As the adiabatic geometric phase approaches its twentieth birthday \[1, 2\] it continues to impact research in physics and chemistry. The original scenario considered a quantum system \(Q\) with a discrete, nondegenerate energy spectrum whose dynamics was driven by a classical set of control parameters \(B(t)\). It was shown that when \(B(t)\) was cycled adiabatically through a loop in parameter space, and \(Q\) was initially prepared in an eigenstate \(\ket{E(0)}\) of the initial Hamiltonian \(H(0)\), its quantum state would return to the initial state \(\ket{E(0)}\) at the end of the cycle, to within a phase factor \(\exp[\imath \phi]\). The phase shift \(\phi\) contained a geometric contribution now known as the adiabatic geometric phase (AGP) which had been discarded in prior treatments of quantum adiabatic dynamics. Since then, this original scenario has been generalized in a number of ways: (1) the requirement of nondegeneracy was removed \[3\]; (2) the adiabatic restriction was removed \[4\]; (3) the control field \(B(t)\) was allowed to have its own dynamics \[5\]; and (4) \(Q\) was allowed to interact with a quantum environment as well as with the control field \(B(t)\) \[6, 7, 8, 9\]. In this Letter we return to the original scenario, though we allow the classical control field to contain a noise component \(B_n(t)\) along with the (deterministic) adiabatically varying component \(B_a(t): B(t) = B_a(t) + B_n(t)\). We will show that \(B_n(t)\) causes a stochastic phase shift to appear in the off-diagonal elements of \(Q\)’s density matrix which, for sufficiently strong noise, causes decoherence. We derive the condition for onset of decoherence, and identify the noise properties that drive decoherence. We show that this decoherence mechanism causes all physical consequences of the AGP to become unobservable.

The AGP has also had an impact on the new research area of quantum computation. In 2000, a proposal was put forth for a geometric quantum computer (GQC) \[7\] which uses the AGP to encode conditional phase shifts into the quantum state of a GQC. It was argued that the AGP gave the GQC a modest degree of fault tolerance to uncertainty in the control parameters, though the need to examine the robustness of a GQC to decoherence was also noted. In the second part of this Letter we carry out such a decoherence analysis. We show that the above decoherence mechanism arising from noisy control severely impacts the performance of a quantum algorithm run on a noisy GQC. Specifically, we show how noisy control causes the Shor algorithm to loose its computational efficiency so that the time to factor an integer \((N)\) grows exponentially with the size of the integer \((\log N)\). We establish the condition for the onset of decoherence, and show that the success probability for the algorithm is smoothly degraded as the noise becomes stronger, reaching the point at onset where the computational efficiency of the Shor algorithm has been completely destroyed. We also show that at onset of decoherence, the entanglement fidelity of a 2-qubit entangled state is degraded to that of a mixed state upon output from a sufficiently noisy controlled phase gate of the type used in a GQC. This noise-induced loss of entanglement confirms that the GQC is no longer behaving as a quantum computer. To the best of our knowledge, this Letter provides the first quantitative demonstration of how the computational efficiency of a landmark quantum algorithm such as Shor’s is destroyed when it is run on a sufficiently noisy GQC. A detailed presentation of all arguments will be reported elsewhere \[10\].

1. General Analysis: We consider the situation where the dynamics of an \(N\)-qubit system \(Q\) is driven by an external control field \(B(t)\). The control field is composed of a deterministic component \(B_a(t)\) and a noise component \(B_n(t)\). We will be interested in cases where \(B_a(t)\) executes a cyclic evolution with period \(T\). We take the noise component \(B_n(t)\) to be a stationary stochastic process with zero mean \((\langle B_n(t) \rangle = 0)\) and noise correlation time \(\tau_c\). To simplify the notation we assume the noise is isotropic. This assumption is not essential, and the following analysis can easily be repeated without it. For isotropic noise, the noise correlation function \(\langle B_n(t)B_n(t') \rangle = \delta_{ij} \sigma^2 f(\tau)\).

Here: (1) \(\delta_{ij}\) is the Kronecker delta; (2) \(\sigma^2\) is the variance of the noise \((\langle B_n(t) \rangle)^2\); (3) \(\tau = t - t'\); and (4) \(f(\tau)\) is...
the normalized noise fluctuation profile with peak value $f(0) = 1$ and temporal width of order $\tau_c$. Because of the stochastic nature of $B_n(t)$, each application $k$ of the control field contains a different realization of the noise $B_n(t; k)$. To account for the dynamical effects of noise, it is necessary to introduce an ensemble of identical quantum systems $\mathcal{Q}_k$, all prepared in the same initial state $|\psi(0)\rangle$, but whose evolution is subject to a different noise realization $B_n(t; k)$.

