Non-Fermi liquid behavior in U and Ce intermetallics

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Abstract

In this paper we review the current experimental and theoretical situation of the description of non-Fermi liquid behavior (NFL) in U and Ce intermetallics. We focus on the magnetic and thermodynamic properties. We also discuss a recent theoretical interpretation of this behavior in terms of Griffiths-McCoy singularities close to the magnetic quantum critical point (QCP). We show how an effective Hamiltonian which contains both the RKKY coupling and the Kondo interaction can be written after high energy degrees of freedom away from the Fermi surface are traced out. We argue that dissipation due to particle-hole excitations close to the Fermi surface is a relevant perturbation at low temperatures and we estimate the crossover temperature $T^*$ above which power law behavior in specific heat and magnetic response occurs ($C_V/T \sim \chi(T) \propto T^{-1+\lambda}$ with $\lambda < 1$). Below $T^*$ a new regime dominated by dissipation is found and deviations from power law behavior are expected.
I. INTRODUCTION

The basis for the study of metals was set by Landau almost 50 years ago in his studies of He$^3$. During all these years the Landau theory has been a paradigm used to explain the experimental behavior and electronic properties of quantum Fermi liquids $[1,2]$. Initially the theory appeared as a phenomenological framework with a few parameters fixed by experiments. The presence of unknown parameters reflected, at that time, the lack of a microscopic theory. However, it was an extraordinary and necessary first step. Landau himself also established the route for the microscopic explanation for the validity of the theory. The Landau theory became the main tool for the study of the effects of correlations in electronic systems, and its foundation was eventually established on microscopic grounds using field theoretic methods $[3,4]$. The theory is supposed to hold at temperatures much lower than the Fermi temperature of the system. It is based mostly on the assumption that the interaction among electrons is short ranged (due to screening) and they are such that a perturbative expansion in the interaction converges. Thus there is a one-to-one correspondence between the interacting system of electrons and a weakly interacting system of quasiparticles. Moreover, the physics of the quasiparticles is completely determined by the Fermi surface.

As consequences of Landau’s theory the thermodynamic and response functions of the electron fluid are smooth functions of the temperature. One would have for instance a temperature independent Pauli susceptibility, $\chi(T) \propto$ constant, a temperature independent specific heat coefficient, $\gamma(T) = C_V/T \propto$ constant, and a Korringa law for the NMR relaxation rate, $1/(T_1 T) \propto$ constant. Furthermore, at low temperatures one expects the electronic resistivity to behave like $\rho(T) = \rho_0 + AT^2$, where $\rho_0$ is the resistivity due to impurities and $A$ is a coefficient which comes from three different sources: electron-electron Umklapp processes $[5]$, electron-electron interactions mediated by phonons $[6]$, and the inelastic scattering of electrons by impurities $[7]$. These predictions have been confirmed in a wide class of metals and are considered the trademark of Fermi liquid behavior.

Violations of Fermi liquid behavior have been expected for a long time in the context of one-dimensional conductors $[8]$ due to strong restrictions in phase space for electron-electron scattering. It turns out, however, that it is very hard to experimentally observe NFL behavior in one-dimensional systems. In the case of organic conductors $[9]$, which are considered the prototype of one-dimensional metals, there is always a crossover to higher dimensional behavior (that is, Fermi liquid behavior) due to coupling between chains at low temperatures. The only clear observation of NFL behavior in low dimensional systems appears in the very special case of the edge states of quantum Hall bars $[10]$ where NFL behavior is due to the Landau level degeneracy. Indeed, there is a strong controversy about the possibility of NFL behavior in dimensions higher than one. The subject was raised by Anderson in the context of high temperature superconductors $[11]$. Although the possibility of NFL behavior in 2 dimensions has not been discarded there are today strong arguments against it $[12–16]$. In 3 dimensions Landau’s Fermi liquid theory is assumed to be the correct starting point. We stress that even in the presence of disorder Fermi liquid theory should be valid in 3 D (at least when the disorder is weak enough to be treated in perturbation theory) $[17]$. Thus, it is indeed very surprising that for such a broad class of U and Ce alloys (which clearly show three-dimensional behavior) deviations from Landau’s theory are so abundant.
Actually, there are nowadays so many examples of alloys presenting NFL behavior that the discovery of a new compound which exhibits such a behavior is not a surprise.

We organize this paper as follows: in the next section we give a brief overview of the theoretical and experimental situation on the problem of NFL behavior in U and Ce intermetallics. We apologize in advance to any whose work we have unintentionally left out. We concentrate on the situation of the thermodynamic and magnetic response in these systems and leave the important problem of transport for a later publication. In Section II we discuss the problem of Griffiths-McCoy singularities in insulating magnets; in Section IV we discuss the Kondo lattice problem and show how the RKKY interaction and the Kondo effect appear at the Hamiltonian level when high energy degrees of freedom are eliminated from the Hamiltonian; in Section V we discuss the differences between the insulating case and the metallic case for the formation of Griffiths-McCoy singularities; finally Section VI contains our conclusions.

II. OVERVIEW OF THE NFL BEHAVIOR IN U AND CE INTERMETALLICS

The systems we are considering in this paper are metallic alloys of rare earths or actinides which can be classified as (1) Kondo hole systems, in which the rare earth or actinide (R) is substituted by a non-magnetic metallic atom (M) with a chemical formula $\text{R}_{1-x}\text{M}_x$ (a typical example is $\text{U}_{1-x}\text{Th}_x\text{Pd}_2\text{Al}_3$); (2) Ligand systems, where one of the metallic atoms (M1) is replaced by another (M2) but the rare earths or actinides are not touched and thus have the formula $\text{R(M1)}_{1-x}\text{(M2)}_x$ (as, for instance, $\text{UCu}_{5-x}\text{Pd}_x$). Often these alloys order magnetically at $x = 0$ (ordered Kondo lattices) and long range order is lost at some $x = x^*$ as shown in Fig.1. For $x > x^*$ the ordered state is replaced by a metallic state which shows physical properties which deviate strongly from the predictions of Fermi liquid theory. This state is called non-Fermi liquid state (NFL).

