Research Article

Calculation on Bending Stiffness of RC Short Beam Strengthened by CFRP

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Based on the bending tests of seven reinforced concrete (RC) short beams strengthened with carbon fiber reinforced polymer (CFRP), the bending stiffness curves of the whole process of the short beams strengthened with CFRP were obtained. The variation law of bending stiffness curve of short beam in the whole loading process was analyzed. Based on the reasonable calculation assumption, the calculation method of flexural rigidity of short reinforced concrete beams strengthened with CFRP sheets in the whole loading process was put forward. The comparison between the calculated value and the test value of bending stiffness showed that the calculation method of bending stiffness was reasonable and had high calculation accuracy. This calculation method can be used not only in the calculation of flexural rigidity of short reinforced concrete beams strengthened with CFRP sheets but also in the calculation of flexural rigidity of ordinary short reinforced concrete beams. The calculation method in this paper can provide a theoretical basis for the deformation calculation of reinforced concrete short beams strengthened with CFRP sheets.

1. Introduction

In China, a large number of structural engineering has been damaged or destroyed due to the damage of natural disasters such as earthquake, the increase of service load, the long construction time, or the lower original design standard. It is necessary to reinforce and reconstruct the existing engineering structure. In the existing engineering structure, there is a kind of simply supported beam whose span height ratio is between 2 and 5, which is called short beam. Short beam is a common horizontal component in the existing engineering structure. It is not only widely used in construction engineering but also widely used in hydraulic engineering, port engineering, railway, highway, municipal engineering, and other fields [1]. Because of the small span height ratio, the short beam has a large bearing capacity. The failure of reinforced concrete short beam belongs to bad brittle failure. In order to improve the ductility and bearing capacity of reinforced concrete short beams, it can be realized by adding a certain amount of short fiber or by fiber reinforced polymer (FRP) reinforcement. Steel fiber, polypropylene fiber, and polyvinyl alcohol fiber are the short fibers to be added. This method is often used in new structure [2–4]. FRP is more suitable for the reinforcement of existing reinforced concrete structures. Among the commonly used FRP materials, the CFRP material has the best performance and is most commonly used in structural strengthening [5].

Scholars at home and abroad have done a lot of experimental and theoretical research on the flexural behavior of reinforced concrete shallow beams strengthened with FRP materials and achieved many research results [6–10]. There are many research studies on the calculation method of bearing capacity of beams strengthened with CFRP in the existing literature [11–15]. There are few studies on the stiffness of beams strengthened with CFRP in the existing literature [16–18]. In the existing literature, the research object of flexural rigidity is mostly reinforced concrete
shallow beam [19–21]. The stiffness analysis method is often used in the existing literature [22]. The obtained stiffness calculation formula is cumbersome or needs to be discussed [23–25]. The stress stage of the existing literature is the stage before the steel bars yield, and the stiffness after the steel bars yield has not been studied. In view of the importance of beam stiffness to deformation calculation and the lack of current research, based on the effective moment of inertia method, this paper further discusses the calculation method of short-term stiffness of reinforced concrete short beams strengthened with CFRP sheets in the whole loading process. In order to study the flexural rigidity of short RC beams strengthened with CFRP sheets, the flexural rigidity expression consistent with that of ordinary reinforced concrete beams is established. In this paper, the influence of CFRP layers, concrete strength grade, and longitudinal reinforcement ratio on the flexural rigidity of reinforced concrete short beams is studied through 7 members. In this paper, the calculation formula of flexural rigidity for short reinforced concrete beams strengthened with CFRP sheets and ordinary short reinforced concrete beams is put forward.

2. Experimental Program

Seven beams were designed in this experiment. The cross section of the beams was 150 mm in width and 500 mm in height. The clear span was 2000 mm with the span ratio of 4. The geometrical dimensions and reinforcement of the beams are shown in Figure 1. One beam was an ordinary RC short beam without CFRP sheets and the rest were RC short beams with CFRP sheets. As shown in Figure 2, CFRP sheets were pasted on the beam bottom to improve bending strength, and two strips of CFRP sheet with 100 mm width were set up in the bending shear zone of each side to prevent the occurrence of stripping at the ends. The detailed design parameters of beams are shown in Table 1. The measured elastic modulus of CFRP sheet was 246 GPa, the tensile strength was 3512 MPa, the thickness was 0.167 mm, and the elongation was 1.71%. The measured mechanical properties of concrete and reinforcement are shown in Tables 2 and 3.

