Inflation driven by exponential non-minimal coupling of inflaton with gravity

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We consider a modified gravity model of the form \( f(R, \phi) = R e^{h(\phi) R} \), where \( \phi \) is a non-interacting massive scalar field. We show that the conformal transformation of this model to Einstein frame leads to non-canonical kinetic term and negates the advantage of the Einstein frame. We obtain exact solutions for the background and show that the model leads to inflation with exit. We obtain scalar and tensor power-spectrum in Jordan frame and show that the model leads to blue-tilt. We discuss the implications of the same in the light of cosmological observations.

I. INTRODUCTION

Cosmological inflation remains a successful paradigm during the high energy phase of the Universe [1–5]. The problems of the standard cosmology, like flatness/horizon, are elegantly solved if the comoving scales grow quasi-exponentially. The prediction of the simplest models of inflation is a flat Universe, i.e. \( \Omega_{\text{total}} = 1 \). Importantly, it provides a natural mechanism for the generation of primordial density perturbations for understanding the initial conditions for structure formation and for Cosmic Microwave Background (CMB) anisotropy. The metric perturbations at the last scattering surface are observable as the temperature anisotropy in the CMB [1–5].

Despite the simplicity of the inflationary paradigm, obtaining a canonical scalar field (\( \phi \)) driven inflationary model within General relativity (GR) requires highly fine tuned potential [1–5]. Since inflation takes place at high energies, we need to consider quantum corrections to both gravity and matter sectors which result in modifications to general relativity [6–9]. A possible way to introduce modifications to GR is by adding higher order curvature terms like contracted Ricci/Riemann tensors or higher powers of Ricci scalar to the Einstein-Hilbert action. \( f(R) \) models, where Lagrangian density is a general function of Ricci scalar, are the simplest among them [10–13]. These models are general enough to include the modifications to the GR in high energy limit, and more importantly, they do not suffer from Ostriodrasky instability [14].

It has been shown that \( f(R) \) models can successfully describe the inflationary phase of the universe [11]. However, the field equations associated with these models are higher order and non-trivial. To circumvent this problem, one performs conformal transformation so that the resultant action corresponds to Einstein-Hilbert action with a canonical scalar field. Although the two — original (Jordan frame) and the conformally transformed (Einstein frame) — actions are related by conformal transformations, it is unclear how the observable quantities are related to the physical quantities computed in the two frames [15–17].

As we show in Sec. (II), in many cases, the action in the conformally transformed frame contains non-canonical kinetic term and it negates the advantage of transforming to Einstein frame. Recently, the current authors have devised an analytical method to study the background equations and the first order perturbed equations in the Jordan frame [18]. The approach reduces the higher order perturbed equations to a set of second order differential equations and provides a way to directly relate the derived quantities to the observed quantities. The method was applied to a simple case of \( f(\phi)(R + \alpha R^2) \) [18].

In this work, we use the above analytical approach to investigate a more non-trivial modified gravity model \( R \exp[h(\phi)R] \). The physical motivation for such a scenario comes from the fact that the quantum corrections to the gravity and scalar field can have a scale dependent corrections [6–9] and the first order term in the above exponent leads to

\[
R \exp[h(\phi)R] \simeq R + h(\phi)R^2
\] (1)

Thus, the modification to general relativity appears as a non-minimal coupling term to the \( R^2 \) term only and not to the Einstein-Hilbert action. Since the Einstein-Hilbert action does not contain any non-minimal coupling term, there

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is no motivation for the conformal transformation. Thus, we can not rely on the conformal transformation techniques used in the literature, and we use the new analytical technique developed by us in Ref. [18].

The paper is organized as the following. In the next section, we introduce the model in detail and show that it is convenient to study the model in Jordan frame. We then obtain the exact analytical solution for the de Sitter case and obtain the quasi-exponential inflationary solution with exit numerically for a range of initial conditions. In section (III), we compute the scalar and tensor power spectra. In the last section, we present the results and conclusions.

We use $(-+,+,+)$ metric signature and natural units, $c = 1$, $\hbar = 1$ and $\kappa \equiv \frac{1}{M_{Pl}^2}$, where $M_{Pl}$ is the reduced Planck mass. Lower Latin alphabets denote the 4-dimensional space-time, and lower Greek letters are used for the 3-dimensional space. Dot represents the derivative with respect to cosmic time $t$, while prime denotes derivative with respect to conformal time $\eta$. $H \equiv \frac{a}{a}$ is the Hubble parameter.

