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String field theory

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Abstract
This elementary introduction to string field theory highlights the features and the limitations of this approach to quantum gravity as it is currently understood. String field theory is a formulation of string theory as a field theory in space-time with an infinite number of massive fields. Although existing constructions of string field theory require expanding around a fixed choice of space-time background, the theory is in principle background-independent, in the sense that different backgrounds can be realized as different field configurations in the theory. String field theory is the only string formalism developed so far which, in principle, has the potential to systematically address questions involving multiple asymptotically distinct string backgrounds. Thus, although it is not yet well defined as a quantum theory, string field theory may eventually be helpful for understanding questions related to cosmology in string theory.

1.1 Introduction
In the early days of the subject, string theory was understood only as a perturbative theory. The theory arose from the study of S-matrices and was conceived of as a new class of theory describing perturbative interactions of massless particles including the gravitational quanta, as well as an infinite family of massive particles associated with excited string states. In string theory, instead of the one-dimensional world line
of a pointlike particle tracing out a path through space-time, a two-dimensional surface describes the trajectory of an oscillating loop of string, which appears pointlike only to an observer much larger than the string.

As the theory developed further, the need for a nonperturbative description of the theory became clear. The M(atrix) model of M-theory, and the AdS/CFT correspondence, each of which is reviewed in another chapter of this volume, are nonperturbative descriptions of string theory in space-time backgrounds with fixed asymptotic forms. These approaches to string theory give true nonperturbative formulations of the theory, which fulfill in some sense one of the primary theoretical goals of string theory: the formulation of a nonperturbative theory of quantum gravity.

There are a number of questions, however, which cannot–even in principle–be answered using perturbative methods or the nonperturbative M(atrix) and AdS/CFT descriptions. Recent experimental evidence points strongly to the conclusion that the space-time in which we live has a small but nonzero positive cosmological constant. None of the existing formulations of string theory can be used to describe physics in such a space-time, however, existing tools in string theory and field theory suggest that string theory has a large number of metastable local minima with positive cosmological constants. The term “string landscape” (see, e.g., Susskind, 2003) is often used to describe the space of string theory configurations which includes all these metastable local minima. We currently have no tools to rigorously define this space of string theory configurations, however, or to understand the dynamics of string theory in a cosmological context–a formalism capable of describing the string landscape would presumably need to be a background-independent formulation of the theory such as string field theory.

The traditional perturbative approach to string theory involves constructing a field theory on the two-dimensional string “world-sheet” $\Sigma$, which is mapped into the “target” space-time through a function $X: \Sigma \to \text{space-time}$; this function is locally described by a set of coordinates $X^\mu$. The theory on the world-sheet is quantized, and the excitations of the resulting string become associated with massless and massive particles moving in space-time. The states of the string live in a Fock space similar to the state space of a quantized simple harmonic oscillator. The ground state of the string at momentum $p$, denoted $|p\rangle$,
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is associated with a space-time scalar particle† of momentum \( p \). There are two kinds of raising operators acting on the single-string Fock space, analogous to the raising operator \( a_\dagger \) which adds a unit of energy to a simple harmonic oscillator. The operators \( \alpha_{\mu n} \equiv (\alpha_{\mu n}^\dagger) \) and \( \tilde{\alpha}^\mu_{\mu n} \equiv (\tilde{\alpha}^\mu_{\mu n}^\dagger) \) each add a unit of excitation to the \( n \)th oscillation modes of the \( \mu \) coordinate of the string. There are two operators for each \( n \) because there are two such oscillation modes, which can be thought of as sin and cos modes or as right- and left-moving modes. The excited states of the string correspond to different particles in space-time. For example, the state

\[
(\alpha^\mu_{\mu 1} \tilde{\alpha}^\nu_{\nu 1} + \alpha^\nu_{\nu 1} \tilde{\alpha}^\mu_{\mu 1}) |p\rangle
\]

(1.1)
corresponds to a symmetric spin 2 particle of momentum \( p \). These states satisfy a physical state condition \( p^2 = 0 \), so that this excitation state of the string can be associated with a quantum of the gravitational field—a graviton. Acting with more raising operators on the string state produces a series of more and more highly excited strings corresponding to a tower of massive particle states in space-time. In perturbative string theory, interactions between the massless and massive particles of the theory are computed by calculating correlation functions on the string world-sheet using techniques of two-dimensional conformal field theory.

