SINR Maximization Beam Selection for mmWave Beamspace MIMO Systems

IGBAFE ORIKUMHI, (Member, IEEE),
JEONGWAN KANG, (Graduate Student Member, IEEE),
HYEKYUNG JWA, JEE-HYEON NA, (Member, IEEE),
AND SUNWOO KIM, (Senior Member, IEEE)

1Department of Electronics and Computer Engineering, Hanyang University, Seoul 04763, South Korea
2Electronics and Telecommunications Research Institute, Daejeon 34129, South Korea

Corresponding author: Sunwoo Kim (remero@hanyang.ac.kr)

This work was supported in part by the Korea Institute of Energy Technology Evaluation and Planning (KETEP)
Grant funded by the Korea Government Ministry of Trade, Industry and Energy through the Human Resources Development Program under
Grant 20194010201860, in part by the Institute of Information and Communications Technology Planning Evaluation (IITP)
Grant funded by the Korea Government [Ministry of Science and ICT (MSIT)] (5G Open Intelligence-Defined RAN (ID-RAN)
Technique based on 5G New Radio) under Grant 2018-0-01659, and in part by the National Research Foundation of Korea (NRF) funded by the Ministry of Education through the Basic Science Research Program under Grant 2020R1H1A1A01073438.

ABSTRACT Beamspace multiple-input multiple-output (MIMO) systems employ beam selection algorithms
based on the sparsity of the beamspace channel to reduce the number of radio-frequency chains required
in multi-user systems. In this article, we address the problem of selecting multiple beams to each user.
Specifically, we propose a beam selection scheme subject to signal to interference plus noise ratio (SINR)
maximization. The proposed beamspace beam selection algorithm is developed in two stages. In the first
stage, each user is assigned a non-overlapping beam determined by Kuhn-Munkres assignment algorithm.
In the second stage, the selected beams in the previous stage are augmented with additional beams from an
M-dominant beam set subject to a sum-rate maximization criterion. The performance of the proposed scheme
is verified by simulations, and the results show a near-optimal performance in terms of the achievable sum-rate. Also, a significant improvement in terms of energy efficiency is achieved compared to the conventional
beam selection scheme.

INDEX TERMS Beamspace, massive MIMO, millimeter wave communications, multi-user interference,
beam selection, beamforming.

I. INTRODUCTION
In recent years, there has been an increasing demand for broadband which has led both researchers and industries to
explore new solutions for the fifth-generation (5G) wireless communication systems. As part of the solutions, the millimeter wave (mmWave) band has received a great deal of consideration due to the large bandwidth available at this frequency band. The mmWave band provides large swaths of bandwidth at higher frequencies spectrum with carrier frequencies of 30-300 GHz.

Notwithstanding the great potential benefits, several issues need to be addressed before the benefits of the mmWave band can be fully harnessed. One major issue is that the mmWave band suffers from severe penetration and path-loss [1], [2]. A key solution to overcoming the high loss and blockage effect is the beamforming techniques in which large array antennas are employed at the base station (BS). The use of large array antennas which is also termed as massive MIMO (m-MIMO) systems allows a good angular resolution, such that the transmitted radio signals can be spatially resolved. The benefits of the angular resolution obtained from multiple antennas systems has been exploited in localization and detection of multiple user equipment (UE) such as swarm of unmanned aerial vehicle [3]. In addition to localization and detection, the m-MIMO system enables multiple users to be served from a transmitter in the same resource block [4]. Hence, m-MIMO has been considered as a viable technology to meet the increasing broadband demand in 5G cellular communication.

The mmWave m-MIMO technology combines the prospects of the huge available mmWave bandwidth on the one hand, and the large array gains from m-MIMO antenna arrays on the other hand to support high-speed services and
applications requiring large bandwidth, making it suitable for industrial big data transmission [5]. Besides, the large antenna array allows the use of highly directional beamforming, hence, multiple beams can be employed for communication between the transmitter and receiver. As shown in [6], [7], mmWave systems can provide high-dimensional MIMO operations. However, the full potential of multiple beams transmission in mmWave systems is not exploited in most of the current results. For instance, in the schemes proposed in [8], [9], the authors focused on single beam transmission.

The use of multiple beams at the BS is considered in [10], [11], where exhaustive beam search is performed to determine the best set of beams to the UE. The exhaustive search beams selection poses some challenges of high computation and transceiver design complexity [12]. To reduce the complexity, several schemes have been proposed such as antenna selection [13], [14] and random beamforming [15], but these solutions are sub-optimal and therefore lead to system performance degradation. In [16], a beam selection scheme for multi-user mmWave MIMO was proposed based on compressed sensing approach with analog beamformers. The computational complexity of the exhaustive search beam selection can also be reduced by estimating and tracking the channel parameters such as the direction of arrival and direction of departure [17]. The estimation of angular information is essential in radar communications [18], [19] to enable big data transmission. Reducing the complexity of the exhaustive search algorithm based on parameter estimation are beyond the scope of this article. Specifically, we seek to reduce the complexity of the exhaustive search algorithm by exploiting the sparsity of the mmWave channel. Recently, new attempts have been made to reduce the beam selection complexity in the angular domain [20], [21].