In NFL systems it is usually observed that even in the paramagnetic phase the specific heat coefficient and the magnetic susceptibility do not saturate as expected from the Landau scenario. The theoretical reason for this anomalous behavior is still not completely understood; many different theories have been proposed and the subject is very controversial. One possible reason for singular behavior in the thermodynamic and response functions of the system is due to closeness of these systems to long-range order. The idea that a quantum critical point (QCP) could be responsible for NFL behavior was proposed by Hertz [18] and later extended by Millis and Continentino [19]. Indeed, there is strong evidence that QCP physics is responsible for NFL behavior in $\text{CeCu}_{6-x}\text{Au}_x$, where NFL behavior can be fine tuned via magnetic fields or pressure to the QCP [20]. It turns out, however, that even for this compound there is controversy about the correct description of the QCP [21,22]. Another system recently studied which seems to be in this category is $\text{CeNi}_2\text{Ge}_2$, which has been shown to have a minimum amount of disorder [23]. Hertz also studied the problem of disorder in a XY magnet and found disorder to be a relevant perturbation [24]. In this context Hertz conjectured that disorder could lead to clustering of magnetic moments. More recently it has been shown that quenched disorder has a strong effect on the properties of quantum antiferromagnets and leads to very unconventional critical behavior [25].

In all the cases studied so far the data for the susceptibility and specific heat have
been fitted to weak power laws or logarithmic functions \[26\]. The resistivity of the systems discussed here can be fitted with \(\rho(T) = \rho_0 + A T^\alpha\) where \(\alpha < 2\). Neutron scattering experiments in \(\text{UCu}_{5-x}\text{Pd}_x\) \[27\] show that the imaginary part of the frequency dependent susceptibility, \(\Im(\chi(\omega))\), has power law behavior, that is, \(\Im(\chi(\omega)) \propto \omega^{1-\lambda}\) with \(\lambda \approx 0.7\), over a wide range of frequencies (for a Fermi liquid one expects \(\lambda = 1\)). Moreover, consistent with this behavior the static magnetic susceptibility seems to diverge with \(T^{-1+\lambda}\) at low temperatures \[27\]. What is interesting about \(\text{UCu}_{5-x}\text{Pd}_x\) is that it has been shown in recent EXAFS experiments that this compound has a large amount of disorder \[28\] consistent with early NMR and \(\mu\)SR experiments \[29\]. Even in stoichiometric systems like \(\text{CeAl}_3\) there is evidence of spatial inhomogeneity \[30\]. In \(\text{CePd}_2\text{Al}_3\) it has been shown that while polycrystalline samples show a magnetic phase transition at finite temperatures the critical temperature is driven to zero in single crystals due to internal stresses which suppress the moment formation \[31\]. Moreover, \(\text{UCu}_4\text{Pd}\) is supposed to be exactly at the QCP for antiferromagnetic order. All these properties have also been seen in a similar alloys such as \(\text{UCu}_{5-x}\text{Al}_x\) and \(\text{UCu}_{5-x}\text{Ag}_x\) \[32\]. Another system which is also close to a magnetic order is \(U_{1-x}Y_x\text{Pd}_3\) which shows spin glass order \[33\]. It was the study of this system which lead Andraka and Tsvelik to propose that the NFL behavior observed in this compound was due to the spin glass transition at the QCP \[34\]. This point of view has also been explored by other researchers in the field \[35\]. Another interesting example where NFL behavior happens close to a QCP is in the system \(U_{1-x}\text{Th}_x\text{Cu}_2\text{Si}_2\) which shows a ferromagnetic QCP \[36\]. Thus, NFL behavior has been observed in systems with very different types of magnetic ordering. Indeed, the magnetic behavior in these systems is very rich. Very recent frequency dependent susceptibility measurements have found signs of super-paramagnetism (that is, cluster physics \[37\]) close to the QCP of \(\text{UCu}_{5-x}\text{Pd}_x\) \[38\]. Fluctuating magnetic moments were also found in \(\mu\)SR experiments in \(\text{Ce}(\text{Ru}_{1-x}\text{Rh}_x)_2\text{Si}_2\) close to the QCP \[39\]. Actually, pressure experiments in the same compound have revealed the importance of disorder for the appearance of NFL behavior \[40\]. The same type of physics is found in very complex systems such as \(\text{U}_{3-x}\text{Ni}_3\text{Sn}_{1-y}\) \[41\] and even in stoichiometric alloys like \(\text{Yb}_2\text{Ni}_2\text{Al}\) where a coexistence of magnetic and paramagnetic phases have been observed \[42\]. The phenomena observed here are indeed very similar to those observed in simpler magnetic alloys close to a magnetic phase transition as has been shown in recent experiments in \(\text{Ni}_x\text{Pd}_{1-x}\) close to a ferromagnetic QCP \[43\]. Indeed, the phenomena of clustering, superparamagnetism, and magnetic fluctuations has been discussed long ago in simple alloys such as \(\text{Ni}_x\text{Cu}_{1-x}\) both theoretically and experimentally \[44\]. As in their f-electron counterparts, the specific heat and magnetic susceptibility of these systems deviate strongly from Fermi liquid behavior in a relatively broad region around the quantum critical point. Indeed, the phenomenon of NFL behavior is not limited to f-electron systems but is also very common in d-electron compounds \[45\].

Another source of NFL behavior is of single impurity nature. Single impurity approaches for the NFL problem are very important because the Kondo effect \[46\] is known to happen in the dilute limit of Kondo hole systems (e.g., in \(U_{1-x}\text{Th}_x\text{Pd}_3\) for \(x \approx 1\)) and has been suggested as the source of heavy fermion behavior \[48\] in undiluted ligand systems (e.g., \(\text{CeAl}_3\)) \[49\]. The Kondo effect is probably one of the most studied problems in many-body theory. It has been understood from many different points of view, from renormalization group (RG) calculations \[52\] to the exact analytic solution \[53\] and from the conformal field.
theory point of view [54]. In the anisotropic case the Kondo Hamiltonian can be mapped via bosonization into the dissipative two level system (DTLS) [55,56].

