Charmed hadron production via equal-velocity quark combination in ultra-relativistic heavy ion collisions

Rui-Qin Wang,¹ Jun Song,² Feng-Lan Shao,³ and Zuo-Tang Liang⁴

¹School of Physics and Physical Engineering, Qufu Normal University, Shandong 273165, China
²Department of Physics, Jining University, Shandong 273155, China
³Key Laboratory of Particle Physics and Particle Irradiation (MOE), Institute of Frontier and Interdisciplinary Science, Shandong University, Qingdao, Shandong 266237, China

Recent data on the production of D mesons and Λ⁺ baryons in heavy ion collisions at the Relativistic Heavy Ion Collider and the Large Hadron Collider exhibit a number of striking characteristics such as enhanced yield ratios \( D_s^+ / D^0 \), \( \Lambda^+ / D^0 \) and their transverse momentum dependences. In this paper, we derive the momentum dependence of open charm mesons and singly charmed baryons produced in ultra-relativistic heavy ion collisions via the equal-velocity quark combination. We present analytic expressions and numerical results of yield ratios and compare them with the experimental data available. We make predictions for other charmed hadrons.

PACS numbers: 25.75.-q, 25.75.Dw, 25.75.Gz

I. INTRODUCTION

In ultra-relativistic heavy ion collisions, heavy quarks and anti-quarks are produced predominantly via initial hard scatterings and then experience the entire evolution of a violently interacting medium of deconfined quarks and gluons — the quark gluon plasma (QGP). These heavy flavor quarks and anti-quarks interact strongly with constituents of the QGP medium [1, 2], exchange energy and momentum in the partonic evolution process [3–5] and combine with them to form heavy flavor hadrons that can be observed experimentally. Therefore, the production of heavy flavor hadrons is usually believed to play a very special role in studying the hadronization mechanism and properties of the QGP matter [6–11].

Recently, experimental data on open charm mesons and \( \Lambda^+ \) baryons with improved precision and extended transverse momentum (\( p_T \)) coverage at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) have been published [12–18]. These data show indeed a number of fascinating features. The most striking ones might be the enhancement of the strange to non-strange meson yield ratio \( D_s^+ / D^0 \) and that of the baryon-to-meson ratio \( \Lambda^+ / D^0 \). It has been observed [15, 16] that the \( D_s^+ / D^0 \) ratio in ultra-relativistic heavy ion collisions is much higher than predictions of fragmentation models [19, 20]. The \( \Lambda^+ / D^0 \) ratio exhibits also a very strong enhancement trend at intermediate \( p_T \) not only in RHIC Au-Au and LHC Pb-Pb collisions but also in \( pp \) and \( p-Pb \) reactions at the extremely high LHC energies [18]. They are much higher than the predictions of perturbative QCD and event generators based on fragmentation models [18, 21].

Theoretically, much effort has also been made to the hadronization mechanism of charm quarks in high energy heavy ion collisions [22–34]. It seems in particular that the coalescence or (re-)combination mechanism should play an irreplaceable role in describing the production of charmed hadrons with low and intermediate transverse momenta [24–33]. In this connection, we note in particular that the quark combination model under equal-velocity combination (hereafter referred as EVC) [35] provides a very simple and elegant way to describe the \( p_T \) dependence of production rates of hadrons and has been successfully applied [36, 37] to \( pp \) and \( p-Pb \) collisions to describe the enhancement of \( \Lambda^+ / D^0 \). It is therefore interesting to see whether it can also be applied to heavy ion collisions to describe the heavy flavor hadron production as well.

In this paper, we apply the quark combination via EVC to ultra-relativistic heavy ion collisions to study the production of open charm D mesons and singly charmed baryons in the low and intermediate \( p_T \) regions. We present the detailed derivations and give analytic expressions for \( p_T \)-dependences of ratios of production rates of different hadrons. We compare the results with the experimental data available [12, 14, 15] and make predictions for other types of hadrons.

The rest of the paper is organized as follows. In Sec. II, we present the derivation of the momentum-dependent production of singly charmed hadrons in the quark combination model via EVC in ultra-relativistic heavy ion collisions. We present in particular the analytic expressions for two kinds of yield ratios measured by RHIC and LHC experiments [14, 17] as the function of \( p_T \) and discuss their qualitative properties. In Sec. III, we apply the results obtained in Sec. II to Au-Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV. In Sec. IV, we present a summary.

II. THE CHARMED HADRON PRODUCTION IN THE QUARK COMBINATION VIA EVC

The basic idea of the quark combination mechanism and formulae for the momentum dependence have been pre-
sented in many different literatures. An example for the
derivation for light flavor hadrons starting from the gen-
eral formulae has been given in [38] where \( p_T \)-integrated
yield correlations have been calculated. Here, in this sec-
tion, for explicitness, we just follow the same procedure as
that in [38] and present results of momentum dependence
for singly charmed hadrons. After that, we present the cor-
responding results obtained under EVC.

A. The general formalism

To derive the momentum-dependent production rates of
mesons and baryons in ultra-relativistic heavy ion colli-
sions, we just as in [38] start with a color-neutral quark-
 anti-quark system with \( N_q \) quarks of flavor \( q \) and \( N_{\bar{q}} \)
anti-quarks of flavor \( \bar{q} \) and suppose they hadronize via
the quark combination mechanism. The momentum distri-
butions \( f_M(p; N_q, N_{\bar{q}}) \) and \( f_B(p; N_q, N_{\bar{q}}) \) for the directly
produced meson \( M \) and baryon \( B \) are given by,

\[
f_M(p; N_q, N_{\bar{q}}) = \sum_{q_1q_2} \int dp_1 dp_2 N_{q_1q_2} f^{(n)}_{q_1q_2} (p_1, p_2; N_q, N_{\bar{q}}) \times R_M(q_1q_2; p, p_1, p_2; N_q, N_{\bar{q}}),
\]

\[
f_B(p; N_q, N_{\bar{q}}) = \sum_{q_1q_2q_3} \int dp_1 dp_2 dp_3 \times N_{q_1q_2q_3} f^{(n)}_{q_1q_2q_3} (p_1, p_2, p_3; N_q, N_{\bar{q}}) \times R_B(q_1q_2q_3; p, p_1, p_2, p_3; N_q, N_{\bar{q}}),
\]

where \( f^{(n)}_{q_1q_2} \) and \( f^{(n)}_{q_1q_2q_3} \) are normalized joint momentum dis-
tributions for \( q_1q_2 \) and \( q_1q_2q_3 \) pairs respectively; \( N_{q_1q_2} = N_q N_{\bar{q}} \)
is the number of \( q_1q_2 \) pairs; \( N_{q_1q_2q_3} \) is the number of \( q_1q_2q_3 \) clusters in the system, and it takes \( N_q N_{\bar{q}} \) for \( q_1 \neq q_2 \neq q_3 \) and \( N_q \) \( N_{\bar{q}} \) for \( q_1 = q_2 \neq q_3 \) and \( N_q \) \( N_{\bar{q}} \) \( (N_q - 1) \) \( N_{\bar{q}} - 2 \) for \( q_1 = q_2 = q_3 \); kernel functions
\( R_M(q_1q_2) \) and \( R_B(q_1q_2q_3) \) stand for the probability density for a
\( q_1q_2 \) pair with momenta \( p_1 \) and \( p_2 \) to combine into a meson
\( M \) of momentum \( p \) and that for a \( q_1q_2q_3 \) cluster with \( p_1 \),
\( p_2 \) and \( p_3 \) to combine into a baryon \( B \) of \( p \) respectively.