II. MODEL AND BACKGROUND SOLUTION

We consider the following action in Jordan frame:

$$ S_J = \int d^4x \sqrt{-g} \left[ \frac{1}{2} f(R, \phi) - \frac{\omega}{2} g^{ab} \nabla_a \phi \nabla_b \phi - V(\phi) \right], $$

where $V(\phi)$ is the scalar field potential and $\omega = +1$ corresponds to canonical scalar field, $f(R, \phi)$ is taken to be

$$ f(R, \phi) = \frac{1}{\kappa} R e^{h(\phi) R}, $$

and $h(\phi)$ is the scalar field coupling function. For the sake of clarity, we expand the exponential up to the first order in Ricci scalar i.e.,

$$ f(R, \phi) \approx \frac{1}{\kappa} \left[ R + h(\phi) R^2 \right]. $$

A. Conformal transformation in $f(R, \phi)$ models

As mentioned in the Introduction, in the case of Einstein gravity with non-minimally coupled scalar field, or $f(R)$ models, field equations and equations of motions are of higher order [5, 11, 19]. One way to get around this problem is to do a conformal transformation to bring the action to the Einstein frame. For Einstein gravity with non-minimally coupled scalar field, this along with redefinition of the scalar fields leads to Einstein gravity with minimally coupled scalar fields [19, 20]. However in our model, or generally in $f(R, \phi)$ models, this is not the case [21].

To see this, the conformal transformation $\tilde{g}_{\mu\nu} = F g_{\mu\nu}$ on the above action, leads to:

$$ \tilde{S}_E = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{\tilde{R}}{2\kappa} - \frac{1}{2e^{\zeta R}} \tilde{g}^{ab} \partial_a \phi \partial_b \phi - \frac{1}{2} \tilde{g}^{ab} \partial_a \zeta \partial_b \zeta - \tilde{W} \right], $$

where

$$ F = \frac{\partial f}{\partial R}, \quad \zeta = \sqrt{\frac{3}{2\kappa}} \ln F, \quad \tilde{W} = \frac{FR - f}{\kappa F^2} + \frac{V}{F^2}. $$

As it is evident, the conformal transformation leads to non-canonical kinetic term in the Einstein frame which leads to more complicated equations of motion. This negates the advantage of the Einstein frame action. In order to avoid these, we do all the calculations in Jordan frame.

Field equations and equation of motion of the scalar field for a general $f(R, \phi)$ model in the Jordan frame are given by [22]

$$ \Box \phi + \frac{1}{2\omega} \left( \omega, \phi \phi^a_a + f, \phi - 2V, \phi \right) = 0 $$

$$ FG^p_q = \omega \left( \phi^p \phi_q - \frac{1}{2} \delta^p_q \phi^c \phi_{,c} \right) - \frac{1}{2} \delta^p_q (RF - f + 2V) + F^{,p}_{,q} - \delta^p_q \Box F. $$

In the rest of this section, we obtain the exact de Sitter solution analytically and then extend the analysis and numerically obtain a quasi-exponential solution with an exit.
B. Background inflationary solution

Consider the spatially flat FRW metric given by the line element

\[ ds^2 = -dt^2 + a(t)^2 \left( dx^2 + dy^2 + dz^2 \right), \]  

(8)

where \( a(t) \) is the scale factor. For the above background, the field equations (7) and equation of motion of \( \phi \) (6) reduce to:

\[-\omega \dot{\phi}^2 - 6F \left( \dot{H} + H^2 \right) + 6\dot{F}H - (2V - f) = 0, \]

(9)

\[ 4FH^2 + 2F \left( \dot{H} + H^2 \right) - \omega \dot{\phi}^2 - 2\ddot{F} - 4\dot{F}H + 2V - f = 0, \]

(10)

\[ 2\omega \ddot{\phi} + 6\omega \dot{\phi} H - \frac{h_{,\phi} f}{h} + \frac{12h_{,\phi} H^2}{h\kappa} + \frac{6h_{,\phi} \dot{H}}{h\kappa} + 2V_{,\phi} = 0, \]

(11)

where \( F = \frac{\partial f}{\partial R} \), and \( h = h(\phi) \) is the coupling function. Let us now consider the following ansatz:

\[ \phi = \phi_0 e^{-\frac{n}{H_D}t}, \]

(12)

where \( n, \phi_0, \) and \( H_D \) are constants. Substituting the above ansatz in Eqs. (9, 10, 11) and solving the equations, we obtain the following functional form for the coupling function:

\[ h(\phi) = -\lambda \phi^2, \quad \text{where} \quad \lambda = \frac{1}{48} \frac{\omega n \kappa}{(2n + 1)H_D^2}. \]

