A family of double-beauty tetraquarks: Axial-vector state $T^b_{bb\pi\pi}$

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The spectroscopic parameters and decay channels of the axial-vector tetraquark $T^b_{bb\pi\pi}$ (in what follows $T^b_{AV}$) are explored by means of the QCD sum rule method. The mass and coupling of this state are calculated using the two-point sum rules by taking into account various vacuum condensates up to dimension 10. Our prediction for the mass of this state $m = (10215\pm 250)$ MeV confirms that it is stable against strong and electromagnetic decays, and can dissociate to conventional mesons only through weak transformations.

We investigate the dominant semileptonic $T^b_{AV} \to Z^b_{b\pi}M$ and nonleptonic $T^b_{AV} \to Z^b_{b\pi}M$ decays of $T^b_{AV}$. In these processes the $Z^b_{b\pi}$ is a scalar tetraquark $[be]\overline{[md]}$ built of a color-triplet diquark and antidiquark, whereas $M$ is one of the vector mesons $\rho^0$, $K^*\overline{c}(892), D^*\overline{c}(2010)^-, and D^{*0}\overline{c}^-$. In order to calculate partial widths of these decays, we make use of the QCD three-point sum rule approach and evaluate weak transition form factors. Obtained information on parameters of $T^b_{AV}$ and $Z^b_{b\pi}$ is useful for experimental investigations of these double-heavy exotic mesons.

I. INTRODUCTION

Recently, double-beauty tetraquarks composed of $bb$ diquark and light antidiquark $\overline{ms}$ became an object of intensive theoretical studies [1–6]. The interest to these states was inspired by experimental observation of the baryon $\Xi^+_c$ and measurements of its parameters [7]. The latter were used as an input information in phenomenological models to estimate masses of double-beauty states $[1]$. Investigations performed in this paper demonstrated that an axial-vector tetraquark $T^b_{bb\pi\pi}$ (hereafter $T^b_{bb}$) with the mass $m = (10389\pm 12)$ MeV is stable against strong and electromagnetic decays, and can dissociate to conventional meson only through weak transformation. A similar conclusion on a stable nature some of tetraquarks $bb\overline{ms}$ was made in Ref. [2] as well, where the authors used methods of heavy-quark symmetry analysis.

The double-heavy tetraquarks $QQ\overline{ms}$, in fairness, were studied already in classical articles [8,12], in which they were examined as candidates to stable four-quark compounds. The main qualitative conclusion drawn in these works was constraint on masses of constituent quarks: It was found that tetraquarks $QQ\overline{ms}$ may form strong-interaction stable exotic mesons provided the ratio $m_Q/m_q$ is large. Therefore, tetraquarks $bb\overline{ms}$ are most promising candidates to stable four-quark mesons.

Quantitative analyses of these problems were continued in following years in frameworks of various models and using different methods of high energy physics. Thus, tetraquarks $T_{QQ}$ were explored using the chiral, dynamical and relativistic quark models [13–17]. The axial-vector states $T_{QQ\overline{ms}}$ were considered in the context of the sum rule method [18,19]. Processes in which the tetraquarks $T_{cc}$ may be produced, namely electron-positron annihilations, heavy ion and proton-proton collisions, $B_s$ meson and $\Xi^0_{bc}$ baryon decays also attracted interests of researches [20–24].

The axial-vector particle $T^b_{bb}$ was studied in our work as well [3]. We employed the QCD sum rule method and evaluated the mass of this state $m = (10035\pm 260)$ MeV. This means that $m$ is below both the $B^-\overline{B}^0$ and $B^-\overline{B}^0\gamma$ thresholds, and hence this state is strong- and electromagnetic-interaction stable tetraquark. We also explored the semileptonic decays $T^b_{bb} \to Z^b_{b\pi}M$, where $Z^b_{b\pi}$ is the scalar tetraquark $[be]\overline{[md]}$ composed of color-triplet diquarks, and calculated their partial widths. The predictions for the full width and mean lifetime of $T^b_{bb}$ obtained in Ref. [3] are useful for experimental investigation of double-beauty exotic mesons.

Other members of the $bb\overline{ms}$ family, studied in a rather detailed form, are the scalar tetraquarks $T^b_{bb\overline{ms}}$ and $T^{-}_{bb\overline{ms}}$ (in short forms, $T^b_{b\pi}$ and $T^{-}_{b\pi}$, respectively). The mass and coupling of $T^b_{b\pi}$ and $T^{-}_{b\pi}$ were calculated in Refs. [25,26], in which we demonstrated that they cannot decay to ordinary mesons through strong and electromagnetic processes. We also investigated dominant semileptonic and nonleptonic weak decays of these tetraquarks, and estimated their full width and lifetime.

In the present article we extend our analysis and investigate the axial-vector partner of the $T^b_{b\pi}$ with the same quark content $bb\overline{ms}$. It can be treated also as "s" member of the axial-vector multiplet of the states $bb\overline{ms}$. We denote this tetraquark by $T^b_{AV}$ and compute its spectroscopic parameters using the two-point QCD sum rule method. Calculations are carried out by taking into account various vacuum condensates up to dimension 10. The obtained result for its mass $m = (10215\pm 250)$ MeV proves that this state is stable against strong and electromagnetic decays. In fact, $T^b_{AV}$
in $S$-wave can fall-apart to pairs of conventional mesons $B^* B_{s}^*$ and $B^* B_{s}^*$, provided $m$ exceeds the corresponding thresholds 10695/10692 MeV, respectively. For the electromagnetic decay to a final state $B^* B_{s}^* \gamma$ the threshold equals to 10646 MeV. It is seen that even the maximum allowed value of the mass 10465 MeV is below all of these limits.

Therefore, to evaluate the full width and lifetime of $T_{b,\pi}^{AV}$, we analyze semileptonic and nonleptonic weak decays $T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0 \pi_l$ and $T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0 M$, respectively. Here, $Z_{b,\pi}^0$ is scalar tetraquark $bc[\bar{c}b\pi]$ built of a color-triplet diquark and antiquark, and $M$ is one of the vector mesons $\rho^\pm$, $K^*(892)$, $D^*(2010)^-$, and $D_s^{\pm}$. The weak transitions of $T_{b,\pi}^{AV}$ can be described by means of the form factors $G_i(q^2)$ which determine differential rates $d\Gamma/dq^2$ of the semileptonic and partial widths of the nonleptonic processes. These weak form factors will be extracted from the QCD three-point sum rules in Section III.

