Acceptance sampling plans under two-parameter Quasi Shanker distribution assuring mean life with an application to manufacturing data

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Abstract
The acceptance sampling plan (ASP) is a statistical tool used in industry for quality control to determine the quality of products by selecting a specified number for testing in order to accept or reject the lot. The main objective is to develop a new ASP based on truncated life tests assuming that the lifetime follows the two parameters Quasi Shanker distribution, since this distribution showed its superiority in providing a better model for some applications than the exponential distribution. The ASP steps are carried out to find the minimum sample sizes needed to assert the certain life mean that are calculated under a given customer’s risk. The operating characteristic values of the sampling plan and the producer risk values are obtained. The efficiency of the suggested plans is analyzed based on real data that is fitted to the Quasi Shanker distribution. For various values of the Quasi Shanker distribution parameters, numerical examples are presented for illustrative purposes. The results indicate that the suggested ASP provides smaller sample sizes than other competitors considered in this study. The suggested ASP has been found to provide a substantial sampling economy in terms of reducing the sample. Hence, it is recommended that the ASP can be used in industry and for future research works as double and group ASP.

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**Introduction**
Shanker\(^1\) suggested one parameter lifetime distribution known as Shanker distribution (SD) for modeling lifetime data from engineering and biomedical sciences with a cumulative distribution function (CDF) and probability density function (PDF), respectively, given by

\[
F_{SD}(x; \theta) = 1 - \left[ 1 + \frac{\theta x}{\theta^2 + 1} \right] e^{-\theta x}, \ x > 0, \theta > 0,
\]

and

\[
f_{SD}(x; \theta) = \frac{\theta^2}{\theta^2 + 1} (\theta + x) e^{-\theta x}, \ x > 0, \theta > 0.
\]

Due to the importance of Shanker distribution, a two-parameter Quasi Shanker distribution\(^2\) (TPQSD) is suggested as a modification of SD and to be more flexible than SD. The QSD is a special case of the TPQSD. The latter showed its superiority in providing a better model for some applications than the exponential distribution.\(^2\) The probability density function of TPQSD is given as

\[
f_{TPQSD}(x; \theta, \alpha) = \frac{\theta^3}{\theta^3 + \theta + 2\alpha} (\theta + x + \alpha x^2) e^{-\theta x}, \ x > 0, \alpha > 0, \theta > 0.
\]

The corresponding CDF of the TPQSD is

\[
F_{TPQSD}(x; \theta, \alpha) = 1 - \left[ 1 + \frac{\alpha \theta^2 x^2 + \theta x (\theta + 2\alpha)}{\theta^3 + \theta + 2\alpha} \right] e^{-\theta x}, \ x > 0, \theta > 0, \alpha > 0.
\]

The \(r\)th non-central moment about the origin is given by

\[
\mu'_{rTPQSD} = E(X^r) = \frac{r! \left[ \theta^3 + (r + 1)\theta + (r + 1)(r + 2)\alpha \right]}{\theta^3 + \theta + 2\alpha}, \ r = 1, 2, 3, ...
\]

Therefore, for \(r = 1\) in (3) the mean of the TPQSD is

\[
\mu_{TPQSD} = E(X) = \frac{\theta^3 + 2\theta + 6\alpha}{\theta^3 + \theta + 2\alpha}.
\]

The reliability, hazard rate and the mean residual life functions of the TPQSD distribution, respectively, are

\[
R_{TPQSD}(x; \theta, \alpha) = 1 - F_{TPQSD}(x; \theta, \alpha) = 1 + \frac{\alpha \theta^2 x^2 + \theta x (\theta + 2\alpha)}{\theta^3 + \theta + 2\alpha} e^{-\theta x},
\]
\[ h_{TPQSD}(x; \theta, \alpha) = \frac{f_{TPQSD}(x; \theta, \alpha)}{1 - F_{TPQSD}(x; \theta, \alpha)} = \frac{\theta^3(\theta + x + \alpha x^2)}{\theta^3 + \theta + 2\alpha + \alpha \theta^2 x^2 + \theta x(\theta + 2\alpha)}, \]  

and

\[ m_{TPQSD}(x; \theta, \alpha) = \frac{\alpha \theta^2 x^2 + \theta(\theta + 4\alpha)x + (\theta^3 + 2\theta + 6\alpha)}{\theta[\alpha \theta^2 x^2 + \theta(\theta + 2\alpha)x + (\theta^3 + \theta + 2\alpha)]}. \]

