Vibratory finishing, end phase surface topography, resolving current issues

Karl Walton, Damian Conroy and Liam Blunt
EPSRC Future Metrology Hub, University of Huddersfield, Huddersfield, United Kingdom
1 Author to whom any correspondence should be addressed.
E-mail: k.walton@hud.ac.uk

Abstract
Vibratory finishing belongs to the wider group of near ubiquitous mass finishing processes. Components are typically bulk finished in a fluidised media bed and applications vary from performance critical polishing, to cosmetic surface preparation. The established model of surface topography development has been one of initial rapid change during a ‘transient’ process phase, with subsequent slowing and a final ‘steady state’ phase in which surface character is uniform with no dependence on process duration or initial surface finish.

Recent reassessment of the steady state phase has suggested that its surface roughness oscillates with process duration about a mean value and redefines it as the ‘equilibrium’ phase. Though this new model is considered to incorporate the existing model as a special case, it is argued that it represents a fundamental shift in the process mechanics. In addition, it is argued that the new findings have potential relevance to a much wider group of stochastic processes. Thus, the current paper seeks to clarify the underlying development process by investigating the nature of the end phase surface and testing these models by examining new and existing data. Results show no statistical evidence to support any periodic oscillations in the end phase surface and a number of issues with the oscillating model are noted. In addition, it is suggested that there is no clear physical mechanism to underpin the oscillating model and that the saturating model is sufficient to describe all the presented data. In conclusion a qualitative description of the end phase topography and its dynamic but temporally uniform character is given. The idea of ‘process bandwidth’ relating to the range of surface-media interactions is introduced to help describe some of the behavioural and control aspects of this stochastic process and the surface topography it produces.

1. Introduction

One of a wider group of ‘mass finishing’ processes, vibratory finishing of components is widespread in manufacturing industries. Distinguished from other stochastic abrasive processes like grinding and lapping by being ‘force bounded’ (limited) [1] due to the abrasive media being loose. The process is typically carried out in a vessel containing a fluidized media bed excited by rotating eccentric masses on a motor driven shaft. Figure 1 shows a schematic of the small-scale laboratory vibratory finishing machine used in the current work.

In many cases surfactant flushing compound is used to remove waste debris and prevent the clogging of cutting and component surfaces. In many cases components are processed loose within the media bed, but fixturing and robotic guiding (drag finishing) bring increased flexibility and control to the process.

Applications of vibratory finishing are numerous and include; polishing, deburring, and radiusing. Practical process fundamentals are considered in the handbook of Gillespie 2007 [2] and a series of publications by Kittredge [3–6].

Mediratta et al [7] offer a useful wide ranging review of vibratory finishing. They detail key process variables for the vibratory finishing process group including the central aspects of media: size, shape and material composition. They note that three main media-surface interaction types are seen; free impact, rolling of individual media and media rolling over a stationary piece of media (three body
interaction). They also concur with other published findings ‘plastic deformation occurs by media impacts and material removal by relative motion of media and work piece’. More detailed consideration of the mechanics of surface-media interaction, material removal and displacement mechanisms are offered in [8–12].

The important consideration of surface-media impact energy has been investigated by a number of groups. Yabuki et al [10] measure surface stress, at normal and tangential media impacts, they report the well supported finding that the former is approximately an order of magnitude greater than the later. The relationship between media/surface interaction orientation and polishing effect is further investigated by groups including Srivastava et al [17]. They report an associated increasing in polishing effect on faces fixtured parallel to media flow compared to perpendicular to the flow due to the predominance of sliding interactions.

Significant work has been carried out to model the development of surfaces due to this process type. Early modelling was carried out by Sofronas and Taraman [13] regression fitting to empirical data using the response surface methodology with process variables of; process duration, component hardness, media size and vibratory frequency. However, when the rates of change of stock removal and surface roughness were considered Hashimoto and DeBra [14] developed the seminal (‘saturating’) model for vibratory finishing. This model indicates two distinct underlying stages, an initial and end phase as illustrated in figure 2 for a polishing process.

Much work has been carried out to enhance this basic model with other process parameters to generalise its application, though it’s overall structure remains unchanged. Wan et al [15] began with tribological wear laws to incorporate point sliding velocity, apparent pressure and hardness, and extended modelling beyond a 2D derivation and gave applicability to freeform surfaces. Vijayaraghavan et al [16] develop a further numerical model and offer a brief review of modelling in the field and highlight significant process variables.