This work is structured in the following way: In Section II we calculate the mass and coupling of the tetraquarks $T_{b,\pi}^{AV}$ and $Z_{b,\pi}^{0}$ For these purposes, we derive the sum rules for their masses and couplings from analysis of the corresponding two-point correlation functions. Numerical computations are performed by taking into account quark, gluon and mixed condensates up to dimension ten. In Section III we compute the weak form factors $G_i(q^2)$ from the three-point sum rules at momentum transfers $q^2$, where this method is applicable. In this section we determine also model functions $G_i(q^2)$ and find the partial widths of the semileptonic decays $T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0 \pi_l$. The weak nonleptonic processes $T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0 M$ are investigated in Section IV This section contains also our final results for the full width and mean lifetime of the tetraquark $T_{b,\pi}^{AV}$. Section V is reserved for discussion of obtained results, and concluding notes.

II. SPECTROSCOPIC PARAMETERS OF THE AXIAL-VECTOR $T_{b,\pi}^{AV}$ AND SCALAR $Z_{b,\pi}^{0}$ TETRAQUARKS

In this section we calculate the mass $m_{AV}$ and coupling $f_{AV}$ of the axial-vector tetraquark $T_{b,\pi}^{AV}$, which are necessary to clarify its nature and conclude whether this particle is stable against strong- and electromagnetic decays or not. Another tetraquark considered here is the scalar exotic meson $Z_{b,\pi}^{0}$ that appears in a final state of the master particle’s decays: Spectroscopic parameters of this state enter to expressions for partial widths of $T_{b,\pi}^{AV}$ tetraquark’s decay channels. The $Z_{b,\pi}^{0}$ is a member of the $bc\pi\pi$ family, and is of interest from this point of view as well.

The sum rules to evaluate the mass and coupling of the axial-vector tetraquark $T_{b,\pi}^{AV}$ can be obtained from analysis of the two-point correlation function

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0| T\{ J_\mu(x), J^\dagger_\nu(0) \}|0 \rangle,$$

where $J_\mu(x)$ is the corresponding interpolating current. We suggest that $T_{b,\pi}^{AV}$ is composed of the axial-vector diquark and scalar antidiquark, and the current $J_\mu(x)$ has the form

$$J_\mu(x) = [b_\mu^T(x) C \gamma_\mu b_b(x)] [\bar{\pi}_a(x) \gamma_\mu C \bar{\pi}_b^T(x)].$$

To solve the same problems in the case of the scalar tetraquark $Z_{b,\pi}^{0}$, we start from the correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0| T\{ J_\mu(x), J^\dagger_\nu(0) \}|0 \rangle.$$

Here $J_\mu(x)$ is the interpolating current for $Z_{b,\pi}^{0}$

$$J_\mu(x) = [b_\mu^T(x) C \gamma_\mu b_b(x)] [\bar{\pi}_a(x) \gamma_\mu C \bar{\pi}_b^T(x)] - \bar{\pi}_a(x) \gamma_\mu C \bar{\pi}_b^T(x).$$

In expressions above $a$ and $b$ are the color indices and $C$ is the charge conjugation operator.

The current $[29]$ is composed of diquarks which belong to sextet representation of the color group $[6_c]_{bb} \otimes [3_c]_{\pi\pi}$. Contrary, $J_\mu(x)$ describes the scalar tetraquark made of color-triplet $[3_c]_{bc} \otimes [\bar{3}_c]_{\pi\pi}$ diquarks.

Now, we concentrate on calculations of the parameters $m_{AV}$ and $f_{AV}$. Following standard prescriptions of the sum rule method, we express $\Pi_{\mu\nu}(p)$ using spectroscopic parameters of $T_{b,\pi}^{AV}$ These manipulations generate the physical or phenomenological side of the sum rules

$$\Pi_{\mu\nu}^{\text{Phys}}(p) = \frac{\langle 0| J_\mu T_{b,\pi}^{AV}(p) \rangle \langle T_{b,\pi}^{AV}(p) | J^\dagger_\nu(0) \rangle}{m_{AV}^2 - p^2} + \ldots$$

Here, we isolate the ground-state contribution to $\Pi_{\mu\nu}^{\text{Phys}}(p)$ from effects due to higher resonances and continuum states, which are denoted by the dots. In our study, we assume that the phenomenological side of the sum rules $\Pi_{\mu\nu}^{\text{Phys}}(p)$ can be approximated by a zero-width single pole term. In the case of four-quark systems the physical side, however, receive contributions also from two-meson reducible terms $[27, 25]$. Interaction of $J_\mu(x)$ with such two-meson continuum generates a finite width $\Gamma(p^2)$ of the tetraquark and results in the modification

$$\frac{1}{m^2 - p^2} \rightarrow \frac{1}{m^2 - p^2 - i\sqrt{\Gamma(p^2)}}.$$

The contribution of two-meson continuum can be properly taken into account by rescaling the coupling $f$, whereas the mass of the tetraquark $m$ preserves its initial value $[30]$. These effects may be essential for strong-interaction unstable tetraquarks, because their
full widths are around of a few 100 MeV. Stated differently, a two-meson continuum is important provided a mass of a tetraquark is higher than a relevant threshold. But even in the case of unstable tetraquarks these effects are numerically small, therefore it is convenient for the phenomenological side use Eq. (5), and a posteriori check self-consistency of obtained results by estimating two-meson contributions [30]. As we shall see later, the tetraquark \(T_{b\bar{b}}\) is strong-interaction stable particle and \(m_{AV}\) resides below the two-meson continuum, which justifies a zero-width single-pole approximation for \(\Pi^{\text{phys}}_{\mu\nu}(p)\).

The correlator \(\Pi^{\text{phys}}_{\mu\nu}(p)\) can be simplified further by defining the matrix element \(\langle 0|J_{\mu}|T_{b\bar{b}}^\text{AV}(p)\rangle\)

\[
\langle 0|J_{\mu}|T_{b\bar{b}}^\text{AV}(p)\rangle = m_{AV}\delta^{\mu}\epsilon_{\mu},
\]

where \(\epsilon_{\mu}\) is the polarization vector of the state \(T_{b\bar{b}}^\text{AV}\). In terms of \(m_{AV}\) and \(f_{AV}\) the function \(\Pi^{\text{phys}}_{\mu\nu}(p)\) takes the form

\[
\Pi^{\text{phys}}_{\mu\nu}(p) = \frac{m_{AV}^2 f_{AV}^2}{m_{AV}^2 - p^2} \left(-\sigma_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{m_{AV}^2} + \ldots\right)\tag{8}
\]

The QCD side of the sum rules can be found by substituting \(J_{\mu}(x)\) into the correlation function (11) and contracting the relevant quark fields, which yields

\[
\Pi_{\mu\nu}^{\text{OPE}}(p) = i \int d^4x e^{ipx} \text{Tr} \left[\gamma_5 S_{b\bar{b}}^\prime(x)\gamma_\mu S_{\bar{b}b}^\prime(x)\right] - \text{Tr} \left[\gamma_\mu S_{b\bar{b}}^\prime(x)\gamma_\mu S_{\bar{b}b}^\prime(x)\right]
\]

where \(S_{a\bar{b}}^\prime(x)\) is the quark propagators. Explicit expressions of the light and heavy quark propagators were presented in Refs. [31, 32], respectively. In Eq. (9) we introduce the notation

\[
\tilde{S}_{\mu}(x) = CS^T_{\mu}(x).\tag{10}
\]

It is seen that the correlator \(\Pi^{\text{phys}}_{\mu\nu}(p)\) contains the Lorentz structure of the vector particle. To derive the sum rules, we choose to work with invariant amplitudes \(\Pi^{\text{phys}}(p^2)\) and \(\Pi^{\text{OPE}}(p^2)\) corresponding to terms \(\sim g_{\mu\nu}\), because they are free of scalar particles’ contributions.

