New Physics in $\bar{b} \to \bar{s}$ Transitions and the 
$B_{d,s}^0 \to V_1V_2$ Angular Analysis

Alakabha Datta $^a$, Murugeswaran Duraisamy $^a$ and David London $^b$

Abstract

We suppose that there is new physics (NP) in $\bar{b} \to \bar{s}$ transitions, and examine its effect on the angular distribution of $B_{q}^0 \to V_1V_2 \ (q = d, s)$, where $V_{1,2}$ are vector mesons. We find that, in the presence of such NP, the formulae relating the parameters of the untagged, time-integrated angular distribution to certain observables (polarization fractions, CP-violating triple-product asymmetries, CP-conserving interference term) must be modified from their standard-model forms. This modification is due in part to a nonzero $B_s^0$-$\bar{B}_s^0$ width difference, which is significant only for $B_s^0$ decays. We re-analyze the $B_s^0 \to \phi\phi$ data to see the effect of these modifications. As $\Delta\Gamma_s/2\Gamma_s \sim 10\%$, there are $O(10\%)$ changes in the derived observables. These are not large, but may be important given that one is looking for signals of NP. In addition, if the NP contributes to the $\bar{b} \to \bar{s}$ decay, the measurement of the untagged time-dependent angular distribution provides enough information to extract all the NP parameters.

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1 Introduction

As recently as a year ago, there were several hints of physics beyond the standard model (SM) in $\bar{b} \to \bar{s}$ transitions. For example, the CDF [11] and DØ [2] Collaborations measured the CP asymmetry in $B^0_s \to J/\psi \phi$, and found a hint for indirect (mixing-induced) CP violation. This is counter to the expectation of the SM, which predicts this CP violation to be $\approx 0$. In general, this result was interpreted as evidence for a nonzero value of the weak phase of $B^0_s - \bar{B}^0_s$ mixing ($2\beta_s$), and the contributions of various new-physics (NP) models to the $B_s$ mixing phase were explored [3, 14, 6, 7, 8, 9]. It was also pointed out that NP in the decay $\bar{b} \to \bar{s}c\bar{c}$ could also play an important role [10]. In addition, the SM predicts that the measured indirect CP asymmetry in $\bar{b} \to \bar{s}s\bar{s}$ penguin decays should generally be equal to that found in charmonium decays such as $B^0_d \to J/\psi K_S$. However, it was found that these two quantities were not identical for several decays [11]. As a third example, the CDF Collaboration reported the measurement of $B(B^0_s \to \mu^+\mu^-) = (1.8_{-0.4}^{+1.4}) \times 10^{-8}$ [12]. This is larger than the SM prediction for this branching ratio, which is $B(B^0_s \to \mu^+\mu^-) = (3.35 \pm 0.32) \times 10^{-9}$ [13].

However, with recent measurements, these effects have largely disappeared, or at least been reduced. First, LHCb has measured the indirect CP asymmetry in $B^0_s \to J/\psi \phi$, and finds $2\beta_s \approx 0$, in agreement with the SM [14]. Specifically, they measure $2\beta_s = (-0.06 \pm 5.77 \text{ (stat)} \pm 1.54 \text{ (syst)})^\circ$. Still, the errors are large enough that NP cannot be excluded. Second, with the latest indirect CP asymmetry data, the Heavy Flavor Averaging Group (HFAG) [15] finds that the $B^0_s - \bar{B}^0_s$ mixing phase $\sin 2\beta$ is measured to be (i) $0.68 \pm 0.02$ in charmonium decays, and (ii) $0.64 \pm 0.04$ in $\bar{b} \to \bar{s}s\bar{s}$ penguin decays. These numbers are quite similar, so that no real discrepancy can be claimed. On the other hand, several of the $\bar{b} \to \bar{s}s\bar{s}$ decays have additional contributions with a different weak phase, and so HFAG urges that the “naive average” in (ii) be used with extreme caution. Third, the recent LHCb update does not confirm the CDF $B^0_s \to \mu^+\mu^-$ result [16]. They improve the present upper bound to $B(B^0_s \to \mu^+\mu^-) \leq 1.3 \times 10^{-8}$ (90% C.L.), in agreement with the SM. Most of the other effects have similarly gone away, or are simply not large enough to be compelling.

On the other hand, there is one discrepancy with the SM which has not disappeared. The DØ Collaboration has reported an anomalously large CP-violating like-sign dimuon charge asymmetry in the $B$ system. In Ref. [17], the asymmetry was found to be

$$A_{sl}^b = -(9.57 \pm 2.51 \pm 1.46) \times 10^{-3} ,$$

which is a $3.2\sigma$ deviation from the SM prediction, $A_{sl}^{b,SM} = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}$ [18].
In fact, the updated measurement \cite{19} exhibits an even larger discrepancy:

$$A_{sl}^b = -(7.87 \pm 1.72 \pm 0.93) \times 10^{-3},$$

(2)
a 3.9\sigma deviation. Now, it has been shown that this anomaly is due mainly to $B_s$ decays, i.e. a $\bar{b} \to \bar{s}$ transition. So this is a solid indication of $\bar{b} \to \bar{s}$ NP.

Thus, at present the status of $\bar{b} \to \bar{s}$ NP is uncertain. It seems unlikely that the effect of such NP can be very large, but a smaller effect is still possible. In this paper we make the assumption that NP is present in $\bar{b} \to \bar{s}$ transitions. However, in addition to taking into account its effect on $B_s$ mixing, which is what is conventionally done, we also consider its effect on $\bar{b} \to \bar{s}$ decays. The main aim is to examine the effect of $\bar{b} \to \bar{s}$ NP on the angular distribution of $B_q^0 \to V_1 V_2$ ($q = d, s$), where $V_{1,2}$ are vector mesons. In particular, we consider final states which are self-conjugate, so that both $B_q^0$ and $\bar{B}_q^0$ can decay to $V_1 V_2$, generating indirect (mixing-induced) CP-violating effects.

There are three classes of $B_q^0$ decays which can be affected by $\bar{b} \to \bar{s}$ NP:

1. $B_s^0$ decays with $\bar{b} \to \bar{s}$,
2. $B_d^0$ decays with $\bar{b} \to \bar{s}$,
3. $B_s^0$ decays with $\bar{b} \to \bar{d}$.

Our analysis is completely general and can be applied to any of these classes. However, we also focus specifically on $B_s^0 \to \phi \phi$. There are two reasons. First, this is a pure $\bar{b} \to \bar{s}$ penguin decay, and so there can well be NP contributions to any of the loop-level penguin decay amplitudes\footnote{$B_s^0 \to \phi \phi$ and $B_q^0 \to J/\psi \phi$ were examined in Ref. \cite{20}, but only NP in $B_q^0$-$\bar{B}_q^0$ mixing was considered, not NP in the decay.}. Second, the untagged angular distribution of the decay has already been measured by the CDF \cite{21} and LHCb \cite{22} Collaborations, and so their results can be (re)interpreted in the context of $\bar{b} \to \bar{s}$ NP contributions.

The result of this analysis – and this is the main point of the paper – is as follows. The parameters of the untagged, time-integrated angular distribution can be measured experimentally. Certain observables can be derived from these parameters. However, in the presence of NP, the formulae which relate the observables and parameters are modified compared to their SM forms. There are six terms ($i = 1-6$) in the angular distribution, and we correspondingly find six observables for which the relation between the experimental data and theoretical parameters must be modified. For $i = 1-3$ they are the polarization fractions, for $i = 4,6$ they are the CP-violating triple-product asymmetries, and $i = 5$ corresponds to a CP-conserving observable. The modifications for the polarization fractions are particularly striking. Here there are corrections to the SM formulae that are proportional to the width difference in the $B_q^0$-$\bar{B}_q^0$ system. Now, the width difference $\Delta \Gamma$ is sizeable only for $B_s^0$.
decays. Thus, the formulae modifications due to NP are important only for class-(1) and (3) decays, which include $B^0 \to \phi \phi$. $\Delta \Gamma_s/2\Gamma_s \sim 10\%$, so that the modifications lead to $O(10\%)$ changes in the derived observables. These are not large, but may be important given that one is looking for signals of NP.

Another result is that, if the untagged, time-dependent angular distribution can be measured, 12 observables can be obtained. If the NP contributes to the $\bar{b} \to \bar{s}$ decay, there are fewer than 12 unknown NP parameters. Thus, all of these parameters can be extracted from the angular distribution. This may allow the identification of the NP.

We begin in Sec. 2 by presenting the most general $B^0_{d,s} \to V_1 V_2$ angular distribution, allowing for NP in the mixing and/or the decay. We consider the angular distribution for several different scenarios: at $t = 0$ (Sec. 2.2.1), time-dependent (Sec. 2.2.2), untagged time-dependent (Sec. 2.3), untagged time-integrated (Sec. 2.4). In Sec. 3 we examine the untagged time-dependent and time-integrated distributions for $B^0_s \to \phi \phi$ within the SM. The study of $B^0_s \to \phi \phi$ is extended to the SM + NP in Sec. 4. We discuss observables such as the polarization fractions, CP-violating triple-product asymmetries and the CP-conserving interference term, and note the changes in the formulae used for their extraction necessitated by the inclusion of $\bar{b} \to \bar{s}$ NP. We also show that all the unknown NP parameters in the $\bar{b} \to \bar{s}$ decay can be determined from the measurement of the untagged, time-dependent angular distribution. In Sec. 5 we present a numerical reanalysis of the $B^0_s \to \phi \phi$ data allowing for the possibility of $\bar{b} \to \bar{s}$ NP contributions in the decay. We conclude in Sec. 6.

## 2 $B \to V_1 V_2$ Angular Distribution

### 2.1 Generalities

The most general Lorentz-covariant amplitude for the decay $B(p) \to V_1(k_1, \epsilon_1) + V_2(k_2, \epsilon_2)$ is given by [23] [24]

$$M = a \epsilon_1^* \cdot \epsilon_2 + \frac{b}{m_B^2} (p \cdot \epsilon_1^*) (p \cdot \epsilon_2^*) + i \frac{c}{m_B^2} \epsilon_{\mu \nu \rho \sigma} q^\mu q'^\nu \epsilon_1^{* \rho} \epsilon_2^{* \sigma},$$

(3)

where $q \equiv k_1 - k_2$. The quantities $a$, $b$ and $c$ are complex and contain in general both CP-conserving strong phases and CP-violating weak phases. In $B \to V_1 V_2$ decays, the final state can have total spin 0, 1 or 2, which correspond to the $V_1$ and $V_2$ having relative orbital angular momentum $l = 0$ (s wave), $l = 1$ (p wave), or $l = 2$ (d wave), respectively. The $a$ and $b$ terms correspond to combinations of the parity-even s- and d-wave amplitudes, while the $c$ term corresponds to the parity-odd p-wave amplitude.

