Frequency Stability Using Inverter Power Control in Low-Inertia Power Systems

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Abstract—The electrical grid is evolving from a network consisting of mostly synchronous machines to a mixture of synchronous machines and inverter-based resources such as wind, solar, and energy storage. This transformation has led to a decrease in mechanical inertia, which necessitate a need for the new resources to provide frequency responses by controlling their inverter interfaces. In this paper we proposed a new strategy based on model predictive control to determine the the optimal active-power set-point for inverters in the event of a disturbance in the system. In contrast to existing methods, our framework explicitly takes the hard constraints in power and energy into account. We show that it is also robust to measurement noise and limited communications by using an observer to estimate the model mismatches in real-time. We demonstrate that our proposed controller significantly outperforms an optimally tuned virtual synchronous machine on standard IEEE 9-bus and 39-bus systems under a number of scenarios. In turn, this implies optimized inverter-based resources can provide better frequency responses compared to conventional synchronous machines.

I. INTRODUCTION

The electric grid has been undergoing a transition from a network with dynamics fully governed by synchronous machines to a mixed-source network with dynamics governed by both synchronous machines and inverter-based resources (IBRs). This transition is marked by a reduction of the amount of mechanical inertia in the system, which has led to more pronounced frequency responses to disturbances and faults in the grid [1], [2]. At the same time, by the virtue of the speed of power electronic circuits, IBRs such as solar, wind and energy storage have the capability to respond to frequency changes in the grid at a much faster rate then traditional generator with rotating masses. The challenge of how to best utilize these new capabilities has spurred much research interest in the last few years (e.g., see [3] and the references within).

Various control strategies that utilizes the IBRs has been proposed. The goal of these strategies is to design the active power response of the IBRs to changes in frequency, such that some objective is minimized. For example, standard objectives of interests are the magnitude of the frequency deviation, the rate of change of frequency (ROCOF) and the settling time. A unique challenge in the control of IBRs is that they tend to face much tighter limits than conventional machines. For example, solar and wind resources cannot increase their power output beyond the maximum power tracking point, which introduces a hard (and asymmetrical) constraint on the action of the inverters. For a storage unit, it has only a limited amount of energy that can be used to respond to a disturbance.

Of the varying control strategies proposed for IBRs, Droop Control [4]–[6] and Virtual Synchronous Machines (VSMs) [7]–[9] are the most popular as they function by mimicking the frequency-power dynamic response of a synchronous machine. As suggested by their names, droop control injects/absorbs an amount of active power in proportion to the frequency deviation, and VMS act as a second order oscillator to provide inertia and damping to the grid. The parameters (droop slope, inertia and damping constants) used in these strategies can be optimized using a number of techniques [10]–[12].

The structural simplicity of VSMs also lead to a fundamental limitation [10], [13]. Since there are only two parameters to tune (inertia and damping), there is an inherent trade-off between different objectives and there is no choice of parameters that will make both the frequency deviation and ROCOF small at the same time [10]. In addition, it is difficult to include hard constraints, since simply thresholding the output once the constraints are reached tend to lead to very poor performances [14]. Adaptive rules can be used to alleviate this drawback somewhat, and works in [13], [15], [16] change the parameter based on the measured frequency deviation and ROCOF values. However, it is difficult to find an optimal rule to update these parameters in real-time.

In this work, we propose a novel control strategy called the Inverter Power Control (IPC) based on model predictive control (MPC). We explicitly formulate the problem of finding the optimal active power set-point of an IBR to minimize the frequency deviation and the ROCOF. More specifically, at any timestep, we simulate the dynamics of the systems for a finite horizon, then find the best set-points that optimizes the objective over that horizon. The first action is then adopted for the current timestep, and the process repeats. Our approach is similar in spirit to the ones in [13], [15], [16] since an objective is optimized in an online fashion. However, instead of optimizing the parameters, we directly find the best power set-points. This approach turns out to provide both an easier optimization problem and better control performances. Namely, the hard constraints on the IBRs are explicitly included in the optimization process.

A requirement of MPC is that the IBR must have a model of the system to be optimized. If wide-area measurements are available, then the system states can be obtained from these.
measures [17]. In some systems, only a limited buses are equipped with these measurement devices (e.g., PMUs). We show that our proposed IPC framework is still applicable to these systems by building an observer to estimate unmeasured disturbances and states. Through simulation studies, we show that the IPC strictly outperforms optimally tuned VSMs for the IEEE 9-bus and 39-bus systems, even under limited communication and large measurement noises.

The remainder of this paper is organized as follows: Section II defines the models used in this paper. Section III presents the design and formulation of the IPC algorithm. Section IV presents the state and disturbance observer design. Section V compares the performances of IPC to VSMs in two standard test systems. Section VI concludes the paper.