The basic idea of string field theory is to reformulate string theory in the target space-time, rather than on the world sheet, as an off-shell theory of the infinite number of fields associated with the states in the string Fock space. The degrees of freedom in string field theory are encoded in a “string field”, which can be thought of in several equivalent ways. Conceptually, the simplest way to think of a string field is as a functional \( \Psi[X(\sigma)] \), which associates a complex number with every possible configuration \( X(\sigma) \) of a one-dimensional string with coordinate \( \sigma \). This is the natural generalization to a string of the standard quantum mechanical wave function \( \psi(x) \), which associates a complex number with every possible position \( x \) of a pointlike particle in space. Mathematically, however, dealing directly with functionals like \( \Psi[X(\sigma)] \) is difficult and awkward. In most cases it is more convenient to use a Fock space representation of the the string field. Just as a wave function \( \psi(x) \in L^2(\mathbb{R}) \) for a single particle can be represented in a basis of harmonic oscillator

† Actually, this ground state is associated with a scalar tachyon field describing a particle with negative mass squared \( m^2 < 0 \). The presence of such a tachyon indicates that the vacuum around which the theory is being expanded is unstable. This tachyon is removed from the spectrum when we consider supersymmetric string theory.
eigenstates $|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$, the string field $\Psi[X(\sigma)]$ representing a string moving in $D$ space-time dimensions can be equivalently represented in the string Fock space through

$$
\Psi = \int d^Dp \phi(p)|p\rangle + g_{\mu\nu}(p)(\alpha_{-1}^\mu \tilde{\alpha}_{-1}^\nu + \alpha_{-1}^\nu \tilde{\alpha}_{-1}^\mu)|p\rangle + \cdots
$$

(1.2)

where the sum includes contributions from the infinite tower of massive string states. Because in this case the states carry a continuously varying momentum, the coefficient of each state, which was just a constant $c_n$ in the case of the harmonic oscillator, becomes a field in space-time written in the Fourier representation. Thus, we see that the string field contains within it an infinite family of space-time fields, including the scalar field $\phi$, the graviton field (metric) $g_{\mu\nu}$, and an infinite family of massive fields.

String field theory is defined by giving an action functional $L(\Psi)$ depending on the string field. When written in terms of the individual component fields $\phi(x), g_{\mu\nu}(x), \ldots$, this then gives a fairly conventional-looking action for a quantum field theory, although the number of fields is infinite and the interactions may contain higher derivatives and appear nonlocal. To be a consistent description of a known perturbative string theory, the action must be chosen carefully so that the perturbative string field theory diagrams precisely reproduce the string amplitudes computed from the perturbative string theory. This requirement puts a highly constraining algebraic structure on the theory (Zwiebach, 1993; Gaberdiel and Zwiebach, 1997a, 1997b). Generally, it is necessary to include an infinite series of terms in the action to meet this requirement, although in the case of the bosonic open string Witten has given an elegant formulation of string field theory which includes only cubic interaction terms for the string field $\Psi$. We will describe this simplest and best-understood string field theory in the next section.

Once a string field theory has been defined through an action, the next question is whether it can be used as a tool to usefully compute new results in string theory which extend beyond those accessible to the perturbative formulation of the theory. Although work on string field theory began over 30 years ago, until 7 years ago there was no clear example of a calculation in which string field theory gave results which go beyond perturbation theory. In 1999, however, Ashoke Sen (1999) made an insightful conjecture that two distinct open string backgrounds, one with a space filling D-brane and one without, could be explicitly realized as different solutions of the same open string field theory. Subsequent work on this conjecture has brought new impetus to
the study of string field theory, and has conclusively demonstrated the nonperturbative background-independence of the theory. Despite these advances, however, there are still enormous technical challenges for the theory. The theory is not completely well-defined even at the classical level, and a full definition of the quantum theory seems very difficult. Analytic calculations are difficult and involve subtle issues of limits and divergences, and numerical computations, while possible in many cases, are cumbersome and often difficult to interpret. Even for the simpler open string field theory many conceptual challenges exist, and although there has been recent progress on formulating closed string field theories, using these theories to describe the landscape of string vacua is still well beyond our technical capacity.

In the remainder of this paper we describe in some further detail the state of knowledge in this subject. In section 2 we give a somewhat more explicit description of Witten’s open bosonic string field theory; we describe the recent work in which this theory was shown to describe distinct string backgrounds, and we discuss some outstanding issues for this theory. In section 3 we review the state of the art in closed string field theory. Section 4 contains a summary of successes and challenges for this formulation of string theory and some speculation about possible future directions for this area of research.

1.2 Open string field theory (OSFT)

We now introduce the simplest covariant string field theory. A very simple cubic form for the off-shell open bosonic string field theory action was proposed by Witten (1986). In subsection 1.2.1 we briefly summarize the string field theory described by this action. In subsection 1.2.2 we review the recent work applying this theory to the study of Sen’s conjecture and discuss the progress which has been made. For a more detailed review of this subject see Taylor & Zwiebach (2001). In subsection 1.2.3 we discuss some problems and outstanding issues for open string field theory.

