A Self-paced Regularization Framework for Partial-Label Learning

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Abstract—Partial label learning (PLL) aims to solve the problem where each training instance is associated with a set of candidate labels, one of which is the correct label. Most PLL algorithms try to disambiguate the candidate label set, by either simply treating each candidate label equally or iteratively identifying the true label. Nonetheless, existing algorithms usually treat all labels and instances equally, and the complexities of both labels and instances are not taken into consideration during the learning stage. Inspired by the successful application of self-paced learning strategy in machine learning field, we integrate the self-paced regime into the partial label learning framework and propose a novel Self-Paced Partial-Label Learning (SP-PLL) algorithm, which could control the learning process to alleviate the problem by ranking the priorities of the training examples together with their candidate labels during each learning iteration. Extensive experiments and comparisons with other baseline methods demonstrate the effectiveness and robustness of the proposed method.

Index Terms—Partial Label Learning, Self-Paced Regime, Label Disambiguation, Maximum Margin

I. INTRODUCTION

As a weakly-supervised learning framework, partial-label learning (PLL) learns from ambiguous labeling information where each training example is associated with a candidate label set instead of a uniquely explicit label. It aims at disambiguating the ground-truth label from the candidate label set among which the other labels are incorrect.

In recent years, such learning mechanism has been widely used in many real-world scenarios. For example, in crowdsourcing online annotation system (Figure 1(A)), users with different knowledge background probably annotate the same image with different labels. Thus, it is necessary to find the correspondence between each image and its ground-truth label which is resided in the candidate annotations. Another representative application is naming faces in images using text captions (Figure 1(B)), users with different knowledge background probably annotate the same image with different labels. In this setting, since the name of the depicted people typically appears in the caption, the resulting set of images is ambiguously labeled if more than one name appears in the caption. In other words, the specific correspondence between the faces and their names are unknown. Partial label learning provides an effective solution to resolve such weakly supervision problem by disambiguating the correct label from large number of ambiguous labels. In addition to the applications mentioned above, PLL has also been widely used in many other scenarios, including web mining, facial age estimation, multimedia content analysis, ecoinformatics, etc.

A. Related Work

Existing PLL algorithms can be roughly grouped into the following three categories: Average Disambiguation Strategy, Identification Disambiguation Strategy and Disambiguation-Free Strategy, respectively.

1) Average Disambiguation Strategy (ADS): ADS based PLL methods assume that each candidate label contributes equally to the modeling process and make prediction for unseen instances by averaging the output from all candidate labels. Following such strategy, [14] and [15] adopt an instance-based model following \( \arg \max_{y \in Y} \sum_{i \in N(y)} \mathbb{I}(y \in S_i) \) to predict the label \( y^* \) of unseen instance \( x^* \). [6] disambiguates the ground-truth label by averaging the outputs from all candidate labels, i.e. \( \frac{1}{S} \sum_{y \in S} F(x, \Theta, y) \). [16] and [17] also adopt an instance-based model and they make prediction via k-nearest neighbors weighted voting and minimum error reconstruction criterion. [10] proposes the so-called PL-LEAF algorithm, which facilitates the disambiguation process by taking the local topological information from feature space into consideration. Obviously, the ADS based PLL methods mentioned above are intuitive and easy to implement. However, these algorithms share a critical shortcoming that the output of the ground-truth label could be overwhelmed by the outputs of the other false positive labels, which will enforce negative influence on the effectiveness of the final model.

2) Identification Disambiguation Strategy (IDS): IDS based PLL methods are proposed to alleviate the shortcoming of ADS based methods mentioned in Section A1. This strategy aims at directly identifying the ground-truth label from corresponding candidate label set instead of averaging the output from all candidate labels. Existing PLL algorithms following this strategy often regard the ground-truth label as
a latent variable first, identified as $\arg\max_{y \in S_i} F(x, \Theta, y)$, and then refine the model parameter $\Theta$ iteratively by utilizing some specific criterions. For example, considering the fact that treating each candidate label equally is inappropriate, Jin et al. [18] utilizes Expectation-Maximization (EM) procedure to optimize the latent variable $y$, which is based on maximum likelihood criterion: $\sum_{i=1}^{n} \log(\sum_{y \in S_i} F(x, \Theta, y))$. Similarly, [2], [13], [19], [20], [18] and [21] also adopt the maximum likelihood criterion to refine the latent variable. Moreover, Maximum margin technique is also widely employed as objective function in the PLL problem. For example, $\max_{y \in S_i} F(x, \Theta, y)$ maximizes the margin between the output from candidate labels and non-candidate labels to train a multi-class classifier, which utilizes Error-Correcting Output Codes (ECOC) coding [22], which utilizes $\sum_{i=1}^{n} \sum_{y \notin S_i} F(x, \Theta, y)$, while [23] directly maximizes the margin between the ground-truth label and the other candidate labels: $\sum_{i=1}^{n} (F(x, \Theta, y) - \max_{y \neq \hat{y}} F(x, \Theta, y))$. Although these IDS based PLL methods have achieved satisfactory performances in many scenarios, they suffer from the common shortcomings that training instances may be assigned with incorrect labels during each iterative optimization, especially for the PL data whose instances or candidate labels are difficult to disambiguate, and it will affect the optimization of classifier parameters in the next iteration.

