Exploration of multiphoton entangled states by using weak nonlinearities

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We propose a fruitful scheme for exploring multiphoton entangled states based on linear optics and weak nonlinearities. Compared with the previous schemes the present method is more feasible because there are only small phase shifts instead of a series of related functions of photon numbers in the process of interaction with Kerr nonlinearities. In the absence of decoherence we analyze the error probabilities induced by homodyne measurement and show that the maximal error probability can be made small enough even when the number of photons is large. This implies that the present scheme is quite tractable and it is possible to produce entangled states involving a large number of photons.

Undoubtedly, entanglement\textsuperscript{1–4} is one of the most crucial elements in quantum information processing, e.g. quantum cryptography\textsuperscript{5}, quantum secure direct communication\textsuperscript{7,8}, etc. In recent years, quantum entanglement has been extensively investigated in various candidate physical systems\textsuperscript{9–16}, in particular, one can prepare and manipulate multipartite entanglement in optical systems\textsuperscript{17–24}.

Generally, a spontaneous parametric down-conversion (PDC) source\textsuperscript{29,30} is capable of emitting pairs of strongly time-correlated photons in two spatial modes. As extensions of interest, with linear optics and nonlinear optical materials several schemes for creating multiphoton entangled states have been proposed\textsuperscript{31–37}. For a large number of photons, however, there are some technological challenges such as probabilistic emission of PDC sources and imperfect detectors. A feasible approach is to use the simple single-photon sources, instead of waiting the successive pairs, and quantum nondemolition (QND) measurement\textsuperscript{38–45} with weak Kerr nonlinearities. Note that the Kerr nonlinearities\textsuperscript{46–48} are extremely weak and the order of magnitude of them is only $10^{-2}$ even by using electromagnetically induced transparency\textsuperscript{49,50}. More recently, Shapiro et al.\textsuperscript{51,52} showed that the causality-induced phase noise will preclude high-fidelity $\pi$-radian conditional phase shifts created by the cross-Kerr effect. In these cases, with the increase of the number of photons it is usually more and more difficult to study multiphoton entanglement in the regime of weak nonlinearities.

In this paper, we focus on the exploration of multiphoton entangled states with linear optics and weak nonlinearities. We show a quantum circuit to evolve multimode signal photons fed by a group of arbitrary single-photon states and the coherent probe beam. Particularly, there are only two specified but small phase shifts induced in the process of interaction with weak nonlinearities. This fruitful architecture allows us to explore multiphoton entangled states with a large number of photons but still in the regime of weak nonlinearities.

Kerr nonlinearities

Before describing the proposed scheme, let us first give a brief introduction of the Kerr nonlinearities. The nonlinear Kerr media can be used to induce a cross phase modulation with Hamiltonian of the form $H = \hbar \chi a^\dagger a a^\dagger p a p$, where $\chi$ is the coupling constant and $a (a^\dagger)$ represents the annihilation operator for photons in the signal (probe) mode. If we assume that the signal mode is initially described by the state $|\psi\rangle_s = c_0 |0\rangle_s + c_1 |1\rangle_s$ and the coherent probe beam is $|\alpha\rangle_p$, then after the Kerr interaction the whole system evolves as

$$e^{iHt} |\psi\rangle_s |\alpha\rangle_p = c_0 |0\rangle_s |\alpha\rangle_p + c_1 |1\rangle_s |\alpha e^{i\chi t}\rangle_p,$$

(1)

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where $\theta = \chi t$ with interaction time $t$. In order to distinguish different cases, one may perform a homodyne measurement on the probe beam with quadrature operator $\hat{\phi}(\phi) = a_{\phi} e^{i\phi} + a_{\phi}^* e^{-i\phi}$, where $\phi$ is a real constant.

Especially for $\phi = 0$, this operation is conventionally referred to as $X$ homodyne measurement; while for $\phi = \pi/2$, it is called $P$ homodyne measurement.

Creation of multiphoton entangled states with linear optics and weak nonlinearities

Let $a_i$, $i = 1, 2, \ldots, n$ represent input ports with respective spatial modes, namely signal modes, and $\alpha$ is a coherent beam in probe mode. The setup of creating multiphoton entangled states is shown in Fig. 1.

