Three-dimensional thermal calculations of the radiation chamber of a cylindrical heating tube furnace

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Abstract. The paper briefly describes the method of thermal calculations of radiation chambers of tube furnaces on the basis of modelling interconnected processes taking place in the furnace by three-dimensional differential energy equations, the transfer of momentum components, mass concentrations of methane and air, continuity, a two-parameter model of turbulence and the equation of energy transfer by radiation. A software package using this method is applied to carry out numerical analysis of the spatial temperature distribution, the averaged values of velocities of the combustion products turbulent flow in the combustion chamber of a cylindrical heating tube furnace of the oil-processing industry. A furnace diagram with diffusion burners on the radiation chamber floor and differential equations describing physical processes taking place in the furnace are given. Methods for solution of the initial equations are described. The obtained distributions of the surface densities of heat fluxes to the coils along the chamber perimeter at different heights of the furnace are demonstrated. Graphs of the temperature change of the combustion products and the tubular reactor walls for different values of the angular coordinate are given.

In oil-processing and chemical production, cylindrical tube furnaces are often used to preheat raw materials or to perform necessary transformations of hydrocarbons. The paper demonstrates results of three-dimensional calculations of complex process of heat and mass transfer in the furnace chamber of a heating cylindrical tube furnace. The discrete location of the burners and a range of other factors account for the three-dimensional fuel combustion products flow in the combustion chamber. To predict detailed thermal and aerodynamic parameters and to determine the possible uneven heat flows distribution along the radiation chamber perimeter, it is essential to perform three-dimensional calculations.

The object of study is a vertically flaring gas fuel furnace, with a vertical arrangement of coil pipes around the radiation chamber perimeter (figure 1). The height of the radiation chamber is 12.89 m. A single-row tubular screen consisting of 4 coils is placed at a distance of 2.074 m from the chamber axis. Each coil is placed in the fourth section of the radiation chamber. The heated product is supplied in four streams to the coils. Eight diffusion burners are located symmetrically on the furnace floor at a distance of 1,035 m from the axis of the radiation chamber. The walls inside are lined with kaolin wool. The axial symmetry and the burners’ symmetrical arrangement allow us to divide the radiation chamber volume into 4 equal parts with 2 burners and a coil. Therefore, it suffices to carry out calculations for one such part.
In the performed calculations, methane is taken as fuel and air is taken as the oxidizer. The mass consumption of the fuel mixture is equal to 4,797 kg/s. Figure 1 demonstrates a simplified diagram of the radiation chamber of the tube furnace and the coordinate system.

Figure 1. The layout of the combustion chamber and the coordinate system: 1 – lined wall, 2 – tubular screen, 3 – burners, 4 – flame.

Local temperature values in the combustion chamber volume are obtained by solving the energy conservation equation written relative to the temperature.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r p c_p v T \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( r p c_p w T \right) + \frac{\partial}{\partial z} \left( r p c_p w T \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_{ef} \frac{\partial T}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \phi} \left( \lambda_{ef} \frac{\partial T}{\partial \phi} \right) + \frac{\partial}{\partial z} \left( \lambda_{ef} \frac{\partial T}{\partial z} \right) + S_T, \quad S_T = Q_v - \text{div} \mathbf{q}_R.
\]

where \( \lambda_{ef} = \lambda + \lambda_\alpha \), \( \lambda_\alpha \), \( \lambda_t \) - effective coefficient of flue gas thermal conductivity and molecular and turbulent thermal conductivity, respectively; \( p c_p \) - specific isobaric heat capacity of combustion products; \( u, v, w \) - axial, radial, circumferential components of the gas velocity vector \( \mathbf{v} \); \( \rho \) - gas mixture density; \( T \) is local temperature value of the combustion products at a point \( \mathbf{M} \); \( Q_v \) is the volumetric heat capacity in an elementary volume; \( \mathbf{q}_R \) - vector of surface density of the integral heat flux of radiation.

The divergence of radiation fluxes is calculated by solving the equation of energy transfer by radiation. In our case, this equation is applied in the \( S_2 \)-approximation of the method of discrete ordinate:

\[
\frac{\mu_m}{r} \frac{\partial}{\partial r} \left( r I_m^k \right) - \frac{1}{r} \frac{\partial}{\partial \psi} \left( I_m^k \right) + \frac{\partial}{\partial z} \frac{\partial I_m^k}{\partial z} + \frac{\eta_m}{r} \frac{\partial I_m^k}{\partial \phi} = \alpha_k \int_{\lambda_{a-1}}^{\lambda} I_{bh} \, d\lambda - (\alpha_k + \beta_k) I_m^k
\]

