Quantum Dissipation in a Neutrino System Propagating in Vacuum and in Matter

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Considering the neutrinos like an open quantum state, we analyse the evolution of this state when it is propagating in vacuum or in matter. Complete positivity constraint gives us all of the most effective quantum dissipators and from them we can study decoherence and relaxation effects. We show how decoherence effects can be studied with or without matter effects and, in our approach, avoid ambiguities that arise due to some specific supposition about the quantum dissipative medium. We conclude this analysis discussing an adiabatic approximation in this approach. We show that when the neutrino source is far away from its detection, only relaxation effects rather than decoherence effects can act in neutrino oscillations.

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I. INTRODUCTION

We present a study about decoherence and relaxation effects in neutrino oscillations. These effects are obtained when we consider neutrinos like an open quantum system\textsuperscript{1,2}. In this approach, neutrinos are treated as a subsystem that is free to interact with the quantum environment. The neutrinos together with quantum environment, that is considered as reservoir, form a global state.

In general terms, we carefully apply the open quantum system theory to neutrino oscillations\textsuperscript{4,5}. From the main statements of this approach, we briefly review how the Lindblad Master Equation arises naturally from usual quantum mechanics\textsuperscript{6,7}. Using this master equation, we can evolve neutrinos including phenomenological dissipative effects by means of a quantum dissipator. Each kind of quantum dissipator includes one or more dissipative effects in the interest subsystem. We emphasize that the quantum environment can always be the same. However, the interactions between it and the interest subsystem are not always the same and depend only on how we define the quantum dissipator.

Decoherence is the most usual dissipative effect, but here we will discuss decoherence and relaxation effects as well. In the neutrino oscillation phenomenon, the decoherence effect acts on the quantum interference which is responsible by neutrino oscillation that changes the neutrino flavor. On the other hand, the relaxation effect changes the oscillation behavior in a different way and even when there is not coherence it is still possible to change the neutrino flavor. We observe that the pure initial state is converted into a ratio of $1/n$ where $n$ is the number of families when decoherence and relaxation effect act together on the interest subsystem\textsuperscript{8}. We emphasize that relaxation effects can seem to be decoherence effects and we must be careful in our interpretation.

Sometimes, the composition between the interest subsystem and the environment can make the dynamic equation very complicated to be solved\textsuperscript{2,6,8}. In general, this occurs when the matter potential is taken into account. In this work we show that the dynamic equation is considerably simplified if we use the effective mass basis to solve it and, as is well known, the dynamic equation in vacuum and in constant matter can have the same form. We also study the neutrino propagation in varying matter density in the case where the adiabatic limit can be satisfied. We show that decoherence effect cannot happen and only the relaxation effects can occur in this limit.

We finished this work showing that the decoherence limit in the channel $\nu_e \rightarrow \nu_\mu$ can be different from the limit obtained in Ref.\textsuperscript{8} and an unambiguous limit to decoherence could be obtained from other sources.

II. NEUTRINOS LIKE OPEN QUANTUM SYSTEM

In open quantum systems approach, we must define a global state formed by an interest subsystem and an environment. The interest subsystem can be represented by $S$ states which are associated with the Hilbert space $\mathbb{H}_S$, while the environment can be represented by $R$ states which are associated with the Hilbert space $\mathbb{H}_R$. The product tensor from these spaces form the total Hilbert space $\mathbb{H}_G = \mathbb{H}_S \otimes \mathbb{H}_R$. In
other words, we can write a global state as
\[ \rho_G = \rho_S \otimes \omega_R, \] (1)

where \( \rho_S \) is the interest subsystem and \( \omega_R \) is the reservoir state. The evolution of this system is obtained using the following transformation:
\[ \rho_G(t) = U(\rho_S \otimes \omega_R)U^\dagger, \] (2)
such that \( U = \exp[-iH_{\text{tot}}t] \) is the unitary operator and the time evolution is governed by the total Hamiltonian that can be defined as \( H_{\text{tot}} = H_S + H_R + H_{\text{int}} \), where \( H_{\text{int}} \) is the interact Hamiltonian.

