Abstract—Intelligent reflecting surface (IRS) is envisioned as a promising solution for controlling radio propagation environments in future wireless systems. In this paper, IRS is exploited to extend the millimeter wave (mmW) signal coverage to blind spots, thereby reducing power consumption while improving communication performance. To merge IRS with mmW communications enjoying gigabit data-rate, we introduce a distributed IRS aided mmW system to support multi-user transmission. Taking into account the multi-user interference, we study a joint active and passive beamforming problem for weighted sum-rate maximization. Then, an alternating iterative algorithm with closed-form expressions is proposed to tackle the non-convex problem. Moreover, we design a quantitative projection method for the IRS with discrete phase shifts. Finally, the simulation results demonstrate that the distributed IRS can effectively support multi-user mmW transmissions based on our proposed algorithm.

Index Terms—Intelligent reflecting surface, millimeter wave, distributed IRS deployment, beamforming.

I. INTRODUCTION

The proliferation of applications and mobile data traffic in fifth generation (5G) use cases is driving the paradigm shift for evolving wireless technologies, e.g., millimeter wave (mmW), and massive multiple-input multiple-output (MIMO). Out of these desired physical layer technologies, however, providing the logical control of the radio propagation remains as an unsettled challenge [1], [2]. For the envisioned mmW systems with unrivalled data rates [3], signals are highly susceptible to blocking, thus causing sparse and low-rank channel structures. To enhance the practical feasibility, such scheme as ultra-dense deployment [4] also brings high cost and interference issues.

Recently, intelligent reflecting surface (IRS), as an emerging concept, has great potential for cost-effectively improving the spectral efficiency in 5G and beyond system. Unlike the MIMO antenna, IRS is composed of reconfigurable and nearly passive reflecting elements, which has its origin in software-defined metamaterials [5]. Specifically, IRS can adaptively change the signal propagation by judiciously adjusting the phase shifter according to the dynamic wireless environment. In addition, instead of introducing additional power consumption and noise in amplify-and-forward (AF) relay assisted communications, the passive IRS elements can be easily integrated into the physical planar surface.

The beamforming design for IRS-enhanced wireless networks has been extensively studied. The reflect beamforming by the phase shifters at the IRS is referred to as passive beamforming (PBF). In contrast, the precoding operation at the base station (BS) can be termed as active beamforming (ABF). In [1], the ABF and PBF problems are jointly optimized to reduce the transmit power while satisfying the received signal-to-interference-plus-noise ratio (SINR) requirement for each user. Besides this, the insights of the IRS deployment optimization are also provided. Previous theoretical studies have assumed the ideal resolution [6] of IRS reflecting elements without considering hardware limitations. Accordingly, the authors in [5], [7]–[10] have conducted the comparative performance analysis between the continuous-phase and discrete-phase cases. An interesting trade-off between reflecting elements number and the phase resolution is observed by [7]. To investigate the suitability of IRS in terms of energy efficiency (EE), the authors of [5], [10] propose the EE maximization algorithms for the multiple-input single-output (MISO) scenario. Most existing studies assume the rich-scattering environment between BS and IRS [8], [9], but the low-rank BS-IRS channel [2] should be considered when it comes to mmW transmissions. Very recently, a joint ABF and PBF study is revisited in [11] from a mmW communication perspective. Moreover, it is shown that the average received power according to the method in [11] scales quadratically with the number of reflector elements. Although interesting, these research conclusions are closely relying on the rank-one assumption of BS-IRS channel, which may be impractical and difficult to generalize. The study domain of IRS-enhanced mmW system is still in its infancy.

In this paper, we investigate the joint ABF and PBF strategies for multi-user mmW downlink systems. Notably, we assume that there exist sparse multipath components between BS and IRS, and take the multi-user interference into account. Meanwhile, in order to enhance the mmW coverage and multi-user transmission services, we propose a distributed IRS deployment solution. The main contributions of our work are as follows:
• For mmW systems without direct links where IRS acts as an electromagnetic relay, a distributed IRS deployment scheme is proposed to avoid low-rank BS-IRS mmW channel. For one thing, this multi-IRS scenario breaks the constraint of rank-one channel, thus leading to high spatial multiplexing gain. For another, the IRS controller is capable of coordinating with multiple IRS units and BS to achieve global optimization of wireless propagation control through the channel state information (CSI) feedback.

