DARK MATTER, MUON G - 2 AND OTHER ACCELERATOR CONSTRAINTS

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We review the current status of the Brookhaven muon g - 2 experiment, and it’s effects on the SUSY parameter space when combined with dark matter relic density bounds, $b \to s\gamma$ and Higgs mass constraints. If the $3\sigma$ deviation of g - 2 from the Standard Model value is correct, these data constrain the mSUGRA parameter space strongly, i.e. $300 \text{ GeV} \lesssim m_{1/2} \lesssim 850 \text{ GeV}$, and $m_0$ (at fixed $\tan \beta$, $A_0$) is tightly constrained (except at very large $\tan \beta$). Dark matter detection cross sections lie within the range accessible to future planned experiments. A non-universal gluino soft breaking mass however can greatly reduce the lower bound on $m_{1/2}$ (arising from the $b \to s\gamma$ constraint) allowing for relatively light neutralinos, while non-universal Higgs $H_2$ mass can lead to new regions of allowed relic density where the detection cross sections can be increased by a factor of 10 or more.

1. Introduction

While current Tevatron and LEP measurements have not greatly constrained the SUSY particle spectra, there are a number of quantities, which if accurately measured and if accurate theoretical calculations existed, could greatly limit the SUSY parameter space of a given model, and thus allow significant predictions of what might be expected at the LHC and what might occur in the next round of dark matter detector experiments. We consider here the following quantities: the muon g - 2, the light Higgs mass $m_h$, the $b \to s\gamma$ branching ratio and the amount of dark matter. If accurately determined, these would greatly restrict the SUSY parameter space for a variety of models. We will first examine these within the framework of mSUGRA models (with R parity invariance) and then show some non-universal models which could moderate somewhat the mSUGRA constraints.
2. The Muon $g - 2$ Anomaly: The Saga Continues!

We review the current situation with the muon $g - 2$ magnetic moment anomaly. Recall that in 2001, the Brookhaven E821 experiment reported their high precision measurement of $a_{\mu} = (g - 2)/2$. Based on the then best theoretical calculation of the Standard Model value $^1$, they reported a 2.6$\sigma$ deviation. Unfortunately, a sign error was subsequently found in the “scattering of light by light” (LbL) diagrams, which reduced the effect to only 1.6sigma. Since that time the following has happened:

(i) New E821 data (the $\mu^+ 2000$ data) has been analyzed reducing the experimental error in $a_{\mu}$ by a factor of two$^2$. The current world average is now

$$a_{\mu}^{\exp} = 11659203(8) \times 10^{-10}$$

i. e. a measurement at the level of 0.7ppm! One interesting feature of the Brookhaven measurements is the stability of their central value (with the error flag successively being reduced).

(ii) New data has come from Novosibirsk (CMD-2) and Beijing (BES) on $e^++e^- \rightarrow$ hadrons, and from ALEPH and CLEO on tau decay into two and four pions. These may be used to calculate the hadronic contributions to the SM $a_{\mu}$ prediction to get a more accurate determination of any deviation between theory and experiment that may exist.

Two groups $^3, 4$ have now used this new data to reevaluate $a_{\mu}^{SM}$, and we briefly review their results. $a_{\mu}^{SM}$ can be divided into the following parts:

$$a_{\mu}^{SM} = a_{\mu}^{QED} + a_{\mu}^{weak} + a_{\mu}^{had}$$

(2)

where

$$a_{\mu}^{had} = a_{\mu}^{LO} + a_{\mu}^{LbL} + a_{\mu}^{HO}$$

(3)

The QED and weak contributions to $a_{\mu}$ are well established, and the higher order (HO) hadronic contribution appears to be in good shape. With the corrected sign, the light by light (LbL) contribution has been evaluated by several groups $^5$ with general agreement. We use here the value $a_{\mu}^{LbL} = [8.6 \pm 3.5] \times 10^{-10}$. Current difficulties arise from the leading order in alpha (LO) hadronic contributions. This quantity can be calculated using a dispersion relation:

$$a_{\mu}^{LO} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_e^2}^{\infty} ds \frac{sK(s)}{s} R(s)$$

(4)

where $K(s)$ is the QED kernel and $R(s) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. The integral is strongly weighted at low energy, i. e. about 90% comes from $\sqrt{s} < 1.8\text{GeV}$ and about 75% from $e^+e^- \rightarrow \pi\pi$ (from the $\rho$).
Two procedures have been used to evaluate the dispersion integral. The first method uses the $e^+e^-$ cross sections of CMD-2, BES and a large amount of earlier data to calculate $R(s)$. However, it should be noted that the $\sigma(e^+e^- \rightarrow \text{hadrons})$ to be used in Eq.(4) is a “bare” cross section, i.e. the experimental data must be corrected for initial state radiation, photon vacuum polarization and electron vertex loop effects (and not carrying this out correctly has led to some errors in past analyses.) Carrying out this analysis, Ref.3 finds a discrepancy between the experiment and the SM of $\Delta a_\mu = 33.9(11.2) \times 10^{-10}$ a $3.0\sigma$ effect, while Ref.4 finds $\Delta_\mu = 35.13(10.63) \times 10^{-10}$ i.e a $3.3\sigma$ effect. Thus the two analyses give consistent results.

