We investigate the multiphoton quantum dynamics of a leaking single-mode quantized cavity field coupled with a resonantly driven two-level qubit possessing permanent dipoles. The frequencies of the interacting subsystems, i.e. cavity and the emitter, are being considered very different, e.g. microwave and optical domains, respectively, and therefore the latter one couples to the resonator mode via its parallel dipole moments. As well, the generalized Rabi frequency resulting form the external coherent driving of the two-level qubit is assumed different from the resonator’s frequency.

As a consequence, this highly dispersive interaction regime is responsible for the cavity multiphoton quantum dynamics and photon conversion from optical to microwave ranges, respectively.

I. INTRODUCTION

Frequency conversion processes where an input light beam can be converted at will into an output beam of a different frequency are very relevant nowadays due to various feasible quantum applications [1–4]. Among the first demonstrations of this effect is the experiment reported in [5] promising developments of tunable sources of quantum light. From this reason, single-photon upconversion from a quantum dot preserving the quantum features was demonstrated in [6]. Experimental demonstration of strong coupling between telecom (1550 nm) and visible (775 nm) optical modes on an aluminum nitride photonic chip was demonstrated as well, in Ref. [7]. Even bigger frequency differences can be generated. For instance, an experimental demonstration of converting a microwave field to an optical field via frequency mixing in a cloud of cold $^{87}$Rb atoms was reported in [8]. Earlier theoretical studies have demonstrated frequency downconversion in pumped two-level systems with broken inversion symmetry [9, 10]. Furthermore, single- and multiphoton frequency conversion via ultra-strong coupling of a two-level qubit to two resonators was theoretically predicted in [11]. Although multiquanta processes are being investigated already for a long period of time, recently have attracted considerable attention as well. This is mainly due to potential application of these processes to quantum technologies related to quantum lithography [12] or novel sources of light [13, etc. 14–15]. Additionally, optomechanically multiphonon induced transparency of x-rays via optical control was demonstrated in [16] while strongly correlated multiphonon emission in an acoustic cavity coupled to a driven two-level quantum dot was demonstrated in Ref. [17], respectively.

However, most of the frequency conversion investigations refer to resonant processes. In this context, here, we shall demonstrate a photon conversion scheme involving non-resonant multiphoton effects, respectively. Actually, we investigate frequency downconversion processes via a resonantly laser-pumped two-level emitter possessing permanent diagonal dipoles, $d_{\alpha \alpha} \neq 0$ with $\alpha \in \{1,2\}$, and embedded in a quantized microwave resonator, see Fig. 1. The frequencies of the two-level qubit and the single-mode cavity are significantly different from each other, e.g. optical and microwave domains. Therefore, the two-level emitter naturally couples to the resonator through its permanent parallel dipoles only. The cavity’s frequency differs as well from the generalized Rabi frequency arising due to resonant and coherent external driving of the two-level emitter. As a result, this highly dispersive interaction regime leads to multiphoton absorption-emission processes in the resonator mode mediated by the corresponding damping effects, i.e., qubit’s spontaneous emission and the photon leaking through the cavity, respectively. We have obtained the corresponding cavity photon quantum dynamics in the steady state and demonstrated the feasibility to generate a certain multiphoton superposition state with high probability, and at different frequencies than that of the input external coherent pumping. The advantage of our scheme consists in availability of its constituents, having $d_{22} \neq d_{11}$, such as asymmetrical two-level quantum dots [14, 17] and molecules [22–24], or, equivalently, spin or quantum circuits [25–26], together with the technological progress towards their coupling to various resonators.

The article is organized as follows. In Sec. II we ap-

$$\begin{pmatrix}
|2\rangle & \mathcal{G} & d_{22} \neq 0 \\
|1\rangle & \mathcal{G} & d_{11} \neq 0
\end{pmatrix}$$

FIG. 1: The schematic of the model: A coherently pumped two-level qubit couples with a single-mode resonator of frequency $\omega$ via its non-zero parallel dipoles, $d_{\alpha \alpha}$, with $\alpha \in \{1,2\}$. Here, $\Omega$ is the corresponding Rabi frequency due to the off-diagonal dipole moment $d_{21}$ whereas $\omega_L$, $\omega_L \gg \omega$, is the frequency of the resonantly applied external field. The qubit–resonator coupling strength is denoted by $g$. 

