We calculate the flux of ultra high energy photons from individual ordinary (i.e. non-superconducting) cosmic strings and compare the results with the sensitivity of current and proposed TeV and EeV telescopes. Our calculations give only upper limits for the gamma ray flux, since the source of the photons, jets from particle production at cusps, may be weakened by back reaction effects. For the usual cosmic distribution of strings, the predicted bursts from strings with the value of mass per unit length associated with galaxy formation or light strings may just be detectable. A diffuse gamma ray background from light strings may also be seen by the Fly’s
Eye detector at above $7 \times 10^{10}$ GeV.
I. INTRODUCTION

This work is motivated by the need for direct evidence of cosmic strings\(^1\). Even if cosmic string theory succeeds in describing the large scale structure of the universe\(^2\), its validity will ultimately rest on the detection of direct string signatures. Here we discuss the chance of detecting ultra high energy photon radiation from the cusp regions of individual nearby ordinary (i.e. non-superconducting) cosmic strings.

In a previous paper [Ref. 3; see also Refs. 4 & 5], we investigated the possibility of detecting ultra high energy (UHE) neutrino radiation from the cusps. We considered the case in which the total energy in the cusp region is released as extremely energetic particles almost instantaneously, about once per oscillation of the string loop. These primary emitted particles then decay down to some superheavy fermion scale \(Q_f\), at which point we apply an extrapolation of the QCD multiplicity function to determine the energy distribution of the final particles. The value of \(\mu\), the mass per unit string length, required for galaxy and large-scale structure formation is \(G\mu/c^2 \approx 10^{-6}\). We found that the UHE neutrino background from a distribution of strings with this value lies below present observational bounds\(^6,7\) in the energy range \(10^8 \text{ GeV} < E < 10^{11} \text{ GeV}\), even if the cusp mechanism is maximally effective. It is also smaller than the photoproduced flux expected below \(E \approx 10^{11} \text{ GeV}\) from cosmic ray collisions with the microwave background. As \(G\mu\) decreases, the cusp background increases until \(G\mu/c^2 \approx 10^{-15}\), due to a greater number of small loops, and then decreases\(^4\). For \(G\mu/c^2 \approx 10^{-15}\) it may exceed the observational bounds if \(Q_f \gtrsim 10^{15} \text{ GeV}\) and cusp annihilation works at maximal strength\(^3\). Cusp neutrinos may be more easily seen above \(E \approx 10^{11} \text{ GeV}\); or by detectors whose sensitivity matches the expected photoproduced background around \(10^{10} \text{ GeV}\) (if \(10^{-15} \lesssim G\mu/c^2 \lesssim 10^{-13}\)
and $Q_f \gtrsim 10^{15} \text{ GeV}$). Note, however, that the final energy distribution of decay neutrinos is highly uncertain. At the energies we are concerned with, the true neutrino flux may be either higher or lower than our approximation by a couple of orders of magnitude. In all cases, we found it extremely unlikely that neutrino bursts from individual cusps would ever be observed.

The photon emission from cusp decays, integrated over the strings in the Universe, should be very similar in shape and magnitude to the neutrino emission. The neutrino background $E^3 F(E)$ peaks at about $E = 10^{-1} Q_f$ and then decreases as $E$ decreases. The particles with lower energies today were emitted by strings at higher redshifts. However, photons emitted with energy $E \gtrsim 10^6 z^{-1} \text{ GeV}$ where $z$ is the redshift at the epoch of emission will have been affected by pair production off the cosmic microwave and radio backgrounds\textsuperscript{8}. Because of this, the predicted photon background is less likely to be observed than the neutrino background except above $10^{10} \text{ GeV}$. On the other hand, the conclusions with regard to the burst from an individual cusp are reversed. Present TeV ($= 10^3 \text{ GeV}$) detectors are much more sensitive to a photon burst from strings because an incoming photon has a much greater chance of interacting with the Earth’s atmosphere than an incoming neutrino. In this paper, we calculate the maximum photon cusp radiation from an individual string or background of strings and compare it with the detection capabilities of existing and proposed air shower array and Čerenkov telescopes. Our main assumption is that back reaction effects do not prevent cusps from forming, and that once formed cusps copiously produce particles. We conclude that searching for TeV gamma-ray bursts or a $10^{11} - 10^{12} \text{ GeV}$ gamma-ray background from strings probably represents the most likely way of detecting cusp radiation from strings, if it occurs.
Note that the recent detection \(^{44}\) of anisotropies in the microwave background by the COBE experiment is compatible \(^{45}\) with cosmic strings being the source of the seeds for structure formation and \(G\mu/c^2 \simeq 10^{-6}\). However, many particle physics models predict strings with a smaller mass per unit length. Such strings would be irrelevant for structure formation, but might have other observable consequences. It is important to look for independent constraints on such models. This is an additional motivation for our work.

In Section II, we recount cosmic string emission. In Section III we discuss the probability of detecting local strings or a cosmic background of strings in light of present and proposed TeV and EeV telescopes. \(c\) and \(G\) represents the speed of light and Newton’s constant, respectively, \(t_0\) denotes the present time and \(h\) is the Hubble parameter in units of \(100 \, \text{kms}^{-1} \, \text{Mpc}^{-1}\). The scenario \(^{9-11}\) in which superconducting cosmic strings \(^{12,13}\) emit UHE cosmic rays once the current in the string reaches a critical value, is unrelated to our process of cusp annihilation.

II. EMISSION FROM COSMIC STRINGS

Ordinary cosmic strings decay predominantly by gravitational radiation, losing energy at a rate \(^{14-16}\)

\[
P_g = \gamma G\mu c
\]  

where \(\mu\) is the mass per unit length of the string and \(\gamma\) is a constant of order \(40–100\). There are also two mechanisms for particle emission from ordinary strings. Both are suppressed by a large factor with respect to \(P_g\), at all but the final stages of string life. In the first mechanism, particle-antiparticle pairs are produced in the
background of a moving string loop. Applying lowest order perturbation theory, a string of microscopical width \( w \sim (\hbar/\mu c)^{1/2} \) radiates a power of \( P_a \sim \frac{wR}{\mu c^3} \).

\[ P_a \sim \frac{wR}{\mu c^3} \quad (2.2) \]

\( R \), the radius of the loop, is typically a cosmological length.

The second mechanism, “cusp annihilation”\(^{18,19} \), can be summarized as follows. Ignoring the small but finite string width and describing the loop trajectory by a world sheet \( \bar{x}(s, \tau) \), we can choose a gauge in Minkowski space for which \( \tau \) is the coordinate time \( t \) and \( s \) parametrizes the length along the string. The trajectories \( \bar{x}(s, \tau) \) are then solutions of the equations which follow from the Nambu action. These solutions are periodic in time and typically contain one or more pairs of cusps per oscillation - a cusp being a point \( (s, t) \) on the string world sheet where \( |\dot{\bar{x}}| = 1 \) and \( \bar{x}' = 0 \) (' denotes the derivative with respect to \( s \)). At a cusp, the assumptions under which one can show that string evolution is described by the Nambu action break down. Since two segments of the string overlap there, the microphysical forces are very strong and should lead to a smoothing out of the cusp by particle emission. Similar particle emission has been shown\(^{20} \) to occur close to the interaction point between two long straight intercommuting strings. In this paper, we will assume that the entire energy in the cusp region is released as particles. If we neglect back reaction, this assumption seems reasonable. However, back reaction may play a crucial role and prevent or diminish the chance of cusps forming. (In which case, the effect we are discussing would be much weaker. We shall see, though, that a background from strings may be detectable even if cusp annihilation does not work at full strength.)

