Circuit simulation of two-degree-of-freedom unilateral impact dynamics system with gap

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Abstract. In a two-degree-of-freedom unilateral collision system with a gap, the two oscillators will produce nonlinear motion. A dynamic model of the two-degree-of-freedom collision system is established, and the two oscillators are analyzed separately by numerical simulation. On this basis, an electronic circuit composed of TL074CN operational amplifier, capacitance and resistance is designed, which is equivalent to the mathematical model. The electronic circuit is simulated by multisim12.0, and the result is consistent with that of numerical simulation. The equivalent electronic circuit model not only has dramatically shortened the calculation time, but also has the advantage of adjustable dynamic parameters, which makes up for the lack of numerical simulation and provides a reference tool for the simulation analysis of multi-degree-of-freedom nonlinear mechanical vibration systems.

1. Introduction
In mechanical manufacturing and processing, some devices have gaps in parts due to installation errors and unavoidable wear in using. Due to the existence of the gap, the equipment will cause collision and friction between parts under external excitation during operation, and cause wear and noise, which would further reduce the production efficiency and even damage the equipment. Therefore, the study on the collision, friction and other issues has great significance to the reliability analysis of mechanical system dynamics and noise control. In recent years, many scholars begin to study multi-degree-of-freedom collision dynamics systems. Compared with single-degree-of-freedom collision dynamics systems, multi-degree-of-freedom collision dynamics systems are closer to actual applications and have more complex dynamic characteristics. With the advancement and development of computers, numerical simulation methods are generally used to analyze multi-degree-of-freedom nonlinear dynamic systems.

Numerical simulation of nonlinear vibration systems requires efficient programs and algorithms to shorten the calculation time. Since numerical simulation uses numerical iteration to solve differential and integral equations, when the accuracy is very high, the calculation would take a long time and a huge amount of calculation. In order to quickly complete the calculation process, researchers usually optimize the algorithm to improve the efficiency of numerical calculations, or use expensive high-performance computers to shorten the calculation time. Besides consuming long time and high requirements of computer performance, using numerical simulation to study the dynamics of nonlinear vibration systems also requires a high level of programming ability for researchers. Moreover, numerical simulation couldn’t adjust dynamic parameters, which greatly limits application in actual engineering. Multisim, a Windows-based circuit simulation tool, is mainly used for circuit teaching, circuit diagram design and SPICE simulation. Multisim has rich functions and excellent simulation.
computing capabilities. Users can build circuit schematic diagrams and circuit hardware description language input.

The paper designs a fully equivalent circuit model of a two-degree-of-freedom unilateral collision system with gaps. The circuit model uses several simple circuit components such as TL074CN operational amplifiers, resistors and capacitors, and simulates them in Multisim12.0. The results are consistent with those obtained by numerical simulation. More importantly, the simulation time is greatly shortened. It is easily found that the equivalent circuit can analyze the dynamics of nonlinear vibration systems, and have the advantages of high speed and adjustable dynamic parameters that are not available in numerical simulation.

2. Dynamic model and numerical analysis

2.1. mathematical model

Fig.1 shows a two-degree-of-freedom unilateral collision dynamic system with a gap.

![Figure 1. dynamic model](image)

According to the dynamic model, the dynamic equations of the masses $M_1$ and $M_2$ are established as follows:

\[
\begin{align*}
M_1\ddot{X}_1 + C_1(\dot{X}_1 - \dot{X}_2) + K_1(X_1 - X_2) + F(X_1) &= 0 \\
M_2\ddot{X}_2 + C_2\dot{X}_2 + K_2X_2 - C_1(\dot{X}_1 - \dot{X}_2) - K_1(X_1 - X_2) &= P_2\sin(\Omega T + \tau)
\end{align*}
\]

(1)

