Chiral waves in metamaterial medium

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Abstract: In this paper we will present a brief overview of electromagnetic properties of the media with negative permittivity and permeability, and then we will discuss some ideas about possible applications of these materials. In this work we discuss the chiral waves in metamaterial media. Here we study the conditions for strong chirality which can give situations of chiral metamaterial and we propose to investigate the conditions to obtain a metamaterial having simultaneously negative $\varepsilon$ and negative $\mu$ and very low eddy current loss.

Keywords: Electromagnetic theory, Refraction, optical pulse propagation, chirality, metamaterial.

1. Introduction:

Composite materials in which both permittivity and permeability possess negative values at some frequencies have recently gained considerable attention [1-5]. This idea was originally initiated by Veselago in 1967, who theoretically studied plane wave propagation in a material whose permittivity and permeability were assumed to be simultaneously negative [4]. Recently, Shelby [3], Smith-Schultz [5] constructed such a composite medium for the microwave regime, and experimentally showed the presence of anomalous refraction in this medium [3]. Previous theoretical study of electromagnetic wave interaction with omega
media using the circuit-model approach had also revealed the possibility of having negative permittivity and permeability in omega media for certain range of frequencies [2].

The anomalous refraction at the boundary of such a medium with a conventional medium, and the fact that for a time-harmonic monochromatic plane wave the direction of the Poynting vector is antiparallel with the direction of phase velocity, can lead to exciting features that can be advantageous in design of novel devices and components. For instance, as a potential application of this material, the idea of compact cavity resonators in which a combination of a slab of conventional material and a slab of metamaterial with negative permittivity and permeability. The problems of radiation, scattering, and guidance of electromagnetic waves in metamaterials with negative permittivity and permeability, and in media in which the combined paired layers of such media together with the conventional media are present, can possess very interesting features leading to various ideas for future potential applications such as phase conjugators, unconventional guided-wave structures, compact thin cavities, thin absorbing layers, high-impedance surfaces, to name a few. In this talk, we will first present a brief overview of electromagnetic properties of the media with negative permittivity and permeability, and then we will discuss some ideas about possible applications of these materials. In this work we discuss the chiral waves in metamaterial media.

Chirality is first referred to a kind of asymmetry in geometry and group theory in mathematics. The asymmetry exists broadly in organic molecules, crystal lattices, and liquid crystals, leading to two stereoisomers, dextrorotatory and levorotatory, as a hot research domain in stereochemistry. If the two stereoisomers coexist in one molecule (mesomer), or equally different steroisomers get mixed (raceme), there will be no special characters other than the common magneto-dielectric. When we get one pure steroisomer, however, interesting phenomena occur with an incident linearly-polarized wave, which can be seen as a superposition of two dual circularly-polarized waves. In case of perpendicular incidence, the two different circularly polarized waves have different phase velocities and their polarized planes rotate oppositely. As a result, the output polarization direction gets rotated, also known as optical activity or natural optical rotation phenomenon. Moreover, in elementary particle physics, chirality and asymmetry also play important roles, but they are out of the range of this paper. Here we study the conditions for strong chirality which can give situations of chiral metamaterial, so we relate the behaviour of two representations to describe a chiral medium: the Born- Fedorov and the Paster -Tellegen relation between electromagnetic fields.

2. Chiral Waves:
In classical electrodynamics, the response (typically frequency dependent) of a material to electric and magnetic fields is characterized by two fundamental quantities, the permittivity $\varepsilon$ and the permeability $\mu$. The permittivity relates the electric displacement field $\vec{D}$ to the electric field $\vec{E}$ through $\vec{D} = \varepsilon \vec{E}$, and the permeability $\mu$ relates the magnetic field $\vec{B}$ and $\vec{H}$ by $\vec{B} = \mu \vec{H}$. If we do not take losses into account and treat $\varepsilon$ and $\mu$ as real numbers, according to Maxwell’s equations, electromagnetic waves can propagate through a material only if the index of refraction $n$, is real. Dissipation will add imaginary components to $\varepsilon$ and $\mu$ cause losses, but for a qualitative picture, one can ignore losses and treat $\varepsilon$ and $\mu$ as real numbers. Also, strictly speaking, $\varepsilon$ and $\mu$ are second-rank tensors, but they reduce to scalars for isotropic materials. In a medium with $\varepsilon$ and $\mu$ both positive, the index of refraction is real and electromagnetic waves can propagate. All our everyday transparent materials are such kind of media. In a medium where one of the $\varepsilon$ and $\mu$ is negative but the other is positive, the index of refraction is imaginary and electromagnetic waves cannot propagate. Metals and Earth’s ionosphere are such kind of media. In fact, the electromagnetic response of metals in the visible and near-ultraviolet regions is dominated by the negative epsilon concept [1-3].