The mapping between the Kondo problem and DTLS was developed in order to understand the problem of a quantum phase transition in the DTLS [57,58] and was believed to be valid close to the QCP. Nowadays, extensive numerical simulations have shown that the mapping is valid over most of the parameter space [59]. In the single channel Kondo problem up and down spin electrons spin-flip scatter against the magnetic moment. Nozières and Blandin proposed that when the number of scattering channels is increased one can obtain a local NFL ground state [60]. This is the so-called multichannel Kondo effect. In 1986 D. Cox proposed an elegant mechanism for NFL behavior in U systems based on a multichannel effect of quadrupolar origin [61]. The same mechanism has been studied in the context of the Kondo lattice [62]. There is still controversy about the applicability of the quadrupolar Kondo effect to the compounds we are discussing [63]. The quadrupolar Kondo effect requires a non-magnetic $\Gamma_3$ ground state which has been confirmed to exist in PrInAg$_2$ [64]. It turns out that the experimental situation in this compound is far from clear: while the magnetic susceptibility seems to show NFL behavior the specific heat is well described by Fermi liquid theory [65]. Moreover, the exponents predicted by multichannel effects are not consistent with the experimental data in many compounds. Another source of NFL behavior based on single impurity physics is the so-called Kondo disorder approach in which it is assumed that due to the intrinsic disorder in the Kondo lattice there is a broad distribution of Kondo temperatures going down to a vanishing one [29]. This kind of approach has been applied to UCu$_{5-x}$Pd$_x$ [66]. Naturally, the main criticism to the single impurity approaches is that the systems where NFL behavior is observed are concentrated. Furthermore, NFL behavior usually occurs close to a QCP where interactions among the moments and tendency to magnetic ordering is very important. Thus, the problem of single impurity versus QCP physics as a source of NFL behavior remains a very controversial one and debate among researchers is still in progress.

In the next sections we discuss a possible explanation for the NFL observed in these alloys which is due to Griffiths-McCoy singularities close to the QCP point [67]. The origin of these singularities is due to the competition between the RKKY interaction, which leads to magnetic order, and the Kondo effect, which leads to magnetic quenching in the presence of disorder due to alloying.

### III. GRIFFITHS-MCCOY SINGULARITIES

The simplest way to understand the nature of Griffiths singularities is to imagine the dilution of a magnetic lattice by non-magnetic atoms. Long range order is lost at percolation threshold when the last infinite cluster of magnetic moments seize to exist. Above the threshold the system is composed of finite clusters of magnetic atoms. Griffiths showed that when a magnetic field is applied to the percolating lattice there is a non-analytic contribution of the clusters to the free energy [68]. This contribution comes from rare large clusters. The classical problem was studied in great detail by many researchers in the 70’s [69]. An important special model related with the problem of Griffiths singularities was proposed by McCoy and Wu [70] and studied more recently by Shankar and Murthy [71].
The McCoy-Wu model is a rectangular Ising model with disorder in only one direction. The importance of this model relies on the fact that it is the only known exactly solvable model with disorder. Moreover, it was shown that in this model while the system orders magnetically at some temperature $T_c$ the magnetic susceptibility diverges before the system reaches $T_c$. This strange behavior is again due to Griffiths singularities. Although classical Griffiths singularities are rather weak (and for a long time researchers believed only the singularity coming from the infinite cluster would be observable experimentally) there is recent experimental evidence for their existence in some Ising magnets \([72,73]\).

It turns out that from a statistical mechanical point of view a classical 2D Ising problem is equivalent to a 1D quantum Ising model at zero temperature. Thus, the McCoy-Wu problem maps, at zero temperature, into the random transverse field Ising chain. The random transverse field Ising model has also been extensively studied analytically \([73,74]\) and numerically \([74]\) in one and higher dimensions and evidence for Griffiths-like singularities was obtained. In particular, Thill and Huse proposed a quantum droplet model for the problem where the magnetic cluster is treated like a single degree of freedom (or two level system). This kind of treatment agrees very well with the exact 1D calculation. It has been shown that the same type of singularities can also happen in more complicated random magnetic systems such as in the XY model \([75]\). One of the main characteristics of the Griffiths-McCoy singularities is the divergence of the physical quantities at zero temperature with non-universal power laws. The magnetic susceptibility, for instance, behaves like $\chi(T) \propto T^{-1+\lambda}$ and diverges in the paramagnetic phase if $\lambda < 1$, and the nonlinear susceptibility diverges with an even stronger power law, $\chi_{nl}(T) \propto T^{-3+\lambda}$ (indeed it has been shown that for the pure Ising model $\lambda \to 0$ at the QCP \([74]\)). Experimental evidence for a divergent non-linear susceptibility was found in the compound $U_{1-x}Th_xBe_{13}$ \([79]\).

That Griffiths singularities should be important in some single band correlated systems was proposed by Bhatt and Fisher \([80]\) and extended more recently by Sachdev \([81]\). A similar type of phenomenon happens also in magnetically doped semiconductors where a singlet phase was proposed by Bhatt and Lee \([82]\). It was shown that for a single band Hubbard model in a disordered environment the magnetic properties could be explained in terms of a quantum spin glass ground state \([83]\). We have proposed recently that the Kondo lattice problem is in the same class of problems as the random transverse Ising model \([84]\). Dipolar interactions are too small to account for the ordering temperature in these systems (which range from 100 K down to 10 K in the pure compounds), and the direct exchange between f orbitals is very weak since the spatial extent of the f orbitals is small. As is well-known the magnetism in Kondo lattices comes from the localized f-moments which are weakly hybridized with the conduction band. In the pure compound (say, UPd$_2$Al$_3$ or UCu$_5$) the moments interact with each other via the RKKY interaction which is propagated by the conduction band \([84]\) and in the presence of disorder and spin-orbit effects the RKKY interaction becomes short ranged \([85]\). Moreover, most of the heavy fermion alloys are magnetically anisotropic because of crystal field effects or spin-orbit coupling which are known to be very important in these systems. In this case the magnetic phase diagram can be very rich since an anisotropic spin exchange interaction, or Dzyaloshinsky-Moriya (DM)
exchange interaction \[80\], is generated. These anisotropies have been observed long ago in alloys of rare earths of the form R-CrO$_3$ \[81,88\] where R is a rare earth.

