Non-Hermitian topological microwave photonics with synthetic non-Abelian gauges

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Topological phases in spinless non-Hermitian models have been widely studied both theoretically and experimentally in some artificial materials using photonics and photonics. In this work, we investigate the interplay between non-Hermitian loss and gain and non-Abelian gauge potential realized in a two-component superconducting circuit. In our model, the non-Hermiticity along only gives rise to trivial gain and loss to the states; while the non-Abelian gauge along gives rise to flying butterfly spectra and associated edge modes, which in photonics can be directly measured by the intensity of photons at the boundaries. These two terms do not commute, and their interplay can give rise to several intriguing non-Hermitian phases, including the fully gapped quantum spin Hall (QSH) phase, gapless QSH phase, trivial gapped phase and gapless metallic phase. The bulk-edge correspondence is absent, and we find that during the closing of energy gap in the gapped QSH phase, the system enters the gapless QSH phase regime which still supports topological edge modes. Our proposed model in this work has potential to be realized immediately, in regarding of the huge progresses achieved in quantum computation based on superconducting circuits.

The study of topological matters in physics, initiated from quantum Hall effect [1–3], is of fundamental importance in modern physics. A lot of topological insulators and superconductors have been realized using solid materials [4–6], which may lead to interesting applications such as spintronics and topological quantum computation [6–11]. Moreover, the state-of-art technologies still allow us to simulate these novel phases in the context of quantum simulation in a more controllable manner [12]. Recently, this idea has been implemented in a lot of systems, including photonics [13–17], magnons [18, 19], ultracold atoms [20–24], linear circuits [25, 26], and superconducting circuits [27]. Due to the experimental advantages in these systems, the details of these topological protected edge modes, such as the zero modes, Fermi points and even Fermi arcs, can been seen much clearer, as compared with that in solid materials [28, 29]. These achievements can greatly advance our understanding of topological matters and may seek new applications of these materials [30, 31].

Here we are mainly interested in these topological phases in the superconducting circuit, due to its long coherence time [32] and its scalability in industrial fabrication. To date, a superconducting circuit up to 92 physical qubits was reported recently to achieve quantum supremacy [33]. The topological phases with these circuits can be realized with bigger size due to the much lower requirement of uniformity. This platform is also intriguing for its controllability in lost and gain [34, 35], which enables us to simulate the non-Hermitian physics.

In previous literature, the topological non-Hermitian models are generally considered with some spinless models [40–43]. Here we are interested in the basic problem how non-Abelian potential interacts with the non-Hermiticity in a realistic system, and what will happen to the corresponding topological phases [44, 45]. To this end, we propose a method to realize a non-Abelian model with controllable lost and gain based on Lindblad formalism in a superconducting circuit. We find that: (I) Without lost and gain, this model can be used to simulate the non-Hermitian physics.

FIG. 1. The proposed experimental setup. (a) The square lattice consisting of two kinds of sites, each of which support two different modes. (b) and (c) show the coupling between different modes. In (b) the off-diagonals coupling between different modes is realized by a SQUID, as shown in (d). Detailed simulation for two coupled cavities is presented in Ref. [32].
Hofstadter effect and associated edge modes, satisfying bulk-edge correspondence; while without non-Abelian gauge potential, the lost and gain only play trivial roles to the topological phases. (II) The non-Abelian gauge potential can break the U(1) symmetry of each component and introduces coupling to the non-Hermitian interaction, thus influences the fate of topological phases. We find that with non-Hermiticity, our model can host four different non-Hermitian phases: the gapped QSH phase, trivial gapped phase, gapless QSH phase and metallic phase. (III) The bulk-edge correspondence is explicitly broken in the non-Abelian model. In our model, by closing the energy gap, the model can enter the gapless QSH regime, which also supports topological edge modes. The resonant couplings between the extended modes and localized edge modes are not allowed for their different energies in the complex plane. The detection of these phases, their band structures and associated edge modes will also be discussed.

Model and Hamiltonian We aim to realize the Hamiltonian of the following general form,

\[ \mathcal{H}_0 = - \sum_{\mathbf{r}, \mathbf{s} = x, y} t_{\mathbf{r}s} \psi^\dagger_{\mathbf{r}1} U^x_{\mathbf{r}}, \psi_{\mathbf{s}1} + \text{h.c.} \]  

