Longitudinal waves in a cylinder with active external friction in a limited area

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Abstract. The problem of longitudinal wave propagation in a cylindrical body in the presence of an incompressible external medium in a length limited area is solved in the paper in a one-dimensional statement. Along with the non-uniformity of motion, the friction force arising on the contact surface directly depends on the parameters of propagating waves. Numerical results are obtained using the finite-difference method according to the Wilkins scheme for axisymmetric problems. The results of wave propagation in a cylindrical body with a rigid external medium frame limited in length are presented. According to the calculation results, an increase in wave parameters (stresses and particle velocity) over time and with distance are observed as well as the causes of stress increase mechanism in the cylinder; the effect of external load parameters and physical-mechanical characteristics of the cylinder on the parameters of propagating waves are determined. Waves arising from active friction and reflected from the boundaries are revealed and the effect of location and length of external medium on changes in wave parameters in the cylinder is revealed.

1. Introduction

In engineering and machine building, shaft-sleeve structures or two-layer in radius rod systems are used. In such structures (systems), the motion of external medium (sleeves, frames, etc.) is possible [1, 2]. Of particular interest are the studies of external medium motion effect on the parameters of the waves propagating in the cylinder under Coulomb friction with external medium. In [1-3], the problems of longitudinal wave propagation in two compressed rods in a one-dimensional statement are considered. In practice, the structures are often used in which an external body is applied on a limited area of the cylinder. Strength calculations of such structures (cylinder) for dynamic loads, in some cases, lead to the problems of wave propagation in an elastic cylindrical rod with a limited area of external friction (i.e., when the friction force acts on some limited in length area of the cylinder’s side surface). Consider the most dangerous case when the external body (frame) moves with a certain velocity in the direction of wave propagation in the cylinder. It should be emphasized that in the considered options of the problem, the velocity of the external body is assumed to be 1 m/s and the process under study has a duration of about 1 ms. During this time, the external medium has a displacement of up to 0.1 sm. Assume that this value of external body displacement does not significantly affect the wave process in the cylinder and neglect it. Otherwise, it is necessary to solve the problem with a variable moving section of external friction.

2. Statement and method of problem solution

Consider a semi-infinite elastic cylinder of radius $R_0$ surrounded by a rough non-deformable body in a
limited area \{x_1; x_2\} along the cylinder length that moves at velocity \(U_{ext}\). The general layout of the cylinder with the medium is shown in figure 1. Note that the effect of medium motion on the cylinder behavior begins only at lateral expansion of the cylinder. Therefore, we are dealing with a cylinder compressed by an external body and pressure-free. The front end of the cylinder \((x=0)\) is hit, the value of this impact load is determined by the relations

\[
\sigma_{xx} = \sigma_{max} H(t-T), \quad \tau_{xx} = 0, \quad \text{at } t > 0.
\]

(1)

Naturally, before the load is applied, the cylinder in question is considered to be at rest and stress-free. From the moment of load application (1), a longitudinal wave propagates through the cylinder along the axis of symmetry. Behind the wave front, longitudinal, radial, and annular stresses arise in the cylinder, respectively, and due to the undeformability of external body on the side surface of the limited area \{x_1; x_2\} a friction force appears, acting counter the direction of relative velocity. We believe that in a limited area \{x_1; x_2\} along the length of cylinder outer surface, the following boundary conditions are true

\[
\tau = \kappa \cdot f \cdot |\sigma_{rr}(x,R_0,t)| \quad \text{and} \quad U_s(x,R_0,t) = 0,
\]

(2)

shear stress (friction force) arises from the amplitude of radial stresses determined in solution; \(\kappa = -\text{Sign}(U_{ext} - U_s(x,R_0,t))\), at \(U_{ext} = U_s(x,R_0,t)\), according to [3], \(\kappa = \pm 1\). In the remaining sections of the cylinder side surface, we accept the stress-free conditions, i.e. \(\tau = 0\) and \(\sigma_{rr} = 0\).

The equations of the cylinder dynamics correspond to the system of equations

\[
\rho \frac{dU_s}{dt} = \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{rr}}{\partial r} + \tau_{xx}, \quad \rho \frac{dU_r}{dr} = \frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\sigma_{rr} - \sigma_{qq}}{r}, \quad \frac{\partial U_s}{\partial x} + \frac{\partial U_r}{\partial r} + \frac{U_s}{r} = \frac{V}{V},
\]

(3)

\[
\dot{\varepsilon}_{xx} = \frac{\partial U_s}{\partial x}, \quad \dot{\varepsilon}_{rr} = \frac{\partial U_r}{\partial r}, \quad \dot{\varepsilon}_{qq} = \frac{\partial U_s}{\partial x} + \frac{\partial U_r}{\partial r}/2.
\]

