Thermodynamics of Charged AdS Black Holes in Extended Phases Space via M2-branes Background

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Abstract

Motivated by a recent work on asymptotically AdS\textsubscript{4} black holes in M-theory, we investigate both thermodynamics and thermodynamical geometry of Raissner-Nordstrom-AdS black holes from M2-branes. More precisely, we study AdS black holes in AdS\textsubscript{4} × S\textsuperscript{7}, with the number of M2-branes interpreted as a thermodynamical variable. In this context, we calculate various thermodynamical quantities including the chemical potential, and examine their phase transitions along with the corresponding stability behaviors. In addition, we also evaluate the thermodynamical curvatures of the Weinhold, Ruppeiner and Quevedo metrics for M2-branes geometry to study the stability of such black object. We show that the singularities of these scalar curvature’s metrics reproduce similar stability results obtained by the phase transition program via the heat capacities in different ensembles either when the number of the M2 branes or the charge are held fixed. Also, we note that all results derived in [1] are recovered in the limit of the vanishing charge.
1 Introduction

A more increasing interest has been recently devoted to the black hole physics and the connection with both string theory and thermodynamical models. Particularly, many studies focus on the relationship between the gravity theories and the thermodynamical physics using Anti-De Sitter geometries. In this context, the laws of thermodynamics have been translated into laws of black holes [2–5]. Hence, the phase transitions along with various critical phenomena for AdS black holes have been extensively analysed in the framework of different approaches [7–11]. Also, the equations of state describing rotating black holes have been interpreted by confronting them to some known thermodynamical ones, as Van der Waals gas [12–19]. Emphasis has also been put on the free energy and its behavior in the fixed charge ensemble. These studies shed some light on the thermodynamical critical-ity, free energy, first order phase transition and on understanding of the behaviors near the critical points with respect to the liquid-gas systems.

In this context, very recently the thermodynamics and thermodynamical geometry for five dimensional AdS black hole in type IIB superstring background known by $AdS_5 \times S^5$ [20–22] have been scrutinezed. this geometry has been studied in many places in connection with AdS/CFT correspondence provides a very useful framework to investigate such geometry via the equivalence between gravitational theories in d-dimensional AdS space and the conformal field theories (CFT) in a (d-1)-dimensional boundary of such AdS spaces [23–26].

The number of colors has been interpreted as a thermodynamical variable in these works. In this respect, various thermodynamical quantities have been computed and the stability problem of $AdS_5 \times S^5$ black holes analysed by identifying the cosmological constant in the bulk with the number of colors.

All these recent inspiring works on asymptotically AdS$_4$ black holes in M-theory [27–30] motivate us to study the thermodynamics and thermodynamical geometry of $AdS_4 \times S^7$, from the physics of M2-branes, where we interpret the number of M2 as a thermodynamical variable as in [1]. We then discuss the stability of such solutions and examine the corresponding first phase transition by analysing various relevant quantities including the chemical potential, free energy and heat capacitites. Besides, we also evaluate the thermodynamical curvatures from the Weinhold, Ruppeiner and Quevedo metrics for M2-branes geometry and study the corresponding stability problems via their singularities.

The paper is arranged as follows: In section 2 we discuss thermodynamic properties and stability of the charged black holes in $AdS_4 \times S^7$, by assuming the number of M2-branes as a thermodynamical variable. Section 3 and 4 are devoted to show that similar results are recovered through thermodynamical curvature calculations associated with the Weinhold, Ruppeiner and Quevedo metrics. Our conclusion is drawn in section 5.
2 Thermodynamics of black holes in $AdS_4 \times S^7$ space

In this section, we investigate the phase transition of the Reissner Nordstrom-AdS black holes in M-theory in the presence of solitonic objects. Here we recall that, at low energy, M-theory describes an eleven dimensional supergravity. This theory, as proposed by Witten, can produce some nonperturbative limits of superstring models after its compactification on particular geometries [31].

First, let us consider the case of M2-brane. The corresponding geometry is $AdS_4 \times S^7$. In such a geometric background, the line element of the black M2-brane metric is given by [32, 34]

$$ds^2 = \frac{r^4}{L^4} \left( -f dt^2 + \sum_{i=1}^{2} dx_i^2 \right) + \frac{L^2}{r^2} f^{-1} dr^2 + L^2 d\Omega_7^2, \quad (1)$$

where $d\Omega_7^2$ is the metric of seven-dimensional sphere with unit radius. In this solution, the metric function reads as follows

$$f = 1 - m \frac{r}{r} + q \frac{r^2}{r^2} + \frac{r^2}{L^2}, \quad (2)$$

where $L$ is the AdS radius and $m$ and $q$ are integration constants. The cosmological constant is $\Lambda = -6/L^2$. From M-theory point of view, the eleven-dimensional spacetime in Eq. (1) can be interpreted as the near horizon geometry of $N$ coincident configurations of M2-branes. In this background, the AdS radius $L$ is linked to the M2-brane number $N$ via the relation [1, 32, 35]

$$L^9 = N^3 / 2 \sqrt[2]{\frac{\pi^5}{\pi^5}}. \quad (3)$$

According to the proposition reported in [1, 20–22], we consider the cosmological constant as the number of M2-branes in the M theory background and its conjugate quantity as the associated chemical potential.

