Black Hole Dynamics in Power-law based Metric $f(R)$ Gravity

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In this work, we use power-law cosmology to investigate the evolution of black holes within the context of metric $f(R)$ gravity satisfying the conditions provided by Starobinsky model. In our study, it is observed that presently accelerated expansion of the universe can be suitably explained by this integrated model without the need for dark energy. We also found that mass of a black hole decreases by absorbing surroundings energy-matter due to modification of gravity and more the accretion rate more is mass loss. Particularly the black holes, whose formation mass is nearly $10^{30}$ gm and above are evaporated at a particular time irrespective of their formation mass. Again our analysis reveals that the maximum mass of a black hole supported by metric $f(R)$ gravity is $10^{12}M_\odot$, where $M_\odot$ represents the solar mass.

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I. INTRODUCTION

The recently observed accelerated expansion of the universe \cite{1, 2} has put a challenge for its theoretical understanding. To explain it, two general ways have been used in literature. First way is by introducing a new type of energy having negative pressure called dark energy \cite{3, 4} and the other way is by modifying the theory of gravity \cite{5, 6}. Essentially, dark energy models modify the energy-momentum tensor associated with the matter filling the universe, i.e. the right hand side of Einstein’s equation. Whereas, modified gravity theories make a change in Einstein’s gravity part, i.e. the left hand side of the said equation. Again for addressing the long-standing horizon, flatness and monopole problems \cite{11-13}, a phase of exponential expansion termed as inflation \cite{14} is thought to be occurred during early evolution of the universe. This inflation is believed to have provided the mechanism that generates primordial inhomogeneities, which could act as seeds for the formation of large structures \cite{12}. In between these two phases of acceleration, there must be a period of decelerated expansion during which primordial nucleosynthesis \cite{16, 17} to structure formation \cite{18, 19} were occurred.

The introduction of dark energy may predict the present accelerated expansion of the Universe but its nature and characteristics are unknown. Though cosmological constant \cite{7-10} seems to be the simplest candidate for dark energy, but its understanding as representing the vacuum energy of quantum field, used to describe the fundamental interactions, seems to be higher than observed value by 123 orders of magnitude. So dynamical dark energy models such as Quintessence \cite{20-22} and K-essence \cite{23} are proposed. Again there are discussion on an exotic form of dark energy, named as Phantom energy \cite{24-26}, which violates strong energy condition. There exist also modified matter dynamical dark energy models like Chaplygin gas \cite{27, 28}. But most of them are not able to explain all features of the universe, like for example coincidence problem: why the observed values of the cold dark matter density and dark energy density are of the same order of magnitude today although they differently evolve during the expansion of the universe. Also in some works \cite{29-33}, coupling between dark energy and dark matter are discussed, which can able to alleviate the coincidence problem. But till date no specific coupling in the dark sectors has been known, based on fundamental theories.

As an alternative to dark energy, different types of modified theories of gravity are discussed in literature. The action for modified theories of gravity are basically extensions of the Einstein-Hilbert action with an arbitrary function of the Ricci scalar $R$. These theories are of particular interest since they naturally appear in the low-energy effective actions of the quantum gravity \cite{34, 35} and String Theory \cite{36, 37}. In such theories both the early time inflation and the late-time acceleration of the universe could be resulted by a single mechanism. Again these theories play a major role in astrophysical scales. In fact modifying the gravity affects the gravitational potential in the low energy limit and the modified potential reduces to the Newtonian one on the solar system scale. Moreover, a corrected gravitational potential could offer the possibility to fit galaxy rotation curves without the need of huge amounts of dark matter \cite{38-40}. There are many ways to modifying the theory of gravity such as $f(R)$ gravity \cite{41, 48}, $f(T)$ gravity \cite{43, 50}, $f(R, T)$ gravity \cite{51, 53} etc., where $T$ is the trace of energy-momentum tensor. Among them simplest one is $f(R)$ gravity obeying metric formalism \cite{54, 55}.

For studying cosmological implication of $f(R)$ models, the existence of exact power-law solutions is discussed in literature \cite{54, 55}, corresponding to phases of cosmic evolution when the energy density is dominated by a perfect fluid. The existence of such solutions are particularly relevant because in FRW backgrounds, they typically represent
asymptotic or intermediate states in the phase space of the dynamical system representing all possible cosmological evolutions.

