A Consensus-based Nonlinear and Lipchitz-continuous Distributed Cooperative Secondary Control Method for Islanded AC Microgrids

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Abstract. In this paper, a consensus-based, nonlinear, and Lipchitz-continuous distributed secondary control (DSC) method is proposed to coordinate all the distributed generators (DGs) in an islanded AC microgrids (MGs). This proposed DSC strategy can not only guarantee the restoration control for frequency and voltage, but also realize an accurate active power sharing control for the whole microgrid system. Through introducing a nonlinear, Lipchitz-continuous dynamic from Beta Probability Distribution Function (Beta PDF), the convergence speed of DSC is accelerated, the finite-time convergence of DSC is ensured, and the transient overshoot of DSC is diminished comparing with traditional DSC. Moreover, the common chattering phenomenon in non-Lipchitz DSC scheme is eliminated. The stability and performance of the proposed DSC are also analysed in this paper. An islanded AC microgrid test system with four inverter-based DGs is built in MATLAB/SIMULINK to further validate the effectiveness of the proposed DSC strategy.

1. Introduction

Microgrid is a localized group of Distributed Generation (DG), Energy Storage System (ESS) and local loads that normally connect to and synchronous with the main grid, but can also intentionally or unintentionally disconnect to islanded mode [1]. In this way, a microgrid can effectively supply emergency power, changing between islanded and connected modes. In islanded mode, the microgrid is easier to suffer from frequency and voltage deviation due to its low power capacity, weak network structure and frequent fluctuations of loads. Thus, a wide-used operation scheme called hierarchical control is implemented in islanded microgrid, and it usually has three control levels [2-4].

The primary control level normally adopts the $P$-$\omega$ and $Q$-$V$ droop control strategy, or some improved forms in special application scenarios. The control effect is stabilizing voltage and frequency with some inherent deviations between reference and practical values. Then the secondary control level is implemented to compensate these deviations and further realize the active or reactive power sharing. Finally, the tertiary level can determine the optimizing and economic operation. In this paper, we mainly focus on the secondary control level.

In general terms, secondary control structures can be centralized [5-7] or distributed [8-17], which are two commonly utilized secondary control methods in microgrid. In a centralized way, the microgrid possesses a central controller, called MGCC, to extract the frequency and voltage of the PCC and transmit these signals to each DG through a unidirectional communication link. However,
the high redundancy of communication structure and heavy dependency of MGCC do reduce the system’s economic efficiency and reliability. Thus, a distributed way is proposed in recent years, which only requires information exchange among several neighbouring local controllers, and the computation burden can be shared through all the local controllers rather than a centralized MGCC. In this way the problems caused by single-point failure and communication congestion can be much more relieved.

Consensus-based control methods of multi-agent system (MAS) are widely used as a distributed secondary control in microgrid [10]- [17]. In [10]- [12], the traditional linear consensus feedback control scheme is used, where all the agents reach consensus in an asymptotical convergence time. However, considering the intermittent of DGs and constantly changing of local loads, the traditional linear asymptotical convergence property may not suitable for the fast-changing operating conditions.

Base on this, some nonlinear consensus-based finite-time convergence control methods are proposed to achieve a finite settling time for the secondary control [13]- [17]. In [13] and [14], the voltage and frequency restoration control with accurate real power sharing are achieved in finite-time. In [15], a finite-time nonlinear convergence control is proposed, which considers the synchronization control for the frequency and voltage and active power sharing when microgrid having switching communication topologies. Given that there is a tradeoff between voltage regulation and reactive power sharing, a novel observer-based distributed voltage regulator involving certain reactive power sharing constraints is proposed in [16], which can realize reactive power sharing with a bounded voltage. Still a novel nonlinear consensus-based DSC method is also proposed to restore the frequency and achieve the active power sharing. Moreover, to accommodate the intermittent renewable generations and constantly changing load demands, a distributed, bounded and finite-time convergence secondary control is proposed in [17] by using the inverse hyperbolic tangent (tanh) nonlinear dynamic technique.

