Electromechanical impedance instrumented circular piezoelectric-metal transducer for corrosion monitoring: modeling and validation

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Abstract
Corrosion induced thickness loss of metallic structures is one of the most common issues across multiple industries. In our previous work, a new type of corrosion sensor based on lead zirconate titanate (PZT) using electromechanical impedance (EMI) technique was proposed. The sensor is fabricated by bonding a PZT patch onto a metal plate. The previous work has demonstrated that the peak frequencies in the conductance signatures decrease linearly with the increase of the corrosion induced thickness loss. However, a theoretical model that fully describe the coupled vibration between piezoelectric element and the metal plate, and the EMI characteristics has not been established. This paper presents the theoretical modeling of the EMI instrumented circular piezoelectric-metal transducer for corrosion monitoring purpose. Based on electro-elastic and Kirchhoff plate theory, the EMI responses of the transducer operated in transverse bending modes with free boundary conditions were modeled. Finite element modeling calculations and experimental measurement were conducted to validate the theoretical results with good agreement.

Keywords: piezoelectric-metal transducer, electromechanical impedance (EMI), corrosion monitoring, electro-elastic modeling, finite element modeling

(Some figures may appear in colour only in the online journal)

1. Introduction

Corrosion is the deterioration of metallic materials as a result of chemical or electrochemical reaction between them and their environment. Corrosion is one of the most common issues in multiple industries, including oil and gas, civil, mechanical, aerospace, mining, and processing. Generally, corrosion can be classified into two types, the uniform corrosion and localized corrosion. The uniform corrosion leads to the thinning of metallic materials, which is the most common form of corrosion and represents the greatest destruction on metals. The corrosion induced thickness loss on the metallic structures poses great threat to the safety and serviceability of the structures, and in severe situations, it is responsible for the huge amount of economic loss and even the loss of human life. It is reported that the global cost of corrosion is about 2.5 trillion US dollars each year (Koch et al...
Various methods are under development for corrosion monitoring. For example, the development of guided wave tomography (Huthwaite and Simonetti 2013, Rao et al 2016a, 2016b, Brath et al 2017, Rao et al 2017a) has enabled the quantification of corrosion in plates and pipes (Nagy et al 2014, Rao et al 2017b, 2017c). On the other hand, the piezoelectric based structural health monitoring (SHM) has received increasing research attention over the past decades. A piezoelectric transducer acts as both sensor and actuator due to the direct and inverse piezoelectric effects. Piezoelectric material, the lead zirconate titanate (PZT) in particular, offers many advantages such as noninvasive, low cost, high sensitivity, wide bandwidth, quick response, and online monitoring capability. The piezoelectric based SHM methods have been adopted in the monitoring of damages in various engineering structures. For example, the rock bolt health monitoring (Song et al 2017, Wang et al 2017b), interlayer slide detection (Wu et al 2017), water seepage monitoring in concrete structures (Liu et al 2013, Zou et al 2014), concrete crack monitoring (Dumoulin et al 2014, Tsangouris et al 2015), debonding detection (Xu et al 2013, Li et al 2016a, Luo et al 2016, Xu et al 2018, Chen et al 2019), crack detection in aluminum plate (Kudela et al 2018), concrete hydration and strength gain monitoring (Dumoulin et al 2012, Kong et al 2013, Lu et al 2018b, Lu et al 2019), novel smart aggregate based health monitoring methods for concrete structures (Li et al 2016b, Zou et al 2017, Du et al 2018), and novel signal processing approaches were also proposed for extracting useful damage related information (Fan et al 2016, 2018b). The electromechanical impedance (EMI) technique has been extensively recognized as a promising and powerful tool for damage detection and evaluation of a variety of engineering structures, such as pre-stressed structures (Wang et al 2017a, Fan et al 2018a, Huyhn et al 2018, Ryu et al 2019), concrete structures (Liang et al 2016, Li et al 2018, Lu et al 2018a, Shi et al 2018, Talakokula et al 2018), metal plates (Giurgiutiu and Zagari 2005, Vieira Filho et al 2011, Fan et al 2018c), and interlayer slide (Wu et al 2018). In this technique, the PZT patch is either surface-bonded on or embedded into the structure to be examined. The vibration of the PZT patch and the host structure are thus coupled. The PZT patch is collocated as sensor and actuator and the EMI signals are measured by an impedance analyzer. If variations presented in the EMI signals of current condition as compared to those of pristine condition, structural damages are detected. The EMI method is fast, accurate, and has the potential to provide real-time, remote and autonomous monitoring at low cost with large scale application capability.

