The energies of the $\eta_c d$ and $\eta_c ^3$He bound states are calculated on the basis of exact three- and four-body AGS equations. For the $\eta_c N$ interaction a Yukawa-type potential has been adopted. The calculations are done for a certain range of its strength parameter. The results obtained are quite different from calculations based on the folding model.

PACS numbers: 21.45.+v, 13.75.Gx, 13.85.-t, Jz

I. INTRODUCTION

The behavior of mesons in a nuclear medium has attracted much attention during the last years [1]. We mention the existence and properties of deeply bound meson-nuclear states [2] as well as the possible restoration of chiral symmetry in the nuclear environment [8], a point closely related to modifications of widths and masses of the mesons propagating through nuclear matter [4].

It should be emphasized that all these phenomena are usually described in terms of effective degrees of freedom including mesons, nucleons, and isobars. The manifestation of more fundamental quark-gluon degrees of freedom at low energies is very unlikely because of the strong mismatch between the nuclear energy scale (few tenth of MeV) and the QCD energy scale (few hundreds or thousands of MeV).

However, as pointed out in [5] with regard to charmonium-like mesons, for example $\eta_c$, which do not contain $u$- and $d$-quarks, the main contribution to the meson-nucleon interaction is expected to originate not from effective, but from fundamental QCD degrees of freedom, namely from the few-gluon exchange. In this way, using the Pomeron exchange model, one can deduce a Yukawa-type potential for the $\eta_c$-nucleon system [5]. Following these ideas, the authors of [6] were able to explain the pp-spin correlation at the threshold energy of charm production. Such an analysis allows one to restrict the uncertainties of the parameters of the charmonium-nucleon potential. Independent theoretical indication on the existence of attraction between charmonium and nucleons has been obtained in [7]. In case of an infinite-mass limit of the heavy quark, these authors have obtained a binding energy of $J/\psi$ in nuclear matter of the order of 10 MeV. For the (2S) charmonium state this approach predicts an even stronger binding. Despite the above indications, the data of the charmonium-$N$ interaction are still rather scarce. Bearing this in mind, it appears reasonable to look for possible formations of the corresponding mesic nuclei, similar to the $\eta$-nuclear systems which were extensively discussed in the literature (see e.g. [8] and references therein). In particular the study of the reactions

$$\bar{p} + A \rightarrow \bar{p} + p + A' \rightarrow \eta_c + A'$$

(1)

has been planned within the PANDA project [9]. The production of $\eta_c$ in electromagnetic processes has already been started in the CLIO collaboration [10]. As explained in [5], the precise knowledge of the binding energy of meson-nuclear systems is of crucial importance for their experimental observation.

First attempts to calculate the binding of $\eta_c$-mesons with light nuclei have been made in Ref. [5] on the basis of variational calculations, and in Ref. [11] where the folding model was used. Both calculations involve a Yukawa form for the basic $\eta_c N$ interaction. Our results for the binding energies of $\eta_c d$ and $\eta_c ^3$He, presented below, are obtained within the AGS theory [12, 13]. Similar to [5, 11] we use the Yukawa type $\eta_c N$ potential

$$V_{\eta_c N}(r) = -a \frac{e^{-\alpha r}}{r}$$

(2)
with $\alpha = 0.6$ GeV. The parameter $a$ is varied between 0.4 and 0.6. Since the two-body potentials act essentially via their $s$-waves contributions, we take into account only $s$-wave parts of the corresponding $\eta_c N$ and $NN$ scattering amplitudes entering the kernel of the AGS equations.

II. FORMALISM

Let us start by giving a brief description of the 3-body $\eta_c NN$ formalism. Its main ingredient is a separable representation of the driving $\eta_c N$ and $NN$ potentials

$$V_\alpha = \sum_{i,j}^N |\chi^i_\alpha \rangle \langle \chi^j_\alpha|, \quad \alpha \in \{\eta_c N, NN\}. \quad (3)$$

To outline the important aspects of this approach, we use for the moment an oversimplified representation of the $\eta_c N$ potential keeping only the first term in the sum \[1\] and taking $\eta_c = 1$ and $\eta_0 = 0$. While the actual calculations are performed with a more realistic representation containing six terms (we will discuss the sensitivity of our results to the number of terms kept in the separable expansion). For the $s$-wave triplet state $^3S_1$ of two nucleons we employ the separable rank-one version of the Paris potential of Ref. \[1\].