In this article, we focus on exploiting the concept of beamspace MIMO [6], where multi-user data are multiplexed onto orthogonal spatial beams. The use of beamspace MIMO allows the conventional channel to be transformed into the angular domain that captures the mmWave channel sparsity [22]. Furthermore, exploiting beamspace MIMO enables beams to be selected according to the beamspace channel sparsity, leading to a dramatic reduction in transceiver and hardware complexity. In [23], [24], beamspace MIMO is proposed for a single user and multi-users scenario respectively. The authors focused on the number of beams used for transmission and reception rather than the number of antennas at the transmitter and receiver. In a multi-user scenario, most of the existing schemes select beams with large power magnitude to each user without considering the effect of multi-user interference which may lead to non-negligible performance degradation since several users may select overlapping beams. [20], [25].

To this end, we focus on the angular domain beam selection in a multiple user scenario and analyze its performance. The contributions of this article are presented as follows

- Firstly, a multi-user beamspace beam selection is proposed subject to the sum-rate maximization constraints. Specifically, we propose a beamspace beam selection in a potential multi-user interference scenario. The beam selection scheme employs the use of zero-forcing (ZF) precoding at the BS to simultaneously serve multiple users. The proposed beam selection scheme is designed in two stages.
  - In the first stage, a single non-overlapping beam is selected for each user based on Munkres assignment algorithm [26].
  - In the second stage, unassigned beams whose contribution do not degrade the performance of the system are selected from a set of $M$-dominant beams to argument the beams selected in the previous stage subject to the criterion of sum-rate maximization.

- Secondly, the computational complexity of the proposed beam selection scheme is analyzed with regards to the conventional beamspace MIMO.
- Finally, the trade-off between the complexity and performance achieved by the proposed beam selection scheme is modelled and analyzed in terms of the number of selected radio frequency (RF) chains and the energy efficiency of the system.

The proposed scheme is designed to achieve near-optimal performance with a reduced RF complexity transceivers. This work is different from existing work [20], [27] in that, the channel rank deficiency associated with multiple UEs selecting the same beam and the multi-user interference between the users are considered in the proposed beam selection algorithm. The multi-user interference is not considered in [20], while [27] focus on a single beam transmission to each UE and hence, cannot support multiple beam transmission. Simulation results are provided to verify the performance of the proposed beam selection scheme.

**NOTATIONS**

Throughout this article, matrices and vector symbols are represented by uppercase and lowercase boldface respectively. $A^T$ and $A^H$ represent the transpose and Hermitian transpose of the matrix $A$ respectively. $\text{tr}(A)$ represent the trace of matrix $A$.

**II. RECEIVED SIGNAL AND CHANNEL MODELS**

We focus on a downlink mmWave massive MIMO system, where the BS is equipped with $N$ antennas and $N_{RF}$ RF chains serving $K$ distributed single-antenna UEs as shown in Fig. 1. Note that in Fig. 1, the number of RF chains required for a traditional system is $N_{RF} = N$ which is very large in m-MIMO mmWave systems [28]. Hence, we aim to design a reduce complexity transceiver that achieves a near-optimal performance with a low number of RF chains.

For the downlink transmission to multiple UEs, the BS jointly precodes the multi-user signal. Specifically, using a linear precoder, the BS first precodes the data as follows

$$s = Gx = \sum_{k=1}^{K} g_{k} x_{k}. \tag{1}$$
where \( \mathbf{g}_k \in \mathbb{C}^{N \times 1} \) is the precoding vector for the \( k \)-th UE, \( \mathbf{x} = [x_1, x_2, \ldots, x_K]^T \in \mathbb{C}^{K \times 1} \) is the vector of independent symbols to different UEs, \( \mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \ldots, \mathbf{g}_K] \in \mathbb{C}^{N \times K} \) is the transmit precoding matrix, \( \text{tr} \left( \mathbf{G} \mathbf{G}^H \right) \leq \rho \), where the inequality represent the constraints on total transmit power \( \rho \). Define \( \Lambda = E[\mathbf{x}\mathbf{x}^H] \) and \( \mathbf{s} \in \mathbb{C}^{N \times 1} \) is the transmitted signal vector.

The received signal at the \( k \)-th UE can be expressed as [29]

\[
y_k = \mathbf{h}_k^H \mathbf{s} + n_k, \tag{2}
\]

where \( n_k \sim \mathcal{CN}(0, \sigma_k^2) \) is the additive white Gaussian noise with 0 mean and variance \( \sigma_k^2 \) at the receiver of the \( k \)-th UE and \( \mathbf{h}_k \in \mathbb{C}^{N \times 1} \) is the channel vectors of the \( k \)-th UE. Due to the sparsity and high directional nature of the mmWave channel, the line of sight (LOS) path is the predominant mode of propagation with possible few multi-path components. Hence, we assume that the LOS component for each of the \( K \) UEs exists. Define \( \theta_{k,0} \) as the angle corresponding to the LOS path for the \( k \)-th UE, where \( k \in \{1, 2, \ldots, K\} \), then the sparse channel \( \mathbf{h}_k \) in (2) can be modelled as [27]

\[
\mathbf{h}_k = \sqrt{N} \left[ \alpha_{k,0} a(\Psi_{k,0}) + \sum_{l=1}^{L} \alpha_{k,l} a(\Psi_{k,l}) \right], \tag{3}
\]

