Topological Solitons  
from DeConstrucTed Extra Dimensions

Christopher T. Hill  
Fermi National Accelerator Laboratory  
P.O. Box 500, Batavia, Illinois 60510, USA *

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Abstract

A topological monopole-like field configuration exists for Yang-Mills gauge fields in a $4 + 1$ dimensions. When the extra dimension is compactified to $3 + 1$ dimensions with periodic lattice boundary conditions, these objects reappear in the low energy effective theory as a novel solution, a gauged-bosonic Skyrmion. When the low energy theory spontaneously breaks, the Nambu-Goldstone mode develops a VEV, and the gauged-bosonic Skyrmion morphs into a ‘t Hooft-Polyakov monopole.

*e-mail: hill@fnal.gov
1 Introduction

This is a tale of three well-known topological solitons: the instanton, the Skyrmion, and the ‘t Hooft-Polyakov monopole. All three of these objects arise from a common source when Yang-Mills fields propagate in a $4+1$ bulk compactified by periodic boundary conditions to $3+1$ dimensions. A consistent description of this dimensional descent is most readily obtained through deconstruction, or latticization, of the extra compactified dimension. The structures of the conserved Chern-Simons currents neatly match, as they must, between the effective descriptions.

We start in $4+1$ dimensions with an $SU(2)$ Yang-Mills theory and note that there are “instantonic monopoles” (IM). These are static, topologically stable solutions of the pure Yang-Mills gauge theory and represent nontrivial homotopy of $\Pi_3(SU(2))$, the winding of the field configuration on the surface $S_3$ at infinity in four spatial dimensions. These objects were considered about a year ago by Ramond and the present author [1, 2], and they are evidently the anticipated pure-Yang-Mills solitons that can exist only in $4+1$ by Deser [3]. These are essentially instantons [4, 5] “lifted” to become the spatial configurations of a static object. For the Instantonic Monopole we can choose in $4+1$ the (noncompactified) vector potentials (where $A, B, ..$ run from 0 to 4, $x^4$ is our 5th dimension; time is $x^0$): 

\[
A_0^a = 0 \quad A_4^a \tau^a = -\frac{1}{g} \vec{x} \cdot \tau \quad A_i^a \tau^a = \frac{1}{g} \left( x_4 \tau_i + \vec{x} \epsilon^{ijk} \tau_k \right)
\]

This field configuration has an associated conserved topological current [1]:

\[
Q_A = \frac{g^2}{16\pi^2} \epsilon_{ABCDE} \text{Tr}(F^{BC} F^{DE})
\]

The resulting field strength is self–dual as a static configuration, i.e., $F_{AB} = \tilde{F}_{0AB}$. It has a mass given by $8\pi^2/g^2$ where $g$ is a $4+1$ coupling constant with dimension (mass)$^{-1/2}$. This mass is essentially $M_{KK}/\alpha$ where $M_{KK}$ is the lowest KK-mode mass when the theory is compactified.

If we compactify the 5th dimension and “deconstruct,” or latticize the compactified dimension, we obtain an equivalent low energy effective theory in $3+1$ dimensions [6, 7, 8]. With periodic boundary conditions in our compactification, the $A_4^a$ vector potential becomes a Nambu-Goldstone zero mode, and the product of Wilson links in the $x^4$ dimension becomes a low energy chiral field $U$, the exponentiated Nambu-Goldstone zero
mode. Keeping only a single lattice brane as an approximation, the effective low energy 3 + 1 theory is then the gauged chiral Lagrangian:

$$L = \frac{1}{2} v^2 \text{Tr}[D\mu, U^\dagger][D\mu, U] - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu}$$

(1.3)

where $F_{\mu\nu} \equiv F^a_{\mu\nu} \tau^a/2$, and with $U = \exp(i\phi/v)$, $\phi = \phi^a \tau^a/2$, where $\phi$ is essentially the Wilson line $\sim ig \int dx^4 A^4$ in 4 + 1 with $A_\mu = A^a_\mu \tau^a/2$. The action of the covariant derivative upon $U$ is $[D_\mu, U] = \partial_\mu U - ig[A_\mu, U]$.

