Confinement of pulsar wind nebulae by their supernova remnants and magnetic dissipation

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Abstract. The standard model of the pulsar wind nebula states that the pulsar wind should be low-magnetization just upstream the termination shock, called the ‘sigma-problem’. Low-magnetization is required in order to confine the pulsar wind nebula inside the slowly expanding supernova remnant whose expansion velocity is much smaller than the speed of light. Although the standard model is based on the ideal magnetohydrodynamic approximation, the recent studies indicate that the non-ideal magnetohydrodynamic effects would be important to resolve the sigma-problem. In this study, we extend the standard model including the turbulent magnetic field and the magnetic dissipation. The conversion of the toroidal magnetic field to the turbulent magnetic field and the magnetic dissipation terms are treated phenomenologically. We find that the conversion of the toroidal magnetic field to the turbulent magnetic field decelerates the nebular flow to fit the observed value of the expansion velocity. On the other hand, the magnetic dissipation hardly contributes the flow deceleration, although the magnetic dissipation is important to reproduce the observed emission properties of the PWNe.

1. Introduction
Pulsar wind nebulae (PWNe) are composed of relativistic magnetized plasma supplied by the central pulsar. The relativistic magnetized outflow is called pulsar wind and PWNe are the shocked pulsar wind. The pulsar wind is originally the rotational energy of the central pulsar and spins down the pulsar. Because the extraction of the rotational energy is mediated by the strong magnetic field of the pulsar itself [1], the magnetization (the ratio of the magnetic to particle energy fluxes) of the pulsar wind should be much larger than unity [2,3].

The observed luminosity of the pulsed emission from the pulsar has only a fraction of spin-down power, while the PWN around the pulsar has the luminosity comparable with the spin-down power, i.e., the PWN is more luminous and larger astrophysical object than the pulsar itself. Important observational properties of the Crab Nebula, which is a prototypical example of PWNe, are the small expansion velocity $v_{\text{PWN}} \ll c$ and the large ratio of the synchrotron to the inverse Compton fluxes. These observations indicate that the pulsar wind of the Crab Nebula has the magnetization of much less than unity at the immediately upstream the pulsar wind shock and also at the post shock region [4–7]. This mismatch of the magnetization at the inner magnetosphere and that at around the shock is called the ‘sigma-problem’.

A lot of efforts have been made to resolve the sigma-problem. Most works consider the magnetic dissipation at the pre-shock pulsar wind [8–10] or that at the shock [11,12]. The recent 3D relativistic MHD simulation of a PWN states that the magnetic dissipation and
the randomization of the magnetic field at the post shock (PWN) region are important [13]. Recently, the magnetic dissipation at the PWN region is also studied analytically [14,15].

Here, we extend the model of PWN by [5] including the magnetic dissipation and the randomization of the magnetic field. It will be found that only the randomization of the magnetic field is enough to reproduce the observed property $v_{\text{PWN}} \ll c$. In section 2, we describe our model of PWN outflow. In section 3, we show the flow profile based on our model and demonstrate that the outflow is decelerated by the randomization of the magnetic field. In section 4, we discuss the obtained results and summarise this paper. We are preparing a paper by adding greater details of our model and more results than the contents of this proceeding. The readers who are interested in this study will be able to find our paper in near future.

2. Model

Here, we extend the model of [5] (KC model) by including the turbulent component of the magnetic field and the magnetic dissipation. The upstream properties of the nebular flow is set to exactly the same as the KC model. We assume that the pulsar spin-down power is the same as the power of the pulsar wind,

$$L_{\text{spin}} = L_{\text{wind}} = \kappa \dot{N}_{\text{GJ}} \gamma_w m_e c^2 (1 + \sigma_w),$$  \hspace{1cm} (1)

where $\kappa$ is the pair multiplicity (the particle number flux normalized by the Goldreich-Julian number flux $\dot{N}_{\text{GJ}} \equiv \sqrt{6cL_{\text{spin}}}/e$), $\gamma_w$ is the wind bulk Lorentz factor and $\sigma_w$ is the wind magnetization parameter (the ratio of the Poynting to the kinetic energy fluxes). At the termination shock radius $r_{TS}$, we impose the Rankin-Hugoniot relations for the strong perpendicular shock derived by [5]. The termination shock is set to the standing shock in the observer (pulsar) frame.

