About actual contradiction in geotechnical design and optimal way of it resolution

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Abstract. The paper describes serious and fundamental contradiction in geotechnical design, which manifested itself to the greatest extent when designing the foundations of high-rise buildings in the third quarter of the 20th century has begun to form from the late of the 19th century to the 20th years of the 20th century. Prime cause of this contradiction is in complex physical structure of soils, complex nature of their formation and, therefore, in complex form of their deformation. The article also notes the high technical and economic efficiency of using realistic physically nonlinear soil models in the design of geotechnical parts of buildings and structures. In this case such a designing is most effective when using the parameters of nonlinear models determined from data of in-situ tests.

1. Introduction
Serious and fundamental contradiction in geotechnical design, which manifested itself to the greatest extent when designing the foundations of high-rise buildings in the third quarter of the 20th century has begun to form from the late of the 19th century to the 20th years of the 20th century. Prime cause of this contradiction is in complex physical structure of soils, complex nature of their formation and, therefore, in complex form of their deformation. Once a well-known physicist Lorenz said that if he was asked to make an equation for soil deformation, he would have run away in fright [1]. However, the needs of building design and status of soil base as a supporting element of all structure made the problem of studying the mechanical properties of soils and firstly the problem of studying the character of soil deformation practically and theoretically very relevant. Certainly, as in other technical branches, researches have begun with the simplest assumption of a proportional ratio between a load on the soil (pressure $P$) and soil settlement $S$: $S = P/C_z$ (here $C_z = \text{Const}$ is proportionality coefficient called "coefficient of soil reaction" «or "bed coefficient"). Graph of the ration between $P$ and $S$ is shown in Figure 1. This ratio proposed by Fuss in 1798 [2], but sometimes not exactly is called as Winkler model [3,4,5], must be more correctly defined as a "contact model of the soil", because it does not reflect the relationship between deformations $\varepsilon$ and stresses $\sigma$ inside soil massif, and provides designers only with a ratio between the surface pressure $P$ on the soil beneath the foundation and settlement $S$ of the part of the surface of soil under foundation. But such a primitive inclusion of the factor of soil massif in designing of buildings (including software) has a number of serious negative consequences: firstly, results of the pressure transfer to the underlying soil layers are not taken into account, but these layers may be weaker than the bearing layer under the foundation;
secondly, there is not reflected transmission of the pressure to soil bases of neighboring buildings, that not allowing calculation of additional deformations of these buildings, while it is very important in conditions of dense urban development; thirdly, many other important features of loading the soil massif are not reflected, for example, the dependence of the coefficient $C_z$ on the size and shape of the loaded area – phenomenon, which has been well studied experimentally [5, 6, 7, 8, 9, 10, 11]: an approximate diagram of the dependence of the settlement ($w$) of a rigid square plate at the same pressure under it on the width of the plate ($a'$) is shown in Figure 2; in Figures 3 and 4 are shown similar diagrams for the dependence of settlement ($w$) on the ratio of the area ($A$) of the rigid square plate to its perimeter ($Pr$), i.e. on its shape [8, 9]. This model also does not allow to reflect another very important, well-known and characteristic phenomenon for the soil base, that distinguishes soil from all structural (i.e., artificial) materials, namely, an increase in the rigidity of even a homogeneous soil base with increase in depth [14, 15, 13, 5, 15], as shown in Figure 5 and the consequence of which is a stiffer soil base compared to the case of constant stiffness of soil [14, 5, 15]; later it became clear that all these features of soils are result of significant physical nonlinearity of soils, i.e. dependence of their rigidity on the stress state [15] and relationship of deformation with soil strength [16]. Requirement to take into account in geotechnical designing physically nonlinear type of soil deformation is included in all regulatory documents of the Russian Federation on the design of soil bases of buildings and structures, starting in 1975 [16, 17, 18, 19]. Moreover, in the Russian Federation this requirement, due to its special importance for the safety and reliability of buildings, is included in Article 16 “Mechanical Safety” of Federal Law № 384-ФЗ “Technical Regulations on the Safety of Buildings and Structures” [20]. Attempts to improve physically linear models, in particular, Fuss-Winkler contact model, by adding in its later version additional coefficient to take into account soil distribution ability [20, 21], did not solve even this problem because essence of the problem was in absence of rations between stresses (σ) and strains (ε) in “contact model” framework and as a result of which new coefficient had the same disadvantages as the coefficient $C_z$.

