A Framework of Randomized Selection Based Certified Defenses Against Data Poisoning Attacks

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Abstract

Neural network classifiers are vulnerable to data poisoning attacks, as attackers can degrade or even manipulate their predictions thorough poisoning only a few training samples. However, the robustness of heuristic defenses is hard to measure. Random selection based defenses can achieve certified robustness by averaging the classifiers’ predictions on the sub-datasets sampled from the training set. This paper proposes a framework of random selection based certified defenses against data poisoning attacks. Specifically, we prove that the random selection schemes that satisfy certain conditions are robust against data poisoning attacks. We also derive the analytical form of the certified radius for the qualified random selection schemes. The certified radius of bagging derived by our framework is tighter than the previous work. Our framework allows users to improve robustness by leveraging prior knowledge about the training set and the poisoning model. Given higher level of prior knowledge, we can achieve higher certified accuracy both theoretically and practically. According to the experiments on three benchmark datasets: MNIST 1/7, MNIST, and CIFAR-10, our method outperforms the state-of-the-art.

Introduction

Neural network classifiers are vulnerable to designed training samples added in the training set or the testing data (Biggio, Nelson, and Laskov 2012; Szegedy et al. 2014). Manipulating 1% of the dataset can cause the target image to be misclassified at a 90% success rate (Huang et al. 2020). Inserting less than 5% poisoned training samples can make the classifier’s feature selection almost randomly (Xiao et al. 2015). There exists many kinds of data poisoning attacks (Chen et al. 2017; Liu et al. 2018; Turner, Tsipras, and Madry 2018; Zhao et al. 2020; Turner, Tsipras, and Madry 2019; Ji, Zhang, and Wang 2017; Yao et al. 2019b; Zhang et al. 2020; Yao et al. 2019a). Therefore, it is urgent to develop defenses against data poisoning attacks. Many heuristic defenses have been proposed (Wang et al. 2019; Chen et al. 2018; Chen et al. 2019; Gao et al. 2019; Tran, Li, and Madry 2018; Liu et al. 2019; Qiao, Yang, and Li 2019; Steinhardt, Koh, and Liang 2017) against data poisoning attacks, but the security level of those defenses is hard to measure. To achieve certified robustness, various certified defenses have been proposed, including randomized smoothing-based defenses (Wang et al. 2020; Rosenfeld et al. 2020; Jia, Cao, and Gong 2020), loss based defenses (Steinhardt, Koh, and Liang 2017) and differential privacy based defenses (Ma, Zhu, and Hsu 2019). Compared with other defenses, randomized smoothing has no limitation on the training algorithms and is able to certify robustness for each testing data. To our knowledge, previous works of randomized smoothing defenses only considered one specific smooth scheme. There is a lack of a framework of randomized smoothing based defenses in defending data poisoning attacks.

In this paper, we propose a framework of random selection based defenses against data poisoning attacks, which is a subset of randomized smoothing. Our framework shows that any random selection scheme satisfying certain conditions is robust to data poisoning attacks. Through our framework, we can derive the certified radius for all qualified random selection schemes, including bagging. We also propose a method to enhance robustness by leveraging 2 types of prior knowledge: the prior knowledge about the training set and the poisoning model. We divide the poisoning model into 6 cases and the prior knowledge about the training set into 3 cases. We can improve the derived certified radius theoretically when the poisoning model is weak. When we have high level of prior knowledge about the training set, we propose 2-phase classification to enhance robustness empirically. In addition, our training algorithms use a weight balance method to solve the problem of low certified accuracy when the size of the selection is small. Our contributions are as follow:

\begin{itemize}
  \item We propose a framework of random selection based defenses against data poisoning attacks. The certified radius derived by our framework is tighter than previous works.
  \item We propose a method to enhance robustness by leveraging prior knowledge about the training set and the poisoning model, which makes our framework more practical in real applications.
  \item We evaluate 3 random selection based defenses across three classification tasks: MNIST 1/7, MNIST and CIFAR-10. Our certified accuracy outperforms previous works.
\end{itemize}
Related Works

Previous works have investigated in using randomized smoothing to defend data poisoning attacks. The main challenges in applying randomized smoothing are choosing an appropriate smooth scheme and deriving its certified radius. Rosenfeld et al. (Rosenfeld et al. 2019) proposed the random flip scheme to defend label flipping attacks, but Rosenfeld has not considered the attacks of modifying training data. Wang et al. (Wang et al. 2020) proposed the smooth scheme of adding noise into each training sample to defend backdoor attacks, but the certified radius derived by Wang is loose. Jia et al. (Jia, Cao, and Gong 2020) proposed the bagging smooth scheme, which is a variant of random selection schemes. However, Jia has not considered other random selection schemes in defending data poisoning attacks. We also find that the certified radius derived by Jia still has room for improvement. The randomized smoothing defenses also have their intrinsic limitation. Weber and Kumar et al. (Weber et al. 2020; Kumar et al. 2020) have proved the no-go theorem for randomized smoothing independently: without more information than label statistics under random input perturbations, the largest \( \rho_p \) certified radius decreases as \( O(d^{1/p-1/2}) \) with dimension \( d \) for \( p > 2 \). This theorem proves the upper bound for the robustness of randomized smoothing methods. To bypass this limitation, leveraging prior knowledge is necessary. Jia et al. (Jia, Cao, and Gong 2020) propose to use transfer learning to improve robustness. However, the improvement of transfer learning methods is defending on the pre-train models. In many cases, a satisfactory pre-train model is not available. Instead of pre-train models, we propose to use prior knowledge about the poisoning model and the training set to improve robustness. The prior knowledge about the training set is the clean training samples known in advance, which is more practical than pre-train models.

