Gear shape optimization for non-circular gearing

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Abstract. The quality of gearings is largely decided by their geometric design. If the geometric design is incorrect, the reliability of the transmission will not be ensured even by the use of the highest quality materials. Conversely, sometimes excellent geometric gear design can save expensive material costs. The work describes the procedure of optimizing the geometric model of a non-standard elliptical gear eccentrically mounted with a continuously changing gear number for specific parameters.

1. Introduction
Toothed gears are among the most widely used transmission mechanisms. They are the basic element by which the transmission and transformation of mechanical energy and motion is realized in machines [1]. The basic part of the transmission is gears, which consist of a pair of meshing gears - driving and driven [2-4]. Gears have become a symbol of engineering. Gears were known and used by mankind even before the current era (BCE), mainly in the propulsion of water mills, for pumping water or lifting heavy loads. The evidence provides also the knowledge in the works of Aristotle (384 - 322 BCE). In the schemes of Leonardo da Vinci (1452 - 1519) there are many toothed gears that are currently used in machines and devices. Leonardo da Vinci sketched circular as well as non-circular gears. In the past, these non-circular gears have found use in many types of mechanical devices, such as clocks and toys.

“Standard” gear transmissions are characterized by a constant gear ratio. The teeth of these “standard” gears have the same shape on one gear and the teeth profile is symmetrical (in exceptional cases also asymmetrical) [5-7]. In practice, gear transmissions units whose gear ratio is not constant are also used [8,9]. Such gear transmissions are used, for example in the drives of window shade panels, in order to generate vibrations, which interfere with the natural oscillations and cancel them out [10-12]. Or in high-powered starters – mechanical systems which provide a progressive torque for easier starting of the machines, where the progressive torque helps to overcome the start-up inertia. Non-circular gears have their application in oval gear flowmeters [13]. In the automotive industry, for example in VW diesel engines, where the manufacturer reduced the load of the belt by the use of multiple atypical design elements, which include also a non-circular gear with "non-identical" teeth [14,15]

The quality of gear transmissions is largely decided by their geometric design [16,17]. If the geometric design is incorrect, the use of the highest quality materials will not ensure the reliability of the transmission [18,19]. Conversely, sometimes excellent geometric gear design can save expensive material costs. The work describes the procedure of creating a geometric model of a non-standard elliptical gear eccentrically stored with gas is changed by a gear number for specific parameters.
2. Defining non-circular transmission requirements

Gear shape optimization is based on the requirements of practice. The client also supplied gears made, so to speak, "roughly" to illustrate the problem (Figure 1). The supplied gear did not meet the conditions for correct meshing.

![Figure 1. Model of the supplied gear set.](image)

It was necessary to design the shape of the gear transmission for the specified parameters. The gear set was to consist of two identical gear wheels. The gear ratio had to change harmonically in the range from \( u = 0.5 \) through 1.0 to 2.0 and back during one revolution of the inter-meshing gears. The number of teeth of the gears are \( z_1 = z_2 = 24 \) and the standardized value of the gearing module is \( m_n = 3.75 \) mm. The axial distance is \( a = 90 \) mm, the pressure angles is \( \alpha_n = 20^\circ \) and gear set is intended for one sense of rotation.

3. Design of circular gears with eccentric centers of rotation

As the basis of the gear transmission model (Figure 1), two circular gears with eccentrically placed centres of rotation and with the pitch circles with the diameter \( d = 90 \) mm \((d = z.m_n)\) – see Figure 2 were provided.

![Figure 2. Pitch circles of the two inter-meshing gears according to the original model of the gear set.](image)

The centres of rotation of the gears \( O_1 \) and \( O_2 \) were determined according to the demand to create a gear transmission with the time-variable gear ratio in the range from 0.5 to 2.0. The solution – using the eccentrically placed circular gears – does not meet the basic condition of pitch circles rolling [20]. An intersection of both circles occurred and the greatest value of the intersection was at the meshing of teeth denoted by the number 6 (as shown in Figure 3), what resulted in the idea, using the elliptical shaped gears.
4. Design of non-circular gears with eccentric centers of rotation

The first step was the correct design of the pitch curve. This was followed by the design and optimization of the shape of the gear teeth.

