CFD simulations of a suspension bridge deck for different deck shapes with railings and vortex mitigating devices

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Abstract. Aerodynamic response of a single-box suspension bridge girder is investigated using CFD simulations. The importance of including railings and vortex mitigating devices such as guide vanes and a spoiler in the simulations is explored since they alter the flow field around the deck greatly. The paper compares results from a bare deck section and a section with mitigating devices. A clear vortex shedding observed for the bare deck section is suppressed by the use of mitigating devices and the aerodynamic force coefficients are very different between these cases. The effect of varying deck width is also studied. Flutter derivatives are defined based on quasi-steady formulation and flutter velocity is computed for each section. The section with the largest width to depth ratio has the best performance against flutter. This study is an initial phase of multi-fidelity optimization of bridge deck shape considering aerodynamic constraints.

1. Introduction
With the advance of technologies, the achievable span of suspension bridges becomes longer and longer. The Great Belt Bridge and Akashi Bridge, which both opened in 1998, have main span of more than 1.6 km. Other suspension bridges, such as Xihoumen Bridge in China, which opened in 2009 and the recent construction of Osman Gazi Bridge in Turkey follow this trend. As the structures become longer and more flexible, they are more vulnerable to wind-induced phenomena such as flutter, buffeting and vortex shedding.

For the design of such structures, it is essential to consider their aerodynamic response, since it may lead to collapse. The shape of a bridge deck is one of the key factors influencing the structural response under wind load. Among different deck shapes, the stream-lined single box girder has become popular after its successful application to the Severn Bridge in UK in 1966. It possesses good performance against vortex shedding and flutter instabilities. Since the box section provides larger torsional inertia than open sections, the deck can be designed shorter in height, which results in more slender and aesthetic appearances. The box section is also more economic in construction as well as corrosion maintenance [1].

The characterization of aerodynamic response of a bridge deck has been historically performed in wind tunnel tests. However, according to Mannini at al. [2], Sarkic et al [3] among others, the improvement in Computational Fluid Dynamics (CFD) techniques in the last decade has made this method more increasingly reliable to study aerodynamic behaviour of bridge deck cross sections. For instance, Ge and Xian [4] used this technique to obtain flutter response of a bridge numerically.
This work focuses on the use of 2D unsteady Reynolds Averaged Navier-Stokes (URANS) CFD simulations to obtain aerodynamic coefficients for different deck shapes. Although this method presents some limitations such as perfect correlation of the flow along the span-wise direction, it can accurately provide the mean pressure distribution around the geometry and consequently force coefficients as demonstrated by researchers such as Vairo [5], Brusiani [6] and Nieto [7].

The aerodynamic coefficients are essential to characterize bridge deck section since they can be used to carry out flutter and buffeting analyses based on quasi-steady theory. This formulation is appropriate for high reduced wind velocities and streamlined deck cross sections according to Wu and Kareem [8] and Tubino [9].

In this work, the aerodynamic characteristics of different deck sections are studied. First of all, two deck sections, one with a bare deck and a second one with vortex mitigating devices are studied and compared. Then four sections of different deck width and a constant height are investigated. Flutter derivatives based on the evaluated force coefficients are compared as well as the flutter speed for each section. Although many researchers have worked with CFD simulation of a bare deck [10,11,12], there are are limited number of numerical investigations of deck sections with vortex mitigating devices [13,14,15]. Some previous studies have addressed the influence of aerodynamic appendages such as flaps on flutter velocity [16] or porosity of railings on critical flutter velocity [17].

This study is an initial phase of multi-fidelity optimization of a suspension bridge deck shape under flutter constraint. Julsundet Bridge in Norway was used as an application example.

2. Formulation

2.1 Force coefficients

The aerodynamic force coefficients and Strouhal number of a bridge deck section under wind load are expressed as:

\[
C_L = \frac{L}{0.5 \rho U^2 B}; \quad C_D = \frac{D}{0.5 \rho U^2 H}; \quad C_M = \frac{M}{0.5 \rho U^2 B^2}; \quad S_f = \frac{jH}{U}
\]

where \(C_L\), \(C_D\) and \(C_M\) are lift, drag and moment coefficients, \(L\), \(D\) and \(M\) are the time-averaged lift and drag forces and moment per unit of length, \(\rho\) is the flow density, \(U\) is the undisturbed wind velocity, \(f\) is the vortex shedding frequency and \(B\) and \(H\) are the width and the height of the deck cross-section. The force and sign convention used for the study is shown in Figure 1.