• Based on the distributed IRS-enhanced multi-user mmW system, we formulate the joint ABF and PBF design as a weighted sum-rate maximization (WSM) problem. To tackle this non-convex problem, we propose a alternating iterative method by exploiting quadratic transform. Furthermore, we provide closed-form expressions for the proposed method.

• To extend to the discrete-phase cases, a quantized phase projection approach is devised. The performance of our scheme is evaluated compared to the baseline method, and the impact of the number of reflector elements and users on the sum-rate is analyzed.

Notations: The lower-case and upper-case boldface letters denote vectors and matrices, respectively; (·)*, (·)T, and (·)H represent the conjugate, the transpose, and the conjugate transpose; tr(·), vec(·) and diag(·) return the trace, vectorization and diagonalization; [·]i,j represents the (i,j)-th entry of a matrix; j = √−1, ℜ(·), and arg(·) denote the imaginary unit, the real part and the phase; ⊙ and ⊗ denote the Kronecker and Hadamard products, respectively.

II. SYSTEM MODEL

A. IRS-assisted Downlink MmW MIMO System

Let us start with an IRS-assisted downlink mmW system, as shown in Fig. 1. The BS is equipped with a uniform linear array (ULA) composed of N elements. The IRS consists of G IRS units to serve K single-antenna mobile users (MUs), and each IRS unit is assumed to be with Maz elements in horizon and Mel elements in vertical. Here, let M = Maz × Mel. In a typical technical scenario of IRS, the direct mmW links between BS and MUs are severely blocked by obstacles, while the MUs are located in the hotspot area served by IRS. Thus, we assume that BS is a dedicated AP that only communicates with IRS since the mmW links between BS and MUs are highly susceptible to environmental blockages and dynamics. Note that these distributed IRS units are managed by a smart controller [1], [12], [13], which exchanges CSI and coordinates the reflecting modes for all IRS units.

The received baseband signal at the k-th MU can be expressed as

\[ y_k = \sum_{g=1}^{G} h_{g,k}^H \Phi_g^H W_g p_k s_k + \sum_{j=1,j \neq k}^{K} p_j s_j + u_k, \tag{1} \]

where \( W_g \in \mathbb{C}^{M \times 1} \) is the mmW channel matrix for the channel between BS and the g-th IRS unit, and \( h_{g,k} \in \mathbb{C}^{M \times 1} \) represents the channel between the g-th IRS unit and the k-th MU; the phase shift matrix of the g-th IRS unit is denoted by \( \Phi_g = \sqrt{\eta} \text{diag}(\{\theta_{g,1}, \ldots, \theta_{g,M}\}) \) where \( \eta \) indicates the reflection coefficient \( 0 \leq \eta \leq 1 \) and \( \theta_{g,m} \in \mathbb{C} \) with \( \varphi_{g,m} \) being the reflection phase shift; \( s_k \) is the transmit signal from BS for the k-th MU with zero mean and normalized power \( \mathbb{E}\{|s_k|^2\} = 1 \), and \( u_k \) is the noise vector which follows the circularly symmetric complex Gaussian (CSCG) distribution \( \mathcal{CN}(0, \sigma_n^2) \); \( P = [p_1, \ldots, p_K] \) is the precoding matrix where \( p_k \in \mathbb{C}^{N \times 1} \) is used by BS to transmit signal \( s_k \). The total transmit power constraint at the BS can be expressed as \( \text{tr}(PP^H) \leq P_{\text{max}} \). In addition, note that the mmW system is undermined by mixed-signal processing and hardware constraints in practice, which enables the hybrid precoding [14] scheme more feasible. The specific hybrid precoding structure can be expressed as

\[ P = F_{\text{RF}}F_{\text{BB}}, \tag{2} \]

where \( F_{\text{RF}} \) indicates the radio frequency (RF) precoder, and \( F_{\text{BB}} \) indicates the baseband precoder. The subsequent hybrid precoding problem can be tackled by the existing spatially sparse precoding solutions [14]. However, this is out of the scope of current manuscript.