The sum rules for \(m_{AV}\) and \(f_{AV}\) can be derived by equating these two invariant amplitudes and carrying out all standard manipulations of the method. At the first stage, we apply the Borel transformation to both sides of this equality, which suppresses contributions of higher resonances and continuum states. At the next step, by using the quark-hadron duality hypothesis, we subtract higher resonance and continuum terms from the physical side of the equality. As a result, the sum rule equality gains a dependence on the Borel \(M^2\) and continuum threshold \(s_0\) parameters. The second equality necessary to derive required sum rules is obtained by applying the operator \(d/d(-1/M^2)\) to the first expression. Then the sum rules for \(m_{AV}\) and \(f_{AV}\) read

\[
m_{AV}^2 = \frac{\int_{s_0}^{\infty} ds \rho^{\text{OPE}}(s)e^{-s/M^2}}{\int_{s_0}^{\infty} ds \rho^{\text{OPE}}(s)e^{-s/M^2}},\tag{11}
\]

and

\[
f_{AV}^2 = \frac{1}{m_{AV}^2} \int_{s_0}^{\infty} ds \rho^{\text{OPE}}(s)e^{(m_{AV}^2-s)/M^2},\tag{12}
\]

where \(M = 2m_b + m_s\).

The two-point spectral density \(\rho^{\text{OPE}}(s)\) is a key component of computations. It is given as an imaginary part of the correlation function \(\Pi^{\text{OPE}}_{\mu\nu}(p)\). In the present work, we calculate \(\rho^{\text{OPE}}(s)\) by taking into account nonperturbative terms up to dimension 10.

The sum rules for the mass \(m_{AV}\), and coupling \(f_{AV}\) of the scalar tetraquark \(Z_{b\bar{b}}^0\) can be found by the same manner. The correlator \(\Pi^{\text{phys}}(p)\) contains only a trivial Lorentz structure proportional to \(I\), and relevant invariant amplitude has the simple form \(\Pi^{\text{phys}}(p^2) = m_{AV}^2 f_{AV}^2/(m_{AV}^2 - p^2)\). The QCD side of sum rules is determined by the formula

\[
\Pi^{\text{OPE}}(p) = i \int d^4x e^{ipx} \text{Tr} \left[\gamma_5 S_{b\bar{b}}^{a\prime}(x)\gamma_\mu S_{\bar{b}b}^{a\prime}(x)\right]
\]

\[
	imes \left\{\text{Tr} \left[\gamma_5 S_{b\bar{b}}^{b\prime}(x)\gamma_s S_{\bar{b}b}^{a\prime}(x)\right] - \text{Tr} \left[\gamma_5 S_{b\bar{b}}^{a\prime}(x)\gamma_s S_{\bar{b}b}^{b\prime}(x)\right]\right\}
\]

\[
+ \left[\text{Tr} \left[\gamma_5 S_{b\bar{b}}^{a\prime}(x)\gamma_s S_{\bar{b}b}^{b\prime}(x)\right] - \text{Tr} \left[\gamma_5 S_{b\bar{b}}^{b\prime}(x)\gamma_s S_{\bar{b}b}^{a\prime}(x)\right]\right]\}.\tag{13}
\]

The parameters of \(Z_{b\bar{b}}^0\) after evident replacements \(\rho^{\text{OPE}}(s) \rightarrow \rho^{\text{OPE}}(s)\) and \(M \rightarrow \bar{M} = m_b + m_c + m_s\) are determined by Eqs. (11) and (12). Here, \(\rho^{\text{OPE}}(s)\) is the spectral density corresponding to the correlation function \(\Pi^{\text{OPE}}_{\mu\nu}(p)\).

The sum rules through the propagators depend on different vacuum condensates. These condensates are universal parameters of computations and do not depend on a problem under analysis. It is worth noting that
light quark propagator contains various quark, gluon and mixed condensates of different dimensions \[31\]. Some of these terms, for example, \((\bar{q}q, \sigma Gq)\) and \((\bar{q}q, \sigma GS)\), \((\bar{q}q)^2\) and \((\bar{s}s)^2\), \((\bar{q}q)^2(\alpha_s G^2/\pi)\) and \((\bar{s}s)^2(\alpha_s G^2/\pi)\), and other ones were obtained from higher dimensional condensates using the factorization hypothesis. But the factorization assumption is not precise and violates in the case of higher dimensional condensates \[32\]: for the condensates of dimension 10 even an order of magnitude of such a violation is unclear. But, contributions coming from such terms are small, therefore in what follows, we ignore uncertainties generated by this violation. Below we list the vacuum condensates and masses of \(b\), \(c\), and \(s\) quarks used in numerical analysis:

\[
\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3, \quad \langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle, \\
\langle \bar{q}g, \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle, \quad \langle \bar{q}g, \sigma GS \rangle = m_0^2 \langle \bar{s}s \rangle, \\
\langle \alpha_s G^2 \rangle = (0.012 \pm 0.004) \text{GeV}^4, \\
\langle g_s^3 \rangle = (0.57 \pm 0.29) \text{GeV}^6, \quad m_s = 93^{+11}_{-5} \text{MeV}, \\
m_c = 1.27 \pm 0.2 \text{GeV}, \quad m_b = 4.18^{+0.03}_{-0.02} \text{GeV}. \quad (14)
\]

In Eq. \[14\], we introduce the short-hand notations

\[
G^2 = G^{A}_{\alpha \beta} G^{A}_{\alpha \beta}, \quad G^3 = f^{ABC} G^{A}_{\alpha \beta} G^{B}_{\beta \delta} G^{C}_{\delta \alpha}, \quad (15)
\]

where \(G^{A}_{\alpha \beta}\) is the gluon field strength tensor, \(f^{ABC}\) are the structure constants of the color group \(SU_c(3)\), and \(A, B, C = 1, 2, \ldots 8\).

