On 15-component theory of a charged spin-1 particle
with polarizability in Coulomb and Dirac monopole fields

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The problem of a spin 1 charged particle with electromagnetic polarizability,
obeying a generalized 15-component quantum mechanical equation, is investi-
gated in presence of the external Coulomb potential. With the use of the Wigner’s
functions techniques, separation of variables in the spherical tetrad basis is done
and the 15-component radial system is given. It is shown that there exists a
class of quantum states for which the additional characteristics, polarizability,
does not manifest itself anyhow; at this the energy spectrum of the system coin-
cides with the known spectrum of the scalar particle. For \( j = 0 \) states, a 2-order
differential equation is derived, it contains an additional potential term \( r^{-4} \).

In analogous approach wave functions of the generalized particle are examined
in presence of external Dirac monopole field. It is shown that there exists one
special state with minimal conserved quantum number \( j_{\text{min}} \). It this solution, first,
the polarizability does not exhibits itself, and second, this solution provides us
with analogue of the known peculiar solution in the theory of the Dirac particle in
external monopole field, which can describe certain bound quantum mechanical
state.

Analysis of the usual vector particle in external Coulomb potential, additional
 to that existing in the literature, is given. It is shown that at \( j = 0 \) some bound
states will arise. The corresponding energy spectrum is found.

1 Basic equation and notation

Initial equation for a 15-component wave function of the particle with electric charge and
polarizability [1-8] has in matrix formalism the form

\[
(\Gamma^a \partial_n - m) \Psi = 0, \quad \Psi = \begin{bmatrix} C \\ C_l \\ \Phi_t \\ \Phi_{mn} \end{bmatrix}, \quad \Gamma^a = \begin{bmatrix} 0 & G^a & 0 & 0 \\ 0 & 0 & 0 & K^a \\ 0 & 0 & K^a & 0 \\ 0 & 0 & \Lambda^a & 0 \end{bmatrix}
\]

(1)

here blocks of dimensions \( 1 \times 4, 4 \times 1, 4 \times 6, 6 \times 4 \) respectively are used:

\[
(G^a)^{(0)}_n^k = g^a^k, \quad (\Delta^a)^{(0)}_n = \delta^n_a.
\]

\[
(K^a)_n^{kl} = -g^a^k \delta_l^a + g^a_l \delta^a_k, \quad (\Lambda^a)_n^{mb} = \delta_a^m \delta_b^n - \delta_a^n \delta_b^m.
\]

The \( \sigma \) stands for a free parameter of the model. It concerns with an additional characteristic
of \( S = 1 \) particle manifesting itself in external electromagnetic and gravitational fields.

As was shown in [1], equation (1.1) can be extended to general relativity case [9-12] in
the following way

\[
[ \Gamma^a (x) (\partial_\alpha + B_\alpha (x)) - m ] \Psi(x) = 0,
\]

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\[ \Gamma^\alpha(x) = \Gamma^\alpha e^{\alpha}_{(a)}(x), \quad B_\alpha(x) = \frac{1}{2} J^{ab} e^\beta_{(a)} \nabla_\alpha (e_{(b)}^\beta). \] (3)

In the present work, some technical possibilities in equations (2) underlying by the used tetrad formalism will be widely exploited in studying behavior of the generalized vector particle in presence of Coulomb and Dirac monopole fields. In that sense, present Section helps us prepare for detailed treatment of these two problems.

Let us consider equation (2) in Minkowski space assuming the use of spherical coordinates and a diagonal tetrad:

\[ dS^2 = c^2 dt^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2) = g_{\alpha\beta} dx^\alpha dx^\beta, \]

\[ x^\alpha = (ct, r, \theta, \phi), \quad g_{\alpha\beta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \theta \end{bmatrix}, \]

\[ e^{0}_{(0)} = (1, 0, 0, 0), \quad e^{\alpha}_{(3)} = (0, 1, 0, 0), \]

\[ e^{\alpha}_{(1)} = (0, 0, \frac{1}{r}, 0), \quad e^{\alpha}_{(2)} = (1, 0, 0, \frac{1}{r \sin \theta}). \] (4)

In tetrad (4), equation (1.3) will looks

\[ \left[ \Gamma^0 \partial_0 + \Gamma^3 \partial_r + \frac{\Gamma^1 J^{31} + \Gamma^2 J^{32}}{r} + \frac{1}{r} \Sigma_{\theta,\phi} - m \right] \psi(x) = 0, \] (5)

where \( \Sigma_{\theta,\phi} \) designates \( \theta, \phi \) - dependent operator

\[ \Sigma_{\theta,\phi} = \Gamma^1 \partial_\theta + \Gamma^2 \frac{\partial_\phi + \cos \theta J^{12}}{\sin \theta}. \] (6)

We will need explicit representation for \( \Gamma^a \)-matrices. With the use of notation

\[ \bar{e}_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad \bar{e}_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \quad \bar{e}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \]

\[ \bar{e}_1^t = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{e}_2^t = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \bar{e}_3^t = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \]

\[ \tau_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \tau_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad \tau_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \]

the \( \Gamma^a \) look as

\[ \Gamma^0 = \begin{bmatrix} 0 & 1 & 0 & 0 & \bar{e} & \bar{e} & \bar{e} & \bar{e} \\ 0 & 0 & 0 & 0 & 0 & 0 & \bar{e} & \bar{e} \\ 0 & 0 & 0 & 0 & 0 & 0 & \bar{e} & \bar{e} \\ 0 & 0 & \bar{e}^t & \bar{e}^t & 0 & 0 & -I & 0 \\ 0 & 0 & \bar{e}^t & \bar{e}^t & 0 & 0 & 0 & -I \\ 0 & 0 & \bar{e}^t & \bar{e}^t & 0 & 0 & I & 0 \\ 0 & 0 & \bar{e}^t & \bar{e}^t & 0 & 0 & 0 & 0 \end{bmatrix}, \]
we arrive at the way to connect Cartesian wave function $\Psi$ with spherical wave function $\Psi$ and so on. Total momentum operators in Cartesian basis has the conventional form

$$\Gamma^i = \begin{vmatrix} 0 & 0 & -\vec{e}_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tau_i & 0 \\ 0 & 0 & -\vec{e}_i^t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tau_i & 0 \\ 0 & 0 & -\vec{e}_i^t & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tau_i & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tau_i & 0 \end{vmatrix}.$$  

Also we will be needing generators of the Lorentz group representations involved:

$$J^{ab} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & V^{ab} & 0 & 0 \\ 0 & 0 & V^{ab} & 0 \\ 0 & 0 & 0 & (V \otimes V)^{ab} \end{vmatrix},$$

Vector generators are

$$(V^{ab})^I = -g^{al}\delta^b_k + g^{bl}\delta^a_k, \quad (V^{23})^I = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ \tau_1 & 0 \end{vmatrix},$$

and so on. Tensor generators are (index combinations 01, 02, 03, 23, 31, 12 are used)

$$[(V \otimes V)^{ab}]_{mn}^{sp} = (-g^{as}\delta^b_m + g^{bs}\delta^a_m)\delta^p_n + \delta^s_m (-g^{ap}\delta^b_n + g^{bp}\delta^a_n)$$

$$[(V \otimes V)^{23}]_{mn}^{sp} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} \tau_1 & 0 \\ 0 & \tau_1 \end{vmatrix},$$

and so on. Total momentum operators in Cartesian basis has the conventional form

$$J_k = l_k + S_k, \quad S_1 = iJ^{23}, \quad S_2 = iJ^{31}, \quad S_3 = iJ^{12}, \quad S_k = i \left[ 0 \oplus (0 \oplus \tau_k) \oplus (0 \oplus \tau_k) \oplus (\tau_k \oplus \tau_k) \right]$$

