Open charm and bottom meson-nucleon potentials à la the nuclear force

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We discuss the interaction of an open heavy meson ( $D$ and $D^*$ for charm or $B$ and $B^*$ for bottom) and a nucleon ( $N$) by considering the $\pi$, $\sigma$, $\rho$, and $\omega$ exchange potentials. We construct a potential model by respecting chiral symmetry for light quarks and spin symmetry for heavy quarks. Model parameters are adjusted by referring the phenomenological nuclear (CD-Bonn) potentials reproducing the low-energy $NN$ scattering. We show that the resulting interaction may accommodate $DN$ and $BN$ bound states with quantum numbers $I(J^P) = 0(1/2^-)$, and $1(1/2^-)$. We find that, in the present potential model, the $\pi$ exchange potential plays an important role for the isosinglet channel, while the $\sigma$ exchange potential does for the isotriplet one.

I. INTRODUCTION

Studies of exotic hadrons, such as X, Y, Z, $P_c$, $X_{cc}$, $T_{cc}$, and so on, have revealed novel properties of multi-quark systems with heavy flavors of charm and bottom [1-15]. One of the most important problems in exotic hadrons is the inter-hadron interactions. In the present paper, we focus on the interaction between a nucleon $N$ and an open-heavy meson, a $D$ ($D^*$) meson or a $B$ ($B^*$) meson, which is intimately related to the formation of pentaquarks. Such an interaction is also relevant for heavy-flavored exotic nuclei as bound states formed by a multiple number of baryons [16]. Recently the ALICE collaboration in LHCb has reported the first experimental study of the $DN$ interaction which was measured through the correlation functions from proton-proton collisions [17]. Further development of studying the interaction between a nucleon $N$ and an open-heavy meson should be awaited.

One of the efficient theoretical analyses can be performed systematically with the basis on the heavy-quark effective theory. This is an effective theory of QCD, where a charm (bottom) quark is approximately regarded as a particle with an infinitely heavy mass $m_Q \rightarrow \infty$. In this limit, there appears the heavy-quark spin (HQS) symmetry, i.e., the SU(2) spin symmetry, as in the non-relativistic limit. This symmetry stems from the decoupling of the heavy quark from light degrees of freedom with the suppressed magnetic interaction, i.e., the spin-flip interaction. The HQS symmetry puts conditions on the spin structure of interaction vertices not only in the quark-gluon dynamics but also in the hadron dynamics.

The HQS symmetry is seen in the observed approximate degeneracy in masses of $D$ and $D^*$ ($B$ and $B^*$) mesons. Also, the HQS symmetry constrains the structure of the inter-hadron interaction in the channel-coupled $DN$ and $D^*N$ ($BN$ and $B^*N$) systems. For example, it was shown that the approximate degeneracy in $D$ and $D^*$ mesons increases the attractive interaction strength between a nucleon and a $D$ meson through the box diagram $DN \rightarrow D^*N \rightarrow DN$ in the second-order perturbative process [18]. This mechanism is different from the conventional approach based on the SU(4) flavor symmetry [19, 20] and the quark-meson coupling model [18, 21, 22]. The role of the HQS symmetry is shown to be important by including all the coupled channels of $DN$ and $D^*N$ ($BN$ and $B^*N$). Hereafter we will introduce the short notations $P$ and $P^*$ corresponding to $D$ and $D^*$ ($B$ and $B^*$), respectively. We employ $P^{(*)}$ to denote either $P$ or $P^*$. In such a framework, we consider the coupled channels of $PN$ and $P^*N$ and study the interaction between a $P^{(*)}$ meson and a nucleon, denoted by $PN$-$P^*N$.

In the literature, the $PN$-$P^*N$ interactions were introduced by the one-pion exchange potential (OPEP) with the constraint conditions induced by the HQS symmetry [23, 24]. The analysis of the $PN$-$P^*N$ systems showed the possible existence of composite states: bound states below the $DN$ ($BN$) threshold [24, 25], and Fesbach resonant states in the continuum region slightly below the $D^*N$ ($B^*N$) threshold [26, 27]. In the heavy quark limit, these two states are regarded as the doublet

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In the present work, we reconstruct the $PN-P^* N$ potential, where we refer to the phenomenological nuclear potential, the CD-Bonn potential [20]. In the framework of the CD-Bonn potential, the nuclear force is described by the $\pi$, $\rho$, $\omega$, and $\sigma$ exchanges. It is known that the $\sigma$ exchange is important to reproduce the phase shifts in $NN$ scatterings for isospin singlet and triplet channels simultaneously. In fact, the $\pi$, $\rho$, and $\omega$-exchange potentials are not enough for the fitting to the observed data of $NN$ scatterings. In reference to the CD-Bonn potential, thus we also introduce the middle-range force by the $\sigma$ exchange potential in addition to the $\pi$, $\rho$, and $\omega$ potentials in the $PN-P^* N$ interaction which were discussed by the previous studies [25,29]. As introduced in the CD-Bonn potential, the parameters of the $\sigma$ exchange have different values between the isosinglet and isotriplet channels. Considering $DN-D^* N$ and $BN-B^* N$ systems with the reconstructed potentials, we discuss the possible existence of bound states, as discussed in [25,27].

The paper is organized as the followings. In Sec. II A, we introduce the potentials for $PN$ and $P^* N$ in terms of the $\pi$, $\sigma$, $\rho$, and $\omega$ exchanges. We give an analysis for the $\sigma$ exchange potential which is newly introduced in the present study. We present in details the calculation process of the derivation of the potential, because we include some corrections for the potential forms derived in our previous works. In Sec. II B, we present the numerical results for the scattering lengths in the $PN$ and $P^* N$ potentials and the binding energies for the bound states. The final section is devoted to our conclusion and prospects for future studies.

II. FORMALISM

A. Construction of $PN$ and $P^* N$ potentials

1. OPEP

Let us consider the $PN-P^* N$ states of $J^P = 1/2^-$ with a total angular momentum $J$ and parity $P$. $PN$ and $P^* N$ components in those states are represented by

$$PN(^2S_{1/2}), P^* N(^2S_{1/2}), P^* N(^4D_{1/2}).$$

Here the notation $^{2S+1}L_J$ in the parentheses stands for the combination of the total spin $S$ and the relative angular momentum $L$ for a given $J$. In view of the HQS symmetry, the wave functions given above are decomposed into the product of a heavy antiquark $Q$ and a light component “$v$”. Here “$v$” is nonperturbatively composed of the light quarks ($q$) and gluons ($g$) inside the $PN-P^* N$ state. Such a light component may be schematically denoted by $qqqq$, because it should be a composite state of the light quark $q$ in $P$ or $P^*$ and the three quarks $qqq$ in the nucleon $N$. This is the special case of the so-called brown muck which was introduced in the early days when the heavy quark effective theory (HQET) was constructed.

The idea of the light composite state leads to the mass degeneracy of the $PN-P^* N$ states with different $J^P$, such as $J^P = 1/2^-$ and $3/2^-$ by taking the heavy quark limit, because the spin-dependent interaction between the heavy antiquark ($Q$) and the brown muck ($qqqq$) is suppressed by $1/m_Q$ with the heavy quark mass $m_Q$. The mass degeneracy of the $PN-P^* N$ states have been studied in Refs. [16,29,29].

For the interaction in the $PN-P^* N$ systems, we adopt the meson-exchange potential between $P^{(*)}$ and $N$. We consider the one-pion exchange potential (OPEP) as the long-range force. We also consider the $\sigma$-meson exchange potentials and the $\rho$ and $\omega$-meson exchange potentials as the middle-range force.

Let us first explain the derivation of the OPEP in details as an illustration. In constructing the OPEP, we need the information of the interaction vertices of $\pi$ and $P^{(*)}$ and those of $\pi$ and $N$. For the $\pi PP^*$ and $\pi P^* P^*$ vertices, we employ the heavy meson effective theory (HMET) satisfying the HQS as well as chiral symmetry [31,32]. Notice the absence of the $\pi PP$ vertex due to the parity conservation.

For heavy mesons $P$ and $P^*$, we define the effective field $H_\alpha$ being a superposition of a heavy pseudoscalar meson and a vector meson as

$$H_\alpha = (P_\alpha^{\mu} \gamma_\mu + P_\alpha \gamma_5) \frac{1-\gamma_\mu}{2},$$

where the subscripts $\alpha = \pm 1/2$ represent the isospin components (up and down) in the light quark components. $P_\alpha$ and $P_\alpha^{\mu}$ denote the pseudoscalar and vector meson fields, respectively. The relative phase of $P_\alpha^{\mu}$ and $P_\alpha$ is arbitrary, and the present choice is adopted for the convenience in representing the $PN-P^* N$ potential as it will be shown later. Here $\nu^\mu (\mu = 0, 1, 2, 3)$ is the four

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1 In the present setting, the brown muck is regarded to have the special component $q^4 N$ in $qqqqq$. 
velocity of the heavy meson (heavy antiquark) satisfying \( v_\mu v^\mu = 1 \) and \( v^0 > 0 \). We notice that \( (1 - \hat{v})/2 \) is the operator for projecting out the positive-energy component in the heavy antiquark \( \bar{Q} \) and discarding the negative-energy component. The complex conjugate of \( H_\alpha \) is defined by \( \bar{H}_\alpha = \gamma_0 H_\alpha^\dagger \gamma_0 \). The effective field \( H_\alpha \) transforms as \( H_\alpha \to U_{\alpha\beta} H_\beta S^\dagger \) under the heavy-quark spin and chiral symmetries. Here \( S \in SU(2)_\text{spin} \) represents the transformation operator for the heavy-quark spin and \( U_{\alpha\beta} = U_{\alpha\beta}(L, R) \) is a function in the nonlinear representation of chiral symmetry with \( L \in SU(2)_L \) and \( R \in SU(2)_R \) for light up and down flavors.

