Design learning in mathematics education: Engaging early childhood students in geometrical activities to enhance geometry and spatial reasoning

R Novita¹, M Putra², E Rosayanti³ and F Fitriati¹
¹Mathematics Department, STKIP Bina Bangsa Getsempena Jl. Tanggul Krueng Aceh No.34, Rukoh, Darussalam, Banda Aceh 23112, Indonesia.
²Hiroshima University, Higashihiroshima, 〒739-0046 Hiroshima Prefecture, Higashihiroshima, Kagamiyama, 1 丁目 3-2, Japan
³TK Al-Washliyah Alue Naga, Banda Aceh, Indonesia

E-mail: rita@stkipgetsempena.ac.id

Abstract. Understanding geometry and spatial reasoning is an essential area of mathematics learning. Besides, geometry and spatial reasoning serve as the foundation for most learning of mathematics and other subjects, especially for early childhood students. Unfortunately, these aspects are often ignored or minimized in early education and the professional development of early childhood teachers. This may be due to a lack of activities which involve students to reason about geometric phenomena in their surrounding world. Therefore, this study aimed to design an instructional activity to guide the early childhood students in understanding geometry and spatial reasoning by doing some activities related to shapes. Therefore, this study used van den Akker et al. design research model including three phases, namely: Preparing for the experiment, Experiment in the classroom, and Retrospective analysis. The study involved 13 children ages 4-5 in one of the kindergartens in Banda Aceh, Indonesia. Teaching experiment results showed that the learning trajectory developed is potential to make a significant difference in the geometry learning of early childhood helps children to connect their experience about various types of form in their life with mathematical knowledge (i.e., geometry and spatial reasoning) that allows them to reinvent and reinforce the geometry concept.

1. Introduction
Learning mathematics is not only oriented to the mastery of numbers, but some other aspects are equally important to be understood by young children. The aspects are geometric concepts and relationships. Understanding geometry and spatial reasoning is an essential area of mathematics learning. Besides, for early childhood students, geometry and spatial reasoning serve as the foundation for the learning of mathematics and other subjects. Furthermore, knowledge of geometry concepts is essential in many real-life contexts [1]; not only in and of themselves but also for the supporting roles they play in learning algebra and the concepts and skills of number and arithmetic [1,2,3]. Geometric models also contribute to learn and solve problems in measurement, graphing, ratio, and proportion, probability, as well as algebra. Other studies have also mentioned that high achieving students' numerical ability connect to their spatial and measurement abilities. Furthermore, low achieving
students in mathematics generally show little growth in geometry. Therefore, educators should give full attention to the development and understanding of geometry and spatial reasoning [2,4,5,6].

Geometry includes a wide range of skills - from recognizing simple shapes to create complicated proofs, from understanding above and beside navigating complex spaces with maps and graphing algebraic functions. By learning geometric principles, students develop reasoning and justification skills and learn how to interpret and describe physical environments [3]. Therefore, it is essential to introduce geometry at every age, grade, and year of education [1], especially for early childhood. In the early childhood education, understanding of geometry is defined as the ability of children to recognize, point, mention and collect objects around them based on geometric shapes [7].

Thus, the construction of the geometry concepts in children starting from identifying forms, investigating shapes and separating the usual pictures such as, quadrilateral, circle, and triangle; learning the concepts of location, such as below, above, left, right are the basics of understanding geometry [8]. Furthermore, NCTM's Curriculum Focal Points [9] described areas of geometry that children should learn by age and education level from pre-kindergarten to 12-year-olds.

In the Indonesia standard curriculum of early childhood education, it also described the indicators of geometry learning achievement for children aged 4-6 years [10]. It expected that children could: (1) Know and show geometric shapes based on the size of the objects; (2) Classify more objects into similar groups, or groups in pairs; (3) Classifying objects by colour, shape, and size; (4) Sorting objects from the smallest size to the largest or vice versa. Young children have already recognized shapes before they come to school because they see all kind of shapes in their surroundings. They might experience with shapes, shadow or other geometric phenomena in informal ways. For example, they know that the form of a door is rectangular and the wheel is round. However, having that knowledge does not guarantee that they understand geometry and can explain why such things happen. Therefore, the learning activities focusing on understanding geometry and spatial reasoning need to be given and taught to the children.

However, some studies [2,4,5,8,11,12] show that students are less involved in geometry learning that is rich in concepts and depictions and students are less well-versed with geometric phenomena in their surrounding world. Also, some international studies like TIMSS and PISA also indicate a weakness in student's geometry skills [1]. These deficits have been identified in the earliest years of life which caused by the teachers who do not always provide with adequate preparation in geometry as well as the teaching and learning of geometry [1,2,4]. Besides, it could be due to the lack of teachers' abilities in developing activities that appeal to students to learn geometry.

