Fano–Kondo and the Kondo box regimes crossover in a quantum dot coupled to a quantum box

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Abstract

In this work, we study the Kondo effect of a quantum dot (QD) connected to leads and to a discrete set of one-particle states provided by a quantum box represented by a quantum ring (QR) pierced by a magnetic flux side attached to the QD. The interplay between the bulk Kondo effect and the so-called Kondo box regime is studied. In this system the QR energies can be continuously modified by the application of the magnetic field. The crossover between these two regimes is analyzed by changing the connection of the QD to the QR from the weak to the strong coupling regime. In the weak coupling regime, the differential conductance develops a sequence of Fano–Kondo anti-resonances due to destructive interference between the discrete quantum ring levels and the conducting Kondo channel provided by the leads. In the strong coupling regime the differential conductance has very sharp resonances when one of the Kondo discrete sub-levels characterizing the Kondo box is tuned by the applied potential. The conductance, the current fluctuations and the Fano coefficient result as being the relevant physical magnitudes to be analyzed to reveal the physical properties of these two Kondo regimes and the crossover region between them. The results were obtained by using the slave boson mean field theory (SBMFT).

(Some figures may appear in colour only in the online journal)

1. Introduction

Electronic transport through nanoscopic systems, as quantum dots (QDs), has been extensively studied in the past decade [1–4]. The QDs allow the systematic study of quantum-coherent effects such as the Kondo, Fano and Aharonov–Bohm effects due to the possibility of continuously tuning the relevant parameters governing the properties of these regimes, in equilibrium and nonequilibrium situations [1, 4–10].

The Kondo effect in QDs dominates the electronic transport properties due to the strong many-body correlations between the conduction band electron spins in the leads and the localized spin in the QD, when the temperature $T$ is below the Kondo temperature $T_K$ [2, 11]. The main signature of the Kondo state in nanosystems as a QD connected to two leads is the enhancement of the conductance below $T_K$ up to the unitary limit $(2e^2/h)$ [2]. In this configuration electrons transmitted from one electrode to the other necessarily pass through the QD. It is presently known that coupled quantum dots exhibit the electronic counterpart of the Fano and Dicke effects and that they can be controlled by the magnetic flux and quantum-dot asymmetry [12–15]. The emergence of Fano resonances requires two scattering channels: a discrete level and a broad continuum band [5]. The present understanding of electron transport properties of quantum dots is based mainly on direct current measurements. However, additional information can be obtained from fluctuations of the current [16–24]. The electronic current through any conductor fluctuates with time, reflecting charge granularity. This phenomenon is referred in the literature as shot noise. It has been demonstrated
that electron shot noise provides a useful tool to detect the role played by electron coherence and Coulomb interactions and correlation in electronic transport through quantum dots [17, 24]. Also, it provides information about current fluctuations that cannot be extracted from the average current alone [22, 23].

There have been theoretical [25–27] and experimental studies [28, 29] of the Kondo phenomena when the QD is connected to a quantum box, where the uncorrelated conducting electrons are described by a series of discrete energy levels. The finite system density of states consists of a series of peaks separated by an energy $\Delta$ inversely proportional to the number of sites $N$ and, in a tight binding representation of the discrete system, proportional to the hopping matrix elements $v$ that connects the sites. The universal exponential dependence with the system parameters of the Kondo temperature is established in this case in the limit when the energy separation of the discrete levels of the quantum box is much less than the characteristic Kondo temperature that this system would have if the impurity were connected to a continuum conduction band. The behavior of a Kondo box, from the discrete to the continuum limit when the size of the box tends to infinity, has been studied [25, 30]. For the case of a QD connected to a quantum box able to be represented in the Fermi region by only one semi-occupied level, the energy of the singlet ground state is less than the energy of the excited triplet state by an amount that can be associated with a $T_k$ of this discrete system. This $T_k$ can be expanded in powers of the parameter $y = \frac{t}{\epsilon_{L,R}}$, where $t$ is the non-diagonal hopping matrix elements connecting the QD to the quantum box and $\epsilon_{L,R}$ is the energy of the local QD level [31].

