Reply to “Comment on ‘Conductance scaling in Kondo-correlated quantum dots: Role of level asymmetry and charging energy’”

L. Merker,1 S. Kirchner,2,3 E. Muñoz,4 and T. A. Costi1

1 Peter Grünberg Institut and Institute for Advanced Simulation, Research Centre Jülich, 52425 Jülich, Germany
2 Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany
3 Max Planck Institute for Chemical Physics of Solids, 01187 Dresden, Germany
4 Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile

(Received 8 April 2014; revised manuscript received 22 July 2014; published 13 August 2014)

The Comment of A. A. Aligia claims that the superperturbation theory (SPT) approach [E. Muñoz, C. J. Bolech, and S. Kirchner, Phys. Rev. Lett. 110, 016601 (2013)] formulated using dual fermions [A. N. Rubtsov, M. I. Katsnelson, and A. I. Lichtenstein, Phys. Rev. B 77, 033101 (2008)] and used by us to compare with numerical renormalization group (NRG) results for the conductance [L. Merker, S. Kirchner, E. Muñoz, and T. A. Costi, Phys. Rev. B 87, 165132 (2013)], fails to correctly extend the results of the symmetric Anderson impurity model (SIAM) for general values of the local level $E_d$ in the Kondo regime. We answer this criticism. We also compare new NRG results for $c_B$, with $c_B$ calculated directly from the low-field conductance, with new higher-order SPT calculations for this quantity, finding excellent agreement for all $E_d$ and for $U/\pi\Delta$ extending into the strong coupling regime.

DOI: 10.1103/PhysRevB.90.077102

PACS number(s): 75.20.Hr, 71.27.+a, 72.15.Qm, 73.63.Kv

Motivated by recent experiments on conductance scaling in correlated quantum dots exhibiting the Kondo effect [1–3], we recently presented a detailed study of the low-temperature and low-field scaling properties of the linear conductance of a quantum dot described by the single level Anderson impurity model [4]. Scaling in physical properties is a hallmark of a quantum dot described by the single level Anderson model on the values of $c_T$ and $c_B$ by using the detailed Supplemental Material of Ref. [9]. Muñoz et al. [16] showed that the source of this controversy lies in a Ward identity that is not satisfied in Refs. [13,15], as can be explicitly checked from Refs. [16,17].

In the preceding Comment [14], Aligia makes two claims on our Ref. [4], to which we respond below. Specifically, these claims are that:

1. “the results presented in Ref. [10] (of the preceding comment) as coming from NRG are misleading, because one expects that they are highly accurate, but since they were obtained indirectly neglecting the last term in Eq. (2), they should be corrected.”; and

2. the SPT of Ref. [9] “fails to correctly extend the results for the SIAM for general values of $E_d$ in the Kondo regime.”

We will address these claims in turn.

The expression that we used for calculating $c_B = \pi^2/16 \left[1 - \cot^2(\pi n_d/2)\right]$ in Ref. [4] from a numerical renormalization group calculation of the local level occupancy $n_d$ made use of a Fermi liquid argument where we took only the linear in $B$ corrections to the local level occupancy $n_d$, resulting in an approximate expression for $c_B$. Aligia points out that there is an additional contribution to $c_B$ that results from a $B^2$ correction to $n_d$. Taking this into account results in a modification of our expression for $c_B$ given by Eq. (7) of Ref. [14].

$$c_B = \frac{\pi^2}{16} \left[1 - \cot^2(\pi n_d/2)\right] - \frac{\pi}{2} \cot(\pi n_d/2) T_0^2 \frac{\partial^2 n_d}{\partial B^2}. \quad (1)$$

The last term in Eq. (1) is, in general, finite and vanishes only for the symmetric Anderson impurity model. In order to address this point in more detail, we compare the results of Fig. 8 of Ref. [4] with full NRG calculations in which $c_B$ is calculated directly from the conductance and thus includes the second derivative of the local occupation with respect to the applied field in Eq. (1). The results are shown in Fig. 1. The old and new NRG results differ significantly only for local level positions far from the Kondo regime and become identical in the symmetric Kondo regime. Note that the inclusion of the

References [16,17]. Interested readers can follow explicitly the latter by using the detailed Supplemental Material of Ref. [9]. Muñoz et al. [16] made use of a Fermi liquid argument where we took only the linear in $B$ corrections to the local level occupancy $n_d$ made use of a Fermi liquid argument where we took only the linear in $B$ corrections to the local level occupancy $n_d$, resulting in an approximate expression for $c_B$. Aligia points out that there is an additional contribution to $c_B$ that results from a $B^2$ correction to $n_d$. Taking this into account results in a modification of our expression for $c_B$ given by Eq. (7) of Ref. [14].

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The SPT is perturbative in the deviation from particle-hole symmetry around the strong coupling (or Kondo) fixed point. This can be seen, e.g., in Figs. 3 and 4 of Ref. [4], where agreement is found for all values of $U/\Delta \approx 1$. As we already discussed on p. 6 of our paper, “Although we show comparisons also in the region $\tilde{\varepsilon}_d \gtrsim 1$, by construction the SPT calculation is perturbative in $\tilde{\varepsilon}_d$ and the renormalized interaction. For small $U/\Delta$, the resulting $\tilde{\varepsilon}_d$ is small and considering terms up to only order $O(\tilde{\varepsilon}_d^2)$ in the SPT works well for all $-U/2 < \varepsilon_d < 0$.

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