Maxwell’s $(D, H)$ excitation fields: lessons from permanent magnets

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Abstract
Macroscopic Maxwellian electrodynamics consists of four field quantities along with electric charges and electric currents. The fields occur in pairs, the primary ones being the electric and magnetic fields $(E, B)$, and the other the excitation fields $(D, H)$. The link between the two pairs of field is provided by constitutive relations, which specify $(D, H)$ in terms of $(E, B)$; this last connection enabling Maxwell’s (differential) equations to be combined in a way that supports waves. In this paper we examine the role played by the excitation fields $(D, H)$, showing that they can be regarded as not having a physical existence, and are merely playing a mathematically convenient role. This point of view is made particularly relevant when we consider competing constitutive models of permanent magnets, which although having the same measurable magnetic properties, have startlingly different behaviours for the magnetic excitation field $H$.

Keywords: Maxwell’s equations, magnetism, gauge freedom, measurability

(Some figures may appear in colour only in the online journal)
1. Introduction

The role and meaning of the Maxwell excitation fields $(D, H)$ has been a subject of debate, with arguments for and against their independent existence and measurability [1–6]. However, a recent work has demonstrated that in certain physical circumstances, such as when a black hole forms then evaporates, $(D, H)$ cannot be guaranteed to be uniquely defined [7]. This reduces $(D, H)$ into the role of a gauge field for the current, meaning that there is no role for attempting to measure them.

Here we advance the argument that $(D, H)$ are not physical fields equal in status to $(E, B)$, and do so in a way intended to be more concrete and comprehensible to a wider audience. In the textbook by Jackson [8] for example, the author states clearly that $(D, H)$ are ‘derived fields’ used as a ‘convenience’, but does not discuss their measurability. There is also plenty of valid electromagnetic theory in which all four fields are treated as having the same status, ranging over traditional textbooks [9, 8], pre-metric electrodynamics [10], and others [11]. Generally, these rely on the existence of agreed constitutive relations, although from the pre-metric electrodynamics [10] point of view, the constitutive relations include the role of a metric, rather than being only the material responses. Further, in some aspects of optics and engineering, $H$ is treated as if it were the fundamental magnetic field instead of $B$, although this is based more on a long-standing calculational convenience, rather than the result of taking a deliberate theoretical stance.

Beyond an examination of Maxwell’s equations themselves, we consider three possible constitutive models that can be used to describe magnetism: bulk magnetic response, surface currents, and a distribution of magnetic dipoles. Although an initial response might be to expect the models to differ, and consider the fact unremarkable, here we use it to drive home the necessarily ambiguous interpretation of the $(D, H)$ fields.

In what follows, we follow an undergraduate-level presentation of electromagnetism and the role of its excitation fields $(D, H)$. We do this with a particular focus on the treatment of magnetisation using different constitutive models. In section 2 we start by briefly discussing Maxwell’s equations and the role of gauge transformations, and then in section 3 consider the role of measurement and electromagnetic constitutive relations. Next, in section 4 we compare three different models for the constitutive relations of a permanent magnet, and show how although each gives the same result exterior to the magnet, the specific $H$ required is not the same, and it can even point in opposing directions. These inconsistencies between models and the difficulties that arise with surface properties are emphasised by considering infinitely large magnetic slabs in section 5, after which we conclude in section 6.

2. Maxwell’s equations

Maxwell’s macroscopic equations for electromagnetism [9, 8] are well known, and in their Heaviside (vectorial) form, with an over-dot denoting a time derivative, are

$$\nabla \cdot B = 0, \quad \nabla \times E + \dot{B} = 0,$$

$$\nabla \cdot D = \rho, \quad \nabla \times H - \dot{D} = J.$$

Here the fields $E$ and $B$ are the fundamental electric and magnetic fields, whereas $D$ and $H$ are their related excitation fields. The excitation field $D$ is called the ’displacement field’, and
\( \mathbf{H} \) is often\(^5\) known as the ‘magnetic field strength’. Lastly, \( \rho_f \) and \( J_f \) are the free charge and current densities.