The event horizon $r_h$ of the corresponding black hole is determined by solving the equation $f = 0$. From Eq. (2), the mass of the black hole can be written as

$$M = \frac{m \omega_2}{8 \pi G_4} = \frac{r \omega_2 \left( L^2 + r^2 \right)}{8 \pi G_4 L^2} + \frac{2 \pi G_4 Q^2}{r \omega_2} \quad \{1\}$$

where the charge of the black hole $Q$ is related to the constant $q$ through the formula,

$$Q = \frac{\omega_2}{4 \pi G_4} q. \quad (5)$$

$^1$ where $\omega_d = \frac{2\pi^{d/2}}{\Gamma(\frac{d}{2})}$. 

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The Bekenstein-Hawking entropy formula of the black hole reads as,

\[ S = \frac{A}{4G_4} = \frac{\omega_2 r^2}{4G_4}. \]  

Here we recall that four-dimensional Newton gravitational constant is related to the eleven-dimensional one as

\[ G_4 = \frac{3G_{11}}{2\pi \omega_2 L^4}. \]  

For simplicity reason, we use \( G_{11} = \kappa_{11} = 1 \) in the remainder of the paper. In this way, the black hole mass can be expressed as a function of \( N \) and \( S \),

\[ M(S, N) = \frac{3\sqrt{2}\pi^{11/9} \sqrt{N} Q^2 + 3\sqrt{\pi} S^2 + 8\sqrt{2}NS}{4 2^{13/18} 3\pi^{11/18} N^{2/3} \sqrt{S}}. \]  

Using the standard thermodynamic relation \( dM = TdS + \mu dN + \Phi dQ \), the corresponding temperature takes the following form

\[ T = \frac{\partial M(S, N)}{\partial S} \bigg|_N = \frac{-3\sqrt{2}\pi^{11/9} \sqrt{N} Q^2 + 9\sqrt{\pi} S^2 + 8\sqrt{2}NS}{8 2^{13/18} 3\pi^{11/18} N^{2/3} S^{3/2}}. \]  

This quantity can be identified with the Hawking temperature of the black hole. Using eq. (8) the chemical potential \( \mu \) conjugate to the number of M2-branes is given by

\[ \mu = \frac{\partial M_4(S, N)}{\partial N} \bigg|_S = \frac{-3\sqrt{2}\pi^{11/9} \sqrt{N} Q^2 - 6\sqrt{\pi} S^2 + 8\sqrt{2}NS}{12 2^{13/18} 3\pi^{11/18} N^{5/3} \sqrt{S}}. \]  

It defines the measure of the energy cost to the system when one increases the variable \( N \).

while the electric potential reads as

\[ \Phi = \frac{\partial M(S, N)}{\partial Q} \bigg|_S = \frac{\sqrt{3}\pi^{11/18} Q}{2 2^{11/18} \sqrt{N} \sqrt{S}}. \]  

In terms of these quantities, the Helmholtz free energy is expressed by,

\[ \mathcal{F}(T, N) = M - T S = \frac{9\sqrt{2}\pi^{11/9} \sqrt{N} Q^2 - 3\sqrt{\pi} S^2 + 8\sqrt{2}NS}{8 2^{13/18} 3\pi^{11/18} N^{2/3} \sqrt{S}}. \]  

Having calculated the relevant thermodynamical quantities, we turn now to the analysis of the corresponding phase transition. For this, we study the variation of the Hawking temperature as a function of the entropy.

This variation plotted in figure 1 shows that Hawking temperature is a monotonic function if \( Q > Q_c \), but when \( Q \leq Q_c \), it presents a critical point to be determined by solving the
The solution of this equation is easily derived,

\[ Q_c = \frac{4}{9} \frac{2^{5/18} N^{5/6}}{\pi^{7/9}} \], \quad S_c = \frac{4}{9} \sqrt{\frac{2}{\pi}} N. (14) 

In figure 2 we illustrate the Helmholtz free energy as function of the Hawking temperature \( T \) for some fixed values of \( N \).

The sign change of the free energy indicates Hawking-Page phase transition, which occurs at

\[ S_{HP} = \frac{\sqrt{2}}{3} \left( 4 \frac{2^{5/18} N}{\pi^{7/9}} + \sqrt{16 \frac{2^{5/9} N^2}{\pi} + 27 \pi^{14/9} \sqrt{N Q^2}} \right). \] (15)

It can be seen that the “swallow tail”, a type signal for the first phase transition, between small black hole and large one.
To study the phase transition, we vary the chemical potential in terms of the entropy, and plot in figure 3 such a variation for a fixed value of $N$.

Figure 3: The chemical potential $\mu$ as function of the entropy for $N = 3$.

From the figure we can see that the chemical potential becomes positive when the entropy lies within the interval $S_- \leq S \leq S_+$ with

$$S_\pm = \frac{4\sqrt{2}N \pm 2^{5/9} \sqrt{8^{25/9} N^2 - 9\pi^{14/9} \sqrt{N} Q^2}}{6\sqrt{\pi}}$$

(16)

Furthermore, we also plot in figure 4 the behavior of the chemical potential as a function of temperature $T$ for a fixed $N$.