The mass and coupling of the tetraquarks \[11\] and \[12\] depend also on the Borel and continuum threshold parameters \(M^2\) and \(s_0\). The \(M^2\) and \(s_0\) are the auxiliary quantities, and their correct choice is one of important problems of sum rule studies. Proper working regions for \(M^2\) and \(s_0\) must satisfy restrictions imposed on the pole contribution (PC) and convergence of the operator product expansion measured by the ratio \(R(M^2)\), which we define respectively by the expressions

\[
PC = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)}, \quad (16)
\]

and

\[
R(M^2) = \frac{\Pi^{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)}. \quad (17)
\]

Here, \(\Pi(M^2, s_0)\) is the Borel-transformed and subtracted invariant amplitude, and \(\Pi^{\text{DimN}}(M^2, s_0)\) is a contribution to the correlation function of a last term (or a sum of last few terms) in the operator product expansion. In our analysis, we use a sum of last three terms, hence \(\text{DimN} = \text{Dim}(8 + 9 + 10)\). In present work, we use the following restrictions imposed on these parameters: at maximum edge of \(M^2\) the pole contribution should obey \(PC > 0.2\), and at minimum value of \(M^2\), we require fulfillment of \(R(M^2) \leq 0.01\).

Variations of \(M^2\) and \(s_0\) within the allowed working regions are main sources of theoretical errors in sum rule computations. Therefore, the Borel parameter \(M^2\) should be fixed in such a way that to minimize dependence of extracted physical quantities on its variations. A situation with \(s_0\) is more subtle, because it bears a physical information on excited states of the tetraquark \(T_{AV}^{\Delta V}\). In fact, the continuum threshold parameter \(s_0\) separates a ground-state contribution from the ones of higher resonances and continuum states, hence \(s_0\) should be below a first excitation of \(T_{AV}^{\Delta V}\). But available information on excited states of tetraquarks is limited by few theoretical studies \[34\]–\[36\]. As a result, one fixes \(s_0\) to achieve a maximum for PC ensuring, at the same time, fulfillments of other constraints, and keeping under control a self-consistency of computations. The latter means a gap \(\sqrt{s_0} - m_{AV}\) in the case of heavy tetraquarks should be around \(\sim 600\) MeV, which serves as a measure of an excitation.

Performed numerical analyses demonstrate that regions

\[
M^2 \in [9, 12] \text{ GeV}^2, \quad s_0 \in [115, 120] \text{ GeV}^2, \quad (18)
\]
satisfy all aforementioned constraints on \(M^2\) and \(s_0\). Thus, at \(M^2 = 12\) GeV\(^2\) the pole contribution is 0.23, and at \(M^2 = 9\) GeV\(^2\) it amounts to 0.62. These values of \(M^2\) limit the boundaries of a region in which the Borel parameter can be changed. At the minimum of \(M^2 = 9\) GeV\(^2\) we get \(R \approx 0.005\). Additionally, at minimum of the Borel parameter the perturbative contribution forms 79\% of the result overshooting the nonperturbative effects.

For \(m_{AV}\) and \(f_{AV}\) we have obtained

\[
m_{AV} = (10215 \pm 250) \text{ MeV}, \\
f_{AV} = (2.26 \pm 0.57) \times 10^{-2} \text{ GeV}^4. \quad (19)
\]

In Eq. \[19\] theoretical uncertainties of computations are shown as well. For the mass \(m_{AV}\) these uncertainties equal to \(\pm 2.4\%\) of the central value, and for the coupling \(f_{AV}\) amount to \(\pm 25\%\), but in both cases they remain within limits accepted in sum rule computations. In Fig. \[4\] we plot our prediction for \(m_{AV}\) as a function of \(M^2\) and \(s_0\): one can see a mild dependence of \(m_{AV}\) on these parameters. It is also not difficult to find that

\[
\sqrt{s_0} - m_{AV} = [510, 740] \text{ MeV}, \quad (20)
\]

which is a reasonable mass gap between ground-state and excited heavy tetraquarks.

Returning to the issue of two-meson continuum, we can now compare the mass of the tetraquark \(T_{AV}^{\Delta V}\) with energy level of this continuum. It is clear that, the two-meson continuum may be populated by pairs \(B^- B^+_s\) and \(B^0 \bar{B}^0\), and that \(T_{AV}^{\Delta V}\) is \(\approx 480\) MeV below it. This difference is comparable with \[20\], hence one can ignore the two-meson continuum impact on physical parameters of \(T_{AV}^{\Delta V}\).
The mass $m_Z$ and coupling $f_Z$ of the state $Z^0_{b\tau}$ are found from the sum rules by utilizing for $M^2$ and $s_0$ the following working windows

$$M^2 \in [5.5, 6.5] \text{ GeV}^2, \quad s_0 \in [52, 54] \text{ GeV}^2.$$  \quad \quad (21)

The regions $[21]$ meet standard restrictions of the sum rule computations. In fact, at $M^2 = 5.5 \text{ GeV}^2$ the ratio $R$ is 0.009, hence the convergence of the sum rules is satisfied. The pole contribution PC at $M^2 = 6.5 \text{ GeV}^2$ and $M^2 = 5.5 \text{ GeV}^2$ equals to 0.23 and 0.61, respectively. At minimum of $M^2$ the perturbative contribution constitutes 72% of the whole result, and exceeds considerably nonperturbative terms.

For $m_Z$ and $f_Z$ our computations yield

$$m_Z = (6770 \pm 150) \text{ MeV},$$

$$f_Z = (6,3 \pm 1.3) \times 10^{-3} \text{ GeV}^4.$$  \quad \quad (22)

In Fig. 2 we depict the mass of the tetraquark $Z^0_{b\tau}$ and demonstrate its dependence on $M^2$ and $s_0$.

![Diagram](attachment:image.png)

**FIG. 2:** The mass $m_Z$ of the tetraquark $Z^0_{b\tau}$ as a function of the parameters $M^2$ and $s_0$.

### III. WEAK FORM FACTORS $G_i(q^2)$ AND SEMILEPTONIC DECAYS $T_{b\tau}^{AV} \rightarrow Z^0_{b\tau} p^i$.

The analysis performed in the previous section confirms that the tetraquark $T_{b\tau}^{AV}$ is stable against the strong and electromagnetic decays. Indeed, the mass of this state $m_{AV} = 10215 \text{ MeV}$ is 480/477 MeV below the thresholds 10695/10692 MeV for its strong decays to mesons $B^- B^*$ and $B^{*-} B^{0*}$, respectively. The maximum of the mass 10465 MeV is still lower these limits. The threshold 10646 MeV for the process $T_{b\tau}^{AV} \rightarrow B^- B^{0*}$ also exceeds the maximum allowed value of $m_{AV}$ which forbids this electromagnetic decay. Therefore, the full width and mean lifetime of $T_{b\tau}^{AV}$ are determined by its weak decays.

