A Study of Excited $\Omega_b^-$ States in Hypercentral Constituent Quark Model via Artificial Neural Network

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In this work, we have obtained mass spectra, radiative decay widths and strong decay widths of newly observed excited $\Omega_b^-$ states, i.e. $\Omega_b^-(6316)$, $\Omega_b^-(6330)$, $\Omega_b^-(6340)$, and $\Omega_b^-(6350)$. Mass spectrum is obtained in Hypercentral Constituent Quark Model (hCQM) by solving six-dimensional nonrelativistic Schrödinger equation via Artificial Neural Network (ANN). In this respect, radiative decay widths are calculated by a generalization of a framework from meson to baryon. Also, strong decay widths of the low-lying $\Omega_b$ states within $^3P_0$ model are calculated. Obtained results are presented with the comparison of available experimental data and other theoretical studies.

I. INTRODUCTION

In the past years, significant experimental progress has been made in heavy baryon physics. In resemblance with the atomic spectroscopy, where transitions of the lines in the spectrum gives information about the interaction between the atomic core and the electrons, energy of the resonances gives information about interaction inside the nucleons. According to quark model, hadrons are strongly interacting particles that are made of quarks.

In this context, the heavy baryons have been got much attention theoretically and experimentally in the last decade. Single heavy baryon system ($Qqq$) is composed of a heavy ($Q$) quark and two light quarks ($qq$) which is bound via gluon interactions. This system is the QCD analogue of the helium atom in which two electrons (light particles) are orbiting around a fixed proton (heavy particle) bounded by the electromagnetic interactions.

Very recently, the LHCb collaboration reported four narrow peaks in the $\Xi_b^0 K^-$ mass spectrum [1]

$$\Omega_b^-(6316) : M = 6315.64 \pm 0.31 \pm 0.07 \pm 0.50 \text{ MeV},$$
$$\Gamma < 2.8 \text{ MeV} \quad (1)$$

$$\Omega_b^-(6330) : M = 6330.30 \pm 0.28 \pm 0.07 \pm 0.50 \text{ MeV},$$
$$\Gamma < 3.1 \text{ MeV} \quad (2)$$

$$\Omega_b^-(6340) : M = 6339.71 \pm 0.26 \pm 0.05 \pm 0.50 \text{ MeV},$$
$$\Gamma < 1.5 \text{ MeV} \quad (3)$$

$$\Omega_b^-(6350) : M = 6349.88 \pm 0.35 \pm 0.07 \pm 0.50 \text{ MeV},$$
$$\Gamma = 1.4^{+1.0}_{-0.8} \pm 0.1 \text{ MeV} \quad (4)$$

where the uncertainties are statistical, systematic and the last is due to the knowledge of $\Xi_b^0$ mass. The significances of these states are $2.1\sigma$ for $\Omega_b^-(6316)$, $2.6\sigma$ and exceed $5\sigma$ for $\Omega_b^-(6340)$ and $\Omega_b^-(6350)$.

Before the LHCb’s measurement there is a vast literature for the mass spectrum of the excited $\Omega_b$ states by various quark models [2,7], QCD Sum Rules [8-12] and Regge phenomenology [13-15]. The experimental result has already stimulated theoretical works. In Ref. [16], Chen et al. systematically studied the internal structure of $P$-wave $\Omega_b$ baryons, and calculated their mass spectra and strong decay properties by QCD Sum Rules method. They concluded that all the four excited $\Omega_b$ baryons recently discovered by LHCb can be well explained as $P$-wave $\Omega_b$ baryons. In Ref. [17], Liang and Li studied the strong decays of the low-lying $\Omega_b$ states within the $^3P_0$ model and pointed out that these excited states can be reasonably assigned as $\lambda$-mode $\Omega_b(1P)$ states. The possible interpretations of the four peaks as the meson-baryon molecular states were studied by Liang and Oset [18]. Wang used explicit $P$-wave between the two $s$-quarks to construct the current operators to study the $P$-wave $\Omega_b$ states with the full QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 10 [19]. Xiao et al. elaborated OZI allowed two-body strong decays of the low-lying $\lambda$-mode $\Omega_b$ baryons up to $N=2$ shell using the chiral quark model within the $j_1$ coupling scheme [20].