By expanding the solutions of the string equations of motion about the cusp\(^9 \),
one can show\textsuperscript{18} that the s-parameter (comoving) length of the region where the two string segments overlap is

\[ \ell_c \sim w^{1/3}R^{2/3} \]  

The corresponding physical length obtained by evaluating \( x(s,t) \) at \( s = l_c \) is \( l_p \propto l_c^2R^{-1} \). The energy per unit comoving length is independent of the string velocity, whereas the energy per unit physical length contains the usual relativistic Lorentz factor, \( \gamma_L = R/l_c \) (when evaluated at \( s = l_c \)). Since the period of loop motion is \( R/c \), cusp annihilation produces a radiated power of

\[ P_c \sim \frac{\mu \ell_c c^3}{R} \sim \frac{\mu w^{1/3}c^3}{R^{1/3}} \]  

averaged over the loop period. Each annihilation should occur on the time scale associated with the energy scale of the string\textsuperscript{18}, i.e. \( \Delta t_{\text{cusp}} \simeq \ell_c/c \) in the frame comoving with the loop, while the time scale between each cusp forming is \( R/c \). In the inertial frame, the initial particles produced at the cusp will be beamed into a solid angle \( \gamma_L^{-2} \).

The primary particles emitted from the cusps will be the scalar and gauge particles associated with the fields which make up the cosmic string. These high energy particles then decay rapidly into jets of lower mass products. By conservation of quantum numbers, cusp annihilations should produce equal numbers of superheavy fermions and antifermions (up to CP violation effects\textsuperscript{21}) which decay into equal numbers of particles and antiparticles (up to the initial charge of the superheavy fermion and CP violation effects). In order to calculate the photon flux, we need to know how
many photons of energy $E$ are generated in the decay. In the final decay stages, we assume that the fragmentation proceeds via quarks, gluons and leptons and model it in a way consistent with current QCD multiplicity data. However, we should be cautious in extrapolating the empirical QCD multiplicity functions to arbitrarily high energies: at energies above the symmetry breaking scale of the field theory which gives rise to strings

$$\sigma = (\mu \hbar c^3)^{1/2} \simeq 3.1 \times 10^{19} \left( \frac{G \mu}{c^2} \right)^{1/2} \text{GeV},$$

a QCD-like extrapolation probably gives a poor description of fragmentation. The uncertainty in the fragmentation pattern at energies above $\sigma$ introduces a large uncertainty in our calculations. We could proceed thus in two ways.

In the first approach, the approach taken in this paper, we assume that the initial particles are emitted from the cusp with energy $\gg m_{pl}$ in the center of mass frame of the loop. The particles then fragment after a number of steps into particles with energy $Q_f \ll m_{pl}$, at which point we apply the extrapolation of the QCD multiplicity function for a jet of initial energy $Q_f$. We consider various values of $Q_f$ around $\sigma$. For simplicity the initial jet energy distribution is also assumed to be monochromatic - extension to a more general distribution is straightforward. The number of jets with initial energy $Q_f$ emitted from a loop per cusp annihilation is thus

$$N = \frac{\mu \ell c^2}{Q_f}$$

Or per unit time and averaged over the period,
\[ \hat{N} = \frac{P_c}{Q_f} \simeq \frac{\mu^{5/6} \bar{h}^{1/6} c^{17/6}}{Q_f R^{1/3}} \quad (2.7) \]

In the second approach, the initial jet energy is the energy of a Higgs particle emitted from the cusp in the center of mass frame, i.e. \( \sigma \gamma L \propto \sigma (\sigma R)^{1/3} \). We would then extrapolate the QCD fragmentation functions to that energy. This has the effect of substantially increasing \( Q_f \) in (3.13), thereby decreasing the chances of detecting a cusp annihilation. However, since it involves extending QCD-like behavior to energies above the Planck scale, we regard it as less realistic and do not consider it further here.

The observable photon spectrum, in both approaches, is dominated by the photons created by decaying neutral pions in the quark and gluon jets\(^2\). Any fragmentation function should continue down to at least energies around the pion rest mass \( m_{\pi^0} \simeq 135 \text{ MeV} \). Following Ref. 10 (and noting that QCD jets generate roughly equal numbers of each charged and neutral pion species), we describe the pion multiplicity distribution in the jets by

\[ dN'/dx = \frac{15}{16} x^{-3/2} (1 - x)^2 \quad (2.8) \]

for simplicity, where \( N'(x) \) gives the probability of finding a pion with energy \( E_{\pi^0} = xQ_f \) in the jet. Precise modelling of the multiplicity function even up to initial jet energies of \( Q_f \) is not possible from first principles. (A Drell-Yan-West approximation can be applied as \( x \to 1 \). Here however we are mainly interested in the \( x << 1 \) region since \( Q_f \) is usually much greater than the energy thresholds of detectors.) (2.8) implies that the final number of pions in the jet scales as \( \sqrt{Q_f} \). This roughly
equals the multiplicity growth seen in GeV-TeV collider experiments\textsuperscript{10}. The function also matches well the more stringent $x << 1$ approximation derived in Ref. 23. Since each pion decays into 2 photons, the final photon distribution is found by integrating (2.8) with invariant measure $dx/x$ over all $E_{\pi^0}$ greater than the photon energy $E$.

Using (2.6), this gives a photon distribution of

$$
\frac{dN}{dE} = \frac{15}{16} \frac{\mu_c^2}{Q_f^2} \left( \frac{16}{3} - 2x^{1/2} - 4x^{-1/2} + \frac{2}{3}x^{-3/2} \right) \bigg|_{x=E/Q_f} \quad (2.9)
$$

All loops will be emitting photons with energies between 0 and $Q_f$.

In both approaches, the final decay products will be spread over a solid angle $\Theta^2$ around the initial direction of the primary particle. If $< p_T >$ is the average transverse momentum in a jet whose initial energy is $Q$, $< N_{TOT} >$ is the mean total multiplicity and $p_{TOT}$ is the total momentum ($p_{TOT} \simeq Q/c$ in the relativistic limit), then\textsuperscript{24} $\Theta \simeq < N_{TOT} > < p_T > / p_{TOT}$. Using the results of Ref. 24 derived for QCD jets, $\Theta \simeq 1.29\alpha_s(Q^2)$ radians to first order in the strong coupling “constant” $\alpha_s(Q^2)$ where

$$
\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \ln (Q^2/\Lambda^2)} \left( 1 - \frac{6}{(33 - 2n_f)^2} \ln \left( \frac{Q^2/\Lambda^2}{(33 - 2n_f)^2} \right) \right) + \ldots \quad (2.10)
$$

In (2.10), $n_f$ is the number of quark flavors with mass less than $Q$ and $\Lambda$ is an empirically derived energy scale\textsuperscript{25}. $\Lambda$ depends on $n_f$ in such a way that (2.10) remains valid for all values of $Q$ at collider scales. Substituting $\Lambda^{(n_f=4)} = 238 \pm 43$ MeV, we then have
\[ \Theta \simeq \frac{0.97}{\ln(Q/\text{GeV}) + 1.44} \left( 1 - 0.74 \frac{\ln(\ln(Q^2/5.7 \times 10^{-2} \text{ GeV}^2))}{\ln(Q^2/5.7 \times 10^{-2} \text{ GeV}^2)} \right) \text{ radians} \quad (2.11) \]

Again, the true form of \( \Theta \) at initial jet energies \( Q_f \) is uncertain. However, because \( \Theta^2 \gg \gamma^{-2}_L \), we can say that after decay the emission from an annihilating cosmic string cusp will be beamed into a solid angle \( \Theta^2 \), not \( \gamma^{-2}_L \). During the annihilation process, the direction of the beam will continuously change at the rate given by \( \gamma_L \).

