Algorithm for operating an ordinary engineering system as a quantum bit

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\section*{ABSTRACT}
We show an algorithm for the previously proposed quantum computer using classical apparatus. The target system is not limited to mass points but is a general engineering system that has a more manageable form in equipping terminal edge. After we define such system as “ordinary engineering system,” the method of quantum mechanical optimum feedback control of the nonlinear system is applied. The method makes an ordinary engineering system into a quantum one. As an example that is as stail as possible, a temperature control system is taken. Simulation studies show that our system works as a 1-qubit quantum computer without the special large-scale peripheral equipment required to maintain quantumness. The proposed system is a classic system that is everywhere, so it is easy for anyone to understand. Another advantage is that easy-to-use one-dimensional system can be applied.

\section*{1. Introduction}
Is “edge quantum computer” possible? In other words, is it possible to install a quantum computer, which is expected to be a very high-speed computer, in the terminal of a machine? Such terminal computers will have the potential to dramatically improve the capabilities of machines: robots with the higher cognitive ability \cite{1,2}, vehicles that immediately avoid collisions, etc. Here, we first outline each of the edge \cite{3–5} and the quantum computer \cite{6–9}. Next, we will look at the difficulty of combining these two and what ideas the authors have tried \cite{10} to overcome. After that, we give an overview of the contents of the paper on how to lower the development hurdles for edge quantum computers.

“Edge computing is a distributed computing paradigm that brings computation and data storage closer to the sources of data. This is expected to improve response times and save bandwidth ” \cite{11}. The problems of cloud networks are as follows. For example, car collision avoidance has too much delay over the cloud. For analysis of large amounts of video data on the cloud, it is necessary to reduce the amount of communication data in order to prevent pressure on the network bandwidth. The computational speed will eventually reach a plateau. Quantum computer that fully uses linear parallelism becomes one powerful possibility of a breakthrough in high-speed computing. Under these circumstances, the idea of mounting quantum computers on the edge naturally emerges. Quantum mechanical linearity is vulnerable to noise. Peripheral equipment that maintains the superposition is indispensable for the computation system. For example, the superconducting type requires a cooling device. The equipment has to be large-scale. In another model using light, since the light goes straight, the circuit is inevitably large-scale apart from the question of whether to keep the overlay. It would be just a dream that is difficult to realize to install such large-scale quantum computer to the edge.

In this awareness of the problem, we set the main challenge in this paper to lowering the development hurdles in quantum computation using classical mass particles \cite{10}. Quantum computation consists of setting the calculation input as the initial wave function, time evolution of the wave function according to Schrödinger equation, and the output determination by measuring the wave function at an appropriate time. In our scheme, there is a time evolution of the particle coordinate due to the feedback determined by the wave function. Quantum calculation was thus positioned as the extraction of wave function parameters. Monitoring the mass particle trajectory gives the extraction. Even classical mass particles move quantum mechanically if they are driven by ingenious feedback. This system only needs a particle moving as a quantum bit, qubit, and a control loop for each qubit. Our proposed system will be a configuration that excludes large-scale peripheral equipment from the current quantum computer system. To control by velocity as input, a mobile robot is assumed as a mass particle to be engineered. If we use a two-dimensional plane, an omni-directional mobile

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robot is applied. However, to realize the edge computing system, it is necessary to manufacture as small robots and drive motors as possible. This is a very difficult task. We will consider what to do in order to lower the development hurdles such as making very small motors. As shown in Subsection 2.2, an appropriate feedback input makes control system, not limited to the classical mass point, quantum one. If so, we can find a system that is as simple as possible and mount it on the edge as a quantum computer. By the way, we solved the Deutsch problem by looking at the trajectories generated by the wave functions $\phi_\pm$ in Figure 4 in [10]. Such “visual” must be replaced by a computational algorithm. That is, there were two problems in our previous study: we had to make a very small motor and we had a visual monitoring of the trajectories not based on algorithms.

