Realization of a high quality factor resonator with hollow dielectric cylinders for axion searches

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Abstract

We discuss the realization and characterization of a high quality factor resonator composed of two hollow-dielectric cylinders with its pseudo-TM030 mode resonating at 10.9 GHz frequency. We measured the quality factor at the temperatures 300 K and 4 K obtaining $Q_{300\,K} = 150,000$ and $Q_{4\,K} = 720,000$ respectively, the latter corresponding to a gain of one order of magnitude with respect to a traditional copper cylindrical-cavity with the corresponding TM010 mode resonating at the same frequency. We discuss the implications to dark-matter axion-searches with cavity experiments and show that the gain in quality factor is not spoiled by a reduced coupling to the axion field, estimating a reduction effect at most of 20%. We show with numerical simulations that frequency tuning of several hundreds MHz is feasible.

Keywords: axion, dielectric cavity, haloscope

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1. Introduction

The operation in DC magnetic-field of three-dimensional microwave-cavities with high quality-factor plays an important role in cavity quantum-electrodynamycs experiments [1, 2] and in the search of dark-matter (DM) axions [3, 4, 5, 6, 7]. While losses at high frequency limit the use of copper cavities in these experiments, an external magnetic field limits the high performances achievable with superconductive cavities by causing losses due to the presence of fluxons [8] or a transition to the normal, resistive, state. Moreover, in quantum-electrodynamycs experiments, partial or complete screening by the superconductive walls prevent controlling superconducting qubits with external magnetic fields. Although encouraging results were obtained with NbTi and YBCO cavities [9, 10, 11], the possibility of using dielectric structures in axion experiments aroused considerable interest [12, 13, 14, 15, 16]. In this paper we describe the design, fabrication and test of a pseudo-cylindrical cavity with dielectric shells made of sapphire as sketched in Fig. 1 with an extremely high quality factor (potentially larger than $10^6$ in the $X$-band frequency range at cryogenic temperature) and with a relatively simple and tunable geometry. These types of geometries have been already implemented by other authors for different applications [17, 18, 19, 20, 21]. In particular, the realization of a resonant cavity with hollow dielectric-cylinders for axion haloscopes was first proposed in [12] and a proof-of-concept demonstration was done in [15]. As pointed out by these authors, if properly designed, the two cylindrical sapphire shells act as shielding that strongly reduce the magnetic field amplitude on the outer wall of the cavity and, therefore, the power losses. More precisely, the cavity working mode in this type of structures is a pseudo-$\text{TM}_{000}$ mode where the dielectric shells, acting as electromagnetic screens, reduce the amplitude of the secondary lobes of the field with respect to the main one. On the other hand, losses due to sapphire are negligible thanks to its very low loss-tangent, going from about $10^{-5}$, at room temperature, down to a fraction of $10^{-7}$ at cryogenic temperatures [22].
In the following we focus on the application of our dielectric cavity to DM-axion searches. Many haloscope experiments are taking data or have been proposed in recent years: ADMX [23], HAYSTAC [24], ORGAN [25], CAPP [26], KLASH [27], RADES [28] and QUAX [9]. When the resonant frequency of the cavity is tuned to the corresponding axion mass, $\nu_c = m_a c^2 / h$, the expected power generated by DM axions is given by [24]:

$$P_{\text{sig}} = \left( g_\gamma^2 \frac{\alpha^2}{\pi^2} \frac{\hbar^3 c^3 \rho_a}{\Lambda^4} \right) \times \left( \frac{\beta}{1 + \beta} \frac{1}{\mu_0} B_0^2 V C_{mnl} Q_L \right)$$

(1)

where $\rho_a = 0.45$ GeV/cm$^3$ is the local DM density, $\alpha$ is the fine-structure constant, $\mu_0$ the vacuum permeability, $\Lambda = 78$ MeV is a scale parameter related to hadronic physics, $g_\gamma$ the photon-axion coupling constant equal to $-0.97(0.36)$ in the KSVZ (DFSZ) model [29, 30, 31, 32]. It is related to the coupling appearing in the Lagrangian $g_{a\gamma\gamma} = (g_\gamma \alpha / \pi \Lambda^2) m_a$. The second parentheses contain the magnetic field strength $B_0$, the cavity volume $V$, its angular frequency $\omega_c = 2\pi \nu_c$, the coupling between cavity and receiver $\beta$ and the loaded quality factor $Q_L = Q_0 / (1 + \beta)$, where $Q_0$ is the unloaded quality factor. $C_{mnl}$ is a geometrical factor depending on the cavity mode:

