THE RADIO AFTERGLOW OF GRB 030329 AT CENTIMETER WAVELENGTHS:
EVIDENCE FOR A STRUCTURED JET OR NONRELATIVISTIC EXPANSION

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ABSTRACT

We present our centimeter wavelength (1.4, 2.3, and 4.8 GHz) light curves of the afterglow of GRB 030329, which were obtained with the Westerbork Synthesis Radio Telescope. Modeling the data according to a collimated afterglow results in a jet-break time of 10 days. This is in accordance with earlier results obtained at higher radio frequencies. However, with respect to the afterglow model, some additional flux at the lower frequencies is present when these light curves reach their maximum after 40–80 days. We show that this additional flux can be modeled with two or more components with progressively later jet breaks. From these results we infer that the jet is in fact a structured or a layered jet, where the ejecta with lower Lorentz factors produce additional flux that becomes visible at late times in the lowest frequency bands. We show that a transition to nonrelativistic expansion of the fireball at late times can also account for the observed flux excess, except for the lowest frequency (1.4 GHz) data.

Subject headings: gamma rays: bursts — radiation mechanisms: nonthermal — radio continuum: general

1. INTRODUCTION

Since the discovery of afterglow emission of gamma-ray bursts (GRBs) at X-ray, optical, and radio wavelengths (Costa et al. 1997; van Paradijs et al. 1997; Frail et al. 1997), it has become clear that broadband observations are needed to determine the physical processes producing the afterglow emission in the context of the available models, the most popular being the fireball model (e.g., Rees & Mészáros 1992; Mészáros & Rees 1997). Obtaining the overall shape of the energy distribution and the time evolution of the GRB afterglow provides information about the intrinsic energy, both in electrons and in magnetic fields, as well as about the matter into which the GRB blasted its ejecta (see, e.g., Wijers & Galama 1999). Although optical observations alone can constrain the value of some of these physical parameters, observations covering the radio to X-ray wavelength regions are required to determine all of them.

The self-absorption frequency, \( \nu_a \), of the afterglow broadband spectrum can often be constrained by radio observations at centimeter wavelengths (e.g., Wijers & Galama 1999). As the afterglow spectrum evolves, the two other characteristic break frequencies in its broadband spectrum (the frequency at the peak flux, \( \nu_m \), and the cooling frequency, \( \nu_c \)) enter the radio regime as well, although in practice the flux level at \( \nu_c \) is below the detection limit of current radio telescopes. The latter frequency can usually be determined from the available optical and X-ray data, which span the frequency range in which the cooling frequency is found during the first days of the afterglow. These break frequencies and their time evolution uniquely determine the parameters that make up the fireball model and its evolution.

GRB 030329 is the closest gamma-ray burst discovered so far for which an afterglow has been found. 5 At a redshift of \( z = 0.1685 \) (Greiner et al. 2003), its afterglow was discovered in \( R = 12.4 \) mag, just 67 minutes after the GRB itself (Sato et al. 2003), about 100 times brighter than the average GRB afterglow. The brightness of the afterglow made it possible to study its evolution for a long time and in detail over a broad range of frequencies, from X-ray to centimeter wavelengths. Furthermore, its proximity provided an excellent opportunity to look for a supernova signature in both the light curve and the spectrum, as predicted by the collapsar model (Woosley 1993; MacFadyen & Woosley 1999), the currently favored progenitor model for long-duration gamma-ray bursts. The resemblance between the supernova spectrum distilled from the GRB 030329 afterglow and that of the energetic Type Ic supernova SN 1998bw (associated with GRB 980425; Galama et al. 1998) provides strong support for the core collapse of massive stars as the cause for GRBs (Hjorth et al. 2003; Stanek et al. 2003).

Several authors have modeled the broadband afterglow behavior with a standard fireball model for the afterglow. A first approximation shows excess flux (on top of the already bumpy light curve) after the first few days, most noticeably at the lower frequencies. Willingale et al. (2004) attribute the excess flux to the underlyng supernova, but most authors (e.g., Berger et al. 2003; Sheth et al. 2003; Tiengo et al. 2004) prefer a two-component jet model, where a slower jet is responsible for the extra emission appearing at optical wavelengths around 10 days after the burst. Even later time observations show a likely transition to the nonrelativistic regime, estimated around 40–50 days after the burst (Tiengo et al. 2004; Frail et al. 2005).