In order to obtain the angular distribution for $B \to V_1 V_2$, one uses the linear polarization basis. Here, one decomposes the decay amplitude into components in
which the polarizations of the final-state vector mesons are either longitudinal \((A_0)\), or transverse to their directions of motion and parallel \((A_{||})\) or perpendicular \((A_{\perp})\) to one another. The transversity amplitudes \(A_h\) \((h = 0, ||, \perp)\) are related to \(a, b\) and \(c\) of Eq. \((3)\) via \([24]\)

\[
A_{||} = \sqrt{2}a, \quad A_0 = -ax - \frac{m_1 m_2}{m_B^2}b(x^2 - 1), \quad A_{\perp} = 2\sqrt{2} \frac{m_1 m_2}{m_B^2} c \sqrt{x^2 - 1},
\]

where \(x = k_1 \cdot k_2 / (m_1 m_2)\) \((m_1\) and \(m_2\) are the masses of \(V_1\) and \(V_2\), respectively.\)

The amplitude for \(B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)\) can be obtained by operating on Eq. \((3)\) with \(\text{CP}\). This yields

\[
\tilde{M} = \bar{a} \varepsilon_1^* \cdot \varepsilon_2 + \frac{\bar{b}}{m_B^2} (p \cdot \varepsilon_1^*) (p \cdot \varepsilon_2^*) - i \frac{\bar{c}}{m_B^2} \varepsilon_{\mu \nu \rho \sigma} p^\mu q^\nu \varepsilon_1^* \varepsilon_2^* \varepsilon^\rho \varepsilon^\sigma,
\]

in which \(\bar{a}, \bar{b}\) and \(\bar{c}\) are equal to \(a, b\) and \(c\), respectively, except that the weak phases are of opposite sign. The above equation can be obtained from Eq. \((3)\) by changing \(a \to \bar{a}, b \to \bar{b}\) and \(c \to -\bar{c}\). Similarly, one defines \(\bar{A}_0, \bar{A}_{||}, \) and \(\bar{A}_{\perp}\), which are equal to \(A_0, A_{||}, \) and \(A_{\perp}\), respectively, but with weak phases of opposite sign.

### 2.2 \(B_{d,s}^0 \to V_1 V_2\)

As mentioned in the introduction, we are interested in the decays \(B_{d,s}^0 \to V_1 V_2\) \((q = d, s)\), in which both \(B_q^0\) and \(\bar{B}_q^0\) can decay to \(V_1 V_2\). Due to \(B_q^0-\bar{B}_q^0\) mixing, the amplitude is time dependent. Assuming that \(V_{1,2}\) both decay into two pseudoscalars, i.e. \(V_1 \to P_1 P_1', \) \(V_2 \to P_2 P_2'\), the angular distribution is given in terms of the vector \(\vec{\omega} = (\cos \theta_1, \cos \theta_2, \Phi)\) \([25, 26]\):

\[
\frac{d^4 \Gamma(t)}{dt d\vec{\omega}} = \frac{9}{32 \pi} \sum_{i=1}^{6} K_i(t) f_i(\vec{\omega}).
\]

Here, \(\theta_1 (\theta_2)\) is the angle between the directions of motion of the \(P_1 (P_2)\) in the \(V_1 (V_2)\) rest frame and the \(V_1 (V_2)\) in the \(B\) rest frame, and \(\Phi\) is the angle between the normals to the planes defined by \(P_1 P_1'\) and \(P_2 P_2'\) in the \(B\) rest frame. The angular-dependent terms are given by

\[
\begin{align*}
 f_1(\vec{\omega}) &= 4 \cos^2 \theta_1 \cos^2 \theta_2, & f_2(\vec{\omega}) &= 2 \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \Phi, \\
 f_3(\vec{\omega}) &= 2 \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \Phi, & f_4(\vec{\omega}) &= -2 \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\Phi, \\
 f_5(\vec{\omega}) &= \sqrt{2} \sin 2\theta_1 \sin 2\theta_2 \cos \Phi, & f_6(\vec{\omega}) &= -\sqrt{2} \sin 2\theta_1 \sin 2\theta_2 \sin \Phi.
\end{align*}
\]


2.2.1 \( t = 0 \)

At \( t = 0 \), the \( K_i \) are

\[
\begin{align*}
K_1 &= |A_0|^2, & K_2 &= |A_\parallel|^2, & K_3 &= |A_\perp|^2, \\
K_4 &= \text{Im}(A_\perp A_\parallel^*), & K_5 &= \text{Re}(A_\parallel A_0^*), & K_6 &= \text{Im}(A_\perp A_0^*). 
\end{align*}
\]

(8)

The angular distribution for the CP-conjugate decay \( \bar{B}_0^0 \rightarrow V_1 V_2 \) is the same as that given above with the replacements \( K_i \rightarrow \bar{K}_i \), and \( A_h \rightarrow \bar{A}_h \).

The quantities \( K_4 \) and \( K_6 \) are particularly interesting. They are related to the \( \varepsilon_{\mu
u\rho\sigma} p^{\mu} q^{\nu} \varepsilon_1^\rho \varepsilon_2^\sigma \) term of Eq. (3), which is proportional to \( \vec{q} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2) \) in the rest frame of the \( B \). This is the triple product (TP). The TP is odd under both parity and time reversal, and thus constitutes a potential signal of CP violation. However, here one has to be a bit careful. As noted above, the \( A_h \) possess both weak (CP-odd) and strong (CP-even) phases. Thus, \( K_4 \) and/or \( K_6 \) can be nonzero even if the weak phases vanish. In order to obtain a true signal of CP violation, one has to compare the \( B \) and \( \bar{B} \) decays. Now, \( \bar{K}_4 \) is the same as \( K_4 \), except that (i) the weak phases change sign, and (ii) there is an overall relative minus sign due to the presence of \( \bar{A}_\perp / A_\perp \), and similarly for \( \bar{K}_6 \) and \( K_6 \). This implies that the true (CP-violating) TP’s are given by the untagged observables \( K_4 + \bar{K}_4 \) and \( K_6 + \bar{K}_6 \). There are also fake (CP-conserving) TP’s, due only to strong phases of the the \( A_h \)’s, given by \( K_4 - \bar{K}_4 \) and \( K_6 - \bar{K}_6 \).

2.2.2 Time dependence

In order to calculate the \( K_i(t) \), one proceeds as follows. Due to \( B^0_q \bar{B}^0_q \) mixing, the time evolution of the states \( |B^0_q(t)\rangle \) and \( |\bar{B}^0_q(t)\rangle \) can be described by the relations

\[
\begin{align*}
|B^0_q(t)\rangle &= g_+(t) \left| B^0_q \right\rangle + \frac{q}{p} g_-(t) \left| \bar{B}^0_q \right\rangle, \\
|\bar{B}^0_q(t)\rangle &= \frac{p}{q} g_-(t) \left| B^0_q \right\rangle + g_+(t) \left| \bar{B}^0_q \right\rangle,
\end{align*}
\]

(9)

where \( q/p = e^{-i\phi_q} \). Here, \( \phi_q \) is the phase of \( B^0_q \bar{B}^0_q \) mixing. In the SM, we have \( \phi_q = 2\beta = (42.8 \pm 1.6)^\circ \) from charmonium decays [15]. Also, assuming no NP in the decay, the LHCb Collaboration measures \( \phi_s = (-0.06 \pm 5.77 \, \text{(stat)} \pm 1.54 \, \text{(syst)})^\circ \) in \( B^0_s \rightarrow J/\psi \phi \) [14]. Although this agrees with the SM prediction of \( \phi_s \approx 0 \), the errors are still large enough that NP in the decay and/or mixing cannot be excluded.

In the above, we have

\[
g_+(t) = \frac{1}{2} \left( e^{-(iM_L+\Gamma_L/2)t} + e^{-(iM_H+\Gamma_H/2)t} \right),
\]

6
\[ g_-(t) = \frac{1}{2} \left( e^{-i(M_L+\Gamma_L/2)t} - e^{-i(M_H+\Gamma_H/2)t} \right), \]  

(10)

where \( L \) and \( H \) indicate the light and heavy states, respectively. The average mass and width, and the mass and width differences of the \( B \)-meson eigenstates are defined by

\[
\begin{align*}
m &= \frac{M_H + M_L}{2}, & \Gamma &= \frac{\Gamma_L + \Gamma_H}{2}, \\
\Delta m &= M_H - M_L, & \Delta \Gamma &= \Gamma_L - \Gamma_H.
\end{align*}
\]

(11)

\( \Delta m \) is positive by definition. For \( B^0_d \) mesons, \( \Gamma_L \simeq \Gamma_H \), so that \( \Delta \Gamma_d = 0 \). However, for \( B^0_s \) mesons, \( \Delta \Gamma_s \) is reasonably large: \(|\Delta \Gamma_s| = 0.123 \pm 0.029 \) (stat) \( \pm 0.011 \) (syst) ps\(^{-1} \) [28]. In our convention the SM prediction for \( \Delta \Gamma_s \) is positive, and it has been recently confirmed experimentally that \( \Delta \Gamma_s > 0 \) [29].