The nebular flow of the KC model is based on the following assumptions; (i) steady state, (ii) spherical symmetry, (iii) ideal MHD, (iv) pure radial flow velocity, and (v) pure toroidal magnetic field. Adopting the same assumptions, we add the following two effects inside the nebula; (vi) the conversion of the toroidal magnetic field to the turbulent one and (vii) the decay of the turbulent magnetic field. We adopt phenomenological descriptions of these effects referring to the studies by [16,17].

The nebular flow has six variables, the radial four-velocity $u$, the proper density $n$, the proper enthalpy density $w$, the pressure $p$, the proper toroidal magnetic field $\bar{b}$ and the proper turbulent magnetic field $\delta b$. The turbulent magnetic field is set to isotropic in the proper frame and then the magnetic field three-vector in the proper frame would be written as $\mathbf{b} = \bar{b} \mathbf{e}_\phi + \delta \mathbf{b}$ satisfying $<\mathbf{b}> = \bar{b} e_\phi$ and $<\mathbf{b}^2> = \bar{b}^2 + \delta \mathbf{b}^2$, where $<>$ represents ensemble average. The corresponding six equations are two algebraic equations,

$$w = nm_e c^2 + \frac{\dot{\Gamma}}{\Gamma - 1} p,$$

$$4\pi n u c r^2 = \kappa \dot{N}_{\text{GJ}} = \text{const.},$$ \hspace{1cm} (3)

and four differential equations,

$$\frac{d}{dr} \left[ r^2 \gamma u \left( w + \bar{b}^2 + \frac{2}{3} \delta b^2 \right) \right] = -r^2 n \gamma \frac{p_{\text{syn}}}{c},$$ \hspace{1cm} (4)

$$\frac{d}{dr} (w - p) + w \frac{d}{dr} \ln u r^2 = \frac{\delta b^2 / 2}{u c \tau_{\text{diss}}} - \frac{n p_{\text{syn}}}{u c},$$ \hspace{1cm} (5)

$$\frac{d}{dr} \frac{\bar{b}^2}{2} + \frac{\bar{b}^2}{2} \frac{d}{dr} \ln u r^2 = -\frac{\bar{b}^2 / 2}{u c \tau_{\text{conv}}},$$ \hspace{1cm} (6)
Table 1. Summary of the parameters.

| Parameter | Value          |
|-----------|----------------|
| $L_{\text{spin}}$ [erg s$^{-1}$] | $4.6 \times 10^{38}$ |
| $\kappa$ | $10^4$ |
| $\tau_{TS}$ [yr] | 10 |
| $r_{\text{PWN}}$ [pc] | 2.0 |
| $\sigma_w$ | 0.1 |
| $\tau_{\text{diss}}$ [yr] | 0.028 |
| $\tau_{\text{conv}}$ [yr] | 0.228 |
| $\eta_{\text{syn}}$ | 0.308 |

Equations (2) and (3) correspond to the equation of state and the conservation of the particle number, respectively. If we ignore the right-hand sides of Equations (4) – (7), these equations represent the conservation of the total energy, the internal energy of the fluid, the toroidal magnetic energy and the turbulent magnetic energy, respectively, and exactly correspond to the equations of the KC model for $\delta b = 0$. We use the adiabatic index of $\hat{\Gamma} = 4/3$ throughout this paper.

In the right-hand sides of Equations (4) – (7), the terms proportional to the synchrotron power per particle $p_{\text{syn}}$, to the inverse of the conversion time-scale $\tau_{\text{conv}}^{-1}$ and to the inverse of the dissipation time-scale $\tau_{\text{diss}}^{-1}$ represent the synchrotron cooling effect, the conversion of the toroidal magnetic field to the turbulent magnetic field, and the dissipation of the turbulent magnetic field, respectively. Introducing the thermal Lorentz factor $\gamma_{\text{th}} \equiv (w - p)/(nm_e c^2)$, for simplicity, we assume

$$p_{\text{syn}} = \frac{4}{3} \sigma_T c \gamma_{\text{th}}^2 \frac{\vec{b}^2 + \delta b^2}{2}. \tag{8}$$

where $\sigma_T$ is the Thomson cross section. $\tau_{\text{conv}}$ and $\tau_{\text{diss}}$ are the parameters of the basic equations and are set to constants in the proper frame.