Here we describe our radio monitoring campaign of this extraordinarily bright afterglow with the Westerbork Synthesis Radio Telescope (WSRT) in the centimeter waveband. In § 2 we describe the data we obtained. In § 3 we apply an afterglow model to the data, and in § 4 we compare our results with radio data obtained by other groups. Finally, in § 5, we summarize our findings and draw our conclusions.

2. DATA REDUCTION AND ANALYSIS

Data were obtained with the WSRT, at 1.4, 2.3, and 4.8 GHz. We used the Multi Frequency Front Ends (Tan 1991) in combination
with the IVC+DZB back end in continuum mode, with a bandwidth of $8 \times 20$ MHz. Gain and phase calibrations were performed with the calibrator 3C 286, although sometimes 3C 147 or 3C 48 were used. Table 1 lists the log of the observations, all done in 2003. VLBI observations prevented us from using the WSRT in the second half of May, and observations were resumed in June, mostly at 4.8 GHz. At 2.3 and 1.4 GHz the observations suffered from confusion from nearby bright sources, causing the noise to be at least a factor of 2 above the theoretical limit.

We checked our results for consistency by measuring the flux of several nearby point sources, which were assumed to be constant. In a few observations, we found these sources to be

\[ \text{See } \S 5.2 \text{ at http://www.astron.nl/wsrt/wsrtGuide/node6.html.} \]
systematically dimmer, as indicated in the observation log; we therefore suspect that the flux derived in these observations for the afterglow is also below its real value. Although we could in principle scale these fluxes upward, we decided to ignore these observations in our analysis, as the cause of these low flux levels is not clear.

The 1.4, 2.3, and 4.8 GHz light curves are presented in Figure 1. The general trend of the light curves is that expected for the low-frequency part of a GRB afterglow: as long as the self-absorption frequency, $\nu_a$, is higher than the observed frequency interval, the light curve rises since the frequency of the minimum electron injection energy $\nu_m$ moves toward lower frequencies. When both $\nu_a$ and $\nu_m$ pass the observed frequency interval (not necessarily at the same time), a turnover in the light curve occurs and the flux falls off steeply. We have listed the precise temporal dependencies of $\nu_a$, $\nu_m$, $\nu_c$, and the peak flux $F_m$ in Table 2 for a homogeneous circumburst medium, as well as for a massive stellar wind, in which the external density $\rho$ depends on distance $r$ to the center as $\rho \propto r^{-2}$. The evolution in time of $\nu_a$, $\nu_m$ and $F_m$ is plotted in Figure 2. Table 3 lists the dependencies of the...
The dotted lines show where the breaks in the temporal behavior of the parameters occur at the fireball expands into a massive stellar wind, with a jet-break time \( t_{j} \) of medium, with a jet-break time. At earlier and later times the characteristic frequencies and the peak flux evolve in time according to Table 2. The resulting light curves are given in Table 3. The transitions between the different regimes, marked by the jet-break time \( t_{j} \) and the time \( t_{nr} \) at which the fireball becomes nonrelativistic, are treated as smoothly broken power laws as described in the Appendix.

3. APPLYING THE FIREBALL MODEL TO THE DATA

We have modeled the data simultaneously in time and frequency, using a general broadband afterglow model that includes a jet break and a transition to nonrelativistic expansion of the fireball. The several power-law segments of the broadband spectrum were connected smoothly in a way described in the Appendix. Because \( F_{\nu}, \nu_{a}, \nu_{m}, \) and \( \nu_{c} \) are functions of time, we need to quote them at some fixed moment, for which we choose the jet-break time. At earlier and later times the characteristic frequencies and the peak flux evolve in time according to Table 2. The resulting light curves are given in Table 3. The transitions between the different regimes, marked by the jet-break time \( t_{j} \) and the time \( t_{nr} \) at which the fireball becomes nonrelativistic, are treated as smoothly broken power laws as described in the Appendix.

