Escape of cosmic-ray electrons from supernova remnants

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ABSTRACT

We investigate escape of cosmic ray (CR) electrons from a supernova remnant (SNR) to interstellar space. We show that CR electrons escape in order from high energies to low energies like CR nuclei, while the escape starts later than the beginning of the Sedov phase at an SNR age of $10^3\sim7\times10^3$ yrs and the maximum energy of runaway CR electrons is below the knee about $0.3\sim50$ TeV because unlike CR nuclei, CR electrons lose their energy due to synchrotron radiation. Highest energy CR electrons will be directly probed by AMS-02, CALET, CTA and LHAASO experiments, or have been already detected by H.E.S.S. and MAGIC as a cutoff in the CR electron spectrum. Furthermore, we also calculate the spatial distribution of runaway CR electrons and their radiation spectra around SNRs. Contrary to common belief, maximum-energy photons of synchrotron radiation around 1 keV are emitted by runaway CR electrons which have been caught up by the shock. Inverse Compton scattering by runaway CR electrons can dominate the gamma-ray emission from runaway CR nuclei via pion decay, and both are detectable by CTA and LHAASO as clues to the CR origin and the amplification of magnetic fluctuations around the SNR. We also discuss middle-aged and/or old SNRs as unidentified very-high-energy gamma-ray sources.

Key words: acceleration of particles – cosmic rays – gamma rays – shock waves – supernova remnants.

1 INTRODUCTION

The origin of cosmic rays (CRs) is a longstanding problem in astrophysics. Supernova remnants (SNRs) are thought to be the origin of Galactic CR nuclei and electrons. The most popular SNR acceleration mechanism is the diffusive shock acceleration (Axford et al. 1977; Krymsky 1977; Bell 1978; Blandford & Ostriker 1978). In fact, Fermi and AGILE show that middle-aged SNRs interacting with molecular clouds emit GeV gamma rays (e.g., Abdo et al. 2009; Tavani et al. 2010) and the origin of the GeV gamma rays can be interpreted as the decay of neutral pions produced by CR nuclei (e.g., Ohira et al. 2011). In addition, X-ray observations provide an evidence that electrons are accelerated to highly relativistic energies in SNR shocks (Kovama et al. 1993). SNRs have been also observed by ground-based Cherenkov telescopes (e.g., Muraishi et al. 2000; Aharonian et al. 2006a). However, it is still unclear whether the origin of TeV gamma rays is inverse Compton scattering by CR electrons or the decay of neutral pions produced by inelastic collisions between CR protons and ambient thermal nuclei. In addition, there are many unidentified very-high-energy gamma-ray sources in our Galaxy, and their emission mechanism is also still unclear.

Escape of CR nuclei from SNRs has been investigated by several authors (e.g., Ptuskin & Zirakashvili 2003; Ohira et al. 2010; Caprioli et al. 2010; Ohira & Ioka 2011; Drury 2011), and emission from runaway CR nuclei has also been investigated (Aharonian & Atoyan 1996; Gabici et al. 2009; Ohira et al. 2011, 2012; Ellison & Bykov 2011). Runaway CR nuclei can emit gamma rays from the exterior of accelerators and its radiation spectrum is softer than that from the interior because of energy-dependent diffusion of CRs (Aharonian & Atoyan 1996). The escape process of CRs from accelerators is important because it changes the CR spectrum. The runaway CR spectrum depends not only on the CR spectrum at accelerators but also on the evolution of the maximum energy and the evolution of the number of accelerated CRs (Ohira et al. 2010). In addition, considering the escape effect, one can explain the spectral difference between CR protons and helium observed by CREAM (Ahn et al. 2010; Ohira & Ioka 2011).

However, the same processes for CR electrons suffering cooling effects have never been investigated so far. Cooling
processes via synchrotron radiation and inverse Compton scattering are important for CR electrons because their cooling times can be smaller than other characteristic timescales. Therefore, it is unclear whether CR electrons can escape from SNRs or not, and whether the electron cooling affects the spectrum of runaway CRs electrons or not, as in the energy spectrum of CR electrons inside the SNRs. Runaway CR electrons can produce gamma rays outside accelerators.

In the low density region, these gamma rays probably dominate over those from runaway CR nuclei and may be observed as unidentified very-high-energy gamma-ray sources. This is because CR nuclei can not produce sufficient gamma rays there electrons can always produce enough gamma rays via inverse Compton scattering of CMB photons (see equation (7) of Katz & Waxmann 2008). Furthermore, distant SNRs can not be easily identified by radio observations because of brighter emission from molecular clouds than synchrotron radiation.

Direct measurements of the CR electron spectrum may also bring us important information on escape of CR electrons. So far, Fermi, H.E.S.S. and MAGIC have revealed the spectrum of CR electrons and positrons up to a few TeV (Aharonian et al. 2009; Ackermann et al. 2010; Tridon et al. 2011). Future experiments, such as AMS-02, CALET, CTA, and LHAAO, will measure the spectrum of CR electrons up to 1−100 TeV with good accuracy (Komjai 2010; Torii et al. 2008; CTA consortium 2010; Cao 2011; Kobayashi et al. 2011) pointed out that a few nearby sources like Vela and Cygnus loop may leave their own signatures in the TeV energy band, and that we will be able to see a spectral shape of CR electrons and positrons from a single source. Moreover, Kawanaka et al. (2011) have pointed out that escape of CR electrons can be investigated by future observations of CR electrons, in particular, with a low-energy spectral cutoff.

In this paper, we investigate escape of CR electrons from an SNR. Our stance in this paper is that because the evolution of the magnetic field has not been understood theoretically, we predict observable quantities by using phenomenological approaches and restrict the phenomenological models by comparing the predicted values and observations. Assuming the evolution of the maximum energy of CR protons (hereafter we treat nuclei as protons) during the Sedov phase (Gabici et al. 2009; Ohira et al. 2010), we obtain the evolution of the diffusion coefficient, the magnetic field, and the maximum energy limited by synchrotron cooling (section 2). Moreover, we calculate spatial distributions of runaway CR electrons around the SNR (section 3), and radiation spectra from runaway CR electrons (section 4). Finally, we discuss our results (section 5).

2 ESCAPE OF CR ELECTRONS

2.1 Evolution of SNRs

In this paper, in order to understand essential features of escape of CR electrons, we assume simple evolutions of the shock radius, $R_{sh}(t)$, and the shock velocity, $u_{sh}(t)$, as in the following forms:

$$R_{sh}(t) = R_S \times \left\{ \begin{array}{ll}
\left( \frac{t}{t_S} \right)^{\frac{2}{3}} & (t \leq t_S)
\left( \frac{t}{t_S} \right)^{\frac{1}{3}} & (t_S < t)
\end{array} \right.$$, (1)

$$u_{sh}(t) = \frac{R_S}{t_S} \times \left\{ \begin{array}{ll}
1 & (t \leq t_S)
\left( \frac{t}{t_S} \right)^{\frac{2}{3}} & (t_S < t)
\end{array} \right.$$, (2)

where $t$ is the SNR age, and $R_S$ and $t_S$ are the SNR shock radius and the SNR age at the beginning of the Sedov phase, respectively (see Table 1 for summary). The Sedov phase starts when the swept-up mass, $4\pi n_{ISM}m_pR_S^3/3$, becomes comparable with the ejecta mass $M_{ej}$, where $n_{ISM}$ and $m_p$ are the number density of the interstellar medium (ISM) and the proton mass, respectively. The free expansion velocity, $u_{free} = R_S/t_S$, is obtained from $E_{SN} = M_{ej}u_{free}^2/2$, where $E_{SN}$ is the explosion energy of a supernova. Then, $R_S$ and $t_S$ are represented by

$$R_S = 2.13 \text{ pc} \left( \frac{M_{ej}}{1M_{\odot}} \right)^{\frac{1}{3}} \left( \frac{n_{ISM}}{1 \text{ cm}^{-3}} \right)^{-\frac{1}{3}},$$

$$t_S = 209 \text{ yr} \left( \frac{E_{SN}}{10^{51} \text{ erg}} \right)^{\frac{1}{3}} \left( \frac{M_{ej}}{1M_{\odot}} \right)^{\frac{1}{3}} \left( \frac{n_{ISM}}{1 \text{ cm}^{-3}} \right)^{-\frac{1}{3}}.$$ (3)

For simplicity, we assume $R_S = 2$ pc and $t_S = 200$ yr in this paper.

