X-Rule’s Precursor is also Logically Universal

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Dedicated to the memory of Harold V. McIntosh our friend and teacher.
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Abstract

We re-examine the isotropic Precursor-Rule (of the anisotropic X-Rule[6]) and show that it is also logically universal. The Precursor-Rule was selected from a sample of biased cellular automata rules classified by input-entropy[11]. These biases followed most “Life-Like” constraints — in particular isotropy, but not simple birth/survival logic. The Precursor-Rule was chosen for its spontaneously emergent mobile and stable patterns, gliders and eaters/reflectors, but glider-guns, originally absent, have recently been discovered, as well as other complex structures from the Game-of-Life lexicon. We demonstrate these newly discovered structures, and build the logical gates required for universality in the logical sense.

keywords: universality, cellular automata, glider-gun, logical gates.

1 Introduction

Since the publication of Conway’s Game-of-Life[5], many rules have been found with, to a degree, similarly interesting behavior[2]. Most of these rules are Game-of-Life variants and “Life-Like” in that they follow a simple birth/survival logic based on the total of 1s in the outer neighborhood, which for Life is defined as birth=3, survival=2 or 3 (B3S23). The variants are useful to study the nature and context of the Game-of-Life, to underline why the Game-of-Life itself is so special, and why the birth/survival scheme is able in some cases to produce gliders, glider-guns, logic gates, and universal computation.

To generalise these questions, another approach is to consider rules without the birth/survival scheme, to study their characteristics, and thus to enrich the landscape that makes universal computation possible in binary 2D cellular
automata with a Moore neighborhood. Rules have been found that do not follow simple birth/survival but are nevertheless candidates for universality. To mention two examples, the isotropic R-Rule discovered by Sapin and the anisotropic X-Rule discovered by the authors of this paper. The Precursor-Rule, defined in figures 4 to 7, belongs to this latter class of cellular automata, not following birth/survival, but still isotropic, where all rotations/flips of a given Moore neighborhood map to the same output.

Glers and stable “eaters” emerge spontaneously in the Game-of-Life, but a glider-gun was originally absent and only subsequently discovered by Gosper. In a curious imitation of this order of events, glider-guns in the Precursor-Rule have only recently been discovered. Thanks to these glider-guns, it’s possible to build the logical gates for negation, conjunction and disjunction and satisfy the third of Conway’s three conditions for universality to demonstrate universal computation in the logical sense.

The Precursor-Rule was selected from a sample of biased rules classified by the input-entropy method, giving the scatter-plot in figure 2. These biases followed “Life-Like” constraints though not simple birth/survival logic, to the extent that the rules are binary, with a Moore neighborhood, and in particular that they are isotropic, and where the parameter, the density of 1s in the look-up table, is similar to the Game-of-Life where $\lambda = 0.273$. Excluding the chaotic sector of the sample (the most heavily populated) a short list of 71 rules with spontaneously emergent gliders and eaters (also called eaters/reflectors)
Figure 2: The scatter-plot of a sample of 93000+ rules, plotting min-max entropy variability against mean entropy, coarse-grained on a 256×256 grid where the dot colors represent rule frequency intervals, +256,128,...,4,2,1. The left panel shows the location of the shortlist of 71 rules in the ordered region with low mean and max-min entropy. The Precursor-Rule is indicated.

were selected from the ordered sector which has low entropy variability.

The Precursor-Rule itself was selected from this short list, firstly because it featured two spontaneously emergent glider types, moving orthogonally (Gc, figure 10) and diagonally (Ga, figure 8), and secondly because it was possible to construct oscillating behavior where glider Gc was made to bounce between stable reflectors (figures 22, 23). This became the basis for the design of the glider-guns in the anisotropic X-rule[6], a close mutant of its isotropic precursor. Isotropic behaviour, where gliders and any other dynamical mechanisms operate equivalently in any direction, has arguably an advantage over anisotropy in that it simplifies and makes the design of the mechanisms more flexible.

Glider-guns are the key components for logical gates and thus universality. However, at the time we were unable to discover or construct glider-guns in the Precursor-Rule. Lately, with the collaboration of members of the ConwayLife forum[19], glider-guns have now been created for gliders Gc and Ga (figure 1). In addition, the forum contributed a plethora of complex structures from the Game-of-Life lexicon, including other glider-guns, oscillators, ships, puffer-trains, rakes, and breeders, which enrich the Precursor-Rule’s behavior and complexity. There is another orthogonal glider (Gb, figure 9) less likely to emerge spontaneously because of its more complicated phases, but as yet a glider-gun forGb has not been discovered.

