DOES MAGNETIC-FIELD–ROTATION MISALIGNMENT SOLVE THE MAGNETIC BRAKING CATASTROPHE IN PROTOSTELLAR DISK FORMATION?

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ABSTRACT
Stars form in dense cores of molecular clouds that are observed to be significantly magnetized. In the simplest case of a laminar (non-turbulent) core with the magnetic field aligned with the rotation axis, both analytic considerations and numerical simulations have shown that the formation of a large, $10^2$ AU scale, rotationally supported protostellar disk is suppressed by magnetic braking in the ideal MHD limit. This theoretical difficulty in forming protostellar disks is termed the “magnetic braking catastrophe.” A possible resolution to this problem, proposed by Hennebelle & Ciardi and Joos et al., is that misalignment between the magnetic field and rotation axis may weaken the magnetic braking enough to enable disk formation. We evaluate this possibility quantitatively through numerical simulations. We confirm the basic result of Joos et al. that the misalignment is indeed conducive to disk formation. In relatively weakly magnetized cores with dimensionless mass-to-flux ratio $\lambda \gtrsim 4$, it enabled the formation of rotationally supported disks that would otherwise be suppressed if the magnetic field and rotation axis are aligned. For more strongly magnetized cores, disk formation remains suppressed, however, even for the maximum tilt angle of $90^\circ$. If dense cores are as strongly magnetized as indicated by OH Zeeman observations (with a mean dimensionless mass-to-flux ratio $\sim 2$), it would be difficult for the misalignment alone to enable disk formation in the majority of them. We conclude that, while beneficial to disk formation, especially for the relatively weak field case, misalignment does not completely solve the problem of catastrophic magnetic braking in general.

Key words: accretion, accretion disks – ISM: clouds – ISM: magnetic fields – magnetohydrodynamics (MHD)

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1. INTRODUCTION
Star and planet formation are connected through disks. Disk formation, long thought to be a trivial consequence of angular momentum conservation during core collapse and star formation (e.g., Bodenheimer 1995), turned out to be much more complicated than originally envisioned. The complication comes from magnetic fields, which are observed in dense, star-forming cores of molecular clouds (see Crutcher 2012 for a recent review). The field can strongly affect the angular momentum evolution of core collapse and disk formation through magnetic braking.

There have been a number of studies aimed at quantifying the effects of magnetic field on disk formation. In the ideal MHD limit, both analytic considerations and numerical simulations have shown that the formation of a rotationally supported disk (RSD hereafter) is suppressed by a realistic magnetic field (corresponding to a dimensionless mass-to-flux ratio of $\lambda \sim$ a few; Troland & Crutcher 2008) during the protostellar mass accretion phase in the simplest case of a non-turbulent core with the magnetic field aligned with the rotation axis (Allen et al. 2003; Galli et al. 2006; Price & Bate 2007; Mellon & Li 2008; Hennebelle & Fromang 2008; Dapp & Basu 2010; Seifried et al. 2011; Santos-Lima et al. 2012). The suppression of RSDs by excessive magnetic braking is termed “magnetic braking catastrophe” in star formation.

RSDs are routinely observed, however, around evolved Class II young stellar objects (YSOs; see Williams & Cieza 2011 for a review), and increasingly around Class I sources (e.g., Jørgensen et al. 2009; Lee 2011; Takakuwa et al. 2012) and even one Class 0 source (Tobin et al. 2012). When and how such disks form in view of magnetic braking catastrophe is unclear. The current attempts to overcome catastrophic braking fall into three categories: (1) non-ideal MHD effects, including ambipolar diffusion, ohmic dissipation and Hall effect, (2) misalignment between magnetic and rotation axes, and (3) turbulence. Ambipolar diffusion does not appear to weaken the braking enough to enable large-scale RSD formation under realistic conditions (Krasnopolsky & Königl 2002; Mellon & Li 2009; Duffin & Pudritz 2009; Li et al. 2011). Ohmic dissipation can produce small, AU scale, RSD in the early protostellar accretion phase (Machida et al. 2011; Dapp & Basu 2010; Dapp et al. 2012; Tomida et al. 2013). Larger, $10^2$ AU scale RSDs can be produced if the resistivity or the Hall coefficient of the dense core is much larger than the classical (microscopic) value (Krasnopolsky et al. 2010, 2011; see also Braiding & Wardle 2012a, 2012b). Joos et al. (2012) explored the effects of tilting the magnetic field away from the rotation axis on disk formation (see also Machida et al. 2006; Price & Bate 2007; Hennebelle & Ciardi 2009). They concluded that Keplerian disks can form for a mass-to-flux ratio $\lambda$ as low as 3, as long as the tilt angle is close to $90^\circ$ (see their Figure 14). The effects of turbulence were explored by Santos-Lima et al. (2012, 2013), who concluded that a strong enough turbulence can induce enough magnetic diffusion to enable the formation of a $10^2$ AU scale RSD. Seifried et al. (2012) and Myers et al. (2013) considered supersonically turbulent massive cores. They found rotationally dominated disks around low-mass stars, although in both cases the turbulence-induced rotation is misaligned with the initial magnetic field by a large angle, which
may have contributed to the disk formation (see also Joos et al. 2013).

The goal of this paper is to revisit the role of magnetic-field–rotation misalignment in disk formation. The misalignment is expected if the angular momenta of dense cores are generated through turbulent motions (e.g., Burkert & Bodenheimer 2000; Myers et al. 2013). It is also inferred from the misalignment between the field direction traced by polarized dust emission and the outflow axis, which is taken as a proxy for the direction of rotation (Hull et al. 2013). Indeed, in the CARMA sample of Hull et al., the distribution of the angle $\theta_0$ between the magnetic field and jet/rotation axis is consistent with being random. If true, it would indicate that in half of the sources the two axes are misaligned by a large angle of $\theta_0 > 60^\circ$ (see however Chapman et al. 2013 and Davidson et al. 2011; and the discussion in Section 5.2). Such a large misalignment would be enough to allow disk formation in dense cores magnetized to a realistic level (with $\lambda$ of a few; Troland & Crutcher 2008) according to Joos et al. (2012). If the alignment angle $\theta_0$ is indeed random and Joos et al.’s conclusions are generally true, the magnetic braking catastrophe would be largely solved. Given their far-reaching implications, it is prudent to check Joos et al.’s conclusions using a different numerical code. That is the task of this paper.

We carry out numerical experiments of disk formation in dense cores with misaligned magnetic and rotation axes using non-ideal MHD code Zeus-TW that includes self-gravity. We find that a large misalignment angle does indeed enable the formation of RSDs in weakly magnetized dense cores with dimensionless mass-to-flux ratios $\gtrsim 4$, but not in dense cores magnetized to higher, more typical levels. Our conclusion is that while the misalignment helps with disk formation, especially in relatively weakly magnetized cores, it may not provide a complete resolution to the magnetic braking catastrophe by itself.

