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Study of COVID-19 mathematical model of fractional order via modified Euler method

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Abstract Our main goal is to develop some results for transmission of COVID-19 disease through Bats-Hosts-Reservoir-People (BHRP) mathematical model under the Caputo fractional order derivative (CFOD). In first step, the feasible region and boundedness of the model are derived. Also, we derive the disease free equilibrium points (DFE) and basic reproductive number for the model. Next, we establish theoretical results for the considered model via fixed point theory. Further, the condition for Hyers-Ulam’s (H-U) type stability for the approximate solution is also established. Then, we compute numerical solution for the concerned model by applying the modified Euler’s method (MEM). For the demonstration of our proposed method, we provide graphical representation of the concerned results using some real values for the parameters involved in our considered model.

1. Introduction

Recently a threatful disease which is called COVID-19 another form of SARS has started to spread in globe from Wuhan a big city of China during the end of 2019. Up-to date more than 0.8 millions people all around the world have been died. Further, more than fifteen million people have been infected around the globe. According to World Health Organization (WHO) in China a medical office was identified the cases of pneumonia of unknown etiology in Wuhan City of Hubei Province of China on 31 December 2019. WHO informed that a COVID-19 was detected. Further it was declared most dangerous virus by Chinese authorities on 7 January 2020 [1,2]. Tiamen et al. on 19 January 2020 developed a Bats-Hosts-Reservoir-People (BHRP) model for transmission from the infectious source to the human. They assumed that virus spread in the Bats population, then virus transmitted to an unknown wild animals (hosts). After hunting the hosts (defined as revivor), the virus spread in a seafood market which became cause of infection in some people. Also it has been considered that the source of COVID-19 is the transmission from animal to human. Some other researchers guaranteed that

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transmission also occurs from person to person. Therefore various countries implemented strict lock down in their states and advised the public to keep social distance. Such policy have controlled the disease in some countries but this is not a permanent way to save people. Because such policies have very badly destroyed the economies of low income countries all over the world. Now, the proper vaccine has been prepared for the cure of the COVID-19.

On the other hand, bioengineers, mathematicians and researchers are also trying to make such procedure which may reduced or controlled the spreading of disease in our society further. As it is well known that mathematical models are powerful tools to study the transmission of infectious disease. Also mathematical models of infectious disease have been studied in last few decades very well [3,4,18–27,24,25,27]. In this regards very recently many researchers developed various models to investigate the transmission of COVID-19. Many researcher worked on the COVID-19 model using the data for different countries [32–36]. Therefore, BHRP model was considered model though fixed point approach. Also, we derive feasibility, bounded of solution and reproduction number.

Further, we established a numerical algorithm to provide graphical representation of the result to the model under consideration. We considered the COVID-19 model (1) under the CFOD with order \( \eta \) such that \( 0 < \eta \leq 1 \) as

\[
\begin{align*}
\mathcal{D}^\eta S_p(t) &= \lambda_p - m_p S_p - \beta_p S_p(I_p + k A_p) - \beta_p W S_p W, \\
\mathcal{D}^\eta E_p(t) &= \beta_p S_p(I_p + k A_p) + \beta_p W S_p W - (1 - \delta_p)\omega_p E_p - \delta_p \omega_p E_p - m_p E_p, \\
\mathcal{D}^\eta I_p(t) &= (1 - \delta_p)\omega_p E_p - (\gamma_p + m_p) I_p, \\
\mathcal{D}^\eta A_p(t) &= \delta_p \omega_p E_p - (\gamma_p + m_p) A_p, \\
\mathcal{D}^\eta R_p(t) &= \gamma_p I_p + \gamma_p A_p - m_p R_p, \\
\mathcal{D}^\eta W(t) &= \mu_p I_p + \mu_p A_p - \varepsilon W,
\end{align*}
\]

with initial conditions as

\[
S_p(0) = S_{p_0}, \quad E_p(0) = E_{p_0}, \quad I_p(0) = I_{p_0}, \quad A_p(0) = A_{p_0}, \quad R_p(0) = R_{p_0}, \quad W(0) = W_0.
\]

Since it is important that to check whether a model of real problem exists or not. This thing is guaranteed by applying fixed point theory. Therefore, we establish existence theory for the considered model (2) under CFOD by fixed point theory. Here, we have established the H-U stability mostly computed for numerical results. Since, our work address numerical computation of COVID-19 model. Also, we have established the feasible region, bounded ness and reproductive number for the model. In this work, we have extended modified Euler method (MEM) to simulate the results. The concerned procedure has been used in the past for very simple nonlinear problems. Here, we have derived an algorithm to simulate our results for the considered nonlinear systems. We exhibit the results against distinct values of fractional order with graphs by using computational software like Matlab.