2. Distinction between linear and nonlinear soil models

Thus, further development of geotechnical designing was possible only based on models having relationships between stresses (σ) and strains (ε). But such a model at the turn of the nineteenth and
twentieth centuries was only the physically linear Hook’s model that he proposed at the end of seventeenth century for calculating deformations of metal, rubber and other similar materials. Nevertheless, by a willed decision, which was not entirely justified, but in absence of other models and with notes about inconsistencies Hook’s model to the physical nature of the soil [22, 23] in the 30s…40s of the 20th century for calculating deformations of soil bases exactly Hook’s model was adopted [24, 25, 26, 27, 28]. The justification of the volitional decision to apply Hooke’s model for soils was not very convincing because, in addition to unconfirmed by more accurate nonlinear calculations [29, 30, 15], of previously obtained with use of Hooke's linear theory of contact stresses beneath foundations and stresses in soil massifs [31, 32, 33], this decision was based mainly on the statement that if in experiments [34, 35, 11, 28] relation between load (P) and a settlement (S) is directly proportional (linear), then relation between stress (σ) and strain (ε) should be also proportional (linear), as it is defined in Hooke's theory. However, such a logical conclusion is not entirely fair and can be approximately satisfied only under influence of a certain parametric relation linearizing ratio between S and P. Wherein inverse logical relation (ε ∼ σ ⇒ S ∼ P) is certainly true. Semantically linear Hooke's model has following analytical form: ε = σ/E (here E=Const, as in the case of Fuss-Winkler model is a proportionality coefficient which is called "Young's modulus"). Graph of physically linear Hooke’s model (E = Const) in figure 6 is similar to the graphic image of also physically linear (Cz=Const) Fuss-Winkler model in figure 1. However, differential, essentially, Hooke's dependence ε = f (σ) and integral, essentially, Fuss-Winkler relationship S = f (P) differ because strain ε (in general case strain tensor εi) can be decomposed into volumetric strain εi de and shape (shear) εs deformation in the framework of a continuous medium concept: εi = ε + εs with corresponding decomposition of the stress tensor: σii=σ+σi.

Figure. 3. Diagram of dependence of the settlement of a rigid square plate on its dimensions at the same pressure beneath it (for sand) [10]

Graphically, in coordinates of invariants of stress and strain tensors, such a separation of stresses and strains in the case of a physically linear Hooke model looks like that shown in Figure 7; in this case dependences for both parts of the deformation remain physically linear, independent of the stress-strain state: volumetric stiffness is characterized by a constant bulk modulus K=σ/ε {here σ and ε are first invariants of stress tensor and strain tensor (volume deformation); shaping stiffness is characterized by a constant shear modulus G=σ/εs (here σi and εs are second invariants of deviatoric parts of stress tensor and strain tensor). This mechanical model most realistically describes the deformation of elastic-linear (physically linear) materials such as metals, rubber and concrete. Of cause, adoption of such a model for calculating soil deformations was extremely contradictory. By the way, by-effect (philological) consequences of this contradictory was replacement, in the case of soil, of the name “Young's modulus” E with “modulus of deformation E” [28,37].
Below are approximating relations for the bulk modulus $K$ and the shear modulus $G$:

$$K = \frac{\sigma}{\varepsilon} = \frac{\sigma^{1-\alpha}}{A_0},$$

$$G = \frac{\sigma_i}{\varepsilon_i} = \frac{\sigma_i}{\varepsilon_i} = \frac{\sigma_i}{B + \varepsilon_i}.$$  \(1\)

Here $\sigma_{\mu} = A\sigma + C$ – strength condition of Mises-Botkin for soil; $A_0$, $\alpha$, $A$, $B$, $C$ – parameters dependent on the soil.

Despite the fruitful technical potential of the Hooke’s model, which allowed developing on its basis a large number of geotechnical design methods, both analytical [27, 13, 28, 37, 38, 39, 40, 41, 42, 43, 17, 18, 19] and numerical [39, 5, 43], rather quickly serious contradictions appeared from the application of this physically linear model to a substantially physically nonlinear soil. Unfortunately, in a view of the beginning war in 1941, results of investigations of physically nonlinear soil deformation [44, 45], the essence of which as follows from the graphs in Figure 8 and their approximating by formulas (1) consists in dependence of the stiffness characteristics on the stress-strain state, have not been applied in design practice.
Figure 7. Diagram of linear deformation of structural materials (metals, as well as rock: :
bulk deformation modulus $K = \sigma / \varepsilon$; shear modulus $G = \sigma_i / \varepsilon_i$; $\sigma$, $\varepsilon$ – first invariants of stress and strain tensors (volumetric deformation); $\sigma_i$, $\varepsilon_i$ – second invariants of the deviator parts of the tensors of stresses and deformations (shape change – shear).