Figure 4: The chemical potential $\mu$ as function of the temperature $T$, with $N = 3$.

From figure 4 we can see that there exists a multivalued region, which just corresponds to the unstable region of the black hole with a negative heat capacity (red line in figure 1).
To illustrate the effect of the number of the M2-branes, we discuss the behavior of the chemical potential $\mu$ in terms of such a variable as shown in figure 12.

![Chemical potential plot](image)

Figure 5: The chemical potential $\mu$ as function of $N$, we have set $S_4 = 4$.

We clearly see that the chemical potential $\mu$ presents a maximum at

$$N_{max} = \frac{3\sqrt{\pi} \left( \frac{2\pi^2 Q^6}{\sqrt{15Q^6 S^{5/2} + 225Q^{12}S^5 - 4\pi^3 Q^{18}}} + \sqrt{2\pi} \sqrt{15Q^6 S^{5/2} + 225Q^{12}S^5 - 4\pi^3 Q^{18}} + 5 2^{2/3} S^{5/2} \right)}{16S^{3/2}}$$

(17)

We remark that this is quite different from the classical gas having a negative chemical potential. In the case where the chemical potential approaches to zero and becomes positive, quantum effects should be considered and become relevant in the discussion [22].

In the subsequent sections, we consider thermodynamical geometry of the M2-branes black holes in the extended phase space and study the stability problem when either $N$ or the charge is held fixed.

3 Geothermodynamics and phase transition of charged AdS black holes with fixed $N$ case

Here we discuss the geothermodynamics of the charged AdS black holes in $AdS_4 \times S^7$: Our analysis will focus on the singular limits of certain thermodynamical quantities, including the heat capacities and scalar curvatures, which are relevant in the study of the stability of such black hole solution.

To do this, the number of branes $N$ should be held fixed to consider the thermodynamics in the canonical ensemble. For a fixed $N$, the heat capacities for M2-branes AdS black hole are given respectively by,
\[ C_{Q,N} = T \left( \frac{\partial S}{\partial T} \right)_{Q,N} = T_4 \left( \frac{8\sqrt{2}N + 18\sqrt{\pi S}}{-3\sqrt{2\pi^{11/9}}\sqrt{NQ^2} + 9\sqrt{\pi S^2} + 8\sqrt{2NS}} - \frac{3}{2S} \right)^{-1} \]  \tag{18}

\[ C_{\Phi,N} = T \left( \frac{\partial S}{\partial T} \right)_{\Phi,N} = \left( \frac{9S}{-3\sqrt{2\pi^{8/9}}\sqrt{NQ^2} + 9S^2 + 8\sqrt{2\pi S}NS} - \frac{1}{2S} \right)^{-1} \]  \tag{19}

In the canonical ensemble with fixed \( N \), a critical point exists and is given by the Eq. (14). The behavior of the \( C_{\Phi,N} \) as function of the entropy is plotted in the figure 6.

Figure 6: The heat capacity in the case with a fixed \( N = 3 \) as a function of entropy \( S \) for \( Q = 0.5 \ Q_c \).

From the figure we can see that the \( C_{\Phi,N} \) presents two singularities at

\[ S_{\Phi,\pm} = \frac{1}{9} \left( 4\sqrt{\frac{2}{\pi}}N \pm \sqrt{16 \left( \frac{2}{\pi} \right)^{2/3} N^2 - 27\sqrt{2\pi^{8/9}}\sqrt{NQ^2}} \right) \]  \tag{20}

The heat capacity \( C_{Q,N} \) is plotted in figure 7.

Figure 7: The heat capacity in the case with a fixed \( N = 3 \) as a function of entropy \( S \).

We see that it is consistent with the temperature shown in the figure 1 (red line). For small and large black hole the heat capacity is always positive, while for the intermediate
black holes it is negative when $Q$ is less than the critical value, whereas it is always positive in the case when the charge is larger than the critical point.

The heat capacity $C_{Q,N}$ under the critical case has two singularities at

$$S_{Q,\pm} = \frac{8\sqrt{2}N \pm 2\sqrt{16 \cdot 2^{2/3}N^2 - 81\sqrt{2}\pi^{14/9}\sqrt{N}Q^2}}{18\sqrt{\pi}}$$

(21)

these two singularities coincide for $Q = Q_c$, and (21) becomes

$$S_Q = \frac{4}{9} \sqrt{\frac{2}{\pi}} \left( \sqrt{N^2 - 3 \cdot 3^{2/3} \sqrt{N} + N} \right)$$

(22)

We turn now our attention to the thermodynamical geometry of the black hole to see whether the thermodynamical curvature can reveal the singularities of these two specific heats. The Weinhold metric [36] is defined as the second derivative of the internal energy with respect to the entropy and other extensive quantities in the energy representation, while the Ruppeiner metric [37] is related the Weinhold metric by a conformal factor of the temperature [38].

$$ds^2_{R} = \frac{1}{T}ds^2_{W}$$

(23)

Notice that the Weinhold and Ruppeiner metrics, which depend on the choice of thermodynamic potentials, are not Legendre invariant. The Quevedo metric defined as [39–42]

$$g = \left( E^c \frac{\partial \phi}{\partial E^c} \right) \left( \eta_{ab} \delta_{bc} \frac{\partial^2 \phi}{\partial E^c \partial E^d} \right), \quad \eta_{cd} = \text{diag}(-1, 1, \cdots, 1)$$

(24)

is a Legendre invariant. $\phi$ denotes the thermodynamic potential, $E^a$ and $I^a$ represent respectively the set of extensive variables and the set of the intensive variable, while $a = 1, 2, \cdots, n$.