2. Background mmaterial

Here, in this section we recall some preliminaries from fractional calculus. For more detailed study, we refer to [6,7,10].

Definition 2.1. The fractional integral of Riemann–Liouville type of order \( \eta \in \mathbb{R}^+ \) of a function \( f \in L^1([0, \infty), \mathbb{R}) \) is defined as

\[
P^\eta f(t) = \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta - 1} f(\xi) d\xi,
\]

provided that the integral on the right side is point-wise converges on \((0, \infty)\).

Definition 2.2. The CFOD of a function \( f \) is defined by

\[
\mathcal{D}^\eta f(t) = \frac{1}{\Gamma(\eta - m)} \int_0^t (t - \xi)^{\eta - m - 1} f^{(m)}(\xi) d\xi,
\]

where \( m = [\eta] + 1 \) and \([\eta]\) represents the integer part of \( \eta \). Through out this paper, we use CFOD for Caputo fractional order derivatives.

Lemma 2.3. The following result holds:

\[
P^\eta [\mathcal{D}^\eta f](t) = f(t) + a_0 + a_1 t + a_2 t^2 + \ldots + a_{m-1} t^{m-1},
\]
for arbitrary \( a_i \in \mathbb{R} \), and \( i = 0, 1, 2, \ldots, m - 1 \), where \( m = \lfloor \eta \rfloor + 1 \) and \( \lfloor \eta \rfloor \) represents the integer part of \( \eta \).

**Definition 2.4.** [16] The “generalized Taylor formula” for \( f(t) \) can be written as

\[
f(t) = \sum_{i=0}^{n} \frac{\mu_i}{\Gamma(n+1)} D^n f(0) + \frac{D^{(m+1)}f(\xi)}{\Gamma((m+1)\eta+1)},
\]

such that \( \xi \in [0, t] \), at all \( t \in (0, a] \), \( \eta \in (0, 1] \). We establish MEM using (4).

**Lemma 2.5.** The solution of the problem for \( 0 < \eta \leq 1 \)

\[
\frac{d^\eta \phi(t)}{dt^\eta} = g(t), \quad t \in [0, T] = \mathcal{F},
\]

\[
\phi(0) = \phi_0,
\]

is provided by

\[
\phi(t) = \phi_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t-s)^{\eta-1} g(s) ds.
\]

We represent Banach space by \( Z = Y \times Y \times Y \times Y \times Y \), where \( Y = C(\mathcal{F}) \), \( 0 < t < T < \infty \) under the norm

\[
\| V \| = \| (S_\eta, E_\eta, I_\eta, A_\eta, R_\eta, W_\eta) \| = \max_{m \in \mathbb{N}} \| S_m(t) \| + \| E_m(t) \| + \| I_m(t) \| + \| A_m(t) \| + \| R_m(t) \| + \| W_m(t) \|.
\]

3. Feasibility, boundedness and computation for reproductive number

Here first, we derive feasible region and bounded-ness of the model (1).

**Theorem 3.1.** The boundedness and feasible region of solution to the proposed model (1) is given by

\[
\Gamma = \{ (S_\eta, E_\eta, I_\eta, A_\eta, R_\eta, W_\eta) \in \mathbb{R}^6_+ : V(t) \leq \frac{\Lambda_\rho}{(\epsilon + \mu_\rho - (m_\rho + \mu_\rho))} \}.
\]

