The minimum cost flow finding under fuzzy intuitionistic conditions

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Abstract. Present paper deals with the minimum cost flow finding task with fuzzy trapezoidal intuitionistic values of arc capacities and costs. Nowadays, flow tasks in networks are leading tasks in flow modelling and urban logistics. Peculiarity of the problem is in optimal paths finding, which allows decreasing the total cost of transportation and choosing the best paths for transportation. Fuzzy logic as a powerful tool for dealing with uncertainty, vagueness and inaccurate data influenced the way of presenting characteristics of networks. Arc capacities and costs can be presented as fuzzy numbers of the different form enabling researchers to get more reliable solutions. However, during the simulation a researcher is often faced with difficulties in exact specifying of arc capacities and costs. It is a commonplace that experts hesitate or have doubts choosing a specific value for parameters of the network. In such circumstances arc capacities and costs can be presented in a fuzzy intuitionistic form, in particular, as fuzzy intuitionistic triangular or trapezoidal numbers. A fuzzy intuitionistic number allows taking into account a level of hesitation by including degree of membership, non-membership and indeterminacy margin. Incorporating fuzzy intuitionistic sets into conventional flow patterns enables researchers to solve tasks in fuzzy, vague conditions even when there is a lack of necessary data for problem statement. The main contribution to the paper is the proposal for the fuzzy intuitionistic minimum cost flow algorithm using fuzzy intuitionistic flow patterns based on the ranking technique and arithmetic operations with fuzzy intuitionistic trapezoidal numbers. A case study numerical is presented to illustrate the proposed algorithm.

1. Introduction

The task of the minimum cost flow finding is one of the most important in the theory of flows because it allows optimizing the cost of the transported cargo. It is particularly significant while determining the paths of the minimum cost in the tasks of determining the maximum or specified volume of cargo transportation. This problem is based on finding the shortest path that can be found using various algorithms named after authors E. Dijkstra, R. Bellman and L. Ford, R. Floyd and S. Warshall, B. Levit, D. Johnson [1]. The most popular algorithm for finding the shortest path is Dijkstra's algorithm, which was proposed in 1959. The essence of the algorithm is to find the shortest path from a given vertex to all other vertices of the graph; time efficiency in the general case is \(O(X^2)\) [2]. To find the shortest path, B. Levit's algorithm can also be used, which in the general case has exponential time. The algorithm operates with sets of vertices similar to Dijkstra's algorithm, however, the marked vertices are divided into the main and priority queues [1]. The main disadvantage of the algorithm is
the necessity to reprocess the vertices. The considered algorithms, despite the obvious advantages associated with time complexity, cannot be used for negative edge weights.

One of the first methods of the shortest path finding was the method of R. Bellman and L. Ford [1] that enables finding the shortest path from one vertex to the rest in time $O(XA)$. The advantage of the algorithm is the ability to deal with negative edges weights of the graph, as well as the simultaneous search for cycles of negative weight. A feature of the algorithm of R. Floyd and S. Warshall [1], by analogy with the Bellman-Ford method, is the ability to find the shortest paths between all pairs of graph vertices without preserving the paths themselves, while simultaneously searching for cycles of negative weight.

The algorithms described above underlie the algorithms for finding the minimum cost flow and the key algorithms of the flow theory. One of the most common algorithms for finding the flow of the minimal cost is R. Basaker-P. Gowan's algorithm [3] based on sequential search for shortest paths, as well as M. Klein's algorithm [4], which aims to identify and remove cycles of negative weight. The first mentioned algorithm works as follows: at the first stage, we look for the shortest path, then let the flow pass along it sequentially, starting from the zero flow value and until the path stops being the shortest. On the contrary, for M. Klein's algorithm it is necessary to choose an arbitrary value with its subsequent optimization and check for the absence of negative value cycles as an optimality criterion. R. Floyd's algorithm can be used as one of the algorithms for determining the cycle of the negative cost.