Like the fields \( \mathbf{E}, \mathbf{B} \), the excitation fields \( \mathbf{D}, \mathbf{H} \) only appear in Maxwell’s equations with a derivative operator applied to them. This means that \( \mathbf{D}, \mathbf{H} \) are invariant under the addition of a total derivative, i.e.

\[
\mathbf{H} \rightarrow \mathbf{H} + \nabla \psi + \varphi,
\]

\[
\mathbf{D} \rightarrow \mathbf{D} + \nabla \times \mathbf{v},
\]

where \( \psi \) and \( \mathbf{v} \) are arbitrary scalar and vector fields. As far as Maxwell’s equations are concerned we may regard the excitation fields \( \mathbf{D}, \mathbf{H} \) as potentials for \( \mathbf{J}, \mathbf{f} \). This is not dissimilar to the way we can regard quantities \( \phi, \mathbf{A} \) as potentials for \( \mathbf{E}, \mathbf{B} \), where \( \mathbf{E}, \mathbf{B} \) are invariant under the gauge transformation

\[
\phi \rightarrow \phi + \varphi,
\]

\[
\mathbf{A} \rightarrow \mathbf{A} + \nabla \varphi,
\]

where \( \varphi \) is an arbitrary scalar field. It is worth noting that although many do not regard the electromagnetic potentials \( \phi, \mathbf{A} \) as being real physical (measurable) quantities, they are often treated as exactly that—notably, in quantum electrodynamics [12], where it is the electromagnetic potential \( \mathbf{A} \) that is quantised, not the fields \( \mathbf{E}, \mathbf{B} \). Of course, in some cases, e.g. in the field of quantum optics, there can be a preference to instead quantise the dual potential [13–15], as a technical device to assist in the handling of the material response.

One way of avoiding the indeterminacy of \( \mathbf{D}, \mathbf{H} \) is to choose to start with the microscopic Maxwell’s equations. In this case we can express the constitutive relations for the material properties solely in terms of bound charges \( \rho_b \) and currents \( \mathbf{J}_b \). The physical picture attached to these properties is that they are due to charges attached (‘bound’) to an atom, molecule, or some other similarly localised object. Thus if an applied electric field polarises an atom, displacing its orbiting electrons, the atom is no longer a simple neutral object, but instead appears as a bound electric dipole. Similarly, an electron orbiting an atom can appear as a bound current loop. We treat these bound charges and currents \( (\rho_b, \mathbf{J}_b) \) separately from the free charges and currents \( (\rho_f, \mathbf{J}_f) \). We have therefore that

\[
\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0,
\]

\[
\varepsilon_0 \nabla \cdot \mathbf{E} = \rho_f + \rho_b \quad \text{and} \quad \frac{1}{\mu_0} \nabla \times \mathbf{B} - \varepsilon_0 \dot{\mathbf{E}} = \mathbf{J}_f + \mathbf{J}_b.
\]

Next, changing to a macroscopic view, we can relate these bound charges and currents to polarisation fields \( \mathbf{P} \) and \( \mathbf{M} \) in the usual way, i.e.

\[
\rho_b = -\nabla \cdot \mathbf{P}, \quad \mathbf{J}_b = \nabla \times \mathbf{M} + \dot{\mathbf{P}},
\]

with \( \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}, \quad \mathbf{H} = \frac{1}{\mu_0} \mathbf{B} - \mathbf{M} \).

\(^5\) Further, \( \mathbf{H} \) might also be called, depending on context, the ‘magnetic field intensity’ or the ‘magnetising field’. Sometimes \( \mathbf{H} \) is even called the ‘magnetic field’, which is why \( \mathbf{B} \) also has alternative names, notably ‘magnetic flux density’ or ‘magnetic induction’. This ambiguity is why we prefer the strict naming of ‘fields \( \mathbf{E}, \mathbf{B} \)’ and ‘excitation fields \( \mathbf{D}, \mathbf{H} \)’.
We can immediately see that since $P$ and $M$ only appear when subject to derivative operators, they are not uniquely determined by $\rho_b$ and $J_b$. Of course, although the polarisation $P$ and magnetisation $M$ have the same gauge freedom as (3), (4),

$$M \rightarrow M - \nabla \psi - \hat{v} \quad \text{and} \quad P \rightarrow P + \nabla \times \hat{v}, \quad (11)$$

the bound charge and current are gauge invariant. Thus we can see that the attempt (or choice) we make to use $P$ and $M$ instead of $\rho_b$ and $J_b$ necessarily introduces an ambiguity into our description. Our subsequent choice of polarisation gauge or detailed constitutive model is then driven by the form in which we find that the constitutive relations are best expressed, and not by any objective physical reasons. Notably, if we assume that the polarisation and magnetisation depend on the $(E, B)$ fields, e.g. $P = \hat{P}(E)$ for some function $\hat{P}$, then the constitutive relations give $\rho_b$ and $J_b$ in terms of the derivatives of $E$ and $B$.

