Research Article

Energy Dissipation and Damage Evolution Characteristics of Shale under Triaxial Cyclic Loading and Unloading

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1. Introduction

In the process of shale gas exploitation, the underground stress condition is very complex, and the shale is constantly damaged in the deformation process, which is accompanied by the absorption, accumulation, and release of energy, and the release of energy inside the shale is the main cause of engineering accidents. Therefore, from the perspective of mechanical behavior, the deformation and failure of rock under different loading states is a process from crack initiation, plastic deformation, to global failure. According to the law of thermodynamics, energy transformation is the essential characteristic of physical change, and material destruction is the state instability phenomenon driven by energy [1]. Therefore, rock failure is also a kind of instability phenomenon driven by energy; energy plays a fundamental role in rock deformation and failure process; the ability of rocks to absorb, accumulate, dissipate, and release energy is also a way to evaluate the rock-bearing capacity.

The deep rock mass is in a state of high in-situ stress, and the stress state of the rock mass is redistributed under the influence of excavation. Taking shale gas extraction as an
example, the instability of shale gas wellbore surrounding rock, such as wellbore collapse, sticking, and borehole expansion, has become a prominent and significant problem in the exploration and development of shale gas resources. In the process of shale gas extraction, the wellbore surrounding rock experiences multiple cycles of loading and unloading, and its mechanical properties and deformation mechanism are very complex. Since the fundamental cause of rock damage and yielding is energy-driven [2], studying the damage evolution and deformation characteristics of shale under cyclic loading and unloading from the perspective of dissipated energy can more essentially reveal its strength failure and destabilization mechanism, and can provide a new perspective for stability analysis of wellbore surrounding rock.

Gong et al. [3] investigate the energy evolution characteristics of rock materials under uniaxial compression. For rock materials, input energy density, elastic energy density, and dissipated energy density non-linearly increase with increasing unloading stress levels. The input energy is mainly converted into elastic energy and dissipated energy during the experiment. Elastic energy density and dissipated energy density linearly increase with an increase in input energy density. Meanwhile, the ratio of the elastic energy to dissipation energy of rock materials can reflect the damage of the rock. The ratio of dissipated energy to elastic energy increases gradually with the increase in cycle times and reaches the maximum at peak stress.

Xie et al. [1, 2, 4] pointed out that the deformation and failure of rock is a process of exchanging energy with the external environment, and revealed the intrinsic connection between rock deformation and failure, energy dissipation, and energy release. Singh [5] proposed the burst energy release index to characterize the degree of rock damage. It was found that the stronger the energy storage capacity of the rock, the greater the energy release index when reaching the peak strength, and the greater the damage degree. Mikhailuk and Zakharov [6] considered the elastic limit of the rock and carried out relevant experiments to study the energy dissipation of the rock during the elastic deformation stage. Munoz et al. [7] analyzed the energy characteristics of the rock pre-peak stage based on conventional uniaxial and triaxial rock compression tests. Meng et al. [8, 9] carried out triaxial cyclic loading and unloading tests on sandstone, marble, and limestone, and revealed the relationship between the energy evolution of rock samples and the confining pressure, cycle times, and stress. Ai et al. [10] analyzed the axial stress-axial strain curves and proposed a brittleness evaluation standard based on the energy principle to reveal the correlation between rock brittleness and energy evolution. Li et al. [11] conducted triaxial compression tests on different types of granite specimens under different loading and unloading stress paths, and found that the total strain energy, elastic strain energy, and circumferential strain energy all increased with the increase in the initial confining pressures, while the dissipated energy decreases. During unloading, hard rock is more prone to rock burst than soft rock under conventional triaxial loading conditions. Zhang et al. [12] did triaxial cyclic loading and unloading tests on sandstone under six different confining pressures. The total absorbed energy density, elastic energy density, and dissipated energy density of rock specimens under different confining pressures were obtained. The influence mechanism of confining pressure on energy evolution was analyzed. Zhou et al. [13] studied the post-peak energy evolution characteristics of Beishan granite under cyclic loading and unloading based on the acoustic emission test. The above researches revealed the basic characteristics of energy evolution of rocks under load to failure: a large amount of elastic energy is accumulated in the rock before the peak strength, while the stored elastic energy in the rock after the peak strength is rapidly released and transformed into the energy consumed by rock breakage.