Up to now, literature survey showed that limited studies have been carried out on corrosion monitoring using the EMI technique. Talakokula et al evaluated the corrosion process of rebars of reinforced concrete using the surface-bonded PZT patches and the EMI technique (Talakokula et al 2013). It was shown that the equivalent parameters derived from the admittance signatures were correlated to the corrosion amount of the reinforced concrete. Zhu et al introduced the concept of structural mechanical impedance for corrosion detection of steel structures using the EMI method (Zhu et al 2016). The experimental results showed that the structural mechanical impedance is sensitive to corrosion damage despite the limited sensing range. Na investigated the possibility of detecting wall thickness loss of metal based pipeline facilities using the EMI technique (Na 2017). It was found that the resonance peaks in the impedance signatures is shifted due to the reduction in wall thickness. However, the common issue with these studies is that the PZT patch is usually attached to the host structures which have complex geometry, loading conditions, and boundary conditions. Thus, it is difficult to build up the physical relation between the resonant peaks in the EMI signatures and the structural parameters. Also, it is difficult to isolate the other influencing factors, such as variations in loading and boundary conditions, from the corrosion induced damages. Since the resonant frequency range are deeply affected by the geometry shape, loading conditions, and boundary conditions of the host structure, choosing the appropriate frequency range for the EMI signatures would be a challenging task. Furthermore, the damage induced variations in the EMI signatures are quantified by the statistical metrics, such as root-mean-square deviation and correlation coefficient, which may not work well if the selected frequency range is inadequate.

In our previous studies, a new type of EMI based corrosion sensor was proposed to overcome the shortcomings of the abovementioned EMI corrosion monitoring methods (Li et al 2019a, 2019b). The proposed corrosion sensor is essentially a piezoelectric-metal transducer, which consists of a metal plate with a PZT patch attached onto it. Previous studies demonstrated that the peak frequencies decrease linearly with the increase of the corrosion induced thickness loss. And the finite element modeling (FEM) based modal analysis showed that the vibration modes of the piezoelectric-metal transducer corresponding to the shifted peak frequencies are the bending modes. However, a theoretical model that fully describe the coupling vibration between piezoelectric element and the metal plate and the EMI characteristics has not been established. In the present study, the theoretical model accounts for the transverse bending vibration of the piezoelectric-metal transducer was established. The EMI characteristics of the transducer was investigated and the validity of using the proposed transducer as corrosion sensor was verified theoretically. The theoretical calculations were compared with FEM and experimental results. The results showed that theoretical predictions, FEM calculations and experimental measurements agree well with each other.

2. Theoretical modeling

The EMI based corrosion sensor to be modeled is considered as a circular piezoelectric-metal transducer fabricated by bonding a piezoelectric disk to a metal plate, as shown in figure 1. The piezoelectric disk is polarized in the thickness direction, and its major surfaces are completely covered with
electrodes with negligible thickness. When the electrodes are connected to an sinusoidal actuating voltage of magnitude $V_0$ and circular frequency $\omega$, the in-plane extension of the piezoelectric disk excites the bending vibration of the transducer. The thickness reduction of the metal plate will reduce the bending resonant frequencies of the transducer, and thus the transducer can be used for corrosion monitoring. In the following analysis, the piezoelectric-metal transducer is treated as a single electromechanical body operated in transverse bending modes.

2.1. Basic equations

The thicknesses of the piezoelectric disk and the metal plate are $h_p$ and $h_m$, respectively, and their radii are both $a$. The total thickness is then expressed as $h_{total} = h_p + h_m$. Subscripts $p$ and $m$ indicate the piezoelectric disk and the metal plate, respectively. The neutral surface of the whole sensor is chosen as the coordinate $z = 0$. Thus, the distance from the top surface to the neutral surface is $h_0$ and the distance from the bottom surface to the neutral surface is $h_1$. The total thickness is also expressed as $h_{total} = h_0 + h_1$.

Based on Kirchhoff’s hypotheses, the strain-displacement relationships can be expressed as (Mo et al 2006, Deshpande and Sagare 2007, Li et al 2009)

$$\varepsilon_r = -z \frac{dw}{dr}, \quad \varepsilon_\theta = -z \frac{dv}{dr},$$

where $\varepsilon_r$ and $\varepsilon_\theta$ are the radial and circumferential strains, respectively; $w$ is the transverse displacement.

