A Mathematical Model for Fracture Networks with Considering Fracture Propagation and Closure in Shale

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Abstract. This study develops a semi-analytical model for fracture networks with considering fracture propagation and closure. To consider the volumetric fracturing, Warren-Root dual-porosity model is used to divide the reservoir into a fracture system and a matrix system. In addition, the discrete secondary fractures connected by hydraulic fractures are also taken into account. After model solution, it is found that flow stages of the theoretical well test curve of the multi-stage fractured horizontal well in shale reservoirs can be divided into: stage before the fracture closes, stage after the fracture closes, supply flow stage between secondary fractures and hydraulic fractures, linear flow stage, early dual radial flow stage, interporosity flow stage, late dual radial flow, and pseudo radial flow. The pressure and pressure conduction curves coincide with a straight line with a slope of 1 at stage before the fracture closes, which is similar to the wellbore storage effect. The shape of the derivative curve of dimensionless pressure and dimensionless pressure is similar to the behaviour of variable well-storage, during stage after the fracture closes. At the supply flow stage, there is a slight "dip", which means that the reservoir fluid is supplied from the secondary fractures activated by fracturing to the main hydraulic fractures.

1. Introduction

With the development of fracturing technology, complex fracture network system is formed in shale reservoirs after fracturing [1], including matrix, natural fracture, fracturing secondary fracture and hydraulic main fracture. Different media have great differences in properties resulting in strong heterogeneity. Coupling flow between fractures of different scales is the key problem to be solved in shale reservoirs. Therefore, it is necessary to carry out parameter evaluation of fracture network.

At present, the understanding of complex fracture network can be classified into the following six kinds: (1) geological analysis of fractures in natural outcrops, (2) analysis of micro-seismic monitoring data, (3) core analysis, (4) logging identification, interpretation and analysis, (5) interwell tracer method, (6) well test interpretation and analysis [2-14].
Natural outcrop geological analysis of fractures is the most direct method to describe fractures. This method is mainly based on the measurement statistics and classification of the fractures on the surface of outcrop rocks, meanwhile, it has certain limitations: the number of natural outcrop samples and the accuracy of analytical instruments has a great influence for fracture parameters. Due to reservoir depth differentiation effect, formation stress environment is different. According to the data analysis, some natural outcrop can get fractures shape and characteristics, but some findings are not related. For the credibility and accuracy of the results, it’s necessary to find outcrop fractures which have stronger correlation of underground fractures, to summarize and to get the mathematical statistics law so as to carry on the further in-depth research. This method also has certain advantages. The geological analysis of natural outcrop fractures can obtain more comprehensive fracture parameters, including stratum structure type and rock composition type, stratum occurrence, stratum thickness, fracture length, width, spacing, occurrence and so on.

The analysis of micro-seismic monitoring data refers to setting up a micro-seismic monitoring system in the well or on the ground. When fracturing fluid is injected into the formation, the formation pressure becomes higher, causing the formation rupture, and then the micro-seismic monitoring signals are generated and transmitted through the monitoring system, and the inversion technology can obtain the specific parameters of the main fracture or natural fracture. Of course, high-accuracy micro-seismic monitoring and inversion data can be used to verify the accuracy of fracture model.

Core analysis description method is generally used to study the core by experimental methods. Common coring methods include vertical well coring, inclined well coring, horizontal well coring and directional coring. Through qualitative observation, the fractures parameters include fracture type, fracture surface properties and roughness, and the relationship between fracture distribution and fracture distribution depth. In addition, directional coring can also obtain fracture strike, fracture azimuth, fracture inclination and other parameters. Due to the difficulty and the high cost of directional coring, vertical well coring is generally used to obtain rock samples for the next quantitative measurement, including gas permeability, liquid permeability, electron microscope analysis and CT scan, in order to obtain accurate fracture parameters.

