CORRECTING CAMERA SHAKE BY INCREMENTAL SPARSE APPROXIMATION

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ABSTRACT

The problem of deburring an image when the blur kernel is unknown remains challenging after decades of work. Recently there has been rapid progress on correcting irregular blur patterns caused by camera shake, but there is still much room for improvement. We propose a new blind deconvolution method using incremental sparse edge approximation to recover images blurred by camera shake. We estimate the blur kernel first from only the strongest edges in the image, then gradually refine this estimate by allowing for weaker and weaker edges. Our method competes with the benchmark deblurring performance of the state-of-the-art while being significantly faster and easier to generalize.

1. INTRODUCTION

In the problem of blind image deconvolution, we are given a blurry image \( y \) and challenged to determine an estimate \( x \) of the unknown sharp image \( x^\text{true} \) without knowledge of the blur kernel \( k^\text{true} \). In the simplest model of blur, \( y \) is formed by convolving \( x^\text{true} \) with \( k^\text{true} \) and adding noise \( n \):

\[
y = k^\text{true} \ast x^\text{true} + n.
\]

This convolution model assumes spatially uniform blur, which is frequently violated due to slight camera rotations and out-of-plane effects [1]. Still, the uniform model works surprisingly well and methods for it can be extended to handle nonuniform blur [2, 3].

Even with uniform blur by a single kernel, the blind deconvolution problem is highly underdetermined and additional assumptions must be made to obtain a solution. These assumptions are often imposed most conveniently by moving the problem into a filter space. We define filters \( \{ f_\gamma \}_{\gamma=1}^L \) and set \( y_\gamma = f_\gamma \ast y \) and \( x^\text{true}_\gamma = f_\gamma \ast x^\text{true} \), so that

\[
y_\gamma = k^\text{true}_\gamma \ast x^\text{true}_\gamma + n_\gamma
\]

for \( \gamma \in [L] = \{1, \ldots, L\} \). Defining \( x^\text{true} = \{x^\text{true}_\gamma\}_{\gamma=1}^L \), \( y = \{y_\gamma\}_{\gamma=1}^L \), and \( (k \ast x^\text{true})_\gamma = k \ast x^\text{true}_\gamma \), we can write the filter space problem compactly as

\[
y = k^\text{true} \ast x^\text{true} + n.
\]

The simplest nontrivial filter space is gradient space, where \( L = 2 \) and \( f_1 = [1, -1] \), \( f_2 = [1, -1]^T \), but there are many other possibilities. Determining \( x \) from a filter space representation \( x \) often does not work well, so typically one obtains an estimate \( k \) of \( k^\text{true} \) and deconvolves \( y \) with \( k \) to get \( x \) [1].

Bayesian inference is a convenient framework for imposing prior assumptions to regularize blind deconvolution [4]. By assuming some distribution of \( n \) we obtain a likelihood function \( p(y | k \ast x) \) which gives the probability that the data \( y \) arose from a given pair \((k, x)\). We then choose priors \( p(k) \) and \( p(x) \) and compute the posterior distribution

\[
p(k, x | y) \propto p(y | k \ast x) p(x) p(k).
\]

Estimates of \( x \) and \( k \) may be obtained by summary statistics on \( p(k, x | y) \). We call the mode of \( p(k, x | y) \) the joint maximum a posteriori (MAP) estimator, while the mode of the marginal \( p(k | y) = \int p(k, x | y) dx \) is the kernel MAP estimator. Most blind deconvolution methods are nominally MAP estimators but do not actually find a global minimizer, as this is typically intractable and may even be counterproductive. We refer to any method organized around optimizing a posterior as a MAP method, while methods that actually find a global minimum will be called ideal MAP methods. Joint MAP methods typically attempt to minimize the cost function

\[
F(k, x) = -\log p(k, x | y),
\]

which may be written (up to an irrelevant additive constant) as the sum of a data misfit and two regularization terms,

\[
F(k, x) = L(k \ast x) + R_x(x) + R_k(k),
\]

where each of these functions may take the value \( +\infty \) to represent a hard constraint. Kernel MAP estimation is more difficult as it involves a high-dimensional marginalization, and it is typically approximated by variational Bayes or MCMC sampling [5].

Joint MAP estimation is the oldest, simplest, and most versatile approach to blind deconvolution [6-8], but initial joint MAP efforts on the camera shake problem met with failure [9], even when \( \ell_p \) regularizers for \( p < 1 \) were used. In [1], Levin et al. showed that the \( \ell_p \) regularizer generally prefers blurry images to sharp ones: \( ||y||_p < ||x^\text{true}||_p \), so that ideal joint MAP typically gives the trivial no-blur solution \((k, x) = (\delta_0, y)\), where \( \delta_0 \) is the Kronecker delta kernel. The non-ideal joint MAP methods [10, 11] somewhat compensate for the defects of ideal joint MAP by dynamic edge prediction and likelihood weighting, but benchmarking in [11, 12] showed that these heuristics sometimes fail.
In [9] Fergus et al developed a kernel MAP method with a sparse edge prior which was very effective for correcting camera shake. In [1] it was noted that marginalization over x seems to immunize ideal kernel MAP against the blur-favoring prior problem. More refined kernel MAP methods were recently reported in [12] and [13], and to our knowledge these two methods are the top performers on the benchmark test set from [1]. While these efforts have made kernel MAP much more tractable, it remains harder to understand and generalize than joint MAP, so it would be useful to find a joint MAP method that is competitive with kernel MAP on the camera shake problem.