In a Kondo hole system (say, U$_{1-x}$Th$_x$Pd$_2$Al$_3$) the destruction of magnetism occurs mainly by the dilution of the magnetic lattice and the QCP is the percolation threshold for the lattice. In the ligand systems (like, say, in UCu$_{5-x}$Pd$_x$) the dilution is a more subtle effect because the magnetic atoms remain on the lattice. In order to understand how dilution quenches a magnetic moment in a ligand system one has to look at what happens in the related heavy fermion materials which do not show long range order. In heavy fermions the magnetic moments are quenched by the Kondo effect \[50\] as described, for instance, in the dynamical mean field (or \(d \rightarrow \infty\)) theories of the Anderson lattice \[89\]. Thus, it is reasonable to assume that in ligand systems the effect of doping is to affect locally the hybridization between localized moments and conduction electrons. This naturally leads us to the picture proposed long ago by Doniach \[90\], namely, there are two relevant energy scales in the problem: the Kondo temperature of the moment \(T_K\) which is an exponential function of the exchange \(J\) between local moments and the itinerant electrons \((J \propto V^2\) where \(V\) is the hybridization matrix element and \(T_K \propto \exp[-1/(N(0)J)]\) where \(N(0)\) is the density of states at the Fermi surface\) and the RKKY temperature scale, \(T_{RKKY}\), associated with magnetic ordering of the moments which scales with \(J^2\). As shown in Fig.2 there is a critical value of \(J\) (say, \(J_c\)) for which these two energy scales become of the same order of magnitude. For \(J < J_c\) we have \(T_K < T_{RKKY}\) and therefore, as the temperature is lowered the system orders magnetically before the moment is quenched. If \(J > J_c\), that is, if \(T_K > T_{RKKY}\), as the temperature is lowered the moment is quenched before it has the chance to order. Although this picture is quite naive it has been confirmed in mean field theories \[91,92\], numerical calculations \[93\] and pressure experiments in ordered \[94\] and disordered Kondo lattices \[95\]. In the absence of disorder this competition leads to a finite ordering temperature \(T_N\) which vanishes at a QCP at \(J_c\) as shown in Fig.2. Moreover, the competition appears explicitly in a simple problem of two interacting moments in the presence of a Fermi sea which is the two impurity Kondo problem \[96\]. Numerical works on this problem have confirmed the theoretical expectations \[97\]. Thus, in our scenario long range order is lost in a ligand system by the local quenching of the magnetic moments. Again one has to deal with a quantum percolation problem. The QCP is again the percolation point of the magnetic lattice. Away from the QCP only magnetic clusters can exist. As one further dopes the system away from the magnetic phase one eventually finds a heavy fermion ground state where all the moments are quenched \[50\] (unless, of course, there is a structural or another magnetic phase transition in the intermediate region).

In order to describe mathematically the competition between the Kondo effect and the RKKY interaction we may start with the mean field description of the ordered phase (this is a good first approximation since there is true long range order in the system at finite temperatures and quantum fluctuations are small). On the one hand the conduction electron band is renormalized by the average field created by the ordered magnetic moments. Unless there are commensuration effects between the magnetic ordering vector and the Fermi momentum the electrons remain gapless (such commensuration effects are very unlikely in these systems with complex unit cells). On the other hand the conduction electrons produce an effective medium for the propagation of the RKKY interaction. Suppose one dilutes slightly the ordered state by changing a local exchange constant \(J\) to a value much larger than the
average. In this case, like in the Doniach argument, the local moment instead of participating in the collective magnetic state will prefer to form a local singlet. That is, one has again a simple Kondo effect with a renormalized conduction band. The magnetization of the system has to drop. Mathematically, this quenching of the magnetic moment can be described in terms of the anisotropic Kondo problem. Actually, as we discussed, the magnetism in these systems is anisotropic and therefore the Kondo effect does not have SU(2) symmetry. Indeed, there are very recent inelastic neutron scattering experiments in Ce$_{1-x}$La$_x$Al$_3$ which show evidence for anisotropic Kondo behavior [98]. Therefore, as mentioned previously, the Kondo effect can be mapped into the DTLS. As we discuss in the next section the XY part of the Kondo problem becomes a transverse field - the origin of this term can be thought as a transverse magnetic field applied by the electron spin on the magnetic moment - and the Ising component of the Kondo exchange describes the coupling of the magnetic moment to a heat bath - which represents the fact that each time the magnetic moment flips it produces particle-hole excitations at the Fermi surface. It turns out that the coupling to the heat bath becomes small in the limit of large anisotropy and in zeroth order can be disregarded. Thus, we have argued [67] that in this extreme limit the Kondo lattice problem maps into the random transverse field Ising model which, as we said previously, has been shown to present Griffiths-McCoy singularities with power law behavior at low temperatures. Recent experiments have shown that power law behavior is consistent with measurements of magnetic susceptibility and specific heat in these systems [99]. In particular, for the Griffiths phase one has $\chi(T) \sim \gamma(T) \propto T^{-1+\lambda}$ with $\lambda < 1$. In the next section we show that the residual coupling with the particle-hole bath changes the behavior of the response functions below a certain energy scale.

The Griffiths phase picture has been very successful in describing the power law behavior of the physical quantities in some of the systems mentioned above, especially, UCu$_{5-x}$Pd$_x$ where structural disorder was clearly measured [29] and the exponents measured from specific heat and susceptibility agree well with each other ($\lambda \approx 0.72$) [99]. In other alloys such as U$_{1-x}$Th$_x$Pd$_2$Al$_3$ ($\lambda \approx 0.8$ from specific heat and $\lambda \approx 0.63$ from susceptibility data) [99] or Ce(Pd$_{1-x}$Ni$_x$)$_2$Ge$_2$ ($\lambda \approx 0.7$ from specific heat and $\lambda \approx 0.84$ from susceptibility data) [100] the agreement between exponents is not as good (although, we should stress, power law behavior is clearly observed). Moreover, in systems like U$_{0.2}$Y$_{0.8}$Pd$_3$ the divergence seems to be stronger than power law or logarithm [99]. One possible explanation for these stronger divergences might be related with the possibility that the disorder in these systems is correlated and not random. In this case one expects stronger divergences [101]. Furthermore, in some of the systems described above there may be a return to Fermi liquid behavior at very low temperatures (for instance, in UCu$_4$Pd is evidence of that below 0.1 K [98] while in CeRhRuSi$_2$ it seems to occur below 1 K [102]). We argue below that some of these problems can be resolved if the metallic character of the electronic environment is taken into consideration.

IV. GRIFFITHS SINGULARITIES AND THE KONDO LATTICE PROBLEM

It is intuitively obvious that the Doniach argument lead to an inhomogeneous behavior when it is taken locally instead of globally. The main problem here is how this argument can
be tested at the Hamiltonian level. In this section we show how this can be accomplished for the Kondo lattice model from the renormalization group point of view [102].

