Pretension design method for cable-beam structure

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Abstract
A pretension design method is proposed for the cablenet system of large span cable-beam structure. This method can improve computational efficiency of pretension design under influence of beams’ deformations. In this method, cablenet system is divided into two parts—inner cablenet and edge cablenet. Inner cablenet is a pure cablenet area, its optimal pretension distribution can be easily found by using balance matrix analysis method. Edge cablenet is the other cablenet area connected with beams, and for this cablenet system, an iterative calculation combined with balance matrix analysis method and nonlinear finite element method is used to calculate its reasonable pretension distributions. In the iteration process, the shape of edge cablenet is iteratively updated according to displacements of connection joints between edge cablenet and beams, and based on these shape changes, pretension distributions of edge cablenet is recalculated iteratively. And by using this method, influence of beams’ deformations can be added to cablenet pretension design. Because above complicated iterative calculation is just limited to the edge cablenet, scale of nonlinear finite element calculation in the iteration will be much smaller than calculation for the whole cable-beam structure, and the efficiency and accuracy of this calculation can be also improved at the same time. At last, this method is programmed for a numerical example, and the results indicate that the method is feasible.

Key words: Cable-beam structure, Pretension distribution, Balance matrix, Nonlinear finite element method, Iterative algorithm

1. Introduction

These days, large span cable-beam structure has been widely used in areas such as cable dome structure, large radio telescope, cable supported bridge and deployable antenna (Legg, 1998, Leonhardt and Schlaich, 1972, Meguroa, et al., 2009, Karoumi, 1999). This type of structure is often lightweight and attractive. The cable-beam structure studied in this paper is a deployable antenna (as shown in Fig. 1), it consists of two parts—flexible cablenet system and folding rigid beams system. For this type of cable-beam structure, the most important thing is finding a feasible method to achieve...
reasonable cablenet system’s pretension distribution that can ensure an ideal designated shape.

Common methods for cablenet system’s design include non-linear finite element method, force density method, dynamic relaxation method and balance matrix analysis method.

Non-linear finite element method (Argyris, 1974, Ahmadi-Kashani and Bell, 1988, Bathe, 1996, Tibert, 1999) is used to design cablenet system with nodal displacements as unknowns. For this method, an initial shape and pretension distributions of cablenet system should be given at first. Then, the finite element model of structure can be established, and elements’ tangent stiffness matrix can also be assembled to generate total stiffness matrix. Based on this, structural non-linear finite element equations can be established at last, and by solving these equations, we can obtain ultimate results such as nodal displacements, cablenet system’s shape and pretension distributions.

Force density method is first proposed by Schek (Schek, 1974, Maurin and Motro, 1997). Force density is the ratio of cable force to cable length. For this method, first give a set of initial force densities of cables, then establish balance equations with unrestrained nodes as free nodes. And by solving these equations, free nodes’ location and cablenet’s shape can be obtained at last.

Dynamic relaxation method is a type of numerical method for non-linear problems. Barnes studied the subject since the late seventies (Barnes, 1999), and successfully applied this method to the form-finding of cablenet and membrane structure. For dynamic relaxation method, cablenet system’s static problems are considered dynamic problems. First, the static loads applied to cablenet are treated as dynamic loads, and all nodal speeds and displacements are set to zero. Nodes’ unbalance forces will cause oscillations of nodal displacements. And then, kinetic energy change of cablenet is tracked, and when it reaches its max value, all nodal speeds are set to zero again. Corresponding iteration will continue in this way, and ultimate results will be obtained when nodes’ unbalance forces fall to their convergence values.

Balance matrix analysis method provides a linear solving technique for cablenet (Pellegrino, 1993, Tibert, 2002). For this method, cable elements are considered bar elements first, and based on this, the whole structure’s balance equations can be established. Next, by using singular value division method to analyze balance matrix of above equations, pretension model of cablenet can also be obtained. And based on pretension model results, balance equations can be solved with optimum methods then, and ultimate pretension results can be obtained under conditions with a given optimization criterion at last.