Just as discussed in [38], \( R_M(q_1q_2) \) and \( R_B(q_1q_2q_3) \) carry the
kinematical and dynamical information of the quark combi-
nation. Their precise forms can not be derived from the
first principles due to their complicated non-perturbative
nature. Nevertheless, they are constrained by a number of
symmetry laws and rules such as the momentum conserva-
tion, constraints due to intrinsic quantum numbers such as
spin and flavor, the requirement of the hadronization unitari-
y so that the production of all the open and hidden charm
hadrons should exhaust all charm quarks and anti-
quarks in the system, and the meson-baryon production
competition and so on. To take these constraints into ac-
count explicitly, we re-write them in the following forms,

\[
R_M(q_1q_2; p, p_1, p_2; N_q, N_{\bar{q}}) = C_M f_M^{(f)}_{q_1q_2},
\]

\[
R_B(q_1q_2q_3; p, p_1, p_2, p_3; N_q, N_{\bar{q}}) = C_B R_B^{(f)}_{q_1q_2q_3},
\]

\[
\times R_B(q_1q_2q_3; (p_1, p_2, p_3; N_q, N_{\bar{q}})) \delta(p_1 + p_2 - p),
\]

Here the \( \delta \)-functions are used to guarantee the momentum
conservation. The factors \( R_M^{(f)}_{q_1q_2} \) and \( R_B^{(f)}_{q_1q_2} \) contain
Kronecker \( \delta \)'s to guarantee the quark flavor conservation,
e.g., if \( M_j \) is a D-meson with constituent quark content \( q_c \),
\( R_B^{(f)}_{q_1q_2} = \delta_{q_1q_2q_3} \delta_{q_1q_2q_3} \). For \( f_M^{(f)}_{q_1q_2} \), we need to include a number
\( N_{iter} \) to account for the fact that there are different itera-
tions for the flavors of the three quarks and \( N_{iter} = 1, 3 \) or
6 for \( q_1 = q_2 = q_3 \), \( q_1 = q_2 \neq q_3 \) or \( q_1 \neq q_2 = q_3 \).

The factor \( C_{M_f} \) is the probability for \( M \) to be \( M_f \) if
the quark content of \( M \) is the same as \( M_f \) and similar for
\( C_{B_f} \). In the case that only \( J^P = 0^- \) and \( 1^- \) mesons
and \( J^P = (1/2)^+ \) and \( (3/2)^+ \) baryons are considered, they are
just determined completely by the production ratio of vec-
to pseudo-scalar mesons and that of \( J^P = (1/2)^+ \) to
\( J^P = (3/2)^+ \) baryons with the same flavor content.

The remaining factor \( R_{M,q_1q_2} \) now stands for the proba-
bility of a \( q_1q_2 \) pair with momenta \( p_1 \) and \( p_2 \) to combine
into a meson \( M \) with any momentum and other quantum
numbers and similar for \( R_{B,q_1q_2q_3} \). They depend on the
momenta of the (anti-)quarks and their situated environments
represented by \( N_q \) and \( N_{\bar{q}} \) and should be determined by
the dynamics in the combination process.

In this way, we obtain the momentum distribution for a
charged meson \( D_j \) with quark flavor content \( q_1c \) and
that for a singly charged baryon \( B_j \) with \( q_1c \),

\[
f_D(p; N_q, N_{\bar{q}}) = \int dp_1 dp_2 N_{q_1c} f^{(n)}_{q_1c} (p_1, p_2; N_q, N_{\bar{q}}) \times C_D R_{M,q_1c}; (p_1, p_2, N_q, N_{\bar{q}})) \delta(p_1 + p_2 - p),
\]

\[
f_B(p; N_q, N_{\bar{q}}) = \int dp_1 dp_2 dp_3 \times N_{q_1c} f^{(n)}_{q_1c} (p_1, p_2, p_3; N_q, N_{\bar{q}}) C_{B_f} N_{iter} \times R_{B,q_1c}; (p_1, p_2, N_q, N_{\bar{q}})) \delta(p_1 + p_2 + p_3 - p). \]

Here, as well as in the following of this paper, when talking
about charmed mesons \( D_j \)'s and singly charmed baryons
\( B_j \)'s, \( q_1 \) and \( q_2 \) denote u, d and s flavors of quarks or
anti-quarks.

B. The momentum distribution under EVC

It has been shown that [35] EVC seems to work well
for light and strange hadron production in low and inter-
mediate \( p_T \) regions. For charmed hadrons, the mass dif-
fences between the charm and other quarks are large. It is
therefore much more sensitive and more interesting to see
whether the combination proceeds via EVC or other rules.
Under EVC, \( R_{M,q_1q_2} \) and \( R_{B,q_1q_2q_3} \) take,

\[
R_{M,q_1q_2}; (p_1, p_2; N_q, N_{\bar{q}}) = R_{M,q_1q_2}; (N_q, N_{\bar{q}}) \times \frac{m_{q_1} + m_{q_2}}{m_{q_1} m_{q_2}} \delta(p_1 - p_2),
\]

\[
R_{B,q_1q_2q_3}; (p_1, p_2, p_3; N_q, N_{\bar{q}}) = R_{B,q_1q_2q_3}; (N_q, N_{\bar{q}}) \times \frac{m_{q_1} + m_{q_2} + m_{q_3}}{m_{q_1} m_{q_2} m_{q_3}} \delta(p_1 - p_2, p_3 = p_1 - p_2). \]
where \(m_q\) is the constituent quark mass and is taken as \(m_{u} = m_{d} = 0.33\) GeV, \(m_s = 0.5\) GeV and \(m_c = 1.5\) GeV in the numerical calculations in this paper; \(A_{M,q,\bar{q}}\) and \(A_{B,q,\bar{q}}\) are factors denoting the meson-baryon production competition as well as guaranteeing the unitarity. They depend on the numbers of different flavor quarks and anti-quarks in the considered system. The mass term is introduced for normalization, so that the original kernel functions in Eqs. \(1\) and \(3\) take the form,

\[
R_{M,q,\bar{q}}(p_1, p_2; N_q, N_{\bar{q}}) = C_M R^{(f)}_{q,\bar{q}}(N_q, N_{\bar{q}}) \times \delta(p_1 - x^{(0)}_{q,\bar{q}}(p) \delta(p_2 - x^{(0)}_{q,\bar{q}}(p)),
\]

\[
R_{B,q,\bar{q}}(p_1, p_2, p_3; N_q, N_{\bar{q}}) = C_B R^{(f)}_{q,\bar{q}}(N_q, N_{\bar{q}}) \times \delta(p_1 - x^{(0)}_{q,\bar{q}}(p) \delta(p_2 - x^{(0)}_{q,\bar{q}}(p)),
\]

where \(x^{(0)}_{q,\bar{q}} = m_q/(m_q + m_{\bar{q}})\) and \(x^{(0)}_{q,\bar{q}} = m_q/(m_q + m_{\bar{q}} + m_q)\) are the fraction of the momentum of the produced hadron carried by \(q\).

