BANDWIDTH REDUCTION IN COGNITIVE RADIO

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Abstract. Due to mushroom development of wireless devices cognitive radio is used to resolve the bandwidth utilization and sacristy problem. The crafty usage of bandwidth in cognitive radio based on error correcting codes is ensured to accommodate an unauthorized user. This study proposes a transmission model by which a finite sequence of binary cyclic codes constructed by a binary BCH code of length \( n = 2^s - 1 \), in which all codes have same error correction capability and code rate but sequentially increasing code lengths greater than \( n \). Initially all these codes are carrying data of their corresponding primary users. A transmission pattern is planned in the sprit of interweave model deals the transmission parameters; modulation scheme, bandwidth and code rate. Whenever, any of the primary users having mod of transmission, the binary cyclic code, is not using its allocated bandwidth, the user having its data built by binary BCH code enter and exploit the free path as a secondary user. Eventually whenever the primary user with \( W \) bandwidth having binary BCH code for its data transmission, change its status as a secondary user, it just requires the bandwidth less than \( W \).

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1. Introduction

Cognitive radio is a most recent technology in wireless communication by which the spectrum is vigorously utilized when the primary user, the authorized holder of the spectrum, is not in. The idea of cognitive radio is introduced in \cite{9}. Interpreting to this notion the cognitive radio has the capability to evaluate the radio surroundings and boost the decision permitting to the transmission parameters such as modulation scheme, power, carrier frequency, bandwidth and code rate. By \cite{3} power is allocated to entire bandwidth for upsurge in capacity and keep the interference at the primary user at the given start and endure the complete transmission power inside properties.

Alternative inking of the interference temperature model of \cite{4} is necessary for the primary receiver to close the interference boundary, henceforth the secondary user can transmit beneath the set level. The fundamental plan in \cite{20} is to matter license spectrum to secondary users and guaranteed the interference perceived through primary users. To protect the primary user commencing the interference triggered by the secondary user through transmission, Srinivasa and Jafar \cite{16} offered an organization of transmission models as: Interweave, overlay and underlay.

By \cite{9}, in the interweave model the secondary user utilized the primary under utilized spectrum opportunistically and draw out when the primary wants to in again. Thus the sensing is necessary for the interweave scheme. However, dissimilar type of the sensing procedures are used to identify the primary user and elude the interference shaped by the secondary user. By \cite{9} for primary user detection, three prominent methods known as: energy detection, feature detection and match filter are getting attention. In cognitive radio secondary user, permitted to use a spectrum of primary user devoid of making interference to the primary user. However, the secondary users need to recurrently monitor the management of the spectrum to repel from interfering with the primary user(see \cite{8}). Overlay and underlay stay with spectrally modulated and spectrally encoding (SMSE) procedure all beside with code division multiple access (CDMA) and orthogonal frequency division multiple access (OFDMA). According to \cite{15}, in underlay, primary and secondary user transmit the data concurrently...
under the situation that the secondary user interferes less than an optimistic beginning with the primary user. Spectrum sensing is not mandatory for underlay. Only the interference limit is compulsory for the secondary user for talented utilization of the spectrum. Henceforth, this interference limitation confines the communication of secondary user to small range. Whereas by [3], some range of the power should be endorsed to the subcarrier of underlay as they also craft extra interference to the primary user. The request of wireless devices is increasing every day. Therefore, efficient utilization of spectrum is a burning subject to decrease the spectrum over crowdedness. By [2], the overlay model permits the concurrent transmission of primary and secondary users. The secondary transmitter is supposed to have awareness of the primary message and use for sinking the interference at its receiver. Precisely, in underlay and overlay models the secondary users can transmit their data at once with the primary users under some limitations: In underlay, secondary users can transmit with enough power due to interference bound fixed at the primary receiver whereas in the case of overlay transmission of secondary users are solitary promising if secondary transmitter recognizes the code-books and channel information. Besides, both of these models do not guarantee that secondary user will not produce interference for primary user during simultaneous transmission. Similarly, the allowing surroundings for these models may damage the transmission of both primary and secondary user.

In [11] wireless mesh network is practiced for terminal to terminal bandwidth allocation which used the routing and scheduling algorithms. The Max-min model is used for allocation of a fair bandwidth. In the [19] TV band is utilized for the cognitive radio over sensing and opportunistically use the vacant frequencies. The diverse protocols are used for centralized and decentralized spectrum distribution.