Without loss of generality we may suppose that each input port is supplied with an arbitrary single-photon state. Then, the total input state reads

$$|\Psi_m\rangle\!\langle\alpha| = \left( a_{\theta_0} |H\rangle^\otimes n + \sum_{\text{Perm}} a_{\theta_1} |H\rangle^\otimes (n-1) |V\rangle + \cdots \right. + \left. \sum_{\text{Perm}} a_{\theta_2} |H\rangle^\otimes (n-2) |V\rangle^\otimes 2 + \cdots \right)$$

$$+ \left. \sum_{\text{Perm}} a_{\theta_3} |H\rangle^\otimes (n-3) |V\rangle^\otimes 3 + \cdots \right)$$

$$+ \sum_{\text{Perm}} a_{\theta_4} |H\rangle^\otimes (n-4) |V\rangle^\otimes 4 + \sum_{\text{Perm}} a_{\theta_5} |H\rangle^\otimes (n-5) |V\rangle^\otimes 5 + \sum_{\text{Perm}} a_{\theta_6} |H\rangle^\otimes (n-6) |V\rangle^\otimes 6 + \cdots$$

where $a_{\theta_0}, a_{\theta_1}, a_{\theta_2}, \ldots, a_{\theta_n}$ are complex coefficients satisfying the normalization condition $\sum_{i_1, i_2, \ldots, i_n=0}^{1} |a_{i_1, i_2, \ldots, i_n}|^2 = 1$ and $\sum_{\text{Perm}}$ denotes the sum over all possible permutations of the signal modes, for example,

$$\sum_{\text{Perm}} a_{\theta_0} |H\rangle^\otimes (n-1) |V\rangle = a_{\theta_0} |H\rangle^\otimes (n-1) |V\rangle$$

$$+ a_{\theta_1} |H\rangle^\otimes (n-2) |V\rangle^\otimes 2 + \cdots$$

$$+ a_{\theta_2} |H\rangle^\otimes (n-3) |V\rangle^\otimes 3 + \cdots$$

Each polarizing beam splitter (PBS) is used to transmit $|H\rangle$ polarization photons and reflect $|V\rangle$ polarization photons. When the signal photons travel to the PBSs, they will be individually split into two spatial modes and then interact with the nonlinear media so that pairs of phase shifts $\theta$ and $2\theta$ are induced on the coherent probe beam, respectively. We here introduce a single phase gate $R_n(\theta) = -3n\theta/2$ so as to implement the next $X$ homodyne measurement on the probe beam. $\varphi_m(x)$, $m = 1, 2, \ldots, n/2$ for even $n$ and $m = 1/2, 3/2, \ldots, n/2$ for odd $n$, are phase shifts on the signal photons based on the measured values of $x$ via the classical feed-forward information.

At last, at the ports $b_i$, $i = 1, 2, \ldots, n$ one may obtain $n/2$ output states for even $n$ or $(n+1)/2$ states for odd $n$.

We describe our method in details. For $n$ is even, after the interaction between the photons with Kerr media and followed by the action of the phase gate, the combined system evolves as...
\[ |\Psi_{ck}\rangle = a_{00 \ldots 0}|H|^{\otimes n}|\alpha e^{-(n/2)i\theta}) + (\sum_{\text{Perm}} a_{100 \ldots 0}|H|^{\otimes (n-1)}|V\rangle) |\alpha e^{-(n/2-1)i\theta}) + (\sum_{\text{Perm}} a_{110 \ldots 0}|H|^{\otimes (n-2)}|V\rangle^{\otimes 2}) |\alpha e^{-(n/2-2)i\theta}) + \cdots + (\sum_{\text{Perm}} a_{11 \ldots 00}|H|^{\otimes (n/2)}|V\rangle^{\otimes n/2}) |\alpha e^{(n/2)i\theta}) + (\sum_{\text{Perm}} a_{11 \ldots 10}|H|^{\otimes (n-1)}|V\rangle^{\otimes (n-1)}) |\alpha e^{(n/2-1)i\theta}) + (\sum_{\text{Perm}} a_{11 \ldots 11}|H|^{\otimes n}|V\rangle^{\otimes n}) |\alpha e^{(n/2)i\theta}). \] (4)

