Third order effects generated by refractive lenses on sub 20 femtosecond optical pulses.

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Abstract. When using lenses to focus ultra-short pulses, chromatic aberration produces pulse spreading, after propagation through the lens. The focusing of ultra-short pulses has been analyzed by using Fourier optics where the field amplitude of the pulse is evaluated around the focal region of the lens by performing a third order expansion on the wave number around the central frequency of the carrier. In the literature, the pulse focusing in the neighborhood of the focal region of the lens has been calculated by expanding the wave number up to second order. The second order approximation works for pulses with a duration greater than 20 fs, or pulses propagating through low dispersion materials; but, it is necessary to do third order approximation for pulses with a shorter duration, or propagating through highly dispersive materials. In this paper we analyze 15 fs and 20 fs pulses, with a carrier wavelength of 810 nm, at the paraxial focal plane of singlets and achromatic doublets. The analysis includes the third order GVD and the results are compared with those obtained when the wave number is expanded up to second order.

Keywords: ultra-short pulses, group velocity dispersion, lenses.

1. Theory.
The field distribution, in the focal region of a lens, is evaluated by the diffraction integral [1],

\[ U(x_2, y_2, z; \Delta \omega) \approx \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx_1 dy_1 P(x_1, y_1) U_0(x_1, y_1) A(\Delta \omega) \Phi(x_1, y_1) \times \exp \left\{ -i \Theta(x_1, y_1; w_0) \right\} \times \exp \left\{ \frac{i}{2z} \left[ (x_2 - x_1)^2 + (y_2 - y_1)^2 \right] \right\} \]

where the term \( A(\Delta \omega) \) refers to the incident pulse which we assume has a Gaussian temporal envelope expressed as: \( A_0 \exp \left[ -\left( \frac{T \Delta \omega}{2} \right)^2 \right] \), \( U_0(x_1, y_1) \) is Gaussian illumination, and \( \Phi(x_1, y_1) \) is the phase contribution produced by the lens expressed as:
\[
\Phi(x_i,y_i) = \exp[i(k_x x_i + k_y y_i)]\times \exp\left[-i(k_{x_0} - k_x)^2 + i(k_{y_0} - k_y)^2\right]
\]

\[
= \exp[i(k_x x_i + k_y y_i)]\times \exp\left[-i(k_{x_0} - k_x)^2 + i(k_{y_0} - k_y)^2\right]
\]

\[
\Theta(x_i,y_i;w_0) \text{ is the primary spherical aberration produced by the lens expressed in terms of the Seidel coefficient, } S_{T_0}, \text{ and given by [1]:}
\]

\[
\Theta(x_i,y_i;w_0) = -\frac{k_0}{8} S_{T_0} (w_0) (x_i + y_i)^4
\]

The pupil function, \(P(x_i,y_i)\), is given by:

\[
P(x_i,y_i) = \begin{cases} 
1, & \text{if } x_i^2 + y_i^2 = r^2 = (\rho r)^2, \quad r \in [0,1] \\
0, & \text{any other case}
\end{cases}
\]

The wave number is defined as \(k_x = k_0 \left(1 + \frac{\Delta \omega}{\omega_0}\right)\), where \(\Delta \omega = \omega - \omega_0\) and \(k_0 = \frac{\omega_0}{c}\), \(\omega_0\) is the frequency of the carrier wave, or central frequency, and \(c\) is the speed of light in vacuum.

The integral given by eq. (1) is evaluated by performing a Taylor series expansion of the wave number around the carrier frequency \(\omega_0\), up to third order, that is:

\[
k_i = \frac{\omega_0}{c} n(\omega_0) \left[1 + \frac{1}{\omega_0} + \frac{1}{n_0} \frac{d n}{d \omega_0} \right] \Delta \omega + \left( \frac{1}{\omega_0} \frac{d n}{d \omega_0} + \frac{1}{2 n_0^2} \frac{d^2 n}{d \omega_0^2} \right) \Delta \omega^2 + \left( \frac{1}{2 \omega_0 n_0} \frac{d^2 n}{d \omega_0^2} + \frac{1}{6 n_0} \frac{d^3 n}{d \omega_0^3} \right) \Delta \omega^3
\]

