Multiple Regression Model for Determining and Predicting the Viscosity of Crude Oils Mixture

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Abstract. The article presents development stages of a reliable multiple regression model for determining and predicting the oils mixture viscosity as a multifactor parameter. On the data of the laboratory experiment, a correlation and regression analysis was performed to select significant factors in the model. A fractional factorial experiment was carried out. A matrix of regression coefficients and a multiple linear regression equation were obtained. Estimation of the model significance has shown that the equation obtained describes empirical data with a high degree of reliability. The conducted studies showed that the known dependencies adequately describe the viscosity of crude oils mixture only when the content of a high-sulfur component is less than 10% or more than 90%. On a wider range of concentrations (20-80%), the viscosity of the mixture becomes a multifactor parameter and is more accurately described by the regression equation. The obtained dependence has a wide field of application in the practice of operating pipelines transporting compounded oil.

1. Introduction
The main oil pipeline system operation efficiency is directly related to the exact prediction of technological parameters, while compliance with the requirements for the transported oil quality. Compounding technologies are used to ensure the specified annual volumes of crude oil pumped to consumers [1]. In this case, oils from different deposits are mixed. They may have rather different physical properties and composition [2, 3, 4]. For example, West Siberian oil may be added to high-sulfur oil from other oil production regions.

One of the most significant factors affecting the hydrodynamic parameters of pumping is a viscosity of the oil. Actual viscosity of the mixture after the transition to compounded oil pumping can significantly deviate from the value accepted in calculations at the design stage [5, 6, 7]. During the operation, the viscosity is determined by the actual sampling of oil in the pumping stations laboratory. To predict the oil rheological properties in order to optimize the planning of operating practices it is quite necessary to have the possibility of an analytical calculation of the mixture viscosity. Therefore, becomes urgent the question of obtaining a mathematical model of the viscosity of compounded oil with a high degree of reliability [8, 9].

2. Analytical and experimental analysis
The main problem is that the liquids mixture viscosity is not an additive property [10]. From the current dependences for determining the liquids mixture viscosity there were selected those, that could...
be used to calculate the crude oils mixture viscosity [11, 12, 13, 14]. The selected dependencies are shown in Table 1.

**Table 1.** Calculated dependencies for determining the liquids mixture viscosity [2, 3, 4].

| Method name | Calculating formula |
|-------------|---------------------|
| Kendall-Monroe | $\mu_{mix}^{1/3} = x_1 \cdot \mu_1^{1/3} + x_2 \cdot \mu_2^{1/3}$ |
| Arrhenius | $\lg \mu_{mix} = x_1 \cdot \lg \mu_1 + x_2 \cdot \lg \mu_2$ |
| Walter | $\lg \lg (\theta_{cm} + 0.6) = \sum x_i \cdot \lg \theta(\theta_i + 0.6)$ |
| Akhatov Sh N, Abramzon L S, Iskhakov R G and Tugunov P I | $\mu_{mix} = \mu_n \cdot e^{ak+bk^2}$ |

\[ a = \frac{k_1^2 \cdot \ln \mu_n - \ln \mu_{1c}}{k_1 \cdot (1 - k_1)} \]

\[ b = -\left(a + \ln \frac{\mu_n}{\mu}\right) \]

$\mu_{mix}$ - dynamic viscosity coefficient of the mixture;
$\mu_1$, $\mu_n$ - dynamic viscosity coefficient of the first and second components, respectively;
$x_i$, $x_1$, $x_2$ - molar fractions of the mixture components;
$\theta_{cm}$ - coefficient of kinematic viscosity coefficient of the mixture;
$\theta_i$ - coefficient of kinematic viscosity coefficient of the i-th component;
$\mu_n$, $\mu_p$ - dynamic viscosity coefficient of oil and diluent, respectively;
k - diluent concentration;
$\mu_{1c}$ - dynamic viscosity coefficient of the mixture with concentration of diluent $k_1$.

In order to determine the possibility of application these dependencies to calculate compounded oil viscosity, an experiment was conducted in industrial laboratory to determine the actual kinematic viscosity coefficient of a mixture of West Siberian sweet crude and high-sulfur Arlan-Chekmagush crude at various concentrations of the components under isothermal conditions. The viscosity of the mixture for the corresponding concentrations of the initial components was calculated from the existing dependences (Table 1). An experimental curve describing the actual mixture viscosity and analytical curves constructed from the dependences of Akhatov Sh N, Abramzon L S, Iskhakov R G and Tugunov P I are shown in Figure 1.

A comparison of the obtained results shows that the known dependences describe viscosity of the crude oil mixture with a significant error. Arrhenius and Kendall-Monroe formulas demonstrate the smallest error and they also have the identical results. The deviation fluctuates within ten percent in the direction of underestimation of the viscosity coefficient relatively to its actual value. Akhatov Sh N formula has an error of up to 22%. As can be seen from Figure 1, an increase in the error is not monotonous, but is typical for a certain concentration interval of the high-sulfur component. Walters formula gives the greatest deviation from the experimental values.

It should be noted that the relationships presented in Table 1 describe the mixture viscosity as a function of viscosity coefficients of the initial components and their concentrations only. Then for
explanation of the deviations in the viscosity estimation, it is logical to assume that the mixture viscosity, with respect to molecular theory of viscosity, is a more complex functional dependence [15].

Indeed, when mixing such complex natural hydrocarbon components, such as crude oils from different deposits, the viscosity of their mixture can be influenced by many other chemical and physical factors [16, 17]. Therefore the problem arises to determine the coefficient of kinematic viscosity of oils mixture as a dependence on the number of parameters.

To solve this problem an application of multiple correlation and regression analysis is proposed [18].

