Observation of the $\eta_c(2S)$ in exclusive $B \to KK_SK^-\pi^+$ decays

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Abstract

We report the observation of a narrow peak in the $K_S K^- \pi^+$ invariant mass distribution in a sample of exclusive $B \to KK_S K^- \pi^+$ decays collected with the Belle detector at the KEKB asymmetric $e^+e^-$ collider. The measured mass of the peak is $M = 3654 \pm 6\text{(stat)} \pm 8\text{(syst)}$ MeV$/c^2$ and we place a 90% confidence level upper limit on the width of $\Gamma < 55$ MeV$/c^2$. The properties agree with heavy-quark potential model expectations for the $\eta_c(2S)$ meson, the $n = 2$ singlet $S$ charmonium state.
Major experimental issues for the charmed-quark anticharmed-quark \((c\bar{c})\) charmonium particle system are the two \(c\bar{c}\) states that are expected to be below open charm threshold but are still not well established: the radially excited \(n = 2\) singlet \(S\) state, the \(\eta_c(2S)\) meson, and the \(n = 1\) singlet \(P\) state, the \(h_c(1P)\). The observation of these states and the determination of their masses would complete the below-threshold charmonium particle spectrum and provide useful information about the spin-spin part of the charmonium potential \([1]\).

\(B\) meson decays provide an excellent opportunity for searching for the \(\eta_c(2S)\) and clarifying its properties. They are a copious \(\eta_c(1S)\) source: the decays \(B \to K\eta_c(1S)\) have been observed by CLEO \([2]\), BaBar \([3]\), and Belle \([4]\) with relatively large branching fractions: \(\mathcal{B}(B \to K\eta_c(1S)) \simeq \mathcal{B}(B \to KJ/\psi) \simeq 1 \times 10^{-3}\). (In the following, we use \(\eta_c\) to denote the \(\eta_c(1S)\).) Moreover, in the case of the triplet charmonium \(S\) states, \(B\) meson decays to the radially excited \(\psi(2S)\) are nearly as common as those to the \(n = 1\) \(J/\psi\) radial ground state: \(\mathcal{B}(B^+ \to K^+\psi(2S))/\mathcal{B}(B^+ \to K^+J/\psi) \sim 0.6\) \([4]\). Thus, it is reasonable to expect the decays \(B \to K\eta_c(2S)\) to occur at a rate comparable to that for \(B \to K\eta_c\). Unlike the \(J/\psi\) and \(\psi(2S)\), where hadronic decays proceed via highly suppressed three-gluon intermediate states, the \(\eta_c\) and \(\eta_c(2S)\) decay via less-suppressed two-gluon processes. As a result, intercharmonium transitions are not very important and the hadronic decay branching fractions for the two states are expected to be similar \([4]\). Thus, any final state that shows a strong \(B \to K\eta_c\) signal is a promising channel for an \(\eta_c(2S)\) search.

A simple application of heavy-quark potential models \([4]\) predicts a \(\psi(2S)-\eta_c(2S)\) mass splitting that is smaller than that for the ground-state \(J/\psi-\eta_c\) splitting because of the smaller value of the wave function at zero \(c\bar{c}\) separations and the running of the QCD coupling strength \(\alpha_s(M^2)\). These models predict an \(\eta_c(2S)\) mass in the range \(3625 < M_{\eta_c(2S)} < 3645\) MeV/c\(^2\). Similar factors result in the expectation that the \(\eta_c(2S)\) total width is somewhat smaller than that of the \(\eta_c\).

The Crystal Ball group \([8]\) reported an excess of \(E_\gamma \simeq 91\) MeV gamma rays from inclusive \(\psi(2S) \to \gamma X\) decays, and interpreted this as possible evidence for the \(\eta_c(2S)\) with mass \(3594 \pm 5\) MeV/c\(^2\). This result implies a \(\psi(2S)-\eta_c(2S)\) mass splitting that is considerably larger than heavy-quark model expectations. The result has not been confirmed by other experiments \([4]\).