(13)

and the scalar field potential as

\[ V(\phi) = m^2 \phi^2 + V_0, \quad \text{where} \quad m^2 = \frac{\omega n^2 H_D^2}{(2n + 1)\left(\frac{5}{2} - n\right)}, \quad V_0 = \frac{3H_D^2}{\kappa}. \]

(14)

Substituting these expressions in Eqs. (9, 10, 11) and solving for \( \ddot{\phi} \) and \( \ddot{H} \), we get

\[ \ddot{\phi} = \frac{72h_{,\phi} H^4}{\omega \kappa} + \frac{72h_{,\phi} \dot{H} H^2}{\omega \kappa} + \frac{18h_{,\phi} \dot{H}^2}{\omega \kappa} - \frac{V_{,\phi}}{\omega} - 3\ddot{\phi}H. \]

(15)

\[ \ddot{H} = \frac{h_{,\phi} \dot{H}^2}{72H h} + \frac{\dot{H}^2}{2H} - \frac{H}{12h} - \frac{2H^2 h}{h} - \frac{h\dot{H}}{h} - \frac{3H \dot{H}}{h} + \frac{\kappa V}{36H h}. \]

(16)

It is difficult to analytically solve these equations for a general \( H \), however, we can obtain an exact analytical solution for the exact de Sitter, i.e., \( H = H_D \). In this case we obtain the evolution of the scalar field to be exponentially decaying:

\[ \phi = \phi_0 e^{-nH_D t}. \]

To find out a solution for an arbitrary \( H \), we can solve these equations numerically and study the evolution of Hubble parameter \( H \) and scalar field \( \phi \). The procedure we adopt is the following: Fix all except one parameter and the initial values of variables. Vary the unfixed parameter to obtain the evolution of \( H \) and \( \phi \). Unless otherwise specified, the numerical values of parameters are as given below.

\[ H_D = 4 \times 10^{-4}, \quad n = 0.01, \quad \phi_0 = 100, \quad m^2 = 3.90588 \times 10^{-11}, \quad \lambda = 1276.5522, \quad \kappa = 1. \]

(17)

The plot for evolution of slow-roll parameter \( \epsilon \) and scalar field \( \phi \) for various initial values of \( \dot{\phi} \) is give below.
As we can see from these plots, $\epsilon$ remains zero for $\dot{\phi}_i = \dot{\phi}_D$. The initial condition $|\dot{\phi}_i| < |\dot{\phi}_D|$ leads to inflation with exit $\epsilon > 1$. In these cases, the number of e-foldings is proportional to the value of $|\dot{\phi}_i|/|\dot{\phi}_D|$. In other words, smaller the value of $|\dot{\phi}_i|/|\dot{\phi}_D|$, the number of e-foldings is less. However, in the case of $|\dot{\phi}_i| > |\dot{\phi}_D|$, it does not lead to exit, which is indicated by $\epsilon$ diverging to $-\infty$. In this case, rate of inflation increases with increase in the value of $|\dot{\phi}_i|$. This shows that deviation of initial values from de Sitter value leads to either inflation with exit or super inflation, and that the de Sitter solution is a saddle point. This is similar to the case discussed in Ref. [18], and similar solutions have been considered in [23–25].

### III. FIRST ORDER SCALAR AND TENSOR PERTURBATIONS

The first order perturbations about the FRW background is given by

$$
ds^2 = -(1 + 2\theta)dt^2 - a(\beta, t + B_\alpha)dx^\alpha + a^2[\alpha_\beta(1 - 2\psi) + 2\gamma_{,\beta} + 2C_{\alpha|\beta} + 2C_{\alpha\beta}].$$

where $a(t)$ is the cosmic scale factor with $dt \equiv ad\eta$. $\theta(x, t), \beta(x, t), \psi(x, t)$, and $\gamma(x, t)$ denote the scalar perturbation. $B_\alpha(x, t)$ and $C_\alpha(x, t)$ are tracefree vector perturbation and $C_{\alpha\beta}(x, t)$ is the transverse and tracefree tensor perturbation. The vertical bar denotes the covariant derivative on 3-D spatial hypersurface characterized by $\gamma^{(3)}_{\alpha\beta}$. We decompose the scalar field as $\phi(x, t) = \tilde{\phi}(t) + \delta\phi(x, t)$.