The coefficient of variation \( CV_{TPQSD} \), and the index of dispersion \( ID_{TPQSD} \) of the TPQSD, respectively, are given by

\[ CV_{TPQSD} = \frac{\sqrt{\theta^6 + 4\theta^4 + 16\theta^3 + 2\theta^2 + 12\theta + 12\alpha}}{\theta^3 + 2\theta + 6\alpha}, \]

and

\[ ID_{TPQSD} = \frac{\theta^6 + 4\theta^4 + 16\theta^3 + 2\theta^2 + 12\theta + 12\alpha}{\theta(\theta^3 + \theta + 2\alpha)(\theta^3 + \theta + 6\alpha)}. \]

The maximum likelihood estimates (MLE) of distribution parameters \( \theta \) and \( \alpha \) can be determined by solving the following non-linear equations:

\[ \frac{\partial \ln L}{\partial \theta} = \frac{3n}{\theta} - \frac{n(3\theta^2 + 1)}{\theta^3 + \theta + 2\alpha} + \sum_{i=1}^{n} \frac{1}{\theta + x_i + \alpha x_i^2} - \sum_{i=1}^{n} x_i = 0, \]

and

\[ \frac{\partial \ln L}{\partial \alpha} = \frac{-2n}{\theta^3 + \theta + 2\alpha} + \sum_{i=1}^{n} \frac{x_i^2}{\theta + x_i + \alpha x_i^2} = 0. \]

The TPQSD is considered in this paper to suggest new single acceptance sampling plans. The acceptance sampling plan (APS) is a valuable methodology for the producers to accept or reject the lot based on the quality of the samples’ inspection. It is useful in reducing the cost of full inspection since the latter is costly and time-consuming.

The ASPs focusing on the truncated lifetime tests are considered in literature by many researchers in many cases. See, for example, a list of applications with the corresponding references: the Tsallis \( q \)-exponential distribution,\(^3\) the generalized exponential and exponential distributions,\(^4,5\) the generalized inverted exponential distribution,\(^6\) the weighted exponential distribution,\(^7\) the three-parameter Kappa distribution,\(^8\) the Birnbaum Saunders model,\(^9\) the Garima distribution,\(^10\) the transmuted generalized inverse Weibull distribution,\(^11\) the extended Exponential distribution,\(^12\) the new Weibull-Pareto distribution,\(^13\) finite and infinite lot size under power Lindley distribution,\(^14\) double ASP based on truncated life tests for the inverse Rayleigh distribution,\(^15\) double ASP based on the Burr type X distribution,\(^16\) the generalized inverse Weibull distribution,\(^17\) the transmuted inverse Rayleigh distribution,\(^18\) the Sushila distribution,\(^19\) double ASP for transmuted
generalized inverse Weibull distribution, the generalized exponential distribution, the Weibull product distribution, the Gamma distribution, the log-logistic model, the inverse-gamma distribution, ASP for Akash distribution, length-biased weighted Lomax distribution, three-parameter Lindley distribution, double ASP for Half-Normal distribution, and exponentiated generalized inverse Rayleigh distribution. Also, see a mixed double sampling plan based on Cpk, design of variables sampling plans based on the lifetime-performance index in the presence of hybrid censoring scheme, determination of multiple dependent state repetitive group sampling plan based on the process capability index, and selecting better process based on difference statistic using double sampling plan. To the best of our knowledge, there are no studies about the ASPs for the TPQSD.

The structure of this paper is as follows. “METHOD” section reveals the methodology and the new suggested ASP under the TPQSD distribution. Also, we analyzed the minimum sample size, the operating characteristic function values and the producer’s risk in the same section. Descriptions of the tables and illustrated examples are discussed in the “RESULTS AND DISCUSSIONS” section. An investigation of a real data is given in the “REAL DATA APPLICATION” section and, finally, the conclusion of the paper in the “CONCLUSION” section.

**Method**

**Design of the ASP**

This section provides the new proposed ASP based on the assumptions that the lifetime follows the two-parameter Quasi Shanker distribution. An ASP-based on truncated life tests consists of:

Step 1: The number of items $n$ to be selected from the lot.
Step 2: The acceptance number $c$, and if $c$ or fewer failures out of sample size $n$ occur within the test time ($t$), the lot is accepted.
Step 3: The ratio $t/\mu_0$, where $\mu_0$ is the identified mean lifetime and $t_0$ is the predetermined testing time.