Following the method of Rybicki and Lightman (Ref. 26; see also Ref. 27) and noting that \( \Delta t_{\text{cusp}} \simeq \gamma^{-1}_L R/c \), one can show that an observer at Earth will remain in the cone of the beam defined by \( \Theta \) for a time

\[ \Delta t_{\text{beam}} \simeq \Theta \gamma^{-1}_L \Delta t_{\text{cusp}} \quad (2.12) \]

if \( \gamma_L \gg 1 \) and the timescale for the decay of the emission is ignored. However, \( \Delta t_{\text{beam}} \) and \( \gamma^{-1}_L \Delta t_{\text{cusp}} \) (the duration of cusp annihilation in the inertial frame) are much smaller than the timescale over which the emission decays. Thus the duration of the signal at the detector should be determined by the spreading out in arrival times of particles due to the decay process. Given the uncertainties in the process, this is extremely difficult to estimate. In the cascade process used to describe QCD jet decay prior to hadronization, the lowest momentum non-relativistic decay products remain closest to the creation point of the jet while the highest momentum ultra-relativistic products travel farthest, reaching a distance \( \hbar c/\Lambda \) in the center of mass frame from the creation point before hadronizing. Thus an estimate of the spread in arrival times for particles created in a QCD jet would \( \hbar/\Lambda \) appropriately Lorentz...
transformed into the observer’s frame. If a similar analysis can be applied in the case of cusp emission, an estimate of the duration of the burst observed at Earth could be

$$\Delta t_{\text{burst}} \simeq \frac{\hbar Q_f}{\Lambda^2}.$$  \hspace{1cm} (2.13)

### III. EXPECTED SIGNAL AT DETECTOR

Let us now consider the number density of cosmic strings given by the usual scale invariant distribution.\textsuperscript{1,29} We also include the effect of gravitational and cusp radiation\textsuperscript{18,19} on loops of small radius. The number density $n(R) dR$ of loops at the present time $t_0$ with radius in the interval $[R, R + dR]$ is then given by\textsuperscript{3,29}

$$n(R) = \begin{cases} 
\nu (ct_0)^{-2} R^{-2}, & \text{max} (R_{\text{eq}}, R_*) < R < ct_0 \\
\nu (ct_0)^{-2} R_{eq}^{1/2} R^{-5/2}, & R_* < R < R_{eq}, \ R_* < R_{eq} \\
\nu (ct_0)^{-2} R_*^{-2}, & R_{eq} < R < R_* , \ R_{eq} < R_* \\
\nu (ct_0)^{-2} R_{eq}^{1/2} R_*^{-5/2}, & R_{min} < R < \min (R_{eq}, R_*) \\
0, & R < R_{min} 
\end{cases}$$  \hspace{1cm} (3.1)

provided the following condition on $G\mu$ is met: $t_0 > 5.4 \times 10^{-44} \gamma^{-4} (G\mu)^{-9/2}$ sec, where

$$R_* = \gamma G\mu t_0 / c \simeq 1.2 \times 10^{30} \left( \frac{\hbar}{0.5} \right)^{-1} \left( \frac{\gamma}{10^2} \right) \left( \frac{G\mu}{c^2} \right) \text{ cm}$$

$$R_{eq} = ct_{eq} \simeq 5.8 \times 10^{21} \left( \frac{\hbar}{0.5} \right)^{-4} \text{ cm}$$  \hspace{1cm} (3.2)

$$R_{min} = ct_{eq}^{3/4} (\sigma / \hbar)^{-1/4} \simeq 5.9 \times 10^{12} \left( \frac{\hbar}{0.5} \right)^{-3/4} \left( \frac{G\mu}{c^2} \right)^{-1/8} \text{ cm}$$

for an $\Omega = 1$ Friedmann Universe. Here $t_{eq}$ is the time of equal radiation and matter.
density in the Universe, \( h \simeq 0.3 - 1 \) is the value of the Hubble parameter today, \( \gamma \simeq 40 - 100 \) and \( \nu \) is a constant of order 0.01 whose value must be determined in numerical simulations. Currently, \( \nu \) is still uncertain by a factor of at least 10 so we write

\[
\nu = \nu_{0.01} 10^{-2}.
\]  

(3.3)

**A. BURST FROM INDIVIDUAL STRING**

To estimate the probability of observing a burst from an individual nearby string, we first note that the observation of a cusp burst will be characterized by the almost simultaneous arrival of UHE photons from one position in the sky. Recall that an estimate of the burst timescale at observation may be

\[
\Delta t_{\text{burst}} \simeq \frac{\hbar}{\Lambda} \frac{Q_f}{\Lambda} \simeq 10^{-8} \left( \frac{Q_f}{10^{15} \text{ GeV}} \right) \text{ sec}
\]  

(3.4)

arising from the jet decay process. The photons will have the spectrum determined by the decay of the initial superheavy fermions. In the case of our approximation to the multiplicity function (2.9), we would predict a slope of \( E^{-3/2} \) at TeV energies (\( \ll Q_f \)).

Consider now a string loop of radius \( R \) which is located a distance \( d \) from Earth. Noting that the radiation after decay will be beamed into solid angle \( \Theta^2 \), the number per unit energy per unit area of photons of energy \( E \) expected at Earth from a single cusp annihilation is
\[ N_{\text{burst}}(E) \simeq \frac{1}{\Theta^2 d^2} \frac{\mu l c^2}{Q_f^2} \left( \frac{16}{3} - 2 \left( \frac{E}{Q_f} \right)^{1/2} - 4 \left( \frac{E}{Q_f} \right)^{-1/2} + \frac{2}{3} \left( \frac{E}{Q_f} \right)^{-3/2} \right) \] (3.5)

Here we have included the multiplicity approximation (2.9). The predicted number of air showers at the detector above an energy threshold \( E_D \) is then

\[ S_{\text{burst}}(> E_D) = \int_{E_D}^{\infty} N_{\text{burst}}(E) A_D \, dE \] (3.6)

where \( A_D \) is the effective area of the detector.

The telescope may also see the background of cosmic-ray induced showers\(^{30}\),

\[ \dot{N}_{CR}(> E) \simeq 0.2 \left( \frac{E}{\text{GeV}} \right)^{-1.5} \text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1} \] (3.7)

This would produce a background in the detector of

\[ S_{\text{bgnd}}(> E_D) \simeq 7 \times 10^{-2} \Delta t_D \left( \frac{\theta}{10^{-3} \text{sr}} \right) \left( \frac{A_D}{10^{10} \text{cm}^2} \right) \left( \frac{E_D}{10^8 \text{GeV}} \right)^{-1.5} \] (3.8)

showers per angular resolution \( \theta \) of the detector over the time \( \Delta t_D \gg \Delta t_{\text{burst}} \). The new generation of atmospheric Čerenkov and air shower array telescopes\(^{31}\), currently under construction or proposed, typically have energy thresholds of \( E_D \simeq 10 - 100 \text{ TeV} \), collection areas in excess of \( A_D \simeq 10^8 - 10^{10} \text{ cm}^2 \) and angular resolutions less than \( 4 \times 10^{-4} \text{ sr} \). They should be able to reject hadron-induced showers from photon-induced showers down to photon/hadron ratios of \( 10^{-4} - 10^{-5} \). For these
specifications, the expected cosmic-ray or extragalactic photon background in the telescope over the burst timescale $\Delta t_{\text{burst}}$ is negligible. Hence, to detect a cusp burst, we simply require that

$$S_{\text{burst}}(> E_D) \gtrsim n_\gamma$$

(3.9)

where $n_\gamma \approx 1 - 5$ is the minimum number of showers required to register as a burst.