One should have their representation in spherical tetrad basis. To this end, taking the tetrad transformation law

$$e'_{(a)} = \frac{\partial x'^{\alpha}}{\partial x^\beta} L_{a}^{b} e_{(b)}^{\beta},$$

where Lorentz transformation is reduced to a pure rotation

$$O_1^j(\theta, \phi) = \begin{vmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{vmatrix}. $$

we arrive at the way to connect Cartesian wave function $\Psi$ with spherical wave function $\Psi'$:

$$\Psi'(x) = S \Psi(x), \quad S(\theta, \phi) = 1 \oplus (1 \oplus O)(1 \oplus O) \oplus (O \oplus O) \]$$
With the help of the same $S$ one transforms operators $J_a = l_a + S_a$, $a = 1, 2, 3$:

$$J'_a = S J_a S^{-1}. \quad (1.20)$$

Taking in mind the known formulas

$$l_1 = i (\sin \phi \partial_\theta + \tan \theta \cos \phi \partial_\phi),$$
$$l_2 = i (-\cos \phi \partial_\theta + \tan \theta \sin \phi \partial_\phi), \quad l_3 = -i \partial_\phi$$

and two intermediate ones

$$O i \partial_\theta O^{-1} = i \tau_2, \quad O i \partial_\phi O^{-1} = i (\cos \theta \tau_3 - \sin \theta \tau_1),$$

for $l'_a = O l_a O^{-1}$ one derives

$$l'_1 = l_1 - i (\cos \theta \cos \phi \tau_1 + \sin \phi \tau_2 + \frac{\cos^2 \theta}{\sin \theta} \cos \phi \tau_3),$$
$$l'_2 = l_2 - i (\cos \theta \sin \phi \tau_1 - \cos \phi \tau_2 + \frac{\cos^2 \theta}{\sin \theta} \sin \phi \tau_3),$$
$$l'_3 = l_3 + i (\sin \phi \tau_1 - \cos \phi \tau_3),$$

and additionally for $\tau'_a = O \tau_a O^{-1}$:

$$\tau'_1 = \cos \theta \cos \phi \tau_1 - \sin \phi \tau_2 + \sin \theta \cos \phi \tau_3,$$
$$\tau'_2 = \cos \theta \sin \phi \tau_1 - \cos \phi \tau_2 + \sin \theta \sin \phi \tau_3,$$
$$\tau'_3 = -\sin \phi \tau_1 + \cos \theta \tau_3.$$

Thus, $J'_a$ turns out to be

$$J'_1 = l_1 + \frac{\cos \phi}{\sin \theta} S_3, \quad J'_2 = l_2 + \frac{\sin \phi}{\sin \theta} S_3, \quad J'_3 = l_3.$$

$$\vec{J}'^2 = -\frac{1}{\sin \theta} \sin \theta \partial_\theta + \frac{-\partial_\phi^2 + 2 i \partial_\phi S_3 \cos \theta + S_3^2}{\sin^2 \theta}. \quad (7)$$

For the following it will be convenient to have $S_3$ diagonal which can be achieved through going to the so-called cyclic basis [2]:

$$\Psi'' = U \Psi', \quad U = 1 \oplus (1 \oplus U_3) \oplus (1 \oplus U_3) \oplus (U_3 \oplus U_3)$$

$$U_3 = \begin{vmatrix} -1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & i/\sqrt{2} & 0 \end{vmatrix}, \quad U_3^{-1} = U_3^+ = \begin{vmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -i/\sqrt{2} & 0 & -i/\sqrt{2} \\ 0 & 1 & 0 \end{vmatrix}.$$

It is easily verified

$$U_3 \tau_1 U_3^{-1} = \frac{1}{\sqrt{2}} \begin{vmatrix} 0 & -i & 0 \\ -i & 0 & -i \\ 0 & -i & 0 \end{vmatrix} = \tau'_1,$$
$$U_3 \tau_2 U_3^{-1} = \frac{1}{\sqrt{2}} \begin{vmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{vmatrix} = \tau'_2,$$
$$U_3 \tau_3 U_3^{-1} = -i \begin{vmatrix} +1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{vmatrix} = \tau'_3.$$
In this cyclic representation the matrix $S_3$ is diagonal indeed:

$$S_3 = \text{diag} \begin{pmatrix} 0 & 0, 1 & 0, -1; 0, 1, 0, -1; 1, 0, -1; 1, 0, -1; 1, 0, -1 \end{pmatrix}.$$  

Also we will be needing cyclic quantities (all marks reminding about the use of spherical tetrad and cyclic representation basis will be omitted below)

$$\vec{e}_1 = \left( -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right), \quad \vec{e}_2 = \left( -\frac{i}{\sqrt{2}}, 0, -\frac{i}{\sqrt{2}} \right), \quad \vec{e}_3 = (0, 1, 0).$$

$$\vec{e}_1^t = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \vec{e}_2^t = \begin{bmatrix} \frac{i}{\sqrt{2}} \\ 0 \\ -\frac{i}{\sqrt{2}} \end{bmatrix}, \quad \vec{e}_3^t = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$  

## Separation of variables for a free particle

Now we are going to construct proper functions of $\vec{J}^2, J_3$-operators. In accordance with the known differential relations [13] for Wigner’s functions $D_{-m,s}^j(\phi, \theta, 0)$

$$-i \partial_\phi D_{-m,s}^j = m D_{-m,s}^j,$$

$$\left( -\frac{1}{\sin \theta} \partial_\theta \sin \theta \partial_\theta + \frac{m^2 + 2m \sigma \cos \theta + \sigma^2}{\sin^2 \theta} \right) D_{-m,s}^j = j(j+1) D_{-m,s}^j,$$

the most general form of those proper functions is (see [14])

$$\Psi(x) = \{ C(x), C_0(x), \vec{C}(x), \Phi_0(x), \vec{\Phi}(x), \vec{E}(x), \vec{H}(x) \},$$

$$C(x) = e^{-iMt} C(r) D_0, \quad C_0(x) = e^{-iMt} C_0(r) D_0, \quad \Phi_0(x) = e^{-iMt} \Phi_0(x) D_0,$$

$$\vec{C}(x) = e^{-iMt} \begin{bmatrix} C_1(r) D_{-1} \\ C_2(3) D_0 \\ C_3(r) D_{+1} \end{bmatrix}, \quad \vec{\Phi}(x) = e^{-iMt} \begin{bmatrix} \Phi_1(r) D_{-1} \\ \Phi_2(r) D_0 \\ \Phi_3(r) D_{+1} \end{bmatrix},$$

$$\vec{E}(x) = e^{-iMt} \begin{bmatrix} E_1(r) D_{-1} \\ E_2(r) D_0 \\ E_3(r) D_{+1} \end{bmatrix}, \quad \vec{H}(x) = e^{-iMt} \begin{bmatrix} H_1(r) D_{-1} \\ H_2(r) D_0 \\ H_3(r) D_{+1} \end{bmatrix},$$

where the notation is used

$$D_s = D_{-m,s}^j(\phi, \theta, 0), \quad s = 0, +1, -1.$$  

We will need recurrence relations [13]

$$\partial_\phi D_{-1} = (1/2) \left( a D_{-2} - \nu D_0 \right),$$

$$\frac{-m + \cos \theta}{\sin \theta} \quad D_{-1} = (1/2) \left( -a D_{-2} - \nu D_0 \right),$$

$$\partial_\theta D_0 = (1/2) \left( \nu D_{-1} - \nu D_{+1} \right),$$

$$\frac{-m}{\sin \theta} \quad D_0 = (1/2) \left( -\nu D_{-1} - \nu D_{+1} \right),$$

$$\partial_\theta D_{+1} = (1/2) \left( \nu D_0 - a D_{+2} \right),$$

$$\frac{-m - \cos \theta}{\sin \theta} \quad D_{+1} = (1/2) \left( -\nu D_0 - a D_{+2} \right).$$

\[9\]
where \( \nu = \sqrt{j(j+1)} \), \( a = \sqrt{(j-1)(j+2)} \).

