Composite Higgs Bosons and Mini Black Holes

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Pairs of standard model fermions can annihilate to produce mini black holes with gauge quantum numbers of the Higgs boson at $M_{Planck}$. This leads to a Nambu-Jona-Lasinio model at the Planck scale with strong coupling which binds fermion pairs into Higgs fields. At critical coupling the renormalization group dresses these objects, which then descend in scale to emerge as bound-state Higgs bosons at low energies. We obtain the multi-Higgs spectrum of a “scalar democracy.” The observed Higgs boson is a gravitationally bound $\tilde{t}$ composite. Sequential states may be seen at the LHC, and/or its upgrades.

I. INTRODUCTION

Standard arguments suggest that a sufficiently energetic collision between, e.g., a left-handed electron ($e_L$) and an anti-right-handed electron ($\bar{\tau}_R$), can produce a mini black hole $B$:

$$e_L + \bar{\tau}_R \rightarrow B$$

Production of $B$ requires $M_B = \sqrt{s}$ and that the collision have an impact parameter, $b$, where $b \lesssim 2G\sqrt{s}$, hence $b$ is the Schwarzschild radius for the corresponding black hole [1, 2].

Let us assume the total angular momentum of the initial state is $s$-wave and spin zero. The incident charged electrons have the standard model weak isospin, and hypercharge, $[I_3, Y]$, $e_L \sim [-1/2, -1]$ and $\bar{\tau}_R \sim [0, 2]$. These will produce an electrically neutral black hole, $B \sim [-1/2, 1]$, where the electric charge is $Q = I_3 + \frac{Y}{2} = 0$. If we replace the incident $e_L$ by the left-handed neutrino, $\nu_L \sim [1/2, -1]$, we obtain the charged black hole, $B \sim [1/2, 1]$, with $Q = 1$.

These are the quantum numbers of the neutral and charged components of the Higgs doublet in the standard model (SM). Therefore, SM fermions and gravity, alone, automatically imply scalar “Higgs bosons” that are gravitationally bound pairs of fermions, alas with masses $\sim M_P$! Classically we describe these by the Reissner-Nordstrom (RN) metric and the Higgs black hole is an electroweak isodoublet, has “hair,” with external gauge fields, $W^\pm$, $Z^0$ and $\gamma$. By conventional wisdom they are guaranteed to exist.

In the present paper we consider the possibility that there is a deeper connection between the existence of these “Higgs black holes” (HBH) and the physically observed Higgs boson(s) of the SM. We consider the virtual effects of the threshold HBH and find that these imply a strong interaction as one approaches the Planck scale. Integrating out the holes leads to an effective Nambu-Jona-Lasinio model that drives the formation of composite states. We find that, by renormalization group (RG) effects, the black holes may form composite Higgs fields in the infrared. One can view this as black holes becoming dressed by the renormalization group i.e., becoming “Wilsonian black holes” as cores of Higgs bosons [3].

The main issue is, how far into the infrared can this composite spectrum extend? At larger distance the composite Higgs bosons are mainly loops of SM particles and the HBH is virtual. Fermion loops bind, subtracting from the bare mass of an HBH and pull it into the infrared.

We require an exact critical coupling of fermions to the HBH to make the composite Higgs states massless. This is analogous to criticality in second order phase transitions. However, in the present Nambu-Jona-Lasinio model this involves a drastic fine-tuning (this is identical to what happens in top-condensation models [4, 5]).

For critical coupling the fine tuning is a symmetry, an effective scale invariance of the bound-state with respect to the Planck mass. Conceivably this might arise dynamically, i.e., the bound-state system may internally self-adjust dimensionless parameters to minimize its energy, and find the cancellation that realizes the symmetry. This would likely be sensitive to the quantum numbers of the composites, e.g., favoring light color singlets and leaving colored states at very large masses. The resulting small masses for the composite Higgs bosons would then arise from infrared scale breaking effects, of order $10^2$ GeV to $10^6$ TeV. We’ll assume something like this works, fine tune, and proceed.

We find that multiple Higgs scalars occur, at least one for any $s$-wave fermion bilinear channel present at the Planck scale. If all SM fermions are present near $M_P$ then we can form 1176 complex scalars, the symmetric bilinear representation of $SU(48)$. This leads to 18 Higgs doublets in the quark sector and 18 in the lepton sector. This is an idea proposed recently of “scalar democracy” [6–8]. It is consistent with, and in principle “explains,” flavor physics. It is testable at the LHC upgrades and it implies a plethora of new states for a $\sim 100$ TeV machine.

We are mainly interested in the physics near the threshold of a spectrum of black holes. Most analyses of black hole production focus on large $\sqrt{s} \gg M_P$ (typically in compactified extra dimensional schemes with
low effective $M_P$), and in this limit we reliably recover the geometrical picture of black holes. However, for the quantum theory near threshold we expect a breakdown of classical intuition, just as is the case of the Hydrogen atom. Here we find the ideas of Dvali and Gomez (DG) et al. to be compelling and yield a useful “portrait” of the threshold theory [9–11] (see also [12]).

