Significant effects of weak gravitational lensing on determinations of the cosmology from Type Ia supernovae

Andrew J. Barber

Astronomy Centre, University of Sussex, Falmer, Brighton BN1 9QJ

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ABSTRACT

Significant adjustments to the values of the cosmological parameters estimated from high-redshift Type Ia supernovae data are reported, almost an order of magnitude greater than previously found. They arise from the effects of weak gravitational lensing on observations of high-redshift sources. The lensing statistics used have been obtained from computations of the three-dimensional shear in a range of cosmological N-body simulations, from which it is estimated that cosmologies with an underlying deceleration parameter $q_0 = -0.51^{+0.03}_{-0.24}$ may be interpreted as having $q_0 = -0.55$ (appropriate to the currently popular cosmology with density parameter $\Omega = 0.3$ and vacuum energy density parameter $\Omega = 0.7$). In addition, the standard deviation expected from weak lensing for the peak magnitudes of Type Ia supernovae at redshifts of $z = 1$ is expected to be approximately 0.078 mag, and 0.185 mag at redshift $z = 2$. This latter value is greater than the accepted intrinsic dispersion of 0.17 mag. Consequently, the effects of weak lensing in observations of high-redshift sources must be taken properly into account.

Key words: supernovae: general – galaxies: clusters: general – cosmology: miscellaneous – cosmology: observations – gravitational lensing – large-scale structure of Universe.

1 INTRODUCTION

1.1 Background

The weak gravitational lensing of light from distant sources by the large-scale structure in the Universe results in the appearance of shear and convergence in images. The application of ‘ray-tracing’ methods, and other programmes, to cosmological N-body simulations has enabled weak lensing statistics to be recorded for imaginary sources at various redshifts. These statistics in general show significant differences between different cosmological models, and, of course, differences for sources at different redshifts.

From weak lensing magnifications computed from ‘ray tracing’ in N-body simulations, Wambsganss et al. (1997) were able to estimate the likely magnification biases for sources at redshifts $z = 0.5$ and $z = 1$, and suggested that these biases should be applied to the observed magnitudes of high-redshift Type Ia supernovae. They found that if the computed median demagnification for $z = 1$ was applied to the observed supernovae at this redshift, it would shift their positions on the Hubble diagram indicative of a slightly different cosmology.

Riess et al. (1998) and Perlmutter et al. (1999) have independently used data sets of Type Ia supernovae extending to a redshift of $z = 0.97$ to estimate values of the cosmological parameters from the Hubble diagram. In discussion of their results in the light of the work by Wambsganss et al. (1997), Riess et al. (1998) have stated that the effects on values of the cosmological parameters should be negligible. Perlmutter et al. (1999) have assumed that the effects of magnification or demagnification average out, and that the most overdense (or high-magnification) lines of sight should be rare for their set of 42 high-redshift supernovae.

However, Barber et al. (2000) have now applied a new algorithm developed by Couchman, Barber & Thomas (1999) for the three-dimensional shear to obtain weak lensing statistics in simulations with cosmological parameters similar to those estimated from the high-redshift supernovae data. The algorithm has a number of key features, described in Section 2.2, which make the procedure very different from traditional ‘ray-tracing’ methods. Their results show that the median demagnification values are considerably less and the dispersions in the probability distributions for the magnification are greater than found by Wambsganss et al. (1997) for redshifts of $z = 1$ and 0.5 (although from different cosmologies). This suggests that the work of Wambsganss et al. (1997) should be reviewed in the light of the new weak lensing data now available from a suitable cosmology.

In this paper I apply the new data from Barber et al. (2000) in an attempt to re-estimate the effects of weak lensing on the cosmological parameters, and find the effects to be far from negligible.

* E-mail: abarber@star.cpes.susx.ac.uk

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1.2 Outline of paper

A brief outline of this paper is as follows. In Section 1.3, work by others on adjustments to the cosmological parameters resulting from weak gravitational lensing is summarized. Section 2 summarizes the method followed by Barber et al. (2000) for obtaining the weak lensing statistics, and describes how the integrated shear values are used to determine the probability distributions for the magnifications of sources at high redshift. Section 3 summarizes the relevant weak lensing statistics that are used in the present work. In Section 4, I describe the form of the Hubble diagram and its dependence on the cosmological parameters, and also summarize the important work by Riess et al. (1998) and Perlmutter et al. (1999) in estimating the cosmological parameters from high-redshift Type Ia supernovae data. In Section 5, I discuss how the dispersions in the magnification distributions for different source redshifts may affect the observed distance moduli. For the specific case of the cosmology with density parameter \( \Omega_M = 0.3 \) and vacuum energy density parameter \( \Omega_\Lambda = 0.7 \), I then consider by how much the supernovae data are displaced on the Hubble diagram, and how this translates into effective changes to the perceived values of the cosmological parameters. Section 6 summarizes and discusses the results obtained for the deceleration parameter.