In the current work the term ‘saturating model’ and its formulation due to Hashimoto and DeBra [14] will be used. Basic process rules were developed in conjunction with the model in [14] and are stated as follows;

1. The end phase (steady state) surface has an inherent surface texture with a constant ‘roughness limitation value’ (nothing of the original surface remains)
2. The greater the difference between the initial surface roughness and the ‘roughness limitation value’ the quicker the change of surface roughness
3. The stock removal rate in the steady state phase is constant

Hashimoto and DeBra [14] model only considered the variables of process time and initial surface roughness. A model coefficient ‘T’ to account for other process variables for a given process was determined empirically.

Figure 1. Cross section schematic of a small capacity bench top vibratory finishing machine.
\[ Ra = (I_r - D_r) e^{-T} + D_r. \] (1)

Where
- \( Ra \) = arithmetic surface roughness (surface roughness)
- \( I_r \) = initial surface roughness
- \( D_r \) = roughness limitation (finished roughness)
- \( T \) = process duration

Thus by interpolation of the measured data,

\[ T = t \text{ when } Ra(T) = Ra(t). \]

Re-evaluating existing data sets form Hashimoto and DeBra [14] and Domblesky et al [19], Wan et al [18] recently reported that surface roughness in the end phase varies cyclically with duration. They describe the characteristic period and amplitude of this oscillation about an equilibrium roughness and this model will be referred to as the ‘oscillating’ model. In this model, the end phase of processing is described as the equilibrium phase to reflect the oscillation about a mean surface value.

\[ Ra = (R_i - R_E) e^{-\frac{kT p_g v_s H}{k_E}} + k_E \sin(\omega t) + R_E. \] (2)

The individual terms in the expression in equation (2a) from (2) cannot be assigned independently for the processes in the current analysis and are collectively considered equivalent to \( 1/T \) from equation (1) and will be referred as the response time.

Wan et al [18] also distinguish between loose abrasive (mass finishing) processes by reference to a ‘controlled’ quality, relating this to the magnitude of the oscillation in the equilibrium phase. What ‘controlled’ means precisely is not clearly stated though they describe: ‘very aggressive roughening experimental runs from Domblesky et al [19] versus more ‘subtle’, controlled ones Hashimoto and DeBra [14].’ and offer the examples of abrasive flow machining and magnetorheological finishing as well-controlled processes, both of which have strongly directional media flow.

Thus, Wan et al [18] state that, for well-controlled processes, equations (1) and (2) become equal at the limit as \( k_E \) (amplitude) in equation (2) approaches zero. In addition, they note that the oscillating model includes the saturating model as a special case when the oscillation amplitude approaches zero.

In the current work it is argued that the oscillating and saturating models represent fundamentally different underlying principles. Though not specifically claimed by the authors in [18] their findings are considered to have fundamental significance for a range of stochastic production processes and their resulting surface topography. It is also noted that any oscillations in surface topography would be seen to some extent in the later stages of the transient phase. Thus, the physical implications of the oscillating model are significant and warrant further investigation.

Figure 2. Schematic of a typical vibratory finishing (polishing) process for surface roughness with duration.
The aim of the current work is to test the veracity of the saturating and oscillating models thus improving understanding of the end phase of the vibratory finishing process, particularly in respect of the temporal character of its surface topography. This will involve the following objectives:

- Examine the formulation of the saturating and oscillating models
- Assess and/or statistically test the saturating and oscillating models against:
  - Pertinent data from [14]
  - Pertinent data from [19]
  - New data collected for the current work
- Highlight the variance in the spatial and temporal aspects of surface-media interactions and the surfaces resulting from them.
- Offer a simplified qualitative description of the complex nature of surface-media interactions in vibratory processing and how this relates to the surface produced.

2. Materials and methods

2.1. Titanium sample processing with relocated measurement

To investigate the temporal character of the end phase surface and test the saturating and oscillating models a Titanium coupon was processed by vibratory finishing (see figure 1) over a total of 48 h in 3 h increments from approximately $S_a 0.15 \mu m$ to $0.34 \mu m$. With no clear means to predict the frequency of any surface oscillation and relate this to measurement interval, an approximation was made. A 3 h measurement interval was selected on the basis that it would give approximately the same relationship between the measurement interval and the transient phase duration for the Titanium sample as that seen for the samples in [19].

