Hall potentiometer in the ballistic regime

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We demonstrate theoretically how a two-dimensional electron gas can be used to probe local potential profiles using the Hall effect. For small magnetic fields, the Hall resistance is inversely proportional to the average potential profile in the Hall cross and is independent of the shape and the position of this profile in the junction. The bend resistance, on the other hand, is much more sensitive on the exact details of the local potential profile in the cross junction.

A conductive atomic force microscope (AFM) tip has been used, in contact and non-contact mode, as a local voltage probe in order to measure the distribution of the electrical potential on a surface. This technique is called nanopotentiometry and allows two-dimensional potential mapping. The sensitivity and the spatial resolution are limited by the finite size of the conductive probe and by the quality of the surface preparation. On the other hand such tips can also be used to induce potential variations in the sample in order to influence the conduction locally. Measuring the change in the resistance of the device gives information on the local transport properties.

To describe the transport properties in the Hall cross we will use the billiard model. In this model the electrons are considered as point particles which is justified when the Fermi wavelength $\lambda_F \ll W, d$ where $2W$ is the width of the Hall probes and $d$ the radius of the potential profile which acts as a scatterer for the electrons. The electron motion is taken ballistic and governed by Newton’s law which is justified at low temperatures and in case of high mobility samples where the mean free path $l_c \gg W, d$. We assume that the temperature is not extremely low such that interference effects are averaged out due to thermal smearing. In a typical GaAs-heterostructure the electron density is $n \sim 3 \times 10^{11} \text{cm}^{-2}$ with typical low temperature mobility $\mu \sim 10^6 \text{cm}^2/\text{Vs}$, which gives $\lambda_F = 450 \AA$ and $l_c = 5.4 \mu\text{m}$. This billiard model has been used successfully to describe e.g. the experiments of Ref and to explain the working of a ballistic magnetometer.

Using the Landauer-Büttiker formalism for the Hall geometry with identical leads, the Hall $R_H$ and the bend $R_B$ resistances are given by

$$R_H = \frac{\mu_4 - \mu_2}{e I_{1\to 3}} = \frac{h}{2e^2} \frac{T_R^2 - T_L^2}{Z}, \quad (1)$$
$$R_B = \frac{\mu_2 - \mu_3}{e I_{1\to 4}} = \frac{h}{2e^2} \frac{T_R^2 - T_L T_R}{Z}, \quad (2)$$

where $Z = \left[T_R^2 + T_L^2 + 2T_F (T_R + T_F + T_L) (T_R + T_L)\right]$ ($T_R$ and $T_L$, $T_F$ are the probabilities for an electron to turn to the right probe, to the left probe and to the forward probe, respectively. These probabilities will be calculated using the ballistic billiard model).

In the following we will express the magnetic field in units of $B_0 = m v_F^{\text{LIM}}/2eW$, and the resistance in $R_0 = (h/2e^2) \pi/2k_F W$, where $W$ is the half width of the leads, $m$ is the mass of the electron, $k_F = \sqrt{2mE_F/\hbar^2}$, and $v_F = \hbar k_F/m$ the Fermi velocity. For electrons moving in GaAs ($m = 0.067 m_e$) and for a typical channel width of $2W = 1 \mu\text{m}$ and a Fermi energy of $E_F = 10 \text{meV}$ ($n_e = 2.8 \times 10^{11} \text{cm}^{-2}$), we obtain $B_0 = 0.087 T$ and $R_0 = 0.308 k\Omega$. In order to demonstrate the main physics involved, we consider first a mathematically simple potential profile, namely a rectangular potential barrier with radius $d$ and height $V_0$ placed in the center of the cross junction: $V(r) = V_0$ if $r < d$ and $V(r) = 0$ if $r > d$. This potential barrier is schematically shown in the inset of Fig. 1(a) by the shaded circular area. Inside the potential barrier the kinetic energy of the electrons is reduced to $E_F - V_0$ with $E_F$ the kinetic energy of the electrons outside this region. Hence, the electron velocity $v$, the density $n$ of the electrons and also the radius $R_c$ of the cyclotron orbit are reduced inside the potential region. For $V_0 < 0$ the opposite occurs. This
will result in changes in the transmission probabilities $T_R$, $T_F$, $T_L$ and consequently it will alter the Hall and bend resistances.

