Introduction to Open Superstring Field Theory

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Abstract. We review briefly open superstring theory, and then provide a very short introduction to some basic aspects of open superstring field theory and comment some applications.

1. Introduction
The first quantized approach of open superstring theory has gauge group $SO(32)$ with critical space-time dimension $D = 10$. This theory is space-time supersymmetric, tachyon free, and describes all known massless bosonic and fermionic particles. Superstring theory has great properties to build better phenomenological models in high energy physics than string theory.

Since our observable physical world is four dimensional we have to consider some compactification of the six extra internal coordinates. The principles that control this compactification is not completely understood yet. In order to make progress in the unification we need to develop non perturbative methods and go beyond the first quantized description of strings. By implementing supersymmetry in string theory we obtain better convergence of scattering amplitudes and vacuum. The purpose of this article is to provide a short introduction to superstring field theory.

2. Open Superstring
Supersymmetry is a beautiful and useful extension of Poincare algebra that relates fermions and bosons. Supersymmetry makes strong restrictions when we applied it to fundamental strings. In this paper we only discuss descriptions of superstring that have supersymmetry manifest in the world sheet because they are simple to quantize. A great textbook to learn more details about superstrings is [1].

2.1. Ramond Neveu Schwarz Superstring
To the bosonic worldsheet coordinates $X^\mu(\tau, \sigma)$ we incorporate the fermionic coordinates $\psi^\mu(\tau, \sigma)$, where now $\mu = 0, 1, \ldots, 9$. The theory is consistent at quantum level only for $D = 10$ space-time dimensions. Using complex coordinates in the worldsheet by $z = (\sigma + i\tau)/2$, $\bar{z} = (\sigma - i\tau)/2$, the RNS action is

$$S_{RNS}[X, \psi] = \int d^2 z (\partial X^\mu \bar{\partial} \bar{X}_\mu + \psi^\mu \bar{\partial} \psi_\mu + \bar{\psi}^\mu \partial \bar{\psi}_\mu)$$ (1)

In superstring theory we have two sectors. The first one is the Neveu-Schwarz sector that corresponds to antiperiodic boundary conditions in $\psi$ and contains the space-time bosonic states.
The second one is the **Ramond** sector that corresponds to periodic boundary conditions in \( \psi \) and contains the space-time bosonic states.

### 2.2. Symmetries

Classically, the RNS action has manifest space-time Poincare symmetry and \( N = 1 \) world-sheet superconformal symmetry. In RNS superstring the space-time supersymmetry is not manifest, but it this symmetry can be restored at after an appropriate truncation of the quantum spectrum. In this paper we are not consider the Green-Schwarz description that has manifest space-time supersymmetry because its action is nonlinear and hard to quantize.

### 2.3. Quantization

From the point of view of the world-sheet the variables \( X^\mu \) and \( \psi^\mu \) are two dimensional fields with an infinite dimensional superconformal symmetry. In order to quantize such highly constrained system we need both the conformal anti-commuting \((b,c)\) ghost system and the superconformal commuting \((\beta,\gamma)\) superghost system. The generators of this superconformal symmetry correspond to the **Supervirasoro** constraints and satisfy a superalgebra. The matter contribution of these generators are the world sheet energy momentum tensor \( T \) and the world-sheet super-current \( G \). The lowest level of the spectrum is massless and contain the bosonic Yang-Mills gluon and its supersymmetric partner the fermionic gluino.

In 1986, a more convenient superghost variables to build fermionic vertex operators was discovered by Friedan, Martinec, and Shenker [2]. This new system is made of an anti-commuting \((\eta,\xi)\) superghost system and a chiral boson \( \phi \). The relation of these variables with the previous one is \( \beta = e^{-\phi} \partial \xi, \gamma = \eta e^{\phi} \).

The BRST nilpotent operator in terms of the FMS variables is

\[
Q = \int dz [c(T + b \partial c - \frac{1}{2} \partial \phi \partial \phi - \partial^2 \phi - \eta \partial \xi) + \eta e^{\phi} G^m - b \eta \partial \eta e^{2\phi}]
\]

All physical vertex operator \( V \) belongs to the Q cohomology. Furthermore, in order to remove the tachyon from the physical spectrum we have to apply the **GSO projection**. The **Space-time supersymmetry** current \( q^\alpha \) involve bosonized expressions of \( \phi \) and \( \psi^\mu \).

In the first quantized approach we use vertex operators associated to any particular set of superstring states to compute on-shell scattering amplitudes. The prescription to do this computation is essentially perturbative and has been done only for flat Minkowski space-time background.

One subtle aspect in superstring theory is the fact that fermionic vertex operator for Ramond states needs bosonized expresions of \( \phi \) and \( \psi^\mu \). Furthermore for each state of the superstring we have an infinite number vertex operator that are physically equivalent. Each vertex operators is characterized by one picture number. Ramond vertex operators have half integer picture values and Neveu-Schwarz vertex operators have integer picture values. Picture number is defined by \( n_p = \int dz(\xi \eta - \partial \phi) \) and Picture raising operator is defined by \( Z = \{Q, \xi\} \). The Hilbert space of the FMS variables is called the large Hilbert space because it is bigger than the Hilbert space of the \((\beta,\gamma)\) system which do not depend of the zero mode \( \xi_0 \).