The process of rock loading is a gradual process of damage, and the dissipated energy is mainly used for the generation of new fractures. The results are closer to the nature of rock damage by exploring the damage evolution during rock loading from the perspective of energy dissipation. He et al. [14] introduced the dissipated energy coefficient to study the energy evolution characteristics of rocks and proposed a method to determine the rock burst proneness and crack propagation in rocks. Liu et al. [15] established a damage constitutive model of rocks from the perspective of dissipated energy to describe the behavior of rocks under cyclic loading. Wang et al. [16] used the constitutive model and multi-criteria model established for rock failure from the perspective of energy. Zhao et al. [17] found that the variation of dissipated energy can be used to define the damage variable of the rock, and the damage variable also increases with the increase in loading rate. Li et al. [18] established a damage model of fractured rocks related to dissipated energy. Chen and Guo [19] analyzed the fracture closure effect and the relationship between fracture parameters and height-diameter ratio (H/D) through the uniaxial compression test of sandstone. A nonlinear model of energy dissipation based on the crack closure effect was established. Pei et al. [20] carried out cyclic loading and unloading tests on granites under six different confining pressures, and proposed four energy parameters, namely energy storage rock, energy storage limit, energy storage ratio, and energy dissipation ratio, to describe the energy storage and energy dissipation characteristics of rocks. Duan and Yang [21] conducted cyclic loading and unloading tests on sandstone, and found that dissipated energy can represent fatigue deformation of rock samples, and the curves of dissipated energy-cycle numbers present a U-shaped curve. Wang et al. [22] carried out triaxial compression tests on marble with different bedding angles, and found that compared with conventional triaxial tests, more energy was consumed for complete failure of rock samples under stress disturbance. Wang et al. [23] performed fatigue tests under different maximum cycling stresses on salt rock specimens to study the microstructural variations and damage evolution of salt rock under cyclic loading; the fatigue damage evolution equation of salt rock under cyclic loading was established based on the change in porosity. Hou et al. [24] combined microscopic tests and a thermal damage simulation to evaluate the changes in the internal structure of the
coal caused by the LN2 cooling. Xue et al. [25] conducted triaxial compression tests of coal under different gas pressure conditions, and the mechanical properties, acoustic emission (AE) energy characteristics, and nonlinear characteristics of the energy evolution of coal were obtained.

When the external force applied to the rock is in the elastic range (below the lower threshold), there will be no plastic deformation or crack initiation. When the force reaches the upper threshold of strength, failure will occur. When the force is between the upper and lower thresholds, each loading will cause damage to the rock. With the accumulation of damage caused by cyclic loading, when a certain damage threshold is reached, instability failure will occur. The physical manifestations of damage include plastic deformation, crack propagation, and crack friction, which can be expressed as the weakening of the elastic modulus in mechanical properties and energy dissipation in energy level. Therefore, how to quantitatively describe the damage characteristics of rocks subjected to cyclic loading and evaluate its stability (such as shale gas reservoir rock) has always been the focus of research.

In this paper, the variation trend of plastic strain under cyclic loading was firstly discussed. Considering damage mechanics, a plastic strain damage model including confining pressure parameters, damage accumulation factor, and average damage factor was established. Based on the conservation law of energy, the evolution law of total energy, elastic energy, and dissipated energy under cyclic loading was analyzed, and the relationship between the energy dissipation ratio and plastic strain was discussed. The criterion of rock failure is established. This research provides theoretical guidance for the quantitative analysis of the mechanical response and parameters (strength, elastic modulus, etc.), degradation of rocks during the excavation and unloading process of underground engineering (such as drilling, underground chamber, carbon sequestration, tunnel excavation), and the failure mechanism of rock under cyclic load is studied from the perspective of energy. These results can provide a theoretical basis for support design, stability analysis, and risk prediction in the process of underground engineering development.

2. Experimental Design

2.1. Sample Preparation. The shale samples were taken from Jiangjin District, Chongqing, and were collected in strict accordance with the relevant test procedures of the International Society for Rock Mechanics [26] to prepare $50 \text{mm} \times 100 \text{mm}$ cylindrical samples. Before the test, in order to ensure the flatness of the upper and lower faces of the rock samples, the samples were drilled perpendicular to the bedding plane of shale. Parts of the specimens are shown in Figure 1.

