Towards Holographic QCD
AdS/CFT, Confinement Deformation, and DIS at Small-x

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Rencontres de Moriond QCD and High Energy Interactions
To Appear

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Outline

• Background and Motivations

• Soft Wall Model

• Numerical Analysis

• Conclusions and Future
Physical Motivations

- QCD has been a resounding success for describing some areas of strong-force physics: Flavor, Color, Asymptotic Freedom ($\beta < 0$), etc.. But there are still physical regimes that are not well understood: n-particle scattering (amplitudes), strong coupling, confinement, etc.

- Deep Inelastic Scattering (DIS) allows us to probe hadronic processes using a relatively simple probe. In the regge limit ($s \gg |t|$) the scattering is non-perturbative. The scattering process is affected by color confinement.

- By taking a holographic approach where we have a dual description involving quantum gravity we can use perturbative string theory in 10 dimensions to describe strongly coupled four dimensional physics.
Questions To Keep in Mind

**Question**

Is strong coupling appropriate?

**Guide**

In many regimes, DIS can be treated perturbatively, but at small enough $x$ (for fixed $Q^2$), particularly in the forward or near forward limits, the process becomes generically nonperturbative.

**Question**

Is confinement important?

**Guide**

Even for single pomeron exchange, we will see confinement playing a role in determining the onset of saturation.
The AdS/CFT is a holographic duality that equates a string theory (gravity) in high dimension with a conformal field theory (gauge) in 4 dimensions. Specifically, compactified 10 dimensional super string theory is conjectured to correspond to $\mathcal{N} = 4$ Super Yang Mills theory in 4 dimensions in the limit of large 't Hooft coupling: $\lambda = g_s N = g_{ym}^2 N_c = R^4 / \alpha'^2 \gg 1$.

The compactified geometry is a negatively curved space times a sphere: $AdS_5 \times S^5$

$$ds^2 = \frac{R^2}{z^2} \left[ dz^2 + dx \cdot dx \right] + R^2 d\Omega_5 \rightarrow e^{2A(z)} \left[ dz^2 + dx \cdot dx \right] + R^2 d\Omega_5$$

As the function $A(z)$ changes, the space is deformed away from pure AdS
The correspondence relates string modes to CFT states via the correspondence:

$$\left\langle e^{\int d^4x \phi_i(x) \mathcal{O}_i(x)} \right\rangle_{CFT} = Z_{\text{string}} [\phi_i(x, z) | z \to 0]$$

- At high energies QCD is asymptotically free and approximately conformal

- The similarity between QCD and $\mathcal{N} = 4$ SYM allows us to study the strong interaction using string theory

- The AdS/CFT conjecture has passed many (theoretical) tests (spectra of operators, wilson loops, correlation functions, etc.)

- The AdS/CFT is robust: the correspondance should allow for relevant deformations, finite temperatures, etc.

- A lot of success has been found for the AdS/CFT in heavy ion collisions, calculating entanglement entropies, and describing conformal field theories.
• Where are the single quarks? Naively, this could be explained by a quark-quark energy that grows with separation. At large distance it becomes energetically favorable to create new quarks.

• Wilson originally used wilson loops $W = \frac{1}{N} tr P \exp \left( ig \oint_C A \right)$ to try and describe confinement. In the limit of large times, a square path for a quark corresponds to the energy of two static quarks.

• In a confining theory one expects the expectation of the wilson loop to have an area dependence: $\langle W \rangle \sim \exp(-\sigma \text{Area})$

• In AdS Wilson loops in $\mathcal{N} = 4$ SYM are dual to minimal surfaces that extend into the bulk AdS.[Maldacena],[Polyakov] Note, in pure AdS, distances diverge at the boundary (small $z$) and become small in the interior of the bulk (large $z$).
Soft Wall Basics

In order to confine the theory one must effectively deform the AdS geometry. This can be done via:

- Sharp cutoff – \( z = z_0 \approx \frac{1}{\Lambda_{QCD}} \) (Hard Wall Model) [Polchinski, Strassler], [Brower, Djuric, Sarcevic, Tan]