In terms of \( H_\alpha \) defined by Eq. (3), the interaction Lagrangian for the \( \pi P^{(s)} P^{(s)} \) vertex is given by

\[
\mathcal{L}_{\pi HH} = i g_\pi \text{tr} \left( H_\alpha \bar{H}_\beta \epsilon_{\mu\nu\rho} A_{\mu\rho A}^{\dagger} \right),
\]

where the axial current \( A_{\mu\rho A}^{\dagger} \) by pions is defined by \( A^\mu = (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)/2 \) with the nonlinear representation

\[
\xi = \exp \left( \frac{i \tau \cdot \pi}{2 f_\pi} \right),
\]

with the pion decay constant \( f_\pi = 94 \text{ MeV} \). The pion field is defined by \( \pi = (\pi_1, \pi_2, \pi_3) \) with \( \pi^\pm_\tau = (\pi_1 + i \pi_2)/\sqrt{2} \) for charged pions and \( \pi^0 = \pi_3 \) for a neutral pion. Notice that the matrix \( A^\mu \) is transformed by \( A^\mu \to U A^\mu U^\dagger \) in the nonlinear representation of chiral symmetry. Thus we confirm that the interaction Lagrangian (3) is invariant under both the HQS and chiral symmetries. The coupling constant \( g_\pi = 0.59 \) in Eq. (3) is determined from the decay width of \( D^{(*)} \to D^{(*)} \pi \) observed by experiments [33]. We note that \( g_\pi \) is nothing but the quark axial coupling \( g_A^q \) whose value looks smaller than what is naively expected, \( g_A^q = 1 \) [34]. The small value is understood by considering corrections due to quark’s relativistic motion inside hadrons as discussed in detail for baryon decays [35]. There are uncertainties for choosing the signs of the coupling constants in \( D \) (\( D^{(*)} \)) and \( B \) (\( B^{(*)} \)). In the present study, we assume that the \( \pi, \sigma, \rho, \) and \( \omega \) mesons couple to the light constituent quarks in the heavy mesons as well as in the nucleons. In this scheme, we can consider that the signs of these meson couplings for the light mesons are the same as for the nucleon, because both have the same light (up and down) constituent quarks according to the conventional quark model.

Below we consider the frame in which the heavy meson is at rest and set \( v^\mu = (1, 0) \) in Eq. (3). Thus we obtain the \( \pi P^{(s)} P^{(s)} \) vertices:

\[
\mathcal{L}_{\pi P^* P^*} = \frac{i g_\pi}{f_\pi} \epsilon_{\mu\nu\rho} v^\nu P^{\dagger \rho} (\tau \cdot \partial^\mu \pi)_{\beta\alpha} P^{*\sigma}_{\alpha},
\]

\[
\mathcal{L}_{\pi P^* P} = \frac{i g_\pi}{f_\pi} P^{\dagger \rho} (\tau \cdot \partial^\mu \pi)_{\beta\alpha} P_{\alpha},
\]

\[
\mathcal{L}_{\pi P^* P} = \frac{i g_\pi}{f_\pi} P^{\dagger \rho} (\tau \cdot \partial^\mu \pi)_{\beta\alpha} P^{*\sigma}_{\alpha \beta}.
\]

We introduce the interaction Lagrangian of a pion and a nucleon in the axial-vector coupling

\[
\mathcal{L}_{\pi NN} = \frac{g_N^A}{2 f_\pi} \bar{\psi} \gamma_\mu \gamma_5 \tau \cdot \partial^\mu \pi \psi.
\]

Here \( \psi = (\psi_{+1/2}, \psi_{-1/2})^T \) with the isospin components \( \psi_{+1/2} \) and \( \psi_{-1/2} \) for a proton and a neutron, respectively. The value of \( g_N^A \) is given by the Goldberger-Treiman relation

\[
\frac{g_N^A}{f_\pi} = \frac{g_{\pi NN}}{m_N},
\]

and \( g_{\pi NN}/4\pi = 13.6 \) from the phenomenological nuclear potential in Ref. [30] (see also Ref. [36]). We adopt the values of the coupling constants and the cutoff parameters by referring the parameters in the CD-Bonn potential. The nuclear potentials used in the present study are explained in Appendix A.

With the interaction vertices (3) and (8), we construct the OPEP between \( P^{(*)} \) and \( N \) [25–27]. We show the demonstration to derive the potential for the simple model in Appendix B. The OPEP includes three channels: \( P^* N \to P^* N, P^* N \to PN, \) and \( PN \to P^* N \). We notice that the \( PN \to PN \) process is absent as a direct process due to the prohibition of the \( \pi PP \) vertex, and that the \( PN-PN \) interaction is indirectly supplied by multi-step process stemming from the mixing of \( PN \) and \( P^* N \) [25–27]. The OPEPs for \( P^* N-P^* N, P^* N-PN, \) and \( PN-P^* N \) are given by

\[
V_{\pi P^* N-P^* N}(r) = G_{\pi} \left( T(r; m_\pi) \left( 3 (T \cdot \hat{r}) (\sigma \cdot \hat{r}) - T \cdot \sigma \right) + C(r; m_\pi) T \cdot \sigma \right) \tau^H \cdot \tau^N,
\]

\[
V_{\pi P^* N-PN}(r) = -G_{\pi} \left( T(r; m_\pi) \left( 3 (\epsilon^* \cdot \hat{r}) (\sigma \cdot \hat{r}) - \epsilon^* \cdot \sigma \right) + C(r; m_\pi) \epsilon^* \cdot \sigma \right) \tau^H \cdot \tau^N,
\]

\[
V_{\pi PN-P^* N}(r) = -G_{\pi} \left( T(r; m_\pi) \left( 3 (\epsilon \cdot \hat{r}) (\sigma \cdot \hat{r}) - \epsilon \cdot \sigma \right) + C(r; m_\pi) \epsilon \cdot \sigma \right) \tau^H \cdot \tau^N,
\]

with the coefficient

\[
G_{\pi} = \frac{1}{3} \frac{g_{\pi NN}}{2} \frac{g_{\pi}}{f_\pi}.
\]

We notice that the coefficient \( 1/2 \) is necessary due to the
normalization factor of the wave functions, which was missing in Refs. [16, 28, 29]. The derivation of the OPEP is shown in Appendix C in details. The functions \( C(r; m) \) and \( T(r; m) \) are defined by

\[
C(r; m) = \frac{m^2}{4\pi r} \left( e^{-mr} + \frac{\Lambda_N^2 - m^2}{\Lambda_N^2 - \Lambda_H^2} e^{-\Lambda_N r} + \frac{\Lambda_H^2 - m^2}{\Lambda_H^2 - \Lambda_N^2} e^{-\Lambda_H r} \right),
\]

\[
T(r; m) = \frac{1}{4\pi} \left( m^2 \left( \frac{1}{r} \right) + \frac{3}{m^2 r^3} \right) e^{-mr}
+ \Lambda_N^2 \left( \frac{1}{r} + \frac{3}{\Lambda_N^2 r^3} \right) e^{-\Lambda_N r}
+ \Lambda_H^2 \left( \frac{1}{r} + \frac{3}{\Lambda_H^2 r^3} \right) e^{-\Lambda_H r},
\]

with \( m = m_\pi \), respectively, as functions of an interdistance \( r = |r| \) for \( r \) being the relative coordinate vector between \( P^{(*)} \) and \( N \). The detailed information to derive the potentials are presented in Appendix C. Notice that the values of the cutoff parameters \( \Lambda_H \) (\( H = D, B \)) and \( \Lambda_N \) are dependent on the properties of the exchanged light meson, e.g., the \( \pi \) meson. Originally, \( C(r, m) \) and \( V(r, m) \) are defined by

\[
C(r; m) = \int \frac{d^3q}{(2\pi)^3} \frac{m^2}{q^2 + m^2} e^{iqr} F(q; m),
\]

\[
S_\sigma \mathcal{T}(r; m) = \int \frac{d^3q}{(2\pi)^3} \frac{-q^2}{q^2 + m^2} S_\sigma(q) e^{iqr} F(q; m),
\]

for the central and tensor parts, respectively, with \( q = q/|q| \). We note that the contact term in the central part is neglected. The dipole-type form factor is given by

\[
F(q; m) = \frac{\Lambda_H^2 - m^2}{\Lambda_H^2 + q^2/\Lambda_N^2 + |q|^2},
\]

which is normalized at \( q^2 = m^2 \) with a four-momentum \( q \). The cutoff parameters \( \Lambda_H \) and \( \Lambda_N \) would correspond to the inverse of the spatial sizes of hadrons. See the derivations in Appendix C for more details. In Eqs. (11) and (12), we define the polarization vectors \( \epsilon^{(\lambda)} \) (\( \epsilon^{(*)} \)) for the incoming (outgoing) \( P^* \) meson with the polarization \( \lambda = 0, \pm 1 \). The explicit forms of \( \epsilon^{(\lambda)} \) can be represented by

\[
\epsilon^{(\pm)} = \frac{1}{\sqrt{2}} (\mp 1, -i, 0), \quad \epsilon^{(0)} = (0, 0, 1),
\]

by choosing the positive direction in the \( z \) axis for the helicity \( \lambda = 0 \). As for the spin-one operator for the \( P^* \) meson in Eq. (10), we define \( T = (T_1, T_2, T_3) \) by \( T_j \lambda \lambda \equiv -i \varepsilon_{ijk} \lambda_j \epsilon^{(\lambda)}_k \) \((i, j, k = 1, 2, 3)\):

\[
T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}, \quad T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & -i & 0 \\
i & 0 & -i \\
0 & 0 & 0
\end{pmatrix},
\]

\[
T_3 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{pmatrix},
\]

satisfying the commutation relation \([T_i, T_j] = i \varepsilon_{ijk} T_k\) as the generators of the spin symmetry. We define the tensor operators \( S_\sigma (\hat{r}) \) and \( S_\tau (\hat{r}) \) by \( S_\sigma (\hat{r}) = 3(\mathcal{O} \cdot \hat{r}) (\sigma \cdot \hat{r}) \) and \( S_\tau (\hat{r}) = \mathcal{O} \sigma \) with \( \hat{r} = r/r \) for \( \mathcal{O} = \epsilon \) and \( T \). Here \( \sigma \) are the Pauli matrices acting on the nucleon spin, and \( \tau_{\beta_1 \alpha_2} \) with \( \alpha_i, \beta_i = \pm 1/2 \) are the isospin Pauli operators for \( \sigma^{(*)} \) \((i = 1)\) and \( \tau_{\beta_1 \alpha_2} \) \((i = 2)\). For short notations. In Eq. (21), we confirm that the mixing between \( PN \) and \( P^* N \) are represented by the off-diagonal parts including the tensor potentials. These tensor potentials induce the strong mixing by different angular momenta, leading to the strong attractions at short-range scales. Thus, the mixing of \( PN \) and \( P^* N \) is important to switch on the strong attraction. This is analogous to the OPEP in the nucleon-nucleon interaction.

