Deep Reinforcement Learning for Foreign Exchange Trading

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Abstract—Reinforcement learning can interact with the environment and is suitable for applications in decision control systems. Therefore, we used the reinforcement learning method to establish a foreign exchange transaction, avoiding the long-standing problem of unstable trends in deep learning predictions. In the system design, we optimized the Sure-Fire statistical arbitrage policy, set three different actions, encoded the continuous price over a period of time into a heat-map view of the Gramian Angular Field (GAF) and compared the Deep Q Learning (DQN) and Proximal Policy Optimization (PPO) algorithms. To test feasibility, we analysed three currency pairs, namely EUR/USD, GBP/USD, and AUD/USD. We trained the data in units of four hours from 1 August 2018 to 30 November 2018 and tested model performance using data between 1 December 2018 and 31 December 2018. The test results of the various models indicated that favourable investment performance was achieved as long as the model was able to handle complex and random processes and the state was able to describe the environment, validating the feasibility of reinforcement learning in the development of trading strategies.

Index Terms—Gramian Angular Field (GAF), Deep Q Learning (DQN), Proximal Policy Optimization (PPO), Reinforcement Learning, Foreign Exchange Trading

I. INTRODUCTION

As the FinTech revolution sweeps through the banking industry, there are increasing opportunities to apply robotics. However, whether robotic investment can truly benefit investors remains unclear. Generally, robotic investments contain two major processes. First, financial institutions use programs to understand customer traits and risk orientations (this is sometimes called Know Your Customer (KYC)). Second, trading strategies are converted into algorithms, leaving little to no room for human intervention and providing unmanned or automated services. Robotic investments only control risk within the investor’s acceptable scope and automatically adjust investment portfolios based on their risk tolerance during market fluctuation. Moreover, many robot-advisers focus on applying artificial intelligence (AI) to customer service rather than targeting investment, and do not usually incorporate reinforcement learning or deep learning algorithms.

Quantitative trading is one of the most popular projects among investors. In the past, people used statistical methods or machine learning methods to obtain profitable investment information. Statistical methods are basically the application of technical indicators, such as the Moving Average (MA) or Bollinger Band (BB). Machine learning is the extraction of characteristics through supervised or unsupervised learning and the use of Support Vector Machines (SVMs) for classification and prediction. Alternatively, artificial neural networks, such as Long-Short-Term Memory (LSTM), can be applied to predict prices and obtain trading information.

If the provided feedback or pricing information is not current or accurate, even the most robust algorithms may produce erroneous information. That is, the garbage in, garbage out principle applies to algorithms. However, what if we simply set stop-loss and stop-gain points and focus on gaining returns from stable trading? In this study, we aim to apply reinforcement learning in foreign exchange trading.

We plan to use deep-enhanced learning to mimic how humans make decisions, using the state of the current environment to execute actions and obtain rewards from the environment. Moreover, people’s actions impact the environment, causing the environment to enter a new state. To check the feasibility of this approach, we adopted the approach of training four-hour units of EUR/USD, GBP/USD, and AUD/USD data between 1 August 2018 and 30 November 2018. We then applied the trained data to the period between 1 December 2018 and 31 December 2018 to validate the system performance.

II. PRELIMINARY

A. Foreign Exchange Trading

There were a number of reasons for selecting foreign exchange as the trading product in this study. One of the reasons was that it was beneficial for data analysis as foreign exchange can be traded within 24 hours and trading volumes are extremely high. Excluding weekends, foreign currency transactions can be regularly carried out, so numerical data are stable and continuous, and daily opening and closing prices do not need to be accounted for, unlike for the Taiwan Stock Exchange. Moreover, as long as governments do not collapse, foreign currencies can be held indefinitely. Therefore, there is
no need to consider daily cash settlements, unlike for futures trading in Taiwan.