Quite generally, the system Hamiltonian $H(t)$ can be written as the sum of two terms: $H_s(t)$ and $H_a(t)$. Here $H_s(t)$ is the Hamiltonian in the absence of noise and depends solely on $B_n(t)$. The stochastic interaction term $H_a(t)$ depends explicitly on the noise component $B_n(t)$ and its action varies from one application of the control field to another. We introduce the instantaneous energy eigenvalues $E_n(t)$ and eigenstates $|E_n(t)\rangle$ of the noiseless Hamiltonian $H_s(t): H_s(t)|E_n(t)\rangle = E_n(t)|E_n(t)\rangle$. Note that the periodicity of $B_n(t)$ implies that $H_a(t + mT) = H_a(t)$. As in the original AGP scenario, the instantaneous energies $E_n(t)$ are assumed to be discrete and nondegenerate for all $t$ of interest so that $|E_n(t + mT)\rangle = |E_n(t)\rangle$. Introducing the minimum energy level separation $\Delta = \min|E_n(t) - E_m(t)||$, the noiseless dynamics will be adiabatic if $h/\tau_T \ll \Delta$. Furthermore, as shown in Ref. [1], the dynamics associated with the noise will also be adiabatic if $h/\tau_c \ll \Delta$. Both of these conditions are assumed in the remainder of this Letter.

As noted above, each ensemble element $\mathcal{Q}_k$ sees a different noise realization $B_n(t; k)$ added to $H_a(t)$. The dynamics of $\mathcal{Q}_k$ is driven by $H(t; k) = H_s(t) + H_a(t; k)$ which generates the final state $|\psi(t; k)\rangle = U(t_f, t_0; k)|\psi(0)\rangle$, where $U(t_f, t_0; k) = \exp \left[-i \int_{t_0}^{t_f} dt H(t; k)\right]$. Thus the final state produced by the noisy control field varies over the ensemble. Consequently, calculation of the expectation value of an observable requires an ensemble average over the noise, along with the usual quantum mechanical averaging.

We divide the time interval $(t_0, t_f)$ into $J$ subintervals of duration $\epsilon = (t_f - t_0)/J$ by introducing intermediate times $t_j = t_0 + j\epsilon$ ($j = 0, \ldots, J$). Eventually we let $J \rightarrow \infty$. The propagator $U(t_f, t_0)$ factors into a product of propagators: $U(t_f, t_0) = U(J) \cdots U(0)$, where $U(j) = U(t_j, t_{j-1})$. One can show [12] that in the adiabatic limit the matrix elements $U_{lm}(j) = \langle E_l(t_j)|U(j)|E_m(t_{j-1})\rangle$ give:

$$|\psi(t_f)\rangle = \exp \left[-i \left\{ \Gamma_a(k) + \Gamma_s(k) \right\} \right] |E_k(t_0)\rangle. \tag{2}$$

Here $\Gamma_a(k) = \langle 1/\hbar \int_{t_0}^{t_f} dt \{E_k(t) - \hbar\gamma_k(t)\} \rangle$ and $\Gamma_s(k) = (1/\hbar \int_{t_0}^{t_f} dt H_s^k(t))$. Eq. (2) shows that noise introduces a stochastic phase shift $\Gamma_s(k)$ into the final state $|\psi(t_f)\rangle$ which is sensitive to the initial energy eigenstate $|E_k(t_0)\rangle$ through $H_s^k(t)$.

To bring out the physical consequences of the stochastic phase $\Gamma_s(k)$ we consider an initial superposition state $|\psi(0)\rangle = \sum_k c_k |E_k(t_0)\rangle$. From the linearity of quantum mechanics: $|\psi(t_f)\rangle = \sum_k c_k U(t_f, t_0)|E_k(t_0)\rangle$. Information about the phase coherence of the final superposition is carried in the off-diagonal elements of the final density operator $\rho(t_f) = |\psi(t_f)\rangle \langle \psi(t_f)|$. One can show [11] that the noise averaged density matrix elements are:

$$\overline{\rho_{kj}}(t_f) = c_k c_j^* \exp \left[-i \Gamma_a(k, j) \right] D(k, j), \tag{3}$$

