Robust Designs of Beamforming and Power Splitting for Distributed Antenna Systems with Wireless Energy Harvesting

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Abstract—In this paper, we investigate a multiuser distributed antenna system with simultaneous wireless information and power transmission under the assumption of imperfect channel state information (CSI). In this system, a distributed antenna port with multiple antennas supports a set of mobile stations who can decode information and harvest energy simultaneously via a power splitter. To design robust transmit beamforming vectors and the power splitting (PS) factors in the presence of CSI errors, we maximize the average worst-case signal-to-interference-plus-noise ratio (SINR) while achieving individual energy harvesting constraint for each mobile station. First, we develop an efficient algorithm to convert the max-min SINR problem to a set of “dual” min-max power balancing problems. Then, motivated by the penalty function method, an iterative algorithm based on semi-definite programming (SDP) is proposed to achieve a local optimal rank-one solution. Also, to reduce the computational complexity, we present another iterative scheme based on the Lagrangian method and the successive convex approximation (SCA) technique to yield a suboptimal solution. Simulation results are shown to validate the robustness and effectiveness of the proposed algorithms.

I. INTRODUCTION

For the past decade, there has been a considerable evolution of wireless networks to satisfy demands on high speed data. Since resources shared among users are limited, a capacity increase is technically challenging in the wireless networks. Recently, a distributed antenna system (DAS) has received a lot of attentions as a new cellular communication structure to expand coverage and increase sum rates [1]–[3].

Unlike conventional cellular systems where all antennas are co-located at the cell center, distributed antenna (DA) ports of the DAS are separated geographically in a cell and are connected with each other by backhaul links [4]. Each DA port in the DAS is usually equipped with its own power amplifier at the analog front-end [4] [5]. Thus, individual power constraint at each antenna should be considered for the DAS unlike the conventional systems which normally impose sum power constraint [6].

In the meantime, one of the limits in current cellular communication systems is the short lifetime of batteries. To combat the battery problem of mobile users, simultaneous wireless information and power transmission (SWIPT) has been studied in [6]–[13]. With the aid of the SWIPT, users can charge their devices based on the received signal [8] [9]. To realize the SWIPT, a co-located receiver has been proposed [10], which employs a power splitter to perform energy harvesting (EH) and information decoding (ID) at the same time [11]. By adopting the power splitting (PS) receiver, the SWIPT scheme for multiple-input single-output (MISO) downlink systems has been examined in [8] and [11] where perfect channel state information at the transmitter (CSIT) was assumed. In practice, however, due to channel estimation errors and feedback delays, it is not possible to obtain perfect CSIT [14]–[17].

On the other hand, some recent works have investigated SWIPT in DAS [18]–[25]. [18] has provided several intuitions and revealed the challenges and opportunities in DAS SWIPT systems. In order to improve energy efficiency of SWIPT, the application of advanced smart antenna technologies has been focused in [19]. In [20], a power management strategy has been studied to supply maximum wireless information transfer (WIT) with minimum wireless energy transfer (WET) constraint for adopting PS. Moreover, a tradeoff between the power transfer efficiency and the information transfer capacity has been introduced in [21]. The work in [22] examined a design of robust beamforming and PS for multiuser downlink DAS SWIPT. However, only one antenna was considered in each DA port. The authors in [23] investigated resource allocation for DAS SWIPT systems based on the worst-case model, where per-DA port power constraint was adopted. In [24], a few open issues and promising research trends in the wireless powered communications area with DAS were introduced. In addition, to achieve a balance between transmission power and circuit power, [25] studies a system utility minimization problem in a DAS SWIPT system via joint design of remote radio heads selection and beamforming. However, joint optimal design of transmit beamforming and the receive PS factor for SWIPT in DAS PS-based systems with multiple transmit antennas of each DA port, has not been considered in the literature yet.
Motivated by the existing literature [18–25], in this paper, we study a joint design of robust transmit beamforming at the DA port and the receive PS factors at mobile stations (MSs) in multiuser DAS SWIPT systems with imperfect CSI. Channel uncertainties are modeled by the worst-case model as in [22]. Our aim is to maximize the worst-case signal-to-interference-and-noise ratio (SINR) subject to EH constraint and per-DA port power constraint. The contributions of this work are summarized as follows:

- For a given SINR target, the original problem is decomposed into a sequence of min-max per-DA port power balancing problems. In order to convert the non-convex constraint into linear matrix inequality (LMI), Schur complement is used to derive the equivalent forms of the SINR constraint and the EH constraint. Furthermore, we prove that a solution of the relaxed semi-definite program (SDP) is always rank-two. Also, to recover a near-optimal rank-one solution, we employ a penalty function method instead of the conventional Gaussian randomization (GR) technique.
- To reduce the computational complexity, another formulation is expressed for the minimum SINR maximization problem. By employing the Lagrangian multiplier method and the first order Taylor expansion, the SINR constraint can be approximately reformulated into two convex forms with linear constraints. Then, we propose an iterative algorithm based on the successive convex approximation (SCA) to find a suboptimal solution.

Simulations evaluation have been conducted to provide the robustness and effectiveness of the proposed algorithms. The performance is also compared with other recent conventional schemes in this area. We show that the proposed algorithms have the superior performances in terms of average worst-case rate by extensive simulation results.

The remainder of this paper is organized as follows: in Section II, we describe a system model for the multiuser DAS SWIPT and formulate the worst-case SINR maximization problem subject to per-DA port power and EH constraint. Section III derives the proposed robust joint designs. In Section IV, we present the computational complexity of the proposed algorithms. Simulation results are presented in Section V. Finally, Section VI concludes this paper.

**Notation:** Lower-case letters are denoted by scalars, boldface lower-case letters are used for vectors, and boldface upper-case letters means matrices. \( \|x\| \) represents the Euclidean norm of a complex vector \( x \) and \( \text{diag}(x) \) denotes the diagonal matrix whose diagonal element vector is \( x \). \( |z| \) stands for the norm of a complex number \( z \). For a matrix \( M \), \( M^T \), \( M^H \), \( \text{rank}(M) \), and \( |M|_{i,j} \) are defined as trace, transpose, conjugate transpose, rank, and the \((i,j)\)-th element, respectively. \( \lambda_{\text{max}}(M) \) denotes the maximum eigenvalue of \( M \), and \( \text{vec}(M) \) stacks the elements of \( M \) in a column vector. \( \mathbf{I} \) defines an identity matrix, \( \mathbb{C}^{M \times N} \), \( \mathbb{R}^{M \times N} \), and \( \mathbb{R}^{M \times N} \) are the set of complex matrices, Hermitian matrices and real matrices of size \( M \times N \), respectively. \( \mathbb{H}^+ \) equals the set of positive semi-definite (PSD) Hermitian matrices. \( 0_{M \times L} \) is a null matrix with size \( M \times L \).

