Steady three-dimensional dark state entanglement in dissipative Rydberg atoms

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Scheme to prepare three-dimensional entangled state between a pair of Rydberg atoms is proposed via dissipative dynamics and Electromagnetic Induced Transparency (EIT) associated with the single-atom dark state. The prepared entangled state is the dark state of the whole system. The schemes are feasible no matter the system initially in arbitrary purity or mixed states and do not have accurate requirements on evolution time. In contrast to most of the former Rydberg-atom-based dissipative schemes, the Rydberg-Rydberg interaction (RRI) strength do not need to satisfy a certain relation with laser detuning since it works in the blockade as well as intermediate regimes.

I. INTRODUCTION

Rydberg atoms are the neutral atoms with high lying excited state, and they would exhibit strong Rydberg-Rydberg interaction (RRI) when close enough [1–3]. The most interesting thing caused by RRI is the Rydberg blockade, which has been observed between two Rydberg atoms through sequent [4] and collective driving [5], respectively. Besides, the RRI has also been directly measured in experiment [6]. For quantum information processing (QIP) tasks, quantum logic gates and entangled states are basic building block and resource, respectively [7]. The pioneering works relevant to Rydberg quantum logic gate were proposed by Jaksch et al. [8], in which the model conditions $V$ (RRI strength) $\gg \Omega$ (Rabi frequency) and $u \ll \Omega$ are considered, respectively. Lukin et al. [9] then describe a method for the coherent manipulation of quantum information stored in collective excitations of mesoscopic many-Rydberg-atom ensembles. These two works [8, 9] open a new chapter for Rydberg-atom-based QIP and has been followed by a variety of interesting works, typically including preparation of quantum entanglement [10–12], construction of quantum logic gate [13–16], quantum logic gates [17], quantum algorithms [18], and quantum repeaters [19].

Dissipation induced by the coupling between quantum system and its environment is inevitable and always be considered detrimental for QIP tasks. Commonly, there are three methods to deal with the dissipation: (i) Quantum error correction method [20], which relies on the high-fidelity quantum gate for detecting and correcting errors. (ii) Dynamical decoupling method [21], which seeks to minimize the unwanted system-bath interactions in an open quantum system but can never completely avoid all unitary errors [22]. (iii) Decoherence-free subspace method [23], which requires the symmetric coupling between system and its bath. Different from the above methods that are trying to avoid the influence of dissipation, dissipative dynamics method [24] opens a new path to deal with decoherence since the dissipation plays a significant role to realize QIP tasks [25].

The dissipative-dynamics-based schemes in Rydberg atoms are interesting since they combine the advantages of Rydberg atom and of dissipative dynamics together. Recently, Petrosyan and Mølmer found that the atomic spontaneous emission of the intermediate excited state facilitates a single excitation of the Rydberg atom ensemble with nearly unit probability [26], which is meaningful for the construction of Rydberg superatom. And Lì, Ates, and Lesanovsky studied the dissipative blockade for excited Rydberg atoms [27]. Then, Rao and Mølmer [28], and Carr and Saffman [29] proposed pioneering works to prepare steady entangled state in Rydberg atoms via the dissipation. The scheme proposed in Ref. [28] works under the blockade regime ($V \gg \Omega$) as well as intermediate regime ($V \sim \Omega$) based on the Electromagnetic Induced Transparency (EIT). The steady entangled state contains the dark state induced by the EIT regime. Then the many-body steady entangled state is studied [30, 31] based on the RRI. On the other hand, the scheme proposed in Ref. [29] works under the Rydberg antiblockade regime [32] which requires the Rydberg pumping condition $V = 2\Delta \gg \Omega$ ($\Delta$ denotes laser detuning) and followed by many works [33].

In this manuscript, inspired by the pioneering work in Ref [28], we design a scheme to prepare the three-dimensional entangled state, which can enhance the security and capacity of QIP [34] and violate the local realism more strongly than the two-dimensional entangled state [35]. The prepared state is the dark state of the whole system which involves the single-atom dark state induced by the dissipative EIT and thus robust on RRI-induced mechanical effect. In addition, the scheme is insensitive to the RRI because it works under the blockade as well as the intermediate regimes and has short convergence time approaches steady state. Finally, we try to translate the dark state entangled state to the ground state entangled state via the adiabatic passage method.