B. Wireless Channel Model

In this paper, according to the widely used 3D Saleh-Valenzuela channel model [15], [16] for mmW communicat-
tions, $\mathbf{W}_g$ can be mathematically expressed as
\[ \mathbf{W}_g = \sum_{\ell=0}^{N_{\nu}} \nu^{(\ell)} \mathbf{a}_B \left( \phi_B^{(\ell)} \right) \mathbf{a}_I^H \left( \phi_I^{(\ell)}, \theta_I^{(\ell)} \right), \]
(3)
where $N_{\nu}$ denotes the number of non-line-of-sight (NLoS) paths and $\nu = 0$ represents the line-of-sight (LoS) path, and $\nu^{(\ell)}$ expresses the complex gain of the $\ell$-th path. Here, the elevation and azimuth angles are denoted by $\theta_I^{(\ell)}$ and $\phi_I^{(\ell)}$. In (3),
\[ \mathbf{a}_B (\phi) = \frac{1}{\sqrt{N}} \left[ e^{-j2\pi d \phi} \right]_{i \in \mathcal{I}(N)} \]
(4)
and
\[ \mathbf{a}_I (\phi, \theta) = \mathbf{a}_I^{\text{re}} (\phi) \otimes \mathbf{a}_I^{\text{im}} (\theta) \]
(5)
are the array steering vectors of ULA and IRS, respectively. The array steering vectors $\mathbf{a}_I^{\text{re}} (\phi)$ and $\mathbf{a}_I^{\text{im}} (\theta)$ are defined in the same manner as $\mathbf{a}_B (\phi)$, where $\lambda$ is the mmW wavelength, $d$ is the antenna spacing, and $\mathcal{I}(N) = \{ n - (N-1)/2, n = 0, 1, \ldots, N - 1 \}$.

For the mmW channel between IRS and MUs, the IRS is densely distributed in the hotspot spaces, which gives rise to a high probability of LoS propagation. Due to the severe path loss, the transmit power of 2 or more reflections can be ignored so that only LoS is considered. Thus, the channel representation between the $g$-th IRS unit and the $k$-th MU can be obtained as
\[ \mathbf{h}_{g,k} = \sqrt{\nu_k} g_k \mathbf{a}_k (\phi_k), \]
(6)
in which $\nu_k$ indicate the channel gain; $g_k$ and $\mathbf{a}_k$ are the receive and transmit antenna element gains, respectively; $\mathbf{a}_k$ is the normalized array steering vector of IRS.

In the following sections, we assume that the CSI knowledge of all channels involved is perfectly estimated by BS and the controller attached to IRS.

III. JOINT ACTIVE AND PASSIVE BEAMFORMING DESIGN

A. Problem Formulation

Let us concentrate on the joint design of the active and passive beamforming to achieve the downlink system sum-rate maximization. To be specific, the precoding matrix $\mathbf{P}$ and phase shift matrix $\Phi_g$ should be designed using the SINR metric. The individual rate of the $k$-th MU is given by
\[ R_k = \log_2 (1 + \gamma_k), \]
(7)
where the SINR of the $k$-th MU is computed by
\[ \gamma_k = \frac{\left| \sum_{g=1}^{G} \mathbf{h}_{g,k}^H \Phi_g^H \mathbf{W}_g \mathbf{b}_k \right|^2}{\sum_{j=1,j \neq k}^{G} \left| \sum_{g=1}^{G} \mathbf{h}_{g,j}^H \Phi_g^H \mathbf{W}_g \mathbf{b}_j \right|^2 + \sigma_n^2}. \]
(8)
The corresponding WSM problem can be formulated as
\[ \max_{\mathbf{P}, \Phi_g} f_1 (\mathbf{P}, \Phi_g) = \sum_{k=1}^{K} \omega_k \mathcal{R}_k \]
(9a)
s.t. \[ \text{tr} (\mathbf{P} \mathbf{P}^H) \leq P_{\text{max}}, \quad \theta_{g,m} \in \mathcal{F}, \quad \forall g, \forall m, \]
(9b)
where the weight $\omega_k$ is the required service priority of the $k$-th MU, and the feasible set for $\theta_{g,m}$ is $\mathcal{F} = \mathcal{F}_c$ or $\mathcal{F}_d$. Here, $\mathcal{F}_c = \{ \theta_{g,m} = e^{i \varphi_{g,m}} | \varphi_{g,m} \in [0, 2\pi) \}$ denotes the feasible set of the continuous phase shifts, while the feasible set of the infinite-resolution phase shifts $[5]$ at each element is given by
\[ \mathcal{F}_d = \{ \theta_{g,m} = e^{i \varphi_{g,m}} | \varphi_{g,m} \in \left\{ \frac{2\pi i}{2^p} \right\}_{i=0}^{2^p-1} \}. \]
(10)
where $B$ denotes the phase resolution in number of bits.