Now let us proceed to separate variables in the main equation (1). Accounting block-
structure of all quantities involved after relevant calculation we arrive at

\[
\begin{align*}
\partial_t C_0 - (\partial_r + \frac{2}{r}) \tilde{e}_3 \tilde{C} - \frac{1}{r} \left( \tilde{e}_1 \partial_\theta + \tilde{e}_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} \right) \tilde{C} - m C &= 0 , \\
-(\partial_r + \frac{2}{r}) \tilde{e}_3 \tilde{E} - \frac{1}{r} \left( \tilde{e}_1 \partial_\theta + \tilde{e}_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} \right) \tilde{E} - m C_0 &= 0 , \\
- \partial_t \tilde{E} - (\partial_r + \frac{1}{r}) \tau_3 \tilde{H} - \frac{1}{r} \left( \tau_1 \partial_\theta + \tau_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} \right) \tilde{H} - m \tilde{C} &= 0 , \\
\sigma \partial_t C - (\partial_r + \frac{2}{r}) \tilde{e}_3 \tilde{E} - \frac{1}{r} \left( \tilde{e}_1 \partial_\theta + \tilde{e}_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} \right) \tilde{E} - m \Phi_0 &= 0 , \\
\partial_t \tilde{E} - (\partial_r + \frac{1}{r}) \tau_3 \tilde{H} + \sigma \hat{e}_3^t \partial_r C + \\
+ \sigma \frac{1}{r} \left( \tilde{e}_1^t \partial_\theta + \tilde{e}_2^t \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} \right) C - \frac{1}{r} \left( \tau_1 \partial_\theta + \tau_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} \right) \tilde{H} - m \tilde{\Phi} &= 0 , \\
\partial_t \tilde{\Phi} - (\tilde{e}_3^t \partial_r \Phi_0 - \frac{1}{r} \left( \tilde{e}_1^t \partial_\theta + \tilde{e}_2^t \frac{\partial \phi}{\sin \theta} \right) \Phi_0 - m \tilde{E} &= 0 , \\
(\partial_r + \frac{1}{r}) \tau_3 \tilde{\Phi} + \frac{1}{r} \left( \tau_1 \partial_\theta + \tau_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} \right) \Phi - m \tilde{H} &= 0 .
\end{align*}
\]

Further with the use of intermediate results

\[
( \tilde{e}_1 \partial_\theta + \tilde{e}_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} ) \tilde{C}(x) = e^{-i \nu t} \left\{ \begin{array}{c}
\tilde{e}_1 \\
\tilde{e}_2 \\
C_0 \partial_\theta D_{-1} \\
C_0 \partial_\theta D_0 \\
C_0 \partial_\theta D_{+1}
\end{array} \right\}
\]

\[
+ (-i) \tilde{e}_2 \left\{ \begin{array}{c}
C_1 \frac{-m \cos \theta}{\sin \theta} D_{-1} \\
C_2 \frac{-m \cos \theta}{\sin \theta} D_0 \\
C_3 \frac{-m \cos \theta}{\sin \theta} D_{+1}
\end{array} \right\} = e^{-i \nu t} \sqrt{2} \left\{ \begin{array}{c}
\tau_1 \Phi_0 \\
\tau_2 \tilde{H}(x) = e^{-i \nu t} \left\{ \begin{array}{c}
H_1 \partial_\theta D_{-1} \\
H_2 \partial_\theta D_0 \\
H_3 \partial_\theta D_{+1}
\end{array} \right\}
\right\}
\]

\[
( \tau_1 \partial_\theta + \tau_2 \frac{\partial \phi + \tau_3 \cos \theta}{\sin \theta} ) \tilde{H}(x) = e^{-i \nu t} \sqrt{2} \left\{ \begin{array}{c}
H_1 \partial_\theta D_{-1} \\
H_2 \partial_\theta D_0 \\
H_3 \partial_\theta D_{+1}
\end{array} \right\}
\]

\[
+ \tau_2 \left\{ \begin{array}{c}
H_1 \frac{-m \cos \theta}{\sin \theta} D_{-1} \\
H_2 \frac{-m \cos \theta}{\sin \theta} D_0 \\
H_3 \frac{-m \cos \theta}{\sin \theta} D_{+1}
\end{array} \right\} = e^{-i \nu t} \sqrt{2} \left\{ \begin{array}{c}
\tau_1 \Phi_0 \\
\tau_2 \tilde{H}(x) = e^{-i \nu t} \left\{ \begin{array}{c}
H_1 \partial_\theta D_{-1} \\
H_2 \partial_\theta D_0 \\
H_3 \partial_\theta D_{+1}
\end{array} \right\}
\right\}
\]

\[
( \tilde{e}_1^t \partial_\theta + \tilde{e}_2^t \frac{\partial \phi}{\sin \theta} ) C(x) = e^{-i \nu t} \sqrt{2} \left\{ \begin{array}{c}
C \partial_\theta D_{-1} \\
C \partial_\theta D_0 \\
C \partial_\theta D_{+1}
\end{array} \right\}
\]

we get to a radial equation system (the notation \( \nu = \sqrt{j(j+1)} / \sqrt{2} \) is used)

\[
-i \epsilon C_0 - \left( \frac{d}{dr} + \frac{2}{r} \right) C_2 - \frac{\nu}{r} \left( C_1 + C_3 \right) = m C .
\]

\[
- \left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - \frac{\nu}{r} \left( E_1 + E_3 \right) = m C_0 ,
\]

\[
+i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + i \frac{\nu}{r} H_2 = m C_1 ,
\]

\[
+i \epsilon E_2 - i \frac{\nu}{r} \left( H_1 - H_3 \right) = m C_2 ,
\]

\[
+i \epsilon E_3 - i \left( \frac{d}{dr} + \frac{1}{r} \right) H_3 - i \frac{\nu}{r} H_2 = m C_3 .
\]
\(-i \epsilon \sigma C - \left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - \frac{\nu}{r} (E_1 + E_3) = m \Phi_0 \),
\[+i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + i \frac{\nu}{r} H_2 - \sigma \frac{\nu}{r} C = m \Phi_1 ,
\[+i \epsilon E_2 + \sigma \frac{d}{dr} C - i \frac{\nu}{r} (H_1 - H_3) = m \Phi_2 ,
\[+i \epsilon E_3 - i \left( \frac{d}{dr} + \frac{1}{r} \right) H_3 - i \frac{\nu}{r} H_2 - \sigma \frac{\nu}{r} C = m \Phi_3 . \tag{13}\]

Concurrently we will diagonalize \(P\)-inversion operator. In Cartesian tetrad it has a conventional form
\[
\Pi = \begin{bmatrix} 1 & \ominus (1 \ominus -I) & \ominus (1 \ominus -I) & (\ominus I \oplus +I) \end{bmatrix} \hat{P} , \quad \hat{P} \Psi(\vec{r}) = \Psi(-\vec{r}) . \tag{15}\]
which after translating to the spherical-cyclic basis will become
\[
\hat{P}' = \begin{bmatrix} 1 \oplus (1 \oplus \Pi_3) \oplus (1 \oplus \Pi_3) \oplus (\Pi_3 \oplus -\Pi_3) \end{bmatrix} \hat{P} , \quad \Pi_3 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} . \tag{16}\]
The eigenvalue equation \(\hat{P} \Psi = P \Psi\), with the use of \(\hat{P} D_s = (-1)^j D_{-s}\), supplies us with solutions of two sorts:
\[
P = (-1)^{j+1} , \quad C = 0 , \quad C_0 = 0 , \quad C_3 = -C_1 , \quad C_2 = 0 , \quad \Phi_0 = 0 , \quad \Phi_3 = -\Phi_1 , \quad \Phi_2 = 0 , \quad E_3 = -E_1 , \quad E_2 = 0 , \quad H_3 = H_1 ; \tag{17}\]
\[
P = (-1)^j , \quad C_3 = +C_1 , \quad \Phi_3 = +\Phi_1 , \quad E_3 = +E_1 , \quad H_3 = -H_1 , \quad H_2 = 0 . \tag{18}\]
It is easily verified that both (17) and (18) are consistent with the above radial system (2.6); at this we will get two sub-systems respectively:
\[
P = (-1)^{j+1} , \quad +i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + i \frac{\nu}{r} H_2 = m C_1 ,
\[+i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + i \frac{\nu}{r} H_2 = m \Phi_1 , \quad -i \epsilon \Phi_1 = m E_1 ,
\[-i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_1 = m H_1 , \quad 2i \frac{\nu}{r} \Phi_1 = m H_2 ; \tag{19}\]
\[ P = (-1)^j, \quad -i \epsilon C_0 - \left( \frac{d}{dr} + \frac{2}{r} \right) C_2 - 2 \frac{\nu}{r} C_1 = m C, \]
\[ -\left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - 2 \frac{\nu}{r} E_1 = m C_0, \]
\[ +i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 = m C_1, \]
\[ +i \epsilon E_2 - 2i \frac{\nu}{r} H_1 = m C_2, \]
\[ -i \epsilon \sigma C - \left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - 2 \frac{\nu}{r} E_1 = m \Phi_0, \]
\[ +i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 - \sigma \frac{\nu}{r} C = m \Phi_1, \]
\[ +i \epsilon E_2 - 2i \frac{\nu}{r} H_1 + \sigma \frac{d}{dr} C = m \Phi_2, \]
\[ -i \epsilon \Phi_1 + \frac{\nu}{r} \Phi_0 = m E_1, \]
\[ -i \epsilon \Phi_2 - \frac{d}{dr} \Phi_0 = m E_2, \]
\[ -i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_1 - i \frac{\nu}{r} \Phi_2 = m H_1. \] (20)

