Diffractive-Like (or Parametric-Resonance-Like?) Enhancement of the Earth (Day-Night) Effect for Solar Neutrinos Crossing the Earth Core

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Abstract

It is shown that the strong enhancement of the Earth (day-night) effect for solar neutrinos crossing the Earth core in the case of the small mixing angle MSW $\nu_e \to \nu_\mu (\tau)$ transition solution of the solar neutrino problem is due to a new resonance effect in the solar neutrino transitions in the Earth and not just to the MSW effect in the core. The effect is in many respects similar to the electron paramagnetic resonance. The conditions for existence of this new resonance effect are discussed. They include specific constraints on the neutrino oscillation lengths in the Earth mantle and in the Earth core, thus the resonance is a “neutrino oscillation length resonance”. The effect exhibits strong dependence on the neutrino energy. Analytic expression for the probability accounting for the solar neutrino transitions in the Earth, which provides a high precision description of the transitions, including the new resonance effect, is derived. The implications of our results for the searches of the day-night asymmetry in the solar neutrino experiments are also briefly discussed. The new resonance effect is operative also in the $\nu_\mu \to \nu_e (\nu_e \to \nu_\mu)$ transitions of atmospheric neutrinos crossing the Earth core.

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1. Introduction

In the present article we show that the strong enhancement of the Earth effect in the transitions of the solar neutrinos crossing the Earth core in comparison with the effect in the transitions of the only mantle crossing solar neutrinos, found [1–3] to occur in the case of the matter-enhanced small mixing angle $\nu_e \rightarrow \nu_{\mu(\tau)}$ solution of the solar neutrino problem, is due to a new resonance effect in the transitions and not just to the MSW enhancement of the neutrino mixing caused by the matter in the core. The effect is in many respects analogous to the electron paramagnetic resonance.

The existence of the core enhancement of the Earth matter effect was established numerically in the rather detailed and high precision studies of the day-night (D-N) effect for the Super-Kamiokande detector, performed in [1–3] for the $\nu_e \rightarrow \nu_{\mu(\tau)}$ and $\nu_e \rightarrow \nu_s$ transition solutions. We have found, in particular, that due to this enhancement the D-N asymmetry in the sample of events whose night fraction is caused by the solar neutrinos which cross the Earth core before reaching the detector, is in the case of the small mixing $\nu_e \rightarrow \nu_{\mu(\tau)}$ solution by a factor of up to six bigger than the D-N asymmetry determined using the whole night event sample produced by the solar neutrinos which cross only the Earth mantle or the mantle and the core. Such a strong enhancement was interpreted to be purely due to the resonance in the neutrino transitions, generated by the effect of the core matter on the neutrino mixing and taking place in the Earth core. We show in what follows that the origin of this enhancement is the presence of a new type of resonance in the transitions of the solar neutrinos crossing the Earth core. For reasons to become clear later we will use the term “neutrino oscillation length resonance” for it.

In the analyses which follow we use the Stacey model from 1977 [4] as a reference Earth model. The Earth radius in the Stacey model is $R_{\oplus} = 6371$ km. As in all Earth models known to us, the density distribution is spherically symmetric and there are two major density structures - the core and the mantle, and a large number of substructures (shells or layers). The core has a radius $R_c = 3485.7$ km, so the Earth mantle depth is approximately $R_{\text{man}} = 2885.3$ km. The mean matter densities in the core and in the mantle read, respectively: $\bar{\rho}_c \cong 11.5$ g/cm$^3$ and $\bar{\rho}_{\text{man}} \cong 4.5$ g/cm$^3$. Let us note that the density distribution in the 1977 Stacey model practically coincides with the density distribution in the more recent PREM model [5].
We will assume in the present study that the simplest two-neutrino $\nu_e - \nu_\mu(\tau)$ or $\nu_e - \nu_s$ mixing (with nonzero mass neutrinos), $\nu_s$ being a sterile neutrino, takes place in vacuum and that it is at the origin of the solar $\nu_e$ matter-enhanced transitions into $\nu_\mu(\tau)$ or $\nu_s$ in the Sun, producing the observed solar neutrino deficit.

2. Enhancement of the Transitions of Solar Neutrinos Crossing the Earth Core

Because of the spherical symmetry of the Earth, the path of a neutrino in the Earth mantle before the neutrino reaches the Earth core is identical to the path in the mantle after the neutrino exits the core; in particular, the lengths of the two paths in the mantle are equal. For the same reason a given neutrino trajectory is completely specified by its Nadir angle [2].

It proves convenient to analyse the Earth effect in the transitions of solar neutrinos which cross the Earth core by using the two-layer model of the Earth: all the interesting features of the transitions can be understood quantitatively in the framework of this rather simple model. The density profile of the Earth in the two-layer model is assumed to consist of two structures - the mantle and the core, having different densities, $\rho_{\text{man}}$ and $\rho_c$, and different electron fraction numbers, $Y_{e\text{man}}$ and $Y_e^c$, none of which however vary within a given structure. The core radius and the depth of the mantle are known with a rather good precision and these data are incorporated in the Earth models [4,5]. The densities $\rho_{\text{man}}$ and $\rho_c$ in the case of interest should be considered as mean effective densities along the neutrino trajectories, which can vary somewhat with the change of the trajectory: $\rho_{\text{man}} = \bar{\rho}_{\text{man}}$ and $\rho_c = \bar{\rho}_c$. In the Stacey model one has: $\bar{\rho}_{\text{man}} \cong (4 - 5) \text{ g/cm}^3$ and $\bar{\rho}_c \cong (11 - 12) \text{ g/cm}^3$. For the electron fraction numbers in the mantle and in the core one can use the standard values [4–6] (see also [1]) $Y_{e\text{man}} = 0.49$ and $Y_e^c = 0.467$. Numerical calculations show [7] that, e.g., the time-averaged probability $P_{e2}$ calculated within the two-layer model of the Earth with $\bar{\rho}_{\text{man}}$ and $\bar{\rho}_c$ taken from the Stacey 1977 model [4] reproduces with a remarkably high precision the probability calculated by solving numerically the relevant system of evolution equations with the much more sophisticated Earth density profile of the Stacey model [4].

The term which accounts for the Earth matter effect in the solar $\nu_e$ survival probability
in the case of interest is the probability of the $\nu_2 \rightarrow \nu_e$ transition in the Earth, $P_{e2}$, where $\nu_2$ is the heavier of the two mass eigenstate neutrinos in vacuum. The probability amplitude of interest $A(\nu_2 \rightarrow \nu_e)$, $P_{e2} = |A(\nu_2 \rightarrow \nu_e)|^2$, is given by the following simple expression in the two-layer model:

$$A(\nu_2 \rightarrow \nu_e) = \sin \theta + \left( e^{-i \Delta E' X'} - 1 \right) \left[ 1 + \left( e^{-i \Delta E'' X''} - 1 \right) \cos^2 (\theta'_{\text{m}} - \theta''_{\text{m}}) \right] \cos (\theta - \theta'_{\text{m}}) \sin \theta'_{\text{m}}$$

$$+ \left( e^{-i \Delta E'' X''} - 1 \right) \cos (\theta - \theta''_{\text{m}}) \sin \theta''_{\text{m}}$$

$$+ \frac{1}{2} \left( e^{-i \Delta E' X'} - 1 \right) \left( e^{-i \Delta E' X'} - 1 \right) \sin (2 \theta'_{\text{m}} - 2 \theta'_{\text{m}}) \cos (\theta - 2 \theta'_{\text{m}}).$$  \(1\)