2.2. Experimental Equipment. To explore the energy evolution characteristics of loaded rock samples under different confining pressures, the experimental stress path of constant confining pressure and unloading axial pressure was designed by applying a constant axial displacement increment at each loading cycle, and triaxial cyclic loading and unloading tests under four confining pressures were carried out. It is worth noting that this study mainly focuses on the energy evolution characteristics of shale before failure; the tests were ended after the sample reaches its peak strength.

The tests were conducted on TFD–2000 (as shown in Figure 2). The equipment comprises axial loading, confining pressure loading, hydraulic pressure loading, numerical control, and a measuring system. The maximum axial load is 2000 kN and the measurement resolution is 10 N. The maximum confining pressure is 100 MPa and the measurement resolution is 0.001 MPa. The maximum hydraulic pressure is 70 MPa, the maximum axial deformation is 10 mm, the measurement accuracy is ±0.5%, and the temperature range is from atmospheric temperature to 200°C. This equipment can perform controlled tests and data analysis by computerized and robotized operations, which ensures the accuracy, timeliness, and safety of the test results.

2.3. Experimental Procedure. The procedure of triaxial cyclic loading and unloading tests can be described as follows. Firstly, an initial axial load of 1.0 kN was applied to fix the rock sample; then the confining pressure was applied to the design value (10, 15, 20, 30 MPa) by stress control at a speed of 0.5 kN/s. The confining pressure was kept constant during the whole test process. The displacement control rate of 0.05 mm/min was used to apply the axial stress up to each unloading point (10, 20, 30, 40, 50 MPa), and then the axial stress was unloaded to 1 MPa and reloaded to the previous unloading stress at the unloading and reloading rate of 0.5 kN/s. Upon reaching the unloading stress value of the last cycle, the displacement method was used to continue the next loading cycle at a rate of 0.05 mm/min. The schematic diagram of the loading path is shown in Figure 3, and the blue and green lines are displacement loading and stress loading paths, respectively. The recording frequency of stress and strain during the tests was 0.1 s/time and the test scheme is shown in Table 1.

2.4. Energy Calculation Method. According to the first law of thermodynamics, thermal energy can be transferred, its form can be transformed, and total energy in various forms...
is constant in the process of transfer and conversion. In the process of cyclic loading and unloading, the work input by the testing machine is transformed into mechanical energy and heat energy \[27, 28\]. Therefore, the work input by the testing machine is the main source of mechanical energy of the rock sample. For the reason that rock is a typical elastic-plastic material, the input mechanical energy is mainly divided into two types: one is stored in the form of elastic energy, and the other is the dissipated energy used for crack expansion, friction, and plastic deformation. In the triaxial cyclic loading and unloading experiments, axial stress \(\sigma_1\) exerts positive work on rock samples. Under the condition of triaxial constant confining pressure, for a unit volume of rock sample sheet, the stress state is biaxial compression, and there are both axial strain \(\varepsilon_1\) and radial strain \(\varepsilon_2\) and \(\varepsilon_3\). To simplify the calculation, we assumed that \(\varepsilon_2\) was equal to \(\varepsilon_3\);
thus, when we evaluated \( u_3 \), the value of \( u_3 \) was twice of the unilateral radial diffusion energy. Confining pressure \( \sigma_3 \) exerted negative work by restraining lateral deformation on rock samples. In this study, it was assumed that the whole test process was a closed system and there was no heat exchange with external environment, according to the law of conservation of energy,

\[
u = u_1 + u_3 = \int_0^{\varepsilon_1} \sigma_1 \, d \varepsilon_1 + 2 \int_0^{\varepsilon_3} \sigma_3 \, d \varepsilon_3, \tag{1}\]

\( u \) is the total energy input from the electrohydraulic servo testing machine, \( u_1 \) is the axial strain energy, and \( u_3 \) is the energy consumed for radial deformation; \( u_1^e \) is the elastic energy accumulated in the rock, and \( u_3^d \) is the energy dissipated due to pore compaction, plastic deformation, and crack propagation. \( \sigma_1 \) is the axial stress and \( \sigma_3 \) is the confining pressure, \( \varepsilon_1 \) is the axial strain, and \( \varepsilon_3 \) is the radial strain; considering that the distribution of the annular deformation along the rock sample is not uniform during the compression process, especially after the rock sample was damaged, this effect was more obvious. Therefore, when calculating according to (1), it was assumed that \( \varepsilon_3 \) is 1/2 of the central annular deformation value of the measured rock sample.

