Angular dependences in electroweak semi-inclusive leptoproduction

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We present the leading order unpolarized and polarized cross sections in electroweak semi-inclusive deep inelastic leptoproduction. The azimuthal dependences in the cross section differential in the transverse momentum of the vector boson arise due to intrinsic transverse momenta of the quarks. However, the presented asymmetries are not suppressed by inverse powers of the hard scale. We discuss the different opportunities to measure specific asymmetries as offered by neutral compared to charged current processes and point out the optimal kinematical regions. The present and (proposed) future HERA collider experiments would be most suitable for measuring some of the asymmetries discussed here, especially in case of Λ production.

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In this article we extend results of Refs. \cite{1}--\cite{3} on semi-inclusive deep inelastic lepton-hadron scattering to include contributions from Z-boson exchange and γ-Z interference terms in neutral current processes as well as contributions from W-boson exchange in charged current processes. The azimuthal dependences in the cross section differential in the transverse momentum of the vector boson arise due to intrinsic transverse momenta of the quarks. Only leading order $(1/Q)^0$ effects are discussed, since higher twist contributions like those of Refs. \cite{4}--\cite{6} are expected to be negligible at energies for which electroweak contributions are relevant. Also, we will focus on tree level, i.e., order $(\alpha_s)^0$. A rich structure nevertheless arises when taking into account the polarization of the initial or final state particles. At the end of this article we will discuss the experimental opportunities to study specific terms in the cross sections.

For details of the calculation and the formalism we refer to \cite{1,2}. We shortly repeat the essentials. It is convenient to use the hadron momenta in the process $\ell H \rightarrow \ell' h X$ to define two lightlike vectors $n_+ \equiv p \cdot n_+$ and $n_- \equiv -q$, satisfying $n_+ \cdot n_- = 1$. These vectors then define the lightcone components of a vector as $p^\pm \equiv p \cdot n_{\pm}$ and we use the component notation $p = [p^-, p^+, p_z]$. Up to mass terms the momentum $P$ of the target hadron ($H$) is along $n_+$, the momentum $P_h$ of the outgoing hadron ($h$) is along $n_-$. We assume here that we are discussing current quark fragmentation, for which one requires $P \cdot P_h \sim Q^2$, where $q^2 = -Q^2$ is the momentum transfer squared.

In leading order in $1/Q$ the process factorizes into a product of a hard perturbative partonic subprocess and two soft nonperturbative parts, which describe the distribution of quarks in the target and the final fragmentation of the struck quark into hadrons, respectively.

The neutral current cross section for unpolarized and polarized electroweak semi-inclusive lepton-hadron scattering is given by

$$\frac{d\sigma(\ell H \rightarrow \ell' h X)}{d\Omega \, dx \, dz \, dq_T} = \frac{\alpha_s \, y}{8 \, Q^4} \sum_{ij} T_{ij}^{\mu \nu} 2M W_{ij}^{\mu \nu} \chi_{ij}. \quad (1)$$

We use invariants $x = Q^2/(2P \cdot q)$, $z = P \cdot P_h/(P \cdot l)$ and $y = (P \cdot q)/(P \cdot l) \approx q^-/l^-$ ($l$ being the momentum of the beam lepton). The cross section is differential in $dx$, $dz$, $d\Omega = 2dy \, d\phi$, and in $dq_T$ where $q_T = q + x \, P - P_h/z =$...
[0, 0, \mathbf{q}_T]$. The indices $i, j$ can be $\gamma$ for the photon or $Z$ for the $Z$-boson. The relative propagator factors $\chi_{ij}$ are given by
\[ \chi_{\gamma\gamma} = 1, \quad \chi_{\gamma z} = \chi_{z\gamma} = \frac{1}{\sin^2(2\theta_W)} \frac{Q^2}{Q^2 + M_Z^2}, \quad \chi_{zz} = (\sin^2 \theta_W). \] (2)

Here we note that in this process the scale $Q$ is defined by the *spacelike* vector boson momentum $q$ (with $Q^2 = -q^2$), hence the width of the $Z$-boson plays a negligible role and we have taken it to be zero. Also, we will consider the cross section differential in the transverse momentum of the vector boson, but the factorized expression \( \frac{|q_T|^2}{Q^2} \) that we will consider will require \( |q_T|^2 \ll Q^2 \), to insure that one is sensitive to the region of intrinsic transverse momenta.

The lepton tensor (neglecting the lepton masses) is given by
\[ L^{ij}_{\mu\nu}(l, l') = C^{ij}[2\nu_{l}\nu_{l'} + 2\nu_{l}\nu_{l'} - Q^2 g_{\mu\nu}] + D^{ij} 2i \epsilon_{\mu\nu\rho\sigma} l^{\rho} P^{\sigma}, \]
where the incoming lepton has momentum $l$ and the back-scattered lepton momentum $l'$. We have defined
\[ C^{\gamma\gamma} = 1, \quad C^{\gamma Z} = C^{Z\gamma} = e^l(g_V^l - g_A^l \lambda_e), \quad C^{ZZ} = (g_V^l + g_A^l)^2 - (2g_V^l g_A^l) \lambda_e, \]
\[ D^{\gamma\gamma} = \lambda_e, \quad D^{\gamma Z} = D^{Z\gamma} = e^l(g_V^l \lambda_e - g_A^l), \quad D^{ZZ} = (g_V^l + g_A^l)^2 \lambda_e - (2g_V^l g_A^l), \]
where $e^l$ denotes the coupling of the photon to the leptons in units of the positron charge; $g_V^l, g_A^l$ denote the vector and axial-vector couplings of the $Z$-boson to the leptons, respectively and $\lambda_e$ is the helicity of the incoming lepton.

