PROSPECTS FOR DELENSING THE COSMIC MICROWAVE BACKGROUND FOR STUDYING INFLATION

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ABSTRACT

A detection of excess cosmic microwave background (CMB) B-mode polarization on large scales allows the possibility of measuring not only the amplitude of these fluctuations but also their scale dependence, which can be parametrized as the tensor tilt $n_T$. Measurements of this scale dependence will be hindered by the secondary B-mode polarization anisotropy induced by gravitational lensing. Fortunately, these contaminating B modes can be estimated and removed with a sufficiently good estimate of the intervening gravitational potential and a good map of CMB E-mode polarization. We present forecasts for how well these gravitational lensing B modes can be removed, assuming that the lensing potential can be estimated either internally from CMB data or using maps of the cosmic infrared background (CIB) as a tracer. We find that CIB maps are as effective as CMB maps for delensing at the noise levels of the current generation of CMB experiments, while the CMB maps themselves will ultimately be best for delensing at polarization noise below $\Delta P = 1\,\mu$Karcmin. At this sensitivity level, CMB delensing will be able to measure $n_T$ to an accuracy of 0.02 or better, which corresponds to the tensor tilt predicted by the consistency relation for single-field slow-roll models of inflation with $r = 0.2$. However, CIB-based delensing will not be sufficient for constraining $n_T$ in simple inflationary models.

Key words: cosmic background radiation – inflation

1. INTRODUCTION

The initial BICEP2 high-sensitivity measurements of B-mode polarization in the microwave sky on large angular scales ($\ell \lesssim 300$) generated a great deal of excitement (BICEP2 Collaboration 2014). This result was originally seen as a clear detection of primordial B-modes, which can be interpreted as the imprint of gravitational radiation from the epoch of inflation (see, for example, BICEP2 Collaboration 2014 and Martin et al. 2014, for a review of different models). These potential primordial B-modes could also be interpreted as the signature of alternatives to inflation (Brandenberger et al. 2014; Cai et al. 2014; Gerbino et al. 2014; Wang & Xue 2014) or as the signature of topological defects (Lizarraga et al. 2014; Moss & Pogosian 2014). Subsequent work has shown the strong possibility that the BICEP2 signal can be explained by Galactic dust (Flauger et al. 2014; Mortonson & Seljak 2014; Planck Collaboration Int. XXX 2014). While the excess B-modes seen by BICEP2 have been confirmed by deeper measurements from the Keck Array (BICEP2/Keck Array Collaborations V 2015) over the same patch of sky, a joint analysis of the BICEP2, Keck Array, and Planck maps finds that the significance of the residual signal after subtraction of the Galactic dust emission is too low to constitute a robust detection of primordial B-modes (BICEP2/Keck and Planck Collaborations 2015).

It is not yet clear whether the BICEP2 signal is due entirely to dust or has a significant primordial contribution. If the signal really is primordial, it would be interesting to measure the scale dependence of these tensor fluctuations (henceforth called “tensor tilt”) using future polarization measurements. Figure 1 shows the effect of varying the tensor tilt $n_\nu$ on the theoretical power spectrum of the primordial B modes for a fixed value of the tensor-to-scalar ratio $r$. The $r = 0.2$ power spectra have been normalized at $\ell = 100$, an angular scale well probed by the BICEP2 experiment, although for the results presented in Section 3 the pivot scale at which $r$ is taken will be held to $k_0 = 0.002\,\text{Mpc}^{-1}$. This choice of pivot scale affects the quoted values of $r$ but not the $r$-marginalized constraints on $n_T$ (Boyle et al. 2014).

A serious impediment to measuring the tensor tilt will be noise coming from gravitational lensing of the cosmic microwave background (CMB) polarization fluctuations generated by scalar fluctuations. Removing this signal is possible, using a procedure known as “delensing” (Kesden et al. 2002; Knox & Song 2002). In principle, with sufficiently high signal-to-noise measurements and in the absence of any primordial B-mode signal this delensing procedure can be done with arbitrarily high precision (Seljak & Hirata 2004). The ability to remove the lensing signal requires a good map of the polarization fluctuations from the scalars (E-mode polarization anisotropies) and a good estimate of the intervening projected gravitational potential. E-mode polarization maps can be obtained only from sensitive CMB polarization measurements on the relevant angular scales, while the gravitational potential can either be estimated from the CMB data or obtained using astronomical sources as tracers of the potential.