The logic of the burst analysis is as follows. In order for a burst to be seen, the probability of a loop producing a cusp over a given period of time must be sufficiently large. This leads to a condition $R < R_D$ for loops to be observable (see below). Next, we note that the closest loop with radius $R < R_D$ must lie within a distance $d_c(R, E_D)$ for its signal to be strong enough in the detector. Thus, the mean separation $d(R)$ of loops of radius $R$ must be smaller than $d_c(R, E_D)$. As we shall see, for $R < R_D$ the ratio $d_c(R, E_D)/d(R)$ decreases as $R$ decreases. Hence, the condition which must be satisfied if we are to observe any burst is

$$\frac{d_c(R_D, E_D)}{d(R_D)} > 1.$$  

Equations (2.3), (3.6) and (3.9) imply that a string closer than
\[
d_c(R, E_D) \approx \left( \frac{A_D}{4\pi n_\gamma} \right)^{1/2} \Theta^{-1} \left( \frac{\mu c}{Q_f} \right)^{1/2} \left( \frac{E_D}{Q_f} \right)^{1/2} \\
\left( \frac{-16}{3} + \frac{4}{3} \left( \frac{E_D}{Q_f} \right)^{1/2} + 8 \left( \frac{E_D}{Q_f} \right)^{-1/2} + 4 \left( \frac{E_D}{Q_f} \right)^{-3/2} \right)^{1/2} \\
\simeq 1.0 \times 10^9 \left( \frac{A_D/n_\gamma}{\text{cm}^2} \right)^{1/2} \Theta^{-1} \left( \frac{\sigma}{Q_f} \right)^{1/2} \left( \frac{\sigma}{10^{15} \text{ GeV}} \right)^{1/3} \left( \frac{R}{\text{cm}} \right)^{1/3} \left( \frac{E_D}{Q_f} \right)^{1/2} \\
\cdot \left( \frac{-16}{3} + \frac{4}{3} \left( \frac{E_D}{Q_f} \right)^{1/2} + 8 \left( \frac{E_D}{Q_f} \right)^{-1/2} + 4 \left( \frac{E_D}{Q_f} \right)^{-3/2} \right)^{1/2} \text{cm}
\] (3.10)

will be seen by the detector. On the other hand, the average frequency of cusp annihilations, \( f(R) = cR^{-1} \). Hence strings with radius \( R_D \simeq 10^{18} - 10^{19} \, \text{cm} \) should produce cusps at a rate \( 0.1 - 1 \, \text{yr}^{-1} \) (which we regard as a minimum detectable rate).

For all values of \( G\mu/c^2 \), \( R_D \) is much less than \( R_{eq} \) and much greater than \( R_{min} \). If

\[
\frac{G\mu}{c^2} \gtrsim 8.2 \times 10^{-13} \left( \frac{h}{0.5} \right) \left( \frac{\gamma}{10^2} \right)^{-1} \left( \frac{R_D}{10^{18} \, \text{cm}} \right),
\] (3.11)

then \( R_D \) is less than \( R_* \). Because the chance of being in the beam from an individual cusp is \( \Theta^2/4\pi \), the closest observed loop of radius less than or equal to \( R_D \) should lie within a distance

\[
d(R_D) \simeq \left( \frac{\Theta^2}{3} \int_{R_{min}}^{R_D} n(R) dR \right)^{-1/3} \\
\begin{cases} 
1.4\Theta^{-2/3} \nu^{-1/3} (ct_0)^{2/3} R_{eq}^{-1/6} R_*^{5/6} R_D^{-1/3}, & R_{min} < R_D < \min (R_{eq}, R_*) \\
1.4\Theta^{-2/3} \nu^{-1/3} (ct_0)^{2/3} R_{eq}^{-1/6} \left( \frac{5}{3} R_*^{-3/2} - \frac{2}{3} R_D^{3/2} \right)^{-1/3}, & R_* < R_D < R_{eq}
\end{cases}
\] (3.12)

using (3.1). The ratio of the distances given in (3.10) and (3.12) becomes
are large uncertainties in our knowledge of $\nu$

At first glance this ratio lies well below 1. This however may not be so. There
Evaluating the fragmentation function at $R = R_0$, $E_D$ dominates and so we finally have

$$
\frac{d_e(R_D, E_D)}{d(R_D)} \approx \begin{cases}
1 \times 10^{-12} \left( \frac{A_D/n_{\gamma}}{10^{10} \text{ cm}^2} \right)^{1/2} \left( \frac{R_D}{10^{15} \text{ cm}} \right)^{2/3} \left( \frac{\nu}{10^{-2}} \right)^{1/3} \left( \frac{h}{0.5} \right)^{5/6} \\
\cdot \left( \frac{\gamma}{10^7} \right)^{-5/6} \left( \frac{10^3 \sigma}{\text{GeV}} \right)^{-5/6} \left( \frac{Q_f}{10^{15} \text{ GeV}} \right)^{-1} \left( \frac{E_D}{10^9 \text{ GeV}} \right)^{1/2} \\
\cdot \left( -\frac{16}{3} + \frac{4}{3} \left( \frac{E_D}{Q_f} \right)^{1/2} + 8 \left( \frac{E_D}{Q_f} \right)^{-1/2} + \frac{4}{3} \left( \frac{E_D}{Q_f} \right)^{-3/2} \right)^{1/2}, \quad R_{\text{min}} < R_D < \min (R_{\text{eq}}, R_*)
\end{cases}
$$

(3.13)

If we evaluate the fragmentation function at $E_D = 10$ TeV, the $(E_D/Q_f)^{-3/2}$ term dominates and so we finally have

$$
\frac{d_e(R_D, E_D)}{d(R_D)} \approx \begin{cases}
7 \times 10^{-9} \left( \frac{A_D/n_{\gamma}}{10^{10} \text{ cm}^2} \right)^{1/2} \left( \frac{R_D}{10^{15} \text{ cm}} \right)^{2/3} \left( \frac{\nu}{10^{-2}} \right)^{1/3} \left( \frac{h}{0.5} \right)^{5/6} \\
\cdot \left( \frac{\gamma}{10^7} \right)^{-5/6} \left( \frac{G_{\mu}}{c^2} \right)^{-5/6} \left( \frac{Q_f}{10^{15} \text{ GeV}} \right)^{-1/4} \left( \frac{E_D}{10^9 \text{ GeV}} \right)^{-1/4}, \quad R_{\text{min}} < R_D < \min (R_{\text{eq}}, R_*)
\end{cases}
$$

(3.14)

At first glance this ratio lies well below 1. This however may not be so. There
are large uncertainties in our knowledge of $\nu$, $\gamma$ and the Hubble constant (which is
probably $> 0.5$). More importantly, the true form of the fragmentation function at
these energies is unknown, as is the value of $Q_f$ at which we can apply a QCD-like extrapolation. With regard to the detector, the ratio can be increased by increasing the observing time of the detector, thereby increasing $R_D$ in (3.14); by increasing the effective area of the detector $A_D$; or by decreasing the threshold energy $E_D$ of the detector. In summary, it is a possibility that TeV detectors may register cusp bursts from an individual cosmic string. We also stress that the ratio (3.14) is derived assuming that the cusp annihilation mechanism works at full efficiency.

