Constraint on the magnetic moment of the top quark

R. Martínez and J.-Alexis Rodríguez

1. Depto. de Física, Universidad Nacional, Bogotá, Colombia
2. Centro Internacional de Física, Bogotá, Colombia

We derive a bound on the magnetic dipole moment of the top quark in the context of the effective Lagrangian approach by using the ratios $R_b = \Gamma(Z \rightarrow b\bar{b}) / \Gamma(Z \rightarrow \text{hadron})$, $R_l = \Gamma(Z \rightarrow l\bar{l}) / \Gamma(Z \rightarrow \text{hadron})$ and the $Z$ width. We take into account the vertex and oblique corrections.

The most recent analyses of precision measurements at the Large Electron Positron (LEP) collider lead to the conclusion that the predictions of the Standard Model (SM) of electroweak interactions, based on the gauge group $SU(2)_L \otimes U(1)_Y$ are in excellent agreement with the experimental results. Recently the discovery of the top quark has been announced by the Collider Detector at Fermilab (CDF) and D0 collaborations. The direct measurement of the top quark mass $m_t$ is in agreement with the indirect estimates derived by confronting the SM $m_t$ dependent higher order corrections with the LEP and other experimental results. The measurement of the top quark mass reduced the number of free parameters of the SM. A precise knowledge of the value of the top mass will improve the sensitivity of searches of new physics through small indirect effects.

The precise measurements of the $g - 2$ value of the electron provides a test of its point-like character. Similarly, measurements of the electric- and chromo- magnetic moments of the quarks can be important to study physics beyond the SM. In particular, the chromomagnetic moment of the top quark can affect its production in the $pp$ and $e^+e^-$ reactions.

The SM predicts how the top quark should behave under these interactions, so any deviation from this behaviour would provide us with a probe of new physics beyond the SM. If new physics is found in this sector, it could probably originate from a non standard symmetry breaking mechanism. This is because the top mass is of the order of the electroweak (EW) breaking scale, and hence it is conceivable that the top-quark properties are sensitive to unsuppressed EW breaking effects.

The aim of the present work is to extract indirect information on the magnetic dipole moment of the top quark from LEP data, specifically we use the ratios $R_b$ and $R_l$ defined by

$$R_b = \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow \text{hadron})},$$

$$R_l = \frac{\Gamma(Z \rightarrow l\bar{l})}{\Gamma(Z \rightarrow \text{hadron})}$$

and the $Z$ width, in the context of an effective Lagrangian approach. The oblique and QCD corrections to the $b$ quark and hadronic $Z$ decay widths cancel off in the ratio $R_b$. This property makes $R_b$ very sensitive to direct corrections to the $Zb\bar{b}$ vertex, specially those involving the heavy top quark, while $\Gamma_Z$ and $R_l$ are more sensitive to oblique corrections.

The effective Lagrangian approach is a convenient model independent parametrization of the low-energy effects of the new physics that may show up at high energies. Effective Lagrangians, employed to study processes at a typical energy scale $E$ can be written as a power series in $1/\Lambda$, where the scale $\Lambda$ is associated with the heavy particles masses of the underlying theory. The coefficients of the different terms in the effective Lagrangian arise from integrating out the heavy degrees of freedom that are characteristic of a particular model for new physics.

In order to define an effective Lagrangian it is necessary to specify the symmetry and the particle content of the low-energy theory. In our case, we consider a Lagrangian in the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \sum_n \alpha_n \mathcal{O}^n$$

where the operators $\mathcal{O}^n$ are of dimension greater than four. In the present work, we consider the following dimension six and $CP$-conserving operators,

$$O_{uW}^{ab} = \bar{Q}_L^a \sigma^{\mu\nu} W_{\mu\nu}^{i} \tau^i \tilde{\phi} U_R^b,$$

$$O_{uB}^{ab} = \bar{Q}_L^a \sigma^{\mu\nu} Y_{\mu\nu} \phi U_R^b.$$
where $Q^a_L$ is the quark isodoublet, $U^a_R$ is the up quark isosinglet, $a$, $b$ are the family indices, $B_{\mu \nu}$ and $W_{\mu \nu}$ are the $U(1)_Y$ and $SU(2)_L$ field strengths, respectively, and $\phi = i \tau_2 \phi^*$. We use the notation introduced by Buchmüller and Wyler \[1\]. In the case of the operators $O^{ab}_{ub L}$ and $O^{ab}_{uW}$, some degrees of family mixing is made explicit (corresponding to $a \neq b$) without breaking SM gauge invariance. After spontaneous symmetry breaking, these fermionic operators generate also effective vertices proportional to the anomalous magnetic moments of quarks. The above operators for the third family give rise to the anomalous $tt\gamma$ vertex and the unknown coefficients $\epsilon^{ab}_{uB}$ and $\epsilon^{ab}_{uW}$ are related respectively with the anomalous magnetic moment of the top quark through

$$\delta \kappa_t = - \sqrt{2} \frac{m_t}{m_W} \frac{g}{\epsilon Q_t} (s_W \epsilon^{ab}_{uW} + c_W \epsilon^{ab}_{uB}) \ . \quad (0.4)$$

where $s_W$ denotes the sine of the weak mixing angle.