The rest part of the manuscript is organized as follows. In Sec. II, the basic model of the scheme, including the dissipative EIT dark state and the approximations based on the dressed state, are introduced. In Sec. III, the basic dynamics and the performance of the scheme (including fidelity, purity and negativity) are shown based on one group of specific parameters. In Sec. IV, we discuss the robustness of the scheme for a wide range of parameters, try to transfer the dark-state-basis-based entangled state to ground state subspace, and consider experimental feasibility in some ways. The conclusions are given in Sec. V.
If \( \omega \) is much smaller than \( \Omega_1 \) and \( \Omega_2 \), Hamiltonian (1) constitutes dressed states as

\[ |D\rangle, |\zeta\rangle = (2\Omega_1 |1\rangle + (\Delta \pm \tilde{\Delta})|p\rangle + 2\Omega_2 |R\rangle)/N_{\pm}, \]

with the corresponding eigenvalues \( E_0 = \Delta \), \( E_{\pm} = (\Delta \pm \tilde{\Delta})/2 \), where \( \tilde{\Delta} = \sqrt{\Delta^2 + 4\Omega_1^2 + 4\Omega_2^2} \). Thus, the whole Hamiltonian of single atom \( \hat{H}_\text{Dr} + \hat{H}_\omega \) could be rewritten as

\[ \hat{H}_\omega = \omega(\langle 1|D|0\rangle|D\rangle e^{iE_0 t} + \langle 1|\zeta\rangle|0\rangle|\zeta\rangle e^{iE_\pm t}) + \text{H.c.} \]

after rotating with respect to the dressed states. If \( E_\pm \gg E_0 \), after discarding the high-frequency oscillating terms, one can approximately get

\[ \hat{H}'_\omega = \frac{\omega\Omega_2}{\sqrt{\Omega_1^2 + \Omega_2^2}} |D\rangle + \text{H.c..} \]

Sec. II.1 shows that the steady state of the dressed state space is the dark state \( |D\rangle \), and sec. II.2 shows that replacing the coupling \( |0\rangle \leftrightarrow |1\rangle \) with \( |0\rangle \leftrightarrow |D\rangle \) is reasonable. In the following subsection we would describe the basic dynamics of the system.

III. PREPARATION OF THREE-DIMENSIONAL ENTANGLED DARK STATE

III.1. Desired Entangled State

The three-dimensional entangled state has the form \([34]\)

\[ |\Psi\rangle = (|00\rangle + |11\rangle + |22\rangle)/\sqrt{3}, \]

which can enhance the security of QIP \([34]\) and violate the local realism more strongly than the two-dimensional entangled state \([35]\). In this manuscript, combing with the single-atom EIT dark state, we aim to prepare

\[ |\mathcal{D}\rangle = (\langle D_0| |D_0\rangle + \langle D_1| |1\rangle + \langle D_2| |D_2\rangle)/\sqrt{3}, \]

where \( |D_j\rangle = (\Omega_2 |j\rangle - \Omega_1 |R_j\rangle)/\sqrt{\Omega_1^2 + \Omega_2^2} \) is the dark state of the Hamiltonian \( \hat{H} = \Delta |p_j\rangle\langle p_j| + \Omega_1 |j\rangle\langle p_j| + \Omega_2 |p_j\rangle\langle R_j| + \text{H.c..} \)

III.2. Configuration and Hamiltonian

We consider two Rydberg atoms as shown in Fig. 2. Both of the atoms have two Rydberg states, two metastable states and three ground states. These two atoms interact with each other through the van-der-Waals-type RRI. The Hamiltonian of the whole system can be written as \( \hat{H} = \hat{H}_\Omega + \hat{H}_P + V \), in which

\[ \hat{H}_\Omega = \sum_{m=0,1} (\Omega_1 |m\rangle\langle p_m| + \Omega_2 |p_m\rangle\langle R_m| + \text{H.c.}) \]
III.3. Effective Dynamics

We now analyze the effective dynamics of the whole system. Based on the process similar to that discussed in Sec. II.2, the Hamiltonian of atom 1 can be approximated to

\[ \hat{H}_{\text{eff}1} = \omega_{\text{eff}1}|D_0\rangle\langle 2| + \omega_{\text{eff}2}|D_0\rangle\langle D_1| + \text{H.c.}, \]

and of atom 2 can be approximated to

\[ \hat{H}_{\text{eff}2} = \omega_{\text{eff}3}|D_0\rangle\langle 1| + \omega_{\text{eff}4}|D_0\rangle\langle D_2| + \text{H.c.}, \]

as shown in Fig. 3(a). The effective microwave coupling can be calculated through

