A study of homogeneous turbulence within a baroclinic context

Alexandre Pieri, Claude Cambon, Fabien Godeferd
Laboratoire de Mécanique des Fluides et d’Acoustique, Université de Lyon
E-mail: alexandre.pieri@ec-lyon.fr

Abstract. Investigations on homogeneous turbulent flows submitted to both uniform shear and stable vertical buoyancy gradient or rotation have been made during the past decade. This paper discusses the results obtained by coupling the three previous distortions: the so-called ‘baroclinic’ context representative of situations in geophysical flows. We compare the predictions of linear theory (Rapid distortion) and nonlinear simulations (Direct Numerical Simulations) is done. Depending on the values of the multiple parameters, optimally reduced to the Richardson number and a baroclinic parameter, the baroclinic instability is triggered and modifies the dynamics and structure of the flow. We characterize turbulence in these different states, which present specific anisotropic features using physical-space quantities (energies, two point velocity correlation lengths) and spectral statistics (energy spectra).

1 Introduction

The ‘baroclinic context’ means that there is superposition of three phenomena involved in the baroclinic instability. The first one is the Coriolis force, caused by Earth rotation. Secondly, we add the buoyancy effects, a consequence of non-zero density gradients in the atmosphere. Thirdly, homogeneous shear is included in the model, present in the atmosphere under the form of jet stream near the tropopause, a thermally stable layer that defines the limit between the troposphere and the stratosphere. In our model, the troposphere is characterized by constant values of vertical and horizontal density gradients and a constant growth of the zonal wind with height. In addition to the vertical density gradient we have to take into account density variation along the spanwise direction in order for the mean flow to be admissible i.e. solution of mean fields Euler equations. Single vertical stratification leads to non-homogeneous Euler equations due to mean vorticity creation in the streamwise direction in presence of rotation and shear. The configuration is schematically presented in fig. 1. A physical explanation of the baroclinic instability in terms of particle displacement can be found in Drazin & Reid (1981). In particular the angle between the tilted isopycnal surfaces (having an angle equal to $\tan(\varepsilon_B)$ with the horizontal plane) and the particles is taken as a stability criterion.

The first works on baroclinic instability were done by Charney (1947) using quasi-geostrophic equations in the $\beta$-plane approximation. He derived a necessary condition for instability to occur formulated in terms of the Rossby radius of deformation. Further contribution by Eady (1949) using the $f$-plane approximation was later added. Eady considered a simpler model — a Couette-like configuration — of the atmosphere and obtained an instability condition resting on a critical wavenumber (Pedlosky, 1987). Following Eady, we also place the study within the $f$-plane approximation. We replace the two-boundary plates approximation by an infinite Couette flow to model shear effects in agreement with
statistical homogeneity for fluctuations.

Regarding Direct Numerical Simulations, the turbulence box is built under the \( f \)-plane approximation, \textit{i.e.} we assume that the Coriolis frequency is constant so that the latitude is weakly varying. This approximation, which relies on the idea that the Earth’s curvature can be neglected remains true for wavelengths small compared to the circumference of the corresponding zonal circle. The mean motion is then considered planar (zonal) and a cartesian system of coordinate can be introduced. Periodicity is assumed in the three directions. Indeed, the use of Fourier spectral methods lies on the assumption that the parts of fluid at the ends of the computational domain move at the same velocity, which is not true in the presence of mean velocity field. Therefore, to allow periodic boundary conditions, it is necessary to adopt a Lagrangian viewpoint. So, under the action of shear the mesh is deformed and a time-periodic remeshing is needed using the algorithm by Rogallo (1981) (the anisotropic adaptation of the Orszag-Patterson algorithm) in spectral space \((p, q, r)\). De-aliasing is done following the method by Delorme (1985) and doubling the number of modes \(M\) in the \((p, y, r)\) space (physical space in the \(y\)-spanwise direction) and then by discarding wave-numbers \(k_2 > M/2\) (for more details, see Canuto \textit{et al.}, 2007).

