Improved Exponential model with pairing attenuation and the bakbending phenomenon

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A modified version of the exponential model with pairing attenuation is proposed, and used to describe successfully the backbending of the moment of inertia, in even-even nuclei, not only in well-deformed nuclei but also in slightly deformed nuclei. The model remarkably fits good the experimental observation with a few parameters.

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Several models have been remarkably successful in the description of the low lying collective states in many light, medium and heavy even-even nuclei. In the last three decades, a large amount of high-spin states of nuclei has been accumulated. Among these data, an interesting effect has been observed in the ground state rotational band (yrast states) of some even-even nuclei. It is called the backbending which occurs as one plots the moment of inertia versus the square of the rotational frequency. Many efforts within the framework of different models have been attempted to understand the mechanism of the rapid change of the moment of inertia. Three types of explanations have been proposed for this change. The essential features of these explanations are: pairing collapse, rotation alignment and centrifugal stretching. Most of these features could be described in terms of band mixing or band crossing.

The rapid increase of the moment of inertia with the rotational frequency indicates a major modification of the intrinsic structure at the point where backbending occurs. The fact that the moment of inertia is almost doubled and is approaching the value of a rigid rotation suggests that the transition is associated with a considerable reduction in the pair correlation. The dependence of the nuclear moments of inertia on pairing correlations (in an exponential form) has been established previously. In an attempt to provide a theoretical basis for this relation, Sood and Jain found that the underlying physical process is attributed to a transition from quasiparticles in the presence of pairing, to independent particles as the pairing disappears. The analogy between the destruction of the pair correlation by coupling at large orbital angular momentum and the destruction of the superconductivity by a magnetic field led them to draw a relation between the pairing gap and the angular momentum in the following form:

\[ \Delta (I) = \Delta (0) \left( 1 - \frac{I}{I_c} \right) ^{\frac{1}{2}} \]  

with \( \nu = 2 \), such that the pairing gap \( \Delta (I) \) vanishes at the angular momentum \( I_c \). This relation was previously obtained for nuclear problems by Moretto in his statistical model calculations. Accordingly, Sood and Jain gave the following expression for rotational energy levels of ground state band:

\[ E (I) = \frac{\hbar^2}{2 \varphi_0} I (I + 1) e^{\Delta_0 \sqrt{1 - \frac{I}{I_c}}} \]  

They applied their approach to describe the backbending phenomena in the well-deformed nuclei region (150 < A < 190), with \( \Delta_0 \) and \( \varphi_0 \) as free parameters and \( I_c \) is selected around 18\( \hbar \). Satisfactory results have been obtained below the point where backbending occurs. Making use of their formula it was not however possible to describe the forward or down-bending region of the \( \varphi - \omega^2 \) plot.

Further applications of the exponential model with pairing attenuation have been recently carried out to the superdeformed nuclei in the \( A \sim 190 \) region and satisfactory results have been obtained for superdeformed bands.

Old and recent theoretical investigations of backbending mechanism have led to the use of multiparameter models, such as the various extensions of VMI, and have yielded qualitatively different results at different mass regions where the corresponding deformations change considerably from slightly deformed to superdeformed nuclei.

We present, here, a simple three parameter modified formula based on the exponential model with pairing attenuation for the description of the backbending phenomena, by letting \( \nu \) to be a free parameter. We take this model as a
phenomenological one without reverting to its original derivation. This model not only reasonably describes the backbending of the moment of inertia in the well-deformed nuclei but its excellent validity also extends to include the slightly deformed nuclei at $A \sim 100$, and actinide nuclei where the interplay between single particle and collective aspects maybe expected at $A^{\sim}240$ as well.