2. \( \sigma \) exchange potential

The interaction Lagrangian for a \( \sigma \) meson and a \( P^{(*)} \) meson is given by

\[
\mathcal{L}_{\sigma, HH} = -g_{\sigma_1} \text{tr} (\hat{H} \sigma_1 \hat{H}),
\]

which leads to the \( \sigma P^{(*)} P^{(*)} \) vertices,

\[
\mathcal{L}_{\sigma_1 PP} = 2g_{\sigma_1} (\sigma^* \pi P \sigma P) ,
\]

\[
\mathcal{L}_{\sigma_1 P^* P^*} = -2g_{\sigma_1} (\sigma^* \pi^* \sigma \pi P^*). \tag{24}
\]

Here we introduce the channel-dependent \( \sigma_1 \) meson for isospin-singlet \((I = 0)\) and isospin-triplet \((I = 1)\) channels for the \( PN-P^* N \) scatterings, as introduced in the CD-Bonn potential [20]. The parameter of the \( \sigma \) exchange potential in the CD-Bonn potential [30] has the different value for each partial waves, i.e., isospin channels. Thus, \( \sigma_1 \) in the present work also has a channel-dependent mass \((m_{\sigma_1})\), coupling constant \((g_{\sigma_1})\), and cut-off parameter \((\Lambda_{\sigma_1})\). Using the \( \sigma NN \) vertices given by

\[
\mathcal{L}_{\sigma_1 NN} = g_{\sigma_1 NN} \bar{\psi} \sigma_1 \psi,
\]

(25)
we find that the $\sigma$ potentials for $PN$ and $P^*N$ are obtained by

\[
\begin{align*}
V_{\sigma_1}^{PN-PN}(r) &= -\frac{g_{\sigma_1NN}g_{\sigma_1}}{m_{\sigma_1}^2}C(r; m_{\sigma_1}), \\
V_{\sigma_1}^{P^*-N^*P}(r) &= -\frac{g_{\sigma_1NN}g_{\sigma_1}}{m_{\sigma_1}^2}C(r; m_{\sigma_1}),
\end{align*}
\]  

(26)  

(27)

where we employ the values of $m_{\sigma_1}$ and $g_{\sigma_1NN}$ in the CD-Bonn potential, see Appendix A. Concerning the values of $g_{\sigma_1}$, we choose $g_{\sigma_1} = g_{\sigma_1NN}/3$ by assuming that the coupling of a $\sigma$ meson and a hadron $h = P^{(*)}$, $N$ is proportional to the number of the light quarks in the hadron $h$: one light quark in $P^{(*)}$ and three light quarks in $N$. The $\sigma$-exchange potentials are expressed explicitly by

\[
V_{3/2}^{\sigma_1} = \begin{pmatrix}
C_{\sigma_1} & 0 & 0 \\
0 & C_{\sigma_1} & 0 \\
0 & 0 & C_{\sigma_1}
\end{pmatrix},
\]

(28)

for the basis by Eq. [1], where we define the function

\[
C_{\sigma_1} = -\frac{g_{\sigma_1NN}g_{\sigma_1}}{m_{\sigma_1}^2}C(r; m_{\sigma_1})
\]

(29)

for short notations.

3. $\rho$ and $\omega$ exchanges potential

Finally, we consider the exchange of the vector mesons, $\rho$ and $\omega$, at shorter range. The $\rho$ and $\omega$ potentials can be constructed from the $\nu P^{(*)}P^{(*)}$ vertices for light vector meson $\nu$ ($\nu = \rho, \omega$). Following the previous papers, we consider the interaction Lagrangian

\[
\mathcal{L}_{vPH} = i\beta \tr(\hat{H}_\beta \nu^{\mu}M\rho_{\beta\alpha}H_\alpha) \\
+ i\lambda \tr(\hat{H}_\beta \sigma^{\mu\nu}(F_{\mu\nu}(\rho))\beta\alpha H_\alpha),
\]

(30)

by respecting the HQS symmetry. The vector meson field is defined by $\rho_{\mu} = ig_{\nu^{\mu}}\hat{\rho}_{\mu}/\sqrt{2}$ with $\hat{\rho}_{\mu}$,

\[
\hat{\rho}_{\mu} = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} \\
\rho^+ \\
-\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}}
\end{pmatrix},
\]

(31)

and $g_{\nu^{\mu}} \simeq 5.8$ the universal vector-meson coupling. In Eq. (30), the tensor field is given by $F_{\mu\nu}(\rho) = \partial_\mu \rho_{\nu} - \partial_\nu \rho_{\mu} + [\rho_{\mu}, \rho_{\nu}]$. The coupling constants are given by $\beta = 0.9$ and $\lambda = 0.56 \text{ GeV}^{-1}$ by following Refs. [32, 33]. In Ref. [34], $\beta$ was determined by the vector-meson dominance, and $\lambda$ was evaluated by the long distance charm-inenguin diagrams in the $B$ meson decay process. The $\nu P^{(*)}P^{(*)}$ vertices are obtained by the Lagrangians (30) as

\[
\begin{align*}
\mathcal{L}_{vP^{*}P^{*}} &= -\beta g_{\nu} \nu^{\mu}P_{\beta\alpha}^{+}(\tau \cdot \rho^\mu)_{\beta\alpha}P_{\alpha}^{*\nu} \\
&+ 2\lambda g_{\nu} \left(\nu^{\mu}P_{\beta\alpha}^{+}(\tau \cdot \rho^\mu)_{\beta\alpha}P_{\alpha}^{*\mu} \\
&- P_{\beta}^{*\mu}(\tau \cdot \rho)_{\beta\alpha}P_{\alpha}^{*\nu}\right),
\end{align*}
\]

(32)

\[
\mathcal{L}_{vPP} = 2\lambda g_{\nu} \nu^{\mu\nu}P_{\beta}(\tau \cdot \rho^\nu)_{\beta\alpha}P_{\alpha}^{*\rho},
\]

(33)

\[
\mathcal{L}_{vPP} = \beta g_{\nu} \nu^{\mu}P_{\beta}(\tau \cdot \rho^\mu)_{\beta\alpha}P_{\alpha}^{*\rho},
\]

(34)

for the $\nu NN$ vertex, we use the interaction Lagrangian

\[
\begin{align*}
\mathcal{L}_{\nu NN} &= g_{\nu}NN\nu^{\gamma\mu}\nu^{\rho}\psi \left(\frac{f_{\rho NN}}{2m_N} + g_{\omega NN}\nu^{\gamma\rho}\mu\psi \nu^{\rho}\psi \right) \\
&+ g_{\omega NN}\nu^{\gamma\rho}\mu\psi \nu^{\rho}\psi \nu^{\rho}\psi
\end{align*}
\]

(36)

for $\rho^\mu = (\rho^0, \rho^i, \rho^3)$ and $\rho^\mu = (\rho^0, \pm i\rho^i)/\sqrt{2}$ and $\rho^\mu = \rho^3$. The coupling constants are given by $g_{\rho NN}/4\pi = 20.0$, $f_{\rho NN}/g_{\rho NN} = 6.1$, and $g_{\omega NN}/g_{\omega NN} = 0.0$ [30] (see also Ref. [35]). We leave a comment that the coupling strengths in Eqs. (30) and (36) reflect the number of constituent quarks inside the hadrons. This can be easily checked by the nonrelativistic quark model. We should notice, however, that the tensor parts, $\lambda$ and $f_{\rho NN}$ ($\nu = \rho, \omega$), could be different by some factors from the naive expectations, which would be understood from the composite structures of the constituent quarks.

From Eqs. (30) and (36), the one-boson exchange potentials are obtained as

\[
V_{1/2}^{\nu} = \begin{pmatrix}
C'_{\nu} & 2\sqrt{3}C_{\nu} & \sqrt{6}T_{\nu} \\
2\sqrt{3}C_{\nu} & C'_{\nu} - 4C_{\nu} & \sqrt{2}T_{\nu} \\
\sqrt{6}T_{\nu} & \sqrt{2}T_{\nu} & C'_{\nu} + 2C_{\nu} + 2T_{\nu}
\end{pmatrix},
\]

(37)

with $\nu = \rho, \omega$ for the $1/2^{-}$ state in Eq. [1]. The functions $C'_{\nu}$, $C_{\nu}$, and $T_{\nu}$ are defined by

\[
\begin{align*}
C'_{\rho} &= \frac{g_{\nu}NN\beta}{2m_\rho^2}C(r; m_\rho)\tau^H \cdot \tau^N, \\
C_{\rho} &= \frac{g_{\nu}(g_{\rho NN} + f_{\rho NN})\lambda_1}{2m_\rho}T(r; m_\rho)\tau^H \cdot \tau^N, \\
T_{\rho} &= \frac{g_{\nu}(g_{\rho NN} + f_{\rho NN})\lambda_1}{2m_\rho}T(r; m_\rho)\tau^H \cdot \tau^N, \\
C'_{\omega} &= \frac{g_{\nu}NN\beta}{2m_\omega^2}C(r; m_\omega), \\
C_{\omega} &= \frac{g_{\nu}(g_{\omega NN} + f_{\omega NN})\lambda_1}{2m_\omega}C(r; m_\omega), \\
T_{\omega} &= \frac{g_{\nu}(g_{\omega NN} + f_{\omega NN})\lambda_1}{2m_\omega}T(r; m_\omega),
\end{align*}
\]