Here

$$\Delta E' (\Delta E'') = \frac{\Delta m^2}{2E} \sqrt{\left( 1 - \frac{\tilde{\rho}_{\text{man}} (c)}{\rho_{\text{res}} (c)} \right)^2 \cos^2 2\theta + \sin^2 2\theta},$$  \(2\)

$\theta'_{\text{m}}$ and $\theta''_{\text{m}}$ are the mixing angles in matter in the mantle and in the core, respectively.

$$\sin^2 2\theta'_{\text{m}} \left( \sin^2 2\theta''_{\text{m}} \right) = \frac{\sin^2 2\theta}{(1 - \frac{\tilde{\rho}_{\text{man}} (c)}{\rho_{\text{res}} (c)})^2 \cos^2 2\theta + \sin^2 2\theta},$$  \(3\)

$X'$ is half of the distance the neutrino travels in the mantle and $X''$ is the length of the path of the neutrino in the core, and $\rho_{\text{res}}^{\text{man}}$ and $\rho_{\text{res}}^{\text{core}}$ are the resonance densities in the mantle and in the core. The latter can be obtained from the expressions

$$\rho_{\text{res}}^{\text{man}} = \frac{\Delta m^2 \cos 2\theta}{2E \sqrt{2G_F Y_e}} m_N, \quad \nu_e \rightarrow \nu_{\mu(\tau)} \text{ transitions},$$  \(4\)

$$\rho_{\text{res}}^{\text{core}} = \frac{\Delta m^2 \cos 2\theta}{2E \sqrt{2G_F \frac{1}{2} (3Y_e - 1)}} m_N, \quad \nu_e \rightarrow \nu_s \text{ transitions},$$  \(5\)

$m_N$ being the nucleon mass, by using the specific values of $Y_e$ in the mantle and in the core. We have $\rho_{\text{res}}^{\text{man}} \neq \rho_{\text{res}}^{\text{core}}$ (but $\rho_{\text{res}}^{\text{man}} \sim \rho_{\text{res}}^{\text{core}}$) because $Y_e = 0.467$ and $Y_e^{\text{man}} = 0.49$. Obviously,

\[1\]This is the probability that a solar $\nu_e$ will not be converted into $\nu_{\mu(\tau)}$ (or $\nu_s$) when it travels from the central part of the Sun to the surface of the Earth and further crosses the Earth to reach the detector.
\[ Y_{e}^{\text{man}} \rho_{c}^{\text{res}} = Y_{e}^{\text{man}} \rho_{c}^{\text{res}}. \] For a neutrino trajectory which is specified by a given Nadir angle \( h \), the following relations hold true:

\[ X' = R_{e} \cosh - \sqrt{R_{e}^{2} - R_{e}^{2} \sin^{2} h}, \quad X'' = 2 \sqrt{R_{e}^{2} - R_{e}^{2} \sin^{2} h}. \] (6)

It is not difficult to find from eq. (1) that for any \( \theta \) the probability of interest \( P_{e2} \) has the form:

\[
P_{e2} = \sin^{2} \theta + \frac{1}{2} \left[ \sin^{2}(2\theta_{m}' - \theta) - \sin^{2}\theta \right]
+ \frac{1}{4} \left[ 1 - \cos \Delta E'X' \right] \sin^{2}(2\theta_{m}' - \theta) \cos(2\theta_{m}' - \theta)
+ \frac{1}{2} \left[ \sin \Delta E''X'' \sin 2\Delta E'X' \right] \sin^{2}(2\theta_{m}' - \theta) \cos(2\theta_{m}' - \theta)
+ \frac{1}{4} \left[ \cos(\Delta E'X' + \Delta E''X'') \sin(2\theta_{m}' - 2\theta) \sin(2\theta_{m}' - 2\theta) \right]. \] (7)

The effect of the core enhancement of the probability \( P_{e2} \) was established to be dramatic [1–3] at small mixing angles, \( \sin^{2} \theta \lesssim 0.10 \). For \( \sin^{2} \theta \lesssim 0.10 \), the probability \( P_{e2} \), considered as a function of \( E/\Delta m^{2} \), or equivalently of the density parameter [1–3]

\[ \rho_{r} = \frac{\Delta m^{2} \cos 2\theta}{2E\sqrt{2}G_{F}C_{a(s)}m_{N}}, \] (8)

where the constant \( C_{a(s)} = 0.50 \) (0.25) for the \( \nu_{e} \rightarrow \nu_{\mu(r)} \) (\( \nu_{e} \rightarrow \nu_{s} \)) transitions \(^2\), can have several prominent local maxima in its bulk region. When the MSW resonance (i.e., the resonance in \( \sin^{2} 2\theta_{m}' \) or \( \sin^{2} 2\theta_{m}' \)) takes place in the Earth core, we have \( \rho_{c}^{\text{res}} \approx \bar{\rho}_{c} \), and

\(^{2}\)The parameter \( \rho_{c} \) would coincide with the resonance density if \( Y_{e} \) were equal to \( 1/2 \) both in the mantle and in the core; it is equivalent to \( E/\Delta m^{2} \), but gives an idea about the densities at which one has an enhancement of the Earth matter effect and we have used it for the latter purpose in [1–3].
correspondingly $2\theta'' \cong \pi/2$, while $\theta' \cong 1.7\theta \ll \theta''$. In the resonance region the probability $P_{e2}$, as it follows from eq. (7), is given by the expression:

$$P_{e2}^{\text{res}} \cong \frac{1}{2} \left[ 1 - \cos \Delta E'' X'' \right] \sin^2 (2\theta'' - \theta),$$

as long as the oscillating factor $0.5(1 - \cos \Delta E'' X'')$ is not small. Obviously, $P_{e2}^{\text{res}}$ can be suppressed even when $\sin^2 2\theta' \cong 1$ if in the resonance region of $\sin^2 2\theta''$ one has $\cos \Delta E'' X'' \cong 1$. In the latter case the other terms in eq. (7), notably the last one with the factor $\sin(4\theta' - 2\theta) \sin(2\theta' - 2\theta')$, can give substantial contribution in $P_{e2}$ in the region where $\sin^2 2\theta''$ is strongly enhanced by the matter effect.

If the MSW resonance occurs in the mantle, i.e., if $\rho_{\text{man}}^{\text{res}} \cong \bar{\rho}_{\text{man}}$, then $2\theta' \cong \pi/2$, $\pi/2 - \theta'' \ll \theta'$ and at small mixing angles we expect $P_{e2}$ to be given in the resonance region of $\sin^2 2\theta'$ approximately by the expression

$$P_{e2}^{\text{res}} \cong \frac{1}{4} \left[ 1 + \cos \Delta E'' X'' \right] \left[ 1 - \cos 2\Delta E' X' \right] \sin^2 2\theta',$$

The effect of the core will not suppress the probability $P_{e2}^{\text{res}}$ in this case only if $\cos \Delta E'' X'' \cong 1$. However, if $\cos \Delta E'' X'' \cong -1$ we get $P_{e2}^{\text{res}} \cong 0$ in spite of the resonance in $\sin^2 2\theta'$ in the mantle. The probability $P_{e2}^{\text{res}}$ can also be suppressed if in the resonance region of $\sin^2 2\theta'$ one has, e.g., $(2\Delta E' X')^2 \ll 1$. In these cases $P_{e2}$ may turn out to be determined in the region of interest not by $P_{e2}^{\text{res}}$, but rather by other terms in eq. (7) (see further).

