Inflationary cosmology
of the extreme cosmic string

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Starting with a study of the cosmological solution to the Einstein equations for the internal
spacetime of an extreme supermassive cosmic string kink, and by evaluating the probability measure
for the formation of such a kink in semiclassical approximation using a minisuperspace with the
appropriate symmetry, we have found a set of arguments in favour of the claim that the kinked
extreme string can actually be regarded as an unbounded chain of pairs of Planck-sized universes.
Once one such universe pairs is created along a primordial phase transition at the Planck scale, it
undergoes an endless process of continuous self-regeneration driven by chaotic inflation in each of
the universes forming the pair.

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I. INTRODUCTION

It has been currently believed that extreme supermassive strings with linear energy density as large as \( \mu = \frac{1}{2} \)
could not exist because they would drive all the exterior broken-symmetry phase to collapse into the core, leaving
a pure false-vacuum phase in which the picture of a cosmic string with a core region of trapped energy is lost [1].
However, it has been recently argued [2] that the gravity coupling of supermassive strings is so large that the field-theory defect would also satisfy the conservation laws of a gravitational topological defect, that is it should become a cosmic string kink. Then, the picture of a cosmic string is somehow retained even for the extreme case \( G\mu = \frac{1}{2} \), as the conical singularity is transformed into the apparent singularity of a cosmological event horizon which turns out to be surrounded by a shell of the broken-symmetry phase with width
\( r_* = (\sqrt{2} - 1)(8\pi G \epsilon)^{-\frac{1}{2}} \) (\( \epsilon \) being the uniform string density) that prevents the string from disappearing.

What is most interesting about the resulting extreme string kink is that it meets all the necessary properties to
drive an inflationary process in its core region [2]. On each of its cross-sectional surfaces, the extreme string kink
exactly possesses the symmetry of a hemispherical section of the de Sitter kink [3], with the radius of the string
exceeding the size of the corresponding cosmological horizon. A de Sitter-like inflationary process could then be spontaneous in the string core, without any fine tuning of the initial conditions. It is the main aim of the present work to explore this rather intriguing possibility by considering both, the semiclassical approximation to the quantum cosmological model that results from the minisuperspace that describes a string whose cross sections satisfy the symmetry of a de Sitter space, and the physical constraints that must be imposed to the parameters that define the extreme string.

Our main conclusion is that an extreme string kink drives the creation of pairs of de Sitter universes. In each of
these pairs, the universes are connected to each other by a tunnel at their largest surfaces, and inflate according
to the Linde’s chaotic inflationary scenario [4]. This model may quite naturally be accommodated to a process of
continually self-reproducing inflating-universe pairs [4,5], and is made possible by the fact that a stringy topological
defect frozen in the phase transition of a field theory with large gravity coupling, \( G\mu \sim 1 \), is subject to the back
reaction of the gravitational kink. This action would shrink the cross-sectional area of the string, down to the Planck
scale, \( r_* \sim M_p^{-1} \), with \( M_p \) the Planck mass, and hence implies an initial value for the field potential
\( V_0 \sim M_p^4 \), i.e. just the initial conditions for chaotic inflation [4].

The paper is organized as follows. Sec. II briefly reviews the geometry of the extreme string kink and discusses some
of its topological properties. In Sec. III we extend the discussion on the geometry of the kinked string and show that
it can be visualized by the embedding of a six-hyperboloid in space \( E^6 \). The instantons that can be associated to the
Euclidean metrics of the extreme string kink are dealt with in Sec. IV. It is seen that the instantons can be created by
two different types of continuation: the usual Wick rotation and the rotation of the spacelike quantities characterizing
the kinked string geometry. In Sec. V we consider a minisuperspace model that represents the extreme cosmic string.
We use the Hartle-Hawking no boundary [6] and Vilenkin “tunneling” [7] proposals to obtain the classical solution,
and this is in turn employed to get a semiclassical probability measure that predicts inflation only for the case of the
tunneling wave function. Sec. VI deals in some detail with the nature and properties of the inflationary process that
is driven in the string core. It turns out that Linde's chaotic inflation is the model that fits best with the estimated parameters of the kinked string. We close in Sec. VII, with a brief summary of the results, and some comments on the considered model in relation with other scenarios for string-driven inflation, so as on the notion of universe pairs.

II. THE EXTREME STRING KINK

The motivation to consider an extreme cosmic string kink in the cosmological context is the physical expectation that, when realized in a given spacetime, the vacuum manifold, $M$, of the underlying field model with large gravity coupling should map, through the Einstein equations, into the associated gravitational manifold, $M_g$. Since the vacuum manifold for a cosmic string is not simply connected and has, therefore, nontrivial loops characterized by a group which is isomorphic to the group of integers (winding numbers) $\mathbb{Z}$, each of these noncontractible loops would then map into a noncontractible loop in each of the resulting, mutually disconnected components, which is isomorphic to the group of integers (winding numbers) $\mathbb{Z}$, each of these noncontractible loops would then map into a noncontractible loop in each of the resulting, mutually disconnected components, $M_g \equiv M^\infty$, characterized with an integer topological charge, $\kappa = 0, \pm 1, \pm 2, \ldots$, of a gravitational kink $[9]$; i.e.: one would expect that a string (a topological defect in the vacuum manifold of the field model signaled by the third homotopy group of the projective sphere, $\pi_3(M^3)$). As a result from this mapping, the gravitational kink would back-react onto the geometry of the cosmic string, which becomes consequently distorted.

This mapping can occur only for $G\mu = \frac{1}{2}$ in which case all of the existing spacetime is in the string interior; otherwise, the gravitational submanifold corresponding to the exterior broken-symmetry phase has to not possess any nontrivial loops and, therefore, $M_g$ cannot in general be divided into disconnected pieces $M^3$ each with a nontrivial loop, so preventing the gravitational kink to exist (i.e. the existing exterior region with a conical singularity of cosmic strings with $G\mu \ll 1$ does not allow the mapping-induced creation of a compact region supporting the kink.)

In what follows, let us briefly first review the topological properties of the kinked extreme string, and then comment on some aspects of its geometry. The general concept of a gravitational kink can be introduced by starting with the Lorentz metric $g_{ab}$ of a four-dimensional spacetime as given by a map, $P$, from any connected three-manifold, $\partial M$, of the spacetime four-manifold, $M$, into the set of timelike directions in $M$ $[10]$. Metric homotopy can then be classified by the degree of this map, and the kink number (or topological charge) of the Lorentz metric, with respect to a hypersurface $\Sigma$, can be defined by $[10]$

$$Kink(\Sigma; g_{ab}) = \text{deg}(P),$$

so that the gravitational kink can be viewed as a measure of how many times the light cones rotate around as one moves along hypersurface $\Sigma$.

