Faraday tomography: a new, three-dimensional probe of the interstellar magnetic field

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Abstract. Much of our present observational knowledge of the interstellar magnetic field of our Galaxy comes from two different sources, both of which involve mechanisms operating at radio wavelengths: the first one is the Faraday rotation of linearly-polarized radio waves propagating through the magneto-ionic interstellar medium, and the second one is the diffuse synchrotron emission from our Galaxy. I will review what these two classical probes have taught us about the strength, the direction/orientation, and the spatial distribution of the interstellar magnetic field. I will then present a recent method, known as Faraday tomography or rotation measure synthesis, which relies on a combination of Faraday rotation and synchrotron emission and which makes it possible to probe the interstellar magnetic field in three dimensions.

1. Introduction
The interstellar magnetic field (ISMF), $\vec{B}$, is an important component of the interstellar medium (ISM) of our Galaxy. It plays a crucial role in a variety of physical processes, including the acceleration, propagation, and confinement of cosmic rays, the spatial distribution, dynamics, and energetics of interstellar matter, the vital process of star formation. However, its properties remain poorly understood.

One of the main difficulties is the lack of direct measurements, apart from Zeeman-splitting measurements. In addition, all existing observational methods provide only partial information: Zeeman-splitting measurements yield $B_\parallel$, the line-of-sight component of $\vec{B}$, in neutral (molecular and atomic) clouds. Faraday rotation measures (RMs), in combination with a model for the thermal-electron density, lead to $B_\parallel$ in ionized regions. Synchrotron emission data, in combination with assumptions on the relativistic-electron density, lead to $B_\perp$, the plane-of-sky projection of $\vec{B}$, in the general, cosmic-ray filled ISM. Finally, linear polarization of starlight and of dust thermal emission yields the orientation of $B_\perp$ in the general, dusty ISM.

In most cases, magnetic field observations are difficult to interpret, because the observed emission (or Faraday rotation) along any line of sight is generally produced by a number of structures, whose contributions are often hard to disentangle and locate along the line of sight. One way to overcome this intrinsic problem is to perform Faraday tomography, a recent method also known as rotation measure synthesis, which exploits the Faraday rotation of the Galactic diffuse synchrotron emission.

Because understanding Faraday rotation and synchrotron emission is prerequisite to understanding Faraday tomography, I will start with a brief overview of Faraday rotation in...
Section 2 and synchrotron emission in Section 3. I will then proceed with a discussion of Faraday tomography in Section 4.

2. Faraday rotation

Faraday rotation is the rotation of the polarization direction of a linearly-polarized radio wave that propagates through a magneto-ionic medium, where it interacts with the free (thermal) electrons gyrating about magnetic field lines. The polarization direction rotates by an angle \( \Delta \theta = R_M \lambda^2 \), where \( \lambda \) is the observing wavelength and \( R_M \) is the so-called rotation measure, given by

\[
R_M = C \int_0^L n_e B_{||} \, ds ,
\]

with \( C \) a numerical constant, \( n_e \) the thermal-electron density, \( B_{||} \) the line-of-sight component of the magnetic field (positive/negative when the field points toward/away from the observer), and \( L \) the path length between the source and the observer. In practice, the RM of a given radio source can be determined by measuring the polarization direction of the incoming radiation at at least two different wavelengths.

RMs have now been derived for about 1 100 Galactic pulsars [1] and 42 000 extragalactic radio point sources (see Figure 1). Used in conjunction either with a model for the spatial distribution of thermal electrons or, in the case of Galactic pulsars, with their distances and dispersion measures (DMs), these numerous RMs have made it possible to gather a wealth of information on the strength, direction, and spatial configuration of the ISMF in the magneto-ionic ISM. Here are the key points that have emerged from Faraday rotation studies.

![Figure 1. All-sky map in Galactic coordinates (with the Galactic center in the middle, longitude increasing to the left, and latitude increasing upward) of the RMs of \( \sim \) 42 000 extragalactic radio point sources from the NVSS (\( \delta > -40^\circ \)) and S-PASS (\( \delta < 0^\circ \)) surveys. Positive/negative RMs, which correspond to a magnetic field pointing on average toward/away from the observer, are plotted in blue/red. Figure credit: Dominic Schnitzeler.](image)