**B. Trading Strategy**

In this study, we adopted the Sure-Fire arbitrage strategy, which is a variant of the Martingale. It involves increasing bets after every loss so that the first win recovers all previous losses plus a small profit. After entering the market and initiating trading, the investor uses the margin between the stop-loss and stop-gain prices as the raise margin. As long as the price fluctuates within the raise margin and touches on the raise price, the Sure-Fire Strategy urges investors to continue raising the stakes until they surpass the margin in order to profit.

First, as illustrated in Fig. 2, we purchase one unit at any price and set a stop-gain price of $+k$ and a stop-loss price of $-2k$. At the same time, we select a price with a difference of $-k$ to the buy price and $+k$ to the stop-loss price and set a backhand limit order for three units. Backhand refers to engaging in the opposite behaviour. The backhand of buying is selling and the backhand of selling is buying. A limit order refers to the automatic acquisition of corresponding units.

As illustrated in Fig. 3, when a limit order is triggered, and three units are successfully sold backhand, we place an additional backhand limit order, where the buy price is $+k$ to the sell price and $-k$ to the stop-loss price. We set the stop-gain point as the difference of $+k$ and the stop-loss point as the difference of $-2k$, after which an additional six units are bought.

As illustrated in Fig. 4, the limit order is triggered in the third transaction. The final price exceeded the stop-gain price of the first transaction, the stop-loss price of the second transaction, and the stop-gain price of the third transaction. In this instance, the transaction is complete. The calculation in the right block shows that the profit is $+1k$.

**C. Reinforcement Learning**

The reinforcement learning model comprises an agent. The agent performs an action based on the current state. The action is received by the environment and returns feedback to the agent. The feedback can be either a reward or a penalty [12].

Once the agent receives the reward, they adjust the relative function between the state and the action to maximize the overall expected return. The function could be a value function or a policy function.

A value function refers to the reward obtained from a certain action in a certain state. Therefore, accurately estimating the value function is an important component of the model.
Underestimating or overestimating the value of certain states or actions would influence learning performance.

A policy function refers to the ideal action to achieve a maximum expected return in a certain state. In the reinforcement learning model, actions that maximize expected return (value) in a certain state are called policies. In a number of advanced models, policy functions are directly applied to maximize expected return.

D. Deep Q Network (DQN)

The Deep Q Network (DQN) is a deep reinforcement learning framework. It is a modified version of Q-Learning. In the most basic reinforcement learning frameworks, or Q-Learning frameworks, a two-dimensional array called a Q-Table is adopted as the value function. However, an excessive number of actions and states runs the risk of the curse of dimensionality. Therefore, artificial neural networks (ANNs) were used to replace Q-Tables in the DQN. The historical rewards within a specific period of time were used to update the ANNs (experience replay).

For the selection of actions, if the greedy method alone is applied to select the action with the highest expected return every round, then the chance of selecting other actions would be lost. Therefore, DQN adopts the $\epsilon$-greedy exploration method. Specifically, each time an action is selected, the system provides a small probability ($\epsilon$) for exploration, in which the best action is abandoned and other new actions are executed. The probability of exploration increases concurrently with the $\epsilon$ value, making the agent more likely to try a new state or action. This process improves the learning effects. The disadvantage is that the convergence time increases exponentially with the duration of exploration.

The DQN algorithm is as follows (the updated command is highlighted):

E. Proximal Policy Optimization (PPO)

Proximal policy optimization (PPO) is a modified version of the policy gradient method. It is used to address the refresh rate problem of policy functions. The algorithms used in the reinforcement learning method can be categorized into dynamic programming (DP), the Monte Carlo method, and temporal-difference (TD) \cite{12}. The policy gradient method uses the Monte Carlo method, wherein samples are collected for learning. However, a single step in an episode could completely alter the outcomes. Therefore, the method produces poor learning accuracy. The advantage of this method is that it can be applied to unknown environments. The policy gradient method abandons value functions. Rather, it directly uses rewards to update the policy function, which outputs the probability of taking a specific action in a certain state. Thus, it can effectively facilitate decision making for non-discrete actions \cite{13}.