A monotone static loading test was carried out for all beams. The concrete strain, deflection, inclination, and bearing capacity of beams are measured. The arrangement of test loading and measuring equipment is shown in Figure 3. The concrete strains in the compression zone, tension zone, and beam height range of the pure bending span were measured using the strain gauges of \( \pi \) shape. Strain-type displacement sensors were arranged in the upper part of the support, the lower part of the loading point, and the middle of the span to measure the deflection. Ten inclinometers were placed at 25 mm intervals on the top of the beam to measure the rotation angle of the top of the beam. All test data were collected by a data acquisition instrument. The cracking load, yield load, and ultimate load of 7 test pieces measured in the test are listed in Table 4. The failure modes of CFRP strengthened beams include three modes: first tensile failure of CFRP, first crushing failure of concrete, and almost simultaneous boundary failure of both. The specific failure modes are also listed in Table 4.

The values of curvature and stiffness for the pure bending section of beam under different bending moments were calculated by equations (1) and (2). All bending moment-curvature curves and bending moment-stiffness curves are drawn in Figures 4–10.

\[ \phi = \frac{1}{\rho} = \frac{\bar{T}_c + \bar{T}_s}{h} \]  
\[ B = EI = \frac{M}{\phi} \]

where \( \rho \) is the radius of curvature; \( \bar{T}_c \) is the average concrete strain in the compression zone of pure bending section; \( \bar{T}_s \) is average concrete strain in the tension zone of pure bending section; \( h \) is the average distance between strain gauge in the compression zone and the tension zone of the pure bending section; \( E \) is the elastic modulus of concrete; \( I \) is inertia moment of beam; and \( M \) is the bending moment of the midspan section.

The general characteristics of bending moment-curvature and bending moment-stiffness curves are shown in Figures 11 and 12, respectively. Curves are divided into three stages by the cracking moment, yielding moment, and ultimate bending moment:

1. Stage of prcracking of concrete: the curvature is linear, and linear slope is a maximum value. The stiffness is a constant value in general. However, the measured bending stiffness changes greatly because of the small curvature in this stage, which is affected by the precision of \( \pi \) strain gauge, and there will be small fluctuation in curvature. From formula (2), it can be seen that the small fluctuation of curvature will cause great fluctuation in bending stiffness. Therefore, the bending stiffness value close to the cracking load should be taken as the fixed value in this stage as far as possible.

2. Stage of concrete cracking to the yield-steel: the curvature is approximately linear, the slope decreases, and the stiffness decreases sharply. A nonlinear regulation is presented in this stage.

3. Stage of the yield-steel to the ultimate bearing state: the curvature is also approximately linear, and the slope and stiffness continuously decrease. A nonlinear change regulation is also presented in this stage.

3. Calculation of Bending Stiffness

3.1. Calculation Assumption. There are three assumptions for the calculation of bending stiffness:

The concrete is elastic in the prcracking stage.

In the stage of concrete cracking to the yield-steel, the compressive strain of concrete is smaller than the peak strain. Concrete is assumed as a linear elastic material,
Table 1: Design parameters of the specimen.

| No. | Beams ID  | CFRP sheet layers | Concrete strength | Reinforcement ratio (%) |
|-----|-----------|-------------------|-------------------|-------------------------|
| 1   | B-0-30-4  | 0                 | C30               | 0.42                    |
| 2   | B-1-30-4  | 1                 | C30               | 0.42                    |
| 3   | B-2-30-4  | 2                 | C30               | 0.42                    |
| 4   | B-1-20-4  | 1                 | C20               | 0.42                    |
| 5   | B-1-40-4  | 1                 | C40               | 0.42                    |
| 6   | B-1-30-6  | 1                 | C30               | 0.60                    |
| 7   | B-1-30-8  | 1                 | C30               | 0.82                    |