(38)  

(39)  

(40)  

(41)  

(42)  

(43)  

with $\tau^H$ and $\tau^N$ being the abbreviations of $\tau^H_{\beta\alpha}$ and $\tau^N_{\beta\alpha}$ for the isospin Pauli operators acting on $P^{(*)}$ and $N$, respectively.
B. Total Hamiltonian

The total Hamiltonian for the $P^{(*)}N$ states is given as a sum of the kinetic term and the $\pi$, $\sigma$, $\rho$, and $\omega$ potentials as

$$H_{IJP} = K_{JP} + V_{JP} + V_{JP}^* + V_{JP}^* + V_{JP}^*.$$  \hspace{1cm} (44)

Here $K_{JP}$ is the diagonal matrix for the kinetic terms given by

$$K_{1/2} = \text{diag}(K_0, K_0^*, K_2^*),$$  \hspace{1cm} (45)

where each component is defined by

$$K_L = -\frac{1}{2\mu} \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{L(L+1)}{r^2} \right),$$  \hspace{1cm} (46)

$$K_L^* = -\frac{1}{2\mu^*} \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{L(L+1)}{r^2} \right),$$  \hspace{1cm} (47)

for angular momenta $L = 0$ and $L = 2$. The reduced masses $\mu = m_N m_P/(m_N + m_P)$ and $\mu^* = m_N m_P^*/(m_N + m_P^*)$ are defined with $m_P$ and $m_P^*$, being the masses of $P$ and $P^*$ mesons, respectively.

Concerning the cutoff parameters in the potentials, we consider $\Lambda_H$ in Eq. (13) to be expressed by $\Lambda_H = \kappa_{HN} \Lambda_N$ where $\kappa_{HN}$ is the ratio stemming from inverse hadron size. In Refs. [25-27], we obtained $\kappa_{DHN} = 1.35$ for the $D^{(*)}N$ potential and $\kappa_{BN} = 1.29$ for the $B^{(*)}N$ potential. The same ratios were adopted for the $\rho$ and $\omega$ exchange potentials, and can be applied also to the $\sigma$ exchange potential. In the present study, however, we regard $\kappa_{HN}$ as a free parameter in order to investigate the dependence of the results on the choice of $\kappa_{HN}$ within a range around $\kappa_{DHN} = 1.35$ and $\kappa_{BN} = 1.29$. The value of $\Lambda_N$ is determined by modifying the cutoffs in the CD-Bonn potential by another scale parameter $\kappa_1 (I = 0, 1)$ for each isospin channels. The scale parameter is determined by reproducing the scattering lengths of the $NN$ scatterings for $I = 1$ and the binding energy of a deuteron for $I = 0$, where we employ the simplified nuclear potential neglecting the massive scalar meson, non-local effects and so on in the CD-Bonn potential, see Appendix A in details. The obtained cutoffs are summarized in Table I.

III. NUMERICAL RESULTS

First let us show the phase shifts for $\bar{D}^{(*)}N$ and $B^{(*)}N$ scatterings with $I = 0$ and $I = 1$ in Fig. 1. In the case of $DN$, the $I = 0$ channel has a bound state below the $DN$ mass threshold as the phase shift starts at $\delta = \pi$ and it decreases to zero as the scattering energy increases [Fig. 1(a)]. We notice that the $\bar{D}^*N$ component exhibits repulsion due to the existence of the shallow bound state. At first sight, if we look at the phase shift of the $DN$ component in the $I = 1$ channel, then we may notice that the interaction is repulsive and therefore no bound state exists. However, if we turn our attention to the phase shift of the $\bar{D}^*N(2S_{1/2})$ channel, it starts at $\delta = \pi$, indicating the presence of a bound state [Fig. 1(b)]. As a result, we find a bound state that is formed below the $DN$ threshold. In the bottom case, the $BN$ interaction in the $I = 0$ channel has a bound state below the $BN$ mass threshold, and the $B^*N$ component feels repulsion due to this bound state [Fig. 1(c)]. For $I = 1$, the $B^*N(2S_{1/2})$ phase shift also starts as $\delta = \pi$ [Fig. 1(d)], as well as the $\bar{D}^*N(2S_{1/2})$ one, indicating that there is a bound state driven by the $B^*N$ component.

In Table II, we summarize the binding energies and the mixing ratios of $PN$ and $P^*N$ components. The bound $DN$ state in $I = 0$ has the binding energy 1.38 MeV. The state is almost dominated by $\bar{D}N(2S_{1/2})$ with a small mixture of $\bar{D}^*N(2S_{1/2})$ and $\bar{D}^*N(4D_{1/2})$. Even when the amount of $D^*$-wave component is small, it plays an important role to provide attraction by the tensor interaction in the OPEP as emphasized in our previous papers [25-27]. In $I = 1$, we also obtain the bound state with the binding energy 5.99 MeV. In contrast to the isosinglet state, the bound state has a few amount of the $D^*(4D_{1/2})$ component. This suggests that the $DN$ bound state with $I = 1$ is generated mainly not by the OPEP but by the other potentials. In the present model setting, in fact, the $\sigma$ exchange potential provides a strong attraction in the $P^{(*)}N$ systems as the $\sigma$ exchange potential is strongly attractive for the $NN$ system with $I = 1$ in the CD-Bonn potential. In the bottom case, the $BN$ states with $I = 0$ and $I = 1$ give deeply bound states with the binding energies 29.7 and 66.0 MeV, respectively. In $I = 0$, the main component is $BN(2S_{1/2})$ with a small amount of $B^*N(2S_{1/2})$ and $B^*N(4D_{1/2})$ components. The existence of the $D$-wave component indicates again the importance of the OPEP. In $I = 1$, the $D$-wave component is negligible as seen in the $DN$ bound state. Interestingly, the $B^*N(2S_{1/2})$ channel dominates in the isosinglet bound state, which will be discussed in Sec. IV. The scattering lengths in each state are summarized in Table III.

The phase shifts for $I = 1$ in Figs. 1(b) and 1(d), starting at $\delta = \pi$, imply the existence of the $\bar{D}^*N$ and $B^*N$ bound states. In order to confirm this idea, we have performed a bound-state analysis considering only the $\bar{D}^*N$ ($B^*N$) channels, when the $DN$ ($BN$) channel is switched off. As a result, we find $\bar{D}^*N$ and $B^*N$ bound states with the binding energies 29.1 and 51.0 MeV, respectively, measured from the $D^*(B^*)$ threshold.

We investigate the parameter dependence of the attraction in $P^{(*)}N$, where the values of these parameters have some ambiguity in the present model setting. In Fig. 2, we show the dependence of the scattering lengths on the cutoff-ratio parameters, $\kappa_{DHN}$ and $\kappa_{BN}$. In the $DN$ case, we find that the attraction in $I = 0$ is provided for $\kappa_{DHN} \gtrsim 1.1$ whose values are consistent with the one estimated by the ratio of the different hadron sizes of a $D$ meson and a nucleon, as previously discussed in Refs. [25-27]. The strength of attraction in $I = 1$ is not so dependent on the choice of $\kappa_{DHN}$. In the $BN$ case,
TABLE I. Parameters of the meson exchange potentials. The meson masses are given as the isospin-averaged values. $g_{\pi}$, $\beta$, $\lambda$, and $g_{\sigma_I}$ are the coupling constants of heavy mesons (see text in details), while $g_{\alpha NN}$ and $f_{\alpha NN}$ are those of a nucleon taken from the CD-Bonn potential [30]. The cutoffs $\Lambda_B$ and $\Lambda_B$ are shown as typical values for $\Lambda_B = 1.35\Lambda_N$ and $\Lambda_B = 1.29\Lambda_N$, where $\Lambda_N$ is the nucleon cutoff which is scaled by the parameter $\kappa_1$ ($\kappa_0 = 0.804$ and $\kappa_1 = 0.773$) from the CD-Bonn potential (see Appendix A in details).

| Mesons | $\alpha$ | Masses [MeV] | $g_{\pi}$ | $\beta$ [GeV$^{-1}$] | $\lambda$ | $g_{\sigma_I}$ | $g_{\alpha NN}$ | $f_{\alpha NN}$ | $\Lambda_B$ [MeV] | $\Lambda_B$ [MeV] | $\Lambda_N$ [MeV] |
|--------|---------|-------------|-----------|----------------|---------|-------------|-------------|-------------|----------------|----------------|----------------|
| $\pi$  | 138.04  | 0.59        | —         | —              | —       | —           | 13.6        | 1686        | 1795          | 1785          | 1715          | 1384          | 1330         |
| $\rho$ | 769.68  | —           | 0.9       | 0.56           | —       | 0.84        | 6.1         | 1359        | 1306          | 1423          | 1367          | 1054          | 1013         |
| $\omega$ | 781.94  | —           | 0.9       | 0.56           | —       | 20          | 0.0         | 1629        | 1565          | 1557          | 1496          | 1207          | 1159         |
| $\sigma_0$ | 350    | —           | —         | —              | —       | 0.849406    | 0.51673     | —           | —              | —              | —              | 2715          | —            |
| $\sigma_1$ | 452    | —           | —         | —              | —       | 2.35276     | 3.96451     | —           | —              | —              | —              | 2609          | 2493         | 1932        |

FIG. 1. The phase shifts of $\bar{D}N$ [(a) and (b)] and $BN$ [(c) and (d)] as functions of the scattering energy. Panels (a) and (c) are for $I = 0$, and panels (b) and (d) are for $I = 1$.

The attraction in $I = 0$ has only weak dependence on the choice of $\kappa_{BN}$ in the range of $\kappa_{BN} \gtrsim 1.0$. This result would tell us a confidence for the existence of the $BN$ bound state in $I = 0$. In comparison with $I = 0$, the attraction in $I = 1$ is more sensitive to choice of the value of $\kappa_{BN}$. Thus the deeply $BN$ bound state in $I = 1$ needs to be carefully considered in terms of its model dependence.