Where $M_1$ and $M_2$ are, $M_1$ and $M_2$ are the masses of mass 1 and mass 2 respectively. Where $C_1$ and $C_2$ are, $C_1$ and $C_2$ are the damping coefficients of two linear damping respectively. Where $K_0, K_1$ and $K_2$ are, $K_0, K_1$ and $K_2$ are the elastic coefficients of three linear springs respectively. Where $X_1$ and $X_2$ are, $X_1$ and $X_2$ are the displacements of mass 1 and mass 2 respectively. Where $\dot{X}_1$ and $\dot{X}_2$ are, $\dot{X}_1$ and $\dot{X}_2$ are the speeds of mass 1 and mass 2 respectively. Where $\ddot{X}_1$ and $\ddot{X}_2$ are, $\ddot{X}_1$ and $\ddot{X}_2$ are the accelerations of mass 1 and mass 2 respectively. Where $P_2$ is, $P_2$ is the amplitude of harmonic excitation force. Where $\Omega$ is, $\Omega$ is the harmonic excitation force frequency. Where $\tau$ is, $\tau$ is the initial phase.

The nonlinear part is as follows:

\[
F(X_1) = \begin{cases}
K_0(X_1 - B) & X_1 > B \\
0 & X_1 \leq B
\end{cases}
\]

Where B is, B is the gap between mass 1 and the massless collision surface. Where $F(X_1)$ is, $F(X_1)$ is the spring force of the mass 1 on the collision surface of the connecting spring $K_0$.

The dimensionless coefficients are as follows:

\[
x_i = \frac{K_iX_i}{P_2}, \delta = \frac{K_iB}{P_2}, \omega = \Omega \sqrt{\frac{M_1}{K_1}}, t = T \sqrt{\frac{K_1}{M_1}}, \varepsilon = \frac{C_1}{2\sqrt{K_iM_1}}
\]
\[
\mu_m = \frac{M_2}{M_1}, \mu_{k0} = \frac{K_2}{K_1}, \mu_{k1} = \frac{C_2}{C_1}, \mu_c = \frac{C_k}{C_k}
\]

Then the dimensionless form is:
\[
\begin{align*}
\dot{x}_1 + 2\xi (\dot{x}_1 - \dot{x}_2) + (x_1 - x_2) + f(x_1) &= 0 \\
\mu_m \ddot{x}_1 + 2\xi \mu_c \dot{x}_2 + \mu_{k1} x_2 - 2\xi (\dot{x}_1 - \dot{x}_2) - (x_1 - x_2) &= \sin(\omega t + \tau)
\end{align*}
\]

(2)

The nonlinear part is as follows:
\[
f(x_1) = \begin{cases} 
\mu_{k0} (x_1 - \delta) & x_1 > \delta \\
0 & x_1 \leq \delta 
\end{cases}
\]

2.2. Bifurcation diagram and phase diagram

There are 5 parameters in formula (2), and different parameters will lead to different motion characteristics. Respectively taking values, \(\mu_m = \mu_c = \mu_{k1} = 1, \mu_{k0} = 25, \xi = 0.2, \delta = 0.02\). Bring the above parameters into equation (2), and letting \(v_1 = x_1, v_1 = x_2, v_2 = \dot{x}_1, v_3 = \dot{x}_2\), converted to the state equation is as follows:
\[
\begin{align*}
\dot{v}_2 &= \dot{v}_1 \\
\dot{v}_3 &= 2\xi (v_2 - v_3) + (v_1 - v_3) + f(v_1) \\
\dot{v}_4 &= \dot{v}_3 \\
\mu_m \dot{v}_4 &= 2\xi \mu_c v_4 + \mu_{k1} v_3 - 2\xi (v_2 - v_3) - (v_1 - v_3) = \sin(\omega t + \tau)
\end{align*}
\]

(3)

The nonlinear part is as follows:
\[
f(v_1) = \begin{cases} 
\mu_{k0} (v_1 - \delta) & v_1 > \delta \\
0 & v_1 \leq \delta 
\end{cases}
\]

According to the state equation (3), the system is programmed by ODE45 algorithm, and the bifurcation diagram is drawn, as shown in Fig.2. In the figure, the abscissa is the frequency \(\omega\), and the ordinate is the speed.

Fig.2(a) shows the velocity \(v_2\) of the M1, and Fig.2(b) shows the velocity \(v_4\) of the M2. It can be found from the figure that the system is incredibly sensitive to the change of \(\omega\). With the change of frequency, the system has complex nonlinear motion characteristics including a single period, two periods, four periods, eight periods and chaos.