In Fig. 1 we show the Hall resistance $R_H$ and the bend resistance $R_B$ as function of the external applied magnetic field for different sizes and heights of the rectangular potential profiles. Fig. 1(a) shows the Hall resistance and Fig. 1(b) the bend resistance for different potential heights $V_0$ but fixed radius $d = 0.5W$, while in Fig. 1(c) and 1(d) the radius $d$ is varied and the potential height $V_0 = 0.2E_F$ is kept fixed. Notice that there exists a critical magnetic field $B_c = B_c(d)$, such that for $B > B_c$ no electrons are entering the area of the potential barrier, because their cyclotron radius is so small that they skip along the edge of the probe without 'feeling' the potential barrier. For $B > B_c$ the diameter of the cyclotron orbit, $2R_c = 2E_F/\omega_c$, is less than the distance between the edge of the rectangular potential barrier and the corner of the cross junction. Therefore, the electrons do not feel the presence of the potential profiles (Fig. 1(b)) the bend resistance for different potential heights $B$ magnetic fields (i.e. $V_0 = E_F$). We found that, even in the presence of an inhomogeneous potential profile, the Hall resistance is linear for small $T_F$, $T_L$, and $T_R$ which are only weakly bend. And consequently they sample the potential barrier away from the center of the junction. For a rectangular potential profile we find for the average electron density in the cross region, and is independent of the detailed potential profile as long as $\langle V \rangle \neq 0$. The latter region increases (decreases) the turning probability $T_R$ and hence reduces (enhances) $R_B$ and enhances (reduces) $R_H$ as clearly observed in Fig. 1 for $V_0 > 0$ ($V_0 < 0$).

For low potential barriers, i.e. $|V_0| < E_F$, we have $R_B \approx 0$ for $B/B_0 > 2$ which is substantially below $B_c$. The reason is that almost no electrons finish in probe 2 and 3, i.e. $T_L \approx 0 \approx T_F$, and consequently $R_B = 0$. At the magnetic field $B = 2B_0$ the cyclotron diameter, $2R_c$, equals the probe width, $2W$. For higher potential barriers some of the electrons are deflected on the potential barrier into probe 2 and hence $R_B < 0$. This is clearly observed in Fig. 1(b).

We found that, even in the presence of an inhomogeneous potential profile, the Hall resistance is linear for small magnetic fields (i.e. $B \ll B_c$). The slope increases with the radius and with the height of the rectangular potential barrier. We analyzed the Hall factor $\alpha = R_H/B$ for $B \ll B_c$ in Fig. 2(a) as function of the radius $d$ for $V_0 = 0.2E_F$ and in Fig. 2(b) as function of the height $V_0$ for $d = 0.4W$. Notice that the Hall factor increases with $d$ and $V_0$. Increasing $d$ or $V_0$ results in an increase of the average potential $\langle V \rangle$ or in a reduction of the average electron density $\langle n_e \rangle$ in the cross junction. For a rectangular potential profile we find for the average electron density in the cross junction $\langle n_e \rangle = n_e \left[1 - \pi (d/2W)^2 V_0/E_F\right]$ from which we define an effective Hall coefficient $\alpha^* = (R_H/B) \langle n_e \rangle / n_e$ which is shown by the dashed curve in Fig. 2. Notice that $\alpha^*$ is almost independent (within 2%) of $d$ for $d/W < 1.0$ if $V_0$ is constant and practically independent of $V_0$ for $V_0/E_F < 0.5$ if $d$ is constant. For $V_0/E_F > 0.5$ the potential profile is no longer a weak perturbation on the electron motion and consequently the Hall resistance can no longer be described in terms of an average electron density in the cross junction. The fact that $\alpha^*$ is practically independent of $V_0$ and $d$ indicates that for low magnetic fields the Hall resistance is completely determined by the average potential in the cross region, and is independent of the detailed potential profile as long as $\langle V \rangle / E_F < 0.5$. This is our major conclusion of this work, which will be confirmed further.

The bend resistance $R_B$, on the other hand, is much more sensitive to the exact form of the potential barrier. For example: the bend resistance for a wide, but low barrier ($d = 0.9W$, $V_0 = 0.2 E_F$) does not equal to the bend resistance for a narrow, but high barrier ($d = 0.5W$, $V_0 = 0.648 E_F$), even though the average potential in the junction region is the same (see Figs. 1(b,d)).

Next, we investigate the effect of the functional form of the potential by considering, as an example, a gaussian potential profile in the center of the junction: $V_g(r) = V_{g,0} \exp\left(-r^2/2d_g^2\right)$ with $V_{g,0}$ the height and $2d_g$ the width at half height. In Fig. 3 we compare the Hall factors $\alpha$ and $\alpha^*$ of the rectangular potential (solid curves) with height $V_0 = 0.2 E_F$ and radius $d$, and the gaussian potential (symbols) with width $d_g = d$ and height $V_{g,0}$ as a function of the radius $d$. $V_{g,0}$ is varied with $d$ such that the average potential inside the cross junction is the same for the two potential profiles (again $\langle V \rangle = \langle V_g \rangle$). The differences between the effective Hall factor $\alpha^*$ in the two cases is negligible and hence this illustrates again that the Hall resistance is completely determined by the average potential in the cross region, and is independent of the detailed potential profile. In the inset of Fig. 3 we show, as an example, the rectangular potential with $d = 0.5W$ and $V_0 = 0.2 E_F$ and the corresponding gaussian potential with $d_g = 0.5W$ and $V_{g,0} \approx 0.202 E_F$ which has the same $\langle V \rangle$.