In the Newtonian gauge, scalar perturbations in the Fourier space satisfies the following equations [22]:

$$
-F\psi + F\theta + \delta F = 0. \quad (19)
$$

$$
-2F\dot{\psi} - 2FH\theta - \dot{F}\theta + \dot{\phi}\delta\phi + \delta\dot{F} - H\delta F = 0. \quad (20)
$$

$$
6FH\dot{\psi} + 6FH^2\theta - 2F\frac{k^2\psi}{a^2} - \dot{\phi}^2 + 3\dot{F}\psi + 6\dot{F}H\theta + 6F\psi\phi - 3H\delta\phi - 3H\delta\psi - 3\dot{H}\delta F + 3H^2\delta F + \frac{k^2\delta F}{a^2} = 0. \quad (21)
$$

$$
6F\ddot{\psi} + 12FH\dot{\theta} + 6FH^2\theta + 12H^2\psi + 2F\frac{k^2\theta}{a^2} + 3\dot{F}\psi + 6\dot{F}H\theta + 3\dot{F}\dot{\theta} + 4\dot{\phi}^2 + 6\dot{\psi}\phi - 2\dot{\phi}\delta\phi - 2\dot{\psi}\delta\phi - 6H\dot{\psi}\phi - 3\dot{H}\delta\psi - 3H^2\delta F + 6H^2\delta F + \frac{k^2\delta F}{a^2} = 0. \quad (22)
$$

$$
\ddot{\phi} + 3H\delta\phi - \frac{1}{2} f_{\phi\phi}\delta\phi + V_{\phi\phi}\delta\phi + \frac{k^2}{a^2}\delta\phi - 3\dot{\phi}\psi - 6H\dot{\phi}\theta - \dot{\phi}\theta - 2\dot{\phi}\theta + 3F_\phi\ddot{\psi} + 6F_\phi\dot{H}\theta + 3F_\psi\dot{H}\theta + 12F_\theta\dot{H}\psi + 12F_\psi\dot{H}^2\theta + 2F_\phi\frac{k^2}{a^2}\psi - F_{\phi\phi}\frac{k^2}{a^2}\theta = 0. \quad (24)
$$

$$
\delta F - F_\phi\delta\phi - F_R\delta R = 0. \quad (25)
$$
And the tensor perturbations satisfy the following differential equation\cite{22}:

\[
\dddot{C}_\beta^\alpha + \left( \frac{\dot{F}}{F} + 3H \right) \ddot{C}_\beta^\alpha + \frac{k^2}{a^2} C_\beta^\alpha = 0.
\] (26)

### A. Scalar power spectrum

In this section, we derive the equation satisfied by the 3-curvature perturbation $R$ and calculate corresponding power spectrum. We follow the procedure used in Ref. \cite{18}. In Jordan frame, $R$ is defined as

\[
R = \psi + \frac{H}{\phi} \delta \phi.
\] (27)

For single scalar field driven inflationary models, $R$ is time-independent at super-Hubble scale. Hence, 3-Curvature directly relates CMB observations to the inflationary dynamics at the time of horizon crossing \cite{26}. Since equations governing the scalar perturbations are highly complicated, instead of trying to solve for $R$ directly, we solve the equations for other perturbed quantities and use those solutions to derive the equation satisfied by $R$.

First, we define new variables so that it will reduce the total no. of variables and make the equations simpler. We define a new variable $\Theta = \theta + \psi$. Using Eqs. (19, 20, 21), we get

\[
\left( -6 F \ddot{\phi} \dot{\phi} + 4 \dddot{F} + 2 F \dot{\phi} \phi \right) + \left( 2 F \ddot{\phi} \dot{\phi} + 4 \dddot{F} - 2 F \dddot{\phi} \dot{\phi} - \dddot{F} + \dddot{F} + F k^2 a^2 \right) \Theta = 0.
\] (28)

Substituting the background quantities corresponding to de Sitter case, for very large values of $k$ and assuming $n \ll 1$, $\Theta$ satisfies the following differential equation:

\[
\ddot{\Theta} + H_D \dot{\Theta} + \frac{k^2}{a^2} \Theta = 0.
\] (29)

Rewriting $\delta \phi$ in terms of $\Theta$, we get,

\[
\delta \phi = \frac{1}{2} \frac{(\phi_0 n k - 2)(\dot{\Theta} + H_D \Theta)}{\phi_0 n H_D k}.
\] (30)

Now rewrite the perturbation equations in terms of $R$. Using Eqs. (24, 29) and the rewritten perturbation equations, after lengthy calculation, we can rewrite $\Theta$ in terms of $R$ and its derivatives as:

\[
\Theta = -\frac{(3a^2 \dddot{R} + 7H_D a^2 \dddot{R} + 3k^2 R) \phi_0^2 n^2 k}{k^2 (\phi_0^2 n k - 2)}.
\] (31)

