On Indirect Excitation of Lateral Vibrations of the Table of the Electrodynamic Stand Suspended on Viscoelastic Shock Absorbers

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Abstract. Modern machines and aerospace facilities are equipped with rather complex, multi-link high-level vibration protection systems. Therefore, it is of interest to carry out work on the formation of dynamic models of vibration-protective properties of the apparatus, taking into account the properties of their real design features. The plane-parallel motion of a viscoelastic suspended rigid body with an axis of symmetry and imitating the motion of the table of an electrodynamic stand is considered. In the calculated dynamic scheme, the action of the suspension membranes and the hanging shock absorber is represented as the action of some linear springs. It is assumed that a periodic perturbing force (the action of a pulsating magnetic field) directed vertically is applied at the body’s center of gravity. Nonlinear integro-differential equations of motion are compiled, in expansions of trigonometric functions and power series, only small second order are kept. The method of averaging is used to reduce to normal coordinates. The conditions for indirect excitation of axial lateral vibrations (torsional vibrations about the horizontal axis) under the stability of the parametric resonance of the vertical are obtained. Formulas providing the suspended object are obtained.

Keywords: oscillations, plane-parallel motion, integro-differential equations, resonance phenomenon, rigid body, averaging method

1. Introduction
The increase in body weight, making vertical oscillations by a deformable spring, the stiffness of which corresponds to the stiffness of the suspension. A spatial single mass model is convenient for research, and therefore is often used for dynamic analysis of various structures [1,2,3,4]. For example, in [1], various types of single mass design schemes are considered. The analysis showed that calculations using such simplified schemes make it possible to determine only the lowest frequency, at which the effect of the mass is insignificant, and the damping effect of the shock absorber can be neglected [5, 6]. In work [7] it is shown that one mass system, in a vehicle, is about 12 - 20% of the total mass. Thus, on the basis of the results obtained, it was found that the natural frequency of vibrations of the suspended masses is significantly lower than the natural frequency of vibrations of other elements. In works [7, 8, 9, 10, 11],...
when one mass model was studied, in addition to the above factors, the factors random kinematic action. Thus, forced vibrations of a suspended one mass system are analyzed.

The paper deals with the problem of indirect excitation of lateral vibrations of the table of an electrodynamic stand suspended on viscoelastic shock absorbers. The main movements of the electrodynamic table are in the vertical direction. But, experiments show [12] that in addition to this basic movement, there are still significant oscillations of the table in lateral directions, which introduces additional overloads and distorts the test result. It is important to know in advance the dangerous frequencies and conditions for the excitation of "lateral" oscillations of the table in order to correctly evaluate the results of experiments.

2. Methods

2.1. Problem statement and solution methods

Let us assume that the vibrating table is a rigid body, symmetrical about the vertical axis passing through the center of gravity $O$. We neglect the static and dynamic deformations of the table and will consider, by virtue of symmetry, the vibrations of the latter as a rigid body in the vertical plane and its torsional vibrations about the horizontal axis. As a result, we have to study the plane-parallel motion of a rigid body with viscoelastic constraints. In the linear formulation of the problem, when reduced to normal coordinates, the equations of motion of the table are divided along the principal coordinates and are integrated independently of each other. It turns out that the frequency and amplitude of lateral vibrations does not explicitly depend on the frequency and amplitude of vertical vibrations. Let us accept the dynamic design scheme shown in figure 1, where the vibrating table is represented in the form of a rigid body suspended by three elastic springs, the lengths of which $l_1, l_2, l_3$ respectively, while

![Figure 1. Dynamic design diagram of the vibration table suspension stiffness coefficients (operator stiffness), respectively](image-url)
\[ \tilde{k}_1[f(t)] = k_{01} \left[ f(t) - \int_{-\infty}^{t} R_{01}(t-s)f(s)ds \right], \]
\[ \tilde{k}_2[f(t)] = k_{02} \left[ f(t) - \int_{-\infty}^{t} R_{02}(t-s)f(s)ds \right], \]
\[ \tilde{k}_3[f(t)] = k_{03} \left[ f(t) - \int_{-\infty}^{t} R_{03}(t-s)f(s)ds \right], \]

where \( f(t) \) - arbitrary function of time, \( R_{01}(t-s) \) and \( R_{02}(t-s) \) are relaxation nuclei.

