Competitive Online Peak-Demand Minimization
Using Energy Storage
Yanfang Mo, Member, IEEE, Qiulin Lin, Minghua Chen, Senior Member, IEEE
and Si-Zhao Joe Qin, Fellow, IEEE

Abstract—We study the problem of online peak-demand minimization under energy storage constraints. It is motivated by an increasingly popular scenario where large-load customers utilize energy storage to reduce the peak procurement from the grid, which accounts for up to 90% of their electric bills. The problem is uniquely challenging due to (i) the coupling of online decisions across time imposed by the inventory constraints and (ii) the noncumulative nature of the peak procurement. In this paper, we develop an optimal online algorithm for the problem, attaining the best possible competitive ratio (CR) among all deterministic and randomized algorithms. We show that the optimal CR can be computed in polynomial time, by solving a linear number of linear-fractional problems. More importantly, we generalize our approach to develop an anytime-optimal online algorithm that achieves the best possible CR at any epoch, given the inputs and online decisions so far. The algorithm retains the optimal worst-case performance and achieves adaptive average-case performance. Simulation results based on real-world traces show that, under typical settings, our algorithms improve peak reduction by over 19% as compared to baseline alternatives.

Index Terms—Energy storage management, peak-demand charge, online competitive algorithms.

NOMENCLATURE

| Symbol     | Description                                      |
|------------|--------------------------------------------------|
| c          | Storage capacity (kWh).                         |
| δ          | Maximum discharge amount per time slot (kWh).   |
| T          | Number of time slots.                           |
| d, d′       | Net demand bounds per time slot (kWh).          |
| d          | Net demand profile in $\mathbb{R}^T$.           |
| δt         | Discharging quantity at time slot $t$, $\delta t \geq 0$ (kWh). |
| $v(d)$     | Peak usage of an optimal offline solution under $d$. |
| $v^{\pi}(d)$ | Peak usage of online algorithm $\pi$ under $d$. |
| $d^\pi$    | $\{d_1, d_2, \ldots, d_1, d_\pi, \ldots, d_d\}$ in $\mathbb{R}^T$ derived from $d$. |
| $\delta(\pi, d)$ | Discharge vector in $\mathbb{R}^T$ under pCR-PMD($\pi$) and $d$. |
| $\pi^*$    | Best CR among all online algorithms for PMD.     |
| $\pi^*_t$  | Anytime-optimal CR at time slot $t \in [T]$.     |
| [n]        | The set $\{1, 2, \ldots, n\}$ for $n \in \mathbb{N}$. |
| [x]⁺        | The maximum of $x$ and 0 for $x \in \mathbb{R}$. |
| [x]         | The largest integer no more than $x \in \mathbb{R}$. |
| $\mathcal{X} \setminus \mathcal{Y}$ | $\{x \mid x \in \mathcal{X} \land x \notin \mathcal{Y}\}$ for two sets $\mathcal{X}$ and $\mathcal{Y}$. |

Figure 1: Real-time energy consumption in two on-peak periods (left) and an illustrative scenario of utilizing energy storage systems to reduce peak usages for large-load consumers with behind-the-meter renewable generations (right). We consider general energy storage systems, including fuel cells and fly-wheels, which may not admit charging during their operations.

I. INTRODUCTION

Demand side management (DSM) promotes desirable changes in load curves to moderate the demand uncertainty and ease the supply side burden [2]. A popular DSM mechanism is that utilities use pricing schemes to motivate customers to improve their consumption patterns. Many pricing schemes are designed to reflect the cost of energy over time, e.g., time-of-use (TOU) and real-time pricing [3], [4]. These schemes use price differences to incentivize load shifting from on-peak hours, when the supply is scarce or expensive, to off-peak hours with abundant or cheap supply. Meanwhile, utilities further introduce peak-demand charges, to encourage large-load customers, e.g., data centers and shopping malls, to flatten their load curves (e.g., during the on-peak hours) [3], [5]. Here the peak-demand charge is the maximum rate at which a customer pays for the amount of power (demand) during a billing period. The demand is often measured in 15-minute intervals, with the highest measured value setting the “peak” for which the customer is billed. The unit of measure is typically the kilowatt (kW). Notably, the peak-demand charge can be 80%–90% of the total electricity bill of a large-load customer like a charging station [6] or a data center [7].

It is thus financially attractive to invest in technologies for reducing the peak electricity usage [7]–[9]. See Fig. 1 for the illustration of an increasingly popular scenario, where large-load customers, e.g., the commercial, the industrial, and the municipal, utilize energy storage to reduce the peak power procurement from the grid, in the presence of self-owned renewable generations like wind turbines and solar panels. While certain types of energy storage systems, e.g., batteries, allow real-time charging and discharging, we consider the general setting where energy storage systems, e.g., fuel cells [10]...
and flywheels, may not admit charging by the grid or the renewable during operation.\footnote{The focus on the discharging-only setting is of interest for the following reasons. First, it is a simple yet popular strategy to charge the storage systems during off-peak hours and discharge in on-peak periods to reduce the peak procurement. Second, certain storage systems, e.g., fuel cells and pumped storage, do not allow charging during the operation. Finally, solutions for the discharging-only setting can serve as benchmarks for those designed for the charging/discharging setting.}

Customers’ peak-minimizing endeavors also benefit the power system because flattened load curves reduce the risk of overloading transmission lines or transformers, the need of calling expensive and carbon-intensive reserve generators, and the pressure of managing curtailable electricity services\footnote{There are many results under the clairvoyant/stochastic setting where the complete or distributional information of the load and renewable generation are available [13]–[17]. While usually achieving strong performance, they may not be practical due to the difficulty of estimating loads and the renewable.} to serve surging loads.

However, it is algorithmically non-trivial to realize such a win-win benefit in practice. It is hard to forecast the fluctuating loads, especially peak usages for individual customers. Even worse, the user-owned (behind-the-meter) intermittent renewable generations aggravate the unpredictability of net demands to be satisfied [12], leading to online (peak) optimization with little or no future information.\footnote{While online optimization is a well-studied topic, versions with inventory constraints and peak charges are known to be difficult. The coupling of online decisions across time imposed by the inventory constraints and the noncumulative nature of the peak usage make the problem uniquely challenging and little understanding is available in the literature; see a discussion on the related work in Section II.}

While online optimization is a well-studied topic, versions with inventory constraints and peak charges are known to be difficult. The coupling of online decisions across time imposed by the inventory constraints and the noncumulative nature of the peak usage make the problem uniquely challenging and little understanding is available in the literature; see a discussion on the related work in Section II.