The code used in this study is a MPI-parallel code: the turbulence box is cut into several slices following the algorithm by Coleman & Kim & Spalart (2000). Time-advancing is done with a third order Runge-Kutta (RK3) method. Lastly, the rotational form of the non-linear term has been implemented. Figure 1 presents a global view of the problem and the direction of the distortions. The terminology we will use in the following corresponds to geophysical community’s lexicon: the streamwise flow will be said to be ‘zonal’ and the spanwise flow said to be ‘meridional’. In spite of the limited Reynolds number in DNS and the very large range of scales interacting in the atmosphere, this paper aims at describing as well as possible the dynamics of homogeneous baroclinic turbulence. We will show that this approach gives a good understanding of structure emergence and energetic cascade that can be observed in geophysical flows.

2 Equations of motion

We assume a mean uniform shear \( S \) along the vertical direction. For the flow to be homogeneous, we assume a linear dependency of the mean velocity field with the spatial coordinates. One has to keep in mind that the homogeneous assumption (no feed-back from the fluctuating field because the Reynolds stress tensor is space-uniform) remains valid only theoretically due to the fact that in practice the turbulent field will modify the mean velocity profile.

Due to the simultaneous presence of shear and rotation, geostrophic adjustment leads to a spanwise density gradient in order to compensate mean vorticity production in the streamwise direction giving the mean buoyancy field \( b \) defined in the following. We consider a Boussinesq fluid, \textit{i.e} temperature
fluctuations are considered to be small compared to the mean temperature or density $\rho$, where the link between these two variables is done through the thermal expansion coefficient $\beta$ by the relation $\rho = \beta T \rho_0$. The Brunt-Väisälä frequency $N$ is such that $N = (-g \partial_\theta \rho_0/\rho_0)^{1/2}$. The Coriolis frequency $f = 2\Omega$ is twice the rotation rate. In the following, $\delta_{ij}$ is the Kronecker symbol. We denote mean background quantities by an overbar. The conservation equations for fluctuating velocity and buoyancy fields are:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \cdot \nabla u + u \cdot \nabla \bar{u} + u \cdot \nabla u = -\nabla p^* + \nu \nabla^2 u - 2\Omega \times u + bn$$

$$\nabla \cdot u = 0$$

$$\nabla p^* = \nabla p + \Omega \times (\Omega \times r)$$

$$\frac{\partial b}{\partial t} + u \cdot \nabla b + \bar{u} \cdot \nabla b = -u \cdot \nabla b + \chi \nabla^2 b$$

with

$$\bar{b} = N^2 y + \varepsilon_B N^2 z$$

$$\bar{u} = Sy e_x$$

and where $\chi$ is the thermal diffusivity of the scalar responsible for buoyancy forces, $\nu$ is the kinematic viscosity and $\bar{b}$ is the mean buoyancy field, at hydrostatic equilibrium. The centrifugal term of the Coriolis force has been included into the pressure gradient. The resulting Coriolis force in the right hand side of the equation acts as a source or sink of angular momentum depending on the orientation of the velocity field respectively to the rotation axis $\Omega y$. The presence of additional shear and stratification means that three forces are in competition. On the one hand, the Coriolis force does not work but acts on energy transfers in a way that inhibits the energy cascade and reduces the dissipation rate (Cambon et al., 1997). On the other hand, the shear intensifies the turbulence, acting as permanent and natural forcing. Finally, buoyancy forces, on the contrary, tend to reduce the turbulence (Godeferd et al., 1994) by converting turbulent kinetic energy into potential energy but are also a natural reservoir of available potential energy which may be released and transferred to fluctuations through baroclinic instability.

### 2.1 Dimensionless numbers

The usual practice in stratified flows is to make the parameters a function of the Richardson number $Ri = N^2/S^2$ (or Froude number $Fr = u^*/(NL)$). This non-dimensional parameter represents the relative intensity of stratification and shear. Here, because of the presence of horizontal density gradient, we can define both a vertical and horizontal Richardson number, taking respectively the vertical or horizontal Brunt-Väisälä frequency, $N_v$ or $N_h^2 = \varepsilon_B N_v^2$ respectively.