**Proof.** Since

\[
V(t) = (S_\eta(t) + E_\eta(t) + I_\eta(t) + A_\eta(t) + R_\eta(t) + W_\eta(t)),
\]

we have

\[
\frac{dV}{dt} = \frac{\partial V}{\partial S_\eta} \frac{dS_\eta}{dt} + \frac{\partial V}{\partial E_\eta} \frac{dE_\eta}{dt} + \frac{\partial V}{\partial I_\eta} \frac{dI_\eta}{dt} + \frac{\partial V}{\partial A_\eta} \frac{dA_\eta}{dt} + \frac{\partial V}{\partial R_\eta} \frac{dR_\eta}{dt} + \frac{\partial V}{\partial W_\eta} \frac{dW_\eta}{dt},
\]

where \( C \) is the constant of integration. Since from (7), one has, when \( t \to \infty \),

\[
V(t) \leq \frac{\Lambda_\rho}{(\epsilon + \mu_\rho - (m_\rho + \mu_\rho))},
\]

which is our required result.

Now, we are going to compute disease free equilibrium (DFE) point and reproductive number of the model (1). For computing the equilibrium point of the model (1), we have

\[
\frac{dS_\eta}{dt} = \frac{dE_\eta}{dt} = \frac{dI_\eta}{dt} = \frac{dA_\eta}{dt} = \frac{dR_\eta}{dt} = \frac{dW_\eta}{dt} = 0.
\]

The disease free equilibrium point (DFE) is denoted as \( \theta = (S_\eta^0, E_\eta^0, I_\eta^0, A_\eta^0, R_\eta^0, W_\eta^0) \) given as

\[
\theta = \left( \frac{\Lambda_\rho}{m_\rho}, 0, 0, 0, 0, 0 \right).
\]

**Theorem 3.2.** The reproduction number of the model (1) is

\[
R_0 = \frac{\beta_\rho \Lambda_\rho ((1 - \delta_\rho)\omega_p + \delta_\rho \omega_p) + \beta_\omega \Lambda_\omega ((1 - \delta_\omega)\omega_p + \delta_\omega \omega_p) - \mu_\eta I_\eta + \mu_\rho A_\rho + \epsilon W}{(1 - \delta_\rho)\omega_p + \delta_\rho \omega_p + \mu_\rho A_\rho + \epsilon W}.
\]

**Proof.** We take the 2nd, 3rd and 6th equations of the model (1) for finding the reproduction number. With necessary computations for the \( F \) and \( V \) matrices are given as

\[
F = \begin{pmatrix}
0 & \beta_\rho S_\eta \omega_p + \beta_\omega S_\eta \omega_p & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
\]

\[
V = \begin{pmatrix}
(1 - \delta_\rho)\omega_p + \delta_\rho \omega_p + \mu_\rho A_\rho & 0 & 0 \\
0 & (1 - \delta_\rho)\omega_p + \delta_\rho \omega_p + \mu_\rho A_\rho
\end{pmatrix}.
\]

Next, we have to find the generation matrix as \( FV^{-1} \) by letting by letting \( a = (1 - \delta_\rho)\omega_p + \delta_\rho \omega_p + \mu_\rho A_\rho \) and \( c = (\gamma_\rho + m_\rho) \), as

\[
FV^{-1} = \begin{pmatrix}
\frac{1}{\epsilon} & 0 & 0 \\
0 & \frac{1}{\epsilon}
\end{pmatrix},
\]

and

\[
FV^{-1} = \begin{pmatrix}
\frac{\beta_\rho S_\eta \omega_p}{\epsilon} & \frac{\beta_\rho S_\eta \omega_p}{\epsilon} & \frac{\beta_\rho S_\eta \omega_p}{\epsilon} & \frac{\beta_\rho S_\eta \omega_p}{\epsilon} \\
0 & 0 & 0 & 0
\end{pmatrix}.
\]
Spectral radius at equation at (9), $\theta = \left(\frac{n}{m^2}, 0, 0, 0, 0, 0\right)$, is

$$\rho(FV^{-1}) = \frac{\rho_0 S_p}{\rho_0 S_p + \rho_0 S_p A_t / \rho_0 S_p W},$$

$$R_0 = \frac{\rho_0 S_p}{\rho_0 S_p + \rho_0 S_p A_t / \rho_0 S_p W},$$

$$= \frac{\rho_0 S_p A_t}{\rho_0 S_p A_t / \rho_0 S_p W},$$

$$= \frac{\rho_0 S_p A_t}{\rho_0 S_p A_t / \rho_0 S_p W},$$

$$R_0 = \frac{\rho_0 S_p A_t}{\rho_0 S_p A_t / \rho_0 S_p W}. \quad (14)$$

Hence the required result is proved. If $R_0 < 1$, then the model (1) is locally asymptotically stable (out break will go to end). If $R_0 > 1$, then the model (1) is unstable (outbreak will spread).