In the case of the spacetime of an extreme cosmic string, whose interior geometry can be visualized as that of a sphere when the corresponding two-metric is embedded in an Euclidean three-sphere $[11]$, the pair $(\Sigma; g)$ will describe a gravitational kink with topological charge $\kappa = +1$ if $\text{Kink}(\Sigma; g) = 1$. From the above discussion, one may also visualize the internal geometry of the extreme string by enforcing the constant-time sections, $\tau = \tau_0$, of the interior metric of the string $[2,11]$

$$ds^2 = \frac{dr^2}{1 - \frac{r^2}{r_*^2}} + dz^2 + r^2 d\phi^2,$$

where

$$r = r_* \sin \frac{\rho}{r_*}$$

and $-\infty < z < \infty, 0 \leq \phi \leq 2\pi, 0 \leq \rho \leq r_*, \arccos(1 - 4G\mu),$ to be isometrically embedded in the kinked spacetime. The corresponding cylindrically-symmetric standard, kinked metric is given by $[2,12]$

$$ds^2 = -\cos 2\alpha dt^2 \mp 2k dt dr + dz^2 + r^2 d\phi^2,$$

where the upper/lower sign of the second term corresponds to a positive/negative topological charge, $k = \pm 1$, depending on which of the two coordinate patches required for a complete description of the kink is being considered $[2]$, and $\alpha$ is the tilt angle of the light cones in the kink, $0 \leq \alpha \leq \pi$. The isometric embedding will hold if in metric (2.3) we have furthermore.
\[ \cos 2\alpha = 1 - \frac{r^2}{r_*^2} \]  

(2.4)

and

\[ \dot{t} = \tau_0 - k \int \frac{dr}{\cos 2\alpha}. \]  

(2.5)

Actually, a gravitational kink depends only on D-1 of the D spacetime coordinates, and is spherically symmetric on them [9]. However, the cylindric coordinate \( z \) in metric (2.1) and (2.3) is not going to play any role in the analysis to follow and, therefore, one could reduce these metrics just to their hemispherical \( z = \text{const.} \) sections. On the other hand, one can also embed the \( z = \text{const.} \) sections of metric (2.3) in an Euclidean space and, hence re-express that metric in an explicit spherically-symmetric form:

\[ ds^2 = -\cos 2\alpha d\tau^2 - k d\sigma + r_*^2 d\Omega_2^2, \]

where we have specialized to the case of a gravitational kink with positive topological charge, \( k = +1, d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2 \) is the metric on the unit two-sphere, and we have used Eq. (2.5).

The metric (2.3) can be then regarded as the metric for the embedding of metric (2.1), and the kinked time \( \dot{t} \), as the corresponding embedding function. Hence, one can obtain an embedding "rate"

\[ \frac{d^2 r}{dt^2} = \frac{2r}{r_*} \left( \frac{r_*^2}{r^2} - 1 \right), \]  

(2.6)

which tells us that the embedding surface would flare either outward if \( r < r_* \), or inward if \( r > r_* \). The string metric (2.1) should now be interpreted as a kinked boundary in the space with kinked spacetime (2.3).

If the isometric embedding of metric (2.1) in metric (2.3) holds, from (2.2) and (2.4) we have \( \cos^2 \theta = \cos 2\alpha \), with \( \theta = \frac{\pi}{2} \), and if the one-kink is conserved, then \( G\mu \) is enforced to be \( \frac{1}{2} \) and \( r \) should be analytically continued beyond \( r_* \), up to \( \sqrt{2}r_* \) [2]. This extension creates a spherical shell filled with broken phase at each \( z = \text{const.} \) section, preventing the extreme string with \( G\mu = \frac{1}{2} \) from disappearing, and converts the conical singularity at \( r = r_* \) into a de Sitter-like cosmological singularity (horizon) [2]. All of the topological charge of the kink would then be confined within the shell, that is within a finite compact region beyond the cosmological horizon that extends up to \( r = \sqrt{2}r_* \). Inside the horizon all hypersurfaces \( \Sigma \) are everywhere spacelike. Thus, as a consequence from the back reaction of the gravitational field of the one-kink, the lost picture of a cosmic string with a core region of trapped energy would be somehow recovered for the extreme string with \( G\mu = \frac{1}{2} \). In Sec. VI it will be argued that this back reaction would also shift the symmetry-breaking scale, \( \eta \), to a value much larger than that is implied by \( \eta^2 \sim \mu \), and such that \( r_* \sim M_\text{P}^{-1} \).

We have established a consistent and regular embedding of the extreme string metric in a kinked spacetime whose surfaces would, according to expression (2.6), flare outward at \( \sqrt{2}r_* \), with a maximum "rate"

\[ \left. \frac{d^2 r}{dt^2} \right|_{r = \sqrt{2}r_*} = \frac{2 \sqrt{2}}{r_*}. \]

To stationary observers at the center of the sphere corresponding to each surface \( z = \text{const.}, \tau = \text{const.} \), the compact shell containing all the topological charge of the kink [13] locally coincides with a finite region of the exterior of either a de Sitter space when the light cones rotate away from the observers (positive topological charge), or the time-reverse to de Sitter space if the observers see light cones rotating in the opposite direction (negative topological charge). In the latter case, only the region outside the cosmological horizon would be accessible to stationary observers.

Topological changes can occur in the compact region [13] of the shell supporting the kink, but not inside the cosmological horizon. Since all topologies are allowed to occur in such a compact region, it would be regarded as an essentially quantum-mechanical bounded region. Actually, in order to preserve an integer topological charge for the kink, the description of the spacetime of a kinked string requires two coordinate patches, with a string spacetime being fully described in each of these patches. Besides, continuity of rotation of the light cones implies that the external boundaries of the compact regions supporting the kink in both patches be identified to each other and, therefore, a topological change could be induced in the compact external regions of the hemispherical sections of the two de Sitter spaces - one in each patch - which would thereby be "bridged" to each other along a connection at
\( r = \sqrt{2} r_* = (16\pi G\epsilon)^{-1} \). This is what allows the emergence of tunneling processes between the cosmological horizons of the two de Sitter spaces (see Sec. VII.)

Since the physical theories involved in their definition are all time-symmetric, de Sitter and time-reversed de Sitter spaces must be physically indistinguishable quantum-mechanically [14]. Therefore, the sign \( \mp \) of the second term of the nonstatic metric (2.3) should be regarded as an unphysical artifact coming from a bad choice of coordinates, such as it is shown by the fact that metric (2.3) is still geodesically incomplete at the apparent singularity at \( r = r_* \). In fact, it will be seen in the next section that the maximally-extended line element obtained from (2.3) using the Kruskal technique no longer contains any sign ambiguity other than that is related with the choice of coordinate patch [2]. Throughout this paper we shall then restrict ourselves to consider only the extreme cosmic string kink with positive topological charge.

