Composite fermions in the neighborhood of $\nu = 1/3$

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We present extensive comparisons of the composite fermion theory with exact results in the filling factor range $2/5 > \nu > 1/3$, which affirm that the composite fermion theory correctly describes the qualitative reorganization of the low energy Hilbert space of the strongly correlated electrons, and predicts eigenenergies with an accuracy of $\sim 0.1\%$. These facts establish the basic validity of the composite fermion description in this filling factor region.

This article concerns the applicability of the composite fermion theory in the filling factor region $2/5 > \nu > 1/3$, which has come into focus because of the recent observation\cite{1,2} of fractional quantum Hall effect (FQHE)\cite{3} at several fractions in this range, for example $\nu = 4/11$. Given that the CF theory\cite{1,2,3} provides a unified and quantitatively accurate description of the FQHE liquid in a large range of lowest Landau level (LL) fillings, it might seem natural that it also captures the physics in the filling factor range $2/5 > \nu > 1/3$. This article will review earlier theoretical studies and present more extensive recent theoretical tests which confirm the validity of the CF description in this parameter region.

COMPOSITE FERMI ON BASICS

The central principle of the composite fermion theory is that interacting electrons at filling factor $\nu$ minimize their interaction energy by transforming into weakly interacting composite fermions. The composite fermions experience an effective magnetic field, and have a filling factor $\nu^*$, where $\nu$ and $\nu^*$ are related by $\nu = \nu^*/(2p\nu^* \pm 1)$, with the even integer $2p$ denoting the vorticity of the composite fermion. The formation of composite fermions leads to a profound reorganization of the low energy Hilbert space of the strongly correlated electrons. In particular, a gap opens up at $\nu = n/(2p\nu^* \pm 1)$, with the even integer $2p$ denoting the vorticity of the composite fermion. The formation of composite fermions leads to a profound reorganization of the low energy Hilbert space of the strongly correlated electrons. In particular, a gap opens up at $\nu = n/(2p\nu^* \pm 1)$, with the even integer $2p$ denoting the vorticity of the composite fermion.

Briefly, our method for calculating the low energy spectrum is as follows. We will denote the total number of composite fermions (which is the same as the total number of electrons) by $N$, the CF filling factor by $\nu^* = n + \bar{\nu}$, and the number of composite fermions in the topmost partially occupied level by $\bar{N}$. We take all basis states of electrons at filling $\nu^* = n + \bar{\nu}$ in which the $n$ lowest electronic LLs are fully occupied and the $(n + 1)^{st}$ LL has filling $\bar{\nu}$. (States involving excitations of electrons across a LL are neglected at the lowest order approximation.) From this basis, we construct a basis for composite fermions at $\nu^* = n + \bar{\nu}$ following the standard procedure\cite{8}. The advantage of going to the CF description is that the dimension of the Fock space in the CF theory is exponentially small compared to the dimension of the Fock space of original electron problem. Indeed, when $\bar{\nu} = 0$, i.e., $\nu^* = n$, there is only a single state in the low energy band, and nothing further remains to be done (except, of course, to evaluate its properties). This state is the $\nu = n/(2p\nu^* + 1)$ FQHE ground state. When there are many basis states, we diagonalize the Coulomb Hamiltonian in the CF basis. We will only give the results here, referring the reader to the literature for the construction of the wave functions and the diagonalization procedure\cite{8,9}. It is stressed that all energies obtained in our calculations are exact variational upper bounds. (A related approach for treating interacting composite fermions\cite{10,11,12} formulates the problem in terms of fermions with pair-wise interaction, which is deduced either from exact diagonalization or from CF theory by considering a system which has only two composite fermions in the second CF level.)

Our focus in this article will be on the filling factor range $2/5 > \nu > 1/3$. It has been confirmed in the past that the states at $\nu = 1/3$ and $2/5$ are well described as one and two filled level of composite fermions. In this article, we ask if the composite fermion description continues to be valid at intermediate filling factors, where the composite fermions filling factor is $\nu^* = 1 + \bar{\nu}$. The underlying physical picture is shown in Fig. 1: as the filling factor is increased from $\nu = 1/3$ ($\nu^* = 1$), the second level of composite fermions gets increasingly more densely populated, eventually becoming completely full and producing incompressibility at $\nu = 2/5$ ($\nu^* = 2$).