4. Theoretical results for model (2)

The existence of solution to a physical problem is verified by using fixed point approach. We use the theorem published in [14,15] to derive the intended results. The right sides of model (2) can be expressed as:

$$\Theta_1 = A_p - m_S S_p - \beta_p S_p (I_p + k A_t) - \beta_p S_p W,$$

$$\Theta_2 = \beta_p S_p (I_p + k A_t) + \beta_p S_p W - (1 - \gamma_p) \omega_p E_p - \delta_p \omega_p E_p - m_p E_p,$$

$$\Theta_3 = (1 - \delta_p) \omega_p E_p - (\gamma_p + m_p) I_p,$$

$$\Theta_4 = \delta_p \omega_p E_p - (\gamma_p + m_p) I_p,$$

$$\Theta_5 = \gamma_p I_p + (\gamma_p + m_p) A_t,$$

$$\Theta_6 = \beta_p \omega_p E_p + \epsilon W. \quad (15)$$

Using (15), model (2) becomes

$$D^\eta V(t) = \Psi(t, V(t)), \quad 0 < \eta \leq 1,$$

$$V(0) = V_0. \quad (16)$$

In view of Lemma 2.5, (16) yields

$$V(t) = V_0(t) + \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi))d\xi, \quad (17)$$

where $V(t) = \begin{bmatrix} S_p(t) \\ E_p(t) \\ I_p(t) \\ A_p(t) \\ R_p(t) \\ W(t) \end{bmatrix}$, $V_0(t) = \begin{bmatrix} S_{p0} \\ E_{p0} \\ I_{p0} \\ A_{p0} \\ R_{p0} \\ W_{p0} \end{bmatrix}$, $\Psi(t, V(t)) = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \\ \Theta_4 \\ \Theta_5 \\ \Theta_6 \end{bmatrix}$.

To derive required results, some assumptions need to be hold:

(i) There exists constant $K_p > 0$, such that for each $V(t), \Psi(t)$, with

$$|\Psi(t, V(t)) - \Psi(t, \Psi(t))| = K_p |V(t) - \Psi(t)|;$$

(ii) There exists constants $C_p > 0$ and $M_p > 0$, such that

$$|\Psi(t, V(t))| = C_p |V| + M_p.$$

Theorem 4.1. Let $Y$ be the Banach space and $\Omega \subset Y$ be the convex and compact subset, then there exist operator $S : \Omega \rightarrow \Omega$, which has at least one fixed point.

Proof. Considered a compact and closed set $\Omega$ denoted by $\Omega = \{V \in Y : |V| \leq r\}$. Also define an operator as

$$S(V(t)) = V_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi))d\xi. \quad (18)$$

To show the operator $S$ in (18) is contraction, let $V, \Psi \in Y$, we have

$$|S(V) - S(\Psi)| = |S(V(t)) - S(\Psi(t))|,$$

$$= \left| \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi))d\xi - \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, \Psi(\xi))d\xi \right|,$$

$$= \left| \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi) - \Psi(\xi, \Psi(\xi))d\xi \right|.$$

From which we have

$$|S(V) - S(\Psi)| \leq \frac{1}{\Gamma(\eta + 1)} K_p |V - \Psi|. \quad (19)$$

Which shows that $S$ is contraction if $K_p < 1$.

