INVESTIGATION OF THE NEUTRON FORM FACTORS BY INCLUSIVE QUASI-ELASTIC SCATTERING OF POLARIZED ELECTRONS OFF POLARIZED $^3$He: A THEORETICAL OVERVIEW

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ABSTRACT

The theory of quasi-elastic inclusive scattering of polarized leptons off polarized $^3$He is critically reviewed and the origin of different expressions for the polarized nuclear response function appearing in the literature is explained. The sensitivity of the longitudinal asymmetry upon the neutron form factors is thoroughly investigated and the role played by the polarization angle for minimizing the proton contribution is illustrated.

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1. Introduction

The scattering of polarized electrons by polarized targets represents a valuable tool for investigating nucleon and nuclear properties in great detail [1]. In particular quasi-elastic (qe) inclusive experiments involving polarized $^3$He are viewed as possible sources of information on the neutron form factors [2, 3, 4]. As a matter of fact, if a naive model of $^3$He, including only the main spatially symmetric S component of the three-body wave function with the two protons with opposite spins, is considered, a polarized $^3$He can be regarded to a large extent as an effective neutron target. However a proton contribution, arises from the S' and D-waves of the three body wave function. Such a contribution has been investigated in Ref.[5] within the closure approximation, i.e., by describing nuclear effects through spin-dependent momentum distributions. Adopting the general formalism of Ref.[5], the effects of nucleon binding have been analysed in Ref.[5], where the concept of the spin dependent spectral function has been introduced and applied to the calculation of the $^3$He asymmetry. An analysis of the asymmetry in plane wave impulse approximation (PWIA) has been performed also in Ref.[6], where, besides studying the effects of binding: i) a new expression for the polarized nuclear structure functions has been obtained and the formalism of Ref.[5] has been shown to suffer from severe inconsistencies, and ii) doubts have been raised as to the possibility of obtaining reliable information on the neutron form factors by the measurement of the inclusive asymmetry, due to the large proton contribution.

In view of the relevance of this two points it is our aim: i) to elucidate in detail the origin of the differences between the expression of the polarized structure functions used in Refs.[5, 6] and the one obtained in Ref.[7] by presenting a comprehensive derivation of the inclusive cross section in PWIA, and ii) to show that the proton contribution depends upon the kinematics in such a way that one can choose a proper kinematics in order to make the qe asymmetry very sensitive to neutron properties, including the neutron electric form factor.

The paper is organized as follows: in Sect. 2 the antisymmetric part of the hadronic tensor will be analyzed and the different methods for obtaining the nuclear polarized structure functions will be discussed; in Sect. 3 the nuclear polarized structure functions will be obtained within the PWIA by using the nucleon spin-dependent spectral function; in Sect. 4 the comparison with the experimental asymmetries of Ref.[2, 3] will be discussed; in Sect. 5 a proposal for minimizing the proton contribution will be illustrated and in Sect. 6 conclusions will be drawn.

2. The hadronic tensor and the inclusive cross section

In what follows we will consider the inclusive cross section describing the scattering of a longitudinally polarized lepton of helicity $h = \pm 1$ by a polarized hadron of spin $J = 1/2$; in one photon exchange approximation one gets [4]

$$\frac{d^2\sigma(h)}{d\Omega_2 d\nu} \equiv \sigma_2 \left(\nu, Q^2, \vec{S}_A, h\right) = \frac{4\alpha^2}{Q^4} \frac{\epsilon_2}{\epsilon_1} m^2 L^{\mu\nu} W_{\mu\nu} =$$
\[ = \frac{4\alpha^2}{Q^4} \frac{\epsilon_2}{\epsilon_1} m^2 \left[ L_s^{\mu\nu} W_s^{\mu\nu} + L_a^{\mu\nu} W_a^{\mu\nu} \right] \] (1)

where the symmetric \((s)\) and antisymmetric \((a)\) leptonic tensors are

\[
L_s^{\mu\nu} = -(g^{\mu\nu} \frac{q^\mu q^\nu}{Q^2}) \frac{Q^2}{4m^2} + \frac{1}{m^2} \left( k_1^{\mu} - \frac{q^\mu}{2} \right) \left( k_1^{\nu} - \frac{q^\nu}{2} \right) \] (2)

\[
L_a^{\mu\nu} = i \frac{\epsilon^{\mu\nu\rho\sigma} q_\rho k_{1\sigma}}{2m^2} \] (3)

and the corresponding hadronic tensors are

\[
W_s^{\mu\nu} = -(g_{\mu\nu} + q_\mu q_\nu) W_1^{A} + (P_{A\mu} + \frac{P_A \cdot q}{Q^2} q_\mu) (P_{A\nu} + \frac{P_A \cdot q}{Q^2} q_\nu) \frac{W_2^{A}}{M_A^2} \] (4)

\[
W_a^{\mu\nu} = i \epsilon_{\mu\nu\rho\sigma} q_\rho V^\sigma \] (5)

The pseudovector \(V^\sigma\) appearing in Eq.(5) can be expressed as follows

\[
V^\sigma \equiv S_A^\sigma \frac{G_A^1}{M_A^1} + (P_A \cdot q S_A^\sigma - S_A \cdot q P_A^\sigma) \frac{G_A^2}{M_A^2} \] (6)

In the above equations, the index \(A\) denotes the number of nucleons composing the target, \(k_1^{\mu(2)} \equiv (\epsilon_1(2), \tilde{k}_1(2))\) and \(P_A^\mu \equiv (M_A, 0)\) are electron and target four-momenta, \(q^\mu \equiv (\nu, \vec{q})\) is the four-momentum transfer, \(Q^2 = -q^2 = |\vec{q}|^2 - \nu^2\), \(g_{\mu\nu}\) is the symmetric metric tensor, \(\epsilon_{\mu\nu\rho\sigma}\) the fully antisymmetric tensor, \(S_A^\mu\) the polarization four-vector (in the nucleus rest frame \(S_A^\mu \equiv (0, \vec{S}_A)\)) and \(W_1^{A}\) and \(G_1^{A}\) are the nuclear unpolarized and polarized structure functions, respectively.

In polarized scattering, both the symmetric \((W_s^{\mu\nu})\) and the antisymmetric \((W_a^{\mu\nu})\) part of the hadronic tensor are involved, but in what follows we will focus on the antisymmetric one, since it contains the relevant physical quantities we will investigate. To this end the following comments are in order:

i) to obtain the general form of \(W_a^{\mu\nu}\) one follows a procedure analogous to the one adopted to obtain the symmetric tensor \(W_s^{\mu\nu}\), namely one imposes Lorentz, gauge, parity and time reversal invariances on the weighted sum of all the available antisymmetric tensors;

ii) terms proportional to \(q^\sigma\), which in principle could appear in the definition of \(V^\sigma\), were not included in Eq. (5) since the antisymmetric tensor \(\epsilon_{\mu\nu\rho\sigma}\) in Eq. (5) cancels out the contribution to \(W_a^{\mu\nu}\) arising from any term of this kind (\(\epsilon_{\mu\nu\rho\sigma} q^\alpha q^\beta = 0\)).