2.2 Relevant timescales

The maximum energy of accelerated particles is limited by a finite SNR age, their cooling, or escape. Hence it is obtained by comparisons of timescales, which are given as functions of a CR energy, $E$, and the SNR age, $t$, (see Table 1). The acceleration time of DSA, $t_{acc}(E,t)$, is represented by

$$t_{acc}(E,t) = \frac{\eta_{acc}}{u_{sh}(t)} \frac{D(E,t)}{D(E,t) \times u_{sh}(t)^2},$$

where $D(E,t)$ is the diffusion coefficient around the shock, and $\eta_{acc} \approx 10$ is a numerical factor which depends on the shock compression ratio and the spatial dependence of $D(E,t)$ (Drury 1983). First, we assume the Bohm-type diffusion of relativistic particles although the diffusion coefficient is still unclear when the magnetic field is strongly amplified (e.g. Reville et al. 2005). The Bohm-type diffusion means that the mean free path of CRs is proportional to the gyroradius of CRs, where the constant of proportionality, $\eta_k$, is the gyrofactor and $\eta_k = 1$ for the Bohm limit. Then, the diffusion coefficient of CRs is represented by

$$D(E,t) = \eta_k \frac{cE}{3eB(t)},$$

where $c$, $E$, $e$, and $B(t)$ are the velocity of light, the energy of CRs, the elementary charge and the magnetic field strength in the upstream region, respectively. In section 2.3 we will give explicit forms of time dependence of $B(t)$ and $\eta_k(t)$ [see Eqs. (18) and (19)]. Then, the acceleration time can be expressed by

$$t_{acc}(E,t) = \frac{cE}{3eB(t)u_{sh}(t)^2} \eta_{acc} \eta_k(t).$$

The escape time due to diffusion, $t_{esc}(E,t)$, is written by
$t_{\text{esc}}(E, t) = \eta_{\text{esc}} \frac{R_{\text{sh}}(t)^2}{D(E, t)} = \frac{\eta_{\text{esc}} 3eB(t)R_{\text{sh}}(t)^2}{\eta_{b}(t) cE}$. \hspace{1cm} (8)

where $\eta_{\text{esc}} < 1$ is a numerical factor.

The energy loss of CR protons above 1 GeV is due to the pion production by inelastic collisions, and the cooling time of CR protons, $t_{\text{cool, p}}(t)$, is represented by

$$t_{\text{cool, p}}(t) \approx \frac{1}{0.5 n_{\text{pp}} e c^2},$$ \hspace{1cm} (9)

where $n$ and $n_{\text{pp}} \approx 3 \times 10^{-26}$ cm$^{-3}$ are the number density ($\sim 4 \text{cm}^3$ in the downstream region) and the cross section of the nuclear interaction, respectively. A factor of 0.5 is the inelasticity of the nuclear interaction. Cooling of CR electrons above 1 GeV is due to synchrotron emission, inverse Compton scattering, or bremsstrahlung emission. For typical SNRs, the energy loss is dominated by synchrotron emission in the downstream region when the maximum energy is limited by cooling. The cooling time of CR protons due to synchrotron emission in the downstream region, $t_{\text{cool, e}}(E, t)$, is represented by

$$t_{\text{cool, e}}(E, t) = \frac{9m_e c^7}{4e^3 B(t)^2 E},$$ \hspace{1cm} (10)

where $m_e$ is the electron mass and $B(t)$ is the magnetic field strength in the downstream region. In this paper, we assume $B(t) = 4B(t)$ because of the shock compression.

### 2.3 Maximum energy of CR protons

For CR protons, cooling is usually not important for a determination of the maximum energy. One can obtain the age-limited maximum energy, $E_{m, \text{age}}(t)$, from the condition $t_{\text{acc}}(E, t) = t$ as

$$E_{m, \text{age}}(t) = \frac{3eB(t)R_{\text{sh}}(t)^2}{\eta_{\text{acc}} \eta_{b}(t) c^2} \times \left\{ \left( \frac{t}{t_{\text{esc}}} \right)^{-\alpha} \quad \text{for } t \leq t_{S} \right\}, \hspace{1cm} (11)$$

while the escape-limited maximum energy, $E_{m, \text{esc}}(t)$, is obtained from the condition $t_{\text{esc}}(E, t) = t_{\text{cool}}(E, t)$ as

$$E_{m, \text{esc}}(t) = \sqrt{\frac{\eta_{\text{esc}}}{\eta_{\text{acc}}}} E_{m, \text{age}}(t). \hspace{1cm} (12)$$

Note that $E_{m, \text{age}}(t)$ is comparable to $E_{m, \text{esc}}(t)$ because $\sqrt{\frac{\eta_{\text{esc}}}{\eta_{\text{acc}}}}$ is on the order of unity \cite{Drury2011}. In this paper, we assume $E_{m, \text{age}}(t) = E_{m, \text{esc}}(t)$ for simplicity (that is, $\sqrt{\frac{\eta_{\text{esc}}}{\eta_{\text{acc}}}} = 1$). If the maximum energy is limited by escape, CRs above $E_{m, \text{esc}}(t)$ leave the shock front upstream. However, they are caught up by the shock during the free expansion phase because the diffusion length is proportional to $t^{1/2}$ but the shock radius is proportional to $t$. Therefore, the maximum energy is limited by the SNR age during the free expansion phase. On the other hand, CRs can leave the shock front during the Sedov phase because $R_{\text{sh}} \propto t^{2/3}$, which is slower than diffusion. Therefore, the maximum energy is limited by escape during the Sedov phase.