Next show that $S$ is compact and continuous operator. To get this goal, consider

$$S(V(t)) = |V_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi))d\xi|,$$

$$\leq |V_0| + \frac{1}{\Gamma(\eta + 1)} \left| \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi))d\xi \right|.$$

Hence $S$ is bounded in (19). Let $t_1 < t_2 \in \mathbb{J}$, one has

$$|S(V(t_2)) - S(V(t_1))| = \left| \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(t_2))d\xi - \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(t_1))d\xi \right|,$$

$$\leq \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} |\Psi(\xi, V(t_2)) - \Psi(\xi, V(t_1))|d\xi,$$

$$\leq \frac{1}{\Gamma(\eta)} |C_p| |V(t_2) - V(t_1)|^l. \quad (20)$$

Since if $t_2 \rightarrow t_1$, then right side of (20) goes to zero. Hence $t_2 \rightarrow t_1$, led us that

$$|S(V(t_2)) - S(V(t_1))| \rightarrow 0.$$

Hence $S$ is equi-continuous, so $S$ is compact continuous. Therefore $S$ is completely continues operator. Thus all the condition of Theorem 4.1 are satisfied so the model (2) has at least one solution.

Theorem 4.2. Under the continuity of $\Theta_i$, for $i = 1, 2, 3, 4, 5, 6$, and if the condition $\frac{K_p}{\Gamma(\eta + 1)} < 1$ holds, then the system (2) has a unique solution.

Proof. Let $S : Y \rightarrow Y$, be the operator defined by

$$S(V(t)) = V_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi))d\xi. \quad (21)$$

Let $V, \Psi \in Y$, then we have

$$|S(V) - S(\Psi)| = |S(V(t)) - S(\Psi(t))|,$$

$$= \left| \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(\xi))d\xi - \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, \Psi(\xi))d\xi \right|,$$

$$\leq \left| \frac{1}{\Gamma(\eta)} \int_0^t (t - \xi)^{\eta-1} \Psi(\xi, V(t) - \Psi(t, \Psi(\xi)))d\xi \right|.$$
5. H-U stability

Here, we derive H-U type stability for (16), which lead us to the stability of system (2). Consider a small perturbation \( \phi \in C(\mathcal{F}) \) with \( \phi(0) = 0 \).

**Lemma 5.1.** The perturbed problem

\[
\begin{align*}
\mathcal{D}^{\alpha} V(t) &= \Psi(t, V(t)) + \phi(t), \\
V(0) &= V_0,
\end{align*}
\]

solution obeys

\[
\left| V(t) - \left( V_0(t) + \int_0^t (t-\xi)^{\alpha-1} \Psi(\xi, V(\xi)) \, d\xi \right) \right| \leq C_{\gamma} \delta. 
\]

**Proof.** The proof is easy.

**Theorem 5.2.** Under hypothesis (\( \mathcal{A}_2 \)) together with result (23) in Lemma 5.1, the solution of the integral Eq. (17) is H-U stable and consequently, the numerical results of the considered system are H-U stable if \( \Lambda = \frac{\beta}{1-\alpha} K_{\Phi} < 1 \).

**Proof.** Let \( \Phi \in \Omega \) be a unique solution and \( V \in \Omega \) be any solution of (17), then

\[
\left| V(t) - \Phi(t) \right| = \left| V(t) - \left( V_0(t) + \int_0^t (t-\xi)^{\alpha-1} \Psi(\xi, \Phi(\xi)) \, d\xi \right) \right| \leq \left| V(t) - \left( V_0(t) + \int_0^t (t-\xi)^{\alpha-1} \Psi(\xi, \Phi(\xi)) \, d\xi \right) \right| + C_{\gamma} \delta + \frac{\beta}{1-\alpha} K_{\Phi} \left| V - \Phi \right|. 
\]

From which we have

\[
\left| V - \Phi \right| \leq C_{\gamma} \delta + \Lambda \left| V - U \right|. 
\]

From (25), we can write

\[
\left| V - \Phi \right| \leq \frac{C_{\gamma}}{1 - \Lambda} \delta. 
\]

Hence the required results about H-U stability.

6. Numerical algorithm and discussion

In this part of the paper, we have to evaluate approximate solutions of the model (2) under CFOD. Then the numerical simulations are acquired via the suggested scheme. To this aim, we employ the CFOD to establish a numerical procedure for the simulation of our considered model (2).