Next, we examine the transformation steps for our proposed alternating iterative algorithm applied to the original WSM problem. By introducing auxiliary variables $\alpha = [\alpha_1, \ldots, \alpha_K]^T$ for SINR, the objective function of problem (9a) can be equivalently represented as
\[ f_2 (\mathbf{P}, \Phi_g, \alpha) = \max_{\mathbf{P}, \Phi_g, \alpha} \frac{1}{\ln 2} \sum_{k=1}^{K} \omega_k \ln (1 + \alpha_k) \]
\[ - \omega_k \alpha_k + \frac{\omega_k (1 + \alpha_k) \gamma_k}{1 + \gamma_k}. \]
(11)
Fundamentally, this problem cannot be solved globally because it is nonconvex. We observe that the optimal $\alpha_k$ for $f_2 (\mathbf{P}, \Phi_g, \alpha)$, when $\mathbf{P}$ and $\Phi_g$ hold fixed, is $\hat{\alpha}_k = \gamma_k$. This means that in each iteration step, $\alpha_k$ is initially updated by $[8]$. Then, for the fixed $\alpha_k$ attained in current iteration, only the last term of $f_2 (\mathbf{P}, \Phi_g, \alpha)$ is involved in optimizing $\mathbf{P}$ and $\Phi_g$. Therefore, (11) is further recast to
\[ \max_{\mathbf{P}, \Phi_g} f_3 (\mathbf{P}, \Phi_g) = \sum_{k=1}^{K} \frac{\omega_k (1 + \alpha_k) \gamma_k}{1 + \gamma_k} \]
(12)
s.t. (9b), (9c).
Note that this alternating iterative optimization is a tractable solution for dealing with the logarithm in (11). In addition, the convergence of the alternating iterative approach to the stationary point is established in Section IV B.

B. Active Beamforming Scheme

Now, we intend to find the solutions for precoding matrix $\mathbf{P}$ given fixed passive beamforming matrix set $\{ \Phi_1, \ldots, \Phi_G \}$. For notational brevity, let
\[ \mathbf{h}_k^H = \sum_{g=1}^{G} \mathbf{h}_{g,k}^H \Phi_g^H \mathbf{W}_g. \]
(13)
Substituting (13) into (8), (12) can be rewritten as
\[
\max_P f_4(P) = \sum_{k=1}^{K} \frac{\alpha_k |\hat{h}_k^H p_k|^2}{\sum_{j=1}^{K} |\hat{h}_j^H p_j|^2 + \sigma_u^2} \tag{14}
\]
s.t. (9b),
where \(\alpha_k = \omega_k(1 + \alpha_k)\). Note that (14) is a multi-ratio fractional programming problem. Based on the quadratic transform (8), (17), we reformulate (14) as
\[
f_5(P, \beta) = \sum_{k=1}^{K} 2\sqrt{\alpha_k} \Re\{\beta_k^* \hat{h}_k^H p_k\}
- |\beta_k|^2 \left( \sum_{j=1}^{K} |\hat{h}_j^H p_j|^2 + \sigma_u^2 \right), \tag{15}
\]
where \(\beta = [\beta_1, \cdots, \beta_K]^T \in \mathbb{C}^{K \times 1}\) is a collection of the auxiliary variables introduced during the quadratic transform. At this point, the function (15) becomes a biconvex optimization problem. Next, we update \(P\) and \(\beta\) by fixing one of them. Performing the square on each term in the summation of (15) and setting \(\partial f_5/\partial \beta_k\) to zero, the optimal \(\beta_k\) for a given \(P\) is given by
\[
\hat{\beta}_k = \frac{\sqrt{\alpha_k} \hat{h}_k^H p_k}{\sum_{j=1}^{K} |\hat{h}_j^H p_j|^2 + \sigma_u^2}. \tag{16}
\]
Then, by introducing the Lagrange multiplier \(\mu \geq 0\) for power constraint (9b), the optimal \(p_k\) for the Lagrangian standard form of (15) is updated as
\[
p_k = \sqrt{\alpha_k} \hat{\beta}_k \left( \mu I_N + \sum_{i=1}^{K} |\beta_i|^2 \hat{h}_i^H \hat{h}_i^H \right)^{-1} \hat{h}_k. \tag{17}
\]