Since our data show a large scatter, especially at early times due to scintillation, where they do not follow a smooth curve, we did not apply a \( \chi^{2} \) fit, but merely tried to obtain a best fit by eye. This ignores the scintillation, and it will also put some more emphasis on the 1.4 GHz light curve: a \( \chi^{2} \) fit tends to follow the 4.8 GHz data points since they are more numerous, and will hence ignore the global trend seen in the 1.4 GHz light curve. The result of such an eyeball broadband fit is shown in Figure 1. Note that the scintillation amplitude is largest in the first few days, after which it quickly declines (the large decrease in the 2.3 GHz light curve around 20 days could be an artifact in the data, since it does not show up in the other light curves).

In this way, we obtained a value of 35 GHz for the electron injection frequency, \( \nu_{a} \), and a value of 13 GHz for the self-absorption

| Temporal Range | \( \nu_{a} (\nu_{a} < \nu_{m} < \nu_{c}) \) | \( \nu_{m} (\nu_{m} < \nu_{c} < \nu_{c}) \) | \( \nu_{c} \) | \( F_{\nu} \) |
|---------------|---------------------------------|---------------------------------|---------|-------|
| \( t < t_{j} < t_{nr} \) (homogeneous) | \( t^{0} \) | \( t^{-3/2} \) | \( t^{-1/2} \) | \( t^{0} \) |
| \( t < t_{j} < t_{nr} \) (stellar wind) | \( t^{-3/5} \) | \( t^{-3} \) | \( t^{-1} \) | \( t^{-3} \) |
| \( t_{j} < t < t_{nr} \) | \( t^{-1} \) | \( t^{-2} \) | \( t^{-1} \) | \( t^{-3} \) |
| \( t_{j} < t_{nr} \) (homogeneous) | \( t^{-6/5} \) | \( t^{-1} \) | \( t^{-1/5} \) | \( t^{-3/5} \) |
| \( t_{j} < t_{nr} \) (stellar wind) | \( t^{-2/5} \) | \( t^{-7/3} \) | \( t^{-1/2} \) | \( t^{-3/5} \) |

Notes.—Before the jet-break time \( t_{j} \) and after the nonrelativistic timescale \( t_{nr} \), different scalings arise from a homogeneous circumburst medium or a stellar wind, when the external density \( \rho \) depends on distance \( r \) to the center as \( \rho \propto r^{-2} \). Between \( t_{j} \) and \( t_{nr} \), the external density profile does not influence the scalings.

Fig. 2.—Temporal evolution of the electron injection frequency, \( \nu_{a} \), the self-absorption frequency, \( \nu_{a} \), and the peak flux, \( F_{\nu} \). The top panels show the evolution of \( \nu_{a} \) (solid line) and \( \nu_{a} \) (dash-dotted line), the bottom panels the evolution of \( F_{\nu} \). The left panels show a model in which the fireball expands into a homogeneous medium, with a jet-break time \( t_{j} = 10 \) days, and the nonrelativistic phase of the fireball evolution starts after \( t_{nr} = 80 \) days. The right panels show a model in which the fireball expands into a massive stellar wind, with a jet-break time \( t_{j} = 10 \) days, and the nonrelativistic phase of the fireball evolution starts after \( t_{nr} = 80 \) days. The dotted lines show where the breaks in the temporal behavior of the parameters occur at \( t_{j} \) and \( t_{nr} \).
frequency, $\nu_{\alpha}$, at 10 days after the burst, which is the jet-break time. The flux at $\nu_{\alpha}$ is about 61 mJy at that time. We find the electron index to be $p = 2.1$. The value of $\nu_{\alpha}$ can only be determined with observations at higher frequencies; our data set indicates that $\nu_{\alpha} \gtrsim 10^{12}$ Hz, which is in agreement with the findings by Smith et al. (2005), who find that the rapid fall in their 350 GHz light curve can be attributed to the cooling frequency passing through their observing band. These results are somewhat at odds with the findings by Berger et al. (2003). They obtain higher values for the characteristic frequencies $\nu_{\alpha}$ (19 GHz) and $\nu_{\nu}$ (43 GHz), and the peak flux $F_{\nu m}$ (96 mJy) at the jet-break time $t_j \approx 9.8$ days. To investigate this further, we performed a fit that includes their data, as well as radio data from Sheh et al. (2003). This fit gives similar results to those obtained from our previous fit.

