White dwarf - main sequence star collisions from wide triples in the field

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ABSTRACT

Multiple star systems interact strongly with Galactic field stars when the outer semi-major axis of a triple or multiple star is $>10^3$ AU. Stable triples composed of two white-dwarfs (WD) and a low mass main sequence (MS) star in a wide outer orbit can thus be destabilized by gravitational interactions with random field stars. Such interactions excite the eccentricity of the distant third star sufficiently so that it begins to interact significantly with the inner binary. When this occurs the triple undergoes multiple binary-single resonant encounters. These encounters may result either in a collision between the nondegenerate component and a WD, or the breakup of the triple into a compact binary and a third object which is ejected. The compact binary can be either a MS-WD pair which survives, or collides, or a double WD which may inspiral through gravitational wave emission. We calculate the collision rate between a MS and WD star, and the merger rate of double WDs. Additionally, we describe the prospects of detectability of such a collision, which may resemble a sub-luminous SN event.

Key words: Keyword – Keyword – Keyword

1 INTRODUCTION

In recent years it had been shown that for ultrawide stellar systems the host galactic field is collisional (Kaib & Raymond 2014; Antognini & Thompson 2016; Michaely & Perets 2016, 2019b; Michaely & Perets 2020; Michaely 2020), that is one cannot regard the system as isolated from its environment. Therefore, when considering the evolution of wide stellar systems one needs to account for random gravitational interactions with passing stars, including close flybys, even in low density environments like the field of the host galaxy. In turn, the interactions between flybys and wide systems change the wide system’s orbital characteristics, both the semi-major axis (SMA) and its orbital eccentricity. The change in eccentricity is usually the more pronounced effect. (Lightman & Shapiro 1977; Merritt 2013).

Here we consider ultra wide systems to be either wide binaries or wide hierarchical triples. Kaib & Raymond (2014) calculated the rate of stellar collision in the Milky-Way (MW) Galaxy, caused by wide (SMA > 1000AU) binaries interacting with random flyby stars in the field. They found that a stellar collision between two main sequence (MS) stars happens every 1000 – 7500yrs in the MW. Expanding on that work Michaely & Perets (2016) described the formation of low-mass X-ray binaries (LMXBs) from wide binary systems composed of a stellar compact object, i.e. a black-hole (BH) or a neutron star (NS) and a low mass companion. They found that if BHs are born with little to no natal kicks then the formation rate of LMXBs is consistent with the inferred rate from observations. Additionally, Michaely & Perets (2019b) found that gravitational waves (GW) sources are also formed from wide binary BHs interacting with flyby stars in their host galaxies and reported a rate of $1 – 10$ Gpc$^{-3}$yr$^{-1}$.

Hierarchical triples are composed of an inner binary and an outer binary, where the inner binary center of mass (COM) is consider one component of the outer binary and the distant, third object is the second component. Michaely & Perets (2020) calculated the GW merger rate for wide BH triples that become unstable due to interactions with random field stars that excite their outer eccentricity. As a result the triple becomes unstable and a GW merger can happen either in the instability phase or at the endstate phase. Under their assumptions they found an extremely high merger rate of $100 – 250$ Gpc$^{-3}$yr$^{-1}$, which indicates that while the formation channel is robust the underlying assumptions are very generous. Recently (Michaely 2020, hereafter: Paper I) calculated via similar dynamical processes the Type Ia supernova (SN) rate of triple white-
dwarf (WD) systems. The SN originate either by a direct collision from double WDs (Raskin et al. 2009; Rosswog et al. 2009; Thompson 2011; Kushnir et al. 2013) or by a GW inspiral that leads to double WDs coalescing in a Hubble time, a process called “the double-degenerate (DD) channel” (Iben & Tutukov 1984; Webbink 1984).

Recently several studies were done on the dynamics of scattering events trying to calculate, among others, the rates of DWD mergers or collisions as a source for Type Ia SNe. Antognini & Thompson (2016) performed an extensive numerical study on the nature of scattering events: binary-binary, triple-single, and triple-single (the focus of this manuscript). In Antognini & Thompson (2016) only triples with outer SMA < 1000AU were considered, unlike paper I and this manuscript. They reported that the collision rates of DWD in the field is $\sim 2 \times 10^{-7}$ yr$^{-1}$. Population synthesis studies were carried out in order to quantify the Type Ia SN originating from triples, due to secular evolution Hamers et al. (2013); Toonen et al. (2018). They showed that the fine tuning of the mutual orbital inclination combined with stellar evolution lower SN rates down to $0.1 \sim 1\%$ of the total rate. Additionally, Hamers (2018) calculated the Type Ia SN originating from quadruple star systems, due to mergers and collisions. It was found the combined rate for all the possible channels that lead to a SN is $3$ order of magnitude lower than the observed rate. In Leigh et al. (2018) it was shown that gravitational scattering is dominated by triple-single scattering events, similar to paper I and this manuscript, rather than binary-binary or triple-binary scattering.

