SO(8) Colour as possible origin of generations

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Abstract

A possible connection between the existence of three quark-lepton generations and the triality property of SO(8) group (the equality between 8-dimensional vectors and spinors) is investigated.

1 Introduction

One of the most striking features of quark-lepton spectrum is its cloning property: \( \mu \) and \( \tau \) families seem to be just heavy copies of electron family. Actually we have two questions to be answered: what is an origin of family formation and how many generations do exist. Recent LEP data [1] strongly suggests three quark-lepton generations. Although Calabi-Yau compactifications of the heterotic string model can lead to three generations [2], there are many such Calabi-Yau manifolds, and additional assumptions are needed to argue why the number three is preferred [3].

There is another well-known example of particle cloning (doubling of states): the existence of antiparticles. Algebraically the charge conjugation operator defines an (outer) automorphism of underlying symmetry group [4, 5] and reflects the symmetry of the corresponding Dynkin diagram. We can thought that the observed triplication of states can have the same origin.

The most symmetric Dynkin diagram is associated with SO(8) group. So it is the richest in automorphisms and if SO(8) plays some dynamical role we
can hope that its greatly symmetrical internal structure naturally lead to the desired multiplication of states in elementary particle spectrum. what follows is an elaboration of this idea.

Although the relevant mathematical properties of SO(8) are known for a long time [6], they have not been discussed in context of the generation problem, to my knowledge.

2 Peculiarities of the SO(8) group

It is well known [7, 8] that the structure of a simple Lie algebra is uniquely defined by the length and angle relations among simple roots. This information is compactly represented by the Dynkin diagram. On such a diagram each simple root is depicted by a small circle, which is made black, if the root is a short one. Each pair of vertexes on the Dynkin diagram is connected by lines, number of which equals to $4 \cos^2 \varphi$, $\varphi$ being the angle between corresponding simple roots.

The main classification theorem for simple Lie algebras states that there exists only four infinite series and five exceptional algebras [7]. Among them $D_4$, the Lie algebra of the SO(8) group, really has the most symmetric Dynkin diagram

![Dynkin Diagram]

Actually only the symmetry with regard to the cyclic permutations of the $(\alpha_1, \alpha_3, \alpha_4)$ simple roots (which we call triality symmetry) is new, because the symmetry with regard to the interchange $\alpha_3 \leftrightarrow \alpha_4$ (last two simple roots) is shared by other $D_n$ Lie algebras also.
Due to this triality symmetry, $8_v = (1000), 8_c = (0010)$ and $8_s = (0001)$ basic irreps ( $(a_1, a_2, ..., a_r)$ being the highest weight in the Dynkin coordinates $[8]$) all have the same dimensionality $8$ - the remarkable fact valid only for the $D_4$ Lie algebra. The corresponding highest weights are connected by above mentioned triality symmetry. For other orthogonal groups $(10...0)$ is a vector representation, $(00...01)$ - a first kind spinor and $(00...10)$ - a second kind spinor. So there is no intrinsic difference between (complex) vectors and spinors in the eight-dimensional space $[9]$, which object is vector and which ones are spinors depends simply on how we have enumerated symmetric simple roots and so is a mere convention.

It is tempting to use this peculiarity of the SO(8) group to justify observed triplication of quark-lepton degrees of freedom. This possible connection between generations and SO(8) can be formulated most naturally in terms of octonions.

3 Octonions and triality

Eight-dimensional vectors and spinors can be realized through octonions $[10, 11]$, which can be viewed as a generalization of the complex numbers: instead of one imaginary unit we have seven imaginary units $e^2_A = -1, \ A = 1 \div 7$, in the octonionic algebra. The multiplication table between them can be found in $[11]$.