Evolution of the four-velocity is obtained from Equations (2) – (7),

$$\epsilon (\beta^2 - \beta_c^2) \frac{du}{dr} = \frac{2u}{r} \left( \hat{\Gamma} p + \frac{2}{9} \delta b^2 \right) + \left( \hat{\Gamma} - 1 \right)n \frac{p_{\text{syn}}}{c} + \frac{\vec{b}^2}{3 c \tau_{\text{conv}}} + \left( \frac{4}{3} - \hat{\Gamma} \right) \frac{\delta b^2}{2 c \tau_{\text{diss}}}, \tag{9}$$

where $\beta_c = (\hat{\Gamma} p + \vec{b}^2 + 2/9 \delta b^2)/(w + \vec{b}^2 + 2/3 \delta b^2)$ is the characteristic velocity corresponding to a phase velocity of a fast magnetosonic wave for the case of the pure toroidal magnetic field. The right-hand side of Equation (9) is positive so that the nebular flow always decelerates with $r$ for the post shock flow ($\beta < \beta_c$).

3. Results

We apply the model described in section 2 to the Crab Nebula. We set $L_{\text{spin}} = 4.6 \times 10^{38}$ erg s$^{-1}$, $\kappa = 10^4$, $r_{\text{pc}} = 0.1$ pc and $r_{\text{PWN}} = 2.0$ pc. We consider three different values of the wind magnetization $\sigma_w = (0.1, 10, 10^3)$ and two different dissipation time-scale $\tau_{\text{diss}} = (10 \text{ yr}, \infty)$, and look for the values of $\tau_{\text{conv}}$ which satisfies $v_{\text{PWN}} = 1500$ km s$^{-1}$. The parameters are also summarised in Table 1.

Figure shows the resultant flow profiles. The four-velocity hardly depends on $\tau_{\text{diss}}$, i.e., the thick and thin lines are overlapped in the top panel and the obtained $\tau_{\text{conv}}$ are almost the same.
for the cases of $\tau_{\text{diss}} = 10$ yr and $\infty$. On the other hand, $\sigma$ in the bottom panels are different between thick and thin lines because the magnetic energy is converted to the internal energy of the plasma by $\tau_{\text{diss}}$. The definition of the magnetization is

$$\sigma(r) \equiv \frac{\overline{b^2} + 2/3 \delta b^2}{w},$$

and this is different from the definition of $\sigma_w$, while those are almost the same for the case of $\gamma_w \gg 1$.

The magnetic dissipation heats up the plasma and changes the synchrotron cooling efficiency $\eta_{\text{syn}}$ (the fraction of the energy flux radiated away from the system by the synchrotron cooling $p_{\text{syn}}$). However, the synchrotron cooling hardly changes the flow profile even for the KC model ($\eta_{\text{syn}} \sim 0.36$). The thick and thin lines of the top panel (four-velocity) is not exactly the same but are overlapped.

4. Discussion & Conclusions
We extend the KC model including the turbulent magnetic field and the magnetic dissipation. The conversion of the toroidal magnetic field to the turbulent magnetic field and the magnetic dissipation terms are treated phenomenologically. The time-scales of the conversion and dissipation are the parameters of the system and we take them as constants in the proper frame for simplicity. We find that the conversion of the toroidal magnetic field to the turbulent magnetic field decelerates the nebular flow to fit the observed value of the expansion velocity. On the other hand, the magnetic dissipation hardly contributes the flow deceleration.

Here, we did not consider the emission spectrum of the PWN. Equation (8) is too simple description of the radiation effect. The low-magnetization of the Crab Nebula is required in both
Dynamical and spectral point of views, i.e., we should reproduce not only the observed expansion velocity of the Crab Nebula but also the observed synchrotron to the inverse Compton flux ratio. In order to reproduce the flux ratio, $\tau_{\text{diss}}$ plays an important role. In addition, the values of $\eta_{\text{syn}}$ in this study are much smaller than the observations of the Crab Nebula. $\tau_{\text{diss}}$ is also used to control the synchrotron cooling efficiency and it should be studied with a broadband emission model. The studies of the radial profiles of the radiation spectrum are also important to constrain the parameters [18].

In addition to $\tau_{\text{diss}}$, $\kappa$ changes the synchrotron cooling efficiency. In this paper, we adopted $\kappa = 10^4$, although the studies of the broadband spectral modeling of the Crab Nebula suggest $\kappa > 10^6$ [19–21], however, see also [22]. The larger particle number $\kappa$ leads to the smaller temperature $\gamma_{\text{th}}$ of the post shock flow because the total energy flux is limited by Equation (1). We need a larger $\gamma_{\text{th}}$ (smaller $\kappa$) in order to make the synchrotron cooling more effective than this study. Adopting $\kappa > 10^4$ does not change the results of this paper.

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