To leading order the expression for the hadron tensor, including quarks and anti-quarks, is
\[ 2M W_{ij}^{\mu\nu} = \int dp^+ \frac{d^2 p}{(2\pi)^3} \frac{d^2 k}{(2\pi)^3} \delta^4(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T) \text{Tr} \left( \Phi(p) V_i^{\mu} \Delta(k) V_j^{\nu} \right) \bigg|_{p^+, k^-} + \left( \frac{q^{+}}{\mu} \leftrightarrow -\frac{q^{+}}{\nu} \right), \]
where $p$ and $k$ represent the quark momentum before and after the interaction with the vector boson. The vertices $V_i^{\mu}$ can be either the photon vertex $V_i^{\mu} = \epsilon^{\mu} P$ or the $Z$-boson vertex $V_i^{\mu} = g_V \gamma^{\mu} + g_A \gamma_5 \gamma^{\mu}$. The vector and axial-vector couplings to the $Z$ boson are given by:
\[ g_V^l = \frac{3k_T^l - 2e^k \sin^2 \theta_W}{T^l}, \]
\[ g_A^l = \frac{T^l}{3}, \]
where $e^k$ denotes the charge and $T^l$ the weak isospin of particle $k$ (i.e., $T^l = +1/2$ for $k = +\mu, +\tau, +\nu$ and $T^l = -1/2$ for $k = -\mu, -\tau, -\bar{\nu}, d, s$). We have omitted flavor indices and summation. The correlation functions $\Phi, \Delta$ comprise information on the hadronic structure of the target in terms of quark distributions and on the quark hadronization process, respectively. They are given by (path-ordered exponentials are suppressed):
\[ \Phi_{mn}(P, S; p) = \int \frac{d^4 x}{(2\pi)^4} e^{i p \cdot x} \langle P, S| \bar{\psi}_n(x) \psi_m(x)|P, S \rangle, \]
\[ \Delta_{mn}(P_h, S_h; k) = \sum_X \int \frac{d^4 x}{(2\pi)^4} e^{i k \cdot x} \langle 0| \bar{\psi}_n(x) |P_h, S_h, X \rangle \langle P_h, S_h, X| \bar{\psi}_m(0) |0 \rangle. \]

Using Lorentz invariance, hermiticity, and parity invariance, the (partly integrated) correlation function
\[ \Phi(x, \mathbf{p}_T) \equiv \int dp^+ \Phi(P, S; p) \bigg|_{p^+ = x P^+} = \Phi^{(O)}(x, \mathbf{p}_T) + \Phi^{(L)}(x, \mathbf{p}_T) + \Phi^{(T)}(x, \mathbf{p}_T), \]
(with indices $O, L, T$ indicating the polarization of the target: unpolarized, longitudinally and transversely polarized, respectively) is parametrized in terms of distribution functions as:
\[ \Phi^{(O)}(x, \mathbf{p}_T) = \frac{M}{2P_T} \left\{ f_1(x, \mathbf{p}_T^2) \frac{\mathbf{p}_T}{M} + h_1(x, \mathbf{p}_T^2) \frac{\sigma_{\mu\nu} \gamma^\rho P^\rho P^\nu}{M^2} \right\}, \]
\[ \Phi^{(L)}(x, \mathbf{p}_T) = \frac{M}{2P_T} \left\{ -\frac{\lambda g_{1L}(x, \mathbf{p}_T^2)}{M} \frac{\mathbf{p}_T}{M} - \lambda h_{1L}(x, \mathbf{p}_T^2) \frac{i\sigma_{\mu\nu} \gamma^\rho P^\rho S^\nu}{M^2} \right\}, \]
\[ \Phi^{(T)}(x, \mathbf{p}_T) = \frac{M}{2P_T} \left\{ f_{1T}(x, \mathbf{p}_T^2) \epsilon_{\mu\nu\rho\sigma} P^\rho P^\nu S^\sigma - \frac{P_T \cdot S_T}{M} g_{1T}(x, \mathbf{p}_T^2) \frac{\mathbf{p}_T}{M} - h_{1T}(x, \mathbf{p}_T^2) \frac{i\sigma_{\mu\nu} \gamma_5 S^\nu P^\rho}{M} \right\}, \]
\[ + h_{1T}(x, \mathbf{p}_T^2) \frac{i\sigma_{\mu\nu} \gamma_5 S^\nu P^\rho}{M} - \frac{P_T \cdot S_T}{M} h_{1T}(x, \mathbf{p}_T^2) \frac{i\sigma_{\mu\nu} \gamma_5 P^\rho P^\nu}{M^2} \right\}, \]
(12)
Time reversal invariance is expected to eliminate the T-odd functions \(s\); there is a possibility that they might be effectively generated by a gluonic background field, cf. e.g. [9], hence inclusive deep inelastic scattering due to the absence of initial state interactions, cf. Refs. [7,8,2]. Nevertheless, there is a possibility that they might be effectively generated by a gluonic background field, cf. e.g. [3], hence we keep these functions for completeness.