In order to create the desired multiphoton entangled states, we here perform an X homodyne measurement\(^{38,45}\) on the probe beam. If the value \(x\) of the X homodyne measurement is obtained, then the signal photons become

\[ |\Psi_x\rangle = f(x, \alpha) \times \sum_{\text{Perm}} a_{11 \ldots 100 \ldots 0}|H|^{\otimes n/2}|V\rangle^{\otimes n/2} + f(x, \alpha \cos \theta) \times (e^{-i\theta} \sum_{\text{Perm}} a_{11 \ldots 100 \ldots 0}|H|^{\otimes (n/2+1)}|V\rangle^{\otimes (n/2+1)}) + \cdots + f(x, \alpha \cos [(n/2 - 1) \theta] \times (e^{-i\theta} \sum_{\text{Perm}} a_{11 \ldots 10}|H|^{\otimes (n/2-1)}|V\rangle^{\otimes (n/2-1)}) + e^{i\phi_m(x)} \sum_{\text{Perm}} a_{11 \ldots 11}|H|^{\otimes n}|V\rangle^{\otimes n}; \] (5)

where

\[ f(x, \alpha \cos (m\theta)) = (2\pi)^{-1/4} e^{-(x - 2\alpha \cos (m\theta))}; \] (6)

\[ \phi_m(x) = \alpha \sin (m\theta) \mod 2\pi, \] (7)

\( m = 0, 1, 2, \ldots, n/2 \), are respectively Gaussian curves which are associated with the probability amplitudes of the outputs, and

\[ \phi_m(x) = \alpha \sin (m\theta) \mod 2\pi, \] (7)

\( m = 1, 2, \ldots, n/2 \), are respectively phase factors based on the values of the X homodyne measurement. Note that the peaks of these Gaussian distributions locate at \(2\alpha \cos (m\theta)\) and the distances of two nearby peaks \(x_m = 2\alpha \cos (\theta/2) \cos (k - 1/2)\) with \(k = 0, 1, 2, \ldots, n/2\). Obviously, with these \(n/2\) midpoints, there exist \(n/2 + 1\) intervals and each interval corresponds to an output state.

We now consider the phase shifts \(\varphi_m(x)\). The signal photon evolves as \(\hat{b}_m^\dagger = e^{i\varphi_m(x)} \hat{a}_m^\dagger\), \(i = 1, 2, \ldots, n\). A straightforward calculation shows that

\[ \varphi_m(x) = \alpha \sin (m\theta) \mod 2\pi. \] (8)

After these feedforward phase shifts have been implemented and the signal photons pass through the PBSs, one can obtain the desired states as follows. Clearly, for \(x < x_{m/2}\) we have

\[ a_{00 \ldots 0}|H|^{\otimes n} + a_{11 \ldots 1}|V\rangle^{\otimes n}, \] (9)

for \(x_{m/2} < x < x_{m+1/2}\) we have

\[ \sum_{\text{Perm}} a_{11 \ldots 100 \ldots 0}|H|^{\otimes (n/2+k)}|V\rangle^{\otimes (n/2-k)} + \sum_{\text{Perm}} a_{11 \ldots 100 \ldots 0}|H|^{\otimes (n/2-k)}|V\rangle^{\otimes (n/2+k)}; \] (10)

and for \(x > x_m\) we obtain the state

\[ \sum_{\text{Perm}} a_{11 \ldots 100 \ldots 0}|H|^{\otimes n/2}|V\rangle^{\otimes n/2}. \] (11)
Similarly, for odd $n$, we have

$$|\psi_{ek}\rangle = a_{00\cdots 0}|H\rangle^{\otimes n}|\alpha e^{-(n/2)\theta}| + \left( \sum_{\text{Perm}} a_{100\cdots 0}|H\rangle^{\otimes (n-2)}|V\rangle^{\otimes 2}|\alpha e^{-(n/2-1)\theta} \right) + \cdots $$