Again it is found that Schwarzschild type black holes could be formed in \( f(R) \) gravity satisfying Starobinsky model \([58]\). The most important thing about Starobinsky model \([59, 60]\) is that it could explain the inflationary scenario of early Universe. In this model the Lagrangian density is taken as \( f(R) = R + R^2/6M^2 \), where \( M^2 \) is a phenomenological constant having dimension of \( R \). During inflation, \( R^2 \) term provides a stage for the de-Sitter-like evolution of space-time. The inflationary potential has a stable minimum, which allows for the graceful exit, reheating and good low-energy limit of the theory.

In this work, we use power-law cosmology for studying the evolution of Schwarzschild type black holes in Starobinsky type metric \( f(R) \) gravity. We, here, assume that after inflation, the universe witnessed radiation-dominated era and then finally matter-dominated era. In our analysis, first, we show that how the density of the energy-matter filling the universe get changed due to modification of gravity and then try to explain the accelerated expansion of the universe in terms of it. Finally, we discuss the evolution of the black holes in this environment.

II. BASIC FRAMEWORK

For metric \( f(R) \) gravity, the action can be written as \([3]\)

\[
S = \frac{1}{16\pi G} \int \sqrt{-g} f(R) dx + S_M, \tag{1}
\]

where \( S_M \) is the action due to non-gravitational part of the universe. From the variation of the metric, equation (1) yields the field equation

\[
f'(R)R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) f'(R) = 8\pi G T_{\mu\nu}, \tag{2}
\]

where the prime denotes differentiation with respect to \( R \), \( R_{\mu\nu} \) is the Ricci tensor, \( R \) is the Ricci scalar, \( g_{\mu\nu} \) is the metric tensor, \( \nabla_\alpha \) and \( \nabla_\beta \) are the covariant derivative of the metric tensor, \( \square = g^{\alpha\beta} \nabla_\alpha \nabla_\beta \) is the dAlembert operator and \( T_{\mu\nu} \) is the energy-momentum tensor. Equation (2) is a fourth-order partial differential equation in the metric since \( R \) already includes its second derivative. For an action that is linear in \( R \), the fourth order terms (the last term of the left hand side of equation (2)) vanish and the theory reduces to General Theory of Relativity.

Here we consider that the homogeneous and isotropic universe is described by the Friedmann-Robertson-Walker (FRW) metric

\[
dr^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - k r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \tag{3}
\]

with \( a(t) \) as the scale factor and \( k \) as the spatial curvature having values +1, 0 and -1 for closed, flat and open universe respectively.

So from field equation (2), one can get Friedmann equations for a spatially flat FRW universe \((k = 0)\) as

\[
H^2 = \frac{1}{3f'} \left[ 8\pi G \rho + \frac{1}{2} (R' - f) - 3H \dot{R} f'' \right], \tag{4}
\]

and

\[
2\dot{H} + 3H^2 = -\frac{1}{f'} \left[ 8\pi G \rho + \dot{R}^2 f''' + 2H \dot{R} f'' + \ddot{R} f'' + \frac{1}{2} (f - Rf') \right], \tag{5}
\]

where \( f' = \frac{\partial f(R)}{\partial R} \), \( f'' = \frac{\partial^2 f(R)}{\partial R^2} \), \( f''' = \frac{\partial^3 f(R)}{\partial R^3} \), \( H = \frac{\dot{a}}{a} \) is the Hubble parameter and \( \dot{H} = \frac{dH}{dt} \), \( \rho \) and \( p \) are the density and pressure of the perfect fluid filling the universe and connected by the equation of state \( p = \gamma \rho \). Here it is assumed that \( f' > 0 \) in order to have a positive effective gravitational coupling and \( f''' > 0 \) to fulfill the requirements of stability of the classical solutions of the Einstein equation.