The (partly integrated) correlation function

\[
\Delta(z, k_T) = \frac{1}{z} \int dk^+ \frac{\Delta(P_h, S_h; k)}{k_T} = \Delta^{(O)}(z, k_T) + \Delta^{(L)}(z, k_T) + \Delta^{(T)}(z, k_T),
\]

(now \(O, L, T\) indicating the polarization of the observed final state hadron) is parametrized in terms of fragmentation functions as:

\[
\Delta^{(O)}(z, k_T) = \frac{M_h}{P_h} \left\{ D_1(z, z^2 k_T^2) \frac{p_h^+}{M_h} + H_1^+(z, z^2 k_T^2) \frac{\sigma_{\mu\nu} k_T^\mu P_{\perp}}{M_h^2} \right\},
\]

\[
\Delta^{(L)}(z, k_T) = \frac{M_h}{P_h} \left\{ -\lambda_h G_{1L}(z, z^2 k_T^2) \frac{p_h^+}{M_h} - \lambda_h H_1^+(z, z^2 k_T^2) \frac{i\sigma_{\mu\nu} \gamma_5 k_T^\mu P_{\perp}}{M_h^2} \right\},
\]

\[
\Delta^{(T)}(z, k_T) = \frac{M_h}{P_h} \left\{ D_{1T}(z, z^2 k_T^2) \frac{\epsilon_{\mu\nu\rho\sigma} k_T^\rho S_{hT}^{\nu\sigma}}{M_h^2} - \frac{\langle k_T \cdot S_{hT} \rangle}{M_h} G_{1T}(z, z^2 k_T^2) \frac{p_h^+}{M_h} \right. \\
- H_{1T}(z, z^2 k_T^2) \frac{i\sigma_{\mu\nu} \gamma_5 S_{hT}^{\nu\mu} P_{\perp}}{M_h} - \left\{ \langle k_T \cdot S_{hT} \rangle \right\} H_{1T}(z, z^2 k_T^2) \frac{i\sigma_{\mu\nu} \gamma_5 k_T^\mu P_{\perp}}{M_h^2} \right\},
\]

with \(M_h\) the mass, \(\lambda_h = M_h S_{hL}^\mu P_{\perp\mu}\) the lightcone helicity, and \(S_{hT}\) the transverse spin of the produced spin-1/2 hadron. The choice of factors in the definition of fragmentation functions is such that \(\sum_a \int dz z^2 d^2k_T D_a(z, z^2 k_T^2) = N_h\), where \(N_h\) is the total number of produced hadrons. The fragmentation functions \(H_1^+\) and \(D_{1T}\) are called T-odd functions, which in contrast to the T-odd distribution functions are expected to be present, since they are not forbidden by time reversal invariance due to the presence of final state interactions, cf. Ref. [3].

We would like to emphasize that the distribution and fragmentation functions as defined above parametrize the soft nonperturbative parts of the scattering process. At present a complete calculation of these nonperturbative objects from first principles, as for instance by a lattice calculation, is not available. Estimates can be made using QCD sum rule or model calculations. Apart from the known functions \(f_1(x), g_1(x)\) and \(D_1(z)\), which are functions integrated over the transverse momentum (there exists also marginal information on \(G_1(z)\) [10]), recently first experimental hints on the size of some asymmetries concerning the so-called Collins function \(H_1^+\) have been presented [11]. None of the other functions has been experimentally accessed. Therefore, many of the observables that we will discuss contain unknown functions, which however are not just parametrizations of ignorance, but represent essential information on the structure of hadrons as can be probed in hard scattering processes and on the hadronization process. For the interpretation of the various functions we refer to Refs. [3,10].

In processes with at least two hadrons one needs to be careful with the notion of transverse. In the definition of distribution and fragmentation functions (which one wants to be independent of the specifics of the process), transverse momentum components denoted by a subscript \(T\) are defined with respect to the momenta \(P\) and \(P_h\) such that \(P_T = 0\) and \(P_{hT} = 0\), respectively. Consequently vectors are decomposed in plus, minus and transverse components defined by the lightlike vectors \(n_+\) and \(n_-\), constructed from \(P\) and \(P_h\), and the transverse projector \(q^\mu_T = g^\mu\nu - n_+^\mu n_-^\nu\).

For defining the (process dependent) azimuthal angles with respect to the scattering plane, on the other hand, it is more convenient to use a frame where the virtual boson and the target, i.e. the momenta \(q\) and \(P\), are collinear. We indicate transverse components in the latter frame with the subscript \(\perp\) (and call them perpendicular henceforth). Thus depending on the choice of frame the covariantly defined vector \(q^\mu_T = q^\mu + x P^\mu - P_h^\mu / z\) can be the transverse component of \(q\) with respect to the collinear hadrons, or it is up to a factor \(1/z\) the component of \(P_h\) perpendicular to the scattering plane, \(q_T = -P_{h\perp} / z\). The kinematics in the frame where \(q\) and \(P\) can be expressed by the set of normalized vectors:

\[q_T = -P_{h\perp} / z, \quad k_T^\perp = k_T / z, \quad k_T^\parallel = k_T - k_T^\perp, \quad k_T^\parallel / z, \quad P_{h\perp} / z, \quad P_{h\parallel} / z.\]
\[ i = \frac{2x}{Q} \hat{P}, \]  
\[ \hat{z} = -q/Q, \]  
\[ \hat{h} = -q_x/Q_T = -(q_x P - P_h/z)/Q_T, \]  
where \( Q_T^2 = -q_x^2 \) and \( \hat{P} \equiv P - (P \cdot q) q/q^2 \), such that:

\[ n_+^\mu = \frac{1}{\sqrt{2}} \left( \hat{\mu}^\mu + \hat{\nu}^\nu \right), \]  
\[ n_-^\mu = \frac{1}{\sqrt{2}} \left( \hat{\mu}^\mu - \hat{\nu}^\nu + 2 \frac{Q_T}{Q} \hat{h}^\mu \right). \]

