Numerical simulation on quantum turbulence created by an oscillating object

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Abstract.
We have conducted a numerical simulation of vortex dynamics in superfluid $^4$He in the presence of an oscillating sphere. The experiment on a vibrating wire that measured the transition from laminar to turbulent flow is modelled in our simulations. The simulation exhibits the details of vortex growth by the oscillating sphere. Our result also shows that a more realistic modelling may change the destiny of the vortex rings detached from the sphere. We have evaluated the force driven by the sphere in the simulation and have confirmed the onset of the quantum turbulence.

1. Introduction
Moving objects in superfluid such as a sphere, grid, and wire, can create turbulence. Its transition velocity $v_c \sim 50$ mm/s [1] from laminar to turbulent flow is about 1/1000 of the velocity at which vortex nucleation occurs. Quantum turbulence is believed to be created as a result of the vortex growth rather than the vortex nucleation. Remnant vortices, which are nucleated during cooling liquid $^4$He down below the transition temperature, survive even at very low temperatures, and become "seeds" for the vortex tangle. It is widely believed that the remnant vortices are developed to turbulence by moving objects. Despite that there are a variety of experiments that manifest quantum turbulence caused from the remnant vortices [2, 3, 4], it was impossible to eliminate the remnant vortices from the system.

Recently Hashimoto et al. have succeeded in making the state free from remnant vortices on a vibrating wire [1]. They furthermore have succeeded in making two wires coexist in the same cell, one without remnant vortices (wire A), and the other with them (wire B) [5]. Using this cell, they have conducted an interesting experiment as follows. First, only wire B is vibrated beyond $v_c$, and it is confirmed that the flow around wire B enters the turbulent state. Second, only wire A is vibrated beyond $v_c$ up to about 1m/s, and then the flow around the wire keeps laminar. Keeping wire A vibrating, they start to vibrate wire B. After a while, wire A around which the flow is laminar makes turbulence. To understand this experimental result, the following scenario can be considered. Turbulence created in wire B emits a lot of vortex rings. These vortex rings propagate to wire A and collide with it. These vortices are stretched by the vibration of wire A and they finally form turbulence (hereafter we call wire A detector, and wire B generator).

To confirm this scenario, we have performed numerical simulation on vortex dynamics around an oscillating object.
2. Formulation

In modeling the experimental situation, an oscillating sphere is considered rather than a vibrating wire; this is because it is easier to calculate the case of a sphere, while it still reflects the essence of the physics that we are interested in. The problems we desire to solve numerically are what occurs when vortex rings coming from the generator collide with an oscillating sphere, and whether after collision these vortices form turbulence or not.

To formulate the vortex motion, the vortex filament model is used in the simulation [6, 7]. The superfluid velocity $v_{vor}$ generated by the vortices is calculated using Biot-Savart integration. An additional velocity field $v_b$ is imposed in order to satisfy the boundary condition on the surface of the sphere for a superfluid $(v_s - v_p) \cdot n = 0$, where $v_s = v_{vor} + v_b$ is the total superfluid velocity, $v_p$ is the sphere velocity, and $n$ is the unit normal vector to the surface of the sphere. The additional velocity $v_b$ takes the form

$$ v_b = \nabla \Phi_b + \nabla \Phi_u, $$

where $\Phi_b$ is the velocity potential that cancels the normal velocity component to the sphere made by the vortices [6] and

$$ \Phi_u = \frac{1}{2} \left( \frac{R}{r} \right)^3 v_p \cdot r $$

is the scalar velocity potential made by the moving sphere. Here $R$ denotes the radius of the sphere, and $r$ is the position from the center of the sphere. The reconnection is assumed to occur when a vortex becomes close to another vortex or a spherical boundary within the computational resolution. The parameters used in the calculation are chosen to match those of the experiments, such as a sphere radius of 3 $\mu$m and an oscillation frequency of 1590 Hz. The velocity of the oscillation is chosen to be 140 mm/s which is much higher than the observed critical velocity and 50 mm/s close to the velocity. The experiments were performed at a very low temperature of 30 mK, where the normal fluid component is negligible. Thus, we have performed the numerical simulation at the zero temperature limit to neglect mutual friction. Vortex rings all having a

![Figure 1](image1.png)

Figure 1. Time development of the vortex rings. (a) $t = 0.12$ ms, (b) $t = 0.24$ ms, (c) $t = 3.01$ ms. Horizontal lines in these figure indicate the injected vortex rings.