Motivated by these observations, we study the problem of online peak minimization under inventory constraints. We focus on designing competitive algorithms under the online setting, where the loads and renewable generations are revealed sequentially in time but the algorithm has to make irrevocable decisions at current epoch with little or no future information. We use competitive ratio (CR) as the performance metric, which is the worst-case ratio between the peak procurement attained by the online/real-time algorithm with little future information and that obtained by an optimal offline/clairvoyant solution with complete knowledge of future input [13], [19]. CR takes a value at least one and we prefer online algorithms with small CRs. We make the following contributions.

▷ After formulating the problem of peak minimization under inventory constraints in Section III, we design an optimal online algorithm with the best CR among all deterministic and randomized algorithms in Section IV. We show the best CR can be computed in polynomial time, by solving a linear number of linear-fractional programs. The best CR also captures the fundamental price of uncertainty for the problem. To our best knowledge, these are the first (and optimal) results on this theoretically challenging and practically relevant problem.

▷ In Section V, we generalize our approach to develop an anytime-optimal online algorithm. It achieves the best possible CR at any epoch, given the inputs and online decisions so far. The idea is to progressively prune the input space based on the inputs observed so far and adjust the online decisions for the best competitiveness with respect to the residual uncertainty.

We carry out simulations based on real-world traces in Section VII to evaluate the empirical performance of our algorithms. Under typical settings, our anytime-optimal algorithm improves the average peak reduction by over 19\% as compared to baseline alternatives. It achieves up to 77\% of the best possible peak reduction attained by the optimal offline algorithm. Note that the optimal offline algorithm is impractical since it is non-causal and assumes full knowledge of future demand and renewable generation. Our algorithm outperforms the receding horizon control (RHC) [20], [21] with a quarter of the operation period as the look-ahead window size. This highlights the usefulness of competitive algorithm design in decision making under uncertainty.

We analyze our algorithms for the case of minimizing the sum of peak-demand and volume charges in Section VI. We obtain the first online algorithms, to our best knowledge, for this general setting with strong worst-case performance guarantee under a mild assumption. We conclude the work in Section VIII. Proofs are deferred to the appendix.

II. RELATED WORK

A. Electricity Cost Saving by Energy Storage

Ref. [22] studied the economics of energy storage systems in New York state’s electricity market by applications like energy arbitrage and regulation services. Ref. [23] used UPS units to reduce the time average electric utility bill for a single data center via a Lyapunov optimization approach. Ref. [22] further investigated the extended case with multiple data centers, under time-varying and location-varying electricity costs. Ref. [25] explored the financial incentives for electric-vehicle (EV) owners to provide vehicle-to-grid services. An economic analysis was given in [26] on integrating used EV batteries into commercial building microgrids. Moreover, in [27] and [28], the authors respectively discussed the value of storage in a residential home and the impact of shared energy storage systems in apartment buildings.

B. Peak-Demand Charge

Ref. [29] justified the usefulness of peak-demand charges for price discrimination given the competition among isolated industrial customers. Ref. [30] studied the demand charge incurred by the largest accumulated demand over several slots. Knowing the maximum gross demand, a client respectively used lossy and lossless buffering to minimize the maximum resource request in [31] and [32]. Ref. [9] used battery storage for peak shaving together with frequency regulation, based on day-ahead load prediction. Ref. [33] applied the economic model predictive control for time-varying cost and demand charge optimization. Given a tariff with peak-demand charges, Ref. [34] studied the scheduling of EV charging jobs, while Ref. [6] studied the pricing of EV charging services. Our work specifically focuses on reducing the peak-demand charge in a bill by discharging storage in an online fashion.
C. Competitive Analysis and Optimization under Uncertainty

Online algorithm design with competitive analysis is widely used in smart grids, e.g., EV charging [34]-[37] and economic dispatching [38]-[41]. Online peak-demand minimization is uniquely hard by the noncumulative nature. Prior to this work, Refs. [34] and [40] respectively designed competitive online algorithms for EV charging and economic dispatching for microgrids, to minimize the peak consumption. Online optimization with inventory constraints challenges researchers by the coupling of online decisions across time. Classic instances include the one-way trading problem [42] and variations of the knapsack problem [43]. Particularly, Ref. [44] concerned the online revenue maximization under inventory constraints and Ref. [45] studied the online linear optimization with inventory management constraints. Our study differs from them and complements the literature by considering both the noncumulative peak minimization objective function and inventory constraints. Furthermore, there is a useful algorithmic framework, called CR-Pursuit. It leads to competitive algorithms with given CRs in [34], [36], [37], [44] and [46]. These pioneering results motivate us to design online peak-minimizing algorithms parameterized by “pursuing” carefully designed CR at each decision-making epoch.

There are also other popular approaches for optimization under uncertainty, e.g., robust optimization [13], stochastic optimization [15], [14], and model predictive control (MPC) [20], [33]. Similarly to these methods, our approach adopts the worst-case analysis as robust optimization, concerns the average-case performance as stochastic optimization, and makes the decision at each round by virtue of a simple optimization problem as MPC. Meanwhile, as compared to competitive online optimization, robust optimization usually does not tackle sequential decisions, stochastic optimization suffers from inaccurate estimations, and MPC seldom has optimality guarantees. Moreover, these three approaches do not directly consider fairness in performance evaluation under different inputs. In this paper, we develop online algorithm for minimizing peak demand using energy storage with the optimal worst-case competitive performance guarantee.

III. Problem Formulation

We consider the scenario where large electricity customers, e.g., shopping malls and data centers, utilize energy storage to reduce their total electricity bills in a month. In a typical tariff [47], the converted demand-charge rate is over 300 times of the TOU rates. In view of the drastic rate difference and the significance of peak-demand charge, we first focus on minimizing the peak procurement from the grid. In a later section, we will extend the solutions for peak minimization to the case with both peak-demand and volume charges. For our purpose, we focus on an on-peak operation period, e.g., from 7 a.m. to 10 p.m. for a mall [38]. We divide the period into $T$ time slots, each of 15 minutes, matching the power measurement interval used in determining the peak-demand charge in practice. We convert the power demand (in kW) into energy consumption (in kWh) of a time slot.