Therefore, this study aimed to design an instructional activity to guide the early childhood students in understanding Geometry and spatial reasoning related to shape. The idea of this research was to design a learning trajectory in geometry domain that giving children the meaningful experience in geometry and providing them the opportunity to develop geometric and spatial reasoning in particular shapes. The learning-teaching trajectory description for geometry in the lower grades emphasizes three aspects of geometry including orienting, constructing and operating with shapes and figures [8]. This research focused on learning trajectory in orienting or recognizing the variety of shapes. Utilizing the constructing activities, children will develop their skills to know and describe types of shapes in term of their color, form, and size. Consequently, the primary goal of this research was to develop children's learning process in geometry properties of shapes by constructing activities and providing meaningful learning for children by discussion. We hope this research can provide input for teachers in developing activities on geometry and spatial learning.

2. Research method

This research used design research methods as a tool to achieve the research objective of developing the Local Instructional Theory (LIT). Design research is a research method aiming to develop local learning theory or Local Instruction Theory (LIT) that can be done by researchers together with teachers to improve the quality of learning [13]. Furthermore, this study used authentic classroom assessment in the process of developing LIT that has been prepared. Research subjects were 13
children aged 4-5 years in one of the kindergartens in Banda Aceh, Indonesia.

![Design Research Procedure](image1.png)  
**Figure 1.** The design research procedure [14].

![Developmental Research](image2.png)  
**Figure 2.** Developmental research, a cumulative cyclic process [15].

According to van Den Akker [15], research design is divided into three main phases, namely preparing for the experiment, teaching experiment, and retrospective analysis. Those can repeatedly be conducted to discover a new theory as the result of a revision of the learning theory tested. It also related to Gravemeijer’s idea in [14] arguing that design research is a cyclical process (Figure 1) of thought and instruction experiments. The cycle process (recurring) is intended from the experiment of thought (thought experiment) and then experimental learning (instruction experiments). So the procedure of this study is described as Figure 2.

Data collection in this study was conducted using documentation, interviews, and observation. The data were analyzed descriptively by considering the validity and reliability of the data. The validity of the research took into account the Hypothetical Learning Trajectory and the trackability. Furthermore, a reliability test was conducted in two ways, namely data triangulation technique and cross interpretation.

The research begins with a study of literature on the concept of LIT geometry that will be developed before designing the learning activities. Based on the review of the literature, the first Hypothetical Learning Trajectory (HLT) which is the hypothesis of the LIT will then be designed. According to Gravemeijer & Cobb [12], HLT consists of three components, including (a) the purpose of mathematics learning for students; (b) the learning activities and tools or media used in the learning process; (c) the conjecture learning process, how to examine the understanding and strategies of students arisen and developed when the learning activities are conducted in the classroom. At the preliminary design stage, HLT serves as a guideline for instructional materials to be developed. While, at the teaching test stage, HLT serves as a guideline for teachers and researchers in teaching, interviewing, and observing [9]. While LIT aims at "... the description of, the rationale for, the envisioned learning route as it relates to a set of instructional activities for a specific domain." [15]. Therefore, we can define LIT as a theory of the process in which students learn a mathematical topic and theory about media or learning material used in helping the learning process of the topic. In addition, Gravemeijer & Van Eerde [2] said that LIT also contains conjecture about the possibilities occurred in the learning process and the chances of ways that can be done to support the learning process.
3. Result and discussion
The first step of this research was to design learning trajectory and Hypothetical Learning Trajectory (HLT) of geometry learning based on literature study of LIT for geometry and spatial reasoning in early childhood (4-5 years). The developed HLT contains three components, namely: (a) the learning objectives of the geometry for the children; (b) The learning activities that are accompanied by devices or media used in the learning process of geometry and spatial reasoning; and (c) The conjecture learning process occurs in the classroom. The preparation of this HLT was based on preliminary study results on the child’s ability and limited discussion with the classroom teachers about the experiences of teaching the concepts of geometric and spatial reasoning. The results of this discussion showed that the teacher only introduced geometric forms classically in front of the class during the geometric learning. This activity was conducted by indicating if this object was a circle or a rectangle or just a triangle without continuing with directional activity. On the other hand, students were directed to construct blocks and play with blocks; however, it was unclear what skills and competencies they should master. Teachers made the lesson plans before teaching geometry yet often the planning they made was useless because they did not implement them in their teaching where the teachers only write a lesson plan to address a requirement by the school’s principal. Besides, based on interviews conducted with teachers it was found that the lack of teachers’ knowledge about the concept of geometry was also a constraint in designing the learning activities for geometric and spatial reasoning. Consequently, the research concerning activities related to geometric and spatial reasoning in school is insufficient.

| No | Learning objectives/competencies | Activities |
|----|----------------------------------|------------|
| 1  | Children can interpret the physical world with geometric ideas and describe it with corresponding vocabularies | Nature exploration |
| 2  | Children can find shapes in their environments and describe them in their own words. | Recognize shape |
| 3  | Children can build figures and designs by combining two- and three-dimensional shapes | Create the picture |
|   |                                                                                 | Shape picture |
| 4  | Children can discuss the relative positions of objects with vocabulary such as “above,” “below,” and “next to.” | Put the ball |
| 5  | Children can identify names and describe a variety of shapes with different sizes and orientations. | Identify the shape |
| 6  | Children can classify more objects into similar groups, or groups in pairs | Find the similar shape |
|   |                                                                                 | Coloring the similar shape |

