AC-driven double quantum dots as spin pumps and spin filters.

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We propose and analyze a new scheme of realizing both spin filtering and spin pumping by using ac-driven double quantum dots in the Coulomb blockade regime. By calculating the current through the system in the sequential tunneling regime, we demonstrate that the spin polarization of the current can be controlled by tuning the parameters (amplitude and frequency) of the ac field. We also discuss spin relaxation and decoherence effects in the pumped current.

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The emerging field of spintronics aims at creating devices based on the spin of electrons.\textsuperscript{1} One of the most important requirements for any spin-based electronics is the ability to generate a spin current. Proposals for generating spin-polarized currents include spin injection by using ferromagnetic metals\textsuperscript{2} or magnetic semiconductors\textsuperscript{3}. Alternatively, one may use quantum dots (QDs) as spin filters or spin pumps\textsuperscript{4, 5}. For QD spin filters, one needs a single spin-polarized electron\textsuperscript{6} that can be guided through the device even with no dc voltage applied.\textsuperscript{7, 8} Spin current rectification has also been realized\textsuperscript{9, 10}.

Spin current in QDs can be controlled by tuning the parameters (amplitude and frequency) of the ac field. We illustrate how by tuning the external ac field one can drive the driven DQD as a bipolar spin filter with no dc voltage applied.

In this Letter we propose and analyze a new scheme of realizing both spin filtering and spin pumping by using a double quantum dot (DQD), in the Coulomb blockade regime, with time-dependent gate voltages and in the presence of a uniform magnetic field. The periodic variation of the gate potentials allows for a net dc current through the device even with no dc voltage applied (PAT)\textsuperscript{11, 12}. If the system is driven at a frequency (or subharmonic) corresponding to the energy difference between two time-independent eigenstates, the electrons become completely delocalized\textsuperscript{13, 14}. If the left reservoir (chemical potential \(\mu_L\)) is set to the level below the Fermi energy, and the right reservoir (chemical potential \(\mu_R\)) is set to the level above the Fermi energy, the system in the sequential tunneling regime, we demonstrate that the spin polarization of the pumped current is 100\% spin-down (up) dependent on the applied frequency, such that the degree of spin polarization can be tuned by means of the ac field. For example, if one drives the system in a state with \(n_L+n_R=3\) electrons, at a frequency corresponding to the energy difference between the singlets in both dots, the electron with spin \(\downarrow\) becomes delocalized in the DQD system. If now the chemical potential for taking \((\uparrow)\) electrons out of the right dot is above (below) \(\mu_R\), a spin-polarized current is generated. The above conditions for the chemical potentials can be achieved by breaking the spin-degeneracy through a Zeeman term \(\Delta_z = g\mu_B B\), where \(B\) is the external magnetic field (which is applied parallel to the sample in order to minimize orbital effects), \(g\) is the effective g-factor and \(\mu_B\) the Bohr magneton.

Our main findings can be summarized in Fig. 2 where we present a plot of the pumped current as a function of the applied frequency for a particular choice of parameters. Importantly, the current presents a series of peaks which are uniquely associated with a definite spin polarization: the pumped current is 100\% spin-down (up) dependent on the applied frequency for a particular choice of parameters (amplitude and frequency) of the ac field.

Formalism.— Our system consists of an asymmetric DQD connected to two reservoirs kept at the chemical potentials \(\mu_\alpha, \alpha = L, R\). Using a standard tunneling Hamiltonian approach, we write for the full Hamiltonian \(\mathcal{H}_L + \mathcal{H}_{DQD} + \mathcal{H}_R\), where \(\mathcal{H}_L = \sum_\alpha \sum_{k,\sigma} \epsilon_k c_{k,\sigma}^{\dagger} c_{k,\sigma}\) describes the leads and \(\mathcal{H}_{DQD} = \mathcal{H}_{QD}^L + \mathcal{H}_{QD}^R + \mathcal{H}_{L \leftrightarrow R}\) describes the DQD. It is assumed that only one orbital in the left dot participates in the spin-polarized pumping process whereas two orbitals in the right dot (energy separation \(\Delta E\)) have to be considered. The isolated left dot is thus modelled as a one–level Anderson impurity: \(\mathcal{H}_{QD}^L = \sum_{\alpha} E_{\alpha}^L d_{\alpha}^{\dagger} d_{\alpha} + U_L n_{\alpha\uparrow} n_{\alpha\downarrow}\), whereas the isolated right dot is modelled as: \(\mathcal{H}_{QD}^R = \sum_{\alpha} E_{\alpha}^R d_{\alpha}^{\dagger} d_{\alpha} + U_R \sum_{\beta} n_{\beta\uparrow} n_{\beta\downarrow} + \sum_{\alpha,\sigma} n_{\alpha\sigma} n_{\beta\sigma} + J S_0 S_1\). The index \(i = 0, 1\) denotes the two levels. In practice, we take...
is higher in energy than the triplets states with current is spin-unpolarized. b) Pumping through two particle diagrams of the double dot in energy than the singlet states. As a consequence of Hund’s rule, this asymmetry can be realized by making the right dot smaller. c) Pumping involving a triplet (case b) or up (case c) become delocalized by the microwaves. Dashed arrows denote delocalized spins whereas solid arrows denote spins that remain localized on each dot.