Among above methods, balance matrix analysis method can calculate pretension distributions of cablenet more efficiently, but for cable-beam structure, this method is ineffective (Tibert, 2002). On the other hand, nonlinear finite element method can solve cable-beam structure model, but when the model is too complex, there will be convergence troubles, and it will be difficult to optimize pretension distribution by conditions like too many cable-bar elements. Through comparisons of advantages and disadvantages of these methods, an iterative algorithm combined with balance matrix analysis method and nonlinear finite element method is proposed in this paper for calculating pretension distributions of cable-beam structure.

2. Division of cablenet system

Balance matrix analysis method can efficiently calculate pretension distributions of cablenet system with no beam elements contained. If we substitute pretension values obtained by balance matrix analysis method into cable-beam structure’s FEM model, the beams will be changed in shape under influence of cables’ tension, and this, in turn, will make cablenet system deviate from its original ideal shape. In order to solve above problem, the cablenet system of cable-beam structure is divided into two parts—inner cablenet and edge cablenet, and an iterative algorithm combined with balance matrix analysis method and nonlinear finite element method is proposed here for them.

The first step of division is to classify cable nodes according to types of connections between cable nodes and beam nodes. The cable nodes connected with beam elements are classed as outside cable nodes of cablenet system, the nodes of cable elements with outside cable nodes as their the other end nodes are classed as edge cable nodes, and after outside cable nodes and edge cable nodes, the rest cable nodes of cablenet system are classed as inner cable nodes.

Next, according to above cable nodes classifications, the cable elements of cablenet system can be classified as follows. Firstly, the cable elements with outside cable nodes as their one end nodes and edge cable nodes as their the other end nodes are classed as edge cable elements, and obviously, this type of cable elements are directly connected with beam elements. Secondly, the cable elements with edge cable nodes as their all end nodes are also classed as edge cable elements. Thirdly, the rest cable elements of cablenet system are classed as inner cable elements, and it is clear
that inner cable elements contain all edge cable nodes and inner cable nodes.

When cablenet system is in its self-balance state, on any edge cable nodes, the resultant force of inner cable elements must be balanced by resultant force of edge cable elements. Based on this relationship, two restrictive conditions of the iterative algorithm for cable-beam structure are proposed as shown in Eq. (1) and Eq. (2).

\[ u_{\text{edge}} = 0 \quad (1) \]

Where \( u_{\text{edge}} \) is displacements of edge cable nodes that are relative to their original ideal locations, and Eq. (1) indicates that edge cable nodes should stay at their ideal locations for the iterative algorithm.

\[ F_{\text{edge}}' \sim F_{\text{edge}} \quad (2) \]

Where \( F_{\text{edge}}' \) is edge cable elements resultant force on edge cable nodes under influence of beams’ deformations, \( F_{\text{edge}} \) is the same type of resultant force calculated according to the cablenet system with no beam elements contained. Eq. (2) indicates that when the iterative algorithm is used for pretension design of cablenet system under influence of beams’ deformations, it is required that cable-beam structure’s edge cable elements resultant force on edge cable nodes should be equivalent to the resultant force calculated by the cablenet system with no beam elements contained.

Once these two conditions are met, we can substitute inner cable elements’ pretension values calculated by balance matrix analysis method into the cable-beam structure’s FEM model. And after nonlinear finite element calculating, the inner cablenet system will remain its ideal shape even under influence of beams’ deformations.

Based on above analysis, it is clear that when cablenet pretension values calculated by balance matrix analysis method are substituted into cable-beam structure’s FEM model, cablenet system will deviate from its original ideal shape under influence of beams’ deformations. But if cablenet pretension distribution can be redesigned according to the changes of outside cable nodes’ locations, and this new pretension distribution can just ensure above two equations’ validity, then even under the influence of beams’ deformations, the inner cablenet system can also return to its ideal shape with its original pretension distribution. Because inner cablenet system contains all cable nodes with demand for the shape constrain, above method can ensure cablenet’s shape precision is good enough. And because only edge cable elements’ pretension values are needed to be recalculated, scale of calculation in this method is much smaller than common methods.