where $\Gamma_a(k, j) = \Gamma_a(k) - \Gamma_a(j)$, and the decoherence factor $D(k, j)$ is:

$$D(k, j) = \exp \left[-i \overline{\Gamma_s(k, j)} \right], \tag{4}$$

with $\Gamma_s(k, j) = \Gamma_s(k) - \Gamma_s(j)$. We see that the decoherence factor $D(k, j)$ is a direct consequence of the noise-induced phase shift $\Gamma_s$. Often, $H_s(t)$ is linear in the noise component $B_n(t)$ so that $H_s(t) = (\gamma/2) B_n(t) \cdot \hat{O}$. Here $\gamma$ is the coupling constant and $\hat{O}$ is a vector operator. In the adiabatic limit, $\tau_c \ll (t_f - t_0) \equiv T$. Thus, if we partition the integration interval $(t_0, t_f)$ that appears in the definition of $\Gamma_s(k, j)$ into $M = T/\tau_c$ subintervals of duration $\tau_c$, we render $\Gamma_s(k, j)$ into a sum of uncorrelated random variables $\Gamma^m_s(k, j)$:

$$\Gamma_s(k, j) = \sum_{m=1}^{M} \Gamma^m_s(k, j).$$

Because the noise is stationary, the $\Gamma^m_s(k, j)$ have identical probability distributions. If the $\Gamma^m_s(k, j)$ are not only uncorrelated, but also statistically independent, it follows from the Central Limit Theorem that $\Gamma_s(k, j)$ will have a Gaussian probability distribution. In this case, one can show [11] that $\Gamma_a(k, j) = 0$ and

$$\overline{\Gamma^2_s(k, j)} = \frac{\eta^2}{4} I_{kj}, \tag{5}$$

where: (1) $\eta$ cycles of the control field have been applied ($\eta T = T$); (2) $I_{kj} = \int_0^T dt dt' \overline{\hat{O}_{k,j}(t) \cdot \hat{O}_{k,j}(t')}$, with $\overline{\hat{O}_{k,j}(t) = \overline{\hat{O}_{k,j}(t) \cdot \hat{O}_{k,j}(t')}}$, and $\overline{\hat{O}_{k,j}(t) = \overline{\hat{O}_{k,j}(t) \cdot \hat{O}_{k,j}(t')}}$; (3) $f(t - t')$ is the normalized noise fluctuation profile introduced earlier. Having the mean and variance of the Gaussian probability distribution for $\Gamma_s(k, j)$, we can evaluate the noise average in eq. (4). The result is $D(k, j) = \exp \left[-\overline{\Gamma^2_s(k, j)}/2 \right]$. One expects that when the phase uncertainty $\sqrt{\overline{\Gamma^2_s(k, j)}} \sim 2\pi$, the superposition in $|\psi(t_f)\rangle$ will have been dephased/decphored by the noise. Inserting $\overline{\Gamma^2_s(k, j)} \sim (2\pi)^2$ into our expression for $D(k, j)$.
gives \( D(k,j) \sim 3 \times 10^{-9} \) so that the off-diagonal elements of \( \rho(t_f) \) (see eq. (3)) are effectively zero and the noise has in fact caused an effective collapse of the wavefunction \( |\psi(t_f)\rangle \). One can show that the noise variance \( \sigma^2 \), the average noise power absorbed by \( Q \) per unit volume \( \overline{P}/V \), and the effective bandwidth \( \Delta \omega \) of the absorbed noise power are related: \( \overline{P}/V = \sigma^2 \Delta \omega \). Using this relation, together with eq. (5), one can re-express the condition for onset of decoherence \( \sqrt{\Gamma_x^2(k,j)} \sim 2\pi \) as

\[
\frac{\eta}{16\pi^2} \left( \frac{\sigma^2}{\Delta \omega} \right) \left( \frac{\overline{P}}{V} \right) I_{jk} \sim 1.
\]  

We consider a number of applications of this general analysis in the remainder of this Letter.