What then is the fate of the instantonic monopole in this low energy theory viewed as a dimensional deconstruction? In an attempt to clarify what the instantonic monopole is in a compactified theory, the present author was originally motivated to consider latticizing the extra dimension [6, 7]. As we will see, a remarkable correspondence emerges.

The first problem is to ask whether a compactified IM solution exists? This question was approached in ref.[1], but see [2]. One employs the method of images and exact multi-instanton solutions to construct a solution satisfying the periodic boundary conditions. This is well known from finite temperature applications of instantons [9]. For compactification with periodic boundary conditions, the low energy pseudoscalar $A^4_\mu$ remains as a zero mode, while with orbifold boundary conditions this mode is absent. Correspondingly, while it is straightforward to compactify the IM with periodic boundary conditions, it is not with the orbifold boundary conditions. This is a consequence of topology; the topology is determined by the winding of field $U = \exp(ig \int dx^4 A_4)$ throughout the manifold on large distances, requiring the $A_4$ zero mode. This will form the basis of the correspondence with a low energy effective Lagrangian description below.

One important consequence of compactification of the IM is the following [9]. In an infinite bulk the IM is conformally invariant. The solution has a scale parameter $\lambda$ but the action is independent of $\lambda$. The action density is concentrated in an arbitrarily large region $r \lesssim \lambda$, ergo arbitrarily large instantons exist. When a dimension is compactified with a length scale $\delta$, however, the field strength configuration changes, and the action density has appreciable values only over $r \lesssim \delta$. Hence, compactification effectively cuts-off the large instantonic monopoles and gives them a size of order the compactification scale.

For the effective description of the 4 + 1 IM in the 3 + 1 effective Lagrangian we note that the theory of eq.(1.3), which is just a conventional gauged chiral Lagrangian, does indeed contain a novel soliton, a “bosonic gauged Skyrmion.” This, we will argue, is the 3 + 1 correspondence of the instantonic monopole of 4 + 1. This object is an “inverted Skyrmion” built out of the $\exp(i\phi/v)$ Wilson link chiral field. At infinity $\phi/v \to \pi \hat{x} \cdot \vec{\tau}$ is
a hedgehog, while at the origin \( \phi/v \to 0 \). This is inverted from the usual Skyrmion, but still trivially represents the nontrivial \( \Pi_3(SU(2)) \) mapping into the 3 + 1 spatial volume (which, of course, corresponds to the spatial \( S_3 \) surface of 4+1). There are, however, other key differences between the Bosonic Gauged Skyrmion BGS and the usual Skyrmion.

The usual Skyrmion has a nontrivial Wess-Zumino (WZ) term which gives it unusual spin and statistics. Choosing the quantized WZ term coefficient to match to \( N_c = 3 \) QCD, the WZ term makes the Skyrmion into a spin\(-\frac{1}{2}\) baryon. In the present case the gauging by \( SU(2) \) forbids the WZ term, and the gauged Skyrmion is a bosonic object of spin\(-0\).

The usual Skyrmion carries a nontrivial topological charge determined from a Chern-Simons current. This current is nontrivially modified in the present case, and is seen to involve a new term which matches the current of eq.(1.2) under dimensional descent.

### 2 Gauge Invariant Chern-Simons Current

The usual Skyrmion is associated with the conserved, normalized Chern-Simons current, and carries a unit charge:

\[
Q^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( U^\dagger (\partial_\nu U) U^\dagger (\partial_\rho U) U^\dagger (\partial_\sigma U) \right)
\]

The index for the usual Skyrmion anzatz, \( U = \cos(f(r)) + \vec{x} \cdot \tau \sin(f(r)) \), is then

\[
\int d^3x \frac{1}{24\pi^2} \epsilon^{ijk} \text{Tr} \left( U^\dagger \partial_i U U^\dagger \partial_j U U^\dagger \partial_k U \right) = \frac{1}{2\pi} \left[ 2(f(\infty) - f(0)) + \sin(2f(\infty)) - \sin(2f(0)) \right]
\]

\( f(r) = \phi(r)/v \) is a kink-like configuration that runs from \( f(0) = 0 \) to \( f(\infty) = \pi \), and thus has unit charge. Note that \( f \to \pm f + N\pi \) is a discrete symmetry (with charge conjugation), so the usual QCD Skyrmion with \( f(0) = \pi \) to \( f(\infty) = 0 \) is equivalent.