A. Mathematical Model

Renewable Generation and Net Electricity Demand: We consider user-owned (behind-the-meter) renewable generations; the associated generation cost is thus negligible. We adopt the general setting where the renewable generation may not be able to charge the storage system, like fuel cells and flywheel systems. Let $d_t$ (in kWh) be the residual electricity demand not balanced by the local renewable generation at time slot $t$. Then, we obtain the net demand profile $d \in \mathbb{R}^T$. We assume the minimum prior knowledge of the net demand, namely $d_t \in [\underline{d}, \bar{d}]$ for all $t \in [T]$, where $\underline{d} \in \mathbb{R}$ and $\bar{d} \in \mathbb{R}$ are respectively the lower and upper bounds of the net demand in a single time slot $t$. Note that we do not rely on any specific stochastic model of $d_t$, $t \in [T]$, which can be difficult to obtain because of the fluctuating loads of individual customers and the small-scale renewable generations.

Energy Storage: The large-load customer has a dedicated energy storage system, e.g., batteries or fuel cells, which has a capacity $c$ (in kWh) and a discharging rate limit $\delta$, i.e., the maximum discharge amount per time slot $t$. Customers may hybridize storage technologies with complementary characteristics of power and energy density for desired $c$ and $\delta$ [49]. For example, we can use supercapacitors for short-term power needs and fuel cells for long-term energy needs in a hybrid system. Note that we consider the general scenario where the system is fully charged at the beginning of the on-peak period and will not allow charging in the $T$ slots. This is due to charging by the grid is not only costly in an on-peak period but also impossible for fuel cells, e.g., Bloom Energy servers [50].

Let $\delta_t \geq 0$ (in kWh) be the discharge amount at slot $t$ and $\delta$ be the discharge vector in $\mathbb{R}^T$. Given that the storage system is fully charged initially, the capacity constraint is $\sum_{t=1}^T \delta_t \leq c$ and the discharging constraint is $\delta_t \leq \delta$, for all $t \in [T]$.

For large-load users, we can assume that the storage capacity never exceeds the total demand (even at its minimum):

$$c \leq T \cdot \underline{d}. \quad (1)$$

B. Problem Formulation and Optimal Offline Solution

As motivated in the introduction, we formulate the peak-minimizing storage-discharging (PMD) problem:

$$\text{PMD:} \quad \min_{d \in \mathbb{R}^T} \quad \max_{t \in [T]} \left( d_t - \delta_t \right)$$

subject to

$$\sum_{t=1}^T \delta_t \leq c; \quad \text{(Inventory Constraint)}$$

$$0 \leq \delta_t \leq \min\{\delta, d_t\}, \text{ for all } t \in [T].$$

This objective function is the peak usage. The constraints are due to the storage capacity, the maximum discharging rate, and that there is no need to discharge more than the net demand.

3The approach and analysis herein can be generalized to the case where each slot relates to a different pair of lower and upper bounds of net demand.

4Here the capacity $c$ is defined as the product of the maximum depth of discharge and the inverse of the discharging efficiency ratio (in $[0,1]$). The maximum depth of discharge specifies the maximum fraction of the capacity that can be withdrawn from a storage system without causing irreparable damage to the storage system. The discharging efficiency defines the ratio of the energy received by the demand and that taken out of the system. Similarly, the rate limit $\delta$ is factored in the consideration of the discharging efficiency.
Under the clairvoyant/offline setting where the demand profile \(d\) is known beforehand, the PMD problem can be reformulated as a linear program and it is easy to obtain the optimal offline solution as shown in the following proposition.

**Proposition 1.** The unique optimal offline solution to the PMD problem is given by

\[
\delta^*_t = \left[ \max_{t \in [T]} \left( d_t - \delta - v \right) + v \right]^+, \text{ for all } t \in [T],
\]

where \(v\) satisfies \(\sum_{t=1}^{T} (d_t - v)^+ = c\).

The above proposition shows that the optimal offline solution can be generated by a constant threshold – the maximum of \(\max_{t \in [T]} d_t - \delta\) and \(v\) satisfying \(\sum_{t=1}^{T} (d_t - v)^+ = c\).

In practice, however, the net demand profiles of individual customers are usually not given beforehand, as it is challenging to forecast the fluctuating demand and the volatile renewable generation accurately [12]. This observation motivates us to consider the more practical online setting where the net demands are revealed sequentially in time, yet one has to make irrevocable discharging decisions, without knowing future loads \(\{d_t\}_{t \in [T]}\).

In this section, we develop an online algorithm for the PMD problem with optimal performance guarantee.

### IV. AN OPTIMAL ONLINE ALGORITHM

In this section, we develop an online algorithm for the PMD problem with optimal performance guarantee.

#### A. Online/Real-Time Setting

Under the online/real-time setting, as shown in Fig. 2, the loads \(\{d_t\}_{t \in [T]}\) are revealed chronologically. At each time \(t\), the demand \(d_t \in [\underline{d}, \overline{d}]\) is revealed and one has to make causal and irrevocable discharging decisions, without knowing future demands. Let \(D = \{x \in \mathbb{R}^T \mid \underline{d} \leq x_t \leq \overline{d}, \forall t \in [T]\}\).

We employ CR as the performance metric for online algorithms [18]. For a deterministic online algorithm \(\mathcal{A}\) for PMD, CR is defined as the worst-case ratio between the peak performance gap obtained by the algorithm \(\mathcal{A}\) and that by an optimal offline solution, under all possible input sequences (or demand profiles), i.e., \(CR_{\mathcal{A}} = \max_{d \in D} \frac{\sqrt{\mathcal{A}(d)}}{\sqrt{\mathcal{A}(\mathcal{O}(d))}}\), where \(\mathcal{O}(d)\) and \(\mathcal{O}(d)\) are respectively the optimal offline objective value of PMD under the online algorithm and an optimal offline solution with the same demand profile \(d\). If randomized strategies are allowed, then we can simply extend the definition of CR for a randomized algorithm \(\mathcal{A}\) by substituting \(\mathcal{O}(d)\) with \(\mathbb{E}[\mathcal{O}(d)]\), where the expectation is taken with respect to the distributions used by \(\mathcal{A}\).

Clearly, we have \(CR_{\mathcal{A}} \geq 1\) and prefer online algorithms with smaller CRs. The smallest achievable CR also captures the price of uncertainty that quantifies the fundamental performance gap between online and offline decision-making [18].

It is known that randomization may help to improve the competitiveness of online algorithms. However, the following proposition suggests that it suffices to focus on deterministic algorithms for attaining the best CR for the online PMD.