1.3 Other work

Wambsganss et al. (1997) have used the same ray-tracing method as described by Wambsganss, Cen & Ostriker (1998) for cosmological simulations with \( \Omega_M = 0.4 \), \( \Omega_\Lambda = 0.6 \) and normalization \( s_8 = 0.79 \). The magnification values above and below which 97.5 per cent of all of their lines of sight fall were \( \mu_{\text{low}} \) and \( \mu_{\text{high}} \) for source redshifts of \( z = 1 \), and \( \mu_{\text{low}} = 0.978 \) and \( \mu_{\text{high}} = 1.034 \) for \( z = 0.5 \). The median values for the magnification were 0.983 at \( z = 1 \), and 0.993 at \( z = 0.5 \). The authors claim that these values would give rise to observed values of \( q_0 = -0.395 \pm 0.095 \) for the \( z = 1 \) data, and \( q_0 = -0.398 \pm 0.095 \) for the \( z = 0.5 \) data, rather than the assumed value of \( -0.4 \). (The quoted errors arise from the magnification ranges described above in the asymmetrical distributions.) Wang (1999) was able to derive empirical formulae for the fitting of Wambsganss et al.’s (1997) magnification probability distributions.

Fluke, Webster & Mortlock (1999, 2000) have used a ‘ray-bundle’ method in which a discrete bundle of light rays is traced. The method allows a direct comparison between the shape and size of the bundle at the observer and at the source plane, so that the magnification, ellipticity and rotation can be determined straightforwardly. The cosmological models investigated were similar to those of Barber et al. (2000). However, the particles were considered as point masses, so that very high magnification values could be achieved in principle; however, to alleviate this, the authors do not include bundles that pass within \( \sqrt{2} \) of the Einstein radius of any particle. Because of their use of point particles, they use the empty beam approximation, which gives rise to magnification probability distributions with minimum magnifications of unity (and therefore mean and median values greater than, or equal to, unity), high-magnification tails, and broad dispersions in magnification. Because of the high-magnification tails they discard those high-magnification lines of sight which occur with low probability before defining the \( \mu_{\text{low}} \) and \( \mu_{\text{high}} \) values.

These magnification probability distributions are then used by Fluke & Webster (2000) to examine the effects of the weak lensing dispersion on measurements of \( q_0 \) in both the empty beam (more appropriate for point particles) and full beam limits. They consider whether the resulting shifts in the Hubble diagram can be fitted with various cosmological models. With input data from the \( z = 1 \) magnification distribution for the flat LCDM simulations, they find that \( q_0 = -0.53 \pm 0.06 \) from the empty beam distribution. To convert their empty beam magnification values to filled beam values, Fluke & Webster (2000) use a simple scaling relationship. However, since the filled beam approach is inappropriate to their method, the resulting values should be treated with caution. They find the revised value \( q_0 = -0.61 \pm 0.09 \), i.e. a much larger departure than for the empty beam approximation, but a similar dispersion. Again the dispersion can be partly explained in terms of their use of point particles.

It should be mentioned that Wambsganss et al. (1997) have assumed that the observed supernovae have magnifications of unity, and then have assessed departures from the best-fitting cosmology, rather than assuming that the best-fitting cosmology comes from the most likely demagnification at the peak of the magnification distributions, which is the approach I take in this paper. Although Fluke & Webster (2000) use the median magnification values, the values are always close to unity and greater than, or equal to, unity for the empty beam case.

A novel approach to weak gravitational lensing has been used by Holz & Wald (1998) in a range of cosmological models. Each model contains an individual probability distribution of matter for each redshift (variable with cosmological time). They lay down a set of spheres between the observer and the desired redshift, in which the underlying Robertson–Walker metric is assumed to be perturbed by the matter distribution. A scalar potential and corresponding curvature, related to these perturbations, can then be evaluated. This then allows integration of the geodesic deviation equation (based on departures from the null Robertson–Walker geodesics) along straight lines with random impact parameters through each sphere, to determine the accumulated angular deviations and shear. For each cosmology, about 2000 ‘runs’ are made to produce a large body of statistical data, which include the magnification probability distributions.

Holz (1998) has used the results of this work to consider the effects on the cosmological parameters estimated from the Type Ia supernovae data. He finds that the cosmological model fitting the data best is unchanged, but that the confidence contours in the \( \Omega_M–\Omega_\Lambda \) parameter space are enlarged. For continuously distributed matter, the effects are only slight; but for matter distributed in the form of compact objects, the effects become considerably more important.