We briefly summarize the ideas of Dvali and Gomez (DG). Here black holes are composed of “condensates” of a large number, $N$, of gravitons and perhaps other objects such as the fermion pair that creates an HBH. The behavior becomes classical as $N \gg 1$ and we would expect the geometrical aspects of black holes are then emergent.

On the other hand, for small $N \to 1$ we approach the quantum limit, and the behavior is radically different than the classical picture. Here many classical theorems about black holes break down (such as the “no-hair” theorem; moreover the viability of global symmetries, such as flavor symmetries is maintained). For small $N$ the states have quantized masses (modulo widths) and form a tower of resonances with schematic decay chains that cause transitions $N \to N - 1$ (Hawking radiation). The RN-black hole “remembers” the global charges that produced it. Near threshold, the decay width of small $N$ black holes approaches $\sim M_P$. The effective coupling of matter to threshold black holes is strong.

A threshold Schwarzschild black hole consists of a single graviton with mass $\mu \sim \pi/2R$, localized within the Schwarzschild radius $R$. The graviton can be thought of as a half-wave “lump” within the (effective) horizon of size $\sim 2R$, and corresponding to a full wavelength of $\sim 4R$. If we consider a Fock state with $N$ quanta in this mode, we will have a black hole mass $M = N\mu = N\pi/2R$, which will form a horizon as:

$$1 = 2GM/R = GN\pi/R^2,$$  

hence $R = \sqrt{\pi N M_P^{-1}}$, (2)

where $G = 1/M_P^2$, and therefore:

$$M_N = N\pi/2R = \sqrt{N\pi M_P}/2.$$  

(3)

A key feature of the DG theory is that it has an effective smallest quantum wavelength and corresponding momentum cutoff. For concreteness, we will define these to be, respectively:\footnote{Note, we have inserted the necessary factors of $\pi$ into the DG discussion when one relates “wavelength” or “Schwarzschild radius” to “mass” or “momentum” when one sets $\hbar = 1$.}

$$\lambda_0 \sim \sqrt{\pi M_P^{-1}}, \quad p_0 \sim 2\pi/\lambda_0 = \sqrt{4\pi M_P}. \quad (4)$$

We’ve defined $\lambda_0$ as the Schwarzschild radius of the single graviton black hole in the DB picture, $N = 1$. At shorter distances the gravitational interaction is so strong that ordinary space-time becomes unthinkable. Anything with a quantum wavelength $\lesssim \lambda_0$ will be self-cloaked in gravitons, e.g., if one imagines boosting an electron above the cutoff momentum, say to $\sim 2p_0$, one will have a point-like electron with momentum $\sim p_0$ and collinear gravitons with $\sim p_0$. Hence at short distances we can never resolve a point-like electron with momentum component in excess of the cut-off.

Therefore, the smallest threshold black hole has a Schwarzschild radius $R = \sqrt{\pi M_P^{-1}}$, and a constituent quantum wavelength $\lambda = 4\sqrt{\pi M_P^{-1}}$, safely larger than the fundamental wavelength cut-off $\lambda > \lambda_0$. As $N$ increases, the black hole size does as well, $\propto \sqrt{N}$. Higher modes then become accessible, never exceeding the fundamental cutoff momentum $p_0$.

It is important to keep in mind that $N$ is the occupancy of a mode, and not a “principle quantum number” of the modes. DG refer to large $N$ as a “Bose-Einstein condensate”; these are actually Fock states, until the black hole Schwarzschild radius gets large and more available modes with wavelength greater than the cut-off open up. As we excite a black hole its radius grows as $\sqrt{N}$ and we produce more gravitons in the lowest mode, and the wavelengths of these constituents is never smaller than $\lambda_0$. Conversely, we see that $N \propto R^2$ which is an affirmation of Bekenstein entropy in the classical limit. This is also the basis of the claim of DG that Einstein gravity is self-healing and “classicalizes” in the far UV, and does not require a UV completion theory.

II. MINI BLACK HOLE INDUCED HIGGS COMPOSITENESS

We can extend the DG model to Reissner-Nordstrom black holes by including the incident fermions as components of the black hole. The ground-state then consists of the pair of incident fermions that produced it, $f_1 f_2$. The $N$th excitation (occupancy) above the ground-state will have these two fermions plus $N$ gravitons. Each is assumed to have an energy $\mu \sim \hbar\pi/2R$ where $R$ is the Schwarzschild radius. The system self-binds into a black hole with mass $M_N = (2 + N)\pi/2R_N$. Hence:

$$\frac{G_N(2 + N)\pi}{R_N^2} = 1 \quad R_N = \sqrt{(2 + N)\pi/M_P}$$

$$M_N = \sqrt{(2 + N)\pi M_P}/2. \quad (5)$$

To expedite the discussion we focus on a single flavor channel, and only the ground-state black hole of mass $M_0 = (\sqrt{\pi/2})M_P$ and Schwarzschild radius $R_0 = \sqrt{2\pi}/M_P$. 

