Spontaneous R-Parity Violation, $A_4$ Flavor Symmetry and Tribimaximal Mixing

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Abstract

We explore the possibility of spontaneous R parity violation in the context of $A_4$ flavor symmetry. Our model contains $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet matter chiral superfields which are arranged as triplet of $A_4$ and as well as few additional Higgs chiral superfields which are singlet under MSSM gauge group and belong to triplet and singlet representation under the $A_4$ flavor symmetry. R parity is broken spontaneously by the vacuum expectation values of the different sneutrino fields and hence we have neutrino-neutralino as well as neutrino-MSSM gauge singlet higgsino mixings in our model, in addition to the standard model neutrino- gauge singlet neutrino, gaugino-higgsino and higgsino-higgsino mixings. Because all of these mixings we have an extended neutral fermion mass matrix. We explore the low energy neutrino mass matrix for our model and point out that with some specific constraints between the sneutrino vacuum expectation values as well as the MSSM gauge singlet Higgs vacuum expectation values, the low energy neutrino mass matrix will lead to a tribimaximal mixing matrix. We also analyze the potential minimization for our model and show that one can realize a higher vacuum expectation value of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ singlet sneutrino fields even when the other sneutrino vacuum expectation values are extremely small or even zero.
1 Introduction

Various experimental evidences on neutrino mass and mixing have opened up a window to physics beyond the standard model of particle physics. Experiments like SNO, KamLAND, K2K and MINOS [1–4] provide information on the two mass square differences $\Delta m_{21}^2$ and $\Delta m_{31}^2$ and on the two mixing angles $\theta_{12}$ and $\theta_{23}$. The third mixing angle $\theta_{13}$ is yet not determined, but certainly is known to be small [5]. The current $3\sigma$ allowed intervals of the oscillation parameters are given as [6]

$$7.1 \times 10^{-5} \text{eV}^2 < \Delta m_{21}^2 < 8.3 \times 10^{-5} \text{eV}^2, \quad 2.0 \times 10^{-3} \text{eV}^2 < \Delta m_{31}^2 < 2.8 \times 10^{-3} \text{eV}^2$$ (1)

$$0.26 < \sin^2 \theta_{12} < 0.42, \quad 0.34 < \sin^2 \theta_{23} < 0.67, \quad \sin^2 \theta_{13} < 0.05$$ (2)

To explain the above mentioned very precise data of neutrino mass and mixing and without bringing any additional fine tuning problem into the theory, one has to look for beyond standard model physics. In beyond standard model physics very small Majorana neutrino masses can be generated by the dimension 5 operator $\frac{1}{\Lambda}LLHH$ [7], where the masses are suppressed naturally by the scale of new physics $\Lambda$. Note that this term breaks lepton number which is mandatory for the generation of Majorana masses. The seesaw [8] in its simplest version could be Type I seesaw [8], Type II [9] or Type III [10–13] depending on the heavy particles which would be integrated out and generate the above mentioned dimension 5 operator are standard model singlet, standard model triplet with hypercharge $Y = 2$ and standard model triplet with hypercharge $Y = 0$ respectively. Observed neutrino mixing can be obtained very naturally by imposing flavor symmetry. Among the numerous viable flavor symmetry models [14], the models based on the group $A_4$ are the most popular ones [15–18].

Among all the different models of beyond standard model physics supersymmetry is probably the most attractive one, for its capability to solve the Higgs mass hierarchy problem very naturally. The most general superpotential which respects the standard model gauge group also allows lepton number and baryon number violating bilinear $\hat{e}\hat{L}\hat{H}_u$ and trilinear $\lambda, \lambda', \lambda''$ terms [19–21]. The R-Parity is a discrete symmetry defined as $R_p = (-1)^{3(B-L)+2S}$ and for all matter chiral superfields it is $-1$ while for Higgs chiral superfields it is $+1$. Defined in this way R-parity conservation forbids all the baryon and lepton number violating terms in the superpotential. However minimal supersymmetric standard model with R parity violation [22–28] opens up the possibility of neutrino mass generation. If one sticks to the basic MSSM gauge group and to the MSSM particle contents and breaks R parity spontaneously, then eventually one will encounter with the problem of Majoron [23], which could be evaded if one extends the MSSM particle contents and/or extends the gauge group suitably.

The neutrino masses could be generated from the bilinear as well as from the trilinear R-Parity violating terms of the superpotential. While the bilinear R-Parity violating $\hat{L}\hat{H}_u$ term generates the neutrino mass via neutrino-higgsino mixing [24, 25], neutrino masses
could also be generated from the lepton number violating trilinear terms via loop effect [29–31]. However severe bounds [32] on the lepton and baryon number violating $\lambda$, $\lambda'$ and $\lambda''$ terms of the superpotential come from the non-observation of proton decay which essentially constraint the simultaneous presence of lepton number and baryon number violation in the superpotential.

The spontaneous R parity violation provides a natural explanation for the absence of baryon number violating $\lambda''$ term in the superpotential, as long as one sticks to the renormalizable field theory. In the scheme of spontaneous R parity violation R parity is a symmetry of the theory and once different sneutrino fields get vacuum expectation values, R parity is being violated spontaneously. In addition to the field contents of MSSM if one has a MSSM gauge singlet/triplet sneutrino state, then the bilinear R parity violating term involving leptons and Higgs would be generated once this gauge singlet/triplet sneutrino state gets vacuum expectation value. It is possible to realize the other terms $\lambda$ and $\lambda'$ from the R parity conserving MSSM superpotential only after redefinition of the basis.