\[
+ \frac{\beta_k}{4\pi} \sum_{m=1}^{N_k} w_{m} \Phi_{m} I_{m}^k ; \quad m = 1, 2, 3, \ldots, N_\alpha; k = 1, 2, 3, \ldots, N_\kappa .
\]
where $I_n^k$ - the integrated radiation intensity of the $k$-th spectral range $[\lambda_{k-1}, \lambda_k]$ along the direction $S_n$ ($m = 1, 2, 3, ..., N_o$) ($N_o = 8$ for the $S_2$-approximation); $I_{\lambda k}$ - Planck function; $w_m$ are the weight coefficients; $\alpha_k, \beta_k$ are the averaged spectral absorption and scattering coefficients in the $k$-th spectral range; $N_s$ - number of spectral bands.

The density vector components of the resulting radiation flux $q_R$ at the nodal points of the computational domain are calculated by summing over the selected directions and over all spectral ranges.

To determine the local values of the combustion products averaged velocity at each point of the furnace volume, it is necessary to solve the motion equations in a three-dimensional formulation and the continuity equation:

$$
\frac{1}{r} \frac{\partial (\rho u)}{\partial r} + \frac{1}{r} \frac{\partial (\rho w)}{\partial \varphi} + \frac{1}{r} \frac{\partial (\rho v)}{\partial z} = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left[ \eta_{\text{eff}} \left( \frac{\partial u}{\partial r} + \frac{\partial v}{\partial z} \right) \right]
$$

$$
+ \frac{1}{r} \frac{\partial}{\partial \varphi} \left[ \mu_{\text{eff}} \left( \frac{\partial u}{\partial \varphi} + \frac{\partial w}{\partial \varphi} \right) \right] + \frac{\partial}{\partial z} \left( 2 \eta_{\text{eff}} \frac{\partial u}{\partial z} \right)
$$

$$
\frac{1}{r} \frac{\partial (\rho w)}{\partial r} + \frac{1}{r} \frac{\partial (\rho v)}{\partial \varphi} + \frac{1}{r} \frac{\partial (\rho w)}{\partial z} = -\frac{\partial p}{\partial r} - \frac{\rho w^2}{r} + \frac{1}{r} \frac{\partial}{\partial r} \left( 2 \eta_{\text{eff}} \frac{\partial \omega}{\partial r} \right)
$$

$$
+ \frac{1}{r} \frac{\partial}{\partial \varphi} \left[ \mu_{\text{eff}} \left( \frac{\partial (w/r)}{\partial \varphi} + \frac{\partial \omega}{\partial r} \right) \right] - \frac{2}{r} \frac{1}{r} \frac{\partial (w/v)}{\partial \varphi} + \frac{\partial}{\partial z} \left[ \mu_{\text{eff}} \left( \frac{\partial (\omega/v)}{\partial z} \right) \right]
$$

$$
+ \frac{\mu_{\text{eff}}}{r} \left[ \frac{\partial (w/r)}{\partial r} + \frac{1}{r} \frac{\partial \omega}{\partial \varphi} + \frac{\partial (p u)}{\partial z} \right] = 0.
$$

$$
\mu_{\text{eff}} = \mu + \mu_t.
$$

The coefficients of turbulent viscosity and thermal conductivity of the flue gas mixture are calculated using the following relations: $\mu = c_{\mu} f_{\mu} \cdot \rho k^2/\epsilon$, $\lambda = c_{\rho} \mu_t / Pr_t$, where $k, \epsilon$ - kinetic energy and the rate of dissipation of turbulent pulsations of the averaged flow rate, respectively, which are calculated by solving the two-parameter turbulence model; $Pr_t$ - the turbulent Prandtl number.

To determine the distribution of molar fractions of H$_2$O, CO$_2$, and CO in the radiation chamber volume, the equations of the two-stage methane combustion model are solved. Component transport equations have the form

$$
\frac{1}{r} \frac{\partial (\rho m_i)}{\partial r} + \frac{1}{r} \frac{\partial (\rho w m_i)}{\partial \varphi} + \frac{\partial (\rho u m_i)}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \Gamma_i \frac{\partial m_i}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( r \Gamma_i \frac{\partial m_i}{\partial \varphi} \right) + \frac{\partial}{\partial z} \left( r \Gamma_i \frac{\partial m_i}{\partial z} \right) + S_i.
$$

Here $m_i$ - mass concentrations of CH$_4$ and CO, respectively; $\Gamma_i = \mu_{\text{eff}} / \sigma_i$, where $\sigma_i$ - Schmidt number; $S_i$ - source member.

