Resonant CP violation induced by particle mixing in transition amplitudes

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ABSTRACT

We analyze CP violation in resonant transitions involving scalar as well as fermionic intermediate states, using a gauge-invariant resummation approach implemented by the pinch technique. We derive the necessary conditions for resonantly enhanced CP violation induced by particle mixing, by paying special attention to CPT invariance, and apply the results of our analysis to two representative new-physics scenarios: (i) the indirect mixing of a CP-even Higgs scalar with a CP-odd Higgs particle in two-Higgs doublet models and (ii) the CP-asymmetric mixing between the top quark and a new heavy up-type quark, $t'$, in mirror fermion models. Furthermore, we explicitly demonstrate the equivalence of our scattering-amplitude formalism with that based on the effective Hamiltonian and discuss the possibility of maximal CP violation in the limiting case of two degenerate particles.

PACS nos.: 11.30.Er, 13.10.+q, 14.80.Bn, 14.80.Cp
1 Introduction

The first experimental observation, which has shown conclusively that nature laws are not invariant under charge and parity (CP) transformations, took place in the kaon complex some time ago [1]. The CP-violating phenomenon observed originates from the CP-asymmetric particle mixing of a $K^0$ with its CP-conjugate state, $\bar{K}^0$ [2]. So far, this has been the only example of CP violation in nature, which has been established experimentally.

One of the most phenomenologically successful theories describing the time evolution and the mixing of unstable particles is the known approximation due to Weisskopf and Wigner (WW) [3]. In the WW approximation, an unstable particle system is described through an effective Hamiltonian and the whole time dependence of the system may well be determined by an effective Schrödinger equation [4]. Another interesting approach is the one formulated by Sachs [5], which makes use of the dynamical properties of the complex pole of the propagator. However, both formulations mentioned above are, to some extend, phenomenologically oriented and face serious difficulties to accommodate important quantum field-theoretic properties for theories with enhanced predictive power, such as renormalizable gauge field theories. Apart from developing a well-defined renormalization scheme to cope with ultra-violet (UV) divergences at high quantum orders [6], one has to worry that gauge symmetries [7,8] and other required properties, such as unitarity, analyticity, etc., are preserved within such theories [8].

In our analysis, we shall consider a manifestly gauge-invariant approach for resonant transitions [8], which is implemented by the pinch technique (PT) [9]. This approach is free from CP-odd gauge artifacts, since it preserves all the discrete symmetries of the classical action after quantization. For example, it reassures the absence of off-mass shell transitions between particles with different CP quantum numbers in a CP-invariant and anomaly-free theory.

Over the last years, there has been an increasing interest in CP asymmetries induced by finite widths of unstable particles [10,11,12,13]. For example, CP-violating effects may arise from the interference of the top quark with a new up-type quark $t'$ in many extensions of the Standard Model (SM) [10,11] or from similar interference width effects in the top-quark decay [13] in a two-Higgs doublet model. In Ref. [12], the authors considered CP violation through scalar-quark mixing in supersymmetric (SUSY) models. Since SUSY imposes an approximate mass degeneracy between the scalar strange, $\tilde{s}$, and down, $\tilde{d}$, quarks [13], it was found that there is a resonant phenomenon of CP violation due to $\tilde{s}$-$\tilde{d}$

*Most recently, we became aware of studies, which have extended this idea to the scalar lepton sector in SUSY theories [14].
oscillations, which can be rather large, i.e., of order 20%. All these CP-violating phenomena may have dynamical features similar to those of the $K^0\bar{K}^0$ system. We will exemplify this connection for certain models beyond the SM.

Recently, we have studied resonant CP-violating transitions of a CP-even Higgs scalar, $H$, into a CP-odd Higgs particle, $A$ [16]. These transitions, which come from a non-vanishing $HA$ mixing, exhibit the very same dynamics known from the $K^0\bar{K}^0$ system [17]. The size of CP violation has been estimated to be fairly large, i.e., of order one, for a range of kinematic parameters, preferred by SUSY.

In this paper, we shall present a comprehensive field-theoretic formalism for resonant CP violation through particle mixing in scattering amplitudes. The mixed particles, which occur in the intermediate state as resonances, are either bosons or fermions. As will be seen, a resonant enhancement of CP violation can only take place within this formalism, if at least two of the intermediate particles are nearly degenerate, i.e., their mass difference is comparable to their widths. This requirement of approximate mass degeneracy, which appears to be a fine-tuning of the kinematic parameters, may be the result of some enforced symmetry in the low-energy limit of the theory, such as SUSY, compositeness, or a general horizontal symmetry. Even though we will not specify all the details of our CP-violating models at very high-energies, e.g., at the grand unification scale, it is, however, conceivable to assume that there exist heavy degrees of freedom, which can break such a low-energy symmetry and hence induce a small splitting in the masses of the intermediate particles. Consequently, this effect is quite analogous to what happens in the $K^0\bar{K}^0$ system, in which the exact mass degeneracy of the strong states $K^0$ and $\bar{K}^0$ becomes an approximate one when the weak interactions are taken into account.

The paper has the following structure: In Section 2, we consider the mixing of bosonic particles, where the PT resummation approach for particle mixing in scatterings is presented. For illustration, we assume a $2 \to 2$ scattering process involving resonant transitions of two Higgs scalars $H$ and $A$, with opposite CP parities: $\text{CP}(H) = +1$ and $\text{CP}(A) = -1$. If the Higgs particles $H$ and $A$ mix, then this mixing phenomenon is described by a non-diagonal $2 \times 2$ propagator matrix in the transition amplitude, which results from summing up a geometric series of $HH$, $AA$ and $HA$ self-energies. Furthermore, issues of mixing and mass renormalization in scalar field theories are discussed in Appendices A and B, respectively. In Section 3, we derive the necessary conditions for resonantly enhanced CP violation in the $HA$ system and argue that such a mixing-induced CP violation shares common features with the CP-violating phenomenon through indirect mixing in the kaon system.

It is known that invariance of all interactions under the combined action of CP and
time reversal (T) is a fundamental property of the underlying Hamiltonian. Therefore, in Section 1, we discuss possible constraints on transition amplitudes, which may originate from CPT invariance. We find that CPT symmetry generally leads to a modest reduction of the size of the CP asymmetries.

In Section 2, we briefly review the effective Hamiltonian approach and explicitly demonstrate the equivalence of that approach with our formalism. Then, we investigate the possibility of maximal CP violation for the limiting case of two degenerate particles. We observe that, if the effective Hamiltonian expressed in a \( K^0 \bar{K}^0 \)-like basis has the Jordan form, the two mass-eigenvalues of the effective Hamiltonian are exactly equal and, in this extreme case, CP violation through mixing takes its maximum allowed value.

In Section 3, we discuss models that predict a potentially large \( HA \) mixing. Such a mixing can naturally occur within two-Higgs doublet models either at the tree level, if one adds softly CP-violating breaking terms to the Higgs potential, or at one loop, after integrating out heavy degrees of freedom that break the CP invariance of the Higgs sector, such as heavy Majorana neutrinos. In Section 4, we investigate phenomenological examples of resonant CP violation through \( HA \) mixing. Numerical estimates reveal that the CP-violating phenomenon could be of order one and may hence be observed at future high-energy \( pp, e^+e^- \) or \( \mu^+\mu^- \) machines. Bounds from electric dipole moments (EDM’s) of the neutron and electron are also implemented in this analysis.

In Section 5, we examine fermionic mixing, which is more involved in comparison with that of bosons owing to the spinorial structure of fermions, resulting in four degrees of freedom, i.e., left- and right-handed fields for particles and anti-particles. As for the phenomenon of CP violation through mixing, we find that it is conceptually similar to that of bosons. If the mass difference of the two mixed fermions is of order of their widths, the CP-violating phenomenon becomes resonant. As an example, in Section 6, we analyze a simple new-physics CP-violating scenario, which predicts an asymmetric mixing between the top-quark and a new up-type quark, \( t' \). Resonant CP-violating \( tt' \) transitions may be probed at the CERN \( pp \) Large Hadron Collider (LHC). Section 10 contains our conclusions.

## 2 Bosonic case

In this section, we shall analyze CP violation induced by the mixing of two bosons with opposite CP quantum numbers in scattering amplitudes. First, we shall focus our attention on an example with two-scalar mixing and then extend our discussion to scalar-vector mixing, such as the mixing of a Higgs particle, \( H \), with the \( Z \) boson. Finally, we will...
comment on transitions involving two vector particles with two different CP quantum numbers.

\[ \text{Fig. 1: Resonant CP violation induced by } HA \text{ mixing.} \]

Let us consider the resonant prototype process \( ab \rightarrow H^*, A^* \rightarrow cd \) in Fig. 1, where \( H \) and \( A \) are CP-even and CP-odd (Higgs) scalars, respectively. The asymptotic states \( a, b, c, d \) could be either fermions, \textit{e.g.}, \( b \) or \( t \) quarks, or vector bosons, \textit{e.g.}, \( W \) or \( Z \) bosons. The transition amplitude of such a process may conveniently be expressed as

\[ \mathcal{T} = \mathcal{T}^{\text{res}} + \mathcal{T}^{\text{box}} = V^P_i \left( \frac{1}{s - \mathcal{H}(s)} \right)_ij V^D_j + \mathcal{T}^{\text{box}}, \tag{2.1} \]

where

\[ s - \mathcal{H}(s) = \hat{\Delta}^{-1}(s) = s1 - \begin{bmatrix} M_A^2 - \hat{\Pi}^{AA}(s) & -\hat{\Pi}^{AH}(s) \\ -\hat{\Pi}^{HA}(s) & M_H^2 - \hat{\Pi}^{HH}(s) \end{bmatrix} \tag{2.2} \]

is the inverse propagator matrix, which describes the dynamics of the \( HA \)-mixing system. In fact, the propagator matrix \( \hat{\Delta}(s) \) arises from summing up a geometric series of \( HH, AA, HA \) and \( AH \) self-energies. In this resummation formalism based on transition amplitudes, \( \mathcal{H}(s) \) is closely related with the effective Hamiltonian, obtained in the WW approximation. As will be discussed in Section 5, there is only a minor difference between \( \mathcal{H}(s) \) and the effective Hamiltonian. The latter may equivalently be evaluated from the former, \( \mathcal{H}(s) \), at the resonant region \( s \approx M_H^2 \approx M_A^2 \), and is therefore \( s \)-independent. Furthermore, \( V^P_i \) and \( V^D_i \) are the production and decay amplitudes of the process, as shown in Fig. 1(a). In addition, \( \mathcal{T}^{\text{box}} \) refers to the non-resonant part of the amplitude depicted in Fig. 1(b), \textit{i.e.}, \( t \)-channel or box graphs.

In Eq. (2.2), the symbol hat on the self-energies, \textit{i.e.}, \( \hat{\Pi}^{ij}(s) \) with \( i, j = H, A \), has two meanings. First, it indicates that the diagonal self-energies are renormalized in some natural scheme, \textit{e.g.}, on-mass-shell (OS) renormalization. In our calculations, we consider the OS scheme, since it can be implemented much easier than the pole-mass renormalization scheme. More details on mixing and mass renormalization in scalar theories are relegated
The off-diagonal self-energies, $\hat{\Pi}^HA(s)$ and $\hat{\Pi}^{AH}(s)$, are UV safe; such CP-violating transitions occur either at the tree level or are generated radiatively. The second meaning of the symbol hat refers to the fact that the resummed self-energies should be gauge independent. Throughout our analysis, we shall adopt a gauge-invariant resummation approach implemented by the PT, which respects the gauge symmetries of classical action at the tree level \[\text{[8]}\]. The advantages of this method will be seen later on, when we will discuss the $HZ$ mixing.

The CP-conjugate amplitude, $\mathcal{T}^{CP}$, may be written down as follows:

$$\mathcal{T}^{CP} = \mathcal{T}^{res} + \mathcal{T}^{box} = \mathcal{V}^P_i \left( \frac{1}{s - \mathcal{H}(s)} \right)_{ij} \mathcal{V}^D_j + \mathcal{T}^{box}, \quad (2.3)$$

where the CP transformation on the production and decay amplitudes may generally given by

$$V_{i}^{P,D} \rightarrow V_{i}^{P,D} e^{i\delta_f} e^{i\delta_w} \quad \mathcal{CP} \quad V_{i}^{P,D} = |V_{i}^{P,D}| e^{i\delta_f} e^{-i\delta_w}. \quad (2.4)$$

Here, $\delta_f$ denotes the absorptive or final state phase coming from OS unitarity cuts and $\delta_w$ represents the weak phase. Under a CP transformation, only $\delta_w$ changes sign. CP-violating effects in $V_{i}^{P,D}$ are sometimes called $\epsilon'$-type effects in analogy with the $K^0\bar{K}^0$ system. In our study, we shall ignore that kind of effects, since they are generally small, unless it is stated otherwise (see also discussion of the fermionic case in Section \[8\]). Moreover, we shall neglect the non-resonant part $\mathcal{T}^{box}$. In fact, for sufficiently narrow resonances, the small value of $\mathcal{T}^{box} \ll \mathcal{T}^{res}$ may be justified near the resonant region. This is equivalent to the first assumption in the WW approximation \[3\], where direct transitions between the asymptotic states are omitted. Here, we shall concentrate on CP-violating effects resulting from the effective Hamiltonian (mass) matrix, $\mathcal{H}(s)$. In a $K^0\bar{K}^0$-like basis, $\mathcal{H}(s)$ transforms as

$$\mathcal{H}(s) \xrightarrow{\mathcal{CP}} \mathcal{H}(s) = \mathcal{H}^T(s). \quad (2.5)$$

Note that the effective Hamiltonian in Eq. (2.2) is written in a different basis. We shall derive the necessary conditions for resonantly enhanced CP violation in Section \[3\].

We must emphasize again that CP violation coming from the CP-asymmetric mixing in $\mathcal{H}(s)$ can be much larger than that from the decay and production amplitudes, $V_{i}^{P,D}$ and $\mathcal{T}^{box}$. To quantify the size of CP violation for a generic transition, $\mathcal{T}_{FI}$, of some initial state $I$ (e.g., $I = a, b$), into the final state $F$ (e.g., $F = c, d$), one should consider the observable

$$a_{CP} = \frac{|\mathcal{T}_{FI}|^2 - |\mathcal{T}_{FI}|^2}{|\mathcal{T}_{FI}|^2 + |\mathcal{T}_{FI}|^2} \approx \frac{|\mathcal{T}^{res}|^2 - |\mathcal{T}^{res}|^2}{|\mathcal{T}^{res}|^2 + |\mathcal{T}^{res}|^2}, \quad (2.6)$$

where the state $\bar{I}$ ($\bar{F}$) is the CP transform of $I$ ($F$), and hence $\mathcal{T}^{CP}_{FI} \equiv \mathcal{T}_{\bar{F}I}$. In Eq. (2.6), only the resonant contribution to $\mathcal{T}$ is taken into account. For our purposes, only the
s-valued matrix $\hat{\Delta}^{-1}(s)$ in Eq. (2.2) needs to be inverted. It is then easy to find the entries of the matrix $\hat{\Delta}(s)$, viz.

\[
\hat{\Delta}_{AA}(s) = \left[ s - M_A^2 + \hat{\Pi}^{AA}(s) - \frac{\hat{\Pi}^{AH}(s)\hat{\Pi}^{HA}(s)}{s - M_H^2 + \hat{\Pi}^{HH}(s)} \right]^{-1}, \tag{2.7}
\]

\[
\hat{\Delta}_{HH}(s) = \left[ s - M_H^2 + \hat{\Pi}^{HH}(s) - \frac{\hat{\Pi}^{HA}(s)\hat{\Pi}^{AH}(s)}{s - M_A^2 + \hat{\Pi}^{AA}(s)} \right]^{-1}, \tag{2.8}
\]

\[
\hat{\Delta}_{HA}(s) = -\hat{\Delta}_{AH}(s) = -\hat{\Pi}^{AH}(s)\left[ (s - M_H^2 + \hat{\Pi}^{HH}(s))(s - M_A^2 + \hat{\Pi}^{AA}(s)) \right. \\
\left. -\hat{\Pi}^{AH}(s)\hat{\Pi}^{HA}(s) \right]^{-1}. \tag{2.9}
\]

Moreover, the Hermiticity condition of the Lagrangian on the real scalar fields $H$ and $A$ implies that $\hat{\Pi}^{AH}(s) = \hat{\Pi}^{HA}(s)$.

\[
p^\mu \hat{\Pi}_\mu^{ZH}(p) - iM_Z \hat{\Pi}^{G^0H}(p^2) = 0, \\
p^\mu \hat{\Pi}_\mu^{ZZ}(p) + iM_Z \hat{\Pi}^{G^0G^0}(p^2) = 0,
\]

**Fig. 2:** PT WI’s involving the $HZ$ mixing.

We will now discuss the mixing of a CP-even (Higgs) scalar, $H$, with a massive (gauge) vector particle, e.g., the $Z$ boson. It is obvious that the scalar, $H$, can only couple to the longitudinal component of the gauge particle due to angular momentum conservation. In spontaneous symmetry breaking (SSB) theories, the longitudinal component of a gauge boson, $Z$, may be represented equally well by the respective would-be Goldstone boson, $G^0$, which is a CP-odd scalar. The advantages of our gauge-invariant resummation approach may be seen in the description of the $HZ$ system. In fact, there are PT Ward identities (WI’s) that can be used to convert $ZH$ and $ZZ$ strings into $G^0H$ and $G^0G^0$ ones before resummation occurs [18]. As shown in Fig. 2, these identities are
\[ p^\mu p^\nu \hat{\Pi}_{\mu\nu}^{ZZ}(p) - M_Z^2 \hat{\Pi}^{G^0 G^0}(p^2) = 0, \quad p^\mu \Gamma_{\mu}^{ZZ} = -i M_Z \Gamma^{G^0 G^0}. \]  

(2.10)

Considering the fact that

\[ \hat{\Pi}_{\mu}^{ZH}(p) = p_\mu \hat{\Pi}^{ZH}(p^2) \quad \text{and} \quad \hat{\Pi}_{\mu \nu}^{ZZ}(p) = t_{\mu \nu}(p) \hat{\Pi}_{\mu}^{ZZ}(p^2) + \ell_{\mu \nu}(p) \hat{\Pi}_{L}^{ZZ}, \]  

(2.11)

with \( s = p^2 \) and

\[ t_{\mu \nu}(p) = -g_{\mu \nu} + \frac{p_\mu p_\nu}{p^2}, \quad \ell_{\mu \nu}(p) = \frac{p_\mu p_\nu}{p^2}, \]

one obtains the relations

\[ p^2 \hat{\Pi}^{ZH}(p^2) = -i M_Z \hat{\Pi}^{G^0 H}(p^2), \quad p^2 \hat{\Pi}_{L}^{ZZ}(p^2) = M_Z^2 \hat{\Pi}^{G^0 G^0}(p^2). \]  

(2.12)

Also, it is important to stress that the vacuum polarizations \( \gamma H \) and \( \gamma G^0 \) are completely absent within the PT framework \[16\], i.e.,

\[ \hat{\Pi}_{\mu}^{\gamma G^0}(p) = \hat{\Pi}_{\mu}^{\gamma H}(p) = 0, \]  

(2.13)

independently of whether CP violation is present in the theory. This must be contrasted with the conventional S-matrix prediction in the \( R_\xi \) gauges, in which the \( \gamma G^0 \) and \( \gamma H \) self-energies do not vanish in general.

Taking the PT WI’s in Eq. (2.10) into account, one then ends up with a simple coupled Dyson-Schwinger equation system similar to Eq. (2.2), in which only \( H \) and \( G^0 \) mix. This system may be described by the following inverse propagator matrix:

\[ \hat{\Delta}^{-1}(s) = \begin{pmatrix} s + \hat{\Pi}^{G^0 G^0}(s) & \hat{\Pi}^{G^0 H}(s) \\ \hat{\Pi}^{H G^0}(s) & s - M_H^2 + \hat{\Pi}^{H H}(s) \end{pmatrix}. \]  

(2.14)

It is interesting to notice in Eq. (2.14) that the would-be Goldstone boson, \( G^0 \), has the desirable property to be massless in the PT. The inversion of \( \hat{\Delta}^{-1}(s) \) proceeds analogously, by making the identifications \( A \equiv G^0 \) and \( M_A = 0 \) in Eqs. (2.7)–(2.9).