Almost all the nonlinear finite-time DSC methods proposed in the literatures above adopt some sort combining techniques of sign function, saturation function, and fractional power integrator. In most cases, these types of control algorithms are called right side non-Lipchitz derivative functions [18-20]. Adopting these types of DSC methods can realize a fast finite time convergence. However, in the meantime, an inherent, non-negligible chattering phenomenon happened at steady state will be introduced in the system dynamic [21], which are rarely discuss in recent research papers.

In this paper, a nonlinear, distributed, and bounded convergence secondary control is proposed in an islanded AC microgrid to restore the frequency and voltage and meantime realize an accurate active power sharing for each DG without chattering phenomenon. Using a Beta Probability Distribution Function technique combining with sign function and saturation function, the convergence speed is highly improved and the transient overshot is greatly reduced. The Lyapunov function is presented to certify the Lyapunov stability of this proposed DSC method. Moreover, the chattering phenomenon in non-Lipchitz dynamic system is discussed, the Lipchitz continuity is ensured to eliminating the chattering in steady state. An islanded AC microgrid test system with 4 inverter-based DGs in Matlab/Simulink is presented to validate the effectiveness of the proposed distributed control strategy.

The rest of the paper is organized as follows. Section II presents the preliminaries of graph theory and the conventional droop-based primary control scheme. Section III introduces the proposed nonlinear distributed control algorithm for frequency restoration and active power sharing control. Section IV provides the Lyapunov stability proof and discusses the chattering phenomenon in non-Lipchitz dynamic system and its elimination method to improve the dynamic performance. Section V discusses the simulation results, and Section VI provides the conclusion.

2. Problem Formation and Preliminaries

2.1. Problem Formulation

In this paper, we mainly focus on an islanded AC microgrid with N inverter-based DGs. Each DG consists of an ideal DC source, a DC/AC inverter, and an Inductance-Capacitance-Inductance (LCL)
filter. Its’ basic control diagram include inner voltage and current control loops as well as the primary and proposed secondary control scheme, which can be presented in Fig. 1.

As seen in Fig. 1, the current, voltage, and power control loops are employed in each DG. The primary control procedure is implemented during the power control loop with the nominal set points \( V_{ni} \) and \( \omega_{ni} \) generated by the secondary control procedure. Then, with the reference values, \( V_{odi}^{ref} \) and \( V_{oqi}^{ref} \), provided by the power loop, the outer voltage loop generates the current reference, \( i_{odi}^{ref} \) and \( i_{oqi}^{ref} \), for the inner current loop. Finally, the current error is calculated and further used to regulate the output of the inverter by the sinusoidal pulse width modulation (SPWM) mode [1].

![Figure 1. The control loops for an multi inverter-based DG microgrid.](image)

In this paper, we propose a nonlinear consensus-based DSC method through the information exchanges among the several neighboring DGs in a sparse communication network to update \( V_{ni} \) and \( \omega_{ni} \) in each primary control process, and further restore the output voltage \( V_i \) and frequency \( \omega_i \), to their reference values. And the reference values, \( V_{ref} \) and \( \omega_{ref} \), are provided by a so-called virtual leader DG\(_{0}\), which can be offered by the main grid in grid-connected mode or obtained by a specific command DG in islanded mode.

The output impedance characteristic of each DG is highly inductive due to the LCL filter circuit, so that the following conventional \( P-\omega \) and \( Q-V \) droop-based primary control strategy can be used in the power control loop [1], [3]:

\[
\begin{aligned}
\omega_i &= \omega_{ni} - K_p P_i \\
V_i &= V_{ni} - K_Q Q_i \\
V_{odi} &= V_{odi}^{ref}
\end{aligned}
\]

(1)

where \( V_{odi} \) and \( \omega_i \) are the output voltage magnitude and frequency respectively, here we choose the d-axis output voltage orientation principle. \( P_i \) and \( Q_i \) are the average active and reactive powers after low-pass filters, \( K_p \) and \( K_Q \) are the corresponded droop coefficients, and their magnitudes are commonly designed according to their rating power \( P_{i,\max} \) and \( Q_{i,\max} \).