The policy gradient algorithm is as follows (the updated command is highlighted):

To update the policy function:

$$\theta_{\text{new}} = \theta_{\text{old}} + \alpha \nabla J.$$
PPO adopts a clipping method to restrict the update range of the policy function:

\[ L_{CLIP}(\theta) = \hat{E}_t \left[ \min \left( r_t(\theta) \hat{A}_t, \text{clip} \left( r_t(\theta), 1 - \epsilon, 1 + \epsilon \right) \hat{A}_t \right) \right]. \]

By avoiding the policy gradient, identifying the optimal refresh rate becomes extremely difficult, which improves learning effects. The following is a schematic diagram of clipping for the loss function:

![Fig. 9. Clipping of the loss function.](image)

In Fig. 9, A represents the value calculated by the advantage function, where \( A > 0 \) represents the selection of a beneficial action. However, in this instance, the probability of selecting this action should not be infinitely expanded. Therefore, a clipping method is used to restrict extreme update ranges. Similarly, \( A < 0 \) represents the poor selection of an action. However, the policy function should not drastically reduce the selection probability of this action [11].

The PPO algorithm is as follows:

```
Input: initial policy parameters \( \theta_0 \), clipping threshold \( \epsilon \\
for k = 0, 1, 2, \ldots \ do 
    Collect set of partial trajectories \( D_k \) on policy \( \pi_k = \pi(\theta_k) \) 
    Estimate advantages \( \hat{A}_k^t \) using any advantage estimation algorithm 
    Compute policy update \( \theta_{k+1} = \arg \max_\theta L_{CLIP}^t(\theta) \) by taking K steps of minibatch SGD (via Adam), where 
    \[ L_{CLIP}^t(\theta) = \hat{E}_t \left[ \sum_{t=0}^{T-1} \min(\epsilon(\theta)\hat{A}_t, \text{clip}(\epsilon(\theta), 1 - \epsilon, 1 + \epsilon)\hat{A}_t) \right] \] 
end for
```

![Fig. 10. PPO algorithm.](image)

**E. Convolutional Neural Networks**

Compared to conventional neural networks, convolutional neural networks (CNNs) include an additional convolutional layer and pooling layer. They also perform convolution computation using filters, or kernels, to reduce model complexity, as well as pooling to reduce dimensionality and overall computation space.

Input data pass through the convolutional layer for convolution computation. The weights serve as inner products. The filters scan specific characteristics in the images, such as vertical and horizontal lines. Using images in convolution computation enhances image features and reduces noise. After computation, the network generates numerous vectors called feature maps. The number of features that can be extracted increases concurrently with the number of filters applied.

The pooling layer uses a fixed matrix to move on the image. Max pooling refers to adopting the maximum pixel value of a specific range as the output. Then, the matrix moves in steps to select the maximum pixel value of the next range. Finally, the output image matrix is composed of the maximum pixel values of various ranges. The pooling layer reduces the spatial dimension by down-sampling the image using local correlation. In addition to max pooling, there are also overlapping pooling and mean pooling methods.

The final layers of CNNs are all connected to the fully-connected layer. The fully-connected layer is similar to the neural pathways of conventional neural networks, where each neuron is connected to all the neurons in the previous layer [9].

**G. Gramian Angular Field (GAF)**

Foreign exchange prices can be viewed as a time series. Therefore, it is necessary to consider prices over time rather than adopting a set of opening and closing prices as a single state.