3. Analysis and calculation for inner cablenet system

Based on above division, the cablenet system of a triangular facet cablenet deployable antenna studied in this paper is divided into two parts, and its inner cablenet system is shown in Fig. 2. Because antenna’s reflector consists of all upper cable nodes of inner cablenet system, this part must keep a given paraboloid shape to meet antenna’s shape precision demand. And since balance matrix analysis method can efficiently calculate pretension distributions of cablenet structure with given shape and topology, it is used here for inner cablenet system’s shape analysis and pretension optimization.

\[ A \textbf{T} = 0 \quad (3) \]

When a cablenet structure with given shape and topology is in its self-balance state, without cables’ gravity and sags taken into account, the force balance equations of its unrestrained nodes can be written as (Pellegrino, 1993),
Where $A$ is structural balance matrix of inner cablenet, and its rows’ number is equal to number of degrees of freedom of unrestrained nodes, its columns’ number is equal to elements’ number of inner cablenet. $T$ is pretension column vectors of cable elements of inner cablenet.

For a complex cablenet structure, Eq. (3) will have more than one pretension solution coefficient. And since too many coefficients will affect calculating efficiency, balance matrix $A$ should be simplified according to axial symmetry of cablenet to reduce number of solutions coefficients. And after this simplification, it will be easier to use optimization technique to calculate Eq. (3).

Based on axial symmetry, inner cablenet can be divided into the same twelve groups (as shown in Fig. 2). In each group, the number of cable elements is $p$, and pretension values of cable elements with the same axial symmetry locations are equal to each other. Then according to above division, Eq. (3) can be rewritten as,

$$A E T' = B T' = 0$$  \hspace{1cm} (4)

Where $E$ is pretension transforming matrix, its rows’ number is equal to the total number of cable elements, and its columns’ number is equal to the number of cable elements of grouped inner cablenet. $T'$ is pretension column vectors of cable elements of grouped inner cablenet. $B$ is simplified structural balance matrix, its rows’ number is equal to the number of degrees of freedom of unrestrained nodes, and its columns’ number is equal to the number of cable elements of grouped inner cablenet.

Because the number of dimensions of matrix $B$ is much less than matrix $A$, the number of pretension solutions coefficients could also be reduced, and this can make optimization for pretension solutions coefficients much easier.

When the number of pretension solutions coefficients of Eq. (4) is greater than 1, there will be more than one pretension distribution’s pattern in the divided cablenet group. And then it is needed to use optimization method with solutions coefficients as design variables to calculate the optimum pretension solutions. In order to maximize performance of cablenet system, the evenness of pretension distribution is considered as objective function. And the optimum mathematic model for grouped inner cablenet can be written as,

**Find** \hspace{1cm} $X = (k_1, k_2, \ldots, k_w)^T$

**Min** \hspace{1cm} $f(X) = \sum_{i=1}^{w} (T_i - T_a)^2$  \hspace{1cm} (5)

**s.t.** \hspace{1cm} $B T' = 0$  \hspace{1cm} (6)

$$g(X) = T_a = \frac{\sum T_i}{p} \geq T_L$$  \hspace{1cm} (7)

$$T_i > 0$$  \hspace{1cm} (8)

$$X_L \leq X \leq X_U$$  \hspace{1cm} (9)

Where design variables $X$ are the set of pretension solutions coefficients, its total number is $w$. Objective function $f(X)$ is squares sum of deviations between pretension value of each cable element and the average pretension value, $p$ is number of cable elements of grouped inner cablenet, and $f(X)$ is used to reflect evenness of pretension distribution in grouped inner cablenet. Constraint function $g(X)$ is the average pretension value of grouped inner cable elements, $T_L$ is lower limit of the average pretension value, $X_U$ and $X_L$ are upper and lower limit of design variables.

By solving above optimum mathematic model, the optimum pretension solutions coefficients under constraints (6)-(9) can be obtained, and based on these coefficients, the optimum pretension distribution of inner cablenet system can be calculated at last.