By introducing the expansion for the wave number into equation (1), making a change to polar coordinates and several algebraic manipulations, the field is given by:

\[
U(u,v,z;t) \propto \exp[ik_0(\mu d)]\exp\left[i \frac{v^2}{4N} \int_0^\infty d(\Delta \omega) A_0 \exp\left[-(\Delta \omega)^2 \left( \frac{T^2}{4} - i(\delta^2 - r^2) \right) \right] \times \exp\left[-i(\Delta \omega)(r - \delta^2 + r^2 \delta) \right] \times \exp\left[i(\Delta \omega) \left( \frac{r^2 - r^2 \gamma}{} \right) \right] \times \int_0^r rdr U_0(r) \exp[-i \Theta(r;w_0)] J_0(\mu r) \times \exp\left[-i \frac{\mu r^2}{2} \right]
\]

where:

\[
u = \frac{\rho r^2 k_0}{f_0}, \quad N = \frac{\rho^2 k_0}{2 f_0} \quad \text{and} \quad v = \frac{\rho r^2 k_0}{f_0}
\]
the terms:

\[
\delta = \frac{k_0 \rho^2}{2} \left( \frac{(n-1)b_2}{R_1} - \frac{(n-1)b_3}{R_2} \right) \tag{4}
\]

\[
\gamma = \frac{k_0 \rho^2}{2} \left( \frac{(n-1)b_2}{R_1} - \frac{(n-1)b_3}{R_2} \right) \tag{5}
\]

\[
\delta' = k_0 [nda_2] \tag{6}
\]

\[
\gamma' = k_0 [nda_3] \tag{7}
\]

\[
\tau' = k_0 [nda_1] \tag{8}
\]

represent group velocity dispersion (GVD) at the second-order (Eq. 4 and Eq. 6) and the third-order (Eq. 5 and Eq. 7) approximation. Eq. (8) gives only a displacement of the pulse.

The term:

\[
\tau = \frac{k_0 \rho^2}{2} \left( \frac{(n-1)b_2}{R_1} - \frac{(n-1)b_3}{R_2} \right) - \left( \frac{k_0 \rho^2}{2 f_\omega \omega_0} - \frac{u}{2 \omega_0} \right) \tag{9}
\]

is related to the propagation time difference (PTD).

The \(a_i\) coefficients in equations (6), (7) and (8) are given in terms of refraction index derivatives as:

\[
a_1 = \left( \frac{1}{\omega_0} + \frac{1}{n_0} \frac{dn}{d\omega} \right) \quad a_2 = \left( \frac{1}{\omega_0 n_0} \frac{dn}{d\omega} + \frac{1}{2n_0^2} \frac{d^2n}{d\omega^2} \right) \quad a_3 = \left( \frac{1}{2\omega_0 n_0} \frac{d^2n}{d\omega^2} + \frac{1}{6n_0^3} \frac{d^3n}{d\omega^3} \right) \tag{10}
\]

The \(b_i\) coefficients, where \(i=1,2,3\), can be obtained by changing \(n\) for \(n-1\), in eq. (10).

2. Pulse width.

The full width half maximum, FWHM, is commonly adopted as the quantity representative of pulse duration. Nevertheless, when pulses have substructures or wings, where energy is redistributed, a better measure is to consider the average values derived from the appropriate second-order moments, as proposed in [2]. In this paper we use the mean square (MSQ) deviation as the criterion for the width distribution given by,

\[
< \tau_p > = \Delta t = \left[ \frac{1}{W} \int_{-\infty}^{\infty} t^2 I(t)dt - \frac{1}{W^2} \left( \int_{-\infty}^{\infty} t I(t)dt \right)^2 \right]^{1/2} \tag{11}
\]
where \( W = \int_{-\infty}^{\infty} I(t) dt \), and the intensity is given by

For a Gaussian pulse without chirp, the value of \( \langle \tau_p \rangle \) is normalized to one.

3. Results

By using eq. (3) the pulses were evaluated on the paraxial focal plane of the lenses. The recursive method presented by Rosete-Aguilar, et.al., [3] has not been used for the cases in the present paper due to numerical errors in the method which do not allow its use in the propagation of pulses through real single lenses and achromats. Instead, the integration of eq. (3) was performed by using the rectangle method. Five real single lenses with different focal lengths taken from the Edmund Optics catalogue were analyzed. The glasses of the lenses were: BK7 and SF5, low and high dispersion materials, respectively. The incident pulse beam with a carrier wavelength of 810nm was Gaussian with \( w_0 = 6\, \text{mm}, \quad r = 6\, \text{mm}, \) (i.e., the Gaussian beam intensity falls to \( 1/e \) at the edge of the lens).

Table 1 shows the value of the mean square deviation, MSQ, calculated with eq. (11). For those calculations, we considered an incident pulse without chirp. We also assumed the lenses are ideal so the propagation time difference, the second-order group velocity dispersion and the primary spherical aberration of the single lenses were set equal to zero, so that only the third-order group velocity dispersion generated in the lens can be observed in the pulse; for the second-order approximation, the values of MSQ are equal to one; this means that the pulse is not affected when passing through the single lens.