**II. SYSTEM MODEL AND PROBLEM FORMULATION**

In Fig. 1, we describe a single cell system model for the multiuser downlink DAS scenario with SWIPT. The DAS consists of \( M \) DA ports and \( K \) single-antenna MSs. It is assumed that each DA port is equipped with \( N_T \) antennas, which have individual power constraint. All DA ports are physically connected to the main processing unit (MPU) through fiber optics or an exclusive radio frequency (RF) link. Furthermore, all DA ports share the information of user distance and user data, but do not require CSI of all MSs as in [4]. The MS distance information can be simply obtained by measuring the received signal strength indicator [5]. Note that one MS can be supported by several DA ports.

We consider the channel model for DAS which contains both small scale and large scale fading [5]. We denote the channel between the \( m \)-th DA port \((m = 1, \ldots, M)\) and the \( k \)-th MS \((k = 1, \ldots, K)\) as \( h_{m,k} \equiv d_{m,k}^{-\gamma/2} h_{m,k} \), where \( d_{m,k} \) stands for the distance between the \( m \)-th DA port and the \( k \)-th MS, \( \gamma \) indicates the path loss exponent, and \( h_{m,k} \in \mathbb{C}^{N_T \times 1} \) equals the channel vector for small scale fading. For the \( k \)-th MS, the channel vector is given as \( h_k = \{ h_{1,k}^T, \ldots, h_{M,k}^T \}^T \).

Due to channel estimation and quantization errors, CSI is imperfect at each DA port and we assume that the uncertainty of the channel vectors is determined by \( \mathcal{H}_k \) as an Euclidean ball [10] [13] as

\[
\mathcal{H}_k = \left\{ \bar{h}_k + \Delta h_k \mid \Phi_k \Delta h_k \leq \varepsilon_k^2 \right\}, \quad k = 1, 2, \ldots, K
\]  

(1)

where the ball is centered around the actual value of the estimated CSI vector \( \bar{h}_k \) from \( M \) DA ports to the \( k \)-th MS, \( \Delta h_k \in \mathbb{C}^{M N_T \times 1} \) is the norm-bounded uncertainty vector, \( \Phi_k \in \mathbb{C}^{M N_T \times M N_T} \) defines the orientation of the region, and \( \varepsilon_k \) represents the radius of the ball.

During one time slot, \( K \) independent signal streams are conveyed simultaneously to \( K \) MSs. Specifically, the transmit beamforming vector \( \mathbf{v}_m^k \in \mathbb{C}^{N_T \times 1} \) is allocated for the \( k \)-th
MS at the $m$-th DA port. Thus, we denote the joint transmit beamformer vector $\mathbf{v}_k \in \mathbb{C}^{MN_T \times 1}$ used by $M$ DS ports for the $k$-th MS as $\mathbf{v}_k = \text{vec} \left( \begin{bmatrix} \mathbf{v}_{1,k} & \mathbf{v}_{2,k} & \ldots & \mathbf{v}_{M,k} \end{bmatrix} \right)$. Then, the transmitted signal to the $k$-th MS is obtained by

$$\mathbf{x}_k = \mathbf{v}_k s_k, \quad \forall k,$$

where $s_k \sim \mathcal{CN}(0, 1)$ indicates the corresponding transmitted data symbol for the $k$-th MS, which is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) random variable with zero mean and unit variance. We assume that each DA port has its own power constraint $P_m (m = 1, \ldots, M)$. Let us define an $M N_T \times M N_T$ square matrix $\mathbf{D}_m \triangleq \text{diag}(0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0)$. Then, the PS divides the received signal power into two parts using a square matrix $\mathbf{G}$ obtained as $\mathbf{G} = \frac{\mathbf{D}_m}{\sqrt{\text{tr} (\mathbf{D}_m)}}$, where $\mathbf{G}$ is the constant that accounts for the energy conversion efficiency for the EH of the $k$-th MS. It is also assumed that each DA port should be larger than a given threshold, and each DA port needs to satisfy per-DA port power constraint. Hence, our aim is to jointly optimize the transmit beamforming vector and the PS factor by maximizing the minimum SINR subject to EH constraint and per-DA power constraint. Then, by incorporating the norm-bounded channel uncertainty model in (1), the robust optimization problem is expressed as

$$\begin{align*}
\max_{\{\mathbf{v}_k\}_k, \{\rho_k\}_k} & \quad \text{SINR}_k\{\{\mathbf{v}_k\}_k, \{\rho_k\}_k\} \\
\text{s.t.} & \quad \zeta_k (1 - \rho_k) \sum_{j=1}^K |h_k^H v_j|^2 + \sigma_k^2 \leq e_k, \quad \forall k, \quad (3b) \\
& \quad \sum_{k=1}^K \text{tr} (\mathbf{D}_m \mathbf{v}_k \mathbf{v}_k^H) \leq P_m, \quad \forall m, \quad (3c) \\
& \quad 0 < \rho_k \leq 1, \quad \forall k, \quad (3d)
\end{align*}$$

where $e_k$ represents the required harvested power of the $k$-th MS. Problem (3) is non-convex due to coupled variables $\{\rho_k\}_k$ and $\{\mathbf{v}_k\}_k$ in both the objective function and the EH constraint, and thus, is difficult to solve efficiently.

### III. Proposed Robust Joint Designs

In this section, we propose two robust joint design algorithms for problem (3). First, we present a bisection search method which generates a local optimal rank-one solution. To reduce the computational complexity, we then introduce an SCA based algorithm to achieve a suboptimal solution.

#### A. Proposed Method Based on Bisection Search

To make problem (3) tractable, we decompose the problem into a set of the min-max per-DA port power balancing problems, one for each given SINR target $\Gamma > 0$ [15]. Using bisection search over $\Gamma$, the optimal solution to problem (3) can be obtained by solving the corresponding min-max per-DA port power balancing problem with different $\Gamma$. Then, for a given $\Gamma$, we focus on the following min-max per-DA port power balancing problem as

$$\begin{align*}
\min_{\{\mathbf{v}_k\}_k, \{\rho_k\}_k} & \quad \max_{1 \leq m \leq M} \sum_{k=1}^K \frac{\text{tr}(\mathbf{D}_m \mathbf{v}_k \mathbf{v}_k^H)}{P_m} \\
\text{s.t.} & \quad \zeta_k (1 - \rho_k) \sum_{j=1}^K |h_k^H v_j|^2 + \sigma_k^2 \geq e_k, \quad \forall k, \quad (4b) \\
& \quad \text{SINR}_k\{\{\mathbf{v}_k\}_k, \{\rho_k\}_k\} \geq \Gamma, \quad \forall k, \quad (4c) \\
& \quad 0 < \rho_k \leq 1, \quad \forall k, \quad (4d)
\end{align*}$$