2.2. Preliminaries of Communication Network

The communication structure of an microgrid with multi DGs is a typical multi-agent system, where each DG follows the consensus-based convergence rule. Here the algebraic digraph theory is introduced [21], and the communication network can be modeled as a directed graph \( G(\mathbf{v}, \mathbf{e}, \mathbf{A}) \), where \( \mathbf{v} = \{v_1, v_2, \ldots, v_N\} \) denotes the node set of DG, \( \mathbf{e} \subseteq \mathbf{v} \times \mathbf{v} \) denotes the communication link, and \( \mathbf{A} = (a_{ij}) \) \( N \times N \) denotes the weighted adjacency matrix. What’s more, the adjacent weight \( a_{ij} = 0 \), \( a_{ij} \geq 0 \), and \( a_{ij} > 0 \) if and only if the link \( (v_i, v_j) \in \mathbf{e} \).

The set of DG’s neighbors is given by \( N_i = \{v_j \in \mathbf{v} : (v_i, v_j) \in \mathbf{e} \} \). The Laplacian matrix of \( \mathbf{A} \) is
defined as \( L(A)=l_{ij} \) \( N \times N \), \( l_{ij}=-a_{ij}, i \neq j \), and \( l_{ij} = \sum_{k=1}^{N} a_{ik} \) for all \( i \), which satisfies \( L(A)1_{N}=0 \) with \( 1_{N}=(1, \ldots, 1)^{T} \in \mathbb{R}^{N} \). Diagonal matrix \( D=\text{diag}[a_{00}, \ldots, a_{NM}] \) is called the virtual leader adjacency matrix, where \( a_{ii}>0 \) if follower \( DGI \) is connected to the leader \( DG0 \) through the link \((v_{0}, v_{i})\), otherwise \( a_{ii}=0 \).

3. Proposed Distributed Secondary Control

Much researches have concluded that the dynamics of the voltage and current control loops are much faster than the dynamics of the power control loop if the whole control parameters are well set. In other words, a typical AC microgrid with multi inverter-based DGs is a multi-time scale system and processes a high time-separating dynamics. Thus, in this paper, we mainly focus on the stability and dynamic performance issue of the power control loop, and simply implement the traditional PI controllers for the voltage and current inner control loops.

Differentiating the equations in (1) yields

\[
\begin{cases}
\dot{\omega}_{ni} = \dot{\omega}_{i} + K_{P_i} P_{i} = u_{n_i} + u_{p_i} \\
V_{ni} = V_{i} + K_{Q_i} Q_{i} = u_{v_i} + u_{q_i}
\end{cases}
\]

(2)

where \( u_{n_i} = \dot{\omega}_{i} \), \( u_{v_i} = V_{i} \), \( u_{p_i} = K_{P_i} P_{i} \), and \( u_{q_i} = K_{Q_i} Q_{i} \) are the DSC controllers outputs for the frequency, voltage, active power and reactive power, respectively.

Now that the nominal set points for the primary power controller can be designed as,

\[
\begin{cases}
\omega_{ni} = \int (u_{n_i} + u_{p_i}) dt \\
V_{ni} = \int (u_{v_i} + u_{q_i}) dt
\end{cases}
\]

(3)

Noticing that in an islanded AC microgrid, the frequency is a global variable, but the output voltages of DGs are local variables and may be somehow different. So there is always an inherent contradiction existing between the precise voltage regulation and reactive power sharing especially in low-voltage MGs [1, 12]. In this paper, we mainly focus on the precise voltage regulation.

Thus we use a traditional DSC controller to restore the output voltages of all DGs, and meanwhile design a proposed nonlinear DSC method to faster restore the frequency and realize an accurate active power sharing. The control objectives can be expressed as follows.

1. All DGs’ frequency and voltage regulation can be achieved asymptotically, i.e,

\[
\lim_{t \to \infty} |\dot{\omega}_i(t) - \omega_{ref}| = 0, \quad \lim_{t \to \infty} |V_i(t) - V_{ref}| = 0, \quad \forall i.
\]

(4)

2. The accurate active power sharing can also be achieved as below,

\[
\lim_{t \to \infty} |K_{P_i} P_i - K_{P_j} P_j| = 0, \quad \forall i \neq j
\]

(5)

3. The reactive power control inputs, \( u_{q_i} \), can be adopted from [12], as \( u_{q_i} = -K_{Q_i}(\omega_i Q_i + \omega_c q_i) \), where \( \omega_c \) is the cutoff frequency of the low-pass filters, \( K_{Q_i} \) is the reactive droop coefficient and \( q_i \) is the instantaneous reactive power component.

