Supporting mathematics learning in situational-referential phase with emergent modeling

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Abstract. This study aims to describe learning activities that can stimulate student activity in learning in this case their activities in submitting arguments related to mathematics in two important phases, namely the situational and referential phases. Improvement at this stage will greatly help students progress in the next stage, namely the general stage and formal mathematics. If each activity in this phase can be categorized as good, then students will not experience problems in building a complete understanding of mathematics. The study specifically answer the question "How can the student learn to build understanding about the problem in situational and referential phase?" The description on how emergent models are used in stimulating the progress of the movement from model of to model for is elaborated. The specific model used was the percentage bar as a tool to understand as well as to solve the design activities. The method of this study is design research, started with preparation phase in formulating and revising the Hypothetical Learning Trajectory, preliminary teaching, teaching experiment, and retrospective analysis, the study showed that the lesson using the percentage as a model can support students’ understanding about the situation of the problem, this finding need to be proved for the next two stages, general and formal phase.

1. Introduction

From students perspective, the goal of learning instruction which involve argumentation in a mathematics classroom is describing what they know about a mathematical topic and challenging the peers to have particular discussion. From teachers perspective, the use of argumentation in mathematics classroom help them in collecting information about students learning progress. The information is very useful to evaluate students individually and to design a correct and proper instructional activities. A good context will also help students to understand the mathematical concept better. Another benefit of using context that student can relate their prior knowledge and experience relating to the context that later in the next stage of learning they know how condition and situation in which the knowledge is applicable [1, 2].

The study highlight the core activities done by the students in important phases, situational phase and referential phase. The situational phase is the level in which the student interpret the problem in their own world, that is why, in this phase, starting point of the student will influence the performance for the next three phases [1, 3]. The next is the referential phase is the level in which the students use their own model to visualize the problem, in this study, the role of percentage bar is described. This two phases is essential for students to understand what the known and unknown elements of the problem. Without sufficient knowledge to perform better in this two phases, the student might have difficulty in continuing to the next to phase, general phase and formal phase. This study is aimed at describing the instructional activities which can stimulate students to reason and understand the core of a mathematical problem and give argument related to the problem.
The students are expected to explore and investigate each part of the problem thoroughly because the good skill in exploration can lead to better performance in finishing the problem in the formal level [4-6]. This study focus on elaborating how the emergent modeling is being applied in a specific context of mathematical problem to stimulate the shifting from model of to model for. The word ‘model’ use in many contexts, in mathematics learning contexts, model is the tool for the learner in solving the problem that bridge the knowledge about the situation of the problem and the principles in finishing the mathematical tasks [7-9].

The model plays important roles, the first role is for showing the mathematical process, for instance in determining the result of an equation, the second role is for mathematical manipulation, the example of using the manipulatives is the use of blocks, chains, or any other object for demonstrating the process of addition and subtraction, also the third role is for explaining the techniques used in mathematics such as the repeated addition for multiplication [10-12]. The type of the learning chosen by the teachers can optimize the roles of model in solving mathematical problems. This study emphasize the roles of the model in some specific tasks given to students. However, the highlight is describing the model to understand the problem in situational level and general level. In this study, the topic is about the use of percentage in daily life and how the percentage bar was used as a model to show the mathematical process. The situational phase and the referential phase in the lesson is shown in the excerpt of students’ discussion with the researcher.

2. Method
The method used in this study is design research to develop instruction theory in using the model as a tool to understand the problem situationally. Design research combines theoretical thoughts of the researchers and empiric-based analysis to build the instruction theory. Three phases of design research are preparation phase, teaching experiment, and retrospective analysis. Research subjects are students in grade 7 of Junior High School, the students were selected based on their abilities in mathematics generally. Preparation phase is conducted to obtain students prior knowledge and their response toward the learning, it will be useful to support the socio-mathematical norms during the learning. Also, in this phase, the Hypothetical Learning Trajectory is formulated and refined based on the results of the preliminary stage of the teaching experiment.

During the teaching experiment, the data is also used to describe the socio-mathematical norms occur in the classroom while the students learn. The method of data collection are written test followed by interview to clarify the pupils answer, classroom observation, and then pupils individual work. The activities designed for the study had been processed through face validity by the expert.

3. Result and Discussion
The focus of this lesson was that to introduce the percentage bar as a model of mathematical tool to understand and to solve the problem. The researcher also acted as the teacher. By considering the result of the interview, the researcher designed a teaching learning on percentage focusing in encouraging the use of percentage bar. It is required to select the problem which can be solved by both of them. The problems chosen were the simple context with the number which is easy to calculate. We want to focus on how they can use the percentage bar to solve some contextual problem. The first problem is the problem about uploading picture in which the students must determine the percentage of the file which has been uploaded. The second problem is about the discount problem. The first lesson was designed with the aim that students with different ability can understand the use of percentage bar.

