Generative-Discriminative Complementary Learning

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Abstract

Majority of state-of-the-art deep learning methods for vision applications are discriminative approaches, which model the conditional distribution. The success of such approaches heavily depends on high-quality labeled instances, which are not easy to obtain, especially as the number of candidate classes increases. In this paper, we study the complementary learning problem. Unlike ordinary labels, complementary labels are easy to obtain because an annotator only needs to provide a yes/no answer to a randomly chosen candidate class for each instance. We propose a generative-discriminative complementary learning method that estimates the ordinary labels by modeling both the conditional (discriminative) and instance (generative) distributions. Our method, we call Complementary Conditional GAN (CCGAN), improves the accuracy of predicting ordinary labels and is able to generate high-quality instances in spite of weak supervision. In addition to the extensive empirical studies, we also theoretically show that our model can retrieve the true conditional distribution from the complementarily-labeled data.

1. Introduction

Deep supervised learning has achieved great success in various vision applications such as object recognition and detection [33, 11], semantic segmentation [7, 6], 3D geometry [4], etc. Despite the effectiveness of supervised classifiers, acquiring labeled data is often expensive and time-consuming. As a result, learning from weak supervision has been studied extensively in recent decades, including but not limited to semi-supervised learning [19, 8], positive-unlabeled learning [21], multi-instance learning [14], learning from side information [13], and learning from data with noisy labels [28, 17].

In this paper, we consider a recently proposed weakly supervised classification scenario, i.e., learning from complementary labels [15, 35]. Unlike an ordinary label, a complementary label specifies a class that an object does not belong to. Given an image from a class, it is laborious to choose the correct class label from many candidate classes, especially when the number of classes is relatively large or the annotator is not familiar with the visual appearance of all candidate classes. However, it is less demanding and inexpensive to choose one of the incorrect class as a complementary label for an image. For example, when an annotator is labelling an image containing an animal that she has never seen before, she can easily identify that this animal does not belong the usual animal classes she can see in daily life, such as, “not dogs”.

Existing complementary learning methods modified the ordinary classification loss functions to enable learning from complementary labels. Ishida et al. [15] proposed a method that provides consistent estimate of the classifier from complementarily-labeled data where the loss function satisfies a certain symmetric condition. However, this method only allows classification loss functions with certain non-convex binary losses for one-versus-all and pairwise comparison. Later, Yu et al. [35] proposed to use the forward loss correction technique [31, 34] that learns the conditional, \( P_{Y|X} \), from complementary labels, where \( X \) denote the features and \( Y \) denote labels. This method can be applied to cross-entropy loss, which is commonly used for training deep neural networks.

To clarify the differences between learning with ordinary and complementary labels, we define the notion of “effective sample size”, which is the number of instances with ordinary labels that carries the same amount of information as instances with complementary labels of given size. Since the complementary labels are weak labels, they carry only partial information about the ordinary labels. Hence, the effective sample size \( n_t \) for complementary learning is much smaller than the given sample size \( n \) (i.e., \( n_t << n \)). Current methods for learning with complementary labels need a relatively large training set to ensure low vari-
ance for predicting ordinary label.

Although $n_i$ is small under complementary learning settings, the instance distribution $P_X$ can be estimated from the all samples with size $n$. However, current complementary methods focus on modeling conditional $P_{Y|X}$ and thus fail to account for information hidden in $P_X$ which is essential in complementary learning.

To improve the prediction performance, we propose a generative-discriminative complementary learning approach that learns $P_{Y|X}$ and $P_{X|Y}$ in a unified framework. Our main contributions can be summarized as follows:

- We propose a Complementary Conditional Generative Adversarial Net (CCGAN) which can simultaneously learn $P_{Y|X}$ and $P_{X|Y}$ from complementary labels. Because the estimate of $P_{X|Y}$ benefits from $P_X$, it provides constraints on $P_{Y|X}$ and helps reduce its estimation variance.

- Theoretically, we show that the proposed CCGAN model is guaranteed to learn $P_{X|Y}$ from complementarily-labeled data.

- Empirically, we conduct comprehensive experiments on benchmark datasets, including MNIST, CIFAR, and VGG Face; demonstrating that our model is able to improve the classification accuracy while generating high-quality images.