**4. Analysis and calculation for edge cablenet system**

The edge cablenet system obtained by division method is located between inner cablenet system and beams, as is shown in Fig. 3. The purpose of analysis and calculation for edge cablenet system is to obtain a set of edge cablenet pretension values that can make inner cablenet system, which is under influence of beams’ deformations, keep its original ideal...
shape and pretension distribution. And therefore, an iterative algorithm combined with balance matrix analysis method and nonlinear finite element method is proposed here to calculate the optimum pretension distribution of edge cablenet system. And in this method, the influence of beams’ deformations is reflected by an iteration algorithm.

![Fig. 3 Edge cablenet and grouped edge cablenet](image)

### 4.1 Analysis for balance matrix of edge cablenet system

Firstly, balance matrix analysis method is used to calculate pretension values of edge cablenet with its ideal shape.

Assuming that only the influence of inner cablenet’s force is taken into account, in order to simplify calculation model of edge cablenet, all its outside cable nodes and edge cable nodes are separately considered as restrained nodes and unrestrained nodes, and the influence of inner cablenet’s force is just simplified as inner cable elements’ resultant force on unrestrained nodes. And force balance equations of unrestrained nodes of edge cablenet, in its self-balance state, can be written as,

$$\bar{A} \bar{T} = \bar{F}$$  \hspace{1cm} (10)

Where $\bar{A}$ is structural balance matrix of edge cablenet, its rows’ number is equal to the number of degrees of freedom of unrestrained nodes, and its columns’ number is equal to the number of edge cablenet elements. $\bar{T}$ is pretension column vectors of cable elements of edge cablenet. $\bar{F}$ is inner cablenet’s resultant pretension column vectors on edge cable nodes.

By using inner cablenet’s grouping method, the edge cablenet also can be divided into the same twelve groups, as is shown in Fig. 3. And then Eq. (10) can be further written as,

$$\bar{A} \bar{E} \bar{T} = \bar{B} \bar{T} = \bar{F}$$  \hspace{1cm} (11)

Where $\bar{E}$ is pretension transforming matrix, its rows’ number is equal to the total number of cable elements of edge cablenet, and its columns’ number is equal to the number of cable elements of grouped edge cablenet. $\bar{T}$ is pretension column vectors of cable elements of grouped edge cablenet. $\bar{B}$ is simplified structural balance matrix of edge cablenet.

The optimum pretension distribution of edge cablenet can be calculated by using the same optimization method in Section 3, so there is no need to introduce it again.

### 4.2 Iterative algorithm for edge cablenet calculation under influence of beams’ deformations

Balance matrix analysis method can efficiently calculate pretension distributions of cablenet, but for large span cable-beam structure, when we substitute the pretension values obtained by balance matrix analysis method into its FEM model, beams will be changed in shape under influence of cables tension, and this, in turn, will make inner cablenet system deviate from its original ideal shape. In order to solve this problem, an iterative algorithm combined with balance matrix analysis method and nonlinear finite element method is used here to make sure Eq. (1) and Eq. (2) is valid.

Concrete steps of iterative algorithm can be listed as,

1. The first step is linking beams and edge cablenet on edge cable nodes (as shown in Fig. 4), and set original restrained nodes on beams and connection joints between edge cablenet and inner cablenet as new restrained nodes. Then by substituting edge cablenet pretension values $\bar{T}$ obtained by balance matrix analysis method into the FEM model of this edge cablenet-beam structure, we can solve this FEM model with nonlinear finite element method, and obtain new pretension values $\bar{T}^*$ of edge cablenet (the difference between $\bar{T}$ and $\bar{T}^*$ is $\Delta \bar{T}^*$).
(2) The second step is determining whether \( \Delta \bar{T}_s \) has met convergence conditions of iteration. If \( \Delta \bar{T}_s \) has met the conditions, the iteration should jump to step (3), and it is considered that the edge cablenet pretension values have returned to its ideal levels. On the other hand, if \( \Delta \bar{T}_s \) has not met the convergence conditions, \( \bar{T} + \Delta \bar{T}_s \) will be set as new initial pretension values of edge cablenet, and the iteration should jump to step (1) to solve the FEM model again.