Substituting Eqs. \(2\) and \(4\) into Eqs. \(5\) and \(6\) and carrying out the integration over the quark momenta, we obtain

\[
f_{D}(p) = C_{D} \mathcal{A}_{M,q,\bar{q}}(N_q, N_{\bar{q}}) \times f^{(n)}_{q,\bar{q}}(x^{(0)}_{q,\bar{q}}(p), x_{q,\bar{q}}^{(1)}(p)),
\]

\[
f_{B}(p) = C_{B} \mathcal{A}_{B,q,\bar{q}}(N_q, N_{\bar{q}}) N_{iter} \times f^{(n)}_{q,\bar{q}}(x^{(0)}_{q,\bar{q}}(p), x_{q,\bar{q}}^{(1)}(p)).
\]

Here and from now on in this paper, we suppress the arguments \(N_q, N_{\bar{q}}\) in the momentum distribution functions for explicitness.

The factor \(\mathcal{A}_{M,q,\bar{q}}(N_q, N_{\bar{q}})\) is the probability for a charm quark to capture a specific anti-quark \(\bar{q}\) to form a meson in the bulk quark-anti-quark system, it should be inversely proportional to the total number of quarks and anti-quarks \(N_q + N_{\bar{q}}\). Similarly, \(\mathcal{A}_{B,q,\bar{q}}(N_q, N_{\bar{q}})\) should be proportional to \(1/(N_q + N_{\bar{q}})^2\). Therefore, one can write

\[
\mathcal{A}_{M,q,\bar{q}}(N_q, N_{\bar{q}}) = \mathcal{A}_M/(N_q + N_{\bar{q}}),
\]

\[
\mathcal{A}_{B,q,\bar{q}}(N_q, N_{\bar{q}}) = \mathcal{A}_B/(N_q + N_{\bar{q}})^2,
\]

where \(\mathcal{A}_M\) and \(\mathcal{A}_B\) are proportionality coefficients and they closely relate to the unitarity and the meson-baryon production competition. For a given quark-anti-quark system, \(\mathcal{A}_M\) and \(\mathcal{A}_B\) should be universal for all different \(D\) mesons and singly charmed baryons due to the quark flavor blindness of the strong interaction. Substituting Eqs. \(13\) and \(14\) into Eqs. \(11\) and \(12\), respectively, we have

\[
f_{D}(p) = N_c \mathcal{A}_M C_D \bar{A}_{q}\mathcal{f}^{(n)}_{q,\bar{q}}(x^{(0)}_{q,\bar{q}}(p), x_{q,\bar{q}}^{(1)}(p)),
\]

\[
f_{B}(p) = N_c \mathcal{A}_B C_B \bar{A}_{q}\mathcal{f}^{(n)}_{q,\bar{q}}(x^{(0)}_{q,\bar{q}}(p), x_{q,\bar{q}}^{(1)}(p)),
\]

where \(\lambda_{q} \equiv N_{q}/(N_q + N_{\bar{q}})\) and \(\lambda_{q,\bar{q}} \equiv N_{q,\bar{q}}/(N_q + N_{\bar{q}})^2\). If we consider a quark-anti-quark system in the mid-rapidity region at high energies so that the influence of net quarks from the colliding nuclei can be neglected, the ratios \(\lambda_{q}\) and \(\lambda_{q,\bar{q}}\) are both determined completely by the strangeness suppression factor \(\lambda_{s}\).

If we neglect correlations of momentum distributions of quarks and/or anti-quarks of different flavors in the system, i.e. we take,

\[
f_{q,\bar{q}}^{(n)}(p_1, p_2) = f_{q,\bar{q}}^{(n)}(p_1) f_{q,\bar{q}}^{(n)}(p_2),
\]

\[
f_{q,\bar{q}}^{(n)}(p_1, p_2, p_3) = f_{q,\bar{q}}^{(n)}(p_1) f_{q,\bar{q}}^{(n)}(p_2) f_{q,\bar{q}}^{(n)}(p_3).
\]

In this case, we obtain

\[
f_{D}(p) = N_c \mathcal{A}_M C_D \bar{A}_{q}\mathcal{f}^{(n)}_{q,\bar{q}}(x^{(0)}_{q,\bar{q}}(p), x_{q,\bar{q}}^{(1)}(p)),
\]

\[
f_{B}(p) = N_c \mathcal{A}_B C_B \bar{A}_{q}\mathcal{f}^{(n)}_{q,\bar{q}}(x^{(0)}_{q,\bar{q}}(p), x_{q,\bar{q}}^{(1)}(p)).
\]

By using Eqs. \(19\) and \(20\), we can calculate momentum distributions and ratios for the production of different charmed hadrons. The factors \(\mathcal{A}_M\) and \(\mathcal{A}_B\) are determined by charm quark number conservation in the combination process and the charmed baryon-to-meson production ratio \(N_B/N_D\). Here \(N_B\) and \(N_D\) are the total number of all the produced \(D\) mesons and that of singly charmed baryons. For the charm quark number conservation, we need in principle to consider the production of hadrons besides singly charmed ones such as charmonia, doubly and triply charmed baryons and even exotic states. However, the production rates for them are very small and they exhaust about less than 5% of total charm quark. In the numerical calculations in the following of this paper, we will just neglect them so the charm quark number conservation just takes \(N_B + N_{\bar{q}} \approx N_c\). In this approximation, \(\mathcal{A}_M\) and \(\mathcal{A}_B\) are just determined by the ratio \(R^{(c)}_{B/D} \equiv N_B/N_D\) that is taken as a parameter fixed by the experimental data of one yield ratio such as \(\Lambda_c^+ / D^0\) in the calculations.

