Probing the non-locality of Majorana fermions via quantum correlations

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Majorana fermions (MFs) are exotic particles that are their own anti-particles. Recently, the search for the MFs occurring as quasi-particle excitations in solid-state systems has attracted widespread interest, because of their fundamental importance in fundamental physics and potential applications in topological quantum computation based on solid-state devices. Here we study the quantum correlations between two spatially separate quantum dots induced by a pair of MFs emerging at the two ends of a semiconductor nanowire, in order to develop a new method for probing the MFs. We find that without the tunnel coupling between these paired MFs, quantum entanglement cannot be induced from an unentangled (i.e., product) state, but quantum discord is observed due to the intrinsic nonlocal correlations of the paired MFs. This finding reveals that quantum discord can indeed demonstrate the intrinsic non-locality of the MFs formed in the nanowire. Also, quantum discord can be employed to discriminate the MFs from the regular fermions. Furthermore, we propose an experimental setup to measure the onset of quantum discord due to the nonlocal correlations. Our approach provides a new, and experimentally accessible, method to study the Majorana bound states by probing their intrinsic non-locality signature.

Quantum entanglement is a quantum correlation that has no classical analog. Dynamical entanglement has exhibited some exotic properties such as the early-stage decoherence¹ and revival². Recently, it was found that there exists another type of nonclassical correlation coined as quantum discord³⁴. A remarkable feature of the quantum discord is that it can characterize some fundamental processes of physics, including the quantum phase transitions⁵ and the discrimination of quantum and classical Maxwell’s demons⁶⁷. In addition, it can be used as a new resource for quantum information processing protocols⁸–¹⁰, and be harnessed as a measurable quantity to implement quantum-enhanced tasks in the absence of quantum entanglement¹¹–¹³.

Majorana fermions (MFs) as quasi-particle excitations were anticipated to occur in different solid-state systems, such as the 5/2 fractional quantum Hall system¹⁴, the p-wave superconductor¹⁵, and the hybrid systems of a topological insulator¹⁶. Recently, semiconductor nanowires with strong spin-orbit coupling are attracting increasing interest. In addition to the application in spin-orbit qubit¹⁷,¹⁸, these semiconductor nanowires provide a platform for demonstrating MFs¹⁹–²². Theoretically, it is shown that MFs can occur at the ends of a semiconductor nanowire with strong spin-orbit coupling when the nanowire is placed in proximity to an s-wave superconductor¹⁹–²⁰. It is expected that the nanowire-superconductor system based on conventional materials is more accessible to experimentally detecting the MFs²¹–²². In solid-state systems, MFs are coherent superpositions of electron and hole excitations. Two spatially separated MFs can form one fermionic level, which can be either occupied or empty, and thus defines a nonlocal qubit. The paired MFs may exhibit intrinsic nonlocal properties, so the search for the quantum effects induced by such nonlocal paired MFs can provide an important signature for the existence of MFs. Several schemes have been proposed to test the non-locality of MFs by studying the correlation behaviors of electron tunneling through individual MFs²³–²⁵. However, for the completely separated MFs, the tunneling events through each MF are shown to be fully independent²³–²⁵. Therefore, the intrinsic non-locality of the paired MFs cannot be demonstrated in such a context.

A hybrid quantum system combining two or more physical systems can often utilize the strengths of different systems to better explore new phenomena and potentially bring about novel quantum technologies²⁶. For the two MFs emerging at the ends of a semiconductor nanowire, because their energies are zero, it is difficult to directly demonstrate their intrinsic non-locality, particularly when the tunnel coupling between them becomes zero. Thus, to probe the emerged two MFs in the nanowire, in this work, we consider a hybrid quantum system consisting of two quantum dots (QDs) interacting with these two MFs. To characterize the quantum correlations between the two QDs mediated by the paired MFs, we use both quantum entanglement and quantum discord.
show that when the paired MFs are tunnel-coupled via the wave-function overlapping, the maximally entangled states can be induced from an unentangled (i.e., product) state at specific dynamical time points. However, when the paired MFs are completely separated (i.e., no tunneling between them), quantum entanglement cannot be induced from this product state, but remarkably, quantum discord is shown to still persist owing to the intrinsic nonlocal correlation of MFs. This indicates that the quantum discord can indeed reflect the non-locality of the paired MFs. Furthermore, we propose an experimentally accessible approach to measuring the onset of quantum discord. Our protocol provides a new method to probe the MFs emerging at the two ends of the nanowire.