where \( \alpha_{k,0} \) is the complex path gain of the LOS path to the \( k \)-th UE, \( \Psi_{k,l} = 1/2 \sin \theta_{k,l} \) is the discrete physical pointing angle and \( \theta_{k,l} \in [-\pi/2, \pi/2], \theta_{k,l} \) and \( \alpha_{k,l} \) denote the path angle and complex path gain of the \( l \)-th non-LOS path to the \( k \)-th UE, and \( L \) denotes the number of non-LOS paths. Note that the amplitude of the multi-path components \( \alpha_{k,l} \) are usually 5 dB to 10 dB weaker than the LOS component [30]. In this article, we assume a uniform linear array (ULA), hence, the steering vector \( a(\Psi_{k,l}) \) is given by

\[
a(\Psi_{k,l}) = \frac{1}{\sqrt{N}} \left[ e^{-j2\pi(\frac{N-1}{2N})\Psi_{k,l}}, \ldots, e^{j2\pi(\frac{N-1}{2N})\Psi_{k,l}} \right]^T. \tag{4}
\]

Define \( \mathbf{V} \) as the \( N \times N \) beamspace transformation matrix, where the column of \( \mathbf{V} \) correspond to a uniformly sampling spatial angle designed as [31]

\[
\mathbf{V} = \left[ a \left( \frac{N-1}{2N} \right), \ldots, a \left( \frac{N-1}{2N} \right) \right]. \tag{5}
\]

By stacking the received signal \( \mathbf{y} \in \mathbb{C}^{K \times 1} \) for all \( K \) UEs and substituting for \( \mathbf{s} \) (defined in (1)), we obtain the following

\[
\mathbf{y} = \mathbf{H}^H \mathbf{s} + \mathbf{n} = \mathbf{H}^H \mathbf{G} \mathbf{x} + \mathbf{n}, \tag{6}
\]

where \( \mathbf{y} = [y_1, y_2, \ldots, y_K]^T \in \mathbb{C}^{K \times 1} \) is the received signal vectors at the \( K \) distributed UEs, \( \mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K] \in \mathbb{C}^{N \times K} \), and \( \mathbf{n} = [n_1, n_2, \ldots, n_K]^T \in \mathbb{C}^{K \times 1} \). By choosing \( \mathbf{G} = \mathbf{V} \mathbf{G}_b \), the received signal can be represented in the beamspace as follows

\[
\mathbf{y} = \mathbf{H}_b^H \mathbf{G}_b \mathbf{x} + \mathbf{n}, \tag{7}
\]

where \( \mathbf{H}_b = \mathbf{V}^H \mathbf{H} = [\mathbf{h}_{b,1}, \ldots, \mathbf{h}_{b,K}] \in \mathbb{C}^{N \times K} \) is the full-dimensional beamspace matrix containing the beamspace channel of all the \( K \) UEs and \( \mathbf{G}_b \in \mathbb{C}^{N \times K} \) is the beamspace precoder. Since \( \mathbf{V} \) is a unitary matrix, the beamspace channel matrix \( \mathbf{H}_b \) is an equivalent representation of \( \mathbf{H} \). Hence, the rows of \( \mathbf{H}_b \) corresponds to the \( N \) orthogonal beams with different spatial directions.

Note that in mmWave propagation environment, the number of paths in (3) is much smaller than the number of antennas \( N \) which makes the number of dominant elements in \( \mathbf{h}_{b,k} \) much less than \( N \) [22]. As such, we can select a small number of beams that achieve near-optimal performance to reduce the dimension of the MIMO system and the number of RF chains in the transceiver design as shown in Fig. 2. The \( k \)-th UE’s SINR can be expressed as

\[
\text{SINR}_k = \frac{|\mathbf{h}_{b,k}^H \mathbf{g}_{b,k}|^2}{\sum_{k' \neq k} |\mathbf{h}_{b,k}^H \mathbf{g}_{b,k'}|^2 + \sigma_k^2}. \tag{8}
\]

### III. CONVENTIONAL BEAMSPACE BEAM SELECTION

As a reference to the proposed beam selection scheme, the maximum magnitude beam selection scheme based on projected power [20] is first presented, after which the proposed scheme is discussed.

#### A. PER-USER BEAM SELECTION BASED ON PROJECTED POWER (PP-BS)

Considering the complexity of the exhaustive search algorithm in beam selection, the beam selection scheme based on power level selects the \( M \)-dominant elements (beams) of \( \mathbf{h}_{b,k} \) for the \( k \)-th UE, where the magnitude of \( \mathbf{h}_{b,k} \) reflects the \( k \)
UE’s projected power level on the LOS path of the selected beams [20].

The scheme exploits the sparsity structure of the beamspace channel $H_b$ to design a reduced-complexity beamspace precoder matrix $G_b$. This selection algorithm selects the $M$-dominant beams as follows [20]

$$
M_k = \left\{ i \in J(N) : |h_{b,k}(i)|^2 \geq \xi_k \max_i |h_{b,k}(i)|^2 \right\},
$$

(9)

where $J(N) = \{c - (N - 1)/2 : c = 0, 1, \ldots, N - 1\}$, and

$$
M = \bigcup_{k=1,2,\ldots,K} M_k,
$$

(10)

where $M_k$ is a sparsity mask for the $k$-th UE, determined by the threshold $\xi_k \in \{0, 1\}$.

However, the projected power beam selection scheme does not guaranty the orthogonality of the effective LOS path for different UEs mainly because the same beam may be selected for more that one UE. This, in turn, causes multi-user interference and consequently impact the system performance.