2. Related Works

Complementary Learning Currently, there is very limited work on complementary learning. To the best our knowledge, learning with complementary labels was first proposed by Ishida et al. [15]. They assumed that the complementary labels are uniformly selected and they modified the one-vs-all and pairwise comparison classification losses to provide an unbiased estimation for the true classification risk. Yu et al. [35] considered biased complementary label selection, and provided a forward loss correction approach that can be applied to cross-entropy loss. Because two these methods only work for specific losses such as the one-vs-all loss or cross-entropy loss, Ishida et al. [16] derived an unbiased risk estimator for arbitrary losses and models. All the existing three methods are discriminative methods that ignore the information contained in $P_X$. Unlike these methods, our proposed CCGAN takes advantage of $P_X$ to help infer $P_{Y|X}$.

Generative Adversarial Nets Generative Adversarial Nets (GANs) are a class of implicit generative models learned by adversarial training [9]. With the development of new network architectures (e.g., [32]) and stabilizing techniques (e.g., [1] [10] [20]), GANs is able to generate high-quality images that are indistinguishable from real images. Conditional GANs (CGANs) [25] extend the GAN models to generate images given specific labels, which can be used to model the class conditional $P_{X|Y}$ (e.g., ACGAN [30] and Projection GAN[27]). However, training of CGANs requires ordinary labels for the images, which are not available under the complementary learning settings. To the best of our knowledge, our proposed CCGAN is the first conditional GAN that can be trained with complementary labels.

Semi-Supervised Learning Under semi-supervised learning settings, we are provided a relatively small number of labeled data and plenty of unlabeled data. The basic assumption for the semi-supervised methods is that the knowledge on $P_X$ gained from unlabeled data carries information that is useful in inferring $P_{Y|X}$. This principle has been implemented in various forms, such as co-training [3], manifold regularization [2], generative modeling [20] [29] [23], etc. Inspired by the commonalities between complementary learning and semi-supervised learning, i.e., incomplete supervision information; we propose to make use of $P_X$ to help infer $P_{Y|X}$ in complementary learning.

3. Background

In this section, we first introduce the concept of learning from so-called complementary labels. Then, we discuss a state-of-the-art discriminative complementary learning approach, [35], which is the most relevant to our method.

3.1. Problem Setup

Let two random variables $X$ and $Y$ denote the features and the label, respectively. The goal of discriminative learning is to infer a decision function (classifier) from independent and identically distributed training set \{$(x_i, y_i) \}_{i=1}^{n} \subseteq \mathcal{X} \times \mathcal{Y}$ drawn from an unknown joint distribution $P_{XY}$, where $X \in \mathcal{X} = \mathbb{R}^d$ and $Y \in \mathcal{Y} = \{1, \ldots, K\}$. The optimal function, $f^*$, can be learned by minimizing the expected risk $R(f) = \mathbb{E}_{(X,Y) \sim P_{XY}} \ell(f(X), Y)$, where $\mathbb{E}$ denotes the expectation and $\ell$ denotes a classification loss function. Because $P_{XY}$ is unknown, we usually approximate $R(f)$ using its empirical estimation $R_n(f) = \frac{1}{n} \sum_{i=1}^{n} \ell(f(x_i), y_i)$.

In the complementary learning setting, for each sample $x$, we are given only a complementary label $\bar{y} \in \mathcal{Y} \setminus \{y\}$ which specifies a class that $x$ does not belong to. That is to say, our goal is to learn $f$ that minimizes the classification risk $R(f)$ from complementarily-labeled data \{$(x_i, \bar{y}_i) \}_{i=1}^{n} \subseteq \mathcal{X} \times \mathcal{Y}$ drawn from an unknown distribution $P_{\bar{XY}}$, where $\bar{Y}$ denote the random variable for complementary label. The ordinary loss function, $\ell(\cdot, \cdot)$, cannot be used since we do not have access to the ordinary labels ($y_i$’s). In the following, we explain how discriminative learning can be extended in such scenarios.
3.2. Discriminative Complementary Learning

Existing Discriminative Complementary Learning (DCL) methods modified the ordinary classification loss function $\ell$ to the complementary classification loss $\ell'$ to provide a consistent estimation of $f$. Various loss functions have been considered in the literature, such as one-vs-all ramp/sigmoid loss [15], pair-comparison ramp/sigmoid loss [15], and cross-entropy loss [35]. Here we briefly review a recent method that modifies the cross-entropy loss for deep learning with complementary labels [35].