In this letter we report a search for the \(\eta_c(2S)\) produced via the processes \(B^+ \to K^+\eta_c(2S)\) and \(B^0 \to K_S\eta_c(2S)\), where \(\eta_c(2S) \to K_SK^-\pi^+\) \([1]\). We concentrate on this final state because it is a strong decay channel for the \(\eta_c\) \((B \simeq 1.8\%)\), has low combinatorial backgrounds, and, since the final state contains all charged particles, is reconstructed with good resolution. Moreover, the process \(\psi(2S) \to K_SK^-\pi^+\) is strongly suppressed, and the background in this channel from \(B \to K\psi(2S)\) decays is expected to be less than 0.1 events.

The search uses a 41.8 fb\(^{-1}\) data sample collected with the Belle detector \([1]\) at the KEKB \(e^+e^-\) collider \([12]\) operating at the \(\Upsilon(4S)\) resonance \((\sqrt{s} = 10.58\) GeV\). The data sample contains 44.8 million \(B\bar{B}\) meson pairs.

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a three-layer silicon vertex detector, a 50-layer central drift chamber (CDC), an array of aerogel threshold Čerenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals located...
inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux- 
return located outside of the coil is instrumented to detect $K_L$ mesons and to identify muons. 
The detector is described in detail elsewhere [11].

We select events with $K^+K_S^0K^{±\pi^±}$ or $K^0\bar{K}S\pi^±\pi^±$ combinations. 
Here the charged 
kaon (pion) tracks are required to originate from within $\delta r < 0.3$ cm and $|\delta z| < 2.2$ cm 
of the run-by-run determined interaction point (IP) in the transverse ($r\phi$) and beamline ($z$) 
directions, respectively. In addition, they must be positively identified as kaons (pions) by 
the combined information from the ACC, TOF and CDC $dE/dx$ measurement. Candidate 
$K_S^0 \rightarrow \pi^+\pi^-$ decays correspond to pairs of oppositely charged tracks with invariant mass 
within 12 MeV/$c^2$ (3$\sigma$) of $M_{K^0}$ that originate from a common vertex that is displaced 
by more than 0.3 cm from the IP. The direction of the $K_S^0$ momentum vector is required to be 
within 0.2 rad of the direction between the IP and the position of the displaced vertex.

Candidate $B$ mesons are reconstructed using the energy difference $\Delta E \equiv E_{\text{cm}}^B - E_{\text{beam}}^B$ 
and the beam-energy constrained mass $M_{bc} \equiv \sqrt{(E_{\text{cm}}^B)^2 - (p_B^B)^2}$, where $E_{\text{beam}}^B$ is 
the center of mass (cms) beam energy, and $E_{\text{cm}}^B$ and $p_B^B$ are the cms energy and momentum of 
the $B$ candidate. The signal region is defined as $5.271 < M_{bc} < 5.287$ GeV/$c^2$ and $|\Delta E| < 
0.040$ GeV, which correspond to $±3\sigma$ from the central values for both quantities.

In order to suppress background from the $e^+e^- \rightarrow q\bar{q}$ continuum ($q = u, \ d, \ s \ & \ c$), 
we form a likelihood ratio from two variables. One is a Fisher discriminant determined 
from five modified Fox-Wolfram moments [13], the cosine of the angle formed by the thrust 
axis of the candidate $B \rightarrow KK_S\pi^±$ tracks and that of the remaining tracks in the 
event, and the sum of the absolute values of transverse momenta of particles relative to the 
$B$ candidate’s thrust axis with angle larger than 60° normalized by the sum of the total 
momenta. The coefficients of the Fisher discriminant are chosen to optimize the separation 
between signal and continuum Monte Carlo (MC) events [14]. The other is the cosine of the 
angle between the $B$ candidate flight direction and the beam axis in the $\Upsilon(4S)$ rest 
frame ($\cos \theta_B$). Normalized probability density functions (pdfs) formed from the Fisher 
discriminants and the $\cos \theta_B$ distribution are multiplied to form likelihood functions for 
the signal ($L_{\text{sig}}$) and continuum ($L_{\text{cont}}$) processes. We select events with a likelihood ratio 
$LR \equiv L_{\text{sig}}/(L_{\text{sig}}+L_{\text{cont}}) > 0.6$, which was determined by optimizing $S/\sqrt{S+\bar{B}}$ ($S$ and $B$ are 
signal and background, respectively) for Monte Carlo simulations of the process $B \rightarrow K\eta_c$, 
where $\eta_c \rightarrow K_SK^-\pi^+$.