As shown in Equations (11) and (12), the maximum energy depends on the evolution of the magnetic field around the shock \cite{Ptuskin2003}. Although the magnetic field amplification around the shock is studied by linear analyses \cite{Bell2004, Reville2007, Ohira2009b, Ohira2009a, Ohira2014, Bykov2011, Schure2011}, and numerical simulations \cite{Lucek2003, Giacalone2003}, the escape radius of CRs, that is, the SNR radius at which CRs with an energy of $E$ escape, can be expressed by

$$R_{\text{esc}}(E) = R_{S} \left( \frac{E}{E_{\text{knee}}} \right)^{-\frac{1}{2m}}. \hspace{1cm} (17)$$

\hspace{1cm}

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References: Niemiec et al. 2003, Riquelme & Spitkovsky 2003, Inoue et al. 2009, Ohira et al. 2009a, Vladimirov et al. 2009, Gargáte et al. 2010, the saturation of the magnetic-field amplification and the diffusion coefficient have not been understood yet in detail. Here we use a phenomenological approach based on the assumption that young SNRs are responsible for observed CRs below the knee \cite{Gabici2009}. The maximum energy of CR protons, $E_{m, p}(t)$, is expected to increase up to the knee energy $E_{\text{knee}} = 10^{15.5}$ eV until the beginning of the Sedov phase $t_{S}$ and decreases from that epoch. We may assume a functional form of $E_{m, p}(t)$ to be

$$E_{m, p}(t) = E_{\text{knee}} \times \left\{ \left( \frac{t}{t_{S}} \right)^{-\alpha} \quad \text{for } t \leq t_{S} \right\}, \hspace{1cm} (13)$$

where $\alpha$ is a parameter to describe the evolution of the maximum energy during the Sedov phase. In this paper, we assume that $E_{m, p} = 1$ GeV at the end of the Sedov phase $t = t_{S}$ to reproduce Galactic CRs, then $\alpha = 2.6$. Hereafter, we adopt $\alpha = 2.6$. This assumption is consistent with the following fact. So far, about 300 radio SNRs have been observed in a part of our Galaxy \cite{Case1998}. From \cite{Case1998}, we expect that there are about 5 × 300 radio SNRs in our Galaxy. Assuming the supernova rate of 0.03 yr$^{-1}$, the lifetime of radio SNRs is about 5 × 10$^7$ yr which is comparable to the end time of the Sedov phase. Radio synchrotron photons are emitted by electrons with energies of a few GeV, so that we can expect that the lifetime of radio SNRs corresponds to the escape time of 1 GeV particles. Therefore, the assumption, $E_{m, p}(t_{S}) = 10^{2.5}t_{S}$, is reasonable. The transition time from the Sedov phase to the radiative phase is somewhat ambiguous \cite{Truelove1999} and \cite{Petruk2002} thought the end of the Sedov phase is $1.2 \times 10^3$ yr and $3 \times 10^3$ yr, respectively. In these cases, $\alpha$ becomes 3.66 and 2.99, respectively. Future direct observations of CR electrons, AMS-02, CALET, CTA and LHAASO will be able to provide useful informations about $\alpha$ \cite{Kawanaka2011, Thoudam2012}.
2.4 Magnetic field

Equations (11), (12), and (13) determine the evolution of \( B(t)/\eta_g(t) \). During the free-expansion phase, both the upstream magnetic field and the gyrofactor are constant with time: \( B(t < t_s) = B_{\text{free}} \) and \( \eta_g(t < t_s) = \eta_g,\text{free} \), because these quantities may depend on the shock velocity which is constant in this phase. For the Sedov phase, we assume that the upstream magnetic field strength is \( B \propto t^{-\alpha_B} \) as long as it is larger than the value of the ISM, \( B_{\text{ISM}} \), and that after the end time of the magnetic field amplification \( t_B \) at which \( B(t_B) = B_{\text{ISM}} \), \( B \) is equal to \( B_{\text{ISM}} \). Then, the upstream magnetic field, \( B(t) \), and the gyrofactor, \( \eta_g(t) \), are given by

\[
B(t) = \begin{cases} 
B_{\text{free}} & (t \leq t_s) \\
B_{\text{free}} \left( \frac{t}{t_s} \right)^{-\alpha_B} & (t_s \leq t \leq t_B) \\
B_{\text{ISM}} & (t_B \leq t)
\end{cases}
\]

and

\[
\eta_g(t) = \eta_g,\text{free} \times \begin{cases} 
1 & (t \leq t_s) \\
\left( \frac{t}{t_s} \right)^{\alpha - \alpha_B - \frac{3}{2}} & (t_s \leq t \leq t_B) \\
\left( \frac{t}{t_B} \right)^{\alpha - \frac{3}{2}} & (t_B \leq t)
\end{cases}
\]

where \( B_{\text{free}} \) is the amplified magnetic field during the free expansion phase and given by

\[
B_{\text{free}} = \frac{\eta_g,\text{free} \eta_g,\text{acc} C S E_{\text{knee}}}{3eR_S^2} = \frac{174 \mu G \left( \frac{\eta_g,\text{free}}{1} \right) \left( \frac{\eta_g,\text{acc}}{10} \right)}{\left( \frac{E_{\text{knee}}}{10^{15.5} \text{ eV}} \right) \left( \frac{t_s}{200 \text{ yr}} \right) \left( \frac{R_S}{2 \text{ pc}} \right)^{-2}}
\]

and \( \eta_g,\text{free} \approx 1 \) is the gyrofactor during the free expansion phase, and the end time of the magnetic field amplification \( t_B \) is given by

\[
t_B = t_s \left( \frac{B_{\text{free}}}{B_{\text{ISM}}} \right)^{\frac{1}{\alpha_B}}.
\]

The downstream magnetic field is \( B_d = 4B_{\text{free}} = 697 \mu G \) during the free expansion phase. In this paper, we consider the following three evolution models of \( B(t) \) for \( t_s < t < t_B \),

\[
\alpha_B = \begin{cases} 
\alpha - \frac{1}{2} & (\text{for } \eta_g = \eta_g,\text{free}) \\
\frac{\alpha}{2} & (\text{for } B^2 \propto u_{sh}^3) \\
\frac{\alpha}{3} & (\text{for } B^2 \propto u_{sh}^3)
\end{cases}
\]

The first model originates from the assumption that the gyrofactor, \( \eta_g \), is constant during \( B > B_{\text{ISM}} \). In this case, \( B^2 \propto t^{-\alpha} \propto u_{sh}^3 \) for \( \alpha = 2.6 \). The second model is proposed by Bell (2004). The third model originates from the assumption that the pressure of the amplified magnetic field is proportional to the shock ram pressure (e.g. Volk et al. 2005). Observations of the velocity dependence of the magnetic field can be found in Vink (2008) that suggests \( B^2 \propto u_{sh}^3 \).

From Equations (20) - (22), the end time of the magnetic field amplification, \( t_B \), are given by

\[
t_B = \begin{cases} 
1.09 \times 10^3 \text{ yr} & (\text{for } \eta_g = \eta_g,\text{free}) \\
1.82 \times 10^4 \text{ yr} & (\text{for } B^2 \propto u_{sh}^3) \\
1.74 \times 10^5 \text{ yr} & (\text{for } B^2 \propto u_{sh}^3)
\end{cases}
\]

where we assume \( \alpha = 2.6 \) and \( B_{\text{ISM}} = 3 \mu G \). Note that we do not discuss the density dependence of the magnetic field in this paper. Furthermore, there is no guarantee that the magnetic field evolution is powered by a power law form \( \eta_g \propto t^{\alpha-1/2} \), as expected by Ptuskin & Zirakashvili (2003), Yan et al. (2012). However, the Galactic CR spectrum is approximated by a single power law below the knee energy, that is, CR observations suggest absence of characteristic scale. Moreover, the power-law behavior of the gyrofactor, \( \eta_g \propto t^{\alpha-1/2} \), is expected by Ptuskin & Zirakashvili (2003), Yan et al. (2012). Therefore, we assume that all evolutions have power law dependences. We believe that this approximation is useful to extract essential features.
and Equations (18) and (19), we obtain the cooling-limited maximum energy. From the condition at early stages. As the maximum energy increases, synchrotron cooling becomes significant and limits the maximum energy. From the condition t_{acc}(E,t) = t_{cool,\alpha}(E,t) and Equations (18) and (19), we obtain the cooling-limited maximum energy, $E_{m,cool}(t)$, as