The Precursor-Rule has a number of glider-guns, any of which can be used to build logical gates, however we have chosen to use the basic glider-gun in figure 1(b) to demonstrate logical universality, using analogous methods to Conway[1] and the X-Rule[6].
Figure 3: A typical evolution emerging after 99 time-steps from a 50x50 central random zone within a 150x150 space. Gliders Ga and Gc emerge, leaving stable eaters at the center. Dynamic trails=20.

The paper is organised into the following further sections. (2) the Precursor-Rule definition, (3) a description of gliders, eaters, and collisions, (4) the basic glider-guns for gliders Gc and Ga, (5) logical universality by logical gates using glider-gun GGa, (6) a review of alternative glider-guns and other dynamical structures discovered to date, and (7) the concluding remarks.

2 The Precursor-Rule definition

Figures 4 to 7 define the Precursor-Rule in four ways; the rule-table, the rule-table expanded to show all 512 neighborhoods, as a 102-bit isotropic rule-table, and in terms of birth/survival where a simple logic is not evident.

Figure 4: The precursor's rule-table in descending order of their values

1Ongoing investigation shows that a small but significant proportion of rule-table outputs are quasi-neutral (wildcards) — their mutations have little or no effect on most glider-guns featured in this paper, making the Precursor-Rule part of a cluster of very similar rules.
Figure 5: The layout in figure 4 expanded to show all 512 neighborhoods. 134 black neighborhoods map to 1, 378 blue neighborhoods map to 0.

Figure 6: The rule-table based on 102 isotropic neighborhoods — one (maximum value) prototype of each spin/flip group shown in descending order of their values with outputs (squares) below — empty=0, color=1. Blue or red depends on 0 or 1 as the neighborhood’s central cell value, so a blue output for a blue neighborhood signals “birth”, a red output for a red neighborhoods signals “survival”. The size of each group [1, 2, 4, 8] is shown above each prototype.

Figure 7: The 28 isotropic prototypes with output 1 are shown here separated into 11 cases of “birth” and 17 of “survival” (the rest output 0). A birth/survival logic is not discernible. The size of each group [1, 2, 4, 8] is shown above each prototype.
3 Gliders, Eaters, and Collisions

A glider is a special kind of oscillator, a mobile pattern that recovers its form but in a displaced position, thus moving at a given velocity. A rule with the ability to support a glider, together with a stable eater/reflector, and a diversity of interactions between gliders and eaters, provides the first hint of potential universality.

From a typical chaotic initial condition as in figure 3, and evolution subject to the Precursor-Rule, its easy to detect the spontaneous emergence of two eater types, and (and their spins/flips), and two glider types, glider Ga (Figure 8) and glider Gc (Figure 10). A combined glider G2a, two Ga gliders joined together with a one cell overlap, can also emerge (figure 11). Glider Gb (Figure 9) is not detected immediately but with a more patient search it can be found. Figures 12 to 15 describe some of the collision results between Ga and Gc gliders, and between these gliders and eaters. Similar experiments could include G2a gliders and also the oscillators in sections 6.1 and 6.2 to provide a more thorough collision catalog.

![Figure 8: The 4 phases of glider Ga, moving SouthWest with speed c/4.](image)

![Figure 9: The 4 phases of glider Gb, moving East with speed c/2.](image)

![Figure 10: The 4 phases of glider Gc, moving East with speed c/2.](image)

![Figure 11: Glider Ga (left) and combined Ga gliders of increasing size 2, 3, 4, 5 and 6, orientated SE. They move in 4 phases at a speed of c/4. There is a one cell overlap between adjacent Ga gliders.](image)
3.1 Gliders colliding with gliders

The outcomes of collisions between gliders are very diverse, depending on the phase, angle, and point of impact, and include the destruction of either or both gliders, a bounce, or a transformation to different or combined glider types. A residual pattern of eaters/reflectors may also be created.

