Optimization of floating wind turbine support structures using frequency-domain analysis and analytical gradients

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Abstract. A framework for conceptual optimal design of floating wind turbine support structures including mooring system is proposed. A four degree-of-freedom frequency-domain model is used for the dynamic response of the floating wind turbine subjected to wind and wave loads. The framework allows for integrated design optimization involving the geometrical properties of the floater and the mooring system and inclusion of long realizations of multiple load cases in the analysis. Analytical design sensitivities of the governing frequency-domain equations and the design requirements are developed. This ensures that modern numerical optimization methods can efficiently be used to solve the design problem. The framework is applied to sizing optimization of a spar-buoy floater including the mooring system. The inclusion of dynamic constraints in the design optimization is demonstrated. The optimization provides designs accurately satisfying optimality conditions in minutes.

1. Introduction

The design of substructures for offshore floating wind turbines combines requirements of stability, dynamic response levels for nacelle accelerations, tower inclination and mooring loads, all subject to the forcing from wind, waves and the controller actions. For a given design, validation over a large set of design load cases are needed. This makes the application of traditional aero-elastic tools in the time domain too expensive in the context of numerical optimization.

In this work, we therefore aim to combine a frequency domain response model with gradient based optimization to achieve a fast tool for conceptual and preliminary design of floating wind turbine support structures. The floater response is calculated with the QuLAF (Quick Load Analysis of Floating wind turbines) frequency domain model [1, 2, 3]. The analysis capabilities are extended to provide analytical design sensitivity analysis for the considered design requirements. This capability enables fast optimization using modern algorithms based on e.g. Sequential Quadratic Programming (SQP) [4].

A number of authors have investigated the loads and response of floating wind turbines using simplified frequency-domain models. Lupton [5] developed a model for a spar with eight degrees of freedom (DoFs): rigid-body surge, heave and pitch, two tower fore-aft modes, and one normal
mode per blade. The model included linear hydrodynamics from a potential-flow panel code, and aerodynamic loads through harmonic linearization. The work of Wang et al. [6] involved a model of a semi-submersible with two rigid-body DoFs: floater surge and pitch. Linear potential-flow hydrodynamics, as well as linearized drag and drift forces were considered. Aerodynamic loads were modelled as a constant force and a damping term, thus neglecting stochastic wind forcing. In the study of Lemmer et al. [7], a reduced-order time-domain model of a semi-submersible floater was investigated. The model, with six DoFs (floater surge, pitch and heave, first tower fore-aft mode, rotor speed and blade pitch angle), included linear hydrodynamics with slow-drift forces through Newman’s approximation, and quasi-static representations of the mooring and aerodynamic loads. The reduced-order time-domain model was further linearized to obtain a frequency-domain version.

The QuLAF model of Pegalajar-Jurado et al. [2] combines the four important planer degrees of freedom (floater surge, heave, pitch and first tower fore-aft mode) with irregular wave forcing and realistic rotor forcing through pre-computed fixed-rotor thrust and moment time series and pre-computed aerodynamic damping. Hereby stochastic response to wind and wave forcing can be obtained rapidly, through solution of linear response equations in the frequency domain. The original model included linear radiation-diffraction hydrodynamics, viscous damping, and a linearized mooring matrix (obtained for each wind speed). The model approach was validated for fatigue loads in [2] and a selection of design load cases in Madsen et al. [3] for a semi-submersible floater. A similar approach was published by Hegseth and Bachynski [8] to model two different spar designs with three DoFs: floater surge and pitch, and first tower bending mode. In their case, linear hydrodynamic loads were modelled through the Morison equation (including MacCamy-Fuchs correction), and the mooring system was linearized around the equilibrium position without wind. Recently, Karimi et al. [9] has investigated three different floaters by developing frequency-domain models with six rigid-body DoFs. In their case, the aerodynamic and structural (including mooring) models were obtained through the linearization capabilities of the time-domain aeroelastic tool FAST [10], whereas the hydrodynamic properties were calculated in a radiation-diffraction solver.

Fylling and Berthelsen [11] presented an early study of gradient based optimization for offshore wind turbine spar floater. The work included design of the mooring system and power cable. The objective function modelled the material cost and the design variables represented geometrical properties of the spar-buoy and mooring system. Constraints were included on e.g. tower inclination, nacelle acceleration, floater heave and pitch natural periods and maximum tensions in mooring lines. The optimization method was a variant of SQP with the application of finite differences for the sensitivity analysis.

