Anthropic Explanation of the Dark Matter Abundance

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Abstract

I use Bousso’s causal diamond measure to make a statistical prediction for the dark matter abundance, assuming an axion with a large decay constant $f_a \gg 10^{12}$ GeV. Using a crude approximation for observer formation, the prediction agrees well with observation: 30% of observers form in regions with less dark matter than we observe, while 70% of observers form in regions with more dark matter. Large values of the dark matter ratio are disfavored by an elementary effect: increasing the amount of dark matter while holding fixed the baryon to photon ratio decreases the number of baryons inside one horizon volume. Thus the prediction is rather insensitive to assumptions about observer formation in universes with much more dark matter than our own. The key assumption is that the number of observers per baryon is roughly independent of the dark matter ratio for ratios near the observed value.
1 Introduction

Our current understanding of string theory suggests a vast landscape, with eternal inflation leading to an infinite number of pocket universes containing different low energy physics [1–3]. In this context, a prediction requires a combination of landscape statistics, the dynamics of eternal inflation, and anthropic considerations. Some quantities may still have a conventionally natural explanation, while for others anthropic arguments are crucial. The most famous example of a quantity whose observed value is well explained by anthropics is the cosmological constant [4].

It is natural to wonder whether the dark matter abundance has an anthropic explanation. For example, models with a large axion decay constant, \( f_a \gg 10^{12} \) GeV, are attractive from the particle physics point of view [5] and arise naturally in string theory. However, in these theories the dark matter abundance is naturally much larger than what we observe [6]. Many authors [7–10], beginning with Linde, have examined the question of whether the dark matter abundance is typical once anthropic weighting is taken into account. These authors have reached a variety of conclusions. Due to the difficulty in simulating universes different from our own and our ignorance about the conditions necessary for life, it is unclear whether a dark matter abundance of even 100 times the observed value prevents observers from forming. Furthermore, even the most stringent assumptions about the requirements for life [9] lead to the conclusion that the observed dark matter abundance is unusually small.

It is not possible to make a prediction in the landscape without regulating the infinities. Eternal inflation produces an infinite number of pocket universes of every type, and each pocket universe is spatially infinite. One attractive recipe for regulating the infinities is the causal diamond measure of Bousso [11], which explains the observed cosmological constant well [12, 13].

Here I focus on the simplest possible scenario: I fix all parameters to their observed values and fix the axion decay constant to a large value, \( f_a \gg 10^{12} \) GeV. I assume that axions make up all of the dark matter. The only parameter which is allowed to vary is the initial axion misalignment angle \( \theta_i \). In terms of observable quantities, this means that the baryon to photon ratio is held fixed, while the ratio of dark matter to ordinary matter is allowed to vary. In this simple context, the causal diamond measure just tells us to count

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1I apologize for missing references throughout this note. Please let me know if your work should be cited.
the number of observations within the backward lightcone of one geodesic. In other words, we can get a finite answer from spatially infinite universes by restricting our attention to one horizon volume. (The “horizon” throughout this paper refers to everything within the backward lightcone of one geodesic, and never to the apparent horizon.)

In this note, I show that dark matter abundances much larger than what we observe are disfavored in the causal diamond measure by an elementary effect: increasing the amount of dark matter while holding fixed the baryon to photon ratio decreases the number of baryons inside our causal patch. Quantitatively, if ζ is the ratio of dark matter to ordinary matter, the number of baryons $N_b$ in the causal patch is

$$N_b \propto \frac{1}{1 + \zeta}.$$  \hspace{1cm} (1.1)

This may be surprising, because it seems that we are not subtracting any baryons when we increase the amount of dark matter. To see that the above formula is correct at least at one time, consider the time $t_\Lambda$ when the cosmological constant begins to dominate the energy density. The horizon volume at this time is set by the cosmological constant, independent of the dark matter ratio $\zeta$. By definition, at the time $t_\Lambda$ the total matter density is equal to the vacuum energy density, $\rho_\Lambda(t_\Lambda) = \rho_m(t_\Lambda)$, so the total amount of matter inside the horizon at the time $t_\Lambda$ is independent of $\zeta$. Therefore, the number of baryons inside the horizon is $N_b(t_\Lambda) \propto 1/(1 + \zeta)$. Incidentally, the time $t_\Lambda$ has only a tiny dependence on $\zeta$.