Although all our everyday transparent materials have both positive $\varepsilon$ and positive $\mu$, from the theoretical point of view, in a medium with $\varepsilon$ and $\mu$ both negative, electromagnetic waves can also propagate through. Moreover, if such media exist, the propagation of waves through them should give rise to several peculiar properties. This was first pointed out by Veselago over 30 years ago when no material with simultaneously negative $\varepsilon$ and $\mu$ was known [4]. For example, the cross product of $\vec{E}$ and $\vec{H}$ for a plane wave in regular media gives the direction of both propagation and energy flow, and the electric field $\vec{E}$, the magnetic field $\vec{H}$, and the wave vector $\vec{k}$ form a right-handed triplet of vectors. In contrast, in a medium with $\varepsilon$ and $\mu$ both negative, $\vec{E} \times \vec{H}$ for a plane wave still gives the direction of energy flow, but the wave itself that is, the phase velocity propagates in the opposite direction, i.e., wave vector $\vec{k}$ lies in the opposite direction of $\vec{E} \times \vec{H}$ for propagating waves. In this case, electric field $\vec{E}$, magnetic field $\vec{H}$, and wave vector $\vec{k}$ form a left-handed triplet of vectors.

Such a medium is therefore termed left-handed medium [5]. In addition to this ‘‘left-handed’’ characteristic, there are a number of other dramatically different propagation characteristics stemming from a simultaneous change of the signs of $\varepsilon$ and $\mu$, including reversal of both the Doppler shift and the Cerenkov radiation, anomalous refraction, and even reversal of radiation pressure to radiation tension. However, although these counterintuitive properties follow directly from Maxwell’s equations, which still hold in these unusual materials. Such type of left-handed materials have never been found in nature but such media can be prepared artificially, they will offer exciting opportunities to explore new physics and potential applications in the area of radiation-material interactions. Following the suggestion of Pendry et al [1], Smith and co-workers reported [5] that a medium made up of an array of conducting...
nonmagnetic split ring resonators and continuous thin wires can have both an effective negative permittivity $\varepsilon$ and negative permeability $\mu$ for electromagnetic waves propagating in some special direction and special polarization at microwave frequencies [5]. This is the first experimental realization of an artificial preparation of a left-handed material, where on the one hand, the permittivity of metallic particles is automatically negative at frequencies less than the plasma frequency, and on the other hand, the effective permeability of ferromagnetic materials for electromagnetic waves propagating in some particular direction and polarization can be negative at frequency in the vicinity of the ferromagnetic resonance frequency, which is usually in the frequency region of microwaves. However, this configuration exhibits chirality and a rotation of the polarization so the analysis of metamaterial presented by several authors provides a good but not exact characterization of the metamaterial [6]. The evidence of chirality behavior suggests that if it is included in the conditions to obtain a metamaterial behavior of a medium further progress will be obtained.

In this short paper, we propose to investigate the conditions to obtain a metamaterials having simultaneously negative $\varepsilon$ and negative $\mu$ and very low eddy current loss. As a initial point, we considerer a media where the electric polarization $\vec{P}$ depends not only on the electric field $\vec{E}$, and the magnetization $\vec{M}$ depends not only on the magnetic field $\vec{H}$, and we may have, for example, constitutive relations given by the Born-Federov formalism [7]

\begin{align}
\vec{D}(\vec{r}, \omega) &= \varepsilon(\omega)(\vec{E}(\vec{r}, \omega) + T(\omega)\nabla \times \vec{E}(\vec{r}, \omega)) \\
\vec{B}(\vec{r}, \omega) &= \mu(\omega)(\vec{H}(\vec{r}, \omega) + T(\omega)\nabla \times \vec{H}(\vec{r}, \omega))
\end{align}

The pseudoscalar $T$ represents the chirality of the material and it has length units [7]. In the limit $T \to 0$, the constitutive relations (1) and (2) for a standard linear isotropic lossless dielectric with permittivity $\varepsilon$ and permeability $\mu$ are recovered.