### 3 Vector particle in Coulomb field

In presence of external electromagnetic fields the original equation (2) will become
\[
\left[ \Gamma^\alpha(x) \left( \partial_\alpha + B_\alpha(x) - i \frac{e}{\hbar c} A_\alpha \right) - \frac{mc}{\hbar} \right] \Psi(x) = 0. \] (21)

As Coulomb field is described by the potential \( A_\alpha(x) = (ze/r, 0, 0, 0) \), then most of the calculation in previous Sec. 2 and Sec. 3 need not to be repeated – you can immediately turn to radial equations with one formal change (\( ze^2 = \alpha \))

\[ \epsilon \quad \Longrightarrow \quad (\epsilon + \frac{ze^2}{r}). \]

Thus, we will obtain
\[ P = (-1)^{j+1}, \quad +i(\epsilon + \frac{\alpha}{r}) E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + \frac{\nu}{r} H_2 = m C_1, \]
\[ +i(\epsilon + \frac{\alpha}{r}) E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + \frac{\nu}{r} H_2 = m \Phi_1, \]
\[ -i (\epsilon + \frac{\alpha}{r}) \Phi_1 = m E_1, \quad -i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_1 = m H_1, \quad 2i \frac{\nu}{r} \Phi_1 = m H_2; \] (22)

\[ P = (-1)^j, \quad -i(\epsilon + \frac{\alpha}{r}) C_0 - \left( \frac{d}{dr} + \frac{2}{r} \right) C_2 - 2 \frac{\nu}{r} C_1 = m C, \]
\[ -\left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - 2 \frac{\nu}{r} E_1 = m C_0, \]
\[ +i(\epsilon + \frac{\alpha}{r}) E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 = m C_1, \]
\[ +i(\epsilon + \frac{\alpha}{r}) E_2 - 2i \frac{\nu}{r} H_1 = m C_2. \]
\[
- i(\epsilon + \frac{\alpha}{r}) \sigma C - \left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - 2\nu \frac{r}{E_1} = m \Phi_0 , \\
+ i(\epsilon + \frac{\alpha}{r}) E_1 + i\left( \frac{d}{dr} + \frac{1}{r} \right) H_1 - \sigma \frac{r}{E_1} \Phi_0 = m \Phi_1 , \\
+ i(\epsilon + \frac{\alpha}{r}) E_2 - 2i\nu H_1 + \sigma \frac{d}{dr} \Phi_0 = m \Phi_2 , \\
-i(\epsilon + \frac{\alpha}{r}) \Phi_1 + \frac{\nu}{r} \Phi_0 = m E_1 , \\
i(\epsilon + \frac{\alpha}{r}) \Phi_2 - \frac{d}{dr} \Phi_0 = m E_2 , \\
-i(\frac{d}{dr} + \frac{1}{r}) \Phi_1 - i\nu \frac{r}{H_1} = m H_1 .
\]

(23)

Concerning eqs. (22) for \( P = (-1)^{j+1} \) not much need to be done to arrive at a final second-order differential relation. Actually, from (22) it follows

\[
C_1(r) = \Phi_1(r) , \quad m E_1 = - i(\epsilon + \frac{\alpha}{r}) \Phi_1 , \\
m H_1 = - i\left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_1 , \quad m H_2 = 2i\nu \Phi_1 
\]

(24)

and for \( \Phi_1 \)

\[
\left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + \left( \epsilon + \frac{\alpha}{r} \right)^2 - \frac{j(j+1)}{r^2} \right] \Phi_1 = 0
\]

(25)

what is the common equation in the theory of a usual scalar particle in Coulomb field. Its solutions are well-known.

So, in part, the extended theory under consideration behaves like an ordinary scalar particle in the Coulomb field. Remaining case when \( P = (-1)^{j+1} \) turns out to be much more involved. For instance, even the simplest case \( j = 0 \) leads us to an equation too difficult for analytical treatment.

Now let us turn to (23). Taking \( C_0, C_1, C_2 \) from the first equation for \( m^2 C(r) \) it follows

\[
m^2 C = - i(\epsilon + \frac{\alpha}{r}) \left[ - \left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - 2\nu \frac{r}{E_1} \right] - \\
- \left( \frac{d}{dr} + \frac{2}{r} \right) \left[ i(\epsilon + \frac{\alpha}{r}) E_2 - 2i\nu H_1 \right] - 2\nu \left[ i(\epsilon + \frac{\alpha}{r}) \Phi_1 + i\left( \frac{d}{dr} + \frac{1}{r} \right) H_1 \right] ,
\]

that is

\[
C(r) = + \frac{i\alpha}{m^2 r^2} E_2(r) .
\]

(26)

Substitution this expression into remaining equation in (23), we get to

\[
\frac{\sigma \alpha}{m^2 r^2} \left( \epsilon + \frac{\alpha}{r} \right) E_2 - \left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - 2\nu \frac{r}{E_1} = m \Phi_0 , \\
i(\epsilon + \frac{\alpha}{r}) E_1 + i\left( \frac{d}{dr} + \frac{1}{r} \right) H_1 - \frac{i\sigma \alpha \nu}{m^2 r^3} E_2 = m \Phi_1 , \\
i(\epsilon + \frac{\alpha}{r}) E_2 - 2\frac{r}{E_1} H_1 + i\sigma \frac{d}{dr} \left( \frac{\alpha}{m^2 r^2} E_2 \right) = m \Phi_2 ,
\]

(27)
\[-i(\epsilon + \frac{\alpha}{r}) \Phi_1 + \nu \Phi_0 = m E_1 , \]
\[-(\epsilon + \frac{\alpha}{r}) \Phi_2 - \frac{d}{dr} \Phi_0 = m E_2 , \]
\[-i(\frac{d}{dr} + \frac{1}{r}) \Phi_1 - i \nu \Phi_2 = m H_1 . \] (27)

Unfortunately, we have not been able to proceed with these equations successfully.

For instance, let us consider in some detail the simplest case of \( j = 0 \). For this case a special initial form of the wave function must be used (compare it with (8))

\[ \Psi_{j=0}(x) = \{ C(x), C_0(x), \bar{C}(x), \Phi_0(x), \bar{\Phi}(x), \bar{E}(x), \bar{H}(x) \} , \]

\[ C(x) = e^{-i\epsilon t} C(r) D_0 , \quad C_0(x) = e^{-i\epsilon t} C_0(r) D_0 , \quad \Phi_0(x) = e^{-i\epsilon t} \Phi_0(x) D_0 , \]

\[ \bar{C}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 & C_2(3) \\ 0 & 0 \end{vmatrix} , \quad \bar{\Phi}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ 0 \end{vmatrix} , \]

\[ \bar{E}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 & E_2(r) \\ 0 & 0 \end{vmatrix} , \quad \bar{H}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ 0 \end{vmatrix} . \] (28)

Operator \( \Sigma_{\theta_0} \) being applied to that functions vanishes identically; \( P \)-operator takes on such solutions the proper value \(-1\). So setting \( \nu = 0 \) in (27) and taking in mind eqs. (28) we get only three nontrivial equations:

\[ \frac{\sigma \alpha}{m^2 r^2} (\epsilon + \frac{\alpha}{r}) E_2 - (\frac{d}{dr} + \frac{2}{r}) E_2 = m \Phi_0 , \quad 0 = 0 , \]
\[ i(\epsilon + \frac{\alpha}{r}) E_2 + i\sigma \frac{d}{dr} (\frac{\alpha}{m^2 r^2} E_2) = m \Phi_2 , \quad 0 = 0 , \]
\[ -(\epsilon + \frac{\alpha}{r}) \Phi_2 - \frac{d}{dr} \Phi_0 = m E_2 , \quad 0 = 0 . \] (29)

Excluding variables \( \Phi_0, \Phi_2 \), we arrive at an equation for \( E_2 \):

\[ m^2 E_2 = -(\epsilon + \frac{\alpha}{r}) \left[ i(\epsilon + \frac{\alpha}{r}) + i\sigma \frac{d}{dr} \frac{\alpha}{m^2 r^2} \right] E_2 - \]
\[ -\frac{d}{dr} \left[ \frac{\sigma \alpha}{m^2 r^2} (\epsilon + \frac{\alpha}{r}) E_2 - (\frac{d}{dr} + \frac{2}{r}) E_2 \right] , \]

from which it follows a 2-order differential equation

\[ \left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + (\epsilon + \frac{\alpha}{r})^2 - m^2 - \frac{2}{r^2} + \frac{\sigma \alpha^2}{m^2 r^4} \right] E_2(r) = 0 . \] (30)

