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LETTER

Single-particle versus many-body phase coherence in an interacting Fermi gas

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Abstract

In quantum mechanics, each particle is described by a complex valued wave-function characterized by amplitude and phase. When many particles interact each other, cooperative phenomena give rise to a quantum many-body state with a specific quantum coherence. What is the interplay between single-particle’s phase coherence and many-body quantum coherence? Over the years, such question has been object of profound analysis in quantum physics. Here, we demonstrate how the time-dependent interference formed by releasing an interacting degenerate Fermi gas from a specific matter-wave circuit in an effective magnetic field can tell apart the two notions. Single-particle phase coherence, indicated by the first-order correlator, and many-body quantum coherence, indicated by the density–density correlator, are displayed as distinct features of the interferogram. Single particle phase coherence produces spiral interference of the Fermi orbitals at intermediate times. Many-body quantum coherence emerges as long times interference. The interplay between single-particle coherence and many-body coherence is reflected in a stepwise dependence of the interference pattern on the effective magnetic field.

Phase coherence is the ability of the quantum wave-function to retain its phase information. For free particles, such notion is operatively inferred, for example, through the interference pattern in two slits experiments. When it comes to be referred to interacting quantum many particles systems, though, the notion of phase coherence is more involved. While all quantum particles can generally cooperate to establish a macroscopic coherence [1, 2], bosons and fermions display two different behaviors. Bosonic systems can form a Bose–Einstein condensate and be described by a macroscopic wave-function \( \Psi \sim e^{i\phi} \), with a phase \( \phi \) coinciding with the single-particle phase. With the spectacular advances on atom trapping and cooling such phase coherence has been studied with unprecedented degree of control and precision of physical conditions. Although cold atoms systems are made of large but finite number of particles (\( \sim 10^5 \)), Bose–Einstein condensate (BEC) provide a meaningful interference pattern as a result of self-averaging [3, 4]. Fundamental limits on fringes visibility are provided by phase diffusion [5, 6]. Bose Josephson effect is a direct test for BEC coherence [7–10]. The phase portrait of ultracold bosonic systems has been analyzed recently through a series of interferometric experiments (heterodyne phase detection protocol) [11–13], within the emerging field of atomtronics [14, 15].

For fermionic systems, the Pauli principle prevents the occupancy of a single quantum level by particles with the same spin, and therefore the many-body coherence is achieved from the coherence of each single particle through more complicated mechanisms [16]. The persistent current [17–19], a frictionless flow occurring e.g. in metallic or superconducting small rings, provides a characteristic trait of quantum coherence in fermionic systems. The nature of the many-body phase coherence in degenerate fermions, though, depends on particles interaction. Ultracold atoms experiments provide an ideal platform to explore...
Expansion protocol at different times. After the release of the trap, the particles in the center and on the ring interfere. We shall see the single-particle coherence to emerge at intermediate times, while in the long-time limit this protocol provides information about the many-body quantum coherence.

these effects, with interactions that can be adjusted from repulsive to attractive cases [20]. For repulsive interactions, the effects of the ferromagnetic correlations have been observed [21, 22]. For attractive interactions, bound states of Fermi pairs can condense, experiencing the Bardeen–Cooper–Schrieffer (BCS)-BEC crossover [23]. Relevant information on the system coherence in the crossover can be extracted through the study of the Josephson effect [24–26].