When we go over to the gauged case, we might guess that the gauge invariant generalization of the Chern-Simons current is:

\[
Q_1^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( U^\dagger [D_\nu, U] U^\dagger [D_\rho, U] U^\dagger [D_\sigma, U] \right)
\]

However, \( Q_1 \) is not conserved, as seen by explicit calculation:

\[
\partial_\mu Q_1^\mu = \frac{ig}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( F_{\mu\nu} [D_\rho, U] [D_\sigma, U^\dagger] - F_{\mu\nu} [D_\rho, U^\dagger] [D_\sigma, U] \right)
\]

Note as a technical aside that \( Q_1^\mu = V + A \) is constructed from purely right-handed (or left-handed) chiral currents. One cannot build a conserved Chern-Simons current.
out of the product of mixed vector \( V = (1/2)(U[D_\rho, U] + U[D_\rho, U]) \) and axial vector \( A = (1/2)(U[D_\rho, U] - U[D_\rho, U]) \) currents, even in the ungauged case. \( Q^\mu_1 \) transforms, however, as a vector under parity, since \( \phi \rightarrow -\phi \) hence \( U \leftrightarrow U^\dagger \), \( \epsilon^{\mu\nu\rho\sigma} \leftrightarrow -\epsilon^{\mu\nu\rho\sigma} \) and \( D_\mu \rightarrow -D_\mu \), so \( Q^\mu \rightarrow -Q^\mu \). Note also,

\[
Q^\mu_1 = -\frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( U[D_\nu, U][D_\rho, U][D_\sigma, U] \right) \tag{2.8}
\]

using \( U[D_\nu, U] = -[D_\nu, U^\dagger]U \) and cyclicity of the trace, hence \( Q_1 \) is equivalent to a current built out of pure left-handed chiral currents, i.e., the Chern-Simons current is unique.

Does there exist a conserved current to match to the 4+1 conserved current of eq. (1.2)? With the nontrivial gauge fields we can presently introduce two new currents:

\[
Q^\mu_2 = \frac{ig}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( F_{\nu\rho}U[D_\sigma, U] - F_{\nu\rho}U[D_\sigma, U] \right) \tag{2.9}
\]

\[
Q^\mu_3 = \frac{ig}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( F_{\nu\rho}U[D_\sigma, U] + F_{\nu\rho}U[D_\sigma, U] \right) \tag{2.10}
\]

These latter currents are indeed expected to play a role in the matching because in 4+1 dimensions we had the conserved current of the instantonic-monopole:

\[
\epsilon_{ABCDE} \text{Tr}(F_{BC}F_{DE}) \sim \epsilon^{\mu\nu\rho\sigma} \text{Tr}(F_{\mu\nu}U[D_\sigma, U]) \tag{2.11}
\]

and \( U[D_\sigma, U] \sim F_{\sigma4} \) is the appropriate dimensional descent correspondence of \( A_4 \) to the Nambu-Goldstone boson.

We see that \( Q_2 \) has normal vectorial parity, and it can thus form a vector combination with \( Q_1 \). \( Q_3 \) is an axial vector under parity. Computing the divergence of \( Q_2 \) we obtain the opposite of the rhs eq. (2.7), and we thus arrive at the conclusion that there is a new conserved current:

\[
\bar{Q}^\mu = Q^\mu_1 + Q^\mu_2 \quad \partial_\mu \left( \bar{Q}^\mu \right) = 0 \tag{2.12}
\]

\( \bar{Q}^\mu \) is thus the 3+1 current corresponding to the 4+1 eq. (1.2) under dimensional descent. The new index remains an exact differential and is discussed below in eq. (4.31).