Later studies have applied genetic algorithms for the floater optimization, which fall into the category of gradient free optimization methods. Hall et al. [12] combined this with a frequency domain model to optimize the geometry of the floater cylinders, heave plates and mooring system. The constraints included limits on the static and dynamic pitch angle. An extension to multi-objective optimization was later presented by Karimi et al. [9], while an alternative representation of the design variables was proposed by Hall et al. [13]. Instead of using geometric sizing variables a set of basis platform geometries could be combined.

Genetic algorithms based optimizations have also been combined with other methods. In the study of Pillai et al. [14], a multi-objective mooring system optimization for a semi-submersible floater was combined with a random forest based surrogate model. In another recent work by Barbanti et al. [15], mooring system optimization for a spar-buoy turbine was achieved by combination with a pattern-search method as a local solver. Another way to make the optimization more feasible was presented by Myhr and Nygaard [16] for the design of a tension-leg-buoy platform. They applied a two phase optimization consisting of first determining the mooring line stiffness and layout under eigenvalue limitations followed by time-domain
optimization of the overall geometry of the floater.

From the existing work on floating wind turbine optimization, it is quite clear that model efficiency and low computational cost are of high importance to achieve a feasible design tool. We pursue this in the present study by a combination of the QuLAF model, the SQP optimization method, and the analytical design sensitivities of the objective and constraint functions.

The combination of the chosen analysis model and optimization method allows for many load cases and long realization times to be considered. We apply this approach to the geometric design of a spar floater and a simple mooring system. A similar approach, with further extension to optimal control parameters, has very recently been presented by Hegseth et al. [17] and thus confirms the relevance of this idea. Our future work will pursue similar developments and further application to other floater configurations.

2. Modelling and analysis

The analysis model employed is based on QuLAF [2, 3], a frequency-domain model for the dynamic response of floating wind turbines to wind and wave loads. The model considers four degrees of freedom: the floater surge, heave and pitch, and the first tower fore-aft bending mode. The model includes:

- Pre-computed aerodynamic loads and damping, which, for a given wind turbine, are extracted only once for each mean wind speed from time-domain aeroelastic rotor computations. These aerodynamic properties are thus assumed to be independent of the design variables for floater and mooring system.
- Linear hydrodynamic loads, either by radiation-diffraction analysis (as in [2, 3]) or through the inertia term of the Morison equation [18]. The latter is used in the present study in an efficient transfer-function format to speed up the optimization framework. The added mass is constant and Morison-based, and radiation damping is excluded. Viscous damping effects are included through a damping matrix, but viscous excitation is not included due to the loads being inertia-dominated.
- A linearized mooring system represented by a stiffness matrix.

The QuLAF model has been extensively compared to FAST in [2, 3] for a 10 MW semi-submersible configuration. Generally a good agreement was found for the damage-equivalent bending moment at the tower base. The floater motion was also well estimated, with some under-prediction for strong sea states (due to the missing viscous excitation). The nacelle acceleration, on the other hand, was systematically under-predicted, mainly due to an overestimation of the aerodynamic damping applied to the tower vibrations. This overestimation is a consequence of the tower frequency shift when coupled to the floater. It can be fixed by adding frequency dependency for aerodynamic damping. This fix, though, was not pursued in the present study.

2.1. Equations of motion

Throughout the manuscript it is assumed that a vector \( \mathbf{v} \) collecting the design variables is provided. The design variables represent geometric properties of the floater and the mooring system. The linear equation of motion of floating wind turbine is written as

\[
(-\omega^2 (M(\mathbf{v}) + A(\mathbf{v})) + i \omega B(\mathbf{v}) + C(\mathbf{v})) \hat{\xi}_j(\omega) = \hat{F}_h^j(\mathbf{v}, \omega) + \hat{F}_a^j(\omega)
\]  

(1)

where \( \hat{\xi}_j(\omega) \) denotes the complex Fourier amplitudes of the motion in the frequency domain for the \( j \)th environmental condition. The right hand side contains the aero- and the hydrodynamic loads. \( \hat{F}_h^j(\mathbf{v}, \omega) \) are the hydrodynamic loads. These are computed internally through the Morison equation for each candidate design and sea state. \( \hat{F}_a^j(\omega) \) are the aerodynamic loads. These
are pre-computed beforehand through rigid-structure rotor computations in FAST [10]. The aerodynamic loads are thus independent on the design variables. \( \mathbf{M}(\mathbf{v}) \) is the structural mass matrix which is computed internally for each candidate design from the structural properties of the wind turbine, the tower, and the floating substructure. The hydrodynamic added mass matrix \( \mathbf{A}(\mathbf{v}) \) is computed internally for each candidate design based on the Morison equation. The damping matrix \( \mathbf{B}(\mathbf{v}) \) contains contributions from the tower structural damping, the aerodynamic damping (pre-computed) and the hydrodynamic viscous damping (computed internally for each candidate design). \( \mathbf{C}(\mathbf{v}) \) is the restoring matrix, including hydrostatic and mooring contributions. Both are computed internally for each candidate design.