Let us stress that, according to remark ii), only the component of \(V^\sigma\) orthogonal to \(q^\sigma\) can be determined by the knowledge of the tensor \(W_a^{\mu\nu}\), and therefore only such a component represents a physically relevant quantity, while the one parallel to \(q^\sigma\) is completely undetermined. Indeed, by "inverting" Eq.(5) one obtains
\[ \tilde{V}^\sigma \equiv V^\sigma + \frac{q^\sigma}{Q^2} (V \cdot q) = i \frac{1}{2} \frac{1}{Q^2} \epsilon^{\sigma \mu \nu \omega} q_\omega W^\sigma_{\mu \nu} \] (7)

where the four-vector \( \tilde{V}^\sigma \) results to be orthogonal to \( q^\sigma \), i.e. \( \tilde{V} \cdot q = 0 \). The relevance of this last comment will be clear later on, when the method for obtaining the polarized structure functions within the PWIA will be discussed in detail.

In total analogy with the case of the unpolarized structure functions \( W^A_{1(2)} \), which are determined by the symmetric part of the hadronic tensor, the polarized structure functions \( G^A_{1(2)} \) are obtained by expressing them in terms of the components of \( W^\sigma_{\mu \nu} \), or equivalently, they can be expressed in terms of the components of the pseudovector \( \tilde{V}^\sigma \). In the rest frame of the target, using Eqs. (5), (6) and (7) and choosing the z-axis along the momentum transfer \( \hat{q} \equiv \hat{u}_z \), one gets the following expressions for the polarized structure functions \( G^A_1 \) and \( G^A_2 \) (cf. Refs. [7] and [8]):

\[ \frac{G^A_1}{M^2_A} = - Q^2 \frac{\tilde{V}_z}{|q|^2} \frac{S^2_A}{S^2_{A\perp}} - \frac{\tilde{V}_z}{S^2_A} \left( \frac{\tilde{V}_z}{S^2_A} - \frac{\tilde{V}_z}{S^2_{A\perp}} \right) = - i \frac{1}{|q|^2} \left( \frac{Q^2}{|q|^2} \frac{W^\sigma_{02}}{S^2_A} + \frac{\nu}{|q|^2} \frac{W^\sigma_{12}}{S^2_A} \right) \] (8)

\[ \frac{G^A_2}{M^2_A} = \tilde{V}_0 \frac{S^2_A}{S^2_{A\perp}} + \nu \frac{1}{|q|^2} \left( \frac{\tilde{V}_z}{S^2_A} - \frac{\tilde{V}_z}{S^2_{A\perp}} \right) = - i \frac{1}{|q|^2} \left( \frac{\nu}{|q|^2} \frac{W^\sigma_{02}}{S^2_A} - \frac{W^\sigma_{12}}{S^2_A} \right) \] (9)

where \( S^2_{A\perp} = \sqrt{S^2_{Ax} + S^2_{Ay}} \) and \( \tilde{V} = (\tilde{V} \cdot \hat{S}_A - \tilde{V} \cdot S_{A\perp})/S_{A\perp} \).

In line with remark ii), Eqs. (8) and (9) are not affected, because of Eq. (5), by any arbitrary term proportional to \( q^\sigma \) which could be added to \( V^\sigma \). In Ref. [7] a different extraction method was proposed, namely \( G^A_{1(2)} \) were obtained by expressing them in terms of the components of the pseudovector \( V^\sigma \). From Eq. (5) one has

\[ \frac{G^A_1}{M^2_A} = - \frac{(V \cdot q)}{|q| S_A} \] (10)

\[ \frac{G^A_2}{M^2_A} = \frac{V_0}{|q| S_A} \] (11)

Given the form (3) for \( V^\sigma \), Eqs. (8) and (9) are totally equivalent to Eqs. (10) and (11). However, such an equivalence will break down if a term proportional to \( q^\sigma \) is explicitly added to the r.h.s. of Eq. (3), since, as already noted, Eqs. (8) and (9) will be unaffected by the added term, whereas Eqs. (10) and (11) will be; therefore \( G^A_1 \) and \( G^A_2 \) obtained from Eqs. (10) and (11) will be incorrect in this case. This remark will be very relevant for the discussion of the evaluation of \( G^A_1 \) and \( G^A_2 \) within the PWIA, which will be presented in the next Section. To sum up, unlike the unpolarized case, two different procedures have been followed to obtain the polarized structure functions \( G^A_{1(2)} \); they lead to Eqs. (8) and (9) and Eqs. (10) and (11), respectively; however the latter are correct only in so far as \( V^\sigma \) is a linear combination of only \( S^\sigma_A \) and \( P^\sigma_A \) and terms proportional to \( q^\sigma \) are absent. We will
call the correct prescription leading to Eqs. (8) and (9) prescription I (corresponding to the prescription A of Ref. [7]) and the one leading to Eqs. (10) and (11) prescription II (corresponding to the prescription C of Ref. [7] and originally proposed in Ref. [5]).

3. The polarized structure functions in PWIA

The equations given in Sect.2 are general ones, relying only on the one photon exchange approximation. When comparing with experimental data, one has to adopt models for the nuclear structure functions. In particular all papers so far published [5, 6, 7, 8] use the PWIA. Within such an approximation, one can obtain the following expression for the antisymmetric hadronic tensor

$$w_{\mu \nu}^\alpha = i\epsilon_{\mu \nu \alpha \beta} q^\beta R^3$$

where the four-pseudovector $R^3$ is given by

$$R^3 = \sum_{N=p,n} \left[ \frac{\tilde{G}_1^N(Q^2)}{M} \langle S^3 \rangle_N + \frac{\tilde{G}_2^N(Q^2)}{M^3} q_\alpha \left( \langle p^\alpha S^3 \rangle_N - \langle p^3 S^\alpha \rangle_N \right) \right]$$

In Eq. (13) $p^\alpha \equiv (\sqrt{M^2 + |\vec{p}|^2}, \vec{p})$ is the on-shell nucleon momentum, $\tilde{G}_1^N(Q^2)$ and $\tilde{G}_2^N(Q^2)$ are the nucleon polarized form factors, related to the nucleon Sachs form factors by the following equations [5]

$$\tilde{G}_1^N(Q^2) = -\frac{G_M^N}{2} \frac{(G_E^N + \tau G_M^N)}{1 + \tau}$$

$$\tilde{G}_2^N(Q^2) = \frac{G_M^N}{4} \frac{(G_M^N - G_E^N)}{1 + \tau}$$

with $\tau = Q^2/(4M^2)$, and the mean values $\langle S^3 \rangle_N$ and $\langle p^\alpha S^3 \rangle_N$ read as follows

$$\langle [p^\alpha] S^3 \rangle_N = \int dE \int d\vec{p} \frac{M^2}{E_p E_{p+q}} [p^\alpha] \sum_{\ell=x,y,z} f_M^N(p, E) S_{\ell}^3 \delta(\nu + M_A - \sqrt{(M (A-1) + E_{f(A-1)}^2) + |\vec{p}|^2 - E_{p+q}})$$