The expression for $R_b$ is given by

$$R_b = R_b^{SM} (1 + (1 - R_b^{SM}) \delta_b) \ , \quad (0.5)$$

where $R_b^{SM}$ is the value predicted by the SM and $\delta_b$ is the factor which contains the new physics contribution, and it is defined as follows

$$\delta_b = 2 \frac{g_V^{SM} g_V^{NP} + g_A^{SM} g_A^{NP}}{(g_V^{SM})^2 + (g_A^{SM})^2} \quad (0.6)$$

and $g_V^{SM}$ and $g_A^{SM}$ are the vector and axial vector couplings of the $Z\bar{b}b$ vertex normalized as $g_V^{SM} = -1/2 + 2 s_W^2/3$ and $g_A^{SM} = -1/2$. The contributions from new physics, eq. (0.3), to $R_t$ and $\Gamma_Z$ are of two classes. One from vertex correction to $Z\bar{b}b$ in the $\Gamma_{had}$ and the other from the oblique correction through $\Delta \kappa$ in the $\sin^2 \theta_W$. These can be written as

$$R_t = R_t^{SM} (1 - 0.1851 \Delta \kappa + 0.2157 \delta_b) \ ,$$

$$\Gamma_Z = \Gamma_Z^{SM} (1 - 0.2351 \Delta \kappa + 1.506 \delta_b) \quad (0.7)$$

where $\Delta \rho$ is equal to zero for the operators that we are considering.

The contribution of the above effective operators to the $Z\bar{b}b$ vertex is given by the Feynman diagrams shown in fig. 1, where a heavy dot denotes an effective vertex. After evaluating the Feynman diagrams, with insertions of the effective operators $O^{ab}_{ub L}$ and $O^{ab}_{uW}$ we obtain

$$g_V^{NP} = 4 \sqrt{2} \epsilon^{33}_{uW} G_F m_W^3 m_t \{ 3 c_W (\tilde{C}_{12} - \tilde{C}_{11}) - \frac{m_t^2}{\sqrt{2} m_W^2} (C_{12} - C_{11} + C_0)$$

$$+ \frac{1 + a}{8 c_W} (C_{11} + C_{12} + C_0) + \frac{1}{\sqrt{2}} (C_{12} - C_{11} - C_0)$$

$$- 3 a \frac{m_t^2}{4 c_W m_W^2} (B_1 - B_0) \} \ , \quad (0.8)$$

$$g_A^{NP} = 4 \sqrt{2} \epsilon^{33}_{uW} G_F m_W^3 m_t \{ - \frac{a}{2 c_W} (C_0 + C_{12} - C_{11}) - \frac{1}{\sqrt{2}} (C_{12} - C_0 - C_{11})$$

$$+ \frac{m_t^2}{\sqrt{2} m_W^2} (C_{12} - C_{11} + C_0) - \frac{2 m_t s_W}{m_W} (\tilde{C}_0 + \tilde{C}_{12} - \tilde{C}_{11})$$

$$- \frac{3 m_t^2}{4 c_W m_W^2} (B_1 - B_0) \} \quad (0.9)$$

for the operator $O_{uW}$ and,

$$g_V^{NP} = g_A^{NP} = \frac{4 \sqrt{2} \epsilon^{33}_{uB} G_F m_W^3 m_t}{3 c_W} \{ \frac{m_t^2}{\sqrt{2} m_W^2} (C_{12} - C_{11} + C_0)$$

$$- \frac{1}{\sqrt{2}} (-C_{11} + C_{12} - C_0) \} \ , \quad (0.10)$$

for the operator $O_{uB}$. In the above equations $a = 1 - \frac{s_W^2}{2}$ while $C_{ij} = C_{ij}(m_W, m_t, m_t)$, $\tilde{C}_{ij} = \tilde{C}_{ij}(m_t, m_W, m_W)$ and $B_i = B_i(0, m_t, m_W)$ are the Passarino-Veltman scalar integral functions \[8\]. The combination $B_0 - B_1$ has a pole
in $d=4$ dimensions that is identified with the logarithmic dependence on the cutoff. Using the prescription given in ref. [3], the pole can be replaced by $\ln \Lambda^2/m_Z^2$.

The operators (11) contribute to the fermion processes at one loop level, giving oblique corrections to the gauge boson self energies. The contribution is essentially coming from the $\Sigma_{\gamma Z}(m_Z^2)$ self energy. Therefore these operators only contribute to $\Delta \kappa$ parameter [4]. For $\Delta \kappa$ we have obtained the same results of the eqs. (50) and (51) of ref. [1].

To obtain the physical quantities $R_b$, $R_l$ and $\Gamma_Z$ as a function of $\delta \kappa_i$ instead of two parameters $c_{uW}^{33}$ and $c_{uB}^{33}$, we consider that only one coefficient at the time is different from zero at the scale $\Lambda$. Then we proceed to sum the value of the contribution of each operator in order to avoid cancellation between them. With this prescription we get an optimal bound.