The speed of a glider (or other periodic mobile structure) relative to the speed of light \( c \), is measured by the number of squares advanced within its period. In general, orthogonal gliders based on Gc advance 2 squares in a period of 4 giving a speed of \( c/2 \), whereas diagonal gliders based on Ga advance 1 square (on both axes) in a period of 4 giving an speed of \( c/4 \).

Figures 12 to 16 show gliders about to collide with each other at various points of impact (top panel), and the outcomes after a given number of time-steps (lower panel), giving just a flavour of the diversity of behaviour, with dynamic trails=20.
Figure 14: Two Gc gliders colliding head-on, +19 time-steps.

Figure 15: Two Gc gliders colliding at 90°, +37 time-steps.

Figure 16: Glider Gc colliding with Ga, +26 time-steps.
3.2 Gliders colliding with eaters/reflectors

Stable structures emerge spontaneously in the Precursor-Rule which may destroy and/or reflect colliding gliders. The two basic eaters/reflectors (also known as “still life”) are isolated patterns consisting of 3 cells in an “L” shape, and two adjoining cells, giving the following with all rotations/flips:

The eaters/reflectors may themselves be destroyed or transformed in the collision, and the glider may be destroyed, bounce, and transform to a different or combined glider. As with collisions between gliders, the outcomes of collisions between a glider and an eater/-reflector are very diverse, depending on the phase, angle, and point of impact.

Figures [17] and [18] show gliders about to collide with eaters at various points of impact (top panel), and the outcomes after a given number of time-steps (lower panel), showing just a flavour of the diversity of behaviour, with dynamic trails=20.

Figure 17: Glider Ga colliding with eaters/reflectors, +33 time-steps.

Figure 18: Glider Gc colliding with eaters, +38 time-steps.
4 Basic glider-guns

Although a diversity of interactions between gliders and eaters provides the first hint of potential universality, the essential ingredient is a glider-gun, a dynamic structure that ejects gliders periodically into space. A glider-gun can also be seen as an oscillator that adds to its form periodically to shed gliders. In some rules a glider-gun may emerge spontaneously\cite{8,13}, but not in the Game-of-Life, the X-Rule, or the Precursor-Rule — in these cases the glider-gun is a complex structure with a negligible probability of emerging from a random pattern — it has to be found, discovered or somehow constructed.

Gosper found the game-of-Life glider-gun\cite{5,1}. The anisotropic X-Rule gliders-guns were constructed from reflecting/bouncing oscillators in its isotropic precursor by Gómez\cite{6}, the search for a glider-gun in the Precursor-Rule itself having been abandoned at that time. However, since the publication of \cite{6} and its announcement on the ConwayLife forum\cite{19}, a member, Arie Paap\cite{26}, discovered the first two glider-guns in the Precursor-Rule — GG2a shooting the G2a glider (two Ga’s combined) shown in figure \ref{fig:26}, followed by the “basic” GGc in figure \ref{fig:1}(a). A number of other Gc and Ga glider-guns were later announced in the forum, described in section \ref{sec:6.3} together with a diversity of other complex structures. However, the “basic” glider-gun that we apply to demonstrate logical gates is GGa — latterly constructed by Gómez\cite{21} by colliding two GGc glider-streams head on (figure \ref{fig:1}(b)). For a while this was the smallest Ga glider-gun, but a comparably compact gun with double the period has lately been found\cite{17} by colliding two GGc glider-streams at 90° (figure \ref{fig:45}).

In the next section the basic glider-gun GGa will be harnessed to demonstrate the logical gates, NOT, AND, and OR, to show that the Precursor-Rule is logically universal.

5 Logical Universality

Traditionally the proof for universality in cellular automata is based on the Turing Machine or an equivalent mechanism, but in another approach by Conway\cite{1}, a cellular automata is universal in the full sense if it is capable of the following,

1. Data storage or memory.

2. Data transmission requiring wires and an internal clock.

3. Data processing requiring a universal set of logic gates NOT, AND, and OR, to satisfy negation, conjunction and disjunction.

This paper is confined to proving condition 3 only, for universality in the logical sense. To demonstrate universality in the full sense as for the Game-of-Life, it would be necessary to also prove conditions 1 and 2, or to prove universality in terms of the Turing Machine, as was done by Randall\cite{7} for the Game-of-Life.
5.1 Logical Gates

Logical universality in the Precursor-Rule, as in the Game-of-Life, is based on Post’s Functional Completeness Theorem (FCT)\[4\]. This theorem guarantees that it is possible to construct a conjunctive (or disjunctive) normal form formula using only the logical gates NOT, AND and OR.