II. MODELING

We denote the real line by $\mathbb{R}$, the cardinality of a set $\mathcal{S}$ as $|\mathcal{S}|$, the $n \times n$ identity and zero matrices as $I_n$ and $0_n$, respectively. Matrices and vectors are denoted by a bold-faced variables. 

A. System Structure

Steady state conditions in a power systems is achieved when there is a balance between the power produced by the generating sources and the power consumed by loads and lossy components. For stability analysis, the entire system can be reduced to an equivalent network via Kron reduction [18]. This eliminates passive and non-dynamic load buses and leaves only buses with at least one generating source connected. With this in place, frequency stability analysis can be carried out, with the frequency dynamics governed by the reactions of buses to active power imbalances in the system.

In this work, we assume the availability of state variables and network information for control purposes. In a later section, we will relax this assumption to partial availability of state variables from some generators.

Because the generators and IBRs had different dynamics, we denote their sets by $\mathcal{G}$ and $\mathcal{I}$, respectively. Note that the total number of generating sources in the network is $\mathcal{N} := \mathcal{G} \cup \mathcal{I}$.

B. Synchronous Machines

The rotor dynamics of each synchronous generator in a given power system is governed by the well-known swing equation [19]. Here we adopt a discretized version of the equations, which in per unit (p.u.) system is:

$$
\omega_i^{t+1} = \omega_i^t + \frac{h}{m_i} \left( P_{m,i}^t - P_{e,i}^t - d_i \omega_i^t \right),
$$

$$
\delta_i^{t+1} = \delta_i^t + h \left( \omega_i^t + \omega_b \right),
$$

\forall i \in \mathcal{G}, \text{ where } h \text{ is the step size for the discrete simulation, } \delta_i \text{ (rad) is the rotor angle, } \omega = \omega_i - \omega_b \text{ is the rotor speed deviation, } \omega_b \text{ is the base speed of the system, } m_i \text{ is the inertia constant, } d_i \text{ is the damping constant, } P_{m,i} \text{ is the mechanical input power and } P_{e,i} \text{ is the electric power output of the } i^{th} \text{ machine.}

The electrical output power $P_{e,i}$ is given by the AC power flow equation in terms of the internal emf $|E_i|$ and rotor angle $\delta_i$:

$$
P_{e,i} = \sum_{j} (E_i E_j) [g_{ij} \cos(\delta_i^t - \delta_j^t) + b_{ij} \sin(\delta_i^t - \delta_j^t)],
$$

\forall i, j \in \mathcal{G}, \text{ where } g_{ij} + j b_{ij} \text{ is the modified admittance between nodes } i \text{ and } j. \text{ We assume the internal emf are constant because of the actions of the exciter systems.}

The nonlinearity of the AC power flow in (2) makes it difficult to use for control applications. Using DC power flow [20], the bus dynamics become:

$$
\Delta \omega_i^{t+1} = \Delta \omega_i^t + \frac{h}{m_i} \left( \Delta P_{m,i}^t - \Delta P_{e,i}^t - d_i \Delta \omega_i^t \right),
$$

$$
\Delta \delta_i^{t+1} = \omega_b \left( \Delta \delta_i^t + h \Delta \omega_i^{t+1} \right),
$$

\text{ where } \Delta P_{e,i} = \sum_{j} b_{ij} \delta_{ij} \text{ which is the dc power flow between 2 buses.}

We model changes to the mechanical input power $\Delta P_{m,i}$ by a combination of droop and automatic governor control (AGC) actions [20] according to the discretized equation:

$$
\Delta P_{m,i}^{t+1} = \frac{1}{1 + h k_1} \left( 2 + h k_2 - \frac{h^2 k_3}{m_i} \right) \Delta P_{m,i}^t - \Delta P_{m,i}^{t-1} + h^2 \left( \frac{k_2 d_i}{m_i} - k_3 \right) \Delta \omega_i^t - \frac{k_2 k_3}{m_i} (\Delta P_{e,i}^t),
$$

\text{ where } k_1, k_2, k_3 \text{ are the gain coefficients of the droop and AGC controller.}

C. Inverter Model and Control

From the network point of view, the grid-connected IBRs is seen as producing a constant power according to its predetermined set-point and fast dynamics governed by closed controls actions [21], which helps maintain the output power while remaining synchronized to the terminal voltage set by the grid. For system analysis, the the inverter can be modeled as a voltage source behind a reactance, much like a synchronous machine, as shown in Fig [1].

In the event of a power imbalance in the network reflected by a frequency deviation, an inverter does not have an "natural" response to as synchronous machines do since they are made of power electronics components and have no rotating mass. To elicit some response, an additional control loop is therefore needed to enable the inverters participate in frequency control by changing the power set-point of the inverter based on frequency measurements.