In this context we can evaluate the thermodynamical curvature of the black hole. For the Weinhold metric,

$$g^W = \begin{pmatrix} M_{SS} & M_{SQ} \\ M_{QS} & M_{QQ} \end{pmatrix}$$

(25)

where $M_{ij}$ stands for $\partial^2 M / \partial x^i \partial x^j$, and $x^1 = S$, $x^2 = Q$, we can see that its scalar curvature is derived via a direct calculation, simply by substituting Eq. (8) into Eq. (25),

$$R^W_1 = -\frac{64 \sqrt{2} \sqrt{3\pi^{11/18}N^{5/3}S^{3/2}}}{\left(3\sqrt{2}\pi^{11/9}\sqrt{N}Q^2 + 9\sqrt{\pi}S^2 - 8\sqrt{2}NS\right)^2}$$

(26)
While the Ruppeiner metric, deduced from Eq. (23), is given by

\[ g^R = \frac{1}{T} \begin{pmatrix} M_{SS} & M_{SQ} \\ M_{QS} & M_{QQ} \end{pmatrix}, \tag{27} \]

with the following curvature,

\[ R^R_1 = \frac{A_1}{B_2} \tag{28} \]

where,

\[
A_1 = 162\pi^{2/3} S \left[ -486 \ 2^{4/9} \pi^7 N^2 Q^{12} + 6480 \ 2^{2/3} \pi^{52/9} N^{8/3} Q^{10} S + 1296 \ 2^{2/9} \pi^{2/3} N^{4/3} S^9 \
+ 144\sqrt{2}\pi^{10/3} N^{7/3} Q^6 S^3 \left( 1280 N^{5/3} - 261 \ 2^{4/9} \pi^{14/9} Q^2 \right) + 18\sqrt{2} N^{2/3} S^8 \
+ 90112\sqrt{2} N^{10/3} + 5103\pi^{28/9} Q^4 - 62208 \ 2^{5/9} \pi^{14/9} N^{5/3} Q^2 \right] - 96\sqrt{2} N^{5/3} S^7 \\
\times \left( -4096\sqrt{2} N^{10/3} - 3645\pi^{28/9} Q^4 + 8640 \ 2^{5/9} \pi^{14/9} N^{5/3} Q^2 \right) + 48\sqrt{2}\pi^{11/9} N Q^2 S^6 \\
\times \left( -4096\sqrt{2} N^{10/3} - 729\pi^{28/9} Q^4 + 5184 \ 2^{5/9} \pi^{14/9} N^{5/3} Q^2 \right) - 32\pi^{8/9} N^2 Q^2 S^5 \\
\times \left( -4096 \ 2^{5/9} N^{10/3} + 243 \ 2^{4/9} \pi^{28/9} Q^4 + 8064\pi^{14/9} N^{5/3} Q^2 \right) + 6 \ 2^{2/9} \pi^{19/9} N^{4/3} Q^4 S^4 \\
\times \left( -4096 \ 2^{5/9} N^{10/3} - 729\pi^{28/9} Q^4 + 20736 \ 2^{5/9} \pi^{14/9} N^{5/3} Q^2 \right) - 59049 \ 2^{7/9} \pi^{5/3} S^{12} \\
- 104976\sqrt{2}\pi^{4/3} N S^{11} + 559872 \ 2^{4/9} \pi N^2 S^{10} \right] \tag{29} \\

B_1 = \left( 3\sqrt{2}\pi^{11/9} \sqrt{N} Q^2 - 9\sqrt{2} N S^2 - 8\sqrt{2} N S \right)^3 \tag{30} \\
\times \left( -18 \ 2^{2/9} \pi^{22/9} N^{2/3} Q^4 + 96\sqrt{2}\pi^{11/9} N^{4/3} Q^2 S + 81 \ 2^{8/9} \pi^{2/3} S^4 - 128 \ 2^{5/9} N^2 S^2 \right)^2
\]

In the figure we plot the scalar curvatures of the Weinhold and Ruppeiner metrics where the charge is less than critical one.
Figure 8: The scalar curvatures of Weinhold and Ruppeiner metrics vs entropy with $N = 3$ and $Q = 0.5 Q_c$.

From Fig 8 we see that scalar curvature of Weinhold and Ruppeiner metrics reveal both the same singularities $S_{\phi \pm}$ of the heat capacity $C_{\Phi, N}$. The Ruppeiner metric presents a further singularity in $S_0$ where the black hole is extremal $T = 0$. Hence both Weinhold and Ruppeiner metric are able to show phase transition of the black hole in the fixed $\Phi$ ensemble.