Since it may seem unnatural to fix $Q_f$, the mass of the initial superheavy particles emitted from the cusp, while varying $G\mu$, we also set $Q_f = \sigma$ in which case (3.14) becomes

$$
\frac{d_c(R_D, E_D)}{d(R_D)} \approx \left\{ \begin{array}{ll}
6 \times 10^{-10} \left( \frac{A_D/n}{10^{10} \text{ cm}^2} \right)^{1/2} \Theta^{-1/3} \left( \frac{R_D}{10^{18} \text{ cm}} \right)^{2/3} \left( \frac{\nu}{10^{-2}} \right)^{1/3} \left( \frac{h}{0.5} \right)^{5/6} \\
\cdot \left( \frac{\gamma}{10^2} \right)^{-5/6} \left( \frac{G\mu}{c^2} \right)^{-13/24} \left( \frac{E_D}{10^5 \text{ GeV}} \right)^{-1/4}
\end{array} \right.
$$

$$
R_{\text{min}} < R_D < \min (R_{\text{eq}}, R_*)
$$

$$
7 \times 10^{-6} \left( \frac{A_D/n}{10^{10} \text{ cm}^2} \right)^{1/2} \Theta^{-1/3} \left( \frac{R_D}{10^{18} \text{ cm}} \right)^{1/3} \left( \frac{\nu}{10^{-2}} \right)^{1/3} \left( \frac{h}{0.5} \right)^{1/2} \\
\cdot \left( \frac{\gamma}{10^2} \right)^{-1/2} \left( \frac{G\mu}{c^2} \right)^{-5/24} \left( \frac{E_D}{10^5 \text{ GeV}} \right)^{-1/4}
$$

$$
R_* < R_D < R_{\text{eq}}
$$

(3.15)

If we chose $\gamma = 100$, $h = 0.75$, $\nu = 0.03$ and $G\mu/c^2 = 3 \times 10^{-7}$ (since these resemble the values used in the cosmic string model of galaxy formation), the ratio becomes

$$
\frac{d_c(R_D, E_D)}{d(R_D)} \approx 4 \times 10^{-6} \left( \frac{A_D/n\gamma}{10^{10} \text{ cm}^2} \right)^{1/2} \Theta^{-1/3} \left( \frac{R_D}{10^{18} \text{ cm}} \right)^{2/3} \left( \frac{E_D}{10^5 \text{ GeV}} \right)^{-1/4}
$$

(3.16)
Similarly for $G\mu/c^2 \simeq 10^{-12}$, we have

$$\frac{d_c(R_D, E_D)}{d(R_D)} \simeq 2 \times 10^{-3} \left( \frac{A_D/n_\gamma}{10^{10} \text{ cm}^2} \right)^{1/2} \Theta^{-1/3} \left( \frac{R_D}{10^{18} \text{ cm}} \right)^{1/3} \left( \frac{E_D}{10^5 \text{ GeV}} \right)^{-1/4}$$

(3.17)

for a typical TeV detector. Thus strings with small $G\mu$ may be more easily seen.

We must check that the cusp photons are not cut off by pair-production off the cosmic background photons in their travel to the detector. The distance to the nearest string of radius $R_D$ should be

$$d(R_D) \simeq \begin{cases} 
3 \times 10^{10} \Theta^{-2/3} \left( \frac{R_D}{10^{18} \text{ cm}} \right)^{-1/3} \left( \frac{\nu}{0.03} \right)^{-1/3} \left( \frac{h}{0.5} \right)^{-5/6} \left( \frac{\gamma}{10^2} \right)^{5/6} \left( \frac{G\mu}{c^2} \right)^{5/6} \text{ Mpc,} \\
R_{\text{min}} < R_D < \min (R_{\text{eq}}, R_*) \\
3 \times 10^6 \Theta^{-2/3} \left( \frac{\nu}{0.03} \right)^{-1/3} \left( \frac{h}{0.5} \right)^{-1/2} \left( \frac{\gamma}{10^2} \right)^{1/2} \left( \frac{G\mu}{c^2} \right)^{1/2} \text{ Mpc,} \\
R_* < R_D < R_{\text{eq}} 
\end{cases}$$

(3.18)

from (3.12). This compares to an absorption probability of $\kappa_{\gamma\gamma} \simeq 10^{-3} \text{ Mpc}^{-1}$ for $E \simeq 10^5 \text{ GeV}$ photons in intergalactic space. (The absorption rises quickly above $10^5 \text{ GeV}$ to $\kappa_{\gamma\gamma} \lesssim 10^2 \text{ Mpc}^{-1}$ for $E \simeq 10^6 \text{ GeV}$ and then falls off with a slope of about $E^{-1}$. Below $10^5 \text{ GeV}$, the absorption rises to $\kappa_{\gamma\gamma} \lesssim 5 \times 10^{-3} \text{ Mpc}^{-1}$ at $10^3 - 10^4 \text{ GeV}$, due to pair-production off light from Population II stars, and then falls off steeply.) Thus the strings at a distance $d(R_D)$ should be just within the visible Universe at $10^5 \text{ GeV}$ if $G\mu/c^2 \simeq 3 \times 10^{-7}$. Strings with smaller values of $G\mu$ lie further within the observable Universe at $10^5 \text{ GeV}$. It is also relevant to mention the Fly’s Eye detector. The Fly’s Eye detector
is capable of seeing showers induced by ultra-high energy photons, although they have not yet been observed. $10^9 - 10^{10} \text{ GeV}$ photons should be visible out to $1 - 10 \text{ Mpc}$, while $E \gtrsim 10^{11} \text{ GeV}$ photons will be affected by pair-production off the Earth’s magnetic field depending on their angle of incidence$^{32}$. The effective area of the detector is $A_D \simeq 10^{13} \text{ cm}^2$ at $10^9 \text{ GeV}$ and increases slightly at higher energies. Thus, if $G\mu/c^2 \simeq 10^{-10} - 10^{-11}$, (3.14) implies that the Fly’s Eye detector may offer a $2 - 3$ times greater chance of detecting a burst at $10^9 - 10^{10} \text{ GeV}$ than do TeV telescopes at lower energies.

**B. GAMMA RAY BACKGROUND FROM STRINGS**

Regardless of whether individual bursts from cusps may be detected, the combined radiation from cusps which have annihilated over the history of the Universe will contribute a diffuse component to the cosmic gamma ray background. To calculate the number density $F(E)dE$ of photons in the energy range $[E, E+dE]$, we must first integrate over all times $t$ when photons with present energy $E$ were emitted. At each $t$ we must also integrate over all loops contributing to the emission. The number density of loops at an earlier epoch is given by (3.1) and (3.2) with $t_0$ replaced by $t$, for $R_{\text{min}}(t) \leq R_*(t)$ i.e. $t \geq t_B$ where

$$t_B = 5.4 \times 10^{-44} \gamma^{-4} \left(G\mu/c^2\right)^{-9/2} \text{ sec; }$$

and

$$n(R) = \begin{cases} \nu (ct)^{-2} R^{-2}, & R_{\text{eq}} < R < ct \\ \nu (ct)^{-2} R_{\text{eq}}^{1/2} R^{-5/2}, & R_{\text{min}} < R < R_{\text{eq}} \end{cases}$$

(3.19)

for $t \leq t_B \text{ sec}$. Thus using (2.7), we can write
\[ F(E) = \int z^{-3}(t)f(z(t)E, t) \, dt \]  \hspace{1cm} (3.20)

where

\[ f(zE, t) = \frac{z}{Q_f} \frac{dN}{dx} \bigg|_{x=zE/Q_f} \int_{R_{\text{min}}}^{c t} n(R, t) \frac{\mu^{5/6} h^{1/6} c^{17/6}}{Q_f R^{1/3}} \, dR \]  \hspace{1cm} (3.21)

and the redshift \( z \) has been included in the number density and energy. (\( z = 1 \) in the present epoch.) It is also convenient to change variables from \( t \) to \( z \). Hence we obtain

\[ F(E) = \frac{\mu^{5/6} h^{1/6} c^{17/6}}{Q_f^2} \int dz \frac{dt}{dz} z^{-2} \bigg|_{x=zE/Q_f} \int_0^{t(z)} n(R, z) \frac{\mu^{5/6} h^{1/6} c^{17/6}}{Q_f R^{1/3}} \, dR \]  \hspace{1cm} (3.22)

The \( z \) integral runs over \([1, \min(Q_f/E, z_{CO})]\) where \( z_{CO} \) is the redshift at which a photon of energy \( z_{CO} E \) is cut off by interactions with the ambient matter or radiation in the Universe. (\( z_{CO} \) is discussed below.)