**Minimum sample size**

Within the experiment, assuming that the lot size is sufficiently large, the binomial distribution can be used to find the probability of accepting the lot. Also, it is assumed that the consumer risk is specified to be $1 - P^*$ at most, meaning that the probability of the real average life $\mu$ (lot quality parameter) is smaller than $\mu_0$. The objective of the experimenter, in this case, is to find the minimum sample size $n$ desirable to satisfy
\[
\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - P^*, \quad (11)
\]

where \( P^* \in (0, 1) \) is given and indicates the consumer confidence level. Also, the probability of a failure detected in the time \( t \), \( p = F(t; \mu_0) \), depends only on \( t/\mu_0 \), where \( \mu_0 = \frac{\theta_0^3 + 2\theta_0 + 6\alpha_0}{\theta_0 (\theta_0^3 + \theta_0 + 2\alpha_0)} \). If the number of detected failure items during the time \( t \) is at most \( c \), then, with a reference to equation (11), we can assert with a probability \( P^* \) that \( F(t; \mu) \leq F(t; \mu_0) \), which indicates \( \mu_0 \leq \mu \).

In this study, the ASP values are \( t/\mu_0 = 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927, 4.712 \), and \( P^* = 0.75, 0.9, 0.95, 0.99 \). These values allow us to compare our results with those given for the Gamma distribution, Garima distribution, transmuted generalized inverse Weibull distribution, and the log-logistic distribution. The smallest sample sizes that sustaining inequality (11) are presented in Table 1 for \( FTPQSD(x; \theta = 3, \alpha = 2) \) and in Table 4 for \( FTPQSD(x; \theta = 2, \alpha = 3) \).

**Operating characteristic value**

The operating characteristic \( OC(p) \) of the sampling plan \( (n, c, t/\mu_0) \) gives the probability of accepting the lot and it is given by

\[
OC(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i},
\]

where \( p = F(t; \mu) \) is a function of the mean \( \mu \). The operating characteristic function values, for the acceptance sampling plan \( (n, c = 2, t/\mu_0) \), are presented in Table 2 for \( FTPQSD(x; \theta = 3, \alpha = 2) \) and in Table 5 for \( FTPQSD(x; \theta = 2, \alpha = 3) \).

**Producer’s risk**

The producer risk is the probability of rejecting a good lot when \( \mu \geq \mu_0 \), and it is defined as

\[
PR(p) = \sum_{i=c+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}.
\]

For a given value of producer’s risk, say \( \varphi = 0.05 \), the researcher is concerned in determining the value of \( \mu/\mu_0 \) that asserts the producer risk will not exceed \( \varphi \). The value of \( \mu/\mu_0 \) is the minimum positive number for which \( p = F\left(\frac{\mu}{\mu_0} \mu\right) \) satisfies the inequality

\[
\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq 1 - \varphi.
\]
Table 1. Minimum sample sizes necessary when $\theta = 3$ and $\alpha = 2$ in the TPQSD.

