Pseudo-Differential \((2 + \alpha)\)-Order
Butterworth Frequency Filter

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ABSTRACT This paper describes the design of analog pseudo-differential fractional frequency filter with the order of \((2 + \alpha)\), where \(0 < \alpha < 1\). The filter operates in a mixed-transadmittance mode (voltage input, current output) and provides a low-pass frequency response according to Butterworth approximation. General formulas to determine the required transfer function coefficients for desired value of fractional order \(\alpha\) are also introduced. The designed filter provides the beneficial features of fully-differential solutions but with a less complex circuit topology. It is canonical, i.e. it employs a minimum number of passive elements, whereas all are grounded, and current conveyors as active elements. The proposed structure offers high input impedance, high output impedance, and high common-mode rejection ratio. By simple modification, voltage response can also be obtained. The performance of the proposed frequency filter is verified both by simulations and experimental measurements proving the validity of theory and the advantageous features of the filter.

INDEX TERMS Current conveyor, fractional Butterworth transfer function, fractional-order filter, pseudo-differential filter.

I. INTRODUCTION

Within the last decades, there is a significantly rising attention being paid to fractional-order (FO) calculus due to its promising utilization in various research areas such as economy and finance [1]–[3], medical and health science [4]–[6], agriculture and food processing [7]–[10], automotive [11]–[13], and also in electrical engineering [14]–[43] to design, describe, model and/or control various systems and function blocks. Basically, the presence of fractional order \((\alpha)\) represents another degree of freedom to mathematically describe the behavior of a function block. This enables to provide characteristics in between integer-orders in comparison to classic (integer-order) circuits, which may become beneficial while more accurate signal generation and measurement, and/or system modeling and control is needed.

In the discipline of electrical engineering and analog signal processing, FO frequency filters [14]–[28], oscillators [29]–[32], controllers [33]–[36], and FO element emulators and converters [37]–[43] are mostly discussed.

In case of analog circuit design and signal processing, the presence of the fractional-order element (FOE), having impedance with a non-integer power law dependence on the Laplace operator \(s\), is generally necessary. However, FOEs are currently not readily available as discrete elements, as it is the case of classic resistors, capacitors and inductors, although a number of promising technologies is being investigated as has been recently summarized in [44]. Currently, there are generally two approaches to overcome the absence of discrete FOEs, both approximating the behavior of FOE in a specific frequency range and with required accuracy. The first approach approximates directly the term \(s^\alpha\) being present in the transfer function by integer-order polynomial function. The originally FO transfer function turns into an integer-order one, which results in a more complex circuit counting more active and passive elements, see
TABLE 1. Comparison of relevant fractional-order filters.

| Ref. | Filter order | ABB type/count | Passive el. type/count | FOE impl. | Transfer function | Mode | Sim/Meas |
|------|--------------|----------------|------------------------|-----------|--------------------|------|----------|
| [14] | α            | OTA:3          | FC:2                   | Foster II | LP, HP, BP, BS, AP | voltage | yes/no   |
| [15] | α            | OTA:10         | C:2                    | sα approx. | LP                 | voltage | yes/no   |
| [16] | α            | OTA:10         | C:2                    | sα approx. | AP                 | current | yes/no   |
| [17] | α            | OTA:2          | FC:1                   | Foster I  | current            | yes/no   |
| [18] | 1 + α        | opamp:2        | R:10, C:3              | sα approx. | LP, HP              | voltage | yes/yes  |
| [19] | 1 + α        | CFOA:4         | R:9/8, C:3/4           | sα approx. | LP, HP              | voltage | yes/yes  |
| [20] | 1 + α        | CFOA:8         | R:16, C:3              | sα approx. | LP, HP, BP, BS     | voltage | yes/no   |
| [21] | 1 + α        | opamp:3        | FC:1, R:6, C:1         | Foster I  | LP                 | voltage | yes/no   |
| [22] | 1 + α        | CG-CCDDCC:4, VGA:1 | FC:1, R:2, C:1   | Foster I  | LP                 | voltage | yes/no   |
| [23] | α + β        | DVCC:1         | FC:1, FL:1, R:1        | Foster I  | LP                 | voltage | yes/no   |
| [24] | α + β + γ    | opamp:2        | PC:3, R:6              | Foster I  | LP                 | voltage | yes/yes  |
| [25] | 5 + α        | opamp:10       | C:7, R22               | sα approx. | LP                 | voltage | yes/yes  |
| [26] | 1 + α, 2 + α | OTA:4, IOGC-CA:1 | FC:1-4, C:0-4          | Foster I  | LP                 | voltage | yes/no   |
| [27] | 2 + α        | DDCC:1, DVCC:2, CCII:1 | FC:1, C:2, R:4/6   | Foster II | LP                 | mixed, voltage | yes/yes  |