This Hamiltonian can be defined in a square lattice, as shown in Fig. 1(a), where site \( \mathbf{r} = (m, n) \), and the two-component field operators are \( \psi_{\mathbf{r}} = (a_{\mathbf{r}1}, a_{\mathbf{r}2})^T \), with \( a^\dagger_{\mathbf{r}s} \) and \( a_{\mathbf{r}s} \) being the creation and annihilation operators for each component \( s \). Only the hopping between nearest neighboring is allowed, thus \( e_{\mathbf{x}} = (1, 0) \) and \( e_{\mathbf{y}} = (0, 1) \). The above model is quite general and different types of topological phases can be realized in this platform by carefully choosing the hopping matrix \( U^x_{\mathbf{r}} \) and \( U^y_{\mathbf{r}} \). In superconducting circuits, it can be realized in the following way by considering two different modes in a 3D cavity. In recent years, this 3D cavity has been utilized by several different groups [24, 33, 46–49], in which the energy difference between these two modes is of the order of GHz [32]. Direct hops between each modes in two neighboring cavities are allowed naturally, which correspond to the diagonal terms in \( U^x_{\mathbf{r}} \). To realize the off-diagonal couplings between the energy mismatched components, which yields the non-Abelian gauge potential, one needs to compensate their energy differences by periodically driving the superconducting quantum interference device (SQUID) connected between the two neighboring cavities (see Fig. 1(d) and Ref. [32]). In this way, we can realize different non-Abelian gauge potentials by controlling \( U^x \) and \( U^y \). Details for the simulation of this model can be found in Ref. [32]. Generalizing this model to multiply components is also straightforward.

Let us focus on the following non-Abelian gauge,

\[ U^x_{\mathbf{r}} = U^x = e^{i \Theta_{\mathbf{x}}} \quad U^y_{\mathbf{r}} = U^y_{\mathbf{m}} = e^{i \Theta_{\mathbf{y}}} \]

with \( \Theta_{\mathbf{x}} = \gamma \cdot 2 \pi \sigma_x \), and \( \Theta_{\mathbf{y}} = \alpha \cdot 2 \pi m \sigma_z \). This model, after Fourier transformation, will be reduced to the conventional models with Rashba spin-orbit coupling (SOC) studied in various systems [21, 50, 52], in which the coefficient \( \gamma \) can be used to engineer the SOC strength. Other types of non-Abelian potentials can also be realized, which is one of the advantages of this platform; while in previous literature, including that in solid materials and ultracold atoms, the engineering of this interaction is always challenging.

Flying Hofstadter butterfly and edge modes We first consider the Hofstadter butterfly effect [53–56] with this gauge potential. For \( \alpha = p/q \), where \( p, q \) are coprime numbers, we can diagonalize the model in a reduced magnetic unit cell, yielding the following Harper equation,

\[ -t_x \tilde{U}^{k_0}_{x} u_n - t_y (\tilde{U}^{k_y} u_{n+1} + \tilde{U}^{-k_y} u_{n-1}) = E u_n, \]

where \( \tilde{U}^{k_y} = e^{-i k_y U^x + \text{h.c.}} \), \( \tilde{U}^{k_y} = e^{-i k_y \sigma_y} \), and \( u_n = (u_{n,\uparrow}, u_{n,\downarrow})^T \), with \( u_{n,\sigma} \) being the amplitude of the wave functions in each site and \( n = 1, 2, \cdots, q - 1 \).

The boundary condition is \( u_0 = u_q \) and \( k^0_x = \left[-\frac{2\pi}{q}, \frac{2\pi}{q}\right] \) with \( k_y \in [\pi, \pi] \). For a cylindrical geometry along \( k_y \) direction, we have:

\[ -t_x [U^x u_{m+1} - (U^x)^\dagger u_{m-1}] - t_y (e^{i k_y U^y_{m}} \text{h.c.}) u_m = E u_m \]

The butterfly diagram is presented in Fig. 2, which exhibits some features that are totally different from the Abelian case. In the latter case, the flux parameterized by \( k_y \) only slightly modify the edge modes [32]; however, here it strongly influences the structure of the butterfly, exhibiting some flying effect, that is, by tuning \( k_y \) the butterfly pattern may oscillate periodically [32]. This arises from the coupling between the butterflies in each components, where the non-Abelian gauge breaks the degeneracy at the crossing points. This butterfly has been studied by Osterloh et al [51], which does not exhibit this effect by fixing \( k_y \) and tuning \( \gamma \).

The novel things caused by the non-Abelian gauge field
are the quantum spin Hall (QSH) effect and associated edge modes in a finite system. When $\gamma = 0$, the model is decoupled into two spinless copies with opposite magnetic field, which thus have opposite Chern numbers (see Fig. 3(a)). In this case, the spin is conserved, and the spin Chern number can be well defined [57, 58]. This potential can be easily implemented in the SQUID engineering by $K_{\alpha\beta} = 2\kappa\delta_{\alpha\beta}\sigma_z$. Similar technique has been used in circuit QED system by engineering the qubit-cavity interaction [30]. The loss and gain in these schemes can be controlled in the experiments. The connection of Eq. 6 to the non-Hermitian is realized by noticing that its dynamics is equivalent to that in the following non-Hermitian Hamiltonian,

$$H = H_0 + \sum_{m,n} i\kappa^\dagger\sigma_m^n\psi_m^n\psi_m^n, \quad (7)$$

where $\Psi$ is the vector consisting of all the $\psi_{\alpha}$ and $H = \Psi^\dagger M \Psi$. The matrix $K$ consists of all the dissipative parameter $\kappa_m$ engineered by $K_{\alpha\beta} = 2\kappa\delta_{\alpha\beta}\sigma_z$. Similar technique has been used in circuit QED system by engineering the qubit-cavity interaction [30]. The loss and gain in these schemes can be controlled in the experiments. The connection of Eq. 6 to the non-Hermitian is realized by noticing that its dynamics is equivalent to that in the following non-Hermitian Hamiltonian,

