Kinematics Rods of Simulator-Hexapod

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Authors' contributions

This work was performed in cooperation between all authors. Author VY directed researches, offered and proved universal system model of kinematics of platform. Author AAO offered and proved application of model of kinematics rods of simulator-hexapod. Author GK offered and proved application an algorithm for research of kinematics of the rods of simulator-hexapod. Author NM executed necessary researches and developed formulas for the trajectory of the of type "pitch". All authors read and approved the final manuscript.

ABSTRACT

**Purpose:** To develop mathematical model of the kinematic parameters of motion rods of hexapod at the maneuver of type “pitch”.

**Methodology:** The motion of hexapod platform was generated based on the pole translational equation of motion and rotational motion via angles of Euler-Krylov. Modelling the platform’s trajectory of type “pitch” was done based on control of coordinate positions. The movement of each rods of hexapod around a motionless axis was presented by two angles of Euler.

**Results:** Are obtained the mathematically models of kinematic parameters of rods the hexapod, when performing by a platform of any complex maneuver. Angular velocity and angular acceleration is attached as projections on the fixed coordinate system and on the coordinate system is attached to the moving rods. Was presented the algorithm for research of kinematics the rods of a hexapod. Calculations for design of the standard avia-exercise machine of firm “ANTK Antonov” have confirmed a possibility of obtaining kinematic parameters of the rods of hexapod when performing maneuver like “pitch”. Has been set the maximum of accelerations load on the
crew at the maximum admissible angles of maneuver types “pitch” of the simulator-hexapod.

Conclusion: Was confirmed possibility of modelling a complex maneuver of simulator-hexapod at permissible acceleration load on the crew. The process of obtaining kinematic parameters has been necessary for dynamics problems.

Keywords: Motion; matrixes; mechanism of parallel structure; maneuver; training.

1. INTRODUCTION

The control of mobile transports objects is highly complex and dangerous operation, requiring taking fast decisions on change of a course and instant reactions drivers for motion changes. Technologies of training have gained the greatest distribution where mistakes when training at real objects lead to extraordinary consequences, and their elimination — to big financial expenses: in aircraft and space, in military science, in medicine, at natural disaster response, in nuclear power, hi-tech production. The perfection of simulators of modern mobile machine (MM) is highly dependent on the degree of reliability of kinematic simulation. For studying dynamic loads of simulator it is necessary to do comprehensive research about the kinematic parameters legs of hexapod. Therefore, the problem of research kinematics of legs of simulator-hexapod, simulation of control process of the driving system, performing motion of the simulator’s cockpit, is an actual problem.

In modern researches besides the fact that it combines the new formulae for mobility connectivity, redundancy and over constraints, and the evolutionary morphology in a unified approach of structural synthesis giving interesting innovative solutions for parallel mechanisms [1]. The most effective is design of parallel structure machines (PSM), researches of forward and inverse kinematic problem, workspace and singularity, done by Merlet J, Bonev I, Liu XJ, Wang J. [2-4]. Matrix method calculation of PSM represented in Reza N. Jazar [5]. Deserve attention of achievement in researches of designing of PSM of Yu. N. Kuznetsov DA. Dmitriyev [6,7]. Questions of kinematics of PSM as a part of industrial platforms was considered in works of Lenarces J, Bajd T, Stanisic M. [8]. In kinematics field and of improving the mobility of the combined model of simulator-hexapod are known works, in which was obtained results the level of acceleration effects on the crew [9,10].

However, in a well-known kinematic researches PSM was not considered the kinematic motion of rods hexapod and is completely missing the methodology for studying their kinematic parameters. Comprehensive investigation kinematic parameters of the hexapods are necessary not only for determining their orientation, but for dynamic problem solution for studying the loads on the simulator. The solution of the problem of comprehensive preparation of crews of mobile equipment for action in emergency situations demands creation and improvement of systems of modeling of the movement MM at various maneuvering conditions.

2. METHODOLOGY

Nowadays all well-known high level simulators are dynamic platforms based on PSM of type «hexapod» (Stewart platform) [1,2]. Modern simulator of mobile machines based Stewart platform for drivers has 6-degree of freedom and are consist of mobile platform (MP) 1 and fixed base 2 (Fig. 1). Six universal joints \( A_i \) on the base and six spherical joints \( B_i \) of the moving platform are connected with six legs 3 which can change its length \( A_i A_j \). Motion platforms of simulator-hexapod relatively the fixed base \( OX_0Y_0Z_0 \) was defined with six independent parameters: Translational movement with pole P of platform (coordinates – \( x_P, y_P, z_P \) ) and rotation (angles Euler-Krylov – \( \psi, \theta, \phi \) ) [9].