Results

Quantum dynamics. We study a hybrid structure shown in Fig. 1, where a semiconductor nanowire with strong spin-orbit coupling is placed in proximity to an s-wave superconductor. This nanowire is predicted to be driven into a topological superconducting phase\textsuperscript{19,20}. A pair of zero-energy MFs, $\gamma_1$ and $\gamma_2$, are anticipated to appear at the two ends of the nanowire. The Hamiltonian of the paired MFs is described by $H_M = \kappa m \gamma_1 \gamma_2 / 2$, where $\kappa m \sim e^{-L/\xi}$ describes the tunnel coupling between the two MFs due to the overlap of wavefunctions. This inter-MF coupling dampens exponentially with $L$ (the length of the nanowire) and $\xi$ (the superconducting coherent length). Because the paired MFs have zero energies, it is difficult to directly demonstrate their quantum correlations, particularly when the tunnel coupling $\kappa m$ becomes zero. Thus, we introduce two QDs (i.e., QD$_1$ and QD$_2$) on both sides of the nanowire, each of which is tunnel-coupled to a MF neighboring to it. In addition, we consider the strong Coulomb blockade regime for each QD, so that only one electron is allowed therein when the tunneling process takes place. The Hamiltonian of the two spatially separated quantum dots is written as $H_{QD} = \sum_{j=1,2} \varepsilon_j d_j^\dagger d_j$, where $d_j^\dagger$ ($d_j$) is the electron creation (annihilation) operator of the dot $j$, with the corresponding energy $\varepsilon_j$. The tunneling Hamiltonian between the QDs and the MFs is described by\textsuperscript{23-25} $H_{TM} = \sum_{j=1,2} \lambda_j (d_j^\dagger - d_j) \gamma_j$, with $\lambda_j$ ($j = 1, 2$) denoting the tunnel coupling of the $j$th dot to its neighboring MF.

The total Hamiltonian of the system is a sum of the above three parts: $H_{sys} = H_M + H_{QD} + H_{TM}$. For practical calculations, it is more convenient to switch from the Majorana representation to the regular fermion-one via the transformations: $\gamma_1 = i(f - f')$ and $\gamma_2 = f + f'$, where $f$ is the regular fermion operator, satisfying the anticommutation relation $\{f, f'\} = 1$. After an additional local gauge transformation $d_i \rightarrow id_i$, the total Hamiltonian of the system can be written in the new representation as\textsuperscript{23-25}

$$H_{sys} = \varepsilon_m (f_1 f - 1/2) + \sum_{j=1,2} \left[ \varepsilon_j d_j^\dagger d_j + \lambda_j (d_j^\dagger f + f' d_j) \right] - \lambda_1 (d_1^\dagger f + f d_1) + \lambda_2 (d_2^\dagger f' + f d_2).$$

(1)

For simplicity, we adopt a symmetric setup with $\lambda_1 = \lambda_2 = \lambda$. Moreover, the two QDs are adjusted in resonance with the two MFs (i.e., $\varepsilon_1 = \varepsilon_2 = 0$) via the gate voltages. Also, we use $\lambda (1/\lambda)$ as the energy (time) unit throughout the paper.

The QD-MF system considered in this paper includes the two QDs (i.e., QD$_1$ and QD$_2$) and the MFs. For the new fermion representation, it is convenient to use the state basis $|n_1, n_2, n_3\rangle$, where $n_i = 0$ or 1, with $l = 1, 2, M$, denoting the electron occupation number in the left QD (i.e., QD$_1$), the right QD (i.e., QD$_2$) and the paired MFs, respectively. From Hamiltonian (1), one may notice that the electron numbers in both QDs and the paired MFs are not conserved, because a pair of electrons can be extracted out from the superconductor or absorbed by it. As a consequence, the purity of the isolated QD-MF system becomes conserved, and the Hilbert space can be split into two subspaces: $|1, 1, 1\rangle$, $|1, 0, 0\rangle$, $|0, 1, 0\rangle$, and $|0, 0, 0\rangle$ with odd parity; $|1, 0, 1\rangle$, $|0, 1, 1\rangle$, and $|0, 0, 0\rangle$ with even parity. Thus, starting from a product state $|0, 0, 1\rangle$, only odd-parity states are involved in the state evolution:

$$|\Psi(t)\rangle = C_1(t)|1, 1, 1\rangle + C_2(t)|1, 0, 0\rangle + C_3(t)|0, 1, 0\rangle + C_4(t)|0, 0, 1\rangle.$$  