In [14], Krishnan et al addressed the blur-favoring prior problem in joint MAP by changing the prior, proposing the scale-invariant \( \ell_1/\ell_2 \) ratio as a ‘normalized’ sparse edge penalty. The \( \ell_2 \) normalization compensates for the way that blur reduces total \( \ell_1 \) edge mass, causing the \( \ell_1/\ell_2 \) penalty to prefer sharp images and eliminating the need for additional heuristics. While their algorithm does not quite match the performance of [9] on the benchmark test set from [1], it comes fairly close while being significantly simpler, faster, and in some cases more robust. Other promising joint MAP methods include [15–17], but we are not aware of public code and in some cases more robust. Other promising joint MAP methods include [15–17], but we are not aware of public code with full benchmark results for these methods.

### 1.1. Our approach

We propose a new approach to joint MAP blind deconvolution in which the kernel is estimated from a sparse approximation of the sharp gradient map \( x^{\text{true}} \). Initially we constrain \( x \) to be very sparse, so it contains only the few strongest edges in the image, and we determine \( k \) such that \( k \ast x \approx y \). Because \( x \) is so much sparser than \( y \), the solution \( k = \delta_{0} \) is very unlikely. But generally this initial \( k \) overestimates \( k^{\text{true}} \), so we refine \( k \) by letting weaker edges into \( x \).

To present this approach formally, we set \( f_1 = [1, -1], f_2 = [1, -1]^T \), so that \( x(p) = (x_1(p), x_2(p)) \) is the discrete image gradient vector at each pixel \( p \). We set \( L(k, x) = \frac{1}{2} \| k \ast x - y \|_2^2 \) and impose the usual positivity and unit sum constraints on \( k \). We measure gradient sparsity using the \( \ell_{2,0} \) norm: \( |x(p)| \) is the \( \ell_2 \) length of \( x(p) \) and \( \| x \|_{2,0} = \| x \|_0 \) the number of nonzero gradient vectors. The joint MAP optimization problem is then

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{2} \| k \ast x - y \|_2^2 \\
\text{subject to} & \quad k \geq 0, \quad 1^T k = 1, \quad \| x \|_{2,0} \leq \tau,
\end{align*}
\]

where the expression \( a^T b \) denotes the dot product of the arrays \( a \) and \( b \) when considered as vectors, and the 1 in \( 1^T k \) is an all-ones array.

We solve this problem with an iterative optimizer described in [2] and slowly increase \( \tau \) as the iterations proceed. To initialize \( \tau \) we use the \( \ell_1/\ell_2 \) ratio, a robust lower bound on a signal’s sparsity [18]. The sharp \( x \) should be significantly sparser than \( y \), so initially we set \( \tau = \beta_0 \tau_y \), where \( \tau_y = \| y \|_1 / \| y \|_2 \) and \( \beta_0 < 1 \) is a small constant. After an initial burn-in period of \( I_0 \) iterations we multiply \( \tau \) by a constant growth factor \( \gamma > 1 \), an action we repeat every \( I_s \) iterations thereafter.

We use a standard multiscale seeding technique to accelerate the kernel estimation step [9,13]. We begin by solving (6) with a heavily downsampled \( y \), giving a cheap, low-resolution approximation to \( k \) and \( x \). We then upsample this approximation and use it as an initial guess to solve (6) with a higher resolution \( y \), repeating the upsample-and-seed cycle until we reach the full resolution \( y \). At each scale we use the same \( \tau \) increase schedule. After kernel estimation we use non-blind deconvolution of \( y \) with \( k \) to get the sharp image \( x \).

The easiest way to understand how our kernel estimation works is to watch \( k \) and \( x \) evolve as the iterations progress. In Fig. 1 the state of \( k \) and \( x \) is shown at iterations 2, 32, and 150 of the final full-resolution scale, with \( k^{\text{true}} \) and \( x^{\text{true}} \) at far right. Initially \( x \) is quite sparse, so \( k \) cannot be a trivial kernel because the parts of \( y \) not in \( x \) must be attributed to blur. But this initial approximation is crude, so as \( \tau \) increases with iteration, \( x \) is allowed to have more and more edges so that \( k \) can be refined.

