SU(3) MIXING FOR EXCITED MESONS

W.S. Carvalho§*, A.S. de Castro§§† and A.C.B. Antunes§§§‡

§Instituto de Geociências
Universidade Federal do Rio de Janeiro
21949-000 Rio de Janeiro RJ - Brasil

§§ UNESP/Campus de Guaratinguetá - DFQ
C.P. 205
12516-410 Guaratinguetá SP - Brasil

§§§ Instituto de Física
Universidade Federal do Rio de Janeiro
C.P. 68528
21945-970 Rio de Janeiro RJ - Brasil

Abstract

The SU(3)-flavor symmetry breaking and the quark-antiquark annihilation mechanism are taken into account for describing the singlet-octet mixing for several nonets assigned by Particle Data Group(PDG). This task is approached with the mass matrix formalism.

*weuber@acd.ufrj.br
†castro@feg.unesp.br
‡antunes@if.ufrj.br
1 Introduction

In the constituent quark model the mesons are considered as bound states of a quark and an antiquark. Taking into account the SU(3)-flavor symmetry the mesons are either in an SU(3) singlets or octets: $3 \otimes 3 = 1 \oplus 8$. Nonetheless, due to the SU(3)-symmetry breaking the isoscalar physical states appear as mixtures of the singlet and octet members. This singlet-octet mixing is also called SU(3) mixing. The inability of the Gell-Mann-Okubo mass formula \[1\] to adjust the masses of the pseudoscalar mesons has been considered as a suggestion for the inclusion of other effects such as the quark-antiquark annihilation into gluons. The fail of an SU(3)-invariant annihilation amplitude in attempting to solve the $\eta-\eta'$ mass splitting \[2\], \[3\] led De Rujula et al. \[4\] to propose that the quark-antiquark annihilation mechanism might not be SU(3)-invariant.

In a previous paper \[5\] the $\eta-\eta'$ mass splitting was explained in a SU(3)-symmetry breaking framework. The physical states are mixtures of the isoscalar singlet and octet states and the amplitudes of quark-antiquark annihilation into gluons as well as the binding energies are supposed to be flavor dependent. Within this formulation an extended expression for Schwinger's sum rule is satisfied. Also the SU(3) mixing angle obtained, $\theta = -19.51^\circ$, is consistent with the experimental data ($\theta \simeq -20^\circ$) from $\eta$ and $\eta'$ decays into pions \[6\]. As a very natural extension of the previous paper we assume the SU(2)-symmetry breaking in the SU(3) mixing framework \[7\]. In this way the pseudoscalar neutral mesons are mixtures of isoscalar and isovector states and the neutral pion takes part in the mixing scheme. This model works well, but the result gives a hint that some significant effect possibly has not been considered. The strange result is that the ratio $m_s/m_u \simeq 2$ takes a somewhat large value, in comparison with those used in the constituent quark models ($m_s/m_u \simeq 1.3\ldots 1.8$). Our formulation is incompatible with fundamental models. If current quark masses were used the free parameters of the model would not be able to fit the masses of $\eta$ and $\eta'$. In addition, the correct singlet-octet mixing angle would not be obtained.

The $\eta-\eta'$ mixing scheme could be enlarged by the inclusion of gluonic degrees of freedom. The $\iota(1440)$ was interpreted as a strong glueball candidate due to its enhanced production in a gluon-rich channel \[8\]. The $\iota(1440)$, with the same quantum numbers as the $\eta$ and $\eta'$ system, motivated the study of the $\eta-\eta'-\iota$ mixing scheme \[9\]-\[13\]. Recently, the mass region near to $\iota(1440)$ has been resolved into two states $\eta''(1410)$ and $\eta(1490)$ \[14\]. The first one has
been interpreted as being mainly a glueball mixed with $q\bar{q}$ and the second one as mainly a $s\bar{s}$ radially excited state [13], [16]. Therefore one is tempted to identify $\eta''(1410)$ as the remaining physical state in this extended mixing scheme [13]–[17] for ground states. On the other hand, the state $\eta(1490)$ is interpreted as a partner of the radially excited state $\eta(1295)$ [16]. The states $\eta(1295)$ and $\eta(1490)$ are the physical manifestations of mixtures among 2S excited states including solely light and strange quarks [15]. In a recent paper [18] we describe the $\eta$-$\eta'$-$\eta''$ and $\eta(1295)$-$\eta(1490)$ systems with the same formalism used in Ref. [5] but enlarging the mixing scheme to include glueballs. The small overlapping of the respective mass intervals suggests the possibility of mixing among ground states and radial excitations as considered by [19], however, in a first approximation, we assume that this 1S-2S mixing may be neglected. In searching for the best results of the branching ratios and of the decay widths involving the $\eta$, $\eta'$ and $\eta''$ mesons we have fixed all the parameters of the problem. This enlarged mixing scheme furnishes satisfactory results for the experimental data and improves the high value for the ratio $m_s/m_u$ obtained in Ref. [18]. We obtained $m_s/m_u = 1.772$. Finally we extend the mixing scheme to the excited states using the value of $m_s/m_u$ determined for the ground state.