In this study we follow-up the work of paper I and relax the assumption that all three components of the triples are WDs. Instead we focus on triples wherein two of the components are WDs and the third is a low mass MS star. Instead we focus on triples wherein two of the components are WDs. Instead we focus on triples wherein two of the components are WDs.

The outcomes of a collision between a WD and a low mass MS star have been explored in the literature (Shara & Shaviv 1977, 1978; Shara & Regev 1986; Regev & Shara 1987; Ruffert 1992; Shara 1999). Other stellar collisions have been studied as well (Soker & Tylenda 2006; Katz & Dong 2012; Aznar-Sigué et al. 2013). Here we consider the energetics, timescales and the potential detectability of the expected transient generated by a WD-MS collision and reserve a detailed study for future research.

In section 2 we describe the interaction of ultra-wide triples in the field of a host galaxy. In section 3 we discuss the dynamics in the unstable phase of the triple as multiple binary-single encounters. In section 4 we calculate the expected rates of collisions and DD mergers in large spiral and elliptical galaxies. We discuss the energetics and detectability of WD-MS collisions in section 5. We estimate the delay-time distribution of WD-MS collisions in section 6. We summarize our results in section 7.

2 WIDE TRIPLES IN THE FIELD

In what follows we briefly describe the gravitational interaction between a wide hierarchical system, illustrated in Fig. 1, and a flyby star in the field of the host galaxy. A complete mathematical description can be found in paper I and references therein.

For simplicity we assume throughout this paper that all three components in the triple have the same mass $m_3 = m_2 = m_3 = 0.6 M_\odot$. The inner binary (masses $m_1$ and $m_2$) of the triple is characterized by a circular orbit with a SMA $a_1$. The outer binary, composed of the third object $m_3$ and the center of mass (COM) of the inner binary is characterized by a SMA $a_2 > 1000AU$ and an eccentricity $e_2$. The nature of the interaction between the triple and a random flyby star is determined by the local environment of the host galaxy. In particular, the encounter is determined by the relative speed between the flyby star and the triple, $v_{\text{enc}}$ and the interaction rate, $f$. We identify the relative speed to be the local velocity dispersion, $v_{\text{enc}} = \sigma$. The interaction rate is related to the local number stellar density $n$, by $f = n_{\text{CS}} v_{\text{enc}}$ where $\sigma_{\text{CS}}$ is the geometrical cross-section of the flyby interaction.

In paper I it was shown that for an ensemble of wide triple systems with a thermal distribution, $f(e_2) = 2e_2$ of outer eccentricities, there is a non-negligible probability that the third component’s pericenter passage, $q = a_2 (1 - e_2)$, is sufficiently close to the inner binary COM that the systems becomes unstable. Specifically, the instability condition is $q \leq a_1$.

The instability of the triple can be described as follows. The triple undergoes multiple binary-single encounters, during which a temporary binary is formed from a random pair of the three system components, with a SMA $a_{\text{IMS}}$ and eccentricity $e_{\text{IMS}}$, where the subindex IMS stands for “intermediate state”. The values of $a_{\text{IMS}}$ are drawn uniformly between $(a_1, 2a_1)$ and the $e_{\text{IMS}}$ are drawn from a thermal distribution (for a complete derivation see eq. 20-24 in paper I). The third object is bound to the temporary binary on a wide Keplerian orbit with $a_{\text{IMS}}$ determined by conservation of orbital energy. The relatively wide outer orbit allows the inner binary to revolve around its COM multiple times before the next pericenter passage, namely before the next binary-single interaction where this process repeats.

For point-like particles the average number of binary-single encounters is $(N) = 20$ (Michaely & Perets 2020). The end state is the formation of a compact binary with SMA $a_{\text{ES}} < a_1$, and eccentricity $e_{\text{ES}}$ which follows a thermal distribution. The end state binary is composed of two objects randomly chosen out of the triple. The third object is ejected to infinity.