$$C_{m,n,l} = \frac{\int dV \vec{E} \cdot \vec{B}_0}{\sqrt{\int dV c_r |E|^2}}$$

(2)
2. Cavity Design

In the proposed configuration, with two shells, the selected mode is the pseudo-TM$_{030}$, as given in Fig. 2, where we show the electric field amplitude in one quarter of the cavity. The cavity resonant-frequency $f_{res}$ was tuned to 10.9 GHz and the dielectric shells geometrical parameters ($R_{cyl1}$, $R_{cyl2}$, $t_{cyl1}$, $t_{cyl2}$ as defined in Fig. 1) were optimized to minimize the losses in the outer walls. The choice of this frequency was mainly given by the possibility of incorporating the resonator inside the detection chain developed within the QUAX haloscope [33].

![Figure 2: Electric field amplitude of the pseudo TM$_{030}$ mode.](image)

The longitudinal electric field and the azimuthal magnetic field are shown in Fig. 3 as a function of the transverse coordinate, compared with the results expected for an empty ideal cylindrical cavity operating in the TM$_{030}$ mode. The presence of the two sapphire shells reduces the amplitude of the outer field lobes and simultaneously concentrates the mode in the internal cylinder. This results in a larger form factor $C_{030}$ of the mode [34] and in reduced losses on the cavity outer-walls and therefore in a higher quality factor. The ratio of the two H fields on the outer wall of the cavity is about 9 and this gives a decrease of the losses of the order of one hundred. The final design of the cavity with its main dimensions is given in Fig. 4.

![Sapphire cylindrical shells](image)
Figure 3: Longitudinal electric field and azimuthal magnetic field as a function of the transverse coordinate for the cavity with sapphire shells (green lines) and for an ideal cylindrical cavity operating in the TM$_{030}$ mode (red lines).

Figure 4: Final design of the fabricated cavity with its main dimensions in millimeters. Both end-plates have a conical shape to further reduce losses.
The electromagnetic design was done using the electromagnetic code ANSYS Electronics [35]. Since the sapphire dielectric constant varies in a wide range depending on the crystal orientations, impurities and temperature, we considered in the simulations a sapphire dielectric constant equal to 11.2 and a loss tangent at cryogenic temperature of $10^{-7}$ [22]. The copper surface resistance was set to 5.5 mΩ as expected in the anomalous regime at this frequency [36]. The two end-plates, shown in the figure [4], were designed in order to reduce the power losses on the plates themselves. In particular, as already done in [9, 10], two conical shapes were used. The length of the cones was chosen to have enough attenuation of the electromagnetic field on the cones themselves, thus reducing the losses on the copper endplates. To excite and detect the resonant modes, two coaxial antennas were inserted at the end of the cones.

![Figure 5](image)

Figure 5: Magnitude of the electric field of the first two TM resonant modes: TM$_{030}$-like (a) and TM$_{031}$-like (b).

The simulated magnitude of the electric field of the two TM resonant modes (TM$_{030}$-like and TM$_{031}$-like) is shown in Fig. 5. Because of the cavity symmetry, we simulated a small sector of the cavity with perfect magnetic boundary conditions. The first mode is expected to have a Q-factor of about $1.9 \times 10^6$ at cryogenic temperature, essentially limited by the losses on the cavity walls and endplates. Fig. 6 shows for completeness the simulated transmission coefficient between the two coupled antennas whose peaks correspond to the two
mentioned resonant modes. We calculated, for several configurations, the ratio $R_{CV}$ of the product $C_{030} \times V$ between the form factor and the volume of the dielectric cavity and the product $C_{010} \times V$ for an ideal cylindrical-cavity of the same length with the TM$_{010}$ mode resonating at the same frequency. According to equation 1, the gain in signal power is proportional to the ratio $R_{CV}$ and to the ratio of quality factors. For a cavity without cones in the endplates and sapphires geometry as reported in Fig. 4, we obtained $R_{CV} = 72\%$. The decrease is due to the field on the second lobe that has an opposite sign with respect to the main one. For a cavity with the conic endplates, this value decreases depending on the length of the cones themselves with respect to the volume of the body. With our design parameters it reaches the value 66\%. We verified that with optimized dimensions of the sapphire cylinders (for instance $t_{cyl1}=2.2$ mm and $t_{cyl2}=1$ mm) the mentioned ratio of 72\% increases up to almost 100\%, and for a cavity with conic endplates it increases up to 90\%. Cavities with these dimensions will be implemented in future realizations.