The Rossby number $Ro = S/f$ characterize the relative dominance of rotation effects over shear effects. The baroclinic parameter $\varepsilon_B = Sf/N^2 = (Ro Ri)^{-1}$ is a combination of the two previous dimensionless number $Ri$ and $Ro$. Physically, it can be seen as the tangent of the angle formed by the isopycnal surfaces with the horizontal plane. It is interesting to note that if one sets $S = 0$ or $f = 0$ then this angle becomes zero; this clearly shows that baroclinicity requires the simultaneous combination of the three effects. We shall see later how this parameter influences the stability properties of the flow. At this point, one remark may be useful for the reader to understand the subtle effect of rotation in baroclinic flows. In the absence of shear, rotation enters into account only in the third-order transfer equations (triple correlations) because the Coriolis force is not a production term for energy. In homogeneous baroclinic turbulence, the simultaneous presence of shear and rotation renders the Coriolis effects more direct, say, in the sense that it appears explicitly in second-order correlations equations (Lin equation for example). So, at the second-order (in the statistical sense) rotation behaves like a horizontal restoring force. The effect of rotation is thus more subtle than usual, because shear couples it also to second-order correlation terms.
For the potential energy spectrum:

that nonlinearities are not fully developed yet. The results we present here concern kinetic energy imposed immediately allows us to keep higher level of initial energy in the flow with the counterpart

Table 1 shows the parameters of the different numerical simulations. The fact that distortions are

2.2 Transfers between kinetic and potential energy

In this section, we present the complete equations for energy transfers (Lin equations) in homogeneous baroclinic turbulence to obtain second order correlations that play an important role in the energy balance of baroclinic turbulence. For the spectral kinetic energy density,

\[ \frac{\partial e_K(k)}{\partial t} + 2\nu k^2 e_K(k) + SU_{12}(k) - B_2(k) = \tau_{NL}(uuu(k)) \]  

(1)

with \( e_K(k) \approx \tilde{u}(k) \cdot \tilde{u}(k), U_{12}(k) \approx \tilde{u}_1(k)\tilde{u}_1(k) + \tilde{u}_1(k)\tilde{u}_2(k), B_2(k) \approx \tilde{b}(k)\tilde{u}_2(k) + \tilde{b}^*(k)\tilde{u}_2(k) \).

For the potential energy spectrum:

\[ \frac{\partial e_P(k)}{\partial t} + 2\chi k^2 e_P(k) + N^2 B_2(k) + \varepsilon_B N^2 B_3(k) = \tau_{NL}(ubb(k)) \]  

(2)

Integrating over the whole spectral space gives the equations for time-evolution of total kinetic and potential energy

\[ \frac{\partial E_K}{\partial t} = P_K + B - \varepsilon_K \]  

(3)

with \( P_K = -S\tilde{u}_1\tilde{u}_2, B = \bar{b}\bar{u}_2, \) and

\[ \frac{\partial E_P}{\partial t} = -P_P - \varepsilon_P \]  

(4)

with \( P_P = N^2(\varepsilon_B \bar{b}\bar{u}_3 + \bar{b}\bar{u}_2) = N^2(\varepsilon_B \bar{b}\bar{u}_3 + B) \). Note that rotation arises in potential energy transfer via the product \( \varepsilon_B N^2 = SF \) and the correlation \( B = \bar{b}\bar{u}_2 \). From this equation, we derive a simplified equation for kinetic and potential energy. In the following, we will be interested in time-evolution of the right hand side of the kinetic energy equation (3) and its dimensionless equivalent denoted \( \gamma \), defined by

\[ \gamma = \frac{1}{SK} \frac{\partial E_K}{\partial t} \]  

(5)

3 Parametric analysis by coarse DNS

In this section, we follow the approach used by Jacobitz et al. (1997) on shear-stratified flows. The initial conditions are given by synthetic velocity field having an exponential shaped energy spectrum of the form \( E(k) = k^2 e^{-k^4} \). The three distortions are imposed immediately without isotropic pre-calculation. The consequence of isotropic pre-calculation would be a loss of energy during turbulence decay that may change the critical Richardson number \( Ri_c \).

