The conversion of phase structure of singular beams spreading in uniaxial crystal

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Abstract. The transformation of the intensity and phase of paraxial optical beams passed uniaxial crystal strictly orthogonal to the optical axis is analysed. Imbedded optical vortex in such case imputes structural disturbance to the phase and intensity distribution after anisotropic media. Considering Left and Right circular polarized components of light, we theoretically and numerically shown the dynamics of phase shaping within the rotating birefringent crystal due to anisotropic diffraction effect. Off-axial vortex experienced tangential shift at the beam component and stimulates appearance of topological pairs in vicinity of beam axis.

1. Introduction
An investigation of optical vortex beams propagation and their transformation in anisotropic media plays a special role in singular optics of anisotropic threads [1 – 4]; it allows to form the beam shape and phase by the anisotropy of crystals [3 – 5] provoking appearance of polarization singularities. Changing of crystal properties and geometry makes possible to control the number and relative position of singular beams in front of observation plane, which can be widely used for modulation devices, optical tweezers and microscopy [6]. As it was shown in related articles, named to the orthogonal beam propagation in anisotropic media [7], the beams experienced elliptical deformation of cross-section, the value of ellipticity is proportional to the relation between the refractive indices for the ordinary and extraordinary beams. Nevertheless, the question about the features of the spatial phase structure was not fully disclosed. In particular, the forming of singular beam conoscopic patterns deserves special consideration due to the rich variety of effects caused by the interference between ordinary and extraordinary beams.

Physical mechanisms of the formation of complex vector fields in crystals were previously studied as propagation of uniformly polarized paraxial beams along the crystal optical axis [4]. In our case, the rotation of vortex around perpendicular to the optical axis allows to specify required configuration of phase singularities in the field after crystal. Thus, the question about beam shaping and phase control of optical vortices is a topical problem of modern singular optics.

2. Paraxial wave equations for singular beam spreading through the uniaxial crystal
Let us consider the uniaxial crystal with optical axis directed strictly perpendicular to the beam direction. The crystal can be represented as uniaxial homogeneous and unlimited media, where main crystallographic axes are tied to the laboratory axes linked with beam direction.
The dielectric permittivity tensor can be written as \([1]\):

\[
\varepsilon = \begin{pmatrix}
\varepsilon_1 & 0 & 0 \\
0 & \varepsilon_2 & 0 \\
0 & 0 & \varepsilon_3
\end{pmatrix},
\]

(1)

where \(\varepsilon_1\) is a relative dielectric permittivity coefficient for the ordinary \(E_x\) beam and \(\varepsilon_2\) is for the extraordinary \(E_y\) beam respectively. The beam components are orthogonally polarized fundamental Gaussian modes with linear polarization and, in this case, do not interfere with each other. In paraxial approximation the beam components can be expressed as:

\[
E_x = \tilde{E}_x(x, y, z) e^{i(k_x x - \omega t)} ,
\]

(2)

\[
E_y = \tilde{E}_y(x, y, z) e^{i(k_y y - \omega t)} .
\]

(3)

Then, for each transverse polarized component we can write wave equation in a form:

\[
\partial_x^2 \tilde{E}_x + \partial_y^2 \tilde{E}_x - 2i k_x \partial_z \tilde{E}_x = 0 ,
\]

(4)

\[
\partial_x^2 \tilde{E}_y + \partial_y^2 \tilde{E}_y - 2i k_x \partial_z \tilde{E}_y = 0 ,
\]

(5)

where \(k_x = \frac{k^{(1)}_x}{\varepsilon_1}\) and \(k_y = \frac{k^{(1)}_y}{\varepsilon_3}\). Let crystal rotate and introduce new variables: \(\psi\) – is the precession angle and \(\varphi\) – is an angle of internal vortex rotation around beam axis. We restrict ourselves to the case when initial beam has circular cross-section \(w_o = w_e\) at the plane \(z = 0\). The solutions of wave equations (4) and (5) can be written as electric field components:

\[
\tilde{E}_x = \left( \frac{X - i \xi Y}{w_x^{(1)} \sigma_y} - a e^{-i(\psi - \varphi)} \right) \times \exp \left[ - \left( \frac{X^2 + Y^2}{w_x^{(1)} \sigma_y} \right) / \sigma_y \right] ,
\]

(6)

\[
\tilde{E}_y = \frac{i}{\sqrt{\sigma_x \sigma_y}} \left( \frac{X - i \xi Y}{w_x^{(1)} \sigma_y} - a e^{-i(\psi - \varphi)} \right) \times \exp \left[ - \left( \frac{X^2}{w_x^{(1)} \sigma_x} - \frac{Y^2}{w_y^{(1)} \sigma_y} \right) / \sigma_y \right] ,
\]

(7)

where we introduced:

\[
\begin{align*}
z_o &= k_x w_o^2 / 2 , \\
z_e &= k_x w_e^2 / 2 , \\
z_y &= k_x w_y^2 / 2 , \\
\sigma_o &= 1 - i z / z_o , \\
\sigma_y &= 1 - i z / z_y , \\
X &= x \cos \psi - y \sin \psi , \\
\xi &= \pm 1.
\end{align*}
\]

Obviously, the shapes of ordinary and extraordinary beams due to the crystal’s anisotropy and unequal Rayleigh length will be different and, as result, we get displacement of optical vortex in the beam cross section of each circularly polarized component: \(E_+ = E_x - i E_y, E_- = E_x + i E_y\). The phase portrait shown in figure 1 (a,b) indicates the presence of a vortex in the beam central region where direction of the helix refers to the sign of topological charge.

