Determining $H_0$ with a model-independent method

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ABSTRACT

In this letter, by using the type Ia supernovae (SNIa) to provide the luminosity distance (LD) directly, which is dependent on the value of the Hubble constant $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, and the angular diameter distance from galaxy clusters or baryon acoustic oscillations (BAOs) to give the derived LD according to the distance duality relation, we propose a model-independent method to determine $h$ from the fact that different observations should give the same LD at a redshift. Combining the Union 2.1 SNIa and galaxy cluster data, we obtain that at the 1σ confidence level (CL) $h = 0.589 \pm 0.030$ for the sample of the elliptical $\beta$ model for galaxy clusters, and $h = 0.635 \pm 0.029$ for that of the spherical $\beta$ model. The former is smaller than the values from other observations, while the latter is consistent with the Planck result at the 1σ CL and agrees very well with the value reconstructed directly from the $H(z)$ data. With the Union 2.1 SNIa and BAO measurements, a tighter constraint: $h = 0.681 \pm 0.014$, a 2% determination, is obtained, which is very well consistent with the results from the Planck, the BAOs, as well as the local measurement from Cepheids and very low redshift SNIa.

Subject headings: cosmological parameters—cosmology: observations—cosmology: distance scale
1. Introduction

The determination of Hubble constant $H_0$ from astronomical observations is very important in cosmology since it gives the present cosmic expansion speed and relates with the cosmic components, and the size and age of our Universe. Its first accurate value, $H_0 = 72 \pm 8 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$ at the 1$\sigma$ confidence level (CL) (Freedman et al. 2001), was given by the local measurements from Hubble Space Telescope. About ten years later, Riess et al. (2011) improved this result significantly from Cepheids and very low redshift Type Ia supernovae (SNIa) and obtained $H_0 = 73.8 \pm 2.4 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$ with the uncertainty being reduced from $\sim 11\%$ to $\sim 3\%$. Besides the cosmic distance ladder, the Hubble constant can also be determined by using the cosmic microwave background (CMB) measurement. Based on a six parameters $\Lambda$CDM model, the nine-year Wilkinson Microwave Anisotropy Probe (WMAP9) data gives $H_0 = 70.0 \pm 2.2 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$, a 3% determination (Bennett et al. 2013; Hinshaw et al. 2013). Combining WMAP9 with smaller angular scale CMB data from the South Pole Telescope (SPT) and the Atacama Cosmology Telescope (ACT) as well as the distance measurements derived from baryon acoustic oscillation (BAO) observations can improve this determination to be 1.2% and give $H_0 = 68.76 \pm 0.84 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$ (Bennett et al. 2014). Recently, in the frame of a six parameters $\Lambda$CDM model, too, the Planck Collaboration found that $H_0 = 67.3 \pm 1.2 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$ (Ade et al. 2014), which is slight less than the WMAP9 result and has an about 2.5$\sigma$ tension with local measurements given by Riess et al. (2011).

This tension might indicate the need of new physics, on the one hand, but, on the other hand, it may also owe to some systematic errors. For example, revising the geometric maser distance to NGC 4258 from Humphreys et al. (2013) and using this indicator to calibrate the Riess et al. (2011) data, Efstathiou (2014) got $H_0 = 70.6 \pm 3.3 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$. Considering predominately star-forming environments, Rigault et al. (2014) found $H_0 = 70.6 \pm 2.6 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$. These results are consistent with that of the Planck. In addition, the systematic errors from the CMB analysis can also be responsible for some part of the tension. Removing the $217 \times 217 \text{ GHz}$ detector set spectrum used in the Planck analysis, Spiegel et al. (2013) found $H_0 = 68.0 \pm 1.1 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$.

