CM3: Cooperative Multi-goal Multi-stage Multi-agent Reinforcement Learning

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Abstract

A variety of cooperative multi-agent control problems require agents to achieve individual goals while contributing to collective success. This multi-goal multi-agent setting poses difficulties for recent algorithms, which primarily target settings with a single global reward, due to two new challenges: efficient exploration for learning both individual goal attainment and cooperation for others’ success, and credit-assignment for interactions between actions and goals of different agents. To address both challenges, we restructure the problem into a novel two-stage curriculum, in which single-agent goal attainment is learned prior to learning multi-agent cooperation, and we derive a new multi-goal multi-agent policy gradient with a credit function for fine-grained credit assignment. We further demonstrate a function augmentation scheme to reduce computational cost of learning value and policy functions in synergy with the curriculum. The complete architecture, called CM3, learns significantly faster than direct adaptations of existing algorithms on three challenging multi-goal multi-agent problems: cooperative navigation in difficult formations, negotiating multi-vehicle lane changes in the SUMO traffic simulator, and strategic cooperation in a Checkers environment.

1 Introduction

Cooperative multi-agent control problems that are pervasive in real-world settings, such as strategic cooperation in social dilemmas \cite{42} and coordination among autonomous vehicles \cite{4}, often can be viewed as a multi-goal problem: each agent is associated with a different goal, and the global optimum where all agents succeed is only attained when agents simultaneously accomplish their individual goals and enable the success of other agents. In autonomous driving for example, multiple vehicles may encounter scenarios where their individual goal locations and nominal trajectories are in conflict (e.g. double lane merges), such that all vehicles succeed only by simultaneous cooperative maneuvers. Moreover, even settings with a team objective that seem unfactorizable at first glance are in fact multi-goal problems: Starcraft II agents must achieve specific goals (e.g. scout enemy base, attack units); players in team soccer have different simultaneous roles (pass to striker, cover opponent). While the framework of multi-agent reinforcement learning (MARL) \cite{40, 17, 35, 9, 30} has been equipped with successful methods in deep reinforcement learning (RL) \cite{20, 15} and shown promise on high-dimensional problems with complex agent interactions \cite{19, 21, 13, 6, 16, 34}, learning multi-agent cooperation in the multi-goal scenario involves significant open challenges.

First, given that exploration is crucial for RL \cite{41} and even more so in MARL with larger state and joint action spaces, how should agents explore to learn both individual goal attainment and cooperation for others’ success? Uniform random exploration is common in deep MARL \cite{8} but can be highly inefficient as the value of joint cooperative actions may be discovered only in specific regions of state space where cooperation is needed. For instance, physical proximity is required to discover optimal actions for collision avoidance during lane merges, but agent vehicles who have not learned to change lanes will rarely be in such proximal states. Furthermore, the conceptual difference between attaining one’s own goal and cooperating for others’ success poses an open challenge for designing better modularized and targeted approaches. Second, while there are methods for multi-agent credit assignment when all agents share a single goal (i.e. a global reward) \cite{5, 26, 6, 22}, and while one could treat the cooperative multi-goal scenario as a problem with a single joint goal, this coarse approach makes it extremely difficult to isolate the impact of any agent’s action on any
other agent’s success. Instead, the multi-goal scenario can benefit from fine-grained credit assignment that leverages available structure in action-goal interactions, such as local interactions where few agents significantly affect another agent’s goal attainment at any time.

Given these open challenges, our paper focuses on the cooperative multi-goal multi-agent setting where each agent is assigned a goal and must learn to cooperate with other agents with possibly different goals. To tackle the problems of efficient exploration and credit assignment in this complex problem setting, we develop CM3, a novel general framework involving three synergistic components:

1. We approach the difficulty of multi-agent exploration from a novel curriculum learning perspective, by first training an actor-critic pair to achieve different goals in an induced single-agent setting (Stage 1), then using them to initialize all agents for further training in the real multi-agent environment (Stage 2). The key insight is that agents who have learned to achieve individual objectives in the absence of other agents are better prepared to produce state configurations where cooperative solutions can be easily discovered with additional exploration in the full multi-agent environment. In contrast to hierarchical learning where sub-goals are selected sequentially in time, all agents act toward their goals simultaneously in Stage 2 of our curriculum learning approach.

2. Next, we observe that multi-agent environments permit a decomposition of an agent’s observation into a representation of the agent’s own state (e.g., position and velocity) and a representation of other agents. The global state can also be decomposed into agent-specific and agent-independent dimensions. This motivates a new function augmentation scheme to bridge Stages 1-2: we improve the computational efficiency of training Stage 1 actor and critic functions by limiting their input space to the part that is sufficient for a single-agent environment, then augment the architecture in Stage 2 with additional inputs and trainable parameters for learning in the multi-agent environment.

3. In synergy with the two-stage curriculum, we address credit assignment in multi-goal MARL by using a new credit function for localized credit assignment between action-goal pairs in a multi-agent policy gradient in Stage 2. The credit function is constructed via function augmentation from the Stage 1 critic, further benefiting from latent similarity between achieving one’s own goal and enabling others’ success in agents populations whose goals are assigned from a common set.

We evaluated our method on three challenging multi-goal multi-agent environments with high-dimensional state spaces: cooperative navigation with difficult formations in a multi-agent particle environment, double lane merges in the SUMO simulator [18], and strategic teamwork in a Checkers game. CM3 solved all domains significantly faster than existing MARL methods, none of which were uniformly robust across all domains. Exhaustive ablation experiments show that the synergistic combination of all three components is crucial for CM3’s overall high performance.

2 Related work

Early theoretical work analyzed MARL in discrete state and action spaces [40, 17, 9]. Recent work have leveraged techniques from deep RL to develop general algorithms for high dimensional environments requiring complex agent interactions [39, 21, 19], in contrast to traditional planning methods that require full system information and do not generalize by learning interactions [3].

Cooperative multi-agent learning is important since many real-world problems can be formulated as distributed systems in which decentralized agents must coordinate to achieve shared objectives [26]. Austerweil et al. [1] showed that agents whose rewards depend on all agents’ success perform better than agents who optimize for their own success, which motivates the multi-agent credit assignment problem. In the special case when all agents have a single goal and share a global reward, COMA [6] uses a counterfactual baseline, while Nguyen et al. [22] employs count-based variance reduction limited to discrete-state environments. However, the centralized critic in these methods does not evaluate the specific impact of an agent’s action on another’s success in the general multi-goal setting. When a global objective is the sum of agents’ individual objectives, value-decomposition methods optimize a centralized Q-function

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1The hard problem of goal assignment is an open question for recent deep MARL [8]. However, many practical multi-agent problems have clear goal assignments, such as in autonomous driving and team soccer. Our use of “multi-goal” pertains to known goal assignment, without precluding the more general case.
We derive its variance in Appendix C.1. However, COMA is insufficient for credit assignment in multi-goal MARL, as while transfer learning [25] is complementary to our novel curriculum equipped with modularized augmentation of while preserving scalable decentralized execution [36, 27, 33], but do not address credit assignment. While MADDPG [19] and M3DDPG [14] apply to agents with different reward, they do not specifically address the need for cooperation; in fact, they do not distinguish the problems of cooperation and competition, despite the fundamental difference.

Multi-goal MARL was considered in Zhang et al. [45], who analyzed convergence in a special networked setting restricted to fully-decentralized training, while we conduct centralized training with decentralized execution [23]. In contrast to multi-task MARL, which aims for generalization among non-simultaneous tasks [24], and in contrast to hierarchical methods with top-level managers that sequentially select subtasks [43, 31], our decentralized agents must cooperate in parallel to attain all goals. To the best of our knowledge, few have investigated the benefits of curriculum learning [2] for MARL. Gupta et al. [7] solved a single cooperative task defined by the number of agents, via gradually increasing agent count. Rusu et al. [28] instantiate new neural network columns for task transfer in single-agent RL, while transfer learning [25] is complementary to our novel curriculum equipped with modularized augmentation of function approximators.

3 Preliminaries

In multi-goal MARL, each agent should learn to accomplish any goal within a finite set, cooperate with other agents for collective success, and act independently with limited local observations. We formalize the problem as an episodic multi-goal Markov game, where all agents are assigned randomly sampled goals in each episode. We also review an actor-critic approach to centralized training of decentralized policies, and summarize counterfactual-based multi-agent credit assignment.

Markov Games. A multi-goal Markov game is a tuple \( \langle S, \{O^n\}, \{A^n\}, P, R, G, N, \gamma \rangle \) with \( N \) agents labeled by \( n \in [1..N] \). Each agent \( n \) has one fixed goal \( g^n \in G \) in each episode. At time \( t \) and global state \( s_t \in S \), each agent \( n \) receives an observation \( o^n_t := o^n(s_t) \in O^n \) and chooses an action \( a^n_t \in A^n \). The environment moves to \( s_{t+1} \) due to joint action \( a_t := \{a^1_t, \ldots, a^N_t\} \), according to transition probability \( P(s_{t+1} | s_t, a_t) \). Each agent receives a reward \( R^n_t := R(s_t, a_t, g^n) \), and the learning task is to find stochastic decentralized policies \( \pi^n : O^n \times G \times A^n \to [0, 1] \), conditioned only on local observations and goals, to maximize \( J(\pi) := \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t \sum_{n=1}^{N} R(s_t, a^n_t, g^n) \right] \), where \( \gamma \in (0, 1) \) and joint policy \( \pi \) factorizes as \( \pi(a|s, g) := \prod_{n=1}^{N} \pi^n(a^n|o^n, g^n) \) due to decentralization. Let \( a^n \) and \( g^n \) denote all agents’ actions and goals, respectively, except that of agent \( n \). Let boldface \( a \) and \( g \) denote the joint action and joint goals, respectively. For brevity, let \( \pi(a^n) := \pi^n(a^n|o^n, g^n) \).