List of previously unexplained abbreviations used in this table:
CFOA: Current Feedback Operational Amplifier, CG-CCDDCC: controlled gain current-controlled differential difference current conveyor, VGA: Variable Gain Amplifier, IOGC-CA: individual output gain controlled current amplifier
C: capacitor, R: resistor, FC: capacitive FOE, FL: inductive FOE
LP: low-pass, HP: high-pass, BP: band-pass, BS: band-stop, AP: all-pass

e.g., [15], [16]. The second approach approximates primarily the required FOE, most commonly by a RC ladder network, such as Foster I, Foster II, Cauer I and Cauer II [45]. The advantage of this second approach is that the complexity of the final circuit solution increases only by the count of the resistors and capacitors that are used to approximate the FOE, see e.g., [14], [24]. Additionally, once FOEs become readily available as solid-state, it will be only necessary to replace the RC ladder network by the specific FOE. This is not the case of the FO circuits being designed using the first approach (i.e. designed by direct approximation of sα), whereas these circuit solutions will become basically obsolete. An overview of FO frequency filters designed by both above described approaches can be found in Table 1. The different types of active building blocks (ABBs) are used to design fractional filters of α- or (1 + α)-order, primarily operational transconductance amplifiers (OTAs) and operational amplifiers (opamps). Fractional-order solutions of higher-order filters are rare as presented in [25]–[27], and are based on Butterworth approximation. However, the issue of circuits described in [25]–[27] is that both partial transfer functions are determined by using Butterworth approximation. Thus, the resulted (N + α)-order transfer function does not correspond to Butterworth approximation anymore, see e.g., [25] and [26] featuring at the pole-frequency the decrease in magnitude of 6 dB and not the commonly expected 3 dB, or showing significant peaking for some values of α [27]. Design of FO higher-order filters is also discussed in [28], where the proposed filter offers a wide variety of possible order combinations and to design up to (3 + α)-order filter. Although the experimental results are presented, the obtained frequency responses do not follow any common approximation type. Therefore, in this paper we primarily provide general formulas to determine (2 + α)-order transfer function coefficients for Butterworth approximation.

Next to FO filter design, in this paper we also contribute to the design of pseudo-differential frequency filters. Compared to “true” fully-differential circuits, whose circuit topology is also fully differential (e.g., [46]–[51]), the pseudo-differential structures as presented e.g., in [52]–[62] also provide the advantages of higher ability to reject the common-mode noise signals, suppress power supply noise, or feature higher dynamic range together with reduced harmonic distortion of the processed signal. However, pseudo-differential filters keep the simplicity of the circuit solutions being comparable to single-ended filters [63]–[65]. The solutions of pseudo-differential filters from [52]–[62] use different types of ABBs, primarily from the family of current conveyors (CCs). Table 2 summarizes the key features of such pseudo-differential filters. Basically, the proposed circuit solution may be used for the design of 3rd-order filter, where the fractional capacitor (FC) is replaced by classic capacitor. However, within further circuit analyses, simulations and experimental measurements, its FO version is assumed.

In this paper, we merge the two research topics and contribute both to pseudo-differential and FO frequency filter design. The filter operates in mixed (transadmittance) mode and provides a low-pass (2 + α)-order response according to Butterworth approximation. The current conveyors are advantageously used as active elements to maintain both high input and output (infinite in ideal case) impedance. By a simple modification, the filter can operate also in voltage mode. The performance of the proposed filter is verified both by simulation and experimental measurements to show its proper behavior.

The paper is organized as follows: Section II provides the theory on pseudo-differential filters, while the description...
of fractional \((2 + \alpha)\)-order transfer function approximated according to Butterworth is presented in Section III. The proposed filter is described in Section IV, where both the transadmittance- and voltage-mode transfer functions are given. Designing two prototypes of FOEs \((\alpha = 0.3\) and \(\alpha = 0.6)\) the simulation and experimental measurement results are presented in Section V, where not only the magnitude and phase frequency responses, but also the low total harmonic distortion (THD) and the common-mode rejection ratio (CMRR) were determined. Section VI concludes the paper.