The outer ends of the burins (points D) are attached to a fixed base, the inner ends (points A, B, C) are attached to a solid. Distances are also given: \( OA = b_1, OA_2 = b_2, OE_1 = a_1, OE = a_2 \). The ends of the springs are considered ideal hinges. The reaction of the spring is considered proportional to its elongation, and the direction of the reaction vector coincides with the axis of the spring. The stiffnesses of the introduced springs are the stiffnesses of the membranes in their plane for the considered plane-parallel motion of the stage. The table suspension structures of most electrodynamic stands are reduced to this model. Let a periodic perturbing force (action of a pulsating magnetic field) be applied at the center of gravity, directed along the axis \( y \) [13]. To derive the differential equations of motion of the investigated body, we introduce the following coordinate systems (see Figure 1): \( Ox'y' \) - stationary coordinate system associated with the base of the stand, \( O_1\bar{x}\bar{y} \) - coordinate system invariably associated with a moving body (shaker table). Dot \( O_1 \) coincides with the center of gravity of the body, \( J \) - moment of inertia of a body about an axis perpendicular to the plane \( Ox'y' \) and passing through the center of gravity, \( M \) - body mass. In the body balance position, the point \( O_1 \) coincides with point \( O \) and axes \( O_1\bar{x}, O_1\bar{y} \) coincide respectively with the axes \( Ox \) and \( Oy \). We will consider oscillations in a fixed coordinate system \( Ox'y' \), in which its position is determined by three generalized coordinates: two linear displacements of the center of gravity \( x \) and \( y \) and the angle of rotation about the center of gravity. In deriving the equations, we restrict ourselves to the expansions of trigonometric functions by the following expressions:

\[ \sin \varphi = \varphi, \cos \varphi = 1 - \frac{1}{2} \varphi^2. \] (2)

In all expansions in power series, we will keep only up to terms of the second order with respect to coordinates and their derivatives. We will assume that the forces of resistance \( H \) and their moments \( N \) satisfy the integral relations, that is, [14]

\[ H_x = H_{0x} \left[ x(t,s) - \int_{-\infty}^{t} R_{0x}(t-\tau)x(\tau,s)ds \right], \]
\[ N_\varphi = N_{0\varphi} \left[ \varphi(t,s) - \int_{-\infty}^{t} R_{0\varphi}(t-\tau)\varphi(\tau,s)ds \right], \]
\[ H_y = H_{0y} \left[ y(t,s) - \int_{-\infty}^{t} R_{0y}(t-\tau)y(\tau,s)ds \right], \] (3)

where \( H_{0x}, N_{0\varphi}, H_{0y} \) - instantaneous values of the modulus of elasticity, determined from experiments, \( R_{0x}(t-\tau), R_{0\varphi}(t-\tau) \) and \( R_{0y}(t-\tau) \) - relaxation kernels of the material determined from experiments [15]. The equations of motion, taking into account the above, are as follows:

\[ M \frac{d^2x}{dt^2} + c_1(x - \int_{-\infty}^{t} R_{1x}(t-s)x(s)ds) + c_{1\varphi}(\varphi - \int_{-\infty}^{t} R_{1\varphi}(t-s)\varphi(s)ds) + \]

\[ N_\varphi = N_{0\varphi} \left[ \varphi(t,s) - \int_{-\infty}^{t} R_{0\varphi}(t-\tau)\varphi(\tau,s)ds \right], \]

\[ H_y = H_{0y} \left[ y(t,s) - \int_{-\infty}^{t} R_{0y}(t-\tau)y(\tau,s)ds \right], \]
\[ +c_{12}(y^2 - \int_{-\infty}^{t} R_{12y}(t-s)y(s)ds) + c_{13}(\varphi^2 - \int_{-\infty}^{t} R_{13\varphi}(t-s)\varphi^2(s)ds) + \]

\[ +S_{11}(xy - \int_{-\infty}^{t} R_{11xy}(t-s)x(s)y(s)ds) + S_{13}(y\varphi - \int_{-\infty}^{t} R_{13y\varphi}(t-s)y(s)\varphi(t)ds) = 0, \]

\[ f\frac{d^2\varphi}{dt^2} + c_{3}(\varphi - \int_{-\infty}^{t} R_{3\varphi}(t-s)\varphi(s)ds) + c_{30}(x - \int_{-\infty}^{t} R_{30x}(t-s)x(s)ds)x + \]

\[ +c_{32}(y^2 - \int_{-\infty}^{t} R_{32y}(t-s)y^2(s)ds) + c_{33}(\varphi^2 - \int_{-\infty}^{t} R_{33\varphi}(t-s)\varphi^2(s)ds) + \]

\[ +S_{31}(xy - \int_{-\infty}^{t} R_{31xy}(t-s)x(s)y(s)ds) + S_{32}(x\varphi - \int_{-\infty}^{t} R_{32x\varphi}(t-s)x(s)\varphi(s)ds) + \]

\[ +S_{34}(y\varphi - \int_{-\infty}^{t} R_{34y\varphi}(t-s)y(s)\varphi(s)ds)yY + M(x\ddot{y} + y\ddot{x}) = 0, \quad (4) \]

\[ \frac{M\ddot{y}}{dt^2} + c_{2}(y - \int_{-\infty}^{t} R_{2y}(t-s)y(s)ds) + c_{21}(x^2 - \int_{-\infty}^{t} R_{21x}(t-s)x^2(s)ds) + \]

\[ +c_{23}(\varphi^2 - \int_{-\infty}^{t} R_{23\varphi}(t-s)\varphi^2(s)ds) + S_{21}(xy - \int_{-\infty}^{t} R_{21xy}(t-s)xy(s)ds) + \]

