Similar Single-Difference Model and Its Algorithm for Solving GPS Monitoring Deformation Directly at Single Epoch

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1 Introduction

Presently, GPS techniques are widely used to monitor the deformation of all kinds of buildings and constructions. For the high-precision GPS deformation monitoring, the carrier phases are usually used as the basic observational techniques. Detecting and repairing the cycle slips correctly and solving the ambiguities are the keys to obtain the high-precision positioning results. In this paper, a new similar single-difference mathematical model (SSDM) and its algorithm are advanced to solve the GPS monitoring information directly in single epoch.

2 Mathematical model

In order to obtain reliable deformation data and ensure the accuracy of deformation monitoring, the following three characteristics can be usually found in the high-precision GPS deformation monitoring networks: ① The base points are built in the places where the geologic condition is steady and easily preserved. That is to say, the deformations of fiducial points, compared with that of monitoring points, can be neglected. ② In order to avoid the instrument error affecting the deformation monitoring accuracy, the monitoring distance (distance between fiducial point and monitoring point) commonly is less than 3 km. ③ In order to obtain reliable deformation data, the monitoring network should be linked with the adjacent
high level GPS points and use precise baseline resolution software to process baselines in the first observation. According to these characteristics, the SSDM will be introduced in detail.

The coordinates in WGS-84 system of fiducial point $p_1$ and monitoring point $p_2$ have been obtained through the first observation. During the course of surveying the monitoring network, the fiducial point $p_1$ is immovable and the monitoring point $p_2$ has displacement. The deformed position of $p_1$ is $p_3$, and the deformation value is $d$.

At epoch $t$, the carrier phase for the satellite $i$ on the fiducial point $p_1$ and the deformed monitoring point $p_3$ can be expressed using the following two formulae:

$$
\ddot{p}_i = \lambda_0 \phi_i + \lambda N_i - c \Delta t_1 + c \dot{t}_i + h_1 \sin \theta_1 + \frac{p_1}{c} + \frac{p_3}{c} \Delta t_2 - \Delta \lambda_{i,2}
$$

$$
\ddot{p}_3 = \lambda_0 \phi_i + \lambda N_i - c \Delta t_1 + c \dot{t}_i + h_1 \sin \theta_1 + \frac{p_1}{c} + \frac{p_3}{c} \Delta t_2 - \Delta \lambda_{i,2}
$$

where $\lambda$ is the wavelength of carrier wave; $\phi$ is the carrier phase observation; $N$ is the initial integer cycles; $\rho$ is the geometric distance between satellite and station; $\dot{p}$ is the distance varying ratio; $h$ is the height of the antenna; $\theta$ is the altitude angle; $h \cdot \sin \theta$ is the correction that the distance between satellite and the antenna phase center is corrected to the distance between satellite and the station’s center.

In Eq. (5), besides the unknown deformation, there are the initial integer cycles of carrier phase, so Eq. (5) can not be used to calculate the deformation directly. The first term on the right side of Eq. (5) can be written as

$$
\Delta \lambda_{i,2} = \Delta \lambda_{i,2,\text{temp}} + \Delta \lambda_{i,2,\text{ion}} + \Delta \lambda_{i,2,\text{mult}} + \Delta \lambda_{i,2}
$$

In Eq. (6), besides the unknown deformation, there are the initial integer cycles of carrier phase, so Eq. (5) can not be used to calculate the deformation directly. The first term on the right side of Eq. (5) can be written as
The single-difference integer cycle unknowns (SD ambiguities) remain in Eq. (7). At epoch, if all the double-difference integer cycle unknowns (DD ambiguities) relative to the reference satellite (assuming its number is 1)

\[ N_i = N_{i,1} \]

have been calculated, as

\[ N_{i,1} = N_{i,1} + N_{i,2} \]

Substituting Eq. (8) into Eq. (7), we have

\[ \Delta N_i = N_{i,1} - N_{i,2} - \hat{N}_{i,2} \]

By substituting Eq. (9) into Eq. (5), Eq. (5) can be further rewritten as

\[ dX = -\lambda N_{i,1} \]

Eq. (10) is just the mathematical model of solving the monitoring point deformation information directly in single epoch. This model is similar to the single-difference model of carrier phase observations, and the difference lies in considering the direction cosine. So, the mathematical model is called similar single-difference model (SSDM), and the corresponding algorithm is called similar single-difference method.

3 Qualitative analysis for model errors

In Eq. (10), the SD ambiguity \( N_{i,1} \) of the reference satellite and the deformations \( dX \) of the monitoring points will be estimated at every epoch.