In the optimal design problem we consider constraints limiting the behaviour of the system. For this purpose we introduce \( \theta_j \) as the maximum computational pitch motion (over the simulation time) of the floater under load case \( j \). In the model, the nacelle acceleration is a linear combination of floater surge, pitch and tower deflection. We approximate \( \theta_j \) by the mean plus three times the standard deviation, i.e. \( \theta_j \approx \theta_{j,\text{mean}} + 3 \sigma_{j,\theta} \). The standard deviation is computed through \( \sigma_{j,\theta}^2 = \frac{1}{2} \hat{\theta}_j^2 \), where \( \hat{\theta}_j \) is a vector collecting the frequency-domain pitch motion extracted from \( \xi_j(\omega) \) for all frequency components. The mean floater pitch is calculated using the mean aerodynamic loads and the stiffness matrix. Similarly, the maximum nacelle acceleration \( a_n \) under the \( j \)th dynamic wind and wave force is also approximated by using three times the standard deviation, i.e. \( a_n \approx 3\sigma_{i,a} \).

The optimal design problem also contains limits to the system eigenvalues \( \lambda_i \) which become functions of the design variables. These are obtained through the eigenvalue problem

\[
(C(v) - \lambda_i(M(v) + A(v)))\phi_i = 0
\]

where \( \phi_i \) are the corresponding eigenmodes.

Quasi-static analysis for the given static load vector \( f^s \) is also used to define two constraints of maximum pitch angle and surge, see Table 2. The governing equations are

\[
C(v)u = f^s
\]

### 2.2. Mooring system model

We assume that the mooring system is symmetric and comprises three (or more) identical and uniformly distributed catenary lines in its configuration. The work by Al-Solihat and Nahon [19] provides an analytical derivation of the mooring stiffness matrix. In this study, the sensitivities of the mooring stiffness matrix are derived and implemented in the optimization. Considering surge, heave and pitch motions, the design-dependent stiffness matrix for the catenary mooring system and its entries are given as

\[
C_m(v) = \begin{pmatrix}
K_{11} & 0 & K_{15} \\
0 & K_{33} & 0 \\
K_{51} & 0 & K_{55}
\end{pmatrix}
\]

\[
K_{11} = \frac{1}{2}n(K_{11}^p + \frac{H}{2}), \quad K_{33} = nK_{33}^p, \\
K_{55} = n(-DK_{11}^p + \frac{D^2}{2}K_{11}^p + \frac{DH}{2}), \\
K_{15} = K_{51} = -n\left(-\frac{R}{2}K_{12}^p + \frac{D}{2}K_{11}^p + \frac{DV}{2} + \frac{HR}{2} + \frac{D^2H}{2}\right)
\]

Here \( l \) and \( h \) denote the given horizontal and vertical projection of the mooring line, \( H \) and \( V \) denote the horizontal and vertical force at the fairlead obtained by solving the catenary equation [19] using a Newton-Raphson method. \( D \) and \( R \) denote the vertical position \( (z = -D) \) and radius of the fairlead. \( K_{11}^p, K_{12}^p \) and \( K_{22}^p \) denote mooring line stiffness, i.e. \( \frac{\partial H}{\partial H}, \frac{\partial H}{\partial z}, \frac{\partial V}{\partial H} \). All these terms are implicit functions of the design variables \( \mathbf{v} \).
2.3. Objective function
The objective function is chosen as the mass-proportional cost of the steel for the floater plus the mass-proportional cost of the ballast plus the cost per meter of mooring line multiplied by the number of lines and the un-stretched length of the line.

3. Optimal design problem
The optimization problem is formulated as a single-objective design optimization. The considered objective function is the economical cost of the floater, ballast, and mooring system. The constraints are formulated to account for the static and dynamic performance of the wind turbine as well as geometric ranges that may influence the optimized design. The static performance is represented by the static surge and pitch motion, whereas the dynamic performance is represented by the eigen-frequencies and the dynamic responses, e.g. floater pitch motion and tower top acceleration. The geometric ranges are imposed on mooring lines concerning the maximum and minimum length as well as the maximum percentage of suspended mooring line multiplied by the number of lines and the un-stretched length of the line.