![Fig. 4 Edge cablenet-beam structure](image)

![Fig. 5 Flowchart of iteration](image)
Step (1) and step (2) can make edge cablenet pretension values return to their original ideal levels. But at the same time, the beams have also been changed in shape under influence of cables tension, and these shape changes, in turn, will make outside cable nodes deviate from their original locations. So in this case, the edge cablenet pretension values obtained by above iteration cannot yet ensure validity of Eq. (2).

(3) In order to solve above problem, the third step is updating shape of edge cablenet according to the new locations of outside cable nodes, and recalculating pretension values of edge cablenet based on its new shape to make sure Eq. (2) is valid.

After updating the shape of edge cablenet, structural balance matrix of edge cablenet in Eq. (10) will be transformed into \( \Delta T \). Based on the optimum edge cablenet pretension values \( T \), least square method is used for solving new Eq. (10), and as result, we can obtain a set of new edge cablenet pretension values \( \Delta T \) that can make sure Eq. (2) is valid (the difference between \( T \) and \( \Delta T \) is \( \Delta T \)). On the other hand, if new Eq. (10) has no solution, the iteration will be terminated in this case.

(4) The fourth step is determining whether \( \Delta T \) has met the convergence conditions of iteration. If \( \Delta T \) has met the conditions, it is considered that all iteration has finished, and the edge cablenet pretension values \( T \) obtained by step (3) can ensure edge cablenet keep its original ideal shape and pretension distribution under influence of beams’ deformations. And on the other hand, if \( \Delta T \) has not met the convergence conditions, \( T \) will be set as the new initial pretension values of edge cablenet, and the iteration should jump to step (1) to solve the FEM model again.

In above iteration, the adjustments of edge cablenet pretension values virtually correspond to the adjustments of edge cables’ lengths. And based on these, inner cablenet can return to its ideal shape at last. Additionally, because edge cablenet-beam structure is very simple, there are no convergence troubles in solving its FEM model by using nonlinear finite element method.

Fig. 5 shows the flowchart of above iteration.

5. An example

The cable-beam structure shown in Fig. 1 is analyzed as an example here.

This cable-beam structure’s diameter is 10m, its upper cablenet and lower cablenet are exactly symmetrical.

Firstly, balance matrix analysis method is used for analyzing cable-beam structure’s inner cablenet shown in Fig. 2. The inner cablenet contains 439 cable elements, and the analyzing results show that the number of pretension solutions coefficients of inner cablenet is 109. Based on axial symmetry of triangular facet cablenet, the inner cablenet is divided into the same twelve cablenet groups (as shown in Fig. 2). Each group has the same structure and contains 27 independent cable pretension variables, and the numbers in Fig. 2 indicate the ordinals of cable pretension variables. By using balance matrix analysis method to analyze grouped cablenet, we can obtain that its number of pretension solutions coefficients is 10. Then the optimum mathematic model of grouped inner cablenet is established and solved with these 10 coefficients as design variables and \( T \geq 15N \) as constraint. As result, the optimum pretension solutions are listed in Table 1 (numbers 20-27 represent ordinals of longitudinal cable pretension).

Secondly, balance matrix analysis method is used again for analyzing the cable-beam structure’s edge cablenet

| ordinal | pretension value[N] | ordinal | pretension value[N] | ordinal | pretension value[N] |
|---------|---------------------|---------|---------------------|---------|---------------------|
| 1       | 20.4549             | 10      | 19.0685             | 19      | 17.7977             |
| 2       | 19.0576             | 11      | 19.9018             | 20      | 5.1093              |
| 3       | 19.3006             | 12      | 19.4049             | 21      | 4.7330              |
| 4       | 19.3534             | 13      | 19.5276             | 22      | 4.7275              |
| 5       | 19.5910             | 14      | 18.9221             | 23      | 4.7962              |
| 6       | 19.3397             | 15      | 18.8906             | 24      | 4.7253              |
| 7       | 19.9841             | 16      | 18.2632             | 25      | 4.7325              |
| 8       | 20.2953             | 17      | 18.9969             | 26      | 4.7376              |
| 9       | 19.3914             | 18      | 19.0630             | 27      | 4.8345              |
shown in Fig. 3. The edge cablenet contains 222 cable elements. By using inner cablenet grouping method, the edge cablenet also can be divided into the same twelve groups (as shown in Fig. 3), and each group contains 12 independent cable pretension variables. By using balance matrix analysis method to analyze grouped edge cablenet, we can obtain that its number of pretension solutions coefficients is 4. Then the optimum mathematic model of grouped edge cablenet is established and solved with these 4 coefficients as design variables and \( T_i \geq 15N \) as constraint. As result, the optimum pretension solutions are listed in Table 2 (numbers 10-12 represent ordinals of longitudinal cable pretension).