It is useful to recall here the difference between open and closed strings. A closed string forms a one-dimensional loop. Parameterizing the string by $\sigma \in [0, 1]$ we form a closed string by identifying the endpoints $\sigma = 0, \sigma = 1$. Because fields on a closed string take periodic boundary conditions, there are separate right- and left-moving modes. This is what allows us to construct a graviton state from a closed string as in (1.1). An open string, on the other hand, has Dirichlet ($X = 0$) or
Neumann ($\partial_r X = 0$) boundary conditions at the endpoints, and therefore only has one set of oscillation modes, which are associated with a single family of raising operators $\alpha^\mu_n$. For the bosonic open string, the string field can then be expanded as

$$\Psi = \int d^{26}p \left[ \varphi(p) |p\rangle + A_\mu(p) \alpha^{-1}_\mu |p\rangle + \cdots \right]. \quad (1.3)$$

The leading fields in this expansion are a space-time tachyon field $\varphi(p)$ and a massless space-time vector field $A_\mu(p)$.

### 1.2.1 Witten’s cubic OSFT action

The action proposed by Witten for the open bosonic string field theory takes the simple cubic form

$$S = -\frac{1}{2} \int \Psi \ast Q \Psi - \frac{g}{3} \int \Psi \ast \Psi \ast \Psi. \quad (1.4)$$

In this action, $g$ is the (open) string coupling constant. The field $\Psi$ is the open string field. Abstractly, this field can be considered to take values in an algebra $A$. Associated with the algebra $A$ there is a star product

$$\ast : A \otimes A \rightarrow A, \quad (1.5)$$

The algebra $A$ is graded, such that the open string field has degree $G = 1$, and the degree $G$ is additive under the star product ($G_{\Psi \ast \Phi} = G_\Psi + G_\Phi$). There is also an operator

$$Q : A \rightarrow A, \quad (1.6)$$

called the BRST operator, which is of degree one ($G_{Q \Psi} = 1 + G_\Psi$).

String fields can be integrated using

$$\int : A \rightarrow \mathbb{C}. \quad (1.7)$$

This integral vanishes for all $\Psi$ with degree $G_\Psi \neq 3$. Thus, the action (1.4) is only nonvanishing for a string field $\Psi$ of degree 1. The action (1.4) thus has the general form of a Chern-Simons theory on a 3-manifold, although for string field theory there is no explicit interpretation of the integration in terms of a concrete 3-manifold.
The elements $Q, \star, \int$ that define the string field theory are assumed to satisfy the following axioms:

(a) Nilpotency of $Q$: $Q^2 \Psi = 0, \ \forall \Psi \in \mathcal{A}$.

(b) $\int Q \Psi = 0, \ \forall \Psi \in \mathcal{A}$.

(c) Derivation property of $Q$: $Q(\Psi \star \Phi) = (Q \Psi) \star \Phi + (-1)^{G \Psi} \Psi \star (Q \Phi), \ \forall \Psi, \Phi \in \mathcal{A}$.

(d) Cyclicity: $\int \Psi \star \Phi = (-1)^{G \Psi + G \Phi} \int \Phi \star \Psi, \ \forall \Psi, \Phi \in \mathcal{A}$.

(e) Associativity: $(\Phi \star \Psi) \star \Xi = \Phi \star (\Psi \star \Xi), \ \forall \Phi, \Psi, \Xi \in \mathcal{A}$.

When these axioms are satisfied, the action (1.4) is invariant under the gauge transformations

$$
\delta \Psi = Q \Lambda + \Psi \star \Lambda - \Lambda \star \Psi,
$$

for any gauge parameter $\Lambda \in \mathcal{A}$ with degree 0.

When the string coupling $g$ is taken to vanish, the equation of motion for the theory defined by (1.4) simply becomes $Q \Psi = 0$, and the gauge transformations (1.8) simply become

$$
\delta \Psi = Q \Lambda.
$$

This structure at $g = 0$ is precisely what is needed to describe a free bosonic string in the BRST formalism, where physical states live in the cohomology of the BRST operator $Q$, which acts on the string Fock space†. The motivation for introducing the extra structure in (1.4) was to find a simple interacting extension of the free theory, consistent with the perturbative expansion of open bosonic string theory.