The nonet of axial (1$^{++}$, 1$^3P_1$) and tensor (2$^{++}$, 1$^3P_2$) mesons are well established [20]. The axial nonet consists of the isodoublet $K_{1A}(1340)$, the isovector $a_1(1260)$ and the isoscalars $f_1(1285)$ and $f_1(1510)$. The $K_{1A}$ is a mixture of $K_1(1270)$ and $K_1(1470)$ with a close to 45° mixing angle [21]. The tensor nonet is formed by the isodoublet $K^*_2(1430)$, the isovector $a_2(1320)$ and the isoscalars $f_2(1270)$ and $f_2'(1525)$. Nonetheless, there are extra isoscalar states with quantum numbers and masses permitting that they can be interpreted as partners of the nonets of axial and tensor mesons. The axial state $f_1(1420)$, observed in two experiments [22], has been considered by some authors [23] as a possible candidate to exotic. On the other side, there are two candidate to exotic tensor states: $f_2(1640)$ [24] and $f_J(1710)$ [25]. There is a controversy about the value of the spin of the $f_J(1710)$: it may be a scalar or a tensor state [26]. In other paper [27] we approached the problem of axial and tensor mesons where the candidates to exotics $f_1(1420)$ and $f_2(1640)$, or $f_2(1710)$, are supposed to be components of a quarkonia-gluonia mixing scheme similar to that previously applied to the pseudoscalar mesons [3]. In this last paper $m_s/m_u = 1.772$ determined in Ref. [18] has been used as an input. The predictions of the model for branching ratios and electromagnetic decays are incompatible with the experimental results. These facts
suggest the absence of gluonic components in the axial and tensor isosinglet mesons analyzed. On the other hand, the interpretations of the states $f_1(1420)$, $f_1(1510)$, $f_2(1640)$ and $f_J(1710)$ are controversial and, moreover, some of them need confirmation. The same mixing scheme was not applied to the scalar states because only the assignment for the scalar isodoublet is well-established.

Here we analyze the mixing scheme for the nonets listed in Table 13.2 of Particle Data Group (PDG) [28] which have all the members suggested, including the scalar states and excepting the lowest pseudoscalar states ($\pi$, $K$, $\eta$, $\eta'$). For all intents and purposes we ignore any quarkonia-gluonia interference. We also assume the SU(2) invariance which is justified by a preceding work [7] in which we have shown that the SU(2)-symmetry breaking is important to the mass splitting between the $\pi^0$ and $\pi^\pm$, but it has negligible effects in the $\eta$-$\eta'$ mixing. We will suppose that the isospin symmetry breaking causes no mixing between the isoscalar members of the excited nonets.

2 The mass matrix formalism

Several kinds of the mixing schemes has been proposed to give account of the peculiar properties of the isoscalar mesons. In some schemes the physical states are written as linear combinations of pure quarkonia and gluonia states. The linear coefficients are generally related to the rotation angles and may be determined by the decay properties of, or into, the physical mesons [12], [13], [14], [17], [29], [30]. Another approach, in which the interference is considered at a more fundamental level, consists in writing a mass matrix for the physical states in the basis of the pure quarkonia and gluonia states. The elements of this mass matrix are obtained from a model that describes the process of interference. The mixtures of the basic states are induced by the off-diagonal elements. Thus, these elements must contain the amplitudes for transitions from one to another states of the basis. The eigenvalues of that matrix give the masses of the physical states and the corresponding eigenvectors give the proportion of quarkonia and gluonia in each meson [10], [15], [31].

In Ref. [5], [7], [18], [27] we have adopted a mixing scheme based on a mass matrix approach. The flavor-dependent annihilation amplitudes and binding energies are the responsible mechanisms for the quarkonia-gluonia mixing. Here a brief review of the mass matrix formalism we have used in previous papers is outlined only for the quarkonia mixing. The mass matrix
in the basis $|u\bar{u}>$, $|d\bar{d}>$ and $|s\bar{s}>$, including flavor-dependent binding
energies and annihilation amplitudes, has matrix elements given by

$$\mathcal{M}_{ij} = (2m_i + E_{ij})\delta_{ij} + A_{ij}$$

(1)

where $i,j = u,d,s$. The contribution to the elements of the mass matrix are:
the rest masses of the quarks $m_i$, the eigenvalues $E_{ij}$ of the Hamiltonian for
the stationary bound state $(ij)$ and the amplitudes $A_{ij}$, that account for the
possibility of quarkonia-gluonia transitions. As in the previous papers we
assume that $E_{ij}$ and $A_{ij}$ are not SU(3)-invariant quantities. Another basis
also used consists of the isoscalar singlet and octet of the SU(3)

$$|1> = \frac{1}{\sqrt{3}}(\sqrt{2}|N> + |S>)$$

(2)

$$|8> = \frac{1}{\sqrt{6}}(\sqrt{2}|N> - 2|S>)$$

(3)

where this basis is written in a form that presents a segregation of strange
and nonstrange quarks,

$$|N> = \frac{1}{\sqrt{2}}(|u\bar{u}> + |d\bar{d}>)$$

(4)

$$|S> = |s\bar{s}>$$

(5)

Besides these states we need also the isovector states

$$|\tilde{\pi}^0> = \frac{1}{\sqrt{2}}(|u\bar{u}> - |d\bar{d}>)$$

(6)

