A broadband electromagnetic energy harvester with a coupled bistable structure

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Abstract. This paper investigates a broadband electromagnetic energy harvester with a coupled bistable structure. Both analytical model and experimental results showed that the coupled bistable structure requires lower excitation force to trigger bistable operation than conventional bistable structures. A compact electromagnetic vibration energy harvester with a coupled bistable structure was implemented and tested. It was excited under white noise vibrations. Experimental results showed that the coupled bistable energy harvester can achieve bistable operation with lower excitation amplitude and generate more output power than both conventional bistable and linear energy harvesters under white noise excitation.

1. Introduction
Mechanical energy from ambient vibrations can be converted into electrical energy and this is often an attractive approach for powering wireless sensors. A linear energy harvester generates maximum power when its resonant frequency matches the ambient vibration frequency and they are typically designed to have a high Q-factor [1]. Its output power drops dramatically if the ambient and device frequencies do not match. However, real world vibrations do not normally consist of only one peak at a certain frequency [2]. Performance of linear energy harvesters will be affected significantly when excited at broadband vibrations or vibrations with multiple peaks [3]. This drawback severely limits the practical applications of linear vibration energy harvesters.

Many methods have been developed to increase operational frequency range of vibration energy harvesting [4], one of which is to use bistable structures. A bistable structure is a type of nonlinear structure that has two potential wells. These two potential wells allow the inertial mass in the system to travel between the two equilibrium positions under external mechanical vibrations regardless of their frequencies. It can be realised with a pre-stressed structure or by applying an external nonlinear force to a linear structure. Conventional bistable structures have been found to have broader operating frequency ranges than linear structures and harvest more energy than linear structures under white noise excitation [5].

This paper presents a vibration energy harvester with a coupled bistable structure. A prototype device was implemented and tested under white noise excitation of various amplitudes. Its output power was compared to those of a linear energy harvester and a conventional bistable energy harvester.
2. Coupled bistable structure

2.1. Overview
A coupled bistable structure proposed in [6] consists of two cantilevers, the main cantilever and the assisting cantilever as shown in Figure 1(a). Two magnet sets are placed at free ends of the two cantilevers to produce a repelling force in-between and thus create two potential wells. The cantilevers jump between the two equilibrium positions to give bistable operation. In conventional bistable structures one magnet is fixed as shown in Figure 1(b), whereas in the device presented here both sets of magnet sets are free to move in the direction of the vibrations.

![Figure 1](image)

**Figure 1.** (a) Coupled bistable structure (b) Conventional bistable structure.

2.2. Theory
Figure 2 shows the dynamic model of the coupled bistable structure. As shown in Figure 2(a), the main cantilever and the assisting cantilever have initial displacements of \(x_m\) and \(x_a\), respectively. The interaction between the two sets of magnets can be regarded in such way that the two magnets are connected via a magnetic spring, \(k_m\), whose initial length is \(l_0\). When an external force, \(F\), is applied to the base, \(m_1\) has a displacement of \(\Delta x_m\) and the length of the spring \(k_1\) becomes \(x_m\). \(m_2\) has a displacement of \(\Delta x_a\), the length of the spring \(k_2\) becomes \(x_a\) and the length of the magnetic spring becomes \(l'\). Due to limited space in this conference paper, only final equations are presented here. Detailed derivation will be presented in a future journal paper.

![Figure 2](image)

**Figure 2.** Dynamic model for the coupled bistable structure (a) equilibrium position (b) force acting on the spring.

The potential energy of the two linear springs, \(E_{ls1}\) and \(E_{ls2}\), are:

\[
E_{ls1} = \frac{1}{2} k_1 (x_{m0} - x_m)^2
\]

\[
E_{ls2} = \frac{1}{2} k_2 (x_{a0} - x_a)^2
\]
The potential energy of the virtual magnetic spring, $E_{ms}$, is:

$$E_{ms} = -C \left[ \frac{1}{\sqrt{d^2 + (x_m + x_a)^2}} \right] \left[ \frac{1}{\sqrt{d^2 + (x_m - x_a)^2}} \right]$$

where $C$ is a constant related to the magnetic force.