While the features mentioned above do provide specific aspects for single particle and many-body quantum coherence, their interplay in interacting many-body systems remains unclear [16, 27]. In this paper, we operatively track the aforementioned interplay through a single protocol (see figure 1), the expansion dynamics of an interacting Fermi gas, that is well within the current state of the art of the cold atoms research field [28, 29]. To this end, we study a degenerate interacting Fermi gas confined in a ring-shaped potential and pierced by an effective magnetic flux. Because of the effective magnetic flux, a current is imprinted on the degenerate gas. In analogy with bosonic protocols [4, 11–13, 30], such system is let to interfere with a second degenerate gas placed at the center of the ring. In the following, we consider the gas in the center to be composed by two fermions with opposite spins. We stress that the density of the gas in the center does not affect qualitatively our results. Next, we study the entire time evolution of such expansion. Despite the similarity in the schemes, we shall see that the fermionic interferograms are markedly different from the bosonic ones. We show that the particles phase coherence emerges in the intermediate times interference images, displaying characteristic dislocations (see dashed lines in figure 2(d)) due to Fermi sphere effects. On the other hand, the many-body coherence, exemplified by pairing correlations and off-diagonal-long-range order (ODLRO), i.e. long-ranged spatial coherence [2], emerge at long times in the density–density correlators. The response of the many-body coherence to magnetic field arises as a step-wise dependence of the density–density correlators on the magnetic field.

1. Model

We consider a gas of $N$ degenerate fermions confined in a ring lattice of radius $R$ comprised of $N_s$ sites and pierced by an effective magnetic flux $\Omega$ induced by an artificial gauge field. Such effective magnetic flux can be applied in several ways, for example by stirring the gas, by phase imprinting or by two-photon Raman transitions [31]. The system is described by the Hubbard Hamiltonian in a one-dimensional (1d) ring-shaped spatial geometry

$$
\hat{H}_{\text{BH}} = -J\sum_{j=1}^{N_s} \sum_{\sigma=\uparrow,\downarrow} \left( \hat{c}_{j,\sigma}^\dagger \hat{c}_{j+1,\sigma} + H.c. \right) + U \sum_{j=1}^{N_s} \hat{n}_{j,\uparrow} \hat{n}_{j,\downarrow}
$$

where $U$ is the particles interaction and $J$ is the tunnel amplitude. The applied gauge field is taken into account through the Peierls phase factors $e^{i\Omega}$, in which $\Omega = 2\pi n \Omega = \frac{2\pi}{N_s} \phi_0$, with $\Omega_0 = \frac{\hbar}{2m}$ being the typical frequency of the ring and $m$ the particle mass. In analogy with electronic systems, we also introduced $\phi = \pi R^2 \Omega$ as the artificial gauge flux and $\phi_0 = \frac{\hbar}{2m}$ as the flux quantum. We remark that fermions in continuous rings with delta-interaction can be described with Hubbard rings lattices in the small fillings $N/N_s$ regimes (see supplementary material).

For $U > 0$, the ground-state many-body wave-function is made of extended states characterized by real wave-momenta [32]. For $U < 0$, the ground state is characterized by bound pairs, with complex wave-momenta. For weak attractions, the ground state of the system is a BCS-like state with the wave-function of the pair decaying on distance larger than the mean interparticle separation [33–35].
Intermediate times density–density correlations. The Bose (left column) and the Fermi (right column) case are compared. In the upper panels, the overall correlation $G(r, r'; t)$ for $\omega t = 3$ is presented. In (c) and (d), the interference $G^{(C,R)}(r, r'; t)$ between the ring and the central site is displayed (see also text). Calculation performed by DMRG on a system of $N = 14$ particles on $N_c = 20$ sites, with interaction $U/J = 0.2$ and $\Omega = 1.4$. In every plot, $r' = (R, 0)$.

For stronger attractions, the bound states are formed by tightly bound particle pairs. The persistent currents in Hubbard models was studied for repulsive interactions in [36] and for attractive interactions in [37].