2 METHOD

2.1 The Hydra N-body simulations

The cosmological N-body simulations used in the study by Barber et al. (2000) were provided by the Hydra Consortium\(^1\) and produced using the ‘Hydra’ N-body hydrodynamics code (Couchman, Thomas & Pearce 1995). Results are reported on simulations from four different cosmologies, referred to as the SCDM, TCDM, OCDM and LCDM cosmologies. Each of the simulations uses a cold dark matter (CDM) like spectrum, and the parameters used in

\(^1\) http://hydra.mcmaster.ca/hydra/index.html
Table 1. Parameters used in the generation of the four different cosmological simulations.

| Cosmology | \( \Omega_M \) | \( \Omega_{\Lambda} \) | \( \Gamma \) | \( \sigma_8 \) | No. of particles | Box side (\( h^{-1} \) Mpc) |
|-----------|----------------|----------------|-----------|-------------|-----------------|-----------------|
| SCDM      | 1.0            | 0.0            | 0.50      | 0.64        | 128\(^3\)       | 100             |
| TCDM      | 1.0            | 0.0            | 0.25      | 0.64        | 128\(^3\)       | 100             |
| OCDM      | 0.3            | 0.0            | 0.25      | 1.06        | 86\(^3\)        | 100             |
| LCDM      | 0.3            | 0.7            | 0.25      | 1.22        | 86\(^3\)        | 100             |

The generation and specification of these simulations are listed in Table 1. \( \Omega_M \) and \( \Omega_{\Lambda} \) are the present-day values of the density parameter and the vacuum energy density parameter, respectively. The power spectrum shape parameter, \( \Gamma \), is set to 0.5 in the SCDM cosmology, but the empirical determination (Peacock & Dodds 1994) of 0.25 for cluster scales has been used in the other cosmologies. In each case, the normalization, \( \sigma_8 \), on scales of \( 8 h^{-1} \) Mpc has been set to reproduce the number density of clusters (Viana & Liddle 1996); \( h \) is the Hubble parameter expressed in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). In the SCDM and TCDM cosmologies, the number of particles was 128\(^3\), leading to individual dark matter particle masses of \( 1.29 \times 10^{11} \) h\(^{-1}\) solar masses. In the low-density universes, the number of particles was 0.3 times the number in the critical-density universes, leading to the same individual particle masses. The simulation output times were chosen so that consecutive simulation volumes could be snugly abutted; the side dimensions were 100 h\(^{-1}\) Mpc in every case.

Since each time output in a given simulation run is generated using the same initial conditions, particular structures (although evolving) occur at the same locations in all the boxes, and are therefore repeated with the periodicity of the box. To avoid these correlations, each simulation box was arbitrarily translated, rotated (by multiples of 90\(^\circ\)) and reflected about each coordinate axis, before linking together to form the continuous depiction of the universe back to the redshift.

2.2 The three-dimensional shear algorithm

Couchman et al. (1999) describe in detail the algorithm for the three-dimensional shear which has been applied by Barber et al. (2000) to the simulations for the different cosmologies. The main features of the code are as follows.

First, the algorithm uses a fast Fourier transform method in the particle-mesh part of the code, in which the density distribution is smoothed. The procedure makes use of the periodicity of the fundamental volume, so that the effects of matter effectively stretching to infinity are included in the computations. Couchman et al. (1999) pointed particularly to the importance of including the effects of matter beyond a single period orthogonal to the direction of the line of sight.

Secondly, in order to evaluate the shear components correctly, the algorithm works with the ‘peculiar gravitational potential’, which describes departures in the potential from homogeneity, and which results in net zero mass for the universe.

Thirdly, a key feature of the algorithm is the variable particle softening. The feature enables particles in low-density regions to have extended softening, so that nearby evaluation positions for the shear register a density rather than a complete absence of matter. By contrast, particles in highly clustered regions are assigned low softening values, and a selected minimum value is introduced that limits the possibility of strong lensing behaviour.

The minimum value of the softening in the work by Barber et al. (2000) was selected to be \( 10^{-3} \) in box units, equivalent to \( 0.1 h^{-1} \) Mpc at \( z = 0 \). The variable softening feature enables a much more realistic depiction of the large-scale structure within a simulation to be made.

The output from the algorithm is the six independent components of the three-dimensional shear evaluated at a large number of positions within each simulation cube.

2.3 Method

A regular rectangular grid of 100 \( \times \) 100 directions through each box was established. (Since there were only small deflections, and the point of interest was the statistics of output values, each light ray was considered to follow one of the lines defined by these directions through the boxes.) Each ‘ray’ was then connected with the corresponding line of sight through subsequent boxes in order to obtain the required statistics of weak lensing. In the application of the code, the shear was evaluated at 1000 positions along each of the lines of sight, forming a regular grid of evaluation positions in each box.

The evaluation of the three-dimensional shear within the volume of the boxes used the appropriate angular diameter distances at every evaluation position, representing a distinct advantage over two-dimensional (planar) methods. Furthermore, the variable softening facility within the algorithm led naturally to the assumption that the universe may be described in terms of the filled beam approach, and this was the approach adopted by Barber et al. (2000). This is different from the assumptions of many other workers, who frequently use point particles, or a limited form of fixed softening, and use the empty beam approximation. The two approaches, elegantly discussed by Pei (1993), give rise to quite different expectations and results, the most obvious being the following.