A Taylor Hobson coherence correlation interferometer (see ISO 22178-604 [20] for nominal characteristics) was used with a $20 \times$ lens, with sample spacing of $0.87 \mu m$ and sub nm vertical quantization. Areal data files were robust spline filtered with S-L nesting index of $0.008 mm$ and $0.8 mm$. Sampling scale was selected to be 25 instrument fields, each $890 \times 890 \mu m$, approximately $4.5 \times 4.5 mm$ in total, this is significantly in excess of the sampling scale suggested as default in ISO 4288 [21] (for profile measurements); $0.1 < Ra \leq 2$, roughness sampling length $= 0.8 mm$, roughness evaluation length $= 4 mm$.

To optimise the resolution of any oscillations in the end phase, surface spatial variation and non-uniformity [22] were minimized by using a large centrally relocated measurement region. Figure 3 shows a schematic of the test coupon with measurement region consisting of the 25 contiguous instrument fields and relocation fixture.

Values of the arithmetic mean surface height (roughness) $S_a$ were computed in line with ISO 25178-2 [23]. $S_a$ is the areal (calculated over an area) equivalent of the arithmetic mean surface height, $Ra$, for a profile measurement (see ISO 4287 [24]). $S_a$ and $Ra$ though not necessarily equivalent are strongly correlated.

2.2. F test for goodness of fit

The saturating model is a special case of the oscillating model, thus, in a statistical sense the models are nested and an F test can be used to compare their goodness of fit to the data. The models are however based of physical process principals and are strictly speaking...
not a true regression fit to the data. Though this type of fitting may not be the best possible fit for a given data set the F test is considered to be a fair test of the models’ prediction of the data.

To test the statistical significance of the additional parameters $\omega$ and $k_E$ and thus the validity of oscillating model, an F test is applied with the null hypothesis; $H_0; \omega = k_E = 0$ (or; the oscillating model does not improve the saturating model’s fit to the data)

$$F = \frac{(SSR_o - SSR_s) / (Dofs - Dof_o)}{SSR_s / Dof_s} \rightarrow H_0 \quad (3)$$

where

- $SSR = \text{sum of squared residuals}$
- $Dof = \text{degrees of freedom} = N - V$
- $N = \text{number of data points}$
- $V = \text{number of parameters in the model}$
- $a = \text{the significance level selected for the specific test (decimal%)}$

It is expected that for valid nested models increasing the number of parameters will improve the fit. For any such improvement to be ‘significant’ it is expected that it be greater than any increase simply due to the relative increase in the number of parameters (degrees of freedom). If the test does not result in a clear margin, no conclusion can be made, but applying the principle of Occam’s razor the simplest model would be selected.

3. Results and discussion

3.1. Model formulation issue

Figure 4 shows the schematic behaviour of the oscillating model for an arbitrary process for three different initial levels of roughness. The equilibrium roughness level $R_E$ is assumed to be constant for all cases, (if different $R_E$ values are possible the phase relationship remains unchanged).

A specific issue of the oscillating model is most apparent at ‘A’ in figure 4 the initial roughness of the surface is equal to the equilibrium roughness ($R_i = R_E$)

From equation (2)

$$R_a = (R_i - R_E) e^{-\frac{k_T p_g}{H} t} + k_E \sin \omega t + R_E.$$  

The slope of this function is

$$\frac{dR_a}{dt} = k_E \omega \cos \omega t \quad (5)$$

At the start of processing $t = 0$

$$\frac{dR_a}{dt} \quad \text{has a positive value}$$

Thus, for all surfaces processed under these conditions the gradient of the function is positive and all surfaces must get rougher initially. However, for a nominally ‘stochastic’ process such as vibratory finishing, initial roughening or smoothing with equal probability would be more logical. The same apparent inconsistency is evident for a surface of any initial roughness where processing is stopped and restarted (for measurement perhaps) at or close to $R_E$. Thus, the feasibility of measuring such an oscillating surface type is brought into question along with the formulation of the oscillating model.

3.2. Hashimoto and DeBra [14] data

Figures 5–7 show empirical data for the vibratory finishing of a steel sample due to Hashimoto and DeBra [14], with the saturating and oscillating
process models represented by equations (1) and (2) respectively. The coefficients in equations (1) and (2) as assigned by the respective model authors are shown in table 1.