Let’s reconsider at this time. What does it mean to use quantum law for calculations in the first place? In this paper, the discussion will proceed on the premise that quantum computers have higher performance than classical computers. So what is amazing about quantum computer? The basic principle of quantum computers is quantum mechanics. In quantum mechanics, physical state, such as the motion of mass particles, is regarded as a linear wave. The particle-wave duality elucidated the physical phenomena of the microscopic world. Linear vectors can be overlaid. Use of two orthogonal unit vectors $\mathbf{U}$ and $\mathbf{D}$ allows us to take a basic element of quantum computers for one qubit system represented by $\psi = \alpha \mathbf{U} + \beta \mathbf{D}$ with complex coefficients $\alpha$ and $\beta$ satisfying $|\alpha|^2 + |\beta|^2 = 1$. To perform calculations, quantum computers use superpositions of various states. Many calculations are executed at the same time. This simultaneity speeds up the calculation. Of course, it is necessary to extract desired results from the superposition by making full use of a clever algorithm [12]. Even the cooling device for quantum computer systems may become smaller if efforts are made. But before we push forward with technological innovation in that direction, we have a thing to check first. What does it mean to “use” the “quantum superposition” that we want to keep even through such technological effort? What do we extract from quantum computing systems? Quantum computing itself does not use enough energy for large-scale cooling. What needs to be retrieved is just the “signal” in a form of superposition. So what is a quantum signal? It is information on complex weights of two states of qubit. Quantum computing can be done, macro or micro, as long as the state weights are known. It would be desirable to create a system that can understand the qubit weighting factor using, for example, a familiar temperature control system. This paper proposes quantum computer system using such ordinary engineering systems. In Subsection 2.1 we define what “ordinary engineering systems” means. Our proposed system consists of an ordinary engineering system and a loop to control it. Feedback law is, as shown in detail in Subsection 3.1, an equal time input–output relation written in elementary functions in state variable $x$ and time $t$. No special peripheral equipment is required to keep the quantum superposition. On the other hand, the current quantum computer has a quantum system and a loop for controlling it. These two are the same in the current and proposal systems. But in the current systems, in addition, large-scale peripheral equipment is required to maintain quantum superposition.

Table 1 compares quantum computers using ordinary engineering systems with the current ones, as we propose here. In other words, our proposed system is, after all, a form in which large-scale peripheral equipment is deleted from the current system. Anyone can easily understand the proposed system because it is a classic system that is seen everywhere. Another advantage is that engineers can use a one-dimensional system.

By the way, there are also calculation methods [13–16], including quantum neural network [17–19], in which only the algorithm is quantum. But software that produces definitive results, be it a simulator or an algorithm, cannot express “unpredictability.” We consider that unpredictability is related to emotions and consciousness [20]. For that purpose, it is necessary to use hardware in which disturbance is inevitable. Therefore, we propose in this paper the method that is easier to manufacture. Certainly, there are also attempts to use hardware [21] for acceleration in simulators. However, even there, the essential unpredictability of quantum mechanics, probability interpretation, has been processed as an algorithm by the “projection postulate” [22].

The report presented in [23] uses the cost of harmonic oscillators for a brief and concise explanation. We checked “correspondence principle” or “classical limit” that makes $H_R$ zero using the simplicity of calculation. The basic component was general fluid control. But in this paper, more important thing is to estimate the probabilities of the $\mathbf{U}$ and $\mathbf{D}$ states while preserving the qubit properties (two levels of properties). So, we adopt square well potential and stopped looking at zero $H_R$ limit. We emphasize that the system used in this paper is commonplace. Here we specifically use water to control the temperature $T[K]$ with electric power $P[W]$. Also, specific water property values are applied
for the purpose of concretely showing what the actual system looks like. Wave function parameter determination by monitoring state trajectory is actually calculated in this paper.

In Section 2, after clarifying the meaning “ordinary engineering systems” in Subsection 2.1, we show in Subsection 2.2 methods how to treat ordinary engineering systems in quantum manner. Feedback law is given. The method of estimating the wave function parameters from the monitoring of the output of the engineering system is given as an algorithm in Subsection 3.1, after a simple explanation on quantum gates. According to the algorithm, we first give in Subsection 3.2 the results of Deutsch problem in Figure 4 [10]. After that, we simulate a system that quantumly feedback controls of water in an ideally insulated box near room temperature. It must be emphasized once again that this is just an example of using an ordinary engineering system. Section 4 gives conclusion and discussion for future research.