Interwell tracer method is to add tracer into the injection well, and as the injected fluid flows in the reservoir, the production of the tracer is monitored and sampled in the production well around the injection well. The location of the fracture and the sealing property of the formation can be analyzed by monitoring the production of the tracer. The interwell tracers include trace substance tracers, gas tracers, chemical tracers and radioactive tracers. Through collecting and analyzing tracer monitoring data, and combined with the drilling, logging, core and production performance data, tracer curve can be divided into single slit, independent multi-peak and consecutive multi-peak type. Single slit type indicates there is a single flow channel such as a single fracture. The independent multi-peak tracer curve indicates that there are multiple fractures with different flow capacity between the test Wells. The continuous multi-fracture tracer curve indicates that there are multiple fractures with little difference in flow capacity between the test Wells.

Logging identification interpretation analysis method is applied to fracture identification, mainly including the following methods: Formation micro-resistivity scanning imaging log (FMS), Formation micro-resistivity imaging log (FMI), electromagnetic propagation tool (EPT), acoustic imaging log (UBO), ultrasonic imaging log (UBI), dipole shear wave imaging log (DSI), nuclear magnetic resonance imaging log (MRI), etc. These methods can be used to visually display and evaluate fracture reservoirs, and a series of fracture parameters can be obtained, including fracture development depth, fracture occurrence, fracture width, fracture length, fracture filling information and other information characteristics. Fracture opening and occurrence have the most obvious effect on logging response curve.

Well test interpretation analysis combined with the seepage mechanics means that various parameters of reservoir including fractures can be obtained by inversion of well test curve or production dynamic
data combining with different seepage mechanics laws. Well test methods can be divided into transient well test methods and productivity well test methods. Transient well test methods include pressure drop, pressure recovery, multi-well interference, pulse well test and so on. Productivity well test methods include systematic well test (for oil well), isochronous well test, backpressure well test (for gas well), etc. Fracture parameters such as fracture half-length, fracture conductivity, fracture angle and fracture occurrence can be obtained by well test inversion method. For instance, some well test methods are used to evaluate the fracture parameters during Diagnostic fracture-injection/falloff tests (DFITs), with considering the fracture propagation and fracture closure. In their model, only reactivated natural/secondary fractures were considered, while the natural fracture distributed in the whole formation was ignored [15-16].

Based on the above methods and the actual situation of the oil field, the majority of oil fields mainly use micro seismic monitoring method for fracture network parameter inversion currently. Micro seismic monitoring method can only evaluate the shape and position of fracture network macroscopically. In this paper, a well test method is adopted to develop a semi-analytical model for fracture networks with considering fracture propagation and closure, which is helpful for fracture evaluation of hydraulic fractures.

2. Physical Model

Shale reservoirs have a high degree of development of natural fractures, and the combination of horizontal wells and large hydraulic fracturing technology makes the exploitation of shale oil reservoirs economically beneficial. In the process of hydraulic fracturing, there is a process of fracturing and shutting. During this process, there is the expansion and closure of fractures. In this process, some natural fractures are always in the open state, and the other part were opened after the horizontal well was fractured by large-scale hydraulic fracturing. Compared with the previous two models, the complex fracture network model takes into account the reservoir volume increase area, which has a better degree of reservoir reconstruction. Both the volumetric fracturing model and the composite fracturing model use the Warren-Root dual media model to simulate the reservoir stimulation volume zone, dividing the reservoir into a fracture zone dominated by seepage fluid and a matrix system dominated by storage fluid [17-20]. Moreover, the complex fracture network model also takes the discrete secondary fractures in the reservoir stimulation volume area into account. In addition to the reservoir stimulation volume, the reservoir physical properties are poor, and it does not contribute to the oil well production and is regarded as the original reservoir. Establish the physical model of the complex fracture network model, as shown in Figure 1. The assumptions of the complex fracture network model well model are as follows:

1) The reservoir is homogeneous, horizontal, and infinite with uniform thickness.
2) The conductivity of the fracture network is finite. The reservoir is completely penetrated by the fractures, and the flow at the endpoints of the fracture is ignored.
3) Fluid in reservoir only flows from fractures to wellbore.
4) Influence of pressure drop loss and gravity are ignored.
5) Based on fluid storage, the method of Craig’s work [21-22] to consider the propagation and closure of fractures in complex fracture network.
Fig. 1 Schematic diagram of the complex fracture network model.