In Eq. (16) $E_{p+q} = \sqrt{M^2 + (\vec{p} + \vec{q})^2}$, $E = E_{f(A-1)} - E_A$ is the nucleon removal energy, $E_{f(A-1)}$ and $E_A$ are the eigenvalues of the energy of the spectator system and of the target nucleus, respectively, and $S_{\ell}^3 \equiv (\hat{u}_\ell \cdot \vec{p}/M, \hat{u}_\ell + \vec{p} \hat{u}_\ell \cdot \vec{p}/(M(E_p + M)))$ is a four-vector, which in the rest frame of the nucleon has the direction of the $\ell$-axis ( $\hat{u}_\ell$ is the versor corresponding to the $\ell$-axis ($\ell = x, y, z$)). Eq. (16) is a generalization [5, 8] of the expressions of Ref. [5] to the case where both the nucleon momentum and energy distributions are considered.
In Eq. (13) the three-dimensional pseudovector \( \vec{f}_M(p, E) \) describes the nuclear structure and is defined as follows

\[
\vec{f}_M(p, E) = \text{Tr} \left( \hat{P}_M(p, E) \hat{\sigma} \right)
\] (17)

where the 2x2 matrix \( \hat{P}_M(p, E) \) is the spin dependent spectral function of a nucleon inside a nucleus with polarization \( \vec{S}_A \) oriented, in general, along a direction different from the z-axis, and \( M \) is the component of the total angular momentum along \( \vec{S}_A \). The elements of the matrix \( \hat{P}_M(p, E) \) are given by

\[
P^N_{\sigma, \sigma', M}(\vec{p}, E) = \sum_{f(A-1)} N\langle \vec{p}, \sigma; \psi_{f(A-1)} | \psi_{JM} \rangle \langle \psi_{JM} | \psi_{f(A-1)}; \vec{p}, \sigma' \rangle_N \delta(E - E_{f(A-1)} + E_A) \] (18)

where \( |\psi_{JM}\rangle \) is the ground state of the target nucleus polarized along \( \vec{S}_A \), \( |\psi_{f(A-1)}\rangle \) an eigenstate of the (A-1) nucleon system interacting with the same two-body potential of the target nucleus, \( |\vec{p}, \sigma\rangle_N \) the plane wave for the nucleon \( N \) with the spin along the z-axis equal to \( \sigma \).

In a more compact form, for \( J = 1/2 \), \( \hat{P}_M(p, E) \) is given by

\[
\hat{P}_M(p, E) = \frac{1}{2} \left[ B^N_{0,M}(|p|, E) + \hat{\sigma} \cdot \vec{f}_M(p, E) \right]
\] (19)

where the function \( B^N_{0,M}(|p|, E) \) is the trace of \( \hat{P}_M(p, E) \) and yields the usual unpolarized spectral function \( P^N(|p|, E) \) \[^{[6]}\] . It should be noticed that the matrix \( \hat{P}_M(p, E) \) and the pseudovector \( \vec{f}_M(p, E) \) depend on the direction of the polarization vector \( \vec{S}_A \). Since \( \vec{f}_M(p, E) \) is a pseudovector, it is a linear combination of the pseudovectors at our disposal, viz. \( \vec{S}_A \) and \( \hat{p} \cdot (\hat{p} \cdot \vec{S}_A) \), and therefore it can be put in the following form, where any angular dependence is explicitly given,

\[
\vec{f}_M(p, E) = \vec{S}_A B^N_{1,M}(|p|, E) + \hat{p} \cdot (\hat{p} \cdot \vec{S}_A) B^N_{2,M}(|p|, E)
\] (20)

The functions \( B^N_{0,M}(|p|, E) \), \( B^N_{1,M}(|p|, E) \) and \( B^N_{2,M}(|p|, E) \) satisfy the following relations

\[
\begin{align*}
B^N_{0,1/2}(|p|, E) &= B^N_{0,-1/2}(|p|, E) \\
B^N_{1,1/2}(|p|, E) &= -B^N_{1,-1/2}(|p|, E) \\
B^N_{2,1/2}(|p|, E) &= -B^N_{2,-1/2}(|p|, E)
\end{align*}
\] (21)

The explicit expressions of the functions \( B^N_{0(1,2),M} \) for a nucleus with an arbitrary value of the total angular momentum are given in the Appendix A in terms of the overlap integrals \( N\langle \vec{p}, \sigma; \psi_{f(A-1)} | \psi_{JM} \rangle \), where \( \rho \) is the Jacobi coordinate of the nucleon \( N \) with respect to the (A-1) system.
The spin dependent spectral function of $^3$He has been first obtained \[1\] from the overlap integrals, calculated with a variational three body wave function corresponding to the Reid soft-core interaction \[11\]. The same quantity has been calculated in Ref. \[4\], but using a Faddeev wave function and the Paris potential \[11\], obtaining a similar dependence. In Fig. 1 (2), the unpolarized spectral function \(P^{\text{unpol}}(\{\vec{p}\}, E) \equiv B^{\text{pol}(n)}_{0,\frac{1}{2}}(\{\vec{p}\}, E)\) and the functions \(|B^{\text{pol}(n)}_{1,\frac{1}{2}}(\{\vec{p}\}, E)|\) and \(|B^{\text{pol}(n)}_{2,\frac{1}{2}}(\{\vec{p}\}, E)|\) are shown (in the case of the proton the curve corresponding to a spectator deuteron is not presented). The relations between \(B^{N}_{1,M}, B^{N}_{2,M}\) and the quantities \(P^{\parallel}(\{\vec{p}\}, E, \alpha)\) and \(P^{\perp}(\{\vec{p}\}, E, \alpha)\) used in our previous paper \[4\] can be found from Eqs. (17), (18), (19) and (20) by assuming \(\vec{S}_A \equiv \hat{\gamma} \equiv \hat{u}_z\) and \(M = 1/2\). Indeed, from the z-component of \(f^N_M(\vec{p}, E)\) one has

\[
P^{\parallel}(\{\vec{p}\}, E, \alpha) = \sum_{\nu = \frac{1}{2}, \frac{3}{2}} P^N_{\frac{1}{2}+\frac{1}{2}}(\vec{p}, E) - P^N_{\frac{1}{2}-\frac{1}{2}}(\vec{p}, E) = B^N_{1\frac{1}{2}}(\{\vec{p}\}, E) + B^N_{1\frac{3}{2}}(\{\vec{p}\}, E) \cos^2 \alpha \tag{22}
\]

while from the other two components of \(f^N_M(\vec{p}, E)\) one gets

\[
P^{\perp}(\{\vec{p}\}, E, \alpha) = 2 P^N_{\frac{1}{2}+\frac{3}{2}}(\vec{p}, E) \cos \alpha \sin \alpha \tag{23}
\]

where \(\cos \alpha = \hat{p} \cdot \hat{\gamma}\). It should be pointed out that for a nucleus with an arbitrary \(J\) the function \(B^{N}_{0,M}\) (for \(J \geq 1\)) and \(B^{N}_{1(2),M}\) (for \(J > 1\)) depend upon even powers of the pseudoscalar quantity \((\vec{S}_A \cdot \hat{p})\) as well (see Appendix A).