The set $V$ contains the constraints which do not require any frequency-domain simulations, i.e.

$$V = \{ v \mid v^\text{min} \leq v \leq v^\text{max}, z_b(v) \geq z_{cm}(v), l^\text{min}(v) \leq l_0(v) \leq l^\text{max}(v), V(v) \leq \eta w(v) \}$$

where $u$ denotes the static response, $p_k$ is a unit vector with a one in the position corresponding to the quantity of interest, and $\alpha_k^\text{max}$ is the prescribed limit, see the maximum static surge and pitch in Table 2. $\omega^\text{max}$ and $\alpha_k^\text{max}$ are the user supplied limits for the pitch and nacelle acceleration. The eigenvalues are similarly bounded by the limits $\lambda_i^\text{min}$ and $\lambda_i^\text{max}$ for the relevant frequencies. The set $\mathcal{V}$ contains the constraints which do not require any frequency-domain simulations, i.e.

$$\mathcal{V} = \{ v \mid v^\text{min} \leq v \leq v^\text{max}, z_b(v) \geq z_{cm}(v), l^\text{min}(v) \leq l_0(v) \leq l^\text{max}(v), V(v) \leq \eta w(v) \}$$

where $z_b(v)$ is the buoyancy center of the floater and $z_{cm}(v)$ is the center of mass of the floater and wind turbine, $V(v)$ and $w(v)$ denote the vertical force at the fairlead and the total weight in water of mooring line, and $\eta$ denotes a prescribed ratio. Here $l^\text{min} = \sqrt{l_0^2 + h^2}$ and $l^\text{max} = l + h$ with $l$ and $h$ given in Section 2.2. Problem (P) is a nonlinear optimization problem with continuous variables and a non-convex objective function and non-convex feasible set.

4. Design sensitivity analysis
Here we detail the required design sensitivity analysis for all the analysis which is involved in the optimization problem (P). All sensitivities make use of partial derivatives of the matrices $A$, $B$, $C$, and $M$ which are analytical in the design variables for the chosen modelling assumptions. Analytical design sensitivities of $\hat{\xi}_j(\omega)$, are provided through

$$\frac{\partial \hat{\xi}_j(\omega)}{\partial v_k} = \left( -\omega^2(M+A) + i\omega B + C \right)^{-1} \left( \frac{\partial F_k^\text{M}(\omega)}{\partial v_k} + i \omega^2 \frac{\partial(M+A)}{\partial v_k} - i \omega \frac{\partial B}{\partial v_k} - \frac{\partial C}{\partial v_k} \right) \hat{\xi}_j(\omega) \quad (4)$$

Analytical design sensitivities of the eigenvalues $\lambda_i$ are readily available, see [20], under the condition that the eigenvalues are distinct. This situation is expected for the considered system,
and we thus have

$$\frac{\partial \lambda_i(v)}{\partial v_k} = \phi_i^T \left( \frac{\partial C(v)}{\partial v_k} - \lambda_i \frac{\partial (M(v) + A(v))}{\partial v_k} \right) \phi_i$$

(5)

where $\phi_i$ is the normalized vibration mode linked to the eigenvalue $\lambda_i$, i.e. $\phi_i^T(M + A)\phi_i = 1$.

Design sensitivity of static analysis with design-independent load follows from e.g. [20]

$$\frac{\partial u(v)}{\partial v_k} = \frac{\partial C^{-1}(v)}{\partial v_k} f_s = -C^{-1}(v) \frac{\partial C(v)}{\partial v_k} C^{-1}(v) f_s = -C^{-1}(v) \frac{\partial C(v)}{\partial v_k} u(v)$$

(6)

The computational cost of the design sensitivity analysis can be estimated. The partial derivatives of the mass, damping, stiffness and added mass matrices are analytical and can be quickly computed for the given choices of design variables. The cost of the design sensitivities for the static analysis and the eigenvalues are negligible given the sizes of the matrices. The major computational effort is in the derivatives of the equations of motions. They require essentially one analysis per design variable, but with the advantage that the matrices are already assembled and possibly also factorized from the solution of the equation (1) in the response analysis. This direct design sensitivity analysis will therefore be faster than a finite difference scheme.