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$\ell(f(X), Y) = |P(Y = y) - P_y|$, where $g = (g_1(X), \ldots, g_K(X))^\top$ and $M$ is the transition matrix satisfying

$$
P(\hat{Y} = j|X) = \sum_{i \neq j} p(\hat{Y} = j|Y = i)P(Y = i|X). \quad (1)$$

If the complementary labels are assigned uniformly, then $M_{i,j} = 1/(K - 1), i \neq j; M_{i,i} = 0, i = j$. It has been shown that the classifier $\bar{f}_n$ that minimizes the empirical estimation of $\bar{R}(f)$, i.e.,

$$
\bar{R}_n(f) = \frac{1}{n} \sum_{i=1}^n \bar{\ell}(f(x_i), y_i), \quad (2)
$$

converges to the optimal classifier $f^*$ as $n \to \infty$ [35].

4. Proposed Method

In this section, we will present the motivation and details of our generative-discriminative complementary learning method. First, we demonstrate why generative modeling is valuable for learning from complementary labels. Second, we present our Complementary Conditional GAN (CCGAN) model that can be learned from complementarily-labeled data and provide theoretical guarantees. Finally, we discuss several practical factors that are crucial for reliably training our model.

4.1. Motivation

It is guaranteed that existing discriminative complementary learning approaches lead to optimal classifiers, given sufficiently large sample size. However, due to the uncertainty introduced by the complementary labels, the effective sample size is much smaller than the sample size $n$. If we have access to samples with ordinary labels $\{x_i, y_i\}_{i=1}^n$, we can learn the classifier $f_n$ by minimizing $R_n(f)$. Since knowing the ordinary labels is equivalent to having all the $K - 1$ complementary labels, we can also learn $f_n$ with ordinary labels by minimizing the empirical risk

$$
\bar{R}_n(f) = \frac{1}{n(K - 1)} \sum_{i=1}^n \sum_{k=1}^{K-1} \bar{\ell}(f(x_i), y_{ik}), \quad (3)
$$

where $y_{ik}$ is the $k$-th complementary label for the $i$-th example. In practice, since we only have one complementary label for each instance, we are minimizing $\bar{R}_n(f)$ as shown in Eq. (2), rather than $\bar{R}_n(f)$. It can be seen that $\bar{R}_n(f)$ approximates $\bar{R}_n(f)$ by randomly picking up one complementary label for the $i$-th example, which implies that the effective sample size is roughly $n/(K - 1)$. That is to say, although we provide each instance a complementary label, the accuracy of the classifier learned by minimizing $\bar{R}_n$ is close to that of a classifier learned with $n/(K - 1)$ examples with ordinary labels.

Because the effective sample size could be much smaller than the actual sample size, complementary learning resembles semi-supervised learning, where only a small proportion of instances have ordinary labels and the rest are unlabeled. In semi-supervised learning, $P_X$ can be estimated with more unlabeled samples compared to $P_{Y|X}$, which requires labels to estimate. Therefore, modeling $P_X$ is beneficial because it allows us to take advantage of unlabeled data. This justifies having a generative term for complementary learning. A natural way to utilize $P_X$ is to model the class-discriminative, $P_{X|Y}$. $P_X$ imposes a constraint on $P_{X|Y}$ indirectly since $P_X = \int P(X|Y = y)P(y)dy$. Therefore, a more accurate estimation of $P_X$ will improve the estimation of $P_{X|Y}$ and thus $P_{Y|X}$.

4.2. Complementary Conditional GAN (CCGAN)

Given the recent advances in generative modeling using (conditional) GANs, we propose to use conditional GAN to model $P_{X|Y}$ in the paper. A conditional GAN learns a function $G(Y, Z)$ that generates samples from a conditional distribution $Q_{X|Y}$, neural network is used to parameterize the generator function, and $Z$ is a random sample drawn from a canonical distribution $P_Z$. To learn the parameters, we can minimize certain divergence between $Q_{X,Y}$ and $P_{X,Y}$ by solving the following optimization:

$$
\min_{G} \max_{D, (X,Y) \sim P_{X,Y}} \mathbb{E}_{(X,Y) \sim P_{X,Y}} [\phi(D(X,Y))] + \mathbb{E}_{Z \sim P_Z, Y \sim P_Y} [\phi(1 - D(G(Z,Y), Y))], \quad (4)
$$

where $\phi$ is a function of choice and $D$ is the discriminator.

However, the conditional GAN framework cannot be directly used for our purpose for the following two reasons: 1) the first term in Eq. (4) cannot be evaluated directly, because we do not have access to the ordinary labels. 2) the conditional GAN only generates $X$’s and does not infer the ordinary labels. A straightforward solution would be to generate $(x, y)$ from the learned conditional GAN model and
to train a separate classifier on the generated data. We will show that such two-step solution results in a sub-optimal performance.

