Gamow states and continua in the cluster-orbital shell model approach

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Abstract. Importance of the unbound states in loosely bound systems by comparing to the stable nuclei is investigated. We use the cluster-orbital shell model (COSM) approach and expand the wave function using the complete set of the single-particle states. The completeness relation is constructed by the Berggren metrics, which includes bound, resonant and anti-bound states, and continua. We precisely investigated such the contributions of the resonant states (Gamow states) and continua in the helium isotopes and compare them those obtained by the Gamow shell model.

1. Introduction

The front-line of the research field of nuclear physics has been extended toward the drip-line of the neutron and proton, and now it goes on to beyond the line. In the drip-line regions, it has been revealed that the knowledge which constructs the nuclear physics mainly on the stable nuclei cannot be directly applied to the study of drip-line nuclei, since the mechanism of nuclear system in such nuclei becomes different very much from the stable one. Further, due to the weakness of the binding energy, the role of unbound states becomes important and coupling to the unbound states should be taken into account to describe the systems. In order to understand new phenomena and properties of such the drip-line nuclei, it is necessary to develop a new approach, which is suitable and has an ability to treat the typical features of these nuclei.

One of the important feature to treat the nuclear many-body system is the nucleon-nucleon correlation in the system. In the ground and low-lying excited states, a mean-field description has been employed as a zeroth order approximation to treat nuclear many-body correlations. An explicit treatment of bound-continuum and continuum-continuum couplings has been developed. The Gamow shell mode [1, 2, 3, 4, 5, 6, 7] is one of the most successful approaches to treat the coupling to the continuum. In GSM, the completeness relation is defined by using the Berggren metric [8, 9] on the complex momentum plane with the bound, resonant, and continuum states. The two-body matrix elements of GSM involving the Gamow states can be calculated with a careful treatment of the integration path of the coordinates. However, it becomes a problem that the number of valence nucleon increase the very rapid increase of number of basis sets as a function of the valence nucleon.

Recently, we proposed an extended cluster-orbital shell model approach [10, 11], and study oxygen and helium isotopes with the COSM formalism in a core+XN model space. In our
extended COSM approach, we performed a stochastic variational calculation to solve the motion of the valence nucleons around the core. The important point of our approach is that the contributions of the unbound states are correctly taken into account to the wave function in terms of the superposition of the Gaussian functions. In order to show the above feature of contributions of the unbound states are correctly taken into account to the wave function in order that the valence nucleons around the core. The important point of our approach is that the Gaussian products with different width parameters as follows:

\[ \Phi_V = \sum_m c^{(m)} A \left\{ F^{(m)}(r_1, \ldots, r_N) \cdot |JMTM_t^{(m)}| \right\}. \]

Here, the radial part \( F^{(m)}(r_1, r_2, \cdots) \) is the products of the Gaussian functions as follows:

\[ F^{(m)}(r_1, \ldots, r_N) \equiv g_1^{(m)}(r_1) \cdots g_N^{(m)}(r_N). \]  

The angular momentum part is constructed by the normal coupling scheme. The width parameter of the Gaussian and angular momenta of the single-particle basis are generated as the parameters, and the parameter sets are chosen by using a sort of the stochastic variational technique, in order to reduce the number of basis sets. Since the function \( F^{(m)}(r_1, r_2, \cdots) \) is the products of the Gaussian functions, the components of the continuum states of single-particle wave functions are implicitly included. In the viewpoint of the asymptotic condition, the linear combination of \( F^{(m)}(r_1, r_2, \cdots) \) has the \( N \)-body bound state boundary condition.

3. Energies and r.m.s. radii

3.1. Oxygen isotopes

We construct the interaction between the core and a valence nucleon (the core-\( N \) interaction) microscopically by folding a nucleon-nucleon interaction with the core wave function. We assume the lowest configuration of the harmonic oscillator wave function of \( ^{16}O \) and use the core-size parameter \( b_C = 1.723 \text{ fm} \) so as to reproduce the r.m.s. radius of \( ^{16}O \) within the experimental error bars. The Pauli principle between nucleons in the core and a valence one is treated in terms of the orthogonality condition model (OCM). As an conventional method to eliminate the forbidden states, we use a pseudo potential \( \lambda \Lambda_i = \lambda |F.S.\rangle \langle F.S.| \) where \( |F.S.| \) stands for the Pauli forbidden states. As taking \( \lambda \rightarrow \infty \) in the calculation, the forbidden states are effectively eliminated. Additionally, we introduce an effective \( LS \) potential \( \hat{V}^{ls} \) to reproduce the energy splitting of the low-lying positive parity states \( 5/2^+ \) and \( 3/2^+ \) of \( ^{17}O \). The Hamiltonian of the core-\( N \) part for the \( i \)th valence nucleon \( \hat{h}_i \) becomes as follows:

\[ \hat{h}_i = \hat{\mathcal{T}}_i + \hat{V}_i^{nd} + \hat{V}_i^{ex} + \hat{V}_i^{ls} + \lambda \Lambda_i. \]  