Since the response of the inverter is entirely digital, it can be programmed with almost arbitrary functions [8], [22]. VSMs
adopt control laws that allows IBRs to mimic the synchronous machines through the following dynamics:

\[
\begin{align*}
\omega^{t+1}_{ibr} &= \omega^{t}_{ibr} + \frac{h}{k_m} (P_{ref} - P_{ibr}^{t} - k_d (\omega^{t}_{ibr} - \omega^{t})) \\
\delta^{t+1}_{ibr} &= \delta^{t}_{ibr} + h \omega^{t+1}_{ibr}
\end{align*}
\]

where \(k_m\) and \(k_d\) are the inertia and damping gains coefficients, respectively. In contrast to synchronous machines where the constants are decided by the physical parameters, the constants of the VSM can be optimized over \([10]\).

As stated in the introduction, even though the constants in the VSM can be adjusted, they do not provide enough degrees of freedom to optimize the active power output of the inverter. In the next section, we fully leverage the flexibility of the power electronic interfaces using a MPC framework.

### III. INVERTER POWER CONTROL (IPC)

In this work, we propose a novel method for controlling the output power of the IBR, called the Inverter Power Control (IPC). This controller functions by modifying the real power set-point as shown in Fig. 2 at each time step such that a weighted sum of the frequency deviation and ROCOF is minimized. Due to the timescale difference between IBRs and synchronous machines, the real power set-points of an IBR can be set almost instantaneously. Therefore, the important question becomes how to solve the optimization problem at each time step fast enough to find the real power set-point and how much communication is required in performing these calculations. In this section, we describe how to formulate the optimization problem and provide an efficient algorithm, assuming all of the information are known at the IBR. The next section then discusses how to deal with limited and noisy measurements, as well as incomplete communication.

For \(k\)th IBR, let \(u_k\) denote its angle (referenced to the slack-bus). We think of this \(u_k\) as the control variable in the optimization problem. Note that the actual control of the IBR is not done via angle control, rather, we use the optimized \(u_k\) to find the corresponding active power output of the inverter, then set the inverter to that power. To determine this real power set-point at a given time step, consider the swing equation in (1) and we write out the \(i\)th generator’s power output \(P_{e,i}\) into two parts: power flowing from the \(i\)th generator to another generator denoted as \(P_{e,G,i}\) and from the \(i\)th generator to an IBR denoted as \(P_{e,i,IBR}\), such that:

\[
P_{e,i} = P_{e,G,i} + P_{e,i,IBR},
\]

and the output power from the \(k\)th IBR denoted as \(P_{ibr,k}\) can be written as:

\[
P_{ibr,k} = \sum_{k \sim j, k \in I, i \in G} |E_{k}E_{i}| [g_{ij}\cos(\delta_{i} - \delta_{j}) + b_{ij}\sin(\delta_{i} - \delta_{j})]
\]

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\]

\[
\sum_{k \sim j, k \in I, i \in G} |E_{k}E_{i}| [g_{ij}\cos(u_{k} - \delta_{i}) + b_{ij}\sin(u_{k} - \delta_{i})],
\]

depending on whether the \(k\)th IBR is connected to a synchronous machine or another IBR.

### A. Nonlinear Optimization Problem

At any timestep, we consider the behavior of the system \(N\) steps ahead. Without loss of generality, we start the problem at time \(t = 0\). The control variables are the inverter angles, which we denote as \(u^0, u^1, \ldots, u^{N-1}\). Once these are set, the rest of the system are governed by their swing equations. As stated before, the objective is to minimize a function of the frequency deviation and the ROCOF, and the IPC problem is given by:

\[
\begin{align*}
\text{Min.} & \quad \sum_{t=0}^{N-1} \left\{ \|\omega^{t+1}\|_2^2 + \frac{1}{h} \|\omega^{t+1} - \omega^{t}\|_2^2 \right\} \\
\text{s.t.} & \quad \omega^{t+1} = \omega^{t} + \frac{h}{m_i} (P_{m,i}^{t} - P_{e,i}^{t} - d_i \omega^{t} - \Delta P_i^{t}), \; \forall i \in G \\
P_{e,i}^{t} &= \text{Equation (6)}, \; \forall i \in G \\
P_{ibr,k}^{t} &= \text{Equation (7),} \; \forall k \in I \\
P_{ibr,min,k}^{t} &\leq P_{ibr,k}^{t} \leq P_{ibr,max,k}^{t} \\
\sum_{t} P_{ibr,k}^{t} &\leq E_{ibr, max, k},
\end{align*}
\]

where \(\omega^{t+1} \in \mathbb{R}^{|G|}\) is a vector of all machine frequency deviations at the next time step and \(\omega^{t+1} - \omega^{t}\) is a vector of all machine ROCOF between the current and next time step. The evolution of \(\omega\) is given in (8b) (swing equations) with the added \(\Delta P_i\) used to denote disturbances to the network which can be either a loss in generation or load, the power constraints are given in (8e) and the energy constraints are in (8f).
Here we take the frequency deviation and the ROCOF to be equally weighted for simplicity, but their weighting can be adjusted as needed for different practical scenarios.