The evaluation of \(G^A_1\) and \(G^A_2\) within the PWIA can be carried out by substituting in Eqs. (8) and (9) the elements of the PWIA hadronic tensor \(w^a_{\mu \nu}\) obtained from Eqs. (12), (13) and (16), and choosing \(\hat{\gamma} \equiv \vec{S}_A \equiv \hat{u}_z\), since the polarized structure functions do not depend upon the direction of both \(\vec{q}\) and \(\vec{S}_A\). In order to use \(P^{N}_{\parallel,\perp}(\{\vec{p}\}, E, \alpha)\) instead of the functions \(B^{N}_{1(2),\frac{1}{2}}\) one can invert Eqs. (22) and (23); then one gets (in what follows \(p \equiv |\vec{p}|\))

\[
\frac{G^A_1(Q^2, \nu)}{M_A} = 2\pi \sum_{N=p,n} E_{\text{max}}(Q^2, \nu) \int_{E_{\text{min}}} E_{\text{max}}(Q^2, \nu, E) \frac{p}{|q|E_p} dp \left\{ \tilde{G}^N_1(Q^2) \left[ M P^N_{\parallel}(p, E, \alpha) + 
right.
\left. -p \left( \nu \frac{p \cos \alpha}{M + E_p} \right) P^N(p, E, \alpha) \right] - \frac{Q^2}{|q|^2} \mathcal{L}^N \right\} \tag{24}
\]

\[
\frac{G^A_2(Q^2, \nu)}{M_A^2} = 2\pi \sum_{N=p,n} E_{\text{max}}(Q^2, \nu) \int_{E_{\text{min}}} E_{\text{max}}(Q^2, \nu, E) \frac{p}{|q|E_p} dp \left\{ \tilde{G}^N_1(Q^2) \frac{p}{|q|} P^N(p, E, \alpha) + 
right.
\left. -p \left( \nu \frac{p \cos \alpha}{M + E_p} \right) P^N(p, E, \alpha) \right\}
\]

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\[ + \frac{G_2^N(Q^2)}{M} \left( E_p \, P_\parallel(p, E, \alpha) - \frac{p^2 \cos \alpha}{M + E_p} \, P_N(p, E, \alpha) \right) - \frac{\nu}{|q|^2} \mathcal{L}^N \]  

(25)

with

\[
\mathcal{L}^N = \left[ \tilde{G}_1^N(Q^2) \, \mathcal{H}_1^N + |\vec{q}| \, \frac{G_2^N(Q^2)}{M} \, \mathcal{H}_2^N \right]
\]

(26)

\[
\mathcal{H}_1^N = \frac{1}{2} \left( \frac{3 \cos^2 \alpha - 1}{\cos \alpha} \right) \left[ \frac{p^2}{M + E_p} P_N(p, E, \alpha) + M \frac{P_{N\perp}(p, E, \alpha)}{\sin \alpha} \right]
\]

(27)

\[
\mathcal{H}_2^N = p \left[ P_N(p, E, \alpha) - \frac{P_{N\perp}(p, E, \alpha)}{\sin \alpha} \right] + \frac{\nu}{2|\vec{q}|} \left( \frac{3 \cos^2 \alpha - 1}{\cos \alpha} \right) \left[ \frac{p^2}{M + E_p} P_N(p, E, \alpha) - E_p \frac{P_{N\perp}(p, E, \alpha)}{\sin \alpha} \right]
\]

(28)

and \( P_N(p, E, \alpha) = \cos \alpha \, P_\parallel(p, E, \alpha) + \sin \alpha \, P_{N\perp}(p, E, \alpha) \). In Eqs. (24) and (25) the integration limits and \( \cos \alpha \) are determined, as usual, by energy conservation [12]. The polarized structure functions \( G_{1(2)}^A \), given by Eqs. (24) and (25), coincide with the ones corresponding to the extraction scheme (A) of Ref. [7], once the off-shell effects are neglected and \( \tilde{P}_{\parallel(\perp)}(p, E, \alpha) \) are expressed in terms of the scalar functions \( f_1 \) and \( f_2 \) introduced in Ref. [7].

It should be pointed out that the expression for \( G_{1(2)}^A \), obtained in Ref. [8], differ from Eqs. (24) and (25) in that they do not contain the term \( \mathcal{L}^N \). The origin of such a difference can be traced back to the procedure of Ref. [3], used in Ref. [4], according to which the polarized structure functions are obtained from Eqs. (10) and (11) by replacing the four-vector \( V^\sigma \) with the four-vector \( R^\sigma \), i.e. according to prescription II. As already explained in Sect.2, such a procedure is a correct one only if the functional dependence of the two four-vectors is the same, i.e. if \( R^\sigma \) is a linear combination of only \( S^\sigma_A \) and \( P^\sigma_A \). It turns out by an explicit evaluation of Eq. (13) that this is not the case, for \( \tilde{R}^\sigma \) is a linear combination of \( S^\sigma_A \), \( P^\sigma_A \) and a term proportional to \( q^\sigma \). A different situation occurs when, instead of the vectors \( V^\sigma \) and \( R^\sigma \), the tensors \( W^\sigma_{\mu\nu} \) and \( w^\sigma_{\mu\nu} \) are considered and Eqs. (8) and (9) are used. As a matter of fact, as already observed, Eqs. (8) and (9) hold independently of the presence of a term proportional to \( q^\sigma \), since it is washed out by \( \epsilon_{\mu\nu\sigma} q^\sigma \) when the hadronic tensor is evaluated. Then the tensor \( W^\sigma_{\mu\nu} \) can be safely replaced by \( w^\sigma_{\mu\nu} \) in Eqs. (8) and (9).

Let us now analyze in more detail the differences between prescription I and II. By recalling that from the knowledge of the hadronic tensor follows the knowledge of the four-vector \( \tilde{V}^\sigma \equiv V^\sigma + \frac{\sigma^\sigma}{Q^2} (V \cdot q) \) only, we can identify such a four-vector with its PWIA counterpart, i.e. \( \tilde{R}^\sigma \equiv R^\sigma + \frac{\sigma^\sigma}{Q^2} (R \cdot q) \). Then we can express Eqs. (8) and (9) in terms of
\( \tilde{R}^\sigma \) and eventually in terms of \( R^\sigma \). One gets

\[
\frac{G_1^1}{M_A} = -\left( \frac{R \cdot q}{|q| S_{A_z}} - \frac{Q^2}{|q|^2} \left( \frac{R_z}{S_{A_z}} - \frac{R_\perp}{S_{A_\perp}} \right) \right) \tag{29}
\]

\[
\frac{G_2^2}{M_A^2} = \frac{R_0}{|q| S_{A_z}} - \frac{\nu}{|q|^2} \left( \frac{R_z}{S_{A_z}} - \frac{R_\perp}{S_{A_\perp}} \right) \tag{30}
\]

where \( R_\perp = (\tilde{R} \cdot \vec{S}_A - R_z S_{A_z})/S_{A_\perp} \). From these equations, after inserting the actual expression for \( R^\sigma \) (Eq. (13)), one gets again Eqs. (24) and (25). It should be pointed out that Eqs. (29) and (30) reduce to Eqs. (10) and (11) if the term \( (R_z/S_{A_z} - R_\perp/S_{A_\perp}) \) vanishes. However this is not the case, because \( \vec{R} \) does not result to be parallel to \( \vec{S}_A \).

4. The asymmetry in the quasi-elastic region

The contraction of the two tensors in Eq. (1) yields

\[
d^2\sigma(h) \frac{d\Omega_2 d\nu}{d\Omega_2 d\nu} = \Sigma + h \Delta \tag{31}
\]

where

\[
\Sigma = \sigma_{\text{Mott}} \left[ W_A^2(Q^2, \nu) + 2 \tan^2 \theta_e \frac{1}{2} W_1^1(Q^2, \nu) \right] \tag{32}
\]

\[
\Delta = \sigma_{\text{Mott}} 2 \tan^2 \frac{\theta_e}{2} \left[ \frac{G_1^1(Q^2, \nu)}{M_A^2} (k_1 + k_2) + \frac{G_2^2(Q^2, \nu)}{M_A^2} (\epsilon_1 \epsilon_2 - \epsilon_1 \epsilon_2) \right] \cdot \vec{S}_A \tag{33}
\]

with \( \theta_e \) being the scattering angle. In what follows, the target polarization vector \( \vec{S}_A \) is supposed to lie within the scattering plane formed by \( \vec{k}_1 \) and \( \vec{k}_2 \).