(4)

where \(V = \rho_0 / \rho\) is the relative volume; \(V = dV/dt\); \(\rho_0\) is the initial density of the medium. The equation of state is considered elastic, i.e., written in the form

\[
\frac{dP}{dt} = -K \frac{\dot{V}}{V}, \quad \frac{dS_{ij}}{dt} = 2G \left[ \dot{\varepsilon}_{ij} - \frac{1}{3} \frac{\dot{V}}{V} \right],
\]

(6)

here the components of the stress tensor are related to the corresponding components of stress deviators by the relationships (6), which have the form:

\[
\sigma_{xx} = S_{xx} - P, \quad \sigma_{rr} = S_{rr} - P, \quad \sigma_{qq} = S_{qq} - P, \quad \tau_{xx} = S_{xx}.
\]

(7)

Thus, the problem under consideration is reduced to solving the systems of equations (3)-(7) with zero initial and boundary conditions (1)-(2). The problem is solved by the finite difference method using the Wilkins scheme for two-dimensional axisymmetric problems. The finite-difference relations of the considered equations are available in [4]. Note that the method [4] is modified for various boundary conditions [5, 6] and successfully applied in [6–8].

3. Results and their analysis

Consider the results of calculations. Initial data are taken as follows: \(\rho_0 = 7800 \text{ kg/m}^3\); \(E = 2.1 \cdot 10^5 \text{ MPa}\); \(v = 0.3\); \(R = 0.2 \text{ m}\); \(\sigma_{max} = 11.163 \text{ MPa}\); \(U_{ext} = 0.5 \text{ m/s}\); \(f = 0.2\). Depending on the place and size of the area location of external body and the duration of acting load, several calculation options are
considered, shown in Table 1.

First, consider the cases when the external medium in the form of a frame is located at the initial section of the cylinder (options 1 and 2). Figures 2-4 show the changes in longitudinal stresses, velocities and particle displacements in time at fixed points of the cylinder

| No | $x_1$ (m) | $x_2$ (m) | $T$ (msec) |
|----|-----------|-----------|------------|
| 1  | 0         | 0.5       | 1          |
| 2  | 0         | 1         | 1          |
| 3  | 1         | 1.5       | 1          |
| 4  | 1         | 2         | 1          |
| 5  | 1         | 2         | 0.2        |
| 6  | 1         | 2         | 0.2        |
| 7  | 0         | 2.5       | 1          |
| 8  | 0         | 2.5       | 0.2        |

Curves 1-4 in these figures correspond to the points $x=0; 0.4$ m; 1.2 m and 2 m. As seen from figure 2, the stresses in cylinder particles also increase, but less intensively than at similar lengths of the cylinder and external medium [8]. Here, the course of curves 1 and 2, located inside the area of external medium, shows an increase in stresses in the cylinder. Further, an increase in stresses is observed behind the friction area (curves 3 and 4, figure 2). This increase in stresses is most likely due to the waves propagating from the boundaries of external body. Indeed, the stresses appear in the cylinder particles with the appearance of waves initiated by load action on the front end of the cylinder. As this wave passes through the friction area, radial displacements appear, and by virtue of the accepted stress-free condition on the side surface of the cylinder, the radial stresses on the side boundary tend to zero. In this regard, together with the front of the incident wave, a wave propagates from the boundaries of external medium along the cylinder. Therefore, in particles outside the area with friction, stresses increase. Naturally the stress increase is less than the one observed in [2,8].

An increase in the length of moving external medium insignificantly affects further increase in longitudinal stress. However, the effect of the wave from the friction boundaries is clearly observed in the case of length extension of external medium (curves 3 and 4, figure 2(b)). Therefore, the resulting “tails” after unloading are a consequence of unloading wave reflection from the initial boundary, where the friction force acts. Note that in the experiment on checking the dry friction law [9], the stress “tails” were obtained during stepwise pulse propagation in a rod. The term “tail” is taken from...
that work.

![Graph](image1)

**Figure 3.** Change in longitudinal velocities in time in the fixed points of cylinder in the presence of an area with friction \{0; 0.5m\} (a) and \{0; 1m\} (b).

![Graph](image2)

**Figure 4.** Change in longitudinal displacements in time in the fixed points of cylinder in the presence of an area with friction \{0; 0.5m\} (a) and \{0; 1m\} (b).

From figures 3-4 it is seen that a small area with active external friction leads to an increase in wave parameters in a cylindrical body. Particle velocities beyond the friction area intensively increase to a certain value, and then, over time, they remain practically constant. In the second option, i.e. when the length of external medium is equal to 1 m, and the particle velocities at the initial sections of the cylinder do not reach the value of external medium velocity, behind the friction area (curves 3, figure 3) the velocities do reach the value of external medium velocity. Calculations showed that the process of particle velocity increase depends on the motion of external medium and other parameters, as well as on the length of friction area, if it is limited. Longitudinal displacements of particles beyond the effect of external medium are delayed by the time of wave arrival (figure 3), and the slope of their change in time for all points after a certain time becomes almost the same. This should have been expected from the behavior of particle velocities over time.