By using the undisclosed messages for cognitive radio channels first of all fixed the boundaries for channel capacity. Two transmitters having the primary and secondary messages go through the channel which was received at the receivers with distinct primary and secondary messages. By [18], improvement of spectral efficiency in cognitive radio can be achieved by the secondary user through the consent to utilize the untrustworthy part of the spectrum which is assigned to the primary user. Henceforth the optimal bandwidth is required from the numerous bandwidth allocated to the primary and secondary users. Additionally, the secondary users effort to choose the best possible bandwidth out of the different many collections of bandwidths. Furthermore in [18], different spectrum sharing procedure is considered to improve the cognitive radio networks.

In [11] it is established that for a given binary BCH code $C^0_n$ of length $n$ generated by a polynomial $g(x)$ of $\mathbb{F}_2[x]$ of degree $r$ there exists a sequence of binary cyclic codes $\{C^j_{2^j-1(n+1)n}\}_{j \geq 1}$ such that for each $j \geq 1$, the binary cyclic code $C^j_{2^j-1(n+1)n}$ has length $2^j-1(n+1)n$ and generated by $2^r$ degree generalized polynomial $g(x^{2^r})$ in $\mathbb{F}_2[x, \frac{1}{x}]\mathbb{Z}_2$. Furthermore $C^0_n$ is embedded in $C^j_{2^j-1(n+1)n}$ for each $j \geq 1$ and every code of the family $\{C^j_{2^j-1(n+1)n}\}_{j \geq 1}$ have same code rate and higher than of the binary BCH code $C^0_n$. Furthermore, in [11] a decoding algorithm is proposed, by which the binary BCH code $C^0_n$ can be transmit and decode through any of binary cyclic codes of the family $\{C^j_{2^j-1(n+1)n}\}_{j \geq 1}$.

In the interweave model the secondary user exploited the primary under utilized spectrum opportunistically and pull out when the primary wants to in once again. Thus the sensing is necessary for the interweave scheme. Though, dissimilar kind of the sensing processes is used to detect the primary user and escape the interference formed by the secondary user. For primary user uncovering, bulging approaches, energy detection, feature detection and match filter are receiving consideration.

Like interweave scheme, in this study we designed a transmission model built on error correcting codes. Cognitive radio has the ability to gauge the radio environs and boost the decision allowing to the transmission parameters such as modulation scheme, bandwidth, code rate, power, carrier frequency. In this study due to our model formation we address the parameters; modulation scheme, bandwidth and code rate. This model uses a finite embeddings sequence $\{C^j_{2^j-1(n+1)n}\}_{j \geq 1}$ of binary cyclic codes introduced in [11] against a binary BCH code $C^0_n$ in which all codes have same error correction capability and code rate but sequentially increasing code lengths. Initially all these codes including the binary BCH code are carrying data of their corresponding primary users. The procedure advances as, whenever, any of the primary users having mod of transmission,
the binary cyclic code, is not using its allocated bandwidth, the user having its data configured by binary BCH code enter and utilize the free path as a secondary user.

2. Basic facts

This section covers some of the basic results associated to monoid ring, cyclic codes and specific on transmission parameters.

To construct a polynomial \((n,k)\)-code \(C\) over a finite Galois field \(F_q\), where \(q\) is power of some prime, we choose a polynomial \(g(x)\) of degree \(n-k=r\) from \(F_q[x]\). A message is represented by a polynomial, called the message polynomial, \(j(x)\) of degree less than or equal to \(k-1\). The code polynomial corresponding to this \(j(x)\) is \(v(x)\) and is equal to \(r(x) + x^{n-k}j(x)\), where \(r(x)\) is the remainder of \(x^{n-k}j(x)\) after dividing it by \(g(x)\). A polynomial-code is an error correcting code whose codewords consists of multiple of a given fixed polynomial \(g(x)\) known as the generator polynomial.