Preliminaries

Neural Network

A neural network \( h(x|D_n) : \mathbb{R}^d \rightarrow \mathcal{Y} \) is a mapping from the input feature space to the output space. We refer to the input space as \( \mathbb{R}^d \) and the output space as \( \mathcal{Y} \). The confidence for each class is \( h^c(x) : \mathbb{R}^d \rightarrow [0, 1] \) \((c = 1, 2, \ldots, k)\). We rank each class by its confidence in decreasing order \( c_1(x), c_2(x), \ldots, c_k(x) \). We denote the confidence of \( c_i(x) \) as \( p_i(x) \equiv h^c(x) \).

Classifier

We refer to \( D_n \equiv \{(x_i, y_i)_{i=1}^n : x_i \in \mathbb{R}^d, y_i \in \mathcal{Y}\} \) as the training set. \( h(x|D_n) \) is the classifier trained on the dataset \( D_n \). In this paper, we mainly focus on the relation between the predictions and the training set. We express \( h(x|D_n) \) as \( f_x(D_n) \), as a function of \( D_n \) when \( x \) is fixed. \( f_x(D_n) \) can be any deterministic or random function.

Poisoning model

Poisoning model is prior knowledge about the attacker’s capability. We denote the poisoning model as \( P_\rho : D_n \rightarrow \{D'_m\} \) that maps a dataset to a set of all possible poisoned datasets. \( \rho \) is the intensity of the poisoning attacks. In this paper, we consider 6 cases of the poisoning model:

1. \( (P^1_\rho) \) Insert at most \( \rho \) arbitrary samples into \( D_n \).
2. \( (P^2_\rho) \) Delete at most \( \rho \) arbitrary samples in \( D_n \).
3. \( (P^3_\rho) \) Modify at most \( \rho \) arbitrary samples in \( D_n \).
4. \( (P^4_\rho) \) Insert and/or modify at most \( \rho \) arbitrary samples in \( D_n \).
5. \( (P^5_\rho) \) Delete and/or modify at most \( \rho \) arbitrary samples in \( D_n \).
6. \( (P^6_\rho) \) Insert, delete and/or modify at most \( \rho \) arbitrary samples in \( D_n \).

We do not consider the case of \textit{Insertion and deletion} because the effect of deleting \( k \) training sample and inserting \( k \) new sample simultaneously is equivalent to modifying \( k \) training sample. In the later of this paper, we will show how to leverage the information about the poisoning model to improve robustness.

Random selection scheme

A random selection scheme is a scheme of selecting a subset from the dataset \( \mu : D_n \rightarrow D_{sub} \). Here \( D_{sub} = \{(x_i, y_i)_{i \in S} : S \subseteq \{1, 2, \ldots, n\}\} \). Random selection schemes will not modify any sample in \( D_n \).

Certified radius

Given a classifier \( f_x() \) and the poisoning model \( P_\rho \), the certified radius of \( f_x() \) at \( x \) under \( P_\rho \) is the maximum value of \( \rho \) that satisfies:

\[
f_x(D'_m) = f_x(D_n), \forall D'_m \in P_\rho(D_n)
\]

Certified radius means the maximum attack intensity that the defense can defend.

Proposed Framework

In this paper, our goal is to solve the following 2 problems:

1. Which random selection schemes are robust to the poisoning model?

2. How to compute the certified radius of the random selection schemes that are robust to the poisoning model?

In this section, we solve the above 2 problems by Theorem 1. To show the applicability of Theorem 1, we propose 3 commonly used variants of random selection schemes. All 3 schemes satisfy the conditions proposed in theorem 1. Finally we propose 2 training algorithms for different cases of prior knowledge about the training set to apply random selection schemes into practice.

To achieve certified robustness against the attack where training samples are arbitrarily changed, we first assume that the prediction will change if the training set contains greater...
and if $p_1, p_2$ satisfy:

$$
\pi_1 p_1 - \pi_1 \sum_{D_{sub} \in \Omega} Pr(\mu(D_n) = D_{sub}) \geq 0
$$

and

$$
\pi_2 p_2 + \sum_{D_{sub} \in \Omega} Pr(\mu(D'_m) = D_{sub}) \geq 0
$$

Then:

$$
g_x(D'_m) = c_1
$$

According to Theorem 1, if we can find $\pi_1, \pi_2$ for the random selection scheme $\mu$, $\mu$ is robust to data poisoning attacks. Given the poisoning model $P_\rho$, we can compute the tight certified radius according to the equation 5. To show the applicability of Theorem 1, we give 3 typical random selection schemes and prove that they all satisfy the conditions.