The Quevedo metric is defined by,

$$g^Q = (S T + Q \Phi) \begin{pmatrix} -M_{SS} & 0 \\ 0 & M_{QQ} \end{pmatrix}, \quad (31)$$

Using Eqs. (39) and Eq. (31) we show that the scalar curvature reads as,

$$R^Q_1 = \frac{A_2}{B_2} \quad (32)$$

with,

$$A_2 = -768 \sqrt{2} \pi^{11/9} N^{4/3} S \left[ 4374 \sqrt{2} \pi^{44/9} N^{4/3} Q^8 + 131220 \pi^4 N Q^6 S^2 + 19440 \sqrt{2} \pi^{11/3} N^2 Q^6 S \right.
+ 104976 2^{8/9} \pi^{28/9} N^{2/3} Q^4 S^4 + 85536 \pi^{2/9} N^{25/9} N^{5/3} Q^4 S^3 + 17280 2^{5/9} \pi^{22/9} N^{8/3} Q^4 S^2
+ 21870 2^{7/9} \pi^{20/9} \sqrt{N} Q^2 S^6 + 19440 \sqrt{2} \pi^{17/9} N^{4/3} Q^2 S^5 - 34560 2^{4/9} \pi^{14/9} N^{7/3} Q^2 S^4
+ 21504 2^{7/9} \pi^{11/9} N^{10/3} Q^2 S^3 - 19683 2^{2/3} \pi^{4/3} S^8 - 46656 \pi N S^7 - 10368 \sqrt{2} \pi^{2/3} N^2 S^6
+ 18432 2^{2/3} \sqrt{\pi N} S^5 + 32768 N^{4/3} S^4 \right] \quad (33)$$

$$B_2 = \left( 9 \sqrt{2} \pi^{11/9} N^2 Q^2 + 9 \sqrt{2} \pi^2 S^2 + 16 N S \right)^2 \left( 9 \sqrt{2} \pi^{11/9} \sqrt{N} Q^2 + 9 \sqrt{2} \pi S^2 - 8 \sqrt{2} N S \right)^2
\times \left( 9 \sqrt{2} \pi^{11/9} \sqrt{N} Q^2 + 9 \sqrt{2} \pi S^2 + 8 \sqrt{2} N S \right)^2 \quad (34)$$
In the next figure, we plot $R_1^Q$ in terms of the entropy for a fixed $N$ (here $N = 3$).

![Figure 9: The scalar curvature vs entropy for the Quevedo metric case with $N = 3$. Right side: $Q = Q_c$, Left side: $Q < Q_c$.](image)

Under the critical scheme $Q < Q_c$ the scalar curvature of the Quevedo metric presents two singularities at $S_{Q, \pm}$ which are the same as those of the heat capacity $C_{Q,N}$ shown in figure 7 (red line). When $Q = Q_c$, the two singularities $S_{Q, \pm}$ coincide to one $S_Q$ (represented by dashed black line). That mean that the Quevedo metric can reveal the phase transition in the fixed charge ensemble.

4 Geothermodynamics and phase transition of charged AdS black holes with fixed charge $Q$

In this section we study the thermodynamics geometry of the M2-branes black holes in the canonical ensemble (fixed charge). That means that the charge of the black hole is not treated as thermodynamical variable but as a fixed external parameter. The critical number of the M2-brane reads as,

$$N_c = \frac{9 \cdot 3^{2/5} \pi^{14/15} Q^{6/5}}{4 \cdot 2^{11/15}}$$  \hspace{1cm} (35)

The heat capacity $C_{\mu,Q}$ with a fixed chemical potential is given by

$$C_{\mu,Q} = T \left( \frac{\partial T}{\partial S} \right)_{\mu,Q}^{-1} \hspace{1cm} (36)$$

The full expression of the $C_{\mu,Q}$ quite lengthy, is not given here. Instead we plot, in figure 10 $C_{\mu,Q}$ in terms of the entropy in the critical sector.
From the left panel, we see that the heat capacity $C_{\mu,\bar{Q}}$ presents two divergencies up to the critical regime, given numerically by $S_{\mu,-} \simeq 0.5$ and $S_{\mu,+} \simeq 1.27$ for $Q = 0.55$. When $N = N_c$, these two singularities coincide, as shown in the right panel, to only one singularity.

In the fixed charge case the Weinhold metric can be expressed as

$$g^W = \begin{pmatrix} M_{SS} & M_{SN} \\ M_{NS} & M_{NN} \end{pmatrix},$$

(37)

From a treatment similar to the calculation performed in the previous section, one can derive the full expression of the scalar curvatures of both the Weinhold and Ruppeiner metrics respectively. For the former one finds,

$$R^W_2 = \frac{A_3}{B_3},$$

(38)

with,

$$A_3 = 120 \, 2^{5/6} \sqrt{3} \pi^{11/6} N Q^2 S^{3/2} \left[ -408 \, 2^{4/9} \pi^{11/9} N^{4/3} Q^2 + 837 \sqrt{2} \pi^{14/9} \sqrt{N} Q^2 S + 1404 \pi^{2/3} S^3 \right] + 64 \sqrt{2} N S \left( 7 \sqrt{2} N - 27 \sqrt{\pi} S \right)$$