Two possible kinematical conditions can be considered:

\( \beta \)-kinematics. The target polarization angle is measured with respect to the direction of the incident electron, i.e. \( \cos \beta = \vec{S}_A \cdot \vec{k}_1/|\vec{k}_1| \), (this is a natural choice from the experimental point of view). In this case, one gets

\[
\Delta \equiv \Delta_\beta = \sigma_{\text{Mott}} 2 \tan^2 \frac{\theta_e}{2} \left\{ \frac{G_1^1(Q^2, \nu)}{M_A^2} \left[ \epsilon_1 \cos \beta + \epsilon_2 \cos(\theta_e - \beta) \right] + \frac{G_2^2(Q^2, \nu)}{M_A^2} \epsilon_1 \epsilon_2 \left[ \cos \beta - \cos(\theta_e - \beta) \right] \right\} \tag{34}
\]

\( \theta^* \)-kinematics. The target polarization angle is measured with respect to the direction of the momentum transfer, i.e. \( \cos \theta^* = \vec{S}_A \cdot \vec{q}/|\vec{q}| \). Then one can write
\[ \Delta \equiv \Delta_{\theta^*} = -\sigma_{\text{Mott}} \tan \frac{\theta_e}{2} \left\{ \cos \theta^* R^A_{T'}(Q^2, \nu) \left[ \frac{Q^2}{|q|^2} + \tan^2 \frac{\theta_e}{2} \right]^{1/2} + \frac{Q^2}{|q|^2} \sqrt{2} \sin \theta^* R^A_{T L'}(Q^2, \nu) \right\} \]  

where

\[ R^A_{T'}(Q^2, \nu) = -2 \left( \frac{G_1^A(Q^2, \nu)}{M_A} \nu - Q^2 \frac{G_2^A(Q^2, \nu)}{M_A^2} \right) = i 2 \frac{W_{12}^a}{S_{Az}} \]  

\[ R^A_{T L'}(Q^2, \nu) = 2 \sqrt{2} |q| \left( \frac{G_1^A(Q^2, \nu)}{M_A} + \nu \frac{G_2^A(Q^2, \nu)}{M_A^2} \right) = -i 2\sqrt{2} \frac{W_{02}^a}{S_{Ax}} \]

In principle the \( \theta^* \) - kinematics is very appealing, since by performing experiments at \( \theta^* = 0 \) and \( 90^o \) one can disentangle \( R^A_{T'} \) and \( R^A_{T L'} \), which, at the top of the qe peak, are proportional to \( (G^n_M)^2 \) and \( G^n_E G^n_M \), respectively, provided the proton contribution can be disregarded \[2,3\]. Let us analyze in what follows to what extent such a condition really occurs.

Experimentally one measures the longitudinal asymmetry defined as

\[ A = \frac{\sigma_2(\nu, Q^2, \bar{S}_A, +1) - \sigma_2(\nu, Q^2, \bar{S}_A, -1)}{\sigma_2(\nu, Q^2, \bar{S}_A, +1) + \sigma_2(\nu, Q^2, \bar{S}_A, -1)} = \frac{\Delta}{\Sigma} \]  

If the naive model of \(^3\)He holds, this quantity is in principle very sensitive to the neutron properties, since the numerator should be essentially given by the neutron with its spin aligned along \( \bar{S}_A \). With this simple picture in mind, let us consider the comparison between our results based on Eqs. (24) and (25), with the experimental data obtained at MIT-Bates \[2,3\].

In Fig. 3 the asymmetry corresponding to \( \epsilon_1 = 574 \text{ MeV} \) and \( \theta_e = 44^o \), measured by the MIT-Caltech collaboration \[3\] is shown. The experimental data were obtained in a large interval of energy transfer after averaging over three different values of the \( \beta \) angle (\( \beta = 44.5^o, 51.5^o, 135.5^o \), with the corresponding azimuthal angles being: \( \phi = 180^o, 180^o, 0^o \)). It is worth noting that in these kinematical conditions one has \( \theta^* \approx 90^o \) only at the top of the qe peak, and therefore only there the measured asymmetry reduces to \( R_{T L'} \). In the figure the neutron (dotted line) and proton (dashed line) contributions are separately shown, and the relevance of the proton contribution can be noticed particularly at the top of the qe peak. There, one has

\[ A^{th} = 3.74 \% \]
\[ A^{th}_p = 2.20 \% \]
for the Gari-Kruempelmann\textsuperscript{13} form factors of the nucleon. Similar results hold for other models of the nucleon form factors, such as the Blatnick-Zovko\textsuperscript{14} and the Hoehler\textsuperscript{15} ones. In order to compare the theoretical prediction with the experimental data at the qe peak, one has to perform a further averaging over an interval of the energy transfer of about 100 MeV, as it was done in the experiment of Ref.\textsuperscript{2}; then one gets

\[
A_{\text{qe}} = 2.41 \pm 1.29 \mp 0.51 \% \quad \text{MIT - Caltech [2]}
\]
\[
A^{\text{th}} = 1.65 \%
\]
\[
A_{\text{p}}^{\text{th}} \approx 0
\]

The aim of this kind of experiments is the extraction of information on the neutron form factors $G_E^n$ and $G_M^n$ from the experimental asymmetry. The last comparison and the theoretical curves shown in Fig. 3 suggest that, since the asymmetry drastically changes when the energy transfer varies, the averaging procedure has to be considered with some care. Indeed, the above mentioned proportionality between $R_{TL'}$ and $G_E^n G_M^n$ holds only at the top of the qe peak and provided the proton contribution is negligible, whereas the theoretical calculations show that the proton contribution is relevant at the top of the qe peak. This result is completely hidden by the averaging procedure and therefore the extraction of information on the neutron form factors from the experimental data averaged on a large energy range could be questionable.

For the sake of completeness, herebelow we compare our results for the asymmetry at the qe peak with the experimental results of Ref.\textsuperscript{3} ($\epsilon_1 = 578$ MeV, $\theta_e = 51.1^\circ$ and $\theta^* = 90.2^\circ$):

\[
A_{\text{qe}}^{\text{exp}} = 1.75 \pm 1.20 \mp 0.31 \% \quad \text{MIT - Harvard [3]}
\]
\[
A^{\text{th}} = 4.00 \%
\]
\[
A_{\text{p}}^{\text{th}} = 2.78 \%
\]

It should be pointed out that for this experiment the energy transfer range of the experimental averaging has not been specified.