### B. Overview of the CR-Pursuit Framework and Challenges

Before proceeding, we review the useful CR-Pursuit algorithmic framework. Usually, one first designs an online algorithm and then computes its CR for evaluating the performance. However, the CR-Pursuit framework makes decisions with the goal of “pursuing” a specified CR. Specifically, in each decision-making round, the algorithm characterized by a ratio \(\pi\) will choose actions to maintain the online-to-offline objective ratio to be no more than \(\pi\), given the inputs observed so far. Clearly, if we can always choose the actions to maintain the given ratio \(\pi\) under any input sequence, then the ratio \(\pi\) will be the CR of the algorithm. Although the idea behind CR-Pursuit is simple and intuitive, designing an online algorithm under the framework is non-trivial [37], [44].

Each algorithm under CR-Pursuit is characterized by its CR, and we expect to identify the smallest CR which can be maintained under CR-Pursuit. Here are two key steps of CR-Pursuit: identifying the best “pursued” CR and finding a proper way to maintain the CR in each round. How to carry out these steps are problem-specific and unique challenges of applying the CR-Pursuit framework. More critically, it is not clear whether the best CR-Pursuit algorithm is also optimal among all online algorithms for the considered problem. If so, CR-Pursuit significantly reduces the search space for the optimal online algorithm to a one-dimensional space over the ratios to maintain. Bearing these challenges in mind, we next devise online algorithms for PMD.

### C. First Optimal Online Algorithm for PMD

Recall that \(\mathcal{O}(d)\) denotes the optimal offline objective value of PMD under the demand \(d\). To apply the CR-Pursuit framework, we exploit the revealed inputs and identify a reference input sequence \(d^* \in \mathbb{R}^T\) as \([d_1, d_2, \ldots, d_T, d_T]\) for each time slot \(t \in [T]\). Then, we define a class of algorithms, named after pCR-PMD(\(\pi\)). Each pCR-PMD(\(\pi\)) is characterized by a ratio \(\pi \geq 1\) specified before we observe any inputs. We denote by \(\delta(\pi, d) \in \mathbb{R}^T\) the output sequence under pCR-PMD(\(\pi\)) and the input sequence \(d\), whose \(t\)th element is obtained by the formula in Algorithm [1] for all \(t \in [T]\). Observe that in each
Algorithm 1: pCR-PMD(\(\pi\))

\[
\text{for } t = 1, 2, \ldots, T \text{ do} \\
\quad \text{The discharge amount at time slot } t \text{ is given by } \delta_t(\pi, d) = |d_t - \pi \cdot v(d_t)|^{+};
\]

decision making, we solve an offline PMD under the demand profile \(d^t\), while we pursue the CR \(\pi\) by maintaining the online-to-offline objective ratio under the reference input \(d^t\) to be no more than \(\pi\), for all \(t \in [T]\). However, the question arises whether \(\delta(\pi, d)\) is always a feasible solution to PMD for any demand profile \(d \in D\). If so, then pCR-PMD(\(\pi\)) attains the CR \(\pi\) by definition; otherwise, the CR of pCR-PMD(\(\pi\)) exceeds \(\pi\). This puzzle motivates us to define and characterize the feasibility of pCR-PMD algorithms in the following.

**Definition 1.** pCR-PMD(\(\pi\)) is feasible if \(\delta(\pi, d)\) is a feasible solution to PMD for any \(d \in D\).

pCR-PMD(\(\pi\)) is feasible if and only if it generates a feasible solution to PMD for any possible input sequence. Then, identifying the best pCR-PMD algorithm is equivalent to finding the smallest ratio \(\pi\) such that pCR-PMD(\(\pi\)) is feasible. For this purpose, we derive the following proposition, which characterizes the feasibility of pCR-PMD(\(\pi\)) by a threshold condition involving the storage capacity.

**Proposition 3.** pCR-PMD(\(\pi\)) is feasible if and only if

\[
\Phi(\pi) \leq c, \text{ where } \Phi(\pi) = \max_{d \in D} \sum_{t=1}^{T} \delta_t(\pi, d).
\]

Moreover, the lemma below implies that the smallest \(\pi\) with \(\Phi(\pi) \leq c\) gives the best pCR-PMD algorithm for PMD.

**Lemma 1.** \(\Phi(\pi)\) strictly decreases in \(\pi\) over \([1, \bar{\pi}]\), where \(\bar{\pi}\) is the smallest ratio with \(\Phi(\bar{\pi}) = 0\).

With Lemma 1 we derive a main result of this work.

**Theorem 1.** Given \(\bar{\delta}, \bar{T}, \bar{D}\), and \(\bar{\pi}\), the unique solution \(\pi^*\) to \(\Phi(\pi) = c\) is the best CR of all online algorithms for PMD.

The above theorem shows that the best pCR-PMD algorithm is also the best among all online (deterministic and randomized) algorithms for PMD. As a result, we can reduce the search of an optimal online algorithm for PMD to the search of a feasible pCR-PMD algorithm with the smallest possible CR \(\pi^*\). Theorem 1 also characterizes the price of necessity as the unique solution to the univariate equation \(\Phi(\pi) = c\). Nevertheless, the function \(\Phi(\pi)\) seems complicated at first sight and it is unclear how to characterize the optimal CR \(\pi^*\).

**D. Computing the Optimal Competitive Ratio \(\pi^*\)**

Owing to the complexity of \(\Phi(\pi)\), it is hard to derive the analytic expression of the optimal CR \(\pi^*\). As a key contribution, we transform the computation of \(\pi^*\) into solving a sequence of linear-fractional problems, each of which is as simple as solving linear programming problems [51, Section 4.3.2]. As discussed later, a direct approach leads to an exponential number of linear-fractional problems to solve, regarding the time-slot number \(T\). Interestingly, we carefully exploit the problem structure and show that it suffices to consider at most \(T\) such linear-fractional problems.

First, we observe that if pCR-PMD(\(\pi\)) is feasible, then for any set \(\mathcal{I} \subseteq [T]\) and any input sequence \(d \in D\), we have \(\sum_{i \in \mathcal{I}} (d_i - \pi \cdot v(d_i)) \leq \sum_{t=1}^{T} |d_t - \pi \cdot v(d_t)|^{+} \leq c\), implying

\[
\sum_{i \in \mathcal{I}} d_i - c \leq \sum_{i \in \mathcal{I}} v(d_i) \leq \pi. \tag{2}
\]

In other words, the optimal CR \(\pi^*\) is lower bounded by the smallest ratio \(\pi\) satisfying (2) for any set \(\mathcal{I} \subseteq [T]\) and any demand profile \(d \in D\). Moreover, there exist an index set \(\mathcal{I}^*\) and a demand profile \(d^*\) such that the equality of (2) holds for \(\mathcal{I}^*, d^*\) and \(\pi^*\); thus the lower bound is also tight. This observation, together with Lemma 1 and Proposition 3 leads to the following proposition.

