Online Correction of Dispersion Error in 2D Waveguide Meshes

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Abstract

An elastic ideal 2D propagation medium, i.e., a membrane, can be modeled by a model simulating the wave equation on the time-space grid (finite difference methods), or locally discretizing the solution of the wave equation (waveguide meshes). The two approaches provide equivalent computational structures, and introduce numerical dispersion that induces a misalignment of the modes from their theoretical positions. Prior literature shows that dispersion can be arbitrarily reduced by oversizing and over-sampling the mesh, or by adopting on-line warping techniques. In this paper we propose to reduce numerical dispersion by embedding warping elements, i.e., properly tuned all-pass filters, in the structure. The resulting model exhibits a significant reduction in dispersion, and requires less computational resources than a regular mesh structure having comparable accuracy.

1 Introduction

Membranes are the crucial component of most percussion instruments. Their response to an excitation, and their interaction with the rest of the musical instrument and with the environment, strongly affect the sound quality of a percussion. Physical modeling of membranes has drawn the attention of the computer music community when a new model based on the digital waveguide was designed, called 2-D Digital Waveguide Mesh (Van Duyne and Smith, 1993). The model was proved to provide a computational structure equivalent to the Finite Difference Scheme (FDS). In particular, it was shown that the numerical artifacts introduced by the model cause a phenomenon called dispersion. This means that, even in an excitable medium, different spatial frequency components travel at different speeds, and the speed is direction- and frequency-dependent (Stikweerd, 1989, Van Duyne and Smith, 1993).

Different mesh geometries have been studied: each of them have an equivalent FDS, and exhibits its peculiar dispersion error function (Fontana and Rocchesso, 2000). The triangular geometry exhibits two valuable properties: the dispersion error is, with good approximation, independent from the direction of propagation of the spatial components; at the same time, the Triangular Waveguide Mesh (TW Mesh) de-nets, from a signal-theoretical point of view, the most efficient sampling scheme among the geometries that can be derived from mesh models used in practice (Savioja, 2000). Fontana and Rocchesso, 2000). The independency from direction has been successfully exploited (Savioja, 2000) to warp the signals produced by the model, using on-line warping techniques (Ham et al., 2000). In this paper we work on a similar idea, but warping is performed online by cascading each unit delay in the TW Mesh with a first-order all-pass filter. By properly tuning the filter parameter, we will prove that a considerable reduction of the dispersion error can be achieved in a wide range around dc.

This result is then compared with the performance of a TW Mesh, oversized in order to reduce dispersion in the first modes. It will be shown that the warped mesh is less expensive in terms of memory and computational resources. This evidence holds both for the straight waveguide and the FDS implementation. Our conclusion is that the most efficient, low-dispersion computational scheme for membrane modeling is a triangular FDS where the unit delays are cascaded with properly tuned all-pass filters.

Having an efficient and accurate membrane model is a key step toward the construction of an affordable, tunable, and realistic models of complete percussion instruments. In particular, the coupling between air and membrane (Fontana and Rocchesso, 1998), and the interface with resonating structures are fundamental components that should be added to the membrane model in order to achieve better realism (Arditi et al., 2000).

2 Online Warping

For a wave traveling along the waveguide mesh, the numerical dispersion error is a function of the two spatial frequency components. In the TW Mesh, this function is symmetric around the origin of the spatial frequency axes, with good approximation. Con-
sequentially, it makes sense to plot the dispersion factor as a single-variable function of spatial frequency, moving from the center of the surface to the absolute band edge along one of the three directions deduced by the waveguide orientation.\footnote{In [Savioja, 2000] a function averaging the surface magnitude around the origin is constructed, resulting in a slight difference respect to the curve adopted here.} Assuming the waveguides to have unit length, the spatial band edge results to be equal to $2p = \frac{\pi}{3}$ \textit{[rad=spatial sample]} [Fontana and Roccress, 2000]. A plot of the dispersion factor versus temporal frequency is then calculated recalling the nominal propagation speed factor, equal to $2p = \frac{\pi}{3}$ \textit{[rad=spatial sample]}, affecting any finite difference model [Utkovski, 1989], that yes the edge of the temporal frequency at the value $2 = \frac{\pi}{3}$ \textit{[rad=spatial sample]}, Figure 1 depicts a plot of the dispersion factor.

This analysis is conducted by simulations conducted over a TWM modeling a square membrane of size 24 24 waveguide sections, clamped at the four edges, excited at the central junctor by an impulse. In fact, the impulse response taken at the central junctor shows that the positions of its modes match well with the theoretical frequencies of the odd modes of the membrane, each one of them being shifted by its own dispersion and by the nominal propagation speed factor. The results are depicted in Figure 2, where the frequency response of the model is plotted together with \textit{(a)} the theoretical positions compressed by the nominal propagation speed factor of the modes resonating in the membrane below $2 = \frac{\pi}{3}$ \textit{[rad=spatial sample]}, and \textit{(b)} the real positions of the same modes, a rolled by dispersion. Overall, dispersion introduces a modal coloration that increases with frequency. The careful reader will note a slight difference between the calculated frequency cut and the band-

width of the signal coming from the simulation. This difference is probably due to the simplifying assumption of considering the dispersion function as direction independent. Moreover, some modes show up as twin peaks. This may be due to the actual irregular shape of the resonator model, caused by the impossibility to design a perfectly square geometry using a TWM model. In order to conduct a controlled analytical study we avoided using interpolation along the edge, even though this is recommended in practical implementations [Airdal., 2000].