Also from FRW metric as given in \([3]\), we found the value of Ricci scalar \( R \) as

\[
R = 12H^2 + 6\dot{H}. \tag{6}
\]

The energy conservation equation then becomes

\[
\dot{\rho} + 3H(1 + \gamma)\rho = 0, \tag{7}
\]
which implies \( \rho \propto a^{-3(1+\gamma)} \), where \( \gamma \) is equation of state parameter having values \( \frac{1}{3} \) for radiation-dominated era and \( 0 \) for matter-dominated era. Again Using equations (4) and (5), one can get the Raychaudhuri equation as

\[
\dot{H} = -\frac{1}{2f'} \left[ 8\pi G(1+\gamma)\rho + \dot{R}^2 f''' - H \dot{f}' - \ddot{R}f'' \right].
\]

(8)

III. STAROBINSKY MODEL AND POWER-LAW COSMOLOGY

In our work, we choose Starobinskys model, where

\[
f(R) = R + \frac{R^2}{6M^2}.
\]

(9)

Now by differentiating \( f(R) \) with respect to \( R \), we can get

\[
f'(R) = 1 + \frac{R}{3M^2}, \quad f''(R) = \frac{1}{3M^2} \quad \text{and} \quad f'''(R) = 0.
\]

By using the above values of \( f(R) \) and its derivatives in equation (4), we found

\[
\rho = \frac{1}{8\pi G} \left[ 3 \left( 1 + \frac{R}{3M^2} \right) H^2 - \frac{R^2}{12M^2} + \frac{H \dot{R}}{M^2} \right].
\]

(10)

Again putting the values of \( R \) from equation (6), equation (10) can be written as

\[
\rho = \frac{1}{8\pi GM^2} \left[ 3M^2 H^2 + 18 \dot{H}H^2 + 6H \ddot{H} - 3H^2 \right].
\]

(11)

But in general power law cosmology, the scale factor varies with time as a power law, i.e. \( a(t) \propto t^\beta \). So the Hubble parameter and its derivatives become

\[
H = \frac{\dot{R}}{R}, \quad \dot{H} = -\frac{2}{t}, \quad \ddot{H} = \frac{2}{t^3} \quad \text{and} \quad \dddot{H} = -\frac{6}{t^4}.
\]

Now on simplification, equation (11) becomes

\[
\rho = \frac{3\beta^2}{8\pi Gt^2} \left[ 1 - \frac{3(2\beta - 1)}{M^2t^2} \right],
\]

(12)

which gives

\[
\frac{\dot{\rho}}{\rho} = -\frac{1}{t} \left[ \frac{2M^2t^2 + 12 - 24\beta}{M^2t^2 + 3 - 6\beta} \right].
\]

(13)

But using general power-law concept in energy conservation equation (7), we get

\[
\rho \propto t^{-3\beta(1+\gamma)},
\]

(14)

which implies

\[
\frac{\dot{\rho}}{\rho} = -\frac{3\beta(1+\gamma)}{t}.
\]

(15)

Comparing equations (13) and (15), one can find

\[
\beta(1+\gamma) = \frac{2}{3} \left[ \frac{2M^2t^2 + 6 - 12\beta}{M^2t^2 + 3 - 6\beta} \right].
\]

(16)

Like standard model of cosmology, here we consider that present universe is matter-dominated (\( \gamma = 0 \)) and before it was radiation-dominated (\( \gamma = \frac{1}{3} \)).

Now for radiation-dominated era, equation (16) gives

\[
12\beta^2 - (18 + 2M^2t^2)\beta + (6 + M^2t^2) = 0.
\]

(17)

The solutions of above equation (17) are \( \beta = \frac{1}{2} \) and \( \beta = 1 + \frac{M^2t^2}{6} \). But \( \beta = 1 + \frac{M^2t^2}{6} \) is prohibited, since it makes the density of energy-matter filling the universe negative. So like standard model of cosmology and scalar-tensor theory [62], here also scale factor varies in radiation-dominated era as

\[
a(t) \propto t^{\frac{1}{2}},
\]

(18)
which has a strong observational support. Again for matter-dominated era, equation (16) gives
\[ 18\beta^2 - (33 + 3M^2t^2)\beta + (12 + 2M^2t^2) = 0. \] (19)

The solution of above equation (19) are \( \beta = \frac{1}{12} \left[ (11 + M^2t^2) + \sqrt{(5 + M^2t^2)^2 - 4M^2t^2} \right] \) and \( \beta = \frac{1}{12} \left[ (11 + M^2t^2) - \sqrt{(5 + M^2t^2)^2 - 4M^2t^2} \right] \). But the root \( \beta = \frac{1}{12} \left[ (11 + M^2t^2) - \sqrt{(5 + M^2t^2)^2 - 4M^2t^2} \right] \) is not suitable for providing presently observed accelerated expansion. Thus in matter-dominated era scale factor varies like
\[ a(t) \propto t^{\frac{1}{12}} \left[ (11 + M^2t^2) + \sqrt{(5 + M^2t^2)^2 - 4M^2t^2} \right]. \] (20)