According to Maxwell’s equations, electromagnetic waves propagating in a homogeneous dielectric magnetic material are either positive or negative transverse circularly polarized waves, and can be expressed as

\begin{align}
\vec{E}^\pm(\vec{r}, t) &= \vec{E}_0^n(\vec{x} \pm \vec{y}) e^{-i(k_z z - \omega t)} \\
\vec{H}^\pm(\vec{r}, t) &= \vec{H}_0^n(\vec{x} \pm \vec{y}) e^{-i(k_z z - \omega t)}
\end{align}

where $E_0^n = E_0(\hat{x} \pm \hat{y})$, and $\nabla \times \vec{E}^\pm(\vec{r}, t) = \mp k_z \vec{E}^\pm$, $k_z \geq 0$ is the chiral wave number.

In this case of right polarized wave we can see that the effective permittivity $\varepsilon_{\text{eff}}$ and the effective permeability $\mu_{\text{eff}}$ are obtained from

\[
\int \vec{D}(\vec{r}, \omega) e^{-i(k_z z - \omega t)} d\vec{r} = \varepsilon \int (\vec{E}(\vec{r}, \omega) + T\nabla \times \vec{E}) e^{-i(k_z z - \omega t)} d\vec{r} = \varepsilon \int (1 - k_z T) \vec{E} e^{-i(k_z z - \omega t)} d\vec{r}
\]

\[
\int \vec{B}(\vec{r}, \omega) e^{-i(k_z z - \omega t)} d\vec{r} = \mu \int (\vec{H}(\vec{r}, \omega) + T\nabla \times \vec{H}) e^{-i(k_z z - \omega t)} d\vec{r} = \mu \int (1 + k_z T) \vec{H} e^{-i(k_z z - \omega t)} d\vec{r}
\]
with $\varepsilon_{\text{eff}} = \varepsilon(1-k,T)$ and $k,T \geq 1$.

Similarly, we have

$$
\int \vec{B}(\vec{r},\omega) e^{-i\beta z} d\vec{r} = \mu_{\text{eff}} \int \vec{H}(\vec{r},\omega) e^{-i\beta z} d\vec{r} = \mu(1-k,T) \int \vec{H}(\vec{r},\omega) e^{-i\beta z} d\vec{r}
$$

with $\mu_{\text{eff}} = \mu(1-k,T)$ and where $k_{\text{eff}}$ and $\omega$ are related by $k_{\text{eff}}^2 = \omega^2 (\varepsilon_{\text{eff}} \mu_{\text{eff}})$.

Equations (5) and (6) are exact, in principle, assuming that nonlocal effects can be neglected. This assumption is appropriate in many cases. But in some cases, nonlocal effects can be significant and cannot be neglected, as has been shown in the past. In such cases, Eqs. (5) and (6) shall be not exact. For simplicity, in this paper we have assumed that nonlocal effects can be neglected and hence Eqs. (5) and (6) shall be valid. In Eqs. (3) and (4) the sign of the effective wave number can be positive or negative depending on the product $k,T$ and the energy flow. For convenience we assume that the direction of energy flow is in the positive direction of the $z$ axis, but the sign of $k_{\text{eff}}$ still can be positive or negative. In the case of right polarization, if $1 \geq k,T \geq 0$, the phase velocity and energy flow are in the same directions, and from Maxwell’s equation, one can see that the electric $\vec{E}$ and magnetic field and $\vec{H}$ and the wave vector $\vec{k}_{\text{eff}}$ shall form a right-handed triplet of vectors. This is the usual case for right-handed materials. In contrast, if $k,T \geq 1$ the phase velocity and energy flow are in opposite directions, and $\vec{E}$, $\vec{H}$, and $\vec{k}_{\text{eff}}$ shall form a left-handed triplet of vectors. This is just the peculiar case for left handed materials where the effective permittivity $\varepsilon_{\text{eff}}$ and the effective permeability $\mu_{\text{eff}}$ are simultaneously negative. So, for incident waves of a given frequency $\nu$, we can determine whether wave propagation in the composite is right handed or left handed through the relative sign changes of $k_{\text{eff}}$.