Witten presented this formal structure and argued that all the needed axioms are satisfied when $\mathcal{A}$ is taken to be the space of string fields of the form (1.3). In this realization, the star product $\star$ acts on a pair of functionals $\Psi, \Phi$ by gluing the right half of one string to the left half of the other using a delta function interaction

† For a detailed introduction to BRST string quantization, see Polchinski (1998)
Similarly, the integral over a string field corresponds to gluing the left and right halves of the string together with a delta function interaction

\[ \Psi \]

Combining these pictures, the three-string vertex \( \int \Psi_1 \star \Psi_2 \star \Psi_3 \) corresponds to a three-string overlap

\[ \Psi_1 \quad \Psi_2 \quad \Psi_3 \]

While these pictures may seem rather abstract, they can be given explicit meaning in terms of the oscillator raising and lowering operators \( \alpha_n^\alpha \) (Cremmer et al., 1986; Ohta, 1986; Samuel, 1986; Gross and Jevicki, 1987a, 1987b). Given an explicit representation of the terms in the string field action in terms of these raising and lowering operators, the contribution to the action from any set of component fields in the full string field can be worked out. The quadratic terms for the string fields \( \phi(p), A_\mu(p) \) are the standard kinetic and mass terms for a tachyon field and a massless gauge field. The massive string fields similarly have kinetic terms and positive mass squared terms. The interaction terms for the component fields coming from the term \( \int \Psi \star \Psi \star \Psi \) in the action, however, seem more exotic from the point of view of conventional field theory. These terms contain exponentials of derivatives, which appear as nonlocal interactions from the point of view of field theory. For example, the cubic interaction term for the scalar tachyon field \( \varphi(p) \) takes the momentum space form

\[
\int d^{26}p d^{26}q \frac{\kappa q}{3} e^{(\ln(16/27)(p^2 + q^2 + p \cdot q))} \varphi(-p)\varphi(-q)\varphi(p + q). \tag{1.10}
\]

where \( \kappa \) is a constant. There are similar interaction terms between general sets of 3 component fields in the string field.

The appearance of an infinite number of fields and arbitrary numbers of derivatives (powers of momentum) in the action make the target space string field theory into a very unusual field theory. There are a
number of obstacles to having a complete definition of this theory as a quantum field theory. Even at the classical level, it is not clear precisely what range of fields is allowed for the string field. In particular, due to the presence of ghosts, there is no positive definite inner product on the string Fock space, so there is no natural finite norm condition to constrain the class of allowed string fields. Determining precisely what normalization condition should be satisfied by physical states is an important problem which may need to be solved to make substantial progress with the theory as a nonperturbative formulation of string theory. Beyond this issue the unbounded number of derivatives makes even the classical time-dependence of the string field difficult to pin down. The string field seems to obey a differential equation of infinite order, suggesting an infinite number of boundary conditions are needed. Some recent progress on these problems has been made (Moeller & Zwiebach, 2002; Erler & Gross, 2004; Coletti et al., 2005), but even in this simplest case of Witten’s open cubic bosonic string field theory, it seems clear that we are far from a complete understanding of how the theory should be defined. Despite these difficulties, however, the action (1.4) gives rise to a well-defined perturbative theory which can be used to calculate scattering amplitudes of on-shell string states associated with particles in the string Fock space. Furthermore, it was shown that these amplitudes agree with the perturbative formulation of string theory, as desired (Giddings & Martinec, 1986; Giddings, Martinec, & Witten, 1986; Zwiebach, 1991).

1.2.2 The Sen conjectures

Despite our limited understanding of the full definition of quantum string field theory, in the last few years a great deal of progress has been made in understanding the nature of the classical open string field theory described in the previous subsection.

One apparent problem for the open bosonic string and the associated string field theory is the open string tachyon. This tachyon indicates that the vacuum of the theory is unstable and can decay. Ashoke Sen (1999) conjectured that a precise understanding of the nature of this instability and decay process could be attained through open string field theory. He argued that the unstable vacuum is one with a space-filling “D-brane” carrying positive energy density. D-branes have been a major subject of study in string theory over the last decade. D-branes are higher-dimensional extended objects on which open strings can end. In
supersymmetric string theories, D-branes of some dimensions can be stable and supersymmetric. In the bosonic string theory, however, all D-branes are unstable. Sen suggested that the instability of the space-filling D-brane in bosonic string theory is manifested by the open bosonic string tachyon. He further suggested that string field theory should contain another nonzero field configuration $\Psi^*$ which would satisfy the classical equation of motion $Q\Psi^* + g\Psi^* \ast \Psi^* = 0$. Sen argued that this nontrivial vacuum field configuration should have several specific properties. It should have a vacuum energy which is lower than the initial unstable vacuum by precisely the volume of space-time times the energy density (tension) $T$ of the unstable D-brane. The stable vacuum should also have no open string excitations. This latter condition is highly nontrivial and states that at the linearized level all open string fluctuations around the nontrivial vacuum become unphysical. To realize this change of backgrounds, the degrees of freedom of the theory must reorganize completely in going from one background to another. The ability of a single set of degrees of freedom to rearrange themselves to form the physical degrees of freedom associated with fluctuations around different backgrounds is perhaps the most striking feature of background-independent theories, and presents the greatest challenge in constructing and understanding such theories.