In the present work, we investigate the transport properties of a strongly correlated QD attached to two leads and to a quantum ring (QR) pierced by a magnetic flux, which in fact acts as a quantum box, whose energies can be continuously modified by the application of the magnetic field. This system is conditioned by the interplay between two different Kondo effects, the bulk Kondo regime that results from the connection of the QD to the leads and the Kondo box regime due to the interaction between the QD and the ring. Manipulating the parameters, the system presents very interesting unexplored crossover behavior. A schematic model of the structure proposed is shown in figure 1. We draw attention to the fact that this system possesses similarities to the one proposed to study the properties of a two-channel Kondo system [28]. However, the physics we are analyzing is completely different because our system does not have two independent channels, as in this case the electrons can hop from the continuous to the discrete reservoir without any restrictions. There is no Coulomb correlation among the electrons within the QR to impede the free entrance of an electron when one of the discrete levels is in the vicinity of the Fermi energy [28].

The crossover between the continuous Kondo and the Kondo box regimes can be studied by manipulating the connection of the QD to the leads and to the QR. The conductance, the current fluctuations and the Fano coefficient result as being the relevant physical magnitudes to be analyzed in the parameter space to reveal the physical properties of these two Kondo regimes and the crossover region between them. The physics of the QR acting as a quantum box and the crossover between the two regimes depend upon the spacing of the QR states in comparison with the Kondo temperature and on the quantum box being at resonance (off-resonance) when one of its states is (is not) in the neighborhood of the Fermi level. This last condition can be changed continuously by simply modifying the magnetic flux threading the ring.

The results are obtained using the SBMFT [8, 32, 33] approach, which is able to adequately describe the properties of the Kondo regime at very low energies, far away from the mixed-valence regime.

The paper is organized as follows. In section 2, we present the model, the Hamiltonian we used to study it, the central concepts regarding the SBMFA and the derivation of the analytical expression for the current $I$ and shot noise $S$. In section 3, we briefly discuss the results for the local density of state (LDOS) and the transport properties corresponding to the different regimes of the system. Finally, we elaborate a summary of the paper in section 4.

2. Model

We consider a QD with e–e interactions, connected with left (L) and right (R) leads, and a QR with $N$ sites. The Hamiltonian of the system outlined in figure 1 can be written as

$$H = H_{\text{Lead}} + H_{\text{QR}} + H_{\text{QD}} + H_{\text{L–QD}} + H_{\text{QD–QR}}. \quad (1)$$

where the different sub-Hamiltonian $H_{\beta}$ are given by

$$H_{\text{Lead}} = \sum_{k_{\mu \sigma} \alpha \in L,R} \epsilon_{k_{\mu \sigma} \alpha} c_{k_{\mu \sigma} \alpha}^\dagger c_{k_{\mu \sigma} \alpha} + \text{h.c.}$$

$$H_{\text{QR}} = \sum_{\alpha \sigma \rho} \sum_{m \sigma} \epsilon_{m} (\Phi) a_{m \sigma}^\dagger a_{m \rho} \sigma$$

$$H_{\text{QD}} = \epsilon_{0}(n_{\uparrow} + n_{\downarrow}) + \sum_{\alpha} U n_{\alpha} n_{\bar{\alpha}} \quad (2)$$

$$H_{\text{L–QD}} = \sum_{k_{\mu \alpha} \in L,R} \sum_{\sigma} t_{\alpha \sigma} c_{k_{\mu \alpha} \sigma}^\dagger d_{\sigma} + \text{h.c.}$$

$$H_{\text{QD–QR}} = \frac{t}{\sqrt{N}} \sum_{m,\sigma} a_{m \sigma}^\dagger d_{\sigma} + \text{h.c.}$$