There are also some other representations to describe a chiral medium, among which the most common one is deduced by Pasteur and Tellegen [8] as

$$
\vec{D} = \varepsilon_{\nu} \vec{E} + (\chi + i\kappa)\vec{H}, \quad (7)
$$

$$
\vec{B} = \mu_{\nu} \vec{H} + (\chi - i\kappa)\vec{E}, \quad (8)
$$

in which electromagnetic coupling terms are added to the basic terms. Bi-isotropy or bianisotropy is used for calling such constitutive equations, according to the parameters to be scalars or tensors. If $\kappa = 0$ and $\chi \neq 0$, it is the Tellegen medium; if $\kappa \neq 0$ and $\chi = 0$, as the requirement of reciprocity, it is the Pasteur medium:

$$
\vec{D} = \varepsilon_{\nu} \vec{E} + i\kappa \vec{H}, \quad (9)
$$

$$
\vec{B} = \mu_{\nu} \vec{H} + i\kappa \vec{E}, \quad (10)
$$
We pay more attention to such a chiral medium. Positive and negative $\kappa$ values differentiate two conjugated stereoisomer structures. We assume $\kappa > 0$ in the following analysis. Actually, the constitutive relations above are essentially equivalent, with corresponding parameters to be [7,8]

\[
\varepsilon = \varepsilon_p (1 - \frac{\kappa^2}{\mu_p \varepsilon_p}) \quad (11)
\]
\[
\mu = \mu_p (1 - \frac{\kappa^2}{\mu_p \varepsilon_p}) \quad (12)
\]
\[
T = \frac{\kappa}{\mu_p \varepsilon_p} \quad (13)
\]

It is clear that the parameters are different in such two representations. Then a question may rise up: which are the “true” material permittivity and permeability? The answer is, both. The concepts of permittivity and permeability are effective coefficients derived from a mathematical model. We actually have different mathematical models describing the same physical material. Thus there are different effective parameters describing the proportion of $\mathbf{D}$ to $\mathbf{E}$, and $\mathbf{B}$ to $\mathbf{H}$.

The rotation terms in BF model include both real and imaginary parts, resulting in a change in the real part and creating the imaginary chiral terms in the Pasteur model, vice versa. In other words, the difference in representations of coupling terms lead to different permittivity and permeability formulations.

It should be noticed that Faraday gyratory medium can also lead to optical rotation within the plasma or ferrite under an additional DC magnetic field [8]. Hence it is not natural, and is usually referred as “gyratory”, “Faraday optical rotation”, “magneto-optical effect”, etc. However, sometimes people do not differentiate “chiral” and “gyratory”. We need pay attention that such two types of optical rotation have different essence and different characters. Only natural optical activity is discussed here.

3. **Energy and spatial dispersion in strong chiral medium with $\mathbf{E} \perp \mathbf{H}$:**

There is a long dispute on strong chiral medium since it was introduced theoretically. Traditional electromagnetic conclusions have limited us to understand strong chirality, i.e. $k_s T \geq 1$ or $\mu_p \varepsilon_p < \kappa^2$, until we see the fact that artificial Veselago’s medium [9] was successfully realized in certain frequency bands [3]. For the normal case, $\mathbf{E} \perp \mathbf{H}$, we have to ask the following question: can strong chiral medium exist? In Ref. [3], the reason for traditional restriction of chirality parameters was concluded as: i) The wave vector of one
eigenwave will be negative; 2) The requirement of a positive definite matrix to keep positive energy:

\[
\begin{pmatrix}
\epsilon_r & i\kappa \\
-i\kappa & \mu_r
\end{pmatrix}
\]  

(14)

With the exploration of backward-wave medium, we know that negative wave vector, or opposite phase and group velocities, are actually realizable. And there is an unfortunate mathematical error in the second reason: in linear algebra, only if it is real and symmetric, positive definite matrix is equivalent to that all eigenvalues should be positive. The matrix (14) is a complex one, making the analysis on restriction of positive energy meaningless. Actually, in a strong bi-isotropic medium with constitutive relations as Eqs. (1) and (2), the energy can be drawn as

\[
w = w_u + w_a = \frac{\mathcal{D} \cdot \mathcal{E}}{2} + \frac{\mathcal{B} \cdot \mathcal{H}}{2} = \epsilon \frac{|\mathcal{E}|^2}{2} + \mu \frac{|\mathcal{H}|^2}{2}
\]  

(15)

Mathematically, the amount of energy density propagated is proportional to the magnitude of the Poynting vector $\mathbf{S}$, where $\mathbf{S} = \mathbf{E} \times \mathbf{H}$. With the condition $\mathbf{E} \perp \mathbf{H}$, $S_{r,t} \neq 0$ so we find right circularly polarized wave or left circularly polarized wave.

