Measurement of the $B^\pm \to \rho^\pm \pi^0$ branching fraction and direct CP asymmetry

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MEASUREMENT OF THE $B^+ \to \rho^+ \pi^0 \ldots$

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Branching fraction and CP asymmetry measurements of charmless B meson decays provide valuable constraints for the determination of the unitarity triangle. In particular, the angle \( \alpha \equiv \arg\left[ -V_{ub}V_{cb}^*/V_{cd}V_{ud}^* \right] \) of the Unitarity Triangle can be extracted from decays of the B meson to \( \rho \pi \) final states [3]. However, the extraction is complicated by the interference of decay amplitudes with differing weak and strong phases. One strategy to overcome this problem is to perform an SU(2) analysis that uses all \( \rho \pi \) final states [4]. Assuming isospin symmetry, the angle \( \alpha \) can be determined free of hadronic uncertainties from a pentagon relation formed in the complex plane by the five B \( \rightarrow \rho \pi \) decay amplitudes \( B^0 \rightarrow \rho^{-} \pi^+ \), \( B^0 \rightarrow \rho^{+} \pi^- \), \( B^0 \rightarrow \rho^{0} \pi^0 \), \( B^+ \rightarrow \rho^+ \pi^0 \), and \( B^- \rightarrow \rho^0 \pi^+ \). These amplitudes can be determined from measurements of the corresponding decay rates and CP asymmetries. While all these modes have been measured [5,6], the current experimental uncertainties need to be reduced substantially for a determination of \( \alpha \). Here we present an update to previous measurements of the \( B^\pm \rightarrow \rho^\pm \pi^0 \) branching fraction and CP asymmetry

\[
\mathcal{A}_{CP} = \frac{N(B^+ \rightarrow \rho^- \pi^0) - N(B^0 \rightarrow \rho^+ \pi^0)}{N(B^- \rightarrow \rho^+ \pi^0) + N(B^+ \rightarrow \rho^- \pi^0)},
\]

The main additions compared to our previous analysis [5] are larger data sets, a study of possible backgrounds from higher \( \rho \) resonances and the use of the \( \rho \) mass in the maximum likelihood fit.

The data were collected with the BABAR detector [7] at the PEP-II asymmetric-energy \( e^+e^- \) storage ring at SLAC. Charged-particle trajectories are measured by a five-layer double-sided silicon vertex tracker and a 40-layer drift chamber located within a 1.5-T magnetic field. Charged hadrons are identified by combining energy-loss information from tracking (dE/dx) with the measurements from a ring-imaging Cherenkov detector. Photons are detected by a CsI(Tl) crystal electromagnetic calorimeter with an energy resolution of \( \sigma_E/E = 0.023(E/{\text{GeV}})^{-1/4} \oplus 0.014 \). The magnetic flux return is instrumented for muon and \( K^0_L \) identification. The data sample includes 232 \pm 3 million \( B\bar{B} \) pairs collected at the \( Y(4S) \) resonance, corresponding to an integrated luminosity of 211 fb\(^{-1}\). In addition, 22 fb\(^{-1}\) of data collected 40 MeV below the \( Y(4S) \) resonance mass are used for background studies. We perform full detector Monte Carlo (MC) simulations equivalent to 460 fb\(^{-1}\) of generic \( B\bar{B} \) decays and 140 fb\(^{-1}\) of continuum quark-antiquark events (e\(^+\)e\(^-\) \rightarrow q\bar{q} \), \( q = u, d, s, c \)). In addition, we simulate over 50 exclusive charmless B meson decay modes, including \( 1.4 \times 10^6 \) signal \( B^\pm \rightarrow \rho^\pm \pi^0 \) decays.

B meson candidates are reconstructed from one charged track and two neutral pions. The charged track used to form the \( B^\pm \rightarrow \rho^\pm \pi^0 \) candidate is required to have at least 12 hits in the drift chamber, to have a transverse momentum greater than 0.1 GeV/c, and to be consistent with originating from the beam-spot. It must have ionization-energy loss and Cherenkov angle signatures consistent with those expected for a pion. We remove charged tracks that pass electron selection criteria based on dE/dx and calorimeter information. Neutral pion candidates are formed from two photon candidates, each with a minimum energy of 0.03 GeV and which are required to exhibit a lateral profile of energy deposition in the electromagnetic calorimeter consistent with an electromagnetic shower [7]. The angular acceptance of photon candidates is restricted to exclude parts of the calorimeter where showers are not fully contained. We require the photon clusters forming the \( \pi^0 \) to be separated in space, with a \( \pi^0 \) energy of at least 0.2 GeV and an invariant mass between 0.10 and 0.16 GeV/c\(^2\).