Besides the very low redshift data from local measurements and the CMB data at $z = 1089$, the intermediate redshift data provide a complementary tool to determine the Hubble constant and they help to reduce the uncertainties of $H_0$ significantly if combined with CMB data. Combining BAO data from 6dF Galaxy Survey (Beutler et al. 2011), Baryon Oscillation Spectroscopic Survey (BOSS) Data Release 11 (DR11) clustering of galaxies (Anderson et al. 2014), WiggleZ (Kazin et al. 2013) and $z = 2.34$ from BOSS DR11 quasar Lyman-α forest lines (Delubac et al. 2014) as well as the simultaneous measurements from the two-dimensional two-point correlation function from BOSS DR9 CMASS sample (Chuang et al. 2013) and two-dimensional matter power spectrum from SDSS DR7 sample (Hemantha et al. 2013), Cheng & Huang (2014) obtained that in a $\Lambda$CDM model $H_0 = 68.0 \pm 1.1 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$, a 1.3% determination. Using the WMAP9+SPT+ACT+6dFGS+BOSS/DR11+$H_0$/Riess and basing on the six-parameter $\Lambda$CDM cosmology, Bennett et al. (2014) got $H_0 = 69.6 \pm 0.7 \text{ km} \text{s}^{-1} \text{ Mpc}^{-1}$, a 1% determination, which is the best accurate result up to now. Through a non-parametric reconstruction of $H(z)$ data,
Busti et al. (2014) obtained $H_0 = 64.9 \pm 4.2$ km s$^{-1}$ Mpc$^{-1}$ by extrapolating the reconstruction to redshift 0. All these results seem to be consistent with the Planck one. However, using other intermediate redshift data including angular diameter distances from the Bonamente et al. (2006) galaxy clusters sample, 11 ages of old high-redshift galaxies (Ferreras et al. 2009; Longhetti et al. 2007), 18 $H(z)$ data point (Simon et al. 2005; Gaztanaga et al. 2009; Stern et al. 2010) and the BAO peak at $z = 0.35$ (Eisenstein et al. 2005), Lima & Cunha (2014) found $H_0 = 74.1 \pm 2.2$ km s$^{-1}$ Mpc$^{-1}$ in a ΛCDM model, which is a consistent value with the Riess et al. (2011) result. Later, Holanda et al. (2014) considered the effects of different galaxy cluster samples and found that different samples give different results, i.e., $H_0 = 70 \pm 4$ km s$^{-1}$ Mpc$^{-1}$ if the Bonamente et al. (2006) galaxy clusters sample is replaced by the Filippis et al. (2005) one. Therefore, not only the local and global measurements but also different intermediate redshift data may give the controversial results in the value of Hubble constant.

From the above discussions, one can see that, except for the direct measurement from the cosmic distance ladder and the non-parametric reconstruction from $H(z)$ data, all other results are based on the ΛCDM model. In this letter, we propose a new model-independent method to determine $H_0$ by combining the observed luminosity distances from SNIa data and angular diameter distances from galaxy clusters or baryon acoustic oscillations (BAOs).

2. Method

The measurement of SNIa is very important in modern cosmology and it first indicates that the present expansion of our Universe is speeding up. The observed data of SNIa are released usually in the form of the distance modulus $\mu$, which connects with the LD $D_L$ through the relation

$$\mu = 5 \log_{10} \frac{D_L}{\text{Mpc}} + 25.$$  \hspace{1cm} (1)

Using SALT2 (Guy et al. 2007) to fit supernova light curves, the observed value of the distance modulus can be obtained through

$$\mu = m_B^{\text{max}} - M_B + \alpha x - \beta c,$$  \hspace{1cm} (2)

where $m_B^{\text{max}}$ and $M_B$ are the rest-frame peak magnitude and the peak absolute magnitude of B bands, respectively, $x$ is the stretch factor, which describes the effects of shapes of light curves on $\mu$, and $c$ is the color parameter, which denotes the influences of the intrinsic color and reddening by dust. $\alpha$, $\beta$ and $M_B$ are are fitted by minimizing the residuals in the Hubble diagram. Since $M_B$ is degenerate with $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$, we must choose a value of $h$ to obtain the distance modulus. So, the obtained $M_B$ is $h$-dependent (Astier et al. 2006). For example, SNLS data gives (Astier et al. 2006)

$$M_B = -19.31 \pm 0.03 + 5 \log_{10} h_{70}.$$  \hspace{1cm} (3)
Here, \( h_{70} = h / 0.7 \). Substituting the obtained \( \alpha, \beta \) and \( M_B \) into Eq. (2) and using \( h_0 \) to denote the chosen value of \( h \) to determine the distance modulus, one finds that

\[
\mu = \mu_0 - 5 \log_{10} \frac{h}{h_0},
\]