$$E_{m,cool}(t) = E_{m,S} \times \begin{cases} \frac{1}{(\frac{t}{t_S})^{2\alpha - \alpha - 1}} & (t < t_S) \\ \left(\frac{1}{\frac{t}{t_S}}\right)^{\alpha B} \left(\frac{1}{\frac{t}{t_S}}\right)^{-\alpha + \beta} & (t_S < t < t_B) \\ \left(\frac{1}{\frac{t}{t_S}}\right)^{\alpha(\beta/\alpha)} & (t_S < t) \end{cases}$$

where $E_{m,S}$ is the cooling-limited maximum energy during the free expansion phase ($t < t_S$) and given by

$$E_{m,S} = \frac{9m_e^2 c^5/2 R_S^2}{8\eta_b \Delta E_{knee} c^2} F^{1/2}_{knee}$$

2.5 Maximum energy of CR electrons

The maximum energy of CR electrons is limited by the SNR age at early stages. As the maximum energy increases, synchrotron cooling becomes significant and limits the maximum energy. From the condition $t_{acc}(E,t) = t_{cool,\alpha}(E,t)$ and Equations (18) and (19), we obtain the cooling-limited maximum energy, $E_{m,cool}(t)$, as

$$E_{m,cool}(t) = E_{m,S} \times \begin{cases} \frac{1}{(\frac{t}{t_S})^{2\alpha - \alpha - 1}} & (t < t_S) \\ \left(\frac{1}{\frac{t}{t_S}}\right)^{\alpha B} \left(\frac{1}{\frac{t}{t_S}}\right)^{-\alpha + \beta} & (t_S < t < t_B) \\ \left(\frac{1}{\frac{t}{t_S}}\right)^{\alpha(\beta/\alpha)} & (t_S < t) \end{cases}$$

where $E_{m,S}$ is the cooling-limited maximum energy during the free expansion phase ($t < t_S$) and given by

$$E_{m,S} = \frac{9m_e^2 c^5/2 R_S^2}{8\eta_b \Delta E_{knee} c^2} F^{1/2}_{knee}$$
\[ E_{m, o}(t) = \begin{cases} E_{m,S}(t) & (t \leq t_c) \\ E_{m,S}(t) \frac{2 \alpha - 1}{2} & (t_c < t \leq t_S) \\ E_{m,S}(t) \frac{2}{1} & (t < t_S) \end{cases} \]

where \( t_c \) is given by
\[ t_c = t_S \left( \frac{E_{m,S}}{E_{knee}} \right)^{\frac{2 - \alpha}{2}} = 1.28 \text{ yr} \left( \frac{\eta_{\text{free}}}{1} \right)^{-1} \left( \frac{\eta_{\text{cool}}}{10} \right)^{-1} \]
\times \left( \frac{E_{knee}}{10^{15.5} \text{ eV}} \right)^{-\frac{2}{1}} \left( \frac{t_S}{200 \text{ yr}} \right)^{-\frac{2}{1}} \left( \frac{R_S}{2 \text{ pc}} \right)^{2} \]  

and \( t_S \) is the start time of escape of CR electrons and given by
\[ t_S = t_S \left( \frac{E_{m,S}}{E_{knee}} \right)^{-\frac{2 - \alpha}{2}} = \left( \frac{9\eta_{\text{free}}^{2}C^{2/3}R_{S}^{3}}{8\eta_{\text{free}}^{2}C^{2/3}E_{knee}^{3/2}} \right)^{-\frac{2 - \alpha}{2}} = \left( \frac{9 \times 10^{2} \text{ yr}}{8 \eta_{\text{free}}^{2}C^{2/3}E_{knee}^{3/2}} \right)^{-\frac{2 - \alpha}{2}} = 9.70 \times 10^{12} \text{ yr} \left( \frac{\eta_{\text{free}}}{\eta_{\text{free}}^{2}} \right) \]
\[ = 3.91 \times 10^{12} \text{ yr} \left( \frac{B^{2} \times u_{sh}^{3}}{u_{sh}^{3}} \right) \]
\[ = 7.39 \times 10^{12} \text{ yr} \left( \frac{B^{2} \times u_{sh}^{3}}{u_{sh}^{3}} \right) \]  

where we assume \( \alpha = 2.6, \) Equation (22) for \( \alpha B \) and normalization of Equation (29).

In Figure 1 we show the schematic picture of the evolution of the maximum energy as a function of time (red dashed line) for \( \eta_{k} = \eta_{\text{free}} \) (see also Table 1). In Figure 2 we show the same figure as Figure 1 but for \( B^{2} \times u_{sh}^{3} \) or \( u_{sh}^{3} \). Initially, the maximum energy increases linearly with time \((t < t_c)\), then the maximum energy is constant until the beginning of the Sedov phase \((t < t_S)\). During \( t_S < t < t_e \), the maximum energy of CR electrons increases with time for \( \eta_{k} = \eta_{\text{free}} \) (Figure 1) and decreases for \( B^{2} \times u_{sh}^{3} \) or \( u_{sh}^{3} \) (Figure 2). Finally, CR electrons start to escape from the SNR at \( t = t_e \). The maximum energy of runaway CR electrons is given by
\[ E_{m,e}(t_e) = \begin{cases} E_{knee} \left( \frac{t_e}{t_S} \right)^{\frac{2}{1}} & (t \leq t_c) \\ E_{m,S} \left( \frac{t_e}{t_S} \right)^{2} & (t_c < t \leq t_S) \\ E_{m,S} \left( \frac{t_e}{t_S} \right)^{2} & (t_S < t \leq t_e) \end{cases} \]

where we assume \( \alpha = 2.6, \) Equation (22) for \( \alpha B \) and normalization of Equation (29). Therefore, SNRs can produce the Galactic CR electrons up to about 0.3 – 50 TeV which depends on the evolution of the magnetic field. Interestingly, the maximum energy of runaway CR electrons becomes smaller as the maximum energy of CR protons \( (E_{knee}) \) becomes larger.

