Approaching Lambda without fine-tuning

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We address the fine-tuning problem of dark energy cosmologies which arises when the dark energy density needs to initially lie in a narrow range in order for its present value to be consistent with observations. As recently noticed, this problem becomes particularly severe in canonical Quintessence scenarios, when trying to reproduce the behavior of a cosmological constant, i.e. when the dark energy equation of state \( w_Q \) approaches \(-1\): these models may be reconciled with a large basin of attraction only by requiring a rapid evolution of \( w_Q \) at low redshifts, which is in conflict with the most recent estimates from type Ia Supernovae discovered by Hubble Space Telescope. Next, we focus on scalar-tensor theories of gravity, discussing the implications of a coupling between the Quintessence scalar field and the Ricci scalar (“Extended Quintessence”). We show that, even if the equation of state today is very close to \(-1\), by virtue of the scalar-tensor-coupling the quintessence trajectories still possess the attractive feature which allows to reach the present level of cosmic acceleration starting by a set of initial conditions which covers tens of orders of magnitude; this effect, entirely of gravitational origin, represents a new important consequence of the possible coupling between dark energy and gravity. We illustrate this effect in typical Extended Quintessence scenarios.

Since the observations of distant type Ia Supernovae suggested that the Universe expansion is accelerating \cite{1, 2}, a great deal of cosmological models has been proposed in order to describe possible mechanisms to speed up the expansion rate. Most of the models in the literature focus on “Quintessence” cosmologies, where a classical, minimally-coupled scalar field evolves along a shallow potential, while its energy density and pressure combine to produce a negative equation of state, thus making the field act as a repulsive force. The fine tuning problem of many Quintessence models, i.e. the need to set the field initial conditions in a very tiny range in order to get the observed energy density and equation of state today (as distinguished from the “coincidence problem”, i.e. the similarity between the dark and the critical energy densities today), motivated the search for models where the equations of motion admits attractor solutions \cite{3, 10}. The main property of attractor solutions is that a very wide range of initial conditions rapidly converge to a common evolutionary track. In particular, “tracking” scalar fields will eventually evolve into a “tracking solution” in the background-dominated era, where they have almost constant equation of state \( w_Q \) lying between \(-1\) and the background equation of state \( Q \). Attractor solutions can be obtained from different potentials; the most popular being the exponential potential \cite{3}, the inverse power-law potential \cite{10}, and the SUGRA model \cite{4}, where the exponential function is modulated by an inverse power law. Despite the appeal of these models, originally introduced to overcome the fine-tuning problem while allowing for a negative equation of state, it has been recently pointed out in \cite{12} that they are affected by a serious drawback when analyzed in the post-tracking regime. The most recent analysis, combining Cosmic Microwave Background (CMB) observations, Large-Scale Structure (LSS) data, Hubble parameter estimation and distant type Ia Supernovae, give, for a fiducial model with constant Quintessence equation of state, \( w_Q = -0.98 \pm 0.12 \) \cite{13}, while other authors give \( w_Q = -0.91 \pm 0.15 \) \cite{4}. A completely orthogonal information comes from the age of globular clusters, giving \( w_Q < -0.8 \) (68\% confidence level) \cite{13}. The problem with tracking Quintessence is that, once we fix the present Quintessence energy density to a value consistent with its estimate in a flat universe, \( \Omega_Q = 0.73 \pm 0.04 \) \cite{13}, the observational constraints on \( w_Q \) allow only “crawling” quintessence, or potentials with large current curvature. The tracking solutions are, indeed, defined in the background dominated epoch, where the background is either radia-
tion or non-relativistic matter; in the present Quintessence-dominated era, the scalar field has already passed the tracking phase. When tracking models are analyzed in detail, including the post-tracking behavior of the field, it can be shown that “good trackers”, i.e. attractors having a large basin of attraction, end up in the present Quintessence dominated era with a value of the equation of state which, being too different from −1, is ruled out by the observational constraints. More plausible values of \( w_Q \) today can be obtained by flattening the potential in which the field evolves (“crawling” Quintessence), but this requires to shrink the basin of attraction (“poor trackers”), making the Universe too sensitive to the Dark Energy (DE) initial conditions. This shrinking becomes increasingly dramatic the more the present \( w_Q \) approaches the cosmological constant case is realized only as we approach the limiting case \( w \equiv Q = 1; \) in this case, the trajectories of the field are almost flat, so that a real tracking never occurs (“crawling” Quintessence). The range of initial values of the scalar field energy density is increasingly narrow as we approach the limiting case \( w_Q = −1; \) the cosmological constant case is realized only starting from a single value which is exactly tuned to the present one \( (\rho_Q \sim 26 \text{ meV}^4) \) for a flat model with \( h = 0.7 \). This can be easily understood recalling that, in the tracking regime, the value of the DE equation of state is strictly related to the shape of the potential; generally, the closer \( w_Q \) is to −1, the flatter the potential needs to be. In practice, the efficiency of tracking is lost when requiring such low values of the equation of state.