The octonion algebra is an alternative algebra (but not associative). This means that the associator $(x, y, z) = x \cdot (y \cdot z) - (x \cdot y) \cdot z$ is a skew symmetric function of the $x, y, z$ octonions.

the conjugate octonion $\bar{q}$ and the scalar product of octonions are defined as

$$\bar{q} = q_0 - q_A e_A \quad (p, q) = \frac{1}{2}(p \cdot \bar{q} + q \cdot \bar{p}) = (\bar{p}, \bar{q})$$

Let us consider eight linear operators $\Gamma_m, \ m = 0 \div 7$, acting in the 16-
dimensional bioctonionic space:

\[
\Gamma_m \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 0 & e_m \\ e_m & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} e_m \cdot q_2 \\ e_m \cdot q_1 \end{pmatrix}
\] (2)

Using the alternativity property of octonions, it can be tested that these operators generate a Clifford algebra

\[
\Gamma_m \Gamma_n + \Gamma_n \Gamma_m = 2 \delta_{mn} .
\]

(Note, that, because of nonassociativity, the operator product is not equivalent to the product of the corresponding octonionic matrices).

The eight-dimensional vectors and spinors can be constructed in the standard way from this Clifford algebra [12]. Namely, the infinitesimal rotation in the (k,l)-plane by an angle \(\theta\) is represented by the operator

\[
R_{kl} = 1 + \frac{1}{2} \theta \Gamma_k \Gamma_l ,
\]

and the transformation law for the (bi)spinor \(\Psi = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}\) is \(\Psi' = R_{kl} \Psi\).

For \(\Gamma_m\) given by Eq.2 the upper and lower octonionic components of \(\Psi\) transform independently under the 8-dimensional rotations:

\[
q'_1 = q_1 + \frac{1}{2} \theta e_k \cdot (\bar{e}_l \cdot q_1) \equiv q_1 + \theta F_{kl}(q_1)
\]

\[
q'_2 = q_2 + \frac{1}{2} \theta \bar{e}_k \cdot (e_l \cdot q_2) \equiv q_1 + \theta C_{kl}(q_2)
\] (3)

while a vector transformation law can be represented in the form

\[
x' = x + \theta \{ e_k(e_l, x) - e_l(e_k, x) \} \equiv x + G_{kl}(x)
\] (4)

One more manifestation of the equality between 8-dimensional vectors and spinors is the fact [9] that each spinor transformation from Eq.3 can be represented as a sum of four vector rotations

\[
F_{0A} = \frac{1}{2} (G_{0A} + G_{A_1B_1} + G_{A_2B_2} + G_{A_3B_3})
\] (5)
where $A_i, B_i$ are defined through the condition $e_{A_i} \cdot e_{B_i} = e_A$, and

$$F_{A_1 B_1} = \frac{1}{2}(G_{A_1 B_1} + G_{0A} - G_{A_2 B_2} - G_{A_3 B_3})$$

(6)

An algebraic expression of the equality between vectors and spinors in the eight-dimensional space is the following equation, valid for any two $x, y$ octonions [11]:

$$S_{kl}(\overline{x \cdot y}) = G_{kl}(x) \cdot y + x \cdot C_{kl}(y)$$

(7)

where $S_{kl} = KF_{kl}K$, $K$ being the (octonionic) conjugation operator $K(q) = \overline{q}$.

Eq.7 remains valid under any cyclic permutations of $(S_{kl}, G_{kl}, C_{kl})$. Note that

$$S_{kl} = \tau(G_{kl}), \quad C_{kl} = \tau(S_{kl}) = \tau^2(G_{kl})$$

(8)

where $\tau$ is an automorphism of the $D_4$ Lie algebra. We can call it the triality automorphism, because it performs a cyclic interchange between vector and spinors: $G_{kl}$ operators realize the (1000) vector representation, $S_{kl}$ - a first kind spinor (0001), and $C_{kl}$ - a second kind spinor (0010).

In general, vector and spinors transform differently under 8-dimensional rotations, because $G_{kl} \neq S_{kl} \neq C_{kl}$. But it follows from Eq.6 that $G_{A_1 B_1} - G_{A_2 B_2}$ and $G_{A_1 B_1} - G_{A_3 B_3}$ are invariant with regard to the triality automorphism, and so under such rotations 8-dimensional vector and both kinds of spinors transform in the same way. These transformations are automorphisms of the octonion algebra, because their generators act as derivations, as the principle of triality (Eq.7) shows. We can construct 14 linearly independent derivations of the octonion algebra, because the method described above gives two independent rotations per one imaginary octonionic unit $e_A = e_{A_i} \cdot e_{B_i}$. It is well known [10] that the derivations of the octonion algebra form $G_2$ exceptional Lie algebra. It was suggested [11, 13, 14] that the subgroup of this $G_2$ , which leaves the seventh imaginary unit invariant, can be identified with the colour
SU(3) group. If we define the split octonionic units \[11\]