![Figure 1. Force and sign convention of a bridge deck under constant velocity.](image)

2.2 Computation of flutter speed based on quasi-steady theory

Fully-computational approach of flutter velocity computation permits the substitution of wind tunnel tests by equivalent numerical simulations [18]. This fully computational method is briefly described next.

Scanlan and Tomko [19] established three aeroelastic force components based on frequency dependent function called flutter derivatives as follows.
where $B$ is the deck width, $U$ is the acting wind speed, $K = \frac{B \omega}{U}$ is the reduced frequency with $\omega$ as the response frequency, whereas $A'_i$, $H'_i$ and $P'_i$ ($i=1,\ldots,6$) are the flutter derivatives.

The quasi-steady model relates the displacements and rotations of the deck and velocities with the relative wind velocity components. The flutter derivatives may be expressed in terms of force coefficients by comparing the quasi-steady and Scanlan’s model as:

$$
H'_i = -\frac{C'_{L,0} + C'_{D,0}}{K}; \quad H'_2 = \frac{C'_{L,0} + C'_{D,0}}{K} \cdot \mu_H; \quad H'_3 = -\frac{C'_{L,0}}{K^2};
$$

$$
A'_i = \frac{C'_{M,0}}{K}; \quad A'_2 = \frac{C'_{M,0}}{K} \cdot \mu_A; \quad A'_3 = -\frac{C'_{M,0}}{K^2};
$$

where $C_{L,0}$, $C_{D,0}$, $C_{M,0}$, are lift, drag, and moment coefficients at 0 degree of angle of attack, $C'_{L,0}$, $C'_{D,0}$, $C'_{M,0}$, are their derivatives while $\mu_H$ and $\mu_A$ can be estimated according to Larose and Livesey [20] as:

$$
\mu_H \approx \frac{A'_1}{H'_1} \quad \text{and} \quad \mu_A \approx \frac{A'_1}{H'_3};
$$

The dynamic equilibrium of a deck under aeroelastic forces, $f_a$ can be written in a matrix form as:

$$
\mathbf{M} \ddot{\mathbf{u}} + (\mathbf{C} + \mathbf{C}_a) \dot{\mathbf{u}} + (\mathbf{K} + \mathbf{K}_a) \mathbf{u} = 0
$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are mass, damping, and stiffness matrices, $\mathbf{K}_a$ and $\mathbf{C}_a$ are aeroelastic stiffness and damping matrices. By applying modal analysis to Eq. (4), the solution is a combination of the most relevant $m$ modes. This leads to the following eigenvalue problem:

$$
(\mathbf{A} - \mu \mathbf{I}) \mathbf{w}_\mu e^{\mu t} = 0
$$

where the matrix $\mathbf{A}$ is a function of the matrices, $\mathbf{K}_a$ and $\mathbf{C}_a$, and $\mathbf{w}_\mu$ is an eigenvector. In order to solve this problem, we need mode shapes and natural frequencies of the bridge under study, which can be obtained by a finite element model. The solution to Eq. (5) is expressed as $\mu = \alpha + i\beta$ where $\alpha$ is related to structural damping and $\beta$ is the damped frequency ($j = 1,\ldots, m$ where $m$ is number of frequencies). Flutter occurs when $\alpha$ becomes null with increasing wind speed.

3. CFD analysis of the bridge deck

CFD simulations were performed to obtain force coefficients of the bridge deck for varying shapes. 2D URANS approach was adapted to perform the analyses. The open source CFD code of OpenFoam v.2.4 [21] was used to carry out CFD analyses in this study.

The $k$-$\omega$ Shear-Stress Transport (SST) turbulence model was selected among many available turbulence models because of its good performance and wide acceptance within the bridge engineering field. This model was initially developed by Menter [22] and later improved by Menter and Esch [23]. It takes advantage of robust and accurate formulation of Wilcox $k$-$\omega$ model [24] at the near wall region as well as a good performance of the $k$-$\epsilon$ turbulence model [25] in the far field. See for example, Sarkic et al. [3], De Miranda et al. [26] and Montoya [27].