After (8) is solved, the control variable \( u^0 \) is substituted into the power flow equations (8) to find the active power set-points of the IBRs. Then the IBRs hold their power at these set-points until the next time the optimization problem is solved. Note, even though only the first control variable \( u^0 \) is used, we need to solve for all of the other control variables because of the time coupling in the dynamics of the system. It turns out that the AC power flow equations in (8) and (7) makes the problem nonlinear and difficult to solve in real-time. Therefore, the next two sections uses DC power flow to obtain an approximate problem that is much easier to solve.

### B. Unconstrained Linearized Problem

The main source of non-linearity comes from the AC power flow equations in (8c) and (8d) and we use the standard DC power flow model from (3) to approximate these equations.

Therefore, at bus \( i \in G \) (synchronous machines), we have:

\[
\triangle P_{e,i} = \triangle P_{eG,i} + \triangle P_{eI,i} = \sum_{i,j \in G} b_{ij}(\triangle \delta_i - \triangle \delta_j) + \sum_{i,k \in I} b_{ik}(\triangle \delta_i - u_k),
\]

which can be written in matrix form as:

\[
\begin{bmatrix}
\Delta P_e \\
\Delta \delta^t
\end{bmatrix}_{t+1} =
\begin{bmatrix}
\bar{B}_{ee} & -\bar{B}_{ei} \\
-\bar{B}_{ie} & B_{ei} \\
\end{bmatrix}
\begin{bmatrix}
\Delta \omega^t \\
\Delta \delta^t
\end{bmatrix}_t
+ \begin{bmatrix}
b_{ii} & -b_{ij} \\
-b_{ji} & b_{jj}
\end{bmatrix} u_k,
\]

where \( B_{ee} \) contains the connection between synchronous generators and \( B_{ei} \) contains the connection between a synchronous generator and IBRs. In state space form, it becomes

\[
\begin{bmatrix}
\Delta \omega^t \\
\Delta \delta^t
\end{bmatrix}_{t+1} =
\begin{bmatrix}
\bar{A} & \bar{B}_u \\
\bar{B}_d & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \omega^t \\
\Delta \delta^t
\end{bmatrix}_t
+ \begin{bmatrix}
0_n \\
0_m
\end{bmatrix} u^t
\]

where \( \Delta \delta \in \mathbb{R}^n \) is the rotor angles deviation, \( \Delta \omega \in \mathbb{R}^n \) is the rotor speed deviation, \( M = \text{diag}(m_1, \ldots, m_n) \in \mathbb{R}^{n \times n} \), \( D = \text{diag}(d_1, \ldots, d_n) \in \mathbb{R}^{n \times n} \in \mathbb{R}^n \), \( \Delta P \in \mathbb{R}^n \) is vector of all power deviations which comes from the disturbances and noises in the system, denoted by \( d^t \).

Since the IPC does not know the disturbance or noise impacting the system, we use a two step process to solve the optimization problem. First, we ignore the disturbance term, and the IPC’s model of the system is:

\[
\begin{bmatrix}
\Delta \omega^t \\
\Delta \delta^t
\end{bmatrix}_{t+1} =
\begin{bmatrix}
\bar{A} & \bar{B}_u \\
\bar{B}_d & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \omega^t \\
\Delta \delta^t
\end{bmatrix}_t
+ \begin{bmatrix}
0_n \\
0_m
\end{bmatrix} u^t.
\]

Note, in this case the IPC’s model of the system is actually wrong since the disturbances are not modeled. It turns out that this model is still useful, since the measurements are updated every time the MPC problem is solved, and this compensates for using a wrong model. In the rest of this section, we focus on solving the optimization problem using the model in (12) since it illustrates our methodology. Of course, when the measurement noise in the system is large or not every bus is equipped with wide-area measurement devices, it becomes necessary to explicitly estimate the mismatch between the model and the actual system. We do so in section IV.