The lepton momentum reads

\[ l^\mu = \frac{2 - y}{2y} \hat{\mu} - \frac{Q}{2} \hat{\nu} + Q \sqrt{1 - y} \frac{\hat{\mu}}{y} \hat{\nu}, \]

and we define the tensors

\[ g_1^{\mu \nu} \equiv g^{\mu \nu} - \hat{\mu}^\mu \hat{\nu}^\nu + \hat{\nu}^\nu \hat{\mu}^\mu, \]  
\[ e_1^{\mu \nu} \equiv -\epsilon^{\mu \nu \rho \sigma} \hat{\rho} \hat{\sigma}. \]

The cross sections are obtained from the hadron tensor after contraction with the lepton tensor Eq. (3), which to leading order in \( 1/Q \) is

\[ I_{ij}^{\mu \nu} = C_{ij} \frac{Q^2}{y^2} \left[ -2A(y) g_1^{\mu \nu} + 4B(y) \left( \frac{I_1^{\mu}}{I_1^{\nu}} + \frac{1}{2} g_1^{\mu \nu} \right) \right] - D_{ij} \frac{Q^2}{y^2} I(C(y)) e_1^{\mu \nu}, \]

expressed in terms of the functions

\[ A(y) = \left( 1 - y + \frac{1}{2} y^2 \right), \]  
\[ B(y) = (1 - y), \]  
\[ C(y) = y(2 - y). \]

Azimuthal angles inside the perpendicular plane are defined with respect to \( \hat{\mu}_1 \), defined to be the normalized perpendicular part of the incoming lepton momentum \( \hat{l} \),

\[ \hat{l}_1 \cdot a_\perp = -|a_\perp| \cos \phi_a, \]  
\[ e_1^{\mu \nu} \hat{l}_1 \mu a_\perp \nu = |a_\perp| \sin \phi_a, \]

for a generic vector \( a \). In the cross sections we will encounter dependences on the three azimuthal angles \( \phi, \phi_S \) and \( \phi_{S_h} \) of the produced hadron momentum, its spin vector, and of the target hadron spin vector, respectively (cf. Fig. 1). We would like to note that at leading order the spin vector \( S_x \) is identical to the spin vector perpendicular to \( \hat{z} \), i.e. \( S_\perp \), and also \( \lambda = M (S \cdot \hat{z})/(P \cdot \hat{z}) \) (and analogously for \( \lambda_h \)). This does not hold at order \( 1/Q \), cf. Ref. [2].

In order to present our results on cross sections in a compact form we define the following combinations of the couplings and Z-boson propagators

\[ K_1^a(y) = A(y) \left[ C_{1}^{\gamma \gamma} e_a^{2} \lambda_{\alpha \gamma} + 2 C_{1}^{\gamma \lambda} e_a^{2} g_\lambda^{\alpha} \lambda_{\gamma} + C_{1}^{\gamma \lambda} e_a^{2} \lambda_{\lambda \gamma} \right], \]  
\[ K_2^a(y) = B(y) \left[ 2 C_{1}^{\gamma \lambda} e_a^{2} g_\lambda^{\alpha} \lambda_{\gamma} + C_{1}^{\gamma \lambda} e_a^{2} \lambda_{\lambda \gamma} \right], \]  
\[ K_3^a(y) = -B(y) \left[ C_{1}^{\gamma \lambda} e_a^{2} \lambda_{\gamma} + 2 C_{1}^{\gamma \lambda} e_a^{2} g_\lambda^{\alpha} \lambda_{\gamma} + C_{1}^{\gamma \lambda} e_a^{2} \lambda_{\lambda \gamma} \right], \]

which contain the combinations of the Z-boson-to-quark couplings

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where we have not included the unpolarized cross section terms again, i.e. parts which cancel from differences of cross sections with reversed polarization are suppressed here and in the following.

\[ c_1 = (g_V^2 + g_A^2) \]
\[ c_2 = (g_V^2 - g_A^2) \]
\[ c_3 = 2g_Y^2g_A^2 \]

Furthermore, we use the convolution notation (where \( w \) denotes a weight function)

\[ \mathcal{F} [w(p_x, k_y) f (D)] = \int d^2 p_x d^2 k_y \delta^2(p_x + q_y - k_y) w(p_x, k_y) f^a(x, p_x^2)D^a(z, z^2k_y^2) \] (35)

