Suppression of Rayleigh-Taylor turbulence by time-periodic acceleration

G. Boffetta
Dipartimento di Fisica and INFN, Università di Torino, via P. Giuria 1, 10125 Torino, Italy

M. Magnani
Dipartimento di Fisica, Università di Torino, via P. Giuria 1, 10125 Torino, Italy

S. Musacchio
Université Côte d’Azur, CNRS, LJAD, Nice 06108, France

The dynamics of Rayleigh-Taylor turbulence convection in presence of an alternating, time periodic acceleration is studied by means of extensive direct numerical simulations of the Boussinesq equations. Within this framework, we discover a new mechanism of relaminarization of turbulence: The alternating acceleration, which initially produces a growing turbulent mixing layer, at longer times suppresses turbulent fluctuation and drives the system toward an asymptotic stationary configuration. Dimensional arguments and linear stability theory are used to predicts the width of the mixing layer in the asymptotic state as a function of the period of the acceleration. Our results provide an example of simple control and suppression of turbulent convection with potential applications in different fields.

The acceleration of a mass of fluid against another one with lower density gives rise to the Rayleigh-Taylor (RT) instability at the interface between the two fluids [1–3]. If the acceleration persists, the unstable interface evolves into a turbulent mixing layer which broadens in time. RT turbulence is one of the paradigmatic examples of turbulent mixing with a large variety of applications, ranging from astrophysical and geophysical phenomena (e.g. supernovae flames [4, 5], Earth’s mantle motions 9, ionosphere irregularities 10) to confined nuclear fusion 11, plasma physics 12, and laser matter interactions 13.

Although in many instances the driving acceleration may be considered constant (as in the case of gravity), there are important applications where acceleration changes in time and may even reverse sign. Examples include Inertial Confined Fusion 14, pulsating stars and supernovae explosions 16. RT instability and turbulence with complex acceleration history have been studied experimentally 17 and numerically 18–20. In particular, the effects of a single stabilizing phase (corresponding to gravity reversal) has been shown to produce a slowing down of the turbulent mixing layer 17 and an increase of small scale mixing 21.

In this Letter we address the question of what happens to RT turbulence in presence of a time periodic acceleration which continuously alternates phases of unstable and stable stratification. By extensive numerical simulations of the Boussinesq model, we find that the turbulent mixing layer, which initially develops and grows quadratically in time, eventually stops for any period of the acceleration modulation. The mechanism at the basis of this surprising suppression of RT mixing is the decay of the turbulent fluctuations during the stable phase, with a reduction of the temperature/density fluctuations in the mixing layer. We find simple scaling laws for the arresting time and the asymptotic size of the mixing layer, and we show how the suppression of turbulence can be understood in terms of linear stability theory.

We consider the Boussinesq model of an incompressible velocity field \( \mathbf{u}(\mathbf{x}, t) \) coupled with a temperature (density) field \( \theta(\mathbf{x}, t) \) in the presence of a time dependent acceleration \( \mathbf{g}(t) = -g(t)\hat{z} \) in the vertical direction

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} - \beta \mathbf{g} \theta
\] (1)
\[
\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \kappa \nabla^2 \theta
\]  

(2)

where \( \nu \) is the kinematic viscosity, \( \kappa \) the thermal diffusivity and \( \beta \) the thermal expansion coefficient. We assume standard initial conditions for RT, with \( \mathbf{u}(\mathbf{x}, t) = 0 \) and \( \theta(\mathbf{x}, 0) = -(\theta_0/2) \text{sgn}(z) \) where \( \theta_0 \) is the temperature jump which defines the Atwood number \( A = \beta \theta_0/2 \).