\[ E_L^1 = E_R^1 = 0 \quad (E_L^2 = E_R^2 = \Delta_z) \], so all the asymmetry is included in the charging energies \( U_R > U_L \). Experimentally, this asymmetry can be realized by making the dot right smaller. \( S_i = (1/2) \sum_{\sigma} d_{Ri\sigma} d_{Ri\sigma}^* \) are the spins of the two levels. As a consequence of Hund’s rule, the intra-dot exchange, \( J < 0 \) such that the singlet \( |S_1\rangle = (1/\sqrt{2})(d_{R0\uparrow}^d d_{R1\downarrow} - d_{R0\downarrow}^d d_{R1\uparrow})/0) \) is higher in energy than the triplets \( |T_+\rangle = d_{R0\uparrow}^d d_{R1\uparrow}^*|0\rangle, \quad |T_0\rangle = (1/\sqrt{2})(d_{R0\uparrow}^d d_{R1\uparrow} + d_{R0\downarrow}^d d_{R1\downarrow})/0) \) and \( |T_-\rangle = d_{R0\downarrow}^d d_{R1\uparrow}^*|0\rangle \). Due to the Zeeman splitting \( E^{T_+} > E^{T_0} > E^{T_-} = \Delta \chi + U_R - J/4 \). Finally, we consider the case where \( \Delta \chi > \Delta z + J/4 \) such that the triplet \( |T_+\rangle \) is higher in energy than the singlet \( |S_0\rangle = (1/\sqrt{2})(d_{R0\uparrow}^d d_{R1\uparrow}^* - d_{R0\downarrow}^d d_{R1\downarrow}^*)/0) \).

\[ \mathcal{H}_{LR} = \sum_{i,J} t_{LR}(d_{La}^d d_{LJ} + h.c.) \] describes tunneling between dots. The tunneling between leads and each QD is described by the perturbation \( \mathcal{H}_T = \sum_{kL,J} V_L(c_{kLJ}^d d_{La} + h.c.) + \sum_{ijR,K} V_R(c_{KRL}^d d_{RJa} + h.c.) \). \( \Gamma_{L,R} = 2\pi D_{L,R}|V_{L,R}|^2 \) are the tunneling rates. It is assumed that the density of states in both leads \( D_{L,R} \) and the tunneling couplings are energy-independent.

We study the system by a reduced density matrix (RDM), \( \rho = Tr_L|\chi\rangle\langle\chi| \), where \( \chi \) is the full density matrix, and \( Tr_L \) is the trace over the leads. The dynamics of the RDM is formulated in terms of the eigenstates and eigenenergies of each isolated QD. We concentrate on the Coulomb blockade regime (with up to two electrons per dot, which defines a basis of 20 QD states) and study the sequential tunneling regime (Born-Markov approximation). For example, the diagonal elements of the RDM read

\[ \dot{\rho}_{ss} = -\frac{i}{\hbar}[\mathcal{H}_{LR}, \rho]_{ss} + \sum_{m\neq s} W_{sm} \rho_{sm} - \sum_{k\neq s} W_{ks} \rho_{ss} \] (1)

where \( W_{ij} \) are the transition rates (calculated using a standard Fermi Golden Rule). In addition we consider an ac field acting on the dots, such that the single particle energy levels become \( \epsilon_L(R) \rightarrow \epsilon_L(R)(t) = \epsilon_L(R) + \frac{eV_{AC}}{2}\cos \omega t \), where \( eV_{AC} \) and \( \omega \) are the amplitude and frequency, respectively, of the applied field. We include spin relaxation and decoherence phenomenologically in the corresponding elements of the equation for the RDM. Relaxation processes are described by the spin relaxation time \( T_1 = (W_{1\uparrow} + W_{1\downarrow})^{-1} \), where \( W_{1\uparrow} \) and \( W_{1\downarrow} \) are spin-flip relaxation rates fulfilling a detailed balance.