There are different weak decay channels of $T_{b\tau}^{AV}$, which can be generated by sub-processes $b \rightarrow W^- c$ and $b \rightarrow W^- u$. The decays triggered by the transition $b \rightarrow W^- c$ are dominant processes relative to ones connected with $b \rightarrow W^- u$: The latter are suppressed relative to dominant decays by a factor $|V_{ub}|^2/|V_{ub}|^2 \approx 0.01$ with $V_{tb}$ being the Cabibbo-Kobayasi-Maskawa (CKM) matrix elements. In the present work we restrict ourselves by analysis of the dominant weak decays of $T_{b\tau}^{AV}$.

The dominant processes themselves can be collected into two groups: the first group contains semileptonic decays $T_{b\tau}^{AV} \rightarrow Z^0_{b\tau} p^i$, whereas nonleptonic transitions $T_{b\tau}^{AV} \rightarrow Z^0_{b\tau} M$ are processes of the second type. In this section we consider the semileptonic decays and calculate the partial widths of the processes $T_{b\tau}^{AV} \rightarrow Z^0_{b\tau} p^i$, where $l$ is one of the lepton species $e, \mu$ and $\tau$. Due to large mass difference between the initial and final tetraquarks 3445 MeV, all of these semileptonic decays are kinematically allowed ones.

The effective Hamiltonian to describe the subprocess $b \rightarrow W^- c$ at the tree-level is given by the expression

$$\mathcal{H}^{eff} = \frac{G_F}{\sqrt{2}} V_{bc} \gamma_{\mu}(1 - \gamma_5) b \gamma^{\mu}(1 - \gamma_5) \nu_l,$$  \quad \quad (23)

with $G_F$ and $V_{bc}$ being the Fermi coupling constant and CKM matrix element, respectively. A matrix element of $\mathcal{H}^{eff}$ between the initial and final tetraquarks is equal to

$$\langle Z^0_{b\tau} (p') | \mathcal{H}^{eff} | T_{b\tau}^{AV} (p) \rangle = L_{\mu} H_{\mu},$$  \quad \quad (24)

where $L_{\mu}$ and $H_{\mu}$ are leptonic and hadronic factors, respectively. A treatment of $L_{\mu}$ is trivial, therefore we consider in a detailed form the matrix element $H_{\mu}$, which depends on parameters of the tetraquarks. After factoring out the constant factors, $H_{\mu}$ is the matrix element of the current

$$J_{\mu}^{ir} = \tau \gamma_{\mu}(1 - \gamma_5) b,$$  \quad \quad (25)

The matrix element $\langle Z^0_{b\tau} (p') | J_{\mu}^{ir} | T_{b\tau}^{AV} (p) \rangle$ describes the weak decays of the initial axial-vector tetraquark to the final scalar particle, and is expressible in terms of weak form factors $G_i(q^2)$ that parametrize long-distance dynamical effects of these transformations. It has the form

$$\langle Z^0_{b\tau} (p') | J_{\mu}^{ir} | T_{b\tau}^{AV} (p) \rangle = \mathcal{G}_0(q^2) \epsilon_{\mu} + \mathcal{G}_1(q^2) (\epsilon^{\gamma}) P_{\mu}$$

$$+ i \mathcal{G}_2(q^2) (\epsilon^{\mu} \epsilon^{\gamma}) q_{\mu} + i \mathcal{G}_3(q^2) \epsilon_{\mu \alpha \beta \gamma} \epsilon^{\nu} p^{\alpha} p^{\beta}.$$  \quad \quad (26)

The scaled functions $\mathcal{G}_i(q^2)$ are connected with the dimensionless form factors $G_i(q^2)$ by the equalities

$$\mathcal{G}_0(q^2) = \frac{m G_0(q^2)}{m}, \quad \mathcal{G}_j(q^2) = \frac{G_j(q^2)}{m}, \quad j = 1, 2, 3. \quad (27)$$

Here, $m = m_{AV} + m_Z$, $p_i$ and $\epsilon_i$ are the momentum and polarization vector of the tetraquark $T_{b\tau}^{AV}$, $p'$ is the momentum of the scalar state $Z^0_{b\tau}$. We use also $P_{\mu} = p'_\mu + p_{\mu}$, and $q_{\mu} = p_{\mu} - p'_{\mu}$, the latter being a momentum transferred to the leptons. It is evident that $q^2$ changes within the limits $m_i^2 \lesssim q^2 \lesssim (m_{AV} - m_i)^2$, where $m_i$ is the mass of a lepton $l$. 


The sum rules for the form factors $G_i(q^2)$ can be obtained from analysis of the three-point correlation function

$$\Pi_{\mu\nu}(p, p') = i^2 \int d^4x d^4y e^{i(p'y - px)} \times \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle.$$  \hspace{1cm} (28)

To this end, we have to express $\Pi_{\mu\nu}(p, p')$ using the masses and couplings of the tetraquarks, and by this way to determine the physical side of the sum rules $\Pi^{\text{phys}}_{\mu\nu}(p, p')$. The function $\Pi^{\text{phys}}_{\mu\nu}(p, p')$ can be presented in the form

$$\Pi^{\text{phys}}_{\mu\nu}(p, p') = \langle 0 | J_\mu(z) | Z^{0}_{b\tau}(p') \rangle \langle Z^{0}_{b\tau}(p') | J_\nu^\dagger | T^{A\nu}_{b\tau} (p, \epsilon) \rangle \times (p^2 - m^2_{\text{AV}})/(p^2 - m^2_{\tilde{z}})$$

$$\times \bigg\{ (T^{A\nu}_{b\tau} (p, \epsilon) | J_\mu^\dagger | 0 \rangle + \ldots \bigg\}.$$  \hspace{1cm} (29)

where we take into account contribution arising only from the ground-state particles, and denote effects of the excited and continuum states by dots.

