Towards Searching for Entangled Photons in the CMB Sky

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(Dated: September 3, 2018)

We explore the possibility of detecting entangled photon pairs from cosmic microwave background or other cosmological sources coming from two patches of the sky. The measurements use two detectors with different photon polarizer directions. When two photon sources are separated by a large angle relative to the earth, such that each detector has only one photon source in its field of view, the signal for entanglement can be separated from the background by changing the polarizer directions. When the angle between two photon sources is small enough such that both sources are in the fields of view of both detectors, then the background becomes more complicated which makes it more difficult to be subtracted.

PACS numbers: 98.70.Vc; 42.50.Xa; 03.65.Ud

INTRODUCTION

Quantum fluctuations, which are usually of atomic scales, can be amplified to cosmological scales in an inflationary scenario (see e.g. [1, 2]), and their traces can be seen from cosmological microwave background (CMB) [3, 4]. However, some alternative theories can also produce similar fluctuations that are classical in nature. To rule out those theories, it is important to show that the fluctuations are indeed quantum mechanical. Therefore, entanglement, which can be demonstrated by the violation of Bell inequality, becomes the natural choice to show that the fluctuations cannot come from classical theories [5–13]. Theoretically, it is an interesting question how quantum entanglement can survive the inflation such that two particles from two patches of the sky can still be entangled. This involves how the quantum fluctuations are produced and how part of the entanglement is destroyed by decoherence during the evolution of the universe. This type of analyses usually depend on the models and observables, and we leave this point to the final discussion.

In this work, we ask ourselves a more elementary question, that is, if there are entangled photon pairs coming from two different patches of the sky, how do we tell they are entangled? And if we do not know how large the signal and background are a priori, how do we isolate the signal from background? For the entanglement, we make use of the Bell inequality test based on polarization correlations. And since we have two photon sources, the set up of Hanbury Brown and Twiss (HBT) intensity interferometer [14–16] is a reasonable choice. Combining the two, we arrive at two different type of scenarios.

The first scenario is shown in Fig. 1 with the two photons, which might be causally connected in the far past, coming from two very close patches of the sky, such that both sources are in the fields of view of both detectors. The second scenario is shown in Fig. 2 with two sources in a relatively large angle to the earth such that each detector has one source in its field of view. The main difference is that the interference between different photon paths only happens in the first scenario but not in the second one. It turns out that the second scenario allows simpler isolation of the signal to background by adjusting the orientation of the polarizers. This is an encouraging result. We leave the estimation of the size of the signal for a future work.

BELL INEQUALITY FOR TWO PHOTON SYSTEMS

In the helicity basis, a photon state with $±\hbar$ helicity can be denoted as $|\varepsilon_1\rangle \pm |\varepsilon_2\rangle$, with $\varepsilon_1$ and $\varepsilon_2$ the two orthogonal linear polarizations perpendicular to the direction of propagation. Then there are four independent two photons spin eigen wave functions: $|\varepsilon_+\rangle \otimes |\varepsilon_+\rangle$, $|\varepsilon_-\rangle \otimes |\varepsilon_-\rangle$, $|\varepsilon_+\rangle \otimes |\varepsilon_-\rangle + |\varepsilon_-\rangle \otimes |\varepsilon_+\rangle$ and $|\varepsilon_+\rangle \otimes |\varepsilon_-\rangle - |\varepsilon_-\rangle \otimes |\varepsilon_+\rangle$. The latter two are entangled states which can be rewritten in linear polarization basis as

$$|\psi_1\rangle = \frac{1}{\sqrt{2}} (|\varepsilon_1\rangle \otimes |\varepsilon_1\rangle + |\varepsilon_2\rangle \otimes |\varepsilon_2\rangle),$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}} (|\varepsilon_1\rangle \otimes |\varepsilon_2\rangle - |\varepsilon_2\rangle \otimes |\varepsilon_1\rangle).$$

(1)

Suppose the measurement of photons takes place at two spatially separated locations labelled by Alice (ab-
breviated by A) and Bob (abbreviated by B). $\Pi_A$ and $\Pi_{A'}$ are projections applied at detector A, while $\Pi_B$ and $\Pi_{B'}$ at detector B. Consider the combination spin-spin operators proposed by CHSH [17]