First, strong lensing can occur with particles of small radii, leading to high-magnification tails in the probability distributions. Secondly, magnification distributions in the empty beam approximation all have minimum magnifications of \( \mu_{\text{min}} = 1 \), whilst in the filled beam, \( \mu_{\text{min}} \ll 1 \); this may alter the dispersions and the median values in the two distributions, essential for an understanding of the magnitudes expected for high-redshift Type Ia supernovae. Finally, the mean values for the magnifications can be calculated from the respective angular diameter distances in the different cosmologies, for the empty beam approximation; however, the mean values in the filled beam approximation are always unity. [Interestingly, Metcalf & Silk (1999) claim that observations of a large number of Type Ia supernovae should have the potential to distinguish between smoothly distributed dark matter in the form of weakly interacting elementary particles, and dark matter consisting of microscopic compact objects, thereby helping to resolve the issue of the underlying form of dark matter. Seljak & Holz (1999) reach a similar conclusion from studies of magnification distributions generated using many values for the fraction of matter in the form of compact objects.] The six independent second derivatives of the peculiar gravitational potential were calculated by the code at each of the selected evaluation positions throughout each box. These were then integrated in small steps along each line, forming, essentially, a large number of deflection sites through each simulation box. The integrated values formed the input data to establish the elements of the two-dimensional Jacobian matrix on each of the
lines of sight for each of the deflection sites. The Jacobian matrix elements were then used together with the multiple lens-plane theory (summarized by Schneider, Ehlers & Falco 1992) and the appropriate angular diameter distances to obtain values for the magnification, source ellipticity, shear and convergence from each of the 10 000 lines of sight throughout all the simulation boxes linked back to the required redshift.

In the procedure to obtain the two-dimensional second derivatives of the effective lensing potentials, it was assumed that: (a) the angular diameter distances varied linearly between the evaluation step points (although they were evaluated exactly at each step point); (b) the angular diameter distances varied continuously through the depth of each simulation box, as they would in the real universe; (c) the shearing of light was weak, so that ‘rays’ were considered to follow the straight lines of sight defined by the grid of evaluation positions within each simulation box; and (d) the accumulating component values within the developing Jacobian matrices were computed using the assumption that the shear was weak.

### 3 WEAK LENSING IN DIFFERENT COSMOLOGIES

Barber et al. (2000) have reported significant ranges of magnifications in all the cosmologies for nominal source redshifts of 0.5, 1.0, 2.0, 3.0 and 4.0. From the magnification distributions they have computed the median values, $\mu_{\text{low}}$, the values, $\mu_{\text{peak}}$, and $\mu_{\text{high}}$, above and below which 97.5 per cent of all lines of sight fall, and also the rms deviations from unity for the magnifications. All the values mentioned are displayed in Table 2, which is reproduced from Barber et al. (2000). It is interesting to compare the distributions in the different cosmologies. Figs 1 and 2 (also reproduced from Barber et al. 2000) show the magnification distributions for all the cosmologies for source redshifts of 4 and 1, respectively. The distributions are all broader in the SCDM cosmology when compared with the TCDM cosmology, as a result of its more clumpy character. For source redshifts of 4, the OCDM and SCDM cosmologies have very similar distributions even though the angular diameter distance multiplying factors are larger in the OCDM cosmology. The most significant feature for our purposes here is that for high source redshifts the magnification distributions are broadest in the LCDM cosmology (and the maximum values of the magnification are greatest here), but for lower source redshifts the width of the distribution falls below that for the SCDM and OCDM cosmologies. In the present work, I shall be mostly concerned with the magnification values for sources at $z = 1$.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** The magnification probability distributions for all the cosmologies, assuming $z = 4$.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The magnification probability distributions for all the cosmologies, assuming $z = 1$.

| $z$  | $\mu_{\text{low}}$ | $\mu_{\text{peak}}$ | $\mu_{\text{high}}$ | rms deviation |
|-----|-------------------|---------------------|---------------------|--------------|
| SCDM |                   |                     |                     |              |
| 3.9  | 0.835 0.933 1.420 | 0.852 0.949 1.367   | 0.885 0.947 1.277   |              |
| 3.0  | 0.930 0.973 1.181 | 0.969 0.985 1.089   | 0.991 0.997 1.115   |              |
| 1.9  | 0.858 0.919 1.469 | 0.884 0.939 1.469   | 0.905 0.942 1.283   |              |
| 1.0  | 0.941 0.972 1.144 | 0.974 0.986 1.067   | 0.974 0.989 1.062   |              |
| 0.5  | 0.858 0.919 1.469 | 0.884 0.939 1.469   | 0.905 0.942 1.283   |              |
| TCDM |                   |                     |                     |              |
| 4.0  | 0.858 0.919 1.459 | 0.884 0.939 1.469   | 0.905 0.942 1.283   |              |
| 2.9  | 0.915 0.942 1.283 | 0.944 0.972 1.147   | 0.974 0.986 1.062   |              |
| 2.0  | 0.870 0.934 1.453 | 0.904 0.966 1.191   | 0.934 0.964 1.070   |              |
| 1.0  | 0.944 0.966 1.191 | 0.974 0.986 1.062   | 0.974 0.989 1.062   |              |
| 0.5  | 0.981 0.987 1.070 | 0.981 0.987 1.070   | 0.981 0.987 1.070   |              |
| OCDM |                   |                     |                     |              |
| 4.0  | 0.858 0.919 1.459 | 0.884 0.939 1.469   | 0.905 0.942 1.283   |              |
| 2.9  | 0.915 0.942 1.283 | 0.944 0.972 1.147   | 0.974 0.986 1.062   |              |
| 2.0  | 0.870 0.934 1.453 | 0.904 0.966 1.191   | 0.934 0.964 1.070   |              |
| 1.0  | 0.944 0.966 1.191 | 0.974 0.986 1.062   | 0.974 0.989 1.062   |              |
| 0.5  | 0.981 0.987 1.070 | 0.981 0.987 1.070   | 0.981 0.987 1.070   |              |
| LCDM |                   |                     |                     |              |
| 3.6  | 0.789 0.885 1.850 | 0.870 0.934 1.453   | 0.904 0.966 1.191   |              |
| 2.0  | 0.870 0.934 1.453 | 0.904 0.966 1.191   | 0.934 0.964 1.070   |              |
| 1.0  | 0.944 0.966 1.191 | 0.974 0.986 1.062   | 0.974 0.989 1.062   |              |
| 0.5  | 0.981 0.987 1.070 | 0.981 0.987 1.070   | 0.981 0.987 1.070   |              |
4 DETERMINATIONS OF THE COSMOLOGY FROM TYPE I A SUPERNOVAE