![Figure 1. Schematic diagram of quantum dot attached to two leads and coupled to a quantum ring (quantum box) pierced by a magnetic flux (color online).](image-url)
where \( a_{\sigma m} (a_{m \sigma}) \) creates (destroys) an electron in the QR with energy \( \epsilon_m \) with spin \( \sigma \), with \( \epsilon(\Phi) = -2v \cos(\sqrt{2}\Phi/\Phi_0 + m) \) \((m = 1, \ldots, N)\) and where \( \Phi_0 = \hbar/e \) is the quantum of flux. The operator \( c_{k,\sigma}^\dagger (c_{k,\sigma}) \) creates (destroys) an electron with momentum \( k \) and spin \( \sigma \) in the lead \( \alpha \) and \( d_{\sigma}^\dagger (d_{\sigma}) \) creates (destroys) an electron with spin \( \sigma \) in the quantum dot. The parameter \( \epsilon_0 \) represents the QD energy level and \( U \) the Coulomb repulsion between the electrons in the QD. The coupling between the dot and the lead \( \alpha \) is \( t_{\alpha} \) and \( t \) is the QD–QR tunneling interaction.

We consider the density of states describing the left (right) lead \( \rho_{L(R)} \) as being constant and equal to \( 1/D \), where \( D \) is the lead bandwidth. The coupling strength between the QD and the leads is given by \( \Gamma_L(R) = \pi \rho_{L(R)} \frac{\pi \hbar^2}{2m} \).

In the wide-band limit (\( D \gg 1 \)) and for infinite Coulomb repulsion (\( U \to \infty \)), \( T_k = e^{-\pi |n|/\Gamma} \) [33]. It is necessary that \( T < T_k \) for the system to have a Kondo behavior. When the temperature \( T > T_k \), the spin correlations between the localized electron and the conduction electrons are eliminated by thermal fluctuation, driving the system out of the Kondo regime. The Kondo ground state corresponds to a situation in which the QD is occupied by one electron, which requires \( \epsilon_0 < -\Gamma \) and \( \epsilon_0 + U > \Gamma \). Otherwise, charge fluctuations are significant and the system is in a mixed valence regime.

The intra-dot Coulomb interaction \( U \) is not a relevant parameter for the Kondo effect if the QD is in the one-electron Coulomb blockade regime. Therefore, we consider the \( U \to \infty \) limit as it simplifies the self-consistent calculations, because the QD in this case has only three available states: the empty state or the spin up or down electron occupied states [33]. It is convenient to rewrite the Hamiltonian \( H \), projecting out the double QD occupancies. In order to do this, we use the slave boson approach [32]. We define the operators \( f_{\sigma}^\dagger (f_{\sigma}) \) that create (destroys) the pseudo-fermion of \( \sigma \) spin in the QD and \( b^\dagger (b) \) that creates (destroys) an empty pseudo-Bose state and impose a constraint that explicitly eliminates the double occupied state,

\[
b^\dagger b + \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} = 1. \tag{3}\]

The use of an auxiliary field \( \lambda \) which acts as a Lagrange multiplier permits one to impose the condition equation (3). The Hamiltonian takes the form

\[
H = \sum_{k,\sigma,\alpha} \epsilon_{k,\sigma} c_{k,\sigma}^\dagger c_{k,\sigma}\sigma + \sum_{\sigma} \epsilon_0 f_{\sigma}^\dagger f_{\sigma}
+ \sum_{m} \epsilon_m(\Phi) a_{m \sigma}^\dagger a_{m \sigma} + \frac{1}{2} \sum_{k,\sigma,\alpha=L,R} t_{\alpha} c_{k,\sigma}^\dagger b^\dagger f_{\sigma}
+ \frac{\lambda}{\sqrt{2N}} \sum_{m} a_{m \sigma}^\dagger b^\dagger f_{\sigma} + \lambda \left( b^\dagger b + \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} - 1 \right). \tag{4}\]