Let \((S, \ast)\) and \((R, +, \cdot)\) be a commutative semigroup and an associative unitary commutative ring respectively. The set \(SR\) of all finitely nonzero functions \(f\) from \(S\) into \(R\) is a ring with respect to binary operations addition and multiplication defined as \((f + g)(s) = f(s) + g(s)\) and \((fg)(s) = \sum_{t \ast u = s} f(t)g(u)\), where the symbol \(\sum\) indicates that the sum is taken over all pairs \((t, u)\) of elements of \(S\) such that \(t \ast u = s\) and it is settled that in the situation where \(s\) is not expressible in the form \(t \ast u\) for any \(t, u \in S\), \((fg)(s) = 0\). The set \(SR\) is called a unitary commutative semigroup ring of \(S\) over \(R\). If \(S\) is a monoid, then \(SR\) is called monoid ring. This ring \(SR\) is represented as \(R[S]\) whenever \(S\) is a multiplicative semigroup and elements of \(T\) are written either as \(\sum f(s)s\) or as \(\sum_{i=1}^{n} f(s_i)s_i\). The representation of \(SR\) will be \(R[x; S]\) whenever \(S\) is an additive semigroup. A nonzero element \(f\) of \(R[x; S]\) is uniquely represented in the canonical form \(\sum_{i=1}^{n} f(s_i)x^{s_i} = \sum_{i=1}^{n} f_i x^{s_i}\), where \(f_i \neq 0\) and \(s_i \neq s_j\) for \(i \neq j\). Of course, the monoid ring \(R[x; S]\) is a polynomial ring in one indeterminate if \(S\) is the additive monoid \(Z_0\) of non-negative integers.

The concept of degree and order are not generally defined in a semigroup ring. Though if \(S\) is a totally ordered semigroup, then the degree and order of an element of semigroup ring \(R[x; S]\) is defined as: if \(f = \sum_{i=1}^{n} f_i x^{s_i}\) is the canonical form of the nonzero element \(f \in R[x; S]\), where \(s_1 < s_2 < \cdots < s_n\), then \(s_n\) is called the degree of \(f\) and we write \(\text{deg}(f) = s_n\) and similarly the order of \(f\) written as \(\text{ord}(f) = s_1\). Now, if \(R\) is an integral domain, then for \(f, g \in R[x; S]\), it follows that \(\text{deg}(fg) = \text{deg}(f) + \text{deg}(g)\) and \(\text{ord}(fg) = \text{ord}(f) + \text{ord}(g)\).

We start by an observation that, for a field \(F\) and an integer \(j \geq 0\), the structures of a polynomial ring \(F[x]\) and a monoid ring \(F[x; \frac{1}{j}Z_0]\) have many interconnections, for instance, for an ordered monoid \(S\), the monoid ring \(F[x, S]\) is a Euclidean domain if \(F\) is a field and \(S \cong Z\) or \(S \cong Z_0\) [5 Theorem 8.4]. Of course here \(\frac{1}{2}Z_0\) is totally ordered and and has an isomorphism with \(Z_0\).

Let \(F\) be any field and \(\frac{1}{j}Z_0\) is the additive monoid, then \(F[x; \frac{1}{j}Z_0]\) is a monoid ring. A generalized polynomial \(g(x^\frac{1}{j})\) of arbitrary degree \(r\) in \(F[x; \frac{1}{j}Z_0]\) is represented as

\[
g(x^\frac{1}{j}) = g_0 + g_1 x^\frac{1}{j} + g_2 x^{2\frac{1}{j}} + \cdots + g_r x^{r\frac{1}{j}}.
\]

Andrade and Shah has constructed cyclic codes over a local finite commutative ring \(R_0\), through the monoid rings \(R[x; \frac{1}{3}Z_0]\), \(R[x; \frac{1}{2}Z_0]\) and \(R[x; \frac{1}{27}Z_0]\) in [11], [12] and [13], respectively. However, in [14] the cyclic codes of certain types are discussed corresponding to the ascending chain of monoid rings.