There have been several attempts to realize R-parity violation spontaneously. In the context of a GUT theory one can relate the spontaneous R-parity violation with the gauge symmetry breaking [33] and it is possible to realize some of the sneutrino vacuum expectation values to be in the TEV scale. In these kind of models the low energy neutrino masses would be generated via double seesaw mechanism. However if one sticks to the basic MSSM gauge group and explore the possibility of spontaneous R-parity violation without invoking any problem of Majoron, one has to extend the particle content of the model. One can introduce the standard model singlet/triplet matter chiral superfield into the theory. In this kind of model the low energy neutrino mass would be generated via the Type-I/Type-III seesaw and it is possible to get proper mass splitting between the low energy neutrino masses and the correct mixing even with one generation of heavy neutrino matter chiral superfield [34]. The different sneutrino vacuum expectation values share a proportionality relation in this kind of model.

In this work we explore the possibility of spontaneous R-parity violation in the context of $A_4$ flavor symmetry while we stick to the MSSM gauge group. The best fit value of the neutrino oscillation parameters points towards a tribimaximal neutrino mixing matrix, which is possible to achieve very naturally for the R parity conserving scenario if one imposes a flavor symmetry such as $A_4$ in the theory [16]. In our model we have few additional standard model singlet Higgs superfields $\hat{\phi}_T$, $\hat{\phi}_S$ and $\hat{\xi}$ along with the standard model singlet matter chiral superfields $\hat{\nu}$. In addition to $A_4$ we have also implemented one $Z_3$ symmetry in our model. The different Higgs chiral superfields and matter chiral superfields belong to the triplet as well as singlet representation under the flavor symmetry group $A_4$ and have suitable $Z_3$ charges. The symmetry group $A_4$ would be broken by the vacuum expectation value of the $A_4$ triplet fields while R-parity would be broken spontaneously by the vacuum expectation values of the different sneutrino $\tilde{\nu}$ and $\tilde{\nu}$ fields. The neutral fermion mass matrix in our model is enlarged compared to that of minimal supersymmetric standard model and in addition to the conventional Dirac mixing between the standard model neutrino $\nu$ and the gauge singlet neutrino $\hat{N}$, R-parity violation brings mixing between the different neutrino and neutralino states. In our model because of R parity violation we also have
other mixings between the different neutrino states and the MSSM gauge singlet higgsino fields along with the different gaugino-higgsino and higgsino-higgsino mixings. With all these mixings we explore the possibility of generating tribimaximal mixing in our model. We point out that although the neutral fermion mass matrix is enlarged and has many parameters, however the low energy neutrino mass matrix would still lead to a tribimaximal mixing matrix, provided the vacuum expectation values of the different sneutrino and gauge singlet Higgs satisfy some specific constraints. We also show that in this model it is possible to get a higher vacuum expectation value of the gauge singlet sneutrino fields $\tilde{N}$ even if the other sneutrino vacuum expectation values $\langle \tilde{\nu} \rangle$ are extremely small and even $\langle \tilde{\nu} \rangle \to 0$.

The paper is organized as follows. In section 2 we describe the model and in section 3 we discuss the neutrino phenomenology where we explore the possibility of getting tribimaximal mixing for our model. In section 4 we discuss the symmetry breaking part where we show that even in the $\langle \tilde{\nu} \rangle \to 0$ limit one expects a higher value of the sneutrino vacuum expectation value $\langle \tilde{N} \rangle$. In section 4 we present our conclusion. Discussion on the soft supersymmetry breaking Lagrangian for this model has been presented in the Appendix.

2 Model

In this section we present our model. We stick to the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group of MSSM and impose $A_4$ flavor symmetry in our model. In addition to the $A_4$ triplet and standard model singlet Majorana superfields $\tilde{N}$, our model contains one standard model and $A_4$ singlet superfield $\tilde{\xi}$ and two $A_4$ triplets and standard model singlet superfields $\tilde{\phi}_T$ and $\tilde{\phi}_S$. The particle contents of our model has explicitly been given in Table. (1). The details about the discrete symmetry group $A_4$ could be found in [16, 18]. With these superfield contents the superpotential of our model is,

$$W = y_1 \bar{e} (\Phi_T L) + y_2 \bar{\mu} (\Phi_T L)'' + y_3 \bar{\tau} (\Phi_T L)' + y_4 \tilde{N} \hat{L} \hat{H}_u$$

$$+ x_A \xi (\tilde{N} \hat{N}) + x_B (\tilde{\phi}_S \tilde{N} \hat{N}) + \alpha \xi \hat{\xi} + \beta' \tilde{\phi}_T \tilde{\phi}_S + \beta \tilde{\phi}_S \tilde{\phi}_S \hat{\xi}$$

$$+ \gamma \tilde{\phi}_T \tilde{\phi}_T + \gamma' \tilde{\phi}_T \tilde{\phi}_T + \mu \tilde{H}_u \tilde{H}_d. \quad (3)$$

Note that apart from $A_4$, one more discrete $Z_3$ symmetry has been implemented in our model so that $\hat{\phi}_T$ does not contribute to the standard model neutrino mass generation, $\hat{\xi}$ and $\hat{\phi}_S$ do not contribute to the charged lepton masses, although $\hat{\xi}$ and $\hat{\phi}_S$ contribute to the neutrino sector significantly. We represent the superfields $\hat{\xi}$ and $\hat{N}$ as follows,

$$\hat{\xi} = \xi + \sqrt{2} \theta \xi + \theta \theta F_\xi, \quad (4)$$

$$\hat{N}_i = \tilde{N}_i + \sqrt{2} \theta \tilde{N}_i + \theta \theta F_{N_i}, \quad (5)$$

where $i$ represents the $A_4$ index and varies from $i = 1, 2, 3$. The other superfields $\hat{\phi}_T$ and $\hat{\phi}_S$ will have the same structure as of $\hat{\xi}$ given in Eq. (4) i.e., $\phi_{T,S}$ denote the scalar partners
Table 1: Field transformation under $A_4$ and $Z_3$