### 1.2. Novelty and relations with existing methods

Direct \( \ell_0 \) optimization is well-established in the compressed sensing community [19,20] but we are not aware of any effective \( \ell_0 \) approaches to blind deconvolution. In [14] the \( \ell_1/\ell_2 \) ratio was deliberately chosen over \( \ell_0 \) because while both have the desired scale invariance, the graph of \( \ell_1/\ell_2 \) is smoother and looks more ‘optimizable’ than \( \ell_0 \). We contend that \( \ell_0 \) may be difficult to use as a cost function, but very effective as a constraint. Gradient and kernel thresholding are commonly used [10] and these can be interpreted as \( \ell_0 \) projections, but they are typically used as auxiliary heuristics, not as the central modeling idea. Our technique of slowly increasing the sparsity constraint \( \tau \) is reminiscent of matching pursuit algorithms for sparse approximation [21,22]. It is also related to the likelihood reweighting technique of [10], which may be seen by considering the Lagrangian of (6). However, our initialization strategy requires that we use the constrained formulation rather than the Lagrangian.

### 2. Alternating Projected Gradient Method

To solve problem (6) at a given scale, we use a standard alternating descent strategy: starting from some initial \( k \) and \( x \), we reduce the cost function by updating \( x \) with \( k \) fixed, then \( k \) with \( x \) fixed, cycling until a stopping criterion is met. Each cycle, or outer iteration, consists of \( I_k \) inner iterations updating \( x \) and \( I_k \) inner iterations updating \( k \). All updates are
Barzilai-Borwein step size

$$\alpha_k = \frac{(g_k - g_k^{\text{old}})^T (g_k - g_k^{\text{old}})}{(g_k - g_k^{\text{old}})^T (k - k^{\text{old}})}$$

where $g_k^{\text{old}}$ and $k^{\text{old}}$ denote the values of $g_k$ and $k$ at the previous SPG iteration.

3. IMPLEMENTATION AND EXPERIMENTS

We implemented our method in MATLAB by modifying the code of [14], which uses a similar strategy of alternating minimization with multiscale seeding. The full-resolution kernel size was set to $35 \times 35$ for all experiments. The initial stage of the multiscale algorithm downsamples $y$ by a factor of $5/35$ in each direction, so that the kernel is of size $5 \times 5$, and each upsample cycle increases the size of $k$, $x$, and $y$ by a factor of roughly $\sqrt{2}$ until full resolution is reached. The parameters of the core single-scale algorithm from [2] were set to $\beta_0 = 0.15$, $\gamma = 1.10$, $I_0 = 20$, $I_0 = 10$, $I_0 = 1$, $I_0 = 6$. We do 30 iterations of the alternating projected gradient method for all scales except the final, full-resolution scale, which uses 180 iterations. Non-blind deconvolution with the estimated kernel was performed using the method of [27], using the parameter settings chosen in the code for [12].

In [1] a test set of 32 blurry images with known ground truth was created for benchmarking blind deconvolution methods. Each blurry image was formed by taking a picture of a sharp image with a camera that shook in-plane, and bright points outside the image were used to obtain ground truth blur kernels. A total of 32 blurry images were formed by blurring 4 sharp images on 8 different shake trajectories. This test set has become the de facto standard for objectively comparing different methods.

We ran our algorithm on this test set and compared its performance against the methods of [12] and [13]. We compare against these methods because they have published implementations which match or exceed the performance of the state-of-the-art methods in [9–11, 14], and we know of no
We have proposed a blind deconvolution method in which the blur kernel is estimated by incremental sparse edge approximation. A rough global blur kernel is first estimated from only the strongest edges in the image, then it is refined as we allow the image edge map to gradually become less and less sparse. Ours is the first simple, fast joint MAP method to match the state-of-the-art kernel MAP methods in \cite{b12, b13} on an objective benchmark. The success of the methods in \cite{b14} and this paper suggest that the downsides of ideal joint MAP described in \cite{b1} can be robustly avoided without resort to a more complex kernel MAP estimation.

There are many potential avenues for improving and extending our method. The edge sparsity relaxation schedule we use is slow and conservative, and a more adaptive schedule could make the method faster. Our initialization of the edge map sparsity does not take noise into account, and may need to be modified for very noisy images. Extension to nonuniform blur models, nonquadratic likelihoods, and fast parallel or GPU implementations are possible. The speed of our kernel estimation may make it useful as an input to high-quality non-blind methods such as \cite{b13}.

4. CONCLUSION

We have proposed a blind deconvolution method in which the blur kernel is estimated by incremental sparse edge approximation. A rough global blur kernel is first estimated from only the strongest edges in the image, then it is refined as we allow the image edge map to gradually become less and less sparse. Ours is the first simple, fast joint MAP method to match the state-of-the-art kernel MAP methods in \cite{b12, b13} on an objective benchmark. The success of the methods in \cite{b14} and this paper suggest that the downsides of ideal joint MAP described in \cite{b1} can be robustly avoided without resort to a more complex kernel MAP estimation.

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Fig. 2. Sample results from our method, \cite{b12, b13} on the benchmark set of \cite{b1}. True and recovered kernels inset.

Fig. 3. Cumulative deblurring performance our method, \cite{b12}, and \cite{b13} on the 32 image test set of \cite{b1}. The vertical axis is the percentage of the 32 runs having at most a given SSE.
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