The rest of the paper is organized as follows. In Section 2, we describe the model setup. The numerical results are described in Sections 3 and 4. We compare our results to those of Joos et al. and discuss their implications in Section 5 and conclude with a short summary in Section 5.3.

2. PROBLEM SETUP

We follow Li et al. (2011) and Krasnopolsky et al. (2012) and start our simulations from a uniform, spherical core of $1 \ M_\odot$ and radius $10^{17}$ cm in a spherical coordinate system $(r, \theta, \phi)$. The initial density $\rho_0 = 4.77 \times 10^{-19}$ g cm$^{-3}$ corresponds to a molecular hydrogen number density of $10^5$ cm$^{-3}$. We adopt an isothermal equation of state with a sound speed $a = 0.2$ km s$^{-1}$ below a critical density $\rho_c = 10^{-11}$ g cm$^{-3}$, and a polytropic equation of state $p \propto \rho^{5/3}$ above it. At the beginning of the simulation, we impose a solid-body rotation of angular speed $\Omega_0 = 10^{-13}$ s$^{-1}$ on the core, with axis along the north pole ($\theta = 0$). It corresponds to a ratio of rotational to gravitational binding energy of 0.025, which is typical of the values inferred for NH$_3$ cores (Goodman et al. 1993). The initial magnetic field is uniform, tilting away from the rotation axis by an angle $\theta_0$. We consider three values for the initial field: $B_0 = 10.6, 21.3$ and $35.4 \mu G$, corresponding to dimensionless mass-to-flux ratio, in units of $(2 \pi G^{1/2})^{-1}$, $\lambda = 9.72, 4.86$ and 2.92, respectively, for the core as a whole. The mass-to-flux ratio for the central flux tube $\lambda_c$ is higher than the global value $\lambda$ by 50%, so that $\lambda_c = 14.6, 7.29$ and 4.38 for the three cases respectively. The effective mass-to-flux ratio $\lambda_{\text{eff}}$ should lie between these two limits. If the star formation efficiency per core is $\sim 1/3$ (e.g., Alves et al. 2007), then one way to estimate $\lambda_{\text{eff}}$ is to consider the (cylindrical) magnetic flux surface that encloses $1/3$ of the core mass, which yields $\lambda_{\text{eff}} = 1.41\lambda$, corresponding to 13.7, 6.85, and 4.12 for the three cases respectively; the fraction $1/3$ is also not far from the typical fraction of core mass that has accreted onto the central object at the end of our simulations (see Table 1). For the tilt angle, we also consider three values: $\theta_0 = 0^\circ, 45^\circ$, and $90^\circ$. The $\theta_0 = 0^\circ$ case, with the magnetic field and rotation axis both along the $z$-axis ($\theta = 0$). The $\theta_0 = 90^\circ$ case corresponds to the orthogonal case, with the magnetic field along the $x$-axis ($\theta = 90^\circ, \phi = 0$). Models with these nine combinations of parameters are listed in Table 1; additional models are discussed below.

As in Krasnopolsky et al. (2012), we choose a non-uniform grid of $96 \times 64 \times 60$. In the radial direction, the inner and outer boundaries are located at $r = 10^{15}$ and $10^{18}$ cm, respectively. The radial cell size is smallest near the inner boundary ($5 \times 10^{12}$ cm or $\sim 0.3$ AU). It increases outward by a constant factor $\sim 1.08$ between adjacent cells. In the polar direction, we choose a relatively large cell size ($7:5$) near the polar axes, to prevent the azimuthal cell size from becoming prohibitively small; it decreases smoothly to a minimum of $\sim 0:63$ near the equator, where RSDs may form. The grid is uniform in the azimuthal direction.

The boundary conditions in the azimuthal direction are periodic. In the radial direction, we impose the standard outflow boundary conditions. Material leaving the inner radial boundary is collected as a point mass (protostar) at the center. It acts on the matter in the computational domain through gravity. On the polar axes, the boundary condition is chosen to be reflective. Although this is not strictly valid, we expect its effect to be limited to a small region near the axis.

| Model | $\lambda^a$ | $\lambda_{\text{eff}}^b$ | $\theta_0$ | $\eta$ $(10^{17}$ cm$^2$ s$^{-1})$ | $M_*^c$ $(M_\odot)$ | RSD$^d$ |
|-------|-------------|----------------|-----------|----------------|------------------|-------|
| A     | 9.72        | 13.7           | 0$^\circ$ | 1              | 0.24             | No    |
| B     | 4.86        | 6.85           | 0$^\circ$ | 1              | 0.22             | No    |
| C     | 2.92        | 4.12           | 0$^\circ$ | 1              | 0.33             | No    |
| D     | 9.72        | 13.7           | 45$^\circ$| 1              | 0.21             | Yes/porous |
| E     | 4.86        | 6.85           | 45$^\circ$| 1              | 0.35             | No    |
| F     | 2.92        | 4.12           | 45$^\circ$| 1              | 0.27             | No    |
| G     | 9.72        | 13.7           | 90$^\circ$| 1              | 0.38             | Yes/robust |
| H     | 4.86        | 6.85           | 90$^\circ$| 1              | 0.46             | Yes/porous |
| I     | 2.92        | 4.12           | 90$^\circ$| 1              | 0.47             | No    |
| M     | 9.72        | 13.7           | 90$^\circ$| 0              | 0.10             | Yes/robust |
| N     | 9.72        | 13.7           | 90$^\circ$| 0.1            | 0.26             | Yes/robust |
| P     | 4.03        | 6.85           | 90$^\circ$| 1              | 0.32             | Yes/porous |
| Q     | 3.44        | 4.85           | 90$^\circ$| 1              | 0.22             | No    |
| X     | 4.00        | 4.00           | 90$^\circ$| 1              | 0.31             | No    |
| Y     | 5.00        | 5.00           | 90$^\circ$| 1              | 0.33             | Yes/porous |
| Z     | 6.00        | 6.00           | 90$^\circ$| 1              | 0.30             | Yes/porous |

Notes:  

$^a$ The average dimensionless mass-to-flux ratio for the core as a whole.  

$^b$ The effective mass-to-flux ratio for the central 1/3 of the core mass (see Section 2 for discussion).  

$^c$ Mass of the central object when the simulation is stopped.  

$^d$ “Robust” disks are persistent, rotationally supported structures that rarely display large deviations from smooth Keplerian motions, whereas “porous” disks are highly active, rotationally dominated structures with large distortions and may occasionally be completely disrupted.