Following Sen’s conjectures, a substantial body of work was carried out which confirmed these conjectures in detail. A primary tool used in analyzing these conjectures using string field theory was the notion of “level truncation”. The idea of level truncation is to reduce the infinite number of string fields to a finite number by throwing out all fields above a fixed mass cutoff. By performing such a truncation and restricting attention to the constant modes with $p = 0$, the infinite number of string field component equations reduces to a finite system of cubic equations. These equations were solved numerically at various levels of truncation, and confirmed to 99.99% accuracy the conjecture that there is a nontrivial vacuum solution with the predicted energy (Sen & Zwiebach, 2000; Moeller & Taylor, 2000; Gaiotto & Rastelli, 2003; Taylor, 2003). The conjecture that the nontrivial vacuum has no physical open string excitations was also tested numerically and found to hold to high accuracy (Ellwood & Taylor, 2001a; Ellwood et al., 2001). The effective potential $V(\varphi)$ for the tachyon field can be computed using this approach; this potential is graphed in Figure 1.1. This figure clearly illustrates
the unstable perturbative vacuum as well as the stable nonperturbative vacuum.

The results of numerical analysis have confirmed Sen’s conjectures very clearly. Perhaps the most important consequence of this confirmation is that we have for the first time concrete evidence that string field theory can describe multiple disconnected† string vacua in terms of a common set of variables. This is in principle the kind of construction which is needed to describe the disparate string vacua of the closed string landscape. Indeed, Figure 1.1 can be seen as a piece of the “open string landscape”. To extrapolate from the results achieved so far in classical open string field theory to the picture we desire of a set of independent solutions of a quantum closed string field theory, however, a number of significant further steps must be taken. We discuss some of the issues which must be resolved in the following subsection.

1.2.3 Outstanding problems and issues in OSFT

In order to improve our understanding of OSFT so that we can better understand the space of solutions of the theory, one very important first step is to develop analytic tools to describe the nontrivial open string vacuum described in the previous subsection. One approach to

† By disconnected we mean that there is no continuous family of vacuum solutions interpolating between the distinct vacua.
this problem was to try to reformulate string field theory around this vacuum using “vacuum string field theory” (Rastelli et al., 2001). This approach led to the development of some powerful analytic tools for understanding the star algebra and projectors in the theory; recently these tools were used to make an important step forward by Schnabl (2005), who has found an analytic form for the nontrivial vacuum of Witten’s open string field theory. The presentation of this vacuum state has interesting analytic properties related to Bernoulli numbers. It seems to have a part which is well-behaved under level truncation, and another part which involves an infinite sequence of massive string fields. The second part of this state has vanishing inner product with all states which appear in level truncation, and is not yet completely understood (for further discussion of this state see Okawa, 2006). This construction seems to be a promising step towards developing analytic machinery to describe solutions of classical string field theory; it seems likely that in the reasonably near future this may lead to significant new developments in this area.

Another important issue, relevant for understanding string field theory analytically and for describing a disparate family of solutions to the theory, even at the classical level, is the problem of field redefinitions. The issue here is that the fields appearing in the string field, such as $\varphi$ and $A_\mu$, are only identified at linear order with the usual space-time fields of conformal field theory. At higher order, these fields are related by a highly nontrivial field redefinition which can include arbitrary numbers of derivatives (Ghoshal & Sen, 1992). For example, the SFT $A_\mu$ (after integrating out the massive fields) is related to the CFT $\tilde{A}_\mu$ by a field redefinition

$$\tilde{A}_\mu = A_\mu + \alpha A^2 A_\mu + \beta A^2 \partial^2 A_\mu + \cdots$$

(1.11)

where arbitrarily complicated terms appear on the RHS (Coletti et al., 2003). Because of these field redefinitions, simple physical properties such as turning on a constant deformation $A_\mu$, corresponding to the simple translation of a D-brane in flat space in a dual picture, are difficult to understand in the variables natural to SFT (Sen & Zwiebach, 2000). Similar field redefinitions, involving arbitrary numbers of time derivatives, take a reasonably well-behaved time-dependent tachyon solution which classically rolls down the hill depicted in Figure 1.1 in the CFT description to a string field theory solution which has wild exponentially increasing oscillations (Moeller & Zwiebach, 2002; Coletti et al., 2005). These field redefinitions make it very difficult to interpret simple
physical properties of a system in the variables natural to string field theory. This is a generic problem for background-independent theories, but some systematic way of dealing with these different descriptions of physics needs to be found for us to sensibly interpret and analyze multiple vacua within a single formulation of string field theory.