\[
\begin{align*}
    u_0 &= \frac{1}{2}(e_0 + ie_7) & u_0^* &= \frac{1}{2}(e_0 - ie_7) \\
    u_k &= \frac{1}{2}(e_k + ie_{k+3}) & u_k^* &= \frac{1}{2}(e_k - ie_{k+3})
\end{align*}
\]  

where \(k = 1:\div 3\), then with regard to this SU(3) \(u_k\) transforms as triplet, \(u_k^*\) as antitriplet and \(u_0, u_0^*\) are singlets \[11\]. Therefore, all one-flavour quark-lepton degrees of freedom can be represented as one octonionic (super)field

\[
q(x) = l(x)u_0 + q_k(x)u_k + q_k^C(x)u_k^* + l^C(x)u_0^*
\]  

(10)

here \(l(x), q_k(x)\) are lepton and (three coloured) quark fields and \(l^C(x), q_k^C\) - their charge conjugates.

Note that it doesn’t matter what an octonion, first kind spinor, second kind spinor or vector we have in Eq.10, because they all transform identically under SU(3).

So SO(8) can be considered as a natural one-flavour quark-lepton unification group. We can call it also a generalized colour group in the Pati-Salam sense, remembering their idea about the lepton number as the fourth colour \[15\]. Then the triality property of the SO(8) gives a natural reason why the number of flavours should be triplicated.

4 Family formation and SO(10)

Unfortunately, SO(8) is not large enough to be used as a grand unification group: there is no room for weak interactions in it. This is not surprising, because weak interactions connect two different flavours and we are considering SO(8) as a one-flavour unification group.

The following observation points out the way how SO(8) can be extended to include the weak interactions. Because \(C_{AB} = F_{AB}\) and \(C_{A0} = -F_{A0}\) for
A, B = 1 ÷ 7, the SO(8) (Hermitian) generators for the (bi)spinor transformation Eq.3 can be represented as \( M_{AB} = -iF_{AB} \) and \( M_{A0} = -M_{0A} = -i\sigma_3 F_{A0} \). The last equation suggests to consider \( M_{A,7+k} = -i\sigma_k F_{A0} \) generators, where \( k = 1 ÷ 3 \) and summation to the modulus 10 is assumed, i.e. 7+3=0. So we have two new operators \(-i\sigma_1 F_{A0}\) and \(-i\sigma_2 F_{A0}\) which mix the upper and lower (bi)spinor octonionic components. Besides, if we consider these operators as rotations, then we have to add two extra dimensions and it is expected that SO(8) will be enlarged to SO(10) in this way and two different SO(8) spinors (two different flavours) will join in one SO(10) spinor (family formation).

Indeed, the following generators

\[
M_{AB} = -iF_{AB} \quad M_{7+i,7+j} = \frac{1}{2}\varepsilon_{ijk}\sigma_k
\]

\[
M_{A,7+k} = -i\sigma_k F_{A0} \quad M_{7+k,A} = -M_{A,7+k} ,
\]

where \( A, B = 1 ÷ 7 \) and \( i, j, k = 1 ÷ 3 \), really satisfy the SO(10) commutation relations

\[
[M_{\mu\nu}, M_{\tau\rho}] = -i(\delta_{\nu\tau}M_{\mu\rho} + \delta_{\mu\rho}M_{\nu\tau} - \delta_{\mu\tau}M_{\nu\rho} - \delta_{\nu\rho}M_{\mu\tau} ) .
\]

It is clear from Eq.12, that \( M_{\alpha\beta} (\alpha, \beta = 0, 7, 8, 9) \) and \( M_{mn} (m, n = 1 ÷ 6) \) subsets of generators are closed under commutation and commute to each other. They correspond to \( SU_L(2) \otimes SU_R(2) \) and \( SU(4) \) subgroups of SO(10). The generators of the \( SU_L(2) \otimes SU_R(2) \) can be represented as

\[
T^i_L = \frac{1}{2}\sigma_i u_0 \quad T^i_R = \frac{1}{2}\sigma_i u_0^* .
\]

So multiplication by \( u_0 \) or \( u_0^* \) split octonionic units plays the role of projection operator on the left and right weak isospin, respectively.