This study used the Gramian Angular Field method, which is a time series coding method consisting of many Gram matrices. A Gram matrix is a useful tool in linear algebra and geometry. It is often used to calculate the linear correlation of a set of vectors. A Gram matrix containing n vectors is composed of inner products of vector pairs, and can be expressed as follows: If we define \( \phi(i,j) \) as the angle between the \( i \)th and \( j \)th vectors, the following matrix is derived:

\[
G = \begin{pmatrix}
\langle v_1, v_1 \rangle & \langle v_1, v_2 \rangle & \ldots & \langle v_1, v_n \rangle \\
\langle v_2, v_1 \rangle & \langle v_2, v_2 \rangle & \ldots & \langle v_2, v_n \rangle \\
\vdots & \vdots & \ddots & \vdots \\
\langle v_n, v_1 \rangle & \langle v_n, v_2 \rangle & \ldots & \langle v_n, v_n \rangle
\end{pmatrix}
\]

![Fig. 11. Gram matrix 1.](image)

In a Gram matrix, time increases as the position shifts from the top left corner to the bottom right corner. Therefore, time is encoded into the relative position of a value in the matrix. Gram matrices preserve time dependence in the data [1].

GAF is formulated as follows:

1) Normalize sequence \( X = \{x_1, x_2, \ldots, x_n\} \) to \([-1, 1]\) or \([0, 1]\).
2) Derive the polar coordinate angle $\phi_i$ of value $x_i$ according to $\phi_i = \arccos(x_i)$.

3) The angles in Fig. 12 are the angle differences of each value converted to a polar coordinate. Therefore, the formula can be converted by incorporating $\phi_i$:

$$G = \begin{pmatrix}
\cos(\phi_1 + \phi_1) & \cos(\phi_1 + \phi_2) & \ldots & \cos(\phi_1 + \phi_n) \\
\cos(\phi_2 + \phi_1) & \cos(\phi_2 + \phi_2) & \ldots & \cos(\phi_2 + \phi_n) \\
\vdots & \vdots & \ddots & \vdots \\
\cos(\phi_n + \phi_1) & \cos(\phi_n + \phi_2) & \ldots & \cos(\phi_n + \phi_n)
\end{pmatrix}$$

Fig. 13. Gram matrix 3.

In Fig. 14 the left image is a heat-map view of the GAF, and the right image is the corresponding numerical line chart. The chart shows high values.

H. Financial Quantitative Trading

Currently, quantitative domestic and overseas transactions largely target financial products. The most common approach directly measures the quantitative values of financial prices, interest rates, exchange rates, or indicators and incorporates deep learning methods, such as recurrent neural networks (RNN) and long short-term memory (LSTM). However, these methods cannot imitate the thinking logic that people generate when they look directly at trading graphs.

To accurately reflect people’s views when directly analysing trade graphs rather than generating rule-based decisions, the use of the CNNs for image recognition to predict financial asset prices is inadequate. Persio et al. attempted to use artificial neural networks (ANNs) to predict stock market indices in classification-based models. They compared three common neural network models, namely, the multi-layer perceptron (MLP), CNNs, and LSTM. They found that a new framework built on wavelets and a CNN achieved 83% accuracy, far exceeding the 4% of RNNs.

The breakthrough of this study was the application of reinforcement learning to identify the optimal actions for implementing statistical arbitrage policies.

III. METHODOLOGY

A. System Design

1) State Design: States are derived from an agent’s observations of the environment. They are used to describe or represent environments. The agents then perform actions corresponding to the perceived state. Therefore, defining the state is key to learning performance. In this study, we use the sliding window method on the “opening price, highest point, lowest point, and closing price” of the foreign exchange state in units of four hours to obtain sets of 12. After GAF encoding, the data were inputted as a state with dimension $12 \times 12 \times 4$.

2) Action Design: The purpose of this study was to optimize the Sure-Fire policy. There were two optimization objectives:

1) to reduce overall buy-ins and
2) to set favorable stop-loss and stop-gain points.

The former is to avoid the lack of funding while the latter is to prevent the fluctuation of prices and unnecessary placement of investments.

After pragmatic consideration, three discrete actions were developed:

- Upper limit to additional buy-ins after entering the market: $\{1, 2, 3\}$.
- First buy or sell after entering the market: $\{BUY, SELL\}$.
- Stop-gain (stop-loss is double stop-gain): $\{20, 25, 30\}$.

