Temperature of rock formation and fracturing fluid during the hydraulic fracturing process

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Abstract. The difference in temperature at the surface and at the depth of hydraulic fracturing is of the order of 80 degrees, which imposes higher requirements on the rheological properties of hydraulic fracturing fluid. Under the influence of temperature, the viscosity changes that affects the final shape of the fracture and the dynamics of its propagation. It is necessary to build a model that makes it possible to calculate the temperature of the liquid and the rock layer in the vicinity of the crack, consuming minimum of computational resources and using available data on the thermal properties of the rock and hydraulic fracturing fluid.

1. Introduction
The temperature of the liquid $T_F(t)$ in the fracture and the temperature of the rock layer $T_R(t)$ are assumed to be homogeneous throughout the entire area under the consideration. The fluid exchanges thermal energy with the rock due to leakage into the rock and heat transfer according to the Newton-Richman law [1,2] through the growing crack surface area with a variable coefficient of heat transfer. The crack surface is assumed to be known and in the simplest case can be taken from the PKN model [3,4,5]. It is assumed that the rock is uniform. It is assumed that the rock is uniform. Thickness of the layer with which the liquid exchanges heat is found by using the approximate assumption proposed in [6] and it grows as a function of time as follows:

$$D(t) = \sqrt{\lambda_R t/(C_V^R \rho_R)}$$

where $\lambda_R$ is the heat conductivity of rock formation, $t$ is the time counted from the beginning of the pumping, $C_V^R$ is the specific heat of rock, $\rho_R$ is the rock density.

The crack into which the liquid is pumped is considered to be one cell and so the coordinate “x” is reduced, that is, the approximation used considers temperature to be equilibrium both in the direction of the “x” axis and in the direction of the “y” axis. The liquid enters the entire volume of the crack at the initial moment of time.

2. Thermomechanical data used in the work
The following are the thermomechanical parameters used in the calculation as an example. The average values are taken for sandstone and water at normal pressure and temperature [3,4].
Table 1. Example of possible thermomechanical data used in the present work for illustrative purposes.

| Parameter                              | Value                           |
|----------------------------------------|---------------------------------|
| Young’s modulus, $E$                   | $40 \times 10^9 \text{ Pa}$     |
| Plane strain Young’s modulus, $E_l$    | $E_l/(1-\nu^2) \text{ Pa}$      |
| Poisson’s ratio, $\nu$                | 0.25                            |
| Carter’s leakoff coefficient, $C_l$    | $3.54 \times 10^{-4} \text{ m/s}$ |
| Viscosity of fracturing fluid, $\mu$  | 0.001 $\text{ Pa} \cdot \text{s}$ |
| Density of fracturing fluid, $\rho_F$ | 1000 $\text{ kg/m}^3$          |
| Specific heat of fracturing fluid, $C_v^F$ | 4200 $\text{ J/(kg} \cdot \text{K)}$ |
| Density of the rock formation, $\rho_R$ | 2968 $\text{ kg/m}^3$          |
| Specific heat of rock formation, $C_v^R$ | 850 $\text{ J/(kg} \cdot \text{K)}$ |
| Thermal conductivity of rock formation, $\lambda_R$ | 3 $\text{ W/(m} \cdot \text{K)}$ |
| Newton-Richman heat exchange coefficient, $L$ | 100 $\text{ W/(m}^2 \cdot \text{K)}$ |
| Fracturing fluid pumping time, $t_P$   | 1000 s                          |

The heat transfer coefficient $L$ depends on the physical properties of the fluid and the physical conditions in which heat exchange takes place. Therefore, for each system, this coefficient must be obtained or found experimentally. For laminar flows, the heat transfer coefficient is rather small in comparison with one for turbulent flows [7].

The Perkins-Kern-Nordgren model (PKN) is another well-known two-dimensional classical hydraulic fracture model [5] with a long fracture length (hundreds of meters in length along the “x” axis), limited but constant height (tens of meters along the “y”) and a small width (measured in millimeters along the “z” axis) propagating in an infinite homogeneous isotropic linear elastic reservoir that is characterized by Young’s modulus $E$, Poisson’s ratio $\nu$ and viscosity coefficient $\mu$. Then the change of the crack length with time will have the following form [5]:

$$ l = \left( \frac{625}{4096\pi^3} \right)^{\frac{1}{5}} \left( \frac{Q_0^3 E_l}{\mu H^4} \right)^{\frac{1}{5}} t^{\frac{4}{5}}, $$

where the crack height $H$ is chosen to be 40m. Since the length of the crack is large in comparison with two other dimensions, the approximate condition of plane strain can be taken in all the planes orthogonal to the length (directed along the x axis).