In real life cases, when it is necessary to calculate the optimal transportation routes and the value of the minimum cost flow, difficulties arise when specifying the arc capacities and transportation costs. Sometimes the exact assigning network parameters is limited by measurement inaccuracy, errors, and imprecision. Changes in prices, renovations, new road construction and sign installation also affect road capacity and network parameters. In this regard, a number of authors [1, 5-6] conduct research in the field of fuzzy flow theory. Fuzzy logic is a powerful tool for modeling uncertainty in transportation problems. Fuzzy logic is based on the concept of a fuzzy set, a fuzzy number, a membership function. Thus, by setting network parameters in the form of fuzzy numbers of various shapes, researchers obtain more reliable solutions.

In recent years, when solving flow problems in fuzzy terms, researchers are faced with the impossibility of accurate representing the membership function by one number. For instance, an expert hesitates or doubts when choosing a membership function. K. Atanassov developed a concept of intuitionistic fuzzy sets [7] so that one could express a doubt and uncertainty of a decision-maker in a mathematical form. These numbers operate with a membership function, a non-membership function, which sum must be less than or equal to 1. Thus, in the theory of intuitionistic fuzzy sets, the degree of uncertainty is allowed, which can be interpreted as a level of indeterminacy. Papers [8-9] survey the approaches to the shortest path finding in fuzzy intuitionistic terms. However, the problem of finding the minimum flow value has not been fully addressed in the literature. In this regard, this article will be devoted to solving the task of the minimum cost flow finding with fuzzy intuitionistic values of parameters of a network.

The paper is organized as follows. Section 2 is devoted to the preliminaries and problem statement. Proposed algorithm for the minimum cost flow finding in intuitionistic conditions is given in the Section 3. Section 4 describes a numerical example. Finally, Section 5 concludes the paper.

2. Preliminaries and problem statement

Let us give main definitions of the proposed method.

Definition of a fuzzy set.

Let us consider universal set $X=\{x\}$. Fuzzy set on the set $X$ is the set of the pairs $\tilde{A}=<x, \mu_{\tilde{A}}(x), x \in X>$, where $\mu_{\tilde{A}}: X \rightarrow [0,1]$ is mapping of the set $X$ into the unit interval $[0,1]$, called membership function of the fuzzy set $\tilde{A}$. The value of the membership function $\mu_{\tilde{A}}(x)$ for the element $x \in X$ is called the membership degree. Interpretation of the membership degree is a subjective
measure of the element corresponding to the concept, which meaning is formalized by the fuzzy set [1].

Definition of an intuitionistic fuzzy set.

Let X be a nonempty set. An intuitionistic fuzzy set \( \tilde{A} \) in X is defined as \( \tilde{A} = \langle x, \mu_A(x), \nu_A(x) : x \in X \rangle \), where the functions \( \mu_A(x), \nu_A(x) : X \to [0,1] \) are the degrees of membership and non-membership of the element \( x \in X \), such as for each \( x \in X \): 0 \( \leq \) \( \mu_A(x) \) + \( \nu_A(x) \) \( \leq \) 1. Furthermore, we have \( \pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \) called the intuitionistic fuzzy set index or hesitation margin of \( x \) in \( X \). It represents the degree of indeterminacy or hesitation of \( x \in A \). For each \( x \in X \): 0 \( \leq \) \( \pi_A(x) \) \( \leq \) 1. The pair \( \alpha = \mu_A(x), \nu_A(x) \) is called an intuitionistic fuzzy value (IFV). The main operations with intuitionistic fuzzy values used in the algorithm, are performed in [10].

Definition of a trapezoidal intuitionistic fuzzy number.