Thus, the interest subsystem changes in time due to its internal dynamic as well as the interaction with the environment [4, 5]. On the other hand, the environment state does not change in time, because it is considered a reservoir and its dynamics is not interesting. Then, the interest subsystem dynamics is obtained taking the trace over the environment states in Eq. (2) [4, 5, 12], i.e.,
\[ \rho_S(0) \rightarrow \rho_s(t) = \Lambda \rho_s(0) = \text{Tr}_R U(\rho_S \otimes \omega_R)U^\dagger. \] (3)

The Eq. (3) is known as dynamic reduced of \( S \), where \( \Lambda \) is a dynamic map. Solving the partial trace in the Eq. (3), we can rewrite this transformation as
\[ \Lambda \rho_S(0) = \sum_{\alpha} W_\alpha \rho_S W_\alpha^\dagger, \] (4)

where \( W_\alpha \in \mathbb{H}_S \) and \( \sum_\alpha W_\alpha W_\alpha^\dagger = 1 \) [12]. In order to evolve the state, this map must satisfy complete positivity constraint. Besides, we need a family of linear maps that satisfies the semigroup properties [7, 10, 12] and thus, we can obtain a dynamical generator, which can be written as
\[ \frac{d\rho_\nu(t)}{dt} = -i[H_S, \rho_\nu(t)] + D[\rho_\nu(t)]. \] (5)

This equation has been studied in literature and more information about it and its proprieties can be found in Refs. [4, 5, 7, 10, 12]. This equation is called by Lindblad Master Equation and it is composed by an usual Hamiltonian and a non-Hamiltoniana piece, which gives origin to the dissipative effects. The dissipator in (5) can be defined as
\[ D[\rho_\nu] = \frac{1}{2} \sum_{k=1}^{N^2-1} \left( [V_k, \rho_\nu V_k^\dagger] + [V_k^\dagger, \rho_\nu V_k^\dagger] \right), \] (6)

where \( V_k \) are dissipative operators which act only on \( \mathbb{H}_S \) space that is \( N \)-dimensional. The trace preservation of \( \rho_\nu \) occurs only if \( \sum_k V_k^\dagger V_k = 1 \) is satisfied. The \( V \) operators arise from the interaction of the interest subsystem with the environment and the evolution from the equation (5) leads an initial density matrix state into a new density matrix state [1]. The evolution is complete positive, transforming pure states into mixed states due to dissipation effects [4, 5, 10, 12]. The Von Neumann entropy of the interest subsystem, \( S = -\text{Tr} [\rho_\nu \ln \rho_\nu] \), must be increasing in time and this occurs if we impose \( V_k^\dagger = V_k \) [15].

In general, neutrinos propagate in vacuum or in matter and in both situations are possible to evolve the neutrino like an open quantum system. To this end, we can use the concepts discussed before and by means of straight application of the Eqs. (5) and (6). However, it is important to remember that this equations were developed taking into account as the interest subsystem was composed. Thus, we can have a complete knowledge from the Hamiltonian of the interest subsystem that is going to used before of applying the Eqs. (5) and (6).

In fact, the quantum dissipator written in Eq. (6) is different for neutrinos propagating in vacuum or in matter, but it can have the same form in both cases. It is easy to prove this statement since we can always write \( H_S \) in Eq. (2) as being diagonal even in matter propagation. However, the parameter values in operator \( V_k \) are different in each case.

Another important point is to know the differences of each kind of dissipation effects [3, 8, 16]. In order to facilitate the discussion as a whole and by concreteness of our aim, we are going to make some specific procedures and constraint those ones which can leave the explanation simpler. Let us start considering two neutrino oscillations and thus, the relation between the mass and flavor bases in vacuum is given by
\[ \rho_m = U^\dagger \rho_f U, \] (7)

where \( \rho_m \) is mass basis, \( \rho_f \) is the flavor basis and \( U \) is usual \( 2 \times 2 \) unitary mixing matrix. The same transformation between the effective mass basis and flavor basis can be written changing \( \rho_m \rightarrow \bar{\rho}_m \) and \( U \rightarrow \bar{U} \), where \( \bar{\rho}_m \) as effective mass basis and \( \bar{U} \) is composed by effective mixing angles [17, 18].

