SMALL-X BEHAVIOR OF PARTON DENSITIES AT LOW Q2 VALUES

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Abstract

In the leading twist approximation of the Wilson operator product expansion with standard and “frozen” versions of strong coupling constant we show that the Bessel-inspired behavior of the structure function $F_2$ at small $x$, obtained for a flat initial condition in the DGLAP evolution, leads to very good agreement with the deep inelastic scattering experimental data from HERA.

1 Introduction

The measurements of the deep-inelastic scattering structure function (SF) $F_2$ at HERA \cite{1} have permitted the access to a very interesting kinematical range for testing the theoretical ideas on the behavior of quarks and gluons carrying a very low fraction of proton momentum, the so-called small $x$ region. In this limit one expects that the non-perturbative effects may give an essential contributions. However, the reasonable agreement between the HERA data and the next-to-leading order (NLO) approximation of perturbative QCD, which has been observed for $Q^2 > 1\text{GeV}^2$ (see the reviews in \cite{2}), indicates that the perturbative QCD could describe the SF evolution up to very low $Q^2$ values, traditionally explained by soft processes. It is of fundamental importance to find out the kinematical region where the well-established perturbative QCD formalism can be safely applied at small $x$.

The standard program to study the small $x$ behavior of quarks and gluons is carried out by comparison of the data with the numerical solution of the DGLAP equations fitting the parameters of the $x$ profile of partons at some initial $Q^2_0$ and the QCD energy scale $\Lambda$ (see, for instance, \cite{3,4}). However, in analyzing exclusively the small $x$ region ($x \leq 0.01$), there is the alternative of doing a simpler analysis by using some of the existing analytical solutions of DGLAP in the small $x$ limit (see \cite{2} for review). It was done in Refs. \cite{5}-\cite{8}, where it was pointed out that the HERA small $x$ data can be interpreted in the so called doubled asymptotic scaling (DAS) approximation related to the asymptotic behavior of the DGLAP evolution discovered in \cite{9} many years ago.

Here we illustrate results obtained in \cite{6}-\cite{8}, which are the extension of previous leading order (LO) studies \cite{9,5} to the NLO QCD approximation. The main ingredients are:

1. Both, the gluon and quark singlet densities are presented in terms of two components (‘+’ and ‘−’) which are obtained from the analytical $Q^2$ dependent expressions of the corresponding (‘+’ and ‘−’) parton distribution (PD) moments.

2. The ‘−’ component is constant at small $x$, whereas the ‘+’ component grows at $Q^2 \geq Q^2_0$ as $\sim \exp(\sigma_{NLO})$, where

$$
\sigma_{NLO} = 2\sqrt{(\hat{d} s + \hat{D} p) \ln(x)},
$$



the LO term \( \dot{d}_+ = -12/\beta_0 \) and the NLO one \( \dot{D}_+ = \dot{d}_{++} + \dot{d}_+ \beta_1/\beta_0 \) with \( \dot{d}_{++} = 412 f/(27\beta_0) \). Here the coupling constant \( \alpha_s = \alpha_s/(4\pi) \), \( s = \ln[a_s(Q_0^2)/a_s(Q^2)] \) and \( p = a_s(Q_0^2) - a_s(Q^2) \), \( \beta_0 \) and \( \beta_1 \) are the first two coefficients of QCD \( \beta \)-function and \( f \) is the number of active flavors.

2 Basic formulae

Our purpose is to extract the small \( x \) asymptotic PD form in the framework of the DGLAP equation starting at some \( Q_0^2 \) with the flat function:

\[
f_a^\tau(\tilde{Q}_0^2) = A_a \quad (\text{hereafter } a = q, g),
\]

where \( f_a^\tau \) are the leading-twist parts of parton (quark and gluon) distributions, multiplied by \( x \), and \( A_a \) are unknown parameters that have to be determined from data. We neglect the non-singlet quark component at small \( x \).

We would like to note that HERA data \(^1\) show a rise of \( F_2 \) at low \( Q^2 \) values \( (Q^2 < 1 \text{GeV}^2) \) when \( Q \to 0 \) (see Fig.1 below). This rise can be explained naturally by incorporation of higher-twist terms in the analysis (see \(^2\) and Fig.1).