It is imperative in the context of the systems discussed in this paper to take into account spin-orbit effects since these are very important for f-electron magnetism. In this case the exchange between local moments and conduction electrons is not isotropic in spin space and can be generally written as [103]:

\[
H = \sum_{k,\kappa} \epsilon_\kappa(k)c_{k,\kappa}^\dagger c_{k,\kappa} + \sum_i \sum_{a,b,\kappa,\kappa'} J_{a,b}(i)S^a(i)c_{\kappa}^\dagger(i)\tau_{a,\kappa,\kappa'}^b c_{\kappa'}(i)
\]  

(1)

where \(\kappa = 1,2\) labels the spin states in the diagonal basis, \(J_{a,b}\) are the effective exchange constants between the localized spins, \(S^a(i)\), and the conduction electron spin, \(\sum_{\kappa,\kappa'} c_{\kappa}^\dagger(i)\tau_{a,\kappa,\kappa'}^b c_{\kappa'}(i)\). In the simplest case of uniaxial symmetry (which is going to be discussed throughout this paper) one has \(J_{a,b} = J_a \delta_{a,b}\) where \(J_z > J_x = J_y = J_\perp\).

The main problem in studying the competition of RKKY and Kondo effect in the Hamiltonian (1) is related with the fact that both the RKKY and the Kondo effect have origin on the same magnetic coupling between spins and electrons. What allows us to treat this problem is the fact that the RKKY interaction is perturbative in \(J/E_F\) while the Kondo effect is not. Moreover, the RKKY interaction depends on electronic states deep inside the Fermi sea while the Kondo effect is a Fermi surface effect. Thus, it seems to be possible to use perturbative renormalization group approach to treat the RKKY interaction while for the Kondo effect one needs to do a better job. This kind of treatment was proposed recently in the context of the two impurity Kondo problem [104].

We will consider, for simplicity, the case where the Fermi surface for the electrons is spherical (non-nested, non-spherical Fermi surfaces can be treated in an analogous way). The local electron operator can be written in momentum space as

\[
c_\kappa(r) = \frac{1}{\sqrt{N}} \sum_k e^{i k \cdot r} c_{k,\kappa}.
\]

(2)

We now separate the states in momentum space into three different regions of energy as shown in Fig. 3, namely, \(\Omega_0\) where \(k_F - \Lambda < k < k_F + \Lambda\); \(\Omega_1\) where \(k < k_F - \Lambda\); and \(\Omega_2\) where \(k > k_F + \Lambda\) where \(\Lambda\) is an arbitrary cut-off. Observe that in this case the sum in (2) can now also be split into these three different regions. The problem we want to address is how the states in region \(\Omega_0\) close to the Fermi surface renormalize as one traces out high energy degrees of freedom which are present in regions \(\Omega_1\) and \(\Omega_2\). We can perform this calculation perturbatively in \(J/E_F\). For that purpose it is more convenient to use a path integral representation for the problem and write the quantum partition function as

\[
Z = \int DS(n,t)D\bar{\psi}(r,t)D\psi(r,t) \exp \left\{ \frac{i}{\hbar} \mathcal{S}[\bar{\psi},\psi] \right\}
\]

(3)
in terms of Grassman variables \(\bar{\psi}\) and \(\psi\) and where the path integral over the localized spins also contains the constraint that \(S^2(n,t) = S(S + 1)\). The quantum action in (3) can be separated into three different pieces, \(\mathcal{S} = \mathcal{S}_0[S] + \mathcal{S}_0[\bar{\psi},\psi] + \mathcal{S}_f[S,\bar{\psi},\psi]\), where \(\mathcal{S}_0[S]\) is the free actions of the spins (which can be written, for instance, in terms of spin coherent states [105]).
\[ S_0[\bar{\psi}, \psi] = \sum_{\alpha, \gamma} \int d\omega \int d\mathbf{k} \bar{\psi}_\alpha(\mathbf{k}, \omega) (\omega + \mu - \epsilon_\alpha(\mathbf{k})) \delta_{\alpha, \gamma} \psi_\gamma(\mathbf{k}, \omega) \] (4)

is the free action for the conduction electrons and

\[ S_I[\bar{\psi}, \psi, \psi] = \sum_{\alpha, \gamma} \sum_{n, a, b} \int dt J_{a, b}(n) S_a(n, t) \tau^b_{\alpha, \gamma} \bar{\psi}_\alpha(n, t) \psi_\gamma(n, t) \] (5)

is the exchange interaction between conduction electrons and localized moments.

We can now split the Grassman fields into the momentum shells defined above, that is, we rewrite the path integral as

\[ Z = \int DS(n, t) \prod_{i=0}^2 D\bar{\psi}_i(r, t)D\psi_i(r, t) \exp \left\{ \frac{i}{\hbar} S[\bar{\psi}_i, \psi_i] \right\} \] (6)

where the index 0, 1, 2 refers to the degrees of freedom which reside in the momentum regions \( \Omega_0, \Omega_1 \) and \( \Omega_2 \), respectively. In this case the action of the problem can be rewritten as \( S = S_0[\bar{\psi}_0, \psi_0] + \sum_{i=0}^2 S_0[\bar{\psi}_i, \psi_i] + S_I[\bar{\psi}_i, \psi_i] \). Notice the free part of the electron action is just a sum of three terms (essentially by definition since the non-interacting problem is diagonal in momentum space). Moreover, the exchange part mixes electrons in all three regions defined above:

\[ S_I = \sum_n \sum_{i, i'}^2 \int dt J_{a, b}(r_n) S_a(r_n, t) \tau^b_{\alpha, \gamma} \bar{\psi}_\alpha, i(r_n, t) \psi_\gamma, i'(r_n, t) . \] (7)