The significant ranges in the magnifications (dependent on the cosmology) which might apply to distant sources have been summarized in the previous section. In the absence of magnification (or demagnification) from the large-scale structure, it would be possible to determine the cosmological parameters, $\Omega_M$ and $\Omega_\Lambda$, from the departures from linearity in the Hubble diagram, provided ‘standard candle’ sources together with good calibration were available for measurement at high redshift. This is precisely the route taken by a number of authors, most importantly Riess et al. (1998) and Perlmutter et al. (1999), both of whom have used high-redshift Type Ia supernovae data of redshifts up to 0.97.

Carroll, Press & Turner (1992) give the general expression for the distance measure to redshift, $z$:

$$
\frac{H_0 D_L}{c(1 + z)} = \frac{1}{|\Omega_k|^{1/2}} \sin \left\{ \sqrt{\Omega_k} \left[ \left(1 + z^2\right)^{1/2} - \left(1 + \Omega_M z^2 + \Omega_\Lambda z^4 \right)^{1/2} \right] \right\},
$$

where $\sinh$ means the hyperbolic sine, sinh, if $\Omega_k > 0$, or sine if $\Omega_k < 0$; $H_0$ is the Hubble parameter, $D_L$ is the luminosity distance, related to the angular distance diameter through $D_L = (1 + z)^2 D$, and $c$ is the velocity of light. The curvature density parameter, $\Omega_k$, is defined in terms of the matter density parameter, $\Omega_M$, and the vacuum energy density parameter, $\Omega_\Lambda$:

$$1 = \Omega_M + \Omega_\Lambda + \Omega_k.
$$

However, if $\Omega_k = 0$ (which I have assumed for this work), then

$$
\frac{H_0 D_L}{c(1 + z)} = \int_0^z \left[ (1 + z')^2 (1 + \Omega_M z' + \Omega_\Lambda z'^2)^{-1/2} \right] dz' = 1,
$$

and this distance measure, $I$, can be evaluated straightforwardly by numerical integration for any cosmology and any redshift. At low redshifts the expression reduces to the well-known Hubble law, but departures from the linear Hubble relation become evident as $z$ is increased, and then different cosmologies may be differentiated.

Observationally, the distance modulus $m - M$ (where $m$ is the apparent magnitude and $M$ is the absolute magnitude) is determined, and this is related to the distance measure through (see e.g. Peebles 1993):

$$m - M = 25 + 5 \log_{10} [3000(1 + z)I] - 5 \log_{10} h.
$$

(In practice, the absolute magnitudes are determined by employing empirical correction methods based on the light-curve data as mentioned below, and by making the appropriate $k$-corrections in each bandpass.)

Consequently, the value of the distance modulus is fixed for any given redshift, cosmology and Hubble parameter, but if the source is magnified (or demagnified) by weak gravitational lensing effects, then it will appear to be closer (or further away), according to

$$\Delta m = -2.5 \log_{10} \mu,
$$

where $\mu$ is the magnification. Alternatively, the observational evidence could be interpreted as pointing to different values of the cosmological parameters, $\Omega_M$ and $\Omega_\Lambda$, since the curve on the Hubble diagram will be slightly displaced.

I now briefly summarize the work of Riess et al. (1998) and Perlmutter et al. (1999), who have been able to estimate the cosmological parameters from data sets of high-redshift Type Ia supernovae.

