Radius Stabilization
by Two-Loop Casimir Energy

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Abstract

It is well known that the Casimir energy of bulk fields induces a non-trivial potential for the compactification radius of higher-dimensional field theories. On dimensional grounds, the 1-loop potential is \( \sim 1/R^4 \). Since the 5d gauge coupling constant \( g^2 \) has the dimension of length, the two-loop correction is \( \sim g^2/R^5 \). The interplay of these two terms leads, under very general circumstances (including other interacting theories and more compact dimensions), to a stabilization at finite radius. Perturbative control or, equivalently, a parametrically large compact radius is ensured if the 1-loop coefficient is small because of an approximate fermion-boson cancellation. This is similar to the perturbativity argument underlying the Banks-Zaks fixed point proposal. Our analysis includes a scalar toy model, 5d Yang-Mills theory with charged matter, the examination of \( S^1 \) and \( S^1/Z_2 \) geometries, as well as a brief discussion of the supersymmetric case with Scherk-Schwarz SUSY breaking. 2-Loop calculability in the \( S^1/Z_2 \) case relies on the log-enhancement of boundary kinetic terms at the 1-loop level.
1 Introduction

Higher-dimensional field theories arise in the low energy limit of string- or M-theory, which is our best candidate for a theory of quantum gravity. Independently, compactified higher-dimensional models provide many interesting possibilities for the unification of known fields and interactions. Familiar examples are the appearance of 4-dimensional gauge theories as a manifestation of higher-dimensional diffeomorphism invariance or the unification of known bosons and fermions in higher-dimensional multiplets of supersymmetry (SUSY). Thus, we consider higher-dimensional compactified models a promising ingredient in possible physics beyond the standard model, which makes the further investigation of stabilization mechanisms for the compactification radius an interesting and potentially important subject.

One very generic ingredient in the dynamics of the compactification radius is the Casimir energy of massless bulk fields [1]. As a simple example, consider 5d general relativity with a vanishing cosmological constant and a set of massless 5d fermions and bosons. Compactifying to 4d on an $S^1$ with physical volume $2\pi R$, one finds a flat 4d effective potential for the radion $R$. This flatness is lifted by the 1-loop Casimir energy which, on dimensional grounds, is $\sim 1/R^4$ and does not lead to a stable finite-$R$ solution. Clearly, to overcome the $1/R^4$ behaviour which is too simple for stabilization, one has to introduce a mass scale into the potential, which can be achieved, for example, by considering warped compactifications [2], massive bulk matter or brane localized kinetic terms for bulk fields [3].

In this paper, we point out that the required mass scale is, in fact, generically present in the simplest interacting higher-dimensional field theories, such as $\lambda \phi^4$ theories, gauge theories with coupling constant $g$, or models with Yukawa interactions. In 5d the above coupling constants are dimensionful, leading to a 2-loop Casimir energy contribution $\sim \lambda/R^5$ or $g^2/R^5$. Thus, radius stabilization will generically arise at the 2-loop level by a balancing of the $1/R^4$ and the $1/R^5$ contributions without the need to invoke any extra effects or operators. Generically, the compactification scale is set by the lowest of the strong interaction scales of various 5d field theories present in a given model.

We note that a different 2-loop stabilization mechanism was previously considered in the context of 6d $\lambda \phi^3$ theory, where the coupling is dimensionless and a logarithmic $R$-dependence arises at the 2-loop level [5]. Furthermore, the possibility of 2-loop stabilization based on the vanishing of the 1-loop contribution $1/R^4$ and a balancing of the $1/R^5$ and the $\ln(R)/R^5$ terms has been pointed out in [6]. Two-loop corrections to the 4d Casimir effect have been considered by many authors (see, e.g., [7]).

The paper is organized as follows. In Sect. 2 the above idea is illustrated using the simple example of 5d $\lambda \phi^4$ theory. We emphasize in particular that, by a judicious choice

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1 Introducing a non-zero 5d cosmological constant $\Lambda_5$, a stable solution can be found by balancing the resulting $2\pi R \Lambda_5$ contribution against the Casimir energy. However, a positive 4d cosmological constant results whose scale is set by $R$ and which is therefore generically too large.

2 Note that this differs qualitatively from the results of the early discussion of higher-loop Casimir stabilization in [4] (see below for more details).
of the field content, stabilization at moderately large radii, \( R \gg \lambda \), can be achieved, such that higher-loop corrections are negligible. This is similar to the way in which a perturbatively controlled non-trivial fixed point arises in the proposal of Banks and Zaks [8].

Section 3 extends the analysis to a 5d Yang-Mills theory with charged bosons and fermions. Amusingly, all 2-loop integrals reduce straightforwardly to the simple scalar case. Controlled 2-loop stabilization at large radius can be achieved, e.g., in the large \( N \) limit of SU(\( N \)) gauge theory with appropriate matter content. Furthermore, it is shown that our 2-loop stabilization mechanism extends straightforwardly to SUSY models with Scherk-Schwarz SUSY breaking.

In Sect. 4, we provide a qualitative discussion of the phenomenologically more interesting cases of \( S^1/Z_2 \) and of the \( S^1 \) with 3-branes. Since the finite and calculable 2-loop bulk contribution mixes with the 1-loop effect induced by brane operators with unknown coefficients, a complete predictivity just on the basis of the field content can not be achieved. In the generic case, the 2-loop effect considered here represents an \( \mathcal{O}(1) \) correction to the previously discussed stabilization by brane-kinetic terms. We point out the interesting and natural limit of logarithmically enhanced brane-localized gauge-kinetic terms, which allows one to neglect the bulk-2-loop effect and to achieve full predictivity just on the basis of the particle spectrum.