Two kinematic variables, \( \Delta E = E_B^+ - \sqrt{s}/2 \) and the beam-energy substituted mass of the B meson \( m^*_{\text{ES}} = \sqrt{(s/2) + p_0 \cdot p_B}/E_B^+ - p_B^2 \), are used for the final selection of events. Here \( E_B^+ \) is the energy of the B meson candidate in the center-of-mass frame, \( E_B^+ \) and \( \sqrt{s} \) are the total energies of the \( e^+e^- \) system in the laboratory and center-of-mass frames, respectively, and \( p_0 \) and \( p_B \) are the three-momenta of the \( e^+e^- \) system and the B meson candidate in the laboratory frame, respectively. For correctly reconstructed \( \rho^\pm \pi^0 \) candidates \( \Delta E \) peaks at zero, while for final states with a charged kaon, such as \( B^\pm \rightarrow K^{\pm} \pi^0 \), \( \Delta E \) is shifted by approximately 80 MeV on average. Events are selected with \( 5.20 < m^*_{\text{ES}} < 5.29 \) GeV/c\(^2\) and \( |\Delta E| < 0.20 \) GeV. The \( \Delta E \) limits remove background from two- and four-body B meson decays with a small loss in signal efficiency.

Continuum events are the dominant background. To suppress this background, we select only those events where the angle \( \theta^S_{\text{Sph}} \) in the center-of-mass frame between the sphericity axis [8] of the B meson candidate’s decay products and the sphericity axis of the rest of the event.
satisfies $|\cos \theta_{Sph}| < 0.9$. In addition, we construct a non-linear discriminant, implemented as an artificial neural network (ANN) that uses three input parameters: the zeroth- and second-order Legendre event shape polynomials $L_0$ and $L_2$ calculated from the momenta and polar angles, with respect to the $B$ meson thrust axis, of all charged-particle and photon candidates not associated with the $B$ meson candidate, and the output of a multivariate, nonlinear $B$ meson candidate flavor tagging algorithm [9]. The output $A_{NN}$ of the artificial neural network peaks at 0.5 for continuumlike events and at 1.0 for $B$ meson decays. We require $A_{NN} > 0.63$ which reduces the continuum background by half for a 5% loss in signal MC efficiency. To further improve the signal-to-background ratio we restrict the invariant mass of the $\rho$ candidate to $0.55 < m_{\pi\pi} < 0.95 \text{ GeV}/c^2$.

The average $B$ meson candidate multiplicity per event is 1.8 as neutral and charged pion combinatorics can lead to more than one $B$ meson candidate. We choose the best candidate based on a $\chi^2$ formed from the measured masses of the two $\pi^0$ candidates within the event compared to the known $\pi^0$ mass [10]. In the case of multiple charged pion candidates the choice is random so as not to bias the fit distributions. This random selection has a negligible impact on the systematic uncertainty. The total $B^+ \rightarrow \rho^{\pm} \pi^0$ selection efficiency is $15.4 \pm 0.1\%$. In signal MC studies, the candidate is correctly reconstructed 54.9% of the time. The remaining candidates come from self-cross-feed (SCF, 37.5%) and mistag events (7.6%). SCF events stem primarily from swapping the low energy $\pi^0$ from the resonance with another from the rest of the event. Signal events reconstructed with the wrong charge are classified as mistag events. Both SCF and mistag events emulate signal events, however the resolution in $m_{ES}$ and $\Delta E$ tends to be worse.

We use MC events to study the backgrounds from other $B$ meson decays. The dominant contribution comes from $b \rightarrow c$ transitions; the next most important is from charmless $B$ meson decays. Seventeen individual charmless modes show a significant contribution once the event selection has been applied. These modes are added into the fit (described below) fixed at the yield and asymmetry determined by the simulation, based on their measured values [10]. The largest contributions come from $B^0 \rightarrow \rho^{\pm} \rho^{\mp}$ and $B^0 \rightarrow \rho^{-}\pi^+$. For $B^{+} \rightarrow \eta^{'} \pi^{0}$ and $B^0 \rightarrow K^{*0} \rho^{\mp}$ we use half the measured upper limit [10]. We estimate the $B^0 \rightarrow \pi^0 \pi^0$ branching fraction from that of $B^0 \rightarrow a_1^{0} \pi^-$ [11] using isospin relations. If no charge asymmetry measurement is available, we assume zero asymmetry.