Next, consider the cases when the boundaries of the contact area of the cylinder with an external moving medium are at some distances from the initial section of the cylinder (options 3-4, table 1). For these calculation options, the results are shown in figures 5-7. The curves numbers are the same as in figures 2-4. In these cases, a slight increase in wave parameters is observed behind the friction area (curves 4, figures 5-6). Here the increase in wave parameters is less intense. However, in such cases, the fact of wave reflection from the front boundary of the friction area was discovered. The reflected wave leads to a decrease in longitudinal stress in the particles up to the friction area (curves 2, figure 5). In other words, the reflected wave is a rarefaction wave. Particle velocities in these areas (up to the friction boundary) increase with the arrival of reflected waves.

Similar results, but with inverse properties, were obtained in the experiment in [9], i.e. in case of immobile external medium, stresses increased, and velocities decreased. In the case of external
medium motion in the direction of cylinder motion, behind the front of the propagating waves from the action of an external load on the front end of the cylinder, from the superiority of external medium velocity, the friction force becomes active. These forces draw the cylinder into motion, resulting in an increase in cylinder particles velocity. These forces draw the cylinder into motion, resulting in an increase in cylinder particles velocity. This process expands in the direction opposite to the cylinder motion. The result of an increase in longitudinal velocities of cylinder particles leads to tensile stresses. As a result, there is a decrease in compressive stresses at the initial sections of the cylinder.

![Image of longitudinal stresses](image1.png)

**Figure 5.** Change in longitudinal stresses in time in the fixed points of cylinder in the presence of an area with friction \{1; 1.5 m\} (a) and \{1m; 2m\} (b).

![Image of longitudinal velocities](image2.png)

**Figure 6.** Change in longitudinal velocities in time in the fixed points of cylinder in the presence of an area with friction \{1; 1.5m\} (a) and \{1m; 2m\} (b).

Analyzing the results of solution for the particles inside the friction area (curves 3, figures 5-6), we conclude that the wave arising from the front boundary of the friction area also propagates in the direction of the cylinder motion. As a result, over time, the longitudinal velocities of particles increase and the compressive stresses of the cylinder decrease. The behavior of particles beyond the friction area is similar to the previous calculation options (figures 2-3). The behavior of particle displacements is also qualitatively similar to the ones in figure 4. However, the location of the external medium plays an important role in the changes in cylinder kinematic parameters. If the external medium is located at a certain distance from the initial section, the particle velocities increase with time (figure 3), therefore, the maximum values of longitudinal displacements (figure 7) at the fixed points of time are always less than the displacements in the previous calculation options. The length of the friction area affects the behavior of wave parameters in the cylinder. An increase in external medium length leads to an increase in the amplitude of wave parameters.

Figure 8 shows the change in radial stresses at fixed points along the axis of symmetry of a cylindrical body. The dashed curves correspond to option 1, i.e. friction area\{x_1=0; x_2=0.5m\}, and solid curves to option 2 \{x_1=0; x_2=1m\}. This also applies to figure 8 (b) for options 3 and 4 (the dashed curves correspond to the friction area\{x_1=1; x_2=1.5m\}, and the solid curves to \{-x_1=1; x_2=2m\}). Curves 1-4 correspond to the points \(x = 0; 0.4 \text{ m}; 1.2 \text{ m} \text{ and } 2 \text{ m}\) of the axis of symmetry of
the cylinder. This shows that due to the absence of radial strains, the corresponding prevailing stress values arise in the areas of external medium (curves 1, 2 (a) and curves 3 (b)). Further, behind the friction area, due to the free side surface, the radial stress values decrease (curve 3 and 4 for options 2 and 4, respectively, figure 8). Radial stresses before the friction area for option 4 (curve 2, figure 8 (b)) acquire positive values. This is due to the appearance of tensile longitudinal stresses with wave reflection from the front boundary of the friction area.

**Figure 7.** Change in longitudinal displacements in time in the fixed points of cylinder in the presence of an area with friction {1; 1.5m} (a) and {1m; 2m} (b).

**Figure 8.** Change in cylinder radial stresses with time.

**Figure 9.** Change in friction forces appearing on the side surface of the cylinder with time.

Figure 9 shows the changes in tangential stresses arising on the side surface of the cylinder in the contact area with the external body. Here, the curve numbers correspond to the numbers of calculation options. For the first two options, the dependence \( \tau(t) \) is shown for \( x=0.4 \) m, for the third and fourth options for \( x=1.2 \) m and \( x=2 \) m, respectively. The friction force acting in cylinder motion direction also depends on the location of external medium. The maximum values of the friction force arise when the medium is located in the initial sections of the cylinder. In this case, the wave propagation from the boundaries of external medium also affects the parameters of active friction force, reducing their amplitude.