Within the SBMFT approach [33], we replace the bosonic operator by its expectation value \( \langle b \rangle = \sqrt{2\lambda} \) and assume that \( \langle b^\dagger b \rangle \simeq \langle b^\dagger \rangle^2 = 2\lambda^2 \). At temperature \( T = 0 \), SBMFT describes quite well the Kondo regime characterized by strong spin fluctuation, but it is not reliable in the charged fluctuating mixed valence region. This confines our analysis to \( \epsilon_0/\Gamma_L + \Gamma_R < -0.5 \). Within this formalism, the Hamiltonian becomes a one-body effective one, \( \tilde{H} \).

\[
\tilde{H} = \sum_{k,\sigma,\alpha} \epsilon_{k,\sigma} c_{k,\sigma}^\dagger c_{k,\sigma}\sigma + \sum_{\sigma} \epsilon_0 f_{\sigma}^\dagger f_{\sigma}
+ \sum_{m} (t_{\alpha} c_{k,\sigma}^\dagger f_{\sigma} + \text{h.c.}) + \frac{i}{\sqrt{N}} \sum_{m,\sigma} (a_{m \sigma}^\dagger f_{\sigma} + \text{h.c.})
+ \lambda (2\lambda^2 - 1) \tag{5}\]

where \( \tilde{\Gamma}_{L(R)} = t_{L(R)} \tilde{t} = i\tilde{t} \) and \( \tilde{\epsilon}_0 = \epsilon_0 + \lambda \) are renormalized parameters. By minimizing the mean value \( \langle \tilde{H} \rangle \) with respect to \( \lambda \) and \( \tilde{t} \), we obtain the following equations for the parameters,

\[
\tilde{b}^2 = \frac{1}{2} - \frac{1}{2} \sum_{\sigma} \langle f_{\sigma}^\dagger f_{\sigma} \rangle,
4\lambda \tilde{b}^2 + \frac{i}{\sqrt{N}} \sum_{m,\sigma} \langle (a_{m \sigma}^\dagger f_{\sigma}) + (f_{\sigma}^\dagger a_{m \sigma}) \rangle \tag{6}\]

By using the Keldysh–Langreth [8, 34, 35] formalism, we obtain the QD lesser Green function \( G_{d,\sigma}^<(t_2 - t_1) = i \langle f_{\sigma}^\dagger f_{\sigma}(t_2) f_{\sigma}(t_1) \rangle \) and close the equations (6). In the Fourier space the above equations are given by

\[
\tilde{b}^2 = \frac{1}{2} - \frac{i}{4\pi} \sum_{n} \int_{-D}^{D} G_{d,\sigma}^<(\omega - \tilde{\epsilon}_0) G_{d,\sigma}^<(\omega) d\omega \tag{7}\]

which are solved self-consistently. The function \( G_{d,\sigma}^< \) and the retarded Green’s function \( G_{d,\sigma}^r \) are obtained using Langreth analytic continuation and the D'yon equation respectively [35]

\[
G_{d,\sigma}^<(\omega) = \frac{2i}{\omega - \tilde{\epsilon}_0 - \frac{\langle \tilde{\epsilon}(\omega) \rangle}{N}} + (\tilde{\Gamma}_L + \tilde{\Gamma}_R)^2 \tag{8}\]

where \( \tilde{\Gamma}_{L(R)} = \tilde{\lambda} \tilde{\Gamma}_{L(R)} \) and following [36], we obtain a closed relation for \( Q(\omega) \),

\[
Q(\omega) = \sum_{n=1}^{N} \frac{n^2}{\omega - \epsilon_n(\Phi)}
\frac{\sin(N) \sqrt{2v \sin k \cos(2\pi \Phi/\Phi_0) - \cos(N)}}{2v} \tag{9}\]

with \( k = \cos(\omega/2v) \).