To enable generative-discriminative complementary learning, we propose a complementary conditional GAN (CCGAN) by extending the ACGAN framework to deal with complementarily-labeled data. The model structures of GAN, ACGAN, and our CCGAN are shown in Figure 1. ACGAN decomposes the joint distributions as $P_{XY} = P_{Y|X}P_{X}$ and $Q_{XY} = Q_{Y|X}Q_{X}$ and match the conditional distributions and marginal distributions separately. The marginals $P_{X}$ and $Q_{X}$ are matched using adversarial loss $\ell$, and $P_{Y|X}$ and $Q_{Y|X}$ are matched by sharing a classifier with probabilistic outputs. However, $P_{Y|X}$ is not accessible in a complementary setting since the ordinary labels are not observed. Therefore, $P_{Y|X}$ and $Q_{Y|X}$ cannot be directly matched as in ACGAN. Fortunately, we make use of the relation between $P_{Y|X}$ and $P_{Y|X}$ (Eq. (1)) and propose a new loss matching $P_{Y|X}$ and $Q_{Y|X}$ in a complementary setting. Specifically, we learn our CCGAN using the following objective

$$
\min_G \max_D \min_C \mathbb{E}_X \phi(D(X)) + \mathbb{E}_{Z \sim P_{Z,Y \sim P_Y}} \phi(1 - D(G(Z,Y))) + \mathbb{E}_{(X,Y) \sim P_{XY}} \ell(\hat{Y}, \hat{C}(X)) + \mathbb{E}_{(X,Y) \sim Q_{XY}} \ell(Y, C(G(Z,Y)))
$$

From the objective function, we can see that our method naturally combines generative and discriminative components in a unified framework. Specifically, the component 6 performs pure discriminative complementary learning on the complementarily-labeled data (only learns $C$), and the components 4 and 5 perform generative and discriminative learning simultaneously (learn both $G$ and $C$).

The three components in Eq. (5) correspond to the following three divergences: 1) Component 4 corresponds to Jensen-Shannon divergence between $P_{X}$ and $Q_{X}$. 2) Component 6 represents KL divergence between $Q'_{Y|X}$ and $P_{Y|X}$, and 3) Component 5 corresponds to KL divergence between $Q_{Y|X}$ and $Q'_{Y|X}$, where $Q'_{Y|X}$ is a conditional distribution of ordinary labels given features modeled by $C$ and $Q'_{Y|X}$ is a conditional distribution of complementary labels given features implied by $Q'_{Y|X}$ through the relation $Q'_{Y|X} = MQ_{Y|X}$. The following theorem demonstrates that minimizing these three divergences in our objective can effectively reduce the divergence between $Q_{Y|X}$ and $P_{Y|X}$.

**Theorem 1** Let $P_{Y|X}$ and $Q_{Y|X}$ denote the data distribution and the distribution implied by our model, respectively. Let $Q'_{Y|X}$ denote the conditional distribution of ordinary (complementary) labels given features induced by the parametric model $C$. If $M$ is full rank, we have

$$
d_{TV}(P_{XY}, Q_{XY}) \leq 2c_1 \sqrt{d_{JS}(P_{X}, Q_{X})} + c_2 \|M^{-1}\|_{\infty} \sqrt{d_{KL}(P_{Y|X}, Q'_{Y|X})} + c_3 \sqrt{d_{KL}(Q'_{Y|X}, Q_{Y|X})},
$$

where $d_{TV}$ is the total variation distance, $d_{JS}$ is the Jensen-
Shannon divergence, $d_{KL}$ is the KL divergence, and $c_1$ and $c_2$ are two constants.

A proof of Theorem[1] is provided in Section S1 of the supplementary file. An illustrative figure that shows the relations between the quantities in Theorem[1] is also provided in Section S2 of the supplementary file.

4.3. Practical Considerations

Estimating Prior $P_Y$ In our CCGAN model, we need to sample the ordinary labels $y$ from the prior distribution $P_Y$, which needs to be estimated from complementary labels. Let $P_Y = [P_Y(Y = 1),\ldots,P_Y(Y = K)]^\top$ be the vector containing complementary label probabilities and $\bar{P}_Y = [P_Y(Y = 1),\ldots,P_Y(Y = K)]^\top$ be true label probabilities. We estimate $\bar{P}_Y$ by solving the following optimization:

$$\min_{\bar{P}_Y} \|\bar{P}_Y - M^\top \bar{P}_Y\|^2,$$

s.t. $\|\bar{P}_Y\|_1 = 1$ and $\bar{P}_Y[i] \geq 0$. (7)

This is a standard quadratic programming (QP) problem and can be easily solved using a QP solver.