\section{C. Decay contributions}

To compare with the experimental data, we need to include strong and electromagnetic decay contributions from short-lived charmed resonances. In our case, we need only to consider decays of \(D^*\) mesons, \(J^P = (3/2)^+\) singly charmed baryons and \(\Sigma\) baryons. They all decay into a \(D\) meson or a \(J^P = (1/2)^+\) singly charmed baryon with a light particle such as a pion or a photon. In such a decay process, the momentum of the light daughter particle is so small that can be neglected compared to that of the heavy daughter charmed hadron. We can approximately take the momentum of the daughter charmed hadron equal to that of the mother charmed hadron. In this approximation, and take vector to pseudo-scalar meson production ratio as 1.5 \([36, 41]\), we find the results for the final \(D\) mesons as,

\[
f_{D^0}^{(fin)}(p) \approx 3.516 f_{D^0}(p),
\]

\[
f_{D^+}^{(fin)}(p) \approx 1.485 f_{D^+}(p),
\]

\[
f_{D^+}^{(fin)}(p) \approx 2.5 f_{D^+}(p).
\]
Similarly, we take $J^P = (1/2)^+$ to $J^P = (3/2)^+$ singly charmed baryon ratio as 2 [36], and obtain,

\[
\begin{align*}
    f_{\Omega^+}^{(fm)}(p) & \approx 5 f_{\Xi^+}(p), \\
    f_{\Xi^+}^{(fm)}(p) & \approx f_{\Omega^+}(p), \\
    f_{\Sigma^0}^{(fm)}(p) & \approx f_{\Sigma^+}(p), \\
    f_{\Xi^-}^{(fm)}(p) & \approx f_{\Xi^0}(p), \\
    f_{\Xi^0}^{(fm)}(p) & \approx 2.5 f_{\Xi^-}(p), \\
    f_{\Xi^+}^{(fm)}(p) & \approx 2.5 f_{\Xi^0}(p), \\
    f_{\Omega^-}^{(fm)}(p) & \approx 1.5 f_{\Omega^0}(p).
\end{align*}
\]

From these results, we see clearly that contributions from resonance decays are important for most of the charmed hadrons. For example, for $\Lambda^+_c$ baryons, about 80% are from decay contributions and only about 20% are directly produced ones.

### D. Ratios of different hadrons

We consider the production of charmed hadrons at midrapidity $y = 0$ and apply Eqs. (19) and (20) to obtain the $p_T$ dependence. From them, we calculate ratios of different charmed hadrons. In this case, the numbers of different flavor quarks are just replaced by the number densities $dN_{ji}/dy$ at $y = 0$.

We first consider yield ratios of strange to non-strange hadrons in the charm sector. They are given by,

\[
\begin{align*}
    \frac{D_+^s}{D_0^s} & = 0.711 \lambda_s f_{d_s}^s(x_{d_s}^f) f_{c_s}^s(x_{c_s}^f), \\
    \frac{\Sigma_+^s}{\Lambda_+^s} & = 0.5 \lambda_s f_{d_s}^s(x_{d_s}^f) f_{c_s}^s(x_{c_s}^f) f_{\Lambda^+_c}^s(x_{\Lambda^+_c}^f), \\
    \frac{\Omega_0^s}{\Sigma_0^s} & = 0.5 \lambda_s f_{d_s}^s(x_{d_s}^f) f_{c_s}^s(x_{c_s}^f) f_{\Xi^0}^s(x_{\Xi^0}^f), \\
    \frac{\Omega_+^s}{\Lambda_+^s} & = 0.25 \lambda_s^2 f_{d_s}^s(x_{d_s}^f) f_{c_s}^s(x_{c_s}^f) f_{\Xi^+}^s(x_{\Xi^+}^f), \\
    \frac{\Omega^-}{\Omega^+} & = 0.25 \lambda_s^2 f_{d_s}^s(x_{d_s}^f) f_{c_s}^s(x_{c_s}^f) f_{\Xi^-}^s(x_{\Xi^-}^f).
\end{align*}
\]

We can use them to calculate these yield ratios numerically. Here we can see clearly that these ratios depend not only on the strangeness suppression factor $\lambda_s$ but also on the ratios of $p_T$-distributions of different flavors of (anti-)quarks.

To see the qualitative features more explicitly, we note that, since $m_s$ is much larger than $m_d$ and $m_c$, it dominates the sum of $m$, with other quark mass. We have that $\bar{q}_{\bar{q}_{u_d}} \sim x_{\bar{q}_{u_d}} \sim x_{\bar{q}_{d}}$, $x_{\bar{q}_{c}} \sim x_{\bar{q}_{d}}$ and the latter should be much larger than the former. We take, in a rough approximation, the former as the same, and re-write Eqs. (31-34) in the form,

\[
\begin{align*}
    \frac{D_+^s}{D_0^s} & = 0.711 \lambda_s \frac{f_{d_s}^s(x_{d_s}^f)}{f_{c_s}^s(x_{c_s}^f)} \frac{f_{\Lambda^+_c}^s(x_{\Lambda^+_c}^f)}{f_{\Xi^0}^s(x_{\Xi^0}^f)}, \\
    \frac{\Sigma_+^s}{\Lambda_+^s} & = 0.5 \lambda_s \frac{f_{d_s}^s(x_{d_s}^f)}{f_{c_s}^s(x_{c_s}^f)} \frac{f_{\Xi^0}^s(x_{\Xi^0}^f)}{f_{\Xi^+}^s(x_{\Xi^+}^f)}, \\
    \frac{\Omega_0^s}{\Sigma_0^s} & = 0.5 \lambda_s \frac{f_{d_s}^s(x_{d_s}^f)}{f_{c_s}^s(x_{c_s}^f)} \frac{f_{\Xi^+}^s(x_{\Xi^+}^f)}{f_{\Xi^-}^s(x_{\Xi^-}^f)}, \\
    \frac{\Omega_+^s}{\Lambda_+^s} & = 0.25 \lambda_s^2 \frac{f_{d_s}^s(x_{d_s}^f)}{f_{c_s}^s(x_{c_s}^f)} \frac{f_{c_s}^s(x_{\Xi^+}^f)}{f_{c_s}^s(x_{\Xi^-}^f)}, \\
    \frac{\Omega^-}{\Omega^+} & = 0.25 \lambda_s^2 \frac{f_{d_s}^s(x_{d_s}^f)}{f_{c_s}^s(x_{c_s}^f)} \frac{f_{c_s}^s(x_{\Xi^-}^f)}{f_{c_s}^s(x_{\Xi^0}^f)}.
\end{align*}
\]

Here, we see clearly that, besides $\lambda_s$ (or $\lambda_c^2$), these ratios are proportional to the ratio of $s$ to $d$-quark spectrum (or squared) and the ratio of $c$-quark spectrum at slightly different $p_T$ values. Since the values of $x$’s involved here are all quite small for $d$ and $s$ quarks, they should be sensitive to the $d$- and $s$-quark $p_T$ spectra in the relatively low $p_T$ regions. The observed enhancement of strange to non-strange charged hadron ratios does not necessarily come from the enhancement of $\lambda_s$ but can also from the influence of the $p_T$-spectra of quarks.