Finally, the mixing of two vector particles, e.g., \( Z \) and \( Z' \), can be described by an inverse propagator matrix very analogous to Eq. (2.2), in which only the transverse parts of the vacuum polarizations \( ZZ, ZZ', \) and \( Z'Z' \) are involved. However, if the transverse \( Z' \) boson is assumed to be CP-odd, it should only have a coupling of EDM type to fermions. In gauge theories, such a five dimensional operator cannot be present at the tree level without spoiling renormalizability. However, a CP-odd scalar \( Z' \) boson can have tree-level couplings with Higgs scalars \[19\]. Since the essential features of a CP-asymmetric mixing between two vector particles will be identical to those of the scalars, the building of a particular model that predicts a CP-violating \( ZZ' \) mixing may be studied elsewhere \[20\].
Necessary conditions for resonant CP violation

We shall derive the necessary conditions under which CP violation can be resonantly enhanced in the \(HA\) system. We shall perform our analysis in a \(K^0\bar{K}^0\)-like basis. The results of our study may also carry over to cases involving vector-scalar and vector-vector particle mixing.

The relation between the \(K^0\bar{K}^0\) and \(HA\) bases is given by the following transformations:

\[
iA = \frac{1}{\sqrt{2}} \left( K^0 - \bar{K}^0 \right),
H = \frac{1}{\sqrt{2}} \left( K^0 + \bar{K}^0 \right),
\]

(3.1)

where \(\bar{K}^0\) is the Hermitian- and CP-conjugate state of \(K^0\). Expressing the effective Hamiltonian \(H(s)\) in Eq. (2.2) in the \(K^0\bar{K}^0\) basis, we obtain

\[
(K^{0*}, \bar{K}^{0*}) \tilde{H} \left( \begin{array}{c} K^0 \\ \bar{K}^0 \end{array} \right) =
\frac{1}{2} \left[ \begin{array}{cc}
M_H^2 + M_A^2 - \hat{\Pi}^{HH} - \hat{\Pi}^{AA} & M_H^2 - M_A^2 - \hat{\Pi}^{HH} + \hat{\Pi}^{AA} + 2i\hat{\Pi}^{AH} \\
M_H^2 - M_A^2 - \hat{\Pi}^{HH} + \hat{\Pi}^{AA} - 2i\hat{\Pi}^{AH} & M_H^2 + M_A^2 - \hat{\Pi}^{HH} - \hat{\Pi}^{AA} 
\end{array} \right],
\]

(3.2)

where dependence of the self-energies on \(s\) is implied. From Eq. (3.2), we see that \(\tilde{H}\) shares all that properties known from the kaon system. More explicitly, CPT invariance requires that

\[
\tilde{H}_{11}(s) = \tilde{H}_{22}(s),
\]

(3.3)

which holds true in Eq. (3.2), as it should. In addition, CP invariance prescribes the equality

\[
\tilde{H}_{12}(s) = \tilde{H}_{21}(s).
\]

(3.4)

This is only valid if \(\hat{\Pi}^{AH}(s) = 0\). Thus, the \(HA\) mixing gives rise to CP violation through the effective Hamiltonian (mass) matrix \(\tilde{H}(s)\).

As will be seen in Section 4, the basic parameter in the WW approximation that quantifies CP violation through indirect mixing in the kaon system [4] is given by

\[
\left| \frac{q}{p} \right|^2 = \frac{|\tilde{H}_{21}|}{|\tilde{H}_{12}|} = \left( \frac{[M_H^2 - M_A^2 - \Re(\hat{\Pi}^{HH} - \hat{\Pi}^{AA}) - 2\Im\hat{\Pi}^{HA}]^2 + [\Im(\hat{\Pi}^{HH} - \hat{\Pi}^{AA}) + 2\Re\hat{\Pi}^{HA}]^2}{[M_H^2 - M_A^2 - \Re(\hat{\Pi}^{HH} - \hat{\Pi}^{AA}) + 2\Im\hat{\Pi}^{HA}]^2 + [\Im(\hat{\Pi}^{HH} - \hat{\Pi}^{AA}) - 2\Re\hat{\Pi}^{HA}]^2} \right)^{1/2},
\]

(3.5)
which is a rephasing invariant quantity and hence physically meaningful. In the OS
renormalization scheme, $\Re(\hat{\Pi}^{HH} - \hat{\Pi}^{AA})(s)$ is a negligible term, near the resonant region
$s \approx M_H^2 \approx M_A^2$, which is formally of order $g^6$ in the coupling constant. Indeed, expanding
the self-energy $\Re\hat{\Pi}^{HH}(s)$ in a Taylor series about $M_H^2$ yields

$$\Re\hat{\Pi}^{HH}(s) = \Re\hat{\Pi}^{HH}(M_H^2) + (s - M_H^2)\Re\hat{\Pi}^{HH'}(M_H^2) + \ldots,$$

where the prime denotes differentiation with respect to the variable $s$. The first two terms on
the RHS of Eq. (3.6) vanish in the OS scheme (see also Eqs. (A.11) and (A.12) in Appendix
A). The remaining terms are of order $g^6$ and higher, because $s - M_H^2 = O(\Im\hat{\Pi}^{HH}(M_H^2)) = O(g^2)$. Likewise, one can show that $\Re\hat{\Pi}^{AA}(s) = O(g^6)$ near the resonant region of our
interest. Therefore, in what follows, we can safely neglect these high-order terms, after OS
renormalization has been carried out. It is now instructive to consider the following two
cases:

(i) The $HA$ mixing may be predicted at the tree level or induced radiatively after inte-
grating out heavy degrees of freedom, i.e., $\Re\hat{\Pi}^{HA} \neq 0$ and it is UV safe. Examples of
the kind will be discussed in Section [3]. In addition, we assume $\Im\hat{\Pi}^{HA} = 0$. Then,
in the region of interest, $M_H \approx M_A$, the CP-violating mixing parameter behaves as

$$\frac{|q/p|}{2} \sim \left| \frac{\Im(\hat{\Pi}^{HH} - \hat{\Pi}^{AA}) + 2\Re\hat{\Pi}^{HA}}{\Im(\hat{\Pi}^{HH} - \hat{\Pi}^{AA}) - 2\Re\hat{\Pi}^{HA}} \right|.$$

Evidently, if

$$\Im(\hat{\Pi}^{HH} - \hat{\Pi}^{AA}) \sim \pm 2\Re\hat{\Pi}^{HA},$$

$|q/p|$ takes either very small or very large values, giving rise to a resonant enhancement
of CP violation. Note that for large mass differences between $H$ and $A$, one has
$|q/p| \approx 1$, near the resonant region, as can be seen from Eq. (3.7).

(ii) Another interesting case and, perhaps, equivalent to (i), arises when $\Im\hat{\Pi}^{HA} \neq 0$ and
$\Re\hat{\Pi}^{HA} = 0$. This possibility may emerge from rotating or renormalizing away the
finite term $\Re\hat{\Pi}^{HA}$ in case (i). For instance, if $\Re\hat{\Pi}^{HA}$ is an $s$-independent squared
mass term and $\Im\hat{\Pi}^{HA} = 0$, one can perform an orthogonal transformation in the
Hilbert space spanned by the fields $H$ and $A$, such that $\Re\hat{\Pi}^{HA} = 0$. This newly
deﬁned basis for the fields $H$ and $A$ is usually called mass basis. In the mass basis, the
absorptive self-energy transition $\Im\hat{\Pi}^{HA}$ is in general different from zero. Note that
the transition matrix elements $T$ and $T^{res}$ in Eq. (2.1) are invariant under such basis
transformations of the intermediate scalar states and, hence, uniquely determined. For small width differences, \( \Im m \hat{\Pi}^{HH} \approx \Im m \hat{\Pi}^{AA} \), one then finds

\[
|q/p|^2 \sim \frac{|M_H^2 - M_A^2 - 2\Im m \hat{\Pi}^{HA}|}{|M_H^2 - M_A^2 + 2\Im m \hat{\Pi}^{HA}|},
\]

and the condition for resonant CP violation reads

\[
M_H^2 - M_A^2 \sim \pm 2\Im m \hat{\Pi}^{HA}.
\]

Again, we remark that large mass differences, \( M_H - M_A \), lead to small CP-violating effects through particle mixing, \( i.e., |q/p| \approx 1 \), near the region of the \( H \) and \( A \) resonances, \( e.g., \) for \( s \approx M_H^2 \) or \( s \approx M_A^2 \).

It is important to note that for maximal CP violation, \( i.e., \) of order unity, the value of \( |q/p| \) should either vanish or tend to infinity. This implies that either \( \hat{H}_{12} = 0 \) or \( \hat{H}_{21} = 0 \), but not both. As will be discussed in Section 3, this limiting case reflects the fact that the two (non-free) particles, \( H \) and \( A \), are exactly degenerate, \( i.e., \overline{M}_H = \overline{M}_A \) and \( \overline{\Gamma}_H = \overline{\Gamma}_A \), where \( M_{H,A}^2 - i\overline{M}_{H,A}\overline{\Gamma}_{H,A} \) are the two complex pole-mass eigenvalues of the effective Hamiltonian \( \hat{H}(s) (\mathcal{H}(s)) \) in Eq. (3.2) (Eq. (2.2)).

Although our derivation of the necessary conditions for resonant CP violation through particle mixing has been performed in a \( K^0 \bar{K}^0 \)-like basis, we should stress again that these results can apply equally well to any other orthogonal weak basis. In general, the necessary conditions for CP invariance within perturbation field theories may be expressed in terms of CP-odd invariants, which are flavour-basis independent. For example, there is only one such CP-odd invariant in the minimal SM [21,22]. In particular, the authors in [22] have devised a systematic approach to constructing CP-odd invariants, by making use of generalized CP transformations. Under generalized CP transformations, the fields are mapped into their CP conjugates and, on the same footing, their weak basis is changed by an orthogonal or unitary rotation, so as to leave the pure gauge sector of the underlying Lagrangian invariant. This approach has found numerous applications to many extensions of the SM, including those to multi-Higgs doublet models [23]. In Ref. [23], it has been concluded that there is no mixing-induced CP violation at the tree-level of the two-Higgs doublet models, if two Higgs particles have equal masses, \( e.g., M_H = M_A \). This result is valid in the mass basis, in which \( \Re e \hat{\Pi}^{HA} = 0 \), \( i.e., \) for the case (ii) discussed above. Indeed, our conclusions are consistent with this condition. As can be seen from Eqs. (3.3) and (3.9), we find that \( |q/p| \to 1 \), in the mass-degenerate limit \( M_H^2 \to M_A^2 \), at the resonant region \( s \approx M_H^2 \approx M_A^2 \). However, studying CP-odd invariants only is not sufficient to obtain definite theoretical predictions about the magnitude of CP violation. According to the necessary conditions
and (3.10), even a small mass splitting between $H$ and $A$ comparable to their widths can produce large CP violation. A phenomenological example of the kind will be analyzed in Section 4.

In the above discussion of CP-odd invariants, it has been very crucial to differentiate the OS-renormalized masses $M_H$ and $M_A$ present in our Hermitian Lagrangian from the mass eigenvalues $\bar{M}_H$ and $\bar{M}_A$ of the non-Hermitian effective Hamiltonian (3.2). The former correspond to states defined in an orthogonal or unitary Hilbert space, upon which our perturbation field theory is based, whereas the resulting eigenvectors of the latter are in general non-unitary or non-orthogonal among themselves. Since the afore-mentioned generalized CP transformations refer to orthogonal and/or unitary states, depending on whether the fields are real or complex, this property of orthogonality and/or unitarity must be maintained for the renormalized fields as well. Therefore, the CP-odd invariants derived with the method in [22] can only involve masses that are renormalized within a well-defined field-theoretic framework, which respects the property of unitarity, such as the OS renormalization scheme [3]. The relation between OS and pole-mass renormalization in the presence of a large particle mixing is discussed in Appendix B. Moreover, CP-odd invariants have been barely investigated beyond the Born approximation. In fact, not only the masses of particles but also the mixing matrices must be renormalized [24, 25]. Within the context of the approach in [22], it is necessary to have a renormalization scheme that consistently preserves the orthogonality or the unitarity of the weak-basis transformations order by order in perturbation theory. In Appendix A, we discuss how mixing-matrix renormalization applies to scalar theories at one loop.

4 Constraints from CPT invariance

So far, we have not taken into account constraints on transition amplitudes that may arise from CPT invariance. The fact that the underlying Lagrangian of the theory is CPT invariant gives rise to additional relations between kinematic parameters and interactions. These relations can, in principle, affect the size of the CP-violating mixing phenomenon mentioned in Section 3. In general, they have the tendency to reduce the actual magnitude of $a_{CP}$ in Eq. (2.6), even though $|q/p|$ takes its extreme value. In this section, we shall briefly illustrate how CPT restoration takes place.

Consider the transition of some initial state $i$ to a final state $f$. Then, up to an insignificant overall phase, CPT invariance requires

$$\langle f | T | i \rangle = \langle i_{CP} | T | f_{CP} \rangle,$$  \hspace{1cm} (4.1)
where the subscript CPT indicates that the states $i$ and $f$ are transformed under CPT. For some state $\beta$ with three-momentum $\vec{p}$ and spin $\vec{s}$, CPT acts as $|\beta_{\text{CPT}}(\vec{p}, \vec{s})\rangle = |\bar{\beta}(\vec{p}, -\vec{s})\rangle$, with $\bar{\beta}$ being the anti-particle of $\beta$. Equation (4.1) holds for the Hermitian and anti-Hermitian part of $T_{fi} = \langle f|\mathcal{T}|i\rangle$ independently. As a consequence of (4.1), using the optical theorem,

$$T_{fi} - T_{if}^* = i \sum_k T_{ki}T_{kf}^*,$$

(4.2)

where the sum is over all possible intermediate states $k$ including phase-space integration, and taking $|i\rangle = |f\rangle \equiv |\alpha\rangle$, one obtains the relation

$$\Im m\langle\alpha|T|\alpha\rangle = \Im m\langle\alpha_{\text{CPT}}|T|\alpha_{\text{CPT}}\rangle = m_\alpha \sum_X \Gamma(\alpha \to X),$$

(4.3)

where $m_\alpha$ is the mass of the decaying particle $\alpha$ and the summation is understood over all possible final states $X$ that $\alpha$ can decay. Equation (4.3) together with the fact that the Hamiltonian of the theory is Hermitian represent the known corollary emanating from CPT invariance, which states that the mass and lifetime of a particle is equal with that of its anti-particle.

It is now straightforward to extend our considerations to scattering processes. Analogously with Eq. (4.3), taking as asymptotic states $|i\rangle = |f\rangle = |a(\vec{p}_a, \vec{s}_a)b(\vec{p}_b, \vec{s}_b)\rangle$, and making use of the optical theorem in Eq. (4.2) for the forward scattering, viz.

$$\Im m\langle a(\vec{p}_a, \vec{s}_a)b(\vec{p}_b, \vec{s}_b)|T|a(\vec{p}_a, \vec{s}_a)b(\vec{p}_b, \vec{s}_b)\rangle = \lambda^{1/2}(s, m_a^2, m_b^2) \sum_X \sigma(a(\vec{p}_a, \vec{s}_a)b(\vec{p}_b, \vec{s}_b) \to X),$$

(4.4)

and for its CPT-conjugate counterpart, with $\lambda(x, y, z) = (x - y - z)^2 - 4yz$, we arrive at the CPT constraint involving total cross sections

$$\sum_X \sigma(a(\vec{p}_a, \vec{s}_a)b(\vec{p}_b, \vec{s}_b) \to X) = \sum_X \sigma(a(\vec{p}_a, \vec{s}_a)b(\vec{p}_b, \vec{s}_b) \to X).$$

(4.5)

Note that Eqs. (4.3) and (4.5) are still valid within the perturbation theory through the order considered.

![Fig. 3: CPT invariance in the squared transition amplitudes.](image-url)
It is now interesting to see how CPT applies to our resummation formalism. To facilitate our task, we assume that there are some asymptotic states, \( a \) and \( b \) say, that couple to \( K^0 \) only and produce it with an effective vertex \( V_K \). Then, the CPT-conjugate states, \( a_{CPT} \) and \( b_{CPT} \), will only couple to \( \bar{K}^0 \) with a production amplitude \( \bar{V}_K \). We also assume that \( a \) and \( b \) do not introduce \( \varepsilon' \) effects into the vertex \( V_K \). Subsequently, the so-produced kaons, \( K^0 \) and \( \bar{K}^0 \), decay into all possible final states \( X \) with couplings \( V_i^X \) and \( V_2^X \), respectively, as shown in Fig. 3. Taking only the dominant \( s \)-channel contributions into account, the two squared amplitudes can be cast into the form

\[
|T_K|^2 = V_K (s - \hat{H})^{-1}_{ii} V_i^X V_j^X (s - \hat{H}^\dagger)^{-1}_{jj} V_j^*,
\]

\[
|T_\bar{K}|^2 = \bar{V}_K (s - \hat{H})^{-1}_{jj} V_j^X V_\bar{j}^X (s - \hat{H}^\dagger)^{-1}_{ii} V_i^*,
\]

where summation over the repeated indices \( i, j = 1, 2 \) and integration over the phase space of the final states \( X \) must be understood. Employing the optical theorem in Eq. (4.2) at the level of the effective Hamiltonian \( \tilde{H} \), one has through the order considered

\[
(\tilde{H} - \tilde{H}^\dagger)_{ij} = -i V_i^X V_j^{X*}.
\]

Substituting Eq. (4.8) into Eqs. (4.6) and (4.7), it is not difficult to find

\[
|T_K|^2 = -2 |V_K|^2 \Im (s - \hat{H})^{-1}_{ii}, \quad |T_\bar{K}|^2 = -2 |\bar{V}_K|^2 \Im (s - \hat{H})^{-1}_{jj}.
\]

As has been assumed above, absence of CP violation in the production vertices implies that \( |V_K| = |\bar{V}_K| \). Since the effective Hamiltonian is CPT invariant, \( i.e., \hat{H}_{11} = \bar{H}_{22} \) (cf. Eq. (3.3)), one easily concludes that \( |T_K|^2 = |T_\bar{K}|^2 \), as it should be in agreement with Eq. (4.3).

It is evident that CPT symmetry yields relations among the CP-violating parts of the squared transition amplitudes similar to those found for the partial decay rates in Ref. [26]. To make this explicit, let us consider the example mentioned above and assume that the set of all final states \( X \) can be divided into two sub-sets. The first sub-set involves the states \( a, b \), \( i.e., a, b \to K^{0*} \to a, b \), while the second one, \( A \) say, does not contain the states \( a, b \). Thus, the squared amplitudes governing the reaction \( a, b \to K^{0*} \to A \) and its CP-conjugate process are given by

\[
|T_{AK}|^2 = |V_K|^2 (s - \hat{H})^{-1}_{ii} V_i^A V_j^{A*} (s - \hat{H}^\dagger)^{-1}_{jj},
\]

\[
|T_{\bar{A}K}|^2 = |\bar{V}_K|^2 (s - \hat{H})^{-1}_{jj} V_j^A V_\bar{i}^{A*} (s - \hat{H}^\dagger)^{-1}_{ii}
\]

\[
= |V_K|^2 (s - \hat{H}^T)^{-1}_{ii} V_i^A V_j^{A*} (s - \hat{H}^\dagger)^{-1}_{jj}.
\]

In the last equality of Eq. (4.11), we have assumed that CP violation is completely absent in the production and decay vertices, \( i.e., V_K = \bar{V}_K, V_i^A = \bar{V}_i^A \) and \( V_2^A = \bar{V}_1^A \), which
amounts to $\Im m \Pi^{HA} = 0$ in the $HA$ basis. In the subsequent sections, we shall see that expressions analogous to Eqs. (4.10) and (4.11) will be used in explicit calculations of CP violation. For this purpose, we introduce the following difference based on matrix elements squared:

$$\Delta_{AK} = |T_{AK}|^2 - |T_{\bar{A} \bar{K}}|^2,$$

which is a genuine CP-violating quantity. Similarly, one can define the CP-violating difference $\Delta_{KK}$ as

$$\Delta_{KK} = |T_{KK}|^2 - |T_{\bar{K} \bar{K}}|^2,$$

for the process $a, b \rightarrow K^0_s \rightarrow a, b$ and its CP-conjugate reaction. As a consequence of CPT invariance in Eq. (4.9), one has that

$$\Delta_{KK} + \Delta_{KA} = 0.$$

The equality (4.14) demonstrates explicitly how the sum of all partial CP-violating differences vanishes \cite{26} within this formalism.