Estimating $M$ If the annotator is allowed to choose to assign either an ordinary label or a complementary label for each instance, the matrix $M$ will be unknown because of the possible non-uniform selection of the complementary labels. In [35], the authors provided an anchor-based method to estimate $M$. In our paper, we estimate $M$ by optimizing it together with other parameters in Eq. (5) in an end-to-end fashion.

Incorporating Unlabeled Data In practice, we may have access to additional unlabeled data. We can readily incorporate such unlabeled data to improve the estimation of the first term in Eq. (5), which further improves the learning of $G$ through the second term in Eq. (5) and eventually improves the classification performance.

5. Experiments

To demonstrate the effectiveness of our method, we present a number of experiments examining different aspects of our method. After introducing the implementation details, we evaluate our methods on three datasets, including MNIST [24], CIFAR10 [22], and VGGFACE2 [5]. We compare classification accuracy of our CCGAN with the state-of-the-art Discriminative Complementary Learning (DCL) method [35] and show the capability of CCGAN to generate good quality class-conditioned images from complementarily-labeled data. In addition, ablation studies based on MNIST are presented to give a more detailed analysis for our method.

5.1. Implementation Details

Complementary Label Generation All the three datasets have ordinary class labels, which allows to generate complementary labels to evaluate our method. Following the procedure in [15], the complementary label for each image was obtained by randomly picking a candidate class and asking the labeler to answer “yes” or “no” questions. In this case, The candidate classes are uniformly assigned to each image, and therefore the transition matrix $M$ satisfies $M_{i,j} = 1/(K-1), i \neq j; M_{i,i} = 0, i = j$. Theoretically, $n/K$ images will have ordinary labels and $n(K-1)/K$ images will have complementary labels. In the ablation study, we will also consider the biased complementary label generation procedure in [35] and evaluate our the performance of our method when $M$ is unknown.

Training Details We implemented our CCGAN model in Pytorch. We trained our CCGAN model in a two-stage manner, including a warming up stage and an end-to-end iterative update stage. In the warming up stage, we first optimized component $\bar{G}$ in Eq. (5) w.r.t. $C$ for 40 epochs using stochastic gradient descent (SGD) with batch size 128, weight decay $5 \epsilon - 4$, momentum $\gamma = 0.9$, and learning rate $1 \epsilon - 2$. In the second stage, we optimized the whole objective using Adam [13] with learning rate $2 \epsilon - 4$, $\beta_1 = 0.5$, $\beta_2 = 0.999$. For data augmentation, we employed random cropping of size $28 \times 28$ for MNIST and CIFAR10 and $60 \times 60$ for VGGFACE2.

5.2. MNIST

We first evaluate our model on MNIST, which is a handwritten digit recognition dataset that contains 60K training images and 10K testing images, with size $32 \times 32$. We chose Lenet-5 [24] as the network structure for the DCL method and the $C$ network in our CCGAN. We employed the DCGAN network [32] for the generator $G$ and discriminator $D$ of our CCGAN. Due to the simplicity of MNIST data, the accuracy of complementary learning is close to that of learning with ordinary labels if we use all 60K training samples. Therefore, we sample a subset of 6K images as our basic sampling set $S$ for training.

In the experiments, we evaluate all the methods under different sample sizes. Specifically, we randomly re-sampled subsets with $r_l \times 6K$ samples, where $r_l = 0.1, 0.2, \ldots, 1$; and trained all the methods on these subsets. The classification accuracy was evaluated on the 10k test set. We report the results under the following three settings: 1) We only use samples with complementary labels, ignoring all ordinary labels. In this case, the proportion of images with complementary labels is $r_c = 1.0$. 2) We use samples with either complementary labels or ordinary labels ($r_c = 0.9$). 3) And we also train ordinary classifier such that all labeled data are provided with ordinary labels.
When we use complementary labels v.s. the number of all labelled samples (with either complementary or ordinary labels). When we use complementary labels only, we have \( r_c = 1.0 \). In this figure we test DCL and our proposed model CCGAN in two cases, where \( r_c = 1.0 \) and \( r_c = 0.9 \). We also show the performance of ordinary classifier trained on ordinary labeled data.

\( r_c = 0.0 \). This classifier is trained with the strongest supervision possible, representing the best achievable classification performance. The results are shown in Figure 2 (a).