For the two-body interaction \( \hat{v}_{ik} \), we use the Volkov No.2 interaction [12]. The parameters of the interaction is determined so that the energies of the lowest three positive parity states of \( ^{17}O \); \( 5/2^+ \), \( 1/2^+ \) and \( 3/2^+ \), and the gourd state of \( ^{18}O \) are reproduced. Calculated one-neutron separation energies \( (S_n) \) and r.m.s. radii \( (R_{rms}) \) are shown in Table 1. Both \( S_n \) and \( R_n \) are in a good agreement with the observed values.
Table 1. Calculated $S_n$ and $R_{rms}$ of oxygen isotopes and experimental data.

|       | $^{17}$O | $^{18}$O | $^{19}$O | $^{20}$O |
|-------|----------|----------|----------|----------|
| $S_n$ (MeV) |          |          |          |          |
| Calc. | 4.143    | 8.048    | 4.204    | 7.331    |
| Exp. [13, 14] | 4.143    | 8.044    | 3.957    | 7.607    |
| $R_{rms}$ (fm) |          |          |          |          |
| Calc. | 2.59     | 2.64     | 2.67     | 2.71     |
| Exp. [15] | 2.59 ± 0.05 | 2.61 ± 0.08 | 2.68 ± 0.03 | 2.69 ± 0.03 |

3.2. Helium isotopes

Next, we show calculated energies and r.m.s. radii for helium isotopes as an example of weakly bound nuclei. In this calculation the $^4$He-core is considered to be inert and has the $(0s)^4$ configuration. As the core-N interaction, we use the so called “KKNN”-potential [16]. The Pauli principle between the core-nucleons and a valence one is treated by OCM same as the oxygen isotope case. The Minnesota effective potential [17] with the exchange parameter $u = 1.0$ is used for the two-body interaction, and an effective three-body interaction between the $^4$He and two valence nucleons is employed [18], which is introduced to simulate the effect of the excitation of the core and to fit the binding energy of $^6$He.

Calculated energies from the $^4$He$+Xn$ threshold are shown in Table 2. Due to the lack of the model space in COSM, where the rearrangement coordinate system is not introduced, the energy of $^6$He ground state is obtained as $-0.79$ (MeV), while the experimentally observed one is $-0.98$ (MeV). Nevertheless, the calculated energies almost agree with the experimental data within $\sim 0.3$ (MeV) of the average deviation.

Table 2. Calculated $S_n$ and $R_{rms}$ of helium isotopes and experimental data.

|       | $^5$He | $^6$He | $^7$He | $^8$He |
|-------|--------|--------|--------|--------|
| $E$ (MeV) |        |        |        |        |
| Calc. | 0.74   | -0.79  | -0.89  | -2.94  |
| Exp. [19] | 0.89   | -0.98  | -0.53  | -3.11  |
| $R_{rms}$ (fm) |        |        |        |        |
| Calc. |        | 2.48   |        |        |
| Exp. [20] |        | 2.48 ± 0.03 |        |        |
| Exp. [21] |        | 2.30 ± 0.07 |        |        |

4. Importance of the Gamow states

4.1. Completeness relation in the complex scaling method

We employ an efficient way to define the complete set [22], by using the complex scaling method (CSM) [23, 24]. Due to the finiteness of the basis set, the continuum states are obtained as the eigen vectors with discrete complex energies along the $2\theta$-line on the complex energy plane, and hence, all states are obtained as a discrete set of eigen vectors as:

$$1_i = \sum_{m=0,r} |\phi_{\theta,i}^{(m)}\rangle\langle\phi_{\theta,i}^{(m)}| + \oint_{L_\theta} dk |\phi_{\theta,i}(k)\rangle\langle\phi_{\theta,i}(k)|$$

$$= \sum_{\mu} |\phi_{\theta,i}[\mu]\rangle\langle\phi_{\theta,i}[\mu]| . \quad (3)$$
Here, $|\phi_\mu[\mu]\rangle$ are $|\phi_\mu^{(\mu)}\rangle$ for bound and resonant states (pole states) and $|\phi_\mu(k_\mu^{(\mu)}\rangle$ for continuum states. $\mu$ is the label of the eigen states representing the bound, resonant and discretized continuum states.

