Weak-decay rates of pf-shell nuclei in stellar scenarios

Pedro Sarriguren
Instituto de Estructura de la Materia, IEM-CSIC, Serrano 123, E-28006 Madrid, Spain
E-mail: p.sarriguren@csic.es

Abstract.
Weak decay rates under various stellar density and temperature conditions are studied in selected pf-shell nuclei of special relevance as constituents in presupernova formations. The rates are relevant to understand late stages of stellar evolution leading to supernova explosions and the nucleosynthesis of heavier nuclei. The nuclear structure part of the problem is described within a quasiparticle random-phase approximation. Results are compared to benchmark shell-model calculations and experimental data extracted from charge-exchange reactions.

1. Introduction
Weak \(\beta\)-decay and electron-capture (EC) processes are very important mechanisms to understand the late stages of the stellar evolution, playing a critical role to determine both the presupernova stellar structure and the nucleosynthesis of heavier nuclei. These processes are dominated by Gamow-Teller (GT) transitions and therefore, the GT properties of pf-shell nuclei are of special importance because these nuclei are the main constituents of the stellar core in presupernova formations leading to core-collapse (type II) or thermonuclear (type Ia) supernovae. An accurate understanding of those astrophysical processes requires input from nuclear physics that in most cases cannot be measured directly because of the extreme conditions of density (\(\rho\)) and temperature (\(T\)) that hold in stellar scenarios and therefore, the GT strength distributions must be estimated in many cases by model calculations. Obviously, nuclear physics uncertainties will finally affect the reliability of the description of those astrophysical processes.

A systematic evaluation of the ability to reproduce the measured GT strength distributions of various theoretical models based on shell model (SM) and QRPA was done recently in Ref. [1]. EC rates were also derived from those models at relevant \(\rho\) and \(T\). While several sets of SM calculations using different effective interactions were compared, namely KB3G [2] and GXPF1a [3], in the case of QRPA only the formalism developed in [4] using deformations and masses obtained from the finite range droplet model [5] was considered in Ref. [1]. In this work, we study the dependence of the EC rates on both \(\rho\) and \(T\) with GT strength distributions calculated within a QRPA approach based on a selfconsistent deformed Hartree-Fock (HF) mean field with Skyrme interactions including pairing correlations and residual separable forces in both particle-hole (\(ph\)) and particle-particle (\(pp\)) channels. We compare our calculations with the benchmark calculations in Ref. [1]. This formalism represents an improvement over the QRPA-Möller approach in several aspects. First, instead of a phenomenological approach, the deformed mean field is obtained selfconsistently and secondly, a separable residual GT interaction in the \(pp\) channel is included. The present nuclear model has been tested successfully reproducing...
very reasonably the experimental information available on both bulk and decay properties of medium-mass nuclei [6, 7, 8, 9].

2. Theoretical approach
The decay rate corresponding to an initial nuclear parent state \( i \) is given by

\[
\lambda_i = \sum_f \lambda_{if} = \frac{\ln 2}{D} \sum_f B_{if} \Phi_{if}(\rho, T),
\]

where the sum extends over all the states in the final nucleus reached in the process and \( D = 6146 \) s. This expression is decomposed into a phase space factor \( \Phi_{if} \), which is a function of \( \rho \) and \( T \) and a nuclear structure part \( B_{if} \) that contains the transition probabilities for allowed GT transitions.

In the astrophysical scenarios of our study, nuclei are fully ionized and continuum EC from the degenerate electron plasma are possible. The phase space factor for EC is given by

\[
\Phi_{if}^{EC} = \int_0^\infty \omega p(Q_{if} + \omega)^2 F(Z, \omega) S_e(\omega) d\omega,
\]

where \( \omega \) is the total energy of the electron, \( p = \sqrt{\omega^2 - 1} \) is the momentum, and \( Q_{if} \) is given by \( Q_{if} = (Q_{EC} - m_e c^2 - E_f)/m_e c^2 \). \( F(Z, \omega) \) is the Fermi function that takes into account the distortion of the electron wave function due to the Coulomb interaction. \( S_e \) is the electron distribution function given by a Fermi-Dirac distribution. The lower integration limit is given by \( \omega_l = 1 \) if \( Q_{if} > -1 \), or \( \omega_l = |Q_{if}| \) if \( Q_{if} < -1 \).

The nuclear structure involved in the weak rates is described within a microscopic deformed quasiparticle random-phase approximation (QRPA) based on a selfconsistent mean field obtained from Skyrme (SLy4 and SG2) Hartree-Fock + BCS calculations. A residual separable interaction includes those ranges relevant for astrophysical scenarios related to the silicon-burning stage in a presupernovae star \( (\rho Y_e = 10^7 \text{ mol/cm}^3 \text{ and } T_9 = 3) \), as well as scenarios related to pre-collapse of the core and thermonuclear runaway type Ia supernovae \( (\rho Y_e = 10^9 \text{ mol/cm}^3 \text{ and } T_9 = 10) \). The calculated total strength, including results from SM calculations, overestimates always the experiment.