3.1 Critical Richardson number in the baroclinic case for \( \varepsilon_B = 1 \) at low \( Re \)

Table 1 shows the parameters of the different numerical simulations. The fact that distortions are imposed immediately allows us to keep higher level of initial energy in the flow with the counterpart that nonlinearities are not fully developed yet. The results we present here concern kinetic energy
growth rate and second order correlations present in the Lin equations established in section 2.2. We show that the critical Richardson number is of the order of unity (fig. 2) that corresponds to the limit found by linear analysis in Salhi & Cambon (2006). We also observe that high transient growth can occur for $Ri > 1$ that is in accordance with the analysis by Mamatsashvili et al. (2010). Looking at

![Figure 2](image1.png)

**Figure 2.** (a) Kinetic energy evolution; (b) Kinetic energy growth rate $\gamma$ as a function of $Ri$ for $Re_{\lambda}(0) = 16.3$ and $SK/\epsilon(0) = 4.0$.

The buoyancy fluxes (fig. 3), we note that in the unstable cases $Ri < 1$ the global flux (fig.3a) remains negative while for the transient cases $Ri \geq 1$ it can have positive values indicating that kinetic energy produced by shear via the correlation $u_1u_2$ is stored into potential energy by the flow. The analysis of the vertical buoyancy flux $\bar{b}u_2$ (fig.3b) shows that it contributes most to the evolution of the global buoyancy flux. In fact, the baroclinicity parameter $\varepsilon_B$ acts as a partitioner of potential energy fluxes between vertical and horizontal buoyancy-velocity correlations. When $\varepsilon_B = 1$, the same importance is given to both fluxes.

![Figure 3](image2.png)

**Figure 3.** (a) Total potential energy production; (b) contribution of the vertical flux as a function of $Ri$ for $Re_{\lambda}(0) = 16.3$ and $SK/\epsilon(0) = 4.0$.

3.2 A first step towards anisotropy characterization and 2D-3D transition

Here we analyse the results obtained for the transient case $\varepsilon_B = 1$ and $Ri = 2$. We consider second-order statistics such as dimensionality, componentality and circulicity based on the works by Cambon et al. (1992) and Kassinos et al. (2001) and also present statistics regarding velocity correlation length scales. We show that starting from 3D isotropy the flow becomes two-dimensional due to shear effects,
and exhibits elongated structures (vortical tubes) aligned with the mean flow (with a dimensionality tensor component $d_{11} \approx 0$, fig.4a). After this initial phase, longitudinal structures break up and two-dimensionality is killed, so that the flow reverts back to a more isotropic three-dimensional state with e.g. characteristic ‘worms’ of vorticity (fig. 5).

![Figure 4](image)

**Figure 4.** (a) Time evolution of the components of KRR tensor; (b) streamwise correlation length scales. $Ri = 2, \varepsilon_B = 1$.

![Figure 5](image)

**Figure 5.** Streamwise vorticity iso-surfaces at $\omega_x = -79.0$, at two different times for the case $Ri = 2, \varepsilon_B = 1$.

4 Direct Numerical Simulation of homogeneous baroclinic turbulence

In this section we propose some new results of Direct Numerical Simulation (DNS) of a baroclinic turbulent flow. We investigate the dynamics in both stable and unstable cases, focusing particularly on the transition from isotropy to anisotropy. Energy transfers are also investigated via the analysis of significant second order correlations. Table 2 shows the different parameters used in our DNS.

