Precision Predictions

Ulf-G. Meißner, Univ. Bonn & FZ Jülich

supported by DFG, SFB/TR-110

by CAS, PIFI

by VolkswagenStiftung
CONTENTS

• Entrée:
  – What is a precision prediction?
  – Two time-honored examples

• Plat principal:
  – From Schwinger’s tombstone to ultraprecision
  – Precision sigma-term physics
  – Shapiro delay precision physics
  – Precision meets anthropics: Life in the multiverse

• Dessert:
  – Precision physics - quo vadis?
Entrée
WHAT is a PRECISION PREDICTION?

My definition:

A prediction is considered **precise**, if it has a **small (relative) theoretical uncertainty**.

Remarks

- this does not imply that it agrees with experiment (cf. Popper)
- “small uncertainty” can be best quantified if we have an underlying counting rule
- a prediction without uncertainty makes little sense
- I will mostly consider the interplay of precision predictions with the corresponding precise experiments
**EX. 1: MASS of the TOP QUARK and the HIGGS BOSON**

- Virtual particles can leave traces
  - a cornerstone of precision physics

- This requires precision predictions

- Radiative corrections to $e^+e^-$ collisions (LEP, SLD)
  - $m_{\text{top}}$ and $M_{\text{Higgs}}$ could be inferred

- Direct measurements of the top/Higgs 1995/2012 at Fermilab/CERN
EX. 2: The HULSE-TAYLOR PULSAR

- General Relativity predicts the slowing down of pulsar period \( P_b \) due to the radiation of gravitational waves.

- Binary system with masses \( m_1, m_2 \) and eccentricity \( e \):

\[
\dot{P}_{b,GR} = -\frac{192\pi G^{5/3}}{5c^5} \left( \frac{P_b}{2\pi} \right)^{-5/3} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) (1 - e^2)^{-7/2} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}}
\]

Peters, Phys. Rev. 136, B1224 (1964)

- A triumph of GR:

\[
P = 0.059029995269(2) \, s \\
\dot{P} = 8.64(2) \cdot 10^{-18} \, s^{-1}
\]

Taylor, Fowler, McCulloch, Nature 277, 437 (1979)
EX. 2: The HULSE-TAYLOR PULSAR continued

- This has become a true precision test of GR:
  - Gravitational waves exist
  - Energy as predicted by GR
  - Gravity propagates with the speed of light
  - GR holds for strongly self-gravitating masses
  - Theoretical uncertainty in $P_b$:
    \[ \delta \dot{P}_b = 0.002\% \]
- Gravitational waves detected directly by LIGO/VIRGO in 2015 → talk by Giovanni Losourdo
Plat principal
EX. 1: From Schwinger’s tombstone to ultrahigh precision

• Dirac’s prediction: a spin-1/2 particle has a Landé factor of $g = 2$

• The dawn of the precision era:

$$a_e = \frac{1}{2} (g_e - 2) = \frac{1}{2} \frac{\alpha}{2 \pi} \simeq 1.1 \cdot 10^{-3}$$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137.03599...}$$

J. Schwinger, Phys. Rev. 73 (1948) 416 [triggered by: Kusch, Foley, Phys. Rev. 72 (1947) 1256]

• The most precise Standard Model prediction $[SU(3)_C \times SU(2)_L \times U(1)_Y]$:

$$a_e = a_e(\text{QED}) + a_e(\text{weak}) + a_e(\text{strong})$$

$$a_e(\text{QED}) = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau)$$

$$A_n = \sum_{i=1,2,...} \left( \frac{\alpha}{\pi} \right)^i A_n^{(2i)}$$

→ must know $A_1$ to **tenth order** to achieve sub-ppb precision
EX. 1: From Schwinger’s tombstone to ultrahigh precision

• Completion of the 10\textsuperscript{th} order (12762 diagrams):

\[ A_{1}^{(10)} = 6.675(192) \times (\alpha/\pi)^5 \simeq 0.07 \cdot 10^{-12} \]

Aoyama, Kinoshita, Nio, Phys. Rev. D97 (2018) 036001

• Complete SM prediction:

\[
\begin{align*}
    a_e(\text{th'\,y}) &= 1\,159\,652\,182.037 \pm (11) \times 10^{-12} \text{ using } \alpha(\text{Rb}) \\
    a_e(\text{th'\,y}) &= 1\,159\,652\,181.606 \pm (12) \times 10^{-12} \text{ using } \alpha(\text{Cs})
\end{align*}
\]

\[\delta_{\text{QED}} \delta_{\text{strong}} \delta\alpha\]