A summary of our results as well as a discussion of possible further research directions, in particular the applicability to SUSY models on \( S^1/Z_2 \) and to the case of more than 5 dimensions, can be found in Sect. 5.

The evaluation of the relevant loop integrals and non-trivial SUSY-based checks of the 2-loop gauge theory calculation are described in two appendices.

## 2 A \( \lambda \phi^4 \) example

Consider the 5d theory of Einstein gravity and massless real scalar with classical action

\[
S = \int d^5 x \sqrt{-g} \left( \frac{1}{2} \bar{M}_{P5}^2 R_5 + \frac{1}{2} \left( \partial \phi \right)^2 - \frac{\lambda}{4!} \phi^4 \right)
\]

compactified on an \( S^1 \) with radius \( R \). Clearly, a variation of the metric background field \( g_{55} \) is equivalently described by a variation of the volume \( 2\pi R \). In the following, we will always use a 5d Minkowski metric treating \( R \) as our volume or radion degree of freedom. It will not be necessary to perform a Weyl rescaling of the metric to manifestly separate graviton and radion degrees of freedom in the 4d effective theory.

Assuming that \( \bar{M}_{P5} \gg 1/\lambda \), we can consistently neglect gravitational interactions. On dimensional grounds, the effective potential for \( R \) then reads

\[
V(R) = \frac{1}{R^4} \left( c^{(1)} + c^{(2)} \frac{\lambda}{R} + c^{(3)} \frac{\lambda^2}{R^2} + \ldots \right).
\]
where the $c^{(n)}$ are $n$-loop coefficients. Note that, even in the limit of large 5d Planck mass, $c^{(1)}$ has to include the 5d graviton contribution.

Radius stabilization can be achieved already at the 2-loop level. Indeed, if $c^{(1)}$ is negative and $c^{(2)}$ positive, the 2-loop potential is minimized at

$$ R = -\frac{5}{4} \frac{c^{(2)}}{c^{(1)}} \lambda . $$

However, the 4d cosmological constant at the minimum is negative. This can be remedied either by adding a 3-brane with appropriately tuned positive tension or a 5d bulk cosmological constant. In the second case, an extra $R$-dependent contribution to the 4d effective potential results,

$$ V_{cc}(R) = 2\pi \Lambda_5 R. $$

Requiring both $V'(R)$ and $V(R)$ to vanish at the same point determines the precise value of $\Lambda_5$ and gives rise to a slightly shifted minimum at

$$ R = -\frac{6}{5} \frac{c^{(2)}}{c^{(1)}} \lambda . $$

Unfortunately, assuming that $c^{(n)} = O(1)$, it is immediately clear that higher-loop terms cannot be neglected in the vicinity of the above 2-loop minimum. This situation may, in fact, be generic. In this case we can not do more than express the justified hope that, in many models, higher-loop effects will significantly change but not destroy the minimum found at the 2-loop level. Our first conclusion is therefore that radius stabilization by higher-loop Casimir energy is presumably generic (in the sense of occurring in a large fraction of models) and the resulting compactification scale is of the order of the strong interaction scale of the most strongly coupled of the bulk field theories.

However, in specific examples a quantitatively controlled minimum based on the 2-loop approximation can occur. Indeed, in models where $|c^{(1)}| \ll |c^{(2)}|$, the 2-loop minimum is at $R \gg \lambda$ and higher-loop effects are suppressed (assuming that no undue enhancement of the coefficients $c^{(n)}$ with $n \geq 3$ occurs). As a concrete realization, consider the $O(N_s)$ symmetric generalization of the above scalar $\lambda \phi^4$ lagrangian,

$$ \frac{\lambda}{4!} \phi^4 \rightarrow \frac{\lambda}{4!} \left( \sum_{i=1}^{N_s} \phi_i^2 \right)^2, $$

together with $N_f$ fermions which are not (or only very weakly) coupled to the scalars. It is clear that, in this case,

$$ c^{(1)} \sim 4N_f - N_s - 5. $$

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3Strictly speaking, since we are not working in the Einstein frame, the equation of motion for $R$ is $V' + \pi \bar{M}_5^3 \mathcal{R}_4 = 0$, where $\mathcal{R}_4$ is the 4d curvature. Demanding $V = 0$ at the minimum also ensures that minimization of $V$ is equivalent to solving the equation of motion.

4We could, of course, improve our estimates by extracting appropriate loop suppression factors from the coefficients $c^{(n)}$ using naive dimensional analysis (see, e.g., [9]). However, since this would not affect our argument qualitatively, we do not enter such a more detailed discussion. Alternatively, one can imagine that these factors have already been absorbed in a redefined coupling.
(which is the difference of fermionic and bosonic degrees of freedom, including the 5 graviton polarizations) while

$$c^{(n)} \sim N_s^n \quad \text{for} \quad n \geq 2.$$  

(8)

Thus, taking $N_s$ large while keeping $4N_f - N_s - 5 \sim \mathcal{O}(1)$ and negative, one has $R \sim \lambda N_s^2$ at the 2-loop minimum and all higher-loop terms are suppressed.