The AGS equations for the $\eta_c NN$ system read \[8\]

$$X_{11}(z) = 2 Z_{21}(z) \tau_2(z - \frac{p_2^2}{2\mu_2}) X_{21}(z) \quad (4)$$
$$X_{21}(z) = Z_{21} + Z_{21}(z) \tau_1(z - \frac{p_1^2}{2\mu_1}) X_{11}(z) + Z_{23}(z) \tau_2(z - \frac{p_2^2}{2\mu_2}) X_{21}(z).$$

The functions $\tau_\alpha(z)$, $Z_{\alpha\beta}$, and $X_{\alpha\beta}$ ($\alpha, \beta = 1, 2$) can be expressed in terms of the AGS transition operators $U_{\beta\alpha}$ and form factors $|\chi_\alpha\rangle$ appearing in \[3\] via

$$Z_{\beta\alpha}(z) \equiv (1 - \delta_{\alpha\beta}) \langle \chi_\beta | G_0(z) | \chi_\alpha \rangle \quad (5)$$
$$X_{\beta\alpha}(z) \equiv \langle \chi_\beta | U_{\beta\alpha} G_0(z) | \chi_\alpha \rangle \quad (6)$$

where $G_0(z)$ is the free $\eta_c NN$ propagator. As mentioned, for the $NN$ subsystem we use a separable representation of the Paris potential

$$V_{NN}(k, k') = -g(k) g(k'), \quad \text{where } g(k) = \sum_{i=1}^6 \frac{C_i}{k^2 + \beta_i^2}, \quad (7)$$

with parameters $C_i$ and $\beta_i$ listed for the $^3S_1$ configuration in \[1\].

For the $\eta_c N$ interaction we first consider the rank-one representation of the $\eta_c N$ potential \[8\] within the Bateman method \[12\]. The corresponding general expression reads

$$V_0^{[N]}(k, k') = \sum_{i,j}^N V_0(k, s_i) d_{ij}^{-1} V_0(s_j, k'), \quad (8)$$

with $d_{ij} = V_0(s_i, s_j)$. Here $s_i$ is an expansion parameter and

$$V_0(k, k') = \frac{1}{2\pi^2} \int_0^\infty j_0(k r) V(r) j_0(k'r) r^2 dr \quad (9)$$

the $s$-wave Fourier transform of the potential $V(r)$. Using expression \[2\] for the potential $V(r)$ in the integral \[9\] and taking $N = 1$ and $s_1 = 0$ in the expansion \[8\] one can readily obtain

$$V_0^{[1]}(k, k') = \frac{V(k, 0)V(0, k')}{V(0, 0)} = -4\pi a \frac{\alpha^2}{(k^2 + \alpha^2)(k'^2 + \alpha^2)}. \quad (10)$$

If the Bateman method is applied to a negative potential, the next terms in the separable expansion tend to increase the attraction. Therefore, the neglect of the higher order terms in the sum \[8\] will result in underestimation of the
FIG. 1: Binding energy of $\eta_c d$ system, as function of the strength $a$ of the Yukawa potential. The results obtained with the Bateman and Hilbert-Schmidt representation of the $\eta_c N$ potential are presented by the dashed and the solid line, respectively. The absolute value of the $\eta_c d$ binding energy. An obvious advantage of the Bateman method is that it provides analytical expressions for all terms in (8). It is worthwhile to note that the potential predicts an antibound (virtual) state in the $\eta_c N$ system lying at 

$$\epsilon = -\frac{1}{2\mu} \left( \sqrt{\frac{a\mu\alpha^3}{4\pi}} - a \right)^2 = -6.1 \text{ MeV},$$

(11)

where $\mu$ is $\eta_c N$ reduced mass. The corresponding scattering length

$$a_{\eta_c N} = \frac{2}{4\mu - \alpha} \approx 0.97 \text{ fm}$$

(12)

is a bit larger than the $\eta N$ scattering length predicted by the modern analyses (see e.g. compilation in [10]). Furthermore, there is no absorptive part in the $\eta_c N$ potential, which in the $\eta N$ case effectively increases the repulsive properties of the interaction. The most important difference between $\eta N$ and $\eta_c N$ is the larger mass of the $\eta_c$ meson, which should increase the role of attraction in the $\eta_c$ few-nucleon dynamics. According to this, one can expect that the $\eta_c$ meson should be more tightly bound to the nucleons than the $\eta$ meson. The energy of the $\eta_c d$ bound state, calculated with the potential as a function of the strength parameter $a$, is presented in Fig. 1 by the dashed line.