Although all other states that decay like the $\rho$ to $\pi \pi^0$ — the $\rho(1450)$ and the $\rho(1700)$, subsequently referred to collectively as $\rho^*$ — lie outside our $\rho(770)$ mass cut, a contribution to our signal cannot be ruled out a priori. To account for the possible presence of these modes, an unbinned maximum likelihood fit to the $B^\pm \rightarrow \rho^{\pm} \pi^0$ yield is performed in a sideband of the $m_{\pi\pi}$ invariant mass. This fit uses the same algorithm as described below but with only the three input variables $m_{ES}$, $\Delta E$, and $A_{NN}$. The mass window is chosen to be as far as possible from the $\rho(770)$ mass, centered near the pole of the $\rho(1700)$ at $1.5 < m_{\pi\pi} < 2.0 \text{ GeV}/c^2$. The fitted yield for the $B^\pm \rightarrow \rho^{\pm} \pi^0$ decay is then extrapolated into the $\rho(770)$ region, $0.55 < m_{\pi\pi} < 0.95 \text{ GeV}/c^2$, using a nonrelativistic Breit-Wigner line shape. Although the choice of mass range is motivated by the $\rho(1700)$, any yield seen is attributed entirely to the $\rho(1450)$, which is the closer of the two resonances to the signal. From the $B^\pm \rightarrow \rho^{\pm}(1450)\pi^0$ MC, the ratio of the number of candidates in the sideband to candidates in the signal mass region is approximately 12.6:1. The fit in the sideband yields $101 \pm 32$ events, resulting in an estimate of the $\rho^*$ background of 8 events.

We investigate possible interference effects by using an analytical model for the line shapes of the $\rho(770)$ and the $\rho^*$. We compare the use of relativistic and nonrelativistic Breit-Wigner line shapes and vary the widths of the line shapes by their uncertainties [10]. We also scan the relative phase between the two resonances from $-\pi$ to $\pi$. We assign a conservative systematic uncertainty of 100% for the $\rho^*$ background based on the largest change in the number of events in the range $0.55 < m_{\pi\pi} < 0.95 \text{ GeV}/c^2$ from these tests. The $\rho^*$ then enters into the nominal fit with probability density functions (PDFs) constructed from $B^\pm \rightarrow \rho^{\pm}(1450)\pi^0$ MC simulation.

The nonresonant $B^\pm \rightarrow \pi^\pm \pi^0\pi^0$ branching fraction has, to date, not been measured. To estimate the size of its contribution we select a region of the Dalitz plot—defined by the triangle $(m_{\pi^+\pi^-\pi^0}^2, m_{\pi^+\pi^0\pi^-}^2) =$ (6, 6), (6, 15), (11, 11) GeV$^2$/c$^4$—that is far from the signal as well as the $\rho(1450)$ and higher resonances and which has low levels of continuum background. The unbinned maximum likelihood fit with only three input variables ($m_{ES}$, $\Delta E$, and $A_{NN}$) is applied in this region. The only significant backgrounds expected are from generic $B$ and continuum events. The yields of the generic $B$ decays are fixed to values expected from MC simulation while the continuum and nonresonant yields are allowed to float. There are 1100 data events in the selected Dalitz region and the fit yields $-5.1 \pm 7.6$ nonresonant events. This is consistent with zero and the nonresonant contribution is therefore not considered as a background to our signal.

An unbinned maximum likelihood fit to the variables $m_{ES}$, $\Delta E$, $A_{NN}$, and $m_{\pi\pi}$ is used to extract the total number of signal $B^\pm \rightarrow \rho^{\pm} \pi^0$ and continuum background events and their respective charge asymmetries. The likelihood for the selected sample is given by the product of the PDFs for each individual candidate, multiplied by the Poisson factor:

$$L = \frac{1}{N!} e^{-N'} (N')^N \prod_{i=1}^{N} P_{\rho}$$
where $N$ and $N'$ are the number of observed and expected events, respectively. The PDF $P_i$ for a given event $i$ is a sum of the signal and background terms:

$$P_i = \frac{1}{2}[1 - (1 - Q_i A_{i}^{\text{Sig}})] a_{\text{Sig}} P_{1}^{\text{Sig}} + (1 - Q_i A_{i}^{\text{Sig}})] f_{\text{SCF}} P_{\text{SCF},i}^{\text{Sig}} + (1 + Q_i A_{i}^{\text{Sig}})] f_{\text{Mis}} P_{\text{Mis},i}^{\text{Sig}}$$

$$+ \sum_j N_{j}^{\text{Bkg}} \times \frac{1}{2}[1 - (1 - Q_i A_{i}^{\text{Bkg}})] f_{j}^{\text{Bkg}}$$