![Figure 2. bifurcation diagram](image_url)
Fig. 2 shows the period doubling bifurcation of the velocity $v$ as the frequency $\omega$ increases. It can be found from Fig. 2(a) that the system has the first doubling bifurcation near $\omega = 1.61$, changing from period 1 motion to period 2 motion (Fig. 3(a) $\omega = 1.65$). Then a second doubling bifurcation occurs near $\omega = 1.7$, changing from period 2 motion to period 4 motion (Fig. 3(a) $\omega = 1.72$). Then the third doubling bifurcation occurred, changing from period 4 motion to period 8 motion (Fig. 3(a) $\omega = 1.723$), and most from period 8 motion to chaos (Figure 3(a) $\omega = 1.74$).

Fig. 2(b) is the same as Fig. 2(a). After three times of doubling and bifurcation, the movement gradually changes from period 1 motion to chaos. The phase diagrams is shown in Fig. 3(b).

3. Equivalent circuit model and simulation

The numerical analysis and calculation of the vibration system are all integrated calculations under discrete conditions with a computer. Usually Runge-Kutta calculations are used. The algorithm requires several iterations. Therefore, there is a big contradiction between speed and accuracy. Especially when the parameters are changed, the calculation has to be restarted, which severely restricts the calculation speed and the calculation is static calculation[8].

The differential equation of the equivalent circuit and the differential equation of the vibration system are completely equivalent, so that the simulation speed of the system is greatly improved. The circuit is composed of only the TL074CN amplifier, resistors and capacitors to fulfill the above requirements.

3.1. Equivalent circuit design[9]

In order to realize equation (2) using the circuit, the state equation (3) needs to be converted into a circuit equation. In order to realize the circuit equation, we can choose the differential or integral circuit. Here, select the integrated circuit and convert the equation (3) into the corresponding integral equation, as follows:
The nonlinear part is as follows:

\[ f(v_i) = 0.5 \mu_40 (|v_i - \delta| + v_i - \delta) \]

To realize the formula (4), choose four integral modules to realize it, among which the operational amplifier chooses TL074CN. The design is shown in Fig.4. The first equation is implemented by U1A and U1B; the second equation is composed of U2C; the third equation is composed of U1C and U1D; the fourth equation is composed of U3A, U3B, and U3C. U4A and U2A are inverters, U4B and U4C are two adders, where \( v_6 = v_1 - v_2, \ v_7 = v_3 - v_4 \). Oscilloscope 1 shows the phase diagram of M1 movement, and oscilloscope 2 shows the phase diagram of M2 movement.

(a) The first and third equations in formula (4)
The second and fourth equations in formula (4)

The nonlinear function is realized by the NONLINEAR-DEPENDENT module provided by the software, where the power supply voltage is ±18V, and the equation (4) is converted into the second-order differential equation \[^{[10-11]}\], as shown in equation (5).

\[
\begin{align*}
R_1C_1\dot{v}_1 &+ \frac{1}{R_3C_1}(R_3C_1\dot{v}_1 - R_2C_2\dot{v}_2) + \frac{1}{R_4C_3}(v_1 - v_3) + \frac{1}{R_4C_3}f(v_1) = 0 \\
R_1C_1\dot{v}_1 &+ \frac{1}{R_3C_1}(R_3C_1\dot{v}_1 - R_2C_2\dot{v}_2) + \frac{1}{R_4C_3}(v_1 - v_3) + \frac{1}{R_4C_3}(v_1 - v_3) - \frac{V_m}{R_4C_3}\sin(\omega t + \tau) = 0
\end{align*}
\]  

Equations (5) and (2) are exactly in the same form. In order to make the formulas mathematically equivalent, the coefficients must be equal. The parameters selected according to the dimensionless process, namely, \(\mu_m = \mu_c = \mu_k = 1, \mu_{v0} = 25, \xi = 0.2, \delta = 0.02\). Taking into account the impact of resistance and capacitance on electronic circuits, select \(R_1 = R_2 = 10k\Omega, R_{12} = 100k\Omega, C_1 = C_2 = C_3 = C_4 = 22nf\), then get \(R_4 = R_9 = 30k\Omega, R_5 = R_{11} = 30k\Omega, R_3 = R_8 = R_{10} = 43.3k\Omega\), the remaining resistors are only used in the inverter circuit, so they are all 100k\Omega.