The SU(4) generators can be also expressed via split octonionic units:

\[
E_{ij} = -u_i \cdot (u_j^* ) \quad E_{0i} = -u_j \cdot (u_k) \quad E_{i0} = u_j^* \cdot (u_k^* ) .
\]
In the last two equations \((i, j, k)\) is a cyclic permutation of \((1, 2, 3)\) and it is assumed that, for example, \(E_{ij}(q) = -u_i \cdot (u_j^* \cdot q)\).

Under SU(4) \(u_\alpha\), \(\alpha = 0 \div 3\), transforms as a 4 fundamental representation and \(u_\alpha^*\) - as its conjugate \(4^*\). So SU(4) unifies \(u_0\) colour singlet and \(u_k\) colour triplet in one single object, and therefore plays the role of the Pati-Salam group \[15\].

Note that all one-family (left-handed) quark-lepton degrees of freedom are unified in one bioctonionic (super)field (16-dimensional SO(10) spinor) \[16\]

\[
\Psi_L = \left( \begin{array}{c} \nu(x) \\ l(x) \end{array} \right)_L u_0 + \left( \begin{array}{c} q_i^u(x) \\ q_i^d(x) \end{array} \right)_L u_i + \left( \begin{array}{c} l^C(x) \\ \nu^C(x) \end{array} \right)_L u_0^* + \left( \begin{array}{c} q_i^{dC}(x) \\ q_i^{uC}(x) \end{array} \right)_L u_i^*. \tag{15}
\]

The fact that we should take the Weyl (left-handed) spinors instead of Dirac (that the weak interactions are flavour chiral) indicates close interplay between space-time (space inversion) and internal symmetries \[17\].

Thus our construction leads to SO(10) as a natural one – family unification group. But doing so, we have broken the triality symmetry: only the spinoric octonions take part in family formation and the vectoric octonion is singled out. Can we in some way restore equivalence between vector and spinor octonions?

First of all we need to realize vector octonion in terms of the SO(10) representation and this can be done by means of \(2 \times 2\) octonionic Hermitian matrices, which together with the symmetric product \(X \circ Y = \frac{1}{2}\left(XY + YX\right)\) form the \(M_2^8\) Jordan algebra \[18\]. SO(10) appears as a (reduced) structure group of this Jordan algebra \[18\] and 10-dimensional complex vector space generated by the \(M_2^8\) basic elements (the complexification of \(M_2^8\)), gives \((10000)\) irreducible representation of its \(D_5\) Lie algebra.

Thus, now we have at hand the realization of spinoric octonions as a 16-dimensional SO(10) spinor \(\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}\) and vectoric octonion as a 10-dimensional SO(10) vector \(\begin{pmatrix} \alpha \\ \bar{q} \\ \beta \end{pmatrix}\). How to unify them? The familiar unitary symmetry example how to unify an isodublet and an isotriplet in the \(3 \times 3\) complex
Hermitian matrix can give a hint and so let us consider $3 \times 3$ octonion Hermitian matrices.

5  \textit{E}$_6$, \textit{triality and family triplication}

Together with the symmetric product, $3 \times 3$ octonion Hermitian matrices form the $M_3^8$ exceptional Jordan algebra [10]. A general element from it has a form

$$X = \begin{pmatrix} \alpha & x_3 & \bar{x}_2 \\ \bar{x}_3 & \beta & x_1 \\ x_2 & \bar{x}_1 & \gamma \end{pmatrix}$$

and can be uniquely represented as $X = \alpha E_1 + \beta E_2 + \gamma E_3 + F_1^{x_1} + F_2^{x_2} + F_3^{x_3}$. This is the Peirce decomposition [19] of $M_3^8$ relative to the mutually orthogonal idempotents $E_i$.

A reduced structure group of $M_3^8$ is $E_6$ exceptional Lie group [18]. Its Lie algebra consists of the following transformations:

1) 24 linearly independent $\{a_1, a_2, a_3\}$ generators, which are defined as $\{a_1, a_2, a_3\} X = [A, X]$, where

$$A = \begin{pmatrix} 0 & a_3 & \bar{a}_2 \\ -\bar{a}_3 & 0 & a_1 \\ -\bar{a}_2 & -\bar{a}_1 & 0 \end{pmatrix}$$

is a $3 \times 3$ octonion anti-Hermitian matrix with zero diagonal elements.