3) Disambiguation-Free Strategy (DFS): More recently, different from the above two PLL strategies, some attempts have been made to learn from PL data by fitting the PL data to existing learning techniques instead of disambiguation. [4] proposes a disambiguation-free algorithm named PL-ECOC, which utilizes Error-Correcting Output Codes (ECOC) coding matrix [24] and transfers the PLL problem into binary learning problem. [25] proposes another disambiguation-free algorithm called PALOC, which enables binary decomposition for PL data in a more concise manner without relying on extra manipulations such as coding matrix. However, the performance of these two algorithms are inferior to IDS based methods in some scenarios.

B. Our Motivation

Although the algorithms mentioned above have obtained desirable performance in many real-world scenarios, they still suffer from several common drawbacks. For example, the PLL methods mentioned above treat all the training examples equally together with their candidate labels, but none of them take the complexity of training examples and that of the labels into consideration. However, in real-world scenarios, examples with different backgrounds and labels in different candidate label sets often express varying difficulties. For example, as shown in Figure 2 we can easily see that images (B,D) is harder than images (A,C) not only from the complexity of instance but also from the number of candidate labels. In particular, when we utilize the iterative optimization method to refine the model parameters, the label $cat$ may be assigned to the image $B$ in an iteration, but obviously such assignment is a large noise in the subsequent iterations, which has bad effect on the classifier model. Thus, to improve the effectiveness of model, the complexity of the training instances together with the candidate labels should be taken into consideration.

In recent years, inspired by the cognitive process of human, Self-Paced Learning (SPL) is proposed to deal with the above problem, which automatically leads the learning process from easy to hard [26] [27]. Concretely, during the optimization process of SPL, the ‘easy’ samples will be selected and learned first in the previous iterations, and then the ‘hard’ samples can be gradually selected in the subsequent iterations. The learning mechanism can smoothly guide the learning process to pay more attention to the reliable discriminative data rather than the confusing ones [28]. So far, SPL has obtained empirical success in many research fields, such as multi-instance learning [29] [30], multi-label learning [31], multi-task learning [32], multi-view learning [33], matrix-factorization [34], face identification [35] and so on.

In light of this observation, in this paper, we build a connection between the PLL and the SPL, and propose a novel unified framework Self-Paced Partial-Label Learning (SP-PLL). With an adaptive step from ‘easy’ to ‘hard’, SP-PLL can dynamically adjust the learning order of the training data (i.e. examples together with their candidate labels) and guide the learning to focus more on the data with high-confidence. Benefiting from the self-controlled sample-selected scheme, SP-PLL can effectively capture the valuable label information from the true label and minimize the negative impact of other candidate labels. Experimental results on UCI data sets and real-world data sets demonstrate the effectiveness of the proposed approach.

II. Background

In the following two sections, we separately give brief introduction about the work of partial-label learning [23] and self-paced learning [34], which our approach originates from.

A. Partial-label learning (PLL)

Formally speaking, we denote by $\mathcal{X} = \mathbb{R}^d$ the $d$-dimensional input space, and $\mathcal{Y} = \{1, 2, \ldots, q\}$ the output space with $q$ class labels. PLL aims to learn a classifier $f : \mathcal{X} \rightarrow \mathcal{Y}$ from the PL training data $\mathcal{D} = \{(x_i, S_i)\}_{1 \leq i \leq n}$, where the instance $x_i \in \mathcal{X}$ is described as a $d$-dimensional feature vector and the candidate label set $S_i \subseteq \mathcal{Y}$ is associated with the instance.
self-paced parameter for controlling the learning process. The empirical loss for the $i$-th training example, and the training instances assigned with false candidate labels will damage the corresponding candidate labels, which have varying complexity, often contribute differently to the learning result. The instances assigned with false candidate labels will damage the effectiveness and robustness of the learning model.

To overcome the potential shortcoming mentioned above, self-paced scheme is incorporated into our framework, which has the following advantages: 1) it can avoid the negative effect produced by the instances assigned with unreliable labels and it can make the instances assigned with high-confidence labels contribute more to the learning model. Specifically, during each learning iteration, we fix the assigned labels $y = \{y_1, y_2, \ldots, y_n\}$ to update the classifier $\Theta$. The instances assigned with high-confidence labels (i.e. the loss is smaller) can be learned first, and then the instances assigned with low-confidence labels (i.e. the loss is larger) can be admitted into the learning process, when the model has already become mature and the unreliable labels associated with the untrained instances have also become more reliable.

Following the proposed method, we design SP-PLL according to the steps as follows: Firstly, we define the loss $L(x_i, w, b, y_i)$ by deforming the hinge loss, which is defined as:

$$L(x_i, w, b, y_i) = \begin{cases} 0, & L(x_i, w, b, y_i) \leq 0 \\ 1 - \xi_i, & 0 < L(x_i, w, b, y_i) < 1 \\ 1, & L(x_i, w, b, y_i) \geq 1 \end{cases}$$

III. THE SP-PLL APPROACH

Here, we first introduce how we integrate the self-paced scheme into the task of PLL, and then present the formulation of SP-PLL. After that, we give an efficient algorithm to solve the optimization problem of our proposed method.

