Creep mathematical model on the example of early age concrete

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Abstract. The objective of the study is development of a creep kernel recording form, which allows obtaining representations for creep curves calculation. Based on the elastic-creeping body theory a possibility of the high-rate creep movement analytical study has been shown. A creep kernel, which contains a formula to describe the aging material properties, has been built. Essential stages of the proposed creep kernel formation have been given. The correspondence of the proposed calculations to real processes has been proved by a comparison with experimental data. In the numerical implementation, decomposition of integration elements into power series has been used. A possibility of calculating functions for a long time interval has been shown. Creep kernel parameters have been determined on the basis of experimental data of early age concrete samples by minimizing the standard deviation of theoretical and empirical values.

Keywords: composite structures, creep kernel, viscoelastic properties, aging material.

1. Introduction
The modern stage of the industry development is characterized by the improvement of technical parameters of building constructions, increasing their reliability and durability. Under market conditions, there is a fast substitution of structural materials. This is due to a reduction of material consumption while increasing strength, durability and efficiency. The solution to the problem of designing facilities is to improve general structural schemes and their calculation methods, as well as to develop effective structural elements and to use new high-strength materials. One of the types of modern efficient structures is composite shells, plates and beams [1, 2]. Individual elements of such structures are made of different materials: polymer, composite, etc. Concrete is used as the cheapest material. Connection between the layers of composite plates can be provided with adhesive joint or anchors [3, 4].

Practice shows that plastic deformations and creep occur when using composite systems. In viscoelastic materials deformation increases (accumulates) in time under a constant load. This leads to a qualitative and significantly quantitative change in the stress-strain state (SSS): stiffness reduction, redistribution of forces between structural elements, etc. In contrast to elasticity problem solutions [5, 6], where the SSS only depends on spatial coordinates, the viscoelastic material SSS components change in time [5, 7-9]. To describe the above phenomena building special physical and mathematical models and methods is required.
In the article a notation of the relationships between the aging material stresses and strains has been proposed. On the basis of the elastic-creeping body theory a possibility of a high-rate creep movement analytical description has been shown. Using the Arutyunyan - Zevin method [9, 10] new creep kernels with respect to aging material have been built. Kernel numerical implementation has been considered on the example of experimental A. A. Ross data [2]. Error estimation has been given.

2. Materials and Methods

It is assumed that structure material has creeping and aging properties. Then, in the linear theory of the elastic - creeping body at a uniaxial stress state, the relationship between stresses $\sigma(t)$ and strains $\varepsilon(t)$ in such a structure is as follows [2]:

$$
\varepsilon(t) = \frac{\sigma(t)}{E(t)} + \frac{1}{E(t)} \int_{\tau_0}^{t} K(t, \tau) \sigma(\tau) d\tau,
$$

(1)

where $\varepsilon(t)$ – strain components at an instant of time $t$, $\sigma(t)$ – stress components at an instant of time $t$, $E(t)$ – modulus of elasticity of instantaneous strain; $K(t, \tau)$ – creep kernel, $t$ – an instant of time under consideration, $\tau$ – an intermediate instant of time, $\tau_0$ – an instant of stress application $\sigma(t)$.

The objective of the study is development of a creep kernel recording form, which allows obtaining "formulization" for creep curves calculation. Using method [2], new kernels for a description of aging material creep have been determined. The possibility of calculating the obtained functions using power series for a sufficiently long time interval has been shown [2, 8].

To describe the relations between stresses and strains in accordance with the Arutyunyan - Zevin method formation of new kernels has been carried out taking into account material aging [2, 10, 11]:

$$
K_1(t, \tau) = Q(t, \tau) + B(t - \tau) + \int_{\tau}^{t} Q(s)B(s - \tau) ds;
$$

$$
K_2(t, \tau) = Q(t, \tau) + B(t - \tau) + Q(\tau) \int_{\tau}^{t} B(t - s) ds.
$$

(2)

In formulas (2) the following notations have been adopted: $K_1(t, \tau), K_2(t, \tau)$ – aging material creep kernels, $Q(t, \tau)$ – regular part of the creep kernel; $B(t - \tau)$ – differential part of the creep kernel, $t - \tau$ – difference argument, $t$ – an instant of time under consideration, $\tau$ – an intermediate instant of time, $s$ – intermediate argument.