Another important consequence of CPT symmetry is the decrease of CP violation, as observed by $a_{CP}$ in Eq. (2.4), despite the fact that the CP-asymmetric mixing as expressed by the parameter $|q/p|$ is maximal. To exemplify this point, we consider the effective Hamiltonian

$$\tilde{H} = \begin{pmatrix} M - i\Gamma & -2i\Gamma_{12} \\ 0 & M - i\bar{\Gamma} \end{pmatrix},$$

which corresponds to the extreme value of $|q/p| = 0$. Clearly, the entries of the effective Hamiltonian in Eq. (4.15) are given in units of energy squared. Defining as

$$iV_1^AV_1^{A*} = iV_2^AV_2^{A*} = 2i\Gamma = 2i\Gamma - |V_K|^2,$$

$$iV_1^AV_2^{A*} = iV_2^AV_1^{A*} = 2i\Gamma_{12} = 2i\Gamma_{12} - iV_K \bar{V_K}^*,$$

we find at energies $s \approx M$ that for the reaction $a, b \rightarrow K^0_s \rightarrow A$, $a_{CP}$ takes the simple form

$$a_{CP} = \frac{\Delta_{AK}}{|T_{AK}|^2 + |T_{\bar{A} \bar{K}}|^2} = \frac{2\Gamma_{12}(\Gamma_{12} - \Gamma_{12})}{\Gamma^2 + 2\Gamma_{12}(\Gamma_{12} - \Gamma_{12})}.$$

From Eq. (4.17), one can readily see that $a_{CP}$ vanishes if $A$ represents the sum over all final states $X$, as a result of CPT. Furthermore, it is obvious that the value of $a_{CP}$ is generally smaller than unity even in the extreme particle-mixing limit. In fact, $a_{CP}$ can take its maximum value only in the limit $\Gamma \rightarrow 0$, with $\Gamma_{12} \neq 0$. In such a case, the process $\bar{a}\bar{b} \rightarrow \bar{K}^0_s \rightarrow \bar{A}$ is forbidden, whereas the transition $ab \rightarrow K^0_s \rightarrow A$ is allowed. Hence, we can conclude that our resummation formalism respects the CPT symmetry and takes automatically account of all possible CPT constraints.
5 Effective Hamiltonian approach

We shall briefly review the main features of the effective Hamiltonian approach, which was formulated by Weisskopf and Wigner \cite{3} and applied to describe CP violation in the kaon system by Lee, Oehme and Yang \cite{4}. Our approach presented in Sections 2 and 3 is equivalent with that based on diagonalizing the effective Hamiltonian \cite{27}. In this section, we shall pay special attention to the limitations of the latter that may arise from describing CP violation in a system where the two mixed particles become degenerate. In fact, we will show that there exist extreme cases, for which the effective Hamiltonian cannot be diagonalized via a similarity transformation. Such non-diagonalizable effective Hamiltonians represent situations, in which the mixing-induced CP violation reaches its maximum allowed value and the complex mass eigenvalues of the effective Hamiltonian are exactly degenerate. In addition, we shall show that non-diagonalizable effective Hamiltonians are admissible forms, for which the Lee-Wolfenstein \cite{28} inequality due to unitarity is saturated. In this context, we find that our resummation approach based on transition amplitudes constitutes a self-consistent field-theoretic framework for dealing even with such singular cases.

To elucidate our points, we shall consider the $HA$ system from the viewpoint of the effective Hamiltonian formalism. In the WW approximation, the time evolution of the $HA$ system can be described by means of an effective Schrödinger equation, in which the Hamilton operator is a $2 \times 2$ non-Hermitian matrix \cite{4}. Taking dispersive and absorptive quantum effects into account, this effective Hamiltonian is given by

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} M_{11} - i\Gamma_{11} & M_{12} - i\Gamma_{12} \\ M_{12}^* - i\Gamma_{22} & M_{22} - i\Gamma_{22} \end{bmatrix}, \quad (5.1)$$

where the index 1 (2) refers to $A$ ($H$) and all entries of $H$ are given in units of energy squared. As we have seen in Section 2, the effective Hamiltonian in Eq. (5.1) may be approximated by the matrix $H(s)$ defined in Eq. (2.2), for $s \approx M_H^2 \approx M_A^2$. In the last equality of Eq. (5.1), we have decomposed the non-Hermitian Hamiltonian in terms of two Hermitian matrices as follows:

$$M_{ij} = \frac{1}{2} (H + H^\dagger), \quad \Gamma_{ij} = \frac{i}{2} (H - H^\dagger). \quad (5.2)$$

Furthermore, CPT invariance requires that $|H_{12}| = |H_{21}|$ for the case of neutral-scalar mixing, which in turn implies that $\Im(m(M_{12}\Gamma_{12})) = 0$. Since $M_{12}$ is real as being the dispersive part of transitions among real scalars (see also Appendix A), $\Gamma_{12}$ should be a real function as well, as a consequence of CPT invariance. However, we must caution the reader that this CPT constraint does not apply for mixing between charged scalars.
Assuming the existence of a non-unitary matrix $X$, $H$ can be brought into a diagonal form through a similarity transformation

$$XHX^{-1} = \begin{pmatrix} \overline{M}_{A}^{2} - i\overline{\Gamma}_{A}\overline{M}_{A} & 0 \\ 0 & \overline{M}_{H}^{2} - i\overline{\Gamma}_{H}\overline{M}_{H} \end{pmatrix},$$

where the two complex mass eigenvalues (or the two pole masses of the transition amplitude $\mathcal{F}$) are given by

$$\overline{M}_{A}^{2} - i\overline{\Gamma}_{A}\overline{M}_{A} = \frac{1}{2}(H_{11} + H_{22} - \Delta),$$

$$\overline{M}_{H}^{2} - i\overline{\Gamma}_{H}\overline{M}_{H} = \frac{1}{2}(H_{11} + H_{22} + \Delta),$$

where $\Delta^{2} = (H_{11} - H_{22})^{2} + 4H_{21}H_{12}$ and the square root is taken on the first sheet. Clearly, the masses $\overline{M}_{H}$ and $\overline{M}_{A}$ differ from those that the $H$ and $A$ particles would have if they were free, i.e., if all couplings were switched off. The matrices $X$ and $X^{-1}$ that diagonalize $H$ have the explicit forms:

$$X = \left(\frac{\Delta - H_{11} + H_{22}}{2\Delta}\right)^{1/2} \begin{pmatrix} 1 & -\frac{1}{2H_{21}}(H_{11} - H_{22} + \Delta) \\ \frac{1}{2H_{21}}(H_{11} - H_{22} + \Delta) & 1 \end{pmatrix},$$

$$X^{-1} = \left(\frac{\Delta - H_{11} + H_{22}}{2\Delta}\right)^{1/2} \begin{pmatrix} 1 & \frac{1}{2H_{21}}(H_{11} - H_{22} + \Delta) \\ -\frac{1}{2H_{21}}(H_{11} - H_{22} + \Delta) & 1 \end{pmatrix}.$$

From Eqs. (5.7) and (5.6), one may easily deduce that the matrices $X$ or $X^{-1}$ do exist unless both $\Delta = 0$ and the off-diagonal elements $H_{12}, H_{21}$ are different from zero. Of course, if $H_{12} = H_{21} = 0$, $X$ becomes the unity matrix. However, one could think of an effective Hamiltonian with $\Delta = 0$ and $H_{12} = H_{21} \neq 0$, for which $X$ becomes singular, e.g.

$$\mathcal{H} = \begin{pmatrix} A - ib & -b/2 \\ -b/2 & A - 2ib \end{pmatrix},$$

where $A$ and $b$ are some positive numbers. Most importantly, the two pole-mass eigenvalues, for $H$ and $A$ say, are exactly equal, since $\Delta = 0$ in Eq. (5.4). As a consequence, the two non-free particles $H$ and $A$ are exactly degenerate. As has been found in [16] and will be further analyzed in Section 4, such a singular case can occur in two-Higgs doublet models, when one studies resonant CP violation due to a $HA$ mixing. For example, if the Higgs self-energies in Eq. (2.2) satisfy the relation: $b = \Im m\tilde{\Pi}^{AA} = \frac{1}{2}\Im m\tilde{\Pi}^{HH} = 2\tilde{\Pi}^{HA}$ for $s = M_{H}^{2} = M_{A}^{2}$, this gives rise to the non-diagonalizable effective Hamiltonian of the form (5.7).

Apart from the difficulty of diagonalizing $\mathcal{H}$ for a degenerate system of bound states, one may now raise the following question: What is the actual magnitude of CP violation
through particle mixing in such a degenerate case? To successfully address this question, one should perform the above analysis in a $\bar{K}_0 - K_0$-like basis. Taking the basis rotations in Eq. (3.1) into account, we obtain the new effective Hamiltonian $\tilde{H}$ (equivalent to Eq. (3.2)) and the respective non-unitary matrix $\tilde{X}$

$$\tilde{H} = \frac{1}{2} \begin{pmatrix}
H_{11} + H_{22} & H_{22} - H_{11} + 2iH_{12} \\
H_{22} - H_{11} - 2iH_{12} & H_{11} + H_{22}
\end{pmatrix}, \quad \tilde{X} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -p/q \\
q/p & 1
\end{pmatrix},$$

where $q/p$ is the basic parameter, which determines the CP-violating phenomenon induced by the CP-asymmetric mixing of $K^0$ with its CP-conjugate state, $\bar{K}_0$ [2]. This parameter is given by

$$q/p = \left( \frac{\tilde{H}_{21}}{\tilde{H}_{12}} \right)^{1/2}. \quad (5.9)$$

In the effective Hamiltonian formalism, the two (right) mass eigenstates, $|K_L\rangle$ and $|K_S\rangle$, are then expressed in terms of the strong states, $K^0$ and $\bar{K}_0$, as

$$|K_L\rangle = q|K^0\rangle - p|\bar{K}^0\rangle \quad \text{and} \quad |K_S\rangle = q|K^0\rangle + p|\bar{K}^0\rangle,$$

with the normalization $|p|^2 + |q|^2 = 1$. In this phase convention, the CP-violating mixing parameter $\varepsilon = \langle K_L|K_S\rangle$ is a real number. Obviously, the anomalously degenerate $HA$ or $K^0\bar{K}^0$ system mentioned above, which leads to a singular $\tilde{X}$, corresponds to the case where either $\tilde{H}_{21}$ or $\tilde{H}_{12}$ vanish but not both of them, i.e., the effective Hamiltonian $\tilde{H}$ takes mathematically the Jordan form [29]. In such a case, $q/p$ is either zero or tends to infinity, which shows explicitly that CP violation through particle mixing takes its maximum attainable value. Apart from CPT constraints, CP violation could be of order unity, when the non-free $H$ and $A$ states are exactly degenerate.

It is now important to investigate whether the non-diagonalizable Jordan form of the effective Hamiltonian satisfies unitarity. Using the optical theorem in Eq. (4.2) for a general effective Hamiltonian $\tilde{H}$, one can easily obtain the Bell-Steinberger unitarity relation [30]

$$\langle K_L|(\tilde{H} - \tilde{H}^\dagger)|K_S\rangle = [\bar{M}_H^2 - \bar{M}_A^2 - i(\bar{M}_A\bar{\Gamma}_A + \bar{M}_H\bar{\Gamma}_H)] \langle K_L|K_S\rangle = i \sum_\xi \langle \xi| T |K_L\rangle^* \langle \xi| T |K_S\rangle,$$

where $\xi$ represents all intermediate states that $K_L$ and $K_S$ can decay. In this notation, $K_L$ and $K_S$ formally denote the two non-free $A$ and $H$ mass-eigenstates of $\tilde{H}$, respectively. Employing Schwartz’s inequality

$$\left| \sum_\xi \langle \xi| T |K_L\rangle^* \langle \xi| T |K_S\rangle \right|^2 \leq \left| \sum_m \langle m| T |K_L\rangle \right|^2 \left| \sum_n \langle n| T |K_S\rangle \right|^2,$$

*Eq. (5.10) differs from the standard convention, since we have defined $CP(K^0) = \bar{K}_0$ instead of $-\bar{K}_0$ and assigned the two pole-mass eigenstates $K_L$ and $K_S$ in the reversed order.
and the fact that \( \sum_\xi |\langle \xi | T | X \rangle|^2 = 2 M_X \Gamma_X \), for \( X = K_L, K_S \), we find the following inequality:

\[
|\varepsilon|^2 = |\langle K_L | K_S \rangle|^2 \leq \frac{4 M_H \Gamma_H M_A \Gamma_A}{(M_H^2 - M_A^2)^2 + (M_H \Gamma_H + M_A \Gamma_A)^2},
\]

which is known as the Lee-Wolfenstein inequality [28]. The relation between the CP-violating mixing parameters \( \varepsilon \) and \( |q/p| \) is given by \( |q/p|^2 = (1 + \varepsilon)/(1 - \varepsilon) \). Clearly, the non-diagonalizable Jordan form of \( \mathcal{H} \) corresponds to the values of \( \varepsilon = \pm 1 \). For the extreme case of exact mass degeneracy, \( i.e., M_H = M_A \) and \( \Gamma_H = \Gamma_A \), we obtain the unitarity restriction \( |\varepsilon|^2 \leq 1 \), as is derived from Eq. (5.13). It is obvious that the Lee-Wolfenstein unitarity bound gets saturated in the non-diagonalizable limit of the effective Hamiltonian \( \mathcal{H} \).

Our resummation approach is free from the kind of pathological singularities mentioned above, which arise whenever one attempts to diagonalize the effective Hamiltonian \( \mathcal{H} \) or the inverse propagator \( \hat{\Delta}^{-1}(s) \) in Eq. (2.2) [4] for an anomalously degenerate system. Since we always consider physical transition amplitudes, in which the resummed propagators are sandwiched between the matrix elements related to the initial and final states of the process, the diagonalization of the propagator \( \hat{\Delta}(s) \) is no more necessary. Thus, our scattering-amplitude approach is reminiscent of the known density matrix formalism [31]. In fact, as the two non-free particles approach the anomalously degenerate limit, the mass eigenvectors obtained from \( \mathcal{H} \) or \( \hat{\mathcal{H}} \), such as \( |K_L\rangle \) and \( |K_S\rangle \) as well as their dual (left) eigenvectors [4,32], may not bear any physical meaning [29]. Only the complex pole positions of a transition amplitude can be considered as physical (observable) quantities. Of course, far away from the anomalous situation, the traditional description of diagonalizing the effective Hamiltonian \( \mathcal{H} \) can equally well represent the known experimental data, namely, CP violation in the ordinary \( K^0\bar{K}^0 \) system [2].

### 6 HA mixing in two-Higgs doublet models

New-physics scenarios that could predict a potentially large \( HA \) mixing should have two ingredients: First, they should extend the field content of the SM by an extra Higgs doublet and, second, they must violate the CP symmetry of the Higgs sector. There are two representative paths that can lead to a CP-violating \( HA \) term. The first possibility is to produce such a term at the tree-level. We shall see that this purpose can naturally be achieved within a two-Higgs doublet model, in which the CP invariance of the Higgs potential is explicitly broken by adding soft mass terms. Another alternative is to induce a \( HA \) mixing radiatively after integrating out heavy degrees of freedom. These heavy degrees
of freedom will explicitly violate the CP symmetry of the Lagrangian. As such, one may think of heavy Majorana fermions, such as heavy Majorana neutrinos or neutralinos in the minimal supersymmetric SM (MSSM), that couple to $H$ and $A$ scalars with both scalar and pseudo-scalar couplings. In the following, we shall determine the size of the CP-violating $HA$ term in the two models mentioned above.

(i) $HA$ mixing in the two-Higgs doublet model with an explicitly CP-violating Higgs potential. First, we shall briefly describe the basic structure of this minimally extended model, which may be considered as the simplest realization of CP violation in the neutral Higgs sector. The presence of two-Higgs doublets, $\Phi_1$ and $\Phi_2$, in the Lagrangian can give rise to large flavour-changing neutral currents (FCNC) in the Higgs coupling to fermions. In order to avoid FCNC at the tree level, one usually imposes the discrete symmetry $D$: $\Phi_1 \rightarrow -\Phi_1$, $\Phi_2 \rightarrow \Phi_2$ and $d_i^{\prime R} \rightarrow -d_i^{\prime R}$, where $d_i^{\prime R}$ are the right-handed down-type quarks in the flavour basis. Invariance under the discrete symmetry $D$ entails that $\Phi_1$ will couple to down-type quarks and $\Phi_2$ to up-type quarks only, thus leading to diagonal couplings of the neutral Higgs particles to quarks [33,34]. Furthermore, imposing the $D$ symmetry on the Higgs potential leads to a CP-invariant two-Higgs doublet model. The model predicts four neutral scalar fields: the three massive fields $h$, $H$, $A$ and the massless would-be Goldstone boson of the $Z$ boson, $G^0$. In the $D$-symmetric limit, the neutral Higgs scalars $h$ and $H$ are exactly CP even, whereas the third neutral Higgs particle, $A$, is CP odd. The Lagrangians describing the interactions of the neutral Higgs scalars to quarks are given by

$$L_{G^0} = \frac{ig_m f}{2M_W} G^0 \bar{f} T_z^f \gamma_5 f,$$
$$L_A = \frac{ig}{2M_W} A \left[ \cot \beta m_u \bar{u}_i u_i + \tan \beta m_d \bar{d}_i \gamma_5 d_i \right],$$
$$L_{H,h} = -\frac{g}{2M_W} \left[ (h \chi_h^u + H \chi_H^u) m_u \bar{u}_i u_i + (h \chi_h^d + H \chi_H^d) m_d \bar{d}_i d_i \right],$$

with

$$\chi_h^u = -\frac{\sin \theta}{\sin \beta}, \quad \chi_H^u = \frac{\cos \theta}{\sin \beta}, \quad \chi_h^d = \frac{\cos \theta}{\cos \beta}, \quad \chi_H^d = \frac{\sin \theta}{\cos \beta}.$$  

Here, $g$ is the weak coupling constant, $\tan \beta = v_2/v_1$ is the ratio of the $\Phi_2$ to $\Phi_1$ vacuum expectation values (VEV’s), the angle $\theta$ relates the weak states to the physical states $h$ and $H$, and $T_z^f$ is the $z$-component of the weak isospin of the fermion $f$ ($T_z^u = 1$, $T_z^d = -1$). The absence of FCNC in the Higgs couplings can easily be extended to the lepton sector, by requiring in addition to symmetry $D$ that $l_i^{\prime R} \rightarrow -l_i^{\prime R}$, where $l_i^{\prime R}$ are the right-handed charged leptons. The interaction Lagrangians for the Higgs particles with the charged leptons $l_i = e$, $\mu$ and $\tau$ may then be obtained from Eqs. (6.1)–(6.3), after making the obvious replacements: $d \rightarrow l$ and $m_{d_i} \rightarrow m_{l_i}$.
Up to now, CP has been a good symmetry of the whole Lagrangian provided the discrete symmetry D remains unbroken. In order to introduce CP violation in the theory, we must break the symmetry D in some part of the Lagrangian. The most convenient way is to write down soft-breaking mass terms in the Higgs sector that violate the symmetry D explicitly [35]. In this way, the Higgs potential $V_H$ may be decomposed into two terms as follows:

$$V_H(\Phi_1, \Phi_2) = V_D(\Phi_1, \Phi_2) + \Delta V(\Phi_1, \Phi_2),$$

(6.5)

where $V_D$ respects the D symmetry and has the general Hermitian form

$$V_D(\Phi_1, \Phi_2) = \mu_1^2(\Phi_1^\dagger \Phi_1) + \mu_2^2(\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2(\Phi_2^\dagger \Phi_2)^2$$

$$+ \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \frac{1}{2} \lambda_5(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_6^*(\Phi_2^\dagger \Phi_1)^2,$$