Table 2: Mechanical performance index of concrete.

| Beams ID  | Concrete strength | Modulus of elasticity (MPa) |
|-----------|-------------------|----------------------------|
| B-0-30-4  | Cube strength (MPa) | 32.95 | 22.67 | 1.72 | 16073.9 |
| B-1-30-4  | Axial compressive strength (MPa) | 38.11 | 32.62 | 2.40 | 23023.6 |
| B-2-30-4  | Tensile strength (MPa) | 36.38 | 29.63 | 2.11 | 20325.7 |
| B-1-20-4  | 2.06 | 19490.8 |
| B-1-40-4  | 2.69 | 24078.7 |
| B-1-30-6  | 2.00 | 20898.2 |
| B-1-30-8  | 2.00 | 16495.0 |

Table 3: Mechanical performance index of steel bar.

| Rebar grade | Diameter (mm) | Yield strength (MPa) | Tensile strength (MPa) | Modulus of elasticity (GPa) |
|-------------|---------------|----------------------|------------------------|----------------------------|
| HRB400      | 8             | 418                  | 642                    | 200                        |
|             | 10            | 411                  | 641                    | 200                        |
|             | 12            | 520                  | 616                    | 200                        |
|             | 14            | 421                  | 543                    | 200                        |
and the tensile steel bar is still elastic. The stress-strain relationship conforms to Hooke’s law. The average strain of the beam section is assumed to be in line with the plane section assumption. The correction coefficient of internal force arm is taken into account for the short beam, and the calculation method is conformed to reference [16]. The resistance to tension of concrete in the tensile zone of cracking section is ignored.

The basic assumption in the stage of the steel yield to the ultimate bearing state is in accordance to reference [16].

### 3.2. Calculation of the Precracking Stage

In this stage, as the concrete has not cracked yet, the steel bar and CFRP sheet can be equivalent to concrete according to the elastic theory.

![Figure 3: Layout of test measuring equipment (unit: mm).](image)

**Table 4: Main test results of the specimen.**

| Beams ID | Cracking load (kN) | Yield load (kN) | Ultimate load (kN) | Failure mode               |
|----------|--------------------|-----------------|--------------------|---------------------------|
| B-0-30-4 | 66.5               | 153.0           | 223.0              | Failure mode of suitable reinforced beam |
| B-1-30-4 | 100.0              | 188.9           | 282.0              | CFRP sheet pulled         |
| B-2-30-4 | 98.0               | 190.3           | 355.0              | Boundary failure          |
| B-1-20-4 | 83.0               | 154.4           | 283.0              | CFRP sheet pulled         |
| B-1-40-4 | 115.0              | 170.5           | 303.0              | CFRP sheet pulled         |
| B-1-30-6 | 82.5               | 294.8           | 358.9              | Concrete crushed          |
| B-1-30-8 | 92.0               | 325.0           | 371.4              | Concrete crushed          |

![Figure 4: B-0-30-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.](image)
Figure 5: B-1-30-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 6: B-2-30-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 7: B-1-20-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.
Figure 8: B-1-40-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 9: B-1-30-6. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 10: B-1-30-8. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.
So, the calculation equations of parameters for beam section are established.

The height of neutral axis from the edge of concrete in the compressive zone \( x_0 \) and the inertia moment of section \( I_0 \) can be calculated by the following two equations:

\[
x_0 = \frac{(bh^2/2) + (\alpha_E - 1)A_s h_0 + \alpha_f A_f h}{bh + (\alpha_E - 1)A_s + \alpha_f A_f}.
\]

\[
I_0 = \frac{bx_0^3}{3} + \frac{b(h - x_0)^3}{3} + (\alpha_E - 1)A_s(h_0 - x_0)^2 + \alpha_f A_f(h - x_0)^2,
\]

where \( b, h, \) and \( h_0 \) are beam width, height, and the distance from the joint action point of tensile steel bars to the compression edge of the beam, respectively. \( A_s \) and \( A_f \) are section areas of tensile steel bar and CFRP sheet, respectively. \( \alpha_E \) is the ratio of elastic modulus of steel bar to the elastic modulus of concrete \( (E_s/E_c) \). \( \alpha_f \) is the ratio of elastic modulus of CFRP sheet to the elastic modulus of concrete \( (E_f/E_c) \). \( E_s, E_c, \) and \( E_f \) are elastic moduli of steel bar, concrete, and CFRP sheet, respectively. \( I_0 \) is the inertia moment of the uncracked beam section.