In order to determine the rate at which wide triple systems become unstable we focus on the outer binary of the wide triple, with $a_2$ and $e_2$. This wide binary interacts with flyby stars, of mass $m_p$, which perturb the outer binary. Lightman & Shapiro (1977) and Merritt & Poon (2004) showed that the perturbation primarily torques the outer binary, changing its eccentricity. We use the “loss-cone” formalism to determine the loss rate of wide stable triple star systems.

We consider an ensemble of wide systems with a thermal distribution of the outer eccentricity. The loss cone is the fraction of systems from the ensemble that becomes unsta-
able, namely the outer pericenter distance of the outer binary is within the inner binary SMA, \( q \leq a_1 \). From paper I we get:

\[
F_q = \frac{2a_1}{a_2}
\]

which for hierarchical triples \( F_q \ll 1 \). We compare this value to the smear-cone, \( F_\pi \) (paper I) which essentially represents the change in eccentricity due to the impulse interaction with the perturber.

\[
F_\pi = 27 \left( \frac{m_3}{M} \right)^2 \left( \frac{GM}{a_2 v_\text{esc}^2} \right) \left( \frac{a_2}{b} \right)^4
\]

where \( M = m_1 + m_2 + m_3 \) is the total mass of the triple and \( b \) is the closest approach of the perturber to the triple COM. These quantities naturally create two regimes, the full loss cone and the empty loss cone regimes. In the full loss cone regime, where \( F_q \geq F_\pi \), the loss cone is continuously full, namely the interactions are frequent and strong enough to keep the loss cone full. In the empty loss cone regime, \( F_q < F_\pi \) the loss cone is primarily empty until a weak interaction with a flyby star occurs which kicks a system into the loss cone.

Additionally, we account for binary ionization processes in the field. Because the field is collisional for wide binaries we need to account for the loss of wide binaries due to the disruption of wide binaries from random gravitational processes. We use the standard half-life calculation from (Bahcall et al. 1985)

\[
t_{1/2} = \frac{V_\text{esc}}{GM_{\text{ planets}}/a_2},
\]

The loss probabilities, the probability of a system with inner SMA \( a_1 \), outer SMA \( a_2 \), located in a stellar environment with number density \( n_\star \) at time \( t \) to become unstable, are given by the following equations: for the empty loss cone regime

\[
L(a_1, a_2, n_\star)_{\text{empty}} = \frac{2a_1}{a_2} n_\star \pi \sqrt{\frac{27}{8} \left( \frac{m_3}{M} \right)^2 \left( \frac{GM a_1^2}{a_2} \right)} (1 - e^{-t/\tau})
\]

where \( \tau = t_{1/2}/\ln 2 \). For the full loss cone regime we get

\[
L(a_1, a_2, n_\star)_{\text{full}} = \frac{2a_1}{a_2} \left( \frac{GM}{4\pi^2 a_1^2} \right)^{1/2} \left( 1 - e^{-t/\tau} \right).
\]

## 3 BINARY-SINGLE ENCOUNTERS

In this section we focus on the dynamics of the triple system when it becomes unstable and enters the multiple binary-single interaction phase. In subsection 3.1 we focus on the intermediate phase while in subsection 3.2 we explore the end state of the intermediate phase, specifically the compact binary that forms as a result of the intermediate phase.

The case where one of the components is a low mass star, in its MS phase, is fundamentally different from the case presented in paper I and in Michaely & Perets (2020). The reason is that the radius of a low mass MS star is roughly 50 times larger than the radius of a WD. Hence a direct collision between a WD and the MS star becomes probable.

3.1 The intermediate phase

In order to calculate the probability of this scenario, a collision between a MS star and a WD from a triple with two WDs and one MS, star we perform a numerical calculation. We sample 20 values of the inner SMA \( a_1 \) from \( (10^{-2} \text{AU}, 10^5 \text{AU}) \) equally distributed in log space. For each value of \( a_1 \) we perform \( N_{\text{tot}} = 10^5 \) “scattering experiments”. In this context a single “scattering experiment” is \( N_{\text{IMS}} = 20 \) binary-single interactions, where for each one a temporary binary is formed with \( a_{\text{IMS}} \) and \( e_{\text{IMS}} \). \( a_{\text{IMS}} \) is drawn uniformly from \( [a_1, a_1'] \) and \( e_{\text{IMS}} \) is drawn from a thermal distribution. The boundary values \( (a_1', a_1) \) are determined by the individual masses of the triple (see eq. 18-24 in paper I). Moreover, each temporary binary is composed of two random components, such that either a double WD is formed or a WD-MS binary is formed. The temporary pericenter is \( q = a_{\text{IMS}} (1 - e_{\text{IMS}}) \) and we compare it to the sum of the radii of the temporary binary, \( R = r_1 + r_2 \). We calculate the radius of the WD using Hamada & Salpeter (1961):