### 4 Particle in magnetic monopole field

Now we will consider the vector particle in external Dirac monopole field [15]. Initial wave equation is

\[ \left[ \Gamma^\alpha(x) \left( \partial_\alpha + B_\alpha - \frac{e}{\hbar c} A_\alpha \right) - \frac{mc}{\hbar} \right] \Psi(x) = 0 . \]
To describe monopole field the Schwinger monopole potential will be used; the latter after translating to spherical coordinates will have the explicit form \( A_\alpha = (0, 0, A_\phi = g \cos \theta) \); \( g \) stands for a magnetic charge. According to this you are to make one substitution

\[
\partial_\phi \Longrightarrow (\partial_\phi + i k \cos \theta), \quad \text{where} \quad k = \frac{e g}{\hbar c}.
\]

Correspondingly, the main equation can be read as

\[
\left[ \Gamma^0 \partial_0 + \Gamma^3 \partial_r + \frac{\Gamma^1 J^{31} + \Gamma^2 J^{32}}{r} + \frac{1}{r} \Sigma^k_{\theta, \phi} - m \right] \Psi = 0, \tag{31}
\]

where

\[
\Sigma^k_{\theta, \phi} = \Gamma^1 \partial_\theta + \frac{\Gamma^2 \partial_\phi + (J^{12} + i k) \cos \theta}{\sin \theta}. \tag{32}
\]

You may immediately pass to equations of the type [10]:

\[
\begin{align*}
\partial_t C_0 & - \left( \partial_r + \frac{2}{r} \right) \vec{e}_3 \vec{C} - \frac{1}{r} \left( \vec{e}_1 \partial_\theta + \vec{e}_2 \frac{\partial_\phi + (\tau_3 + i k) \cos \theta}{\sin \theta} \right) \vec{C} = m C, \\
- \partial_t \vec{E} & - \left( \partial_r + \frac{1}{r} \right) \vec{e}_3 \vec{H} - \frac{1}{r} \left( \vec{e}_1 \partial_\theta + \vec{e}_2 \frac{\partial_\phi + (\tau_3 + i k) \cos \theta}{\sin \theta} \right) \vec{H} = m \vec{C}, \\
\sigma \partial_t C & - \left( \partial_r + \frac{2}{r} \right) \vec{e}_3 \vec{E} - \frac{1}{r} \left( \vec{e}_1 \partial_\theta + \vec{e}_2 \frac{\partial_\phi + (\tau_3 + i k) \cos \theta}{\sin \theta} \right) \vec{E} = m \Phi_0, \\
- \partial_t \vec{E} & - \left( \partial_r + \frac{1}{r} \right) \tau_3 \vec{H} + \sigma \vec{e}_3^t \partial_r C + \sigma \frac{1}{r} \left( \vec{e}_1^t \partial_\theta + \vec{e}_2^t \frac{\partial_\phi + i k \cos \theta}{\sin \theta} \right) C - \\
- \frac{1}{r} \left( \tau_1 \partial_\theta + \tau_2 \frac{\partial_\phi + (\tau_3 + i k) \cos \theta}{\sin \theta} \right) \vec{H} = m \vec{F}, \\
\partial_t \vec{F} & - \vec{e}_3^t \partial_r \Phi_0 - \frac{1}{r} \left( \vec{e}_1^t \partial_\theta + \vec{e}_2^t \frac{\partial_\phi + i k \cos \theta}{\sin \theta} \right) \Phi_0 = m \vec{E}, \\
( \partial_r + \frac{1}{r} ) \tau_3 \vec{F} & + \frac{1}{r} \left( \tau_1 \partial_\theta + \tau_2 \frac{\partial_\phi + (\tau_3 + i k) \cos \theta}{\sin \theta} \right) \vec{F} = m \vec{H}. \tag{33}
\end{align*}
\]

It is the point to establish a suitable form of the wave function being proper one of rotation symmetry-based operators. In presence monopole background there exist three relevant ones:

\[
J^{(k)}_1 = l_1 + \frac{\cos \phi}{\sin \theta} (S_3 - k), \quad J^{(k)}_2 = l_2 + \frac{\sin \phi}{\sin \theta} (S_3 - k), \quad J^{(k)}_3 = l_3. \tag{34}
\]

Therefore, two operators to be diagonalized are

\[
J^{(k)}_3 = l_3, \quad \vec{J}^{(k)}_2 = - \frac{1}{\sin \theta} \partial_\theta \sin \theta \partial_\theta + \\
- \frac{\partial_\phi^2 + 2 i \partial_\phi (S_3 - k) \cos \theta + (S - k)^2_3}{\sin^2 \theta}. \tag{35}
\]

So, the most general form of 15-component wave function with quantum numbers \( \epsilon, j, m \) will look as

\[
\Psi(x) = \{ C(x), \ C_0(x), \ \vec{C}(x), \ \Phi_0(x), \ \vec{\Phi}(x), \ \vec{E}(x), \ \vec{H}(x) \}. 
\]
where \( D_s = D_{-m,s}(\phi, \theta, 0) \), \( s = k, k+1, k-1 \).

Now you should substitute the wave function into (33) and exclude variable \( t, \theta, \phi \). At this you will need some known recurrence relations [13]

\[
\begin{align*}
\partial_\theta D_{k-1} &= (a D_{k-2} - c D_k), \\
\frac{-m - (k-1) \cos \theta}{\sin \theta} D_{k-1} &= (-a D_{k-2} - c D_k), \\
\partial_\theta D_k &= (c D_{k-1} - d D_{k+1}), \\
\frac{-m - k \cos \theta}{\sin \theta} D_k &= (-c D_{k-1} - d D_{k+1}), \\
\partial_\theta D_{k+1} &= (d D_k - b D_{k+2}), \\
\frac{-m - (k + 1) \cos \theta}{\sin \theta} D_{k+1} &= (-d D_k - b D_{k+2}),
\end{align*}
\]

(37)

where

\[
\begin{align*}
a &= \frac{1}{2} \sqrt{(j + k - 1)(j - k + 2)}, & b &= \frac{1}{2} \sqrt{(j - k - 1)(j + k + 2)}, \\
c &= \frac{1}{2} \sqrt{(j + k)(j - k + 1)}, & d &= \frac{1}{2} \sqrt{(j - k)(j + k + 1)}.
\end{align*}
\]

Eqs. (33) rewritten as

\[
\begin{align*}
\partial_t C_0 &= (\partial_\tau + \frac{2}{r}) \tilde{e}_3 \tilde{C} - \frac{1}{r} \left[ \tilde{e}_1 \partial_\theta + \tilde{e}_2 (-i) \frac{-m - (k - s_3) \cos \theta}{\sin \theta} \right] \tilde{C} = m \tilde{C}, \\
-(\partial_\tau + \frac{2}{r}) \tilde{e}_3 \tilde{E} - \frac{1}{r} \left[ \tilde{e}_1 \partial_\theta + \tilde{e}_2 (-i) \frac{-m - (k - s_3) \cos \theta}{\sin \theta} \right] \tilde{E} &= m \tilde{C}_0, \\
-\tilde{\partial}_t \tilde{E} &= (\partial_\tau + \frac{1}{r}) \tau_3 \tilde{H} - \frac{1}{r} \left[ \tau_1 \partial_\theta + \tau_2 (-i) \frac{-m - (k - s_3) \cos \theta}{\sin \theta} \right] \tilde{H} = m \tilde{C}, \\
\sigma \partial_\tau C &= (\partial_\tau + \frac{2}{r}) \tilde{e}_3 \tilde{E} - \frac{1}{r} \left[ \tilde{e}_1 \partial_\theta + \tilde{e}_2 (-i) \frac{-m - (k - s_3) \cos \theta}{\sin \theta} \right] \tilde{E} = m \Phi_0, \\
-\tilde{\partial}_t \tilde{E} &= (\partial_\tau + \frac{1}{r}) \tau_3 \tilde{H} + \sigma \tilde{e}_3^t \partial_\tau C + \sigma \frac{1}{r} \left[ \tilde{e}_1^t \partial_\theta + \tilde{e}_2^t (-i) \frac{-m - k \cos \theta}{\sin \theta} \right] C - \\
- \frac{1}{r} \left[ \tau_1 \partial_\theta + \tau_2 (-i) \frac{-m - (k - s_3) \cos \theta}{\sin \theta} \right] \tilde{H} = m \tilde{\Phi}, \\
\partial_\tau \tilde{\Phi} &= \tilde{e}_3 \partial_\tau \Phi_0 - \frac{1}{r} \left[ \tilde{e}_1 \partial_\theta + \tilde{e}_2 (-i) \frac{-m - k \cos \theta}{\sin \theta} \right] \Phi_0 = m \tilde{E}, \\
(\partial_\tau + \frac{1}{r}) \tau_3 \tilde{\Phi} + \frac{1}{r} \left[ \tau_1 \partial_\theta + \tau_2 (-i) \frac{-m - (k - s_3) \cos \theta}{\sin \theta} \right] \tilde{\Phi} = m \tilde{H}.
\end{align*}
\]