Let us assume that \( R_*(t) < R_{eq} \), \( zE << Q_f \) (so that the \( x^{-3/2} \) term in the \( dN/dx \) approximation, (2.9), dominates) and \( z_{CO} << z_{eq} \). The condition \( t \leq, \geq t_B \) implies that three cases must be considered when integrating (3.22): \( z_{CO} \leq z_B; z_{CO} \geq z_B \geq 1; \) and \( z_{CO} > 1 \geq z_B \) where \( z_B = (t_0/t_B)^{2/3} = 5.2 \times 10^{45} (\gamma/10^2)^{8/3} (G\mu/c^2)^3 \) is the redshift corresponding to \( t_B \).

Firstly if \( z_{CO} \leq z_B \), (3.22) becomes
\[
E^3 F(E) \simeq \frac{45}{16} h^{1/2} c^{1/2} Q_f \left( \frac{E}{Q_f} \right)^{3/2} \nu t_0 \mu^{5/6} \left[ \frac{10}{11} \left( \frac{\gamma G \mu}{c^2} \right)^{-11/6} t_{eq}^{1/2} t_0^{1/2} t_{eq}^{-23/6} z^{3/4} \right.
\]

\[
- \frac{1}{4} t_0^{-10/3} \ln z - \frac{3}{88} t_{eq}^{-4/3} t_0^{-2} z^{-2}
\]

\[
- \frac{1}{2} h^{1/12} c^{-1/4} \mu^{-1/12} \left( \frac{\gamma G \mu}{c^2} \right)^{-5/2} t_{eq}^{1/2} t_0^{-4} z
\]

\[
z_{CO}(E) \left( \frac{E}{0.1 \text{ GeV}} \right)^{3/2} \left( \frac{Q_f}{10^{15} \text{ GeV}} \right)^{-1/2} \left( \frac{\nu}{10^{-2}} \right)
\]

\[
\cdot \left[ 2.9 \times 10^{-10} \left( \frac{\gamma}{10^2} \right)^{-11/6} \left( \frac{G \mu}{c^2} \right)^{-1} \left( \frac{h}{0.5} \right)^{5/6} z^{3/4} \right.
\]

\[
- 2.0 \times 10^4 \left( \frac{G \mu}{c^2} \right)^{5/6} \left( \frac{h}{0.5} \right)^{19/3} z^{-2}
\]

\[
- 5.3 \times 10^{-22} \left( \frac{\gamma}{10^2} \right)^{-5/2} \left( \frac{G \mu}{c^2} \right)^{-7/4} \left( \frac{h}{0.5} \right)^z
\]

\[
z_{CO}(E) \left( \frac{E}{E_0} \right) \left( \frac{E_0}{1 \text{ eV}} \right)^2 m^{-2} \text{ sec}^{-1}
\]

(3.23)

If \( z_{CO} \geq z_B \geq 1 \),

\[
E^3 F(E) \simeq \frac{45}{16} h^{1/2} c^{1/2} Q_f \left( \frac{E}{Q_f} \right)^{3/2} \nu t_0 \mu^{5/6} \left[ \frac{10}{11} \left( \frac{\gamma G \mu}{c^2} \right)^{-11/6} t_{eq}^{1/2} t_0^{1/2} t_{eq}^{-23/6} z^{3/4} \right.
\]

\[
- \frac{1}{4} t_0^{-10/3} \ln z - \frac{3}{88} t_{eq}^{-4/3} t_0^{-2} z^{-2}
\]

\[
- \frac{1}{2} h^{1/12} c^{-1/4} \mu^{-1/12} \left( \frac{\gamma G \mu}{c^2} \right)^{-5/2} t_{eq}^{1/2} t_0^{-4} z
\]

\[
\left. \frac{32}{11} h^{-11/48} G^{-11/48} c^{-55/48} \left( \frac{\gamma G \mu}{c^2} \right)^{11/48} t_{eq}^{1/2} t_0^{-27/8} z^{1/16} \right]_{z_B}
\]

\[
- \frac{1}{4} t_0^{-10/3} \ln z - \frac{3}{88} t_{eq}^{-4/3} t_0^{-2} z^{-2}
\]

\[
z_{CO}(E) \left( \frac{E}{E_0} \right) \left( \frac{E_0}{1 \text{ eV}} \right)^2 m^{-2} \text{ sec}^{-1}
\]

(3.23)
\[
\begin{align*}
& \simeq \left( \frac{E}{0.1 \text{ GeV}} \right)^{3/2} \left( \frac{Q_f}{10^{15} \text{ GeV}} \right)^{-1/2} \left( \frac{\nu}{10^{-2}} \right) \\
& \cdot \left\{ 
2.9 \times 10^{-10} \left( \frac{\gamma}{10^2} \right)^{-11/6} \left( \frac{G\mu}{c^2} \right)^{-1} \left( \frac{h}{0.5} \right)^{5/6} z^{3/4} \\
& -5.3 \times 10^{-22} \left( \frac{\gamma}{10^2} \right)^{-5/2} \left( \frac{G\mu}{c^2} \right)^{-7/4} \left( \frac{h}{0.5} \right) z \right\}^{z_B} \\
& - \left[ 2.0 \times 10^4 \left( \frac{G\mu}{c^2} \right)^{5/6} \left( \frac{h}{0.5} \right)^{19/3} z^{-2} \right]^{z_B} \\
& + \left[ 8.9 \times 10^{18} \left( \frac{G\mu}{c^2} \right)^{17/16} \left( \frac{h}{0.5} \right)^{1/8} z^{1/16} \right]^{z_B} \right\} \text{eV}^2 \text{ m}^{-2} \text{ sec}^{-1}
\end{align*}
\]

and if \( z_{CO} > 1 \geq z_B \),

\[
E^3 F(E) \simeq \frac{45}{16} h^{1/6} c^{1/2} Q_f \left( \frac{E}{Q_f} \right)^{3/2} \nu t_0 t^{5/6}
\]

\[
\cdot \left[ \frac{32}{11} h^{-11/48} G^{-11/48} c^{-55/48} \left( \frac{G\mu}{c^2} \right)^{11/48} t_{eq}^{1/2} t_0^{-27/8} z^{1/16} \\
- \frac{1}{4} t_0^{-10/3} \ln z - \frac{3}{88} t_{eq}^{-4/3} t_0^{-2} z^{-2} \right]^{z_{CO}(E)} \\
\simeq \left( \frac{E}{0.1 \text{ GeV}} \right)^{3/2} \left( \frac{Q_f}{10^{15} \text{ GeV}} \right)^{-1/2} \left( \frac{\nu}{10^{-2}} \right) \\
\cdot \left[ 8.9 \times 10^{18} \left( \frac{G\mu}{c^2} \right)^{17/16} \left( \frac{h}{0.5} \right)^{1/8} z^{1/16} \right]^{z_{CO}(E)} \text{ eV}^2 \text{ m}^{-2} \text{ sec}^{-1}
\]

The dominant interaction suffered by the extragalactic photons is pair production off nuclei, if the photon energy at the relevant epoch is 65 MeV \( \lesssim E' \lesssim 100 \text{ GeV} \), or pair production off cosmic background photons if \( E' \gtrsim 100 \text{ GeV} \). The former process cuts off the emitted photons at a redshift\(^{33}\)
\[ z_{CO} \simeq 3.5 \times 10^2 h^{-2/3} \left( \frac{\Omega_p}{0.2} \right)^{-2/3} \]  

