Linear defects are generic in continuous media. In quantum systems they appear as topological line defects which are associated with a circulating persistent current. In relativistic quantum vacuum they are known as cosmic strings, in superconductors as quantized flux lines, and in superfluids and low-density atomic Bose-Einstein condensates as quantized vortex lines. We discuss unconventional vortices in unconventional superfluids and superconductors, which have been observed or have to be observed, such as continuous singly and doubly quantized vortices in $^3$He-A and chiral Bose condensates; half-quantum vortices (Alice strings) in $^3$He-A and in nonchiral Bose condensates; Abrikosov vortices with fractional magnetic flux in chiral and d-wave superconductors; vortex sheets in $^3$He-A and chiral superconductors; the nexus – combined object formed by vortices and monopoles. Some properties of vortices related to the fermionic quasiparticles living in the vortex core are also discussed.

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by the gap nodes. $^3$He-A is the first example of superfluids with the gap nodes in the energy spectrum for quasiparticles. Though existence of nodes in $^3$He-A was proven only recently in experiments with the AB interface where the $T^4$ behavior has been verified, these nodes influence the investigation of low-$T$ properties of unconventional superconductors. In particular, the observed low-$T$ and low-$B$ scaling for the specific heat in a mixed state of high-$T_c$ superconductors related to the gap nodes and vortices has its history started in 1981, when it was found that gap nodes in $^3$He-A lead to finite nonanalytic density of states at $T = 0$ in the presence of the order parameter texture. The intermediate steps were the Ref., where the superfluid velocity field has been used instead of a texture, and Ref. where this was extended for the superflow around the vortex in $d$-wave superconductor.

Another example of influence of $^3$He-A is the observation in high-$T_c$ superconductors of the fractional flux attached to the tricrystal line, which is the junction of three grain boundaries. This was inspired by the discussion of the flux quantization with the half of conventional magnitude in a heavy fermionic superconductors, which in turn has been inspired by $^3$He-A where the possibility of a vortex with half-integer circulation has been shown.

The two examples above are the two sides of the same phenomenon: the nontrivial topology. The gap nodes in momentum space and vortices – the nodes in position space – are topological objects, which can be continuously transformed to each other by transformation in the extended momentum+position space, as also was first discussed in $^3$He-A. The same topology is also in the basis of the Standard Model of electroweak and strong interactions in particle physics, (see Review). As a result, in the vicinity of the gap nodes the $^3$He-A liquid as well as the high-temperature superconductors and other similar systems acquire some attributes of relativistic quantum field theory such as Lorentz invariance, gauge invariance, chirality and chiral fermions, etc. The conceptual similarity between these condensed matter systems and the quantum vacuum makes them an ideal laboratory for simulating many effects in high energy physics and cosmology.

Now it is believed that the tetragonal layered superconductor Sr$_2$RuO$_4$ has the order parameter of the same symmetry class as $^3$He-A and thus shares many unusual properties of the latter. In particular, it is assumed that the superconducting state of Sr$_2$RuO$_4$ is chiral, i.e. reflection and time reversal symmetries are spontaneously broken. There are other systems which share some of unusual properties of $^3$He-A: the multicomponent atomic Bose-condensates; neutron superfluids; quark matter with colour superconductivity ... It is worthwhile to consider where we are now: what is observed from these exotic properties and what is to be observed.
2. CONTINUOUS VORTICITY

According to the Landau picture of superfluidity, the superfluid flow is potential: its velocity $v_s$ is curl-free: $\nabla \times v_s$. Later Onsager and Feynman found that this statement must be generalized: $\nabla \times v_s \neq 0$ at singular lines, the quantized vortices, around which the phase of the order parameter winds by $2\pi N$. Discovery of superfluid $^3$He-A made further weakening of the rule: The nonsingular vorticity can be produced by the regular texture of the order parameter. The order parameter – the wave function of Cooper pair with spin $S = 1$ and orbital momentum $L = 1$ – is determined by two vectors

$$\Delta(k) = \Delta_0 \left( \sigma \cdot \mathbf{d} \right) \left( k \cdot (\mathbf{e}_1 + i\mathbf{e}_2) \right).$$