Table 1: Models
We initially intended to carry out simulations in the ideal MHD limit, so that they can be compared more directly with other work, especially Joos et al. (2012). However, ideal MHD simulations tend to produce numerical “hot zones” that force the calculation to stop early in the protostellar mass accretion phase, a tendency we noted in our previous two-dimensional (Mellon & Li 2008) and three-dimensional (3D) simulations (Krasnopolsky et al. 2012). To lengthen the simulation, we include a small, spatially uniform resistivity $\eta = 10^{17} \text{cm}^2 \text{s}^{-1}$. We have verified that, in the particular case of Model G ($\lambda = 9.72$ and $\theta_0 = 90^\circ$), this resistivity changes the flow structure little compared to either the ideal MHD Model M (before the latter stops) or Model N, where the resistivity is reduced by a factor of 10, to $10^{16} \text{cm}^2 \text{s}^{-1}$.

3. WEAK-FIELD CASE: DISK FORMATION ENABLED BY FIELD–ROTATION MISALIGNMENT

3.1. Equatorial Pseudodisk versus Magnetically Induced Curtain

To illustrate the effect of the misalignment between the magnetic field direction and rotation axis, we first consider an extreme case where the magnetic field is rather weak (with a mass-to-flux ratio $\lambda = 9.72$ for the core as a whole and $\lambda_{\text{eff}} = 13.7$ for the inner 1/3 of the core mass). In this case, a well-formed RSD is present in the orthogonal case with $\theta_0 = 90^\circ$ (Model G in Table 1). Such a disk is absent in the aligned case (with $\theta_0 = 0^\circ$, Model A). The contrast is illustrated in Figure 1, where we plot snapshots of the aligned and orthogonal cases at a representative time $t = 3.9 \times 10^{12} \text{s}$, when a central mass of 0.11 and 0.12 $M_\odot$, respectively, has formed. The flow structures in the two cases are very different in both the equatorial (panels (a) and (b)) and meridional (panels (c) and (d)) planes. In the equatorial plane, the aligned case has a relatively large (with radius $\sim 10^{16} \text{cm}$) over-dense region where material spirals rapidly inward. On the (smaller) scale of $10^{15} \text{cm}$, the structure is dominated by expanding, low-density lobes; they are the decoupling enabled magnetic structures (DEMS for short) that have been studied in detail by Zhao et al. (2011) and Krasnopolsky et al. (2012). No RSD is evident. The equatorial structure on the $10^{16} \text{cm}$ scale in the orthogonal case is dominated by a pair of spirals instead. The spirals merge, on the $10^{15} \text{cm}$ scale, into a more or less continuous, rapidly rotating structure—an RSD. Clearly, the accretion flow in the orthogonal case was able to retain more angular momentum than in the aligned case. Why is this the case?

A clue comes from the meridian view of the two cases (panels (c) and (d) of Figure 1). In the aligned case, there is a strong bipolar outflow extending beyond $3 \times 10^{16} \text{cm}$ at the relatively early time shown. The outflow forces most of the infalling material to accrete through a flattened equatorial structure—an over-dense pseudodisk (Galli & Shu 1993; see panel (a) for a face-on view of the pseudodisk, noting the difference in scale between panels (c) and (a)). It is the winding of the magnetic field lines by the rotating material in the pseudodisk that drives the bipolar outflow in the first place. The wound-up field lines act back on the pseudodisk material, braking its rotation. The efficient magnetic braking in the pseudodisk plays a key role in suppressing the formation of an RSD in the aligned case.

The prominent bipolar outflow indicative of efficient magnetic braking is absent in the orthogonal case, as was emphasized by Ciardi & Hennebelle (2010). It is replaced by a much smaller, shell-like structure inside which the $10^{15} \text{cm}$ scale RSD is encased (panel (d)). To understand this difference in flow structure pictorially, we plot in Figure 2 the 3D structure of the magnetic field lines on the scale of 1000 AU (or $1.5 \times 10^{16} \text{cm}$), which is 50% larger than the size of panels (a) and (b) of Figure 1, but half of that of panels (c) and (d). Clearly, in the aligned case, the relatively weak initial magnetic field (corresponding to $\lambda \approx 10$) has been wound up many turns by the material in the equatorial pseudodisk, building up a magnetic pressure in the equatorial region that is released along the polar directions (see the top panel of Figure 2). The magnetic pressure gradient drives a bipolar outflow, which is evident in panel (c) and in many previous simulations of magnetized core collapse, including the early ones such as Tomisaka (1998) and Allen et al. (2003). In contrast, in the orthogonal case, the equatorial region is no longer the region of the magnetically induced pseudodisk. In the absence of rotation (along the $z$-axis), the dense core material would preferentially contract along the field lines (that are initially along the $x$-axis) to form a dense sheet in the $y$–$z$ plane that passes through the origin. The twisting of this sheet by rotation along the $z$-axis produces two curved “curtains” that spiral smoothly into the disk at small distances, somewhat analogous in shape to two snail shells (see the bottom panel of Figure 2).

The snail-shaped dense curtain in the orthogonal case naturally explains the morphology of the density maps shown in panels (b) and (d) of Figure 1. First, the two prominent spiral arms in panel (a) are simply the equatorial ($x$–$y$) cut of the curved curtains. An interesting feature of the spirals (and the snail-shaped dense curtain as a whole) is that they are the region where the magnetic field lines change directions sharply. This is illustrated in Figure 3, which is similar to panel (b) of Figure 1, except that the magnetic vectors (rather than velocity vectors) are plotted on top of the density map. Clearly, the spirals separate the field lines rotating counter clockwise (lower-right part of the figure) from those rotating clockwise (upper left). The sharp kink is analogous to the well-known field line kink across the equatorial pseudodisk in the aligned case, where the radial component of the magnetic field changes direction. It supports our interpretation of the spirals and, by extension, the curtain as a magnetically induced feature, as is the case of pseudodisk. In other words, the spirals are not produced by gravitational instability in a rotationally supported structure; they are “pseudospirals” in the same sense as the “pseudodisks” of Galli & Shu (1993). The field line kinks are also evident across the dense curtain in the 3D structure shown in the bottom panel of Figure 2.

The 3D topology of the magnetic field and the dense structures that it induces lie at the heart of the difference in the magnetic braking efficiency between the aligned and orthogonal case. In particular, a flattened, rotating, equatorial pseudodisk threaded by an ordered magnetic field with an appreciable vertical component (along the rotation axis) is more conducive to driving an outflow than a warped curtain with a magnetic field predominantly tangential to its surface. Together with the magnetic torque, the outflow plays a key role in angular momentum removal and the suppression of RSDs, as we demonstrate next.