(2)

With this initial condition, the time-dependent coefficients can be obtained, via directly solving Schrödinger equation, as follows:

$$C_1(t) = \frac{-\Delta e^{-\Delta t} + \lambda \cos(\Delta t) - i\Omega \sin(\Delta t)}{2\Delta},$$

$$C_2(t) = \frac{i\Omega \sin(\Delta t)}{\Delta},$$

$$C_3(t) = \frac{-i\Omega \sin(\Delta t)}{\Delta},$$

$$C_4(t) = \frac{-\Delta e^{-\Delta t} + \lambda \cos(\Delta t) + i\Omega \sin(\Delta t)}{2\Delta},$$

(3)

where $\Delta = \sqrt{\varepsilon_m^2 + 16\lambda^2} / 2$, and $\Omega = \varepsilon_m / 2$. From equation (2), we can easily obtain the density operator $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$. After tracing over the degrees of freedom of the MFs, the reduced density operator $\rho_0(t)$ for the two QDs is obtained. This reduced density operator contains the nonlocal information of the paired MFs and serves as the starting point for calculating the quantum correlations (see Methods).

In order to investigate the decoherence effect on quantum correlations, we also introduce two electrode leads, each weakly coupled to its neighboring QD. The two electrode leads can be treated as electron reservoirs described by the Hamiltonian $H_{leads} = \sum_{j=1,2} \sum_k \varepsilon_{jk} c_{jk}^\dagger c_{jk}$, where $c_{jk}^\dagger$ ($c_{jk}$) is the creation (annihilation) operator for the electron with wave vector $k$ in the $j$th electrode lead and $\varepsilon_{jk}$ is the corresponding energy. The tunneling Hamiltonian between the electrode leads and the QDs is

![Figure 1](https://www.nature.com/scientificreports/)
We assume a symmetric setup with $\lambda_1 = \lambda_2 = \lambda$, and $\epsilon_1 = \epsilon_2 = 0$. (a) Two QDs mediated by a pair of MFs with an inter-MF coupling $\epsilon_m = 0.5 \lambda$. (b) Two QDs mediated by a pair of MFs without an inter-MF coupling (i.e., $\epsilon_m = 0$). (c) Two QDs mediated by a regular fermion with energy $\epsilon = \lambda$. (d) Two QDs mediated by a regular fermion with $\epsilon = \lambda$. (e) Two QDs mediated by a pair of non-interacting regular fermions with energies $\epsilon_1 = \epsilon_2 = 0$.

$H_{\text{TL}} = \sum_{j=1,2} \sum_k \left[ t_j d_j^\dagger c_k + h.c. \right]$. Here we employ the master equation approach to solving this problem. For the whole setup, the leads are regarded as an environment, and the central QD-MF device is described by a reduced density operator $\rho(t)$. We assume that the chemical potentials of the two leads, both denoted by $\mu$, are well below the energy levels of the dots, i.e., the energy difference $\epsilon_1 - \mu$ and $\epsilon_2 - \mu$ are much larger than the level broadening of the QDs. In this case, electrons can only jump outwards from the central device to the electrodes. In addition, we consider the regime with a weak tunnel coupling between the QDs and the leads. Thus, we can use the standard Born-Markov master equation$^{27}$. In the present case, the master equation can be written as

$$\dot{\rho}(t) = -i[H_{\text{sys}}, \rho] + \Gamma(D[d_1]\rho(t) + D[d_2]\rho(t)),$$

(4)

where $D[A] = A\rho A^\dagger - \frac{1}{2}\{A^\dagger A, \rho\}$, and the tunneling rate $\Gamma$ is associated with both the tunnel-coupling strengths and the densities of states of the two electrode leads. With the density operator $\rho(t)$ solved from equation (4), we can then obtain the reduced density operator $\rho_{\text{sys}}(t)$ of the two QDs by tracing over the degrees of freedom of the MFs.