### 4.2. Poles and continua contributions: comparison between COSM and GSM

The essential difference between COSM and GSM is the treatment of the unbound components of the single-particle states. Our COSM approach describes the components of unbound states as the linear combination of the Gaussian basis sets. Each basis set has convergent asymptotic behavior and is not an eigenstate of the single-particle Hamiltonian. Hence, the components of unbound states in the wave function are treated implicitly. In GSM, on the other hand, the components of the unbound states are included to the wave function in terms of the explicit treatment of the continuum states for the basis sets.

First, we calculate the contributions of poles and continua in $^{18}\text{O}$. Both approaches, COSM and GSM show almost the same contributions for poles and continua even though the interaction and model are different each other. This result indicate that the main component of the $^{18}\text{O}$ wave function is the well-bound-state poles, $(0d_{5/2})$ and $(1s_{1/2})$, and the contribution to the total wave function does not depend on the choice of the nucleon-nucleon interaction.

| $(C_k)^2$ | COSM $|\langle l = 1 \rangle$ | GSM $[7]$ |
|-----------|----------------|-------------|
| $(0p_{3/2})^2$ | 1.211 $\pm$ 0.666 | 1.139 $\pm$ 0.742 | 1.105 $\pm$ 0.832 |
| $(S1)_{p_{3/2}}$ | $-0.252 + i0.692$ | $-0.119 + i0.773$ | $-0.060 + i0.881$ |
| $(S2)_{p_{3/2}}$ | $-0.042 - i0.026$ | $-0.060 - i0.031$ | $-0.097 - i0.050$ |
| sum | 0.917 | 0.960 | 0.948 |
| $(0p_{1/2})^2$ | 1.447 $+ i0.007$ | 0.353 $- i0.077$ | 0.226 $- i0.161$ |
| $(S1)_{p_{1/2}}$ | $-0.658 - i0.042$ | $-0.534 + i0.065$ | $-0.198 + i0.224$ |
| $(S2)_{p_{1/2}}$ | $1.249 + i0.034$ | $0.221 + i0.012$ | $0.025 - i0.063$ |
| sum | 0.038 | 0.040 | 0.053 |

Next, we calculate the pole and continuum contributions in $^6\text{He}$. For the continuum contributions, we calculate $p_{3/2}$ and $p_{1/2}$ components separately, and summarize each partial wave in Table 3. Contributions of $p_{3/2}$: $(0p_{3/2})^2$-pole, $(S1)_{p_{3/2}}$ and $(S2)_{p_{3/2}}$, which are considered to be the main contributions in $^6\text{He}$, do not change from the “full” $(l \leq 5)$ calculation. On the other hand, contributions of $p_{1/2}$ drastically change from those of full calculation, and the values become close to those obtained by GSM $[7]$. Here, we notice that the sum of the $p_{1/2}$ contributions does not change from the full calculation to the restricted one, which are 0.038 and 0.040. Hence, components of higher partial waves in the full calculation are renormalized to the $p_{3/2}$ contributions in the restricted model space calculation, since the sum of $p_{3/2}$ contributions changes from 0.917 to 0.960. The higher partial wave components are pure continua in the $^4\text{He}+n$ system. Therefore, in the restricted model space, continua of higher waves, which are very important to describe the halo structure, are pushed to the $p_{3/2}$-wave contribution, and smeared to the $(0p_{3/2})^2$-pole and continua.

Furthermore, in the restricted model space, the calculated r.m.s. radius of $^6\text{He}$ becomes much smaller than that of the full calculation. The radius changes from 2.48 (fm) in the full calculation...
to 2.40 (fm). Hence, if we try to reproduce both the binding energy and r.m.s.radius in the restricted model space, we need to modify interactions between the core and a valence nucleon.