3. Measurement of the electromagnetic and excitation fields

Our ability to directly measure the properties of a system is a primary test of whether or not we consider those properties real and physical—if we cannot directly measure them, it becomes possible to debate whether they have an independent physical existence at all.

The most direct way of measuring $(E, B)$ is using the Lorentz force law, which lets us calculate the acceleration of a known charge $q$ due to those $(E, B)$ electromagnetic fields, i.e.

$$F = qE + q\hat{v} \times B. \quad (12)$$

Even the generalisation of this to the case of hybrid electric–magnetic charged dyons depends on $(E, B)$ and not $(D, H)$ [16]. Another point to note that using (12) is really only practical in free space, as inside a medium the path of an electron is too dependent on the material’s atomic or molecular structure.

Alternatively, we might use the Aharonov–Bohm effect [17–20] where we track the phase of electrons around a block of material by relating a closed electron path to properties on a surface bounded by that path. The advantage of this is that while the electrons may pass around the outside of a block of matter, the surface $S$ can pass though the medium. This enables one to measure the magnetic flux—i.e. the integrated $B$—inside the medium. For some surface $S$ whose boundary $\partial S$ matches the integrated trajectory of a particle with charge $q$, the phase shift $\theta$ induced on that particle is determined by the integral

$$\theta = \frac{q}{\hbar} \oint_{\partial S} A \cdot d\ell = \frac{q}{\hbar} \int_{S} B \cdot dS. \quad (13)$$

Thus we can directly measure $B$ in a medium, at least as it is averaged over the cross-sectional surface $S$. A complementary experiment enables us to measure $E$ in the medium [20].

In contrast, whether $(D, H)$ might also be directly measurable fields in the manner of $(E, B)$ is at best uncertain. Firstly, there is no well established force law involving $(D, H)$, and even those that are specify the forces on magnetic multipoles [21]. Secondly, no Aharonov–Bohm-type concept exists for $(D, H)$. Of course if the $E$ or $B$ has been measured at a point in vacuum, we can use the defined vacuum constitutive relations of $D = \varepsilon_0 E$ and $B = \mu_0 H$ to also determine $(D, H)$. However this inference of $(D, H)$ is entirely dependent on measurements taken of $(E, B)$, and then converted with the help of the vacuum constitutive relationship.

Indeed, close examination of all the cases we have seen where $(D, H)$ is claimed to be measured (e.g. note in particular the careful theoretical arguments in [10], or the more practical ones in [3]) it turns out that they are not directly measured, but inferred from a measurement of $(E, B)$ and a model of the constitutive relations, i.e. the permittivity and
permeability is posited and the free parameters of the model determined by experiment. For example the permeability of iron in a static field $\mu_{\text{iron}}$ can be determined by measuring the effect in the vacuum of an external magnetic field when a piece of iron is introduced. However, the interpretation of this result depends on a model of the magnetic constitutive relation, i.e. $B = \mu_{\text{iron}}H$. It is this model that ties $H$ to $B$, and so prevents one performing the gauge transformation (3), (4) on $(D, H)$.

As a result, if we are unable or unwilling to assume a constitutive model, we are forced to then claim it is not possible to measure $(D, H)$, even indirectly. This is important, for example when we consider homogenisation procedures [22] which claim to determine effective constitutive relations. Therefore we need to identify what properties of the effective constitutive relations are being assumed. The freedom is even larger when we permit the effective constitutive relations to have more than just a temporal response, i.e. if they can also be spatially dispersive [23–30].