**Certified robustness of 3 random selection schemes**

We introduce 3 typical random selection schemes: bagging without replacement, bagging with replacement and binomial selection. Those 3 schemes all have the property $\pi_1 = \pi_2$, so the inequality 5 can be simplified to be $p_1 - p_2 \geq \delta(\rho)$ where $\delta(\rho)$ is

$$
\sum_{D_{sub} \in \Omega} Pr(\mu(D_n) = D_{sub}) + \frac{1}{\pi_1} \sum_{D_{sub} \in \Omega} Pr(\mu(D'_m) = D_{sub})
$$

$\delta(\rho)$ is the minimum value of $p_1 - p_2$ for $x$ to be robust against poisoning $n$ training samples. $\delta(\rho)$ increases in proportion to $\rho$. $\delta(\rho) = 0$ when $\rho = 0$. We will give analytical forms of $\delta(\rho)$ for 3 schemes.

1. (Bagging without replacement). Given $D_n$ and let $n_s$ be the size of $D_{sub}$, bagging without replacement $\mu$ is:

$$
Pr(\mu(D_n) = D_{sub}) = 1/n_s
$$

(7)

2. (Bagging with replacement). Given $D_n$ and $n_s$, bagging with replacement $\mu$ is.

$$
Pr(\mu(D_n) = D_{sub}) = (\frac{1}{n_s})^{n_s}
$$

(8)

3. (Binomial selection). For each sample $(x_i, y_i)_{i=1}^n \in D_n$, the probability of $(x_i, y_i)$ being selected is $p = n_s/n$.

$$
Pr(\mu(D_n) = D_{sub}) = p^{n_s} (1 - p)^{n - n_s}
$$

(9)

$\delta(\rho)$ of 3 random selection schemes against the poisoning model $P_\rho$ are as follow:

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**Figure 1:** A sketch map of random selection based defenses. We sample $T$ sub-datasets $\{D_{sub}^i\}_{i=1}^T = \mu(D^p)$ from the poisoned part $D^p$ of the training set. $\mu$ is the random selection scheme. We combine $\{D_{sub}^i\}_{i=1}^T$ with the clean part $D^c$ of the training set to generate the training set for the base classifiers $D_{sub} \cup D^c$. We train $T$ base classifiers on $T$ datasets $\{D_{sub}^i \cup D^c\}_{i=1}^T$. The smooth classifier $g$ is the aggregation of those $T$ base classifiers by majority rule.
Corollary 1. (Certified robustness of bagging without replacement). \(\pi_1, \pi_2\) of bagging without replacement is

\[
\pi_1 = \pi_2 = \left(\frac{m}{n}\right)^{n_s} \quad (10)
\]

\(\delta(\rho)\) of bagging without replacement is

\[
\max_{m \in [n - \rho, n + \rho]} 1 - \left(\frac{\max(m, n) - \rho}{n_s}\right)^{n_s} + \left(\frac{n}{n_s}\right)^{n_s} - \left(\frac{\max(m, n) - \rho}{n_s}\right)^{n_s} \quad (11)
\]

Corollary 2. (Certified robustness of bagging with replacement). \(\pi_1, \pi_2\) of bagging with replacement is

\[
\pi_1 = \pi_2 = \left(\frac{n}{m}\right)^{n_s} \quad (12)
\]

\(\delta(\rho)\) of bagging with replacement is

\[
\max_{m \in [n -\rho, n+\rho]} 1 - \left(\frac{\max(m, n) - \rho}{n_s}\right)^{n_s} + 2\left(\frac{\max(m, n) - \rho}{n_s}\right)^{n_s} \quad (13)
\]

Corollary 3. (Certified robustness of binomial selection). \(\pi_1, \pi_2\) of binomial selection is

\[
\pi_1 = \pi_2 = (1 - p)^{m-n} \quad (14)
\]

\(\delta(\rho)\) of binomial selection is

\[
\max_{m \in [n -\rho, n+\rho]} 1 + (1 - p)^{n-m} - 2(1 - p)^{\min(n-m, 0) + \rho} (p = n_s/n) \quad (15)
\]

Comparison with previous works To our knowledge, only bagging with replacement (Jia, Cao, and Gong 2020) and differentially-private learners (Ma, Zhu, and Hsu 2019) achieve certified robustness against general data poisoning attacks. The assumption of differentially-private learners is different from us, so it is hard to compare with it theoretically. Therefore, we only compare our theory with (Jia, Cao, and Gong 2020). Compared with (Jia, Cao, and Gong 2020), \(\delta\) derived by Jia contains 2 additional non-negative terms \(p_1 - (p_1 \cdot n_k)/n_k^k\) and \(p_2^2 - (p_2^2 \cdot n_k^k)/n_k^k\), In almost all cases, these 2 terms are greater than 0. Therefore, the certified robustness derived by us is tighter than Jia.