Using a specific right-angle collision, two Ga gliders can self-destruct leaving no residue. Applying this between GGa glider-gun streams, and a Ga glider/gap sequence with the correct spacing and phases representing a “string” of information, it’s possible to build logical gates. Gates NOT, AND and OR are illustrated in figures\[19\] to \[21\]. Note that the AND and OR gates include intermediate NOT and NOR gates\[3\], explained in the captions.

Gaps in a string are indicated by grey circles, dynamic trails=10 are included, and eaters are positioned to eventually stop gliders.

5.1.1 Logical Gate NOT

The logical gate NOT (1→0 and 0→1), also called an “inverter”, requires one GGa glider-gun interacting with a string of Ga gliders/gaps, illustrated as before/after snapshots in figure \[19\].

![Diagram of NOT gate](image)

Figure 19: An example of the NOT gate: input string A (11001) moving SE interacts with a GGa glider-stream moving NE, resulting in NOT-A (00110) moving NE after 102 time-steps. Any Ga input string can be substituted for A.
5.1.2 Logical gate AND (also NOR)

The logical gate AND (1+1→1, else→0), also a NOR gate (0+0→1, else→0), requires one GGa glider-gun interacting with two input strings of Ga gliders/gaps, illustrated as before/after snapshots in figure 20. Note that gate AND contains gate NOT.

Figure 20: An example of the AND gate: input strings A (11001) and B (10101) both moving SE interact with a GGa glider-stream moving NE, resulting in A-AND-B (10001) moving SE after 208 time-steps. Any two Ga input strings can be substituted for A and B. The dynamics making this AND gate first makes an intermediate NOT-A string 00110 (as in figure 19) which then interacts with string B to simultaneously produce both the A-AND-B string moving SE described above, and also the A-NOR-B string 00010 moving NE.

5.1.3 Logical Gate OR

The logical gate OR (0+0→0, else→1) requires two GGa glider-guns interacting with two input strings of Ga gliders/gaps, illustrated as before/after snapshots in figure 21. Note that gate OR contains both NOT and AND/NOR gates.
Figure 21: An example of the OR gate: input string A (11001) and B (10101) both moving SE interact with a GGa glider-stream moving NE, and subsequently with GGa glider-stream moving SE, resulting in A-OR-B (11101) moving SE after 302 time-steps. Any two Ga input strings can be substituted for A and B. The dynamics making this OR gate first makes an intermediate NOT-A string 00110 (as in figure 19), which then interacts with string B to simultaneously produce both the AND and the NOR string as in figure 20. The intermediate A-NOR-B string 00010 is inverted by the upper glider-gun stream to make NOT(A-NOR-B) which is the same as A-OR-B.
6 Precursor-Rule Universe

As well as the gliders, eaters, and collisions in section 3, the basic glider-guns in section 4 and the logical gates built from some of these components in section 5, the Precursor-Rule is capable of an astonishing diversity of dynamical behaviour. This section gives examples, starting with oscillators, then various complex dynamical structures — glider-guns, space ships, puffer-trains, rakes, and breeders, named from the Game-of-Life lexicon, and discovered by members of the ConwayLife forum[19]. These kinds of structures may eventually provide the components for universality in the full sense discussed in section 5.

6.1 Variable length/period oscillators

Here we show oscillators between stable double reflectors, where the gap can be increased by modular intervals, which increases the period (figures 22, 23 and 24). The double reflectors can be made of , or interchangeably. The dynamics are indicated by green trails.

Figure 22: Simple reflecting oscillators (SRO) — a Gc glider bouncing between stable reflectors. The period \( p \) depends on the gap between reflectors \( g \). For the SROs above, \( g/p = 3/2, 4/6, 6/14, 8/22, \ldots \). From \( g/p=4/6 \), as \( g \) increases by 2, \( p \) increases by 8.

Figure 23: Reflecting/bouncing oscillators (RBO) — two Gc gliders bouncing off each other while reflecting between stable reflectors. The period \( p \) depends on the gap between reflectors \( g \). For the RBOs above, \( g/p = 16/22, 20/30, 24/38, 28/46 \). As \( g \) increases by 4, \( p \) increases by 8.

