On higher order computations and synaptic meta-plasticity in the human brain: IT point of view

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Abstract

Glia modify neuronal connectivity by creating structural changes in the neuronal connectome. Glia also influence the functional connectome by modifying the flow of information through neural networks (Fields et al. 2015 [8]). There are strong experimental evidences that glia are responsible for synaptic meta-plasticity. Synaptic plasticity is the modification of the strength of connections between neurons. Meta-plasticity, i.e. plasticity of synaptic plasticity, may be viewed as mechanisms for dynamic reconfiguration of neural circuits. First order computations in the brain are done by static neural circuits, whereas higher order computations are done by dynamic reconfigurations of the links (synapses) between the neural circuits. Static neural circuits correspond to first order computable functions. Synapse creation correspond to the mathematical notion of function composition. Functionals are higher order functions that take functions as their arguments. The construction of functionals is based on dynamic reconfigurations of the function composition. Perhaps the functionals correspond to the meta-plasticity in the human brain.

1 Introduction

Gedankenexperiment: a backward time travel of a computer. A contemporary computer was moved into the XIX-th century so that scientists could make experimental research. Actually, the idea underlining the functioning of a computer is extremely simple; it is the von Neumann computer architecture. Would it be possible for the scientists of nineteenth century to discover the idea by examining the electric circuits and their complex functioning of the working computer system consisting of monitor, a motherboard, a CPU, a RAM, graphic cards, expansion cards, a power supply, an optical disc drive, a hard disk drive, a keyboard and a mouse? What about BIOS and operating system as well as many applications installed?

Perhaps the Gedankenexperiment may serve as a metaphor of the research on (the human) brain functioning. Although great achievements have been made in the brain research, the basic mechanisms (idea) underling the human brain functioning are still a great mystery.
A short review of the current research on higher order computations in the brain is presented below. Astrocytes are a kind of glial cells (simply glia). Let us cite the recent views of the role of glia and meta-plasticity in the brain.

Fields et al. 2015 [8]: “Astrocytes have anatomical and physiological properties that can impose a higher order organization on information processing and integration in the neuronal connectome. Neurons compute via membrane voltage, but how do astrocytes compute? What do glia contribute to information processing that neurons cannot accomplish? ... In comparison to neurons, glia communicate slowly and over broader spatial scales. This may make glia particularly well suited for involvement in integration, in homeostatic regulation, and alterations in structural or functional connectivity of neural networks taking place over periods of weeks or months.”

Min et al. 2015 [16]: “Many studies have shown that astrocytes can dynamically modulate neuronal excitability and synaptic plasticity, and might participate in higher brain functions like learning and memory. ... mathematical modeling will prove crucial for testing predictions on the possible functions of astrocytes in neuronal networks, and to generate novel ideas as to how astrocytes can contribute to the complexity of the brain. ...”

Gilson et al. 2015 [9]: “Experiments have revealed a plethora of synaptic and cellular plasticity mechanisms acting simultaneously in neural circuits. How such diverse forms of plasticity collectively give rise to neural computation remains poorly understood. ... To learn how neuronal circuits self-organize and how computation emerges in the brain it is therefore vital to focus on interacting forms of plasticity.”

Park and Friston 2013 [19]: “... the emergence of dynamic functional connectivity, from static structural connections, calls for formal (computational) approaches to neuronal information processing ...”

According to Bertolero, Yeo, and DEsposito (2015) [6], so called “connector hubs” are responsible for composition of modules (neuronal circuits) implementing cognitive functions.

Braun et al. (2015) [7]: “... dynamic network reconfiguration forms a fundamental neurophysiological mechanism for executive function.”

The research on computational models of neural circuits is well established starting with McCulloch-Pitts networks [15] via the Hopfield model (11 and [12]) to recurrent neural networks (RNNs). It seems that RNNs adequately represent the computations done in the human brain by the real neuron networks. From the Computer Science point of view, RNNs are Turing complete (Siegelmann and Sontag [21]), i.e., every computable function may be represented as a RNN. However, Turing machine is a flat model of computation. There are also higher order computations, i.e. computable functionals where arguments (input) as well as values (output) are functions. For a comprehensive review of higher order computations, see Longley and Norman 2015 [?].