As shown in Figure 4, the area of the loading stress-strain curve projected onto the X coordinate axes is the total energy input to the rock sample, i.e., the work done by the testing machine on the rock sample. The area of the unloading stress-strain curve projected onto the X coordinate axes is the elastic energy accumulated in the rock sample, and the difference between the total energy and the elastic energy is the dissipated energy.

\[
u_1^d = u_1 - u_1^e = \int_0^{\varepsilon_1} \sigma_1 \, d \varepsilon_1 - \int_0^{\varepsilon_1^d} \sigma_1 \, d \varepsilon_1, \tag{2}\]

where \( u_1 \) is the total energy of a single loading and unloading, including the energy input by the machine and the energy consumed by radial strain expansion, \( u_1^e \) is the elastic energy accumulated in the rock sample, \( u_1^d \) is the dissipated energy, \( \varepsilon_1 \) is the strain corresponding to the unloading point, \( \varepsilon_1^d \) is the corresponding strain when unloading is completed, and \( \sigma_1 \) is the stress.

As a discontinuous, elastic-plastic material with initial defects, the rock dissipated energy irreversibly closes the micropores and natural cracks, reduces plastic deformation, and propagates cracks during the loading process; thus, the dissipated energy exists in the whole process of rock deformation from compaction to failure. Obviously, the damage of the rock is positively correlated with the amount of dissipated energy; therefore, the energy dissipation ratio (the ratio of dissipated energy to input energy) is proposed to characterize the damage inside the rock and is calculated as:

\[
\eta = \frac{1 - u_1^e}{u} = \frac{u_1^d}{u}. \tag{3}\]

As a parameter to describe the damage characteristics of the rock such as plastic deformation and crack propagation, the ratio of dissipated energy to total input energy reflects the degree of rock damage under different stress states to a certain extent.

3. Experimental Results

3.1. Mechanical behavior. The triaxial cyclic loading and unloading tests were carried out on shale, and the confining pressure was kept constant in the loading process to restore the in-situ stress state of the rock. The axial load was imposed on the sample, and the rock’s deformation occurred in each cycle (including the axial strain and radial strain). When the bearing capacity of the rock sample was reached, the rock sample was destroyed. Through this test, the influence of reciprocating load on the rock mass was simulated. The axial stress-axial strain curves of typical shale samples obtained from the experiments were presented in Figure 5. It can be found that the peak strength and peak strain increase with the increase in confining pressures. The greater the confining pressure, the greater the peak strength of rock samples. Meanwhile, the rock changed from brittleness to ductility under higher confining pressures, and the peak strain of rock samples increases with the increase in confining pressure. It can be seen from Figure 5 that in the process of a single cyclic loading and unloading, the unloading curve does not completely coincide with the loading curve and is lower than the loading curve due to damage and elastic hysteresis. As a geological material, micropores and natural cracks contained in the rock make the unloading curves unable to completely coincide with the loading curves. As an experimental phenomenon, hysteresis loop is not only a manifestation of stress path, but also a manifestation of energy dissipation. With the increase in loading times, the area of the hysteresis loop gradually increases, which means that the dissipated energy is larger. Before failure, the area of the hysteresis loop is proportional to the number of cycles. After reaching the peak strength, shear failure occurs in the rock sample with internal cracks converging to a main fracture surface, and the dissipated energy reaches the largest.
3.2. Energy Evolution Characteristics. According to the triaxial cyclic loading and unloading test curves and the energy calculation method in section 2.4, the evolution law of total energy, elastic energy, and dissipated energy of shale under cyclic loading and unloading under different confining pressures can be obtained. As can be seen from Figure 6, energy evolution of rock samples under different confining pressures is similar, and the energy of rock samples under different confining pressures shows nonlinear growth with an increase in strain. In the elastic stage, the growth rate of elastic energy is slow, and in the plastic stage, the elastic energy increases rapidly. When the peak strength is reached, elastic energy is released rapidly, resulting in macroscopic fractures of the sample. The dissipated energy also presents a nonlinear growth trend with the increase in strain, with slow growth in the elastic stage and sharp growth in the plastic stage. The maximum values of input energy, dissipated energy, and energy dissipation ratio all appear at the peak strength point. In the pre-peak strength stage, the difference between the input energy and the dissipated energy increases gradually with the increase in the number of cycles, indicating that most of the energy input by the testing machine is accumulated in the form of elastic energy. When reaching the peak strength point, a large amount of energy is consumed due to the expansion of a large number of fractures and the bearing capacity of rock samples decreases gradually until failure.