Closely related to the issue of field redefinitions is the issue of gauge fixing. To perform explicit calculations in string field theory, the infinite gauge symmetry must be fixed. One standard approach to this is the “Feynman-Siegel” gauge, where all states are taken to be annihilated by a certain ghost field. For string fields near $\Psi = 0$ this is a good gauge fixing. For larger string fields, however, this gauge fixing is not valid (Ellwood & Taylor, 2001b). Some string field configurations have no representative in this gauge, and some have several (Gribov ambiguities). If for example one tries to continue the potential graphed in Figure 1.1 to negative $\varphi$ much below the perturbative unstable vacuum or to positive $\varphi$ much past the stable vacuum, the calculation cannot be done in Feynman-Siegel gauge. Currently no systematic way of globally fixing the gauge is known. This issue must be better understood to fully analyze the space of vacua classically and to define the quantum theory. For example, it should be possible in principle to describe a two-D-brane state in the Witten OSFT starting in the background with a single D-brane. This would correspond to a configuration satisfying the equation of motion, but with energy above the perturbative vacuum by the same amount as the stable vacuum is below it. In this 2 D-brane vacuum there would be 4 copies of each of the perturbative open string states in the original model. No state of this kind has yet been found, and it seems likely that such a state cannot be identified without a better approach to global gauge fixing. It is interesting to note that the analytic solution by Schnabl uses a different gauge choice than Feynman-Siegel gauge; it will be interesting to see if this gauge has better features with regard to some of the problems mentioned here.

The open string field theory we have discussed here is a theory of bosonic strings. Attempting to quantize this theory is problematic because of the bosonic closed string tachyon, which leads to divergences and which is still poorly understood. To discuss the quantum theory we should shift attention to supersymmetric open string field theory, which is tachyon free. Witten’s approach to describing OSFT through a cubic action encounters problems for the superstring due to technical

† Recent work suggests, however, that even this tachyon may condense to a physically sensible vacuum (Yang & Zwiebach, 2005)
issues with “picture changing” operators. Although it may be possible to resolve these issues in the context of Witten’s cubic formulation (Arefeva et al., 1990), an approach which may be more promising was taken by Berkovits (2001a, 2001b), where he developed an alternative formulation of the open superstring field theory. This formulation is more like a Wess-Zumino-Witten model than the Chern-Simons model on which (1.4) is based. The action has an infinite number of terms but can be written in closed form. Some analysis of this model using level truncation (see Ohmori, 2003 for a review) gives evidence that this framework can be used to carry out a parallel analysis to that of the bosonic theory, and that disconnected open superstring vacua can be described using this approach, at least numerically. At the classical level, the same problems of field redefinition, lack of analytic tools, and gauge fixing must be tackled. But in principle this is a promising model to extend to a quantum theory. In principle, a complete quantum theory of open strings must include closed strings, since closed strings appear as intermediate states in open string scattering diagrams (indeed in some sense this is how closed strings were first discovered, as poles in open string scattering amplitudes). It should then in principle be possible to compute closed string scattering amplitudes using OSFT. A much more challenging problem, however, is turning on nonperturbative deformations of closed string fields in the open string language. The simple version of this would be to deform a modulus such as the dilaton by a constant value. Much more challenging would be to identify topologically distinct closed string vacua as quantum states in a single OSFT. Such a construction is well beyond any tools currently available. Since open string field theory seems better understood than closed string field theory this is perhaps a goal worth aiming at. In the next section, however, we describe the current state of direct constructions of closed string field theory.

1.3 Closed string field theory

A direct formulation of closed string field theory is more complicated than the theory for open strings. In closed string field theory, the string field $\Psi[X(\sigma)]$ has a field expansion (1.2) analogous to the open string field expansion (1.3). Writing an action for this string field which reproduces the perturbative amplitudes of conformal field theory is, however, much more complicated even in the bosonic theory than the simple Witten action (1.4).
Using a generalization of the BRST formalism, Zwiebach (1992) developed a systematic way of organizing the terms in a closed bosonic string field theory action. Unlike the Witten action, which has only cubic interactions, Zwiebach’s closed string field theory action contains interaction terms at all orders. The key to organizing this action and making sure that it reproduces the standard closed string perturbative expansion from CFT was finding a way of systematically cutting apart Riemann surfaces (using “Strebel differentials”) so that each Riemann surface can be written in a unique way in terms of propagators and vertices. This approach is based very closely on the geometry of the string world-sheet and it seems to give a complete formulation of the bosonic theory, at least to the same extent that Witten’s theory describes the open bosonic string.