2) $\{\Delta_1, \Delta_2, \Delta_3\}$ triality triplets (Eq.7 and Eq.8), which annihilate $E_i$ idempotents and in the $F_i$ Peirce components act according to

$$\{\Delta_1, \Delta_2, \Delta_3\} F_i^a = F_i^{\Delta_i(a)}.$$

Because a triality triplet is uniquely defined by its first element: $\Delta_2 = \tau(\Delta_1)$ and $\Delta_3 = \tau(\Delta_2)$, this gives extra 28 linearly independent generators. Together with $\{a_1, a_2, a_3\}$ type operators, they form 52-dimensional $F_4$ exceptional Lie algebra [10, 20].
3) $T^\wedge$ linear transformations of $M^8_3$, defined as $T^\wedge X = T \circ X$, where $T$ is any element from $M^8_3$ with zero trace.

The way how $E_6$ exceptional Lie algebra was constructed shows the close relationship between $D_5$ and $E_6$: the latter is connected to the exceptional Jordan algebra $M^8_3$ and the former - to the $M^8_2$ Jordan algebra [21]. But $M^8_3$ has three $M^8_2$ (Jordan) subalgebras, consisting correspondingly from elements

$$
\begin{pmatrix}
\alpha & a & 0 \\
\bar{a} & \beta & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad
\begin{pmatrix}
\alpha & 0 & \bar{a} \\
0 & 0 & 0 \\
a & 0 & \beta
\end{pmatrix}, \quad
\begin{pmatrix}
0 & 0 & 0 \\
0 & \alpha & a \\
0 & \bar{a} & \beta
\end{pmatrix},
$$

therefore $E_6$ has three equivalent $D_5$ subalgebras. Let $D^i_5$ be that $D_5$ subalgebra of $E_6$ which acts in the $M^8_2$ Jordan algebra, formed from the $F^a_i$, $E_j$, $E_k$ elements. It consists from $\{\Delta_1, \Delta_2, \Delta_3\}, (F^a_i)^\wedge, (E_j - E_k)^\wedge, \{\delta_{i1}a_1, \delta_{i2}a_2, \delta_{i3}a_3\}$ operators and their (complex) linear combinations. Therefore the intersection of these $D^i_5$ subalgebras is $D_4$ formed from the $\{\Delta_1, \Delta_2, \Delta_3\}$ triality triplets, and their unification gives the whole $E_6$ algebra.

The triality automorphism for $D_4$ can be continued on $E_6$: 

![Diagram](image.png)
It can be verified [22] that Eq.16 actually gives an $E_6$ automorphism. This $\tau$ automorphism causes a cyclic permutation of the $D_5^i$ subalgebras

\[
\tau : \quad D_5^1 \to D_5^2 \to D_5^3 \to D_5^1
\]

So $E_6$ exceptional Lie group is very closely related to triality. Firstly, it unifies the spinoric and vectoric octonions in one 27-dimensional irreducible representation (algebraically they unify in the $M_8^3$ exceptional Jordan algebra). Secondly, its internal structure also reveals a very interesting triality picture:

The equality between $SO(8)$ spinors and vectors now results in the equality of three $SO(10)$ subgroups (in the existence of the triality automorphism $\tau$, which interchanges these subgroups).

To form a quark-lepton family, we have to select one of these $SO(10)$ subgroups. But a priori there is no reason to prefer any of them. The simplest possibility to have family formation which respects this equality between various $SO(10)$ subgroups ($E_6$ triality symmetry) is to take three copies of $M_3^8$ and arrange matters in such a way that in the first $M_3^8$ the first $SO(10)$ subgroup
acts as a family formatting group, in the second $M_3^8$ - the second SO(10) and in the third one - the third SO(10):

More formally, we have $27 + 27 + 27$ reducible representation of $E_6$, such that when we go from one irreducible subspace to another, representation matrices are rotated by the triality automorphism $\tau$.