Table 1. Coefficient values assigned for equations (1) and (2).

| Coefficient (oscillating model) | $R_t$ | $R_g$ | $k_R/s$ | $k_P/s$ | $\omega$ | $k_f$ |
|---------------------------------|-------|-------|---------|---------|---------|-------|
| Units (Wan)                     | $Ra \mu m$ | $Ra \mu m$ | hours$^{-1}$ | hours$^{-1}$ | $Ra \mu m$ |
| Value (Wan)                     | 0.053 | 0.01  | 3       | 0.72    | 0.07    |
| Value (reassigned)              | 0.053 | 0.07  | 3       | 0.72    | 0.01    |

| Coefficient (saturating model)  | $I_r$ | $D_r$ | $T$ | — | — |
|---------------------------------|-------|-------|----|---|---|
| Value (Hashimoto)               | 0.053 | 0.064 | 0.25 | — | — |

Figure 5 shows the data and the models described and plotted over the original process duration from [14]. Wan et al [18] use the plot in figure 5 to demonstrate that their model uniquely reproduces the initial decrease in surface roughness seen at the second data point. Thus, they argue that this data from [14] provides evidence to support the oscillating model.

While the oscillating model is an adequate fit to the data over the original process range, extending the model beyond that range reveals a clear contradiction. Wan et al [18] do not explicitly state how the values of the coefficient terms in equation (2) are determined. However, it is assumed this is done by direct observation of trends in the data. It is suggested that the values of $R_g$ and $k_f$ were misassigned in this case, as the amplitude of the oscillation is larger than the mean and must result in a negative roughness. These values are reassigned (see table 1) given that $R_g$ should be approximately the mean value of the surface roughness in the equilibrium phase and that $k_f$ should be the amplitude of the variation from $R_g$. Figure 7 shows the oscillating model with the reassigned values of $R_g$ and $k_f$.

With this reassignment of coefficients, the oscillating model no longer reproduces the initial reduction in surface roughness seen in the data. It is further noted that with valid coefficients of any value in equation (2) the initial dip in question could not be reproduced. Consequently, this data from [14] does not provide evidence for the oscillating model in the way described by Wan et al [18]. Given the significant variance of this data and the limited goodness of fit to either model, no further fitting and F testing was attempted.

3.3. Domblesky et al [19] data

Wan et al [18] also applied the oscillating model to a selection of data from Domblesky et al [19] in which Aluminium, Brass and Steel samples were roughened by vibratory finishing with machine bowl acceleration as a process variable. The bowl acceleration in the current work only serves to identify the original data. These data are shown in figure 8 through 11 and the saturating model is added for visual and statistical comparison. Model function coefficient values for equations (1) and (2) are assigned as in table 2 based on interpretation from [19] that the surfaces had reached the end phase at 2 h duration.

By inspection, the plotted data in figure 8 through 11 show varying degrees of goodness of fit to the oscillating model. With the exception of figure 11 the oscillating model appears to offer a better fit to the data than the saturating model and this is borne out by the ‘positive’ F statistics in table 3. However, outliers from the fit of the oscillating model and the small number of data points available mean that the F test is failed in all cases. This indicates that the expected improvement in fit with the addition of 2 variables to the saturating model is not met and the noted improvements should only be attributed to random variations. Thus the data sets in [19] provide no statistical evidence to support the oscillating model as an improvement on the saturating one.

It is implicit within the saturating model that for a given process (set of process variables) the saturated surface roughness $D_r$ is not dependent on the initial surface roughness $I_r$. This is a logical conclusion as nothing of the original surface remains after sufficient processing whether this finished surface is considered as steady state or equilibrium. However, Wan et al [18] attribute the difference in the equilibrium roughness $R_g$ values for the steel samples in table 2 and seen in figures 10 and 11 to the difference in the initial roughness of the samples. The bowl accelerations for these steel samples are clearly different but in [19] it is considered that ‘surface roughness does not appear to be influenced by resultant bowl acceleration’. It should also be noted that saturating model roughness ($D_r$) is also different for the steel samples in table 2. This is because they were calculated individually as the mean of the surface roughnesses over the end phase (2 h onward) as defined in [19]. If bowl acceleration and initial roughness do not influence final surface roughness it could be argued that ($D_r$) should be calculated as an average of both sets of steel data after 2 h. The F tests in table 3 were repeated using this alternative value for ($D_r$) this did not change the test outcome. In addition, there is some evidence from the Brass and Aluminium data in [19] that it may take 5 to 6 h for the surfaces to reach equilibrium, though it is not clear why this might be the case, though Domblesky et al [19] did not note this.