5. Implementation and benchmark of the design sensitivity analysis

The proposed optimization framework was implemented in MATLAB. The chosen optimization algorithm is the SQP method implemented in the function `fmincon` which is part of the MATLAB Optimization Toolbox. The solver parameters are generally set to default values. Analytical design sensitivities of the objective function and the constraints are provided.

The aerodynamic loads and damping are pre-computed using FAST [10]. The free-surface elevation is computed internally for each given sea state. The hydrodynamic properties, including added mass, viscous damping and hydrostatic stiffness, are all design-dependent, and are therefore calculated in the optimization process. The mooring stiffness is also design-dependent. In each constraint function evaluation, the mooring stiffness is evaluated at the equilibrium state in still water. The ballast is also determined in this equilibrium state.

5.1. Benchmark of the design sensitivity analysis

The computational efficiency of using analytical design sensitivities is evaluated in comparison with the numerical calculation of design sensitivities using the central finite difference scheme. The computation time is measured on a PC with an Intel Core i7-8650 CPU running at 1.90 GHz with 16 GB memory, and using MATLAB R2019b. In order to average out the fluctuation of the computer performance, all constraints are called 100 times. The computation without and with design sensitivities takes 1667.4 s and 1710.2 s, respectively. It can be seen that the time spent on the calculation of analytical design sensitivities is marginal compared to the calculation of the constraints. For numerically estimated design sensitivities, we assume that the central finite difference scheme is used because it provides higher accuracy over the forward or backward difference scheme. In this study, there are six design variables. Computing the analytical design sensitivities costs only an additional 2.57% in time, whereas computing of numerically estimated design sensitivities costs an additional 12 times in time. As a result, the speedup is more than 400 using analytical rather than numerical design sensitivities. The speedup becomes even larger when the number of design variables increases. However, note that the above analysis of the speedup depends on the implementation. Besides the computational efficiency, the accuracy of design sensitivities is important for the robustness of the optimization. In terms of accuracy, the analytical ones are advantageous compared to the numerical estimates. The latter’s accuracy may be impacted by the step size in the finite difference scheme.
6. Numerical experiments
For the numerical experiments, the wind turbine is chosen as the DTU 10 MW reference turbine [21]. The water depth is chosen as 320 m. The environmental conditions (ECs) have been extracted from [22]. Following the work of [23], twelve combinations of mean wind speed \( WS \), significant wave height \( H_s \) and wave peak period \( T_p \) have been used, representative of Design Load Case (DLC) 1.2 (see Table 1). The turbulent wind corresponds to a Kaimal spectrum, Class A and normal turbulence, and a Pierson-Moskowitz spectrum was used for the irregular sea states. Each EC in Table 1 was simulated for one hour.

| EC  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|
| \( WS \) | 5.00 | 7.00 | 9.00 | 11.00 | 11.40 | 13.00 | 15.00 | 17.00 | 19.00 | 21.00 | 23.00 | 25.00 |
| \( H_s \) | 1.38 | 1.66 | 1.98 | 2.36 | 2.46 | 2.83 | 3.38 | 4.01 | 4.79 | 5.70 | 6.85 | 8.31 |
| \( T_p \) | 7.00 | 7.95 | 8.00 | 8.29 | 8.46 | 9.13 | 9.64 | 9.89 | 10.65 | 11.85 | 12.34 | 12.00 |

6.1. Spar-buoy floater
In the case study, a spar-buoy floater anchored with a catenary mooring system is optimized. The geometry and the definitions of the design variables are illustrated in Figure 1. The design variables for the spar-buoy floater represent the draft \( l_1 \) and the two diameters \( d_1 \) and \( d_2 \). Additionally, three design variables allow for changes of the mooring system. These are the anchor radius \( r_a \), the vertical fairlead position \( z_m \), and the un-stretched length \( l_0 \). The wall thickness of the steel tubes are fixed to 0.06m. This assumption of a fixed wall thickness simplifies the optimization problem, see e.g. [24]. The mooring line effective mass in water is 300 kg/m. The price for steel and mooring line is set to 2.5 €/kg and 40.0 €/m, respectively. These parameters are in line with values from e.g. [25]. The price for ballast is set to zero. The density of steel, ballast and water is set to 7850, 2600 and 1025 kg/m\(^3\), respectively.

The constraints for the optimal design problem and their limits are described in Table 2. The limits and the optimized values of the design variables are shown in Table 3.