(6.6)

whereas $\Delta V$,

$$\Delta V(\Phi_1, \Phi_2) = \lambda_6(\Phi_1^\dagger \Phi_2) + \lambda_6^*(\Phi_2^\dagger \Phi_1),$$

(6.7)

violates the symmetry D softly. Invariance of $V_H$ under CP is only reassured if

$$\Im m(\lambda_5 \lambda_6^*) = 0.$$  

(6.8)

After SSB, the VEV’s of $\Phi_1$ and $\Phi_2$, $v_1$ and $v_2$, are generally complex relative to one another, i.e., $\Im m(v_1 v_2) \neq 0$, if CP is not conserved. Nevertheless, using the freedom of the phase redefinitions, $\Phi_i \rightarrow e^{i \alpha_i} \Phi_i$, in $V_H$, we can always make $v_1$ and $v_2$ real at the cost of both $\lambda_5$ and $\lambda_6$ being complex. As a result, any CP-noninvariant term will be proportional either to $\Im m(\lambda_5 \lambda_6^*)$ or $\Im m \lambda_5$. The latter can, however, be expressed in terms of the former by making use of the conditions obtained from minimizing the Higgs potential $V_H$. This D-broken two-Higgs doublet model predicts CP-violating mass terms, such as $H A$ and $h A$, already at the tree level. Specifically, we find

$$\Pi^{HA} = \frac{1}{\kappa} \Im m(\lambda_5 \lambda_6^*) (\cos \theta \sin \beta - \sin \theta \cos \beta),$$

(6.9)

$$\Pi^{hA} = - \frac{1}{\kappa} \Im m(\lambda_5 \lambda_6^*) (\sin \theta \sin \beta + \cos \theta \cos \beta),$$

(6.10)

where $\kappa$ is some squared mass combination which depends entirely on the VEV’s $v_1$, $v_2$, and $\Re \lambda_5$ and $\Re \lambda_6$. In our analysis, we shall treat the CP-violating squared mass terms, $\Pi^{HA}$ and $\Pi^{hA}$, as small parameters compared to the Higgs particle masses, i.e. $\Pi^{HA}/M_H^2 \ll 1$ and $\Pi^{hA}/M_h^2 \ll 1$. In Section 7, we will find that this mass pattern is also compatible with experimental upper bounds on EDM’s.

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(ii) HA mixing in a two-Higgs doublet model with heavy Majorana neutrinos. In this two-Higgs doublet model, the discrete symmetry D is broken explicitly by the Majorana terms of two isosinglet neutrinos, call them $S_{1R}$ and $S_{2R}$. In addition, we introduce a sequential weak isodoublet in the model, $(\nu_4, E_L)$. To avoid possible phenomenological limits coming from the presently observed sector, we assume the complete absence of inter-family mixings with the three light generations. The Yukawa sector containing the neutrino mass matrix $M^\nu$ of our CP-violating scenario reads:

$$- \mathcal{L}^\nu_Y = \frac{1}{2} \begin{pmatrix} \tilde{\nu}_4 L, (S_{1R})^C, (S_{2R})^C \end{pmatrix} \begin{bmatrix} 0 & a & b \\ a & A & 0 \\ b & 0 & B \end{bmatrix} \begin{pmatrix} (\nu_4 L)^C \\ S_{1R} \\ S_{2R} \end{pmatrix} + \text{H.c.} \quad (6.11)$$

In Eq. (6.11), the parameters $A$ and $B$ can always be chosen to be real, whereas $a$ and $b$ are in general complex. CP is violated only if the rephasing invariant quantity $\Im m(ab^*) \neq 0$, which is only possible with the presence of two heavy singlets in the model. The symmetric mass matrix in Eq. (6.11) can be diagonalized by the unitary transformation:

$$U^T M^\nu U = \hat{M}^\nu,$$

where $\hat{M}^\nu$ is a diagonal matrix containing the physical heavy neutrino masses, $m_i$ $(i = 1, 2, 3)$. From the three heavy Majorana neutrinos, denoted by $N_1$, $N_2$, and $N_3$, that the model predicts, $N_1$ is predominantly a SU(2)$_L$ isodoublet, and $N_2$ and $N_3$ are mainly singlets in the limit of $A, B \gg a, b$. Moreover, the Lagrangians governing the interactions between $N_i$ and $h, H, A$ are given by [36]:

$$\mathcal{L}_A = \frac{ig}{4M_W} A \chi_A^u \sum_{i,j=1}^{3} \bar{N}_i \left[ \gamma_5 (m_i + m_j) \Re C_{ij} + i(m_j - m_i) \Im m C_{ij} \right] N_j, \quad (6.12)$$

$$\mathcal{L}_{h,H} = - \frac{g}{4M_W} (h \chi_h^u + H \chi_H^u) \times \sum_{i,j=1}^{3} \bar{N}_i \left[ (m_i + m_j) \Re C_{ij} + i\gamma_5 (m_j - m_i) \Im m C_{ij} \right] N_j, \quad (6.13)$$

where $\chi_A^u = \cot \beta$, $\chi_h^u$ and $\chi_H^u$ are given in Eq. (6.4), and $C_{ij}$ is a $3 \times 3$ mixing matrix defined as $C_{ij} \equiv U_{1i} U_{1j}^*$. Thus, the rephasing invariant and CP-violating quantity mentioned above may equivalently be expressed as

$$\Im m C_{12}^2 = \sin \delta_{CP} |C_{12}|^2. \quad (6.14)$$

We shall assume that $\Im m C_{12}^2$ in Eq. (6.14) takes the maximum possible value, i.e., it is of order one. Further details on that model may be found in [37]. Note that a CP-violating HA mixing may also be induced within the MSSM, in which neutralinos and charginos may assume the rôle of heavy Majorana neutrinos. In the MSSM, the SU(2)$_L \times U(1)_Y$-singlet bilinear term $\mu$ and the tri-linear soft-SUSY-breaking couplings $A$ may contain non-trivial
CP-violating phases, which lead to complex chargino- and neutralino-mass matrices \[38\]. Consequently, interaction Lagrangians of the form given in Eqs. (6.12) and (6.13) may naturally occur in the MSSM.

\[
\begin{align*}
\hat{\Pi}_{AH}(s) &= \frac{\alpha_w}{4\pi} \chi_A^u \chi_H^u \sum_{j>i} 3 m_c^2 \sqrt{\lambda_i \lambda_j} \left[ B_0(s/M_W^2, \lambda_i, \lambda_j) + 2B_1(s/M_W^2, \lambda_i, \lambda_j) \right], \quad (6.15)
\end{align*}
\]

where \(\hat{\Pi}_{AH}(s) = \hat{\Pi}_{HA}(s)\), \(\lambda_i = m_i^2/M_W^2\), and \(B_0\) and \(B_1\) are the usual Veltman-Passarino loop functions, evaluated in the conventions of Ref. \[39\]. The transition \(G^0H\) is easily recovered from Eq. (6.15) by setting \(\chi_A^u = 1\). Correspondingly, the other CP-violating transitions, \(G^0h\) and \(Ah\), may be obtained by replacing \(\chi_H^u\) with \(\chi_H^u\) in Eq. (6.13). All these CP-violating self-energies are UV finite, and hence, do not require renormalization (see also Appendix A on that matter).

Since there is an established equivalence of the one-loop PT \(n\)-point correlation functions with those obtained in the background field method for the gauge-fixing-parameter value \(\xi_Q = 1\) \[10,11\], we shall calculate the one-loop PT self-energies, using the latter method. Furthermore, UV divergences occurring in the dispersive parts of the self-energies are absorbed by mass and wave-function renormalization constants. However, near the resonant region, \(s \approx M_H^2 \approx M_A^2\), OS renormalization renders the dispersive self-energies, \(\text{Re}\hat{\Pi}_{HH}(s)\) and \(\text{Re}\hat{\Pi}_{AA}(s)\), very small of order \(g^6\) (cf. Eq. (3.6)), so that these high-order terms can be neglected. Therefore, only one-loop absorptive \(HH\) and \(AA\) self-energies are of interest here. Such an approach may be viewed as an improved Born approximation. It is then straightforward to obtain for the different decay modes

\[
\Im m\hat{\Pi}_{(ff)}^{HH}(s) = \frac{\alpha_w N_c}{8} (\lambda_H^f)^2 s \frac{m^2_f}{M_W^2} \left( 1 - \frac{4m^2_f}{s} \right)^{3/2} \theta(s - 4m^2_f), \quad (6.16)
\]

As shown in Fig. 4, the \(HA\) mixing may be induced radiatively by heavy Majorana neutrino loops. Taking the Lagrangians (6.12) and (6.13) into account, we calculate the CP-violating \(HA\) mixing in our two-Higgs doublet model, \textit{viz}.

\[
\begin{align*}
\hat{\Pi}_{AH}(s) &= -\frac{\alpha_w}{4\pi} \chi_A^u \chi_H^u \sum_{j>i} 3 m_c^2 \sqrt{\lambda_i \lambda_j} \left[ B_0(s/M_W^2, \lambda_i, \lambda_j) + 2B_1(s/M_W^2, \lambda_i, \lambda_j) \right], \quad (6.15)
\end{align*}
\]
\( \Im \hat{\Pi}^{AA}_{(ff)}(s) = \frac{\alpha_w N_c}{8} (\chi_{A}^f)^2 s \frac{m_f^2}{M_W^2} \left( 1 - \frac{4m_f^2}{s} \right)^{1/2} \theta(s - 4m_f^2), \)  
(6.17)

\( \Im \hat{\Pi}^{HH}_{(VV)}(s) = \frac{n_V \alpha_w}{32} (\chi_{W}^V)^2 \frac{M_H^2}{M_W^2} \left( 1 - \frac{4M_V^2}{s} \right)^{1/2} \times \left[ 1 + \frac{4M_V^2}{M_H^2} - \frac{4M_V^2}{M_H^2}(2s - 3M_V^2) \right] \theta(s - 4M_V^2). \)  
(6.18)

Here, \( \alpha_w = g^2/4\pi, n_V = 2, 1 \) for \( V \equiv W, Z, \) respectively, and \( N_c^f = 1 \) for leptons and 3 for quarks. In Eqs. (6.16)–(6.18), \( \chi_{H,A}^f \) are the model-dependent factors in Eq. (6.4), and \( \chi_{W,V}^H,H \) are similar factors that multiply the couplings \( HWW \) and \( HZZ \) of the SM.

The explicit dependence of \( \chi_{W,H}^V \) on the angles \( \theta \) and \( \beta \) is given by

\[
\chi_{W}^H = \cos \theta \cos \beta + \sin \theta \sin \beta, \quad \chi_{W}^V = -\sin \theta \cos \beta + \cos \theta \sin \beta. 
\]  
(6.19)

In addition, one has \( \chi_{V}^A = 0 \) because of CP invariance. Thus, only fermions can contribute to \( \Im \hat{\Pi}^{AA}(s) \). Other channels involving the \( HZA \) vertex may also give contributions to the absorptive self-energies. However, for the kinematic range of our interest, \( M_H \simeq M_A \), relevant for resonant CP violation, these absorptive channels are considered to be phase-space suppressed and hence have not been taken into account.

## 7 Resonant CP violation through HA mixing

In this section, we shall study CP-violating phenomena which are induced by resonant transitions of a CP-even Higgs particle, \( H \), into a CP-odd Higgs scalar, \( A \). The importance of the resonant enhancement of CP violation through \( HA \) particle mixing has been addressed in an earlier communication [10]. Here, we shall present further details of the calculation and analyze possible low-energy constraints on \( HA \) mixing, such as bounds coming from electric dipole moments (EDM’s) of neutron, electron and muon.

The most ideal place to look for resonant CP-violating \( HA \) transitions is at \( e^+e^- \) and, most interestingly, at muon colliders [12]. In general, there are many observables suggested at high-energy colliders [13,14,15,16,17,18] that may be formed to project out different CP/T-noninvariant contributions. All the CP-violating observables, however, may fall into two categories, depending on whether they are even or odd under naive CPT transformations. For instance, typical CP-odd and CPT-even observables in a process, such as \( p\bar{p} \rightarrow t\bar{t}X \) [13,14] or \( e^+e^- \rightarrow t\bar{t}X \) [14], are triple-product correlations of the type, e.g., \( \langle k_p \cdot k_{\bar{t}} \times k_{\bar{t}} \rangle \) or \( \langle k_e \cdot k_{\bar{t}} \times k_{\bar{t}} \rangle \), based on the three-momenta of the initial beam particles \( p \) or \( e \) and top quarks in the final state. In this class of observables, CP violation may
occur already in the Born approximation \cite{43,44,45}. The other category comprises CP- and CPT-odd observables of the form, $\langle \vec{s}_t \vec{k}_t \rangle$ or $\langle \vec{s}_{\bar{t}} \vec{k}_{\bar{t}} \rangle$ \cite{34,47,49}, where $\vec{s}_t$ and $\vec{k}_t$ are respectively the spin and the three-momentum of the top-quark in the $t\bar{t}$ centre of mass (c.m.) system. These observables, being odd under naive CPT transformations, can only combine with absorptive loops, which are also naively odd under CPT, to produce a real and CPT-invariant contribution to the matrix element squared.

Our quantitative analysis of $HA$ mixing phenomena will rely on CP-violating quantities of the second class, mentioned above. For definiteness, assuming that having longitudinally polarized muon beams will be feasible without much loss of luminosity, we shall consider the CP asymmetry \cite{50}

$$A_{CP}^{(\mu)} = \frac{\sigma(\mu_L^+ \mu_L^- \rightarrow \bar{f} \bar{f}) - \sigma(\mu_R^+ \mu_R^- \rightarrow \bar{f} \bar{f})}{\sigma(\mu_L^+ \mu_L^- \rightarrow \bar{f} \bar{f}) + \sigma(\mu_R^+ \mu_R^- \rightarrow \bar{f} \bar{f})}.$$  \hspace{1cm} (7.1)

If one is able to tag on the final fermion pair $\bar{f} \bar{f}$ (e.g., $\tau^+ \tau^-$, $b\bar{b}$, or $t\bar{t}$), $A_{CP}^{(\mu)}$ is then a genuine observable of CP violation, as the helicity states $\mu_L^+ \mu_L^-$ transform into $\mu_R^+ \mu_R^-$ under CP in the c.m. system. Similarly, at $e^+e^-$ or $p\bar{p}$ machines, one can define the CP asymmetry

$$A_{CP}^{(e)} = \frac{\sigma(e^- e^+ \rightarrow \bar{f} \bar{f} \bar{L} X) - \sigma(e^{-} e^{+} \rightarrow \bar{f} \bar{f} \bar{R} X)}{\sigma(e^{-} e^{+} \rightarrow \bar{f} \bar{f} \bar{L} X) + \sigma(e^{-} e^{+} \rightarrow \bar{f} \bar{f} \bar{R} X)},$$  \hspace{1cm} (7.2)

and an analogous observable $A_{CP}^{(p)}$ involving $p\bar{p}$ beams. In Eq. (7.2), the chirality of fermions, such as the top quark, may not be directly observed. However, the decay characteristics of a left-handed top quark differ substantially from those of its right-handed component, giving rise to distinct angular-momentum distributions and energy asymmetries of the produced charged leptons and jets \cite{47,48}.

Since our main interest is in resonant $HA$ or $hA$ transitions, we shall assume for simplicity that only one CP-even Higgs particle, $H$, say, has a mass quite close to $A$, i.e., $M_H - M_A \ll M_H, M_A$, while the other CP-even Higgs, $h$, is much lighter than $H, A$, and vice versa. As has been noted in Section 3, $h$ will effectively decouple from the mixing system, having negligible contributions to both cross section and CP asymmetry at c.m. energies $s \approx M_H, M_A$. This is also the main reason accounting for the fact that CP violation through $HZ \ (C^0H)$ mixing has been found to be small \cite{16}, as $H$ only couples to the longitudinal component of the $Z$ boson, the massless would-be Goldstone $G^0$. To give an estimate, in the two-Higgs model with heavy Majorana neutrinos, we find that $A_{CP}^{(e)} \approx 2 \times 10^{-2}$ for $M_H = 500$ GeV and $m_{1,2,3} = 0.5, 1.5, 3$ TeV, while the production cross-section is $\sigma \simeq 1$ fb. It is therefore unlikely to observe $HZ$-mixing effects, even if one assumes a high integrated luminosity of 50 fb$^{-1}$, designed for $e^+e^-$ and $\mu^+\mu^-$ colliders.
We shall now focus our attention on the resonant transition amplitudes \( \mathcal{T}_L(\mu_L^+ \mu_L^- \to H^*, A^* \to f\bar{f}) \) and \( \mathcal{T}_R(\mu_R^+ \mu_R^- \to H^*, A^* \to f\bar{f}) \). Therefore, it will prove useful to further decompose these amplitudes as follows:

\[
\begin{align*}
\mathcal{T}_L & = \mathcal{T}_L^H(\mu_L^+ \mu_L^- \to H^*H^*, A^*H^* \to f\bar{f}) + \mathcal{T}_L^A(\mu_L^+ \mu_L^- \to A^*A^*, H^*A^* \to f\bar{f}), \\
\mathcal{T}_R & = \mathcal{T}_R^H(\mu_R^+ \mu_R^- \to H^*H^*, A^*H^* \to f\bar{f}) + \mathcal{T}_R^A(\mu_R^+ \mu_R^- \to A^*A^*, H^*A^* \to f\bar{f}).
\end{align*}
\] (7.3)

In Eq. (7.3), the individual amplitudes \( \mathcal{T}_{L,R}^H, \mathcal{T}_{L,R}^A \) are uniquely specified by the tree-level coupling of the virtual Higgs particle \( H(A) \) to the fermions \( f \) in the final state. In this way, we can evaluate the following CP-violating differences of the matrix elements squared:

\[
\Delta \mathcal{T}^A = |\mathcal{T}_L^A|^2 - |\mathcal{T}_R^A|^2 = -4r_d \Im \frac{\Pi^{HH}(s)}{\Pi^{AH}(s)} |\Delta_{AH}(s)|^2 |\mathcal{M}(A^* \to \mu^- \mu^+_L)|^2 |\mathcal{M}(A^* \to f\bar{f})|^2,
\] (7.4)

\[
\Delta \mathcal{T}^H = |\mathcal{T}_L^H|^2 - |\mathcal{T}_R^H|^2 = 4\Im \frac{\Pi^{AA}(s)}{r_d \Pi^{AH}(s)} |\Delta_{AH}(s)|^2 |\mathcal{M}(H^* \to \mu^- \mu^+_L)|^2 |\mathcal{M}(H^* \to f\bar{f})|^2,
\] (7.5)

where \( r_d = \chi_H^H/\chi_A^A \) and \( \Pi^{AH}(s) \) is assumed to be real (dispersive), for the two-Higgs doublet models discussed in Section 4. In Eqs. (7.4) and (7.5), the tree-level squared amplitudes involving the \( Hf\bar{f} \) and \( Af\bar{f} \) couplings obey the relation: \( |\mathcal{M}(H^* \to f\bar{f})|^2 = x_f(s) |\mathcal{M}(A^* \to f\bar{f})|^2 \), where \( x_f(s) = r_f^2(1 - 4m_f^2/s) \) with \( r_f = \chi_f^H/\chi_f^A \). Furthermore, the CP-conserving squared amplitudes are given by

\[
|\mathcal{T}^A|^2 = |\mathcal{T}_L^A|^2 + |\mathcal{T}_R^A|^2 = 2 \left[ r_d^2 + \frac{(s - M_H^2)^2 + (\Im \Pi^{HH}(s))^2}{(\Pi^{AH}(s))^2} \right] |\Delta_{AH}(s)|^2 |\mathcal{M}(A^* \to \mu^- \mu^+_L)|^2 |\mathcal{M}(A^* \to f\bar{f})|^2,
\] (7.6)

\[
|\mathcal{T}^H|^2 = |\mathcal{T}_L^H|^2 + |\mathcal{T}_R^H|^2 = 2 \left[ r_d^2 + \frac{(s - M_L^2)^2 + (\Im \Pi^{AA}(s))^2}{(\Pi^{AH}(s))^2} \right] |\Delta_{AH}(s)|^2 |\mathcal{M}(H^* \to \mu^- \mu^+_L)|^2 |\mathcal{M}(H^* \to f\bar{f})|^2.
\] (7.7)

Considering Eqs. (7.4)–(7.7), it is not difficult to calculate the CP asymmetry

\[
\mathcal{A}_{CP}^{(\mu)}(s) = \frac{\Delta \mathcal{T}^A + \Delta \mathcal{T}^H}{|\mathcal{T}^A|^2 + |\mathcal{T}^H|^2} = \frac{2r_d \Im \Pi^{AH}(s_f \Im \Pi^{AA} - \Im \Pi^{HH})}{s_f r_d^2 [(s - M_A^2)^2 + (\Im \Pi^{AA})^2] + s_f (\Pi^{AH})^2 + (s - M_H^2)^2 + (\Im \Pi^{HH})^2 + r_d^2 (\Pi^{AH})^2}.
\] (7.8)

The analytic result of \( \mathcal{A}_{CP}^{(\mu)}(s) \) in Eq. (7.8) simplifies to the qualitative estimate presented in [10], if finite mass effects of the asymptotic states are neglected, the value \( r_f \approx 1 \) is taken,
(i.e., $x_f \approx 1$), and $(\hat{\Pi}^{AH})^2$ terms are omitted in the CP-conserving part of the squared amplitude. In the present analysis, we shall include the high-order $(\hat{\Pi}^{AH})^2$ terms and take into account all those refinements inherent to the two-Higgs doublet model.