It is not difficult to convince oneself treating the phases $\Delta E' X'$ and $\Delta E'' X''$ as independent parameters and $\theta, \theta'$, and $\theta''$ as having fixed values that for any $\theta$ the probability $P_{e2}$ has a local maximum if the following conditions are fulfilled:

$$\Delta E' X' = \pi(2k + 1), \quad \Delta E'' X'' = \pi(2k' + 1), \quad k, k' = 0, 1, 2, \ldots,$$

and

$$\sin^2 (2\theta'' - 4\theta' + \theta) - \sin^2 \theta > 0,$$

$$\sin^2 (2\theta'' - 4\theta' + \theta) \sin (2\theta'' - 2\theta') \sin (2\theta' - \theta) \cos (2\theta'' - 4\theta' + \theta)$$

$$+ \frac{1}{4} \sin^2 \theta \sin (4\theta'' - 8\theta' + 2\theta) \sin (4\theta'' - 4\theta') < 0.$$

Conditions (11) are written in the most general form. As we shall see, the relevant conditions for the problem of interest - transitions of solar neutrinos crossing the Earth core, correspond
to \( k = k' = 0 \). At the maximum one has:

\[
P_{e2}^{\text{max}} = \sin^2(2\theta''_m - 4\theta'_m + \theta).
\]  

(13)

This local maximum will dominate in \( P_{e2} \) at small mixing angles provided

\[
P_{e2}^{\text{max}} > (\gg) \max P_{e2}^{\text{res}}, \max P_{e2}^{\text{res}},
\]  

(14)

and if it is sufficiently wide. It is not clear a priori whether inequalities (14) will hold in the case of interest even if all the other conditions for the presence of the maximum in \( P_{e2} \) are fulfilled.

The requirements (12a) and (12b) are the two supplementary conditions which ensure that \( P_{e2} \) has a maximum when the equalities (11) hold. At small mixing angles condition (12a) is fulfilled for \( \rho_{\text{res}}^{\text{man}} < 17.4 \text{ g/cm}^3, \rho_{\text{res}}^{\text{c}} \neq \rho_{\text{man}}, \) and in this region condition (12b) reduces to the following simple constraint:

\[
\cos(2\theta''_m - 4\theta'_m + \theta) < 0
\]  

(15)

It can be shown that conditions (12a) and (12b) (or condition (15)) can be satisfied if

\[
\bar{\rho}_{\text{man}} < \rho_{\text{res}}^{\text{man},c} < \rho_c,
\]  

(16)

i.e., when \( 0 < 2\theta'_m < \pi/2 \) and \( \pi/2 < 2\theta''_m \leq \pi \). They are always fulfilled, in particular, when \( \bar{\rho}_{\text{man}} \ll \rho_{\text{res}}^{\text{man}} (2\theta''_m \approx \pi) \) and \( \rho_c/\rho_{\text{res}}^{\text{c}} \gg 1, \tan^2 2\theta (2\theta''_m \approx \pi) \), but these inequalities are not realized for the Earth as \( \bar{\rho}_c \approx 2.6 \bar{\rho}_{\text{man}} \); moreover in this case \( \sin^2(2\theta''_m - 4\theta'_m + \theta) \approx \sin^2(3\theta) \) and at small mixing angles \( P_{e2}^{\text{max}} \) is suppressed. Actually, this observation indicates that the ratio between \( \bar{\rho}_c \) and \( \bar{\rho}_{\text{man}} \) for the Earth is favorable for the effect under discussion. If, however, \( \bar{\rho}_{\text{man}}, \bar{\rho}_c < \rho_{\text{res}}^{\text{man},c} (2\theta'_m, 2\theta''_m < \pi/2, 2\theta'_m < 2\theta''_m) \), the inequalities (12a) and (12b) will not hold. They do not hold also, e.g., for \( \bar{\rho}_{\text{man}}, \bar{\rho}_c \ll \rho_{\text{res}}^{\text{man},c} \), as well as when \( \rho_{\text{res}}^{\text{man}} = \bar{\rho}_{\text{man}}, \rho_{\text{res}}^{\text{c}} = \bar{\rho}_c \).

\footnote{If the sign of the inequality in eq. (12a) is opposite, \( P_{e2} \) would have a minimum.}

\footnote{It is not difficult to convince oneself that in the problem of interest one always has: \( \sin(2\theta''_m - 2\theta'_m) > 0 \) and \( \sin(2\theta'_m - \theta) > 0 \).}
If only $\Delta E'X' = \pi(2k + 1)$, one finds from eq. (7) that for any $\theta$

$$P_{e2} = \sin^2 \theta + \frac{1}{2} \left[ 1 - \cos \Delta E''X'' \right] \left[ \sin^2 (2\theta''_m - 2\theta' + \theta) - \sin^2 \theta \right], \quad \Delta E'X' = \pi(2k + 1). \quad (17)$$

Note that at small mixing angles this expression differs from the one in eq. (9). Similarly, when $\Delta E''X'' = \pi(2k' + 1)$, for any $\theta$ we obtain:

$$P_{e2} = \sin^2 (2\theta''_m - \theta) + \frac{1}{2} \left[ 1 - \cos \Delta E'X' \right] \left[ \sin^2 (2\theta''_m - 4\theta' + \theta) - \sin^2 (2\theta''_m - \theta) \right]$$

$$- \frac{1}{2} [1 - \cos 2\Delta E'X' \left[ \sin^2 (2\theta''_m - \theta) - \sin^2 \theta \right] \cos^2 (2\theta''_m - 2\theta'_m), \quad \Delta E''X'' = \pi(2k' + 1). \quad (18)$$

Let us discuss next the relevance and the implications of the above results for the transitions of the solar neutrinos in the Earth.

\[ \text{2.1 MSW Resonance in the Earth Core} \]

Consider first the case of $\nu_e \rightarrow \nu_{\mu(r)}$ transitions. If the MSW resonance takes place in the core we have $\sin^2 2\theta''_m \simeq 1$ for $\rho_{e}^{\text{res}} \simeq \bar{\rho}_e \simeq (11.0 - 11.5) \text{ g/cm}^3$. However, the position of the maximum of $P_{e2} \simeq P_{e2}^{\text{res}}$ (eq. (9)) in the resonance region of $\sin^2 2\theta''_m$ is determined by the condition

$$\Delta E''X'' = \pi(2k' + 1), \quad k' = 0, 1, \ldots, \quad (19)$$

for which the oscillating term in eq. (9), $0.5(1 - \cos \Delta E''X'')$, has a maximal value. For $k = 0$ this condition is fulfilled for the core-crossing neutrinos for $\rho_{e}^{\text{res}} \simeq (13.3 - 14.5) \text{ g/cm}^3$. Thus, the influence of the oscillating term in eq. (9) shifts the position of the maximum of $P_{e2}^{\text{res}}$ to values of $\rho_{e}^{\text{res}}$ which differ from those at which the maximum of $\sin^2 2\theta''_m$ is located.