Riess et al. (1998) have analysed the spectral and photometric observational data of a total of 50 Type Ia supernovae with redshifts $0.01 \leq z \leq 0.97$. They have made use of two different correction methods for the peak magnitudes, namely the multi-colour light curve shape (MLCS) method and a template method. The MLCS method, due to Riess, Press & Kirshner (1996), is an extension of Riess, Press & Kirshner’s (1995) empirical method, described above, and makes use of up to four colours in the supernovae photometry to quantify the amount of reddening by interstellar extinction. In addition, Riess et al. (1998) have improved the method by using a larger ‘training set’ of supernovae and including a quadratic term in the algorithm describing the light-curve shapes. The template method is that of Hamuy et al. (1995) combined with Phillips’ (1993) light-curve decline parameter, $\Delta m_{15}(B)$. The fitting of the data has been done using a $\chi^2$ statistic for a wide range of the parameters, $H_0$, $\Omega_M$ and $\Omega_\Lambda$, and including both the dispersions in the host galaxy redshifts (in terms of the distance moduli) and the intrinsic dispersions (after correction) in the supernovae peak magnitudes. With the constraint $\Omega_M + \Omega_\Lambda = 1$, Riess et al. (1998) find that $\Omega_\Lambda > 0$ at the 7$\sigma$ and 9$\sigma$ confidence levels for the two correction methods, and formally quote, with this constraint,

$$\Omega_M = 0.28 \pm 0.10, \quad \Omega_\Lambda = 0.72 \pm 0.10
$$

using the MLCS method, and

$$\Omega_M = 0.16 \pm 0.09, \quad \Omega_\Lambda = 0.84 \pm 0.09
$$

using the template method.

Perlmutter et al. (1999) have based their analysis on 42 high-redshift Type Ia supernovae having redshifts $0.18 \leq z \leq 0.83$, and combined their results with 18 Type Ia supernovae from the Calán/ Tololo Supernova Survey having redshifts less than 0.1. Their method for determining the peak magnitudes relies on an empirical relationship between the light-curve width and the luminosity for the various template light curves used. First, the time axis of the template is dilated by the factor $(1 + z)$ to allow for the cosmological lengthening of the time-scale. Then the observed light curve is fitted to the template by introducing a ‘stretch factor’, $s$, on the time axis. Finally, the peak magnitude is corrected to

$$m_{corr} = m + \Delta m_{corr}(s),
$$

where the correction term, $\Delta m_{corr}(s)$, is a simple monotonic function of $s$. The correction terms (different for different wavebands) were determined by Perlmutter et al. (1997) by relationship to the decline $\Delta m_{15}$, and the assessment of the luminosity by Hamuy et al.’s (1995) method. They claim that the residual dispersion in the peak magnitudes for all the supernovae, after applying the correction based on the light-curve width–luminosity relationship, is within about 0.17 mag. They state, however, that it is not clear whether the dispersion is best modelled as a normal distribution in terms of the flux, or a log-normal distribution (Gaussian in magnitude terms). This point is worth bearing in mind if Monte Carlo simulations are performed to sample both the intrinsic dispersion and the magnification distribution arising from weak lensing. The preferred fitting of the data by Perlmutter et al. (1999) was performed by minimizing $\chi^2$ using the magnitude
residuals in the Hubble diagram, from which they find:

$$0.8 \Omega_M - 0.6 \Omega_\Lambda = -0.2 \pm 0.1.$$  

(9)

With the constraint, $$\Omega_M + \Omega_\Lambda = 1$$, they then find:

$$\Omega_M = 0.28^{+0.09}_{-0.08}(1\sigma \text{ statistical})^{+0.05}_{-0.05}(\text{systematic}).$$  

(10)

It is worth noting, however, that the above results are far from conclusive. Goodwin et al. (1999), for example, have pointed to the consistency of the data with a zero cosmological constant, provided the Hubble parameter has a higher local value compared with the global value; Riess et al. (1999) have pointed to the possible evolution of Type Ia supernovae light-curve shapes arising from differences in the rise-times, which may affect the estimation of the peak magnitudes, although this has been questioned by Aldering, Knop & Nugent (2000).

### 5 THE EFFECTS OF WEAK LENSING ON DETERMINATIONS OF THE COSMOLOGICAL PARAMETERS

Both groups of workers, i.e. Riess et al. (1998) and Perlmutter et al. (1999), point to cosmologies that are close to the $$\Omega_M = 0.3$$, $$\Omega_\Lambda = 0.7$$ cosmological simulation that Barber et al. (2000) analysed in terms of weak lensing. I shall therefore discuss the dispersions in magnification and the impact on determinations of the deceleration parameter, $$q_0$$, with regard to this assumed cosmology.