For completeness, notice that the axial current is not conserved:

\[
\partial_\mu Q^\mu_3 = -\frac{ig}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( F_{\mu\nu}[D_\rho, U][D_\sigma, U] + F_{\mu\nu}[D_\rho, U][D_\sigma, U] \right) + \frac{ig}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( F_{\mu\nu}U[D_\rho, U^\dagger] - F_{\mu\nu}F_{\rho\sigma} \right) \tag{2.13}
\]
The latter term resembles an anomaly. If, for the usual Skyrmion, we gauged only the left-handed or right-handed pieces of \( U \), e.g., as in the electroweak theory, then we would obtain the normal current algebra anomalies through these manipulations, but our baryon number current must be modified as in eq. (2.12).

3 Energetics

The existence of the conserved current \( \tilde{Q}^\mu \) guarantees that there are nontrivial Skyrmionic configurations including the gauge fields. The core profile of the Skyrmion must, moreover, act as a source to Yang-Mills fields.

The Skyrmion, however, is unstable to core collapse, even in the gauged case. It’s core is stabilized in the usual ungauged case by adding the “Skyrme term” which must be viewed as short-distance correction to the action:

\[
S_0 = \frac{1}{32} \text{Tr} \left( [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U] \right)^2
\]

In the present case there are indeed gauge invariant generalizations of the Skyrme-term,

\[
S_1 = \frac{1}{32} \text{Tr} \left( [U^\dagger [D_\mu, U], U^\dagger [D_\nu, U]] \right)^2 = \frac{1}{32} \text{Tr} \left( [[D_\mu, U^\dagger], [D_\nu, U]] \right)^2
\]

The gauge invariant Skyrme term \( S_1 \), with positive coefficient in the energy, stabilizes the solution on distance scales \( \delta \sim 1/v \).

The stable Skyrmion solution necessarily involves the nontrivial near-zone gauge field configuration in the core. These can be seen to be identical to the short-distance core of a BPS monopole \([10]\). The chiral field \( \phi/v \) is identified with the Wilson line \( \int dx^4 A_4 \) and we thus choose the anzatz:

\[
\int dx^4 A_4 \sim \phi/v = f(r) \hat{x} \cdot \vec{\tau}
\]

For the vector potential we choose:

\[
A_i^a \hat{\tau}_a = \frac{h(r)}{g} \vec{x} \varepsilon^{ijk} \tau_k
\]

The energy ((−1)× action for static configurations) then takes the form:

\[
E = \frac{4\pi}{g^2} \int_0^\infty dr \left[ r^2 \left( h'(r) + \frac{h}{r} \right)^2 + \frac{1}{2} r^2 \left( h^2(r) - \frac{2h}{r} \right) \right] \\
+ \frac{1}{2} \nu^2 \int_0^\infty dr \left[ r^2 (f'(r))^2 + 2(H(r))^2 \sin^2(f(r)) \right]
\]
where it is convenient to introduce the combination:

\[ H(r) = 1 - rh(r) \]  \hspace{1cm} (3.19)

Note that \( h(r) = 2/r \) is a pure gauge configuration. If we substitute any particular anzatz into eq.(3.18) we obtain:

\[ E = \frac{4\pi}{\lambda g^2} c_0 + \frac{1}{2} v^2 \lambda c_1 \]  \hspace{1cm} (3.20)

where \( c_0 \) and \( c_1 \) are determined from the Yang-Mills and Skyrmonic energies respectively. The energy of the particular anzatz is then relaxed to the minimum of eq.(3.20) with the choice:

\[ \lambda^2 = \frac{8\pi c_0}{g^2 v^2 c_1} \] \hspace{1cm} (a); \hspace{1cm} \[ E = \frac{2v}{g} \sqrt{2\pi c_0 c_1} \] \hspace{1cm} (b)  \hspace{1cm} (3.21)

The energy is equipartitioned between the two terms of eq.(3.20), which accounts for the factor of 2 in eq.(3.21.b).