One of the most promising CMB lensing potential tracers is the cosmic infrared background (CIB). The CIB is an extragalactic radiation field generated by the unresolved emission from star-forming galaxies (Dole et al. 2006, and references therein). It is generated by dust which is heated by the UV light from young stars and then reradiates thermally in the infrared with a graybody spectrum of $T \sim 30$ K. Due to its higher temperature, fluctuations in the CIB dominate over the CMB on most angular scales at frequencies $\nu \gtrsim 300$ GHz. The CIB contains approximately half of the total extragalactic stellar flux, and has long been predicted to have excellent redshift overlap with the CMB lensing potential (Song et al. 2003). Recent measurements of the cross-correlation between the lensing potential and high signal-to-noise measurements of the CIB fluctuations from the Planck and Herschel satellites have upheld these predictions (Hanson et al. 2013; Holder et al. 2013; Planck Collaboration XVIII 2014; POLARBEAR Collaboration 2014b).
In this paper, we evaluate prospective constraints on $n_T$ using both the CMB and the CIB to estimate the lensing potential. A similar study uses delensing to obtain constraints on the tensor-to-scalar ratio, for $r$ values much smaller than the BICEP2 initial claim of $r = 0.2$ (Sherwin & Schmidtfull 2014). We refer the reader to this work for a thorough investigation of the optimal use of CIB data for delensing purposes, alone or in combination with the CMB. The paper is divided as follow: in Section 2 we quantify the noise levels associated with these tracers of the lensing potential and E modes and describe the forecasting method used to delens the B-mode power. In Section 3 we present the resulting constraints and our conclusions are summarized in Section 4.

Throughout this work, we use the Planck/WP/highL fiducial cosmology (Planck Collaboration XVI 2014) for all cosmological parameters except $r$ and $n_T$. We take the fiducial values of the tensor power spectrum parameters to follow the consistency relation $n_T = -r/8$, characteristic of the single-field slow-roll inflation models (Liddle & Lyth 1992). The CMB temperature, CMB polarization and lensing potential power spectrum have been computed using the CLASS Boltzmann code (Blas et al. 2011).

2. FORECASTING METHOD

We discuss here the tools we used to achieve CMB delensing and to compute forecasted errors on the parameters describing the primordial B modes. Gravitational lensing deflects CMB temperature and polarization primordial anisotropies according to (Lewis & Challinor 2006):

$$T^\text{lens}(\hat{n}) = T^\text{unl}(\hat{n} + \nabla \phi(\hat{n}))$$

$$(Q \pm iU)^\text{lens}(\hat{n}) = (Q \pm iU)^\text{unl}(\hat{n} + \nabla \phi(\hat{n})).$$

The lensing potential $\phi(\hat{n})$ can be expressed as a line of sight integral:

$$\phi(\hat{n}) = -2 \int_0^{\z_d} \frac{dz}{H(z)} \Psi(z, D(z) \hat{n}) \times \left\{ \frac{1}{D(z)} - \frac{1}{D(\z_d)} \right\},$$

where $H(z)$ is the Hubble factor, $\Psi(z, x)$ the Newtonian gravitational potential, and $D(z)$ the comoving distance to the redshift $z$. A flat universe has been assumed in writing expression (2). Rewriting the right-hand side of Equation (1) in the E modes and B modes CMB polarization formalism (Zaldarriaga & Seljak 1997) and expanding it up to first order in the lensing potential, we get the following approximation for the lensed B modes (Hu 2000):

$$B^{\text{lens}}_{lm} \approx B^{\text{unl}}_{lm} + \sum_{\ell_0 m_0 \ell M} f_{\ell L}^{\ell_0} \frac{E^{\text{unl}}_{\ell_0 m_0} \ell M}{D(z)} \phi_{\ell M}^{*}.$$  

The $f_{\ell L}$ couplings are given by

$$f_{\ell L} = \frac{F_{\ell M}^{\ell L}}{2i}.$$  

where

$$F_{\ell M}^{\ell L} = \frac{1}{2} \left[ -\ell (\ell+1) + \ell (\ell' + 1) + L(L+1) \right] \times \sqrt{\frac{(2\ell + 1)(2\ell' + 1)(2L+1)}{4\pi}} \begin{pmatrix} \ell \\ \ell' \\ L \end{pmatrix}.$$  

Taking the power spectrum of Equation (3), we obtain:

$$C_{\ell}^{B,\text{lens}} \approx C_{\ell}^{B,\text{unl}} + \frac{1}{2\ell + 1} \sum_{\ell' L} [f_{\ell L}]^2 C_{\ell'}^{E,\text{unl}} C_{\ell}.$$  