In this basis the mixing among the isoscalar and isovector states is caused
by isospin symmetry breaking terms. Therefore, assuming the exact SU(2)-
flavor symmetry, one needs only consider the subspace spanned by the isoscalar
states when the mass matrix reduces to a 2x2 matrix $\mathcal{M}_0$:

$\mathcal{M}_0 = \begin{pmatrix}
    m_8 & m_{18} \\
    m_{18} & m_1
\end{pmatrix}$

(7)

where
\[ m_1 = \frac{2}{3} (2m_u + m_s) + \frac{1}{3} (2E_{uu} + E_{ss}) + A_{11} \]  \hspace{1cm} (8)

\[ m_8 = \frac{2}{3} (m_u + 2m_s) + \frac{1}{3} (E_{uu} + 2E_{ss}) + A_{88} \]  \hspace{1cm} (9)

\[ m_{18} = \frac{2\sqrt{2}}{3} (m_u - m_s) + \frac{\sqrt{2}}{3} (E_{uu} - E_{ss}) + A_{18} \]  \hspace{1cm} (10)

and

\[ A_{88} = \frac{2}{3} (A_{uu} - 2A_{us} + A_{ss}) \]  \hspace{1cm} (11)

\[ A_{11} = \frac{1}{3} (4A_{uu} + 4A_{us} + A_{ss}) \]  \hspace{1cm} (12)

\[ A_{18} = \frac{\sqrt{2}}{3} (2A_{uu} - A_{us} - A_{ss}) \]  \hspace{1cm} (13)

Using the mass relations for the isovector and isodoublet members,

\[ M_1 = 2m_u + E_{uu} \]  \hspace{1cm} (14)

\[ M_{1/2} = m_u + m_s + E_{us} \]  \hspace{1cm} (15)

where the annihilation effects are absent, only the rest masses of the quarks and the binding energies contribute to the physical masses. The notation uses subscripts in \( M \) to identify the isospin. Defining

\[ M_{1/2}^{(\varepsilon)} = M_{1/2} + \varepsilon \]  \hspace{1cm} (16)

where

\[ \varepsilon = \frac{E_{uu} + E_{ss}}{2} - E_{us} \]  \hspace{1cm} (17)

the elements of the mass matrix \( \mathcal{M}_0 \) are found to be
The above results show that the SU(3)-symmetry breaking gives rise to off-diagonal elements in the mass matrix. These elements are generated not only by the gluon annihilation amplitudes but also by influences due to the differences in the binding energies. These off-diagonal elements are responsible for the mixing effects among the states composing the physical mesons. We adopt an expression for the amplitude of the process $q\bar{q} \leftrightarrow gg \leftrightarrow q'\bar{q}'$ similar to that of Cohen et al. [32] and Isgur [33], where the numerator of the two-gluon annihilation amplitude expression is assumed to be a SU(3)-invariant parameter, which means that we parameterize the annihilation amplitude in the form

$$A_{qq'} = \frac{\Lambda}{m_q m_{q'}}$$

The phenomenological parameter $\Lambda$ is to be determined. Then, the amplitudes become

$$A_{11} = \frac{1}{2}(2 + r_1)^2 r_2$$  \hspace{1cm} (22)

$$A_{88} = \frac{2}{3}(1 - r_1)^2 r_2$$  \hspace{1cm} (23)

$$A_{18} = \frac{\sqrt{2}}{3} (2 + r_1)(1 - r_1) r_2$$  \hspace{1cm} (24)

where

$$\frac{1}{r_1} = \frac{m_s}{m_u}$$  \hspace{1cm} (25)
\[ r_2 = \frac{\Lambda}{m_u^2} \]  

The invariants of the mass matrix \( M_0 \) under a unitary transformation give the following mass relations for the isoscalar physical states:

\[ M + \tilde{M} = \text{tr}(M_0) \]  
\[ M \times \tilde{M} = \text{det}(M_0) \]  

where \( M \) and \( \tilde{M} \) are the eigenvalues of the mass matrix \( M_0 \) (masses of the isoscalar physical states). Their corresponding eigenvectors are the physical states \( |M> \) and \( |\tilde{M}> \) which are mixtures of \( |1> \) and \( |8> \):

\[ |M> = \cos(\theta) |8> - \sin(\theta) |1> \]  
\[ |\tilde{M}> = \sin(\theta) |8> + \cos(\theta) |1> \]  

where the coefficients of the eigenvectors are written in terms of the singlet-octet mixing angle given by

\[ \theta = \arctan \left( \frac{m_8 - M}{m_{18}} \right) \]  

In terms of strange and nonstrange quarks (29)-(30) can be written as

\[ |M> = X|N> + \tilde{Y}|S> \]  
\[ |\tilde{M}> = \tilde{X}|N> + Y|S> \]  

where

\[ X = Y = \frac{\cos(\theta) - \sqrt{2}\sin(\theta)}{\sqrt{3}}, \quad Y = -\tilde{X} = -\frac{\sqrt{2}\cos(\theta) + \sin(\theta)}{\sqrt{3}} \]
Eliminating $A_{11}$ from (27) and (28) we obtain the generalized Schwinger sum rule:

$$(M + \tilde{M})(4M_{1/2}^{(e)} - M_1) - 3M\tilde{M} = 4 \left[ 2M_{1/2}^{(e)} - (1 - \epsilon_1^2)\epsilon_1 \right] (M_{1/2}^{(e)} - M_1) + 3M_1^2$$