To verify that a nontrivial Yang-Mills field is part of a stable solution we check that it is required for binding. We can compare to the energy of the same Skyrme profile in the case that the Yang-Mills field is switched off:

\[ E_{\text{off}} = \frac{1}{2} v^2 \int_0^\infty dr \left[ r^2 (f'(r))^2 + 2 \sin^2(f(r)) \right] \equiv \frac{1}{2} v^2 c_2 \]  \hspace{1cm} (3.22)

Thus, we must have:

\[ \frac{E}{E_{\text{off}}} < 1, \quad \text{or,} \quad \frac{c_1}{c_2} < \frac{1}{2}. \]  \hspace{1cm} (3.23)

Various choices of anzatze for the GBS have been explored numerically. One is inspired from the instantonic monopole. Matching \( f(r) \) to \( f \ dx^4 A_4 \) and \( h(r) \) to the \( x^4 = 0 \) behavior of \( A_4 \) we obtain:

\[ f(r) = \frac{\pi r}{\lambda^2 + r^2}, \quad h(r) = \frac{2r}{\sqrt{\lambda^2 + r^2}}, \]  \hspace{1cm} (3.24)

In fact, we find that this anzatz is not bound, and numerically \( E/E_{\text{off}} = 1.4 \), not close to a binding a solution. The reason is as follows; we can easily see that for small \( r \), and \( f(r) \), our action is equivalent to that of a BPS monopole (e.g., see the analysis of [10]).

The core structure of the previous anzatz is far from that of a BPS monopole. After some numerical experimentation we are led to the following:

\[ \tilde{f}(r) = \frac{\pi \sqrt{r}}{\sqrt{\epsilon \lambda + r}}, \quad \tilde{h}(r) = \frac{2}{\lambda + r}. \]  \hspace{1cm} (3.25)
Let us initially choose $\epsilon = 1$. Then we find:

$$\tilde{c}_1 = \int_0^\infty dx \left[ \frac{\pi^2 x}{4(1+x)^3} + 2 \sin^2 \left( \frac{\pi \sqrt{x}}{\sqrt{1+x}} \right) \left( 1 - \frac{x}{1+x} \right)^2 \right]$$

$$\tilde{c}_2 = \int_0^\infty dx \left[ \frac{\pi^2 x}{4(1+x)^3} + 2 \sin^2 \left( \frac{\pi \sqrt{x}}{\sqrt{1+x}} \right) \right]$$

and this leads to a net binding:

$$\frac{E}{E_{\text{off}}} = \frac{2\tilde{c}_1}{\tilde{c}_2} = 0.883 < 1$$

While the form of eq.(3.20) suggests stability of the core of a solution supported by the Yang-Mills field, generally we find that the Skyrme profile can be deformed to collapse and reduce the energy in the absence of the Skyrme term $S_1$. We can, for example, deform the above solution by choosing $\epsilon \neq 1$. We find that the energy is reduced, and the Skyrme core is unstable.

The Skyrme term can be added and takes the form in an anzatz:

$$S_1 = 2\pi \int_0^\infty dr \left[ \sin^2(f)H^2(r) \left( 2(f')^2 + \frac{1}{r^2} \sin^2(f)H^2(r) \right) \right]$$

$S_1$ must enter the energy with a positive coefficient and always dominates at extreme core collapse. The result is a stable object with a mass given by eq.(3.21) of order $v/g \sim M_{KK}/\alpha$, as in the case of the IM.

The original conformal invariance of the IM is lost with the GBS. One can also examine duality and see that it, too, has become only approximate. These are no doubt a consequence of compactification and the truncation on the extreme low energy physics. The Skyrme term is a measure of the truncation of theory; the effective deconstructed theory is not expected to match the short-distance physics but to capture only the large distance aspects. It would be instructive to evaluate the Skyrme term coefficient, as well as the effects of other operators, such as:

$$\mathcal{O}^\pm = ig \text{Tr} F_{\mu\nu}(D^\mu U^\dagger)(D^\nu U) \pm (D^\mu U)(D^\nu U^\dagger)$$

which are analogues of the new currents of eq.(2.9) and eq.(2.10).
4 Spontaneously Broken $SU(2)$

We can add terms to the Lagrangian that are consistent with the $SU(2)$ symmetry of the form $\sum_p c_p (\text{Tr}(U))^p + h.c.$ Indeed, such terms must arise at the quantum level, as in a computation of the Coleman-Weinberg potential (see ref.[8]). We presently add them by hand. With such terms we can then destabilize the vacuum; $\phi$ becomes a Higgs-field which breaks $SU(2) \rightarrow U(1)$, (as in the recent model of ref.[8], though we do not presently want an $I = \frac{1}{2}$ Higgs). This means that an arbitrary VEV of $\phi$ can be engineered, $\langle \phi \rangle = (1 - \epsilon) v$ where $\epsilon \neq 0$, and is not gauge equivalent to the unbroken vacuum.