**Figure 10.** Change in strain energy of the cylinder with time.

In the course of solving the problem, the strain energy of the cylinder was checked. The change in
cylinder energy with time for options 1-6 (curves 1-6, respectively) is shown in figure 10. As seen from figure 10, the action of active external friction on the side surface of the cylinder leads to an increase in the slope of strain energy accumulation. An increase in friction area length leads to an increase in energy, and a decrease in load duration leads to a decrease in strain energy increase and, hence, in the wave parameters. In case of propagation of a wave limited in pulse length, after passing through the friction area, the energy of this wave remains constant over time (curves 5 and 6 of figure 10), i.e. the law of energy conservation is satisfied. The effect of external medium location on the change in strain energy, i.e. wave parameters, can be seen in curves 1-4. Distances from the front end of the cylinder to the boundaries of friction area affects the intensity of energy increase.

Figures 11-13 show the changes in longitudinal stresses, velocities and displacements of particles in time at fixed points x=0; 0.4 m; 1.2 m and 2 m (curves 1-4, respectively) of the axis of symmetry of the cylinder during the stepwise pulse propagation (5th calculation option). In this case, the shape of the propagating pulse is preserved. Note that the smearing of the fronts of loading and unloading waves is a consequence of the lack of a solution method. In this case, an increase in the pulse amplitude is observed in the friction area and beyond this area, then it propagates without attenuation in a linear elastic cylindrical body. This leads to another important conclusion for the problems in which there is a need to transmit the maximum pulse to the other end of the cylinder by using the external medium motion with the minimum transmitted load to one end of the cylinder. The structures of such type are found in mechanical engineering and in other industries.

![Figure 11. Change in longitudinal stresses in time in the fixed points of cylinder.](image1)

![Figure 12. Change in longitudinal velocities in time in the fixed points of cylinder.](image2)

![Figure 13. Change in longitudinal displacements of cylinder particles at stepwise pulse propagation.](image3)

![Figure 14. Change in friction forces in time arising on the side surface of the cylinder.](image4)

Referring again to figures 11-13, note the effect of reflected waves from the ends of the friction area, which leads to the reflection of a triangular shape from the front boundary and in the form of “tails” from the other end (figure 12). The tangential stresses arising in the friction areas in time are
shown in figure 14. Curves 1-2 correspond to the distances \( x = 1.2 \text{ m} \) and \( x = 2 \text{ m} \). Here, the propagation of a slight tangential force from the boundaries of the friction area and a decrease in the force friction on the end section of the moving external medium location are observed.

The lengths of external medium taken in options 1 and 2 are insufficient until the joint motion of the cylinder particles with the external medium. Therefore, the cases are considered where the velocities of the cylinder particles have time to align with the velocity of external medium (variants 7 and 8). For this option from the previous task, we have \( t_s = 0.384 \text{ ms} \) and \( x_s = 2.34 \text{ m} \) for the initial data of option 1 of table 1. Figure 15 shows the changes in longitudinal stresses in time. Curves 1-4 correspond to points \( x = 1.2 \text{ m}; 2 \text{ m}; 3 \text{ m} \) and \( 4 \text{ m} \) of the axis of symmetry of the cylinder.

![Figure 15](image)

**Figure 15.** Change in longitudinal stresses in time in the fixed points of cylinder in the presence of an area with friction \{0; 2.5m\} for option 7 (a) and 8 (b).

A significant increase in the length of external medium leads to the joint motion of cylinder particles with the external medium. A slight decrease is observed in the change of longitudinal stresses of the cylinder particles after the cylinder is released from the external medium. This is most likely due to the wave propagation in the cylinder with free side surfaces and due to a wave propagating from the end boundary of the external medium. The last circumstance, i.e. the reflected wave significantly affects the stepwise pulse propagation (figure 15 (b)).

4. **Conclusion**

The distinctive feature of the considered problem is the fact that, firstly, the friction force values are determined from the wave parameters and stress states behind the wave front, and, secondly, the direction of force action depends on the kinematic parameters of the cylindrical body taken together with the external medium. The results on wave propagation in a cylindrical body with a rigid frame limited in length are presented. From the calculation results, an increase in active friction of the propagating waves in the cylinder in time and distance in the case of interaction with an undeformable external medium is revealed. The mechanism of stress increase in the cylinder is explained and the effect of load parameters and interaction conditions, as well as the physical and mechanical characteristics of the cylinder on the parameters of propagating waves are determined. Waves arising from active friction and reflected from the boundaries are observed. The effect of location and length of the external frame on the change in wave parameters in the cylinder is revealed.

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