A. Effective Field Theory

We presently assume that the incident flavors are a pair consisting of the electron doublet $E_L = (\nu, e)_L$ and anti-right-handed singlet $\bar{\tau}_R$. Therefore the produced RN black hole, $B_0$, will be an HBH weak isodoublet with quantum numbers of the SM Higgs doublet $B_0 \sim \tau_R E_L$.

Consider an effective field theory of the coupling of the leptons to the threshold HBH $B_0$:

$$\mathcal{L}_0 = DB_0^\dagger DB_0 - g (\overline{E}_L B_0 e_R + h.c.) - M_0^2 B_0^\dagger B_0. \quad (6)$$

While this is a local approximation, which cannot be an exact description of the production process, the purpose of this effective field theory is only to roughly estimate the coupling constant $g$.

We compute the field theory cross-section for $E_L + \bar{\tau}_R \rightarrow B$. Calculating the cross-section with the usual rules, as in Bjorken and Drell [13], for a $2 \rightarrow 1$ process, there occurs an un-integrated $2\pi\delta(E_f - E_i)$ where $E_f - E_i = 0$. This is interpreted as $2\pi\delta(0) \sim T$ where $T$ is the lifetime of the final state, i.e., the inverse width $\Gamma$. The cross-section is then:

$$\sigma = \frac{g^2}{2M_0 \Gamma_0} \quad (7)$$

Likewise, the field theory calculation of the width via the allowed process $B_0 \rightarrow \overline{E}e$ is:

$$\Gamma_0 = \frac{g^2}{8\pi} M_0 \quad (8)$$

Note that $g^2/\Gamma_0 = 8\pi/M_0$ is now determined and therefore the cross-section is:

$$\sigma = \frac{4\pi}{M_0^2} = \frac{4}{\pi} R_0^2 \quad (9)$$

This is slightly smaller than the usual presumed geometric cross-section, $\sim \pi R_0^2$, [1, 2], which owes to the point-like approximation. Nonetheless, these are comparable.

To calibrate $g^2$ we require an input for $\Gamma_0$. For small $N$ we are far from a Hawking thermal decay process, and there are expected to be large $1/N$ corrections. In ref.[9] the ground-state decay width for small $N$ is estimated to be of order the Planck scale $M_P \sim M_0$. We will introduce an order-unity parameter $\eta$ and define:

$$\Gamma_0 \sim \frac{1}{4\eta} M_0 \quad \text{hence,} \quad g^2 \sim 2\pi \eta. \quad (10)$$

Hence our crude field theory fit to the properties of the quantum black hole suggests, with $\eta \sim 1$, there is reasonably strong coupling to the fermions with large $g^2$.

We note that this width is considerably larger than a computation using the Hawking temperature $T \sim M_P^2/8\pi M_0$, which is $T \sim a/2\pi$ with the acceleration, $a \sim GM/R^2$, redshifted to infinity. However, the decay is nonthermal, and the mini black hole decay process is happening promptly, at extremely short distances, and on the horizon $a$, hence $T$, is infinite. Once the constituents have escaped to a distance of a few $\sim \lambda_0$ the system is unbound, and $T$ at infinity is irrelevant.

If we go beyond the lowest mass threshold HBH, we will have a tower of states, each labeled by $N$. Higher $N$ states are expected to decay via coupled channel processes such as $B_N \rightarrow B_{N-1} + X$, or a “balding process” as $B_N \rightarrow S_N + f_1 f_2$ where $S_N$ is a Schwarzschild black hole, and $S_N \rightarrow S_{N-1} + X$. The exclusive process $B_N \rightarrow f_1 f_2$ characterized by an effective coupling $g_{f_1 f_2}^2$ also exists.

Integrating out the HBH tower in our crude field theory yields an effective Nambu–Jona-Lasinio interaction that is applicable below the threshold at a scale $M \lesssim M_0$:

$$\mathcal{L}_M = -\sum_N \left( \frac{g_{N}^2}{M_N^2} \right) E_L e_R \overline{\tau}_R E_L \quad (11)$$

Note that width effects, $\sim iN \Gamma N/2$ in the denominator, are suppressed since we are at momenta $p^2 \ll M_0^2$ and the width vanishes below threshold.

In principle many black holes contribute to this interaction in any given channel. DG observe, however, that the lifetime of the $N$th occupancy state is $\sim N^{3/2} M_P^{-1}$ (see eq.(9) of [9]). For the HBH this implies $\Gamma_N \sim (2 + N)^{3/2} M_P$ and hence, $g_{N}^2 \sim (2 + N)^{-2}$ and $g_{N}^2/M_N^2 \sim (2 + N)^{-3}$. This therefore suggests that the sum converges quickly, and may be reliably approximated by the ground-state term. However, this is a large $N$ limit, and we might expect $g_{N}^2/M_N \sim (\text{constant})$ for small $N$, and hence there may be enhancements from several states lowest in the tower.