**Lemma 1:** The optimal \(\mu\) corresponding to the optimal \(p_k\) is given by
\[
\hat{\mu} = \left\{ \mu \geq 0 : \text{tr} (PP^H) = P_{\text{max}} \right\}. \tag{18}
\]

**Proof:** See Appendix A

### IV. Passive Beamforming Scheme

#### A. Lagrange Dual Problem Transformation

Initially, we carry out some manipulations to (13) as
\[
\hat{h}_k^H p_j = \sqrt{\tau} \sum_{g=1}^{G} \theta_g^* \text{diag} (h_{tg,k}^H) W_g p_j, \tag{19}
\]
where \(\theta_g = [e^{j\varphi_{g,1}}, \cdots, e^{j\varphi_{g,M}}]^T\). With these notations, we define
\[
v_{g,k,j} = \sqrt{\tau} \text{diag} (h_{tg,k}^H) W_g p_j. \tag{20}
\]

**Algorithm 1** The proposed alternating iterative framework.

**Initialization:** Set feasible values of \(\{P^{(0)}, \Phi_g^{(0)}\}\) and iteration index \(t = 0\).
1: **repeat**
2: Set \(t \leftarrow t + 1\);
3: Update \(\alpha_k^{(t)} = \gamma_k\) by (8);
4: Update \(\beta_k^{(t)}\) and \(p_k^{(t)}\) by (16) and (17), respectively;
5: Update \(\gamma^{(t)}\) by (22) and (23) for the multi-IRS case;
6: Update \(\rho_k^{(t)}\) by (26) to compute \(A^t\) and \(b^t\);
7: Update \(\zeta^{(t)}\) by solving problem (37) to obtain \(\theta^{(t)}\) by (14);
8: Perform the constant-modulus adjustment by (38);
9: With given \(\theta^{(t)}\), update \(\Phi_g^{(t)}\);
10: **until** The function (11) converges.
11: Perform the quantized phase projection by using (39).

For given \(\alpha\) and \(P\), (14) can be rewritten as
\[
\max_{\theta_g} f_6(\theta_g) = \sum_{k=1}^{K} \frac{\alpha_k \sum_{g=1}^{G} \theta_{g}^* \text{diag} (h_{tg,k}^H) W_g v_{g,k,j}}{\sum_{g=1}^{G} \theta_{g}^* \text{diag} (h_{tg,k}^H) W_g v_{g,k,j}^2 + \sigma_u^2} \tag{21}
\]
s.t. (9c).

To facilitate the subsequent manipulations, we first construct
\[
\Theta = [\theta_1, \theta_2, \cdots, \theta_G], \tag{22}
\]
and
\[
V_{k,j} = [v_{1,k,j}, v_{2,k,j}, \cdots, v_{G,k,j}]. \tag{23}
\]
Hence, (21) can be reformulated as
\[
\max_{\theta_g} f_7(\theta_g) = \sum_{k=1}^{K} \frac{\alpha_k \text{tr} (\Theta^H V_{k,k})}{\sum_{j=1}^{K} \text{tr} (\Theta^H V_{j,k})^2 + \sigma_u^2} \tag{24}
\]
\[
= \frac{\alpha_k \text{tr} (\Theta^H V_{k,k})}{\sum_{j=1}^{K} \text{tr} (\Theta^H V_{j,k})^2 + \sigma_u^2} \tag{24}
\]
in which \(\Theta = \text{vec}(\Theta)\) and \(v_{g,k,j} = \text{vec}(V_{g,k,j})\). The corresponding quadratic transform for (24) is
\[
\max_{\rho, \theta} f_8(\rho, \theta) = \sum_{k=1}^{K} 2\sqrt{\alpha_k} \Re \left\{ \rho_k \text{diag} (h_{tg,k}^H) W_g v_{g,k,j} \right\}
- |\rho_k|^2 \left( \sum_{j=1}^{K} |\theta_{g}^* v_{g,j,k}|^2 + \sigma_u^2 \right), \tag{25}
\]
where \(\rho\) refers to the auxiliary variable vector \([\rho_1, \cdots, \rho_K]^T\).

