Abstract—A strong motivation of charging depleted battery can be an enabler for network capacity increase. In this light we propose a spatial attraction cellular network (SAN) consisting of macro cells overlaid with small cell base stations that wirelessly charge user batteries. Such a network makes battery depleting users move toward the vicinity of small cell base stations. With a fine adjustment of charging power, this user spatial attraction (SA) yields improvement in spectral efficiency (SE) as well as load balancing (LB). We jointly optimize both enhancements thanks to SA, and derive the corresponding optimal charging power in a closed form by using a stochastic geometric approach.

Index Terms—Wireless power transfer, spatial attraction, spectral efficiency, load balancing, rate coverage, stochastic geometry.

I. INTRODUCTION

Seeking for charging depleted batteries even makes mobile users move to a nearby place providing power outlets [1], [2]. Motivated by such behavior, we propose a spatial attraction cellular network (SAN) comprising macro cells overlaid with small cell base stations (BS) that are capable of wireless power transfer (WPT). Its wireless battery charging yields the spatial attraction (SA) of users in macro cells toward the vicinity of small cell BSs, of which the attraction amount is determined by the charging power.

This physical displacement from macro to small cells can be utilized for the improvement in macro-to-small cell load balancing (LB). From the small cell user perspective, in addition, this attraction of users toward their associated BSs not only increases their received powers of the desired signals but also coincidentally decreases interference, leading to the improvement in spectral efficiency (SE).

In an SAN, these LB and SE improvements vary with charging power, but their corresponding behaviors are non-trivial. From the LB perspective, increasing charging power for instance may incur too much associations at small cell BSs. To mitigate such an issue, it requires an optimal charging power that balances the associations between macro and small cell BSs. From the SE perspective, on the contrary, high charging power increases the SA users’ distances from the associated BSs, at which the charged powers are still satisfactory (compare (a) and (b) in Fig. 1). As a result, increasing charging power decreases SE improvement. In this paper we thus provide the SAN rate maximizing charging power such that the LB and SE improvements are jointly optimized.

Feasibility of SAN – SAN implementation hinges on whether SA is viable in practice. Although mobile device proliferation is relentless, improving battery capacity has still been a major technical bottleneck [3]. It makes users more prone to be in danger of battery depletion, leading to a strong battery charging motivation in reality [4]. Such a motivation results in SA toward battery charging places, for example, when determining where to go for coffee [1], [5] or even when charging battery is the sole reason for movements [2].

Another key enabler is to implement WPT capable small cells. Microwave WPT has a potential to provide battery charging for most of small cell coverage ranges while guaranteeing user safety. When utilizing omnidirectional antennas, for instance, it theoretically yields safe charging at the distance of 2 meters with the charging amount three times higher than the average consuming amount of a mobile phone [6].

Related Works – WPT aided cellular networks have recently attracted much attention, accompanied by rapid advancement in WPT technologies [7], [8]. These networks allow more communicating users in that WPT mitigates battery depletion, leading to network capacity enhancement [9], [10]. Along with such improvement, we also capture SE increase owing to SA. The SA also provides improvement via macro-to-small cell LB without degrading SE unlike preceding schemes such as cell range expansion (CRE) [11], [12] which balance the load in return for the SE decrement.

Contributions – This study examines the ramifications of WPT in downlink cellular networks in terms of rate coverage. The main contributions are listed below.

• We propose an SAN that spatially attracts users to small cell BSs by the aid of battery charging WPT and thereby elevates the rate coverage. We use a stochastic geometric approach to derive its rate coverage and demonstrate that significant improvements are feasible only with short SA distance, e.g. 125% rate coverage when maximum SA
An optimal charging power is derived in a closed form in an asymptotic case where user density is much higher than that of BSs (see Corollary 1 in Section III).

To analyze SAN rate coverage, we consider data usage pattern dependent on user’s residual energy. The analytic result shows that the more users’ downlink (DL) usages are affected by their residual energies, the more rate coverage SAN can achieve (see Fig. 4).