Similarly, for baryon-to-meson ratios, we obtain from Eqs. (21-30) that,

\[
\begin{align*}
    \frac{\Lambda^+_c}{D^0} & = \frac{4.267}{2 + \lambda_s} \frac{\bar{A}_B \int f_{d}^{(n)}(x_{d}^{f})^2 f_{c}^{(n)}(x_{c}^{f}) \, d^2x} {f_{d}^{(n)}(x_{d}^{f}) f_{c}^{(n)}(x_{c}^{f})}, \\
    \frac{\Sigma_0^s}{D^0} & = \frac{0.711}{2 + \lambda_s} \frac{\bar{A}_B \int f_{d}^{(n)}(x_{d}^{f})^2 f_{c}^{(n)}(x_{c}^{f}) \, d^2x} {f_{d}^{(n)}(x_{d}^{f}) f_{c}^{(n)}(x_{c}^{f})}, \\
    \frac{\Omega_0^s}{D^0} & = \frac{1.684}{2 + \lambda_s} \frac{\bar{A}_B \int f_{d}^{(n)}(x_{d}^{f})^2 f_{c}^{(n)}(x_{c}^{f}) \, d^2x} {f_{d}^{(n)}(x_{d}^{f}) f_{c}^{(n)}(x_{c}^{f})}, \\
    \frac{\Omega^s}{D^0} & = \frac{2.133 \lambda_s}{2 + \lambda_s} \frac{\bar{A}_B \int f_{d}^{(n)}(x_{d}^{f})^2 f_{c}^{(n)}(x_{c}^{f}) \, d^2x} {f_{d}^{(n)}(x_{d}^{f}) f_{c}^{(n)}(x_{c}^{f})}, \\
    \frac{\Xi^0}{D^0} & = \frac{3}{2 + \lambda_s} \frac{\bar{A}_B \int f_{d}^{(n)}(x_{d}^{f})^2 f_{c}^{(n)}(x_{c}^{f}) \, d^2x} {f_{d}^{(n)}(x_{d}^{f}) f_{c}^{(n)}(x_{c}^{f})}, \\
    \frac{\Xi^-}{D^0} & = \frac{1.067 \lambda_s^2}{2 + \lambda_s} \frac{\bar{A}_B \int f_{d}^{(n)}(x_{d}^{f})^2 f_{c}^{(n)}(x_{c}^{f}) \, d^2x} {f_{d}^{(n)}(x_{d}^{f}) f_{c}^{(n)}(x_{c}^{f})}, \\
    \frac{\Omega^-}{D^0} & = \frac{1.5 \lambda_s}{2 + \lambda_s} \frac{\bar{A}_B \int f_{d}^{(n)}(x_{d}^{f})^2 f_{c}^{(n)}(x_{c}^{f}) \, d^2x} {f_{d}^{(n)}(x_{d}^{f}) f_{c}^{(n)}(x_{c}^{f})}.
\end{align*}
\]

We see again that, besides strangeness suppression factor $\lambda_s$, and $\bar{A}_B/\bar{A}_M$ (determined by the charmed baryon-to-meson ratio $R_{B/M}^{(c)}$), these ratios depend also on the $p_T$ spectra of quarks. Similarly, in the rough estimation with
\[ \frac{\Lambda^+}{D^0} \approx \frac{4.267}{\sqrt{\frac{1}{2} + \lambda_s A_M}} \frac{\lambda}{d} f^{(s)}(x_{ddc}^P T) \frac{f^{(s)}(x_{sdc}^P T)}{f^{(s)}(x_{n}^P T)}, \]
\[ \frac{\Sigma^0}{D^0} \approx \frac{0.711}{\sqrt{\frac{1}{2} + \lambda_s A_M}} \frac{\lambda}{d} f^{(s)}(x_{ddc}^P T) \frac{f^{(s)}(x_{sdc}^P T)}{f^{(s)}(x_{n}^P T)}, \]
\[ \frac{\Sigma^+}{D^*} \approx \frac{1.684}{\sqrt{\frac{1}{2} + \lambda_s A_M}} \frac{\lambda}{d} f^{(s)}(x_{ddc}^P T) \frac{f^{(s)}(x_{sdc}^P T)}{f^{(s)}(x_{n}^P T)}, \]
\[ \frac{\Xi_c^+}{D^*} \approx \frac{2.133}{\sqrt{\frac{1}{2} + \lambda_s A_M}} \frac{\lambda}{d} f^{(s)}(x_{ddc}^P T) \frac{f^{(s)}(x_{sdc}^P T)}{f^{(s)}(x_{n}^P T)}, \]
\[ \frac{\Omega^0}{D^*} \approx \frac{1.067}{\sqrt{\frac{1}{2} + \lambda_s A_M}} \frac{\lambda}{d} f^{(s)}(x_{ddc}^P T) \frac{f^{(s)}(x_{sdc}^P T)}{f^{(s)}(x_{n}^P T)} \times \frac{f^{(s)}(x_{scc}^P T)}{f^{(s)}(x_{ddc}^P T) \frac{f^{(s)}(x_{sdc}^P T)}{f^{(s)}(x_{n}^P T)}}, \]
\[ \frac{\Omega_c^0}{D^*} \approx \frac{1.54}{\sqrt{\frac{1}{2} + \lambda_s A_M}} \frac{\lambda}{d} f^{(s)}(x_{ddc}^P T) \frac{f^{(s)}(x_{scc}^P T)}{f^{(s)}(x_{sdc}^P T) \frac{f^{(s)}(x_{n}^P T)}{f^{(s)}(x_{n}^P T)}}. \]

Here we see clearly that these ratios, besides \( \lambda \), and \( A_b / A_M \), should be sensitive to the \( p_T \) spectrum of \( d \) or \( s \)-quarks in the relatively small \( p_T \) region. Since the \( p_T \) distribution \( f^{(s)}(p_T) \) of \( d \) or \( s \)-quarks in such \( p_T \) regions typically exhibits “rise-peak-fall” behaviors, we expect that these charged baryon-to-mesons ratios should have similar rise-peak-fall behaviors. We also expect that these ratios should have much stronger \( p_T \)-dependences than those of strange to non-strange ratios given by Eqs. (43,44) since the latter depends only on ratios of \( p_T \)-spectra of quarks but the former depends the spectrum itself. We would like to emphasize that Eqs. (43,44) are characteristic results in the equal-velocity quark combination (EVC). They can be directly used to test the combination hadronization mechanism of charm quarks and the validity of the EVC. They may also provide special ways to probe the properties of the QGP due to their close relationships to low \( p_T \) spectra of \( d \) and \( s \)-quarks.