We represent $\alpha^*(\Gamma)$ as the optimal objective value of problem (4). Note that based on the equation $\alpha^*(\Gamma) = 1$ [22, Lemma 2], we can obtain the optimal beamforming solution for problem (3). Problem (4) is still non-convex in terms of the non-convex objective function (4a). First, we tackle the objective function (4a) by introducing an auxiliary variable $\alpha$. Then, the min-max per-DA port power balancing problem (4) can be rewritten as

$$\begin{align*}
\min_{\{\mathbf{v}_k\}_k, \{\rho_k\}_k, \alpha, \{\mathbf{h}_k\}_k} & \quad \alpha \\
\text{s.t.} & \quad \sum_{k=1}^K \text{tr}(\mathbf{D}_m \mathbf{v}_k \mathbf{v}_k^H) \leq \alpha P_m, \quad \forall m, \quad (5b)
\end{align*}$$

We can see that problem (5) has semi-infinite constraints (4b) and (4c), which are non-convex. To make the constraint (4b)
tractable, the following lemma is introduced to convert \((45)\) into a quadratic matrix inequality (QMI).

**Lemma 1:** (Schur complement \([26]\)) Let \(N\) be a complex Hermitian matrix as
\[
N = N^H = \begin{bmatrix} Y_1 & Y_2 & Y_3 \\ Y_2 & Y_4 & Y_5 \\ Y_3 & Y_5 & Y_6 \end{bmatrix}.
\]
Then, we have \(N \succ 0\) if and only if \(Y_1 - Y_2 Y_3^{-1} Y_2 \succeq 0\) with \(Y_3 > 0\), or \(Y_4 - Y_2 Y_3^{-1} Y_2 \succeq 0\) with \(Y_1 > 0\).

Let us define an \(MN_T \times MN_T\) square matrix \(V_k\) as \(V_k = v_k v_k^H\). By utilizing Lemma 1, the constraint \((45)\) can be converted into
\[
\begin{bmatrix} \zeta_k (1 - \rho_k) \\ \sqrt{\zeta_k} \end{bmatrix} (\hat{h}_k + \Delta h_k)^H R (\hat{h}_k + \Delta h_k) + \sigma_k^2 \succeq 0, \tag{6}
\]
where \(R \triangleq \sum_{k=1}^{K} V_k\). Note that \((6)\) is still non-convex. In order to remove the channel uncertainty in \((6)\), the following lemma is required to convert the constraint \((6)\) into linear matrix inequality (LMI).

**Lemma 2:** \([30, \text{Theorem } 3.5]\) Let us denote \(U_k \in \mathbb{C}, \) for \(k \in [1, 6]\). If \(T_i \succeq 0\) for \(i = 1, 2\), then the following QMI
\[
\begin{align*}
U_1 & = U_2 + U_1 X \\
(U_2 + U_3 X)^H & U_4 + X^H U_5 + U_5^H X + X^H U_6 X \succeq 0, \\
I - X^H T_i X & \succeq 0, \quad \text{for } \forall X
\end{align*}
\]
are equivalent to the LMI
\[
\begin{align*}
\begin{bmatrix} U_1 & U_2 & U_3 \\ U_2^H & U_4 & U_5^H \\ U_3^H & U_5 & U_6 \end{bmatrix} & + \lambda_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & T_1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & T_2 \end{bmatrix} \succeq 0,
\end{align*}
\]
where \(\lambda_i \geq 0\) \((i = 1, 2)\).

To proceed, we set \(X = \Delta h_k, T_1 = 1/\varepsilon_k I, T_2 = 0, U_1 = 1 - \rho_k, U_2 = \sqrt{\varepsilon_k}, U_3 = 0_{1 \times MN_T}, U_4 = \hat{h}_k^H R h_k + \sigma_k^2 - t_k, U_5 = \hat{h}_k^H R, U_6 = R\). Then, by exploiting Lemma 2, the constraint \((6)\) can be equivalently modified as the following convex LMI
\[
A_k = \begin{bmatrix} \zeta_k (1 - \rho_k) \\ \sqrt{\zeta_k} \end{bmatrix} (\hat{h}_k + \Delta h_k)^H R (\hat{h}_k + \Delta h_k) + \sigma_k^2 \succeq 0, \tag{7}
\]
where \(t_k \geq 0\) is a slack variable.

Next, we transform the constraint \((45)\) to the convex one. Due to the definition of SINRk and \(H_k,\) the constraint \((45)\) can be recast as
\[
\rho_k \left\| (\hat{h}_k + \Delta h_k)^H v_k \right\|^2 \geq \Gamma \left( \rho_k \sum_{j \neq k} \| (\hat{h}_k + \Delta h_k)^H v_j \|^2 + \rho_k \sigma_k^2 + \delta_k^2 \right),
\]
and thus, it follows
\[
\rho_k \left( (\hat{h}_k + \Delta h_k)^H M_k (\hat{h}_k + \Delta h_k) + \sigma_k^2 \right) \geq \delta_k^2, \tag{8}
\]
where \(M_k = \frac{1}{\rho_k} V_k - \sum_{j \neq k} V_j\).

Also, we utilize a similar methodology for \((8)\) as follows. By applying Lemma 1, the constraint \((8)\) can be changed into
\[
\begin{bmatrix} \rho_k \\ \delta_k \end{bmatrix} (\hat{h}_k + \Delta h_k)^H M_k (\hat{h}_k + \Delta h_k) + \sigma_k^2 \succeq 0. \tag{9}
\]
In order to get rid of the channel uncertainty \(\Delta h_k\) in \((9)\), Lemma 2 is adopted, and the constraint \((9)\) is equivalently modified as
\[
B_k = \begin{bmatrix} \rho_k \sigma_k^2 + \delta_k \\ \delta_k \end{bmatrix} M_k (\hat{h}_k + \Delta h_k) \succeq 0, \tag{10}
\]
where \(r_k \geq 0\) is a slack variable.

Defining \(\hat{v}_{m,k}\) as \(V_{m,k} = D_m V_k\), problem \((5)\) is thus reformulated as
\[
\begin{align*}
\min_{\hat{v}_{m,k}} \quad & \min_{\rho_k, \sigma_k, t_k, r_k} \alpha \\
\text{s.t.} \quad & \sum_{k=1}^{K} \text{tr}(\hat{v}_{m,k}) \leq \alpha P_m, \quad \forall m, \\
& A_k \succeq 0, \quad B_k \succeq 0, \quad V_k \succeq 0, \quad (4a), \\
& t_k \geq 0, \quad r_k \geq 0, \quad \text{rank}(V_k) = 1, \forall k.
\end{align*}
\]

The above optimization problem is difficult to solve in general due to the rank-one constraint. Therefore, we employ the semi-definite relaxation (SDR) technique \([27]\) which simply drops the constraints \(\text{rank}(V_k) = 1\) for all \(V_k\)’s. Then, problem \((11)\) becomes a convex problem which can be solved efficiently by a convex programming solver such as CVX \([28]\). In the following theorem, we show that a solution \(V_k^*\) to problem \((11)\) satisfies \(\text{rank}(V_k^*) \leq 2\).