Now, we expand the Eqs. (5) and (6) in SU(2) basis matrices. Thus, we write the Eq. (5) as
\[ \frac{d}{dx} \rho_{\nu}(x) \sigma_\mu = 2 \epsilon_{ijk} H_{i\nu}(x) \sigma_\mu \delta_{jk} + D_{\nu\mu}(x) \sigma_\mu, \] (8)

with \( D_{\nu\mu} = D_{\mu\nu} = 0 \) to keep the probability conservation and we used the usual space dependence in the equation above. The matrix \( D_{\mu\nu} \) can be parametrized as
\[ D_{\mu\nu} = -\begin{pmatrix} \gamma_1 & \alpha & \beta \\ \alpha & \gamma_2 & \delta \\ \beta & \delta & \gamma_3 \end{pmatrix}, \] (9)

where the complete positivity constrains each parameter in the following form
\[ 2R \equiv \gamma_1 + \gamma_2 - \gamma_3 \geq 0; \quad RS - \alpha^2 \geq 0; \quad 2S \equiv \gamma_1 + \gamma_3 - \gamma_2 \geq 0; \quad RT - \beta^2 \geq 0; \quad 2T \equiv \gamma_2 + \gamma_3 - \gamma_1 \geq 0; \quad ST - \delta^2 \geq 0; \]
\[ RST \geq 2\alpha\beta\delta + T\delta^2 + S\beta^2 + R\alpha^2. \] (10)
It is possible to study how each entry in the matrix \( D \) changes the neutrino probabilities. Here for achieving our aim, we will use only two models where one of them will have two different parameters which will be associated with the decoherence and relaxation effects.

The most usual quantum dissipation is given by

\[
D_{km} = -\{\gamma_1, \gamma_1, 0\} \quad (11)
\]

where this dissipator is obtained imposing energy conservation, \([H_S, V_k] = 0\), on the interest subsystem. That is, a strong constraint, because as the system is considered open, naturally we should hope that the interest subsystem does not conserve its energy. So, we can obtain another quantum dissipation making null the entry \( D_{33} \). Thus we can rewrite a new dissipator as

\[
D_{km} = -\{\gamma_1, \gamma_1, \gamma_2\}, \quad (12)
\]

which violates the following condition, \([H_S, V_k] = 0\) and in fact, the last dissipator is more general than the previous. These physical constraints over the quantum dissipators are also supported by complete positivity, because as we can see in Eq. (11), to satisfy the complete positivity the diagonal parameters must be bigger in magnitude than the parameters off diagonal. Therefore, if some new physics could appear in terms of this approach, it must be already contained in terms of these quantum dissipators.

If we consider that all parameter in diagonal of Eq. (9) are equal, then \( \gamma_1 = \gamma_2 = \gamma \) and there would be two other diagonal models, but we do not treat them here because these models only brought redundancies in our argument. There would be still three interesting models with diagonal and off-diagonal parameters. However, as we have mentioned before, we will disregard of-off diagonal parameters here because they give origin to terms proportional to \( \gamma/2 \) times odd (or even) oscillatory function in the oscillation probability and these terms are suppressed by dumping terms created by diagonal parameters in the probabilities.

Now, we are able to identify the differences between two specific quantum dissipation effects, which are known as decoherence, \( D_{11} = D_{22} = \gamma_1 \), and relaxation, \( D_{33} = \gamma_2 \).

### III. Usual Dissipation

Using Lindblad Master Equation we can study many dissipative effects in neutrino oscillations. A detailed study of the behavior of each dissipative effect can help us in understanding how the state changes during its propagation as well as the effects change the oscillation probabilities. Decoherence effect is the most usual dissipative effect, but it is not the only one and then, it is important to know how it acts on the oscillation probability. This must also be applied to the other effects. We should have in mind that the decoherence effect occurs only when we impose that \([H_S, V_k] = 0\). Thus, when the interest subsystem Hamiltonian is changed, the form of the quantum dissipator must be changed too. Otherwise, we would not have only decoherence in the dynamic of the interest subsystem.