Figure 1. Comparison of the actual kinematic viscosity of the mixture with the calculated values using formulas of Monroe-Kendall, Arrhenius, Walter and Akhatov Sh N.

The multiple linear regression equation, which describes the dependent variable Y as a function of explanatory variables $X_i$, has the form [18]:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_m X_m$$

(1)

where $\beta_0$ – regression constant, representing the point of intersection of the regression line with the ordinate axis; $\beta$ – regression parameters to be determined.
Factors that can influence the magnitude of the kinematic viscosity coefficient of the crude oil mixture include:

- kinematic viscosity coefficient each of the component;
- density each of the component;
- volume concentration each of the component;
- chloride salts concentration;
- mass fraction of water;
- mass fraction of mechanical impurities;
- mass fraction of ballast;
- mass fraction of organic chlorides;
- mass fraction of sulfur;
- mass fraction of hydrogen sulphide;
- mass fraction of ethyl mercaptans;
- mass fraction of paraffins;
- crude oil vapor pressure;
- yield of fractions (at 200 and 300 degrees).

To identify the most significant factors and to reject the least significant factors, a correlation and regression analysis of all the listed factors was made for two-year laboratory industrial data. It was found that such factors as the concentration of chloride salts, mass fractions of water, mechanical impurities, ballast, organic chlorides, hydrogen sulphide, ethyl mercaptans, paraffins, saturated vapor pressure, the yield of fractions (at 200 and 300 degrees) mix, in a concrete situation, practically do not influence. This is due either to their extremely small value or to a negligible difference in the mixed components. It should be specially noted that the significance of the factors affecting the viscosity of the mixture can be quite different for crude oils from other deposits as, for example, for paraffin-asphalt or paraffin-resinous oils.

Thus, the number of explanatory factors reduced to seven:

- viscosity of high sulfur crude oil;
- viscosity of sweet crude oil;
- density of high sulfur crude oil;
- density of sweet crude oil;
- volume concentration of high sulfur crude oil;
- volume concentration of sweet crude oil;
- mass fraction of sulfur in the mixture of oils.

Further, a fractional factor experiment was carried out under isothermal conditions, at a constant temperature of components and a mixture of 20 °C [19]. A planning matrix corresponding to a given type of fractional factor experiment was used. To ensure high accuracy of processing the results of the experiment by correlation and regression analysis, it is necessary that the number of measurements exceeds the number of factors to be examined, at least 3 times. Then 25 measurements were performed [19]. The measurements were carried out in accordance with the normative documents [20-24].

Based on the experimental data, regression coefficients were evaluated by a standard procedure of regression analysis. Substituting these coefficients into equation (1), we obtain the regression equation for determining the coefficient of kinematic viscosity of a mixture of West Siberian crude oil and high-sulfur Arlan-Chekmagush oil:

\[
Y = 7,7 \times 10^{-3} + 0,4678 \cdot X_1 + 0,4234 \cdot X_2 + 8,6964 \cdot X_3 + 13,2447 \cdot X_4 + 0,08349 \cdot X_5 - 2,8347 \cdot X_6 + 4,2685 \cdot X_7,
\]

\(X_1\) – volume concentration of sulfurous oil in the mixture, fraction; \(X_2\) – volume fraction of high sulfur oil in the mixture, fraction; \(X_3\) – kinematic viscosity coefficient of sweet oil, cSt \(\cdot 10^{-2}\); \(X_4\) –
coefficient of kinematic viscosity of high sulfur crude oil, cSt \cdot 10^{-2}; X_5 – mass fraction of sulfur in the mixture, %; X_6 – density of sweet crude oil, g/m^3; X_7 – density of high sulfur crude oil, g/m^3.

Statistical analysis of the regression equation carried out to check the significance of this equation and its coefficients showed that the variance estimate $s_e^2 = 0.000307$, and the coefficient of multiple determination $R^2 = 0.9965$. Thus, the obtained value of the multiple determination coefficient indicates that this regression model corresponds to empirical data with a high degree of reliability. Only 3.5% of the total variance is due to other factors not included in this model.

For an even more reliable estimate of significance, the corrected determination coefficient was calculated [18], which takes into account the introduction of correction for the number of degrees of freedom of the total variance at relatively small sample sizes:

$$R^2 = 1 - (1 - R^2) \cdot \frac{n - 1}{n - m - 1} = 1 - (1 - 0.9965) \cdot \frac{25 - 1}{25 - 8 - 1} = 0.995.$$ \hspace{1cm} (3)

3. Results and discussion

Graphical interpretation of results, obtained from the regression equation with respect to the actual experimental values is shown on Figure 2.

![Figure 2. Kinematic viscosity coefficient of crude oils mixture versus the high-sulfur crude oil concentration by experimental data and multiple regression model.](image)

The volume concentration of high-sulfur component, fraction

- Coefficient of kinematic viscosity by regression equation, cSt
- Coefficient of kinematic viscosity of the mixture, cSt

It can be seen from the figure that the results of calculations performed by the regression equation not only enter into the confidence interval of the actual values of the oil mixture viscosity, but also quite accurately describe its behavior.

The conducted studies showed that the mixture viscosity is described quite accurately by the formulas presented in Table 1 only if the content of the high sulfur component is less than 10% or more than 90%. On a broader range of concentrations (20-80%) the regression equation (2) gives significantly more accurate results. A comparative analysis of the results allows for the conclusion that in the range of 20-80% of the high-sulfur component content, mixture viscosity becomes a multifactor
parameter, influenced not only by the viscosity of the initial components, but their density and sulfur content as well.

The resulting multiple linear regression equation has a wide field of application, since pumping of compounded sulfur crude oils is usually carried out in the concentration range of 20-45% of the high-sulfur component. That is the range of mixture viscosity determination where the regression model demonstrates better result with respect to known formulas.

4. References

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