The spherical geometry is used in all our calculations below, in which electrons move on the surface of a sphere, with the total flux through the surface of the sphere given by $2Q$ in units of the quantum of flux $\phi_0 = hc/e$, where $Q$ is either an integer or a half integer. The Landau levels are angular momentum shells in this geometry, with the lowest LL corresponding to angular momentum $Q$, the second to $Q + 1$, and so on. The many body eigenstates are labeled by their total orbital angular moment-
FIG. 1: The physical picture for the evolution of the many-body state as the filling factor varies between $\nu = 1/3$ and $\nu = 2/5$ (a and d, respectively). Panel b shows a solitary composite fermion in the second level, which is interpreted as a “quasiparticle” of $\nu = 1/3$. Panel c shows an intermediate filling factor in the range $2/5 > \nu > 1/3$.

Excitations at $\nu = 1/3$

The electron filling factor $\nu = 1/3$ maps into filling factor $\nu^* = 1$ of composite fermions: the ground state is interpreted as one filled Landau level of composite fermions (Fig. 1a) and excited states are obtained by creating particle hole pairs of composite fermions.

The effective flux at $\nu^* = 1$ is $2Q^* = N - 1$. For the lowest energy excitation, one particle is promoted to the next level, thus creating a particle hole pair of composite fermions. The particle and the hole have angular momenta $Q^* + 1 = (N + 1)/2$ and $Q^* = (N - 1)/2$, respectively, giving the total angular momentum values of $L = 1, 2, \cdots, N$ for the pair. It turns out that the particle hole state with $L = 1$ at $\nu^* = 1$ does not produce any $L = 1$ state for composite fermions; the wave function is annihilated upon lowest LL projection [13]. Thus the CF theory predicts single multiplets at $L = 2, \cdots, N$, which is in agreement with exact results. The energies (see $E^{(1)}$ in Table I) are in good agreement with the exact energies, especially in view of the fact that the wave functions contain no adjustable parameters.

To understand higher energy excitations we consider a mixture of states containing zero, one or two excitons of composite fermions. In this case, the basis at $\nu^* = 1$ contains several states at each $L$, producing, in turn, many states at each $L$ for $\nu = 1/3$. Diagonalization in this basis produces the spectrum shown in Fig. 2. With this expanded basis, the lowest energy in each $L$ sector predicted by the CF theory, given in Table I, is practically exact.

It is noted that finite thickness, disorder, and Landau level mixing, neglected in this article, must be taken into account for an accurate quantitative comparison with experiment. The excitation energies observed in Raman experiments are in qualitative and semi-quantitative agreement with the CF theory at $\nu = 1/3$ and 2/5, and nicely demonstrate the formation of Landau-like levels in the intermediate fillings [14].
TABLE I: The second column gives the exact energy per particle for the lowest energy at orbital angular momenta \( L = 0, 1, \ldots, 10 \) for 10 particles at \( \nu = 1/3 \). \( E_{CF}^{(1)} \) is the energy of the state with a single particle hole pair of composite fermion, obtained with the help of a wave function with no adjustable parameters. \( E_{CF}^{(2)} \) is the energy from Fig. (2), which is obtained by diagonalization in the Fock space of all states containing 0, 1, or 2 pairs of particle hole excitations of composite fermions. The last column gives the % error for cases where \( E_{CF}^{(2)} \) differs from \( E_{exact} \) significantly. All energies are quoted in units of \( e^2/\epsilon l \).

| \( L \) | \( E_{exact} \) | \( E_{CF}^{(1)} \) | \( E_{CF}^{(2)} \) | error (%) |
|-------|-------------|-------------|-------------|-----------|
| 0     | -0.41062897 | -0.41039(2) | -0.41062(3) | -         |
| 1     | -0.39609058 | -0.3957(1)  | 0.1         |           |
| 2     | -0.39836847 | -0.3981(1)  | 0.07        |           |
| 3     | -0.39985548 | -0.3996(1)  | 0.06        |           |
| 4     | -0.40203892 | -0.4017(1)  | 0.02        |           |
| 5     | -0.40354097 | -0.40343(4) | 0.03        |           |
| 6     | -0.40268431 | -0.40261(5) | 0.02        |           |
| 7     | -0.40123511 | -0.40127(5) | 0.014(1)    |           |
| 8     | -0.40115314 | -0.40115(5) | -           |           |
| 9     | -0.40157815 | -0.40145(1) | -           |           |
| 10    | -0.40120351 | -0.40127(5) | -           |           |

FIG. 3: The density profile of a CF-particle at the South pole and a CF-hole at the North pole at \( \nu = 1/3 \) (solid line). For comparison, the density profile for an electron in the second LL (South pole) and a hole in the lowest LL (North pole) at \( \nu = 1 \) is also given (dotted line). The distance is measured in units of the magnetic length, from the North pole to the South pole. Source: Ref. [15].