In order to reduce the large number of independent parameters that may vary independently, we fix the angles $\beta$ and $\theta$ to the values such that $\tan \beta = 2$ and $\tan \theta = 1$. In this scheme, the heaviest CP-even Higgs, $H$, has a significant coupling to fermions, i.e., $\chi^W_H \approx \chi^Z_H \ll \chi^u_H$, whereas $(\chi^W_H)^2 = (\chi^Z_H)^2 \approx 0.1$. For $M_A > 2M_Z$, such a scheme is motivated by the MSSM and leads to nearly degenerate $H$ and $A$ scalars, i.e., $M_H \approx M_A$ [31]. This mass relation also turns out to be very typical within SUSY unified models [32]. Nevertheless, we shall not consider here that $M_H$ is very strongly correlated with $M_A$. In the two-Higgs doublet model with heavy Majorana neutrinos, the three heavy neutrino masses are also fixed to $m_1 = 0.5$ TeV, $m_2 = 1$ TeV and $m_3 = 1.5$ TeV. In the two-Higgs doublet model with broken D symmetry, the tree-level $HA$ or $hA$ mixings, $\Pi^{HA}$ and $\Pi^{hA}$, are considered to be small phenomenological parameters, compared to the squared masses of all three Higgs particles, which is a result of constraints coming from the neutron EDM, as we will briefly discuss at the end of this section. For the top quark mass, we use $m_t = 170$ GeV close to its experimental mean value [33].

Assuming that tuning the collider c.m. energy to the mass of $H$ or $h$ is possible, i.e., $\sqrt{s} = M_H$ or $M_h$, we analyze the following two reactions:

(a) $\mu_L^+ \mu_L^- \to h^*, A^* \to \bar{b}b$, with $M_A = 170$ GeV,
(b) $\mu_L^+ \mu_L^- \to H^*, A^* \to t\bar{t}$, with $M_A = 400$ GeV.

In Fig. 5(a), we display cross sections (solid lines) and CP asymmetries (dotted lines) as a function of the c.m. energy, $\sqrt{s}$, for the two reactions (a) and (b) in the two-Higgs doublet model with heavy Majorana neutrinos. In reaction (a), one can have a significant CP-violating signal for $M_H \gg M_h$ and $M_A$, if $M_h = 170\pm 8$ GeV. As can also be seen from Fig. 5(a), we observe the resonant enhancement of $A_{CP}^{(u)}(M_h^2)$ when $M_h \approx M_A$. CP violation in reaction (b) may be probed for a wider range of Higgs-boson masses, i.e., for $M_H = 380$ GeV $- 420$ GeV. According to the discussion in Section 3, CP violation becomes maximal when the necessary condition (3.8) for $M_H \approx M_A$ is met. Furthermore, CPT symmetry preserves the constraint for the resonant cross sections (cf. Eq. (4.5))

$$\sigma(\mu_L^+ \mu_L^- \to H^*(h^*), A^* \to \text{all}) = \sigma(\mu_R^+ \mu_R^- \to H^*(h^*), A^* \to \text{all}),$$

which may generally reduce the magnitude of CP violation and so influence the actual dependence of $A_{CP}$ on the $HA$ or $hA$ mixing. However, our resummation approach takes automatically account of such CPT constraints, as has been explicitly demonstrated in
Section 4 (cf. Eq. (4.17)). In Fig. 5(b), we show how $|A_{CP}|$ varies as a function of the parameter $x_A = \Pi^{HA}/\Im(\hat{\Pi}^{HH} - \hat{\Pi}^{AA})$ or $\Pi^{hA}/\Im(\hat{\Pi}^{hh} - \hat{\Pi}^{AA})$, in the two-Higgs doublet model with a D-symmetry breaking. We consider the kinematic region for resonantly enhanced CP violation, i.e., $M_A = M_h$ $(M_H)$ for the reaction (a) (reaction (b)). We find that CP-violating effects could become very large, if the parameter $x_A$ was tuned to the value $x_A = 1$ for the process (a) and $x_A = 3$ for the process (b).

At the next linear $e^+e^-$ colliders (NLC’s), Higgs bosons may copiously be produced either via the Bjorken process for c.m. energies up to 0.5 TeV or through $WW$ fusion at higher energies [54]. The most convenient way is to study CP violation in the kinematic range of the Higgs production and decay [34,55]. Therefore, we shall be interested in the observable formed by differential cross sections

$$A_{CP}^{(e)}(\hat{s}) = \frac{d\sigma(e^-e^+ \rightarrow f_L\bar{f}_L X)/d\hat{s} - d\sigma(e^-e^+ \rightarrow f_R\bar{f}_R X)/d\hat{s}}{d\sigma(e^-e^+ \rightarrow f_L\bar{f}_L X)/d\hat{s} + d\sigma(e^-e^+ \rightarrow f_R\bar{f}_R X)/d\hat{s}},$$

(7.10)

where $\hat{s}$ is the invariant mass energy of the produced final fermions $f$. As final states, we may take bottom or top quarks. Since $A$ does not couple to $WW$ or $ZZ$, the CP asymmetry $	ilde{A}_{CP}^{(e)}$ in Eq. (7.10) takes the simple form

$$\tilde{A}_{CP}^{(e)}(\hat{s}) = -\frac{2r_f\hat{\Pi}^{AH}(\hat{s})\Im\hat{\Pi}^{AA}(\hat{s})}{r_f^2[(\hat{s} - M_A^2)^2 + (\Im\hat{\Pi}^{AA}(\hat{s}))^2] + (\hat{\Pi}^{AH}(\hat{s}))^2}. $$

(7.11)

In Fig. 6(a), we display the dependence of $\tilde{A}_{CP}^{(e)}$ as a function of $\hat{s}$. As expected, we find a resonant enhancement of CP violation when $M_A \approx M_h$ or $M_H$. Since the destructive term, $\Im\hat{\Pi}^{HH}$, is absent in Eq. (7.11), CP violation may become even larger, i.e., of order unity for specific values of the parameter $x_A$. Indeed, we see from Fig. 6(b) that $\tilde{A}_{CP}^{(e)} \approx 1$, if $x_A = 0.07 (3)$ for the reaction with longitudinally polarized $b$ $(t)$ quarks in the final state. Estimates of the CP asymmetry based on totally integrated cross sections, $A_{CP}^{(e)}$ in Eq. (7.2), are presented in [34]. The authors [33] find that $A_{CP}^{(e)} < 15\%$ for Higgs masses $M_H < 600$ GeV and cross sections $\sigma(e^-e^+ \rightarrow H^*(h^*), A^* \rightarrow t\bar{t} X) \approx 10 – 100$ fb for c.m. energies of 1 – 2 TeV. Such CP-violating effects have high chances to be detected at future NLC’s.
At the LHC, the respective CP asymmetry $A_{CP}(\hat{s})$ may be obtained from Eq. (7.11), for Higgs particles that have a production mechanism similar to that at the NLC. If the Higgs is produced via gluon fusion, one has to use the analytic expression of $A_{CP}(\hat{s})$ and replace $r_d$ with $r_u$ in Eq. (7.8). Unless CP violation is resonantly amplified, i.e., of order one, the chances to detect CP-violating phenomena on the Higgs-resonance line after removing the contributing background appear to be quite limited at the LHC. It is therefore worth stressing that a large CP-violating signal at the Higgs-boson peak will certainly point towards the existence of an almost degenerate $HA$ mixing system.

The presence of a $HA$ operator may also contribute to other low-energy CP-violating observables. CP-violating quantities sensitive to $HA$ terms, for which there exist quite strict experimental upper bounds, are the EDM’s of the neutron, electron and muon. From the one-loop flavour diagrams shown in Fig. 7, it is straightforward to calculate

$$d_f/e \approx -Q_f \frac{\alpha_w}{4\pi} \frac{m_f}{M^2} \frac{m_f^2}{M_W^2} \xi_{HA} \left[ \ln \left( \frac{m_f^2}{M^2} \right) + \frac{3}{2} \right],$$

(7.12)

with $M = (M_H + M_A)/2$, $Q_f$ denoting the charge of fermion in units of $|e|$ and

$$\xi_{HA} = \chi_{Af} \chi_{Hf} \frac{\hat{\Pi}^{HA}(M^2)}{M^2}.$$

(7.13)

The most severe constraint comes from the EDM of the neutron, which has the experimental upper bound $(d_n/e) < 1.1 \times 10^{-25}$ cm, at 95% of confidence level (CL) [53]. The contribution of the one-loop graphs in Fig. 7 is much smaller. Taking the typical values of $m_d = 10$ MeV and $M_W \approx M \approx 100$ GeV, Eq. (7.12) predicts $(d_n/e) < 3 \times 10^{-30} \xi_{HA}$ cm, far beyond the
Nevertheless, the two-loop Barr-Zee (BZ) mechanism [56] shown in Fig. 8 may have a significant impact on the actual size of the EDM, thus leading to tighter bounds on the $HA$ mixing parameter $\xi_{HA}$. In general, the theoretical prediction for the EDM is enhanced by the absence of chirality suppressed terms of order $m_f^2/M_W^2$, despite the fact that the BZ mechanism occurs at the two-loop order. Thus, the net effect is to increase the value of $(d_n/e)$ in Eq. (7.12) by a factor $\alpha(M_W^2/m_d^2) \approx 10^6$, where $\alpha = 1/137$ is the fine-structure constant [56]. A recent analysis of constraints from the electron and neutron EDM’s is presented in [57] for the two-Higgs doublet model with maximal CP violation, where the Weinberg’s unitarity bound is almost saturated [58]. Even though the prediction for the electron EDM is below the experimental limit, the neutron EDM gives theoretical bounds that may be evaded if the mass difference, $\Delta M$, between $A$ and one of the CP-even Higgs scalars $H$ (or $h$) is sufficiently small. In this way, the authors of [57] find

$$\frac{\Delta M}{M} \approx \frac{\tilde{\eta}_{HA}}{M^2} < 0.10, 0.13, 0.24,$$

(7.14)

for $M = 200$, 400, 600 GeV, respectively. Note that the constraints on $\Delta M/M$, obtained in [57] for the diagonal Higgs-mass basis, have been adapted to our off-diagonal $HA$ basis, to leading order in $(M_H^2 - M_A^2)/M^2$. The limits due to the neutron EDM in Eq. (7.14) are automatically satisfied by our new-physics scenarios. Of course, for large $\tan \beta$ values, e.g., $\tan \beta > 3$, the above limits on the mass difference between $H$ and $A$ will be much weaker. As has been discussed above however, our resonant CP-violating phenomena through particle mixing can still be very large as soon as the necessary conditions (3.8) and/or (3.10) are fulfilled.

8 Fermionic case

We shall consider CP violation induced by the mixing of two fermions. Issues of fermionic mixing renormalization are discussed in Ref. [25] and hence will not be repeated here. For our illustrations, we shall assume the mixing of two Dirac particles, which have a small mass difference compared to their masses. To be specific, one may think of scatterings involving the mixing of a top quark with a new sequential up-type fermion, $t'$, as shown in Fig. 9. Such resonant transitions were also discussed in [10]. Here, we follow a theoretically more rigorous approach.

Assuming that box or non-resonant graphs are very small, we can write down the matrix element for our prototype transition, $d\phi^+ \rightarrow t^*, t'^* \rightarrow s\phi^+$, in the following compact
form:

$$T \approx T_{ds}^{res} = V_i^s \left( \frac{1}{\not{p} - \mathcal{H}(\not{p})} \right)_{ij} V_j^d,$$

(8.1)

where $V_i^d$ and $V_i^s$ are production and decay vertices involving the quarks $t$, $t'$ and the charged scalar $\phi^+$. The propagator matrix $[\not{p} - \mathcal{H}(\not{p})]^{-1}$, which is defined in the sub-space formed by $t$ and $t'$, describes the resonant dynamics of the $tt'$-mixing system. Its explicit form will be given in this section later on. The scalar field $\phi^+$ may represent either a physical Higgs boson in new-physics scenarios or the would-be Goldstone boson of $W^+$, $G^+$, which is a good approximation for the longitudinal $W$ boson at high energies. Also, the CP-conjugate process is given by

$$T^{CP} \approx T_{ds}^{res} = \overline{V}_i^s \left( \frac{1}{\not{p} - \mathcal{H}(\not{p})} \right)_{ij} \overline{V}_j^d,$$

(8.2)

where $\overline{V}_i^{d,s}$ and $\overline{\mathcal{H}(\not{p})}$ are the CP transforms of $V_i^{d,s}$ and $\mathcal{H}(\not{p})$, respectively. These CP transformations are very analogous to those given in Eqs. (2.4) and (2.5) for the scalar case.

Fig. 9: Resonant CP-violating $tt'$ transitions.

It is now worth emphasizing that we consider CP violation that originates mainly from the mixing of the intermediate up-type fermionic states $t$ and $t'$. However, one should bear in mind that the tree-level production and decay vertices $\overline{V}_i^{d,s}$, e.g., $Wu_i d_j$, contain extra flavour rotations which result from diagonalizing the mass matrix of the up-quark family by means of bi-unitary transformations. In the SM, these rotations are parameterized by the well-known Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix in the flavour basis where the down-quark mass matrix is diagonal. Therefore, one must always think of these CKM rotations as being part of the fermionic effective Hamiltonian $\mathcal{H}(\not{p})$. Besides the afore-mentioned CKM rotations, the tree-level couplings $\phi^+ u_i d_j$ depend on the up- and/or down-family mass matrices, which can also introduce CP violation in the vertices. Formally speaking, such a CP violation is of $\varepsilon'$ type and may therefore be competitive with
our $\varepsilon$-type effects. Since it may be difficult to disentangle the $\varepsilon$-type mass parameters from the $\varepsilon'$-type couplings, we shall include both contributions in the Born approximation and ignore possible high order $\varepsilon'$-type effects in $\overline{V}_i^{ds}$, as they are expected to be quite small near the resonant region.

Within the context of our resummation approach, the propagator matrix for two fermions, $\hat{S}_{ij}(\not{p})$, results from summing up PT self-energies, $\hat{\Sigma}_{ij}(\not{p})$, i.e.,

$$[\not{p} - \mathcal{H}(\not{p})]^{-1} \equiv \hat{S}_{ij}(\not{p}) = \delta_{ij} S_i^0(\not{p}) - S_i^0(\not{p}) \hat{\Sigma}_{ij}(\not{p}) S_j^0(\not{p}) + \ldots$$

$$= [S_i^{0-1}(\not{p}) + \hat{\Sigma}(\not{p})]^{-1} = \begin{pmatrix} \not{p} - m_i + \hat{\Sigma}_{tt}(\not{p}) & \hat{\Sigma}_{t't'}(\not{p}) \\ \hat{\Sigma}_{t't}(\not{p}) & \not{p} - m_{t'} + \hat{\Sigma}_{t't'}(\not{p}) \end{pmatrix}^{-1}, \quad (8.3)$$

where

$$S_i^0(\not{p}) = \frac{1}{\not{p} - m_i + i\varepsilon}, \quad i = t, t'$$

$$\hat{\Sigma}_{ij}(\not{p}) = \not{p} P_L \Sigma_{ij}^L(s) + \not{p} P_R \Sigma_{ij}^R(s) + P_L \Sigma_{ij}^D(s) + P_R \bar{\Sigma}_{ij}^R(s), \quad (8.4)$$

with $s = p^2$. In Eq. (8.4), $P_{L(R)} = [1 - (\pm)\gamma_5]/2$ is the chirality projection operator, while the symbol hat on the individual components of $\hat{\Sigma}_{ij}(\not{p})$ has been dropped for simplicity. Nevertheless, it must be kept in mind that the self-energies are OS renormalized within the PT framework [8].

Decomposing the different self-energy components into dispersive and absorptive parts,

$$\Sigma_{ij}^L(s) = \Sigma_{ij}^{L,\text{disp}}(s) + i\Sigma_{ij}^{L,\text{abs}}(s), \quad \Sigma_{ij}^R(s) = \Sigma_{ij}^{R,\text{disp}}(s) + i\Sigma_{ij}^{R,\text{abs}}(s),$$

$$\Sigma_{ij}^D(s) = \Sigma_{ij}^{D,\text{disp}}(s) + i\Sigma_{ij}^{D,\text{abs}}(s), \quad \Sigma_{ji}^{D,\text{disp}}(s) + i\Sigma_{ji}^{D,\text{abs}}(s), \quad (8.5)$$

one derives from the Hermiticity property of the Lagrangian

$$\Sigma_{ij}^{L,\text{disp}}(s) = \Sigma_{ji}^{L,\text{disp}}(s), \quad \Sigma_{ij}^{L,\text{abs}}(s) = \Sigma_{ji}^{L,\text{abs}}(s). \quad (8.6)$$

Similar equalities also hold for $\Sigma_{ij}^{R,\text{disp}}(s)$ and $\Sigma_{ij}^{R,\text{abs}}(s)$. At one loop, only the dispersive parts of the self-energies can participate in the renormalization, since the bare as well as counter-term (CT) Lagrangian is Hermitian [25], whereas the absorptive self-energy parts, $i\hat{\Sigma}_{ij}^{\text{abs}}(\not{p})$, are anti-Hermitian.