In this present work, the bridge planned for crossing of the Julsundet Fjord in Norway is taken as a case study. It is a suspension bridge with a main span of 1.6 km and the deck section is 32 m wide and 4 m high aerodynamic single box girder. The baseline deck geometry in this study follows that of the 1:50 scale model of the planned bridge shown in Figure 2. The further description of the bridge can be found in a report from the Norwegian Public Road Administration [28].

The flow domain and the boundary conditions are illustrated in Figure 3. The flow domain is $30B \times 41B$ where $B$ is the deck width. The simulation was performed at $Re = 3.5 \times 10^5$. Uniform velocity of 8 m/s at the inlet boundary was imposed while atmospheric pressure was imposed at the outlet.
Turbulent intensity of 5% was considered for the incoming flow with 0.1 \( B \) turbulent length scale. This length scale is also adopted by Ribeiro [29] and Nieto [7]. Both upper and lower walls have slip wall condition while the wall around the deck was non-slip. Linear interpolation of values from cell centres to face centres are carried out. The gradient terms are discretized by the Gauss scheme with linear interpolation. For divergence, the Gauss scheme with linear upwind and the limited linear interpolation schemes were applied. The 2D unstructured mesh was generated using software PointWise [30].

A boundary layer mesh was applied around the deck wall in a way that gave a maximum \( y^+ \) value less than 4 where \( y^+ = u_\tau h_1 / \nu = \sqrt{\tau_w / \rho h_1 / \nu} \), \( u_\tau \) is wall friction velocity, \( h_1 \), the distance from the wall to the first cell centre, \( \tau_w \), wall shear stress, \( \nu \), kinematic viscosity. The first element height was set to \( h_1 / B = 1.14 \times 10^{-4} \) while Courant number was set to 2 for the simulation time-step. The time integration was verified by Courant number 1 and 0.5 prior to the study, and no significant difference in results were found. A mesh convergence study was conducted using a bare deck section with three different mesh densities in order to verify the discretization. Three types of meshes were considered: a coarse mesh with 112,564 cells, a medium mesh with 196,126 cells and a fine mesh with 321,435 cells. The coarse mesh underestimates the lift coefficient while the medium and fine mesh gave similar results. The medium mesh scheme was therefore selected for the simulations.
3.1 Case study 1

In order to assess the importance of mitigating devices and railings on the force coefficients, the bare deck model with original deck shape in Figure 2 was compared with the model with railings and the mitigating devices shown in Figure 4. The railings geometry is a simplified representation of that adopted in wind tunnel tests [28]. The model has various railings, two guide vanes as well as a spoiler at the bottom center part of the deck. The model scale is 1:50 with respect to the actual bridge dimension of B=32 m. Figure 5 shows the details of mesh for the section with aerodynamic devices. The number of cells for this model is approximately 240,000.

![Figure 4. Deck geometry with mitigating devices and railings.](image)

![Figure 5. Meshing details of the model (a) with railings (b) and guide vanes (c).](image)

The evaluated force coefficients are summarized in Table 1, along with standard deviation, Strouhal number and the gradient with respect to the angle of attack. Figure 6 shows the force coefficient history of both sections. The bare deck presents periodic oscillations with a single peak frequency while the amplitude of oscillation for the section with mitigating devices are one order of magnitude smaller.

| Geometry       | $C_L$  | $C_D$ | $C_M$ | $\sigma_{C_L}$ | $\sigma_{C_D}$ | $\sigma_{C_M}$ | $C_L'$ | $C_D'$ | $C_M'$ | $St$ |
|----------------|--------|-------|-------|----------------|----------------|----------------|--------|--------|--------|------|
| Bare deck      | -0.00043 | 0.391 | 0.0234 | 0.016          | 0.0035         | 0.0034         | 5.98   | 0.20   | 1.45   | 0.25 |
| Deck & devices | -0.148  | 0.650 | 0.0021 | 0.002          | 0.0126         | 0.0005         | 4.64   | 0.43   | 1.29   | -    |
Figure 6. Force coefficients history of bare deck (left) and the section with mitigating devices (right).