In Fig.4 the theoretical asymmetry, corresponding to $\epsilon_1 = 574$ MeV and $\theta_e = 51.1^\circ$ and averaged over the same values of the polarization angle as in Fig. 3, is shown. Such a kinematics was chosen\textsuperscript{2} with the aim of extracting $R_{TL'}$ at the qe peak. Only one experimental point has been obtained for the averaged asymmetry around the top of the qe peak, where $\theta^* \approx 0^\circ$ ($A_{\text{qe}}^{\text{exp}} \propto R_{TL'}^{\exp}$). The comparison between the experimental result (obtained after a further averaging over an interval of the energy transfer of 48 MeV) and our calculations is as follows

\[
A_{\text{qe}}^{\text{exp}} = -3.79 \pm 1.37 \mp 0.67 \% \quad \text{MIT - Caltech [2]}
\]
\[
A^{\text{th}} = -4.30 \% \quad [\text{without averaging} \quad -3.43 \%]
\]
\[
A_{\text{p}}^{\text{th}} = 1.05 \% \quad [\text{without averaging} \quad 1.30 \%]
\]
Our calculation without averaging compares to the response function \( R_T' \) corresponding to the kinematics of the experiment of Ref. \( [3] \) (\( \epsilon_1 = 578 \text{ MeV} \) and \( \theta_e = 51.1^\circ \), and two values of the polarization angles: \( \theta^* = 3.2^\circ, \phi^* = 0 \) and \( \theta^* = 176.8^\circ, \phi^* = 180^\circ \)) as follows

\[
A_{\text{exp}}^{qe} = -2.60 \mp 0.90 \mp 0.46 \% \quad MIT - Harvard [3]
\]

\[
A^{th} = -3.68 \%
\]

\[
A_p^{th} = 1.18 \%
\]

The same comments and warnings made for the case \( \theta^* = 90^\circ \) should be extended to \( \theta^* = 0^\circ \) as well. It should be pointed out that our numerical results, as shown in Fig. 5, only slightly differ from the ones obtained in Ref. \( [7] \), where a spin-dependent Faddeev spectral function has been used.

In Figs. 6a and 6b the results based upon prescription I are compared with our previous calculations \( [4] \), based upon prescription II (the corresponding explicit expressions for \( G_{1(2)}^A \) are given in Ref. \( [3] \) and coincide, as already mentioned, with Eqs. \( (24) \) and \( (25) \) with the term \( \mathcal{L}^N \) dropped out). The results of the comparison show that at \( \theta^* \approx 90^\circ \) prescription II yields results very different from the correct ones, whereas at \( \theta^* \approx 0^\circ \) such a difference is not present. This is due to the fact that in procedure II, based on Eqs. \( (11) \) and \( (12) \), \( R_{TL'} \), Eq. \( (37) \), results to be proportional to the component of \( \vec{R} \) along \( \hat{q} \), instead of being proportional to its transverse part (i.e. \( W_{02}^A \propto R_{\bot}/S_{\bot} \), see Eqs. \( (20) \), \( (21) \) and \( (37) \)), as it should be. Therefore \( R_{TL'} \), within prescription II, is affected by the presence of a term proportional to \( q^\sigma \) in \( R_T' \). For \( \theta^* \approx 0^\circ \) the differences, as shown in Fig. 6b, are very small over the whole range of the energy transfer considered, since \( R_T' \) is the same for the two prescriptions, and therefore it is unaffected by the extra term in \( R_T' \). Moreover it is easy to explain the small differences on the wings of the asymmetry, since the asymmetry is proportional to \( R_T' \) only at the top of the \( q_e \) peak, while on the wings there is a mixing with \( R_{TL'} \).

From the above comparisons it turns out that the difference between the two procedures is almost entirely due to the proton contribution; nevertheless, the correctness of our conclusion, reached in \( [6] \) using prescription II, about the possibility of obtaining information on the neutron form factors by properly minimizing the proton contribution is not affected by the use of prescription I. This will be illustrated in the next Section.

5. Minimizing the proton contribution

As shown in Figs. 3 and 4, the proton contribution to the asymmetry, corresponding to the polarization angle of the actual experiments is sizeable. Following our previous paper \( [6] \), in this Section the possibility to minimize or even to make vanishing the proton contribution will be investigated, using prescription I. To this end we have analyzed the proton contribution to the asymmetry at the top of the \( q_e \) peak, for different values of \( \beta \), different values of the energy of the incident electron and different models for the proton.
form factors. The results are presented in Fig. 7. It can be seen that for \( \theta_e = 75^\circ \) the proton contribution is almost vanishing around \( \beta \equiv \beta_c = 105^\circ \), in a large spectrum of values of incident electron energy; moreover, such a feature weakly depends upon the model for the nucleon form factors. In order to understand the behaviour of the proton contribution, let us consider the asymmetry given by Eq. (38). The proton contribution vanishes in (38) if in \( \Delta \beta \), Eq. (34), a polarization angle \( \beta = \beta_c \) is chosen such that

\[
\tan \beta_c = -\frac{1}{\tan \theta_e} - \frac{1}{\sin \theta_e} \gamma
\]  

(39)

with

\[
\gamma = \frac{\left( G_A^{(p)} M_A / (2 \epsilon_2 G_A^{(p)}) - 1 \right)}{\left( G_A^{(p)} M_A / (2 \epsilon_1 G_A^{(p)}) + 1 \right)}
\]

(40)

The critical polarization angle does not change too much, even if \( \gamma \) changes by orders of magnitude, since the equation determining \( \beta_c \) involves the tangent function, which always results to be \( |\tan \beta_c| \gg 1 \). As a matter of fact, we have checked that for \( 10^\circ \leq \theta_e \leq 80^\circ \) and \( .5 \) (GeV) \( \leq \epsilon_1 \leq 3 \) (GeV), \( \beta_c \) varies between \( 89^\circ \) and \( 110^\circ \). This is due to the fact that \( \gamma \) is always large (\( |\gamma| > 1 \)), since \( G_A^{(p)} M_A / G_A^{(p)} \) is negative and the denominator in Eq. (40) is small for all nucleon form factor models we have used. Therefore the presence of the tangent function in Eq. (39) explains the weak dependence of \( \beta_c \) upon sizeable changes of both the incident energy, the scattering angle and the models of the nucleon form factors. In Fig. 8, the asymmetry and the proton contribution, vs \( Q^2 \), at fixed values of \( \beta_c = 95^\circ \) and \( \theta_e = 75^\circ \) are presented for three different models of the nucleon form factors (Refs. [13, 14, 15]). It can be seen that the asymmetry is sensitive to the neutron form factors; moreover the calculations have shown that the neutron asymmetry is essentially given by the terms in \( G_{1(2)}^A \), containing \( P_{||}(|\vec{p}|, E, \alpha) \). However the results presented in Fig. 8 do not tell us whether the differences in the asymmetry are given by the differences in \( G_E^n \) or in \( G_M^n \), since both of them vary within the models we have considered. In order to make our analysis a more stringent one, we have repeated the calculation by using the Galster model of the nucleon form factors [16], since within such a model \( G_E^n \) can be changed independently of \( G_M^n \). In fact one has

\[
G_M^n = \mu_n G_E^n \\
G_E^n = -\frac{\tau \mu_n}{(1 + \eta \tau)} G_E^p
\]

(41)

where \( G_E^p = 1/(1 + Q^2/B)^2 \), \( B = 0.71 \) (GeV/c)^2 and \( \eta \) is a parameter. The resulting asymmetry and proton contribution are shown in Fig. 9 for different values of \( \eta \). Fig.

13
9 illustrates how the total asymmetry can depend upon $G^n_E$, having, at the same time, a vanishing proton contribution. It should moreover be stressed that the proposed kinematics, which minimizes the proton contribution, corresponds to the qe peak, where the final state interaction is expected to play a minor role.