As a next step we employ the Hilbert-Schmidt method for the separable representation of the potential. The corresponding expansion reads

$$V_{\eta_c N}(k, k') = \sum_{n=1}^{N} \lambda_n v_n(k) v_n(k'),$$

(13)

where the form factors $v_n(k)$ and strength parameters $\lambda_n$ obey the homogeneous integral equation

$$v_n(k) = \frac{1}{\lambda_n} \int_0^\infty V_{\eta_c N}(k, k') \frac{v_n(k')}{B - \frac{k'^2}{2\mu}} \frac{dk'}{2\pi^2}.$$  

(14)

$B = 0$ was chosen in the actual calculation.

III. RESULTS

The $\eta_c d$ binding energy calculated for the potential with $N = 6$ terms is presented in Fig. 1. As expected, the last model predicts stronger binding than the leading term in the Bateman expansion. Furthermore as one can
see, in contrast to the results of [5] and [11], we find only one bound state lying in the region $-2 \, \text{MeV} \leq E_b \leq -5 \, \text{MeV}$, when the parameter $a$ is varied between 0.5 and 0.6 GeV.

**TABLE I:** Dependence of the $\eta_c d$ binding energy $E_b$ on the number of separable terms $N$ in the expansion [13]. Parameter $a$ is equal to 0.55.

| $N$  | 1    | 2    | 3    | 6     |
|------|------|------|------|-------|
| $E_b$ [MeV] | -3.429 | -3.251 | -3.217 | -3.196 |

The results in Table I show the accuracy of the Hilbert-Schmidt method for the $\eta_c d$ binding energy. Evidently, already the first two terms in the sum [13] provide quite a satisfactory approximation. Going up to $N = 6$ results in only 1.7% increase of $|E_b|$.

Now we turn to the four-body $\eta_c ^3 \text{He}$ system. For the calculation we use the formalism developed in Ref. [17] and Ref. [18] for the interaction of isoscalar-pseudoscalar mesons with three-body nuclei. Since the formal part of the problem is quite involved we do not present it here and refer the reader to these references. It should only be recalled that the calculations are based on separable representations of the two- and three-body kernels of the basic AGS equations so that, as a consequence, the resulting integral equations have the same structure as the three-body equations [1].

**TABLE II:** The binding energy of $\eta_c ^3 \text{He}$ system (in MeV).

| $a$ | 4-Body calculations | Variational calculation [5] | Folding model [11] |
|-----|---------------------|-----------------------------|-------------------|
| 0.4 | -1.3                | -3.0                        |                   |
| 0.6 | -14.5               | -19.0                       | -0.8              |

Let us compare our results for the binding energy of $\eta_c ^3 \text{He}$ system with variational [5] and folding model calculations [11]. One should emphasize that the variational method used in [5] is not an ab initio many body treatment. Rather it is a model of meson-nuclear interaction, postulating a Yukawa form for the meson-nuclear potential with a range depending on the number of nucleons in the nucleus. This model, as well as the folding model, takes into account the important effect of smearing the meson-nucleon interaction over the nuclear volume. However, the multiple scattering of mesons in the nucleus is not accurately accounted for. Our results of the microscopic four-body calculations of the $\eta_c ^3 \text{He}$ binding energy are presented in Fig. 2 and compared with the alternative ones in Table II. The calculations are performed with the value $a = 0.6$ GeV for the range parameter of the $\eta_c N$ potential.

In summary, we calculated the binding energies of $\eta_c d$ and $\eta_c ^3 \text{He}$ using three- and four-body equations of the AGS theory. The results exhibit essential differences to the predictions of the folding model as well as to the variational calculations. The characteristic values of the binding energies are about $-3 \, \text{MeV}$ for the $\eta_c d$ system and amount to $-14.5 \, \text{MeV}$ for $a = 0.6$ in the $\eta_c ^3 \text{He}$ case.

**Acknowledgment**

This work was supported by the Deutsche Forschungsgemeinschaft and the Russian Foundation for Basic Research, project 436RUS113/761.

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FIG. 2: Dependence of the $\eta_c^3\text{He}$ binding energy on the parameter $a$ in \cite{5}.