A thermo-mechanical model for pile-soil interface behavior

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Abstract. This study proposes a constitutive model of the pile-soil interface based on a thermodynamic approach. Firstly, the thermo-elastic coupled nonlinear hyperelastic relation of the pile-soil interface is derived by defining the interface potential energy function of the pile-soil interface subjected to normal and tangential forces. The plastic interface constitutive relations are then developed using non-equilibrium thermodynamics, based on the energy dissipations caused by the pile-soil interface’s shear-induced slipping behavior. The stress- and temperature-dependent pile-soil interface behavior is described using the concept of interface density and the normal elastic thermal expansion coefficient.

1. Introduction
A pile also serves as a heat exchanger connected to a geothermal source heat pump system in the field of energy-pile foundation[1-3]; therefore, the thermo-mechanical behavior of the pile-soil interface becomes an essential issue in the energy-pile structural design. Due to the thermally-induced pore pressure development and the non-isothermal consolidation, the mechanical properties of the soil surrounding piles can be significantly changed when subjected to temperature variations[4-5], leading to the change in the pile-soil behavior. Meanwhile, the pile-soil interactions are also influenced by temperature variations due to the thermal effects of the absorbed bound water film between the soil particles and the pile surface and the difference in the thermal expansion coefficient between the soil particles and the pile material. Depending on the interface’s stress state and stress history, the thermo-mechanical behavior of the pile-soil interface should be the net effect of these different physical mechanisms.

Research on some experimental studies on the non-isothermal pile-soil interface behavior had been reported, mainly based on the temperature-controlled direct shearing tests of the interface. In these studies, pile-soil interface specimens compressed under a given normal stress were sheared at different temperatures after heating or cooling processes[6-8]. Wang, et al. (2020)[9] reported a series of non-isothermal direct shearing tests for concrete-clay interface subjected to monotonic and cyclic temperature loads, revealing a significant OCR (over-consolidation ratio) effect of the clay on the non-isothermal interface behavior. When subjected to monotonic heating, the shear strength can be increased for normally consolidated or slightly over-consolidated clay interfaces and decreased for those in highly over-consolidated clay[9]. Such thermally-induced change in the interface shear strength is usually irreversible, and thus the thermo-plastic interface behavior should be considered.

Despite the existence of several constitutive models for the thermo-mechanical behavior of soils[10-13], constitutive modeling of the pile-soil interface behavior under non-isothermal conditions remains...
an issue requiring more attention. Some existing thermo-mechanical interface models were developed based on the theory of hypo-plasticity\cite{14} or the critical state soil mechanics\cite{15}. Unlike these models, this study proposes a thermodynamics-based thermo-mechanical model for a pile-soil interface, validated by predicting the non-isothermal direct shearing tests reported in Ref. [9]. The thermo-elasto-plastic constitutive relations for the pile-soil interface are obtained using the interface energy potential and dissipation described based on a thermodynamic approach\cite{16-18}. Thus, this study provides an alternative approach for modeling the pile-soil interface behavior under non-isothermal conditions.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{The schematic diagram of the physical model of the pile-soil interface shearing.}
\end{figure}

\section{Theory development: Interface hyperelasticity}

In this section, the nonlinear thermo-hyperelastic model for the pile-soil interface will be developed by defining the interface elastic potential function using the theory of hyperelasticity for soils proposed by\cite{16}. Figure 1 shows the schematic diagram of the physical model of the pile-soil interface shearing. Hereafter, the normal pressure on the pile-soil interface is denoted as $P$, and the shear stress on the shear plane $Q$ is decomposed into $Q_x$ and $Q_y$ along the coordinate axes of $x$ and $y$ on the shearing plane. Neglecting the bonding effect of the pile-soil interface, the elastic free energy of the interface can be defined as a function of the normal compressive elastic displacement $\Delta_{e}$ and the shearing elastic displacement $u_e$. Following the hyperelastic model of saturated soils\cite{16}, the elastic potential energy function of the pile-soil interface can be defined as:

\begin{equation}
F_{i} = \frac{1}{m+2} B \Delta_{e}^{m+2} + A \Delta_{e}^{n} u_{e}^{2}
\end{equation}

\begin{equation}
B = B_{0} \tilde{\rho}^{\tilde{k}}, A = A_{0} \tilde{\rho}^{\tilde{k}}
\end{equation}

where $A_0$ and $B_0$ are two elastic constants, $\tilde{\rho}$ is the interface density, and $\tilde{k}$ is a material parameter determining the density-dependency of the interface elasticity; $m$ and $n$ are two nonlinear indexes; $\Delta_{e}$ is a thermally equivalent normal compressive elastic displacement defined as
\[ \bar{A}_e = \Delta_e + \beta \Delta T \]  \hspace{1cm} (1c)

\( \beta \) is the normal thermal expansion coefficient of the interface and \( \Delta T = (T - T_0) \) is the temperature change with respect to a reference temperature \( T_0 \). In this study, \( \beta \) is simply the sum of the thermal expansion coefficients of the soil and the pile material, i.e., \( \beta = \beta_1 + \beta_2 \), where \( \beta_1 \) is the thermal expansion coefficient of the clay particles (e.g., \( \beta_1 = 5 \times 10^{-6} \) for Kaolin clay), \( \beta_2 \) is the thermal expansion coefficient of the pile material (e.g., \( \beta_2 = 10 \times 10^{-6} \) for concrete).

The interface density \( \rho \) can be regarded as the mass of the soil particles contacting the pile surface per unit area of the pile-soil interface, employed to consider the influence of the soil density surrounding the pile and the roughness of the pile surface on the pile-soil interface behavior. The non-isothermal consolidation of the soil, the thermally-induced pore water migration, and the rearrangement of soil particles near the interface can all induce change in \( \rho \) when the interface is subjected to temperature variations. Therefore, the variation of \( \rho \) with temperature should be sensitive to the OCR value of the soil surrounding the pile.

The relationship between the normal pressure \( P \), the shear stress \( Q \), and the interfacial elastic free energy \( F_e \) can be then expressed as:

\[ P = \frac{\partial F_e}{\partial \Delta_e} = B \Delta_e^m + An\Delta_e^{m-1}u_e^2 \]  \hspace{1cm} (2a)

\[ Q = \frac{\partial F_e}{\partial u_e} = 2A\Delta_e^nu_e \]  \hspace{1cm} (2b)

The shearing elastic displacement \( u_e \) can also be decomposed into components along the \( x \)- (\( u_{ex} \)) and \( y \)-directions (\( u_{ey} \)), i.e., \( u_e = \sqrt{u_{ex}^2 + u_{ey}^2} \). Similar formulations for \( Q_x \) and \( Q_y \) can then be derived as follows:

\[ Q_x = \frac{\partial F_e}{\partial u_{ex}} = 2A\Delta_e^nu_{ex} \]  \hspace{1cm} (3a)

\[ Q_y = \frac{\partial F_e}{\partial u_{ey}} = 2A\Delta_e^nu_{ey} \]  \hspace{1cm} (3b)

Given that the direction of the applied external tangential load in an interface direct shearing test is parallel to the \( x \)-axis, \( Q = Q_x \), \( Q_y = 0 \), \( u_e = u_{ex} \) and \( u_{ey} = 0 \).