Table 1  Values of MSQ \( \langle \tau_p \rangle \) for 15 fs and 20 fs incident pulses through the singlet lens.

| Focal length (mm) | Diameter (mm) | Glass | NA | Second order \( < \tau_p > \) | Third order \( < \tau_p > \) |
|------------------|---------------|-------|----|-----------------|-----------------|
| 18               | 12            | SF5   | 0.33 | 1               | 1.07            | 1.04            |
| 20               | 12            | BK7   | 0.3  | 1               | 1.05            | 1.03            |
| 24               | 12            | BK7   | 0.24 | 1               | 1.07            | 1.03            |
| 30               | 12            | BK7   | 0.2  | 1               | 1.07            | 1.03            |
| 42               | 12            | BK7   | 0.15 | 1               | 1.05            | 1.03            |

Table 1 shows that for 15 fs and 20 fs pulses the third order GVD is negligible after propagating through the singlet lenses. This result can be verified in figure 1, where the incident pulse is shown with dotted lines and the pulse at the paraxial focus of the lens is shown by a continuous line.
Figure 1. Pulse intensity at the paraxial focus of two single lenses: f=18mm (left column) and f=30mm (right column) given in table 1. All effects are zero except third order GVD (continuous line), incident pulse (dotted line).

Table 2 shows calculated values for pulses at the paraxial focus of the single lenses, but now considering PTD and primary spherical aberration. The terms, $\delta'$, $\delta''$ and $\tau'$ given by Eqs. (4), (6) and (8) are set equal to zero.

Table 2 MSQ for the second and third order GVD approximations, for 15fs and 20fs taking into account PTD and spherical aberration.

| Focal length (mm) | Diameter (mm) | Glass | NA  | Second order $<\tau_p>$ | Third order $<\tau_p>$ |
|------------------|---------------|-------|-----|------------------------|-----------------------|
|                  |               |       |     | 15fs   | 20fs       | 15fs   | 20fs       |
| 18               | 12            | SF5   | 0.33| 5.40   | 4.06       | 8.80   | 6.20       |
| 20               | 12            | BK7   | 0.30| 4.71   | 3.66       | 4.91   | 3.82       |
| 24               | 12            | BK7   | 0.24| 3.67   | 2.83       | 3.73   | 2.90       |
| 30               | 12            | BK7   | 0.20| 2.58   | 2.04       | 2.53   | 1.99       |
| 42               | 12            | BK7   | 0.15| 1.42   | 1.25       | 1.88   | 1.56       |

In figures 2 and 3 the pulse intensity in the paraxial focal plane of the singlet lens is shown for 15fs and 20fs pulses, respectively. The PTD effect and the primary spherical aberration are included in the calculations. It can be seen that the pulse loses its Gaussian shape. In both cases the focal length is 18mm and the diameter is 12mm, and the glass of the lens is SF5.
Figure 2. Pulse intensity on the focal plane of a single lens with focal length 18mm, for 15fs and third order, PTD and spherical aberration. The dotted line represents the incident pulse.

Figure 3. Pulse intensity at the focal plane of a single lens with focal length 18mm, for 20fs incident pulse. The third order, PTD and spherical aberration are included in the calculation (solid line). The dotted line shows the incident pulse.
Real achromatic doublets were also analyzed. Figure 4 (5) shows the intensity for a 15fs (20fs) Gaussian pulse at the paraxial focal region of an achromatic doublet with a focal length 18mm, diameter 12mm and glasses: LAKN22-SFL6. The Gaussian beam waist is $w_0 = 6\text{mm}$. We can see that the temporal spreading of the pulse can be corrected by using an achromatic doublet designed in the IR region between 700nm and 1100nm. The third order dispersion produces an additional temporal spreading of the pulse of 9fs and 6fs for the 15fs and 20 fs shown in figures 4 and 5, respectively.

**Figure 4.** Intensity of a pulse with 15fs initial duration, at the paraxial focal focus of an achromatic doublet with focal length 18mm and diameter 12mm.

**Figure 5.** Intensity of a pulse with 20fs initial duration, at the paraxial focal region of an achromatic doublet with focal length 18mm and diameter 12mm.
The pulse intensity for a 20fs Gaussian pulse propagating through a doublet with focal length of 30mm, diameter of 12mm and glasses: LAKN22-SFL6 is shown in figure 6. The additional temporal spreading of the pulse due to third-order group velocity dispersion is 4fs. An error in a previous algorithm led us to an erroneous conclusion about the propagation time difference effect produced by these achromats [4].

| Second order | Third order |
|--------------|-------------|

![Figure 6](image)

Figure 6. Intensity of a pulse with 20fs initial duration, at the paraxial focus of an achromatic doublet with focal length 30mm and diameter 12mm.

4. Conclusions

Ultra-short pulses, with initial duration 20fs and 15fs and a carrier wavelength of 810nm, have been evaluated at the paraxial focus of single and achromatic doublets with different focal lengths. The temporal spreading of the pulses produced by single lenses, with numerical aperture up to 0.33, can be corrected by using achromatic doublets designed in the IR region between 700nm and 1100nm but the third-order dispersion begins to contribute in the temporal spreading of the pulse, with an additional temporal spreading of 9fs for a 20fs @810nm pulse at the paraxial focal region of an achromat with focal length of 18mm and diameter of 12mm.

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5. References

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