A. Formulation

As is shown in Section III-A, compared with other methods based on margin criterion, M3PL is an effective algorithm to alleviate the noise produced by the false labels in candidate label sets. Nonetheless, during the optimization iterations, M3PL ignores the fact that each training instance and the corresponding candidate labels, which have varying complexity, often contribute differently to the learning result. And the instances assigned with false candidate labels will damage the effectiveness and robustness of the learning model.

In this subsection, we first define notations and then introduce the self-paced function.

We denote $v = \{v_1, v_2, \ldots, v_n\}$ as $n$-dimensional weight vector for the $n$ training examples, $L(x_i, w, b, y_i)$ as the empirical loss for the $i$-th training example, and $\lambda$ as the self-paced parameter for controlling the learning process. The general self-paced framework could be designed as: 

$$v^* = \arg \min_{v \in [0, 1]} \sum_{i=1}^{n} v_i \cdot L(x_i, w, b, y_i) + f(v, \lambda)$$

Here, $f(\lambda, v)$ is the self-paced regularization, and it satisfies the three following constraints:

- $f(\lambda, v)$ is convex with respect to $v \in [0, 1]$;
- $v^*$ is monotonically increasing with respect to $\lambda$, and it holds that $\lim_{\lambda \to 0} v^* = 0$, and $\lim_{\lambda \to \infty} v^* \leq 1$;
- $v^*$ is monotonically decreasing with respect to $L(x_i, w, b, y_i)$, and it holds that $\lim_{L(x_i, w, b, y_i) \to 0} v^* \leq 1$, and $\lim_{L(x_i, w, b, y_i) \to \infty} v^* = 0$;

From the three constraints mentioned above, we can easily find how the self-paced function works: by controlling the self-growth variable $\lambda$, SPL tends to select several easy examples (the loss is smaller) to learn first, and then gradually takes more, probably complex (the loss is larger), into the learning process. After all the instances are fed into the training model, SPL can get a more ‘mature’ model than the algorithm without self-paced learning scheme.

Learning from the two strategies above, we incorporate the SPL strategy into the PLL framework and propose the SP-PLL algorithm, which is introduced in the following section in a more detailed manner.
where \( \xi_i = [(\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i}) - \max_{y \neq y_i} (\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i})] \), which is the margin between modeling outputs from ground-truth label and that from all other labels.

Next, we choose a suitable self-paced regularizer \( f(\mathbf{v}, \lambda) \) for SP-PLL framework, which is associated with the weight values of the training examples. Learning from [34], soft weighting can assign real-valued weights, which tends to reflect the latent importance of training samples more faithfully. Meanwhile, soft weighting has also been demonstrated that it is more effective than hard weighting in many real applications. Thus, we choose the soft SP-regularizer as follows:

\[
f(\mathbf{v}, \lambda) = \sum_{i=1}^{n} \frac{\lambda}{2} (v_i^2 - 2v_i)
\]

Finally, we integrate the loss Eq. (2) and SP-regularizer Eq. (3) into the partial-label learning framework, and then get the framework of SP-PLL corresponds to the following optimization problem \( \text{OP}(2) \):

\[
\min_{\mathbf{w}, \mathbf{b}, \mathbf{v}} \sum_{i=1}^{n} v_i \cdot L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) + \frac{1}{2} \sum_{p=1}^{q} \| \mathbf{w}_{p} \|^2 + \frac{1}{2} \sum_{i=1}^{n} \lambda \cdot (v_i^2 - 2v_i)
\]

with

\[
\begin{align*}
& L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) = 1 - [(\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i}) - \max_{y \neq y_i} (\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i})] \\
& \text{s.t.} \quad L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) \geq 0, \forall i \in \{1, 2, \ldots, n\} \\
& \sum_{i=1}^{n} \| \mathbf{y} \| = n_p, \forall p \in \{1, 2, \ldots, q\}
\end{align*}
\]

where \( n_p \) is the prior number of training instances belonging to the \( p \)-th class label in \( \mathcal{Y} \) and \( \sum_{p=1}^{q} n_p = m \). The \( n_p \) is defined as:

\[
n_p = \begin{cases} 
\lceil \bar{n}_p \rceil + 1, & p \in \{1, 2, \ldots, r\}, \text{and } n_r > \bar{n}_p \\
\bar{n}_p, & \text{otherwise}
\end{cases}
\]

here, \( \bar{n}_p \) is the integer part of \( \bar{n}_p \) and \( \bar{n}_p = \sum_{i=1}^{m} \| \mathbf{p} \| \in S_i \cdot \frac{1}{n_i} \cdot r \) is the residual number after the rounding operation and \( r = m - \sum_{p=1}^{q} \bar{n}_p \).

Since \( \text{OP}(2) \) is a optimization problem with mixed-type variables, alternating optimization procedure is a good choice to solve the problem. We give the optimization details in the following subsection.

### B. Optimization

During the process of alternating optimization, we first improve the optimization algorithm of [22] to make it suitable for our method to update the variables \( \mathbf{y} \) and \( \Theta(\mathbf{w}, \mathbf{b}) \), which is briefly introduced in Part (A) and (B), respectively. Then, we give the optimization algorithm of self-paced learning to update \( \mathbf{v} \) that is used to control the weight of different instances in each iterative process, which is introduced in part (C). And finally we summarize the procedure of SP-PLL at the end of the section.