### III. THE SPACETIME OF THE EXTREME COSMIC STRING

It was obtained in Ref. [2] that the maximally-extended metric describing the spacetime of an extreme string kink can be written as

\[
\begin{align*}
\text{(3.1)}
\end{align*}
\]

where again \( k(= \pm 1) \) labels the two coordinate patches required to describe a complete one-kink,

\( r_* = (8\pi G\epsilon)^{-\frac{1}{2}}, \)  

with \( \epsilon \) the uniform string density, out to some cylindrical radius; \( U \) and \( V \) are the Kruskal coordinates [2]

\[
\begin{align*}
U = \mp e^{-\frac{kr_*}{2}} \left( \frac{r_* - r}{r + r_*} \right), & \quad V = \pm e^{\frac{rk}{2r_*}} ,
\end{align*}
\]

in terms of which the radial coordinate can be defined as

\[
\begin{align*}
\text{(3.4)}
\end{align*}
\]

with the time \( \hat{t} \) given by

\[
\begin{align*}
\text{(3.5)}
\end{align*}
\]

where \( t \) is the metrical kinked time which is related to the time entering the metric of the kinkless cosmic string [2].

As it was already pointed out in Sec. II, the interesting feature of metric (3.1) is that its \( z \)-const. sections coincide exactly with the metric which describes a hemispherical section of the de Sitter spacetime kink [3] for a positive cosmological constant \( \Lambda = \frac{3}{r_*^2} \). Hence, by suitably redefining the time entering the metric, we can re-express it as the geodesically-incomplete metric of a static de Sitter space with cylindrical symmetry, or as the corresponding Robertson-Walker metric, which are both induced in the embedding of a six-hyperboloid

\[
\begin{align*}
\text{(3.6)}
\end{align*}
\]

in \( E^6 \). In this embedding we choose for the metric of the hyperboloid

\[
\begin{align*}
\text{(3.7)}
\end{align*}
\]
with the topology $R_1 \times R_5 \times S^4$.

In coordinates $T \in (-\infty, \infty), r \in (0, H^{-1}), \Psi_3 \in (-\infty, \infty), \Psi_2 \in (0, \pi)$ and $\Psi_1 \in (0, 2\pi)$, defined by

$$x_5 = H^{-1} \sinh \Psi_3, \quad x_4 = H^{-1} \cosh \Psi_3$$

$$x_3 = \sqrt{k}H^{-1} \sin \Psi_2 \cos(\sqrt{k}\Psi_1)$$

$$x_2 = \sqrt{k}H^{-1} \sin \Psi_2 \sin(\sqrt{k}\Psi_1) \quad (3.8)$$

$$x_1 = \sqrt{k}H^{-1} \cos \Psi_2 \cosh(HT)$$

$$x_0 = \sqrt{k}H^{-1} \cos \Psi_2 \sinh(HT),$$

we can write the static, geodesically-incomplete metric that corresponds to the kinked metric (3.1) in the form:

$$ds^2 = -k \left(1 - H^2 r^2\right) dt^2 + \frac{k dr^2}{(1 - H^2 r^2)} + dz^2 + r^2 d\phi^2,$$  

where we have set $r = H^{-1} \sin \Psi_2, \Psi_1 = \phi$ and $\Psi_3 = H z$. Of course, using Kruskal coordinates similar, but not equal to (3.3), one can recover a maximally-extended metric with the same form as (3.1) from (3.9).

If we now analytically continue the time $T$ so that $T = i\tau$, then we obtain an Euclidean metric with signature $++$ in the first coordinate patch $k = +1$, and a Kleinian metric with signature $- + + +$ in the second coordinate patch $k = -1$.

It is worth noting that the coordinates defined in (3.8) cover only the portion of space with $x_1 > 0$ and $\sum_{a=2}^{3} x_a^2 < H^{-2}$; i.e. the region inside the particle and event horizons of an observer moving on $r = 0$, along the entire axis $z$ of the cosmic string. The event horizon occurs at the apparent singularity of metric (3.9), at $r = H^{-1} \equiv r_*$.

In coordinates $T' \in (-\infty, \infty), \Psi_3 \in (-\infty, \infty), \Psi_2 \in (0, \pi)$ and $\Psi_1 \in (0, 2\pi)$, defined by

$$x_5 = H^{-1} \sinh \Psi_3, \quad x_4 = H^{-1} \cosh \Psi_3$$

$$x_3 = \sqrt{k}H^{-1} \cosh(HT') \sin \Psi_2 \cos(\sqrt{k}\Psi_1)$$

$$x_2 = \sqrt{k}H^{-1} \cosh(HT') \sin \Psi_2 \sin(\sqrt{k}\Psi_1) \quad (3.10)$$

$$x_1 = \sqrt{k}H^{-1} \cos \Psi_2 \cosh(HT')$$

$$x_0 = \sqrt{k}H^{-1} \sinh(HT'),$$

metric (3.7) becomes the line element describing the geometry of a homogeneous and cylindrically-isotropic kinked spacetime, that is

$$ds^2 = -kdT'^2$$

$$+ H^{-2} \left[d\Psi_3^2 + \cosh^2(HT') \left(kd\Psi_2^2 + \sin^2 \Psi_2 d\Psi_1^2\right)\right].$$  

(3.11)

Under the analytical continuation $T' = i\tau'$, metric (3.11) is converted into again an Euclidean definite positive line element for $k = +1$ and a Kleinian metric for $k = -1$. The $z = \text{const.}$ sections of (3.11) for $k = +1$ correspond to a three-dimensional Robertson-Walker metric whose spatial sections have topology $S^2$ with radius $H^{-2} \cosh(HT')$; i.e. they are de Sitter space in three dimensions. Spatial sections of (3.11) correspond to the hyperboloid $R \times S^2$.

The coordinates (3.10) cover entirely this cylindrically-deformed de Sitter space which would first contract around the entire axis $\Psi_3$ until $T' = 0$, and expand thereafter from that axis.
The hyperboloid described by metric (3.7) is defined for a constant \((1 + k)H^{-2}\) which becomes \(2H^{-2}\) in the patch \(k = +1\) and vanishes in the patch \(k = -1\). In order to keep the same absolute value for the constant in the two patches while preserving the same Euclidean signature, we should introduce a new analytical continuation for the second patch such that, instead of the usual Euclidean continuation, we make \(z = -i\Theta, H = -i\Pi, ds = -id\sigma\) (and hence \(r = -i\rho = -i\Pi^{-1}\sin\psi_2\)), while changing sign of the parameter \(k\) in (3.8) and (3.10). We then obtain

\[
d\sigma^2 = -k\left(1 - \Pi^2\rho^2\right)dT^2 - \frac{k d\rho^2}{(1 - \Pi^2\rho^2)} + d\Theta^2 + \rho^2 d\phi^2,
\]

and

\[
d\sigma^2 = -kdT'^2 + d\Theta^2 + \Pi^{-2}\cos^2(\Pi T') \left(-k d\Psi^2 + \sin^2\Psi d\Psi^2\right),
\]

respectively. These metrics are now both Euclidean for \(k = -1\) and both Kleinian for \(k = +1\).

It would then appear that a positive definite metric can only be achieved if we make the usual Wick rotation in patch \(k = +1\), and the new continuation, where one rotates \(z, H\) and the metric itself, in the patch \(k = -1\). In the following section we shall discuss in more detail this choice and the physical reason supporting it.