Let us describe a notion of a trapezoidal fuzzy number \( \tilde{A} \). Trapezoidal intuitionistic fuzzy number \( \tilde{A} \) is the number of the form \( \tilde{A} = \langle a_1, a_2, a_3, a_4 \rangle, \langle a_1', a_2, a_3, a_4' \rangle \), where \( a_1' < a_1 < a_2 < a_3 < a_4 < a_4' \); \( a_1', a_2, a_3, a_4, a_4' \in R \). The membership and the non-membership functions can be represented as follows [8]:

\[
\tilde{\mu}_A(x) = \begin{cases} 
\frac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2, \\
1, & \text{for } a_2 \leq x \leq a_3, \\
\frac{a_4 - x}{a_4 - a_3}, & \text{for } a_3 \leq x \leq a_4, \\
0, & \text{otherwise}.
\end{cases}
\]

\[
\tilde{\nu}_A(x) = \begin{cases} 
\frac{a_2 - x}{a_2 - a_1'}, & \text{for } a_1' \leq x \leq a_2, \\
1, & \text{for } a_2 \leq x \leq a_3, \\
\frac{x - a_3}{a_4' - a_3}, & \text{for } a_3 \leq x \leq a_4', \\
0, & \text{otherwise}.
\end{cases}
\]

Main operations on the on trapezoidal intuitionistic fuzzy numbers.

Let \( \tilde{A} = \langle a_1, a_2, a_3, a_4 \rangle, \langle a_1', a_2, a_3, a_4' \rangle \) and \( \tilde{B} = \langle b_1, b_2, b_3, b_4 \rangle, \langle b_1', b_2, a_3, b_4' \rangle \) be two trapezoidal intuitionistic fuzzy number [1]. Adding operation of trapezoidal intuitionistic fuzzy numbers can be represented as \( \tilde{A} \oplus \tilde{B} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4 \rangle, \langle a_1' + b_1', a_2 + b_2, a_3 + b_3, a_4' + b_4' \rangle \) (figure 1).

![Figure 1. Trapezoidal intuitionistic fuzzy number.](image)

The ranking technique for intuitionistic fuzzy numbers.
Let $\bar{A}$ and $\bar{B}$ be two trapezoidal numbers, then $\bar{A} \leq \bar{B} \iff R^\Delta(\bar{A}) \leq R^\Delta(\bar{B})$ and $R^{\Delta\Delta}(\bar{A}) \leq R^{\Delta\Delta}(\bar{B})$. 

$\alpha$-cut ranking technique can be represented as:

$$R(\bar{A}) = \left(\frac{a_1 + 2(a_2 + a_3) + a_4}{6}, \frac{2(a_1' + a_3') + (a_2 + a_3)}{6}\right) = (R^\Delta(\bar{A}), R^{\Delta\Delta}(\bar{A})).$$

Summing up, let us give a model for the minimum cost flow determining with fuzzy intuitionistic values of arc capacities and costs. Let the arc capacities and arc costs are fuzzy intuitionistic numbers of the form $(\tilde{c}_{ij}, c_{ij})$, $(i, j) \in \bar{A} = (\bar{A}_1, \bar{A}_2)$. $\bar{A}_1$ is the arc weight of membership function. $\bar{A}_2$ – non-membership function. The model for the minimum cost flow determining in fuzzy intuitionistic terms is given as a model (1)-(3).

$$Minimize\ \bar{W} = (\min \bar{D}, \max \bar{d}) = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}, \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \tilde{x}_{ij}\right)$$

Subject to:

$$\sum_{x \in \Gamma^+(x)} \tilde{x}_{ij} = \sum_{x \in \Gamma^-(x)} \tilde{x}_{ki} = \begin{cases} \tilde{\rho}, x_i = s, \\
-\tilde{\rho}, x_i = t, \\
0, x_i \neq s, t. \end{cases}$$

$$0 \leq \tilde{x}_{ij} \leq \bar{u}_{ij}, \forall (x_i, x_j) \in \bar{A}. \quad (3)$$

3. Proposed algorithm

Let us represent the proposed method for the minimum cost flow determining in fuzzy intuitionistic conditions.

