Preparation of Knill-Lafamme-Milburn states based on superconducting qudits

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We propose two schemes for generating the Knill-Lafamme-Milburn (KLM) states of two distant polar molecules (PMs) ensembles respectively in two transmission-line resonators (TLRs) connected by a superconducting charge qutrit (SCQ), and of two SCQs in a TLR, respectively. Both of the schemes are robust against photon decay due to the virtual excitations of the microwave photons of the TLRs, and the spontaneous emission can be suppressed owing to the virtual transitions of the SCQs in the second scheme. In addition, the schemes have high controllability and feasibility under the current available techniques.

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OCIS codes: 230.5750, 270.5585

1. INTRODUCTION

Quantum entanglement, an interesting and attractive phenomenon in quantum mechanics, plays a significant role not only in testing quantum nonlocality, but also in processing a variety of quantum information tasks [1–5]. Therefore, preparation of various quantum entangled states has been being an important subject in quantum information science since a few decades ago [9, 20]. In 2001, Knill, Lafamme and Milburn proposed a specific class of partially entangled states [21], KLM states, and they derived that employing the KLM states as ancillary resources can improve the success probability of teleportation gradually up to unity with the increase of the particle number in these
ancillary states, and thereafter, Modlawska and Grudka showed that multiple adaptive teleportation in the KLM scheme can also increase the probability of faithful teleportation [22], which can elevate the efficiency of quantum computing significantly. In the past dozen years, the investigation of the KLM states preparation has attracted a great deal of attention [23–27]. The first explicit scheme to prepare the KLM states was proposed by Franson et al. in 2004 by using elementary linear optics gates and solid-state approach, respectively [23]. In 2008, Lemr et al. proposed a scheme to prepare the two-photon four-mode KLM states using linear optical elements [24]. Soon afterwards, preparing high fidelity two-photon KLM states experimentally was implemented using spontaneous parametric down-conversion photon source and linear optical components in 2010 [25]. In 2011, Lemr proposed a scheme to prepare KLM states by using a tunable controlled phase gate and optimized the scheme for the framework of linear optics [26]. In 2012, Cheng et al. proposed two schemes to prepare the two-atom KLM states with a strong coupling cavity-fiber system and the cavity-assisted single-photon input-output process, respectively [27].

Although it has been verified that photonic qubits and atomic qubits can be used to realize the KLM-type quantum computation [28], single-photon detectors needed for photon KLM computation are inefficient due to photon losses or dark counts, while the manipulation of the atom is still a severe challenge in the state of the art though great progress has been made in recent decades. So it is hoped to find an alternative candidate of photon and atom, which not only has high usability and feasibility, but also has better controllability and enormous superiority. Fortunately, the SCQ concerning the interaction with the TLR in microwave cavities provides a promising candidate of the physical system for quantum information processing. In this paper, we put forward two schemes to generate the KLM states of two distant PMs ensembles via a hybrid device and of two SCQs in a TLR, respectively. Our schemes have following advantages: 1) With the help of the SCQ and TLR, our schemes have longer coherence time and storage time than that in Refs. [23–27]. 2) Compared with the Ref. [27], it is needless to take the coding qubits out of the TLR in our schemes, which
avoids the decoherence induced by the environment. 3) The controllability and feasibility of our schemes are higher than that in Refs. [23–27] in the current techniques, because the strong coupling in our schemes can be obtained by increasing the number of PMs in each ensemble [29,30] in the first scheme or locating the SCQs at the antinodes of the voltage [31] in the second scheme. 4) Both of the schemes are robust against photon decay due to the virtual excitations of the microwave photons of the TLRs, and the spontaneous emission can be suppressed owing to the virtual transitions of the SCQs in the second scheme. 5) The second scheme can be generalized to prepare $N$ qubits KLM states straightforward.

The rest of the present paper is organized as follows. In Section 2, we give a scheme to prepare the two qubits KLM states of two PMs, then in Section 3 we give another scheme to prepare the two qubits KLM states of two SCQs and generalize the scheme to $N$ qubits. The feasibility analysis and conclusion are given in Section 4.