In the general case, the evolution can be made by taking the dissipator in Eq. (12) and, after by cancelling the entry \( D_{33} \), we obtain the evolution to the case of the dissipator from Eq. (11). In all situations, we always consider the oscillation Hamiltonian in its diagonal form and it represents the interest subsystem Hamiltonian, \( H_S \). Then, when the oscillation happens in vacuum, \( H_S \) is written as \( H_S = \text{diag}\{E_1, E_2\} \) in the mass basis and when the oscillation occurs in matter, it is possible to write the Hamiltonian as \( H_S = \text{diag}\{\tilde{E}_1, \tilde{E}_2\} \) using the effective mass basis. We are going to use the approximation \( E_i = E + m_i/2E \) and the same is valid to \( \tilde{E}_i = E + m_i/2E \). Thus, we can solve the Eq. (9) writing it as

\[
\begin{pmatrix}
\dot{\rho}_1(x) \\
\dot{\rho}_2(x) \\
\dot{\rho}_3(x)
\end{pmatrix} =
\begin{pmatrix}
-\gamma_1 & -\Delta & 0 \\
\Delta & -\gamma_1 & 0 \\
0 & 0 & -\gamma_2
\end{pmatrix}
\begin{pmatrix}
\rho_1(x) \\
\rho_2(x) \\
\rho_3(x)
\end{pmatrix}, \quad (13)
\]

where \( \Delta = \Delta m^2/2E \). If the propagation is in matter, we can evoke the effective quantities, i.e., \( \Delta \rightarrow \tilde{\Delta} = \Delta m^2/2E, \gamma_i \rightarrow \tilde{\gamma}_i \) and \( \rho_i \rightarrow \tilde{\rho}_i \). Of course, this changes nothing from the point of view of the equation solution and from now on, we cannot mention more this similarity.

Further, the component \( \rho_0 \) has a trivial differential equation given by \( \dot{\rho}_0(t) = 0 \) and its solution is \( \rho_0(t) = \rho_0(0) \) that in two neutrino oscillation means \( \rho_0(t) = 1/2 \). The Eq. (13) can be written in short form as

\[
\dot{R}(t) = \mathbb{H} R(t), \quad (14)
\]

where the eigenvalues are \( \lambda_0 = -\gamma_2, \lambda_1 = -\gamma_1 - i\Delta \) and \( \lambda_2 = -\gamma_1 + i\Delta \). For each eigenvalues is possible to obtain a correspondent eigenvector, \( u_0, u_1, u_2 \) that compose the matrix \( \mathbb{A} = [u_0, u_1, u_2] \) that diagonalizes the matrix \( \mathbb{H} \) by performing the following similarity transformation \( \mathbb{A}^{-1} \mathbb{H} \mathbb{A} \). The solution of the Eq. (14) is given by

\[
R(x) = \mathbb{M}(x) R(0), \quad (15)
\]

where \( \mathbb{M}(x) \) is obtained making

\[
\mathbb{M}(x) = \mathbb{A} \text{diag}\{e^{\lambda_0 x}, e^{\lambda_1 x}, e^{\lambda_2 x}\} \mathbb{A}^{-1}. \quad (16)
\]

Furthermore, it is useful to write the state propagated. It is given by

\[
\rho(x) = \begin{pmatrix}
\rho_0(x) + \rho_3(x) & \rho_1(x) - i\rho_2(x) \\
\rho_1(x) + i\rho_2(x) & \rho_0(x) - \rho_3(x)
\end{pmatrix}. \quad (17)
\]

From the Eq. (13), one can see that the state propagated is written as

\[
\rho(x) = \begin{pmatrix}
\frac{1}{2}e^{-\gamma_2 x} \cos 2\theta & \frac{1}{2}e^{-(\gamma_1 - i\Delta)x} \sin 2\theta \\
rac{1}{2}e^{-(\gamma_1 + i\Delta)x} \sin 2\theta & -\frac{1}{2}e^{-\gamma_2 x} \cos 2\theta
\end{pmatrix}, \quad (18)
\]
where it is possible to identify two unusual behavior. The off-diagonal entries are called coherence elements. There is a damping term that eliminates the quantum coherence during the propagation. Therefore, this is the decoherence effect and we can see clearly as the decoherence effect is associated to the matrix elements \( M_{ij}(x) \propto \exp[\lambda_1(x)] \). While, in the Eq. (18), the diagonal elements are known as population elements and thus, they are the quantum probabilities.