3. Bandwidths of \(n\) length BCH code and \(2^{j-1}(n+1)n\) lengths cyclic codes

3.1. Cyclic codes through \(F_q[x]\) and \(F_q[x; \frac{1}{27}Z_0]\). For any positive integer \(j\), there is a following ascending chain of monoid rings given by

\[
F_q[x] \subset F_q[x; \frac{1}{2}Z_0] \subset F_q[x; \frac{1}{22}Z_0] \subset \cdots \subset F_q[x; \frac{1}{27}Z_0] \subset \cdots
\]
Let $n = 2^s - 1$, where $s \in \mathbb{Z}^+$ and take $(n + 1)n = n'$. Thus, $a_0 + a_1 \zeta + a_2 \zeta^2 + \cdots + a_{(2^s - 1)n - 1} \zeta^{2^s - 1} - 1$ is a typical element of $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]/((x^{2^s})^{2^s} - 1)$, where $a_0, a_1, a_2, \cdots, a_{(2^s - 1)n - 1} \in \mathbb{F}_2$ and $\zeta$ is the coset \((x^{2^s})^{2^s} - 1\). So $f(\zeta) = 0$, where $\zeta$ satisfies the relation $\zeta^{2^s - 1} - 1 = 0$.

Now, replace $x^{2^s}$ at the place of $\zeta$. Thus, the ring $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]/((x^{2^s})^{2^s} - 1)$ becomes $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$, in which the relation $(x^{2^s})^{2^s} - 1$ holds. The binary operation multiplications $*$ in the ring $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$ is modulo the ideal $((x^{2^s})^{2^s} - 1)$. So, given $a(x^{2^s}), b(x^{2^s}) \in \mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$, we write $a(x^{2^s}) * b(x^{2^s})$ to denote their product in the ring $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$ and $a(x^{2^s}) * b(x^{2^s})$ to denote their product in the ring $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]$. If deg$(a(x^{2^s})) + \text{deg}(b(x^{2^s})) < 2^{s-1}$, then $a(x^{2^s}) * b(x^{2^s}) = a(x^{2^s})b(x^{2^s})$. Otherwise, $a(x^{2^s}) * b(x^{2^s})$ is the remainder left on dividing $a(x^{2^s})b(x^{2^s})$ by $(x^{2^s})^{2^s} - 1$. In other words, if $a(x^{2^s}) * b(x^{2^s}) = r(x^{2^s})$, then $a(x^{2^s})b(x^{2^s}) = r(x^{2^s}) + ((x^{2^s})^{2^s} - 1)q(x^{2^s})$ for some generalized polynomial $q(x^{2^s})$. Practically, to get $a(x^{2^s}) * b(x^{2^s})$, we only compute the ordinary product $a(x^{2^s})b(x^{2^s})$ and then put $(x^{2^s})^{2^s} - 1 = 1$, $(x^{2^s})^{2^s-1}(n+1)n + \frac{1}{2} = x^{2^s}$, and so on. Now, consider $x^{2^s} * a(x^{2^s})$, and it would be

$$a_{(2^s-1)n+1} + a_0 x^{2^s} + a_1 (x^{2^s})^2 + \cdots + a_{(2^s-1)n-1} (x^{2^s})^{2^s-1}.$$ 

Particularly, the product $x^{2^s} * a(x^{2^s})$ in $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$: as the $\mathbb{F}_2$-space $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$ is isomorphic to $\mathbb{F}_2$-space $\mathbb{F}_2^{2^s-1}$, indeed, $(x^{2^s})^{2^s} - 1 = y^{2^s-1}(n+1)n - 1$, where $x^{2^s} = y$. In fact, we deal the coefficients of generalized polynomials $a(x^{2^s}) = a_0 + a_1 x^{2^s} + \cdots + a_{(2^s-1)n-1} (x^{2^s})^{2^s-1} - 1$ of $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]$, so $c(x^{2^s})$ has $2^s - 1$ terms and hence the coefficients in $\mathbb{F}_2$. Corresponding to the polynomial $a(x^{2^s})$ of $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$, there is a $2^s - 1$-tupled vector $(a_0, a_1, \cdots, a_{(2^s-1)n-1})$ in $\mathbb{F}_2^{2^s-1}$. Thus, there is an isomorphism between the vector spaces $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$ and $\mathbb{F}_2^{2^s-1}$, defined by $a \mapsto a(x^{2^s})$.

We observed that, multiplication by $x^{2^s}$ in the ring $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$ corresponds to cyclic shift $\sigma$ in $\mathbb{F}_2^{2^s-1}$, that is, $x^{2^s} * a(x^{2^s}) = \sigma(a(x^{2^s}))$.