**B. LIMITATIONS OF THE CONVENTIONAL PP-BS METHOD**

As can be observed from the PP-BS scheme first proposed in [20], there are two key challenges with the method namely: inter-beam interference and users sharing the same beam. These issues are detailed below. In an ideal scenario, there exists no interference between the beams selected for different users. However, the ideal scenario cannot be guaranteed. For instance, consider a scenario where each UE selects one strongest beam which is different from other UEs, such scenario is equivalent to selecting $K$ out of $N$ total beams. The probability that there exists users sharing the same strongest beam is given as [27]

$$
P = 1 - \frac{N!}{N^K (N-K)!}.
$$

(11)

For a mmWave system with $N = 256$ beams and $K = 32$ UEs, $P \approx 87\%$ which cannot be ignored.

The challenges with user distribution and beam selection are described from Fig 3. As shown in Fig. 3a, the strongest channel gain for UE1 and UE2 is obtained from beam $i_2$, where UE$k$ is used to denote the $k$-th UE. However, selecting the strongest beam $i_2$ for both UEs will result in a rank-deficient dimension-reduced beamspace channel matrix $\tilde{H}_b$. This implies that several UEs cannot be served simultaneously, resulting in an obvious performance loss. To address this issue, the BS can select beam $i_1$ for UE1 and beam $i_2$ for UE2.

The issue of inter-beam interference is illustrated in Fig. 3b. From the figure, the best beams to UE3 and UE4 are beam $i_1$ and beam $i_2$ respectively. However, the inter beam interference between UE3 and UE4 cannot be ignored. To this end, the BS can serve UE3 with beam $i_1$ while simultaneously serving UE4 with beam $i_3$. Fig. 3b indicates that the best beam to the two UEs may not be the optimal beam if the presence of inter beam interference is considered. Due to these limitations, poor performance is observed with the conventional PP-BS scheme as will be shown in the numerical result section.

Unlike [27], where these issues are addressed with a single beam transmission to each UE, in our proposed scheme, each UE may be served by more than one beam if the additional beam does not degrade the total system performance. Hence, we focus on choosing $M$-dominant beams to each UE under a total power constraint $\rho$. Considering the multi-user interference, we aim to select beams for each UE such that the interference is minimized under the constraints that useful power is delivered to all UEs.

**IV. PROPOSED SINR MAXIMIZATION BEAM SELECTION**

The proposed beam selection scheme utilizes the concept of Munkres assignment algorithm. Hence, we first present the algorithm and then refer to it in the proposed beam selection.

**A. BRIEF DESCRIPTION OF MUNKRES ALGORITHM**

Consider the full dimensional beamspace channel matrix $H_b$, since the Munkres algorithm solves a minimization

**FIGURE 3. User distribution scenario (a) users sharing a single beam; (b) inter-beam interference.**
problem [26], the cost matrix can be obtained by taking the inverse of each element in \( \mathbf{H}_b \), i.e., \( \mathbf{C} = \mathbf{1}/\mathbf{H}_b = [c_{n,k}] \), for \( n = 1, \ldots, N; k = 1, \ldots, K \). Note that for the proposed beam selection scheme the number of available beams at the BS should be greater or equal to the number of UEs. In the latter case when the number of beams is equal to the number of UEs (i.e., \( N = K \)), the full dimensional beamspace matrix \( \mathbf{H}_b \) is a squared matrix from which the selection can be performed. When the number of beams is more than the number of UEs (i.e., \( N > K \)), a square matrix must be created to implement the Munkres algorithm. This can be achieve by adding \( N - K \) columns to \( \mathbf{C} \) with cost function \( c_{n,j} = 0 \), \( \forall j > K \). Now define

\[
da_{n,k} = \begin{cases} 1 & \text{if beam } n \text{ has been assigned to the } k\text{-th UE,} \\ 0 & \text{otherwise,} \end{cases}
\]

then the problem can be defined as

\[
\text{minimize} \sum_{n=1}^{N} \sum_{j=1}^{N} c_{n,j}a_{n,j}
\]

s. t. \( \sum_{n=1}^{N} a_{n,j} = 1; \sum_{j=1}^{N} a_{n,j} = 1. \) (13)

Note that for the new problem where the cost matrix \( \mathbf{C} \) is transformed into a square matrix by padding with zeros, if \( a_{n,j} \) for \( n = 1, \ldots, N \) and \( j = 1, \ldots, N \) are optimal assignments, then the restriction \( a_{n,k} \) for \( n = 1, \ldots, N \) and \( k = 1, \ldots, K \) is the optimal solution for the original problem.

**Proof:** Recall that \( c_{n,j} = 0 \), \( \forall j > K \), hence,

\[
\sum_{n=1}^{N} \sum_{j=1}^{N} c_{n,j}a_{n,j} = 0,
\]

then

\[
\sum_{n=1}^{N} \sum_{j=1}^{K} c_{n,j}a_{n,j} = \sum_{n=1}^{N} \sum_{j=1}^{K} \sum_{j=1}^{K} c_{n,j}a_{n,j}.
\]

given the minimum cost of the beam selection problem for any optimal choice of \( a_{n,j} \) for the new problem. \( \square \)

We refer our readers to [26] and reference therein for the steps on the Munkres assignment algorithm. The input of the algorithm is the cost matrix \( \mathbf{C} \) and the number of UEs. The output returns the optimal assignment matrix \( \mathbf{A} \) and the corresponding cost \( \mathbf{C}_0 \) of the assignment. This will be exploited in the design of the proposed beam selection algorithm.