**IV. KINKED EXTREME STRING INSTANTONS**

The Euclidean continuation of the extreme string metric which contains one-kink should correspond to making the Wick rotation

\[
\hat{t} = i\hat{\tau},
\]

where \(\hat{t}\) is the kinky time defined by Eq. (3.5). Using this continuation, we have [2]

\[
d\hat{\tau} = -idt - i\left(\tan 2\alpha - \frac{k}{\cos 2\alpha}\right)dr,
\]

with \(\alpha\) again being the angle of tilt of the light cones on the hypersurfaces and [2]

\[
\sin \alpha = \frac{r}{\sqrt{2r_*}}.
\]

The Euclidean continuation (4.1) gives rise to metrics which are positive definite only if we choose either the usual continuation \(t = i\tau\) in the coordinate patch \(k = +1\), or the new continuation implying \(z = -\Theta, r = -i\rho, r_* = -i\rho_*\) and \(ds = -id\sigma\) in the coordinate patch \(k = -1\), which were considered in Sec. III.

In order to investigate the instanton structure of the extreme string kink, let us first re-write metric (3.1) in the form:

\[
ds^2 = -k(r + r_*)^2dUdV + dz^2 + r^2d\phi^2,
\]

where use has been made of (3.4). Introducing then the new variables \(x + y = U\) and \(x - y = V\) in (4.4), we get

\[
ds^2 = -k(r + r_*)^2(dx^2 - dy^2) + dz^2 + r^2d\phi^2.
\]

The following relations will then hold

\[
UV = x^2 - y^2 = k \left(\frac{r - r_*}{r + r_*}\right)
\]

\[
\frac{U}{V} = \frac{x + y}{x - y} = ke^{-\frac{2i\alpha}{r + r_*}} \left(\frac{r - r_*}{r + r_*}\right)
\]

The origin of radial coordinate \(r = 0\) lies on the surfaces \(y^2 - x^2 = k\), and the cosmological event horizon \(r = r_*\) lies on the surfaces \(x^2 - y^2 = 0\), on both coordinate patches. One can avoid the region either beyond the horizon or
inside the horizon by defining new coordinates, \( \zeta = ix \) or \( \xi = iy \), respectively. For the first choice, the metric (4.5) takes the form

\[
ds^2 = k(r + r_*)^2(\xi^2 + dy^2) + dz^2 + r^2 d\phi^2,
\]

which is in fact positive definite in the patch \( k = +1 \), and has Kleinian signature in the patch \( k = -1 \). For the choice \( \zeta = ix \), the radial coordinate is defined by

\[
\zeta^2 + y^2 = k \left( \frac{r_* - r}{r_* + r} \right).
\]

Then, on the section on which \( y \) and \( \zeta \) are both real (the usual Euclidean section for patch \( k = +1 \) or the Kleinian section for patch \( k = -1 \)), \( \frac{1}{r_*} \) will be real and smaller or equal to 1 on patch \( k = +1 \), and take on values in the interval \( \sqrt{2} \geq \frac{1}{r_*} \geq 1 \), on the patch \( k = -1 \); the upper limit \( \sqrt{2} \) of this interval being imposed by the continuity of the light-cone tipping on the surfaces at \( \alpha = \frac{\pi}{2} \) [2].

We define now the imaginary time by \( t = i\tau \). This continuation leaves invariant the form of the metric (4.8) and is therefore compatible with the choice \( \zeta = ix \). Then, from Eq. (4.7) we can obtain

\[
y - i\zeta = \sqrt{k}F(r, r_*)(y^2 + \zeta^2)^{\frac{1}{2}}e^{-\frac{ik\tau}{2}},
\]

where

\[
F(r, r_*) = \exp \left( \sqrt{2} \left( 1 - \frac{r^2}{2r_*^2} \right) \right) \frac{1 - 2\sqrt{1 - \frac{r^2}{2r_*^2}}}{1 + 2\sqrt{1 - \frac{r^2}{2r_*^2}}}.
\]

It follows that for this time continuation \( \tau \) is periodic with period \( 2\pi kr_* \), which exactly corresponds to the inverse to the temperature of the isotropic background of thermal radiation that is emitted in the space of an extreme string kink [2]. On the considered Euclidean section, \( \tau \) has then the character of an angular coordinate which rotates about the "axis" \( r = 0 \) clockwise in patch \( k = +1 \), and anti-clockwise about the "axis" \( r = r_* \) in patch \( k = -1 \).

Since for the spacetime being considered the boundary term in the action must vanish (see Sec. V), the action of the instanton will be evaluated using the scalar curvature \( R \) only. This action turns out to be

\[
I_{k=\pm 1} = \frac{i\pi M_p^2}{2\Lambda},
\]

where \( M_p \) is the Planck mass.

For the second choice of coordinate, \( \xi = iy \), the metric (4.5) takes the form:

\[
ds^2 = -k(r + r_*)^2(dx^2 + d\xi^2) + dz^2 + r^2 d\phi^2,
\]

which is positive definite in patch \( k = -1 \) and Kleinian in patch \( k = +1 \). For this coordinate choice, the radial coordinate becomes defined by

\[
x^2 + \xi^2 = k \left( \frac{r - r_*}{r + r_*} \right);
\]

so, on the section on which \( x \) and \( \xi \) are both real (the usual Euclidean section for patch \( k = -1 \) and the Kleinian section for patch \( k = +1 \)), the ratio \( \frac{1}{r_*} \) will now take on values within the interval \( (\sqrt{2}, 1) \) for patch \( k = +1 \), and will be smaller or equal to unity in patch \( k = -1 \).

Since we have defined the imaginary time \( \tau \) for the first choice \( \zeta = ix \), according to Eq. (4.2), we should now define the imaginary quantities that make the term \( (\sin 2\alpha - k)dr/\cos 2\alpha \) imaginary. Using (4.3) one can see that such quantities are the imaginary of radial coordinate \( r \) and the imaginary of the extreme string parameter \( r_* \). We then define \( r = -ip \) and \( r_* = -i\rho_* \), while keeping time \( t \) real. In order for this definition to be compatible with the coordinate transformation \( \xi = iy \), one should require that this definition leaves metric (4.13) formally unchanged, and this can only be accomplished if coordinate \( z \) and the metric itself, \( ds \), are also continued into their imaginary values, so that \( dz = -id\Theta \) and \( ds = -id\sigma \), while keeping the angular coordinate \( \phi \) real, such as it was done in Sec. III.

From Eq. (4.7) we get then
\[ x - i\xi = \sqrt{kF(\rho, \rho_*)(x^2 + \xi^2)^{\frac{3}{2}}} e^{-\frac{ikt}{\rho_*}}. \]  

(4.15)

It is now the Lorentzian time \( t \) which becomes periodic with period \( 2\pi k\rho_* \), on the new instantonic section. Therefore, \( t \) would have the character of an angular coordinate on this section: it will rotate clockwise about the "axis" \( \rho = \rho_* \) on the coordinate patch \( k = +1 \), and anti-clockwise about the "axis" \( \rho = 0 \) on the coordinate patch \( k = -1 \). The instantonic action on this new Euclidean section can again be computed from the scalar curvature only. It is:

\[ I_{k\pm 1} = \frac{i\pi M_p^2}{2\Lambda E}, \]  

(4.16)

with \( \Lambda_E = \frac{3}{\rho_*^2} \), on both coordinate patches.