II. SAN System Model

A. Information Transmission

Consider a two-tier downlink cellular network comprising macro BSs $B_1$ and small cell BSs $B_2$. Let the subscript $k \in \{1, 2\}$ hereafter denote the corresponding tier. The $k$-th tier BS coordinates $\Phi_k$ follow a homogeneous Poisson point process (PPP) with density $\lambda_k$ where $\Phi_1$ and $\Phi_2$ are independent of each other. User locations $\Phi_u$ also follow a homogeneous PPP with density $\lambda_u$, independent of $\Phi_1$ and $\Phi_2$. For simplicity without loss of generality, we consider a unity time slot in between the realizations of the spatial locations.

A BS in the $k$-th tier $B_k$ transmits a downlink information signal with power $P_k$. The transmitted signal experiences path loss attenuation with the exponent $\alpha > 2$ and Rayleigh fading with unity mean. Both $B_1$ and $B_2$ utilize the same frequency, whose bandwidth is denoted by $W$, resulting in inter-tier interference. Consider an interference-limited environment where noise power is negligible. By the aid of Slyvnak’s theorem [13], the signal-to-interference ratio (SIR) at a typical user associated with a $B_k$ and located at the origin is given as

$$\text{SIR}_k := \frac{P_k h_k^{(0)}}{\sum_{j=1}^{\infty} \sum_{B_j^{(i)} \in \Phi_j} P_j h_j^{(i)}}$$

where $B_k^{(0)}$ denotes the coordinates of the BS associated with the typical user, $B_k^{(i)}$ for $i \in \{1, 2, \ldots\}$ the $i$-th nearest interfering BS coordinates from the typical user, and $h_k^{(0)}$ and $h_k^{(i)}$ the corresponding channel fading gains independently following exponential distribution with unity mean. The notation $|| \cdot ||$ indicates the Euclidian norm.

Each user associates with a BS providing the strongest received power. Multiple users of a BS are served according to a uniformly random scheduler [14].

B. User Energy Consumption

Users attempt to download information with probability $u$. Each information reception consumes energy amount $P_u$ per unity time slot. For a user with residual energy amount $L$, it allows at most $i$ times consecutive information receptions when $iP_u \leq L < (i+1)P_u$ for an integer $i \geq 0$. We discretize $L$ into $L_i$’s according to the maximum number of information receptions $i$ in a way that $i = \left\lfloor \frac{L}{L_i} \right\rfloor$ for all $L \geq 0$ where $\left\lfloor \cdot \right\rfloor$ is a floor function. For more clarity without loss of generality, we consider battery capacity is $3P_u$, i.e. $L \in \{L_0, L_1, L_2, L_3\}$.

Battery depleting users are assumed to attempt less frequent information receptions. This leads to specify the information download attempt probability $u$ as

$$u = \begin{cases} 0 & \text{if } L \in L_0 \text{ (Empty)} \\ u_t & \text{if } L \in L_1 \text{ (Low)} \\ u_h & \text{if } L \in \{L_2, L_3\} \text{ (High)} \end{cases}$$

where $u_h \geq u_t \geq 0$. For notational brevity, let $U_t$ denotes battery depleting users whose residual battery energy $L \in L_1$. Similarly, $U_h$ indicates the users having $L \in \{L_2, L_3\}$.

C. WPT Charging

Small cell BS $B_2$’s are equipped with additional omnidirectional antennas dedicated for charging battery via microwave WPT. Each $B_2$ transmits a charging signal with power $P_{c_k}$ through a power transfer dedicated frequency whose bandwidth is assumed to be unity without loss of generality. The charging transmission bandwidth is separated from the information transmission’s, yielding no mutual interference. The transmitted charging signal experiences path loss attenuation with the exponent $\beta > 2$ and Rayleigh fading. Users associate with their nearest $B_2$. The received charging power is

$$P_{c_k} := P_{c_k} \max \left\{ \left| B_{2_k}^{(0)} \right|, 1 \right\}^{-\beta}$$

where $B_{2_k}^{(0)}$ denotes the nearest $B_2$ and $h$ a channel fading gain following exponential distribution with unity mean.