3.2. Torque Analysis

To quantify the outflow effect, we follow Zhao & Li (2013; see also Joos et al. 2012) and compare the rates of angular momentum change inside a finite volume $V$ through its surface $S$ due to infall and outflow to that due to magnetic torque. The total magnetic torque relative to the origin (from which a radius
Figure 1. Snapshots of the logarithm of density (color map, in units of g cm$^{-3}$) and velocity field (white arrows) for the weakly magnetized core of $\lambda = 9.72$ at a representative time $t = 3.9 \times 10^{12}$ s. The left (right) panels are for the aligned (orthogonal) case, and the top (bottom) panels are for the equatorial $x$–$y$ (meridian $y$–$z$) plane. Note the powerful bipolar outflow in panel (c) driven by the pseudodisk in panel (a). The lack of powerful outflow in panel (d) is indicative of a weaker magnetic braking, which is consistent with the presence of a rotationally supported disk in panel (b). The length unit is in centimeters. The scale in (c) and (d) is three times larger than that in (a) and (b).

(A color version of this figure is available in the online journal.)

The vector $\mathbf{r}$ is defined by

$$\mathbf{r} = \frac{1}{4\pi} \int [\mathbf{r} \times ((\nabla \times \mathbf{B}) \times \mathbf{B})] \, dV,$$

(1)

where the integration is over the volume $V$. Typically, the magnetic torque comes mainly from the magnetic tension rather than pressure force. The dominant magnetic tension term can be simplified to a surface integral (Matsumoto & Tomisaka 2004)

$$\mathbf{N}_m = \frac{1}{4\pi} \int (\mathbf{r} \times \mathbf{B})(\mathbf{B} \cdot d\mathbf{S}),$$

(2)

over the surface $S$ of the volume. This volume-integrated magnetic torque is to be compared with the rate of angular momentum advected into the volume through fluid motion,

$$\mathbf{N}_a = -\int \rho(\mathbf{r} \times \mathbf{v})(\mathbf{v} \cdot d\mathbf{S}),$$

(3)

which will be referred to as the advective torque below.

Since the initial angular momentum of the dense core is along the $z$-axis, we will be mainly concerned with the $z$-component of the magnetic and advective torque which, for a spherical volume
Figure 2. 3D view of representative magnetic field lines and isodensity surfaces at $\rho = 10^{-16}$ (red) and $10^{-15}$ g cm$^{-3}$ (blue) for the aligned Model A and orthogonal Model G. For clarity, only field lines originated from the bottom $x$–$y$ and left $y$–$z$ plane are plotted, respectively. Note that the magnetically induced equatorial pseudodisk in the aligned case is warped by rotation into two snail-shaped curtains that spiral inward to form a disk (blue surface) in the orthogonal case. The length is in units of AU.

Figure 3. Same as panel (b) of Figure 1 but with magnetic unit vectors instead of velocity vectors plotted. Note that the field lines kink sharply in the spirals. The kinks demonstrate that the spirals (and the curtain that contains them) are the counterparts to the equatorial pseudodisk in the aligned case.

(1) The advective torque consists of two parts: the rates of angular momentum advected into and out of the sphere by infall and outflow respectively:

$$N_{\text{a}, z} = -\int \rho \sigma \nu_{\phi} \nu_{r} dS.$$  

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$$N_{\text{a}, z} = -\int \rho \sigma \nu_{\phi} \nu_{r} dS.$$  

and

$$N_{\text{a}, z} = -\int \rho \sigma \nu_{\phi} \nu_{r} dS.$$  

An example of the magnetic and advective torques is shown in Figure 4. The torques are evaluated on spherical surfaces of different radii, at the representative time $t = 3.9 \times 10^{12}$ s. For the aligned case, the net torque close to the central object is nearly zero up to a radius of $\sim 2 \times 10^{15}$ cm, indicating that the angular momentum advected inward is nearly completely removed by magnetic braking there. At larger distances, between $\sim 2 \times 10^{15}$ and $\sim 10^{16}$ cm, the net torque $N_{\text{a}, z} + N_{\text{m}, z}$ is negative, indicating that the angular momentum of the material inside a sphere of radius in this range decreases with time (the decrease is a transient; the time-integrated net torque stays positive; see Figure 5 below). This is in sharp contrast with the orthogonal case, where the net torque is positive in that radial range, with the angular momentum there increasing (rather than decreasing) with time.

One may think that the difference is mainly due to a significantly larger magnetic torque $N_{\text{m}, z}$ in the aligned case than in the orthogonal case. Although this is typically the case at early times, the magnetic torques in the two cases become comparable at later times (see the lowest solid lines in the left and right panels of Figure 4; an animation of the torques is available on request from the authors). A bigger difference comes from the total (or net)
Figure 4. Magnetic (black solid line, \(N_{\text{m}} \)) and total advective (magenta dash-dot-dot-dot, \(N_{a,z} \)) torque acting on spheres of different radii for the aligned (left panel) and orthogonal (right) case at the representative time \(t = 3.9 \times 10^{12} \) s. Also plotted are the net torque (red dash-dotted, \(N_{\text{m}} + N_{a,z} \)), and the contributions to the advective torque by infall (dotted, \(N_{\text{in}}^{a,z} \)) and outflow (dashed, \(N_{\text{out}}^{a,z} \)). Note that the net torque is negative between \(\sim 2 \times 10^{15} \) and \(\sim 10^{16} \) cm for the aligned case but positive for the orthogonal case, mainly because the outflow removes more angular momentum in the former than in the latter. The torques are plotted in units of \(10^{41} \) dyn cm. (A color version of this figure is available in the online journal.)

Figure 5. The angular momenta transported by magnetic torque (solid lines), infall (dotted), outflow (dashed) and all torques (dash-dotted) for the aligned (black, thick) and orthogonal (red, thin) case, in units of \(10^{51} \) g cm\(^2\) s\(^{-1}\). (A color version of this figure is available in the online journal.)