In the case of constant \( g \), the usual phenomenology of RT turbulence is expected. The initial condition is unstable with respect to perturbation of the interface \( z = 0 \) and, for a single-mode perturbation at wavenumber \( k \), the linear stability analysis for an inviscid potential flow gives the growth rate of the amplitude \( \lambda(k) = \sqrt{Agk} \), while high wavenumber are stabilized by viscosity and diffusivity \( [6, 22] \). After this linear phase, the system enters in a nonlinear regime in which a turbulent mixing layer grows starting from the interface at \( z = 0 \). Within the mixing layer, potential energy is converted in turbulent kinetic energy, with large-scale velocity fluctuations increasing dimensionally as \( u_{\text{rms}} \simeq Agt \). Consequently, the width of the mixing layer \( h \) grows as \( h(t) \simeq Agt^2 [3] \).

In this Letter we consider the case of periodic acceleration \( g(t) \). To be specific, we study the case of a square wave of amplitude \( g_0 \) in which the sign of \( g(t) \) is reversed every half period \( T/2 \) (starting from the positive, unstable value). We also studied the different protocol of sinusoidal function \( g(t) = g_0 \cos(2\pi t/T) \) (not reported here) to verify the generality of our results.

We performed extensive numerical simulations of the Boussinesq equations \( [12] \) by means of fully parallel pseudo-spectral code in a 3D box of dimension \( L_x = L_y = L_z/4 \) at resolutions up to \( 512 \times 512 \times 2048 \) with uniform grid. The boundary conditions for velocity are periodic in the lateral directions and no-slip in the vertical direction. Time evolution is obtained by a second-order Runge-Kutta scheme with explicit linear part. For all runs we fix \( Pr = \nu/\kappa = 1 \) and viscosity is sufficiently large to resolve the small scales during all the phases of the simulations \( [25] \). In all runs, the value of the period \( T \) is taken sufficiently long to allow for the development of the turbulent mixing layer before inverting the acceleration, i.e. \( \lambda(k)T > 1 \). Numerical results are made dimensionless by using \( L_x, T = \sqrt{L_x/(Ag)} \) and \( \theta_0 \) as unit length, time and temperature respectively.

RT instability is seeded by perturbing the initial temperature field with respect to the unstable step profile. Two different perturbations were implemented. In the first case, we perturbed the initial condition by adding 5% of white noise to the value of \( \theta(x, 0) \) in a narrow layer around the interface \( z = 0 \). In the second set of simulations, we perturb the interface by a superposition of 2D linear waves of small amplitudes and narrow range of wavenumbers around \( k = 30 \). The two sets of simulations gave similar results for long time statistics. The results reported in this Letter are relative to the first type of perturbation, and they are averaged over 10 independent realizations of the white noise for each value of the parameters.

Figure 2 shows the evolution of the vertical rms velocity for a simulation with period \( T = 3 \pi \). At the beginning we observe an average growth of \( w_{\text{rms}} \) with large oscillations within each period \( T \), which are due to the acceleration in the unstable phases (grey region) and deceleration in the stable phases (white regions). Nonetheless, after few periods (the first 4 periods in this case), the amplitude of the oscillations reduces and vertical velocity fluctuations asymptotically decay. During the unstable phases there is on average a growth of the velocity fluctuations, initially in agreement with the dimensional linear law. After the first periods, the growth of \( w_{\text{rms}} \) is preceded by a transient decrease. This is due to the fact that, immediately after each acceleration reversal, the plumes must invert the direction of their motion before accelerating again. Fig. 2 also shows a secondary peak of vertical velocity in the stable phases (white regions), which is caused by the exchange between kinetic and potential energy in stably stratified turbulence. The period of these secondary oscillations \( T_{BV} = 2 \pi / N \) is determined by the Brunt-Väisälä frequency \( N = \sqrt{g \beta |\partial(\theta)/\partial z|} \) where \( \langle \theta(z, t) \rangle \equiv 1/(L_y L_y) \int dx \int dy \theta(x, t) \) is the mean temperature profile. The initial strong gradient of temperature at the interface gives large values of \( N \) and therefore a short secondary period \( T_{BV} \ll T \). By approximating the mean temperature gradient as \( |\partial(\theta)/\partial z| \simeq \theta_0/h \), we obtain \( N \simeq 2Ag/h \). Therefore the growth of the mixing layer causes a decrease of the Brunt-Väisälä frequency.