The distance between the fiducial point and the monitoring point is less than 3 km, then the direction cosines of this two stations to the same satellite can be considered as the same, so the common errors, such as clock offset of the satellite, offsets of ephemerides, the delay errors of the atmosphere etc. can be eliminated or reduced well.

The performance of the two receiver clocks is different, and the relative clock error may be very big, so the errors of the receiver clocks can not be ignored. This is one of the main error sources that affect the accuracy of deformations. Moreover, the offsets of the receiver clock must be calculated precisely.

The errors of the antenna height affect the accuracy of deformations directly, and must be considered in the design and observation. As for the high-precise GPS deformation monitoring, especially the vertical deformation monitoring, some observation measures, such as adopting fixed antenna height and forcible center device, should be taken.

The fifth and sixth terms on the right of Eq. (10) can not be ignored in the model, but the calculated errors have not notable influence upon the deformations.

The accuracy of the known baseline vectors \( X_{i,2} \) between the fiducial point and the monitoring point lies on the data processing accuracy of the first observation. For the high-precision GPS deformation monitoring, the network should be linked with the adjacent high level GPS points and use precise baseline resolution software, such as GAMIT software, to process baseline vectors in the first observation. Then the accuracy of the baseline vectors can reach the mm level.

The validity of the DD ambiguity \( N_{i,1} \) is another main factor that affects the reliability of the deformation value. From Eq. (10), the effects of the DD ambiguity error on the deformation value relate to the direction cosine from the fiducial point to the non-reference satellite, that is to say, the effects relate to the geometry of the fiducial point and the satellite. In the worst circumstances, one cycle error in ambiguity will cause one cycle destructive effect on the deformation value. Hence, in order to ensure the accuracy of deformations, the appropriate measures should be taken to solve the DD ambiguity.
The initial integer cycle unknown $N^i_{p_1}$ of the vector which is between the non-reference satellite and the monitoring point affects the accuracy of deformations as well. Since the distance between the fiducial point and the monitoring point is less than 3 km, and the factor $N^i_{p_1}$ is very small, its computational accuracy will be ensured by use of pseudorange and carrier phase observation.

In a word, the main factors affecting the model accuracy are the relative clock error of receivers and the validity of the DD ambiguities.

4 Basic algorithm

At epoch $t$, the offsets of the receiver clock and satellite clock, atmospheric delay corrections, the DD ambiguities etc. should be computed firstly. Then the error equations are formed to solve the deformations.

4.1 Computing the DD ambiguities

At epoch $t$, the observations for satellite $i$ and the reference satellite (assuming its number is 1) is carried out on the fiducial point $p_1$ and the monitoring point $p_2$, then the linearized DD observation equation can be written as

$$\varphi_{p_1,p_2}^{i,i} = -\frac{\delta X}{c} \left[ l_{p_1}^i m_{p_2}^i n_{p_2}^i \right] \delta X + \delta Y + \delta Z,$$

where

$$\varphi_{p_1,p_2}^{i,i} = \varphi_{p_1}^{i,i} - \varphi_{p_2}^{i,i} + \varphi_{p_1}^{i,i}$$

The coordinates of the fiducial point $p_1$ and the monitoring point $p_2$ have been obtained from the first observation, and the fiducial point $p_1$ is steady. When the deformation of monitoring point $p_2$ is small, Eq. (11) can be written as

$$N^i_{p_1,p_2} = f \times p_{p_1,p_2}^{i,i}$$

In Eq. (13), $p_{p_1,p_2}^{i,i}$ and $p_{p_1}^{i,i}$ are the computational pseudoranges in the station coordinates from the first observation and the satellite coordinates at $t$ epoch. The DD ambiguities can be obtained from Eq. (13) and Eq. (14).

4.2 Calculation of the deformation

At epoch $t$, the fiducial point $p_1$ is regarded as the reference station, the number of the reference satellite is 1, then Eq. (10) for the satellite $i$ can be rewritten as.

$$\varphi_{X,i} = \delta X + \lambda l_{p_1}^i \delta N_{p_1,p_2}^i - \left( l_{p_1}^i \lambda \delta \rho_{p_1} - \delta \rho_{p_1} + \delta \rho_{p_1} \right)$$