Let $H(z)$ be the transfer function of a TWM, regardless of the excitation (input) and acquisition (output) positions. The transfer function can be handled by conformal mapping, a method consisting in the application of a particular map $T$ to the z-domain, in order to obtain a new, warped domain $z = T(z)$ [Hoover, 1981; Hamm a et al., 2004]. The frequency response $H(2\pi s)$, calculated from $H(z)\mid_{z=2\pi s}$, changes according with the map.

Practical implementations of transfer functions obtained by conformal mapping are often acted by non computable loops, that can sometimes be resolved [Hamm a, 1998]. In a TWM, delay free loops appear whenever the map does not allow to extract an explicit unit delay. However, if we change the number of unit delays in each waveguide section of a waveguide mesh, we only change the number of Fourier images of the frequency response, by simply compressing each single image. Now, inagine a map that translates each unit delay into the cascade of a first-order allpass

\footnote{This occurs whenever a map $z = 2^n$ is applied to a discrete-time linear filter.}
Choosing one such traveling into the mesh respecting to the higher more layers by the all pass are the lower frequencies traveling into the mesh respect to the higher ones.

Choosing $\alpha = 0.45$ (curve in the middle), the warping so introduced in its the modal dispersion to very low values. Figure 3 shows the frequency response of the same TW M simulated before, after warping. Using the same notation of figures 2, the response is compared with the ideal positions of the odd modes, re-scaled to align the fundam entals ( ), and with the real positions of the same modes, acted by the residual dispersion ( ). The improvement in terms of precision in the alignment of the modes is evident by comparison of crosses and circles in figures 2 and 4.

### 3 Computational Performance

Figure 5 shows a plot of the dispersion factor after warping. Dispersion is below 2% in a range equal to 75% of the whole band, then it climbs to the maximum. From a perceptual viewpoint, it is not clear how much tolerance we might admit in the frequency positions of high order partials of a drum. Frequency deviation thresholds should be derived from subjective experimentation, as it was done for piano sounds [Rocco eos and Sollon, 1999]. Certainly, figure 5 shows an unnatural compression of modes that results in a decrease in brightness. Moreover, the frequency distribution of resonances brings us some information about the shape of the resonating object, for instance making it possible to discriminate a circular from a square drum. The warped TW M preserves the correct distribution of modes quite well up to 75% of the frequency band.

By comparison of figures 5 and 6, we can note that a similar precision is achieved by a TW M if the waveguides are reduced to one third of the original length (the mesh is nine times denser). Then, the fundaments can be aligned in the two models by multiplying times 1.75 the sampling rate of the warped TW M. Finally, both the output signal is an be low pass filtered.

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**Figure 3:** Mapping functions $z^\alpha = z^A(z)$ for equally-spaced values of the parameter of the all pass $A(z)$. Top line: $\alpha = 0.9$. Bottom line: $\alpha = 0.9$. Choosing one such traveling into the mesh respecting to the higher more layers by the all pass are the lower frequencies traveling into the mesh respect to the higher ones.

**Figure 4:** Impulse response taken at the center of a warped TW M (size 24x24) excited by an impulse at the same point. Theoretical positions of the odd modes resonating in a membrane, re-scaled to align the fundam entals ( ), Positions of the same modes acted by residual dispersion ( ).
The coefficient of the embedded all-pass filters is also a parameter that can be controlled to introduce tension modulation or other more exotic effects.

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From these considerations, a comparison of the TWM versus its warped version (W) in terms of needed sum s, multiplies and memory locations. Both the TWM (FDS) and its warped version allow the same dispersion tolerance.

Table 1: Performance of the TWM (FDS) vs. warped version (W) in terms of needed sum s, multiplies and memory locations. Both the TWM (FDS) and its warped version allow the same dispersion tolerance.

|        | Sum s | Mult | Memory |
|--------|-------|------|--------|
| TWM    | 99    | 9    | 54     |
| W TWM  | 40:25 | 22:75| 22:75  |
| FDS    | 54    | 9    | 18     |
| W FDS  | 175   | 8:75 | 7      |

Figure 5: Plot of the dispersion error versus temporal frequency magnitude in the warped TWM.

A new technique to reduce modal dispersion in a wide frequency range in TWM and triangular FDS models of 2D resonators has been presented. This technique is based on first-order all-pass filters embedded in the mesh, and it requires an increase in temporal sampling rate accompanied by lowpass filtering of the output signal. The resulting warped TWM is shown to be less expensive in terms of computation resources and memory consumption than oversampling a TWM or FDS model. The coefficient of the embedded all-pass filters is also a parameter that can be controlled to introduce tension modulation or other more exotic effects.

4 CONCLUSION

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