Coordinates of the universal joints \( A_i \) in the fixed base \( OX_0Y_0Z_0 \) and coordinates of spherical joints \( B_i \) in moving platform \( PX_PY_PZ_P \) don’t change, when the platform moves (Fig. 2) [10]:

\[
0 A_i = (x_{A_i}, y_{A_i}, z_{A_i}), \quad P B_i = (B_{x_i}, B_{y_i}, B_{z_i}),
\]

\( i = 1...6 \). (1)

From the geometry of the location of the joints \( A_i \) determined this formula for angles

\[
y_1 = 2 \arcsin \left( \frac{A_{3}A_{4}}{2R_A} \right) \quad \text{and} \quad y_2 = \frac{2\pi}{3} - y_1; \quad R_A \quad \text{and} \quad R_B \end{align*}
\]

\( R_B \) – radii of joints center \( A_i \) and \( B_i \) respectively. In accordance with Euler theorem
about the final displacement of a rigid body with one fixed point on the platform 1 from start position $PX'Y'Z'$ to the final one $PX_YZ_P$ can be represented by three rotations (Fig. 1). First rotation is around axis $PZ'$ on the angle $\psi$ and with angular velocity $\bar{\omega}_\psi$ from $PX'Y'Z'$ to the position $cPX_YZ_i$. Second rotation is around the line of nodes $PX_iYP_1Z_1$ on the angle $\theta$ degree and with angular velocity $\bar{\omega}_\theta$ from $PX_iYP_1Z_1$ to the position $PX_iYP_2Z_2$. Third rotation is around the axis $PY_i$ of moving platform's on the angle $\phi$ and with angular velocity $\bar{\omega}_\phi$ from $PX_iYP_2Z_2$ to the final provision of a platform $PX_iYP_2Z_P$.

![Fig. 1. Scheme rotation of platform around pole $P$ and rotation the rod $A_iB_i$ around joint $A_i$.](image)

Considering the matrix of transformations, was obtained the coordinates of the center of joint $B_i$ in the fixed system $OX_0Y_0Z_0$ as [9]:

\[
\begin{align*}
\begin{pmatrix}
 x_{Bi} \\
 y_{Bi} \\
 z_{Bi}
\end{pmatrix}
&= \begin{pmatrix}
 B_{xi}(c_\psi c_\phi - s_\psi s_\phi) + B_{yi}(-s_\psi c_\phi) + B_{zi}(c_\psi s_\phi + s_\psi c_\phi) + x_P \\
 B_{xi}(s_\psi c_\phi + c_\psi s_\phi) + B_{yi}c_\phi c_\phi + B_{zi}(s_\psi s_\phi - s_\psi c_\phi) + y_P \\
 B_{xi}(-c_\phi s_\phi) + B_{yi}c_\phi + B_{zi}c_\phi c_\phi + z_P
\end{pmatrix},
\end{align*}
\]

where $c_\psi = \cos \psi$, $s_\psi = \sin \psi$, $c_\phi = \cos \phi$, $s_\phi = \sin \phi$, $c_\theta = \cos \theta$, $s_\theta = \sin \theta$, $c_\phi = \cos \phi$, $s_\phi = \sin \phi$. 

3
When the simulator is moving, the rod is rotates around the fixed joint \( A_i \). The motion of the \( i \)-th rods is obtained in its own coordinate system \( A_i X_i Y_i Z_i \). Orientation of system \( A_i X_i Y_i Z_i \) is determined with the coordinates of the joint \( A_i \{x_{Ai}, y_{Ai}, z_{Ai}\} \) and with the angle \( \theta_i \) of line \( OA_i \) (axis \( A_i X_i \)) as (Figs. 1, 2):

\[
\begin{align*}
\theta_1 &= 0.5y_2; \\
\theta_2 &= 0.5y_2 + y_3; \\
\theta_3 &= 1.5y_2 + y_3; \\
\theta_4 &= 1.5y_2 + 2y_3; \\
\theta_5 &= 2.5y_2 + 2y_3; \\
\theta_6 &= 2.5y_2 + 3y_3.
\end{align*}
\]

(3)