**Proposition 4.** Given \(c, \bar{\delta}, \bar{T}, \bar{D}\), and \(\bar{\pi}\), the best CR of all online algorithms for PMD is \(\pi^* = \max_{\mathcal{I} \subseteq [T]} \max_{d \in D} \sum_{i \in \mathcal{I}} d_i - c \left/ \sum_{i \in \mathcal{I}} v(d_i) \right.\).

The above proposition suggests that we can find the optimal CR \(\pi^*\) by solving a sequence of linear-fractional programs. Specifically, we define a class of linear-fractional problems, each of which is parameterized by a nonempty subset \(\mathcal{I}\) of \([T]\) and named after CR-Compute(\(\mathcal{I}\)):

\[
\text{CR-Compute(}\mathcal{I}\text{): } \max_{d \in \mathcal{D}, \delta, u, \delta_{ij}} \sum_{i \in \mathcal{I}} d_i - c \left/ \sum_{i \in \mathcal{I}} v(d_i) \right. \text{ subject to } \sum_{j=1}^{T} \delta_{ij} = c, \text{ for all } i \in [T];
\]

\[
0 \leq \delta_{ij} \leq \bar{\delta}, \text{ for all } i, j \in [T];
\]

\[
d_j - \delta_{ij} \leq u_i, \text{ for all } 1 \leq j \leq i \leq T;
\]

\[
d_j - \delta_{ij} \leq u_i, \text{ for all } 1 \leq i < j \leq T.
\]

The constraints above are due to the offline PMD problem solved in each time slot under any pCR-PMD algorithm. Specifically, the variables \(\delta_{ij}, j \in [T]\) and the variable \(u_i\) are respectively associated with the optimal solution and the optimal objective of PMD under the demand profile \(d^t\), for all \(i \in [T]\). The objective function comes from Proposition 3. By abuse of notation, let CR-Compute(\(\mathcal{I}\)) denote the resulting optimal objective value. Then, we attain the following lemma.

**Lemma 2.** For any nonempty set \(\mathcal{I} \subseteq [T]\), it follows that

\[
\text{CR-Compute(}\mathcal{I}\text{)} = \max_{d \in [T]} \sum_{i \in \mathcal{I}} d_i - c \left/ \sum_{i \in \mathcal{I}} v(d_i) \right.
\]

We conclude from Proposition 4 and Lemma 2 that the best CR \(\pi^*\) of all online algorithms for PMD is given by \(\max_{\mathcal{I} \subseteq [T]} \text{CR-Compute(}\mathcal{I}\text{)}\). That is, we can convert the computation of \(\pi^*\) into solving a sequence of linear-fractional programs CR-Compute(\(\mathcal{I}\)) parameterized by \(\mathcal{I} \subseteq [T]\). Note that, for any \(\mathcal{I} \subset [T]\), CR-Compute(\(\mathcal{I}\)) can be tackled by transforming to a linear program [51, Section 4.3.2]. Although Proposition 4 and Lemma 2 suggest a numerical way to obtain the best CR \(\pi^*\), we have to solve an exponential number (\(2^T\)) of linear programs for the best CR \(\pi^*\). That is undesirable and we expect to exclude as many redundant CR-Compute(\(\mathcal{I}\)) programs as possible. To this end, we exploit the worst-case input sequences of pCR-PMD(\(\pi^*\)), which will use up the storage under the algorithm.
The best CR of all online algorithms for PMD is to a linear number (less than CR-Compute of the following theorem and significantly reduce the number of CR-Compute of the online algorithm). Theorem 2 indicates that the anytime CR at any epoch to improve online decisions, given the inputs of future uncertainty and pursues the adaptive best possible performance and an adaptive average-case performance. To this end, we extend the pCR-PMD(π*) algorithm to an anytime-optimal one by adaptively pruning the input space based on the observations so far and adjusting the online decisions for the best competitiveness regarding the residual uncertainty. That is, the anytime-optimal algorithm captures the varying prices of future uncertainty and pursues the adaptive best possible CR at any epoch to improve online decisions, given the inputs and online actions so far. In this way, we not only retain the optimal worst-case performance but also achieve an adaptive worst-case performance. These results suggest the potential of combining efficiency and robustness, leading to more practical competitive online algorithms.

Before proceeding, we extend the concept of CR to a sequence of anytime CRs. Recall that the CR of an algorithm implies the worst-case online-to-offline ratio of the objective value attained by the algorithm over all possible inputs. Intuitively, we say that the anytime-optimal CR at slot t under the anytime-optimal pCR-PMD: π∗, πub, q = maxπ∈AOCR-THR(π, T), if q ≥ (c − ∑t−1 k=1 δk) then πub = πub, otherwise;

Algorithm 2: Anytime-Optimal pCR-PMD Algorithm for t = 1, 2, . . . , T do
The discharge amount at time slot t is given by δt = (d∗ − πub ∗ v(d∗))∗.

Algorithm 3: A Bisection Method for πub in the Anytime-Optimal pCR-PMD Algorithm
Input: c, δ, T, d, d, observed inputs dk, k ∈ [t], implemented actions δk, k ∈ [t − 1], and πub − 1;
Output: The anytime-optimal CR at time slot t under Anytime-Optimal pCR-PMD: πub, q = maxπ∈AOCR-THR(π, T), if q ≥ (c − ∑t−1 k=1 δk) then πub = πub, otherwise;

combining efficiency and robustness, leading to more practical competitive online algorithms.

Figure 3: The best CR π∗ varies as c, T, d, and d. By default, we set c = 80, T = 12, d = 100, d = 300 and δ = ∞.
While the best possible CR $\pi^*$ captures the price of uncertainty before we know any inputs, the anytime-optimal CR $\pi_\tau^*$ at slot $t$ characterizes the instantaneous price of uncertainty at time slot $t$, given the observed inputs $d_k$, $k \in [t]$ and implemented actions $\delta_k$, $k \in [t-1]$. That is, an online algorithm can at best maintain the online-to-offline ratio of peak purchased demand to be $\pi_\tau^*$, for all possible future inputs. It is clear that $\pi^*_t$ is subject to the observed inputs and online actions, for all $t \in [T]$. Based on the introduction of anytime-optimal CRs, we next introduce the adaptive extension of the pCR-PMD($\pi^*$) algorithm.