In this work, we study the co-expansion of the initially ring-trapped Fermi gas and the one located at the ring's center, i.e. two fermionic atoms with opposite spin, after the two confining potentials are suddenly switched off simultaneously [30, 38]. We integrate the density profiles along the $z$ axis and focus on the dependence on $r = (x, y)$ in the ring plane. The field operator of the whole system (ring plus center) is

$$\hat{\Psi}_\alpha(r, t) = w_C(r, t)\hat{c}_{C,\alpha} + \sum_{j=1}^{N_c} w_j(r, t)\hat{c}_{j,\alpha},$$

where $\alpha = \uparrow, \downarrow$, $C$ indicates the central site and

$$w_\Lambda(r, t) = \frac{\exp\left\{-(r - r_\Lambda)^2/(2\sigma^2(1 + i\omega_0 t))\right\}}{\sqrt{\pi}\sigma(1 + i\omega_0 t)}$$

is the time-dependent Wannier function centered at position $r_\Lambda$ in the Gaussian approximation [39], $\omega_0$ being the frequency of each lattice well in the harmonic approximation. This allows for an explicit solution for the dynamics of the field operator following a sudden turn-off of the lattice [40, 41]. In the experiment with weakly interacting bosonic condensates, a spiral interferogram emerges in a single co-expansion. However, in our theoretical approach, the particle density $\langle n(r, t) \rangle = \sum_{\alpha=\uparrow,\downarrow} \langle n_{\alpha}(r, t) \rangle = \langle \hat{\Psi}_\uparrow^\dagger(r, t)\hat{\Psi}_\alpha(r, t) \rangle$ is reconstructed as an expectation value, corresponding to an average over different realizations of the co-expansion protocol. Since each co-expansion is characterized by a well-defined, yet randomly distributed relative phase between the particles released from the ring and from the central site, the interference pattern is washed out in $n(r, t)$. We shall see that non trivial phase information on the system is captured by the density–density correlator:

$$G(r, r'; t) = \sum_{\alpha,\beta=\uparrow,\downarrow} G_{\alpha,\beta}(r, r'; t) \text{ where}$$

$$G_{\alpha,\beta}(r, r'; t) = \langle \hat{\Psi}_{\alpha}^\dagger(r, t)\hat{\Psi}_{\alpha}(r, t)\hat{\Psi}_{\beta}^\dagger(r', t)\hat{\Psi}_{\beta}(r', t) \rangle.$$

For our combined center-ring system, such correlation can be broken down as $G(r, r', t) = G^{(C,C)} + G^{(R,R)} + G^{(C,R)}$, where $C$ refers to central site and $R$ to the ring. Since the ring lattice and the central site are disconnected, we have $|\psi(t = 0)\rangle = |\psi(t = 0)\rangle_C \otimes |\psi(t = 0)\rangle_R$. Assuming a free
expansion for \( t \geq 0 \), the ring-center correlations read \( G^{(C_R)}(r, r', t) = \langle \hat{n}^{(R)}(r, t) \rangle \langle \hat{n}^{(C)}(r', t) \rangle + \langle \hat{n}^{(R)}(r', t) \rangle \langle \hat{n}^{(C)}(r, t) \rangle + \tilde{G}^{(C,R)}(r, r', t) \), where

\[
\tilde{G}^{(C,R)}(r, r', t) = \sum_{\alpha} \sum_{ij} I_{ij}(r, r', t) \left( \delta_{ij} - \langle \hat{c}^{\dagger}_{j,\alpha} \hat{c}_{i,\alpha} \rangle \right)
\]

with \( I_{ij}(r, r', t) = \omega^{\alpha}_{c}(r, t) \omega^{\alpha}_{c}(r', t) \omega^{\alpha}_{c}(r', t) \omega^{\alpha}_{c}(r, t) \). From equation (6) we see that the interference between the center and the ring only affects the particles belonging to the same spin species and depends on the first-order correlator \( \langle \hat{c}^{\dagger}_{j,\alpha} \hat{c}_{i,\alpha} \rangle \). We note that the \( G^{(C,R)} \) correlator can be accessed by measuring the full correlator \( G(r, r', t) \), and by subtracting the ring–ring \( G^{(R,R)} \) and center–center contributions \( G^{(C,C)} \), as well as the densities \( \langle \hat{n}^{(R)}(r, t) \rangle \), \( \langle \hat{n}^{(C)}(r, t) \rangle \), measured independently by eliminating either the ring lattice or the central site.