To reformulate the objective function in terms of the network model in (12) by defining the output of the linearized model as the frequency deviation \( \Delta \omega^t \):

\[
y^t = \left[ I_n \ 0_n \right] x^t = \Delta \omega^t
\]

such that the ROCOF becomes:

\[
\Delta y^t = \frac{1}{h} \left[ y^t - y^{t-1} \right] = \frac{1}{h} \left[ \Delta \omega^t - \Delta \omega^{t-1} \right]
\]

The IPC optimization algorithm in (9) without the power limit constraint (8e) and total energy constraints (8f) can now be written as a linear quadratic programming problem:

\[
\begin{align*}
\text{Min.} \quad & J_u = \frac{1}{2} \sum_{t=1}^{N-1} \left[ y^{tT} Q_1 y^t + \Delta y^{tT} Q_2 \Delta y^t \right] \\
\text{s.t.} \quad & x^{t+1} = A x^t + B u^t \\
& y^t = C x^t.
\end{align*}
\]

Not surprisingly, the optimal solution to this unconstrained problem is linear in the starting point \( x^0 \):

\[
u^* = -H^{-1}F^T x^0,
\]

where \( H \) and \( F \) are constant matrices depending on \( A \) and \( B \) (see Appendix). This solution can be interpreted as a linear policy, where the optimal action is determined as a linear function of the current state information.

### C. Constrained Linearized Optimization Problem

In the presence of constraints, we need to solve a quadratic programming optimization problem with linear constraints of the form:

\[
\begin{align*}
\text{Min.} \quad & J_u = \frac{1}{2} x^T G x^0 + x^T F u + \frac{1}{2} u^T H u \\
\text{s.t.} \quad & L u \leq W + V x^0,
\end{align*}
\]

where \( L, W \) and \( V \) are constant matrices depending on the constraint being considered. In this paper, we consider constraints on the power output at each time step (8e) and constraints on the total energy available to provide frequency control (8f).

1) **Power Output Constraint**: In practical considerations, there can be a limit on the amount of instantaneous power that can be drawn from the IBR due to factors such as the distance to the maximum power tracking operating point, the current ratings and switching speed of some power electronics components, and also power capability or C-rate of a battery.

The transformation of the minimum and maximum instantaneous power limit from \( P^t_{\text{ibr,min,k}} \leq P^t_{\text{ibr,k}} \leq P^t_{\text{ibr,max,k}} \) to the
The output power $P_{ibr,k}^t$ of the $k$th IBR at time step $t$ involves writing the linearized power output of the $k$th IBR at time step $t$ in terms of the control variable $u^t$ and states $x^t$, and then stacking them in matrix form for the $N$ control horizon.

The output power $P_{ibr,k}^t$ is written in terms of the power flow to generators and to other IBRs as:

$$P_{ibr,k}^t = \sum_{k=1,i\in G} b_{ki}(u^t_k - \triangle y_i^t) + \sum_{k-j,i\in G} b_{kj}(u^t_k - u^t_j)$$

$$= - \sum_{k-j,i\in G} b_{kj}u^t_j + \sum_{k-j,i\in G} b_{kj}u^t_k - \sum_{k-i\in G} b_{ki} \triangle y_i^t,$$

which can be written in matrix form as:

$$P_{ibr,k}^t = -[B]_{jk} [B]_{kk} u^t + [0_n - [B]_{ki}] x^t,$$

Stacking (19) for a $N$ time horizon and writing the linear system dynamics in terms of the initial state results in a form:

$$P_{ibr,k} = B_{p1} u + B_{p2} x^0,$$

which can finally be written in the linear constraint form of (17) as

$$\begin{bmatrix} -B_{p1} \\ B_{p2} \end{bmatrix} u \leq \begin{bmatrix} -P_{ibr, min} \\ P_{ibr, max} \end{bmatrix} + \begin{bmatrix} B_{p2} \\ -B_{p2} \end{bmatrix} x^0.$$

2) Total Energy Constraint: This constraint occurs when there is a limit on the energy capacity of the IBR as in the case of a battery. For this constraint to be fully satisfied, the total energy not only at the end of the control horizon but also at each rolling sum of the consecutive time step should be less than the maximum energy capacity.

As with the power output constraint, the total energy constraint

$$\sum_{t=1}^T P_{ibr,k}^t t \leq E_{ibr, max,k}$$

can also be written in the linear constraint form in (17) by taking the rolling sum over the inverter power output matrix in (20). This results in another matrix of the form:

$$E_{ibr,k} = B_{e1} u + B_{e2} x^0.$$

To avoid a sudden decline in the power output when the maximum available energy limit is reached, a rate constraint can be added to the power output decline between a specified consecutive time step. This can also be represent in the form of (17) by taking a one time step difference of the IBR power output matrix in (20), that, is, a difference between the next time step and current time step IBR power output. The results of this is a matrix that can be written in the form:

$$\triangle P_{ibr,k} = \epsilon = B_{r1} u + B_{r2} x^0.$$

Equation (23) and (22) can finally be written in the linear constraint form of (17) as

$$\begin{bmatrix} B_{e1} \\ B_{e2} \end{bmatrix} u \leq \begin{bmatrix} E_{ibr, max} \\ \epsilon \end{bmatrix} \begin{bmatrix} -B_{r2} \\ -B_{r2} \end{bmatrix} x^0.$$

where $\epsilon$ is a vector of IBR power output rate limit for each one time step difference.