For the piezoelectric disk polarized along its thickness direction, its constitutive equations can be expressed as (Mo et al 2006, Dong et al 2007, Wang and Shi 2013, Wang et al 2018)

$$\sigma_p = \frac{E_p}{1 - \nu_p^2} (\varepsilon_r + \nu_p \varepsilon_\theta) - \varepsilon_{31} E_z, \quad \sigma_m = \frac{E_m}{1 - \nu_m^2} (\varepsilon_r + \nu_m \varepsilon_\theta) - \varepsilon_{31} E_z,$$

where $\sigma_p$ and $\sigma_m$ are the radial and circumferential stresses, respectively; $D_z$ and $E_z$ are the electric displacement and electric field, respectively; $E_p = 1/s_{11}^p$ is Young’s modulus of the piezoelectric disk; $\nu_p = -s_{12}^p/s_{11}^p$ is Poisson’s ratio; $e_{31} = d_{31}/(s_{11}^p + s_{12}^p)$ is the effective piezoelectric stress constant; $\varepsilon_{33} = \varepsilon_{33}^s - 2d_{31}^s/(s_{11}^p + s_{12}^p)$ is the effective dielectric permittivity at constant strain; $s_{11}^p, s_{12}^p, d_{31}^p$ and $\varepsilon_{33}^s$ are the elastic compliances at constant electric field, the piezoelectric strain constant, the dielectric permittivity at constant stress, respectively.

For the metal plate, its constitutive equations can be expressed as (Mo et al 2006, Dong et al 2007)

$$\sigma_m = \frac{E_m}{1 - \nu_m^2} (\varepsilon_r + \nu_m \varepsilon_\theta), \quad \sigma_m = \frac{E_m}{1 - \nu_m^2} (\varepsilon_r + \nu_m \varepsilon_\theta),$$

where $E_m$ and $\nu_m$ are Young’s modulus and Poisson’s ratio of the metal plate, respectively.

The resultant radial and circumferential bending moments $M_r$ and $M_\theta$ are

$$M_r = \int_{-h_1}^{h_0} \sigma_m dz + \int_{h_0}^{h_1} \sigma_m dz, \quad M_\theta = \int_{-h_1}^{h_0} \sigma_m dz + \int_{h_0}^{h_1} \sigma_m dz.$$  

The equilibrium equations for the axisymmetric bending problem are

$$Q_r = \frac{dM_r}{dr} + \frac{M_r - M_\theta}{r}, \quad \frac{dQ_\theta}{dr} + \frac{Q_\theta}{r} = m \frac{d^2w}{dt^2},$$

where $Q_r$ is the resultant shear force; $\rho_p$ and $\rho_m$ are the densities of the piezoelectric disk and the metal plate, respectively; $m = \rho_p h_p + \rho_m h_m$ is the mass per unit area; $t$ is the time.

2.2. Solution

In order to solve the equilibrium equations together with constitutive equations in view of strain-displacement relationships, the location of the neutral surface needs to be determined based on the force equilibrium of the resultant radial force $N_r$, that is

$$N_r = \int_{-h_1}^{h_0} \sigma_m dz + \int_{h_0}^{h_1} \sigma_m dz = 0. \quad (12)$$

To simply the problem, the electrical effect is usually assumed to be negligible when determining the neutral surface. Under this condition, the second term in equation (3) is neglected for the piezoelectric disk. Upon solving equation (12), the distances $h_0$ and $h_1$ can be derived as (Dong et al 2007, Li et al 2009)

$$h_0 = \frac{Y_m h_m^2 + Y_p h_p^2 + 2Y_m h_m h_p}{2(Y_m h_m + Y_p h_p)},$$

$$h_1 = \frac{Y_m h_m^2 + Y_p h_p^2 + 2Y_m h_m h_p}{2(Y_m h_m + Y_p h_p)},$$

where $Y_m = E_m/(1 - \nu_m^2), Y_p = E_p/(1 - \nu_p^2).$
Substituting equations (1)–(4), (6), (7), (13) and (14) into equations (8) and (9), the resultant radial and circumferential bending moments $M_r$ and $M_	heta$ can be expressed in terms of the transverse displacement $w$ as follows:

$$M_r = \bar{D} \left[ - \frac{d^2 w}{dr^2} \right] + \bar{D}_r \left( \frac{1}{r} \frac{dw}{dr} \right) - e_{31} E_z h_{pr},$$

(15)

$$M_{\theta} = \bar{D} \left[ - \frac{d^2 w}{dr^2} \right] + \bar{D}_\theta \left( \frac{1}{r} \frac{dw}{dr} \right) - e_{31} E_z h_{pr},$$

(16)

where

$$\bar{Y}_n = \frac{1}{3} Y_m [ (h_m - h_i)^3 + h_i^3 ],$$

$$\bar{Y}_p = \frac{1}{3} Y_p [ h_0^3 - (h_m - h_i)^3 ];$$

$\bar{D} = \bar{Y}_m + \bar{Y}_p$ is the flexural rigidity of the whole structure; $\bar{D}_r = \bar{Y}_m v_m + \bar{Y}_p v_{pr}$