**II. THEORY ON PSEUDO-DIFFERENTIAL FILTERS**

Dealing generally with any differential circuit, the following notation for differential input voltage \((v_{id})\), common-mode input voltage \((v_{ic})\) and differential output voltage \((v_{2d})\) is assumed [66]:

\[
v_{id} = v_{1+} - v_{1-}, \quad v_{ic} = \frac{v_{1+} + v_{1-}}{2}, \quad v_{2d} = v_{2+} - v_{2-},
\]

whereas similarly a set of relations valid for current-mode differential circuits is specified:

\[
i_{id} = i_{1+} - i_{1-}, \quad i_{ic} = \frac{i_{1+} + i_{1-}}{2}, \quad i_{2d} = i_{2+} - i_{2-}.
\]

The differential input signal \(v_{1d}\) (or \(i_{1d}\)) is simply the difference between the two input signals \(v_{1+}\) and \(v_{1-}\) (or \(i_{1+}\) and \(i_{1-}\)), whereas the common-mode input signal \(v_{1c}\) (or \(i_{1c}\)) is the average of the two input signals.

In the view of (1), the differential-output voltage \(v_{2d}\) is then defined as:

\[
v_{2d} = A_{dm}v_{1d} + A_{cm}v_{1c},
\]

where \(A_{dm}\) and \(A_{cm}\) are the differential and the common-mode voltage gains, respectively. The capability of the differential circuit to reject the common-mode signal in voltage-mode signal processing is determined by the common-mode rejection ratio (CMRR) defined as [66]:

\[
CMRR = 20 \log \left(\frac{A_{dm}}{A_{cm}}\right).
\]

Similarly to (3) and using (2), the differential output current \(i_{2d}\) can be defined as:

\[
i_{2d} = B_{dm}i_{1d} + B_{cm}i_{1c},
\]

where \(B_{dm}\) and \(B_{cm}\) are the differential and the common-mode current gains, respectively, and similarly to (4) the common-mode rejection ratio of a current-mode differential circuit can be determined:

\[
CMRR = 20 \log \left(\frac{B_{dm}}{B_{cm}}\right).
\]

From the viewpoint of general mathematical description of differential circuits using (1)–(6), it is evident that the inner circuit topology is never taken into account. Hence, dealing with differential circuits, it is not necessary to absolutely assume the circuit topology to be differential also. As a result, the “pseudo” fully-differential (or simply pseudo-differential) filters can be designed being specific with rather single-ended circuit topology but still providing differential input and output. Compared to “true” fully-differential filters, the main benefit of pseudo-differential filters is their significantly reduced complexity in the count of required active and passive elements of the final structure, which is usually twice smaller. At the same time, they keep the positive properties of “true” fully-differential filters, such as high common-mode rejection ratio and low harmonic distortion.

**III. \((2 + \alpha)\) BUTTERWORTH APPROXIMATION**

For the purpose of \((2 + \alpha)\)-order frequency filter design, the following general low-pass transfer function is assumed as:

\[
H_{2+\alpha}^{LP}(s) = \frac{1}{s^{2+\alpha}k_1 + s^{2}k_2 + sk_3 + k_4},
\]

where \(0 < \alpha < 1\), and \(k_1, k_2, k_3,\) and \(k_4\) are the transfer function coefficients that were determined using an optimization algorithm to match the target Butterworth fractional low-pass magnitude response.

The relation for the magnitude of the target Butterworth FO low-pass transfer function generalized to fractional order
(2 + \alpha) can be written as follows:

\[ |H_{2+\alpha}^{LP}(\omega)| = \frac{1}{\sqrt{1 + \omega^{2(2+\alpha)}}} \]  

(8)

that provides unity pass-band gain, magnitude of \(-3\) dB at cut-off angular frequency \(1\) rad/s, and stop-band roll-off \(-20(2 + \alpha)\) dB/dec.