**Quantum discord versus quantum entanglement.** To measure quantum entanglement, here we use both quantum entanglement and quantum discord (see Methods). We show the evolutions of quantum entanglement and quantum discord for the two QDs (i.e., QD$_1$ and QD$_2$) in Fig. 2. As a typical example, we choose $\epsilon_m = 0.5 \lambda$ in Fig. 2a. In this case, the two MFs are tunnel-coupled, in addition to the intrinsic nonlocal correlation of the paired MFs. Equation (3) shows that there are two frequencies determining the oscillating behaviors of the quantum state $|\Psi(t)\rangle$. The fast frequency $\Delta/2\pi$ determines the period of the oscillating cycles. From equations (2) and (3), it can be seen that the state of the system becomes $C(t)|1,1,1\rangle + C(t)|0,0,0\rangle$ at $t = \pi \Delta/2\pi \Delta(n = 1,2,3 \ldots)$, i.e., the state of the two QDs is just an entangled state $C(t)|1,1\rangle + C(t)|0,0\rangle$. As shown in Fig. 2a, the quantum entanglement peaks at $t = n\pi/\Delta$. The slow frequency $\Omega/2\pi$ modulates the height of oscillating peaks and determines the period of the modulating cycles. It can be seen that the quantum discord exhibits similar behaviors of oscillations except for the very short initial and final periods of each cycle, where the quantum entanglement becomes close to zero. Also, as shown in Fig. 2a, the quantum entanglement (discord) can reach its maximal value $C_{\text{d}} = 1$ ($D_{\text{d}} = 1$) at some specific time points. When $\epsilon_m = 0$, which corresponds to the long-length limit of the nanowire, the two MFs are completely separated in space. In this case, there is no direct tunnel coupling between the two MFs, but their intrinsic nonlocal correlation still exists. As shown in Fig. 2b, the temporal behaviors of quantum entanglement and quantum discord are very different at $\epsilon_m = 0$. While the quantum entanglement remains zero with time, the quantum discord oscillates with a smaller amplitude. Previous studies have found it hard to demonstrate the intrinsic nonlocal correlation between the two MFs in the electrical measurement signals, including the electrical current and the current noise$^{23-25}$. Thus, it will be important to select a physical quantity that can significantly reflect the non-locality of the MFs. Interestingly, our results reveal that the quantum discord can clearly serve as an indicator showing the intrinsic nonlocal correlation of the MFs in contrast to the insensitive quantum entanglement considered in our system.

For the purpose of discriminating MFs from regular fermions, we also show the quantum correlations of the two QDs (i.e., QD$_1$ and QD$_2$) induced by regular fermions that couple to them. We first consider the case with the two QDs coupled by a common regular fermion. Different from equation (1), the Hamiltonian of this QD-fermion system is given by

$$H_{\text{sys}} = \epsilon_c c^\dagger c + \sum_{j=1,2} \left[ \epsilon_j d_j^\dagger d_j + \lambda_j \left( d_j^\dagger c + c^\dagger d_j \right) \right],$$

where $c^\dagger$ ($c$) is the electron creation (annihilation) operator of the regular fermion with energy $\epsilon_c$, and $\lambda_j$ ($j = 1, 2$) denotes the tunnel coupling between the $j$th dot and the regular fermion. We assume that the two QDs are initially empty, but the
fermionic level $e_1$ is occupied by a single regular fermion such as the fermionic quasi-particle excitation in the nanowire. When the fermionic level $e_1$ is resonant to the energy levels $e_2$ of the two QDs, both quantum discord and quantum entanglement of the two QDs induced by the fermion have identical dynamical evolutions (see Fig. 2c); when $e_1$ is off-resonant to $e_2$, these quantum discord and quantum entanglement have different amplitudes, but still exhibit very similar temporal behaviors. Moreover, we consider the case involving a pair of non-interacting regular fermions, where each fermion couples to a QD. The Hamiltonian of this QD-fermion system is

$$H_{FD} = \sum_{j=1,2} \left[ e_j d_j^+ d_j + \lambda_j (d_j^+ c_j + c_j d_j) \right],$$

where $e_j (j = 1, 2)$ is the electron creation (annihilation) operator of the regular fermion with energy $e_j$, and $\lambda_j (j = 1, 2)$ denotes the tunnel coupling between the jth quantum dot and the regular fermion that couples to it. As we assume that the two QDs are initially empty, but each fermionic level $e_1$ is occupied by a single regular fermion. Because there is no intrinsic non-locality between these two non-interacting regular fermions, both quantum discord and quantum entanglement of the two QDs remains zero with time (see Fig. 2e). As compared with Figs. 2a and 2b, these very different temporal behaviors in Figs. 2c–2d can be used to discriminate the MFs from the regular fermions.