The long-time decay of velocity fluctuations indicates that asymptotically the unstable phase is unable to sustain turbulence in the mixing layer which is eventually arrested. This surprising result is indeed observed in
our simulations, as shown qualitatively in Fig. 4. More quantitatively, in Fig. 4 we show the time evolution of the width $h(t)$ of the mixing layer, which is defined from the mean temperature profiles as the difference between the two heights $z_{\pm}$ at which $\langle \theta(z_{\pm}, t) \rangle = \pm 0.95(\theta_0/2)$. In all the case investigated, after few oscillations the width of the mixing layer reaches an asymptotic value $h_\infty$. Strictly speaking, a truly asymptotic value cannot be reached because of the presence of a diffusive terms in (2) but, since the value of $\kappa$ in our simulations is very small, its effect is observable only on much longer timescales.

The period of the accelerations is the only external time-scale in the dynamics, therefore one is tempted to rescale time in Fig. 4 with $T$ and correspondingly the spatial scale with the dimensional expression $AgT^2$. With this rescaling we observe an almost perfect collapse of the curves $h(t)$ at different periods of oscillation. The asymptotic width of the mixing layer is found to collapse to $h_\infty \approx 0.18AgT^2$ and the corresponding period of the Brunt-Väisälä oscillations is $T_{BV} \approx 1.9T$.

Figure 4 shows the mean temperature profiles $\langle \theta(z, t) \rangle$ and the temperature standard deviation $\sigma_\theta(z, t) = (\langle \theta^2 \rangle - \langle \theta \rangle^2)^{1/2}$ at different times in the evolution of the mixing layer. As in usual RT turbulence, the average temperature $\langle \theta \rangle$ develops a mean linear profile which evolves in self-similar way until it is stopped (at $t \geq 8T$, see Fig. 4). At late times the mean temperature profile remains frozen, while temperature fluctuations, represented by the standard deviation in Fig. 4 decay in time, following the decay of velocity fluctuations shown in Fig. 2. All together, these results show that the mechanism which initially produces turbulence is unable to sustain the flow for long times. Such novel phenomenon of asymptotic relaminarization of turbulence within the
mixing layer in the time-periodic RT system is reminiscent of the relaminarization observed in pipe flows \[25\].

The physical interpretation of turbulence suppression and of the observed rescaling is the following. During the stable phase turbulence decays and velocity and temperature fluctuations are reduced, as shown in Fig. [1] and Fig. [2]. When the acceleration is switched back to the unstable phase, it takes some time for the available potential energy to produce new turbulent fluctuations and associate kinetic energy, and this time increases with the width of the mixing layer. Although the rate of this non-linear instability cannot be analytically computed, one can use the results for linear instability growth rate, an approximation which is better and better justified with time since turbulent fluctuations are decaying. For the approximation which is better and better justified with time since turbulent fluctuations are decaying, we observe the recovery of the quadratic growth of the unstable condition after the asymptotic stage is reached by stopping the periodic acceleration reversal during the finite extension of amplitude proportional to \[A_h\] as \[26\].

\[
\lambda^2(k) = \frac{Agk}{A + (1 - A)hk[1 - e^{-hk}]^{-1}} \tag{3}
\]

which recovers the standard result \(\lambda^2 = Agk\) in the limit of very steep gradient \(hk \rightarrow 0\). In the opposite limit of wide mixing layer, which is relevant for the present situation, \(hk \gg 1\) in \[3\] gives

\[
\lambda^2(k) = \frac{Ag}{(1 - A)h} \tag{4}
\]

which shows that the growth rate become independent on \(k\) and decays as \(1/\sqrt{h}\). Therefore by increasing \(h\), \(1/\lambda\) becomes eventually longer than \(T/2\) and the perturbation has not sufficient time to grow. From \[4\] this happens at a scale which is proportional to \(T^2\), in agreement with the phenomenological rescaling used in Fig. [3]. A numerical confirmation of this argument is given by the fact that by stopping the periodic acceleration reversal during the unstable condition after the asymptotic stage is reached we observe the recovery of the quadratic growth of the mixing layer after a time longer than \(T/2\).