The voltage amplitude of the sinusoidal excitation signal is related to the gap \(\delta\), because \(\delta = 0.02\), so:

\[
\delta = B \frac{K_1}{P_2} = 0.064 \frac{1}{R_3C_3} = \frac{V_m}{R_4C_4} = 0.064 \frac{R_{12}}{R_4}V_m = 0.02
\]

From the above formula: \(V_m = 10.67V\)

From the dimensionless process, the relationship between the excitation frequency \(f\) and \(\omega\) can be obtained as:

\[
\omega = \Omega \sqrt{\frac{M_1}{K_1}} = 2\pi \sqrt{\frac{R_9C_1C_2C_3}
\]

3.2. MultiSim12.0 simulation analysis

The circuit model is simulated by MultiSim12.0. The amplitude of the sine wave of the function generator is 10.67V. The excitation frequencies of \(M_1\) movement are 689.16Hz(1.65\omega)、718.39Hz(1.72\omega)、719.65(1.723\omega)、726.75Hz(1.74\omega), and the excitation frequencies for the movement of \(M_2\) are 697.51Hz(1.67\omega)、705.87Hz(1.69\omega)、714.22Hz(1.71\omega)、731.35Hz(1.751\omega). As shown in figure (5), the abscissa channel 1 of oscilloscope 1 is displacement,
and the scale is 1V/Div; the ordinate channel 2 is speed, and the scale is 500mV/Div; the abscissa channel 1 of oscilloscope 2 is displacement, and the scale is 1V/Div; ordinate channel 2 is speed, and the scale is 1V/Div.

1.65ω 1.72ω 1.723ω 1.74ω
(a) Phase diagram of M1 at different frequencies

1.67ω 1.69ω 1.71ω 1.751ω
(b) Phase diagram of M2 at different frequencies

Figure 5. Phase diagram of simulation in Multisim12.0 software

By comparing Fig.3 and Fig.5, the same results are obtained. When the frequency ω of the numerical simulation is transformed into the frequency f of the Multisim simulation, considering that the errors of the resistance and capacitance are both ±1%, and the offset of the operational amplifier is 5%, the error is less than 10%. When the integration step accuracy is selected as 0.01, the numerical simulation time is about 5.2 seconds, and the Multisim simulation time is about 1.8 seconds; when the integration step accuracy is 0.001, the numerical simulation time is about 26.5 seconds, and the Multisim simulation time is about 9.3 seconds. Therefore, the phase diagram drawn by Multisim12.0 is exactly the same as the phase diagram drawn by programming. The running time of self-programming is three times that of equivalent circuit simulation, which shows that the program needs to be further optimized. Of course, it also shows that programming has too much influence on nonlinear research. Circuit simulation can reduce its influence and improve research efficiency. At the same time, it has the advantage of adjusting dynamic parameters. Using equivalent circuits to analyze nonlinear dynamics can reduce programming requirements and improve research efficiency.

4. Conclusions
The thesis established a dynamic model of a two-degree-of-freedom unilateral collision system with a gap, carried out a numerical simulation analysis on the two vibrators in the system, and obtained the phase diagrams of the vibrators at different frequencies. However, numerical simulation analysis has the shortcomings of long operation time and inability to adjust dynamic parameters. At the same time, it has higher requirements for computer performance and programming ability, which limits its application in actual engineering.

According to the above shortcomings, the thesis uses several TL074CN operational amplifiers and capacitor-resistance components to design a circuit model equivalent to the dynamics model. After the circuit simulation software MultiSim12.0 runs, the same result as the numerical simulation is obtained. Therefore, it is feasible to use the equivalent circuit to analyze nonlinear dynamics problems. The equivalent circuit has the advantages of fast simulation speed and adjustable dynamic parameters; besides, it reduces the requirements of numerical simulation on algorithms, programming and computer performance. The equivalent circuit does not need to be programmed and has low requirements on the computer. Only a few simple circuit components can be used to achieve the same results as the numerical simulation. During the simulation operation, the parameters in Multisim can be adjusted to observe the impact of the nonlinear system when the parameters change in real time, which has greater practicability and implementability in actual engineering applications.
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