Since we are interested only on the physics close to the Fermi surface we trace out the fast electronic modes in the regions \( \Omega_1 \) and \( \Omega_2 \) assuming that \( J_{a, b} \ll \mu \). In this case, as we show elsewhere [103], besides the renormalization of the parameters in free action of the electrons in the region \( \Omega_0 \), we get the RKKY interaction between localized moments, that is, the effective action of the problem becomes:

\[ S_{\text{eff}}[\bar{\psi}_0, \psi_0] = S_0[\bar{\psi}_0, \psi_0] + \sum_n \sum_{\alpha, \gamma, a, b} \int dt J^R_{a, b}(r_n) S_a(r_n, t) \tau^b_{\alpha, \gamma} \bar{\psi}_\alpha, 0(r_n, t) \psi_\gamma, 0(r_n, t) + S_0[S] + \sum_{n, m, a, b} \int dt \Gamma^R_{a, b}(r_n - r_m) S_a(r_n, t) S_b(r_m, t) \] (8)

where \( \Gamma^R_{a, b}(r_n - r_m) \) is the cut-off dependent RKKY interaction between the local moments and \( J^R_{a, b}(n) \) is the Kondo electron-spin coupling renormalized by the high energy degrees of freedom. The renormalization can be calculated order by order in perturbation theory [103]. We observe further that the perturbation theory here is well behaved and there are no infrared singularities in the perturbative expansion. Thus, the limit of \( \Lambda \to 0 \) is well-defined. In this limit \( \Gamma_{a, b}(r_n - r_m, \Lambda \to 0) \) becomes the usual RKKY interaction one would calculate by tracing all the energy shells of the problem. Observe that there are no retardation effects in tracing this high energy degrees of freedom since they are much faster than the electrons close to the Fermi surface and therefore adapt adiabatically to their motion (the situation here is somewhat similar to the one in the Born-Oppenheimer approximation where the ions are much slower than the electrons and therefore can only renormalize the coupling
constants). Hamiltonian (8) is the basic starting point of our discussion and contains the basic elements for the discussion of magnetic order in the system. As discussed in the Doniach’s argument, the RKKY interaction tends to order the magnetic moments while the Kondo coupling tends to quench it. It is the interplay of these two interaction which leads to the physics we discuss here. The way this quenching occurs is fundamental for the understanding of physics of this problem and it is discussed in the next section.

V. DISSIPATION IN METALLIC MAGNETIC ALLOYS

In order to understand our line of argument it is important to study a very simple case of (8) where RKKY interactions are not present, that is, the single impurity Kondo problem. As we have said previously this problem is well understood and here we will just quote a few of the important results for our discussion. Notice that the single impurity Kondo effect should occur in the case of Kondo hole systems when \( x \ll 1 \) or in moment quenching in ordered ligand systems when \( x \approx 1 \).

For the single impurity the mathematical description simplifies greatly because we just have to solve a scattering problem in terms of incoming and outgoing waves. Thus, the problem is effectively one dimensional with a boundary condition at the impurity position. In this case we can use the technique of bosonization to understand the basic physics. First of all we can show that the renormalization of the Ising component of the Kondo interaction (8) is given in terms of the phase shift \( \delta \) of the electrons by the impurity:

\[
J_z^R = 8v_F \delta(J_z)
\]

where \( v_F \) is the Fermi velocity and

\[
\delta(J_z) = \arctan \left( \frac{\pi N(0)J_z}{4} \right).
\]

Since we are treating states very close to the Fermi surface we linearize the electron dispersion close to the Fermi surface:

\[
E_k = E_F + v_F(|k| - k_F),
\]

in which case the conduction band Hamiltonian is written as

\[
H_C = \sum_{p,\sigma} v_F p c_{p,\sigma}^\dagger c_{p,\sigma}
\]

where \( c_{p,\sigma} \) creates an electron with spin \( \sigma \), momentum \( |k| = p + k_F \) and angular momentum \( l = 0 \). Thus, in writing (12) we have reduced the problem to an effective one-dimensional problem. We introduce right, \( R \), and left, \( L \), moving electron operators

\[
\psi_{R(L),\sigma}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\Lambda}^{\Lambda} dk e^{ikx} e^{ikx} e^{ikx}
\]

which are used to express the electron operator as
\[ \psi_\sigma(x) = \psi_{R,\sigma}(x)e^{ik_F x} + \psi_{L,\sigma}(x)e^{-ik_F x}. \] (14)

In any impurity problem the right and left moving operators produce a redundant description of the problem since they are actually equivalent to in-coming or out-going waves out of the impurity. Therefore we have two options: either we work with right and left movers in half of the line or we work in the full line but we impose the condition \( \psi_{R,\sigma}(x) = \psi_{L,\sigma}(-x) \). We will use the last option. Thus, from now on we drop the symbol \( R \) from the problem and work with left movers only. The left mover fermion can be bosonized as

\[ \psi_\sigma(x) = \frac{K_\sigma}{\sqrt{2\pi a}}e^{i\Phi_\sigma(x)} \] (15)

where

\[ \Phi_\sigma(x) = \sum_{p>0} \sqrt{\frac{\pi}{pL}} \left( (b_p + \sigma a_p)e^{i\sigma px} - h.c. \right) \] (16)

and \( K_\sigma \) is a factor which preserves the correct commutation relations between electrons, that is, \( \{ \psi_\sigma(x), \psi_{\sigma'}(y) \} = \delta(x-y)\delta_{\sigma,\sigma'} \). The basic operators in bosonization are the charge and spin densities \( (k > 0) \):

\[ \rho(k) = \sum_{p,\sigma} c_{p+k,\sigma}^\dagger c_{p,\sigma} \]
\[ \sigma(k) = \sum_{p,\sigma} \sigma c_{p+k,\sigma}^\dagger c_{p,\sigma} \] (17)

which are written as bosonic operators \( b_k \) and \( a_k \),

\[ b_k = \sqrt{\frac{\pi}{kL}}\rho(-k) \]
\[ a_k = \sqrt{\frac{\pi}{kL}}\sigma(-k) \] (18)

and obey canonical commutation relations \( [a_k, a_{k'}^\dagger] = [b_k, b_{k'}^\dagger] = \delta_{p,k} \).