Using numerical optimization algorithm, the coefficients \(k_1, k_2, k_3,\) and \(k_4\) from (7) were found such that the maximum absolute error between magnitude in dB of (7) and (8) is minimized. For this purpose, the MATLAB function \textit{fminsearch} was used with the following fitness function:

\[ f = \max_i 20\log|H_{2+\alpha}^{LP}(\omega)| - 20\log|H_{2+\alpha}^{LP}(\omega)| \]  

(9)

where \(x\) is the sought vector of the coefficients \(k_1, k_2, k_3,\) and \(k_4\). Each search used 100 frequency points logarithmically spaced in the wide frequency range from \(\omega_1 = 0.01\) rad/s to \(\omega_{100} = 100\) rad/s to cover both pass- and stop-band of (8). The individual runs of \textit{fminsearch} function were performed for the fractional component \(\alpha\) decreasing from 0.99 to 0.01 with a linear step 0.01.

As a result, the transfer function coefficients \(k_1, k_2, k_3,\) and \(k_4\) that yield the lowest error according to (9) for specific value of fractional order \(\alpha\) are shown in Fig. 1. For better convenience, to determine the transfer function coefficients \(k_1, k_2, k_3,\) and \(k_4\) in (7) according to Butterworth approximation for specific values of fractional order \(\alpha\), the following interpolation matrix can be used:

\[
\begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4 \\
\end{bmatrix} = \begin{bmatrix}
0.357 & 0.138 & -0.026 & 0.519 \\
0.630 & 1.051 & -0.507 & 0.820 \\
1.415 & 0.542 & -0.108 & 0.151 \\
1.000 & -0.004 & -0.001 & 0.005 \\
\end{bmatrix} \begin{bmatrix}
1 \\
\alpha \\
\alpha^2 \\
\alpha^3 \\
\end{bmatrix},
\]

(10)

and in Fig. 1 shown as \(k_{1aprx}, k_{2aprx}, k_{3aprx},\) and \(k_{4aprx}\) are compared to coefficients values of \(k_1, k_2, k_3,\) and \(k_4\) determined using the fitness function (9).

To prove sufficient accuracy while determining the transfer function coefficients using the interpolation matrix (10), the relative error between coefficients \(k_1, k_2, k_3,\) and \(k_4\) determined using the MATLAB \textit{fminsearch} function and the interpolated coefficients using (10) is shown in Fig. 2. It can be seen that for \(0.03 < \alpha < 1\) the relative error is always below 2%.

Before the coefficients \(k_1, k_2, k_3,\) and \(k_4\) determined by (10) can be utilized to compute the values of the capacitors and resistors in the filter structure, the frequency shifting to the cut-off frequency \(\omega_0\) should be carried out, as the target function (8) has the \(-3\) dB cut-off frequency \(1\) rad/s, which is not practical. Thus the transfer function coefficients should be modified by dividing them by the respective power of \(\omega_0\) as follows:

\[ H_{2+\alpha,\omega_0}^{LP}(s) = \frac{1}{s^{2+\alpha} \frac{k_1}{\omega_0^{2\alpha}} + s^{2} \frac{k_2}{\omega_0^{2}} + s \frac{k_3}{\omega_0} + k_4}. \]  

(11)

\[ i_{Z+} = i_X, \quad i_{Z-} = -i_X. \]  

(12)
The only significant difference between the assumed current conveyor types is the formula specifying the voltage at X terminal:

\[
\begin{align*}
&\text{for DVCC: } v_X = v_{Y1} - v_{Y2} \\
&\text{for DDCC: } v_X = v_{Y1} - v_{Y2} + v_Y \\
&\text{for CCII: } v_X = v_Y
\end{align*}
\]  
(13)

Using the current conveyors, the proposed pseudo-differential filter suitable to provide a \((2 + \alpha)\)-order low-pass frequency response operating in transadmittance- or voltage-mode is shown in Fig. 4(a) and Fig. 4(b), respectively. The principal structure of the filter is based on inverse follower (IFLF). The DDCC, DVCC and CCII together with the corresponding passive elements operate as integrators. Taking the advantage of the DDCC and its voltage terminals, the differential input voltage \(v_{id}\) is directly applied to the Y1 and Y2 terminals and hence high input impedance of the proposed filters is ensured. The CCII primarily serves as voltage-to-current converter and provides the differential current output (Fig. 4(a)).