The Virtual Brain (TVB [20], www.thevirtualbrain.org) project aims at building a large-scale simulation model of the human brain. It is supposed that brain function may emerge from the interaction of large numbers of neurons, so that, the research on
TVB may contribute essentially to our understanding of the spatiotemporal dynamics of the brain’s electrical activity. However, it is unclear how this activity may contribute to the comprehension of the principles of the human mind functioning.

Adolphs 2015 [1]: “Some argue that we can only understand the brain once we know how it could be built. Both evolution and development describe temporally sequenced processes whose final expression looks very complex indeed, but the underlying generative rules may be relatively simple … ”

Another interesting approach is due to Juergen Schmidhuber: “The human brain is a recurrent neural network (RNN): a network of neurons with feedback connections”; see http://people.idsia.ch/ juergen/rnn.html. Indeed, real neural circuits can be modeled as (continuous time) RNNs. Despite the enormous complexity of a hypothetical RNN modeling the human brain, there is a paradox here because (continuous time) RNNs are nonlinear dynamic systems. It means that RNNs are high level mathematical abstractions (of human mind) involving the notion of space-time Continuum that comprises actual infinity. These very abstractions are created in the human brain (consisting of a finite number of cells), i.e. the notions related to space-time continuum are represented (in the brain) in a finitary way as finite structures.

Some parts of the connectome may and should be considered as modules responsible for particular (elementary) cognitive functions of the brain. This very modularity reduces considerable the complexity. Once the modules are distinguished as functions with clearly defined input and output, it gives rise to compose them. The composition is, in turn, the basic mechanism for constructing higher order functionals. However, it seems that RNNs still lack the modularity and ability to compose the modules. Perhaps, if the notions of modularity and computable functionals were introduced to RNNs, they could model the higher order computations as dynamic formation and reconfigurations of the links (synapses) between the neurons.

Let us shortly review (in the form of citations) the current literature on the modularity in the human brain.

Bertolero et al. 2015 [6]: “The principle of modularity, in which a system or process is mostly decomposable into distinct units or modules, explains the architecture of many complex systems. Biological systems, including the human brain, are particularly well explained by the principle of modularity.”

Sporns et al. 2016 [22]: “Behavior and cognition are associated with neuronal activity in distributed networks of neuronal populations and brain regions. These brain networks are linked by anatomical connections and engage in complex patterns of neuronal communication and signaling.”

Gu et al. 2015 [10]: “Cognitive function is driven by dynamic interactions between large-scale neural circuits or networks, enabling behaviour. However, fundamental principles constraining these dynamic network processes have remained elusive.”

Braun et al. 2015 [7]: “The brain is an inherently dynamic system, and executive cognition requires dynamically reconfiguring, highly evolving networks of brain regions that interact in complex and transient communication patterns. However, a precise
characterization of these reconfiguration processes during cognitive function in humans remains elusive."

To summarize the review. The foundations of the mind functioning might be ingenious in its simplicity although the underlying biological mechanism are extremely complex and sophisticated. Hence, in order to model neural circuits and the mechanisms responsible for structural changes in the neuronal connectome, let us use much more simple (than RNN) primitive notions from Mathematics and Computer Science, i.e. the computable functions and computable functionals. Since Mathematics is a creation of the human mind, the Foundations of Mathematics may shed some light on the principles of the brain functioning. That is, the basic mathematical notions can be recognized as concrete mental structures, and then the corresponding mechanisms of the human brain can be discovered.

2 Neural circuits, computable functions and functionals

Before going into details, several assumptions are to be made. The first one is that elementary neural circuits (corresponding to functional units of the brain) can be distinguished. The second assumption is that any such circuits (at least temporary) has clearly identified input (dendrite spines of some postsynaptic neurons) and output (axons of some presynaptic neurons). It means that the output is exactly determined by the input. The third assumption is that such circuits can be composed by a linking the output of one circuit to the input of another circuit; it may be done by creating a synapse connecting an axon (of the output of one circuits) to a dendrite spine of the input of the other circuit. If the above assumptions can be verified experimentally, then the following considerations make sense. However, from the conceptual point of view, they may also be of some interest to Neurobiology.

If the above assumption are taken as granted, then a neural circuit can be represented as a first order function defined on natural numbers. That is, spike sequences (bursts), generated by a neuron, may be interpreted as natural numbers in the unary code, input of the circuit as arguments whereas output as values of the function. Note, that this is a static (one shot) representation of neural circuits. It means that one output is produced form one input.