Uncertainty in the current model is also brought by the sigma coupling. In general, the coupling constants of the meson exchange potential are fixed by the experimental data, such as the nucleon-nucleon scattering data and heavy meson decays. However, the sigma coupling to the heavy meson is difficult to be determined uniquely only by the currently existing experimental data. In our present calculation framework, we have adopted $g_{\sigma} = g_{\sigma NN}/3$ (see Sec. II A2). In order to investigate the uncertainty from the ambiguity of the sigma coupling value, we estimate the dependence of binding energies and scattering lengths on the $g_{\sigma}$ coupling con-
TABLE II. Binding energies (B.E.) and mixing ratios of the $\bar{D}^{(*)}N$ and $B^{(*)}N$ states with $I(J^P)$ quantum numbers. The binding energies are measured from the mass thresholds of $\bar{D}N$ or $BN$.

| $\bar{D}N$ | B.E. [MeV] | Mixing ratio [%] |
|------------|------------|-----------------|
| $0(1/2^-)$ | 1.38       | $\bar{D}N(\bar{2}S_{1/2})$ 96.1 |
|           |            | $\bar{D}N^*(\bar{2}S_{1/2})$ 1.94 |
|           |            | $\bar{D}^*N(\bar{4}D_{1/2})$ 1.93 |
| $1(1/2^-)$ | 5.99       | $\bar{D}N(\bar{2}S_{1/2})$ 88.9 |
|           |            | $\bar{D}N^*(\bar{2}S_{1/2})$ 10.9 |
|           |            | $\bar{D}^*N(\bar{4}D_{1/2})$ 0.11 |
| $BN$      | B.E. [MeV] | Mixing ratio [%] |
| $0(1/2^-)$ | 29.7       | $BN(\bar{2}S_{1/2})$ 76.4 |
|           |            | $B^*N(\bar{2}S_{1/2})$ 14.1 |
|           |            | $B^*N(\bar{4}D_{1/2})$ 9.46 |
| $1(1/2^-)$ | 66.0       | $BN(\bar{2}S_{1/2})$ 38.5 |
|           |            | $B^*N(\bar{2}S_{1/2})$ 61.5 |
|           |            | $B^*N(\bar{4}D_{1/2})$ $1.82 \times 10^{-2}$ |

stant as shown in Fig 3. Here we show (a) the binding energies, (b) the scattering lengths for $PN$, and (c) the coupling strengths of the meson-exchange potentials were incorrectly overestimated by a factor of 2 due to the incorrect normalization of wave functions in Refs. 25 and 27. In the present analysis for $I = 0$, we have also found that similar bound states exist by reconstructing the $PN$ interaction model newly including the $\sigma$ exchange. Again, the $\pi$ exchange potential plays the dominate role to produce the attraction. In contrast, the bound states in $I = 1$ have been obtained in the $PN$ states, where the main attraction is provided by the $\sigma$ potential whose strength in $I = 1$ is set to be larger than that in $I = 0$.

IV. DISCUSSION

We discuss the internal spin structures of the bound $\bar{D}N$ and $BN$ states in a view of the HQS symmetry. As already discussed in detail in Ref. 29, the $P^{(*)}N$ state can be decomposed into product states of the heavy antiquark $\bar{Q}$ and the light quarks $qqqq$ in the heavy quark limit. The latter component is called the light spin complex, instead of the brown muck, because it makes a specific structure composed of $q$ and $N$ which is denoted by $[qN]_P$ with total spin $j$ and parity $P$ of the light quark components. These are a conserved quantities due to the spin decoupling from the heavy quark. The important property in the heavy quark limit is that the ratio of the fractions of the amount of $P\bar{N}(2S+1L_J)$ and $P^*\bar{N}(2'S+1L_J')$ wave functions is determined uniquely. Here $S'$ and $L'$ can be different from $S$ and $L$, respectively, in general. As shown explicitly in Ref. 29, we obtain the fractions

$$P\bar{N}(2S_{1/2}) : P^*\bar{N}(2S_{1/2}) = 1 : 3, \quad (48)$$

for $j_P = 0^+$ and

$$P\bar{N}(2S_{1/2}) : P^*\bar{N}(2S_{1/2}) = 3 : 1, \quad (49)$$

for $j_P = 1^+$, which hold irrespectively of the choice of the $PN$-$P^*N$ potential. Although these ratios are exact only in the heavy quark limit, they provide us with a guideline to understand the internal spin structures of the obtained $\bar{D}N$ and $BN$ bound states.

In Table IV, for example, we show that the mixing ratios of $BN(\bar{2}S_{1/2})$ and $B^*N(\bar{2}S_{1/2})$ in $I = 0$ are 76.4 % and 14.4%, respectively, which are close to the ratio in Eq. (48) rather than that in Eq. (49). Thus, it is suggested that the $BN$ bound state in $I = 0$ is dominated by the light spin complex with $j_P = 1^+$. In contrast, the mixing ratios $BN(\bar{2}S_{1/2})$ and $B^*N(\bar{2}S_{1/2})$ in $I = 1$ are 38.5 % and 61.5 %, respectively, are close to the ratio in Eq. (48) rather than that in Eq. (49). Thus, it is suggested that the $BN$ bound state in $I = 1$ includes the light spin complex with $j_P = 0^+$ as a major component.

One may wonder that the ratios in bottom sector are not the same as the ratios in Eqs. (48) and (49) in spite of the sufficient heaviness of the bottom quark mass. This would be simply due to the violation of the heavy quark spin symmetry stemming from the difference of the $B$ meson mass and the $B^*$ meson mass, as noted in Ref. 29.

We should notice that the existence of the $j_P = 0^+$ state is new because only the $j_P = 1^+$ state was reported for the $\pi$, $\rho$, and $\omega$ potentials in Ref. 29. We can understand this new result in terms of the fact that the $j_P = 0^+$ state is provided mainly by the $\sigma$ potential because of the sufficient attraction in the $\sigma$ exchange stemming from the characteristic property of the CD-Bonn potential (see Table IV in Appendix A).

V. CONCLUSION

We have discussed the $\bar{D}^{(*)}N$ and $B^{(*)}N$ bound states in terms of the $\pi$, $\sigma$, $\rho$, and $\omega$ meson-exchange potentials by considering the heavy-quark spin symmetry and the
chiral symmetry. By referring the CD-Bonn potential for the nuclear force, we have constructed the $PN-P^*N$ potential with the $\sigma$ exchanges as new degrees of freedom at middle-range interaction. We have carefully calculated the potentials with appropriate factors stemming from the normalization of the wave function which were underestimated in our previous studies [25–27]. As results, we have found that the interaction is largely attractive to hold the $\bar{D}N$ bound state and the $BN$ bound state below the lowest mass threshold for each in $I(J^P) = 0(1/2^-)$ channel. Their binding energies are close to the values that were obtained by our previous works. With the present potential including $\sigma$ exchange, interestingly, we have found that the $\sigma$ exchange as well as the $\pi$ exchange still plays an important role. We also have found the $\bar{D}N$ and $BN$ bound states in $I(J^P) = 1(1/2^-)$ as a new state which has not been discussed so far. It is expected that those states are relevant to the $D^-p$ interaction researched in LHCb [17].

The attraction in $PN-P^*N$ systems would open a new way to understand the inter-hadron interaction in heavy flavors. It is important that these systems are made of
TABLE III. S-wave scattering lengths \( a \) of the \( D^{(s)}N \) and \( B^{(s)}N \) states. An attractive scattering length is given by the negative sign \( (a < 0) \), and a repulsive scattering length and the scattering length for a bound state are given by the positive sign \( (a > 0) \).

| \( \bar{D}N \) | \( a [\text{fm}] \) |
|----------------|--------------------|
| \( 0(1/2^-) \) | \( \bar{D}N(2^{1/2}) \) 5.21 |
|                  | \( \bar{D}N^*(2^{1/2}) \) 0.868 – i3.72 \times 10^{-2} |
| \( 1(1/2^-) \) | \( \bar{D}N(2^{1/2}) \) 2.60 |
|                  | \( \bar{D}N^*(2^{1/2}) \) 0.944 – i0.722 |
| \( B\bar{N} \)  | \( a [\text{fm}] \) |
| \( 0(1/2^-) \) | \( B\bar{N}(2^{1/2}) \) 1.25 |
|                  | \( B\bar{N}^*(2^{1/2}) \) 1.03 – i1.07 \times 10^{-2} |
| \( 1(1/2^-) \) | \( B\bar{N}(2^{1/2}) \) 3.84 \times 10^{-2} |
|                  | \( B\bar{N}^*(2^{1/2}) \) 0.263 – i0.585 |

genuinely five-quark components due to the absence of the annihilation channels. It may help us to understand the new channels of exotic hadrons. Furthermore, the many-body dynamics would be an interesting subject, because the many-body dynamics would be an interesting subject, and so on would also be interesting. In theoretical study, the cross sections for producing such exotic nuclei have been studied theoretically for some possible exotic light nuclei \[\text{[10]}.\] Few-body systems such as \( DNN \) (\( BNN \) \[\text{[38]}\) and \( D\bar{N} \) (\( B\bar{N} \) \[\text{[38]}\) are also interesting, which can be accessed through the relativistic heavy ion collisions in LHC and RHIC \[\text{[29, 41]}\]. The nuclear structure of charm and bottom nuclei has been studied theoretically for some possible exotic light nuclei \[\text{[12]}\]. Experiments at J-PARC, GSI-FAIR, NICA, and so on would also be interesting. In theoretical study, the cross sections for producing such exotic nuclei have been discussed \[\text{[33]}\]. As one of the advanced topics related to heavy-flavored nuclei, the isospin Kondo effect is interesting as it exhibits the “confinement” of isospin charge \[\text{[44, 48]}\]. Many subjects are awaiting to be discussed in the future.