In the present work the energy levels of the ground state band, for even-even nuclei, are obtained from the following expression:

$$E(I) = \frac{\hbar^2}{2\varphi_0} I (I + 1) e^{\Delta_0 (1 - \frac{I}{I_c})^\nu},$$  \hspace{1cm} (3)

where $\varphi_0, \Delta_0$ and $\nu$ are the three parameters of the model which are adjusted to give a least squares fit to the experimental data for low and high angular momenta. To fit these parameters we have carried out an exponential model least squares fit to the observed energy levels up to $I = 30$. For better predictive power we scanned for optimum $I_c$ by keeping $\varphi_0, \Delta_0$ and $\nu$ as free parameters, where $I_c$ corresponds to the minimum value of the root mean square deviation value $\sigma$ given by:

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(1 - \frac{E_{cal}}{E_{exp}}\right)^2},$$ \hspace{1cm} (4)

In the case where $\sigma$ changes exponentially with no minimum we adopt $I_c$ as the value where the variation of $I_c$ does not affect $\sigma$ significantly.

Next we undertake a comparative study between our calculations and the experimental data through $\varphi - \omega^2$ plots. The moment of inertia $\varphi$ and the squared rotational frequency $\omega^2$ are related to the spin derivatives of the energy through the relations

$$2\varphi/\hbar^2 = (4I - 2)/\Delta E_{\gamma},$$ \hspace{1cm} (5)

$$(\hbar\omega)^2 = (I^2 - I + 1)/[\Delta E_{\gamma}/(2I - 1)]^2$$ \hspace{1cm} (6)

where

$$\Delta E_{\gamma} = E(I) - E(I - 2)$$

The results of the ground state energy levels up to spin $I = 30$ for a set of representative nuclei are presented in Table I, with the corresponding experimental energy levels,[17] and the calculated parameters are given in Table II.

In Figs. 1, 2, and 3, we present the $\varphi - \omega^2$ plots for 24 nuclei covering the mass region from 100 to 242. The backbending displayed in Figs. 1, 2, and 3 shows an excellent agreement between the model predictions and experimental data even in the forward or downbending regions.

These results show that the present improved version of the exponential model with paring attenuation is a successful tool in studying ground state energy levels in slightly deformed and deformed nuclei up to high spin. The restriction imposed in reference [14] in the energy ratio $E(4^+)/E(2^+)$ to exceed 3.0 does not arise here since the exponent $\nu$ is a free parameter (see last column of table II). Moreover, our improved model overcomes the inapplicability found in previous works concerning the description of the forward or downbending region of the $\varphi - \omega^2$ plot.
| Table I: Experimental and calculated energies (in MeV) of levels of ground state bands of even-even Nuclei |
|---------------------------------------------------------------|
| Isotope | Experimental | Calculated |
| Zr | 0.21254 0.56486 | 0.20871 0.56075 |
| Pd | 0.15178 0.47828 | 0.14888 0.47777 |
| Cd | 0.3738 0.92077 | 0.3636 0.92497 |
| Ba | 0.186 0.544 | 0.180 0.544 |
| Xe | 0.32118 0.8283 | 0.30529 0.8330 |
| Nd | 0.15905 0.4855 | 0.16598 0.4987 |
| Gd | 0.51537 1.20907 | 0.50906 1.24572 |
| Os | 0.11072 1.12058 | 0.11586 1.21058 |
| Dy | 0.07251 0.23662 | 0.07251 0.23708 |
| Er | 0.10354 0.33250 | 0.10354 0.33250 |
| Os | 0.2467 1.13059 | 0.2486 1.14657 |
| Th | 0.0532 0.1741 | 0.0532 0.1741 |
| Pu | 0.0486 0.1473 | 0.0486 0.1473 |

**Notes:**
- Table values in MeV.
- Experimental and calculated energies are presented for various isotopes, including Zr, Pd, Cd, Ba, Xe, Nd, Gd, Os, Dy, Er, and Th.
- The table provides both experimental and calculated energy values for each isotope, allowing for comparison between observed and predicted energies. 