As a difference component, a weakly singular kernel meeting all the necessary requirements is used [2, 12]:

$$
B(t - \tau) = \chi e^{-\rho(t - \tau)^\alpha} (t - \tau)^{\theta-1} \frac{1}{\Gamma(\theta)}.
$$

(3)

where $B(t - \tau)$ – differential part of the creep kernel, $\Gamma(\theta)$ – gamma-function, $\chi$, $\rho$, $\theta$ – kernel characteristics, $\chi > 0$; $\rho > 0$; $\theta \epsilon (0;1)$; $\alpha \epsilon (0;1)$, $t - \tau$ – difference argument, $t$ – an instant of time under consideration, $\tau$ – an intermediate instant of time.

The introduction of $\alpha$ parameter extends the possibility of calculating the function using power series in a longer time interval in comparison with the variant when $\alpha = 1$ (the Rzhanitsyn kernel) [12].

As a regular component, the kernel, based on the creep measure of the following form is used [2, 11]:
\[ Q(t, \tau) = -\frac{\partial}{\partial \tau} \left[ C_0 + Ae^{-\beta \tau} \right] \left[ 1 - e^{-\gamma (t-\tau)} \right] = \sum_{k=1}^{3} a_ke^{-\gamma k\tau}e^{\alpha_k \tau}, \]

\[ a_1 = \beta A, \quad a_2 = C_0 \gamma, \quad a_3 = A(\gamma - \beta), \]

\[ a_1 = -\beta, \quad a_2 = \gamma, \quad a_3 = \gamma - \beta, \quad \gamma_1 = 0, \quad \gamma_2 = \gamma_3 = \gamma \]

where \( Q(t, \tau) \) – regular part of the creep kernel, \( t - \tau \) – difference argument, \( t \) – an instant of time under consideration, \( \tau \) – an intermediate instant of time.

According to paper [10], the resolvent for the regular kernel has the form:

\[ G(t, \tau) = Q(t, \tau)e^{(-\int_0^\tau q(t,s)ds)} = \sum_{k=1}^{3} a_ke^{\alpha_k t - \gamma k\tau}e^{-\left[ C_0 + Ae^{-\beta \tau} \right] \left[ 1 - e^{-\gamma (t - \tau)} \right]}, \]

Here \( G(t, \tau) \) – regular kernel resolvent, \( Q(t, \tau) \) – regular part of the creep kernel, \( t - \tau \) – difference argument, \( t \) – an instant of time under consideration, \( \tau \) – an intermediate instant of time, \( s \) – intermediate argument.

Substituting formulas (3) and (4) into the formulas for the kernel (2), we obtain:

\[ K_r(t, \tau) = Q(t, \tau) + B(t - \tau) + \frac{\chi}{\Gamma(\theta)} \sum_{k=1}^{3} a_k e^{\alpha_k t - \gamma k\tau} \int_0^{\tau-\tau} e^{\eta_{rk} z - \rho z^\theta} z^{\theta-1} dz, \]

where \( r = l, 2; \eta_{1k} = \alpha_k; \eta_{2k} = \gamma_k; \alpha_k, \chi \) – required parameters, \( Q(t, \tau) \) – regular part of the creep kernel, \( B(t - \tau) \) – differential part of the creep kernel, \( \Gamma(\theta) \) – gamma-function, \( t - \tau \) – difference argument, \( t \) – an instant of time under consideration, \( \tau \) – an intermediate instant of time, \( z \) – intermediate argument.

As a result of integrating formula (6) we obtain:

\[ V_r(t, \tau) = \int_{\tau}^{t} K_r(t, s)ds = -\frac{\partial}{\partial s} \int_{\tau}^{t} \left[ C_0 + Ae^{-\beta s} \right] \left[ 1 - e^{-\gamma (t-s)} \right] ds + \]

\[ + \frac{\chi}{\Gamma(\theta)} \int_{\tau}^{t} e^{-\rho (t-s)^\theta} (t-s)^{\theta-1} ds + \]