1) **Update \( \mathbf{w}, \mathbf{b} \) with other variables fixed:** After initializing the weight vector \( \mathbf{v} \) and the ground-truth labels \( \mathbf{y} \) of training examples, \( \text{OP}(2) \) turns to be the following optimization problem \( \text{OP}(3) \):

\[
\min_{\mathbf{w}, \mathbf{b}, \mathbf{v}} \sum_{i=1}^{n} v_i \cdot L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) + \frac{1}{2} \sum_{p=1}^{q} \| \mathbf{w}_{p} \|^2
\]

with

\[
\begin{align*}
& L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) = 1 - [(\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i}) - \max_{y \neq y_i} (\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i})] \\
& \text{s.t.} \quad L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) \geq 0, \forall i \in \{1, 2, \ldots, n\} \\
& \sum_{i=1}^{n} \| \mathbf{y} \| = n_p, \forall p \in \{1, 2, \ldots, q\}
\end{align*}
\]

As is described in \( \text{OP}(3) \), fixing the variables \( \mathbf{y} \), \( \text{OP}(3) \) is a typical single-label multi-class maximum margin optimization problem [37] [38], which can be solved by utilizing the multi-class SVM implementations, such as liblinear toolbox [39].

2) **Update \( \mathbf{y} \) with other variables fixed:** By fixing the classification model \( \Theta = \{\mathbf{w}, \mathbf{b}\} \) and the weight vector \( \mathbf{v} \), \( \text{OP}(2) \) can turn to be the following optimization problem \( \text{OP}(4) \):

\[
\min_{\mathbf{y}} \sum_{i=1}^{n} v_i \cdot L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i)
\]

with

\[
\begin{align*}
& L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) = 1 - [(\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i}) - \max_{y \neq y_i} (\mathbf{w}^\top y_i \cdot \mathbf{x}_i + b_{y_i})] \\
& \text{s.t.} \quad L(\mathbf{x}_i, \mathbf{w}, \mathbf{b}, y_i) \geq 0, \forall i \in \{1, 2, \ldots, n\} \\
& \sum_{i=1}^{n} \| \mathbf{y} \| = n_p, \forall p \in \{1, 2, \ldots, q\}
\end{align*}
\]

To simplify the problem \( \text{OP}(4) \), inspired by [22], we first replace...
TABLE I

| Real World data sets | EXP* | FEA* | CL* | AVG-CL* | TASK DOMAIN |
|----------------------|------|------|-----|---------|-------------|
| Lost                 | 1122 | 108  | 16  | 2.33    | Automatic Face Naming [5] |
| BirdSong             | 4998 | 38   | 13  | 2.18    | Bird Sound Classification [13] |
| MSRCv2               | 1758 | 48   | 23  | 3.16    | Image Classification [40] |
| Soccer Player        | 17472| 279  | 171 | 2.09    | Automatic Face Naming [41] |
| FG-NET               | 1002 | 262  | 99  | 7.48    | Facial Age Estimation [42] |

\[ L(x_i, w, b, y_i) = \max(a_i = 1 - \left[w_i^\top (x - y_i) - \max_{a_i \neq y_i} (w_i^\top x_i + b_i)\right]) \]

where \( w_i \) are the weight vectors. The weight vector \( w_i \) is obtained by solving the optimization problem \( \text{OP}(5) \):

\[
\min_{Z} \sum_{p=1}^{q} \sum_{i=1}^{n} v_i \cdot c_{pi} \cdot z_{pi} \\
\text{s.t.} : \begin{cases} 
\sum_{i=1}^{q} z_{pi} = 1, & \forall i \in \{1, 2, \ldots, n\} \\
\sum_{i=1}^{n} z_{pi} = n_p, & \forall p \in \{1, 2, \ldots, q\} \\
z_{pi} \in [0, 1] 
\end{cases}
\]

\( \text{OP}(5) \) is an easy linear programming problem, which can be solved by utilizing the standard LP solver.

3) Update \( v \) with other variables fixed: By fixing the classification model \( w, b, y \), and the ground-truth labels \( y \), we update the weight vector \( v \) by solving the following optimization problem \( \text{OP}(6) \):

\[
\min_{v} \sum_{i=1}^{n} v_i \cdot L(x_i, w, b, y_i) + \frac{\lambda}{2} \cdot (v_i^2 - 2v_i)
\]

According to \( \text{OP}(6) \), \( v_i \) in the SP-PLL model could be computed as

\[
v^*(\lambda, L) = \begin{cases} 
1 - \frac{L(x_i, w, b, y_i)}{x}, & L(x_i, w, b, y_i) \leq \lambda \\
0, & L(x_i, w, b, y_i) > \lambda
\end{cases}
\]

here, it is easy to see that examples assigned with higher-confidence labels (i.e. \( L(x_i, w, b, y_i) \) is smaller) can get higher weight values than the examples assigned with lower-confidence labels (i.e. \( L(x_i, w, b, y_i) \) is larger), while examples assigned with extremely unreliable labels (i.e. \( L(x_i, w, b, y_i) \) is larger than \( \lambda \)) even can not be chosen in previous iterative flow, which is the so called ‘learning from easy to hard’ self-paced scheme.