We shortly compile below the LO results (the NLO results may be found in \(^6\)). The full small \( x \) asymptotic results for PD and SF \( F_2 \) at LO are:

\[
F_2(x, Q^2) = e f_q(x, Q^2),
\]

\[
f_a(x, Q^2) = f_a^+(x, Q^2) + f_a^-(x, Q^2),
\]

where \( e = (\sum_{i=1}^f e_i^2)/f \) is the average charge square. The ’+’ and ’−’ components \( f_a^\pm(x, Q^2) \) are given by the sum

\[
f_a^\pm(x, Q^2) = f_a^{\tau \pm}(x, Q^2) + f_a^{h \tau \pm}(x, Q^2)
\]

of the leading-twist parts \( f_a^{\tau \pm}(x, Q^2) \) and the higher-twist parts \( f_a^{h \tau \pm}(x, Q^2) \), respectively.

The small \( x \) asymptotic results for PD \( f_a^{\tau \pm} \) are

\[
f_q^{\tau +}(x, Q^2) = \left(A_g + \frac{4}{9} A_q\right) \tilde{I}_0(\sigma) e^{\tilde{d}_+}(1)s + O(\rho),
\]

\[
f_q^{\tau +}(x, Q^2) = \frac{f}{9} \left(A_g + \frac{4}{9} A_q\right) \rho \tilde{I}_1(\sigma) e^{-\tilde{d}_+}(1)s + O(\rho),
\]

\[
f_g^{\tau -}(x, Q^2) = \frac{4}{9} A_\tilde{s} e^{d_-(1)s} + O(x),
\]

\[
f_g^{\tau -}(x, Q^2) = A_\tilde{s} e^{d_-(1)s} + O(x),
\]

where \( \tilde{d}_+(1) = 1 + 20 f/(27\beta_0) \) and \( d_-(1) = 16 f/(27\beta_0) \) are the regular parts of \( d_+ \) and \( d_- \) anomalous dimensions, respectively, in the limit \( n \to 1 \). The functions \( \tilde{I}_\nu \) \( (\nu = 0, 1) \) are related to the modified Bessel function \( I_\nu \) and to the Bessel function \( J_\nu \) by:

\[
\tilde{I}_\nu(\sigma) = \begin{cases} I_\nu(\sigma), & \text{if } s \geq 0 \\ J_\nu(\sigma), & \text{if } s < 0 \end{cases}
\]

The variables \( \sigma \) and \( \rho \) are given by

\[
\sigma = 2\sqrt{d_+ s \ln(x)}, \quad \rho = \sqrt{\frac{d_+ s}{\ln(x)}} = \frac{\sigma}{2 \ln(1/x)}
\]

\(^1\) For a quantity \( k(n) \) we use the notation \( \hat{k}(n) \) for the singular part when \( n \to 1 \) and \( \overline{k}(n) \) for the corresponding regular part.
3 Effective slopes

As it has been shown in Refs. [6]-[8], the PD and $F_2$ behavior, given in the Bessel-like form by generalized DAS approach, can mimic a power law shape over a limited region of $x$ and $Q^2$:

$$f_a(x, Q^2) \sim x^{-\lambda_{eff}^a(x,Q^2)} \quad \text{and} \quad F_2(x, Q^2) \sim x^{-\lambda_{eff}^2(x,Q^2)}.$$

At the twist-two LO approximation, they have the following form

$$\lambda_{eff}^a(x, Q^2) = \frac{f_a^+(x, Q^2)}{f_a(x, Q^2)} \rho \frac{I_1(\sigma)}{I_0(\sigma)},$$

$$\lambda_{eff}^{F_2}(x, Q^2) = \lambda_{eff}^{q}(x, Q^2) = \frac{f_2^+(x, Q^2)}{f_2(x, Q^2)} \rho \frac{I_2(\sigma)}{I_2(\sigma)}.$$ (11)

The corresponding NLO expressions and the higher-twist terms can be found in Refs. [6]-[8].

The effective slopes $\lambda_{eff}^a$ and $\lambda_{eff}^{F_2}$ depend on the magnitudes $A_a$ of the initial PD and also on the chosen input values of $Q_0^2$ and $\Lambda$. To compare with the experimental data it is necessary to use the exact expressions (11) but for qualitative analysis one can use some appropriate approximations.