angular momentum \(N_{a,z} = N_{\text{in}}^{a,z} + N_{\text{out}}^{a,z} \) advected inward, which is substantially smaller in the aligned case than in the orthogonal case (see the uppermost solid lines in the two panels). The main reason for the difference is that a good fraction of the angular momentum advected inward by infall \(N_{\text{in}}^{a,z} \) is advected back out by outflow \(N_{\text{out}}^{a,z} \) in the former, but not the latter. This is helped by the fact that \(N_{\text{in}}^{a,z} \) is somewhat smaller in the aligned case to begin with (compare the dotted lines in the two panels), perhaps because more efficient angular momentum removal by a combination of magnetic torque and outflow has already reduced the angular momentum of the accreted material by a larger factor in the aligned case compared to the orthogonal case. The lack of appreciable outward advection of angular momentum by outflow, which is itself a product of field-winding and magnetic braking in the aligned case, appears to be a key reason for the orthogonal case to retain more angular momentum at small radii and form an RSD in this particular case of relatively weak magnetic field. To quantify the effects of magnetic torque, inflow, and outflow further, we plot in Figure 5 the angular momenta that they transport as a function of radius between the time \(t = 3.5 \times 10^{12} \) s when the central mass is still tiny and the representative time \(t = 3.9 \times 10^{12} \) s shown in Figure 4. The figure shows more clearly that the angular momenta removed by magnetic torque are not that different for the aligned and orthogonal cases, especially on scales less than \(\sim 10^{16} \) cm that are more directly tied to disk formation. The difference between the angular momenta removed by outflow is much more prominent. In the region where angular momentum removal by magnetic torque is most efficient (between \(\sim 10^{15} \) and \(10^{16} \) cm), the angular momentum removed by outflow is \(\sim 40\%–70\% \) of that by magnetic torque in the aligned case, but less than \(10\% \) in the orthogonal case. Another key difference lies in the angular momentum advected inward by the infalling material inside \(\sim 10^{16} \) cm, which is significantly lower in the aligned case than in the orthogonal case. As mentioned earlier, this difference comes about presumably because the combination of magnetic torque and outflow has already reduced the angular momentum of the infalling material by a larger factor in the aligned case. The fact that the inwardly advected angular momenta start to diverge around the location where angular momentum removal by outflow starts to become significant in one case (aligned) but not the other (orthogonal, \(\sim 10^{16} \) cm) indicates that the outflow is at least partially responsible for this difference.
The formation of an RSD can be seen most clearly in Figure 6, where we plot the infall and rotation speed, as well as specific angular momentum as a function of radius along four $(\pm x$ and $\pm y)$ directions in the equatorial plane. In the orthogonal case, the infall and rotation speeds display the two tell-tale signs of RSDs: (1) a slow, subsonic (although nonzero) infall speed much smaller than the free-fall value, and (2) a much faster rotation speed close to the Keplerian value inside a radius of $\sim 100$ AU. The absence of an RSD in the aligned case is just as obvious. It has a rotation speed well below the Keplerian value and an infall speed close to the free-fall value, especially at small radii up to $\sim 100$ AU. This corresponds to the region dominated by the low-density, strongly magnetized, expanding lobes (i.e., DEMS; see panel (a) of Figure 1) where the angular momentum is almost completely removed by a combination of magnetic torque and outflow (see the right panel in Figure 6). Also evident from the panel is that the specific angular momentum of the equatorial inflow drops significantly twice: near $\sim 10^{15}$ and $\sim 10^{16}$ cm respectively. The former corresponds to the DEMS-dominated region, and the latter the pseudodisk (see panel (a) of Figure 1). The relatively slow infall inside the pseudodisk allows more time for magnetic braking to remove angular momentum. It is the pseudodisk (and its associated outflow) working in tandem with the DEMS that suppresses the formation of an RSD in the aligned case. Interestingly, there is a bump near $\sim 10^{16}$ cm for the specific angular momentum of the orthogonal case, indicating that the angular momentum in the equatorial plane is transported radially outward along the spiraling field lines from small to large distances (see Figure 3).

The spiraling equatorial field lines in the orthogonal Model G have an interesting property: they consist of two strains of opposite polarity. As the strains get wound up more and more tightly by rotation at smaller and smaller radii, field lines of opposite polarity are pressed closer and closer together, creating a situation that is conducive to reconnection, either of physical or numerical origin (see the left panel of Figure 7). Model G contains a small but finite resistivity ($\eta = 10^{17}$ cm$^2$/s$^{-1}$). It does not appear to be responsible for the formation and survival of the Keplerian disk, because a similar disk is also formed at the same (relatively early) time for a smaller resistivity of $\eta = 10^{16}$ cm$^2$/s$^{-1}$ (Model N) and even without any explicit resistivity (Model M). Numerical resistivity may have played a role here, but it is difficult to quantify at the moment. In any case, the magnetic field on the Keplerian disk appears to be rather weak, as can be seen from the right panel of Figure 7, where the plasma $\beta$ is plotted along four $(\pm x$ and $\pm y)$ directions in the equatorial plane. On the Keplerian disk in the orthogonal Model G (inside $\sim 100$ AU), $\beta$ is of order $10^2$ or more, indicating that there is more matter accumulating in the disk than magnetic field, either because the matter slides along the field lines into the disk (increasing density but not the field strength) or because of numerical reconnection that weakens the field, or both. This situation is drastically different from the aligned case, where the inner 100 AU region is heavily influenced by the magnetically dominated low-density lobes.

4. MODERATELY STRONG FIELD CASES: DIFFICULTY WITH DISK FORMATION

We have seen from the preceding section that, in the weakly magnetized case of $\lambda = 9.72$, the $10^2$ AU scale inner part of the protostellar accretion flow is dominated by two very different types of structures: a weakly magnetized, dense, RSD in the orthogonal case ($\theta_0 = 90^\circ$, Model G) and magnetically dominated, low-density lobes or DEMS in the aligned case ($\theta_0 = 0^\circ$, Model A), at least at the relatively early time discussed in Section 3, when the central mass reaches $\sim 0.1 M_\odot$. This dichotomy persists to later times for these two models and for other models as well, as illustrated by Figure 8, where we plot Models A–I at a time when the central mass reaches 0.2 $M_\odot$. It is clear that the RSD for Model G becomes even more prominent at the later time, although a small magnetically dominated, low-density lobe is evident close to the center of the disk; it is a trapped DEMS that is too weak to disrupt the disk. In this case, the identification of a robust RSD is secure, at even later times (up to the end of the simulation, when the central mass reaches 0.38 $M_\odot$ or 38% of the initial core mass). In the aligned case (Model A), the inner accretion flow remains dominated by the highly dynamic DEMS at late times, with no sign of RSD formation.