4.1 Initial conditions

The numerical simulations are initialized with a fully developed isotropic spectrum. At time $t = -T_0$ we start with an exponential shaped spectrum $E(k) = E_0k^4\exp(-2k^2/k_0^2)$ with $k_0 = 7$ and $E_0 = 16\sqrt{2/\pi}/k_0^5$. Turbulence is forced during this initial phase: we inject energy in a certain interval of small wavenumbers $[k_1, k_2]$. In this range, spectral modes have their modulus multiplied by a given constant $\beta$.
Table 2. Numerical parameters for the high resolution runs.

| Run   | \(Re\) | \(Re_\lambda(0)\) | \(\frac{\nu k}{\epsilon}(0)\) | \(dt\) | \(S\) | \(\Omega\) | \(N\) | Resolution       |
|-------|--------|-----------------|-----------------|-------|------|--------|------|----------------|
| 512Inst | 6000   | 122             | 4.46            | 10\(^{-3}\) | 7.07 | 0.106  | 2.738 | \(512 \times 768 \times 512\) |
| 512Stab | 6000   | 122             | 4.46            | 10\(^{-3}\) | 7.07 | 0.354  | 5    | \(512 \times 768 \times 512\) |

(that sets the rate of forcing). Then the turbulence adapts progressively to this energy injection. When the required Reynolds number \(Re_\lambda\) is reached, after several eddy turnover times, the preliminary isotropic simulation is stopped. The corresponding field, at time \(t = 0\), is then submitted to the baroclinic effects. We use forcing in the precomputation stage to keep enough kinetic energy in the flow and to let nonlinearities develop before linear distortions are applied.

# 5 Departure from isotropy in the stable case

## 5.1 Energy spectra

In this section, we present results concerning the dynamics of energy exchanges in the flows. Fig. 6 (a) shows the time evolution of the kinetic energy spectrum. It clearly shows that the energy spectra tends towards a \(k^{-1}\) scaling, representative of flows dominated by shear. We explain this hereafter shortly. If one looks at the energy equation for sheared turbulence only:

\[
\frac{dK}{dt} = -S u_1 u_2 + \epsilon. \tag{6}
\]

At the energetic equilibrium, the production term due to shear balances the dissipation term and we have:

\[
\epsilon \sim S^3 k_S^{-2} \sim \frac{E_S}{1/S} \tag{7}
\]

meaning that all the energy produced at the shear scales \(k_S\) is transfered through the energy cascade to dissipation scales. This defines the shear scales as:

\[
k_S = S^2 \epsilon^{-\frac{1}{2}} \tag{8}
\]

From dimensional analysis, \(E(k_S) \sim (\frac{\nu}{\epsilon})^{-1} k_S^{-1} \sim (S\epsilon)^{-1} k_S^{-1}\). This analysis assumes that \(\tau_s \ll \tau_{turb}\) i.e. that the linear effects are dominants at the scales \(k_S\). Otherwise, taking \(\tau_{turb} = \epsilon^{-1/3} k^{-2/3}\) as characteristic time instead of \(\tau_S\) leads to the \(-\frac{5}{3}\) Kolmogorov law.

## 5.2 Vortex tracking by NAM filtering

We investigate further the type of structures that evolve in homogeneous baroclinic turbulence. We propose to use a data-filtering method called ‘Normalized angular momentum’. This method was first used by Michard et al. (1997) for post-processing PIV measurements. This method allows to localize vortical structures independently on their intensity. It consists in averaging the angular momentum over a finite domain whose size characterizes the cut-off scales (scales that will be filtered). At each point of the domain, the momentum is normalized and then a gaussian weight based on the distance from the center is applied. The results concerning the \(x\)-component of the angular momentum show vortical structures elongated in the streamwise (zonal) direction and aligned in the vertical-meridional plane with the tilted isodensity (isopycnal) surfaces (fig. 7).
$	au \uparrow \ k - 5 \ 3$

$t_S = 0 \quad t_S = 14$

Figure 6. Kinetic energy spectra evolution for (a) $\dot{Ri} = 0.5$, $\varepsilon_B = 0.2$; (b) $\dot{Ri} = 0.15$, $\varepsilon_B = 0.2$.