FIG. 4: \( \Delta E \) is the energy difference between two wave functions for the \( \nu = 1/3 \) quasiparticle (Laughlin’s [20], and that based on composite fermions [4, 19]) as a function of \( N \), the total number of particles. The energy difference in the thermodynamic limit is 0.012(2)\( e^2/\epsilon l \), which is approximately 15% of the energy of the quasiparticle (\( \sim 0.07 e^2/\epsilon l \)).

already provides a strong hint that the state in the filling factor range \( 2/5 > \nu > 1/3 \) is to be interpreted in terms of composite fermions partially occupying the second level.

\[ 2/5 > \nu > 1/3 \]

The CF theory has been tested in the past at \( \nu = n/(2pm + 1) \), as well as at fractions in between. For the region of our interest, where \( 2 > \nu^* > 1 \), the problem...
maps into $\tilde{N} = N - (2Q^* + 1)$ composite fermions in $L^* = Q^* + 1$ shell (i.e., the second level), with $Q^* = Q - N + 1$.

Fig. 5 shows a study in the filling factor range $2/5 > \nu > 1/3$ for $N = 8$ particles. As the flux changes from $2Q = 20$ to $2Q = 17$, the number of composite fermions in the second level, $\tilde{N}$, changes from $\tilde{N} = 1$ to $\tilde{N} = 4$. Fig. 5 presents results for $N = 12$ particles at flux $2Q = 29$, which maps into $N = 12$ composite fermions at $2Q^* = 7$. The lowest level can accommodate 8 composite fermions and the second level has $\tilde{N} = 4$ composite fermions. In all of these figures, the dashes are the exact energies and the dots are the energies predicted by the CF theory. The CF energies in Fig. 5 are essentially the same as those in Fig. 3 of Ref. 9, but calculated with greater accuracy. (Slightly lower energies than those quoted in Ref. 9 are obtained by varying the function used for importance sampling in our Monte Carlo.) The following points are noteworthy.

The CF theory makes the following prediction for the total angular momentum quantum numbers for the low energy states:

- $(N, Q) = (8, 10)$ maps into $\tilde{N} = 1$ at $L^* = 4$. Here we have only one multiplet with total angular momentum $L = 4$.
- $(N, Q) = (8, 14)$ maps into $\tilde{N} = 2$ at $L^* = \frac{7}{2}$. The possible total angular momenta, within the constraints of the Pauli principle, are given by
  \[
  \frac{7}{2} \otimes \frac{7}{2} = 0 \oplus 2 \oplus 4 \oplus 6
  \]
- $(N, Q) = (8, 9)$ maps into $\tilde{N} = 3$ at $L^* = 3$. The possible total angular momenta, incorporating the Pauli principle, are
  \[
  3 \otimes 3 \otimes 3 = 0 \oplus 2 \oplus 3 \oplus 4 \oplus 6
  \]
- $(N, Q) = (8, 17)$ maps into $\tilde{N} = 4$ at $L^* = \frac{5}{2}$. The possible total angular momenta now are
  \[
  \frac{5}{2} \otimes \frac{5}{2} \otimes \frac{5}{2} \otimes \frac{5}{2} = 0 \oplus 2 \oplus 4
  \]

In this case, the degeneracy of the angular momentum shell is $2L^* + 1 = 6$, so it would have been easier to consider two fermion holes in the $L^* = \frac{5}{2}$ shell.