(38)

where

\[
s_3 = \begin{vmatrix}
+1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{vmatrix}
\]
with the use of several intermediate formulas

\[
\begin{align*}
\partial_\theta \vec{C} &= e^{-i\epsilon t} \begin{vmatrix} C_1 \partial_\theta D_{k-1} & C_1(a D_{k-2} - c D_k) \\ C_2 \partial_\theta D_k & C_2(c D_{k-1} - d D_{k+1}) \\ C_3 \partial_\theta D_{k+1} & C_3(d D_k - b D_{k+2}) \end{vmatrix}, \\
\frac{-m - (k - s_3) \cos \theta}{\sin \theta} \vec{C} &= e^{-i\epsilon t} \begin{vmatrix} C_1(-a D_{k-2} - c D_k) \\ C_2(-c D_{k-1} - d D_{k+1}) \\ C_3(-d D_k - b D_{k+2}) \end{vmatrix}, \\
\vec{e}_1 \partial_\theta \vec{C} &= e^{-i\epsilon t} \frac{1}{\sqrt{2}} \left[ -C_1(a D_{k-2} - c D_k) + C_3(d D_k - b D_{k+2}) \right], \\
-i\vec{e}_2 \frac{-m - (k - s_3) \cos \theta}{\sin \theta} \vec{C} &= e^{-i\epsilon t} \frac{\sqrt{2}}{2} \begin{vmatrix} -c H_2 D_{k-1} \\ (c H_1 - d H_3) D_k \\ +d H_2 D_{k+1} \end{vmatrix}, \\
(\vec{e}_1 \partial_\theta - i \vec{e}_2 \frac{-m - (k - s_3) \cos \theta}{\sin \theta}) \vec{C} &= e^{-i\epsilon t} \frac{\sqrt{2}}{2} \begin{vmatrix} -c C(r) D_{k-1} \\ 0 \\ -d C(r) D_{k+1} \end{vmatrix}.
\end{align*}
\]

from (38) you arrive at

\[
\begin{align*}
-\epsilon C_0 - \left( \frac{d}{dr} + \frac{2}{r} \right) C_2 - \frac{\sqrt{2}}{r} (c C_1 + d C_3) &= m C, \\
-\left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - \frac{\sqrt{2}}{r} (c E_1 + d E_3) &= m C_0, \\
+i\epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + \frac{i\sqrt{2} c}{r} H_2 &= m C_1, \\
+i\epsilon E_2 - \frac{i\sqrt{2}}{r} (c H_1 - d H_3) &= m C_2, \\
+i\epsilon E_3 - i \left( \frac{d}{dr} + \frac{1}{r} \right) H_3 - \frac{i\sqrt{2} d}{r} H_2 &= m C_3,
\end{align*}
\]

(39)

\[
\begin{align*}
-\epsilon \sigma C - \left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - \frac{\sqrt{2}}{r} (c E_1 + d E_3) &= m \Phi_0, \\
+i\epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 - \sigma \frac{\sqrt{2} c}{r} C + \frac{i\sqrt{2} c}{r} H_2 &= m \Phi_1, \\
+i\epsilon E_2 + \sigma \frac{d}{dr} C - \frac{i\sqrt{2}}{r} (c H_1 - d H_3) &= m \Phi_2, \\
+i\epsilon E_3 - i \left( \frac{d}{dr} + \frac{1}{r} \right) H_3 - \sigma \frac{\sqrt{2} d}{r} C - \frac{i\sqrt{2} d}{r} H_2 &= m \Phi_3,
\end{align*}
\]

(41)
\[-i \epsilon \Phi_1 + \frac{\sqrt{2} c}{r} \Phi_0 = m E_1, \quad -i \epsilon \Phi_2 - \frac{d}{dr} \Phi_2 = m E_2,\]
\[-i \epsilon \Phi_3 + \frac{\sqrt{2} d}{r} \Phi_0 = m E_3, \quad -i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_1 - \frac{i \sqrt{2} c}{r} \Phi_2 = m H_1,\]
\[i \sqrt{2} \left( c \Phi_1 - d \Phi_3 \right) = m H_2, \quad +i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_3 + \frac{i \sqrt{2} d}{r} \Phi_2 = m H_3.\] (42)

5 States with $j = j_{\text{min}}$ in monopole field

As known, in the monopole problem there arises a very peculiar situation for states with minimal value of $j$. Now we are going to consider those in our case. In accordance with quantization conditions by Dirac-Schwinger the parameter $k = eg/\hbar c$ involved will take on the values

\[k = \pm 1, \pm 3/2, \pm 2, ... \quad \text{and} \quad j = |k| - 1, |k|, |k| + 1, ...\] (43)

One should consider with more details several particular cases. Firstly, let $k = 1, j_{\text{min}} = 0$.

Correspondingly, a wave function should be taken in the form

\[\Psi_{k=+1}^{j=0}(x): \quad C(r) = 0, \quad C_0(r) = 0, \quad \Phi_0(r) = 0,\]
\[\tilde{C}(x) = e^{-i \epsilon t}, \quad \tilde{\Phi}(x) = e^{-i \epsilon t}, \quad \Phi_1(r) = 0, \quad 0,\]
\[\tilde{E}(x) = e^{-i \epsilon t}, \quad H(x) = e^{-i \epsilon t}, \quad H_1(r) = 0, \quad 0.\] (44)

On those $\Psi_{k=+1}^{j=0}(x)$ the angular operator vanishes identically $\Sigma_{\theta, \phi} \Psi_{j=0}^{k=+1}(x) = 0$, and corresponding radial equations are as follows

\[+i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 = m C_1,\]
\[+i \epsilon E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 = m \Phi_1,\]
\[-i \epsilon \Phi_1 = m E_1, \quad -i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_1 = m H_1.\]

Excluding $H_1, E_1$ for $C_1 = \Phi_1$ one gets

\[\left( \frac{d^2}{dr^2} + \epsilon^2 - m^2 \right) \frac{\Phi_1}{r} = 0 \quad \Rightarrow \quad \frac{\Phi_1}{r} = \exp \left[ \pm \sqrt{m^2 - \epsilon^2} \right].\] (45)

One of these solutions might be associated with a "bound state", this is what makes them very interesting from physical standpoint.

Another variant

\[k = -1, \quad j_{\text{min}} = 0\]
will look the same. Now an initial substitution is

\[
\Psi_{k=\pm 1}^{j=0}(x) : \quad C(r) = 0 , \quad C_0(r) = 0 , \quad \Phi_0(r) = 0 ,
\]

\[
\tilde{C}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ 0 \\ C_3(r) \end{vmatrix} , \quad \tilde{\Phi}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ 0 \\ \Phi_3(r) \end{vmatrix} ,
\]

\[
\tilde{E}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ 0 \\ E_3(r) \end{vmatrix} , \quad \tilde{H}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ 0 \\ H_3(r) \end{vmatrix} . \quad (46)
\]

Again on those functions \( \Psi^{j=0}(x), \Sigma_{\theta,\phi}^{k} \Psi^{j=0}(x) = 0 \), and radial system will be

\[
+i \epsilon E_3 - i \left( \frac{d}{dr} + \frac{1}{r} \right) H_3 = m C_3 ,
\]

\[
+i \epsilon E_3 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_3 = m \Phi_3 ,
\]

\[
-i \epsilon \Phi_3 = m E_3 , \quad i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_3 = m H_3 .
\]

which after excluding \( H_3, E \) gives

\[
\left( \frac{d^2}{dr^2} + \epsilon^2 - m^2 \right) \frac{\Phi_3}{r} = 0 \quad \Rightarrow \quad \frac{\Phi_3}{r} = \exp[ \pm \sqrt{m^2 - \epsilon^2} ] . \quad (47)
\]

Now we consider states with minimal \( j_{\text{min}} = |k| - 1 \) when \( k = \pm 3/2, \pm 2, \pm 5/2, \ldots \).