A similar effect is likely to be present in measures other than the causal diamond measure. For example, the scale factor measure [14] in the homogeneous approximation effectively weights vacua by the physical density of observers [15]. The density of baryons at fixed time is proportional to $1/(1 + \zeta)$, so the same weighting factor is recovered.

While this effect is independent of the details of the dark matter, the prediction for the dark matter abundance depends on the prior distribution for the dark matter density. The amount of dark matter is determined by the initial axion misalignment angle $\theta_i$, and symmetry guarantees that $\theta_i$ is randomly distributed. For “unnaturally” small dark matter ratios, $\theta_i$ is near its minimum, and $\zeta \propto \theta_i^2$. This leads to a prior distribution for the ratio of dark matter to ordinary matter

$$\frac{dP}{d\zeta} \propto \frac{1}{\sqrt{\zeta}} \quad \text{(prior).}$$  \hspace{1cm} (1.2)

In principle, after computing the prior probability distribution for $\zeta$ and the number of baryons as a function of $\zeta$, one should compute the number of observations per baryon
as a function of $\zeta$. Here I follow Tegmark, Aguirre, Rees, and Wilczek [9] and assume that the number of observations per baryon is roughly constant for dark matter ratios in the range $2.5 < \zeta < 100$, while using the approximation that outside of this range there are almost no observations per baryon. I find that 30% of observers see less dark matter than we do, while 70% see more. Therefore, our observations are quite typical. The distribution is rather broad: in our approximation, 68% of observers see a dark matter ratio $\zeta \lesssim 15$, while 95% of observers see $\zeta \lesssim 65$.

The results are rather robust, which is good because the conclusions of [9] are controversial. For example, Hellerman and Walcher [8] conclude that, to the best of our current understanding, observers may form up to $\zeta \approx 10^5$. I show that the statistical prediction here is only mildly sensitive to assumptions about observer formation at large $\zeta$, unlike in the conventional anthropic analysis: assuming that observers form efficiently all the way up to $\zeta = \infty$ only has the effect of changing the number of observers who see less dark matter than we do from 30% to 25%.

While the prediction for the dark matter abundance agrees well with observation, it may not be absolutely persuasive to antianthropicists because the probability distribution is rather broad and the dark matter ratio has already been measured. Confirmation of these ideas could come from observation, because axionic dark matter has distinctive observational signals in the form of isocurvature perturbations [10, 18].

In the next section, I review the prior distribution for the dark matter abundance in high-scale axion models. In section 3, I compute the number of baryons inside the horizon as a function of $\zeta$ and describe the approximation for the number of observers per baryon. In section 4, I combine these results to get a statistical prediction for the dark matter abundance. In section 5, I mention several interesting future directions.

## 2 Prior Distribution

We focus on a small part of the landscape. We fix the axion decay constant to a large value, $f_a \gg 10^{12}$ GeV, and assume that the energy scale of inflation is significantly smaller than $f_a$. We consider the set of vacua exactly like ours, but with varying initial axion misalignment angle $\theta_i$. To be concrete, we could imagine that slow roll inflation begins by a tunneling event from a metastable false vacuum, and that the shift symmetry of the axion is unbroken in the entire regime of interest. In other words, the axion is basically a spectator in the tunneling event. Then every time a bubble forms the axion is roughly
homogeneous throughout the interior of the bubble, with a random initial misalignment angle.