Another important point pertains to the issue of gauge invariance of resonant processes involving heavy fermions, e.g., the heavy $t$ and $t'$ quarks, after resummation has been completed. Apart from the gauge-fixing parameter independence within the PT, one has to worry about preserving additional gauge symmetries, when a gauge boson, such as the
$W$ or $Z$ bosons, couples to the fermionic line. In the conventional formalism, these gauge symmetries may get distorted by high-order quantum effects, after resummation has been carried out. In the PT, these extra gauge symmetries are reassured by the following PT WI's:

\[ p^\mu \tilde{\Gamma}^{W+\, ud}_{\mu}(p, p_u, p_d) + M_W \tilde{\Gamma}^{G+\, ud}_{\mu}(p, p_u, p_d) = -\frac{ig}{\sqrt{2}} V_{ud} \left[ \Sigma_{uu}(\not{p}_u) P_L - P_R \Sigma_{dd}(\not{p}_d) \right], \tag{8.7} \]
\[ p^\mu \tilde{\Gamma}^{W-\, du}_{\mu}(p, p_d, p_u) - M_W \tilde{\Gamma}^{G-\, du}_{\mu}(p, p_d, p_u) = -\frac{ig}{\sqrt{2}} V_{ud}^* \left[ \Sigma_{dd}(\not{p}_d) P_L - P_R \Sigma_{uu}(\not{p}_u) \right], \tag{8.8} \]
\[ p^\mu \tilde{\Gamma}^{Zff}_{\mu}(p, p_f, p_f) - iM_Z \tilde{\Gamma}^{G^0ff}_{\mu}(p, p_f, p_f) = \frac{ig}{2 \cos \theta_w} \left[ \Sigma_{ff}(\not{p}_f)(T_z^f P_L - 2Q_f \sin^2 \theta_w) - (T_z^f P_R - 2Q_f \sin^2 \theta_w) \Sigma_{ff}(\not{p}_f) \right], \tag{8.9} \]

where $V_{ud}$ is the CKM matrix, $\theta_w$ is the weak mixing angle, $Q_f$ is the fractional charge of quarks (i.e., $Q_u = 2/3$, $Q_d = -1/3$) and $T_z^f$ is the $z$-component of the weak isospin of the fermion $f$, defined after Eq. (5.4). In Eqs. (8.7)–(8.9), the momentum $p_\mu$ of the gauge bosons flows into the vertex, while the four-momenta of the fermions point to the same direction with the fermion-number arrow. The PT WI’s (8.7) and (8.8) may be of interest here, since the PT WI in Eq. (8.9) is only crucial when heavy fermions are produced via $s$-channel $Z$-boson interactions. It is obvious that possible absorptive parts in the vertices, e.g., $Wtb$ and $Wt'b$, will now communicate with the respective absorptive parts of the self-energies by means of the PT WI’s of the $W$ and $Z$ bosons, couples to the fermionic line. In the conventional formalism, these gauge symmetries remain the same. Inverting the matrix, $\tilde{S}_{ij}^{-1}(\not{p})$ in Eq. (8.3), we arrive at the resummed fermionic propagators

\[ \tilde{S}_{tt}(\not{p}) = \left[ \not{p} - m_t + \Sigma_{tt}(\not{p}) - \frac{1}{\not{p} - m_t + \Sigma_{tt}(\not{p})} \Sigma_{tt}(\not{p}) \right]^{-1}, \tag{8.10} \]

* Similar WI’s have been derived in [31] in a semi-phenomenological manner, by studying the structure of the gauge cancellations for different Breit-Wigner propagators. The PT WI’s in Eqs. (8.7)–(8.9) coincide with those obtained by the background field method in [40].
\( \tilde{S}_{\nu\nu}(\nu') = \left[ \nu' - m_{\nu'} + \Sigma_{\nu\nu}(\nu') - \Sigma_{\nu\nu}(\nu) \frac{1}{\nu' - m_{\nu} + \Sigma_{\nu\nu}(\nu)} \right]^{-1}, \quad (8.11) \)

\( \tilde{S}_{uu}(\nu') = - \tilde{S}_{uu}(\nu) \Sigma_{\nu\nu}(\nu') \left[ \nu' - m_{\nu'} + \Sigma_{\nu\nu}(\nu) \right]^{-1} \)

\( = - \left[ \nu' - m_{\nu'} + \Sigma_{\nu\nu}(\nu) \right]^{-1} \Sigma_{\nu\nu}(\nu') \tilde{S}_{\nu\nu}(\nu') \), \quad (8.12) \)

\( \tilde{S}_{\nu\nu}(\nu') = - \tilde{S}_{\nu\nu}(\nu) \Sigma_{\nu\nu}(\nu') \left[ \nu' - m_{\nu'} + \Sigma_{\nu\nu}(\nu) \right]^{-1} \)

\( = - \left[ \nu' - m_{\nu'} + \Sigma_{\nu\nu}(\nu) \right]^{-1} \Sigma_{\nu\nu}(\nu') \tilde{S}_{\nu\nu}(\nu') . \quad (8.13) \)

As has been mentioned above, it may be difficult to find a rotational invariant weak basis that quantifies the magnitude of CP violation coming entirely from \( tt' \) mixing, compared to the case of two neutral scalars. One possibility could therefore be to factor out mass-matrix and CKM-type mixing elements from the vertices \( V_{id}^d \) and \( V_{is}^s \), and re-absorb them into the definition of effective Hamiltonian. Another way, and perhaps the most consistent one, is to consider the complete production and decay amplitudes at the tree level, since \( \varepsilon \)- and tree-level \( \varepsilon' \)-type effects may not be easily separated in most natural extensions of the SM. Following the latter, we may define the flavour-dependent CP-violating parameter

\[
\delta_{ds} = \left| \frac{T_{ds}^{res}}{T_{ds}} \right| = \left| \frac{V_{is}^0 [\nu' - \mathcal{H}(\nu')]^{-1} V_{jd}^{0*}}{V_{is}^0 [\nu' - \mathcal{H}(\nu')]^{-1} V_{jd}^0} \right| ,
\]

where \( \mathcal{H}(\nu') = \mathcal{H}^T(\nu') \), and \( V_{id}^0 \) and \( V_{is}^0 \) are the respective couplings \( \phi^+ dj \) and \( \phi^+ sj \) \((j = t, t')\) in the Born approximation. Clearly, instead of \( d \) and \( s \) quarks, one could consider another pair of down-type quarks as asymptotic states, \( e.g. \ s \) and \( b \) or \( b \) and \( b' \). Because of fermion-number conservation, the production, mixing, and decay phenomena in the up-quark family will unavoidably become manifest in the down-quark sector.

It may be worth mentioning that the diagonal elements of the flavour-matrix \( \delta \) satisfy the requirement

\[
\delta_{d_i d_i} = 1 \quad (8.15)
\]

as a result of CPT invariance. Thus, we expect effects of CP violation through mixing only from off-diagonal transitions. If an observable is sensitive to the handedness or the helicity of the asymptotic quarks, one is then able to define CP-violating mixing parameters, such as \( \delta_{d_L d_i R} \), which may generally deviate from unity and consistently respect CPT invariance. Furthermore, necessary conditions for resonant CP violation may be derived from Eq. \( (8.14) \). We will see a specific example in the next section.

Finally, we briefly comment on the case of transitions that involve heavy Majorana fermions as intermediate states. In particular, a heavy Majorana neutrino can decay into a charged lepton, \( l \), as well as into an anti-lepton \( l^C \) with the emission of a charged Higgs. Such
scenarios have received much attention, since they can account for the baryon asymmetry in the universe \cite{60}. Within our formulation of CP violation, it is evident that one could still have
\[ \delta_{\text{H}} = \delta_{\text{C}}^{-1} \neq 1, \] (8.16)
in agreement with CPT invariance. Detailed study of the latter may be given elsewhere.

9 Resonant CP violation via a $tt'$ mixing

New-physics scenarios that give rise to a CP-asymmetric $tt'$ mixing should extend the fermionic sector of the SM in a non-trivial manner. For instance, adding one sequential weak isodoublet, $(t', b')_L$, and two right-handed weak iso-singlets, $t'_R$ and $b'_R$, appears to be the most straightforward way to accomplish that purpose. In Ref. \cite{10}, we have analyzed CP-violating effects originating from $tt'$ transitions in four-generation extensions with one, two and three Higgs doublets. Unlike \cite{10}, we shall study the phenomenological implications of a strong $tt'$ mixing related to the kinematic range $m_t \approx m_{t'}$ for the LHC. Since direct experimental searches at Tevatron find that $b'$ should be quite heavy, $m_{b'} > 85$ GeV \cite{53}, absorptive phases related with the opening of intermediate decay modes, such as $t \rightarrow W^+ b'$ or $t' \rightarrow W^+ b'$, are therefore suppressed. The CP asymmetries strongly depend on these $b'$-dependent absorptive phases, thus rendering CP violation difficult to detect in these models. In order to demonstrate that resonant CP violation via fermionic mixing can still take place in the top quark sector, we shall consider a simple CP-violating new-physics model, in which a mirror iso-doublet is added to the field content of the SM, apart from the sequential doublet mentioned above.

The model of our interest extends the third family of quarks of the SM in the following way:

\[
U_{1L} = \begin{pmatrix} t_1 \\ b_1 \end{pmatrix}_L, \quad U_{2L} = \begin{pmatrix} t_2 \\ b_2 \end{pmatrix}_L, \quad U_{C2L} = \begin{pmatrix} b_C^2 \\ t_C^2 \end{pmatrix}_L, \quad t_{1R}, \quad b_{1R}, \quad (9.1)
\]

with hypercharge assignments $Y(U_{1L}) = Y(U_{2L}) = 1/3$, $Y(U_{C2L}) = -1/3$, $Y(t_{1R}) = 4/3$, $Y(b_{1R}) = -2/3$ and weak isospins $T(U_{1L}) = T(U_{2L}) = T(U_{C2L}) = 1/2$, $T(t_{1R}) = T(b_{1R}) = 0$. Since the only difference relative to the SM is the addition of two weak iso-doublets with opposite hypercharges, the above scenario is anomaly free as well; one does not need to include extra charged lepton fields. Such new-physics extensions may even be motivated by E$_6$ unified models \cite{61,62}. After SSB, the Yukawa sector of the model reads

\[
- \mathcal{L}^\text{mass}_Y = \langle \Phi \rangle [f_1^t \bar{t}_{1L} t_{1R} + f_2^t \bar{t}_{2L} t_{1R}] + \langle \Phi \rangle [f_1^b \bar{b}_{1L} b_{1R} + f_2^b \bar{b}_{2L} b_{1R}]
+ M_1(U_{C2L}^T i \tau_2 C^{-1} U_{1L} + M_2(U_{2L}^C)^T i \tau_2 C^{-1} U_{2L} + \text{H.c.}}, \quad (9.2)
\]
with $i\tau_2$ representing the Levi-Civita antisymmetric tensor $\varepsilon_{ij}$ and $\langle \Phi^0 \rangle = \langle \overline{\Phi} \rangle$ being the VEV of the SM Higgs doublet. If we denote with $m^b_{1,2} = f^b_{1,2}\langle \Phi^0 \rangle$ and $m^b_t = f^b_t\langle \Phi^0 \rangle$, the $t_1t_2$- and $b_1b_2$-mass matrices are respectively given by

$$M^t = \begin{bmatrix} m^t_1 & M^*_1 \\ m^t_2 & M^*_2 \end{bmatrix}, \quad M^b = \begin{bmatrix} m^b_1 & -M^*_1 \\ m^b_2 & -M^*_2 \end{bmatrix}.$$ \hspace{1cm} (9.3)

The physical states for the $b$-quark sector, $b, b'$, and the $t$-quark system, $t, t'$, are obtained through the bi-unitary transformations

$$U^b_L M^b U^b_R = \tilde{M}^b, \quad U^t_L M^t U^t_R = \tilde{M}^t,$$ \hspace{1cm} (9.4)

with

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{L,R} = U^b_{L,R} \begin{pmatrix} b \\ b' \end{pmatrix}_{L,R}, \quad \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}_{L,R} = U^t_{L,R} \begin{pmatrix} t \\ t' \end{pmatrix}_{L,R},$$ \hspace{1cm} (9.5)

where $U^b_{L,R}$ and $U^t_{L,R}$ are two-dimensional unitary matrices.

We are now in a position to write down the Lagrangian, $\mathcal{L}_W$, for the charged current interactions in this sequential mirror fermion model. The Lagrangian $\mathcal{L}_W$ is given by

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W^+ \mu \left( \bar{t}_{1L} \gamma_\mu b_{1L} + \bar{t}_{2L} \gamma_\mu b_{2L} + \bar{b}^C_{2L} \gamma_\mu t^C_{2L} \right) + \text{H.c.}$$

$$= -\frac{g}{\sqrt{2}} W^+ \mu \left( \bar{t}, \bar{t}' \right) \left( V^L_{ij} \gamma_\mu P_L + V^R_{ij} \gamma_\mu P_R \right) \begin{pmatrix} b \\ b' \end{pmatrix} + \text{H.c.},$$ \hspace{1cm} (9.6)

with the mixing matrices

$$V^L_{ij} = (U^t_{L})^*_i(U^b_{L})^*_j, \quad V^R_{ij} = - (U^t_{R})^*_i(U^b_{R})^*_j.$$ \hspace{1cm} (9.7)

In the last equality of Eq. (9.6), we have used the property $\bar{b}^C_{2L} \gamma_\mu t^C_{2L} = -\bar{t}_{2R} \gamma_\mu b_{2R}$. From Eq. (9.7), we see that $V^L$ is a unitary matrix but $V^R$ is not. These mixing matrices can mediate CP violation. In general, there are many rephasing-invariant CP-odd quantities in this model, which can be formed as follows:

$$\Im m(V^L_{tb} V^{R*}_{tb}), \quad \Im m(V^L_{tb} V^{L*}_{tb} V^{R*}_{tb} V^{R*}_{tb}), \quad \Im m(V^L_{tb} V^{L*}_{tb} V^{R*}_{tb} V^{R*}_{tb}), \quad \text{etc.}$$ \hspace{1cm} (9.8)

Note that this scenario is different from a usual two-generation mixing model in the SM, in which judicious phase rotations of the left- and right-handed chiral fermions can be used to eliminate all trivial CP-odd phases, thus giving rise to a real mixing matrix. For instance, phase re-definations of the left-handed weak states, $t_{1L}, b_{1L}, t_{2L}$ and $b_{2L}$, will depend on those of the right-handed states, $b^C_{2L}$ and $t^C_{2L}$. The reason is that such field re-phasings will
simultaneously affect the gauge interactions of the $W$ boson with the heavy quarks in Eq. (9.6) and the positivity of the diagonal mass matrices $\hat{M}^b$ and $\hat{M}^t$.

To avoid the tight phenomenological limits, we have implicitly assumed that the heavy quarks $t_1$, $t_2$, $b_1$ and $b_2$ couple feebly to the two lighter families, i.e., to $u$, $d$, $s$, and $c$ quarks. The most significant constraint arises from the $Zb_R\bar{b}_R$ coupling, which may affect the longitudinal polarization asymmetry of the produced $b$ quarks measured at the CERN Large $e^+e^-$ Collider (LEP1) [63]. The present experimental sensitivity can only give the upper bound $|(U^b_R)_{21}|^2 < 0.10$, which is not very restrictive. In fact, mirror fermion models may be appealing scenarios, since they can produce a positive shift to the observable $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ in agreement with experiments at the LEP1 [64].

In models with additional sequential doublets, one can generally induce a $tt'$ transition amplitude as shown in Fig. 10. It is therefore useful to know the absorptive parts of all $tt$, $tt'$ and $t't'$ quark self-energies within the PT. The best way to evaluate the PT self-energies is to choose a gauge, in which there are no pinching $k_\mu k_\nu$ momenta coming from the virtual $W$ propagator [4,47]. Consequently, we shall adopt the Feynman-'t Hooft gauge, i.e., $\xi = 1$ in $R_\xi$ gauges. To avoid large effects in the electroweak oblique parameters, we assume the mass pattern $m_{b'} \approx m_t \approx m_{t'}$, which leads to vanishing $b'$-absorptive contributions. Taking the above assumptions into account, the analytic results of the quark self-energies may be cast into the form

$$\hat{\Sigma}^\text{abs}_{ij}(p)\Big|_{(a)} = \frac{\alpha_w}{8} \left(1 - \frac{M^2_W}{p^2}\right)^2 \left(V^L_{ib}V^L_{jb}\not{p}P_L + V^R_{ib}V^R_{jb}\not{p}P_R\right), \quad (9.9)$$

$$\hat{\Sigma}^\text{abs}_{ij}(p)\Big|_{(b)} = \frac{\alpha_w m_i m_j}{16 M^2_W} \left(1 - \frac{M^2_W}{p^2}\right)^2 \left(V^L_{ib}V^L_{jb}\not{p}P_L + V^R_{ib}V^R_{jb}\not{p}P_R\right), \quad (9.10)$$

where the indices $i, j$ run over $t, t'$. As can be seen from Eqs. (9.9) and (9.10), the self-energy graph in Fig. 10(a) has different chirality structure from that shown in Fig. 10(b).
The main reason is that the longitudinal and transverse degrees of a gauge boson give rise to different chirality flips. To be precise, the longitudinal $W$ boson behaves as a scalar current which will change a left-handed chiral fermion into a right-handed one and vice versa, whereas the transverse component of the $W$ boson will not. In the SM with an effective two-generation mixing, the respective analytic results may be easily recovered by dropping all those terms that are proportional to $V_{ij}^R$ in Eqs. (9.9) and (9.10).