| Field | $L$ | $\tilde{e}^c$ | $\tilde{\mu}^c$ | $\tilde{\tau}^c$ | $N$ | $H_{u,d}$ | $\xi$ | $\tilde{\phi}_T$ | $\tilde{\phi}_S$ |
|-------|-----|--------------|----------------|----------------|-----|----------|-----|---------------|---------------|
| $A_4$ | 3   | 1            | 1$'$          | 1$''$         | 3   | 1        | 1   | 3            | 3            |
| $Z_3$ | $\omega$ | $\omega^2$  | $\omega^2$   | $\omega^2$   | 1   | $\omega^2$ | 1   | $\omega^2$   |               |
| $R_p$ | -1  | -1           | -1            | -1            | -1  | +1       | +1  | +1           | +1           |

and $\tilde{\phi}_{T,S}$ denote the fermionic partners and $F_{T,S_i}$ are the auxiliary components. Note that the matter chiral superfields $\tilde{N}, \tilde{L}, \tilde{e}^c, \tilde{\mu}^c$ and $\tilde{\tau}^c$ are odd under R-parity while the two usual MSSM Higgs chiral superfields $\tilde{H}_{u,d}$ as well as the other MSSM singlet Higgs chiral superfields $\tilde{\phi}_T, \tilde{\phi}_S$ and $\tilde{\xi}$ are even under R parity. The superpotential given in Eq. (3) as well as the Kähler potential given in Eq. (63) and Eq. (64) conserve R-parity, however R-parity will be broken spontaneously by the vacuum expectation values of the different sneutrino fields $\tilde{N}_i$ and $\tilde{\nu}_i$. We would like to point out here that the $y_1\tilde{e}^c(\tilde{\phi}_TL)$ term in the superpotential Eq. (3) actually represents $\frac{1}{3}H_d(\tilde{\phi}_TL)e^c$ and similarly for the other operators $y_2\tilde{\mu}^c(\tilde{\phi}_TL)''$ and $y_3\tilde{\tau}^c(\tilde{\phi}_TL)'$. We would also like to stress here that in our model the lepton number is explicitly broken in the superpotential by few of the the trilinear terms. Hence the spontaneous R parity violation is not associated with any global $U(1)$ symmetry breaking. Therefore in our model spontaneous R parity violation does not bring any problem of Majoron.

In the R parity conserving scenario, the MSSM neutralinos $\tilde{\lambda}^{0,3}, \tilde{H}_{u,d}$ as well as the higgsinos of the other MSSM gauge singlet Higgs chiral superfields i.e $\tilde{\phi}_T, \tilde{\phi}_S$ and $\tilde{\xi}$ decoouple from the neutrino sector $\nu$ and $N$. The low energy neutrino mass matrix for R-parity conserving scenario would be,

$$M_\nu \sim m_D^T \tilde{M}^{-1} m_D,$$

where

$$m_D = y_\nu v_2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

and

$$\tilde{M} = 2 \begin{pmatrix} a + \frac{2b_1}{3} & -\frac{b_3}{3} & -\frac{b_3}{3} \\ -\frac{b_3}{3} & \frac{2b_2}{3} & a - \frac{b_1}{3} \\ -\frac{b_3}{3} & a - \frac{b_1}{3} & \frac{2b_3}{3} \end{pmatrix}.$$
where $b = x_B u$ and the low energy neutrino mass matrix becomes,

$$M_\nu = \frac{y_\nu v_2^2}{3a(b^2 - a^2)} \begin{pmatrix} b^2 + 2ab - 3a^2 & b^2 - ab & b^2 - ab \\ b^2 - ab & b^2 + 2ab & b^2 - ab - 3a^2 \\ b^2 - ab & b^2 - ab - 3a^2 & b^2 + 2ab \end{pmatrix}. \tag{10}$$

The mixing matrix $U_\nu$ which diagonalizes the mass matrix $M_\nu$ and satisfies the diagonalizing relation $U_\nu^T M_\nu U_\nu^* = D_k$ has this tribimaximal form,

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{\sqrt{\frac{1}{3}}}{\sqrt{2}} & 0 \\ \frac{\sqrt{\frac{1}{3}}}{\sqrt{2}} & \sqrt{\frac{1}{3}} & -\frac{\sqrt{\frac{1}{2}}}{\sqrt{2}} \\ \frac{\sqrt{\frac{1}{3}}}{\sqrt{2}} & \frac{\sqrt{\frac{1}{3}}}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \tag{11}$$

and the low energy neutrino masses have this following form,

$$D_k = y_\nu v_2^2 \text{diag}(\frac{1}{a + b}, \frac{1}{a}, \frac{1}{b - a}). \tag{12}$$

Since in our model R parity is violated spontaneously, the neutral fermion sector of our model will change significantly. Because of R parity violation the standard model neutrinos $\nu_i$ as well as the gauge singlet Majorana neutrinos $N_i$ mix with the different neutralinos $\tilde{\lambda}_0, 3$ and $H_{du}^0$. In addition to this we also have neutrino-gauge singlet higgsino mixings and the usual conventional standard model neutrino-gauge singlet neutrino, MSSM gaugino-higgsino and gauge singlet higgsino-higgsino mixings. In our model the neutral fermion basis is $\psi = (\nu', N', \Phi_1, \Phi_2, \tilde{\chi})$, where $\nu' = (\nu_e, \nu_\mu, \nu_\tau)$, $N' = (N_1, N_2, N_3)$, $\Phi_1 = (\xi, \tilde{\phi}_{S_1}, \tilde{\phi}_{S_2}, \tilde{\phi}_{S_3})$, $\Phi_2 = (\tilde{\phi}_{T_1}, \tilde{\phi}_{T_2}, \tilde{\phi}_{T_3})$ and $\tilde{\chi} = (\tilde{\lambda}_0, \tilde{\lambda}_3, H_{du}^0)$. We denote the vacuum expectation values of the different Higgs and the sneutrino fields by $\langle \phi_{S_i} \rangle = u_i$, $\langle \phi_{T_i} \rangle = t_i$, $\langle \xi \rangle = s$, $\langle H_{du}^0 \rangle = v_{1,2}$, $\langle \tilde{N}_i \rangle = w_i$ and $\langle \tilde{\nu}_{L_i} \rangle = x_i$ respectively. The neutral fermion mass matrix is,

$$L = -\frac{1}{2} \psi^T M \psi + h.c., \tag{13}$$

where

$$M = \begin{pmatrix} 0 & A & 0 & 0 & B \\ A^T & C & D & 0 & P \\ 0 & D^T & K & 0 & 0 \\ 0 & 0 & 0 & F & 0 \\ B^T & P^T & 0 & 0 & G \end{pmatrix}. \tag{14}$$