where $Q_i$ is the charge of the pion in the event, $N_{i}^{\text{Sig}}(A_{i}^{\text{Bkg}})$ and $A_{i}^{\text{Sig}}(A_{i}^{\text{Bkg}})$ are the yield and asymmetry for signal (background) component $j$, respectively. The fractions of true signal $(f_{\text{Sig}})$, SCF signal $(f_{\text{SCF}})$, and wrong-charge mistag events $(f_{\text{Mis}})$ are fixed to the numbers obtained from MC simulations. The $j$ individual background terms comprise continuum, $b \rightarrow c$ decays, $\rho^+$, and 17 other exclusive charmless $B$ meson decay modes. Signal and continuum yields are allowed to float in the fit, with the generic $B$ yields fixed to values expected from MC simulation. The PDF for each component, in turn, is the product of the PDFs for each of the fit input variables, $P = P(m_{\text{ES}}, \Delta E) P(A_{\text{NN}}) P(m_{\pi^\pm})$. Because of correlations between $\Delta E$ and $m_{\text{ES}}$, the $P(m_{\text{ES}}, \Delta E)$ for signal and all background from $B$ meson decays are described by two-dimensional nonparametric PDFs [12] obtained from MC events. For continuum background, $P(m_{\text{ES}}, \Delta E)$ is the product of two one-dimensional nonparametric PDFs; $m_{\text{ES}}$ is well described by an empirical phase-space threshold function [13] and $\Delta E$ is parametrized with a second degree polynomial. The parameters of the continuum PDFs are allowed to float in the fit except for the endpoint of the empirical phase-space threshold function which is fixed at 5.29 GeV/c$^2$. $A_{\text{NN}}$ is described by the product of an exponential and a polynomial function for continuum background and by a Gaussian with a power-law tail on one side [14] for all other modes. For $P(m_{\pi^\pm})$, one-dimensional nonparametric PDFs obtained from MC events are used to describe all modes except the signal mode itself, which is described by a nonrelativistic Breit-Wigner line shape. The parameters for this PDF are held fixed to the MC values and varied within errors to estimate systematic uncertainties. The covariance matrix from the fit to data confirms that correlations between all fit variables are small.

A number of cross checks confirm that the fit is unbiased. Using a double Gaussian PDF instead of a Breit-Wigner or omitting $m_{\pi^\pm}$ altogether as a fit variable has no significant effect on the measured branching fraction. In 1000 MC pseudoexperiments, we use the maximum likelihood fit to extract the yields and asymmetries. The distributions for each component are generated from the component’s PDF, giving values for the fit variables $m_{\text{ES}}$, $\Delta E$, $A_{\text{NN}}$, and $m_{\pi^\pm}$. The expected number of events is calculated from the branching fraction and efficiency for each individual mode. The generated number of events for each fit component is determined by varying the expected number according to a Poisson distribution. The test is repeated using samples with different asymmetry values. We repeat these MC studies using fully simulated signal $B^\pm \rightarrow \rho^+ \pi^0$ events instead of generating the signal component from the PDFs. This verifies that the signal component is correctly modeled, including correlations between the fit variables. We also compare the MC continuum distributions with the data collected below the $Y(4S)$ resonance and confirm that the PDFs model the data correctly within statistics. As another cross check we compare the distribution of the helicity angle $\theta_{hel}$ between the momenta of the charged pion and the $B$ meson in the $\rho$ rest frame in data with that modeled in MC samples for a variety of selection criteria. To investigate the possible effects of interference, we repeat the analysis excluding events where both $m_{\pi^+\pi^-}$ combinations are in the range 0.55 to 0.95 GeV/c$^2$; the branching fraction decreases by 0.1%.

Individual contributions to the systematic uncertainty are summarized in Table I. For each contributing exclusive $B$ meson decay mode, we vary the number of events in the fit by its measured uncertainty, or by $\pm 100\%$ if derived from an upper limit. For the $b \rightarrow c$ component, we fix the rate based on the number calculated from MC samples and vary the amount based on the statistical uncertainty on this number. The shifts in the fitted yields are calculated for each mode in turn and then added in quadrature to find the total systematic effect. To take into account the variation of

| Source | $\sigma_{\text{Syst}}^{\text{Yield}}$ (Events) |
|--------|------------------------------------------|
| $B$ background normalization | $\pm 0.9$ |
| PDF shapes | $\pm 0.2$ |
| SCF fraction | $\pm 0.12$ |
| Mistag fraction | $\pm 0.2$ |
| $\Delta E$ shift | $\pm 0.6$ |
| Total | $\pm 1.5$ |