$M_0^{2}$ is renormalized by fermion loop contributions extending from $\Lambda$ down to $M$, which we treat in the block-spin approximation which keeps quadratic running [4, 5]:

$$M_0^{2} = M_0^{2} - \frac{g^2}{8\pi^2} (\Lambda^2 - M^2). \quad (12)$$

Here $\Lambda \sim p_0 = \sqrt{4\pi} M_P$ is the momentum space cut-off of the theory associated with the fundamental length cut-
VEVs may lead to the relaxation of the black hole mass if \( dM_0^2/dv = 0 \). This may be interpreted as a condensate of dilatons localized around the black hole. At present we do not know how to implement these ideas and will content ourselves with the fine-tuning, which is equivalent to the usual fine tuning in the SM.

We now introduce a weak isodoublet auxiliary field \( H \) that factorizes the interaction of eq.\((11)\):

\[
\mathcal{L}_M = -g(E_L e_R H + h.c.) - M_0^2 H^1 H \tag{16}
\]

Solving the equations of motion for \( H \) and substituting back into eq.\((16)\) yields eq.\((11)\). This is our main point, that the HBH’s can be virtual yet induce a strong interaction below the scale \( M_0 \). \( H \) is the induced composite scalar state due to these strong interactions from virtual HBH’s.

We can now integrate the theory down to lower mass scales. It useful to consider just the fermion loops by themselves at one-loop order, to obtain, \([4, 5]\):

\[
\mathcal{L}_m = -g(E_L e_R H + h.c.) - M_m^2 H^1 H + ZDH^1 DH - \frac{\lambda}{2}(H^1 H)^2. \tag{17}
\]

Here we have displayed the induced relevant operator terms. The “block spin renormalization group” keeps both the logarithmic and the quadratic running of the mass induced by fermion loops.

We obtain from the fermion loops \([4, 5]\):

\[
M_m^2 = M_0^2 - \frac{g^2}{8\pi^2}(M_2 - m^2) = M_0^2 - \frac{g^2}{8\pi^2}(\Lambda^2 - m^2) \tag{18}
\]

With critical coupling we see that \( M_m^2 \rightarrow 0 \) with \( m^2 \rightarrow 0 \). The running of \( M_m^2 \) to zero will be cut-off by an explicit scale breaking mass term, that specifies the physical composite Higgs doublet mass in the infrared, \( \sim 10^2 \text{ GeV} \) to \( 10^6 \text{ TeV} \) range. We do not have a theory of these infrared masses at present but fit them to the observed infrared physics.

Likewise, we have the induced wave-function renormalization constant and the quartic coupling \([4, 5]\):

\[
Z = \frac{g^2}{16\pi^2} \ln \left( \frac{\Lambda^2}{m^2} \right) \quad \lambda = \frac{g^4}{8\pi^2} \ln \left( \frac{\Lambda^2}{m^2} \right). \tag{19}
\]

The renormalized theory is then:

\[
\mathcal{L}_m = -g(E_L e_R H + h.c.) - \bar{M}_m^2 H^1 H + \bar{D}H^1 DH - \frac{\lambda}{2}(H^1 H)^2. \tag{20}
\]

where,

\[
\bar{g} \sim \frac{g}{\sqrt{Z}} \quad \bar{\lambda} = \frac{\lambda}{Z^2} \quad \bar{M}_m^2 = \frac{M_m^2}{Z}. \tag{21}
\]

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4 Most notably, in the broken phase of a Nambu-Jona-Lasinio model the constituent fermion develops a mass \( m \) while the “Higgs boson,” which is composed of two fermions, has a mass of \( 2m \). It would be wrong to conclude that the Higgs is then unbound; the “binding” starts at the scale \( \Lambda >> m \).
We have only used the fermion loops, which is technically justified in a large \( g^2 \) limit. From this we can infer the behavior of the renormalized couplings as \( m \to \Lambda \):

\[
\bar{g}^2 = \frac{1}{2} \chi \sim \frac{16 \pi^2}{\ln(\Lambda^2/m^2)} \to \infty \tag{22}
\]

which is a behavior identical to the top condensation models \([4, 5]\). Note the critical coupling, \( g^2 = g_c^2 \) cancels in the running couplings. This corresponds to the RG running of these couplings in the limit of retaining only the fermion loops.