(35)

To our knowledge this generalized sum rule was obtained for the first time in Ref. [5]. Note that the ordinary Schwinger sum rule [2] can be recovered doing $r_1 = 1$ in (35). Equations (27) and (28) can also be solved for $r_1$ and $r_2$ giving

$$\frac{m_s}{m_u} = \frac{\sqrt{2}}{2} \sqrt{\frac{(M - M_1)(\tilde{M} - M_1)}{\tilde{M} + M_1 - 2M_{1/2}^{(e)}} \left( \frac{2M_{1/2}^{(e)} - M - M_1}{2M_{1/2}^{(e)} - M_1} \right)}$$

(36)

$$\frac{\Lambda}{m_u^2} = \frac{(\tilde{M} - M_1)(M - M_1)}{4 \left( M_{1/2}^{(e)} - M_1 \right)}$$

(37)

The invariants of the mass matrix are functions of $m_s/m_u$, $\Lambda/m_u^2$ and $\epsilon$. These quantities are not all free. The equations (27) and (28) impose some constraints among them. The equations are to be solved for $\Lambda/m_u^2$ and $\epsilon$ by considering $m_s/m_u$ in a range of values consistent with those usually adopted when using constituent quark masses in nonrelativistic quark model ($m_s/m_u = 1.3 \ldots 1.8$). For finding the solutions one needs to solve a second degree algebraic equation. One of those solutions is an extraneous root and the criterion to get rid of it is the comparison with the solution obtained for the SU(3) mixing angle (31) in the case of SU(3)-invariant amplitudes and binding energies. Our choice consists in the mixing angle nearest to that SU(3)-invariant mixing angle.

3 Mixing in excited states

The mixing scheme briefly presented in the preceding section, ignoring any quarkonia-ghuonia mixing, is now applied to the excited members of the nonets. The attention will be paid to the referred assignments in the Table 13.2 of the PDG [28], even for the cases which are controversial. These results, corresponding to the range $m_s/m_u = 1.3 \ldots 1.8$, are summarized in Table 1.
3.1 \( 1^1S_0 \ (0^{-+}) \)

The ground-state pseudoscalar nonet \((\pi, K, \eta, \eta')\) has already been considered in Ref. \cite{18}, where an enlarged mixing scheme including gluonia has been shown to be necessary. Putting to test the present mixing scheme for this nonet without gluonic degrees of freedom ends in a complete fiasco in the range of \(m_s/m_u\) considered.

3.2 \( 1^3S_1 \ (1^{--}) \)

The ground-state vector nonet \((\rho, K^*(892), \omega, \phi)\) is well established since a long time ago. It presents a SU(3) mixing angle near to ideal \(\omega - \phi\). It can be found that \(\phi\) presents \(99.9\% \ldots 100\%\) of strange quarks and mixing angles in the range \(36.9^\circ \ldots 36.4^\circ\). These values are to be compared with this one listed by PDG \((\theta = 36^\circ)\).

3.3 \( 1^1P_1 \ (1^{++}) \)

We found that the content of strange quarks in \(h_1(1380)\) is much higher than in its isoscalar partner. This result is supported by the experimental data which show \(h_1(1380) \to K K^*(892) + c.c.\) and \(h_1(1170) \to \rho\pi\) being the only ones decay modes seen, at least up to now.

3.4 \( 1^3P_0 \ (0^{++}) \)

For this nonet we found that \(f_0(1370)\) presents \(89.7\% \pm 18.5\% \ldots 94.1\% \pm 19.9\%\) of strange quarks and \(\theta = -73.4^\circ \pm 13.8^\circ \ldots -68.8^\circ \pm 10.2^\circ\). These values were found taking into account that the broad resonance \(f_0(1370)\) has mass equal to \((1.35 \pm 0.15)\) GeV. It is worthwhile to remark that among the two candidates for the \(I = 1\) \((a_0(980), a_0(1450))\) states and the four ones for \(I = 0\) \((f_0(400 - 1200), f_0(980), f_0(1370), f_0(1710))\) acceptable results were found only for the isovector \(a_0(1450)\) and for the isoscalars \(f_0(1370)\) and \(f_0(1710)\), namely the states listed in Table 13.2 of PDG. It should be highlighted, though, that \(f_0(1710)\) contains only a small fraction of strange quarks in contrast to the indication of the PDG based on the naive quark model. In addition, it is observed that \(f_0(1710)\) has a dominant \(K\overline{K}\) decay mode and \(f_0(1370)\) couples more strongly to \(\pi\pi\) than to \(K\overline{K}\).
3.5 $1^3P_1$ $(1^{++})$