Substituting equations (15) and (16) into (10), the resultant shear force $Q_r$ can also be expressed in terms of the transverse displacement $w$ as follows:

$$Q_r = \bar{D} \left[ - \frac{d^3 w}{dr^3} \right] + \left( \frac{1}{r^2} \frac{d^2 w}{dr^2} \right) + \left( \frac{1}{r} \frac{dw}{dr} \right).$$

(17)

Substituting equation (17) into (11) yields

$$-\bar{D} \left[ \frac{d^2 w}{dr^3} + 2 \frac{d^2 w}{r \, dr^2} - \left( \frac{1}{r} \frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right) \right] = m \frac{d^2 w}{dr^2}.$$  

(18)

Considering a harmonic actuating voltage $V(t) = V_0 e^{j \omega t}$ ($E = -(V_0 / h_p) e^{j \omega t}$) applied to the piezoelectric disk, the transverse displacement $w$ can be written as

$$w = w(r) e^{j \omega t},$$

(19)

where $V_0$ is the actuating voltage amplitude, $j = \sqrt{-1}$ is the imaginary unit, $f$ is the excitation frequency, and $\omega = 2 \pi f$ is the circular frequency.

Substituting equation (19) into (18), results in the following governing equation

$$\frac{d^3 w(r)}{dr^3} + 2 \frac{d^2 w(r)}{r \, dr^2} - \left( \frac{1}{r^2} \frac{d^2 w(r)}{dr^2} \right) + \frac{1}{r} \frac{dw(r)}{dr} - k^4 w(r) = 0,$$

(20)

where $k^4 = \omega^2 m / \bar{D}$.

Solving equation (20), yields

$$w(r) = A_1 J_0 (kr) + B_1 Y_0 (kr) + C_1 I_0 (kr) + D_1 K_0 (kr),$$

(21)

where $A_1$, $B_1$, $C_1$ and $D_1$ are the undetermined coefficients; $J_0 (kr)$ and $Y_0 (kr)$ are Bessel functions of the first and second kinds, respectively; $I_0 (kr)$ and $K_0 (kr)$ are Modified Bessel functions of the first and second kinds, respectively. When $r \to 0$, $Y_0 (0)$, $K_0 (0) \to \infty$. Thus, $B_1 = 0$, $D_1 = 0$. Then equation (21) becomes

$$w(r) = A_1 J_0 (kr) + C_1 I_0 (kr).$$

(22)

At $r = a$, the free boundary conditions must be satisfied, which are expressed as

$$M_r |_{r=a} = 0,$$

(23)

$$Q_r |_{r=a} = 0.$$  

(24)

Combining equations (23) and (24), yields

$$A_1 \delta_1 - C_1 \delta_2 = -e_{31} V_0,$$

(25)

$$A_1 \delta_3 - C_1 \delta_4 = 0,$$

(26)

where

$$\delta_1 = (\bar{D} + \bar{D}_r) (1 / a) k J_1 (ka) - \bar{D} k^2 J_2 (ka),$$

(27)

$$\delta_2 = (\bar{D} + \bar{D}_r) (1 / a) k I_1 (ka) + \bar{D} k^2 I_2 (ka),$$

(28)

$$\delta_3 = \bar{D} k^2 [ - (4 / a) J_2 (ka) + k J_3 (ka) ],$$

(29)

$$\delta_4 = \bar{D} k^2 [ (4 / a) I_2 (ka) + k I_3 (ka) ].$$

(30)

Solving equations (25) and (26), the undetermined coefficients $A_1$ and $C_1$ are

$$A_1 = \delta_5 V_0,$$

(31)

$$C_1 = \delta_6 V_0,$$

(32)

where

$$\delta_5 = -e_{31} \delta_3 / (\delta_1 \delta_4 - \delta_2 \delta_3),$$

(33)

$$\delta_6 = -e_{31} \delta_4 / (\delta_1 \delta_3 - \delta_2 \delta_4).$$

(34)

Further, combining equations (1), (2), (5), (22), (31), (32), and performing the integration over the upper electrode area of the piezoelectric disk gives the electrical charge

$$Q(t) = \int_0^{2 \pi} \int_0^a D_2 \left[ -h_0 r dr d\theta \right] = \tilde{C}_0 V_0 e^{j \omega t},$$

(35)

where $\tilde{C}_0$ is the effective electric capacitance, and is defined as

$$\tilde{C}_0 = 2 \pi \varepsilon_3 \varepsilon_0 k a [ \delta_5 J_1 (ka) - \delta_6 \delta_2 (ka) ] - C_0.$$  

(36)

In equation (36), $C_0 = \pi a^2 \varepsilon_{33} / h_p$ is the clamped electric capacitance of the piezoelectric disk. Thus, the currents $I(t)$ can be solved as

$$I(t) = - \frac{dQ(t)}{dt} = -j \omega \tilde{C}_0 V_0 e^{j \omega t}. $$

(37)

Then, the electrical admittance $Y$, which is the inverse of impedance $Z$, can be expressed as

$$Y = \frac{1}{Z} = G + jB = \frac{I(t)}{V(t)} = -j \omega \tilde{C}_0,$$

(38)

where $G$ and $B$ are the conductance (real part of admittance) and susceptance (imaginary part of admittance), respectively. The local maxima appearing in the admittance curve or conductance curve corresponds to resonance.