6. Summary and conclusion

The qe spin-dependent structure functions for a nucleus with $J = 1/2$ have been obtained by a proper procedure, based on the replacement of the exact hadronic tensor with its PWIA version. Our formal results are in agreement with the ones of Ref. [7], and the numerical calculations only slightly differ from them, which demonstrates that the spin dependent spectral functions used in Ref. [8] and the one used Ref. [7] are essentially equivalent.

The differences between the predictions of the correct procedure and the ones [4, 9] based upon the replacement of the hadronic pseudovector $V^\sigma$, Eq.(19), with its PWIA version $R^\sigma$, Eq.(13), have been shown to be produced by the presence of a contribution proportional to the momentum transfer $q^\sigma$ in the four-vector $R^\sigma$. This contribution affects only the response function $R_{TL'}$.

Our analysis of the asymmetry, based on the correct expression of $G_1^A$ and $G_2^A$ given by Eqs. (24) and (25), respectively, has fully confirmed the main conclusions of our previous paper [6], concerning: i) the relevance of the proton contribution for the experimental kinematics considered till now, and ii) the possibility to select a polarization angle, which leads at the qe peak to an almost vanishing proton contribution for a wide range of kinematical variables; within such a kinematical condition, the sensitivity of the asymmetry to the electric neutron form factor has been thoroughly investigated.

Calculations of the final state effects are in progress.

7. Acknowledgment

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Appendix A

For the sake of completeness, let us repeat here Eqs.(19) and (21) for a nucleus with an arbitrary value of the total angular momentum $J_A$

$$\hat P^N_M(\vec p, E) = \frac{1}{2} \left[ B^N_{0,M}(|\vec p|, E, (\hat p \cdot \vec S_A)^2) + \vec s \cdot \vec f^N_M(\vec p, E) \right]$$

(42)

$$\vec f^N_M(\vec p, E) = \vec S_A \ B^N_{1,M}(|\vec p|, E, (\hat p \cdot \vec S_A)^2) + \hat p \ (\hat p \cdot \vec S_A) \ B^N_{2,M}(|\vec p|, E, (\hat p \cdot \vec S_A)^2)$$

(43)
In this Appendix the expressions of the functions $B_{0(1,2),M}$, will be given in terms of the overlap integrals [3]

$$N\langle \vec{p}, \sigma ; \psi_{f(A-1),\eta}^{J_{A-1},M_{A-1}} | \Psi_{J_{A},M_{A}} \rangle$$

(44)

where $\psi_{f(A-1),\eta}^{J_{A-1},M_{A-1}}$ is the wave function of the (A-1) spectator system, $\Psi_{J_{A},M_{A}}$ the wave function of the target nucleus, $\sigma$, $M_{A-1}$ and $M_{A}$ are the components of the angular momentum of the nucleon $N$, the (A-1) system and the target system along $\hat{q}$, respectively, and $\vec{p}$ is the Jacobi coordinate of the nucleon $N$ with respect to the (A-1) system, while $\eta$ represents all of the quantum numbers of the (A-1) system, but the energy $E_{f(A-1)} = E + E_{A}$ and the total angular momentum $J_{A-1}$ with its third component $M_{A-1}$.

The starting point is the usual transformation property from a wave function with a given third component of the total angular momentum with respect to a certain axis (e.g. with respect to the $\hat{S}_{A}$-axis) to the wave function where the third component of the angular momentum is defined with respect to another axis (e.g. the $\hat{q}$-axis). By using the Wigner D-function $D_{M_{A},M_A}^{J_A}$ one has [17]

$$\langle \theta | J_{A} M \rangle \hat{S}_{A} = \langle \theta | D | J_{A} M \rangle \hat{q} = \sum_{M_{A}} \langle \theta | J_{A} M_{A} \rangle \hat{q} D_{M_{A},M_A}^{J_A}(\alpha, \beta, \gamma)$$

(45)

where the subscripts $\hat{q}$ and $\hat{S}_{A}$ indicate the angular momentum quantization axis, and $\alpha$, $\beta$, $\gamma$ are the Euler angles describing the proper rotation from $\hat{q}$ to $\hat{S}_{A}$. This transformation has to be applied to the overlap integrals $N\langle \vec{p}, \sigma; \psi_{f(A-1)} | \Psi_{J_{A},M_{A}} \rangle$, which appear in the definition of the spin dependent spectral function (see Eq. (18)). Then, after a lengthy algebraic manipulations and using the following properties of the bipolar spherical harmonics, which hold for odd values of $J$,

$$\sum_{\mu\lambda} \langle 1a, (J+1)\lambda | J \mu \rangle \mathcal{Y}_j^{\mu} (\hat{S}_{A}) \mathcal{Y}_{j+1}^{\lambda}(\vec{p}) = -\frac{j}{\sqrt{3}(j+1)} \cdot$$

$$\mathcal{Y}_{1}^{\mu*}(\vec{p}) \sum_{l=0}^{\frac{j-1}{2}} (2l+1) \mathcal{Y}_{2l+1}^{\mu*}(\vec{p} \cdot \hat{S}_{A}) - \mathcal{Y}_{1}^{\mu*}(\vec{p} \cdot \hat{S}_{A}) \sum_{l=0}^{\frac{j-1}{2}} (2l) \mathcal{Y}_{2l}^{\mu*}(\vec{p} \cdot \hat{S}_{A})$$

$$\sum_{\mu\lambda} \langle 1a, (J-1)\lambda | J \mu \rangle \mathcal{Y}_j^{\mu} (\hat{S}_{A}) \mathcal{Y}_{j-1}^{\lambda}(\vec{p}) = -\frac{j}{\sqrt{3}j} \cdot$$

$$\mathcal{Y}_{1}^{\mu*}(\vec{p}) \sum_{l=0}^{\frac{j-3}{2}} (2l+1) \mathcal{Y}_{2l+1}^{\mu*}(\vec{p} \cdot \hat{S}_{A}) - \mathcal{Y}_{1}^{\mu*}(\vec{p} \cdot \hat{S}_{A}) \sum_{l=0}^{\frac{j-3}{2}} (2l) \mathcal{Y}_{2l}^{\mu*}(\vec{p} \cdot \hat{S}_{A})$$

(46)

one gets

$$B_{0,M}^{N}(|\vec{p}|, E, (\vec{p} \cdot \hat{S}_{A})^2) = \frac{J_{A}\sqrt{2}}{\pi^{3/2}} \sum_{j_{even}=0}^{2J_{A}} \hat{j} \langle J_{A} M_{A} | J_{A} M_{A} \rangle \mathcal{Y}_j^{0}(\vec{p} \cdot \hat{S}_{A}) F_{N}^{J_{A}}(j, j, 0)$$