(3.26)

where \( \Omega_p \) is the present cosmological proton density as a fraction of the critical 
density. Above \( E' \simeq 100 \) GeV, we follow the method of Refs. 8 to calculate \( z_{CO} \). The optical depth of the universe to a photon emitted at a redshift \( z' \) with an energy corresponding to a redshifted energy today of \( E \) is

\[ \tau (E, z') = \int_{1}^{z'} \kappa_{\gamma \gamma} (E, z) \frac{dl}{dz} \, dz \]  

(3.27)

Here \( \kappa_{\gamma \gamma} (E, z) \) is the absorption probability per unit length and

\[ \frac{dl}{dz} = c H_0^{-1} z^{-5/2} \]

for an \( \Omega = 1 \) Friedmann universe, \( H_0 = 100h \) km sec\(^{-1}\) Mpc\(^{-1}\) and \( z' < z_{eq} \). Focussing on \( z = 1 \) for the moment, we can write

\[ \kappa_{\gamma \gamma} (E) = \frac{\pi e^4 m_e^2 c^4}{E^2} \int_{m_e^2 c^4/E}^{\infty} \epsilon^{-2} n(\epsilon) \phi(\epsilon) \, d\epsilon \]  

(3.28)

where

\[ \phi(\epsilon) = \frac{\epsilon E/(m_e^2 c^4)}{\int_{1}^{\infty} \frac{2m_e^2 c^4 \sigma_{\gamma \gamma}(s)}{\pi e^4} \, ds} \]

In (3.28), \( \epsilon \) is the energy of the cosmic background photon, \( n(\epsilon) \) is the number
density per unit energy of cosmic background photons and $\sigma_{\gamma\gamma}(s)$ is the total cross-section for the process $\gamma + \gamma \rightarrow e^+ + e^-$ as a function of the electron or positron velocity in the centre of mass frame, $\beta = (1 - 1/s)^{1/2}$. The threshold for $e^+e^-$ pair production is $\epsilon E = m_e^2c^4$. We then find the cutoff redshift $z_{CO}(E)$ by incorporating the relevant redshift dependence into $\epsilon$, $n(\epsilon)$ and $\epsilon$ in $\phi(\epsilon)$ in (3.28) and solving (3.27) for $\tau(E, z_{CO}) = 1$.

The cosmic microwave background in the present era extends between $2 \times 10^{-6}$ eV $< \epsilon < 6 \times 10^{-3}$ eV and is accurately described\textsuperscript{34} by $n(\epsilon) = (hc)^{-3}(\epsilon/\pi)^2 (\exp^{\epsilon/kT} - 1)^{-1}$ with $T_0 = 2.735(\pm 0.06)$ K. At earlier epochs, $\epsilon$ should be replaced by $z\epsilon$ and $T_0$ by $zT_0$. A cosmic radio background has been observed\textsuperscript{35} below $\epsilon \simeq 2 \times 10^{-6}$ eV. The $\epsilon n(\epsilon)$ radio spectrum peaks at $\epsilon \simeq 10^{-8}$ eV and extends down to $\epsilon \simeq 10^{-9}$ eV. At lower energies, the cosmic background can not be seen because of inverse bremsstrahlung (free-free) absorption of radio photons by electrons in the interstellar medium. We will assume that the extragalactic radio spectrum continues to fall off with the same slope down to $10^{-11}$ eV. The origin of the radio background is not known but it is postulated to be the integrated emission of all unresolved extragalactic radio sources\textsuperscript{36} and to be modified below the peak by free-free absorption by intergalactic gas\textsuperscript{8}. Since the evolution of intergalactic gas is unknown, we will also assume for simplicity that $\kappa_{\gamma\gamma}(E, z) \propto z$ at radio frequencies. The true redshift dependence at these frequencies can effect our results little since $z_{CO} \simeq 1$.

Figure 1 plots $z_{CO}(E)$, the cutoff redshift, as found from (3.27) using a variation of Simpson’s rule for numerical integration\textsuperscript{37}. $z_{CO}$ decreases sharply to a minimum at $E \simeq 10^6$ GeV due to pair production off the microwave background and then falls off gradually. Pair production off the radio background dominates at about $5 \times 10^9$
GeV with the greatest effect at \( E \simeq 4 \times 10^{10} \) GeV. (The maximum radio absorption and the energy at which it occurs are two orders of magnitude smaller than the values presented in Refs. 8. In those References, the radio background was inexactely modelled prior to its observation.) There is also an absorption component due to double pair production\(^{38}\) off the microwave background, \( \gamma + \gamma \rightarrow e^+ e^- e^+ e^- \). In the \( s \rightarrow \infty \) limit, the cross-section for double pair production is approximately constant, \( \sigma'_{\gamma\gamma} \simeq 6.5 \times 10^{-30} \) cm\(^2\), and corresponds to an absorption probability of \( \kappa'_{\gamma\gamma} \simeq 6 \times 10^{-27} \) cm\(^{-1}\). This is considerably weaker than the absorption probability for single pair production if \( 10^5 < E < 10^{13} \) GeV. Applying the results of the \( z_{CO} \) calculation, we can restate in terms of \( E \) the conditions given after (3.25) for the dominant interaction: the photons are cut off by pair production off nuclei if their present energy would lie in the range \( 1.8 \times 10^{-4} h^{2/3} (\Omega_p/0.2)^{2/3} \lesssim E \lesssim 0.2 - 0.4 \) GeV or by pair production off the microwave background if \( E \gtrsim 0.2 - 0.4 \) GeV.

In Figure 2, we plot \( E^3 F(E) \) for various values of \( G\mu/c^2 \) and the superheavy fermion scale \( Q_f \). We can see from the curves that the flux is greatest if \( G\mu/c^2 \simeq 10^{-15} \) and falls off quickly for smaller values of \( G\mu/c^2 \) (due to the evaporation of string loops by cusp and gravitational radiation). It also increases as \( Q_f \) decreases. The \( G\mu/c^2 = 10^{-15} \) flux is displayed in greater detail in Figure 3. Since again it may be unnatural not to set \( Q_f \) by the symmetry breaking scale associated with the string, Figure 4 shows \( E^3 F(E) \) for \( Q_f = \sigma \). In this case, the flux is maximized at highest energies if \( G\mu/c^2 \simeq 10^{-15} \).

In all Figures, the dip at \( E \simeq 10^5 \) GeV is produced by absorption off the microwave background. This effect weakens above \( 10^6 \) GeV. Absorption off the radio background becomes important at \( E \simeq 10^{10} \) GeV, as marked by the kink in the
spectra. Because of our approximation to the multiplicity function (2.9), all spectra cut off abruptly at $E = Q_f$. The true multiplicity function should approach zero sharply\(^\text{10}\) between $0.8Q_f < E < Q_f$. However, given the uncertainty in extrapolating the collider multiplicity function to high energies and the uncertainty in the cusp emission process and the initial energy of the particles coming off the cusp (which we assumed to be monochromatic for simplicity), further modelling of this region is not justified.