2. Engineering system under feedback

Why did we fall into the trap of making a very small mobile robot that resembles a mass point? That is because, when trying to express quantum mechanics classically, the simplest theory was based on the quantum mechanics of mass points. We wish we could make the classic correspondence of spins and lights used in quantum computers. However, spin and light are very complicated forms when (forcibly) treated in classical theory [24]. As long as we are faithful to the correspondence of spins and lights used in the quantum theory of the system that is seen on a daily basis? Feedback optimization [25] of nonlinear control systems can be applied to such attempts.

2.1. Ordinary engineering systems

Let state vector \( x \in \mathbb{R}^n \) move under input vector \( u \in \mathbb{R}^m \) according to the state equation

\[
\dot{x} = f(x, u).
\]

With positive definite \( R \) and \( V(x) \), input vector \( u \) is determined so as to meet the following minimization

\[
\delta \int dtL(x, u) = 0, \quad L(x, u) = uR u - V(x).
\]

We call a system represented by (1) and (2) an “ordinary engineering system.” We can add mass point that flies around in space due to classical mechanics to a typical one of ordinary engineering systems in the above sense, when we especially set \( f = u \) in (1) and \( R = \frac{m}{2} I \) (\( m \): mass, \( I \): identity matrix). On the other hand, the flow velocity and pressure of fluid dynamics according to the Navier–Stokes equation cannot be derived from the principle of minimization, so it cannot be said to be such a system. Whether it is a robot arm that follows strict theoretical mechanical mechanics or a temperature or concentration based on the phenomenological causal relationship of a process, we will treat it according to these set of equations. We would like to give two comments. The handling of the actual physical system is constrained by its spatial dimensions. In mass mechanics, the variable \( \hat{x} \) has three-dimensional freedom. Two-dimensional and one-dimensional can only be realized in special environments such as material surfaces and fine lines. However, we are free to choose the dimensions according to the mathematical model of the system. For example, one-dimensional for lumped constant approximation. How much approximation we should take is determined, ultimately, by the calculation accuracy of quantum computation. The most simple one to implement is 1 dimension. Therefore, another comment is explained with \( n = m = 1 \). As a one-dimensional version of (2) we take

\[
L(x, u) = \frac{m}{2} u^2 - V(x).
\]

The way of signing \( V \) in (2) is the opposite of the case of optimal control [26]. For example, take \( \dot{x} = u \). LQ theory tells us to take

\[
L(x; u) = \frac{m}{2} u^2 + V(x), \quad V(x) = \frac{m\omega^2}{2} x^2.
\]

to guarantee asymptotic convergence \( x \sim e^{-\omega t} \). However, for the same \( V = \frac{m\omega^2}{2} x^2 \) in the (3) formula, \( x \) will oscillate around its initial value as \( x - x(0) \sim e^{\pm \omega t} \). Since we are not aiming for zero control error \( x \rightarrow 0 \), the oscillation does not lead problems as long as it does not diverge.

2.2. Feedback to make system quantum

We continue to take \( n = m = 1 \) throughout all the following. To avoid unnecessary complexity, take affine form of (1) with a constant coefficient \( g \) of input

\[
\dot{x} = f(x, u) = gu + F(x).
\]

In the following, we show how deterministic state variable \( x \) quantum mechanically describes our ordinary engineering system including mass mechanics. The following three steps are enough to quantize the system. The first step is describing control specification as (3).