As a result of the studies mentioned above, these newly observed excited $\Omega_b^-$ states can be viewed as $P$-wave states. In quark models, the $\Omega_b$ states have three valence quarks, $ssb$. If we don’t include an additional $P$-wave to this system, we can obtain the ground states of $\Omega_b$ as $\Omega_b(J^P = \frac{1^+}{2})$ and $\Omega_b(J^P = \frac{3^+}{2})$. For the time being, only the $\Omega_b(\frac{1^+}{2})$ is observed [21]. But if there exist a relative $P$-wave between two $s$-quarks or between $ss$ diquark and $b$ quark, five negative parity $\Omega_b$ states can be obtained. Furthermore, if this relative $P$-wave excitation takes about an energy of 300-500 MeV, then $P$-wave $\Omega_b$ states have the mass in the range of 6300-6400 MeV. Quark models and diquark-quark models finds similar mass values [22, 23].

In the present work, we will study $\Omega_b^-$ states in quark model via artificial neural network. In Section II Hypercentral Constituent Quark Model (hCQM) is presented. Section III is devoted to the Artificial Neural Network and application of it to quantum mechanical systems. In Section IV we present our results for mass spectrum, radiative and strong decay widths comparing with available experimental data and theoretical studies. A short summary is concluded in the last section.

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II. QUARK MODEL OF $\Omega_b$ STATE

A baryon is an example of quantum mechanical version of classical three-body system. Three-quark systems with a heavy quark ($Q$) of mass $m_Q$ and two light quarks ($qq$) of equal mass $m_q$ can be studied in this prescribed way. There are two kind of excitations for the $\Omega_b$ states, the $\lambda$-mode and $\rho$-mode which can be seen in Figure 1. The $\lambda$-mode is the excitation between the strange quark subsystem $(ss)$ and bottom quark $b$ while the $\rho$-mode is the excitation between two strange quarks, $(ss)$.

FIG. 1: The $\rho$-mode and $\lambda$-mode excitations of $\Omega_b$ system. $s$ stands for strange quark and $b$ stands for the bottom quark.

The $\rho$-mode excited energy is larger than the $\lambda$- mode excited energy. Therefore $\lambda$-mode excitations should be more easily formed than the $\rho$-mode energy [24]. So in general, all the works are concentrated to the $\lambda$- modes. Besides that single heavy baryons with one heavy quark ($Q$) ($c$ or $b$) and two light quarks ($u$, $d$ and $s$) provide a laboratory for studying the dynamics of the light quarks in the presence of a heavy quark. Two states which are in the same mode but have different spin numbers may mix in the heavy quark limit since light-quark spin-spin force is still on work in this limit.

In this present work, we use non-relativistic framework of hypercentral Constituent Quark Model (hCQM) which is well established for the study of the properties of baryons [25][25].

A. Hypercentral Constituent Quark Model (hCQM)

The fundamental idea of the hypercentral approach to three-body systems is simple. The coordinates of the bodies in the system are rewritten in terms of the relative coordinates. In this case, two relative coordinates $(\vec{\rho}, \vec{\lambda})$ are rewritten into a non-relativistic Schrödinger equation in the six-dimensional space. The hyperradius $x$ and hyperangle $\xi$ is given as

$$x = \sqrt{\vec{\rho}^2 + \vec{\lambda}^2}, \quad \xi = \arctan \frac{\vec{\rho}}{\vec{\lambda}}.$$  \hspace{1cm} (5)

To remove the center of mass motion of the system, Jacobi coordinates can be used [27]

$$\vec{\rho} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2),$$  \hspace{1cm} (6)

$$\vec{\lambda} = m_1 \vec{r}_1 + m_2 \vec{r}_2 - (m_1 + m_2) \vec{r}_3, \quad (m_1 + m_2)^2,$$  \hspace{1cm} (7)

where $r_i$ and $m_i = (1, 2, 3)$ denote the spatial and the mass coordinate of the $i$-th constituent quark. The reduced mass corresponding to Jacobi coordinates $\vec{\rho}$ and $\vec{\lambda}$ are

$$m_\rho = \frac{2m_1m_2}{m_1 + m_2},$$  \hspace{1cm} (8)

$$m_\lambda = \frac{2m_3(m_1^2 + m_2^2 + m_1m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)},$$  \hspace{1cm} (9)

where $m_i = (1, 2, 3)$ are the constituent quark masses. The angles of the hyperspherical coordinates are given by $\Omega_p = (\theta_p, \phi_p)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$.