Step 1. Construct a fuzzy residual network $\tilde{G}^\mu$, which arcs are assigned arc costs in the form of the fuzzy trapezoidal intuitionistic numbers $\tilde{c}_{ij} = \langle c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4 \rangle$, and arc capacities in the form of the fuzzy trapezoidal intuitionistic numbers $\bar{u}_{ij} = \langle u_{ij}^1, u_{ij}^2, u_{ij}^3, u_{ij}^4 \rangle$, depending on the flow values of the initial network according to the following rule:

1) decrease the residual arc capacity $\bar{u}_{ij}^\mu$ of the direct arc by the value of current flow along the corresponding arc $\bar{u}_{ij}^\mu = \bar{u}_{ij} - \tilde{x}_{ij}$.

2) increase the residual arc capacity $\bar{u}_{ij}^\mu$ of the opposite artificial arc by the value of current flow along the corresponding arc $\bar{u}_{ij}^\mu = \tilde{x}_{ij}$.

Step 2. Search the shortest path $\tilde{p}^\mu$ as the minimum cost path in the fuzzy residual network $\tilde{G}^\mu$ by Dijkstra's algorithm [11-12].

a) Initialization.

The label of the source node is considered as 0, the labels of the other vertices are set to infinity.

All vertices of the graph are marked as unvisited.

b) Algorithm step.

If all vertices have been visited, the algorithm stops.

Otherwise, the vertex $x$ with the minimum label is selected from the unvisited vertices.

We consider all possible routes in which $x$ is the penultimate item. The vertices to which the edges from $u$ lead are called neighbours of this vertex. For each neighbour of the vertex $x$, except those marked as visited, consider the new path cost equal to the sum of the values of the current label $x$ and the cost of the edge connecting $x$ with this neighbour.

If the obtained cost value is less than the label value of the neighbour, replace the label value with the obtained cost value. Considering all neighbours, mark the vertex $x$ as visited and repeat the step of the algorithm.

2.1. If a path $p^\mu$ is found, turn to the step 3.

2.2. If a path $p^\mu$ is not found, the maximum flow of the minimum cost is determined, exit.

Step 3. Convey the flow value according to the minimal from arc capacities of the path: equal to the arc with minimal arc capacity in the residual network $\bar{c}_{ij}^\mu = min\{\bar{u}_{ij}^\mu\}$.

Step 4. Recalculate the flow values and turn to the step 1.
1) for arcs in the opposite direction between nodes \((x_j, \theta)\) and \((x_i, \theta)\) modify the flow value \(\hat{\xi}_{ij}\) and decrease it on the value \(\delta^\mu\). Final flow value will be \(\hat{\xi}_{ij} - \delta^\mu\).

2) for directed arcs between nodes \((x_i, \theta)\) and \((x_j, \theta)\) modify the flow value \(\hat{\xi}_{ij}\) and increase it on the value \(\delta^\mu\). Final flow value will be \(\hat{\xi}_{ij} + \delta^\mu\).

4. Numerical example

In this section, a numerical example is considered to justify the proposed method of the minimum cost flow finding in fuzzy intuitionistic terms. Figure 2 presents a transportation network with assigned values of transmission costs and arc capacities in the form of fuzzy intuitionistic trapezoidal numbers. The former parameters are given on the arc of the network, while the latter are presented in the table 1.

Let us demonstrate the proposed method. The goal is to find the minimum cost maximum flow value.

Step 1. Initial flow values are equal to zero, therefore, a residual network coincides with the initial network in figure 2.

Table 1. Arc capacities of the initial network.

| The arc  | Fuzzy trapezoidal intuitionistic value of arc capacity |
|---------|------------------------------------------------------|
| \(\tilde{u}_{12}\) | \(<11, 15, 19, 24>, <9, 15, 19, 28>\) |
| \(\tilde{u}_{13}\) | \(<8, 10, 13, 15>, <6, 10, 13, 18>\) |
| \(\tilde{u}_{24}\) | \(<22, 30, 39, 47>, <19, 30, 39, 50>\) |
| \(\tilde{u}_{34}\) | \(<30, 45, 59, 74>, <25, 45, 59, 80>\) |
| \(\tilde{u}_{35}\) | \(<27, 40, 52, 64>, <22, 40, 52, 67>\) |
| \(\tilde{u}_{45}\) | \(<41, 60, 79, 98>, <38, 60, 79, 103>\) |

Step 2a. The first iteration. Apply Dijkstra’s modified algorithm for intuitionistic fuzzy conditions to search for the shortest path. The node 1 has a minimal label as it is the source node.