We consider the change of these details of the $p_{1/2}$-component shows the importance of the correlation as following reasons. First, oxygen isotopes $^{17-20}$O are normal nuclei and are not weakly bound systems. In $^{18}$O, even though both the nucleon-nucleon interaction and those model space are different, poles and continuum contributions of $s_{1/2}$-, $d_{5/2}$- and $d_{3/2}$-components are almost the same between COSM and GSM. This is because $^{17}$O has two bound states; $1s_{1/2}$ and $0d_{5/2}$, and these contributions are the main part of the $^{18}$O ground state. Hence, the continuum contributions are not important and the effect of pole-continuum and continuum-continuum couplings become small. Second, different from $^{18}$O, $^6$He is a typical weakly bound and halo nucleus. In the $^4$He+$2n$ model space, there is no bound state in each sub-system, which is called the “Borromean” system. Therefore, components of continuum-continuum coupling and (resonant) pole-continuum coupling become much important. In the “full” ($l \leq 5$) calculation, $(0p_{1/2})^2$-pole, which is resonant pole, can couple other angular momentum states. When we change the model space to $l = 1$ from the full one, the $(0p_{1/2})^2$-pole has no chance to make a correlation to the higher angular momentum states. Therefore, the $(0p_{1/2})^2$-pole contribution becomes small and is close to that of GSM [7], in which the model space is restricted to $l = 1$.

5. Discussions
We also calculated contributions of poles and continua of $^6$He by changing the number of basis functions. Results are shown in Figs.1 and 2. In Fig. 2, as we increase the maximum angular momenta for the basis function, the pole contribution $(0p_{1/2})^2$ drastically increases. The small contribution of the total $p_{1/2}$-partial waves comes from the large cancellation between the pole component and continua. Then, if we employ the large model space, in other words, many partial waves are employed in the basis sets, total wave function has the large $(0p_{1/2})^2$-pole contributions and this may causes a problem in the case that the wave function is constructed by the products of the single-particle eigen states. Because a same amount of large continuum contribution is needed in order to cancel the pole contribution, that makes the numerical difficulty in actual calculations. On the other hand, our COSM approach does not construct the total wave function from the products of the single-particle states. Therefore, we do not face such the problem related to the large cancellation, and do have the exact wave function, which has the large pole and continua contributions, simultaneously.
6. Summary
We studied the weakly bound and normal nuclei using an extended COSM approach. The binding energies and r.m.s. radii are well reproduced. In $^6$He, the Gamow state, $0p_1/2$-resonant pole has a large contribution in our model. This shows the importance of the exact treatment of the unbound state in the weakly bound systems.

References
[1] Michel N, Nazarewicz W, Ploszajczak M and Bennaceur K, Phys. Rev. Lett. 89 042502
[2] Michel N, Nazarewicz W, Ploszajczak M and Okołowicz J, Phys. Rev. C 67 054311
[3] Michel N, Nazarewicz W and Ploszajczak M, Phys. Rev. C 70 064313
[4] Betan R Id, Liotta R J, Sandulescu N and Vertse T, Phys. Rev. Lett. 89 042501
[5] Betan R Id, Liotta R J, Sandulescu N and Vertse T, Phys. Rev. C 67 014322
[6] Betan R Id, Liotta R J, Sandulescu N and Vertse T, Phys. Lett. B 584 48
[7] Hagen G, Hjorth-Jensen M and Vaagen J S, Phys. Rev. C 71 044314
[8] Berggren T, Nucl. Phys. A 109 265
[9] Berggren T and Lind P, Phys. Rev. C 47 768
[10] Masui H, Katō K, and Ikeda K, Phys. Rev. C 73 034318
[11] Masui H, Katō K, and Ikeda K, Phys. Rev. C 75 034316
[12] Volkov A B, Nucl. Phys. 74 33
[13] Ajzenberg-Selove F, Nucl. Phys. A 460 1
[14] Ajzenberg-Selove F, Nucl. Phys. A 475 1
[15] Ozawa A et al., Nucl. Phys. A 691 599
[16] Kanada H, Kaneko T, Nagata S and Nomoto M, Prog. Theor. Phys. 61 1327
[17] Thompson D R, LeMere M and Tang Y C, Nucl. Phys. A 286 53
[18] Myo T, Katō K, Aoyama S and Ikeda K, Phys. Rev. C 63 054313
[19] Ajzenberg-Selove F, Nucl. Phys. A A490 1
[20] Tanihata I, Nucl. Phys. A 478 795c
[21] Alkhazov C D et al., Phys. Rev. Lett. 78 2313
[22] Myo T, Ohnishi A and Katō K, Prog. Theor. Phys. 99 801
[23] Aguilar J and Combes J M, Commun. Math. Phys. 22 269
[24] Balslev E and Combes J M, Commun. Math. Phys. 22 280