A stationary observer at the origin of the radial coordinate \( r \) in the patch \( k = +1 \) would interpret the above two instantonic sections as providing the probability of the occurrence in the vacuum state,

\[ P \sim \exp(-2I_{k=\pm 1}), \]

of an extreme string with positive energy and internal radius \( r_* \) on the coordinate patch \( k = +1 \), and an extreme string with negative energy and radius \( \rho_* \) on the coordinate patch \( k = -1 \). If the stationary observers were on the patch \( k = -1 \), then it would get the same interpretation, but now the positive-energy string with radius \( r_* \) would occur in patch \( k = -1 \) and the negative-energy string with radius \( \rho_* \) in patch \( k = +1 \), provided that the topological charge of the kink continues being positive with respect to the observer. In both cases, the spacetimes of the two strings should join to each other on the surfaces \( \sqrt{2r_*} \) (or \( \sqrt{2\rho_*} \)), beyond their horizons. The resulting whole geometrical construct could then be regarded as a kinked extreme string pair.

V. CREATION OF UNIVERSE PAIRS

In this section we consider the vacuum solution of the Euclidean Einstein equations with a cosmological constant \( \Lambda = \frac{3}{\rho_*^2} \), describing the interior of a kinked cylindric extreme cosmic string. Assuming that the no boundary condition [6] is satisfied at the initial time, the Euclidean action for the system can be written

\[ I_E = -\frac{M_p^2}{16\pi} \int_M d^4x \sqrt{g}(R - 2\Lambda) \]

\[ -\frac{M_p^2}{8\pi} \left( \int_{\partial M} d^3x \sqrt{h}K - \int_{\tau=0} d^3x \sqrt{h}K \right), \]  

(5.1)

where \( g \) and \( h \) refer to the four- and three-metric, respectively, \( R \) is the scalar curvature, and \( K \) is the trace of the second fundamental form, both on the chosen boundary \( \partial M \) and at the initial time \( \tau = 0 \). The latter term must be added because it is an essential prescription of the no boundary proposal [6] that there should be no boundary at the initial time \( \tau = 0 \), so this term explicitly adds the contribution from \( \tau = 0 \) back in. Such a term could, in principle, be nonzero for the kind of topologies we are going to use. If we introduce any other boundary conditions, the last term in (5.1) should not be added.

According to the characteristics of the spacetime dealt with in the previous sections, we choose a minisuperspace model given by the Euclidean metric

\[ ds^2 = kd\tau^2 + dz^2 + b^2 \left( kd\Psi_2^2 + \sin^2 \Psi_2 d\Psi_2^2 \right), \]  

(5.2)

with \( b \equiv b(\tau) \) the scale factor.

From (5.2) we obtain for the Ricci-scalar

\[ R = -k \left( 3\frac{\dot{b}^2}{b^2} + 4\frac{\ddot{b}}{b} - \frac{1}{b^2} \right), \]  

(5.3)

where the dot means time derivative, \( \dot{\cdot} = \frac{d}{d\tau} \). In order to derive an expression for the Euclidean action in our minisuperspace model, we inscribe each spatial section \( S^2 \) of the metric in a cylinder so that the origin of the angular coordinate \( \Psi_2 \) lies on the \( z \)-axis of the cylinder. If, for a two-sphere located at the origin of the cylindrical coordinates,
we choose as the metric the Euclidean continuation of (3.11), then we should take \( z = r \cos \Psi \), since \( \Psi_1 = \phi \) and \( r = H^{-1} \cos(H\tau) \), with \( H = \sqrt{\Lambda} \). It follows that in the Euclidean sector that corresponds to just one inscribed sphere, \(-H^{-1} \leq z \leq H^{-1}\), and hence we obtain for the action on that sector in the two coordinate patches

\[
I_E = -\frac{M_p^2}{H} \left( \int Nd\tau \left( \frac{\dot{b}^2}{N^2} + 1 - \Lambda b^2 \right) + \left[ \frac{\dot{b}}{N} \right]_{\tau=0} \right), \tag{5.4}
\]

where \( N \) is the lapse function and the second term corresponds to the last surface term of Eq. (5.1).

In the gauge \( N = 1 \), the equation of motion for \( b \) and the Hamiltonian constraint will be:

\[
\ddot{b} = -b\Lambda \tag{5.5}
\]

\[
1 - \dot{b}^2 - \Lambda b^2 = 0. \tag{5.6}
\]

A solution to these equations is

\[
b(\tau) = H^{-1} \sin(H\tau), \tag{5.7}
\]

which looks like Nariai spacetime [15]. This solution naturally satisfies the no boundary condition as, at \( \tau = 0 \)

\[
b = 0, \quad \dot{b} = 1. \tag{5.8}
\]

Note that the boundary term in Eq. (5.4) vanishes for this solution. Now, if we choose a path along the \( \text{Re} \tau \) axis from 0 to \( \frac{\pi}{2}H \), the solution (5.7) will describe twice half of the Euclidean \( R \times S^3 \) instanton, each time in a coordinate patch. If the path is continued from \( \text{Re} \tau = \frac{\pi}{2} \) parallel to the imaginary axis \( \text{Im} \tau \), then \( b \) will still remain real, with

\[
b(\text{Im} \tau)\big|_{\text{Re} \tau = \frac{\pi}{2}} = H^{-1} \cosh(H\text{Im} \tau). \tag{5.9}
\]

The scale factor (5.9) describes twice half of a Lorentzian universe, each time on a coordinate patch. The spacetime sections of this universe can be visualized as being formed by two spheres (one in each patch) growing up from the original size of the extreme string kink. The physical interpretation would be that of a pair of physical universes spontaneously created out from the extreme string kink, the two universes in each pair being formed at the same time, and joined to each other at the surfaces \( \sqrt{2}b \), beyond their respective observable regions of radius \( b \). (Note that for \( G\mu = \frac{1}{4} \) there is no exterior space to the string shell in one coordinate patch, except that of the string shell in the other coordinate patch.) Thus, the observable parts of the two universes inside the cosmological horizon accelerate away from each other as \( b \), and hence their mutual separation \( 2(\sqrt{2}-1)b \), grows.