2. PREPARATION OF THE KLM STATES OF DISTANT POLAR MOLECULES ENSEMBLES VIA A HYBRID DEVICE

We consider a hybrid device with two cold PMs ensembles, two TLRs and a SCQ, as shown in Fig. 1. Here, the SCQ capacitively coupling to the two TLRs can be viewed as an artificial three-level atom with two stable ground states $|i \rangle_s$, $|g \rangle_s$ and an excited state $|e \rangle_s$. [32,33]. The transition between $|g \rangle_s$ and $|e \rangle_s$ with frequency $\omega_s$ is dispersively coupled to the TLR mode. The cold PMs ensembles are placed respectively at the antinodes of the two TLRs and possess stable rotational states and can be well controlled by microwave fields [34,35]. When the PMs are cooled to the ground states and vibrational states of electrons, the PMs can be viewed as two-level system with excited state $|e \rangle_m$ and ground state $|g \rangle_m$ under external weak electric fields. Assume that there are $N$ identical PMs in each PMs ensemble and they have no interaction each other. Under driven by a classical microwave field with frequency $\omega_d$ and Rabi frequency $\Omega$, the whole dynamics of the combined system involving
a cold PMs ensemble, a TLR and a SCQ are governed by the Hamiltonian ( $\hbar = 1$ )

$$H_s = \frac{\omega_s}{2} \sigma^z + \Omega (\sigma^- e^{i\omega_d t} + \sigma^+ e^{-i\omega_d t}) + \frac{\omega_m}{2} S^z + \omega_c a^\dagger a + \sigma_s (\sigma^+ a + \sigma^- a^\dagger) + g_m (S^+ a + S^- a^\dagger),$$  

(1)

where $\sigma^z = |e\rangle_s \langle e| - |g\rangle_s \langle g|$, $\sigma^+ = |e\rangle_s \langle g|$, $\sigma^- = |g\rangle_s \langle e|$. $S^z = \sum_{j=1}^N s_j^z (s_j^z = |e\rangle_{m,j} \langle e| - |g\rangle_{m,j} \langle g|)$, $S^+ = \sum_{j=1}^N s_j^+ (s_j^+ = |e\rangle_{m,j} \langle g|)$ and $S^- = \sum_{j=1}^N s_j^- (s_j^- = |g\rangle_{m,j} \langle e|)$ are the collective spin operators for the PMs. $a^\dagger$ and $a$ are the creation and annihilation operators of the microwave photon with frequency $\omega_c$. $g_s (g_m)$ is the coupling strength between the SCQ (PMs) and the TLR. Under the conditions $|\Delta_s| = |\omega_s - \omega_c| \gg g_s$, $|\Delta_m| = |\omega_m - \omega_c| \gg g_m$ and $|\Delta_a| = |\omega_s - \omega_d| \gg \Omega$, the dispersive interaction and classical field induce the Stark shifts, respectively. Letting $b = (1/\sqrt{N}) S^-$, $b^\dagger = (1/\sqrt{N}) S^+$ and $n_b = \sum_{j=1}^N |e\rangle_{m,j} \langle e|$, then $[b, b^\dagger] = 1 - (2/N) n_b$, $[n_b, b^\dagger] = b^\dagger$ and $[n_b, b] = -b$. In the low-excitation case, $b^\dagger$ and $b$ can be regarded as the bosonic operators and the PMs ensemble can be regarded as a bosonic system. If the TLR is initially in the vacuum state, the Hamiltonian $H_s$ can be reduced to

$$H_{eff} = \frac{1}{2} (2\lambda_{sd} + \lambda_{sc}) \sigma^z + g (\sigma^+ b + \sigma^- b^\dagger) + N\lambda_{mc} b^\dagger b,$$  

(2)

where $\lambda_{sd} = \Omega^2 / \Delta_d$, $\lambda_{sc} = g_s^2 / \Delta_s$, $\lambda_{mc} = g_m^2 / \Delta_m$, $g = \sqrt{N} \lambda_{sm}$ ($\lambda_{sm} = g_m g_s / 2 (1/\Delta_m + 1/\Delta_s)$) and the term of the constant energy has been discarded. Under the resonant condition $2\lambda_{sd} + \lambda_{sc} = N\lambda_{mc}$, the effective Hamiltonian $H_{eff}$ leads to the resonant interaction between the SCQ and the PMs ensemble. We can have the following transitions:

$$|e\rangle_s |n\rangle \rightarrow \cos (g \sqrt{n+1} t) |e\rangle_s |n\rangle - i \sin (g \sqrt{n+1} t) |g\rangle_s |n+1\rangle,$$  

(3)

$$|g\rangle_s |n+1\rangle \rightarrow \cos (g \sqrt{n+1} t) |g\rangle_s |n+1\rangle - i \sin (g \sqrt{n+1} t) |e\rangle_s |n\rangle,$$  

(4)

where $|n\rangle$ represents the $n$-excitation Fock state of the PMs mode. Here the common phase factor is discarded.