Once the universe cools sufficiently, the potential for the axion becomes important. The axion field is approximately homogeneous in one horizon volume and acts like dark matter [6]. The amount of dark matter relative to baryonic matter is determined by the misalignment angle. In models with a large axion decay constant, the natural value for the dark matter abundance is much larger than what we observe, so values near the observed value correspond to the axion being rather close to the minimum of its potential. In this regime, the potential is approximately quadratic, so the dark matter abundance is

$$\zeta \propto \theta^2. \quad (2.3)$$

Therefore, the prior probability distribution for $\sqrt{\zeta}$ is flat between zero and some large value where our quadratic approximation breaks down. Changing variables, the prior distribution for the dark matter ratio $\zeta$ in the scenario with a large axion decay constant is

$$\frac{dP}{d\zeta} \propto \frac{1}{\sqrt{\zeta}} \quad \text{(prior).} \quad (2.4)$$

3 Anthropic Considerations

We now compute the effect of increasing the dark matter abundance on the number of observers. The analysis in this section is independent of the details of the dark matter. However, inspired by axion models, we hold fixed the baryon to photon ratio, and simply increase the amount of dark matter. This means that at decoupling, the density of baryons is independent of the dark matter abundance, while the total matter density at decoupling increases. Since we are assuming a flat universe, the Hubble parameter at decoupling will increase as the dark matter abundance increases.

We first compute the number of baryons inside the horizon as a function of time, then discuss the number of observations per baryon.

3.1 Number of baryons inside the horizon

We are simply counting the number of baryons in the backward lightcone of future infinity as a function of time. The number of baryons inside the horizon can be computed in the

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2 Smaller causal diamonds have been used in [12], but the fundamental definition of the causal diamond measure is to count everything within a large causal diamond which extends beyond our own bubble. This
homogeneous approximation. Assuming a flat universe, the metric is

\[ ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) \] (3.5)

We begin at cosmological constant domination and work backwards. By definition, \( t = t_\Lambda \) when \( \rho_m = \rho_\Lambda \). Therefore,

\[ \rho_b(t_\Lambda) = \rho_\Lambda / (1 + \zeta) . \] (3.6)

The horizon volume at the time \( t_\Lambda \) is set by the de Sitter radius, independent of the dark matter abundance. Therefore the baryon mass inside the horizon at the time \( t_\Lambda \) is

\[ M_b(t_\Lambda) \propto 1/(1 + \zeta) . \] (3.7)

More generally, since observations can be made at any time, we need the number of baryons inside the horizon as a function of time. Note that both the Hubble parameter at vacuum domination and the total matter energy density \( \rho_m \) at vacuum domination are independent of the dark matter ratio \( \zeta \) in the approximation that radiation makes up a negligible fraction of the energy density at \( t_\Lambda \). Therefore, the scale factor as a function of time from vacuum domination, \( a(t - t_\Lambda) \), and the total matter density, \( \rho_m(t - t_\Lambda) \), are independent of the ratio \( \zeta \). Since the scale factor is unaffected by \( \zeta \), the volume inside the horizon, \( V(t - t_\Lambda) \) is also independent of \( \zeta \). On the other hand, the energy density in radiation, \( \rho_\gamma \), does depend on \( \zeta \).

Therefore, as long as radiation is a negligible fraction of the energy density, the total mass inside the horizon, \( M(t-t_\Lambda) \), is independent of the dark matter ratio \( \zeta \). So the number of baryons inside the horizon, for all times with negligible energy density in radiation, is given by

\[ N_b(t - t_\Lambda) = \frac{1}{1 + \zeta} N_0(t - t_\Lambda) \] (3.8)

where \( N_0(t - t_\Lambda) \) is a universal function which does not depend on \( \zeta \).

Of course the dark matter ratio does affect the early universe. The baryon to photon ratio is fixed, so at decoupling the ratio \( \rho_b/\rho_\gamma \) is independent of \( \zeta \). So the total matter density at decoupling depends on \( \zeta \),

\[ \rho_m(t_{DC}) \propto 1 + \zeta \] (3.9)

is equivalent, within our bubble, to counting everything in the backward lightcone of future infinity.
Therefore matter-radiation equality happens at a higher temperature as $\zeta$ is increased. Quantitatively, the matter-radiation ratio redshifts as
\[
\frac{\rho_m}{\rho_\gamma} \sim a \quad (3.10)
\]
and the temperature redshifts as $T \sim 1/a$, so
\[
T_{eq} \propto 1 + \zeta \quad (3.11)
\]
The matter density is also higher at equality,
\[
\rho_m(t_{eq}) \propto T_{eq}^4 \propto (1 + \zeta)^4 \quad (3.12)
\]