The behaviour of the SQP method is illustrated in Table 4 below. The column labelled \( \text{Itn.} \) represents the number of SQP iterations (i.e. the number of subproblems solved), whereas the column labelled \( \text{F.evals.} \) represents the number of objective and constraint function and design sensitivity evaluations required. For all problem instances, the SQP method reports that a feasible point satisfying the first-order optimality conditions has been found. For the optimized
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Table 2. A list of constraints in the optimization problem of the spar-buoy floater

| Type                  | Number | Description of the targeting function | Values                        |
|-----------------------|--------|----------------------------------------|-------------------------------|
| Stability             | 1      | buoyancy center higher than mass center| 0 (minimum)                   |
| Static responses      | 2      | maximum pitch angle \(\alpha_1^{\text{max}}\) | 10°                           |
|                       |        | maximum surge \(\alpha_2^{\text{max}}\) | 50 m                          |
| Eigenvalues           | 5      | maximum surge eigenvalue \(\lambda_1^{\text{max}}\) | \(\frac{(2\pi)^2}{100}\)     |
|                       |        | \(\lambda_i^{\text{min}}, \lambda_i^{\text{max}}, i = 2, 3\) | \(\frac{(2\pi)^2}{35}\), \(\frac{(2\pi)^2}{25}\) |
| Mooring line          | 3      | mix/max mooring line \(l_0\)           |                                |
|                       |        | maximum percentage of suspended line \(\eta\) | 75%                           |
| Dynamic responses     | 24     | maximum pitch angle \(\theta^{\text{max}}\) | 10°                           |
|                       |        | maximum nacelle acceleration \(a^{\text{max}}\) | 2 m/s²                        |

Table 3. The limits, initial and optimized values of design variables.

| Variables          | Min | Max | Initial | Optimized 11 constraints | Optimized 35 constraints |
|--------------------|-----|-----|---------|--------------------------|-------------------------|
| Length \(l_1\)    | 60  | 150 | 130     | 72.93                    | 120.45                  |
| Diameter \(d_1\)  | 5   | 20  | 18      | 20.00                    | 14.48                   |
| Diameter \(d_2\)  | 5   | 20  | 10      | 9.78                     | 8.96                    |
| Fairlead position \(z_m\) | -160 | -12 | -20     | -12.00                   | -73.03                  |
| Anchor radius \(r_a\) | 600  | 2000 | 650     | 819.37                   | 726.98                  |
| Cable length \(l_0\) | 600  | 2100 | 750     | 906.58                   | 790.87                  |

design, its cost increases by 13% when the dynamic constraints are included, signalling that some of these constraints are design driving. Further analysis shows that the tower-top acceleration is design driving. This agrees with the work in [24], where a multi-objective design optimization is studied in terms of the economical cost of floater and mooring system as well as the wind turbine fore-aft nacelle acceleration. The other active constraints for the optimized designs involve maximum static pitch angle and surge, the lower bounds of eigenvalues of heave and pitch modes, and the maximum percentage of suspended mooring line.

The dynamic responses of the two optimized designs to the same ECs are compared in Figure 2 for rated wind speed and Figure 3 for cut-out wind speed. In each figure, the left column shows a portion of the time series of (from top to bottom): horizontal wind speed at the hub, free-surface elevation, floater surge, heave and pitch, and nacelle acceleration. The middle column shows Power Spectral Density (PSD) plots of the corresponding signals. The right column shows the exceedance probability, obtained by extracting the peaks of each time series, sorting them in ascending order and assigning an exceedance probability to each value based on its position in the list. As seen in the plots (especially in terms of exceedance probability), including the dynamic constraints in the optimization leads to a design with smaller responses, both for rated and cut-out environmental conditions. The two designs also have slightly different natural frequencies, as seen in the PSD plots. Since the constraints are prescribed through 3 times standard deviation, the maximum nacelle acceleration in the responses may exceed the prescribed threshold with a low probability, see Figures 2 and 3, and Table 5. The maximum tower top acceleration is under-estimated by using the proposed estimation (mean+3\(\sigma\)). A factor may be applied in the
Table 4. Optimization statistics. The objective is normalized with respect to the cost of the same initial design as shown in Table 3.

| Problem     | Dynamic | Itn. | F.evals. | CPU (s) | Objective |
|-------------|---------|------|----------|---------|-----------|
| Spar-buoy   | No      | 35   | 79       | 41      | 0.67      |
|             | Yes     | 28   | 61       | 513     | 0.76      |

Table 5. Comparison of the estimated maximum values (mean+3σ) with the maximum values in the response calculation.