We also study the temporal behaviors of both quantum entanglement and quantum discord for the QD-MF system in the presence of two electrode leads. Here we choose a tunneling rate $\Gamma = 0.05 \lambda$ to ensure a weak coupling between the central QD-MF system and the two electrode leads. These two leads play the role of a fermionic environment which induces decoherence on the evolutions of quantum correlations. With tunnel coupling between the paired MFs, e.g., $e_m = 0.5 \lambda$ in Figs. 3a, both quantum entanglement and quantum discord can be generated from the product state, but the oscillating quantum entanglement decays faster with time than the quantum discord. This implies that the quantum discord, as a new resource of quantum information processing, is more robust than the quantum entanglement in this model system. Moreover, without tunnel coupling between the paired MFs, i.e., $e_m = 0$, the quantum entanglement still remains zero with time, but the oscillating quantum discord decays due to the decoherence induced by the electric leads (see Fig. 3b).

**A measurement scheme.** Note that equation (2) can be rewritten as

$$|\Psi(t)\rangle = b_1(t) e^{i\phi_1(t)} |11,1\rangle + b_2(t) |01,0\rangle + b_3(t) e^{i\phi_3(t)} |00,1\rangle,$$

where the real parameters $b_1(t)$, $b_2(t)$, $b_3(t)$, $\phi_1(t)$, and $\phi_3(t)$ are determined by $C_i(t)$, $i = 1, 2, 3, 4$, in equation (3). After tracing over the degrees of freedom of the MFs, the reduced density operator of the two QDs (i.e., QD1 and QD2) can be obtained via equation (5) as

$$\rho(t) = \begin{pmatrix}
 b_1^2(t) & 0 & 0 & b_1(t) b_3(t) e^{i\Delta \phi(t)} \\
 0 & b_2^2(t) & b_2^2(t) & 0 \\
 0 & b_2(t) b_3(t) & b_3^2(t) & 0 \\
 b_1(t) b_3(t) e^{-i\Delta \phi(t)} & 0 & 0 & b_3^2(t)
\end{pmatrix},$$

with $\Delta \phi(t) = \phi_1(t) - \phi_3(t)$. Here $b_1^2(t)$, $b_2^2(t)$, and $b_3^2(t)$, i.e., the diagonal matrix elements in equation (6), correspond to the probabilities of both dots occupied, either dot occupied, and both dots empty, respectively. These probabilities can be directly obtained by joint measurements on the electron occupation of QD1 and QD2 (see, e.g., Ref. 30). In order to obtain the values of the phase difference $\Delta \phi(t)$, we introduce, as Fig. 1b shows, two auxiliary QDs (denoted by QD3 and QD4) adjacent to QD1 and QD2, respectively. First, instead of $|0\rangle$, these two auxiliary QDs are initially prepared in the superposition state: $|\psi_{LR}(t)\rangle = (|1\rangle + |1\rangle) / \sqrt{2}$. Second, the QD3 (QD4) is adjusted in resonance with the QD1 (QD2) (i.e., $e_1 = e_2 = 0$, and $e_{L} = e_{R} = 0$) at the moment $t$, via tuning the gate voltages. Simultaneously, the tunnel coupling between QD1 (QD2) and its adjoining MF is switched off by tuning the voltages of the electrical gates that control the tunnel barrier between them. The Hamiltonian of the total system becomes $H_{TD} = T_1 \left( d_1^+ d_1 + d_2^+ d_2 \right) + T_2 \left( d_3^+ d_3 + d_4^+ d_4 \right)$, where $T_1$, $T_2$ is the tunnel coupling between QD1 (QD2) and QD3 (QD4) and assume $T_1 = T_2 = T$. Therefore, the system further evolves under the Hamiltonian $H_{TD}$, from the initial product state $|\psi_{LR}\rangle \otimes |\psi_{LR}\rangle \otimes |\psi(t)\rangle$. Third, along with the evolution of the total system, we choose a specific moment, $t + \Delta t$, to jointly measure the occupation probabilities of both QD1 and QD2, i.e., $p_{11}$, $p_{10}$, $p_{01}$, and $p_{00}$ which denote the probabilities of both dots occupied, single QD1 occupied, single QD2 occupied, and both dots empty, respectively. When $\Delta t = \pi/4T$, these probabilities can be obtained as

$$p_{11}(t) = \frac{1}{16} [9b_1^2(t) + 8b_2^2(t) + 2b_1(t)b_3(t) \cos \Delta \phi(t)],$$

$$p_{00}(t) = \frac{1}{16} [b_1^2(t) + 9b_2^2(t) + 2b_1(t)b_3(t) \cos \Delta \phi(t)],$$