3. Interface strength criterion based on hyperelasticity

Following the concept of elastic stability from thermodynamics\cite{16-18}, the strength criterion of the pile-soil interface can be derived by the following interface state boundary equation:

\[ \frac{\partial^2 F_e}{\partial \Delta_e^2} \frac{\partial^2 F_e}{\partial u_e^2} - \left( \frac{\partial^2 F_e}{\partial \Delta_e \partial u_e} \right)^2 = 0 \]  \hspace{1cm} (4)

By substituting Eq. (1) into Eq. (4) and then substituting Eq. (2) into Eq. (4), the strength criterion equation of the pile-soil interface can be expressed as:

\[ Q = 2 \sqrt{AB} \frac{m+1}{n(n+1)} \left( \frac{1}{B} \right) \frac{n+1}{m+n+2} P_{\text{clay}} \]  \hspace{1cm} (5)

Eq. (5) gives the maximum allowable shearing stress \( Q \) at a given normal pressure \( P \). Since Eq. (1b) determines the variables \( A \) and \( B \), the interface strength criterion shown in Eq. (5) should be dependent on the interface density \( \rho \). The value of \( \rho \) can be determined as a function of the previous consolidation pressure \( P_c \) and the current normal pressure \( P \). Based on the concepts of normally compression line (NCL) and rebound line (RBL) defined in the compression and rebound tests of soils,
one can define similar concepts of NCL and RBL for a pile-soil interface. Assuming that both NCL and RBL are straight lines in the \( \ln P - \ln \bar{P} \) space, we have

\[
\text{NCL}: \ln P = a \ln \bar{P} - b \quad (6a)
\]

\[
\text{RBL}: \ln P = \ln P_c + c(\ln \bar{P}) \quad (6b)
\]

where \( a \) and \( c \) are the slopes of NCL and RBL on the \( \ln P - \ln \bar{P} \) space, respectively; \( b \) is the intercept of the NCL on the \( \ln P \) axis, and \( \bar{P} \) is the interface density at \( P = P_c \). Thus, from Eq. (6), the interface density can then be expressed as:

\[
\bar{P} = \frac{\ln p + b}{a} + \frac{\ln p - \ln p_c}{c} \quad (7)
\]

Substituting Eq. (7) into Eq. (1b) leads to

\[
B = \bar{B}_0 P_c^{k_2} P^k, A = \bar{A}_0 P_c^{k_1} - k_2 P^k \quad (8)
\]

where \( \bar{B}_0 = B_0 e^{k/a}, \bar{A}_0 = A_0 e^{k/a}, k_1 = k / a \) and \( k_2 = k / c \). Inspection of Eq. (8) reveals that only the parameters \( \bar{B}_0, \bar{A}_0, k_1 \) and \( k_2 \) are needed to be determined in the model simulations. Substituting Eq. (8) into Eq. (5) further yields

\[
Q = 2 \sqrt{\frac{A_0 B_0}{n(n+1)}} \left( \frac{1}{B_0} \right)^{\frac{m+n+2}{2(n+1)}} P_c^{\frac{m+n+2}{2(n+1)}} \left( k_1 - k_2 \right)^{\frac{m-n}{2(n+1)}} \left( k_1 - k_2 \right)^{\frac{m+n+2}{2(n+1)}(1-k_1)+k_2} \quad (9)
\]

According to the experimental studies, the failure line of the pile-soil interface with normally consolidated clay surrounding the pile (i.e., \( P = P_c \)) can be regarded as a straight line passing through the origin in the \( P - Q \) space. Therefore, from Eq. (9), \( k_1 = 1 \). When the current consolidation pressure \( P_c \) is fixed, \( Q \propto P^{0.5(m+n+2)(1-k_2)/(m+1)+k_2} \), which can be utilized to determine the values of \( k_2, m, \) and \( n \). Then, the values of \( \bar{B}_0 \) and \( \bar{A}_0 \) can be further determined according to a stress-displacement curve measured in the pile-soil interface shearing test.