During the entire process of alternating optimization, we first initialize the required variables, and then repeat the above process until the algorithm converges. Finally, we get the predicted labels of the unseen instances according to the trained classifier. The detail process of SP-PLL is summarized in Algorithm 1

IV. Experiments

A. Experimental Setup

To evaluate the performance of the proposed SP-PLL algorithm, we implement experiments on two controlled UCI data sets and five real-world data sets: (1) UCI data sets. Under different configurations of two controlling parameters (i.e. \( p \) and \( r \)), the two UCI data sets generate 84 partial-labeled data sets with different configurations [5, 2]. Here, \( p \in \{0.1, 0.2, \ldots, 0.7\} \) is the proportion of partial-labeled examples and \( r \in \{1, 2, 3\} \) is the number of candidate labels instead of the correct one. (2) Real-World (RW) data sets. These data sets are collected from the following task domains: (A) Facial Age Estimation; (B) Automatic Face Naming; (C) Image Classification; (D) Bird Sound Classification;

Table I and Table II separately summarizes the characteristics of the above UCI data sets and real world data sets, including the number of examples (EXP*), the number of the feature (FEA*), and the whole number of class labels (CL*).

Meanwhile, we employ eight state-of-the-art partial label learning algorithms2 for comparative studies, where the configured parameters of each method are utilized according to that suggested in the respective literatures:

- **PL-SVM [22]**: Based on the maximum-margin strategy, it gets the predicted-label according to calculating the maximum values of model outputs. [suggested configuration: \( \lambda \in \{10^{-3}, 10^{-2}, \ldots, 10^3\} \) ];
- **PL-KNN [14]**: Based on k-nearest neighbor method, it gets the predicted-label according to averaging the outputs of the k-nearest neighbors. [suggested configuration: \( k=10 \) ];
- **CLPL [5]**: A convex optimization partial-label learning method via averaging-based disambiguation [suggested configuration: SVM with hinge loss];
- **LSB-CMM [13]**: Based on maximum-likelihood strategy, it gets the predicted-label according to calculating the maximum-likelihood value of the model with unseen

2 We partially use the open source codes from Zhang Minling’s homepage: http://cse.seu.edu.cn/PersonalPage/zhangml/
instances input. [suggested configuration: $q$ mixture components];

- **M3PL** [23]: Originated from PL-SVM, it is also based on the maximum-margin strategy, and it gets the predicted-label according to calculating the maximum values of model outputs. [suggested configuration: $C_{\text{max}} \in \{0.01, 0.1, \ldots, 1.0\}$];

- **PL-LEAF** [10]: A partial-label learning method via feature-aware disambiguation [suggested configuration: $k=10$, $C_1 = 10$, $C_2 = 1$];

- **IPAL** [16]: it disambiguates the candidate label set by utilizing instance-based techniques [suggested configuration: $k=10$];

- **PL-ECOC** [4]: Based on a coding-decoding procedure, it learns from partial-label training examples in a disambiguation-free manner [suggested configuration: the codeword length $L = \lceil \log_2(q) \rceil$];

Inspired by [22] and [23], we set $C_{\text{max}}$ among $\{0.01, \ldots, 1.0\}$ via cross-validation. And the initial value of $\lambda$ is empirically set to more than 0.5 to guarantee that at least half of the training instances can be learned during the first iterative optimization process. Furthermore, the other variables are set as $\Delta = 0.5$, $loss_{\text{max}} = 10^{-4}$, and $\mu = 1.05$. After initializing the above variables, we adopt ten-fold cross-validation to train each data set and report the average classification accuracy on each data set.

**B. Experimental Results**

In our paper, the experimental results of the comparing algorithms originate from two aspects: one is from the results we implement by using the source codes provided by the authors; another is from the results shown in the respective literatures.

1) **UCI data sets**: We compare the SP-PLL with the comparing methods PL-SVM and M3PL, which SP-PLL originates from, to evaluate the effect of SP-regularizer in SP-PLL. Meanwhile, we also compare the proposed method (SP-PLL) with other baseline methods that are not based on maximum-margin strategy. The classification accuracy on the four UCI data sets (glass, segment, vehicle and letter, each data set with $7\times3=21$ configurations) are shown in Figure 2.

- **A)** SP-PLL achieves superior performance against M3PL in 95.24% cases and PL-SVM in 97.62% cases respectively (total cases: $3\times7\times4 = 84$);

- **B)** SP-PLL outperforms PL-KNN in 60.72% cases and is inferior to PL-KNN in 39.28% cases;

- **C)** SP-PLL has been outperformed by CLPL in only 3 cases and by LSB-CMM in only 4 cases, and it outperforms them in the rest cases.

As is described in Figure 3, SP-PLL achieves superior performance against the two algorithms (i.e. PL-SVM and M3PL) that our methods originate from and obtains competitive performance against than other comparing methods, which
is embodied in the following aspects:

- **Average Classification Accuracy** With the increasing of $p$ (the proportion of partial-label examples) and $r$ (the number of extra labels in candidate label set), more noisy labels are added to the training data. As shown in Figure 3, M3PL and PL-SVM are greatly influenced by these noises and the classification accuracy decreases significantly. In contrast, SP-PLL still performs well on disambiguating candidate labels, where the average classification accuracy of SP-PLL is 4.66% higher than that of M3PL on the glass data set, 0.71% higher on the segment data set, 0.53% higher on the vehicle data set and 0.49% higher on the letter, respectively.