The first lesson emphasize on the use of percentage bar to solve percentage problem. The context be used in the problem was uploading context. The reason of choosing this context was that because the context enabled the students to model the problem easily and because the representation of uploading context more or less the same as the percentage bar. The tasks were designed to guide the students in understanding the percentage problem. The focus of the discussion is the Student 1 who was a lower achiever and Student 1 who performed better than the other, the researcher also tried not to include complicated number and choose the number which can be easily calculated by the students. The way of the two students understand the problem in situational level was emphasized since this level is the core of solving the mathematical problem. The data that had been analyzed were the result of
written test, performance of each student during the interview and the lesson together. But in this description, the researcher will only focus on two students.

3.1. The hypothetical learning trajectory

Hypothetical Learning Trajectory is sort of prediction on what will students during the learning process. HLT is designed after the learning components such as model of didactical analysis and conceptual analysis has been elaborated. Gravemeijer stated that HLT is the ways of teacher to envision the activities of students in the classroom and anticipate how students’ argument might lead to expected explanation and justification from the teacher [13].

Three components of HLT are the learning goal to keep the activity focus and specific on the particular topic. Second component is the learning activities which is organized and structured properly considering students ability, the level of importance of the topic, and time allocation. Third, the hypothetical learning process which is a prediction of how the students thinking and understanding in learning activities. The most important which must be underlined is the part of making HLT as an prediction on what students do with this sort of problem, how students respond certain instruction, what kind of arguments they will deliver and how the discussion among them will reflect their understanding.

HLT include learning goals, learning activities, and assumption about how students thinking and understand will evolve [13,14] HLT must describe clearly the envision of what you hope the students will do and what you do not expect from them to do. HLT must contain a learning trajectory, a route through concepts and context, an outline of the intended learning process, a learning line, a road map through the field of didactical obstacles.

3.1.1. HLT for Lesson 1: Discount Problem

The lesson ask the student “How much does this treatment normally cost at Beauty Salon?”. The teacher ask the students what they know about the problem. Some of the possible responses are: (1) the students may react that 50% off means that the price becomes half of the normal price; (2) students may calculate the price by doubling the price with reduction. $25 + $25 = $50;(3) students make a percent bar relating to the problem; (4) students understand that if 50% of something is $25, 100% should be double of 25% becomes $50.

3.1.2. HLT for Lesson 2: Uploading Problem

The mathematical goal of this lesson is that students can use the percentage bar as mathematical tool to understand the problem. The teacher briefly introduce the context of uploading the files and ask...
whether students familiar with the situation, the teacher explain about the percentage bar. The Figure 2 ask what is the percentage of the file being uploaded. Some possible responses of the students are: (1) the uploading process is not completed yet; (2) it only uploaded 50 MB; or maybe they connected it with fraction like less than a half of the file have been uploaded, (3) there is still 200 MB again to be uploaded; or maybe they answer more than half of the files still need to be uploaded, the teacher may have some follow up questions towards this.

The teacher ask the students different problem, "How many percent if the file uploaded is 120 MB? If the remaining file is 30 MB, what is the percentage of it?" and The Figure below show some possible response of the students using the percentage bar.

![Figure 3. The percentage bar: Possible response of student.](image1)

![Figure 4. The percentage bar: Possible response of student.](image2)

In the interview about discount and price reduction the student 1 was asked to clarify her answer and she said that the number is not the percentage, but she was not able to explain why, she was again asked what she knew about price reduction.

**Teacher (T)** : When you visit a stationary store for example, the price of this pencil is labelled with 10%. What does it mean?

**Students 1 (S1)** : It means that 10% of

**S1** : Yeah, if the price of the pencil is...Yeah, the price is bigger like the price is more but you take 10% from it.

**T** : And how about 15% reduction from 700 euros?

**S1** : Hmm. I don’t know how to take away 15 from those two zeros. Because I learn it in different way if it is a big number.

From this conversation, Student 1 knew that when you have 10% reduction for example, you must take away 10% from the price of the item. However, still, the problem was probably too difficult for her because she seemed need support for basic mathematics like subtracting bigger number. This can be seen from this conversation when she was asked to solve the problem number 9, to estimate the price of bicycle, which was 700 euro with 15% price reduction. Her intention was that she wanted to subtract the reduction 15% directly from the original price. However, by thinking that way she got more confusion because she had difficulty in subtracting those numbers.