Thus, the bending stiffness \( B_0 \) of the uncracked beam section is calculated using the following equation:

\[
B = B_0 = E_c I_0.
\]

The cracking moment is calculated using the following equation:

\[
M_{cr} = \frac{\gamma_m f_t I_0}{(h - x_0)},
\]

where \( f_t \) is tensile strength of concrete and \( \gamma_m \) is the plastic coefficient of section resistance moment. The value of \( \gamma_m \) is 1.75, referring to the ordinary RC beam.

### 3.3. Calculation of Concrete Cracking to the Yield-Steel

It is assumed that the concrete in the compression zone is a linear elastic material, and the cross-sectional force diagram at this stage is shown in Figure 13. \( C \) is the resultant force of compression concrete. \( \sigma_s \) and \( A_s \) are tensile stress and area of steel bars, respectively. \( \sigma_f \) and \( A_f \) are the tensile stress and effective area of CFRP sheets. \( A_{fe} \) is the actual area of the CFRP sheet multiplied by the thickness reduction coefficient \( K_m \). The value of \( K_m \) is determined according to Chinese specification GB50367-2013 [26].
When the tensile steel yields, equations (7) and (8) can be obtained according to the equilibrium condition of the force.

\[ f_y A_s + \sigma_c A_{te} - \frac{1}{2} b x_c \sigma_c = 0, \quad (7) \]

\[ E_s \varepsilon_y A_s + E_f \varepsilon_f A_{te} - \frac{1}{2} b x_c E_c \varepsilon_c = 0, \quad (8) \]

where \( f_y, \sigma_c, \) and \( x_c \) are yield strengths of steel bar, compressive stress of concrete, and height of the concrete compression zone, respectively.

The yield strain \( \varepsilon_y \) of steel bar is obtained from the similar triangle relation of strain.

\[ \varepsilon_c = \frac{\varepsilon_y x_c}{h_0 - x_c}, \quad (9) \]

\[ \varepsilon_f = \frac{\varepsilon_y (h - x_c)}{h_0 - x_c}. \quad (10) \]

Equations (9) and (10) are substituted into equation (8) to obtain the following equation:

\[ E_s \varepsilon_y A_s + E_f \varepsilon_f (h - x_c) A_{te} - \frac{1}{2} b x_c E_c \varepsilon_c (h - x_c) = 0. \quad (11) \]

The height of the compression zone \( x_c \) can be obtained by solving equation (11), and then the yield bending moment is obtained as follows:

\[ M_f = E_s \varepsilon_y A_s \sigma_c \left( \frac{h_0 - x_c}{2} \right) + E_f \varepsilon_f A_{te} \sigma_c \left( h - \frac{x_c}{3} \right). \quad (12) \]

When the steel yields, the inertia moment of the cracked section \( I_{cr} \) can be calculated by the following equation:

\[ I_{cr} = \frac{1}{3} b x_c^3 + (\alpha_E - 1) A_s (h_0 - x_c)^2 + \alpha_f A_f (h - x_c)^2. \quad (13) \]

The inertia moment of the cracked section \( I_{cr} \) is the minimum inertia moment of each section along the component axis. The stiffness of the cracked section is calculated using the following equation:

\[ B_{cr} = E_c I_{cr}. \quad (14) \]

In this stage, the value of average section stiffness \( B \) is a value between \( B_0 \) and \( B_{cr} \), which decreases with the increase of bending moment.