In the above matrix $A, C, D, F, K, G, B$ and $P$ are these following matrices,

$$A = \begin{pmatrix} y_\nu v_2 & 0 & 0 \\ 0 & 0 & y_\nu v_2 \\ 0 & y_\nu v_2 & 0 \end{pmatrix}, \tag{15}$$

1Being gauge singlet, $N_i$ does not mix with the gauginos $\tilde{\lambda}_0, 3$, however mix with the higgsino $H_{du}^0$. 

6
\[ C = 2 \begin{pmatrix} a + \frac{2b_1}{3} & -\frac{b_1}{3} & -\frac{b_2}{3} \\ -\frac{b_1}{3} & \frac{2b_2}{3} & a - \frac{2b_1}{3} \\ -\frac{b_2}{3} & a - \frac{b_1}{3} & \frac{2b_2}{3} \end{pmatrix} \], \quad (16)

\[ D = 2 \begin{pmatrix} x_A w_1 & \frac{2}{3} x_B w_1 & -\frac{1}{3} w_3 x_B & -\frac{1}{3} w_2 x_B \\ x_A w_2 & -\frac{1}{3} x_B w_3 & \frac{2}{3} w_2 x_B & -\frac{1}{3} w_1 x_B \end{pmatrix}, \quad (17) \]

\[ F = 2 \begin{pmatrix} \gamma + 2\gamma' t_1 & -\gamma' t_3 & -\gamma' t_2 \\ -\gamma' t_3 & 2\gamma' t_2 & \gamma - \gamma' t_1 \\ -\gamma' t_2 & \gamma - \gamma' t_1 & 2\gamma' t_3 \end{pmatrix}, \quad (18) \]

\[ K = 2 \begin{pmatrix} 3\alpha s & \beta u_1 & \beta u_3 & \beta u_2 \\ \beta u_1 & \beta s + 2\beta' u_1 & -\beta' u_3 & -\beta' u_2 \\ \beta u_3 & -\beta' u_3 & 2\beta' u_2 & \beta s - \beta' u_1 \\ \beta u_2 & -\beta' u_2 & \beta s - \beta' u_1 & 2\beta' u_3 \end{pmatrix}, \quad (19) \]

\[ G = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} M_1 & 0 & -g_1 v_1 & g_1 v_2 \\ 0 & \sqrt{2} M_2 & g_2 v_1 & -g_2 v_2 \\ -g_1 v_1 & g_2 v_1 & 0 & -\sqrt{2} \mu \\ g_1 v_2 & -g_2 v_2 & -\sqrt{2} \mu & 0 \end{pmatrix}, \quad (20) \]

\[ B = \frac{1}{\sqrt{2}} \begin{pmatrix} -g_1 x_1 & g_2 x_1 & 0 & \sqrt{2} y_\nu w_1 \\ -g_1 x_2 & g_2 x_2 & 0 & \sqrt{2} y_\nu w_3 \\ -g_1 x_3 & g_2 x_3 & 0 & \sqrt{2} y_\nu w_2 \end{pmatrix}, \quad (21) \]

and

\[ P = \begin{pmatrix} 0 & 0 & 0 & y_\nu x_1 \\ 0 & 0 & 0 & y_\nu x_3 \\ 0 & 0 & 0 & y_\nu x_2 \end{pmatrix}. \quad (22) \]

Note that in the R parity conserving scenario the sneutrino vacuum expectation values \( x_i = 0 \) and \( w_i = 0 \) and hence the matrices \( D, B \) and \( P \) vanish. As a result the standard model light neutrinos \( \nu_i \) as well as the gauge singlet neutrinos \( N_i \) decouple from the the MSSM neutralino states \( \tilde{\chi} = (\tilde{\lambda}_0^3, \tilde{H}^0_{u,d}) \) and as well as from the gauge singlet higgsino states \( \tilde{\phi}_T, \tilde{\phi}_S, \tilde{\chi} \), as already have been mentioned previously.

### 3 Neutrino Mass and Tribimaximal Mixing

In this section we discuss about the low energy neutrino phenomenology. We would like to stress that even with R-parity violation and enlargement of the neutrino sector as shown in
Eq. (14) the low energy neutrino mixing matrix will still be the tribimaximal one, provided the different sneutrino vacuum expectation values of $\tilde{N}_i$ and $\tilde{\nu}_i$ satisfies a particular equality relation, i.e $w_1 = w_2 = w_3$ and $x_1 = x_2 = x_3$ along with the other necessary condition $u_1 = u_2 = u_3$. The mass matrix given in Eq. (14) could be written as,

$$ M = \begin{pmatrix} 0 & M_D \\ M_D^T & M' \end{pmatrix}, $$

(23)

where $M_D$ is the $3 \times 14$ block, $M'$ is a $14 \times 14$ matrix. Written in this way, the $M_D$ and $M'$ are these following two matrices,

$$ M_D = (A \ 0 \ 0 \ B), $$

(24)

and

$$ M' = \begin{pmatrix} C & D & 0 & P \\ DT & K & 0 & 0 \\ 0 & 0 & F & 0 \\ PT & 0 & 0 & G \end{pmatrix}. $$

(25)

The low energy neutrino mass matrix is,

$$ M_\nu \sim M_D M'^{-1} M_D^T. $$

(26)