Here, though the \( \theta, \phi \)-variables enters \( \Psi^{j_{\text{min}}}(x) \), however the angular operator acts on these functions as zero-like. Actually, as

\[
k = +3/2, +2, +5/2, \ldots , \quad j_{\text{min}} = |k| - 1 , \quad \Psi^{j_{\text{min}}}(x) : \quad C(r) = 0 , \quad C_0(r) = 0 , \quad \Phi_0(r) = 0 ,
\]

\[
\tilde{C}(x) = e^{-i\epsilon t} \begin{vmatrix} C_1(r)D_{k-1} \\ 0 \\ 0 \end{vmatrix} , \quad \tilde{\Phi}(x) = e^{-i\epsilon t} \begin{vmatrix} \Phi_1(r)D_{k-1} \\ 0 \\ 0 \end{vmatrix} ,
\]

\[
\tilde{E}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ E_1(r)D_{k-1} \\ 0 \end{vmatrix} , \quad \tilde{H}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ H_1(r)D_{k-1} \\ 0 \end{vmatrix} . \quad (48)
\]

Here you need the recurrence formulas [2]

\[
\partial_{\theta} D_{k-1} = \sqrt{\frac{k-1}{2}} D_{k-2} , \quad -m - (k-1) \cos \theta \sin \theta D_{k-1} = -\sqrt{\frac{k-1}{2}} D_{k-2} .
\]

with the help of which you easy can prove \( \Sigma_{\theta,\phi} \Psi^{j_{\text{min}}}(x) = 0 \) and then again we arrive at eqs. (14).

Another variant

\[
k = -3/2, -2, -5/2, \ldots , \quad j_{\text{min}} = |k| - 1 ;
\]
looks the same:

\[ \Psi^{j_{\text{min}}}(x) : C(r) = 0, \quad C_0(r) = 0, \quad \Phi_0(r) = 0, \]

\[
\bar{C}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 & 0 \\ C_3(r)D_{k+1} & 0 \end{vmatrix}, \quad \bar{\Phi}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ \Phi_3(r)D_{k+1} \end{vmatrix},
\]

\[
\bar{E}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ E_+(r)D_{k+1} \end{vmatrix}, \quad \bar{H}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ H_3(r)D_{k+1} \end{vmatrix},
\]

\[
\partial_0 D_{k+1} = -\sqrt{-\frac{(k+1)}{2}} D_{k+2},
\]

\[
-\frac{m - (k+1) \cos \theta}{\sin \theta} D_{k+1} = -\sqrt{-\frac{(k+1)}{2}} D_{k+2},
\]

with the help of which you easy can prove \( \Sigma_{\theta,\phi} \Psi^{j_{\text{min}}}(x) = 0 \) and then again we arrive at eqs. (17).

### 6 On the Coulomb problem of an ordinary \( S = 1 \) particle

Quantum mechanical study of a vector particle in Coulomb field has a long history. We are going to review this old problem else one time in the frame of the approach developed above. So, let us consider a 10-component Duffin-Kemmer equation for a \( S=1 \) particle in Coulomn

\[
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\]

\[
\text{field all details on the working technic used here and based on the tetrad formalism).}
\]

In the spherical-cyclic basis the most general form of 10-component wave function with quantum numbers \( \epsilon, j, m \) is

\[
\Psi(x) = \{ \Phi_0(x), \bar{\Phi}(x), \bar{E}(x), \bar{H}(x) \},
\]

\[
\Phi_0(x) = e^{-i\epsilon t} \Phi_0(x) D_0, \quad \bar{\Phi}(x) = e^{-i\epsilon t} \begin{vmatrix} \Phi_1(r) D_{-1} \\ \Phi_2(r) D_0 \\ \Phi_3(r) D_{+1} \end{vmatrix},
\]

\[
\bar{E} = e^{-i\epsilon t} \begin{vmatrix} E_1(r) D_{-1} \\ E_2(r) D_0 \\ E_3(r) D_{+1} \end{vmatrix}, \quad \bar{H} = e^{-i\epsilon t} \begin{vmatrix} H_1(r) D_{-1} \\ H_2(r) D_0 \\ H_3(r) D_{+1} \end{vmatrix},
\]

\[
(50)
\]

where \( D_s = D_{-m,s}(\phi, \theta, 0), \quad s = 0, +1, -1 \). Omitting details of separation of variables let us immediately write down a radial system (that can be derived directly from 15-component above by setting \( \sigma = 0 \) and so on)

\[
-\left( \frac{d}{dr} + \frac{2}{r} \right) E_2 - \frac{\nu}{r} (E_1 + E_3) = m \Phi_0,
\]

\[
+i(\epsilon + \frac{\alpha}{r}) E_1 + i \left( \frac{d}{dr} + \frac{1}{r} \right) H_1 + i \frac{\nu}{r} H_2 = m \Phi_1,
\]

\[
+i(\epsilon + \frac{\alpha}{r}) E_2 - i \frac{\nu}{r} (H_1 - H_3) = m \Phi_2,
\]

\[
+i(\epsilon + \frac{\alpha}{r}) E_3 - i \left( \frac{d}{dr} + \frac{1}{r} \right) H_3 - i \frac{\nu}{r} H_2 = m \Phi_3,
\]

\[
(51)
\]
−i(ε + \frac{α}{r}) \Phi_1 + \frac{ε}{r} \Phi_0 - mE_1 = 0 ,
−i(ε + \frac{α}{r}) \Phi_2 - \frac{d}{dr} \Phi_0 - mE_2 = 0 ,
−i(ε + \frac{α}{r}) \Phi_3 + \frac{ν}{r} \Phi_0 - mE_3 = 0 ,
−i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_1 - iν \frac{Φ_2}{r} - mH_1 = 0 ,
+iν \frac{Φ_1}{r} - mH_2 = 0 ,
+i \left( \frac{d}{dr} + \frac{1}{r} \right) \Phi_3 + iν \frac{Φ_2}{r} - mH_3 = 0 .
(52)

Concurrently with \vec{J}_2, J_3 one may diagonalize the operator of the space inversion \hat{Π}. That in chosen representation has the form
\hat{Π} = \begin{bmatrix} (1 \oplus Π_3) \oplus (Π_3 \oplus -Π_3) \end{bmatrix} \hat{P} , \quad Π_3 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} .

The eigenvalue equation \hat{Π}Ψ = P Ψ gives solutions of two types:
\begin{align*}
P = (-1)^{j+1} , & \quad Φ_0 = 0 , \quad Φ_3 = -Φ_1 , \quad Φ_2 = 0 , \\
E_3 = -E_1 , \quad E_2 = 0 , \quad H_3 = H_1 ;
\end{align*}
\begin{align*}
P = (-1)^{j} , & \quad E_3 = +E_1 , \quad H_3 = -H_1 , \quad H_2 = 0 .
(53)
\end{align*}

As for the case \( P = (-1)^{j+1} \) you have the system
\begin{align*}
+i(ε + \frac{α}{r}) E_1 + i(\frac{d}{dr} + \frac{1}{r}) H_1 + iν \frac{H_2}{r} = mΦ_1 ,
-i(ε + \frac{α}{r}) Φ_1 = mE_1 , \quad -i(\frac{d}{dr} + \frac{1}{r}) Φ_1 = mH_1 , \quad 2iν \frac{Φ_1}{r} = mH_2 ,
\end{align*}
which after excluding \( E_1, H_1, H_2 \) leads to
\begin{align*}
\left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + (ε + \frac{α}{r})^2 - \frac{j(j+1)}{r^2} \right] Φ_1 = 0 .
(54)
\end{align*}

This exactly the same equation that arises in the theory of a scalar particle in Coulomb field. Its energy spectrum and wave functions are well known.

States with \( P = (-1)^{j} \) are characterized by the system
\begin{align*}
(\frac{d}{dr} + \frac{2}{r}) E_2 - 2ν \frac{E_1}{r} = mΦ_0 ,
+i(ε + \frac{α}{r}) E_1 + i(\frac{d}{dr} + \frac{1}{r}) H_1 = mΦ_1 ,
+i(ε + \frac{α}{r}) E_2 - 2iν \frac{H_1}{r} = mΦ_2 ,
-i(ε + \frac{α}{r}) Φ_1 + \frac{ν}{r} Φ_0 = mE_1 ,
-i(ε + \frac{α}{r}) Φ_2 - \frac{d}{dr} Φ_0 = mE_2 ,
-i(\frac{d}{dr} + \frac{1}{r}) Φ_1 - iν \frac{Φ_2}{r} = mH_1
(55)
\end{align*}
in solving which we have not been able to succeed.