The diagonal elements in the Eq. (18) state us which the probability of obtaining the eigenvalue \( E_1 \) or \( E_2 \) of the observable \( H \). In the usual case, the observable is diagonal in the mass basis and the diagonal elements of the state are space independent, but in the state in Eq. (18) the probability elements change with the propagation. This effect implies that the neutrinos may change their flavor without using the oscillating mechanism. As the asymptotical state is a complete mixing, the \( \gamma_2 \) in diagonal elements is called as relaxation effect and this effect is associated to the matrix element \( M_{ij}(x) \propto \exp[\lambda_0] \).

The flavor oscillation probabilities can be obtained from the Eq. (7) and \( \rho_{ij}^{\nu} \) element is the survival probability that is written as

\[
P_{\nu_\alpha \rightarrow \nu_\alpha} = \frac{1}{2} \left[ 1 + e^{-\gamma_1 x} \cos 2\theta + e^{-\gamma_2 x} \sin 2\theta \sin^2 \left( \frac{\Delta}{2} \right) \right],
\]

From the Eq. (19) we see the asymptotic probability, \( x \to \infty \), goes to a maximal mixing, \( P_{\nu_\alpha \rightarrow \nu_\alpha} = 1/2 \), and it happens for any mixing angle. Thus, by mean of this approach, the neutrino is able to change its flavor and it does not need to use the oscillatory mechanism to this end (10).

When the propagation is performed with the dissipator in Eq. (11), we obtain some important differences. In this case, \( \mathbb{H} \) has only two non-trivial eigenvalues which are equal to \( \lambda_1 \) and \( \lambda_2 \) which were written before. Then, the matrix \( \mathbb{M}(t) \) is changed to

\[
\mathbb{M}(t) = \mathbb{A} \text{diag} \{1, e^{\lambda_1 t}, e^{\lambda_2 t}\} \mathbb{A}^\dagger,
\]

and consequently, the state is written as

\[
\rho(x) = \begin{pmatrix}
\frac{1}{2} + \frac{1}{2} \cos^2 \theta & \frac{1}{2} e^{-\gamma_1 x} \sin 2\theta \\
\frac{1}{2} e^{-\gamma_1 x} \sin 2\theta & \frac{1}{2} - \frac{1}{2} \cos^2 \theta
\end{pmatrix}.
\]

Now, we can see clearly when the subsystem is only under influence of the decoherence effects and the coherent elements are eliminated during the propagation. In this case, the survival oscillation probability is written as

\[
P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \frac{1}{2} \sin^2(2\theta) \left[ 1 - e^{-\gamma_1 x} \cos(\Delta x) \right].
\]

This probability was discussed in Refs. 1, 13, 22. Indeed, we can see that the decoherence effects only are described by \( \gamma_1 \) term associated to the eigenvalues \( \lambda_1 \) and \( \lambda_2 \).

Physically, when there is energy conservation in interest subsystem, \( T_{\mathbb{H}}[H_S \rho_m(t)] = 0 \), the asymptotical probability, \( x \to \infty \), still depends on the mixing angle as

\[
P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \frac{1}{2} \sin^2(2\theta)
\]

and we have the maximal mixing only if \( \theta = \pi/4 \).

From mathematical point of view, when neutrino is considered an open quantum system there are not differences to obtain its evolution in vacuum or in constant matter. It is possible to understand this since we can always take the usual Hamiltonian in its diagonal form even when the neutrinos are propagating in vacuum or in constant matter. The connection between the approach of the vacuum and the constant matter is made by changing each parameter obtained to the vacuum propagation in their corresponding effective parameters, where they keep diagonal the oscillation Hamiltonian. It is supported by dynamics that is made from Eq. (3), where one can decide how is the interest subsystem Hamiltonian, \( H_S \) and thus, the evolution operator \( U \).