**Theorem 1:** If problem \((11)\) is feasible, the rank of a solution \(V_k^*\) to problem \((11)\) via rank relaxation is less than or equal to 2.

**Proof:** See Appendix A.

After \(V_k^*\) is obtained, if \(\text{rank}(V_k^*) = 1\), we can compute an optimal transmit beamforming solution \(v_k\) by eigenvalue decomposition (EVD) of \(V_k^*\). If \(\text{rank}(V_k^*) = 2\), we use the conventional Gaussian randomization (GR) technique \([27]\) to find \(v_k\) for \(k = 1, \ldots, K\). In particular, the GR technique generates a suboptimal solution. Hence, when \(\text{rank}(V_k^*) = 2\), we will propose an iterative algorithm to recover the optimal rank-one solution by following the approach in \([34]\).

First, since \(V_{m,k}\) is always semi-positive definite, we have \(\text{tr}(\hat{v}_{m,k}) \geq \lambda_{\max}(\hat{v}_{m,k})\). Thus, we can prove that \(\text{rank}(V_{m,k}) = 1\) if \(\text{tr}(\hat{v}_{m,k}) \leq \lambda_{\max}(\hat{v}_{m,k})\). Then, we can transform the constraint \(\text{rank}(V_{m,k}) = 1\) into the single reverse convex constraint as
\[
\begin{align*}
\sum_{k=1}^{K} \left( \text{tr}(\hat{v}_{m,k}) - \lambda_{\max}(\hat{v}_{m,k}) \right) & \leq 0.
\end{align*}
\]
Note that the function \(\lambda_{\max}(\hat{v}_{m,k})\) on the set of Hermitian matrices is convex. When \(\sum_{k=1}^{K} \left( \text{tr}(\hat{v}_{m,k}) - \lambda_{\max}(\hat{v}_{m,k}) \right)\) is small enough, \(\hat{v}_{m,k}\) will approach \(\lambda_{\max}(\hat{v}_{m,k}) v_{m,k}^\max(\hat{v}_{m,k}) H\), where \(v_{m,k}^\max\) represents the eigenvector corresponding to the maximum eigenvalue \(\lambda_{\max}(\hat{v}_{m,k})\) with \(\| v_{m,k}^\max \| = 1\). Then the optimal transmit beamformer vector can be expressed by
\[
\begin{align*}
v_{m,k} & = \sqrt{\lambda_{\max}(\hat{v}_{m,k})} v_{m,k}^\max,
\end{align*}
\]
which satisfies the rank-one constraint.
Thus, in order to make $\sum_{k=1}^{K} (\text{tr}(\hat{V}_{m,k}) - \lambda_{\text{max}}(\hat{V}_{m,k}))$ as small as possible, we adopt the exact penalty method. First, introducing a sufficiently large penalty ratio $\theta > 0$, the alternative formulation is considered as

$$\min_{\{v_k\}, \rho_k, n, t_k, r_k} \alpha$$ \hspace{1cm} (13a)

subject to

$$A_k \geq 0, \ B_k \geq 0, \ V_k \geq 0, \ \text{and} \ k = 1, \ldots, K,$$

$$\sum_{k=1}^{K} (\text{tr}(\hat{V}_{m,k}) + \theta (\text{tr}(\hat{V}_{m,k}) - \lambda_{\text{max}}(\hat{V}_{m,k}))) \leq \alpha P_m, \quad (13c)$$

$$t_k \geq 0, \ r_k \geq 0, \ \forall k.$$ \hspace{1cm} (13d)

We can find from (13c) that the difference $\text{tr}(\hat{V}_{m,k}) - \lambda_{\text{max}}(\hat{V}_{m,k})$ will be minimized when $\theta$ is large enough. Clearly, (13c) is to minimize $\text{tr}(\hat{V}_{m,k}) - \lambda_{\text{max}}(\hat{V}_{m,k})$. Note that (13c) is non-convex due to the coupled $\theta$ and $\hat{V}_{m,k}$. To eliminate the coupling between $\theta$ and $\hat{V}_{m,k}$, we apply the following lemma to provide an effective approximation of (13c).

**Lemma 3:** Let us define $C \in \mathbb{H}_+$ and $E \in \mathbb{H}_+$. Then, it always follows $\lambda_{\text{max}}(C) - \lambda_{\text{max}}(E) \geq e_{\text{max}}^H(C - E)e_{\text{max}}$, where $e_{\text{max}}$ denotes the eigenvector corresponding to the maximum eigenvalue of $E$.

According to Lemma 3, we propose an iterative algorithm to recover a local optimal solution. For given some feasible $\{\hat{V}_{m,k}\}$ to problem (13), we get

$$\text{tr}(\hat{V}_{m,k}^{(n+1)}) + \theta \left[ \text{tr}(\hat{V}_{m,k}^{(n)}) - \lambda_{\text{max}}(\hat{V}_{m,k}^{(n)}) \right] - (\hat{V}_{m,k}^{(n)})^H (\hat{V}_{m,k}^{(n)} - \hat{V}_{m,k}^{(n)}) \hat{V}_{m,k}^{(n)}$$

$$\leq \text{tr}(\hat{V}_{m,k}^{(n)}) + \theta (\text{tr}(\hat{V}_{m,k}^{(n)}) - \lambda_{\text{max}}(\hat{V}_{m,k}^{(n)})),$$

where the superscript $n$ represents the $n$-th iteration.