Next, we introduce the extended form of Lagrangian as

\[
L'(x, u; \lambda; \dot{x}) = L(x, u) + \lambda \cdot (f(x, u) - \dot{x}).
\]

that gives (5) in the variation of \( \lambda \). Finally, we quantize a system with such constraints by Dirac’s method.
Let us proceed. Calculating canonical momenta of \(x, u\) and \(\lambda\) as \(p = \frac{\partial L}{\partial \dot{u}} = -\lambda, p_u = \frac{\partial L}{\partial \dot{u}} = 0\) and \(p_\lambda = \frac{\partial L}{\partial \dot{\lambda}} = 0\) give three constraints, \(p = -\lambda, p_u = 0\) and \(p_\lambda = 0\). Subsequently, we calculate Hamiltonian as

\[
H = \dot{x}p + up_u + \dot{\lambda}p_\lambda - L.
\] (7)

Equations of motion are given by \(H\) and Dirac bracket [27, \{\}, \{DB\}], consistent with the constraints as

\[
\dot{\Theta} = \{\Theta, H\}_{DB} + \frac{\partial \Theta}{\partial t}.
\] (8)

We introduce a design constant \(H_R\) to set up quantization as

\[
iH_R(\Theta, \Sigma)_{DB} \longrightarrow [\Theta, \Sigma] \equiv \Theta \Sigma - \Sigma \Theta.
\] (9)

Proceed with specific calculations.

\[
H = -L - \lambda \cdot f = -\left(\frac{m}{2} u^2 - V\right) + p(gu + F).
\] (10)

We require optimality of control \(\frac{\partial H}{\partial u} = 0\) that gives \(u = \frac{\ddot{\phi}}{m}\). We calculate Hamiltonian as

\[
H = \frac{\ddot{\phi}^2}{2m} + V + mp^2 + pF = \frac{\ddot{\phi}^2}{2m} + V + pF.
\] (11)

We construct quantum mechanics over our engineering systems by requiring the following commutator (9),

\[
iH_R(x, p)_{DB} = iH_R \longrightarrow [x, p]
\] (12)

that leads to \(\dot{\phi} = -iH\). Symmetrizing the term \(pF\) in (11), \(\dot{\phi} = \frac{pF + pF^*}{2}\), gives us Schrödinger equation

\[
iH_R \frac{\partial \psi}{\partial t} = \hat{H}\psi = -\frac{\ddot{\phi}^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi + iH_R \left( \frac{\partial \psi}{\partial x} + \frac{1}{2} \frac{\partial F}{\partial x} \psi \right).
\] (13)

Start with a radial representation [28,29] of the complex valued wave function : \(\psi = \Re e^{i\frac{\phi}{m}}\). Schrödinger equation allows us to obtain a picture in which a “classical” system (5) moves under feedback with quantum fluctuation

\[
u = \frac{\ddot{\phi}}{m} \frac{\partial S}{\partial x} = \frac{\ddot{\phi}^2}{2m} \frac{\partial S}{\partial x} - \frac{\partial S}{\partial x} \frac{\partial \phi^*}{\partial x} \frac{\partial \phi}{\partial x} - \frac{\partial \phi^*}{\partial x} \frac{\partial \phi}{\partial x} |\psi|^2.
\] (14)

3. One qubit quantum computation

Any quantum computation can be done with a combination of unitary gate for 1-qubit and CNOT gate for 2-qubits [8]. In a 2-qubits system, feedback input \(u_1\) that drives \(x_1\) depends not only on \(x_1\) itself but also on another \(x_2\). The situation is the same for \(u_2\) driving \(x_2\). Discussing 2-qubits in detail is done in the near future. We focus here on 1-qubit given as a vector \(\psi = \alpha \vec{U} + \beta \vec{D}\), where \(\alpha\) and \(\beta\) are to be estimated.

3.1. Two-state quantum mechanical system

We take \(V\) in (3), which has two energy eigenvalues. The two energy levels correspond to \(\vec{U}\) and \(\vec{D}\) of the qubit. We assume, for simplicity, \(F(x) = 0\) in (5). In a temperature control system, this is equivalent to insulating the system. As a result, unnecessary complexity is avoided due to the reality of the eigenfunctions corresponding to \(\vec{U}\) and \(\vec{D}\).