In our approach, we monitor the complete correlator \( G(r, r', t) \), the interference term \( \tilde{G}^{(C,R)} \) equation (6) and the spin resolved correlator \( G_{\uparrow, \downarrow} \) \[42\]. We shall see that at intermediate times \( \tilde{G}^{(C,R)} \) contains direct information on the single-particle phase coherence. Instead, at long times, \( G_{\uparrow, \downarrow}(r, r', t) \) probes the many-body phase coherence in momentum space with \( k = mt/\hbar \), \( k' = mr'/\hbar \) with \( t_{\exp} \) being the expansion time. In the present work we employ DMRG simulations to obtain the correlator matrix. For a lattice with \( N \) sites, this accounts to calculate \( N^2 \) terms.

2. Bosons versus fermions

For bosons, the spiral interference pattern arises because of the simple coupling between the effective gauge field and the phase of the Bose condensate. The quantized circulation reflects the effective magnetic flux quantization. The complete phase structure of the bosonic field emerges as a characteristic spiral interferogram in the expanding density \[11–13\] as well as in the interference term \( G(r, r'; t) \) \[30\] (see figure 2(c)).

For fermionic systems, the relation between the imparted phase and the density–density correlator is more involved. The difference traces back to the symmetry properties of the many-body wave-functions of the two systems resulting in a different momentum distribution. Bosonic wave-functions yield a momentum distribution peaked at \( k = 0 \) \[43–45\], while Fermi systems are characterized by a broader momentum distribution. Then, when fermionic particles are put in motion by an effective magnetic flux, each momentum component of the distribution is characterized by a different phase factor. As a consequence, phases recombination occurs, and the time-of-flight image of the density results to be suppressed at \( |k| = 0 \) only after half of the Fermi sphere is displaced by the effective magnetic flux (see supplementary material). In \( G(r, r', t) \), at intermediate expansion times, specific dislocations are found in the interference pattern (see figure 2(d)), deforming the smooth spiral-like picture we observe in the bosonic case (figure 2(c)). This is again due to the distinct particle orbitals characterizing the fermionic state. For \( U = 0 \), such orbitals are strictly single-particle, and each of them yields a spiral-like interference (see supplementary material). The dislocations, just \( N_\uparrow - 1 \) (or equivalently \( N_\downarrow - 1 \)) in number, are due to the interference of the \( N_\uparrow (N_\downarrow) \) independent orbitals. Remarkably, the dislocations are clearly visible at small and moderate interactions. By increasing interactions we find that dislocations disappear (see supplementary material), revealing that the system cannot be described in terms of independent quasi-particles.

3. Repulsive vs. attractive interactions

In the following, we demonstrate that the long time expansion of our setup allows us to access to many-body coherence through the connected spin–resolved correlator \( G_{\uparrow, \downarrow}(r, r'; t_{\exp}) - \langle \hat{n}_\uparrow(r, t_{\exp}) \rangle \langle \hat{n}_\downarrow(r', t_{\exp}) \rangle \). Notably, we consider the connected part to eliminate the background from the total correlations. We shall see that the different nature of the many-body state for repulsive and attractive interactions is clearly reflected in our pictures (see figure 3). We note that, within our model, the presence of a reference site does not affect the correlator \( G_{\uparrow, \downarrow} \), since the coupling between the ring and the center is nonzero only for particles with the same spin.