The anytime-optimal pCR-PMD algorithm improves pCR-PMD($\pi^*$) following the anytime-optimal competitive framework, where we always pursue the best possible worst-case performance guarantee at any time slot based on previous inputs and online decisions. That is, at each time slot $t$, instead of the optimal CR $\pi^*$, we maintain the online-to-offline ratio of the peak purchased demand in future time slots to be no more than the anytime-optimal CR $\pi_\tau^*$, since $\pi_\tau^*$ essentially captures the price of future uncertainty given the revealed inputs and implemented actions so far. We present the pseudocodes of the anytime-optimal pCR-PMD in Algorithm 3. Note that it is unnecessary to decrease the purchased demand of the current time slot to be less than the peak purchased demand in previous time slots. This fact brings additional difficulty in characterizing the anytime-optimal CRs, compared to the best CR $\pi^*$, as elaborated in the sequel.

First, we observe that the anytime-optimal CR $\pi_\tau^*$ is no more than the best CR $\pi^*$, because we exploit the additional information $d_1$. Second, since the anytime-optimal pCR-PMD algorithm makes decisions by pursuing the anytime-optimal CR at each time slot, we conclude that the sequence $\pi_\tau^*, t \in [T]$ is nonincreasing in $t$, whatever the inputs are. Third, we see that $\pi_\tau^* \geq \max_{k \in [t-1]} (d_k - \delta_k) / \nu(d')$, because the ultimate online peak usage under the input sequence $d'$ is no less than $\max_{k \in [t-1]} (d_k - \delta_k)$. If the equality holds, we have $\delta_t = [d_t - \max_{k \in [t-1]} (d_k - \delta_k)]_+$. Based on the three observations, we shall show how to search for $\pi_\tau^*$ by a bisection method. To this end, in the following, given observed inputs $d_k$, $k \in [t]$ and actions $\delta_k$, $k \in [t-1]$, we define a linear program parameterized by a ratio $\pi \in [\max_{k \in [t-1]} (d_k - \delta_k) / \nu(d'), \pi_{\tau-1}^*]$ and a set $I \in I_t$, where $I_t = \{ [k] \setminus [t] | k = t, t+1, \ldots, T \}$, denoted as AOCR-THR($\pi, I$), as follows:

$$\max_{u_{ij}, \delta_{ij}, d_t, i \in I} \left[ d_t - \max_{k \in [t-1]} \left( \pi \nu(d') \right) \right] + \sum_{i \in I} \left( d_i - \pi u_{ii} \right)$$

subject to

$$\sum_{j=1}^{T} \delta_{ij} = c_i, \text{ for all } i \in I;$$
$$0 \leq \delta_{ij} \leq \delta, \text{ for all } i \in I \text{ and } j \in [T];$$
$$d_j - \delta_{ij} \leq u_{ij}, \text{ for all } i \in I \text{ and } j \leq i;$$
$$d - \delta_{ij} \leq u_{ij}, \text{ for all } t \leq i < j \leq T;$$
$$\max_{k \in [t-1]} (d_k - \delta_k) \leq d_t, d \leq d_t \leq \delta;$$
$$\max_{k \in [t-1]} (d_k - \delta_k) \leq \pi u_{ii}, \text{ for all } i \in I.$$

The above objective function corresponds to the sum of discharge amounts over a set of time slots assuming that we uniformly maintain the online-to-offline ratio of peak purchased demand to be $\pi$ in all future time slots. Similar to CR-Compute($I$), the constraints of AOCR-THR($\pi, I$) are due to the offline PMD problem solved in each time slot under the anytime-optimal pCR-PMD, while the difference lies in that we never decrease the purchased demand of a future time slot to be less than the observed peak purchased demand, as described by the inequality constraints on the last line. By similar arguments for computing the best CR $\pi^*$, we conclude that the anytime-optimal CR at time slot $t$ should be the smallest ratio $\pi$ in $[\max_{k \in [t-1]} (d_k - \delta_k) / \nu(d'), \pi_{\tau-1}^*]$ such that $\max_{I \in I_t} \text{AOCR-THR}(\pi, I)$ does not exceed a threshold, namely, the inventory $c - \sum_{k=1}^{T-1} \delta_k$. Thus, we can search for $\pi_\tau^*$ by the bisection method as shown in Algorithm 3.

If the input sequence $d$ is in the worst case regarding pCR-PMD($\pi^*$), then $\pi_\tau^* = \pi^*$, for all $t \in [T]$. Otherwise, there is an index $\tau \in [T]$ such that $\pi_\tau^* < \pi^*$, for all $t \geq \tau$. From this perspective, we show that the anytime-optimal pCR-PMD still achieves the optimal CR among all online algorithms for PMD, and performs better than pCR-PMD($\pi^*$) under general cases. As a whole, extending pCR-PMD($\pi^*$) to the anytime-optimal pCR-PMD can improve the practical utilization of storage for minimizing the peak demand, which will be further verified by real-world traces later.

VI. ELECTRICITY BILL REDUCTION

In this section, we show the the solutions obtained by the pCR-PMD($\pi^*$) and anytime-optimal pCR-PMD algorithms are also competitive for minimizing the electricity bill that consists of the peak-demand and volume charges [9].

Let $p^m$ and $p^e_i$ (in $$/kWh) respectively be the peak-demand charge rate and the volume charge rate of each slot $t \in [T]$. Given $p^m$ and $p^e_i$, $t \in [T]$, we formulate the bill-minimizing storage-discharging (BMD) problem as follows:

$$\text{BMD: } \min_{\delta \in \mathbb{R}^T} \sum_{t=1}^{T} p^e_i (d_t - \delta_t) + p^m \max_{t \in [T]} (d_t - \delta_t)$$

subject to

$$\sum_{t=1}^{T} \delta_t \leq c;$$
$$0 \leq \delta_t \leq \min\{\delta_i, d_i\}, \text{ for all } t \in [T].$$

The above problem BMD differs from problem PMD in Sec. III-B only in the objective function. Thus it is not surprising that they share the same optimal (offline) solution when the peak-demand charge dominates the overall bill. In particular, BMD and PMD share the same optimal offline solution given in in Proposition 1 when

$$p^m \geq T \cdot \max_{i,j \in [T]} \left( p^e_i - p^e_j \right).$$

The above inequality usually holds in practice since a demand-charge rate (around 20 $$/kW$$) is typically significantly higher than the ToU rate differences (around 0.1 $$/kW$$) and the on-peak period is around 15 hours, i.e., $T = 60$ when the measurement interval is 15 minutes [48]. We thus assume that the condition in (3) is satisfied in the following discussion.
Given the similarity between problem BMD and PMD, one way to solve problem BMD in an online fashion is to leverage the schemes developed for problem PMD. Specifically, given the same demands revealed sequentially in time, one can apply the pCR-PMD($\pi^*$) and anytime-optimal pCR-PMD algorithms to obtain an online solution for problem PMD, for minimizing the peak-demand charge, and then use the same solution for solving BMD, for minimizing the total electricity bill. Interestingly, the following lemma shows that doing so gives us the first set of online algorithms for minimizing the total bill with strong performance guarantee.