1) The ISMF has a regular component, \( \vec{B}_{\text{reg}} \), and a randomly fluctuating component, \( \vec{B}_{\text{fluct}} \). In the large-scale vicinity of the Sun, \( B_{\text{reg}} \approx 1.5 \mu \text{G} \) and \( B_{\text{fluct}} \approx 5 \mu \text{G} \) [2]. \( B_{\text{reg}} \) increases toward the Galactic center, to \( \gtrsim 3 \mu \text{G} \) at Galactocentric radius \( R = 3 \text{ kpc} \) [3], i.e., with an exponential scale length \( \lesssim 7.2 \text{ kpc} \). Moreover, \( B_{\text{reg}} \) decreases away from the Galactic plane, albeit at a very uncertain rate; for reference, the exponential scale height inferred from the RMs of extragalactic radio point sources is \( \sim 1.4 \text{ kpc} \) [4].
2) In the Galactic disk, $\vec{B}_{\text{reg}}$ is nearly horizontal and generally dominated by its azimuthal component. In the large-scale vicinity of the Sun, $\vec{B}_{\text{reg}}$ runs clockwise at an angle $\simeq -8^\circ$ to the azimuthal direction [5], which is very close to the pitch angle $\simeq -7^\circ$ inferred from starlight polarization [6]. $\vec{B}_{\text{reg}}$ reverses direction at least a couple of times with decreasing Galactocentric radius, but the exact number and radial locations of the field reversals are still highly controversial [3; 5; 7–10]. These field reversals have often been interpreted as evidence that $\vec{B}_{\text{reg}}$ is bisymmetric (azimuthal wavenumber $m = 1$), while an axisymmetric ($m = 0$) field would be expected from dynamo theory. In reality, [11] showed that neither the axisymmetric nor the bisymmetric picture is consistent with the existing pulsar RMs, and they concluded that $\vec{B}_{\text{reg}}$ must have a more complex pattern.

3) In the Galactic halo, $\vec{B}_{\text{reg}}$ could have a significant vertical component. Near the horizontal position of the Sun, [12] obtained $(B_{\text{reg}})_z \simeq -0.14$ $\mu$G above the Galactic midplane ($z > 0$) and $(B_{\text{reg}})_z \simeq +0.30$ $\mu$G below the midplane ($z < 0$), whereas [13] obtained $(B_{\text{reg}})_z \simeq 0.00$ $\mu$G toward the north Galactic pole and $(B_{\text{reg}})_z \simeq +0.31$ $\mu$G toward the south Galactic pole. In contrast to the situation prevailing in the Galactic disk, the horizontal component of $\vec{B}_{\text{reg}}$ shows no sign of reversal with decreasing Galactocentric radius.

4) $\vec{B}_{\text{reg}}$ exhibits some symmetry properties with respect to the Galactic midplane. At low latitudes (basically, in the disk), $\vec{B}_{\text{reg}}$ appears to be roughly symmetric in $z$ [7; 14], while at high latitudes (in the halo), the RM sky features a rather striking antisymmetry/symmetry in $z$ in the inner/outter Galactic quadrants [5; 15; 16], which suggests that $\vec{B}_{\text{reg}}$ is roughly antisymmetric in $z$ inside the solar circle [5; 16, but see also [14]]. Finding $\vec{B}_{\text{reg}}$ to be symmetric in the disk and antisymmetric in the inner halo is consistent with the predictions of dynamo theory and with the results of galactic dynamo calculations [e.g., 17–19].

3. Synchrotron emission

Synchrotron emission is produced by relativistic electrons gyrating about magnetic field lines. The synchrotron emissivity at frequency $\nu$ due to a power-law energy spectrum of relativistic electrons, $f(E) = K_e E^{-\gamma}$, is given by

$$\mathcal{E}_{\nu} = \mathcal{F}(\gamma) \, K_e \, B_{\perp}^{\frac{\gamma + 1}{2}} \, \nu^{-\frac{\gamma + 1}{2}},$$

(2)

where $\mathcal{F}(\gamma)$ is a known function of the electron spectral index and $B_{\perp}$ is the strength of the sky-projected magnetic field.

The spatial distribution of the Galactic synchrotron emissivity was modeled by [20], based on the all-sky 408 MHz radio continuum map of [21] (see Figure 2). Several authors then used this synchrotron distribution model to derive the ISMF distribution in our Galaxy. To do so, the vast majority of them resorted to the standard double assumption that (1) relativistic electrons represent a fixed fraction of the cosmic-ray population and (2) cosmic rays and magnetic fields are in (energy or pressure) equipartition. Relying on the cosmic-ray ion and electron spectra directly measured by the Voyager spacecraft, [22] verified that, near the Sun, cosmic rays and magnetic fields are indeed close to (pressure) equipartition, with a total magnetic field strength $B \simeq 5 \mu$G. She also found that the total field strength has a radial scale length $\simeq 12$ kpc and a vertical scale height near the horizontal position of the Sun $\simeq 4.5$ kpc.