Now the method for preparing the KLM states of the collective modes of two distant PMs ensembles is given based on the resonant interaction discussed above. Assume that the SCQ is initially
in the state $|i\rangle_s$ and the two PMs ensembles are initially in the vacuum state $|0\rangle_1|0\rangle_2$, where the subscript 1 and 2 indicate different PMs ensembles. The operations for realizing the KLM states are described as below:

**Step 1:** Adjust the energy level gap of the SCQ \[38–40\] so that the SCQ does not interact with two PMs ensembles and apply a classical microwave pulse with the Rabi frequency $\Omega'$ to the SCQ, then the transition $|i\rangle_s \rightarrow \cos (\Omega' t_0) |i\rangle_s - ie^{-i\phi} \sin (\Omega' t_0) |e\rangle_s$ will be performed. Hence the state of the combined system is

$$|\Psi_1\rangle = \cos (\Omega' t_0) |i\rangle_s |0\rangle_1 |0\rangle_2 - ie^{-i\phi} \sin (\Omega' t_0) |e\rangle_s |0\rangle_1 |0\rangle_2. \quad (5)$$

**Step 2:** Let the SCQ interact with ensemble 1 and set the interaction time $t_1 = \pi/(2g)$. The state of the combined system evolves to

$$|\Psi_2\rangle = \cos (\Omega' t_0) |i\rangle_s |0\rangle_1 - e^{-i\phi} \sin (\Omega' t_0) |g\rangle_s |1\rangle_1 |0\rangle_2. \quad (6)$$

**Step 3:** Adjust the energy level gap of the SCQ again to decouple the SCQ from the TLRs and apply another classical microwave pulse with the Rabi frequency $\Omega''$ to the SCQ for implementing the transition $|g\rangle_s \rightarrow \cos (\Omega'' t_2) |g\rangle_s - ie^{-i\phi'} \sin (\Omega'' t_2) |e\rangle_s$. Thus the state of the combined system evolves into

$$|\Psi_3\rangle = \cos (\Omega' t_0) |i\rangle_s |0\rangle_1 |0\rangle_2 - e^{-i\phi} \sin (\Omega' t_0) \cos (\Omega'' t_2) |g\rangle_s |1\rangle_1 |0\rangle_2$$

$$+ ie^{-i(\phi+\phi')} \sin (\Omega' t_0) \sin (\Omega'' t_2) |e\rangle_s |1\rangle_1 |0\rangle_2. \quad (7)$$

**Step 4:** Let the SCQ interact with ensemble 2 and choose the interaction time $t_3 = \pi/(2g)$, after that the two collective modes of two PMs ensembles evolve to

$$|\Psi_4\rangle = \cos (\Omega' t_0) |i\rangle_s |0\rangle_1 |0\rangle_2 - e^{-i\phi} \sin (\Omega' t_0) \cos (\Omega'' t_2) |g\rangle_s |1\rangle_1 |0\rangle_2$$

$$+ e^{-i(\phi+\phi')} \sin (\Omega' t_0) \sin (\Omega'' t_2) |g\rangle_s |1\rangle_1 |1\rangle_2. \quad (8)$$

**Step 5:** Apply a classical microwave pulse to the SCQ for the transition $|i\rangle_s \rightarrow |e\rangle_s$, so the state...
\[ |\Psi\rangle_5 = \cos (\Omega' t_0)|e\rangle_s|0\rangle_1|0\rangle_2 - e^{-i\phi} \sin (\Omega' t_0) \cos (\Omega'' t_2)|g\rangle_s|1\rangle_1|0\rangle_2 \\
+ e^{-i(\phi+\phi')} \sin (\Omega' t_0) \sin (\Omega'' t_2)|g\rangle_s|1\rangle_1|1\rangle_2. \]  