Centralized learning of decentralized policies. The actor-critic method in single-agent RL [37, 38, 11] is suitable for adaptation to centralized multi-agent learning of decentralized policies. A centralized critic that receives full state-action information can speed up training of decentralized actors that receive only local information, and only the actors are retained for test execution [19, 6]. For each \( n \in [1..N] \), the global value function is \( V^n(s) := \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R^n_t \mid s_0 = s \right] \) and the global action-value function is \( Q^n(s, a) := \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R^n_t \mid s_0 = s, a_0 = a \right] \). Note that expectations are taken w.r.t. the joint policy \( \pi \). In other words, for each goal \( g^n \), we can restrict attention to the reward \( R^n \) and evaluate the joint policy (treated as a single centralized policy) against this reward.

Multi-agent credit assignment. Extending the policy gradient [38], COMA uses a counterfactual baseline for credit assignment in MARL with a single global reward via the gradient [6, Lemma 1]

\[
\nabla \theta J(\pi) = \mathbb{E}_\pi \left[ \sum_n \nabla \theta \log \pi^n(a^n|o^n) (Q^n(s, a) - \sum_{a^n} \pi^n(a^n|o^n)Q^n(s, (a^n, a^{-n}))) \right]
\]

The advantage function \( A^n(s, a) \) evaluates the contribution of chosen action \( a^n \) versus the average of all possible counterparts \( \tilde{a}^n \), keeping \( a^{n^{-}} \) fixed. Since factorizing a multi-agent joint policy across agents is formally equivalent to factorizing a single-agent policy across action dimensions, Wu et al. [44] suggests that (1) is a low-variance estimator. We derive its variance in Appendix C.1. However, COMA is insufficient for credit assignment in multi-goal MARL, as it would treat the collection of goals \( g \) as a global goal and only learn from total reward, making it extremely difficult to disentangle each agent’s impact on other agents’ goal attainment. Furthermore, a global Q-function does not explicitly capture structure in agents’ interactions, such as local interactions involving a limited number of agents. We substantiate...
these arguments by experimental results in Section 6. Hence multi-goal MARL demands additional techniques for faster learning and credit assignment.

4 Methods

We describe the complete CM3 learning framework as follows. First we define a new credit function as a mechanism for credit assignment in multi-goal MARL, then derive a new cooperative multi-goal policy gradient with localized credit assignment. Next we motivate the possibility of significant training speedup via a two-stage curriculum for multi-goal MARL. We describe function augmentation as a mechanism for efficiently bridging policy and value functions across the curriculum stages, and finally synthesize all three components into a synergistic learning framework.

4.1 Credit assignment in multi-goal MARL

If all agents take greedy goal-directed actions that are individually optimal in the absence of other agents, the joint action can be sub-optimal (e.g. straight-line trajectory towards target in traffic). Instead rewarding agents for both individual and collective success can avoid such bad local optima. A na"ive implementation based on previous works [6, 19] would evaluate the joint action $a$ via a global Q-function $Q_{\pi_n}(s, a)$ for each goal $g_n$, thereby decomposing the problem in the space of goals and training all agents for each goal independently. However, collective success in multi-goal MARL fundamentally depends on each agent’s contribution to another agent’s attainment of its goal, but this is not represented explicitly by a monolithic $Q_{\pi_n}(s, a)$. Instead, we propose an explicit mechanism for credit assignment by learning an additional function $Q_{\pi_n}(s, a^m)$ that evaluates pairs of action $a^m$ and goal $g_n$, for use in a multi-goal actor-critic algorithm. First we define a new credit function and show that it satisfies the classical relation needed for sample-based model-free learning.

Definition 1. For $n, m \in \{1..N\}$, $s \in S$, the credit function for goal $g_n$ and $a^m \in A^m$ by agent $m$ is:

$$Q_{\pi_n}(s, a^m) := \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t R^m_t \mid s_0 = s, a^m_0 = a^m \right]$$  \hspace{1cm} (2)

Proposition 1. For all $m, n \in \{1..N\}$, the credit function (2) satisfies the following relations:

$$Q_{\pi_n}(s, a^m) = \mathbb{E}_{\pi} \left[ R^m_t + \gamma Q_{\pi_n}(s_{t+1}, a^m_{t+1}) \mid s_t = s, a^m_t = a^m \right]$$ \hspace{1cm} (3)

$$V_{\pi_n}(s) = \sum_{a^m} \pi^m(a^m|o^m, g^m)Q_{\pi_n}(s, a^m)$$ \hspace{1cm} (4)

Derivations are given in Appendix B.1, including the relation between $Q_{\pi_n}(s, a^m)$ and $Q_{\pi_n}(s, a)$. Equation (3) takes the form of the Bellman expectation equation, which justifies learning the credit function, parameterized by $\theta_{Q_{\pi}}$, by optimizing the standard loss function in deep RL:

$$L(\theta_{Q_{\pi}}) = \mathbb{E}_{\pi} \left[ (R(s_t, a_t, g^n) + \gamma Q_{\pi_n}(s_{t+1}, a^m_{t+1}; \theta_{Q_{\pi}}) - Q_{\pi_n}(s_t, a^m_t; \theta_{Q_{\pi}}))^2 \right]$$ \hspace{1cm} (5)

While centralized training means the input space scales linearly with agent count, many practical environments involving only local interactions between agents allows centralized training with few agents while retaining decentralized performance when deployed at scale (evidenced in Appendix D).

4.2 Cooperative multi-goal multi-agent policy gradient

We use the credit function as a critic within a policy gradient for multi-goal MARL. Letting $\theta$ parameterize $\pi$, the overall objective $J(\pi)$ is maximized by ascending the following gradient:
We propose a new MARL curriculum that first solves a single-agent Markov decision processes (MDP) for significant $\epsilon$ (when the other agent is absent). Case 1: Suppose that for $\epsilon$ (Appendix F gives several examples). While the reduction from $\epsilon$ (Section 6). More significantly, as the credit function takes in a single agent’s action, it synergizes with both CM3’s $M^H$ exploration, as shown by a simple thought experiment. Consider a two-player $M^H$ defined by a $4 \times 3$ gridworld with unit actions (up, down, left, right). Agent $A$ starts at (1,2) with goal (4,2), while agent $B$ starts at (4,2) with goal (1,2). The greedy policy for each agent in $M^H$ is to move horizontally toward its target, since this is optimal in the induced $M$ (when the other agent is absent). Case 1: Suppose that for $\epsilon \in (0, 1)$, $A$ and $B$ follow greedy policies with probability $1 - \epsilon$, and take random actions ($p(a) = 1/4$) with probability $\epsilon$. Then the probability of a symmetric optimal trajectory is $P($cooperate$) = 2\epsilon^2((1 - \epsilon) + \epsilon/4)^8$. For $\epsilon = 0.5$, $P($cooperate$) \approx 0.01$. Case 2: If agents execute uniform random exploration, then $P($cooperate$) = 3.05e-5 \ll 0.01$.

**Proposition 2.** The cooperative multi-goal credit function based MARL policy gradient is

$$
\nabla_\theta J(\pi) = \mathbb{E}_\pi \left[ \sum_{m, n=1}^{N} (\nabla_\theta \log \pi^m(a^m | o^m, g^m)) \pi^m_n(s, a) \right]
$$

(6)

$$
A^\pi_{n, m}(s, a) := Q^\pi_n(s, a) - \sum_{\hat{a}^m} \pi^m(\hat{a}^m | o^m, g^m)Q^\pi_n(s, \hat{a}^m)
$$

(7)

This is derived in Appendix B.2. For a fixed agent $m$, the inner summation over $n$ considers all agents’ goals $g^n$ and updates $m$’s policy based on the advantage of $a^m$ over all counterfactual actions $\hat{a}^m$, as measured by the credit function for $g^n$. The strength of interaction between action-goal pairs is captured by the extent to which $Q^\pi_n(s, \hat{a}^m)$ varies with $\hat{a}^m$, which directly impacts the magnitude of the gradient on agent $m$’s policy. For example, strong interaction results in non-constant $Q^\pi_n(s, \cdot)$, which implies larger magnitude of $A^\pi_{n, m}$ and larger weight on $\nabla_\theta \log \pi(a^m)$. As the second term in $A^\pi_{n, m}$ is a baseline, the reduction of variance can be analyzed similarly to Appendix C.1. While $A^\pi_{n, m} = Q^\pi_n(s, a) - V^\pi_n(s)$ (due to (4)), ablation results show clear stability improvement due to the credit function (Section 6). More significantly, as the credit function takes in a single agent’s action, it synergizes with both CM3’s curriculum and function augmentation as described in Section 4.5.

### 4.3 Curriculum for multi-goal MARL

Multi-goal MARL poses a significant challenge for exploration, since random exploration to learn both individual task completion and cooperative behavior concurrently can be highly inefficient. Agents who cannot make progress toward individual goals may rarely encounter the region of state space where cooperation is needed, rendering any exploration useless for learning cooperative behavior. On the other extreme, exploratory actions taken in situations that require precise coordination can easily lead to penalties that cause agents to avoid the coordination problem and fail to achieve individual goals. Instead, we hypothesize that agents who can achieve individual goals in the absence of other agents can more reliably produce state configurations where cooperative solutions are easily discovered with additional random exploration in the full multi-agent environment.

We propose a new MARL curriculum that first solves a single-agent Markov decision processes (MDP) for significant speedup in multi-goal MARL. Given a cooperative multi-goal Markov game $M^{G}$, we induce an MDP $M$ to be the tuple $(S^n, O^n, A^n, P^n, R, \gamma)$, where an agent $n$ is selected to be the single agent in $M$. Entities $S^n$, $P^n$, and $R$ are defined by removing all dependencies on agent interactions, so that only components depending on agent $n$ remain (Appendix F gives several examples). While the reduction from $M^{G}$ to $M$ must be defined based on context, it can easily be applied to many multi-goal Markov game models of real-world systems: e.g. multi-vehicle maneuvers in autonomous driving (removing all but one car from a road) and traffic light control on a network (removing all but one intersection controller). Based on $M$, we define a greedy policy for $M^{G}$.