It is useful to gain an intuitive understanding of which scales in E-mode polarization and the lensing potential source the lensing B modes. For this purpose, in Figure 2, we have plotted the kernel for the lensing B-mode power spectrum, broken into contributions from $C_{\ell}^{E,\text{unl}}$ and $C_{\ell}^{E}$. It can be seen that in general the important E modes contributing to a given $\ell$ in lensing B modes are those from a scale that is just slightly smaller (larger in $\ell$). Thus, to delens B modes up to $\ell \sim 500$ it is only required to accurately measure E modes up to $\ell \sim 1000$, given a good map of the gravitational potential. Obtaining accurate E-mode measurements on these scales is relatively straightforward: current ground-based CMB experiments such as SPTpol (Austermann et al. 2012), ACTpol (Niemack et al. 2010), and PolarBear (Kermish et al. 2012) make sample-variance limited measurements of the E-mode polarization on these scales.

For the lensing potential, most of the lensing B modes are generated by gravitational potential fluctuations on half-degree scales or larger ($\ell \lesssim 500$). There are again a number of

Figure 1. Theoretical power spectrum of the gravitational waves B-mode polarization and lensing B-mode polarization along with the BICEP2/Keck Array (BICEP2/Keck Array Collaborations V 2015), SPTpol (Keisler et al. 2015) and POLARBEAR (POLARBEAR Collaboration 2014a) measurements. The shaded region illustrates the tilt imprinted on a $r = 0.2$ primordial power spectrum from $n_T = -0.5$ (purple) to $n_T = 0.5$ (green). All tensor power spectra are for $r$ defined at $\ell = 100$. 

[Figure 1]

In this paper, we evaluate prospective constraints on $n_T$ using both the CMB and the CIB to estimate the lensing potential. A similar study uses delensing to obtain constraints on the tensor-to-scalar ratio, for $r$ values much smaller than the BICEP2 initial claim of $r = 0.2$ (Sherwin & Schmidtfull 2014). We refer the reader to this work for a thorough investigation of the optimal use of CIB data for delensing purposes, alone or in combination with the CMB. The paper is divided as follow: in Section 2 we quantify the noise levels associated with these tracers of the lensing potential and E modes and describe the forecasting method used to delens the B-mode power. In Section 3 we present the resulting constraints and our conclusions are summarized in Section 4.

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possible tracers for these modes. CMB lensing measurements can be used, although polarization-based lensing measurements require high sensitivity and sensitivity to smaller angular scales than are required for large-scale tensor B-mode experiments (such as BICEP2). Alternatively, astrophysical tracers could be used to estimate the potential, provided they are sufficiently correlated with the lensing potential.

The idea behind delensing is to build a noisy estimate of the lensing B modes using Wiener-filtered E modes and lensing potential $\phi$. This estimate is then subtracted from the perfectly reconstructed B modes, leaving as a difference the residual B modes:

$$C_\ell^{B,\text{res}} = \frac{1}{2\ell + 1} \sum_{\ell_1} [c_{\ell_1}]^2 \left[ C_{\ell_1}^{E,\text{len}} C_{\ell_2}^{\phi} - \frac{(C_{\ell_1}^{E,\text{len}})^2}{C_{\ell_1}^{E,\text{len}} + N_{\ell_1}^E} \right] \left[ \frac{(C_{\ell_2}^{\phi})^2}{C_{\ell_2}^{\phi} + N_{\ell_2}^\phi} \right].$$ (7)

A formal derivation of this expression can be found in Smith et al. (2009). In writing Equation (7), we have substituted the unlensed E-mode power spectrum coming from the derivation of Equation (6) with the lensed E-mode power spectrum, which has been shown by Lewis et al. (2011) to produce an estimated lensed B-mode power spectrum effectively accurate to higher order. The experimental noise power spectrum can be written as (Knox 1995):

$$N_{\ell}^E = N_{\ell}^B = \frac{\Delta_p^2 \exp \left( \frac{\ell (\ell + 1) \theta_{\text{FWHM}}^2}{8 \ln 2} \right)}{8 \ln 2},$$ (8)

where $\Delta_p$ is the polarization pixel noise in [$\mu$K radian] and $\theta_{\text{FWHM}}$ is the FWHM of the beam, assuming this beam is Gaussian. We do not consider an additional foreground noise power spectrum.