| $p^*$ | $c$  | $t/\mu_0$ |
|-------|------|-----------|
|       |      | 0.628  | 0.942  | 1.257  | 1.571  | 2.356  | 3.141  | 3.927  | 4.712  |
| 0.75  | 0    | 3      | 2      | 2      | 1      | 1      | 1      | 1      | 1      |
| 0.75  | 1    | 5      | 4      | 3      | 3      | 2      | 2      | 2      | 2      |
| 0.75  | 2    | 8      | 6      | 5      | 4      | 4      | 3      | 3      | 3      |
| 0.75  | 3    | 10     | 8      | 6      | 6      | 5      | 4      | 4      | 4      |
| 0.75  | 4    | 13     | 9      | 8      | 7      | 6      | 5      | 5      | 5      |
| 0.75  | 5    | 15     | 11     | 9      | 8      | 7      | 6      | 6      | 6      |
| 0.75  | 6    | 17     | 13     | 11     | 10     | 8      | 8      | 7      | 7      |
| 0.75  | 7    | 20     | 15     | 12     | 11     | 9      | 9      | 8      | 8      |
| 0.75  | 8    | 22     | 16     | 14     | 12     | 11     | 10     | 9      | 9      |
| 0.75  | 9    | 24     | 18     | 15     | 14     | 12     | 11     | 10     | 10     |
| 0.75  | 10   | 26     | 20     | 17     | 15     | 13     | 12     | 11     | 11     |
| 0.90  | 0    | 4      | 3      | 2      | 2      | 1      | 1      | 1      | 1      |
| 0.90  | 1    | 7      | 5      | 4      | 4      | 3      | 2      | 2      | 2      |
| 0.90  | 2    | 10     | 7      | 6      | 5      | 4      | 4      | 3      | 3      |
| 0.90  | 3    | 13     | 9      | 8      | 7      | 6      | 5      | 4      | 4      |
| 0.90  | 4    | 15     | 11     | 9      | 8      | 7      | 6      | 5      | 5      |
| 0.90  | 5    | 18     | 13     | 11     | 9      | 8      | 7      | 6      | 6      |
| 0.90  | 6    | 20     | 15     | 12     | 11     | 9      | 8      | 8      | 7      |
| 0.90  | 7    | 23     | 17     | 14     | 12     | 10     | 9      | 9      | 8      |
| 0.90  | 8    | 25     | 19     | 16     | 14     | 11     | 10     | 10     | 9      |
| 0.90  | 9    | 28     | 21     | 17     | 15     | 13     | 11     | 11     | 10     |
| 0.90  | 10   | 30     | 22     | 19     | 16     | 14     | 12     | 12     | 11     |
| 0.95  | 0    | 5      | 4      | 3      | 2      | 2      | 1      | 1      | 1      |
| 0.95  | 1    | 8      | 6      | 5      | 4      | 3      | 3      | 2      | 2      |
| 0.95  | 2    | 11     | 8      | 7      | 6      | 5      | 4      | 4      | 3      |
| 0.95  | 3    | 14     | 10     | 8      | 7      | 6      | 5      | 5      | 4      |
| 0.95  | 4    | 17     | 12     | 10     | 9      | 7      | 6      | 6      | 5      |
| 0.95  | 5    | 20     | 14     | 12     | 10     | 8      | 7      | 7      | 7      |
| 0.95  | 6    | 22     | 16     | 13     | 12     | 10     | 8      | 8      | 8      |
| 0.95  | 7    | 25     | 18     | 15     | 13     | 11     | 10     | 9      | 9      |
| 0.95  | 8    | 27     | 20     | 17     | 15     | 12     | 11     | 10     | 10     |
| 0.95  | 9    | 30     | 22     | 18     | 16     | 13     | 12     | 11     | 11     |
| 0.95  | 10   | 32     | 24     | 20     | 17     | 14     | 13     | 12     | 12     |
| 0.99  | 0    | 8      | 5      | 4      | 3      | 2      | 2      | 1      | 1      |
| 0.99  | 1    | 11     | 8      | 6      | 5      | 4      | 3      | 3      | 3      |
| 0.99  | 2    | 15     | 10     | 8      | 7      | 5      | 5      | 4      | 4      |
| 0.99  | 3    | 18     | 13     | 10     | 9      | 7      | 6      | 5      | 5      |
| 0.99  | 4    | 21     | 15     | 12     | 10     | 8      | 7      | 6      | 6      |
| 0.99  | 5    | 24     | 17     | 14     | 12     | 9      | 8      | 7      | 7      |
| 0.99  | 6    | 27     | 19     | 16     | 13     | 11     | 9      | 9      | 8      |
| 0.99  | 7    | 29     | 21     | 17     | 15     | 12     | 10     | 10     | 9      |
| 0.99  | 8    | 32     | 23     | 19     | 16     | 13     | 12     | 11     | 10     |
| 0.99  | 9    | 35     | 25     | 21     | 18     | 14     | 13     | 12     | 11     |
| 0.99  | 10   | 37     | 27     | 22     | 19     | 16     | 14     | 13     | 12     |

$p^*$ is the consumer confidence level.
For a given ASP $n$, $c$, $t/m_0$ under the TPQSD at an identified confidence level $P^*$, the smallest values of $m_0$ satisfying (14), are presented in Table 3 for $FTPQSD(x; \theta = 3, \alpha = 2)$ and in Table 5 for $FTPQSD(x; \theta = 2, \alpha = 3)$.

### Results and discussions

In this section, we discuss the results obtained based on the suggested ASP. Recall that, the results for $FTPQSD(x; \theta = 3, \alpha = 2)$ are presented in Tables 1 to 3 and for $FTPQSD(x; \theta = 2, \alpha = 3)$ are reported in Tables 4 to 6.
Table 3. Minimum ratio of $\mu / \mu_0$ for the acceptability of a lot with $\theta = 3$ and $\alpha = 2$ in the TPQSD and producer's risk 0.05.