Finally, we now want to fill in the explicit numbers for the first two loop coefficients used above. As already mentioned, $c^{(1)}$ is proportional to the difference of on-shell fermionic and bosonic degrees of freedom of the 5d theory,

$$c^{(1)} = (N_{\text{fermions}} - N_{\text{bosons}}) c^{(1)}_0,$$  

(9)

where $[1,3]$ (see also Appendix A)

$$c^{(1)}_0 \equiv \frac{3 \zeta(5)}{(2\pi)^6}. $$  

(10)

The 2-loop coefficient for a single scalar is due to the “figure-8” diagram, which has previously been derived using the winding mode expansion [6]. In our context, it is crucial that the tree-level masslessness in 5d is maintained by an appropriate 1-loop counterterm.\(^5\) The result reads (for explicit calculations see Appendix A)

$$c^{(2)} = \frac{\zeta(3)^2}{8(2\pi)^9}. $$  

(11)

Going from a single scalar to the $O(N_s)$-symmetric model, the coefficient $c^{(2)}$ of Eq. (11) has to be multiplied by $(N_s^2 + 2N_s)/3$. For our simple scalar example to work, it is important that $c^{(2)} > 0$.

Thus, a quantitatively controlled 2-loop minimum arises from the potential

$$V(R) = \frac{1}{(2\pi)^6 R^4} \left\{ 3 \zeta(5)(4N_f - N_s - 5) + \frac{\zeta(3)^2}{24(2\pi)^3} (N_s^2 + 2N_s) \frac{\lambda}{R} \right\}, $$  

(12)

in the specific large-$N_s$ limit described above.

While this simple analysis shows that it is quite easy to achieve stabilization in a scalar $\phi^4$ theory, a more interesting and realistic case is that of a 5d gauge theory. In particular, the masslessness assumption introduced above, which is unnatural for an interacting scalar, will be natural for gauge bosons and charged fermions.

### 3 Gauge theory

We now turn to the case of a 5d gauge theory compactified on $S^1$ with the action

$$S = \int d^5x \sqrt{-g} \left( \frac{1}{2} \bar{M}_{p,5}^3 R_5 - \frac{1}{2g^2} \text{tr}_f \left( F_{MNP} F^{MNP} \right) + (D_M \Phi)^\dagger (D^M \Phi) + \bar{\Psi} i \slashed{D} \Psi \right). $$  

(13)

\(^5\)There is however a finite (nonlocal) positive correction to the scalar mass squared of the 4d zero mode.
Here the bosonic and fermionic matter fields transform in some representation $r$ of the gauge group $G$, the field strength is $F_{MN} = -i[D_M, D_N]$ with $D_M = \partial_M + iA_M$, and the trace is in the fundamental representation with generators normalized by $2 \text{tr}_f(T^a T^b) = \delta^{ab}$.

The 2-loop vacuum energy contributions come from the diagrams depicted in Figs. 1 and 2. It turns out that, by simple algebraic manipulations, they can all be reduced to the 2-loop integral encountered in the scalar model above. Therefore, the complete result can be expressed in terms of the constant

$$c_0^{(2)} = \frac{\zeta(3)^2}{(2\pi)^9} \dim(G) \tag{14}$$

where $\dim(G)$ is the dimension of the gauge group (the $1/8$ of Eq. (11) is a symmetry factor characteristic of the $\lambda\phi^4$ model).

Including the symmetry factors and numerator algebra of the various diagrams and accounting for the trace normalization through the constant $T(r)$, defined by $\text{tr}_r(T^a T^b) = T(r)\delta^{ab}$ (which is also known as the Dynkin index of the representation $r$), one finds

$$\frac{c_{\text{vector}}^{(2)}}{c_0^{(2)}} = \frac{1}{4} d(d-1) T(a) - \frac{3}{4} (d-1) T(a) + \frac{1}{4} T(a) = +\frac{9}{4} T(a),$$

$$\frac{c_{\text{scalar}}^{(2)}}{c_0^{(2)}} = d T(r) - \frac{3}{2} T(r) = +\frac{7}{2} T(r), \tag{15}$$

$$\frac{c_{\text{fermion}}^{(2)}}{c_0^{(2)}} = (2-d) T(r) = -3 T(r).$$
The sum of these coefficients defines $c^{(2)}$ of Eq. (2) (with $\lambda$ replaced by $g^2$) and thereby the 2-loop contribution to the Casimir energy arising from gauge fields and gauged matter. To facilitate comparison with other calculations, we have made explicit the $d$-dependence before setting $d = 5$ and specified the contributions arising from the separate Feynman gauge diagrams. Specifically, the three contributions to $c^{(2)}_{\text{vector}}$ come from the “figure 8”, the “setting sun” diagram, and the “setting sun” diagram with ghosts (in this order, cf. Fig. 1). Analogously, the two contributions to $c^{(2)}_{\text{scalar}}$ arise from the “figure 8” and the “setting sun” diagram (cf. Fig. 2). A non-trivial check of these results based on the vanishing of the Casimir energy in models with unbroken SUSY can be found in Appendix B.

One can see that in a pure gauge theory, one again encounters precisely the situation outlined above: $c^{(1)}$ is negative while $c^{(2)}$ is positive so that 2-loop stabilization is automatic. In order to maintain perturbativity, we need to reduce the one-loop coefficient without affecting the other $c^{(n)}$. This is most easily achieved by considering large groups (e.g. $\text{SU}(N)$ with $N$ large, where $c^{(n)} \sim N^{1+n}$) and adding fermions uncharged under the gauge group to reduce $c^{(1)}$ to $O(1)$. This will lead to $R \sim g^2 N^3$ at the 2-loop minimum and result in a relative suppression of the $n$-loop ($n > 2$) contribution near this minimum by $1/N^{2n-4}$. In fact, this situation may arise fairly naturally since higher-loop coefficients are dominated by the most strongly coupled gauge group factor, so that it is sufficient to require the relevant fermions to be neutral only under this part of the group. We have thus seen that it is easy to stabilize the $S^1$ radius $R$ in a controlled fashion. Of course, to build a realistic model one would have to include branes or to consider more sophisticated geometries allowing for chiral fermions.