The lensing reconstruction noise power spectrum $N^\phi$ can be obtained through the prevalent quadratic estimator method (Hu 2000). This technique builds an estimate of the lensing potential from the optimally filtered off-diagonal correlations between two lensed CMB fields $X, X' \in \{T, E, B\}$. We use the EB estimator which yields the best signal to noise ratio at high experimental sensitivities. The resulting quadratic lens reconstruction noise level can be expressed as (Okamoto & Hu 2003):

$$N_{\ell}^\phi = \left[ \frac{1}{2\ell + 1} \sum_{\ell_1} [c_{\ell_1}^{EB}]^2 \right] \times \left[ \frac{1}{C_{\ell_1}^{B,\text{len}} + N_{\ell_1}^B} \right] \left[ \frac{(C_{\ell_2}^{E,\text{len}})^2}{C_{\ell_2}^{E,\text{len}} + N_{\ell_2}^E} \right]^{-1}.$$ (9)

We used the fast real-space algorithm proposed by Smith et al. (2012) to compute the geometrical factors described in Equations (4) and (5) necessary for the numerical implementation of Equations (7) and (9).

It can be seen from Equation (9) that the lensing B modes act as a contaminant for the lensing reconstruction process. Substituting the signal+noise lensed B modes of Equation (9) by delensed B modes, defined in this forecast as

$$C_{\ell}^{B,\text{del}} \approx C_{\ell}^{B,\text{unl}} + C_{\ell}^{B,\text{res}} + N_{\ell}^B,$$ (10)

one can estimate the lensing potential with higher signal to noise and then produce lower residual B modes. Repeating these steps until convergence of $C_{\ell}^{B,\text{del}}$ is referred to as iterative delensing (Seljak & Hirata 2004; Smith et al. 2009). Quadratic delensing will indicate the use of a delensed power spectrum $C_{\ell}^{B,\text{del}}$ computed through only one iteration.
Lensing reconstruction can also be achieved using large-scale structure (Smith et al. 2012). We explore here the possibility of estimating the lensing potential by using its cross-correlation with the CIB, its best known tracer (Song et al. 2003; Hanson et al. 2013; Holder et al. 2013; Planck Collaboration XVIII 2014; POLARBEAR Collaboration 2014b). In this work, we assume that high signal-to-noise CIB measurements are available, and that the cross-correlation between the CIB and the lensing potential can be well-approximated as flat with a constant correlation coefficient

\[ \hat{f}_{\text{corr}} = \frac{C_\ell^{\text{CIB} \times \phi}}{\sqrt{C_\ell^{\text{CIB}} C_\ell^\phi}}. \]  

This simple approximation is quite accurate (see for example Figure 13 of Planck Collaboration XVIII 2014). Including the Poisson noise component to the CIB power spectrum in Equation (11) causes \( \hat{f}_{\text{corr}} \) to fall off on small scales, but on scales larger than \( \ell \sim 500 \) of interest for delensing, the flatness of the correlation factor is not substantially affected. Although recent measurements have shown a correlation at the \( \hat{f}_{\text{corr}} = 0.8 \) level, we will pessimistically also consider delensing with correlation coefficients as low as \( \hat{f}_{\text{corr}} = 0.4 \). This could effectively be the case, for example, in a region with comparable foreground and CIB power. The introduction of this correlation coefficient results in a simple expression for the lensing estimate signal+noise power spectrum used in Equation (7):

\[ C_\ell^\phi + N_\ell^\phi = C_\ell^\phi / \hat{f}_{\text{corr}}^2. \]  

Assuming Gaussianity of the likelihood function \( \mathcal{L}(d|\theta) \) which gives the probability distribution of a cosmological model \( \theta \) given some set of independent observations \( d \), we use the Fisher matrix formalism to compute forecasted errors on the tensor tilt. For the present analysis, the data covariance matrix will be reduced to the B-mode power spectrum since the constraining power on cosmological parameters such as \( r \) and \( n_T \) comes mainly from the B-mode signal. Each element of the Fisher matrix reduces to (Jungman et al. 1996)

\[ F_{ij} = \sum_{\ell_{\text{min}}}^{\ell_{\text{max}}} \frac{(C_\ell^B)_{ij} (C_\ell^B)}{(\delta C_\ell^B)^2}, \]  

where the expected 1σ error on the measurement of the B-modes power spectrum is

\[ \delta C_\ell^B = \sqrt{\frac{2}{2\ell + 1} f_{\text{sky}}^2} C_\ell^B. \]  

The marginalized error on a given parameter of the model corresponds to \( \sigma(\theta) = \sqrt{\mathcal{F}^{-1}} \).