![Fig. 2. Scheme of the position of cardan joints \( A_i \) and of spherical joints \( B_i \)](image)

Auxiliary intermediate system coordinate \( A_i X_i' Y_i' Z_i' \) is located in parallel with the fixed reference coordinate system \( OX_0 Y_0 Z_0 \). \( B_{i,xy} \) is the projection of the center of joint \( B_i \) on plan \( OX_0 Y_0 \). With every of \( i \)-th rod \( A_i B_i \) has been attached coordinate system \( A_i \xi_i \eta_i \zeta_i \), which is rotating around the fixed joint \( A_i \) together with the rod. Orientation of the moving axes \( A_i \xi_i \eta_i \zeta_i \) is obtained relative to the fixed axes \( A_i X_i Y_i Z_i \) at the angles \( \alpha_i \) and \( \beta_i \). \( A_i \) is cardan joint, so the leg rotation around its longitudinal axis \( A_i \eta_i \) is impossible. At initial position ( \( \alpha_i = 0 \) and \( \beta_i = 0 \) ) the axes of system \( A_i \xi_0 \eta_0 \zeta_0 \) and of \( A_i X_i Y_i Z_i \) are coincide, so the legs are in horizontal position \( A_i B_i \) (on the plan \( OX_0 Y_0 \)) as a perpendicular to \( OA_i \). First rotation is around joint \( A_i \) about fixed axis \( A_i Z_i \) on precession angle \( \alpha_i \) and with angular velocity \( \overset{\cdot}{\alpha}_i \). After the first rotation is disposed the axis \( A_i \xi_i \) in the final position, axis \( A_i \eta_i \) and rod \( A_i B_i \) in the intermediate positions. The second rotation is around line of nodes (axis \( A_i \xi_i \)) by angle \( \beta_i \) and with angular velocity \( \overset{\cdot}{\beta}_i \). After second rotation coordinate system \( A_i \xi_i \eta_i \zeta_i \) and rod \( A_i B_i \) are disposed in the final positions.

By the known coordinates (2) of the center of joint \( B_i \) in system \( OX_0 Y_0 Z_0 \) are defined coordinates of this center in the (local) motionless system of coordinates \( A_i X_i Y_i Z_i \) with the matrixes of transformation of coordinates: translational motion from point \( O \) in the position \( A_i \) and the rotation around axis \( A_i \zeta_i \) on the angle \( \theta_i \). After transformation was denoted coordinates of the joint \( B_i \) on system \( A_i X_i Y_i Z_i \) as \( B_i \{x_i, y_i, z_i\} \). As a result, was obtained:

\[
\begin{align*}
|x_i| &= (x_{Bi} - x_{Ai})\sin\theta_i + (y_{Bi} - y_{Ai})\cos\theta_i \\
y_i &= (x_{Bi} - x_{Ai})\cos\theta_i - (y_{Bi} - y_{Ai})\sin\theta_i \\
z_i &= z_{Bi}.
\end{align*}
\]

(4)

From right triangles \( A_i B_i \xi_{i,xy} \) and \( A_i B_i \eta_{i,xy} \) (Fig. 1.) was obtained:

\[
tg\alpha_i = \frac{B_{i,xy}}{A_{i,xy}} = \frac{x_i}{y_i}; \quad \tg\beta_i = \frac{B_{i,xy}}{A_{i,xy}} = \frac{z_i}{h_i},
\]

(5)

where \( h_i \) – horizontal projection of rod \( A_i B_i \) and \( x_i \perp y_i \) (Fig. 1.).

\[
h_i = A_i B_{i,xy} = y_i / \cos\alpha_i.
\]

(6)

The equations (5) can be written as:

\[
y_i \cdot \tg\alpha_i = -x_i; \quad h_i \cdot \tg\beta_i = z_i.
\]

(5,a)

After differentiation on time both parts of equalities (5, a) was received:

\[
\begin{align*}
\dot{y}_i \cdot \tg\alpha_i + \frac{y_i \cdot \overset{\cdot}{\alpha}_i}{\cos^2\alpha_i} &= -x_i; \\
h_i \cdot \tg\beta_i + \frac{h_i \cdot \overset{\cdot}{\beta}_i}{\cos^2\beta_i} &= z_i.
\end{align*}
\]
The angular velocity of the i-th rod:
\[
\omega_{i} = \dot{\alpha}_{i} = -\frac{1}{y_i} (\dot{x}_i + \dot{y}_i \tan \alpha_i) \cos^2 \alpha_i;
\]
\[
\omega_{2i} = \dot{\beta}_i = \frac{1}{h_i} [z_i - h_i \tan \beta_i] \cos^2 \beta_i.
\]  
(7)

Dots on the angles means differentiating with respect to time.