3. Numerical results

3.1. Theoretical calculations

Using the theoretical analysis described in the previous section, the EMI responses of the piezoelectric-metal transducer with different metal thicknesses can be calculated numerically. The piezoelectric-metal transducers considered in this study are listed in table 1. Transducers with two different diameters are considered, that is, 30 and 40 mm. The thickness of the piezoelectric disk is 0.5 mm. The thickness of the metal plate is reduced from 3.0 to 1.0 mm with an interval...
Table 1. Sizes of piezoelectric-metal transducer.

| Specimen | Metal plate (mm) | PZT thickness (mm) | Diameter (mm) |
|----------|-----------------|-------------------|---------------|
| D30T30   | 3.0             | 0.5               | 30            |
| D30T25   | 2.5             |                   |               |
| D30T20   | 2.0             |                   |               |
| D30T15   | 1.5             |                   |               |
| D30T10   | 1.0             |                   |               |
| D40T30   | 3.0             | 0.5               | 40            |
| D40T25   | 2.5             |                   |               |
| D40T20   | 2.0             |                   |               |
| D40T15   | 1.5             |                   |               |
| D40T10   | 1.0             |                   |               |

Table 2. Material properties of the piezoelectric disk and the metal plate.

| Material | Properties          | Symbols | Values  |
|----------|---------------------|---------|---------|
| PZT      | Density (kg m$^{-3}$) | $\rho$  | 7450    |
|          | Dielectric loss factor | $\tan \delta$ | 0.023  |
|          | Compliance (10$^{-12}$ m$^2$ N$^{-1}$) | $s_{ii}$ | 13     |
|          | Relative permittivity | $\varepsilon_{ii}/\varepsilon_0$ | 5800   |
|          | Piezoelectric strain coefficients (10$^{-12}$ C/N) | $d_{31}$, $d_{32}$ | -186   |
|          | Damping ratio       | $\zeta$ | 0.014  |
| Metal plate | Density (kg m$^{-3}$) | $\rho$  | 7800    |
|          | Poisson’s Ratio     | $\nu$   | 0.26    |
|          | Young’s modulus (GPa) | $E$     | 200     |

of 0.5 mm, so as to simulate thickness loss and observe the corresponding EMI responses. The diameter-to-thickness ratio is larger than 10 except for specimen D30T30. The material for the piezoelectric disk is PZT-5H and those for metal plate is mild steel. The material properties are listed in table 2.

The conductance signatures of transducers with different metal plate thickness in the frequency range 1–200 kHz are shown in figure 2. Since some of the peaks for different metal thickness are overlapped, the conductance signatures for different metal plate thickness are plotted in separate subplots. The first peak corresponds to the resonant frequency of first bending mode and the same goes for the second and the third. In this study, only the first and the second bending modes were investigated since these modes are within 200 kHz frequency range that can be easily measured using a common impedance analyzer. It can be observed that the peak frequency shows leftward movement due to the reduction in metal plate thickness. That is to say, the resonant frequency reduced with the decrease in metal thickness, or the increase in thickness loss of the metal plate. Such property in the EMI response can be utilized for corrosion monitoring. Such phenomenon were also observed in our previous studies (Li et al 2019a, 2019b). The peak frequencies calculated from the theory are listed in table 3 for comparison.

The peak frequencies of the first two bending modes as a function of thickness loss of the metal plate, together with the linear fitted curves, are shown in figure 3. The expressions and the coefficients of determination ($R^2$) of the fitted curves are also presented. The linearity between the peak frequency and the thickness loss is very good as evidenced by the fact that all the coefficients of determination of the fitted curves are very close to one. The linear relationship between the peak frequency of bending modes and the thickness loss of the metal plate is an advantageous feature of the proposed transducer. The slope of the fitted curve is the sensitivity of the transducer. For the 30 mm diameter transducer, the sensitivity for the first and the second bending modes are 9.12 kHz mm$^{-1}$ and 39.30 kHz mm$^{-1}$, respectively. And for the 40 mm diameter transducer, the sensitivity for the first peak and the second peak are 5.10 kHz mm$^{-1}$ and 22.12 kHz mm$^{-1}$, respectively. As can be seen, the second mode is much more sensitive than the first mode. In addition, when comparing the same mode, that of the 30 mm diameter transducer is more sensitive than that of the 40 mm diameter transducer. Consequently, higher order modes and transducer with smaller diameter, are more sensitive to the thickness loss of the metal plate. Thus, it is possible to adjust the sensitivity of the transducer by selecting the proper mode as well as the size of the transducer. Given that the sampling resolution of the impedance analyzer can be as high as 1 Hz/sample, the proposed transducer is able to achieve sub-micrometer accuracy technically. The results demonstrate that the EMI instrumented piezoelectric-metal transducer can be utilized as corrosion sensor for quantitative determination of corrosion induced thickness loss.