3.1. Preliminaries of Communication Network

In a traditional droop-based AC microgrid, the primary control can automatically achieve the active power sharing but having the inherent frequency deviation. However, simply implementing the secondary frequency control without adding the auxiliary active power control may lead to inaccurate active power sharing. The restoration control of frequency and active power sharing control should be considered thoroughly as an integral.

1) Proposed Nonlinear DSC method for frequency restoration and accurate active power sharing

In order to faster restore the system’s frequency and realize an accurate active power sharing, a nonlinear consensus-based DSC method is designed as follow.

We first calculate the global errors of frequency and active power for \( DGI \) through the communication networks, \( G(A^N) \) and \( G(A^N) \),
\[ e_{vi} = \sum_{j \in N_v} a^v_{ij} (V_i - V_j) + a^v_{i0} (V_i - V_{r_{ref}}) \]

\[ e_{pi} = \sum_{j \in N_p} a^p_{ij} (K_{pi} - K_{pj} P_j) \]

\[ \epsilon_{ei} = \sum_{j \in N_e} a^{e0}_{ij} (\omega_i - \omega_j) + a^{e0}_{i0} (\omega_i - \omega_{r_{ref}}) \]

Theorem 1 If digraph \( G(A^v) \) and \( G(A^w) \) are strong connected, and there is at least one DG can access to the virtual leader reference \( \omega_{r_{ref}} \), then the droop control law (1) combining with the proposed DSC method, (7) and (8), can guide all DGs’ frequencies to their reference values in a finite time while maintaining the active power sharing accuracy.

We simply use the traditional linear DSC method to solve the leader-followers’ consensus for voltage [11], so the continuous-time distributed controllers can be constructed as

\[ u_{vi} = -C_v e_{vi} \]

where \( C_v > 0 \), and \( e_{vi} \) can be derived as,

\[ e_{vi} = \sum_{j \in N_v} a^v_{ij} (V_i - V_j) + a^v_{i0} (V_i - V_{r_{ref}}) \]

The corresponding adjacent weights for voltage leader is \( a^v_{i0} \), and we also assume only one DG can access to the voltage reference \( V_{r_{ref}} \).

Theorem 2 Let \( G(A^v) \) be connected and at least one agent can receive information from the leader node. Then the distributed controllers in (10) ensures global stability of the voltage dynamics system.

The stability certification of theorem 2 is quite simple, which can be found in [11] and [12], so it is not presented in this paper.

4. Stability and Performance Analysis
In this section, the Lyapunov method is used at first to illustrate the stability issue of Theorem 1 for frequency and active power. Then, we further discuss a common problem happened in non-Lipschitz dynamic control algorithm called chattering phenomenon and analyse its negative effects to the system performance.

4.1. Proof of Theorem 1
In order to better derive the stability of (7), first let \( \tilde{\omega} = [\delta_1, \delta_2, \ldots, \delta_n]^T \), with \( \tilde{\omega}_i = \omega_{ri} - \omega_{ef} \), \( x = [x_1, x_2, \ldots, x_n]^T \), with \( x_i = K_{pi} p_i \), and \( y = [y_1, y_2, \ldots, y_n]^T \), with \( y_i = e_{ui} \), \( z = [z_1, z_2, \ldots, z_n]^T \), with \( z_i = e_{pi} \). So here we can get

\[
y_i = \sum_{i \in N_i} a_i^n (\tilde{\omega}_i - \tilde{\omega}_j) + a_i^n \tilde{\vartheta}_i , \quad \text{and} \quad z_i = \sum_{i \in N_i} a_i^n (x_i - x_j),
\]

further presented in matrix form as

\[
y = (L^n + D^n) \tilde{\omega}, \quad z = L^n x.
\]

We differential the two side of the expressions above as,

\[
\begin{align*}
\dot{y} &= -\text{sign}(y) C_{\omega} (L^n + D^n)(1 - (1 - r_s \text{sat}_{ui}(\dot{y})))^{\beta_n} \\
\dot{z} &= -\text{sign}(z) C_{\rho} L^n (1 - (1 - r_s \text{sat}_{ui}(\dot{z})))^{\beta_p}
\end{align*}
\]

