An Approximate Analytical Solution to the Problem of Heat Exchange in Porous Material

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Abstract. The article consists some researches of heat exchange in porous medium. The general formulations of the heat exchange task are presented. The physical meaning and its mathematical interpretation of the task are described. The analytical solution of differential system equation for simply boundary conditions are found. The field of simple mathematical formulation application is detected.

1. Introduction
A flow regime of filterable fluid plays a significant role in the research of heat and mass transfer in porous medium. During the laminar filtration one can concede that heat exchange happens instantly and as a result a fluid and a structure (solid skeleton of the porous medium) exchange heat until the temperature equalization in every cross-section. Under transitional and turbulent regimes distribution of the fluid and the structure temperatures is different, and modeling is performed by using a system that consists of two differential equations. The solution of this system is dependence of the fluid and the structure temperatures on a coordinate. The studies of transitional and turbulent filtration are often experimental and done with the definition of dependence between the Re and Nu numbers [1-7]. Along with this, a large number of theoretical studies are also known [10-13]. The aim of the article is an analytical study of these processes.

Figure 1. Porous slab which is blown by the fluid. Red line is a possible distribution of temperature in the structure; blue line is a fluid temperature.
Averaged flow properties are usually used in such kind of calculations [14, 15]. It is connected with the small size of pores in comparison to the size of area, studying during the modeling. In view of the fact that mechanically the fluid behavior doesn’t fundamentally differ from a gas behavior at subsonic speeds, we will consider that such notions as “fluid” and “gas” are equivalent [16]. The temperature distribution under laminar flow can be found from the differential equation [17, 18]

$$\lambda_s (1 - \gamma) \cdot \Delta T - \gamma C_f \rho_f w \cdot \nabla T = 0$$

(1)

$$\lambda_s$$ - heat conductivity coefficient of the structure, Wt/(m·K),
$$\gamma$$ - porosity,
$$C_f$$ - specific heat capacity of the fluid, J/(kg·K),
$$\rho_f$$ - fluid density, kg/m³,
$$w$$- velocity of the fluid in the cross-section (without porosity accounting), m/s.

A non-dimensional parameter which is called cooling number is entered for this equation

$$Kn = \frac{\gamma C_s \rho_s w \delta}{\lambda_s (1 - \gamma)}.$$  

The system of differential equations is used for the calculation of transitional and turbulent regimes [19]

$$\gamma \frac{\partial \rho \cdot H_f}{\partial \tau} = -\gamma \nabla \left( \nabla \cdot \rho \cdot H_f \right) + Q_{fs},$$

$$\gamma_s \frac{\partial \rho_s \cdot C_s \cdot T_s}{\partial \tau} = \gamma_s \nabla \left( \lambda_s \cdot \nabla T_s \right) + Q_{sf}.$$  

$$H_f$$–fluid enthalpy, J/kg
$$\rho_f$$ – density of the structure, kg/m³
$$C_s$$ – specific heat capacity of the structure, J/(kg·K)
$$\gamma_s$$ – proportion occupied by the structure in the general volume of porous medium,
$$\gamma = 1 - \gamma_s$$

$$Q_{fs}$$–heat flow from the fluid to the structure, Wt

$$Q_{fs} = -Q_{sf} = \alpha A_{fs} (T_s - T_f)$$

$$\alpha$$– heat transmission coefficient from the structure to the fluid, Wt/(m²·K)

$$A_{fs}$$– surface density of the heat exchange, m⁻¹

$$T_s, T_f$$–the structure and the fluid temperatures, K

$$A_{fs} = \frac{A}{V}$$

$$A$$– contact area of the fluid and the structure, m²

$$V$$–general volume of porous filling (with pores), m³.