We close this section by commenting on a possible SUSY version of this scenario. Consider a model containing a 5d supergravity multiplet, $N_V = \dim(G)$ vector multiplets defining a super Yang-Mills theory with gauge group $G$, and $N_H$ hypermultiplets in a representation $r$ of $G$. Let us break SUSY from $N = 2$ to $N = 0$ by introducing Scherk-Schwarz boundary conditions on the $S^1$ [10]. The effect of this breaking is a mass shift for gauginos, gravitinos and hyperscalars. Their Kaluza-Klein (KK) masses become $m_n R = n + \omega$, where the real parameter $\omega$ is known as the Scherk-Schwarz parameter. Already at this point it is clear that our stabilization mechanism is qualitatively unchanged: The Scherk-Schwarz parameter is dimensionless and our basic formula, Eq. (2), remains valid. Of course, the coefficients $c^{(i)}$ are now functions of $\omega$ (which vanish for $\omega = 0$).

The 1-loop contribution to the Casimir energy is specified by [4, 11, 12]

$$c^{(1)}_{\text{SS}} = \frac{12}{(2\pi)^6} (N_H - N_V - 2) \{\zeta(5) - \zeta_\omega(5)\},$$

(16)

where we have defined the function $\zeta_\omega(n)$ by

$$\zeta_\omega(n) = \sum_{k=1}^{\infty} \frac{\cos(2\pi k \omega)}{k^n}. \quad (17)$$

Since $\zeta_\omega(n) \leq \zeta_0(n) = \zeta(n)$, this contribution is negative as long as $N_V + 2 > N_H$. We can now balance matter and gauge multiplets in exactly the same way as in the non-SUSY
case to ensure $c^{(1)} = \mathcal{O}(1)$. For the 2-loop contribution of the vectormultiplet, we use the results of Appendix B but evaluate the corresponding gaugino integrals with the shifted masses to obtain

$$c_{\text{SS vector}}^{(2)} = \frac{4}{(2\pi)^9} N_V T(a) \{\zeta(3) - \zeta(3)\}^2.$$  

(18)

Likewise, the hypermultiplet contribution reads

$$c_{\text{SS hyper}}^{(2)} = -\frac{4}{(2\pi)^9} N_V T(r) \{\zeta(3) - \zeta(3)\}^2.$$  

(19)

Since, as in the non-SUSY case, the vector multiplet contribution is positive, our 2-loop stabilization mechanism remains effective in this simple SUSY model as long as there is not too much matter charged under the most strongly coupled gauge group factor.

### 4 Compactifications with branes and fixed-points

In this section, we focus on the 5d gauge theory case discussed in Sect. 3 since it is presumably more likely to be part of phenomenologically interesting theories than the scalar toy model of Sect. 2. However, most of our discussion applies, qualitatively, also to the scalar case and presumably to many other 5d models with dimensionful couplings and corresponding 2-loop Casimir stabilization.

As already mentioned above, the pure $S^1$ case discussed up to now can not give rise to realistic models since the 4d particle spectrum is necessarily vector-like. A simple way of embedding this type of $S^1$ stabilization in a realistic construction is to add a 3-brane on which matter fields can live. To apply our stabilization mechanism without any modification, one could simply assume that the brane fields are not charged under the bulk gauge theory responsible for the stabilizing 2-loop Casimir energy.

If one does not make this assumption, the charged brane fields generate, at the 1-loop level, brane localized gauge-kinetic terms for the bulk gauge theory. Such brane-kinetic terms contribute to the 1-loop Casimir energy induced by bulk fields. We can view this as a 2-loop effect including both brane and bulk fields. It is easy to convince oneself that this contribution is not parametrically suppressed relative to the bulk 2-loop effect calculated in Sect. 3 Thus, we are precisely in the situation of 1-loop Casimir stabilization with brane kinetic terms considered in [3]. Our 2-loop calculation is then simply a finite $\mathcal{O}(1)$ correction to this stabilization mechanism. However, since the coefficient of the relevant

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6The relation $c_{\text{SS vector}}^{(i)} + c_{\text{SS hyper}}^{(i)} = 0$ for an adjoint hypermultiplet representation reflects the fact these $\mathcal{N} = 2$ multiplets combine into $\mathcal{N} = 4$ vector multiplets. The Scherk-Schwarz mechanism leaves $\mathcal{N} = 2$ SUSY unbroken and the Casimir energy remains zero.

7As already mentioned earlier, a brane localized cosmological constant can be used to tune the effective 4d cosmological constant to zero. Its interplay with a possible bulk cosmological constant can also affect stabilization (for more options along these lines see, e.g., [13]). Here, we require the bulk cosmological constant to be small because of some symmetry principle (e.g. bulk SUSY) and treat the sum of brane cosmological constants as an unknown parameter to be fixed by the requirement of 4d flatness.
brane-kinetic term is UV-sensitive, its value after renormalization is in essence just a new parameter of the model. The precise result of the 2-loop calculation is then only meaningful if one has some first-principles knowledge about the coefficient of the brane kinetic term. This will, in general, require a UV completion as it is provided, e.g., by a string orbifold model.

A very similar situation arises in an $S^1/Z_2$ geometry. Here, not only brane-localized charged fields but also the bulk gauge and matter fields themselves induce brane-localized gauge-kinetic terms. This is, in fact, a familiar and well-studied effect in the context of higher-dimensional grand unified theories (GUTs) with symmetry breaking by brane-localized Higgs fields and in orbifold GUTs \[14,15\]. It corresponds to the statement that a (modified) logarithmic running of the gauge couplings continues above the compactification scale, which can be understood as the running of the coefficient of a brane localized $F^2$ term \[15\].