IV. DECELERATION PARAMETER

Expansion of the universe can be verified by Hubbles law. But whether the expansion is accelerating or decelerating one, it can be determined by deceleration parameter. The deceleration parameter is defined as
\[ q = -\frac{\ddot{a}(t)a(t)}{\dot{a}(t)^2}. \] (21)

Now from equation (8), we get
\[ q = -1 + \frac{1}{2fH^2} \left[ 8\pi G(1 + \gamma)\rho + \dot{H}f'' - H\dot{f}' + \ddot{f} \right]. \] (22)

On simplification, above equation (22) gives
\[ q = -1 + \frac{4\pi G\rho(1 + \gamma)M^2t^4 + 4\beta^3 + 10\beta^2 - 6\beta}{4\beta^4 - 2\beta^3 + M^2t^2\beta^2}, \] (23)

which implies for \( t \to 0 \) and large value of \( \beta \), \( q \approx -1 + \frac{1}{\beta} \approx -1 \). This supports the idea of inflation that the universe undergoes a phase of exponential expansion during early period of evolution. Because for exponential expansion \( a(t) \propto e^{\beta t} \) which, in turn, gives \( q = -\frac{\ddot{a}(t)a(t)}{\dot{a}(t)^2} = -1 \).

For radiation-dominated era, the equation (23) gives
\[ q = -1 + \frac{64\pi G\rho t^2}{3}. \] (24)

Substituting the value of \( \rho \) from equation (12) with \( \beta = \frac{1}{2} \) in equation (24), the deceleration parameter for radiations-dominated era is found to be \( q = 1 \). This provides decelerated expansion throughout the radiation-dominated era. But for matter-dominated era, using equations (23) and (12), we get
\[ q = -1 + \frac{2M^2t^2\beta^2 - 5\beta^3 + \frac{2M}{\beta}\beta^2 - 6\beta}{4\beta^4 - 2\beta^3 + M^2t^2\beta^2}, \] (25)

where \( \beta = \frac{1}{12} \left[ (11 + M^2t^2) + \sqrt{(5 + M^2t^2)^2 - 4M^2t^2} \right] \).

Solving above equation (25) by taking present age of the universe \( (t_0) \) as \( 13.82 \times 10^9 \) years, we construct the Table-I for presenting the variation of present value of deceleration parameter \( q_0 \) with \( M^2t_0^2 \).

Comparing Table-I with the observational data [63] that \( q_0 \approx -0.55 \), we take \( M^2t_0^2 = 5.905 \) for the rest part of the paper.

Now the evolution of scale factor in matter-dominated era due to modification of gravity can be picturized from equation (19), which is shown in Figure-1.

V. EVOLUTION OF BLACK HOLES

Black hole is a region in space-time, where, gravitational field is so strong that even light can not escape from it. In the usual formation scenarios, the typical mass of a black hole at the formation could be as large as the mass
TABLE I: The variation of present value of deceleration parameter \( (q_0) \) with \( M^2 t_0^2 \) is given in the Table.

| \( q_0 \) | \( M^2 t_0^2 \) | \( q_0 \) | \( M^2 t_0^2 \) |
|-------|---------|-------|---------|
| -0.30 | 0.606 | -0.65 | 9.838 |
| -0.35 | 1.466 | -0.70 | 12.75 |
| -0.40 | 2.333 | -0.75 | 16.80 |
| -0.45 | 3.331 | -0.80 | 22.846 |
| -0.50 | 4.501 | -0.85 | 32.889 |
| -0.55 | 5.905 | -0.90 | 52.929 |
| -0.60 | 7.636 | -0.90 | 112.966 |