**IV. NEUTRINOS IN NON-UNIFORM MATTER**

From the above discussion, we can study the neutrino propagation when the density matter is not constant. There are many sources of these types, but we do not make a review on them. We are going, nevertheless, to assume that neutrino propagation in ordinary matter where the adiabatic limit is valid [17, 18]. Then, the adiabatic limit is valid [17, 18]. Thus, the main focus now is to understand which dissipative effects act in neutrinos supposing that the source is far away from the Earth.

Using the same point of view from the previous section, we can write a diagonal Hamiltonian using the effective mass basis. We start with the quantum dissipator of Eq. (12). Thus, we have to solve the same evolution equation given in Eq. (13), but on the right side, the elements of the first matrix are space dependent as well. So, the Eq. (14) is written now as

\[
\dot{\mathbb{H}}(x) = \mathbb{H}(x) \mathbb{R}(x),
\]

that has solution identical to Eq. (15), but \( \mathbb{M}(x) \) is proportional to

\[
\mathbb{M}(x) \propto \text{diag} \{e^{\int_{r}^{R } \lambda_0(x) dz}, e^{\int_{r}^{R } \lambda_1(x) dz}, e^{\int_{r}^{R } \lambda_2(x) dz} \},
\]

where \( r \) and \( R \) are the creation and detection point, respectively. \( \mathbb{A} \) is defined in the same way as in the previous section, but now \( \lambda_0 \) may have a space dependence, because it is possible to define the dissipative effects with such a dependence [9]. \( \gamma_1 \to \gamma_1(x) \). The state in this situation is given by

\[
\dot{\rho}(x) = \begin{pmatrix}
\frac{1}{2} + \frac{1}{2} e^{-T^*} \cos 2\theta & \frac{1}{2} e^{-T^*} \sin 2\theta \\
\frac{1}{2} e^{-T^*} \sin 2\theta & \frac{1}{2} - \frac{1}{2} e^{-T^*} \cos 2\theta
\end{pmatrix},
\]
where we have defined
\[ \Gamma = -\int_r^{R_\odot} \tilde{\gamma}_2(x)dx \] (27)
and
\[ \Gamma_1 = -\int_r^{R_\odot} \tilde{\gamma}_1(x)dx + i \int_r^{R_\odot} \tilde{\Delta}(x)dx \] (28)
where, in both case, \( r \) is the creation point and \( R_\odot \) is the detection point.

In general, the second term in Eq. (28) gives rise to fast oscillation terms in the off-diagonal elements and can be eliminated. Thus, the state has the following form
\[ \tilde{\rho}_m(x) = \begin{pmatrix} \frac{1}{2} + \frac{i}{2} e^{-\Gamma} \cos 2\tilde{\theta} & 0 \\ 0 & \frac{1}{2} - \frac{i}{2} e^{-\Gamma} \cos 2\tilde{\theta} \end{pmatrix}, \] (29)
where we can see clearly that the decoherence effects cannot happen in this situation.

It is possible to obtain the usual adiabatic probability. As is well known, the effective mixing angle changes during the neutrino propagation and, therefore, the mixing angle in the detection point must be different. We defined the initial mass state from the Eq. (7), where in the creation point we used the effective mixing angles written as \( \tilde{\theta} \). So, we can change the representation by applying another mixing matrix that we can define with other mixing angle. Defining these angles in the detection point as \( \tilde{\theta}_d \), we have
\[ \rho_f(x) = U_d \tilde{\rho}_m(x) U_d^\dagger, \] (30)
where \( U_d \) is the usual mixing matrix, but with mixing angle \( \tilde{\theta}_d \). Then, the adiabatic survival probability, \( \rho_{11}^f(x) \), is given by
\[ P_{\nu_e\rightarrow\nu_e}^{\text{adiab.}} = \frac{1}{2} + \frac{1}{2} e^{-\Gamma} \cos 2\tilde{\theta} \cos 2\tilde{\theta}_d. \] (31)

It is easy to see that if we set \( \Gamma = 0 \) in the survival above probability, we recover the usual survival probability in the adiabatic limit case [17, 18]. Furthermore, if we consider the dissipator in Eq. (11) to perform the evolution, we are going to achieve the usual survival probability that does not have any dissipative effect. Therefore, it is not possible put some bound on the decoherence effects in this hypothetical situation and there is only one dissipative effect in the state (30) or in the probability (31), that is the relaxation effect.

