Efficient Compression Technique for Sparse Sets

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ABSTRACT
Recent technological advancements have led to the generation of huge amounts of data over the web, such as text, image, audio and video. Needless to say, most of this data is high dimensional and sparse, consider, for instance, the bag-of-words representation used for representing text. Often, an efficient search for similar data points needs to be performed in many applications like clustering, nearest neighbour search, ranking and indexing. Even though there have been significant increases in computational power, a simple brute-force similarity-search on such datasets is inefficient and at times impossible. Thus, it is desirable to get a compressed representation which preserves the similarity between data points. In this work, we consider the data points as sets and use Jaccard similarity as the similarity measure. Compression techniques are generally evaluated on the following parameters – 1) Randomness required for compression, 2) Time required for compression, 3) Dimension of the data after compression, and 4) Space required to store the compressed data. Ideally, the compressed representation of the data should be such that the similarity between each pair of data points is preserved, while keeping the time and the randomness required for compression as low as possible.

Recently, Pratap and Kulkarni [11], suggested a compression technique for compressing high dimensional, sparse, binary data while preserving the inner product and Hamming distance between each pair of data points. In this work, we show that their compression technique also works well for Jaccard similarity. We present a theoretical proof of the same and complement it with rigorous experiments on synthetic as well as real-world datasets. We also compare our results with the state-of-the-art “min-wise independent permutation”, and show that our compression algorithm achieves almost equal accuracy while significantly reducing the compression time and the randomness. Moreover, after compression our representational compression is in binary form as opposed to integer in case of min-wise permutation, which leads to a significant reduction in search-time on the compressed data.

1 INTRODUCTION
We are at the dawn of a new age. An age in which the availability of raw computational power and massive data sets gives machines the ability to learn, leading to the first practical applications of Artificial Intelligence. The human race has generated more amount of data in the last 2 years than in the last couple of decades, and it seems like just the beginning. As we can see, practically everything we use on a daily basis generates enormous amounts of data and in order to build smarter, more personalised products, it is required to analyse these datasets and draw logical conclusions from it. Therefore, performing computations on big data is inevitable, and efficient algorithms that are able to deal with large amounts of data, are the need of the day.

We would like to emphasize that most of these datasets are high dimensional and sparse – the number of possible attributes in the dataset are large however only a small number of them are present in most of the data points. For example: micro-blogging site Twitter can have each tweet of maximum 140 characters. If we consider only English tweets, considering the vocabulary size is of 171, 476 words, each tweet can be represented as a sparse binary vector in 171, 476 dimension, where 1 indicates that a word is present, 0 otherwise. Also, large variety of short and irregular forms in tweets add further sparseness. Sparsity is also quite common in web documents, text, audio, video data as well.

Therefore, it is desirable to investigate the compression techniques that can compress the dimension of the data while preserving the similarity between data points. In this work, we focus on sparse, binary data, which can also be considered as sets, and the underlying similarity measure as Jaccard similarity. Given two sets \( A \) and \( B \) the Jaccard similarity between them is denoted as \( JS(A, B) \) and is defined as \( JS(A, B) = |A \cap B| / |A \cup B| \). Jaccard Similarity is popularly used to determine whether two documents are similar. [2] showed that this problem can be reduced to set intersection problem via shingling\(^1\). For example: two documents \( A \) and \( B \) first get converted into two shingles \( S_A \) and \( S_B \), then similarity between these two documents is defined as \( JS(A, B) = |S_A \cap S_B| / |S_A \cup S_B| \). Experiments validate that high Jaccard similarity implies that two documents are similar.

Broder et al. [5, 6] suggested a technique to compress a collection of sets while preserving the Jaccard similarity between every pair of sets. For a set \( U \) of binary vectors \( \{u_i\}_{i=1}^{n} \subseteq \{0, 1\}^{d} \), their technique includes taking a random permutation of \( \{1, 2, \ldots, d\} \) and assigning a value to each set which maps to minimum under that permutation.