The real unit vector $\mathbf{d}$ enters the spin part of the order parameter, where $\sigma$ are the Pauli matrices. The orbital part, which shows the momentum $k$ dependence, is described by the complex vector $\mathbf{e}_1 + i\mathbf{e}_2$, which mutually orthogonal real and imaginary parts. The order parameter is intrinsically complex, i.e. its phase cannot be eliminated by a gauge transformation. This violates the time reversal symmetry and leads to the spontaneous angular momentum along the unit vector $\hat{l} = \mathbf{e}^{(1)} \times \mathbf{e}^{(2)}$. The superfluid velocity of the chiral condensate is $v_s = \frac{\hbar}{2m} \mathbf{e}_i^{(1)} \nabla \mathbf{e}_i^{(2)}$ (where $2m$ is the mass of the Cooper pair) with continuous vorticity satisfying the Merm-in-Ho relation: $\nabla \times v_s = \frac{\hbar}{4m} \epsilon_{ijk} \nabla \mathbf{l}_j \times \nabla \mathbf{l}_k$.

Quantization of vorticity is provided by topology of textures: The texture which is homogeneous at large distances has $4\pi$ winding (i.e. $N = 2$) if the $\hat{l}$ field covers all $4\pi$ orientations in the soft core of continuous vortex. If the order parameter is a spinor (the “half of vector”), as it occurs in spin-1/2 Bose condensates and in the Standard Model of electroweak interactions, the continuous vortices and correspondingly continuous cosmic strings have twice less winding number, $N = 1$.

The NMR measurements already in 1983 provided the indication for existence of the continuous $4\pi$ lines in rotating $^3$He-A (see Review). However, the first proof that the isolated continuous vortex contains $N = 2$ circulation quanta has been obtained only recently. Lines with continuous vorticity have been used for “cosmological” experiments. Investigation of the dynamics of these vortices presented the first demonstration of the condensed matter analog of the axial anomaly in relativistic field theory, which is believed responsible for the present excess of matter over antimatter. Another cosmological phenomenon is related to nucleation of these vortices investigated in. Vortex formation marks the helical instability of the normal/superfluid counterflow in $^3$He-A (the counterflow is the flow of the normal component with respect to the superfluid one). The instability is described by the same
G.E. Volovik

equations and actually represents the same physics (see Review\cite{12}) as the helical instability of the bath of right-handed electrons towards formation of the helical hypermagnetic field discussed in\cite{27,28}. This experiment thus supported the Joyce-Shaposhnikov scenario of formation of primordial cosmological magnetic field.

Another example of continuous vorticity observed in $^3\text{He}$-A is the vortex sheet\cite{29}. This is the chain of the so called Mermin-Ho continuous vortices with $N = 1$ trapped by the soliton. Each vortex is the kink in the soliton structure. The kink can live only within the soliton, precisely as the Bloch line within the Bloch wall in magnets.

3. FRACTIONAL VORTICES IN SUPERFLUIDS AND SUPERCONDUCTORS

In $^3\text{He}$-A the fractional vorticity is still to be observed. The discrete symmetry, which supports the half-quantum vortex comes from the identification of points $\hat{d}, \hat{e}_1 + i\hat{e}_2$ and $-\hat{d}, -(\hat{e}_1 + i\hat{e}_2)$. In a half-quantum vortex both vectors change sign after circling around the vortex while the order parameter in Eq.(1) returns to its initial value

$$\Delta = \Delta_0 (\hat{d} \cdot \sigma) (\sin k \cdot a + i \sin k \cdot b) e^{i\theta},$$

where $\theta$ is the phase of the order parameter; $a$ and $b$ are the elementary vectors of the crystal lattice within the layer.