For more brevity, we henceforth consider $P_{c_k}$ is normalized by $P_u$ and always bigger than $P_u$. Since residual battery level is elevated when $P_{c_k} \geq P_u$, the maximum charging range $r_c$ becomes $(P_{c_k}/P_u)^{1/\beta}$ which makes $P_{c_k}$ be equivalent to $P_u$.

D. User Spatial Attraction

All users tend to keep their positions, but the battery depleting users are willing to move for charging a unit energy amount $P_u$ with the SA distance $r_a$ of which the maximum value is $\tilde{r}_a$. Specifically, assuming $B_2$’s broadcast their locations, a user is spatially attracted toward his nearest $B_2$ if the following two SA conditions hold: (i) $L \in \{L_0, L_1\}$ and (ii) $r_a \leq \tilde{r}_a$. The former indicates users are battery depleting (or $U_t$), and the latter represents SA distance to the destination should be no greater than the users’ maximum feasible SA distance. The required amount of energy to receive the $B_2$ location information is assumed to be negligible; in other words, even $L_0$ users can successfully receive.
Such SA affects user locations, and thus results in an SAN user mobility model specified by the following two phases. For each unity time slot, they occur in order.

1. (Uniform Distribution): Users are uniformly distributed.
2. (Spatial Attraction): SA users move toward their nearest BS through the shortest paths until reaching BS’s charging rim providing $P_u$ (see the yellow thick circle in Fig. 1); Non-SA users keep their locations.

III. SAN RATE COVERAGE MAXIMIZATION

In this section we focus on WPT charging power $P_k$ that maximizes average rate coverage $R$ in an SAN, the probability that a typical user’s average downlink rate exceeds a target threshold $\theta$. We rigorously specify the definition as follows.

$$R := \mathbb{E}_{\theta_k} \left[ \frac{n}{N_k + 1} \mathbb{P} \left( \log [1 + \text{SIR}_k] > \theta \right) \right]$$  \hspace{1cm} (5)

where $N_k$ represents the number of users in a $B_k$ associated with a typical user.

A. Association Probability

SA impacts on the BS association, utilized to derive rate coverage. Specifically, a typical user is associated with a BS based on his residual energy and the received power strength.

**Lemma 1.** The probability $A_k^h$ (or $A_k^f$) that $U_h$ (or $U_f$) associates with $B_k$ is given as

$$A_k^h = \frac{\lambda_k^{-1} \sum_{i=1}^{2} \lambda_i \left( \frac{P_i}{P_k} \right)^{\frac{2}{a}}}{1},$$  \hspace{1cm} (6)

$$A_k^f = (-1)^k \left( A_k^h e^{-\pi \lambda_k r_s^2} - e^{-\pi \lambda_2 r_s^2} \right) + (k - 1),$$  \hspace{1cm} (7)

where $r_s = r_c + r_a$.

**Proof.** See Appendix-A.

Such a BS association changes BS association distance as well as the number of associated users. They are derived by utilizing the above association probabilities in the sequel, to produce the SAN rate coverage.

B. Rate Coverage and Its Optimal Charging Power

To state our main result, rate coverage, let $q$ denote the steady-state probability vector of each residual battery energy level, $q = \{q_0, q_1, q_2, q_3\}$, whose states and transition probabilities are shown as a Markov chain in Fig. 2. By exploiting a stochastic geometric approach combined with Lemma 1, SAN rate coverage $R$ is represented in the following proposition.