Rb: Bouchendira et al., Phys. Rev. Lett. 120 (2018) 183001; Cs: Parker et al., Science 360 (2018) 191

• Best measurement: \[ a_e = 1\,159\,652\,182.73(28) \times 10^{-12} \]

Hanneke, Fogwell, Gabrielse, Phys. Rev. Lett. 100 (2008) 120801

• Truely amazing!
EX. 1: From Schwinger’s tombstone to ultrahigh precision

- But a small tension!
  ↩ a sign of BSM physics?

- Improved measurements of $R_\infty$ and $a_e$ planned
  Gabrielse et al., 1904.06174 [quant-ph]

- Effects of heavy mass particles are enhanced by $(m_\mu/m_e)^2 \approx 43.000$ in $(g - 2)_\mu$

- Hadronic contribution is the culprit:
  data-driven (VP) and Lattice QCD (LbL) evaluations (many groups world-wide)

- There is a tension → ???

Keshavarzi, Nomura, Teubner, Phys. Rev. D 97 (2018) 114025
EX. 2: Precision $\sigma$-term physics

- Massless classical QCD is scale-invariant (dilatations)

- Scale invariance broken by quantization: **dimensional transmutation** $\sim \Lambda_{\text{QCD}}$

- **Trace anomaly** = generation of hadron masses (central result of QCD)

$$m_N = \langle N(p)|\theta_{\mu}^{\mu}|N(p)\rangle \quad [\theta_{\mu\nu} = \text{energy-momentum tensor}]$$

$$= \langle N(p)|\frac{\beta_{\text{QCD}}}{2g}G_{\mu\nu}^{a}G_{\mu\nu}^{a} + m_\text{u} \bar{u}u + m_\text{d} \bar{d}d + m_\text{s} \bar{s}s|N(p)\rangle$$

- Need to determine

$$\langle N(p)|m_\text{u} \bar{u}u + m_\text{d} \bar{d}d|N(p)\rangle = \sigma_{\pi N}$$

$$\langle N(p)|m_\text{s} \bar{s}s|N(p)\rangle = \sigma_s$$

- The pion-nucleon sigma-term $\sigma_{\pi N}$ can be obtained to high-precision from a Roy-Steiner analysis of pion-nucleon scattering data

\[Crewther (1972), \text{Chanowitz, Ellis (1972), Collins, Duncan, Joglekar(1977)}\]
EX. 2: Precision $\sigma$-term physics II

- Role of the pion-nucleon $\sigma$-term:
  
  $\star$ Scalar couplings of the nucleon
  
  $$\langle N | m_q \bar{q} q | N \rangle = f^N_q m_N \quad (N = p, n)$$
  
  $$\quad (q = u, d, s)$$

  $\hookrightarrow$ Dark Matter detection

  $\hookrightarrow \mu \rightarrow e$ conversion in nuclei

  $\star$ Condensates in nuclear matter

  $$\frac{\langle \bar{q} q \rangle (\rho)}{\langle 0 | \bar{q} q | 0 \rangle} = 1 - \frac{\rho \sigma_{\pi N}}{F^2 \pi M^2_{\pi}} + \ldots$$

  $\star$ CP-violating $\pi N$ couplings

  $\hookrightarrow$ hadronic EDMs (nucleon, nuclei)

Crivellin, Hoferichter, Procura (2014)
EX. 2: Precision $\sigma$-term physics III

- Roy-Steiner analysis of $\pi N \rightarrow \pi N$
- Important input: precision pionic atom data from PSI
  $\rightarrow$ accurate $\pi N$ scattering lengths (w/ theory)
- First ever dispersive analysis with error bars!
- High-precision determination of $\sigma_{\pi N}$:

$$\sigma_{\pi N} = (59.1 \pm 1.9_{RS} \pm 3.0_{LET}) \text{ MeV}$$
$$\quad = (59.1 \pm 3.5) \text{ MeV}$$

- Strangeness sigma-term: $\sigma_s \simeq (30 \pm 30) \text{ MeV}$
- $m_N$ can be calculated to high precision in lattice QCD
  $\Rightarrow$ Only about 100 MeV of the nucleon mass $a$
due to the Higgs! [“mass without mass”]