We reduce potential backgrounds from $B \rightarrow D(D_s)X$ decays by rejecting $D$ and $D_s$ 
mesons with the requirements $|M_{K_S^0}-M_D| > 10$ MeV/$c^2$ and $|M_{K_S^0K^±}-M_{Ds}| > 10$ MeV/$c^2$.
The decay $\eta_c(nS) \rightarrow K^*(890)K$ is suppressed by an angular momentum barrier; in order to 
reduce backgrounds from other $B$ meson decay modes with minimal loss in signal, we reject 
events with a $K^*$ candidate with the requirement $|M_{K_S^0}-M_{K^*}| > 50$ MeV/$c^2$.

Figure [14] shows the $M_{bc}$ projections of events in the $|\Delta E| \leq 0.040$ GeV signal region for 
twenty five $M_{K_S^0K^±}$ mass bins, each 40 MeV/$c^2$-wide and with central values ranging from 
2840 through 3800 MeV/$c^2$. The mass bins of Figs. [14](d) and (e) straddle $M_{\eta_c}$ and clear 
peaks corresponding to $B \rightarrow K\eta_c, \eta_c \rightarrow K_SK^-\pi^+$ decays are apparent. Figures [14](u) and 
(v), which cover a region near the expected mass of the $\eta_c(2S)$, also show distinct $B$ meson 
signals.
FIG. 1. The $M_{bc}$ projections for 40 MeV/c$^2$ bins of $M_{K_S K\pi}$, with central values ranging from 2840 (a) to 3800 MeV/c$^2$ (y). Only events with $|\Delta E| < 40$ MeV are included; the charged and neutral $B$ decay modes are combined.

We perform simultaneous fits to each of the $M_{bc}$ distributions of Fig. 1 and the corresponding $\Delta E$ distributions for events in the $M_{bc}$ signal region (not shown). The fits use Gaussian functions with MC-determined widths to represent the signals; the areas of the $M_{bc}$ and $\Delta E$ signal functions are constrained to be equal. The $M_{bc}$ background is modeled by a smooth function that behaves like phase-space near the kinematic end point [15]; for the $\Delta E$ background, we use a second-order polynomial. As an example, the results of the fit to the $M_{bc}$ and $\Delta E$ distributions of the $M_{K_S K\pi} = 3640$ MeV/c$^2$ bin are shown in Figs. 2(a) and (b), respectively.
The signal yields extracted from the simultaneous fits to the different $K_SK^-\pi^+$ mass bins are plotted vs. $M_{K_SK\pi}$ in Fig. 2 where, in addition to a prominent $\eta_c$ peak and a hint of a $J/\psi$, a clear peak at higher mass is evident. We identify this as a candidate for the $\eta_c(2S)$. Between the peaks is a non-zero, non-resonant contribution. The curve in Fig. 2 is the result of a fit with simple Breit-Wigner functions that represent the $\eta_c$ and candidate $\eta_c(2S)$, a Gaussian function with mass and width fixed at the $J/\psi$ values, and a second-order polynomial to represent the non-resonant contribution. These functions are convolved with a Gaussian resolution function with a MC-determined width of $\sigma = 15\text{ MeV}/c^2$.