To make our points on resonant CP violation more explicit, we shall consider the simple $2 \rightarrow 2$ scattering process $\phi^+ s \rightarrow \phi^+ b$, which involves $tt'$ mixing as displayed in Fig. 9. Since the dominant contribution to such a process will originate from longitudinal $W$ bosons, we will approximate the $W^+$ boson with its unphysical would-be Goldstone boson, denoted here as $\phi^+$. In two-Higgs doublet models or supersymmetrized versions of this model, $\phi^+$ could also be a physical charged Higgs boson. For our illustrations, we shall also assume that only the top quark couples to the $s$ quark, i.e., $V_{ts}^L = V_{ts}^R = 0$, and the c.m. energy is somehow tuned to $m_t$. Employing the relations (8.12) and (8.13) between the resummed propagators $\tilde{S}_{tt}(p)$ and $\tilde{S}_{tt'}(p)$, $\tilde{S}_{tt'}(p)$, we may conveniently factorize the matrix elements of our resonant process and its CP-conjugate counterpart as follows:

$$
T_{sb}^{res}(\phi^+ s \rightarrow \phi^+ b) \approx \frac{1}{\imath m_t \Gamma_t} \mathcal{T}^0(\phi^+ s \rightarrow t) \mathcal{T}(t \rightarrow \phi^+ b),
$$

$$
T_{sb}^{res}(\phi^+ s \rightarrow \phi^+ b) \approx \frac{1}{\imath m_t \Gamma_t} \mathcal{T}^0(\phi^- s \rightarrow t) \mathcal{T}(t \rightarrow \phi^+ b),
$$

(9.11)

where $\mathcal{T}^0$ denotes the tree-level amplitude. In Eq. (9.11), $\Gamma_t$ takes account of width effects near the top-quark production. Even though one can always work with the exact propagator expressions, the above factorization of the resonant amplitudes will not significantly affect our quantitative discussion as far as the phenomenon of CP violation is concerned. To make this obvious, we define the CP asymmetry

$$
a_{CP} = \frac{|T_{sb}^{res}|^2 - |T_{sb}^{res}|^2}{|T_{sb}^{res}|^2 + |T_{sb}^{res}|^2} = \frac{\delta_{sb}^2 - 1}{1 + \delta_{sb}^2} \approx \frac{|\mathcal{T}(t \rightarrow \phi^+ b)|^2 - |\mathcal{T}(t \rightarrow \phi^- b)|^2}{|\mathcal{T}(t \rightarrow \phi^+ b)|^2 + |\mathcal{T}(t \rightarrow \phi^- b)|^2}.
$$

(9.12)

As may be seen from Eq. (9.12), the main advantage of the above simplifications is that the CP asymmetry $a_{CP}$ reduces to calculating CP violation in the partial decay rate of the top quark into the charged scalar $\phi^+ \phi$ and the $b$ quark. Up to overall coupling constants, the matrix element for $t \rightarrow \phi^+ b$ may then be given by

$$
\mathcal{T}(t \rightarrow \phi^+ b) \sim \frac{m_t}{M_W} \bar{u}_b \left( V_{tb}^{L*} P_R + V_{tb}^{R*} P_L \right) u_t - \imath \frac{m_{t'}}{M_W} \bar{u}_b \left( V_{tb}^{L*} P_R + V_{tb}^{R*} P_L \right)
$$

$$
\times \left[ \not\!p - m_{t'} + \imath \not\!\Sigma^{abs}_{tt'}(\not\!p) \right]^{-1} \Sigma^{abs}_{tt'}(\not\!p) u_t.
$$

(9.13)
Similarly, the CP-conjugate decay matrix element may be written down

\[
\overline{T}(\bar{t} \to \phi \bar{b}) \sim \frac{m_t}{M_W} \bar{v}_t \left( V_{tb}^L P_L + V_{tb}^R P_R \right) v_b - i \frac{m_{t'}}{M_W} \bar{v}_{t'} \hat{\Sigma}_{t'i}^{\text{abs}} (\phi) \\
\times \left[ \hat{\Sigma}_{t'i}^{\text{abs}} (\phi) \right]^{-1} \left( V_{tb}^L P_L + V_{tb}^R P_R \right) v_b
\]

\[
= \frac{m_t}{M_W} \bar{u}_b \left( V_{tb}^L P_L + V_{tb}^R P_R \right) u_t - i \frac{m_{t'}}{M_W} \bar{u}_{t'} \left( V_{tb}^L P_L + V_{tb}^R P_R \right) u_t
\]

\[
\times \left[ \hat{\Sigma}_{t'i}^{\text{abs}} (\phi) \right]^{-1} \hat{\Sigma}_{t'i}^{\text{abs}, C} (\phi) \ u_t .
\]

(9.14)

In the derivation of the last step of Eq. (9.14), we have used the known identities: \( u(p, s) = C\bar{v}^T(p, s) \) and \( C\gamma^T \mu \mu^{-1} = -\gamma^T \mu \). If we now define the absorptive \( j \to i \) self-energies \( (i, j = t, t') \) as

\[
\hat{\Sigma}_{ij}^{\text{abs}} (\phi) = A_{ij}^L \phi P_R + A_{ij}^R \phi P_L ,
\]

(9.15)

with \( A_{ij}^L = V_{tb}^L V_{tb}^{L*} A_{ij} \) and \( A_{ij}^R = V_{tb}^R V_{tb}^{R*} A_{ij} \)

\[
A_{ij} = \frac{\alpha_w}{16} \frac{m_i m_j}{M_W^2} \left( 1 - \frac{M_W^2}{p^2} \right)^2 ,
\]

(9.16)

then the absorptive part of the charge-transform transition, \( \bar{j} \to \bar{i} \), is given by

\[
\hat{\Sigma}_{ij}^{\text{abs}, C} (\phi) = A_{ij}^L \phi P_L + A_{ij}^R \phi P_R .
\]

(9.17)

Comparing Eq. (9.13) with Eq. (9.14), it is easy to observe that, under a CP transformation, the mixing matrices \( V^L, V^R \) and the chirality projector \( P_L \) are mapped into the mixing matrices \( V^{L*}, V^{R*} \) and \( P_R \), respectively, while all the absorptive phases remain unchanged. By virtue of the Dirac equation of motion, the matrix element \( T(t \to \phi^+ b) \) simplifies to the expression

\[
T(t \to \phi^+ b) = T_L(t \to \phi^+ b) + T_R(t \to \phi^+ b) ,
\]

(9.18)

where

\[
T_L(t \to \phi^+ b) \sim \bar{u}_b P_L u_t \\
\times \left[ V_{tb}^{R*} \frac{m_t}{M_W} - i V_{tb}^{R*} \frac{m_{t'}}{M_W} \frac{m_i^2 (1 + i A_{i'i'}^R) A_{i'i'}^R + m_i m_{t'} A_{i'i'}^L}{m_i^2 (1 + i A_{i'i'}^R)(1 + i A_{i'i'}^R) - m_{t'}^2} \right].
\]

(9.19)

and

\[
T_R(t \to \phi^+ b) \sim \bar{u}_b P_R u_t \\
\times \left[ V_{tb}^{L*} \frac{m_t}{M_W} - i V_{tb}^{L*} \frac{m_{t'}}{M_W} \frac{m_i^2 (1 + i A_{i'i'}^L) A_{i'i'}^L + m_i m_{t'} A_{i'i'}^R}{m_i^2 (1 + i A_{i'i'}^L)(1 + i A_{i'i'}^L) - m_{t'}^2} \right].
\]

(9.20)

Neglecting small \( m_b \)-dependent terms, one can now verify that the CP-violating contributions of \( |T_L|^2 \) and \( |T_R|^2 \) to \( a_{CP} \) defined in Eq. (9.12) cancel in the sum. The underlying
reason responsible for such a cancellation is the invariance of the process under CPT transformations. Evidently, if there was an extra charged Higgs, such as $\phi^+$, which preferred to couple to right-handed $b$ quarks and/or left-handed top quarks only, then $a_{CP}$ would not vanish.

Motivated by the recent studies of probing CP violation in the production of polarized top decays at the LHC [17,18], we shall focus our attention on the decay $t_L \to \phi^+ b$ and the CP-conjugate decay $\bar{t}_R \to \phi^- \bar{b}$. In our prototype scattering $\phi^+ s \to \phi^+ b$, this would amount to assuming that the $s$ quark couples to the left-handed top quark only. The matrix element for the decay $t_L \to \phi^+ b$ may be obtained from $\mathcal{T}_L(t \to \phi^+ b)$ in Eq. (9.19). In this way, we find for the CP asymmetry $A_{CP}^{pol}$,

$$A_{CP}^{pol} = \frac{\Gamma(t_L \to \phi^+ b) - \Gamma(\bar{t}_R \to \phi^- \bar{b})}{\Gamma(t_L \to \phi^+ b) + \Gamma(\bar{t}_R \to \phi^- \bar{b})}. \quad (9.21)$$

By analogy, one could define the CP asymmetry $\overline{A}_{CP}^{pol}$ based on the decays $t_R \to \phi^+ b$ and the CP transforms, $\bar{t}_L \to \phi^- \bar{b}$. Since our primary interest lies in the region for resonant CP-violating $tt'$ transitions, we may simplify calculation by making the approximations $\Delta m_t^2 = m_t^2 - m_{t'}^2 \ll m_t^2, m_{t'}^2$. Defining the ratio $r_t = \Delta m_t^2/m_{t'}^2$ and neglecting terms of order $A_{ij}^2$ where possible, we derive the simple expression for the CP asymmetry

$$A_{CP}^{pol} \approx - \frac{2\Im(V_{tb}^L V_{tb}^{L\ast} V_{tb}^{R\ast} V_{tb}^R)}{|V_{tb}^R|^2} \frac{r_t A_{tt}}{r_t^2 + (|V_{tb}^R|^2 + |V_{tb}^L|^2)^2 A_{tt}^2}. \quad (9.22)$$

The analytic form for $\overline{A}_{CP}^{pol}$ may also be obtained from Eq. (9.22), if one replaces $|V_{tb}^R|^2$ with $|V_{tb}^{L\ast}|^2$ and changes the overall sign. In Fig. 11, we have plotted the dependence of $A_{CP}^{pol}$ as a function of $r_t$ for different values of the mixing matrix combination $|V_{tb}^R|^2 + |V_{tb}^L|^2$. The CP asymmetry $\overline{A}_{CP}^{pol}$ is given in units of $3\Im(V_{tb}^L V_{tb}^{L\ast} V_{tb}^{R\ast} V_{tb}^R)/|V_{tb}^R|^2$, which is generally smaller than 0.5. As can be seen from Fig. 11, we recover the known feature of resonant CP violation through particle mixing, namely, CP violation may become maximal for small values of the parameter $r_t$, i.e., $10^{-4} < r_t < 10^{-1}$.

This mixing mechanism, through which CP violation is resonantly amplified, may take place in scatterings where top quarks are produced either singly or in pairs via gluon fusion processes. In fact, one expects to be able to analyze about $10^6 - 10^7$ top decays at the LHC for an integrated luminosity of 100 fb$^{-1}$. Obviously, various techniques in analyzing top quark polarization have been suggested in the literature [14,17,18], which may help to study resonant CP-violating effects at high-energy colliders. As has been shown in this section, CP violation may be of order one due to $tt'$ resonant transitions, especially when the mass difference $m_t - m_{t'}$ lies in the vicinity of the widths of the $t$ and $t'$ quarks. This gives a unique chance to probe such effects in future experiments and so unravel the underlying CP nature of the top-quark sector.
10 Conclusions

The CP-violating dynamics known from the $K^0\bar{K}^0$ system has been extended to processes that can take place at high-energy colliders. At low energies, one may carry out measurements based on the time evolution of the unstable kaons, whereas, at high energies, one is compelled to consider reactions that can only be described by scattering amplitudes. This constitutes a non-trivial generalization of the $K^0\bar{K}^0$ dynamics. In transition amplitudes, the role of time assumes its Fourier-conjugate variable, the energy. At high energies, it is therefore crucial to study the dependence of CP violation as a function of invariant mass energies and/or momenta of the asymptotic final states, such as charged leptons and jets.

As has been analyzed in Sections 2 and 8, CP-violating phenomena can be significantly enhanced through the mixing of two resonant particles that behave differently under CP and whose mass difference is comparable to their widths. In particular, the underlying mechanism for large CP violation induced by resonant bosonic as well as fermionic transitions has been clarified and studied carefully on a more rigorous field-theoretic basis. In this context, we have considered a resummation approach, which is implemented by the PT \cite{8} and hence preserves the gauge symmetries of the theory.

Models that may give rise to non-negligible bosonic $HA$ and/or fermionic $tt'$ mixings are discussed in Sections 6 and 9. In Sections 7 and 9, we have further analyzed the phenomenological implications of our mechanism for large CP-violating phenomena in the production, mixing and decay of a top quark or a Higgs particle at planned high-energy machines, such as the LHC, NLC and/or muon collider. Since high order $\varepsilon'$-type effects are generally suppressed near the resonant region, possible large CP-violating phenomena can naturally be accounted for by the mixing mechanism presented in this paper.

Our analysis given in Section 3 has shown that the CP-violating phenomenon becomes maximal, i.e., it could be of order one, when the two non-free particles are degenerate but possess an anomalous non-vanishing mixing. To the best of our knowledge, this is a novel aspect, which has not been addressed properly in the literature before. Moreover, we have paid special attention to possible constraints imposed by CPT invariance on the actual magnitude of CP violation in Section 4. On the other hand, approaches based on diagonalizing the effective Hamiltonian by means of a similarity transformation are inadequate to deal with an anomalously degenerate mixing system. For instance, in a $K^0\bar{K}^0$-like basis, such an anomalously degenerate mixing system is manifested by a non-diagonalizable effective Hamiltonian of the Jordan form. As has been demonstrated in Section 5, the transformation matrix $X$ becomes singular in such a case. However, our formalism does not display this kind of singularity, since it makes use of the well-defined
properties of the transition amplitudes, for which such similarity transformations are not needed. Therefore, it may be fair to say that our field-theoretic approach unifies features of the effective Hamiltonian [3] and/or the propagator formalism by Sachs [5] with those pertaining to the density matrix [31]. The significance of this resonantly amplified CP-violating mechanism for other anomalously degenerate physical systems appears to be an open challenge for further investigations.

Acknowledgments. I wish to thank Emmanuel Paschos for enlightening conversations and for his continuous encouragement to complete this work. I am also very much benefited from discussions with Pasha Kabir, Joannis Papavassiliou, Leo Stodolsky and Arkady Veinshtein.
A Mixing renormalization in scalar theories

If the $HA$ mixing occurs at the tree-level, one may then have to worry about the appearance of UV infinities that arise generally at higher loops \cite{24}. Facing this problem is unavoidable, since one-loop absorptive corrections must be considered in the calculation, otherwise our CP asymmetries (see, e.g., Eq. (4.17)) will vanish identically. Therefore, renormalization of the UV divergences in the presence of a $HA$ mixing requires particular care. Here, we shall extend the mixing renormalization programme, presented in \cite{25} for fermions, to the mixing of scalar particles.

Let us consider the kinetic part of the bare Lagrangian governing the mixing of $N$ real scalars $S_i^0\ (i = 1, 2, \ldots, N)$:

$$L_{\text{kin}}^0 = \frac{1}{2}(\partial_\mu S^0\partial^{\mu} S^0) - \frac{1}{2}S^0 (M^0)^2 S^0. \quad (A.1)$$

Here and in the following, we use the convention to denote quantities that are expressed in the flavour basis by a prime, e.g., $S_i'$, $M'_{ij}$, while we attach the superscript ‘0’ for all unrenormalized quantities. In general, the mass matrix $M^0$ in Eq. (A.1) is a $N \times N$ dimensional real, positive semi-definite and symmetric matrix. This matrix can then be diagonalized by performing an orthogonal rotation, $O^0$, of the weak fields $S_i^0$, i.e.,

$$(M^0)^2 = O^0 (M^0)^2 O^{0T}, \quad S^0 = O^0 S^0, \quad (A.2)$$

where the absence of a prime on the fields and the kinematic parameters indicates that these quantities are written in the mass basis. As a result, the bare mass matrix $M^0$ is a non-negative diagonal matrix of $N \times N$ dimensions. Following \cite{25}, we introduce counter-terms (CT’s) and express the bare quantities in terms of the renormalized ones:

$$O^0 = O + \delta O, \quad (A.3)$$
$$M^0 = M^0 + \delta M^0, \quad (A.4)$$
$$M^0 = M^0 + \delta M^0, \quad (A.5)$$
$$S^0 = \frac{Z^{1/2}}{\sqrt{2}} S' = (1 + \frac{1}{2} \delta Z') S', \quad (A.6)$$
$$S^0 = Z^{1/2} S = (1 + \frac{1}{2} \delta Z) S, \quad (A.7)$$

where $\delta M^2$ ($\delta M^2$) and $Z^{1/2}$ ($Z^{1/2}$) are the mass and wave-function renormalization constants in the flavour (mass) basis. Note that a novel CT for the orthogonal matrix $O$, $\delta O$, is induced by this procedure. If we impose that unitarity, precisely speaking orthogonality, of the theory should hold order by order in perturbation theory for the bare as well as
renormalized orthogonal matrix $O$, i.e., $OO^T = O^0O^0^T = 1$, it is then easy to find that the matrix $O\delta O^T$ is anti-orthogonal, viz.

$$O\delta O^T = -\delta OO^T.$$  \hspace{1cm} (A.8)

Taking Eqs. (A.3)-(A.7) into account, the one-loop CT Lagrangian reads:

$$\delta L_{\text{kin}} \equiv \frac{1}{4} \left[ (\partial_\mu S^T)(\delta Z + \delta Z^T)(\partial^\mu S) - S^T (2\delta M^2 + M^2\delta Z + \delta Z^T M^2) S \right].$$  \hspace{1cm} (A.9)

Since both the bare and CT Lagrangians are Hermitian, it is evident that only the dispersive parts of the two-point correlation functions should enter the renormalization.

We can now proceed renormalizing the one-loop transitions, $S_j \to S_i$. In general, these transitions are described by the unrenormalized self-energy functions $\Pi_{ij}(p^2)$. Considering the CT Lagrangian in Eq. (A.9), the renormalized self-energies $\hat{\Pi}_{ij}(p^2)$ may be written down

$$\hat{\Pi}_{ij}(p^2) = \Pi_{ij}(p^2) + \frac{p^2}{2} (\delta Z_{ij} + \delta Z_{ji}) - \delta_{ij} \delta M^2_{ij} - \frac{1}{2} (M^2_i \delta Z_{ij} + \delta Z_{ji} M^2_j).$$  \hspace{1cm} (A.10)

Note that $\Pi_{ij}(p^2) = \Pi_{ji}(p^2)$, which also implies that $\hat{\Pi}_{ij}(p^2) = \hat{\Pi}_{ji}(p^2)$. In addition, the renormalized self-energies satisfy the following OS renormalization conditions:

$$\Re e \hat{\Pi}_{ij}(M^2_j) = \Re e \hat{\Pi}_{ji}(M^2_i) = 0,$$  \hspace{1cm} (A.11)

$$\lim_{p^2 \to M^2_i} \frac{1}{p^2 - M^2_i} \Re e \hat{\Pi}_{ii}(p^2) = 0.$$  \hspace{1cm} (A.12)

We could also choose another renormalization scheme, in which the self-energies are renormalized by requiring that the complex pole positions of the matrix elements are not shifted. Even though such a scheme is shown to be gauge independent in the weak mixing limit, it is, however, more involved than OS renormalization, if the mixing becomes strong. In Appendix B, we will show that the difference between OS and pole-mass renormalization is of higher order in the absence of particle mixing; they differ at two loops. In the presence of a large mixing, OS renormalization constitutes a more natural scheme, since the respective eigenvectors of the OS-renormalized masses form an orthogonal (in general unitary) Hilbert space, whereas the eigenvectors of the pole masses do not. Furthermore, we calculate all the one-loop OS renormalized self-energies within the PT framework, so as to avoid problems arising from possible violations of gauge symmetries when resummation is considered.

From the OS conditions in Eqs. (A.11) and (A.12), we can now calculate the mass and wave-function CT’s in terms of bare self-energies:

$$\delta M^2_i = \Re e \Pi_{ii}(M^2_i),$$  \hspace{1cm} (A.13)
\[
\begin{align*}
\delta Z_{ii} &= -\Re \Pi_i^i(M_i^2), \quad (A.14) \\
\delta Z_{ij} &= \frac{2\Re \Pi_{ij}(M_j^2)}{M_i^2 - M_j^2}. \quad (A.15)
\end{align*}
\]

In Eq. (A.14), the prime on the self-energy denotes differentiation with respect to \(p^2\) at the position \(p^2 = M_i^2\). The remaining CT, \(\delta O\), is indispensable when one considers the renormalization of vertex interactions. For example, let us assume that the scalars \(S_i^0\) couple to the fermion field \(f^0\), e.g.,

\[
\mathcal{L}_{\text{int}}^0 = g^0 S_i^0 \bar{f}^0 f^0, \quad (A.16)
\]

where \(g^0\) is the bare coupling constant. As we will see in a moment, our derivation of mixing renormalization will not depend upon other model details, e.g., if additional fermions with scalar and/or pseudo-scalar couplings are present in the interaction Lagrangian (A.16).

Expressing \(\mathcal{L}_{\text{kin}}\) in terms of mass-eigenfields and renormalized quantities, we obtain the CT Lagrangian for \(\mathcal{L}_{\text{int}}\),

\[
\delta \mathcal{L}_{\text{int}} = g \delta O_{ji} S_j \bar{f} f + g O_{ki} \left( \frac{1}{2} \delta Z_{kj} + \delta_{kj} \frac{\delta g}{g} + \delta_{kj} \delta Z_f \right) S_j \bar{f} f. \quad (A.17)
\]

Since the mixing renormalization should not depend on the coupling constant CT, \(\delta g\), and the fermion wave-function renormalization, \(\delta Z_f\), one is then left with the fact that \(\delta O\) must only depend on \(O_{ij}\) and \(\delta Z_{ij}\). Employing Eq. (A.8), we can rewrite \(\delta \mathcal{L}_{\text{int}}\) in matrix notation as follows:

\[
\delta \mathcal{L}_{\text{int}} = g O^T \left( -\delta O O^T + \frac{1}{2} \delta Z \right) S \bar{f} f + \ldots, \quad (A.18)
\]

where the ellipses denote the other terms omitted in Eq. (A.18). One can now take advantage of the fact that the matrix \(\delta O O^T\) is an anti-orthogonal matrix and so re-express the latter in terms of the wave-function CT’s as

\[
\delta O_{ij} = \frac{1}{4} (\delta Z_{il} - \delta Z_{li}) O_{ij}. \quad (A.19)
\]

In this way, the anti-symmetric parts of \(\delta Z_{ij}\) are completely absorbed by the CT’s \((\delta O O^T)_{ij}\) in Eq. (A.18), whereas the remaining symmetric CT’s, \((\delta Z_{ij} + \delta Z_{ji})/4\), are necessary for the renormalization of the one-loop irreducible \(S_i \bar{f} f\) couplings. As has already been pointed out in [23] for the case of fermionic mixing which is here true as well, Eq. (A.19) is unique up to finite anti-orthogonal terms, \(\kappa_{ij}\), which quantify possible deviations of different mixing renormalization schemes. Nevertheless, the scheme used here is characterized by its simplicity and may hence be considered as quite natural. We will not pursue this issue any further. Instead, we remark that the above one-loop renormalization analysis for real scalars can equally well apply to complex or charged scalars, by making the replacements \(Z^T \to Z^\dagger\) and \(O^T \to O^\dagger\), where appropriate.