\[ +S_{23}(xy - \int_{-\infty}^{t} R_{23xy}(t-s)x(s)y(s)ds) + S_{24}(x\varphi - \int_{-\infty}^{t} R_{24x\varphi}(t-s)x(s)\varphi(s)ds) = P_0e^{-ipt}, \]

where

\[ c_1 = k_{02} + k_{03}, c_{10} = -k_{02}b_1 + k_{03}b_2, c_{12} = -\frac{k_{02}}{2l_2} - \frac{k_{03}}{2l_3}, \]

\[ k_{02}a_1(l_2 + a_1) - \frac{k_{03}a_2(l_3 + a_2)}{2l_3}, S_{11} = -\frac{k_{01}}{l_1}, \]

\[ S_{1s} = \frac{k_{01}b_1}{l_1} - \frac{k_{02}a_1}{l_2} - \frac{k_{03}a_2}{l_2}, c_2 = k_{04}, c_{2s} = -\frac{N_4}{2l_1}, \]

\[ c_{2s} = -\frac{k_{01}(b_1 + l_1)b_1}{2l_1} + \frac{k_{02}a_1b_1}{l_2} - \frac{k_{03}a_2b_2}{l_2}, \]

\[ S_{2s} = \frac{k_{02}b_2}{l_2} - \frac{k_{03}b_2}{l_2}, c_3 = b_1^2k_0^2 + b_2^2k_0a, \]

\[ c_{3s} = -b_1k_{02} + b_2k_{03}, c_{3s} = k_{02}b_1 - \frac{k_{03}b_2}{2l_2}, \]

\[ c_{1s} = \frac{3k_{02}a_1b_1}{2l_1} + \frac{3a_1b_1k_{02}}{2l_2} - \frac{3k_{03}a_2b_2}{2l_2} - \frac{3a_2b_1k_{03}}{2l_2}, \]

\[ S_{3s} = k_{01} + \frac{k_{01}b_1}{l_1} - \frac{k_{02}a_1}{l_2} - \frac{k_{03}a_2}{l_2} - k_{02} - k_{03}, \]

\[ S_{3s} = -\frac{k_{01}a_1}{l_2} - \frac{k_{03}a_2}{l_2} - k_{02}a_1 - k_{02}a_2. \]
\[ S_{34} = b_1 k_0 l_1 - \frac{k_0 b_1^2}{l_1} + \frac{2k_0 a_1 b_1}{l_2} - \frac{2k_0 a_2 b_2}{l_2} + k_0 a_1 - k_0 a_2. \]  

(5)

Oscillations of the body are assumed such that the integral nonlinear terms containing the products of coordinates and their derivatives are considered small in comparison with the main linear terms. We reflect the smallness of the terms by introducing a small positive parameter \( \mu \) in the following way:

\[ \mu h_1 = \frac{H_{0x}}{M}, \mu h_2 = \frac{H_{0y}}{M}, \mu h_3 = \frac{H_{0p}}{M}, \]

where

\[ \mu \varepsilon_{12} = \frac{c_{012}}{M}, \mu \varepsilon_{13} = \frac{c_{013}}{M}, \mu \varepsilon_{14} = \frac{c_{014}}{M}, \mu \varepsilon_{15} = \frac{c_{015}}{M}. \]

Further denoting

\[ \lambda_{11}^2 = \frac{c_{1}}{M}, \lambda_{21}^2 = \frac{c_{2}}{M}, \lambda_{31}^2 = \frac{c_{3}}{I}, \lambda_{12}^2 = \frac{c_{10}}{M}, \lambda_{32}^2 = \frac{c_{30}}{I}, \rho' = \frac{L}{M}. \]

we rewrite system (1) as

\[
\frac{\partial^2 x}{\partial t^2} + \lambda_{11}^2 \left[ x - \int_{-\infty}^{t} R_{1x}(t-s)x(s)ds \right] + \lambda_{12}^2 \left[ \varphi - \int_{-\infty}^{t} R_{10\varphi}(t-s)\varphi(s)ds \right] = \\
= \mu \psi_1(x, y, \varphi, y^2, \varphi^2, xy, t-s) \int_{-\infty}^{t} R_{12y}(t-s)y^2(s)ds, \\
\int_{-\infty}^{t} R_{13\varphi}(t-s)\varphi^2(s)ds, \int_{-\infty}^{t} R_{11xy}(t-s)x(s)y(s)ds, \int_{-\infty}^{t} R_{13y\varphi}(t-s)y(s)\varphi(t)ds \\
\int_{-\infty}^{t} R_{13\varphi}(t-s)\varphi^2(s)ds, \int_{-\infty}^{t} R_{12xy}(t-s)xy(s)ds, \int_{-\infty}^{t} R_{13y\varphi}(t-s)y(s)\varphi(t)ds \\
\int_{-\infty}^{t} R_{13\varphi}(t-s)\varphi^2(s)ds, \int_{-\infty}^{t} R_{12xy}(t-s)xy(s)ds, \int_{-\infty}^{t} R_{13y\varphi}(t-s)y(s)\varphi(t)ds 
\]

(6)

(7)

at \( \mu = 0 \) and zero values of the relaxation kernels of viscoelastic elements, system (6) admits a particular solution:

\[ x = \varphi = 0, y = qe^{i\omega t}, q = \frac{p'}{\lambda_{21}^2 - \omega^2}. \]  

(7)
This means that under the action of an external periodic force, oscillations of a rigid body will take place only in the direction of the y coordinate, i.e., along the coordinate in relation to which the external force acts. Other coordinates \((x, \varphi)\) while staying alone.