### 3. Open Superstring Field

In 1986 Witten proposed a action for interacting open superstring field [3]. This cibic action is Chern-Simons-like \( S_W[V, \Psi] = \langle V | QV \rangle + \langle V | ZVV \rangle + \langle \Psi | YQ \Psi \rangle + \langle V | \Psi \Psi \rangle \), where \( V \) and \( \Psi \) are the NS and R superstring fields. But this action turns out to be inconsistent because it requires the use of picture changing operators that are singular when they coincide. Many years physicist were looking for a consistent action in order to fix this technical problem.
3.2. Symmetries

This action is a generalization of the WZW Wess Zumino Witten Action \( \langle \langle GSO \rangle \rangle \) is considered the prescription is correctly reproduced.

\[ A \] Vertices), (b) Channel-t \( A \) vertex worldsheet toward the complex upper-half plane UHP.

Then we need to expand the action up to quartic order because they all contributes for the four string amplitudes.

\[ \langle \langle \ldots \rangle \rangle \] is correlation in the large Hilbert space, and \( G_0^+ = Q \) and \( \tilde{G}_0^+ = \eta_0 \)

3.1. Berkovits Superstring Field Action

In 1995 Berkovits proposed a WZW-like open superstring field theory action restricted to the Neveu-Schwarz sector \[ 4 \]. This theory is based on the FMS variables an uses a single superstring field \( \Phi[X, \psi, b, c, \xi, \eta, \phi] \) The action is

\[ S_B[\Phi] = \frac{1}{2g^2} \left[ \langle \langle e^{-\Phi} G_0^+ e^\Phi | e^{-\Phi} \tilde{G}_0^+ e^\Phi \rangle \rangle \right] - \int_0^1 dt \langle \langle e^{-t\Phi} \partial_t e^{t\Phi} | \left\{ e^{-t\Phi} G_0^+ e^{t\Phi}, e^{-t\Phi} \tilde{G}_0^+ e^{t\Phi} \right\} \rangle \rangle \]

\[ \langle \langle \ldots \rangle \rangle \] is correlation in the large Hilbert space, and \( G_0^+ = Q \) and \( \tilde{G}_0^+ = \eta_0 \)

The equation of motion is \( \eta_0 \left( e^{-\Phi} Q e^\Phi \right) = 0 \).

The string field \( \Phi \) satisfy the following relations

\[ \langle \langle \Phi_1 \Phi_2 \ldots \Phi_{n-1} \Phi_n \rangle \rangle = \langle \langle \Phi_n \Phi_1 \ldots \Phi_{n-2} \Phi_{n-1} \rangle \rangle \]
\[ \langle \langle \Phi_1 \Phi_2 \ldots \Phi_{n-1} Q \Phi_n \rangle \rangle = -\langle \langle Q \Phi_n \Phi_1 \ldots \Phi_{n-2} \Phi_{n-1} \rangle \rangle \]
\[ \langle \langle Q \ldots \rangle \rangle \rangle = 0 \]
\[ Q(\Phi_1 \Phi_2) = (Q \Phi_1) \Phi_2 + \Phi_1 (Q \Phi_2) \]

The quantization of the WZW-like action is currently in development using the BV methods. The gauge fixing procedure of the free action has been developed, but the full interacting action needs to be completed.

4. Applications

We sketch here the computation of tree level on-shell amplitudes of four NS superstrings \[ 5 \].

4.1. Superstring Amplitudes

The linearized gauge transformation is \( \delta \Phi = G_0^+ \Omega + \tilde{G}_0^+ \tilde{\Omega} \). Two gauge fixing conditions \( G_0^+ \Phi = b_0 \Phi = 0 \), \( \tilde{G}_0^+ \Phi = \xi_0 \Phi = 0 \). Using this condition we build the appropriate propagator.

Then we need to expand the action up to quartic order because they all contributes for the four string amplitudes.

\[ S[\Phi] = \langle \langle \frac{1}{2} G_0^+ \Phi \tilde{G}_0^+ \Phi - \frac{1}{6} \Phi \left\{ G_0^+ \Phi, \tilde{G}_0^+ \Phi \right\} - \frac{1}{24} [\Phi, G_0^+ \Phi][\Phi, \tilde{G}_0^+ \Phi] + \ldots \rangle \]

The Giddings conformal transformation is also required to map from the conic-folded Witten vertex worldsheet toward the complex upper-half plane UHP.

The total amplitude has three contributions: (a) Channel-s \( A_s \) (one Propagator + two cubic Vertices), (b) Channel-t \( A_t \) (one Propagator + two cubic Vertices), and (c) Quartic Vertex \( A_q \) (without Propagator) such that \( A_4 = A_s + A_t + A_q \). We verified that the first quantized prescription is correctly reproduced.

The WZW-Like action has generalized to describe non-BPS D-branes. In order to do that it is considered the GSO(-) projected out states of the superstring which includes the tachyon. This allowed to calculate aproximated computation of tachyon condensation \[ 6 \].
5. Conclusions
We introduced some basic ideas about superstring and superstring field theory. The main
mathematical tools are superconformal field theory BRST quantization, gauge symmetry, and
supersymmetry. After a long time dealing with technical problems related with picture changing
operators, the classical superstring field theory action for Neveu Schwarzs has been clearly
identified and it is currently being tested.

We expect that a recent proposed open superstring field theory actions that include the
Ramond sector will give us better understanding of dualities and other non perturbative effects.
And soon we will able to compute better low energy effective actions to super Yang Mills Actions.

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References
[1] Polchinski J 1998 String Theory vol 2 (Cambridge University Press)
[2] Friedan D, Martinec E, and Shenker S 1986 Nucl. Phys. B271 93
[3] Witten E 1986 Nucl. Phys. B276 291
[4] Berkovits N 1985 Nucl. Phys. B450 90
[5] Berkovits N and Tello C 2000 Phys. Lett. B478 343
[6] Berkovits N, Sen A, and Zwiebach B 2000 Nucl. Phys. B587 147