![Figure 2. Failure envelops of concrete-Kaolin clay clay interfaces with different OCR values.](image-url)
Figure 2 depicts the failure envelops of pile-clay interfaces with different OCR values of Kaolin clay surrounding the pile concrete, where the markers are measured data for concrete-Kaolin clay interfaces[9]. $A_0 = 2$, $B_0 = 2$, $k_2 = 0.1$, $m = 1.5$ and $n = 0.5$ are used in Figure 3 according to the interface direct shearing tests[9]. It can be seen that the larger the OCR value, the greater the interface shearing strength at a given normal pressure. Figure 3 presents the shearing failure lines of the concrete-clay interface with different pre-consolidation pressures. Under the same normal pressure, the larger the initial consolidation pressure, the greater the strength of the interface. Figure 3 shows that the failure line is a curved power line passing through the origin for interfaces with the same pre-consolidation pressure. It is worth noting that the cohesion of the interface is not considered in this study, which should be improved for interfaces with bonding effects between the soil and pile surface. When the interface is heated at a constant normal pressure $P$, the interface density $\rho$ should change in response to the temperature variation. Therefore, thermal effects should be considered in Eqs. (6–8). To this end, the NCL and RBL should be defined as a function of temperature and OCR value.

Figure 3. Failure envelops of concrete-Kaolin clay interfaces with different pre-consolidation pressures.

4. Interface energy dissipation and plastic constitutive relations

For a thermodynamic system at equilibrium, the physical quantities describing the macroscopic properties of the system are not completely independent, according to thermodynamics. There is a set of independent thermodynamic state variables, and all other state variables can be expressed in terms of the independent ones. the normal elastic compression $\Delta_e$, the shearing elastic displacement in the $x$-direction ($u_{ex}$), the shearing elastic displacement in the $y$-direction ($u_{ey}$), and the entropy density $s$ are the independent thermodynamic state variables for a pile-soil interface. As a result, the total energy density of the interface, denoted as $\omega$, has the following total differential relationship with these state variables:
\[ d\omega = Pd\Delta_e + Q_x du_{ex} + Q_y du_{ey} + Tds \]  
(10a)
\[ F_s = \omega - Ts \]  
(10b)
\[ dF_s = Pd\Delta_e + Q_x du_{ex} + Q_y du_{ey} - sdT \]  
(10c)

where the term \( Tds \) represents the increment of the thermal energy induced by the energy dissipation.

In non-equilibrium thermodynamics, the concepts of dissipative force and flow are used to describe the energy dissipation in a thermodynamic system[16-18]. Neglecting the entropy variation resulted from the mass and heat transfers, the energy dissipation rate of the pile-soil interface can be governed by the following entropy balance equation:

\[ R = T\dot{s} = Y_1 P + Y_2 Q_x + Y_3 Q_y \]  
(11)

where \( \dot{s} = ds/dt \) is the rate of entropy density; \( P \), \( Q_x \) and \( Q_y \) are defined as dissipation forces resulting in energy dissipations; \( Y_1 \), \( Y_2 \) and \( Y_3 \) are the corresponding dissipative flows. The dissipative flows can be expressed as a function of their corresponding dissipative forces as:

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix} =
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{bmatrix}
\begin{bmatrix}
P \\
Q_x \\
Q_y
\end{bmatrix}
\]
(12)

where \( \lambda_1 \) and \( \lambda_2 \) are called migration coefficients in thermodynamics.

On the other hand, the following energy conservation equation should hold according to the energy conservation law:

\[ \dot{\omega} = P\dot{\Delta} + Q\dot{u}_x + Q\dot{u}_y \]  
(13)

where \( \Delta \) is the total normal compression of the interface, \( u_x \) and \( u_y \) are the total shearing displacements in the x-direction and y-direction, respectively. The total displacements defined above can be further decomposed into elastic and plastic components, i.e.,

\[
\Delta = \Delta_e + \Delta_p
\]  
(14a)
\[
\dot{u}_x = \dot{u}_{ex} + \dot{u}_{px}
\]  
(14b)
\[
\dot{u}_y = \dot{u}_{ey} + \dot{u}_{py}
\]  
(14c)

Combining Eqs. (10–14) leads to the following thermodynamic identity that should hold for arbitrary values of \( P, Q_x \), and \( Q_y \):

\[
(Y_1 - \Delta_p)P + (Y_2 - \dot{u}_{px})Q_x + (Y_3 - \dot{u}_{py})Q_y = 0
\]  
(15)