On the other hand, since for the considered solution the second term of (5.4) vanishes, the real part of the action comes entirely from the first term. Besides, the Lorentzian segment of the chosen path only contributes to \( \text{Im} I_E \), so we obtain finally for the real part of the action corresponding to the two universes in a pair

\[
\text{Re} I_E = -\frac{2M_p^2}{H} \int_0^{\frac{\pi}{2}} d\tau \cos^2 H\tau = -\frac{\pi M_p^2}{2\Lambda}, \tag{5.10}
\]

and the semiclassical probability measure for creation of a pair of such universes will be given by

\[
P_{HH} \sim \exp \left( \frac{\pi M_p^2}{\Lambda} \right). \tag{5.11}
\]

One should compare (5.11) with the corresponding probability for a single spherically-symmetric de Sitter universe satisfying the no boundary proposal [16]

\[
P_{dS} \sim \exp \left( \frac{3\pi M_p^2}{\Lambda} \right). \tag{5.12}
\]

One can conclude that, if the no boundary holds, then the creation of a single de Sitter universe is by far a more probable process than the creation of a universe pair according to the mechanism considered in this work. On the other hand, larger values of the matter field \( \phi \) entering the potential
with \( \Lambda = 2\pi G\lambda \phi^4 \), will be strongly suppressed according to (5.11). Therefore, also when applied to a universe pair created out from an extreme string kink, the no boundary proposal would prevent inflation to occur in each universe of the pair. It is worth noting that the main objection raised by Linde [17] and Vilenkin [7] against the no boundary proposal is that it is unable to predict inflation. This result was to be expected as the universe pairs created from an extreme string kink with finite size cannot be self-contained, but is a consequence from the existence of a previous physical spacetime reality that necessarily includes some phase transition.

It appears most appropriate that the boundary conditions for these universe pairs be related with a "tunneling" condition [7], where the wave function \( \psi(b) \) does not vanish for \( b = 0 \). The reason is that in the case under consideration, all of the space is confined within the finite interior region of the kinked extreme string, and thereby all three-geometries, defined on the hypersurfaces, of the corresponding superspace should map onto the same compact region. Now, since the kink makes the light cones to continuously rotate on the hypersurfaces, the time direction would rotate as well, pointing always toward the superspace’s boundary and covering all possible directions only if the two coordinate patches and both, positive and negative gravitational topological charges are used. Actually, the time direction of the light cones point toward the interior of superspace in the second coordinate patch, \( k = -1 \) [3]. However, because the energy of the modes becomes negative in this patch [2], their time direction will be equivalent to that of a positive-energy flux for outgoing modes, just as in the first coordinate patch, \( k = +1 \). Therefore, the wave functional representing the quantum state of the kinked extreme string should include only all the outgoing modes carrying positive flux out of superspace, and this just expresses the Vilenkin’s tunneling boundary condition [7]; i.e.: the quantum creation of the extreme string from nothing, meaning by “nothing” the absence of an exterior spacetime.

The Wheeler-DeWitt equation for \( \psi(b) \) can be written

\[
\left\{ x^{-p} \frac{\partial}{\partial x} x^p \frac{\partial}{\partial x} + \left[ \left( \frac{M_p}{H} \right)^2 - \frac{1}{4} x^2 \right] \right\} \psi(x) = 0,
\]

(5.13)

where \( x = 2M_pb \) and the parameter \( p \) represents the factor-ordering ambiguity. For the choice \( p = 0 \) and the general boundary condition \( \psi(x) \to 0 \), as \( b \to \infty \), we obtain the general solution

\[
\psi(b) = D \frac{M_p}{p^{1/2}} \left( 2M_pb \right),
\]

(5.14)

with \( D \) being the parabolic cylinder function for \( b > 0 \).

However, for the purposes of this work, it suffices obtaining the WKB solutions to (5.13). In the most natural case that corresponds to a tunneling boundary condition, the real part of the underbarrier (\( b < H^{-1} \)) solution to (5.13) has the form [7]

\[
\psi_T \sim \psi_{HH}^{-1} \sim \exp \left( -\frac{\pi M_p^2}{2\Lambda} \right),
\]

(5.15)

where \( \psi_{HH} \) is the (underbarrier) WKB solution satisfying the no boundary condition and we have disregarded the pre-exponential factor. It follows that the tunneling semiclassical probability measure for the creation of a pair of universes must be given by

\[
P_T \sim \exp \left( -\frac{\pi M_p^2}{\Lambda} \right),
\]

(5.16)

for which larger values of the field \( \phi \) will be strongly favoured, rather than suppressed. Moreover, by comparing (5.16) with the corresponding probability for a single de Sitter universe satisfying the tunneling proposal, obtained from (5.12) by the same approximate procedure, that is

\[
P_{dS} \sim \exp \left( -\frac{3\pi M_p^2}{\Lambda} \right),
\]

(5.17)

we deduce that, in this case the process of pair creation turns out to be by far more likely than the creation of a single de Sitter universe.

Thus, if one insists in an inflationary cosmological model in which the universes are created by the mechanism of pair formation considered in this work, it appears that one should choose as initial condition an original tunneling from nothing, where the outgoing modes of the wave function point toward the complete singular boundary of superspace only if two coordinate patches and both, positive and negative topological charges are used to describe the kinked string.
VI. INFLATION IN UNIVERSE PAIRS

There are two main reasons in favour of the idea that, once simultaneously created, the two universes forming a pair along the cosmic string kink immediately undergo a separate, but equivalent inflationary process. First of all, it is the fact that the radius of a spherical extreme string kink exceeds the size of its corresponding cosmological horizon. This is a straightforward consequence from the de Sitter structure of the $z=\text{const.}$ sections of the extreme-string kink interior, which should induce it to quite naturally drive an exponential expansion, without fine tuning of the initial conditions. This implication agrees with the proposal by Linde and Linde [18] and Vilenkin [19] that inflation may be generated in the core of topological defects for sufficiently large gravity coupling. The second reason for an inflationary process in each of the spacetimes of the pair has been discussed at the end of Sec. V; i.e.: the quantum-cosmological prediction of an increase of the semiclassical probability measure as the involved matter field $\varphi$ becomes larger, provided that the initial state satisfies a tunneling boundary condition. It is well-known that inflation can only be driven for high initial values of the matter field $\varphi$.

In order for the event horizon at $r = r_*$ of an extreme string kink (which has linear energy density $\mu = \frac{1}{2}M_p^2$ and possesses $z=\text{const.}$ sections with the symmetry of the hemispherical section of a de Sitter kink [2,3]) to be a cosmological horizon with size

$$H^{-1} = r_* = \left(\frac{3}{8\pi G V_0}\right)^{\frac{1}{2}},$$

one must choose the uniform string density to be $\epsilon = \frac{V_0}{3}$, so that the cosmological constant becomes $\Lambda = 8\pi GV_0 = \frac{3}{\gamma^2}$. The crucial point now is that, if we use the approximate expression [20] $\mu \sim \eta^2$, where $\eta \sim M_p$ is the symmetry-breaking scale of the underlying model

$$V(\varphi) = \frac{1}{4}\lambda(\varphi_0^a \varphi_a - \eta^2)^2, \quad a = 1, 2,$$

then $V_0 = \frac{1}{4}\lambda\eta^4 \sim \lambda M_p^4$ and, since $\lambda \ll 1$, $V_0 \ll M_p^4$ and $r_* = (8\pi G\epsilon)^{-\frac{1}{2}} \sim \lambda^{-\frac{1}{2}} M_p^{-1} \gg M_p^{-1}$.