Now substituting this in to the perturbation equations, we obtain the following differential equation of $R$:

\[
\dddot{R} + 3H_D \ddot{R} + \frac{k^2}{a^2} R = 0.
\] (32)

This is one of the main result. It is important to note that the procedure allows us to reduce the higher order differential equation to second order. To obtain the power-spectrum, we obtain the solution to the above differential equation in the short wavelength limit and use the Bunch-Davies vacuum at the initial epoch of inflation. This leads to:

\[
R_\text{<} = \frac{H_D}{2a \sqrt{k}} e^{-i k \eta}.
\] (33)

In the long wavelength limit, we get $R_\text{>} = C$. By matching $R_\text{<}$ and $R_\text{>}$ at horizon crossing ($|k \eta| = 2 \pi$), we get

\[
C = \frac{\sqrt{2} H_D \pi}{k^{3/2}}
\]
leading to scale-invariant scalar power spectrum given by

$$P_R = H_D^2. \quad (34)$$

Following points are important to note regarding this result: The above analysis is analytical expression in the limit of $n \ll 1$. In the other cases, it is only possible to obtain a semi-analytical expression which will lead to tilt in the spectrum. In the next subsection, we evaluate the tensor power-spectrum without taking $n \ll 1$ limit and show that it leads to blue tilt.

**B. Tensor power spectrum**

Substituting the background quantities and using the approximation $2h(\phi)R \gg 1$, Eq. (26) takes the following form:

$$\ddot{C}_\beta^\alpha + (3 - 2n) H_D \dot{C}_\beta^\alpha + \frac{k^2}{a^2} C_\beta^\alpha = 0. \quad (35)$$

We can simplify this equation by rewriting $C_\beta^\alpha = \nu_g/z_g$, where $z_g = ae^{-nH_D t}$. Eq. (35) becomes,

$$\nu_g'' + \left( k^2 - \frac{z_g''}{z_g} \right) \nu_g = 0. \quad (36)$$

Then solution to the above equation is given by

$$\nu_g = \sqrt{-\eta} \left( C_1 H^{(1)}_{3/2-n} (-k\eta) + C_2 H^{(2)}_{3/2-\nu} (-k\eta) \right). \quad (37)$$

At the initial epoch of inflation, setting the field to be in the Bunch-Davies vacuum, we can see that $C_2 = 0$ and $C_1 = \sqrt{\frac{\pi}{4}}$. We then get

$$\nu_g = \sqrt{\frac{\pi}{4}} \sqrt{-\eta} H^{(1)}_{3/2-n} (-k\eta), \quad (38)$$

with tensor power spectrum given by

$$P_g = 8 \left( \frac{k}{k_*} \right)^{2n} \frac{2^{-2n} 2^n}{4\pi^2} H_D^2 \left( \frac{\Gamma(3/2 - n)}{\Gamma(3/2)} \right)^2 e^{2nH_D t}. \quad (39)$$

Thus the tensorial spectral index $n_T = 2n$ and is positive. In other words, the tensor power-spectrum in this case is blue tilted.

**IV. RESULTS AND CONCLUSION**

In this work, we proposed an inflationary model in which a massive scalar field is non-minimally coupled to $f(R)$ gravity. Even though it is common to study the $f(R)$ models in Einstein frame, we showed explicitly that in the Einstein frame, action contains non-canonical kinetic term. Thus, the advantage of the conformal transformation is negated. Thus, we performed the background and the first order perturbation analysis in the Jordan frame.

We showed explicitly that the model supports inflationary solution with an exit and the number of e-foldings depends on the deviation of initial values from de Sitter scenario. We obtained first order scalar and tensor perturbation equations explicitly. We have used the new analytical method devised in Ref. [18] to reduce the higher order scalar perturbation equations to second order in 3-curvature perturbation. We analytically obtained the scalar power-spectrum in $n \ll 1$ limit and showed that the scalar power spectrum is scale invariant. We obtained the tensor power-spectrum for any $n$ and showed that the spectrum is blue tilted.

It is important to note that the inflationary models within general relativity lead to red-tilt [3, 4]. Our analysis in Ref. [18] and here clearly show that the distinguishing feature between the modified gravity theories and general relativity is that the spectrum is blue-tilted. We plan to extend the analysis to higher order, specifically to evaluate the non-Gaussianity and find the distinguishing features between inflationary models in modified gravity and general relativity.
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