We find for the leading order unpolarized cross section, taking into account both photon and Z-boson contributions,

\[
\frac{d\sigma(\ell H \rightarrow \ell' h X)}{d\Omega dx dz d^2q_x} = \frac{\alpha^2 x z^2 s}{Q^4} \sum_{a,\lambda} \left\{ K_1^a(y) \mathcal{F}[f_1D_1] + K_3^a(y) \cos(2\phi) \mathcal{F}\left[ (2 \hat{\lambda} \cdot p_x \cdot \hat{k}_T - p_x \cdot k_y) h_1^\perp H_1^\perp M_1 M_2 \right] \right\},
\]

(36)

where the sum runs over all quark (and anti-quark) flavors. Perturbative QCD corrections to the term proportional to \( f_1D_1 \), which is independent of the azimuthal angle at tree level, will also produce a \( \cos(2\phi) \) term, next to a \( \cos(\phi) \) term and the time-reversal odd \( \sin(\phi) \) and \( \sin(2\phi) \) terms, but all these will be suppressed by inverse powers of the hard scale \( \ell^2 \). In order to differentiate between the perturbatively generated and the above given \( \cos(2\phi) \) asymmetry, one could for instance apply a transverse momentum cut-off to exclude the contributions from intrinsic transverse momentum \( \ell^2 \) or one can study the analogous charged current exchange process, since the \( \cos(2\phi) \) term of Eq. (36) will then be absent \( (K_3 = 0) \) as we will observe below.

In case the target hadron is polarized the cross section is found to be

\[
\frac{d\sigma(\ell H \rightarrow \ell' h X)}{d\Omega dx dz d^2q_x} = \frac{\alpha^2 x z^2 s}{Q^4} \sum_{a,\lambda} \left\{ \lambda K_2^a(y) \mathcal{F}[g_1D_1] - \lambda K_3^a(y) \sin(\phi) \mathcal{F}\left[ (2 \hat{\lambda} \cdot p_x \cdot \hat{k}_T - p_x \cdot k_y) \frac{h_1^\perp H_1^\perp}{M_1 M_2} \right] \right\}
\]

(37)
The cross section for an un polarized target, but with spin vector of the final state (spin-1/2) hadron being determined is

\[ \frac{d\sigma(\ell H \rightarrow \ell' H X)}{d\Omega dx dz d^2q_T} = \frac{\alpha^2 x z s}{Q^4} \sum_{a,\lambda} \left\{ \lambda \lambda_h K^a_3(y) \mathcal{F} \left[ f_1 G_1 \right] + \lambda_h K^a_3(y) \sin(2\phi) \mathcal{F} \left[ \frac{(2 \hat{h} \cdot p_T \hat{h} \cdot k_T - p_T \cdot k_T) h_{1L}^{\perp} H_{1L}^{\perp}}{MM_h} \right] + |S_{hT}| \frac{K^a_3(y) \sin(\phi - \phi_{Sh})}{2} \mathcal{F} \left[ \hat{h} \cdot p_T \frac{h_{1L}^{\perp}}{M} \right] + |S_{hT}| \frac{K^a_3(y) \sin(3\phi - \phi_{Sh})}{2} \mathcal{F} \left[ \frac{(4 \hat{h} \cdot k_T)^2 \hat{h} \cdot p_T - 2 \hat{h} \cdot k_T p_T \cdot k_T - \hat{h} \cdot p_T k_T^2 h_{1L}^{\perp} H_{1L}^{\perp}}{2MM_h} \right] \right\} . \] (38)

The Eqs. (37) and (38) show some similarity; the latter is obtained from the former by the set of replacements (distribution functions ↔ fragmentation functions, \( M \leftrightarrow M_h, k \leftrightarrow p, \lambda \rightarrow \lambda_h, S_{hT} \rightarrow S_{hT}, \phi_S \rightarrow \phi_{Sh} \)) together with an additional minus sign for each time-reversal odd function \( f_1, h_{1L}^{\perp}, H_{1L}^{\perp}, \) where the replacement of the distribution functions by the fragmentation functions means that \( f, g, h \) functions are replaced by \( D, G, H \) functions, respectively (and vice versa).