$$C = \Pi_A\Pi_B + \Pi_{A'}\Pi_B + \Pi_A\Pi_{B'} - \Pi_{A'}\Pi_{B'}, \tag{2}$$

in which the projection operator $\Pi_A$ gives a value +1 when a photon with polarization in the $\vec{n}_A$ direction is detected, and −1 when a photon with polarization perpendicular to $\vec{n}_A$ (denoted as $\vec{n}_{A1}$) is measured, i.e. $\Pi_A = |n_A\rangle\langle n_A| - |n_{A1}\rangle\langle n_{A1}|$. In other words, each photon registers a +1 or −1 at the detector with polarizer $\Pi_i$ ($i = A, B, A', B'$). Therefore, if $\{\Pi_A, \Pi_{A'}\}$ register $\{+1,+1\}$ or $\{-1,-1\}$, then the last two terms in Eq.(2) cancel. If $\{\Pi_A, \Pi_{A'}\}$ register $\{+1,-1\}$ or $\{-1,+1\}$, then the first two terms in Eq.(2) cancel. This leads to the Bell inequality $|\langle C \rangle| \leq 2$ appropriate for for a local classical hidden variable theory.

On the other hand, in quantum mechanics, the Bell inequality can be violated [18, 19], and the expectation value of $C$ can be bigger, satisfying $|\langle C \rangle| \leq 2\sqrt{2}$ [20], as $C^2 = 4 - \|\Pi_A, \Pi_{A'}\Pi_B, \Pi_{B'}\| = 4(1 + \sin 2\theta_{AB} \sin 2\theta_{B'}) \|$, with $\theta_{AB}$ the angle between $\vec{n}_A$ and $\vec{n}_B$.

Since

$$\langle \psi_i | \Pi_A\Pi_B | \psi_i \rangle = (-1)^{i+1} \cos 2\theta_{AB}, \tag{3}$$

for $i = 1, 2$, then for both $|\psi_1\rangle$ and $|\psi_2\rangle$, the quantum mechanical bound can be saturated by choosing

$$\theta_{AB} = \theta_{AB'} = \theta_{A'B'} = \pi/8, \quad \theta_{A'B} = 3\pi/8. \tag{4}$$

For unentangled states, $\langle \psi_i | \Pi_A\Pi_B | \psi_i \rangle = 0$.

An important lesson we have learned above is that the angular dependence of Eq.(3) can be considered as a signature of entanglement, since this angular dependence implies the violation of Bell Inequality.

**DETECTION OF ENTANGLED CMB PHOTONS**

In this section, we explore the question that, if two photons coming from two different patches of the sky are entangled, whether we can isolate the entangled signal from the background of unentangle photon pairs. Our set-ups are similar to the Bell’s test, except that the photons are coming from two patches of the CMB sky as two sources rather than from a single one.

**Scenario I: Sources from small angles—interference**

The first scenario is when the two photons, which might be causally connected in the far past, are coming from two patches of the sky (called sources 1 and 2) that are close enough to each other, such that sources 1 and 2 are both in the fields of view of the detectors Alice (A) and Bob (B). This is a Hanbury Brown and Twiss (HBT) intensity interferometer [14–16] as shown in Fig. 1.

![Entangled fluctuations](image)

**FIG. 1.** Assuming a pair of entangled photons were sent to two regions 1 & 2 in the sky after inflation, then the two photons are detected by two detectors Alice and Bob. Since both 1 & 2 are in the fields of view for the two detectors, the photons can be detected by going through the black or red paths. Correlation of photon polarizations are measured to detect entanglement.

In HBT, we consider the process with two photons emitted from source 1 and 2 in the initial state and those two photons are detected by detectors A and B in the final state. The propagation of photons is described by Feynman’s path integral. The amplitude can be written as

$$A = D_{1A}D_{2B} + D_{2A}D_{1B}, \tag{5}$$

where $D_{1A}$ denotes the propagator from source 1 to detector A, and so on. The two different terms correspond to two different paths shown in Fig. 1, and higher order loop diagrams are neglected. Then the transition probability is proportional to

$$|A|^2 = |D_{1A}|^2|D_{2B}|^2 + |D_{2A}|^2|D_{1B}|^2 + 2\text{Re}D_{1A}D_{2B}D_{2A}D_{1B}. \tag{6}$$

Since the photon from each source appears both in a propagator and a conjugate one, the random phases associated with each of the sources 1 and 2 cancel in the interference term. Therefore, the coherent source is not required. For simplicity, we assume the two photons are produced and emitted at the same time, thus the interference term depends only on the relative phase factor arising from the geometry of the setup. The longer the baseline (i.e. the distance between the two detectors), the better resolution for the interferometry. One can generalize the analysis to cases with the two sources coming from an extended object such as a star or the CMB sky.
Signal for Entanglement

Now we add the ingredient of Bell inequality to the HBT set up. Suppose the two photons emitted from the sources 1 and 2 are entangled to form the $|\Psi_1\rangle$ state of Eq.(1), then after propagating to the detectors, the photon state becomes