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Appendix A: The \( NN \) potential

We construct the nuclear potential by considering the \( \pi, \sigma, \rho, \) and \( \omega \) exchanges. Their interaction Lagrangians for the vertices with a nucleon are given by

\[
\mathcal{L}_{\pi NN} = g_{\pi NN} \bar{\psi} \gamma_5 \tau \cdot \pi \psi, \\
\mathcal{L}_{\sigma_1 NN} = g_{\sigma_1 NN} \bar{\psi} \sigma_1 \psi, \\
\mathcal{L}_{\rho NN} = g_{\rho NN} \bar{\psi} \gamma_\mu \rho^\mu \psi + \frac{f_{\rho NN}}{4m_N} \bar{\psi} \sigma_{\mu \nu} \cdot (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) \psi, \\
\mathcal{L}_{\omega NN} = g_{\omega NN} \bar{\psi} \gamma_\mu \omega^\mu \psi,
\]

with the appropriate coupling constants. We use different \( \sigma \) mesons: the \( \sigma_0 \) meson for the isosinglet \( (I = 0) \) \( NN \) scatterings and the \( \sigma_1 \) meson for the isotriplet \( (I = 1) \) \( NN \) scatterings. Their difference appears not only in the coupling constants but also in their masses. We sometimes omit the underscript \( I \) if unnecessary. From the Lagrangians \( \text{[A1]-[A4]} \), we obtain the \( NN \) potentials:

\[
V_\pi(r) = \left( \frac{g_{\pi NN}}{2m_N} \right)^2 \left( \frac{1}{3} \sigma_1 \cdot \sigma_2 C_\pi(r) + S_{12}(\bar{r})T_\pi(r) \right) \tau_1 \cdot \tau_2, \\
V_\rho(r) = g_{\rho NN}^2 \left( \frac{1}{m_\rho^2} + \frac{1}{g_{\rho NN}} \right) \left( \frac{1}{2m_N^2} \right)^2 \pi_1 \cdot \pi_2 \left( \sigma_1 \cdot \sigma_2 C_\rho(r) \right), \\
V_\omega(r) = g_{\omega NN}^2 \left( \frac{1}{m_\omega^2} + \frac{1}{g_{\omega NN}} \right) \left( \frac{1}{2m_N^2} \right)^2 \pi_1 \cdot \pi_2 \left( \sigma_1 \cdot \sigma_2 C_\omega(r) \right),
\]

with \( v = \rho, \omega \), where the functions \( C_\pi, T_\pi, C_\sigma_1, C_\rho, C_\omega \) are defined as above. More concretely, the \( NN \) potentials are expressed by

\[
\tilde{V}_{\pi NN}^{S_1}(r) = \tilde{V}_{\pi NN}^{S_0}(r) + \tilde{V}_{\rho NN}^{S_0}(r) + \tilde{V}_{\omega NN}(r),
\]

with

\[
\tilde{V}_{\pi NN}(r) = \left( \begin{array}{cc} -3C_\pi^{NN} & -6\sqrt{2}T_\pi^{NN} \\ 6\sqrt{2}T_\pi^{NN} & -3C_\pi^{NN} + 6T_\pi^{NN} \end{array} \right), \\
\tilde{V}_{\rho NN}(r) = \left( \begin{array}{cc} C_\pi^{NN} + 2C_\rho^{NN} & -2\sqrt{2}T_\pi^{NN} \\ -2\sqrt{2}T_\pi^{NN} & C_\pi^{NN} + 2C_\rho^{NN} + 2T_\pi^{NN} \end{array} \right), \\
\tilde{V}_{\omega NN}(r) = \left( \begin{array}{cc} -C_\sigma_1^{NN} & 0 \\ 0 & -C_\sigma_0^{NN} \end{array} \right).
\]
in the $^3S_1$ channel, where the $^3S_1$ and $^3D_1$ components are coupled, and

\[ V_{1S_0}^{NN}(r) = V_{s}^{NN}(r) + V_{\sigma}^{NN}(r) + V_{\rho}^{NN}(r) + V_{\omega}^{NN}(r), \]

(A12)

with

\[ V_{s}^{NN}(r) = -3C_s^{NN}(r), \]

(A13)

\[ V_{\sigma}^{NN}(r) = C_{\sigma}^{NN}(r) - 6C_s^{NN}, \]

(A14)

\[ V_{\omega}^{NN} = C_{\omega}^{NN}. \]

(A15)

in the $^1S_0$ channel. Notice that the tensor potentials are switched on due to the spin-1 property in the $I = 0$ channel.

We choose the values of the coupling constants to be the same values as those in the CD-Bonn potential [30] as summarized in Table IV. We notice that the CD-Bonn potential originally includes the nonlocal potentials in the $\pi$, $\sigma$, $\rho$, and $\omega$ exchanges, and contact terms stemming from the short-range part in the meson exchange. In the present study, however, we neglect the nonlocal potentials, the contact terms and massive $\sigma$ mesons, and so on, because we are interested only in the low-energy parts in the $NN$ scatterings.

In order to compensate the difference from the CD-Bonn potential, we rescale the cutoff parameter by introducing $\kappa_I$ providing the new cutoffs $\Lambda_N = \kappa_I \Lambda_N^{CD-Bonn}$. Here $\Lambda_N^{CD-Bonn}$ is the original cutoff parameter in the CD-Bonn potential [30], whose values depend on the exchanged mesons, $\pi$, $\sigma$, $\rho$, and $\omega$. $\kappa_I$ $(I = 0$ and $I = 1)$ are the scale parameter, introduced newly for the adjustment to reproduce the low-energy $NN$ scatterings in the present simple model of nuclear force. Notice the values of $\kappa_I$ are dependent only on the isospin channels $I = 0$ and $I = 1$, while they are common to the $\pi$, $\sigma$, $\rho$, and $\omega$ exchanges. We use the values in proton-neutron channel in $I = 1$ in the CD-Bonn potential, because the electric Coulomb force is not included in our potential. We determine the values of $\kappa_I$ to reproduce the binding energy of a deuteron $B_d$ in the $^3S_1 (I = 0)$ channel as well as the $NN$ scattering length in the $^1S_0 (I = 1)$ channel. As the best fitting, we obtain $\kappa_0 = 0.804$ for $I = 0$ and $\kappa_1 = 0.773$ for $I = 1$. Roughly, we consider that those values would represent the “effective” cutoff parameters when the higher-energy dynamics is renormalized at lower energy near thresholds. Similar values are obtained also when the $NN$ scattering length in the $^3S_1 (I = 0)$ channel is chosen instead of $B_d$. As shown in Table IV, the obtained values of the scattering lengths and the effective ranges are well consistent with those obtained from the original CD-Bonn potential, $a(^3S_1) = 5.419 \pm 0.007$ fm, $r_c(^3S_1) = 1.753 \pm 0.008$ fm, $a(^1S_0) = -23.740 \pm 0.020$ fm, $r_c(^1S_0) = 2.77 \pm 0.05$ fm, and $B_d = 2.225$ MeV, see Ref. [30] for details.

| Mesons (α) Masses [MeV] | $\frac{g_{2NN}}{\alpha}$ | $\frac{g_{\sigma NN}}{\sigma_{NN}}$ | $\Lambda_N [\text{MeV}]$ |
|-------------------------|---------------------------|---------------------------|------------------|
| $\pi$                   | 138.04                    | 13.6                      | 1384             |
| $\rho$                  | 769.68                    | 0.84                      | 1054             |
| $\omega$                | 781.94                    | 0.0                       | 1207             |
| $\sigma_0$              | 350                       | 0.51673                   | 2011             |
| $\sigma_1$              | 452                       | 3.96451                   | 1932             |

| Channel | $\kappa_I (I = 0, 1)$ | $a [\text{fm}]$ | $r_c [\text{fm}]$ | $B_d [\text{MeV}]$ |
|---------|-----------------------|-----------------|------------------|------------------|
| $^3S_1 (I = 0)$ | 0.804 | 5.296 | 1.562 | 2.225* |
| $^1S_0 (I = 1)$ | 0.773 | -23.740* | 2.337 |  |

### Appendix B: Potential in a simple model

As an illustration of deriving a potential, we consider a simple model where a potential is provided by the boson exchange interaction ($\Phi$) between two heavy particles ($\Phi$). We consider the Lagrangian

\[ \mathcal{L}[\Phi, \Phi] = \frac{1}{2} \left( \partial_{\mu} \phi \partial^{\mu} \phi - m^2 \phi^2 \right) - g \phi \Phi^\dagger \Phi \]

\[ + \partial_{\mu} \Phi^\dagger \partial^{\mu} \Phi - M^2 \Phi^\dagger \Phi, \]

(B1)

with the masses $m$ and $M$ for $\phi$ and $\Phi$, respectively. From the equation of motion for $\phi$, $(\partial^2 + m^2) \phi = -g \Phi^\dagger \Phi$, we obtain the solution

\[ \phi(x) = g \int \! d^4 y \langle x | \left( \frac{1}{\partial^2 + m^2} \right)_{xy} | y \rangle \Phi^\dagger(y) \Phi(y), \]

(B2)

for given $\Phi(y)$. As a nonrelativistic limit, making the approximation $\partial^2 = \partial_0^2 - \partial^2 \approx -\partial^2$, we find that the solution is expressed by

\[ \phi(x) = g \int \! d^3 y \langle x | \left( \frac{1}{\partial^2 - m^2} \right)_{xy} | y \rangle \Phi^\dagger(y) \Phi(y), \]

(B3)

by dropping the temporal dependence in $x^\mu = (x_0, x)$ and $y^\mu = (y_0, y)$. The states $|x\rangle$ and $|y\rangle$ are also changed
to $|x\rangle$ and $|y\rangle$, respectively. Hereafter, we omit $x_0$ and $y_0$ unless required for specification.