3) Reward Design: Reward is defined as follows:

$$\text{Reward} = \text{Profit} \times \text{Discount},$$

where profit is the net income of the current transaction and the discount is $1.0 - 0.1 \times (\text{number of additional buy-ins})$.

This calculation is a variant of the system quality number (SQN). SQNs are one of the indicators used to score trading strategies. The formula is: $(\text{expected profit/standard deviation}) \times (\text{square root of the number of transactions})$.

The discount factor is a variable that decreases concurrently with the increase in transactions. The reason for adding a discount factor to profit was to inform the agent that the reward decreased concurrently with the increase in the number of additional buy-ins. That is, risk increases concurrently with the number of additional buy-ins. Therefore, excessive buy-ins should be avoided.

B. Experiment Design

1) Experiment Data: In this study, we used the EUR/USD, GBP/USD, and AUD/USD exchange data in units of four hours between 1 August 2018 and 31 December 2018 as a reference for the environment design and performance calculation of the system. The period between 1 August 2018 and 30 November 2018 was the training period for the agent. In this period, the agent repeatedly applied the data to learn, eventually obtaining an optimal investment strategy. The period between 1 December 2018 and 31 December 2018 was the agent’s performance evaluation period. The agent uses a trading strategy to form decisions. The system accumulates the rewards obtained during the evaluation period, which served as a reference for performance.
2) Trade Environment Settings: In this study, EUR/USD, GBP/USD, and AUD/USD were adopted as investment targets. The smallest unit was 1 pip (0.00001 of the price). The default earnings for an action was set to at least 20 pips, which would be greater than the slip price and transaction fee. Therefore, the slip price and transaction fees were not taken into account. Whenever a transaction was tested, the closing price was used as the data point rather than considering whether the transactions were triggered at a high point or low point in the data (that is, of the four values for each day—opening prices, high point, low point, and closing price—the final one was chosen). Time scales smaller than one day rarely contained stop-loss and stop-gain points.

3) Experiment Model Design: In addition to DQN and PPO, a constant agent was added to the experiment model, serving as a reference value for performance. The input of the three different algorithms was a GAF-encoded time series of dimension $12 \times 12 \times 4$. A CNN was used as the policy network or Q-network.

| Model Code | Algorithm | Currencies |
|------------|-----------|------------|
| CEU        | Constant  | EUR/USD    |
| CGU        | Constant  | GBP/USD    |
| CAU        | Constant  | AUD/USD    |
| DEU        | DQN       | EUR/USD    |
| DGU        | DQN       | GBP/USD    |
| DAU        | DQN       | EUR/USD    |
| PEU        | PPO       | EUR/USD    |
| PGU        | PPO       | GBP/USD    |
| PAU        | PPO       | EUR/USD    |

TABLE I

IV. EXPERIMENT RESULTS AND ANALYSIS

A. Model Training

Fig. 15. Blue line is DAU; Purple line is CAU.

In Figures 15—20, the left image illustrates the net earnings after the complete training of one episode, and the right image illustrates the cumulative earnings after the final training process. The purple line represents the corresponding constant agent. The data shows that although PPO did not converge as easily as DQN, its performance was far better than the reference value and DQN. The performance of the DQN did not meet expectations. This may be because it was an algorithm built on a value function and it used the greedy method to form a policy action. Therefore, the learned policy was always a deterministic policy and, thus, rapidly converged. The DQN could not learn random policies like the PPO.