\[ \omega_{\text{eff}}^{(1,2)} = \langle D_0|\hat{H}_{\omega k}|2(D_1)\rangle, \quad \omega_{\text{eff}}^{(2,4)} = \langle D_0|\hat{H}_{\omega k}|1(D_2)\rangle, \]

where \( \hat{H}_{\omega k} = \omega(|1\rangle k\langle 0| + |1\rangle k\langle 0|) + \text{H.c.} \) denotes the microwave field Hamiltonian of the \( k \)-th Rydberg atom. And the effective interaction strength between \( |D\rangle_m \) of atom 1 and \( |D\rangle_n \) of atom 2 can be calculated through

\[ V_{mn}^{\text{eff}} = \langle D_m|\hat{H}|D_n\rangle. \]

where we define \( \langle D_m|D_n\rangle \) is the conjugate transpose of \( |D_m\rangle\langle D_n| \) with \( m = 0,1 \) and \( n = 0,2 \). Fig. 3(b) describes the effective dynamics of the whole system. The effective coupling between \( |D_m\rangle\langle D_n| \) and \( |R_m\rangle\langle R_n| \), \( \Omega_{mn}^{\text{eff}} \), can be calculated as

\[ \Omega_{mn}^{\text{eff}} = \langle R_n|\langle R_m|\hat{H}|D_m\rangle|D_n\rangle = \Omega_{mn}^{V}V_{mn}. \]

The effective dynamics of the whole system can be illustrated as follows. Without consideration of RRs, the two-atom Hamiltonian has three dark states \( |D\rangle \), \( |D_1\rangle \), and \( |D_2\rangle \) and six bright eigenstates denoted by the set \( S \). \( |D_1\rangle \), \( |D_2\rangle \), and \( S \) contain at least one of the states \( |D_1\rangle|D_0\rangle \), \( |D_0\rangle|D_2\rangle \), and \( |D_1\rangle|D_2\rangle \) and maybe contain \( |D_0\rangle|D_0\rangle \) among the state \( |D_m\rangle\langle D_n| \). While \( |D_1\rangle \) only contain \( |D_0\rangle|D_0\rangle \) among the state \( |D_m\rangle\langle D_n| \). As mentioned above, the two-atom dark state \( |D_m\rangle\langle D_n| \) couples to the two-excitation Rydberg state \( |R_m\rangle\langle R_n| \) via the strength \( \Omega_{mn}^{\text{eff}1} \). And \( |R_m\rangle\langle R_n| \) couples with the metastable states which would decay to the space \( \mathcal{S} \equiv \{ |D\rangle, |D_1\rangle, |D_2\rangle, S \} \). If \( \Omega_{mn}^{\text{eff}} \ll \Omega_{10}, \Omega_{12}, \Omega_{21}, \Omega_{22}^{\text{eff}} \) is satisfied, \( |D\rangle \) would be the steady state of the whole system under the cooperation of unitary and dissipative dynamics.
Fidelity (Purity) can be calculated as \[38\] 
Purity is a measure of how pure a quantum state is and through \[37\] 

dissipative terms. 

Numerically solving the master equation basis are used for calculation. 

The pa- 

FIG. 4. Fidelity to prepare \(|D\rangle\) and purity of the system versus evolution time. The parameters are chosen as \(\Omega_1/2\pi = 30\) MHz, \(\Omega_2 = 3.85\Omega_1\), \(V_{12} = 2\Omega_1\), \(V_{10} = 0.8V_{12}\), \(V_{00} = 0.001V_{12}\), \(\gamma_p/2\pi = 10\) MHz, \(\gamma_R = 1\) KHz, \(\omega_1 = 0.004\Omega_1\), and \(\Delta = 0\). Suppose the system is initially in the mixed state \((\sum_j=0,1,2 \sum_{j'=}0,1,2 |j⟩⟨j'| J⟩)/9\).

FIG. 5. Negativity of practical systematic state versus evolu- 

tion time and the negativity of the ideal state. The inset displays the variation of the logarithmic negativity. The pa- 

rameters are the same as that in Fig. 4. And the bare state basis are used for calculation.