Figure 7. Distribution of the streamwise component of the filtered angular momentum, exhibiting small scale elongated structures, along the streamwise (zonal) direction.

5.3 Integral length scales
A good indicator of the structure’s aspect-ratio is given by the integral length scales, or two-points velocity correlation length scales $L_{ij}$ whose definition is recalled here:

$$L_{ij}^{(n)} = \int u_i(x) u_j(x + r^{(n)}) \, dr$$

with $r^{(n)} = r \cdot i_n$, $n = 1, 3$. Typically, in flows dominated by shear, the ratio $L_{11}/L_{22}$ indicates how the structures are elongated in the zonal direction (streaks) and spaced in the vertical one. Statistics predict in an efficient way that the flow is dominated by tube-like structures with axis aligned with the streamwise direction (fig. 8 left). This is the novel feature observed when adding the Coriolis effect to shear-stratified turbulence. The presence of a spanwise density gradient inhibits the spreading of structures in the meridional direction and the flow structures evolves from including streak-like or elongated pancakes to including vortical tubes.

6 Departure from isotropy in the unstable case
The following results concern baroclinic turbulence at low $\dot{Ri}$ in order to achieve baroclinic instability. From the results obtained in the section 3 about parametric analysis by coarse DNS, we choose to set the Richardson number equal to 0.15 and $\varepsilon_B = 0.2$. 
6.1 Energy spectra
The results concerning the energy cascade show a tendency toward a $k^{-5/3}$ scaling (see fig. 6(b)) hence to a dynamics typical of forced homogeneous turbulence. In fact, the production of kinetic energy due to baroclinic instability act as a forcing source in the flow. In addition, the eventual breakdown of longitudinal structures tends to isotropize the flow. Since the dynamics is forced by baroclinic instability the inertial range of motion is not dominated by shear effects anymore.

6.2 Correlation length scales
Looking at the evolution of the correlation length scales (fig. 8(b)), we see that the baroclinic instability tends to inhibit the growth of the structures in the longitudinal direction by shearing, another clue of the decrease of the shear effect. After a phase of important initial growth we observe on fig. 8(b) the slow evolution of $L_{11}^X$ and saturation of $L_{22}^X$ and $L_{33}^X$. This also attests the breakdown of the streak-like structures.

Conclusion and perspectives
In the first part of this paper we have done a brief parametrical analysis we at large baroclinic parameter $\varepsilon_B = \frac{S_f}{N^2} = 1$ and shown that in this case the critical Richardson number $Ri_c$ that corresponds to the transition from stable to unstable baroclinic turbulence is of the order of 1 (Jacobitz et al. (1997) found $Ri_c \approx 0.16$ for shear-stratified turbulence), in agreement with the linear stability analysis in Salhi & Cambon (2006) for axisymmetric modes. We also observe that for Richardson numbers larger than 1, important transitional growth of $K$ can however be observed, that is also in accordance with the analysis exposed in Mamatsashvili et al. (2010) for non-axisymmetric modes. The analysis of second-order correlations is also shown to be relevant to understand the dynamics of baroclinic turbulence, and the following conclusions have been drawn: the correlation $\overline{u_1 u_2}$ is always negative and exhibit a more important decrease in the stable cases with transitional growth. Therefore, the following evolution is determined by the horizontal and vertical buoyancy fluxes. The predominance of vertical or horizontal fluxes is fixed by the baroclinicity parameter: a parameter $\varepsilon_B > 1$ enhances the horizontal flux while $\varepsilon_B < 1$ enhances the vertical one.