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ply the developed analytical approach to the system of interest and describe it, while in Sec. III we analyze the obtained results. The summary is given in Sec. IV.

II. ANALYTICAL FRAMEWORK

The master equation describing the interaction of a two-level qubit, possessing permanent diagonal dipoles, with a classical coherent electromagnetic field of frequency $\omega_k$, as well as with a quantized single mode resonator of frequency $\omega$, with $\omega \ll \omega_k$ (see Fig. 1), and damped via the corresponding environmental reservoirs in the Born-Markov approximations [27–29], is:

$$\frac{d}{dt}\rho(t) + i\hbar^{-1}[H, \rho] = -\gamma \frac{\hbar}{2} [S^+, S^-] \rho - \frac{\hbar}{2}(1 + \bar{n})[b^\dagger, b]\rho - \frac{\hbar}{2} \hat{n}[b, b^\dagger]\rho + H.c..$$

(1)

Here, $\gamma$ is the single-qubit spontaneous decay rate, whereas $\bar{n}$ is the corresponding boson-mode’s leaking rate with $\bar{n} = \exp[h\omega/(k_BT)] - 1$ being the mean resonator photons number due to the environmental thermostat at temperature $T$, and $k_B$ is the Boltzmann constant. The two-level system may have the transition frequency in the optical domain, whereas the single-mode cavity frequency may lay in the microwave range, respectively. The wavevector of the coherent applied field is perpendicular to the cavity axis. In the Eq. (1), the bare-state qubit’s operators $S^+ = |2\rangle \langle 1|$ and $S^- = |S^+\rangle^\dagger$ obey the commutation relations for su(2) algebra, namely, $[S^+, S^-] = 2S_z$ and $[S_z, S^\pm] = \pm S^\pm$, where $S_z = (|2\rangle \langle 2| - |1\rangle \langle 1|)/2$ is the bare-state inversion operator. $|2\rangle$ and $|1\rangle$ are the excited and ground state of the qubit, respectively, while $b^\dagger$ and $b$ are the creation and the annihilation operator of the electromagnetic field (EMF) in the resonator, and satisfy the standard bosonic commutation relations, i.e., $[b, b^\dagger] = 1$, and $[b, b^\dagger, b] = 0$. The Hamiltonian characterizing the respective coherent evolution of the considered compound system is:

$$H = h\omega b^\dagger b + h\Delta S_z - h\Omega(S^+ + S^-) + hgS_z(b^\dagger + b).$$

(2)

In the Hamiltonian (2), the first two components describe the free energies of the cavity electromagnetic field and the two-level qubit, respectively, with $\Delta = \omega_{21} - \omega_L$ being the detuning of the qubit transition frequency $\omega_{21}$ from the laser one. The last two terms depict, respectively, the laser interaction with the two-level qubit and the qubit-cavity interaction. Correspondingly, $\Omega$ and $g$ are the respective coupling strengths, see Fig. 1. Note at this stage that while the Rabi frequency $\Omega$ is proportional to the off-diagonal dipole moment $d_{21}$, the qubit-cavity coupling is proportional to the diagonal dipole moments, i.e. $g \propto (d_{22} - d_{11})$. The interaction of the external coherent electromagnetic field with permanent dipoles is omitted here as being rapidly oscillating. From the same reason, the qubit-cavity interaction described by the usual Jaynes-Cummings Hamiltonian, proportional to $d_{21}$, is neglected as well here.

In what follows, we perform a spin rotation [30, 32],

$$U(\chi) = \exp[2i\chi S_y],$$

(3)

where $S_y = (S^+ - S^-)/2i$ and $2\chi = \arctan[2\Omega/\Delta]$ with $\Delta = \Delta + g(b^\dagger + b)$, diagonalizing the last three terms of the Hamiltonian (2). This action will lead to new quasi-spin operators, i.e. $R_z$ and $R^\pm$, defined via the old qubit’s operators in the following way

$$R_z = S_z \cos 2\chi - (S^+ + S^-) \sin 2\chi/2,$n

$$R^+ = S^+ \cos^2 \chi - S^- \sin^2 \chi + S_z \sin 2\chi,$n

$$R^- = [R^+]^\dagger.$$n

(4)