Figure 7 shows flow fields for the two cases. The flow field for the bare deck section shows that there is a flow separation at the first corner after the leading edge both at the top and bottom of the deck, but the flow reattaches again close to the edge. From the Figure 7 b1), it is seen that the suction has similar value at the top as well as the bottom plates resulting in overall small lift coefficient. From Figure 7 c1), a vortex shedding phenomenon is clearly observed for this section, which is verified by the standard deviation of the force coefficients (Table 1).

On the other hand, the flow for the section with mitigating devices is rather complex due to the elements added to the deck. From the pressure plot, it can be seen that the pressures on the bottom part are largely negative while the top part experiences rather positive because the flow is slowed down by the railings. This results in a large negative lift coefficient. The vortex shedding behaviour seen for the bare deck section is suppressed for the deck with mitigating devices as could be expected.

The drag coefficient for the section with devices is 66% higher than for the bare deck section because of the various railings and mitigating devices while the moment coefficients are much smaller. The most notable difference is the lift coefficients; -0.00043 for the bare deck and -0.148 for the section with aerodynamic devices. The gradient of the lift coefficient with respect to the angle of attack is 22% smaller for the section with devices, which is favourable for the aerodynamic response of the bridge deck.
The corresponding force coefficients obtained in the wind tunnel tests are $C_D=0.967$, $C_L=-0.309$, $C_M=-0.017$, $C_{L'}=3.624$, $C_{M'}=1.182$ [28]. The difference between the simulation and the experimental results is considered primarily due to a higher density of railings on the section model in the experimental set up. Experimental results for another aerodynamic box girder [28] with depth to width ratio of 1:6.2 and similar railing configurations as Julsundet Bridge gave $C_D=0.360$ for the bare deck and $C_D=0.845$ for the same section with guard rails. Our drag coefficient for the bare deck model is comparable to these results and the simulation for the section with devices can be improved by additional details of the railings. On the other hand, the objective of this work is a parametric study of a deck section, in which aerodynamic response of the deck is studied for varying the deck width. This concept is later used as a part of multi-fidelity optimization of deck shape considering aerodynamic constraints.

3.2 Case study 2
In this Case study, we have investigated the influence of changing the deck width on the force coefficients while maintaining the deck height. The section with railings and mitigating devices is used for this parametric study. As we change the deck width, we are also changing the angles of the leading and trailing edges. We have considered four cases besides the original design, whose geometries are illustrated in Figure 8 and summarized in Table 2. Each geometry is expressed in terms of a non-dimensional width, $b/0.5B_0$, where $b$ is the horizontal distance from the leading edge to the upper left corner of each deck geometry (Figure 8) and $B_0$ is the initial deck width.
Table 2. Geometrical properties of the simulations (See Figure 2 for the description of $\theta_1$, $\theta_2$, $\theta_T$).

| s. num | $b/0.5B_0$ | $\theta_1$ | $\theta_2$ | $\theta_T$ |
|--------|------------|------------|------------|------------|
| 0      | 0.121      | 33         | 27         | 60         |
| 4      | 0.076      | 46         | 30         | 70         |
| 7      | 0.107      | 36         | 28         | 64         |
| 14     | 0.168      | 25         | 24         | 49         |
| 20     | 0.231      | 19         | 21         | 40         |

Figure 8. The deck shapes considered in case study 2 (the blue line shows the original design, $s_0$).

The force coefficients obtained for different deck shapes are summarized in Table 3 and plotted with respect to the nondimensional width in Figure 9. The lift and moment coefficients refer to the deck width considered in each case. Examples of the evaluated flow fields are shown in Figure 10 for the two extreme section shapes, $s_4$ and $s_{20}$.

For the most bluff section of $s_4$, a large flow separation is observed because of the large angle of the leading edge, while for the most aerodynamic section of $s_{20}$, the flow is more attached to the deck. Large negative pressure is observed on the top left corner of the section $s_4$, which makes the lift coefficient smaller compared to that of section $s_{20}$.

As the deck width increases and the leading-edge angle decreases, the lift coefficient and the moment coefficient decrease. The drag coefficient is also slightly reduced with decreasing leading-edge angle. The reduction in the gradient of the lift coefficient is most notable with increasing deck width.