The fit values for the event yields, masses and total widths of the $\eta_c$ and the $\eta_c(2S)$ candidate state are listed with their statistical errors in Table I. The fit value for the $\eta_c$ mass is in good agreement with the world-average value of $M_{\eta_c} = 2979.8 \pm 1.8\text{ MeV}/c^2$ [5]; the value for the $\eta_c$ width is consistent, within its rather large errors, both with the existing world average of $\Gamma_{\eta_c}^{\text{tot}} = 13.2^{+3.8}_{-3.2}\text{ MeV}/c^2$ [5] and the recent CLEO result of $26\pm6\text{ MeV}/c^2$ [10].

The sum of the observed events in the three mass bins in the signal region (i.e., centered around $M(K_SK\pi) = 3640\text{ MeV}/c^2$) is 56, while the integral of the second-order polynomial over the same interval gives a non-resonant expectation of $21 \pm 2$ events. The probability for this to fluctuate up to 56 events is $\sim 10^{-8}$, which corresponds to a signal significance of more than $6\sigma$.

The fitted mass of the candidate $\eta_c(2S)$ is substantially above the Crystal Ball mass

FIG. 2. The (a) $M_{bc}$ and (b) $\Delta E$ projections for the $M_{K_SK\pi} = 3640\text{ MeV}/c^2$ mass bin. The curves are the results of the simultaneous fit described in the text.

FIG. 3. The distribution of signal events from the simultaneous fits to $M_{bc}$ and $\Delta E$ for each $K_SK\pi$ mass bin. The curve is the result of the fit described in the text.
value and consistent, within errors, with the upper end of potential model expectations. The systematic error on the mass is evaluated by redoing the analysis using different likelihood ratio selection requirements, 50 MeV/c^2-wide bins, bins with central values shifted by half a bin-width, and with different values of the experimental resolution. The maximum change in the fitted mass value is 8 MeV/c^2, which is taken as the systematic error. The limited statistics and the resolution precludes a precise measurement of the width. However, we can establish a 90% confidence level upper limit of \( \Gamma < 55 \text{ MeV}/c^2 \).

Monte Carlo simulations indicate that the acceptance is very nearly constant for the \( K_S K^- \pi^+ \) mass region covered by this measurement \[17\]. Thus, the ratio of product branching fractions for the \( \eta_c \) and \( \eta_c(2S) \) is just the ratio of event yields:

\[
\frac{B(B \rightarrow K\eta_c(2S))B(\eta_c(2S) \rightarrow K_S K^- \pi^+)}{B(B \rightarrow K\eta_c)B(\eta_c \rightarrow K_S K^- \pi^+)} = 0.38 \pm 0.12 \pm 0.05 ,
\]

where the first error is statistical and the second systematic. The systematic error is determined from changes in the ratio observed for different binning, values of resolution, and functions used to model the non-resonant contribution.

In summary, we observe a peak in the \( K_S K^- \pi^+ \) mass from exclusive \( B^+ \rightarrow K^+ K_S K^- \pi^+ \) and \( B^0 \rightarrow K_S K_S K^- \pi^+ \) decays with mass and width values

\[
M = 3654 \pm 6 \pm 8 \text{ MeV}/c^2 \\
\Gamma < 55 \text{ MeV}/c^2 ;
\]

these are consistent with expectations for the \( B \rightarrow K\eta_c(2S) \), where \( \eta_c(2S) \rightarrow K_S K^- \pi^+ \). In addition, the product branching fraction is comparable in magnitude to that for the \( \eta_c \), also in agreement with expectations for the \( \eta_c(2S) \). The observed properties of this system lead us to conclude that we have observed the \( \eta_c(2S) \).

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TABLE I. Results of the fit to the data points in Fig. 3. Only statistical errors are listed.

| Peak   | Events  | Mass (MeV/$c^2$) | $\Gamma^{\text{tot}}$ (MeV/$c^2$) |
|--------|---------|------------------|-----------------------------------|
| $\eta_c$ | $104 \pm 14$ | $2979 \pm 2$ | $11 \pm 11$ |
| $\eta_c(2S)$ | $39 \pm 11$ | $3654 \pm 6$ | $15^{+24}_{-15}$ |
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