It is interesting to note that the quantum dissipative medium spanned by \( \mathbb{H}_R \) cannot change, but the interaction that it has with interest subsystem can. In order to appreciate this phenomenon, we are going to study a system in which the interaction between the quantum dissipative medium and the interest subsystem has different characteristics in each piece of the interest subsystem. We are assuming that neutrinos propagate under matter effects in the adiabatic limit with \( H_S = H_e + H_m, \) where \( H_e \) is the oscillation Hamiltonian in vacuum and \( H_m \) is the matter potential. In addition, we assume energy conservation only in vacuum piece, \( [H_e, V_k] = 0 \), and \( [H_m, V_k] \neq 0 \). Thereby, we must solve the Eq. (24) with \( \mathbb{H} \) given by
\[ \mathbb{H} = \begin{pmatrix} -\gamma_1 & -\Delta & 0 \\ \Delta + A \cos 2\theta & -\gamma_1 & -A \sin 2\theta \\ 0 & A \sin 2\theta & 0 \end{pmatrix}, \] (32)
where, in this case, all parameters are in the vacuum representation.

The characteristic polynomial of the above matrix has a complicated solution, but if we consider that \( \gamma_1 \) is small such that it can be treated as a perturbation, we obtain in first order of approximation the following eigenvalues:
\[ \lambda_0 = -\gamma_1 \frac{A^2}{\Delta} \sin^2 2\tilde{\theta}; \]
\[ \lambda_1 = -\gamma_1 + \gamma_1 \frac{A^2}{\Delta} \sin^2 2\tilde{\theta} - i\Delta; \]
\[ \lambda_2 = -\gamma_1 + \gamma_1 \frac{A^2}{\Delta} \sin^2 2\tilde{\theta} + i\Delta. \] (33)

where \( A = \sqrt{2} G_F n_e \) and, for sake of simplicity, we can rewrite \( \mathbb{H} \) in the effective mass basis, such that it changes to
\[ \mathbb{H} = \begin{pmatrix} -\gamma_1 + \tilde{\gamma}_2 & -\Delta & 0 \\ \Delta & -\gamma_1 + \tilde{\gamma}_2 & 0 \\ 0 & 0 & -\tilde{\gamma}_2 \end{pmatrix}, \] (34)
with \( \tilde{\gamma}_2 = \gamma_1 A^2 \sin^2 2\tilde{\theta}/\Delta^2. \) From \( \mathbb{H} \) given by Eq. (34) we will obtain the same state that was given in Eqs. (29) and (30) and the same survival probability as in Eq. (31), where \( \Gamma \) can be defined again by \( \tilde{\gamma}_2 \).

As we can see the decoherence effects are not important in this condition, but the effective relaxation effect that appear it is important. Notice that even imposing the constraint that gives origin to decoherence effects in neutrino oscillations in vacuum, \( [H_e, V_k] = 0 \), only the relaxation effect, that is another phenomenological effect, appears and may change the neutrino behavior. This keeps the statement that decoherence cannot happen in adiabatic limit. Nevertheless, we can see \( \tilde{\gamma}_2 \propto \gamma_1 \) where \( \gamma_1 \) is the decoherence effect in vacuum, but even there being a relation between these effects, they are different each other. This specific result was obtained in Ref. [9] and \( \tilde{\gamma}_2 \) was limited.

In fact, in Ref. [3] the authors supposed that the interest subsystem could be subdivided such that only vacuum neutrino piece interacts with the environment\(^1\). Although, by means of statements obtained from the

\(^1\) There the authors called the environment as quantum gravity effects.
Eqs. (2) until (5), this hypothesis is not available because the interest subsystem Hamiltonian, $H_S$, is formed by mass Hamiltonian plus effective matter potential. Consequently, the dynamic is developed violating the energy conservation in the interest subsystem, $Tr[H_S, V_k] \neq 0$. This give rise to the correction term in Eq. (33) where $\lambda_0 \neq 0$.