**Proposition 1.** Rate coverage in an SAN is given as

$$R = \sum_{k=1}^{2} \left( \sum_{n=1}^{\infty} \frac{\mathbb{P}(N_k = n)}{n+1} \left[ P_{H} A_k^h R_k^h(\theta) + P_{L} A_k^f R_k^f(\theta) \right] \right).$$  \hspace{1cm} (8)

The residual battery energy dependent information reception probabilities $P_{H} = (q_2 + q_3)u_h$ and $P_{L} = q_1 u_L$ for $q_1$, $q_2$, and $q_3$, which are obtained by solving $qT = q$ where

$$T = \begin{bmatrix}
1 - \sum_{i=1}^{3} c_{\ell}(i) & c_{\ell}(1) & c_{\ell}(2) & c_{\ell}(3)
\end{bmatrix},$$

$$\begin{bmatrix}
u_L & 1 - u_L - \sum_{i=1}^{2} c_{\ell}(i) & c_{\ell}(1) & c_{\ell}(2)
\end{bmatrix},$$

$$\begin{bmatrix}
u_h & 1 - u_h - c_{\ell}(1) c_{\ell}(1)
\end{bmatrix},$$

$$\begin{bmatrix}
u_h & 1 - u_h - c_{\ell}(1) c_{\ell}(1)
\end{bmatrix},$$

$$\begin{bmatrix}
u_h & 1 - u_h - c_{\ell}(1) c_{\ell}(1)
\end{bmatrix}.$$  \hspace{1cm} (9)

The distribution of the $k$-th tier associated number of users is

$$\mathbb{P}(N_k = n) = \sum_{m=0}^{\infty} \mathbb{P}(N_k^h = n - m) \mathbb{P}(N_k^f = m) \mathbb{P}(N_k^a = m)$$

$$\mathbb{P}(N_k^h = n) = \frac{3.5^{\frac{5}{2}} T(\frac{n+4.5}{5}) \left( \frac{\lambda_k P_{H} A_k^h}{\lambda_k} \right)^n \left( 3.5 + \frac{\lambda_k P_{H} A_k^h}{\lambda_k} \right)^{-(n+4.5)}}{n! \Gamma(3.5)},$$

$$\mathbb{P}(N_k^f = n) = \frac{3.5^{\frac{5}{2}} T(\frac{n+4.5}{5}) \left( \frac{\lambda_k P_{H} A_k^f}{\lambda_k} \right)^n \left( 3.5 + \frac{\lambda_k P_{H} A_k^f}{\lambda_k} \right)^{-(n+4.5)}}{n! \Gamma(3.5)}.$$

Given residual battery energy and associations, rate coverages $R_k^h(\theta)$ and $R_k^f(\theta)$ are in Fig. 2. at the bottom of the page.

**Proof.** See Appendix-B.

It is worth noting that SAN rate coverage does not monotonically increase with the charging power $P_c$ as shown in Fig. 3(b). The reason is explained by the following two perspectives.

$$R_k^h(\theta) = \frac{1}{1 + \rho(\theta, \lambda)},$$

$$R_k^f(\theta) = \frac{e^{-\pi \lambda_2 \left( \frac{P_c}{P_k} + \frac{\rho}{\lambda} \right)^{\frac{2}{a}}}}{L_2(\theta) A_k^f} + \frac{e^{-\pi \lambda_2 \left( \frac{P_c}{P_k} + \frac{\rho}{\lambda} \right)^{\frac{2}{a}}}}{A_k^f} \int_0^M e^{-\pi \lambda_2 r_s^2 [1 + \rho(\theta, \lambda) + \rho(\theta, r)]} dr,$$

$$R_k^f(\theta) = \frac{e^{-\pi \lambda_2 \left( \frac{P_c}{P_k} + \frac{\rho}{\lambda} \right)^{\frac{2}{a}}}}{L_2(\theta) A_k^f} + \frac{e^{-\pi \lambda_2 \left( \frac{P_c}{P_k} + \frac{\rho}{\lambda} \right)^{\frac{2}{a}}}}{A_k^f} \left[ \left( \frac{\rho(\theta, \lambda)}{\lambda} \right)^{\frac{2}{a}} + \left( 1 + \frac{\rho}{\lambda} \right)^{\frac{2}{a}} \right].$$

$$\rho(\theta, r) = (e^0 - 1)^2 \int_0^\infty \left( \frac{4}{\pi} \right)^2 (e^0 - 1)^2 \left( 1 + \frac{\rho}{\lambda} \right)^{\frac{2}{a}} d\lambda.$$
(a) For user density $\lambda_u = 2 \times 10^4$ users/km$^2$, $\lambda_2 = 3 \times 10^3$ BSs/km$^2$