Given the boundary conditions on the running couplings as \( m \to \Lambda \) of eq. (22), we can switch to the full RG equations including gauge couplings, \( g^2 \), and \( \lambda \), etc. To apply this to the electron we integrate the full RG equations down to a mass scale of order \( \sim 10^6 \) TeV and stop. There we install an explicit mass for the composite Higgs, \( \sim M_m^2 \sim (10^2)^2 \) TeV\(^2\). This will then be a heavy doublet that does not directly develop a VEV. However, by mass mixing with the SM Higgs boson, \( \sim \mu^2 \sim (10^2)^2 \) TeV\(^2\) the heavy electron Higgs will acquire a tiny “tadpole” VEV, \( \sim v \mu^2/M_m^2 \sim 10^{-6} v \), which determines the electron mass \([6]\).

Essentially, the Higgs boson is the threshold black hole, pulled into the far infrared by the fermion loops and the fine-tuning condition. The black hole is only present at extremely short distances and is in effect virtual. The Higgs wave-function is mainly virtual fermions and gauge fields at large distances, triggered by the binding due to the virtual black hole at the Planck scale.

**B. Quarks**

We assume that the incident flavors are a pair consisting of the top quark doublet \( T^c_L = (t, b)_L \) with \( [I_3 = (1/2, -1/2), Y = 1/3] \) and right-handed singlet \( t_R \sim [0, -4/3] \) where \( i, j \) are color indices.

The Lagrangian is:

\[
DH^\dagger_i D_i^j \equiv \frac{1}{2} \frac{D}{\partial^2} H^a_i D_i^a - g \left( \bar{T}_L^i t_R H_i^a + \text{hc} \right). \tag{23}
\]

Therefore the produced HBH black hole, \( H_{ij} \), will be a weak isodoublet will have \( H_{ij} \sim [I_3 = (1/2, -1/2, -1), 0] \), and its electric charge will be \( Q = [0, -1] \), identical to the SM Higgs doublet. However, it now carries mixed color indices \( i, j \) that we wish to project onto \( SU(3) \) representations.

Define:

\[
H_i^j = H^a_i \frac{\lambda^a_{ij}}{2} + \frac{1}{\sqrt{N_c}} H_\delta^i_j \tag{24}
\]

where \( N_c = 3 \), and we use \( \text{Tr} \left( \frac{\lambda^a_3 \lambda^b_3}{2} \right) = \frac{1}{2} \delta^{ab} \) and \( \left( \frac{\lambda^a_3}{2} \right)^\dagger = \frac{\lambda^a_3}{2} \). The terms in the action become:

\[
DH^\dagger_i D_i^j = \frac{1}{2} D H^a D^a + DH^\dagger D \tag{24}
\]

\[
M_m^2 H^\dagger_i H_i^j = \frac{1}{2} M_m^2 H^a H^a + M_m^2 H^\dagger H \tag{25}
\]

\[
gT_L^i t_R H_i^a = gT_L^i \frac{\lambda^a_3}{2} t_R H^a + gT_L t_R H \tag{25}
\]

and where \( T_L t_R = T_L^i H t_R H_i^a \) and we have:

\[
\frac{g}{\sqrt{N_c}} = g'. \tag{26}
\]

The decay width is now:

\[
\Gamma = \frac{N_c g^2}{8 \pi} M_0 = \frac{g^2}{8 \pi} M_0. \tag{27}
\]

The cross-section is:

\[
\sigma = \frac{g^2}{2 M_0 \Gamma} = \frac{g'^2}{2 M_0 \Gamma N_c}. \tag{28}
\]

per color pair and \( g'^2/2 M_0 \) color averaged. The loop correction to the Higgs mass is as before,

\[
M_m^2 = M_0^2 - \frac{N_c g^2}{8 \pi^2} (\Delta^2 - m^2) \tag{29}
\]

and the critical coupling is \( g_c^2 = \frac{\pi^2}{N_c} = \frac{g'^2}{N_c} \). Hence \( 0 = M_0^2 - N_c g_c^2 \Delta^2 / 8 \pi^2 \). Likewise, we have the induced wave-function renormalization constant and the quartic coupling:

\[
Z = \frac{N_c g^2}{16 \pi^2} \ln \left( \frac{\Lambda^2}{m^2} \right), \quad \lambda = \frac{N_c g'^2}{8 \pi^2} \ln \left( \frac{\Lambda^2}{m^2} \right). \tag{30}
\]

The renormalized parameters are:

\[
\bar{g}' \sim \frac{g'}{\sqrt{Z}}, \quad \bar{\chi} = \frac{\lambda}{Z^2}, \quad \bar{M}_m^2 = \frac{M_m^2}{Z}. \tag{31}
\]

Note the quartic coupling receives a loop factor of \( N_c \), not \( N_c^2 \). Hence the renormalized quartic coupling will be \( \sim 1/N_c \) relative to the lepton case. This preserves the UV relation \( g'^2 \sim \bar{\chi}/2 \).