(9)

**Step 6:** Apply a classical microwave pulse to the SCQ again for the transitions \(|g\rangle_s \rightarrow 1/\sqrt{2}(|g\rangle_s - |e\rangle_s)|, |e\rangle_s \rightarrow 1/\sqrt{2}(|e\rangle_s + |g\rangle_s). So the final state of the combined system turns into

\[ |\Psi\rangle_6 = \frac{1}{\sqrt{2}}|g\rangle_s[\alpha|0\rangle_1|0\rangle_2 - \beta|1\rangle_1|0\rangle_2 + \gamma|1\rangle_1|1\rangle_2] \\
+ \frac{1}{\sqrt{2}}|e\rangle_s[\alpha|0\rangle_1|0\rangle_2 + \beta|1\rangle_1|0\rangle_2 - \gamma|1\rangle_1|1\rangle_2], \]  

(10)

where \(\alpha = \cos (\Omega' t_0), \beta = e^{-i\phi} \sin (\Omega' t_0) \cos (\Omega'' t_2), \gamma = e^{-i(\phi+\phi')} \sin (\Omega' t_0) \sin (\Omega'' t_2). If \(\alpha = \beta = \gamma = 1/\sqrt{3}\) are set, i.e., \(\Omega' t_0 = \arccos(1/\sqrt{3}), \Omega'' t_2 = \pi/4\) and \(\phi = \phi' = 0\), the state becomes

\[ |\Psi\rangle_7 = \frac{1}{\sqrt{6}}|g\rangle_s[|0\rangle_1|0\rangle_2 - |1\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2] \\
+ \frac{1}{\sqrt{6}}|e\rangle_s[|0\rangle_1|0\rangle_2 + |1\rangle_1|0\rangle_2 - |1\rangle_1|1\rangle_2]. \]  

(11)

Then, a detection on the SCQ should be performed. It is worth noting that no matter what the measurement result is, the KLM states can be always achieved. That’s to say, the successful probability of our protocol is unity in the ideal case. In addition, according to the measurement result, a simple local operation \(\sigma^z\) can be performed on the first PMs ensemble via a feedback process to achieve the same KLM states.

Up to now, all the above results are based on the ideal case. In fact, in our scheme, a series of interacting times should be controlled accurately to prepare the KLM states with a high fidelity. So the effect of the time errors on the fidelity should be discussed. Here, the fidelity is defined as

\[ F = \langle \Phi | \varphi \rangle_6 \]  

where \(\varphi_6\) is the final state of system when the time errors \(\Delta t_j\) \((j = 0, 1, 2, 3)\) are introduced (It should be noted that we have ignored the the time errors in Steps 5 and 6, because the precise control of microwave pulse has already been shown by several groups \([41, 42]\)). After
calculation we find $F$ can be evaluated as
\[
F = |\alpha^* \alpha' + \beta^* \beta' + \gamma^* \gamma'|^2, \tag{12}
\]
where $\alpha' = \cos[\Omega'(t_0 + \Delta t_0)]$, $\beta' = e^{-i\phi} \sin[\Omega'(t_0 + \Delta t_0)] \sin[g(t_1 + \Delta t_1)] \cos[\Omega''(t_2 + \Delta t_2)]$, $\gamma' = e^{-i(\phi + \phi')} \sin[\Omega'(t_0 + \Delta t_0)] \sin[g(t_1 + \Delta t_1)] \sin[\Omega''(t_2 + \Delta t_2)] \sin[g(t_3 + \Delta t_3)]$. In order to discuss the effect of time errors on the fidelity, we still let $\Omega' t_0 = \arccos(1/\sqrt{3})$, $\Omega'' t_2 = \pi/4$, $gt_1 = gt_3 = \pi/2$ as above, and define the time error rate as $\eta_j = \Delta t_j/t_j$ ($j = 0, 1, 2, 3$) (Here, $\eta_j (j = 0, 1, 2, 3)$ is the relative error of time, $t_1$ ($t_3$) the interaction time between SCQ and PMs 1 (PMs 2), $t_0$ ($t_2$) the operation time of classical pulse with Rabi frequency $\Omega'$ ($\Omega''$)). Density plots of the fidelity as a function of $\eta_1$ and $\eta_3$ are shown in Fig. 2. It can be seen from Fig. 2(a) the fidelity decreases slightly with the increase of $\eta_1$ and $\eta_3$ when $\eta_0 = \eta_2 = 0$, and the fidelity can still reach 0.975 when $\eta_1 = \eta_3 = 0.1$. Fig. 2(b) shows that the fidelity is still as high as 0.96 when $\eta_1 = \eta_3 = 0.1$ even with $\eta_0 = \eta_2 = 0.1$.