2.5.2 The case of \( t_e > t_B \)
If \( \alpha \) is smaller, \( t_c \) can be larger than \( t_B \). In this case, \( E_{m,e} \) can be expressed by
\[ E_{m,e}(t) = \begin{cases} E_{knee} \left( \frac{t}{t_B} \right) & (t \leq t_c) \\ E_{m,S} \left( \frac{t}{t_B} \right)^{2} & (t_c < t \leq t_S) \\ E_{m,S} \left( \frac{t}{t_B} \right)^{2} & (t_S < t \leq t_e) \end{cases} \]

where we assume \( \alpha > 1, \) otherwise \( E_{m,cool} \) is always smaller than \( E_{m,esc}, \) that is, CR electrons can not escape from the SNR until the SNR shock disappears. The start time of escape of CR electrons, \( t_e, \) for \( t_e > t_B \) is obtained by the condition \( E_{m,esc}(t) = E_{m,cool}(t) \) for \( t_B < t, \)
\[ t_e = t_S \left( \frac{E_{m,S}}{E_{knee}} \right)^{-\frac{2 - \alpha}{2}} = \left( \frac{9 \eta_{\text{free}}^{2}C^{2/3}R_{S}^{3}}{8 \eta_{\text{free}}^{2}C^{2/3}E_{knee}^{3/2}} \right)^{-\frac{2 - \alpha}{2}} \]  

where the evolution of the CR spectrum inside SNRs is necessary to understand those of runaway CRs \( \text{[Ohira et al.} 2010\text{]} \). CR electrons accumulate in SNRs with time, and they cool down due to synchrotron radiation during the accumulation. Hence, the numbers of cooling CR electrons and non-cooling CR electrons are approximately given by \( t_{cool,e}(E)q(E) \propto E^{-\alpha - 1}q(E) \) and \( t_{cool,e}(E)q(E) \), respectively, where \( q(E) \) is an injection spectrum per unit time. Therefore, the CR electron spectrum inside SNRs has a broken power law form (this is, so called, the cooling break). The break energy, \( E_b(t) \), is obtained by \( t_{cool,e}(E_b(t)) = t \). From Equations (10) and (15), the break energy can be represented by
\[ E_b(t) = E_{b,S} \times \begin{cases} \left( \frac{t}{t_S} \right)^{-1} & (t \leq t_S) \\ \left( \frac{t}{t_S} \right)^{2} & (t_S < t \leq t_B) \end{cases} \]

where \( E_{b,S} \) is the break energy at the beginning of the Sedov phase \((t = t_S)\) and given by
\[ E_{b,S} = E_{knee} \left( \frac{E_{m,S}}{E_{knee}} \right)^{2} \]
\[ E_{\text{kin}} = 1.29 \times 10^{11} \text{ eV} \left( \frac{\eta_{\text{free}}}{1} \right)^{-2} \left( \frac{\eta_{\text{acc}}}{10} \right)^{-2} \times \left( \frac{E_{\text{kin}}}{10^{15.2} \text{ eV}} \right)^{-2} \left( \frac{t_S}{200 \text{ yr}} \right)^{-3} \left( \frac{R_s}{2 \text{ pc}} \right)^{4}. \] (34)

In Figures 1 and 2, we show the evolution of the break energy \( E_b(t) \) as a function of time (Cyan line). During \( E_b(t) < E_{m,e}(t) \), i.e., the start of acceleration, the spectrum of CR electrons inside the SNR has the cooling break. Interestingly, the break energy becomes the same as the maximum energy of CR electrons \( E_{m,e} = E_b \) at the end of the injection phase where we consider the\[ t_{\text{cool}}(E, t) \approx t_{\text{cool}, e}(E, t) = t_{\text{cool}, s}(E, t) = t, \]
respectively, and \( t_{\text{cool}, e}(E, t) = t_{\text{cool}, s}(E, t) = t \), respectively (see also Table 1).

The SNR age is approximately the same as the escape time of just escaping CRs \( (t \approx t_{\text{esc}}) \) because of \( E_{m,\text{age}}(t) \approx E_{m,\text{esc}}(t) \) during the Sedov phase. CR electrons start to escape when the cooling-limited maximum energy becomes the same as the escape-limited maximum energy \( E_{m,\text{esc}} = E_{m,\text{esc}} \), that is, the cooling time of CR electrons becomes the same as the escape time \( (t_{\text{cool}, e} \approx t_{\text{esc}}) \). Therefore, the SNR age is approximately the same as the cooling time \( (t \approx t_{\text{cool}, e}) \) at \( t_c \). This is the condition to derive the break energy. Hence, all the characteristic energies have the same energy, \( E_{m,\text{esc}}(t_c) \), at \( t_c \).

Namely, the spectrum of CR electrons inside the SNR has a cooling break before CR electrons escape \( (t < t_c) \). However, it does not appear while CR electrons escape \( (t > t_c) \).

2.7 Spectrum of runaway CR electrons

As mentioned above, the spectrum of runaway CR electrons is unaffected by synchrotron cooling below \( E_{m}(t_c) \). Moreover, the spectrum of runaway CR electrons does not depend on the magnetic field evolution except for the maximum energy of runaway CR electrons.

According to \( \text{Ohira et al. (2010), Ohira \\& Ioka (2011)}, \) the energy spectrum of runaway CR electrons, \( dN_{\text{esc}}/dE \), can be expressed by

\[ dN_{\text{esc}} = AE^{-(s+2)}, \]

where \( A \) is the normalization factor, \( s \approx 2 \) is the index of the energy spectrum of CR electrons inside the SNR and \( \beta \) is a parameter to describe the evolution of the number of CR electrons inside the SNR \( (dN/dE \propto t^\beta E^{-s}) \) where \( dN/dE \) is the CR spectrum inside the SNR. Lower-energy CRs can be produced for a longer time than high-energy CRs, so that the time-integrated spectrum becomes softer than the instantaneous one. Note that \( \beta \) is not understood well for CR electrons and protons because their injection processes for particle acceleration have not been understood well. In this paper, we assume that CR electrons and protons have the same value of \( \beta = 0.6 \) during the Sedov phase and \( \beta = 3 \) during the free expansion phase where we consider the thermal leakage model as the injection model \( \text{Ohira et al. (2011)} \). For \( s = 2.0, \alpha = 2.6 \) and \( \beta = 0.6 \), the spectral index of runaway CRs is \( s + \beta/\alpha = 2.23 \), which is consistent with the index expected as the source of Galactic CRs \( \text{Ohira et al. (2011)} \).

3 SPATIAL DISTRIBUTION OF RUNAWAY CR ELECTRONS AROUND AN SNR

In this section, we derive the distribution of runaway CR electrons at a given distance \( r \) from the SNR center and at a given time \( t \) from the explosion time, \( f(t, r, E) \ [\text{particles cm}^{-3} \text{ eV}^{-1}] \). CRs propagate around SNRs after escaping from the SNRs, so that CR halos are produced around the SNRs. Suppose that the system is spherically symmetric and the diffusion coefficient in the ISM, \( D_{\text{ISM}}(E) \), is spatially uniform. Then, we solve the diffusion equation given by

\[ \frac{\partial}{\partial t} f - D_{\text{ISM}}(E) \Delta f + \frac{\partial}{\partial E}(P(E)f) = q_s(t, E, r), \]

where \( P(E) \) and \( q_s(t, E, r) \) are the energy loss rate of CR electrons and the source term of CR electrons, respectively. The escape process \( \text{Ptuskin \\& Zirakashvili (2003), Ohira et al. (2010)} \), CRs with an energy of \( E \) escape from an SNR at \( t = T_{\text{esc}}(E) \). Let \( dN_{\text{esc}}/dE \) be the total spectrum of runaway CR electrons, that is,

\[ dN_{\text{esc}} = \int dt \int d^3r \ q_s(t, r, E) = AE^{-(s+2)}. \]