In this case, the initial conditions for inflation would correspond to the following set of field parameters:

$$\partial_\mu \varphi \partial^\mu \varphi \ll M_p^4, \quad V_0(\varphi_0) \ll M_p^4, \quad R^2 \ll M_p^4,$$

where $R$ is the Ricci-scalar. Under these conditions, the only region accessible to stationary observers is that with radius $r_* = H^{-1} \gg M_p^{-1}$ and, in order for $H^{-1}$ to recede from $r_*$ slowly enough for any possible particles and other inhomogeneities to not have any effects on events taking place inside the horizon, we should have $H \ll H^* \sim r_*^{-2}$. On the other hand, if the initial values of the potential and scalar field are $V_0 \ll M_p^4$ and $\varphi_0 \sim \eta \sim M_p$, respectively, in a region of size $\ell \sim H^{-1} \gg M_p^{-1}$, the variation of the field $\varphi$ would be $\Delta \varphi \ll \eta \sim \varphi_0 \sim M_p$. This would mean that the given region will be largely homogeneous and isotropic, and therefore describable as a Friedmann spacetime, where

$$H^2 + \frac{s}{b^2} = \frac{\dot{b}^2}{b^2} + \frac{s}{b^2} = \frac{8\pi}{6M_p^2} (\dot{\varphi}^2 + \nabla \varphi^2 + 2V)$$

$$\ddot{b} + 3 \frac{\dot{b} \dot{\varphi}}{b} - \frac{\nabla \varphi}{b^2} = - \frac{dV}{d\varphi},$$

where $s$ is the spatial curvature constant.

However, even though for a sufficiently uniform field $\varphi$ we still may have $(\nabla \varphi)^2 \ll V$ and, if an exponential expansion is assumed, $\ddot{b} \gg 1$ and $\ddot{\varphi} \ll \frac{dV}{d\varphi}$, so that

$$\frac{\dot{b}^2}{b^2} \sim \frac{8\pi}{6M_p^2} (\dot{\varphi}^2 + 2V)$$

$$3H \dot{\varphi} \sim - \frac{dV}{d\varphi},$$

we now had from (6.6)
which, for \( \varphi_0 \sim M_p \), becomes \( \hat{\varphi}^2 \sim V \) and the term with \( \hat{\varphi}^2 \) could not be dropped off from Eq. (6.5). It follows that with the set of values used for the parameters of our model given by (6.2), neither Eq. (6.5) can give rise to a de Sitter exponential expansion of the scale factor, \( b \sim b_0 \exp (Ht) \), nor \( \hat{\varphi}^2 \sim V \) would imply by itself a stress-tensor \( T_{\mu \nu} \sim g_{\mu \nu} V \), predicting a de Sitter state equation \( p \sim -\rho \), with \( p \) the pressure and \( \rho \) the energy density.

This result is clearly contradictory with the fact that we started with a de Sitter space underlying an inflationary process. Any possible way out of this inconsistency would require having \( V_0 \sim M_p^4 \), instead of \( V_0 \sim \lambda M_p^4 \), with \( \lambda \ll 1 \). Actually, the crucial point which decides on these two initial values of the potential of the model is the relation between the symmetry-breaking scale \( \eta = \varphi_0 \) and the string mass per unit length \( \mu \sim M_p^2 \). Choosing \( \eta^2 \sim \mu \), as usual, leads to the above inconsistency, but if we assume a relation \( \eta^2 \gg \mu \), i.e. if \( \varphi_0 \gg M_p \), then we see from the Friedmann equations (6.3) and (6.4) that \( \varphi^2 \) must be much smaller than \( V \), and hence

\[
\frac{\dot{b}^2}{b^2} \approx \frac{8 \pi V}{3 M_p^2},
\]

so that we finally recover the wanted expressions for the scale factor, \( b \sim b_0 \exp (Ht) \), and the state equation, \( p \sim -\rho \), required to keep full consistency in the inflationary model. Besides, if \( \eta^2 \gg \mu \) and \( V \sim M_p^4 \), we have

\[
r_s \sim \mu^{-\frac{1}{2}} \sim M_p^{-1}, \quad \Lambda \sim M_p^2.
\]

But, why should one use the inequality \( \eta^2 \gg \mu \) instead of the approximate relation \( \eta^2 \sim \mu \) of current cosmic string theory [20]? First of all, we remind that the concept of linear energy density (or mass per unit length) \( \mu \), so as the approximate relation \( \eta^2 \sim \mu \) are not unambiguously defined and that they can only be approximately applied to cosmic strings with moderate tension at symmetry-breaking scales quite lower than the Planck scale [20]. More importantly, making a cosmic string obey the conservation laws of the one-kink gravitational defect changes the internal string structure because of the gravitational back-reaction from the spacetime kink.

One would expect that the characteristic radial coordinate \( r \) of the string with \( G \mu = \frac{1}{2} \) classically collapses to a point (Ref. Eq. (2.2)) when the kink is not present, but it would stop shrinking at a minimum, nonzero value of the order the Compton wavelength corresponding to the symmetry-breaking scale \( r \sim \delta \varphi \sim \eta^{-1} \sim M_p^{-1} \) (the maximum energy-scale of the theory) when the quantum structure of the spacetime kink is considered. The observable minimum value of \( r \) in the kinked extreme string cannot correspond to the Compton wavelength of the Higgs boson since all of the observable region of the existing spacetime is filled with false vacuum, with no trace of the broken-symmetry phase, so for stationary observers at \( r = 0 \) there will be not symmetry breaking and, therefore, the Higgs bosons would not exist.

The parameter \( \mu \) is usually defined [20] as a linear energy density, such that

\[
\mu \sim V_0 (\delta \varphi_A)^2,
\]

where \( V_0 \sim \lambda \eta^4 \) and \( \delta \varphi_A \sim m_\varphi^{-1} = \lambda^{-\frac{1}{2}} \eta^{-1} \) is the Compton wavelength of the Higgs boson, so \( \mu \sim \eta^2 \) holds only for the situations in which the Higgs boson is observable; i.e. for cosmic strings with \( G \mu \ll 1 \), which are able to have external observers. For strings with \( G \mu = \frac{1}{2} \) having only internal stationary observers living in the false vacuum, \( \mu \) should instead be defined as

\[
\mu \sim V_0 (\delta \varphi_\eta)^2 \sim \frac{m_\varphi}{\delta \varphi_A} \sim \lambda \eta^2,
\]

so, for \( \mu \sim M_p^2 \) and \( \lambda \ll 1 \), we in fact obtain

\[
\eta^2 \sim \varphi_0^2 \gg \mu \sim M_p^2, \quad \partial_\mu \varphi \partial^\mu \varphi, V_0, R^2 \sim M_p^4,
\]

as the initial conditions for the inflationary process in the kinked extreme string.