However, if a circuit is to be considered in a time extent so that for consecutive inputs it produces a sequence of outputs, then dynamic behavior of the circuit may be represented either as a RNN or as a sequence of interrelated copies of the function representing the circuit. However, this is beyond of the scope of this study.

Simple operations on functions may have their counterparts as operations on circuits. Given two functions \( f \) and \( g \) (from natural numbers into natural numbers), the new function \( h \) defined as \( h(x, y) = f(x) + g(y) \) may serve as an example. If \( f^c \), \( g^c \) and \( +^c \) denote corresponding neural circuits, then the circuit corresponding to function \( h \) may be created by establishing (activating) some synapses between input neurons of \( +^c \), and the output neuron of \( f^c \) and the output neuron of \( g^c \). This may correspond
roughly to the synaptic meta-plasticity. It is interesting (however, not surprising) that this very synapse creation corresponds to a basic notion of Mathematics, i.e. function composition.

Figure 1: Function as input (socket,) body, and output (plug). Simple composition of $f$, $g$, and $+$. Function type as board of sockets and plug, and functions

**Sockets and plugs** are the crucial notions. A function consists of input, body and output, see Fig. 1. Input may consists of multiple sockets, whereas output may consists of multiple plugs. A plug-socket directed link may correspond to synapse as connection of axon and dendrite.

There are also higher order functions (called functionals) where arguments as well as values may be functions. It is also not surprising that these higher level functionals can be constructed by establishing links in the circuits of plugs and sockets.

Each function is of some type. Since the natural numbers (finite sequences (bursts) of spikes) are assumed as the basic type (denoted by $N$), the type of first order functions is of the form $(N^{s_1}; N^{s_2}; ...; N^{s_k}) \rightarrow (N^{p_1}; N^{p_2}; ...; N^{p_m})$, where $(N^{s_1}; N^{s_2}; ...; N^{s_k})$ denotes different sockets of the input, whereas $(N^{p_1}; N^{p_2}; ...; N^{p_m})$ denotes different plugs of the output. This type may be realized as a board consisting of sockets and plugs, see Fig. 1.

It seems that second (and higher) order computations in the brain are done by dynamic (re)configurations of links (synapses) between the neural circuits. Although the links are established between concrete neurons, these neurons belong to fixed circuits, so that (from functional point of view) the links are between circuits and correspond to the circuit composition.

Let us take as granted that glia are responsible for creating synapses and managing their activity. Then, there must be a generic meta-composition process for doing so (corresponding to a functional), where the parameters are: two circuits (to be composed), presynaptic neurons of one circuit, and postsynaptic neurons of the second one.

Hence, such generic process may be represented as a second order function (func-
ational) that takes (as input) two first order functions, a plug of one function and a socket of the second function; then it returns (as the output) a first order function as a composition of these two functions. The problem is how such generic process is realized in the brain. First of all, the circuits to be composed must be discriminated, and then passed, as parameters, to the composition process.

Glia are responsible for higher order computations, i.e. for dynamic creating, composing, and reconfiguring neural circuits. At the bottom level it is realized by creating new synapses; this corresponds to function composition. Since the function composition is the basis for construction of the higher order functions (functionals), the processes of dynamic synapse creation correspond to functionals.

**Hypothesis.** The primitive rules for construction of the computable functionals may have their counterparts in the human brain.

### 2.1 A sketch of formal framework for constructing higher order computation based on functionals

Turing machines and partial recursive functions are not concrete constructions. Their definitions involve actual infinity, i.e. infinite type for Turing machines, and minimization operator $\mu$ for partial recursive functions. This results in possibility of non-terminating computations that are abstract notions and have no grounding in the human brain. The proposed approach is fully constructive, and if restricted only to first order computable functions, it correspond to the general recursive function according to the Herbrand-Gödel definition.

At the basic level it consists of some primitive types, primitive functions and type constructors, i.e. the type of natural numbers, the successor function, constant functions, projections, constructors for product and function type. However, the key princi-
tive functionals correspond to application, composition, copy and iteration. It is crucial that these functionals can be constructed by (dynamic, in the case of iteration) establishing links between plugs (corresponding to output types) and sockets (corresponding to input types).

At the higher level of the approach, types are considered as objects, i.e. constructed as **boards of plugs and sockets**. This gives rise to introduce relations (according to the propositions-as-types correspondence of Curry-Howard), and polymorphism.