For the trajectory with $h = 0^0$, for instance, the maximum of $P_{e2}^{\text{res}}$ occurs at $\rho_{e}^{\text{res}} \simeq 13.4 \text{ g/cm}^3 \left( (E/\Delta m^2)^{\text{res}} \simeq 1.053 \times 10^6 \text{ MeV/eV}^2 \right)$, while for those with $h = 13^0$ and $h = 23^0$ it takes place respectively at $\rho_{e}^{\text{res}} \simeq 14.2 \text{ g/cm}^3 \left( (E/\Delta m^2)^{\text{res}} \simeq 0.993 \times 10^6 \text{ MeV/eV}^2 \right)$ and $\rho_{e}^{\text{res}} \simeq 14.5 \text{ g/cm}^3 \left( (E/\Delta m^2)^{\text{res}} \simeq 0.975 \times 10^6 \text{ MeV/eV}^2 \right)$. At small mixing angles the values of $\rho_{e}^{\text{res}}$ at which $\Delta E''X'' \simeq \pi$ exhibit weak dependence on $\sin^2 2\theta$. At the position of the maximum of $P_{e2}^{\text{res}}$, $\sin^2 2\theta''_m$ is still considerably enhanced by the matter effect: for $\sin^2 2\theta = 0.01$, for instance, on finds for $h = 13^0; 23^0$ that at the maximum $\sin^2 (2\theta''_m - \theta) \simeq 0.15; 0.11$. For the trajectories with $h = 0^0; 13^0; 23^0$ we have $\max P_{e2}^{\text{res}} \simeq 0.20; 0.15; 0.11$, in very good
agreement with the results of our numerical calculations \(^5\) [1,2] (see Figs. 1a - 1c).

If solar neutrinos undergo \(\nu_e \rightarrow \nu_s\) transitions, the value of \(E/\Delta m^2\) corresponding to a given \(\rho^\text{res}_c\) is for \(Y^c_e = 0.467\), as it follows from eqs. (2) and (3), by a factor of 2.33 larger than in the case of \(\nu_e \rightarrow \nu_\mu(\tau)\) transitions: \((E/\Delta m^2)^\text{res}_c \approx 2.33(E/\Delta m^2)^a\). Since \(\Delta E'', \Delta E' \sim (E/\Delta m^2)^{-1}\), condition (20), \(\Delta E''X'' \approx \pi\), is never satisfied for the core-crossing neutrinos in, or in the vicinity of, the resonance region of \(\sin^2 2\theta''_m\). Moreover, in the resonance region of interest \(\Delta E''X''\) is relatively small \(^6\) and the oscillating term in eq. (9) suppresses the probability \(P^\text{res}_{e2}\). As a consequence, \(P_{e2}\) is given in the region of resonance enhancement of \(\sin^2 2\theta''_m\) by the expression:

\[
P^\text{res}_{e2,s} \approx \frac{1}{2} \left[1 - \cos \Delta E''X''\right] \sin^2(2\theta''_m - \theta) + \frac{1}{4} \left[1 + \cos \Delta E''X''\right] \left[1 - \cos 2\Delta E'X'\right] \sin^2 2\theta'_m
+ \frac{1}{4} \cos(\Delta E'X' - \Delta E''X'') \cos(\Delta E'X' + \Delta E''X'') \sin(4\theta'_m - 2\theta) \sin(2\theta''_m - 2\theta'_m). \tag{20}
\]

The maximal value of \(P^\text{res}_{e2,s}\) at small mixing angles occurs for the trajectory with, e.g., \(h = 0^\circ (23^\circ)\) at \(\rho^\text{res}_c \approx 10.9 \text{ g/cm}^3\) \((\rho^\text{res}_c \approx 9.8 \text{ g/cm}^3)\) and its position essentially does not change when \(\sin^2 2\theta\) varies from \(\sim 10^{-3}\) to \(\sim 0.02\). In the region of the maximum of \(P^\text{res}_{e2,s}\), \(\Delta E'X'\) practically does not depend on \(\sin^2 2\theta \approx 0.02\) and for \(h = 0^\circ (23^\circ)\) we have at the maximum: \(\Delta E'X' \approx 0.40\pi\) \((0.38\pi)\). The value of the phase \(\Delta E''X''\) in the region of the maximum of \(P^\text{res}_{e2,s}\) depends on \(\sin^2 2\theta\): if, e.g., \(\sin^2 2\theta = 0.001\); 0.01 we get for \(h = 0^\circ (23^\circ)\) that at the maximum \(\Delta E''X'' \approx 0.12\pi; 0.21\pi\) \((0.15\pi; 0.19\pi)\). The mixing angle factors appearing in eq. (20) have the following values at the maximum for, e.g., \(\sin^2 2\theta = 0.01\) and \(h = 23^\circ\): \(\sin^2(2\theta''_m - \theta) \approx 0.43\), \(\sin 4\theta'_m \approx 0.43\), and \(\sin(2\theta''_m - 2\theta'_m) \approx 0.77\). The last term in eq. (20) is either the dominant one \((h \approx 23^\circ)\) or is comparable in magnitude to the first term and their sum gives the dominant contribution in \(P^\text{res}_{e2,s}\) \((h \approx 0^\circ)\). At the maximum the probability \(P^\text{res}_{e2,s}\) takes the following values for, e.g., \(\sin^2 2\theta = 0.01\) and \(h = 0^\circ; 23^\circ\): max \(P^\text{res}_{e2,s} \approx 0.21; 0.15\), which is in good agreement with the numerical results

\(^5\)Let us note that for the indicated trajectories, at the positions of the corresponding maxima of \(P^\text{res}_{e2,s}\) one has \(\Delta E'X' \approx 3\pi/2\) \((h = 0^\circ)\), \(\Delta E'X' \approx 1.6\pi\) \((h = 13^\circ)\) and \(\Delta E'X' \approx 2\pi\) \((h = 23^\circ)\).

\(^6\)At the point where \(\sin^2(2\theta''_m - \theta) \approx 0.8\) \((\rho^\text{res}_c \approx 10.9 \text{ g/cm}^3)\) for \(\sin^2 2\theta = 0.01\), for instance, we have \(\Delta E''X'' \approx 0.2\pi\).
in ref. [3] (see Figs. 2a - 2c). It is interesting to note that for given $h$ and $\sin^2 2\theta$ one has:

$$\max P_{e2,\text{res}} \sim \max P_{e2}.$$ 

### 2.2 MSW Resonance in the Mantle

Suppose that for the core-crossing solar neutrinos the Earth matter effect leads to enhancement of $\sin^2 2\theta_m'$ due to the MSW resonance in the mantle. We shall assume first that the (solar) $\nu_e$ mix with $\nu_{\mu(\tau)}$ in vacuum. In the region of the enhancement of $\sin^2 2\theta_m'$ the probability of interest $P_{e2} \approx P_{e2}^{\text{mres}}$ and is given approximately by eq. (10) as long as $P_{e2}^{\text{mres}}$ is not rather strongly suppressed. Depending on the neutrino trajectory through the Earth core, we will have $\sin^2 2\theta_m' \approx 1$ if $\rho_m^{\text{res}} \approx (4.0 - 5.0) \, \text{g/cm}^3$. However, the position of the maximum of $P_{e2}^{\text{mres}}$ is determined by the position of the maximum of the oscillating term $0.5(1 + \cos \Delta E''X'')$ in eq. (10), i.e., by the condition