![Figure 1](image1.png)

(a) (b) (c)

**Figure 1.** The intensity distribution of left \(E_-\) (a) and right \(E_+\) (b) circularly polarized components and phase pattern (c) of the beam passed a crystal strictly orthogonal to the optical axis. Beam parameters are: \(a = 0, \omega_b = 20 \mu m, \ z = 20mm\).
A phase shift on the beam periphery indicates presence of constant phase difference between ordinary and extraordinary beams (see figure 1 (c)).

As it shown in figure 2 (a,b), when initial vortex shifts from the beam center on a distance equal to a half of the waist radius, vortex displacement in the ordinary vortex beam after uniaxial crystal corresponds to one in the original beam, while the position of vortex in extraordinary elliptically deformed beam is mealy shifted towards periphery.

![Figure 2](image)

**Figure 2.** The intensity distribution of left $E_-$ (a) and right $E_+$ (b) circularly polarized components and phase pattern (c) of the beam with off-axis singularity. The rectangle highlights a pair of phase dislocations. Beam parameters are: $a = 0.5$, $a_\phi = 10 \mu m$, $z = 20 mm$, $\xi = -1$, $n_o = 1.54$, $n_e = 1.55$.

In other words, elliptical deformation (caused by anisotropy of medium) increases displacement of the singularity in the extraordinary beam.

3. **The dynamics of phase singularities in circularly polarized components after the crystal**

The fine structure of conoscopic pattern contributes an additional distortion to the phase where difference between isoclines has a constant value equal to $\pi$. Accordingly the presence of off-axis vortex at the beam introduces additional phase shift to the conoscopic structure and causes the formation of topological dipole as shown in figure 3 (a).

![Figure 3](image)

**Figure 3.** Phase evolution of left circular polarized component in the vicinity of vortex core: (a) phase patterns of off-axial optical vortex in case of different angles $\phi$ and the trajectory of phase singularities (b): white circles correspond to areas where vortices are born and semi black circles indicates areas of topological reaction.
Spatial orientation of conoscopic pattern and the relative displacement of optical vortex leads to a number of topological reactions and local phase unfolding. By the controlling of initial vortex position with parameter $a$ and azimuthal angle $\phi$, we can increase the slight offset of vortex from the beam axis which is accompanied with misalignment of phase profile between singularity and conoscopic pattern. In this case, the sequential changes in position of optical vortex results on a dynamics of birth and annihilation events for topological dipoles as shown in figure 3 (b). The black arrows indicate a direction of singular points moving from the beam periphery to its center.

The initial optical vortex asymptotically approaches to the beam axis, where it annihilates with vortex of opposite topological charge; meanwhile a new topological pair is born on the beam’s periphery. First vortex is located near the beam axis, while second comes from infinity and replace the original vortex. Upon further vortex rotation, the process repeats. It is worth noting to see the vortex motion appears along the lines marking the families of isoclines where phase difference is discontinuous, and have gap equal to $\pi$. As could be expected, due to the initial phase singularity introduced into the beam, new vortices can appear only in the vicinity of phase gap. This process is illustrated in figure 4.

**Figure 4.** The intensity distribution of left circularly polarized component $E_-$ in case of different rotational angles $\phi$: $\phi = 0.2\pi$ rad (a); $\phi = 0.26\pi$ rad (b); $\phi = 0.3\pi$ rad (c); $\phi = 0.36\pi$ rad (d). Other parameters of the beam are: $a = 0.5$, $\omega_0 = 10\, \mu m$, $z = 20\, mm$, $\xi = -1$, $n_e = 1.54$, $n_o = 1.55$.

All the physical parameters was used in a calculation correspond to the SiO$_2$ crystal and weakly focused Gaussian beam. Thus, such conditions support a paraxial regime and approximately the same beam waists along the crystal. As we can see from the figure 4 (a-d), the initial vortex formed a new topological pair which moves to the beam center and annihilate. The identical process of topological reactions occur at the orthogonal circularly polarized component $E_+$, where local trajectories of topological dipoles depend on the displacement of the initial vortex towards the beam axis.

Topological reaction in component $E_+$ occurs, mainly, due to the interaction between an original vortex and isoclines. Such process results on a splitting of the intensity line with distortion of the phase portrait as illustrated in figure 5.

**Figure 4.** The phase portrait of $E_+$ component in case of different rotational angles $\phi$: $\phi = 0.1\pi$ rad (a); $\phi = 0.16\pi$ rad (b); $\phi = 0.22\pi$ rad (c); $\phi = 0.52\pi$ rad (d) and: $a = 0.5$, $\omega_0 = 10\, \mu m$, $z = 20\, mm$, $\xi = -1$, $n_e = 1.54$, $n_o = 1.55$.  


Here, we can observe a typical "forks", which correspond to the phase singularities and topological dipoles. When angle $\phi$ changes, it results on a splitting of intensity lines localized close to an off-axis vortex path. Thus, the phase singularity brings an excitation of phase in the structure of the conoscopic pattern, causing reaction of the medium and appearance of topological pairs.

4. Conclusion
Classical conoscopic pattern formed by the interference of the ordinary and extraordinary beams has a high sensitivity to phase distortions where optical vortices, edge dislocations, and other inhomogeneities are born.
It is shown the displacement of the optical beam axis relatively to the vortex causes the formation of phase singularities that interact with the original vortex affecting its trajectory. Such effects raise, mainly, due to the spatial divergence of singularities in the ordinary and extraordinary beams.
It is demonstrated that in circularly polarized beam components both ordinary and extraordinary beams interfere and provide complex reorganization of phase structure. Even if initial vortex shifts from the beam center on $0.5w$ it stimulates appearance of topological reactions between vortices in ordinary and extraordinary beams. The birth of topological pairs, their annihilation and spatial reorientation indicates the medium response, and hence, this effects can be used for sensors producing, analyzers and phase modulators.

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