The $f_1(1420)$ competes for a $s\bar{s}$ assignment with percentages of 94.6\% . . . 97.7\% and mixing angles in the range $-41.3^\circ . . . -45.9^\circ$ roughly agreement with 75\% . . . 84\% and $\theta \sim -40^\circ$ obtained by Close et al. [30]. More recently Li et al. [34] obtained 92\% of $s\bar{s}$ in $f_1(1420)$ and $\theta = -38.5^\circ$. As a matter of fact, they obtained $\sim 50^\circ$ and 51.5$^\circ$, respectively, because they changed $|M >$ by $|\tilde{M} >$, and vice versa, in (29)-(30). The ratio of $J/\psi$ radiative branching ratios into $f_1(1285)$ and $f_1(1420)$ and the ratio of the two-photon width of $f_1(1285)$ and $f_1(1420)$ are, using the formulas in Ref. [40], given by:

$$\frac{\Gamma_{\gamma\gamma}(\tilde{f}_1)}{\Gamma_{\gamma\gamma}(f_1)} = \left(\frac{5\tilde{X} + \sqrt{2}\tilde{Y}}{5X + \sqrt{2}Y}\right)^2 \left(\frac{\tilde{M}}{M}\right)^3$$ (38)

$$\frac{\Gamma_{\gamma\gamma}(\tilde{f}_1)}{\Gamma_{\gamma\gamma}(f_1)} = \left(\frac{5\tilde{X} + \sqrt{2}\tilde{Y}}{5X + \sqrt{2}Y}\right)^2 \left(\frac{\tilde{M}}{M}\right)^3$$ (39)

$$\frac{B(J/\psi \rightarrow \gamma \tilde{f}_1)}{B(J/\psi \rightarrow \gamma f_1)} = \left(\frac{\sqrt{2}\tilde{X} + \tilde{Y}}{\sqrt{2}X + Y}\right)^2 \left(\frac{\tilde{P}}{P}\right)^3$$ (40)

$$\frac{B(f_1 \rightarrow \gamma \phi)}{B(f_1 \rightarrow \gamma \rho)} = \frac{4}{9} \left(\frac{P_\phi}{P_\rho}\right)^3 \left(\frac{X}{Y}\right)^2$$ (41)

where $f_1$ and $\tilde{f}_1$ stand for $f_1(1285)$ and $f_1(1420)$, respectively. Our results are summarized in Table 2. In the table one can see that the ratio of (38) and (40) and (39) and (40) yield 0.39 . . . 0.36. On the experimental side these ratios yield 1.03 $\pm$ 0.92 (an inferior limit) and 0.46 $\pm$ 0.40, respectively.

3.6 $1^3P_2$ $(2^{++})$

For this nonet we found mixing angles in the range 30.1$^\circ . . . 31.5^\circ$ which are to be compared with the value 26$^\circ$ presented by PDG and 27.5$^\circ$ found by Li et al. [35]. The ratio of branching ratios, where $f_2$ and $\tilde{f}_2$ stand for $f'_2(1525)$ and $f_2(1270)$, respectively, are given by

$$\frac{B(f_2 \rightarrow \pi\pi)}{B(f_2 \rightarrow K\bar{K})} = \frac{3X^2}{(\sqrt{2}Y + X)^2} \left(\frac{P_\pi}{P_K}\right)^5$$ (42)
\[
\frac{B(f_2 \to K\bar{K})}{B(f_2 \to \pi\pi)} = \frac{\left(\sqrt{2}Y + \bar{X}\right)^2}{3X^2} \frac{P_{\pi}}{P_K}\tag{43}
\]

\[
\frac{B(J/\psi \to \gamma f_2)}{B(J/\psi \to \gamma \bar{f}_2)} = \left(\frac{\sqrt{2}X + Y}{\sqrt{2}X + \bar{Y}}\right)^2 \frac{P}{\bar{P}}\tag{44}
\]

Our results and their comparison with the experimental data for this nonet are summarized in Table 3.

\section*{3.7 \(1^1D_2\) \((2^{-+})\)}

We obtained values consistent with a near to ideal \(\eta_2(1645) - \eta_2(1870)\) mixing and the second isoscalar being dominantly composed of \(s\bar{s}\) as speculated by PDG, although there are some expectations that it may be an hybrid \cite{36}, \cite{37}.

\section*{3.8 \(1^3D_3\) \((3^{--})\)}

For this nonet we found mixing angles in the range \(31.4^\circ \ldots 32.4^\circ\) which are to be compared with the value \(28^\circ\) presented by PDG.

\section*{3.9 \(1^3F_4\) \((4^{++})\)}

We found that \(f_4(2220)\) is mainly a \(s\bar{s}\) state. This result agrees with the suggestion of PDG and had already been conjectured by Godfrey \textit{et al.} \cite{38} and Blundell \textit{et al.} \cite{39}.

\section*{3.10 \(2^1S_0\) \((0^{-+})\)}

For the first radial excitation of the pseudoscalar nonet we found that \(\eta(1440)\) and \(\eta(1295)\) presents almost an ideal mixing with the first isoscalar being a \(s\bar{s}\) state. Nevertheless, the \(\eta(1440)\) is now considered to be composed of two resonances: \(\eta(1410)\) and \(\eta(1490)\) \cite{14}. The first one has been interpreted as being mostly a glueball mixed with \(q\bar{q}\) and the second one as mostly a \(s\bar{s}\) radially excited state \cite{15},\cite{16}. The \(\eta(1410)\) has been identified as the remaining physical state in the quarkonia-gluonia mixing scheme for the pseudoscalar ground states \cite{15}-\cite{18}. On the other hand, the state \(\eta(1490)\) is interpreted
as a partner of the radially excited state $\eta(1295)$ \cite{15}, \cite{13}, \cite{18}. With this point of view we found that $\eta(1490)$ is a $\sim 100%$ $s\bar{s}$ state and the mixing angle is in the range $-55.4^o \ldots -55.2^o$.