3.2. Finite element modeling

The theoretical analysis described in the above section has advantages in terms of using expressions to calculate the EMI signatures. However, such analysis has drawbacks because some assumptions were adopted to simplify the real physical phenomenon. Therefore, the theoretical analysis was validated by FEM and impedance measurement results.

The FEM of the EMI responses of the piezoelectric-metal transducer were performed using ANSYS software. Modal analysis was also carried out to identify the vibration modes corresponding to the peaks in the EMI signatures. The sizes and material properties of the transducers are in accordance with theoretical analysis. The coupled field element SOLID5 was used to model the piezoelectric disk and the structural element SOLID45 was used to model the metal plate. The finite element model of the transducer is shown in figure 4.
The model was meshed with a size of 1 mm. The adjacent nodes between the PZT disk and the metal plate were glued so that their displacement degrees of freedom were coupled. Only one quarter of the transducer was modeled due to symmetry.

Harmonic analysis was carried out to obtain the EMI signatures of the transducer under different thicknesses of the metal plate. The voltage degrees of freedom of the nodes at both the top and bottom surfaces of the PZT were coupled to one master node to simulate the electrodes. A harmonic excitation $V(t) = V_0 e^{j\omega t}$ with $V_0 = 1$ V was applied to the
The conductance signatures of transducers with different metal plate thickness are shown in figure 5. The first peak is not noticeable in the graph, so a refined frequency range was scanned for EMI signatures and the results are shown in figure 6. The refined frequency interval is 25 Hz. The peak frequencies of the first two bending modes as a function of thickness loss of the metal plate obtained by FEM, together with the linear fitted curves, are shown in figure 7. It should be noted that the first two peaks correspond to the first two bending modes of the transducer, while the third peak corresponds to the in-plane extension mode which is confirmed in the following modal analysis. The in-plane extension mode was not presented in the theoretical results since only bending modes were considered in the theoretical modeling. For the 30 mm diameter transducer, the sensitivity for the first and the second bending modes are 8.32 kHz mm\(^{-1}\) and 27.32 kHz mm\(^{-1}\), respectively. And for the 40 mm diameter transducer, the sensitivity for the first and the second bending modes are 4.86 kHz mm\(^{-1}\) and 17.72 kHz mm\(^{-1}\), respectively. The characteristics of the first two peaks is the same as those obtained by theory. The peak frequencies calculated using FEM are listed in table 3 for comparison.

In the modal analysis, the top and bottom electrodes of the PZT were short-circuited by setting voltage to 0 V so as to obtain the resonant frequencies and the corresponding modes. The first two bending mode shapes of the transducer (Mode 1 and Mode 2), along with the in-plane extension mode (Mode 3), are shown in figure 8. The modal frequencies of these three modes versus the thickness loss are shown in figure 9. The modal frequencies match well with the peak frequencies in the conductance signatures. It can also be observed that Mode 1 and Mode 2 are sensitive to thickness loss while Mode 3 is not sensitive to the thickness loss. The physical explanation for such phenomenon is that the thickness loss reduces the plate rigidity of the piezoelectric-metal transducer and therefore the resonant frequencies associate with bending modes.

4. Experimental study

4.1. Impedance measurement

The piezoelectric-metal transducers were fabricated by bonding the PZT disk onto the circular metal plate using epoxy, as shown in figure 10. The size configurations of the transducers were in accordance with the theoretical analysis and FEM. The transducer was placed on a soft foam so that the free boundary conditions were satisfied. The transducer was then connected to an impedance analyzer (PV520A, Beijing Band Era Co., Beijing) to acquire the EMI signatures which were stored in a personal computer for further processing. The frequency range is from 10 to 200 kHz with an interval of 200 Hz. The conductance signatures obtained by the impedance measurement are shown in figure 11. The first peak of specimens D40T15 and D40T10 is below 10 kHz, which is outside the measurement range of impedance analyzer PV520A. The first peak of 40 mm diameter transducer was scanned from 1 to 20 kHz with an interval of 25 Hz using another impedance analyzer (Agilent 4291A, Hewlett Packard, USA). The conductance signatures are shown in figure 12. The peak frequencies of the first two bending modes versus thickness loss of the metal plate obtained by measurement, together with the linear fitted curves, are shown in figure 13. For the 30 mm diameter transducer, the sensitivity for the first and the second bending modes are 8.20 kHz mm\(^{-1}\) and 27.48 kHz mm\(^{-1}\), respectively. And for the 40 mm diameter transducer, the sensitivity for the first and the second bending modes are 4.90 kHz mm\(^{-1}\) and 17.88 kHz mm\(^{-1}\), respectively. The characteristics of the first two peaks is the same as those obtained by theory and FEM. The peak frequencies obtained by measurement are listed in table 3 for comparison.