The time dependence of the transversity amplitudes \( A_h \) is due to \( B^0_q \bar{B}^0_q \) mixing. For the decay to a final state \( f \) we have

\[
\begin{align*}
A_h(t) &= \langle f | B^0_q(t) \rangle_h = \left[ g_+(t)A_h + \eta_h \frac{q/p}{g_-} A_h \right], \\
\bar{A}_h(t) &= \langle f | \bar{B}^0_q(t) \rangle_h = \left[ p/q g_- A_h + \eta_h g_-(t) \bar{A}_h \right],
\end{align*}
\]

(12)

where \( A_h = \langle f | B^0_q \rangle_h, \bar{A}_h = \langle f | \bar{B}^0_q \rangle_h \), and \( \eta_{0,\parallel} = 1, \eta_{\perp} = -1 \). In calculating the \( K_i(t) \), the following relations are useful:

\[
\begin{align*}
|g_\pm(t)|^2 &= \frac{1}{2} e^{-\Gamma t} \left( \cosh (\Delta \Gamma/2)t \pm \cos \Delta mt \right), \\
g_\pm^*(t)g_-^*(t) &= \frac{1}{2} e^{-\Gamma t} \left( -\sinh (\Delta \Gamma/2)t + i\sin \Delta mt \right).
\end{align*}
\]

(13)
\(-2\text{Re}(A^* A) \cos \phi \sinh (\Delta \Gamma /2)t - \sin \phi \sin \Delta mt\)

\(-2\text{Im}(A^* A) \cos \phi \sin \Delta mt + \sin \phi \sinh (\Delta \Gamma /2)t\)

\[K_3(t) = |A_\perp(t)|^2 = \frac{1}{2}e^{-\Gamma t} \left[ (|A_\perp|^2 + |\bar{A}_\perp|^2) \cosh (\Delta \Gamma /2)t + (|A_\perp|^2 - |\bar{A}_\perp|^2) \cos \Delta mt \right.\]

\[+ 2\text{Re}(A^* A) \cos \phi \sinh (\Delta \Gamma /2)t - \sin \phi \sin \Delta mt\]

\[+ 2\text{Im}(A^* A) \cos \phi \sin \Delta mt + \sin \phi \sinh (\Delta \Gamma /2)t\]

\[K_4(t) = \text{Im}(A_\perp(t) A^*_\parallel(t)) = \frac{1}{2}e^{-\Gamma t} \left[ (\text{Im}(A_\perp A^*_\parallel) - \text{Im}(\bar{A}_\perp \bar{A}^*_\parallel)) \cosh (\Delta \Gamma /2)t \right.\]

\[+ (\text{Im}(A_\perp A^*_\parallel) + \text{Im}(\bar{A}_\perp \bar{A}^*_\parallel)) \cos \Delta mt \]

\[+ (\text{Im}(A_\perp A^*_\parallel) - \text{Im}(\bar{A}_\perp \bar{A}^*_\parallel)) (-\sinh (\Delta \Gamma /2)t \cos \phi + \sin \Delta mt \sin \phi) \]

\[+ (\text{Re}(A_\perp A^*_\parallel) + \text{Re}(\bar{A}_\perp \bar{A}^*_\parallel)) (-\sinh (\Delta \Gamma /2)t \sin \phi - \sin \Delta mt \cos \phi)\] \]

\[K_5(t) = \text{Re}(A_\parallel(t) A^*_\perp(t)) = \frac{1}{2}e^{-\Gamma t} \left[ (\text{Re}(A_\parallel A^*_\perp) + \text{Re}(\bar{A}_\parallel \bar{A}^*_\perp)) \cosh (\Delta \Gamma /2)t \right.\]

\[+ (\text{Re}(A_\parallel A^*_\perp) - \text{Re}(\bar{A}_\parallel \bar{A}^*_\perp)) \cos \Delta mt \]

\[+ (\text{Re}(A_\parallel A^*_\perp) + \text{Re}(\bar{A}_\parallel \bar{A}^*_\perp)) (-\sinh (\Delta \Gamma /2)t \cos \phi + \sin \Delta mt \sin \phi) \]

\[+ (\text{Im}(A_\parallel A^*_\perp) - \text{Im}(\bar{A}_\parallel \bar{A}^*_\perp)) (\sinh (\Delta \Gamma /2)t \sin \phi + \sin \Delta mt \cos \phi)\] \]

\[K_6(t) = \text{Im}(A_\perp(t) A^*_\parallel(t)) = \frac{1}{2}e^{-\Gamma t} \left[ (\text{Im}(A_\perp A^*_\parallel) - \text{Im}(\bar{A}_\perp \bar{A}^*_\parallel)) \cosh (\Delta \Gamma /2)t \right.\]

\[+ (\text{Im}(A_\perp A^*_\parallel) + \text{Im}(\bar{A}_\perp \bar{A}^*_\parallel)) \cos \Delta mt \]

\[+ (\text{Im}(A_\perp A^*_\parallel) - \text{Im}(\bar{A}_\perp \bar{A}^*_\parallel)) (-\sinh (\Delta \Gamma /2)t \cos \phi + \sin \Delta mt \sin \phi) \]

\[+ (\text{Re}(A_\perp A^*_\parallel) + \text{Re}(\bar{A}_\perp \bar{A}^*_\parallel)) (-\sinh (\Delta \Gamma /2)t \sin \phi - \sin \Delta mt \cos \phi)\] \]

The expressions for the time-dependent \(\bar{K}_i(t)\)'s can be obtained from the \(K_i(t)\)'s by changing the sign of the weak phases in both the decay \((A_h \leftrightarrow \eta_h \bar{A}_h)\) and the mixing \((\phi_q \rightarrow -\phi_q)\).
2.3 Untagged decays

In the previous subsections, we presented the angular distribution for the case in which the initial decay meson is tagged, so that one can distinguish the $B_q^0$ and $\bar{B}_q^0$ decays. In practice, however, tagging is difficult. Thus, as a first step, experiments will examine the untagged decay, and this is considered here.

The untagged time-dependent angular distribution is given by

$$\frac{d^4(\Gamma_{B_q} + \Gamma_{\bar{B}_q})}{dtd\vec{\omega}} = \frac{9}{32\pi} \sum_{i=1}^{6} (K_i(t) + \bar{K}_i(t)) f_i(\vec{\omega}) ,$$

where the untagged observables can be found using Eq. (14):

- $K_1(t) + \bar{K}_1(t) = e^{-\Gamma t} \left[ \left( |A_0|^2 + |\bar{A}_0|^2 \right) \cosh(\Delta \Gamma/2) t 
  - 2 \left( \text{Re}(A_0^* \bar{A}_0) \cos \phi_q + \text{Im}(A_0^* \bar{A}_0) \sin \phi_q \right) \sinh(\Delta \Gamma/2) t \right] ,$

- $K_2(t) + \bar{K}_2(t) = e^{-\Gamma t} \left[ \left( |A_\parallel|^2 + |\bar{A}_\parallel|^2 \right) \cosh(\Delta \Gamma/2) t 
  - 2 \left( \text{Re}(A_\parallel^* \bar{A}_\parallel) \cos \phi_q + \text{Im}(A_\parallel^* \bar{A}_\parallel) \sin \phi_q \right) \sinh(\Delta \Gamma/2) t \right] ,$

- $K_3(t) + \bar{K}_3(t) = e^{-\Gamma t} \left[ \left( |A_\perp|^2 + |\bar{A}_\perp|^2 \right) \cosh(\Delta \Gamma/2) t 
  + 2 \left( \text{Re}(A_\perp^* \bar{A}_\perp) \cos \phi_q + \text{Im}(A_\perp^* \bar{A}_\perp) \sin \phi_q \right) \sinh(\Delta \Gamma/2) t \right] ,$

- $K_4(t) + \bar{K}_4(t) = e^{-\Gamma t} \left[ \left( \text{Im}(A_\parallel A_0^*) - \text{Im}(\bar{A}_\parallel \bar{A}_0^*) \right) \cosh(\Delta \Gamma/2) t 
  - \left( \text{Im}(A_\perp \bar{A}_0^*) - \text{Im}(\bar{A}_\perp A_0^*) \right) \cos \phi_q 
  + \left( \text{Re}(A_\parallel \bar{A}_0^*) + \text{Re}(\bar{A}_\parallel A_0^*) \right) \sin \phi_q \right) \sinh(\Delta \Gamma/2) t \right] ,$

- $K_5(t) + \bar{K}_5(t) = e^{-\Gamma t} \left[ \left( \text{Re}(A_\parallel A_0^*) + \text{Re}(\bar{A}_\parallel \bar{A}_0^*) \right) \cosh(\Delta \Gamma/2) t 
  - \left( \text{Re}(A_\parallel \bar{A}_0^*) + \text{Re}(\bar{A}_\parallel A_0^*) \right) \cos \phi_q 
  - \left( \text{Im}(A_\parallel A_0^*) - \text{Im}(\bar{A}_\parallel \bar{A}_0^*) \right) \sin \phi_q \right) \sinh(\Delta \Gamma/2) t \right] ,$
\[ K_6(t) + \bar{K}_6(t) = e^{-\Gamma t} \left[ \left( \text{Im}(A_\perp A_6^\ast) - \text{Im}(\bar{A}_\perp \bar{A}_6^\ast) \right) \cosh (\Delta\Gamma/2)t \right. \]
\[ - \left. \left( \text{Im}(A_\perp \bar{A}_6^\ast) - \text{Im}(\bar{A}_\perp A_6^\ast) \right) \cos \phi_q \right) + \left( \text{Re}(A_\perp \bar{A}_6^\ast) + \text{Re}(\bar{A}_\perp A_6^\ast) \right) \sin (\Delta\Gamma/2)t \right] . \tag{16} \]

Note that the CP properties of all the terms are respected. For example, the \( K_i(t) + \bar{K}_i(t) \) \((i = 1, 2, 3, 5)\) are supposed to be CP-even. But they contain terms proportional to \( \sin \phi_q \), which is CP-odd. This is accounted for because, in all cases, \( \sin \phi_q \) is multiplied by a term involving the helicity amplitudes which is also CP-odd. Similarly, \( \cos \phi_q \) (CP-even) is multiplied by a helicity-amplitude term which is also CP-even. The upshot is that the \( K_i(t) + \bar{K}_i(t) \) \((i = 1, 2, 3, 5)\) are indeed CP-even. And it is straightforward to verify that the \( K_i(t) + \bar{K}_i(t) \) \((i = 4, 6)\) are CP-odd.

The key point here is the following. The individual \( K_i \)'s and \( \bar{K}_i \)'s [Eq. (14)] depend on four functions of time: \( e^{-\Gamma t} \cos \Delta mt, e^{-\Gamma t} \sin \Delta mt, e^{-\Gamma t} \cosh (\Delta\Gamma/2)t, \) and \( e^{-\Gamma t} \sinh (\Delta\Gamma/2)t \). However, in the expressions above, the dependence on the functions \( e^{-\Gamma t} \cos \Delta mt \) and \( e^{-\Gamma t} \sin \Delta mt \) cancels, so that the untagged observables depend only on \( e^{-\Gamma t} \cosh (\Delta\Gamma/2)t \) and \( e^{-\Gamma t} \sinh (\Delta\Gamma/2)t \). For \( B_s^0 \) mesons, \( \Delta\Gamma = 0 \), so that the untagged observables are equal to \( e^{-\Gamma t} \times \) simple sums of functions of the \( A_i \) and \( \bar{A}_i \). On the other hand, since \( \Delta\Gamma \neq 0 \) for \( B_s^0 \) mesons, the untagged observables are now complicated functions of the \( A_i \) and \( \bar{A}_i \).