B. Optimal Model Performance Analysis

This was the test phase. That is, Figures 21—26 illustrate the results of the PPO algorithm using the four months of training data repeatedly trained for 800 episodes (the performance of the DQN was lower than the reference value in the training phase; therefore, the results were not compared in the test phase). In Fig. 24, the net earnings of PEU were lower than that of CEU. Notably, the only difference between PEU and CEU in cumulative earnings was a slope of the cumulative earnings graph. The shape of the two graphs was similar, indicating that the low net earnings were attributable to the selection of a conservative encoding method rather than the formation of an erroneous decision. The low maximum drawdown suggests that the PEU may have found a low-risk and stable return policy. In Fig. 25, the performance of PCU and CGU was similar. However, the profit-loss ratio of PGU was far higher than CGU with a lower maximum drawback. Therefore, PGU exhibited high and stable profitability. Figure 26 clearly shows that PGU could form accurate decisions. All items in PGU outperformed CGU. Moreover, the profit graphs for the two systems were similar. The test phase indicated that applying a PPO reinforcement learning algorithm to optimize foreign exchange trading strategies is feasible and that the PPO approach outperformed the DQN.

V. CONCLUSION

In this study, we applied a reinforcement learning method to build a strategic trading system for foreign exchange. We used a GAF-encoded time series as the state, two different algorithms, and three different currency pairs to create six different trading models. We trained and tested data using
| Items       | Descriptions                                                                 |
|------------|-----------------------------------------------------------------------------|
| Total Trade| Sum of all buy-ins and backhand transactions                                 |
| Win Trades | -                                                                           |
| Lose Trades| -                                                                           |
| Net Profit | Sum of all losses and gains, where a higher value is preferable             |
| Profit factor| Absolute value (total positive income/total negative return), where a higher value is preferable |
| Max Drawdown| Maximum loss after maximum earnings/Maximum net earnings, where a lower value is preferable |

**TABLE II**

| Items       | Descriptions                                                                 |
|------------|-----------------------------------------------------------------------------|
| Total Trades| 866                                                                         |
| Win Trades  | 715                                                                         |
| Lose Trades | 151                                                                         |
| Net Profit  | 20641                                                                       |
| Profit Factor| 2.02                                                                         |
| Max Drawdown| -3.30%                                                                      |

**OPTIMAL MODEL PERFORMANCE ANALYSIS DESCRIPTIONS.**

Fig. 18. Blue line is PAU; Purple line is CAU.

Fig. 19. Blue line is PEU; Purple line is CEU.

Fig. 20. Blue line is PGU; Purple line is CGU.

Fig. 21. CEU performance table.

Each model. The test results of the various models indicated that favorable investment performance was achieved as long as the model was able to handle complex and random processes and the state was able to describe the environment, validating the feasibility of reinforcement learning in the development of trading strategies.

We also found that the definition of reward was extremely important. The most difficult aspect was the definition of reward. How do we design a method to accurately evaluate and score the agent’s various actions and decisions? Since reinforcement learning does not allow predictive price behaviors in the learning process, the reward must be completely decoupled from price. Therefore, designs need to be intricately planned.

In this study, we validated the feasibility of applying reinforcement learning to trading strategies. In future research, relevant settings and parameters of the system could be expanded to achieve more stable investment performance. We propose the following four suggestions:

1) Establish a more refined reward formula: The current reward formula is less than ideal. In future, more parameters could be added to develop a more robust reward formula and facilitate reinforcement learning.

2) Provide more states: Only basic price data (opening, price, high point, low point, closing price) after encoding were used as the states in this study. In future, metrics related to volatility, such as the average true range (ATR), could be incorporated.

3) Calculate slip price and transaction fees: Additional costs must be taken into account to more accurately calculate profit and loss.

4) Address GAF normalization: Before the time series is encoded as GAF, it cannot be normalized because the domain of arccos is only $[-1, 1]$. However, normalizing two numbers with a difference of 100 and two numbers with a difference of 1 to $[-1, 1]$ using Min-Max Scalar Normalization produces values between $-1$ and $1$, which is unreasonable. Because of the large difference between the values, no differences were observed after normalization. This is a problem that has yet to be resolved for GAF. We hope to find a more robust coding method in the future.

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