B Issues of mass renormalization

In our calculations, we have used OS renormalized masses for the resonant particles. Another physical renormalization scheme is obtained if one requires that the resummed propagators do not shift the complex pole positions of the S matrix. This renormalization also yields a gauge independent CT for the mass of the unstable particle at higher orders [66]. As has been shown in [8] under plausible assumptions, the very same property shares our PT resummation approach. Apart from being gauge independent, the higher order PT self-energies do not shift the physical complex pole [8,18,67]. Nevertheless, OS renormalization is still a good renormalization framework, since it is simpler than pole-mass renormalization. Here, we shall briefly review the conditions of pole-mass renormalization [66] and show that the difference between these two schemes is beyond one-loop order in the absence of mixing. If mixing among particles is present, pole-mass renormalization becomes more complicated. As we will see, such a complication may be avoided if one expresses the pole masses in terms of OS renormalized masses and self-energies.

For our illustrations, we shall first consider just one un-mixed scalar S. The inverse resummed propagator of S is then given by

\[ \Delta^{-1}(s) = s - (M^0)^2 + \Pi(s), \]  

(B.1)

with \( s = p^2 \). In the pole-mass renormalization, one makes use of the fact that the s-channel exchange transition elements will exhibit a complex pole at the position \( \bar{s} = M^2 - iM\Gamma \). In such a scheme, one imposes the physical condition

\[ \bar{s} - (M^0)^2 + \Pi(\bar{s}) = 0. \]  

(B.2)

In this scheme, \( \bar{M} \) and \( \Gamma \) are the physical pole mass and width of the particle S. The pole-mass CT may be found by writing \( (M^0)^2 = \bar{M}^2 + \delta \bar{M}^2 \) and substituting this expression into Eq. (B.2). Through order \( g^4 \), this leads to

\[ \delta \bar{M}^2 = \Re \Pi(\bar{s}) = \Re \Pi(\bar{M}^2) + \bar{M}\Gamma \Im \Pi' (\bar{M}^2), \]  

(B.3)

\[ \bar{M}\Gamma = \Im m \Pi(\bar{s}) = \Im m \Pi(\bar{M}^2) - \bar{M}\Gamma \Re \Pi'(\bar{M}^2). \]  

(B.4)

The coupled Eqs. (B.3) and (B.4) may be further disentangled perturbatively as

\[ \delta \bar{M}^2 = \Re \Pi(\bar{M}^2) + \Im m \Pi' (\bar{M}^2) \Im m \Pi (\bar{M}^2), \]  

(B.5)

\[ \bar{M}\Gamma = \Im m \Pi(\bar{M}^2) = 1 - \Re \Pi'(\bar{M}^2) \Im m \Pi (\bar{M}^2). \]  

(B.6)

It has been shown in [66] that the Z-pole mass CT \( \delta \bar{M}^2 \) defined in Eq. (B.3) is gauge independent through order \( g^4 \). The latter should also hold true for scalar particles present
in gauge field theories. Moreover, making use of a Taylor series expansion in Eq. (B.3), it is straightforward to find the relation between OS renormalized mass, $M$, and the pole mass $\overline{M}$. This relation is given by

$$M^2 = [1 - \frac{\Gamma}{M} \Im \hat{\Pi}(\overline{M}^2)] \overline{M}^2,$$

(B.7)

where the self-energy $\hat{\Pi}(s)$ is OS renormalized, as has been discussed in Appendix A. Eq. (B.7) explicitly demonstrates that the difference between the two masses enters through order $g^4$. Similarly, one can derive from Eq. (B.4) that the OS and pole renormalized width differ from one another at the three-loop order.

The situation is different if mixing among scalars is present. Let us consider the mixing of two scalars, e.g., $A$ and $H$, which give rise to two complex poles in S-matrix elements at positions $s_A = \overline{M}^2_A - i\overline{M}_A \Gamma_A$ and $s_H = \overline{M}^2_H - i\overline{M}_H \Gamma_H$. A straightforward way to calculate the pole-mass CT's is first to write down the bare masses of $A$ and $H$ as

$$\overline{M}^2_A = (M_A^0)^2, \quad \overline{M}^2_H = (M_H^0)^2,$$

(B.8)

and then impose the no-shift condition of the two complex poles, which is translated into the vanishing of the determinant of the inverse (unrenormalized) propagator, $\Delta^{-1}(s)$ in Eq. (2.2), i.e.

$$\det \begin{pmatrix} s - \overline{M}^2_A - \delta\overline{M}^2_A + \Pi_{AA}(s) & \Pi_{HA}(s) \\ \Pi_{HA}(s) & s - \overline{M}^2_H - \delta\overline{M}^2_H + \Pi_{HH}(s) \end{pmatrix} = 0.$$

(B.9)

Eq. (B.9) is quadratic in the variable $s$, leading to two complex eigenvalue equations:

$$s_A - \lambda_A(s_A) = 0, \quad s_H - \lambda_H(s_H) = 0,$$

(B.10)

In the kinematic range $s \approx \overline{M}^2_H \approx \overline{M}^2_A$, the functions $\lambda_A(s)$ and $\lambda_H(s)$ may be approximated by the two complex mass eigenvalues of the effective Hamiltonian for the $HA$ system (cf. Eq. (5.4)). Retaining the full $s$-dependence of the functions $\lambda_A$ and $\lambda_H$, we find that their explicit form is given by

$$\lambda_{A(H)}(s) = \frac{1}{2} \left\{ (M_A^0)^2 + (M_H^0)^2 - \Pi_{AA}(s) - \Pi_{HH}(s) \\ - (\pm) \left[ ((M_A^0)^2 - (M_H^0)^2 - \Pi_{AA}(s) + \Pi_{HH}(s))^2 + 4\Pi_{HA}(s)^2 \right]^{1/2} \right\},$$

(B.11)

where the bare masses of $A$ and $H$ are given in Eq. (B.8). Unlike Eq. (5.4), Eq. (B.10) is a coupled system of two complex (or four real) equations with four unknown parameters: the two pole-mass CT's, $\delta\overline{M}^2_A$ and $\delta\overline{M}^2_H$, and the two pole widths, $\overline{M}_A \Gamma_A$ and $\overline{M}_A \Gamma_H$. Instead
of solving this $4 \times 4$ system, the best way is to renormalize the $HH$ and $AA$ self-energies in the OS scheme, by decomposing the $H$ and $A$ bare masses into the form given in Eq. (A.5). Then, Eq. (B.10) together with Eq. (B.11) just display the relations between pole masses and OS renormalized masses in the presence of a non-vanishing particle mixing. This is precisely the avenue that has been followed throughout our analysis for the bosonic as well as fermionic case.
References

[1] J.H. Christensen, J.W. Cronin, V.L. Fitch, and R. Turlay, *Phys. Rev. Lett.* **13** (1964) 138.

[2] For reviews, see, *e.g.*, E.A. Paschos and U. Türke, *Phys. Rep.* **178** (1989) 145; W. Grimus, *Fortschr. Phys.* **36** (1988) 201; R. Decker, *Fortschr. Phys.* **37** (1989) 657; P.K. Kabir, “The CP puzzle,” Academic Press, London and New York, 1968; B. Weinstein and L. Wolfenstein, *Rev. Mod. Phys.* **65** (1993) 1113.

[3] V.F. Weisskopf and E.P. Wigner, *Z. Phys.* **63** (1930) 54; **65** (1930) 18.

[4] T.D. Lee, R. Oehme and C.N. Yang, *Phys. Rev.* **106** (1957) 340.

[5] R.G. Sachs, *Ann. Phys.* (N.Y.) **22** (1963) 239.

[6] For reviews, see, *e.g.*, K.-I. Aoki, Z. Hioki, R. Kawabe, M. Konuma and T. Muta, *Prog. Theor. Phys. Suppl.* **73** (1982) 1; M. Böhm, H. Spiesberger and W. Hollik, *Fortschr. Phys.* **34** (1986) 687.

[7] For a phenomenological discussion of the problem of gauge invariance with unstable particles, see, *e.g.*, M. Nowakowski and A. Pilaftsis, *Z. Phys.* **C60** (1993) 121.

[8] J. Papavassiliou and A. Pilaftsis, *Phys. Rev. Lett.* **75** (1995) 3060; *Phys. Rev. D53* (1996) 2128; *Phys. Rev. D54* (1996) 5315, and references therein.

[9] J.M. Cornwall, *Phys. Rev. D26* (1982) 1453; J.M. Cornwall and J. Papavassiliou, *Phys. Rev. D40* (1989) 3474.

[10] A. Pilaftsis, *Z. Phys.* **C47** (1990) 95.

[11] A. Pilaftsis and M. Nowakowski, *Mod. Phys. Lett.* **A6** (1991) 1933.

[12] A. Pilaftsis and M. Nowakowski, *Phys. Lett.* **B245** (1990) 185.

[13] R. Cruz, B. Grzadkowski and J.F. Gunion, *Phys. Lett.* **289**, (1992) 440; D. Atwood, G. Eilam, A. Soni, R.R. Mendel and R. Migneron, *Phys. Rev. Lett.* **70** (1993) 1364; T. Arens and L.M. Sehgal, *Phys. Rev. D51* (1995) 3525.

[14] N. Arkani-Hamed, J.L. Feng, L.J. Hall and H.-C. Cheng, [hep-ph/9704205]; D. Bowser-Chao and W.-Y. Keung, [hep-ph/9704219].
[15] For instance, see, J.M. Gerard, W. Grimus, A. Raychandhuri and G. Zoupanos, Phys. Lett. B128 (1980) 349; J.F. Donoghue, H.P. Nilles and D. Wyler, Phys. Lett. B128 (1983) 55; M.J. Duncan, Nucl. Phys. B221 (1983) 285.

[16] A. Pilaftsis, Phys. Rev. Lett. 77 (1996) 4996.

[17] For a T/CP-noninvariant observable sensitive to $K^0\bar{K}^0$ mixing, see also, P.K. Kabir, Phys. Rev. D2 (1970) 540.

[18] K. Philippides and A. Sirlin, Phys. Lett. B367 (1996) 377.

[19] See, e.g., G. Cvetic, M. Nowakowski and A. Pilaftsis, Phys. Lett. B301 (1993) 77.

[20] For CP violation through ZZ' mixing, see also, A.S. Joshipura and S.D. Rindani, Phys. Rev. D46 (1992) 3008.

[21] C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039; I. Dunietz, O.W. Greenberg and D.-D. Wu, Phys. Rev. Lett. 55 (1985) 2935.

[22] J. Bernabéu, G.C. Branco and M. Gronau, Phys. Lett. B169 (1986) 243.

[23] L. Lavoura and J.P. Silva, Phys. Rev. D50 (1994) 4619.

[24] A. Sirlin, Nucl. Phys. B71 (1974) 29; Rev. Mod. Phys. 50 (1978) 573; A. Denner and T. Sack, Nucl. Phys. B347 (1990) 203.

[25] B.A. Kniehl and A. Pilaftsis, Nucl. Phys. B474 (1996) 286.

[26] For a comprehensive discussion, see, e.g., L. Wolfenstein, Phys. Rev. D43 (1991) 151.

[27] J. Liu and G. Segrè, Phys. Rev. D49 (1994) 1342.

[28] T.D. Lee and L. Wolfenstein, Phys. Rev. B138 (1965) 1490.

[29] For a discussion of Jordan-type Hamiltonians in quantum mechanics, see, A. Bohm et al., quant-ph/9705011, and references therein.

[30] J.S. Bell and J. Steinberger, Proc. Int. Conf. on Elementary Particles, Oxford 1965, ed. T.R. Walsh (Rutherford Lab., 1966).

[31] See, e.g., B. Kayser and L. Stodolsky, Phys. Lett. B359 (1995) 343; MPI-PhT/96-112 (hep-ph/9610522).

[32] L. Stodolsky, Phys. Rev. D1 (1970) 2683.
[33] W. Bernreuther, T. Schröder and T.N. Pham, Phys. Lett. B279 (1992) 389.

[34] A. Pilaftsis and M. Nowakowski, Int. J. Mod. Phys. A9 (1994) 1097; E9 (1994) 5849.

[35] N.G. Deshpande and E. Ma, Phys. Rev. D18 (1978) 2574.

[36] For example, see, A. Pilaftsis, Z. Phys. C55 (1992) 275.

[37] A. Ilakovac, B.A. Kniehl, and A. Pilaftsis, Phys. Lett. B317 (1993) 609.

[38] For example, see, Y. Kizikuri and N. Oshimo, Phys. Rev. D46 (1992) 3025.

[39] B.A. Kniehl, Phys. Rep. 240 (1994) 211.

[40] A. Denner, S. Dittmaier, and G. Weiglein, Nucl. Phys. B440 (1995) 95; S. Hashimoto, J. Kodaira, K. Sasaki and Y. Yasui, Phys. Rev. D50 (1994) 7066; J. Papavassiliou, Phys. Rev. D51 (1995) 856.

[41] A. Pilaftsis, Nucl. Phys. B487 (1997) 467.

[42] V. Barger, M.S. Berger, J.F. Gunion, and T. Han, Phys. Rev. Lett. 75 (1995) 1462; Univ. of California preprint, UCD-96-6, and references therein.

[43] J.F. Donoghue and G. Valencia, Phys. Rev. Lett. 58 (1987) 451; E60 (1988) 243.

[44] M. Nowakowski and A. Pilaftsis, Mod. Phys. Lett. A4 (1989) 821; Z. Phys. C42 (1989) 449.

[45] M.B. Gavela, F. Iddir, A. Le Yaouanc, L. Oliver, O. Pene, and J.C. Raynal, Phys. Rev. D39 (1989) 1870.

[46] W. Bernreuther, O. Nachtmann, P. Overmann and T. Schröder, Nucl. Phys. B388 (1992) 53.

[47] C.R. Schmidt and M.E. Peskin, Phys. Rev. Lett. 69 (1992) 410.

[48] H. Anlauf, W. Bernreuther and A. Brandenburg, Phys. Rev. D52 (1995) 3803; E53 (1996) 1725.

[49] D. Chang and W.-Y. Keung, Phys. Lett. B305 (1993) 261.

[50] B. Grzadkowski and J.F. Gunion, Phys. Lett. B350 (1995) 218; D. Atwood and A. Soni, Phys. Rev. D52 (1995) 6271.
[51] If $M_A > 2M_Z$ in the MSSM, then $\theta \simeq \beta - \pi/2$ and $M_A \simeq M_H$. A detailed analysis of such a scenario may be found by G.L. Kane, C. Kolda, L. Roszkowski, and J.D. Wells, *Phys. Rev. D49* (1994) 6173. For a review, see, J.F. Gunion, H.E. Haber, G. Kane, and S. Dawson, *The Higgs Hunters Guide*, Addison Wesley (1990).

[52] For instance, see, P. Arnowitt and P. Nath, *Phys. Rev. Lett. 69* (1992) 725; G.G. Ross and R.G. Roberts, *Nucl. Phys. B377* (1992) 571.

[53] Particle Data Group, R.M. Barnett *et al.*, *Phys. Rev. D54* (1996) 1.

[54] K. Hagiwara, J. Kanzaki and H. Murayama, KEK report, DTP-91-18 (unpublished); V. Barger, K. Cheung, B.A. Kniehl and R.J.N. Phillips, *Phys. Rev. D46* (1992) 3725.

[55] B. Grzadkowski, *Phys. Lett. B338* (1994) 71; G. Cvetic, *Phys. Rev. D48* (1994) 5280.

[56] S.M. Barr and A. Zee, *Phys. Rev. Lett. 65* (1990) 21; J.F. Gunion and R. Vega, *Phys. Lett. B251* (1990) 157; D. Chang, W.-Y. Keung and T.C. Yuan, *Phys. Rev. D43* (1991) 14; R.G. Leigh, S. Paban and R.-M. Xu, *Nucl. Phys. B352* (1991) 45; S.M. Barr, *Phys. Rev. D47* (1993) 2025.

[57] T. Hayashi, Y. Koide, M. Matsuda, M. Tanimoto and S. Wakaizumi, *Phys. Lett. B348* (1995) 489.

[58] S. Weinberg, *Phys. Rev. D42* (1990) 860.

[59] J.M. Cornwall, D.N. Levin and G. Tiktopoulos, *Phys. Rev. D10* (1974) 1145; C.E. Vayonakis, *Lett. Nuovo Cim. 17* (1976) 383; B.W. Lee, C. Quigg and H. Thacker, *Phys. Rev. D16* (1977) 1519; M.S. Chanowitz and M.K. Gaillard, *Nucl. Phys. B261* (1985) 379.

[60] M. Fukugita and T. Yanagida, *Phys. Lett. B174* (1985) 45; P. Langacker, R.D. Peccei and T. Yanagida, *Mod. Phys. Lett. A1* (1986) 541; M. Flanz, E.A. Paschos and U. Sarkar, *Phys. Lett. B345* (1995) 248; M. Flanz, E.A. Paschos, U. Sarkar and J. Weiss, *Phys. Lett. B389* (1996) 693.

[61] R. Slansky, *Phys. Rep. 79* (1981) 1.

[62] J.L. Hewett and T.G. Rizzo, *Phys. Rep. 183* (1989) 193.

[63] J.G. Körner, A. Pilaftsis and M.M. Tung, *Z. Phys. C63* (1994) 575.

[64] P. Bamert, C.P. Burgess, J.M. Cline, D. London and E. Nardi, *Phys. Rev. D54* (1996) 4275.
[65] J. Papavassiliou, *Phys. Rev.* **D50** (1994) 5958.

[66] A. Sirlin, *Phys. Rev. Lett.* **67** (1991) 2127; *Phys. Lett.* **B267** (1991) 240.

[67] K. Philippides and A. Sirlin, *Nucl. Phys.* **B477** (1996) 59.
Captions of remaining figures

**Fig. 5:** (a) Numerical estimates of production cross-sections and CP violation for \( \mu_{L,R}^+ \mu_{L,R}^- \to h^+ (H^+) \), \( A^* \to f \bar{f} \) as a function of c.m. energy \( \sqrt{s} \). (b) CP asymmetry versus \( x_A = \Pi^{SA}/\Im m(\hat{\Pi}^{SS} - \hat{\Pi}^{AA}) \), with \( S = h, H \).

**Fig. 6:** Numerical estimates of CP asymmetries for \( e^+ e^- \to h^+ (H^+) \), \( A^* \to f_L \bar{f}_L X \) at the NLC as a function of (a) \( f \bar{f} \) invariant mass and (b) \( x_A = \Pi^{SA}/\Im m(\hat{\Pi}^{SS} - \hat{\Pi}^{AA}) \), with \( S = h, H \).

**Fig. 11:** Numerical estimates of CP asymmetries in the production of polarized top quarks at the LHC as a function of the ratio \( r_t = (m_t^2 - m_t'^2)/m_t^2 \) for different values of the mixing matrix combination \( |V_{tb}^R|^2 + |V_{tb}^L|^2 = 1.0, 0.5, 0.1 \).