In terms of the boson operators, the Kondo Hamiltonian (8) for a single impurity becomes

\[ H = v_F \sum_{p>0} p(a_p^\dagger a_p + b_p^\dagger b_p) + J_z S^z \sum_{k>0} \sqrt{\frac{k}{\pi L}}(a_k + a_k^\dagger) \]
\[ + \frac{J_{\perp}}{4\pi a} \left( S^+ \sum_{k>0} \sqrt{\frac{\pi}{kL}}(a_k - a_k^\dagger) + h.c. \right). \] (19)

Moreover, this Hamiltonian can be brought to a simpler form if one performs a unitary transformation

\[ U = \exp \left\{ S^z \sum_{k>0} \sqrt{\frac{\pi}{kL}}(a_k - a_k^\dagger) \right\} \] (20)

which transforms the Hamiltonian to (we drop the \( b_p \) modes since they decouple from the impurity)
\[ H' = U^{-1}HU = v_F \sum_{p>0} p a_p^\dagger a_p + S^z \left( J^R_z - \pi v_F \right) \sum_{k>0} \sqrt{\frac{k}{\pi L}} (a_k + a_k^\dagger) \]
\[ + \frac{J_z}{4\pi a} \left( S^+ + S^- \right). \]  

(21)

An important observation here is that the unitary transformation does not affect \( S^z \). Observe that (21) describes the physics of a two level system coupled to a bosonic environment [55]. The basic physics of the Kondo problem becomes rather simple from the point of view of (21): while the XY component of the Kondo interaction (associated with the coupling \( J_\perp \)) flips the local spin and acts as a transverse field, the Ising coupling (associated with \( J_z \)) leads to a dissipative effect such that each time the spin flips it produces particle-hole excitations at the Fermi surface. It can be shown that the Kondo temperature of the anisotropic Kondo effect can be written as [55,59]:

\[ k_B T_K = \frac{E_c}{\alpha} \left( \frac{\Delta_0}{E_c} \right)^{\frac{1}{1-\alpha}} \]  

(22)

where \( E_c \) is a cut-off energy scale of the order of the bandwidth and

\[ \Delta_0 = \frac{J_\perp}{2\pi a} \]
\[ \alpha = \left( 1 - \frac{J^R_z}{\pi v_F} \right)^2 = \left( 1 - \frac{2}{\pi} \arctan \left( \frac{\pi N(0) J_z}{4} \right) \right)^2. \]  

(23)

Observe that for \( J_z, J_\perp \ll E_c \) the Kondo temperature (22) looks very similar to the SU(2) expression \( k_B T_K \approx E_c \exp \{ -1/(N(0)J) \} \). Indeed, from (22) we have

\[ k_B T_K \approx E_c \exp \left\{ -\ln(1/(N(0)J_\perp)) \right\}/N(0)J_z \].

(24)

Notice that the Kondo temperature of an anisotropic Kondo problem is not a single parameter quantity since it depends on the Ising component \( J_z \) and the XY component given by \( J_\perp \). Moreover, we have \( \alpha < 1(J_z > 0) \) in the case of the antiferromagnetic coupling and \( \alpha > 1(J_z < 0) \) for the ferromagnetic coupling. As is well-known the ferromagnetic Kondo effect is related with the formation of a triplet state and therefore to the freezing of the moment (and not quenching!).

When \( J^z \gg 2v_F \) (the limit of large uniaxial anisotropy) we see from [1] that \( J^R_z \to \pi v_F \) and the Hamiltonian reduces to

\[ H = \frac{J_\perp}{2\pi a} S_x + v_F \sum_{p>0} p a_p^\dagger a_p \]

(25)

with the decoupling of the spin degrees of freedom to the bosonic modes. This is the dissipationless limit of the problem. Observe that in this limit the eigenstates of the system are eigenstates of \( S_x \), that is, the transverse field. We can immediately see from (22) that in this limit
which is a large Kondo temperature.

In ref. [67] we proposed that the Kondo effect which happens in U and Ce alloys has the structure of (25) since most of these systems are not cubic and therefore can be highly anisotropic. Moreover, even in cubic systems the alloying can produce deformations in the unit cell which can produce large local anisotropies. Thus, if we disregard the residual coupling between the conduction electrons and impurities the magnetic problem reduces to the transverse field Ising model:

$$H = \sum_{n,m} \Gamma_z(n,m)S_z(n)S_z(m) + \sum_n \Delta_R(n)S_x(n)$$

(27)

where $\Delta_R \propto k_B T_K$ is the renormalized tunneling splitting of the transverse field. It is obvious that the situation reproduces Doniach’s argument: while the RKKY coupling $\Gamma_z$ works in the direction of making the local spin an eigenstate of $S_z$ (and therefore to order it, $|\langle S_z \rangle| = 1$) the Kondo effect through $\Delta_R$ pushes the local moment to be an eigenstate of $S_x$ and therefore leads $\langle S_z \rangle = 0$. In this picture the results for the case of insulating magnets follow immediately and one expects power law divergences of the physical quantities in the paramagnetic phase.

In one dimension the problem of the Kondo lattice has been studied with the use of bosonization and has been solved exactly at a particularly anisotropic point called the Toulouse point [106] and at half-filling [107]. Moreover, this problem was studied in great detail numerically [108]. Honner and Gulácsi have argued that the Kondo chain indeed maps into the transverse field Ising model, and have shown that in the disordered case that Griffiths singularities appear close to the transition line from ferromagnetic to paramagnetic behavior [109]. This trend seems to be reproduced in other calculations for the same problem [110]. Indeed, power law behavior of the susceptibility was obtained for the Anderson model in one-dimension with exponents very close to the ones obtained experimentally [111]. Since the problem of Griffiths-McCoy singularities is essentially the problem of clusters (zero dimensional objects) surrounded by a metallic environment it seems rather natural that (27) reproduces the magnetic behavior of the Kondo lattice.