5. Comparisons and discussion

The peak frequencies for the first two bending modes obtained by theory, FEM and impedance measurement are plotted in figure 14 and summarized in table 3. The comparison of these peak frequencies are listed in table 4. The FEM calculations agree well with the measurement results, with the discrepancies within 4.29%. The difference between FEM results and impedance measurements may be attributed to the accuracy of the materials constants and defects of the piezoelectric-metal transducer that generated during the fabrication or adhesion process. The theoretical calculations of Mode 1 are in good agreement with the FEM and the measurement results, with the discrepancies within 6.03%. However, those of Mode 2 present large difference with the FEM and the measurement results, with the largest difference of 26.48%. It is worthy to note that for the theoretical calculations, the higher the mode, and/or the smaller the
diameter, and/or the thicker the thickness, the larger the difference with the FEM and measurement results. The reason for such differences is that the Kirchhoff plate hypotheses may not be well satisfied for transducers with small diameter-to-thickness ratio. Such differences can be minimized by using transducers with larger diameter-to-thickness ratio. Another factor that could also contribute to the differences is that the electrical effect is neglected when determining the neutral surface in order to simplify the calculation. These consequences imply that the thickness and diameter of the piezoelectric-metal transducer are important factors for the theoretical investigation.

As can be concluded from the results obtained by theoretical analysis, FFM, and impedance measurement, the EMI instrumented piezoelectric-metal transducer can be utilized as corrosion sensor for quantitative determination of corrosion induced thickness loss. The theoretical analysis was validated by FEM and experimental measurement. Good
agreements among them were achieved. The principle of the transducer for corrosion monitoring is that the thickness loss of the metal plate reduces the bending resonant frequencies of the transducer, and it is measured by the EMI technique. The relationship between the thickness loss and the peak frequency shift is linear, which is an advantageous feature of the proposed transducer. The sensitivity of the transducer is at several kHz mm$^{-1}$, which can be tuned by adjusting the thickness and diameter of the transducer. The transducer is very sensitive and accurate to thickness loss. Considering that the resolution of the most commonly used impedance analyzer can be as high as 1 Hz/sample, the transducer is able to achieve sub-micrometer accuracy technically.

In the theoretical analysis, only the bending vibration of the transducer was considered. The in-plane extension vibration, which was presented in the FEM and impedance measurement analysis, was not modeled. It is showed from the FEM and impedance measurement results that the in-plane extension mode is not sensitive to the thickness loss of the transducer. Such insensitivity of the in-plane extension mode can also be utilized in the design of the transducer. The rule-of-thumb is that the resonant frequency of the selected bending mode should not exceed the resonant frequency of the in-plane extension mode. Otherwise, the resonant frequency of in-plane extension mode may complicate the process of peak frequency identification.

One may notice that the amplitude of conductance signatures from theoretical calculations are in several tens of siemens (s), while those for FEM and experimental measurement are in several hundreds of millisiemens (ms). The difference in amplitude can be attributed to the different calculation interval or sampling interval used. Smaller interval usually results in higher frequency accuracy and higher amplitude. Since the resonant frequency was used to indicate the thickness loss, the variation in the amplitude has negligible effect on the performance of the transducer.

It should be pointed out that the proposed corrosion sensor is composed of a PZT disk and a metal plate of same diameter attached to it. The PZT disk is not meant to be separated from the metal plate and attached onto the structure directly for corrosion monitoring. Instead, the corrosion sensor is installed in distributed and critical locations on the structure. The corrosion sensor measures the corrosion amount on the structure at that specific location.