**B. SINR MAXIMIZATION BEAM SELECTION (SM-BS) ALGORITHM**

In the proposed beam selection scheme, \( M \)-dominant beams are chosen for each of the \( K \) UEs. We begin by sorting the elements (beams) of \( \mathbf{h}_{b,k} \) in descending order of magnitude, and let \( i_{b,m} \) denote the index of the \( m \)-th dominant beam to the \( k \)-th UE. The set of indexes of the \( M \)-dominant beams to the \( k \)-th UE can be expressed as

\[
\mathcal{G}_k = \{i_{k,1}, i_{k,2}, \ldots, i_{k,M}\},
\]

then the set of the dominant beam indexes to all \( K \) UEs can be expressed as follows

\[
\mathcal{G} = \bigcup_{k=1}^{K} \mathcal{G}_k.
\]

The selection of the dominant beams for the \( K \) UEs corresponds to selecting a subset of \( Q = |\mathcal{G}| \leq MK \) rows of the matrix \( \mathbf{H}_b \) resulting in a low-dimensional system, where \( Q \leq N_{RF} \).

Note that if \( \bigcap_{k=1,2,\ldots,K} \mathcal{G}_k = \emptyset \), then the selected dominant beam set for all \( K \) UEs do not overlap and the beam set can achieve a near-optimal performance since the dominant beams contain most of the channel power. Moreover, the multi-user interference is also not severe in such case because the set of dominant beams chosen to each UE are different from the selected beams to other UEs. However, even for single beam selection to each of the \( k \) UEs, the probability of multiple UE sharing the same beam is always non-negligible.

To address this issue, we propose an SM-BS scheme consisting of two stages:

- **Stage 1:** From the set of beams corresponding to the index set \( \mathcal{G} \), choose the best single non-overlapping beam for each UE that minimize the multi-user interference.
- **Stage 2:** From stage 1, since \( K \) non-overlapping beams have been selected to all \( K \) UEs, argument the selected beams to each UE from the remaining \( Q-K \) beams in the index set \( \mathcal{G} \) whose influence on the multi-user interference is minimal.

These stages are discussed as follows:

1) **STAGE 1: SINGLE BEST BEAM SELECTION BASED ON MUNKRES ALGORITHM**

The objective is to select the best non-overlapping beam to each UE from the beam set corresponding to \( \mathcal{G} \). We define a subset \( \mathcal{G}^* \subset \mathcal{G} \) consisting of the index of the single best non-overlapping beams to each UE. In this assignment, we consider that each beam is assigned to only one UE and each UE is assigned to only one beam. To achieve this, the BS selects the best unassigned beam to the UE from the UE’s \( M \)-dominant beams set.

As an example, consider a scenario with \( K = 3 \) UEs, \( N = 8 \) antennas at the BS corresponding to 8 beam set, assume \( m = 3 \) dominant beams are selected for each UE. Let the following beam index set \( \mathcal{G}_1 = \{1, 2, 3\}, \mathcal{G}_2 = \{1, 8, 5\} \) and \( \mathcal{G}_3 = \{8, 1, 5\} \) correspond to the magnitude of the \( 3 \)-dominant beams to each UE in decreasing other. It can be observed that UE1 and UE2 share the same beam with index 1 which will lead to inter-user interference if the signals of both UEs are multiplexed on the same beam. From the proposed scheme, \( \mathcal{G} = \{1, 2, 3, 5, 8\} \), and from stage 1, we can select possible non-overlapping beams to each UE as \( \mathcal{G}^* = \{1, 5, 8\} \text{ or } \mathcal{G}^* = \{1, 8, 5\} \). The non-overlapping beam selection addresses the two limitations highlighted in Section III-B. The assignment problem above correspond to a maximum matching in a bipartite graph, and we address
the problem by employing the Munkres algorithm discussed in Section IV-A with a complexity of $\mathcal{O}(\min(Q^3, K^3))$.

2) STAGE 2: MINIMUM INTERFERENCE BEAM AUGMENTATION

After the non-overlapping beams selection in stage 1, we aim to augment the beam selection by allocating the unassigned $Q - K$ beam set from the index set $\mathcal{G}$ whose contribution does not degrade the overall performance of the system.

In stage 2, the unassigned beams from the index set $\mathcal{G}$ are employed to augment the selected beams such that the multi-user interference is minimized and the SINR is maximized. Hence, the performance of the beam set corresponding to $\mathcal{G}^*$ can be augmented with the unused beam set corresponding to $\mathcal{G}$. Note that $\hat{H}_p \in \mathbb{C}^{K \times K}$ from Algorithm 1 correspond to selecting the rows of $\mathbf{H}_b$ whose index set are in $\mathcal{G}^*$ such that

$$\hat{H}_p = H_b(s, \cdot)_{s \in \mathcal{G}^*}.$$  

The objective is to maximize the SINR which results in minimizing the multi-user interference. In the proposed scheme, beams are added to $\hat{H}_p$ as long as the overall performance is not degraded. This can be expressed as