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}} \left( D_{1A}|\epsilon_1\rangle \otimes D_{2B}|\epsilon_1\rangle + D_{1A}|\epsilon_2\rangle \otimes D_{2B}|\epsilon_2\rangle \right. + D_{2A}|\epsilon_1\rangle \otimes D_{1B}|\epsilon_1\rangle + D_{2A}|\epsilon_2\rangle \otimes D_{1B}|\epsilon_2\rangle)$$

$$= |\psi_1\rangle (D_{1A}D_{2B} + D_{2A}D_{1B}),$$

where the spin wave function is not changed during the propagation and the propagation is not changed by the spin. The same factorization happens for $|\Psi_2\rangle$ as well. As a result, the expectation value of the CHSH quantity reads

$$\langle \Psi_1|C|\Psi_1\rangle = \langle \psi_1|C|\psi_1\rangle |D_{1A}D_{2B} + D_{2A}D_{1B}|^2,$$

with the spin-spin correlation $\langle \psi_1|C|\psi_1\rangle$ and the geometric phase contribution $|D_{1A}D_{2B} + D_{2A}D_{1B}|^2$ completely factorized in the measurement.

Therefore, entanglement between two CMB photons emitted from two sources 1 and 2 (from two patches of the sky) can in principle be detected by observing the $\cos 2\theta_{AB}$ dependence of the polarization correlation between two detectors A and B (see Eq.(3)):

$$\langle \Psi_i|\Pi_A\Pi_B|\Psi_i\rangle \propto \cos 2\theta_{AB},$$

with $i = 1, 2$. This can be achieved by changing $\theta_{AB}$. Note that this conclusion also applies to the special case where the two photons are coming from the same source.

Backgrounds

In a Bell inequality experiment, condition of coincident photons are imposed to reduce the background. In our case, we do not impose coincidence condition but assuming we have a large number of entangled pairs. The entanglement can be detected by varying $\theta_{AB}$ to isolate the entangled signal which has the $\cos 2\theta_{AB}$ dependence. Therefore it is important that the background from unentangled photons has a different angular dependence, such that the signal can be separated from the background without relying on theory prediction of the size of the background.

The possible source of unentangled photons includes: Type (a), two uncorrelated photons coming from two different sources; Type (b), two uncorrelated photons coming from the same source. Type (b) background can be considered as a special case of Type (a). Hence we will consider Type (a) first.

Consider the two unentangled photons are coming from sources 1 and 2 with one photon density matrix $\pi_1$ and $\pi_2$, respectively. By assuming the two photons density matrix can be factorized into two one photon density matrices, the two photons are regarded as unentangled [21]. $\pi_i$ at source $i$ with net polarization $\alpha_i$ in the $\vec{n}_i$ direction can be written by

$$\pi_i = \frac{1}{2 + 2\alpha_i} \left[ (1 + 2\alpha_i)|n_i\rangle\langle n_i| + |n_{i\perp}\rangle\langle n_{i\perp}| \right],$$

where $n_{i\perp}$ is perpendicular to $n_i$ and $i = 1, 2$.

The probability for detectors A and B each detects one photon is [21]

$$\text{Tr} (\Pi_A\pi_1) \text{Tr} (\Pi_B\pi_2) |D_{1A}|^2 |D_{2B}|^2$$

$$+ \text{Tr} (\Pi_A\pi_2) \text{Tr} (\Pi_B\pi_1) |D_{2A}|^2 |D_{1B}|^2$$

$$+ \text{Tr} (\Pi_A\pi_3\Pi_B\pi_2) D_{1A}D_{2B}D_{1B}^*D_{2A}^*$$

$$+ \text{Tr} (\Pi_A\pi_2\Pi_B\pi_1) D_{1A}^*D_{2B}^*D_{2A}D_{1B}.$$ (11)

Then

$$\text{Tr} (\Pi_A\pi_1) \text{Tr} (\Pi_B\pi_2) = \frac{\alpha_1\alpha_2 \cos 2\theta_{A\pi_1} \cos 2\theta_{B\pi_2}}{(1 + \alpha_1)(1 + \alpha_2)}.$$ (12)

where $\theta_{A\pi_i}$ is the angle between the orientation of the polarizer at detector A and $\vec{n}_i$. $\text{Tr} (\Pi_A\pi_2) \text{Tr} (\Pi_B\pi_1)$ can be obtained analogously by interchanging 1 and 2. The geometrical phase dependent interference terms yield

$$\text{Tr} (\Pi_A\pi_1\Pi_B\pi_2) = \text{Tr} (\Pi_A\pi_2\Pi_B\pi_1) \propto \cos 2\theta_{AB}.$$ (13)

up to $O(\alpha_i)$ corrections. When we consider the Type (b) background, we can just set (1,2) to (1,1) or (2,2), although their relative weights are in general different.