To summarize, the bottom line is that $\zeta$ affects the early universe, changing the time and temperature at matter-radiation equality. But for all values of $\zeta$ the universe matches onto a universal late time behavior (in the homogeneous approximation) as soon as the energy density in radiation is much less than than the energy density in matter. During this era, $a(t - t_\Lambda)$ and $\rho_m(t - t_\Lambda)$ are both universal functions which are independent of $\zeta$, so the total mass inside the horizon $M(t - t_\Lambda)$ is also independent of $\zeta$. So the number of baryons at any time with negligible energy density in radiation is a universal function times the factor $1/(1 + \zeta)$.

### 3.2 The number of observations per baryon

Now in principle, we should count the number of observations inside the causal patch by multiplying the number of baryons by the number of observations per baryon per unit time:
\[
N_{obs} = \int dt N_b(t)f(t) \quad (3.13)
\]
where $f(t)$ is the number of observations per baryon per unit time. One estimate for the function $f(t)$ is that it is proportional to the collapsed baryon fraction. We will not try to compute $f$ in detail here, because it is quite difficult and much work has already been done.

To proceed, we make a crude approximation. First we need a lower bound on the amount of dark matter. As the amount of dark matter is decreased, the size of the biggest nonlinear structures decreases. According to Tegmark, Aguirre, Rees, and Wilczek [9], density perturbations at scales similar to our galaxy become nonlinear only if
\[
Q \gtrsim 10^{-5} \left( \frac{1 + \zeta_0}{1 + \zeta} \right)^{4/3} \quad (3.14)
\]
where $\zeta_0 \approx 5$ is the dark matter ratio in our universe and $Q \approx 2 \times 10^{-5}$ is the magnitude of the density perturbations. This criterion gives

$$\zeta_{\text{min}} \approx 2.5.$$  

It is interesting to note that while the above bound depends on $Q$ and $\Lambda$, [9] give other bounds on the dark matter ratio which are more general. For example, structure formation is seriously impeded by Silk damping when $\zeta \lesssim 1$, independent of both $Q$ and $\Lambda$, so a lower bound in the neighborhood of $\zeta \approx 1$ is somewhat robust.

What is the upper bound at which the amount of dark matter starts to seriously impair observer formation? This is a controversial subject, and in any case the most stringent upper bound in the literature is that of [9] who find $\zeta \lesssim 10^2$. On the other hand, Hellerman and Walcher [8] find no compelling evidence for a bound stronger than $\zeta \lesssim 10^5$. One additional consideration in the causal diamond measure is that the number of baryons inside the causal diamond becomes very small after the cosmological constant dominates the vacuum energy, so if for large $\zeta$ observer formation is delayed past the time $t_\Lambda$, the probability of observing such large $\zeta$ will be suppressed.

Here, we will use the results of [9] and approximate the number of observers per baryon as a constant in the range

$$2.5 < \zeta < 100$$

while assuming that there are approximately zero observers per baryon outside this range. We will also demonstrate that the prediction is robust against changing the assumptions. Clearly, there is room for improvement in these considerations.

## 4 Prediction for the Dark Matter Abundance

Now we can combine the known prior distribution for axion dark matter with the anthropic counting of observers to generate the probability distribution for the observed dark matter abundance. To get the final probability distribution for $\zeta$, we multiply the prior distribution

$$\frac{dP}{d\zeta} \propto \frac{1}{\sqrt{\zeta}} \quad (\text{prior})$$

by the number of baryons inside the causal patch,

$$N_b \propto \frac{1}{1 + \zeta}$$

which gives

$$\zeta_{\text{min}} \approx 2.5.$$
and the number of observations per baryon,

\[ \frac{N_{\text{obs}}}{N_b} \approx \text{constant, for } 2.5 < \zeta < 100 \]
\[ \frac{N_{\text{obs}}}{N_b} \approx 0 \quad \text{otherwise.} \]  

(4.19)

Thus the final probability distribution for the dark matter abundance

\[ \frac{dP}{d\zeta} \propto \frac{1}{\sqrt{\zeta} (1 + \zeta)} \quad 2.5 < \zeta < 100 \]
\[ \frac{dP}{d\zeta} \approx 0 \quad \text{otherwise} \]  

(4.20)

This probability distribution is pictured in figure 1, along with the “conventional” anthropic probability distribution, which instead counts the number of observers per baryon.