3. PREPARATION OF THE KLM STATES USING A TUNABLE CONTROLLED PHASE GATE OF SUPERCONDUCTING QUTRITS

In this section, we propose another scheme to prepare the KLM states by using a tunable controlled phase gate of superconducting circuits. The experimental device involving two SCQs capacitively coupling to a TLR for implementing the controlled phase gate and the level configuration of the SCQs are shown in Fig. 3. The SCQ has three levels $|i\rangle$, $|g\rangle$ and $|e\rangle$, among which the two stable ground states $|i\rangle$ and $|g\rangle$ are used to encode qubit information, and the excited state $|e\rangle$ is used for virtual transitions. The TLR with eigenfrequency $\omega_c$ is strongly detuned from SCQ resonance by $\Delta = \omega_c - (\omega_e - \omega_g)$ and differentially detuned from the classical driving field by $\delta = \omega_d - \omega_c$, as illustrated in Fig. 3(b). The SCQs are far apart from each other so that the interaction between
SCQs can be ignored \[43\]. Then the Hamiltonian of the system is \[44\]  
\[
H = \hbar \sum_{j=1,2} \left[ \left( \omega_e - i \frac{\Gamma_j}{2} \right) |e\rangle_j \langle e| + \omega_g |g\rangle_j \langle g| \right] + \frac{1}{2} \sum_{j=1,2} \left( \Omega_j \sigma_j^+ e^{-i\omega d t} + g_j \sigma_j^+ a + \text{h.c} \right) + \hbar (\omega_c - i \kappa) a^\dagger a,
\]
where \(\sigma_j^+ = |e\rangle_j \langle g|, \sigma_j^- = |g\rangle_j \langle e|\) are the \(j\)-th spin operators, \(a^\dagger\) and \(a\) are the creation and annihilation operators of the microwave photon. \(\Omega_j\) is the Rabi frequency of the external microwave driving field and \(g_j\) is the coupling strength between the TLR and SCQs. \(\Gamma_j\) and \(\kappa\) are the decay rates of SCQs and TLR respectively. By combining the coupling of the TLR-SCQ and the effect of a classic microwave driving field, the system will create a dynamical Stark effect for the state \(|g, g\rangle_{12}\), meanwhile, keep the states \(|i, i\rangle_{12}, |i, g\rangle_{12}\) and \(|g, i\rangle_{12}\) unchanged. That is to say, the conditional phase \((\varphi)\) gate between SCQ1 and SCQ2 can be implemented under the action of Hamiltonian Eq. \[13\] as follows \[44\]:  
\[
|i\rangle_1 |i\rangle_2 \rightarrow |i\rangle_1 |i\rangle_2, \quad |i\rangle_1 |g\rangle_2 \rightarrow |i\rangle_1 |g\rangle_2, \\
|g\rangle_1 |i\rangle_2 \rightarrow |g\rangle_1 |i\rangle_2, \quad |g\rangle_1 |g\rangle_2 \rightarrow e^{i\varphi} |g\rangle_1 |g\rangle_2,
\]
where the phase \(\varphi = -t|\Omega|^2/2(\Delta + \delta)\) is tunable. The successful realization of the conditional phase gate is based on the following limits: 1) \(|\Delta| \gg \Gamma_j, \kappa, |\Omega_j|, |g_j|, \text{ and } |\delta| \sim 0; \ 2) \ |g_j| > |\Omega_j| \text{ and } |g_j|^2 \gg \Gamma_j \kappa. In the following calculation, we set the phase \(\varphi = -3\pi/2\) which can be implemented by selecting the appropriate parameters.