3.11 $2^3 S_1 (1-\bar{-})$

The PDG proposes the $\rho(1450)$ to be the isovector partner for this nonet, however we were unable to find consistent results even for the candidate $\rho(1700)$. On the other hand, the state $\rho(1300)$ reported by the LASS detector team \cite{41}, without any entry in the PDG tables, leads to results almost satisfactory. We found that $\phi(1680)$ has a sizeable $s\bar{s}$ component (89.7%...96.1%), but is the $\omega(1420)$ which is mostly octet. This last result is in accord to the experimental data which show that $\phi(1680) \rightarrow K\bar{K}^*(892) + c.c.$ is the dominant decay for $\phi(1680)$ and besides $\omega(1420)$ has no decay to $K\bar{K}$. It is worthwhile to note that is the isoscalar $\omega(1420)$ which is mostly octet instead of the $\phi(1680)$ state. The PDG suggests that the isodoublet $K^*(1410)$ could be replaced by the $K^*(1680)$ in this nonet. Unfortunately, with this replacement we are led to unsatisfactory results for all the $\rho$ candidates.

4 Conclusion

In this paper we have shown that a mixing flavor approach similar to that used to describe the isosinglet states of the pseudoscalar meson nonet \cite{2} can also be used also to describe isosinglet states for several angular momentum and radially excited nonets. In this approach we assumed SU(2) invariance. Moreover, we assumed that the constituent masses of the quarks, the binding energies of the states and the gluon annihilation amplitudes are not SU(3)-invariant quantities. The gluon annihilation amplitudes were parameterized according to the prescriptions of Cohen et al. \cite{32} and Isgur \cite{33}. In addition to these assumptions we disregarded the presence of gluonic components in the physical states. A linear $2\times2$ matrix formulation based in these assumptions was applied to seven orbitally excited nonets and two radially excited S-wave nonets.

The mixing scheme used in this paper works properly for the majority of the isoscalar states listed in the Table 13.2 of the PDG \cite{28}. Ten nonets were analyzed and eight of them appear to be compatible with the experi-
mental predictions for their quark-antiquark contents, branching ratios and radiative decays. Only in two cases our results mismatch the experimental data. In these two cases the isoscalar states are not well established. In the scalar sector there are many resonances competing to be the isoscalar partners of this nonet. The mixing scheme only works using $a_0(1450)$, $f_0(1370)$ and $f_0(1710)$, the states listed in Table 13.2 of PDG, nevertheless we found unsatisfactory results. The current status of the scalar nonet exclude any possibility to achieve a reliable conclusion. For the $2^3S_1$ sector a consistent result was reached using the $\rho(1300)$, contrasting with the candidates listed by the PDG ($\rho(1450)$ and $\rho(1700)$). This point might be considered as a fail of our mixing scheme but the existence of two $\rho$ states and maybe a third one ($\rho(1300)$) would suggest a non-trivial interpretation for this nonet.

To summarize, almost every nonet analyzed in this paper can be satisfactorily described by our mixing scheme without any non-quark mesons. The relative success of this approach suggests that it might be used as a guide to the analyses of quark-antiquark contents of the physical mesons participating of a specific nonet.

**Acknowledgments**

This work was partially supported by CNPq, FAPESP and FINEP.
References

[1] M. Gell-Mann, California Institute of Technology Report CTSL-20 (1961); S. Okubo, Prog. Theoret. Phys. 27, 949 (1962).

[2] J. Schwinger, Phys. Rev. Lett. 12, 273 (1964).

[3] F.E. Close, An Introduction to Quarks and Partons. Academic Press (1979).

[4] A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D 12, 147 (1975).

[5] W.S. Carvalho, A.C.B. Antunes and A.S. de Castro, Mod. Phys. Lett. A 12, 121 (1997).

[6] Particle Data Group, L. Montanet et al., Phys. Rev. D 50, 1173 (1994).

[7] W.S. Carvalho, A.C.B. Antunes and A.S. de Castro, Hadronic J. 22, 105 (1999).

[8] D.L. Scharre et al., Phys. Lett. B 97, 329 (1980); C. Edwards et al., Phys. Rev. Lett. 49, 259 (1982).

[9] N. Aizawa, Z. Maki and I. Umemura, Prog. Theor. Phys. 68, 2120 (1982).

[10] J.L. Rosner, Phys. Rev. D 27, 1101 (1983); J.L. Rosner and S.F. Tuan, Phys. Rev. D 27 1544 (1983).

[11] R.M. Baltrusaitis et al., Phys. Rev. D 32, 2883 (1985).

[12] F. Caruso, E. Predazzi, A.C.B. Antunes and J. Tiommo, Z. Phys. C 30, 493 (1986).

[13] J. Jousset et al., Phys. Rev. D 41, 1389 (1990).

[14] Z. Bai et al., Phys. Rev. Lett. 65, 2057 (1990); C. Amsler et al., Phys. Lett. B 358, 389 (1995); A. Bertin et al., Phys. Lett. B 361, 187 (1995).

[15] I. Kitamura, N. Morisita and T. Teshima, Int. J. Mod. Phys. A 31, 5489 (1994).