Let the initial value of the deformation be $\delta X = 0$, and the initial value of the SD ambiguity $N_{p_1,p_2}^{i,o}$ be

$$N_{p_1,p_2}^{i,o} = \left( \rho_{p_1} - \delta \rho_{p_1} \right) / \lambda - \left( \delta \rho_{p_1} - \delta \rho_{p_1} \right)$$

where $\rho$ is the pseudorange observation; $\varphi$ is the carrier phase observation. The corrections of the deformation and SD ambiguity are $\delta X$ and $\delta N_{p_1,p_2}^{i,o}$, respectively, then Eq. (10) can be rewritten as

$$v_{X,i} = \delta X + \lambda l_{p_1}^i \delta N_{p_1,p_2}^i - \left( l_{p_1}^i \lambda \delta \rho_{p_1} - \delta \rho_{p_1} + \delta \rho_{p_1} \right)$$

where $\left( \ldots \right)$ is the parts of $\left( \ldots \right)$ in Eq. (15). Let $L_{X,i} = \left( \ldots \right) - \delta X - \lambda l_{p_1}^i N_{p_1,p_2}^{i,o}$

then

$$v_{X,i} = \delta X + \lambda l_{p_1}^i \delta N_{p_1,p_2}^i - \left( \ldots \right) + \delta X - \lambda l_{p_1}^i N_{p_1,p_2}^{i,o}$$

where $v_{X,i}$ is the parts of $\left( \ldots \right)$ in Eq. (15). Let

$$L_{X,i} = \left( \ldots \right) - \delta X - \lambda l_{p_1}^i N_{p_1,p_2}^{i,o}$$

then

The forms of the $Y$ and $Z$ components of the deformation are similar to Eq. (19), then

$$v_{Y,i} = \delta Y + \lambda l_{p_1}^i \delta N_{p_1,p_2}^i - L_{Y,i}$$

$$v_{Z,i} = \delta Z + \lambda l_{p_1}^i \delta N_{p_1,p_2}^i - L_{Z,i}$$

where $l$, $m$, and $n$ are the direction cosines from the station to the satellite, and $L$ is the constant term.

At epoch $t$, if $s$ satellites are observed synchronously, then $s$ error equations similar to Eq. (20) can be obtained, and the whole error equations can be written as

$$V = AX - L, \text{ weight } P$$
where

\[ X = [\delta X \ \delta Y \ \delta Z \ \delta N_{s}, \delta p_{2}]^{T} \]  \hspace{1cm} (22)

According to the least-squares estimation theory, the deformations of the monitoring points and SD ambiguity can be obtained from Eq. (21). In order to ensure the deformations accuracy, it is necessary to compute two times iteratively. If the observations are carried out in \( k \) epochs, then the deformation of the monitoring point \( p_{2} \) is the average of these observations in all epochs.

5 Test and analyses

The test was carried out on the slideway in Feb. 2001. Five dual frequency receivers, whose numbers are from 1 to 5, are adopted and in this test the observations are conducted in 5 sessions. The first session is 6 hours long, and the others are 2 hours. The receiver antenna of No. 4 is set up on the slideway and the other 4 antennae are fixed on the mounds. After the first session has ended, the true deformation values of No. 4 (as the monitoring point) can be obtained through moving its antenna and reading the micrometer. The true values of No. 4 from session 2 to session 5 are listed in Table 1. The other four antennae (as the fiducial points) are immovable, so the true deformations are zeros. In Table 1, \( dN \) stands for the deformation along the south-north direction, \( dE \) the east-west, and \( dU \) the zenith.

| Session No. | \( dN \) | \( dE \) | \( dU \) |
|-------------|--------|--------|--------|
| 2           | 15     | -14    | 0      |
| 3           | 18     | -13    | 0      |
| 4           | 21     | -11    | 0      |
| 5           | 27     | -4.5   | 0      |

In order to validate the correctness of the similar single-difference model, on the basis of the results of the first session, the calculation is carried out and using the observations from session 2 to session 5. Firstly, the deformations are computed using \( L_{1} \) and \( L_{2} \) carrier phase observations at every epoch, then the final deformations of each session are the average over all epochs of this session. The final results are listed in Table 2 and Table 3. In order to confirm the correctness of this model in the condition that the monitoring point is immovable, the displacements of No. 2, 3, 4 and 5 fiducial points to No. 1 fiducial point are listed in Table 2 and Table 3. In the two tables, FP stands for the fiducial point, MP the monitoring point, and CP the carrier phase type.