Let us now assume that the relaxation kernels of viscoelastic springs are nonzero. Then \(x = \varphi = 0, y = qe^{iat}\), and the solution takes the following form:

\[
q = \frac{q_{et}}{\sqrt{\lambda_{21}^2 \left( \frac{p^2}{\omega^2} + 1 - \Gamma_c \right)^2 + \Gamma_c^2}} , \quad t g \theta = \frac{\Gamma_c}{\lambda_{21} p^2 - 1 + \Gamma_c}
\]

where \(\Gamma_c = \int_0^\infty R(\tau) \cos p \tau d\tau, \Gamma_s = \int_0^\infty R(\tau) \sin p \tau d\tau\) - cosine and sine are Fourier transforms of relaxation kernels, \(\theta - \) phase shift. This concept of the nature of motion is usually characteristic of the theory of linear systems. Let us now estimate the picture of the oscillations obtained by taking into account the nonlinear terms of system (6).

2.2. Construction of an approximate solution and determination of the conditions for the excitation of coupled oscillations

Let us first find the natural frequencies of the linear part equations (2). Put in (2) \(\mu = 0, l = 0\), and relaxation kernels of viscoelastic elements are equal to zero. The solution is sought in the form [16]

\[
x = A_x^* e^{iat}, y = A_y^* e^{iat}, \varphi = A_\varphi^* e^{iat}, \quad (8)
\]

where \(\Omega = \Omega_R + i \Omega_I\) - complex frequency, \(A_x^*, A_y^*, A_\varphi^*\) - displacement amplitudes, (complex values). Substituting (8) into (6), after which we arrive at a homogeneous system of linear algebraic equations with respect to the quantities \(A_x^*, A_y^*, A_\varphi^*\). In the case under consideration \(\Omega = \Omega_R\) (actual frequency). Equating the determinant of this system to zero, we obtain the frequency equation

\[
[\Omega^4 - (\lambda_{31}^2 + \lambda_{32}^2)\Omega^2 + \lambda_{31}^2 \lambda_{32}^2 - \lambda_{12} \lambda_{32}] (\Omega^2 - \lambda_{21}^2) = 0. \quad (9)
\]

Solving (9), we obtain the natural frequencies of the vibration:

\[
\begin{align*}
\Omega_1 &= \frac{1}{2} (\lambda_{31}^2 + \lambda_{32}^2) - \frac{1}{4} \sqrt{\left( \lambda_{31}^2 - \lambda_{32}^2 \right)^2 + \lambda_{12} \lambda_{32}^2}, \\
\Omega_2 &= \lambda_{21}, \\
\Omega_3 &= \frac{1}{2} (\lambda_{31}^2 + \lambda_{32}^2) + \frac{1}{4} \sqrt{\left( \lambda_{31}^2 - \lambda_{32}^2 \right)^2 + \lambda_{12} \lambda_{32}^2}.
\end{align*} \quad (10)
\]

Amplitude ratio coefficients \(\frac{A_\varphi}{A_x}\) for frequencies \(\Omega_1\) and \(\Omega_2\) have the form:

\[
\alpha_1 = \frac{-a_1^2 + \lambda_{31}^2}{-\lambda_{12}}, \quad \alpha_2 = \frac{-a_1^2 + \lambda_{31}^2}{-\lambda_{12}}. \quad (11)
\]

If the rheological properties of materials are taken into account, then we obtain the following transcendental equations with a complex input parameter

\[
\begin{align*}
[\Omega^4 - (\lambda_{31}^2 (\Omega_R) + \lambda_{32}^2 (\Omega_R)) \Omega^2 + \lambda_{31}^2 (\Omega_R) \lambda_{32}^2 (\Omega_R) - \lambda_{12} (\Omega_R) \lambda_{32} (\Omega_R)] = 0, \\
\Omega^2 - \lambda_{21} (\Omega_R) = 0, \\
\lambda_{11} (\Omega_R) = \lambda_{011} [1 - \Gamma_c (\Omega_R) - i \Gamma_s (\Omega_R)], \\
\lambda_{31} (\Omega_R) = \lambda_{031} [1 - \Gamma_c (\Omega_R) - i \Gamma_s (\Omega_R)], \\
\lambda_{12} (\Omega_R) = \lambda_{012} [1 - \Gamma_c (\Omega_R) - i \Gamma_s (\Omega_R)], \\
\lambda_{32} (\Omega_R) = \lambda_{032} [1 - \Gamma_c (\Omega_R) - i \Gamma_s (\Omega_R)]. \quad (12)
\end{align*}
\]
The calculations used the Koltunov-Rzhanitsyn three-parameter relaxation kernel: \( R(t) = A e^{-\beta t/t^{1-\alpha}} \). Equation (12) is solved numerically by the Muller method [17].