| $p^*$ | $c$ | $t / \mu_0$ |
|-------|-----|-------------|
| 0.75  | 0   | 38.566      |
| 0.75  | 1   | 8.262       |
| 0.75  | 2   | 5.566       |
| 0.75  | 3   | 4.023       |
| 0.75  | 4   | 3.607       |
| 0.75  | 5   | 3.080       |
| 0.75  | 6   | 2.735       |
| 0.75  | 7   | 2.661       |
| 0.75  | 8   | 2.455       |
| 0.75  | 9   | 2.297       |
| 0.75  | 10  | 2.172       |
| 0.90  | 0   | 51.434      |
| 0.90  | 1   | 11.995      |
| 0.90  | 2   | 7.191       |
| 0.90  | 3   | 5.484       |
| 0.90  | 4   | 4.283       |
| 0.90  | 5   | 3.845       |
| 0.90  | 6   | 3.345       |
| 0.90  | 7   | 3.164       |
| 0.90  | 8   | 2.882       |
| 0.90  | 9   | 2.790       |
| 0.90  | 10  | 2.606       |
| 0.95  | 0   | 64.302      |
| 0.95  | 1   | 13.857      |
| 0.95  | 2   | 8.002       |
| 0.95  | 3   | 5.970       |
| 0.95  | 4   | 4.958       |
| 0.95  | 5   | 4.354       |
| 0.95  | 6   | 3.750       |
| 0.95  | 7   | 3.498       |
| 0.95  | 8   | 3.166       |
| 0.95  | 9   | 3.036       |
| 0.95  | 10  | 2.822       |
| 0.99  | 0   | 102.907     |
| 0.99  | 1   | 19.440      |
| 0.99  | 2   | 11.239      |
| 0.99  | 3   | 7.910       |
| 0.99  | 4   | 6.304       |
| 0.99  | 5   | 5.370       |
| 0.99  | 6   | 4.761       |
| 0.99  | 7   | 4.166       |
| 0.99  | 8   | 3.874       |
| 0.99  | 9   | 3.649       |
| 0.99  | 10  | 3.351       |

$p^*$ is the consumer confidence level.
Table 4. Minimum sample sizes necessary when $\theta = 2$ and $\alpha = 3$ in the TPQSD.

| $p^*$  | $c$ | $t/\mu_0$ | 0.628 | 0.942 | 1.257 | 1.571 | 2.356 | 3.141 | 3.927 | 4.712 |
|-------|-----|-----------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.75  | 0   | 3         | 2     | 2     | 1     | 1     | 1     | 1     | 1     | 1     |
| 0.75  | 1   | 6         | 4     | 3     | 3     | 2     | 2     | 2     | 2     |
| 0.75  | 2   | 9         | 6     | 5     | 4     | 3     | 3     | 3     | 3     |
| 0.75  | 3   | 11        | 8     | 7     | 6     | 5     | 4     | 4     | 4     |
| 0.75  | 4   | 14        | 10    | 8     | 7     | 6     | 5     | 5     | 5     |
| 0.75  | 5   | 16        | 12    | 10    | 8     | 7     | 6     | 6     | 6     |
| 0.75  | 6   | 19        | 14    | 11    | 10    | 8     | 7     | 7     | 7     |
| 0.75  | 7   | 22        | 16    | 13    | 11    | 9     | 8     | 8     | 8     |
| 0.75  | 8   | 24        | 18    | 14    | 12    | 10    | 9     | 9     | 9     |
| 0.75  | 9   | 27        | 19    | 16    | 14    | 11    | 11    | 10    | 10    |
| 0.75  | 10  | 29        | 21    | 17    | 15    | 13    | 12    | 11    | 11    |
| 0.75  | 0   | 5         | 3     | 2     | 2     | 1     | 1     | 1     | 1     |
| 0.75  | 1   | 8         | 6     | 4     | 4     | 3     | 2     | 2     | 2     |
| 0.75  | 2   | 11        | 8     | 6     | 5     | 4     | 3     | 3     | 3     |
| 0.75  | 3   | 14        | 10    | 8     | 7     | 5     | 5     | 4     | 4     |
| 0.75  | 4   | 17        | 12    | 10    | 8     | 6     | 6     | 5     | 5     |
| 0.75  | 5   | 20        | 14    | 11    | 10    | 8     | 7     | 6     | 6     |
| 0.75  | 6   | 22        | 16    | 13    | 11    | 9     | 8     | 7     | 7     |
| 0.75  | 7   | 25        | 18    | 15    | 12    | 10    | 9     | 8     | 8     |
| 0.75  | 8   | 28        | 20    | 16    | 14    | 11    | 10    | 9     | 9     |
| 0.75  | 9   | 31        | 22    | 18    | 15    | 12    | 11    | 10    | 10    |
| 0.75  | 10  | 33        | 24    | 19    | 17    | 13    | 12    | 11    | 11    |
| 0.95  | 0   | 6         | 4     | 3     | 2     | 2     | 1     | 1     | 1     |
| 0.95  | 1   | 9         | 7     | 5     | 4     | 3     | 3     | 2     | 2     |
| 0.95  | 2   | 13        | 9     | 7     | 6     | 4     | 4     | 3     | 3     |
| 0.95  | 3   | 16        | 11    | 9     | 7     | 6     | 5     | 4     | 4     |
| 0.95  | 4   | 19        | 13    | 11    | 9     | 7     | 6     | 5     | 5     |
| 0.95  | 5   | 22        | 15    | 12    | 10    | 8     | 7     | 7     | 6     |
| 0.95  | 6   | 25        | 18    | 14    | 12    | 9     | 8     | 8     | 7     |
| 0.95  | 7   | 28        | 20    | 16    | 13    | 10    | 9     | 9     | 8     |
| 0.95  | 8   | 30        | 22    | 17    | 15    | 12    | 10    | 10    | 9     |
| 0.95  | 9   | 33        | 24    | 19    | 16    | 13    | 11    | 11    | 10    |
| 0.95  | 10  | 36        | 26    | 21    | 18    | 14    | 12    | 12    | 11    |
| 0.99  | 0   | 9         | 6     | 4     | 4     | 2     | 2     | 1     | 1     |
| 0.99  | 1   | 13        | 9     | 7     | 5     | 4     | 3     | 3     | 2     |
| 0.99  | 2   | 16        | 11    | 9     | 7     | 5     | 4     | 4     | 3     |
| 0.99  | 3   | 20        | 14    | 11    | 9     | 6     | 5     | 5     | 5     |
| 0.99  | 4   | 23        | 16    | 13    | 10    | 8     | 7     | 6     | 6     |
| 0.99  | 5   | 26        | 18    | 14    | 12    | 9     | 8     | 7     | 7     |
| 0.99  | 6   | 30        | 21    | 16    | 14    | 10    | 9     | 8     | 8     |
| 0.99  | 7   | 33        | 23    | 18    | 15    | 11    | 10    | 9     | 9     |
| 0.99  | 8   | 36        | 25    | 20    | 17    | 13    | 11    | 10    | 10    |
| 0.99  | 9   | 39        | 27    | 22    | 18    | 14    | 12    | 11    | 11    |
| 0.99  | 10  | 42        | 29    | 23    | 20    | 15    | 13    | 12    | 12    |