In fact, it is easy to check that, even in the simple scalar model, the 2-loop vacuum energy in an $S^1/Z_2$ compactification has (in contrast to the pure $S^1$ case) a logarithmic divergence that can be absorbed into a brane-localized $\phi \partial_5^2 \phi$ operator. This is analogous to the boundary term $F_{\mu \nu} F^{\mu \nu}$ (with $\mu, \nu \in \{0, 1, 2, 3\}$) induced in gauge theories with even boundary conditions for $A_\mu$. However, while in the gauge theory case only the 4d-part of the gauge-kinetic term is corrected ($F_{5\mu}$ being zero at the boundary), in the scalar case the correction only affects the $\partial_5$ part (the 1-loop self energy diagram being momentum-independent).

The apparent loss of predictivity associated with the UV sensitive coefficients of brane-localized kinetic terms is not as severe as one might naively think. The reason is the logarithmic enhancement of such terms associated with their logarithmic divergence. Indeed, the bulk gauge theory has a strong interaction scale associated with the 5d gauge coupling, $M \sim 24\pi^3/g^2$ \[9\]. It is natural to assume that, at this scale, the brane-localized $F^2$ term has an $O(1)$ coefficient. Running down to the compactification scale $M_c = 1/R$, one obtains a log-enhanced coefficient $\sim \ln(M/M_c) = \ln(24\pi^3R/g^2)$. This is dominant with respect to the unknown $O(1)$ initial value. The corresponding log-enhanced 1-loop Casimir effect contribution of the brane operator is also dominant with respect to the true 2-loop effect calculated in Sect. 3. Thus, the leading piece of the $g^2/R^5$ contribution is calculable on the basis of the low-energy field content of the model. The parametric behaviour is, in fact, not a pure power of $R$ but includes the logarithmic $R$ dependence of the coefficient. The 4d vacuum energy of a 5d gauge theory compactified on $S^1/Z^2$ thus has the form \[8\]

$$V_4 = \frac{1}{R^4} \left( c^{(1)} + c^{(2)} g^2 \frac{\ln(MR)}{R} \right), \quad (20)$$

\[8\]As already mentioned in the Introduction, higher-loop radius stabilization was also discussed in \[4\], but in a very different approach. We understand that Ref. \[4\] treats the 4d coupling as fundamental and $R$-independent, resulting in loop corrections $\sim \ln(R)/R^4$. By contrast, we consider the 5d coupling as fundamental, which implies a 4d coupling $\sim 1/R$ and hence a vacuum energy loop correction $\sim \ln(R)/R^5$.

Furthermore, a potential similar to Eq. (20) was derived in \[6\] for a scalar model with Yukawa couplings to brane fields. Assuming an exact cancellation of the 1-loop contributions from scalars and fermions, stabilization was achieved by a balancing of the $1/R^5$ and $\ln(R)/R^5$ terms of the 2-loop result.
and can, as explained earlier, give rise to quantitatively controlled radius stabilization at $R \sim g^2 N^3$ in the case of an SU($N$) gauge theory.

To be more specific, focus on 5d SU($N$) gauge theory on $S^1/Z_2$ (where the gauge group is not broken by the orbifolding) with $N_f$ uncharged bulk fermions and with charged brane fermions in a representation $r$ at the $x^5 = 0$ boundary. The logarithmic running above the compactification scale induces an effective brane-localized $F^2$ term

$$\mathcal{L} \supset - \frac{1}{96\pi^2} \text{tr}(F_{\mu\nu} F^{\mu\nu}) \ln(M R) \left[ \delta(x^5) \left( 8 T(r) - \frac{23}{4} T(a) \right) + \delta(x^5 - \pi R) \left( -\frac{23}{4} T(a) \right) \right]$$

(21)

at scale $M_c$. The calculations leading to this formula are explained in detail in Sect. 3 of [16] and can also be extracted from the earlier references [14, 15]. Here the strong interaction scale is $M \simeq \frac{24\pi^3}{g^2 N}$, accounting for the large-$N$ enhancement of higher loops in SU($N$) gauge theory. As far as the 1-loop Casimir energy calculation using a summation of KK modes is concerned, the effect of this contribution can be summarized by an extra contribution to the momentum-space effective action,

$$\frac{\pi R}{2 g^2} (k^2 \delta_{\mu\nu} - k_\mu k_\nu) \rightarrow \left( \frac{\pi R}{2 g^2} + 2b \right) (k^2 \delta_{\mu\nu} - k_\mu k_\nu) ,$$

(22)

to be used for even modes (i.e. the $A_\mu$ modes) only. Here $b$ is defined by minus the sum of the two coefficients of the brane-localized $F^2$ terms as given in Eq. (21) and $k$ is the euclidean 4-momentum. The extra factor of 2 arises since the even higher KK modes are twice more sensitive to brane operators than the zero mode.