- $(N, Q) = (12, 20)$ is equivalent to $\tilde{N} = 4$ fermions in the $9/2$ angular momentum shell. Here we have
  \[
  \frac{9}{2} \otimes \frac{9}{2} \otimes \frac{9}{2} \otimes \frac{9}{2} = 0^2 \oplus 2^2 \oplus 3 \oplus 4^3 \oplus 5 \oplus 6^3 \oplus 7 \oplus 8^2 \oplus 9 \oplus 10 \oplus 12
  \]

In all cases, the angular momentum quantum numbers predicted by the CF theory match perfectly the quantum numbers of the states forming the low energy bands in the exact spectra of Figs. 4 and 5. These results give a clear evidence that the strongly interacting system of electrons in the lowest Landau level resembles a system of weakly interacting fermions at an effective magnetic field.

The most convincing theoretical verification of the CF theory comes from comparing the exact energies (dashes in Figs. 5 and 6) with those predicted by the CF theory (dots). For example, for the 12 particle system the lowest energy in each angular momentum sector is predicted with an accuracy of $\sim 0.1\%$ or better (Table II). (For higher energies the accuracy is somewhat worse, but still very good. The reason for the discrepancy is the neglect of mixing between different CF levels.)

The interpretation of the actual state in the range $2/5 > \nu > 1/3$ in terms of composite fermions (Fig. 1) is thus established. A crucial aspect of composite fermions is that they are weakly interacting, which is what makes them the “true” quasiparticles of the FQHE, in the same sense as the Landau quasiparticles are the true quasiparticles for the normal Fermi liquid or the Cooper pairs for the superconductor. The weakness of the residual interaction can be seen in the above studies. If the composite fermions were non-interacting then all dots would be degenerate. The splitting between various states is therefore a measure of the residual interaction between composite fermions. Take, for example, the 12 particle system at $\nu = 4/11$, the low energy band (dark dashes) of...
which is analogous to that of four composite fermions at \( \nu = 1/3 \). The width of the band in Fig. 6 is \( \sim 0.07e^2/\ell \) (note that the energies shown are single particle energies), which, when compared to \( \sim 0.7e^2/\ell \), the width of the band for 4 electrons at 1/3, indicates that the residual interaction between the composite fermions is an order of magnitude weaker than the interaction between electrons at \( \nu = 1/3 \).

\[ \nu = 4/11 \]

If the composite fermions were completely non-interacting, then there would be no FQHE at \( \nu^* = 1+1/3 = 4/11 \), just as there would be no FQHE for electrons at \( \nu = 1+1/3 \) if the Coulomb interaction were switched off. However, there is a weak residual interaction between composite fermions, which may possibly cause a gap to open, thereby producing a FQHE at \( \nu = 4/11 \). This provides a simple scenario for FQHE at 4/11 (and a host of other new fractions). However, going beyond mere scenarios, the question is whether the residual interaction between composite fermions is sufficiently strongly repulsive at short distances to produce a FQHE of composite fermions here. A quantitative investigation of the issue requires a determination of minute energy differences accurately. Given that we are dealing with a strongly correlated many body state, the situation might seem intractable at first, but, as seen in Fig. 6 and Table II, the wave functions of the CF theory possess the desired accuracy.

The systems at flux \( 2Q = (11/4)N - 4 \) are believed to represent the thermodynamic filling \( \nu = 4/11 \). Thus, the 12 particle system at \( 2Q = 29 \) and the 8 particle system at \( 2Q = 18 \) are relevant for possible FQHE at \( \nu = 4/11 \). (Note that the quantum numbers of the low energy states in exact diagonalization are identical to the quantum numbers of all states of \( \tilde{N} \) fermions at \( \nu = 1/3 \).) For \( N = 12 \), the ground state appears to be incompressible, with an \( L = 0 \) ground state and a gap to excitations. However, incompressibility in the thermodynamic limit requires one to study how the gap behaves as a function of \( N \) and to show that it survives in the \( N^{-1} \rightarrow 0 \) limit. That is particularly crucial here because, as Wójs and Quinn\[24\] have noted, the exact spectrum for \( N = 8 \) particles at \( \nu = 4/11 \) (see Fig. 6) indicates a compressible ground state. Also, while 12 may appear to be a reasonably large number of particles, the relevant number is \( \tilde{N} = 4 \), the number of composite fermions in the second level, just as at filling factor \( \nu = 1+1/3 \) the relevant number is the number of electrons in the second LL, with the electrons filling up the lowest LL being inert. To further investigate if the incompressibility persists in larger systems, Mandal and Jain\[9\] carried out CF diagonalization for \( N = 16, 20, \) and 24 particles, for which \( \tilde{N} = 5, 6, \) and 7. It was found that as \( N \) changes in steps of four, the