3.4. Test sample data

3.4.1. Statistical analysis

Figure 12 shows the data for the Titanium sample vibratory processed over 48 h along with the saturating and oscillating models (full data is plotted in figure 13).
Figure 5. Data and models over the process duration detailed in [14].

Figure 6. Shows the models from figure 5 over an extended process duration.

Figure 7. Showing the oscillating model with reassigned coefficient values of $R_E$ and $k_E$. 
Function coefficient for equations (1) and (2) were assigned as in table 4.

Coefficients were assigned based on: The values of $D_r$ and $R_E$ (limiting roughness value and equilibrium roughness) were both set equal to the mean of the surface roughness measurements over the duration of the end phase. These values have not necessarily been equal for the other data sets quoted where they have been determined by other authors. However, it is assumed that the oscillating model equilibrium roughness in principle would have a value equal to that of a saturating model limiting roughness. The response time value $T$ for the saturating model was estimated as noted at the end of section 1 and the reciprocal response time equation (2a) was assigned to $1/T$ as detailed at the end of section 3.4.1.

Figure 12 shows it is argued that no clear periodicity is apparent in the end phase data in (though selection of measurement interval must be considered in mitigation of this.) Thus, the values of angular frequency and amplitude ($\omega$ and $k_E$) respectively for equation (2) could not be assigned by observation. Hence, these frequency and amplitude coefficients were estimated by allowing them to vary freely over the ranges noted in table 4 while minimising the sum of squared residuals over the whole process data set (non-linear regression for equation (2)).

The frequency and amplitude value ranges for the fitting were selected by considering the available data.

Table 2. Saturating and oscillating model function coefficient values [19].

| Coefficient (saturating model) | Figure | $I_r$ | $D_r$ | $T$ | — | — |
|-------------------------------|--------|------|------|----|---|---|
| Units                         |        | $Ra \ \mu m$ | $Ra \ \mu m$ | hours | — | — |
| Al (23.5 m $s^{-2}$)          | Figure 8 | 25   | 87   | 0.3 | — | — |
| Brass (23.5 m $s^{-2}$)       | Figure 9 | 15   | 58   | 0.4 | — | — |
| Steel (27.4 m $s^{-2}$)       | Figure 10 | 40   | 52   | 0.5 | — | — |
| Steel (32.3 m $s^{-2}$)       | Figure 11 | 28   | 48   | 0.5 | — | — |

Table 3. F test results for saturating and oscillating models of Domblesky data.

| Sample         | Figure | Data points | $F$ statistic (equation (1)) | Critical $F$ value $[F_{6,4}, p < 0.1]$ | $p$ value at $F$ | $H_0$ |
|----------------|--------|-------------|-----------------------------|----------------------------------|----------------|------|
| Al 23.5 m $s^{-2}$ | Figure 8 | 9           | 1.419                       | 4.01                             | 0.383          | Not rejected |
| Brass 23.5 m $s^{-2}$ | Figure 9 | 9           | 2.231                       | 4.01                             | 0.229          | Not rejected |
| Steel 27.4 m $s^{-2}$ | Figure 10 | 9           | 0.549                       | 4.01                             | 0.756          | Not rejected |
| Steel 32.3 m $s^{-2}$ | Figure 11 | 9           | -0.963                      | 4.01                             | —              | Not rejected |
Figure 9. Brass (23.5 m s\(^{-2}\)) [19] data with saturating and oscillating models.

Figure 10. Steel data (27.4 m s\(^{-2}\)) [19] with saturating and oscillating models.

Figure 11. Steel data (32.3 m s\(^{-2}\)) [19] with saturating and oscillating models.
It was noted that for the data in [19] the reciprocal response time (equation (2)) was similar to the value of \( \omega \) in all cases (see table 2). Thus the range of \( \omega \) was set to run from twice the value of the reciprocal response time to a value which equated to a single cycle of the oscillation over the full length of the end phase. This frequency lower limit is somewhat arbitrary and dictated by the overall duration of processing. It is noted that the value of \( \omega \) for the Titanium sample fitting is

![Titanium sample processing](image_url)

**Figure 12.** Titanium sample data with saturating and oscillating models.

![Titanium sample](image_url)