| Property                        | Max | Estimated | Error % | Max | Estimated | Error % |
|---------------------------------|-----|-----------|---------|-----|-----------|---------|
| Floater pitch (deg)             | 6.91| 7.38      | +6.78   | 0.80| 0.71      | -11.47  |
| Nacelle acceleration (m/s²)     | 2.59| 2.00      | -22.78  | 0.80| 0.71      | -11.47  |

Table 6. Static and dynamic properties of the two optimized designs.

| Property of design                        | Optimized 11 constraints | Optimized 35 constraints |
|-------------------------------------------|--------------------------|--------------------------|
| z-coordinate of center of mass (m)        | -55.99                   | -86.08                   |
| z-coordinate of center of buoyancy (m)    | -45.61                   | -68.73                   |
| Eigen-frequency of surge mode (Hz)        | 5.60 × 10⁻³              | 7.15 × 10⁻³              |
| Eigen-frequency of heave mode (Hz)        | 2.86 × 10⁻²              | 2.86 × 10⁻²              |
| Eigen-frequency of pitch mode (Hz)        | 3.18 × 10⁻²              | 2.86 × 10⁻²              |
| Eigen-frequency of coupled tower mode (Hz)| 6.92 × 10⁻¹              | 6.35 × 10⁻¹              |

optimization to address the under-estimation issue in future studies.

A comparison of the static properties and the eigenfrequencies of the two optimized designs are given in Table 6. Note that the eigen-frequency of the coupled tower mode is above the 3P forcing region, which shows that the optimization is able to reduce the dynamic response by altering the design, leading to smaller tower top vibrations. The change of the eigen-frequency of the coupled tower mode is achieved mainly by changing the floater and mooring system. Including tower design into the optimization may improve the design further.

7. Discussions and future work

The load computations are based on simplified models where the aerodynamic loads and damping are pre-computed for a given wind turbine using time-domain aeroelastic rotor computations. These loads are thus assumed to be design-independent and the optimization process is therefore only partially able to take advantage of the couplings between design and loads. One improvement here, may be to include the frequency dependency of the aerodynamic damping of the tower mode. The analysis model for the floater has a very limited number of degrees of freedom and no structural requirements (on e.g. strength, displacement, buckling, and fatigue properties) are included in the design optimization problem.

The mooring system is linearized at its equilibrium position without wind and wave loads. However, the stiffness of the mooring system changes as the structure moves to different mean positions for e.g. different mean wind speeds. The inclusion of this effect is left for future work.
**Figure 2.** Comparison of the dynamic responses at rated wind speed of the optimized designs with (in red) and without (in blue) inclusion of the dynamic constraints. The mean wind speed is 11.4 m/s, significant wave height is 2.46 m, and the wave peak period is 8.46 s.

On the analysis side, QuLAF is currently being extended to 8 DoFs (6 DoF for the floater motion and tower fore-aft and side-side first bending modes), to be able to handle cases with wind-wave misalignment. The hydrodynamic sub-model is being enhanced to compute internally hydrodynamic properties for more complex floater geometries.

The optimization problem and design sensitivity analysis are being extended to include more variables to describe the geometry of multi-components floaters such as semi-submersibles and other types and configurations of mooring systems. The problem is also being extended to include additional analysis capabilities and more constraints, e.g. on the mooring system performance and structural performance.

Our numerical experiments suggest that there are sometimes multiple near-optimal designs with similar cost but diverse configurations. Further studies are required to explore and compare
Figure 3. Comparison of the dynamic responses at cut-out wind speed of the optimized designs with (in red) and without (in blue) inclusion of the dynamic constraints. The mean wind speed is 25 m/s, significant wave height is 8.31 m, and the wave peak period is 12.00 s.

8. Conclusion
An optimization framework for floating wind turbine support structures has been presented. The method builds on frequency domain modelling and analytical design sensitivities to the design variables. We have demonstrated that the design sensitivities can hereby be obtained by only 2.57% additional computational cost of the direct response calculation when 6 design variables are used. The method was applied to DLC 1.2 of combined wind and waves, and provided an optimal design in 513 s. The effect of inclusion of the dynamic constraints was demonstrated and had a clear impact on the optimized design, the coupled tower frequency, and the response...
level for floater pitch and nacelle acceleration.