The above radial equations (50) as well as substitutions (50) are correct only for \( j = 1, 2, \ldots \) However you should consider the \( j = 0 \) case separately and having started with a special wave function:

\[
C_0(x) = e^{-i\epsilon t}C_0(r), \quad \Phi_0(x) = e^{-i\epsilon t}\Phi_0(r),
\]

\[
\bar{C}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ C_2(r) \\ 0 \end{vmatrix}, \quad \bar{\Phi}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ \Phi_2(r) \\ 0 \end{vmatrix},
\]

\[
\bar{E}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ E_2(r) \\ 0 \end{vmatrix}, \quad \bar{H}(x) = e^{-i\epsilon t} \begin{vmatrix} 0 \\ H_2(r) \\ 0 \end{vmatrix}.
\] (56)

This \( \Psi \) is eigenfunction of \( \hat{\Pi} \) with \( \Pi = +1 \). Corresponding radial system is \( (\Phi_0 = \varphi_0, -i\Phi_1 = \varphi_1, -i\Phi_2 = \varphi_2) \)

\[
H_2 = 0, \quad -(\frac{d}{dr} + \frac{2}{r})E_2 = m\varphi_0, \quad (\epsilon + \frac{\alpha}{r})E_2 = m\varphi_2, \quad (\epsilon + \frac{\alpha}{r})\varphi_2 - \frac{d}{dr}\varphi_0 = mE_2,
\]

from where it follows (that corresponds to (30) when \( \sigma = 0 \))

\[
\left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{2}{r^2} + (\epsilon + \frac{\alpha}{r})^2 - m^2 \right] E_2 = 0,
\]

or introducing \( E_2(r) = r^{-1}f(r) \)

\[
\frac{d^2}{dr^2} f + \left( \epsilon^2 - m^2 + \frac{2\alpha\epsilon}{r} - \frac{2 - \alpha^2}{r^2} \right) f = 0.
\] (57)

The latter will read in usual dimension units as

\[
\frac{d^2}{dx^2} f + \left[ \frac{E^2}{c^2h^2} - \frac{M^2c^2}{h^2} + 2Z(e^2/ch)(E/ch) - \frac{2 - Z^2(e^2/ch)^2}{r^2} \right] f = 0.
\]

Now you can pass to a dimensionless coordinate \( x = rE/ch \), then the equation becomes

\[
\frac{d^2}{dx^2} f + \left[ 1 - \frac{M^2c^4}{E^2} + 2Z(e^2/ch) \frac{1}{x} - \frac{2 - Z^2(e^2/ch)^2}{x^2} \right] f(x) = 0.
\]

With the notation

\[
\frac{M^2c^4}{E^2} = \Lambda^2, \quad Z \frac{e^2}{c h} = Z \frac{1}{137} = \gamma < 1
\]

the equation will read as

\[
\frac{d^2}{dx^2} f + \left( 1 - \Lambda^2 + \frac{2\gamma}{x} - \frac{2 - \gamma^2}{x^2} \right) f = 0.
\] (58)

To obtain bound states you should take \( f(x) \) in the form

\[
f(x) = x^{\alpha} e^{-bx} F(x)
\]
where \( a \) and \( b \) are expected to be positive whereas \( F(x) \) be a polynomial in \( x \). From (58) we derive

\[
x F'' + (2a - 2bx) F' + \left[ \frac{a(a - 1) + \gamma^2 - 2}{x} + (b^2 + 1 - \Lambda^2)x + (2\gamma - 2ab) \right] F = 0 .
\]

This equation with demands

\[
a(a - 1) + \gamma^2 - 2 = 0 , \quad b^2 + 1 - \Lambda^2 = 0
\]

leads to

\[
x F'' + 2(a - bx) F' + 2(\gamma - ab) F = 0 . \tag{5.15c}
\]

For \( a \) and \( b \) we have

\[
a = \frac{1 \pm \sqrt{9 - 4\gamma^2}}{2} , \quad b = \pm \sqrt{\Lambda^2 - 1} = \pm \frac{\sqrt{M^2c^4 - E^2}}{E} . \tag{59}
\]

The choice of "upper" signs is what you need to \( a \) and \( b \) be positive.

Further, taking \( F(x) \) as a series

\[
F(x) = \sum_{k=0}^{\infty} C_k x^k , \quad F' = \sum_{k=1}^{\infty} kC_k x^{k-1} , \quad F'' = \sum_{k=2}^{\infty} k(k-1)C_k x^{k-2} ,
\]

you get

\[
\sum_{k=2}^{\infty} k(k-1)C_k x^{k-1} + 2a \sum_{k=1}^{\infty} kC_k x^{k-1} - 2b \sum_{k=1}^{\infty} kC_k x^{k} + 2(\gamma - ab) \sum_{k=0}^{\infty} c_k x^k = 0
\]

or

\[
\sum_{n=1}^{\infty} n(n+1)C_{n+1} x^n + 2a \sum_{n=0}^{\infty} (n+1)C_{n+1} x^n -
- 2b \sum_{n=1}^{\infty} nC_n x^n + 2(\gamma - ab) \sum_{n=0}^{\infty} C_n x^n = 0
\]

or

\[
[ 2aC_1 + 2(\gamma - ab) ] x^0 + [ 2C_2 + 2a 2C_2 - 2b C_1 + 2(\gamma - ab)C_1 ] x +
+ \sum_{n=2}^{\infty} [ n(n+1)C_{n+1} + 2a(n+1)C_{n+1} - 2bnC_n + 2(\gamma - ab)C_n ] x^n = 0 .
\]

Demanding coefficients at all \( x^k \) be equal to zero, we produce recursive relations

\[
C_1 = -(\gamma - ab) \quad C_0 = 0 ,
\]

\[
C_2 (1 + 2a) = 2 \left[ b - (\gamma - ab) \right] C_1 = 0 , \quad n = 2, 3, 4, ... ,
\]

\[
C_{n+1} (n + 1) (n + 2a) = 2 \left[ n b - (\gamma - ab) \right] C_n = 0 . \tag{60}
\]

To terminate an infinite series at a certain place you need impose

\[
C_{N+1} = 0 \quad \implies \quad [ N b - (\gamma - ab) ] = 0
\]
that provides us with a quantization condition to produce an energy spectrum. It looks as

$$\frac{\gamma - ab}{b} = N$$

which having remembered \( a \) and \( b \) (5.16) will take the form (the notation \( 2\Gamma = (1 + \sqrt{9 - 4\gamma^2} \) is used),

$$\frac{2\gamma\epsilon - \Gamma \sqrt{m^2c^4 - \epsilon^2}}{2\sqrt{m^2c^4 - \epsilon^2}} = N$$

from which you arrive at the energy formula

$$\epsilon = mc^2 \left[ 1 + \frac{\gamma^2}{(\Gamma + N)^2} \right]^{-1/2}.$$  \( (61) \)

Some additional remarks might be given. The material of this Section might be of some interest in the light of the well-known old results obtained by I.E. Tamm [...] on behavior of the vector particle wave functions in presence of external Coulomb field.

The most principal part of this work consists in the following. All possible solutions can be divided into two classes. One is reduced to the well known functions of the scalar particle in Coulomb field with usual and expected behavior at the origin \( r = 0 \) and at infinity \( r = \infty \). However another part of wave functions provides us with big surprise – they are singular at the origin, corresponding differential equations have not been resolved up to now; it is generally accepted that they should be associated with a situation of falling down to centre \( r = 0 \).

In essence, results of this Section prove that there exists definite correlation between values of \( j \), of \( P \), and property of wave function to be singular or non-singular; it can be seen in the following table:

| Value of quantum number | \( P = (-1)^j \) singular solution | \( P = (-1)^j \) non-singular solution | \( P = (-1)^{j+1} \) singular solution | \( P = (-1)^{j+1} \) non-singular solution |
|------------------------|-----------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| \( j = 0 \)            |                                   | \( P = +1 \)                         |                                     |                                     |
| \( j = 1 \)            | \( P = -1 \)                       |                                     |                                     |                                     |
| \( j = 2 \)            | \( P = +1 \)                       |                                     |                                     | \( P = +1 \)                       |
| \( j = 3 \)            | \( P = -1 \)                       |                                     |                                     | \( P = -1 \)                       |
| \( j = 4 \)            | \( P = +1 \)                       |                                     |                                     | \( P = +1 \)                       |
| ...                   | ...                               | ...                                 | ...                                 | ...                                 |

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