The overall potential energy of the system, $E_p$, is

$$E_p = E_{ls1} + E_{ls2} + E_{ms}$$

$$E_p = \frac{1}{2} k_1 (x_{m0} - x_m)^2 + \frac{1}{2} k_2 (x_{a0} - x_a)^2 - C \left[ \frac{1}{\sqrt{d^2 + (x_m + x_a)^2}} \right] \left[ \frac{1}{\sqrt{d^2 + (x_m - x_a)^2}} \right]$$

Figure 3 shows an example of the potential energy of a coupled bistable structure with different ratios of $k_2$ to $k_1$. It is obtained using Eq. (5). Total potential energy in each case is normalised by dividing the potential energy of the conventional bistable structure and then compared.

![Figure 3](image-url)

**Figure 3.** Normalised potential energy with variation of ratios of spring constants of the two cantilevers.

It is found that the lower the ratio of $k_2$ to $k_1$ is, the closer the equilibrium position is to the base position and the lower the potential energy barrier is. The potential energy barrier is defined as the energy required to trigger movement of the inertial mass between the two equilibrium positions. In other words, in a coupled bistable structure, the more flexible the assistant cantilever is compared to the main cantilever, the less energy is required to trigger the inertial mass to jump from one equilibrium position to the other.

3. Design of a broadband energy harvester with a coupled bistable structure

3.1. Overview

The proposed coupled bistable energy harvester has three cantilevers shown in Figure 4(a), i.e. two main cantilevers for energy harvesting and a T-shape assisting cantilever in the middle. All cantilevers are made of BeCu. Resonant frequencies of main cantilevers and the assisting cantilever are 28.9 Hz and 16 Hz, respectively. Two 1 mm thick cylinder NdFeB magnets were fixed to the free end of each main cantilever. Same magnets were placed on the both wings of the T-shape assisting cantilever and were axially in line with those magnets on main cantilevers to produce repelling force between them. The distance between magnets on main and assisting cantilevers was 5 mm. Figure 4(b) shows the electromagnetic transduction mechanism. Each main cantilever has a 10 mm thick cylinder NdFeB magnet with a diameter of 10 mm. One coil was placed underneath each main cantilever. Both coils are 5 mm thick and have the outer and inner diameters of 28 mm and 16 mm, respectively. Each coil was wound with 60 µm thick copper wire and has around 5400 turns. The two coils have resistances of 2200 Ω and 2340 Ω, respectively. Magnets travel through coils, which induces a current in coils.
Figure 4. Proposed vibration energy harvester with a coupled bistable structure (a) perspective view (b) side view.

The energy harvester was excited under white noise excitation with three different amplitudes of 6.9 m·s$^{-2}$ \textit{rms} (signal 1), 1.23 m·s$^{-2}$ \textit{rms} (signal 2) and 1.87 m·s$^{-2}$ \textit{rms} (signal 3). Figure 5 shows test setup and one example of the excitation signal spectrum and waveform. Output of the two coils were connected in series and rectified to charge a 1mF electrolytic capacitor. The charging rate of the capacitor was recorded and compared with that of a linear harvester (by removing the magnets on the assisting cantilever) and a conventional bistable harvester (when the tip of the assisting cantilever was clamped).

Figure 5. Test setup and excitation signal.

Figures 6 compares the capacitor charging rates when charged by a coupled bistable energy harvester, a linear energy harvester and a conventional bistable energy harvester under the three excitation signals. When vibration level was low, no bistable operation was triggered in any case. Therefore, the charging rate was the same for the linear and the coupled bistable harvester. However, since cantilevers in the conventional bistable harvester were stiffer than those in the coupled bistable harvester, its charging rate was 56% lower by comparison (Figure 6a). For excitation signal 2, bistable operation occurred occasionally in the coupled bistable harvester. Its charging rate was 2.5% higher than the linear harvester. Meanwhile, the cantilevers in the conventional bistable harvester were still trapped in one potential well, resulting in a 56.7% lower charging rate than the coupled bistable harvester (Figure 6b). When the vibration level reached 18.3 m·s$^{-2}$ \textit{rms}, bistable operation occurred more frequently in the coupled bistable harvester and its charging rate is 46.6% higher than that of the linear harvester. Bistable operation occurred occasionally in the conventional bistable harvester under this excitation but its charging rate was still 45% and 18.8% lower than the coupled bistable harvester and the linear energy harvester, respectively (Figure 6c).
4. Conclusions
This paper presented a compact electromagnetic energy harvester with a coupled bistable structure. Experimental results showed that the coupled bistable energy harvester can achieve bistable operation under lower excitation levels and generate more output power than both conventional bistable and linear energy harvesters under white noise excitation. The next step will be miniaturisation of the couple bistable energy harvester. However, there is a trade-off between miniaturising the harvester and improving its performance, which will be investigated in future work.

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