$$C(\hat{H}_p) = K \log_2 \left(1 + \frac{\rho}{\sigma^2 \text{tr} (F + \rho \mathbf{f}_i^H \mathbf{f}_i^{-1})}\right),$$  

where $\mathbf{f}_i = H_b(s, \cdot)_{s \in \mathcal{G}^*}$, $\mathbf{F} = \hat{H}_p^H \hat{H}_p + \epsilon \mathbf{I}$ and $\epsilon$ is a small positive number to ensure the inversion of the matrix $(\mathbf{F} + \rho \mathbf{f}_i^H \mathbf{f}_i^{-1})$ [32]. The beam augmentation in stage 2 of the proposed scheme is to assign beams to the UEs whose minimum SINR requirement are not satisfied due to the fact that in the selection of the best non-overlapping beams (stage 1), such UEs may not be served by the beam corresponding to its LOS path if the beams are selected by other UEs. The method described above is repeated by adding the corresponding row of $\mathbf{H}_b(s, \cdot)$ until all beams that maximize the SINR are assigned. An example of the proposed beam selection algorithm is shown in Fig. 4. We note that the proposed algorithm may not assign all the beams from the index set $\mathcal{G}$ to the UEs especially when the use of such beam degrades the system performance. This can be observed in Fig. 3b where beam $t_2$ must be ignored due to the inter-beam interference with beam $t_1$. The steps for the proposed beam selection scheme is summarized in Algorithm 1.

### C. COMPLEXITY ANALYSIS

In this section, we present the complexity analysis of the proposed scheme and existing schemes. The complexity analyses are summarized in Table 1.

The table shows that the beamspace projected power beam selection scheme reduces the beam search complexity of the exhaustive search scheme from $N^K$ to $N \times K$. In addition, the scheme achieves the lowest complexity as the selection is based only on the amplitude of the channel power projected to the beams. On the other hand, the proposed SM-BS scheme is affected by higher computational complexity. The increased computational complexity is due to the necessity to take into account the effect of the multi-user interference. The proposed SM-BS stage 1 scheme requires addition computational burden of the Munkres algorithm whose complexity is in the order of $\min(Q^3, K^3)$. Due to beam augmentation in the proposed SM-BS stage 2, an additional computational complexity of $(Q - K)^2$ is required.

It is worth noticing that, although the conventional PP-BS scheme has a lower computational complexity compared to the proposed scheme, the performance in a realistic multi-user scenario is degraded due to the multi-user interference, as will be shown in the results. This consideration makes the proposed beam selection scheme relevant in realistic applications, thanks to the appealing trade-off between the performance and computational cost.

### V. NUMERICAL RESULTS AND DISCUSSION

In this section, we present the numerical results using a mmWave m-MIMO system with $N = 81$ array antenna elements serving 40 UEs simultaneously [20]. We denote
the transmit power $\rho$ as the total transmit SNR for $\sigma_k^2 = 1$ for $\forall k$. The results are averaged over $10^3$ Monte Carlo channel realizations. We consider two scenarios with 1) one LOS component only ($L = 0$) and 2) one LOS and 2 non-LOS components ($L = 2$).

A. ACHIEVABLE SUM-RATE

The achievable sum-rate of the proposed beam selection scheme is compared with the conventional projected power beam selection scheme and the full digital system in this section. In the legends, we denote the conventional projected power beam selection as PP-BS and the proposed maximum SINR beams selection scheme as SM-BS.

In Fig. 5, we compare the achievable sum-rate versus SNR of the proposed scheme and the benchmark schemes. From the figure, it is observed that the proposed SM-BS stage 1 scheme outperforms the conventional PP-BS scheme. Note that the proposed SM-BS stage 1 and conventional PP-BS schemes employ one best non-overlapping beam and 2-dominant beams per UE respectively. Thanks to the multi-user interference reduction achieved by selecting the best non-overlapping beams to each UE in the proposed scheme. By increasing the number of dominant beams per UE, a slight improvement is observed between the proposed SM-BS stage 1 with 2-dominant beams and the proposed SM-BS stage 1 with 3-dominant beams resulting from the increased beam selection diversity.

However, we note that increasing the number of dominant beams for the proposed SM-BS stage 1 scheme only yield slight improvement at high SNR but with an increased selection complexity. To improve the performance, the proposed SM-BS stage 2 scheme argument the beam selection in stage 1 by selecting from the unused beams corresponding to the index set $G$. This implies that by increasing the number of dominant beams for each UE and applying the proposed stage 2 scheme, the multi-user interference could be reduced compared to the conventional PP-BS scheme. Also, as the number of dominant beams per UE increases, a near-optimal performance is achieved at the expense of increased selection complexity.

B. AVERAGE NUMBER OF SELECTED BEAMS

The system performance in terms of the average number of RF chains (beams) selected versus the number of UEs is presented for the LOS and the non-LOS scenario. The number of selected beams is fundamental to this study as it provides an understanding of the improvement in the achievable sum-rate and the complexity reduction of the proposed beam selection.

The average number of beams selected for the LOS and the non-LOS scenarios are presented in Fig. 6 and Fig. 7. In Fig. 6, it can be directly deduced that more beams are required in the conventional PP-BS scheme compared to the proposed SM-BS schemes. Although 3-dominant beams are selected for each UE in the proposed SM-BS schemes, we show that the proposed scheme selects only the set of beams that minimize the multi-user interference from the $M$-dominant beams leading to an overall reduction in the number of RF chains required for transmission. This implies that not all the selected dominant beams are used for transmission.