Based on this, the 1-loop contribution to the 4d potential induced by the brane-kinetic terms reads

$$V_{\text{brane}}^{\text{brane}}(R) = \frac{3 \text{dim}(G)}{2} \sum_{n=1}^{+\infty} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \left\{ \ln[1 + b'] k^2 + (nM_c)^2 \right\}$$

$$\simeq \frac{3 \text{dim}(G)}{2} \sum_{n=1}^{+\infty} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{b'}{k^2 + (nM_c)^2} .$$

(23)

Here $b' = 2b/(\pi R/2g^2)$ and the summation is only over even non-zero (cosine) modes of the KK mode expansion on the $S^1$ compact space. The prefactor of 3 accounts for the 3 on-shell degrees of freedom of a massive vector. Note that we have simply interpreted the boundary operator as a correction to the energy of each separate leading-order KK mode. (This shifted KK mode spectrum has been derived in a different way in Appendix B of [17].) To be more precise, one would have to re-diagonalize the quadratic-order Hamiltonian after inclusion of the boundary terms and then work with the modified KK mode expansion [3]. However, as long as $\ln(M R) \ll M R$, we are entitled to linearize in the boundary term and our simplified treatment is sufficient.

Using the calculational techniques of Appendix A, the above formula is evaluated to give the brane-induced correction (equivalent to the dominant part of the 2-loop Casimir energy)

$$V_{\text{brane}}^{\text{brane}}(R) = \frac{36\zeta(5) g^2 \text{dim}(G) b}{(2\pi)^6 R^5 \pi} ,$$

(24)
leading to the final result\footnote{The factor of $\frac{1}{2}$ in the one-loop contribution is due to the orbifolding, since half of the modes are projected away (cf. also the summation in Eq. (23)).}

\[
V(R) = \frac{3\zeta(5)}{(2\pi)^6 R^4} \left\{ \frac{4N_f - 3(N^2 - 1) - 5}{2} + \frac{N^2 - 1}{(2\pi)^5} \left[ 8T(r) - \frac{23}{2} T(a) \right] \frac{g^2 \ln(MR)}{R} \right\}.
\]

(25)

It is now easy to arrange for the coefficient of the $1/R^4$ piece to be negative and $O(1)$ while keeping the $1/R^5$ term positive and $O(g^2 N^3)$ such that, as before, controlled 2-loop stabilization at a moderately large radius is achieved.\footnote{To turn over the sign in the second term, we need to introduce a certain amount of brane fermions. Note also that assigning negative parities to some gauge fields – in other words, breaking the gauge group by orbifolding – would reduce the contribution of the gauge fields to the brane kinetic terms and can even flip its sign.} Compared with the $S^1$ case, where $MR \sim N^2$, we now have $MR/\ln(MR) \sim N^2$. Using asymptotic properties of the Lambert $W$ function (see, e.g., [18]) the solution for $N$ large can be written as

\[
MR \sim N^2 \nu \left\{ 1 + O(\ln \nu/\nu) \right\}, \quad \nu = \ln N^2,
\]

(26)

thus giving an additional enhancement factor $\ln N^2$.

5 Conclusions

We have presented a mechanism stabilizing the size $R$ of an extra dimension compactified on $S^1$ or on the orbifold $S^1/Z_2$ which is based on the presence of dimensionful couplings – a generic feature of field theoretic models with $d > 4$. We have shown that, by balancing the 1- and 2-loop contributions to the Casimir energy, a perturbatively controlled minimum at moderately large values of $R$ can be realized.

In the case of scalar massless $\phi^4$ theory on $S^1$, the Casimir energy is calculable and finite as long as the masslessness is enforced as a renormalization condition. The 1-loop contribution scales like $\sim -R^{-4}$, while the 2-loop effect of the scalar self-interaction gives $\sim +\lambda R^{-5}$, thus producing a nontrivial minimum. In order to ensure that the result is perturbatively controlled, we have added weakly coupled fermions. Their effect is to reduce the numerical factor of the 1-loop term while only very mildly affecting the higher-loop contributions. This shifts the minimum to larger values of $R$.

The situation is basically the same in the case of a gauge theory. As before, the purely bosonic theory already produces a 2-loop minimum in the radion potential, while the inclusion of fermions uncharged under the most strongly coupled gauge group factor ensures the perturbativity of the result. We have also shown that in a SUSY extension of this scenario (with SUSY-breaking à la Scherk-Schwarz), the mechanism works without modification.

The above calculable two-loop stabilization is modified but not destroyed if, instead of the circle, the orbifold $S^1/Z_2$ is considered. The crucial new feature of this situation are brane-kinetic terms for the gauge fields, which are generated in the 1-loop effective
action. Without the knowledge of the underlying UV physics one cannot predict the corresponding counterterm at the cutoff scale $M$. However, in calculating the potential of $R$ for large values of the radius, $R \gg M^{-1}$, one effectively integrates out the physics down to that scale, and the logarithmic running of the brane-kinetic term dominates over its unknown initial value. The dominant 2-loop effect can then be obtained by evaluating the 1-loop integral in the presence of this log-enhanced brane-kinetic term. The new contribution scales like $\sim g^2 \ln(R/M)/R^5$ and is calculable on the basis of the bulk and brane field content of the model. We emphasize that this log-divergent part of the 2-loop calculation dominates over the remaining (finite) 2-loop effects (scaling as $\sim 1/R^5$) as well as the one-loop effect of the unknown brane-kinetic term (whose leading effect for large $R$ is also $\sim 1/R^5$).

Although we have focussed exclusively on 5d models, we note that our mechanism is equally suitable for other higher-dimensional manifolds with a single unstabilized modulus. This is because such theories look very similar from a 4D point of view, the main difference being a modified KK-mass spectrum which depends on the value of the modulus. Various possibilities for stabilizing such a single modulus have recently been discussed in the context of the KKLT proposal [19], and we believe that Casimir energies will be relevant in this context. This has, in fact, very recently been discussed in the context of the 1-loop effect of massive vector fields in [20], and one can expect that the higher-loop contributions discussed here will also play an important role in the further study of moduli stabilization in more complete, higher-dimensional scenarios.