The damage of rock samples under specific stress or strain conditions can be reflected by the change in the energy dissipation ratio. As shown in Figure 6, the energy dissipation ratio-axial strain curves of rock samples under different confining pressures presents a “scoop” shape. The energy dissipation ratio of rock samples decreases linearly first (the stages of compaction), then develops steadily (the stages of elasticity), and then increases rapidly (the stages of plastic), respectively. In the elastic stage, rock samples are in a stable state, elastic energy and dissipation energy both increase slowly, most cracks in rock samples gradually close, and the energy dissipation ratio decreases relatively slowly. The junction of the elasticity stage and the plastic stage is the...
turning point between the elastic deformation stage and the plastic deformation stage, and the energy dissipation ratio reaches minimum at this turning point. After entering the plastic deformation stage, the growth rate of the dissipated energy increases rapidly, and the energy dissipation ratio increases gradually. In this stage, new cracks and plastic deformation begin to occur inside the rock samples. Before approaching the peak strength, the dissipated energy of rock samples increases due to a large number of new cracks, but the rock sample has not yet failed. At this point, the elastic energy stored in the rock sample reaches the energy storage limit of the rock samples, and the main fracture is about to penetrate. When the peak stress is reached, shear failure occurs in the rock sample with internal cracks converging to a main fracture surface, and the slip of the main fracture dissipates a large amount of energy derived from the accumulated elastic energy as well as the input energy. The rock samples’ failure behavior made the dissipated energy increase sharply, and the energy dissipation ratio reaches its maximum.

3.3. Damage Characteristics of the Specimens Under Cyclic Loading. Damage mechanics is an important way to study rock properties. Density [29], wave velocity [30], elastic modulus [31], and plastic strain [32] are usually used to quantitatively describe the deterioration of rock mechanical properties. The generation of plastic strain is the most intuitive physical manifestation of rock damage, and the accumulation of plastic strain is the cause of rock failure. Triaxial cyclic loading and unloading tests were carried out using the TFD-2000 test system, and the cracks of the rocks were propagated to different degrees and then caused damage. In order to better describe the variation of plastic strain under cyclic loading, the following equation is used to reveal the relationship between plastic strain and axial strain:

$$\varepsilon_c = \varepsilon_m - \beta \log \left( \frac{k}{\varepsilon - \lambda} \right),$$

(4)

$\varepsilon_c$ is the plastic strain under single cyclic loading, $\varepsilon$ is the axial strain corresponding to $\varepsilon_c$, and $\varepsilon_m$ is the average value of the
plastic strain of multiple cyclic loading and unloading, representing the cumulative damage of the whole loading process. $\beta$ is the slope of the stable development stage of the plastic strain curve of rock, $k$ is the confining pressure value, $\lambda$ is the damage accumulation rate factor in the crack development stage, and its value range is $0 - k/\varepsilon$. According to the test results, the variation of plastic strain in single cycle of rock under different confining pressures was obtained, as shown in Figure 7. In the first cycle, the plastic strain increases rapidly under the action of compaction, while the plastic strain value caused by the intermediate cycle increases linearly. In critical failure, the cyclic load accelerates the crack propagation and transfixion, resulting in a sharp increase in plastic strain. With the increase in confining pressure, the number of cycles increases, indicating that the existence of confining pressure enhances the bearing capacity and the deformation capacity of rock samples. Under cyclic loading and unloading conditions, the plastic strain increases with an increase in strain.

As can be seen from Figure 4, input energy is mainly divided into elastic energy and dissipated energy, while dissipated energy is mainly used for the plastic of rock. Plastic deformation occurs in rocks under cyclic loading, and the ultimate macroscopic failure of rocks is caused by accumulative plastic strain exceeding its bearing capacity. Dissipated energy is the energy that causes microcrack propagation, plastic deformation, and stiffness deterioration of rocks. Therefore, the relationship between the energy dissipation ratio and plastic strain is analyzed. The function relation between the energy dissipation ratio and plastic deformation is established:

$$\eta = a\varepsilon_c + \frac{b}{\varepsilon_c} + d,$$

(5)

where $a$, $b$, $c$, and $d$ are fitting parameters.
The energy dissipation ratio-plastic strain curves under different confining pressures were drawn by (5). The fitting curves were compared with the experimental curves (as shown in Figure 8), and the fitting curves were found to be similar to the experimental curves showing a “spoon” shape.