Even with constraints, a linear quadratic program can be solved extremely efficiently for systems with thousands of variables and constraints [23]. Again, to actually implement the controller, we compute and set the power output of the IBRs.

IV. IPC WITH STATE ESTIMATION AND LIMITED COMMUNICATION

In section [11] the IPC controller was designed using the reduced linearized model of the network as in (12) and under the assumption of a full state measurement. When operating this controller in a realistic setting, we would want the controller to be robust against issues such as model mismatch, that is, the difference between the actual system model and the linearized model used by the IPC; noisy measurements, and incomplete measurements because of limited communication between buses.

We address these issues in this section by integrating an observer into the IPC controller system according to Fig. 3 to enable the controller estimate a better model of the system from the received measurements.

![Fig. 3. Block diagram showing the operation of an observer integrated IPC in a power systems.](image)

Let the dynamics of the actual power systems governed by (1) and (4) be represented concisely by:

$$x^{t+1} = f(x^t, u^t)$$

$$y^t = g(x^t, u^t).$$

A simple discrete observer model design for the system in (25) can be written as:

$$\hat{x}^{t+1} = \hat{x}^t + K(y^t - \hat{y}^t),$$

where the notation $\hat{x}^{t+1}$ means the prediction of $x^t$ made at time $t$. Therefore the variables with $\hat{x}^{t+1}$ is the updated observer state prediction based on new measurement $y^t$, $\hat{x}^{t+1}$ is the observer state prediction of the next time step using measurements from the current time step, and $K$ is a gain chosen such that the error between the measured and predicted state $y^t - \hat{y}^t$ is quickly driven to zero.

A. State and Disturbance Estimation

To estimate the state and disturbance in a noisy system with model mismatch and other forms of disturbance, we denote $d^t$ as a vector of all disturbances. We then integrate an input/output constant disturbance model [24] into the IPC system model in (11) to obtain:
\[ \dot{x}^{t+1} = \bar{A}\dot{x}^t + \bar{B}_u u^t + \bar{B}_d d^t \]
\[ d^{t+1} = \bar{d}^t \]
\[ \hat{y}^t = C\hat{x}^t + C_d d^t, \]

where the disturbance $\hat{d}$ is modeled as a constant disturbance for the control period. Equation (27) can then be written in an augmented form as:

\[ \begin{bmatrix} \dot{x}^{t+1} \\ d^{t+1} \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{B}_d \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{x}^t \\ \hat{d}^t \end{bmatrix} + \begin{bmatrix} \bar{B}_u \\ 0 \end{bmatrix} u^t \]
\[ \hat{y}^t = \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} \dot{x}^t \\ \hat{d}^t \end{bmatrix}. \]

The predicted augmented state and disturbance estimate is modeled using the observer model in (27) as:

\[ \begin{bmatrix} \dot{x}^{t|t} \\ \hat{d}^{t|t} \end{bmatrix} = \begin{bmatrix} \dot{x}^{t|-1} \\ \hat{d}^{t|-1} \end{bmatrix} + \begin{bmatrix} y^t - [C & C_d] \begin{bmatrix} \dot{x}^{t|-1} \\ \hat{d}^{t|-1} \end{bmatrix} \end{bmatrix} \]
\[ \begin{bmatrix} \dot{x}^{t|t} \\ \hat{d}^{t|t} \end{bmatrix} = \begin{bmatrix} \dot{x}^{t|-1} \\ \hat{d}^{t|-1} \end{bmatrix} + \begin{bmatrix} y^t - [C & C_d] \begin{bmatrix} \dot{x}^{t|-1} \\ \hat{d}^{t|-1} \end{bmatrix} \end{bmatrix} \]

where $K_x$ and $K_d$ are the gain matrices for the state and disturbance variable respectively. We adopt a rolling window least-square approach in determining these gains where the gain matrix $K$ is the minimizer of \( \| y^t - K \hat{y}^t \|_F \).

This observer integrated IPC model in (28) and (29) replaces the linear model in (12) with the augmented state used in place of the original states $x^t$ and the rest of the algorithm follows through for the constrained and unconstrained case.

### B. Limited Communication

While wide area measurement system (WAMS) data, consisting of sensors and communication infrastructures, is becoming increasingly available in modern power systems, it will still take some time before full communication coverage across the network can be realized. Even with these types of infrastructure, there is always the possibility of communication issues.