Finally, the leading order double polarized cross section is found to be

\[ \frac{d\sigma(\ell H \rightarrow \ell' H X)}{d\Omega dx dz d^2q_T} = \frac{\alpha^2 x z s}{Q^4} \sum_{a,\lambda} \left\{ \lambda \lambda_h \frac{K^a_3(y)}{2} \mathcal{F} \left[ g_1 G_1 \right] + \lambda_h \frac{K^a_3(y)}{2} \cos(2\phi) \mathcal{F} \left[ \frac{(2 \hat{h} \cdot p_T \hat{h} \cdot k_T - p_T \cdot k_T) h_{1L}^{\perp} H_{1L}^{\perp}}{MM_h} \right] + \lambda |S_{hT}| \frac{K^a_3(y) \cos(\phi - \phi_{Sh})}{2} \mathcal{F} \left[ \hat{h} \cdot k_T \frac{g_1 G_{1T}}{M_h} \right] - \lambda |S_{hT}| \frac{K^a_3(y) \sin(\phi - \phi_{Sh})}{2} \mathcal{F} \left[ \hat{h} \cdot k_T \frac{g_1 G_{1T}}{M_h} \right] + \lambda |S_{hT}| \frac{K^a_3(y) \cos(\phi + \phi_{Sh})}{2} \mathcal{F} \left[ \hat{h} \cdot p_T \frac{h_{1L}^{\perp}}{M} \right] + \lambda |S_{hT}| \frac{K^a_3(y) \cos(3\phi - \phi_{Sh})}{2} \mathcal{F} \left[ \frac{(4 \hat{h} \cdot p_T \hat{h} \cdot k_T)^2 \hat{h} \cdot p_T - 2 \hat{h} \cdot k_T p_T \cdot k_T - \hat{h} \cdot p_T k_T^2 h_{1L}^{\perp} H_{1L}^{\perp}}{2MM_h} \right] + |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \cos(2\phi - \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ \hat{h} \cdot p_T \hat{h} \cdot k_T \frac{f_1 G_{1T} + g_1 G_{1T}}{MM_h} \right] - |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \cos(\phi - \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ p_T \cdot k_T \frac{f_1 G_{1T}}{MM_h} \right] + |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \sin(\phi - \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ p_T \cdot k_T \frac{g_1 G_{1T}}{MM_h} \right] + |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \sin(2\phi - \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ \hat{h} \cdot p_T \hat{h} \cdot k_T \frac{f_1 G_{1T}}{MM_h} \right] - |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \cos(\phi - \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ p_T \cdot k_T \frac{f_1 G_{1T}}{MM_h} \right] + |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \cos(\phi + \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ h_1 H_1 \right] + |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \cos(2\phi + \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ \frac{(2(\hat{h} \cdot k_T)^2 - k_T^2) h_{1L}^{\perp}}{2M_h^2} \right] + |S_T| |S_{hT}| \frac{K^a_3(y)}{2} \cos(4\phi - \phi_{S} - \phi_{Sh}) \mathcal{F} \left[ \frac{(8(\hat{h} \cdot p_T)^2(\hat{h} \cdot k_T)^2}{2} \right] \right\} . \]
where the additional term indicated by the parentheses in the last line stands for the set of replacements (distribution functions ↔ fragmentation functions, \( M \leftrightarrow M_h, k \leftrightarrow p, \lambda \leftrightarrow \lambda_h, S_T \leftrightarrow S_{h_T}, \phi_S \leftrightarrow \phi_{S_h} \)) together with an additional minus sign for each time-reversal odd function.

We remark that the appearance of the couplings in the cross sections (36)-(39) shows a clear pattern. All convolutions with chiral-odd distribution functions \( h_{\lambda L}, h_{\lambda T}, h_{\phi S} \) and fragmentation functions \( H_{\lambda L}, H_{\lambda T}, H_{\phi S} \), couple with \( K_{\lambda}^a \). The chiral-even sector involves the couplings \( K_1^\lambda \) and \( K_2^\lambda \). In Eqs. (30) and (33) the convolutions with an even (odd) number of time-reversal odd functions couple with \( K_1^\lambda \) \( (K_2^\lambda) \); in the single polarized cross sections (37) and (38) the situation is reversed, i.e. convolutions with an even (odd) number of time-reversal odd functions couple with \( K_2^\lambda \) \( (K_1^\lambda) \).

\[
-4 \hat{h} \cdot p_y \hat{h} \cdot k_y - 2(\hat{h} \cdot k_y)^2 p_y^2 - 2(\hat{h} \cdot p_y)^2 k_y^2 + p_y^2 k_y^2 \left[ \frac{\hat{h} \cdot k}{2M^2 M_h} \right] + \left\{ \begin{array}{c} 1 \leftrightarrow 2 \\ p \leftrightarrow k \end{array} \right\}, \tag{39}
\]

In Eqs. (30)-(33) we show a clear pattern: All convolutions with chiral-odd distribution functions \( h_{\lambda L}, h_{\lambda T}, h_{\phi S} \) and fragmentation functions \( H_{\lambda L}, H_{\lambda T}, H_{\phi S} \), couple with \( K_{\lambda}^a \). The chiral-even sector involves the couplings \( K_1^\lambda \) and \( K_2^\lambda \). In Eqs. (30) and (33) the convolutions with an even (odd) number of time-reversal odd functions couple with \( K_1^\lambda \) \( (K_2^\lambda) \); in the single polarized cross sections (37) and (38) the situation is reversed, i.e. convolutions with an even (odd) number of time-reversal odd functions couple with \( K_2^\lambda \) \( (K_1^\lambda) \).

To illustrate the kinematical regions where contributions from Z-boson exchange and interference terms become important (for experimental data on azimuthal asymmetries at relatively low values of \( Q^2 \) cf. [16]), we plot the values of the couplings \( K_1^\lambda (y, Q^2) \) as appearing in the neutral current cross sections of scattering with electrons or negatively charged muons, for different fixed values of \( Q^2 \) over the range \( 1 \leq Q^2 \leq 10^5 \text{ GeV}^2 \). Fig. 2 shows the couplings for terms when the struck quark is \( u \)-like, and Fig. 3 when it is a \( d \)-like quark. In both figures the two linear combinations \( K_1^\lambda (y, Q^2, \lambda_c = +1) + K_1^\lambda (y, Q^2, \lambda_c = -1) \) and \( K_1^\lambda (y, Q^2, \lambda_c = +1) - K_1^\lambda (y, Q^2, \lambda_c = -1) \) are plotted as occurring in scattering processes with unpolarized and polarized lepton beams, respectively. In case one scatters with positrons or positively charged muons one has to change the sign of \( \lambda_c \), hence, the resulting plots for unpolarized positron beams are identical to the electron scattering case and for polarized beams the given curves for the couplings simply change sign.