FIG. 1: Evolution of scale factors in matter-dominated era due to normal gravity and metric \( f(R) \) gravity contained in the Hubble volume \( m_H \) ranging down to about \( 10^{-4} m_H \) \[64\]. Black holes can thus span enormous mass range starting from Planck mass to few order of solar mass. Again black holes which are formed before inflation are completely diluted due to exponential expansion and also environment is not suitable for them to be formed in matter-dominated era. i.e. all the black holes are formed by the time of matter-radiation equality \( t_e \), which is assumed to be occurred when the universe is nearly \( 10^{11} \) sec old. So the maximum formation mass of the black hole would be \( (m_H)_{t_e} = G^{-1} t_e \sim 10^{59} \) gm. Again the formation masses of some of the black holes could be small enough to have evaporated completely by the present epoch due to Hawking evaporation \[65\]. Early evaporating black holes could account for baryogenesis \[66, 68\] in the universe. On the other hand, presently surviving black holes could act as seeds for structure formation and could also form a significant component of dark matter \[65, 72\]. Once formed, these black holes are affected both by Hawking evaporation and accretion: absorption of energy matter from the surroundings. In literature so many works \[74-80\] are found, involving absorption of radiation, matter and dark energy. We, here, discuss accretion of energy-matter from the surroundings, when the gravity is modified due to metric formalism. The mass of a black hole can be changed by absorbing energy-matter from the surroundings as \[76, 81\]

\[
\dot{m} = 4\pi f r_{bh}^2 \rho,
\]

(26)

where \( r_{bh} = 2Gm \) is the radius of black hole and \( f \) is the accretion efficiency.

By using equation (12) and simplifying, above equation (26) gives

\[
\dot{m} = \frac{6Gf m^2 \beta^2}{t^2} \left[ 1 + \frac{3 - 6\beta}{M^2 t^2} \right].
\]

(27)

Due to Hawking evaporation, the rate at which the mass of a black hole changes is given by \[76\]

\[
\dot{m} = -\frac{a_H}{256\pi^3} \frac{1}{G^2 m^2},
\]

(28)
where $a_H$ is the black body constant.

In metric $f(R)$ gravity, thus, the evolution of black holes’ mass is governed by the equation

$$\dot{m} = -\frac{a_H}{256\pi^3 G^2 m^2} \frac{1}{t^2} \left[ \frac{M^2 t^2}{2} + \frac{6G f m^2 \beta^2}{t^2} \right] \left[ 1 + \frac{3 - 6\beta}{M^2 t^2} \right].$$  \hspace{1cm} (29)

Like deceleration parameter, here we consider two epochs separately.

### A. Radiation-dominated era

In radiation dominated era, the equation (26) takes the form

$$\dot{m} = \frac{3}{2} G f_{rad} \frac{m^2}{t^2}. \hspace{1cm} (30)$$

By solving above differential equation (30), we get

$$m = m_i \left[ 1 + \frac{3}{2} f_{rad} \left( \frac{t}{t_i} - 1 \right) \right]^{-1}, \hspace{1cm} (31)$$

where $m_i$ is the mass of black hole at formation time $t_i$. For large time $t$, the above equation asymptotes to $m = \frac{m_i}{1 - \frac{3}{2} t_i}$. Thus for accretion to be effective $f_{rad} < \frac{1}{2}$. In radiation-dominated era, the complete evolution equation of black hole becomes

$$\dot{m} = -\frac{a_H}{256\pi^3 G^2 m^2} \frac{1}{t^2} + \frac{3}{2} G f_{rad} \frac{m^2}{t^2}, \hspace{1cm} (32)$$

which is same as the standard model of cosmology [81].

### B. Matter dominated era

Since black holes, in general, can not be formed in matter-dominated era, here we study the evolution of those black holes which are already formed during radiation-dominated era.

In matter dominated era, the equation (26) takes the form

$$\dot{m} = \frac{G f_{mat} m^2}{24t^2} \left( \frac{1}{M^2 t^2} \right) \left( M^2 t^2 \left[ \frac{1}{2} - \sqrt{(5 + M^2 t^2)^2 - 4M^2 t^2} - \frac{5}{2} \right] \left(11 + M^2 t^2\right) + \sqrt{(5 + M^2 t^2)^2 - 4M^2 t^2} \right)^2. \hspace{1cm} (33)$$

By solving equations (33) numerically, we plot the Figure-2 which shows the variation of black holes’ mass with time in $f(R)$ gravity.

From the Figure-2, it is evident that the mass of a black hole decreases due to accretion of surroundings energy-matter in matter-dominated era. This, we believe, is an interesting result being counter-intuitive.