Definition 1.1 (Minhash [5, 6]). Let \( \pi \) be a permutations over \( \{1, \ldots, d\} \), then for a set \( u \subseteq \{1, \ldots, d\} \) \( h_u(u) = \arg \min_{i} \pi(i) \) for

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\(^1\)A document is a string of characters. A \( k\)-shingle for a document is defined as a contiguous substring of length \( k \) found within the document. For example: if our document is \( abcd \), then shingles of size 2 are \( \{ab, bc, cd\} \).
i \in u$. Then,
\[
\Pr[h(u) = h(v)] = \frac{|u \cap v|}{|u \cup v|}.
\]

**Note 1.2 (Representing sets as binary vectors).** Throughout this paper, for convenience of notation we represent sets as binary vectors. Let the cardinality of the universal set is $d$, then each set which is subset of the universal set is represented as a binary vector in $d$-dimension. We mark 1 at position where the corresponding element from universal set is present, and 0 otherwise. We illustrate this with an example: let the universal set is $U = \{1, 2, 3, 4, 5\}$, then we represent the set $\{1, 2\}$ as $11000$, and the set $\{1, 5\}$ as $10001$.

### 1.1 Revisiting Compression Scheme of [11]

Recently, Pratap and Kulkarni [11] suggested a compression scheme for binary data that compress the data while preserving both Hamming distance and Inner product. A major advantage of their scheme is that the compression-length depends only on the sparsity of the data and Inner product. A major advantage of their scheme is that it also works well in the streaming setting. The only prerequisite is an upper bound on the sparsity $\psi$ as well as on the number of data points, which requires to give a bound on the compression length $N$.

#### Parameters for evaluating a compression scheme

The quality of a compression algorithm can be evaluated on the following parameters.

- **Randomness** is the number of random bits required for compression.
- **Compression time** is the time required for compression.
- **Compression length** is the dimension of data after compression.
- **The amount of space required to store the compressed matrix.**

Ideally the compression length and the compression time should be as small as possible while keeping the accuracy as high as possible.

#### 1.3 Comparison between BCS and minhash

We evaluate the quality of our compression scheme with minhash on the parameters stated earlier.

**Randomness.** One of the major advantages of BCS is the reduction in the number of random bits required for compression. We quantify it below.

**Lemma 1.6.** Let a set of $n$ $d$ dimensional binary vectors, which get compressed into a set of $n$ vectors in $N$ dimension via minhash and BCS, respectively. Then, the amount of random bits required for BCS and minhash are $O(d \log N)$ and $O(Nd \log d)$, respectively.

**Proof.** For BCS, it is required to map each bit position from $d$-dimension to $N$-dimension. One bit assignment requires $O(\log N)$ amount of randomness as it needs to generate a number between 1 to $N$ which require $O(\log N)$ bits. Thus, for each bit position in $d$-dimension, the mapping requires $O(d \log N)$ amount of randomness. On the other side, minhash requires creating $N$ permutations in $d$-dimension. One permutation in $d$ dimension requires generating $d$ random numbers each within 1 and $d$. Generating a number between 1 and $d$ requires $O(\log d)$ random bits, and generating $d$ such numbers require $O(d \log d)$ random bits. Thus, generating $N$ such random permutations requires $O(Nd \log d)$ random bits.

**Compression time.** BCS is significantly faster than Minhash algorithm in terms of compression time. This is because, generation of random bits requires a considerable amount of time. Thus, reduction in compression time is proportional to the reduction in the amount of randomness required for compression. Also, for
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Space required for compressed data: Minhash compression generates an integer matrix as opposed to the binary matrix generated by BCS. Therefore, the space required to store the compressed data of BCS is $O(\log d)$ times less as compared to minhash.

Search time. Binary form of our compressed data leads to a significantly faster search as efficient bitwise operations can be used.

In Section 3, we numerically quantify the advantages of our compression on the later three parameters via experiments on synthetic and real-world datasets.

Li et al. [9] presented "b-bit minhash" an improvement over Broder’s minhash by reducing the compression size. They store only a vector of b-bit hash values (of binary representation) of the corresponding integer hash value. However, this approach introduces some error in the accuracy. If we compare BCS with b-bit minhash, then we have same the advantage as of minhash in savings of randomness and compression time. Our search time is again better as we only store one bit instead of b-bits.