It can be seen from the results that our CCGAN method outperforms DCL under different sample sizes, and the gap increases as the sample size reduces. Even when 10% data are labeled with ordinary labels, our method still outperforms DCL by a large margin. The results demonstrate that generative-discriminative modeling is advantageous over discriminative modeling for complementary learning. Figure 3 (a) shows the generated images from ACGAN and our CCGAN. We can see that our CCGAN generates high-quality digit images, suggesting that CCGAN is able to learn \( P_{X|Y} \) very well from complementarily-labeled data.

| Method   | \( r_l \) | 0.1 | 0.3 | 0.5 | 0.7 | 1.0 |
|----------|----------|-----|-----|-----|-----|-----|
|          |          |     |     |     |     |     |
| Ordinary label |        | 0.394 | 0.698 | 0.814 | 0.863 | 0.910 |
| DCL, \( r_c = 1.0 \) |        | 0.108 | 0.459 | 0.677 | 0.75 | 0.830 |
| CCGAN, \( r_c = 1.0 \) |        | **0.150** | **0.509** | **0.683** | **0.769** | **0.833** |

Table 1. This table shows the test accuracy on VGGFACE100 dataset. \( r_l \) denotes the proportion of sampled labeled data for training from the training set \( S \). \( r_c \) denotes the number of samples with complementary labels v.s. the number of all labelled samples (with either complementary or ordinary labels). \( r_c = 1.0 \) indicates that we use complementary labels only.

### 5.3. CIFAR10

We then evaluate our method on the CIFAR10 dataset, which consists of 10 classes of \( 32 \times 32 \) RGB images, including 60K training samples and 10K test samples. We deploy ResNet18 \([12]\) as the structure of the \( C \) network in our model. Since training GANs on the CIFAR10 dataset is unstable, we utilize the latest conditional structure self-attention GANs (SAGAN) \([36]\) for our \( G \) and \( D \) networks. We remove the first two down-sampling layers of SAGAN to adapt to the image size.

We evaluate the methods following the same procedure used in the MNIST dataset. The results are shown in Figure 2 (b). Again our method consistently outperforms the DCL method for different sample sizes. Figure 3 (b) shows the generated images from ACGAN and our CCGAN. It can be seen that our CCGAN successfully learns the appearance of each class from complementary labels.

### 5.4. VGGFACE100

We finally evaluate our method on VGGFACE2 data \([5]\), which is a large-scale face recognition dataset. The face images have large variations in pose, age, illumination, ethnicity, and profession. The number of images for each person (class) varies from 87 to 843, with an average of 362 images for each person. We randomly sampled 100 classes and construct a dataset for evaluation of our method. We selected 80% data as training set \( S \) and the rest 20% as the testing set. Since our CCGAN model can only generate fixed-size images, we re-scaled all training images to the size of \( 64 \times 64 \). Accordingly, we changed the classifier structure to ResNet34 \([12]\) and we removed the top down-sampling layer from SAGAN to adapt to the image size.

Because the number of classes is relatively large, the effective labeled sample size is approximately \( n/99 \), where \( n \) is the total sample size. In case of limited supervision, neither DCL nor our CCGAN can converge. Thus, we ap-
Figure 3. Synthetic results for (a) MNIST (b) CIFAR10 and (c) VGGFACE100. The figures on the left is the real images from training set and the generated samples are put on the right of the dash line. The figures in the middle illustrate images generated by ACGAN [30], which is trained with ordinary labels. The figure on the right show images generated by CCGAN model, which is trained with only complementary labels.

We used the same evaluation procedures used in MNIST and CIFAR10. The classification accuracy is reported in Table 1. It can be seen that our method outperforms DCL by 5% when the proportion of labeled data is smaller than 0.3 and is slightly better than DCL when the proportion is larger than 0.5. Figure 4 (c) shows the generated images from ACGAN and our CCGAN. We can see that CCGAN generates images that are visually similar to the real images for each person.

5.5. Ablation Study

Here we conduct ablation studies on MNIST to study the details and validate possible extensions of our approach.

Multiple Complementary Labels In this experiment, we give an intuitive strategy to verify the effectiveness of generative modeling for complementary learning. In ordinary supervised learning, discriminative models are usually preferred than generative models because estimating the high-dimensional $P_{X|Y}$ is difficult. To demonstrate the importance of generative modeling in complementary learning, we propose to assign multiple complementary labels to each image and observe how the performance changes with the number of complementary labels. The classification accuracy is shown in Figure 4. We can see that the accuracy of our CCGAN and DCL both increases with the number of complementary labels. When the number of complementary labels per image is large, DCL performs better than
our CCGAN because the supervision information is sufficient. However, in practice, the number of complementary labels for each instance is typically small and is usually one. In this case, the advantage of generative modeling is obvious, as demonstrated by the superior performance of our CCGAN compared to DCL.