- Gradual increase in length scales / large effective potential boundary for large \( z \) leads to possible bound states: confinement

For our geometric softwall, the deformation function becomes \( A(z) \to \Lambda^2 z^2 - \log(z/R) \). This leads to a metric

\[
ds^2 \to \frac{e^{2\Lambda^2 z^2} R^2}{z^2} \left[ dz^2 + dx \cdot dx \right]
\]

We wish to use this soft wall model to describe deep inelastic scattering at leading order in the regge-limit. The object of interest is the AdS-pomeron, which was identified to be the Regge trajectory of the graviton [Brower, Polchinski, Strassler, Tan]. For us, it is sufficient to consider a purely geometric confinement deformation. However, to describe mesons it will be required to consider other dynamical fields in the bulk. [Karch, Katz, Son, Stephanov], [de Teramond, Brodsky], [Batell, Gherghetta]

Brower, Djuric, TR, Tan (Brown)
Propagators and Wave functions

In this framework the pomeron propagator obeys:

\[
-\partial_z^2 + 10\Lambda^2 + 4\Lambda^4 z^2 - t + \frac{12 - \alpha^2(j)R^5}{z^2} \chi_P(j, z, z', t) = \delta(z - z')
\]

The solution to this equation can take several forms. For quantized momentum transfer \( t_n \) the solution becomes

\[
\chi_P \sim (\Lambda^2 z z')^{\alpha(j)+1} e^{-\Lambda^2(z^2+z'^2)} L_n^{\alpha}(2\Lambda^2 z^2) L_n^{\alpha}(2\Lambda^2 z'^2)
\] (1)

Where as for a continuous \( t \) spectrum the solution becomes a combination of Whittaker’s functions (generalized hyper geometric functions)

\[
\chi_P \sim \ldots M_{\kappa,\mu}(z_{<}) W_{\kappa,\mu}(z_{>})
\] (2)

for \( \kappa = \kappa(t) \) and \( \mu = \mu(j) \)
**Special Limits, Behavior, and Symmetry**

- $\Lambda$ controls the strength of the soft wall and in the limit $\Lambda \to 0$ one recovers the conformal solution

$$\text{Im} \chi_P^{\text{conformal}}(t = 0) = \frac{g_0^2}{16} \sqrt{\frac{\rho^3}{\pi}} (zz') e^{(1 - \rho)\tau} \frac{e^{(1 - \rho)\tau}}{\tau^{1/2}} \exp \left( \frac{-(\text{Log} z - \text{Log} z')^2}{\rho \tau} \right)$$

where $\tau = \text{Log}(\rho zz' s/2)$ and $\rho = 2 - j_0$. Note: this has a similar behavior to the weak coupling BFKL solution where

$$\text{Im} \chi(p_\perp, p'_\perp, s) \sim \frac{s^{j_0}}{\sqrt{\pi D \text{Log} s}} \exp(- (\text{Log} p'_\perp - \text{Log} p_\perp)^2 / D \text{Log} s)$$

- If we look at the energy dependence of the pomeron propagator, we can see a softened behavior in the regge limit.

$$\chi^{\text{conformal}} \sim -s^{\alpha_0} \text{Log}(s) \to \chi_{HW} \sim -s^{\alpha_0} / \text{Log}(s)$$

Analytically, this corresponded to the softening of a j-plane singularity from $\sqrt{j - j_0} \to 1 / \sqrt{j - j_0}$. Again, we see this same softened behavior in the soft wall model.