Figure 24: Complex Reflecting/bouncing oscillators (cRBO) have additional central reflectors [26], resulting in complex internal dynamics within the green zones. For the cRBOs above, \( g/p = 28/41, 32/49, 36/57, 40/65 \). As \( g \) increases by 4, \( p \) increases by 8.
6.2 Other oscillators

As well as the variable length/period oscillators in section 6.1, there are many other oscillators with various periods, most found at the ConwayLife forum. Some are illustrated in figure 25.

![Figure 25: Oscillators with periods P2 (g and h are extendable) and P3, P4, P5, P6, P8, P12, P25 and P27. Dynamics occurs within the green zones.](image)

6.3 Other glider-guns

![Figure 26: Glider-gun GG2a (discovered by [26]) shoots 4-phase G2a (double Ga) gliders diagonally in 4 directions with a period of 24. The gliders are stopped by strategically placed eaters. Dynamic trails=15](image)

The first two glider-guns in the Precursor-Rule (discovered by [26]), opened the floodgates for further discovery. The second was the basic GGc (figure 1(a)) already discussed. The first “QuadGG2a” shoots G2a, double Ga gliders (figure 26) and is significant because it provides the building blocks for meta-glider-guns made from interacting simpler glider-guns, including the Ga glider-gun discovered by [27], shown in figure 27(b).
6.3.1 Meta-glider-guns

Figure 27 shows 4 examples of meta-glider-guns – they need to be seen in action to appreciate their extraordinary dynamics.

(a) Gc glider-gun “QuadGG2a” [24] shoots Gc gliders North as shown with a period of 408 time-steps. The dynamics are very complex — during this interval three Gc gliders are also shot South at time-steps 133, 307 and 355.

(b) Ga glider-gun [27] shoots Ga gliders with a period of 24 time-steps, shown shooting NW.

(c) Ga glider-gun [21] shoots two Ga gliders with a period of 120, shown shooting NW and NE.

(d) Multiple glider-gun [24] shoots 3 different glider types, Ga, Gc and G2a, with a period of 144, shown shooting NW, North, and NE.

Figure 27: Meta-glider-guns built from two interacting GG2a glider-guns (figure 26), the outcomes of different 90° collisions between two G2a gliders. Dynamic trails=15.
Figure 28: Multiple meta-glider-gun [21] is built from 4 interacting basic GGc glider-guns (figure 1(a)) and produce 2 intermediate G2a glider-streams colliding at 90°. The system’s period is 133 time-steps, but within this interval 2 Ga gliders are released (shown moving NW and NE), and finally two Gc gliders (shown moving North).

6.3.2 Multi-glider-guns

Figure 27(d) demonstrates a multi-glider-gun in that it shoots more than one glider type. In figure 28 we show another multi-glider-gun, also a meta-glider-gun because it is constructed from 4 interacting GGc glider-gun sub-units, discovered by [21]. Sending different glider types simultaneously from the same gun is arguably novel in relation to the Game-of-Life. Of course, any of the glider streams can be blocked by strategically positioned eaters.

6.3.3 Variable period Gc glider-guns

A system of variable period glider-guns, GGcV, for glider Gc was created by [26]. Built from his complex reflecting/bouncing oscillators (cRBOs), the system is demonstrated with the smallest cRBO (gap/period of 28/41 in figure 24). Two cRBOs are positioned as in figure 29(a), with the distance between their centers $d=85$ cells. A Gc glider pointing West is introduced, and its interaction brushing past the pulsating cRBO creates a new Gc glider moving East as in figure 29(b), which interacts with the second cRBO repeating the cycle.
The result is glider-gun GGcV shooting Gc gliders East and West. The whole structure has a period of 328 time-steps. The distance between cRBO centers $d$ can be adjusted by modular amounts to increase the period. To date the following have been demonstrated, $d/p = 85/328, 167/656, 249/984, 331/1312$. The series continues $d+82/p+328$. Similar structures can also be built with the larger cRBOs in figure 24.

Figure 29: Above: snapshots (a) and (b) of the smallest GGcV variable period Gc glider-gun are shown with a continuous green dynamic trace to indicate dynamic regions and glider movement. If (a) is at time-step 0 with a glider moving West, a new glider is created as the Westward glider brushes past the left cRBO. (b) shows the resulting two gliders at time-step 68. The new glider moving East will interact with the right cRBO to continue the cycle, so Gc gliders will be shot West and East — here they are stopped by eaters.