After multiplying both sides of equation (7) on the denominators and after differentiation on time are obtained the expressions for components of angular accelerations of the i-th rod as:
\[
\ddot{\alpha}_i = \frac{1}{y_i} \left\{ (\dot{x}_i + \dot{y}_i \tan \alpha_i) + 2 \frac{\dot{y}_i}{\cos \alpha_i} \sin \alpha_i \right\};
\]
\[
\ddot{\beta}_i = \frac{1}{h_i} \left\{ z_i - h_i \tan \beta_i - 2 h_i \dot{\beta}_i \cos \beta_i - 2 \dot{h}_i \sin \beta_i \right\}.
\]  
(8)

Where
\[
\dot{h}_i = (\dot{y}_i + h_i \dot{\alpha}_i \sin \alpha_i) / \cos \alpha_i,
\]
\[
\ddot{h}_i = \ddot{y}_i / \cos \alpha_i + \left\{ 2 h_i \dot{\alpha}_i + h_i \ddot{\alpha}_i \right\} \tan \alpha_i + h_i \dddot{\alpha}_i.
\]

Vector of the angular velocity of i-th rod is
\[
\vec{\omega}_i = \vec{\omega}_{1i} + \vec{\omega}_{2i}.
\]  
(9)

After projection of the both sides of equation on the fixed \( A_1 X'_1 Y'_1 Z'_1 \) (Fig. 1) it was found:
\[
\omega_{x'i} = \beta \cos \alpha_i; \quad \omega_{y'i} = \beta \sin \alpha_i; \quad \omega_{z'i} = \ddot{\alpha}_i.
\]
\[
\vec{\omega}_i = \dot{\beta} \cos \alpha_i \hat{i} + \dot{\beta} \sin \alpha_i \hat{j} + \ddot{\alpha} \hat{k}.
\]  
(10)

After differentiation on time was obtained the vector of the angular acceleration as:
\[
\ddot{\xi}_i = (\ddot{\beta} \cos \alpha_i - \dot{\alpha} \dot{\beta} \sin \alpha_i) \hat{i} + (\ddot{\beta} \sin \alpha_i + \dot{\alpha} \dot{\beta} \cos \alpha_i) \hat{j} + \dddot{\alpha} \hat{k}.
\]  
(11)

Modules of the angular velocity and of angular acceleration was found as:
\[
\omega_i = \sqrt{\dot{\alpha}_i^2 + \dot{\beta}_i^2 \cos^2 \beta_i}, \quad \varepsilon_i = \sqrt{\dot{\alpha}_i^2 + \dot{\beta}_i^2 \cos^2 \beta_i + \ddot{\alpha}_i^2}.
\]  
(12)

For problems of the dynamics are of the required projection of the angular velocity and angular acceleration in the moving axes \( A_2 \xi_2, \eta_2, \zeta_2 \), associated with the i-th rod. Projecting (10) and (11) on these axes was obtained:
\[
\omega_{x'_2} = \beta \cos \alpha_i; \quad \omega_{y'_2} = \dot{\alpha} \sin \alpha_i; \quad \omega_{z'_2} = \ddot{\alpha} \cos \alpha_i.
\]  
(13)

\[
\varepsilon_{x'_2} = \varepsilon_{x'_2} \cos \alpha_i + \varepsilon_{y'_2} \sin \alpha_i;
\]
\[
\varepsilon_{y'_2} = -\varepsilon_{x'_2} \sin \alpha_i \cos \beta_i + \varepsilon_{y'_2} \cos \alpha_i \cos \beta_i + \varepsilon_{z'_2} \sin \beta_i;
\]
\[
\varepsilon_{z'_2} = \varepsilon_{x'_2} \sin \alpha_i \sin \beta_i - \varepsilon_{y'_2} \cos \alpha_i \sin \beta_i + \varepsilon_{z'_2} \cos \beta_i.
\]  
(14)