- (Possibly) interesting limit $t = 10\Lambda^2$. Here the EOM simplifies and takes the form of a model with 1+1 dimensional conformal symmetry [Fubini]
DIS in AdS

We are interested in deep inelastic scattering (DIS) characterized by a virtual photon off a proton ($\gamma^* p$)

To characterize this process we consider the CM energy $s \approx Q^2 / x$ for $s$ large. In the regge limit, with $Q^2$ fixed, we can treat this process via the exchange of pomerons. (leading order exchange in a sommerfeld-watson decomposition). The primary route to physical relevance is via the optical theorem

$$\sigma_{total} = \frac{1}{s} \text{Im} [A(s, t = 0)] \sim \frac{1}{s} \text{Im} [\chi(s, t = 0)]$$

We can use this to calculate total cross sections and to determine the proton structure function

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}} (\sigma_{trans} + \sigma_{long})$$

Finally we must be wary of saturation where we must consider multipomeron exchange via eikonalization

$$\chi \rightarrow 1 - e^{i\chi}$$

[Cornalba, Costa, Penedones][Brower, Strassler, Tan]
We will consider the combined H1 and Zeus data set published in 2010 [Aaron, et. al.][Chekanov, et. al.], but we restrict ourselves to small-$x$ data, $x < 0.01$. We can write a scattering amplitude as

$$\mathcal{A}(s, t) = s \int_{\text{bulk}} dzdz' P_{13}(z) P_{24}(z') \chi(s, t, z, z')$$

In the conformal limit we can model the photon with a function peaked around $z \sim 1/Q$. To simplify things and to include confining models we will make a local approximation for both the photon and proton where

$$P_{13}(z) \rightarrow \delta(z - 1/Q) \text{ and similarly } P_{24}(z) \rightarrow \delta(z' - 1/Q')$$

Note: The softwall solutions are very numerically slow to handle!
The structure function $F_2(x, Q^2)$ plotted for various values of $Q^2$. The data points are from the H1-Zeus collaboration and the solid lines are the soft wall fit values.
Contour plots of $\text{Im}[\chi]$ as a function of $1/x$ vs $Q^2$ (Gev) for conformal, hardwall, and softwall models. These plots are all in the forward limit, but the impact parameter representation can tell us about the onset of non-linear eikonal effects. The similar behavior for the softwall implies a similar conclusion about confinement vs saturation.
Comparison With Previous Work

| Model       | $\rho$    | $g_0^2$    | $z_0$        | $Q'$       | $\chi^2_{dof}$ |
|-------------|-----------|------------|--------------|------------|----------------|
| conformal   | 0.774*    | 110.13*    | -            | 0.5575* GeV| 11.7 (0.75*)  |
| hard wall   | 0.7792    | 103.14     | 4.96 GeV$^{-1}$ | 0.4333 GeV | 1.07 (0.69*)  |
| softwall    | 0.7774    | 108.3616   | 8.1798 GeV$^{-1}$ | 0.4014 GeV | 1.1035        |
| softwall*   | 0.6741    | 154.6671   | 8.3271 GeV$^{-1}$ | 0.4467 GeV | 1.1245        |

Comparison of the best fit (including a $\chi$ sieve) values for the conformal, hard wall, and soft wall AdS models. The final row includes the soft wall with improved intercept. [Costa, Goncalves, Penedones][Gromov, Levkovich-Maslyuk, Sizov, Valatka] The statistical errors (omitted) are all $\sim 1\%$ of fit parameters.

As expected, best fit values imply

$$
\rho \rightarrow \lambda > 1 \quad \frac{1}{z_0} \sim \Lambda_{QCD} \quad \text{and} \quad Q' \sim m_{proton}
$$
Conclusions and Future Work

Conclusions:
- DIS in small-$x$ regge regimes can be well approximated using the AdS/CFT
- Single pomeron exchange affects a large part of $x$ and $Q^2$ space
- Confinement seems to affect the onset of saturation in a variety of models

Future Directions:
- Softwall eikonal (better numerics or more clever solutions)
- AdS EOM to higher order in $\lambda$ (Hard string calculation!) [Costa, Goncalves, Penedones][Gromov, Levkovich-Maslyuk, Sizov, Valatka]
- Extend to meson exchange. [Karch, Katz, Son, Stephanov] [Brodsky, de Teramond]
- Investigate anomolous dimensions via $\Delta(j)$