In what follows we will show that various different constitutive models for permanent magnets do not and cannot agree on how to infer $H$ from $B$, even though each is widely used and uncontroversial. This demonstrates not only the central role of the constitutive model, but also the lack of any independent meaning in the $(D, H)$ excitation fields.

4. Permanent magnets

The primary reason that a consideration of magnetism exposes the role of constitutive models most clearly is due to the absence of magnetic charges. For the electric response of a material, it is obvious that one can model this by something as straightforward as a polarisation field induced by bound charges. Although one might, as an exercise, model the electric response as loops of magnetic current charges (see e.g. [11]), it would not ordinarily be seen as an approach that represents some underlying microscopic physical model. However, with no magnetic monopoles to work with, it is entirely reasonable to represent a magnetic response as being generated either by loops of electric current or by magnetic dipoles [9, 31, 32].

There are three models of permanent magnets that we wish to consider here. The first two models of the magnetic constitutive relation of a permanent magnets, which we call the bulk and surface current models, are both valid but have contrasting implementations. A third, which we call the microscopic model, uses (8), (9) to replace the $H$-field with the bound current. This model, however, is less useful as the bound current depends on the derivative of the magnetic field $B$.

These models all contain two distinct contributions, the first due to the bulk properties of the material making up the magnet, and its response to the applied magnetic field as defined by a homogeneous permeability $\mu$; and the second defining the magnet’s coercive field intensity $H_c$. The differences between them are primarily due to the modelling of the coercive field intensity, which is the applied field strength at which the magnet’s polarity changes.

4.1. Bulk

In the bulk model the magnetic properties of the material are assumed to be dispersed continuously throughout its volume, along with the coercive field intensity $H_c$. In this case, the magnetic constitutive relation for $H = H^B$ is given by

$$H^B = \frac{1}{\mu} B - H_c,$$

where $\mu > 0$ is the constant permeability of the magnetic material. We see in figure 1 that neither coercive field intensity $H^B = H_c$ nor the remanence field $B = B_c$ can be achieved without an external magnetic field. The remanence field, that which remains after any applied
field is removed, is then \( B_r = \mu H_c \). Away from other external magnetic fields the magnitude of \( H^B \) takes a value between zero and \( \| H_c \| \), i.e. \( 0 < \| H^B \| < \| H_c \| \). Thus from (14) we see that \( H^B \), the field within the magnet, is in the opposite direction to \( B \), as depicted in figure 2.

4.2. Surface current

In the surface current model the magnetic properties of the material are again a result of bulk properties dispersed continuously throughout its volume, but the coercive field intensity \( H_c \) is instead treated as being due to a surface current \( \sigma^S_b \) present on its exposed surfaces. If the bulk permeability is \( \mu \), then the magnetic constitutive relation for \( H = H^S \) is given by

\[
H^S = \frac{1}{\mu} B \quad \text{and} \quad \sigma^S_b = n \times H_c,
\]

where \( n \) is the outward-pointing normal to the surface of the permanent magnet. In this model \( H^S \) is in the same direction as magnetic field \( B \), as depicted on figure 3.

Note that there is a discontinuity in \( H^S \) due to the surface current, i.e.

\[
n \times [H] = \sigma^S_b
\]

where \([H] = H^S - H^{vac}\) with \( H^{vac} \) being the induced magnetic field in the vacuum. When joining two magnets (with the same orientation) together to create a larger magnet, the surface currents on the shared interface simply cancel, as shown in figure 4.

4.3. Microscopic: local dipoles

In the microscopic model the magnetic properties of the material are a result of bulk current-generating properties dispersed continuously throughout its volume, and a coercive field intensity \( H_c \) which is due to a collection of current-loop magnetic dipoles inside the magnet. The dipoles generating \( H_c \) average out inside the bulk, and, in combination with the effect of the discontinuity in \( B \) on the surface of the magnet, gives rise to a net surface current \( \sigma^\text{macro}_b \). If the bulk permeability is \( \mu \), then the magnetic constitutive relation for \( H = H^\text{micro} \) is
$H\text{ micro} = \frac{1}{\mu} B,$

(17)

$J_0\text{ micro} = \left( \frac{1}{\mu_0} - \frac{1}{\mu} \right) \nabla \times B,$

(18)

and $\sigma_b\text{ micro} = n \times H_e + \left( \frac{1}{\mu_0} - \frac{1}{\mu} \right) n \times B,$

(19)

where we have used Maxwell’s microscopic equations along with bound currents and charges. Note that a diagram showing the relative directions of $B$ and $H\text{ micro}$ for this model would be similar to figure 3, because they are straightforwardly proportional to each other, and therefore always in the same direction.