(39)

$$B_3 = \left( -99 \, 2^{2/9} \pi^{22/9} N^{2/3} Q^4 - 486 \sqrt{2} \pi^{14/9} \sqrt{N} Q^2 S^2 + 288 \, 2^{4/9} \pi^{11/9} N^{4/3} Q^2 S + 54 \pi^{2/3} S^4 \right) + 96 \sqrt{2} \pi N S^3 - 64 \, 2^{2/3} N^2 S^2 \right)^2$$

(40)

while the result of the latter metric is,

$$R^R_2 = \frac{A_4}{B_4},$$

(41)

with
\[ A_4 = 15\sqrt{\pi N} \left[ 768\sqrt{\pi N^{8/3}} (27\pi Q^6 - 16S^3) + 99144\sqrt{\pi N^{2/3}} Q^6 S^3 - 432(2\pi)^{2/3} N^{5/3} S^2 \right. \\
\left. \times (261\pi^3 Q^6 + 85^5) + 111372 2^{7/9} \pi^{28/9} \sqrt{\pi N Q^4} S^5 - 126360 2^{5/9} \pi^{25/9} N^{4/3} Q^4 S^4 \\
+ 123264 2^{8/9} \pi^{22/9} N^{7/3} Q^4 S^3 - 55296 2^{2/9} \pi^{19/9} N^{10/3} Q^4 S^2 + 99144 \sqrt{\pi N^{20/9}} Q^2 S^7 \\
- 153600 \sqrt{\pi N^{11/9}} Q^3 S^4 + 49152 2^{4/9} \pi^{8/9} N Q^4 S^3 - 432 2^{7/9} \pi^{14/9} N^2 Q^2 (3\pi^3 Q^6 + 1005^5) \\
+ 81 2^{4/9} \pi^{17/9} N Q^2 S (357\pi^3 Q^6 + 6405^5) + 8192 \sqrt{\pi N^{11/3}} S^5 \right] \] (42)

\[ B_4 = \left( -3\sqrt{\pi N} Q^2 + 9\sqrt{\pi S^2} + 8\sqrt{2} N S \right) \left( -99 2^{2/9} \pi^{22/9} N^{2/3} Q^4 - 486 \sqrt{\pi N Q^2} S^2 + 288 2^{4/9} \pi^{11/9} N^{4/3} Q^2 S + 54\pi^2 S^4 + 96 \sqrt{2} \pi N S^3 - 64 2^{2/3} N^2 S^2 \right)^2 \] (43)

Figure 11: The scalar curvature of the Weinhold and Ruppeiner metrics vs entropy for \( N > N_c \), with \( Q = 0.55 \).

These two scalar curvatures are plotted as functions of the entropy in figure 11 which shows that the two metrics reproduce the results obtained in the previous section regarding the singularities of the heat capacity \( C_{\mu, Q} \). Furthermore, the figure also shows the divergency of the scalar curvature of the Weinhold (left) and Ruppeiner (right) metrics when the entropy tends to the value \( S_0 \) for which the black hole becomes extremal \( T = 0 \).

Next we revisit the Quevedo' metric in the fixed charge case,

\[ g^Q = (ST + N\mu) \begin{pmatrix} -M_{SS} & 0 \\ 0 & M_{NN} \end{pmatrix}, \] (44)

and compute its corresponding scalar curvature given by,

\[ R_2^Q = \frac{A_5}{B_5}, \] (45)
The expression of $A_5$ and $B_5$ are found to be,

$$A_5 = 864\pi^{14/9}N^{5/3}S^2 \left[ 768 \ 2^{4/9} \sqrt[3]{\pi} N^{5/3}S \ (123\pi^3 Q^6 - 88S^5) - 243 \ 2^{7/9} \pi N^{2/3}S^3 \ (953\pi^3 Q^6 - 48S^5) \\
- 144\sqrt[3]{2}\pi^{2/3}N^{5/3}S^2 \ (5817\pi^3 Q^6 + 21765S^5) - 522450 \ 2^{2/3}\pi^{26/9}\sqrt[3]{\pi} N^4S^5 + 490320\pi^{25/9}N^{4/3}Q^4S^4 \\
+ 211624\sqrt[3]{2}\pi^{22/9}N^{7/3}Q^4S^3 - 190464 \ 2^{2/3}\pi^{19/9}N^{10/3}Q^4S^2 - 24786 \ 2^{5/9}\pi^{20/9}Q^2S^7 \\
- 158824 \ 2^{5/9}\pi^{11/9}N^3Q^2S^4 + 114688(2\pi)^{8/9}N^4Q^2S^3 - 864 \ 2^{2/9}\pi^{14/9}N^2Q^2 \ (17\pi^3 Q^6 - 792S^5) \\
+ 81 \ 2^{5/9}\pi^{17/9}NQ^2S \ (459\pi^3 Q^6 + 12520S^5) + 131072 \ 2^{7/9}N^{11/3}S^5 \right]$$

$$B_5 = 5 \left( 3\sqrt[3]{2}\pi^{11/9}\sqrt[3]{\pi} NQ^2 - 3\sqrt[3]{\pi} S^2 - 8\sqrt[3]{2}NS \right)^3 \left( 6\sqrt[3]{2}\pi^{11/9}\sqrt[3]{\pi} NQ^2 + 15\sqrt[3]{\pi} S^2 - 8\sqrt[3]{2}NS \right)^2$$

Illustration of $R^Q_2$ behaviour as a function of the entropy is seen in the next figure when $N > N_c$.