(4)

where \( \mu_0 \) represents the released distance modulus with \( h = h_0 \). Notice that \( h_0 = 0.70 \) is used in Union 2 (Amanullah et al. 2010) and Union 2.1 (Suzuki et al. 2012) SNIa samples. Combining Eq. (4) with Eq. (1) gives the \( h \)-dependent LD from the distance modulus of SNIa:

\[
D_L(h) = \frac{h_0}{h} 10^{\mu_0/5} \text{ Mpc} \equiv \frac{h_0}{h} D_{L0}.
\]

(5)

From the Eq. (5), one can see that, if the LD can be given by other observations and is independent of \( h \), the value of Hubble constant can be determined directly by combining them with the SNIa, and the result is model-independent. However, except for SNIa, no other observations can provide the observed value of LD directly. Fortunately, the angular diameter distance (ADD) \( D_A \) can be obtained from some observations, such as galaxy clusters and BAOs. Using the distance duality relation between the LD and the ADD: \( D_L = (1 + z)^2 D_A \), we can deduce the value of \( D_L \) at the redshift of \( D_A \). Thus, the combination of the SNIa and galaxy clusters (or BAOs) provides a model-independent method to constrain \( h \).

The above analysis shows that finding the observed LD \( (D_{L0}) \) from distance modules of SNIa and ADD from galaxy clusters or BAOs at the same redshift is necessary to constrain \( h \) in a model-independent way. Here, we use the Union 2.1 SNIa data sample (Suzuki et al. 2012), which contains 580 data points in the redshift region \( 0.015 < z < 1.43 \), to provide \( D_{L0} \). So, the data sample of SNIa is much larger than that of galaxy clusters or BAOs. In order to obtain a tight constraint, all ADD data should be kept. Due to the usual absence of SNIa data at the same redshift of ADD data, we use a binning method to give the corresponding LD value of SNIa. This means that all SNIa data available in the range \( \Delta z = |z_L - z_A| < \Delta \) are binned, where \( z_A \) represents the redshift of ADD data and constant \( \Delta \) represents the binned redshift region. If a larger \( \Delta \) is used, the binned \( D_{L0} \) will be more accurate since more SNIa data are included. Thus, we choose \( \Delta = 0.02 \). For all selected data, an inverse variance weighted average is employed. If \( D_{L0i} \) denotes the \( i \)th appropriate SNIa luminosity distance data with \( \sigma_{D_{L0i}}^2 \) representing the corresponding observational uncertainty, one can straightforwardly obtain with the conventional data reduction techniques in (Bevington & Robinson 2003) that

\[
\bar{D}_{L0} = \frac{\sum (D_{L0i}/\sigma_{D_{L0i}}^2)}{\sum 1/\sigma_{D_{L0i}}^2},
\]

(6)

\[
\sigma_{\bar{D}_{L0}}^2 = \frac{1}{\sum 1/\sigma_{D_{L0i}}^2},
\]

(7)

where \( \bar{D}_{L0} \) represents the weighted mean luminosity distance at a given redshift, and \( \sigma_{\bar{D}_{L0}}^2 \) is its
uncertainty. From Eq. (5) and the distance duality relation, the observed \( h_{\text{obs},i} \) can be obtained

\[
h_{\text{obs},i} = \frac{h_0 \bar{D}_{L,0,i}}{D_{A,i}} (1 + z_i)^{-2}.
\] (8)

Minimizing the following \( \chi^2 \) statistics

\[
\chi^2 = \sum_i \frac{(h - h_{\text{obs},i})^2}{\sigma_{h_{\text{obs},i}}^2},
\] (9)

we can get the constraint on \( h \).