A subspace $C$ of $\mathbb{F}_2$-space $\mathbb{F}_2^{2^s-1}$ is a linear code. As already agreed, we recognize every vector $a$ in $\mathbb{F}_2^{2^s-1}$ with the polynomial $a(x^{2^s})$ in $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$, so the ideal $C_{2^s-1,n'}$ in $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0][2^{s-1}]$ is acyclic code. The elements of the codes $C_{2^s-1,n'}$ are now referred as codewords or code generalized polynomials.

Note that if $C_{2^s-1,n'} = (p(x^{2^s}))$ is the ideal generated by $p(x^{2^s})$, then $p(x^{2^s})$ is the generator (generalized) polynomial of $C_{2^s-1,n'}$ if and only if $p(x^{2^s})$ is monic and divides $(x^{2^s})^{2^s - 1} - 1$ in $\mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]$.

In [11], a link is developed between a binary BCH code $(n, n - r)$ and a sequence \(\{2^s-1\}^1, \{2^s-1\}^2, \{2^s-1\}^r\}^j_{j=1}\) of binary cyclic codes.

Following [11], if $C_0^0$ is a binary BCH code $(n, n - r)$ based on the positive integers $c$, $d$, $q = 2$ and $n$ such that $2 \leq d_1 \leq n$ and $n = 2^s - 1$, where $s \in \mathbb{Z}^+$. Thus, the binary BCH code $C_0^0$ has $r$ degree generator polynomial $g(x) = \text{lcm}\{j_i(x) : i = c, c + 1, \cdots, c + d_1 - 2\}$, where $j_i(x)$ are minimal polynomials of $\zeta^i$, for $i = c, c + 1, \cdots, c + d_1 - 2$. Whereas $\zeta$ is the primitive $n$th root of unity in $\mathbb{F}_2$. Since $j_i(x)$ divides $x^n - 1$ for each $i$, it follows that $g(x)$ divides $x^n - 1$. Thus, $C_0^0$ is a principal ideal in the ring $\mathbb{F}_2[x]_n$ which is generated by $g(x)$.

**Theorem 1.** [Theorem 11] For a binary BCH code $s$, let $C_0^0$ be a binary BCH code of length $n = 2^s - 1$ generated by a polynomial $g(x) = g_0 + g_1x + \cdots + g_n x^n \in \mathbb{F}_2[x]$ of degree $r$. Then,

1. there exists a sequence \(\{C_{j}^{2^s-1,n'}\}_{j\geq1}\) of binary cyclic codes such that for each $j \geq 1$, the code $C_{2^s-1,n'}$ has length $2^s - 1(n + 1)n$, generated by $2^s r$ degree generalized polynomial $g(x^{2^s}) = g_0 + g_1(x^{2^s}) + \cdots + g_{2^s-1}(x^{2^s})^{2^s} \in \mathbb{F}_2[x; \frac{1}{2}\mathbb{Z}_0]$.
2. the binary BCH code $C_0^0$ is embedded in each binary cyclic code $C_{j}^{2^s-1,n'}$ for $j \geq 1$,
3. there are embeddings $C_{n'} \hookrightarrow C_{2^s-1,n'} \hookrightarrow \cdots \hookrightarrow C_{2^s-1,n'} \hookrightarrow \cdots$ of binary cyclic codes of the sequence \(\{C_{j}^{2^s-1,n'}\}_{j\geq1}\).

In [11] it is established that for a binary BCH code $C_n^0 = (g(x))$ there does not exist any binary BCH code $C_j^{2^s-1,n'}$ generated by polynomial $g(x^{2^s})$. 
The following table for $C_{2^j-1,n'}^j = (2^j - 1,n', 2^j - 1,n' - 2^j r)$ can be constructed for varying, $n, r, j$, where integer $j \geq 0$.

| $n, r$ | $C_n^0$ | $C_{n'}^1$ | $C_{2n'}^2$ | $C_{2m'}^{3}$ | $C_{2^2m'}^4$ | $\cdots$ |
|--------|---------|------------|-------------|--------------|-------------|-----------|
| 3, 2   | (3, 1)  | (12, 8)    | (24, 16)    | (48, 32)     | (96, 64)    | $\cdots$ |