Good trackers may actually be reconciled with observations only if the potential curvature (and, therefore, \( w_Q \)) is rapidly varying at redshifts \( z \gtrsim 0.5 \), but this appears to further exacerbate the coincidence problem, in that we require the energy density to be small and rapidly changing at recent times. Even accepting an anthropic explanation to this coincidence, another potential problem for good trackers with rapid current evolution of the field may arise from observational constraints on \( \dot{w}_Q = dw_Q/dz; \) indeed, the newest type Ia Supernovae with Hubble Space Telescope \( [16] \), combined with independent constraints from CMB and LSS data, gives \( \dot{w}_Q = 0.6 \pm 0.5 \), for a flat universe, ruling out the possibility of a rapidly-changing equation of state of DE. As a consequence, it is clear that values of \( w_Q \) very close to −1 would seriously challenge the whole class of canonical Quintessence models.

This puzzling problem is however alleviated if we extend the class of DE models, allowing for a non-minimal coupling between the Quintessence scalar field and the Ricci scalar \( R \). We will show that in Extended Quintessence models of Dark Energy \( [21-24] \), the basin of attraction is enlarged by the so-called R-boost, a purely gravitational effect due to the coupling. As a result, non-minimally coupled models of Quintessence can preserve the appealing feature of attractors which is lost in minimally-coupled models, because they can approach the value \( w_Q = −1 \), while still allowing a large range of initial energy density of the field.

As discussed in detail in \( [12] \), if we exclude potentials whose curvature increases rapidly near the present epoch (which would further enhance the coincidence problem), canonical Quintessence scenarios require flat potentials in order to reproduce an equation of state close to −1; in this case, the trajectories of the field are almost flat, so that a real tracking never occurs (“crawling” Quintessence). The range of initial values of the scalar field energy density is increasingly narrow as we approach the limiting case \( w_Q = −1; \) the cosmological constant case is realized only starting from a single value which is exactly tuned to the present one \( (\rho_Q \sim 26 \text{ meV}^4) \) for a flat model with \( h = 0.7 \). This can be easily understood recalling that, in the tracking regime, the value of the DE equation of state is strictly related to the shape of the potential; generally, the closer \( w_Q \) is to −1, the flatter the potential needs to be. In practice, the efficiency of tracking is lost when requiring such low values of the equation of state.

However, this shrinking of the basin of attraction is peculiar of minimally coupled Quintessence fields; the Klein-Gordon equation rules the dynamics of the field, which freezes during most of the Universe history if the potential is nearly flat. The situation is dramatically different in scalar-tensor theories, where the Ricci scalar \( R \) in the gravitational sector of the Lagrangian of General Relativity is replaced by the product of \( R \) with a function \( F(\phi) \). The most important effect of the non-minimal coupling is to enhance the dynamics of the field, while keeping the potential flat. As discussed in \( [24] \), the coupling adds a new source term in the Klein-Gordon equation:

\[
\phi'' + 2\mathcal{H}\phi' = \frac{a^2}{2}F_{,\phi}R - a^2 V_{,\phi}; \tag{1}
\]

in Eq. \( 1 \), primes indicate differentiation with respect to the conformal time, and \( \mathcal{H} \equiv a' / a \). The new term proportional to the Ricci scalar on the R.H.S. adds up to the one produced by the “true” potential of the field, \( V(\phi) \), thus generating an effective potential which is different from the corresponding minimal-coupling one; the difference is especially relevant at high redshifts, when the
coupling term dominates over the true potential. Deep in the radiation era, the quantity $a^2 R$ diverges as $a^{-1}$ imprinting a boost of energy to the scalar field, an effect we named “$R$-boost” \[24\]. This gravitational effect has a deep connection with particle physics, because the onset of the $R$-boost, in the radiation-dominated epoch, occurs as soon as the first particle species of the cosmic plasma becomes non-relativistic \[24\].