3. galaxy cluster sample

Reese et al. (2002) suggested that the angular diameter distance of galaxy cluster can be derived from the measurement of its intrinsic size, which bases on the Sunyaev-Zeldovich effect (SZE) and X-ray surface brightness observations. Here, we employ two different galaxy cluster samples: spherical model (Bonamente et al. 2006) sample and elliptical model (Filippis et al. 2005) sample. The former, compiled by Bonamente et al. (2006), has 38 ADD data points in the redshift region \( 0.14 < z < 0.89 \), which are obtained by using a hydrostatic equilibrium model to analyze the cluster plasma and dark matter distributions. The latter was compiled by Filippis et al. (2005) with an isothermal elliptical \( \beta \) model and consists of 25 ADD data in the redshift region \( 0.02 < z < 0.79 \), in which 18 galaxy clusters were complied by Reese et al. (2002) and 7 galaxy clusters were done by Mason et al. (2001).

The results are shown in Figs. (1, 2). In Fig. (1), we give the distributions of \( h_{\text{obs},i} \) and the red line represents the best fit value. Its left panel and right panel correspond to the Filippis et al. (2005) sample and the Bonamente et al. (2006) sample, respectively. We find the dispersion of \( h_{\text{obs},i} \) is very large and the Filippis et al. (2005) sample has a slightly larger dispersion than that of the Bonamente et al. (2006) although the former has smaller data points than the latter. In addition, the uncertainties of \( h_{\text{obs},i} \) are also very large as a result of the large error bars of galaxy cluster data.

Fig. (2) shows the likelihood distributions of \( h \), in which the solid and dashed lines represent the results from Union 2.1 SNIa+ Filippis et al. (2005) sample and Union 2.1 SNIa+ Bonamente et al. (2006) sample, respectively, and the shaded region is the Planck constraint with the 1\( \sigma \) CL. We find that at the 1\( \sigma \) CL \( h = 0.589 \pm 0.030 \) for Union 2.1 SNIa + 25 galaxy clusters and \( h = 0.635 \pm 0.029 \) for Union 2.1 SNIa + 38 galaxy clusters. The combination of Union 2.1 SNIa and elliptical \( \beta \) model sample seems to favor a smaller Hubble constant, which is consistent with the Planck result (Ade et al. (2014), \( H_0 = 67.3 \pm 1.2 \) km s\(^{-1}\) Mpc\(^{-1}\)) only at the 3\( \sigma \) CL. The Hubble constant from Union 2.1 SNIa + spherical \( \beta \) model is compatible with the one from the Planck at the 1\( \sigma \) CL although the best fit value is smaller than the latter. It is also consistent with the result reconstructed from \( H(z) \) data directly given in (Busti et al. 2014), where \( H_0 = 64.9 \pm \)
4.2 km s$^{-1}$ Mpc$^{-1}$. It is easy to see that different galaxy cluster samples give different $h$ constraints. This is the same as that obtained in (Holanda et al. 2014) based on a ΛCDM model with galaxy cluster samples being combined with other data. But, our values for the Hubble constant are much less than those given in (Holanda et al. 2014).

4. baryon acoustic oscillations

The BAOs (see (Bassett & Hlozek 2009) for a review) arise from the coupling of photons and baryons by Thomson scattering in the early universe. The competition between radiation pressure and gravity gives rise to a system of standing sound waves within the plasma. At recombination the interaction between photons and baryons ends abruptly because the free electrons are quickly captured, which leads to a slight over-density of baryons at the sound horizon. The corresponding scale is the distance traveled by the sound wave in the plasma prior to recombination. This scale has been measured in the clustering distribution of galaxies today and can be used as a standard ruler. Combining measurements of the baryon acoustic peak and the Alcock-Paczynski distortion from galaxy clusters, the ADD data can be obtained. We summarize 5 low redshift data points in Tab. (1). Three of them are determined from the WiggleZ Dark Energy Survey (Balke et al. 2012), and other two data points are from BOSS DR7 (Xu et al. 2013) and DR11 (Samushia et al. 2014).