Algorithm for research of kinematics of the rods of simulator-hexapod

1. Set the geometry of the arrangement of the universal joints of the base and spherical joints of the platform (Fig. 2). For the known radii \( R_A \) and \( R_B \) determine the coordinates of the centers of the joints (1) and angles of orientation (3).
2. It is necessary to form the equations translational ( \( x_p, y_p, z_p \) ) and rotational (angular – \( \psi, \theta, \phi \) ) of the motions of simulator-platform.
3. Determine the coordinates (2) and (4) of the centers \( B_i \) joints of a simulators in fixed systems of base \( OX_pY_pZ_p \) and in local \( A_iX_iY_iZ_i \).
4. Set the level of acceleration load on the crew for a given maneuver of the platform.
5. Compute the angles of rotation of the legs (5) and the values components of the angular velocities (7) and of angular accelerations (8).
6. Determine the projection of the absolute angular velocity and angular acceleration (12) on the stationary axes.
7. It is necessary to form the values for the projections of the angular velocities (10) and angular accelerations (11) of rods on the stationary axes.
8. It is necessary to form the values for the projections of the angular velocities (13) and angular accelerations (14) of rods on the mobile axes.
9. Determine the absolute angular velocities and accelerations of the rods (12) when performing a given maneuver.

3. RESULTS AND DISCUSSION

The trajectory of the type "pitch" of the simulator in the plane \( OX_2Z_2 \) is modeled as circular arcs with maximum pitch angle \( \varphi_0 = 30^\circ \) (Fig. 3).
Along the horizontal axis $OX$, the movement of the platform is simulated as the positional control: set the start and end positions of a pole of platform ($x_{p0} = -2\, m$; $z_{p0} = 3.2\, m$), ($x_{p1} = 2\, m$; $z_{p1} = 3.2\, m$), the maximum values of velocity ($\max \dot{x}_p = 5\, m/s$), acceleration ($\max \ddot{x}_p = 15\, m/s^2$) and snatch ($\rho_x = \bar{x}_{p0} = 30\, m/s^3$) [9]. Coordinate $z_p$ determined from the geometry of the trajectory of the pole $P$ of the platform:

$$z_p = h_0 + R - \left(R^2 - x_p^2\right)^{\frac{1}{2}}; \quad h_0 \quad \text{the minimum height of the pole; } R \quad \text{radius of the trajectory type "pitch".}$$

The vertical axis $PZ_p$ of the platform should deviate at an angle $\varphi$ from the vertical when performing trajectory type "pitch". From the geometry of the trajectory: $\sin \varphi = x_p / R$.

![Fig. 3. The trajectory of the maneuver type "pitch"](image)

For calculations taken source data model design of flight simulators of firm "ANTK Antonov": $R_A = 2.7\, m$; $R_B = 2.45\, m$; $h_0 = 1.3\, m$; $R = 8.0\, m$.

Replacing in (2) the coordinates of the joint at $B_i$ on the coordinates of the center $C$ of gravity $^pC = (C_x \quad C_y \quad C_z) = (0 \quad 0 \quad 1.4)$ of the operator of mobile machines and differentiating twice in time, found the absolute acceleration of the center $C$. The calculations revealed that when a platform performs maneuver of type "pitch" with the given parameters of its trajectory, then the level of acceleration load on the crew is exceed $2.3\, g$ ($g$ – acceleration of free fall). The regulatory limit value of the acceleration load for pilots correspond to the level of $6\, g$. With calculations by the proposed algorithm determined that the maximum value of the angular accelerations of the rods exceed $5\, s^2$ (Fig. 4). Graphs of the accelerations give an idea of the distribution of moments of inertia forces on the rods of hexapod.

![Fig. 4. Graphs of the projections of the angular accelerations of rods hexapod on axis $A\xi_i$ (a) and on the axis $A\zeta_i$ (b) (The numbers indicate on the order number of leg hexapod)](image)

Confirmed the possibility of modelling complex maneuver of the platform of the simulator with a permissible load on the rod hexapod.

4. CONCLUSION

It is developed the mathematical models of the kinematic parameters of rods hexapod when the arbitrary complex maneuver of platform. Installed the formulas of the angular velocities and angular accelerations of the rods in the form of projections on a fixed axes and on the axes associated with mobile legs. Presents the algorithm of research of kinematics of the rods hexapod. Calculations for the typical design of flight simulators of firm "ANTK Antonov" confirmed the possibility of obtaining the limit value of the acceleration load for pilots when performing the maneuver type "pitch". Developed
the process of determining the kinematic parameters of the rods hexapod, which are necessary for problems of the dynamics of the hexapod as a whole.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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