2. Dephasing the AGP: In our first application we reconsider the original AGP scenario [1], allowing for noisy control. Here \( Q \) is a single qubit, and \( B_{a}(t) \) precesses about the z-axis with period \( T \) at an angle \( \theta_0 \). Let \( C \) denote the contour traced out by the tip of \( B_{a}(t) \) in a time \( T \). To observe the AGP, the initial state must be a superposition: \( |\psi(0)\rangle = c_+|E_+(0)\rangle + c_-|E_-(0)\rangle \). Eq. (3) gives the matrix elements of the final density matrix. The diagonal elements are real-valued, and thus contain no information about the AGP. The off-diagonal elements, however, do depend on the AGP through \( \Gamma_{a}(\pm) = \Gamma_{a}(+) - \Gamma_{a}(-) \), where \( \Gamma_{a}(\pm) \) are defined below eq. (2).

In an NMR experiment, observation of the AGP is carried out by measuring the transverse magnetization whose expectation value depends on \( \rho_{+-}(t_f) \) [11, 12]. As the analysis of Section 1 showed, sufficiently noisy control causes \( \rho_{+-}(t_f) \) to effectively vanish so that all physical consequences of the AGP thus become unobservable. The stochastic phase shifts \( \Gamma_{a}(\pm) \) generated by noisy control dephase the final superposition state, reducing it to a mixture of the states \( |E_{\pm}(0)\rangle \) in which all AGP effects are absent. In the NMR setting, the precession of \( B_{a}(t) \) is produced by varying the phase of the rf magnetic field \( B_{r}(t) = B_{r}(t)\hat{x} \). If we imagine the noise is due to rf power fluctuations, \( H_{s}(t) = -(\gamma/2)B_{a}(t)\sigma_{x} \) which identifies \( \hat{O} = \hat{x}\sigma_{x} \). One can show that for this type of noise \( I_{+-} = 4\tau_{c}T\sin^{2}\theta_{0} \). If we denote the static magnetic field that splits the nuclear energy levels by \( B_{0} = B_{0}\hat{z} \), it is well-known [12] that \( \sin^{2}\theta_{0} = B_{r}^{2}/B_{0}^{2} + (B_{0} - B_{r})^{2} \). Using this result for \( I_{+-} \) in eq. (6) gives:

\[
\frac{\eta \gamma T}{4\pi^2} \left[ \frac{B_{r}^{2}}{B_{r}^{2} + (B_{0} - B_{r})^{2}} \right] \left( \frac{\overline{P}}{V} \right) \left( \frac{\tau_{c}}{\Delta \omega} \right) \sim 1.
\]  

Eq. (7) identifies the noise properties that impact decoherence, and allows a quantitative assessment of how much noise can be tolerated before all AGP effects are dephased by noise. We believe that this is the first demonstration of how noisy control causes a dephasing of the AGP, and the first quantitative analysis of the onset condition for this dephasing mechanism.