After having removed the center of mass motion by using Jacobi coordinates, the kinetic energy operator can be written as

$$\frac{\hbar^2}{2m} \frac{L_x^2}{x^2} = -\frac{\hbar^2}{2m} (\Delta_\rho + \Delta_\lambda) - \frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{5}{x} \frac{\partial}{\partial x} + \frac{L^2(\Omega)}{x^2} \right).$$  \hspace{1cm} (10)

where $m = \frac{2m_1m_2}{m_3}$ is the reduced mass and $L^2(\Omega) = L^2(\Omega_\rho, \Omega_\lambda, \xi)$ is the six dimensional generalization of the squared angular momentum operator and is a representation of quadratic Casimir operator of the six-dimensional rotational group $O(6)$. Its eigenfunctions are the hyperspherical harmonics $Y_{\gamma|l_{\rho},l_{\lambda}}(\Omega_\rho, \Omega_\lambda, \xi)$ satisfying the eigenvalue relation

$$L^2 Y_{\gamma|l_{\rho},l_{\lambda}}(\Omega_\rho, \Omega_\lambda, \xi) = -\gamma(\gamma+4) Y_{\gamma|l_{\rho},l_{\lambda}}(\Omega_\rho, \Omega_\lambda, \xi).$$  \hspace{1cm} (11)

Here $\hat{L} = \hat{L}_\rho + \hat{\lambda}$, $l_\rho$ and $l_\lambda$ are the angular momenta associated with the Jacobi coordinates $(\vec{\rho}, \vec{\lambda})$ and $\gamma$ is the hyperangular momentum quantum number satisfying $\gamma = 2n + l_\rho + l_\lambda$ where $n$ is a nonnegative integer.

The potential for the hCQM can be written in terms of hyperradius ($x$) as

$$\sum_{i<j} V(r_{ij}) = V(x) + \cdots.$$  \hspace{1cm} (12)

In this approximation, the three-quark potential is a function of only the hyperradius $x$ and the dependence on the single pair coordinates cannot be disentangled from the third one. Therefore it contains not only two-body interactions but also contains three-body interactions. The case of three-body interactions are important in the study of hadrons since the existence of a direct gluon-gluon interaction—which is the non-abelian feature of QCD—produce three-body forces.
The potential is a key ingredient in the study of the meson and baryon spectra. In the present study, we consider the hypercentral potential \( V(x) \) as the hyper Coulomb plus linear potential with second order correction and spin-dependent interaction, which is given as follows \([31]\):

\[
V(x) = V^0(x) + \left( \frac{1}{m_\rho} + \frac{1}{m_\lambda} \right) V^1(x) + V^2(x) + V_{SD}(x),
\]

where \( V^0(x) \) is given by

\[
V^0(x) = \frac{\tau}{x} + \beta x,
\]

where \( \tau \) is the hyper Coulomb strength corresponding to the strong running coupling constant \( \alpha_s \) and \( \beta \) is the string tension of the confinement part.

The first-order correction reads as:

\[
V^{(1)}(x) = -C_F C_A \frac{\alpha_s^2}{4 \pi x},
\]

where, \( C_F \) and \( C_A \) are the casimir charges of the fundamental and adjoint representation. The strong running coupling constant \( \alpha_s \) is given by

\[
\alpha_s = \frac{\alpha_s(\mu_0)}{1 + \frac{33-2n_f}{12\pi} \alpha_s(\mu_0) \ln \frac{m_1+m_2+m_3}{\mu_0}}.
\]

The second-order correction is given by (with the notation of \( m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda} \) [31]:

\[
\begin{align*}
V^2(x) &= -C_F D^{(2)}_{1,1,s} \left( \frac{1}{x} \frac{\partial^2}{\partial \rho^2} + \frac{C_F D^{(2)}_{1,2,s}}{2m^2} \frac{1}{x^3} \hat{L}^2 \right. \\
&\quad + \frac{\pi C_F D^{(2)}_{1,3,s}}{m^2} \delta^{(3)}(\vec{x}) + \frac{4\pi C_F D^{(2)}_{3,6,s}}{3m^2} \hat{S}^2 \delta^{(3)}(\vec{x}) \\
&\quad + \frac{3C_F D^{(2)}_{1,4,s}}{2m^2} \frac{1}{x^3} \hat{L} \cdot \vec{S} + \frac{C_F D^{(2)}_{3,12,s}}{4m^2} \frac{1}{x^3} \vec{S}_{12}(\vec{x}),
\end{align*}
\]

where \( \vec{S}_{12}(\vec{r}) \equiv 3\vec{\rho} \cdot \vec{\sigma}_1 \vec{\sigma}_2 - \vec{\sigma}_1 \vec{\sigma}_2 - \vec{\sigma}_1 \vec{\sigma}_2 \).