Because synchrotron emission is linearly polarized perpendicular to $\vec{B}_{\perp}$, information can also be gained on the orientation of $\vec{B}_{\perp}$. Evidently, if the observing frequency is low enough to be affected by Faraday rotation, the received polarized signal must somehow be "de-rotated" in order to retrieve the true field orientation. In addition, if the ISMF has a randomly fluctuating component, the contributions from isotropic magnetic fluctuations to the polarized emission cancel out, leaving only the contribution from the ordered (i.e., regular + anisotropic random)
ISMF, $\vec{B}_{\text{ord}}$. Thus, while the total synchrotron intensity yields the strength of the total sky-projected ISMF, the polarized synchrotron intensity yields the strength and the orientation of the ordered sky-projected ISMF.

In the large-scale vicinity of the Sun, the ratio of ordered to total ISMF strengths turns out to be $\simeq 0.6$ [23]. Together with $B \simeq 5 \, \mu G$, this ratio implies an ordered ISMF strength $B_{\text{ord}} \simeq 3 \, \mu G$. Besides, the radio polarization vectors confirm that the large-scale ordered ISMF in the Galactic disk is horizontal.

4. Faraday tomography

An important limitation of the methods described in Sections 2 and 3 is that they provide only line-of-sight integrated quantities, with no details on how the integrands vary along the line of sight. For instance, the synchrotron intensity measured in a given direction tells us only about the total amount of synchrotron emission produced along the entire line of sight through the Galaxy, with no information on the local value of $E_\nu$ (Eq. 2). Similarly, the RM of a given radio source (Eq. 1) tells us only about the total amount of Faraday rotation incurred along the line of sight between the source and the observer, with no information on the local value of $n_e B_\parallel$. For the latter, some large-scale trends can in principle be inferred if the line of sight encounters a sizeable number of well-separated Galactic pulsars with known distances and RMs. In practice, however, there are only $\simeq 1100$ Galactic pulsars with measured RMs [1], and their distances remain quite uncertain.

An alternative, more powerful and more promising, approach to probe the three-dimensional structure of the ISMF is now being increasingly utilized. This approach is also based on Faraday rotation, but instead of considering the Faraday rotation of the linearly-polarized radiation from a background radio source (as explained in Section 2), the idea is to exploit the Faraday rotation of the synchrotron radiation from the Galaxy itself (discussed in Section 3).

As a reminder, in the case of a background radio source, i.e., when the regions of radio emission and Faraday rotation are spatially separated, the Faraday rotation angle, $\Delta \theta$, increases linearly with wavelength squared, $\lambda^2$, and one may define RM as being the slope of the linear relation between $\Delta \theta$ and $\lambda^2$, such that $\Delta \theta = \text{RM} \, \lambda^2$ (see Section 2). Hence, RM is a purely observational
quantity, which can be meaningfully measured only for a background radio source and which can then be related to the physical properties of the foreground Faraday-rotating medium through Eq. (1).

In contrast, when the radio source is the Galaxy itself, the regions of radio emission and Faraday rotation are spatially mixed. In that case, Δθ no longer increases linearly with \( \lambda^2 \) and the very concept of RM becomes meaningless. However, one may resort to the more general concept of Faraday depth (FD), defined as

\[
\Phi = C \int_0^z n_e B_\parallel ds ,
\]

(3)

where \( C, n_e, \) and \( B_\parallel \) have the same meaning as in Eq. (1) and \( z \) is the line-of-sight distance from the observer [24; 25]. \( \Phi \) has basically the same formal expression as RM (Eq. 1), but it differs from RM in the sense that it is a truly physical quantity, which can be defined at any point of the ISM, independent of any background radio source. \( \Phi \) simply corresponds to the line-of-sight depth measured in terms of Faraday rotation – in much the same way as optical depth corresponds to line-of-sight depth measured in terms of opacity.

When radio emission and Faraday rotation are mixed along the line of sight, the polarized intensity measured at a given wavelength \( \lambda \) is the superposition of the polarized emission produced at all line-of-sight distances \( z \), i.e., at all Faraday depths \( \Phi \), and Faraday-rotated by an angle \( \Delta \theta = \Phi \lambda^2 \):

\[
P(\lambda^2) = \int_{-\infty}^{+\infty} F(\Phi) e^{2i\Phi\lambda^2} d\Phi ,
\]

(4)

where \( P(\lambda^2) \) is the complex polarized intensity \( (P = Q + iU) \) at \( \lambda \), and \( F(\Phi) \) is the complex polarized intensity per unit \( \Phi \), also called complex Faraday dispersion function, at \( \Phi \) [24]. Since the Faraday rotation angle varies with wavelength, the polarized intensities measured at different wavelengths correspond to different combinations of all the line-of-sight contributions and, therefore, provide different pieces of information. Thus, the idea is to measure the polarized intensity at a large number of different wavelengths and to convert its variation with \( \lambda^2 \) into a variation with \( \Phi \). Mathematically, this can be done inverting Eq. (4), i.e., by taking the Fourier-like transform of \( P(\lambda^2) \) to obtain \( F(\Phi) \).