In Fig. 2 we display $R_b$ as a function of $\delta \kappa_i$. The horizontal lines represent the experimental measurements $R_b^{exp} = 0.2178 \pm 0.0011$ [2]: our bound can be expressed as $0.38 \leq \delta \kappa_i \leq 1.21$. In Fig. 3 we show $R_l$ fraction versus $\delta \kappa_i$ with the horizontal lines representing the experimental result, $R_l^{exp} = 20.778 \pm 0.029$; for this case the bound can be expresses as $0.02 \leq \delta \kappa_i \leq 0.48$. Finally in Fig. 4 we plot $\Gamma_Z$ versus $\delta \kappa_i$ with the experimental value $\Gamma_Z^{exp} = 2.4946 \pm 0.0027$ GeV; the limit is $0 \leq \delta \kappa_i \leq 0.48$. The SM values for the parameters that we have used are $\Gamma_Z = 2.4972$ GeV, $R_b = 20.747$ GeV, $R_b = 0.2157$, $\Gamma_{hadr} = 174.4$ MeV and $\Gamma_l = 84.03$ MeV; with the input parameters: $m_t = 175$ GeV, $\alpha_s(m_Z) = 0.118$, $m_Z = 91.1861$ GeV, $m_H = 100$ GeV and $\Lambda = 1$ TeV.

In conclusion, the corrections through $R_l$ and $\Gamma_Z$ put a better bound on $\delta \kappa_i$ than the vertex correction from $R_b$. Our results from figs. 3 and 4 are of the same order of the results of the eqs. (56) and (57) of ref. [1], with the appropriate replacement between $f_{IW\phi}$ and $f_{IB\phi}$ and the magnetic dipole moment of the top quark. These bounds agrees also with the one obtained of the same effective operators in reference [2], by using the CLEO result on $B(b \to s\gamma)$.

We would like to thank M. Perez and E. Nardi for their comments. We thank COLCIENCIAS for financial support.

[1] CDF Collaboration, F. Abe et. al., Phys. Rev. Lett. 74, 2626 (1995); D0 Collaboration, S. Abachi et. al., Phys. Rev. Lett. 74, 2636 (1995).
[2] G. Kane, G. Ladinsky, and C.P. Yuan, Phys. Rev. D 45, 1531 (1992); D. Atwood, A. Kagan, and T. Rizzo, Phys. Rev. D 52, 6264 (1995); T. Rizzo, Phys. Rev. D 53, 2326 (1996).
[3] R. D. Peccei and X. Zhang, Nucl. Phys. B 337, 269 (1990); Ehad Malkawi and C.-P. Yuan, Phys. Rev. D 50, 4462 (1994).
[4] S. Mrenna and C.-P. Yuan, Phys. Lett. B 367, 188 (1996); E. Ma, Phys. Rev. D 53, 2276 (1996); P. Bammert, et. al., Phys. Rev. D 54, 4275 (1996).
[5] H. Georgi, Nucl. Phys. B 361, 339 (1991); A. De Rújula et. al., Nucl. Phys. B 369, 3 (1992); J. Wudka, Int. J. Mod. Phys. A9, 2301 (1994).
[6] C. P. Burgess and D. London, Phys. Rev. D 48, 4337 (1993); Phys. Rev. Lett. 69, 3428 (1993).
[7] W. Buchmüller and D. Wyler, Nucl. Phys. B 268, 621 (1986); Phys. Lett. B 197, 379 (1987).
[8] G. Passarino and M. Veltman, Nucl. Phys. B 160, 151 (1979).
[9] K. Hagiwara et. al., Phys. Lett. B 283, 353 (1992); Phys. Rev. D 48, 2182 (1993).
[10] M. Peskin and T. Takeuchi, Phys. Rev. D 46, 381 (1992); V. A. Novikov, L. B. Okun and M. I. Vysotsky, Nucl. Phys. B 397, 35 (1993); P. Altarelli and F. Caravaglios, Nucl. Phys. B 405, 3 (1993); K. Hagiwara et. al., Z. Phys. C 64, 559 (1994).
[11] G. J. Gounaris, F. M. Renard and C. Verzegnassi, Phys. Rev. D 52, 451 (1995).
[12] G. Altarelli, hep-ph/9611239.
[13] R. Martinez, M. A. Perez and J. Toscano, Phys. Lett. B 340, 91 (1994).
Figure Captions

Figure 1. Feynman diagrams contributing to the $Z \to b\bar{b}$ decay. The heavy dots denote an effective vertex.

Figure 2. $R_b$ as a function of $\delta \kappa_t$ for $m_t = 175$ GeV. The horizontal lines are the experimental results.

Figure 3. Same as fig. 2 for $R_l$.

Figure 4. $\Gamma_Z$ as a function of $\delta \kappa_t$ for $m_t = 175$ GeV. The horizontal lines are the experimental results.
Figure 2
Figure 3
