Finite element model of size, shape and blood pressure on rupture of intracranial saccular aneurysms

Jennica Rica Nabong and Guido David
Institute of Mathematics, University of the Philippines, Diliman, Quezon City
E-mail: jennyrica@gmail.com, gdavid@science.upd.edu.ph

Abstract. Rupture of intracranial saccular aneurysms is a primary concern for neurologists and patients because it leads to stroke and permanent disability. This paper examines the role of blood pressure, in connection with size of and wall thickness, in the rupture of saccular aneurysms. A bulb-shaped geometry of a saccular aneurysm is obtained from angiographic images of a patient and modeled using Finite Elements based on the principle of virtual work under the Fung stress-strain relationship. The numerical model is subjected to varying levels of systolic blood pressure. Rupture is assumed to occur when the wall stress exceeded its mechanical strength. The results show which sizes of this class of aneurysms are at high risk of rupture for varying levels of blood pressure.

1. Introduction
Intracranial or cerebral aneurysms are thin-walled, balloon-like dilatations of the arterial wall that occur in an artery located in the front part of the brain that supplies oxygen-rich blood to the brain tissue. Aneurysms occur in one out of twenty people. Cerebral aneurysms, at some stage, may suddenly rupture, causing bleeding within the brain. The incidence of reported aneurysm rupture is about 10 in every 100,000 persons per year in the USA. Aneurysm rupture is one of the primary causes of stroke, and in most cases leads to disability or death. Currently, the two endovascular methods of choice are the deployment of small coils within aneurysms having well-defined necks or the placement of porous intravascular stent for sidewall lesions. The main goal of these treatments is to decrease the risk of subarachnoid hemorrhage, either initially or from a repeated episode of bleeding. However, these treatments are highly dangerous and costly [2],[12],[19]. Depending on the patient’s situation, the doctor will make recommendations for intervention that is appropriate. To help the doctors determine if an aneurysm must undergo treatment, mathematical models of aneurysms are used to predict aneurysm rupture.

In a previous study, an ideal class of spherical aneurysms were modeled using principles of biomechanics applied to biomembranes [3],[15]. The assumption was that rupture occurred when the stresses in the aneurysm wall exceeded the limits of its mechanical strength [1],[2]. The results of the study showed that spherical aneurysms with a diameter of 7 to 8 mm were at risk of rupture when the patient had high blood pressure in the range of 140 to 180 mmHg. Spherical aneurysms with diameter of 9 mm were at risk of rupture for patients with pre-hypertension or blood pressure between 120 to 140 mmHg. Spherical aneurysms with diameter of 10 mm
and above were at risk of rupture for patients with normal blood pressure (120 mmHg and below) [5]. The findings provided general recommendations on which size of aneurysms must be treated under any circumstance, and which aneurysms may be controlled by monitoring the patient’s blood pressure level. The results were in accordance with general recommendations from neurologists.

The present study aims to extend the analytical results of the previous results by considering non-spherical geometry of a saccular aneurysm. The study uses Finite Element methods [6],[4],[17] applied to a model aneurysm obtained from angiographic images, in order to determine the size of aneurysms of this particular shape that are at risk of rupture for various levels of blood pressure.

2. Methods

Let $X$ denote the initial position of a material point in the initial domain $\Omega_0$ and let $x$ denote its position in the deformed configuration $\Omega$. We partition each domain into quadrilateral elements. Let $X^e$ be the vector of the four elemental nodes in the initial domain, and let $x^e$ be the deformed elemental nodes. Then we can write each material point within each element in the initial and deformed configurations as

$$X = N(\eta, \zeta)X^e$$

and

$$x = N(\eta, \zeta)x^e$$

where $N$ is the quadrilateral shape function in terms of local coordinates $\eta, \zeta$ [11],[8]. The local metric tensors in each element are thus given by:

$$G_1 = \frac{\partial N}{\partial \eta}X^e$$

$$G_2 = \frac{\partial N}{\partial \zeta}X^e.$$ (3)

The vectors $G_1$ and $G_2$ are not orthogonal in general. A local orthonormal basis is computed by:

$$t_1 = \frac{G_1}{\|G_1\|}$$

and

$$t_2 = \frac{G_1 \times G_2}{\|G_1 \times G_2\|} \times t_1.$$ (6)

