The use of GeoGebra to help students gain better understanding to definition of definite integral

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Abstract. Many research shows that students have difficulties in understanding definition of definite integration. Therefore, the research is interested in designing classroom activities integrated with design research method to help students gain better understanding. In order to support students, realistic mathematics approach was chosen. Series of instructional activities had been designed to reach mathematical goal by using geogebra software. First year university students were chosen to implement the classroom activities. The results indicate that geogebra software help student gain better understanding to the definition of definite integral by providing dynamic visualization of the sum of inscribed and circumscribed polygon.

1. Introduction
Calculus is considered as the core of mathematics. It serves as foundation for students in university especially majoring natural science, engineering and mathematics in their studies. It has three major topics and one of them is Integration. However, many studies found that students have difficulties in understanding concept of integration in calculus [1]. Serhan found in his research that students in UAE had narrow understanding on topic of definite integral. Their competencies to represent the concept in different ways were limited; they dominant knowledge is the procedural one [2]. This situation is also happened to students in Universitas Negeri Padang (UNP). Lack of understanding of integration concept consider as one of the causes which make them fail [3]. Most of students in UNP might be able to do the integration procedures and find the result of their calculation. But they do not know about the meaning of their calculation. For instance, students are able to count the integration of $x^2$ from zero to two which give them result 8/3 but they do not know what the meaning of the number (8/3) that they found is. This happened because most of them tend to focus on the integrating procedures without building their understanding on the concept of integration [4]. Tasman in his research suggest that educators should not rush to define the procedure without a deep understanding of the concept [5]. So, it is important to design a classroom activities to help students’ gain better understanding on definite integral.

In designing classroom activities, technology plays an important role. Technology has been used to improve capability and success in teaching and learning [6]. Geogebra is one part of technology in form of computer software. It can be used in learning especially in mathematics. It provides dynamic mathematics that can be used to build students understanding. Many research show that this software is powerful software in mathematics. This software not only powerful in geometry [7] and algebra [8] but also in calculus [9].
In learning definite integral, students have to understand about Sigma, Riemann Sum, Limit and Area. Those concepts are used to build the definition of definite integral. However, students found it is difficult for them to understand about that definition [3]. Therefore, we interested to design a series of integrated classroom activities using geogebra software to assist students build their understanding on the definition of definite integral. We are interested to answers our research question on how geogebra software help students gain better understanding on definition of definite integral.

To support the development of students’ understanding on definition of definite integral we tried to apply realistic mathematical approach (RME). This approach has five tenets that adapted to the research. They are (1) Constructions stimulated using concreteness. Which means it does not begin with formal stage, it starts with a situation which is real for students by understanding the notion of area. (2) Mathematical tools are developed from concreteness to abstraction. This tenet connects concreteness to a more formal level by using symbols models, or graphic. (3) Stimulating free production and reflection by students because it assumed meaningful for them. (4) Stimulating social activities of learning which create a good interaction. Classroom social activity provides students opportunity to have interaction between each other. It will trigger them to discuss ideas and have a rich discussion to solve the problems. (5) Intertwining learning strands to get mathematical idea structured. The definition of definite integral has solid relation to Sigma, Riemann Sum, Limit and Area.

2. Design and Method

Design research methodology was applied in this research. There are three reasons to chose this methodology in which in line with [10]; First, this methodology gives productive views to develop theory. Second, it has specific usefulness of its outcomes. Third, it directly makes the researcher in the refinement of mathematics education. Design Research has five characteristics [11] that applied to this research.

2.1. Design and Preparation

This stage focus on determining the goals of mathematical learning and it mixed with anticipatory thought on how to reach the goal. It produce conjectures which consist of three things; Students’ learning goal, Designed instructional activities and Conjectured of learning processes that anticipates how students' thinking and understanding could evolve when instructional activities used in classroom [12].

2.2. Teaching experiment

Instructional activities are tried and data collection explains how a set of the instructional activities could work. Afterward the instructional activities are revised on daily basis to create a well considered and empirically grounded local instructional theory (LIT).

2.3. Retrospective analysis

Data collected are analyzed and the hypothetical learning trajectory (HLT) is compared with students’ actual learning in the classroom. This stage form a new cycle in the emergence of LIT.

3. Finding and Discussion

The learning trajectory is defined as a description of the process of students activities in learning that they can follow to build their understanding on definition of definite integral. It examines the goal, activities and conjecture of learning processes. This trajectory is hypothetical because interpretations, ideas and strategies of students never sure until the students really work on problems. The brief overview HLT of definition of definite integral showed in Table 1.
Students’ worksheets are designed in the design phase. In the worksheets, students were asked about counting area under the curve of $x^2$ which are bounded with the x-axis and $x = 2$, by using inscribed and circumscribed polygon. Students were asked about the meaning of their calculation. After that, they asked to sketch the graph if the $n$ (which indicated the partition of base) equal to 5, and they have to observe the graphs which are draw using geogebra software and to relate with the area under the curve. Then, students were asked to see the visualization using geogebra software, and relate it with infinite limit. Finally, they were given definition of definite integral and were asked that definition in geometric way.