In Fig. 7, we show that the number of beams selected in the proposed SM-BS stage 2 scheme is generally lower compared to the LOS scenario in Fig. 6. On the overall, increasing the number of dominant beams leads to higher sum-rate with an increase in the number of RF chains required for transmission.
I. Orikumhi et al.: SINR Maximization Beam Selection for mmWave Beamspace MIMO Systems

for transmission. However, compared with the full digital system and the conventional PP-BS scheme, the proposed scheme is able to achieve a near-optimal performance with fewer number of RF chains.

Furthermore, it can be observed that as the number of selected dominant beams for each UE increases, the number of beams selected in stage 2 may be more than the number of UEs, the additional beams after assigning the non-overlapping beams in stage 1 can be used to argument the performance of the system. The additional beams can be assigned to the UEs to improve the sum-rate and reduce outages due to blockage.

C. ENERGY EFFICIENCY

Next, we present the results of the energy efficiency of the proposed scheme to provide an evaluation of the trade-off between the number of selected RF chains and the complexity in a practical implementation. The energy efficiency is defined as the ratio of the sum-rate to the total power in watts consumed by the systems. We consider a scenario where the total power consumed by the system include the total transmit power and the total power consumed by the selected RF chains. In this article, the energy efficiency $\eta$ is modelled as follows [33], [34]

$$\eta = \frac{C}{P_t + N_{RF}P_{RF}},$$  

where $P_t$ denotes the transmit power in Watts, $P_{RF}$ is the power consumed by the components per RF chain, $C$ is the achievable sum-rate in bits/s/Hz. We adopt practical values for $P_t = 34.4$mW, $P_{RF} = 32$mW [25]. Note that in (20), an ideal transceiver design is assumed, however, the model can be extended to a non-linear transceiver design with hardware impairment as defined in [35].

In Fig. 8, the energy efficiency versus the number of UEs for the LOS scenario is presented. The best performance is achieved by the proposed SM-BS stage 1 scheme where the number of UEs equal the number of beams selected. The performance of the proposed schemes is attributed to the fewer number of RF chains used compared to the proposed SM-BS stage 2 schemes. On the other hand, the conventional PP-BS scheme uses more RF chains, hence, the energy efficiency is poor. Moreover, from Fig. 5, it is shown that a near-optimal sum-rate performance can be achieved as the number of dominant beams increase. However, the energy efficiency plot shows that the power consumed as the number of RF chains increases can degrade the performance of the system. Overall, the energy efficiency decreases for the proposed SM-BS scheme and the conventional PP-BS scheme as the number of UE increases which is mainly due to the increase in the number of RF chains used for transmission. For the full digital system, the energy efficiency is low due to the use of the full RF chains (i.e., $N_{RF} = N$) for transmission.

Fig. 9 shows the energy efficiency versus transmit power $P_t$ for a fixed sum-rate at SNR of 15 dB. As observed from the figure, the proposed SM-BS outperform the conventional PP-BS scheme and the full digital systems due to the reduced number of RF chains used in the proposed SM-BS. However, as $P_t$ increases, the energy efficiency performance decreases, this is because at high transmit power, the effect of the RF chains and power consumed in the RF chain becomes negligible.

VI. CONCLUSION

In this article, we address the issue of multi-beam selection in a high-dimensional mmWave multi-user MIMO system.
Specifically, we address the issues of rank-deficient dimension-reduced beamspace channel matrix which may result from users selecting overlapping beam set, and inter-beam interference resulting from the multi-user transmission. We proposed a two-stage beam selection algorithm which exploits the sparsity of the mmWave channel in the angular domain to establish a high data rate transmission between the BS and each UE. Simulation results show a near-optimal performance in the achievable sum-rate and a small performance loss is achieved as the number of dominant beams increases. In addition, the results show that the proposed scheme can achieve significant improvement in energy efficiency as a results of the reduced number of RF chains (beams) and increased sum-rate as compared to the conventional projected power beamspace beam selection scheme. The low RF chains, near optimal performance and improved energy efficiency achieved by the proposed scheme makes it suitable for high speed data transmission in mmWave multi-user systems.