Here we present the analytic expression of the low energy neutrino mass matrix. For the sake of simplicity we consider the couplings $x_A$, $x_B$, $\alpha$, $\beta$, $\beta'$ of Eq. (3) to be the same. In addition to this we also consider the vacuum expectation values of $\phi_S$ and $\xi$ fields to be the same, i.e $s = u$, however we have checked explicitly and numerically that neither such a kind of VEV alignment between $s$ and $u$ nor a equality relation between the different couplings of the superpotential is a necessary criteria to get the tribimaximal mixing. With this above mentioned simplified assumption, the low energy neutrino mass matrix has this following form,

$$ M_\nu = \frac{2}{V} \begin{pmatrix} 8Q + 3Z & 3Z - 4Q \ 3Z - 4Q & 3Z + 5Q - \frac{24u^2Q}{v_1^2} \ 3Z - 4Q & 3Z - Q + \frac{24u^2Q}{v_1^2} \ 3Z - 4Q & 3Z + 5Q - \frac{24u^2Q}{v_1^2} \ 3Z - 4Q & 3Z + 5Q - \frac{24u^2Q}{v_1^2} \ 3Z + 5Q - \frac{24u^2Q}{v_1^2} \ 3Z - Q + \frac{24u^2Q}{v_1^2} \ 3Z + 5Q - \frac{24u^2Q}{v_1^2} \end{pmatrix}, $$

(27)

where $Q$, $Z$ and $V$ are,

$$ Q = \mu w_2 y_\nu (\mu M_1 M_2 - (g_2^2 M_1 + g_1^2 M_2) v_1 v_2), $$

(28)

$$ Z = \beta (g_2^2 M_1 + g_1^2 M_2) (8u^2 - w^2)(\mu x + v_1 w_\nu)^2, $$

(29)

$$ V = 12\beta \mu (\mu M_1 M_2 - (g_2^2 M_1 + g_1^2 M_2) v_1 v_2) (8u^2 - w^2). $$

(30)
Clearly $M_\nu(1, 1) + M_\nu(1, 2) = M_\nu(2, 2) + M_\nu(2, 3)$ and hence one expects the mixing matrix to be the tribimaximal one. Since $M_\nu$ is a complex symmetric matrix, it will satisfy the diagonalizing relation $U^T M_\nu U^* = D_k$, where $D_k = \text{diag}(m_1, m_2, m_3)$. The low energy neutrino masses and mixing matrix which will come from the diagonalization of Eq. (27) are as follows,

\begin{align*}
m_1 &= \frac{2uv_\nu^2y_\nu^2}{\beta(8u^2 - w^2)}, \\
m_3 &= -\frac{uv_\nu^2y_\nu^2}{\beta w^2}, \\
m_2 &= \frac{3(g_2^2 M_1 + g_1^2 M_2)(\mu x + v_1 wy_\nu)^2}{2\mu(\mu M_1 M_2 - (g_2^2 M_1 + g_1^2 M_2)v_1 v_2)},
\end{align*}

and the mixing matrix has this tribimaximal form,

\begin{equation}
U = \begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\end{equation}

### 4 Symmetry Breaking

In this section we analyze the potential and the minimization conditions. We show that in this model it is possible to get a higher value of \( \tilde{N} \) vacuum expectation value even for a smaller vacuum expectation value of the \( \tilde{\nu} \) sneutrino field. The small value of \( \langle \tilde{\nu} \rangle \) could be realized for small yukawa \( y_\nu \). In our model the potential is,

\[ V = V_D + V_F + V_{\text{soft}} \]

where $V_D$, $V_F$ represent the D term and F term contributions to the potential respectively and $V_{\text{soft}}$ comes from the soft supersymmetry breaking Lagrangian given in Eq. (61) and in Eq. (62). The F term contribution is $V_F = \sum_i |F_i|^2$, the index $i$ represents the different auxiliary fields of the theory. The neutral component of the potential which would be relevant for the analysis of symmetry breaking is $V_{\text{neutral}} = V_D^n + V_F^n + V_{\text{soft}}^n$ where the D term contribution to $V_{\text{neutral}}$ is,

\[ V_D^n = \frac{1}{8}(g_2^2 + g_1^2)(|H_d^0|^2 - |H_u^0|^2 + \sum_i |\tilde{\nu}_i|^2)^2. \]

Note that the $A_4$ invariant Kähler potential of the gauge singlet matter chiral superfields $\tilde{N}$ is,

\[ \mathcal{K}^N = \int d^4 \theta \tilde{N}^\dagger \tilde{N}. \]

\[ \tilde{N}^\dagger \tilde{N} = \tilde{N}_1^\dagger \tilde{N}_1 + \tilde{N}_2^\dagger \tilde{N}_2 + \tilde{N}_3^\dagger \tilde{N}_3 \] is $A_4$ invariant, hence the Kähler potential is canonical.
Since $\tilde{N}$ is singlet under the MSSM gauge group, hence it does not contributes to the $V_D$. The same argument holds for the other gauge singlets Higgs chiral superfield $\hat{\phi}_{T,S}$ and $\hat{\xi}$. The different auxiliary fields $F_i$ which would be relevant in determining the neutral component of the potential are,

$$-F^*_H = y_\nu (\tilde{N}_1 \tilde{\nu}_e + \tilde{N}_2 \tilde{\nu}_\tau + \tilde{N}_3 \tilde{\nu}_\mu) - \mu H_d^0 + ..., \quad (36)$$

$$-F^*_H = -\mu H_u^0 + ..., \quad (37)$$

$$-F^*_\nu = y_\nu \tilde{N}^0_1 \nu^0 + ..., \quad -F^*_\nu = y_\nu \tilde{N}^0_3 \nu^0 + ..., \quad -F^*_\nu = y_\nu \tilde{N}^0_2 \nu^0 + ..., \quad (38)$$

$$-F^*_\xi = x_A (\tilde{N}_1 \tilde{N}_1 + 2 \tilde{N}_2 \tilde{N}_3) + 3 \alpha \xi^2 + \beta (\phi_S \phi_{S_1} + 2 \phi_{S_2} \phi_{S_3}), \quad (39)$$