3. Mathematical Model

Considering the characteristics of shale oil reservoirs, a mathematical model of complex fracture network model is established. The complex fracture network model applied to segmentation can be divided into two parts: before the fracture is closed and after the fracture is closed. Therefore, a mathematical model is established for these two parts separately, both of which include the flow in the matrix and the flow in hydraulic fractures or secondary fractures.

3.1 Dimensionless Variables

Considering the characteristics of shale oil reservoirs, a mathematical model of a complex fracture network model is established. For the solution of the model, the following dimensionless variables are defined.

Dimensionless pressure, time and flow rate:

\[
P_D = \frac{K_f h (p - p_i)}{1.842 \times 10^{-3} \mu q_{sc} B}, \quad p_{m,f,i,vD} = \frac{K_f h (p_{m,f,i,v} - p_i)}{1.842 \times 10^{-3} \mu q_{sc} B}
\]

\[
t_D = \frac{3.6 K_f t_c}{\mu \phi c_i L_v^2}, \quad t_{vD} = \frac{3.6 K_f t_c}{\mu \phi c_i L_v^2}, \quad t_{fD} = \frac{3.6 K_f t_c}{\mu \phi c_i L_v^2}
\]

\[
q_D = \frac{q_f h}{q_{sc}}, \quad q_{wD} = \frac{q_w}{q_{sc}}
\]

Dimensionless fracture conductivity:

\[
C_{FD} = \frac{K_f W_f}{K L_v}, \quad C_{fD} = \frac{K_f W_f}{K L_v}
\]
In order to simplify the solution process, using the same processing method as the volume fracturing model, the fracture network is discretized into discrete fracture endpoints and fracture units, as shown in Figure 2. The complex fracture network can be expressed by the coordinate of the fracture node and the length of the fracture segment.

\[
C_{pD} = \frac{C_{pf}}{2\pi \phi c_j h L_x^2}, \quad C_{bD} = \frac{C_{bf}}{2\pi \phi c_j h L_y^2}, \quad C_{aD} = \frac{C_{af}}{2\pi \phi c_j h L_z^2} \quad (3)
\]

In Equation (4), \( q_{Dk} \) is the injection volume of the ground, considering the propagation process of the fracture in the unclosed stage, it can be written as:
\[ q_{\text{Dk}} = q_{wD} - U_{(t_{c})} q_{wD} - C_{pD} \frac{dp_{wD}}{dt_{D}} + U_{(t_{c})} \left( C_{pD} - C_{bD} \right) \frac{dp_{wD}}{dt_{D}} \]

\[
U_{a} = \begin{cases} 
0, & t_{D} < a \\
1, & t_{D} > a 
\end{cases}
\]

In Equation (5), \( q_{wD} \) is the injection volume at the wellhead, \( p_{D} \) is bottom hole pressure, \( C_{pD}, C_{bD} \) are the well-storage coefficient of fracture propagation and opening, and \( U_{a} \) is Unit step function.

### 3.2.2 After Fracture Closure.

With considering the dual-porosity media, the mathematical model of fluid flow after fracture closure is

\[
\frac{\partial^{2} p_{\text{Dk}}}{\partial r_{D}^{2}} + \frac{1}{r_{D}} \frac{\partial p_{\text{Dk}}}{\partial r_{D}} = \omega \frac{\partial p_{\text{Dk}}}{\partial t_{D}} + \lambda \left( p_{\text{Dk}} - p_{mD} \right) \\
\left( 1 - \omega \right) \frac{\partial p_{mD}}{\partial t_{D}} = \lambda \left( p_{\text{Dk}} - p_{mD} \right) \\
p_{\text{Dk}} \left( t_{D} = 0 \right) = p_{mD} \left( t_{D} = 0 \right) = 0 \\
p_{\text{Dk}} \left( r_{D} \to \infty \right) = p_{mD} \left( r_{D} \to \infty \right) = 0 \\
r_{D} \frac{\partial p_{\text{Dk}}}{\partial r_{D}} \bigg|_{r_{D} \to 0} = -q_{\text{Dk}}
\]