**Figure 13.** Showing surface roughness data with process duration for the Titanium sample processing, vertical solid lines indicate the range of the 25 measurements at each process increment, one standard deviation error bars are included as horizontal bars. Data for Sdq (RMS surface slope) is included on a secondary scale with arbitrary scaling for trend comparison with Sa.

| Table 4. | Model function coefficient values for Titanium sample figure 12. |
|----------|---------------------------------------------------------------|
| Coefficient (saturating model) | \( L, D, T \) | — | — |
| Units | \( Sa \) \( \mu m \) \( Sa \) \( \mu m \) hours | — | — |
| Coefficient (oscillating model) | \( R_i, R_E, \frac{k_c}{T}, k_E \) | — | — |
| Units | \( Sa \) \( \mu m \) \( Sa \) \( \mu m \) hours \( -1 \) \( Sa \) \( \mu m \) hours \( -1 \) \( Sa \) \( \mu m \) | 0.15 | 0.34 | 1.5789 | 1.342 | 0.0061 | 3.0 | 0 | 0.12 | 0.021 |

| Table 5. | F test results for Titanium sample data fitting to the saturating and oscillating model. |
|----------|---------------------------------------------------------------|
| Process phase | Value/outcome |
| Figure | Figure 12 |
| Data points | 17 |
| Parameters (equation (1)) | 3 |
| Parameters (equation (2)) | 5 |
| F test statistic (equation (3)) | \( F_{14,12}, p < 0.05 \) |
| Critical F value | 2.64 |
| F test value | 1.56 |
| \( P \) value (%) at \( F \) | 22.4 |
| \( H_0 \) | Not rejected |
seen to be close to the value of its reciprocal response time (see table 4).

The value of the oscillation amplitude $k_e$ was allowed to vary from zero to a value equal to the most extreme measured difference from the mean line (at 48 h). The resulting fitted waveform in is, by observation, not a good fit to the data.

Table 5 shows the F test results (see section 2.2) for the Titanium sample process as modelled by the saturating and oscillating models (equations (1) and (2) respectively) with the null hypothesis $H_0: \omega = k_e = 0$.

The oscillating model is a better fit to the data than the saturating model but the difference does not exceed that expected from the two additional parameters. Thus, the null hypothesis is not rejected, the improved fit of the oscillating model is attributed to random effects and there is no evidence in the data to support the validity of the oscillating model.

3.4.2. Surface development
All pre-existing data available for the current study and that quoted by Wan et al [18] is exclusively ‘arithmetic mean of the surface height’ (Ra, Sa). This type of surface metric can show limited functional correlation, however so called ‘hybrid’ parameters; those that incorporate both amplitude and spatial information could be argued to offer better correlation with finishing processes. For this reason, the hybrid areal surface parameter Sdq, the ‘root mean square slope’ of the surface is also considered for the Titanium sample in the current study. Figure 13 shows the Titanium sample Sa data with process duration, with data range and one standard deviation error bars included. In addition, for the well-established equilibrium phase of processing, Sdq data is also plotted with an arbitrary scaling selected to overlay the two plots to facilitate comparison of their trends. For clarity, data range and standard deviation are not included for Sdq, though they are of a similar relative scale to those for Sa.

The plots of the Sa and Sdq in figure 13 are strongly correlated indicating that if Sdq better characterises finishing process development then in this context Sa (Ra) should be a valid metric for the comparisons carried out in the current work. It is also noted that the hybrid parameter Sdr, the ‘developed surface area ratio’ which is a measure of surface complexity, closely follows the trend of Sdq for the Titanium sample.

Significant variation is seen within the groups of 25 measurements at each process increment and across the mean values at each increment. Table 6 shows the coefficients of variation for the end phase data from [19] in section 3.2 and the Titanium sample data. The coefficient of variation is a non-dimensional measure of the variation, in this context it is expressed as a percentage: $[100\% \times \frac{\text{Standard deviation}}{\text{mean}}]$.

It is clear that the roughness scale of the Titanium sample is significantly smaller than the samples from [19]. However, the CV (coefficient of variation) values indicate that the relative variation in end phase roughness is of the same order of magnitude for all samples in table 6. Thus, in the context of the oscillating model, it is reasonable to assume that the processing of the Titanium sample was not sufficiently well ‘controlled’ (see section 1) to result in any oscillation amplitude approaching zero.