Vortices with fractional $N$ can be obtained in two ways. If $\hat{d}$-field is flexible enough, there is analog of $N = 1/2$ vortex in Eq.\cite{32}, in which $\hat{d} \rightarrow -\hat{d}$ and $\theta \rightarrow \theta + \pi$ around the vortex.\cite{33} Another is the M"obius strip geometry produced by twisting the crystal axes $a$ and $b$ around the loop.\cite{33} The closed wire traps the fractional flux, if it is twisted by an angle $\pi/2$ before

Here $\phi$ is the azimuthal angle of the cylindrical coordinate system; the magnetic field is applied along $z$ to keep the $\hat{d}$-vector in $x - y$ plane. $\hat{d}$ is quantization axis for spin. Since it rotates by $\pi$ around the vortex, a “person” living in $^3\text{He}$-A, who moves around the vortex, insensibly finds its spin reversed with respect to the fixed environment. This is an analog of Alice string in particle physics.\cite{30} A particle which moves around an Alice string continuously flips its charge or parity or enters the “shadow” world.\cite{31}

In superconductors the crystalline structure must be taken into account. In simplest representation, which preserves the tetragonal symmetry, the order parameter in Sr$_2$RuO$_4$:

$$\Delta(k) = \Delta_0 (\hat{d} \cdot \sigma) (\sin k \cdot a + i \sin k \cdot b) e^{i\theta},$$

where $\theta$ is the phase of the order parameter; $a$ and $b$ are the elementary vectors of the crystal lattice within the layer.
gluing the ends. Since the local orientation of the crystal lattice continuously changes by $\pi/2$ around the loop, $a \rightarrow b$ and $b \rightarrow -a$, the order parameter is multiplied by $i$ after encircling the loop. The single-valuedness of the order parameter requires that this must be compensated by a change of its phase $\theta$ by $\pi/2$. As a result the phase winding around the loop is $\pi/2$, i.e. $N = 1/4$.

This, however, does not mean that the loop of the chiral $p$-wave superconductor traps $1/4$ of the magnetic flux $\Phi_0$ of conventional Abrikosov vortex. Because of the breaking of time reversal symmetry in chiral crystalline superconductors, persistent electric current arises not only due to the phase coherence but also due to deformations of the crystal.

\[ j = \rho_s \left( \mathbf{v}_s - \frac{e}{mc} \mathbf{A} \right) + Ka_i \nabla b_i, \quad \mathbf{v}_s = \frac{\hbar}{2m} \nabla \theta. \tag{4} \]

Magnetic flux trapped by the loop is obtained from the condition of no current in the wire, $j = 0$ in Eq. (4). Thus, the trapped flux depends on the parameter $K$ in the deformation current. In the limit case of $K = 0$ the flux is $\Phi_0/4$ (or $\Phi_0/6$ if the underlying crystal lattice has hexagonal symmetry).

In the nonchiral $d$-wave superconductor in layered cuprate oxides the order parameter can be represented by:

\[ \Delta(\mathbf{k}) = \Delta_0 \left( \sin^2 \mathbf{k} \cdot \mathbf{a} - \sin^2 \mathbf{k} \cdot \mathbf{b} \right) e^{i\theta}. \tag{5} \]

The same twisted vortex loop with $a \rightarrow b$ and $b \rightarrow -a$, leads to the change of sign of the order parameter, which must be compensated by a change of its phase $\theta$ by $\pi$. This corresponds to the $N = 1/2$ of circulation quantum. The fractional flux trapped by the loop is now exactly $\Phi_0/2$, since the parameter $K$ in Eq. (4) is exactly zero in nonchiral superconductors. The same reasoning gives rise to the $\Phi_0/2$ flux attached to the tricrystal line around this line one has $a \rightarrow b$ and $b \rightarrow -a$. Observation of the fractional flux different from $\Phi_0/2$ would indicate breaking of the time reversal symmetry.\(^{35,36,34}\)

Let us also mention that the vortex sheet with fractional elementary vortices has been predicted to exist in chiral superconductors even before the experimental identification of the vortex sheet in $^3$He-A. The object which traps vorticity is the domain wall separating domains with the opposite orientations of the $\hat{l}$-vector.\(^{37}\) The kink – the Bloch line – in the domain wall is the vortex with the fractional winding number $N = 1/2$. When there are many fractional vortices (kinks) trapped, they form the vortex sheet, which as suggested in can be responsible for the flux flow dynamics in the low-$T$ phase of the heavy fermionic superconductor UPt$_3$.