**III. SCALAR DEMOCRACY**

**A. Counting Higgs Black Holes**

We can count the number of composite scalars produced by threshold RN-black holes. The SM fermionic fields consist of 48 two-component left-handed spinors, \( \psi_i \), including all left-handed and anti-right-handed fermions. \( SU(48) \times U(1) \) is then an approximate dynamical symmetry (neglecting gauge interactions).
The most general non-derivative (s-wave) scalar-field bilinears coupled to RN-black holes takes the form:

\[ g e^{AB} \psi^i_A \psi^j_B \lambda_{ij} + \text{h.c.}, \]  

where \( \lambda_{ij} \) transforms as the symmetric 1176 representation of SU(48). The field \( \lambda_{ij} \) contains many complex scalar fields with assorted quantum numbers, including baryon and lepton number, color, and weak charges. This describes all fermion pair collisions in the SM that can produce a black hole.

The 48 consists of the 24 left-handed quarks and leptons, \( \Psi_{L,i} \), and 24 right-handed counterparts, \( \Psi_{R,i} \). The index \( i \) now runs over the chiral SU(24) \( L \) and \( R \) over the chiral SU(24) \( R \) subgroups of SU(48). We thus have:

\[ \Phi_{ij} \Psi_L^I \Psi_R^J \Psi_{LC} + \Omega_{ij} \Psi_L^I \Psi_R^{JC} + \tilde{\Omega}_{ij} \Psi_R^I \Psi_{LC} + \text{h.c.}, \]  

where \( \Phi_{ij} \) is the (24, 24) complex scalar field with 24\(^2\) = 576 complex degrees of freedom. \( \Omega \) and \( \tilde{\Omega} \) are the symmetric 300 representations of SU(24) \( L \) and SU(24) \( R \) respectively, matching the degrees of freedom of \( \lambda_{ij} \).

Here \( \Omega_{ij} \) and \( \tilde{\Omega}_{ij} \) are the analogues of Majorana masses and carry fermion number, while \( \Phi \) contains fermion number neutral fields, such as Higgs fields, in addition to \( (B-L) \) leptoquark multiplets and colored Higgs doublets.

The resulting spectrum of composite states in the \( \Phi^{ij} \) system becomes:

- \( 18 \times (1, 2, \frac{1}{2}) \sim \overline{Q}_L(U_R, D_R) \); Higgs doublets in quark sector = \( 2 \times 3^2 \times 1 \times 2 = 36 \) dof’s),
- \( 18 \times (1, 2, -\frac{1}{2}) \sim \overline{I}_L(N_R, E_R) \); Higgs doublets in lepton sector = \( 2 \times 3^2 \times 1 \times 2 = 36 \) dof’s),
- \( 9 \times (8, 2, \pm \frac{1}{2}) \sim \overline{Q}_L^I \lambda^I(U_R, D_R) \); color octet, isodoublets, \( 3^2 \times 8 \times 2 \times 2 = 288 \) complex DoFs,
- \( 9 \times (3, 2, \pm \frac{1}{6}[-\frac{5}{6}]) \sim \overline{T}_L(U_R, D_R) \); color triplet, isodoublets, \( 3^2 \times 3 \times 2 \times 2 = 108 \) DoFs,
- \( 9 \times (3, 2, \pm \frac{1}{6}[-\frac{5}{6}]) \sim \overline{Q}_L(N_R, E_R) \); color triplet, isodoublets, \( 3^2 \times 3 \times 2 \times 2 = 108 \) DoFs,

where the brackets denote the SM quantum numbers. The first two entries in the above list are the 36 Higgs doublets, 18 in the quark and 18 in the lepton sectors respectively.

The key feature is that these bound-states will have a universal Higgs-Yukawa coupling \( g \) at the scale \( M_p \). For the picture we have just outlined to work, \( g \) must be sub-critical. Otherwise, with a supercritical coupling, \( \Phi^{ij} \) will condense with a diagonal VEV, \( \langle \Phi^{ij} \rangle = V \delta^{ij} \) and all the fermions would acquire large, diagonal constituent masses of order \( g V \), grossly inconsistent with observation.

We assume that \( \Omega_{ij} \), \( \tilde{\Omega}_{ij} \) and all color-carrying weak doublets have very large positive \( M^2 \) and therefore we will ignore them. They will be inactive in the RG evolution (though they may be welcome when gauge unification is included). With \( g \) taking on a nearly-but-sub-critical value for the color singlets, the Higgs bound-states will generally have positive masses that can be much lighter than \( M_p \). The colored states are presumably more massive owing to the gluon field in the RN solution (this is a long story we’ll not enter into presently).

Small explicit masses are introduced by hand as scale symmetry-breaking effects, required to split the spectroscopy in the infrared and accommodate phenomenology.

In scalar democracy the flavor physics and fermion mass hierarchy problems are flipped out of \( d = 4 \) Higgs-Yukawa (HY) coupling textures and into the structure of the the mass matrix of the many Higgs fields. We have no theory of the small input masses at present, but we can choose these to fit the observed quark and lepton sector masses and CKM physics, as well as maintain consistency with constraints from rare weak decays, etc. It is not obvious a priori that there exists a consistent solution with the flavor constraints, however, it does work [6]. Many of these mass terms are technically natural, protected by the SU(48) symmetry structure which can be seen in a subset model in [7]. The critical theory will thus contain many light composite Higgs doublets with a spectrum of positive \( M^2 \)’s that extends from \( \sim 10^2 \) GeV up to \( \sim 10^6 \) TeV.