In Equation (6), \( q_{\text{Dk}} \) is the amount of injection on the ground, which can be written as follows when considering the propagation process, opening process and closing process of the fractures:

\[ q_{\text{Dk}} = q_{wD} - U_{(t_{c})} q_{wD} - C_{pD} \frac{dp_{wD}}{dt_{D}} + U_{(t_{c})} \left( C_{pD} - C_{bD} \right) \frac{dp_{wD}}{dt_{D}} + U_{(t_{c})} \left( C_{bD} - C_{aD} \right) \frac{dp_{wD}}{dt_{D}} \]  

In Equation (7), \( q_{wD} \) is the injection volume at the wellhead, \( p_{wD} \) is the bottom hole pressure, \( C_{pD}, C_{bD}, C_{aD} \) are the dimensionless well-storage coefficient during fracture propagation and before and after fracture closure.

### 3.3 Model Solution

Firstly, the dimensionless mathematical model before fracture closure is transformed by Laplace transform, we can get:
After solving Equation (8), we can get:

\[
\overline{p}_{fDk} = \left( q_{wD} - q_{wD} \frac{e^{-s_{rD}}}{s} - \int_0^{t_D} e^{-s_{rD}} \left( C_{pD} - C_{bD} \right) p_{wD} \, dt_D \right) \left( r_D \sqrt{sf(s)} \right)
\]

(9)

In Equation (9), \( K_0(x) \) is a modified second-class zero-order Bessel function. Using the superposition principle, the pressure caused by all fracture segments at each fracture node can be written as:

\[
\overline{p}_{fD} = \sum_{m=1}^{n} \overline{q}_{Dm} \times G_m
\]

(10)

In Equation (10), \( G_m \) is Green's function caused by the k-th fracture node of the m-th fracture segment.

\[
G_m = \int_{l_{Dm}/2}^{l_{Dm}/2} K_0 \left( \sqrt{\left( x_{Dm} - x_{Dk} \right)^2 + \left( y_{Dm} - y_{Dk} \right)^2} \right) \sqrt{sf(s)} \, dl
\]

(11)

In Equation (11), \( \overline{q}_{Dm} \) is the flow rate of the m-th (hydraulic/secondary) fracture section, expressed as

\[
q_{Dm} = \begin{cases}
q_{FD} - q_{FD} \frac{e^{-s_{rD}}}{s} - \int_0^{t_D} e^{-s_{rD}} \left( C_{pD} - C_{bD} \right) p_{wD} \, dt_D, & \text{hydraulic fracture} \\
q_{fD} - q_{fD} \frac{e^{-s_{rD}}}{s} - \int_0^{t_D} e^{-s_{rD}} \left( C_{pD} - C_{bD} \right) p_{wD} \, dt_D, & \text{secondary fracture}
\end{cases}
\]

(12)

For the flow in hydraulic fractures or secondary fractures, it is considered as one-dimensional steady flow. The reason why the flow in fractures is assumed to be steady flow is that the fracture permeability is much larger than the matrix permeability, and the flow in fractures is considered to be instantaneous. The dimensionless mathematical model of the flow in the fracture before fracture closure is obtained as follows:

\[
\left\{ \begin{array}{l}
\frac{\partial^2 \rho_{FDk}}{\partial x_D^2} - \frac{2}{C_{FD}} \frac{\partial \rho_{FDk}}{\partial y_D} \bigg|_{y_D = W_{rD}} = 0 \\
p_{FDk}(t_D = 0) = 0 \\
\frac{\partial p_{FDk}}{\partial y_D} \bigg|_{y_D = W_{rD}} = \frac{2\pi}{C_{FD}} q_{Dm} \\
q_{Dk} \bigg|_{y_D = W_{rD} + 1} = q_{Dk} \bigg|_{y_D = W_{rD}} + L_{FDk} q_{FDk}
\end{array} \right.
\]