For the ordinary RC beam, according to the United States code ACI318-05 [27], the effective stiffness of the beam section \( B_{eff} \) in normal service stage is calculated using the following equation:

\[ B_{eff} = E_c I_{eff} = \left( \frac{M_{cr}}{M} \right)^4 E_c I_0 + \left[ 1 - \left( \frac{M_{cr}}{M} \right)^4 \right] E_c I_{cr}. \quad (15) \]

According to the test data of bending moment-bending stiffness curve in this experiment, the equation of bending stiffness for RC short beams strengthened by CFRP sheet is obtained using the nonlinear regression method.

\[ B = \left( \frac{M_{cr}}{M_y} \right)^4 B_0 + \left[ 1 - \left( \frac{M_{cr}}{M_y} \right)^4 \right] B_{cr}. \quad (16) \]

When the bending moment \( M \) is the yield bending moment \( M_y \), the calculated yield stiffness \( B_y \) of the beam is calculated using the following equation:

\[ B_y = \left( \frac{M_{cr}}{M_y} \right)^4 B_0 + \left[ 1 - \left( \frac{M_{cr}}{M_y} \right)^4 \right] B_{cr}. \quad (17) \]

3.4. Calculation of Longitudinal Bars after Yielding. Two bending failure modes as breakage of CFRP sheet and crushing of concrete are observed in this experiment. Two theoretical equations for calculating the bending capacity \( M_u \) in corresponding failure modes have been established in literature [16]. The calculation method in this paper is the same as that in reference [16]. Equations (18) and (19) are calculation equations of section curvature \( \phi_u \), respectively.

For the breakage of CFRP sheet:

\[ \phi_u = \frac{(0.9\varepsilon_{fu} / (h - x_c))}{\alpha_d}. \quad (18) \]

For the crushing of concrete:

\[ \phi_u = \frac{(\varepsilon_{cu} / x_c)}{\alpha_d}. \quad (19) \]

In the ultimate bearing state, the bending stiffness \( B_u \) is calculated using the following equation:
At this stage, the value of bending stiffness $B$ of the beam section is a value between $B_y$ and $B_u$ and decreases with the increase of bending moment. According to the experimental data, a nonlinear regression method is adopted to obtain the calculation equation of bending stiffness of RC beams reinforced by CFRP sheet in the stage of yield-steel to the ultimate bearing state.

$$B = \frac{B_y + (B_u - B_y)(M - M_y)^0.7}{(M_u - M_y)^{0.7}}.$$  \hspace{1cm} (21)

When the bending moment $M$ is equal to the yield bending moment $M_y$, the bending stiffness of the beam is calculated using the following equation:

\[ B_u = \frac{M_u}{\phi_u} \]  \hspace{1cm} (20)
Figure 14: Beam B-0-30-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 15: Beam B-1-30-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 16: Beam B-2-30-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.
Figure 17: Beam B-1-20-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 18: Beam B-1-40-4. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.

Figure 19: Beam B-1-30-6. (a) Bending moment-curvature curve. (b) Bending moment-stiffness curve.
4. Comparison of Calculated and Experimental Data

The test and calculated values of cracking, yield, and ultimate bending moment are shown in Table 5. The ratio in the table is the calculated value divided by the test value. The average values of the ratio of cracking, yield, and ultimate bending moment are 1.148, 0.996, and 1.007, respectively. The coefficient of variation of the ratio of cracking, yield, and ultimate bending moment is 0.061, 0.059, and 0.028, respectively.

The test and calculated values of cracking, yield, and ultimate curvature are shown in Table 6. The ratio in the table is the calculated value divided by the test value. The average values of the ratio of cracking, yield, and ultimate curvature are 1.003, 0.949, and 0.994, respectively. The coefficients of variation of the ratio of cracking, yield, and ultimate curvature are 0.097, 0.158, and 0.139, respectively.

The test and calculated values of cracking, yield, and ultimate bending stiffness are shown in Table 7. The ratio in the table is the calculated value divided by the test value. The average values of the ratio of cracking, yield, and ultimate flexural rigidity are 1.154, 1.035, and 1.029, respectively. The coefficient of variation of the ratio of cracking, yield, and ultimate flexural rigidity is 0.111, 0.108, and 0.121, respectively.

The bending moment-curvature curve and bending moment-flexural rigidity curve are shown in Figures 14–20. It can be seen from the figures that the dotted line is the calculated value and the solid line is the test value. Except for the difference between the calculated value and the test value at the cracking moment, the calculated value and the test value at the rest of the moment are basically consistent.