For a given \(\rho\), the optimization problem for \(\theta\) becomes
\[
\max_{\theta} f_9(\theta) = -\Theta^H A \Theta + 2\Re (\Theta^H b) + C, \tag{27}
\]
where
\[
A = \sum_{k=1}^{K} \left( |\rho_k|^2 \sum_{j=1}^{K} \mathbf{v}_{k,j} \mathbf{v}_{k,j}^H \right),
\]
(28)
\[
b = \sum_{k=1}^{K} \sqrt{\alpha_k} \rho_k \mathbf{v}_{k,k},
\]
(29)
\[
C = -\sum_{k=1}^{K} |\rho_k|^2 \sigma_k^2.
\]
(30)

It is clear that \(A\) is a positive-definite matrix since \(\mathbf{v}_{k,j} \mathbf{v}_{k,j}^H\) is positive-definite for all \(j\) and \(k\). Hence, the objective function (27) is recast into a quadratical constraint quadratic programming (QCQP) problem. However, it is worth noting that the constraint (28) is non-convex. As discussed in the sequel, it is desirable to extend the convex case solution to non-convex case. To be generalized later, let us focus on the simplest case with convex constraint as follows:
\[
\begin{align*}
\max_{\mathbf{z}} & \quad f_{10}(\mathbf{z}) = -\mathbf{z}^H \mathbf{A} \mathbf{z} + \mathbf{2H} \{\mathbf{b}^H \mathbf{z}\}, \\
\text{s.t.} & \quad |z_k|^2 \leq 1, \quad \forall k = 1, 2, \ldots, M_{\text{tot}},
\end{align*}
\]
(31a)
\[
\begin{align*}
\max_{\mathbf{z},e} & \quad \epsilon - \text{tr}(\mathbf{D}(\mathbf{z})\mathbf{b}) + \text{tr}(\text{diag}(\mathbf{z})) , \\
\text{s.t.} & \quad \left[ \mathbf{A} + \text{diag}(\mathbf{z}) \right] \mathbf{b} + \text{tr}(\text{diag}(\mathbf{z})) \geq 0,
\end{align*}
\]
(37a)
where \(M_{\text{tot}} = MG\). According to the Slater’s condition [8], it can be easy to check that the strong duality holds. Hence, the Lagrangian function of (31) can be written as
\[
f_c(\theta, \zeta) = f_{10}(\theta) - \sum_{k=1}^{M_{\text{tot}}} \zeta_k \left( \theta^H e_k e_k^H \theta - 1 \right),
\]
(32)
where \(e_k \in \mathbb{R}^{M_{\text{tot}} \times 1}\) symbolizes the elementary vector with one in the \(k\)-th position and zeros elsewhere, and \(\zeta = [\zeta_1, \zeta_2, \cdots, \zeta_{M_{\text{tot}}}]^T\) denotes the dual variables introduced for constraints (31). In addition, \(f_c(\theta, \zeta)\) is a concave function of \(\theta\). Then, using the dual problem form transformation, (31) can be rewritten as
\[
\begin{align*}
\min_{\zeta} & \quad f_D(\zeta) = \sup_{\theta} f_c(\theta, \zeta), \\
\text{s.t.} & \quad \zeta_k \geq 0, \quad \forall k = 1, 2, \cdots, M_{\text{tot}},
\end{align*}
\]
(33a)
\[
\begin{align*}
\max_{\zeta, \epsilon} & \quad \epsilon - \text{tr}(\mathbf{D}(\mathbf{z})\mathbf{b}) + \text{tr}(\text{diag}(\mathbf{z})) , \\
\text{s.t.} & \quad \left[ \mathbf{A} + \text{diag}(\mathbf{z}) \right] \mathbf{b} + \text{tr}(\text{diag}(\mathbf{z})) \geq 0,
\end{align*}
\]
(37b)
which can be solved by convex tools such as CVX.