\[
\begin{align*}
&\text{for DVCC: } v_X = v_{Y1} - v_{Y2} \\
&\text{for DDCC: } v_X = v_{Y1} - v_{Y2} + v_Y \\
&\text{for CCII: } v_X = v_Y
\end{align*}
\]  
(13)

\[
\begin{align*}
&i_{2+} \text{ and } i_{2-} \text{ of the filter from Fig. 4(a) can be determined as:} \\
i_{2+}(s) &= \frac{1}{R_4} \frac{1}{s^{2+\alpha}u_1 + s^{2}u_2 + s u_3 + 1} v_{id} + 0v_{1c}, \\
i_{2-}(s) &= -\frac{1}{R_4} \frac{1}{s^{2+\alpha}u_1 + s^{2}u_2 + s u_3 + 1} v_{id} + 0v_{1c}, 
\end{align*}
\]  
(14, 15)

where \(u_1 = C_1 C_2 C_0 R_1 R_2 R_3, u_2 = C_1 C_2 R_2 R_3,\) and \(u_3 = C_0\) whereas \(C_0\) is the pseudo-capacitance of the fractional capacitor and \(\alpha\) is its fractional order.

From (14) and (15), the differential transadmittance gain is defined as:

\[
G_{dtu} = \frac{2}{R_4} \frac{1}{s^{2+\alpha}u_1 + s^{2}u_2 + s u_3 + 1}.
\]  
(16)

and the common-mode transmittance gain \(G_{cm}\) is zero.

As shown in Fig. 4(b), adding extra two resistors (\(R_5\) and \(R_6\)) the proposed filter may be operated in voltage-mode. The output voltages \(v_{2+}\) and \(v_{2-}\) then are determined as:

\[
\begin{align*}
v_{2+}(s) &= \frac{1}{R_5} \frac{1}{R_4} \frac{1}{s^{2+\alpha}u_1 + s^{2}u_2 + s u_3 + 1} v_{id} + 0v_{1c} \\
v_{2-}(s) &= \frac{R_6}{R_4} \frac{1}{s^{2+\alpha}u_1 + s^{2}u_2 + s u_3 + 1} v_{id} + 0v_{1c},
\end{align*}
\]  
(17, 18)

and for the differential voltage gain it holds:

\[
A_{du} = \frac{-R_5 + R_6}{R_4} \frac{1}{s^{2+\alpha}u_1 + s^{2}u_2 + s u_3 + 1},
\]  
(19)

whereas the common-mode voltage gain \(A_{cm}\) is zero and the common-mode rejection ratio CMRR is infinite, in theory.

V. SIMULATIONS AND EXPERIMENTAL MEASUREMENTS

The properties of the proposed pseudo-differential FO frequency filter from Fig. 4(b), i.e. operating in voltage-mode, were verified by simulations and mainly also by experimental measurements. Both for simulations and experiments the Universal Current Conveyor UCC-N1B integrated circuit [67] was used to obtain the required types of active elements.

A. FRACTIONAL-ORDER ELEMENT DESIGN

Due to commercial unavailability of fractional capacitors in general, the required \(C_0\) was approximated using the 7th-order Foster II RC network as shown in Fig. 5. Using [45], the values of the resistors and capacitors of the Foster II network were determined, whereas to validate the performance of the proposed filter, two different FOEs were chosen:

- \(\alpha = 0.3, C_0 = 7.038 \, \mu F s^{-0.7}\),
- \(\alpha = 0.6, C_0 = 0.158 \, \mu F s^{-0.4}\).
with central frequency \(f_c\) of 50 kHz and approximation range from 5 kHz to 500 kHz. Both assumed FOEs feature the impedance module of 3184 \(\Omega\) at \(f_c\). The calculated values of the resistors and capacitors used to approximate the FOEs using the Foster II RC network from Fig. 5 are summarized in Table 3, whereas for simulations and experimental measurements, the values for resistors and capacitors were selected from the E24 and E12 series, respectively.

The impedance module and phase shift of the two approximated FOEs obtained by measurements and compared to the simulation results are shown in Fig. 6. From Fig. 6(a), for \(\alpha = 0.3\) the value of module of the approximated FOEs at central frequency 50 kHz is 3176 \(\Omega\), whereas for \(\alpha = 0.6\) the impedance module is and 3198 \(\Omega\), which is very close to the expected value of 3184 \(\Omega\) for ideal FOEs. The results in Fig. 6(b) show the phase shift validate proper approximation in the specified frequency range, i.e. from 5 kHz to 500 kHz.