In typical non-minimal coupling models, where the coupling term is proportional to $\phi^2 R$, the amount of $R$-boost depends on the initial value of the scalar field. Starting from some initial value of the scalar field energy density, the field acquires a new potential energy which will be rapidly converted into kinetic energy; in a very short time, the field accelerates until the friction caused to the Hubble drag term in Eq. \[4\] becomes comparable with the coupling term. After that, the scalar field slowly rolls with a kinetic energy scaling as $a^{-2}$ ($a^{-3}$ in the matter epoch), until the true potential $V$ becomes important (typically, in the matter dominated era), and the evolution occurs along the corresponding attractor trajectory: depending on the value $w_Q$ today, there can be a period of freezing with $w_Q \sim -1$, followed by tracking, or a “crawling” Quintessence without ever really “tracking” if the present $w_Q$ approaches $-1$. For inverse power-law potentials, $V(\phi) \propto \phi^{-\alpha}$, the equation of state in the tracking regime is $w_{Q\text{track}} = -2/(\alpha + 2)$: in order to approach the cosmological constant equation of state, the potential is forced to be extremely flat. What we want to outline here is that, even in the limiting case $w_Q \to -1$, the $R$-boost enlarges the allowed range of initial energy densities $\Omega_{Q\text{init}}$, which can cover several orders of magnitude, as opposite to the minimally-coupled Quintessence case, where a flat potential implies early freezing and narrow basin of attraction. To give a practical example, let us consider the non-minimal coupling described in \[21, 24\], with $F(\phi) = \xi \phi^2 + \text{const.}; \xi$ is the coupling parameter, related to the Jordan-Brans-Dicke parameter, $\omega_{JBD} \equiv (F/F_0)\phi^2$ today. We assumed an inverse power-law potential for the field, and $\omega_{JBD} = 4000$ \[24\]; though recent experiments seem to converge towards larger values of $\omega_{JBD}$ \[37\], this would only anticipate the onset of the $R$-boost, without affecting the substance of our results. We performed a numerical integration of the background equations for a flat universe, requiring $w_Q = -0.999$, $h = 0.7$ and $\Omega_Q = 0.7$ today. Such an equation of state is so close to $-1$ that the potential $V$ closely resembles a cosmological constant. In practice, the potential alone would not be able to induce any dynamics to the field. The energy density of the scalar field vs. redshift is plotted in Fig. \[1\] for four different initial conditions $\Omega_{Q\text{init}}$, spanning several orders of magnitude. In each case, the initial kinetic energy of the field has been taken to vanish, while we changed the initial value of the field in a range of $6$ orders of magnitude; independently of the initial conditions, the Quintessence field is found to finish up with $\dot{w}_Q \sim 0$. The freezing is reached later for higher $\Omega_{Q\text{init}}$ (see \[24\]). In the same figure, we can distinguish the scaling of the matter and radiation components. For comparison, we also plotted in Fig. \[2\] the scaling of the DE component in a minimally-coupled model with the same inverse power-law potential, the same equation of state today and the same initial values of the field as in Fig. \[1\]. In the latter case, because of the potential flatness, very different initial field values span a narrow range of initial energy densities, unlike Extended Quintessence; as discussed in \[12\], the initial energy density has to be tuned in a tiny interval, here covering a range of a few tenths of meV$^4$ at $z \approx 10^8$. As $w_Q \to -1$, this basin of attraction shrinks to a single value, while in non-minimally coupling cases it remains huge, thereby avoiding any fine-tuning of the initial energy density.

From Fig. \[1\] we see how this loss of dependence on the initial conditions is realized in Extended Quintessence: the $R$-boost adds dynamics to the cosmological constant, which can now be reached from a huge range of initial values. The only value which has to be fixed is the present DE energy density $\Omega_Q$. In conclusion, the $R$-boost, a purely gravitational effect characteristic of non-minimally coupled theories, has the attractive feature of enriching the set of initial conditions a field can have to closely mimic the cosmological constant today. In canonical Quintessence scenarios, in order to reach $w_Q = -1$ today one would have to exactly tune the initial value $\Omega_Q$ to the present one; here, the balance between cosmological friction and gravitational coupling plays a fundamental role, enhancing the dynamics of the field down to relatively low redshifts and allowing to approach $w_Q = -1$ without fine-tuning on the initial energy density.

While, given the present experimental uncertainty
on the value of the DE equation of state, canonical Quintessence models are still far from experiencing a real crisis, future precision measurements of $w_Q$ and $\dot{w}_Q$ will be crucial: if experiments will converge towards $w_Q = -1$, and $\dot{w}_Q \to 0$, as it seems to be the case from the latest observations, minimally-coupled Quintessence scenarios would suffer from a serious fine-tuning problem which would call for either an anthropic explanation or for new classes of models, among which Quintessence models based on scalar-tensor theories of gravity appear particularly promising.

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