A lower bound for the spin relaxation time \( T_1 = 50 \mu s \) with a field \( B = 7.3 T \) has been obtained recently in a single electron in a QD using energy spectroscopy and relaxation measurements. In the following, we focus on zero temperature results such that \( W_{1\uparrow} = 0 \) and thus \( T_1 = W_{1\downarrow}^{-1} \). The rate \( T_2^{-1} \) is the spin decoherence time describes intrinsic spin decoherence. We take \( T_2 = 0.1 T_1 \) in all the calculations.

In practice, we integrate numerically the dynamics of the RDM in the chosen basis. In particular, all the results shown in the next paragraphs are obtained by letting the system evolve from the initial state \( |\downarrow\uparrow\uparrow\rangle \) until a stationary state is reached. The dynamical behavior of the system is governed by rates which depend on the electrochemical potentials of the corresponding transitions. The electrochemical potential \( \mu_1(2), \mu_2(3) \) of dot \( L(R) \) is defined as the energy needed to add the \( N_1(2) \) electron to energy level \( i \) of dot \( L(R) \), while having \( N_2(1) \) electrons on dot \( R(L) \). The current from left to right is: \( I_{LR} = \Gamma_R \sum_s \rho_{ss}(t) \), with a similar expression for \( I_{R-L} \). Here, states \( |s\rangle \) are such that the right dot is occupied. For ease in the notation, we take from now on \( h = \epsilon = 1 \), such that \( V_{AC}, \omega \), etc, have units of energy.

Results. - A calculation of the stationary current, for each direction of spin, namely \( I_{LR} = \sum_{\sigma=\pm} I_\sigma \) as a function of \( \omega \) (and fixed intensity \( V_{AC} = V_{AC}^0 = 0.7 \), gives the results shown in Fig. 2. The main peak of \( I_\uparrow \) (con-
FIG. 2: (Color online) Pumped current as a function of the ac frequency. The spin-down component (solid line) shows three peaks corresponding to one (ω = ω↓ = 0.3), two (ω = 0.15) and three (ω = 0.1) photon processes, respectively. The spin-up component (dashed line) shows a main one-photon resonance at ω = ω↑ = 0.7 and up to six more satellites corresponding to multiphoton processes. Parameters: $\Gamma_L = \Gamma_R = 0.001$, $t = 0.005$, $U_L = 1.0$, $U_R = 1.3$, $J = 0.2$, $\mu_L = \mu_R = 1.31$, $\Delta_s = 0.026$, $\Delta_e = 0.45$, $V_{AC} = V_{AC}^0 = 0.7$ (in meV) correspond to typical values in GaAs QDs. In particular, the Zeeman splitting corresponds to a magnetic field $B \approx 1T$. Inset: same as in main figure but with a lower intensity $V_{AC} = V_{AC}^0/5 = 0.14$.

The sequence $(\downarrow, \uparrow) \xrightarrow{\Delta} (\downarrow, \downarrow) \xrightarrow{\Gamma} (\downarrow, \uparrow) \xrightarrow{\Gamma} \ldots$ or $(\downarrow, \uparrow) \xrightarrow{\Delta} (\downarrow, \downarrow) \xrightarrow{\Gamma} (\downarrow, \uparrow) \xrightarrow{\Gamma} \ldots$. At $\omega_{\uparrow}$, $I_\uparrow \approx 0$ such that the spin polarization, defined as

$$P(\omega, V_{AC}) = \frac{I_\uparrow - I_\downarrow}{I_\uparrow + I_\downarrow},$$

has been completely reversed by tuning the frequency of the ac field, namely $P(\omega_{\uparrow}, V_{AC}^0) = 1 = -P(\omega_{\downarrow}, V_{AC}^0)$. Note that the energy difference between both processes, $\omega_{\uparrow} - \omega_{\downarrow} = \Delta_e - J/4$ corresponds to the energy difference between the triplet excited state and the singlet ground state in the right dot at zero magnetic field.