![Figure 6: Simulated transmission coefficient between the two coupled antennas. The peaks correspond to the resonant modes TM$_{030}$ and TM$_{031}$.](image)

As discussed in the next paragraph the cavity was fabricated fixing the two shells on the two endplates. More in detail, the sapphire shells penetrate into the copper for about 10 mm and are fixed to it. The sketch of the geometry is given in the left panel of Fig. 7. To calculate the effect of these penetrations, we
simulated the whole structure. The result is given in the right panel of Fig. 7 where we show the magnitude of the E field. The figure clearly shows that the electromagnetic field does not penetrate into the copper-plates hollows and, as a consequence, the frequency and quality factor are not affected.

This type of geometry allows the implementation of a tuning system to change the resonance frequency of the cavity. The sapphire shells can be cut in two halves, as suggested in [15], and the two half-cylinders can be moved by means of a mechanism embedded into the copper plates. The geometry is sketched in Fig. 8. The resonant frequency, the quality factor, and the factor $C \times V$ are given in table 1 for different position of the two half cylinders. The result put in evidence that it is possible to tune the frequency in a range of more than 500 MHz without affecting significantly the performance of the cavity. The magnitude of the electric field when the distance between the two halves is 3 mm is given in Fig. 9 and clearly shows that the electromagnetic field is well confined within the two half-shells. The mechanical design of this mechanism is still in progress and will be implemented in a further realization of the cavity.
Figure 8: Sketch of the geometry showing the possible implementation of a tuning system to change the resonance frequency of the cavity without affecting its performances.

Figure 9: Magnitude of the electric field when the distance between the two halves is 3 mm.
Table 1: Expected frequency and quality factor.

| $\Delta x$ [mm] | $f_{res}$ [GHz] | $Q \times 10^6$ | $C \times V \times 10^{-6}$ [m$^3$] | $C \times V \times Q$ [m$^3$] |
|-----------------|-----------------|-----------------|----------------------------------|-----------------|
| 0               | 10.92           | 2.01            | 24.75                            | 49.7            |
| 0.25            | 10.81           | 1.766           | 26.23                            | 46.3            |
| 0.5             | 10.71           | 1.80            | 27.62                            | 49.7            |
| 0.75            | 10.62           | 1.69            | 28.94                            | 48.9            |
| 1               | 10.53           | 1.49            | 30.00                            | 44.7            |
| 1.25            | 10.45           | 1.39            | 31.31                            | 43.5            |
| 1.5             | 10.38           | 1.39            | 32.16                            | 44.7            |

3. Cavity fabrication and mechanical tolerance

Two as-grown, fine-grid, polished sapphire-tubes 200 mm long were purchased from ROSTOX-N [37]. We measured tubes diameters with a coordinate-measuring machine at Laboratori Nazionali di Frascati: the smaller one has inner diameter 21.36(1) mm and outer diameter 25.17(1) mm; the larger one has inner diameter 39.71(4) mm and outer radius 42.80(1) mm. The errors reflect the machine precision (about $10 \mu$m) or, if larger, the difference in values measured on the two sides of the tubes. The eccentricity was measured to be within 0.2 mm. Simulations showed that the quality factor is not sensitive to these small variations. The tubes were then sent to Laboratori Nazionali di Legnaro (LNL) for assembly inside the copper cavity. The technical drawing of the copper cavity housing the two sapphire cylinders is shown in Fig. [10]. It is made of four pieces: two end caps and two lateral half sides. On the internal side of each end caps two circular grooves are carved to hold the sapphire cylinders in place. Each grooves width is such so as to avoid compression of the sapphire from the copper when cooling. For the same reason, the depth of the grooves is 1 mm longer than the designed sapphire penetration length. The inner cylindrical volume is formed by combining the two side halves. The resulting cylinder has an inner radius of 29 mm, while from the outside the structure
has a rectangular section. Three 1 mm diameter venting holes are also drilled on one end cap for every separate volume that is formed in the interior of the assembly. All the copper parts were chemically polished before being mounted.