**B. PERFORMANCE ANALYSIS OF THE FILTER**

For the selected values of \(\alpha\), using (10) the coefficients \(k_1\), \(k_2\), \(k_3\) and \(k_4\) of the general transfer function (7) were determined. Choosing the values of capacitors \(C_1 = C_2 = 1\ \text{nF}\) and the filter pole-frequency \(f_0 = 50\ \text{kHz}\) and using (19), the values of resistors \(R_1\), \(R_2\) and \(R_3\) were determined as 5.00 k\(\Omega\), 1.86 k\(\Omega\) and 1.41 k\(\Omega\), respectively, and are the same for both FOEs as their module at the pole-frequency is also the same. The values of the resistors \(R_4\), \(R_5\), and \(R_6\) were selected to be 1 k\(\Omega\).

The magnitude and phase responses of the proposed filter obtained both by simulations and experimental measurements are displayed in Fig. 7(a) and Fig. 7(b), respectively. From the characteristics, it can be clearly seen that both simulation and
experimental measurements follow theoretical presumptions very well. For experimental measurements, the deviation at frequencies above approx. 600 kHz is attributed to parasitic properties and frequency limitations of the active element UCC-N1B [67]. The UCC-N1B is an integrated circuit that was designed as laboratory sample in cooperation with ON Semiconductor company. The chip includes one Universal Current Conveyor (UCC), used to obtain the DDCC and DVCCs, and one CCII+/–.

In addition to the measured magnitude and phase response characteristics, the ability of the pseudo-differential filter to suppress the common-mode signal, the CMRR was experimentally measured. The results are shown in Fig. 8. For both cases of used FOE, the value of CMRR is above 50 dB in the pass band of the filter. The decrease in CMRR above approx. 200 kHz is due to the actual behavior of the UCC-N1B and the mismatch of the voltage gains $\beta_1$ and $\beta_2$ between terminals Y1 and X, and terminals Y2 and X of the active element as we investigated earlier in [58].

The total harmonic distortion (THD) was also evaluated by means of experimental measurements. The reached results are shown in Fig. 9. Similarly, to measuring CMMR, also here the THD is close for both values of fractional order $\alpha$ and is below 2% for the amplitude of the input signal up to 0.7 V at frequency 1 kHz.
The intermodulation distortion (IMD) was determined, whereas using 81150A generator, two fundamental tones $f_1$ and $f_2$ with frequency of 5 kHz and 5.5 kHz were applied. In Fig. 10 the spectrums at the output of the filter are shown for two different input voltages. From Fig. 10(b) the 3rd IMD harmonic suppression is 47.5 dB for $V_{in} = 400$ mVpp. The relationship between the fundamental ($f_1, f_2$) and 3rd-order IMD $(2f_2 - f_1)$ on input signal level is shown in Fig. 11. From Fig. 11 the $-1$ dB gain compression point is at 0.73 dBm of the input power.

Performing the experimental measurements to reach the results as presented above, primarily the network analyser Agilent 4395A was used. To obtain the differential signal and be too able to measure the differential output response, single-to-differential (S/D) and differential-to-single (D/S) converters had to be added and were implemented using AD8476 [68] and AD8429 [69] integrated circuits respectively. For the sake of evaluating also CMRR of the proposed pseudo-differential filter, the AD8271 [70] was used to apply common-mode signal at the input of the filter. The total block connection for experimental measurements is shown in Fig. 12. For sake of completeness the PCB prototype of the measured voltage-mode pseudo-differential filter is also shown in Fig. 13.

VI. CONCLUSION

This article contributes to the design of FO filters, whereas the mathematical calculus enabling to determine the $(2 + \alpha)$-order transfer function coefficients according to Butterworth approximation for arbitrary $\alpha$ was presented. Besides, new circuit solution of the pseudo-differential filter was also proposed and used to prove and support the theory both of the designing pseudo-differential and fractional-order filters. The structure operates in mixed-mode (transadmittance) and thanks to current conveyors, used as active elements, it features high input and output impedance. It was shown that simple modification enables to operate the filter in voltage-mode. Both simulation and experimental measurements show proper behavior of the filter in magnitude and phase and moreover prove high CMRR and dynamic range keeping total harmonic distortion low.

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O. Sladok et al.: Pseudo-Differential \((2 + \alpha)\)-Order Butterworth Frequency Filter

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