1.4 Applications of our result

In cases of high dimensional, sparse data, BCS can be used to improve numerous applications where currently minhash is used.

Faster ranking/de-duplication of documents. Given a corpus of documents and a set of query documents, ranking documents from the corpus based on similarity with the query documents is an important problem in information-retrieval. This also helps in identifying duplicates, as documents that are ranked high with respect to the query documents, share high similarity. Broder [4] suggested an efficient de-duplication technique for documents – by converting documents to shingles; defining the similarity of two documents based on their Jaccard similarity; and then using minhash sketch to efficiently detect near-duplicates. As most the datasets are sparse, BCS can be more effective than minhash on the parameters stated earlier.

Scalable Clustering of documents: Clustering is one of the fundamental information-retrieval problems. [7] suggested an approach to cluster data objects that are similar. The approach is to partition the data into shingles; defining the similarity of two documents based on their Jaccard similarity; and then via minhash generate a sketch of each data object. These sketches preserve the similarity of data objects. Thus, grouping these sketches gives a clustering on the original documents. However, when documents are high dimensional such as webpages, minhash sketching approach might not be efficient. Here also, exploiting the sparsity of documents BCS can be more effective than minhash.

Beyond above applications, minhash compression has been widely used in applications like spam detection [3], compressing social networks [8], all pair similarity [1]. As in most of these cases, data objects are sparse, BCS can provide almost accurate and more efficient solutions to these problems.

We experimentally validate the performance of BCS for ranking experiments on UCI [10] "BoW" dataset and achieved significant improvements over minhash. We discuss this in Subsection 3.2. Similarly, other mentioned results can also be validated.

Organization of the paper. Below, we first present some necessary notations that are used in the paper. In Section 2, we first revisit the results of [11], then building on it we give a proof on the compression bound for Jaccard similarity. In Section 3, we complement our theoretical results via extensive experimentation on synthetic as well as real-world datasets. Finally, in Section 4 we conclude our discussion and state some open questions.

| Notations |
|-----------|
| $N$       | dimension of the compressed data |
| $\psi$    | upper bound on the number of 1’s in binary data. |
| $u[i]$    | i-th bit position of vector $u$. |
| $J_S(u,v)$| Jaccard similarity between binary vectors $u$ and $v$. |
| $d_H(u,v)$| Hamming distance between binary vectors $u$ and $v$. |
| $\langle u,v \rangle$ | Inner product between binary vectors $u$ and $v$. |

2 ANALYSIS

We first revisit the results of [11] which discuss compression bounds for Hamming distance and Inner product, and then building on it, we give a compression bound for Jaccard similarity. We start with discussing the intuition and a proof sketch of their result.

Consider two binary vectors $u, v \in \{0, 1\}^d$, we call a bit position "active" if at least one of the vector between $u$ and $v$ has value 1 in that position. Further, given the sparsity bound $\psi$, there can be at most $2\psi$ active positions between $u$ and $v$. Then let via BCS, they compressed into binary vectors $u', v' \in \{0,1\}^N$. In the compressed version, we call a bit position "pure" if the number of active positions mapped to it is at most one, and "corrupted" otherwise. The contribution of pure bit positions in $u', v'$ towards Hamming distance (Inner product similarity), is exactly equal to the contribution of the bit positions in $u, v$ which get mapped to the pure bit positions. Further, the deviation of Hamming distance (Inner product similarity) between $u'$ and $v'$ from that of $u$ and $v$, corresponds to the number of corrupted bit positions shared between $u'$ and $v'$. Figure 2 illustrate this with an example, and the lemma below analyse it.

**Lemma 2.1 (Lemma 14 of [11]).** Consider two binary vectors $u, v \in \{0, 1\}^d$, which get compressed into vectors $u', v' \in \{0, 1\}^N$ using the BCS, and suppose $\psi$ is the maximum number of 1 in any vector. Then for an integer $r \geq 1$, and $\varepsilon \in (0,1)$, probability that $u'$ and $v'$ share more than $\varepsilon r$ corrupted positions is at most $\left(2\psi/\sqrt{N}\right)^{\varepsilon r}$.