Fig. 3. Rate coverage in an SAN ($\lambda_1 = 10$ BSs/km$^2$, $P_1 = 43$ dBm, $P_2 = 23$ dBm, $W = 10$ MHz, $\theta = 1$ Mbps, $\alpha = 4$, $\beta = 5$, $\eta = 10$ W/m$^2$).

1) The trade-off between LB and SE gains: Increasing $P_{tc}$ (or $r_c$) attracts more users toward small cell areas for LB while reducing the SE improvement that can be achieved by shortening the BS association distance, and vice versa.

2) SA motivation: Too small power $P_{tc}$ motivates few SA that cannot sufficiently operate an SAN so as to provide a rate coverage improvement. Charging too much power, in contrast, retains mobile users’ residual energy enough and may lose their SA motivations, decreasing the improvements in both LB and SE.

This makes us turn our attention to derive the optimal charging power $P_{tc}^*$ maximizing rate coverage. This derivation is not straightforward due to the user safety condition. The received charging power density of a user should not exceed the maximum safe power density $\eta$ [15], delimiting the feasible range of $P_{tc}^*$. Furthermore, the maximum feasible amount of $P_{tc}^*$, as well as its corresponding distance $r_c$, is also restricted by $B_2$ Voronoi cell inscribed circle radius $r$; otherwise, SA does not always increase SE and LB, which may degrade rate coverage. We collectively consider such trade-off and constraints on $P_{tc}$, and yield its optimal value.

Proposition 2. Assuming the load at each BS is equal to its mean, the optimal charging power $P_{tc}^*$ is given as

$$P_{tc}^* = \arg\max_{P_{tc}} \sum_{k=1}^{2} P_k A_k^0 R_k^\theta (\theta) + P_k A_k^\nu R_k^\nu (\theta) \cdot (9)$$

where $P_{tc}^* \leq \min \left\{ \tilde{P}_{tc}^\eta, \tilde{P}_{tc}^\nu \right\}$,

$$\tilde{P}_{tc}^\eta \approx \frac{\eta}{P_{tc}} \left( \frac{1}{4\pi} + \frac{\lambda_2}{2\beta} - \frac{\pi\lambda_2^2}{\beta - 2} \right)^{-1} \cdots \quad \text{and}$$

$$\tilde{P}_{tc}^\nu \approx \left( 16\lambda_2 + 4 \left[ 1 + \left( \frac{P_1}{P_2} \right)^{1/2} \lambda_1 \right] \right)^{-1} \cdots \quad \text{Proof. Appendix-C}$$

Although an optimal charging power $P_{tc}^*$ derivation is not possible in a closed form, it can be discovered through a linear search utilizing the above proposition, which is easily accessible compared to running an exhaustive simulation. To get a closed form optimal value, we consider an asymptotic case where the user density is much higher than BSs’.

Corollary 1. For $\lambda_u \gg \lambda_k$, $P_{tc}^*$ is given as below.

- When $\lambda_1 \ll \lambda_2$,
  $$P_{tc}^* \approx \left[ \frac{1 + \rho(\theta, T)}{6\pi} \frac{\lambda_1}{\lambda_2} \frac{12}{\beta - 2} \right]^{\frac{1}{2}} \frac{1}{\sqrt{2}} \frac{r_a}{2\beta} \cdots \quad (10)$$

- When $\lambda_1 \geq \lambda_2$,
  $$P_{tc}^* \approx \left[ 27 \left( \lambda_1 \frac{P_1}{P_2} + \lambda_2 \right) \frac{\pi\rho(\theta, T)}{2} \right]^{-\frac{1}{2}} \frac{3}{2r_a} \cdots \quad (11)$$

Proof. Appendix-D

The result provides a charging power control guideline in an SAN as follows. For low small cell density $\lambda_2$ and/or the SA distance $r_a$, the SAN is likely to suffer from macro cell traffic congestion, requiring to attract more users to small cells for LB. In such a scenario, $P_{tc}^*$ should be increased, and vice versa for the opposite situation. For a large threshold $T$, SE dominates the rate coverage, and thus $P_{tc}^*$ should be decreased to shorten the association distances, increasing SE.