Hence, the following SDP problem generates an optimal solution $\hat{V}_{m,k}^{(n+1)}$ that is better than $\hat{V}_{m,k}^{(n)}$ to problem (13) as

$$\min_{\{v_k\}, \rho_k, n, t_k, r_k} \alpha$$ \hspace{1cm} (15a)

subject to

$$\sum_{k=1}^{K} \left[ \text{tr}(\hat{V}_{m,k}) + \theta \left[ \text{tr}(\hat{V}_{m,k}) - \lambda_{\text{max}}(\hat{V}_{m,k}) \right] - (\hat{V}_{m,k}^{(n)})^H (\hat{V}_{m,k}^{(n)} - \hat{V}_{m,k}^{(n)}) \hat{V}_{m,k}^{(n)} \right] \leq \alpha P_m. \quad (15c)$$

Now, problem (15) can be further simplified to

$$\min_{\{v_k\}, \rho_k, n, t_k, r_k} \alpha$$ \hspace{1cm} (16a)

subject to

$$\sum_{k=1}^{K} \left[ \text{tr}(\hat{V}_{m,k}) + \theta \left[ \text{tr}(\hat{V}_{m,k}) \right] - (\hat{V}_{m,k}^{(n)})^H (\hat{V}_{m,k}^{(n)} - \hat{V}_{m,k}^{(n)}) \hat{V}_{m,k}^{(n)} \right] \leq \alpha P_m, \forall m. \quad (16c)$$

To summarize, we can solve problem (3) with a given $\Gamma$, and a bisection search algorithm is applied to update $\Gamma$ for the objective value $\alpha^* = 1$. Then, this process is repeated until convergence. For the bisection method, we need to determine an upper bound $\Gamma_{\text{max}}$ as $0 < \Gamma < \Gamma_{\text{max}}$. Then, we can see that

$$\text{SINR}_k (\{v_k\}, \rho_k) = \frac{\rho_k |h_k^T v_k|^2}{\rho_k \sum_{j \neq k} |h_j^T v_j|^2 + \rho_k \sigma_k^2 + \delta_k^2} \leq \frac{\rho_k |h_k^T v_k|^2}{\rho_k \sigma_k^2 + \delta_k^2} \leq \frac{|h_k|^2 \sum_{j=1}^{M} P_m}{\sigma_k^2 + \delta_k^2}.$$

From this, we can set $\Gamma_{\text{max}}$ as $\max_k \left\{ \frac{|h_k|^2 \sum_{j=1}^{M} P_m}{\sigma_k^2 + \delta_k^2} \right\}.$ Due to monotonicity of $\alpha$, the bisection search algorithm needs $O \left( \log_2 \frac{\Gamma_{\text{max}}}{\eta} \right)$ iterations, where $\eta$ is a small positive constant which controls the accuracy of the bisection search algorithm. It is noted that this bisection search algorithm converges to the optimal solution $v_k^*$ for problem (3). The proposed algorithm based on bisection search is summarized in Algorithm 1.

**Algorithm 1** Proposed algorithm based on bisection search

- Set $\Gamma_{\text{min}} = 0$, $\Gamma_{\text{max}} = \max_k \left\{ \frac{|h_k|^2 \sum_{j=1}^{M} P_m}{\sigma_k^2 + \delta_k^2} \right\}$, $n = 0$, $\theta > 0$, a prescribed accuracy tolerance $\epsilon > 0$ and $\eta > 0$.
- Randomly generate an initial value $\{v_k^{(0)}, \rho_k^{(0)}\}, \forall k$ in (16).

Repeat

- Set $\Gamma_{\text{mid}} = (\Gamma_{\text{min}} + \Gamma_{\text{max}})/2$.

Repeat

- Solve problem (16) with $\Gamma_{\text{mid}}$ to obtain a solution $v_k^{(n+1)}$ and $\rho_k^{(n+1)}$.

- If $\text{tr}(\hat{V}_{m,k}^{(n+1)}) = \hat{V}_{m,k}^{(n+1)}$, set $\theta \leftarrow 2\theta$.

- Update $n \leftarrow n+1$.

- Until $|\text{tr}(\hat{V}_{m,k}^{(n+1)}) - \lambda_{\text{max}}(\hat{V}_{m,k}^{(n+1)})| < \epsilon$

- Set $v_k^{(n)} = v_k^{(n)}$, $\rho_k^{(n)} = \rho_k^{(n)}$, and $n = 0$.

Repeat

- Solve problem (16) with $\Gamma_{\text{mid}}$ to obtain a solution $v_k^{(n+1)}$, $\rho_k^{(n+1)}$, and $\alpha^{(n+1)}$.

- Update $n \leftarrow n+1$.

-Until $|\text{tr}(\hat{V}_{m,k}^{(n+1)}) - \lambda_{\text{max}}(\hat{V}_{m,k}^{(n+1)})| < \epsilon$

- If $\alpha^{(n+1)} < 1$,

- set $\Gamma_{\text{min}} = \Gamma_{\text{mid}}$.

- else

- set $\Gamma_{\text{max}} = \Gamma_{\text{mid}}$.

- Until $|\Gamma_{\text{max}} - \Gamma_{\text{min}}| < \eta$

- Calculate $v_k$ according to (12).

**B. Robust Iterative Algorithm Based on Successive Convex Approximation**

To reduce the computational complexity of Algorithm 1, we consider another formulation for the minimum SINR maximization problem. Based on the SCA method, the optimization

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*1The proposed optimization algorithm is performed by MPU. Then, the MPU can send the beamforming solutions to individual transmitters through fiber optics or an exclusive radio frequency (RF) link. Also, it can transmit the PS factor solution to individual receivers through the estimated instantaneous channel.*
can also be reformulated into a convex form with linear constraints. Thus, the robust SINR maximization problem can be rewritten as

$$\min_{\{v_k\}, \rho_k, h_k \in \mathcal{H}_k} \max_{h_k \in \mathcal{H}_k} \frac{|h_k^H v_k|^2}{\sum_{j \neq k} |h_k^H v_j|^2 + \sigma_k^2 + \frac{\delta_k^2}{\rho_k}} \tag{17a}$$

subject to

$$\min_{h_k \in \mathcal{H}_k} \zeta_k (1 - \rho_k) \left( \sum_{j = 1}^K |h_k^H v_j|^2 + \sigma_j^2 \right) \geq \epsilon_k, \forall k \{17b\} \tag{18a}, \{18b\}$$

In this problem, we minimize the numerator of SINR while maximizing the denominator of SINR \[9\]. Based on a tight approximation, the minimum and the maximum for each term can be determined by employing the Lagrangian multiplier method. In addition, to equivalently convert the objective function \[17a\], problem \[17\] is expressed by introducing a slack variable \(\tau\) as

$$\min_{\{v_k\}, \rho_k, \tau, x_k, y_k} \tau \tag{19a}$$

subject to

$$e^{y_k} - e^{x_k} \leq \min_{h_k \in \mathcal{H}_k} |h_k^H v_k|^2, \tag{18a}$$

$$e^{y_k} \geq \max_{h_k \in \mathcal{H}_k} \sum_{j \neq k} |h_k^H v_j|^2 + \sigma_k^2 + \frac{\delta_k^2}{\rho_k}. \tag{18b}$$

Thus, in order to circumvent the non-convex objective function \[17a\], problem \[17\] is expressed by introducing a slack variable \(\tau\) as

$$\min_{\{v_k\}, \rho_k, \tau, x_k, y_k} \tau \tag{19a}$$

subject to

$$e^{y_k} - e^{x_k} \leq \max_{h_k \in \mathcal{H}_k} \sum_{j \neq k} |h_k^H v_j|^2 + \sigma_k^2 + \frac{\delta_k^2}{\rho_k}. \tag{20}$$