### Table II: Comparison of the CF prediction for the lowest energy band at \( \nu = 4/11 \) for \( N = 12 \) particles with the exact eigenenergies from Ref.\[23\]. The number of multiplets at each \( L \) is predicted correctly by the CF theory.

| \( L \) | \( E^{\text{exact}} \) | \( E_{\text{CF}} \) |
|------|-------|-------|
| 0    | -0.441214 | -0.441051(89) |
|      | -0.436440 | -0.435323(69) |
| 2    | -0.440457 | -0.439922(86) |
|      | -0.437646 | -0.437268(59) |
| 3    | -0.438226 | -0.437516(72) |
|      | -0.439280 | -0.438904(58) |
| 4    | -0.439422 | -0.439260(46) |
|      | -0.436844 | -0.435864(49) |
| 5    | -0.438904 | -0.438400(63) |
|      | -0.440547 | -0.440072(75) |
| 6    | -0.439337 | -0.439050(42) |
|      | -0.437093 | -0.436488(80) |
| 7    | -0.439190 | -0.438794(79) |
| 8    | -0.439613 | -0.439404(98) |
|      | -0.437615 | -0.437108(88) |
| 9    | -0.438632 | -0.438215(10) |
| 10   | -0.439507 | -0.439160(50) |
| 12   | -0.439287 | -0.439000(11) |
system alternates between compressible \( N = 8, 16, 24 \) and incompressible \( N = 12, 20 \). The fact that two of the systems suggest an incompressible state makes the interpretation of the results somewhat less certain, but we do not know of any justification for disregarding the other systems. On the basis of the facts that (i) for all securely understood FQHE states to date, the spectrum reflects an incompressible state at all allowed values of \( N \); and (ii) for states that are compressible in the thermodynamic limit, the spectrum can at times resemble an incompressible state for finite systems, it was concluded\(^\[22\]\) that the available results do not support incompressibility at \( \nu = 4/11 \) for the model considered.

This conclusion is at odds with the experimental observation\(^\[1\]\) of FQHE at \( \nu = 4/11 \). At the same time, the comparisons with exact results show that the CF theory is very accurate. How does one reconcile these two facts? Several possibilities come to mind. (i) A partially spin polarized FQHE state at \( \nu = 4/11 \) has been found to be stable theoretically\(^\[26\]\), and one may wonder if the experimental \( 4/11 \) state could be partially polarized. From our calculations, it is possible to obtain a crude estimate of the crossover magnetic field. The Coulomb energy of the partially polarized state is lower by approximately \( 0.005 e^2/\ell l \) per particle compared to the Coulomb energy of the fully polarized state. Which one is the ground state depends on the Zeeman splitting: at sufficiently low Zeeman energies the partially polarized state is the ground state, but as the Zeeman energy is increased, at some point there will be a transition into the fully polarized state. Because the partially polarized state has one-fourth of the electron spins flipped, the crossover magnetic field is given by the formula \( 0.005 e^2/\ell l = E_Z/4 \), where \( E_Z \) is the energy required to flip a single spin. For GaAs parameters, the crossover magnetic field is estimated to be \( \approx 12 \, \text{T} \) for the idealized model, which is fairly high. However, the \( \nu = 4/11 \) FQHE has been observed at higher magnetic fields, which appears to rule out a partially polarized state here. (ii) Another possibility is that the assumption of neglecting finite thickness or Landau level mixing is not sufficiently accurate for the problem of the \( 4/11 \) FQHE. These effects modify the form of the interaction between the electrons. Usually, a slight modification of the interaction only causes a quantitative correction, but because of the extremely tiny energy scales governing the physics here, it is possible that it may alter the ordering of the energy levels to produce incompressibility. Unfortunately, the neglected effects are not understood at the same level of accuracy as the CF theory of the FQHE. (iii) We have explored above a \( 4/11 \) FQHE state that is analogous to \( \nu = 1 + 1/3 \) FQHE of composite fermions. It is, in principle, possible that the \( 4/11 \) FQHE state is an entirely new type of incompressible state of composite fermions.

While we must await the resolution of the \( 4/11 \) enigma, our calculations make a compelling case that the CF theory provides the correct framework for an investigation of the issue.

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