Fig. 3 illustrates the SAN rate coverage that is superior to the case with no SA, and the improvement increases along with SA motivation (or maximum SA distance $r_a$). Moreover, it captures the proposed network outperforms a network with CRE that is beneficial to LB yet is harmful to SE [11], [12]. Specifically, when it comes to LB, CRE resorts to decreasing the received power from an associated BS. In an SAN, on the contrary, its LB coincidentally shortens user association distance, and therefore also increases the received power from an associated BS, leading to the further improvement in rate coverage than CRE. It is worth mentioning that if users are willing to move at least 20 cm for battery charging, an SAN become superior in rate coverage to the network with CRE.
Though, SAN may achieve lower rate coverage than CRE if users are unlikely to be attracted, as shown in Fig. 3(a).

Regarding a user safety requirement, the optimal charging power may exceed the maximum safe charging power (red dotted lines). However, such a situation can be detoured via high user density and/or deploying more small cells as Fig. 3(b) visually validates.

Fig. 4 depicts the maximized SAN rate coverage monotonically increases with small cell BS density. This relentless improvement is expected to be different from the case with CRE whose improvement is saturated for high small cell BS density where LB is not much needed (see the convergence of dotted black and solid green lines in Fig. 3(b)). SAN can increase rate coverage via its SE improvement even viable in such an environment. Focusing on the exponentially decreasing optimal charging power along with small cell BS density, deploying small cells can be a viable solution for ever-growing rate coverage while abiding by a safety requirement.

In addition, the proposed SAN can improve much more rate coverage when DL traffic usage ratio of low-to-high battery users is large, i.e. when users are more sensitive to their residual energy. This represents that SAN effects well along with a growth of booming wireless communication services that require large battery consumptions.

IV. CONCLUSION

In this paper we have analyzed an SAN that spatially attracts battery depleting users by means of providing WPT battery charging. The result sheds light on WPT charging power control in order to maximize rate coverage of the SAN. Our analytic result reveals that charging power increase as much as possible does not guarantees the maximum rate coverage, and thus its optimization is required. Such an optimal charging power is derived in a closed form, and its corresponding design guideline is presented for different network deployment, wireless channel environment, and a user safety requirement.

The weakness of this study is its one-sided focusing on a downlink scenario. SA has a potential to further improve uplink rate coverage by its shrinking association distances. In such a scenario, the uplink optimal charging power may not be in accordance with the downlink optimal charging power. Further extension to this work therefore should involve their joint optimization problem. Another interesting avenue for future work is a network economic analysis on the SAN. In a similar way to [13], the SAN rate coverage optimization will jointly incorporate BS density and spectrum amount for communications, also combined with such values for WPT, providing a network operator’s SAN deployment guideline.

APPENDIX

A. Proof of Lemma 1

For $U_h$, the association probability $A_{1k}^h$ is directly derived by using Lemma 1 in [17]. For $U_{tr}$, the association probability $A_{1k}^{tr}$ should incorporate the SA impact. Let $r_k$ denote the distance between a typical $U_{tr}$ and his nearest $B_k$. Now that a $U_{tr}$ is attracted toward the nearest $B_2$ if $r_2 < r_s$, $A_{1}^{tr}$ is given as

$$A_{1}^{tr} = P(U_{tr} \rightarrow B_1 \text{ and } r_2 \geq r_s) \quad (\text{11})$$

$$= P\left(P_1 r_1^{-\alpha} \geq P_2 r_2^{-\alpha} \text{ and } r_2 \geq r_s\right)$$