For the intermediate tilt angle case of $\theta_0 = 45^\circ$ (Model D), the inner structure of the protostellar accretion flow is shaped by the tussle between RSD and DEMS. Figure 8 shows that, at the plotted time, Model D has several spiral arms that...
The disk is also smaller, less dense, and more dynamic. It appears to merge into a rotating disk. There are, however, at least three low-density “holes” near the center of the disk: they are the magnetically dominated DEMS. Animations show that the highly variable DEMS are generally confined close to the center, although they can occasionally expand to occupy a large fraction of the disk surface. Overall, the circumstellar structure in Model D is more disk-like than DEMS-like. We call it a “porous disk,” to distinguish it from the more filled-in, more robust disk in the orthogonal Model G. Even though most of the porous disk has a rotation speed dominating the infall speed, the infall is highly variable, and often supersonic. The rotation speed also often deviates greatly from the Keplerian value. Such an erratic disk is much more dynamic than the quiescent disks envisioned around relatively mature (e.g., Class II) YSOs. The intermediate tilt angle case drives a powerful bipolar outflow, envisioned around relatively mature (e.g., Class II) YSOs. The intermediate tilt angle case drives a powerful bipolar outflow, unlike the orthogonal case, but similar to the aligned case. This is consistent with the rate of angular momentum removal increasing with decreasing tilt angle (i.e., from 90° to 45°; see also Ciardi & Hennebelle 2010).

As the strength of the initial magnetic field in the core increases, the DEMS becomes more dominant. This is illustrated in the middle column of Figure 8, where the three cases with an intermediate field strength corresponding to \( \lambda = 4.86 \) (and \( \lambda_{\text{eff}} = 6.85 \)) are plotted. In Model H, where the magnetic and rotation axes are orthogonal, a relatively small (with radius of \( \sim 10^2 \) AU) rotationally dominated disk is clearly present at the time shown. As in the weaker field case of Model G, it is fed by prominent “pseudo-spirals” which are part of a magnetically induced curtain in three dimensions (see the bottom panel of Figure 2). Compared to Model G, the curtain here is curved to a lesser degree, which is not surprising because the rotation is slower due to a more efficient braking and the stronger magnetic field embedded in the curtain is harder to bend. The disk is also smaller, less dense, and more dynamic. It is more affected by DEMS, which occasionally disrupt the disk, although it always reforms after disruption. Overall, the circumstellar structure in Model H is more RSD-like than DEMS-like. As in Model D, we classify it as a “porous disk.” As the tilt angle decreases from 90° to 45° (Model E) and further to 0° (Model B), the rotationally dominated circumstellar structure largely disappears; it is replaced by DEMS-dominated structures. Even though there is still a significant amount of rotation in the accretion flow, a dense coherent disk is absent. We conclude that for a moderately strong magnetic field of \( \lambda = 4.86 \), the formation of RSD is suppressed if the tilt angle is moderate.

In the cases of the strongest magnetic field corresponding to \( \lambda = 2.92 \) (and \( \lambda_{\text{eff}} = 4.12 \)), the formation of RSD is suppressed regardless of the tilt angle, as can be seen from the last column of Figure 8. For the orthogonal case (Model I), the prominent “pseudo-spirals” in the weaker field cases of Model G (\( \lambda = 9.72 \)) and H (\( \lambda = 4.86 \)) are replaced by two arms that are only slightly bent. They are part of a well-defined pseudodisk that happens to lie roughly in the \( y = 0 \) (or \( x-z \)) plane (see the bottom panel of Figure 9). In the absence of any initial rotation, one would expect the pseudodisk to form perpendicular to the initial field direction along the \( x \)-axis, i.e., in the \( x = 0 \) (or \( y-z \)) plane. Over the entire course of core evolution and collapse, the rotation has rotated the expected plane of the pseudodisk by nearly 90°. Nevertheless, at the time shown (when the central mass reaches \( 0.2 M_\odot \)), there is apparently little rotation left inside \( 10^3 \) AU to warp the pseudodisk significantly. Except for the orientation, this pseudodisk looks remarkably similar to the familiar one in the aligned case (Model C; see the top panel of Figure 9). In particular, there are low-density “holes” in the inner part of both pseudodisks which are threaded by intense magnetic fields and surrounded by dense filaments: they are the DEMS. In the intermediate tilt angle case of 45° (Model F, not shown in the 3D figure), the pseudodisk is somewhat more
warped than the two other cases, and its inner part is again dominated by DEMS. It is clear that for a magnetic field of \( \lambda \) of a few, the inner circumstellar structure is dominated by the magnetic field, with rotation playing a relatively minor role; the RSD remains suppressed despite the misalignment.

To better estimate the boundary between the cores that produce RSDs and those that do not, we carried out two additional simulations with \( \theta_0 = 90^\circ \) (Models P and Q in Table 1). We found a porous disk in Model P (\( \lambda = 4.03 \) and \( \lambda_{\text{eff}} = 5.68 \)), as in the weaker field case of Model H, but no disk in Model Q (\( \lambda = 3.44 \) and \( \lambda_{\text{eff}} = 4.85 \)), as in the stronger field case of Model I. In addition, we carried out three simulations for the orthogonal case with the initial magnetic field strength proportional to the column density along each field line, so that the local mass-to-flux ratio \( \lambda_I \) (defined as the ratio of either the mass on a given flux tube to the magnetic flux in that tube or the column density to the field strength) is the same for all field lines (Models X, Y and Z in Table 1); in such cases, the local \( \lambda_I \) is the same as \( \lambda_{\text{eff}} \) and the global mass-to-flux ratio \( \lambda \) for the core as a whole. We find that a porous disk forms in Model Y (\( \lambda_I = 5 \)) and Z (\( \lambda_I = 6 \)) but not in Model X (\( \lambda_I = 4 \)). Taken together, these simulations (Model P through Z) indicate that the critical value for the mass-to-flux ratio \( \lambda_{\text{cr}} \) (beyond which at least a porous disk forms) lies between 4 and 5. To be conservative, we adopt \( \lambda_{\text{cr}} \approx 4 \) for the discussion below.

5. DISCUSSION

5.1. Comparison with Joos et al.

Our most important qualitative result is that the misalignment between the magnetic field and rotation axis tends to promote the formation of RSDs, especially in weakly magnetized dense cores. This is in agreement with the conclusion previously reached by Joos et al. (2012; JHC12 hereafter), using a different numerical code and somewhat different problem setup. Their calculations were carried out using an adaptive mesh refinement code in the Cartesian coordinate system, with the central object treated using a stiffening of the equation of state, whereas ours were done using a fixed mesh refinement code in the spherical coordinate system, with an effective sink particle at the origin.
The Astrophysical Journal, 774:82 (12pp), 2013 September 1

Despite the differences, these two distinct sets of calculations yield qualitatively similar results. The case for the misalignment promoting disk formation is therefore strengthened.

Quantitatively, there appears to be a significant discrepancy between our results and theirs. According to their Figure 14, a Keplerian disk is formed in the relatively strongly magnetized case of \( \lambda = 3 \) if the misalignment angle \( \theta_0 = 90^\circ \). Formally, this case corresponds roughly to our Model I \((\lambda = 2.92 \text{ and } \theta_0 = 90^\circ)\), for which we can rule out the formation of an RSD with confidence (see Figure 8).