The method is illustrated in Figure 3, which depicts a situation where the line of sight intersects two Faraday-rotating clouds (shaded in light grey), across which \( \Phi \) increases or decreases according to Eq. (3), and two synchrotron-emitting clouds (shaded in dark blue). The top panel in both Figures 3a and 3b indicates the positions of the four clouds along the line of sight with respect to the observer placed on the far left, as well as the directions of the ISMF (red arrows) in the two Faraday-rotating clouds: in the closer/farther cloud, the ISMF points toward/away from the observer, so that \( B_\parallel \) is positive/negative and \( \Phi \) increases/decreases with increasing \( z \). The corresponding run of \( \Phi \) with \( z \) is plotted in the middle panel, where \( \Phi_1 \) denotes the Faraday thickness of the closer cloud and \( \Phi_2 \) the cumulated Faraday thickness of both Faraday-rotating clouds. The bottom panel displays the Faraday dispersion spectrum, \( |F(\Phi)| \), with the two peaks representing the polarized emissions from the two synchrotron-emitting clouds. In Figure 3a, where the Faraday-rotating and synchrotron-emitting clouds are spatially separated, the closer and farther synchrotron-emitting clouds lie at FDs \( \Phi_1 \) and \( \Phi_2 \), respectively. In Figure 3b, the closer synchrotron-emitting cloud is embedded inside the closer Faraday-rotating cloud, so it has a finite Faraday thickness, i.e., it extends over a range of FDs (up to nearly \( \Phi_1 \)).

In practice, Faraday tomography can be used to separate synchrotron-emitting regions located at different FDs along the line of sight and to estimate their respective polarized synchrotron intensities, which in turn can lead to the strength and the orientation of their \( \vec{B}_\perp \) (see Section 3).
Figure 3. Schematics illustrating the concept of Faraday tomography. The top panel in both (a) and (b) pictures the spatial configuration of the system: two Faraday-rotating clouds (light grey shading, with a red arrow representing the ISMF) and two synchrotron-emitting clouds (dark blue shading), with the observer on the far left. The middle panel shows how the Faraday depth, $\Phi$ (given by Eq. 3), varies with line-of-sight distance from the observer, $z$. The bottom panel provides the Faraday dispersion spectrum, $|F(\Phi)|$. See the main text for more details. Figure credit: Marta Alves.
Faraday tomography can also be used to uncover intervening Faraday screens and to estimate their Faraday thickness, which in turn can lead to their $B_\parallel$. The method is particularly interesting when the uncovered Faraday screens can be identified with known gaseous structures, because it then offers a new way of probing their magnetic field.

As an example of the applicability of the method, [26] performed Faraday tomography of a $140^\circ \times 100^\circ$ field in the northern Galactic hemisphere, based on polarization data from the Global Magneto-Ionic Medium Survey (GMIMS). The observed field turns out to contain a known HI bubble-shell structure, estimated to lie at a distance $\sim 150$ pc. [26] derived $F(\Phi)$ along many lines of sight covering the observed field, and they assembled all their derived $F(\Phi)$ into a so-called RM-synthesis cube, i.e., a three-dimensional cube of $F(\Phi)$ data, with two axes in the plane of the sky and the third axis along $\Phi$. By scanning this RM-synthesis cube, they found that the background polarized radio emission toward the east and west parts of the HI shell is shifted along the $\Phi$ axis, and hence Faraday-rotated, by $\simeq +60$ rad m$^{-2}$ and $-50$ rad m$^{-2}$, respectively. Their interpretation was that the east and west parts of the HI shell act as Faraday screens, with respective Faraday thicknesses $\simeq +60$ rad m$^{-2}$ and $-50$ rad m$^{-2}$, implying a magnetic field pointing toward/away from the observer, respectively. This interpretation, in turn, suggested that the magnetic field is wrapped around the bubble. In addition, the Faraday thicknesses of both parts of the shell enabled the authors to obtain a rough estimate for the magnetic field strength inside the shell.

Within the next few years, Faraday tomography is likely to lead to a big leap in our knowledge and understanding of the ISMF. Polarization data should be acquired at low radio frequencies in order to enhance the effects of Faraday rotation (which increase as $\lambda^2$) and, therefore, to improve sensitivity in FD space. At the same time, broad frequency coverage is needed to achieve fine resolution in FD space [25]. This is why low-frequency, broad-band radio-telescopes, such as the LOw-Frequency ARray (LOFAR) and, in the near future, the Square Kilometer Array (SKA), are ideally suited for the task at hand.

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