In terms of Eqs. (11) and (12), if $\mu_r \epsilon_r < \kappa^2$ for whole frequency range, the energy will still keep positive as long as the permittivity and permeability are positive, under the weak spatial dispersion condition. This is quite different from the Veselago’s medium since there is no bandwidth limitation and the frequency dispersive resonance are no longer required. In another word, the strong chiral medium does not contradict the energy conservation, at least in the weak spatial dispersion model.

Therefore, the real reason for traditional strong-chirality limitation is neither negative wave vector nor energy conversation. Next we will point out two other important reasons. First, with the assumption that $\epsilon_r > 0$, $\mu_r > 0$, $\kappa > 0$ and $\kappa > \sqrt{\mu_r \epsilon_r}$, we easily show that $\epsilon$, $\mu$ and $T$ turn to negative from the transformation between Pasteur constitutive relations and BF relations shown in Eqs. (11)-(13). This is absolutely unacceptable before people realizing Veselago’s medium. Actually, strong chiral medium can be equivalent to Veselago’s medium for the right circularly polarized wave [3, 11]. The negative $\epsilon$ and $\mu$ have shown such a point. Hence the negative sign in the BF model is not strange at all, since we realize effective double-negative with strong chirality parameter instead of simultaneously frequency resonances. For a limiting case, the chiral nihility [10], in which $\mu_r \to 0$ and $\epsilon_r \to 0$ while $\kappa \neq 0$, the parameters in DBF representation become $\epsilon \to \infty$, $\mu \to \infty$, and $T = -1/\omega \kappa$, remaining a finite value after a simple mathematical analysis. There is no evidence that strong chirality
cannot exist in this aspect. Second, it is the effectiveness of linear models. Similar to the case that linear optical and electromagnetic models can no longer deal with very strong optical intensity and electromagnetic field, we introduce nonlinear optics to take into account the higher order terms of polarization. If the spatial dispersion is strong enough, the higher order coupling terms cannot be neglected as before. People used to mistake strong chirality with strong spatial dispersion, hence adding a limitation to chirality parameter, \( \kappa < \sqrt{\mu_\varepsilon} \). We believe that this is the most probable reason. However, the strong spatial dispersion is embodied in the BF model, e.g. the value of \( T \), while the strong chirality is represented by the Pasteur model, e.g. the ratio of \( \kappa \) to \( \sqrt{\mu_\varepsilon} \). That is to say, strong chirality does not necessarily lead to strong spatial dispersion. Based on Eqs. (11)-(13), we have computed \( T \) and \( \varepsilon / \varepsilon_p \) or \( \mu / \mu_p \) versus \( \kappa / \sqrt{\mu_\varepsilon} \), as shown in Figs. 1 and 2. When \( \kappa \) is very close to \( \sqrt{\mu_\varepsilon} \), the value of \( T \) is quite large, indicating a strong spatial dispersion. Hence the singular point is the very point of traditional limitation. However, with \( \kappa \) continuously increasing, the spatial dispersion strength falls down very quickly. Therefore, if \( \kappa \) is not around \( \sqrt{\mu_\varepsilon} \), e.g. \( \kappa < 0.8 \sqrt{\mu_\varepsilon} \), or \( \kappa > 1.2 \sqrt{\mu_\varepsilon} \), we need not take nonlinear terms into consideration at all. Hence the strong spatial dispersion and nonlinearity cannot put the upper limitation to chirality parameters either. When \( \kappa \) is close to \( \sqrt{\mu_\varepsilon} \), where the spatial dispersion is strong, we need to take higher-order terms in the BF relations

\[
\begin{align*}
\tilde{D} &= \varepsilon (\tilde{E} + T_1 \nabla \times \tilde{E} + T_2 \nabla \times \nabla \times \tilde{E} + ...), \\
\tilde{B} &= \mu (\tilde{H} + T_1 \nabla \times \tilde{H} + T_2 \nabla \times \nabla \times \tilde{H} + ...),
\end{align*}
\]

where \( T_n \) stands for the spatial dispersion of the \( n \)th order. We remark that the above is different
Fig. 1. The strength relationship of chirality and spatial dispersion. \( \omega / c \) versus \( \kappa / \sqrt{\mu \varepsilon} \). The point of \( \kappa / \sqrt{\mu \varepsilon} = 1 \) is singularity, corresponding infinite spatial dispersion coefficient \( T \). When \( \kappa / \sqrt{\mu \varepsilon} > 1 \), \( T \) becomes negative for keeping the positive rotation term coefficients with negative \( \mu \) and \( \varepsilon \).