\[
R_{\text{WD}} = 1.3 \times 10^{-2} R_\odot \left( \frac{M_{\text{WD}}}{0.6 M_\odot} \right)^{-1/3},
\]

and for the low mass convective MS star we use

\[
R_\star = 0.6 R_\odot \left( \frac{m_2}{0.6 M_\odot} \right)
\]

taken from Mann et al. (2015).

where \( m_2 \) is the mass of the MS star. In the case where \( q' \leq R \) we flag the scattering experiment as a collision, check the nature of the components that collide and terminate this experiment. In the case where \( q' > R \) we calculate the outer binary orbital period, which is the time of the next interaction and randomize the binary-single interaction again until we reach \( N_{\text{IMS}} \) times.

In order to calculate the fraction of systems that undergo a collision between a WD and a MS star in the intermediate phase, \( f_{\text{WD-MS}}(a_1) \) we divided the number of mergers by \( N_{\text{tot}} \) for each \( a_1 \). The numerical results are presented in Fig. 2. We fit the numerical results with two functions as
a function of the inner SMA $a_1$.

$$
\log f_{\text{WD-MS}} (a_1) = \begin{cases} 
-0.961 \cdot \log a_1 - 1.212, & a_1 > 0.1 \text{AU} \\
-0.542 \cdot \exp \left( \frac{(\log a_1 + 0.49)}{0.682} \right)^2, & a_1 < 0.1 \text{AU} 
\end{cases}
$$

(8)

We note here that we ignore the rate of double WD collisions in the intermediate stage, represented in Fig. 2 with red circles. One can expect the rate of double WD collision scales as the ratio of the radii of WD and the MS star, for $M_{\text{WD}} = m_* = 0.6 M_\odot$

$$
R_{\text{WD}} / R_c \approx 0.02.
$$

(9)

### 3.2 End state

In the case where no collision occurs during the $N_{\text{IMS}}$ times we simulate the endstate configuration. The emerging compact binary has a different SMA distribution than the distribution in the intermediate phase. Specifically, the energy of the final binary is distributed according to the following relation (Stone & Leigh 2019)

$$
E_{\text{ES}} \propto |E_1|^{-4},
$$

(10)

where $E_{\text{ES}}$ is the energy of the endstate binary and $E_1 = -G m_1 m_2 / (2 a_1)$ is the initial binary energy. Similar to the intermediate phase case the eccentricity of the compact binary $e_{\text{ES}}$ is drawn from a thermal distribution (Stone & Leigh 2019). Hence, a compact binary is formed for every system that did not collide during the IMS with a SMA $E_{\text{ES}}$ ($E_{\text{ES}}$) and eccentricity $e_{\text{ES}}$. This endstate binary is composed either of a double WD (DWD) or a MS star with a WD companion. As a result the possible outcomes are either a direct collision between the components or the ensuing evolution of a binary star. Depending on the binary components, the evolution may lead to a merger through gravitational waves in the case of a DWD, or any standard outcome allowed by binary evolution theory, including common envelope evolution (CEE), a cataclysmic variable (CV) etc. In the left plot of Fig. 3 we present the fraction of systems that experienced a WD-MS collision in the post-resonance phase as a function of the initial SMA. The numerical fit presented in the figure is composed of two functions:

$$
\log f_{\text{WD-MS}} (a_1) = \begin{cases} 
-0.94 \cdot \log a_1 - 2.18, & a_1 > 0.1 \text{AU} \\
-0.006 \cdot \exp (-2.98 \log a_1), & a_1 < 0.1 \text{AU} \\
-2.113 \cdot \exp (0.51 \log a_1)
\end{cases}
$$

(11)