Now the KLM states can be prepared using the controlled phase gate based on the above device. It is assumed that the two SCQs are in the steady state \(|i\rangle_1 |i\rangle_2\). We apply a classical microwave pulse to SCQ1, SCQ2 respectively to let the two SCQs in the state \(1/2(|i\rangle_1 + |g\rangle_1)(|i\rangle_2 + |g\rangle_2)\). Subsequently, the interactions between the SCQs and TLR are turned on. Undergoing an appropriate dynamic evolution time, only the state \(|g\rangle_1 |g\rangle_2\) will produce a phase \(\varphi = -3\pi/2\) and the rest states remain unchanged. The state evolution process is  
\[
\frac{1}{2}(|i\rangle_1 + |g\rangle_1)(|i\rangle_2 + |g\rangle_2) \rightarrow \frac{1}{2}(|i, i\rangle_{12} + |i, g\rangle_{12} + |g, i\rangle_{12} + i|g, g\rangle_{12}).
\]
Then apply a classical microwave pulse to SCQ2 for the transformations $|i\rangle_2 \rightarrow 1/\sqrt{2}(|i\rangle_2 - |g\rangle_2)$, $|g\rangle_2 \rightarrow 1/\sqrt{2}(|i\rangle_2 + |g\rangle_2)$. The process is

$$\frac{1}{2}(|i, i\rangle_{12} + |i, g\rangle_{12} + |g, i\rangle_{12} + |g, g\rangle_{12}) \rightarrow \frac{1}{2\sqrt{2}}[2 |i, i\rangle_{12} + (1 + i)|g, i\rangle_{12} + (i - 1)|g, g\rangle_{12}]. \quad (16)$$

Up to now, we have implemented the preparation of the two-qubit KLM states. Moreover, the scheme can be generalized to generate $N$-qubit KLM states straightforward. Assuming that the $N - 1$ qubits have been in the KLM states and the $N$-th SCQ is in the state $|i\rangle_N$ initially, then the procedure for generating the $N$-qubit KLM states is as follows:

**Step 1:** Adjust the previous ($N$-2) SCQs to decouple from the TLR so that the interaction occurs only between ($N$-1)-th and $N$-th SCQs.

**Step 2:** Apply a classical microwave pulse to $N$-th SCQ and implement the transform $|i\rangle_N \rightarrow 1/\sqrt{2}(|i\rangle_N + |g\rangle_N)$, as above.

**Step 3:** Undergoing an appropriate dynamic evolution time, the state $|g\rangle_{N-1}|g\rangle_N$ will produce a phase $\varphi = -3\pi/2$ and the rest of the states will remain unchanged.

**Step 4:** Apply a classical microwave pulse to the $N$-th SCQ once again for implementing the transforms $|i\rangle_N \rightarrow 1/\sqrt{2}(|i\rangle_N - |g\rangle_N)$, $|g\rangle_N \rightarrow 1/\sqrt{2}(|i\rangle_N + |g\rangle_N)$. Then, the KLM states of $N$ qubits can be prepared successfully. Here, we use the notation $|i\rangle^N(|g\rangle^N)$ to denote the number of the SCQs in the state $|i\rangle(|g\rangle)$.

If $N$ is an even number, the KLM states we prepare is

$$|\Psi\rangle_{KLM} = \frac{1}{\sqrt{2}^{N+1}}[\alpha_0 |i\rangle^N + \sum_{j=1}^{N-1} \alpha_j |g\rangle^j |i\rangle^{N-j} + \alpha_N |g\rangle^N], \quad (17)$$

where $\alpha_0 = 2^{N/2}$, $\alpha_j = -2^{N/2-j+1}(i - 1)^{j-2}$, $\alpha_N = -2^{2-N/2-i(i-1)}^{N-3}$.

If $N$ is an odd number, the KLM states we prepare is

$$|\Psi\rangle_{KLM} = \frac{1}{\sqrt{2}^{N}}[\alpha_0 |i\rangle^N + \sum_{j=1}^{N-1} \alpha_j |g\rangle^j |i\rangle^{N-j} + \alpha_N |g\rangle^N], \quad (18)$$

where $\alpha_0 = 2^{(N-1)/2}$, $\alpha_j = -2^{(N+1)/2-j+1}(i - 1)^{j-2}$, $\alpha_N = -2^{(3-N)/2-i(i-1)}^{N-3}$. 