In closed string field theory there are massless fields corresponding to marginal deformations of the closed string background. Such deformations include a modification of the string coupling, which is encoded in the dilaton field $\phi(x)$ through $g = e^{\phi}$. For closed string field theory to be background independent, it needs to be the case that turning on these marginal deformations can be accomplished by simply turning on the fields in the SFT. For example, it must be the case that the string field theory defined with string coupling $g$ has a background described by a certain field configuration $\Psi'$, such that expanding the theory around this background gives a theory equivalent to the SFT defined in a background with a different string coupling $g'$. This background independence was shown for infinitesimal marginal deformations by Sen and Zwiebach (Sen & Zwiebach, 1994a, 1994b). This shows that closed string field theory is indeed background independent. It is more difficult, however, to describe a finite marginal deformation in the theory. This problem is analogous to the problem discussed in open string field theory of describing a finite marginal deformation of the gauge field or position of a D-brane, and there are similar technical obstacles to resolving the problem. This problem was studied for the dilaton and other marginal directions by Yang and Zwiebach (2005a, 2005b). Presumably similar techniques should resolve this type of marginal deformation problem in both the open and closed cases. A resolution of this would make it possible, for example, to describe the moduli space of a Calabi-Yau compactification using closed string field theory. One particularly interesting question is whether a deformation of the dilaton to infinite string coupling, corresponding to the M-theory limit, can be
described by a finite string field configuration; this would show that the background-independence of string field theory includes M-theory.

To go beyond marginal deformations, however, and to identify, for example, topologically distinct or otherwise disjoint vacua in the theory is a much greater challenge. Recently, however, progress has been made in this direction also using closed string field theory. Zwiebach’s closed bosonic string field theory can be used to study the decay of a closed string tachyon in a situation parallel to the open string tachyon discussed in the previous section. It has been shown (Okawa & Zwiebach, 2004a) that the first terms in the bosonic closed string field theory give a nonperturbative description of certain closed string tachyons in accord with physical expectations. The situation here is more subtle than in the case of the open string tachyon, since the tachyon occurs at a point in space where special “twisted” modes are supported, and the tachyon lives in these twisted modes, but as the tachyon condenses, the process affects physics in the bulk of space time further and further from the initial twisted modes. This makes it impossible to identify the new stable vacuum in the same direct way as was done in OSFT, but the results of this analysis do suggest that closed string field theory correctly describes this nonperturbative process and should be capable of describing disconnected vacua. Again, however, presumably similar complications of gauge choice, field redefinitions, and quantum definition will need to be resolved to make progress in this direction.

Because of the closed string bulk tachyon in the bosonic theory, which is not yet known to condense in any natural way, the bosonic theory may not be well-defined quantum mechanically. Again, we must turn to the supersymmetric theory. Until recently, there was no complete description of even a classical supersymmetric closed string field theory. The recent work of Okawa and Zwiebach (2004b) and of Berkovits, Okawa and Zwiebach (2004), however, has led to an apparently complete formulation of a classical string field theory for the heterotic string. This formulation combines the principles underlying the construction by Berkovits of the open superstring field theory with the moduli space decomposition developed by Zwiebach for the bosonic closed string field theory. Interestingly, for apparently somewhat technical reasons, the approach used in constructing this theory does not work in any natural way in the simpler type II theory. The action of the heterotic superstring field theory has a Wess-Zumino-Witten form, and contains an infinite number of interactions at arbitrarily high orders. The development of a SUSY CSFT makes it plausible for the first time that we
could use a background-independent closed string field theory to address questions of string backgrounds and cosmology. Like the open bosonic theory discussed in the previous section, this closed string field theory can be defined in level truncation to give a well-defined set of interaction terms for a finite number of fields, but it is not known in any precise way what the allowed space of fields should be or how to quantize the theory. These are important problems for future work in this area.

1.4 Outlook

We have reviewed here the current state of understanding of string field theory and some recent developments in this area. String field theory is currently the only truly background-independent approach to string theory. We have reviewed some recent successes of this approach, in which it was explicitly shown that distinct vacua of open string field theory, corresponding to dramatically different string backgrounds, appear as solutions of a single theory in terms of a single set of degrees of freedom. While much of the work concretely confirming this picture in string field theory was numerical, it seems likely that further work in the near future will provide a better analytic framework for analyzing these vacua, and for understanding how open string field theory can be more precisely defined, at least at the classical level.

We described open string field theory in some detail, and briefly reviewed the situation for closed string field theory. While gravity certainly requires closed strings, it is not yet clear whether we are better off attempting to directly construct closed string field theory by starting with the closed string fields in a fixed gravity background, or, alternatively, starting with an open string field theory and working with the closed strings which arise as quantum excitations of this theory. On the one hand, open string field theory is better understood, and in principle includes all of closed string physics in a complete quantum formulation. But on the other hand, closed string physics and the space of closed superstring vacua seems much closer in spirit to closed string field theory. Recent advances in closed superstring field theory suggest that perhaps this is the best direction to look in if we want to describe cosmology and the space of closed string vacua using some background-independent formulation of string theory along the lines of SFT.