At the same time, it was found out that a model with constant stiffness parameter (in particular, $E = \text{const}$) cannot, like the Fuss model, reflect increase of the stiffness of a homogeneous soil with increasing depth; the constant value of the modulus $E$, as in the case of the Fuss model, reduces the actual stiffness of the soil base [15] and overestimates its design deformation. It also turned out that value of Young's modulus (modulus of deformation) $E$ as well as the coefficient $C_z$ depends on the shape and size of the loaded area and hence on the type of test of the soil. The desire to explain and take into account this difference, for example, for the results of soil tests with round rigid plates and pressuremeters, led to the adoption of different module models for describing soil deformations [46, 47], which are simplified versions of a physically nonlinear model, such as the Botkin model described above [44], which, due to a more general structure, which includes not only deformation, but also strength parameters of the soil, reflects the non-invariance of the value of the deformation modulus $E$ with respect to the size and shape of the loaded area [15, 48]. It is no coincidence that, as a result of an extensive analysis, it was established [42], a significant discrepancy between the actual and calculated on the base of Hooke’s theory deformations of soil bases of different buildings and structures. In this regard, all soil models that have in their structure a constant Young's modulus $E$, including non-linear ones (for example, elastic-plastic), retain the drawback of the Hooke model in the form of the non-invariance of the modulus $E$ with respect to various stress-strain state factors. From the other hand, numerous experimental and theoretical investigations of soil deformation within the framework of a physically nonlinear model [30, 31, 15, 48] showed a qualitative and quantitative coincidence of actual and theoretical results even in such details as the dependence of the distribution of contact stresses and stresses in the soil mass on the type and condition of the soil, the concentration of stresses under the edges of the foundations, different in different soils, and the consequent increase in deformation at a certain depth along the axis of the foundation. The prognosis of influence of the load onto neighboring buildings is also much more realistic. These successful research results made it possible, in accordance with the Federal Law [20], to use a physically non-linear soil model in real design practice, which ensured both the technical reliability of the structures and significant financial benefits: in the construction of a 70-meter retaining wall, savings amounted to $0.55 million, and in the construction of the foundation of a 32-storey residential building, respectively, $1.3 million.

Figure 8. Simplified nonlinear soil deformation diagram; notation here is the same as in Figure 7.
Such an efficiency of using a physically nonlinear model was ensured, inter alia, by determining its parameters from in-situ tests according to the technique described in [49].

3. Conclusion

Nowadays in geotechnical designing, a serious contradiction has arisen between requirements of regulatory documents on the design of soil bases and foundations (including requirements of Federal Law № 384-ФЗ “Technical Regulations on the Safety of Buildings and Structures” about application in geotechnical calculations of a soil model that reflects real, physically nonlinear features of soil deformation, and existing practice of geotechnical design based on the physically linear Hooke’s theory, reflecting the deformation of structural materials such as metals, rubber and concrete.

Numerous observations of various structures indicate a significant discrepancy between the actual settlements of their soil bases and calculated ones using Hooke theory.

Taking into account in the designing of physically nonlinear features of soil deformation, the essence of which consists in dependence of soil rigidity on stress-strain state and in the link of soil deformation with its strength allows to build economically and technically optimal foundations and other geotechnical structures, for example, retaining walls; moreover, in this case the cost of geotechnical structures is two or more times cheaper than of those developed using the physically linear theory of Hooke; also, the prognosis of influence on neighboring structures is much more realistic than that predicted by Hooke's theory, with its distribution capacity corresponding to the distribution ability of rubber for a long distance.

To widespread implementation of realistic, economically very profitable physically non-linear soil models in design practice should be promoted by examples of their further successful application in practice, as well as the organization of effective scientific and technical monitoring of buildings under construction, with an analysis of geotechnical results. Undoubtedly, in the process of studying the soil of a construction site before designing, the mechanical parameters of physically nonlinear soil models should be determined, mainly by in-situ methods.

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