Prior knowledge about poisoning model \(P\) If we know the poisoning model \(P\) in advance, we can add additional restriction on the variable \(m\) and \(\delta(\rho)\) to derive lower \(\delta(\rho)\). Here we formulate 6 cases of poisoning model into the form that can be used into our theorem 1.

1. (P_1)

\[
\begin{align*}
\text{for } m & \in [n, n + \rho], \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D_{\text{sub}}) = D_{\text{sub}}) = 0
\end{align*}
\]

2. (P_2)

\[
m \in [n - \rho, n], \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D_{\text{m}}) = D_{\text{sub}}) = 1
\]

3. (P_3) \(m = n\)

4. (P_4) \(m \in [n, n + \rho]\)

5. (P_5) \(m \in [n - \rho, n]\)

6. (P_6) \(m \in [n - \rho, n + \rho]\)

A comparison of 3 random selection schemes

Figure 2: \(\delta(\rho)\) of 3 random selection schemes as a function of \(\rho\) under the poisoning scheme \(P_6\). Left: \(n = 13007, n_s = 10\). Right: \(n = 50000, n_s = 1000\).

In Figure 2, we show \(\delta(\rho)\) of 3 random selection schemes respectively. When \(n\) and \(n_s\) are small, \(\delta(\rho)\) of binomial selection is the smallest and \(\delta(\rho)\) of bagging without replacement is the highest. When \(n\) and \(n_s\) are large, \(\delta(\rho)\) of 3 random selection schemes are nearly the same.

Practical Algorithm

In practice, evaluating the precise value of \(p_1, p_2\) needs affordable computation cost. Therefore, we use a Monte-Carlo method to estimate \(p_1, p_2\). We follow (Jia et al. 2020) to use SimuEM for the confidence interval estimation. We estimate \(p_1, p_2\) with confidence level of \(1 - \alpha / 2\). Then the simultaneous confidence level of \(p_1, p_2\) is higher than \(1 - \alpha\).

\[
\frac{p_1}{\overline{p_1}} = \text{BinoCP}(\alpha/2; \mu, count(c_1)) \quad (16)
\]

\[
\frac{p_2}{\overline{p_2}} = \text{BinoCP}(1 - \alpha/2; \mu, count(c_2)) \quad (17)
\]

Our certified prediction 1 is as follow:

Training algorithms and prior knowledge about \(D_n\)

In reality, users may know the clean part \(D^c\) of \(D_n\) in advance. In this case, \(D_n\) can be divided into 2 parts: the clean part \(D^c\) and the potentially poisoned part \(D^p\). \(D^c = \emptyset\) means that all the training samples have the probability of being poisoned. Treating \(D^c\) in the same way as \(D^p\) is not ideal. We regard \(D^c\) as prior knowledge and only apply random selection schemes on \(D^p\). However, how to leverage \(D^c\) efficiently is a problem. We divide prior knowledge into 3 cases as Figure 3.

1. (Case 1), the attacker can poison all the training samples in \(D_n\). \((D^c = \emptyset)\)
(Case 2). The attacker can operate limited number of training samples in the poisoned part of the training set. (\(D^c\) is a set of several training samples)

3. (Case 3). The attacker can operate limited number of training samples of specific classes. (\(D^c\) is all the training samples from the unpoisoned classes)

![Figure 3: A toy example of 3 conditions. We assume \(D_n\) is the 40 training samples and the attacker can poison at most 2 samples. Case 1: the attacker modifies 2 arbitrary samples. Case 2: the attacker poisons 2 samples in the poisoned part \(D^p\). Case 3: the attacker poisons 2 samples of classes 8, 9.](image)

In this section, we propose 2 training algorithms corresponding to the above 3 cases.

### Training algorithm for cases 1,2

We sample \(T\) sub-datasets of size \(n_s\) from \(D_n\) and train \(T\) base classifiers on those \(T\) sub-datasets. In practice, we found that the sampled sub-datasets are usually imbalanced. Especially when \(n_s\) is small, the problem of imbalance is highly severe. The base classifiers tend to predict the most frequent class in the sub-datasets. The most frequent class of each sampled sub-dataset has a high probability to be different, which causes \(T\) base classifiers tend to make different predictions. Therefore, imbalance not only decreases the accuracy of base classifiers, but also lower the robustness of the smooth classifiers. We use the weighted balance method to eliminate the negative influence brought from the imbalance in \(D_n\). The training algorithm for cases 1,2 is showed in Algorithm 2.