Hence, it is important to grasp the constructions of the boards as higher order types. The type of functions from natural numbers into natural numbers (denoted by \( N^s \rightarrow N^p \)) may be realized as a simple board consisting of a socket and a plug, see Fig. 1. Types of higher order are presented in Fig. 1 and Fig. 2. Note that for the type \((A \rightarrow B) \rightarrow C\), the input \(A \rightarrow B\) becomes the socket. For the type \((A \rightarrow B) \rightarrow (C \rightarrow D)\), the output \(C \rightarrow D\) becomes the plug.

![Figure 3: The functional \(\text{Comp}\) of type \(((A \rightarrow B) ; (B \rightarrow C)) \rightarrow (A \rightarrow C)\). Input objects are: \(f\) of type \(A \rightarrow B\), and \(g\) of type \(B \rightarrow C\). When applied to \(\text{Comp}\), the output object is a function of type \(A \rightarrow C\).](image-url)

**Application** of a functional \(F : (A \rightarrow B) \rightarrow C\) to a function \(g : A \rightarrow B\) is realized as follows. \(A \rightarrow B\) is the socket of the functional \(F\). The application is done (see Fig. 2) by establishing appropriate directed connections (links). That is, the link between the socket \(A\) of the socket of \(F\) and the socket \(A\) of \(g\), and the link between the plug \(B\) of \(g\) and the plug of the socket of \(F\).

**Composition functional** (denoted by \(\text{compose}_{A,B,C}\)) for simple composition of two functions (the first function \(f\) of type \(A \rightarrow B\), and the second one \(g\) of type \(B \rightarrow C\)) is realized as two boards with appropriate links shown in Fig. 3. It is easy to check (by following the links) that applying \(\text{compose}_{A,B,C}\) to two functions (see Fig.
Note that a higher order application (i.e. application of a functional to a function), and a functional for composition are constructed just by providing some links between sockets and plugs. Since link corresponds to synapse, it might be interesting whether these functionals have counterparts in the brain.

Each construction, like $F(g)$ and $\text{compose}_{A,B,C}(f; g)$, can be distinguished as an individual object (notion). Perhaps, in the brain, they correspond to concrete regions. This corresponds to a new paradigm called radical embodied neuroscience (REN), see Matyja and Dolega 2015 [14], Kiverstein and Miller 2015 [13].

Generally, discrimination of new notions by the human mind is crucial for reasoning. Once a notion is distinguished, it may be used in more sophisticated reasoning. This evidently corresponds to the reflective abstraction introduced by Piaget, especially if the notions emerge as the results of constructions. Note that here constructions mean dynamic (re)configuration of links between sockets and plugs.

A functional of special interest is $\text{Copy}$. Once an object $a$ is constructed, repeat the construction once again. So that $\text{Copy}(a)$ returns two object: the original $a$, and its copy $a'$. Although the meaning of $\text{Copy}$ seems to be simple, its realization in the brain may be quite complex especially if the object $a$ is of a higher order type.

If it is supposed that the construction of object $a$ occupies some well defined region in the brain, then $\text{Copy}$ may be realized by copying this region into a new “free region”. Since in Biology (living organisms) copying (procreation) is ubiquitous, let us take the implementation of the functional $\text{Copy}$ as granted.

**Iteration as generalization of composition.** That is, compose $n$-times a function $f: A \to A$ with itself. Note that $n$ as a natural number is a parameter. The iteration is denoted by $\text{Iter}_A$ and it is a functional of type $(N; (A \to A)) \to (A \to A)$. So that $\text{Iter}_A(n; f)$ is the function being $n$-time composition of $f$. The realization of $\text{Iter}_A$ requires $\text{Copy}$ for making copies of $f$, and $(n - 1)$ copies of the composition functional, see Fig. 4, where the construction is done for $n$ equal 4. Since natural numbers are involved in the functional, it seems that a hypothetical realization of $\text{Iter}$, in the brain, requires neurons.

Functional $\text{Iter}$ is not the same as feedback loop that occurs when outputs of a circuit are routed back as inputs to the same circuit. The feedback enforces dynamics of the circuits, whereas $\text{Iter}$ is static one shot operation.

Feedback loop can not be realized for higher order functionals where input as well as output are not electrical signals but higher order constructions.

Neural circuits are real dynamic systems where computation is done by consecutive processing signals (spike bursts). The circuits may be represented statically (without dynamics) as first order functions. Functionals are also static constructions operating on first order functions (circuits) by re(configuring) links inside and between the circuits.