The new qubit operators $R^+ = |2\rangle \langle 1|$, $R^- = |1\rangle \langle 2|$ and $R_z = (|2\rangle \langle 2| - |1\rangle \langle 1|)/2$, describing the transitions and populations among the dressed-states $|\{2, 1\}\rangle$, will obey the commutation relations: $[R^+, R^-] = 2R_z$ and $[R_z, R^\pm] = \pm R^\mp$, similarly to the old-basis ones. Respectively, the Hamiltonian (2) transforms to:

$$H = h\omega b^\dagger b + 2h\Omega R_z,$n

(5)

where the operator $\tilde{\Omega} = (\Delta^2/4 + \Omega^2)^{1/2}$, whereas

$$b = \tilde{b} - i\eta S_y \sum_{k=0}^\infty \frac{\eta^k}{k!} (\tilde{b}^\dagger + \tilde{b}) e^{\frac{\Delta^2}{2\Omega}} = \sum_{k=0}^\infty \frac{\eta^k}{k!} (\tilde{b}^\dagger + \tilde{b}) e^{\frac{\Delta^2}{2\Omega}},$$

(6)

with $b^\dagger = [b]^\dagger$, $\tilde{b} = U b U^{-1}$, $\tilde{b}^\dagger = [\tilde{b}]$, and

$$\eta = \frac{g}{2\Omega}, \text{ and } \chi = \frac{\Delta}{2\Omega}.$$n

Now the expressions (1)-(5) have to be introduced in the master equation (1) and the final equation will be somehow cumbersome. It can be simplified if we perform the secular approximation, i.e., neglect all terms from the master equation oscillating at the generalized Rabi frequency $2\Omega_0$, $\Omega_0 = \sqrt{1 + \xi^2}$, and higher one. This is justified if $2\Omega_0 \gg \{\omega, g, \gamma\}$ - the situation considered here.

Thus, in the following, we expand the generalized Rabi frequency $\Omega$ in the Taylor series using the small parameter $\eta$, namely,

$$\Omega = \Omega_0 \left\{ 1 + \frac{\xi \hat{\eta}}{1 + \xi^2} + \frac{\hat{\eta}^2}{2(1 + \xi^2)^2} - \frac{\xi \hat{\eta}^3}{2(1 + \xi^2)^3} + \cdots \right\},$$

where $\hat{\eta} = \eta(\tilde{b}^\dagger + \tilde{b})$. Then perform a unitary transformation $U(t) = \exp[2i\Omega_0 R_z]t$ in the whole master equation and neglect terms oscillating at the Rabi frequency $2\Omega_0$ or higher. Afterwards, perform the operation $\rho_{\alpha\alpha} = \langle \alpha| \rho| \alpha \rangle$, $\alpha \in \{1, 2\}$, and one can arrive then at the following master equation describing the cavity degrees

\[ P(\rho, t) = -i[\rho, H(t)] + \frac{\gamma}{2} (1 + \bar{n})[\rho, \rho^\dagger] - \frac{\hbar}{2} \hat{n} [b, \rho^\dagger], \]

where $H(t) = H_0 + H_{\text{int}}(t)$ is the total Hamiltonian, and $H_{\text{int}}(t)$ accounts for the interaction between the external coherent field and the qubit. The equation $P(\rho, t) = 0$ describes the evolution of the density matrix $\rho$ under the influence of the Hamiltonian $H(t)$. The term $\gamma (1 + \bar{n})[\rho, \rho^\dagger]$ represents the spontaneous emission process, and the term $\frac{\hbar}{2} \hat{n} [b, \rho^\dagger]$ accounts for the decay process due to the interaction with the external field.