$$\Delta E''X'' = 2\pi(2k' + 1), \quad k' = 0, 1, \ldots , \quad (21)$$

and for most of the neutrino trajectories of interest differs from the position of the maximum of $\sin^2 2\theta_m'$. For $h = 0^0 (13^0)$ and $h = 23^0$, for example, the condition $\Delta E''X'' \approx 2\pi$ is realized at $\rho_m^{\text{res}} \approx 5.9 \, (5.1) \, \text{g/cm}^3 ((E/\Delta m^2)^{\text{mres}} \approx 2.270 \, (2.632) \times 10^6 \, \text{MeV/eV}^2)$, and at $\rho_m^{\text{res}} \approx 4.0 \, \text{g/cm}^3 ((E/\Delta m^2)^{\text{mres}} \approx 3.374 \times 10^6 \, \text{MeV/eV}^2)$.

Further, depending on the value of $\sin^2 2\theta$ and on the trajectory, the maximum of $P_{e2}^{\text{mres}}$ can lie outside or inside the resonance region of $\sin^2 2\theta_m'$. For the trajectories for which for given $\sin^2 2\theta \approx 0.02$ the position of $\max P_{e2}^{\text{mres}}$ is within the resonance region of $\sin^2 2\theta_m'$ ($h \gtrsim 15^0$ if $\sin^2 2\theta \approx 0.01$, for instance), we have $\max P_{e2}^{\text{mres}}$ even when both $\Delta E''X'' \approx 2\pi$ and $\sin^2 2\theta_m' \approx 1$. For the same reason $0.5(1 - \cos 2\Delta E'X')$ in the expression for $P_{e2}^{\text{mres}}$ practically does not depend on $\rho_m$. When the maximum of $P_{e2}^{\text{mres}}$ lies outside the resonance region of $\sin^2 2\theta_m'$, the term containing the factor $\sin \Delta E''X'' \sin 2\Delta E'X'$ in eq. (7) typically gives a non-negligible contribution $\approx (15 - 30)\%$ to the value of $P_{e2}^{\text{mres}}$ at the maximum. Even in the latter case $\sin^2 2\theta_m'$ is rather strongly enhanced at the maximum of $P_{e2}^{\text{mres}}$. For the trajectories corresponding to $h = 0^0; 13^0; 23^0$ we get for $\sin^2 2\theta = 0.01$: $\max P_{e2}^{\text{mres}} \approx 0.14; 0.10; 0.07$, in very good agreement with our numerical results [1,2] (Figs. 1a - 1c). Note that at small mixing angles $\max P_{e2}^{\text{mres}}$ is smaller than $\max P_{e2}^{\text{res}}$ typically by a factor of $\sim 1.5$. 

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If $\nu_e - \nu_s$ mixing takes place in vacuum, a given value of $\rho_m^{\text{res}}$ corresponds now to a 2.09 larger value of $E/\Delta m^2$ than in the case of $\nu_e - \nu_{\mu(\tau)}$ mixing: $(E/\Delta m^2)^{\text{mres}} \simeq 2.09(E/\Delta m^2)^{\text{res}}$. For this reason the condition $\Delta E'X' \simeq 2\pi$ is never satisfied for the core-crossing neutrinos for values of $\rho_m^{\text{res}}$ lying in, or relatively close, to the resonance region of $\sin^2 2\theta$. For $h = 0^0 (13^0)$, $\sin^2 2\theta = 0.01$ and $\rho_m^{\text{res}} \simeq 5.9 (5.1) \text{g/cm}^3$, for instance, $\Delta E'X' \simeq \pi$, and $P^{\text{mres}}$ is suppressed. For this value of $\rho_m^{\text{res}}$ we have $(\Delta E'X')^2 \ll 1$ and $\Delta E'X'$ decreases with the decreasing of $\rho_m^{\text{res}}$. As a consequence of these circumstances, in the case of solar $\nu_e \rightarrow \nu_s$ transitions, the probability $P_{e2}$ at small mixing angles does not have a local maximum in the region of the resonance enhancement of $\sin^2 2\theta$: as $\rho_m^{\text{res}}$ increases from $\sim 3.0 \text{g/cm}^3$ to $\sim 6.0 \text{g/cm}^3$, $P_{e2}$ increases monotonically. This is well illustrated in Figs. 2a - 2c.

**2.3 Diffractive-Like Peak (or Parametric Resonance?) in $P_{e2}$: the Maximal Core Enhancement of the Earth Effect**

It is quite remarkable that for the $\nu_e \rightarrow \nu_{\mu(\tau)}$ solution of the solar neutrino problem conditions (11) are approximately fulfilled and the maximal enhancement of the Earth effect in the solar neutrino transitions at small mixing angles takes place for $\rho_m^{\text{res}}$ satisfying inequalities (16) and not in the regions of the two possible MSW resonances (Figs. 1a - 1c). Indeed, for, e.g., $\sin^2 2\theta = 0.01$ and the trajectories corresponding to $h = 0^0; 13^0; 23^0$ we get $\Delta E'X' = \pi$ for $\rho_m^{\text{res}} \simeq 10.0; 9.6; 8.0 \text{g/cm}^3 ((E/\Delta m^2)^{\text{mres}} \simeq 1.646; 1.401; 1.317 \times 10^6 \text{MeV/eV}^2)$. At the indicated values of $\rho_m^{\text{res}}$ the phase $\Delta E''X''$ is smaller than, but for most of the trajectories - rather close to, $\pi$. For the three trajectories indicated above we have: $\Delta E''X'' \simeq 0.55\pi; 0.75\pi; 0.88\pi$. When $\Delta E''X'' = \pi$, which takes place at smaller values of $\rho_m^{\text{res}}$, e.g., in the example considered above at $\rho_m^{\text{res}} \simeq 8.7; 8.4; 7.6 \text{g/cm}^3$, the phase $\Delta E'X'$ is somewhat smaller than $\pi$ (in our example $\Delta E'X' \simeq (0.75 - 0.90)\pi$). The same conclusions are valid for the other trajectories of interest and values of $\sin^2 2\theta \simeq 0.02$. Actually, the requirement that $\Delta E'X' = \Delta E''X'' = \pi$ is equivalent at small mixing angles

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[7] Let us note that the Earth effect in the transitions of the neutrinos crossing only the mantle can be understood very well qualitatively and quantitatively using the one-layer approximation for the density distribution in the Earth mantle (see, e.g., ref. [8]).
in the case of interest to the condition

\[ \pi \left[ \frac{1}{X'} + \frac{1}{X''} \right] \cong \sqrt{2} \frac{G_F}{m_N} (Y_{e}^{c} \bar{\rho}_c - Y_{e}^{\text{man}} \bar{\rho}_{\text{man}}). \]  

(22)

It is not difficult to convince oneself using eq. (6), the values of the core and Earth radii \( R_c \) and \( R_{\oplus} \) and the values of \( Y_{e}^{c}, \bar{\rho}_c, Y_{e}^{\text{man}} \) and \( \bar{\rho}_{\text{man}} \) that the above equality cannot be exactly satisfied for the trajectories of neutrinos crossing the Earth core.