From the obtained characteristic frequencies and the peak flux, we find an isotropic energy $E_{\text{iso}} \approx 4.0 \times 10^{51}$ ergs, a circumburst density $n \approx 0.56 \nu_{c,13}^{-2/3} c^{-3}$, and the fractions of energy in the electrons $e_e$ and magnetic field $e_B$ of 0.25 $\nu_{c,13}$ and 0.49 $\nu_{c,13}^{-4/3}$, respectively. The cooling frequency cannot be determined from the radio observations at centimeter wavelengths, but we take $\nu_{\alpha} \equiv 10^{13} \nu_{c,13}$ to compare our results with those of Berger et al. (2003). The opening angle of the jet can be found to be $\theta_j \approx 0.38 \nu_{c,13}^{-1/2}$ rad (22°) from the jet-break time of 10 days, which gives a beaming-corrected energy of $E_{\text{cor}} \approx 2.9 \times 10^{50} \nu_{c,13}^{2/3}$ ergs. This energy is comparable to Berger et al. (2003), the circumburst medium density we find is smaller, but $e_e$ and $e_B$ are larger.

At the turnover in the late-time light curves at 1.4, 2.3, and 4.8 GHz we find an excess in flux compared to the model. We present two possible explanations for this behavior. The first one is a nonrelativistic phase after $t_{\text{mr}} \approx 80$ days. Table 3 shows that the light curves flatten in the transition to a nonrelativistic phase. In Figure 1 one can see that this extension of the model fits the data at 2.3 and 4.8 GHz well when one assumes a homogeneous external medium. However, the flux at 1.4 GHz is overestimated at the latest times in this case (note that the symbols are larger than the error bars). The model with an external density gradient and a nonrelativistic phase gives a better result at 1.4 GHz but underestimates the flux at higher frequencies.

The second explanation of the late-time flux excess is an extra component, which consists of an afterglow with a jet-break time later than 10 days. The resultant fit is also shown in Figure 1. For the first component, which produces the main flux at higher frequencies, the parameters are set as before except for $F_{\nu m} \approx 48$ mJy. The second component has $t_j \approx 30$ days, $\nu_{\alpha} \approx 20$ GHz, $\nu_{\alpha} \approx 10$ GHz and $F_{\nu m} \approx 16$ mJy at the jet-break time; so at $t = 10$ days the second component has $\nu_{\alpha} \approx 35$ GHz and $F_{\nu m} \approx 48$ mJy. The electron index $p \approx 2.2$ for both jets. The physical parameters we derive from these characteristic frequencies and peak flux are $n \approx 0.82 \nu_{c,13}^{-3/2} c^{-3}$, $e_e \approx 0.28 \nu_{c,13}^{-3/2}$, and $e_B \approx 0.43 \nu_{c,13}^{-1/2}$ for both components; for the first component we find $E_{\text{iso}} \approx 4.0 \times 10^{51}$ ergs, $\theta_j \approx 0.42 \nu_{c,13}^{-1/2}$ rad (24°) and $E_{\text{cor}} \approx 2.4 \times 10^{50} \nu_{c,13}^{2/3}$ ergs, while for the second component $E_{\text{iso}} \approx 9.1 \times 10^{50} \nu_{c,13}^{2/3}$ ergs, $\theta_j \approx 0.73 \nu_{c,13}^{-1/2}$ rad (42°) and $E_{\text{cor}} \approx 2.4 \times 10^{50} \nu_{c,13}^{2/3}$ ergs. These data are well fitted by this two-component jet model. However, data at higher radio frequencies from Berger et al. (2003) cannot be fitted well in this model.