### 3. CONSTRAINTS ON TENSOR TILT

We have computed the projected constraint on the tensor tilt assuming different delensing scenarios; results are shown in Figure 3. The two limiting cases correspond to a situation where no lensing B modes are removed (solid orange curves) and where the total B modes include no lensing B modes (solid light gray curves). The solid and dotted dark gray curves correspond to quadratic and iterative CMB delensing and the shaded regions correspond to delensing using the CIB, where the correlation factor \( \hat{f}_{\text{corr}} \) is comprised between 0.4 and 0.8. For simplicity we will consider that the E-mode estimate and the lensing potential estimate are obtained through the same CMB experiment when considering CMB delensing, which makes the noise power spectra in Equations (7) and (9) parametrized by the same beam width and sensitivity. This forecast could be expanded to the case where a large-scale mission is used for the measurement of the E modes and a
higher resolution experiment is used for estimating the lensing potential.

The beam width $\theta_{\text{WHIM}}$ has been fixed to 4 arcmin in both panels; it has been shown that beam size is not an important factor (Boyle et al. 2014; Dodelson 2014; Wu et al. 2014). We have included modes between $l_{\text{min}} = 10$ and $l_{\text{max}} = 3000$ in the Fisher matrix calculation, although the large scale cutoff value does not affect significantly the projected constraints on $n_T$ (Boyle et al. 2014; Dodelson 2014). The sky coverage has been held to $f_{\text{sky}} = 0.5$, although for a high resolution experiment the results scale with $f_{\text{sky}}$ (Dodelson 2014).

The top panel of Figure 3 shows the forecasted error on $n_T$ as a function of the polarization experimental noise level. As $\Delta_T$ approaches 0, no fundamental floor due to the iterative delensing procedure is found, in conformity with previous works (Seljak & Hirata 2004; Smith et al. 2012; Wu et al. 2014). Better than linear improvement on $\sigma(n_T)$ is observed as polarization noise drops below $\Delta_P = 1 \mu K$ arcmin (Caligiuri & Kosowsky 2014). This noise level also represents the sensitivity required to probe the tensor tilt at a level that is interesting for testing inflationary models; around $\Delta_P = 1 \mu K$ arcmin, $\sigma(n_T)$ falls below the dashed line corresponding to the consistency relation. Given the most optimistic case of correlation level between the CMB and the CIB, it appears that CIB-based delensing will not be sufficient for measuring the tensor tilt. However, for the current next generation of CMB polarization experiments, CIB-based delensing will be of comparable utility.

The error on $n_T$ depends on the value of $r$ with which the B-modes power spectrum $C_P^B$ in Equations (13) and (14) is computed. This dependence is plotted in the bottom panel of Figure 3, for a fixed pixel noise of $\Delta_P = 1 \mu K$ arcmin. The dashed black line shows the absolute value of $n_T$ given that $n_T$ and $r$ are related by the consistency relation. It can be seen that going from $r = 0.2$ down to $r = 0.1$ hinders the ability to distinguish between consistency relation and other models (Caligiuri & Kosowsky 2014; Dodelson 2014).

4. CONCLUSIONS

We have tested the ability of different delensing techniques, namely, EB quadratic delensing, EB iterative delensing, and CIB-based delensing, to constrain the tensor tilt. For low CMB noise levels, CMB-based delensing would be able to probe inflation with a constraint on $n_T$ given the initial BICEP2 signal is entirely of cosmological origin. This would require sensitivities on the order of $1 \mu K$ arcmin in polarized noise over roughly half the sky. While ambitious, this is exactly the scale that is being considered as Stage-IV CMB experiments like CMB-S4 (Abazajian et al. 2013). CIB delensing cannot constrain $n_T$ to a level interesting for probing inflation, although it is competitive with near-future CMB delensing strategies.

While this work was being achieved, the preprint of Boyle et al. (2014) was released, which has strong overlaps with the results presented in this paper. G. S. wishes to thank Ryan Keisler and Elisa G. M. Ferreira for useful discussions and comments. We acknowledge the use of Kendrick Smith’s implementation of the Gauss–Legendre quadrature method and of the Wigner-d matrix. G. H. acknowledges funding from the Canadian Institute for Advanced Research, the National Sciences and Engineering Research Council of Canada and Canada Research Chairs program. D. H. is supported by the Lorne Trotter Chair in Astrophysics and Cosmology at McGill as well as a CITA National Fellowship. G. S. acknowledges support from the Fonds de recherche du Québec—Nature et technologies.

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