In order to characterize the different regimes of the system and its transport properties, it is important to calculate the DOS at the QD. It is given by the expression

\[
\rho_{QD}(\omega) = -\frac{1}{\pi} \text{Im} G_{d,\sigma}(\omega). \tag{10}\]
On the other hand, the transmission probability $T(\omega, V)$ can be written as

$$T(\omega, V) = 4 \frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R} \text{Im}[G_{d,\sigma}(\omega)].$$

(11)

Note that the transmission depends on the applied bias $V$ as it appears in the self-consistent equations (7). The transport properties are studied at temperature $T = 0$. We calculate the current $I$ transmitted through the QD, the shot noise $S$ and the Fano factor $FF = S/2eI$. The current $I$ and the shot noise $S$ are given by [37, 38]

$$I = \frac{2e}{h} \int_{-\sqrt{V/2}}^{\sqrt{V/2}} T(\omega, V) \, d\omega,$$

$$S = \frac{4e^2}{h} \int_{-\sqrt{V/2}}^{\sqrt{V/2}} T(\omega, V) [1 - T(\omega, V)] \, d\omega.$$

(12)

3. Numerical result

In this section we discuss the transport properties at zero temperature $T = 0$. In what follows, we will consider $\Gamma_L = \Gamma_R = \Gamma$ as the energy unit and $E_F = 0$. We have taken the bandwidth to be $D = 35$ and set the QD energy level at $\varepsilon_0 = -3$. For these values $T_K = 1.4 \times 10^{-3}$ and $\sum_{\sigma} (j_f^2,j_0) = 1 - T_K \approx 1$.

As already discussed in the introduction, in the Kondo box regime, the Kondo temperature of the equivalent continuous system is less than, or of the order of the energy separation of the quantum box states. In this regime, supposing an even number of electrons in the system, the energy difference between the ground singlet Kondo-like state and the first excited triplet state is given by $4r^2/\varepsilon_0$ [31], which permits this value to be associated with the Kondo temperature of the quantum box. As a consequence, to characterize the regime of the system it is convenient to compare this typical energy with the Kondo temperature that results from the connection of the QD with the leads. As schematically shown in figure 2, this permits us to define a weak coupling regime when $t \ll \xi$ and strong coupling regime when $t > \xi$, with $\xi = \sqrt{\varepsilon_0 T_K}$. Roughly speaking, $\xi$ is the critical value around which the QD–QR anti-ferromagnetic correlation becomes relevant, leading to a qualitative change in the transport properties of the system.

As far as the quantum box is concerned the transport properties are affected by the QR energies $\varepsilon_n(\Phi)$ and their proximity to the Fermi level. The values of $\varepsilon_n(\Phi)$ are controlled by $\Phi$, as shown in figure 3, while the spacing of the levels is dependent on the magnitude $v/N$, with $N$ being the number of sites of the ring and $v$ the hopping matrix element connecting them. As for numerical reasons it is better to take a small value of $N$, we arbitrary assume $N = 5$ and a value of $v, v = 0.35T_K$, such that the few QR state energies are within the width of the Kondo peak of the QD attached to the leads and unconnected to the ring. This is the condition for the properties of the Kondo regime, derived from the coexistence of a continuous and a discrete bath, to appear. Other possible states of a longer ring, far away from the Fermi level, are not relevant for the physics we analyzed. We assume various magnetic fluxes $\Phi$ so as to manipulate the ring states relative to the Fermi level, studying in particular the situations when one state of the quantum ring is half occupied or the nearest states to the Fermi level are double occupied or unoccupied.