Deviations from the \( Q^2 \) independent behavior at low \( Q^2 \) indicate where interference terms are important. Generally, effects of the weak interaction start to show up above \( Q^2 \sim 300 \text{ GeV}^2 \) and become significant for \( Q^2 \sim 10^3 \text{ GeV}^2 \). It can be read off from the figures that asymmetries involving \( K_1^\lambda \) at low \( Q^2 \) are best measured but this will not affect the relative magnitude of the couplings \( K_1^\lambda \) as a function of \( Q^2 \).

\footnote{In this study we consider only tree level, hence, we do not include logarithmic \( Q^2 \) behavior due to the perturbative running of the strong and electroweak coupling constants. But this will not affect the relative magnitude of the couplings \( K_1^\lambda \) as a function of \( Q^2 \).}
FIG. 3. The couplings $K^a_d(y, Q^2)$ for three different fixed values of $y$: $y = 0.4$ solid line, $y = 0.6$ dashed line, and $y = 0.8$ dotted line. On the left: sum of couplings for $\lambda_e = +1$ and $\lambda_e = -1$ (unpolarized beam). On the right: difference of couplings for $\lambda_e = +1$ and $\lambda_e = -1$ (polarized beam).

with an unpolarized beam at lower values of $y$. In particular for asymmetries proportional to the $K^a_3$ couplings the $y$ dependence is sizable. On the other hand, asymmetries involving the $K^a_2$ couplings are best measured with a polarized beam, preferably at high values of $y$ (although the $y$-dependence here is rather moderate). Moreover, from the r.h.s. of the figures one sees that $K^a_2$ gets an enhancement by a factor 2 for $u$-like quarks and a factor 4 for $d$-like quarks at very high $Q^2$ (the same holds for the coupling $K^a_1$ for unpolarized lepton scattering).

Looking at the definition of $K^a_2$ in Eq. (29) reveals that azimuthal asymmetries involving these particular couplings, which are typical polarization measurements, can be determined even in experiments with an unpolarized beam. The axial vector coupling of the $Z$-boson provides the necessary $\gamma_5$ structure needed to render the involved traces in the calculation non-vanishing, which in the case of a polarized beam is provided by the helicity terms in the lepton tensor. The observation that polarization of quarks can be tested with unpolarized beams using electroweak interference effects was made in the context of electron-positron annihilation [17]. The l.h.s. plots in Figs. 2 and 3, respectively, indicate that azimuthal asymmetries involving $K^a_2$ couplings are accessible with unpolarized lepton beams for $Q^2 > \sim 10^3 \text{GeV}^2$. The strength of the $K^a_2$ couplings has a small $y$ dependence.

Many of the azimuthal asymmetries that arise in the above given cross sections are difficult to measure. Nevertheless, we have presented the complete expressions, since different terms could be accessed in different (future) experiments. First of all, in order to go beyond the photon contributions one needs a relatively high energy experiment. On the other hand, at higher energies effects due to intrinsic transverse momentum are expected to be less important [6], although not power-suppressed. One way around the problem of having to go to very high energies is by studying semi-inclusive deep inelastic leptoproduction involving neutrinos. One can either investigate the case of a neutrino beam scattering off a hadron (for instance in an experiment like NOMAD at CERN) or scattering with an electron or muon beam off a hadron with a neutrino as produced lepton (for instance COMPASS at CERN or ZEUS and H1 at DESY). In both cases no interference with photons occurs. On the other hand, in the case of a produced neutrino a new problem is that one cannot define a lepton scattering plane as given in Fig. 1 (one does not observe $l'$), hence azimuthal angles cannot be defined, unless one can reconstruct the direction of the neutrino by the momentum imbalance [4]. One could also define azimuthal angles with respect to a transverse polarization vector of one initial or final state hadron, but this still limits the number of accessible asymmetries severely and makes the analysis much more difficult.
In order to arrive at the expressions for the cross sections of such charged current processes\(^2\), one can take \(e_a = 0\), \(g'_V = g'_A = 1\), \(c_2^b = 0\) and replace

\[
\chi_{xz} \rightarrow \chi_{ww} = \left( \frac{1}{8 \sin^2 \theta_W} \frac{Q^2}{Q^2 + M_W^2} \right)^2 ,
\]

in the above given couplings \(K^a_\lambda\). In addition, one replaces \(c_1^a = \pm c_2^a = 1\), depending on the chirality of the quark, since \(c_1^a = (g_R^a + g_L^a)/2\) and \(c_2^a = (g_L^a - g_R^a)/2\). Hence, for a left-handed quark one finds \(c_1^a = c_2^a = 1\) and for a right-handed quark one finds \(c_1^a = -c_2^a = 1\). This results in

\[
K^{ab}_1(y) = -K^{ab}_2(y) = 4(1 - \lambda_e) \chi_{ww} |V_{ab}|^2 \left( A(y) \pm \frac{C(y)}{2} \right),
\]

\[
K^{ab}_3(y) = 0 ,
\]