Left: the GGcV glider-gun as a 2d space-time pattern, with about 270 time-steps stacked below each other in an isometric projection showing the paths of gliders and also an impression of the internal cRBO dynamics.
6.4 Spaceships and puffer-trains

A spaceship is a mobile periodic pattern larger than a simple glider — a puffer-train is similar but leaves debris in its wake. The combined Ga gliders in figure 11 could be classified as spaceships. Figures 30 to 33 show examples of spaceships and puffer-trains, some suggested at the ConwayLife forum, and relating to combinations of either Ga or Gc gliders, as well as other patterns. The period \( p \) and speed \( s \) are indicated.

Figure 30: (a) small spaceships built from a subunit between two Ga gliders\(^{26}\), and (b) extend to any length by inserting more subunits\(^{23}\). Shown here moving SE in 4-phases (\( p=4, s=c/4 \)). Dynamic trails=20.

Figure 31: Spaceships and puffer-trains can be built from Gc glider subunits. Two or more Gc gliders separated by one cell (at their widest phase) form stable spaceships (a)\(^{22}\). However, if the Gc gliders touch (b - e) the tail becomes unstable. (b) with two Gc’s forms a spaceship\(^{22}\), (c) with 3 Gc’s forms a spaceship after a Gc glider is first ejected\(^{26}\), and (d, e) with 4 and 5 Gc’s form puffer-trains\(^{26}\). All are shown moving East (\( s=c/2 \)) with dynamic trails=16. The leading edges of Gc gliders transit through the usual 4-phases. The periods of the structures are noted.
Figure 32: Three spaceships. (a) moving SE \(p=6, s=c/6\), (b) moving East \(p=4, s=c/2\), (c) moving East \(p=4, s=c/2\) with dynamic trails=20.

Figure 33: Puffer-trains discovered by \[26\]. (a) is slow moving, advancing just 4 spaces in its period \(p=23, s=c/5.75\), and (b) \(p=12, s=c/2\). Shown moving East with dynamic trails=20.

6.5 Rakes

A rake is a mobile periodic pattern that sheds a succession of gliders in its wake, including Ga or Gc gliders, or a combination of both, a sort of mobile glider-gun. In this sense a rake is an adapted spaceship, or a puffer-train if debris is also left in the wake. A rake can also be a sub-component in building a “meta-rake”. Several rakes of various type and complexity have been discovered at the ConwayLife forum\[19\]. Figures 35 to 41 provide details.

Figure 34: A rake moving North \(p=24, s=c/2\) sheds two Ga glider streams alternately NW and NE. Dynamic trails=24.
Figure 35: Rakes moving East shedding Ga gliders. (a) NE [20] and (b) NW [26] based on three touching Gc subunits, shed intermediate Gc gliders which interact with debris, but also clean it up. (c) [26] is the same as the rake in figure [34] but an accompanying Gc glider cuts off one Ga stream leaving Ga gliders moving NE. Periods as noted. Dynamic trails=20.

Figure 36: A combination between a puffer-train and a rake [16]. p=48, s=c/2, sends Ga gliders SW and leaves debris. Dynamic trails=50.

Figure 37: Two similar rakes based on a core of three touching Gc subunits move East (p=24, s=c/2), shed Ga gliders moving West. Although rake (a) [24] is slightly different from rake (b) [23] — a similar phase is show. Dynamic trails=20.
Figure 38: A rake moving East as in figure 37(a) sheds Gc gliders moving West, which interact with stationary oscillating structures, eventually sending Gc gliders North every 192 time-steps[24]. A similar structure was discovered by [23] based on figure 37(b). Dynamic trails=190.

Figure 39: A rake moving East as in figure 37(b) sheds Gc gliders moving West, which interact with a static structure, eventually sending Ga gliders NW every 144 time-steps[23]. Dynamic trails=100.
Figure 40: A rake with a complicated body moving East \((p=12, s=c/2)\) sheds Gc gliders moving West\cite{26}. Dynamic trails=20.