Since RT turbulence is known to produce an efficient exchange of heat between the two layers at different temperatures, it is interesting to investigate how this is affected by the acceleration reversal. We have therefore computed the evolution of the Rayleigh number \(Ra = Agh^3/(\nu k)\) and of the Nusselt number \(Nu = \langle w\theta\rangle h/\langle w\rho\theta\rangle\) in our numerical simulations. We remind that in standard RT turbulence both \(Ra\) and \(Nu\) grow in time (following the growth of \(h\) and of turbulent velocities) and the dependence of \(Nu\) on \(Ra\) realizes the so-called “ultimate state of thermal convection” for which \(Nu \simeq Ra^{1/2}\) \[27\] \[28\].

In the present case, since the mixing scale \(h\) is arrested, the Rayleigh number reaches an asymptotic value \(Ra_{max} \propto T^6\), while the Nusselt number, shown in Fig. [5] decays in time as a consequence of the reduction of velocity fluctuations (see Fig. [2]). We observe also that \(Nu\) is negative during the stable phase as a consequence of the inversion of the vertical velocity which becomes anticorrelated with temperature fluctuations. The decay of \(Nu\) is not simply due to the reduction of vertical velocity fluctuations: indeed also the correlation \(C(w, \theta) = \langle w\theta\rangle/(w_{rms}\theta_{rms})\) between the velocity and the temperature fields decreases in time, as shown in the inset of Fig. [3]. We also observe an increasing symmetry between the stable and the unstable phases in the oscillations of \(Nu\). As a consequence, after few periods of oscillations the total heat flux over a period is close to zero: the positive flux during the unstable phases is compensated by an equal and opposite flux in the stable phase.

In conclusion, we have studied the phenomenology of Rayleigh-Taylor turbulence in the Boussinesq approximation in presence of a periodic acceleration which alternates phases of unstable and stable stratification. We have found the surprising result that after few periods \(T\) of the acceleration, the turbulent mixing layer reaches a finite extent of amplitude proportional to \(T^2\). In this asymptotic state turbulence is found to decay in time, in spite of the persistence of phases of unstable stratification, and becomes ineffective to develop further instabilities. Extensive numerical simulations show that this result is robust in the sense that the presence of an asymptotic layer is independent on the period \(T\) (in the range of period compatible with the size of the domain) and also on the specific protocol of the acceleration (sinusoidal or square wave).

More in general, our results shed new light on the possibility to control, and even suppress, turbulent convection by periodic modulation of the acceleration field.
This suggests possible applications beyond the specific RT configuration. For example, it is well known that parametric excited Rayleigh-Benard convection, by thermal or acceleration fast modulation, changes the onset of the instability for convection [28, 30, 31]. Our results show that also the nonlinear, turbulent phase could be in principle controlled (and suppressed) by appropriate modulation of the external acceleration. The possibility to observe this effect in other systems would be extremely interesting, encouraging further numerical or experimental investigations.

ACKNOWLEDGMENTS. G.B. acknowledges support by the project CSTO162330 Extreme Events in Turbulent Convection and from the Departments of Excellence grant (MIUR). M.M. thanks the financial support by the project CRT 2015.2697. HPC center CINECA is gratefully acknowledged for computing resources.