\[ + \frac{\chi}{\Gamma(\theta)} \sum_{k=1}^{3} a_k e^{-\gamma k\tau} \int_{\tau}^{t} \left[ \int_{\tau}^{s} e^{\alpha_k s} \int_{0}^{\eta_{rk} z - \rho z^\theta} z^{\theta-1} dz \right] ds = \]

\[ = \left[ C_0 + Ae^{-\beta \tau} \right] \left[ 1 - e^{-\gamma (t - \tau)} \right] + \frac{\chi}{\Gamma(\theta)} \left( \int_{0}^{t-\tau} e^{-\rho z^\theta} z^{\theta-1} dz + \right. \]

\[ + \sum_{k=1}^{3} \frac{a_k e^{-\gamma k\tau} \int_{0}^{t-\tau} e^{\eta_{rk} z - \rho z^\theta} z^{\theta-1} dz - e^{\alpha_k \tau} \int_{0}^{t-\tau} e^{\eta_{rk} z - \rho z^\theta} z^{\theta-1} dz \right), \]

\[ \eta_{1k} = 0, \quad \eta_{2k} = \gamma_k - \alpha_k, \]

where \( V_r(t, \tau) \) – creep kernel integral, \( \Gamma(\theta) \) – gamma-function, \( K_r(t, s) \) – aging material creep kernels \((r = l, 2), t - \tau \) – difference argument, \( t \) – an instant of time under consideration, \( \tau \) – an intermediate instant of time, \( z \) – intermediate argument.
3. Results
For numerical implementation of (4) the exponential functions in integration elements (7) are expanded in the Maclaurin power series. When keeping the first three terms of the series, the following result is obtained:

\[
V_r(t, \tau) = \left[C_0 + Ae^{-\beta \tau}\right]\left[1 - e^{-\gamma(t-\tau)}\right] + \frac{\chi}{\Gamma(\vartheta)}(t - \tau)^{\vartheta} \left[B \frac{1}{\vartheta} - \frac{\rho(t - \tau)^{\alpha+1}}{\vartheta + \alpha + 1} + \frac{(t - \tau)^2}{2 + \vartheta} \sum_{k=1}^{3} \frac{a_k}{\alpha_k} e^{-\gamma_k t}\left[e^{\alpha_k t}N_{2k} - e^{\alpha_k t} \eta_{2k} \right]\right],
\]

(8)

where

\[B = 1 + \sum_{k=1}^{3} \frac{a_k}{\alpha_k} e^{-\gamma_k t}\left(e^{\alpha_k t} - e^{\alpha_k \tau}\right); \quad D_r = \sum_{k=1}^{3} \frac{a_k}{\alpha_k} e^{-\gamma_k t}\left(e^{\alpha_k t}N_{rk} - e^{\alpha_k t} \eta_{rk}\right).\]

4. Discussion
The error of such a transition is determined by the accuracy of the integral count. The numerical implementation of the task showed that with the increase in the number of terms of the Maclaurin power series, the results of the calculations do not go beyond the change of the result by 5%.

The possibility of using the obtained relations to describe the creep of real materials was tested on experimental data of A. A. Ross [2]. Creep curves obtained from the experimental data and their approximation are shown in figure 1.
The creep measure was calculated by formula:

\[
C(t, \tau) = \frac{V_r(t, \tau)}{E(t)} - \frac{1}{E(\tau)} + \frac{1}{E(t)},
\]

(9)

where \(C(t, \tau)\) – creep measure, \(V_r(t, \tau)\) – aging material creep kernel integral, \(E(t)\) – modulus of elasticity, \(E(\tau)\) – stiffness factor of concrete at different age \(\tau\).

On the time interval (Figure 1) experimental curves \(C(t, \tau)\) are shown. The transition from creep measure \(C(t, \tau)\) to integral \(V_r(t, \tau)\) was made using method [10]. The calculations have shown that the stress-strain state of composite structures depends significantly on the age of material \(\tau\) at the instant of loading. There are both quantitative and qualitative changes in the distribution of stresses, forces and moments.

5. Conclusions
Comparison of the results obtained during the implementation of the proposed mathematical model and experimental data showed a discrepancy of no more than 10 %, which confirms the validity of the studies. The obtained results are theoretical justification of solutions of composite plates and shells nonlinear strain problems with account for creep, which allows obtaining predictive estimates of structures behavior under real operating conditions.

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