According to Eq. (15) \( \Delta_p = Y_1, \dot{u}_{px} = Y_2, \dot{u}_{py} = Y_3 \), so that the plastic displacement rates are simply the dissipative flows of the dissipative forces in the corresponding directions. For simplicity, the normal displacement is assumed to be purely elastic below, i.e., \( \Delta_p = 0 \) and \( \Delta = \Delta_e \), thus, only the shearing plastic behavior is considered. From Eqs. (3a and 12), \( Y_2 = \lambda_2 \cdot 2\Delta_e \cdot u_{ex} \) and \( Y_3 = \lambda_3 \cdot 2\Delta_e \cdot u_{ey} \). The shearing dissipative flows \( Y_2 \) and \( Y_3 \) can be further simplified by defining \( \lambda_2 = \lambda_3 = \lambda \). The shearing displacement rate and \( \lambda \) is a material coefficient. Thus, the rates of the plastic shearing displacements are:

\[
\dot{u}_{px} = Y_2 = \lambda \Delta_e \cdot u_{ex}
\]  
(16a)
\[
\dot{u}_{py} = Y_3 = \lambda \Delta_e \cdot u_{ey}
\]  
(16b)

5. Model validation
The model proposed will be validated in this section by predicting the interface direct shearing tests of the concrete-Kaolin clay interface\(^9\). Assuming the shearing load is loaded along the x-direction in the direct shear tests, \( u_{cs} = u_c \) and \( u_s = u \). Thus,

\[
\dot{u}_c = \dot{u}(1 - \frac{\Delta_s u_c}{\bar{A}_2}) \tag{17}
\]

From Eq. (17), as the shear displacement increases, the pile-soil interface will eventually reach a critical state at which \( (\dot{u}_c)_{cr} = 0 \). Hereafter, the subscript “cs” denotes the value of a specified variable at the critical state. Therefore,

\[
(\Delta_s u_c)_{cs} = \frac{1}{\bar{A}_2} \tag{18}
\]

Substituting Eq. (18) into Eq. (2b) and combining with Eq. (8), the critical state shearing strength of the pile-soil interface is:

\[
Q_{cs} = 2\bar{A}_0 P_c \Delta s_c \frac{1}{\bar{A}_2} \tag{19}
\]

Assuming the critical state strength of the pile-soil interface is consistent with the ultimate strength determined by Eq. (9), from Eqs. (19 and 9), the material coefficient \( \bar{A}_2 \) can then be expressed as a function of the pre-consolidation pressure and the current normal pressure as:

\[
\frac{1}{\bar{A}_2} = \hat{\lambda}_2 p_c = \hat{\lambda}_2 p_c \left[ \frac{m+n+2}{m(n+1)} \right] p^{\frac{m+n+2}{2(m+1)} (1-x)} \tag{20a}
\]

\[
\hat{\lambda}_2 = \sqrt{\frac{B_0 (m+1)}{A_0 n (n+1)}} \left( \frac{1}{B_0} \frac{n+1}{m+n+2} \right)^{\frac{m+n+2}{2(m+1)}} \tag{20b}
\]

here \( \hat{\lambda}_2 \) is a constant dependent on the model parameters.

The direct shearing tests of the concrete-Kaolin clay\(^9\) are predicted here using the proposed model for validating the model. The normal stress \( P \) is kept constant in the simulations (i.e., \( \dot{P} = (\partial P / \partial \Delta_s) \Delta_s + (\partial P / \partial u_s) \cdot \dot{u}_s = 0 \)), and the interfaces are sheared at the same shearing displacement rate controlled by the direct shearing tests. Then, combining with Eqs. (2), (17), and (20), the relations between shearing stress and displacement can be simulated. Figure 4 illustrates the comparison between the predicted and measured stress-displacement curves of the interfaces with different OCR values under isothermal conditions. The scattered points are the measured data obtained from the experiments\(^9\). As can be seen, the predicted curve is essentially consistent with the measured data throughout the measured region.