6 Conclusion

If we take seriously that octonions play some underlying dynamical role in particle physics and SO(8) appears as a one-flavour unification group, then the triality property of SO(8) gives a natural reason for the existence of three quark-lepton generations. Family formation from two flavours due to weak interactions can be connected naturally enough to SO(10) group, but with the triality symmetry violated. An attempt to restore this symmetry leads to the exceptional group $E_6$ and three quark-lepton families.
References

[1] ALEPH Collab., Buskulic D. et al., Phys. Lett. B313, 520 (1993); OPAL Collab., Akrawy M. Z. et al., Z. Phys. C50, 373 (1991); L3 Collab., Adriani O. et al., Phys. Lett. B292, 463 (1992).

[2] Candelas P., Lutken C. A. and Schimmrigk R., Nucl. Phys. B306, 113 (1988).

[3] Koca M., Phys. Lett. B271, 377 (1991).

[4] Okubo S. and Mukunda N., Ann. Phys. (N.Y.) 36, 311 (1966); Dothan Y., Nuovo Cimento 30, 399 (1963).

[5] Michel L., Invariance in Quantum Mechanics and Group Extension, Group Theoretical Concepts and Methods in Elementary Particle Physics, edited by F. Gursey (Cordon and Breach, New York, 1964).

[6] Cartan E., The Theory of Spinors, (MIT, Cambridge, MA, 1966).

[7] Jacobson N., Lie Algebras (Wiley-Interscience, New York, 1962); Goto M. and Grosshans F.D., Semisimple Lie algebras, Lecture Notes in Pure and Applied Mathematics, v.38 (Dekker, New York and Basel, 1978); Rowlatt P. A., Group Theory and Elementary Particles, (Longmans, London and Tonbridge, 1966).

[8] Slansky R., Phys. Rep. 79, 1 (1981); Dynkin E. B., Proc. of Moscow Math. Soc. 1, 39 (1952) (in Russian).

[9] Gamba A., J. Math. Phys. 8, 775 (1967).

[10] Freudenthal H., Matematica 1, 117 (1957) (in Russian).

[11] Gunaydin M. and Gursey F., J. Math. Phys. 14, 1651 (1973).
[12] Brauer R. and Weyl H., Amer. J. Math. 57, 447 (1935); Pais A., J. Math. Phys. 3, 1135 (1962).

[13] Gunaydin M. and Gursey F., Phys. Rev. D9, 3387 (1974); Gursey F., in The Johns Hopkins Workshop on Current Problems in High Energy Particle Theory, 1974, p. 15.

[14] Casalbuoni R., Domokos G. and Kovesi-Domokos S., Nuovo Cimento A31, 423 (1976); A33, 432 (1976).

[15] Pati J. C. and Salam A., Phys. Rev. D8, 1240 (1973).

[16] Silagadze Z. K., Proc. of Tbilisi University 220, 34 (1981) (in Russian).

[17] Lee T. D. and Wick G. C., Phys. Rev. 148, 1385 (1966); Silagadze Z. K., Sov. J. Nucl. Phys. 55, 392 (1992).

[18] Jacobson N., Structure and Representations of Jordan Algebras, Amer. Math. Soc. Coloqq. Publ., vol.39 (Amer. Math. Soc., Providence, R.I., 1969); McCrimmon K., Bull. Amer. Math. Soc. 84, 612 (1978).

[19] Schafer R. D., An Introduction to Non-Associative Algebras, (Academic Press, New York, 1966); Zhevlakov K. A. et al., Rings Close to Associative, (Nauka, Moscow, 1978, in Russian).

[20] Freudenthal H., Adv. Math. 1, 145 (1964); Jacobson N., Exceptional Lie Algebras (Dekker, New York, 1971); Ramond P., Introduction to Exceptional Lie Groups and Algebras, Caltech preprint CALT-68-577, 1976.

[21] Buccela F., Falcioni M. and Pugliese A., Lett. Nuovo Cimento 18, 441 (1977); Sudbery A., J. Phys. A17, 939 (1984); Silagadze Z. K., Proc. of Tbilisi University 235, 5 (1982) (in Russian).

[22] Silagadze Z. K., SO(8) Colour as Possible origin of Generations, preprint BUDKERINP 93-93, Novosibirsk, 1993.