$p^*$ is the consumer confidence level.
Results for $FTPQSD(x; \theta = 3, \alpha = 2)$

Assume that the researcher needs to show that the true mean lifetime is at least $\mu_0 = 1000$ h with probability of $P^* = 0.95$ and the life test is truncated at $t_0 = 628$ h. It is shown in Table 1 that for $P^* = 0.95$, $t_0/\mu_0 = 0.628$, and $c = 2$, the minimum sample size is $n = 11$. Hence, out of the 11 items, if no more than two failures within 628 h, the researcher can confirm that the real mean life $\mu$ of the items is at least 1000 h with a confidence level of 95%.

Table 2 shows the $OC(p)$ values for the time truncated based on the ASP parameters given in Table 1. To illustrate the $OC(p)$, as an example for $P^* = 0.95$,
Table 6. Minimum ratio of $\mu/\mu_0$ for the acceptability of a lot with $\theta = 2$ and $\alpha = 3$ in the TPQSD and producer’s risk 0.05.

| $p^*$ | $c$ | $t/\mu_0$ |
|-------|-----|-----------|
| 0.628 | 0   | 34.291    |
| 0.942 | 0.75  | 8.937   |
| 1.257 | 0.75  | 5.599   |
| 1.571 | 0.75  | 3.941   |
| 2.356 | 0.75  | 3.441   |
| 3.141 | 0.75  | 2.902   |
| 3.927 | 0.75  | 2.732   |
| 4.712 | 0.75  | 2.604   |
| 0.75  | 8    | 2.379   |
| 0.75  | 9    | 2.315   |
| 0.75  | 10   | 2.167   |
| 0.90  | 0    | 57.246  |
| 0.90  | 1    | 12.258  |
| 0.90  | 2    | 7.042   |
| 0.90  | 3    | 5.235   |
| 0.90  | 4    | 4.337   |
| 0.90  | 5    | 3.802   |
| 0.90  | 6    | 3.268   |
| 0.90  | 7    | 3.046   |
| 0.90  | 8    | 2.878   |
| 0.90  | 9    | 2.747   |
| 0.90  | 10   | 2.546   |
| 0.95  | 0    | 68.724  |
| 0.95  | 1    | 13.917  |
| 0.95  | 2    | 8.484   |
| 0.95  | 3    | 6.098   |
| 0.95  | 4    | 4.935   |
| 0.95  | 5    | 4.252   |
| 0.95  | 6    | 3.804   |
| 0.95  | 7    | 3.488   |
| 0.95  | 8    | 3.128   |
| 0.95  | 9    | 2.962   |
| 0.95  | 10   | 2.830   |
| 0.99  | 0    | 103.157 |
| 0.99  | 1    | 13.258  |
| 0.99  | 2    | 8.848   |
| 0.99  | 3    | 6.098   |
| 0.99  | 4    | 4.935   |
| 0.99  | 5    | 4.252   |
| 0.99  | 6    | 3.804   |
| 0.99  | 7    | 3.488   |
| 0.99  | 8    | 3.128   |
| 0.99  | 9    | 2.962   |
| 0.99  | 10   | 2.830   |