6.1. General algorithm

Here, we extend the numerical method of Euler for our considered model (2). The aforesaid considered model can be written as

\[
\begin{align*}
\mathcal{D}^{\alpha} S_{\lambda}(t) &= \Theta_{\lambda}(S_{\lambda}(\xi), E_{\lambda}(\xi) - S_{\lambda}(\xi), A_{\lambda}(\xi), R_{\lambda}(\xi), W_{\lambda}(\xi)) + \mu_{\lambda}^{\alpha} S_{\lambda}(\xi), \\
\mathcal{D}^{\alpha} E_{\lambda}(t) &= \Theta_{\lambda}(E_{\lambda}(\xi), A_{\lambda}(\xi), R_{\lambda}(\xi), W_{\lambda}(\xi)) + \mu_{\lambda}^{\alpha} E_{\lambda}(\xi), \\
\mathcal{D}^{\alpha} A_{\lambda}(t) &= \Theta_{\lambda}(E_{\lambda}(\xi), A_{\lambda}(\xi), R_{\lambda}(\xi), W_{\lambda}(\xi)) + \mu_{\lambda}^{\alpha} A_{\lambda}(\xi), \\
\mathcal{D}^{\alpha} R_{\lambda}(t) &= \Theta_{\lambda}(E_{\lambda}(\xi), A_{\lambda}(\xi), R_{\lambda}(\xi), W_{\lambda}(\xi)) + \mu_{\lambda}^{\alpha} R_{\lambda}(\xi), \\
\mathcal{D}^{\alpha} W_{\lambda}(t) &= \Theta_{\lambda}(E_{\lambda}(\xi), A_{\lambda}(\xi), R_{\lambda}(\xi), W_{\lambda}(\xi)) + \mu_{\lambda}^{\alpha} W_{\lambda}(\xi), \\
\end{align*}
\]

Let \( \mathcal{F} \) be the interval of solution for (27). We subdivide the interval \( \mathcal{F} \) into subintervals \( [t_{q}, t_{q+1}] \) with uniform width \( h = T/m \) via using the nodes \( t_{q} = qh \), for \( q = 0, 1, \cdots, m \). Let

\[
\begin{align*}
S_{\lambda}(t_{q}) &= S_{\lambda}(t_{q-1}) + \Theta_{\lambda}(S_{\lambda}(t_{q-1}), E_{\lambda}(t_{q-1}) - S_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} S_{\lambda}(t_{q-1}), \\
E_{\lambda}(t_{q}) &= E_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} E_{\lambda}(t_{q-1}), \\
A_{\lambda}(t_{q}) &= A_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} A_{\lambda}(t_{q-1}), \\
R_{\lambda}(t_{q}) &= R_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} R_{\lambda}(t_{q-1}), \\
W_{\lambda}(t_{q}) &= W_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} W_{\lambda}(t_{q-1}), \\
\end{align*}
\]

up to higher order are continuous on \( \mathcal{F} \). Applying the MEM about \( t = t_{q} = 0 \) the considered model expressed in (27) and for each value \( \lambda \) take value \( \alpha \), the expression for \( t_{q} \), one has

\[
\begin{align*}
S_{\lambda}(t_{q}) &= S_{\lambda}(t_{q-1}) + \Theta_{\lambda}(S_{\lambda}(t_{q-1}), E_{\lambda}(t_{q-1}) - S_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} S_{\lambda}(t_{q-1}), \\
E_{\lambda}(t_{q}) &= E_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} E_{\lambda}(t_{q-1}), \\
A_{\lambda}(t_{q}) &= A_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} A_{\lambda}(t_{q-1}), \\
R_{\lambda}(t_{q}) &= R_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} R_{\lambda}(t_{q-1}), \\
W_{\lambda}(t_{q}) &= W_{\lambda}(t_{q-1}) + \Theta_{\lambda}(E_{\lambda}(t_{q-1}), A_{\lambda}(t_{q-1}), R_{\lambda}(t_{q-1}), W_{\lambda}(t_{q-1})) + \mu_{\lambda}^{\alpha} W_{\lambda}(t_{q-1}), \\
\end{align*}
\]