In addition, we have
\[ e^{-\Gamma t} \cosh (\Delta\Gamma/2)t = \frac{1}{2} \left( e^{-\Gamma L t} + e^{-\Gamma H t} \right) , \quad e^{-\Gamma t} \sinh (\Delta\Gamma/2)t = \frac{1}{2} \left( e^{-\Gamma L t} - e^{-\Gamma H t} \right) . \tag{17} \]

If the \( e^{-\Gamma L t/2} \) and \( e^{-\Gamma H t/2} \) terms can be distinguished experimentally, which is doable for \( B_s^0 \) decays, the untagged time-dependent angular distribution provides 12 observables, 2 for each \( K_i(t) + \bar{K}_i(t) \). Thus, \( B_s^0 \to V_1V_2 \) decays are particularly interesting.

### 2.4 Time-integrated untagged distribution

As noted in the previous subsection, because \( \Delta\Gamma \neq 0 \) for \( B_s^0 \) mesons, \( B_s^0 \) decays can be treated without tagging. The time-integrated untagged angular distribution can be obtained by integrating the \( K_i(t) + \bar{K}_i(t) \) observables over time:
\[ \frac{d^3\langle \Gamma(B_s^0 \to f) \rangle}{d\omega} = \frac{9}{32\pi} \sum_{i=1}^{6} \langle K_i \rangle f_i(\omega) , \tag{18} \]

where
\[ \langle \Gamma(B_s^0 \to f) \rangle = \frac{1}{2} \int_0^\infty dt (\Gamma^{B_s} + \Gamma^{\bar{B}_s}) , \]
and the CP-conserving interference term. Specifically for the polarization fractions, CP-violating triple-product asymmetries, the time-integrated quantities, take a different form than they did at $t = 0$. And this means that, if general $b \to s$ NP is considered, the formulae relating certain observables to the $⟨K_i⟩$ must necessarily include terms proportional to $y_s$. As we will see, this holds specifically for the polarization fractions, CP-violating triple-product asymmetries, and the CP-conserving interference term.

\[ ⟨K_i⟩ = \frac{1}{2} \int_0^\infty dt (K_i(t) + \bar{K}_i(t)) . \]  \tag{19} \]

One can obtain the $⟨K_i⟩$'s from Eq. (16):

\[ ⟨K_1⟩ = \frac{τ_{B_s}}{2(1 - y_s^2)} \left[ (|A_0|^2 + |\bar{A}_0|^2) - 2\left(Re(A_0^*\bar{A}_0)\cos φ_s + Im(A_0^*\bar{A}_0)\sin φ_s\right)y_s \right] , \]

\[ ⟨K_2⟩ = \frac{τ_{B_s}}{2(1 - y_s^2)} \left[ (|A||^2 + |\bar{A}||^2) - 2\left(Re(A^*\bar{A})\cos φ_s + Im(A^*\bar{A})\sin φ_s\right)y_s \right] , \]

\[ ⟨K_3⟩ = \frac{τ_{B_s}}{2(1 - y_s^2)} \left[ (|A|^2 + |\bar{A}|^2) + 2\left(Re(A^*\bar{A})\cos φ_s + Im(A^*\bar{A})\sin φ_s\right)y_s \right] , \]

\[ ⟨K_4⟩ = \frac{τ_{B_s}}{2(1 - y_s^2)} \left[ (Im(A^*A_0^*) - Im(\bar{A}_0^*\bar{A}_0^*)) - (Im(A^*\bar{A}_0) - Im(\bar{A}_0^*\bar{A}_0))\cos φ_s \right. \]

\[ \left. + (Re(A^*\bar{A}_0) + Re(\bar{A}_0^*\bar{A}_0))^2\sin φ_s\right) y_s \right] , \]

\[ ⟨K_5⟩ = \frac{τ_{B_s}}{2(1 - y_s^2)} \left[ (Re(A^*A_0^*) + Re(\bar{A}_0^*\bar{A}_0^*)) - (Re(A^*\bar{A}_0) + Re(\bar{A}_0^*\bar{A}_0))\cos φ_s \right. \]

\[ \left. - (Im(A^*\bar{A}_0) - Im(\bar{A}_0^*\bar{A}_0))^2\sin φ_s\right) y_s \right] , \]

\[ ⟨K_6⟩ = \frac{τ_{B_s}}{2(1 - y_s^2)} \left[ (Im(A^*A_0^*) - Im(\bar{A}_0^*\bar{A}_0^*)) - (Im(A^*\bar{A}_0) - Im(\bar{A}_0^*\bar{A}_0))\cos φ_s \right. \]

\[ \left. + (Re(A^*\bar{A}_0) + Re(\bar{A}_0^*\bar{A}_0))\sin φ_s\right) y_s \right] , \tag{20} \]

where $y_s ≡ ΔΓ_s/2Γ_s$.

At this stage, one clearly sees the effect of a nonzero $ΔΓ_s$ (or $y_s$). For $B_d^0$ decays, $ΔΓ_d = 0$, so there are no terms proportional to $y_d ≡ ΔΓ_d/2Γ_d$ in the $⟨K_i⟩$. Indeed, the $⟨K_i⟩$ take the same form as the $(K_i(t) + \bar{K}_i(t))|_{t=0}$ [Eq. (3)]. However, this does not hold for $B_s^0$ decays. Because of the nonzero $y_s$, the $⟨K_i⟩$, which are time-integrated quantities, take a different form than they did at $t = 0$. And this means that, if general $b \to s$ NP is considered, the formulae relating certain observables to the $⟨K_i⟩$ must necessarily include terms proportional to $y_s$. As we will see, this holds specifically for the polarization fractions, CP-violating triple-product asymmetries, and the CP-conserving interference term.
2.5 Effective lifetime

In general, the expressions for $K_i(t) + \bar{K}_i(t)$ [Eq. (16)] and $\langle K_i \rangle$ [Eq. (20)] have the form

$$
K_i(t) + \bar{K}_i(t) = 2e^{-\Gamma t} \left[ A_{ch}^i \cosh (\Delta \Gamma / 2)t + A_{sh}^i \sinh (\Delta \Gamma / 2)t \right],
$$

$$
\langle K_i \rangle = \frac{\tau_{B_s}}{(1 - y_s^2)} \left[ A_{ch}^i + A_{sh}^i y_s \right],
$$

where the experimental observables (dependent on the $K_i$) are on the left-hand side, and the theoretical expressions (dependent on $A_{ch}^i$ and $A_{sh}^i$) are on the right-hand side. (We have implicitly assumed that $\Delta \Gamma \neq 0$, which implies a $B_0^s$ decay.) $A_{ch}^i$ and $A_{sh}^i$ can be related to the experimental observables via the effective lifetime [30]:

$$
\tau_{eff,i}^{eff,i} = \frac{\int_0^\infty t(K_i(t) + \bar{K}_i(t))dt}{\int_0^\infty (K_i(t) + \bar{K}_i(t))dt} = \frac{\tau_{B_s}}{(1 - y_s^2)} \left( 1 + 2 A_{ch}^i + A_{sh}^i y_s \right),
$$

where $A_{i,\Delta \Gamma}^i \equiv A_{ch}^i / A_{ch}^i$.

Using Eqs. (21) and (22), one can relate the $A_{ch}^i$ to the $\langle K_i \rangle$:

$$
A_{ch}^i = \frac{\langle K_i \rangle}{\tau_{B_s}^{eff,i}} \left( 2 - \frac{\tau_{eff,i}^{eff,i}}{(1 - y_s^2)} (1 - y_s^2) \right) .
$$

The $A_{sh}^i$ can be obtained from $A_{i,\Delta \Gamma}^i$.

3 $B_0^s \rightarrow \phi \phi – SM$

The results of the previous section are completely general. In this section we focus on the angular distribution of the pure $\bar{b} \rightarrow \bar{s}$ penguin decay $B_0^s \rightarrow \phi \phi$ within the SM.

In the SM, the amplitude for $B_0^s \rightarrow \phi \phi$ can be written

$$
A(B_0^s \rightarrow \phi \phi) = \lambda_t^{(s)} P'_t + \lambda_c^{(s)} P'_c + \lambda_u^{(s)} P'_u
$$

$$
= \lambda_t^{(s)} P'_{tc} + \lambda_u^{(s)} P'_{uc} ,
$$

where $\lambda_q^{(s)} \equiv V_{qs}^* V_{qs}$. (As this is a $\bar{b} \rightarrow \bar{s}$ transition, the diagrams are written with primes.) In the second line, we have used the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix ($\lambda_u^{(s)} + \lambda_c^{(s)} + \lambda_t^{(s)} = 0$) to eliminate the $c$-quark contribution: $P'_{tc} \equiv P'_t - P'_c$, $P'_{uc} \equiv P'_u - P'_c$. 

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Now, $|\lambda^{(s)}_t|$ and $|\lambda^{(s)}_u|$ are $O(\lambda^2)$ and $O(\lambda^4)$, respectively, where $\lambda = 0.23$ is the sine of the Cabibbo angle. This suggests that the $\lambda^{(s)}_u P_{uc}$ term can be neglected compared to $\lambda^{(s)}_t P_{tc}$. However, if one does this, one must be consistent and neglect all $O(\lambda^4)$ terms. In particular, $\text{Im}(\lambda^{(s)}_i)$ is $O(\lambda^4)$, and so it too can be neglected. But since $2\beta_s = -\arg((q/p)(\bar{A}/A))$, one also has $\beta_s = 0$ because $(q/p) = (\bar{A}/A) = 1$ in the limit where $\lambda^{(s)}_t$ is real. Thus, in the approximation of neglecting all quantities of $O(\lambda^4)$, there are no nonzero weak phases in $B^0_s \rightarrow \phi\phi$, either in the mixing or in the decay.

### 3.1 Untagged distribution

In the approximation of neglecting all weak phases in $B^0_s \rightarrow \phi\phi$, the untagged observables [Eq. (10)] are

\[
(K_1(t) + \bar{K}_1(t))_{SM} = e^{-\Gamma t} \left[ 2|A_0|^2 \left( \cosh (\Delta \Gamma/2) t - \sinh (\Delta \Gamma/2) t \right) \right],
\]

\[
(K_2(t) + \bar{K}_2(t))_{SM} = e^{-\Gamma t} \left[ 2|A_\parallel|^2 \left( \cosh (\Delta \Gamma/2) t - \sinh (\Delta \Gamma/2) t \right) \right],
\]

\[
(K_3(t) + \bar{K}_3(t))_{SM} = e^{-\Gamma t} \left[ 2|A_\perp|^2 \left( \cosh (\Delta \Gamma/2) t + \sinh (\Delta \Gamma/2) t \right) \right],
\]

\[
(K_4(t) + \bar{K}_4(t))_{SM} = 0,
\]

\[
(K_5(t) + \bar{K}_5(t))_{SM} = e^{-\Gamma t} \left[ 2\text{Re}(A_\parallel^* A_0) \left( \cosh (\Delta \Gamma/2) t - \sinh (\Delta \Gamma/2) t \right) \right],
\]

\[
(K_6(t) + \bar{K}_6(t))_{SM} = 0.
\]

We have $A_1^{ih} = \mp A_1^{ch}$ [Eq. (21)], where the minus sign is for $i = 1, 2, 5$, the plus sign for $i = 3$, and both quantities vanish when $i = 4, 6$. The effective lifetimes are then predicted to be

\[
\tau_{\text{eff},i}^{B_{s,SM}} = \frac{\tau_{B_s}}{(1 + y_s)} , \quad i = 1, 2, 5 \quad \tau_{\text{eff},3}^{B_{s,SM}} = \frac{\tau_{B_s}}{(1 - y_s)} , \quad i = 3 .
\]

If the measurement of an effective lifetime differs from the SM prediction, this will be a sign for NP [30].