III. APPLICATIONS IN Au-Au COLLISIONS AT RHIC

In this section, we apply the deduced results in Sec. III to calculate the charmed hadron production at midrapidity in Au-Au collisions at \( \sqrt{S_{NN}} = 200 \) GeV. We first present the normalized \( p_T \)-spectra of quarks and other related parameters. We then give numerical results of hadron yield ratios. We finally present predictions for \( p_T \) spectra and \( p_T \)-integrated yield densities of different charmed hadrons.

A. The normalized \( p_T \) spectra of quarks

In the midrapidity region of Au-Au collisions at high energies, we can neglect net quark contributions and take isospin symmetric quark distributions. In this case, we only need three parameters \( \lambda_s, \beta_q / M, dN_c / dy \) and normalized \( p_T \) spectra of \( d, s \)- and \( c \)-quarks as inputs. In QGP in heavy-ion collisions, \( \lambda_s \) takes values in the range 0.4 \( \sim \) 0.6 \([38]\). Here, we use yield ratio of anti-baryons, such as \( \Lambda \) to \( \bar{p} \) \([38, 42, 43]\) to fix the value of \( \lambda_s \), and the results in different centralities are shown in Table I. The \( K_{B/M}^{(s)} \) and \( dN_c / dy \) will be given whenever they are needed.

For \( f^{(s)}(p_T) \) and \( f^{(c)}(p_T) \), we take the modified-thermal pattern,
\[ f^{(s)}(p_T) \propto p_T^{\alpha_s} \exp(-\sqrt{p_T^2 + m_s^2}/T_q), \]
and extract the parameters \( T_q \) and \( \alpha_s \) from data \([43, 44]\) on the \( p_T \) spectra of \( \Xi^- \) baryons and \( \phi \) mesons under EVC.

The obtained results are given in Table I. We also plot \( f^{(s)}(p_T) \) and \( f^{(c)}(p_T) \) in Fig. 

| TABLE I: \( \lambda, \) and parameters for quark distributions in different centralities in Au-Au collisions at \( \sqrt{S_{NN}} = 200 \) GeV |
|---|---|---|---|---|---|
| Centrality | 0-10% | 10-20% | 20-40% | 40-60% | 60-80% |
| \( \lambda_s \) | 0.49 | 0.46 | 0.45 | 0.45 | 0.44 |
| \( T_d \) (GeV) | 0.27 | 0.26 | 0.25 | 0.24 | 0.23 |
| \( T_c \) (GeV) | 0.34 | 0.34 | 0.34 | 0.33 | 0.32 |
| \( \alpha_d \) | 0.65 | 0.65 | 0.65 | 0.62 | 0.58 |
| \( \alpha_c \) | 0.65 | 0.65 | 0.65 | 0.62 | 0.58 |
| \( \beta_c \) | 3.65 | 3.45 | 3.20 | 2.85 | 2.80 |

For \( c \)-quark, we adopt the hybrid pattern \([45]\), i.e.,
\[ f^{(c)}(p_T) \propto \alpha_c p_T \exp\left(-\sqrt{p_T^2 + m_c^2}/T_c\right) + \sqrt{p_T^2 + m_c^2}\exp\left(-\sqrt{p_T^2 + m_c^2}/T_c\right)) \]
based on the results after the propagation of charm quarks in the QGP medium in a Boltzmann transport approach \([45]\). The parameters \( \alpha_c, T_c \) and \( \beta_c \) are fixed using the data on the \( p_T \) distribution of \( D^0 \) \([12]\) and are given in Table I. We also plot in Fig. I(c) \( f^{(c)}(p_T) \) in different centralities in Au-Au collisions at \( \sqrt{S_{NN}} = 200 \) GeV. We see that there is a stronger suppression in more central collisions, especially in the region 4 GeV \( < p_T < 8 \) GeV. Shown in Fig. I(d) are the ratios of these distributions in different centralities to that in 60-80% centrality. We see very similar behaviors as that of the nuclear modification factor \( R_{CP} \) of \( D^0 \) mesons measured in \([12]\). In low \( p_T < 2 \)
GeV, $p_T$ distributions of charm quarks are almost the same for different centralities.

Comparing the results in Fig. 1(c) to those given by Fig. 1(a) and 1(b), we see that the $p_T$-dependence of $f_{c}^{(0)}(p_T)$ is much stronger than that of $f_{d}^{(0)}(p_T)$ or $f_{s}^{(0)}(p_T)$. We therefore expect that $f_{c}^{(0)}(p_T)$ should have large influences both on $p_T$-distributions of charmed hadrons and on the ratios given in the last section.

![FIG. 1: (Color online) Normalized $p_T$ spectra of (a) down, (b) strange, (c) charm quarks in different centralities in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. In panel (d), we have ratios of $p_T$ distributions of charm quarks in different centralities to that in 60-80% centrality.](image1)

B. Ratios of strange to non-strange hadrons

With Eqs. (31-34), we calculate yield ratios of strange to non-strange charmed hadrons. The results are shown in Fig. 2. We see that the results are basically consistent with the data [15]. We also see that the enhancement of $D_s^+/D^0$ in Au-Au collisions compared to those in $pp$ reactions at LHC [40] comes mainly from the strangeness suppression factor $\Lambda_0$ of the partonic matter in heavy ion collisions.

We also present the results for $\Xi^+/\Lambda^+$, $\Omega_c^0/\Xi^+$ and $\Omega_c^0/\Lambda^+$ in Fig. 2. To see effects of quark spectra in Eqs. (31-34) on these ratios, we plot also the constant factors in the corresponding panel in Fig. 2 respectively. We see that quark distributions slightly enhance these hadron yield ratios at intermediate $p_T$ region, but just the opposite at low $p_T$ area.

![FIG. 2: (Color online) Ratios of strange to non-strange charmed hadrons as functions of $p_T$ in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The filled cycles and squares are data that are taken from [15]. The filled triangles are $pp$ reaction data obtained at LHC [40]. The dashed-dotted line in each panel represents the constant 0.711$t_1$, 0.5$t_1$, 0.5$t_4$, or 0.25$t_1^2$ at $\Lambda_0 = 0.49$, respectively.](image2)

C. Baryon-to-meson ratios

To calculate baryon-to-meson ratios in the charm sector, we need the parameter $R^{(c)}_{B/M}$ to determine $\mathcal{A}_B/\mathcal{A}_M$ in Eqs. (39,40). It was fixed as $R^{(c)}_{B/M} = 0.43$ in $pp$ and $p$-Pb reactions at LHC energies [41,42]. In heavy ion collisions its value may be larger due to the baryon-beneficial environment. To study the effect of $R^{(c)}_{B/M}$, we present results of calculations on $\Lambda^+/D^0$ in Fig. 3 with $R^{(c)}_{B/M} = 0.43$, 0.60 and 1.00, respectively.