[16] F.E. Close, G.R. Farrar and Z. Li, Phys. Rev. D 55, 5749 (1997).
[17] M. Genovese, D.B. Lichtenberg and E. Predazzi, Z. Phys. C 61, 425 (1994).

[18] W.S. Carvalho, A.C.B. Antunes and A.S. de Castro, Eur. Phys. J. C 7, 95 (1999).

[19] H.J. Lipkin, Phys. Lett. B 67, 65 (1977).

[20] Particle Data Group, R.M. Barnett et al., Phys. Rev. D 54, 1 (1996).

[21] G.M. Brandenburger et al., Phys. Rev. Lett. 36, 703 (1976); R.K. Carnegie et al., Nucl. Phys. B 127, 509 (1977); M.G. Bowler, J. Phys. G 3, 775 (1977).

[22] P. Gavillet et al., Z. Phys. C. 16, 119 (1982); D. Aston et al., Phys. Lett. B 201, 573 (1988).

[23] S.I. Bityukov et al., Phys. Lett B 203, 327 (1988); S. Ishida et al., Prog. Theor. Phys. 82, 119 (1989).

[24] D. Alde et al., Phys. Lett. B 241, 600 (1990); D.V. Bugg et al., Phys. Lett. B 353, 378 (1995).

[25] J.E. Augustin et al., Phys. Rev. Lett. 60, 2238 (1988); T.A. Armstrong et al., Phys. Lett. B 227, 186 (1989).

[26] J.Z. Bai et al., Phys. Rev. Lett. 77, 3959 (1996).

[27] W.S. Carvalho, A.S. de Castro and A.C.B. Antunes, Eur. Phys. J. C 17, 173 (2000).

[28] Particle Data Group, D.E. Groom et al., Eur. Phys. J. C 15, 1 (2000).

[29] H.E. Haber and J. Perrier, Phys. Rev. D 32, 2961 (1985); I. Bediaga, F. Caruso and E. Predazzi, Nuovo Cim. A 91, 306 (1986); F. Caruso and E. Predazzi, Europhys. Lett. 6, 677 (1987); A. Bramon and M.D. Scadron, Phys. Lett. B 234, 346 (1990); C. Amsler et al., Phys. Lett. B 194, 451 (1992); P. Ball et al., Phys. Lett. B 365, 367 (1996); M. Genovese, hep-ph/9608451; G.R. Farrar, hep-ph/9612354; F.E. Close, Nucl. Phys. Proc. Suppl. A 56, 248 (1997); A.V. Anisovich, V.V. Anisovich and A.V. Sarantsev, Phys. Lett. B 395, 123 (1997); A. Bramon et al., Eur. Phys. J. C 7, 271 (1999); L. Burakovsky and T. Goldman, Phys. Rev. D 57, 2879 (1998).
[30] F.E. Close and A. Kirk, Z. Phys. C 76, 469 (1997).

[31] H. Fritzch and P. Minkowsky, Nuovo Cim. A 30, 393 (1975); H. Fritzch and J.D. Jackson, Phys. Lett. B 66, 365 (1977); N. Isgur, Phys. Rev. D 21, 779 (1980); H.J. Schnitzer, Nucl. Phys. B 207, 131 (1982); T. Teshima and S. Oneda, Phys. Rev. D 27, 1551 (1983); S. Godfrey and N. Isgur, Phys. Rev. D 34, 899 (1986); F.J. Gilman and R. Kauffman, Phys. Rev. D 36, 2761 (1987); T. Teshima, I. Kitamura and N. Morisita, Nuovo Cim. A 103, 175 (1990); M. Birkel and H. Fritsch, Phys. Rev. D 53, 6195 (1996); M.M. Brisudova et al., Phys. Rev. D 58, 114015 (1998); D. Weingarten, Nucl. Phys. (Proc. Suppl.) B 53, 232 (1997); H.-M. Choi and C.-R. Ji, Phys. Rev. D 59, 074015 (1999); L. Burakovsky and T. Goldman, Nucl. Phys. A 628, 87 (1998).

[32] I. Cohen and H.J. Lipkin, Nucl. Phys. B 151, 16 (1979).

[33] N. Isgur, Phys. Rev. D 21, 779 (1980).

[34] D.-M. Li, H. Yu and Q.-X. Shen, Chin. Phys. Lett. 17, 558 (2000).

[35] D.-M. Li, H. Yu and Q.-X. Shen, J. Phys. G 27, 807 (2001).

[36] F.E. Close and P.R. Page, Nucl. Phys. 443, 233 (1995).

[37] J. Adomeit et al., Z. Phys. C 71, 227 (1996).

[38] S. Godfrey, R. Kokoski and N. Isgur, Phys. Lett. B 141, 439 (1984).

[39] H. Blundell and S. Godfrey, Phys. Rev. D 53, 3700 (1996).

[40] A. Seiden, H.F.-W. Sadrozinski and H.E. Harber, Phys. Rev. D 38, 824 (1988).