In case when gradients of temperature are small or the fluid and the structure properties are weakly depend on the temperature, some parameters can be taken out of the operator signs. Then for the case of stabilized regime the given system can be analytically solved. We call

$$a = \frac{\gamma w \rho_f C_f}{\alpha A_{fs}^2}, \quad b = \frac{\lambda_s \gamma_s}{\alpha A_{fs}}$$

, $$w$$ – fluid velocity, averaged over the area of cross section (without porosity accounting), m/s.
Also as a matter of convenience let us assume that \( d^2 = \frac{4 \cdot a^2}{b} + 1 \) (2). Then the system of equations assumes an aspect
\[
\begin{cases}
  a \nabla T_f = T_s - T_f, \\
  \frac{4 \cdot a^2}{d^2 - 1} \Delta T_s = T_s - T_f.
\end{cases}
\]
(3)

In case when the geometry of considered area is fairly simple and boundary effects are minor (for example, under calculation of air filtration through the external walls of the buildings) we can limit ourselves to the solution of one-dimensional task. In certain cases the influence of edge can be considered as a correction factor, obtained through the numerical and experimental studies. Let us assume for simplicity that \( T_s(0) = 0, T_f(0) = 0 \),

then the form for the temperature of the structure will have an aspect
\[
T_i(x) = \frac{2Ca(d + 1) \cdot e^{\frac{(d-1)\cdot x}{2a}} - 2Ca \cdot e^{\frac{(d+1)\cdot x}{2a}} - 8Cad}{(d-1)^2(d+1)}. \]

The given function can be expanded as a Taylor series. Let us class a cubic term (the square term is absent) where the coefficient is equal to
\[
T_3 = \frac{C(d + 1)^2}{24a^2} + \frac{C(d + 1)(d - 1)}{24a^2} = \frac{Cd(d + 1)}{12a^2}.
\]

The coefficient is shrink to zero at \( d \rightarrow -1 \), substituting this value into (2) then yields
\[
\frac{4 \gamma \rho_f C_f}{\alpha A_f \lambda_s \gamma_s} \rightarrow 0
\]
and it means that the linear approximation of temperature of the structure distribution holds. For these cases, the second equation in the system of equations (3) can be changed into the linear algebraic equation
\[
T_s = X \cdot \Delta T + t_1
\]
(5)

\( X \) – dimensionless coordinate,
\( \Delta T \) – the temperature of the structure difference on the boundaries, K
\( t_1 \) – the temperature of the structure on the boundary condition from the side of fluid flow in the porous medium (fig. 1), K.

The first differential equation of the system can be solved analytically substituting (5) in (3) and getting the answer in non-dimensional parameters.
\[
\Theta_f = -m \cdot e^m \cdot \left( \frac{X}{e^m - 1} \right) + \frac{t_1}{\Delta T} + X,
\]
\[
\Theta_f = \frac{T_f}{\Delta T}, \quad m = \frac{a}{\delta}.
\]

2. Using the method of integral analogues it can be shown that the parameter \( m \) is a dimensionless criterion of similarity of fluid temperature fields [20]. From all has been written it follows that
\[
m = \frac{\gamma \rho_f C_f}{\alpha A_f \delta}.
\]
Here we can see that the numerator is numerically equal to the quantity of heat transported by fluid under the unit temperature difference. \( \gamma_{wp} \) is a specific fluid flow relative to the unit of the cross-sectional area. The denominator is numerically equal to the quantity of heat transferred through the pores surface area from the structure to the fluid. \( \alpha A_0 \) value is relative to the unit of considered area volume. In order to move from specific characteristics to actual characteristics one ought to multiply numerator by the area of cross section, and to multiply denominator by the volume of calculated area. In case when this area is in the shape of parallelepiped

\[
\frac{A}{V} = \frac{1}{\delta},
\]

\( A \) – area of cross section, \( m^2 \)

\( V \) – volume of porous nozzle, \( m^3 \)

\( \delta \) – thickness of calculated area, \( m \).

Thus \( \delta \) is a characteristic length and its introduction to the denominator is logical. The criterion is an analog of Stanton number, but \( m \) takes into account real tasks properties [21].