(13)
In Equation (13), $x_{Dk}$ is the x-coordinate of the k-th fracture node, $x_{Dk+1}$ is the x coordinate of the k+1-th fracture vertex. After solving Equation (13), the pressure difference of fracture nodes can be expressed as:

$$
- \frac{p_{FDk+1} - p_{FDk}}{r_D} = \frac{2\pi}{C_{FD}} \int_{-L_{FDk}/2}^{L_{FDk}/2} \left[ q_D(x_D + L_{FDk}/2) + q_{wDk} \right] dy_D
$$

In Equation (14), $p_{Dk}$ is the pressure at the k-th fracture vertex, $p_{Dk+1}$ is the pressure of the k+1-th fracture vertex, $q_{Dk}$ is the flow rate of the k-th fracture vertex, $q_{wDk}$ is the flow rate of the k-th fracture vertex.

The dimensionless mathematical model after fracture closure is obtained by Laplace transform:

$$
\frac{1}{r_D} \frac{\partial}{\partial r_D} \left( r_D \frac{\partial \bar{p}_{FDk}}{\partial r_D} \right) = sf(s) \bar{p}_{Dk}
$$

$$
\bar{p}_{FDk}(r_D, t_D = 0) = 0
$$

$$
\lim_{t_D \to 0} \left( r_D \frac{\partial \bar{p}_{FDk}}{\partial r_D} \right) = -\left\{ \frac{q_{wD}}{s} - q_{wD} \frac{e^{-s \tau}}{s} - \int_{0}^{\tau} e^{-s \tau} \left( C_{PD} - C_{BD} \right) p_{wD} \, dt_D + \frac{1}{s} \int_{0}^{\tau} e^{-s \tau} \left( C_{BD} - C_{AD} \right) p_{wD} \, dt_D - s C_{AD} p_{wD} + p_{wD}(0) C_{AD} \right\}
$$

Where $p_{wD}(0)$ is the wellbore pressure at the moment of shut-in. After solving the above equation, $\bar{p}_D$ is expressed as:

$$
\bar{p}_{FDk} = \left\{ \frac{q_{wD}}{s} - \frac{q_{wD} e^{-s \tau}}{s} - \int_{0}^{\tau} e^{-s \tau} \left( C_{PD} - C_{BD} \right) p_{wD} \, dt_D + \frac{1}{s} \int_{0}^{\tau} e^{-s \tau} \left( C_{BD} - C_{AD} \right) p_{wD} \, dt_D - s C_{AD} p_{wD} + p_{wD}(0) C_{AD} \right\} K_0 \left( r_D \sqrt{sf(s)} \right)
$$

Using the superposition principle, the pressure at each fracture node caused by all fracture segments can be written as:

$$
\bar{p}_{FDk} = \sum_{m=1}^{n} \bar{q}_{Dm} \times G_m
$$

In Equation (17), $G_m$ is Green's function caused by the k-th fracture vertex of the m-th fracture segment:

$$
G_m = \int_{-l_{FDm}/2}^{l_{FDm}/2} K_0 \left( \sqrt{(x_{Dm} - x_{Dk})^2 + (y_{Dm} - y_{Dk})^2 + l_m \cos \theta_m} + (y_{Dm} - y_{Dk})^2 + l_m \sin \theta_m \right)^2 \, dl
$$

In Equation (18), $\bar{q}_{Dm}$ is the flow rate of the m-th hydraulic fracture or secondary fracture section, which can be expressed as:
Similarly, for the flow in hydraulic fractures or secondary fractures, the dimensionless mathematical model is:

\[
q_{Dm} = \begin{cases} 
\int_0^{(0)} e^{-st_D} (C_{bd} - C_{ad}) p_{wD} dt_D - sC_{ad} p_{wD} + p_{wb}(0)C_{ad}, \quad \text{hydraulic fracture} \\
\int_0^{(0)} e^{-st_D} (C_{bd} - C_{ad}) p_{wD} dt_D - sC_{ad} p_{wD} + p_{wb}(0)C_{ad}, \quad \text{secondary fracture}
\end{cases}
\]  

(19)