[41] D. Aston et al., Nucl. Phys. B (Proc. Suppl.) 21, 105 (1991).
Table 1: SU(3) mixing angles for excited nonets. As done by PDG \cite{28}, the isosinglets mostly octet are listed first and their percentual contents of strange quarks are also shown. The values presented for $|S>$ and $\theta$ correspond to the range $m_s/m_u = 1.3 \ldots 1.8$. The values for the $1^3P_0$ nonet are found taking into account the central value for the mass of $f_0(1370)$.

| N $2s+1L_J$ | $J^{PC}$ | Nonet members | $|S>$ | $\theta$ |
|-------------|----------|---------------|-------|---------|
| $1^3S_1$    | 1$^-$    | $\rho, K^*(892), \phi, \omega$ | 99.9\% $\ldots$ 100\% | 36.9$^\circ$ $\ldots$ 36.4$^\circ$ |
| $1^1P_1$    | 1$^+$    | $b_1(1235), K_{1B}, h_1(1380), h_1(1170)$ | 98.0\% $\ldots$ 98.9\% | $-62.9^\circ$ $\ldots$ $-60.8^\circ$ |
| $1^3P_0$    | 0$^+$    | $a_0(1450), K^*_0(1430), f_0(1370), f_0(1710)$ | 89.7\% $\ldots$ 94.1\% | $-73.4^\circ$ $\ldots$ $-68.8^\circ$ |
| $1^3P_1$    | 1$^+$    | $a_1(1260), K_{1A}, f_{1}(1285), f_{1}(1420)$ | 5.4\% $\ldots$ 2.3\% | $-41.3^\circ$ $\ldots$ $-45.9^\circ$ |
| $1^3P_2$    | 2$^+$    | $a_2(1320), K_2^*(1430), f'_2(1525), f_2(1270)$ | 99.2\% $\ldots$ 99.6\% | 30.1$^\circ$ $\ldots$ 31.5$^\circ$ |
| $1^1D_2$    | 2$^+$    | $\pi_2(1670), K_2(1770), \phi_2(1870), \eta_2(1645)$ | 99.7\% $\ldots$ 99.8\% | $-59.8^\circ$ $\ldots$ $-59.8^\circ$ |
| $1^3D_3$    | 3$^-$    | $\rho_3(1690), K_3^*(1780), \phi_3(1850), \omega_3(1670)$ | 99.5\% $\ldots$ 99.8\% | 31.4$^\circ$ $\ldots$ 32.4$^\circ$ |
| $1^3F_4$    | 4$^+$    | $a_4(2040), K_4^*(2045), f_4(2050), f_4(2220)$ | 0.3\% $\ldots$ 0.2\% | $-51.5^\circ$ $\ldots$ $-52.4^\circ$ |
| $2^1S_0$    | 0$^-$    | $\pi(1300), K(1460), \eta(1440), \eta(1295)$ | $\sim 100\%$ $\ldots$ $\sim 100\%$ | $-55.4^\circ$ $\ldots$ $-55.2^\circ$ |
| $2^3S_1$    | 1$^-$    | $\rho(1300), K^*(1410), \omega(1420), \phi(1680)$ | 10.3\% $\ldots$ 3.9\% | 54.0$^\circ$ $\ldots$ 46.7$^\circ$ |

Table 2: Branching ratios and electromagnetic decay widths involving the axial mesons. $f_1$ and $\tilde{f}_1$ stand for $f_1(1285)$ and $\tilde{f}_1(1420)$, respectively. The values presented in our model correspond to the range $m_s/m_u = 1.3 \ldots 1.8$.

| Observable | Our model | Experiment \cite{28} |
|------------|-----------|----------------------|
| $\Gamma_{\gamma\gamma}(f_1)$ | 0.43 $\ldots$ 0.29 | $1.4 \pm 0.8$
| $\Gamma_{\gamma\gamma}(f_1)$ | $b(1K \rightarrow K\bar{K}\pi)$ |
| $\Gamma_{\gamma\gamma}(f_1)$ | 0.43 $\ldots$ 0.29 | $0.63 \pm 0.34$
| $\Gamma_{\gamma\gamma}(f_1)$ | $b(1K \rightarrow K\bar{K}\pi)$ |
| $B(J/\psi \rightarrow \gamma f_1)$ | 1.11 $\ldots$ 0.81 | $1.36 \pm 0.44$
| $B(J/\psi \rightarrow \gamma \tilde{f}_1)$ | $b(1K \rightarrow K\bar{K}\pi)$ |
| $B(f_1 \rightarrow \gamma \phi)$ | 0.005 $\ldots$ 0.002 | 0.013 $\pm$ 0.008
| $B(f_1 \rightarrow \gamma \rho)$ | $b(1K \rightarrow K\bar{K}\pi)$ |
Table 3: Branching ratios involving the tensor mesons. $f_2$ and $\tilde{f}_2$ stand for $f'_2(1525)$ and $f_2(1270)$, respectively. The values presented in our model correspond to the range $m_s/m_u = 1.3 \ldots 1.8$.

| Observable                  | Our model | Experiment [28] |
|-----------------------------|-----------|-----------------|
| $B(f_2 \to \pi\pi)$        | 0.024 \ldots 0.012 | 0.0092 \pm 0.0018 |
| $B(f_2 \to KK)$            | 0.18 \ldots 0.17     | 0.055^{+0.005}_{-0.006} |
| $B(J/\psi \to \gamma f_2)$| 0.25 \ldots 0.28     | 0.34 \pm 0.08      |
| $B(J/\psi \to \gamma \tilde{f}_2)$ |          |                  |