Our conjectures in this activity are (1) students would have difficulties in counting the area using inscribed and circumscribed polygon. If this situation happens, we will provoke them by asking their knowledge about area and sigma and the idea of base partition. (2) Some students will have idea to count it and we will ask them to share their ideas in classroom discussion. They are three crucial points that we want our students to aware in our designed activity; (a) their awareness about the result that they found in equation which gives them $n$ as variable represent the number partition of base. Which means that if we give them $n=5$ means the area of 5 polygons for instance. (b) Their understanding about if the partition of base approaching infinity means the area of polygon will be same with the area under the curve. (c) Their ability to relate the formal definition with geometric interpretation of definite integral.

In the teaching experiment, we found that some students find difficulties in counting the area by using inscribed and circumscribed polygon because they do not able to use the special sum of sigma properties which lead them to the wrong calculation. Interestingly, some students were able to found the correct calculation but they are not able to understand the meaning of their calculation. One example comes from Fefen (a first year student, in mathematics department of Universitas Negeri Padang. Therefore, we interviewed him and discuss with him. First we asked him about the meaning of his solution. Fefen starts to explain the process how he comes to the solution which shown in figure 1.
Figure 1. Fefen tried to explain the process to tell his solution

Here is the transcript for the interview:

Researcher : What is the meaning of your calculation (Pointing the result of Fefen calculation)
Fefen : For the inscribed polygon, I know that the area of one polygon is $\int \Delta t$, and I know that the given function is $x$ square then I have $\int \Delta t = \int \Delta t$. My understanding about the partition of the base give me information that the $\Delta t = \frac{2}{n}$. Then I tried to apply the special sum of sigma, therefore I get these answer (pointing his solution).

Based on his explanation, researchers conclude that he is able to count the area of inscribed polygon.

Researcher : That is good explanation, but what is the meaning of your calculation results $\frac{2}{6}(2x^2 - 3x^2 + \frac{1}{3})$ for inscribed polygon and $\frac{6}{6}(2x^2 + 3x^2 + \frac{1}{3})$ for circumscribed polygon?

Fefen : (He quiet for a while), I do not know sir!

Researcher : How about if you determine the $n = 5$? Can you relate with the partition of the base? Can you sketch the graph and count the area of the five polygon?

Fefen : Ooo, I know sir. Let me try sir!

Researcher : Ok.

Fefen started to answer the question and sketched the graph and counted the area of inscribed and circumscribed polygon if the $n = 5$. He tried to answers the question by sketch the graph first and counting its area by adding 5 rectangles. Then the researcher try to provoke him by asking question “so what is the $n$ actually ?”. He was directly aware that the $n$ is the number of partition of the base which means number of polygon. He tried to substitute the $n=5$ to his formula and counted it. Then finally he found a number which is equal with the sum of the 5 rectangle that he count by using the figure that he sketched. Figure 2 shows Fefen answer.
From figure 2, we can see that Fefen found the area of inscribed polygon if the n=5 is $8 \frac{65}{125}$ and the area of circumscribed polygon if the n=5 is $8 \frac{65}{125}$ in two ways; first by adding the rectangle and substituting to the formula that he already got. From Fefen, we see that the students need to know the meaning of their calculation to shows their deep understanding about the given problems to them. After that fruitful discussion with Fefen, the researchers tried to ask the students “how about if the n approaching infinity?”, Then most of students quiet, and some of them said “I do not know”. Then the researcher asked students to see the dynamic visualization of the geogebra to show about the phenomena when the n is approaching infinity on circumscribed and inscribed polygon. That phenomena shown in Figure 3.
After the students observe the visualization of circumscribed and inscribed polygon, then researchers asked Zizi (one of students in the class). Here is the transcript:

**Researcher**: Zizi, what can you said about that? What happened if the \( n \) approaching infinity?

**Zizi**: If we divided the base into the \( n \) segment, then we take the \( n \) approaching infinity then the area of polygon, I mean inscribed and circumscribed polygon will be equal with the area under the curve \( x^2 \) with \( x \)-axis and \( x=2 \).

**Researcher**: What happened with the area of circumscribed and inscribed polygon?

**Zizi**: The area of circumscribed polygon will be decrease until it stop on the curve and the area of inscribed polygon will increase until it stop on the curve too.

**Researcher**: Why do you said that it will stop until the curve?

**Zizi**: I mean that for circumscribed it never be below to the curve and for inscribed it never be upper the curve, I mean the polygon.

**Researcher**: How do you know that?

**Zizi**: I saw on geogebra visualization that you shown to me!

**Researcher**: Can you explain to me how it works?

**Zizi**: In inscribed and circumscribe polygon if the segment you take approaching infinity it means that the length of the base of polygon will approaching zero then the width will be the value of function. This value of function guarantee that the circumscribed polygon will never be below the curve and the inscribed polygon will never be upper the curve.

**Researcher**: Ok. Good.

From the transcript, we see that the dynamic visualization of geogebra gives the student idea that it will never below the curve (for circumscribed polygon) and it will never upper the curve (for inscribed) polygon. Furthermore, our design also helped the students gain better understanding on constructing polygon because they experienced to build their inscribed and circumscribed polygon. When zizi finished explaining her idea, then the formal definition of definite integral (Figure 4) are give to the students.
We asked the students to look to the definition and related with Zizi’ opinion. Most students were able that definition and understand that it also means the area under the curve.

4. Conclusion
The research shows that the use of geogebra by providing dynamic visualization builds students understanding on the definition of definite integral. The students need to see or imagine and understand why the definite integral also means the area under the curve. This research confirm the finding of Monika Dockendorff and Horacio Solar about integrating learning with ICT will give good impact on teacher conceptions about teaching and learning mathematics since our students are perspective teachers [13].

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