To tackle the issue of limited communication, we assume that the initial state measurements of the generators is available. For example, these can be conveyed using the existing SCADA system every two to four seconds. The augmented state and disturbance estimate in (29) can also be used but with a different gain since the structure and dimension of the gains $K_x$ and $K_h$ will change depending on the number of generators with available state information, that is, the dimension of $y^t$. These gains are also selected using the least square approach as in the state and disturbance estimation case. More sophisticated gain structures will be explored in future works.

The key idea here is that the mismatch between the evolved initial state of the generators with limited communication and what the state should be if there was communication is reflected as a disturbance in the network and can be estimated using the measurements from available generators.

### V. Case Studies

In this section, we validate the performance and versatility of the IPC controller by testing it on a 9-bus and a 39-bus system. We study scenarios including constraints on the power and energy output of the inverter, noisy measurements and limited communication. Under each scenario, a large disturbance in form of a partial generating capacity loss is applied to a generator in the network to initiate an event that can lead to a marked frequency decline. The performance metrics for the controller is its ability to maintain the the frequency within a small range, quickly recovering to the nominal frequency value and limiting the ROCOF.

The performance of the proposed controller is compared to that of an optimally tuned VSM controller discussed in Section III-C. Note that VSM controllers are not optimized for power or energy limits and simply saturates if they reach these limits. In our simulations, we include a condition that converts the IBR from a generator bus to a constant impedance load proportional to the active power and energy limit once the computed IBR output power exceeds its limits.

#### A. IEEE 3 Machine 9-Bus System

We validate the proposed IPC controller by first testing it on the WSCC 3-machine 9-bus (3m9b) system, a popular system used in stability studies [19]. We transform the network into a mixed-source network by replacing the third generator with an IBR that has the same output power capacity as the original generator. The network is then reduced to an equivalent network by eliminating the passive and static load buses, that is, buses 4 - 9, using Kron reduction.

The disturbance is applied to the second generator (Gen 2) with its power output starting at 0.85pu, then decreasing to 0.43pu at 0.5 seconds and later increasing to 0.56pu at 2 seconds. The purpose of the variation in disturbance level is to simulate conditions where the dynamics of the generators are well excited.

![Fig. 4. Comparison of IPC and VSM control strategies for an unconstrained scenario in a 3m9b network. The IPC outperforms the VSM in keeping the frequencies within limits while spending much less energy.](image-url)
ditions of unlimited IBR power and energy capacities. The proposed IPC controller is able to optimally determine the the required amount of active power to ensure a suitable frequency response. Specifically, the IPC keeps the frequencies within about 0.2 Hz of nominal, while the frequency varies by more than 0.6 Hz under the VSM controller. This shows that even though unconstrained amount of power is available to both controllers, the look-ahead and adaptive nature of the proposed controller enables it outperform the VSM.

It’s interesting to note that Fig. 4 also implies that the performance of an IBR is strictly better than that of a synchronous generator, since the VSM acts as a synchronous generator with optimized inertia and droop coefficients. Therefore, replacing conventional generators by renewable resource does not necessarily mean the frequency response is worse. Rather, if the resource can be optimized, then much better responses are possible.

1) Power and Energy Constraints: Figs. 5 and 6 shows the frequency in Hz and IBR output power for a power and energy constrained IPC and VSM, respectively. The minimum and maximum power limits at each time step was set to 0.5pu and 2pu respectively while the total energy limit was set to 200pu. The IPC outperforms the VSM in both settings by limiting the frequency deviation to about 0.5 Hz, while the system frequency drops by more than 0.7 Hz and settles above the nominal point when VSM is used. It can also be observed that the IPC uses much less energy for control compared to the VSM which saturates when at the limit. The IPC is able to integrate the resource constraints into its optimization and look-ahead to determine the best control strategy, but similar performance is very hard to achieved using a VSM controller since it lacks an explicit optimization step to deal with hard constraints.

2) Robustness of the Controller: Fig. 7 demonstrates the robustness of the IPC controller to noise, model mismatch and external disturbances to the system with the incorporation of the observer model in [29]. According to PMU standards in [26], the total vector error of a PMU measurement should be < 1%(~ 40dB signal-to-noise ration (SNR)) while [27] suggested that the SNR of PMU measurements can vary between 30 to 65 dB. We therefore model the effect of noisy measurements adding noise to create SNRs of 30dB and 50dB, respectively. These represent the worst-case and an average-case SNR scenarios. Figure 7 shows that noise has very little impact to the performance of the IPC (even under only 30 dB of SNR). Of course, the observer plays an important role in this robustness to noisy measurements.