Since $\phi$ is an isovector field we have all of the conditions required for a nontrivial $\Pi_2(SU(2)/U(1))$. In this case the the Gauged-Bosonic Skyrmion grows into a ’t Hooft-Polyakov monopole. The monopole charge is measured by a Chern-Simons charge in one less dimension, integrated over the surface at infinity. This contains the dual of $F_{ij}$, e.g., $\frac{1}{2} \text{Tr} \tau_k \cdot \epsilon^{ijk} F_{ij}$ which is integrated over the surface $d^2 \Sigma$ at infinity. We have:

$$\int_0^\infty r^2 \sin \theta d\theta d\phi \left. \tilde{F}_r \right|_{r^2 \rightarrow \infty} = 4\pi r^2 \frac{H^2 - 1}{2gr^2} = -\frac{2\pi}{g}$$ (4.30)

where $H(r)$ is defined in eq.(3.19), and we see that asymptotically $H(0) = 0$ and $H(\infty) = 1$ [11]. The Skyrmion terms now play no significant role in the core stability since the nontrivial potential is determining the field value at infinity and it costs energy to shrink the core.

Remarkably, however, we see that our monopole is nontrivially charged under the original $3 + 1$ Chern-Simons charge $\tilde{Q}$ as well. Including the gauge degrees of freedom in the Chern-Simons current in $3 + 1$ we find that the Chern-Simons charge density is an exact differential (see ref.[13]) and the result:

$$\int d^3x \frac{1}{24\pi^2} \tilde{Q}_0 = \frac{1}{2\pi} \left[ 2(f(\infty) - f(0)) - \sin(2f(\infty))H(\infty)^2 + \sin(2f(0))H(0)^2 \right]$$ (4.31)

Note that no manipulations involving the Chern-Simons current rely upon the use of equations of motion. The monopole anzatz for $f(r)$ is similar to the Gauge Bosonic Skyrmion, with $f(0) = 0$ but now the asymptotic value $f(\infty) = (1 - \epsilon) \pi \equiv \theta \neq \pi$. The Chern-Simons charge is now an arbitrary fractional quantity, $(\theta - \sin(\theta) \cos(\theta))/\pi$, a result obtained previously for fermion fractionalization by Goldstone and Wilczek [13]. The reason is that by forcing $f(\infty)$ to a value less than $\pi$ we only partially, map $SU(2)$ into the 3-volume. Essentially, some fraction of the Skyrmion’s charge has flowed out to infinity as the field relaxes into it’s nontrivial VEV.
5 Conclusions

We have explored the dimensional descent of a pure Yang-Mills gauge theory in $4 + 1$ dimensions, via deconstruction, into a $3 + 1$ effective low energy description. We have seen that topologically nontrivial objects, Instantonic Monopoles, exist in $4 + 1$ with nontrivial conserved charges. Under deconstruction these objects morph into Gauged Bosonic Skyrmions, carrying a conserved gauge Chern-Simons charge. The scale size and precise masses are determined by the compactification scale. The masses of these objects are $\sim M_{KK}/\alpha$ with core scale sizes $\sim 1/M_{KK}$. With spontaneous symmetry breaking, the GBS’s further morph into ‘t Hooft–Polyakov monopoles. The latter objects carry the usual magnetic charge, i.e., the magnetic flux crossing the surface at infinity, as well as a fractional Chern-Simons charge in $3 + 1$, measuring the partial mapping of $SU(2)$ into the 3-volume.

All of this occurs with pure $SU(2)$ Yang-Mills theory (it is imbeddible into $SU(N)$) with no explicit Higgs fields, or explicit chiral fields! It is a consequence of dimensional compactification, and deconstruction, which requires the latticeization of the extra dimensions to maintain the explicit manifest gauge invariance. It is an explicit demonstration of the descent cohomology of the classical topological solutions themselves. Moreover, such objects appear to be a necessary consequence of Yang-Mills gauge theories propagating in the bulk with periodic boundary conditions.

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