Now, in (6), we transfer the integral terms of the first, second and third equations to the right-hand sides of the equations. Then we obtain the following systems of equations

\[
\frac{\partial^2 x}{\partial t^2} + \lambda_1^2 x + \lambda_2^2 \varphi = \\
= \mu \varphi_1(x, y, \varphi, y^2, \varphi^2, xy, \int_{-\infty}^{t} R_{1x}^c(t - s) x(s) ds, \\
\int_{-\infty}^{t} R_{10\varphi}^c(t - s) \varphi(s) ds, \int_{-\infty}^{t} R_{12y}^c(t - s) y^2(s) ds, \\
\int_{-\infty}^{t} R_{13\varphi}^c(t - s) \varphi^2(s) ds, \int_{-\infty}^{t} R_{11xy}^s(t - s) x(s)y(s) ds, \int_{-\infty}^{t} R_{13y\varphi}^s(t - s) y(s) \varphi(t) ds)
\]

\[
\frac{\partial^2 y}{\partial t^2} + \lambda_2^2 y = \\
= F_y(t) + \mu \varphi_2(x, y, \varphi, y^2, \varphi^2, xy, \int_{-\infty}^{t} R_{1x}^c(t - s) y(s) ds, \int_{-\infty}^{t} R_{12x}^c(t - s) x^2(s) ds, \\
\int_{-\infty}^{t} R_{23\varphi}^c(t - s) \varphi^2(s) ds, \int_{-\infty}^{t} R_{22xy}^s(t - s) xy(s) ds, \int_{-\infty}^{t} R_{24x\varphi}^s(t - s) x(s) \varphi(s) ds) (13)
\]

\[
\frac{\partial^2 \varphi}{\partial t^2} + \lambda_3^2 \varphi + \lambda_2^2 x = \\
= \mu \varphi_3(x, y, \varphi, y^2, \varphi^2, xy, \int_{-\infty}^{t} R_{5\varphi}^c(t - s) \varphi(s) ds, \\
\int_{-\infty}^{t} R_{50x}^c(t - s) x(s) ds, \int_{-\infty}^{t} R_{52y}^c(t - s) y^2(s) ds, \\
\int_{-\infty}^{t} R_{33\varphi}^c(t - s) \varphi^2(s) ds, \int_{-\infty}^{t} R_{31xy}^c(t - s) x(s)y(s) ds, \\
\int_{-\infty}^{t} R_{32x\varphi}^c(t - s) x(s) \varphi(s) ds, \int_{-\infty}^{t} R_{34y\varphi}^c(t - s) y(s) \varphi(s) ds, M(x\dot{y} + y\dot{x})) (15)
\]

3. Results
To transform system (13) to normal form, we make the change of variables

\[
x = q_\xi + q_\zeta, y = q_\eta, \varphi = \alpha_1 q_\xi + \alpha_2 q_\zeta,
\]

where \( q_\xi, q_\eta, q_\zeta \) - normal coordinates. System (13) in normal coordinates will have the form:

\[
\dot{q}_\xi + \Omega_1^2 q_\xi = -\mu \phi_1^*, \\
\dot{q}_\eta + \Omega_2^2 q_\eta = -\mu \phi_2^* + p'e^{-i\omega t}, \\
\dot{q}_\zeta + \Omega_3^2 q_\zeta = \mu \phi_3^*,
\]

where \( \phi_1^*, \phi_2^*, \phi_3^* \) - linear combinations of functions \( \phi_1, \phi_2, \phi_3 \), and in the latter, replacements are made according to formulas (14). In expanded form, they have the form:

\[
\phi_1^* = \frac{1}{1 + \alpha_1^2} \left( a_{11} \int_{-\infty}^{t} R_{10\xi}^c(t - s) q_\xi(s) ds + a_{12} \int_{-\infty}^{t} R_{10\zeta}^c(t - s) q_\zeta(s) ds + \right.
\]

\[
\left. a_{13} \int_{-\infty}^{t} R_{10\eta}^c(t - s) q_\eta(s) ds \right)
\]

\[
\phi_2^* = \frac{1}{1 + \alpha_2^2} \left( a_{21} \int_{-\infty}^{t} R_{20\xi}^c(t - s) q_\xi(s) ds + a_{22} \int_{-\infty}^{t} R_{20\zeta}^c(t - s) q_\zeta(s) ds + \right.
\]

\[
\left. a_{23} \int_{-\infty}^{t} R_{20\eta}^c(t - s) q_\eta(s) ds \right)
\]