In conclusion of this Section let us mention an exotic combined object in $^3$He-A which still have not been observed – vortex terminated by hedgehog.\(^{35,40}\) Four equally “charged” half-quantum vortices, i.e. each with
G.E. Volovik

\( N = +1/2 \) winding number, can meet each other at one point – the hedgehog in the \( \hat{l} \) field (see Review \[33\]). Such combined object, which reminds the Dirac magnetic monopole with one or several physical Dirac strings, is called a nexus in relativistic theories. In electroweak theory the monopole-antimonopole pair connected by \( Z \) string is called a dumbell \[34\]. These objects can exist in chiral superconductors too, where the nexus has a magnetic charge emanating radially from the hedgehog: this charge is compensated by the flux supplied to the hedgehog by \( N = 1/2 \) Abrikosov vortices. Nexus provides a natural trap for massive magnetic monopole – the ‘t Hooft-Polyakov magnetic monopole. If such a monopole enters the chiral superconductor, it is bound to the hedgehog by two \( N = 1 \) or four \( N = 1/2 \) vortices. \[35\]

4. VORTICES IN ATOMIC BOSE-CONDENSATES

In a vector Bose-condensate with the hyperfine spin \( F = 1 \) an order parameter consists of 3 complex amplitudes of spin projections \( M = (+1, 0, -1) \). They can be organized to form the complex vector \( \mathbf{a} \):

\[
\Psi_\nu = \begin{pmatrix} \Psi_{+1} \\ \Psi_0 \\ \Psi_{-1} \end{pmatrix} = \begin{pmatrix} a_{x+iy} \\ \sqrt{2} a_z \\ a_{x-iy} \end{pmatrix}.
\]

There are two symmetrically distinct phases of the \( F = 1 \) Bose-condensates:

(i) The chiral state, which mimics the orbital part of the \( ^3\)He-A order parameter in Eq.(\[5\]), \( \mathbf{a} = f(\hat{\mathbf{e}}_1 + i\hat{\mathbf{e}}_2), \hat{l} = \hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2 \), occurs when the scattering length \( a_2 \) in the scattering channel of two atoms with the total spin 2 is less than that with the total spin zero, \( a_2 < a_0 \). \[15, 16\] (ii) The nonchiral state, which mimics the spin part of the \( ^3\)He-A order parameter in Eq.(\[5\]), \( \mathbf{a} = f\hat{\mathbf{d}}e^{i\theta} \), occurs for \( a_2 > a_0 \). Each of the two states shares some properties of superfluid \( ^3\)He-A. In particular, the chiral state (i) has continuous \( 4\pi \) vortices \[15, 16\] which are called skyrmions there. An optical method to create half of the full skyrmion – the Mermin-Ho continuous vortices with \( 2\pi \) winding – in the \( F = 1 \) Bose-condensates has been recently discussed in Ref. \[43\]. The nonchiral state (ii) may contain \( N = 1/2 \) vortex – the combination of \( \pi \)-vortex in the phase \( \theta \) and \( \pi \)-disclination in the vector \( \hat{\mathbf{d}} \) as in Eq.(\[5\]).\[44\]

\[
\mathbf{a} = f \left( \hat{\mathbf{x}} \cos \frac{\phi}{2} + \hat{\mathbf{y}} \sin \frac{\phi}{2} \right) e^{i\phi/2}.
\]

In spin projection representation the order parameter far from the core is

\[
\Psi_\nu = fe^{i\phi/2} \begin{pmatrix} e^{i\phi/2} \\ 0 \\ e^{-i\phi/2} \end{pmatrix} = f \begin{pmatrix} e^{i\phi} \\ 0 \\ 1 \end{pmatrix}.
\]
Vortices Observed and to be Observed

This means that the \( N = 1/2 \) vortex can be represented as a vortex in the \( M = +1 \) component, while the \( M = -1 \) component is vortex-free.