3.4. Correlation Between the Energy Dissipation Ratio and Elastic Modulus Evolution. Elastic modulus is a basic mechanical parameter of rock, and the weakening of elastic modulus during loading and unloading reflects the internal structural damage of rocks. Dissipated energy is the intrinsic reflection of rock damage, the elastic modulus of single cycle loading and unloading is calculated by the tangent modulus of the loading stage. To explore the correlation between mechanical characteristic degeneration and the evolution of dissipated energy of shale, the elastic modulus-strain curves (Figure 9) were drawn to compare with the energy dissipation ratio-strain curves, and the equation of correlation was established.

It can be seen from Figure 9 that the elastic modulus of rock samples increases at first, then gently develops, and finally decreases; the change of the elastic modulus reflects the stress state of the rock, namely compaction process, elastic stage, plastic stage, and post-peak stage. Meanwhile, the change of the elastic modulus is also driven by energy; the change in the energy dissipation ratio reflects the damage in the rock, which leads to a change in the elastic modulus.
Figure 9: Elastic modulus—strain curve during cyclic loading and unloading.

(With the progress of the loading, in stages I and II, the elastic modulus increases greatly, and in stage III, the elastic modulus decreases slightly).

The maximum elastic modulus of 10, 15, 20, and 30 MPa is 31.4, 37.8, 37.9, and 41.9 GPa, respectively. Compared with Figure 6, the variation trend of the elastic modulus and the energy dissipation ratio is the opposite; the ratio of the dissipated energy to input energy reflects the weakening characteristics of the elastic modulus to a certain extent. From the energy point of view, the dissipated energy can be used as a factor of the damage and crack propagation of rocks. From the perspective of mechanical characteristics, the decrease of the elastic modulus can also reflect the damage degree of the rocks. Therefore, the energy dissipation ratio is used to characterize the degradation of the elastic modulus, and the relationship between the elastic modulus and the energy dissipation ratio can be expressed as:

\[ E = \varphi E_0 (1 - \eta). \]  

\( E_0 \) is the maximum loading modulus in the cyclic loading and unloading process, \( E \) is the elastic modulus corresponding to the energy dissipated ratio, and the value of \( \varphi \) depends on the confining pressure.

3.5. Strength Failure Criterion Based on Energy Dissipation and Strain Damage. In the underground space, the ultimate instability failure of rocks usually occurs when the internal damage of rocks accumulates to a certain threshold under cyclic loading. In Section 3.3, we discussed the relationship between plastic strain and total strain. Under cyclic loading, as the total strain increases, the plastic strain also gradually increases. When the plastic strain reaches a certain value, the internal damage of rock sample reaches the limit that it can bear, leading to the failure of rock sample.

As shown in Figure 4, the total strain of single loading and unloading consists of recoverable strain and plastic strain:

\[ \varepsilon_i = \varepsilon_i^e + \varepsilon_i^d, \]  

\( \varepsilon_i, \varepsilon_i^e, \) and \( \varepsilon_i^d \) are the total strain, elastic strain, and plastic strain of a single cycle loading and unloading, respectively.

According to (3), the relationship between the total strain, elastic deformation, and energy dissipation ratio of single loading and unloading is as follows:

\[ \varepsilon_i^d = (1 - \eta_i) \varepsilon_i, \]  

\( \eta_i \) is the energy dissipation ratio of a single cycle of loading and unloading.

The stress of the rock sample under load is divided into the effective elastic part and damage part. The effective elastic part satisfies Hooke’s law and can be expressed as:

\[ \sigma = E \varepsilon^e. \]  

The energy dissipation ratio of rock samples can be obtained using the above (8) and (10), and the corresponding stress under the strain can be obtained by the following equation:

\[ \sigma = \varphi E_0 (1 - \eta)^2 \varepsilon, \]  

where \( \varepsilon \) is the strain, and \( \sigma \) is the stress corresponding to \( \varepsilon \).