| ordinal | pretension value[N] | ordinal | pretension value[N] | ordinal | pretension value[N] |
|---------|---------------------|---------|---------------------|---------|---------------------|
| 1       | 21.5360             | 5       | 17.6354             | 9       | 14.0924             |
| 2       | 23.1729             | 6       | 18.7869             | 10      | 4.6239              |
| 3       | 22.2007             | 7       | 18.7855             | 11      | 3.9509              |
| 4       | 20.2066             | 8       | 14.9784             | 12      | 4.0932              |

Next, we establish the FEM model of cable-beam structure. In this FEM model, beam element and cable-bar element are used to simulate beam and cable. The elastic modulus of beam element is 235.0GPa, and its diameter and thickness are 20mm and 1mm. The elastic modulus and diameter of cable-bar element are 124.0GPa and 1mm. By substituting pretension values in Table 1 and Table 2 into the FEM model of cable-beam structure and using nonlinear finite element method to solve this model, we can obtain that the root mean square error of inner cablenet nodes displacements is 0.15119mm, the root mean square value of differences between inner cablenet pretension values obtained by nonlinear finite element method and its ideal pretension values in Table 1 is 1.1564N, and the root mean square value of differences between edge cablenet pretension values obtained by nonlinear finite element method and its ideal pretension values in Table 2 is 2.9778N. After above nonlinear finite element calculation, it is obvious that the shape and pretension distribution of cablenet have deviated from its original ideal shape and pretension distribution.

Thirdly, the iterative algorithm proposed in this paper is used for analyzing the cable-beam structure’s edge cablenet. When convergence values of \( \Delta \bar{T}_v \) and \( \Delta \bar{T}_w \) are set to 0.05N, the iteration is finished after 3 iterative cycles. Then by substituting inner cablenet pretension values in Table 1 and edge cablenet pretension values obtained by iterative algorithm into the FEM model of cable-beam structure and using nonlinear finite element method to solve this model, we can obtain that the root mean square error of inner cablenet nodes displacements is 4.1408×10^{-3}mm, the root mean square value of differences between inner cablenet pretension values obtained by nonlinear finite element method and its ideal pretension values in Table 1 is 0.04075N, the root mean square value of differences between edge cablenet pretension values obtained by nonlinear finite element method and its ideal pretension values in Table 2 is 0.1862N. And the detailed pretension values’ differences are shown in Fig. 6 and Fig. 7. Compared with the results in previous paragraph, the shape error obtained by the iterative algorithm has fallen by an order of magnitude, and the cablenet pretension values are also very close to values in Table 1 and Table 2.
6. Conclusions

In the pretension design method proposed in this paper, the cablenet system of cable-beam structure is first divided into two parts—inner cablenet and edge cablenet, and then an iterative algorithm combined with balance matrix analysis method and nonlinear finite element method is used to calculate the optimum pretension distributions of these two parts. According to results of numerical example, two conclusions can be drawn as follows.

(1) Comparing results obtained by the iterative algorithm with results obtained by nonlinear finite element method, it is clear that the proposed pretension design method can make sure inner cablenet is close to its original ideal shape and pretension distribution under influence of beams’ deformations.

(2) Because edge cablenet-beam structure is very simple, there are no convergence troubles in solving its FEM model by using nonlinear finite element method, and the time spent in nonlinear finite element calculation for edge cablenet-beam structure is a lot less than that for the whole cable-beam structure. Therefore, the proposed pretension design method is more efficient for analyzing cable-beam structure’s cablenet.

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