![Figure 5. The percentage bar as a model used by student 1.](image3)
Figure 5 shows that student 1 tried to use percentage bar. At first, she understood the problem because she knew the answer but when being asked to clarify her answer by using percentage bar did not help her much in solving percentage problem because she still confused how to relate the percent with the numbers. However, she managed to use percentage bar using friendly percentages. Obviously, she needs more exercise in using percentage bar because for her the lesson about the use of percentage bar is not enough only in one meeting.

When the teacher introduced the percentage bar by drawing a similar uploading bar as written on the problem, the students did not directly understand the use of the percentage bar. What they knew was that they were able to put the familiar percentage such as 100%, 75%, 50%, and 25%. Student 1, in this case, already knew how to put this percentage on the percentage bar. She knew it by using half and quarter of the whole. Student 1 was distracted by the picture of the percentage drawn by the teacher on the blackboard.

The problem was what is the percentage of 120MB of files being uploaded if there is 200MB files have to be uploaded? The difficulty the students had in solving this problem was that they did not know how to calculate the number. Student 1 knew how to put 120MB on the number line but she did not the relationship between the number above the percentage bar and the number below the percentage bar.

Student 1 actually knew that the answer should be 60 because she estimate the number by looking at the picture. What she knew that she could make imaginary line in her head/mind to move 10% next 60% which was in her opinion corresponded to 120MB. However, of course this kind of reasoning could not convince the researchers and student 2. Meanwhile student 2 thought differently. What we already had was 100%, 75%, 50%, and 25% above the percentage bar. And 200MB, 150MB, 100MB, and 50MB correspond to the percentage respectively.

Both of the students did not have any idea how to find the percentage of 120MB. The teacher guide the students by giving the clue to find 10% and figure out how many megabytes is 10%. student 1 knew that 10% of 200MB is 20MB by dividing 200MB by 10. However, they could not see directly that the clue could help them to calculate the percentage of 120MB. Student 2 tried to work from 10%, 20%, 30%, and so on. Afterwards, she found the numbers of megabytes corresponded to those percentages. She kept doing so until she found the correct percentage of 120MB.

What the teacher wanted was that the students could think to calculate 120MB by using the existing number, for example adding 100MB and 20MB which were on the percentage bar, and adding the percentages. The teacher showed the students how to do that calculation in order to make clear the use of percentage bar with similar problems. Student 1 agreed on what the teacher did; meanwhile it is difficult for student 2 to accept and understand the calculation done by the teacher. However, when student 1 was asked to explain what her opinion about those calculations to student 2, she could not figure it out. Both student 1 and student 2 did not understand what teacher did. Therefore, student 2 kept doing her own strategy by using the tenth numbers. Student 1 still struggled to understand the bar and the calculation.

4. Conclusion
In conclusion, the study showed that the lesson using the percentage as a model can support students’ understanding about the situation of the problem depending on the starting point of the students, this finding need to be confirmed for the next two stages, general and formal phase.

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6. References

[1] Gravemeijer K, Lehrer R and Van O B 2002 Symbolizing Modeling and Tool Use in Mathematics Education (Dordrecht: Kluwer Academic Publisher)

[2] Doorman L M and Gravemeijer K P E 2009 ZDM 41 199

[3] Rosales A C 2015 Mathematizing: An Emergent Math Curriculum Approach for Young Children St (Paul: Redleaf Press)

[4] Deniz O and Uygur-Kabael T 2017 Hacettepe University Journal of Education 32 123

[5] English L D and Halford G S 2012 Mathematics Education: Models and Processes (London: Routledge)

[6] Gravemeijer K 1999 Mathematical Thinking and Learning 1 155

[7] English L D and Kirshner D 2015 Handbook of International Research in Mathematics Education (New York: Routledge)

[8] Zandieh M and Rasmussen C 2010 The Journal of Mathematical Behavior 29 57

[9] Perminov E, Anakhov S V, Grishin A S and Savitskiy E S 2016 Int. Jour. Environ. Sci. Ed. 11 9339

[10] Cobb P and Bauersfeld H 1995 The Emergence of Mathematical Meaning: Interaction in Classroom Cultures (London: Psychology Press)

[11] MLA 2014 Principles To Actions: Ensuring Mathematical Success for All (Reston VA: NCTM)

[12] Gravemeijer K and Stephan Michelle 2002 Emergent Models as an Instructional Design Heuristic (Dordrecht: Springer)

[13] Bakker A and Van Eerde D 2014 An Introduction to Design-Based Research with an Example From Statistics Education (Springer: New York)

[14] Pratiwi W D et al 2019 J. Phys.: Conf. Ser. 1166 012031