Figure 14 shows false colour height maps A, B (A1, B1) and C of single instrument fields that are typical of the Titanium sample at 0 h, 5 min and 12 h of processing respectively. The series A, B, C are shown with the full false colour range displayed over the height range of the individual surfaces to give full resolution of surface features. The series A, B, C is shown with the false colour range displayed over the height range of each individual surface to give full resolution of surface features. The series A1, B1, C shows the same regions but with false colour ranges all displayed over the same height range.

These maps illustrate the developing topography of the surface: In A (initial surface), the uniform striations of the lapping process are seen. In B (transient phase), this uniform background is still visible but clear marks of surface-media interactions are seen. Relatively energetic low angle interaction marks are seen in the upper middle of the field, while interactions closer to normal are evident in the lower middle and left. Evidence of less energetic interactions is seen elsewhere on the surface. In C (end phase) no evidence of the initial surface remains and the new surface is comprised of pit like features at a range of scales on a more or less uniform plateau.

It is argued that a stochastic process such as vibratory finishing has a characteristic spectrum or bandwidth of surface-media interactions with characteristic intensities and orientations (though these are not

---

**Table 6.** Shows coefficient of variation for the end phase data of the current Titanium sample and the samples of Domblesky et al [19] as seen in section 3.3.

| Sample Source | Data points | Initial Sa or Ra (µm) | End phase mean Sa or Ra (µm) | End phase standard deviation Sa or Ra (µm) | Coefficient of variation (CV) (%) |
|---------------|-------------|-----------------------|-------------------------------|------------------------------------------|------------------------------|
| Titanium      | Current     | 15                    | 0.15                          | 0.34                                     | 0.0098                       | 2.9                          |
| Al 23.5 m s⁻²  | Domblesky [19] | 7                     | 25                            | 85                                        | 2.7                          | 3.2                          |
| Brass 23.5 m s⁻² | 7           | 15                    | 58                            | 2.9                                       | 5.0                          |
| Steel 27.4 m s⁻² | 7           | 40                    | 52                            | 1.9                                       | 3.7                          |
| Steel 32.3 m s⁻² | 7           | 28                    | 47                            | 1.8                                       | 3.8                          |
independent), producing a similarly characteristic distribution of surface modification events. Progressively, a surface composed of features due solely to those events replaces an initial surface. When the surface feature distribution matches that being produced by the process, the surface and the process can be considered as being in dynamic equilibrium. This idea of interaction bandwidth is helpful when considering aspects of vibratory processing such as fixturing and shadowing. As noted in section 1 [17] fixturing parallel to the predominant flow results in a smoother finish due to increased sliding. This is equivalent to skewing the process bandwidth toward oblique interactions. These interactions being lower energy [10] and more polishing [17]. Similarly, surface shadowing can be considered as ‘geometric filtering of the process bandwidth’, where a surface region has its range of surface-media interaction altered by proximity to surface features such as internal corners and edges. Thus a ‘well controlled process’ might well have the following bandwidth limitations; being skewed away from normal and toward low intensity.

Inherently, for a process with such a distribution of surface events, relative roughening and polishing will both be possible. However, it is not clear why these actions would dominate cyclically and uniformly across an entire surface. From a stochastic perspective, they would more logically occur with equal probability at any location, resulting in, on average, a spatially and temporally uniform surface.

This outcome is in clear contradiction of the principles of the oscillating model and given the evidence reported in the current work, the saturating model is considered sufficient for vibratory finishing.

Figure 14. Shows false colour height maps of 3 measurements fields from the titanium coupon at (A) 0 h (B) 5 min, (C) 12 h. The false colour scales have zero height set at the lowest point on the surface.
4. Conclusions

- Data from Hashimoto and DeBra [14] is shown not to support the oscillating model as shown in Wan et al [18]
- With the F test used
  - Data from Domblesky et al [19] shows no statistical evidence for the oscillating model as described in Wan et al [18]
  - New data collected to test the oscillating model shows no statistical evidence for it
- Two paradoxes are noted that challenge the formulation of the oscillating model
  1. Inherently fixed phase
  2. Equilibrium surface mean roughness is dependent on initial surface roughness
- The saturating model is sufficient and there is no apparent evidence or physical principle which necessitates any other model
- A qualitative description of a temporally uniform end phase in terms of media-surface interaction spectrum (bandwidth) is offered
  - This idea can be extended to the wider process group describing process control in terms of bandwidth manipulating
  - The term ‘equilibrium’ phase is considered a useful alternative to end or steady state phase as it relates the sense in which the surface is in equilibrium with the process generating it.