### Table 3

| Flux for Given Frequency Range | $t < t_j < t_{\text{mr}}$ (Homogeneous) | $t_j < t < t_{\text{mr}}$ (Stellar Wind) | $t_j < t < t_{\text{mr}}$ (Homogeneous) | $t_j < t < t_{\text{mr}}$ (Stellar Wind) |
|--------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| $F(\nu < \nu_{\alpha})$       | $\nu^2 t_0^{1/2}$                       | $\nu^2 t_0^1$                          | $\nu^2 t_0^1$                          | $\nu^2 t_0^1$                          |
| $F(\nu_{\alpha} < \nu < \nu_{\nu})$ | $\nu^3 t_0^{1/2}$                      | $\nu^3 t_0^1$                         | $\nu^3 t_0^1$                         | $\nu^3 t_0^1$                         |
| $F(\nu_{\nu} < \nu < \nu_{\alpha})$ | $\nu^4 t_0^{1/2}$                     | $\nu^4 t_0^1$                        | $\nu^4 t_0^4/3$                        | $\nu^4 t_0^4/3$                        |
| $F(\nu_{\alpha} < \nu)$       | $\nu^5 t_0^{1/2}$                       | $\nu^5 t_0^1$                          | $\nu^5 t_0^1$                          | $\nu^5 t_0^1$                          |
| $\nu_{\alpha} < \nu_{\nu} < \nu_{\alpha}$ | $\nu^6 t_0^{1/2}$                     | $\nu^6 t_0^1$                        | $\nu^6 t_0^1$                        | $\nu^6 t_0^1$ |

### 4. DISCUSSION

A similar procedure of fitting two components with different jet breaks was applied by Berger et al. (2003) to explain the break in the early-time optical (and X-ray) light curve. The underlying mechanism involves only the Lorentz factor that produces the early-time light curve (with $t_j \approx 0.5$ days), and the other jet with a larger opening angle and lower Lorentz factor that carries the bulk of the energy and produces the later-time light curve (with $t_j \approx 10$ days). The WSRT observations at 2.4 days after the burst have values for the flux that are well above the theoretical curves (see Fig. 1). We investigated the possibility that these are signatures of the jet that produces the early-time optical light curve. However, with the constraints on the parameters from the optical and X-ray observations, it is not possible to fit the early radio observations with this jet with a jet-break time of 0.5 days. A better explanation for these observations is scintillation.

From the result of our two-component model fit, we can conclude that in addition to the jets with $t_j \approx 0.5$ and $t_j \approx 10$, another jet is present, with an even larger opening angle, that powers the late-time ($t > 50$ days) light curve and is therefore best visible at the very low frequencies observed here. However, it may be that the total jet (which possibly includes the first narrow jet as well) is structured (e.g., Mészáros et al. 1998; Rossi et al. 2002) and that the Lorentz factor $\Gamma$ decreases toward the edge of the jet cone. Alternatively, the outflow consists of a layered jet, where shocks with lower $\Gamma$ follow the faster ones as they run into the surrounding medium. In both cases, one expects that the low Lorentz factors dominate at low frequencies and late times, and that the jet break occurs later at progressively lower frequencies.

The multiple component model is certainly not satisfactory; it does not give a good fit to the data at radio frequencies above 4.8 GHz at late times. Our model in which a transition to a
nonrelativistic phase of the fireball occurs after 80 days gives a better broadband radio fit except for the data at 1.4 GHz. This transition to a nonrelativistic phase is also seen in VLA radio observations by Frail et al. (2005) and at X-ray frequencies by Tiengo et al. (2004), although their estimate for the time at which this transition occurs is lower than ours, i.e., ~50 and ~44 days, respectively. Our low-frequency radio data cannot be fitted well by applying this low value for the nonrelativistic transition.

Although the model in which a transition to a nonrelativistic phase of the fireball occurs gives the best broadband radio fit, the value of $t_{nr}$ we find is much lower than theoretical estimates done by Granot et al. (2005) and Oren et al. (2004), based on determinations of the evolution of the image size of GRB 030329 (Taylor et al. 2005). We estimate $t_{nr} \approx 209[E_{cm}/(10^{51} E)]^{1/2} \approx 168\nu^{-1/3}$ days, which is a factor of 2 higher than the 80 days we get from modeling the centimeter light curves. This discrepancy could be solved by fitting the broadband afterglow light curves of GRB 030329 simultaneously with the evolution of its image size.

5. SUMMARY AND CONCLUSIONS

Our data confirm the picture of a second jet in the afterglow of GRB 030329, which manifests itself around $t_j = 10$ days. However, the flux level around the time when the low-frequency light curve peak is higher than that predicted by the two-component afterglow model (cf. Berger et al. 2003). By adding a third component with a later $t_j$, we account for this excess flux. Taking into account the early jet break, seen most clearly at optical wavelengths, we suggest that one is actually seeing the result of several blast waves with a range in Lorentz factors (Rees & Mészáros 1998; Granot et al. 2003), something that comes quite naturally in the collapsar model for GRBs (MacFadyen et al. 2001; Ramirez-Ruiz et al. 2002; Zhang et al. 2003) and was already suggested by Sheth et al. (2003). However, their high-frequency data were unable to distinguish jet breaks at later times. Our later time low-frequency radio data show such a late-time jet break, corresponding to a lower Lorentz factor, and therefore support a layered or structured jet for the afterglow of GRB 030329.