$$-F^*_{N_1} = 2 x_A \xi \tilde{N}_1 + \frac{2 x_B}{3} (2 \phi_S \tilde{N}_1 - \phi_{S_1} \tilde{N}_3 - \phi_{S_2} \tilde{N}_2) + y_\nu \tilde{\nu}_e H_u^0, \quad (40)$$

$$-F^*_{N_2} = 2 x_A \xi \tilde{N}_3 + \frac{2 x_B}{3} (2 \phi_S \tilde{N}_2 - \phi_{S_1} \tilde{N}_3 - \phi_{S_2} \tilde{N}_1) + y_\nu \tilde{\nu}_e H_u^0, \quad$$

$$-F^*_{N_3} = 2 x_A \xi \tilde{N}_2 + \frac{2 x_B}{3} (2 \phi_S \tilde{N}_3 - \phi_{S_1} \tilde{N}_2 - \phi_{S_2} \tilde{N}_1) + y_\nu \tilde{\nu}_\mu H_u^0,$$

$$-F^*_S_1 = \frac{2 x_B}{3} (\tilde{N}_1 \tilde{N}_1 - \tilde{N}_2 \tilde{N}_3) + 2 \beta \phi_{S_1} \xi + 2 \beta' (\phi^2_{S_1} - \phi_{S_2} \phi_{S_3}), \quad (41)$$

$$-F^*_S_2 = \frac{2 x_B}{3} (\tilde{N}_2 \tilde{N}_2 - \tilde{N}_1 \tilde{N}_3) + 2 \beta \phi_{S_2} \xi + 2 \beta' (\phi^2_{S_2} - \phi_{S_1} \phi_{S_3}), \quad$$

$$-F^*_S_3 = \frac{2 x_B}{3} (\tilde{N}_3 \tilde{N}_3 - \tilde{N}_1 \tilde{N}_2) + 2 \beta \phi_{S_3} \xi + 2 \beta' (\phi^2_{S_3} - \phi_{S_1} \phi_{S_2}),$$

and

$$-F^*_T_1 = 2 \gamma (\phi^2_{T_1} - \phi_{T_2} \phi_{T_3}) + 2 \gamma \phi_{T_1} + ..., \quad (42)$$

$$-F^*_T_2 = 2 \gamma (\phi^2_{T_2} - \phi_{T_1} \phi_{T_3}) + 2 \gamma \phi_{T_2} + ..., \quad$$

$$-F^*_T_3 = 2 \gamma (\phi^2_{T_3} - \phi_{T_1} \phi_{T_2}) + 2 \gamma \phi_{T_3} + ...$$

With these auxiliary fields of the superfields $\hat{H}_{u,d}, \hat{\nu}_i, \hat{\xi}, \hat{N}, \hat{\phi}_S$ and $\hat{\phi}_T$ the F term contribution to the neutral component of the potential will be,

$$V^F_n = V^{F_{H_u^0 H_d^0 \hat{\nu}_i}}_n + V^{F_{\hat{\xi}}}_n + V^{F_{\hat{N}}}_n + V^{F_{\hat{S}}}_n + V^{F_{\hat{T}}}_n, \quad (43)$$
where
\[
\langle V_{F,n}^{F,n} \rangle = \mu^2 v_1^2 + \mu^2 v_2^2 + y_v^2 (w_1 x_1 + w_2 x_2 + w_3 x_2)^2 - 2 \mu y_v v_1 (w_1 x_1 + w_2 x_2 + w_3 x_2) + y_v^2 v_3^2 (w_1^2 + w_2^2 + w_3^2),
\]
(44)

\[
\langle V_{F,n}^{n} \rangle = |x_A (w_1^2 + 2 w_2 w_3) + 3 \alpha s^2 + \beta (u_1^2 + 2 u_2 u_3)|^2,
\]
(45)

\[
\langle V_{N,n}^{F,n} \rangle = |2 x_A s w_1 + \frac{2 x_B}{3} (2 u_1 w_1 - u_2 w_3 - u_3 w_2) + y_v x v_2|^2
\]
(46)

\[
\langle V_{S,n}^{F,n} \rangle = |\frac{2 x_B}{3} (w_1^2 - w_2 w_3) + 2 \beta u_1 s + 2 \beta' (u_1^2 - u_2 u_3)|^2
\]
(47)

and

\[
\langle V_{T,n}^{F,n} \rangle = |2 \gamma' (t_2^2 - t_2 t_3) + 2 \gamma t_2|^2 + |2 \gamma' (t_2^2 - t_2 t_3) + 2 \gamma t_3|^2.
\]
(48)

The soft supersymmetry breaking Lagrangian for this model has been given in Eq. (61) and Eq. (62) in the appendix and below we write the soft supersymmetry breaking contribution to \( \langle V_{\text{neutral}} \rangle \),

\[
\langle V_{\text{soft}}^{n} \rangle = 2 \bar{y}_v (w_1 x_1 + w_2 x_2 + w_3 x_2) v_2 + 2 \bar{x}_A s (w_1^2 + 2 w_2 w_3)
\]
(49)

\[+ \frac{2 x_B}{3} (2 u_1 w_1^2 + 2 u_2 w_2^2 + 2 u_3 w_3^2 - 2 u_1 w_1 w_3 - 2 u_1 w_1 w_3 - 2 u_3 w_1 w_2)\]

\[+ 2 \alpha s^3 + 2 \tilde{\beta} s (u_1^2 + 2 u_2 u_3) + 2 \tilde{\gamma} (t_1^2 + 2 t_2 t_3) + \frac{4 \tilde{\beta}'}{3} (u_1^3 + u_2^3 + u_3^3 - 3 u_1 u_2 u_3)\]

\[+ \frac{4 \tilde{\gamma}'}{3} (t_1^2 + t_2^2 + t_3^3 - 3 t_1 t_2 t_3) + \gamma (t_1^2 + t_2^2 + t_3^2) + r_s (u_1^2 + u_2^2 + u_3^2)\]

\[+ r_N (w_1^2 + w_2^2 + w_3^2) + m_{H_u}^2 v_2^2 + m_{H_d}^2 v_1^2 - 2 b v_1 v_2 + m_s^2 (x_1^2 + x_2^2 + x_3^2)\]
(50)