The same two cases can occur in a mixture of two condensates. The skyrmion and \( N = 1/2 \) vortex can be written in the same form:

\[
\left( \frac{\Psi^+}{\Psi^-} \right) = f \left( e^{i\phi} \frac{\cos \beta(r)}{\sin \beta(r)} \right), \quad \hat{l} = (\sin \beta \cos \phi, -\sin \beta \sin \phi, \cos \beta).
\] (9)

For skyrmion one has \( \beta(\infty) = 0 \) and \( \beta(0) = \pi \). This represents the \( N = 1 \) vortex in condensate \( |\uparrow\rangle \), whose soft core is filled by the condensate \( |\downarrow\rangle \) as was observed in Ref.\[45\] Note that this is the full skyrmion, i.e. the \( \hat{l} \) vector sweeps the whole sphere, however it has \( N = 1 \) winding instead of \( N = 2 \) in a vector Bose-condensate\[21\].

The \( N = 1/2 \) vortex is obtained if \( \beta(\infty) = \pi/2 \): it is the \( N = 1 \) vortex in the \( |\uparrow\rangle \) component, while the \( |\downarrow\rangle \) component is vortex-free.

5. FERMIONS ON EXOTIC VORTICES

Interesting phenomena to be observed are related to behavior of fermionic quasiparticles in topologically nontrivial environments provided by fractional vortices and monopoles. In the presence of a monopole the quantum statistics of particles can change, e.g. isospin degrees of freedom are transformed to spin degrees.\[46\] There are also the fermion zero modes: bound states of fermions at monopole or vortex with exactly zero energy.

The low-energy fermions bound to the vortex core play the main role in the thermodynamics and dynamics of the vortex state in superconductors and Fermi-superfluids. The spectrum of low-energy bound states in the core of the \( N = \pm 1 \) vortex in \( s \)-wave superconductor was obtained in Ref.\[47\]

\[
E_n = \omega_0 \left( n + \frac{1}{2} \right).
\] (10)

This spectrum is two-fold degenerate due to spin degrees of freedom. The integral quantum number \( n \) is related to the angular momentum of the fermions \( n = -NL_z \). The level spacing is small compared to the energy gap of the quasiparticles outside the core, \( \omega_0 \sim \Delta^2/E_F \ll \Delta \), and the discrete nature of the spectrum becomes revealed only at \( T \sim \omega_0 \). There are no quasiparticle states with energies below \( \omega_0/2 \). The latter, however, can appear if the core structure is deformed\[48\] or in some regions of the magnetic field due to Zeeman splitting\[49\] one or several energy levels cross zero as a function of momentum \( p_z \), and one obtains the 1D Fermi liquid(s) living in the vortex core, which can exhibit all the exotic properties of 1D Fermi systems including Peierls instability\[50,49\] and Luttinger liquid physics.\[51\]
N = 1 vortex in 3He-A has also equidistant energy levels but the Maslov index in Bohr-Sommerfeld quantization is 0 instead of 1/2 in Eq.(10):

\[ E_n = \omega_0 n . \]  

The spectrum now contains the state with \( n = 0 \), which has exactly zero energy. This \( E = 0 \) level is doubly degenerate due to spin. Such fermion zero mode is robust to any deformations, which preserve the spin degeneracy.

The most exotic situation will occur for the \( N = 1/2 \) vortex in layered chiral superconductors. As we discussed above, the \( N = 1/2 \) vortex is equivalent to the \( N = 1 \) vortex in only one spin component. That is why each layer of superconductor contains only one energy level with \( E = 0 \). Since the \( E = 0 \) level can be either filled or empty, there is a fractional entropy \((1/2)\ln 2\) per layer per vortex. The factor \((1/2)\) appears because in superconductors the particle excitation coincides with its antiparticle (hole), i.e. the quasiparticle is a Majorana fermion. Majorana fermions at \( E = 0 \) level lead to the non-Abelian statistics of \( N = 1/2 \) vortices: the interchange of two point vortices becomes identical operation (up to an overall phase) only on being repeated four times. This can be used for quantum computing.

Also the spin of the vortex in a chiral superconductor can be fractional, as well as the electric charge per layer per vortex but this is still not conclusive. The problem with the fractional charge, spin and statistics, related to the topological defects in chiral superconductors is still open. This is related also to the problem of the quantization of physical parameters in 2D systems, such as Hall conductivity and spin Hall conductivity.

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