**Definition 2.** A greedy policy $\pi^n$ by agent $n$ for cooperative multi-goal $M^{G}$ is defined as the optimal policy $\pi^*$ for the induced MDP $M$ where only agent $n$ is present.

This naturally leads to a new two-stage MARL curriculum in CM3: Stage 1 trains a single agent in $M$ to achieve an optimal policy, which is then used as a greedy initialization in $M^{G}$ in Stage 2. By sampling all goals for training in both Stages 1 and 2, we significantly mitigate the risk of forgetting.

**Example.** A greedy initialization can provide significant improvement in multi-agent exploration versus naïve random exploration, as shown by a simple thought experiment. Consider a two-player $M^{G}$ defined by a $4 \times 3$ gridworld with unit actions (up, down, left, right). Agent $A$ starts at (1,2) with goal (4,2), while agent $B$ starts at (4,2) with goal (1,2). The greedy policy for each agent in $M^{G}$ is to move horizontally toward its target, since this is optimal in the induced $M$ (when the other agent is absent). Case 1: Suppose that for $\epsilon \in (0, 1)$, $A$ and $B$ follow greedy policies with probability $1 - \epsilon$, and take random actions ($p(a) = 1/4$) with probability $\epsilon$. Then the probability of a symmetric optimal trajectory is $P($cooperate$) = 2\epsilon^2((1 - \epsilon) + \epsilon/4)^8$. For $\epsilon = 0.5$, $P($cooperate$) \approx 0.01$. Case 2: If agents execute uniform random exploration, then $P($cooperate$) = 3.05e-5 \ll 0.01$. 

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Figure 1: In Stage 1, \( Q^1 \) and \( \pi^1 \) learn to achieve multiple goals in a single-agent environment. Between Stage 1 and 2, \( \pi \) is constructed from the trained \( \pi^1 \) and a new module \( \pi^2 \) according to (8) (same construction is done for \( Q_\pi(s, a) \) and \( Q_\pi(s, a^m) \), not shown). In the multi-agent environment of Stage 2, these augmented functions are instantiated for each of \( N \) agents (with parameter-sharing).

4.4 Function augmentation for multi-goal curriculum

Not only can the curriculum provide an exploration speedup, but it also motivates more economical use of function approximation to lower the computational cost of MARL. In many multi-agent environments, an agent’s observation space decomposes into \( O^n = O^n_{\text{self}} \cup O^n_{\text{others}} \), where \( o^n_{\text{self}} \in O^n_{\text{self}} \) captures the agent’s own state (e.g. position) while \( o^n_{\text{others}} \in O^n_{\text{others}} \) is the agent’s observation of surrounding agents. Since the ability to process \( o^n_{\text{others}} \) is unnecessary in Stage 1, we reduce the input space of policy and value functions, thereby reducing the number of trainable parameters. Similarly, global state \( s \) decomposes into \( s := (s_{\text{env}}, s^n, s^{-n}) \), where \( s_{\text{env}} \) is agent-agnostic environment information, and only \( (s_{\text{env}}, s^n) \) are required in Stage 1. In Stage 2, we restore Stage 1 parameters and augment function approximators with new modules to process the additional inputs. This augmentation is especially suitable for efficiently learning the credit function (2) and global Q-function, since a \( Q(s, a) \) can be augmented into both \( Q^n\pi(s, a) \) and \( Q^n\pi(s, a^m) \), as explained below.

4.5 A complete instantiation of CM3

We combine all the preceding components to create the CM3 architecture, using deep neural networks for function approximation (Figure 1 and Algorithm 1). Without loss of generality, we assume parameter-sharing [6] with goals as input [29], instead of \( N \) actor-critics.

**Stage 1.** We train an actor \( \pi^1(a|o, g) \) and critic \( Q^1(s^n, a, g) \) to learn multiple goals in an induced MDP. A goal is uniformly sampled from \( G \) for each episode, to learn all goals over the course of training. The actor and critic are trained according to (6) and (5) with \( N = 1 \) (Appendix I).

**Stage 2.** The Markov game is instantiated with all \( N \) agents. We retain the trained \( \pi^1 \) parameters, instantiate a new neural network \( \pi^2 \) for agents to process \( o^n_{\text{others}} \), and connect the output of \( \pi^2 \) to a selected hidden layer of \( \pi^1 \). For example, let \( h^1_i \in \mathbb{R}^{m_i} \) denote hidden layer \( i \leq L \) with \( m_i \) units in an \( L \)-layer network \( \pi^1 \), connected to layer \( i - 1 \) via \( h^1_i = f(W^{1}_{i} h^{1}_{i-1}) \) with \( W^{1}_{i} \in \mathbb{R}^{m_i \times m_{i-1}} \) and nonlinear activation \( f \). Stage 2 introduces a \( K \)-layer network \( \pi^2(o^n_{\text{others}}) \) with outputs \( h^2_K \in \mathbb{R}^{m_K} \), chooses a particular layer\(^2\) \( i^* \) of \( \pi^1 \), and augments the hidden activations \( h^1_{i^*} \) to be

\[
\hat{h}^1_{i^*} = f(W^{1}_{i^*} \hat{h}^1_{i^*-1} + W^{1:2}_{i^*} h^2_K)
\]

\(^2\)An example of \( i^* \) is a fully-connected layer after convolutional layers have processed agents’ image-representations of their field of view. We found it unnecessary to tune this parameter for our experiments.
We provide strong empirical justification for CM3 versus existing methods on diverse multi-goal MARL environments: we postulate—and find evidence in experiments—that this two-stage construction of actor-critic networks with Algorithm implementations. Without loss of generality, we employed parameter-sharing [6, 36, 27] for all methods; the inhomogeneous case is a simple extension. All architecture and hyperparameter details are in Appendices G and H for full reproducibility. We describe only key points here.

**Stage 1** is defined for each environment as follows: we train the credit function with loss (5), and train the global Q-function with the joint-action analogue of (5). Similar to Stage 1, we assign goals to agents by sampling from a distribution over $G$ during each training episode. We postulate—and find evidence in experiments—that this two-stage construction of actor-critic networks with curriculum learning improves learning speed compared to direct training on the full multi-agent environment. Hidden layers $i < i^*$ that were trained for processing $(o_{\text{self}}^m, g^m)$ in Stage 1 retain the ability to process goal information, while the new module learns the effect of surrounding agents. Higher layers $i \geq i^*$ that can generate goal-directed actions in the single-agent setting of Stage 1 are fine-tuned to generate cooperative actions for joint success of all agents.

**Stage 2** is defined to be a fixed policy component $\pi^+$ that can generate goal-directed actions in the single-agent setting of Stage 1.

### 5 Experimental setup

We provide strong empirical justification for CM3 versus existing methods on diverse multi-goal MARL environments: cooperative navigation in difficult formations, double lane merge in autonomous driving, and strategic cooperation in a Checkers game. We evaluated ablations of CM3 on all domains. We describe key setup here, with full reproducible details in Appendices F to I.

**Cooperative navigation:** We created three variants of the cooperative navigation scenario in Lowe et al. [19], where $N$ agents cooperate to reach a set of targets. We increased the difficulty by giving each agent only an individual reward based on distance to its designated target, instead of a global team reward, but initial and target positions require complex team maneuvers to avoid collision penalties (Figure 3). Agents observe relative positions and velocities (see details in Appendix F.1). **Lane merges in SUMO:** Previous work modeled autonomous driving tasks as MDPs in which all other vehicles do not learn to respond to a single learning agent [10, 12]. However, real-world driving is a multi-goal Markov game requiring deliberate cooperation for different drivers’ goals. Built in the SUMO traffic simulator with fine sublane resolution [18], this experiment requires agent vehicles to learn double-merge maneuvers to reach goal lanes. SUMO controls yellow sedans and trucks. Policy generalization was tested on such traffic conditions.

**Checkers:** While the previous two mostly depend on physical interaction, we also created a strategic game based on Checkers [36] (Appendix F.3). Inside a checkered gridworld with red and yellow squares that disappear when collected (Figure 2), Agent A receives $+1$ for red and $-0.5$ for yellow; Agent B receives $-0.5$ for red and $+1$ for yellow. This challenging combinatorial optimization problem requires agents to cooperate in a precise sequence of moves, as each must clear the path for the other.

**Algorithm implementations.** Without loss of generality, we employed parameter-sharing [6, 36, 27] for all methods; the inhomogeneous case is a simple extension. All architecture and hyperparameter details are in Appendices G and H for full reproducibility. We describe only key points here. **CM3:** Stage 1 is defined for each environment as follows (Appendix F): in cooperative navigation, a single particle learns to reach any specified landmark; in SUMO, a car

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3 For convenience of exposition, we assume the combination $(s^n, s^{-n})$ specifies the full MG state. Agent-independent environment information $s_{\text{env}}$ can be given as extra input for both Stage 1 and 2.

4 Input $s^m$ is needed for disambiguation, so that input action $a^m$ is associated with agent $m$. 

Figure 2: Checkers

![Checkers Game](image_url)

(a) Antipodal  (b) Cross  (c) Merge

Figure 3: Cooperative navigation

![Cooperative Navigation](image_url)

Figure 4: Agent sedans must perform double lane merge to reach goal lanes. SUMO controls yellow sedans and trucks. Policy generalization was tested on such traffic conditions.
learns to reach any specified goal lane; in Checkers, we alternate between training one agent as A and B. Appendix G describes exact details of function augmentation in Stage 2. COMA [6]: In our fully cooperative setting, the joint goal \( g \) and total reward \( \sum_n R^n \) can be directly used in COMA. We give input \((s, o^n, g^n, n, a^{-n}, g^{-n})\) to COMA’s global Q function, so that each output node \( i \) represents \( Q(s, a^n = i, a^{-n}, g) \). IAC [40, 6]: IAC trains each agent’s actor and critic independently, using the agent’s own observation. The TD error of value function \( V(o^n, g^n) \) is used in a standard policy gradient [38]. QMIX [27]: we used the original hypernetwork, giving all goals to the mixer and individual goals to each agent network.