$p^*$ is the consumer confidence level.
$t_0/\mu_0 = 0.628$, $c = 2$, and $\mu/\mu_0 = 2$, the matching entry in Table 2 is 0.375296. That is, for the above example, the lot is accepted if, out of the 11 items, at most two items fail before time $t_0 = 628$ h. Hence, if $\mu \geq 2 \times t_0/0.628 = 3.18 \times t_0 = 2000$ h, the lot will be accepted with at least probability of 0.375.

The minimum ratio of the true mean lifetime to the specified value for the acceptance of a lot is given in Table 3, with producer’s risk of $\phi = 0.05$. For illustration, when $P^* = 0.95$ (the consumer risk is 0.05), $c = 2$, and $t_0/\mu_0 = 0.628$, the values of $\mu/\mu_0$ from Table 3 is $\mu/\mu_0 = 8.002$. That is if $\mu \geq 8.002 \times t_0/0.628 = 12.742 \times t_0 = 8002$ h, then with $c = 2$ and $n = 11$, the lot should be rejected with probability equals to 0.05 or less (the product is accepted with probability of at least 0.95).

**Results for $F_{TPQSD}(x; \theta = 2, \alpha = 3)$**

The tables in this section are for the suggested ASP when the distribution parameters are $\theta = 2, \alpha = 3$. The above discussion can be repeated here. Now, let us investigate the influence of the distribution parameters on the proposed acceptance sampling plan. If we compare the minimum sample sizes shown in Tables 1 and 4, it turns out that the values given in Table 1 are less than their counterparts presented in Table 4.

Comparing to other distributions, the minimum sample sizes, specified in Tables 1 and 4, are less than the corresponding values given for the inverse-gamma distribution$^{24}$ and the transmuted generalized inverse Weibull distribution.$^7$ Therefore, we can say that this paper’s minimum sample sizes are superior to their competitors considered in this study.

**Real data application**

To examine the efficiency of the suggested acceptance sampling plans, we analyzed the data considered by Zimmer et al.$^{42}$ in 2003. The real data consists of the lifetime (in months) to the first failure of $n = 20$ small electric carts used for internal transportation and delivery in a large manufacturing facility. The data are 0.9, 1.5, 2.3, 3.2, 3.9, 5, 6.2, 7.5, 8.3, 10.4, 11.1, 12.6, 15, 16.3, 19.3, 22.6, 24.8, 31.5, 38.1, 53. The same data is considered by some authors as Al-Omari et al.$^{26}$ for ASP when the lifetime follow Akash distribution (AD) with PDF given by

$$f_{AD}(x; \theta) = \frac{\theta^3}{\theta^2 + 2} \left( 1 + x^2 \right) e^{-\theta x}, \quad x > 0, \theta > 0.$$  

Also, the data is fitted to Ishita distribution (ID), with pdf given by

$$f(x) = \frac{\theta^3}{\theta^3 + 2} (\theta + x^2) e^{-\theta x}, \quad x > 0, \theta > 0,$$

and to transmuted generalized inverse Weibull distribution (TGIWD) with pdf
Table 7. The AIC, CAIC, BIC, HQIC, $-2$MLL, K–S ($p$-value), and the MLE (error) for the electric carts data.