An alternative explanation is the transition to a nonrelativistic phase of the fireball. This model gives a good fit to the data at 2.3 and 4.8 GHz but overestimates the flux at 1.4 GHz. This overestimate can be caused by the method of smoothly broken power laws as described in the Appendix. We did not take into account the jet that is pointing away from us, and this can possibly give extra flux at late times when the jet becomes nonrelativistic and spherical. Continuation of observations at late times at low radio frequencies and more detailed physical models can diagnose the cause of this discrepancy more closely.

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APPENDIX

BROADBAND SPECTRUM AND LIGHT-CURVE MODELING

The dominating radiation mechanism for GRB afterglows is synchrotron emission. The broadband synchrotron spectrum is determined by the peak flux and three break frequencies, namely, the synchrotron self-absorption frequency, $\nu_m$, and the cooling frequency, $\nu_c$, which corresponds to electrons that lose their energy quickly by radiation. The time evolution of these four parameters gives the evolution of the spectrum and thus light curves at all observing frequencies.

The relativistic electrons emitting the synchrotron radiation are accelerated at the shock front to a power-law distribution of energies, $N(\gamma_\epsilon) \propto \gamma_\epsilon^{-p} d\gamma_\epsilon$ for $\gamma_\epsilon \geq \gamma_m$. For electrons that cool on a timescale smaller than the dynamical timescale, the energy distribution is steeper, $N(\gamma_\epsilon) \propto \gamma_\epsilon^{-p-1} d\gamma_\epsilon$. The broadband energy spectrum is found by connecting these two energy distributions at the cooling Lorentz factor $\gamma_c$ and then integrating the single-electron synchrotron spectrum over the distribution function.

Synchrotron self-absorption is taken into account by calculating the flux $F_\nu$ as follows:

$$F_\nu = \frac{f_\nu}{D^2} \left( \frac{\alpha_\nu}{\alpha_\nu_0} \right)^{-1} \left[ 1 - \exp\left( \frac{\alpha_\nu}{\alpha_\nu_0} \right) \right].$$

(A1)

with $f_\nu$ the emission coefficient, $\alpha_\nu$, the absorption coefficient, and $D$ the distance. A detailed description of our modeling is presented in A. J. van der Horst et al. (2005, in preparation).

The evolution of the characteristic frequencies and the peak flux is given in Table 2. We assume that after the jet-break time the jet spreads sideways (Rhoads 1999), until it becomes spherical approximately at the same time the fireball becomes nonrelativistic and enters the Sedov–Von Neumann–Taylor phase of the evolution. The transitions between the different regimes, marked by the jet-break time $t_j$ and the time $t_m$ at which the fireball becomes nonrelativistic, are treated as smoothly broken power laws. We introduce a smoothing parameter $s$ and take $\nu_m$ and $\nu_c$ as examples:

$$\nu_m(t) = \nu_m(t_j) \left[ \left( \frac{t}{t_j} \right)^{(3/2)s} + \left( \frac{t}{t_j} \right)^{2s} \left( \frac{t_m}{t_j} \right)^{2s} \left( \frac{t}{t_m} \right)^{3s} \right]^{-1/s},$$

(A2)

$$\nu_c(t) = \nu_c(t_j) \left[ \left( \frac{t}{t_j} \right)^{-(1/2)s} + 1 + \left( \frac{t}{t_m} \right)^{(1/5)s} \right]^{-1}. \quad (A3)$$

Expressions for $\nu_\alpha$ and $F_m$ are similar.
We choose to have a smoothening parameter of $s = 5$ for every transition. However, these transitions will probably be different, and this can only be accounted for in detailed hydrodynamical modeling of the fireball. So this smoothening parameter gives an uncertainty that may account for the discrepancy seen at 1.4 GHz in the case of one jet with a transition to the nonrelativistic phase at $t_{nr} \simeq 80$ days.

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