Further investigation of process bandwidth and its separation is to be carried out by way of component fixturing and shadowing.

Acknowledgment

The authors gratefully acknowledge the UK’s Engineering and Physical Sciences Research Council (EPSRC) funding of the Future Metrology Hub (Grant Ref: EP/P006930/1).

ORCID iDs

Karl Walton  https://orcid.org/0000-0002-1473-6604

References

[1] Klocke F 2009 Manufacturing Processes vol 2 (Berlin: Springer) p 433
[2] Gillespie L R K 2007 Mass Finishing Handbook (New York: Industrial Press)
[3] Kittredge J 1981 Understanding vibratory finishing, II. The compound solution Prod. Finish. 45 62–73
[4] Kittredge J 1981 Understanding vibratory finishing, III. Equipment Prod. Finish. 45 60–9
[5] Kittredge J 1981 Understanding vibratory finishing, IV Prod. Finish. 45 76–84
[6] Kittredge J 1981 Understanding vibratory finishing, I. Media Prod. Finish. 45 94–105
[7] Mediratta R, Abluwalla K and Yeo S 2016 State-of-the-art on vibratory finishing in the aviation industry: an industrial and academic perspective Int. J. Adv. Manuf. Technol. 85 415–29
[8] Uhlmann E, Eulitz A and Dethlefs A 2015 Discrete Element Modelling of Drag Finishing Procedia CIRP 31 369–74
[9] Baghbahan M R, Yabuki A, Timsr R and Spelt J K 2003 Tribological behavior of aluminum alloys in a vibratory finishing process Wear 255 1369–79
[10] Yabuki A, Baghbahan M and Spelt J K 2002 Contact forces and mechanisms in a vibratory finisher Wear 252 635–43
[11] Wang S, Timsr R and Spelt J K 2000 Experimental investigation of vibratory finishing of aluminum Wear 243 147–56
[12] Brocker R, Vits F, Mattfeld P and Klocke F 2015 Contact forces in unguided vibratory finishing ASME 2015 Int. Manufacturing Science and Engineering Conf. (American Society of Mechanical Engineers)
[13] Sofronas A and Taraman S 1979 Model development and optimization of vibratory finishing process Int. J. Prod. Res. 17 23–31
[14] Hashimoto F and DeBra D B 1996 Modelling and optimization of vibratory finishing process CIRP Ann. 45 303–6
[15] Wan S, Liu Y, Woon K S and Tngay G L 2014 A material removal and surface roughness evolution model for loose abrasive polishing of free form surfaces Int. J. Abras. Technol. 6 269–85
[16] Vijayaraghavan V, Castagne S, Srivastava S and Qin C 2016 State-of-the-art in experimental and numerical modeling of surface characterization of components in mass finishing process Int. J. Adv. Manuf. Technol. 90 1–15
[17] Srivastava S, Chua Z Q and Castagne S 2015 Effect of workpiece orientation, lubrication and media geometry on the effectiveness of vibratory finishing of Al6061 MATEC Web of Conf. (EDP Sciences)
[18] Wan S, Liu Y C and Woon K S 2016 A simple general process model for vibratory finishing Int. J. Adv. Manuf. Technol. 86 2393–400
[19] Domblesky J, Cariapa V and Evans R 2003 Investigation of vibratory bowl finishing Int. J. Prod. Res. 41 3943–53
[20] BS EN ISO 25178-604 2013 Surface texture: areal. Nominal characteristics of non-contact (coherence scanning interferometry) instruments Geometrical Product Specifications (GPS) (British Standards Institute)
[21] BS EN ISO 4288 1998 Surface texture. Profile method: rules and procedures for the assessment of surface texture Geometric Product Specification (GPS) (British Standards Institute)
[22] Walton K, Blunt L and Fleming L 2015 The topographic development and areal parametric characterization of a stratified surface polished by mass finishing Surf. Topogr., Metrol. Prop. 3 035003
[23] BS EN ISO 25178-2 2012 Surface texture: areal 2: terms, definitions and surface texture parameters Geometrical Product Specifications (GPS) (British Standards Institute)
[24] BS EN ISO 4287 2000 Surface texture: profile method. Terms, definitions and surface texture parameters Geometrical Product Specification (GPS) (British Standards Institute)