**Lemma 4.** By directly applying the solutions from pCR-PMD($\pi^*$) (for solving problem PMD) to problem BMD, we achieve a CR no larger than $\pi^*$. Similarly, the achieved anytime CR of slot $t$ under the anytime-optimal pCR-PMD (for solving problem BMD) is no larger than $\pi^*_t$, for all $t \in [T]$.

The proof comes from the definition of CR and the fact that BMD and PMD share the same optimal offline solution given the same $d$ under the assumption in [4]. The lemma says that the two peak-minimizing online algorithms can be used to generate online solutions for problem BMD, with strong worst-case performance guarantee. Meanwhile, it is of great interest to design algorithms with better CRs for problem BMD, which we leave for future studies.

In practice, we note that the two algorithms may not use up the storage at the end of the on-peak period. In the case where the energy storage system is charged in off-peak period with a lower cost, it is often preferred to fully discharge the energy storage system by the end of on-peak period. One way to do so is to adapt the anytime-optimal pCR-PMD by discharging the maximum possible amount of stored energy for maintaining the online-to-offline ratio to be no more than $\pi^*_t$, instead of the minimum amount for each slot $t \in [T]$. This will allow us to fully utilize the storage under all input sequences while maintaining the CR in Lemma 4.

## VII. SIMULATION

### A. Data and Evaluation Setup

We conduct simulations by using a 3-month traces from an EV charging station. The raw data consist of a collection of transactions. Each transaction records the starting time, duration (in minutes), and consumed energy (in kWh) of an ordered charging. Assume that the charging rate is constant in the recorded duration and convert the raw data into time series regarding the electricity consumption of the station. Then, we can identify the on-peak period in a day and uniformly divide the period into 15-minute time slots. Note that the demand from EV charging relies on the number of available charging piles and is scalable. Thus, we slightly scale up the demand such that the expected power exceeds 3 MW (750 kWh for a 15-minute duration), which refers to large power in many tariffs [4]. Then, on a daily basis, we preprocess the raw data and evaluate the performance of online algorithms. Specifically, we divide the daily on-peak period into $T = 20$ time slots and set the demand bounds as $(d, \bar{d}) = (442.91, 1020.10)$ kWh. We plot four demand curves in Fig. 4 and observe that the EV charging loads are highly volatile.

![Figure 4: An illustration of load volatility via four representative demand curves from real-world traces.](image)

### Table I: The online-to-offline ratios of peak usage.

| Capacity Rate | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 |
|---------------|------|------|------|------|------|
| $\pi^*$       | 1.3732 | 1.4621 | 1.6031 | 1.7953 | 2.0788 |
| pCR-PMD       | 1.2909 | 1.4216 | 1.5669 | 1.7543 | 2.0244 |
| AO\_pCR       | 1.1960 | 1.2236 | 1.2514 | 1.2912 | 1.3736 |

### B. Empirical Evaluation of Online Algorithms

1) **Anytime-Optimal pCR-PMD vs. pCR-PMD($\pi^*$):**

We have theoretically shown in Section V that the anytime-optimal pCR-PMD has the same CR with pCR-PMD($\pi^*$), while the former in principle presents an adaptive average-case performance. We herein observe from our real-world simulations that the anytime-optimal pCR-PMD (ref. AO\_pCR) attains better empirical performance than pCR-PMD($\pi^*$) (ref. pCR-PMD) under a wide range of storage capacities, as shown in Fig. 5a. The peak usage rate herein refers to the ratio between the peak purchase after storage discharging and the original peak demand. We normalize the storage capacity and the capacity rate refers to the ratio of the storage capacity and the average daily energy consumption (13,083 kWh). We delineate the average peak usage rates and the corresponding standard deviations in Fig. 5a. Specifically, we observe that, on average, the peak usage reduction under the anytime-optimal pCR-PMD is more than twice that under pCR-PMD($\pi^*$), while the peak usage rate under the anytime-optimal pCR-PMD decreases faster than that under pCR-PMD($\pi^*$) as the storage capacity increases. Moreover, we define the empirical performance ratio of an algorithm as the ratio between the average peak usage under the algorithm and that under the optimal offline solution. In the typical setup from real-world traces, we list the best CRs (namely $\pi^*$), the empirical performance ratios of pCR-PMD($\pi^*$), and the empirical performance ratios of the anytime-optimal pCR-PMD under different storage capacities in Table I. As we can see, the anytime-optimal pCR-PMD attains smaller empirical performance ratios compared with pCR-PMD($\pi^*$). These observations substantiate the significance of absorbing real-time information and the advantage of the anytime-optimal pCR-PMD over pCR-PMD($\pi^*$).

We also observe from Table I that the empirical performance ratios of the anytime-optimal pCR-PMD are much smaller than the corresponding best possible CRs. That is because the worst cases regarding the optimal CRs rarely happen. In contrast with pCR-PMD($\pi^*$) which ignores such non-occurrences and keeps maintaining the optimal CR $\pi^*$, the anytime-optimal pCR-PMD adaptively identifies the changing prices of future uncertainty and progressively pursues the anytime-optimal CRs across time instead. Moreover, we
We observe from Fig. 5b and Fig. 5c that our algorithm attains much better empirical performance than the threshold and RHC algorithms under a wide range of storage capacities. Particularly, when the storage capacity rate is 30%, the anytime-optimal pCR-PMD can improve the average peak usage reduction by at least 19% compared to the alternatives. Moreover, as the storage capacity increases, the average peak usage rate of our anytime-optimal pCR-PMD decreases much faster than that of any baseline algorithm.

3) Impact of Discharging Limit: We show the impact of the maximum discharge amount per time slot (δ) in Fig. 6, where the storage capacity is fixed as 3,000 kWh, almost 23% of the average daily demand. The discharging limit (δ/d) refers to the ratio between the maximum discharge amount and the upper bound of the net demand in a time slot. Increasing the limit will improve the optimal offline outcome. However, it may worsen the outcomes of certain online algorithms, e.g., RHC_lb and RHC_half. The peak usage rate of our anytime-optimal pCR-PMD first decreases and then slightly increases as we raise the discharging limit. Since the turning point is 0.434 ≈ d/d, a possible reason for the non-monotonicity may be that the maximum discharge amount exceeds the lower bound of the net demand when the assumption (1) holds. Also, we conclude from Fig. 6 that the discharging rate need not to be very large, because it has less impact on the anytime-optimal pCR-PMD and will no longer reduce the optimal offline usage after exceeding a certain value.