The question that arises in the context of (27) is: what is the effect of the residual interaction of the cluster with the conduction electrons? In the paramagnetic phase we assume that the clusters do not interact with each other. In this case one can focus entirely on the behavior of a single cluster and its metallic environment. This problem is actually very close to the problem of macroscopic quantum tunneling of magnetic grains [112] and it is known that dissipation is a relevant perturbation to this problem especially at low temperatures [113]. Consider for instance the problem of $N$ spins in a cluster. Since we assume the cluster to be in the ordered phase there must be two states of the cluster which are nearly degenerate. For instance, a ferromagnetic state with all the spins up has the same energy of a ferromagnetic state with all the spins down (since the environment is paramagnetic it does not bias any specific configuration). At very low temperatures the only way for the system to relax is to flip all the $N$ spins at once. As we have seen in the case of the single impurity Kondo problem (but we can prove it to be true for the two impurity
Kondo problem as well \cite{103} requires that the \(XY\) component of the Kondo Hamiltonian to act \(N\) times over the ground state wavefunction. Since each spin flip requires an energy of order \(J_\perp\) the total energy in this case is of order \(\Gamma_z (J_\perp/\Gamma_z)^N\) that is, the splitting between the low lying states of the cluster are split by

\[
\Delta_0(N) \approx \Gamma_z \exp \left\{-N \ln (\Gamma_z/J_\perp)\right\}
\]

and therefore is exponentially small — as expected for the insulating case, as well. Each time the cluster flips we expect that particle hole excitations to be created at the Fermi surface. Since the cluster is coupled to the electronic bath by its order parameter (magnetization in the case of a ferromagnetic cluster or staggered magnetization in the case of the antiferromagnetic cluster) we expect that coupling to the bath to be extensive with the cluster size, that is, proportional to \(N\). In this case we see that the dissipation parameter \(\alpha\) has to scale like \(N^2 (\tilde{J}_z/E_c)\) where \(\tilde{J}_z\) is the Ising coupling of the cluster to the bath which is a function of the microscopic couplings and has to be calculated from cluster to cluster \cite{103}. Thus, like in the case of a single impurity Kondo problem we can define a cluster Kondo problem with a characteristic Kondo temperature \(T_K(N)\) or tunneling splitting \(\Delta_R\) given by (using (22) and (28))

\[
\Delta_R(N) = E_c \exp \left\{ -\gamma N + \ln (E_c/\Gamma_z) \right\}
\]

where \(\gamma = \ln (J_\perp/\Gamma_z)\) and \(N_c = E_c/\tilde{J}_z\) depends on the coupling constants of the problem. The importance of \(N_c\) rests of the fact that when \(\alpha > 1\) there is no real Kondo effect. Thus, for \(N > N_c\) the cluster freezes and quantum fluctuations are completely suppressed. Indeed, for \(N = N_c\) the Kondo temperature in (29) vanishes. Therefore, \(N_c\) gives the size of the largest cluster for which the Kondo effect still takes place. We can invert (29) in order to give \(N\) as a function of the splitting as

\[
\frac{N}{N_c} = \frac{1}{2 \ln (E_c/\Delta_R)} \left( \sqrt{(\gamma N_c)^2 + 4 \ln (E_c/\Delta_R) \ln (\Gamma_z/\Delta_R)} - \gamma N_c \right).
\]

Notice that there are two well defined limits of this expression depending whether \(\Delta_R\) is larger or smaller than \(\Delta^*\) where

\[
\Delta^* = \sqrt{E_c \Gamma_z} \exp \left\{ -\frac{1}{2} \ln^2 (E_c/\Gamma_z) + (\gamma N_c)^2 \right\}.
\]

If \(\Delta_R \gg \Delta^*\) we have \(N \approx \ln (\Gamma_z/\Delta_R)/\gamma\) and therefore the splitting is completely determined by \(\gamma\) and we have the same situation as in an insulating magnet. When \(\Delta_R \ll \Delta^*\) and \(N/N_c \approx 1\) the cluster becomes decoherent and the situation is not described by the power law behavior. Thus, \(\Delta^*\) defines an energy scale above which power law singularities should be found and below which a new behavior dominated by dissipation is present. The consequences of this dissipative regime will be discussed elsewhere \cite{103}. But a point we have to make is that since \(\Delta^*\) is exponentially dependent on \(N_c\) the dissipative regime is going to be exponentially small. Above a temperature scale \(T^* = \Delta^*/k_B\) we expect the temperature dependence of the physical quantities to be dominated by power law behavior. We also believe this kind of behavior is responsible for the deviations from power law at \(T < T^*\) which is observed in some U and Ce intermetallics.
VI. CONCLUSIONS

In this paper we have reviewed the theoretical and experimental situation on the NFL behavior in U and Ce intermetallics. We argue that the situation is inconclusive and important issues regarding the description of the problem in terms either of single ion or correlated behavior have not been solved. We argue that the Griffiths-McCoy scenario is, so far, the only one which explains the existence of NFL behavior close to the QCP but not exactly at the QCP. Disorder in these systems is very important and help to pin the pieces of the ordered phase inside of the paramagnetic phase (especially because NFL behavior is only observed in alloys).

We described the problem of Griffiths-McCoy singularities in insulating magnets. This problem is now on a very firm basis, and we know that these singularities lead to power law behavior of the response and thermodynamic functions. We believe that power law can describe quite well the NFL behavior observed in a rather large temperature range in many of the systems discussed here and especially in UCu$_{5-x}$Pd$_x$.

We have shown that the Kondo lattice Hamiltonian can be studied in a renormalization group sense by tracing out the higher energy degrees of freedom deep inside or very far away from the Fermi surface and that the effective Hamiltonian contains the basic ingredients required for the local Doniach description of these systems. We argued on the basis of the mapping of the single impurity Kondo problem into the dissipative two level system that in the limit of high local magnetic anisotropy the Kondo problem indeed maps into a transverse field Ising model, which has been shown to present Griffiths-McCoy singularities in its phase diagram. We also have argued that the power law behavior disappears at temperatures smaller than $T^* = \Delta^*/k_B$ (which is probably quite small) where the situation is dominated by dissipative physics. What we have shown, therefore, is that like in the Kondo disorder picture there is a distribution of Kondo temperatures which is not of single ion character but has to do with the Kondo temperature of isolated clusters. Observe that the distribution is not arbitrary but determined completely by the statistical distribution of clusters in a percolation problem. If a residual interaction between clusters exists close to the QCP then with the lowering of the temperature a quantum spin glass state is a possible ground state. In this case a return to Fermi liquid behavior with a strong temperature crossover is expected. Otherwise, if the clusters are truly non-interacting then the quantum super-paramagnetic state dominated by Griffiths-McCoy singularities can exist and real singularities in the response functions must be observed.

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FIGURES

FIG. 1. Typical phase diagram for heavy fermion alloys. Vertical axis is the critical temperature of the ordered phase and the horizontal axis is chemical composition.

FIG. 2. Doniach phase diagram: long dashed line is the Kondo temperature, $T_K$; short dashed line is the RKKY temperature, $T_{RKKY}$; the continuous line is the ordering temperature $T_N$. 
FIG. 3. Different regions of energy close to the Fermi surface.