The local orthonormal basis $\{t_1, t_2\}$ implicitly define a coordinate system $s_1, s_2$. The Cauchy-Green strain tensor can be then calculated by:

$$C = \begin{pmatrix}
\frac{\partial x}{\partial s_1} & \frac{\partial x}{\partial s_1} & \frac{\partial x}{\partial s_1} & \frac{\partial x}{\partial s_2} \\
\frac{\partial x}{\partial s_2} & \frac{\partial x}{\partial s_1} & \frac{\partial x}{\partial s_2} & \frac{\partial x}{\partial s_2}
\end{pmatrix}$$

(7)

The study used the finite element formulation based on [8]. Consider a constant pressure $pn$ per unit deformed area acting in the direction of the normal vector $n$ acting on the deformed membrane $\Omega$. The principle of virtual work for a finite element analysis of an inflated membrane is given by

$$\int_{\Omega_0} \delta w dA = \int_{\Omega} pn \cdot \delta x da.$$ (8)
where $w$ is the strain energy function per deformed area, and $\delta x$ is the virtual displacement. The strain energy function $w$ is based on the Fung model, which has been shown to be a reasonable model of the mechanical behaviour of soft tissue [7],[9], is given by

$$w = c(e^Q - 1)$$  \hspace{1cm} (9)

where

$$Q = c_1E_{11}^2 + c_2E_{22}^2 + 2c_3E_{11}E_{22}. \hspace{1cm} (10)$$

Here, $c$ is a material parameter having units of force per length, $c_i$’s are non-dimensional material parameters and $E_{ij}$ are the components of the Green Strain Tensor $E = 0.5(C - I)$. After a change of variables and substituting the values of $n$, (8) can be written as

$$g(\mathbf{x}^e) = \int \int ||\mathbf{G}_1 \times \mathbf{G}_2|| \delta w - pN \left( \frac{\partial N}{\partial \eta} \mathbf{x}^e \times \frac{\partial N}{\partial \zeta} \mathbf{x}^e \right) d\eta d\zeta. \hspace{1cm} (11)$$

The function $g(\mathbf{x}^e) = 0$ was then solved numerically by assembling the elemental nodes as a global vector and iterating the Newton-Raphson method using the stiffness matrix $K(\mathbf{x}^e) = \Delta g(\mathbf{x}^e)$ from an initial value of $\mathbf{x}^e$. The integrals in $g(\mathbf{x}^e)$ and $K(\mathbf{x}^e)$ were evaluated at the Gaussian quadrature points $\eta, \zeta = \pm 1/\sqrt{3}$.

3. Results and Discussion

Images of intracranial saccular aneurysms of patients from the Philippine General Hospital (PGH) were gathered and was viewed using Philips DICOM Viewer® c/o Dr. Carmela Sales. Figure 1 presents the angiographic image showing the aneurysm. A numerical model was constructed based on the geometry of the aneurysm under the assumption that the aneurysm was axially symmetric with respect to the $z-$ axis. For the stability of the numerical model, we considered the quarter surface in the first octant, subject to rolling boundary conditions along the planes that bound the first octant. The numerical model consisted of 433 nodes and 380 quadrilateral elements.

The maximum wall strength, or the stresses on the aneurysm wall when rupture occurs, was obtained from studies that tested the mechanical limits of the arterial or aneurysm wall. This value was reported to be in the range of 0.5 to 2.0 MPa [10],[16]. In this paper, the assumed critical wall stress was 1.5 MPa.

The Fung parameter $c$ as measured in force per length was obtained using $c = c_0H$ where $H$ is the reference thickness of the aneurysm and $c_0$ is the material parameter in force per length squared. Based on previous results, $c_0 = 0.28$ MPa, thus $c = 0.0092$ N/mm [13]. For the other
Figure 2. Height vs. Stress Plot of Aneurysm with Uniform Thickness of Height 10 mm with Blood Pressure from 120 mmHg to 180 mmHg.

Material parameters, best fit values were based on [13],[14],[18], giving us $c_1 = 17.58$, $c_2 = 12.19$, $c_3 = 7.57$.

Intracranial aneurysms are assumed to have a reference thickness $H = 0.033$ mm [1]. Three thickness profiles were assumed: constant thickness ($H_0$), linearly varying with thickest at neck ($H_1$) and linearly varying with thickest at the fundus ($H_2$). The minimum and maximum thickness were assumed to be 0.0165 mm and 0.0495 mm, respectively.