B. IEEE New England 10 Machine 39-Bus System

In this section we validate the proposed IPC controller a larger system, the IEEE New England 39-bus system, and verify its ability to function effectively in a limited communication scenario. The network is transformed into a low-inertia network by removing the interconnection to the rest of the US network and replacing the generator at bus 34 with an IBR as shown in Fig. 8 and reduced to an equivalent network using Kron reduction. The disturbance is applied to the fourth generator (G4) located at bus 33. Its power output starts of at 6.32pu, then decreases to 3.16pu at 0.5 seconds and later increases to 4.42pu at 2 seconds.

Figure 9 shows the generator frequencies and IBR output power of a power and energy constrained setting. The simulation setup and result analysis is similar to subsection V-A with the minimum and maximum power limits at each time step set to 2pu and 7pu respectively while the total energy limit was set to 600pu. For a clearer viewing, only the frequency response of the second generator (slack) and fourth generator
In scenario B communication to make up for the limited communication. The difference in the measured and estimated states of the known model in (29) is able to estimate the true system state and uses the communication limitations. This is because the observer frequencies with the IPC still outperforming the VSM despite the large network, integrating model identification techniques and future work explores enhancing the controller to function in a larger and more complex system. We further test the performance of the IPC in a limited communication scenario. In scenario A, we assume that measurements can only be received from the generators colored green (G3, G4, G6, G7) while only initial state measurements is received from the generators colored green (G1, G2, G8, G9) as shown in Fig. 8. This case represent a setting when the faulted generator (G4) is able to communicate with the inverter. Figure 10 shows a comparison of the IPC and the VSM generator frequencies with the IPC still outperforming the VSM despite the communication limitations. This is because the observer model in (29) is able to estimate the true system state and uses difference in the measured and estimated states of the known communication to make up for the limited communication. In scenario B, we do not make the assumption that the fault is among the generators that communicates with the inverter. Even in this case, the IPC still outperforms the VSM. Therefore, by communicating with some buses, the inverter is able to reconstruct enough of the system-level information to make the computations at the IPC useful. The linear observer robustness to communication delays.

VI. CONCLUSION

In this paper, we proposed a novel control strategy called the Inverter Power Control that optimally determines the active power set-point for an inverter-based resource in real-time. Using a model predictive control framework, hard power and energy constraints are considered explicitly in the optimization process. We show via simulation on a small and large test systems the superiority of the proposed controller in comparison to the optimally tuned virtual synchronous machine, under both noisy and limited communication settings. Our future work explores enhancing the controller to function in a large network, integrating model identification techniques and robustness to communication delays.

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The matrices $H$ and $F$ in (11) can be obtained as follows: writing the linear system model in (12) for $N$ time steps ahead in matrix form, we have:

$$
\begin{bmatrix}
\tilde{x}^0 \\
\tilde{x}^1 \\
\vdots \\
\tilde{x}^N \\
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & \ldots & 0 \\
B & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{A}^{-N}B & \tilde{A}^{-N-2}B & \ldots & \tilde{A}^{-N} \\
\end{bmatrix}
\begin{bmatrix}
\tilde{u}^0 \\
\tilde{u}^1 \\
\vdots \\
\tilde{u}^{N-1} \\
\end{bmatrix}
+ \begin{bmatrix}
I \\
\tilde{A} \\
\tilde{A}^2 \\
\vdots \\
\tilde{A}^N \\
\end{bmatrix}
\begin{bmatrix}
x^0 \\
\end{bmatrix}
$$

The objective function in (13) can then be written in terms of the state variable as:

$$
y^T Q_1 y = (C x)^T Q_1 (C x) = x^T C^T Q_1 C x 
$$

Let

$$
\Theta = S[0 : N - 1; 1 : N] - S[1 : N; 1 : N] \\
\Gamma = M[0 : N - 1] - M[1 : N] \\
\Delta x = x[0 : N - 1] - x[1 : N] 
$$

such that

$$
\begin{align*}
\Delta y^T Q_2 \Delta y &= (C \Delta x)^T Q_2 (C \Delta x) \\
&= \Delta x^T C^T Q_2 C \Delta x \\
&= (\Theta u + \Gamma x^0)^T \tilde{Q}_2 (\Theta u + \Gamma x^0)
\end{align*}
$$

Therefore (15) becomes:

$$
J = \frac{1}{2} \{(S u + M x^0)^T \tilde{Q}_1 (S u + M x^0) + (\Theta u + \Gamma x^0)^T \tilde{Q}_2 (\Theta u + \Gamma x^0)\}
$$

$$
= \frac{1}{2} x^0 G x^0 + \frac{1}{2} u^T H u + x^0^T F u
$$