Figure 41: Rake “RGe90” \((p=20, s=c/2)\) sheds Gc gliders at 90°. RGe90 can be started from (a) a seed found by \cite{24}, which evolves to (b) a seed found by \cite{26}. (c) show the transit from (a) to (b) in 53 time-steps. (d) shows the subsequent evolution for 80 time-steps, where the rake moves South shedding Gc gliders moving West. Dynamic trails=53. RGe90 is reproduced by the breeders in section 6.6.
6.6 Breeders

Whereas a glider-gun or a rake ejects a stream of gliders, a breeder is a pattern that ejects a stream of glider-guns, rakes, or puffer-trains, in various combinations. Breeders are said to exhibit unbounded quadratic growth by creating multiple copies of a second object, each of which creates multiple copies of a third object \(^{18}\). The significance of breeders lies in their high level of complexity demonstrating open ended pattern evolution, as well as providing further components for computation and memory. Breeders have been constructed in the Precursor-Rule at the ConwayLife forum\(^ {19}\). In figures 42 to 44 we show four examples of breeders ejecting rakes.

![Figure 42: Two alternative breeders that reproduces the rake “RGc90” (red outline) from figure 41(d), transiting via the seed in figure 41(a), and reproduced every 192 time-steps. The breeders were constructed by \(^{24}\) from interactions between three rakes from figure 37(a) — the rakes on the right are modified to eject Ga gliders. Dynamic trails=20.](image)
Figure 43: A breeder made from meta-glider-guns by [17] for the rake “RGc90” (red outline) from figure 41(d), transiting via the seed in figure 41(a), and reproduced every 409 time-steps. This static breeder is constructed from three interacting “QuadGG2a” glider-guns (figure 27(a)). Dynamic trails=360. Its feasible to represent such long trails because the breeder itself is static (it does not expand as in figure 42). The horizontal trails in the lower-left are the ends of the dynamic trails from the RGc90 rake that was ejected previously.

7 Concluding remarks

The X-Rule’s isotropic precursor was the original cellular automaton that we studied in our search for universal computation, but at the time we found it necessary to modify the rule to find glider-guns — we were then able to demonstrate logical universality in the anisotropic X-rule[6]. After announcing the X-rule and its precursor on ConwayLife[19], members of the forum applied their
A breeder from 8 GGc glider-guns built by [17]. Along the top, 6 GGc’s are aligned horizontally, one slightly above the next, from left to right, sending 6 Gc glider-streams South. The two remaining GGc’s are positioned above each other on the right and eject Gc gliders West. The upper Westward glider-stream is deflected downward (by the 6 top GGc’s) and the two Westward glider-streams finally merge into a complex combined glider-stream, creating a sort of rake that continues to sends Gc glider-streams South (at 90°) all along its length, starting at the tip. An eater below the merging zone is an essential component. Dynamic trails=20.

considerable know-how in Game-of-Life pattern search to discover glider-guns in the isotropic Precursor-Rule. As well as glider-guns, many other important complex dynamical mechanisms have been constructed and we have presented a selection, which incidentally shows the power of diversified search in a dedicated community.

Armed with glider-guns, we were able to construct the logical gates and demonstrate logical universality in the Precursor-Rule, which like the Game-of-Life exhibits an extraordinary diversity of dynamics, but according to a rule not based on birth/survival logic. The results documented in this paper are an initial exploration — further interesting dynamics can be discovered possibly including memory functions required for universality in the Turing sense. The dynamics are open-ended and impossible to pin down within a sufficiently large space-time.

Although the Game-of-Life has accumulated a vast compendium of behaviour, it can be argued that the Precursor-Rule has a more diverse range of basic gliders and glider-guns, providing a richer diversity of the fundamental particles from which more complex structures can be built.
Figure 45: A new lately found compact Ga glider-gun[17], comparable in size to the basic Ga glider-gun in figure 1(b), but with double the period (38 time-steps), is constructed by colliding two GGc glider-streams at 90°. Dynamic trails=20.

The Precursor-Rule belongs to the ordered zone in rule-space within the input-entropy scatter-plot (figure 2), the zone with low values of entropy variability and mean entropy. It seems that the ingredients that enable logical universality — gliders, eaters, and crucially glider-guns — are more likely to occur in this zone, rather than in the “complex” zone with high entropy variability, were activity tends to overwhelm stability. Although 2D binary cellular automata rules supporting glider-guns are exceedingly rare, it appears nonetheless that many such rules are to be found in this zone, which begs the question, what are the underlying principles for the existence of glider-guns?