After solving the equation, the pressure difference of adjacent fracture nodes can be expressed as:

\[
\overline{p}_{FDk+1} - \overline{p}_{FDk} = \frac{2\pi}{C_{FD}} \int_{L_{FDk} / 2}^{L_{FDk} / 2} q_{DK}(x_D + L_{FDk} / 2) + \overline{q}_{wDK} dy_D
\]

(21)

In Equation (22), \(\overline{p}_{FDk}\) is the pressure drop caused by uniform flow rate and variable fracture length, which can be expressed as:

\[
\overline{p}_{FDk} = q_{wD} \overline{P}_{FD} - q_{wD} \overline{P}_{pFD} e^{-x_D} - s\overline{P}_{pFD} \int_0^{L_{FDk}} e^{-st_D} C_{bd} p_{wD} dt_D - s\overline{P}_{FD} C_{bd} \int_0^{L_{FDk}} e^{-st_D} p_{wD} dt_D
\]

(22)

In Equation (23), \(\overline{L}_{FDk}(s)\) is the dimensionless variable length similarly to the power model approximation.

\[
\overline{L}_{FD}(s) = \frac{L_F(s)}{L_F(s_e)} = \left( \frac{s}{s_e} \right)^\alpha
\]

(24)
In (24), $\alpha$ is the fracture storage coefficient, ranging from low loss fracture ($\alpha=1$) to high loss fracture ($\alpha=0.5$). $\overline{P}_{FD}$ is the pressure drop caused by uniform flow rate and variable fracture length, expressed as:

$$
\overline{P}_{FD} = \frac{1}{s} \int_{-\frac{t_{FD}}{2}}^{\frac{t_{FD}}{2}} K_0 \left( \sqrt{(x_D-x_D')^2 + (y_D-y_D')^2 + l \cos \theta} \right) \sqrt{s f(s)} \, dl
$$

(25)

Based on Equation (25), when the fracturing process is short and $t_{cD} = t_D < t_{cD}$, the pressure solution before closure is approximate to Equation (26).

$$
P_{wD} = p_{wD}(0) C_{bd} p_{ad}(t_D)
$$

(26)

In Equation (26), $p_{wD}(t_D)$ is the dimensionless pressure solution before fracture closure, which can be expressed as:

$$
\overline{P}_{bd} = \frac{\overline{P}_{FD}}{1 + s^2 C_{bd} \overline{P}_{FD}}
$$

(27)

where, $\overline{P}_{FD}$ can be obtained by reservoir flow model and fracture flow model. In addition, the flow of the fracture wall meets the following conditions:

$$
(q_{wD})_{in} = (q_{wD})_{out} + L_{FDk} \overline{q}_{Dk}
$$

(28)

In Equation (28), $(q_{wD})_{in}$ is the inflow rate of the k-th fracture node, $(q_{wD})_{out}$ is the outflow rate of the k-th fracture node. We have $2n_v + n_s$ equations composed of Eqs. (10), (21) and (28), and the number of unknowns is: $n_{Fv}$ — fracture node pressure $\overline{P}_{FD}$, $n_{Fv}$ — fracture node outflow rate $\overline{q}_{wD}$, $n_{Fs}$ — Flow rate per unit length of fracture unit $\overline{q}_{D}$.

Based on Equation (22), when the fracture is closed, when $t_{cD} = t_D \ll t_D$, pressure in real space is expressed as:

$$
P_{wD} = \left[ p_{wD}(0) C_{bd} - p_{wD}(t_D) (C_{bd} - C_{ad}) \right] p_{ad}(t_D)
$$

(29)

In Equation (29), $p_{ad}(t_D)$ is the dimensionless pressure solution after fracture closure, which can be written as:

$$
\overline{P}_{ad} = \frac{\overline{P}_{FD}}{1 + s^2 C_{ad} \overline{P}_{FD}}
$$

(30)

We have $2n_v + n_s$ equations composed of Equation (17), (21) and (28), and the number of unknowns is: $n_{Fv}$ — fracture node pressure $\overline{P}_{FD}$, $n_{Fv}$ — fracture node outflow rate $\overline{q}_{wD}$, $n_{Fs}$ — Flow rate per unit length of fracture unit $\overline{q}_{D}$.