Adding Rabi oscillation term to Schrödinger equation, (13) can make qubit change between the two states \(\vec{U}\) and \(\vec{D}\).

\[
iH_R \frac{\partial \psi}{\partial t} \equiv \hat{H}\psi = a \Re \cos(\omega t) \dot{\phi} \psi.
\] (15)

Unitary gate for 1-qubit is a combined physical process of free time development \((a_C = 0)\) and oscillation \((a_C \neq 0)\) among \(\vec{U}\) and \(\vec{D}\). We handle here a single process with \(a_C\) constant in time. Large \(a_C\) brings a high clock of computation. But the strong perturbation makes \(\psi\) extend out of the vector space, \(\psi \neq \alpha \vec{U} + \beta \vec{D}\). The allowable deviation from the vector space \((\vec{U}, \vec{D})\) is determined by the specification of calculation accuracy. The simulation in Subsection 3.2 is a calculation under fairly loose requirements for explanation. Now, let’s proceed. If we take the frequency \(\omega_R\) of vibration be the energy difference \(E_D - E_U\), the system resonates and a transition between states occurs. If the excitation is weak enough, the system will not leave the vector space spanned by \(\vec{U}\) and \(\vec{D}\). Take \(\phi_U(x)\) and \(\phi_D(x)\) as eigenfunctions of Hamiltonian \(\hat{H}\) with eigenvalues \(E_U\) and \(E_D\), respectively. We assume, even during excitation, that the wave function can be expanded as

\[
\psi(x; t) = U(t)\phi_U(x) + D(t)\phi_D(x).
\] (16)

At this time, the feedback law under Rabi vibration can be calculated clearly using the elementary function according to the following steps. First, define a parameter \(\Omega = \frac{m}{\hbar}(E_D - E_U)\alpha_C \int dxU(x)xD(x)\) proportional to \(a_C\). Under the weak vibration condition so that no higher energy excitation excite, \(\dot{\vec{U}} \equiv e^{\frac{E_D}{\hbar}t}U\) and \(\dot{\vec{D}} \equiv e^{\frac{E_D}{\hbar}t}D\) satisfy the following ordinary differential equations [30].

\[
\dot{\vec{U}} = -\Omega \cos(\omega_R t) e^{\frac{E_D x_U}{\hbar}} \vec{D},
\]

\[
\dot{\vec{D}} = \Omega \cos(\omega_R t) e^{\frac{E_D x_D}{\hbar}} \vec{U}.
\] (17)

Furthermore, take the resonance condition

\[
\omega_R = \frac{E_D - E_U}{H_R}.
\] (18)
According to this resonance condition, $\tilde{U}$ and $\tilde{D}$ in (17) are satisfied approximately by the trigonometric functions of time $t$

$$\tilde{U}(t) = -c_D \sin \frac{\Omega}{2} t + c_U \cos \frac{\Omega}{2} t$$

$$\tilde{D}(t) = c_D \cos \frac{\Omega}{2} t + c_U \sin \frac{\Omega}{2} t$$

That is, we approximate the wave function (16) with

$$\psi(x; t) = e^{i \tilde{U}(t) / 2} \tilde{U}(t) \phi_U(x) + e^{i \tilde{D}(t) / 2} \tilde{D}(t) \phi_D(x).$$

The approximation (21) holds under three conditions: $\phi_U$ and $\phi_D$ are eigenfunctions of Hamiltonian $\tilde{H}$ with eigenvalues $E_U$ and $E_D$, respectively, oscillation is sufficiently small ($a_C \to 0$), and resonance condition (18) is satisfied in approximating $\tilde{U}$ and $\tilde{D}$ with $\tilde{U}$ and $\tilde{D}$ respectively. If any of the three does not hold, the Schrödinger equation does not hold, and therefore the superposition principle for quantum computation does not hold.

When the function values are written in the feedback controller, the quantum mechanical motion of the system state, for example temperature, is driven. The simulation in Subsection 3.2 shows how small $a_C$ should be to apply such an approximation. We dare to make a rough calculation, a little strong perturbation, to clarify the calculation method.