**Figure 2: Illustration of pure/corrupted bits in BCS.**
Proof. We first give a bound on the probability that a particular bit position gets corrupted between \(u'\) and \(v'\). As there are at most \(2\sqrt{N}\) active positions shared between vectors \(u\) and \(v\), the number of ways of pairing two active positions from \(2\sqrt{N}\) active positions is at most \((2\sqrt{N})^2\), and this pairing will result in a corrupted bit position in \(u'\) or \(v'\). Then, the probability that a particular bit position in \(u'\) or \(v'\) gets corrupted is at most \((2\sqrt{N})^2/N \leq (4\sqrt{N}/N\) . Further, if the deviation of Hamming distance (or Inner product similarity) between \(u'\) and \(v'\) from that of \(u\) and \(v\) is more than \(\epsilon r\), then at least \(\epsilon r\) corrupted positions are shared between \(u'\) and \(v'\), which implies that at least \(\epsilon r\) pair of active positions in \(u\) and \(v\) got paired up while compression. The number of possible ways of pairing \(\epsilon r\) active positions from \(2\sqrt{N}\) active positions is at most \((2\sqrt{N})^\epsilon r \leq (2\sqrt{N})^r\). Since the probability that a pair of active positions got mapped in the same bit position in the compressed data is \(1/2\), the probability that \(\epsilon r\) pair of active positions got mapped in \(\epsilon r\) distinct bit positions in the compressed data is at most \((1/2)^{\epsilon r}\). Thus, by union bound, the probability that at least \(\epsilon r\) corrupted bit position shared between \(u'\) and \(v'\) is at most \((2\sqrt{N})^r/(N^{1/2}) = (2\sqrt{N}/\sqrt{N})^r\).

In the lemma below generalise the above result for a set of \(n\) binary vectors, and suggest a compression bound so that any pair of compressed vectors share only a very small number of corrupted bits, with high probability.

Lemma 2.2 (Lemma 15 of [11]). Consider a set \(U\) of \(n\) binary vectors \(\{u_i\}_{i=1}^n \subseteq \{0,1\}^d\), which get compressed into a set \(U'\) of binary vectors \(\{u'_i\}_{i=1}^n \subseteq \{0,1\}^{N}\) using the BCS. Then for any positive integer \(r\), and \(\epsilon \in (0,1)\),

- If \(\epsilon r > 3\log n\), and we set \(N = O(\sqrt{\epsilon})\), then probability that for all \(u'_i, u'_j \in U'\) share more than \(\epsilon r\) corrupted positions is at most \(1/n\).
- If \(\epsilon r < 3\log n\), and we set \(N = O(\sqrt{\epsilon} \log^2 n)\), then probability that for all \(u'_i, u'_j \in U'\) share more than \(\epsilon r\) corrupted positions is at most \(1/n\).

Proof. For a fixed pair of compressed vectors \(u'_i\) and \(u'_j\), due to lemma 2.1, probability that they share more than \(\epsilon r\) corrupted positions is at most \((2\sqrt{N}/\sqrt{N})^r\). If \(\epsilon r > 3\log n\), and \(N = 16\sqrt{n}\), then the above probability is at most \((2\sqrt{N}/\sqrt{N})^r < (2\sqrt{N}/4\epsilon)^{3\log n} = (1/2)^{3\log n} < 1/n^3\). As there are at most \(\binom{n}{2}\) pairs of vectors, the required bound follows from union bound of probability.