To determine the effects of blood pressure on the size and geometry of the aneurysm, different values of height from 6 to 12 mm were used. Stresses from the circumferential and meridional directions were determined. Plots of wall stresses vs. height were then obtained using MATLAB®. These plots were used to determine the point at which the aneurysms were at risk of rupture. In the graphs, the maximum wall strength of 1.5 MPa was marked with a solid red line to indicate which aneurysms were at risk of rupture. Figure 2 shows the circumferential and meridional stresses for an aneurysm with height 10 mm at blood pressure of 140 mmHg (prehypertension).
Table 1. Summary of numerical pressure test on model aneurysm with height of 6, 8, 10 and 12 mm. An ‘x’ indicates that the aneurysm was at risk of rupture due to stresses exceeding the maximum wall stress of 1.5 MPa in some region in the aneurysm. $H_0$ assumed a constant wall thickness of 0.033 mm, $H_1$ assumed a linearly decreasing thickness with maximum thickness at the neck, and $H_2$ assumed a linearly decreasing thickness with maximum thickness at the fundus.

| Height 6 mm | $H_0$ | $H_1$ | $H_2$ |
|------------|-------|-------|-------|
| 120 mmHg   | -     | -     | -     |
| 140 mmHg   | -     | -     | -     |
| 160 mmHg   | -     | -     | x     |
| 180 mmHg   | -     | -     | x     |

| Height 8 mm | $H_0$ | $H_1$ | $H_2$ |
|------------|-------|-------|-------|
| 120 mmHg   | -     | -     | x     |
| 140 mmHg   | -     | x     | x     |
| 160 mmHg   | -     | x     | x     |
| 180 mmHg   | x     | x     | x     |

| Height 10 mm | $H_0$ | $H_1$ | $H_2$ |
|--------------|-------|-------|-------|
| 120 mmHg     | -     | x     | x     |
| 140 mmHg     | x     | x     | x     |
| 160 mmHg     | x     | x     | x     |
| 180 mmHg     | x     | x     | x     |

| Height 12 mm | $H_0$ | $H_1$ | $H_2$ |
|--------------|-------|-------|-------|
| 120 mmHg     | x     | x     | x     |
| 140 mmHg     | x     | x     | x     |
| 160 mmHg     | x     | x     | x     |
| 180 mmHg     | x     | x     | x     |

Internal blood pressure of 120 mmHg, 140 mmHg, 160 mmHg and 180 mmHg, with a cerebrospinal fluid pressure of 3 mmHg on the external surface. Table 1 shows the effects of blood pressure on the model aneurysm, for heights of 6, 8, 10 and 12 mm and the three thickness assumptions. The x’s denote risk of rupture, either in the circumferential or meridional direction.

Table 1 shows that a small aneurysm with heigh 6 mm was not at risk of rupture under the constant thickness assumption $H_0$ and assumption of linearly decreasing thickness with maximum thickness at the neck $H_1$. However the aneurysm was at risk of rupture at hypertensive levels of greater than 160 mmHg blood pressure under the assumption of linearly decreasing thickness with maximum thickness at the fundus $H_2$. For an aneurysm with height 8 mm, the aneurysm was at risk of rupture at greater than 180 mmHg under the constant thickness assumption $H_0$, at prehypertensive levels greater than 140 mmHg under the $H_1$ assumption, and at blood pressure levels greater than 120 mmHg under the $H_2$ assumption. An aneurysm with height 10 mm was not at risk of rupture only at normal blood pressure (less than 120 mmHg) only for the constant thickness assumption $H_0$. Finally, an aneurysm with height 12 mm or more was at risk of rupture at any blood pressure level above 120 mmHg.