The feedback rule is given by (14). For sufficiently small $a_C$, the rule is given by a combination of given functions $\phi_U$, $\phi_D$, $\phi_U^*$, and $\phi_D^*$ of $x$ and trigonometric functions of $t$. The feedback function is thus given as an equal time relation of input and output without solving (time integration of) Schrödinger equation. The wavefunction is expressed by the initial condition parameters $c_U$ and $c_D$. Since this parameter is determined by 3 of actual parameters, it can be calculated back by collecting 3 points of feedback input. A simple calculation shows that normalization condition $|c_U|^2 + |c_D|^2 = 1$ leads to the feedback input form

$$u = u(x; t; |c_U|^2, (c_U)^*c_D)$$

in 3 real unknowns, $|c_U|^2$, $\Re(c_U)^*c_D$ and $\Im(c_U)^*c_D$. The three unknowns can be calculated by substituting the input $u$ determined in (14) at three different time points. Both the denominator and the numerator of the feedback law (14) are linear functions of the above three unknowns due to linear dependence of $c_U$ and $c_D$ in $\psi$. So, we emphasize here that unknowns are calculated from linear simultaneous equations. If you want to know the state at time $T$, 3 time points $t_1$, $t_2$ and $t_3$ just before $T$ as shown in Figure 1 are chosen. Since we feedback control the system with the input $u$ of (14), the calculation of these parameters $c_U(T)$ and $c_D(T)$ is accurate. That is, the error of the parameter calculation is derived only from the error of the state equation.

The error of the state equation comes from the situation that, for example, in the case of a temperature control system, lumped constant system approximation is bad, insulated piles cannot ignore heat exchange, etc.

### 3.2. Simulation on one qubit operation

To confirm that our algorithm works well, two examples are given, one without perturbation and the other under perturbation. Parameter estimation in the first one is done by setting $a_C = 0$ in formulas presented in Subsection 3.1. For the time development given in Figure 4 in our previous research [10], $\phi_+$ curve is estimated as $c_U = 0.7071 + i0$, $c_D = 0.7071 + i0$, while $\phi_-$ one is calculated as $c_U = 0.7071 + i0$, $c_D = -0.7071 + i0$. These results give proper calculation for $\phi_+ = \frac{1}{\sqrt{2}} (U \pm D)$.

The second example is given. We would like to emphasize that it targets ordinary engineering systems. For this purpose, a system is adopted to control the temperature of 1[cC] of water in a heat-insulated box near room temperature. It is also emphasized that this is just an example. State equation that describes system dynamics is

$$\dot{x} = gu + Ax,$$

where we write down numerical values with physical dimension to show that we are dealing with a real system: $u = P [1 / s]$, $g = \frac{1}{\epsilon_{pr}} = 0.2372 [K / J]$, $A = \frac{aS}{\epsilon_{pr}} = 0 [1 / s]$. Control specification is given by