\[
\phi_3^* = \frac{1}{1 + \alpha_3^2} \left( a_{31} \int_{-\infty}^{t} R_{30\xi}^c(t - s) q_\xi(s) ds + a_{32} \int_{-\infty}^{t} R_{30\zeta}^c(t - s) q_\zeta(s) ds + \right.
\]

\[
\left. a_{33} \int_{-\infty}^{t} R_{30\eta}^c(t - s) q_\eta(s) ds \right)
\]
\[\begin{align*}
+ a_{14}(q_\eta^2 - \int_{-\infty}^{t} R_{14q_\eta}^c(t-s) q_\eta^2(s)ds) & + a_{15}(q_\zeta^2 - \int_{-\infty}^{t} R_{15q_\zeta}^c(t-s) q_\zeta^2(s)ds) + \\
+ a_{16}(q_\xi q_\eta - \int_{-\infty}^{t} R_{16q\xi q_\eta}^c(t-s) q_\xi(s) q_\eta(s)ds) & + a_{13}(q_\zeta^2 - \int_{-\infty}^{t} R_{13q_\zeta}^c(t-s) q_\zeta^2(s)ds) + \\
+ a_{17}(q_\xi q_\zeta - \int_{-\infty}^{t} R_{17q\xi q_\zeta}^c(t-s) q_\xi(s) q_\zeta(s)ds) & + a_{18}(q_\eta q_\xi - \int_{-\infty}^{t} R_{18q_\eta q_\xi}^c(t-s) q_\eta(s) q_\zeta(s)ds) + \\
+ a_1 \epsilon_1[(q_\xi + q_\zeta)(q_\xi + q_\zeta)] & \],
\end{align*}\]

\[\begin{align*}
\phi_2^* = a_{21} \int_{-\infty}^{t} R_{\xi\eta}^c(t-s) q_\eta(s)ds & + a_{22}(q_\xi^2 - \int_{-\infty}^{t} R_{22q_\xi}^c(t-s) q_\xi^2(s)ds) + \\
+ a_{23}(q_\zeta^2 - \int_{-\infty}^{t} R_{23q_\zeta}^c(t-s) q_\zeta^2(s)ds) & + a_{24}(q_\xi q_\eta - \int_{-\infty}^{t} R_{24q\xi q_\eta}^c(t-s) q_\xi(s) q_\eta(s)ds) + \\
+ a_{25}(q_\xi q_\zeta - \int_{-\infty}^{t} R_{25q\xi q_\zeta}^c(t-s) q_\xi(s) q_\zeta(s)ds) & + a_{26}(q_\eta q_\zeta - \int_{-\infty}^{t} R_{26q_\eta q_\zeta}^c(t-s) q_\eta(s) q_\zeta(s)ds) + \\
\phi_3^* = \frac{1}{1 + \alpha_2^2} & [a_{31} \int_{-\infty}^{t} R_{31q_\eta}^c(t-s) q_\eta(s)ds + a_{32} \int_{-\infty}^{t} R_{32q_\zeta}^c(t-s) q_\zeta(s)ds] + \\
+ a_{33}(q_\xi^2 - \int_{-\infty}^{t} R_{33q_\xi}^c(t-s) q_\xi^2(s)ds) & + a_{35}(q_\zeta^2 - \int_{-\infty}^{t} R_{35q_\zeta}^c(t-s) q_\zeta^2(s)ds) + \\
+ a_{34}(q_\xi q_\eta - \int_{-\infty}^{t} R_{34q\xi q_\eta}^c(t-s) q_\xi(s) q_\eta(s)ds) & + a_{36}(q_\eta q_\zeta - \int_{-\infty}^{t} R_{36q_\eta q_\zeta}^c(t-s) q_\eta(s) q_\zeta(s)ds) + \\
+ a_2[(q_\xi + q_\zeta)(\eta_\xi) - q_\eta(\eta_\xi + \eta_\zeta)] & ,
\end{align*}\]

where

\[\begin{align*}
a_{11} = h_1 + h_3 a_1^2, a_{12} = h_1 + h_3 a_1 a_2, \\
a_{13} = a_1 e_1, a_{14} = a_3 + a_1 e_3, \\
a_{15} = a_2 [a_2 e_1 + a_1 (e_3 a_2 + e_3)], \\
a_{17} = 2 a_1 [a_2 e_1 + a_1 (e_3 a_2 + e_3)], \\
a_{18} = a_1 e_1 + a_1 e_3 a_2 + e_3 (a_1 + a_2), \\
a_{21} = h_2, \\
a_{22} = e_2 a_1 + a_1 (e_2 a_2 + e_3), \\
a_{23} = a_1 e_1 + a_1 e_2 a_2 + e_2 (a_1 + a_2), \\
a_{24} = e_2 a_1 + e_2 a_1 a_2, \\
a_{25} = e_2 a_1 + e_2 a_1 a_2, \\
a_{31} = a_1 e_1 + a_2 (e_3 a_1 + e_3), \\
a_{34} = a_1 e_2 + e_3 a_2, \\
a_{35} = a_1 e_3 a_2 + e_3, \\
a_{36} = a_1 e_1 + e_2 a_1 + a_2 (e_3 a_1 + e_3), \\
a_{37} = a_1 [2 a_1 e_1 + a_2 (e_3 a_2 + e_3)], \\
a_{38} = a_1 e_2 + a_2 (e_3 a_2 + e_3).
\end{align*}\]