4.1. Design of the shape of the pitch curve

The correct design of a pitch curve was the first step in a gearing set design. Based on the results of the solution of given problem using gears with circular shape and eccentrically placed centres of rotation, the elliptical shape of the gears was chosen. The geometrical centre of the gear is not the centre of rotation of the gear. The centre of rotation of the gear was chosen in the focus of the ellipse. The dimensions of the chosen pitch ellipse of the gear are visible in Figure 4.

![Figure 3. Incorrect meshing of chosen circular gears with eccentrically placed centres of rotation.](image1)

![Figure 4. Dimensions of the designed pitch ellipse.](image2)
In Figure 5, the pitch ellipses of the eccentrically placed inter-meshing elliptical gears and also the pitch ellipse division into 24 pitches (according to the number of teeth \( z = 24 \)), which are equal in length, are shown. Both gears are identical, therefore the same marking of teeth was chosen on both inter-meshing gears.

4.2. Design of elliptical gearing

There are two portions on tooth side profile curve, the involute and the non-involute portion. Only the involute portion of the tooth profile is allowed to be active during meshing of a gear. In the first solution, the involute was created by rolling the creating straight line on the base circle despite the fact that in this case we dealt with elliptical gears. The centre of the base circle was always coincident with the centre of rotation in the eccentrically placed gear. For the drawing of the involute portion of the tooth side profile, the trochoidal method of involute construction was used. In Figure 6 is example of involute construction from base circle for the tooth with order number 3.

The diameters of the base circles for the left and the right side of a tooth are different. The resulting shape of the gear created this way is shown in Figure 7 picture a) and it is identical with the shape of
the model provided by the customer. These gears are not functional (Figure 7, b), because the teeth interfere and the conditions of correct meshing are not met.

![Figure 7. a) Designed gearing shape, b) Incorrect meshing.](image)

4.3. Optimization of elliptical gearing

Gear shape optimization was required. For standard involute gears, an involute is used for tooth profiling, the evolute of which is a circle (basic) [21] (Figure 8 - a), as was the case in the first solution. A solution was proposed, where the evolute of the involute was an ellipse, not a basic circle (Figure 8 - b).

![Figure 8. a) The evolute of the involute is a circle, b) The evolute of the involute is an ellipse.](image)

In the Figure 9 is trochoidal method of construction of the involute curve of the tooth side profile if the evolute of the involute is the ellipsis. The evolute for the left and right sides of the teeth is not the same. The lateral curve of the tooth is involute, and is different for the active and passive side of the tooth, the teeth are asymmetric.
Figure 9. Involute construction from base ellipse for the tooth with number 4.

A root transition is the surface between the involute surface of a tooth and the root cylinder. It is a very important region, because it determines considerably the flexural strength and the interference phenomena during meshing. The shape of the root transition curve depends of the manufacturing method of tooting. One of the most common production methods is the production of gearing with a comb tool, where the heel transition curve is an envelope of rounding of the positions of the ridge tool, which rolls along the pitch circle of the gear wheel. Non-involute portions of the tooth profile (its left and right side) were constructed using the same fillet radius \( r_f = 1.425 \) mm. The resulting shape of the designed elliptical eccentrically gear set is shown in Figure 10.

Figure 10. Model of designed non-circular gear set.

Principe difference between the shape of teeth due to differential creation of involute is in the Figure 11. There is the shape of non-circular gear wheel is the evolute of the involute is circle in Figure 11a), and if the evolute of the involute is ellipse in Figure 11 b).
5. Conclusions
The gear set for the specified gearing with a time-varying gear ratio ranging from 0.5 through 1.0 to 2.0 were designed to be elliptical, eccentrically placed so that the conditions of correct meshing. The gear set consists of a pair of identical identical gear wheels, the number of teeth of which is 24 and the normalized value of the module $m_n = 3.75$ mm. Each of the twelve teeth of the gear is different, the other twelve teeth of the same wheel are the same. The lateral curve is involute, and is different for the active and passive side of the tooth. These are gears with an asymmetrical tooth profile.

Unlike standard circular gears, where the angular speed of the driving and also of the driven gear is constant, in this case, the angular velocity of the driven eccentrically placed gear is not constant, but it varies depending on the continuously changing gear ratio.

This designed geometric model of the elliptical gear wheel served as a basis for production of gear set. Elliptical gears were made on an NC machine for electrospark cutting. An NC machine for electrospark cutting (so-called wire cutter) EIR 005 B with RS-ER5 control was used for production. Its accuracy is 0.01mm and roughness Ra 1.6µm. The wheels were tested by the customer and met all his requirements.

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