At large values of $Q^2$, the “−” component of PD is negligible and the dependence of slopes on the PD disappears. In this case the asymptotic behaviors of slopes have the following expressions:

$$\lambda_{eff,as}^a(x, Q^2) = \rho \frac{I_1(\sigma)}{I_0(\sigma)} \approx \rho - \frac{1}{4 \ln (1/x)},$$

$$\lambda_{eff,as}^{F_2}(x, Q^2) = \lambda_{eff,as}^{q}(x, Q^2) = \rho \frac{I_2(\sigma)}{I_1(\sigma)} \approx \rho - \frac{3}{4 \ln (1/x)},$$ (12)

where the symbol $\approx$ marks the approximation obtained by the expansion of the usual and modified Bessel functions in (9).

One can observe from (12), that the gluon effective slope $\lambda_{eff,as}^a$ is larger than the quark slope $\lambda_{eff,as}^{q}$, which is in excellent agreement with global analyses [3].

4 Comparison with experimental data

With the help of the results presented in the previous sections we have analyzed $F_2$ HERA data at small $x$ from the H1 and ZEUS collaborations [1]. In order to keep the analysis as simple as possible we have fixed the number of active flavors $f=4$ and $\Lambda_{\overline{MS}}(n_f = 4) = 292$ MeV in agreement with the more recent H1 results [10].

The typical fits for the SF $F_2(x, Q^2)$ as a function of $x$ for different $Q^2$ bins are presented in Fig. 1. The experimental points are from H1 (open points) and ZEUS (solid points). The solid line represents the NLO fit alone with $\chi^2/n.d.f. = 1.31$. The dashed curve are obtained from the fit at the NLO, when the renormalon contributions of higher-twist terms have been incorporated. The corresponding $\chi^2/n.d.f. = 0.86$. The dash-dotted curve (hardly

2The asymptotic formulae given in Eq. (12) work quite well at any $Q^2 \geq Q_0^2$ values, because at $Q^2 = Q_0^2$ the values of $\lambda_{eff}^a$ and $\lambda_{eff}^{F_2}$ are equal zero. The use of approximations in Eq. (12) instead of the exact results given in Eq. (11) underestimates (overestimates) only slightly the gluon (quark) slope at $Q^2 \geq Q_0^2$.
distinguished from the dashed one) represents the fit at the LO together with the renormalon contributions of higher-twist terms. The corresponding \( \chi^2/n.d.f. = 0.84 \). The results demonstrate excellent agreement between theoretical predictions and experimental data for the region \( Q^2 \geq 0.5 \text{ GeV}^2 \). However, the twist-two approximation is in agreement with the data only for \( Q^2 \geq 2.5 \text{ GeV}^2 \).

Using these results of the fits of the SF \( F_2(x, Q^2) \) we analyze also the HERA data for the slope \( d\ln F_2/d\ln(1/x) \) at small \( x \) from the H1 and ZEUS Collaborations [10]-[12]. The results are shown in Fig. 2. Because the twist-two approximation is reasonable at \( Q^2 \geq 2.5 \text{ GeV}^2 \) only, some modification should be considered at the lower \( Q^2 \) values. In the paper [8] we have added the higher twist corrections and have found a good agreement for \( Q^2 \geq 0.5 \text{ GeV}^2 \).

Here we study another possibility. We modify the QCD coupling constant. From different studies [13, 14] it is known that the effective argument of the coupling constant is higher than \( x \) and \( \lambda \). For the “frozen” coupling constant \( \lambda_{F_2} \) we plan to fit the HERA experimental data for the \( F_2(x, Q^2) \) changing its argument directly with the “frozen” coupling constant \( a_{f_\tau}(Q^2) \). The (renormalon-type) higher-twist terms lead to the natural explanation of the rise of SF as it has been observed earlier with other approaches (see reviews [2]).

The application of the “frozen” coupling constant \( a_{f_\tau}(Q^2) \) leads to good agreement with the recent HERA data [10]-[12] for the slope \( \lambda_{F_2}^{eff}(x, Q^2) \) for \( Q^2 \geq 0.5 \text{ GeV}^2 \). As the next step of our investigations, we plan to fit the HERA experimental data for the \( F_2(x, Q^2) \) SF directly with the “frozen” coupling constant \( a_{f_\tau}(Q^2) \).

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\section{5 Conclusions}

We have shown that the results, developed recently in [6]-[8], have quite simple form and reproduce many PD properties at small \( x \), that have been known from global fits.

We found the very good agreement between our approach, based on QCD, and HERA data, as it has been observed earlier with other approaches (see reviews [2]). The (renormalon-type) higher-twist terms lead to the natural explanation of the rise of SF \( F_2 \) at low values of \( Q^2 \) and \( x \), which has been discovered in recent HERA experiments [1].