$$+ \left( \sum_{\text{Perm}} a_{11\cdots 1 \ 00\cdots 0}|H\rangle^{\otimes (n-1)/2}|V\rangle^{\otimes (n-1)/2}|\alpha e^{-(1/2)\theta} \right) + \cdots $$

$$+ \left( \sum_{\text{Perm}} a_{11\cdots 1 \ 10\cdots 0}|H\rangle^{\otimes (n-1)/2}|V\rangle^{\otimes (n-1)/2}|\alpha e^{-(1/2)\theta} \right) + \cdots $$

$$+ \left( \sum_{\text{Perm}} a_{11\cdots 1 \ 10\cdots 1}|H\rangle^{\otimes (n-1)}|V\rangle^{\otimes n}|\alpha e^{-(n/2-1)\theta} \right).$$

(12)

Also,

$$|\psi_{ek}\rangle = f(x, \alpha \cos(\theta/2))\left( e^{-i\theta e(x)} \sum_{\text{Perm}} a_{11\cdots 1 \ 00\cdots 0}|H\rangle^{\otimes (n+1)/2}|V\rangle^{\otimes (n+1)/2} \right) + e^{i\theta e(x)} \sum_{\text{Perm}} a_{11\cdots 1 \ 00\cdots 0}|H\rangle^{\otimes (n+1)/2}|V\rangle^{\otimes (n+1)/2} + \cdots $$

$$+ f(x, \alpha \cos((n/2 - 1)\theta))\left( e^{-i\theta e(x)} \sum_{\text{Perm}} a_{11\cdots 1 \ 10\cdots 0}|H\rangle^{\otimes (n-1)/2}|V\rangle^{\otimes (n-1)/2} \right) + e^{i\theta e(x)} \sum_{\text{Perm}} a_{11\cdots 1 \ 10\cdots 1}|H\rangle^{\otimes (n-1)}|V\rangle^{\otimes n}$$

$$+ f(x, \alpha \cos(n\theta/2))\left( e^{-i\theta e(x)} a_{00\cdots 0}|H\rangle^{\otimes n} + e^{i\theta e(x)} a_{11\cdots 1}|V\rangle^{\otimes n} \right).$$

(13)

Here, the functions $f(x, \alpha \cos(m\theta))$, phase shifts $\phi_{mk}(x)$ and $\varphi_{mk}(x)$ are approximately the same as those described for even $n$, except for $m = 1/2, 3/2, \ldots, n/2$, and then the similar results hold for the midpoints $x_{mk}$ and the distances $x_{dk}$ with $k = 3/2, 5/2, \ldots, n/2$. Of course, with $(n-1)/2$ midpoint values $x_{mk}$ there may be $(n+1)/2$ output states; that is, one can obtain the states $a_{00\cdots 0}|H\rangle^{\otimes n} + a_{11\cdots 1}|V\rangle^{\otimes n}$ for $x < x_{mk}$:

$$x < x_{mk} < \sum_{\text{Perm}} a_{11\cdots 1 \ 00\cdots 0}|H\rangle^{\otimes (n+2-k)}|V\rangle^{\otimes (n-k+2)} + \sum_{\text{Perm}} a_{11\cdots 1 \ 10\cdots 0}|H\rangle^{\otimes (n-2-k)}|V\rangle^{\otimes (n+2-k)}$$

(14)

for

$$x_{mk} < x < x_{m(k+1)}, k = 3/2, \ldots, n/2 - 1,$$

(15)

and

$$\sum_{\text{Perm}} a_{11\cdots 1 \ 00\cdots 0}|H\rangle^{\otimes (n+1)/2}|V\rangle^{\otimes (n+1)/2} + \sum_{\text{Perm}} a_{11\cdots 1 \ 10\cdots 0}|H\rangle^{\otimes (n-1)/2}|V\rangle^{\otimes (n+1)/2}$$

for $x > x_{m(k+2)}$.