By considering Type Ia supernovae sources at ten evenly spaced redshifts between $$z = 0$$ and $$z = 1$$, the resulting distributions of the magnifications show the expected increasing dispersions, which could be interpreted as observational dispersions in the peak magnitudes. In Table 3, I give the values computed for $$\mu_{\text{low}}$$, $$\mu_{\text{peak}}$$, the rms deviation in magnification and $$\mu_{\text{high}}$$, and, in Table 4, what these values mean in terms of differences in the distance modulus compared with unmagnified sources. The final column in Table 4 gives the total difference in distance modulus between the values derived from $$\mu_{\text{low}}$$ and $$\mu_{\text{high}}$$. The values for the changes in the distance modulus for appropriate values of the (de)magnification are shown in Fig. 3. The asymmetry in the magnification distributions is clearly seen in terms of the distance modulus changes. Riess et al. (1998) have recognized that weak lensing of high-redshift Type Ia supernovae can alter the observed magnitudes, and quote the findings of Wambsganss et al. (1997), i.e. that the light will be on average demagnified by 1.7 per cent at $$z = 1$$ in a universe with a non-negligible cosmological constant. They state that the size of this effect is negligible. Perlmutter et al. (1999) assume that the effects of magnification or demagnification will average out, and that the most overdense lines of sight should be rare for their set of 42 high-redshift supernovae. However, they note that the average (de)amplification bias from integration of the probability distributions is less than 1 per cent for redshifts of $$z \leq 1$$.

My work has shown that, in the LCDM cosmology, a source at $$z = 1$$ will have a median deamplification of 3.4 per cent ($$\mu_{\text{peak}} = 0.966$$), and 1.3 per cent at $$z = 0.5$$. Whilst these demagnification factors are still small, they can result in repositioning of the supernovae data on the Hubble diagram with significantly revised values of the cosmological parameters. In addition, there is a significant probability of observing highly magnified supernovae; 97.5 per cent of all lines of sight display a range of magnifications up to 1.191 at $$z = 1$$, and this range is equivalent to a dispersion in

![Figure 3](https://academic.oup.com/mnras/article-abstract/318/1/195/1143183/26 July 2018)
the magnitudes of 0.252. The standard deviation in the asymmetrical distributions at \( z = 1 \) is 0.078 mag. This is to be compared with the accepted intrinsic dispersion of 0.17 mag for Type Ia supernovae reported by Hamuy et al. (1996) in a set of low-redshift supernovae. For sources at a redshift of 2, however, Barber et al.’s (2000) data predict a standard deviation of 0.185 mag from weak lensing, in excess of the intrinsic dispersion. Clearly, the weak lensing dispersion will become increasingly important as supernovae at greater redshifts are discovered.

In the work by Wambsganss et al. (1997), in the cosmology with \( \Omega_M = 0.4 \) and \( \Omega_\Lambda = 0.6 \) they found \( \mu_{\text{low}} = 0.951 \) and \( \mu_{\text{high}} = 1.101 \) for source redshifts of \( z = 1 \), and \( \mu_{\text{low}} = 0.978 \) and \( \mu_{\text{high}} = 1.034 \) for \( z = 0.5 \). Their median magnification values were 0.983 for \( z = 1 \) and 0.993 for \( z = 0.5 \). These may be compared directly with the values shown in Table 2 for the magnifications in the \( \Omega = 0.3, \Omega_\Lambda = 0.7 \) cosmology that I have investigated, i.e. \( \mu_{\text{low}} = 0.944 \) and \( \mu_{\text{high}} = 1.191 \) for source redshifts of \( z = 1 \), and \( \mu_{\text{low}} = 0.981 \) and \( \mu_{\text{high}} = 1.070 \) for \( z = 0.5 \). The median values are 0.966 for \( z = 1 \) and 0.987 for \( z = 0.5 \).

Since the most likely effect of weak lensing is a slight demagnification of the source, it is probable that the originating data used by Riess et al. (1998) and Perlmutter et al. (1999) may suffer from such a demagnification, and therefore should be repositioned on the Hubble diagram at brighter positions (i.e. lower distance moduli). Holz (1998) concurs with this, since we have to deal with individual supernovae at each redshift and therefore the likelihood of even sampling the probability distribution for a particular redshift is low.] Such repositioning would then point to a slightly different cosmology with a different deceleration parameter. I plot, in Fig. 4, the distance modulus values at \( z = 1 \), for various values of \( \Omega_M \) centred around \( \Omega_M = 0.3 \), and keeping \( \Omega_M + \Omega_\Lambda = 1 \). The plot has been generated by solving equation (4) numerically.

With no magnification, \( \Omega_M = 0.3, \Omega_\Lambda = 0.7, h = 0.65 \) and \( z = 1 \), the distance modulus is 44.2626. Now assume that this cosmology is the best fit to the observed data. Then, by making a redshift \( z = 1 \) supernova brighter than observed (assuming it was observed at the peak of the magnification distribution, with \( \mu_{\text{peak}} = 0.966 \)), the distance modulus would be reduced to 44.2250, equivalent to \( \Omega_M = 0.33 \), and equivalent to changing \( q_0 \) from \(-0.55 \) to \(-0.51 \). For the rms deviations in the magnification, the distance modulus changes by \(+0.0476/-0.0500\), equivalent to revised \( q_0 \) values of \(-0.61 \) and \(-0.49 \). For the \( \mu_{\text{high}} \) and \( \mu_{\text{low}} \) values at \( z = 1 \), \( q_0 \) changes to \(-0.75 \) and \(-0.48 \).