The pressure before fracture closure is expressed in Equation (29). When the wellbore skin exists, Equation (29) can be rewritten as:

$$
\overline{P}_{ad}(S) = \frac{s \overline{P}_{FD} + S}{s \left[ 1 + s C_{ad} [s \overline{P}_{FD} + S] \right]}
$$

(31)
4 Model Results

The dimensionless pressure solution is obtained by solving the mathematical model, and the theoretical pressure and pressure derivative characteristic curves of the complex fracture network well test model for multi-stage fractured horizontal wells are obtained. The basic parameters of the theoretical curve are shown in Table 1.

**Table 1 Basic parameters of theoretical well test model**

| items                      | properties                       | l value | unit  |
|----------------------------|----------------------------------|---------|-------|
| **Reservoir**              |                                   |         |       |
| Thickness                  |                                  | 10      | m     |
| Porosity                   |                                  | 0.1     |       |
| Matrix compressibility     |                                  | $4.35 \times 10^{-4}$ | MPa$^{-1}$ |
| **Well**                   |                                   |         |       |
| Well length                |                                  | 500     | m     |
| Well-storage coefficient   |                                  | 0.1     | m$^3$/MPa |
| **Fracture network**       |                                   |         |       |
| Fracture permeability      |                                  | 0.5     | mD    |
| Matrix permeability        |                                  | 0.001   | mD    |
| Storage coefficient before fracture closure | | 0.01 | m$^3$/MPa |
| Storage coefficient after fracture closure | | 0.015 | m$^3$/MPa |
| Fracture closure time      |                                  | 0.1     | h     |
| Storage ratio              |                                  | 0.1     | /     |
| Interporosity flow coefficient |                                | $1 \times 10^{-6}$ | /     |
| HF stages                  |                                  | 5       | /     |
| HF half length             |                                  | 150     | m     |
| HF conductivity            |                                  | 100     | mDm   |
| NF stages                  |                                  | 100     | /     |
| NF half length             |                                  | 50      | m     |
| NF conductivity            |                                  | 50      | mDm   |
It can be seen from Figure 3 that the flow stages of the theoretical well test curve of the multi-stage fractured horizontal well in shale reservoirs can be divided into:

Stage 1: Stage before the fracture closes. The pressure and pressure conduction curves coincide with a straight line with a slope of 1, which is similar to the wellbore storage effect.

Stage 2: Stage after the fracture closes. At this stage, the shape of the derivative curve of dimensionless pressure and dimensionless pressure is similar to the behaviour of variable well-storage.

Stage 3: Supply flow stage between secondary fractures and hydraulic fractures. At this stage, there is a slight "dip", which means that the reservoir fluid is supplied from the secondary fractures activated by fracturing to the main hydraulic fractures, which is similar to the curve characteristics of dual-porosity media. The flow in secondary fractures is considered to be steady-state flow, regardless of the compressibility of secondary fractures. Therefore, it can be considered that secondary fractures are a stable supply source of hydraulic main fractures. In addition, before the supply flow stage between secondary fractures and hydraulic fractures, there is a fracture bilinear flow stage with a pressure derivative curve slope of 1/4, but this stage is covered by the dual effects of fracture closure and supply flow stages. When fracture propagation and fracture closure are not considered, a brief two-line flow stage should be observed in the pressure response curve, which represents the flow in the hydraulic fracture.

Stage 4: Linear flow stage. The pressure conduction curve is a straight line with a slope of 1/2.
Stage 5: Early dual radial flow stage. After the end of the linear flow, the early bi-radial flow appears, which was manifested as a 1/3 slope of the pressure derivative curve. This indicates that the reservoir fluid flows to a single main fracture in an elliptical shape.

Stage 6: Interporosity flow stage. The pressure conductivity curve is concave with minimal points, which indicates that fluid is channelling from the matrix of the reservoir volume stimulation zone into the fracture network during this stage. The depth and time of concave are controlled by $\omega$ and $\lambda$.