In order to obtain the expressions for the coefficients, one has to perform the matching between nonrelativistic QCD (NRQCD) and potential nonrelativistic QCD (pNRQCD). A procedure for this matching can be found in \([32, 63]\). After matching, one can obtain the relevant contributions as follows \([31]\):

\[
\begin{align*}
\delta D^{(2)}_{1,1,s} &= \alpha_s(x), \\
\delta D^{(2)}_{1,2,s} &= \alpha_s(x), \\
\delta D^{(2)}_{d,s} &= \alpha_s(x), \\
\delta D^{(2)}_{s^{2},s} &= \alpha_s(x) \left\{ 1 + \frac{\alpha_s}{\pi} \left( \frac{2C_F}{3} + \frac{17C_A}{3} \right) \ln mx \right\}, \\
\delta D^{(2)}_{s^{2},s} &= \alpha_s(x) \left\{ 1 - \frac{7C_A \alpha_s}{4 \pi} \ln mx \right\}, \\
\delta D^{(2)}_{L,s} &= \alpha_s(x) \left\{ 1 - \frac{2C_A \alpha_s}{3 \pi} \ln mx \right\}, \\
\delta D^{(2)}_{S^{2},s} &= \alpha_s(x) \left\{ 1 - C_A \frac{\alpha_s}{\pi} \ln mx \right\}.
\end{align*}
\]

One interesting point here is that, spin-dependent potentials do not have \( \ln mx \) contributions. The spin-dependent part of the potential reads as follows:

\[
\begin{align*}
V_{SD}(x) &= V_{SS}(\vec{S}_p \cdot \vec{S}_p) + V_{S}(x)(\vec{\tau} \cdot \vec{S}) \\
&\quad + V_T(x) \left[ S^2 - \frac{3(\vec{S} \cdot \vec{x})(\vec{S} \cdot \vec{x})}{x^2} \right],
\end{align*}
\]

where \( \vec{S} = \vec{S}_p + \vec{S}_p \) with \( \vec{S}_p \) is the spin vector of \( \vec{p} \) and \( \vec{S}_p \) is the spin vector of \( \vec{p} \).

The spin dependent potential contains three types of the interaction: the spin–spin term \( V_{SS} \) which gives spin singlet triplet splittings, the spin–orbit term \( V_S \) and tensor term \( V_T \) give the fine structure of the states \([34]\). Defining \( V_T(x) = \frac{\tau}{x} \) as vector part and \( V_S(x) = \beta x \) as scalar part of the static potential, these spin-dependen terms can be written as follows:

\[
\begin{align*}
V_{SS}(x) &= \frac{1}{3m_\rho m_\lambda} \nabla^2 V, \\
V_{S}(x) &= \frac{1}{2m_\rho m_\lambda x} \left( 3 \left( \frac{dV}{dx} \right) - \frac{dV}{dx} \right), \\
V_T(x) &= \frac{1}{6m_\rho m_\lambda} \left( \frac{d^2 V}{dx^2} - \frac{1}{x} \frac{dV}{dx} \right).
\end{align*}
\]

The Hamiltonian of the three-quark system in the hQCM can be written as

\[
H = \frac{\vec{p}^2}{2m} + V(x),
\]

where \( m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda} \) is the reduced mass and \( x \) is the six-dimensional radial hypercentral coordinate of the three-quark system. Before writing the hyperradial Schrödinger equation corresponding to this Hamiltonian, we have to write a wave function which resemble this three-body system. Since in the hQCM the three-quark interaction is assumed to be hypercentral, i.e.,

\[
V_{3q}(\vec{p}, \vec{\lambda}) = V(x)
\]

the three-quark wave function can be factorized as

\[
\psi_{3q}(\vec{p}, \vec{\lambda}) = \psi_{3q}(x) Y_{\gamma}(\vec{L}_u \lambda)(\Omega_p, \Omega_\lambda, \xi).
\]
Here, hyperradial wave function $\psi_{\gamma \nu}(x)$ is labeled by the grand angular quantum number $\gamma$ and by the number of nodes $\nu$. The angular-hyperangular part of the $3q$-state is completely described by the hyperharmonics and is same for any hypercentral potential. The dynamics is contained in the hyperradial wave function \[27\]:

$$
\psi_{\gamma \nu}(x) = \left[ \frac{v!(2g)!^6}{(2g + 2\nu + 5)(\nu + 2\gamma + 4)!^5} \right]^{1/2} \cdot (2gx)\gamma e^{-gx}L_{2\gamma+4}^2(2gx),
$$

where $g = \frac{\tau m}{\gamma + \nu + 5/2}$.