In the second case, as \(\epsilon r < 3\log n\), we cannot bound the probability as above. Thus, we replicate each bit position \(3\log n\) times, which makes a \(d\) dimensional vector to a \(3d\log n\) dimensional, and as a consequence the Hamming distance (or Inner product similarity) is also scaled up by a multiplicative factor of \(3\log n\). We now apply the compression scheme on these scaled vectors, then for a fixed pair of compressed vectors \(u'_i\) and \(u'_j\), probability that they have more than \(3\epsilon r\log n\) corrupted positions is at most \((6\sqrt{N}/\sqrt{N})^{3\epsilon r\log n}\). As we set \(N = 144\sqrt{N}/2\log n\), the above probability is at most \((6\sqrt{N}/\sqrt{N})^{3\epsilon r\log n} < (1/2)^{3\epsilon r\log n} < 1/n^3\). The final probability follows due to union bound over all \(\binom{n}{2}\) pairs.

After compressing binary data via BCS, the Hamming distance between any pair of binary vectors can not increase. This is due to the fact that compression doesn’t generate any new 1 bit, which could increase the Hamming distance from the uncompressed version. In the following, we recall the main result of [11], which holds due to the above fact and Lemma 2.2.

Theorem 2.3 (Theorem 1, 2 of [11]). Consider a set \(U\) of binary vectors \(\{u_i\}_{i=1}^n \subseteq \{0,1\}^d\), a positive integer \(r\), and \(\epsilon \in (0,1)\). If \(\epsilon r > 3\log n\), we set \(N = O(\sqrt{\epsilon})\); if \(\epsilon r < 3\log n\), we set \(N = O(\sqrt{\epsilon} \log^2 n)\), and compress them into a set \(U'\) of binary vectors \(\{u'_i\}_{i=1}^n \subseteq \{0,1\}^{N}\) via BCS. Then for all \(u_i, u_j \in U\),

- If \(d_H(u_i, u_j) < r\), then \(Pr[d_H(u'_i, u'_j) < r] = 1\).
- If \(d_H(u_i, u_j) \geq (1 + \epsilon) r\), then \(Pr[d_H(u'_i, u'_j) < r] < 1/n\).

For Inner product, if we set \(N = O(\sqrt{\epsilon} \log^2 n)\), then the following is true with probability at least \(1 - 1/n\),

- \((1 - \epsilon)(u_i, u_j) \leq (u'_i, u'_j) < (1 + \epsilon)(u_i, u_j)\).

The following proposition relates Jaccard similarity with Inner product and Hamming distance. The proof follows as for a pair binary vectors their Jaccard similarity is the ratio of the number of positions where 1 is appearing together, with the number of bit positions where 1 is present in either of them. Clearly, numerator is captured by the Inner product between those pair of vectors, and denominator is captured by Inner product plus Hamming distance between them – number of positions where 1 is occurring in both vectors, plus the number of positions where 1 is present in exactly one of them.

Proposition 2.4. For any pair of vectors \(u, v \subseteq \{0,1\}^d\), we have \(JS(u, v) = (u, v)/(u, v) + d_H(u, v)\).

In the following, we complete a proof of the Theorem 1.4 due to Proposition 2.4, and Theorem 2.3.

Proof of Theorem 1.4. Consider a pair of vectors \(u_i, u_j\) from the set \(U \subseteq \{0,1\}^d\), which get compressed into binary vectors \(u'_i, u'_j \subseteq \{0,1\}^{N}\). Due to Proposition 2.4, we have \(JS(u'_i, u'_j) = (u'_i, u'_j)/(u'_i, u'_j) + d_H(u'_i, u'_j))\). Below, we present a lower and upper bound on the expression.

\[
JS(u'_i, u'_j) \geq \frac{(1 - \epsilon)(u'_i, u'_j)}{(1 - \epsilon)(u'_i, u'_j) + d_H(u'_i, u'_j)} \geq \frac{(u'_i, u'_j) + d_H(u'_i, u'_j)}{(1 - \epsilon)JS(u'_i, u'_j)} \geq \frac{1}{1 - \epsilon}JS(u'_i, u'_j)
\]

Equation 1 holds true with probability at least \(1 - 1/n\) due to Theorem 2.3.