![FIG. 3: (Color online) (a) $\Lambda^+/D^0$ as a function of $p_T$ and (b) the $p_T$-integrated $\Lambda^+/D^0$ as a function of $N_{\text{part}}$ in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The data are taken from [14]. In (b), the open circles, squares and rhombuses connected with the dotted, solid and dashed lines to guide the eye are the calculated results with different values of $R^{(c)}_{B/M}$.](image3)
baryon productions, we calculate $p_T$ dependence of $\Lambda_c^+ / D^0$ in different centralities in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The results are given in Fig. 4. We see that they all exhibit similar rise-peak-fall behaviors. From central to peripheral collisions, peak values decrease from about 1.3 to 1.0 and the locations shift to lower $p_T$. This is due to the centrality dependence of $p_T$-distributions of quarks shown in Fig. 1, especially charm quarks. Other models such as the Catania model [30] that includes coalescence and fragmentation gives also the rise-peak-fall behavior of $\Lambda_c^+ / D^0$ as the function of $p_T$. However, the Catania model [30] predicts much flatter rising behavior at low $p_T$ region and almost no shift of peak location from RHIC to LHC energies. Experimental measurements can distinguish these different models and provide important insights into charm quark hadronization in high energy collisions.

Data of the $p_T$-integrated $\Lambda_c^+ / D^0$ from central to peripheral Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV show a decreasing trend [14] that is different from in light sectors [47] where baryon-to-meson ratios show little centrality dependence. We see that the calculated results in the same $p_T$ range (open squares in the figure) exhibit indeed such a trend consistent with the data [14]. However, if we take $p_T$ integrated from 0 GeV to 9 GeV (shown by the open crosses in the figure), this trend disappears and the result is essentially independent of the centrality and lower than those integrated from 3 GeV to 6 GeV. Such properties come from the centrality dependence of $c$-quark distributions given by Fig. 1 where we see that a strong dependence for larger $p_T$ but negligible in the small $p_T$ region. The results for $p_T$ integrated from 0 to 9 GeV are dominated by the small $p_T$ contributions.

Encouraged by the agreements with data available, we make predictions for other similar baryon-to-meson ratios. The results are given in Figs. 5 and 6.

In Fig. 5 we see that all the ratios exhibit similar rise-peak-fall behaviors as functions of $p_T$, and the peak locations change from central to peripheral collisions similar to $\Lambda_c^+ / D^0$. In Fig. 6 we see similar trend for all these $p_T$-integrated ratios, i.e., they all show increasing tendencies for results integrated in the $p_T$ region from 3 to 6 GeV but almost flat if integrated from 0 to 9 GeV.

At the end of this part, we would also like to emphasize that in our calculations, not merely $\Lambda_c^+$, but all charmed baryons are enhanced according to the overall ratio $R_{AA}^{c/J}$ that was taken as 0.60 in the calculations. This is different from other coalescence models where diquarks were introduced to intensely enhance the production of $\Lambda_c^+$, but to less enhance or even not enhance other charmed baryons [26]. Future measurements of different charm baryons should be very helpful in distinguishing different models and understanding the enhancement mechanism of charmed baryon production.

D. The $p_T$ spectra of charmed hadrons

Having the $p_T$ distributions of quarks, we not only calculate the ratios presented above but also the $p_T$-spectra of charmed hadrons obtained under EVC. We present the results in the following. To obtain not only the shape but also the magnitudes, we need the rapidity density of charm quarks $dN_c/dy$ (at $y = 0$) as an input. For this purpose, we estimate it by extrapolating $pp$ reaction data on differential cross section $d\sigma_c^{pp}/dy$ and take $dN_c/dy = \langle T_{AA} \rangle d\sigma_c^{pp}/dy$, where $\langle T_{AA} \rangle$ is the average nuclear overlap function and can be calculated by the Glauber model [48, 49]. We use $d\sigma_c^{pp}/dy = 130 \pm 30 \pm 26 \mu b$ recently measured at midrapidity in $pp$ at $\sqrt{s} = 200$ GeV [14], and obtain $dN_c/dy = 2.945 \pm 0.680 \pm 0.589$ in the most central 0-10% collisions in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Considering the data of $D^0$ [12], we take $dN_c/dy = 2.45$ for the centrality 0-10%. For other centralities 10-20%, 20-40%, 40-60%, 60-80%, we have $dN_c/dy = 1.54, 0.76,$
ratios as the function of $p$. We see that the results agree reasonably with the data. In Fig. 8, we show results for different charmed baryons. We also present $p_T$-integrated yield densities $dN/dy$ of different charmed hadrons at the midrapidity in different centralities in Table II. These results can all be used to test the mechanisms in particular EVC by future experiments.

FIG. 6: (Color online) Different $p_T$-integrated baryon-to-meson ratios as the function of $N_{\text{part}}$ in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Open circles are for $p_T$ integrated from 3 to 6 GeV and open rhombuses are from 0 to 9 GeV.

| Hadron | 0-10% | 10-20% | 20-40% | 40-60% | 60-80% |
|--------|-------|--------|--------|--------|--------|
| $D^0$  | 0.893 | 0.570  | 0.284  | 0.0898 | 0.0207 |
| $D^+$  | 0.377 | 0.241  | 0.120  | 0.0379 | 0.00874|
| $D^{*+}$ | 0.381 | 0.243  | 0.121  | 0.0383 | 0.00883|
| $D_s^+$ | 0.261 | 0.151  | 0.0713 | 0.0223 | 0.00495|
| $\Lambda_c^+$ | 0.636 | 0.412  | 0.207  | 0.0655 | 0.0152 |
| $\Sigma_c^+$ | 0.106 | 0.0687 | 0.0344 | 0.0109 | 0.00253|
| $\Xi_c^+$ | 0.106 | 0.0687 | 0.0344 | 0.0109 | 0.00253|
| $\Omega_c^+$ | 0.129 | 0.0758 | 0.0360 | 0.0113 | 0.00252|
| $\Omega_c^{++}$ | 0.0260 | 0.0139 | 0.00626 | 0.00194 | 0.000419|

FIG. 8: (Color online) Calculated results for $p_T$ spectra of singly charmed baryons in different centralities in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV.

In Fig. 7 we present $p_T$ spectra of different $D$ mesons in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV together with data available [12, 15]. We see that the results agree reasonably with the data. In Fig. 7, we present $p_T$ spectra of open charm mesons in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV.
IV. SUMMARY

Though not much data available yet, charm hadron production seems to provide an important test of different hadronization mechanisms in heavy ion collisions. In this paper, we have derived the $p_T$-dependence of open charm mesons and singly charmed baryons in the quark combination model under the EVC in ultra-relativistic heavy ion collisions. We present in particular analytic expressions of two groups of hadron yield ratios, the strange to non-strange charmed hadron ratios and baryon-to-meson ratios in terms of normalized $p_T$ spectra of quarks. We present normalized $p_T$ spectra of quarks and numerical results for these hadron yield ratios using these quark $p_T$ spectra. We found that the magnitude of the strange to non-strange charmed hadron ratios are mainly determined by the strangeness suppression factor and have weak $p_T$ dependences. In contrast, there is an obvious $p_T$ dependence for baryon-to-meson ratios determined by the quark $p_T$ spectra. The different baryon-to-meson ratios have similar $p_T$ and centrality dependences sensitive to $p_T$ distribution of $c$-quark. We have compared the results obtained with the data available and present predictions for future experiments. Further studies along this line can provide more sensitive tests to charm quark hadronization mechanisms and insight on properties of the QGP in heavy ion collisions.