Then, the hyperradial Schrödinger equation can be written as follows:

\[
\left[ \frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma + 4)}{x^2} \right] \psi_{\gamma \nu}(x) = -2m \left[ E - V(x) \right] \psi_{\gamma \nu}(x).
\]

(27)

In conventional quark model, Schrödinger equation is solved in three-dimensional space. In hCQM, Schrödinger equation lives in six-dimensional space and needs to be handled. This can be done in conventional way such as numerically. In this work, we have solved six-dimensional Schrödinger equation by using Artificial Neural Networks, (ANNs). The main advantage of using ANN is that computational complexity does not increase considerably when the number of dimensions in problem increase. The other point is that the solution is continuous overall the domain of integration.

III. FORMALISM OF ARTIFICIAL NEURAL NETWORK

 Especially the last decade has seen a significant raise of interest in machine learning where the learning part is done by the artificial neural networks. Artificial neural networks (ANNs) are systems of simple processing elements (called "neurons") managing their data and communicating with other elements.

The fundamental ingredient of an artificial neural network is neuron (perceptron in computerized systems). Figure 2 represents a single artificial neuron.

![FIG. 2: A model of single neuron](image)

Each neuron receives any number of input and produces only one output. If this output comes from input layers, it will be an input for the hidden layers. In this manner, the inputs are the outputs of activation functions in where the inputs are multiplied by the connection weights. This activation function (neuron transfer function) determines the output. In practice, one single neuron is not capable of handling problems. That’s why networks composed of neurons are being used. In Figure 3 the architecture of a multilayer perceptron is shown.

![FIG. 3: Multilayer neural network](image)

The most common architecture of ANNs is the multilayer feed forward network. In this study, we consider a feed forward neural network with one input layer, one hidden layer and one output layer. The signals are propagated from the input layer to the output layer where each processing element is responsible for operating the signals coming from preceding layer and sending information to the connected next layer. Fig. 2 is an example of feed forward neural network.
A. Mathematical Model of an Artificial Neural Network

The relationship of the input-output of the layers can be written as follows:

\[ o_i = \sigma(n_i), \]  
\[ o_j = \sigma(n_j), \]  
\[ o_k = \sigma(n_k), \]

where \( i \) is for input, \( j \) is for hidden and \( k \) is for output layers. Input to the perceptrons are given as

\[ n_i = \text{ (Input signal to the NN)}, \]  
\[ n_j = \sum_{i=1}^{N_i} \omega_{ij}o_i + \theta_j, \]  
\[ n_k = \sum_{i=1}^{N_j} \omega_{jk}o_j + \theta_k, \]

where, \( N_i \) and \( N_j \) represents the numbers of the units belonging to input and hidden layers, \( \omega_{ij} \) is the synaptic weight parameter which connects the neurons \( i \) and \( j \) and \( \theta_j \) represents threshold parameter for the neuron \( j \) and \( \theta_k \) is the threshold parameter \[35\]. The output of the network can be written as

\[ o_k = \sum_{j=1}^{b_k} \omega_{jk}\sigma \left( \sum_{i=1}^{a_k} \omega_{ij}n_i + \theta_j \right) + \theta_k. \]  

Derivative of this function is needed in further evaluation of error function. This can be obtained as

\[ \frac{\partial o_k}{\partial \omega_{ij}} = \omega_{jk}\sigma^{(1)}(n_j)n_i, \]  
\[ \frac{\partial o_k}{\partial \omega_{jk}} = \sigma(n_j)\delta_{kk'}, \]  
\[ \frac{\partial o_k}{\partial \theta_j} = \omega_{jk}\sigma^{(1)}(n_j), \]  
\[ \frac{\partial o_k}{\partial \theta_k} = \delta_{kk'}. \]

In this work we use a sigmoid function

\[ \sigma(x) = \frac{1}{1 + e^{-x}} \]

as an activation function since it is possible to derive all the derivatives of \( \sigma(x) \) in terms of itself. This differentiability is an important aspect for the Schrödinger equation.