The second method makes use of the tau decay data of ALEPH and CLEO into $2\pi$ and $4\pi$ final states. Using CVC, the isovector form factor can be used to construct $\sigma(e^+e^- \rightarrow 2\pi, 4\pi)$ for $s \lesssim 3\text{GeV}$ (which is most of the important region). However, in this case, one must include corrections due to the breaking of CVC. Major contributions to this come from the $\pi$ mass differences and the short distance radiative corrections (treated by chiral perturbation theory). Carrying out this analysis, Ref.3 finds from the tau data a discrepancy of $16.7(10.7) \times 10^{-10}$ i.e. only a $1.6\sigma$ effect. Further, one cannot simply average the two procedures of calculating $a_\mu^{\text{LO}}$. To point up the problem, Ref.3 reversed the procedure, and using the $e^+e^-$ data plus the CVC breaking corrections, predicted the tau branching ratios into $2\pi$ and $4\pi$ final states and then compared this to the experimental ALEPH and CLEO data. They found that the prediction fails for the $\pi^-\pi^0$ state by $4.2\sigma$, and fails for the $2\pi^-\pi^+\pi^0$ state by $3.5\sigma$. Thus the two approaches are statistically inconsistent with each other.

At this point, there is no explanation for the disagreement between the two types of analysis. The discrepancy appears to be too large to be attributed to lack of understanding of the CVC breaking effects. Thus the question of whether the muon magnetic moment anomaly implies new physics is once again unclear. There will be more $e^+e^-$ data from CMD-2, BES and also KLOE and BABAR. The B-factories may also be able to measure the tau decays. Finally, the Brookhaven E821 experiment has $3 \times 10^9 \mu^-$ events it is currently analyzing. (Results may be out by early next year.) It also has future plans for running the experiment further.

3. mSUGRA Model

In SUSY models, one generally has a contribution to $a_\mu$ in addition to the SM piece $a_\mu^{\text{SM}}$. This arises from loops with chargino and sneutrino or neutralino and smuon intermediate states, with a magnetic field attached.
to any charged particle. In general these contributions are not small. If the analysis leading to a $3\sigma$ effect is valid, and if one attributes the deviation to the SUSY correction, one gets a significant constraint on the SUSY parameter space, with the SUSY spectrum required to be relatively low and easily within the reach of the LHC. On the other hand, if the 1.6 sigma analysis turns out to be correct implying only a small SUSY contribution is possible, then squark and gluino mass spectrum would be pushed to the TeV domain. Thus the resolution of the current ambiguity in $a_{\mu}^{SM}$ is very important. In this section, we analyze these matters within the framework of the mSUGRA model with R parity invariance.

mSUGRA is the simplest SUGRA model in that it depends only on four parameters and one sign: $m_0$ (the scalar soft breaking mass at $M_G$), $m_{1/2}$ (the gaugino mass at $M_G$), $A_0$ (the cubic soft breaking parameter at $M_G$), $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ (at the electroweak scale), and the sign of $\mu$ (the Higgs mixing parameter in the superpotential: $\mu H_1 H_2$). We examine here the parameter range $m_0 > 0, m_{1/2} < 1$TeV (which corresponds to the gluino mass bound accessible to the LHC: $m_{\tilde{g}} < 2.5$GeV), $2 \leq \tan \beta \leq 55,$
Figure 2. Same as Fig.1 for \( \tan \beta = 40 \), \( A_0 = 0 \), \( \mu > 0 \). The dashed line give contours of \( B_s \to \mu \mu \) branching ratios at the Tevatron. The vertical dotted lines are Higgs mass contours. The short vertical lines are \( \sigma_{\chi_1^0 - p} = 3 \times 10^{-8} \text{pb} \) (lower) and \( 1 \times 10^{-9} \text{pb} \) (upper).

and \( |A_0| \leq 4m_{1/2} \). One starts the analysis at \( M_G \) and uses the renormalization group equations (RGE) to go down to the electroweak scale. Thus all SUSY masses and cross sections are determined in terms of these four parameters and one sign. The details of carrying out this analysis including all coannihilation effects in the relic density analysis can be found in e.g. 8.