When computing the EH constraint in \[17b\] and the SINR constraint in \[20\], we need to calculate \(|h_k^H v_j|^2\). Using \(x^H A x = \text{tr}(Axx^H)\), we can write this as

$$|h_k^H v_j|^2 = |(h_k + \Delta h_k)^H v_j|^2$$

$$= v_j^H (h_k + \Delta h_k)(h_k + \Delta h_k)^H v_j$$

$$= \text{tr}((h_k + \Delta h_k)(h_k + \Delta h_k)^H v_j v_j^H)$$

$$= \text{tr}((h_k + \Delta h_k)^H v_j v_j^H)$$

where \(\hat{h}_k\) is defined as \(\hat{h}_k \triangleq \hat{h}_k^H\), and \(\Delta h_k \triangleq \hat{h}_k \Delta h_k^H + \Delta h_k \hat{h}_k^H + \Delta h_k \Delta h_k^H\) represents the uncertainty in the matrix \(\hat{h}_k\).

It is noted that \(\Delta k\) is a norm-bounded matrix as \(\|\Delta k\|_F \leq \xi_k\). We can straightforwardly find the following relation \[27\] as

$$\|\Delta k\|_F = \|\hat{h}_k \Delta h_k^H + \Delta h_k \hat{h}_k^H + \Delta h_k \Delta h_k^H\|_F$$

$$\leq \|\hat{h}_k \Delta h_k^H\|_F + \|\Delta h_k \hat{h}_k^H\|_F + \|\Delta h_k \Delta h_k^H\|_F$$

$$\leq \|\hat{h}_k\|_F \|\Delta h_k^H\| + \|\Delta h_k\|_F \|\Delta h_k^H\|$$

$$= \epsilon_k^2 + 2\epsilon_k \|\hat{h}_k\|,$$

where the first inequality is based on the triangle inequality, and the second inequality come from the Cauchy-Schwarz inequality. It is possible to choose \(\zeta_k = \epsilon_k^2 + 2\epsilon_k \|\hat{h}_k\|\). It is noted that the bounds of this uncertainty are derived by triangle inequality, Cauchy-Schwarz inequality, and multiplicity of the second norm, which are tight enough.

Adopting the preceding notations, we can rewrite \[19\] at the \(n\)-th iteration as

$$\min_{\{v_k\}, \rho_k, \tau, x_k, y_k} \tau \tag{21a}$$

subject to

$$\max_{h_k \in \mathcal{H}_k} \|\Delta k\|_F \leq \xi_k$$

$$\min_{\|\Delta k\|_F \leq \xi_k} \text{tr}((\hat{h}_k + \Delta h_k) v_j) + \sigma_k^2 + \frac{\delta_k^2}{\rho_k} \leq \epsilon_k \left( y_k - y_k^{(n+1)} \right)$$

$$\min_{\|\Delta k\|_F \leq \xi_k} \sum_{j \neq k} \text{tr}((\hat{h}_k + \Delta h_k) v_j) \geq \frac{\epsilon_k}{\xi_k (1 - \rho_k)} - \sigma_k^2.$$
Algorithm 2 Robust Iterative Algorithm Based on SCA

Initialize \{y_k^{(n)}\} and set n = 0.

Repeat

1. Solve problem (23) with \{y_k^{(n)}\} to obtain \(V_k^{(n)}\) and \(\tau_k^{(n)}\) for \(k = 1, \ldots, K\).
2. Set \(y_k^{(n+1)} = y_k^{(n)}\) for \(k = 1, \ldots, K\).
3. Update \(n \leftarrow n + 1\).

Until Convergence

If rank\((V_k^{(n)})\) = 1,
compute \(\{v_k^*\}\) by EVD of \(V_k^{(n)}\).
else
use the GR technique to find \(\{v_k^*\}\) for \(k = 1, \ldots, K\).

IV. COMPUTATIONAL COMPLEXITY

In this section, we evaluate the computational complexity of the proposed robust design methods. As will be shown in Section V, the proposed algorithms exhibit gains in terms of both computational complexity and performance compared to the conventional SDP scheme in (22) which employs local search. Now, we will present the complexity comparison by adopting the analysis in (21) and (22). The complexities of the proposed algorithms are shown in Table I. Here, we denote \(n\), \(L^\text{max} = \log_2 L^\text{max}\), \(Q^\text{max}\) and \(D^\text{max}\) as the number of decision variables, the bisection search number, the SCA iteration number, and the local search number in (22), respectively.

1) Algorithm 1 in problem (16) involves 2K LMI constraints of size \(M N_T + 2\), K LMI constraints of size \(M N_T\), and \(4K + M\) linear constraints.

2) Algorithm 2 in problem (23) has K second-order cones (SOC) constraints of dimension \(M^2 N_T^2 + 1\), K SOC constraints of dimension \((K - 1)M^2 N_T^2 + 1\), K SOC constraints of dimension \(K M^2 N_T^2 + 1\), K LMI constraints of size \(M N_T\), and \(3K + M\) linear constraints.

3) Conventional scheme in (22) consists of 2K LMI constraints of size \(M N_T + 1\), K LMI constraints of size \(M N_T\), and \(2K + M\) linear constraints.

For example, for a system with \(M = 3\), \(K = 2\), \(N_T = 3\), \(L^\text{max} = Q^\text{max} = 6\), and \(D^\text{max} = 100\), the complexities of the proposed Algorithm 1, Algorithm 2, and the conventional scheme (22) are \(O(1.96 \times 10^9)\), \(O(3.41 \times 10^9)\) and \(O(4.31 \times 10^9)\), respectively. Thus the complexity of the proposed Algorithm 1 and Algorithm 2 are only 4.5% and 0.8% of that of the conventional scheme in (22), respectively.