$$= 2\pi \lambda \int_{r_s}^{\infty} \int \left(r_1^{2/\alpha} + 2\pi \lambda \int_{0}^{r_2} e^{-\pi r_2^2} dr\right) dr$$

$$+ 2\pi \lambda \int_{r_s}^{\infty} e^{-\pi r_2^2} dr$$

where (a) follows from the nearest neighbor distance distribution [13]. Similarly, $A_{2}^{tr}$ is given as

$$A_{2}^{tr} = P(U_{tr} \rightarrow B_2 \text{ or } r_2 < r_s)$$

$$= P\left(P_2 r_2^{-\alpha} \leq P_1 r_1^{-\alpha} \text{ and } r_2 < r_s\right) + P(r_2 < r_s)$$

$$= 2\pi \lambda \int_{r_s}^{\infty} e^{-\pi r_2^2} dr$$

that finalizes the proof.

B. Proof of Proposition 1

The desired result comes from the following four parts.

1) Residual Energy Dependent Information Reception Probabilities $P_{Hi}$ and $P_{E}$. Let $c_i(i)$ denote the probability that $U_{tr}$ is capable of additional $i$ times information receptions thanks to WPT battery charging. Its calculation is followed by the fact that $U_{tr}$ can receive $i$ times information receptions thanks to battery charging if $2^{-\frac{2}{\alpha}} r_i < r_2 \leq r_c + r_a$ for $i = 1$ and if $(i+2)^{-\frac{2}{\alpha}} r_i < r_2 \leq (i+1)^{-\frac{2}{\alpha}} r_c$ for $i = 2, 3$. Let $c_{tr}$ denote the battery charging probability of $U_{tr}$. The event of $c_{tr}$ is identical to the event that $U_{tr}$ is located with the distance closer than $r_c$ from the nearest $B_2$. Applying void probabilities [13] to their spatial regions of $c_i(i)$ and $c_{tr}$ yield the calculations.

2) BS Association Distance Distribution: Now that an SA user goes toward the nearest $B_2$, the distance between them diminishes. For the SA affected association distance for $U_{tr}$, its complementary cumulative distribution function (CCDF) is

$$P(r_{rk} > r | U_{tr} \rightarrow B_k) = P(r_{rk} > r \text{ and } U_{tr} \rightarrow B_k)$$

(14)
where $U \rightarrow B_k$ denotes an event that a user $U$ associates with a $B_k$. For the probability density function (PDF) $f_{U}^i (r)$, calculating the numerator in (14) requires a simple modification of (13) in the proof of Lemma 1. For $f_{U}^i (r)$, the numerator calculation is divided into two cases: (i) if $r_c \leq r < r_s$, an $U_k$ is located at a distance of $r_c$ from $B_2$; (ii) if $r_s < r_c$, $r_c$ is restricted to the $B_2$ cell area, making the calculation become the void probability [13] of a ball with radius $r_s$. Applying Lemma 1 and differentiating (14) derive $f_{r_c}^i (r)$ as below.

$$f_{r_c}^i (r) = \begin{cases} \frac{2\pi\lambda_1}{A_1} r e^{-\pi \lambda_2 r^2} & \text{if } r < r_s \left( \frac{r_s}{r_c} \right)^{3/2} \\ \frac{2\pi\lambda_1}{A_1} e^{-\pi \lambda_2 r^2} r e^{-\pi \lambda_1 r^2} & \text{otherwise} \end{cases}$$ (15)

$$f_{r_c}^i (r) = \begin{cases} \frac{2\pi\lambda_1}{A_1} r e^{-\pi \lambda_2 r^2} & \text{if } r < r_c \\ \delta_{r_c} e^{-\pi \lambda_2 r^2} e^{-\pi \lambda_2 r_s^2} & \text{otherwise} \end{cases}$$ (16)

where $\delta_{r_c}$ is a Dirac delta function yielding 1 for $r = r_c$, otherwise 0. The PDF $f_{U}^i (r)$ for $U_k$ is provided by Lemma 4 in [12] as $2\pi\lambda_2 r e^{-\pi \sum_{i=1}^{k} \lambda_i (\frac{r_i}{r})^{3/2}}$.