In the next section, we will analyze the obtained results and discuss the implications of our findings. The summary is given in Sec. IV.
of freedom only:
\[
\frac{d}{dt}\rho(t) + i\frac{\hbar}{\gamma}(H,\rho) - \frac{\gamma}{4} \left[\cos 2\chi \rho \cos 2\chi + \sin 2\chi \rho \sin 2\chi - \rho\right]
- \frac{\kappa\eta^2}{8} \sum_{k_1,k_2=0}^{\infty} f_{k_1}(\eta,\xi) f_{k_2}(\eta,\xi)
\times \{(\hat{b}^\dagger + \hat{b})^{k_1},(\hat{b}^\dagger + \hat{b})^{k_2} \rho\} + H.c.,
\]
where \(\rho = \rho_{11} + \rho_{22}\). Here,
\[
f_k(\eta,\xi) = \frac{\eta^k}{k!} \frac{\partial^k}{\partial \xi^k} \frac{1}{\sqrt{1 + \xi^2}},
\]
\[
\sin 2\chi = \frac{\Omega}{\Omega} \sum_{k=0}^{\infty} \frac{\eta^k (\hat{b} + \hat{b}^\dagger)^k}{k!} \frac{\partial^k}{\partial \xi^k} \frac{1}{\sqrt{1 + \xi^2}},
\]
\[
\cos 2\chi = \frac{\Delta/\Omega}{\Omega} \sum_{k=0}^{\infty} \frac{\eta^k (\hat{b} + \hat{b}^\dagger)^k}{k!} \frac{\partial^k}{\partial \xi^k} \frac{\xi}{\sqrt{1 + \xi^2}}.
\]
Already at this stage one can recognize the multiphoton nature of the cavity electromagnetic field quantum dynamics.

Using the bosonic operator identity
\[
(A + B)^n = \sum_{k'}^{n} \frac{n!}{k'!(n-k')!} \left(-\frac{C}{2}\right)^{\frac{n-k'}{2}} \sum_{r=0}^{k'} \frac{k'!}{r!(k'-r)!} \times A^r B^{k'-r},
\]
where \([A, B] = C\) and \([A, C] = [B, C] = 0\), whereas \(k'\) is odd for an odd \(n\) and even for an even \(n\), respectively; one can reduce the master equation (7) to a time-independent equation if one further performs a unitary transformation \(V(t) = \exp[i\omega \hat{b}^\dagger \hat{b}]\) and neglects all the terms rotating at frequency \(\omega\) and higher. This would also result in avoiding any resonances in the system, i.e., \(2\Omega - s\omega \neq 0\), \(s \in \{1, 2, \cdots\}\). As a consequence, one can obtain a diagonal equation for \(P_n = \langle n|\rho|n\rangle\), with \(|n\rangle\) being the Fock state and \(n \in \{0, 1, 2, \cdots\}\), describing the cavity multiphoton quantum dynamics, in the presence of corresponding damping effects, which is computed then numerically here. Particularly, for single-photon nonresonant processes one can obtain the following equation for the photon distribution function:
\[
\frac{d}{dt}P_n(t) = -\frac{\gamma\eta^2}{4(1 + \xi^2)^2} \left\{(2n + 1)P_n - (n + 1)P_{n+1}\right\}
- nP_{n-1} - \kappa(1 + \bar{n})\{nP_n - (n + 1)P_{n+1}\}
- \kappa\bar{n}\{(n + 1)P_n - nP_{n-1}\}.
\]

Additional \(N - photon\) non-resonant processes with \(N \in \{2, 3, \cdots\}\) can be incorporated by restricting the equation Eq. (7) to terms up to \(\eta^{2N}\). In order to solve the infinite system of equations for \(P_n\) (see e.g. Eq. (8) for single-photon processes), we truncate it at a certain maximum value \(n = n_{max}\) so that a further increase of its value, i.e. \(n_{max}\), does not modify the obtained results, while \(\eta\) is being fixed.

Thus, the cavity photon dynamics has a multiphoton behavior because of the highly dispersive (non-resonant) nature of the interaction among the asymmetrical two-level qubit and cavity field mode. This way, one obtains an output multiphoton flux of microwave photons, although the two-level system is coherently pumped at a different frequency, i.e. with optical photons.