$$p_{10}(t) = \frac{1}{16} [3b_1^2(t) + 8b_2^2(t) + 3b_3^2(t) - 2b_1(t)b_3(t) \cos \Delta \phi(t)],$$

and $p_{01}(t) = p_{10}(t)$. This measurable occupation probabilities include the phase difference $\Delta \phi(t)$, in addition to $b_1(t)$, $b_3(t)$, and $b_1(t)$ that have previously been obtained via joint measurements as well. Thus, it is experimentally feasible to obtain the reduced density operator $\rho_{\text{re}}(t)$ of the two QDs mediated by the paired MFs. Once the reduced density operator $\rho_{\text{re}}(t)$ is determined experimentally, one can then derive the quantum entanglement and the quantum discord of the two QDs using this reduced density operator (see Methods).

**Figure 3** Quantum dynamics of the quantum correlations between two QDs mediated by a pair of MFs in the presence of quantum decoherence. We assume a symmetric setup with $\lambda_1 = \lambda_2 = \lambda$, and $e_1 = e_2 = 0$. The tunneling rate between the central QD-MF device and the two electrode leads is chosen as $\Gamma = 0.05 \lambda$. (a) $e_m = 0.5 \lambda$, and (b) $e_m = 0$. © 2020 Springer Nature Limited. All Rights Reserved.
Methods

With the reduced density operator \( \rho_B \) of the two QDs (i.e., QD1 and QD2), the concurrence, as a measure of the quantum entanglement, can be obtained as

\[
C_B = \max \left\{ 0, \sqrt{\lambda_1 - \lambda_2} - \sqrt{\lambda_1 - \lambda_3} \right\},
\]

where \( \lambda_1, \lambda_2, \lambda_3, \) and \( \lambda_4 \) are the square roots of the eigenvalues in decreasing order for the matrix

\[
G = \rho_B (\sigma_x \otimes \sigma_x) \rho_B^* (\sigma_x \otimes \sigma_x),
\]

where \( \sigma_x \) is a Pauli matrix, and \( \rho_B^* \) is the complex conjugate of \( \rho_B \).

Quantum discord is a measure of quantum correlations obtained by subtracting the classical correlations from the total correlations\(^{14}\). For a bipartite quantum system including subsystems A and B, the total correlations are defined, in terms of quantum mutual information, as

\[
I(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}),
\]

where \( \rho^{AB} \) denotes the density operator of the bipartite system, with \( \rho^A \) being the reduced density operator of the subsystem A (B), and

\[
S(\rho^{AB}) = \text{Tr}(\rho^{AB} \log_2(\rho^{AB})),
\]

where \( \{ \rho^{AB} \} \) is a complete set of projectors of local measurements performed on the subsystem B; \( \rho^{AB} = \text{Tr}(I \otimes \rho_B)p_k \otimes |\psi\rangle \langle \psi| \) denotes the probability of obtaining the measurement outcome \( k \), with \( \{ \psi \} \) being the identity operator for the subsystem A; \( \rho^A = (I \otimes \rho_B)p_k \otimes |\psi\rangle \langle \psi| \) is the state of the bipartite system, conditioned on the measurement outcome labeled by \( k \). The quantum discord is defined as

\[
D_B(\rho^{AB}) = \min_{\{ \psi \}} \left[ I(\rho^{AB}) - I(\rho_B^{AB} | \{ \psi \}) \right].
\]

For our system, the A and B parts are the two QDs (i.e., QD1 and QD2), and \( \rho_B = \rho_A \). To compute classical correlations, we adopt the specific case of von Neumann local measurements, which are provided by the orthogonal projectors

\[
B_k = R \Pi_k R^T, \quad \text{where } \{ \Pi_k \} = \{|k\rangle \langle k|\}
\]

defines the computational basis \( R \), with \( k = 0, 1 \) corresponding to the empty and occupied states, respectively. As a rotation operator, \( V \) can be parameterized as

\[
V = \left( \begin{array}{cc}
\cos \phi & \sin \phi e^{i \theta} \\
-\sin \phi & \cos \phi \end{array} \right)
\]

where \( \theta \in [0, \pi] \), and \( \phi \in [0, 2\pi] \). Hence, the quantum discord given in equation (10) can be directly obtained by minimizing over all angles \( \theta \) and \( \phi \).

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Author contributions

J.L. carried out all calculations under the guidance of J.Q.Y. and both T.Y. and H.Q.L. participated in the discussions. All authors contributed to the interpretation of the work and the writing of the manuscript.

Additional information

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