For \( U > 0 \) the ground state of the system is made of itinerant correlated particles. Therefore, the interferograms reflect the phase pattern imparted by the effective magnetic flux putting the system in a coherent motion. By the analysis of the long-time behavior of \( G_{\uparrow, \downarrow}(r, r'; t) \), at increasing \( U \) we find that, for fixed \( y \) and \( y' \), the correlation has a clear symmetry with respect to \( x = x' \) (corresponding in momentum space to \( k = k' \)), reflecting the tendency of the system to approach magnetic order at large \( U/J \) \[46, 47\]. The correlations are found to be displaced by a discrete amount as a function of the applied effective magnetic
Long times connected density–density correlator. The correlation for $r = (x, 0)$, $r' = (x', 0)$ and $t_{exp} = 100 \omega^{-1}$ is evaluated. For weak attractive interaction we observe BCS-like correlations at $x = -x' \simeq h\omega_{e}/J$. At strong attractive interactions tightly bound pairs are revealed by the enhancement of the correlations along the whole $x = -x'$ diagonal. In the bottom left panel, the square indicates the size of the Fermi sphere at $x, x' = \pm h\omega_{e}/J$. The calculations are performed using the DMRG method with $N = 14, N_\uparrow = 20$ and $\tilde{\Omega} = 1.4$.

![Figure 3](image1.png)

**Figure 3.** Long times connected density–density correlator. The correlation for $r = (x, 0)$, $r' = (x', 0)$ and $t_{exp} = 100 \omega^{-1}$ is evaluated. For weak attractive interaction we observe BCS-like correlations at $x = -x' \simeq h\omega_{e}/J$. At strong attractive interactions tightly bound pairs are revealed by the enhancement of the correlations along the whole $x = -x'$ diagonal. In the bottom left panel, the square indicates the size of the Fermi sphere at $x, x' = \pm h\omega_{e}/J$. The calculations are performed using the DMRG method with $N = 14, N_\uparrow = 20$ and $\tilde{\Omega} = 1.4$.

![Figure 4](image2.png)

**Figure 4.** Left panel: displacement $\tilde{\ell}/R$ of the correlation peak of figure 3. As a function of the flux for $N = 6, U/J = -5.4$ and $U/J = 0.5$. Right panel: visibility of the correlation peak $V(N, U)$, as a function of interactions for $\tilde{\Omega} = 0$ and various values of number of particles. We observe a markedly different $N$ scaling for repulsive or attractive interactions. In both panels $N_\uparrow = 10$.

flux, reflecting the quantized particle circulation. This can be observed by looking at the displacement $\tilde{\ell}/R$ of the origin $x = x' = 0$ in the direction $x = x'$ as a function of the applied flux (see figure 4).

For $U < 0$, the system is characterized by the off-diagonal (quasi) long-range order due to fermionic pairing [2]. In contrast with repulsive cases, for $U < 0$ the correlator $\tilde{G}_{\uparrow \downarrow}(\mathbf{r}, \mathbf{r}'; t)$ displays a marked structure along the whole anti-diagonal $x = -x'$, reflecting the formation of pairs of smaller and smaller size at increasing $|U|$. We found that in the BCS regime the pairs correspond to enhanced $(k, -k)$ correlations at the Fermi sphere i.e. for wave-vector $|k| = k_F$. On the other hand, at larger interactions, pairs of small spatial size are formed and correlations along the whole anti-diagonal $(k, -k)$ are predicted. Such approach is in line with [48, 49] and further analyzed in the supplementary material. Despite our system can be of small size, the antidiagonal correlations features clearly emerge in our expansion protocol (at long times), thus allowing to probe the nature of pairing in the whole BCS-BEC crossover. The formation of pairs is also reflected in the halving of the period of $\tilde{\ell}$ as a function of the flux. This is due to the doubling of the mass of the components of the system when atoms are bound in diatomic molecules, which reduces by a factor two the value of the flux quantum. Notably, an equivalent phenomenon occurs in superconducting rings [50].