The position of the absolute maximum of \( P_{e2} \), which could be located, e.g., close to one of the two values of \( \rho_{\text{man}}^{\text{res}} \) at which \( \Delta E' X' = \pi \) or \( \Delta E'' X'' = \pi \), is determined primarily by the properties of the function \( \sin^2(2\theta_{m} + 4\theta_{m}' + \theta) \). The latter has a minimum (although a relatively shallow one) at \( \rho_{\text{man}}^{\text{res}} \sim 7.5 \text{ g/cm}^3 \) at small mixing angles, and increases rather steeply monotonically when \( \rho_{\text{man}}^{\text{res}} \) increases from \( \sim 7.5 \text{ g/cm}^3 \) to \( \sim 11.5 \text{ g/cm}^3 \). Since \( \Delta E' X' = \pi \) occurs at larger values of \( \rho_{\text{man}}^{\text{res}} \) than the equality \( \Delta E'' X'' = \pi \), the position of the maximum of \( P_{e2} \) of interest on the \( \rho_{\text{man}}^{\text{res}} \) axis practically coincides with the position of the point where \( \Delta E' X' = \pi \). Correspondingly, the value of \( P_{e2} \) at the maximum is determined by the expression (17). For \( \sin^2 2\theta = 0.01 \) and \( h = 0^0; 13^0; 23^0 \) we get \( \max P_{e2} \cong 0.51; 0.46; 0.40 \), which is in beautiful agreement with the results of our numerical calculations [1] (see Figs. 1a - 1c). Note that, indeed, for a given trajectory through the core and given \( \sin^2 2\theta \lesssim 0.10 \), \( \max P_{e2} \) exceeds \( \max P_{e2}^{\text{res}} \) and \( \max P_{e2}^{\text{man}} \) respectively by the factors of \( \sim 2.5 \) to \( \sim 4.0 \) and of \( \sim 3.0 \) to \( \sim 7.0 \). Moreover, because the MSW resonance in the core (and the corresponding maximum) is located relatively close in \( \rho_{\text{man}}^{\text{res}} \) (or \( E/\Delta m^2 \)), the peak in \( P_{e2} \) under discussion is rather wide \(^8\). In any case it is wider than any of the local maxima corresponding to the MSW enhancement of the mixing at small mixing angles.

It should be noted also that it is the specific combination of mixing angles, \( (2\theta_{m} + 4\theta_{m}' + \theta) \), and the fact that \( \bar{\rho}_c \) and \( \bar{\rho}_{\text{man}} \) do not differ by a large factor for the Earth, which makes \( \sin^2(2\theta_{m} + 4\theta_{m}' + \theta) \) relatively large in the region of interest. The quantities \( \sin^2 2\theta_{m} \) and \( \sin^2 2\theta_{m}' \) associated with the MSW effect can be actually rather small: if, for instance, \( \sin^2 2\theta = 0.01 \), we find that at the position of the maximum of \( P_{e2} \) under discussion, for the

\(^8\)Actually, as Fig. 1 indicates, for \( \sin^2 2\theta \lesssim 0.01 \) and \( 0^0 \leq h \lesssim 15^0 \) the MSW maximum appears just as a “shoulder” on the slope of the maximum related to the conditions (11).
trajectory with $h = 23^0$ one has $\sin^2 2\theta^m_m \simeq 0.08$ and $\sin^2 2\theta'_m \simeq 0.05$, while $\sin^2 (2\theta^m_m - 4\theta'_m + \theta) \simeq 0.40$.

It is quite interesting that, as our analysis shows, for values of $\rho^\text{res}_{\text{man}}$ from the interval $(4.5 - 11.5) \text{g/cm}^3$, conditions (11) are not even approximately fulfilled if solar neutrinos take part in $\nu_e \rightarrow \nu_s$ transitions, and hence the effect of strong enhancement of the probability $P_{e2}$ considered above does not take place in this case. However, as discussed in Section 2.1, when the MSW resonance occurs in the core, the purely MSW-like term (9) in $P_{e2}^\text{res}$, is “assisted” by some of the additional interference terms in $P_{e2}$ (see eq. (20)) and at the corresponding maximum, $P_{e2}$ is by a factor of $\sim (2 - 4)$ bigger than just $P_{e2}^\text{res}$. Still, at small mixing angles, $\sin^2 2\theta \approx 0.02$, the value of $P_{e2}$ at its absolute maximum in the $\nu_e \rightarrow \nu_s$ case is smaller than the corresponding value when $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions take place by a factor of $\sim (2.5 - 4.0)$ (see also [3]).

As to the physical nature of this new type of enhancement of the probability $P_{e2}$ which accounts for the Earth effect in the solar neutrino transitions, it resembles the constructive interference of waves after diffraction, and the corresponding peak in $P_{e2}$ - a diffractive peak. This analogy (although not direct) is related to the fact that the amplitude of interest, $A(\nu_2 \rightarrow \nu_e)$, has the following form in the two-layer model:

$$A(\nu_2 \rightarrow \nu_e) = \sum_{l,l' = e,\mu(\tau)} A^{\text{man}}(\nu_2 \rightarrow \nu_l) A^c(\nu_l \rightarrow \nu'_l) A^{\text{man}}(\nu'_l \rightarrow \nu_e), \quad (23)$$

where $A^{\text{man}}(\nu_2 \rightarrow \nu_l)$ and $A^{\text{man}}(\nu'_l \rightarrow \nu_e)$ are the probability amplitudes of the $\nu_2 \rightarrow \nu_l$ and $\nu'_l \rightarrow \nu_e$ transitions in the Earth mantle (which is crossed twice by the neutrinos which traverse the Earth), while $A^c(\nu_l \rightarrow \nu'_l)$ is the probability amplitude of $\nu_l \rightarrow \nu'_l$ transitions in the core, $l,l' = e, \mu(\tau)$. It is the constructive interference of the different amplitudes in the sum in eq. (23) which leads to the remarkable enhancement of the probability $P_{e2}$ for the core-crossing neutrinos.

Another possible interpretation of the enhancement of the probability $P_{e2}$ discussed above is the existence of a parametric-like resonance in the $\nu_2 \rightarrow \nu_e$ transitions in the Earth for which the conditions eq. (11) are approximately satisfied, although the term “parametric-like” does not suggest any specific analogy. The possibility of a parametric-like enhancement of the $\nu_e \rightarrow \nu_\mu$ transitions in matter with density changing periodically but not continuously along the neutrino path was considered in ref. [9]. Let us note that the case studied in ref.
[9] is different from the case considered by us. Although in [9] one period in the variation of density was assumed to consist of two layers with different finite densities (periodic step function), it was supposed that the two layers have equal spatial dimensions (widths) and that the “matter density is a periodic step function”, i.e. that neutrinos cross an integer number of periods in density while they propagate in matter. This does not correspond to the density profile of the Earth in the two-layer approximation. Further, the results presented in [9] were derived in the specific case of small vacuum mixing angle and for $\bar{\rho}_{\text{man}} \ll \bar{\rho}_{\text{man}}^{\text{res}}$, $\bar{\rho}_{\text{c}} \ll \bar{\rho}_{\text{c}}^{\text{res}}$. It should be clear from the discussion following eq. (16) that this specific case also does not correspond to the one of transitions of neutrinos crossing the Earth core studied by us here [9]. Finally, the relevant probability for the solar neutrino transitions in the Earth is the $\nu_2 \rightarrow \nu_e$ and not the $\nu_e \rightarrow \nu_\mu$ transition probability which actually was considered in [9].