- **Max-Min and Standard deviation of Classification Accuracy** As more noisy candidate labels are gradually fed into the training data, the classification accuracy of M3PL declines dramatically. For the glass data set, Max-Min and standard deviation of M3PL's classification accuracy separately reach to 20% and 0.055, while SP-PLL reaches to only 10% and 0.027. For segment data set, the classification accuracy of SP-PLL and M3PL have the similar Max-Min value but the standard deviation of SP-PLL's classification accuracy is 0.002 smaller than that of M3PL's. And for vehicle data set, the classification accuracy of SP-PLL and M3PL have similar standard deviation value while the Max-Min of SP-PLL's classification accuracy is 0.024 smaller than that of M3PL's.

The above results demonstrate that the proposed SP-PLL is more robust than M3PL.

- **Data Sets with Varying Complexities** According to the statistical comparisons of classification accuracy on the two data sets (accuracy on glass is lower than segment), we can see that examples in glass is much more difficult to be disambiguated than that in segment. However, SP-PLL can express more effective disambiguation ability on such difficult data set. Specifically, the performance on glass is 7% higher than that on segment, which again demonstrates the disambiguation ability of the proposed SP-PLL.

Besides, we note that the performance of SP-PLL is lower than PL-KNN on a few UCI data set (glass and letter). We attribute it to the difference of what the two methods are based on, where the former is based on maximum margin strategy and the latter is based on k-NN strategy. For different data sets, varying learning strategies have difference in the performance of learning results. However, according to the above comparing results, our method (SP-PLL) not only outperforms all existing maximum-margin PLL methods but also obtains competitive performance as compared with most methods based on other strategies.

2) **Real-world (RW) data sets:** We compare the SP-PLL with all above comparing algorithms on the real-world data sets, and the comparison results are reported in Table III, where the recorded results are based on ten-fold cross-
validation.
It is easy to conclude that SP-PLL performs better than most comparing partial-label learning algorithms on these RW data sets. The superiority of SP-PLL can be embodied in the following two aspects:
• Compared with PL-SVM and M3PL, which are also based on maximum margin strategy, SP-PLL outperforms all of them on the whole RW data sets. Especially, the classification accuracy of the proposed method is 7% more than M3PL’s and 20% more than PL-SVM’s respectively on MSRCv2 data set; And on the FG-NET data set, SP-PLL achieves around 100% classification accuracy improvement than M3PL.
• SP-PLL also performs great superiority on some data sets when comparing with the other baseline algorithms. Specifically, for the same data set (such as Lost and SoccerPlayer), in contrast to the M3PL algorithm which has worse performance than the other algorithms, the proposed SP-PLL algorithm usually performs well, which again demonstrates the advantage of incorporating SPL regime in our proposed method.

The two series of experiments mentioned above powerfully demonstrate the effectiveness of SP-PLL, and we attribute the success to the easy to hard self-paced scheme. To learn

### Table III

|               | Lost     | MSRCv2   | BirdSong | SoccerPlayer | FG-NET    |
|---------------|----------|----------|----------|--------------|-----------|
| SP-PLL        | 0.749±0.033| 0.581±0.010| 0.710±0.008| 0.470±0.010 | 0.078±0.022|
| PL-SVM        | 0.639±0.056| 0.417±0.027| 0.671±0.018| 0.430±0.004 | 0.058±0.010|
| M3PL          | 0.732±0.035| 0.546±0.030| 0.709±0.010| 0.446±0.013 | 0.037±0.025|
| CLPL          | 0.670±0.024| 0.375±0.020| 0.624±0.009| 0.347±0.004 | 0.047±0.017|
| PL-KNN        | 0.332±0.030| 0.417±0.012| 0.637±0.009| 0.494±0.004 | 0.037±0.008|
| LSB-CMM       | 0.591±0.019| 0.431±0.008| 0.692±0.015| 0.506±0.006 | 0.056±0.008|
| PL-LEAF       | 0.664±0.020| 0.459±0.013| 0.706±0.012| 0.515±0.004 | 0.072±0.010|
| IPAL          | 0.726±0.041| 0.523±0.025| 0.708±0.014| 0.547±0.014 | 0.057±0.023|
| PL-ECOC       | 0.703±0.052| 0.505±0.027| 0.740±0.016| 0.537±0.020 | 0.040±0.018|
the instances assigned with high-confidence label firstly can make the reliable label information contribute more to the model. Specifically, during each optimization iteration, we first optimize the assigned labels $y$ and then optimize the classifier parameters $\Theta$. When SP-PLL finishes learning from the instances assigned with high-confidence labels in the previous iterations, the unreliable labels associated with the untrained instances have been optimized to become relatively reliable. Thus, the SP scheme can make most data become more reliable before the learning process, which eliminate the noise and increase the reliability of training data to a certain extent. As expected, the experimental results demonstrate the motivation behind our proposed method.