The phenomenological side of the sum rules can be simplified by substituting in Eq. (29) expressions of matrix elements in terms of the tetraquarks’ masses and couplings, and weak transition form factors. For these purposes, we employ Eqs. (1) and (26), and additionally define the matrix element of $Z^{0}_{b\tau}$

$$\langle 0 | J_\mu(z) | Z^{0}_{b\tau}(p') \rangle = f_z m_z.$$  \hspace{1cm} (30)

Then, one gets

$$\Pi^{\text{phys}}_{\mu\nu}(p, p') = \frac{f_{AN} m_{AN} f_z m_z}{(p^2 - m^2_{\text{AV}})/(p^2 - m^2_{\tilde{z}})}$$

$$\times \bigg\{ \tilde{G}_0(q^2) \left[ -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2_{\text{AV}}} \right] + \left[ \tilde{G}_1(q^2) P_\mu \right.$$

$$\left. + \tilde{G}_2(q^2) q_\mu \right] - p'_\mu + \frac{m^2_{\text{AV}} + m^2_{\tilde{z}} - q^2 - 2 m^2_{\text{AV}}}{2 m^2_{\text{AV}}} p_\nu$$

$$- i \tilde{G}_3(q^2) \epsilon_{\mu\nu\alpha\beta} p'_\alpha p_\beta \bigg\} + \ldots.$$  \hspace{1cm} (31)

We should also calculate the correlation function in terms of quark propagators and find $\Pi^{\text{OPE}}_{\mu\nu}(p, p')$. The function $\Pi^{\text{OPE}}_{\mu\nu}(p, p')$ is the second side of the sum rules and has the following form

$$\Pi^{\text{OPE}}_{\mu\nu}(p, p') = \int d^4x d^4y e^{i(p'y - px)} \left\{ \text{Tr} \left[ \gamma_5 \gamma_S \gamma^S_{\alpha} (x - y) \right.$$

$$\times \gamma_5 S_{\alpha}^b (x - y) \right] + \text{Tr} \left[ \gamma_5 \gamma_S \gamma^S_{\alpha} (x - y) \right.$$  

$$\times S_{\alpha}^b (x - y) \right] - \text{Tr} \left[ \gamma_5 \gamma_S \gamma^S_{\alpha} (x - y) \right.$$  

$$\times S_{\alpha}^b (x - y) \right] - \text{Tr} \left[ \gamma_5 \gamma_S \gamma^S_{\alpha} (x - y) \right.$$  

$$\times S_{\alpha}^b (x - y) \right] \right\}.$$  \hspace{1cm} (32)

In order to extract expressions of the form factors $G_i(q^2)$, we equate invariant amplitudes corresponding to the same Lorentz structures both in $\Pi^{\text{phys}}_{\mu\nu}(p, p')$ and $\Pi^{\text{OPE}}_{\mu\nu}(p, p')$, carry out double Borel transformations over the variables $p^2$ and $p'^2$, and perform continuum subtraction. For instance, to extract the sum rule for $G_0(q^2)$ we use the structure $g_{\mu\nu}$, whereas for $G_3(q^2)$ the term $\sim \epsilon_{\mu\nu\alpha\beta} p'_\alpha p_\beta$ can be employed. The sum rules for the scaled form factors $\tilde{G}_i(q^2)$ can be written in a single formula

$$\tilde{G}_i(M^2, s_0, q^2) = \frac{1}{f_{AN} m_{AN} f_z m_z} \int_{M^2}^{s_0} ds e^{(m^2_{\text{AV}} - s)/M^2}$$

$$\times \int_{M^2}^{s_0} ds' \rho_i(s, s') e^{(m^2_{\tilde{z}} - s')/M^2},$$  \hspace{1cm} (33)

where $\rho_i(s, s')$ are spectral densities computed as the imaginary parts of the corresponding terms in $\Pi^{\text{OPE}}_{\mu\nu}(p, p')$. They contain perturbative and nonperturbative contributions, and are found in the present work with dimension-6 accuracy. In Eq. (33) $M^2 = (M^2_1, M^2_2)$ and $s_0 = (s_0, s'_0)$ are the Borel and continuum threshold parameters, respectively. The pair of parameters $(M^2_1, s_0)$ corresponds to the initial tetraquark’s channels, whereas $(M^2_2, s'_0)$ describes the final-state tetraquark.

As usual, the form factors $\tilde{G}_i(M^2, s_0, q^2)$ contain various input parameters, which should be determined before numerical analysis. The vacuum condensates of quark, gluon, and mixed operators are already presented in Eq. (13). The masses and couplings of the tetraquarks $T^{A\nu}_{b\tau}$ and $Z^{0}_{b\tau}$ have been extracted in Section III. The pairs of auxiliary parameters $(M^2_1, s_0)$ and $(M^2_2, s'_0)$ are chosen as in corresponding mass computations.

The form factors $\tilde{G}_i(q^2)$ determine the differential decay rate $dt/dq^2$ of the semileptonic decay $T^{A\nu}_{b\tau} \rightarrow Z^{0}_{b\tau} j_l$, the explicit expression of which can be found in Ref. 3. The partial width of the process is equal to an integral of this rate over the momentum transfer $q^2$ within the limits $m^2_{\tilde{z}} < q^2 \leq (m_{\text{AV}} - m_z)^2$. Our results for the form factors are plotted in Fig. 3. The QCD sum rules lead to reliable predictions at $m^2_1 < q^2 \leq 8$ GeV$^2$. But the latter do not cover the whole integration region $m^2_2 < q^2 \leq 11.87$ GeV$^2$. To solve this problem one has to introduce extrapolating functions $G_i(q^2)$ of relatively simple analytic form that at $q^2$ accessible to QCD sum rules coincide with their predictions, but can be used in the whole region.

For these purposes, we choose to work with the functions

$$G_i(q^2) = G^i_0 \exp \left[ g^i_1 \left( \frac{q^2}{m^2_{\text{AV}}} \right) + g^i_2 \left( \frac{q^2}{m^2_{\text{AV}}} \right)^2 \right],$$  \hspace{1cm} (34)

parameters of which $G^i_0$, $g^i_1$, and $g^i_2$ should be fitted to satisfy sum rules’ predictions. The parameters of the functions $G_i(q^2)$ obtained in numerical analysis are collected in Table I. The functions $G_i(q^2)$ are also shown in
TABLE I: Parameters of the extrapolating functions $G_i(q^2)$.

| $G_i(q^2)$ | $g_1$ | $g_2^*$ |
|------------|-------|---------|
| $G_0(q^2)$ | 4.91  | 19.29  | -15.34 |
| $G_1(q^2)$ | 2.94  | 18.73  | -20.08 |
| $G_2(q^2)$ | -22.67 | 20.50  | -22.95 |
| $G_3(q^2)$ | -21.14 | 20.77  | -23.62 |

Fig. 3: one can see a quite nice agreement between the sum rule predictions and fit functions.

In numerical computations for the Fermi constant, CKM matrix element, and masses of leptons, we use

$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2},$$

$$|V_{bc}| = (42.2 \pm 0.08) \times 10^{-3},$$

and $m_e = 0.511$ MeV, $m_{\mu} = 105.658$ MeV, and $m_{\tau} = (1776.82 \pm 0.16)$ MeV \[^{38}\]. The predictions obtained for the partial width of the semileptonic decays $T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} \tau_i$ are written down below

$$\Gamma(T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} e^- \tau_e) = (5.34 \pm 1.43) \times 10^{-8} \text{ MeV},$$

$$\Gamma(T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} \mu^- \tau_\mu) = (5.32 \pm 1.41) \times 10^{-8} \text{ MeV},$$

$$\Gamma(T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} \tau^- \tau_\tau) = (2.15 \pm 0.54) \times 10^{-8} \text{ MeV},$$

and are main results of this section.