The comparison between Tables 5–7 and Figures 14–20 shows that the bending rigidity calculation method proposed in this paper is reasonable, with high calculation accuracy, and is suitable for the bending rigidity calculation of reinforced concrete short beams strengthened with carbon fiber sheet.

5. Conclusions

Based on the experiment of flexural performance of short reinforced concrete beams strengthened with CFRP sheets, the calculation method of flexural rigidity of short beams in the whole loading process was proposed. The calculated flexural rigidity value was compared with the experimental value, and the following conclusions are drawn:

The characteristics of the bending moment-curvature curve and the bending moment-stiffness curve of the pure bending section in the span of 7 specimens were obtained. The curve of flexural stiffness was divided into three sections by cracking load and yield load. The bending stiffness curve after cracking moment was nonlinear.

According to the characteristics of the three stress stages, the reasonable calculation assumption was adopted. Based on the effective moment of inertia method, the calculation formula of flexural rigidity of reinforced concrete short beams strengthened with CFRP sheets was proposed. The formula was applicable to both short beams strengthened by CFRP and ordinary reinforced concrete beams without reinforcement, and the calculation accuracy was high.

The calculation method of flexural rigidity proposed in this paper can provide theoretical basis for the calculation of the corner of the reinforced concrete short beam strengthened by CFRP and also provide data support and reference for other research studies in the future.

Data Availability

The data used to support the findings of the study are available from the corresponding author upon request.
Conflicts of Interest
The authors declare that they have no conflicts of interest.

Authors’ Contributions
Wang Tingyan did the whole experiment and wrote the article, Zhang Junwei calculated the data, and Zhou Yun revised the article. All authors have read and agreed to the published version of the manuscript.