VIII. CONCLUSIONS AND FUTURE WORK

We study online peak-demand minimization under energy storage constraints and develop an optimal online algorithm pCR-PMD(\(\pi^*\)). It achieves the best possible CR among all deterministic and randomized online algorithms. We show that the optimal CR \(\pi^*\) can be computed by solving a linear number of linear-fractional programs, incurring only polynomial time complexity. To our best knowledge, these are the first (and optimal) results for this theoretically challenging yet practically relevant problem. Furthermore, we generalize our approach to design an anytime-optimal algorithm to retain the optimal worst-case performance and achieve adaptive average-case performance. The idea is to adaptively prune the input space based on the inputs observed so far and adjust the online decisions to achieve the anytime-optimal CR concerning the residual uncertainty. Finally, we demonstrate the empirical efficiency of our algorithms by simulations based on real-world traces. An interesting future direction is to consider real-time charging/discharging and other objective functions such as cost reduction relevant to energy storage management.
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Proof of Proposition 2

Without loss of generality, we can assume that Algorithm $\mathcal{A}$ is a mixed algorithm for PMD, which is a probability distribution $\{\omega(\mathcal{C})\}$ over a collection of deterministic online algorithms: $\mathcal{C} \in C$. Given an arbitrary demand profile $d$, the expected peak purchased demand of Algorithm $\mathcal{A}$ is

$$E[v_\mathcal{A}(d)] = \int \max_{\mathcal{C} \in \mathcal{C}} (d_t - \delta^\mathcal{C}_t) d\omega(\mathcal{C}),$$

where $\delta^\mathcal{C}_t$ is the discharging quantity under Algorithm $\mathcal{C} \in \mathcal{C}$ and the demand profile $d$. With respect to $d$, we consider a deterministic algorithm $\mathcal{B}$, under which the discharging quantity of the $t$th time slot is $\int_{\mathcal{C}} \delta^\mathcal{C}_t d\omega(\mathcal{C})$, for all $t \in [T]$. Since $\delta^\mathcal{C}$ is a feasible solution to PMD for all $\mathcal{C} \in \mathcal{C}$, it follows that Algorithm $\mathcal{B}$ also generates a feasible solution to PMD. Then, the peak purchased demand under Algorithm $\mathcal{B}$ and the last inequality is due to the convexity of the max quantity of the $\pi$ completes the proof.

Proof of Theorem 1

The uniqueness of the solution to $\Phi(\pi) = c$ follows naturally from Lemma 1. In addition, by Proposition 2, it suffices to consider deterministic online algorithms for PMD. Let $\delta(\mathcal{A}, d)$ be an output sequence of any other deterministic online algorithm $\mathcal{A}$ for PMD under the input sequence $d$. Let $\hat{d}$ be a worst-case input sequence of $\text{pCR-PMD}(\pi^*)$, namely, $\sum_{t=1}^T \delta_t(\pi^*, \hat{d}) = c$. Then, we shall construct an input sequence over which the algorithm $\mathcal{A}$ cannot achieve an offline-to-online ratio of peak usage which is strictly smaller than $\pi^*$.

Let $d$ be an input sequence with $d \in \mathcal{D}$ and $d_1 = \hat{d}_1$. If $\delta_1(\mathcal{A}, d) < \delta_1(\pi^*, d)$, then we have

$$\max_{t \in [T]} (d_t - \delta_1(\mathcal{A}, d)) \geq d_1 - \delta_1(\mathcal{A}, d) > d_1 - \delta_1(\pi^*, d).$$

It follows from $\delta_1(\pi^*, d) > 0$ that $d_1 - \delta_1(\pi^*, d) = \pi^* v(d^1)$. Thus, if it happens that $\hat{d} = d^1$, then the peak usage under the online algorithm $\mathcal{A}$ is strictly larger than $\pi^*$ times the optimal offline objective value of PMD. This implies that the CR of the algorithm $\mathcal{A}$ is strictly bigger than $\pi^*$.

By Theorem 1, we can find an input sequence $\hat{d}$ which satisfies

$$\sum_{t=1}^T \delta_t(\pi^*, \hat{d}) = c.$$ 

If there is an index $i$ such that $\delta_i(\pi^*, d) > 0$ whenever $t \in [i]$ and $\delta_i(\pi^*, d) = 0$ whenever $t > i$, then we complete the proof. Otherwise, we observe that there exists an index $i$ such that $\hat{d}_i - \pi^* v(d^i) \leq 0$ and $\hat{d}_{i+1} - \pi^* v(d^{i+1}) > 0$.

Proof of Lemma 1

First, we observe that for any input sequence $d$, the following function indexed by $t \in [T]$ is strictly decreasing over $\pi$ when its function value is positive: $\delta_t(\pi, d) = (d_t - \pi v(d^t))^+$. Then, for any fixed input sequence $d$, we summarize the above functions over $t \in [T]$ and obtain another nonincreasing function over $\pi$: $\sum_{t=1}^T \delta_t(\pi, d)$, which is also strictly decreasing when its function value is positive. Thus, we can conclude that $\Phi(\pi)$ is strictly decreasing before it attains zero.
It follows that
\[ \delta_i(\pi^*, \tilde{d}) \geq \delta_{i+1}(\pi^*, \tilde{d}) > 0 \] and \[ \delta_i(\pi^*, \tilde{d}) = \delta_{i+1}(\pi^*, \tilde{d}) = 0. \]

Moreover, we see that \[ \delta_i(\pi^*, \hat{d}) = \delta_i(\pi^*, \tilde{d}) \] for all \( t \in [T] \) and \( t \notin \{i, i+1\} \). It follows that
\[
c = \Phi(\pi^*) \geq \sum_{t=1}^{T} \delta_t(\pi^*, \tilde{d}) \geq \sum_{t=1}^{T} \delta_t(\pi^*, \hat{d}) = c,
\]
which further implies that \( \sum_{t=1}^{T} \delta_t(\pi^*, \tilde{d}) = c \). Following similar arguments, by a series of exchanges of adjacent elements from \( \hat{d} \), we can finally construct a worse-case input sequence \( \hat{d} \) with an index \( i \) such that \( \delta_t(\pi^*, \hat{d}) > 0 \) if \( t \in [i] \) and \( \delta_t(\pi^*, \hat{d}) = 0 \) otherwise.