IV. NONLEPTONIC DECAYS $T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} M$

The second class of weak decays of the tetraquark $T_{b\tau}^{\text{AV}}$ are the processes $T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} M$, which may affect the full width and lifetime of the tetraquark $T_{b\tau}^{\text{AV}}$. Here, we study the nonleptonic weak decays $T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} M$ of the tetraquark $T_{b\tau}^{\text{AV}}$ in the framework of the QCD factorization method. This approach was applied to investigate nonleptonic decays of the conventional mesons \[^{39, 40}\], but can be used to investigate decays of the tetraquarks as well. Thus, nonleptonic decays of the scalar exotic mesons $T_{b\pi}^{-}, T_{b\pi}^{-}, Z_{b\pi}^{0}$, and $T_{b\pi}^{-}$ were explored by this way in Refs. \[^{25, 24, 11, 42}\], respectively. Weak decays of double- and fully-heavy tetraquarks were analyzed in Refs. \[^{43, 44}\] as well.

We consider processes, where $M$ is one of the vector mesons $\rho^+, K^* (892), D^* (2010)^-, \text{ and } D_s^{*-}$. We provide details of analysis for the decay $T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} \rho^-$, and write down final predictions for other channels. At the quark level, the effective Hamiltonian for this decay is given by the expression

$$\mathcal{H}_{\text{non-lep}} = \frac{G_F}{\sqrt{2}} V_{bc} V^*_{ud} [c_1(\mu) Q_1 + c_2(\mu) Q_2],$$

where

$$Q_1 = \langle \bar{d}_i u_i \rangle_{V-A} \langle \bar{c}_j b_j \rangle_{V-A},$$

$$Q_2 = \langle \bar{d}_i u_i \rangle_{V-A} \langle \bar{c}_j b_j \rangle_{V-A},$$

and $i, j$ are the color indices, and ($\bar{q}_1 q_2$)$_{V-A}$ means

$$\langle \bar{q}_1 q_2 \rangle_{V-A} = \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2.$$

The short-distance Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$ are given at the factorization scale $\mu$.

In the factorization method the amplitude of the decay $T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} \rho^-$ has the form

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{bc} V^*_{ud} a(\mu) \langle \rho^- (q) | \langle \bar{d}_i u_i \rangle_{V-A}|0\rangle$$

$$\times \left( \langle Z_{b\pi}^{0} (p') | \langle \bar{c}_j b_j \rangle_{V-A}|T_{b\tau}^{\text{AV}}(p)\rangle \right),$$

where

$$a(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu),$$

with $N_c = 3$ being the number of quark colors. The only unknown matrix element $\langle \rho^- (q) | \langle \bar{d}_i u_i \rangle_{V-A}|0\rangle$ in $\mathcal{A}$ can be defined in the following form

$$\langle \rho^- (q) | \langle \bar{d}_i u_i \rangle_{V-A}|0\rangle = f_{\rho} m_\rho \epsilon^*_\mu(q).$$

Then it is not difficult to see that

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} f_\rho V_{bc} V^*_{ud} a(\mu) \left[ \tilde{G}_0(q^2) \epsilon_\mu(p) \epsilon^{*\mu}(q) + 2 \tilde{G}_1(q^2) (p' \epsilon(p) | (p' \epsilon^{*}(q)) + i \tilde{G}_2(q^2) \epsilon_{\mu\nu\alpha\beta}(q) \epsilon^*(p) \epsilon^{*\mu}(q) p^\mu p'^\nu \right].$$

The decay modes $T_{b\tau}^{\text{AV}} \rightarrow Z_{b\pi}^{0} K^* (892)[D^* (2010)^-, D_s^{*-}]$ can be analyzed in a similar way. To this end, we have
to replace in relevant expressions the spectroscopic parameters \((m_\rho, f_\rho)\) of the \(\rho\) meson by masses and decay constants of the mesons \(K^*(892), D^*(2010)^-,\) and \(D_s^*-\), and make the substitutions \(V_{cd} \rightarrow V_{cu}, V_{cd},\) and \(V_{cs}\).

The width of the nonleptonic decay \(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0\rho^-\) can be evaluated using the expression

\[
\Gamma = \frac{|A|^2}{24\pi m_{AV}^2} \lambda(m_{AV}, m_Z, m_\rho),
\]

where

\[
\lambda(a, b, c) = \frac{1}{a^2} \left[ a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2) \right]^{1/2}.
\]

The key component in Eq. (44), i.e., \(|A|^2\) has a simple form

\[
|A|^2 = \sum_{j=0,1,2} H_j \tilde{G}_j^2 + H_3 \tilde{G}_0 \tilde{G}_1,
\]

where \(H_j\) are given by the expressions

\[
H_0 = \frac{m_\rho^4 + (m_{AV}^2 - m_Z^2)^2 + 2m_\rho^2(5m_{AV}^2 - m_Z^2)}{4m_\rho^2m_{AV}^2},
\]

\[
H_1 = \frac{\left[m_\rho^4 + (m_{AV}^2 - m_Z^2)^2 - 2m_\rho^2(m_{AV}^2 + m_Z^2)\right]^2}{4m_\rho^2m_{AV}^2},
\]

\[
H_2 = \frac{1}{2} \left[m_\rho^4 + (m_{AV}^2 - m_Z^2)^2 - 2m_\rho^2(m_{AV}^2 + m_Z^2)\right],
\]

\[
H_3 = -\frac{1}{2m_\rho^2m_{AV}^2} \left[m_\rho^6 + (m_{AV}^2 - m_Z^2)^3 - m_\rho^2(m_{AV}^2 + m_Z^2)
+ 3m_Z^2) - m_\rho^2(m_{AV}^2 + 2m_Z^2m_{AV}^2 - 3m_Z^2)\right].
\]

In Eq. (46) we take into account that the weak form factors \(G_j\) are real functions of \(q^2\), and their values for the process \(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0M\) are fixed at \(q^2 = m_M^2\).