2b. The neighbours of the node 1 are nodes 2 and 3. The label of the node 2 is equal to the length of the shortest path: \((1,2)\) or \((1,3)\). Compare current costs \((1,2)\) and \((1,3)\) according to criteria \(R^A, R^{\Delta A}\). 

\[
R^A(2) = \frac{31 + 2 \times 35 + 39 + 46}{6} = 37.5, \quad R^{\Delta A}(2) = -\frac{2 \times 78 + 74}{6} = -38.33, \quad \text{consequently,} \quad R(2) = (37.5, -38.33), \quad R^A(3) = \frac{29 + 2 \times 77 + 50}{6} = 38.83, \quad R^{\Delta A}(2) = -\frac{2 \times 80 + 77}{6} = -39.5, \quad \text{therefore,} \quad R(3) = (38.83, -39.5).
\]

\((37.5, -38.33) \leq (38.83, -39.5)\) and as a result \((1,3) \leq (1,2)\). Finally, the node 3 is selected. All neighbours of the node 1 are examined. A current minimum distance to the node 1 is final and is not subject to revision. We delete it from consideration since it has been visited.
Step 3a. Choose the node with the minimum label \(\Rightarrow\) it is the node 3 with the label \((<12,15,17,21>,<10,15,17,23>)\).

Step 3b. Look through neighbouring nodes for the node 3 and strive to minimize the labels. The label for the node 4 is determined as \((<12,15,17,21>,<10,15,17,23>) + (<17,21,24,29>,<15,21,24,32>) = (<29,36,41,50>,<25,36,41,55>)\). The label for the node 5 is defined as \((<12,15,17,21>,<10,15,17,23>) + (<58,70,80,95>,<53,70,80,99>) = (<70,85,97,116>,<63,85,97,122>)\).

Compare all labels 2, 4 and 5.

All neighbours of the node 3 are viewed, therefore, block distances to it and mark it as visited.

Step 4a. The closest of the visited nodes is the node 2 with the label \((<31,35,39,46>,<28,35,39,50>)\).

Step 4b. Examine its neighbours and strive to decrease distances. The arc leaves the node 2 and enters the node 4. The label of the node is \((<31,35,39,46>,<28,35,39,50>) + (<49,50,55,63>,<43,50,55,70>) = (<80,85,94,109>,<71,85,94,120>)\), which exceeds its current label. Make no changes. All neighbours of the vertex 2 have been scanned, therefore, we block the distance to it and mark it as visited.

Step 5a. Choose the node with minimum label – node 4. One arc leaves it and enters the node 5. Find the label for the node 5: \((<29,36,41,50>,<25,36,41,55> + <52,61,69,82>,<48,61,69,85>) = (<81,97,110,132>,<73,97,110,140>)\). This label exceeds a current one, therefore, a previous label remains. Since we have reached the sink, we go back through predecessors, checking the equality of the labels to the current path, and we get a path 1 \(\rightarrow\) 3 \(\rightarrow\) 5.

Step 3. Conveying the flow

Convey the flow along the path 1 \(\rightarrow\) 3 \(\rightarrow\) 5. Determine the minimum cost path along the arcs (1,3) and (3,5) as well as select the minimum from the fuzzy intuitionistic numbers \((<8,10,13,15>,<6,10,13,18>)\) and \((<24,40,52,64>,<22,40,52,67>)\) according to criteria \(R^A, R^{A\Delta}\). The number \((<27,40,52,64>,<22,40,52,67>)\) is more than the number \((<8,10,13,15>,<6,10,13,18>)\), therefore, pass \((<8,10,13,15>,<6,10,13,18>)\) flow units along this path.

Step 4. Recalculate the flow values: from zero values they turn into \((<8,10,13,15>,<6,10,13,18>)\) as shown in figure 3.