| FP | MP | CP | Session 2 | Session 3 |
|----|----|----|-----------|-----------|
|    |    |    | \( dN \)  | \( dE \)  | \( dU \)  | \( dN \)  | \( dE \)  | \( dU \)  |
| 1  | 4  | L1 | 14.5      | 15.0      | 1.1      | 16.7      | 13.4      | -0.4     |
| 1  | 5  | L2 | 17.1      | 14.8      | 1.4      | 18.3      | 14.1      | -0.4     |
| 2  | 4  | L1 | 13.8      | -14.0     | 0.2      | 16.3      | -13.0     | -1.9     |
| 2  | 5  | L2 | 17.1      | -14.1     | 1.3      | 18.0      | -13.0     | -1.0     |
| 3  | 4  | L1 | 14.5      | -14.9     | 0.9      | 16.6      | -13.9     | -1.1     |
| 3  | 5  | L2 | 17.2      | -14.6     | 1.6      | 17.7      | -13.7     | -0.6     |
| 4  | 4  | L1 | 14.4      | -14.7     | 1.1      | 16.4      | -14.1     | -1.4     |
| 4  | 5  | L2 | 17.1      | -14.7     | 1.7      | 18.2      | -13.9     | -1.2     |

Table 2 Computed average deformations of sessions 2 and 3/mm

| BP | MP | CP | Session 4 | Session 5 |
|----|----|----|-----------|-----------|
|    |    |    | \( dN \)  | \( dE \)  | \( dU \)  | \( dN \)  | \( dE \)  | \( dU \)  |
| 1  | 4  | L1 | 19.8      | -11.8     | -0.6     | 25.6      | -5.7      | 0.9      |
| 1  | 5  | L2 | 21.8      | -12.0     | -0.1     | 27.5      | -6.1      | 1.2      |
| 2  | 4  | L1 | 19.2      | -11.0     | -1.5     | 25.4      | -5.4      | 0.2      |
| 2  | 5  | L2 | 21.7      | -11.5     | -1.0     | 27.2      | -5.4      | 1.3      |
| 3  | 4  | L1 | 19.9      | -12.0     | -1.5     | 25.5      | -5.7      | 0.7      |
| 3  | 5  | L2 | 21.5      | -11.7     | -0.6     | 27.1      | -5.5      | 1.2      |
| 4  | 4  | L1 | 19.8      | -11.7     | -1.5     | 25.6      | -5.8      | 1.0      |
| 4  | 5  | L2 | 21.9      | -12.1     | -0.3     | 27.4      | -5.6      | 1.4      |

Table 3 Computed average deformations of sessions 4 and 5/mm

The deformation rms of No. 4 monitoring point in the four sessions, through the differences between the computed deformations and the actual deformations are shown in Ta-
Table 4  The rms of the monitoring point deformation of sessions 2 and 3/mm

| CP | Session 2 | Session 3 |
|----|-----------|-----------|
| L1 | 0.8       | 0.8       |
| L2 | 2.1       | 0.6       |

Table 5  The rms of the monitoring point deformation of sessions 4 and 5/mm

| CP | Session 4 | Session 5 | All sessions |
|----|-----------|-----------|--------------|
|    | σN        | σE        | σU          | σN        | σE        | σU          |
| L1 | 1.3       | 0.7       | 1.3         | 2.5       | 1.2       | 0.8         | 1.7         | 0.9         | 1.1         |
| L2 | 0.7       | 0.9       | 0.6         | 0.3       | 1.2       | 1.3         | 1.1         | 0.9         | 1.1         |

Practically, Table 2 and Table 3 show that, when using the different fiducial points and different carrier phase observations, the difference of computed deformations in the same session is less than 1.0 mm, in general. From Table 2 to Table 5, it can be seen that there are some differences between the deformations when L1 carrier phase or L2 is used, but the difference is small from “All session” of Table 5.

The accuracy requirement for the plane position is better than 3.0 mm, and that for the vertical direction is superior to 6 mm. In this test, the largest plane position accuracy is about 2.8 mm in session 5, and the vertical direction height accuracy of is 1.5 mm in session 2. Considering the accuracy of GPS height is lower than that of the plane position, and there is no deformation in the vertical in the test, it is predicted that the deformation accuracy in zenith can reach the 6 mm, as required.

**6 Conclusions**

Compared with other methods, the similar single-difference model has the following characteristics.

1. Since the first period baseline vectors between the fiducial points and the monitoring points are used as condition, the ambiguity resolution becomes easier.

2. By adopting the single epoch algorithm, the troublesome problem of detecting and repairing cycle slips is avoided.

3. The unknowns in the model are constant 4 and do not relate to the numbers of satellites.

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