(47)
\[ B^N_{1,M} \left( |\vec{p}|, E, (\vec{p} \cdot \hat{S}_A)^2 \right) = \frac{\mathcal{J}_A}{\pi^{3/2}} \sqrt{6} \sum_{J_{odd}=1}^{2J_A} \mathcal{J} \langle J_A M_J | 0 | J_A M \rangle \cdot \]

\[ \cdot \left[ \frac{F^I_{N}(J,J+1,1)}{\sqrt{J+1}} + \frac{F^I_{N}(J,J-1,1)}{\sqrt{J}} \right] \left[ \mathcal{J} \sum_{\ell=0}^{\tilde{\ell}} (2\ell) \mathcal{J} \frac{0}{2\ell} \left( \hat{p} \cdot \hat{S}_A \right) \right] \]

\[ \cdot \left\{ \frac{F^I_{N}(j,j+1,1)}{\sqrt{j+1}} + \frac{F^I_{N}(j,j-1,1)}{\sqrt{j}} \right\} \left[ \mathcal{J} \sum_{\ell=0}^{\tilde{\ell}} (2\ell+1) \mathcal{J} \frac{0}{2\ell+1} \left( \hat{p} \cdot \hat{S}_A \right) \right] + \]

\[ - \mathcal{J} \frac{0}{2} \left( \hat{p} \cdot \hat{S}_A \right) F^I_{N}(J,J-1,1) \right\} \]

with

\[ F^I_{N}(J,L,a) = (-1)^a \sum_{J_{A-1} = 1}^{J_A} \sum_{\eta = \lambda} \sum_{X,Y} (-1)^{J_A+J_{A-1}} (-1)^{(L+\tilde{L})/2} \hat{X} \hat{X} \hat{L} \hat{L} \cdot \]

\[ \cdot \langle L0 \tilde{L} 0 | 0 \rangle \left\{ \frac{1/2}{X} \frac{1/2}{\tilde{X}} \frac{a}{J_{A-1}} \right\} \left\{ \frac{L}{\tilde{L}} \right\} \sum_{\eta = \lambda} \sum_{X,Y} \frac{X}{J_{A-1}} \eta \langle |\vec{p}|, E \rangle \frac{X}{J_{A-1}} \eta \langle |\vec{p}|, E \rangle \]

where \( \tilde{y} \) means \( \sqrt{2y+1} \) and \( J = 0, ..., 2J_A; L = J - 1, J, J + 1; a = 0, 1 \). The quantity

\[ I^X_{L J_{A-1}} \eta \langle |\vec{p}|, E \rangle \]

is given by

\[ I^X_{L J_{A-1}} \eta \langle |\vec{p}|, E \rangle = \int d\vec{p} \ j_L(|\vec{p}| \rho) \sum_{M_L,M_X} \langle L M_L X M_X | J_A M_A \rangle \mathcal{J} \frac{M_L}{M_X} \eta(|\vec{p}|, E) \cdot \]

\[ \cdot \sum_{M_{A-1} = 1}^{M_{A-1}} \langle J_{A-1} M_{A-1} | \frac{1}{2} \sigma | X M_X \rangle N \langle |\vec{p}|, \sigma; J_{E_{B(A-1)}, \eta} = J_{A,M_A} \rangle. \]
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FIGURE CAPTIONS

Fig. 1a. The proton unpolarized spectral function of $^3$He versus the removal energy $E$ and the nucleon momentum $p \equiv |\vec{p}|$ (see text and appendix A).

Fig. 1b. The function $|B_{1/2}^{p,1}|$ (see Eq. (20) and appendix A) versus the removal energy $E$ and the nucleon momentum $p \equiv |\vec{p}|$.

Fig. 1c. The function $|B_{1/2}^{p,1}|$ (see Eq. (20) and appendix A) versus the removal energy $E$ and the nucleon momentum $p \equiv |\vec{p}|$.

Fig. 2a. The same as Fig. 1a, but for the neutron.

Fig. 2b. The same as Fig. 1b, but for the neutron.

Fig. 2c. The same as Fig. 1c, but for the neutron.

Fig. 3. The asymmetry in $\theta^*$ kinematics corresponding to $\epsilon_1 = 574$ MeV and $\theta_e = 44^\circ$, vs. the energy transfer $\nu$. The experimental data are from Ref. [2] and the theoretical curves were obtained using Eqs. (24) and (25) with the spin-dependent spectral function of Ref. [6]. The continuous line is the total asymmetry, whereas the dotted (dashed) line represents the neutron (proton) contribution. The nucleon form factors of Ref. [13] have been used. The arrow indicates the position of the qe peak.

Fig. 4. The same as in Fig. 3, but for $\theta_e = 51.1^\circ$. The experimental point has been obtained in Ref. [2], after averaging over a 48 MeV interval around the qe peak.

Fig. 5. Comparison of the asymmetry, corresponding to $\epsilon_1 = 574$ MeV and $\theta_e = 44^\circ$ (cfr. Fig. 3), calculated by using Eqs. (24) and (25) and the spin-dependent spectral function of Ref. [6] (solid line) with the one of Ref. [1] (dashed line), based on a Faddeev spin-dependent spectral function. The nucleon form factors of Ref. [13] have been used and the experimental data are from Ref. [2]. The arrow indicates the position of the qe peak.

Fig. 6a. The asymmetry and the neutron contribution for $\epsilon_1 = 574$ MeV and $\theta_e = 44^\circ$ vs. the energy transfer $\nu$ calculated using prescriptions I and II. Solid (dotted) line: the asymmetry (neutron contribution) corresponding to prescription I (Eqs. (24) and (25)); dashed (dot-dashed) line: the asymmetry (neutron contribution) corresponding to prescription II (Eqs. (24) and (25) without the term $\mathcal{L}$). The form factors of Ref. [13] have been used and the experimental data are from Ref. [2]. The arrow indicates the position of the qe peak.

Fig. 6b. The same as in Fig. 4a, but for $\theta_e = 51.1^\circ$. The dotted and dot-dashed lines overlap.

Fig. 7a. The proton contribution to the asymmetry, at the top of the qe peak, vs. $\beta$, for $\theta_e = 75^\circ$. Solid line: $\epsilon_1 = 500$ MeV, long-dashed line: $\epsilon_1 = 1000$ MeV, short-dashed
line: $\epsilon_1 = 1500$ MeV, dotted line: $\epsilon_1 = 2000$ MeV. The nucleon form factors of Ref. [13] have been used.

Fig. 7b. The same as in Fig. 7a, but for the nucleon form factors of Ref. [14].

Fig. 7c. The same as in Fig. 7a, but for the nucleon form factors of Ref. [15].

Fig. 8. The total asymmetry at the top of the $q_\ell$ peak, vs. $Q^2$, for $\theta_e = 75^0$ and $\beta = 95^0$, using Eqs. (24) and (25). Solid line: Gari-Kruempelmann form factors [13]; dashed line: Blatnik-Zovko form factors [14]; dotted line: Hoehler et al. form factors [15]. The curves in the lower part of the figure represent the corresponding proton contributions.

Fig. 9. Sensitivity of the total asymmetry upon variations of the neutron electric form factor. The curves in the upper part were calculated at $\theta_e = 75^0$ and $\beta = 95^0$, by Eqs. (24) and (25) using the Galster form factors [16] for different values of the parameter $\eta$. By this way the neutron electric form factor can be changed leaving unchanged the magnetic form factor. The curves in the lower part of the figure represent the corresponding proton contributions.
Fig. 1a

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 1b

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 1c

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 2a

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 2b

C. CIOFI DEGLI ATTI, E. PACE and G. SALME'
Fig. 2c

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 3

C. CIOFI DEGLI ATTI, E. PACE and G. SALME'
Fig. 4

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 5

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 6a

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 6b

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 7a

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 7b

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 7c

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 8

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
Fig. 9

C. CIOFI DEGLI ATTI, E. PACE and G. SALME’
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