Next, we calculate the fraction of systems that undergo a double WD merger through GW emission similar to the classic DD scenario. In order to do so we calculate the GW merger timescale for eccentric binaries which is (Peters 1964)

$$
t_{\text{merger}} \approx \frac{708}{425} T_e (a_1 (1 - e^2))^{7/2},
$$

(12)

where $T_e = a^4 / \beta$ is the merger timescale for a circular orbit and $\beta = 64 G^2 m_1 m_2 (m_1 + m_2) / (c^2)$. The indices $i, j$ are the indices of the two WDs that ended up as the surviving compact binary and $c$ is the speed of light. If $t_{\text{merger}} < 10^{10} \text{yr}$ we flag this systems as a DD inspiral, and a possible source for a Type Ia SN. The numerical fit is

$$
\log f_{\text{DD}} (a_1) = \begin{cases} 
-1.04 \cdot \log a_1 - 2.647, & a_1 > 0.1 \text{AU} \\
-0.009 \cdot \exp (-2.79 \log a_1), & a_1 < 0.1 \text{AU} \\
-2.77 \cdot \exp (0.56 \log a_1)
\end{cases}
$$

(13)

We emphasize here that the outcome of a WD-MS binary that does not collide is still of interest. These binaries are the progenitors of CEE or CV. Once the MS star companion in the binary evolves to the giant phase it may fill its Roche-lobe and begin mass transfer. If the mass transfer is stable an accretion disk forms around the WD and a CV is formed. In the case where the mass transfer is unstable the system may inspiral to a CEE. The study of the evolution of these binaries has been explored elsewhere (for a recent overview see Beccari & Boffin (2019)).

### 4 GALACTIC RATES

In what follows we use the numerical fits found in the previous section in order to compute rates in galaxies for WD-MS collisions in the intermediate phase and in the endstate. Additionally, we compute the galactic rates of DD inspiral.

#### 4.1 Galaxy models

As described in section 2, the process that destabilizes wide triples depends on the local stellar environment. Specifically these are the local stellar density, $n_*$ and the encounter velocity, $v_{\text{esc}}$ which is determined by the local velocity dispersion. Hence, one needs to model these properties of the host galaxy.

First, we model the MW Galaxy as a prototype of a large spiral galaxies in the local universe, using the same
model described in paper I which follows (Jurić et al. 2008). The number stellar density is given by
\[ n_\star (r) = n_\odot e^{-(r-r_\odot)/R_\odot} \] (14)
where \( n_\star \) represents the number stellar density for and \( n_\odot = 0.1 \text{pc}^{-3} \) is the number stellar density in the solar neighborhood, \( R_\odot = 2.6 \text{kpc} \) is the Galactic length scale and \( r_\odot = 8 \text{kpc} \) is the distance of the Sun from the Galactic center. The velocity dispersion we use is that of the flat rotation curve of the Galaxy, i.e. \( \sigma = 50 \text{km s}^{-1} \) which in turn is set to be the encounter velocity.

We define \( dN_\star (r) \), the number of stars in a volume of the disk at a distant \( r \) from the center of the Galaxy, by
\[ dN_\star (r) = n_\star (r) \cdot 2\pi \cdot r \cdot h \cdot dr \] (15)
where \( h = 1 \text{kpc} \) is the scale height of the disk.

Second, for simplicity we model an elliptical galaxy the same way we did in paper I following Hernquist (1990). The density profile is given by
\[ n_{e_{\text{ell}}} (r) = \frac{M_{\text{galaxy}}}{2\pi r_s} \frac{r_s}{(r+r_s)^3} \] (16)
where \( n_{e_{\text{ell}}} \) is the stellar density for elliptical galaxy and \( r_s = 1 \text{kpc} \) is the scale length of the galaxy, \( M_{\text{galaxy}} = 10^{11} M_\odot \) is the total stellar mass of the galaxy. Hence,
\[ dN_e (r) = \frac{n_{e_{\text{ell}}}}{m} 4\pi \] (17)
is the number of stars within some local volume \( dV \) at a distance \( r \) from the center. \( \langle m \rangle = 0.6 M_\odot \) represents the average stellar mass of the galaxy. The velocity dispersion for a typical elliptical galaxy is \( \sigma = 160 \text{km s}^{-1} \) (Cappellari et al. 2013). In both types of galaxies we set the mass of the perturber to be \( 0.6 M_\odot \), the average mass of a star in the galaxy.

### 4.2 WD-MS collision rate

In this section we calculate the WD-MS collision rate in the intermediate stage and endstate in both model spiral and elliptical galaxies. To do this we must estimate the fraction of triple system which host two WDs and a MS star. There are two possibilities for such systems to exist. The first is a DWD as the inner binary and the stellar companion is the tertiary; we term these systems as WWM. The second, is a WD-MS system as an inner binary and additional WD as the tertiary; we term these systems as WMW.