Side by side with this criterion another one arises

\[
\frac{n}{b} = \frac{\delta}{\delta} = \frac{1}{\frac{\lambda s \gamma_s}{\alpha A_0 \delta}} \cdot \frac{1}{\delta}.
\]

Here \( \delta \) is also the characteristic length. A physical meaning of this criterion is a ratio of moving through the structure the heat flow to transferring through the pores surface area the heat flow. If we set aside the addition of \( \delta^2 \) to the denominator, then the heat flows are considered as specific and zero dimension is not observable. The fraction \( \frac{A}{V} \) consider a possibility of distinction of the thickness of calculated area from 1 \( m \) and is a consequence that comes from Fourier law. As like in the first case, \( \delta \)

\[
\frac{A}{V}
\]

is also appear in the fraction substitution \( \frac{1}{\delta} \), because it is necessary to get not specific but actual heat flows.

3. The assertion that the simplified mathematical formulation of the heat transfer task (1) is permissible only for the laminar conditions is not reasonably sufficient for a number of reasons. First, only an approximate order of \( Re \) numbers under which local whirls are begin to arise in the flow is known for the porous medium and as a result the exact detection of laminar boundary is impossible [22-26]. Secondly, such kind of formulation points to the fact that the necessity of heat rejection ratio accounting does not depend on thermo physical parameters of the process. Although it is not difficult to imagine the combination of parameters under which it does not hold. The heat transmission coefficient is a limiting parameter in the discussed process, but if it is high enough then in practical calculations, the fluid and the structure exchange heat without hindrance. Contingently, the «infinitely high» heat transmission coefficient is fit the case when the temperatures distribution \( T_f \) and \( T_s \) coincide, that’s why knowing the exact solution of the system (3) under boundary conditions (4) it is possible to determine binding between \( m \) and \( n \), under which it is acceptable to ignore the heat-transfer operation and to use a simplified mathematical formulation (1). The coincidence of the curves \( T_f \) and \( T_s \) we can describe by the expression

\[
\frac{T_f}{T_s} = 1
\]

Substituting the solutions of the system (3) into (6) and performing rearrangements we are getting a number of conditions, among which the most optimal is the equality
\[
\frac{4a^3}{b} = 0.
\]

It is necessary to select such kind of \(a\) and \(b\) for the coincidence of \(T_f\) and \(T_s\) that the expression (6) becomes identical. That’s why the most necessary condition with the account of the physical constituent of the tasks the limit as \(a\) and \(b\) nears zero. It follows that \(b\) should be infinitely small and more higher-order then \(a^3\). This infinitesimal order can be taken as a threshold order and as a result the second necessary condition can be formulate

\[b \sim a^3\]

or using the offered similarity criterions

\[n \sim \delta^m\]

2. The conclusion

The analysis of the problem of heat exchange in porous materials is performed. The different formulations of this problem are discussed. The similarity criterions for the case of filtration in turbulent and combined conditions are established. The conditions of the linear approximation are identified and the analytical solution of the problem in this approximation is obtained. The conditions of using the simplified mathematical formulation of the problem are obtained.

References

[1] Jimenez-Islas H, Lopez-Isunza F, Ochoa-Tapia J A 1999 Natural convection in a cylindrical porous cavity with internal heat source: a numerical study with Brinkman-extended Darcy model *International Journal of Heat and Mass Transfer* vol 42 pp 4185–4195

[2] Nield D A, Bejan A 2006 Convection in porous media (New York: Springer) 640 p

[3] Oliveski R D C, Macagnan M H, Copetti J B 2005 Natural convection in a tank of oil: experimental validation of a numerical code with prescribed boundary condition *Experimental Thermal and Fluid Science* vol 29. — P. 671–680.