Then, the Green function, which is the solution in the case of an instantaneous point source, \( q_s(t, E, r) = \delta(r)\delta(t - T_{\text{esc}}(E))dN_{\text{esc}}/dE \), is \( \text{Atovian et al. (1995)} \)

\[ f_{\text{point}}(r, E) = \frac{e^{-\left( T_{\text{esc}}(E) r^2 \right)/D_{\text{diff}}(E) \Delta f + P(E) f}}{\pi^{3/2} R_{\text{diff}}(E) \int dE \ \int dE \ N_{\text{esc}}(E) \ P(E) \ f(E)}, \]

where \( E_0(t, E) \) is the initial energy defined by

\[ \int_E E_0(t, E) \ \frac{dE}{|P(E)|} = t - T_{\text{esc}}(E_0), \]

and the diffusion length while CR electrons cool from \( E_0 \) to \( E \) is
The diffusion coefficient around an SNR has not been understood well so far. Although there is no guarantee that the diffusion coefficient has a power law form, $D_{\text{ISM}} \propto E^\delta$ (e.g. Fujita et al. 2010, 2011), such a diffusion coefficient can explain gamma-ray spectra of middle-aged SNRs observed by Fermi (e.g. Ohira et al. 2011, Li & Chen 2011). Therefore, we assume the diffusion coefficient to be

$$D_{\text{ISM}}(E) = 10^{28} \chi \left(\frac{E}{10 \text{ GeV}}\right)^6 \text{ cm}^2 \text{ s}^{-1}. \quad (41)$$

The values of $\delta$ and $\chi$ are uncertain. The Galactic mean values are $\delta \approx 0.3 - 0.6$ and $\chi \approx 1$ [Berezinskii et al. 1990]. However, $\chi$ is expected to be much smaller than unity ($\chi \approx 0.01$) around SNRs because CRs amplify magnetic fluctuations around SNRs [Fujita et al. 2003, 2010, 2011; Giuliani et al. 2010; Torres et al. 2010; Li & Chen 2010]. We find that the effect of finite source size is important. To see this, we show, in Figure 4, the diffusion length $R_{\text{diff}}(E)$ (color lines) and the escape radius $R_{\text{esc}}(E)$ (black solid line); the former is obtained from Equations (39), (40) and (41), while the latter is obtained from Equation (17), where we consider synchrotron cooling with $B = 3 \mu\text{G}$ and inverse Compton scattering with the Galactic radiation field provided by the 8 kpc model of [Porter et al. 2008]. We fully consider the Klein-Nishina effect for inverse Compton scattering [Blumenthal & Gould 1970]. As shown in Figure 4, $R_{\text{diff}}(E)$ has low- and high-energy cutoffs. The lower cutoff corresponds to the energy of just escaping CR electrons at the SNR age, $t = t_{\text{esc}}(E_0)$. The higher cutoff shows the maximum energy of runaway CR electrons at the SNR age $t$. It is found that for $t = 10^4$ yr and $\chi = 0.01$ (red solid line), CR electrons above 50 TeV escape from the SNR and the diffusion length of runaway CR electrons is smaller than the escape radius. Similarly, for $t = 10^7$ yr and $\chi = 0.01$ (green solid line), CR electrons above $\sim 100$ GeV escape from the SNR and the diffusion length of runaway CR electrons with a few TeV is comparable to the escape radius. Therefore, the point source approximation is not well, that is, the finiteness of the source size is important. We also remind that even if SNRs produce CR electrons with 100 TeV, the SNRs should be younger than about $10^4$ yr (cooling time) and locate within about 500 pc from the Earth to observe the CR electrons directly.

Now, taking into account the fact that CRs escape from the SNR surface [Ohira et al. 2010], the source term is replaced with

$$q_s = \frac{1}{4\pi^2} \frac{dN_{\text{esc}}}{dE} \delta(r - R_{\text{esc}}(E)) \delta(t - t_{\text{esc}}(E)),$$ \quad (42)

where $R_{\text{esc}}(E)$ is the radius where the CRs with an energy of $E$ escape from the SNR. Then, we find the solution to equation (39) as [Ohira et al. 2011]

$$f_{\text{ext}}(t, r, E) = \frac{e^{-\left(\frac{r - R_{\text{esc}}(E)}{R_{\text{diff}}(t, E)}\right)^2} - e^{-\left(\frac{r + R_{\text{esc}}(E)}{R_{\text{diff}}(t, E)}\right)^2}}{4\pi/3 R_{\text{diff}}(t, E) R_{\text{esc}}(E_0) r} \times \frac{P(E_0) dN_{\text{esc}}(E_0)}{P(E)}$$ \quad (43)

The finiteness of the source size is important in the vicinity of the escape radius ($r \approx R_{\text{esc}}$) for $R_{\text{esc}} \gg R_{\text{diff}}$. This means $r - R_{\text{esc}} \ll R_{\text{diff}}$ and $r + R_{\text{esc}} \gg R_{\text{diff}}$, so that $f_{\text{ext}}$ is approximately given by [Ohira et al. 2011]

$$f_{\text{ext}}(t, r, E) \approx \frac{1}{4\pi^{3/2} R_{\text{diff}}(t, E) R_{\text{esc}}(E_0) r} \frac{P(E_0) dN_{\text{esc}}(E_0)}{P(E)}.$$ \quad (44)

4 RADIATION SPECTRA DUE TO RUNAWAY CR ELECTRONS FROM SNRS

In this section, radiation spectra from runaway CR electrons are calculated. They should be calculated using their volume-integrated distribution function. At first, we consider the energy spectrum of runaway CR electrons contributing to the radiation spectrum, $dN_{\text{pro}}/dE$. Emission far outside the SNR is too dim to be detected because the number of emitting electrons is too small. Taking into account an observation sensitivity, we should consider the projected distance $\rho_{\text{size}}$ from the SNR center on the sky. We adopt $\rho_{\text{size}} = 1.2 R_{\text{sh}}$. Then, we obtain

$$\frac{dN_{\text{pro}}}{dE} = \int_0^{\rho_{\text{size}}} dp \int_{-\infty}^{+\infty} dz \ f_{\text{ext}}(t, \sqrt{z^2 + p^2}, E).$$ \quad (45)

where the $z$-direction is along the line of sight. In the energy region satisfying the condition $\rho \ll R_{\text{diff}}(t, E)$, the $z$-integration becomes approximately $f_{\text{point}} R_{\text{diff}}$. Then,
Figure 5. Time evolution of radiation spectra from the SNR ($0 \leq \rho \leq \rho_{\text{size}} = 1.2R_{\text{sh}}$) for $d = 3$ kpc, $U_{\text{CR, e}} = 10^{48}$ ergs, $\chi = 0.01$ and $\delta = 0.6$. The thin and thick color lines show total synchrotron spectra and total inverse Compton spectra, respectively. The red, green, and blue lines correspond to spectra at the age of $10^3$, $10^4$, and $10^5$ yr, respectively. The thick and thin black lines show CTA 50 h (CTA consortium 2014) and LHAASO (where the MAGIC-II-type telescopes improves the low energy LHAASO sensitivity) integral sensitivities of point sources, respectively.

Figure 6. The same as Figure 5 but for $\delta = 0.3$.

Figure 7. The same as Figure 5 but for $\chi = 1$.

dN_{\text{proj}}/dE can be written by
\[
dN_{\text{proj}}/dE \propto \left( \frac{\rho_{\text{size}}}{R_{\text{sh}}(t, E)} \right)^2 \frac{dN_{\text{nsc}}}{dE} \propto \chi^{-1} E^{-(\delta + 1/2)},
\]
where radiative losses are neglected.