![Image](image-url)

**Figure 12:** The heat capacity as a function of entropy $S$ for the two backgrounds in the case with a fixed $Q = 0.55$.

From the figure [12] we can see that the Quevedo metric presents similar singularity’s features, here denoted by $S_Q_{\pm}$, as in the previous analysis of the $C_{Q,N}$ for fixed $N$ shown in Eq[18]. In addition we note that at $S = S_{\mu,0}$, $C_{\mu,Q}$ becomes also singular, while at $S = S_0$ an additional singularity shows up signaling the extremal case.

## 5 Conclusion

In this paper, we have explored the thermodynamics and thermodynamical geometry of charged AdS black holes from M2-branes. More concretely, by assuming the number of M2-
branes as a thermodynamical variable, we have considered AdS black holes in $AdS_4 \times S^7$. Then, we have discussed the corresponding phase transition by computing the relevant quantities. In particular, we have computed the chemical potential and discussed the corresponding stabilities, the critical coordinates and the Helmoltez free energy. In addition, we have also studied the thermodynamical geometry associated with such AdS black holes. More precisely, we have derived the scalar curvatures from the Weinhold, Ruppeiner and Quevedo metrics and demonstrated that these thermodynamical properties are similar to those which show up in the phase transition program. In the limit of the of the vanishing charge we recover all the results of [1]. We aim to extend this work to other geometries and black hole configurations.

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References

[1] A. Belhaj, M. Chabab, H. E. Moumni, K. Masmar and M. B. Sedra, On Thermodynamics of AdS Black Holes in M-Theory, [arXiv:1509.02196 [hep-th]].

[2] S. Hawking, D. N. Page, Thermodynamics of Black Holes in Anti-de Sitter Space, Commun. Math. Phys. 83 (1987) 577.

[3] E. Witten, Anti-de Sitter space, thermal phase transition, and confinement in gauge theories. Adv. Theor. Math. Phys. 2(1998) 505-532.

[4] A. Chamblin, R. Emparan, C. Johnson, R. Myers, Charged AdS black holes and catastrophic holography, Phys. Rev. D 60 (1999) 064018.

[5] A. Chamblin, R. Emparan, C. Johnson, R. Myers, Holography, thermodynamics, and fluctuations of charged AdS black holes, Phys. Rev. D 60, (1999) 104026.

[6] M. Cvetic, G. W. Gibbons, D. Kubiznak, C. N. Pope, Black Hole Enthalpy and an Entropy Inequality for the Thermodynamic Volume, Phys. Rev. D 84 (2011) 024037, [arXiv:1012.2888 [hep-th]].

[7] B. P. Dolan, D. Kastor, D. Kubiznak, R. B. Mann, J. Traschen, Thermodynamic Volumes and Isoperimetric Inequalities for de Sitter Black Holes, [arXiv:1301.5926 [hep-th]].
[8] D. Kastor, S. Ray, J. Traschen, Enthalpy and the Mechanics of AdS Black Holes, Class. Quant. Grav. 261 (2009) 95011, arXiv:0904.2765.

[9] B. P. Dolan, Vacuum energy and the latent heat of AdS-Kerr black holes, Phys. Rev. D 90 (2014) 8, 084002 arXiv:1407.4037 [gr-qc].

[10] C. V. Johnson, Holographic Heat Engines, Class. Quant. Grav. 31 (2014) 205002 arXiv:1404.5982 [hep-th].

[11] N. Altamirano, D. Kubiznak, R. B. Mann and Z. Sherkatghanad, Thermodynamics of rotating black holes and black rings: phase transitions and thermodynamic volume, Galaxies 2 (2014) 89, arXiv:1401.2586 [hep-th].

[12] A. Belhaj, M. Chabab, H. El Moumni, M. B. Sedra, On Thermodynamics of AdS Black Holes in Arbitrary Dimensions, Chin. Phys.Lett. 29 10 (2012)100401.

[13] A. Belhaj, M. Chabab, H. El Moumni, L. Medari, M. B. Sedra, The Thermodynamical Behaviors of Kerr-Newman AdS Black Holes, Chin. Phys. Lett. 30 (2013) 090402.

[14] A. Belhaj, M. Chabab, H. El Moumni, K. Masmar and M. B. Sedra, Critical Behaviors of 3D Black Holes with a Scalar Hair, Int. J. Geom. Meth. Mod. Phys. 12, no. 02, 1550017 (2014) arXiv:1306.2518 [hep-th].

[15] A. Belhaj, M. Chabab, H. El Moumni, K. Masmar and M. B. Sedra, Maxwell’s equal-area law for Gauss-Bonnet-Anti-de Sitter black holes, Eur. Phys. J. C 75 (2015) 2, 71 arXiv:1412.2162 [hep-th].