3. Shor Algorithm on a Noisy GQC: We now consider how noisy control impacts the performance of a GQC. The universal set \( U \) of quantum gates used to construct a GQC contains the 1-qubit Hadamard gate \( H \) and the set of all possible 2-qubit controlled-phase gates \( B(\phi) (\phi \in [0, 2\pi]) \). The action of \( B(\phi) \) on the 2-qubit computational basis states (CBS) \( |xy\rangle \) is: \( B(\phi)|xy\rangle = \exp[\text{i}xy\phi]|xy\rangle \), where \( x, y = 0, 1 \). The AGP is used to encode the conditional phase shift \( \phi \). We focus on the operation of \( B(\phi) \) in the presence of noise as it is the only gate in the set \( U \) that can introduce entanglement into the dynamics of a GQC. The conditional phase shift \( \phi \) is implemented using a four part pulse sequence \( P = P_0P_1P_2P_3 \), with \( P_0 \) (\( P_3 \)) applied first (last). Here \( P_0 = C_{\pi}^{1}; P_1 = \overline{C_{\pi}}^{2}; P_2 = P_0; \) and \( P_3 = P_1 \), where \( C \) is the cyclic evolution of \( B_{a}(t) \) introduced in Section 2; \( \bar{C} \) is the time-reverse of \( C \); and \( \pi_{1} (\pi_{2}) \) is a \( \pi \)-pulse applied to the first (second) qubit. By appropriate choice of \( C \), any phase shift \( \phi \) can be produced. The 1-qubit CBS are: \( |0\rangle = |E_{-}(t)\rangle \) and \( |1\rangle = |E_{+}(t)\rangle \). Let \( k = (i_1, i_2) \) with \( i_1, i_2 = 0, 1 \); then \( |E_{k}(t_0)\rangle \equiv |E_{k,i_1}(t_0)\rangle \otimes |E_{k,i_2}(t_0)\rangle \). The pulse sequence \( P \) maps \( k = k_1 k_2 k_3 k_4 \) to \( k = (i_1 \oplus \sum_{l=1}^{i_2} 1 \text{ mod } 2, i_2 \oplus \sum_{l=0}^{i_1} 1 \text{ mod } 2) \), and \( \oplus \) is addition modulo 2. For \( \psi(t_0) = |E_{k}(t_0)\rangle \), eq. (2) gives the final state \( |\psi(t_f)\rangle \) with \( \Gamma_{a}(k) = (1/\hbar) \sum_{l=0}^{3} \Gamma_{a}(k_l); \Gamma_{a}(k) = (1/\hbar) \sum_{l=0}^{3} \Gamma_{a}(k_l); \Gamma_{a}(k_l) = \int_{t_0(t_f)} dt |\hat{E}_{l}(t_{f}) - \hat{h}_{\phi_{k_{l}}}(t)|; \Gamma_{a}(k_l) = \int_{t_0(t_f)} dt |\hat{H}_{s}^{1} \hat{h}_{k_{l}}(t)|, \) where \( \tau_{c} \ll T \) in the adiabatic limit, the \( \Gamma_{a}(k_l) \) are uncorrelated so that \( \Gamma_{a}^{2}(k,l) = \sum_{i=0}^{3} \Gamma_{a}^{2}(k,l)^{2} \). Because the \( \Gamma^{2}(k,l) \) are also statistically independent, one can show that \( \frac{\overline{P}}{V} = \exp[-\Gamma_{a}^{2}(k,l)/2] \) and \( \Gamma_{a}(k,l) \) is given by eq. (5) with \( I_{k_{l}} \rightarrow \sum_{i=0}^{3} I_{k_{l}} \). Here \( I_{k_{l}} \) has the same form as \( I_{k_{l}} \), except that \( \hat{O}_{k_{l}}(t) = \hat{O}_{k_{l}}(t) \). The condition for onset of decoherence is again \( \Gamma_{a}^{2}(k,l) \sim 4\pi^{2} \). As an example, imagine we input the Bell state \( (1\sqrt{2})(|00\rangle + |11\rangle) \) into \( B(\phi) \). Then \( c_{k} = c_{j} = 1/\sqrt{2}; k = (00); \) and \( j = (11) \). If we assume again that the noise is due to rf power fluctuations, one can show that the noise-averaged entanglement fidelity \( \mathcal{F} \) is \( \langle \psi(t_f)\rangle|\rho(t_f)|\psi(t_f)\rangle = \frac{1}{2} + \frac{1}{2} \cos \Gamma_{a}(k,l)|\hat{D}(t_{f})|, \) with \( \Gamma_{a}(k,l) = \Gamma_{a}(k) - \Gamma_{a}(l) \). Since \( \overline{D}(k,l) = \exp[-\Gamma_{a}^{2}(k,l)/2] \), eq. (4) shows that it goes smoothly to zero with increasing noise variance \( \sigma^{2} \). Thus \( \mathcal{F} = 1/2 \) which is the entanglement fidelity for an initial state which is a uniform mixture of the states \( |00\rangle \) and \( |11\rangle \), indicating that entanglement is de-
stroyed once the control field driving \(B(\phi)\) becomes sufficiently noisy. One can show \(^{1}\) that for this Bell state \(\sum_{l=0}^{A} L_{l} = 32r_{l}T\sin^{2}\theta_{0}\) so that the condition for onset of decoherence (and loss of entanglement) is \((2n/\pi^{2})((\gamma^{2}/\Delta \omega)(|P/V|)(\tau_{c}T\sin^{2}\theta_{0}) \sim 1\). Recent work \(^{13}\) also found that noise will severely impact the performance of \(B(\phi)\).