We reviewed some concrete technical problems which need to be addressed for string field theory, starting with the simpler OSFT, to make the theory better defined and more useful as a tool for analyzing classes
of solutions. Some problems, like gauge fixing and defining the space of allowed states, seem like particular technical problems which come from our current particular formulation of string field theory. Until we can solve these problems, we will not know for sure whether SFT can describe the full range of string backgrounds, and if so how. One might hope that these problems will be resolved as we understand the theory better and can find better formulations. One hope may be that we might find a completely different approach which leads to a complementary description of SFT. For example, the M(atrix) model of M-theory can be understood in two ways: first as a quantum system of D0-branes on which strings moving in 10 dimensions end, and second as a regularized theory of a quantum membrane moving in 11 dimensions. These two derivations give complementary perspectives on the theory; one might hope for a similar alternative approach which would lead to the same structure as SFT, perhaps even starting from M-theory, which might help elucidate the mathematical structure of the theory.

One of the problems we have discussed, however, seems generic to all background-independent theories. This is the problem of field redefinitions. In any background-independent theory which admits numerous solutions corresponding to different perturbative backgrounds, the natural degrees of freedom of each background will tend to be different. Thus, in any particular formulation of the theory, it becomes extremely difficult to extract physics in any background whose natural variables are different. This problem is already very difficult to deal with at the classical level. Relating the degrees of freedom of Witten’s classical open string field theory to the natural fields of conformal field theory in order to describe familiar gauge physics, open string moduli, or the dynamical tachyon condensation process makes it clear that simple physics can be dramatically obscured by the choice of variables natural to string field theory. This problem becomes even more challenging when quantum dynamics are included. QCD is a simple example of this; the physical degrees of freedom we see in mesons and baryons are very difficult to describe precisely in terms of the natural degrees of freedom (the quarks and gluons) in which the fundamental QCD Lagrangian is naturally written. Background independent quantum gravity seems to be a similar problem, but orders of magnitude more difficult.

Any quantum theory of gravity which attempts to deal with the landscape of string vacua by constructing different vacua as solutions of a single theory in terms of a single set of degrees of freedom will face this field-redefinition problem in the worst possible way. Generally, the
degrees of freedom of one vacuum (or metastable vacuum) will be defined in terms of the degrees of freedom natural to another vacuum (or metastable vacuum) through an extremely complicated, generically quantum, field redefinition of this type. This presents a huge obstacle to achieving a full understanding of quantum cosmology. This obstacle is very concrete in the case of string field theory, where it will make it difficult to describe the landscape of string vacua in the language of a common theory. It is also, however a major obstacle for any other attempt to construct a background-independent formulation of quantum gravity (such as loop quantum gravity or other approaches reviewed in this book). Only the future will tell what the best means of grappling with this problem may be, or if in fact this is the right problem to pose. Perhaps there is some radical insight not yet articulated which will make it clear that we are asking the wrong questions, or posing these questions in the wrong way.

Two more fundamental issues which must be confronted if we wish to use string field theory to describe cosmology are the issues of observables and of boundary conditions and initial conditions. These are fundamental and unsolved issues in any framework in which we attempt to describe quantum physics in an asymptotically de Sitter or metastable vacuum. As yet there are no clear ways to resolve these issues in SFT. One interesting possibility, however, is that by considering string field theory on a space-time with all spatial directions compactified, these issues could be somewhat resolved. In particular, one could consider quantum OSFT on an unstable D-brane (or a brane/antibrane pair for the supersymmetric OSFT or the closed heterotic SFT without D-branes) on the background $T^9 \times \mathbb{R}$. The compactification provides an IR cutoff, and by putting in UV cutoffs through level truncation and a momentum cutoff, the theory could be approximated by a finite number of quantum mechanical degrees of freedom. This theory could be studied analytically, or, like lattice QCD, one could imagine simulating this theory and getting some approximation of cosmological dynamics. If SFT is truly background independent, quantum excitations of the closed strings should have states corresponding to other compactification topologies, including for example $T^3 \times X$ where $X$ is any flux compactification of the theory on a Calabi-Yau or other 6D manifold. Quantum fluctuations should also allow the $T^3$ to contain inflating regions where the energy of $X$ is positive, and one could even imagine eternal inflation occurring in such a region, with bubbles of other vacua branching off to populate the string landscape. Or one could imagine some other dynamics
occurring, demonstrating that the landscape picture is incorrect. It is impractical with our current understanding to implement such a computation, and presumably the detailed physics of any inflating region of the universe would require a prohibitive number of degrees of freedom to describe. Nonetheless, if we can sensibly quantize open superstring field theory, or a closed string field theory, on $T^9$ or another completely compact space, it may in some sense be the best-defined background independent formulation of string theory in which to grapple with issues of cosmology.

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