B. Application to Quantum Mechanics

Following the work of \[36\], let us consider the following differential equation:

\[ H\Psi(x) = f(x) \]

where \( H \) is a linear operator, \( f(x) \) is a known function and \( \Psi(x) = 0 \) at the boundaries. In order to solve this differential equation, a trial function

\[ \Psi_t(x) = A(x) + B(x, \eta)N(x, p) \]

of the form can be written which uses a feed forward neural network with parameter vector \( p \) and \( \eta \) to be adjusted. The parameter \( p \) refers to the weights and biases of the neural network. The functions \( A(x) \) and \( B(x, \eta) \) should be specified in a convenient way so that \( \Psi_t(x) \) satisfies the boundary conditions regardless of the \( p \) and \( \eta \) values. To obtain a solution for Eqn. \[40\], the collocation method can be used and the differential equation can be transformed into a minimization problem

\[ \min_{p, \eta} \sum_i \left| H\Psi_t(x_i) - f(x_i) \right|^2. \]

For Schrödinger equation Eqn. \[40\] takes the form

\[ H\Psi(x) = e\Psi(x) \]

with the boundary condition, \( \Psi(x) = 0 \). In this case, the trial solution can be written as

\[ \Psi_t(x) = B(x, \eta)N(x, p) \]

where \( B(x, \eta) = 0 \) at boundary conditions for a range of \( \lambda \) values. By discretizing the domain of the problem, it is transformed into a minimization problem with respect to the parameters \( p \) and \( \eta \)

\[ E(p, \eta) = \sum_i \left| H\Psi_t(x_i, p, \eta) - e\Psi_t(x_i, p, \eta) \right|^2 \]

where \( E \) is the error function and \( \epsilon \) can be computed as

\[ \epsilon = \int \frac{|\Psi_t|^2 d\mathbf{x}}{|\Psi_t|^2 d\mathbf{x}}. \]

We used a trial wave function as

\[ \psi_t(x) = x^{5/2}\psi_{\eta t}(x)N(x, u, w, v), \]

with \( N \) being a feed forward neural network with one hidden layer and \( m \) sigmoid hidden units

\[ N(x, u, w, v) = \sum_{j=1}^{m} v_j \sigma(w_j x + u_j). \]

Step 1 Initialize the weights \( w \) from the input layer to the hidden layer and weights \( v \) from the hidden layer to the output layer. Choose the learning parameter (lies between 0 and 1) and error \( E_{\text{max}} \). Initially error is taken as 0.
Step 2 Train the network.
Step 3 Compute the error value.
Step 4 Compute the error signal terms of the output layer and the hidden layer.
Step 5 Compute components of error gradient vectors.
Step 6 Check the weights if they are properly modified.
Step 7 If $E = E_{\text{max}}$ terminate the training session. If not, go to step 2 with $E \rightarrow 0$ and initiate a new training.

The main point to train the network is to initialize the eigenvalue ($E$) (error function) to zero and train the network with 150 equidistance points in the interval after selecting one hidden unit. The aim is to get energy function to be zero or at least tends to be zero. If the convergence is not achieved, then the eigenvalue would be wrong. In that case, eigenvalue should be changed and tried again. If the error function value did not converge to zero after doing these steps, the number of the hidden units must be changed (increased) and run a new cycle. We keep doing this until the error function converges to zero.

By employing this approach it is possible to obtain energy eigenvalues of the Schrödinger equation. We trained the network with some equidistance points in the interval $0 < x < m$ and solved the Schrödinger equation in the interval $0 < x < 1$ with $\psi(0) = 0$ at the boundaries. We did not analyze the number of hidden units in the present paper. The optimum value was obtained after doing some calculations.

IV. NUMERICAL RESULTS AND DISCUSSION

The numerical parameters and constants used in this work are

$m_s = 0.500 \text{ GeV}$,
$m_b = 4.67 \text{ GeV}$,
$C_F = 2/3$,
$C_A = 3$,
$n_f = 3$,
$\alpha_s = 0.6$ ($\mu_0 = 1 \text{ GeV}$).

The comparison of the obtained mass spectra with other quark models is shown in Table I with the spectroscopic notation ($n^{2s+1}L_j$) except $L$ (angular momentum quantum number) is replaced by $\gamma$ (hyper-angular momentum quantum number) in hCQM.