**Semi-Supervised Complementary Learning** In practice we might have easier access to unlabeled data which can be incorporated into our model to perform semi-supervised complementary learning. On the MNIST dataset, we used the additional 90% data as unlabeled data to improve the estimation of the first term in our objective Eq. (5). We denote the semi-supervised method as Semi-supervised Complementary Conditional GAN (SCCGAN). The classification accuracy w.r.t. different proportion of labeled data is shown in Figure 5. We can see that SCCGAN further improves the accuracy over CCGAN due to the incorporation of unlabeled data.

**Estimation of M** If the labellers are allowed to freely assign labels, the complementary labels will be selected from the candidate classes with bias. In this case, we need to estimate the transition matrix $M$. To demonstrate the ability of our method to estimate $M$, we randomly generated a $M$ matrix with the probability transition constraints and generated complementary labels according to this $M$ matrix. The estimation of $M$ was achieved by optimizing $M$ together with other parameters in Eq. (5). We applied the same procedure to learn $M$ in the DCL method. The classification accuracy and the $M$ estimation accuracy are shown in Figure 6 (a) and Figure 6 (b), respectively. To be notified, the basic training data sampling size is $S = 60K$ here, because estimating $M$ requires sufficient training amount. Clearly, our method performs better than the DCL method with $M$ estimation, indicating the effectiveness of generative modeling for $M$ estimation.

6. **Conclusion**

We study the limitation of complementary learning as a weakly supervised learning problem, where the effective supervised information is much smaller compared to the sample size. To address this problem, we propose a generative-discriminative model to learn a better data distribution, as a strategy to boost the performance of the classifier. We build a conditional GAN model (CCGAN)
which learns a generative model conditioned on ordinary class labels from complementary labeled data, and unify the generative and discriminative modeling in one framework. Our method shows superior classification performance on several dataset, including MNIST, CIFAR10 and VGGFACE100. Also our model are able to generate high quality synthetic image by training with complementary labeled data. In addition, we give theoretical analysis that our model can converge to true conditional distribution learning from complementarily-labeled data.

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Supplementary: “Generative-Discriminative Complementary Learning”

S1. Proof of Theorem 1

According to the triangle inequality of total variation (TV) distance, we have

\[ d_{TV}(P_{XY}, Q_{XY}) \leq d_{TV}(P_{XY}, P_{Y|X}Q_X) + d_{TV}(P_{Y|X}Q_X, Q_{XY}). \]  

(8)

Using the definition of TV distance, we have

\[ d_{TV}(P_{Y|X}P_X, P_{Y|X}Q_X) = \frac{1}{2} \int |p_{Y|X}(y|x)p_X(x) - p_{Y|X}(y|x)q_X(x)|\mu(x, y) \]

\[ \leq \frac{1}{2} \int |p_{Y|X}(y|x)|\mu(x, y) \int |p_X(x) - q_X(x)|\mu(x) \]

\[ \leq c_1d_{TV}(P_X, Q_X), \]  

(9)

where \( p \) and \( q \) are densities, \( \mu \) is a (\( \sigma \)-finite) measure, \( c_1 \) is a constant, and (a) follows from the Hölder inequality. Similarly, we have

\[ d_{TV}(P_{Y|X}Q_X, Q_{Y|X}Q_X) \leq c_2d_{TV}(P_{Y|X}, Q_{Y|X}), \]  

(10)

where \( c_2 \) is a constant. Combining (1), (2), and (3), we have

\[ d_{TV}(P_{XY}, Q_{XY}) \leq c_1d_{TV}(P_{XY}, P_{X}) + c_2d_{TV}(P_{X}, Q_{Y|X}) \]

\[ \leq c_1d_{TV}(P_{XY}, P_{X}) + c_2d_{TV}(P_{Y|X}Q_X, Q_{XY}) + c_2d_{TV}(Q_{Y|X}Q_X, Q_{XY}). \]

(11)