Wheeler (1962)

Gotta, Prog. Part. Nucl. Phys. 52 (2004) 133
Hennebach et al., Eur. Phys. J. A 50 (2014) 190
Gasser et al., Eur. Phys. J. C 26 (2002) 13
Baru et al., Nucl. Phys. A 872 (2011) 69

Hoferichter et al., Phys. Rev. Lett. 115 (2015) 092301
Hoferichter et al., Phys. Rept. 625 (2016) 1
de Elvira et al., J. Phys. G 45 (2018) 024001
EX. 3: Shapiro delay precision physics I

• Shapiro delay = 4th test of GR
  → curved space-time reduces c
  Shapiro, Phys. Rev. Lett. 13 (1964) 789

• Standard approximation (binary):
  consider the potential of the receiving mass
  → 2 post-Keplerian parameters
  \( r_{\text{Sh}} \) and \( s_{\text{Sh}} \) called “range” and “shape”
  \[ r_{\text{Sh}} = \frac{Gm_{\text{companion}}}{c^3}, \quad s_{\text{Sh}} = \sin i \]

• Well tested in the binary pulsar
  PSR J0737-3039A and B
  \( m_A = 1.3381(7)M_\odot \)
  \( m_B = 1.2489(7)M_\odot \)
  Kramer et al., Science 314 (2006) 97

Figures courtesy Michael Kramer & Norbert Wex
EX. 3: Shapiro delay precision physics II

- Retardation
- Light bending
- Pulsar rotation

Blandford & Teukolsky (1976), Wex (1995)
Kopeikin & Schäfer (1999)

Schneider (1990)

Doroshenko & Kopeikin (1995)

⇒ lead to modifications in the delay time (residuals)

Figures courtesy Michael Kramer & Norbert Wex
EX. 3: Shapiro delay precision physics III

- Higher order propagation delays in the double pulsar: Kramer et al., in preparation

- Remarkable agreement between precision theory (GR) and precision experiment

- Theoretical uncertainty in the Shapiro delay prediction: $\delta SD = 0.019\%$
EX. 4: Precision meets anthropics: Life in the multiverse

- Nuclear Lattice Effective Field Theory
  - a new tool to perform nuclear structure and reaction calculations at sub-percent level
    - Lee, Prog. Part. Nucl. Phys. 63 (2009) 117
    - Lähde, Meißner, Lect. Notes Phys. 957 (2019) 1

- First \textit{ab initio} calculation of the Hoyle state in $^{12}\text{C}$
  - Epelbaum et al., Phys.Rev.Lett. 106 (2011) 192501

- Levels calculated with an accuracy of $\pm 300$ keV

- Closeness of the Hoyle state to the $3\alpha$ threshold required to make carbon-based life possible
  - Hoyle (1954)

- "level of life"
  - Linde (2007)

- prime example for the anthropic principle
  - Carter (2006)
EX. 4: Precision meets anthropics: Life in the multiverse II

- How does $\Delta E = E(\text{Hoyle}) - E(3\alpha)$ change, when the fundamental parameters ($m_{\text{quark}}, \alpha$) of the Standard Model are varied?

- Variations of the hadronic/nuclear properties can be computed from chiral EFT plus lattice QCD
  
  Epelbaum et al., Phys. Rev. Lett. 110 (2013) 112502; Lähde et al., arXiv:1906.00607 [nucl-th]

- Variations of $\Delta E$ can be investigated in stellar simulations: $r_{3\alpha} \sim \exp(-\Delta E/k_B T)$
  
  Oberhummer et al., Science 289 (2000) 88; Huang et al., Astropart. Phys. 105 (2019) 13

- Results:
  - fine-tunings in the triple-alpha process are correlated
  - quark masses can be varied by a few percent (lattice)
  - fine structure constant $\alpha$ can be varied by about 7%
Dessert
SUMMARY & OUTLOOK

• Lessons learned / take home:

  Precision predictions rest on scale separations
  $\rightarrow$ Effective Field Theories are the tool

  Precision predictions (physics) are of ever growing importance

  Precision physics might be our best take on discovering BSM physics

  Need to sharpen predictions where the SM gives little contribution
  (e.g. EDMs of nucleons and light nuclei)