From the Lagrangian [B3], we obtain the interaction Hamiltonian $H_{\text{int}} = \int d^4x H_{\text{int}}(x)$ with $H_{\text{int}}(x) = g \phi(x) \Phi^\dagger(x) \Phi(x)$. In the following discussion, we express this term by $H_{\text{int}}(x) = g \phi(x) \Phi^\dagger(x) \Phi(x)$ because the temporal dependence is dropped in the nonrelativistic approximation. The expectation value of $H_{\text{int}}(x)$ leads to the energy shift of the system:

$$\Delta E \equiv \langle 1, 2 | \int d^3x H_{\text{int}}(x) | 1, 2 \rangle,$$

with $|1, 2 \rangle = |1 \rangle \otimes |2 \rangle$ where $|1 \rangle$ and $|2 \rangle$ denote the heavy-particle states at the position 1 and 2, respectively, at the equal time. By using Eq. [B3], we rewrite $\Delta E$ in the following form:

$$\Delta E = g^2 \int d^3x \int d^3y \langle 1, 2 | \Phi^\dagger(x) \Phi(x) | 0 \rangle \langle 0 | \Phi(x) | 1 \rangle \tilde{V}_\phi(x, y) \langle 2 | \Phi^\dagger(y) | 0 \rangle \langle 0 | \Phi(y) | 2 \rangle.$$

In the last equation, we have inserted the vacuum state denoted by $|0\rangle$ normalized by $\langle 0 | 0 \rangle = 1$. We have used $\langle x | p \rangle = e^{ip \cdot x}$ for the plane wave, and defined the potential by

$$\tilde{V}_\phi(x, y) \equiv g^2 \int \frac{d^3p}{(2\pi)^3} \frac{-1}{p^2 + m^2} e^{-ip \cdot (x - y)}, \quad (B6)$$

between $x$ and $y$.

Let us consider the scattering process $p_1 + p_2 \rightarrow p_1' + p_2'$ of two $\Phi$ particles, where the states $|1\rangle$ and $|2\rangle$ (|(1) and (2)|) have the three-dimensional momenta $p_1$ and $p_2$ ($p_1'$ and $p_2'$), respectively. Here we need to evaluate the wave functions, $\langle 0 | \Phi(x) | 1 \rangle$, $\langle 0 | \Phi(y) | 2 \rangle$, $\langle 1 | \Phi^\dagger(x) | 0 \rangle$, and $\langle 2 | \Phi^\dagger(y) | 0 \rangle$, in the plane waves with momentum $p_1$, $p_2$, $p_1'$, and $p_2'$. For this purpose, we expand $\Phi(x)$ by

$$\Phi(x) = \int \frac{d^3p}{(2\pi)^3} \sqrt{2E_p} \left( a_p e^{ip \cdot x} + b_p^\dagger e^{-ip \cdot x} \right), \quad (B7)$$

according to the conventional forms, where $E_p = \sqrt{p^2 + M^2}$ is the energy of the heavy particle, and $a_p$ and $b_p$ ($a_p^\dagger$ and $b_p^\dagger$) are the annihilation (creation) operators for the particle and antiparticle states with three-dimensional momentum $p$. The commutation relations for $a_p$ and $b_p$ ($a_p$ and $b_p^\dagger$) are given by $[a_p, a_p^\dagger] = [b_p, b_p^\dagger] = (2\pi)^3 \delta^{(3)}(p - p')$. In the followings, we consider only the particle state described by $a_p$ and $b_p^\dagger$ by neglecting the antiparticle states.

We consider the state given by $|p\rangle = \sqrt{2E_p} a_p^\dagger |0\rangle$. The normalization of $|p\rangle$ is given by

$$\langle p | p \rangle = 2E_p (2\pi)^3 \delta^3(0) = 2E_p V,$$

which has the factor $2E_p V$, where $V$ is a volume of the whole space. This indicates that the number of the particle in the wave function is $2E_p V$. In Eq. [B5], we calculate $\langle 0 | \Phi(x) | 1 \rangle$, $\langle 0 | \Phi(y) | 2 \rangle$, $\langle 1 | \Phi^\dagger(x) | 0 \rangle$, and $\langle 2 | \Phi^\dagger(y) | 0 \rangle$. The expectation value of $\mathcal{H}_{\text{int}}(x)$ leads to the energy shift of the system:

$$\Delta E \equiv \langle 1, 2 | \int d^3x \mathcal{H}_{\text{int}}(x) | 1, 2 \rangle,$$

and consider $\langle 0 | \Phi(x) | p_1 \rangle$, $\langle 0 | \Phi(y) | p_2 \rangle$, $\langle p_1' | \Phi^\dagger(x) | 0 \rangle$, and $\langle p_2' | \Phi^\dagger(y) | 0 \rangle$. Using Eq. (B7), we obtain

$$\langle 0 | \Phi(x) | p_1 \rangle = e^{ip_1 \cdot x},$$

$$\langle 0 | \Phi(y) | p_2 \rangle = e^{ip_2 \cdot y},$$

$$\langle p_1' | \Phi^\dagger(x) | 0 \rangle = e^{-ip_1' \cdot x},$$

$$\langle p_2' | \Phi^\dagger(y) | 0 \rangle = e^{-ip_2' \cdot y}.$$

Then, we find that $\Delta E$, which stems from $\Delta E$ in the relativistic version of the states, is expressed by

$$\Delta E = \int d^3x \int d^3y \tilde{V}_\phi(x, y) e^{i(p_1 - p_1') \cdot x} e^{i(p_2 - p_2') \cdot y}.$$

When we consider the limit of $p_1, p_2, p_1', p_2' \rightarrow 0$ in the static approximation, we express $\Delta E$ by

$$\Delta E \approx \int d^3x \int d^3y \tilde{V}_\phi(x, y). \quad (B14)$$

From Eq. [B13], we remember that the states $|p_1\rangle$, $|p_2\rangle$, $|p_1'\rangle$, and $|p_2'\rangle$ are normalized to have $2E_p V$, $2E_p V$, $2E_p V$, $2E_p V \approx 2MV$ particles in the nonrelativistic limit. Then, we should regard the quantity $\Delta E/(2MV)^2$ as the potential energy for a pair of particles. Thus, the energy per a pair of particles is given by

$$V_\phi(x, y) \equiv \frac{1}{(2M)^2} \tilde{V}_\phi(x, y), \quad (B15)$$

with $\tilde{V}_\phi(x, y)$ in Eq. [B6]. As a conclusion, $V_\phi$ is the potential between two $\Phi$’s used in the non-relativistic quantum mechanics.
Appendix C: Derivation of OPEP for a $P^{(*)}$ meson and a nucleon

From Eqs. (3) and (8), we obtain the Lagrangian including $\pi$, $N$, and $H$ (= $P$, $P^*$)

$$\mathcal{L}_{\pi N} = \frac{1}{2} (\partial_\mu \pi \partial^\mu \pi - m^2 \pi^2)$$

where the kinetic terms of $H$ and $N$ are not shown. The equation of motion for $\pi$ is

$$(\partial^2 + m^2)\pi = -\frac{ig_\pi}{f_\pi} \partial^\mu \left( \varepsilon_{\mu\nu\rho\sigma} \pi^{\nu\rho} \pi^{\sigma}_\beta (\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) + iP^{\nu}_\beta (\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) + iP^{\nu}_\beta (\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) \right)$$

(C2)

When we consider only the spatial dependence in the fields, we express the solution by

$$\pi_\alpha (x) = \frac{-ig_\pi}{f_\pi} \int \frac{d^3 y}{(\partial^2 + m^2)} \frac{1}{|y|} \partial_\mu \left( \varepsilon_{\mu\nu\rho\sigma} \pi^{\nu\rho}_{\beta\gamma} (\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) + iP^{\nu}_\beta (\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) + iP^{\nu}_\beta (\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) \right)$$

(C3)

with $i, j = 1, 2, 3$ for given $\psi$, $P$, and $P^*$. Then, the interaction energy between $P^{(*)}$ and $N$ is given by

$$\Delta E^{HN} \equiv \langle 1, 2 \rangle \int d^3 x \mathcal{H}_{\text{int}}^{\pi N}(x) |1, 2\rangle$$

$$= \int d^3 x \int d^3 y \langle 1 \left| -\varepsilon_{ijkl} P^{\gamma\delta}_\beta (x)(\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) + iP^{\nu}_\beta (x)(\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) + iP^{\nu}_\beta (x)(\tau_\alpha)_{\beta\gamma} P^{\gamma}_\delta (y) \right| 2 \rangle$$

(C4)

where $\mathcal{H}_{\text{int}}^{\pi N}$ represents the interaction Hamiltonian stemming from Eq. (C1), and $|1\rangle$ and $|2\rangle$ represent a $P^{(*)}$ meson and a nucleon, respectively. For brevity we have defined $\hat{V}^{\pi N}_{ij}(x, y)$ by

$$\hat{V}^{\pi N}_{ij}(x, y) = \frac{g_{\pi N}}{2m_N} \frac{ig_\pi}{f_\pi} \int \frac{d^3 p}{(2\pi)^3} \frac{p_i p_j}{p^2 + m^2} e^{-ip \cdot (x-y)}.$$

(C5)

We consider the matrix element by using the basis states $|1\rangle = |P_\alpha^* (p_1, \lambda_1)\rangle$ or $|P_\alpha (p_1)\rangle$ and $|2\rangle = |P_\beta^* (p_1', \lambda_1')\rangle$ or $|P_\beta (p_1')\rangle$. Here $p_1$ ($p_1'$) is the three-dimensional momentum of the $P^{(*)}$ meson and $\lambda_1$ ($\lambda_1'$) is the helicity of the $P^*$ meson ($\lambda_1, \lambda_1' = 0, \pm$). $\alpha_1, \beta_1 = \pm 1/2$ are the isospin components. Adopting the following channels,

$$\{ |1\rangle, |2\rangle \} = \{ |P_\alpha^* (p_1, \lambda_1)\rangle, |P_\alpha (p_1)\rangle \},$$

$$\{ |P_\beta^* (p_1', \lambda_1')\rangle, |P_\beta (p_1')\rangle \},$$

$$\{ |P_\beta (p_1')\rangle, |P_\alpha^* (p_1, \lambda_1)\rangle \},$$

(C6)

and

$$\{ |2\rangle, |1\rangle \} = \{ |N_{\beta_2} (p_2, s_2)\rangle, |N_{\alpha_2} (p_2, s_2)\rangle \},$$