### 3.2. Bandwidth limitations

Modulation is the process by which information is conveyed by means of an electromagnetic wave. The information is impressed on a sinusoidal carrier wave by varying its amplitude, frequency, or phase. Methods of modulation may be either analog or digital. The power and bandwidth necessary for the transmission of a signal with a given level of quality depends on the method of modulation. There is a classic tradeoff between power and bandwidth that is fundamental to the efficient design of communication systems. There are three types of modulation: Amplitude shift keying (ASK), Frequency shift keying (FSK), and Phase shift keying (PSK). Furthermore, quicker computer processors allow the use of more complex forward error correction coding techniques at high bit rates. Therefore, more spectrum proficient procedures of digital modulation such as 8PSK and 16QAM are more gorgeous, even though the power necessities are higher. Together with powerful coding methods such as concatenated BCH coding, these methods offer the viewpoint of improved spectral efficiency with essentially error-free digital signal transmission.

Let $S$ be the signal set $j$ be the number of signals in the signal set. Suppose $v(t) = (v_0(t), \cdots, v_{n-1}(t)) \in \mathbb{F}_q^n$ is the codeword of an $(n, k)$-code corresponding to a message $u(t) = (u_0(t), \cdots, u_{k-1}(t)) \in \mathbb{F}_q^k$ at time $t$ and we have divided each $v(t)$ into $n/m$ blocks, where $m = \log q^j$ and $j = q^m$ (the case $q = 2$). Thus, modulation is a map $M : \mathbb{F}_q^n \to S$ defined as $s(t) = s(v(t))$, where $s(t) \in S$ and $S$ is a subset of $N$-dimensional real Euclidean space, that is, $S \subset \mathbb{R}^N$ [2].

Following [10], the bandwidth required for an $(n, k)$ code is $W = \frac{R}{m} \left(\frac{1}{r}\right)$, where $m = \log_2 M$, $R_n$ is the source data (transmission) rate and $R = \frac{k}{n}$ the code rate.

The bandwidth may be maximize and minimize, depends upon the minimum and the maximum value of the $n/k = 1/R$ and the value of the, $m$ bits for selection of modulation scheme for different modulation types. These bits may be minimum and maximum for maximum and minimum bandwidth. It can be seen as $W_{\text{max}} = \frac{R}{m_{\text{min}}} \left(\frac{1}{r}\right)_{\text{max}}$ and $W_{\text{min}} = \frac{R}{m_{\text{max}}} \left(\frac{1}{r}\right)_{\text{min}}$. Thus, there are the followings possibilities:

1. $m$ is fixed but $\frac{1}{r}$ is varying, and
2. $m$ and $\frac{1}{r}$ both are varying.

For cognitive radio transformation under the interweave model we may get spectrum corresponding to the given sequence $\{C_{2^j-1,n'}^j\}_{j=1}^j$ of binary cyclic codes for data transfer of the primary users. Now, the setup allow the secondary users having the binary BCH code $C_n^0$ mod for their data transfer. Accordingly the secondary users obtain high speed data transfer as compare to its own scheme of the BCH code $C_n^0$. Furthermore, since there are embeddings $C_n^j \to C_{2n'}^j \to \cdots \to C_{2^j-1,n'}^j$ of binary cyclic codes of the sequence $\{C_{2^j-1,n'}^j\}_{j=1}^j$ and the binary BCH code $C_n^0$ is embedded in each of binary cyclic codes $C_{2^j-1,n'}^j$ for $1 \leq j \leq j_0$.

It is noticed that corresponding to the code rate $R_n^0 = \frac{k}{n}$ of binary BCH code $C_n^0$, the code rate of binary cyclic code $C_{2^j-1,n'}^j$, is $R_{2^j-1,n'}^j = \frac{2^j - 1}{2^j - 1,n'} \cdot \frac{2^j}{2^j - 1,n'}$, for each $1 \leq j \leq j_0$. Moreover, $R_n^0 \leq R_{2^2-1,n'}^j$ and $R_{2^2-1,n'}^j$ is same for each binary cyclic code $C_{2^j-1,n'}^j$, that is, $R_n^1 = R_{2^2-1,n'}^2 = \cdots = R_{2^j-1,n'}^j$. This means $1/R_{2^j-1,n'}^j \leq \frac{1}{R_n^0}$, and therefore, $W_{2^j-1,n'}^j = \frac{R_{2^j-1,n'}}{m} \cdot 1/R_{2^j-1,n'}^j \leq W_n^0 = \frac{R_n^0}{m^0} (1/R_n^0)$. Thus, if we transmit data through any of code of the sequence $\{C_{2^j-1,n'}^{j_0}\}_{j=1}^{j_0}$, the bandwidth $W_{2^j-1,n'}^j$ for each $j \geq 1$ will be lesser the bandwidth $W_n^0$ required for data transmitted through the binary BCH code $C_n^0$. Interestingly the same bandwidth $W_{2^j-1,n'}^j = W$ is required for a user having any type of code from the sequence $\{C_{2^j-1,n'}^j\}_{j=1}^{j_0}$ of binary cyclic codes.