III.4. Performance of the scheme

We get the final state of the whole system through numerically solving the master equation

\[
\dot{\rho} = -i[H, \rho] + \sum_{l=1}^{8} \sum_{j=0}^{2} \left[ \hat{L}_j^l \rho \hat{L}_j^{l\dagger} - \frac{1}{2} (\hat{L}_j^{l\dagger} \hat{L}_j^l + \rho \hat{L}_j^l \hat{L}_j^{l\dagger}) \right],
\]

where \(\hat{L}_j^1 = \sqrt{\gamma_R/3} |j⟩⟨R_0|\), \(\hat{L}_j^2 = \sqrt{\gamma_R/3} |j⟩⟨R_1|\), \(\hat{L}_j^3 = \sqrt{\gamma_p/3} |j⟩⟨p_0|\), \(\hat{L}_j^4 = \sqrt{\gamma_R/3} |j⟩⟨p_1|\), \(\hat{L}_j^5 = \sqrt{\gamma_p/3} |j⟩⟨R_0|\), \(\hat{L}_j^6 = \sqrt{\gamma_R/3} |j⟩⟨R_1|\), \(\hat{L}_j^7 = \sqrt{\gamma_p/3} |j⟩⟨p_0|\), and \(\hat{L}_j^8 = \sqrt{\gamma_p/3} |j⟩⟨p_2|\) denote the dissipative terms.

Fidelity denotes the closeness of states in the natural geometry of Hilbert space, which can be calculated through \[37\] 

\[
F(|D⟩⟨D|, \hat{\rho}(t)) = ⟨D|\hat{\rho}(t)|D⟩. \tag{16}
\]

Purity is a measure of how pure a quantum state is and can be calculated as \[38\] 

\[
P(\hat{\rho}(t)) = \text{Tr}[\hat{\rho}(t)^2]. \tag{17}
\]

One can conclude that the scheme performs well when both of fidelity and purity are very close to 1. In addition, 

negativity, a measure deriving from the PPT criterion for separability \[39\], has shown to be a proper measure of entanglement with the definition \[39\] 

\[
\mathcal{N}(\hat{\rho}) = \frac{\text{Tr}(\rho T_A T_A^\dagger) - 1}{2}. \tag{18}
\]

Then, the logarithmic negativity is proposed with the definition \[40\] 

\[
E_{\mathcal{N}}(\hat{\rho}) \equiv \log_2(\text{Tr}(\rho T_A T_A^\dagger) := \log_2(2\mathcal{N} + 1). \tag{19}
\]

Based on the final state \(\hat{\rho}(t)\), one can calculate the fidelity, purity and negativity of our scheme with the above definitions, as shown in Figs. 4 and 5 with one group of specific parameters. The results show that both of the fidelity and purity are close to 1 and the negativity is close to the ideal value.

IV. DISCUSSIONS

IV.1. Variation of parameters

Experimentally, the obtainable parameters are not unique. Thus, it is necessary to see the performance of the scheme under various parameters. In Fig. 6, we plot the fidelity versus the variations of \(\Omega_2/\Omega_1\) and \(\omega/\Omega_1\), which shows the scheme may have a good performance in a wide range values of parameters \(\Omega_2\) and \(\omega_1\). In Fig. 7, we plot the fidelity and purity versus \(V_{10}, V_{12}, V_{00}, \gamma_p\) and \(\gamma_R\), respectively. The results show a good robustness on the variation of RRI, which is absolutely different to the Rydberg-antiblockade-based schemes and may release the experimental requirements.

IV.2. Transforming the entanglement to ground state subspace

In this subsection, we try to transfer \(|D⟩\) to the three- 

dimensional entangled state in the ground state subspace,
as shown in Eq. (7), via the adiabatic technique. To do this, we should turn off $\Omega_1$ adiabatically. For simplicity, we choose $\Omega_1(\tau) = \cos[\pi \tau/(2T)]$, and the result is shown in Fig. 8, where the beginning of $\tau$ is the end of the time in Fig. 4. Although the fidelity is not higher than 0.99, the performance could be further improved through using the shortcut to adiabaticity method and design the pulse shape more carefully. And we just numerically show that to transfer the entangled state to the ground state subspace is feasible.

IV.3. Experimental considerations

For practical experiments, the ground state energy level of the two atoms could be chosen as $|0\rangle \equiv |F = 1, m_f = 0\rangle$, $|1\rangle \equiv |F = 1, m_f = -1\rangle$ and $|2\rangle \equiv |F = 1, m_f = +1\rangle$ of $5S_{1/2}$. And the metastable energy level of the two atoms could be chosen as $|p_0\rangle \equiv |F = 1, m_f = 0\rangle$, $|p_1\rangle \equiv |F = 1, m_f = -1\rangle$ and $|p_2\rangle \equiv |F = 1, m_f = +1\rangle$ of $5P_{3/2}$. Rydberg state energy level should be chosen carefully because the asymmetric RRI and dipole transition selection rules.