It can be seen from Table I that the obtained mass values are in good agreement with the other theoretical studies. The LHCb collaboration observed four narrow structures of excited $\Omega_b^-$ states, namely $\Omega_b^-$ (6316), $\Omega_b^-$ (6330), $\Omega_b^-$ (6340) and $\Omega_b^-$ (6350) in the $\Xi_b^0 K^-$ mass spectrum. The theory predicts five states whereas experiment observed only four states. The remaining state should have a broad width which can hardly be observed in experiments. The quantum numbers of newly observed excited $\Omega_b^-$ states are not available. Some recent efforts are done and the results can be seen in Table 2.

| State        | $J^P_C$ | $16$ | $17$ | $19$ |
|--------------|---------|------|------|------|
| $\Omega_b^-$ (6316) | $\frac{3}{2}^-$ or $\frac{3}{2}^-$ | $\frac{3}{2}^-$ | $\frac{3}{2}^-$ | $\frac{3}{2}^-$ |
| $\Omega_b^-$ (6330) | $\frac{3}{2}^-$ | $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ |
| $\Omega_b^-$ (6340) | $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ |
| $\Omega_b^-$ (6350) | $\frac{3}{2}^-\gamma$ partner with $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ partner with $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ partner with $\frac{3}{2}^-\gamma$ | $\frac{3}{2}^-\gamma$ partner with $\frac{3}{2}^-\gamma$ |

Since there is no available data up to now on quantum numbers of these excited states, there is no possibility to rule out them.

A. Radiative Decays of Excited $\Omega_b^-$ States

The study of electromagnetic transitions of baryons is an important issue for understanding the internal structure of baryons. To calculate the partial widths of the $E1$ radiative transitions of the $(n^{2s+1}L_j \rightarrow n^{2s'+1}L_j', + \gamma)$ process, we use [38]:

$$\Gamma_{E1} = \frac{4}{3} C_f \delta_{ss'} e_q^2 \alpha |\langle R_f | r | R_i \rangle|^2 E_\gamma^3$$ (49)

where $e_q$ is the quark charge, $\alpha$ is the fine-structure constant, $E_\gamma$ is the final photon energy. The matrix element $C_{f}$ is given as

$$C_{f} = \max(L, L')(2J' + 1) \left\{ \begin{array}{c} \frac{L}{L'} \frac{S'}{J} \frac{J}{L} \end{array} \right\}.$$ (50)

In order to calculate matrix elements, we need $\langle R_f | r | R_i \rangle$, where $R$ represents the radial wave function of the meson. In this work, we generalize radial wave function to hyperradial wave function, $r$ to hyper radius $x$, $L$ (angular momentum quantum number) is replaced by $\gamma$ (hyper-angular momentum quantum number), and meson mass to the baryon mass. For the masses of the initial and final states, we used our calculated values. Apart from hCQM, this generalization from meson to baryon electromagnetic transitions were successfully done in [39] [40]. In Table II we present the partial widths of the radiative decays of excited $\Omega_b^-$ baryons. Our predictions are compatible with the result of Ref. [24]. Radiative decays of $1^2P_{1/2}$ and $1^2P_{3/2}$ to $\Omega_b \gamma$ and $1^4P_{3/2}$ and $1^4P_{3/2}$ to $\Omega_b \gamma$ are very different than others. If these decays are studied in further experiments, this situation can be identified.
TABLE I: Comparison of mass values of excited $\Omega_b^-$ states with other quark models. Results are in MeV.

| State    | This Work | RQM [5] | NQM [6] | hCQM [14] | QM [23] |
|----------|-----------|---------|---------|------------|---------|
| $1^2P_{1/2}$ | 6314      | 6330    | 6333    | 6338       | 6305 ± 15 |
| $1^2P_{3/2}$ | 6330      | 6331    | 6336    | 6328       | 6017 ± 19 |
| $1^4P_{1/2}$ | 6339      | 6339    | 6340    | 6343       | 6013 ± 15 |
| $1^4P_{3/2}$ | 6342      | 6340    | 6344    | 6333       | 6035 ± 19 |
| $1^4P_{5/2}$ | 6352      | 6334    | 6345    | 6329       | 6038 ± 20 |

TABLE III: Partial widths for the radiative decays of the excited $\Omega_b^-$ states. Results are in keV.

| State    | $\Gamma[\Omega_b^-\pi\gamma]$ | $\Gamma[\Omega_b^-\gamma]$ |
|----------|-------------------------------|-----------------------------|
| This work [21] | This work [24] | This work [24] |
| $1^2P_{1/2}$ | 138  | 154  | 1.41 | 1.49 |
| $1^2P_{3/2}$ | 71.8 | 83.4 | 1.45 | 1.51 |
| $1^4P_{1/2}$ | 0.85 | 0.64 | 104.29 | 99.23 |
| $1^4P_{3/2}$ | 2.03 | 1.81 | 73.65 | 70.68 |
| $1^4P_{5/2}$ | 1.02 | 1.21 | 51.57 | 63.26 |

B. Decay Widths of Excited $\Omega_b^-$ States

Besides the mass prediction, the investigation of other physical properties such as radiative decays of the considered resonances would be helpful to identify these resonances more reliably. The study of strong decay of these resonances is also helpful in this respect. The study of the strong decay processes is a challenge task both theoretically and experimentally.