[1] S. Chandrasekhar, Hydrodynamic and hydromagnetic stability (Courier Corporation, 2013).
[2] D. Sharp, Physica D 12, 3 (1984).
[3] S. Abarzhi, Phil. Trans. R. Soc. A 368, 1809 (2010).
[4] M. Chertkov, Phys. Rev. Lett. 91, 115001 (2003).
[5] G. Boffetta and A. Mazzino, Annu. Rev. Fluid Mech. 49, 119 (2017).
[6] Y. Zhou, Phys. Rep. 720, 1 (2017).
[7] W. Hillebrandt and J. Niemeyer, Annu. Rev. Astron. Astrophys. 38, 191 (2000).
[8] J. Bell, M. Day, C. Rendleman, S. Woosley, and M. Zingale, The Astrophysical Journal 608, 883 (2004).
[9] E. Neil and G. Houseman, Geophys. J. Int. 138, 89 (1999).
[10] P. Sultan, Journal of Geophysical Research: Space Physics 101, 26875 (1996).
[11] J. Kilkenny, S. Glendinning, S. Haan, B. Hammel, J. Lindl, D. Munro, B. Remington, S. Weber, J. Knauer, and C. Verdon, Phys. Plasmas 1, 1379 (1994).
[12] M. Tabak, J. Hammer, M. Glinsky, W. Krueer, S. Wilks, J. Woodworth, E. Campbell, M. Perry, and R. Mason, Phys. Plasmas 1, 1626 (1994).
[13] R. Taylor, J. Dahlburg, A. Iwase, J. Gardner, D. Fyfe, and O. Willi, Phys. Rev. Lett. 76, 1643 (1996).
[14] S. Nakai and H. Takabe, Rep. Prog. Phys. 59, 1071 (1996).
[15] R. Betti, C. Zhou, K. Anderson, L. Perkins, W. Theobald, and A. Solodov, Phys. Rev. Lett. 98, 155001 (2007).
[16] S. A. Colgate and R. H. White, Astrophys. J. 143, 626 (1966).
[17] G. Dimonte, P. Ramaprabhu, and M. Andrews, Phys. Rev. E 76, 046313 (2007).
[18] D. Livescu, T. Wei, and M. Petersen, J. Phys.: Conf. Series 318, 082007 (2011).
[19] P. Ramaprabhu, V. Karkhanis, R. Banerjee, H. Varshochi, M. Khan, and A. Lawrie, Phys. Rev. E 93, 013118 (2016).
[20] D. Aslangil, A. Banerjee, and A. Lawrie, Phys. Rev. E 94, 053114 (2016).
[21] P. Ramaprabhu, V. Karkhanis, and A. Lawrie, Phys. Fluids 25, 115104 (2013).
[22] H. Kull, Phys. Rep. 206, 197 (1991).
[23] G. Boffetta, A. Mazzino, S. Musacchio, and L. Vozella, Phys. Fluids 22, 035109 (2010).
[24] G. Boffetta, F. De Lillo, and S. Musacchio, Phys. Rev. Lett. 104, 034505 (2010).
[25] B. Eckhardt, T. M. Schneider, B. Hof, and J. Westerweel, Annu. Rev. Fluid Mech. 39, 447 (2007).
[26] K. Mikaelian, Phys. Rev. A 33, 1216 (1986).
[27] R. H. Kraichnan, Phys. Fluids 5, 1374 (1962).
[28] G. Ahlers, S. Grossmann, and D. Lohse, Rev. Mod. Phys. 81, 503 (2009).
[29] G. Boffetta, A. Mazzino, S. Musacchio, and L. Vozella, Phys. Rev. E 79, 065301 (2009).
[30] J. J. Niemela and R. J. Donnelly, Phys. Rev. Lett. 59, 2431 (1987).
[31] A. Swaminathan, Experimental Investigation of Dynamic Stabilization of the Rayleigh-Bénard Instability by Acceleration Modulation, Ph.D. thesis, The Pennsylvania State University (2017).