**Ablations.** We conducted ablation experiments in all domains. To discover the speedup from the multi-goal curriculum with function augmentation, we trained the full Stage 2 architecture of CM3 (labeled “Direct”) without first training components \( \pi^1 \) and \( Q^1 \) in an induced MDP. To investigate the benefit of the new credit function and multi-goal policy gradient, we trained an ablation (labeled “QV”) with advantage function \( A_n^\pi(s, a) := Q_n^\pi(s, a) - V_n^\pi(s) \), where credit assignment between action-goal pairs is lost. QV uses the same \( \pi^1, Q^1 \), and function augmentation as CM3.

## 6 Results and Discussions

CM3 finds optimal or near-optimal policies significantly faster than IAC and COMA on all domains, and performs significantly higher than QMIX in four out of five. We report absolute runtime in Appendix E and account for CM3’s Stage 1 episodes (Appendix I) when comparing sample efficiency.

**Main comparison** Over all cooperative navigation scenarios (Figures 5a to 5c), CM3 (with 1k episodes in Stage 1) converged more than 15k episodes faster than IAC. IAC reached the same final performance as CM3, because simply moving toward individual goals increases reward despite collisions, but CM3 learns faster using efficient exploration and a mechanism for cooperation. CM3 clearly outperformed both QMIX and COMA, which either settled at bad local optima or diverged, likely due to the difficulty of learning individual goals under a global approach and the fact that cooperation is only learnable in limited regions of state space after agents take goal-directed actions. In SUMO (Figure 5d), only CM3 and QMIX found cooperative solutions, while COMA and IAC could not break out of local optima where vehicles move straight but do not perform merge maneuvers. Since initial states force agents into the region of state space requiring cooperation, credit assignment rather than exploration is the dominant challenge. Hence CM3’s learning progress can be attributed to the credit function. SUMO differs from “Merge” in cooperative navigation: 1. success requires a longer sequence of cooperative actions, proving difficult for IAC; 2. simulator constraints prevent potential divergence of COMA. Appendix D shows generalization results. In Checkers (Figure 5e), CM3 (with 5k episodes in Stage 1) converged 10k episodes faster than COMA to the global optimum (score 24). As exploration of the combinatorially large joint trajectory space is the dominant challenge, CM3’s advantage over COMA can be attributed to its two-stage curriculum. COMA only solved Checkers among all domains, possibly because the bounded
environment alleviates COMA’s difficulty with individual goals in large state spaces. QMIX failed to converge, while IAC was clearly worse than CM3, likely due to lack of centralized information. These results demonstrate CM3’s ability to attain individual goals and find cooperative solutions in diverse multi-agent systems.

**Ablations** The significantly better performance of CM3 versus “Direct” (Figures 6a to 6e) shows that learning individual goal attainment prior to learning multi-agent cooperation is crucial for improving learning speed and stability. It gives evidence that while global action-value and credit functions may be difficult to train from scratch, function augmentation in a two-stage curriculum significantly eases the learning problem. While “QV” initially learns quickly to attain individual goals, it does so at the cost of frequent collisions, higher variance, and inability to maintain a cooperative solution, giving clear evidence for the necessity of the credit function.

7 Conclusion

We presented CM3, a general framework for cooperative multi-goal MARL. CM3 addresses the need for efficient exploration to learn both individual goal attainment and cooperation, via a two-stage curriculum bridged by function augmentation. It achieves local credit assignment between action and goals using a credit function in a multi-goal policy gradient. In diverse experimental domains, CM3 attains significantly higher performance, faster learning, and overall robustness than existing MARL methods, displaying strengths of both independent learning and centralized credit assignment while avoiding shortcomings of existing methods. Ablations demonstrate each component is crucial to the whole framework. Our results motivate future work on analyzing CM3’s theoretical properties and generalizing to inhomogeneous systems or settings without known goal assignments.
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Algorithm 1 Cooperative multi-goal multi-stage multi-agent reinforcement learning (CM3)

1: for curriculum stage \( c = 1 \) to 2 do
2:   if \( c = 1 \) then
3:     Set number of agents \( N = 1 \)
4:     Initialize Stage 1 main networks \( Q_g := Q = Q^1, \pi := \pi^1 \) with parameters \( \theta_{Q^1}, \theta_{\pi^1} \)
5:   Initialize target networks with \( \theta'_{Q^1}, \theta'_{\pi^1} \)
6:   else if \( c = 2 \) then
7:     Instantiate \( N > 1 \) agents
8:     Construct Stage 2 global \( Q_g := Q^\pi_n(s, a) = \{Q^1, Q^2\} \), credit function \( Q_c := Q^\pi_n(s, a^m) = \{Q^1, Q^2\} \) and \( \pi := \{\pi^1, \pi^2\} \) according to (8) with parameters \( \theta_{Q_1}, \theta_{Q_2}, \theta_{\pi} \)
9:     Initialize target networks with \( \theta'_{Q_1}, \theta'_{Q_2}, \theta'_{\pi} \)
10:    Restore values of trained parameters \( \theta_{Q_1}, \theta_{Q_2}, \theta_{\pi} \) into the respective subsets of \( \theta_{Q_g}, \theta_{Q_c}, \theta_{\pi} \)
11: end if
12: end for
13: Set all target network weights to equal main networks weights
14: Initialize exploration parameter \( \epsilon = \epsilon_{\text{start}} \) and empty replay buffer \( B \)
15: for each training episode \( e = 1 \) to \( E \) do
16:     Assign goal(s) \( g^e \) to agent(s) according to given distribution
17:     Get initial state \( s_t \) and observation(s) \( o_t \)
18:     for \( t = 1 \) to \( T \) do // execute policies in environment
19:         Sample action \( a^n_t \sim \pi(a^n_t|o^n_t; \theta_{\pi}, \epsilon) \) for each agent.
20:         Execute action(s) \( a_t \), receive \( \{r^n_t\}_n, s_{t+1}, \text{and } o_{t+1} \)
21:         Store \( (s_t, o_t, g^e, a_t, \{r^n_t\}_n, R^n, s_{t+1}, o_{t+1}) \) into \( B \)
22:         \( s_t \leftarrow s_{t+1}, \text{and } o_t \leftarrow o_{t+1} \)
23:     end for
24:     if \( e \mod E_{\text{rain}} = 0 \) then
25:         for epochs \( 1 \ldots K \) do // conduct training
26:             Sample minibatch of \( S \) transitions \( \{s_t, o_t, g^e, a_t, \{r^n_t\}_n, s_{t+1}, o_{t+1}\} \) from \( B \)
27:             Compute global target for all \( n \): \( x^n_t = r^n_t + \gamma Q(s_{t+1}, a^n_{t+1}, g^n_{t+1}; \theta'_{Q_{g^n}}) | a_{t+1} \sim \pi^n_{a_{t+1}} \)
28:             Gradient descent on \( L(\theta_{Q_g}) = \frac{1}{S} \sum \frac{1}{N} \sum^n_{n=1} (x^n_t - Q(s_t, a^n_t, g^n_t, \theta_{Q_g}))^2 \)
29:             if \( c = 1 \) then
30:                 \( A^\pi(s_t, a_t) := Q^1(s_t, a_t, g_t; \theta_{Q^1}) - \sum_{\pi(a_t)} Q^1(s_t, a_t, g_t; \theta_{Q^1}) \)
31:             else if \( c = 2 \) then
32:                 \( \forall m, n \in [1..N] \), compute target \( y^n_m = r^n_t + \gamma Q(s_{t+1}, a^n_{t+1}, g^n_{t+1}; \theta'_{Q_{g^n}}) | a_{t+1} \sim \pi^n_{a_{t+1}} \)
33:                 Minimize (5): \( L(\theta_{Q_{g^n}}) = \frac{1}{S} \sum \frac{1}{N} \sum^n_{n=1} \sum^N_{m=1} (y^n_m - Q(s_t, a^n_m, g^n_{t}; \theta_{Q_{g^n}}))^2 \)
34:                 \( A^\pi_m(s_t, a_t) := Q(s_t, a^n_m, g^n_{t}; \theta_{Q_{g^n}}) - \sum_{\pi(a^n_m)} Q(s_t, a^n_m, g^n_{t}; \theta_{Q_{g^n}}) \)
35:             end if
36:             \( \nabla_{\theta_{g^n}} J(\pi) = \frac{1}{S} \sum \frac{1}{N} \sum^N_{n=m=1} (\nabla_{\theta_{g^n}} \log \pi(a^n_m|o^n_m, g^n_{m})) A^\pi_m(s_t, a_t) \)
37:             Update policy: \( \theta_{\pi} \leftarrow \theta_{\pi} + \beta \nabla_{\theta_{\pi}} J(\pi) \)
38:     end for
39:     Update all target network parameters using: \( \theta' \leftarrow \tau \theta + (1 - \tau) \theta' \)
40:     Reset buffer \( B \)
41: end if
42: if \( \epsilon > \epsilon_{\text{end}} \), then \( \epsilon \leftarrow \epsilon - \epsilon_{\text{step}} \)
43: end for
44: end for

Off-policy training with a large replay buffer allows RL algorithms to benefit from less correlated transitions [32, 15]. The algorithmic modification for off-policy training is to maintain a circular replay buffer that does not reset (i.e. remove line 38), and conduct training (lines 24-41) while executing policies in the environment (lines 17-22). Despite introducing bias in MARL, we found that off-policy training benefited CM3 in SUMO and Checkers.
B Derivations