To simplify the analysis we assume all the vacuum expectation values and the couplings as real and also assume \( u_1 = u_2 = u_3 = u, w_1 = w_2 = w_3 = w, x_1 = x_2 = x_3 = x \) and
t_1 = t, t_2 = t_3 = 0. Minimizing \( \langle V_{\text{neutral}} \rangle \) given in Eq. (43) w.r.t the different vacuum expectation values \( t, w, x, s, u \) and \( v_{1,2} \) we get these following equations respectively,

\[
8\gamma'^2 t^2 + 4\gamma' + 12\gamma' t + r_T + 2\gamma t + 2\gamma = 0; t \neq 0, \tag{51}
\]

\[
36x_A^2 w^3 + 36x_A \alpha s^2 w + 36x_A \beta w u^2 + 18y_{\nu}^2 x^2 w - 6y_{\nu} \mu v_1 x + 6y_{\nu}^2 v_2^2 w \\
12x_A y_{\nu} v_2 s x + 24x_A^2 s^2 w + 6y_{\nu} v_2 x + 12x_A s w + 6r_N w = 0, \tag{52}
\]

\[
18y_{\nu}^2 w^2 x - 6y_{\nu} \mu v_1 w + 6y_{\nu}^2 v_2^2 x + 6y_{\nu} v_2 w + 6m_L^2 x \\
+ 12x_A y_{\nu} v_2 s w + \left(\frac{g_1^2 + g_2^2}{2}\right) 3x(v_1^2 - v_2^2 + 3x^2) = 0, \tag{53}
\]

\[
36\alpha^2 s^3 + 36x_A \alpha s w^2 + 36\beta \alpha s u^2 + 24x_A^3 s w^2 + 24\beta^2 u^2 s \\
+ 12x_A y_{\nu} v_2 w x + 6x_A^2 w^2 + 6\alpha s^2 + 6\beta u^2 + 2r_s s = 0, \tag{54}
\]

\[
36\beta^2 u^3 + 36\beta \alpha s u^2 + 36x_A \beta w u^2 + 24\beta^2 u s^2 + 12\beta s u + 6r_s u = 0, \tag{55}
\]

\[
2\mu^2 v_1 + 2m_{H_u}^2 v_1 - 2b v_2 - 6y_{\nu} \mu w x + \frac{v_1}{2}(g_1^2 + g_2^2)(v_1^2 - v_2^2 + 3x^2) = 0, \tag{56}
\]

\[
2\mu^2 v_2 + 2m_{H_u}^2 v_2 - 2b v_1 - \frac{v_2}{2}(g_1^2 + g_2^2)(v_1^2 - v_2^2 + 3x^2) \\
+ 12x_A y_{\nu} s w x + 6y_{\nu}^2 w^2 v_2 + 6y_{\nu}^2 x^2 v_2 + 6y_{\nu} w x = 0. \tag{57}
\]

As evident from Eq. (53), the vacuum expectation value \( x \to 0 \) could be naturally realized in the limit \( y_{\nu} \to 0 \). In the limit of small yukawa i.e \( y_{\nu} \to 0 \) and \( x \to 0 \) from Eq. (52) one will get the following relation between the different vacuum expectation values of the sneutrino \( \tilde{N} \) and the Higgs \( \phi_S \) and \( \xi \) fields,

\[
w^2 = \frac{-6x_A \alpha s^2 + 6x_A \beta u^2 + 4x_A^2 s^2 + 2x_A s + r_N}{6x_A^2} \tag{58}
\]

Using Eq. (52) and Eq. (54), the vacuum expectation value \( s \) of the \( \xi \) field would come as,

\[
s = \frac{-\tilde{B} \pm \sqrt{\tilde{B}^2 - 4\tilde{A}\tilde{C}}}{2\tilde{A}} \tag{59}
\]

where \( \tilde{A} = 4x_A \beta(x_A - \beta), \tilde{B} = 2(\beta x_A - \tilde{\beta} x_A) \) and \( \tilde{C} = (\beta r_N - r_S x_A) \). It is clearly evident from Eq. (58) that the vacuum expectation value \( w \) of the sneutrino field \( \tilde{N} \) is related with the vacuum expectation value of the different standard model singlet R-Parity even Higgs fields \( \xi \) and \( \phi_S \), as well as it depends on the soft supersymmetry breaking coupling \( r_N \) and even in the \( (\tilde{\nu}) = x \to 0 \) limit it is possible to get a nonzero and higher value of the sneutrino vacuum expectation value \( \langle \tilde{N} \rangle \).
5 Conclusion

In this work we have explored the possibility of spontaneous R parity violation in the context of a specific flavor model. The flavor symmetry group is $A_4$ and in addition we have implemented another discrete symmetry group $Z_3$. The superpotential given in Eq. (3) conserves R-parity. However R-parity would be broken spontaneously when the different sneutrino fields which are odd under R parity will get the vacuum expectation values. The $A_4$ flavor symmetry will be broken by the vacuum expectation values of the $A_4$ triplet fields. Because of the R parity violation we have mixing between the standard model neutrinos, MSSM higgsinos and gauginos, as well as mixing between the gauge singlet neutrinos and MSSM higgsinos. In our model the Higgs and higgsino sector is enlarged because of the presence of the gauge singlet Higgs chiral superfield $\hat{\phi}_S$, $\hat{\phi}_T$ and $\hat{\xi}$. Hence, in addition to the higgsino-higgsino, gaugino-higgsino and the conventional Dirac type neutrino-neutrino mixings we also have mixings between the neutrino and these gauge singlet higgsinos. As a result in our model the neutral fermion mass matrix is a $17 \times 17$ matrix. However we show that the low energy neutrino mass matrix which will be generated once the different heavier neutral fermionic states are integrated out, can still have a specific form leading to the tribimaximal mixing matrix, provided few constraints between the different sneutrino and Higgs vacuum expectation values are satisfied.