![Figure 3. The network with the fuzzy flow.](image)

Step 1. Reconstruct a residual network with the new flow values as shown in figure 4.

Step 2a. Second iteration. Apply Dijkstra’s algorithm modified for intuitionistic fuzzy conditions to search for the shortest path. The node 1 has a minimal label as it is the source node.

2b. The neighbour of this node is the node 2. The label of the node 2 is equal to the length of the shortest path (1,2). Thus, the label of the node 2 is \((<31,35,39,46>,<28,35,39,50>)\). All neighbours of the node 1 are examined. A current minimum distance to the node 1 is final and is not subject to revision. We delete it from consideration since it has been visited.
Step 3a. Choose the node with the minimum label $\Rightarrow$ it is the node 2.

Step 3b. Look through neighbouring nodes for the node 2 and strive to minimize the labels. The label for the node 4 is determined as $<(31,35,39,46), (28,35,39,50)> +<(49,50,55,63), (43,50,55,70)> =<(80,85,94,109), (71,85,94,120)>$. Select the node with the minimum label – the node 2. All neighbours of the node 2 are viewed, therefore, block distances to it and mark it as visited.

Step 4a. The closest of the visited nodes is the node 4 with the label $<(80,85,94,109), (71,85,94,120)>$.

Step 4b. Examine the neighbours of the node 4 and strive to decrease distances. The arc leaves the node 4 and enters the node 5. The label of the node 5 is $<(80,85,94,109), (71,85,94,120)> +<(52,61,69,82), (48,61,69,85)> =<(132,146,163,191), (119, 146,163,205)>$. Thus, the node 5 has a label and the path $1 \rightarrow 2 \rightarrow 4 \rightarrow 5$ is obtained.

Step 3. Conveying the flow

Convey the flow along the path $1 \rightarrow 2 \rightarrow 4 \rightarrow 5$. Determine the minimum cost path along the arcs $(1,2)$, $(2,4)$ and $(4,5)$ as well as select the minimum from the fuzzy intuitionistic numbers $<(11,15,19,24), (9,15,19,28)>,<(22,30,39,47), (19,30,39,50)>,<(41,60,79,98), (38, 60,79,103)>$ according to criteria $R^A, R^B, R^C$. The number $<(11,15,19,24), (9,15,19,28)>$ is minimal, therefore, pass $(11,15,19,24), (9,15,19,28)$ flow units along this path.

Step 4. Recalculate the flow values: from the value $<(8,10,13,15), (6,10,13,18)>$ they turn to the value $<(19,25,32,39), (15,25,32,46)>$ as shown in figure 5.

Figure 5. The network with the flow distribution.

Step 1. Construct a residual network for the graph in figure 5.
Step 2. There is no path in the constructed network in figure 6. It means that we have found the maximum flow value (<19,25,32,39>,<15,25,32,46>) of the minimum cost presented in figure 5.

5. Conclusion
The paper is devoted to the problem of the minimum cost flow determining in fuzzy intuitionistic terms. The underlying task for the minimum cost selecting is finding the shortest path in a graph. There are many prevalent methods for the shortest path finding; however, it is often necessary to consider input uncertainty and inaccuracy peculiar to parameters of the network. To do this researchers often stick to the fuzzy set theory as the main tool for dealing with imprecise variables. Using a membership function, experts can model the uncertainty, which leads to the flexible solutions. Despite the investigation in the field of fuzzy logic, a researcher is frequently faced with a problem of hesitation during the choice of the membership function. In order to reflect quantitatively the level of hesitation and doubts fuzzy intuitionistic theory was developed. Using this theory, an expert can define a membership, non-membership degree and a level of indeterminacy. The sum of membership and non-membership degrees can be less than 1, which allows researcher introducing a margin of hesitation. Fuzzy intuitionistic numbers, for instance, trapezoidal numbers are used in the proposed algorithm for defining arc capacities and transmission costs. A proposed method searches for the minimum cost flow in fuzzy intuitionistic conditions. A numerical example is presented to illustrate the proposed method.

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