Stage 7: Late dual radial flow. At the end of the interflow stage, the dimensionless pressure derivative curve shows a straight line with a slope of 1/3, indicating that the reservoir fluid flows elliptically to the fracture network. This phenomenon has also been observed by other researchers.

Stage 8: Pseudo radial flow. Over time, the pressure derivative curve shows a horizontal line with a value of 0.5M, indicating a typical pseudo radial flow in the reservoir.

5 Summary and Conclusions

In this paper, the well test method is adopted to develop a semi-analytical model for fracture networks with considering fracture propagation and closure. Flow stages of the theoretical well test curve of the multi-stage fractured horizontal well in shale reservoirs can be divided into: stage before the fracture closes, stage after the fracture closes, supply flow stage between secondary fractures and hydraulic fractures, linear flow stage, early dual radial flow stage, interporosity flow stage, late dual radial flow, and pseudo radial flow. The pressure and pressure conduction curves coincide with a straight line with a slope of 1 at stage before the fracture closes, which is similar to the wellbore storage effect. The shape of the derivative curve of dimensionless pressure and dimensionless pressure is similar to the behaviour of variable well-storage, during stage after the fracture closes.

At the supply flow stage, there is a slight "dip", which means that the reservoir fluid is supplied from the secondary fractures activated by fracturing to the main hydraulic fractures, which is similar to the curve characteristics of dual-porosity media. The flow in secondary fractures is considered to be steady-state flow, regardless of the compressibility of secondary fractures. Therefore, it can be considered that secondary fractures are a stable supply source of hydraulic main fractures. In addition, before the supply flow stage between secondary fractures and hydraulic fractures, there is a fracture bilinear flow stage with a pressure derivative curve slope of 1/4, but this stage is covered by the dual effects of fracture closure and supply flow stages. When fracture propagation and fracture closure are not considered, a brief two-line flow stage should be observed in the pressure response curve, which represents the flow in the hydraulic fracture.

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Nomenclatures

B = volume factor, m³/m³
C = wellbore storage coefficient, m³/MPa
Z = Z factor, dimensionless
cₜ = gas compression coefficient, MPa⁻¹
h = formation thickness, m
K = reservoir permeability, darcy
p = pressure, MPa
ρ = fluid density, Kg/m³
Φ = reservoir porosity, decimal
μ₉ = gas viscosity, mPa.s

qₘₙ = surface flow flux, m³/d
T = temperature, K
qₜₚ = injection volume at the wellhead, m³/d
pₜₚ = bottom hole pressure, MPa
Cₜₚ, Cₜₚ = well-storage coefficient, dimensionless
qₜₚ = amount of injection on the ground, MPa
qₜₚ = injection volume at the wellhead, m³/d
pₜₚ = bottom hole pressure, MPa
Cₜₚ = well-storage coefficient during fracture propagation, dimensionless
Cₜₚ = well-storage coefficient before fracture propagation, dimensionless
Cₜₚ = well-storage coefficient after fracture closure, dimensionless
qₜₚ = flow rate of the m-th (hydraulic/secondary) fracture section, m³/d
pₜₚ = pressure at the k-th fracture vertex, MPa
pₜₚ = pressure of the k+1-th fracture vertex, MPa
qₜₚ = flow rate of the k-th fracture vertex, m³/d
qₜₚ = flow rate of the k-th fracture vertex, m³/d
pₜₚ(0) = wellbore pressure at the moment of shut-in, MPa
\( \bar{P}_{pFD} \) = pressure drop caused by uniform flow rate and variable fracture length, MPa

\( \bar{L}_{FD}(s) \) is the dimensionless variable

\( \alpha \) = fracture storage coefficient, dimensionless

\( \bar{P}_{FD} \) = pressure drop caused by uniform flow rate and variable fracture length, MPa

\( p_{SD}(t_D) \) = dimensionless pressure solution before fracture closure

\( \bar{q}_{wDk}^{in} \) = inflow rate of the k-th fracture node, m\(^3\)/d

\( \bar{q}_{wDk}^{out} \) = outflow rate of the k-th fracture node, m\(^3\)/d

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