Proceeding on aforesaid fashion, a general formula at \( t_{q+1} = t_{q} + h \) is established as

\[
\begin{align*}
S_{\lambda}(t_{q+1}) &= S_{\lambda}(t_{q}) + \Theta_{\lambda}(S_{\lambda}(t_{q}), E_{\lambda}(t_{q}) - S_{\lambda}(t_{q}), A_{\lambda}(t_{q}), R_{\lambda}(t_{q}), W_{\lambda}(t_{q})) + \mu_{\lambda}^{\alpha} S_{\lambda}(t_{q}), \\
E_{\lambda}(t_{q+1}) &= E_{\lambda}(t_{q}) + \Theta_{\lambda}(E_{\lambda}(t_{q}), A_{\lambda}(t_{q}), R_{\lambda}(t_{q}), W_{\lambda}(t_{q})) + \mu_{\lambda}^{\alpha} E_{\lambda}(t_{q}), \\
A_{\lambda}(t_{q+1}) &= A_{\lambda}(t_{q}) + \Theta_{\lambda}(E_{\lambda}(t_{q}), A_{\lambda}(t_{q}), R_{\lambda}(t_{q}), W_{\lambda}(t_{q})) + \mu_{\lambda}^{\alpha} A_{\lambda}(t_{q}), \\
R_{\lambda}(t_{q+1}) &= R_{\lambda}(t_{q}) + \Theta_{\lambda}(E_{\lambda}(t_{q}), A_{\lambda}(t_{q}), R_{\lambda}(t_{q}), W_{\lambda}(t_{q})) + \mu_{\lambda}^{\alpha} R_{\lambda}(t_{q}), \\
W_{\lambda}(t_{q+1}) &= W_{\lambda}(t_{q}) + \Theta_{\lambda}(E_{\lambda}(t_{q}), A_{\lambda}(t_{q}), R_{\lambda}(t_{q}), W_{\lambda}(t_{q})) + \mu_{\lambda}^{\alpha} W_{\lambda}(t_{q}), \\
\end{align*}
\]

where \( q = 0, 1, 2, \cdots, m - 1 \).
Here in this subsection, graphical interpretation of numerical results to the concerned model is given. For this aim, we use the adopted scheme for the numerical simulation. Here, we choose some appropriate values for the parameters used in the model that is given in the Table 1 (see [28]). Graphical presentations are given in Figures 1–6, for various values of $\eta$. We construct an algorithm to simulate the results by using Matlab in Figs. 1–6.

In Figs. 1–6 we have presented the plot for the different compartments of the considered model corresponding to various fractional values order $\eta$. We have presented the numerical results for initial 200 days. Initially the infection in first month that first thirty days was increasingly transmitted but on time control the China government implemented strict precautionary measures which controlled the disease very well in coming two months. In Figs. 1–6, we have given the evolution of COVID-19 in Wuhan city for initial 200 days. Further, From the Figs. 1–6, one can observe that the considered model extremely depends on the order and offers more degree of flexibility. As we increase the values of the $\eta$, we see that the solution tends to integers order solution. The growing and decaying rate of various classes of model is different at different fractional order. Therefore fractional calculus can be helpful in understanding the transmission dynamics of COVID-19. Here we, remark that at smaller fractional order the decay process is faster while the growth rate is slow. Increasing the fractional order the process of decay may become slow while, the grow rate goes on raising. Further, the fractional order has great impact on the transmission dynamics of the proposed model. Also, it helps in better understanding of physical behaviour of spreading of infection in a community. Moreover, the adopted numerical method can be used as a fruitful technique to achieve computational results for such type nonlinear problems. The concerned growth or decay process of various compartments is faster slightly at lower fractional order as compared to greater value of $\eta$.