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FIG. 4. Phase diagram of non-Hermitian model with non-Abelian gauge potential with $\gamma = \frac{1}{4}$ and $\alpha = \frac{1}{6}$. (a) to (c) show the eigenvalues for the red point in (d) with PBC with $\kappa/t = 0.2$ and $\lambda/t = 0.8$. The phase diagram in (d) is plotted by monitoring the gap closing of the bulk and associated edge modes, which is divided into four different phases. The edge modes for the solid symbols are presented in Fig. 5.

in the complex plane, however, these two groups will not be connected to each other. For the gapped phases, these two blocks can be connected through the edge modes. In our calculation, we identify the topological phases with non-Hermiticity by adiabatically switching off the loss and gain, and it is said to be the same phase if and only if the two blocks in free space are connected to each other by the edge modes in an open system. The trivial gapped phase is characterized by absence of edge modes.

Our phase diagram is presented in Fig. 4. When $\kappa = 0$, we are able to find three different phases: a normal insulator (NI) phase, a gapped QSH phase and a normal metallic (NM) phase. These three phases can be understood from the discussions in Fig. 3, in which the phase transition from the QSH and NM satisfies bulk-edge correspondence. Moreover, the fully gapped phases can be characterized by $Z_2$ indexes, following Fukui et al [62]. In the gapped QSH, we find $\nu = +1$; while in the trivial phase, $\nu = 0$. These three limiting phases will help us to identify the phases with non-Hermiticity, since the topological characterization of these phases are still not well understood considering that our model has rank much bigger than two.

To determine the phase boundaries of these phases with non-Hermiticity, we consider the band structure with PBC, in which we can plot the corresponding band structures in momentum space in the same as that in Fig. 4 (a)-(c); and that with OBC, in which we can study their edge modes. The phase with non-Hermiticity is said to be topological only when it can be smoothly connected to the Hermitian case without closing of energy gap in periodic system. This consideration is due to the lack of bulk-edge correspondence, as unveiled in previous literature [63–65]. Starting from a fixed $\lambda$ in the QSH regime and increases $\kappa$, the system will first persist in the gapped QSH regime. In the open boundaries (see Fig. 5 (a1)), it clearly demonstrates two counter-propating gapless edge modes in the two opposite boundaries. More details about the edge modes from the band structure and spin polarization can be found in Ref. [32]. With further increasing $\kappa$, the bulk gap is closed, but will not re-open again. We find another boundary denoted by dashed line in Fig. 4 (c), beyond which the system enters the NM phase without edge modes (see Fig. 5 (a3)). Between the QSH and NM phases, we find an intriguing gapless QSH phase since it supports two edge modes at each open boundary (see Fig. 5 (a2)). The resonance coupling between the edge modes and the extended bulk modes are forbidden since the states with same real eigenvalues may have totally different imaginary parts. Finally, we have also examined the transition from the NM phase to the trivial gapped phase, which is determined merely by the bulk gap closing. In this case, the open and close boundary conditions do not give significant difference in describing this phase transition.

We emphasized that the phase boundary between gapped and gapless QSH and NM phase can not be understood from the simple picture derived in previous literature [63–65]. However, the skin effect [41, 42] is universal since the
eigenvalues for each momentum $k$ can be complex valued. We find that in both spinless QSH and trivial metallic phases, the wave functions will be localized in the left-up and right-down corners, indicating that the this mechanism. The lack of bulk-edge correspondence should be a quite general features in all non-Hermitian models.

**Conclusion** Topological non-Hermitian models have been widely explored based on spinless models. In the future, their realizations in concrete materials should be an important pursuit. Regarding the lack of general principle of topological phase transitions, the knowledge based on toy models needs to be examined more carefully [30]. Along this line, we generalized the spinless models to the realm of multi-component systems, and propose a general way for non-Hermitian models with non-Abelian gauge potentials. These two noncommutative terms and their interplay can lead to various intriguing topological gapped and gapless phases. This platform can even be used to examine a number of important concepts in Hermitian models, such as the butterfly spectra, in which non-Hermiticity can dramatically influence the fate of the Dirac cores [67]. The edge modes are robust against disorder [32], thus it can be used for topological maser with mechanism similar to Ref. [31, 60].

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