Finally, we investigate the effect of displacing the rectangular potential barrier away from the center of the junction. As an example, we consider a rectangular potential barrier with height $V_0 = 0.2 E_F$ and radius $d = 0.5W$ which is displaced at different distances $\rho/W = 0, \pm 0.1, \pm 0.2, \pm 0.3, \pm 0.4, \pm 0.5$ from the center of the cross junction in different directions $\varphi = 0, \pi/4, \pi/2, 3\pi/4$ with regard to the x-axis. Notice that in all these cases the entire potential barrier is inside the cross junction and hence the average potential in the cross junction is the same. The problem is no longer symmetric and we are not allowed to use the reduced Eqs. 3 and 4 but we are forced to retain the original Landauer-Büttiker formula. In Fig. 4 we show that the change in the effective Hall factor $\alpha^*$ as function of the distance $\rho$ for the different directions $\varphi$ is less than 1% for $d = 0.5W$ and $V_0 = 0.2 E_F$. Only when the circular potential barrier is very close to one of the probes the deviation becomes of the order of 1%. This result illustrates
that for low magnetic fields the Hall resistance is completely determined by the average potential in the cross region, and is independent of the detailed position of the potential barrier in the cross junction.

In conclusion, we investigated the Hall and the bend resistances, in the presence of an inhomogeneous potential profile in the junction of a mesoscopic Hall bar. We found that in the low magnetic field regime the Hall resistance is linear in the magnetic field and is determined by the average potential in the cross junction, independent of the shape and the position of the potential barrier as long as \( \langle V \rangle / E_F < 0.5 \). This general result makes such a Hall device a powerful experimental tool for non invasive investigations of induced potential profiles. The bend resistance depends much more sensitively on the detailed shape and position of the potential profile. The resistance at which the classical 2D Hall resistance is recovered gives us information on the size of the potential barrier. The present results are valid in the ballistic regime and are expected, as in the case of the Hall magnetometer\(^9\), to be modified in the diffusive regime.

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FIG. 1. The Hall resistance \( R_H \) (a) and the bend resistance \( R_B \) (b) as function of the magnetic field \( B \) for different heights \( V_0 \) of the rectangular potential barrier and for fixed radius \( d = 0.5W \). The Hall resistance \( R_H \) (c) and the bend resistance \( R_B \) (d) as function of the magnetic field \( B \) for different radii \( d \) and for a fixed height \( V_0 = 0.2E_F \).

FIG. 2. The Hall factor \( \alpha = R_H / B \) in the low magnetic field region (\( B = 0.4B_0 \ll B_0 \)) as a function of (a) the radius \( d \) of the rectangular potential for \( V_0 = 0.2E_F \), and (b) the height \( V_0 \) of this potential for \( d = 0.4W \). The dashed curves are obtained from the effective Hall factor \( \alpha^* = (R_H / B) \langle n_e \rangle / n_e \), where \( \langle n_e \rangle \) is the average electron density in the junction.

FIG. 3. The Hall factors \( \alpha \) en \( \alpha^* \) resulting from a rectangular potential (solid curves) in the center of a Hall cross are compared to these resulting from a gaussian potential (symbols) which has the same average potential in the cross junction. In the inset, the gaussian potential which corresponds to a rectangular potential with \( d = 0.5W \) and \( V_0 = 0.2E_F \) is shown.

FIG. 4. The effective Hall factor \( \alpha^* \) for different positions of the rectangular potential with \( d = 0.5W \) and \( V_0 = 0.2E_F \) for \( B = 0.4B_0 \ll B_0 \). The potential barrier is displaced over a distance \( \rho \) in a direction \( \varphi \) with respect to the direction of the current (see inset). The curves are guides to the eye. The effective Hall factor is scaled to its value for a rectangular potential in the center of the Hall cross.
\( V_0 = 0.2 \, E_F \)

(a) \( \frac{\alpha(d)}{\alpha(d=0)} \)

(b) \( \frac{\alpha(V_0)}{\alpha(V_0=0)} \)

\( \frac{\alpha^*(V_0)}{\alpha^*(V_0=0)} \)
\[ V_0 = 0.2 \, E_F \]
\[ d_g = d \]

- **Hall factor**

\[ \frac{d}{W} = 0.5 \, W \]

- **Inset graph**
  - \( V / E_F \) vs. \( r / W \)
  - \( d = d_g = 0.5 \, W \)

- **Graph**
  - **Black line**: rectangular
  - **Square markers**: gaussian

\[ \alpha / \alpha(0) \]
\[ \alpha^* / \alpha(0) \]
\[ \alpha^*(\rho, \phi) / \alpha^*(\rho=0) \]

- \( \phi = 0 \)
- \( \phi = \pi/4 \)
- \( \phi = \pi/2 \)
- \( \phi = 3\pi/4 \)

\[ d = 0.5 \text{ W} \]
\[ V_0 = 0.2 \text{ } E_F \]