The Contribution of Surface Potential to Diverse Problems in Electrostatics

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Abstract. Electrostatics spans many different subject areas. Some comprise “good electrostatics,” where charge is used for desirable purposes. Such areas include industrial manufacturing, electrophotography, surface modification, precipitators, aerosol control, and MEMS. Other areas comprise “bad electrostatics,” where charge is undesirable. Such areas include hazardous discharges, ESD, health effects, nuisance triboelectrification, particle contamination, and lightning. Conference proceedings such as this one inevitably include papers grouped around these topics. One common thread throughout is the surface potential developed when charge resides on an insulator surface. Often, the charged insulator will be in intimate contact with a ground plane. At other times, the charged insulator will be isolated. In either case, the resulting surface potential is important to such processes as propagating brush discharges, charge along a moving web, electrostatic biasing effects in MEMS, non-contacting voltmeters, field-effect transistor sensors, and the maximum possible charge on a woven fabric.

1. Introduction
Surface potential is a well-known phenomenon. While the surface of a conductor at fixed voltage is an equipotential with field lines normal to the surface, the surface potential of a charged insulator, as in figure 1, can vary with position, and the field lines need not be surface normal. When an insulating sheet charged to \( \rho_s \) covers a ground plane, as in figure 2, a surface potential \( V_S = (\rho_s/\varepsilon)d \) develops with capacitance \( \varepsilon/d \) per m². Unlike a “real” capacitor charged by current, the surface capacitance instead requires on charge deposition. This paper examines the role of such surface potentials in electrostatics.

![Figure 1 – Charged, isolated insulator](image1)

![Figure 2 – Charged sheet over a ground plane](image2)

2. Equilibrium charge distribution along a moving web
When an insulating web passes over a metal roller, triboelectrification and charge accumulation at the peel point can cause arcing, disruptive forces, material damage, and dust precipitation. Understanding how surface charge distributes in equilibrium is important to preventing these nuisances. In the system of figure 3, the charge distribution depends on surface potential, conductivity, capacitance, and transport speed. The discrete model of figure 4 provides insight into the related role of surface potential [1].

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The web, moving at speed $U$, is divided into small segments of area $A$ having height $y_n$ and capacitance $C_n = \varepsilon_0 A / y_n$ to the roller, and surface resistance $R$ to adjacent segments. A uniform surface charge at $t = 0$ is assumed. An iterative calculation then proceeds in small time steps. As a segment leaves the roller at the peel point, its $y_n$ increases and its $C_n$ decreases. The segment voltage $V_n = Q_n / C_n$ thus rises, causing charge to flow back toward the roller via the surface potential difference between segments. After each $\Delta t$, the charge flow between segments is found via Ohm’s law: $i_n = (v_n - v_{n-1}) / R$, and charges are updated by $\Delta Q_n = (i_{n+1} - i_n) \Delta t$. The web is then advanced by $\Delta x = U \Delta t$. After many time steps, the charge distribution reaches equilibrium. A plot of charge per meter web width for $r_s = 10^{10} \text{\Omega/sq}$, $Q_0 = 0.2 \mu\text{C/m}$, and $U = 1 \text{ m/s}$ is shown in figure 5. The back flow of charge toward the peel point is evident. The results compare favourably with the analytical solution of Clum [2] in figure 6.

3. Direct gate field-effect transistor

Measuring DC fields has always been a challenge in electrostatics. Field mills and surface vibrators, which rely on a variable capacitance ($i = v \frac{dC}{dt}$) are reliable but require moving parts. One attempt to build a solid-state field sensor [3] is shown in figure 7. A depletion-mode (internally biased) field-effect transistor (FET) was fabricated with no gate electrode. Its oxide layer was exposed to an incident E-field which provided an equivalent gate field for modulating the channel current $i_D$.

Figure 8 shows the $i_D$ from a 100-kV/m step function, yielding an initial current of 60 $\mu$A. With the field maintained, the current fell $-2$ $\mu$A per hour, rather than holding steady. This drift can be explained by surface potential. Later experiments showed that electrons or negative ions from the
source region slowly flowed via the surface resistance of the oxide layer, as in figure 9. This charge altered the surface potential of the oxide, in turn reducing the modulating field in the FET channel.

4. Measuring surface charge with a non-contacting voltmeter
Non-contacting voltmeters, such as those made by Monroe™ and Trek™, measure surface potential with a variable capacitance probe, detector, and HV amplifier. The probe voltage $V_{\text{PROBE}}$ is raised until its detected field nulls to zero. The surface potential $\rho d/\varepsilon$ of a charged sheet covering a ground plane is easily measured. When the charged insulator is not on top of a ground plane, as in figure 10, the surface potential will vary with the position of both sheet and probe. The NCV, however, will still adjust $V_{\text{PROBE}}$ until the probe sees zero field.

![Figure 10 – Measuring isolated surface charge](image)

The resulting $V_{\text{PROBE}}$ is found by superposition [4]. One field component is from the surface charge with the probe grounded (figure 10, left). The other component (figure 10, right) is due to just $V_{\text{PROBE}}$ with no surface charge present. Both superimposed field components depend on geometry, hence the $V_{\text{PROBE}}$ enforced by the NCV will depend on probe position. The position-dependent NCV reading can be predicted by field modelling. First the surface-charge field is found by finite-element analysis or charge simulation. The probe-field pattern is similarly found, but made proportional to the unknown $V_{\text{PROBE}}$. The unknown $V_{\text{PROBE}}$ is then chosen so as to null the E-field at the probe. Figure 11 shows the predicted and measured NCV readings as a function of axial probe position for a 10-cm disc charged to 10 $\mu$C/m². As the distance increases, $V_{\text{PROBE}}$ doesn’t fall as $1/x$ or $1/x^2$, as one might expect for a charge distribution somewhere between a point and a plane, but rather as something less steep than either. This result suggests that field calculations are indeed needed to interpret the NCV reading.