The second part of our work concerns the departure from isotropy of turbulent baroclinic flows. First, we focused on the stable baroclinic case. Kinetic energy produced by shear is directly converted into potential energy. We show that the flow then behaves as shear dominated flows, exhibiting a $k^{-1}$ cascade in the inertial range and enhancing the formation of longitudinal vortical tubes. In the unstable case, we
find that, after an initial phase of formation of such vortical tubes aligned with the mean flow, correlations are killed by the baroclinic instability that breaks helical eddies and supply the flow with kinetic energy acting as a forcing. As a consequence, the dynamics corresponds to a $k^{-\frac{5}{3}}$ scaling and shear effects are inhibited. This provides an alternative explanation for $k^{-\frac{5}{3}}$ slopes of energy spectra observed in the atmosphere, possibly due to decaying isotropic turbulence or forced baroclinic turbulence. The analysis of potential energy spectrum is less conclusive. Both stable and unstable cases present a slope of the spectrum around $-\frac{5}{3}$ with a slight tendency towards a $-\frac{3}{2}$ cascade in the unstable case.

The dynamics of the buoyancy field thus deserves more investigations, in particular for detailing the link between the kinetic and potential energy cascade. Comparison between active and passive scalar behaviour are also in progress.

References

CAMBON, C., JACQUIN, L. & LUBRANO, J.L. 1992, Toward a new Reynolds stress model for rotating turbulent flows, Phys. Fluids A 4 (4).

CAMBON, C., MANSOUR, N.N. & GODEFERD, F.S. 1997, Energy transfer in rotating turbulence, J. Fluid Mech. 337, 303-332.

CANUTO, C., HUSSAINI, M.Y., QUARTERONI, A. & ZANG, T.A. 2007, Spectral methods, Evolution to Complex Geometries and applications to Fluid Dynamics, Springer.

CHARNEY, J.G. 1947, The dynamics of long waves in baroclinic westerly current, J. Physical Oceanography 4, 135.

CHARNEY, J.G. & STERN, M.E. 1962, On the stability of Internal Baroclinic Jets in a Rotating Atmosphere, J. Atm. Sci. 19, 159-172.

COLEMAN, G.N., KIM, J. & SPALART, P.R. 2000, A numerical study of strained three-dimensional wall-bounded turbulence, J. Fluid Mech. 416, 107.

DELORME, P. 1985, Simulation numérique de turbulence homogène compressible avec ou sans cisaillement imposé, PhD Thesis, ONERA.

DRAZIN, P.G. & REID, W.H. 1981, Hydrodynamic Stability, Cambridge University Press.

EADY, E.T. 1949, Long waves and cyclone waves, Tellus 1, 33.

GODEFERD, F.S. & CAMBON, C. 1994, Detailed investigation of energy transfers in homogeneous stratified turbulence, Phys. Fluids 6, 2084-2100.

JACOBITZ, F.G., SARKAR, S. & VAN ATTA, C.W. 1997, Direct numerical simulations of turbulence evolution in a uniformly sheared and stably stratified flow, J. Fluid Mech. 342, 231-261.

KASSINOS, S.C., REYNOLDS, W.C. & ROGERS, M.M. 2001, One-point turbulence structure tensors, J. Fluid Mech. 428, 213-248.

MAMATSASHVILI, G.R., AVSARKISOV, V.S., CHAGElishvili, G.D. & CHANISHVILI, R.G. 2010, Transient Dynamics of Nonsymmetric Perturbations versus Symmetric Instability in Baroclinic Zonal Shear Flows, J. Atm. Sci. 67, 2972-2989.

MICHAUD, M., GRAFTEAUX, L., LOLLINI, L. & GROSJEAN, N. 1997, Identification of vortical structures by a non-local criterion: Application to P.I.V. measurements and DNS results of turbulent rotating flows, Proceedings of the 11th Symposium on Turbulent Shear Flows.

PEDLOSKY, J. 1987, Geophysical fluid dynamics, Springer.

ROGALLO, R.S. 1981, Numerical Experiments in homogeneous turbulence, NASA Technical Memorandum 81315.

SALHI, A. & CAMBON, C. 2006, Advances in rapid distorsion theory, from rotating shear flows to the baroclinic instability, J. Appl. Mech. 73, 449-460.