V. SIMULATION RESULTS

In this section, we numerically compare the performance of the proposed algorithms for multiuser DAS SWIPT systems. Throughout the simulation, we consider DAS with a circular antenna layout and set \(M = 3\), \(K = 3\), and \(N_T = 4\). The power of each DA port is set to \(P_1 = \frac{P}{6}\), \(P_2 = \frac{P}{3}\), and \(P_3 = \frac{P}{2}\) as in (22). Three DA ports form an equilateral triangle while all MSs are uniformly distributed inside a disc with the cell radius \(R = \sqrt{4\gamma} \) m centered at the centroid of the triangle. The \(j\)-th DA port is located at \(\left(\text{r cos} \left(\frac{2\pi(j-1)}{M}\right), \text{r sin} \left(\frac{2\pi(j-1)}{M}\right)\right)\) for \(j = 1, \ldots, M\) with \(r = \sqrt{2} R\) as in (4). The pathloss exponent \(\gamma\) is set to be 3. According to this setting, a received SNR loss of 23.5 dB is observed at cell edge users compared to cell center users. All channel coefficients \(h_{m,k} \in \mathbb{C}^{N_T \times 1}\) are modelled as Rician fading. The channel vector \(h_{m,k}\) is given as \(h_{m,k} = \sqrt{1 + K_R} h_{m,k}^\text{LOS} + \sqrt{1 + K_R} h_{m,k}^\text{NLOS}\), where \(h_{m,k}^\text{LOS}\) indicates the line-of-sight (LOS) component with \(|h_{m,k}^\text{LOS}|^2 = d_{m,k}^{-\gamma/2}\) and \(h_{m,k}^\text{NLOS}\) represents the Rayleigh fading component as \(h_{m,k}^\text{NLOS} \sim \mathcal{CN}(0, d_{m,k}^{-\gamma/2})\), and \(K_R\) is the Rician factor equal to 3. For the LOS component, we apply the far-field uniform linear antenna array to model the channels in (33). For simplicity, it is assumed that all MSs have the same set of parameters, i.e., \(\zeta_k = \zeta, \sigma_k^2 = \sigma^2, \sigma_k^2 = \sigma^2\), and \(e_k = e\) for \(k = 1, \ldots, K\). In addition, we set \(\sigma^2 = -50\) dBm, \(\delta^2 = -30\) dBm, and \(\zeta = 0.3\). Also, all the channel uncertainties are chosen to be the same as \(e_k = e, \forall k\). In the simulation, the worst-case rate in all the ID users min \(\min_{\forall j} \log_2(1+\text{SNR}_j)\) is plotted by taking an average over 1000 randomly generated channel realizations.

Fig. 2 investigates the convergence performance of the proposed iterative algorithm for various \(P\) for \(e = 3\) dBm and \(e = 0.01\). It is clear that the proposed iterative algorithms indeed converge in all cases. We can see that after 7 iterations, the steady average worst-case rate is achieved for all \(P\).

In Fig. 3, we present the average worst-case rate versus the number of DA ports \(M\) with various channel uncertainty \(e\) with \(P = 60\) dBm, \(e = 5\) dBm and \(e = 0.01\). It is found that our proposed robust algorithms attain substantial worst-case rate improvements over the conventional scheme in (22). It is observed that there is about 0.3 bps/Hz difference between the curves of \(e = 0.01\) and \(0.1\) for the proposed algorithms. Furthermore, our proposed Algorithm 2 achieves about 0.5 bps/Hz and 0.7 bps/Hz gain compared to the conventional scheme (22) for \(e = 0.01\) and \(0.1\), respectively. We also see that our proposed Algorithm 1 outperforms Algorithm 2 at the expense of increased complexity.

Fig. 4 shows the performance comparison among robust algorithms for different number of antennas in each DA port.
with $e = 5 \text{ dBm}$ and $P = 80 \text{ dBm}$. One can see that the conventional algorithm [22] requires more antennas than our proposed robust algorithms. The performance gap between our proposed Algorithm 1 and 2 curves is about 0.3 bps/Hz. Moreover, as $N_T$ increases, the performance gap between our proposed algorithms and the conventional scheme becomes bigger.

Fig. 5 depicts the effect of the channel uncertainty $\varepsilon$ on the average worst-case rate with $e = 0 \text{ dBm}$ and $P = 50 \text{ dBm}$. We can check that as the maximum channel uncertainty $\varepsilon$ decreases, the average worst-case rate becomes enhanced. Clearly, the proposed robust algorithms outperform the conventional scheme [22].

Finally, in Fig. 6, we exhibit the average worst-case rate versus the total transmit power target $P$ for various $\varepsilon$ with $e = 3 \text{ dBm}$. Compared to our proposed Algorithm 1, Algo-

### Table I

**Complexity Analysis of Different Algorithms**

| Algorithms         | Complexity Order |
|--------------------|------------------|
| Algorithm 1        | $O(nL_{\text{max}}^Q\max{\sqrt{2K(MN_T+2)+KMN_T+4K+M(2K(MN_T+2))^2+K(MN_T)^3+4K+M+n^2}}}$ where $n = O(M^2N_T^2+3K+1)$ |
| Algorithm 2        | $O(nQ_{\text{max}}\max{\sqrt{6K+KN_T+3K+M(\{M^2N_T+1\}^2+(K-1)M^2N_T+1)^2+(K^2N_T^2+1)^2+K^2M^2N_T^2+2K+M+n^2}}}$ where $n = O(M^2N_T^2+3K+1)$ |
| Conventional scheme [22] | $O(nD_{\text{max}}^Q\max{\sqrt{3(MN_T+2)+M(2K(MN_T+1))^2+K^2M^2N_T^2+2K+M+n^2}}}$ where $n = O(M^2N_T^2+2K+1)$ |

![Fig. 3. Average worst-case rate versus the number of DA ports](image1)

![Fig. 4. Average worst-case rate versus the number of antennas in each DA port](image2)

![Fig. 5. Average worst-case rate versus channel uncertainty $\varepsilon$](image3)

![Fig. 6. Average worst-case rate versus $P$ for various $\varepsilon$](image4)
Lagrangian dual function of the primal problem (11) is given and corresponding to the constraint comes convex and satisfies the Slater’s condition. Thus, its validity of the proposed algorithms. The maximization problem. Simulation results have demonstrated has been calculated. We have proposed an iterative algorithm and a low-complexity algorithm for the worst-case SINR maximization problem. Simulation results have demonstrated the validity of the proposed algorithms.

**APPENDIX A**

**PROOF OF THEOREM 1**

If the rank-one constraint is ignored, problem (11) becomes convex and satisfies the Slater’s condition. Thus, its dual gap is zero [26]. Assume that the dual variables \( \{C_k\} \in \mathbb{H}^+ \), \( \{Q_k\} \in \mathbb{H}^+ \), \( \{S_k\} \in \mathbb{H}^+ \) and \( \{\mu_m\} \geq 0 \) correspond to the constraint \( A_k \succeq 0 \), \( B_k \succeq 0 \), \( V_k \geq 0 \) and \( \sum_{k=1}^{K} \text{tr}(D_m V_k) \leq \alpha P_m \) in (11), respectively. Then, the Lagrangian dual function of the primal problem (11) is given by

\[
L = \alpha - \sum_{k=1}^{K} (\text{tr}(C_k A_k) + \text{tr}(Q_k B_k) + \text{tr}(S_k V_k)) + \sum_{m=1}^{M} \mu_m \left( \sum_{k=1}^{K} \text{tr}(D_m V_k) - \alpha P_m \right).
\]