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References
[1] T. Wang and J. Zhang, “Experimental study on the flexural performance of reinforced concrete short beams strengthened by CFRP sheets,” Building Structure, vol. 50, no. 2, pp. 76–81, 2020, in Chinese.
[2] P. Zhang, Y. Zheng, K. Wang, and K. Zhang, “Combined influence of nano-CaCO3 and polyvinyl alcohol fibers on fresh and mechanical performance of concrete incorporating fly ash,” Structural Concrete, vol. 21, no. 2, pp. 724–734, 2020.
[3] P. Zhang, Y. Ling, J. Wang, and Y. Shi, “Bending resistance of PVA fiber reinforced cementitious composites containing nano-SiO2,” Nanotechnology Reviews, vol. 8, no. 1, pp. 690–698, 2019.
[4] P. Zhang, L. Kang, J. Wang, J. Guo, S. Hu, and Y. Ling, “Mechanical properties and explosive spalling behavior of steel-fiber-reinforced concrete exposed to high temperature—a review,” Applied Sciences, vol. 10, no. 7, pp. 1–21, 2020.
[5] X. Chen, Guide for the Design and Construction of Fiber Reinforced Plastics in Civil Engineering, pp. 1–4, China Architecture & Building Press, Beijing, China, 2009, in Chinese.
[6] S. Shang, H. Peng, H. Tong, and L. Zhang, “Study of strengthening reinforced concrete beam using prestressed carbon fiber sheet,” Journal of Building Structures, vol. 24, no. 5, pp. 24–30, 2003.
[7] L. Bu, L. Song, and C. Shi, “Experimental and theoretical study on flexural behavior of RC beams strengthened with carbon fiber plates (CFP),” Journal of Building Structures, vol. 28, no. 1, pp. 72–79, 2007.
[8] J. Yang, G. Xiong, Z. Yan et al., “Experimental study on flexural strength of hybrid CF/SFG composite strengthening concrete beams,” China Civil Engineering Journal, vol. 37, no. 7, pp. 18–22, 2004.
[9] Y. Yang, Q. Yue, and L. Ye, “Calculation for flexural debonding bearing capacity of RC beams strengthened with carbon fiber sheets,” China Civil Engineering Journal, vol. 37, no. 2, pp. 33–37, 2004.
[10] M. R. Esfahani, M. R. Kianoush, and A. R. Tajari, “Flexural behaviour of reinforced concrete beams strengthened by CFRP sheets,” Engineering Structures, vol. 29, no. 10, pp. 2428–2444, 2007.
[11] L. I. Xiang and X. GU, “Bending bearing capacity of low strength reinforced concrete beams strengthened with carbon fiber composite sheets,” China Civil Engineering Journal, vol. 45, no. 1, pp. 23–29, 2012, in Chinese.
[12] D. Gao, Pu Zhang, and C. Zhang, “Flexural performance of reinforced concrete one-way slabs strengthened with fiber reinforced polymer sheets,” Journal of Building Structures, vol. 36, no. 7, pp. 51–58, 2015, in Chinese.
[13] Q. Kong and L. Liu, “Experimental investigation of RC beam strengthened with CFRP,” Journal of Zhengzhou University (Engineering Science), vol. 25, no. 4, pp. 24–27, 2004.
[14] W. Zhang, X. Wang, and X. Gu, “Flexural behavior of corroded reinforced concrete beams strengthened with carbon fiber composite sheets,” China Civil Engineering Journal, vol. 43, no. 6, pp. 34–41, 2010.
[15] W. Zheng, W. Chen, and M. Wang, “Experimental study on flexural behavior of concrete beams strengthened with CFRP sheets bonded with an inorganic matrix,” China Civil Engineering Journal, vol. 43, no. 4, pp. 37–45, 2010, in Chinese.
[16] D. Gao, T. Wang, and Y. He, “Flexural test and calculation on capacity of reinforced concrete short beam strengthened by CFRP sheets,” Journal of Building Structures, vol. 38, no. 11, pp. 122–131, 2017.
[17] R. Z. Al-Rousan and M. A. Issa, “Flexural behavior of RC beams externally strengthened with CFRP composites exposed to severe environment conditions,” KSCE Journal of Civil Engineering, vol. 21, pp. 2300–2309, 2017.
[18] G. Spadae, F. Bencardino, and R. N. Swamy, “Structural behavior of composite RC beams with externally bonded CFRP,” Journal of Composites for Construction, vol. 2, no. 3, pp. 132–137, 1998.
[19] X. Chen, H. Li, and X. Zhu, “Experimental and theoretical research on short-term stiffness of reinforced concrete beams strengthened with FRP sheets,” Journal of Building Structures, vol. 39, no. 1, pp. 146–152, 2018.
[20] D. Gao, D. Fang, and Y. Zhu, “Flexural property and calculation method of one-way reinforced concrete slabs externally prestressed with unbonded FRP tendons,” Journal of Basic Science & Engineering, vol. 23, no. 1, pp. 115–126, 2015, in Chinese.
[21] X. Wang, X. Gu, and W. Zhang, “Flexural stiffness of corroded reinforced concrete beams strengthened with carbon fiber composite sheets,” Journal of Building Structures, vol. 30, no. 5, pp. 169–176, 2009, in Chinese.
[22] Y. Yang and Q. Yue, “Calculation of sectional stiffness of RC beams strengthened with carbon fiber sheet,” Industrial Construction, vol. 31, no. 9, pp. 1–4, 2001.
[23] M. Yang and S. Wang, “Calculation of bending stiffness of reinforced concrete beams strengthened with carbon fiber sheets,” Building Science, vol. 21, no. 4, pp. 34–37, 2005.
[24] G. E. Chao, Calculation of Bending Stiffness of Reinforced Concrete Beams Strengthened with Carbon Fiber Sheets, pp. 66–68, Kunming University of Science & Technology, Kunming, China, 2012, in Chinese.
[25] P. Zhang, H. Zhu, S. Meng, and G. Wu, “Calculation of sectional stiffness and deflection of FRP sheets strengthened reinforced concrete beams,” Journal of Building Structures, vol. 32, no. 4, pp. 87–94, 2011, in Chinese.
[26] National Standards of the People’s Republic of China, Code for Design of Strengthening Concrete Structure (GB50367-2013), China Architecture& Building Press, Beijing, China, 2013, in Chinese.
[27] American Concrete Institute Committee: Building Code Requirements for Structural Concrete (ACI318-05).