The application of the “frozen” coupling constant \( a_{f_\tau}(Q^2) \) leads to good agreement with the recent HERA data [10]-[12] for the slope \( \lambda_{F_2}^{eff}(x, Q^2) \) for \( Q^2 \geq 0.5 \text{ GeV}^2 \). As the next step of our investigations, we plan to fit the HERA experimental data for the \( F_2(x, Q^2) \) SF directly with the “frozen” coupling constant \( a_{f_\tau}(Q^2) \).

\section{References}

[1] H1 Collab.: C. Adloff \textit{et al.}, Eur. Phys. J. \textbf{C21}, 33 (2001); \textbf{C13}, 609 (2000); ZEUS Collab.: S. Chekanov \textit{et al.}, Eur. Phys. J. \textbf{C21}, 443 (2001); J. Breitweg \textit{et al.}, Phys.Lett. \textbf{B478}, 53 (2000).
Figure 1: The structure function $F_2$ as a function of $x$ for different $Q^2$ bins.

[2] A. M. Cooper-Sarkar et al., Int.J.Mod.Phys. A13, 3385 (1998); A. V. Kotikov, Phys. Part. Nucl. 38, 1 (2007) [Erratum-ibid. 38, 828 (2007)].

[3] A.D. Martin et al., Eur. Phys. J C23, 73 (2002); CTEQ Collab.: J. Pumplin et al., JHEP 0207, 012 (2002); M. Gluck et al., Eur. Phys. J C5 (1998) 461; C40 (2005) 515.

[4] A.V. Kotikov et al., Z. Phys. C58, 465 (1993); G. Parente et al., Phys.Lett. B333 (1994) 190; A.L. Kataev et al., Phys.Lett. B388 (1996) 179; Phys. Lett. B417 (1998) 374; Nucl. Phys. Proc. Suppl. 64 (1998) 138; Nucl. Phys. B573 (2000) 405; V.G. Krivokhizhin and A.V. Kotikov, Phys.Atom.Nucl. 68, 1873 (2005) [hep-ph/0108224].

[5] R.D. Ball and S. Forte, Phys.Lett. B336 (1994) 77; L. Mankiewicz et al., Phys.Lett. B393 (1997) 175.

[6] A.V. Kotikov and G. Parente, Nucl.Phys. B549 (1999) 242; Nucl. Phys. (Proc. Suppl.) 99A (2001) 196 [hep-ph/0010352].

[7] A.V. Kotikov and G. Parente, J. Exp. Theor. Phys. 97 (2003) 859.

[8] A.Yu. Illarionov et al., Phys.Part.Nucl. 39, 307 (2008); Nucl. Phys. (Proc. Suppl.) 146 (2005) 234.

[9] A. De Rújula et al., Phys.Rev. D10 (1974) 1649.
Figure 2: The slope $\lambda_{F2}^{\text{eff}}(x, Q^2)$ as a function of $Q^2$.

[10] H1 Collab.: C. Adloff et al., Phys. Lett. B520 (2001) 183.

[11] ZEUS Collab.: B. Surrow, hep-ph/0201025.

[12] H1 Collab.: T. Lastovicka, in: Proc. of the Int. Workshop on Deep Inelastic Scattering (2002), Cracow; H1 Collab.: J. Gayler, in: Proc. of the Int. Workshop on Deep Inelastic Scattering (2002), Cracow.

[13] Yu.L. Dokshitzer, D.V. Shirkov, Z. Phys. C67 (1995) 449; A.V. Kotikov, JETP Lett. 59 (1994) 1; Phys. Lett. B338 (1994) 349. W.K. Wong, Phys. Rev. D54 (1996) 1094.

[14] S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov and G.B. Pivovarov, JETP. Lett. 70 (1999) 155; Bo Andersson et al., Eur. Phys. J. C 25 (2002) 77 (hep-ph/0204115).

[15] G.Curci, M.Greco and Y. Srivastava, Phys. Rev. Lett. 43 (1979) 834; Nucl. Phys. B159 (1979) 451; M. Greco, G. Penso and Y. Srivastava, Phys. Rev. D21 (1980) 2520; M. Greco and the PLUTO Collaboration, Phys. Lett. B100 (1981) 351; N.N.Nikolaev and B.M. Zakharov, Z. Phys. C49 (1991) 607; C53 (1992) 331; B.Badelek, J.Kwiecinski and A. Stasto, Z. Phys. C74 (1997) 297.