4.4. Comparison

We see when comparing figures 2 and 3 that the bulk and surface current models give completely different predictions—the direction of the induced magnetic field $H$ in the medium reverses as we switch between models. This stark difference means that if only we could measure $H$ in the bulk, we could tell which model is the physically correct one. However, since we can only measure $(E, B)$, Maxwell’s equations (2) are insufficient to enable us to distinguish between these model.

One ameliorating feature here is that inside the volume of the magnet, the difference in $H$ between the two models can be bridged by a gauge contribution:
where $\psi$ is an inhomogeneous field given by $\psi = -y[N] H_S$ where $y$ is the coordinate along the axis of the magnet, i.e. where $y$ is parallel to $H_S$. However, the surface current model has a bound surface current $\sigma_b$. Thus to make a measurement that determines the correct constituitive model of a permanent magnet, one must...
either measure $H$ inside the medium, or the bound surface current $\sigma_b$. This means that as already discussed in section 3, since we have no mechanism for measuring $H$, we can at best only infer $\sigma_b$ from nearby free-space $B$ measurements, a process which requires us to assume some constitutive mode.

The microscopic model is different in that the bulk bound current $J_{\text{micro}}$ depends on the derivative of the magnetic field $B$; but one may consider this is less practical or aesthetically pleasing. However, the bound currents in the other two models are unrelated to $B$.

It is straightforward to make the empirically-based argument that all these models for the constitutive relations for a permanent magnet are equally valid. This is because it is not possible to distinguish between them by direct measurement, whether of the induced magnetic excitation field $H$, the bound bulk current $J_b$, or the surface current $\sigma_b$. The question as to which to choose is best determined by aesthetics, reference to an underlying quantum model, or practical considerations such as which model is easiest to work with.

### 4.5. Advantages and disadvantages

When considering the advantages and disadvantage of the three models, we note that all three arise naturally from an ‘averaged’ or homogenised view of a magnet’s microscopic properties, namely current loops at the atomic or molecular level. The choice of model therefore depends on the aspect of the underlying microscopic properties the researcher wishes to emphasise.

The bulk model does not suffer from requiring fields which are concentrated in an infinitely thin region at the surface of the magnet. These are typically expressed using delta functions and can produce difficulties when considering self-fields or the total energy/momentum calculated from products. Further, the lack of bound currents in a bulk model tends to make it easier to solve for the fields in vacuum. However, a disadvantage of the bulk model is that one does not homogenise the current loops, but instead the corresponding magnetic moments of the atoms or molecules.

Despite the requirement for delta functions to specify the surface current in the surface current model, it is at least quite easy to understand—it is straightforward to envisage how microscopic current loops representing atomic or molecular properties are averaged to leave just an effective surface current representing the permanent magnetism. Nevertheless, it does retain a somewhat artificial distinction between the response of the magnetic material (expressed as a permeability $\mu$) and the permanent magnetism.

Lastly, the main strength of the microscopic (local dipole) model is that it corresponds to the principle that a medium is really the vacuum and that any responses to an external field are (and should be) represented by bound currents. Further, it does not require us to define the excitation field $H$, although a side effect is that the constitutive relations for the magnet are not constant, and depend on the $B$ field.