We have also explored the potential minimization in detail. One can relate the spontaneous R parity violation with some higher gauge symmetry breaking and in these kind of models [33] one realizes a higher value of one of the the R parity violating sneutrino vacuum expectation value. On the other hand if one sticks to the basic MSSM gauge group and also the MSSM particle content and explores the R parity violation, one would eventually get into the trouble of Majoron [23]. However introducing one MSSM gauge group singlet/triplet matter chiral superfield one can avoid the problem of Majoron because of the explicit breaking of lepton number although breaking R parity spontaneously [34]. In this kind of models the different sneutrino vacuum expectation values share a proportionality relation and the smallness of the neutrino mass forces the sneutrino vacuum expectation values to be small. In this present model although we stick to the basic MSSM gauge group, however it is possible to realize a higher value of the sneutrino VEV $\langle \tilde{N} \rangle$ even in the $\langle \tilde{\nu} \rangle \to 0$ limit. We have analyzed the potential and have shown this particular feature explicitly. For the sake of simplicity we have assumed the different sneutrino vacuum expectation values $u_{1,2,3} = w$ and $x_{1,2,3} = x$ as well as the vacuum expectation values of $\phi_S$ fields $u_{1,2,3} = u$. Note that for this assumption one would also obtain the desired tribimaximal mixing in the low energy neutrino sector, as already been mentioned in section 3 and in the earlier paragraph. We have shown that the sneutrino vacuum expectation value $\langle \tilde{N} \rangle$ is related with the other Higgs vacuum expectation values $\langle \hat{\phi}_S \rangle$ and $\langle \hat{\xi} \rangle$ as well as it depends on the soft supersymmetry breaking parameter $r_N$ given in Eq. (62). Hence for
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Appendix

A Soft Supersymmetry Breaking

Here we write the soft supersymmetry breaking Lagrangian of our model. Following supergravity mediated supersymmetry breaking ansatz \[^{3}^{3}\] the soft supersymmetry breaking lagrangian of this model would be

\[
\mathcal{L}^{\text{soft}} = \mathcal{L}^{\text{soft}}_{1} + \mathcal{L}^{(\phi_T, \phi_S, \xi, \tilde{N})}_{\text{soft}}
\]

where

\[
- \mathcal{L}^{\text{soft}}_{1} = (m_{Q}^{2})_{ij} \tilde{Q}_{i}^{\dagger} \tilde{Q}_{j} + (m_{\tilde{e}}^{2})_{ij} \tilde{e}_{i}^{c} \tilde{e}_{j}^{c} + (m_{\tilde{d}}^{2})_{ij} \tilde{d}_{i}^{c} \tilde{d}_{j}^{c} + (m_{\tilde{l}}^{2})_{ij} \tilde{L}_{i}^{\dagger} \tilde{L}_{j}
\]

\[
+ (m_{H_d}^{2})_{ij} \tilde{H}_{i}^{\dagger} H_{d} + m_{H_u}^{2} H_{u}^{\dagger} H_{u} + (b H_{u} H_{d} + \text{H.c.})
\]

\[
+ \left[-A_{ij}^{u} H_{u} \tilde{Q}_{i} \tilde{e}_{j}^{c} + A_{ij}^{d} H_{d} \tilde{Q}_{i} \tilde{d}_{j}^{c} + \text{H.c.}\right]
\]

\[
+ \left[\tilde{y}_{1} H_{d}(\phi_{T}) \tilde{L} \tilde{e}^{c} + \tilde{y}_{2} H_{d}(\phi_{T}) \tilde{L}^{\prime \prime} \tilde{\mu}^{c} + \tilde{y}_{3} H_{d}(\phi_{T}) \tilde{L}^{\prime} \tilde{\tau}^{c} + \text{H.c.}\right]
\]

\[
+ \frac{1}{2} \left(M_{3} \tilde{g} \tilde{g} + M_{2} \tilde{\lambda} \tilde{\lambda}^{c} + M_{1} \tilde{\lambda}^{0} \tilde{\lambda}^{0} + \text{H.c.}\right).
\]

Note that the trilinear soft supersymmetry breaking terms involving the different slepton fields would be generated from the \( \tilde{y}_{1} H_{d}(\phi_{T}) \tilde{L} \tilde{e}^{c} \), \( \tilde{y}_{2} H_{d}(\phi_{T}) \tilde{L}^{\prime \prime} \tilde{\mu}^{c} \) and \( \tilde{y}_{3} H_{d}(\phi_{T}) \tilde{L}^{\prime} \tilde{\tau}^{c} \) operators once the \( A_{4} \) triplet Higgs \( \phi_{T} \) gets the vacuum expectation value, thereby breaking \( A_{4} \)

[^{3}^{3}]: We consider the canonical Kähler potential and the gauge kinetic function to be \( f_{AB} = \delta_{AB} \)
spontaneously. Since we adopt a supergravity mediated supersymmetry breaking mechanism, \( y_\nu = y_\nu a \) and similar kind of relation would hold for other trilinear and bilinear couplings as well. The Kähler potential involving the gauge singlet superfields \( \hat{\phi}_{S,T} \), \( \hat{\xi} \) and \( \hat{N} \) is,

\[
K^S = \int d^4\theta (\hat{\phi}_{S}^\dagger \hat{\phi}_S + \hat{\phi}_{T}^\dagger \hat{\phi}_T + \hat{\xi}^\dagger \hat{\xi} + \hat{N}^\dagger \hat{N})
\]  

(63)

The Kähler potential for the other superfields which transform nontrivially under standard model gauge group will have this form,

\[
K = \int d^4\theta (\hat{\phi}_{i}^\dagger e^{2gV} \hat{\phi}_i)
\]  

(64)

where \( \hat{\phi}_i \) is any MSSM superfield and \( V \) is the vector superfield.

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