We believe that the discrepancy comes at least partially from the different initial distributions of density and magnetic field adopted in the two studies. Joos et al. adopted a centrally condensed initial mass distribution

\[
\rho(r) = \frac{\rho_0}{1 + (r/r_0)^2}
\]

with the characteristic radius \( r_0 \) set to 1/3 of the core radius \( R_c \), so that the central-to-edge density contrast is 10. The initial magnetic field is unidirectional, with a strength proportional to the column density along each field line (see Ciardi & Hennebelle 2010). These distributions are different from those adopted in our paper, where both the density and magnetic field are initially uniform. As our initially uniform core evolves in time, it passes through a stage where the density distribution resembles that given by Equation (8) (see Figure 1 of Li et al. 2011 for an illustration). At this stage, our initially unidirectional (straight) field lines would not stay unidirectional; they are dragged into an hourglass shape by the core contraction. The hourglass shaped field lines are expected to make magnetic braking more efficient (because of a longer lever arm) and disk formation harder than the straight field lines adopted by JHC12 at a similarly evolved stage. Whether there are other factors that contribute significantly to the discrepancy noted above remains to be determined.

The result that a misalignment between the magnetic field and rotation axis helps disk formation by weakening magnetic braking may be counterintuitive. Mouschovias & Paleologou (1979) showed analytically that, for a uniform rotating cylinder embedded in a uniform static external medium, magnetic braking is much more efficient in the orthogonal case than in the aligned case. This analytic result may not be directly applicable to a collapsing core, however. As emphasized by JHC12, the collapse drags the initially uniform, rotation-aligned magnetic field into a configuration that fans out radially. JHC12 estimated analytically that the collapse-induced field fan-out could in principle make the magnetic braking in the aligned case more efficient than in the orthogonal case. The analytical estimate did not, however, take into account of the angular momentum removal by outflow, which, as we have shown in Section 3.2, is a key difference between the weak-field \((\lambda = 9.72, \lambda_{\text{eff}} = 13.7)\) aligned case (Model A) where disk formation is suppressed and its orthogonal counterpart (Model C) that does produce an RSD (see Figures 1 and 4; and also Ciardi & Hennebelle 2010).

The generation of a powerful outflow in the weak-field aligned case (Model A) is facilitated by the orientation of its pseudodisk, which is perpendicular to the rotation axis (see Figure 2). This configuration is conducive to both the pseudodisk winding up the field lines and the wound-up field escaping above and below the pseudodisk, which drives a bipolar outflow. When the magnetic field is tilted by 90° away from the rotation axis, the pseudodisk is warped by rotation into a snail-shaped curtain that is unfavorable to outflow driving (see Figure 2). The outflow makes it more difficult to form an RSD in the aligned case. Disk formation is further hindered by magnetically dominated, low-density lobes (DEMS), which affect the inner part of the accretion flow of the aligned case more than that of the perpendicular case, at least when the field is relatively weak (see Figures 1 and 8). For more strongly magnetized cases, the DEMS becomes more dynamically important close to the central object, independent of the tilt angle \( \theta_0 \) (see Figure 9).

\( \theta_0 = 90^\circ \)

Figure 9. Same as in Figure 2, but for the stronger field \((\lambda = 2.92)\) cases of Model C (aligned) and Model I (orthogonal) at a time when the central mass reaches 0.2 \( M_\odot \). For clarity, only field lines originated from the bottom \( x-y \) and left \( y-z \) plane are plotted, respectively. Note the difference in orientation of the pseudodisks in the two cases. The inner part of both pseudodisks is dominated by low-density, strongly magnetized regions. The length is in units of AU.

(A color version of this figure is available in the online journal.)

\( \lambda = 3 \)

(see Figures 1 and 8). For more strongly magnetized cases, the DEMS becomes more dynamically important close to the central object, independent of the tilt angle \( \theta_0 \) (see Figure 9).
DEM5-like structures were also seen in some runs of JHC12 (e.g., the case of $\lambda = 2$ and $\theta_0 = 0^\circ$; see their Figure 19) but were not commented upon. As stressed previously by Zhao et al. (2011) and Krasnopol5sky et al. (2012) and confirmed by our calculations, the DEM5 presents a formidable obstacle to the formation and survival of an RSD.

5.2. Misalignment Not Enough for Disk Formation in General

While there is agreement between JHC12 and our calculations that misalignment between the magnetic field and rotation axis is beneficial to disk formation, it is unlikely that the misalignment alone can enable disk formation around the majority of YSOs. The reason is the following. Troland & Crutcher (2008) obtained a mean dimensionless mass-to-flux ratio of $\lambda_{eff} \approx 2$ through OH Zeeman observations for a sample of dense cores in nearby dark clouds. For such a small $\lambda_{eff}$, disk formation is completely suppressed, even for the case of maximum misalignment of $\theta_0 = 90^\circ$. Crutcher et al. (2010) argued, however, that there is a flat distribution of the total field strength in dense cores, from $B_{tot} \approx 0$ to some maximum value $B_{max} \approx 30 \mu G$; the latter corresponds to $\lambda_{eff} \approx 1$, so that the mean $\lambda_{eff}$ stays around 2. If this is the case, some cores could be much more weakly magnetized than others, and disks could form preferentially in these cores. However, to form an RSD, the core material must have (1) an effective mass-to-flux ratio $\lambda_{eff}$ greater than $\sim 4$ and (2) a rather large tilt angle. If one assumes that the core-to-core variation of $\lambda_{eff}$ comes mostly from the field strength rather than the column density (as done in Crutcher et al. 2010), then the probability of a core having $\lambda_{eff} \gtrsim 4$ (or $B_{tot} \lesssim 7.5 \mu G$) is $\sim 1/4$. Since a large tilt angle of $\theta_0 \sim 90^\circ$ is required to form an RSD for $\lambda_{eff} \lesssim 6$–7, the chance of disk formation is reduced from $\sim 1/4$ by at least a factor of two (assuming a random orientation of the magnetic field relative to the rotation axis), to $\sim 12\%$ or less.

The above estimate is necessarily rough, and can easily be off by a factor of two in either direction. It is, however, highly unlikely for the majority of the cores to simultaneously satisfy the conditions on both $\lambda_{eff}$ and $\theta_0$ for disk formation. The condition $\lambda_{eff} \gtrsim 4$ is especially difficult to satisfy because, as noted earlier, it implies that the dense cores probed by OH observations must have a total field strength $B_{tot} \lesssim 7.5 \mu G$, comparable to, or less than, the well-defined median field strength inferred by Heiles & Troland (2005) for the much more diffuse, cold neutral atomic medium (CNM). It is hard to imagine a reasonable scenario in which the majority of dense cores have magnetic fields weaker than the CNM.