Since we have no access to \( P_{Y|X} \), we bound \( d_{TV}(P_{Y|X}, Q'_{Y|X}) \) using complementary conditional probabilities as

\[ d_{TV}(P_{Y|X}, Q'_{Y|X}) = \max_{S_1, \ldots, S_K \subseteq X} \sum_{y \in Y} \{P(y|S_y) - Q'(y|S_y)\} \]

\[ = \max_{S_1, \ldots, S_K \subseteq X} \langle 1, P(\cdot|\{S_y\}_{y \in Y}) - Q'(\cdot|\{S_y\}_{y \in Y}) \rangle \]

\[ = \max_{S_1, \ldots, S_K \subseteq X} \langle 1, M^{-1}(P(\cdot|\{S_y\}_{y \in Y}) - Q'(\cdot|\{S_y\}_{y \in Y}) \rangle \]

\[ \leq \|M^{-1}\|_1 \max_{S_1, \ldots, S_K \subseteq X} \|P(\cdot|\{S_y\}_{y \in Y}) - Q'(\cdot|\{S_y\}_{y \in Y})\|_1 \]

\[ = \|M^{-1}\|_1 d_{TV}(P_{Y|X}, Q'_Y|X), \]  

(12)

where \( P(\cdot|\{S_y\}_{y \in Y}) = [P(Y = 1|S_1), \ldots, P(Y = K|S_K)]^T \), \( P(\cdot|\{S_y\}_{y \in Y}) = [P(Y = 1|S_1), \ldots, P(Y = K|S_K)]^T \). (a) follows from \( P(\cdot|\{S_y\}_{y \in Y}) = MP(\cdot|\{S_y\}_{y \in Y}) \), and (b) follows from the fact that \( 1^TAx \leq \|Ax\|_1 \leq \|A\|_1\|x\|_1 \). By combining (4) and (5), we have

\[ d_{TV}(P_{XY}, Q_{XY}) \leq c_1d_{TV}(P_{XY}, P_{X}) + c_2\|M^{-1}\|_1 d_{TV}(P_{Y|X}, Q'_Y|X) + c_2d_{TV}(Q'_Y|X, Q_{Y|X}). \]

(13)

According to the relations between total variation (TV), KL divergence (\( d_{KL} \)), and Jensen-Shannon divergence (\( d_{JS} \)), we can rewrite (6) as

\[ d_{TV}(P_{XY}, Q_{XY}) \leq 2c_1\sqrt{d_{JS}(P_{X}, Q_X)} + c_2\|M^{-1}\|_1 \sqrt{d_{KL}(P_{Y|X}, Q'_Y|X)} + 2\sqrt{d_{KL}(Q'_Y|X), Q_{Y|X})}, \]

(14)

which follows from the Pinsker’s inequality. By replacing \( 2c_1 \) in (7) with a new constant \( c_1 \) (using the same notation for simplicity), we can obtain the inequality in Theorem 1. From the theorem, we can see that if the complementary labels are highly-biased, it may cause \( M \) to be rank-deficient. In this case, our algorithm may not minimize the distance between \( P_{XY} \) and \( Q_{XY} \) efficiently.

S2. Illustration of Our Objective Function (Eq. (5))
Figure 7. Illustration of the divergence terms that are minimized in Eq. (5). $P_{Y|X}$ ($P'_{Y|X}$) is the conditional distribution of ordinal (complementary) label given features on the real data. $Q'_{Y|X}$ ($Q'_{Y'|X}$) is the conditional distribution of ordinal (complementary) label produced by the classification network $C$ in Eq. (5). $Q_{Y|X}$ is the conditional distribution of ordinal label given features induced by our generator $G$. From the figure, we can see that minimizing $\overset{a}{\|}$ leads to reduced divergence between $P_{Y|X}$ and $Q'_{Y|X}$. Therefore, the objective function minimizes the divergence between $P_{Y|X}$ and $Q_{Y|X}$ further because of $\overset{c}{\|}$. Combined with $\overset{b}{\|}$, our objective minimizes $P_{XY}$ and $Q_{XY}$.

S3, More Generated Images
Figure 8. Synthetic results for MNIST. The figures on the top left is the real images from training set and the generated samples from our model $CCGAN$ are put following dash line. These generated images are sorted by the training data size ratio $r_l$. The larger $r_l$ shows a better generative quality.
Figure 9. Synthetic results for CIFAR10. The figures on the top left is the real images from training set and the generated samples from our model CCGAN are put following dash line. These generated images are sorted by the training data size ratio $r_l$. The larger $r_l$ shows a better generative quality. And the images inside the red box are false generated class by CCGAN.
Figure 10. Synthetic results for VGGFACE100. The figures on the top left is the real images from training set and the generated samples from our model CCGAN are put following dash line. These generated images are sorted by the training data size ratio $r_l$. The larger $r_l$ shows a better generative quality. And the images inside the red box are false generated class by CCGAN.