REFERENCES
[1] R. Chakraborty, N. Kumari, M. Mousam, and A. Mukherjee, “The future of 5G and millimeter waves,” in Proc. 2nd Int. Conf. Electron., Commun. Aerosp. Technol. (ICECA), Mar. 2018, pp. 1679–1683.
[2] M. Xiao, S. Muntau, Y. Huang, D. Dai, Y. Li, M. Matthaiou, G. K. Karagiannidis, E. Björnson, K. Yang, C.-L. I, and A. Ghosh, “Millimeter wave communications for future mobile networks,” IEEE J. Sel. Areas Commun., vol. 35, no. 9, pp. 1909–1935, Sep. 2017.
[3] J. Zheng, T. Yang, H. Liu, T. Su, and L. Wan, “Accurate detection and localization of UAV swarms-enabled MEC system,” IEEE Trans. Ind. Informat., early access, Aug. 11, 2020, doi: 10.1109/TII.2020.3015730.
[4] M. Chowdhury, A. Manolakos, and A. J. Goldsmith, “Multiplexing-diversity tradeoffs in noncoherent massive MIMO systems,” in Proc. 49th Asilomar Conf. Signals, Syst. Comput., Nov. 2015, pp. 312–316.
[5] J. Zheng, T. Yang, H. Liu, and T. Su, “Efficient data transmission strategy for IoIoTs with arbitrary geometrical array,” IEEE Trans. Ind. Informat., early access, May 11, 2020, doi: 10.1109/TII.2020.2993586.
[6] J. Brady, N. Behdad, and A. M. Sayeed, “Beamspace MIMO for millimeter-wave communications: System architecture, modeling, analysis, and measurements,” IEEE Trans. Antennas Propag., vol. 61, no. 7, pp. 3814–3827, Jul. 2013.
[7] A. Sayeed and J. Brady, “Beamspace MIMO channel modeling and measurement: Methodology and results at 28 GHz,” in Proc. IEEE Globecom Workshops (GC Wkshps), Dec. 2016, pp. 1–6.
[8] C. Liu, M. Li, S. V. Hanly, I. B. Collings, and P. Whiting, “Millimeter wave beam alignment: Large deviations analysis and design insights,” IEEE J. Sel. Areas Commun., vol. 35, no. 7, pp. 1619–1631, Jul. 2017.
[9] H. Shokri-Ghadikolaei, L. Gkatzikis, and C. Fischione, “Beam-searching and transmission scheduling in millimeter wave communications,” in Proc. IEEE Int. Conf. Commun. (ICC), Jun. 2015, pp. 1292–1297.
[10] Y. Han, S. Jin, J. Zhang, J. Zhang, and K.-K. Wong, “DFT-based hybrid beamforming multiuser systems: Rate analysis and beam selection,” IEEE J. Sel. Topics Signal Process., vol. 12, no. 3, pp. 514–528, Jun. 2018.
[11] M. Hussain and N. Michelusi, “Throughput optimal beam alignment in millimeter wave networks,” in Proc. Inf. Theory Appl. Workshop (ITA), Feb. 2017, pp. 1–6.
[12] Z. Pi and F. Khan, “An introduction to millimeter-wave mobile broadband systems,” IEEE Commun. Mag., vol. 49, no. 6, pp. 101–107, Jun. 2011.
[13] W. Wang and W. Zhang, “Antenna subset selection for line-of-sight millimeter wave massive MIMO systems,” in Proc. 23rd Asia–Pacific Conf. Commun. (APCC), Dec. 2017, pp. 1–5.
IGBAFE ORIKUMHI (Member, IEEE) received the B.Eng. degree in electrical and computer engineering from the Federal University of Technology Minna, Nigeria, in 2008, and the M.Sc. degree in electrical electronics and telecommunications engineering and the Ph.D. degree in electrical engineering from Universiti Teknologi Malaysia, Skudai, Malaysia, in 2014 and 2017, respectively. From 2018 to 2020, he was a Postdoctoral Fellow with the 5G-Unmanned Aerial Vehicle Laboratory, Hanyang University, Seoul, South Korea. He is currently a Research Professor with Hanyang University. His research interests include relay transmission, cooperative communications, vehicular communication, millimeter wave communication, beam management, nonorthogonal multiple access, and 5G and 6G communications.

JEONGWAN KANG (Graduate Student Member, IEEE) received the B.S. degree in electronic engineering from Hanyang University, Seoul, South Korea, in 2017, where he is currently pursuing the combined master’s and Ph.D. degrees with the Department of Electronics and Computer Engineering. His current research interests include millimeter-wave communications, beam management, MIMO systems, and 5G and B5G communications.

HYEKYUNG JWA received the B.S. degree in electronics engineering from Hanyang University, Seoul, South Korea, in 1999, and the M.S. degree in electrical and electronics engineering from KAIST, Daejeon, South Korea, in 2001. Since 2001, she has been with the Mobile Communication Research, Electronics and Telecommunications Research Institute (ETRI), where she has mainly worked on the modem test bed implementation for WCDMA smart antenna systems, LTE, and LTE-advanced systems, with an emphasis on channel estimation and MIMO detection algorithms. Her research interests include radio resource management algorithms and various design and performance aspects for 5G NR systems.

JEE-HYEON NA (Member, IEEE) received the B.S. degree in computer science from Chonnam National University, and the M.S. and Ph.D. degrees in computer science from Chonnam National University, in 2002 and 2008, respectively. She has been with the Electronics and Telecommunications Research Institute (ETRI), since 1989, where she is currently the Director of the Intelligent Ultra Dense Small Cell Research Section. Her research interests include 4G, 5G small cells, enhanced mobile broadcast and multicast services, self-organizing networks, and location management and paging for mobile communication networks. She is a member of the IEEE Communication Part.

SUNWOO KIM (Senior Member, IEEE) received the B.S. degree from Hanyang University, Seoul, South Korea, in 1999, and the Ph.D. degree from the Department of Electrical and Computer Engineering, University of California at Santa Barbara, in 2005. Since 2005, he has been working with the Department of Electronic Engineering, Hanyang University, where he is currently a Professor. He was a Visiting Scholar with the Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, from 2018 to 2019. He is also the Director of the 5G/Unmanned Vehicle Research Center, funded by the Ministry of Science and ICT, South Korea. His research interests include wireless communication/positioning/localization and statistical signal processing. He is also an Associate Editor of IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.