We refer the reader to [6–8] for more of the phenomenology of this “scalar democracy,” including production and detection at the LHC and upgrades.

**B. RG Solution**

The induced couplings \( \lambda, \lambda', \lambda, \lambda' \) satisfy the RG for the logarithmic running below the Planck scale (we will omit the overline in the following). The boundary conditions are determined by the binding dynamics at the Planck scale:

\[ (g, g', \lambda, \lambda') \rightarrow \infty \]  

Presently we will only sketch very roughly the results for the \( g^2(m) \) and \( g'^2(m) \) RG evolution and leave a more detailed study including the quartic couplings to [19].

At a first glance, note that the HY coupling of the top quark in the SM would be driven to the infrared-quasifixed point of Pendleton-Ross [17] and Hill [18]. The skeletal RG equation for the top quark HY coupling in the SM, \( g_t \), is:

\[ D g_t = g_t \left( N_c + \frac{3}{2} \right) g_t^2 - (N_c^2 - 1) g_t^2 \]  

where \( D = 16 \pi^2 \partial / \partial \ln(m) \), and \( m \) is the running mass scale, \( g_3 \) the QCD coupling. For illustrative purposes we discuss the one-loop RG equation and suppress electroweak corrections, though they are included the figure results.
Starting the running of \(g_t(m)\) at very large mass scales, \(m = M_X\), with large initial values, i.e., \(g_t(M_X) \gg 1\) (effectively a Landau pole at \(M_X\)), it is seen that \(g_t(m)\) flows into an “infrared quasi-fixed point.” This is “quasi” in the sense that, if the QCD coupling, \(g_3\), was a constant then \(g_t\) would flow to an exact conformal fixed point.

The low energy prediction of the top quark HY coupling is very insensitive to its precise, large initial values and mass scales. Starting at \(M_X = M_P\) the result comes in about 16% higher than experiment. This is shown in Fig. (1) where the effective top mass \(m_{top} = g_t(m) v\) (where \(v = 175\) GeV is the electroweak scale) is plotted vs. renormalization scale \(m\); the physical top mass corresponds to \(m \sim v\), or ln(\(m\)) \sim 5.

However, we now expect 18 Higgs doublets in the quark sector, and each doublet coupled to a particular color singlet pair, \(\bar{\psi}_L \psi_R\), where \(a\) counts the 3 LH flavor doublets and \(b\) the 6 RH singlets. Likewise, we have 18 doublets in the lepton sector. The key feature of gravitational binding of these composite Higgs bosons is that the theory has one universal HY coupling \(g'\) in the quark sector, and \(g\) in the lepton sector, defined at the Planck scale by eq. (34). For the quark sector, \(g'\) is determined by the top quark HY coupling at low energies, \(g'(m_t) \sim 1\). This will be different than the SM prediction of Fig. (1) owing to the presence of the 17 other doublets (as we see below). Likewise, the leptons will couple with strength \(g\).

These two subsectors resemble an \(SU(6)_L \times SU(6)_R\) linear \(\Sigma\)-model Lagrangian, where the interaction is sub-critical and ultimately only the SM Higgs condenses. We have only observed the lightest Higgs boson doublet thus far; the remaining doublets are massive but mix with the SM Higgs and thus give power-law suppressed HY couplings to the SM Higgs hence power-law suppressed masses and mixings to the light fermions. The theory is predictive and the sequential massive Higgs, \(H_b\), will couple to \(g' (t, \bar{b})_L H_b \bar{b}_R\) (see below).

In the quark and lepton sectors, each containing 18 doublets, the RG equation for the universal HY couplings take the one-loop form \([6][19]\):

\[
D g' = \left(3 + N_f\right) g'^2 - (N_c^2 - 1) g_4^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2
\]

\[
D g = \left(\frac{5}{2} + N_f\right) g^2 - \frac{9}{4} g_2^2 - \frac{15}{4} g_1^2
\]

where \(N_f = 6\) and \(N_c = 3\). Note the enhanced coefficient of \(g_1^2\) in eq. (36) relative to eq. (34). All other quarks and leptons will couple through power-law suppressed mixing effects via their own Higgds, and receive smaller masses.

We have a limitation in modeling since we need to know the running of the gauge couplings \(g_i^2\), given the large multiplicity of Higgs fields. This also poses challenges for gauge unification. Preliminary studies indicate that gauge unification is possible, but it is likely to be more complex than the usual picture. Here we simply use the SM values for the \(g_i^2\), while with the large multi-Higgs spectrum we expect larger values.