There exists a beautiful analogy between the resonance effect discussed in the present Section and the electron paramagnetic resonance taking place in a specific configuration of magnetic fields [10]: one constant, $\vec{B}_0$, located in the $xoz$ plane of the coordinate system and having a direction which forms an angle $2\theta$ with the $z$-axis, $\cos 2\theta > 0$, and a second, $\vec{B}_1$, along the $z$-axis, whose magnitude can change step-wise in time. In the initial moment $t = t_0$ the electron spin points up along the $z$-axis and in the interval of time $\Delta t_1 = t_1 - t_0 > 0$ it is assumed to precess in the field $\vec{B}_0$ and in the constant field $\vec{B}_1$ pointing down along the $z$-axis and having a relatively small magnitude so that $B_z = B_0 \cos 2\theta - B_1 > 0$, where $B_{0,1} = |\vec{B}_{0,1}|$. At $t = t_1$ the field $\vec{B}_1$ increases considerably (step-wise) in magnitude and in the interval of time $\Delta t_2 = t_2 - t_1 > 0$ we have $B_z = B_0 \cos 2\theta - B_1 < 0$. Finally, at $t = t_2$, $\vec{B}_1$ changes (again step-wise) to its initial value and the precession of the electron spin continues in such a configuration of fields for $\Delta t_3 = t_3 - t_2 = \Delta t_1$. As can be shown, there is one-

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[9] In the earlier version of the present work (hep-ph/9805262 of May 8, 1998) we have incorrectly stated that conditions (11) appear also in the specific case studied in ref. [9].

[10] The author is grateful to L. Wolfenstein who brought this analogy to the author’s attention.

[11] The last set of conditions is redundant in the case of the electron paramagnetic resonance: we have added it to make the analogy with the transitions of the neutrinos crossing the Earth core.
to-one correspondence between the physical quantities characterizing this problem and the ones in the problem of transitions of neutrinos crossing the Earth core. At small values of $\sin^2 2\theta$ the probability $P_{e2}$ corresponds to the probability of the spin flip of the electron. It is known that the latter can be made maximal ($\approx 1$) by choosing $\Delta t_1$, $\Delta t_2$, $B_0$ and the two values of $B_1$ in such a way that the precession angles in the intervals of time $\Delta t_1$ and $\Delta t_2$ obey conditions equivalent to (11) and the analog of the function $\sin^2 (2\theta_m' - 4\theta_m' + \theta)$ for this problem be close to 1. Obviously, in the case of neutrinos crossing the Earth core the parameters are much more constrained because they are predetermined by the properties of the Earth, the limited interval of energies of solar neutrinos and the rather small range of relevant values of $\Delta m^2$. It seems almost a miracle that conditions (11) can be fulfilled even approximately for the requisite values of $\Delta m^2$ and $E$ and that the strong enhancement of the probability $P_{e2}$, associated with the conditions (11), (12a) and (12b) actually takes place.

Since the existence of the resonance effect we have discussed depends crucially on the conditions (11) which for a given neutrino trajectory through the Earth are actually conditions on the neutrino oscillation length in the Earth core, $L^c = \frac{2\pi}{\Delta E''}$, and mantle, $L^{man} = \frac{2\pi}{\Delta E'}$, we shall use the term “neutrino oscillation length resonance” $^{12}$. In contrast, the MSW effect is a resonance effect in the neutrino mixing.

The implications of the oscillation length resonance enhancement of the probability $P_{e2}$ for the core-crossing solar neutrinos for the tests of the $\nu_e \rightarrow \nu_\mu(\tau)$ transition solution of the solar neutrino problem via the measurement of the day-night effect related observables are discussed in detail in ref. [1] (see also [2,3]). It is quite remarkable that for values of $\Delta m^2$ from the small (and part of the large) mixing angle MSW solution region the enhancement takes place for values of the $^8$B neutrino energy lying in the interval $\sim (6 - 12)$ MeV to which the Super-Kamiokande and SNO experiments are sensitive. The peak in $P_{e2}$ at $\sin^2 2\theta = 0.01$ for complete.

$^{12}$In the earlier version of the present article (hep-ph/9805262 of May 8, 1998) we have used conditionally the term “diffractive-like enhancement” for the new resonance effect. We think that “neutrino oscillation length resonance” better describes the essence of the effect.
the trajectory with $h = 23^0$ taking place at $\rho_{\text{man}}^{\text{res}} \sim 8.0 \text{ g/cm}^3$, for instance, corresponds to $E \cong 5.3 (10.5) \text{ MeV}$ if $\Delta m^2 = 4.0 (8.0) \times 10^{-6} \text{ eV}^2$. Correspondingly, at small mixing angles this enhancement leads to a much bigger (by a factor of $\sim 6$) day-night asymmetry in the sample of events due to the core-crossing solar neutrinos in the Super-Kamiokande detector than the asymmetry determined by using the whole night event sample [1]. As a consequence, it may be possible to test a rather large part of the small mixing angle solution region in the $\Delta m^2 - \sin^2 2\theta$ plane by performing selective day-night asymmetry measurements. The new enhancement effect also leads to a rather large day-night asymmetry in the recoil - $e^-$ spectrum which is being measured in the Super-Kamiokande experiment [1]. It is not excluded that some of the future high statistics solar neutrino experiments will be able to observe directly this effect. A detector located at smaller geographical latitudes than the existing ones or those under construction [10] would obviously be better suited for this purpose.

3. Conclusions

We have shown in the present work that the strong enhancement of the Earth effect in the transitions of the solar neutrinos crossing the Earth core in comparison with the effect in the transitions of the only mantle crossing neutrinos in the case of the MSW small mixing angle $\nu_e \rightarrow \nu_\mu(\tau)$ transition solution of the solar neutrino problem [1,2], is due to a new oscillation length resonance effect in the transitions and not just to the MSW enhancement of the neutrino mixing taking place in the Earth core. The effect exhibits strong energy dependence. We have derived analytic expression for the relevant transition probability in the two-layer approximation for the density distribution in the Earth, which reproduces with high precision the probability calculated numerically by using the Earth density profile provided by the Earth model [4]. The general conditions for the existence of the oscillation length resonance in the solar neutrino transitions in the Earth were obtained and it was shown that at small mixing angles they are approximately satisfied in the case of $\nu_e \rightarrow \nu_\mu(\tau)$ transitions. These conditions are not fulfilled if solar neutrinos undergo $\nu_e \rightarrow \nu_s$ transitions. Nevertheless, a similar but weaker (in what regards the maximal value of the relevant transition probability $P_{e2}$) enhancement takes place in the region of the MSW resonance in the core in the latter case. For solar neutrino transitions into active neutrino and for the geographical latitudes at
which the Super-Kamiokande and SNO experiments are located, the resonance enhancement takes place in the neutrino energy interval $\sim (5 - 12) \text{ MeV}$ if $\Delta m^2 \cong (4.0 - 8.0) \times 10^{-6} \text{ eV}^2$, which is just in the region of the small mixing angle MSW solution. If the transitions are into sterile neutrino it occurs at approximately two times larger neutrino energies for the same values of $\Delta m^2$.