Choosing the Lyapunov candidates \( V = V(t, y) + V(t, z) \) where \( V(t, y) \) and \( V(t, z) \) can be chosen respectively as

\[
\begin{align*}
V(y, t) &= \sum_{i=1}^n [r_{\omega} | y_i | - \frac{1}{\beta_n + 1} (1 - (1 - r_s \text{sat}_{ui}(\dot{y})) | y_i |)^{\beta_n + 1}] \\
V(z, t) &= \sum_{i=1}^n [r_\rho | z_i | - \frac{1}{\beta_p + 1} (1 - (1 - r_s \text{sat}_{ui}(\dot{z})) | z_i |)^{\beta_p + 1}]
\end{align*}
\]

Where \( w_i \in \omega \) is a positive column vector element meeting the requirement as \( w^T L(A) = 0 \).

For \( V(t, y) \), it is easy to certify \( V(t, y) \geq 0 \) and is globally positive defined giving that \( \delta V(t, y) \dot{\gamma} \geq 0 \) and \( V(t, 0) = 0 \). Its time derivative function can be further written as

\[
\dot{V}(t, y) = -C_{\omega} r_{\omega} [1 - (1 - r_s \text{sat}_{ui}(\dot{y})) | y |^{\beta_n}]^T
\]

\[
(L^n + D^n)[1 - (1 - r_s \text{sat}_{ui}(\dot{y})) | y |^{\beta_n}]
\]

According to graph theory in [20] and [21], \( L^n + D^n \) is a positive defined matrix so we simply get \( \dot{V}(t, y) < 0 \). Likewise, we get \( V(t, z) \geq 0 \) and is globally positive defined, Its time derivative function can be further written as

\[
\dot{V}(t, z) = -C_{\rho} r_{\rho} [1 - (1 - r_s \text{sat}_{ui}(\dot{z})) | z |^{\beta_p}]^T
\]

\[
dig(w)L^n (1 - (1 - r_s \text{sat}_{ui}(\dot{z})) | z |^{\beta_p})
\]

\[
\leq -C_{\rho} r_{\rho} \lambda_2(M) \sum_{i=1}^n [1 - (1 - r_s \text{sat}_{ui}(\dot{z})) | z_i |^{\beta_p}] \leq 0
\]

where \( M \) is called mirror matrix of \( \text{dig}(w)L^n \), which can be denoted as \( 1/2(\text{dig}(w)L^n + (L^n)^T \text{dig}(w)) \). \( \lambda_2 \) is the second smallest positive eigenvalue of \( M \) [20]. Thus combining (14) and (15) we yield,

\[
\dot{V} = \dot{V}(t, y) + \dot{V}(t, z) \leq 0
\]

For the case of \( \dot{V} = 0 \), we yield \( (y^T, z^T)^T = (0, 0)^T \). Since \( y = (L^n + D^n) \tilde{\omega} \) with the positive definite matrix \( L^n + D^n \), then \( \tilde{\omega} = 0 \) and thus \( \omega_1 = \omega_2 = \cdots = \omega_k = 0 \). Moreover, since \( z = -L^n p \) with rank \( L^n = n - 1 \) due to the strongly connection of \( G(A^n) \), \( z = 0 \) implies that \( K_{pi} P_1 = K_{pi} P_2 = \cdots = K_{pi} P_N \). Therefore, the frequency and active power control problem can be solved within a finite time.

4.2. Chattering Phenomenon Analysis
It is known that there is an undesirable oscillation around the equilibrium, called chattering phenomenon, usually occur in the right-side non-Lipschitz dynamic systems. The further expression of Lipschitz continuity characteristic can be seen in Lemma 1. Note that in [20], it is verified using the
protocols like saturation function (e.g. sat(·), tanh(·) or \( \tan^{-1}(·) \)) does not guarantee accurate finite-time convergence, because the non-Lipchitz characteristic is still exist. In these cases, the system just reaches a region around the stability state in finite-time, which is called as the agreement-boundary; but at the interior of this region, convergence occurs asymptotically. So the accuracy for frequency control and power sharing in [16] and [17] of the proposed method remain in doubt.