For $m = 1, 3$, the relation between bandwidth and code rate is given as $W = w(R_n/m)(1/R) = wR_n/mR$, where $w$ is the bandwidth expansion, $R_n$ is the transmission rate and $R = k/n$ is the code rate. The bandwidth with different code rates is given in the Table III (Chosen codes are from Table 1).
4.1 How the model work. Notions

\( j = 0, 1 \leq j \leq 2^{j_0 - 1} n' \).

\( P_0 \): primary user corresponding to the binary BCH code \( C_0 \).

\( P_{2^{j_0 - 1} n'} \): \( j \)-th primary user corresponding to the binary cyclic code \( C_{2^{j_0 - 1} n'} \).

\( m^j \): information symbols for \( j \)-th user.

\( E^j \): \( j \)-th encoder for \( m^j \).

\( j_P \): modulation for \( E^j \).
4.2. Illustration. Let \( n = 2^2 - 1 = 3 \), \( \delta = 3 \), \( c = 1 \), \( p(x) = x^2 + x + 1 \) a primitive polynomial of degree 2 and \( \mathbb{F}_2 = \mathbb{F}_2[x] / \langle f(x) \rangle \). where \( f(x) \) satisfies the relation \( x^3 + x + 1 = 0 \). Using this relation, we obtain \( \{0, 1, x + 1 \} \). Let \( m_i(x) \) be the minimal polynomial of \( i \), for \( i = c, c + 1, \ldots, c + \delta - 2 \). Thus, \( m_i(x) = x^2 + x + 1 \) and hence \( g(x) = \text{lcm} \{m_i(x) \} = x^2 + x + 1 \). The code \( C_3 = (g(x)) \subset \mathbb{F}_2[x]_3 \) is a binary BCH code based on the positive integers \( c = 1, \delta = 3, q = 2 \) and \( n = 3 \) such that \( 2 \leq \delta \leq n \) with \( \gcd(n, 2) = 1 \). Consequently, \( C_{2^j-1} = \{C_{2^j-1} \} \).
of binary cyclic codes corresponding to the BCH code $C_0^3$. For instance $C_{2^1-1(3+1)+3}^1 = C_{12}^1$ a $(12,8)$ code which is generated by $g(x^2) = (x^2)^2(2) + (x^2)^4 + (x^2)^2 + 1$ in $\mathbb{F}_2[x, \frac{1}{x}Z_0]$ and $C_{2^2-1(3+1)+3}^2 = C_{24}^2$ is a $(24,16)$ code which is generated by $g(x^2) = (x^2)^2(2) + (x^2)^4 + 1$ in $\mathbb{F}_2[x, \frac{1}{x}Z_0]$, and so on.

Follow the Table I and label the corresponding bandwidths as:

\[
\begin{align*}
\text{Form} &= 1 \\
W_3^0 &= 236.4 : \square \square \\
W_{12}^1 &= W_{24}^2 : 118.2 : \\
\text{Form} &= 3 \\
W_3^0 &= 69.78 : \square \square \\
W_{12}^1 &= W_{24}^2 : 34.89 : \\
\end{align*}
\]

If the data of user $P_3^0$ is transmitted through the binary BCH code $C_3^0$ for any modulation scheme, it requires double bandwidth than the bandwidth required for its transmission through the path of any of the binary cyclic codes $C_{12}^0$ and $C_{24}^0$.

In the following we illustrate the decoding steps. Consider the binary BCH code $C_3^0$.