(C7)

we obtain the potential energy in each channel:
\[
\Delta E_{P^N, P^N} = \int d^3x \int d^3y \left( \mathcal{D}_{\beta i}^\dagger (p_1, \lambda_1) \right) \left( -\varepsilon_{ikl} P_{\beta k}^\dagger (x) \langle \tau_\alpha \rangle_{\beta \alpha} P_{\alpha l} (x) \right) \mathcal{D}_{\alpha i} (p_1, \lambda_1) \bar{\nu}_{\pi j}^{HN} (x, y) \\
\times \langle N_{\beta_2} (p_2', s_2') | \bar{\psi} \beta \gamma \gamma_5 \bar{\psi} \alpha | N_{\alpha_2} (p_2, s_2) \rangle,
\]

\[
\Delta E_{P^N, PN} = \int d^3x \int d^3y \left( \mathcal{D}_{\beta i}^\dagger (p_1', \lambda_1') \right) \left( i P_{\beta i} (x) \langle \tau_\alpha \rangle_{\beta \alpha} P_{\alpha l} (x) \right) \mathcal{D}_{\alpha i} (p_1, \lambda_1) \bar{\nu}_{\pi j}^{HN} (x, y) \\
\times \langle N_{\beta_2} (p_2', s_2') | \bar{\psi} \beta \gamma \gamma_5 \bar{\psi} \alpha | N_{\alpha_2} (p_2, s_2) \rangle,
\]

\[
\Delta E_{PN, P^N} = \int d^3x \int d^3y \left( \mathcal{D}_{\beta i}^\dagger (p_1') \right) \left( i P_{\beta i} (x) \langle \tau_\alpha \rangle_{\beta \alpha} P_{\alpha l}^\dagger (x) \right) \mathcal{D}_{\alpha i} (p_1, \lambda_1) \bar{\nu}_{\pi j}^{HN} (x, y) \\
\times \langle N_{\beta_2} (p_2', s_2') | \bar{\psi} \beta \gamma \gamma_5 \bar{\psi} \alpha | N_{\alpha_2} (p_2, s_2) \rangle.
\]

Here \( p_2 \) and \( p_2' \) are the three-dimensional momenta of the nucleon, \( s_2, s_2' = \pm 1/2 \) are the spin components, and \( \alpha_2, \beta_2 = \pm 1/2 \) are the isospin components.

In order to calculate the matrix elements, we expand \( P^\dagger (x) \) and \( P_a (x) \) by plane waves. This is obtained by considering multiplying the mass scale \( M \) to Eq. (B7) and taking the large \( M \) limit. The results are

\[
P^\dagger (x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2}} \left[ a_{p\alpha} e^{ip \cdot x} + b_{p\alpha}^\dagger e^{-ip \cdot x} \right],
\]

\[
P_a (x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2}} \left[ a_{p\alpha} e^{ip \cdot x} + b_{p\alpha}^\dagger e^{-ip \cdot x} \right],
\]

where the factor \( 1/\sqrt{2} \) stems from \( 1/\sqrt{2E_p} \) in the conventional representation multiplied by the factor \( \sqrt{M} \) and taking the large \( M \) limit. Here \( a_{p\alpha}^\dagger \) and \( b_{p\alpha}^\dagger \) \( (a_{p\alpha} \) and \( b_{p\alpha} \) satisfy the commutation relations, \( [a_{p\alpha}, a_{p'\alpha}^\dagger] = [b_{p\alpha}, b_{p'\alpha}^\dagger] = (2\pi)^3 \delta^{ij} \delta^{(3)} (p - p') \) and \( [a_{p\alpha}, b_{p\alpha}^\dagger] = [b_{p\alpha}, b_{p'\alpha}^\dagger] = (2\pi)^3 \delta^{ij} \delta^{(3)} (p - p') \). Let us consider the large \( M \) limit and leave only the leading terms of \( M \). Because the particle states \( | P^\dagger (p, \lambda) \rangle \) and \( | P_a (p) \rangle \) are defined by

\[
| P^\dagger (p, \lambda) \rangle \equiv \sqrt{2} \epsilon_\lambda \langle a_{p\alpha}^\dagger | 0 \rangle, \quad (C13)
\]

\[
| P_a (p) \rangle \equiv \sqrt{2} \langle a_{p\alpha} | 0 \rangle, \quad (C14)
\]

which indicate that the states \( | P^\dagger (p, \lambda) \rangle \) and \( | P_a (p) \rangle \) include \( 2V \) particles in the volume \( V \). The polarization vectors for the \( P^\dagger \) meson are given by Eq. (19). We also use \( T_i (i = 1, 2, 3) \) in Eq. (20). As for the nucleon part, we consider the expansion for \( \psi (x) \) given by

\[
\psi (x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}}
\]

\[
V_{P^N, P^N} (x, y) = - \frac{1}{2} g_{\pi NN} g_{\pi} \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{p^2 + m^2} e^{-ip \cdot (x - y)} \langle \tau_\alpha \rangle_{\lambda_1 \lambda_2} \delta_{\lambda_1}, \delta_{\lambda_2}, \delta_{\alpha_1}, \delta_{\alpha_2}.
\]

Here, \( p_2 \) and \( p_2' \) are the three-dimensional momenta of the nucleon, \( s_2, s_2' = \pm 1/2 \) are the spin components, and \( \alpha_2, \beta_2 = \pm 1/2 \) are the isospin components.

In order to calculate the matrix elements, we expand \( P^\dagger (x) \) and \( P_a (x) \) by plane waves. This is obtained by considering multiplying the mass scale \( M \) to Eq. (B7) and taking the large \( M \) limit. The results are

\[
P^\dagger (x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2}} \left[ a_{p\alpha} e^{ip \cdot x} + b_{p\alpha}^\dagger e^{-ip \cdot x} \right],
\]

\[
P_a (x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2}} \left[ a_{p\alpha} e^{ip \cdot x} + b_{p\alpha}^\dagger e^{-ip \cdot x} \right],
\]

where the factor \( 1/\sqrt{2} \) stems from \( 1/\sqrt{2E_p} \) in the conventional representation multiplied by the factor \( \sqrt{M} \) and taking the large \( M \) limit. Here \( a_{p\alpha}^\dagger \) and \( b_{p\alpha}^\dagger \) \( (a_{p\alpha} \) and \( b_{p\alpha} \) satisfy the commutation relations, \( [a_{p\alpha}, a_{p'\alpha}^\dagger] = [b_{p\alpha}, b_{p'\alpha}^\dagger] = (2\pi)^3 \delta^{ij} \delta^{(3)} (p - p') \) and \( [a_{p\alpha}, b_{p\alpha}^\dagger] = [b_{p\alpha}, b_{p'\alpha}^\dagger] = (2\pi)^3 \delta^{ij} \delta^{(3)} (p - p') \). Let us consider the large \( M \) limit and leave only the leading terms of \( M \). Because the particle states \( | P^\dagger (p, \lambda) \rangle \) and \( | P_a (p) \rangle \) are defined by

\[
| P^\dagger (p, \lambda) \rangle \equiv \sqrt{2} \epsilon_\lambda \langle a_{p\alpha}^\dagger | 0 \rangle, \quad (C13)
\]

\[
| P_a (p) \rangle \equiv \sqrt{2} \langle a_{p\alpha} | 0 \rangle, \quad (C14)
\]

which indicate that the states \( | P^\dagger (p, \lambda) \rangle \) and \( | P_a (p) \rangle \) include \( 2V \) particles in the volume \( V \). The polarization vectors for the \( P^\dagger \) meson are given by Eq. (19). We also use \( T_i (i = 1, 2, 3) \) in Eq. (20). As for the nucleon part, we consider the expansion for \( \psi (x) \) given by

\[
\psi (x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}}
\]

\[
V_{P^N, P^N} (x, y) = \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{p^2 + m^2} e^{-ip \cdot (x - y)} \langle \tau_\alpha \rangle_{\lambda_1 \lambda_2} \delta_{\lambda_1}, \delta_{\lambda_2}, \delta_{\alpha_1}, \delta_{\alpha_2}.
\]
\[ V^{P^+N-PN}(x, y) = \frac{1}{2} \int \frac{d^3 p}{2mN} \frac{p_ip_j}{p^2} e^{-ip \cdot (x-y)} \[ (\sigma_j s_x s_y (\tau_a) \beta_1 \alpha_1 (\tau_a) \beta_2 \alpha_2, \] \] 
\[ V^{PN-P^+N}(x, y) = \frac{1}{2} \int \frac{d^3 p}{2mN} \frac{p_ip_j}{p^2} e^{-ip \cdot (x-y)} \[ (\sigma_j s_x s_y (\tau_a) \beta_1 \alpha_1 (\tau_a) \beta_2 \alpha_2, \] \] 
\]
to be transformed to Eqs. [10], [11], and [12] in the end. Notice that the factor 1/2 in the coefficients have been missed in the previous studies by the authors [25–27]. The calculation of the momentum integrations is easily performed by introducing the form factor (18) in the integrands. In the integrals, it is useful to adopt the formula of the plane-wave expansion
\[ e^{-ip \cdot r} = 4\pi \sum_{l=0, 1, 2, \ldots} (-i)^l j_l(pr) Y_{l_\ell}^*(\hat{p}) Y_{l_\ell}(\hat{r}), \] 
(C23)
with \( l = 0, 1, 2, \ldots \) and \( l_z = -l, -l + 1, \ldots, l - 1, l \). Here \( j_l(x) \) is the spherical Bessel function and \( Y_{l_\ell}(\hat{x}) \) is the spherical harmonic function. As a result, we obtain the explicit forms of the central potential \( C(r; m) \) and the tensor potential \( T(r; m) \) in Eqs. [14] and [15], respectively. In the calculation of the tensor potential, we have used the relationship
\[ a_b S_\ell(j)(\hat{p}) = \sqrt{\frac{24\pi}{5}} \sum_{\mu=-2}^{2} (-1)^\mu (a \times b)^{(2)}_{\mu} Y_{2\mu}(\hat{p}), \] 
(C24)
where \( (a \times b)^{(2)}_{\mu} \) is the rank-2 tensor composed of \( a = (a_1, a_2, a_3) \) and \( b = (b_1, b_2, b_3) \).
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