### Algorithm 2 The training algorithm for cases 1,2

**Input:** The potentially poisoned part of the training set \(D^p\), the clean part of the training set \(D^c\), selection size \(n_s\), the number of base classifiers \(T\);

**Output:** smooth classifier \(g\);

1. for \(i\) in range(\(T\)) do
2. Sample a \(n_s\)-size sub-dataset from \(D^p\): \(D^p_{n_s} = \mu(D^p)\);
3. Construct training set \(D^i_{\text{sub}} = D^i_{n_s} \cup D^c\);
4. \# BALANCE() is a sample balancing method;
5. Train \(f^i\) on the minibatches;
6. \# Majority rule;
7. end for
8. return \(g = Maj(f^1, f^2, \ldots, f^T)\);

### Algorithm 3 2-phase weighted training for case 3

**Input:** The potentially poisoned part of the training set \(D^p\), the clean part of the training set \(D^c\), selection size \(n_s\), the number of base classifiers \(T\);

**Output:** smooth classifier \(g\);

1. for \(i\) in range(\(T\)) do
2. Sample a \(n_s\)-size sub-dataset from \(D^p\): \(D^p_{n_s} = \mu(D^p)\);
3. Construct training set \(D^i_{\text{sub}} = D^i_{n_s} \cup D^c\);
4. \# BALANCE() is a sample balancing method;
5. Train \(f^i\) on the minibatches;
6. \# Majority rule;
7. end for
8. Train \(\phi_2\) on the dataset \(D^c\) for the 2nd phase classification;
9. return \(g = \phi_2(Maj(\phi_1^1, \phi_1^2, \ldots, \phi_1^T))\);

### Training algorithm for case 3

We refer to the set of clean classes and the set of potentially poisoned classes as \(C_{\text{clean}}, C_{\text{poisoned}}\) respectively. \(C_{\text{clean}} \cup C_{\text{poisoned}}\) is the set of all classes. For convenience, we use a virtual class \(\hat{c}\) to represent \(C_{\text{clean}}\). The main idea is to use 2-phase classifiers as the base classifiers for case 3. In the 1st phase, we use the sub-classifier \(\phi_1\) to classify \(x\) to be which class among \(\hat{c} \cup C_{\text{poisoned}}\). If \(\phi_1(x) = \hat{c}\), we use the sub-classifier \(\phi_2\) to classify \(x\) to be which class in \(C_{\text{clean}}\). The base classifier is \(\phi_2(\phi_1(x))\). For the \(i\)th base classifier, the sub-classifier \(\phi_i^1\) is trained on the sub-dataset \(D^i_{\text{sub}} \cup D^c\). \(D^i_{\text{sub}} \in D^p\), \(\phi_2\) is trained on the clean dataset \(D^c\), so \(\phi_2\) would not be affected by poisoning attacks. We let all the base classifiers share the same \(\phi_2\). If two sub-classifiers \(\phi_1^1, \phi_1^2\) predict the same class, the prediction of the base classifiers \(\phi_1^1(\phi_2)\) and \(\phi_1^2(\phi_2)\) will
be the same. Therefore, training \( T \) base classifiers is equal to training \( T \) sub-classifiers \( \{ \phi_i \}_{i=1}^T \) on \( \{ D_{sub} \cup D_{c} \}_{i=1}^T \) and one \( \phi_2 \). The original \( \#C_{\text{clean}} + \#C_{\text{poisoned}} \) classification task is converted into a \( 1 + \#C_{\text{poisoned}} \) classification task. The complexity of neural networks \( \phi_1 \) and \( \phi_2 \) can be much lower than the original base classifiers, which decreases computation cost.

**Experiment**

**Experimental Setup**

To compare fairly, we conduct experiments on three typical datasets: MNIST 1/7 (13,007 training samples), MNIST (60,000 training samples) and CIFAR10 (60,000 training samples), which are the same as previous works. We use LeNet for MNIST 1/7, MNIST and use ResNet 18 for CIFAR-10. We train \( T = 1000 \) base classifiers for MNIST 1/7, \( T = 1000 \) base classifiers for MNIST and \( T = 500 \) base classifiers for CIFAR-10. For all base classifiers, we use the weight balance method to train. We found that when the size of \( D_{sub} \) is small, the I/O cost of loading \( D_{sub} \) is much higher than computation. When \( n_s = 10 \) for MNIST 1/7, the GPU-Util is only 1%. Therefore, for each sampled sub-dataset \( D_{sub} \), we apply WeightedRandomSampler to expand \( D_{sub} \) to a fixed-size dataset.

We achieve convergence after training only a few epochs on the expanded dataset, which vastly accelerates the training. For conciseness, we use scheme 1, 2, 3 to represent the bagging without replacement, the bagging with replacement and the binomial selection respectively.

We evaluate robustness of the defenses by their certified accuracy. Given \( \rho \), the certified accuracy of \( \rho \) is:

\[
CA = \frac{1}{n} \sum_{i=1}^{n} \left \{ g_{x_i}(D_n) = y_i \text{ and } p_1 - p_2 \geq \delta(\rho) \right \}
\]

(15)

where \( p_1, p_2 \) is estimated by SimuEM with confidence level 0.999, which is the same as (Jia, Cao, and Gong 2020).