The direct shearing tests of the concrete-Kaolin clay interface are also simulated using the proposed model. As discussed in previous sections, thermo-elastic coupling terms in the hyperelastic relations (Eqs. (1–2)) and the thermally-induced variations in the interface density \( \tilde{\rho} \) describe the thermal effects of the interface behavior. This study simply determines the values of \( \tilde{\rho} \) at elevated temperatures (\( \Delta T = 25^\circ\text{C} \)) by trial and error using the measured shearing tests\(^9\). Therefore, Figure 5 depicts the relation between the thermally-induced change in \( \tilde{\rho} \) and the OCR value (\( \Delta T = 25^\circ\text{C} \) and \( P_c = 400\text{kPa} \)). Figure 5 also shows that \( \tilde{\rho} \) increases for normally and slightly over-consolidated interfaces (OCR = 1, 2, and 4) and decreases for highly over-consolidated interfaces (OCR = 8). This explains the physical mechanism underlying the responses of the interface shearing behavior at elevated temperature as shown in Figure 6; that is, the interface strength increases for the cases of OCR = 1, 2, and 4, while it decreases for the case of OCR = 8. This is similar to the effect of OCR on the saturated clay shear strength\(^10\). In response to temperature variations, the evolution law of \( \tilde{\rho} \) is needed for a more generalized purpose in simulations. From a practical perspective, it can be described by defining the ICL and RBL equations as a function of temperature and OCR. This will be further studied in the future.
6. Conclusion
This study establishes a thermo-elastoplastic constitutive model of the pile-soil interface based on the theory of thermodynamics. A state-dependent nonlinear thermo-hyperelastic model of the pile-soil interface is proposed by defining the interface potential energy function in terms of the elastic compressive and shearing displacements, interface density, and normal elastic thermal expansion coefficient, leading to a theoretical strength criterion of the pile-soil interface derived using the elastic stability concept. Similar to the strength criterion of saturated clays, the stress state boundary line (or failure) of the pile-soil interface is a curved power function curve and is affected by the OCR value of the soil surrounding the pile. The energy dissipations within the pile-soil interface can then be described based on non-equilibrium thermodynamics, leading to the theoretical derivation of the pile-soil interface plastic constitutive relations. The proposed model is validated by predicting the concrete-Kaolin interface direct shearing tests under isothermal and non-isothermal conditions. The thermo-elastic coupling in the hyperelasticity and the thermally-induced change in the interface density are shown to be the primary physical mechanisms underlying the non-isothermal interface behavior observed in laboratory tests. Therefore, the present study provides a practical approach for modeling the thermo-mechanical behavior of a pile-soil interface.

\begin{figure}[h]
\centering
\subfloat[OCR = 1]{
\includegraphics[width=0.4\textwidth]{figure1a.png}}
\subfloat[OCR = 2]{
\includegraphics[width=0.4\textwidth]{figure1b.png}}
\subfloat[OCR = 4]{
\includegraphics[width=0.4\textwidth]{figure1c.png}}
\subfloat[OCR = 8]{
\includegraphics[width=0.4\textwidth]{figure1d.png}}
\caption{Shear stress vs. displacement for different OCR values.}
\end{figure}
Figure 4. Comparisons of predicted displacement-stress curves with experimental data.

Figure 5. Relationship between OCR and thermally-induced change in interface density ($\Delta T = 25^\circ C$; $\Delta \bar{\rho}_r$ is the thermally-induced variation of $\bar{\rho}$ and $\bar{\rho}_0$ is the value of $\bar{\rho}$ at $T = T_0$).
Figure 6. Simulation results of direct shearing tests for concrete-Kaolin clay interface at different temperatures((a), shearing stress-displacement relations: dots represent the experiment results and lines represent the simulation results; (b). Failure lines in P-Q space).

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