![Figure 2](image2.png)

Figure 2. Vortex line length at different vibration velocity. Solid and dashed lines respectively refer 50 mm/s and 140 mm/s.

radius of 1 $\mu$m [8] are injected every 0.05 ms vertically from the bottom [9] (Fig. 1). The place where the vortex rings is injected is moved sinusoidally in time so that all the rings hit the sphere. Since only vortices in the vicinity of the sphere will affect the result much, the vortices out of a computational box (20 $\mu$m)$^3$ are excluded from the calculation.

3. Results

As shown in Fig. 1, some vortices collide with the sphere and reconnect to it. The vortices attached to the sphere are stretched by the vibration of the sphere. The flow induced by the
sphere motion drives the end points of these vortices to the stagnation point on the sphere. One end of the attached vortex meets another end and makes reconnection. The following vortex rings collide with the sphere one after the other, and again start to be stretched. These series of processes produce larger and more vortex rings than the original ones. The vortex growth is limited by the force exerted by the sphere.

![Figure 3. Force exerted by the sphere.](image1)

![Figure 4. Vortex dynamics when the vortex rings are injected from below uniformly in space.](image2)

seen in the vortex line length (Fig. 2). Both cases of the vibration velocity 50 mm/s and 140 mm/s reached saturation in length because the increase of vortex line length by the vibration is balanced by the decrease due to the escape of the vortex rings out of the computation box. The mean vortex line length in the case of 140 mm/s is about three times larger than that in the case of 50 mm/s.

In the previous calculation, we assumed that the vortices which do not hit but pass by the sphere do not affect the result, that was reflected in the way the vortex rings were injected. Next, we modify the way to inject the vortex rings. In a real system, it is natural that the generator emits vortex rings uniformly in space toward the detector. Then keeping the probability that the injected rings hit the sphere equal to that of the previous simulation, we inject the vortex rings randomly in space (Fig. 4). The destiny after the well-grown vortices are detached becomes different. Since a vortex ring with the radius $R$ has the velocity $\sim \kappa/R$ with a quantum of circulation $\kappa$, the well-grown vortex rings have a smaller velocity than the injected vortex. Then the injected vortices from below overtake the grown vortex rings and form larger vortex rings through the reconnection[3]. Although this way to add vortex rings change the destiny of the detached vortices, it does not seem to change the mechanism that the vortices grow. Moreover this way consumes much computational time because the dynamics of more vortices must be calculated than that of the previous way. Thus we employ the previous way to add vortex rings from now on.

To compare the simulation with the corresponding experiment[5] quantitatively, we focus on the force driven by the sphere. It should be noted that the growth of vortices in length requires energy. In other words, the moving sphere supplies the energy to the vortices, and the sphere gets the reaction force. The reaction force emerges as the sudden drop of the wire velocity in the experiment when the turbulence around the wire is generated[5]. Since the vortex line length is saturated in the simulation, the energy given by the sphere equals to that of the vortices flying out of the computational box. The power from the sphere can be evaluated from the mean of the lost energy per time, and dividing the power by the velocity of the sphere, one can obtain
Table 1. Evaluation of the power and force given by the sphere.

|            | 30 mm/s | 50 mm/s | 70 mm/s | 90 mm/s | 140 mm/s |
|------------|---------|---------|---------|---------|----------|
| \( P \) (pW) | 0.026   | 0.047   | 0.090   | 0.141   | 0.281    |
| \( F \) (pN) | 0.89    | 0.94    | 1.29    | 1.57    | 2.05     |

the force exerted by the sphere on the fluid (table 1 and Fig. 3). To estimate the power due to the sphere motion, the energy of the injected vortices is subtracted from the energy lost by the vortices flying out of the box. Below 50 mm/s, the force exerted on the sphere tends to be constant, whereas above 50 mm/s it explicitly arises. This corresponds to the onset of the quantum turbulence in the experiment. The fact that the force evaluated here is \( 100 \sim 1000 \) times smaller than the actually measured value is explained by the difference in the shapes between the oscillating objects. In the experiment, the vortex tangle will be created over the wire with the length about 1 mm. On the other hand, the sphere has the radius of 3 \( \mu \)m and is expected to create the much less vortex tangle than the wire.

4. Summary
We have numerically followed the vortex dynamics in the presence of an oscillating sphere, and have made the process of the vortex growth clear. Injected vortex rings collide with the sphere and are stretched by the sphere. The rate of the vortex growth changes with the vibration velocity of the sphere. Injecting vortices uniformly in space, we have seen that the injected vortex rings which pass by the sphere do not contribute to the vortex growth but form larger vortices through the reconnection with the grown vortices. Finally, the onset of the quantum turbulence has been shown by evaluating the force exerted by the sphere, which corresponds to the sudden drop of the wire velocity measured in the experiment of the vibrating wire.

References
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