We now show how a GQC containing noisy controlled-phase gates impacts the computational efficiency of the Shor algorithm for factoring an integer \(N\) \(^{14}\). Number theoretic arguments reduce factoring to finding the period \(r\) of the function \(F(a) = y^{a} \mod N\), where \(y\) is co-prime with \(N\). The algorithm begins by preparing the GQC in the state \(|\psi_{0}\rangle = (1/\sqrt{A+1}) \sum_{j=0}^{A} |jr + l\rangle\). Here \(r\) is the period of \(F(a); l\) is the result of a measurement carried out during preparation of \(|\psi_{0}\rangle\); and \(A\) is the largest integer such that \(Ar + l < q\), where \(q\) is chosen such that \(N^{2} \leq q < 2N^{2}\). The algorithm implements a discrete Fourier transform modulo \(q\) (\(DFT_{q}\)) on \(|\psi_{0}\rangle\): \(|\psi_{1}\rangle = DFT_{q}|\psi_{0}\rangle = \sum_{c=0}^{q-1} \hat{f}(c)|c\rangle\), where

\[
\hat{f}(c) = \sqrt{q} \sum_{j=0}^{q/r-1} \exp \left[ \frac{2\pi i}{q} (jr + l)c + i\Gamma_{s}(c) \right].
\]

The \(DFT_{q}\) requires the application of \((L(L-1)/2\) controlled-phase gates, where \(L \equiv \log_{2} q\). Noisy control of this gate causes the stochastic phase shift \(\Gamma_{s}(c)\) to appear in \(\hat{f}(c)\). One can show \(^{10}\) that \(\Gamma_{s}(c) = 0\), and the variance \(\Gamma_{s}^{2}(c)\) is given by \(\eta = L(L-1)/2\); \(\sigma^{2} = \bar{P}/V\Delta \omega\); and \(I_{k} = \sum_{l=0}^{A} L_{l}\). The final step in the algorithm is a measurement which produces the result \(c\) with noise-averaged probability \(P(c) = |\hat{f}(c)|^{2}\). In the absence of noise, constructive interference in eq. \(^{8}\) occurs when \(|rc - cr'q| \leq r/2\), which determines a unique \(c'\), and consequently, \(P(c') = P(c)\). The measurement result \(c\) yields the period \(r\) only if \(c'\) and \(r\) are co-prime. The success probability for the algorithm is then

\[
P_{suc} = \sum_{c'} P(c'),
\]

where in the primed sum only those \(c'\) appear that are both less than, and co-prime with, \(r\). One can show \(^{10}\) that:

\[
|\hat{f}(c)|^{2} = \frac{1}{q} + \frac{2\pi^{2}}{q} \sum_{k=0}^{q/r-1} \sum_{j=k}^{q} \cos \left[ \frac{2\pi(j-k)}{q} (rc \mod q) \right] = D(k,j),
\]

where \(D(k,j)\) is the decoherence factor due to noise. As above, \(D(k,j) = \exp \left[ -T_{2}^{2}(k,j)/2 \right]\) which, by eq. \(^{6}\), goes smoothly to zero as the noise variance \(\sigma^{2}\) increases. In the decoherence limit only the first term on the RHS of eq. \(^{3}\) survives so that \(|\hat{f}(c)|^{2} \sim 1/q \sim 1/N^{2}\). This result is independent of \(c\) so that \(P_{suc} \sim (1/N^{2})\phi(r)\), where \(\phi(r)\) is Euler’s Phi function which gives the number of integers that are less than and co-prime with \(r\). For large

\(N, r \lesssim N\) and \(\phi(r) \sim r/\log r \sim N/\log N\). Thus, in the decoherence limit, \(P_{suc} \sim 1/(N \log N) = 2^{-\log^{2} N/\log N}\). We see that the algorithm must be run on average \(2^{k\log N}\) times to obtain \(P_{suc} \sim 1\). This is exponential in the problem size \(\log N\), indicating that sufficiently strong noise destroys the computational efficiency of the Shor algorithm. This contrasts with the (computationally efficient) noiseless Shor algorithm whose runtime scales linearly with \(\log N\). Onset of decoherence occurs when

\[
\left(\frac{P}{V}\right)(\tau_{c}T\sin^{2}\theta_{0}) \sim \frac{2^{\log N}}{NL(L-1)\gamma^{2}\sin^{2}\theta_{0}}.
\]

Adiabatic operation of a GQC requires \(T\) to be large; and the desire to factor large \(N\) means \(L = \log_{2} q > \log_{2} N\) will also be large. Thus the RHS of eq. \(^{10}\) is expected to be small, so that managing noise will indeed be a significant issue for a GQC after all. Eq. \(^{11}\) allows a quantitative estimate of how much noise can be tolerated by a GQC before decoherence due to noisy control undermines its operation. We believe that the above analysis is the first to quantitatively demonstrate how noisy control destroys the efficiency of the Shor algorithm when run on a sufficiently noisy GQC.

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