| Source | $\sigma_{\text{Syst}}^{B}$ (%) |
|--------|--------------------------------|
| Efficiency estimation | $\pm 7.3$ |
| $B$ counting | $\pm 1.1$ |
| Total | $\pm 7.4$ |

| Source | $\sigma_{\text{Syst}}^{A_{\text{CP}}}$ |
|--------|----------------------------------|
| Background normalization | $\pm 0.006$ |
| Background asymmetry | $\pm 0.024$ |
| Detector asymmetry | $\pm 0.003$ |
| PDF shapes | $\pm 0.001$ |
| Total | $\pm 0.02$ |
the two-dimensional nonparametric PDFs used for $\Delta E$ and $m_{ES}$, we smear the MC-generated distributions from which the PDFs are derived. This is effectively done by varying the kernel bandwidth [12] up to twice its original value. For $m_{\pi\pi}$ and $A_{NN}$, the parametrizations determined from fits to MC events are varied by 1 standard deviation. The systematic uncertainties are determined using the altered PDFs and fitting to the final data sample. The overall shifts in the central value are taken as the size of the systematic uncertainty. We vary the SCF fraction by a conservative estimate of its relative uncertainty (±10%) and assign the shift in the fitted number of signal events as the systematic uncertainty of the SCF fraction. To account for differences in the neutral particle reconstruction between data and MC simulation, the signal PDF distribution in $\Delta E$ is offset by ±5 MeV and the data are then refit. The larger of the two shifts in the central value of the yield is 2.6 events, which is taken as the systematic uncertainty for this effect.

Corrections to the $\pi^0$ energy resolution and efficiency, determined using various data control samples, add a systematic uncertainty of 7.2%. A relative systematic uncertainty of 1% is assumed for the pion identification. A relative systematic uncertainty of 0.8% on the efficiency for a single charged track is applied. Adding all the above contributions in quadrature gives a relative systematic uncertainty on the branching fraction of 7.3%. Another contribution of 1.1% comes from the uncertainty on the total number of $B$ events.

To calculate the effects of systematic shifts in the charge asymmetries of background modes, the asymmetry of each mode is varied by its measured uncertainty. For contributions with no asymmetry measurement, we assume zero asymmetry and assign an uncertainty of 20%, motivated by the largest charge asymmetry measured in any mode so far [15]. The individual shifts are then added in quadrature to find the total systematic uncertainty. In addition, the effect of altering the normalizations of the $B$ backgrounds affects the fitted asymmetry. The size of the shift on the fitted $A_{CP}$ is taken as the size of the systematic uncertainty. Previous studies with particles in the same momentum range [16] found asymmetries from detector effects to be negligible compared to the precision at which we measure $A_{CP}$.

The central value of the signal yield from the maximum likelihood fit is $365 \pm 49$ events, with a background of $44 \pm 840 \pm 217$ continuum events and an expected background of $842 \pm 34$ events from other $B$ decays. The distributions of the input variables as functions of the other input variables confirm that the correlations are small. Figure 1 shows the distributions of $m_{ES}$, $\Delta E$, $A_{NN}$, and $m_{\pi\pi}$. The plots are enhanced in signal by selecting only continuum events and an expected background of 34 events from other $B$ decays. The plots are enhanced in signal by selecting only continuum events and an expected background of 34 events from other $B$ decays.

FIG. 1. Likelihood projection plots for the four fit variables, (a) $m_{ES}$, (b) $\Delta E$, (c) $A_{NN}$, and (d) $m_{\pi\pi}$. Each plot shows the total PDF (solid line), total background (dotted line), continuum contribution (dotted-dashed line), and the signal component (dashed line).

integrated out. This threshold is optimized by maximizing the ratio $S = (N^{\text{Sig}}/\epsilon^{\text{Sig}}) \sqrt{N^{\text{Sig}}\epsilon^{\text{Sig}} + \sum_i N^B_{i}\bar{\epsilon}^B_{i}}$ where $\epsilon$ are the efficiencies after the threshold is applied. The PDF components are then scaled by the appropriate $\epsilon$. The efficiencies for the likelihood ratios vary for each variable and result in a different number of events in each projection. Compared against the null hypothesis, the statistical significance $\sqrt{-2\ln(L_{\text{Null}}/L_{\text{max}})}$ of the signal yield amounts to 8.7 standard deviations. We obtain $B(B^+ \to \rho^+ \pi^0) = [10.2 \pm 1.4 \pm 0.9] \times 10^{-6}$, and $A_{CP} = -0.01 \pm 0.13 \pm 0.02$, where the first error is statistical and the second error systematic. The measurements are consistent with previous results [5] and provide improved constraints for the determination of the angle $\alpha$ from $B \to \rho \pi$ decays.

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