**A comparison on 3 random selection schemes (case 1, 2)**

Figure 4 shows the performances of 3 random selection schemes on MNIST 1/7, full MNIST and CIFAR-10. The certified accuracy of 3 random selection schemes is nearly the same across all the combinations of \( n_s \) and \( n_c \). However, for the dataset MNIST 1/7 when \( n_s = 10, n_c = 0 \), the certified accuracy of scheme 3 decreases to 0% at 868 while scheme 1,2 are at 855, 857 respectively. This is because \( \delta(\rho) \) of scheme 3 is less than the \( \delta(\rho) \) of scheme 1,2 when the size of the training set \( n \) and the selection size \( n_s \) are small, which is showed in Figure 2. Some people may worry that the variation in the sizes of sub-datasets sampled by scheme 3 will cause several base classifiers to perform not well. Figure 5 shows the distribution of the sizes of the sub-datasets and the average accuracy corresponding to the size of training set. Among \( T = 1000 \) sub-datasets, more than 994 datasets are of the sizes larger than 3. The accuracy of 994 base classifiers is higher than 80%. MNIST 1/7 is a simple classification task, so the classifiers trained with few samples can achieve high accuracy. On full MNIST and CIFAR-10, 3 schemes perform almost the same. One reason is showed in Figure 2: when \( n_s + n_c \) is large, \( \delta(\rho) \) of
3 schemes are nearly the same. Another reason is that when the classification task is not simple, the improvement in accuracy needs bigger changes of the dataset. Therefore, the accuracy of 3 schemes is close.

A comparison on case 3
We conduct experiments on MNIST to compare the 2-phase classifiers with the traditional training algorithm used in (Jia, Cao, and Gong 2020). For a fair comparison, we use the random selection scheme 2, which is the same as (Jia, Cao, and Gong 2020). Let 0, 1, 2, 3, 4, 5, 6, 7, 8 to be the clean classes and 9 to be the poisoned class. In our 2-phase classification, the dataset for training the 2nd-phase sub-classifier $\phi_2$ is well balanced. Therefore, we can easily train $\phi_2$ of the accuracy of classifying 0, 1, 2, 3, 4, 5, 6, 7, 8 up to 99.1%. For the 1st-phase sub-classifier $\phi_1$, we use the weight balance method to balance the dataset for the binary classification. In Figure 6, the accuracy of $\phi_1$ is higher than 95% as $n_s = 10$. For all the values of $\rho$, the certified accuracy of the 2-phase classifier outperforms that of the classifier trained by the traditional training algorithm.

A comparison with other methods
We only compare our random selection defenses with bagging method (Jia, Cao, and Gong 2020) because it outperforms other 2 methods (Rosenfeld et al. 2019; Ma, Zhu, and Hsu 2019) largely in the experiments of (Jia, Cao, and Gong 2020). The bagging method is the same as scheme 2. For case 1,2, the $\delta(\rho)$ of scheme 2 is larger than scheme 3 that we propose. Therefore, scheme 3 achieve better certified accuracy in MNIST 17 when $n_s = 10$, $n_c = 0$ as showed in Figure 4. For case 3, we propose 2-phase classifiers, which outperforms previous training algorithms.

Ablation study
Impact of the random selection scheme  When the classification task is simple, which means classifiers trained on a small size sub-dataset can already achieve high accuracy, scheme 3 may be the best choice among 3 random selection schemes. It is because $\delta(\rho)$ of scheme 3 is lower than other 2 schemes when $n$ and $n_s$ are small. When the classification task is difficult, which means only the classifiers trained on large size sub-datasets can achieve ideal accuracy, 3 schemes perform nearly the same.

Impact of $n_s$ $n_s$ is the size of the selection. As $n_s$ increases, each sub-dataset contains more training samples. Therefore, the accuracy of the smooth classifiers increases. Higher $n_s$ also results in higher $\delta(\rho)$, so $\rho_1 - \rho_2$ needs to be larger for the same $\rho$. Therefore, $n_s$ controls the trade-off between the clean accuracy and robustness.

Impact of $n_c$ $n_c$ is the size of the clean part $D^c$. In all experiments, the certified accuracy at all $\rho$ improves as $n_c$ increases. For the random selection schemes at small $n_s$, the degree of the increase in certified accuracy is higher than that of the high $n_s$.