5. Maximum surface charge on a woven fabric
Many textiles exhibit a maximum surface charge limit related to composition and construction. In one experiment [5], cloth samples on a ground plane were corona "sprayed" with positive ions at 4 kV to 9 kV, as in figure 12. From SEM photos, each sample was characterized by a weave pattern, like those in figure 13, and assigned a cloth “pore size” based on the width of interweave air channels and the overall weave pattern. The highest obtainable $V_S$ values under corona charging were measured. Though widely scattered, the results of figure 14 show a trend whereby the maximum possible surface potential $V_S$ occurs on cloth samples having the smallest pores. This trend can be explained by back ionization, well known in the precipitator industry. As ions collect on a highly resistive, porous layer over a ground plane, $V_S$ increases until breakdown occurs in the pores. Ions of both polarities are generated, causing local regions of high conductivity which limit any further charge accumulation. Pore breakdown will follow the classic Paschen discharge curve [6] which can be described (for parallel electrodes) by the curve-fit equation $E_{BK} = (6.2 + 312/g) \text{ V/m}$ [7], where the gap $g$ is in $\mu$m. The maximum sustainable $V_S$ for a tightly woven fabric can thus be estimated by finding $E_{BK}$ for the fabric and computing the resultant $V_{SMAX} = \frac{\rho_0}{\varepsilon_{\text{f}}} T (6.2 + \frac{312}{d})$. Here $T$ is the cloth thickness in $\mu$m.

Figure 14 compares this curve with measured values of $V_{SMAX}$ (white circles) and their averages (black circles.) The near correlation between the measured, charge-limited surface potentials and those predicted by modelling suggests that the Paschen breakdown model for fabric pores is plausible.
6. Electrostatic effects in MEMS devices

A classic dual-cantilever MEMS actuator is shown in figure 15 (upper). Applying 30-300 V causes the polysilicon membrane to deflect downward due to the electrostatic force that is counteracted by the mechanical restoring force of the membrane. The field in the 2 – 5 μm gap falls below the Paschen breakdown minimum. If the structure is modelled as a simple capacitor/spring system, the downward electrostatic force is $F_{elec} \approx -V^2 \varepsilon A/2y^2$, and the mechanical restoring force is $F_{mech} = k(g-y)$. The displacement vs. voltage solution for these equations is shown figure 16 (black and white circles to be explained shortly.) The deflection curve is more-or-less parabolic with increasing voltage until the “snap through” point, where the restoring force can no longer hold back the electrostatic force. Above snap-through, the membrane collapses onto the nitride layer. This is a well-known effect in MEMS.

The plot of figure 17 shows the deflection of a MEMS membrane actuator energized at +28 V DC for several minutes [8]. Over time, deflection increases, even though the voltage is constant. This increase in deflection is caused by a surface potential effect. Specifically, over time, negative charges migrate onto the nitride layer, causing its surface potential to acquire a negative bias, as in figure 15 (lower), thereby increasing the net field in the gap and, in turn, the membrane deflection. Given enough time, the field will be screened entirely out of the nitride layer, effectively reducing the gap distance and increasing deflection for a given applied voltage. The white dot in figure 16 shows the initial operating point of the actuator with +28 V applied. The black dot shows the deflection for the same voltage with the effective gap spacing reduced by the thickness of the nitride layer. Figure 18 demonstrates a different type of surface-potential effect. The plot shows the membrane height with 35 V applied. At 25 seconds, the voltage is increased to 50 V for 10 sec, causing snap-through. Contact charging leaves residual positive charge on the nitride layer, thereby raising its surface potential and reducing the gap field, leading to a smaller deflection for the same voltage after the snap-through event.
7. Modelling propagating brush discharges

Propagating brush discharges can release a lot of energy, making them the most destructive of electrical discharges. They occur when a charged, insulating layer on a ground plane is point-discharged from above. The energy content of such a discharge can be predicted by a model that accounts for surface potential, capacitance, and resistance. The charged surface is divided into segmented capacitor cells (figure 19) bridged by spark gaps to adjacent cells, as shown in figure 20.

Each segment of charge $\rho_s$ is tightly coupled to the ground-plane. The $n^{\text{th}}$ segment’s potential is $(\rho_s/\varepsilon)d$. Charge flows between cells if the mutual spark gap is bridged by a tangential E-field in excess of the 3-kV/mm breakdown of air. Charge can flow by surface breakdown between cells and is what drives the “propagation” of the discharge. Once bridged, a spark gap is modelled as a resistor ($0.1 \ \Omega/mm$) until the gap field falls to 75 V/mm for negative charge and 300 V/mm for positive [10]. The dynamic progression of the discharge is found by time iteration. After $\Delta t$, cell voltages are found, spark gaps are bridged if appropriate, and inter-cell currents computed from Ohm’s law. The charge redistributes with each $\Delta t$. The steps continue until all spark gaps are below breakdown. Figures 21 - 24 compare calculated post-discharge $\rho_s$ distributions with measured results from the literature [11-14].

8. Summary

This paper has addressed the role of surface potential in diverse areas of electrostatics. The effects of surface potential show up in many different situations, from field sensing, to webs, textiles, discharges and MEMS. The charge on insulator surfaces and the resulting fields and forces are often responsible for observed phenomena. At other times, surface potential causes behaviour to differ significantly from the ideal or expected.
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