Let us consider the rest of values used in calculations. The internal crack surface area is given by the expression:

$$ A(t) = 4HL. $$

The total volume of fracturing fluid pumped into the fracture increases linearly with time:

$$ V_P(t) = Q_0 t. $$

The volume of fluid leakoff into the rock formation is determined by Carter’s empirical law:

$$ Q_L = \frac{2C_l}{\sqrt{t+t_0}} A(t). $$

The total volume of leakoff in time $t$ is as follows:

$$ V_L(t) = Q_L t. $$
An estimation of the volume of a rock layer exchanging heat with a fracturing fluid is given by:

\[ V_R(t) = A(t) \sqrt{t} \sqrt{\frac{\lambda_R}{\rho_RC_v}}. \]

The heat transfer coefficient grows along with the surface area of the crack:

\[ c(t) = LA(t) H (- (t - t_P)) + LA(t_P) H ((t - t_P)), \]

where \( H(t) \) is the Heaviside function, used to simulate the termination of the surface area grows. In all of the equations considered below, the heat transfer coefficient enters as a function of time.

### 3. Energy balance equations for the PKN crack

Equations of energy balance for a fluid in a fracture and its surrounding rock formation without taking into account the effect of leakoff will have the form:

\[ \rho_F C_v^F T_F'(t) = -\rho_F C_v^F \frac{V_F'(t)}{V_P(t)} (T_F(t) - T_F^0) H (- (t - t_P)) + c(T_R(t) - T_F(t)), \]

\[ \rho_R C_R^R T_R'(t) = -\rho_R C_R^R \frac{V_R'(t)}{V_R(t)} (T_R(t) - T_R^0) - c(T_R(t) - T_F(t)), \]

where \( C_v^F \) is the specific heat of fracturing fluid, \( \rho_F \) is the density of fracturing fluid, \( C_R^R \) is the specific heat of rock formation, \( \rho_R \) is the density of rock formation. Unknown functions that can be found from the solution of this system: \( T_F(t), T_R(t) \). Initial conditions are given:

\[ T_R(0) = T_R^0, \quad T_F(0) = T_F^0. \]

### 4. The equilibrium temperature

The system of equations in a stationary form (the time derivatives of the temperature vanish) has the following form:

\[ -\rho_F C_v^F \frac{V_F'(t)}{V_P(t)} (T_F(t) - T_F^0) + c(T_R(t) - T_F(t)) = 0, \]

\[ -\rho_R C_R^R \frac{V_R'(t)}{V_R(t)} (T_R(t) - T_R^0) - c(T_R(t) - T_F(t)) = 0. \]

We write out the solution of the system under consideration that the both temperatures do not change, i.e. let us pass to the equilibrium state of temperatures provided that the working fluid is slowly pumped into the fracture:

\[ T_F^S(t) = \frac{cC_v^F T_F^0 \rho_F V_R(t) V_F'(t) + C_v^R \rho_R \left( cT_R^0 V_P(t) + C_v^F T_F^0 \rho_F V_F'(t) \right) V_F'(t)}{cC_v^F \rho_F V_P(t) V_F'(t) + C_v^R \rho_R \left( cV_P(t) + C_v^F \rho_F V_F'(t) \right) V_F'(t)}, \]

\[ T_R^S(t) = T_R^0 + \frac{cC_v^F T_F^0 \left( T_F^0 - T_R^0 \right) \rho_F V_R(t) V_F'(t)}{cC_v^F \rho_F V_P(t) V_F'(t) + C_v^R \rho_R \left( cV_P(t) + C_v^F \rho_F V_F'(t) \right) V_F'(t)}. \]

Let us find the limiting values for the solutions obtained, letting the time go to infinity. The equilibrium solution of the problem of heat exchange between the fluid and the rock layer (2)–(3) under initial conditions (4) is given by the following expression:

\[ T_{eq} = \frac{10C_v^F \rho_F T_F^0 + 13C_v^R \rho_R T_R^0}{10C_v^F \rho_F + 13C_v^R \rho_R}. \]
If the fracturing fluid is water, and the rock formation in which the fracture propagates is the sandstone, the equilibrium temperature can be given by the following formula:

\[ T_{eq} = \frac{(T_R^0 + T_F^0)}{2.2} \quad (6) \]

which agrees well with practical engineering estimates.

The equations, taking into account Carter’s leakoff coefficient, where the corresponding power of heat exchange of the fluid with the reservoir is calculated through the total leakage volume \( V_L(t) \), will take the form:

\[
\rho_F C_v^F T_F'(t) = -\rho_F C_v^F \frac{V_L(t)}{V_F(t)} \left( T_F(t) - T_F^0 \right) H \left( -(t - t_P) \right) + c \left( T_R(t) - T_F(t) \right),
\]

\[
\rho_R C_v^R T_R'(t) = -\rho_R C_v^R \frac{V_R(t)}{V_R(t)} \left( T_R(t) - T_R^0 \right) - c \left( T_R(t) - T_F(t) \right) + \rho_F C_v^L \frac{V_L(t)}{V_R(t)} \left( T_F(t) - T_R(t) \right).
\]

\( \quad (7) \quad (8) \)

**Figure 1.** Temperature of the fracturing fluid (lower curve) and the rock formation (upper curve). Equilibrium temperature is shown by the dashed curve.