Conclusion
We propose a general framework of random selection based certified defenses against data poisoning attacks in this work. We prove the certified robustness of all the random selection schemes that satisfy our proposed conditions. The certified radius derived from our framework is tighter than previous works. To make our framework more practical, we consider the leverage of prior knowledge about the training set and the attacker’s capability. Given prior knowledge about the training set, our certified radius can be tighter. Given prior experience of the poisoning model, our training algorithm can achieve better certified accuracy practically. Our framework shows the potential of random selection schemes in defending data poisoning. Our study highlights the use of prior knowledge for the improvement of randomized smoothing. We hope our framework would inspire the future work on random selection based defenses.
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Appendix

Proof of Theorem 1

Let $P_\rho : D_n \rightarrow D'_m$ as the training model on the dataset $D_n$. For all $D'_m \in P_\rho(D_n)$, let $\Omega$ be $D_n \cap D'_m$. If the random selection scheme $\mu$ satisfies

\begin{align*}
\forall D'_m \in P_\rho(D_n) \\
\exists \text{ constant } \pi_1, \pi_2 \\
\forall D_{\text{sub}} \in \Omega \\
\pi_2 \Pr(\mu(D_n) = D_{\text{sub}}) \geq \Pr(\mu(D'_m) = D_{\text{sub}}) \geq \pi_1 \Pr(\mu(D_n) = D_{\text{sub}})
\end{align*}

We refer to $g_x(D_n) = \mathbb{E}_{D_{\text{sub}} \sim \mu(D_n)} f_x(D_{\text{sub}})$ as the smooth classifier ($x$ is arbitrary). Without loss of generality, we assume $g$ predicts $c_1, c_2$ as the top-2 classes for $x$. The $g'$s confidence of $c_1, c_2$ is $p_1, p_2$.

\begin{align*}
\mathbb{E}_{D_{\text{sub}} \sim \mu(D_n)} \Pr(f_x(D_{\text{sub}}) = c_1) &= p_1 \quad \text{(20)} \\
\mathbb{E}_{D_{\text{sub}} \sim \mu(D_n)} \Pr(f_x(D_{\text{sub}}) = c_2) &= p_2 \quad \text{(21)}
\end{align*}

Our goal is equal to prove $\forall D'_m \in \{P_\rho(D_n)\}$, $\Pr(g_x(D'_m) = c_1) \geq \Pr(g_x(D'_m) = c_2)$.

First we consider the case that the attacker only modify $D_n$, without insertion or deletion. For the poisoned dataset $D'_m$ that different from $D_n$ at most $\rho$ samples. We denote $S_\rho$ as the set of those $\rho$ poisoned samples. Then we have:

\begin{align*}
\Pr(g_x(D'_m) = c_1) &= \mathbb{E}_{D_{\text{sub}} \sim \mu(D'_m)} \Pr(f_x(D_{\text{sub}}) = c_1) \\
= & \sum_{D_{\text{sub}} \subset \Omega} \Pr(\mu(D'_m) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) + \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D'_m) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) \\
\geq & \sum_{D_{\text{sub}} \subset \Omega} \Pr(\mu(D'_m) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) + \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D'_m) = D_{\text{sub}}) \ast 0 \\
= & \sum_{D_{\text{sub}} \subset \Omega} \Pr(\mu(D'_m) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) \\
= & \sum_{D_{\text{sub}} \subset \Omega} \Pr(\mu(D'_m) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}})
\end{align*}

For our assumption about $\mu$, we have:

\begin{align*}
\sum_{D_{\text{sub}} \subset \Omega} \Pr(\mu(D'_m) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) \\
\geq & \pi_1 \sum_{D_{\text{sub}} \subset \Omega} \Pr(\mu(D_n) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) \\
= & \pi_1 \left[ \Pr(g_x(D_n) = c_1) - \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D_n) = \pi D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) \right] \\
= & \pi_1 \left[ p_1 - \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D_n) = D_{\text{sub}}) \Pr(f_x(D_{\text{sub}}) = c_1|D_{\text{sub}}) \right] \\
\geq & \pi_1 \left[ p_1 - \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D_n) = D_{\text{sub}}) \ast 1 \right] \\
= & \pi_1 \left[ p_1 - \sum_{D_{\text{sub}} \not\subset \Omega} \Pr(\mu(D_n) = D_{\text{sub}}) \right]
\end{align*}
Same as the above proof, we have:

\[
Pr(g_x(D'_m) = c_2) = \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) Pr(f_x(D_{sub}) = c_2|D_{sub}) + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) Pr(f_x(D_{sub}) = c_2|D_{sub}) \tag{34}
\]

\[
\leq \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) Pr(f_x(D_{sub}) = c_2|D_{sub}) + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \cdot 1 \tag{35}
\]

\[
= \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) Pr(f_x(D_{sub}) = c_2|D_{sub}) + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{36}
\]

\[
\leq \pi_2 \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{37}
\]

\[
\pi_2 \left[ Pr(g_x(D_n) = c_2) - \sum_{D_{sub} \subset \Omega} Pr(\mu(D_n) = D_{sub}) Pr(f_x(D_{sub}) = c_2|D_{sub}) \right] + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{38}
\]

\[
= \pi_2 \left[ \sum_{D_{sub} \subset \Omega} Pr(\mu(D_n) = D_{sub}) Pr(f_x(D_{sub}) = c_2|D_{sub}) \right] + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{39}
\]

\[
\leq \pi_2 \left[ \sum_{D_{sub} \subset \Omega} Pr(\mu(D_n) = D_{sub}) \cdot 0 \right] + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{40}
\]

\[
= \pi_2 p_2 + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{41}
\]

if

\[
\pi_1 p_1 - \pi_1 \sum_{D_{sub} \subset \Omega} Pr(\mu(D_n) = D_{sub}) \geq \pi_2 p_2 + \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{42}
\]

then we have

\[
Pr(g_x(D'_m) = c_1) \geq Pr(g_x(D'_m) = c_2) \tag{43}
\]

For all the inequalities above 25,29,32,36,38,41, the equalities can hold. Therefore, our bound of the certified radius is tight. The proof of theorem 1 is complete.