(16)

As an example of the applications of interest for the present scheme, we introduce a class of remarkable multipartite entangled states

$$|\psi_{kn}\rangle = \frac{1}{\sqrt{2}} \left( |\psi_{kn}^+\rangle + |\psi_{kn}^-\rangle \right),$$

(17)

where

$$|\psi_{kn}^+\rangle = \frac{1}{\sqrt{n}} \sum_{\text{Perm}} |e\rangle^{\otimes (n/2+k)} |l\rangle^{\otimes (n/2-k)}$$

(18)

and

$$|\psi_{kn}^-\rangle = \frac{1}{\sqrt{n}} \sum_{\text{Perm}} |e\rangle^{\otimes (n/2-k)} |l\rangle^{\otimes (n/2+k)}$$

(19)
are two orthonormal states, namely Dicke states\textsuperscript{53,54}. In view of its “catness”, the state $|\Psi_n\rangle$ can be referred to as a cat-like state, and especially for $k = n/2$ it can be expressed as the canonical $n$-partite Greenberger-Horne-Zeilinger (GHZ) state. In the present scheme, obviously, for $a_{i_1}\cdots a_{i_n} = 1/\sqrt{2^n}, i_1, i_2, \cdots, i_n = 0, 1$, we can obtain these cat-like states with $k = 0, 1, \cdots, n/2$ for even $n$ and $k = 1/2, 3/2, \cdots, n/2$ for odd $n$, where the qubits are encoded with the polarization modes $|H\rangle \equiv |0\rangle$ and $|V\rangle \equiv |1\rangle$. Of course, more generally, we may project out a group of multiphoton entangled states involving generalized Dicke states.

**Discussion**

There are two models commonly employed in the process of Kerr interaction, single-mode model and continuous-time multi-mode model\textsuperscript{11}. The former implies that one may ignore the temporal behavior of the optical pulses but the latter is causal, non-instantaneous model involving phase noise. It has been shown that\textsuperscript{27} this causality-induced phase noise will preclude the possibility of high-fidelity CPHASE gates created by the cross-Kerr effect in optical fiber. To solve this problem, one may need to find an optimum response function for the available medium, or to exploit more favorable systems, such as cavitylike systems\textsuperscript{28}. After all, the ultimate possible performance of Kerr interaction with a larger system is an interesting open issue. More recently, we note that Feizpour et al.\textsuperscript{28} showed the first direct measurement of the cross-phase shift due to single photons. It may be possible to open a door for future studies of nonlinear optics in quantum information processing. In the present scheme, we restrict ourselves to ignoring the phase noise and concentrate mainly on showing a method for exploring multiphoton entangled states in the regime of weak cross-Kerr nonlinearities, i.e. $\theta \ll \pi$.

It is worth noting that, there are only small phase shifts $\theta$ and $\theta/2$ instead of a series of related functions of the number of photons in the process of interaction with Kerr nonlinearities. This implies that the present scheme is quite tractable especially for creating entangled states with a larger number of photons. In addition, the error probabilities $\varepsilon_n$ are $\text{erfc}(x_n/\sqrt{2})/2$, which come from small overlaps between two neighboring curves. Considering the distances of two nearby peaks $x_n \approx (2k - 1)\alpha \theta^2$ with $k = 1, 2, \cdots, n/2$ for even $n$ and $k = 3/2, 5/2, \cdots, n/2$ for odd $n$, the maximal error probability $\varepsilon_{\max} = \text{erfc}(\alpha \theta^2/\sqrt{2})/2$, which is exactly the result described by Nemoto and Munro in\textsuperscript{40}. Obviously, the error probabilities in our scheme are no more than that one even when the number of photons is large. Therefore, by choosing an appropriate coherent probe beam the error probability can be reduced to as low a level as desired and then the present scheme may be realized in a nearly deterministic manner.

In summary, based on linear optics and nonlinearities we have shown a fruitful method for exploring a class of multiphoton entangled states, the generalized cat-like states. Evidently, three aspects are noteworthy in the present framework. First, since there are no large phase shifts in the interacting process with weak Kerr nonlinearities, our scheme is more feasible compared with the previous schemes. Second, the system is measured only once with a small error probability and it means that the present scheme might be realized near deterministically. Finally, the fruitful architecture allows us to explore a group of multiphoton entangled states involving a large number of photons, i.e., to produce entangled states approaching the macroscopic domain.

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Author Contributions

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Additional Information

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