| Model  | K–S ($p$-value) | AIC   | BIC   | CAIC  | HQIC  | $-2$MLL | MLE (std. error) |
|--------|----------------|-------|-------|-------|-------|---------|------------------|
| TPQSD  | 0.15 (0.74)    | 154.414 | 156.41 | 155.12 | 154.80 | 75.21   | $\hat{\theta} = 0.136 (0.034)$, $\hat{\alpha} = 0.001 (0.0342)$ |
| TGIWD  | 0.124 (0.879)  | 159.797 | 163.780 | 162.464 | 160.575 | 75.899   | $\beta = 0.988 (0.159)$, $\gamma = 0.730 (17.725)$, $\lambda = 0.593 (0.456)$ |
| ID     | 0.43 (0.004)   | 162.164 | 163.160 | 162.386 | 162.359 | 80.082   | $\theta = 0.205 (0.026)$ |
| AD     | 0.2 (0.3)      | 160.4  | 161.4  | 160.6  | 160.6  | 79.2     | $\hat{\theta} = 0.202 (0.026)$ |

\[
f_{\text{TGIW}}(x) = \alpha \beta \gamma (\alpha x)^{-\beta - 1} e^{-\gamma (\alpha x)^{-\beta}} \left(1 + \lambda - 2\lambda e^{-\gamma (\alpha x)^{-\beta}}\right),
\]

Now, we have to check whether the two-parameter Quasi Shanker distribution fits the electric carts data. We considered the Bayesian Information Criterion (BIC), the Hannan-Quinn Information Criterion (HQIC), Akaike Information Criterion (AIC), the Consistent Akaike Information Criterion (CAIC), and the maximized log-likelihood (MLL) for selecting the best fit to the data, where

\[
\text{AIC} = -2 \text{MLL} + 2t, \quad \text{CAIC} = -2 \text{MLL} + \frac{2tn}{n - t - 1}, \quad \text{BIC} = -2 \text{MLL} + t \log(n), \quad \text{HQIC} = 2 \log[\log(n)(t - 2\text{MLL})],
\]

where $n$ is the sample size and $t$ is the number of parameters. The outcomes are given in Table 7. Also, we obtain the Kolmogorov-Smirnov statistics (K-S) and the corresponding $P$-Value (shown in brackets) for the TPQLD and the AD, TGIWD and ID. The best distribution with the smallest values of these measures is the better one in fitting these data.

Based on the above results in Table 7, the TPQSD fits the electric carts data adequately and better than the AD, TGIWD and ID. Using the MLE values of $\theta$ and $\alpha$ ($\hat{\theta} = 0.1364$ and $\hat{\alpha} = 0.0009$ respectively), the estimated mean for the electric data is

\[
\hat{\mu}_{\text{TPQSD}}(\text{Data}) = \frac{\hat{\theta}^3 + 2\hat{\theta} + 6\hat{\alpha}}{\hat{\theta}^3 + \hat{\theta} + 2\hat{\alpha}} = 14.623.
\]

Suppose that the lifetime of a product of the electric carts data follows TPQSD distribution. In the suggested ASPs, we assumed various values of the distribution
parameters which are different from the real data’s estimated parameter values. Hence, we re-compute the minimum sample sizes and the operating characteristic functions based on the estimated parameter values. Assume that the specified average lifetime and the testing time are $m_0 = 14.623$ and $t_0 = 17$ months, respectively. Then, for $P^* = 0.90$ and $d = t_0/m_0 = 1.163$, the acceptance number and the corresponding minimum sample sizes are displayed in Table 8, while the OC values and the corresponding producer risk are given in Table 9.

Based on these results, we want to check whether the lot can be accepted or not. From Table 8, for $n = 20$, we found $c = 10$. Hence, the lot is accepted if only the number of failures before $t_0 = 17$ months is equal to 10 or less. Since the number of failures before $t_0 = 17$ is 14, we reject the lot.

**Conclusion**

In this paper, we develop a new acceptance sampling plan when the product’s lifetime follows the two-parameter Quasi Shanker distribution. The minimum sample size needed to guarantee a particular mean life of the test units is provided. The tables of the operating characteristic function values and the associated producer risk values are also obtained. Several illustrated examples of the tables are also provided. An application of real data is presented to dominate the superiority of the proposed acceptance sampling plan.

The TPQSD distribution is a new lifetime distribution and showed its superiority in providing a better lifetime model for some applications than other exponential distributions. It is strongly recommended that industrial practitioners use the suggested sampling plan to test products when the failure time of products follows the TPQSD. The proposed ASP based on truncated lifetime for the TPQSD distribution is essential for industrial companies since the suggested small sample size based on the ASP will save the time and cost of investigation for any product issues.

For future works, the motivation, based on this paper’s results, is to investigate the TPQSD for specific percentile lifetime, double, and group acceptance sampling plans.
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