The problems of setting the boundary conditions for differential equations (1) - (7), the particulars of their discretizing and solving the system of obtained algebraic equations are considered in [1-3] and
in our other publications, references to which are contained in the indicated works. For the energy conservation equation, a 3-kind boundary condition is set on the heating surface. It is believed that the temperature of the heated product inside the pipes varies linearly.

In the performed calculations, the tubular screen is considered as a cylindrical surface with an effective degree of blackness of 0.65, passing along the pipe axes. The calculations take into account the variability of thermophysical properties in the integration field. Moreover, the same empirical dependences were used in the same way as in [1-3]. The chemical reaction rate in the combustion model is calculated by the model of “vortex breakage” [4, 5]. To describe the turbulent flow and heat transfer near a solid boundary, the method of wall functions is used.

Spectral features of water vapor, carbon monoxide, and carbon dioxide emissions are taken into account in the Edwards broad band model [6]. The empirical formula proposed in [7] is used to calculate the spectral absorption coefficient of sooty particles. We used the values of the angular coordinates (\(\mu_m, \xi_m, \eta_m\), \(m = 1, 2, ..., N_o\)) and the \(w_m\) weight coefficients given in [8]. The method considered in [9] was used to discretize equation (2) and to approximate the derivative by the angular variable \(\psi\).

The distributions of the surface densities of radiant heat fluxes \(q_R\) along the perimeter of one fourth of the furnace (0° ≤ \(\varphi\) ≤ 90°) at different heights \(z\) are shown in figure 2. In their distributions, a wave-like character is observed, which is close to periodic. Fluctuations in the surface densities of radiant fluxes reach 7 - 8 kW / m² (figure 2). A similar character with corresponding local extrema is also observed in the temperature distributions of the heat-absorbing surface along the perimeter. The two-dimensional approximation does not allow one to determine such details in the parameter distributions.

![Figure 2. Distributions of surface densities of radiant heat fluxes to a tubular screen around the furnace perimeter at different heights \(z\).](image)

Figure 3 illustrates the temperature change in the combustion products (upper curves) and the pipe walls (lower curves) along the height of the furnace. Here, curves 1 and 2 show the temperature change vertically along the burner axis and along the symmetry axis, respectively. Curves 3, 4 show the temperature change in the heating surface along vertical lines formed by the intersection of the heating surface with planes passing through the 0z axis at angles \(\varphi\) equal to 11.25° and 22.5°, respectively. The plane corresponding to curve 4 (\(\varphi = 22.5^\circ\)) passes through the burner axis. Curve 4 is located below.
To study the effect of the number of burners on the thermal characteristics, an additional calculation was performed for 12 design burners, also evenly placed on the furnace floor at a distance of 1.035 m from the axis of the radiation chamber. In this case, 3 burners are placed in the integration area ($0^\circ \leq \varphi \leq 90^\circ$).

Figure 3. Change in flue gas temperature (upper curves) and pipe walls (lower curves) by the height of the furnace: 1 - vertically along the burner axis; 2 - along the symmetry axis; 3 - $\varphi = 11.25^\circ$; 4 - $\varphi = 22.5^\circ$.

Figure 4 shows the distributions of surface densities of radiant heat flows $q_R$ along the perimeter of the furnace ($0^\circ \leq \varphi \leq 90^\circ$) at different heights $z$. In the $q_R$ distributions, as in the case of 8 burners, there is a periodic character associated with the discrete location of the burners. At low altitudes, local maxima (figure 4) are located opposite the burners and in the middle between them. At altitudes $z > 0.52$ m, the $q_R$ maxima are located opposite the burners. At $z = 1.56$ m, there are small and large peaks in the $q_R$ distributions. In this case, the large peaks are opposite the burners, and the small ones are in the middle between the burners. The largest $q_R$ fluctuations are $5940 \text{ W/m}^2$ at $z = 2.13$ m. As you
move away from the hearth, the amplitude of the oscillations decreases. At $z > 8$ m, the $q_R$ distribution along the perimeter is almost even, with small local maxima opposite the burners and in the middle between them. A similar pattern is observed in the temperature distributions along the perimeter.

The numerical studies carried out in this work show that the strongest temperature changes and changes in the flow rates of the combustion products in the furnace combustion chamber are observed in the initial section (next to the mouths of the burners), at the roof and hearth, at the exit section. When using two-dimensional modeling of processes in the combustion chamber, it is impossible to determine the uneven heating of the reaction pipes along the furnace perimeter. The calculation results obtained in a three-dimensional statement of the problem of complex process of heat transfer in a cylindrical tubular furnace can be used to select the right solutions in the design of such technological installations. The necessary distribution of heat fluxes to the heating surface according to the conditions of the processes, the permissible temperature of the pipes can be achieved by choosing the burners of the appropriate capacity, their number, design, as well as by their correct location.

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