The SM untagged time-dependent angular distribution for $B^0_s \rightarrow \phi\phi$ takes the form

\[
\frac{d^4(\Gamma_{B_s} + \Gamma_{B_s})}{dtd\omega^2} = \frac{9}{32\pi} \left[ F_L(q^2, \omega) K_L(t) + F_H(q^2, \omega) K_H(t) \right],
\]

where the angular and time-dependent terms are

\[
F_L(\omega) = \left[ |A_0|^2 f_1(\omega) + |A_\parallel|^2 f_2(\omega) + |A_\parallel| |A_\parallel| \cos (\delta_\parallel - \delta_0) f_5(\omega) \right],
\]

13
\[ F_H(\vec{\omega}) = |A_\perp|^2, \]
\[ K_L(t) = 2e^{-\Gamma_L t} = 2e^{-\Gamma t} \left( \cosh (\Delta \Gamma/2)t - \sinh (\Delta \Gamma/2)t \right), \]
\[ K_H(t) = 2e^{-\Gamma_H t} = 2e^{-\Gamma t} \left( \cosh (\Delta \Gamma/2)t + \sinh (\Delta \Gamma/2)t \right), \]  
(28)

in which \((\delta_\parallel - \delta_0) = \arg(A_\parallel A_0^*)\).

Thus, if the \(e^{-\Gamma_L t/2}\) and \(e^{-\Gamma_H t/2}\) terms in the time-dependent angular distribution [see Eq. (17)] can be distinguished experimentally, the \(|A_h|\) and \(\cos (\delta_\parallel - \delta_0)\) can be measured. However, as we will see in the next subsection, these observables can be obtained from time-integrated measurements.

### 3.2 Untagged time-integrated distribution

In the SM, the observables in the time-integrated untagged distribution are

\[ \langle K_1 \rangle = \frac{\tau_{B_s}}{1 + y_s} |A_0|^2, \quad \langle K_2 \rangle = \frac{\tau_{B_s}}{1 + y_s} |A_\parallel|^2 \]
\[ \langle K_3 \rangle = \frac{\tau_{B_s}}{1 - y_s} |A_\perp|^2, \quad \langle K_4 \rangle = 0, \]
\[ \langle K_5 \rangle = \frac{\tau_{B_s}}{1 + y_s} |A_0||A_\parallel| \cos (\delta_\parallel - \delta_0), \quad \langle K_6 \rangle = 0. \]
(29)

We have \(y_s = 0.088 \pm 0.014\) and \(\tau_{B_s}^{-1} = (0.6580 \pm 0.0085) \text{ ps}^{-1}\) [30]. With this knowledge, the \(|A_h|\) and \(\cos (\delta_\parallel - \delta_0)\) can be extracted from the above measurements. This is what CDF and LHCb have presented [21, 22].

### 3.3 Polarization Fractions

With no weak phases in the decay, we have \(A_h = \bar{A}_h\), and the \(|A_h|^2\) can be measured in the untagged time-integrated distribution [Eq. (29)]. The polarization fractions are given by

\[ f_0 = \frac{|A_0|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}, \]
\[ f_\parallel = \frac{|A_\parallel|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}, \]
\[ f_\perp = \frac{|A_\perp|^2}{|A_0|^2 + |A_\parallel|^2 + |A_\perp|^2}, \]
(30)
with total polarization \( f_{tot} = f_0 + f_\parallel + f_\perp = 1 \).

Now, in the presence of NP the distribution changes, and so the experimental measurements have to be reinterpreted. We address this issue in the next section.

4 \( B_s^0 \rightarrow \phi\phi - \text{SM + NP} \)

In this section, we consider NP contributions to \( B_s^0 \rightarrow \phi\phi \), in the mixing and/or in the decay.

4.1 Polarization Fractions

The polarization fractions can be written as

\[
\begin{align*}
    f_0 &= \frac{|A_0|^2 + |\bar{A}_0|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_\parallel|^2 + |\bar{A}_\parallel|^2 + |A_\perp|^2 + |\bar{A}_\perp|^2} = \frac{\mathcal{A}_{f0}^h}{\sum_{i=1,2,3} \mathcal{A}_i^{ch}}, \\
    f_\parallel &= \frac{|A_\parallel|^2 + |\bar{A}_\parallel|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_\parallel|^2 + |\bar{A}_\parallel|^2 + |A_\perp|^2 + |\bar{A}_\perp|^2} = \frac{\mathcal{A}_{f\parallel}^h}{\sum_{i=1,2,3} \mathcal{A}_i^{ch}}, \\
    f_\perp &= \frac{|A_\perp|^2 + |\bar{A}_\perp|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_\parallel|^2 + |\bar{A}_\parallel|^2 + |A_\perp|^2 + |\bar{A}_\perp|^2} = \frac{\mathcal{A}_{f\perp}^h}{\sum_{i=1,2,3} \mathcal{A}_i^{ch}}. \tag{31}
\end{align*}
\]

In the above, the \( f_h \) are written in terms of the \(|A_i|^2\) and \(|\bar{A}_i|^2\). However, as noted above, what is measured experimentally in the time-integrated untagged distribution are the \( \langle K_i \rangle \). It is therefore necessary to express the \( f_h \) in terms of the \( \langle K_i \rangle \). This is done as follows. Using Eq. (23), one can write

\[
\mathcal{A}_i^{ch} = \frac{\langle K_i \rangle}{\tau_{B_s}} (1 + \eta_i y_s) Y_i, \quad i = 1, 2, 3, \tag{32}
\]

where the quantity \( Y_i \) is related to \( \tau_{B_s}^{eff,i} \) or \( \mathcal{A}_{\Delta \Gamma}^i \):

\[
Y_i = \frac{1}{1 + \eta_i y_s} \left( 2 - \frac{\tau_{B_s}^{eff,i}}{\tau_{B_s}} (1 - y_s^2) \right) = \frac{(1 - \eta_i y_s)}{(1 + \mathcal{A}_{\Delta \Gamma}^i y_s)}, \tag{33}
\]

with \( \eta_{1,2} = 1 \), and \( \eta_3 = -1 \). From Eq. (20) we have \( \mathcal{A}_{\Delta \Gamma}^i = \mathcal{A}_i^{ch} / \mathcal{A}_i^{ch} =

\[
\begin{cases}
    -2 \left( \text{Re}(A_0^* A_0) \cos \phi_s + \text{Im}(A_0^* A_0) \sin \phi_s \right) / \left( |A_0|^2 + |\bar{A}_0|^2 \right), & i = 1, \\
    -2 \left( \text{Re}(A_\parallel^* A_\parallel) \cos \phi_s + \text{Im}(A_\parallel^* A_\parallel) \sin \phi_s \right) / \left( |A_\parallel|^2 + |\bar{A}_\parallel|^2 \right), & i = 2, \\
    2 \left( \text{Re}(A_\perp^* A_\perp) \cos \phi_s + \text{Im}(A_\perp^* A_\perp) \sin \phi_s \right) / \left( |A_\perp|^2 + |\bar{A}_\perp|^2 \right), & i = 3.
\end{cases} \tag{34}
\]
In the SM, the weak phases of the $A_h$ vanish and $\phi_s = 0$, so that $A_{\Delta \Gamma}^i = \pm 1$ (the minus sign is for $i = 1, 2$, and the plus sign is for $i = 3$). This implies that $Y_{1,2,3} = 1$, so that the polarization fractions are

$$f_0^{SM} = \frac{\langle K_1 \rangle (1 + y_s)}{\langle K_1 \rangle (1 + y_s) + \langle K_2 \rangle (1 + y_s) + \langle K_3 \rangle (1 - y_s)},$$

$$f_\parallel^{SM} = \frac{\langle K_2 \rangle (1 + y_s)}{\langle K_1 \rangle (1 + y_s) + \langle K_2 \rangle (1 + y_s) + \langle K_3 \rangle (1 - y_s)},$$

$$f_\perp^{SM} = \frac{\langle K_3 \rangle (1 - y_s)}{\langle K_1 \rangle (1 + y_s) + \langle K_2 \rangle (1 + y_s) + \langle K_3 \rangle (1 - y_s)}. \tag{35}$$

Note that these are consistent with Eq. (29). However, if there is NP in the mixing and/or the decay, we have $Y_{1,2,3} \neq 1$, so that the polarization fractions take the form

$$f_0 = \frac{\langle K_1 \rangle (1 + y_s)Y_1}{\langle K_1 \rangle (1 + y_s)Y_1 + \langle K_2 \rangle (1 + y_s)Y_2 + \langle K_3 \rangle (1 - y_s)Y_3},$$

$$f_\parallel = \frac{\langle K_2 \rangle (1 + y_s)Y_2}{\langle K_1 \rangle (1 + y_s)Y_1 + \langle K_2 \rangle (1 + y_s)Y_2 + \langle K_3 \rangle (1 - y_s)Y_3},$$

$$f_\perp = \frac{\langle K_3 \rangle (1 - y_s)Y_3}{\langle K_1 \rangle (1 + y_s)Y_1 + \langle K_2 \rangle (1 + y_s)Y_2 + \langle K_3 \rangle (1 - y_s)Y_3}. \tag{36}$$

The $f_h$ are expressed completely in terms of measured quantities. The $\langle K_i \rangle$’s are obtained from the untagged angular distribution, and one can calculate the $Y_i$ using the measured effective lifetimes. If the effective lifetimes have not been measured then, $A_{\Delta \Gamma}^i$ can be varied within a certain range to get a range for the $Y_i$.

Thus, to obtain the correct polarization fractions in the presence of NP, Eq. (36), which includes factors of $Y_i$, must be used. This is one of the main points of the paper. However, experiments have used Eq. (35), so they have effectively excluded NP. If this possibility is allowed, the analysis must be redone and we discuss this in Sec. 5.