\[
JS(u'_i, u'_j) \leq \frac{(u'_i, u'_j)}{(u'_i, u'_j) + d_H(u'_i, u'_j)} \leq \frac{1 + \epsilon}{1 + \epsilon}(u'_i, u'_j) + (1 - \epsilon)d_H(u'_i, u'_j)
\]
We generated 200 pairs whose similarity is high. To generate such pairs, we randomly select those many positions (in dimension) from 1 to 100000, set 1 in both of them, and set remaining to 0. Further, for each of the remaining 600 vectors, we randomly choose a pair of vectors having similarity at least \( \psi \). This gives a pair of vectors having similarity at least \( \frac{1}{e} \) and respecting the sparsity bound. We repeat this step 200 times and obtain 400 vectors. For each of the remaining 600 vectors, we randomly choose an integer from the range 1 to \( \psi \), choose those many positions in the dimension, set them to 1, and set the remaining positions to 0. Thus, we obtained 1000 vectors of dimension 100000 which we used as an input matrix.

\( k \)-NN similarity. A dataset for this experiment consist of a random query vector \( S_q \); 249 vectors whose similarity with \( S_q \) is high; and 750 other vectors. We first generated a query vector \( S_q \) of sparsity between 1 and \( \psi \), then using the procedure mentioned above we generated 249 vectors whose similarity with \( S_q \) is high. Then we generated 750 random vectors of sparsity is at most \( \psi \).

Data representation. We can imagine synthetic dataset as a binary matrix of dimension 100000 \( \times \) 1000. However, for ease and efficiency of implementation, we use a compact representation which consist of a list of lists. The the number of lists is equal to the number of vectors in the binary matrix, and within each list we just store the indices (co-ordinate) where 1s are present. We use this list as an input for both BCS and minhash.

3 Experimental Evaluation

We performed two experiments on synthetic dataset and showed that it preserves both: a) all-pair-similarity, and b) \( k \)-NN similarity. In all-pair-similarity, given a set of \( n \) binary vectors in \( d \)-dimensional space with the sparsity bound \( \psi \), we showed that after compression Jaccard similarity between every pair of vector is preserved. In \( k \)-NN similarity, given is a query vector \( S_q \), we showed that after compression Jaccard similarity between \( S_q \) and the vectors that are similar to \( S_q \), are preserved. We performed experiments on dataset consisted of 1000 vectors in 100000 dimension. Throughout synthetic data experiments, we calculate the accuracy via Jaccard ratio, that is, if the set \( O \) denotes the ground truth result, and the set \( O' \) denotes our result, then the accuracy of our result is calculated by the Jaccard ratio between the sets \( O \) and \( O' \) – that is \( JS(O, O') = |O \cap O'|/|O \cup O'| \). To reduce the effect of randomness we repeat the experiment 10 times and took average.

3.1 Results on Synthetic Data

We performed two experiments on synthetic dataset and showed that it preserves both: a) all-pair-similarity, and b) \( k \)-NN similarity. In all-pair-similarity, given a set of \( n \) binary vectors in \( d \)-dimensional space with the sparsity bound \( \psi \), we showed that after compression Jaccard similarity between every pair of vector is preserved. In \( k \)-NN similarity, given is a query vector \( S_q \), we showed that after compression Jaccard similarity between \( S_q \) and the vectors that are similar to \( S_q \), are preserved. We performed experiments on dataset consisted of 1000 vectors in 100000 dimension. Throughout synthetic data experiments, we calculate the accuracy via Jaccard ratio, that is, if the set \( O \) denotes the ground truth result, and the set \( O' \) denotes our result, then the accuracy of our result is calculated by the Jaccard ratio between the sets \( O \) and \( O' \) – that is \( JS(O, O') = |O \cap O'|/|O \cup O'| \). To reduce the effect of randomness we repeat the experiment 10 times and took average.