We note that Krumholz et al. (2013) independently estimated a range of $\sim 10\%$–$50\%$ for the fraction of dense cores that would produce a Keplerian disk based on Figure 14 of JHC12. Their lower limit of $\sim 10\%$ is in agreement with our estimate. Their upper limit of $\sim 50\%$ is much higher than our estimate, mainly because it includes rather strongly magnetized cores with mass-to-flux ratios as small as 2. Since our calculations show that such strongly magnetized cores do not produce RSDs even for large tilt angles, we believe that this upper limit may be overly generous.

Whether dense cores have large tilt angle $\theta_0$ between the magnetic field and rotation axis that are conducive to disk formation is unclear. Hull et al. (2013) measured the field orientation on the $10^3$ AU scale for a sample of 16 sources using millimeter interferometer CARMA. They found that the field orientation is not tightly correlated with the outflow axis; indeed, the angle between the two is consistent with being random. If the outflow axis is aligned with the core rotation axis and if the field orientation is the same on the core scale as on the smaller, $10^3$ AU scale, then $\theta_0$ would be randomly distributed between $0^\circ$ and $90^\circ$, with half of the sources having $\theta_0 \gtrsim 60^\circ$. However, the outflow axis may not be representative of the core rotation axis. This is because the (fast) outflow is thought to be driven magnetocentrifugally from the inner part of the circumstellar disk (on the AU scale or less; Shu et al. 2000; Königl & Padritz 2000). A parcel of core material would have lost most of its angular momentum on the way to the outflow launching location; the torque (most likely magnetic or gravitational) that removes the angular momentum may also change the direction of the rotation axis. Similarly, the field orientation on the $10^3$ AU scale may not be representative of the initial field orientation on the larger core scale. The magnetic field on the $10^3$ AU scale is more prone to distortion by collapse and rotation than that on the core scale. Indeed, Chapman et al. (2013) found that the field orientation on the core scale measured using the Caltech Submillimeter Observatory single-disk telescope is within $20^\circ$ of the outflow axis for three of the four sources in their sample (see also Davidson et al. 2011); the larger angle measured in the remaining source may be due projection effects because its outflow axis is close to the line-of-sight. If the result of Chapman et al. and Davidson et al. (2011) is valid in general and if the outflow axis reflects the core rotation axis, then dense cores with large tilt angle $\theta_0$ would be rare. In this case, disk formation would be rare according to the calculations presented in this paper and in JHC12, even in the unlikely event that the majority of dense cores are as weakly magnetized as $\lambda_{eff} \gtrsim 4$.

RSDs are observed, however, routinely around evolved Class II YSOs (see Williams & Cieza 2011 for a recent review), and increasingly around younger Class I (e.g., Jørgensen et al. 2009; Lee 2011; Takakuwa et al. 2012) and even Class 0 (Tobin et al. 2012) sources. When and how such disks form remains unclear. Our calculations indicated that the formation of large observable ($10^2$ AU scale) RSDs is difficult even in the presence of a large tilt angle $\theta_0$ during the early protostellar accretion (Class 0) phase. This may not contradict the available observations yet (Maury et al. 2010), since only one Keplerian disk is found around a (late) Class 0 source so far (L1527; Tobin et al. 2012); it could result from a rare combination of an unusually weak magnetic field and a large tilt angle $\theta_0$. If it turns out, through future observations using ALMA and JVLA, that large-scale Keplerian disks are prevalent around Class 0 sources, then some crucial ingredients must be missing from the current calculations. Possible candidates include non-ideal MHD effects and turbulence. The existing calculations indicate that realistic levels of the classical non-ideal MHD effects do not weaken the magnetic braking enough to enable large-scale disk formation (Mellon & Li 2009; Li et al. 2011; Krasnopol5sky et al. 2011).

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4 We assume that the mass-to-flux ratio measured by Troland & Crutcher on the core scale is the same as the effective mass-to-flux ratio near the core center because, for significantly magnetically supercritical cores, ambipolar diffusion is generally ineffective in reducing the value of $\lambda$ near the center relative to that in the envelope.

5 We cannot rule out the existence of small, AU scale disks, because the size of our effective “sink particle” is $6.7$ AU. Small disks may be needed to drive fast outflows during the Class 0 phase, and may form through non-ideal MHD effects (Machida et al. 2011; Dapp & Basu 2010; Dapp et al. 2012).

6 We are unable to follow the collapse until most of the core material is accreted into the central object for numerical reasons. Such a complete accretion may not be realistic, however, if the star formation efficiency in a core is typically as low as 1/3 (e.g., Alves et al. 2007).
et al. 2012; see also Krasnopolsky & Königl 2002 and Braiding & Wardle 2012a, 2012b), although misalignment has yet to be considered in such calculations. Supersonic turbulence was found to be conducive to disk formation (Santos-Lima et al. 2012, 2013; Seifried et al. 2012; Myers et al. 2013; Joos et al. 2013), although dense cores of low-mass star formation typically have subsonic non-thermal line-width and it is unclear whether subsonic turbulence can enable disk formation in dense cores magnetized to a realistic level. If, on the other hand, it turns out that large-scale Keplerian disks are rare among Class 0 sources, then the question of disk growth becomes paramount: How do the mostly undetectable Class 0 disks become detectable in the Class I and II phase? If the magnetic braking plays a role in keeping the early disk undetectable, then its weakening at later times may promote rapid disk growth. One possibility for the late weakening of magnetic braking is the depletion of the protostellar envelope, either by outflow stripping (Mellon & Li 2008) or accretion (Machida et al. 2011). It deserves to be better quantified.

5.3. Summary

We carried out a set of MHD simulations of star formation in dense cores magnetized to different degrees and with different tilt angles between the magnetic field and the rotation axis. We confirmed the qualitative result of Joos et al. (2012) that misalignment between the magnetic field and rotation axis is conducive to disk formation. Quantitatively, we found however that the misalignment enables the formation of an RSD only in dense cores where the star-forming material is rather weakly magnetized, with a dimensionless mass-to-flux ratio $\gamma \gtrsim 4$; large misalignment in such cores allows the rotation to wrap the equatorial pseudodisk in the aligned case into a curved curtain that hinders outflow driving and angular momentum removal, making disk formation easier. In more strongly magnetized cores, disk formation is suppressed independent of the misalignment angle, because the inner part of the protostellar accretion flow is dominated by strongly magnetized, low-density regions. If dense cores are as strongly magnetized as inferred by Troland & Crutcher (2008; with a mean mass-to-flux ratio $\sim 2$), it would be difficult for the misalignment alone to enable disk formation in the majority of them. We conclude that how protostellar disks form remains an open question.

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