The difference between Eqs. (35) and (36) is related to the difference $Y_i - 1$. One can see from Eq. (35) that $Y_i - 1 \to 0$ in the limit that $y_s \to 0$. This indicates that $f_h - f_h^{SM} = O(y_s)$. Since $y_s = 0.088 \pm 0.014$, this corresponds to a correction to the polarization fractions of $O(10\%)$. This is not large, but it may be important given that the measurements hope to identify the presence of NP.

### 4.2 Other Observables

In Sec. 2.2, we noted that the angular distribution of the decay $B^0_q \to V_1 V_2$ ($q = d, s$) is proportional to $
abla_{i=1}^6 K_i(t) f_i(\vec{\omega})$, where $\vec{\omega} = (\cos \theta_1, \cos \theta_2, \Phi)$ [Eq. (1)]. In
the previous subsection, we discussed polarization fractions, observables which are
dependent on $\langle K_i \rangle$, $i = 1, 2, 3$. We now turn to $i = 4, 6$.

In the present case, $K_4$ and $K_6$ are related to the triple products (TP’s) in $B_s^0 \to \phi\phi$. The expressions for the untagged observables in $B_s^0 \to V_1V_2$ are given in Eq. (16). For convenience, $K_i(t) + \bar{K}_i(t)$ ($i = 4, 6$) are repeated below:

\[
K_4(t) + \bar{K}_4(t) = e^{-\Gamma_s t} \left[ \left( \text{Im}(A_{1 A_{||}^\ast}) - \text{Im}(\bar{A}_{1 A_{||}^\ast}) \right) \cosh(\Delta \Gamma_s/2)t - \left( \text{Im}(A_{1 A_{\perp}^\ast}) - \text{Im}(\bar{A}_{1 A_{\perp}^\ast}) \right) \cos \phi_s + \left( \text{Re}(A_{1 A_{\perp}^\ast}) + \text{Re}(\bar{A}_{1 A_{\perp}^\ast}) \right) \sin \phi_s \right] ,
\]

\[
K_6(t) + \bar{K}_6(t) = e^{-\Gamma_s t} \left[ \left( \text{Im}(A_{1 A_{0}^\ast}) - \text{Im}(\bar{A}_{1 A_{0}^\ast}) \right) \cosh(\Delta \Gamma_s/2)t - \left( \text{Im}(A_{1 A_{\perp}^\ast}) - \text{Im}(\bar{A}_{1 A_{\perp}^\ast}) \right) \cos \phi_s + \left( \text{Re}(A_{1 A_{\perp}^\ast}) + \text{Re}(\bar{A}_{1 A_{\perp}^\ast}) \right) \sin \phi_s \right] . \tag{37}
\]

Now, as discussed earlier, in the SM the weak phases in $B_s^0 \to \phi\phi$, both in the mixing
and in the decay, are all approximately zero, so that $K_4(t) + \bar{K}_4(t)$ and $K_6(t) + \bar{K}_6(t)$
vanish. Thus, if one finds evidence for a nonzero TP, this is a clear sign of NP.

Suppose first that there is NP, with a nonzero weak phase, only in the mixing. In this case, the first two terms in each of $K_i(t) + \bar{K}_i(t)$ ($i = 4, 6$) are zero, but the third
is nonzero. This is a particularly interesting situation, as it corresponds to a TP
produced through mixing. It arises only because $\Delta \Gamma_s$ is nonzero; mixing-induced
TP’s cannot be produced in $B_s^0$ decays. And, although $\Delta \Gamma_s \neq 0$, it is still not large,
so that the associated TP is also rather small.

The second possibility is that there is NP, with a nonzero weak phase, only in the decay. In this case, the first two terms in each of $K_i(t) + \bar{K}_i(t)$ ($i = 4, 6$),
proportional to $\cosh(\Delta \Gamma_s/2)t$ and $\cos \phi_s = 1$, are nonzero, but the third is zero.
And of course one can have NP in both the mixing and the decay. If a TP is seen, its source can be determined through its time dependence.

Both $K_i(t) + \bar{K}_i(t)$ ($i = 4, 6$) are CP-violating, so they correspond to true TP’s.
They can be nonzero only if there are two interfering amplitudes with a relative
weak phase. If there is NP in the mixing, the amplitudes are $A(B_s^0 \to \phi\phi)$ and
$A(B_s^0 \to B_s^0 \to \phi\phi)$; if there is NP in the decay, the amplitudes are $A(B_s^0 \to \phi\phi)_{SM}$
and $A(B_s^0 \to \phi\phi)_{NP}$. In addition, in order to produce a TP, the two interfering
amplitudes must be kinematically different [21]. For the case of NP in the decay,
this is satisfied straightforwardly. But for NP in the mixing, how are $B_s^0 \to \phi\phi$ and
$B_s^0 \to B_s^0 \to \phi\phi$ kinematically different? The point is that mixing-induced TP’s are
generated due to a nonzero $\Delta \Gamma_s$. That is, although $B_s^0 \to \phi \phi$ is a penguin decay, $\bar{B}_s^0 \to \phi \phi$ occurs via a mechanism which contributes to $\Delta \Gamma_s$. For example, one possibility is the $B_s^0 \to B_s^0$ transition via the intermediate states $D_s^+D_s^-$, with the $\bar{B}_s^0$ decaying to $\phi \phi$. The $B_s^0$ and $\bar{B}_s^0$ decays are clearly kinematically different.

We now turn to the measurement of TP’s. Here we focus on the time-integrated untagged observables, $\langle K_i \rangle$. We have $\langle K_i \rangle \propto A_i^{th} + A_i^{th} y_s$ [Eq. (21)]. Specifically, the $\langle K_{1,6} \rangle$ are given in Eq. (20):

$$\langle K_4 \rangle = \frac{\tau_{B_s}}{2(1 - y_s^2)} \left[ (\text{Im}(A_\perp A_0^*) - \text{Im}(A_\perp A_0^*)) - \left( (\text{Im}(A_\perp A_0^*) - \text{Im}(A_\perp A_0^*)) \cos \phi_s \right) y_s \right] ,$$

$$\langle K_6 \rangle = \frac{\tau_{B_s}}{2(1 - y_s^2)} \left[ (\text{Im}(A_\perp A_0^*) - \text{Im}(A_\perp A_0^*)) - \left( (\text{Im}(A_\perp A_0^*) - \text{Im}(A_\perp A_0^*)) \cos \phi_s \right) y_s \right] .$$

The TP’s in the untagged distribution can be measured by constructing asymmetries involving the angular variables. We start by integrating Eq. (18) over $\theta_1$ and $\theta_2$ to obtain the differential rate:

$$\frac{d\langle \Gamma(B_q^0 \to V_1 V_2) \rangle}{d\Phi} = \frac{1}{2\pi} \left[ \langle K_1 \rangle + 2\langle K_2 \rangle \cos 2\Phi + 2\langle K_3 \rangle \sin 2\Phi - 2\langle K_4 \rangle \sin 2\Phi \right] .$$

Note that the time-integrated untagged decay rate can be obtained by integrating out the azimuthal angle $\Phi$:

$$\langle \Gamma(B_q^0 \to V_1 V_2) \rangle = \left[ \langle K_1 \rangle + \langle K_2 \rangle + \langle K_3 \rangle \right] .$$

Following Ref. [20] we can define asymmetries to measure the TP’s. We begin with $i = 4$, for which $f_4(\omega) = -2 \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\Phi$. We define $u \equiv \sin 2\Phi$. The TP asymmetry between the number of decays involving positive and negative values of $u$ is given by [20, 21]

$$A_u = \frac{1}{2} \left[ \frac{\langle \Gamma(B_s^0 \to \phi \phi), u > 0 \rangle - \langle \Gamma(B_s^0 \to \phi \phi), u < 0 \rangle}{\langle \Gamma(B_s^0 \to \phi \phi), u > 0 \rangle + \langle \Gamma(B_s^0 \to \phi \phi), u < 0 \rangle} \right]$$

$$= -\frac{2}{\pi} [A_T^{(2)}]_\text{exp} , \quad [A_T^{(2)}]_\text{exp} = \frac{\langle K_4 \rangle}{\langle \Gamma(B_s^0 \to \phi \phi) \rangle} .$$

As noted above, if $A_u \neq 0$ is found, this would clearly indicate NP. However, we would like to know the relation between $[A_T^{(2)}]_\text{exp}$ and the theoretical expression for
the TP in Eq. (38). The measured TP \( A_T^{(2)} \) is related to \( A_T^{(2)} \) via

\[
A_T^{(2)}_{\text{exp}} = A_T^{(2)}_{\text{theo}} \frac{\tau_{B_s}}{\Gamma(B_s^0 \to \phi \phi)} \left( 1 + A_T^{(4)} y_s \right) \left( 1 - y_s^2 \right),
\]

(42)

where \( A_T^{(4)} = A_4^{sh}/A_4^{ch} \) and

\[
A_T^{(2)}_{\text{theo}} = A_4^{ch} = \frac{1}{2} \left( \text{Im}(A_{\perp A_0}^s) - \text{Im}(\bar{A}_{\perp A_0}^s) \right).
\]

(43)

If we define the dimensionless theoretical TP as

\[
\mathcal{T}P_2 \equiv \left[ A_T^{(2)} \right]_{\text{theo}} \frac{\tau_{B_s}}{\Gamma(B_s^0 \to \phi \phi)} ,
\]

(44)

Eq. (42) details the corrections to the naive relation \( A_T^{(2)}_{\text{exp}} = \mathcal{T}P_2 \) due to a nonzero (NP) \( A_4^{sh} \). (In the SM, \( A_T^{(2)} \) = 0, so the relation is trivial.)