**Proof of certified radius bound for 3 random selection schemes.**

Without loss of generality, we assume \(D_{sub}\) is of \(n_s\) size. Given \(m, n\), the size of \(\Omega\) is equal to \(\max(m, n) - \rho\). For all 3 random selection schemes that satisfy \(\pi_1 = \pi_2\), the inequality 43 can be expressed as

\[
p_1 - p_2 \geq \sum_{D_{sub} \subset \Omega} Pr(\mu(D_n) = D_{sub}) + \frac{1}{\pi_1} \sum_{D_{sub} \subset \Omega} Pr(\mu(D'_m) = D_{sub}) \tag{44}
\]
Bagging without replacement  First we prove \( \mu \) satisfies theorem 1

\[
\forall D_{sub} \in \{D_{sub} : D_{sub} \subset \Omega \}
\]
\[
Pr(\mu(D_n) = D_{sub}) = \frac{n}{n_s}
\]
\[
Pr(\mu(D'_m) = D_{sub}) = \frac{m}{n_s}
\]
\[
\pi_1 = \pi_2 = \frac{(m)}{(n_s)} / \frac{(n)}{(n_s)}
\]
\[
\sum_{D_{sub} \not\subset \Omega} Pr(\mu(D_n) = D_{sub}) = 1 - \left( \max(m, n) - \rho \right) / \left( \frac{n}{n_s} \right) + \left( \frac{n}{n_s} \right) - \left( \frac{n}{n_s} \right) \left( \max(m, n) - \rho \right) / \left( \frac{m}{n_s} \right)
\]

The certified radius is \( \max \rho \) that satisfies

\[
\max_{n - \rho \leq m \leq n + \rho} 1 - \left( \max(m, n) - \rho \right) / \left( \frac{n}{n_s} \right) \leq p_1 - p_2
\]

Bagging with replacement  First we prove \( \mu \) satisfies theorem 1

\[
\forall D_{sub} \in \{D_{sub} : D_{sub} \subset \Omega \}
\]
\[
Pr(\mu(D_n) = D_{sub}) = \left( \frac{1}{n} \right)^{n_s}
\]
\[
Pr(\mu(D'_m) = D_{sub}) = \left( \frac{1}{m} \right)^{n_s}
\]
\[
\pi_1 = \pi_2 = \left( \frac{n}{m} \right)^{n_s}
\]
\[
\sum_{D_{sub} \not\subset \Omega} Pr(\mu(D_n) = D_{sub}) = 1 - \left( \frac{\max(m, n) - \rho}{n} \right)^{n_s}
\]
\[
\sum_{D_{sub} \not\subset \Omega} Pr(\mu(D'_m) = D_{sub}) = 1 - \left( \frac{\max(m, n) - \rho}{m} \right)^{n_s}
\]

The certified radius is \( \max \rho \) that satisfies

\[
\max_{n - \rho \leq m \leq n + \rho} 1 + \left( \frac{m}{n} \right)^{n_s} - 2 \left( \frac{\max(m, n) - \rho}{n} \right)^{n_s} \leq p_1 - p_2
\]

Binomial selection \( \mu_p \)  First we prove \( \mu_p \) satisfies theorem 1

\[
\forall D_{sub} \in \{D_{sub} : D_{sub} \subset \Omega \}
\]
\[
Pr(\mu(D_n) = D_{sub}) = p^{n_s}(1 - p)^{n - n_s}
\]
\[
Pr(\mu(D'_m) = D_{sub}) = p^{n_s}(1 - p)^{m - n_s}
\]
\[
\pi_1 = \pi_2 = (1 - p)^{m - n}
\]
\[
\sum_{D_{sub} \not\subset \Omega} Pr(\mu(D_n) = D_{sub}) = 1 - (1 - p)^{\min(n - m, 0) + \rho}
\]
\[
\sum_{D_{sub} \not\subset \Omega} Pr(\mu(D'_m) = D_{sub}) = 1 - (1 - p)^{\min(m - n, 0) + \rho}
\]

The certified radius is \( \max \rho \) that satisfies

\[
\max_{n - \rho \leq m \leq n + \rho} 1 + (1 - p)^{n - m} - 2(1 - p)^{\min(n - m, 0) + \rho} \leq p_1 - p_2
\]