Whether the interaction between the environment and the neutrinos is always the same and it is given by dissipator in Eq. (11), independently of the medium that the neutrinos will go through, then the limit that was given by authors in Ref. [9] to the decoherence effect in the channel $\nu_e \rightarrow \nu_\mu$ is indirect, but reasonable. On the other hand, there is no reason to necessarily think so. Further, in this result still exists the ambiguity between the consideration applied in this problem and the hypothesis that the evolution can be done with dissipator given in Eq. (12), once the decoherence and the relaxation effects do not need to have the same magnitude, i.e., $D_{11} = D_{22} \neq D_{33}$.

Therefore, this is an interest result because we do not have in the literature a reliable limit for decoherence effect in the channel $\nu_e \rightarrow \nu_\mu$. With this discussion, we can understand that solar neutrinos cannot reliably constrain the decoherence effect, but surely the relaxation effect. Other analysis can be done using neutrinos coming from other sources, where the constraint $[H_S, V_k] = 0$ can without any doubt be satisfied and the decoherence effect limited.

V. COMMENTS AND CONCLUSION

In this work we presented a study about neutrino oscillation when the decoherence and the relaxation effects are present. Both effects are obtained in an approach where the neutrinos are treated as an open quantum system. The dynamics evolution is made with the Lindblad Master Equation and we used only two different phenomenological quantum dissipators.

The quantum dissipator in Eq. (11) includes decoherence effects while the quantum dissipator in Eq. (12) includes decoherence plus relaxation effects. In order to simplify the approach, initially we represent both the decoherence and the relaxation effects for $\gamma_1$ and $\gamma_2$ respectively. $\gamma_1$ can be obtained supposing the energy conservation in the interest subsystem is satisfied, $Tr[H_S, V_k] = 0$ and $\gamma_2$ is obtained for violating the condition before.

We studied the evolution in terms of the quantum state and their characteristics were explored when the decoherence and the relaxation effects were acting on the state. We emphasized the differences and similarities between the $\Pi$ eigenvalues that are obtained when we used the dissipators in Eqs. (11) and (12). Thus, we clearly see when the relaxation effect is present in the model and how the behavior of the states is changed in the situation with and without the relaxation effect.

We discussed the evolution in vacuum and in matter with constant density and pointed out how these situations can have similar treatments by means of quantum states. This approach makes the dynamics simpler in the case where the matter potential is important.

Afterwards the situation where the matter density is not constant was analyzed, but in the case when the adiabatic limit is satisfied. We showed that decoherence effects cannot be limited in this case where the source is far way from the detection point. This occurs because the quantum interference oscillates very fast and, as usual, we can neglect it. On the other hand, the relaxation effects can be kept and limited by experimental analysis in several situations.

Nonetheless we discussed a particular case in which it is possible to limit decoherence effects to vacuum propagation in the case where neutrinos propagate by regions with non constant matter density, in the same way it was made in Ref. [9]. This situation is obtained if we suppose that the quantum dissipator is always given by Eq. (11), taking into account the constraint $[H_e, V_k] = 0$, that defines decoherence in vacuum propagation. We pointed out that it can exist ambiguities in this case because one cannot know if this limit is valid for the phenomenological decoherence effect or for phenomenological relaxation effect. Then, in order to avoid the ambiguities, we could think the limit for decoherence effect for the channel $\nu_e \rightarrow \nu_\mu$ should be obtained from other Earth sources, like reactors, for instance. On the other hand, the relaxation surely already has a limit given by the result of the Ref. [9], at least, in two neutrino oscillation approximation.

In summary, we presented an easy and general framework that includes dissipative effects in neutrino oscillations. In particular, we treated the decoherence and the relaxation effects that are the most important dissipative effects. Both effects arise when we treat the neutrinos like an open quantum system and follow the complete positivity constraint. These effects are obtained from the biggest term of the quantum dissipator in Eq. (4) which compose its main diagonal. Extending the study to neutrino propagation in non-constant density matter, we noticed that the decoherence effect cannot be limited by the channel $\nu_e \rightarrow \nu_\mu$ in case where the adiabatic limit is valid and if the source is far way from detection point. A doubtless limit for decoherence effect in the channel $\nu_e \rightarrow \nu_\mu$ is still unknown.

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