[16] D. Kubiznak and R. B. Mann, P-V criticality of charged AdS black holes, J. High Energy Phys. 1207 (2012) 033.

[17] A. Belhaj, M. Chabab, H. El Moumni, K. Masmar, M. B. Sedra and A. Segui, On Heat Properties of AdS Black Holes in Higher Dimensions, JHEP 1505, 149 (2015) arXiv:1503.07308 [hep-th].

[18] A. Belhaj, M. Chabab, H. EL Moumni, K. Masmar and M. B. Sedra, Ehrenfest Scheme of Higher Dimensional Topological AdS Black Holes in The Third Order Lovelock-Born-Infeld Gravity, Int. J. Geom. Meth. Mod. Phys. 12, 1550115 (2015) arXiv:1405.3306 [hep-th].

[19] J. X. Zhao, M. S. Ma, L. C. Zhang, H. H. Zhao and R. Zhao, The equal area law of asymptotically AdS black holes in extended phase space, Astrophys. Space Sci. 352 (2014) 763.

[20] J. L. Zhang, R. G. Cai and H. Yu, Phase transition and thermodynamical geometry for Schwarzschild AdS black hole in AdS$_5 \times S^5$ spacetime, JHEP 1502, 143 (2015) arXiv:1409.5305 [hep-th].
[21] J. L. Zhang, R. G. Cai and H. Yu, Phase transition and thermodynamical geometry of Reissner-Nordstrom-AdS black holes in extended phase space, Phys. Rev. D 91, no. 4, 044028 (2015) [arXiv:1502.01428 [hep-th]].

[22] B. P. Dolan, Bose condensation and branes, JHEP 1410, 179 (2014) [arXiv:1406.7267 [hep-th]].

[23] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, Int. J. Theor. Phys. 38, 1113 (1999) [Adv. Theor. Math. Phys. 2, 231 (1998)] [hep-th/9711200].

[24] E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2, 253 (1998) [hep-th/9802150].

[25] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Gauge theory correlators from noncritical string theory, Phys. Lett. B 428, 105 (1998) [hep-th/9802109].

[26] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Large N field theories, string theory and gravity, Phys. Rept. 323, 183 (2000) [hep-th/9905111].

[27] S. Katmadas and A. Tomasiello, AdS4 black holes from M-theory, [arXiv:1509.00474] [hep-th].

[28] Y. Lozano, N. T. Macpherson and J. Montero, A N = 2 Supersymmetric AdS4 Solution in M-theory with Purely Magnetic Flux, [arXiv:1507.02660] [hep-th].

[29] N. Halmagyi, M. Petrini and A. Zaffaroni, BPS black holes in AdS4 from M-theory, JHEP 1308, 124 (2013) [arXiv:1305.0730 [hep-th]].

[30] I. R. Klebanov, S. S. Pufu and T. Tesileanu, Membranes with Topological Charge and AdS4/CFT3 Correspondence, Phys. Rev. D 81, 125011 (2010) [arXiv:1004.0413 [hep-th]].

[31] E. Witten, Solutions of four-dimensional field theories via M theory, Nucl. Phys. B 500, 3 (1997) [hep-th/9703166].

[32] S. S. Gubser, I. R. Klebanov and A. Tseytlin, Coupling constant dependence in the thermodynamics of N = 4 supersymmetric Yang-Mills theory, Nucl. Phys. B 534, 202 (1998) [hep-th/9805156].

[33] R. Kallosh and A. Rajaraman, Vacua of M theory and string theory, Phys. Rev. D 58, 125003 (1998) [hep-th/9805041].

[34] M. J. Duff, H. Lu and C. N. Pope, The Black branes of M theory, Phys. Lett. B 382, 73 (1996) [hep-th/9604052].
[35] I. R. Klebanov, *World volume approach to absorption by non dilatonic branes*, Nucl. Phys. B 496, 231 (1997) [hep-th/9702076].

[36] F. Weinhold, *Metric geometry of equilibrium thermodynamics*, Journal of Chemical Physics 63 (6), 2479-2483.

[37] G. Ruppeiner, *Thermodynamics: A Riemannian geometric model*, Physical Review A 20 (4), 1608

[38] G. Ruppeiner, *Riemannian geometry in thermodynamic fluctuation theory*, Rev.Mod.Phys. 67 (1995) 605-659

[39] H. Quevedo, *Geometrothermodynamics*, J. Math. Phys. 48 (2007) 013506, [physics/0604164].

[40] H. Quevedo, *Geometrothermodynamics of black holes*, Gen. Rel. Grav. 40 (2008) 971 [arXiv:0704.3102].

[41] H. Quevedo, A. Sanchez, S. Taj and A. Vazquez, *Phase transitions in geometrothermodynamics*, Gen. Rel. Grav. 43 (2011) 1153 [arXiv:1010.5599].

[42] A. Bravetti, D. Momeni, R. Myrzakulov and H. Quevedo, *Geometrothermodynamics of higher dimensional black holes*, Gen. Rel. Grav. 45 (2013) 1603 [arXiv:1211.7134].