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References
[1] Pegalajar-Jurado A, Bredmose H and Borg M 2016 Multi-level hydrodynamic modelling of a scaled 10MW TLP wind turbine Energy Procedia 94 124-32
[2] Pegalajar-Jurado A, Borg M and Bredmose H 2018 An efficient frequency-domain model for quick load analysis of floating offshore wind turbines Wind Energy Science 3 693-712
[3] Madsen F J, Pegalajar-Jurado A and Bredmose H 2019 Performance study of the QuLAF pre-design model for a 10MW floating wind turbine Wind Energy Science 4 527-47
[4] Boggs P T and Tolle J W 1995 Sequential Quadratic Programming Acta Numerica 4 1-51
[5] Lupton R 2014 Frequency-domain modelling of floating wind turbines Ph.D. thesis University of Cambridge
[6] Wang K, Ji C, Xue H and Tang W 2017 Frequency domain approach for the coupled analysis of floating wind turbine system Ships and Offshore Structures 12 767-74
[7] Lemmer F, Yu W and Cheng P W 2018 Iterative frequency-domain response of floating offshore wind turbines with parametric drag Journal of Marine Science and Engineering 6 118
[8] Hegseth J M and Bachynski E E 2019 A semi-analytical frequency domain model for efficient design evaluation of spar floating wind turbines Marine Structures 64 186-210
[9] Karimi M, Buckham B and Crawford C 2019 A fully coupled frequency domain model for floating offshore wind turbines Journal of Ocean Engineering and Marine Energy 5 135-58
[10] Jonkman J M and Jonkman B J 2016 FAST modularization framework for wind turbine simulation: full-system linearization J. Phys.: Conf. Ser. 753 082010
[11] Fylling I and Berthelsen P A 2011 WINDOPT: an optimization tool for floating support structures for deep water wind turbines Proc. Int. Conf. Offshore Mechanics and Arctic Engineering (Rotterdam) Vol 5 pp 767-76.
[12] Hall M, Buckham B and Crawford C 2013 Evolving offshore wind: a genetic algorithm-based support structure optimization framework for floating wind turbines 2013 MTS/IEEE OCEANS-Bergen pp 1-10
[13] Hall M, Buckham B and Crawford C 2014 Hydrodynamics-based floating wind turbine support platform optimization: a basis function approach Renewable Energy 66 559-69
[14] Pillai A, Thies P and Johanning L 2019 Mooring system design optimization using a surrogate assisted multi-objective genetic algorithm Engineering Optimization 51 1370-92
[15] Barbanti G, Marino E and Borri C 2019 Mooring system optimization for a spar-buoy wind turbine in rough wind and sea conditions Proc. XV Conf. of the Italian Association for Wind Engineering ed Ricciardelli F and Avossa A pp 87-98
[16] Myhr A and Nygaard T A 2012 Load reductions and optimizations on tension-leg-buoy offshore wind turbine platforms Proc. 22nd Int. Offshore and Polar Engineering Conf. (Rhodes) pp 232-9
[17] Hegseth J M, Bachynski E E and Martins J R 2020 Integrated design optimization of spar floating wind turbines Marine Structures 72 102771
[18] Morison J R, Johnson J W and Schaar S A 1950 The force exerted by surface waves on piles Journal of Petroleum Technology 2 149-54
[19] Al-Solihat M K and Nahon M 2016 Stiffness of slack and taut moorings Ships and Offshore Structures 11 890-904
[20] Choi K and Kim N H 2005 Structural Sensitivity Analysis and Optimization 1: Linear Systems (Springer)
[21] Bak C, Zahle F, Bitsche R, Kim T, Yde A, Henriksen L C, Hansen M H, Blasques J P, Gaunaa M and Natarajan A 2013 The DTU 10-MW reference wind turbine Danish Wind Power Research 2013
[22] Krieger A, Ramachandran G, Vita L, Alonso P G, Almería G G, Berque J and Aguirre G 2015 LIFES50+ deliverable DT.2: design basis DNVGL Technical Report
[23] Pegalajar-Jurado A, Bredmose H, Borg M, Straume J, Landbo T, Andersen H, Yu W, Müller K and Lemmer F 2018 State-of-the-art model for the LIFES50+-OO-Star Wind Floater Semi 10MW floating wind turbine J. Phys.: Conf. Ser. 1104 p 012024
[24] Karimi M, Hall M, Buckham B and Crawford C 2017 A multi-objective design optimization approach for floating offshore wind turbine support structures Journal of Ocean Engineering and Marine Energy 3 69-87
[25] Myhr A, Bjerkseter C, Ágotnes A and Nygaard T A 2014 Levelised cost of energy for offshore floating wind turbines in a lifecycle perspective Renewable Energy 66 714-28