7.1 SansDomino rule-space

We have recently become aware of “SansDomino” rule-space[25], where rules can support gliders similar to Ga and Gc gliders in the Precursor-Rule, with analogous glider-guns shooting these gliders. These rules are based on modified birth/survival (B2/S13 and B2/S14) with the exception that an adjoining pair of 1s in the outer neighborhood outputs zero. An example of such a modified B2/S14 glider-gun is shown in figure 46. It will be important to investigate the relationship between the Precursor-Rule and ‘SansDomino” rules.

Figure 46: A glider-gun from “SansDomino” rule-space[25] based on birth/survival (B2/S14) featuring identical Ga and similar Gc gliders to the Precursor-Rule. Dynamic trails=20.
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References

[1] Berlekamp E.R., J.H.Conway, R.K.Guy, “Winning Ways for Your Mathematical Plays”, Vol 2. Chapt 25 “What is Life?”, 817-850, Academic Press, New York, 1982.
[2] Eppstein, D. “Growth and Decay in Life-Like Cellular Automata”, in “Game of Life Cellular Automata”, edited by Andrew Adamatzky, Springer Verlag, 2010.
[3] Coxon, E. Private communication, 2016.
[4] Francis Jeffry Pelletier and Norman M. Martin, “Post’s Functional Completeness Theorem”, Notre Dame Journal of Formal Logic, Vol.31, No.2, 1990.
[5] Gardner, M., “Mathematical Games The fantastic combinations of John Conway’s new solitaire game ‘life’ ”. Scientific American 223. pp. 120–123, 1970.
[6] Gomez-Soto, J.M., and A. Wuensche, “The X-rule: universal computation in a non-isotropic Life-like Cellular Automaton”, JCA, Vol 10, No.3-4, 261-294, 2015. preprint: http://arxiv.org/abs/1504.01434/
[7] Randall, Jean-Philippe, “Turing Universality of the Game of Life”, Collision-Based Computing, Andrew Adamatzky Ed. Springer Verlag, 2002.
[8] Sapin, E., O. Bailleux, J.J. Chabrier, and P. Collet. “A new universel automata discovered by evolutionary algorithms”, Gecco2004, Lecture Notes in Computer Science, 3102:175187, 2004.
[9] Eric Weisstein,E., “Weisstein’s Treasure Trove of Life C.A.”, 2008, http://www.ericweisstein.com/encyclopedias/life/Breeder.html
[10] Wolfram,S., “Statistical Mechanics of Cellular Automata”, Reviews of Modern Physics, vol 55, 601-644, 1983.
[11] Wuousche,A., “Classifying Cellular Automata Automatically; Finding gliders, filtering, and relating space-time patterns, attractor basins, and the Z parameter”, COMPLEXITY, Vol.4/no.3, 47-66, 1999.
[12] Wuensche,A., “Glider Dynamics in 3-Value Hexagonal Cellular Automata: The Beehive Rule”, Int. Journ. of Unconventional Computing, Vol.1, No.4, 2005, 375-398, 2005.
[13] Wuensche,A., A.Adamatzky, “On spiral glider-guns in hexagonal cellular automata: activator-inhibitor paradigm”, International Journal of Modern Physics C, Vol. 17, No. 7,1009-1026, 2006.
[14] Wuensche,A., “Exploring Discrete Dynamics – Second Edition”, Luniver Press, 2016.
the ConwayLife forum — Other Cellular Automata — X-Rule

[16] Aidan F. Pierce, “Awesome”.
[17] Charlie Neder, “BlinkerSpawn”.
[18] LifeWiki — Breeder, http://conwaylife.com/w/index.php?title=Breeder.
[19] ConwayLife forum, “A community for Conway’s Game of Life and related cellular automata”, Other Cellular Automata — X-Rule. http://www.conwaylife.com/
[20] “Danieldb”.
[21] Gómez Soto José Manuel, “jmgomez”.
[22] Dary Saka Fitrady, “Saka”.
[23] Dongook Lee, “Scorbie”.
[24] Anonymous.
[25] J. Pystynen, “Tropylium”. http://www.conwaylife.com/forums/viewtopic.php?f=11&t=836
[26] Arie Paap, “Wildmyron”.
[27] Ian Wright, “Wright”.

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