3. Results and discussion
As an example of the results obtained within this scheme [12], we can see in Fig. 1 the GT strength for a transition from an initial state \( i = 0 \) to a final state \( f \) is given by

\[
\frac{g_A}{g_V} \frac{1}{2J_i + 1} \left( \frac{g_A}{g_V} \right)^2_{\text{eff}} \langle f || \sum_j \sigma_j t^+ || i \rangle^2,
\]

where \( \langle g_A/g_V \rangle_{\text{eff}} = 0.7(g_A/g_V)_{\text{bare}} \) is the effective ratio of axial and vector coupling factors.
Figure 1. GT strength distributions for the transition $^{64}$Zn to $^{64}$Cu versus the excitation energy of the daughter nucleus. (a) Experimental data are compared to our QRPA results. Experimental accumulated strengths are compared to QRPA (a) and shell-model calculations (c).

Figure 2. EC rates for $^{64}$Zn obtained from experimental GT strength distributions and from different shell model and QRPA calculations for various densities.

We show the EC rates obtained from the experimental GT strength distributions and from different SM (KB3G and GXPF1a) and QRPA (SLy4, SG2, and Möller) calculations. Some general comments about the sensitivity of the EC rates to $(\rho, T)$, to the Fermi and $Q$ energies, and to the GT distribution are in order to understand better the EC rates. Since the Fermi energy increases with the electron density, it is expected that the EC rates at low densities are mainly sensitive to the GT strength of states at low excitation energies in the daughter nucleus. When $\rho$ increases the rates also increase because the Fermi energy of the electrons is larger and larger allowing to reach higher excitation energies in the daughter nucleus and thus making the GT strength at these energies contribute to the rates in a more significant way. At low $T$, the shape of the electron energy distribution, $S_e$, has a sharp surface at the Fermi energy. When increasing $T$, the rates in general increase because the diffuseness of the electron energy distributions allows to reach more excited states at higher energy.

On the other hand the role of the $Q_{if}$ (and $Q_{EC}$) energy is also very important. It determines the lower integration limit in the EC phase factor (2). Then, the energy of the available electrons must overcome this value to be captured or in other words, the Fermi energy has to be larger than $Q_{if}$ at $T = 0$ where $S_e$ changes abruptly from one to zero at the Fermi energy. When $T$ increases, $S_e$ is smeared out and ECs are possible even for Fermi energies lower than $Q_{if}$. In
general, the larger the $Q_{EC}$ energy (more negative in stable nuclei) the lower the EC rates. This effect is enhanced at low densities where the Fermi energy is small. Similarly, the rates of nuclei with large (negative) $Q_{EC}$ values are mostly sensitive to the GT strength of the ground and lowest excited states whereas the opposite is true for the nuclei with small $Q_{EC}$ values, such as $^{64}$Zn ($Q_{EC} = -0.580\text{ MeV}$), where the rates are sensitive to high excitation energies that can contribute especially at high $\rho$ values.

The agreement with experiment in Fig. 2 is quite reasonable, especially for QRPA-SG2 calculations. The rates are always underestimated.

4. Conclusions
We have evaluated the continuum electron-capture rates in $^{64}$Zn, as an example of a pf-shell nucleus, at different density and temperature conditions holding in stellar scenarios of particular interest. The nuclear structure involved in the calculation of the energy distribution of the Gamow-Teller strength is described within a selfconsistent deformed HF+BCS+QRPA formalism with density-dependent effective Skyrme interactions and spin-isospin residual interactions.

We find that the present QRPA calculation is able to reproduce the main features of the GT distributions extracted in these nuclei from charge-exchange reactions. Comparison of our results [12] to SM calculations and other QRPA results analyzed in Ref. [1] shows that on the overall the agreement with experiment of the present QRPA calculations is comparable to the agreement achieved by SM calculations and that QRPA based on Skyrme forces improve the agreement with experiment with respect to QRPA-Möller [4].

We have studied the sensitivity of the EC rates to both $\rho$ and $T$. At low $\rho$ (low Fermi energies) and low $T$ (sharp shape of the energy distribution of the electrons), the rates are very sensitive to details of the GT strength of the low-lying excitations and therefore to model calculations. On the other hand, when the $\rho$ and $T$ are high enough, the EC rates are sensitive to all the spectrum. Then, the whole description of the GT strength distribution is more important than a detailed description of the low-lying spectrum. Since QRPA reproduces reasonably well the global behavior of the GT strength distributions, it is expected to be a good approach, especially for high $\rho$ and $T$ conditions.

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