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4. THE FEASIBILITY ANALYSIS AND CONCLUSION

Now we discuss briefly the feasibility of the present schemes based on the current available parameters. For the first scheme, we choose the coupling strength between the SCQ and the TLR as $g_s = 2\pi \times 75 \text{ MHz}$ [45], and the coupling strength of molecule-cavity as $g_m = 2\pi \times 20 \text{ MHz}$ [46]. If we set $\Delta_s = 10g_s = 2\pi \times 750 \text{ MHz}$, $\Delta_m = 25g_m = 2\pi \times 0.5 \text{ GHz}$ and $N = 10^4$, then the effective coupling strength between the PMs and the SCQ can reach as $g = 2\pi \times 250 \text{ MHz}$. Furthermore, the effective coupling $g$ can be enhanced greatly with the number $N$ of the PMs of each ensemble increasing. For the choice of the energy relaxation and dephasing times of the SCQ as $\tau_e = 25 \mu s$ and $\tau = 5 \mu s$ [33], the dephasing rate $\gamma = 1/\tau = 2\pi \times 0.032 \text{ MHz}$, and the single-molecule collision rate is demonstrated to be $\gamma_m \leq 2\pi \times 700 \text{ Hz}$ [30], so the decoherence rates induced by the SCQ and the PMs are much smaller than the effective coupling strength. Additionally, the decoherence caused by the decay of the TLRs could be suppressed effectively due to the virtual excitations of microwave photons. When we set $\Omega' = \Omega'' = 0.1g$, the total time used to generate the two-qubit KLM states is less than 29 ns which is far less than the coherence time of the SCQ.

For the second scheme, the strong coupling between the SCQ and the TLR can be yielded by fabricating two SCQs at the maximum of the voltage standing wave [31]. Based on currently available technology, the parameters $\Delta$, $g_j$, $\Omega_j$, $\kappa$, $\Gamma_j \sim 2\pi \times (400, 75, 30, 0.008, 0.0064) \text{ MHz}$ can be achieved [32, 33]. Obviously, the ratio of $\kappa$ ($\Gamma_j$) to $g_j$ is so small that we can safely ignore the effect of the decay rates of TLR (SCQ) (For simplification of analysis, we have assumed $\Gamma_j = \Gamma$ and $g_j = g$). Furthermore, in our scheme, because the decay rates of the TLR and the SCQs are smaller than that of the system in Ref. [44], the conditions for implementing the controlled phase gate are satisfied more easily, so the success probability of the controlled phase gate on SCQs is near determinacy. In addition, we can calculate the gate operation time is about 0.666 $\mu s$ and the total times used to generate two-, $N$-qubit KLM states are about 0.674 $\mu s$ and 0.674($N - 1$) $\mu s$ respectively.

In conclusion, we have proposed two different methods to prepare two- and $N (N \geq 2)$-qubit
KLM states. We have discussed the feasibilities of the two schemes in detail. By using SCQ, TLR and encoding information on the two lower levels of the SCQ, our schemes have longer coherence time and storage time. The controllability and feasibility of our schemes are high in the current techniques. Both of the schemes are robust against photon decay due to the virtual excitations of the microwave photons of the TLRs, and the spontaneous emission can be suppressed owing to the virtual transitions of the SCQs’ internal states in the second scheme. Furthermore, we have generalized the second scheme to prepare N-qubit KLM states. We hope that our work may be useful for the quantum information in the near future.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China under Grant Nos. 11064016 and 61068001.

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Fig. 1. Schematic setup of two PMs ensembles placed in the two separate TLRs coupled by a SCQ and the level diagram for the PM and the SCQ, where $g_s$ is the coupling strength between TLR and SCQ. $\Omega$ is the rabi frequency of microwave pulse.

Fig. 2. Density plots of the fidelity as a function of $\eta_1$ and $\eta_3$ with (a) $\eta_0 = \eta_2 = 0$. (b) $\eta_0 = \eta_2 = 0.1$.

Fig. 3. (a) The schematic circuit of the tunable controlled phase gate. The setup involves two SCQs and a TLR. (b) The level configuration of the SCQ.
Fig. 1.
Fig. 2.
$$\begin{align*}
&\text{(a)} \\
&\text{(b)}
\end{align*}$$

Fig. 3.