The conclusion, on the evidence of these data, is that true underlying cosmologies with \( q_0 = -0.51^{+0.03}_{-0.24} \) may be interpreted as having \( q_0 = -0.55 \), from the use of perfect standard candles (without intrinsic dispersion), arising purely from the effects of weak lensing. This adjustment to the value of \( q_0 \) is almost an order of magnitude larger than that found by Wambsganss et al. (1997) based on a cosmology with \( \Omega_M = 0.4, \Omega_\Lambda = 0.6 \), and the dispersion in the values is slightly larger than their findings.

### 6 SUMMARY AND CONCLUSIONS

I have summarized the results for the magnification distributions for the different cosmologies for different source redshifts as obtained by Barber et al. (2000) in Table 2. At high redshift, the LCDM cosmology produces the highest magnifications, the broadest distribution curves and the lowest peak values. For sources at \( z = 3.6 \) in the LCDM cosmology, 97.5 per cent of all lines of sight have magnification values up to 1.850. (The maximum magnifications, not quoted here, depended on the choice of the minimum softening in the code, although the overall distributions were very insensitive to the softening.) The rms fluctuations in the magnification (about the mean) were as much as 0.191 in this cosmology, for sources at \( z = 3.6 \). Even for sources at \( z = 0.5 \) there is a measurable range of magnifications in all the cosmologies.

The immediate implication is the likely existence of a bias in observed magnitudes of distant objects, and a likely dispersion for standard candles, for example, Type Ia supernovae at high redshift. In particular, the weak lensing dispersion from 97.5 per cent of the lines of sight expected in the peak magnitudes for Type Ia supernovae at redshifts of \( z = 1 \) may be as much as 0.252 mag. The standard deviations in the distributions are 0.078 mag for \( z = 1 \) and 0.185 mag for \( z = 2 \). These values are to be compared with the accepted dispersion of 0.17 mag, so that we should expect to see an increasing dispersion for the peak magnitudes of supernovae as they are discovered at higher redshifts. Wang (1999) concurs with this finding, suggesting that the dispersion in the peak magnitudes due to weak lensing should become comparable with or exceed the intrinsic dispersion ‘at redshifts of a few’.

I have made use of the magnification statistics from the LCDM cosmology, to reanalyse the results for the cosmological parameters determined from the high-redshift Type Ia supernovae data. [Both Riess et al. (1998) and Perlmutter et al. (1999) point to cosmologies with parameters close to those of the LCDM model, i.e. \( \Omega_M = 0.3, \Omega_\Lambda = 0.7 \).]

The dispersions in the magnification are monotonically increasing with redshift, and have rms fluctuations about the mean of 0.045 for \( z = 0.99 \), and 0.016 for \( z = 0.49 \). These have been translated into variations in distance modulus, which, in turn, suggest a different cosmological model for the data. On the assumption that \( \Omega_M = 0.3, \Omega_\Lambda = 0.7 \) and \( q_0 = -0.55 \) is the cosmology that fits the observed data best, my results indicate that the true underlying cosmology could have \( q_0 = -0.51 \) based on the peak magnification values, with a spread of \(+0.03/-0.24\) based on the computed magnification dispersion. This would represent a significant adjustment to the cosmological parameters.

Wambsganss et al. (1997) found a median demagnification value much closer to unity (1.7 per cent below for \( z = 1 \)) in their cosmology with \( \Omega_M = 0.4, \Omega_\Lambda = 0.6 \) than Barber et al. (2000).

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**Figure 4.** The distance modulus for \( z = 1 \) versus \( \Omega_M \), with the constraint \( \Omega_M + \Omega_\Lambda = 1 \).

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have found (3.4 per cent below for $z = 1$) in the $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ cosmology. Consequently, my adjustment to $q_0$ is considerably greater. However, the magnification dispersions give rise to similar dispersions in $q_0$. Wambsganss et al.'s (1997) calculated adjustment to $q_0$ is approximately an order of magnitude smaller than the departure I would suggest.

Fluke et al.'s (2000) estimate for the revised value of the deceleration parameter ($q_0 = 0.53_{-0.02}^{+0.06}$ for the $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$ cosmology) in the empty beam approximation is only half the value of the departure that I suggest. The value of their dispersion cannot be easily compared with mine, because of their use of the empty beam approximation, which gives rise to a different form for the distribution curve, with values of magnification that are always positive. Furthermore, their method to convert the empty beam magnification values (more appropriate to their method) to filled beam values was not rigorous, so that the resulting values should be treated with caution.

Both Wambsganss et al. (1997) and Fluke et al. (2000) have assumed that the observed supernovae have magnifications of unity, so that the median demagnifications when using the filled beam approach are assumed to reposition the supernovae at dimmer magnitudes. Fluke et al. (2000) have to reposition them at brighter magnitudes when using the empty beam approximation. I have adopted the opposite viewpoint. I have assumed that the supernovae are observed at their median demagnification values, and therefore have to be repositioned at brighter magnitudes (i.e. reduced distance moduli) to obtain the correct cosmological parameters. Consequently, the direction of my adjustment to $q_0$ is opposite to those of the above authors for the filled beam approximation.

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