3.1.1 Dataset generation.

All-pair-similarity. We generated 1000 binary vectors in dimension 100000 such that the sparsity of each vector is at most \( \psi \). If we randomly choose binary vectors respecting the sparsity bound, then most likely every pair of vector will have similarity 0. Thus, we had to deliberately generate some vectors having high similarity. We generated 200 pairs whose similarity is high. To generate such a pair, we choose a random number (say \( s \)) between 1 and \( \psi \), then we randomly select those many position (in dimension) from 1 to 100000, set 1 in both of them, and set remaining to 0. Further, for each of the vector in the pair, we choose a random number (say \( s' \)) from the range 1 to \( (\psi - s) \), and again randomly sample those many positions from the remaining positions and set them to 1. This gives a pair of vectors having similarity at least \( \frac{1}{\sqrt{2\pi}} \) and respecting the sparsity bound. We repeat this step 200 times and obtain 400 vectors. For each of the remaining 600 vectors, we randomly choose an integer from the range 1 to \( \psi \), choose those many positions in the dimension, set them to 1, and set the remaining positions to 0. Thus, we obtained 1000 vectors of dimension 100000 which we used as an input matrix.

3.1.2 Evaluation metric. We performed two experiments on synthetic dataset – 1) fixed sparsity while varying compression length, and 2) fixed compression length while varying sparsity. We present these experimental results in Figures 3, 4 respectively. In both of these experiments, we compare and contrast the performance BCS with minhash on accuracy, compression time, and search time parameters. All-pair-similarity experiment result requires a quadratic search – generation of all possible candidate pairs and then pruning those whose similarity score is high, and the corresponding search time is the time required to compute all such pairs. While \( k \)-NN similarity experiment requires a linear search and pruning with respect to the query vector \( S_q \), and the corresponding search time is the time required to compute such vectors. In order to calculate the accuracy on a given support threshold value, we first run a simple brute-force search algorithm on the entire (uncompressed) dataset, and obtain the ground truth result. Then, we calculate the Jaccard ratio between our algorithm’s result/ minhash’s result, with the corresponding exact result, and compute the accuracy. First row of the plots are "accuracy" vs "compression length/sparsity". The second row of the plots are "compression time" vs "compression length/sparsity". Third row of plot shows comparison with respect to "search time" vs "compression length/sparsity".

3.1.3 Insight. In Figure 3, we plot the result of BCS and minhash for all-pair-similarity and \( k \)-NN similarity. For this experiment, we fix the sparsity \( \psi = 200 \) and generate the datasets as stated above. We compress the datasets using BCS and minhash for a range of compression lengths from 50 to 10000. It can be observed that BCS performs remarkably well on the parameters of compression time and search time. Our compression time remains almost constant at 0.2 seconds in contrast to the compression time of minhash, which grows linearly to almost 50 seconds. On an average, BCS is 90 times faster than minhash. Also accuracy for BCS and minhash is almost equal above compression length 300, but in the window of 50 – 300 minhash performs slightly better than BCS. Further, the search-time on BCS is also significantly less than minhash for all compression lengths. On an average search-time is 75 times less than the corresponding minhash search-time. We obtain similar results for \( k \)-NN similarity experiments.
In Figure 4, we plot the result of BCS and minhash for all-pair-similarity. For this experiment, we generate datasets for different values of sparsity ranging from 50 to 10000. We compress these datasets using BCS and minhash to a fixed value of compression length 5000. In all-pair-similarity, when sparsity value is below 2200, average accuracy of BCS is above 0.85. It starts decreasing after that value, at sparsity value is 7500, the accuracy of BCS stays above 0.7, on most of the threshold values. The compression time of BCS is always below 2 seconds while compression time of minhash grows linearly with sparsity – on an average compression time of BCS is around 550 times faster than the corresponding minhash compression time. Further, we again significantly reduce search time – on an average our search-time is 91 times less than minhash. We obtain similar results for k-NN similarity experiments.