### Table 1 Description of the parameters involve in the model (2).

| Parameters | Description of Parameters |
|------------|---------------------------|
| $S_p = 800000$ | Susceptible people |
| $E_p = 200000$ | Exposed people |
| $I_p = 200$ | Infected people |
| $A_p = 250$ | Asymptomatic people |
| $R_p = 0$ | Recovered people |
| $W = 50000$ | Reservoir |
| $m_p = 0.1$ | Rate of death |
| $\Lambda_p = n_p \times N_p = 5000$ | Total population and birth rate |
| $\omega_p = 0.01$ | Incubation period |
| $\omega_p = 0.768$ | Latent period |
| $\gamma_p = 1.05$ | Infectious period of symptomatic infection |
| $\gamma_p = 0.00001$ | Infectious period of asymptomatic infection of people |
| $\beta_p = 0.00006$ | Transmission rate from $I_p$ to $S_p$ |
| $\beta_w = 0.00010$ | Transmission rate from $W$ to $S_p$ |
| $\mu_p = 0.1$ | Shedding coefficients from $I_p$ to $W$ |
| $\mu_p = 0.0003$ | Shedding coefficients from $A_p$ to $W$ |
| $\delta_p = 0.009$ | Proportion of asymptomatic infection rate of people |
| $k = 0.00654$ | Multiple of the transmissibility of $A_p$ to that of $I_p$ |
| $\epsilon = 0.09$ | Lifetime of the virus in $W$ |

6.2. Numerical interpretation

Here in this subsection, graphical interpretation of numerical results to the concerned model is given. For this aim, we use the adopted scheme for the numerical simulation. Here, we choose some appropriate values for the parameters used in the model that is given in the Table 1 (see [28]). Graphical presentations are given in Figures 1–6, for various values of $\eta$. We construct an algorithm to simulate the results by using Matlab in Figs. 1–6.

In Figs. 1–6 we have presented the plot for the different compartments of the considered model corresponding to various fractional values order $\eta$. We have presented the numerical results for initial 200 days. Initially the infection in first month that first thirty days was increasingly transmitted but on time control the China government implemented strict precautionary measures which controlled the disease very well in coming two months. In Figs. 1–6, we have given the evolution of COVID-19 in Wuhan city for initial 200 days. Further, From the Figs. 1–6, one can observe that the considered model extremely depends on the order and offers more degree of flexibility. As we increase the values of the $\eta$, we see that the solution tends to integers order solution. The growing and decaying rate of various classes of model is different at different fractional order. Therefore fractional calculus can be helpful in understanding the transmission dynamics of COVID-19. Here we, remark that at smaller fractional order the decay process is faster while the growth rate is slow. Increasing the fractional order the process of decay may become slow while, the grow rate goes on raising. Further, the fractional order has great impact on the transmission dynamics of the proposed model. Also, it helps in better understanding of physical behaviour of spreading of infection in a community. Moreover, the adopted numerical method can be used as a fruitful technique to achieve computational results for such type nonlinear problems. The concerned growth or decay process of various compartments is faster slightly at lower fractional order as compared to greater value of $\eta$.

![Graph of approximate solution for susceptible class at different fractional values of $\eta$.](image)
Study of COVID-19 mathematical model of fractional order

Fig. 2  Graph of approximate solution for exposed class at different fractional values of $\eta$.

Fig. 3  Graph of approximate solution for symptomatic infected class at different fractional values of $\eta$. 
Fig. 4  Graph of approximate solution for asymptomatic infected class at different fractional values of $\eta$.

Fig. 5  Graph of approximate solution for recovered people at different fractional values of $\eta$.

Fig. 6  Graph of approximate solution for reservoir class at different fractional values of $\eta$. 
7. Conclusion

We have established some qualitative results for the mathematical model (2) involving CFOD. Using the nonlinear analysis, we have derived feasibility of the solution and bounded ness of the result to the concerned model. Also, we derived the reproductive number for the model under study. For the needed results about existence and uniqueness of solution fixed point theory has been used. Also we have developed the Ullam-Hyers stability results for the considered model. Further, we have computed numerical solutions for the considered model via a powerful technique due to Euler. Graphical representations have been given to check the dynamical behavior of solution by using Matlab. We have computed the results regarding to distinct values of fractional orders. The gained results play crucial part in showing the theory of fractional analytical dynamic for the existing outbreak due to COVID –19 which has badly affected the entire globe. From the computations we observed that the increase or decrease in different compartments is faster at higher fractional order of the derivative and we see that fractional calculus has the ability to explain the population dynamics more comprehensively. The presented results may be fruitful for the existing outbreak in a better way and can be used in taking defensive techniques to decrease the infection. In future the proposed scheme can be utilized to investigate more nonlinear problems of FODEs involving CFOD.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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