We further divide runaway CR electrons into three groups as in the following:

- The energy spectrum of runaway CR electrons outside the SNR projected on the sky ($R_{\text{sh}} < \rho \leq \rho_{\text{size}}$), $dN_{\text{proj, out}}/dE$, is obtained by
\[
dN_{\text{proj, out}}/dE = \int_{R_{\text{sh}}}^{R_{\text{size}}} \frac{dN_{\text{proj}}}{dE} \propto 2\pi \rho \int_{-\infty}^{\infty} dz f_{\text{ext}}(t, \sqrt{z^2 + \rho^2}, E),
\]

- Some of runaway CR electrons are caught up by the SNR shock because the SNR shock expands in the halo of runaway CR electrons. Note that most runaway CR electrons are outside the SNR because the diffusion length increases with time faster than the SNR shock. The energy spectrum of runaway CR electrons which have been caught up by the shock, $dN_{\text{caught}}/dE$, is obtained by
\[
dN_{\text{caught}}/dE = \int_0^{R_{\text{sh}}} dr 4\pi r^2 f_{\text{ext}}(t, r, E).
\]

- The energy spectrum of runaway CR electrons which have not been caught up by the shock but are inside the SNR projected on the sky ($\rho \leq R_{\text{sh}}$), $dN_{\text{proj, in}}/dE$, is obtained by
\[
dN_{\text{proj, in}}/dE = \frac{dN_{\text{proj}}}{dE} - \frac{dN_{\text{proj, out}}}{dE} - \frac{dN_{\text{caught}}}{dE}.
\]

Finally, we consider trapped CR electrons. CR electrons with an energy smaller than the escape-limited maximum energy $E_{\text{m, esc}}(t)$ are still trapped in the SNR. Their energy spectrum for $t > t_e$, $dN_{\text{trap}}/dE$ is given by
\[
dN_{\text{trap}}/dE = A(E_{\text{m, esc}}(t)^{-\frac{s+\frac{\beta}{2}}{s}} \left( \frac{E}{E_{\text{m, esc}}(t)} \right)^{-s}.
\]

In order to calculate each energy spectrum described above, several model parameters in section 2 and 3 are fixed. Throughout this section, we only consider the model of $\eta_6 = \eta_{6, \text{free}}$ for the magnetic field evolution, that is, the start time of escape of CR electrons is $t_e = 9.70 \times 10^2$ yr. In our formulation shown in Section 2, the spectral index of runaway CR electrons does not depend on models of the magnetic field evolution, while the maximum energy of runaway CR electrons, $E_{\text{m, esc}}$, and the start time of escape of CR electrons, $t_e$, do depend. We adopt $R_S = 2$ pc, $\ell_S = 200$ yr, $B_{\text{ISM}} = 3 \mu$G, $n_{\text{d, ISM}} = 10$, $s = 2.0$, $\alpha = 2.6$, $\beta = 0.6$, $d = 3$ kpc, $n_{\text{ISM}} = 1$ cm$^{-3}$, and $U_{\text{CR, e}} = 10^{48}$ ergs, where $d$, $n_{\text{ISM}}$, and $U_{\text{CR, e}}$ are the source distance, the number density of the ISM, and the total energy of runaway CR electrons from 1 GeV to $E_{\text{m, esc}}$, respectively. We calculate inverse Compton scattering with the Galactic radiation field provided by the 8 kpc model of Porter et al. (2008) and synchrotron radiation. We fully consider the Klein-Nishina effect for inverse Compton scattering [Blumenthal & Gould 1977]. We use the downstream magnetic field $B_0 = 4B$ for synchrotron radiation emitted inside the SNR.
For given distributions of CR electrons (Equations (47–50)), radiation spectra are calculated. Figure 3 shows the spectral components of four CR electron groups given by Equations (47–50), where \( t = 10^4 \) yr (\( t_{\text{cr}} \)), \( \chi = 0.01 \), and \( \delta = 0.6 \) (\( \chi \) and \( \delta \) are the normalization factor and the energy dependence of the diffusion coefficient: see Equation (31)). We also show in the figure the integral sensitivities of point sources of the future TeV gamma-ray telescopes, CTA 50 h (CTA consortium 2010) and LHAASO (Cao 2010) (black lines). The thin and thick lines show the radiation spectra coming from the interior of the SNR region projected onto the sky (that is \( \rho < R_{\text{sh}} \)) and from the exterior of the SNR \( (R_{\text{sh}} < \rho < \rho_{\text{size}} = 1.2R_{\text{sh}}) \) (Equation (41)), respectively. The thick-solid lines show spectra emitted by trapped CRs which have not escaped from the SNR yet (Equation (50)). The long and short-dashed lines show the spectra emitted by runaway CR electrons which have been caught up by the shock (Equation (48)) and have not been caught up by the shock (Equation (51)), respectively.

One can see from Figure 3 that maximum-energy photons of synchrotron radiation are emitted by runaway CR electrons which have been caught up by the shock. This is completely different from the standard picture that maximum energy synchrotron photons are produced by CR electrons accelerating at the SNR shock. Note that this new picture can be applied after CR electrons start to escape from SNRs later than CR electrons depends on the normalization of the diffusion coefficient, \( \chi \). As shown in Equation (46), the flux of inverse Compton scattering is large for small \( \chi \). Therefore, if \( \chi < 0.1 \), radiation due to runaway CRs from middle-aged and old SNRs can be potentially observed by CTA and LHAASO. Similarly, as shown in Equation (51), the flux of synchrotron radiation from caught-up CR electrons is large for small \( \chi \). It should be noted that if the evolution of the magnetic field is \( B^2 \propto u_{\text{sh}} \) or \( u_{\text{sh}}^2 \), the maximum energy of caught-up CR electrons is too small to produce synchrotron X-ray photons (Equation (59)). However, if the magnetic field at the SNR shock is suddenly amplified by interactions of molecular clouds (Inoue et al. 2009), X-ray photons could be produced by synchrotron radiation from caught-up CR electrons.

![Figure 8. Time evolution of gamma-ray spectra from the SNR](image)

Figure 8. Time evolution of gamma-ray spectra from the SNR (with the projected distance \( \rho_{\text{size}} = 1.2R_{\text{sh}} \) for \( U_{\text{CR, e}} = 10^{48} \) erg \( n_{\text{ISM}} = 1 \) cm\(^{-3} \), \( \chi = 0.01 \), \( \delta = 0.3 \), and other parameters are the same as Figure 3) The thick and thin lines show the total inverse Compton spectra and the gamma rays (\( \pi^0 \to 2\gamma \)) from runaway CR protons, respectively. The red, green, and blue lines correspond to spectra at the age of \( 10^3 \), \( 10^4 \), and \( 10^5 \) yr, respectively. The thick and thin black lines show CTA 50 h (CTA consortium 2010) and LHAASO (Cao 2010) sensitivities, respectively.

### 5 DISCUSSION

In this paper, taking into account escape of CR electrons from SNRs, we have investigated the evolution of the maximum energy of CR electrons, as well as that of the magnetic field and the maximum energy of CR protons. We found the following:

- CR electrons start to escape from SNRs later than CR nuclei (Equation (29)).
- SNRs can produce CR electrons up to about 0.3 – 50 TeV, which depends on the evolution of the magnetic field (Equation (30)).
- If the magnetic field decays slowly, the maximum energy of runaway CR electrons becomes small (Equation (31)).
• The maximum energy of CR electrons is anticorrelated with that of CR nuclei (Equation 29).
• The spectrum of runaway CR electrons is unaffected by the cooling break of synchrotron radiation (Section 2.3).