We also plot an extrapolation of the observed $35 - 150$ MeV extragalactic gamma ray data\(^\text{39}\)

$$E^3F(E) = 3.3 (\pm 0.6) \times 10^{16} \left( \frac{E}{0.1 \text{ GeV}} \right)^{0.6 \pm 0.2} \text{m}^{-2} \text{sec}^{-1} \text{eV}^2$$

on Figures 2-4. No measurements of the diffuse gamma ray background above 150 MeV have been made. The EGRET experiment, currently flying, will be capable of detecting photons up to 20 GeV. It is not known, though, if the extragalactic background extends to energies above 150 MeV or, if it exists, if it can be resolved - even at high Galactic latitudes - out of the Galactic background which falls off less steeply around 100 MeV. Between $10^4 - 10^6$ GeV, there is also a competing predicted diffuse flux arising from the interaction of extragalactic UHE cosmic rays with the microwave background\(^\text{40}\). In this process, cosmic rays pair-produce and inverse Compton scatter off the microwave background creating cascade photons. The postulated cosmic ray-induced flux is of order $E^3F(E) \simeq 10^{20} \text{eV}^2 \text{m}^{-2} \text{sec}^{-1}$ at its peak ($E \simeq 10^5$ GeV) and falls off by more than 3 orders of magnitude between $10^5$ and $3 \times 10^5$ GeV. Thus even for the lowest values of $Q_f$ and $G\mu/c^2 \simeq 10^{-15}$, emission
from cosmic string cusp annihilation is unlikely to be detected between $10^4 - 10^6$ GeV.

We conclude that the most sensitive regime to search for a string background is above $E \simeq 10^{11}$ GeV. At these energies, unlike TeV energies, air shower detectors can not distinguish between photon-induced and cosmic ray-induced showers. However, the diffuse cosmic-ray background is expected to be cut off above the Greisen energy ($E \simeq 7 \times 10^{10}$ GeV) by pair production of charged pions off the microwave background\(^{41}\). The $10^8 - 10^{10}$ GeV cosmic ray data are consistent with this prediction. The Fly’s Eye cosmic ray measurements\(^6\) are shown on Figures 2-4 and lie considerably above the string-generated flux for $10^8 < E < 7 \times 10^{10}$ GeV. Above $E \simeq 7 \times 10^{10}$ GeV, on the other hand, where there should be no competing background, a string signature should stand out provided it is greater than the minimum flux which can be detected by the telescope. If the telescope has an effective area $A_D$, the number of showers seen by the detector in an observing time $\Delta t_D$ is

$$S(> E) \simeq 0.1 \left( \frac{E^3 F(E)}{10^{21} \text{eV}^2 \text{m}^{-2} \text{sec}^{-1}} \right) \left( \frac{E}{10^{11} \text{GeV}} \right)^{-2} \left( \frac{A_D}{10^{14} \text{cm}^2} \right) \left( \frac{\Delta t_D}{1 \text{yr}} \right)$$

(3.30)

(To derive (3.30), note from the Figures that approximately $F(E) \propto E^{-1.25}$ above $E \simeq 10^{10}$ GeV). The current configuration of the Fly’s Eye telescope has an effective aperture of $10^{13}$ cm\(^2\) sr at $E \simeq 10^{11}$ GeV; the High Resolution Fly’s Eye detector\(^{42}\) (HiRes) presently under construction will have an effective aperture of $7 \times 10^{13}$ cm\(^2\) sr at $E \gtrsim 10^{11}$ GeV. We also note that $E \gtrsim 10^{11}$ GeV photons arriving perpendicular to the Earth’s magnetic field will be affected by pair-production off the magnetic field\(^{32}\). Thus, returning to the Figures, a cosmic string background would be detectable at
\( E \simeq 10^{12} \text{ GeV over } \Delta t_D = 1 \text{ yr} \) if, for example, \( 10^{12} \lesssim Q_f \lesssim 10^{14} \text{ GeV} \) and \( G\mu/c^2 \simeq 10^{-13}, \) \( 10^{12} \lesssim Q_f \lesssim 10^{18} \text{ GeV} \) and \( G\mu/c^2 \simeq 10^{-15} \) (see Figures 1 and 3) or if \( Q_f \simeq \sigma \) and \( G\mu/c^2 \simeq 10^{-15} - 10^{-13} \) (see Figure 2). We stress that the predicted flux from cusp annihilation is uncertain, particularly at these energies, due to the inexact knowledge of the cusp annihilation process and the extrapolation of particle decay to ultra high energies. The true flux may be greater or less than shown in the Figures and may have a different spectral index. It is of note, however, that a string background may be detectable even if the annihilation process does not work at full efficiency. For example, taking our approximation in Figure 2, even if the process worked at 1% efficiency (i.e. 1% of the energy in the cusp region is converted into particles), strings with \( G\mu/c^2 \simeq 10^{-15} \) and \( Q_f \simeq \sigma \) would produce one observable shower per year.

### IV. CONCLUSIONS

In the usual cosmic string scenario of galaxy formation with \( G\mu/c^2 \simeq 10^{-6} \), it may be just possible to detect ultra-high energy gamma-ray bursts from the cusp annihilation of nearby strings, if such radiation occurs. If \( G\mu \) is lower, the emission may be more easily seen. Because the probability of detecting the bursts is still small, we cannot yet derive new lower bounds on \( G\mu \), which would complement the upper bounds found by considering the distortion of binary star systems in a string-produced background of gravitational radiation\(^{43}\) and the COBE results for the quadrupole anisotropy of the microwave background\(^{45}\).

If \( G\mu/c^2 \simeq 10^{-15} - 10^{-13} \), the diffuse gamma-ray background from cusp emission may be detected also by EeV telescopes such as the Fly’s Eye and Fly’s Eye HiRes experiments. This signal may be seen even if the cusp evaporation process does not
work at full efficiency.

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FIGURES

Figure 1. The redshift, $z_{CO}$, at which photons with energy $E$ are cut off by pair-production off cosmic background photons. $z = 1$ in the present epoch.

Figures 2 (i) - (vi). The photon background from cusp annihilation as a function of $E$ for $G\mu/c^2 = 10^{-11} - 10^{-16}$ respectively. $\gamma = 100$ and $\nu = 0.03$. Slicing the plot at $E = 10^5$ GeV, the curves represent $Q_f = 10^{18}$ GeV, $10^{14}$ GeV, $10^{10}$ GeV and $10^6$ GeV in order of increasing flux. The dashed triangle up to $10^6$ GeV is an extrapolation of the $35 - 150$ MeV extragalactic photon background. The UHE data points are the cosmic ray measurements from the Fly’s Eye detector which represent an upper limit on the photon flux at those energies.

Figure 3. The predicted photon flux as a function of $E$ for $G\mu/c^2 = 10^{-15}$ and various values of $Q_f$, the mass of the initial superheavy particles emitted from the cusp. $\gamma = 100$ and $\nu = 0.03$ The dashed triangle is an extrapolation of the $35 - 150$ MeV extragalactic photon background. The UHE data points are the upper limit on the photon flux from the Fly’s Eye detector. Slicing the plot at $E = 10^5$ GeV, the curves represent $Q_f = 10^{18}$ GeV, $10^{14}$ GeV, $10^{13}$ GeV, $10^{12}$ GeV, $10^{11}$ GeV, $10^{10}$ GeV and $10^6$ GeV in order of increasing flux.

Figure 4. The predicted photon flux as a function of $E$ for various values of $Q_f = \sigma$ where $\sigma = (\mu h^3)^{1/2}$. $\gamma = 100$ and $\nu = 0.03$. The dashed triangle is an extrapolation of the $35 - 150$ MeV extragalactic photon background. The UHE data points are the upper limit on the photon flux from the Fly’s Eye detector. Slicing the plot at $E = 10^8$ GeV, the curves represent $G\mu/c^2 = 10^{-11}$, $G\mu/c^2 = 10^{-16}$, $G\mu/c^2 = 10^{-12}$, $G\mu/c^2 = 10^{-13}$, $G\mu/c^2 = 10^{-14}$ and $G\mu/c^2 = 10^{-15}$ in order of increasing flux.