Reducing the frequency to $\omega = \omega_{\uparrow}/2$ and $\omega = \omega_{\downarrow}/3$ and so on, peaks corresponding to absorption of up to seven photons appear. Note that each of these peaks has a different width. This remarkable fact can be attributed to a renormalization of the inter-dot hopping induced by the ac potential $V_{AC}$ 

Another interesting feature of the spin pump is that there are frequencies where the one-photon process corresponding to pumping of $\downarrow$ electrons can overlap with multiphoton processes corresponding to pumping of $\uparrow$ electrons. Thus, at these frequencies the current is no longer fully spin-polarized. One can use this to modify the polarization of the current by changing $V_{AC}$ (at fixed $\omega$). We illustrate this with Fig. 3, where the parameters are chosen such that the $N=1$ peak of $I_\uparrow$ is centered at the same frequency ($\omega = \omega_{\downarrow} = 0.3$) as the $N=2$ peak of $I_\downarrow$. At this frequency, the spin polarization can be tuned by modifying the intensity of the ac potential (Fig.3, inset). This result, together with those shown in Fig. 2, demonstrate that the spin polarization of the current $P(\omega, V_{AC})$ can be fully manipulated by tuning either the frequency or the intensity of the external ac field.

Finally, it is important to note also that, contrary to the case for spin-down pumping, the pumping of spin up electrons leaves the double dot in the excited state $|\downarrow, \uparrow\rangle$. This makes the spin-up current extremely sensitive to spin relaxation processes. If the spin $\downarrow$ decays...
density matrix, $\Omega$

three energy scales involved now in the dynamics of the tical Bloch equations [17]. Note, however, that there are regime, a well known phenomenon in the context of op-

fashion. This is reminiscent of the so-called saturation (as a function of $W$)

The full widths (FWHM) of the resonances are plotted now linear with a slope which, interestingly, approaches to minimize nonlinear effects we investigate the low in-

crement can be manipulated (including fully reversing) by just tuning the parameters of the ac field. Our results

also show that the width in frequency of the spin-up pumped current gives information about spin decoher-

ence in the quantum dot. We finish by mentioning that our proposal is within reach with today’s technology for high-frequency experiments in quantum dots [10, 13, 14]. Indeed, PAT with two-electron spin states has recently been reported [20].

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FIG. 3: (Color online) Pumped current as a function of the ac frequency. Same parameters as Fig. 2 except $\Delta \epsilon = 0.35$. The interesting feature here is the overlap between the the one-photon absorption peak of $I_\uparrow$ (solid line) and the two-photon absorption peak of $I_\downarrow$ (dashed line) at $\omega = \omega_\downarrow = 0.3$. The inset shows the spin polarization $P(\omega, V_{AC})$ versus the ac intensity $V_{AC}$ for fixed frequency $\omega = \omega_\downarrow = 0.3$ demonstrating the possibility of controlling the spin polarization of the current by tuning the intensity of the ac field.

before the next electron enters into the left dot, namely if $W_{\uparrow\downarrow} \gtrsim \Gamma_L$, a spin-down current appears through the cycle $(\downarrow \uparrow, \uparrow \downarrow) \Rightarrow$ $(\uparrow \uparrow, \downarrow \uparrow) \Rightarrow$ $(\downarrow \uparrow, \uparrow \downarrow)$ and the pumping cycle is no longer 100% spin-up polarized. We study this effect in Fig. 4, where we plot the main PAT peak at $\omega_\downarrow = 0.6$ for increasing $W_{\uparrow\downarrow}$. As one expects, the peak broadens as $W_{\uparrow\downarrow}$ increases. The full widths (FWHM) of the resonances are plotted as a function of $W_{\uparrow\downarrow}$ in the inset. For large intensities ($V_{AC} = V_{AC}^0$, circles) the FWHMs grow in a nonlinear fashion. This is reminiscent of the so-called saturation regime, a well known phenomenon in the context of optical Bloch equations [17]. Note, however, that there are three energy scales involved now in the dynamics of the density matrix, $\Omega_{Rabi}$, $\Gamma$ and $W_{\uparrow\downarrow}$, such that other sources of nonlinearity (like the ones described when discussing Fig. 2) cannot be completely ruled out [18]. In order to minimize nonlinear effects we investigate the low intensity regime ($V_{AC} = V_{AC}^0/5$, squares) where we expect a FWHM dominated by decoherence. The behavior is now linear with a slope which, interestingly, approaches FWHM $\sim 2W_{\uparrow\downarrow} = 2/T_2$. Thus, experiments along these lines would complement the information about decoherence extracted from other setups [19].

Summary and experimental accessibility.-In summary, we have proposed and analyzed a new scheme of realizing both spin filtering and spin pumping by using ac-driven double quantum dots coupled to unpolarized leads. Our results demonstrate that the spin polarization of the current can be manipulated (including fully reversing) by just tuning the parameters of the ac field.
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