The mSUGRA model allows one to calculate an array of quantities that can be measured and hence can be used to restrict the parameter space. The include the neutralino relic density, the \( b \to s + \gamma \) branching ratio and the Higgs mass bound. For the relic density we take a 2\( \sigma \) range around the current central value 9:

\[
0.07 \leq \Omega_X h^2 \leq 0.21
\]

(5)

For the \( b \to s + \gamma \) decay (which has both systematic and theoretical uncertainties) we take a relatively broad range around the CLEO central value 10 and use the LEP bound for \( m_h \) 11:

\[
1.8 \times 10^{-4} \leq B(b \to s\gamma) \leq 4.5 \times 10^{-4}; \quad m_h > 114 \text{GeV}
\]

(6)
In addition there is the LEP bound on the chargino mass of \( m_{\chi^\pm} > 103\text{GeV} \). 

In spite of the large errors in the experimental data that still are present, Eqs.(5,6) put significant constraints on the SUSY parameter space in that they require that \( m_{1/2} \gtrsim (300 - 400)\text{ GeV} \) across the full parameter space. This is illustrated in Figs. (1-3) for \( \tan \beta = 10, 40, 55 \). If we assume the 3\( \sigma \) deviation for the muon \( a_\mu \) is valid, then the 2\( \sigma \) bound from the central value is \( 11 \times 10^{-10} \). For low \( \tan \beta \), e.g. \( \tan \beta = 10 \) (Fig. 1), the combined constraints then leave very little available parameter space. For higher \( \tan \beta \), e.g. \( \tan \beta = 40 \) (Fig.2), more allowed region exits, but the \( \chi_1^\pm \) mass constraint combined with the \( a_\mu \) constraint eliminates all the “focus point” region 13 of large \( m_0 \) and low \( m_{1/2} \). For very high \( \tan \beta \), e.g. \( \tan \beta = 55 \) (Fig.3) a new region opens up producing a “bulge” at low \( m_{1/2} \) due to rapid s-channel annihilation of the \( A \) and \( H \) Higgs bosons (as their mass is significantly reduced). Throughout the entire range, the \( \chi_1^0 \)- proton cross sections for direct detection of Milky Way dark matter lie in the range of \( (9 \times 10^{-8} - 5 \times 10^{-10})\text{pb} \), a range that is expected to be accessible to future large scale experiments.

Figure 3. Same as Fig. 2 for \( \tan \beta = 55, A_0 = 0, \mu > 0 \) and \( m_t = 175\text{GeV}, m_b = 4.25 \).
Figure 4. Same as Fig. 2 for tan $\beta = 40$, $A_0 = m_{1/2}$, $\mu > 0$ for non-universal soft breaking of the Higgs masses with $\delta_2 = 1$. The lower shaded band is the usual allowed stau-neutralino coannihilation band, and the upper band is the new region arising from the non-universal Higgs masses due to increased annihilation through the Z-channel.

4. Non-Universal Models

We consider here two types of non-universal soft breaking at the GUT scale: for the gaugino masses and for the Higgs masses. The first is of interest in that it can soften significantly the lower bound produced by the $b \to s \gamma$ and $m_h$ constraints. Thus if one assumes at $M_{\text{GUT}}$ that the gluino mass is $m_{1/2}(1 + \delta_3)$ (where $m_{1/2}$ is the universal mass), then for $\delta_3 = 1$, one finds e.g. for $\tan \beta = 50$, that the lower bound on $m_{1/2}$ is reduced to 185 GeV corresponding to a neutralino mass of 75 GeV. (The constraint from $m_h$ is also softened, though the $a_\mu$ constraint becomes somewhat stronger).

If the $H_2$ mass is increased at $M_G$, new effects also occur. Thus writing $m_{H_2}^2 = m_0^2(1 + \delta_2)$, where $m_0$ is the universal scalar mass, then a new annihilation channel in the relic density analysis can open up for low $m_{1/2}$ and high $m_0$ due to rapid annihilation through the s-channel Z pole. This is shown in Fig. 4 for $\tan \beta = 40$. In this new region, the neutralino-proton cross section also increases by a factor of 10 or more, allowing cross sections in the range $(10^{-6} - 10^{-7})\text{pb}$.
5. Conclusions

The muon magnetic moment anomaly can produce strong constraints on the allowed region of SUSY parameter space. Thus if the 3σ deviation analysis is correct, then a 2σ bound from the central value gives an upper bound of $m_{1/2} \lesssim 850$GeV when combined with the relic density constraint. The $b \to s\gamma$ and $m_h$ constraints in the mSUGRA model, produce a lower bound of $m_{1/2} > (300 - 400)$ GeV, thus bounding the parameter space in a region easily accessible to the LHC. Dark matter detection cross sections then lie in a region accessible to future dark matter experiments. However, should the $a_\mu$ anomaly be smaller than $\sim 10 \times 10^{-10}$ the squark and gluino spectrum will be pushed into the TeV domain. Non-universal models can modify these constraints. Thus an increase in the gluino mass at $M_G$ can significantly decrease the the lower bound on $m_{1/2}$ (and hence on the neutralino mass), while an increase of the $H_2$ mass at $M_G$ can give rise to new allowed regions of relic density with detection cross sections increased by a factor of 10 or more.

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