We start by estimating the fraction of WWMs out of the stellar population, \( f_{WWM} \). We assume all stars with mass in the range of \( 1 M_\odot - 8 M_\odot \) become WDs and we count only MS stars in the mass rage of \( 0.4 M_\odot - 1 M_\odot \). We do so to be consistent with our assumption that the triple systems is composed of stars of equal masses. We use the initial mass function (IMF) given by (Kroupa 2001) and find that \( f_{\text{primary}} \approx 0.1 \) of all stars are in the range of \( 1 M_\odot - 8 M_\odot \) which turn into WDs in 10Gyrs. This is an upper limit because binary evolution can hasten WD faster via interactions between the binary stars. The binary companion mass is calculated from a uniform mass ratio distribution (Moe & Di Stefano 2016), \( Q_{\text{outer}} \in (0.1, 1) \). For the tertiary we calculate the outer mass ratio by the following expression, \( Q_{\text{outer}} = m_3 / (m_1 + m_2) \) and its value is drawn from a power law distribution (Moe & Di Stefano 2016) \( f_{Q_{\text{outer}}} \propto Q_{\text{outer}}^{-2} \) where \( Q_{\text{outer}} \in (0.1, 1) \). For WWM systems we count only the systems in which the primary mass is within the range \( 1 M_\odot - 8 M_\odot \), the secondary is in the range \( 1 M_\odot - 8 M_\odot \) and the tertiary is in the range \( 0.4 M_\odot - 1 M_\odot \). Given these distributions we find that the fraction of secondaries in the WD production range is \( f_{\text{secondary}} \approx 0.44 \), while for the tertaries it is \( f_{\text{tertiary}} \approx 0.44 \). The triple fraction is set to be \( f_{\text{triple}} = 0.2 \) (Duchêne & Kraus 2013) and the fraction of wide outer binaries greater than 1000AU from a log-uniform distribution, \( f_{az} \), is \( f_{az} = 0.2 \). Combining these estimates we get
\[ f_{WWM} = f_{\text{primary}} \times f_{\text{secondary}} \times f_{\text{tertiary}} \times f_{\text{triple}} \times f_{az} \approx 7.6 \times 10^{-4}. \] (18)

A similar calculation can be made for the WMW case. Then the primary is a WD, \( f_{\text{primary}} \approx 0.1 \) the secondary is a MS star, \( f_{\text{secondary}} \approx 0.4 \) and the tertiary is a WD, \( f_{\text{tertiary}} \approx 0.14 \).

\[ f_{WWM} \approx 2.2 \times 10^{-4}. \] (19)
We define
\[ f_{\text{model}} = f_{\text{WWM}} + f_{\text{WMW}}. \] (20)

We note that this approach to estimate the fraction of triples is a simplification of a very complex calculation. In order to calculate the wide triple systems out of a certain stellar population more accurately one needs to numerically evolve large numbers of systems and take into account both single and binary stellar evolution. This, in turn, will change the initial SMA and eccentricity distributions while also changing the masses. In particular, common envelope evolution (Igoshev et al. 2020) modifies the inner SMA and even the outer SMA due to mass loss from the inner binary (Michaely & Perets 2019a; Igoshev et al. 2020).

Next, we compute the total galactic rate for WD-MS collisions for both types of galaxies. The rate, \( \Gamma \), is given by integrating the loss cone (4) and (5) for all outer SMAs, \( a_2 \) between \( 10^3 - 10^4\,\text{AU} \), the local stellar density in the galaxy \( n_* \) from equations (14) and (16). In order to integrate the inner binary SMA we use the following limits \( 10^{-1} \) (\( 10^{-2} \)) – \( 10^2\,\text{AU} \). We choose two minimal values of the inner binary in order to roughly estimate the uncertainties caused by our lack of knowledge of the real distribution:
\[ \Gamma = \int \int \frac{L_{\text{collision}}(a_1, a_2, n_*)}{10\,\text{Gyr}} da_1 da_2 dN(r) \] (21)
where \( L_{\text{collision}} \equiv L(a_1, a_2, n_*) f_{a_1} f_{a_2} f_{\text{model}} f_{\text{WD-MS}} \) and we define
\[ dL \equiv \frac{L_{\text{collision}}(a_1, a_2, n_*)}{10\,\text{Gyr}} da_1 da_2 dN(r). \] (22)
Now we plug in the function from eq. (8)
\[ \Gamma_{\text{MW}} = \int_{0.5\,\text{kpc}}^{15\,\text{kpc}} \int_{10^3\,\text{AU}}^{10^7\,\text{AU}} \int_{10^{-1}(10^{-2})\,\text{AU}}^{10^2\,\text{AU}} dL \approx 1.9 \times 10^{-4}\,\text{yr}^{-1} \] (23)
and for a typical elliptical
\[ \Gamma_{\text{elliptical}} = \int_{0.1\,\text{kpc}}^{30\,\text{kpc}} \int_{10^3\,\text{AU}}^{10^7\,\text{AU}} \int_{10^{-1}(10^{-2})\,\text{AU}}^{10^2\,\text{AU}} dL \approx 3.44 \times 10^{-4}\,\text{yr}^{-1}. \] (24)
These results are averaged over a 10 Gyr lifetime of the galaxies. In subsection 6 we approximate the delay-time distribution of these collisions.