### 3.2 Results on Real-world Data

#### 3.2.1 Dataset Description: We compare the performance of BCS with minhash on the task of retrieving top-ranked elements based on Jaccard similarity. We performed this experiment on publicly available high dimensional sparse dataset of UCI machine learning repository [10]. We used four publicly available dataset from UCI repository - namely, NIPS full papers, KOS blog entries, Enron Emails, and NYTimes news articles. These datasets are binary "BoW" representation of the corresponding text corpus. We consider each of these datasets as a binary matrix, where each document corresponds to a binary vector, that is if a particular word is present in the document, then the corresponding entry is 1 in that position, and it is 0 otherwise. For our experiments, we consider the entire corpus of NIPS and KOS dataset, while for ENRON and NYTimes we take a uniform sample of 10000 documents from their corpus. We mention their cardinality, dimension, and sparsity in Table 1.

| Data Set                  | No. of points | Dimension | Sparsity |
|---------------------------|---------------|-----------|----------|
| NYTimes news articles     | 10000         | 102660    | 871      |
| Enron Emails              | 10000         | 28102     | 2021     |
| NIPS full papers:         | 1500          | 12419     | 914      |
| KOS blog entries          | 3430          | 6906      | 457      |

#### 3.2.2 Evaluation metric: We split the dataset in two parts 90% and 10% – the bigger partition is use to compress the data, and is referred as the training partition, while the second one is use to evaluate the quality of compression and is referred as querying partition. We call each vector of the querying partition as query vector. For each query vector, we compute the vectors in the training partition whose Jaccard similarity is higher than a certain threshold (ranging from 0.1 to 0.9). We first do this on the uncompressed data inorder to find the underlying ground truth result – for every query vector compute all vectors that are similar to them. Then we compress the entire data, on various values of compression lengths, using our compression scheme/minhash. For each query vector, we calculate the accuracy of BCS/minhash by taking Jaccard ratio between the set outputed by BCS/minhash, on various values of compression length, with set outputed a simple linear search algorithm on entire
data. This gives us the accuracy of compression of that particular query vector. We repeat this for every vector in the querying partition, and take the average, and we plot the average accuracy for each value in support threshold and compression length. We also note down the corresponding compression time on each of the compression length for both BCS and minhash. Search time is time required to do a linear search on the compressed data, we compute the search time for each of the query vector and take the average in the case of both BCS and minhash. Similar to synthetic dataset result, we plot the comparison between our algorithm with minhash on following three points – 1) accuracy vs compression length, 2) compression time vs compression length, and 3) search time vs compression length.

3.2.3 Insights. We plot experiments of real world dataset [10] in Figure 5, and found that performance of BCS is similar to its performance on synthetic datasets. NYTimes is the sparsest among all other dataset, so the performance of BCS is relatively better as compare to other datasets. For NYT TIMES dataset, on an average BCS is 135 times faster than minhash, and search time for BCS is 25 times less than search time for minhash. For BCS accuracy starts dropping below 0.9 when data is compressed below compression length 300. For minhash, accuracy starts dropping below compression compression length 150. Similar pattern is observed for ENRON dataset as well, where BCS is 268 times faster than minhash, and a search on the compressed data obtained from BCS is 104 times faster than search on data obtained from minhash. KOS and NIPS are dense, low dimensional datasets. However here also, for NIPS, our compression time is 271 times faster and search-time is 90 times faster as compared to minhash. For KOS, our compression time is 162 times faster and search time is 63 times faster than minhash.

To summarise, BCS is significantly faster than minhash in terms of both - compression time and search time while giving almost equal accuracy. Also, the amount of randomness required for BCS is also significantly less as compared to minhash. However, as sparsity is increased, accuracy of BCS starts decreasing slightly as compared to minhash.

4 CONCLUDING REMARKS AND OPEN QUESTIONS

We showed that BCS is able to compress sparse, high-dimensional binary data while preserving the Jaccard similarity. It is considerably faster than the “state-of-the-art” minhash permutation, and also maintains almost equal accuracy while significantly reducing the amount of randomness required. Moreover, the compressed representation obtained from BCS is in binary form, as opposed to integer in case of minhash, due to which the space required to store the compressed data is reduced, and consequently leads to a faster searches on the compressed representation. Another major advantage of BCS is that its compression bound is independent of the dimensions of the data, and only grows polynomially with the sparsity and poly-logarithmically with the number of data points. We present a theoretical proof of the same and complement it with rigorous and extensive experimentations. Our work leaves the possibility of several open questions – improving the compression bound of our result, and extending it to other similarity measures.

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Figure 5: Experiments on Real-world datasets [10].

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