Commonly used EMI corrosion monitoring techniques are the direct bonding of PZT on the metallic structures, which suffer from several issues. Firstly, the metallic structures usually have complex geometry shape, loading
conditions, boundary conditions, and the influences of other environmental factors, and it is very difficult to establish the accurate theoretical model of the electromechanical system. Instead, the over simplified one degree-of-freedom model for electromechanical interaction between a PZT and a host structure by Liang et al. (1994) is adopted. This model states that the variation in the EMI signatures are related to the variation of the mechanical properties of the host structure, such as stiffness, mass, and damping. However, there are many factors can contribute to the variation in the stiffness, mass, and damping of the metallic structure, and therefore the physical meaning between the variation in the EMI signatures and the change in structural properties is not clear. Secondly, since the physical meaning of the peaks in the EMI signatures is not clear, selecting the suitable scanning frequency range is usually done by trial and error. Thirdly, damage assessment is made by quantifying the variations in the EMI signature via statistical metrics, such as root-mean-square deviation and correlation coefficient. The assessment results are qualitative by nature. In a word, there is no direct meaning between the variations in the EMI signatures and the damage of the metallic structure.

In our proposed piezoelectric-metal transducer as corrosion sensor, the direct physical meaning between the peaks in the EMI signature and the thickness of the metal was established. The peak frequencies of bending modes are directly related to the thickness of metal disk. And therefore, for the first time, the theoretical model for the electromechanical coupling of the piezoelectric-metal transducer derived, and the results were validated by FEM calculations and experimental measurement. In this way, the corrosion induced thickness loss of the metal disk is measured by the peak frequency. And more importantly, the relationship between them is linear and the assessment results are quantitative, without needing to find a suitable scanning frequency range and relying on statistical metrics for damage assessment.

The main scope of the current study is the modeling and validation of the EMI instrumented circular piezoelectric-metal transducer as a kind of corrosion sensor. The influence of environmental factors, such as the temperature, humidity, and loading and boundary conditions are not investigated in the current study. It is certain that the temperature will induce small amount frequency shift. If precise determination of corrosion amount is required, the temperature effect needs to be compensated. Further studies will conduct experiments on the practical issues related to the application of the corrosion sensor, such as waterproofing materials, influences of temperature and humidity, encapsulation and isolation schemes for the sensors to satisfy free boundary, and strategies for uneven thickness loss.

6. Conclusions
In this study, theoretical analysis of the EMI responses of the EMI instrumented circular piezoelectric-metal transducer as a new type of corrosion sensor was carried out, and was
validated by FEM and impedance measurement. The piezo-electric-metal transducer was fabricated by bonding a piezo-electric disk to a metal plate. According to the electro-elastic and Kirchhoff plate theory, the EMI responses of the transducer operated in transverse bending modes with free boundary conditions were demonstrated. The results showed that the peak frequency in the conductance signatures is linearly reduced with increase of thickness loss. Transducers using higher order modes or with smaller diameter-to-thickness ratio are generally more sensitive to thickness loss. Therefore, it is possible to adjust the sensitivity of the transducer by selecting proper mode and tuning the size of the transducer. The theoretical calculations were validated by harmonic analysis and modal analysis using FEM, as well as impedance measurement. Comparisons among them were made. The FEM calculations show good agreement with the measurement results, with the discrepancies within 4.29%. The theoretical calculations of the first bending mode are in

| Diameter (mm) | Thickness (mm) | Theory/FEM | Theory/measurement | FEM/measurement |
|---------------|----------------|------------|--------------------|-----------------|
|               |                | Mode 1     | Mode 2             | Mode 1          | Mode 2          |
| 30            | 3.0            | 6.03       | 24.52              | 0.00            | 1.57            |
|               | 2.5            | 4.55       | 18.08              | 20.71           | 0.00            | 2.23            |
|               | 2.0            | 3.00       | 12.03              | 1.98            | 15.23           | −0.99           | 2.86            |
|               | 1.5            | 1.90       | 6.99               | 0.63            | 10.42           | −1.25           | 3.21            |
|               | 1.0            | 0.86       | 2.88               | −0.85           | 7.30            | −1.69           | 4.29            |
| 40            | 3.0            | 3.07       | 14.40              | 3.70            | 15.87           | 0.62            | 1.28            |
|               | 2.5            | 2.16       | 10.51              | 2.90            | 12.13           | 0.72            | 1.47            |
|               | 2.0            | 0.87       | 6.41               | 1.75            | 9.21            | 0.88            | 2.63            |
|               | 1.5            | 1.11       | 3.74               | 2.25            | 7.18            | 1.12            | 3.31            |
|               | 1.0            | 0.00       | 1.08               | 3.12            | 4.85            | 3.12            | 3.73            |

Note. Difference of \((a / b) = 100 \times (a - b) / b\) (%).
good agreement with the FEM and the measurement results, with the discrepancies within 6.03%. Those of the second bending mode show larger discrepancies, which are mainly due to the limitations of Kirchhoff plate hypotheses. The findings of the present study are encouraging. The proposed corrosion sensor has the advantages of low cost, linear, quantitative corrosion assessment, and on-line and remote monitoring. Future works will concentrate on the encapsulation, optimization, and performance under practical conditions.

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