(24)

Since \( C_k \) and \( T_k \) are Hermitian matrices, we have

\[
\text{tr}(C_k A_k) = \text{tr}(C_k G_k^H T_k G_k) + \text{tr}(C_k F_k),
\]

\[
\text{tr}(Q_k B_k) = \text{tr}(Q_k G_k^H M_k G_k) + \text{tr}(Q_k E_k),
\]

where

\[
E_k = \begin{bmatrix}
\rho_k & \delta_k & 0_{1 \times MN_T} \\
0_{MN_T \times 1} & \sigma_k^2 - r_k & 0_{1 \times MN_T} \\
\end{bmatrix},
\]

\[
F_k = \begin{bmatrix}
\zeta_k (1 - \rho_k) & \sqrt{\delta_k} & 0_{1 \times MN_T} \\
\sqrt{\delta_k} & \sigma_k^2 - r_k & 0_{1 \times MN_T} \\
0_{MN_T \times 1} & 0_{MN_T \times 1} & \frac{r_k}{\sigma_k^2} I
\end{bmatrix},
\]

\[
G_k = \begin{bmatrix}
0 & h_k & I \\
\end{bmatrix}.
\]

Taking partial derivative of (24) with respect to \( V_k \) and applying the KKT conditions [26], it follows

\[
\sum_{m=1}^{M} \mu_m D_m - \left( G_k C_k G_k^H + \frac{1}{\lambda} G_k Q_k G_k^H + S_k \right) = 0.
\]

(25)

Let \( \{C_k^*\}, \{Q_k^*\}, \{S_k^*\} \) and \( \{\mu_m^*\} \) be the optimal dual solution to problem (11). Note that \( Q_k^* B_k^* = 0 \) from the complementary slackness conditions of problem (11). Since the size of \( Q_k^* \) and \( B_k^* \) is \( (MN_T + 2) \times (MN_T + 2) \), we have

\[
\text{rank}(Q_k^*) + \text{rank}(B_k^*) \leq MN_T + 2.
\]

Denoting \( r_k^* \) as the optimal solution to problem (11), \( r_k^* \) in \( B_k^* \) in (11) is non-negative. If \( r_k^* > 0 \), \( r_k^* I + M_k^* \) has full rank. We will prove that \( r_k^* \neq 0 \) by contradiction.

If \( r_k^* = 0 \), the constraint \( \|\Delta h_k\|^2 \leq \varepsilon_k^2 \) does not hold since \( r_k^* \) is the dual variable for (11). Note that the condition \( \|\Delta h_k\|^2 \leq \varepsilon_k^2 \) is the only constraint on \( \Delta h_k \).

Thus \( \Delta h_k \) is the worst channel uncertainty which minimizes \( q \triangleq \rho_k |h_k^H v_k|^2 / \left( \rho_k \sum_{j 
eq k} |h_j^H v_j|^2 + \rho_k \sigma_k^2 + \delta_k \right) \), we can always find a scalar \( \omega > 1 \) which satisfies \( \|\Delta h_k\|^2 = \varepsilon_k^2 / \omega^2 \). Substituting the channel uncertainty \( \omega \Delta h_k \) in \( q \), we can find a SINR lower than that obtained by \( \Delta h_k^* \). This is contradictory to the assumption that \( \Delta h_k^* \) minimizes the SINR. Thus, it follows \( r_k^* \neq 0 \), which leads to \( r_k^* > 0 \). As a result, \( r_k^* I + M_k^* \) becomes full rank, and we have \( \text{rank}(B_k^*) \geq MN_T + 2 \). Furthermore, since \( \text{rank}(Q_k^*) \) is non-zero, thus, the rank of \( Q_k^* \) equals 1. Similarly, we can show that \( \text{rank}(C_k^*) = 1 \). Then, it follows

\[
\text{rank}(G_k^H (C_k^* + \frac{1}{\lambda} Q_k^*) G_k^H) + \frac{1}{\lambda} \text{rank}(G_k^H Q_k^* G_k) = 2.
\]

Thus, multiplying both sides of (25) with \( V_k^* \) yields

\[
(\sum_{m=1}^{M} \mu_m^* D_m) V_k^* = (G_k (C_k^* + \frac{1}{\lambda} Q_k^*) G_k^H + S_k^*) V_k^*,
\]

where it is noted that \( S_k^* V_k^* = 0 \). Since \( \sum_{m=1}^{M} \mu_m^* D_m \) has full rank, following the rank inequality \( \text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B)) \), we can finally prove that

\[
\text{rank}(\sum_{m=1}^{M} \mu_m^* D_m) V_k^* \leq \text{rank}(G_k (C_k^* + \frac{1}{\lambda} Q_k^*) G_k^H) + \frac{1}{\lambda} \text{rank}(G_k^H Q_k^* G_k) \leq 2.
\]

**APPENDIX B**

**PROOF OF PROPOSITION 1**

By introducing an arbitrary positive multiplier \( \theta \geq 0 \), the Lagrangian function is given by

\[
L(\Delta_k, \theta) = \text{tr}((\hat{H}_k + \Delta_k) V_k) + \theta (\|\Delta_k\|^2 - \xi_k^2).
\]

We differentiate the Lagrangian function with respect to \( \Delta_k^* \) and equate it to zero [29] as

\[
\nabla \Delta_k^* L(\Delta_k, \theta) = V_k^H + \theta \Delta_k = 0.
\]

Then, we can find the optimal solution \( \Delta_k^{opt} = -\frac{1}{\theta} V_k^H \). In order to remove the role of an arbitrary parameter of \( \theta \), the Lagrangian function is differentiated with respect to \( \theta \) and set to zero as

\[
\nabla \theta L(\Delta_k, \theta) = \|\Delta_k^{opt}\|^2 - \xi_k^2 = 0.
\]

Thus the optimal solution for \( \theta \) is obtained as \( \theta^{opt} = \frac{\|V_k^H\|^2}{\xi_k^2} \).
By combining the above results, we finally get
\[ \Delta_k^{opt} = \pm \xi_k \frac{V^H_k}{\|V_k\|}. \]

Accordingly, the minimum and maximum of \( \Delta_k \) can be expressed as
\[ \Delta_k^{min} = -\xi_k \frac{V^H_k}{\|V_k\|}, \quad \Delta_k^{max} = \xi_k \frac{V^H_k}{\|V_k\|}. \]

To check if this optimal solution is a minimum, we confirm that the second derivative at the optimal solution point \( \Delta_k^{opt} \) is positive semi-definite as
\[ \nabla^2_{\Delta_k} L(\Delta_k^{opt}, \theta^{opt}) = \theta^{opt} (\text{vec} \{ I_{MN_T} \} \text{vec} \{ I_{MN_T} \})^T \geq 0. \]

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