\subsection*{B. Quantized Phase Projection}

To remedy the SDP errors, the \(\theta\) obtained from each iteration through constant-modulus adjustment can be updated as
\[
\hat{\theta} = \exp[j \arg(\theta)],
\]
(38)
where \(\hat{\theta}\) denotes the optimal solution for problem (9) with constraints \(F_c\) when ensuring that \(f_1(P, \Phi, \phi)\) converges after several iteration steps.

However, it is indispensable to consider the discrete phase shifter in terms of IRS hardware implementation. Here, we leverage the quantized phase projection method to obtain the optimal scheme with constraints \(F_c\). To be specific, the optimal solutions corresponding to discrete sets can be found by
\[
\hat{\theta}_k = \arg\min_{\phi \in \mathbb{F}_d}[\phi - \arg(\hat{\theta})], \quad k = 1, 2, \cdots, M_{\text{tot}}.
\]
(39)
In summary, the proposed alternating iterative framework for problem (9) is given in Algorithm 1. To further illustrate the convergence of the whole algorithm, we provide the following proposition.

\textbf{Proposition 1}: The problem in (11) converges when the alternating iterative algorithm is used.

\textbf{Proof:} See Appendix C.

\section*{V. Numerical Results}

For simplicity, the weights \(\omega_k\) are set to be equal in all the simulations. According to [11], [16], the channel gain is taken...
and randomly distributed in a circle centered at antennas is located in the origin point and MUs are uniformly set as $\nu$. It is observed that the sum-rate performance gap between two precoding with random PBF method is selected as the baseline. Clearly, the proposed algorithm in the distributed IRS system outperforms the baseline method in terms of sum-rate. As a solution and that of various quantization bits. In the current performance difference between phase-shifter of infinite resolution case coincides well with that of the infinite resolution elements under the same transmit power increasing from 20 dBm to 45 dBm, all of multi-user scenarios exhibit the same upward trend. Note that as the number of users in the network grows, the sum-rate attained from 1 bit to 8 bits. When $B = 3$, the performance gap can reach 0.17 and 0.26 for 2-user case and 4-user case, respectively. We conclude that only 4-bit phase shifter can achieve the equivalent performance gain as continuous phase-shifter.

In Fig. 5, we assess the impact of transmit power on sum-rate for our proposed alternating iterative algorithm. The settings of IRS size is the same as that considered in Fig. 4. The red, green and blue lines indicate the 2-user, 4-user, and 6-user scenarios, respectively. As depicted, with transmit power increasing from 20 dBm to 45 dBm, all of multi-user scenarios exhibit the same upward trend. Note that as the number of users in the network grows, the sum-rate attained by the infinite resolution elements under the same transmit power condition also increases. Also, the changing tendency of 2-bit resolution case coincides well with that of the infinite resolution case.

At last, the convergence trend of the proposed alternating iterative algorithm is shown in Fig. 6. It can be seen from our simulation results, the convergence of our proposed algorithm is confirmed in multiple simulated cases. When $K = 2$ and $M_{\text{tot}} = 20$, our proposed method converges after about 5 iterations. Meanwhile, we observe that as the number of users or the value of $M_{\text{tot}}$ increases, the upper bound of sum-rate increases after convergence. To conclude, the proposed algorithm owns fast convergence.

![Fig. 3. The sum-rate versus IRS size with $N_p = 2$ and $K = 2$.](image1)

![Fig. 4. The sum-rate versus $B$ with $M_{\text{tot}} = 20$ and $N_p = 2$.](image2)

![Fig. 5. The sum-rate versus $P_{\text{max}}$ with $M_{\text{tot}} = 20$ and $N_p = 2$.](image3)

![Fig. 6. Convergence behavior of the proposed algorithm.](image4)
VI. CONCLUSION

In this paper, an alternating iterative algorithm has been designed for WSM problem in the distributed IRS-aided mmW system. To support multi-user in blind spots area, distributed deployment of IRS units is introduced to avoid the low-rank of mmW channel. For this relay system, we solve the WSM problem by joint optimization of ABF and PBF. Furthermore, each of the steps in our proposed algorithm has a closed-form solution. Simulation results corroborate the feasibility and effectiveness of our proposed schemes.