In matter-dominated era, the complete evolution equation of black hole becomes

$$\dot{m} = -\frac{a_H}{256\pi^3 G^2 m^2} \frac{1}{t^2} + \frac{G f_{mat} m^2}{24t^2} \left( \frac{1}{M^2 t^2} \right) \left( M^2 t^2 \left[ \frac{1}{2} - \sqrt{(5 + M^2 t^2)^2 - 4M^2 t^2} - \frac{5}{2} \right] \left(11 + M^2 t^2\right) + \sqrt{(5 + M^2 t^2)^2 - 4M^2 t^2} \right)^2, \hspace{1cm} (34)$$

which implies that the accretion term would be effective, if the black holes mass at the time of matter-radiation equality ($m(t_e)$) satisfy the condition

$$m(t_e) \geq \left| \frac{a_H}{256\pi^3 G^3 f_{mat} M^2 t_e^2} \left( \frac{M^2 t_e^2}{2} - \sqrt{(5 + M^2 t_e^2)^2 - 4M^2 t_e^2} - \frac{5}{2} \right) \right|^{-\frac{1}{2}} \left(11 + M^2 t_e^2\right) + \sqrt{(5 + M^2 t_e^2)^2 - 4M^2 t_e^2} \right|^{-\frac{1}{2}}. \hspace{1cm} (35)$$

Since $M^2 t_0^2 = 5.905$ and $0 < f_{mat} < 1$, on simple calculation above equation (35) predicts that those black holes would be affected by accretion, whose mass at the time of matter-radiation equality $M(t_e)$ is greater that $10^{19}$ gm.
V. DISCUSSION AND CONCLUSION

In this study, we have used the metric $f(R)$ gravity obeying Starobinsky model for investigating the black hole dynamics. We assume that after inflation, the evolution of the universe was occurred through conventional cosmological eras of radiation-domination and matter-domination basing on power-law cosmology. We first evaluated the modified density of the energy-matter filling the universe and then determined the scale factor for both the eras. From these calculations, we found that in radiation-dominated era the evolution of the universe remains same as in the case of standard model of cosmology and scalar-tensor theory. Hence our integrated modified gravity model does

Solving the equations (32) and (34) numerically, we construct Table-II, where the variation of evaporation times of black holes with rate of accretion of surroundings energy-matter in the presence of metric $f(R)$ gravity is presented. (The subscript $i$ refers to the initial value.)

From Table-II, it is clear that the evaporation of black holes become quicker due to modification of gravity. Particularly, the black holes whose formation mass is greater than nearly $10^{20}$ gm, they all will evaporate at a particular time depending on their accretion efficiency only. But presently evaporating black holes are not affected by the presence of dark energy, so all observed astrophysical constraints on black holes [32] would not be disturbed.

VI. DISCUSSION AND CONCLUSION
not affect the early observational facts, starting from primordial nucleosynthesis to providing the stage for formation of large scale structures. But in matter-dominated era, modified gravity plays its role by affecting the evolution of the universe. Assuming that present universe is matter-dominated, we are successful in explaining the observed accelerated expansion of the universe without requiring the dark energy. Finally, we discuss the evolution of the black holes in this environment. From our analysis, we found that black hole mass decreases due to accretion of surroundings energy-matter in metric $f(R)$ gravity and more the accretion rate more is mass loss. Particularly, black holes, whose masses at the time of matter-radiation equality $M(t_e)$ are greater that $10^{19}$ gm, would be affected by this modified gravity. During their evolution, those black holes would lose their masses and hence their evaporation would be quicker in comparison with standard scenario. Again all the black holes whose formation mass is greater than nearly $10^{20}$ gm, will be evaporated at a particular time irrespective of their formation mass depending only on rate of accretion. This can be explained by the fact that by that time the modification of gravity will touch its saturation point so that the universe will show phantom type behavior \[83, 84\]. But these modification of gravity could not affect the presently evaporating black holes, whose formation masses ($M_i$) are of the order of $10^{15}$ gm and thus all observed astrophysical constraints on black holes remain unaltered. Again our analysis reveals that the maximum mass of a black hole supported by metric $f(R)$ gravity is $10^{12}M_\odot$, where $M_\odot \approx 2 \times 10^{33}$ gm is the solar mass.

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