**Additional Information:**
- The table serves as a foundational tool for understanding the energy levels of ground state bands in even-even nuclei, which is crucial for nuclear physics research.
- The values reflect the dynamic interaction between the nuclear structure and external conditions, offering insights into the stability and transitions within these elements.
TABLE II: The fitting parameters of the present model (Eq. 3). The fifth column gives the root mean square deviation (Eq. 4), and the last column gives the ratio $R_4 (R_4 = E_4^+ / E_2^+)$.  

| Nucleus | $2\omega^2/h^2$ | $\Delta_0$ | $\nu$ | $I_c$ | $\text{rmsd}$ | $E_4^+ / E_2^+$ |
|---------|----------------|-----------|-------|-------|-------------|----------------|
| $^{100}$Zr | 57.1918 | 0.7389 | 0.114596 | 80 | 0.0411068 | 2.66 |
| $^{102}$Zr | 120.206 | 1.1312 | 0.733982 | 80 | 0.0078426 | 2.66 |
| $^{110}$Pd | 48.7823 | 1.4309 | 0.114326 | 64 | 0.0115461 | 2.46 |
| $^{112}$Cd | 37.1679 | 1.7962 | 0.462671 | 16 | 0.0031577 | 2.29 |
| $^{120}$Ba | 87.8122 | 1.0736 | 0.244153 | 80 | 0.0135763 | 2.92 |
| $^{122}$Xe | 55.9335 | 1.2866 | 0.122269 | 80 | 0.0299285 | 2.50 |
| $^{130}$Nd | 75.7929 | 0.8509 | 0.575297 | 26 | 0.0153101 | 3.05 |
| $^{132}$Nd | 77.2153 | 1.1831 | 0.451699 | 20 | 0.0031577 | 2.87 |
| $^{140}$Ba | 34.8359 | 2.2409 | 0.118346 | 30 | 0.0119052 | 1.88 |
| $^{142}$Gd | 51.9782 | 1.833 | 0.451109 | 22 | 0.0131265 | 2.35 |
| $^{150}$Nd | 106.816 | 0.9608 | 0.170712 | 80 | 0.0185105 | 2.82 |
| $^{152}$Nd | 104.880 | 0.2774 | 0.920592 | 20 | 0.0031577 | 3.26 |
| $^{160}$Dy | 442.245 | 1.8949 | 6.008963 | 26 | 0.005267 | 3.27 |
| $^{162}$Er | 597.374 | 2.3679 | 5.397786 | 26 | 0.0067128 | 3.23 |
| $^{170}$Os | 154.772 | 2.1634 | 0.253182 | 80 | 0.0194954 | 2.62 |
| $^{172}$Os | 72.0404 | 1.287 | 0.45591 | 20 | 0.0229337 | 2.66 |
| $^{180}$Os | 510.846 | 2.5195 | 0.742648 | 80 | 0.0061911 | 3.26 |
| $^{182}$Os | 292.084 | 0.7829 | 1.01142 | 80 | 0.0097799 | 3.31 |
| $^{190}$Os | 79930.8 | 6.4192 | 15.3588 | 56 | 0.0059640 | 3.31 |

FIG. 1: The moment of inertia $2\omega^2/h^2$ versus the square of the rotational frequency $(\hbar \omega)^2$ for $^{100,102}$Zr, $^{110,112}$Pd, $^{120,140}$Ba, $^{122}$Xe, $^{130,132}$Nd, and $^{142}$Cd. Dots represent experimental values.
FIG. 2: The moment of inertia $2\phi/\hbar^2$ versus the square of the rotational frequency $(\hbar\omega)^2$ for $^{150,152}$Nd, $^{160}$Dy, $^{162}$Er, and $^{170-192}$Os. Dots represent experimental values.

FIG. 3: The moment of inertia $2\phi/\hbar^2$ versus the square of the rotational frequency $(\hbar\omega)^2$ for $^{230,232}$Th and $^{240,242}$Pu. Dots represent experimental values.

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