B.1 Proposition 1

By stationarity and relabeling $t$, the credit function can be written:

$$Q^\pi_n(s, a^m) := \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a^m_0 = a^m \right] = \mathbb{E}_\pi \left[ \sum_{t=1}^{\infty} \gamma^{t-1} R(s_t, a_t, g^n) \mid s_1 = s, a^m_1 = a^m \right]$$

Using the law of iterated expectation, the credit function satisfies the Bellman expectation equation (3):

$$Q^\pi_n(s, a^m) = \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a^m_0 = a^m \right]$$

$$= \mathbb{E}_\pi \left[ R(s_0, a_0, g^n) + \sum_{t=1}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a^m_0 = a^m \right]$$

$$= \mathbb{E}_{s,a^m} \left[ \pi(a^{-m} \mid s, g^{-m}) R(s, (a^m, a^{-m}), g^n) \right]$$

$$+ \mathbb{E}_\pi \left[ \sum_{t=1}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a^m_0 = a^m, s_1 = s', a^m_1 = \hat{a}^m \right] \mid s_0 = s, a^m_0 = a^m$$

$$= \sum_{a^{-m}} \pi(a^{-m} \mid s, g^{-m}) R(s, (a^m, a^{-m}), g^n)$$

$$+ \sum_{a^{-m}} \pi(a^{-m} \mid s, g^{-m}) \sum_{s'} P(s' \mid s, (a^m, a^{-m})) \sum_{\hat{a}^m} \pi(\hat{a}^m \mid o^m(s')) \mathbb{E}_\pi \left[ \sum_{t=1}^{\infty} \gamma^{t-1} R(s_t, a_t, g^n) \mid s_1 = s', a^m_1 = \hat{a}^m \right]$$

$$= \sum_{a^{-m}} \pi(a^{-m} \mid s, g^{-m}) \left[ R(s, (a^m, a^{-m}), g^n) + \gamma \sum_{s'} P(s' \mid s, (a^m, a^{-m})) \sum_{\hat{a}^m} \pi(\hat{a}^m \mid o^m(s')) Q^\pi_n(s', \hat{a}^m) \right]$$

$$= \mathbb{E}_\pi \left[ R(s_t, a_t, g^n) + \gamma Q^\pi_n(s_{t+1}, a^m_{t+1}) \mid s_t = s, a^m_t = a^m \right]$$

$\square$
We follow the proof of the policy gradient theorem [38]:

The goal-specific joint value function is the marginal of the credit function:

\[
V_{\pi}^n(s) = \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s \right]
\]

\[
= \mathbb{E}_{\pi^n|s_0, \pi} \left[ \mathbb{E}_{\pi} \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a_0^m = a^m \right] \mid s_0 = s \right]
\]

\[
= \sum_{a^m} \pi(a^m | a^m(s), g^m) Q_n^n(s, a^m)
\]

The credit function can be expressed in terms of the goal-specific action-value function:

\[
V_{\pi}^n(s) = \sum_a \pi(a | s, g^m) Q_n^n(s, a)
\]

by (4)

\[
V_{\pi}^n(s) = \sum_a \pi(a | s, g^m) Q_n^n(s, a)
\]

by (10)

\[
\Rightarrow Q_n^n(s, a^m) = \sum_{a^{-m}} \pi(a^{-m} | s, g^{-m}) Q_n^n(s, a)
\]

B.2 Proposition 2

First we state some elementary relations between global functions \(V_{\pi}^n(s)\) and \(Q_n^n(s, a)\). These carry over directly from the case of an MDP, by treating the joint policy \(\pi\) as an effective "single-agent" policy and restricting attention to a single goal \(g^n\) (standard derivations are included at the end of this section).

\[
Q_n^n(s, a) = R(s, a, g^n) + \gamma \sum_{s'} P(s' | s, a) V_{\pi}^n(s')
\]

(9)

\[
V_{\pi}^n(s) = \sum_a \pi(a | s, g) Q_n^n(s, a)
\]

(10)

We follow the proof of the policy gradient theorem [38]:

\[
\nabla_\theta V_{\pi}^n(s) = \nabla_\theta \sum_a \pi(a | s, g) Q_n^n(s, a)
\]

\[
= \sum_a \left[ (\nabla_\theta \pi(a | s, g)) Q_n^n(s, a) + \pi(a | s, g) \nabla_\theta Q_n^n(s, a) \right]
\]

\[
= \sum_a \left[ (\nabla_\theta \pi(a | s, g)) Q_n^n(s, a) + \pi(a | s, g) \nabla_\theta \left( R(s, a, g^n) + \gamma \sum_{s'} P(s' | s, a) V_{\pi}^n(s') \right) \right]
\]

\[
= \sum_a \left[ (\nabla_\theta \pi(a | s, g)) Q_n^n(s, a) + \pi(a | s, g) \gamma \sum_{s'} P(s' | s, a) \nabla_\theta V_{\pi}^n(s') \right]
\]

\[
= \sum_a \sum_{k=0}^{\infty} \gamma^k P(s \rightarrow s', k, \pi) \sum_a (\nabla_\theta \pi(a | s', g)) Q_n^n(s', a) \quad \text{(by recursively unrolling)}
\]

\[
\nabla_\theta J_n(\pi) := \nabla_\theta V_{\pi}^n(s_0) = \sum_{s, k=0}^{\infty} \gamma^k P(s_0 \rightarrow s, k, \pi) \sum_a (\nabla_\theta \pi(a | s, g)) Q_n^n(s, a)
\]

\[
= \sum_s \rho^n(s) \sum_a \pi(a | s, g) (\nabla_\theta \log \pi(a | s, g)) Q_n^n(s, a)
\]

\[
= \mathbb{E}_{\pi} [\nabla_\theta \log \pi(a | s, g) Q_n^n(s, a)]
\]

(11)
We can replace $Q^n(\pi(s, a))$ by the advantage function $A^n(\pi(s, a)) := Q^n(\pi(s, a)) - V^n(\pi(s))$, which does not change the expectation in Equation (11) because:

\begin{align*}
\mathbb{E}_\pi [\nabla_\theta \log \pi(a|s, g) V^n(s)] &= \sum_s \rho^n(s) \sum_a \pi(a|s, g) \nabla_\theta \log \pi(a|s, g) V^n(s) \\
&= \sum_s \rho^n(s) V^n(s) \nabla_\theta \sum_a \pi(a|s, g) = 0
\end{align*}

So the gradient (11) can be written

$$\nabla_\theta J_n(\pi) = \mathbb{E}_\pi \left[ (\nabla_\theta \sum_{m=1}^N \log \pi(a^m|o^m, g^m)) (Q^n(s, a) - V^n(s)) \right] \quad (12)$$

Recall that from (4), for any choice of agent label $k \in [1..N]$:

$$V^n(s) = \sum_{a^k} \pi(a^k|o^k, g^k) Q^n(s, a^k) \quad (13)$$

Then substituting (4) into (12):

$$\nabla_\theta J_n(\pi) = \mathbb{E}_\pi \left[ (\nabla_\theta \sum_{m=1}^N \log \pi(a^m|o^m, g^m)) A^n_{n,k}(s, a) \right] \quad (14)$$

$$A^n_{n,k}(s, a) := Q^n(s, a) - \sum_{\hat{a}^k} \pi(\hat{a}^k|o^k, g^k) Q^n(s, \hat{a}^k) \quad (15)$$

Now notice that the choice of $k$ in (15) is completely arbitrary, since (4) holds for any $k \in [1..N]$. Therefore, it is valid to distribute $A^n_{n,k}(s, a)$ into the summation in (14) using the summation index $m$ instead of $k$. Further summing (14) over all $n$, we arrive at the result of Proposition 2:

$$\nabla_\theta J(\pi) = \mathbb{E}_\pi \left[ \sum_{m=1}^N \sum_{n=1}^N \left( \nabla_\theta \log \pi(a^m|o^m, g^m) \right) A^n_{n,m}(s, a) \right]$$

$$A^n_{n,m}(s, a) := Q^n(s, a) - \sum_{\hat{a}^m} \pi(\hat{a}^m|o^m, g^m) Q^n(s, \hat{a}^m) \quad \square$$
The relation between $V_\pi^n(s)$ and $Q_\pi^n(s, a)$ in (9) and (10) are derived as follows:

$$Q_\pi^n(s, a) := \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a_0 = a \right]$$

$$= \mathbb{E}_\pi \left[ R(s_0, a_0, g^n) + \sum_{t=1}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a_0 = a \right]$$

$$= R(s, a, g^n) + \mathbb{E}_{s_1/s_0, a_0, \pi} \left[ \mathbb{E}_\pi \left[ \sum_{l=1}^{\infty} R(s_l, a_l, g^n) \mid s_0 = s, a_0 = a, s_1 = s' \right] \mid s_0 = s, a_0 = a \right]$$

$$= R(s, a, g^n) + \gamma \sum_{s'} P(s' \mid s, a) Q_\pi^n(s')$$

$$V_\pi^n(s) := \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s \right]$$

$$= \mathbb{E}_{a_0/s_0, \pi} \left[ \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a_0 = a \right] \mid s_0 = s \right]$$

$$= \sum_a \pi(a \mid s, g) \mathbb{E}_\pi \left[ \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t, g^n) \mid s_0 = s, a_0 = a \right]$$

$$= \sum_a \pi(a \mid s, g) Q_\pi^n(s, a)$$

\[\square\]
### C Variance

#### C.1 Variance of COMA gradient.

For convenience, let \( Q := Q^\pi(s, a, g) \) denote the centralized Q function, let \( \pi(a^n) := \pi(a^n|o^n, g^n) \) denote a single agent’s policy, and let \( \pi(a^{-n}) := \pi(a^{-n}|o^{-n}, g^{-n}) \) denote the other agents’ joint policy.