In this work, we adopt the $^3P_0$ model [21] to calculate the two-body strong decay widths of the low-lying $\Omega_b$ states. This model is an efficient model to compute open-flavor strong decays of hadrons in the quark model framework. In this model, a hadron decay occurs in its rest frame and proceeds via the creation of an additional $q\bar{q}$ pair. This quark-antiquark pair is created with vacuum quantum numbers, $J^{PC} = 0^{-+}$. The decay widths can be computed by

$$\Gamma_{A\to BC} = \frac{2\pi\gamma_0^2}{2J_A + 1} \Phi_{A\to BC}(q_0) \sum_{M_{JA}, M_{JB}} |\mathcal{M}_{M_{JA}, M_{JB}}|^2.$$  \hspace{1cm} (51)

Here, $|\mathcal{M}_{M_{JA}, M_{JB}}|$ is the $A \to BC$ amplitude, $\gamma_0$ is the dimensionless pair creation strength, $q_0$ is the relative momentum between $B$ and $C$. The coefficient $\Phi_{A\to BC}(q_0)$ is the phase space factor for the decay. The nonrelativistic expression for this factor reads as

$$\Phi_{A\to BC}(q_0) = 2\pi q_0 \frac{M_B M_C}{M_A}.$$  \hspace{1cm} (52)

The baryon wave function is chosen as $\psi_{3q}(\vec{p}, \vec{\lambda}) = \psi_1(x)Y_1(\vec{\lambda})|0\rangle$. For the wave function and other issues please see Refs. [23] [44]. The obtained decay widths are given in Table IV.

TABLE IV: Strong decay widths of $\Gamma(\Omega_b^- \to \Xi_b K^-)$. Results are in MeV.

| State    | This work | [23] | [24] |
|----------|-----------|------|------|
| $1^2P_{1/2}$ | 0.78     | 0.50 | 49.53 |
| $1^2P_{3/2}$ | 3.18     | 2.79 | 1.90 |
| $1^4P_{1/2}$ | 1.74     | 1.14 | 95.08 |
| $1^4P_{3/2}$ | 0.58     | 0.62 | 0.29 |
| $1^4P_{5/2}$ | 2.83     | 4.28 | 1.66 |

As can be seen from Table IV, our results are in good agreement with Ref. [24] and compatible with Ref. [24]. The discrepancy in two results for the states of $1^2P_{1/2}$ and $1^2P_{1/2}$ of Ref. [24] is interesting. Regarding the strong decays for baryon states, no satisfactory model has been achieved. So the reason for this discrepancy may be as a result of non-existence of a single model for explaining baryon open-flavor strong decays.

V. SUMMARY AND CONCLUDING REMARKS

In this paper, we have employed hypercentral constituent quark model to compute mass spectra by solving six-dimensional Schrödinger equation by using artificial neural network. The obtained mass spectra are in good agreement with other works. Given the uncertainties of different quark models predictions, these structures are good candidates of the $\Omega_b(1P)$ states. As mentioned before, there are five states for $\Omega_b$ in the quark model. Since there are only four observed $\Omega_b$ states and one is remaining, the status and fate of remaining state is still unclear.

We also obtained radiative decay of excited $\Omega_b^-$ states by generalizing the method given in Ref. [35]. The results are appealing for this generalization and we hope this can be used in further studies. The radiative decay widths for $\Omega_b^-$ states is in compatible with the reference study. Within the results of radiative decays, $1^2P_{1/2}$ and $1^2P_{3/2}$ to $\Omega_b\gamma$ and $1^4P_{1/2}$ and $1^2P_{3/2}$ to $\Omega_b^*\gamma$ are very large than the others.

Strong decay widths are calculated via a generalization of $^3P_0$ model. Obtained results are in good agreement with available experimental data and theoretical studies. The results for $1^2P_{1/2}$ and $1^4P_{1/2}$ of Ref. [24] are roughly order of magnitude larger than our results.

We hope future experiments on radiative and strong
decay widths of excited $\Omega^-$ states will clarify the spin assignments and the status of the missing state which have not been observed yet.

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