5. Simulation Validation
In order to verify the effectiveness of the proposed DSC strategy, a 380V/50Hz islanded microgrid test system including four inverted-based DGs and four local loads, is built in Matlab/SimPowerSystems, which can be shown in Fig. 2. The specifications of the DGs, lines, and loads are summarized in Table 1.

![Figure 2. Islanded microgrid test system.](image)

We choose the communication network with 0-1 weight as the unified digraph shown in Fig. 3. Then the associated adjacency matrices for frequency, voltage and active power are \( A^{\omega} = A^{V} = A^{P} = [0, 1, 0, 0; 1, 0, 1, 0; 0, 1, 0, 1; 0, 0, 1, 0] \). And here we assume only DG1 can access to the reference values, then the leader adjacency matrixes for frequency and voltage are \( D^{\omega} = D^{V} = \text{diag} [1, 0, 0, 0] \). The proportion constants, \( C_{\omega}, C_{V} \) and \( C_{P} \), can be selected accordingly. Usually, small constant values may lead to slow convergence speed but have relative good transient response, thus we need to make a tradeoff between these two performance indexes. Here we simply choose \( C_{\omega} = 4, C_{V} = 6, \) and \( C_{P} = 2 \).

The simulation results will be performed in three scenarios: 1) load variation, 2) plug-in and play capability, and 3) performance with communication time delays. Some comparisons among the proposed distributed secondary control (PDSC) scheme, the proposed finite time distributed secondary control (FDSC) scheme in [16], and the traditional asymptotic distributed secondary control (TDSC) scheme will be given under the same scenario.

![Figure 3. Communication digraphs for \( G(A^{\omega}) \), \( G(A^{V}) \) and \( G(A^{P}) \) with fixed topology.](image)
| Parameter | DG1 | DG2 | DG3 | DG4 |
|-----------|-----|-----|-----|-----|
| \(R_{C1}\) | 0.03Ω | 0.03Ω | 0.03Ω | 0.03Ω |
| \(L_{C1}\) | 2mH | 2mH | 2mH | 2mH |
| \(K_{F1}\) | 13e-5 | 9.4e-5 | 13e-5 | 9.4e-5 |
| \(K_{G1}\) | 1e-3 | 0.8e-3 | 1e-3 | 0.8e-3 |

**Table 1.** Parameter values for the test system

| LOAD1 | LOAD2 | LOAD3 | LOAD4 |
|-------|-------|-------|-------|
| \(P_{L1}\) | 19kW | 19kW | 18kW | 18kW |
| \(Q_{L1}\) | 19kVar | 19kVar | 18kVar | 18kVar |
| \(P_{L2}\) | 22kW | 22kW | 17kW |
| \(Q_{L2}\) | 17kVar |

5.1. Load Variation

The effectiveness of the proposed PDSC scheme, especially for the fast convergence speed, the bounded inputs, and the good performance to the chattering phenomenon are verified in case of load changes.

1) General performance: At the first, only the primary control is implemented. Then the PDSC are activated at \(t=3\)s. The results are shown in Fig. 4. Here we set \(r_\omega=0.5, r_P=1\), and \(\beta=3\) for Fig 4 (a1)-(a3), \(\beta=1.5\) for Fig 4(b1)-(b3). As seen, the primary control could guarantee accurate active power sharing according to their droop coefficients, at meantime, keep all the frequencies convergence to a same value 312.5rad/s. But the voltages are distributed between 0.9 to 0.95 p.u. This is because the frequency is a global value but voltage is a local value. And all the output voltages and frequencies are less than the nominal values due to the inherent droop characteristics. After \(t=3\)s, the frequency and voltage of all DGs can be gradually restored to their nominal values (i.e., \(V_{ref}=1\)p.u. and \(\omega_{ref}=314.15\text{rad/s}\)) as well the accurate active power sharing can be maintained in steady state according to the PDSC control strategy. Since the Load 1 is disconnected from the system at \(t=10\)s, and reconnected to the system at \(t=20\)s, Fig. 4 also shows that the PDSC scheme can keep the voltage and frequency of DGs at the nominal values while keeping accurate active power sharing response to both load connection and disconnection.