For \( i = 6 \) we have \( f_6(\varphi) = -\sqrt{2} \sin 2\theta_1 \sin 2\theta_2 \sin \Phi \). We define \( v \equiv \text{sign}(\cos \theta_1 \cos \theta_2) \sin \Phi \), which has the following associated TP asymmetry [20]:

\[
A_v = \frac{1}{\pi} \left[ \frac{\Gamma(B_s^0 \to \phi \phi), v > 0 - \Gamma(B_s^0 \to \phi \phi), v < 0}{\Gamma(B_s^0 \to \phi \phi), v > 0 + \Gamma(B_s^0 \to \phi \phi), v < 0} \right]
\]

\[= -\sqrt{2} \frac{A_T^{(1)}_{\text{exp}}}{K_6} , \quad [A_T^{(1)}]_{\text{exp}} = \frac{\langle K_6 \rangle}{\Gamma(B_s^0 \to \phi \phi)} .
\]

(45)

Then

\[
A_T^{(1)}_{\text{exp}} = A_T^{(1)}_{\text{theo}} \frac{\tau_{B_s}}{\Gamma(B_s^0 \to \phi \phi)} \left( 1 + A_T^{(6)} y_s \right) \left( 1 - y_s^2 \right),
\]

(46)

where \( A_T^{(6)} = A_6^{sh}/A_6^{ch} \) and

\[
A_T^{(1)}_{\text{theo}} = A_6^{ch} = \frac{1}{2} \left( \text{Im}(A_{\perp A_0}^s) - \text{Im}(\bar{A}_{\perp A_0}^s) \right).
\]

(47)

We can again define the dimensionless theoretical TP as

\[
\mathcal{T}P_1 \equiv [A_T^{(1)}]_{\text{theo}} \frac{\tau_{B_s}}{\Gamma(B_s^0 \to \phi \phi)} .
\]

(48)

Eq. (46) gives the corrections to the naive relation \( A_T^{(1)}_{\text{exp}} = \mathcal{T}P_1 \).

Finally, we turn to \( i = 5 \), which corresponds to a CP-conserving observable. From Eq. (20),

\[
\langle K_5 \rangle = \frac{\tau_{B_s}}{2(1 - y_s^2)} \left[ \text{Re}(A_{\parallel A_0}^s) + \text{Re}(\bar{A}_{\parallel A_0}^s) \right] - \left( \text{Re}(A_{\perp A_0}^s) + \text{Re}(\bar{A}_{\perp A_0}^s) \right) \cos \phi_s
\]

(49)
\[- \left( \text{Im}(A_\parallel \bar{A}_0^*) - \text{Im}(\bar{A}_\parallel A_0^*) \right) \sin \phi_s \right] y_s \right). \tag{49}\]

We have $f_5(\bar{\omega}) = \sqrt{2} \sin 2\theta_1 \sin 2\theta_2 \cos \Phi$, so we define $w \equiv \text{sign}(\cos \theta_1 \cos \theta_2) \cos \Phi$. The associated asymmetry is

$$A_w = \frac{1}{2} \left[ \langle \Gamma(B_s^0 \to \phi \phi), w > 0 \rangle - \langle \Gamma(B_s^0 \to \phi \phi), w < 0 \rangle \right]$$

$$= \frac{\sqrt{2}}{\pi} [A^{(5)}]_{\text{exp}}, \quad [A^{(5)}]_{\text{exp}} = \left\langle K_5 \right\rangle / \langle \Gamma(B_s^0 \to \phi \phi) \rangle . \tag{50}\]

We have

$$[A^{(5)}]_{\text{exp}} = [A^{(5)}]_{\text{theo}} \frac{\tau_{B_s}}{\langle \Gamma(B_s^0 \to \phi \phi) \rangle} \left( 1 + A^{(5)}_{\Delta r} y_s \right) / (1 - y_s^2)$$

$$= [A^{(5)}]_{\text{theo}} \frac{\tau_{B_s}}{\langle \Gamma(B_s^0 \to \phi \phi) \rangle} \left( 2 - \frac{\tau_{eff,5}}{\tau_{B_s}} (1 - y_s^2) \right) , \tag{51}\]

where $A^{(5)}_{\Delta r} = A^{ch}_5 / A^{ch}_5$, the effective lifetime $\tau_{eff,5}$ is defined in Eq. (22), and

$$[A^{(5)}]_{\text{theo}} = A^{ch}_5 = \frac{1}{2} \left( \text{Re}(A_\parallel A_0^*) + \text{Re}(\bar{A}_\parallel \bar{A}_0^*) \right) . \tag{52}\]

4.3 NP Parameters

12 observables can be measured from the time-dependent untagged angular distribution (Sec. 2.3). With these, one can identify if NP is present in the mixing and/or the decay. However, we will also want to identify its properties. To be specific, if there is NP in the decay amplitude, it will be important to measure the various NP parameters. With this in mind, the question is: how many theoretical unknowns are there in the most general SM + NP $B_s^0 \to \phi \phi$ amplitude? If there are fewer than 12, then we can extract all the unknowns.

In writing the SM + NP $B_s^0 \to \phi \phi$ amplitude, we have the following points:

- The SM weak phases are $\approx 0$.

- Assuming that the NP amplitudes satisfy $|A_{h}^{NP}| < |A_{h}^{SM}|$, the NP strong phases are negligible [31]. This means that if there are many NP amplitudes, they can all be combined into a single term with an effective magnitude and weak phase.

- In the heavy-quark limit, we have $A_1^{SM} = -A_{2}^{SM}$ [32].
Taking these points into account, the most general SM + NP $B_s^0 \to \phi\phi$ helicity amplitude can then be written

$$A_h = |A_h^{SM}| e^{i\delta_h^{SM}} + |A_h^{NP}| e^{i\phi_h}. \quad (53)$$

There are a total of 11 unknown theoretical parameters – 5 magnitudes (2 SM, 3 NP), 2 SM strong phases, 3 NP weak phases, and the mixing phase $\phi_s$. In principle, these can all be extracted from the 12 observables.

However, note that Eq. (53) includes a different NP weak phase $\phi_h$ for each helicity amplitude. But in many NP models the weak phases are helicity independent. In this case there is only one NP weak phase $\phi$, and the number of theoretical unknowns is reduced to 9. This is a model-dependent result, but it is still very general.

Finally, if the NP is purely left-handed or right-handed, then $A_{NP}^\perp = \mp A_{NP}^\parallel$, which further reduces the number of theoretical unknowns by one.

In all cases, assuming the time-dependent untagged angular distribution can be measured, there are more observables than unknowns, and so we will be able to extract all the NP parameters in the decay. In this way, we may be able to identify the type of NP that is present.

## 5 Numerical Analysis

Recently, the CDF and LHCb Collaborations have reported measurements for the polarization amplitudes, the strong-phase difference between $A_\parallel$ and $A_0$, and the triple-product asymmetries in $B_s^0 \to \phi\phi$. The LHCb results [22] are summarized in Table 1. The values are in good agreement with those reported by the CDF Collaboration [21], except for the TP’s, though all measurements are consistent within errors.

| Observable     | Measurement                                      |
|----------------|--------------------------------------------------|
| $|A_0|_{exp}^2$   | $0.365 \pm 0.022$ (stat) $\pm 0.012$ (syst)       |
| $|A_\perp|_{exp}^2$ | $0.291 \pm 0.024$ (stat) $\pm 0.010$ (syst)       |
| $|A_\parallel|_{exp}^2$ | $0.344 \pm 0.024$ (stat) $\pm 0.014$ (syst)       |
| $\cos(\delta_\parallel - \delta_0)$ | $-0.844 \pm 0.068$ (stat) $\pm 0.029$ (syst) |
| $A_u$           | $-0.055 \pm 0.036$ (stat) $\pm 0.018$ (syst)     |
| $A_v$           | $0.010 \pm 0.036$ (stat) $\pm 0.018$ (syst)      |

Table 1: Measured polarization amplitudes, strong-phase difference, and triple-product asymmetries in $B_s^0 \to \phi\phi$ [22]. The sum of the $|A_h|_{exp}^2$ terms is constrained to unity.

The experiments have measured the $\langle K_i \rangle$ and constructed the polarization fractions assuming the SM. As discussed previously, if one allows for the possibility of
NP in $\bar{b} \to \bar{s}$ transitions, this analysis must be modified. This is done below.

We denote the measured value of $y_s$ as $y_{s0}$. From Eq. (29) we have

$$
\langle K_1 \rangle = \frac{\tau_{B_s}}{1 + y_s} |A_0|^2 = \frac{\tau_{B_s}}{1 + y_{s0}} |A_0|_{y_s = y_{s0}}^2, \\
\langle K_2 \rangle = \frac{\tau_{B_s}}{1 + y_s} |A||^2 = \frac{\tau_{B_s}}{1 + y_{s0}} |A||_{y_s = y_{s0}}^2, \\
\langle K_3 \rangle = \frac{\tau_{B_s}}{1 - y_s} |A_\perp|^2 = \frac{\tau_{B_s}}{1 - y_{s0}} |A_\perp|_{y_s = y_{s0}}^2.
$$

The experimental measurements in Table I are then

$$
|A_0|_{\text{exp}}^2 = \frac{|A_0|_{y_s = y_{s0}}^2}{|A_0|_{y_s = y_{s0}}^2 + |A||_{y_s = y_{s0}}^2 + |A_\perp|_{y_s = y_{s0}}^2}, \\
|A||_{\text{exp}}^2 = \frac{|A||_{y_s = y_{s0}}^2}{|A_0|_{y_s = y_{s0}}^2 + |A||_{y_s = y_{s0}}^2 + |A_\perp|_{y_s = y_{s0}}^2}, \\
|A_\perp|_{\text{exp}}^2 = \frac{|A_\perp|_{y_s = y_{s0}}^2}{|A_0|_{y_s = y_{s0}}^2 + |A||_{y_s = y_{s0}}^2 + |A_\perp|_{y_s = y_{s0}}^2}.
$$

One can now calculate the polarization fractions in the SM as a function of $y_s$. Inputting the expressions for the $\langle K_i \rangle$ from Eq. (54) into Eq. (35), and using Eq. (55), we obtain

$$
f^{SM}_0 = \frac{|A_0|_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}}}{|A_0|_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} + |A||_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} + |A_\perp|_{\text{exp}}^2 \frac{1 - y_s}{1 - y_{s0}}}, \\
f^{SM}_|| = \frac{|A||_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}}}{|A_0|_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} + |A||_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} + |A_\perp|_{\text{exp}}^2 \frac{1 - y_s}{1 - y_{s0}}}, \\
f^{SM}_\perp = \frac{|A_\perp|_{\text{exp}}^2 \frac{1 - y_s}{1 - y_{s0}}}{|A_0|_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} + |A||_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} + |A_\perp|_{\text{exp}}^2 \frac{1 - y_s}{1 - y_{s0}}}.
$$

Hence the $|A_i|_{\text{exp}}^2$ in Table I are just the $f_i^{SM}$ defined in Eq. (56) with $y_s = y_{s0}$.