From the above discussion, we see the geometrical phase dependent background of Eq.(13) has the same angular dependence as the entanglement signal of Eq.(9). This means unless one can compute the background to very high accuracy, one cannot isolate the signal from the background using this geometrical phase dependent term. The geometrical phase independent terms, however, has different angular dependence for the signal. The complication is that one has to first isolate the geometrical phase independent terms from the geometrical phase dependent terms, then subtract the background which includes a linear combination of four terms: Eq.(12) and other terms with the (1,2) indices of Eq.(12) changed to (2,1), (1,1) and (2,2). This is a challenging task. Fortunately, the situation is simpler when we consider another scenario in the next section.

The set up discussed in this section is also discussed in Ref.[21], where it argued that entanglement implies the total density matrix of the two sources cannot be factorized to the direct product of $\pi_1$ and $\pi_2$. This condition is more general than the violation of Bell inequality in the sense that entanglement (non-factorization) does not necessarily implies violation of inequality in this setup with complicated backgrounds, even though violation of Bell inequality indeed implies entanglement.
Scenario II: Sources from large angle—no interference

FIG. 2. This is similar to the set up of Fig. 1, except 1 is in the field of view of Alice only and 2 is only in the field of view of Bob. The photons can only be detected after going via one path. Hence there is no interference terms.

When sources 1 and 2 are far away such that 1 is only in the field of view of A and 2 is only in the field of view of B, then there is no interference between different paths, as shown in Fig. 2. We can simply remove one of the paths in Scenario I by setting $D_{2A} = D_{1B} = 0$ for the signal from Eq. (8) and the background from Eq.(3).

For the signal, we have

$$\langle \psi_i | \Pi_A \Pi_B | \psi_i \rangle = \langle \psi_i | \Pi_A \Pi_B | \psi_i \rangle \propto \cos 2\theta_{AB}. \tag{14}$$

The entanglement signal is still the $\cos 2\theta_{AB}$ dependence.

For the background, we have the prefactor $\text{Tr} (\Pi_A \pi_1) \text{Tr} (\Pi_B \pi_2)$, whose angular dependence is computed in Eq.(12) already:

$$\text{Tr} (\Pi_A \pi_1) \text{Tr} (\Pi_B \pi_2) \propto \cos 2\theta_{An} \cos 2\theta_{Bn}. \tag{15}$$

The background has different angular dependence to the signal, $\cos 2\theta_{AB}$. For example, one could have chosen the direction of polarizer A such that $\cos 2\theta_{An} = 0$ and the background vanishes, then one can subsequently change $\theta_{AB}$ to look for the $\cos 2\theta_{AB}$ signal. This set up does not have the complication of scenario I where the background is a combination of several terms which makes the isolation of signal from background complicated.

The setting of this large angle scenario is similar to a set up in Refs. [22, 23] where lights from two distant sources separated by a large angle are used to determine the polarization directions of the two detectors. This allows pushing back the time for the possible “freedom of choice” loophole to happen all the way to the time when the two distant light sources were in contact.

 SUMMARY

We have explored the possibility of detecting entangled photon pairs from CMB or other cosmological sources coming from two patches of the sky. The measurement uses two detectors with photon polarizers in different directions. When two photon sources are separated by a large angle relative to the earth, such that each detector has only one photon source in its field of view, then the signal for entanglement can be separated from the background by changing the polarizer directions. In some special choice of the polarizer directions, the leading uncorrelated photon background can be completely blocked. When the angle between two photon sources is small enough such that both sources are in the fields of view of both detectors, then the background becomes more complicated with several terms, with different angular dependence and different weights. In all, the large angle scenario is preferred.

One question we do not discuss here in this paper is the different sources for this type of entanglement. Some of the possibilities are: two entangled particles in casually disconnected region are connected by a wormhole [24–26], or they are the decay product of dark matter candidates or some extensive cosmic objects. This entanglement between the CMB photon polarizations is different yet related to the old question that whether the origin of cosmic fluctuations are quantum or classical [11–13], although exact connections require further studies.

Our work is only a first step towards the detection of entangled photons in the sky, even though the result of our initial study is quite encouraging. A critical question is how large the estimated signal is compared with the current detector sensitivity and what would be possible sources of entangled photons. This is left for a future study.

 Acknowledgements

This work is inspired by a lecture of Frank Wilczek on Entanglement Enabled Intensity Interferometry. We also thank Dave Kaiser for interesting discussions. This work is partially supported by the MOST, NTU-CTS and the NTU-CASTS of Taiwan. J.W. Chen is partially supported by MIT MISTI program and the Kenda Foundation. Y.L. Zhang is supported by APCTP and CQUeST.

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SUMMARY

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