In figures 4(a)–(d) we display the LDOS at the QD for weak, intermediate and strong coupling regimes respectively for $\Phi = 0$. In the weak coupling regime the LDOS is slightly perturbed by the ring states appearing as Fano anti-resonances in an otherwise clear Kondo peak that results from the QD connected to its left and right leads. This behavior can be understood by inspection of equation (8). The anti-resonances appear at the ring state energies, which are the poles of $Q(\omega)$. When the interaction with the ring increases, the original Kondo peak is now highly perturbed and the DOS is now characterized by resonances that correspond to the poles of Green’s function and by two side bands with energies that...
get increasingly distant from the Fermi region as $t$ increases. These side bands correspond originally to the continuum Kondo peak that, once split, is pushed away from the Fermi region. Well inside the strong coupling regime in the Fermi region there are several discrete resonances of almost zero width while the continuum has been spread out from the Fermi region. This is clearly depicted in figures 4(c) and (d). In this regime the impurity spin is completely screened by the spin of QR electrons near the Fermi level and there is no Kondo spin–spin correlation with the conducting electrons in the leads. It is very interesting to properly characterize these two regimes and the crossover between them. When the system is in the Kondo regime the renormalized local level of the QD, within the slave boson formalism, is fixed at the Fermi level independently of the gate potential applied to the QD, indicating the existence of a peak, the Abrikosov–Shul resonance. This characteristic of the renormalized $\tilde{\varepsilon}_0$ is the fingerprint of the Kondo regime as far as the SBMFT formalism is concerned [39]. In figure 6 we show the behavior of $\tilde{\varepsilon}_0$ as a function of $\varepsilon_0$. It is clear from the figure that, independently of the system being in the weak or strong coupled regimes, the local level is renormalized to zero when $\varepsilon_0$ is below the Fermi energy. In the weak coupled regime, as is clearly shown by the continuous resonance at the Fermi level of the DOS, the system is in the traditional bulk Kondo regime. In the strong coupling regime, the Kondo peak appears as a bunch of discrete levels highly concentrated at the Fermi level, reflecting the fact that the system is in a Kondo-like regime corresponding to an impurity connected to a quantum box, which has been named a Kondo box. As $\Phi = 0$ does not give rise to semi-occupied state of the QR because no ring state coincides with the Fermi energy, the screening of the QD spin by the QR spins is possible through virtual excitations of one electron of the nearest to the Fermi energy double occupied ring state to the nearest unoccupied one [25]. This argument is not valid in the presence of an external applied potential if one of the level is tuned to be within the left and the right Fermi level, in which case the state is semi-occupied and no virtual excitations are required to screened the QD spin.

We have a quantitatively different situation when $\Phi = \Phi_0/4$, in which case the states in the ring are not degenerate and are symmetrically distributes above and below the Fermi level—with one of them, just at the Fermi level, occupied by only one electron, as depicted in figure 3 and in the corresponding LDOS at the QD shown in figure 5. In this case the emergence of the Kondo box regime is easier, and is reached for smaller values of $t$ [25]. Within the scope of the SBMFT formalism this is the case because the

Figure 4. Density of states as a function of energy with magnetic flux $\Phi = 0$. In panel (a) $t = 0.1\xi$ (solid black line) and for disconnected QR, $t = 0$ (red dashed line). In panel (b) $t = \xi$ and in panel (c) $t = 10\xi$. Panel (d) shows in detail the two central Lorentzian peaks for $t = 10\xi$ (color online).

Figure 5. Density of states as a function of energy with magnetic flux $\Phi = \Phi_0/4$. In panel (a) $t = 0.1\xi$ (black solid line) and for the disconnected QR, $t = 0$ (red dashed line). In panel (b) $t = \xi$ and in panel (c) $t = 10\xi$. Panel (d) shows in detail the two central Lorentzian peaks for $t = 10\xi$ (color online).
renormalized energy $\tilde{\varepsilon}_0$ of the QD state interacts strongly with the QR state at the Fermi level, as both have the same energy, producing a bonding and an anti-bonding state, as can be seen in figure 5(d). The anti-bonding state will be double occupied by two opposite spin electrons. This is the mechanism that can explain the anti-ferromagnetic correlation between the QD spin and the spin of the electron populating the state at the Fermi level of the QR, which gives rise to the Kondo box regime. However, as far as the essential physics we are analyzing is concerned, the behavior of the system in the strong coupling regime is only quantitatively weakly dependent on $\Phi$, as can be concluded by inspection of figures 4 and 5.