The canonical generator matrix of binary cyclic $(12,8)$ code $C_{12}^1$ is given by

\[
G^1 = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1
\end{bmatrix},
\]
which is obtained by the generator polynomial $g(x^{\frac{1}{2}}) = 1 + (x^{\frac{1}{2}})^2 + (x^{\frac{1}{2}})^4$, whereas the parity check matrix with check polynomial $h(x^{\frac{1}{2}}) = 1 + (x^{\frac{1}{2}})^2 + (x^{\frac{1}{2}})^6 + (x^{\frac{1}{2}})^8$ is given

$$H_1 = \begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 
\end{bmatrix}. $$

Syndrome table is given by

| coset leader | syndrome |
|--------------|----------|
| $e_0$        | 000000000000 | 0000 |
| $e_1$        | 100000000000 | 1000 |
| $e_2$        | 010000000000 | 0100 |
| $e_3$        | 001000000000 | 1010 |
| $e_4$        | 000100000000 | 0101 |
| $e_5$        | 000010000000 | 0010 |
| $e_6$        | 000001000000 | 0001 |
| $e_7$        | 110000000000 | 1100 |
| $e_8$        | 100100000000 | 1101 |
| $e_9$        | 100001000000 | 1001 |
| $e_{10}$     | 011000000000 | 1110 |
| $e_{11}$     | 010010000000 | 0110 |
| $e_{12}$     | 001100000000 | 1111 |
| $e_{13}$     | 001001000000 | 1011 |
| $e_{14}$     | 000110000000 | 0111 |
| $e_{15}$     | 000011000000 | 0011 |

Let $b = 101 \in \mathbb{F}_2^3$ be the received vector of binary BCH code $C_{3}^0$. Thus, its polynomial representation is $b(x) = 1 + x^2$ in $\mathbb{F}_2[x]_3$ and corresponding received polynomial in the cyclic code $C_{12}^1$ is given by $b'(x^{\frac{1}{2}}) = 1 + (x^{\frac{1}{2}})^4$ in $\mathbb{F}_2[x; \frac{1}{2} \mathbb{Z}_0]_{12}$ by using [11, Theorem 1], and its vector representation will be $b' = 100010000000$ in $\mathbb{F}_2^2$ and $(b') = b'(H_1)^T = 1010 = S(e_3)$. Hence, the corrected codeword in $C_{12}^1$ is $a' = b' + e_3 = 101010000000$ and its polynomial representation is $a'(x^{\frac{1}{2}}) = 1 + (x^{\frac{1}{2}})^2 + (x^{\frac{1}{2}})^{2(2)}$ in $\mathbb{F}_2[x; \frac{1}{2} \mathbb{Z}_0]_{12}$. Thus, the corresponding corrected codeword in binary BCH code $C_{3}^0$ is $a(x) = 1 + x + x^2$ in $\mathbb{F}_2[x]_3$, i.e., $a = 111$.

In the similar fashion we can decode received vector of $C_{3}^0$ through the decoding of any member of the family \{\textit{C}_{2^j-1}^j(n+1)\}_{j \geq 1}$ instead of $C_{12}^1$.

5. Conclusion

This study proposed a novel interweave inclined transmission model for cognitive radio. The data of primary user $\mathcal{P}_0^n$ is configured and transmitted through the binary BCH code $C_{0}^n$. However, the data of the family \{\textit{P}_{2^j-1}^j\}_{j \geq 1}$ of primary users is configured by the family \{\textit{C}_{2^j-1}^j(n+1)\}_{j \geq 1}$ having sequentially increasing code lengths but with same code rate. Due to the modulation scheme every member of \{\textit{C}_{2^j-1}^j(n)\}_{j \geq 1}$ requires same bandwidth but lesser than required for the binary BCH code $C_{0}^n$.

Initially all these codes are carrying data of their corresponding primary users. A transmission pattern is planned in the spirit of interweave model in such a way that the user $\mathcal{P}_0^n$ observe and opportunistically avail the channel path of any of the Primary users of the family \{\textit{P}_{2^j-1}^j\}_{j \geq 1}$ not utilizing its allotted bandwidth.

This study can also be extended for a set of different $n$, the length of the binary BCH code $C_{0}^n$. Consequently, a multiple transformation model can be designed.

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