In the cooperative multi-goal MARL context, the direct application of COMA has the following gradient.

\[
\nabla_\theta J = \mathbb{E}\left[ \sum_n \nabla_\theta \log \pi(a^n|o^n, g^n) \left( Q - b_n(s, a^{-n}, g) \right) \right]
\]

Define the following:

\[
b_n(s, a^{-n}, g) := \sum \pi(\hat{a}^n|o^n, g^n)Q^\pi(s, \hat{a}^n, a^{-n}, g)
\]

Define:

\[
z_n := \nabla_\theta \log \pi(a^n|o^n, g^n)
\]

\[
f_n := \nabla_\theta \log \pi(a^n|o^n, g^n) \left( Q - b_n(s, a^{-n}, g) \right) = z_n \left( Q - b_n(s, a^{-n}, g) \right)
\]

Define \( M_{nm} := \mathbb{E}_{\pi}[f_n]^T \mathbb{E}_{\pi}[f_m] \) and let \( M := \sum_{n,m} M_{nm} \). Then we have \( M_{nm} = \mathbb{E}_{\pi}[z_n Q]^T \mathbb{E}_{\pi}[z_m Q] \) since

\[
\mathbb{E}_{\pi}[z_n b_n] = \mathbb{E}_{\pi} \left[ \sum_s \rho^\pi(s) \sum_a \pi(a|s, g) \nabla_\theta \log \pi(a^n|o^n, g^n) b_n(s, a^{-n}, g) \right] = \sum_s \rho^\pi(s) \sum_a \pi(a^n|o^{-n}, g^n) \nabla_\theta \log \pi(a^n|o^n, g^n) b_n(s, a^{-n}, g) = 0
\]

Since the COMA gradient is \( \mathbb{E}_{\pi} [\sum_{n=1}^N f_n] \), its variance can be derived to be [44]:

\[
\text{Var}(\sum_{n=1}^N f_n) = \sum_n \mathbb{E}_{\pi} \left[ z_n^T z_n Q^2 - 2b_n z_n^T z_n Q + b_n^2 z_n^T z_n \right] + \sum_n \sum_{m \neq n} \mathbb{E}_{\pi} [z_n^T z_m (Q - b_n)(Q - b_m)] - M
\]

#### C.2 Variance of the CM3 gradient

For convenience, let \( Q_n := Q^\pi_n(s, a) = Q^\pi(s, a, g^n) \) denote the global Q function for goal \( g^n \), and let \( \pi(a^m) := \pi(a^m|o^m, g^m) \). The CM3 gradient can be rewritten as

\[
\nabla_\theta J(\pi) = \mathbb{E}_{\pi} \left[ \sum_{n=1}^N \sum_{m=1}^N \nabla_\theta \log \pi(a^m) \left( Q_n - b_{nm}(s) \right) \right]
\]

Define:

\[
b_{nm}(s) := \sum \pi(\hat{a}^m|o^m)Q^\pi_n(s, \hat{a}^m)
\]

As before, \( z_m := \nabla_\theta \log \pi(a^m) \). Define \( h_{nm} := z_m(Q_n - b_{nm}(s)) \) and let \( h_n := \sum_m h_{nm} \). Then the variance is

\[
\text{Var}(\sum_n h_n) = \sum_n \text{Var}(h_n) + \sum_n \sum_{m \neq n} \text{Cov}(h_n, h_m)
\]

\[
= \sum_n \left( \sum_m \text{Var}(h_{nm}) + \sum_m \sum_{k \neq m} \text{Cov}(h_{nm}, h_{nk}) \right) + \sum_n \sum_{m \neq n} \text{Cov}(h_n, h_m)
\]
D Generalization

Table 1: Test performance with heavy traffic on difficult initial and goal lanes configurations

| Config | Initial lanes | Goal lanes | CM3  | IAC  | COMA |
|--------|---------------|------------|------|------|------|
| C1     | [1, 2]        | [3, 0]     | 16.17| 11.40| 10.00|
| C2     | Unif. random  | Unif. random| 14.93| 12.20| 12.93|
| C3     | [1, 2]        | [2, 1]     | 15.85| 14.32| 15.00|
| C4     | [0, 1]        | [3, 2]     | 16.35| 9.73 | 8.1  |

We investigated whether policies trained with few agent vehicles ($N = 2$) on an empty road can generalize to situations with heavy SUMO-controlled traffic. We also tested on initial and goal lane configurations (C3 and C4) which occur with low probability when training with configurations C1 and C2. Table 1 shows the sum of agents’ reward, averaged over 100 test episodes, on these configurations that require cooperation with each other and with minimally-interactive SUMO-controlled vehicles for success. CM3’s higher performance than IAC and COMA in training is reflected by better generalization performance on these test configurations. There is almost negligible decrease in performance from train Figure 5d to test, giving evidence to our hypothesis that centralized training with few agents is feasible even for deployment in situations with many agents, for certain applications where local interactions are dominant.

E Absolute runtime

CM3’s higher sample efficiency does not come at greater computational cost, as all methods’ runtimes are within an order of magnitude of one another. Test times have no significant difference as all neural networks were similar.

Table 2: Absolute training runtime of all algorithms in seconds

| Environment   | CM3         | IAC         | COMA        | QMIX        |
|---------------|-------------|-------------|-------------|-------------|
| Antipodal     | 1.1e4±348   | 0.9e4±20   | 1.9e4±238   | 1.0e4±19    |
| Cross         | 1.9e4±256   | 1.5e4±26   | 1.3e4±12    | 1.1e4±34    |
| Merge         | 8.5e3±21    | 6.8e3±105  | 9.6e3±294   | 1.2e4±61    |
| SUMO          | 9.6e3±278   | 7.0e3±1.5e3| 8.7e3±1.3e3 | 6.3e3±21    |
| Checkers      | 9.2e3±880   | 8.5e3±568  | 7.7e3±2.2e3 | 11e3±1.4e3  |

F Environment details

The full Markov game for each experimental domain, along with the single-agent MDP induced from the Markov game, are defined in this section. In all domains, each agent’s observation in the Markov game consists of two components, $o_{self}$ and $o_{others}$. CM3 leverages this decomposition for faster training, while IAC and SUMO do not.

F.1 Cooperative navigation

This domain is adapted from the multi-agent particle environment in Lowe et al. [19]. Movable agents and static landmarks are represented as circular objects located in a 2D unbounded world with real-valued position and velocity. Agents experience contact forces during collisions. A simple model of inertia and friction is involved.

State. The global state vector is the concatenation of all agents’ absolute position $(x, y) \in \mathbb{R}^2$ and velocity $(v_x, v_y) \in \mathbb{R}^2$. 

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**Observation.** Each agent’s observation of itself, $o_{\text{self}}$, is its own absolute position and velocity. Each agent’s observation of others, $o_{\text{others}}$, is the concatenation of the relative positions and velocities of all other agents with respect to itself.

**Actions.** Agents take actions from the discrete set do nothing, up, down, left, right, where the movement actions produce an instantaneous velocity (with inertia effects).

**Goals and initial state assignment.** With probability 0.2, landmarks are given uniform random locations in the set $(-1,1)^2$, and agents are assigned initial positions uniformly at random within the set $(-1,1)^2$. With probability 0.8, they are predefined as follows (see Figure 3). In “Antipodal”, landmarks for agents 1 to 4 have $(x, y)$ coordinates $[(0.9,0.9), (-0.9,-0.9), (0.9,-0.9), (-0.9,0.9)]$, while agents 1 to 4 are placed at $[(-0.9,0.9), (0.9,0.9), (-0.9,0.9), (0.9,0.9)]$. In “Intersection”, landmark coordinates are $[(0.9,-0.15), (-0.9,0.15), (0.15,0.9), (-0.15,-0.9)]$, while agents are placed at $[(-0.9,-0.15), (0.9,0.15), (0.15,-0.9), (-0.15,0.9)]$. In “Merge”, landmark coordinates are $[(0.9,-0.2), (0.9,0.2)]$, while agents are $[(-0.9,0.2), (-0.9,-0.2)]$. Each agent’s goal is the assigned landmark position vector.

**Reward.** At each time step, each agent’s individual reward is the negative distance between its position and the position of its assigned landmark. If a collision occurs between any pair of agents, both agents receive an additional -1 penalty. A collision occurs when two agents’ distance is less than the sum of their radius.

**Termination.** Episode terminates when all agents are less than 0.05 distance from assigned landmarks.

**Induced MDP.** This is the $N = 1$ case of the Markov game, used by Stage 1 of CM3. The single agent only receives $o_{\text{self}}$. In each episode, its initial position and the assigned landmark’s initial position are both uniform randomly chosen from $(-1,1)^2$.

### F.2 SUMO

We constructed a straight road of total length 200m and width 12.8m, consisting of four lanes. All lanes have width 3.2m, and vehicles can be aligned along any of four sub-lanes within a lane, with lateral spacing 0.8m. Vehicles are emitted at average speed 30m/s with small deviation. Simulation time resolution was 0.2s per step. Supplementary file `merge_stage3_dense.rou.xml` contains all vehicle parameters, and `merge.net.xml` defines the complete road architecture.

**State.** The global state vector $s$ is the concatenation of all agents’ absolute position $(x, y)$, normalized respectively by the total length and width of the road, and horizontal speed $v$ normalized by 29m/s.

**Observation.** Each agent observation of itself $o_{\text{self}}^n$ is a vector consisting of: agent speed normalized by 29m/s, normalized number of sub-lanes between agent’s current sub-lane and center sub-lane of goal lane, and normalized longitudinal distance to goal position. Each agent’s observation of others $o_{\text{others}}^n$ is discretized observation tensor of shape [13,9,2] centered on the agent, with two channels: binary indicator of vehicle occupancy, and normalized relative speed between agent and other vehicles. Each channel is a matrix with shape [13,9], corresponding to visibility of the total length and width of the road, and horizontal speed.