where \(a, b\) are the incoming and outgoing quark flavor indices, respectively, and \(V_{ab}\) stands for the appropriate CKM matrix element. Needless to say, the sum over flavors in the cross section expressions will now only run over the appropriate flavors (\(u\)-like or \(d\)-like). We have neglected the lepton masses, hence helicity states (\(\lambda_e = \pm 1\)) are equal to the chirality states (\(R/L\)). The term \((1 - \lambda_e)\) reflects the fact that the incoming lepton \((e^-, \mu^-, \nu)\) can only be lefthanded in charged exchange and the incoming anti-lepton \((e^+, \mu^+, \bar{\nu})\) only righthanded \((\lambda_e\) for the incoming lepton is replaced by \(-\lambda_e\) for an incoming anti-lepton). For example, for the elementary process \(\nu d \rightarrow e^- u\), one finds the coupling \(K^{da}_1 = 8 \chi_{ww} |V_{ud}|^2\) and for \(\bar{\nu} \bar{d} \rightarrow e^- \bar{d}\) one finds \(K^{\bar{d}d}_1 = 8 \chi_{ww} |V_{ud}|^2 (1 - y)^2\). The \(y\) dependence of \(K^{da}_1\) is explained by the fact that the total spin of a lefthanded lepton and a lefthanded quark is \(J = 0\) such that the partonic scattering becomes isotropic in the c.m. lepton scattering angle; the coupling \(K^{\bar{d}d}_1\) has a \((1 - y)^2\) dependence characteristic of the \(J = 1\) total spin of a lefthanded neutrino and a righthanded anti-quark (see for instance \([13]\)). Note that these two elementary processes will be accompanied by distribution and fragmentation functions that in general are completely different in magnitude. The same holds for the difference between neutrino/electron versus anti-neutrino/positron scattering, cf. e.g. \([13]\). One also has to be aware that for the charged current cross-sections no averaging over initial lepton polarizations has to be performed.

The present HERA experiments ZEUS and H1 could access asymmetries for which it is important to take into account the \(Z\) boson. However, since in these experiments no initial polarization is present (although longitudinal lepton beam polarization will become disposable also for the collider experiments in the near future), one way to access some of the interesting asymmetries is by focusing on \(\Lambda\) production, since the spin vector of the \(\Lambda\) can be determined from its subsequent decay. From the enhancement of the coupling \(K_2\) at high \(Q^2\) as apparent on the r.h.s. of Figs. 2 and 3 one concludes that it seems quite promising to measure the helicity fragmentation function \(G_1\) by studying for instance \(\Lambda\) production using a polarized lepton beam on an unpolarized target at very high \(Q^2\). This option was investigated in for instance Refs. \([20, 21, 23]\). In this case one is sensitive to the first term in Eq. \([13]\), which does not involve a weight factor. Hence, one can integrate over the transverse momentum of the vector boson, thereby deconvoluting the distribution and fragmentation function. The resulting cross section is proportional to the integrated function \(G_1(z)\) multiplied with the well-known function \(f_1(x)\). As mentioned above, from the l.h.s. of the same figures one sees that azimuthal asymmetries involving \(K_2^a\) couplings are also accessible with unpolarized lepton beams for \(Q^2 \gtrsim 10^3 \text{ GeV}^2\), thus allowing for a measurement of \(G_1\) with unpolarized lepton and proton beams. This is the semi-inclusive deep inelastic scattering analogue of the proposed measurement of \(G_1\) in \(e^+ e^- \rightarrow \Lambda^+ X\) \([22]\). One could also exploit charged current exchange by using a neutrino (or anti-neutrino) beam like for instance in \(\nu p \rightarrow \mu^- \Lambda^+ X\) \([21, 23]\), but we would like to emphasize that one can use the neutral current exchange process without the need for lepton beam polarization, i.e. \(\ell H \rightarrow \ell' \Lambda^+ X\).

\(^2\)A high \(p_T\) azimuthal asymmetry in charged current semi-inclusive deep inelastic scattering arising at order \(\alpha_s\) has been studied in Ref. \([4]\). In Ref. \([15]\) the same asymmetry for the neutral current process was studied, also at low \(p_T\), taking into account purely photon exchange. The mechanism to include intrinsic transverse momentum is the one of \([1]\) and is therefore considered to be of higher twist.
A polarized proton beam, for instance at the proposed polarized HERA collider [24], would give even more opportunities to measure the different asymmetries presented here, i.e. in principle those given in Eqs. (37) and (39), cf. also Ref. [25]. A remark that is relevant here is that measuring the transversity functions \( h_1 \) and \( H_1 \) via the \( \cos(\phi_S + \phi_{SL}) \) term in Eq. (39) cannot be done in the charged current exchange case (e.g. via \( e^- p^+ \to \nu \Lambda^+ X \)), since \( K_{ab}^3(y) = 0 \). The same holds for any other chiral-odd function. But they can of course be accessed in the neutral current processes and at very high energies lepton beam polarization can even be of assistance.

In conclusion, we have presented the leading order unpolarized and polarized cross sections in electroweak semi-inclusive deep inelastic lepton production. We have discussed the present and future possibilities for experimental investigation of some of the asymmetries presented here. In particular, the opportunities offered by neutral and charged current processes were contrasted and the optimal kinematic regions (in \( y \) and \( Q^2 \)) for which one might expect certain asymmetries to be measurable were studied. We have observed that one can measure the helicity fragmentation function \( G_1 \) by \( \Lambda \) production in the neutral current exchange process with both lepton and proton beams unpolarized. Also, we have noted that the transversity distribution and fragmentation functions cannot be measured in charged current exchange semi-inclusive lepton production.

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