In Fig. (3), we show the distribution of $h_{\text{obs},i}$ (left panel) and the likelihood distribution of $h$ (right panel). We find that the uncertainties and dispersion of BAO data are much less than those of galaxy cluster samples, and the WiggleZ Dark Energy Survey favors a slight larger $h$ than the BOSS. In the left panel, the shaded region represents the Planck result at the 1σ CL. It is easy to see that a strong constraint on $h$ is obtained: $h = 0.681 \pm 0.014$ at the 1σ CL, a 2% determination, which is larger than the results from galaxy cluster samples. This result is very well consistent with the result from the Planck with and without the systematic errors of the 217 × 217 GHz detector set spectrum considered (Ade et al. 2014; Spergel et al. 2013), and the combination of all BAO data (Cheng & Huang 2014). It is also compatible with the local measurement from Cepheids and very low redshift SNIa given by Efstathiou (2014); Rigault et al. (2014).

5. conclusions

In this letter, we propose a model-independent method to determine the value of the Hubble constant, which is a very important constant in cosmology. Since the distance modulus of SNIa is dependent on the chosen value of $h$, we obtain a $h$-dependent expression of LD, which is given in Eq. (5). Thus, if the LD at a redshift can be determined by other observation, combining it with the one from SNIa can determine the value of $h$ by use of the fact that two different observations should give the same values of LD at a given redshift. According to the distance duality relation, the LD
can be deduced from the observed ADD data. Using the Union 2.1 SNIa to provide the observed LD data and the galaxy cluster samples to the observed ADD data, we obtain that $h = 0.589 \pm 0.030$ for the elliptical $\beta$ model sample, and $h = 0.635 \pm 0.029$ for the spherical $\beta$ model sample. The former is smaller than the values from other observations, while the latter is consistent with the Planck result (Ade et al. 2014) at the 1$\sigma$ CL and agrees very well with the value directly reconstructed from the $H(z)$ data (Busti et al. 2014). Thus, different galaxy cluster samples give different values of $h$. This is the same as that obtained in Holanda et al. (2014) based on a $\Lambda$CDM model with a combination of galaxy clusters and other observations. However, if the ADD data from BAO measurements are considered, we obtain that $h = 0.681 \pm 0.014$ at the 1$\sigma$ CL, a 2% determination. This result is very well consistent with the values from the Planck (Ade et al. 2014; Spergel et al. 2013), the combination of all BAO data (Cheng & Huang 2014), as well as the local measurement from Cepheids and very low redshift SNIa (Efstathiou 2014; Rigault et al. 2014).

We thank Prof. Shuang-Nan Zhang and Prof. Xiao-Feng Wang for their useful discussions. This work was supported by the National Natural Science Foundation of China under Grants No. 11175093, No. 11222545, No. 11435006, and No. 11375092; the Zhejiang Provincial Natural Science Foundation of China under Grant No. R6110518; the Specialized Research Fund for the Doctoral Program of Higher Education under Grant No. 20124306110001; and the K.C. Wong Magna Fund of Ningbo University.

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This preprint was prepared with the AAS L\TeX macros v5.2.
| $z$ | $D_A(z)$ (Mpc) | Survey          |
|-----|----------------|-----------------|
| 0.44| 1205 ± 114     | WiggleZ(Balke et al. 2012) |
| 0.6 | 1380 ± 95      |                 |
| 0.73| 1534 ± 107     |                 |
| 0.35| 1050 ± 38      | SDSS DR7(Xu et al. 2013) |
| 0.57| 1380 ± 23      | SDSS DR11 CMASS(Samushia et al. 2014) |

Table 1: Summary of the ADD measurements from the BAOs.

Fig. 1.— The distributions of $h_{obs,i}$ from Union 2.1 SNIa + 25 Filippis et al. (2005) sample (left panel) and Union 2.1 SNIa + 38 Bonamente et al. (2006) sample (right panel).
Fig. 2.— The likelihood distributions of $h$. The solid and dashed lines represent the results from Union 2.1 SNIa + 25 Filippis et al. (2005) sample and Union 2.1 SNIa + 38 Bonamente et al. (2006) sample, respectively. The shaded region shows the constraint from the Planck measurement.

Fig. 3.— Left panel: the distributions of $h_{\text{obs},i}$ from Union 2.1 SNIa + BAO data. Right panel: the corresponding likelihood distribution of $h$. The shaded region shows the constraint from the Planck measurement.