Fig. 2. \( \varepsilon / \varepsilon_p, \mu / \mu_p \) versus \( \kappa / \sqrt{\mu \varepsilon} \). With chirality strength increases, \( \varepsilon \) and \( \mu \) reduces quickly from \( \varepsilon_p \) and \( \mu_p \) to \(-\infty\).

4. Energy and spatial dispersion in strong chiral medium with \( \vec{E} \perp \vec{H} \):

The concept of parallel fields is important in the theoretical formulation of: Space electromagnetism and vacuum [11], the classical and quantum gravitational fields [12,13], the study of elementary particles [14], Operator and Dirac matrices, fields and chiral electrodynamics [15-16]. If we put \( \vec{E} = a \vec{B} \) and solving the Maxwell equations with the Born-
Fedorov relations we find $\alpha = i/\sqrt{\mu \varepsilon /4}$, $\bar{E} = i\eta \bar{H} = i\sqrt{\mu /2 \varepsilon /2} \bar{H}$. Equations (11,12,13) are transformed as:

$$\frac{\varepsilon}{2} = \varepsilon_{\rho} \left( 1 - \frac{\kappa^{2}}{\mu_{\rho} \varepsilon_{\rho}} \right)$$

(18)

$$\frac{\mu}{2} = \mu_{\rho} \left( 1 - \frac{\kappa^{2}}{\mu_{\rho} \varepsilon_{\rho}} \right)$$

(19)

$$T = \frac{\kappa}{\alpha(\mu_{\rho} \varepsilon_{\rho} - \kappa^{2})}$$

(20)

In this case the total density energy is

$$w = w_{r} + w_{m} = 0$$

(21)

In this special case where the energy propagated in one direction is equal to that propagated in the opposite direction, there is no net energy flow in the medium and the sum of the two TEM waves form what is generally known as a standing wave. The condition for a standing wave is that the time average of $\bar{S}$ vanishes. This can be achieved if $\bar{S}$ is zero all the time everywhere in the region of space under consideration, i.e., $\bar{S}(r,t) = 0$ (see Fig. 2). Examination of Eq. (21) shows that $\bar{S} = 0$ if $\bar{E} \perp \bar{H}$. In this last case, a particular solution of Eqs. (1) and (2) is when $k_{0}^{2}T^{2} = 1$, where we have the condition $\bar{E} \perp \bar{H}$, and $\bar{E} = i\eta \bar{H}$, so we find the Beltrami force free equation $\bar{E} + 2T V \times \bar{E} = 0$ and the Poynting vector $\bar{S}(r,t)$ vanishes [15,16].

5. Conclusions

From Fig. 1, it is clear that enhancing spatial dispersion will not lead to strong chirality and will reach the traditional limitation point. This is why we have never succeeded in realizing strong chirality no matter how to improve the asymmetry and spatial dispersion. Fortunately, as pointed out earlier, the strong chirality does not require strong spatial dispersion. Hence the most important difference between strong and weak chirality is that $T$ and $\kappa$ have opposite signs, which necessarily leads to negative $\varepsilon$ and $\mu$. Here, $\kappa$ stands for chirality and $T$, is the chiral coefficient of the first order for spatial dispersion. Strong chirality roots from using one type of spatial dispersion to get the conjugate stereoisomer, or chirality. It is an essential condition for supporting the backward eigenwave in strong chiral medium.

In conclusion, a strong chiral medium behaves like Veselago’s medium. Under the weak spatial dispersion, the energy is always positive for chiral medium. We show that strong chirality does not equal strong spatial dispersion, which occurs only around a singular point. Even in this small region with very strong spatial dispersion, the Pasteur model is
meaningful. Neither spatial dispersion nor energy will hinder chirality to be stronger, but we cannot realize strong chirality only by increasing the spatial dispersion. The necessary condition of strong chiral medium is that the chirality and spatial dispersion are of conjugated types. We remark that strong chiral media can find wide applications in the negative refraction and supporting of backward waves.

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