![Technical drawing of the copper cavity housing the two sapphire cylinders. Through holes for assembly and venting holes are not shown](image)

When the two lateral sides are joint together and blocked with M5 non magnetic stainless steel bolts, the two end caps can still be moved independently and fastened. This allows for the easy positioning of the sapphire cylinders while keeping one end cap out. The cavity is normally operated keeping its axis vertical. For this reason no specific holding system has been yet designed for the sapphire tubes, and so they just keep their positions through gravity.

Three photographs of partial assemblies of the dielectric cavity are shown in Fig. [11]. These pictures are just for the record. The final assembly of the sapphire shells is done with only one end-cap removed and the cavity in the vertical position.
Figure 11: Partial assemblies of the dielectric cavity.
4. Experimental characterization of dielectric cavity resonant modes

We characterized the resonant modes of the dielectric cavity at LNL in a LHe-cryostat at about 4 K. We connected the cavity to two fixed antennas subcritically coupled to the pseudo-TM\(_{030}\) mode and placed it inside a vacuum chamber designed to allow operation inside cryogenic dewars. The vacuum chamber is equipped with two rf feedthroughs and a thermometer measuring the temperature of the cavity. The spectrum of the resonant modes, measured with a Vector Network Analyzer (VNA), is shown in Fig. 12. At room temperature the pseudo-TM\(_{030}\) mode had a frequency \(\nu_{030} = 10.886\) GHz with quality factor \(Q_{030} = 150,000\). To prevent damages due to differential contractions of copper and sapphire, we slowed the cooling employing a low exchange-gas pressure, about \(10^{-5}\) mbar. During the cooling, because of thermal contractions, changes in the positions of the sapphire tubes in their housings and variation of the sapphire dielectric constant, we observed drifts and crossings of modes which

![Figure 12: Measured spectrum of the resonant modes of the dielectric cavity. The pseudo-TM\(_{030}\) is the lowest frequency peak, among the strong ones, and the pseudo-TM\(_{031}\) is the subsequent one.](image)
were however followed by continuous measurement of the transmission spectrum. At 40 K the mode frequency reached a plateau at $\nu_{030} = 10.916$ GHz with quality factor $Q_{030} = 320,000$. We then added few mbar of He gas to speed up the cooling that soon stopped at 5.4 K. Transmission and reflection parameters are shown in Fig. 13 as measured from the port with higher coupling to the cavity, while on the other port the reflected signal was barely visible. The measured loaded quality factor is $Q_L = 632,000$. The unloaded quality factor is calculated as $Q_{030} = (1 + k) \times Q_L$ where $k \sim (1 - S_{11}(\nu_{030}))/((1 + S_{11}(\nu_{030}))$ is the coupling to the antenna. We obtain $Q_{030} = 720,000$ a very large quality factor if compared with copper cavities at these frequencies and temperatures with typical quality factor of less than 100,000. The measured quality factor and frequency are summarized in table 2.

Figure 13: Transmission and reflection parameters as a function of frequency for the pseudo-TM$_{030}$ mode at 10.916 GHz at 5.4 K.

5. Conclusions

We realized a dielectric resonance cavity composed of two concentric sapphire hollow-tubes housed in a copper cavity. Placing the sapphire tubes close to the nodes of the TM$_{030}$ mode reduces by an order of magnitude the azimuthal
Table 2: Measured frequency and quality factor.

| T   | \(\nu\)      | Q   |
|-----|--------------|-----|
| 300 K | 10.886 GHz   | 150,000 |
| 5.4 K | 10.916 GHz   | 720,000 |

component of the magnetic field on the cylindrical copper wall, reducing the losses and increasing the quality factor of the pseudo-TM\(_{030}\) mode resonating at 10.9 GHz up to 720,000 at a temperature of 4 K. Electromagnetic simulations show that the frequency mode is tunable in a 500 MHz range. This result improves the one previously obtained by our group with a NbTi cavity \cite{9} and more importantly, it is independent of the applied magnetic field. This quality factor is close to \(10^6\), the limit imposed by the signal linewidth expected from DM axions, and is expected to further improve, up to \(2 \times 10^6\), by properly tuning the thickness of the sapphire tubes.

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