As a simple and specific example, we briefly consider $S^n$ compactifications with the volume undetermined by the Einstein equations. The Casimir energy coming from a self-interacting scalar field will take the same general form as in Eq. (2), with the powers of $R$ modified according to the dimensionality of the coupling and the $c^{(i)}$ to be calculated from the appropriate KK spectrum. The coefficient $c^{(1)}$ has been calculated for the case of a sphere (without additional infinite dimensions) in Ref. [21] for $n = 1, 2, 3, 4$ and found to be negative. Moreover, it is clear that $c^{(2)}$ is always positive since the integral appears squared. Thus, if the inequality $c^{(1)} < 0$ survives the transition from $S^n$ to $M_4 \times S^n$ for $n > 1$ (as it does for $n = 1$), our stabilization mechanism extends straightforwardly to these geometries. In any case, we can be confident that many examples of 2-loop (or higher-loop) Casimir stabilization exist within the rich class of models with $n > 1$ compact dimensions where the total volume is a modulus at tree level.

Our final point concerns SUSY theories on $S^1/Z_2$. The prototype SUSY breaking mechanism on $S^1$, the Scherk-Schwarz mechanism [10], extends to this case. It can equivalently be described by giving a vacuum expectation value to the $F$ component of the radion superfield (the chiral superfield whose lowest component contains $R$) [22, 23]. Indeed, the resulting tree level action corresponds to no-scale supergravity and the potential for $R$ is completely flat. Stabilization at 1-loop has previously been studied by use of bulk mass terms for hypermultiplets [3, 12, 24], brane kinetic terms [3, 25] or massive vector multiplets [20]. The mechanism we described in Sec. 4 seems to be quite suitable for SUSY $S^1/Z_2$ models without 5d masses. The 1-loop contribution is given by $\frac{1}{7}$ of the $S^1$ result obtained in this paper. We expect the dominant 2-loop effect to be the log-enhanced brane-kinetic terms, the same as in the non-SUSY case. However, a detailed
analysis of this scenario, of its interplay with the other stabilization mechanisms mentioned above, or even the construction of a realistic SUSY model go beyond the scope of the present paper and are left to future research.

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**Appendix A: Basic 1- and 2-loop integrals**

We begin by rederiving the known 1-loop vacuum energy for a real scalar field on $R^4 \times S^1$ in dimensional regularization. The 1-loop effective potential of the 5d euclidean theory is given by the $d \to 5$ limit of

\[ V^{(1)}_5(R) = \frac{1}{2R} \sum_{n=-\infty}^{+\infty} \int \frac{d^{d-1}k}{(2\pi)^d} \ln[k^2 + (nM_c)^2], \tag{A.1} \]

where $M_c = 1/R$. Using the fact that the $d$-dimensional integrals of any power of $k^2$ and of $\ln(k^2)$ vanish in dimensional regularization, we have

\[ V^{(1)}_5(R) = \frac{M_c}{2} \int_0^{M_c^2} dM^2 \int \frac{d^{d-1}k}{(2\pi)^d} \sum_{n=-\infty}^{+\infty} \frac{n^2}{k^2 + (nM)^2} \tag{A.2} \]

\[ = -\frac{M_c}{2} \int_0^{M_c^2} dM^2 \int \frac{d^{d-1}k}{(2\pi)^d} \frac{k^2}{M^4} \frac{\pi \coth(\pi |k|/M)}{|k|/M}. \tag{A.3} \]

Appealing again to the fact that the $d$-dimensional integral of a pure power vanishes, $\coth(\pi |k|/M)$ can be replaced by $\coth(\pi |k|/M) - 1$, making the $d^{d-1}k$ integral finite and allowing one to take the limit $d \to 5$. The resulting 4d effective potential is

\[ V^{(1)}_4(R) = 2\pi R V^{(1)}_5(R) = -\frac{3\zeta(5)}{(2\pi)^6 R^4} \quad \text{with} \quad \zeta(5) = 1.0369..., \tag{A.4} \]

in agreement with Eqs. (9) and (10).

Higher-loop corrections to the effective action (in particular to $-V^{(1)}_5$) are given by the sum of all one-particle-irreducible vacuum diagrams of the relevant field theory. As already pointed out in the main text, the 2-loop correction is essentially just the “figure 8” diagram. To be more precise, one has to add to the lagrangian the mass counterterm of the uncompactified 5d theory ensuring that the scalar remains massless at the 1-loop level. This counterterm becomes part of a tadpole-like 1-loop vacuum diagram, which is part of the 2-loop correction. However, in dimensional regularization the mass counterterm vanishes and the 2-loop correction (for a single scalar field) simply reads

\[ V^{(2)}_5 = \frac{\lambda}{8} I^2, \quad \text{where} \quad I = \frac{1}{R} \sum_{n=-\infty}^{+\infty} \int \frac{d^{d-1}k}{(2\pi)^d} \frac{1}{k^2 + (nM_c)^2}. \tag{A.5} \]
Here the prefactor contains a symmetry factor \((1/8)\) and the tadpole integral \(I\) is evaluated as the \(d \to 5\) limit of

\[
I = \int \frac{d^{d-1}k}{(2\pi)^d} \frac{\pi \coth(\pi |k|/M)}{|k|} = \frac{\zeta(3)}{(2\pi)^3 R^3} \quad \text{with} \quad \zeta(3) = 1.2021...
\]

This completes the derivation of Eq. (11) and thereby of Eq. (12).

**Appendix B: SUSY checks of gauge theory results**

To verify the results of our 2-loop gauge theory calculation, we have performed checks based on the vanishing of the Casimir energy in theories with unbroken SUSY in 5 and 4 dimensions.