C. Sensitivity Analysis

The proposed method learns from the PLL examples by utilizing two parameters, i.e. $C_{\text{max}}$ (regularization parameter) and $\lambda$ (the self-growth variable). Figure 6 and Figure 7 respectively illustrates how SP-PLL performs under different $C_{\text{max}}$ and $\lambda$ configurations. We study the sensitivity analysis of SP-PLL in the following subsection.

1) Maximum-Margin regularization parameter $C_{\text{max}}$: The proposed method is based on the maximum-margin strategy, where $C_{\text{max}}$ is the regularization parameter to measure the influence of each sample loss on the learning model. Since the $C_{\text{max}}$ is usually sensitive to the learning model [39], we empirically set the optimal value of $C_{\text{max}}$ on each data set among $\{0.01, \ldots, 100\}$ via cross-validation, which is shown in Table IV.

| The optimal $C_{\text{max}}$ for SP-PLL |
|----------------------------------------|
| The value of $C_{\text{max}}$ | Data Sets |
|--------------------------------|-----------|
| 0.01   | Lost, FG-NET, SoccerPlayer, BirdSong |
| 0.1    | Segment |
| 10     | Glass, Letter |
| 100    | Vehicle, MSRCv2 |

2) Self-Growth variable $\lambda$: As mentioned above, the main contribution of our method is incorporating the Self-Paced Learning (SPL) scheme into Partial-Label Learning (PLL). The SP parameter $\lambda$ plays an important role in controlling the learning process from easy to hard. The larger the initial value of $\lambda$ is, the more training instances can be learned during the first iterative optimization. As described in Figure 6, SP-PLL with a larger value of $\lambda$ tend to achieve poor performance, and we attribute such phenomena to that incorporating more training instances during the first iteration would bring much
noise into the learning process, which has negative effect on the final model. Meanwhile, SP-PLL with a smaller value of $\lambda$, which leads to overfitting and weaken the generalization ability of the learning model, also show poor performance. Thus, we empirically guarantee that half of the training instances can be learned in the first iterative optimization. According to Figure 6, the proposed method achieves desirable performance when the SP parameter $\lambda$ is set to 0.6.

V. CONCLUSION

In this paper, we have proposed a novel self-paced partial-label learning method SP-PLL. To the best of our knowledge, it is the first time to deal with PLL problem by integrating the SPL technique into PLL framework. By simulating the human cognitive process which learns both instances and labels from easy to hard, the proposed SP-PLL algorithm can effectively alleviate the noise produced by the false assignments in PLL setting. Extensive experiments have demonstrated the effectiveness of our proposed method. In the future, we will integrate the SPL technique into PLL framework in a more sophisticated manner to improve the effectiveness and robustness of the model.

REFERENCES

[1] L. Liu and T. Dietterich, “Learnability of the superset label learning problem,” in International Conference on Machine Learning, 2014, pp. 1629–1637.

[2] Y. Chen, V. Patel, R. Chellappa, and P. Phillips, “Ambiguously labeled learning using dictionaries,” IEEE Transactions on Information Forensics and Security, pp. 2076–2088, 2014.

[3] L. Oukhellou, T. Denou, and P. Aknin, “Learning from partially supervised data using mixture models and belief functions,” Pattern Recognition, pp. 334–348, 2009.

[4] M. Zhang, F. Yu, and C. Tang, “Disambiguation-free partial label learning,” IEEE Transactions on Knowledge and Data Engineering, pp. 2155–2167, 2017.

[5] T. Cour, B. Sapp, and B. Taskar, “Learning from partial labels,” IEEE Transactions on Knowledge and Data Engineering, pp. 1501–1536, 2011.

[6] Z.-H. Zhou, “A brief introduction to weakly supervised learning,” National Science Review, pp. 1–1, 2017.

[7] J. Luo and F. Orabona, “Learning from candidate labeling sets,” in Advances in Neural Information Processing Systems, 2010, pp. 1504–1512.

[8] C. Chen, V. Patel, and R. Chellappa, “Learning from ambiguously labeled face images,” IEEE Transactions on Pattern Analysis and Machine Intelligence, pp. 1–1, 2017.

[9] S. Xiao, D. Xu, and J. Wu, “Automatic face naming by learning discriminative affinity matrices from weakly labeled images,” IEEE Transactions on Neural Networks and Learning Systems, pp. 2440–2452, 2015.

[10] M. Zhang, B. Zhou, and X. Liu, “Partial label learning via feature-aware disambiguation,” in International Conference on Knowledge Discovery and Data Mining, 2016, pp. 1335–1344.

[11] Z. Zeng, S. Xiao, K. Jia, T. Chan, S. Gao, D. Xu, and X. Ma, “Learning by associating ambiguously labeled images,” in IEEE Conference on Computer Vision and Pattern Recognition, 2013, pp. 708–715.

[12] M. Xie and S. Huang, “Partial multi-label learning,” in AAAI Conference on Artificial Intelligence, 2018, pp. 1–1.

[13] L. Liu and T. G. Dietterich, “A conditional multinomial mixture model for superset label learning,” in Advances in Neural Information Processing Systems, 2012, pp. 548–556.

[14] E. Hullemeier and J. Beringer, “Learning from ambiguously labeled examples,” International Symposium on Intelligent Data Analysis, pp. 46–57, 2018.