### 5. Paradox? The infinite iron slab

Even though our core argument has been sufficiently demonstrated above, in order to further emphasise our point we can compare and contrast the cases of (a) a slab of dielectric medium between two charged plates, as per figure 5; and (b) a ferromagnetic medium with a zero coercivity between two magnetic poles as per figure 6. We see that at this diagrammatic level the two scenarios are completely equivalent, with the permittivity in figure 5 acting to reduce the electric field $E$ in the dielectric; and the permeability in figure 6 likewise acting to reduce the magnetic excitation $H$ in the medium.
If the slabs are finite in extent, our constitutive models all behave reasonably, but if we extend the slabs out towards infinity, an inconsistency arises when we try to explain the magnetisation in terms of bound electric currents. Of course, in the dielectric slab case the description is straightforward. Figure 7 simply indicates that a bound surface dipole charge distribution \( \mathbf{E}_b = \nabla \cdot \mathbf{b} \) is induced on the dielectric slab. The magnetic version of this has two possible interpretations. As discussed above, and given the absence of any native magnetic sources, Maxwell’s equations (2) insist that the magnetisation instead arises from a bound current, which might be either dispersed throughout the volume with \( \mathbf{J}_b \) being given by (9), or be present on the surface with \( \mathbf{\sigma}_b \) being given by (16).

For magnetisation due to bound currents dispersed throughout the volume, the net effect as seen externally is that the discontinuity in \( \mathbf{H} \) is generated by a surface magnetic monopole.
distribution, as shown on figure 8; which is similar to figure 7. However, this is utterly incompatible with our monopole-free constitutive model of magnetisation, and so as a consequence both the description and the diagram fail to adequately represent the physical situation.

For magnetisation due to bound surface currents, the situation also not straightforward. A depiction in the style of figure 7 is at least possible for the case of a finite magnet, and in figure 9 we can see that the surface currents required to represent the magnetisation can be straightforwardly placed on the top and bottom of the magnetic block. However, if we were to stack such blocks up one after another so as to (eventually) form an infinite slab, the surface currents would cancel on the interfaces, leaving only those on the exterior (see figure 4). These would then be pushed further and further away as blocks were added, and so eventually disappear off to infinity. There would be nowhere on the resulting figure where we could depict the required non-zero surface currents, despite them being a necessary part of the constitutive properties.

The two attempts made here to consistently model the constitutive properties of an infinite slab of magnetised material have failed. Of course, although an infinite slab is not a realistic physical situation, the considerations here are relevant to ‘1D’ calculations in which a 3D system is considered unchanging in the other two.

6. Conclusion

In this work we have discussed how the excitation fields \((\mathbf{D}, \mathbf{H})\) can be (a) seen as gauge fields for the current (section 2), that (b) there is no known way of directly measuring them (section 3), and that (c) they can be given radically different values depending on the chosen constitutive model (section 4). Further, the anomalies in the interpretation of the standard constitutive models for magnetism, as emphasised by section 5, shows how attempts to consider \((\mathbf{D}, \mathbf{H})\) as true physical fields like \((\mathbf{E}, \mathbf{B})\) founder on the chosen details of the constitutive model. This situation is particularly clear in the case of standard Maxwellian magnetism, because unlike electric effects which have their own charges, magnetism does
not. The subsequent necessity of describing magnetic sources in terms of currents then leads directly to the inconsistencies in the constitutive models.

This lack of either measurability or constitutive uniqueness for the excitation field $H$ in these specific magnet examples provides reasons to consider both excitation fields $(D, H)$ as merely gauge fields for the current, rather than quantities with an independent physical existence. This complements the mathematical discussion referring to derivatives or topological considerations [7], which also insist on the non-uniqueness of $(D, H)$. The combination results in compelling arguments that demote $(D, H)$ to a gauge field status.

Figure 8. Bound surface magnetic monopoles (!); this shows a horizontal slice through an infinitely extended slab of magnetisable material (grey), in an applied magnetic field. There are induced magnetic dipoles distributed inside the material, and if such a thing as magnetic monopoles existed, we might say that their constituent magnetic charges all cancel out except on the left and right surfaces. However, although the discontinuity in $H$ experienced as you cross into (or out of) the magnet might be explained in terms of a bound surface magnetic monopole distribution (green), such a situation is utterly incompatible with our constitutive model of magnetisation.

Figure 9. Bound surface currents: a horizontal slice through a finite slab of ferromagnetic material (grey) in an applied magnetic field. Here the discontinuity in $H$ experienced as you cross into (or out of) the slab can be explained in terms of a bound surface current distribution whose direction is indicated by $\circ$ and $\otimes$. However, if we were to stack up these finite slabs, as in figure 4, since the bound surface current $\sigma_3$ only appears on the top and bottom boundaries in the infinitely extended case, where would any bound surface current be located?
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