The HLT developed refers to four geometric and spatial reasoning competencies that must be acquired by children aged 4-5 years based on NCTM [14] and Indonesian curriculum [10]. Table 1 shows a series of geometry learning activities along with the competencies to be achieved after the revision was done (based on the validators’ feedbacks and the findings of the pilot experiment), while the learning trajectory stage can be seen in Figure 3.
In *Nature exploration* activity, the children were taken out of the classroom to explore and observe the school environment. During the activity, children were asked to name objects similar to geometric shapes they can find in the school environments, such as windows and doors that are rectangular, tires of cars and motorcycles that are circular, the roof of a house that is the triangular shape. In this activity, the researcher found that the children are familiar with geometric objects, but they cannot explain correctly like saying a circle on the ball (meaning a circle shape on the ball), side three for a triangle.

In the *Recognize shape & create the picture* activity, the children were asked to draw the objects they found during their browsing activities. The children were free to draw any object, and then they must describe what they have drawn (Figure 4). This activity is excellent to build children's creativity and imagination. They can explore their ability in drawing all the objects that they see.

**Figure 3.** Learning trajectory in geometry & spatial reasoning for early childhood.

**Figure 4.** Children’s attempt to draw the table in their class as a rectangular object.

**Figure 5.** Children design their imaginary picture.

In *Put the ball* activity, the children were given a ball. Teachers asked them to put the ball based on the instructions given such as place the ball on the table, next to the cupboard, under the seat and so on. The other directions were given to the children, such as, "I want Syifa to put her ball inside a ….” “Nila should put his ball under a table.” And this was continued until all the children have had their turn. The reflection gained from this activity is that teachers should use a larger area than in a classroom because it has limited places to play.

In *shape picture* activities, the teacher gives children pieces of color paper cut into basic shapes: circles, squares, triangles, and rectangles. Then ask them to glue the various shapes onto a piece of paper to create their shape picture. Appearing conjecture in this activity was that children were able to
create their imaginary objects creatively, although the color paper they used was coinciding with each other. However, the interesting finding of this activity is that some students tend to be less creative when given shape in a colorful paper (Figure 5a) because they assume that the shapes are separate images from each other.

4. Conclusion
This research created a Local Instructional Theory (LIT) on the learning of geometry and spatial reasoning of early childhood students that can be used by teachers and parents to teach the concept and develop geometry and spatial reasoning abilities of early childhood students, especially on the aspect of orienting or recognizing the variety of shapes for 4-5 year olds. Then, the activities that have been promoted in this LIT can be a good start to help children in developing their understanding about geometry and spatial reasoning for the future.

Acknowledgment
We would like to thank many people who have contributed to this research and supported us through the process. We are very grateful to our institution of STKIP BBG and Hiroshima University that supported and funded us to conduct this research.

References
[1] Sarama J, Clements D H, Parmar R S and Garrison R 2006 Achieving Fluency in Special Education and Mathematics
[2] Clement D H 2008 Geometric and Spatial Thinking in Early Childhood Education (State University of New York: University at Buffalo)
[3] National Mathematics Advisory Panel 2008 Foundations for Success: the Final Report of the National Mathematics Advisory Panel (Washington D.C.: U.S. Department of Education, Office of Planning, Evaluation, and Policy Development)
[4] Clement D H and Sarama J 2011 J. Mathematics Teacher Education. 14(2) 133-48.
[5] Cheng Y L and Mix K S 2014 J. Cognition and Development. 15 2–11.
[6] Cross C T, Taniesha A W and Heidi S 2009 Mathematics Learning in Early Childhood Paths Toward Excellence and Equity (United States of America: Committee on Early Childhood Mathematics)
[7] Lestari, KW 2011 Konsep Matematika Untuk Anak Usia Dini (Jakarta: Direktorat Jendral Pendidikan AUD Nonformal dan Informal)
[8] van den Heuvel-Panhuizen, M. 2005 Young Children Learn Measurement and Geometry: Measurement and Geometry in Line (The Netherlands: Freudenthal Institute, Utrecht University)
[9] National Council of Teachers of Mathematics (NCTM) 2006 Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence (Reston, Va.: NCTM)
[10] Permendikbud 146 Tahun 2014 Kurikulum 2013 PAUD dan Lampirannya. (Jakarta: Peraturan Menteri Pendidikan Dan Kebudayaan)
[11] Acharya A and Ghose A 2015 J. IJEPR. 4(4) 91-8
[12] Jardine R, and Martin N S 1983 Spatial ability and throwing accuracy Behavior Genetics 13 331–40.
[13] Gravemeijer K and Van Eerde D 2009 The Elementary School Journal 109 5
[14] Novita R and Putra M 2017 J. Pendidikan Matematika 11(1) 43-56
[15] Gravemeijer K and Cobb P 2006 Design research from a learning design perspective eds Van Den Akker, K Gravemeijer, S, McKenney, & N. Nieveen Educational Design Research pp. 17 – 51 (New York: Routledge)