![Figure 4](image-url)

Figure 4. State outputs under the PDSC scheme in case of load variation (the red dashed lines show the convergence time of frequency and active power), (a1) and (b1) frequency response. (a2) and (b2) voltage response. (a3) and (b3) active power response.

We further investigate the system dynamic under different shape parameter \(\beta\) by comparing the Fig 4(a1)-(a3) with Fig 4(b1)-(b3). It is obvious that adopting higher \(\beta\) value can have a little fast convergence speed and lower transient overshoot. But higher \(\beta\) means higher convergence acceleration, which will in turn, makes the system more vulnerable to the oscillation in the steady state.

2) **FDSC scheme with chattering phenomenon**: Under the same scenario we simulate the FDSC in [16] by setting the parameters as \(k_1=3, k_2=1, \delta_n=5, \text{and } \delta_d=10\). Fig. 5(a1)-(a3) shows the dynamics of
FDSC in [16] with $\alpha_\omega=0.9$, $\alpha_P=0.9$, and Fig. 5(b1)-(b3) with $\alpha_\omega=0.5$, $\alpha_P=0.5$. As seen, chattering phenomena are not so obvious for frequency response in Fig. 5(a1) and (b1). However, things become different for the active power sharing control. Note that in Fig. 5(a3) and (b3), there are non-negligible chattering phenomena occurring, especially in the case of smaller index ($\alpha_\omega=0.5$, $\alpha_P=0.5$). Though smaller index value means shorter finite convergence time, the larger chattering will occur. Comparing with PDSC method in this paper, the FDSC algorithm in [16] is a time-saving convergence method, but there is always an inherent tradeoff between the chattering and convergence speed for a non-Lipchitz dynamic system.

Figure 5. State outputs under the FDSC scheme in case of load variation (the-red dashed lines show the convergence time). (a1) and (b1) frequency response. (a2) and (b2) frequency response. (a3) and (b3) active power response.

5.2. Plug in and Play

The plug in and play capability of the whole system is analysed in Fig. 6, still using the comparison between PDSC and TDSC scheme. In all cases the switch of DG1 and DG4 are disconnected at t=10s and reconnected at t=20s. As seen in Fig. 6, the graph is still connected with DG1 receiving information from the leader node when DG4 is disconnected. Therefore, the algorithm can still work for the DG1, DG2 and DG3. Fig. 6 also shows that the system may endures dramatic transient overshoot when DG4 is reconnected. This is because no pre-synchronization is implemented. However, the transient frequency is kept in ±5% deviation and system can still keep stable even in this situation under PDSC and TDSC scheme. Still the dynamic performance of PDSC scheme is much better than the TDSC scheme.

Figure 6. State outputs of plug in and play under PDSC scheme and TDSC scheme. (a1) and (b1) frequency response. (a2) and (b2) frequency response. (a3) and (b3) active power response.
5.3. Performance with Time-Delays

![Figure 7](image)

Figure 7. State outputs of PDSC scheme under different communication time-delays. (a1) and (b1) frequency response. (a2) and (b2) frequency response. (a3) and (b3) active power response.

In this section, the dynamic performance of the PDSC scheme, is investigated under different communication time delays. Obviously, the Fig. 7 shows that the more oscillation will happen when the communication time delays increasing. Considering the fact that the stability criteria is based on the Lyapunov theory, larger time delay will definitely diminish the stability region of the whole system. However, in this simulation, we can find that the system can still remain stable under the cases of time-delay $t_d=50\text{ms}$ and $100\text{ms}$. Note that the time-delay of a common communication network will be approximately at milliseconds or tens of milliseconds, it is reasonable to conclude that the PDSC can realize the control requirements under the circumstance of time delay.

6. Conclusion

This paper proposes a novel nonlinear distributed secondary control strategy to achieve frequency restoration and accurate active power sharing in an islanded AC microgrid, while satisfying requirements of the bounded control input, the faster convergence speed and eliminating the chattering phenomenon. The stability issue of proposed DSC method is rigorously proved by using the Lyapunov method. The effectiveness of the control strategy is validated in the simulations with a test microgrid system. The future work will focus on the real-time simulation by using Dspace and further implement the PDSC method in a discrete way.

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