$$L = \frac{m}{2} p^2 - V(x), \quad m = 1[-].$$

where $V$ is a square well potential $V = 0$ for $x \in [-1, 1]$ while $V = 1$ otherwise. We have a little strange dimension $[J^2 / s^2]$ of $V$, $L$ and energy, because we set $m$ dimensionless. Since $V$ is not a potential but an evaluation function of the control specification, such a strange physical dimension does not cause any problem. A designer’s constant for quantization is given as $H_R = 2 [J^2 / s]$, of the physical dimension of time integration of $L$, that leads to two energy levels. We show results of Schrödinger equation and time function in Figure 2, where $P_U$, $P_D$ results are shown in (a), (e) for $a_C = 0.0025$, in (b), (f) for $a_C = 0.005$, in (c), (g) for $a_C = 0.01$ and in (d), (h) for $a_C = 0.02$. Calculation according to Schrödinger’s partial differential
time interval as shown in this Figure, the maximum \( |x| = D \) on the complex Gaussian plane at time \( t = 0 \). If \( \tilde{\mathbf{0}}(14) \) is a probability density that we see \( \mathbf{U} \) and \( \mathbf{D} \) as blue solid lines in (a) to (h). We use finite difference method specialized in unitary time development [31] for numerical calculation. Space and time division are \( dx = 2 \times 4/500 \) (whole space is approximated as \( [-4,4] \)) and \( dt = 1 \times 10^{-5} \), respectively. On the other hand, the red dotted lines in (a) to (h) represent the values of time functions, \( \mathbf{P}_U = |\tilde{\mathbf{U}}(t)|^2 \) and \( \mathbf{P}_D = |\tilde{\mathbf{D}}(t)|^2 \). We see in (a) to (h) that as \( \mathbf{a}_C \) becomes smaller, the difference between the results by p.d.e. and the time function value becomes smaller. The probability value \( \mathbf{P}_{\text{leak}} = 1 - (\mathbf{P}_U + \mathbf{P}_D) \) calculated by time functions represents leakage of our state from the vector space \( (\mathbf{U}, \mathbf{D}) \) in the process of oscillation. The larger \( \mathbf{a}_C \) brings larger leakage. The maximum leakage probabilities \( \max(\mathbf{P}_{\text{leak}}) = 1 - \min(\mathbf{P}_U + \mathbf{P}_D) \) are calculated as 0.0072, 0.0309, 0.1101 and 0.4287 for \( \mathbf{a}_C = 0.0025 \), 0.005, 0.01, and 0.02, respectively. Values \( \mathbf{c}_U \) and \( \mathbf{c}_D \) calculated for \( \mathbf{a}_C = 0.005 \) are shown as vectors in complex Gaussian plane in Figure 3, at time = 200, 600 and 1000. We evaluate it as showing that our estimation, magenta and cyan coloured arrows, well with those, red and blue, by direct time integrals (p.d.e.). In other words, it can be seen that the feedback by the time function works well. For this case of \( \mathbf{a}_C = 0.005 \), we show in Figure 4 time response \( u \) by (14) or (22). Since \( u \), input power \( P(|u|/s) \), is an addition to the design input value, it can take a positive or negative value. The trends \( u \) and \( x \) show the movement of the large wave seen in Figure 2(b,f) with the fast movement, \( \frac{dx}{dt} \sim 30 \), due to (18). At around time \( t \sim 600 \), numerator in (14) is small and denominator takes a large value, so changes in both input \( u \) and state variable \( x \) are small. The denominator of (14) is a probability density that we see state variable at \( x \). So, \( x \) does not reach the point where we have a divergence of \( u \) due to zero division. In fact, in the time interval as shown in this Figure, the maximum \( |u| \) is about \( \sim 12 \).

**Figure 2.** Trends of appearance probability \( \mathbf{P}_U \) and \( \mathbf{P}_D \).

**Figure 3.** Display of the coefficients \( \mathbf{c}_U \) of the state \( \mathbf{U} \) and \( \mathbf{c}_D \) of \( \mathbf{D} \) on the complex Gaussian plane at time = 200, 400 and 600.

Ordinary engineering systems, such as temperature control systems, were considered as qubit. It was constructed using a feedback controller as an equal time, non-causal, input/output relationship written down in elementary functions of time \( t \) and state \( x \). This is more feasible than the previously reported quantum computer using a mobile robot that imitates mass particle governed by classical physics. Calculating a ternary real linear system of equations is the substance of one qubit quantum computation. A quantum computer using the ordinary engineering system has a configuration in which a large-scale peripheral device is removed from the current quantum computer. The proposed system can be used as an “edge quantum computer.” Since it uses a familiar classical engineering system, it is easy for various people involved in system construction to understand. It is also an advantage that a one-dimensional system can be used. We gave a simulation study on how to monitor wave function parameters. Constructing multiple qubits is shown in the near future.

**4. Conclusion**

**Notes**

1. The previous paper [10] uses the symbol \( \mathbf{O} \) instead of \( \mathbf{U} \) and \( \mathbf{T} \) instead of \( \mathbf{D} \). However, \( \mathbf{O} \) representing \( \mathbf{U} \) is confusing with \( \mathbf{O} \) of odd parity of \( \mathbf{D} \). Therefore, \( \mathbf{U} \) and \( \mathbf{D} \) are used in this paper.
2. As we thought in the previous paper [10] on google’s 53-qubits system [9], it is possible to connect systems with two or more energy levels to represent a large number of qubits.

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