The oscillation length resonance enhancement is present at large mixing angles in the transitions of solar neutrinos into an active neutrino [1,2] as well. It can also be present in the $\nu_\mu \to \nu_e$ (and $\nu_e \to \nu_\mu$) transitions of atmospheric neutrinos [11]. The probability of the $\nu_\mu \to \nu_e$ ($\nu_e \to \nu_\mu$) transitions, $P(\nu_\mu \to \nu_e) = P(\nu_e \to \nu_\mu)$, and the analogs of the maximum conditions (12a) and (12b) for this probability (conditions (11) are the same as for $P_{e2}$) can be obtained from eqs. (7), (12a) and (12b) by formally setting $\theta = 0$ while keeping $\theta'_m \neq 0$ and $\theta''_m \neq 0$. Thus, condition (12a) becomes $\sin^2(2\theta''_m - 4\theta'_m) > 0$ and is always fulfilled, while condition (12b) transforms into

$$\cos(2\theta''_m - 4\theta'_m) < 0. \quad (24)$$

At small mixing angles, $\sin^2 2\theta \ll 0.05$, we have $P_{e2} \cong P(\nu_\mu \to \nu_e)$ and thus Fig. 1 illustrates the dependence of $P(\nu_\mu \to \nu_e)$ on $\rho_r$ as well. The new enhancement mechanism can be effective in this case for $\rho^r_{e2}$ and $\rho^r_{m2}$ satisfying inequalities (16). For $\sin^2 2\theta = 0.01$, $\Delta m^2 = 10^{-3} (5 \times 10^{-4}) \text{ eV}^2$, and center-crossing ($h = 0^\circ$), for instance, the absolute maximum of $P(\nu_\mu \to \nu_e)$ takes place at $E \cong 1.6 (0.8) \text{ GeV}$. Thus, for values of $\Delta m^2 \sim (5 \times 10^{-4} - 5 \times 10^{-3}) \text{ eV}^2$ of the region of the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation solution of the atmospheric neutrino problem the oscillation length resonance strongly enhances the $\nu_\mu \to \nu_e$ (and $\nu_e \to \nu_\mu$) transitions of the atmospheric neutrinos crossing the Earth core, making the transition probabilities large (and perhaps the transitions detectable) even at small mixing angles. The transitions under discussion should exist if three-flavour-neutrino mixing takes place in vacuum $^{13}$.

$^{13}$Actually, the excess of e-like events in the region $-1 \leq \cos \theta_z \leq -0.6$, $\theta_z$ being the Zenith angle, in the sub-GeV sample of atmospheric neutrino events observed in the Super-Kamiokande experiment (see, e.g., Y. Fukuda et al., hep-ex/9803006 and submitted to Physics Letters B),
The new enhancement mechanism of $P(\nu_\mu \to \nu_e)$ is operative at large mixing angles as well, at which the probabilities $P_{e2}$ and $P(\nu_\mu \to \nu_e)$ exhibit different dependence on $E/\Delta m^2$ [11].

The effects of the oscillation length resonance enhancement of the solar neutrino transitions in the Earth can be observed in the currently operating high statistics solar neutrino experiments (Super-Kamiokande, SNO) by performing selective day-night asymmetry measurements [1]. A high statistics solar neutrino detector located at smaller geographical latitudes would be better suited for the observation of this enhancement.

can be due to $\nu_\mu \to \nu_e$ small mixing angle, $\sin^2 2\theta_{e\mu} \cong (0.01 - 0.10)$, transitions with $\Delta m^2 \sim 10^{-3}$ eV$^2$, strongly enhanced by the neutrino oscillation length resonance as neutrinos cross the Earth core on the way to the detector. Such transitions are naturally predicted to exist in a three-neutrino mixing scheme, in which, e.g., the small mixing angle MSW $\nu_e \to \nu_\mu$ transitions with $\Delta m^2_{21} \sim (4 - 8) \times 10^{-6}$ eV$^2$, or large mixing angle $\nu_e \leftrightarrow \nu_\mu$ oscillations with $\Delta m^2_{21} \sim 10^{-10}$ eV$^2$, provide the solution of the solar neutrino problem and the atmospheric neutrino anomaly is due to $\nu_\mu \leftrightarrow \nu_\tau$ large mixing angle oscillations with $\Delta m^2_{31} \sim 10^{-3}$ eV$^2$. It should be added that if $\Delta m^2_{31} \cong 5 \times 10^{-3}$ eV$^2$, the excess of e-like atmospheric neutrino events in the Super-Kamiokande data due to the neutrino oscillation length resonance should be present in the multi-GeV sample at $-1 \leq \cos \theta_2 \lesssim -0.8$. 

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**Figure Captions**

**Figure 1.** The dependence of the probability $P_{e2}$ on $\rho_r$ (see eq. (8)) in the case of $\nu_e \rightarrow \nu_\mu(\tau)$ transitions of solar neutrinos for $\sin^2 2\theta = 0.01$ (from ref. [2]). The five plots are obtained for $0.1 \text{ gr/cm}^3 \leq \rho_r \leq 30.0 \text{ gr/cm}^3$ ($Y_{e}^{\text{man}} = 0.49$, $Y_{e}^{c} = 0.467$) and five different solar neutrino trajectories in the Earth determined by the Nadir angle $h$: a) $h = 0^0$ (center crossing), b) $h = 13^0$ (winter solstice for the Super-Kamiokande detector), c) $h = 23^0$ (half core for the Super-Kamiokande detector), d) $h = 33^0$ (core/mantle boundary), e) $h = 51^0$ (half mantle).

**Figure 2.** The dependence of the probability $P_{e2}$ on $\rho_r$ for $\nu_e \rightarrow \nu_s$ transitions of solar neutrinos (from ref. [3]). The five plots were obtained for $\sin^2 2\theta = 0.01$ and the same neutrino trajectories as in figure 1.
$\sin^2(2\theta_v) = 0.0100$

(a) $h = 0^\circ$ Center Crossig
(b) $h = 13^\circ$ SK Winter Solstice
(c) $h = 23^\circ$ Half Core
(d) $h = 33^\circ$ Core/Mantle Boundary
(e) $h = 51^\circ$ Half Mantle
Sterile

\[ P_{\alpha}(\rho) \]

\[ P_{\alpha}(\rho) \]

\[ P_{\alpha}(\rho) \]

\[ P_{\alpha}(\rho) \]

\[ P_{\alpha}(\rho) \]

\[ \sin^2(2\theta) = 0.0100 \]

(a) \( h = 0^\circ \) Center Crossig
(b) \( h = 13^\circ \) SK Winter Solstice
(c) \( h = 23^\circ \) Half Core
(d) \( h = 33^\circ \) Core/Mantle Boundary
(e) \( h = 51^\circ \) Half Mantle