In the case where a direct collision did not occur during the IMS phase the triple is disrupted and a compact binary is formed. We calculate the collision rate by using, \( log f_{\text{WD-MS}} \) from (11) to get for the MW-like Galaxy
\[ \Gamma_{\text{ES, WD}} = \int_{0.5\,\text{pc}}^{15\,\text{kpc}} \int_{10^3\,\text{AU}}^{10^7\,\text{AU}} \int_{10^{-1}(10^{-2})\,\text{AU}}^{10^2\,\text{AU}} dL \approx 1.56 \times 10^{-5}\,\text{yr}^{-1} \] (25)
and for the elliptical galaxy a rate of
\[ \Gamma_{\text{ES, elliptical}} = \int_{0.1\,\text{kpc}}^{30\,\text{kpc}} \int_{10^3\,\text{AU}}^{10^7\,\text{AU}} \int_{10^{-1}(10^{-2})\,\text{AU}}^{10^2\,\text{AU}} dL \approx 2.81 \times 10^{-5}\,\text{yr}^{-1}. \] (26)

4.3 Double degenerate inspiral rate
Similar to the previous subsection here we calculate the galactic rate for an endstate binary WD inspiral the may lead to Type Ia SN. In this case we use eq. (13) and insert it in eq. (22) to get
\[ \Gamma_{\text{ES, DD}}^{\text{MW}} = \int_{0.5\,\text{kpc}}^{15\,\text{kpc}} \int_{10^3\,\text{AU}}^{10^7\,\text{AU}} \int_{10^{-1}(10^{-2})\,\text{AU}}^{10^2\,\text{AU}} dL \approx 7.56 \times 10^{-6}\,\text{yr}^{-1} \] (27)
and for a typical elliptical galaxy
\[ \Gamma_{\text{ES, DD}}^{\text{elliptical}} = \int_{0.5\,\text{kpc}}^{15\,\text{kpc}} \int_{10^3\,\text{AU}}^{10^7\,\text{AU}} \int_{10^{-1}(10^{-2})\,\text{AU}}^{10^2\,\text{AU}} dL \approx 1.34 \times 10^{-5}\,\text{yr}^{-1}. \] (28)
These rates are of order one percent of the total Type Ia rate observed in large galaxies: \( \sim 10^{-3}\,\text{yr}^{-1} \) (Maoz et al. 2014).

5 WD-MS COLLISION
In what follows we discuss the physics and detectability of a WD-MS collision event. Shara & Shaviv (1977) studied the timescales and energetics of WD-MS collisions, while Shara & Shaviv (1978); Shara & Regev (1986); Regev & Shara (1987) carried out 2D hydrodynamics simulations of head-on collisions, including a simple power law prescription to allow for nuclear energy release. In their study a direct collision between a MS star with mass \( M_* \) and radius \( R_* \) and a WD with mass \( M_{\text{WD}} \) and radius \( R_{\text{WD}} \) was considered. The relative velocity, at the collision, was set \( v_{\text{coll}} \sim v_{\text{esc}} \) to the escape velocity of the MS star in Shara & Regev (1986), and to 2000 or 6000 \( \text{km}\,\text{s}^{-1} \) in Regev & Shara (1987).