Solutions (11), in normal coordinates, correspond to

\[q_\xi = q_\zeta = 0, q_\eta = q_1 e^{-i\omega t}, q_1 = \frac{i}{\alpha_2^2 - \omega^2}. \tag{16}\]
Note that in system (4) the coefficient is nonzero for any real values of the shaker parameters. This leads to the fact that in system (4) it is impossible to consider the case of simple indirect excitation (according to the terminology of [18]), but to study the complete system equations (4).

It is seen directly from (15) that for \( \omega = \Omega_j \) \((j = 1, 2, 3)\) in the system under study, the main resonance arises. Because the \( x \) and \( \varphi \) are linear combinations of normal coordinates \( q \xi \) and \( q \zeta \), then we will investigate the behavior of the latter. Due to the nonlinearity of the equations of motion, resonances are possible at combination frequencies. Therefore, we will consider the possibilities of indirect excitation of oscillations in the presence of resonance relations between the frequencies \( \omega, \Omega_1 \) and \( \Omega_3 \).

First, as in [18,19], consider the case when \( \Omega_2 \) satisfies a resonance relation of the form

\[
\Omega_2^2 = \frac{\omega^2}{4} + \mu \omega \varepsilon_1.
\]  

Frequencies \( \Omega_2, \Omega_3 \) non-resonant, \( \varepsilon_1 = \Omega_1 - \frac{\omega}{2} \). Taking into account (17), equations (15) take the form

\[
\ddot{\xi} + \frac{\omega^2}{4} \xi = -\mu (\dot{\phi}_1^* + \omega \varepsilon_1 \xi),
\]
\[
\ddot{\eta} + \Omega_2^2 \eta = -\mu \dot{\phi}_2^* + p' e^{-i\omega t},
\]
\[
\ddot{\zeta} + \Omega_3^2 \zeta = -\mu \dot{\phi}_3^*.
\]

To determine the conditions for the excitation of indirect oscillations, we use the averaging method, as is customary in [20]. The general solution of system (18) in complex form at \( \mu = 0 \) looks like:

\[
q_{\xi_0} = P_1 e^{\frac{i \omega t}{2}} + Q_1 e^{-\frac{i \omega t}{2}},
\]
\[
q_{\eta_0} = P_2 e^{i\Omega_2 t} + Q_1 e^{-i\Omega_2 t},
\]
\[
q_{\zeta_0} = P_3 e^{i\Omega_3 t} + Q_3 e^{-i\Omega_3 t} + \frac{Q_1}{2i} (e^{i\omega t} - e^{-i\omega t}),
\]

where

\[
P_j = \frac{M_j - iN_j}{2}, Q_j = \frac{M_j + iN_j}{2}, \ (j = 1, 2, 3), q_1 = \frac{i}{(\Omega_3^2 - \omega^2)} P_1, P_2, P_3, Q_1, Q_2, Q_3 \] - arbitrary constants. To the right-hand sides of equations (18) we apply the averaging procedure. Then we obtain the following equations of the first approximation:
System (20) admits the trivial solution

\[ P_1 = Q_1 = P_2 = Q_2 = P_3 = Q_3 = 0. \] (21)

If solution (21) turns out to be unstable, which means that solution (16) is also unstable, then it is possible to excite oscillations along coordinates free from the action of external forces. Otherwise, the stability of the solution to equations (16) is determined by the nature of the roots of the fundamental equation of the system of linear equations (20). Solutions

\[ P_2 = Q_2 = P_3 = Q_3 = 0 \]

stable, because

\[ a_{21} > 0, \quad a_{32} > 0, \]

means that the solutions

\[ q_\xi = p'e^{-iat}, \quad q_\zeta = 0. \]

Obviously, the stability of the solution \( q_\zeta = 0 \) is determined from the first two equations of system (20). To obtain the conditions for the stability of the solution \( q_\xi = 0 \), put

\[ P_1 = R_1 e^{i\omega t}, \quad Q_1 = R_2 e^{-i\omega t}, \]

and use the condition for the existence of a nontrivial solution with respect to the quantities \( R_1, R_2 \), what gives:

\[
\begin{align*}
\omega \varepsilon_1 + [1 - \Gamma_{11}^c(\omega R) - i\Gamma_{11}^s(\omega R)] \left[ a_{11} \frac{1}{2\left(1 + a_1^2\right)} + a^* \right] \frac{a_{11}[1 - \Gamma_{11}^c(\omega R) - i\Gamma_{11}^s(\omega R)] q_1}{2i} \omega \varepsilon_1 - [1 - \Gamma_{11}^c(\omega R) - i\Gamma_{11}^s(\omega R)] \left[ a_{11} \frac{1}{2\left(1 + a_1^2\right)} + a^* \right] & = 0 \quad (22)
\end{align*}
\]

or

\[
\begin{align*}
\alpha^* & + \frac{a_{11}}{1 + a_1^2} [1 - \Gamma_{11}^c(\omega R) - i\Gamma_{11}^s(\omega R)] \alpha^* + e_1^2 + \\
& + \frac{a_{11}^2}{4(1 + a_1^2)} [1 - \Gamma_{11}^c(\omega R) - i\Gamma_{11}^s(\omega R)] q_1 & = 0. \quad (23)
\end{align*}
\]