All input information necessary for numerical analysis are collected in Table III it contains spectroscopic parameters of the final-state mesons, and CKM matrix elements. For the masses of the vector mesons we use information from PDG [38]. The decay constants of the mesons \(\rho\) and \(K^*(892)\) are also taken from this source. The decay constants of mesons \(D^*\) and \(D_s^*\) are theoretical predictions obtained in the framework of the Lattice QCD [42]. The coefficients \(c_1(m_b),\) and \(c_2(m_b)\) with next-to-leading order QCD corrections have been borrowed from Refs. [40, 48]

\[
c_1(m_b) = 1.117, \quad c_2(m_b) = -0.257. \quad (48)
\]

For the decay \(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0\rho^-\) calculations yield

\[
\Gamma(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0(\rho^-)) = (3.47 \pm 0.92) \times 10^{-10} \text{ MeV}. \quad (49)
\]

Partial widths of remaining three nonleptonic decays are presented below

\[
\Gamma(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0(\rho^-)) = (1.47 \pm 0.37) \times 10^{-11} \text{ MeV},
\]

\[
\Gamma(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0(D^*(2010)^-)) = (1.54 \pm 0.39) \times 10^{-11} \text{ MeV},
\]

\[
\Gamma(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0(D_s^*-)) = (4.97 \pm 1.32) \times 10^{-10} \text{ MeV}.
\]

It is evident that parameters of the processes \(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0\rho^-\) and \(T_{b,\pi}^{AV} \rightarrow Z_{b,\pi}^0(D^*)^\mp\) are comparable with each other, and may affect predictions for the tetraquark \(T_{b,\pi}^{AV}\); other two decays can be safely neglected in computation of \(\Gamma_{full}\) and \(\tau\). Then using the Eqs. (36), (49) and (50) we find

\[
\Gamma_{full} = (12.9 \pm 2.1) \times 10^{-8} \text{ MeV},
\]

\[
\tau = 5.1^{+0.99}_{-0.71} \times 10^{-15} \text{ s},
\]

which are principally new predictions of the present article.

V. DISCUSSION AND CONCLUDING NOTES

We have calculated the mass, width and lifetime of the stable axial-vector tetraquark \(T_{b,\pi}^{AV}\) with the content \(bb\overline{\pi}\). This particle is a strange partner of the tetraquark \(T_{bb}\), which was explored in Ref. [3]. The width and lifetime of \(T_{bb}\)

\[
\Gamma_{full} = (7.17 \pm 1.23) \times 10^{-8} \text{ MeV},
\]

\[
\tau = 9.18^{+1.90}_{-1.34} \times 10^{-15} \text{ s},
\]

are comparable with ones of the tetraquark \(T_{b,\pi}^{AV}\).

The tetraquark \(T_{b,\pi}^{AV}\) is last of four scalar and axial-vector states \(bb\overline{\pi}\) and \(bb\overline{d}\) considered in our works. The spectroscopic parameters and widths of scalar tetraquarks \(T_{b,\pi}^{AV}\) and \(T_{b,d}^{AV}\) were calculated in Refs. [25, 26].
The widths and lifetimes of these tetraquarks have yielded important insights regarding their dynamical properties. It is worth noting that in forming of their full widths semileptonic decay channels play a crucial role: Our investigations have shown that a partial width of semileptonic decay is enhanced relative to nonleptonic ones by 2 – 3 order of magnitude. The widths of the scalar tetraquarks \( T_{bb} \) and \( T_{b\bar{b}} \) considerably smaller than widths of the axial-vector particles \( T_{bb}^{AV} \) and \( T_{b\bar{b}}^{AV} \). As a result, mean lifetimes of the scalar tetraquarks are around of an one ps, whereas for the axial vector states we get \( \tau \approx 10 \) fs. Stated differently, scalar tetraquarks \( T_{bb}^{AV} \) and \( T_{b\bar{b}}^{AV} \) are heavier and live longer than the corresponding axial-vector particles.

The spectroscopic parameters and lifetimes of the axial-vector states \( T_{bb}^{AV} \) and \( T_{b\bar{b}}^{AV} \) were also explored in Refs. [1] [3]. The lifetime 367 fs of the state \( T_{bb}^{AV} \) predicted in Ref. [1] is considerably larger than our result 9.18 fs. The lifetimes \( \tau \geq 800 \) fs of the tetraquarks \( T_{bb}^{AV} \) and \( T_{b\bar{b}}^{AV} \) obtained in Ref. [3] exceed our predictions as well. Let us note that in Ref. [3] the authors considered only nonleptonic decays of the axial-vector tetraquarks. We have reevaluated the lifetime of \( T_{bb}^{AV} \) using Eqs. (49) and (50) and found \( \tau \approx 753 \) fs. Despite the fact that channels explored in Ref. [3] differ from decays considered in the present work, for \( \tau \) they lead to compatible predictions. One of reasons is that in both cases amplitudes of nonleptonic weak decays contain two CKM matrix elements which suppress their partial widths and branching ratios relative to semileptonic channels. Of course, our results for nonleptonic decays of \( T_{bb}^{AV} \) can be refined by including into analysis relevant channels from Ref. [3]. But in any case, to discover stable exotic mesons their semileptonic decays seem are more promising than other processes.

**TABLE III: Parameters of the scalar and axial-vector tetraquarks composed of the diquark \( bb \) and light antiquarks.**

| Tetraquark \((J^P)\) | Mass (MeV) | Width (MeV) | Lifetime |
|----------------------|------------|-------------|----------|
| \( T_{bb}^{AV}(1^+)\) | 10215 ± 250 | (12.9 ± 2.1) \times 10^{-8} | 5.1^{+0.99}_{-0.71} \text{ fs} |
| \( T_{b\bar{b}}(1^+)\) | 10035 ± 260 | (7.17 ± 1.23) \times 10^{-8} | 9.18^{+1.90}_{-1.34} \text{ fs} |
| \( T_{b\bar{b}}^{0}(1^+)\) | 10250 ± 270 | (15.21 ± 2.59) \times 10^{-10} | 0.433^{+0.080}_{-0.063} \text{ ps} |
| \( T_{b\bar{b}}^{0}(1^+)\) | 10135 ± 240 | (10.80 ± 1.88) \times 10^{-10} | 0.605^{+0.126}_{-0.085} \text{ ps} |

It is seen, that the scalar particles are heavier than their axial-vector counterparts: This effect for tetraquarks \( bb\bar{s}\bar{s} \) is equal to 35 MeV, and for particles with quark content \( bb\bar{d}\bar{d} \) reaches 100 MeV. It is also clear, that the mass splitting of the strange and nonstrange axial-vector tetraquarks 180 MeV exceeds the same parameter for the scalars 115 MeV. These estimates are obtained using central values of various tetraquarks’ masses calculated the framework of the QCD sum rule method. It is known, that predictions of this method suffer from theoretical uncertainties, therefore mass splitting between double-beauty tetraquarks and hierarchy of particles outlined here must be taken with some caution. Nevertheless, we hope the picture described above is a quite reliable image of the real situation.

ACKNOWLEDGEMENTS

The work of K. A, B. B., and H. S was supported in part by the TUBITAK grant under No: 119F050.
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