According to (5) and (10), the stress solution equation can be obtained,

\[ \sigma = \varphi E_0 \left[ 1 - \left( a \varepsilon_c + \frac{b}{c \varepsilon_c} + d \right) \right]^2 \varepsilon, \]

\[ \sigma = \varphi E_0 \left[ 1 - a \left( \varepsilon_m - \beta \log \left( \frac{k + \varepsilon - \lambda}{\varepsilon} \right) \right) \right. \]

\[ + \left. \frac{b}{c \left( \varepsilon_m - \beta \log \left( \frac{k + \varepsilon - \lambda}{\varepsilon} \right) \right) + d \right]^2 \varepsilon. \]

(12) can be used to fit the envelopes of cyclic loading and unloading stress-strain curves under different confining pressures, as shown in Figure 10. When \( \varphi \) is 1.2, 1.15, 1.1, and 1.05, respectively, the envelopes of the stress-strain curves in the pre-peak stage under 10, 15, 20, and 30 MPa confining pressures can be obtained. The equation includes confining pressure-related parameters \( \beta \) and \( k \), plastic strain-related parameters \( \beta \), and damage accumulation factor \( \lambda \) at the critical damage stage. Compared with the experimental curves, the theoretical axial stress-strain curves containing various parameters can well predict the stress-strain variation under cyclic loading and unloading. From Figure 10, it can be seen that the envelope curves calculated by (12) are in good agreement with the experimental curves. Under different confining pressures, the calculated peak strength points of the theoretical curves are basically consistent with the experimental curves. The stress and strain values of the actual strength failure, the stress and strain values of the theoretical...
strength failure, and relative errors are shown in Figure 10. The relative errors of stress and strain are less than 10%, which indicates the equation can predict the strength failure point.

4. Discussion

A large number of studies show that it is feasible to study the dynamic strength variation of rock under external load from the perspective of energy. Many researchers have put forward different viewpoints to define rock damage in terms of energy, and have achieved quite important research results [33–36], which greatly promote the application of the energy principle in geotechnical engineering.

In this study, the relationship between peak strain and plastic strain under cyclic loading was studied, and an equation containing many mechanical parameters was established. The relationship between energy dissipation and plastic deformation was investigated based on the energy principle. In this test, only triaxial cyclic loading and unloading of shale was carried out, and other rocks, such as brittle granite and soft marble, were not tested. Future research needs to be further deepened.

This paper studies the mechanical properties and energy dissipation characteristics of shale under general triaxial quasistatics loading conditions. The second stress and the third principal stress were the same in the test, which cannot reflect the influence of intermediate principal stresses on rock deformation and energy dissipation in the actual situation. In the future, relevant cyclic loading and unloading research will be carried out under true triaxial stress.
5. Conclusions

(1) In the triaxial cyclic loading and unloading experiments of shale, the greater the confining pressure, the greater the peak strength and peak strain values the rock samples can achieve. Due to the existence of confining pressure, the strength and deformation ability of rock samples are enhanced. With the increase in the number of cycles, the energy absorbed and dissipated by the rock sample increases, and the energy dissipated reaches the maximum at the peak strength. After failure, the rock sample still has a certain degree of bearing capacity.

(2) The equation of plastic strain and total strain including confining pressure and damage accumulation factor was established. It is found that plastic strain increases with the increase in total strain and confining pressure. It shows a trend of rapid increase at first, steady development, and sharp increase at last.

(3) The accumulation of damage is the cause of the ultimate failure of shale. The energy dissipated in the test process is mainly used for irreversible plastic deformation and crack propagation. The curve fitting method is used to fit the correlation between the energy dissipation ratio and plastic strain, and it is found that the energy dissipation ratios of shale under different confining pressures all show a “spoon” shape with an increase in strain.

(4) In general, the greater the confining pressure, the greater the elastic modulus. The elastic modulus increases first, then develops steadily, and finally decreases with an increase in strain. The ratio of the dissipated energy to input energy reflects the damage of the rock; therefore, the energy dissipation ratio is available to characterize the degradation of the elastic modulus.

(5) A theoretical stress-strain calculation equation was established based on the evolution characteristic of the dissipated energy, and the results obtained by the equation were used to draw the envelope curves of the stress-strain curves. It is found that the calculated results are basically consistent with the test data, as well as the peak strains and peak stress.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

Ziyun Li wrote the manuscript and administered the project; the original draft was prepared by Song Xie and the author was also responsible for project administration; Dongyan Liu and Qianghui Song were in charge of acquiring resources; Peiyong Wang and Baoyun Zhao acquired the funding for this project; Wei Huang curated the data. All authors have read and agreed to the published version of the manuscript.

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