In order to adequately characterize the different regimes of the system, we calculate the current $I$ and the shot noise $S$ as a function of the potential bias $V$ for different coupling regimes. Figure 7 displays $I$ ($S$) in the upper-left (upper-right) panels restricted to the $\Phi = 0$ case. In the weak and intermediate regimes, the current and shot noise increase smoothly with the bias voltage. However, in the strong coupling regime, $S$ and $I$ have steps for particular values of the external potential, the positions of which depend upon $\Phi$. When the region between the left and right Fermi level includes a discrete local Kondo-like peak, the current is able to flow increasing abruptly. Although this discrete state, localized at the QD, as shown in figures 4 and 5, is a resultant of the screening produced by the free spins in the QR, it provides a path for the electrons to travel along, showing that this state has a superposition with the leads. In the first plateau, the shot noise is $S = 2eI$ and in the last two $S = eI$ (figure 7 (upper-left panel)). The above results show that the measurement of the current and the noise permits one to make a clear distinction and characterization of the regimes of the system. In order to complete the description of the transport properties, we study the Fano factor of the system. Figure 7 (lower panel) displays the Fano factor as a function of bias $V$ for two values of the magnetic flux, (g) $\Phi = 0$ and (h) $\Phi = \Phi_0/4$, for different QD–QR coupling in the range $0.4 \xi < t < 10 \xi$. In the strong coupling regime, the Fano factor has two plateaus and drops from FF = 1 to 1/2 at $V \approx 0.4 T_k$. We observe this behavior for various values of the magnetic flux, showing that the Fano factor is independent of the flux in the Kondo box regime. On the other hand, in the weak coupling regime ($t = 0.1 \xi$) the Fano factor profile depends strongly on the magnetic flux. As is shown in figure 7(g), for $\Phi = 0$ and $V \ll T_k$ the Fano factor is FF $\approx 1$, while in figure 7(h) for $\Phi = \Phi_0/4$, FF $\approx 1$. In the strong coupling regime $S = 2eI$ in the first plateau (see figure 7) and hence FF = 1, while in the last two plateaus $S = eI$, or equivalently FF = 1/2.

In the strong coupling regime, the above behavior is a consequence of the properties of the transmission probability $T(\omega, V)$. In the first plateau $T(\omega, V) \approx 0$, then

$$[1 - T(\omega, V)] \times T(\omega, V) \approx T(\omega, V).$$

From equations (12) and (13), it follows that in the first plateau $S = 2eI$. In the Kondo box regime, we can represent the transmission probability by a superposition of Breit–Wigner resonances,

$$T(\omega, V) = \sum_{\alpha} \frac{\eta^2}{(\omega - \varepsilon_\alpha)^2 + \eta^2},$$

Figure 6. Renormalized energy versus gate voltage for the weak coupling regime, $t = 0.4 \xi$ (black solid line) and the strong coupling regime, $t = 10 \xi$ (red dashed line) for $\Phi = 0$ (color online).