The asymmetric RRI is crucial for our scheme and has been widely used for many Rydberg-atom-based quantum information processing schemes [12, 15, 29, 41]. In Ref. [12], some groups of reasonable asymmetric RRI parameters are used and predicted to satisfy $|\Delta_{sp}, \Delta_{ss}\rangle > \Delta_{pp}$, where $\Delta_{sp}$, $\Delta_{ss}$ and $\Delta_{pp}$ correspond to $V_{10}(V_{01})$, $V_{12}$ and $V_{00}$ of the present scheme, respectively. The Rydberg states considered in Ref. [12] are $|s\rangle = |41s_{1/2}, m = 1/2\rangle$ and $|p\rangle = |40p_{3/2}, m = 1/2\rangle$, and the maximal, minimum and average asymmetry $|\Delta_{sp}(\sim n^4/R^3)/\Delta_{pp}(\sim n^{13}/R^6)|$ are about 1400, 185, 757, respectively. However, we cannot use these results directly because the lifetime $\tau$ of $n = 40$ is about 57$\mu$s which is not enough for our scheme to achieve a high fidelity. From this point, Rydberg states with higher principal quantum numbers are preferred because $\tau \sim n^3$ [1, 2]. Nevertheless, for a fixed distance, the principal quantum number should not be too large because the asymmetry $\sim n^{-7}$. Based on these rules, if we choose $|R_0\rangle \equiv |70p_{3/2} m = 1/2\rangle$ (for two atoms), $|R_1\rangle \equiv |71s_{1/2} m = 3/2\rangle$ (for atom 1) and $|R_2\rangle \equiv |71s_{1/2} m = 3/2\rangle$ (for atom 2), the maximal asymmetry about 27.8523 and the lifetime $\tau \sim 305\mu$s are achieved. It should be noted that the transition between $|p_0\rangle$ and $|R_0\rangle$ could be realized via introducing a large detuning interaction with intermediate $|s\rangle$ or $|d\rangle$ state (similar to the degenerate two-photon process). With these assumptions and assume the other factors which influence the RRI keep invariant, the fidelity and purity are estimated about 0.6047 and 0.507, respectively, at $\tau \sim 305\mu$s with $\Omega_1$, $\Omega_2$ and $\omega$ parameters the same as that in Fig. 4. Similarly, we roughly estimate the performance for different principal quantum numbers at the corresponding lifetime with the maximal asymmetry for different principal quantum numbers, as shown in Fig. 7. We use the dipole-dipole interactions rather than vdW interaction for two different Rydberg states for simulation.

Besides, we could also choose $|R_0\rangle \equiv |75p_{3/2} m = 1/2\rangle$ (for two atoms), $|R_1\rangle \equiv |125s_{1/2} m = 3/2\rangle$ (for atom 1) and $|R_2\rangle \equiv |120s_{1/2} m = 3/2\rangle$ (for atom 2).
We suppose the interaction between states $|R_0\rangle_1$ and $|R_0\rangle_2$ ($|R_1\rangle_1$ and $|R_2\rangle_2$) is van der Waals (vdW) interaction with strength $V_{00}$ ($V_{12}$) while between $|R_1\rangle_1$ and $|R_0\rangle_2$ ($|R_0\rangle_1$ and $|R_2\rangle_2$) is dipole-dipole interaction with strength $D_{10}$ ($D_{00}$). Due to the principal quantum number differences, vdW strength $V_{12}$ could be larger than $V_{00}$. Meanwhile, the Stark tuned Förster resonances have been demonstrated experimentally \cite{42,43} due to the dipole-dipole interaction and the detuning could be adjusted through the electric field. Then, one could achieve the non-resonant dipole-dipole interaction induced vdW between $|R_1\rangle_1$ and $|R_0\rangle_2$ ($|R_0\rangle_1$ and $|R_2\rangle_2$). Under this case, the higher fidelity and purity may be achieved due to the long lifetime Rydberg states.

V. CONCLUSION

In this manuscript, we proposed a scheme to prepare three-dimensional dark state entanglement based on the dissipative EIT and Rydberg blockade regimes. In contrast to the Rydberg-antiblockade-based schemes, the present one is insensitive to the variation of RRI and has a shorter evolution time approaches steady state, which may release the experimental requirements. In addition, the scheme does not have accurate requirement on the evolution time and is feasible even the system is initially in the mixed state.

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