3) Distribution of the $k$-th Tier Associated Number of Users: The probability mass function (PMF) of the user number $N_k$ excluding a typical user associated the $a$-th tier BS is derived by using Lemma 1 and Corollary 1 in [12].

4) Rate Coverages Conditioned on Residual Energy and an Association $R_t^k(\theta)$ and $R_c^k(\theta)$: Applying Theorem 1 in [12] with the preceding results and Lemma 1 enables to calculate the rate coverages. When it comes to averaging interferer locations, it is worth noting that small cell interferer distances at a $B_1$ associated typical $U_t$ range from $r_s$ to $\infty$ if $r_1 < M$; otherwise, the interferer distances range from $0$ to $\infty$. The calculation in detail is omitted for lack of space.

C. Proof of Proposition 2

For analytical tractability, we approximate the number of the $k$-th tier associated users as its average value $E[N_k] = \frac{1 - 2\pi \lambda_2}{\lambda_1} (P_{th} A_k^0 + P_{c} A_k^0)$ as in [12].

The feasible range of charging power is restricted by a safety requirement. For calculating the safe charging power per unit area at a typical user, we only consider $B_{0}^{(i)}$ as the sole charging transmitter since the transmitted charging signals by $B_{i}^{(i)}$’s for $i \geq 1$ are negligible due to their high distance attenuations for large $\beta$ and small $\lambda_2$. The average received charging power per unit area $P_{rc}$ is then represented as

$$P_{rc} = \mathbb{E}_{h,y} \left[ \frac{P_{ch} h y^{-\beta}}{4 \pi y^2} \right] = \lambda_2 \int_{0}^{\gamma} \frac{P_{ch} y^{-\beta}}{y} e^{-\pi \lambda_2 y^2} dy + \frac{P_{c}}{4 \pi}$$

$$\approx \mathbb{E}_{h,y} \left( \frac{1}{4 \pi} + \lambda_2 \frac{\pi \lambda_2^2}{\beta (\beta - 2)} \right)$$ (a)

where (a) follows from Taylor expansion. Applying the safety requirement $P_{rc} \leq \frac{P_{c}}{\lambda_2}$ results in $P_{rc}$. Another factor delimiting the feasible range of charging power is Voronoi cell area, i.e. the maximum charging distance $r_c$ is no larger than the inradius of a $B_2$ cell area $\nu$. To simplify our exposition, we approximate $\nu$ as its expected value $E[\nu]$ represented as $\frac{16 \pi^2}{\lambda_2^2} + \pi \lambda_2 P_{th}/P_{c}$. Applying the maximum charging range requirement $P_{rc} = \mathbb{E}[\nu]^2$ with the Voronoi cell inradius distribution in [16] completes the proof.

D. Proof of Corollary 1

When $\lambda_1 \gg \lambda_2$, $R_c \approx \frac{P_{th} A_k^0}{\lambda_2 (P_{th} A_k^0 + P_{c} A_k^0)}$. Applying Taylor expansion with a linear interpolation by simulation further simplifies $R_c$ as $\frac{1}{\pi \lambda_2 (P_{th} A_k^0 + P_{c} A_k^0)} \left[ 1 + \rho(a, \theta) \left( 4 \pi^2 r_c + 6 \pi \lambda_2 r_c + 4 \pi \lambda_2 r_c^2 + 5 \pi \lambda_2 r_c^3 \right) \right]$ for $\lambda_1 \ll \lambda_2$. In a similar way, the approximation of $R_c$ for $\lambda_1 \geq \lambda_2$ becomes $\frac{\lambda_2 (2 \pi r_c + \pi r_c^2)}{\pi \lambda_2 (2 \pi r_c + \pi r_c^2)}$. Differentiating the approximated $R_c$ with respect to $r_c$ yields the optimal charging power.

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