Figure 7. Current (upper-left panel) and shot noise (upper-right panel) versus bias voltage $V$, for magnetic flux $\Phi = 0$, and (a), (b) $t = 0.1 \xi$, (c), (d) $t = \xi$ and (e), (f) $t = 10 \xi$. Fano factor versus bias (lower panel) for two values of the magnetic flux, (g) $\Phi = 0$ and (h) $\Phi = \Phi_0/4$, for the weak, intermediate and strong coupling regimes, $t = 0.2 \xi$ (red dashed–dotted line), $t = 0.6 \xi$ (blue dotted line), $t = \xi$ (orange dashed line) and (solid black line) $t = 10 \xi$ (color online).
where $\tilde{\epsilon}_n$ and $\tilde{\eta}$ are the positions and width of the Kondo box resonances, respectively. Then, from equations (12) and (14) with $-V < \tilde{\epsilon}_n < V$, the current $I$ and shot noise $S$ can be written as follows,

$$I = \frac{4e}{h} M \tilde{\eta} \arctan \left( \frac{V}{\tilde{\eta}} \right)$$

$$S = \frac{4e^2}{h} M \tilde{\eta} \left[ \frac{(\frac{V}{\tilde{\eta}})^2 + 1}{\eta} \right] \arctan \left( \frac{V}{\tilde{\eta}} \right) - \frac{V}{\eta} \left( \frac{V}{\tilde{\eta}} \right)^2 + 1,$$  \hspace{0.5cm} (15)

where $M$ is the number of resonances in the range of the applied bias. In the limit $V/\tilde{\eta} \gg 1, I \approx 2\pi e \eta / h$ and $S \approx 2\pi e^2 \eta^2 / h$, or equivalently $S = eI$ and $\text{ FF } = 1/2$, in agreement with the numerical results.

When the magnetic flux is $\Phi \approx \Phi_0/4$ the side coupled QR, as can be observed in figure 3, provides a state with energy $\epsilon_n(\Phi_0/4) = E_F = 0$ that can be visited by an electron that interferes destructively with the electron that goes directly through the QD. As a consequence the current intensity decreases and the Fano factor reaches its maximal allowed value ($\text{ FF } = 1$). Away from the maximum, all the QR energy levels are far enough from the Fermi energy and the direct transport through QD predominates without any destructive interference. Then the ratio $S/2eI \ll 1$ and $\text{ FF } \approx 0$. By increasing the bias up $V = T_k$, the Fano factor becomes almost independent of the magnetic flux. As the Fano factor is almost independent of the magnetic flux in the strong coupling Kondo box regime, as discussed above, the experimental measurement of this factor is another clear way of characterizing the system.

The above behavior is reflected in the differential conductance $\text{ d}I/\text{ d}V$, which is depicted in figure 8 as a function of the bias. In the weak coupling regime, we can see that the differential conductance displays Fano anti-resonances any time that a level of the QR enters in the region between the left and the right Fermi energies. However, in the strong coupling regime the differential conductance shows clear peaks corresponding to the steps in the current that were shown in figure 7.

We demonstrate that the differential conductance, the shot noise and the Fano coefficients are all measurable properties that allow a very detailed characterization of the two Kondo regimes and the crossover region that the system goes through by manipulating its parameters.

4. Summary

In this work, we investigate the properties of the very interesting Kondo phenomena that result from the interplay between the traditional bulk Kondo effect and the so-called Kondo box regime. We study the transport properties of a strongly correlated quantum dot attached to two leads and to a quantum ring (QR) pierced by a magnetic flux. In this system the QR acts as a quantum box, whose energies can be continuously modified by the application of the magnetic field. The crossover between these two regimes was studied by changing the connection of the QD to the leads and to the QR.

In the weak coupling regime, we have shown that the differential conductance develops a sequence of Fano–Kondo anti-resonances as a consequence of destructive interference between the $N$ discrete quantum ring levels with the conducting Kondo channel provided by the leads. In the strong coupling regime the differential conductance has very sharp resonances when one of the Kondo discrete sub-levels characterizing the Kondo box is tuned by the applied potential. We were able to demonstrate that the conductance, the current fluctuations and the Fano factor result as being the relevant physical magnitudes to be analyzed in the parameter space to reveal the physical properties of these two Kondo regimes and the crossover region between them.

The transport properties of this system are extremely dependent upon its parameters, which could have interesting potential applications as an active part of a nanoscopic circuit.

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