Figure 1: The probability distribution for the dark matter ratio \( \zeta \) is plotted as a function of \( \sqrt{\zeta} \). The conventional anthropic analysis gives approximately a flat distribution in the range \( 2.5 \lesssim \zeta \lesssim 100 \) (horizontal line), while the causal diamond measure gives an additional factor of \( 1/(1 + \zeta) \) (curved line). The vertical line shows the observed value \( \zeta = 5 \). In the conventional analysis only about 8% of observers form in regions with less dark matter than we observe, even with the most favorable assumptions about observer formation. In the causal diamond measure the observed value is fairly typical: 30% of observers form in regions with less dark matter than we observe. The causal diamond prediction is rather insensitive to assumptions about observer formation at large \( \zeta \).

To address the typicality of our observations, it makes sense to compute how much of the probability distribution is on each side of the observed value. Integrating (4.20), we
find that 30% of the probability is at smaller $\zeta$, while 70% is at larger $\zeta$, so our observations are quite typical. The probability distribution is fairly broad: 68% of observers form in regions with $\zeta \lesssim 15$, and 95% of observers form in regions with $\zeta \lesssim 65$.

Since observer formation at large $\zeta$ is poorly understood, let’s see what happens to our distribution if we assume that the number of observations per baryon is constant all the way up to $\zeta = \infty$. With this assumption, 25% of observers form in regions with less dark matter than us, so our observations are still fairly typical. Since the prediction is not significantly affected, it is not urgent to understand observer formation for very large dark matter ratios $\zeta > 100$.

To probe the sensitivity of our results further, assume for a moment that the number of observers per baryon is constant for $1 < \zeta < 100$. With this assumption, 53% of observers form in regions with less dark matter than we have. While it is probably not realistic to think that structure formation in our universe can proceed effectively down to $\zeta \approx 1$, as we mentioned in the previous section once $Q$ and $\Lambda$ are allowed to vary the lower bound $\zeta_{\text{min}} \approx 2.5$ is not valid in general, while the lower bound $\zeta_{\text{min}} \approx 1$ is valid [9].

5 Future Directions

It will be interesting to extend these considerations to a larger landscape in which more parameters are allowed to vary. It is particularly interesting to ask what happens once the cosmological constant and the density contrast $Q$ are allowed to vary along with the dark matter ratio. A more precise prediction within the small landscape studied here would require an improved analysis of observer formation for a range of dark matter ratios $0 < \zeta \lesssim 1000$, although as we explained large values of $\zeta$ have only a small effect on the prediction.

In supersymmetric field theory and string theory models, one has to worry about cosmological problems arising from the Saxion and other moduli [16]. It is important to verify that the positive results found here survive in a more detailed model. More generally, it would be interesting to repeat the statistical arguments here in the presence of other sources of dark matter in addition to the axion.

An additional interesting direction is the search for observational confirmation of these ideas. As discussed recently by [10, 17, 18] axionic dark matter has observational signatures in the form of isocurvature perturbations; these signals are enhanced when the axion misalignment angle is “unnaturally” small due to anthropic selection [18]. These pertur-
bations can be produced during inflation if the inflation scale is high enough. They can also be produced prior to inflation and survive observationally if inflation does not go on for too long. In fact, Kaplan and Nelson [18] conclude that for some parameter choices, isocurvature perturbations from the preinflationary era will be observable if there are fewer than about 74 efoldings of slow roll inflation. (This is in conventions where the bound coming from the observed flatness of the universe corresponds to 60 efoldings.) Combined with arguments [19] that inflation is quite likely to last for fewer than 74 efoldings, this is a very exciting possibility which deserves further study.

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