Furthermore, we find that the landscape along the anti-diagonal depends on the number of particles with a markedly different scaling for $U > 0$ and $U < 0$. In line with Yang’s criterion for the ODLRO [2], we find that for attractive interactions both the maxima and the minima of the momentum correlator along the antidiagonal scale the same way with $N$. Indeed, in the presence of quasi-ODLRO the momentum correlator is dominated by the pair–pair correlations, and scales as $N^\alpha$ with $0 < \alpha < 1$ for any wave-vector $k$ (see supplementary material). For $U > 0$, instead, the maxima of the momentum correlator are independent on particle number, while the minima increase with $N$. As a result, the visibility defined as

$$V(N, U) = \frac{\text{Max}[\tilde{G}_{\uparrow \downarrow}(x, -x; t)] - \text{Min}[\tilde{G}_{\uparrow \downarrow}(x, -x; t)]}{\text{Max}[\tilde{G}_{\uparrow \downarrow}(x, -x; t)] + \text{Min}[\tilde{G}_{\uparrow \downarrow}(x, -x; t)]}$$  (7)
and presented in figure 4, is independent of $N$ for $U < 0$ and decreasing with $N$ for $U > 0$. We note that the property clearly emerges already at small $N$, providing a further evidence that ring geometries are well suited for minimizing finite size effects [51].

## 4. Conclusions

In this work, we demonstrated how the interplay between the single-particle's phase coherence and the many-body phase coherence of an interacting Fermi gas can be probed with a single protocol inspired by heterodyne phase detection schemes employed in cold atoms laboratories. We described the ring-trapped gas through the Hubbard model with the local interaction $U$ ranging from positive to negative values. We analyzed the dynamics of the density–density correlators. They can be accessed, for example, in cold atoms experiments through state-of-the-art processing of the particles expansion. We note that also continuous (no lattice) ring-shaped degenerate gases can be accessed by our theory in the dilute lattice limit (see supplementary material). For our protocol, we demonstrated that particle’s phase information emerges in the intermediate times interference of the expansion; the many-body phase coherence can be tracked at longer times.

For Fermi systems the effective magnetic flux imparts the phase winding on a broad momentum distribution. We have shown that the relevant information of the phase of fermionic particles can be traced in the response of the orbitals. This analysis can be carried out in our protocol by suitably extrapolating the ring–center correlations from the total correlations (see figure 2). As remarkable spin-off of our analysis on the single orbital interference, we note that our results grants the access to the number of particles $N_{\uparrow}$ ($N_{\downarrow}$). Our prescription is that the number of dislocations obtained in our interferogram figure 2(d) is just $N_{\alpha} - 1$ with $\alpha = \uparrow$ or $\downarrow$.

The proposed protocol provides a step forward towards the realization of the full counting statistics for the system’s particle density fluctuations [49, 52].

For repulsive interactions, we found that the resulting long time image displays enhanced correlations along the diagonal $k = k'$ (reflecting the magnetic ordering). By the application of the effective magnetic flux, the position of the peaks results to be displaced in discrete steps, reflecting the quantized circulation of current along the ring. For attractive interactions a clear broad anti-diagonal $k = -k'$ surfaces in the expansion. Such feature emerges at $k = k_F$ because of the fermionic pairing, leading to a many-body quantum coherence of the BCS type [48, 49] (see supplementary material). We find that anti-diagonal correlations arise also for strong attraction in which the pairs are tightly bound; in this case the peaks dissolve on a broad interval of $k$. As counterpart of the effect found for repulsive fermions, the position of the anti-diagonal results to be displaced by the effective magnetic flux in a quantized fashion. The quantitative analysis shows that for repulsive/attractive interactions the visibility of the anti-diagonal correlations is characterized by a markedly different dependence on the number of particles. Such effect reflects the Yang’s ODLRO scaling of the two-body density matrix.

With our work, we bring conceptually relevant aspects of many-body physics to the domain of what can be operatively tested. Our analysis is timely with the current stage of cold atoms quantum technology: persistent current in toroidal cold fermionic atoms has been achieved in [28, 29] and mesoscopic pairing was experimentally analyzed in [53].

## Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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