Acknowledgements

We thank Zhang-Bu Xu for helpful discussions. This work was supported in part by the National Natural Science Foundation of China under grant 11505104, 11575100, 11675092 and 11975011 and by the Natural Science Foundation of Shandong Province, China under grant ZR2019YQ06.

[1] A. Andronic, et al., Eur. Phys. J. C 76 (3), 107 (2016).
[2] F. Prino and R. Rapp, J. Phys. G 43 (9), 093002 (2016).
[3] R. Rapp and H. van Hees, R.C. Hwa and X.-N. Wang eds., World Scientific (2010), arXiv:0903.1096 [hep-ph].
[4] N. Armesto, A. Dainese, C. A. Salgado, and U. A. Wiedemann, Phys. Rev. D 71 (5), 054027 (2005).
[5] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B 519, 199 (2001).
[6] P. Braun-Mun, Phys. Rep. 621, 76-126 (2016).
[7] S. Batsouli, S. Kelly, M. Gyulassy, and J. L. Nagle, Phys. Lett. B 557, 26 (2003).
[8] Nu Xu and Zhangbu Xu, Nucl. Phys. A 715, 587 (2003).
[9] Z. W. Lin and D. Molnar, Phys. Rev. C 68 (4), 044901 (2003).
[10] G. D. Moore and D. Teaney, Phys. Rev. C 71 (6), 064904 (2005).
[11] H. van Hees, V. Greco, and R. Rapp, Phys. Rev. C 73 (3), 034913 (2006).
[12] J. Adam et al. (STAR Collaboration), Phys. Rev. C 99 (3), 034908 (2019).
[13] S. Acharya et al. (ALICE Collaboration), JHEP 1810, 174 (2018).
[14] Sooraj Radhakrishnan (for the STAR Collaboration), Nucl. Phys. A 982, 659 (2019).
[15] Long Zhou (for the STAR Collaboration), Nucl. Phys. A 967, 620 (2017).
[16] S. Acharya et al. (ALICE Collaboration), arXiv:1906.03425v1 [nucl-ex].
[17] S. Acharya et al. (ALICE Collaboration), Phys. Lett. B 793, 212 (2019).
[18] S. Acharya et al. (ALICE Collaboration), JHEP 1804, 108 (2018).
[19] M. Lisovyi, A. Verbytskyi, O. Zenaiev, Eur. Phys. J. C 76 (7), 397 (2016).
[20] T. Sjostrand, S. Mrenna and P. Skands, JHEP 0605, 026 (2006).
[21] R. Maciuca and A. Szczeurek, Phys. Rev. D 98 (1), 014016 (2018).
[22] A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel, Phys. Lett. B 659, 149 (2008).
[23] I. Kuznetsova and J. Rafelski, Eur. Phys. J. C 51 (1), 113 (2007).
[24] A. Andronic et al., Eur. Phys. J. C 76 (3), 107 (2016).
[25] V. Greco, C. M. Ko, and R. Rapp, Phys. Lett. B 595, 202 (2004).
[26] Su Houng Lee, Kazuaki Ohnishi, Shigehiro Yasui, In-Kwon Yoo, and Che Ming Ko, Phys. Rev. Lett. 100, 222301 (2008).
[27] Y. P. Liu, C. Greiner, and A. Kostyuk, Phys. Rev. C 87 (1), 014910 (2013).
[28] M. He, R. J. Fries, and R. Rapp, Phys. Rev. C 86 (1), 014903 (2012).
[29] H. J. Xu, X. Dong, L. J. Ruan, Q. Wang, Z. B. Xu, and Y. F. Zhang, Phys. Rev. C 89 (2), 024905 (2014).
[30] S. Plumari, V. Minissale, S. K. Das, G. Coci, and V. Greco, Eur. Phys. J. C 78 (4), 348 (2018).
[31] Jiaxing Zhao, Shuzhe Shi, Nu Xu, and Pengfei Zhuang, arXiv:1805.10858v1 [hep-ph].
[32] Min He and Ralf Rapp, arXiv:1905.09216v1 [nucl-th].
[33] Sungtae Cho, Kij-Jia Sun, Che Ming Ko, Su Houng Lee and Yongseok Oh, arXiv:1905.09774v3 [nucl-th].
[34] R. Q. Wang, J. Song, and F. L. Shao, Phys. Rev. C 91 (1), 014909 (2015).
[35] J. Song, X. R. Gou, F. L. Shao, and Z. T. Liang, Phys. Lett. B 774, 516 (2017); X. R. Gou, F. L. Shao, R. Q. Wang, H. H. Li, and J. Song, Phys. Rev. D 96 (9), 094010 (2017).
[36] H.-h. Li, F.-l. Shao, J. Song, and R.-q. Wang, Phys. Rev. C 97 (6), 064915 (2018).
[37] J. Song, H.-h. Li, and F.-l. Shao, Eur. Phys. J. C 78 (4), 344 (2018).
[38] R. Q. Wang, F. L. Shao, J. Song, Q. B. Xie, and Z. T. Liang, Phys. Rev. C 86 (5), 054906 (2012).
[39] Xin Dong, Yen-Jie Lee, and Ralf Rapp, arXiv:1903.07709v1 [nucl-ex].
[40] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98 (3), 030001 (2018).
[41] B. Abelev et al. (ALICE Collaboration), JHEP 1207, 191 (2012).
[42] S. S. Adler et al. (PHENIX Collaboration), Phys. Rev. C 69, 034909 (2004).
[43] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 98, 062301 (2007).
[44] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. 99, 112301 (2007).
[45] F. Scardina, S. K. Das, V. Minissale, S. Plumari, and V. Greco, Phys. Rev. C 96 (4), 044905 (2017).
[46] S. Acharya et al. (ALICE Collaboration), Eur. Phys. J. C 77 (8), 550 (2017).
[47] F.-l. Shao, G.-j. Wang, R.-q. Wang, H.-h. Li, and J. Song, Phys. Rev. C 95 (6), 064911 (2017) and experimental references therein.
[48] R.J. Glauber and G. Matthiae, Nucl. Phys. B 21, 135 (1970).
[49] Constantin Loizides, Jason Kamin, and David dEnterria, Phys. Rev. C 97 (5), 054910 (2018).