[15] C. Tang and M. Zhang, “Confidence-rated discriminative partial label learning,” in AAAI Conference on Artificial Intelligence, 2017, pp. 2611–2617.

[16] M. Zhang and F. Yu, “Solving the partial label learning problem: an instance-based approach,” in International Joint Conference on Artificial Intelligence, 2015, pp. 4048–4054.

[17] G. Chen, T. Liu, Y. Tang, Y. Jian, Y. Jie, and D. Tao, “A regularization approach for instance-based superset label learning,” IEEE Transactions on Cybernetics, pp. 1–12, 2017.

[18] R. Jin and Z. Ghahramani, “Learning with multiple labels,” in Advances in Neural Information Processing Systems, 2003, pp. 921–928.

[19] Y. Grandalet and Y. Bengio, “Learning from partial labels with minimum entropy,” Ciarano Working Papers, pp. 512–517, 2004.

[20] Y. Zhou, J. He, and H. Gu, “Partial label learning via gaussian process,” IEEE Transactions on Cybernetics, pp. 4443–4450, 2016.

[21] P. Vanamoore and P. Smets, “Partially supervised learning by a credal em approach,” in European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty, 2005, pp. 956–967.

[22] N. Nguyen and R. Caruana, “Classification with partial labels,” in ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, 2008, pp. 551–559.

[23] F. Yu and M. Zhang, “Maximum margin partial label learning,” Machine Learning, pp. 573–593, 2017.

[24] T. G. Dietterich and G. Bakiri, “Solving multiclass learning problems via error-correcting output codes,” Journal of Artificial Intelligence Research, pp. 263–286, 1994.

[25] M.-L. Z. Xuan Wu, “Towards enabling binary decomposition for partial label learning,” in International Joint Conference on Artificial Intelligence, 2018, pp. 1–1.

[26] M. Kumar, B. Packer, and D. Koller, “Self-paced learning for latent variable models,” in International Conference on Neural Information Processing Systems, 2010, pp. 1189–1197.

[27] D. Meng and Q. Zhao, “What objective does self-paced learning indeed optimize,” arXiv preprint arXiv:1511.06049, pp. 1–9, 2015.

[28] P. Ti, X. Li, Z. Zhang, D. Meng, F. Wu, J. Xiao, and Y. Zhung, “Self-paced boost learning for classification,” in International Joint Conference on Artificial Intelligence, 2016, pp. 1932–1938.

[29] D. Zhang, D. Meng, and J. Han, “Co-saliency detection via a self-paced multi-instance learning framework,” IEEE Transactions on Pattern Analysis and Machine Intelligence, pp. 865–878, 2017.

[30] E. Sangineto, M. Nabi, D. Culibrk, and N. Sebe, “Self-paced deep learning for weakly supervised object detection,” IEEE Transactions on Pattern Analysis and Machine Intelligence, pp. 1–1, 2018.

[31] C. Li, F. Wei, J. Yan, X. Zhang, Q. Liu, and H. Zha, “A self-paced regularization framework for multilabel learning,” IEEE Transactions on Neural Networks Learning Systems, pp. 1–7, 2016.

[32] C. Li, J. Yan, F. Wei, W. Dong, Q. Liu, and H. Zha, “Self-paced multitask learning,” in AAAI Conference on Artificial Intelligence, 2017, pp. 2175–2181.

[33] C. Xu, D. Tao, and C. Xu, “Multi-view self-paced learning for clustering,” in International Joint Conference on Artificial Intelligence, 2015, pp. 3974–3980.

[34] Q. Zhao, D. Meng, L. Jiang, Q. Xie, Z. Xu, and A. Hauptmann, “Self-paced learning for matrix factorization,” in AAAI Conference on Artificial Intelligence, 2015, pp. 3196–3202.

[35] L. Lin, K. Wang, D. Meng, W. Zuo, and L. Zhang, “Active self-paced learning for cost-effective and progressive face identification,” IEEE Transactions on Pattern Analysis and Machine Intelligence, pp. 7–19, 2017.

[36] D. Meng, Q. Zhao, and L. Jiang, “A theoretical understanding of self-paced learning,” Information Sciences, pp. 319–328, 2017.

[37] K. Crammer and Y. Singer, “On the algorithmic implementation of multiclass kernel-based vector machines,” Journal of Machine Learning Research, pp. 625–629, 2003.

[38] C. Hsu and C. Lin, “A comparison of methods for multiclass support vector machines,” IEEE Transactions Neural Networks, pp. 415–425, 2002.

[39] S. Bengio, J. Weston, and D. Grangier, “Liblinear: A library for large linear classification,” Journal of Machine Learning Research, pp. 1871–1874, 2008.

[40] F. Briggs, X. Fern, and R. Raich, “Rank-loss support instance machines for miml instance annotation,” in International Conference on Knowledge Discovery and Data Mining, 2012, pp. 534–542.

[41] M. Guillaumin, J. Verbeek, and C. Schmid, “Multiple instance metric learning from automatically labeled bags of faces,” in European Conference on Computer Vision, 2010, pp. 634–647.

[42] G. Panis and A. Lianitis, “An overview of research activities in facial age estimation using the fg-net aging database,” Journal of American History, pp. 455–462, 2015.