In addition, we calculated the spatial distribution of runaway CR electrons around SNRs. Using this distribution, we calculated synchrotron radiation and inverse Compton scattering from the runaway CR electrons around SNRs. We found the followings:

• Inverse Compton radiation of the runaway CR electrons can be observed by future ground-based Cherenkov telescopes, such as CTA and LHAASO. Because the cosmic microwave background is inevitably present, CTA and LHAASO can impose tight constraint on the amount of CR electrons.
• Maximum-energy photons of synchrotron radiation are emitted by runaway CR electrons which have been caught up by the shock.
• Radiation spectra from runaway CR electrons depend on the diffusion coefficient of ISM.

Although there are many parameters in determining the evolution of the maximum energy \( B_\text{ISM}, E_{\text{knee}}, n_{\text{esc}} \), \( \eta_{\text{free}}, \eta_{\text{free}}, \alpha \) and \( \eta_B \); see Table 1), all except \( \eta_B \) have typical or appropriate values. The value of \( \eta_B \), which is a parameter to describe the evolution of the magnetic field, is highly uncertain. We have derived some relations for the general \( \eta_B \) and especially discussed three cases. We have assumed that the end of the Sedov phase is 200 yr \( \times 10^{2.5} \) \( \sim 30 \) kyr. On the other hand, Truelove & McKee (1999) and Petruk (2003) thought that it is 12 kyr and 30 kyr, respectively. Even when we take these values, 12 or 30 kyr, our results do not change significantly. In this paper, we have considered only one fiducial model of SNRs \( (t_S = 200 \text{ yr} \quad \text{and} \quad R_S = 2 \text{ pc}) \). It is also an interesting future work to middle-aged and/or old SNRs.

We consider three evolution models of the magnetic field in Section 2. Different models predict different evolutions of the maximum energy limited by synchrotron cooling during \( t_S < t < \min \{ t_e, t_b \} \). Maximum photon energies of synchrotron radiation and inverse Compton scattering depend on evolution models of the magnetic field. Furthermore, the maximum energy of runaway CR electrons also depends on the evolution model of the magnetic field \( (0.3 \sim 50 \text{ TeV}) \). Figures 6 and 7 show that CTA and LHAASO can observe the maximum value. Hence, CTA and LHAASO can provide constraints on evolution models of the magnetic field. To make a more accurate radiation spectrum, we should consider escaping and trapped CR spectra in detail (Caprioli et al. 2009; Reville et al. 2009) and solve SNR dynamics and the diffusion-convection equation (Zirakashvili & Aharonian 2010).

Fermi and H.E.S.S. show the spectral break or the cut off of the CR electron spectrum at around 1 TeV. The magnetic field evolution model of \( B^2 \alpha \nu_h^3 \) could explain the break as the maximum energy of CR electrons that can escape from SNRs (see Equation 30). This model is suggested by other observations (Vink 2008) and by Bell (2004).

A local source may contribute to high energy CR electrons and the maximum energy of CR electrons may be limited by cooling during the propagation (e.g. Kobayashi et al. 2004, Thoudam & Hörandel 2012). Even so, we can rule out any models that predict a smaller maximum energy of CR electrons than that of observed CR electrons. A more tight constraint on the magnetic field evolution will be given by future observations of the CR electron spectrum, AMS-02 and CALET. To make a more realistic prediction, we should perform time-dependent fluid simulations (Schure et al. 2014) and consider the CR back reaction (e.g. Drury et al. 1989, Ellison et al. 1990, Berezhko & Völk 1997, Blasi 2002, Kang et al. 2002).

In this paper, we have focused on escape of accelerated electrons. SNRs produce CR protons as well as CR electrons, so that the hadronic gamma-ray radiation due to the neutral pion decay should be compared to the leptonic gamma rays. In figure 8 we show an example of the gamma-ray spectrum. We adopt \( U_{\text{CR, p}} = 10^{20} \text{ erg, } n_{\text{ISM}} = 1 \text{ cm}^{-3}, \chi = 0.01, \delta = 0.3, \) and other parameters are the same as in the previous section. We use the code provided by Kamae et al. (2006), Karlsson & Kamae (2008) to obtain gamma-ray spectra from CR protons. Note that if the ratio of CR electrons to CR protons is \( U_{\text{CR, e}}/U_{\text{CR, p}} = 10^{-2} \) and the number density of the ISM is \( n_{\text{ISM}} = 1 \text{ cm}^{-3} \), inverse Compton scattering due to runaway CR electrons dominates over the hadronic component at the TeV energy region. The maximum photon energy and its flux decrease with time for synchrotron radiation and inverse Compton scattering because of cooling and diffusion. On the other hand, the maximum energy of gamma rays from CR protons does not change because the cooling time of CR protons is longer than the SNR age. Therefore, radiation from CR protons can dominate over that from CR electrons at the 100 TeV energy region for middle-aged and/or old SNRs.

We comment on a possible origin of unidentified very-high-energy gamma-ray sources (so called, TeV-uniDs) discovered by H.E.S.S. (Aharonian et al. 2003, 2006a, 2008). Figures 3 and 4 tell us that old SNRs \( (t \sim 10^7 \text{ yr}) \) could be the origin of TeV-uniDs. These figures show that inverse Compton scattering from runaway CR electrons can be observed in the TeV region, but synchrotron radiation cannot produce X-ray photons for old SNRs. The maximum energy of runaway CR electrons decreases with time owing to synchrotron and inverse Compton cooling and the maximum energy is a few TeV at \( t = 10^7 \text{ yr} \) (see also Yamazaki et al. 2008, Ioka & Mészáros 2010). Electrons with energies of a few TeV can produce gamma-ray power by inverse Compton scattering with optical photons, but cannot produce X-ray photons by synchrotron radiation with the magnetic field of \( 3 \mu \text{G} \). This is the same reason why pulsar wind nebulae (PWNe) are plausible candidates for TeV-uniDs (de Jager 2008). Radiation from runaway CR electrons depends on the

\[ \text{Note that in Figure 5 the softening around 30 TeV of gamma rays from CR protons is due to the limitation of the code provided by Kamae et al. (2006), Karlsson & Kamae (2008). The code calculates gamma-ray spectra from CR protons with energies up to 512 TeV.} \]
number of CR electrons, the diffusion coefficient around the SNR, the SNR age, and the source distance. For example, if the value of $\chi$ is larger than 0.1, the flux of inverse Compton scattering from runaway CR electrons around an SNR older than $10^7$ yrs with the distance 3 kpc is smaller than the sensitivity of H.E.S.S., so that the number of currently detected TeV unIDs is not so large. However, even in this case, we expect that future observations by CTA will increase the number.

In addition, we comment on escape of CR electrons from PWNe. Kawanaka et al. (2011) investigated CR electron spectra from a young pulsar embedded in the SNR, but did not take into account synchrotron cooling while CR electrons cross the SNR shell. If the diffusion escape time from the SNR is longer than the cooling time, this depends on the evolution of the magnetic field. Kawanaka et al. (2011) investigated CR electrons escaping from nearby sources could be observed in the above TeV band. Therefore, Kawanaka et al. (2011) is complementary to this paper that investigated the high energy cutoff.

Finally, we emphasize that escape of accelerated particles is common in other high energy particle accelerators (e.g. microquasars, gamma ray bursts and active galactic nuclei), so that radiation of runaway accelerated particles is common in other high energy particle accelerators.

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