APPENDIX A

PROOF OF LEMMA 1

For notational brevity, rearranging (17) leads to
\[ \mathbf{P} = (\mu \mathbf{I}_N + \mathbf{X})^{-1} \mathbf{Y} \Lambda, \]
where
\[ \mathbf{X} = \sum_{i=1}^{K} |\beta_i|^2 \mathbf{h}_i \mathbf{h}_i^H, \]
\[ \mathbf{Y} = \begin{bmatrix} \mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_K \end{bmatrix}, \]
\[ \Lambda = \text{diag} \left( \begin{bmatrix} \sqrt{\alpha_1} \beta_1, \cdots, \sqrt{\alpha_K} \beta_K \end{bmatrix} \right). \]

From the derivatives of scalar-valued and matrix-valued functions [13], we can obtain a convenient formula for the derivation with respect to \( \mu \):
\[
\frac{\partial \text{tr} (\mathbf{P}^{(t+1)})}{\partial \mu} = \text{tr} \left( \left( \frac{\partial \text{tr} (\mathbf{P}^{(t)})}{\partial \mathbf{P}} \right)^H \frac{\partial \mathbf{P}}{\partial \mu} \right) = -2 \cdot \text{tr} \left( \mathbf{P}^{(t)} (\mu \mathbf{I}_N + \mathbf{X})^{-1} (\mu \mathbf{I}_N + \mathbf{X})^{-1} \mathbf{Y} \Lambda \right),
\]
\[
= -2 \cdot \text{tr} \left( \mathbf{P}^{(t)} (\mu \mathbf{I}_N + \mathbf{X})^{-1} \mathbf{P} \right). \tag{44}
\]

Since \( (\mu \mathbf{I}_N + \mathbf{X})^{-1} \) is positive-definite, we thus have the following monotonically decreasing proof, as shown by
\[
\frac{\partial \text{tr} (\mathbf{P}^{(t+1)})}{\partial \mu} < 0. \tag{45}
\]
Taking this point into account, the optimal \( \mu \) corresponding to the power constraint criticality is exactly the minimum value of \( \mu \) under the power constraint.

APPENDIX B

PROOF OF LEMMA 2

According to the chain rule in the matrix derivatives [13], we have
\[
\frac{\partial f_{\mathbf{P}}}{\partial \kappa} = 1 - \text{tr} \left[ \mathbf{D} (\zeta) \mathbf{b} \mathbf{b}^H \mathbf{D} (\zeta) \frac{\partial (\mathbf{A} + \text{diag} (\zeta))}{\partial \kappa} \right] = 1 - \left[ \mathbf{D} (\zeta) \mathbf{b} \mathbf{b}^H \mathbf{D} (\zeta) \right]_{\kappa, \kappa}. \tag{46}
\]
Note that setting (46) to zero can be equivalently formulated as
\[
\left[ \mathbf{D} (\zeta) \mathbf{b} \right] \odot \left[ \mathbf{D} (\zeta) \mathbf{b} \right]^T = \mathbf{0} \odot \mathbf{0}^H = 1. \tag{47}
\]
This completes our proof.

APPENDIX C

PROOF OF PROPOSITION 1

Here, we introduce a variable \( t \) as the iteration index in Algorithm 1. Since the optimum solution can be attained at each iteration, we have
\[
f_1 (\mathbf{P}^{(t+1)}, \Phi_g^{(t+1)}) = f_2 (\mathbf{P}^{(t+1)}, \Phi_g^{(t+1)}, \alpha^{(t+1)}) \geq f_2 (\mathbf{P}^{(t)}, \Phi_g^{(t+1)}, \alpha^{(t)}) \geq f_2 (\mathbf{P}^{(t)}, \Phi_g^{(t)}, \alpha^{(t)}) \geq f_1 (\mathbf{P}^{(t)}, \Phi_g^{(t)}). \tag{48}
\]
Note that the WSM problem is bounded above due to the power constraints [17]. Hence, the original objective function is monotonically nondecreasing after each iteration.

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