[4] Nakoryakov V. E., Gorin A. V. Teplomassoperenos v dvyhfaznyh sistemah. — Novosibirsk: Institute of thermal physics, 1994 431 p

[5] Sankar M, Park Y, Lopez J M, Do Y 2011 Numerical study of natural convection in a vertical porous annulus with discrete heating *International Journal of Heat and Mass Transfer* vol 54 pp 1493–1505

[6] Mezentsev I V 2010 Teploobmen v zernistykh sredakh pri reversivnyh rejimakh filtracji Young scientist vol 10 pp 17-20

[7] Ukrainsky V A, Trubaev P A, Grishko B M 2011 Experimental'noe issledovanie teploobmerna pri primuditel'noi filtracii vozduhka cherez sloi klinkernykh granul Innovation materials and technologies pp 186-170

[8] Sankar M, Park Y, Lopez J M, Do Y 2011 Numerical study of natural convection in a vertical porous annulus with discrete heating *International Journal of Heat and Mass Transfer* pp 54 pp 1493–1505

[9] Saeid N H 2007 Conjugate natural convection in a porous enclosure: effect of conduction in one of the vertical walls *International Journal of Thermal Sciences* vol 46 pp 531–539

[10] Saeid N H 2007 Conjugate natural convection in a vertical porous layer sandwiched by finite thickness walls *International Communications in Heat and Mass Transfer* vol 34 pp 210 –216

[11] Mohamad A A 2003 Heat transfer enhancements in heat exchangers fitted with porous media Part I: constant wall temperature *International Journal of Thermal Sciences* 42(4) 385–395 doi:10.1016/s1290-0729(02)00039-x
[12] Diao N, Li Q, & Fang Z 2004 Heat transfer in ground heat exchangers with groundwater advection International Journal of Thermal Sciences 43(12) 1203–1211 doi:10.1016/j.ijthermalsci.2004.04.009

[13] Hayes A M, Khan J A, Shaaban A H & Spearing I G 2008 The thermal modeling of a matrix heat exchanger using a porous medium and the thermal non-equilibrium model International Journal of Thermal Sciences 47(10) pp 1306–1315 doi:10.1016/j.ijthermalsci.2007.11.005

[14] Nield D A, Bejan A 2013 Convection in porous media. N-Y: Springer 778 p doi: 10.1007/978-1-4614-5541-7

[15] Leont'ev N E 2009 Bases of the theory of a filtration (Moscow, MGU Publ.) 87 p

[16] Landau L D, Lifshiz E M 1986 Theoretical physics vol 6 Hydrodynamics (Moscow, Nauka Publ.) 736 p

[17] Lykov A V 1978 Teplomassoobmen (Moscow: Energy) 480 p

[18] Kaviany M 1991 Principles of heat transfer in porous media (Springer-Verlag) 626 p

[19] Konovalov D A 2017 Development and analysis of models of heat transfer in com-pact porous heat exchangers of aero space control systems Vestnik of Samara University Aerospace and Mechanical Engineering vol 16 2 pp 36-46 DOI: 10.18287/2541-7533-2017-16-2-36-46

[20] Venikov V A 1976 Teoria podobia i modelirovania (Moscow: High school) 481 p

[21] Kokorev L S, Subbotin V I, Fedoseev V N, Kharitonov V V, Voskoboinikov V V 1987 O vzaimosvyazi gidravlicheskogo soprotivlenia i teplootdachi v poristykh sredakh High temperature physics 1 pp 92-97

[22] Sheidegger A E 1957 The physics of flow through porous media (Toronto: University of Toronto press) 255 p

[23] Ergun S, Orning A A 1949 Ind. Eng. Chem. 41 1179 p

[24] Fancher G H, Lewis J A 1933 Ind. Eng. Chem. 25 1139 p

[25] Collins R E 1961 Flow of fluids through porous materials (NY.: Reinhold publishing corporation) 351 p

[26] Ibragimov R A, Korolev E V, Deberdeev T R, Leksin V V, Solovev D B 2019 Energy Parameters of the Binder during Activation in the Vortex Layer Apparatus Materials Science Forum 945, pp 98-103 [Online] Available: https://doi.org/10.4028/www.scientific.net/MSF.945.98