Naively applying the \(N_f = 6\) with SM \(g_i^2\) it appears in Fig. (2) that the top mass undershoots the experimental result. However, the effect of decoupling of the heavier Higgs bosons causes the \(g_t\) to come up to concordance with the 174 GeV observed value. The observed top mass
is therefore sensitive to the extended Higgs sector (just as the top and W masses were sensitive to the Higgs boson and predicted its discovery mass).

As stated above, in scalar democracy we explain the origin of mass and CKM mixing in the SM in a novel way—all flavor physics is mapped into the masses and mixings of the array of composite Higgs bosons which have universal couplings to their particular constituent fermion bilinears. The lowest eigenmode is the SM Higgs boson, corresponding to $\tilde{t}t$.

It is somewhat easier to grasp the details of the theory by focusing on the $t-b$ subsector as in \cite{7}. Here we predict the first sequential Higgs $H_b$ with the large $g' \sim O(1)$ coupling to $q'T_L H_b b_R$ (with some additional QCD RG flow to the b-quark mass from 5 TeV this coupling is $g' \sim 1.5$). We expect an upper mass bound on $H_b$ of order 5.5 TeV. This state is accessible at the LHC or its upgrade (see \cite{6} and for the third generation predictions, \cite{7})). Above all, we predict the key result that sequential Higgs bosons couple with a common (modulo renormalization group effects) $O(1)$ coupling, as calibrated by the top quark Higgs-Yukawa coupling constant and dictated by the RG infrared quasi-fixed point. The observation of the $H_b$ with $g \sim g_{ttop}$ would offer significant support to this scenario.

\section*{IV. CONCLUSIONS}

We are thus led to a new idea: Higgs bosons are composite bound-states of standard model fermion pairs driven by threshold black holes at $M_P$ with the corresponding quantum numbers. The black holes of the far UV are quantum mechanical, mini black holes that are dressed by fermion loops to acquire lower energy (multi-TeV scale) masses. There are many bound-state Higgs bosons, at least one per fermion pair at $M_{Planck}$, and a rich spectroscopy of Higgs bosons is expected to emerge. This theory dynamically unifies Planck scale physics with the electroweak and multi-TeV scales. By studying Higgs physics at the LHC one may have a window on the threshold spectrum of black holes at the Planck scale.

The production and decay of the these states has been modeled by effective field theory vertices and masses, and leads to a Nambu-Jona-Lasinio effective field theory for the composite Higgs bosons. We estimate of the critical coupling $g_c^2 \sim \pi^2$ (or $g_c^2 \sim \pi^2/3$). In the NJL model there is fine tuning of $g = g_c$, at the same level as occurs in the SM. This can be stated as a scale invariance condition imposed on the composite mass at the Planck scale. We would hope to someday replace the fine-tuning by a dynamical phenomenon, perhaps involving an underlying scale invariance of the full theory.

This scenario provides an underlying dynamics for the recently proposed “scalar democracy" \cite{6,7}, in which every fermion pair in the SM in an s-wave combination is argued to be associated with a gravitationally bound composite Higgs field. A consequence of this hypothesis is that the many resulting Higgs bosons couple universally to matter. This can explain the masses and mixings of fermions, not by textures, but rather via the masses and mixings of the many Higgs doublets.

Here the observed SM Higgs isodoublet is a $H \sim t_R \times (t, b)_L$ composite. The HY coupling of the top quark calibrates the universal coupling of all Higgs bosons, modulo RG effects. The Higgs-Yukawa universality is a critical prediction of the scenario and its gravitational underpinnings. It can be tested by finding at the LHC (upgrade) the first sequential heavy Higgs doublet, the $H_b$ with a mass of $\lesssim 5.5$ TeV, and confirming its HY coupling to $b\bar{b}$ is $O(1)$ \cite{7,8}. We view the search for sequential Higgs doublets with $O(1)$ HY couplings at LHC to be of high importance. These states will occur in any given channel defined by any SM fermion pair owing to the universality of gravity.

This dynamical picture we’ve presented depends upon the properties of mini black holes in the quantum limit. We have followed a simple schematic model of quantum black holes due to Dvali and Gomez, \cite{9,10,11}, which we find compelling, and similar to the Bohr model of the Hydrogen atom. The DG model is a kind of “bag model” of quantum black holes with a fundamental length cut-off. We extend the model to include fermion pairs. Perhaps this is a more general phenomenon, and a similar picture may arise in string theory \cite{20}.

While it would seem that we are introducing many new scalars, in fact, we are reducing the number of fundamental degrees of freedom of the SM by replacing the existing Higgs by a gravitational bound-state. The rich dynamics of gravity is argued to produce the complexity of multiple scalar fields the infrared. Perhaps this can be extended to explain the origin of flavor and, more radically, composite gauge fields. We will return to these questions elsewhere.
Acknowledgments

We thank W. Bardeen, G. G. Ross and R. Wald for discussions and the Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

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