The true polarization fractions can then be obtained by inputting the expressions for the $\langle K_i \rangle$ from Eq. (34) into Eq. (36), and using Eq. (55):

$$
f_0 = \frac{|A_0|_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} Y_1}{|A_0|_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} Y_1 + |A||_{\text{exp}}^2 \frac{1 + y_s}{1 + y_{s0}} Y_2 + |A_\perp|_{\text{exp}}^2 \frac{1 - y_s}{1 - y_{s0}} Y_3},
$$

22
\[
\begin{align*}
    f_\parallel &= \frac{|A_0|^{2} 1+y_s Y_2}{|A_0|^{2} 1+y_s Y_1 + |A_1|^{2} 1+y_s Y_2 + |A_\perp|^{2} 1-y_s Y_3}, \\
    f_\perp &= \frac{|A_\perp|^{2} 1-y_s Y_3}{|A_0|^{2} 1+y_s Y_1 + |A_1|^{2} 1+y_s Y_2 + |A_\perp|^{2} 1-y_s Y_3}.
\end{align*}
\]

In Fig. 1 we plot the dependence of the polarization fractions \(f_0, f_\parallel\) and \(f_\perp\) as a function of \(y_s\). This figure is read as follows. In all plots the horizontal region represents the experimental result, in which \(|A|_{h=0,\|,\perp}^{2}\) is allowed to vary by \(\pm1\sigma\) (see Table 1). Also, the vertical bands correspond to \(y_s\), with \(\pm1\sigma\) (green) or \(\pm3\sigma\) (yellow) errors. In the SM we have \(Y_i = 1\), corresponding to \((A_{1}\Delta\Gamma = -1, A_{2}\Delta\Gamma = 1)\) [Eq. (34)]. In order to illustrate the effect of NP, we take \((A_{1}\Delta\Gamma = 1, A_{2}\Delta\Gamma = -1, A_{3}\Delta\Gamma = -1)\) (red line) or \((A_{1}\Delta\Gamma = -1, A_{2}\Delta\Gamma = 1, A_{3}\Delta\Gamma = 1)\) (blue line). For these values of \(A_{i}\Delta\Gamma\), we have \(Y_i \neq 1\). Consider first \(f_0\). In the SM the experimental measurement implies \(0.33 \leq f_0 \leq 0.40\). However, with NP, the value of \(f_0\) can lie outside this range – for example, on the red line it can be as small as 0.29. The behavior is similar for \(f_\parallel\) and \(f_\perp\). This shows explicitly that, in the presence of NP, the \(B_s^0 \to \phi\phi\) polarization fractions can be changed from their SM values by \(O(10\%)\) for the current value of \(y_s\).

![Figure 1: The dependence of the theoretical polarization fractions \(f_0, f_\parallel\) and \(f_\perp\) on the decay width parameter \(y_s\) for different values of \(A_{\Delta\Gamma}\). The red line corresponds to \((A_{1}\Delta\Gamma = 1, A_{2}\Delta\Gamma = -1, A_{3}\Delta\Gamma = -1)\), while the blue line has \((A_{1}\Delta\Gamma = -1, A_{2}\Delta\Gamma = 1, A_{3}\Delta\Gamma = 1)\). In all plots the experimental result \(|A|_{h=0,\|,\perp}^{2}\) (horizontal region) is allowed to vary by \(\pm1\sigma\) (see Table 1). The vertical bands correspond to \(y_s\), with \(\pm1\sigma\) (green) or \(\pm3\sigma\) (yellow) errors.](image)

The relation between \(A_n\) and \([A_{T}^{(2)}]_{\text{theo}}\) is given in Eqs. (11) and (12); that between \(A_v\) and \([A_{T}^{(1)}]_{\text{theo}}\) is given in Eqs. (13) and (16). These can be rewritten as

\[
[A_{T}^{(2)}]_{\text{theo}} = \frac{\tau_{B_s}}{\Gamma(B_s^0 \to \phi\phi)} \frac{\pi A_n}{2 A_{\perp}} \frac{(1 - y_s^2)}{(1 + A_{\Delta\Gamma} y_s)},
\]
Figure 2: The dependence of the theoretical TP’s \( [A^{(2)}_T]_{\text{theo}} \tau_{B_s}/\langle \Gamma(B^0_s \rightarrow \phi\phi) \rangle \) (left) and \( [A^{(1)}_T]_{\text{theo}} \tau_{B_s}/\langle \Gamma(B^0_s \rightarrow \phi\phi) \rangle \) (right) on \( y_s \) for different values of \( A^{4(6)}_{\Delta\Gamma} \). In the red and blue regions, we take \( A^{4(6)}_{\Delta\Gamma} = \pm 1 \), respectively. Also, \( A_u \) (left) and \( A_v \) (right) are allowed to vary by \( \pm 1 \sigma \) (see Table I). The dashed black lines correspond to the central values of \( A_u \) (left) and \( A_v \) (right) with \( A^{4(6)}_{\Delta\Gamma} = 0 \). The vertical bands correspond to \( y_s \), with \( \pm 1 \sigma \) (green) or \( \pm 3 \sigma \) (yellow) errors.

\[
[A^{(1)}_T]_{\text{theo}} = \frac{\tau_{B_s}}{\langle \Gamma(B^0_s \rightarrow \phi\phi) \rangle} = -\frac{\pi}{\sqrt{2}} A_v \frac{(1 - y_s^2)}{(1 + A^{(6)}_{\Delta\Gamma} y_s)}. \tag{58}
\]

In Fig. 2 we plot the dependence of the theoretical TP’s \( [A^{(2)}_T]_{\text{theo}} \tau_{B_s}/\langle \Gamma(B^0_s \rightarrow \phi\phi) \rangle \) and \( [A^{(1)}_T]_{\text{theo}} \tau_{B_s}/\langle \Gamma(B^0_s \rightarrow \phi\phi) \rangle \) as a function of \( y_s \). The dashed black lines correspond to the central values of \( A_u \) (left) and \( A_v \) (right) with \( A^{4(6)}_{\Delta\Gamma} = 0 \). The vertical bands correspond to \( y_s \), with \( \pm 1 \sigma \) (green) or \( \pm 3 \sigma \) (yellow) errors. In the red and blue regions, we take \( A^{4(6)}_{\Delta\Gamma} = \pm 1 \), respectively, and allow \( A_u \) (left) and \( A_v \) (right) to vary by \( \pm 1 \sigma \) (see Table I). It is clear from these figures that, in the presence of NP, the values of the theoretical TP’s can differ significantly from the measured asymmetries. (This is not surprising since the TP’s vanish in the SM.)

Finally, for \( i = 5 \), we have estimated the measured value of the CP-conserving observable as follows:

\[
[A^{(5)}]_{\text{exp}} = |A_0|_{\text{exp}} |A_\parallel|_{\text{exp}} \cos(\delta_\parallel - \delta_0) = -0.299 \pm 0.030. \tag{59}
\]

The relation between \( [A^{(5)}]_{\text{exp}} \) and \( [A^{(5)}]_{\text{theo}} \) is given by [see Eq. (51)]

\[
[A^{(5)}]_{\text{theo}} = \frac{\tau_{B_s}}{\langle \Gamma(B^0_s \rightarrow \phi\phi) \rangle} = \frac{[A^{(5)}]_{\text{exp}} (1 - y_s^2)}{(1 + A^{(5)}_{\Delta\Gamma} y_s)}. \tag{60}
\]

In Fig. 3 we plot the dependence of \( [A^{(5)}]_{\text{theo}} \tau_{B_s}/\langle \Gamma(B^0_s \rightarrow \phi\phi) \rangle \) as a function of \( y_s \). As before, the value of this quantity can differ from Eq. (59) by as much as \( O(10\%) \) for the current value of \( y_s \).
Figure 3: The dependence of $[A_T^{(5)}]_{\text{theo} \tau B_s}/\langle \Gamma(B_s^0 \rightarrow \phi \phi) \rangle$ on $y_s$ for different values of $A_{\Delta\Gamma}^s$. In the red and blue regions, we take $A_{\Delta\Gamma}^s = \pm 1$, respectively. Also, $[A^{(5)}]_{\text{exp}}$ is allowed to vary by $\pm 1\sigma$ [see Eq. (59)]. The dashed black line corresponds to the central value of $[A^{(5)}]_{\text{exp}}$ with $A_{\Delta\Gamma}^s = 0$. The vertical bands correspond to $y_s$, with $\pm 1\sigma$ (green) or $\pm 3\sigma$ (yellow) errors.

6 Conclusions

The main goal of studying the $B$ system is to find evidence for physics beyond the standard model (SM). One possibility is new physics (NP) in $b \rightarrow \bar{s}$ transitions. At present its status is uncertain. It seems unlikely that the effect of such NP can be very large, but a smaller effect is still possible. In this paper, we consider $b \rightarrow \bar{s}$ NP. However, in contrast to what is usually done, i.e. considering only NP in $B_s^0$-$\bar{B}_s^0$ mixing, here we also allow NP in the decay. In particular, we examine the effect of such NP on the angular distribution of $B^0_q \rightarrow V_1 V_2$ ($q = d, s$), where $V_{1,2}$ are vector mesons.

Our principal result is the following. The parameters of the untagged, time-integrated angular distribution can be measured experimentally, and certain observables can be derived from these parameters. However, in the presence of NP, the formulae which relate the parameters to the observables must be modified from their SM forms. We find six observables for which the relation between the experimental data and theoretical parameters must be modified, corresponding to the six terms ($i = 1-6$) in the angular distribution. For $i = 1-3$ they are the polarization fractions, for $i = 4, 6$ they are the CP-violating triple-product asymmetries, and $i = 5$ corresponds to a CP-conserving observable. The modifications for the polarization fractions are most interesting. These are due in part to the nonzero width difference in the $B^0_q$-$\bar{B}_q^0$ system, and so are important only for $B^0_s$ decays. In particular, there can be important effects on the pure $b \rightarrow \bar{s}$ penguin decay $B^0_s \rightarrow \phi \phi$.

In light of this, we re-analyze the $B^0_s \rightarrow \phi \phi$ data to see the effect of these modifications. $\Delta \Gamma_s/2 \Gamma_s \sim 10\%$, so that the modifications of the formulae lead to
$O(10\%)$ changes in the polarization fractions. These are not large, but may be
important given that one is looking for signals of NP.

Finally, if the NP contributes to the $\bar{b} \to \bar{s}$ decay, we show that the measurement
of the untagged time-dependent angular distribution provides enough information –
12 observables – to extract all the NP parameters.

Note added: while the paper was being written, DØ produced a direct measure-
ment of the semileptonic charge asymmetry in $B_s^0$ decays [36], and they say that
it agrees with the SM. Technically, this is true. The SM predicts $a_{s\ell}^s \sim 2 \times 10^{-5}$,
and DØ measures $a_{s\ell}^s = [-1.08 \pm 0.72 \text{ (stat)} \pm 0.17 \text{ (syst)}]\%$. While this result is
consistent with 0, the errors are large enough that NP is also a possibility.

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