Specifically, we can consider a 5d super Yang-Mills (SYM) theory which, in addition to the gauge fields, includes a real adjoint scalar \(\Sigma\) and a Dirac gaugino. To calculate the 2-loop effect in this theory, coupled to a hypermultiplet (a Dirac fermion and two complex scalars) in a representation \(r\) of the gauge group, we need the contributions coming from the additional Yukawa- and scalar self-interaction diagrams shown in Fig. 3. For the explicit lagrangian see, e.g., [26].

![Two-loop diagrams from Yukawa couplings and scalar self-interactions.](image)

Figure 3: *Two-loop diagrams from Yukawa couplings and scalar self-interactions.*

The three “setting-sun” diagrams coming from the Yukawa couplings yield

\[
\begin{align*}
    c_{\text{gaugino–fermion–scalar}}^{(2)} &= -8 c_0^{(2)} T(r), \\
    c_{\text{fermion–}\Sigma}^{(2)} &= -c_0^{(2)} T(r), \\
    c_{\text{gaugino–}\Sigma}^{(2)} &= -c_0^{(2)} T(a).
\end{align*}
\]

Furthermore, in the scalar sector there are two new “figure-8” diagrams coming from the coupling of \(\Sigma\) to the scalar and of the scalar to itself, giving two new contributions

\[
\begin{align*}
    c_{\text{scalar–}\Sigma}^{(2)} &= 2 c_0^{(2)} T(r), \\
    c_{\text{scalar–scalar}}^{(2)} &= 3 c_0^{(2)} T(r).
\end{align*}
\]

Finally, the gaugino and \(\Sigma\) as well as the hypermultiplet fermion and scalar are charged under the gauge groups. The corresponding 2-loop corrections can be directly read off...
from Eq. (15). The two bosonic contributions are

\[ c^{(2)}_{\text{vector} - \Sigma} = \frac{7}{4} T(a) c_0^{(2)}, \]
\[ c^{(2)}_{\text{vector} - \text{scalar}} = 7 T(r) c_0^{(2)}, \] (B.3)

where we have included factors \( \frac{1}{2} \) and 2 to account for the reality of \( \Sigma \) and for the presence of two hypermultiplet scalars respectively. The two fermionic contributions are precisely as in the Eq. (15), just with \( T(r) \) replaced by \( T(a) \) in the case of the gaugino.

The contributions of the charged hypermultiplet thus sums up to

\[ c^{(2)}_{\text{vector} - \text{scalar}} + c^{(2)}_{\text{scalar} - \Sigma} + c^{(2)}_{\text{vector} - \text{fermion}} + c^{(2)}_{\text{fermion} - \Sigma} + c^{(2)}_{\text{gaugino} - \text{fermion} - \text{scalar}} = (7 + 2 + 3 - 1 - 8) T(r) c_0^{(2)} = 0. \] (B.4)

Similarly, the self-interactions of the vector-multiplet give rise to

\[ c^{(2)}_{\text{vector}} + c^{(2)}_{\text{scalar} - \Sigma} + c^{(2)}_{\text{vector} - \text{gaugino}} + c^{(2)}_{\text{gaugino} - \Sigma} = \left( \frac{9}{4} + \frac{7}{4} - 3 - 1 \right) T(a) c_0^{(2)} = 0. \] (B.5)

Next, since we have kept the \( d \) dependence in Eq. (15), we can immediately extend our analysis to a 4d \( \mathcal{N} = 1 \) SYM theory. The 2-loop Casimir energy contribution is proportional to

\[ c^{(2)}_{\text{4d vector} - \text{scalar}} + c^{(2)}_{\text{4d scalar} - \text{scalar}} + c^{(2)}_{\text{4d vector} - \text{gaugino}} + c^{(2)}_{\text{4d gaugino} - \text{gaugino}} = (1 - 1) T(a) c_0^{(2)} = 0, \] (B.6)

where the fermionic contribution includes an extra factor \( \frac{1}{2} \) relative to Eq. (15) to account for the chirality of the 4d gaugino.

Finally, consider the contribution of charged 4d matter. The 2-loop effects based on gauge interactions can be inferred from Eq. (15) (with an appropriate factor \( \frac{1}{2} \) for fermion chirality in the loop). The gaugino-fermion-scalar contribution is as in Eq. (B.1), but with an extra factor \( \frac{1}{4} \) for the fermion chirality and half the number of scalars. The scalar “figure 8” diagrams are induced by the \( D \) term potential and give rise to \( \frac{1}{6} \) of the contribution displayed in Eq. (B.2) because of the missing trace over \( SU(2)_R \) degrees of freedom, \( \text{tr} \sigma_i \sigma_i = 6. \) Overall, one finds

\[ c^{(2)}_{\text{4d vector} - \text{scalar}} + c^{(2)}_{\text{4d scalar} - \text{scalar}} + c^{(2)}_{\text{4d vector} - \text{fermion}} + c^{(2)}_{\text{4d gaugino} - \text{fermion} - \text{scalar}} = \left( \frac{5}{2} + \frac{1}{2} - 1 - 2 \right) T(r) c_0^{(2)} = 0, \] (B.7)

concluding our SUSY based checks.

\[ 11 \text{The scalar “figure-8” diagram also contains a piece proportional to } (\text{tr} T^a)^2 \text{ which is zero for non-abelian as well as anomaly-free abelian gauge theories. The reason why the Casimir energy does not vanish in the presence of an anomalous } U(1) \text{ is the associated occurrence of a one-loop Fayet-Iliopoulos term which spontaneously breaks SUSY.} \]
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