III. RESULTS AND DISCUSSION

In the following, we shall describe the cavity multiphoton quantum dynamics based on the Eq. (7). Thus, the resonator’s steady-state mean quanta number can be expressed as:
\[
\langle \hat{b}^\dagger \hat{b}\rangle = \sum_{n=0}^{n_{max}}nP_n.
\]
with
\[
\sum_{n=0}^{n_{max}}P_n = 1.
\]
Respectively, the second-order photon-photon correlation function is defined in the usual way [32], namely,
\[
g^{(2)}(0) = \langle \hat{b}^\dagger \hat{b}^\dagger \hat{b}^\dagger \hat{b}\rangle \sum_{n=0}^{n_{max}}n(n-1)P_n.
\]

![FIG. 2: The cavity photon distribution function \(P_n\) in the steady-state. The green curve is plotted for \(\eta = 0.1\), while the blue one for \(\eta = 0.08\), respectively. Other parameters are: \(\bar{n} = 0.1\), \(\kappa/\gamma = 10^{-3}\) and \(\xi = 0\).](image)
order photon processes would not modify the results significantly. One can observe here that larger ratios of \( \eta = g/(2\Omega) \), with \( \eta < 1 \), lead to population of higher photon states, compare the green and blue curves plotted for \( \eta = 0.1 \) and \( \eta = 0.08 \), respectively, facilitating the generation of multiphoton states when \( \kappa/\gamma \ll 1 \). Correspondingly, \( P_n \) is small for larger \( n \) and smaller \( \eta \), while \( \eta < 1 \), assuring convergence of the results. On the other side, Figure 3(a) depicts the cavity mean photon number in the steady-state. This photon number increases as the qubit-cavity coupling increases too and achieves a maximum value. Analyzing again Fig. 2 one can observe that the probability of a two-photon state, \( n = 2 \), is the same for \( \eta = 0.1 \) and \( \eta = 0.08 \), respectively, and it is higher than 0.1. On the other side, from Fig. 3(a) follows that the range of the mean cavity photon number lies approximately between 1.5 < \( \langle b^\dagger b \rangle \) < 2.5 when \( \eta \) changes from 0.08 to 0.1. One may conjecture then that a multiphoton superposition state around \( n = 2 \) is generated when other parameters are being fixed. Respectively, Figure 3(b) depicts the steady-state behavior of the second-order resonator’s photon-photon correlation function \( g^{(2)}(0) \) against the pumping parameter \( \eta = g/(2\Omega) \). The photon statistics changes from super-Poissonian to quasi-coherent features, respectively, as \( \eta \) increases. Furthermore, the same results, shown in Figs. 2 and 3, will persist for moderate detunings, i.e., would not change significantly when \( \xi \ll 1 \). Concluding here, the presence of diagonal dipole moments, in a resonance coherently pumped two-level qubit, makes possible the coupling to the resonator mode at a completely different frequency than the input one which drives the two-state quantum emitter, and cavity multiphoton state generation, respectively. Furthermore, the developed approach applies equally to a driven two-level quantum dot embedded in an acoustical phonon cavity.

![Graph showing steady-state mean cavity photon number and second-order correlation function](image)

**FIG. 3:** (a) The steady-state mean cavity photon number \( \langle n \rangle \equiv \langle b^\dagger b \rangle \) as well as (b) its second-order correlation function \( g^{(2)}(0) \) as a function of \( \eta = g/(2\Omega) \). Other parameters are as in Fig. 2.

### IV. SUMMARY

We have investigated the possibility to convert photons from, e.g. optical to microwave domains, via a resonantly pumped asymmetrical two-level quantum emitter embedded in a quantized single-mode resonator. The corresponding damping effects due to qubit’s spontaneous emission and cavity’s photon leakage are taking into account as well. The transition frequency of the two-level qubit differs significantly from the cavity’s one, namely, it can lay in the optical range while the resonator’s frequency in the microwave domain, respectively. Therefore, the two-state quantum emitter couples to the cavity mode through its parallel dipole moments. As well, the cavity’s frequency is considering being far off-resonance from the generalized Rabi frequency resulting from the coherent driving of the two-level qubit via its non-diagonal dipole. In these circumstances, multiphoton absorption-emission processes are proper to the cavity quantum dynamics. We have demonstrated the cavity’s multiphoton characteristics and showed the feasibility for a certain output multiphoton superposition state generation. The photon statistics modifies from super-Poissonian to quasi-Poissonian photon statistics as the pumping parameter \( \eta \) is increased from zero. Finally, as a concrete system, where the approach developed here can apply, can serve asymmetrical two-level quantum dots coupled to microwave resonators as well as polar biomolecules, spin or quantum circuit systems, respectively [19, 20]. As well, this analytical approach can be used to study non-resonant multiphoton quantum dynamics when a pumped two-level quantum dot interacts with an acoustical phonon resonator, respectively [21].

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