**Actions.** All agents have the same discrete action space, consisting of five options: no-op (maintain current speed and lane), accelerate $(2.5m/s^2)$, decelerate $(-2.5m/s^2)$, shift one sub-lane to the left, shift one sub-lane to the right. Each agent’s action $a_n$ is represented as a one-hot vector of length 5.

**Goals and initial state assignment.** Each goal vector $g^n$ is a one-hot vector of length 4, indicating the goal lane at which agent $n$ should arrive once it crosses position $x=190m$. With probability 0.2, agents are assigned goals uniformly at random, and agents are assigned initial lanes uniformly at random at position $x=0$. With probability 0.8, agent 1’s goal is lane 2 and agent 2’s goal is lane 1, while agent 1 is initialized at lane 1 and agent 2 is initialized at lane 2 (see Figure 4). Departure times were drawn from a normal distribution with mean 0s and standard deviation 0.5s for each agent.

**Reward.** The reward $R(s_t, a_t, g^n)$ for agent $n$ with goal $g^n$ is given according to the conditions: -1 for a collision; -10 for time-out (exceed 33 simulation steps during an episode); $10(1 - \Delta)$ for reaching the end of the road and having a normalized sub-lane difference of $\Delta$ from the center of the goal lane; and -0.1 if current speed exceeds 35.7m/s.

**Termination.** Episode terminates when 33 simulation steps have elapsed or all agents have $x > 190m$. 

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**Induced MDP.** This is the $N = 1$ case of the Markov game defined above, used by Stage 1 of CM3. The single agent receives only $o_{self}$. For each episode, agent initial and goal lanes are assigned uniformly at random from the available lanes.

**F.3 Checkers**

This domain is adapted from the Checkers environment in Sunehag et al. [36]. It is a gridworld with 5 rows and 13 columns (Figure 2). Agents cannot move to the two highest and lowest rows and the two highest and lowest columns, which are placed for agents’ finite observation grid to be well-defined. Agents cannot be in the same grid location. Red and yellow collectible reward are placed in a checkered pattern in the middle 3x8 region, and they disappear when any agent moves to their location.

**State.** The global state $s$ consists of two components. The first is $s_T$, a tensor of shape [3,9,2], where the two “channels” in the last dimension represents the presence/absence of red and yellow rewards as 1-hot matrices. The second is $s_V$, the concatenation of all agents’ $(x,y)$ location (integer-valued) and the number of red and yellow each agent has collected so far.

**Observation.** Each agent’s observation of others, $o_{other}^n$, is the concatenation of all other agents’ normalized coordinates (normalized by total size of grid). An agent’s observation of itself, $o_{self}^n$, consists of two components. First, $o_{self,V}^n$ is a vector concatenation of agent $n$’s normalized coordinate and the number of red and yellow it has collected so far. Second, $o_{self,T}^n$ is a tensor of shape [5,5,3], centered on its current location in the grid. The tensor has three “channels”, where the first two represent presence/absence of red and yellow rewards as 1-hot matrices, and the last channel indicates the invalid locations as a 1-hot matrix. The agent’s own grid location is a valid location, while other agents’ locations are invalid.

**Actions.** Agents choose from a discrete set of actions do-nothing, up, down, left, right. Movement actions transport the agent one grid cell in the chosen direction.

**Goals.** Agent A’s goal is to collect all red rewards without touching yellow. Agent B’s goal is to collect all yellow without touching red. The goal is represented as a 1-hot vector of length 2.

**Reward.** Agent A gets +1 for red, -0.5 for yellow. Agent B gets -0.5 for red, +1 for yellow.

**Initial state distribution.** Agent A is initialized at (2,8), Agent B is initialized at (4,8). (0,0) is the top-left cell (Figure 2).

**Termination.** Each episode finishes when either 75 time steps have elapsed, or when all rewards have been collected.

**Induced MDP.** For Stage 1 of CM3, the single agent is randomly assigned the role of either Agent A or Agent B in each episode. Everything else is defined as above.

**G Architecture**

For all experiment domains, ReLU nonlinearity was used for all neural network layers unless otherwise specified. All layers are fully-connected feedforward layers, unless otherwise specified. All experiment domains have a discrete action space (with $|A| = 5$ actions), and action probabilities were computed by lower-bounding softmax outputs of all policy networks by $P(a^n = i) = (1 - \epsilon)\text{softmax}(i) + \epsilon/|A|$, where $\epsilon$ is a decaying exploration parameter. To keep neural network architectures as similar as possible among all algorithms, our neural networks for COMA differ from those of Foerster et al. [6] in that we do not use recurrent networks, and we do not feed previous actions into the Q function. For the Q network in all implementations of COMA, the value of each output node $i$ is interpreted as the action-value $Q(s, a^{-n}, a^n = i, g)$ for agent $n$ taking action $i$ and all other agents taking action $a^{-n}$. Also for COMA, agent $n$’s label vector (one-hot indicator vector) and observation $o_{self}$ were used as input to COMA’s global Q function, to differentiate between evaluations of the Q-function for different agents. These were choices in Foerster et al. [6] that we retain.
G.1 Cooperative navigation

CM3. The policy network $\pi^1$ in Stage 1 feeds the concatenation of $o_{\text{self}}$ and goal $g$ to one layer with 64 units, which is connected to the special layer $h^1_s$ with 64 units, then connected to the softmax output layer with 5 units, each corresponding to one discrete action. In Stage 2, $o_{\text{others}}$ is connected to a new layer with 128 units, then connected to $h^1_s$.

The $Q^1$ function in Stage 1 feeds the concatenation of state $s$, goal $g$, and 1-hot action $a$ to one layer with 64 units, which is connected to the special layer $h^1_s$ with 64 units, then to a single linear output unit. In Stage 2, $Q^1$ is augmented into both $Q^1_n(s,a)$ and $Q^1_m(s,a^m)$ as separate networks. For $Q^1_n(s,a)$, $s^{-n}$ (part of state $s$ excluding agent $n$) and $a^{-n}$ are concatenated and connected to a layer with 128 units, then connected to $h^1_s$. For $Q^1_n(s,a^m)$, $s^m$ (agent $m$ portion of state $s$) and $s^{-n}$ are concatenated and connected to a layer with 128 units, then connected to $h^1_s$.

IAC. IAC uses the same policy network as Stage 2 of CM3. The value function of IAC concatenates $o^1_{\text{self}}$ and goal $g^n$, connects to a layer with 64 units, which connects to a second layer $h_2$ with 64 units, then to a single linear output unit. $o^1_{\text{others}}$ is connected to a layer with 128 units, then connected to $h^2$.

COMA. COMA uses the same policy network as Stage 2 of CM3. The global $Q$ function of COMA computes $Q(s,(a^n,a^{-n}))$ for each agent $n$ as follows. Input is the concatenation of state $s$, all other agents’ 1-hot actions $a^{-n}$, agent $n$’s goal $g^n$, all other agent goals $g^{-n}$, agent label $n$, and agent $n$’s observation $o^1_{\text{self}}$. This is passed through two layers of 128 units each, then connected to a linear output layer with 5 units.

QMIX. Individual value functions take input $(o^1_{\text{self}}, o^1_{\text{others}}, g^n)$ and connects to one hidden layer with 64 units, which connects to the output layer. The mixing network follows the exact architecture of Rashid et al. [27] with embedding dimension 64.

G.2 SUMO

CM3. The policy network $\pi^1$ during Stage 1 feeds each of the inputs $o_{\text{self}}$ and goal $g^n$ to a layer with 32 units. The concatenation is then connected to the layer $h^1_s$ with 64 units, and connected to a softmax output layer with 5 units, each corresponding to one discrete action. In Stage 2, the input observation grid $o^1_{\text{others}}$ is processed by a convolutional layer with 4 filters of size 5x3 and stride 1x1, flattened and connected to a layer with 64 units, then connected to the layer $h^1_s$ of $\pi^1$.

The $Q^1$ function in Stage 1 feeds the concatenation of state $s$, goal $g$, and 1-hot action $a$ to one layer with 256 units, which is connected to the special layer $h^1_s$ with 256 units, then to a single linear output unit. In Stage 2, $Q^1$ is augmented into both $Q^1_n(s,a)$ and $Q^1_m(s,a^m)$ as separate networks. For $Q^1_n(s,a)$, $s^{-n}$ (part of state $s$ excluding agent $n$), $a^{-n}$, and $g^{-n}$ are concatenated and connected to a layer with 128 units, then connected to $h^1_s$. For $Q^1_n(s,a^m)$, $s^m$ (agent $m$ portion of state $s$), $s^{-n}$, and $g^{-n}$ are concatenated and connected to a layer with 128 units, then connected to $h^1_s$.

IAC. IAC uses the same policy network as Stage 2 of CM3. The value function of IAC concatenates $o^1_{\text{self}}$ and goal $g^n$, feeds it into a layer with 64 units, which connects to a layer $h_2$ with 64 units, which connects to one linear output unit. $o^1_{\text{others}}$ is processed by a convolutional layer with 4 filters of size 5x3 and stride 1x1, flattened and connected to a layer with 128 units, then connected to $h_2$.

COMA. COMA uses the same policy network as Stage 2 of CM3. The Q function of COMA is exactly the same as the one in COMA for cooperative navigation defined above.

QMIX. Individual value functions take input $(o^1_{\text{self}}, g^n)$ and connects to one hidden layer with 64 units, which connects to layer $h_2$ with 64 units. $o^1_{\text{others}}$ is passed through the same convolutional layer as above and connected to $h_2$. $h_2$ is fully-connected to an output layer. The mixing network follows the exact architecture of Rashid et al. [27] with embedding dimension 64.