Table 3. Force coefficient results of different geometries

| geometry | $b/0.5B_0$ | $C_L$ | $C_D$ | $C_M$ | $C_L'$ | $C_M'$ |
|----------|------------|-------|-------|-------|--------|--------|
| $s_4$    | 0.076      | -0.120| 0.650 | 0.0090| 5.67   | 1.36   |
| $s_7$    | 0.107      | -0.138| 0.643 | 0.0043| 4.55   | 1.23   |
| $s_{14}$ | 0.168      | -0.164| 0.650 | -0.0003| 3.98 | 1.22   |
| $s_{20}$ | 0.231      | -0.181| 0.638 | -0.0023| 3.23 | 1.17   |

4. Computation of flutter velocity based on quasi-steady theory

From the force coefficients obtained in the CFD simulations for different deck sections, flutter derivatives were computed based on quasi-steady formulation. Four of the most important flutter derivatives, $A_2$, $A_3$, $H_2$, $H_3$ are plotted for the sections, $s_4$, $s_0$ and $s_{20}$ in Figure 11. Other sections are omitted for clarity of representation. For $A_2$ and $A_3$ flutter derivative, the gradient $C_M'$ is one of the key parameters and for $A_2$, the ratio of the gradient $C_M'/C_L'$ is important. On the other hand, for $H_2$ and $H_3$, the gradient, $C_L'$ is the dominating parameter.
Figure 9. Force coefficients vs nondimensional width $b/0.5B_0$ ($C_M$ is multiplied by 10).
Flutter velocity of different sections was computed using the first and the second vertical symmetric and the first torsional modes, which are 0.121 Hz, 0.166 Hz and 0.255 Hz respectively [28]. The structural damping ratio was assumed to be 0.3% [31]. The resulting flutter velocity and the corresponding reduced frequency for each section is summarized in Table 4. The flutter velocity of the original design section s0 is 68.95 m/s, which is similar to the experimental results of 71 m/s [28]. As the deck width increases and the deck shape becomes more aerodynamic, flutter velocity increases. Although the vortex mitigating devices such as guide vanes and spoiler are primarily used to suppress vortex shedding, they also contribute favorably to increase flutter velocity by altering the flow field around the deck.

Figure 11. Flutter derivatives, $A_2$, $A_3$, $H_2$ and $H_3$ for different bridge sections.
Table 4. Flutter velocity $V_f$ and the corresponding reduced frequency $K$ for different deck sections.

| geometry | s4 | s7 | s0 | s14 | s20 | bare deck |
|----------|----|----|----|-----|-----|-----------|
| $b/0.5B_0$ | 0.076 | 0.107 | 0.121 | 0.168 | 0.231 | 0.121 |
| $V_f$ (m/s) | 63.44 | 67.71 | 68.95 | 75.39 | 79.82 | 61.21 |
| $K$ | 0.68 | 0.62 | 0.60 | 0.52 | 0.46 | 0.71 |

5. Conclusions

CFD analysis of a single-box deck section was carried out to study the importance of railings and vortex mitigating devices on the bridge response under wind loading. Different deck sections were studied such as a bare deck section and sections with mitigating devices and varying deck width. Aerodynamic force coefficients obtained by CFD simulations were then used to compute flutter derivatives based on quasi-steady formulation. Flutter velocity of different sections was calculated for different sections. The section with the largest width to depth ratio has the best performance against flutter instability, while it is seen that the vortex mitigating devices tend to improve the aerodynamic response of the bridge by altering the flow field around the deck. This CFD study will be used as a part of multi-fidelity optimization of a bridge deck shape considering aerodynamic constraint.

Acknowledgement

This research is financed by the European Union’s Horizon 2020 research and innovation program under Marie Skłodowska-Curie individual fellowship, grant agreement 794646-MFOptBF-H2020-MSCA-IF-2017. The authors would like to thank the Norwegian Public Road Administration and Svend Ole Hansen ApS for the information on the bridge deck geometry and the results of the wind tunnels tests with a section model of Julsundet Bridge. The authors also would like to thank Dr. Guang Yin at the University of Stavanger for his valuable advices on CFD simulations.

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