As the WD approaches the MS star tidal forces act on the MS and stretch its outer envelope. Energy conversion (from kinetic to thermal) and nuclear energy liberation in the collision event begins as the mass from the outer edges of the MS star impacts the surface of the WD, decelerating and being deflected. The collision time is approximated by
\[ \tau_{\text{col}} \approx \frac{R_*}{v_{\text{coll}}} \] (29)
For \( M_* = M_{\text{WD}} = 0.6M_\odot \) and \( R_* = 0.6R_\odot \) this yields, \( \tau_{\text{col}} \approx 900\,\text{sec} \). During this time, during which the WD moves supersonically through the envelope of the MS star, a roughly spherical shock is formed when the collision velocity is lower than the WD’s escape velocity, \( v_{\text{esc, WD}} \approx 7 \times 10^8\,\text{km}\,\text{s}^{-1} \). The shock compresses the envelope by a factor of \( \sim 4 - 10 \) to average values of \( \rho \approx 10^4\,\text{g}\,\text{cm}^{-3} \) and heats it to \( \sim 3 - 5 \times 10^8\,\text{K} \). At this point radiation pressure becomes dominant over gas pressure in the MS star’s envelope. Under these conditions proton capture and the hot CNO cycle are the dominant energy sources in the star, and He burning may also be initiated. The nuclear burning consumes a few percent of the available hydrogen and releases \( E \sim 2 \times 10^{48}\,\text{ergs} \), during \( \tau_{\text{col}} \) and deposits it in the escaping and optically thick stellar envelope. The binding energy of a 0.6\( M_\odot \) star is \( E_C \approx 10^{48}\,\text{ergs} \); this implies that

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6 DELAY TIME DISTRIBUTION (DTD)

In this subsection we calculate the expected delay-time distribution (DTD) of the transients described in section 5. The DTD is the hypothetical rate of transients that follow a brief star formation episode. The main channel of WD-MS collisions originates during the multiple binary-single encounter when the triple becomes unstable. The collision occurs on a dynamical timescale which is extremely short compared to the stellar evolution time that was needed to produce two WDs. Therefore, we only calculate the time since the star formation event which produced the WD and RD stars.

The DTD is determined by the numbers of available triples that become unstable as a function of time. This depends on the initial mass function and the stellar evolution time for each mass. As already assumed, the rate of flyby interaction is constant in time if one disregards binary ionization via other processes (such as mass loss during binary evolution). Therefore we can write the following dependency

\[
\frac{dN}{dt} \propto \frac{ds \, dm}{dt}
\]

The first term is the initial mass function which is similar to those of Kroupa and Salpeter (Kroupa 2001; Salpeter 1955) \(ds/dm = m^{-2.3}\). The second term is just the MS life time, \(t_{\text{MS}}\), i.e. the time it takes a MS star to evolve into a WD, \(dm/dt = t^{-3/3}\). For these simplifying assumptions we get

\[
\frac{dN}{dt} \propto t^{2/3} / t^{-4/3} = t^{-0.56} \approx t^{-3/5}.
\]

The DTD distribution given here is the predicted DTD of the transients discussed in subsection 5. Moreover, due to the functional form of the DTD, we expect to find these events not just in star forming spiral galaxies but also in ellipticals.

7 DISCUSSION AND SUMMARY

7.1 Summary

In this study we described the interactions of triple systems composed of two equal mass WDs and a low mass MS star with field stars of the host galaxy. We found that a significant fraction of these systems become unstable due to interactions with random flyby stars. Those flybys can excite the outer eccentricity of the triple sufficiently to destabilize the triple system. The instability manifests itself via multiple binary-single encounters during which a collision between the MS star and a WD is plausible. In the case where the triple survives the chaotic evolution, i.e. there is no collision between any two components, the systems disrupts to the final endstate of a compact binary, formed of any two object of the triple, and an “escaper” that is ejected to infinity. The newly formed compact binary can, in turn cause a WD-MS collision, or a DWD inspiral through GW emission, similar to the classic DD scenario that leads to a Type Ia SN.

We found that the collision rates are very sensitive to the initial triple population and its characteristics. For a set of plausible assumptions we expect a collision rate of \(\sim 1\) event every \(10^9\)yr in the MW or any similar spiral galaxy, and \(\sim 1\) event every 5000yr in an elliptical galaxy with total mass of \(10^{11}\)\(M_\odot\).

Furthermore, we expect these events to be luminous with \(L \approx 10^7 - 10^9L_\odot\) for weeks to years. We speculate that some sub-luminous SNe or other luminous transients might result. Additionally, we calculated the predicted DTD to be \(t^{-3/5}\), and note that we expect to find these events in both star forming galaxies and ellipticals. Finally, we estimate that the total rate of these events is of order 1% of the SNIa rate in galaxies.

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