It is worth noting that the inflationary scenario that results from the above initial conditions is just that of Linde’s chaotic inflation [4], as applied separately to each of the universes in a pair. For each universe, we then have

\[
b(t) \simeq b_0 \exp \left( \frac{t}{r_s} \right)
\]
\[ \varphi(t) \simeq \eta \exp \left( -\sqrt{\frac{\lambda}{6\pi \eta}} \frac{t}{r_s} \right), \tag{6.11} \]

so that

\[ b \simeq b_0 \exp \left[ \sqrt{\frac{2\pi \lambda}{3M_p^2}} (\eta^2 - \varphi^2) \right]. \tag{6.12} \]

Like in chaotic inflation, here the inflationary regime would end when the field \( \varphi \leq r_s \sim M_p \). Besides, since \( \varphi_0 = \eta \gg M_p \), the overall inflation factor is

\[ E \sim \exp \left[ \sqrt{\frac{2\pi \lambda}{3M_p^2}} \eta^2 \right]. \tag{6.13} \]

One may conclude that most of the physical volume of each pair of universes created out of an kinked extreme string comes into being as a result from the inflation of regions with a size \( \ell \) which, in our case, are confined in a finite interval \( r_s = H^{-1} < \ell \leq \sqrt{2}H^{-1} \), and were initially filled with a sufficiently homogeneous and slowly varying (i.e. \( \Delta \varphi \sim M_p \ll \eta \)), extremely large field \( \varphi = \eta \gg M_p \).

In order to have sufficiently homogeneous and isotropic universes, all the particles and other inhomogeneities initially present within a sphere of radius \( H^{-1} \) should cross the horizon by a time \( H^{-1} \), so that they will have no effect on events taking place inside the horizon (no hair theorem [21]). In our case, however, the exterior region is restricted to extend only up to the surface at \( \sqrt{2}H^{-1} \), and this boundary surface in a coordinate patch is identified with the similar surface of the exterior region of the other universe in the other coordinate patch. Here the no hair theorem is ensured to hold by the mutual annihilation of particles and inhomogeneities coming from the two universes in each pair [2].

Because the parameters of our inflationary model have the values required by chaotic inflation, the global geometry in the two coordinate patches of the infinite string will show remarkable inhomogeneities on the largest scales. One obtains that regions with size \( H^{-1} \) will grow by a factor \( e \) in a typical time \( H^{-1} \) and then subdivide into \( \sim e^3 \) regions of the same size as the original one, each with a field \( \varphi \) which differs from the original one by

\[ \delta \varphi \sim \sqrt{\frac{\lambda \varphi^2}{6\pi M_p^2}} \]

due to quantum fluctuations with wavelength \( \geq H^{-1} \) [4]. Nearly half of these new regions will have fields \( \varphi \) which are larger than \( \varphi_0 \) by \( \delta \varphi \) and hence \( V \gg M_p^4 \), so inflation would be cut short on them. This process would repeat during the next time interval \( H^{-1} \) in the two coordinate patches, and so on, to finally reproduce the picture of an infinite number of self-regenerating pairs of inflationary universes, where no future singularity is needed or possible [4]. On the other hand, since the wave function for each of these regions should satisfy a tunneling initial condition such that the present approach becomes based on a picture where each pair of universes nucleates from an isolated gravitational topological defect in the absence of any exterior spacetime ("nothing"), no past singularity seems to have existed or be needed either.

VII. CONCLUSIONS AND FURTHER COMMENTS

This paper develops and somewhat extends the idea suggested in Ref. [2] that an extreme cosmic string with linear energy density \( \mu = \frac{\lambda}{2\alpha^2} \) must exist as a result of its gravitational kinked structure, and be the seed for an inflationary process.

We began by considering in detail some necessary geometrical aspects of the extreme string kink, and then worked out the structure of the associated instantons. There are two instantonic sections for this kink, one which is achieved by applying the customary Wick rotation on time coordinate, gives a positive definite kinked Kruskal metric only in the first of the two coordinate patches needed for a complete description of the one-kink, and the other, arrived at by analytically continuing on the spacelike coordinates, which produces an Euclidean metric only on the second of these patches. These instaontic sections can be continued from one another by gluing the string metrics of the two patches at their maximum surfaces. This is a requirement from continuity and completeness on the kink, and therefore, the glued instantons represent the probability of creation in the vacuum state of a pair of extreme strings. It has been also shown that the spacetimes on the two coordinate patches must inflate separately.
The inflationary process driven in the kinked extreme string core has been considered under two different points of view. By using the machinery of the semiclassical approximation to quantum cosmology, one can see that inflation can only be driven if one imposes a tunneling boundary condition, and by discussing Einstein equations, we achieve the result that kinked extreme strings undergo a chaotic inflationary process along their core.

We argued that the effect of having a conserved gravitational topological charge in the high energy string is two-fold. On the one hand, it induces the creation of a protecting shell of true vacuum surrounding the string core and, on the other hand, it gives rise to a gravitational back-reaction that fits the string size to be of the order the Planck scale. Whether or not our universe is in a pair created according to the mechanism suggested in this work would be a matter requiring further investigation.

The inflationary scenario proposed in this paper can be thought of as a natural process as far as a phase transition occurred at the Planck time, breaking the unification between gravity and the other forces. If such a transition took place rapidly enough, then topological defects like cosmic strings with the Planck tension had been formed satisfying the conservation laws of a gravitational kink and having the natural conditions for chaotic inflation to be driven in the kinked string core.

The possibility of string-driven inflation was first suggested by Turok [22] who considered the effects of quantum fluctuations on a string network in de Sitter space. He obtained that strings able to produce the wanted effect had to be of the Planck scale. The main criticism that can be raised against models of string-driven inflation is that Planck-energy strings have a deficit angle $\Delta = 8\pi G \mu$ (even exceeding $2\pi$) which would cause strong deviations from homogeneity and isotropy on the horizon scale [20], and so the Friedmann equations will not be applicable in this case. Actually, for the kinked string discussed in this work, there is not anything like a deficit angle. Due to the gravitational back-reaction caused by the conservation of the gravitational topological charge, the geometry of the string becomes no longer conical, but converts into that of a section of the de Sitter space, with the conical singularity replaced for the apparent singularity of a cosmological horizon. Clearly, the picture of a deficit angle is lost, so one should not expect the kinked string to induce any deviations from homogeneity or isotropy in any region of the inflating string interior.

Because the kinked extreme string should be regarded as a quantized geometrical construct [2], it would admit the definition of a maximum Hagedorn temperature of the form $T_H \sim \sqrt{\mu} \sim M_p$. It turns out that this temperature is of the same order as the Gibbons-Hawking temperature $T_{GH} = \frac{1}{2\pi r_*} \sim M_p$, evaluated in Ref. [2] for kinked extreme strings. The same approximate relation, $T_H \sim T_{GH}$, was assumed by Aharonov, Englart and Orloff [23] as the basis for suggesting the possibility of string-driven inflation.

On the other hand, the description of the instantons associated with the kinked spacetimes require two coordinate patches. Generally, these instantons can be regarded as pairs of the corresponding geometrical construct, one in each patch. This interpretation has been recently discussed in the context of neutral black-hole pairs [24]. In the case of the inflating universe pairs considered in this work, the identification of the two patches only at a tilt angle $\alpha = \frac{\pi}{2}$ leads to an interconnection between the two universes taking place at the largest, albeit finite surface beyond the region which is causally connected to stationary observers, with the universe in one patch being the anti-universe to the universe in the other patch. This would give rise to an essentially unobservable bridge between two different cosmological horizons.

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