4. Conclusions
An Finite Element model of a bulb-shaped aneurysm obtained from angiographic images was constructed in order to determine what height and blood pressure level increased its risk of rupture. For this study, three types of thickness profiles were assumed: constant thickness, thickest at the neck and decreasing linearly, or thickest at the fundus and decreasing linearly. The
Finite Element membrane model was inflated with pressure using the virtual work principle and assumed the Fung pseudo-strain energy function for biomaterials. The aneurysm was assumed to be at high risk of rupture when its wall stress exceeded the mechanical strength of the aneurysm wall, assumed to be 1.5 MPa. Internal blood pressure from 120 mmHg to 180 mmHg, less the cerebro spinal fluid pressure of 3 mmHg, was numerically applied and corresponding wall stresses of the aneurysm were computed. The results showed that an aneurysm of this geometry was not at risk of rupture when the height was 6 mm, was at risk of rupture at high blood pressure level of greater than 140 mmHg when the height was 10 mm, and was at risk of rupture even at normal blood pressure when the height exceeded 12 mm. This suggested that for this type of geometry and constant thickness assumption, aneurysms below 10 mm in height could safely be controlled by monitoring blood pressure, but an aneurysm with height at least 12 mm should be treated immediately. The study also showed that changing the thickness assumption increased the risk of rupture for the same aneurysm geometry: an aneurysm thickest at the neck with height 10 mm or higher should be treated immediately, while an aneurysm thickest at the fundus with height 8 mm or more should be treated immediately.

The results of the study may provide a guide for neurologists in order to make recommendations seeking treatment of aneurysms based on the height of the aneurysm for this particular geometry type. Further data would be needed in order to validate the findings of the study. Moreover, the study can be extended further by considering other geometries of aneurysms based on additional angiographic images.

Acknowledgments
The authors would like to thank Dr. Carmela Sales of the Philippine General Hospital for providing images for the aneurysm. This project was supported by Department of Science and Technology.

References
[1] Chaudry H, Lott D, Prestigiacomo C and Findley T 2006 Mathematical model for the rupture of cerebral saccular aneurysms through three dimensional stress distribution in the aneurysm wall Center for Applied Mathematics and Statistics Report
[2] Costalat C, Sanchez M, Ambard D, Thines L and Lonjon N 2011 Biomechanical wall properties of human intracranial aneurysms resected following surgical clipping Journal of Biomechanics
[3] David G and Humphrey J 2003 Further evidence for the dynamic stability of intracranial saccular aneurysms Journal of Biomechanics 36
[4] David G and Humphrey J 2007 Finite element model based of stresses in the anterior lens capsule of the eye Computer Methods in Biomechanics and Biomedical Engineering 10
[5] David G and Nabong J 2014 Rupture model of intracranial saccular aneurysms due to hypertension Journal of Mechanics in Medicine and Biology 15
[6] David G, Pedrigi RM, R.Heistand M and Humphrey J 2007 Regional multiaxial mechanical properties of the porcine anterior lens capsule Journal of Biomedical Engineering 129
[7] Fung Y 1993 Biomechanics: Mechanical Properties of Living Tissues Springer-Verlag, New York
[8] Gruttman F and Taylor RL 1992 Theory and Finite element formulation of rubberlike membrane shells using principal stretches International Journal for Numerical Methods in Engineering 35
[9] Humphrey J 2002 Cardiovascular Solid Mechanics Springer, New York
[10] Kroon M, and Holzapfel G 2008 Modeling of saccular aneurysm growth in a human middle cerebral artery Journal of Biomedical Engineering 130
[11] Kyriacou S and Humphrey J 1996 Influence of size, shape and properties on the mechanics of axisymmetric saccular aneurysms Journal of Biomechanics 29
[12] Parlea L, Fahrig R, Holdsworth D, and Lownie S 1999 An analysis of the geometry of saccular intracranial aneurysms American Journal of Neuroradiology 20
[13] Seshaiyer P, Hsu F, Shah A, Kyriacou S and Humphrey J 2001 Multi-meridional mechanical behavior of human saccular aneurysms Computer Methods in Biomechanics and Biomedical Engineering 4
[14] Seshaiyer P and Humphrey J 2001 On the potentially protective role of contact constraints on saccular aneurysms Journal of Biomechanics 34
[15] Shah A and Humphrey J, 1999 Finite strain elastodynamics of intracranial saccular aneurysms Journal of Biomechanics 32
[16] Sommer G, Mechanical properties of healthy and diseased human arteries Monographic Series TU Graz
[17] Toth B, Raffai G, and Bojtar I 2005 Analysis of the mechanical parameters of human brain aneurysm Acta of Bioengineering and Biomechanics 5
[18] Valencia A, Amd PT, Rivera R, Galvez M and Bravo E 2009 A mechanical study of patient-specific cerebral aneurysm models: The correlations between stress and displacement with geometrical indices Mechanics Research Communications 36
[19] Wolfe S, Baskaya M, Heros R, and Tummala R 2006 Cerebral aneurysms: learning from the past and looking toward the future Clinical Neurosurgery 53