G.3 Checkers

CM3. The policy network $\pi^1$ during Stage 1 feeds $o^1_{\text{self},T}$ to a convolution layer with 6 filters of size 3x3 and stride 1x1, which is flattened and connected to a layer with 32 units, which is concatenated with $o^1_{\text{self},V}$, previous action, and its
goal vector. The concatenation is connected to a layer with 256 units, then to the special layer $h_1$ with 256 units, finally to a softmax output layer with 5 units. In Stage 2, $o_{\text{others}}^n$ is connected to a layer with 256 units, then to the layer $h_1$ of $\pi^1$.

The $Q^1$ function in Stage 1 is defined as: state tensor $s_T$ is fed to a convolutional layer with 4 filters of size 3x5 and stride 1x1 and flattened. $o_{\text{self,T}}^n$ is given to a convolution layer with 6 filters of size 3x3 and stride 1x1 and flattened. Both are concatenated with $s^n$ (agent $n$ part of the $s_V$ vector), goal $g^n$, action $a^n$ and $o_{\text{self,V}}^n$. The concatenation is fed to a layer with 256 units, then to the special layer $h_1$ with 256 units, then to a single linear output unit. In Stage 2, $Q^1$ is augmented into both $Q^\pi_n(s, a)$ and $Q^\pi_n(s, a^m)$ as separate networks. For $Q^\pi_n(s, a)$, $s^{-n}$ (part of state vector $s_V$ excluding agent $n$) and $a^{-n}$ are concatenated and connected to a layer with 32 units, then connected to $h_1$. For $Q^\pi_n(s, a^m)$, $s^m$ (agent $m$ portion of state $s_V$) and $s^{-n}$ are concatenated and connected to a layer with 32 units, then connected to $h_1$.

**IAC.** IAC uses the same policy network as Stage 2 of CM3. The value function of IAC feeds $o_{\text{self,T}}^n$ to a convolutional layer with 6 filters of size 3x3 and stride 1x1, which is flattened and concatenated with $o_{\text{self,V}}^n$ and goal $g^n$. The concatenation is connected to a layer with 256 units, then to a layer $h_2$ with 256 units, then to a single linear output unit. $o_{\text{others}}^n$ is connected to a layer with 32 units, then to the layer $h_2$.

**COMA.** COMA uses the same policy network as Stage 2 of CM3. The global $Q(s, (a^n, a^{-n}))$ function of COMA is defined as follows for each agent $n$. Tensor part of global state $s_T$ is given to a convolutional layer with 4 filters of size 3x5 and stride 1x1. Tensor part of agent $n$’s observation $o_{\text{self,T}}^n$ is given to a convolutional layer with 6 filters of size 3x3 and stride 1x1. Outputs of both convolutional layers are flattened, then concatenated with $s_V$, all other agents’ actions $a^{-n}$, agent $n$’s goal $g^n$, other agents’ goals $g^{-n}$, agent $n$’s label vector, and agent $n$’s vector observation $o_{\text{self,V}}^n$. The concatenation is passed through two layers with 256 units each, then to a linear output layer with 5 units.

**QMix.** Individual value functions are defined as: $o_{\text{self,T}}^n$ is passed through the same convolutional layer as above, connected to hidden layer with 32 units, then concatenated with $o_{\text{self,V}}^n$, $o_{\text{IAC}}^{n-1}$, and $g^n$. This is connected to layer $h_2$ with 64 units. $o_{\text{others}}^n$ is connected to a layer with 64 units then connected to $h_2$. $h_2$ is fully-connected to an output layer. The mixing network feeds $s_T$ into the same convolutional network as above and follows the exact architecture of Rashid et al. [27] with embedding dimension 128.

## H Parameters

We used the Adam optimizer in Tensorflow with hyperparameters the following tables.

| Table 3: Parameters used for CM3, ablations, and baselines in cooperative navigation |
|---------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Parameter                      | Stage 1 | Stage 2 | QV | Direct | IAC | COMA | QMIX |
| Episodes                       | 1k     | 80k    | 80k | 80k    | 80k | 80k  | 80k  |
| $\epsilon_{\text{start}}$     | 1.0    | 0.5    | 0.5 | 1.0    | 1.0 | 1.0  | 1.0  |
| $\epsilon_{\text{end}}$       | 0.01   | 0.05   | 0.05| 0.05   | 0.05| 0.05 | 0.05 |
| $\epsilon_{\text{step}}$      | 9.9e-5 | 2.3e-5 | 2.3e-5| 1.2e-5 | 1.2e-5| 1.2e-5| 1.2e-5 |
| Replay buffer                  | 10k    | 10k    | 10k | 10k    | 10k | 10k  | 10k  |
| Minibatch size                 | 256    | 128    | 128 | 128    | 128 | 128  | 128  |
| Episodes per train             | 10     | 10     | 10  | 10     | 10  | N/A  | N/A  |
| Learning rate $\pi$            | 1e-4   | 1e-4   | 1e-4| 1e-4   | 1e-4| 1e-4 | N/A  |
| Learning rate $Q$              | 1e-3   | 1e-3   | 1e-3| 1e-3   | N/A | 1e-3 | 1e-3 |
| Learning rate $V$              | N/A    | N/A    | 1e-3| N/A    | 1e-3| N/A  | N/A  |
| Epochs                         | 24     | 24     | 24  | 24     | 24  | N/A  | N/A  |
| Steps per train                | N/A    | N/A    | N/A | N/A    | N/A | N/A  | 10   |
| Max env steps                  | 25     | 50     | 50  | 50     | 50  | 50   | 50   |
### Table 4: Parameters used for CM3 and baselines in SUMO

| Parameter | Stage 1 | Stage 2 | QV | Direct | IAC | COMA | QMIX |
|-----------|---------|---------|----|--------|-----|------|------|
| Episodes  | 2.5k    | 50k     | 50k| 50k    | 50k | 50k  | 50k  |
| $\epsilon_{\text{start}}$ | 0.5     | 0.5     | 0.5| 0.5    | 0.5 | 0.5  | 0.5  |
| $\epsilon_{\text{end}}$ | 0.05    | 0.05    | 0.05| 0.05   | 0.05| 0.05 | 0.05 |
| $\epsilon_{\text{step}}$ | 2.25e-4 | 4.5e-4  | 1.13e-5| 1.13e-5| 1.13e-5| 1.13e-5| 1.13e-5|
| Replay buffer | 10k     | 20k     | 20k| 20k    | 10k | 10k  | 10k  |
| Minibatch size | 128     | 128     | 128| 128    | 128 | 128  | 128  |
| Steps per train | 10      | 10      | 10 | 10     | N/A | N/A  | 10   |
| Episodes per train | N/A     | N/A     | N/A| N/A    | 10  | 10   | N/A  |
| Learning rate $\pi$ | 1e-4    | 1e-4    | 1e-4| 1e-4   | 1e-4| 1e-4 | N/A  |
| Learning rate $Q$ | 1e-3    | 1e-3    | 1e-3| 1e-3   | N/A | 1e-3 | 1e-3 |
| Learning rate $V$ | N/A     | N/A     | 1e-3| N/A    | 1e-3| N/A  | N/A  |
| Epochs | N/A     | N/A     | N/A| N/A    | 33  | 33   | 33   |
| Max env steps | 33      | 33      | 33 | 33     | 33  | 33   | 33   |

### Table 5: Parameters used for CM3 and baselines in Checkers

| Parameter | Stage 1 | Stage 2 | QV | Direct | IAC | COMA | QMIX |
|-----------|---------|---------|----|--------|-----|------|------|
| Episodes  | 5k      | 50k     | 50k| 50k    | 50k | 50k  | 50k  |
| $\epsilon_{\text{start}}$ | 1.0     | 0.5     | 0.5| 1.0    | 1.0 | 1.0  | 1.0  |
| $\epsilon_{\text{end}}$ | 0.1     | 0.1     | 0.1| 0.1    | 0.1 | 0.1  | 0.1  |
| $\epsilon_{\text{step}}$ | 1.8e-3  | 4e-4    | 4e-4| 4e-5   | 2e-5| 4e-5 | 9e-5 |
| Replay buffer | 10k     | 10k     | 10k| 10k    | 10k | 10k  | 10k  |
| Minibatch size | 128     | 128     | 128| 128    | 128 | 128  | 128  |
| Steps per train | N/A     | 10      | 10 | 10     | N/A | N/A  | 10   |
| Episodes per train | 10      | N/A     | N/A| N/A    | 10  | 10   | N/A  |
| Learning rate $\pi$ | 1e-4    | 1e-4    | 1e-4| 1e-4   | 1e-4| 1e-4 | N/A  |
| Learning rate $Q$ | 1e-3    | 1e-3    | 1e-3| 1e-3   | N/A | 1e-3 | 1e-3 |
| Learning rate $V$ | N/A     | N/A     | 1e-3| N/A    | 1e-3| N/A  | N/A  |
| Epochs | N/A     | N/A     | N/A| N/A    | 33  | 33   | N/A  |
| Max env steps | 75      | 75      | 75 | 75     | 75  | 75   | 75   |

### I Stage 1

The Stage 1 functions $Q^1$ and $\pi^1$ for a single agent are trained with the $N = 1$ equivalents of (5) and (6):

\[
L(\theta_Q) = \mathbb{E}_\pi \left[ (y_i - Q^1_{\theta_Q}(s_i, a_i))^2 \right] \tag{16}
\]

\[
y_i := R(s_i, a_i, g^n) + \gamma Q^1_{\theta_Q}(s_{i+1}, a_{i+1}) \tag{17}
\]

\[
\nabla_\theta J(\pi^1) = \mathbb{E}_\pi : \nabla_\theta \log \pi(a)(Q^\pi^1(s, a) - \sum_{\hat{a}} \pi^1(\hat{a})Q^\pi^1(s, \hat{a})) \tag{18}
\]

Stage 1 training curves for all three experimental domains are shown in Figure 7.
Figure 7: Stage 1 reward curves for CM3 in cooperative navigation, SUMO and Checkers.