Based on the Routh-Hurwitz criterion, we obtain from (23) the necessary and sufficient conditions for the stability of the solution \( q_\xi = 0 \):
The first of the conditions (24) is always fulfilled in our formulation of the problem, and the second, for certain ratios of the parameters, may not be satisfied. Then towards $q_\xi$ vibrations with frequency are excited $\frac{\omega}{2}$, those fluctuations in coordinates $q_\xi$ and $q_\zeta$ find themselves bound. It can be seen from conditions (24) that increasing the resistance coefficients $h_1$, $h_3$ detuning $\omega \varepsilon_1$ and decreasing the value of expressions $a_{17}q_1$ can satisfy the stability conditions.

Thus, to eliminate the excitation of oscillations along the coordinate $\xi$ it is necessary to select the parameters of the system from conditions (24). Coefficients of friction $h_1$, $h_3$ and detuning $\omega \varepsilon_1$ always contribute to stability, and the amplitude $q_1$. All the results obtained apply to the analysis of the behavior of the coordinate $q_\zeta$ when the resonance relation is fulfilled $\Omega_3^2 = \frac{\omega^2}{4} + \mu \omega \varepsilon_3$. Here $\Omega_1, \Omega_2$- non-resonant frequencies, $\varepsilon_3 = \Omega_3 - \frac{\omega}{2}$.

The stability conditions for the solution $q_\zeta = 0$ there will be inequalities

$$\begin{align*}
\frac{a_{31}}{2} [1 - \Gamma_2^c(\omega_R) - i \Gamma_3^c(\omega_R)] & > 0 \\
\frac{a_{31}}{2} [1 - \Gamma_2^c(\omega_R) - i \Gamma_3^c(\omega_R)] & > \frac{1 + \alpha_2^2}{\omega} \left[ a_{37} [1 - \Gamma_3^c(\omega_R) - i \Gamma_3^c(\omega_R)] q_1 \right]^2 - (\omega \varepsilon_3)^2.
\end{align*}$$

Now let us briefly consider the cases when the resonance relation of the form (17) is simultaneously satisfied by two natural frequencies $\Omega_1$ and $\Omega_3$: $\Omega_2$- not resonant frequency. In this case, it is possible to excite oscillations simultaneously in the directions of two coordinates: $q_\xi$ and $q_\zeta$. This is a case of stronger connectedness of vibrations. Proceeding in a similar way to the cases considered, we obtain the following fourth-degree equation for $a^n$ the nature of the roots of which is determined by the excitation of oscillations along the coordinates $q_\xi$ and $q_\zeta$ at the same time

$$\begin{vmatrix}
\omega \varepsilon_1 + R_{11} \left[ \frac{a_{11}}{2(1 + \alpha_2^2)} + \alpha^* \right] & a_{17} R_{17} p' & R_{11} a_{12} & a_{15} R_{15} p' \\
-a_{17} R_{17} p' & 2i(1 + \alpha_2^2) & 2(1 + \alpha_2^2) i(1 + \alpha_2^2) & -a_{15} R_{15} p' \\
R_{31} a_{31} & a_{37} R_{37} p' & 2(1 + \alpha_2^2) \omega \varepsilon_3 + R_{32} & a_{32} \\
-R_{37} p' a_{37} & R_{31} a_{31} & -R_{35} p' a_{35} & 2(1 + \alpha_2^2) + \alpha^*
\end{vmatrix} = 0$$

here $R_{jk} = 1 - \Gamma_j^k(\omega_R) - i \Gamma_j^k(\omega_R)(j,k = 1,2,3)$. Expanding the determinant and using the Routh-Hurwitz criterion, one can obtain the conditions for the excitation of oscillations along the coordinates $q_\xi$ and $q_\zeta$ simultaneously. However, given the complexity of the resulting conditions, we will not list them here.
The results obtained were compared with the results obtained in [21, 22], which gave an identical result. The results of the work can be applied in the development of energy saving schemes and increase in reliability [23,24], as well as in the development of a new design for drying cotton seeds using heat [25,26].

4. Conclusions
Nonlinear integro-differential equations of motion are compiled in expansions into trigonometric functions and power series, while only infinitesimal up to the second order are retained. The reduction to normal coordinates was used by the averaging method. Conditions for indirect excitation of axial parametric resonance of lateral vibrations (along the horizontal axis, torsional vibrations about the horizontal axis) under the action of forces along the vertical are obtained.

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