
| Richards et al. [23]| Bundle of 7 rods $d_{h} = 4.7$ mm | 4.6–4.7 | 450–1000     | 20–120  | 270–400  |

| **Freon-134a** $p_c = 4.06$ MPa, $t_c = 101.1^{\circ}$C |
| Zhang et al. [24]  | Tube $d = 7.6$ mm | 4.3       | 600          | 20, 30  | 320–410  |

3. Criteria for determining heat exchange intensity

The analysis showed that to obtain generalized dependences describing the experimental data on heat transfer to water and modeling fluids flowing in channels of different configurations, the hydraulic diameter $d_{h}$ can be used as the characteristic geometric dimension. In addition, it was obvious that in the final equation it is necessary to take into account the factors leading to an improvement or deterioration of heat exchange. As mentioned above, in the case of channels of small hydraulic diameter the effect of buoyancy on heat transfer seems to be small, therefore, it enabled main attention to be focused on thermally-bulk flow acceleration, which occurs when the density of the moving medium is significantly reduced with increasing enthalpy near the pseudo-critical temperature $t_{m}$. As a result, it was found that the influence of this factor on heat transfer can be taken into account if we
introduce the correction factor $Y$ into Equation (1) as a function of two dimensionless complexes $K_{A,m}$ and $K_{h}$, including the Reynolds number $Re_m = Gd/h$, the criterion determining the thermal acceleration, $K_{h} = q \beta_m (Gc_{p,m})$, and also the enthalpy factor $(h - h_m)/h_m$ in the form: $K_{A,m} = (Re_m \times 10^{-5})^{0.5}(K_{h} \times 10^{3})$ and $K_{h} = K_{A,m} \Delta h/h_m$, where $\Delta h = (h - h_m)$.

4. Recommendations for design calculations

The experimental data presented in Table 1 were processed in the form of the dependences of the ratio $Nu_{b,Exp}/Nu_{b,N}$ from $K_{h}$ with the criterion $K_{A,m}$ as a parameter. For water (Figure 1) and carbon dioxide (Figure 2), the data are combined into two separate groups, in which $K_{A,m} > 1$ (a) and $K_{A,m} < 1$ (b).

![Figure 1](image1)

**Figure 1.** Results of processing of data [3–13] for water: a) modes with improved heat exchange, $K_{A,m} = 1.53–2.02$; b) modes with deteriorated heat transfer, $K_{A,m} = 0.23–0.64$; • tubes, + annular channels, × rod bundles, dashed lines – calculation according to Equation (3) with $K_{A,m} = 1.67$ and 0.51.

![Figure 2](image2)

**Figure 2.** Results of processing of data [14–22] for carbon dioxide: a) modes with improved heat transfer, $K_{A,m} = 1.36–5.19$; b) modes with deteriorated heat exchange, $K_{A,m} = 0.26–1.07$; • tubes, + annular channels, × rod bundles, dashed lines – calculation by Equation (3) with $K_{A,m} = 2.33$ and 0.70.

The data for Freon-12 and Freon-134a do not reveal any specific features or signs of deterioration of heat transfer, so they are all presented on the same graph (Figure 3).

![Figure 3](image3)

**Figure 3.** Modes of normal and improved heat exchange in experiments with Freon: × rod bundle, Freon-12 [23], $K_{A,m} = 0.30–1.42$; • tube, Freon-134a [24], $K_{A,m} = 1.50–2.25$; dashed line – calculation by Equation (3).

The dependences shown in Figures 1–3 served as the basis for deriving the calculation relations for the main (normal, improved or deteriorated) heat exchange regimes, moreover the possibility of realizing mixed (transient) processes in the system was taken into account. The latter can arise as a consequence of the superposition and interaction of factors that determine the basic regimes. As a result, the correction function $Y$ for Equation (1) was expressed in the form of

$$Y = (1 - \zeta)Y_1 + \zeta Y_2,$$  (3)
where \( Y_{1,2} = 1 + a_{1,2} \exp(b_{1,2} K_h^2 + c_{1,2} K_h) \), and \( \zeta \) is the weight parameter determining the contribution of each of the functions \( Y_1 \) and \( Y_2 \) to the general process and depending on \( K_{A,m} \) according to the equation

\[
\zeta = \exp\left(-0.5 K_{A,m}^s\right).
\] (4)

Coefficients \( a_{1,2}, b_{1,2}, c_{1,2} \) and \( s \) were based on experimental data for a specific medium, their values were obtained mainly in definitely improved (index 1) and clearly deteriorated (index 2) modes of heat exchange. The found values of the coefficients are given in Table 2.

| Fluid          | \( a_1 \) | \( b_1 \) | \( c_1 \) | \( a_2 \) | \( b_2 \) | \( c_2 \) | \( s \) |
|---------------|--------|--------|--------|--------|--------|--------|------|
| Water         | 1.5    | -30    | 3      | -0.15  | -125   | -25    | 2    |
| Carbon dioxide| 1.8    | -60    | 0      | -0.3   | -50    | -15    | 1.5  |
| Freon         | 0.8    | -300   | -15    | -      | -      | -      | -    |

Thus, the final correlation recommended for calculating the heat transfer is

\[
\text{Nu}_b = \text{Nu}_0 \left( \frac{\rho_w}{\rho_b} \right)^{0.25} \left( \frac{c_p}{c_{p,b}} \right)^a Y_c, (5)
\]

where the correction factor \( Y(K_{A,m}, K_h) \) is calculated by the Equations (3) and (4) using the data given in Table 2.

5. Analysis of the results

The results of calculations of heat transfer by the Equations (1) – (5) turned out to be in satisfactory agreement with the data for different heat exchange regimes investigated in experiments with the motion of media of supercritical pressure (water, carbon dioxide, Freon) in round tubes, annular channels and rod bundles. The processing of more than three thousands of experimental points in a wide range of determining parameters showed that the spread of the experimental data near the calculated curves is generally less than \( \pm 15\% - 20\% \) with the exception of the region near the pseudo-phase transition line, where it can reach 25\% or more. For this region, certain instability of heat transfer is characteristic, and at fixed parameters of the process spontaneous transitions from one regime to another can occur. In comparison with the case of flow of a heat-transfer medium inside the pipes, the heat exchange process in bundles is more stable. Intensification of heat transfer in rod bundles is facilitated by devices that spacing the fuel elements.

Statistical analysis of deviations of calculated and experimental data has shown that the relative average arithmetic differences between calculation and experiment do not exceed 3\%, while the root-mean-square deviations are 15\% for water and 15–20\% for modelling media.

6. Conclusion

Thus, to describe heat transfer to media of supercritical parameters under conditions of a strong change in the thermophysical properties of the heat exchange medium, two new complex criteria \( K_{A,m} \) and \( K_h \) are obtained that determine the effect of thermal acceleration of the flow on the heat exchange process in the pseudo-phase transition region. On the basis of these criteria, an original method for calculating heat transfer in mixed heat exchange regimes is constructed. The proposed system of Equations (1) – (5) generalizes the experimental data for water and modelling media when they move inside pipes and annular channels, as well as during external flow of rods in bundles over a wide range of geometric and regime parameters in the supercritical region of states of matter.
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