**Figure 2.** Temperature of the fracturing fluid (lower curve) and the rock formation (upper curve) calculated with regard to Carter’s leakoff coefficient.

Plots in Figures 1 and 2 show the time dependence of the temperature, obtained from the solution of the equations (2)–(3) and (7)–(8) with the initial conditions (4). The upper curve shows the temperature of the formation, the lower curve shows the temperature of the working fluid. The dashed line shows the equilibrium temperature (6). The fluid pumping is stopped at moment of time \( t_P = 1000 \) s. For simplicity, it is assumed that the crack area ceases to increase immediately after the end of pumping, but the Heaviside function in the formula for the heat transfer coefficient (1) can be replaced by any monotonically decreasing function. If leakoff effects are taken into account (Figure 2), the temperatures of the rock formation and the fracturing fluid reach its equilibrium value (5) later than it is shown in Figure 1.
5. Three-dimensional models of crack propagation

The 3D models provide more realistic estimates of the geometry and size of the fracture, which can help to improve the design of hydraulic fracturing process and wellbores. The discussed model can be generalized to Pseudo 3D (P3D) and Planar 3D (PL3D) approaches. In the first case the energy balance equation needs to be formulated for an element of the allocated volume (for a cell).

Let us now calculate the non-stationary temperature of the rock formation and fracturing fluid using data obtained by the P3D algorithm: the power of fracturing fluid supply to each of the cells from its neighboring ones and the area of the internal surface of each cell into which the whole crack is subdivided. The given quantities has both time “t” and cell number “p” dependencies. Since time steps may not be sufficiently robust for the satisfactory convergence of the non stationary heat transfer equations, the table data can be interpolated and a smaller time step can be adopted.

The system of equations without taking into account the leakoff effect can be written as follows:

\[
\rho_F C_V \partial_t T_F(p,t) = -\rho_F C_F \mu(t) \left(T_F(p,t) - T_F^0\right) + c(t) (T_R(p,t) - T_F(p,t)), \quad (9)
\]

\[
\rho_R C_R \partial_t T_R(p,t) = -\rho_R C_R V'_R(t) \left(T_R(p,t) - T_R^0\right) - c(t) (T_R(p,t) - T_F(p,t)). \quad (10)
\]

The solution of system (9)–(10) gives the temperature values in each cell. The initial conditions have the form:

\[
T_R(p,0) = T_R^0, \quad T_F(p,0) = T_F^0. \quad (11)
\]

Let us consider the leakoff effect on the non stationary temperature distribution. Then the modified equations taking into account Carter’s leakoff coefficient will take the form:

\[
\rho_F C_V \partial_t T_F(p,t) = -\rho_F C_F \mu(t+1) \left(T_F(p,t) - T_F^0\right) + c(t) (T_R(p,t) - T_F(p,t)). \quad (12)
\]

\[
\rho_R C_R \partial_t T_R(p,t) = -\rho_R C_R V'_R(t) \left(T_R(p,t) - T_R^0\right) - c(t) (T_R(p,t) - T_F(p,t)) - \rho_F C_V V'_L(t) \left(T_R(p,t) - T_F(p,t)\right) \quad (13)
\]

The solutions of the equations (9)–(10) and (12)–(13) under the initial conditions (11) give the temperature of the fracturing fluid and rock formation as shown in Fig. 3 and Fig. 4. The family of curves from above shows the temperature of the rock formation, the family of curves from below shows the temperature of the fracturing fluid in the cells of the crack (the lower the curve in the plot, the closer the corresponding cell to the wellbore). The dashed line shows the equilibrium temperature (6).

Comparing Figures 3 and 4 one can conclude that under the given thermomechanical parameters, if the leakoff effect is neglected, the equilibrium temperature is reached in the fluid at the tip of the crack after 1000s of pumping. If the leakoff effect is taken into account, the equilibrium temperature is reached in the rock formation at first. This is due to the fact that the layer of rock is cooled more rapidly due to leakoff and the heat exchange power \(c(1)\), depending on the difference in the temperatures of the formation and fluid, decreases proportionally.
Figure 3. Temperature of the fracturing fluid (lower curves) and the rock formation (upper curves) in different fracture cells calculated using data of a P3D model. Equilibrium temperature is shown by the dashed curve.

Figure 4. Temperature of the fracturing fluid (lower curves) and the rock formation (upper curves) in different fracture cells calculated with regard to Carter’s leakoff coefficient.

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