Higher order primitive recursion schema (also known as Grzegorczyk’s iterator) can be constructed as a functional. For arbitrary type $A$, the iterator, denoted by $R^A$, of
Figure 4: The result of application of the functional $Iter_A$ to natural number 4 and function $f$ of type $A \to A$

type $A \to ((N \to (A \to A)) \to (N \to A))$, is defined by the following equations.

for any $a: A, c: N \to (A \to A)$, and $k: N$

\[
(R^A(a))(c)(1) = a \quad \text{and} \quad (R^A(a))(c)(k + 1) = (c(k))((R^A(a))(c)(k))
\]

However, a construction of $R^A$ does not follow from the definition. Actually, it is based on the iteration functional and consists on dynamic formation of links in boards of plugs and sockets. Higher order primitive recursion allows to define a large subclass of general recursive functions, e.g. the famous Ackerman function. This can be done on the basic level of the proposed approach to computable functionals. At higher levels of the approach (where functionals are used) all general recursive functions can be constructed. It seems that higher order computation involving the functionals is useful, especially as efficient and smart organizations of complex and sophisticated first order computations.

Note that there are next orders of constructions of functionals. Functionals operate
on functionals (second order functions) are third order functions that operate on the second order functions by re(configuring) links in the boards of sockets and plugs. By analogy, this may be continued for the next higher orders of constructions.

3 Continuum

It seems that the notion of continuum has a straightforward and natural grounding in the human brain.

Vision sensory nervous system. The retina consists of about 130 million photoreceptor cells, however, there are roughly 1.2 million axons of ganglion cells that transmit information from the retina to the brain. It is interesting that a significant amount of visual preprocessing is done between neurons in the retina. The axons form the optic nerve consisting of fibers (axons). Positions of the fibers in the nerve reflect the spatial and adjacent relations between the corresponding photo-receptors in the retina. In computations, the bundle of spikes in the nerve is considered together with the adjacent relation between the spikes. It is crucial for comprehending the notion of space Continuum.

The somatosensory system. Contrary to the vision system, it is spread through all major parts of a mammal’s body. Spacial and adjacent relations between nerve fibers of the somatosensory system contribute essentially to the notion of space Continuum.

The streams of spikes, in the nerve fibers delivered from sensory receptors to the brain, are not independent from each other; they are structured by causal and adjacent relations. The streams along with the relations are the grounding for the notion of space-time Continuum.

It is interesting that objects of the primitive types are based on neuron spikes. Natural number is just an isolated independent spike burst, whereas object of the type Continuum is a bundle of adjacent spike bursts.

4 Conclusion

Primitive types resulted from the most simple (primitive) and obvious data transfer methods: spike bursts, and bundles of adjacent spike bursts.

Composition (as link creation) is the basic operation for function constructions as well as for construction of higher order functions (functionals). This very composition corresponds to synapse creation in the brain.

The two functionals (Copy and Iter) together with the higher order application, composition, and the primitive types constitute the cornerstone for building a constructive (intuitionistic) part of Arithmetics and Analysis, see [3] and [2]. According to the original meaning of L. E. J. Brouwer, intuitionism is the constructive mental activity of the human mind.
It seems that there are two essential primitive types; the type of natural numbers, and the type of Continuum. Both types have their counterparts in the human brain. The natural numbers may be identified with individual and independent bursts of neuron spikes. The type Continuum has also the straightforward interpretation in the human brain. Sensory nervous systems support this view. The static interpretation of the neural circuits, as first order computable functions, seems to be justified. This may give rise to expect that higher order computable functions (functionals) have counterparts in the human brain.

Let us state the following hypothesis: *Glia seems to be the appropriate place where the mechanisms corresponding to the constructions of computable functionals may be realized.*

Experimental evidences confirming the hypothesis would open a wide area of research in Foundation of Mathematics and Neurobiology. Note that there are already some experimental evidences supporting the hypothesis, i.e. [6], [7], [19].

Since the architecture of human brain is definitely different than von Neumann computer architecture (see von Neumann 1958 [17] and 1966 [18]), the mechanisms of the meta-plasticity may give rise to develop a non-von Neumann computer architecture and a corresponding function-level programming language postulated by John Backus 1977 [5]; for more on this subject see [4].

For a mathematical approach to computable and constructive functionals see Longley and Norman 2016 [7], and (google arXiv Ambroszkiewicz) [3],[2].

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