Dissipation-facilitated molecules in a Fermi gas with non-Hermitian spin-orbit coupling

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We study the impact of non-Hermiticity on the molecule formation in a two-component spin-orbit-coupled Fermi gas near a wide Feshbach resonance. Under an experimentally feasible configuration where the two-photon Raman process is dissipative, the Raman-induced synthetic spin-orbit coupling acquires a complex strength. Remarkably, dissipation of the system facilitates the formation and binding of molecules, which, despite their dissipative nature and finite life time, exist over a wider parameter regime than in the corresponding Hermitian system. These dissipation-facilitated molecules can be probed by the inverse radio-frequency (r.f.) spectroscopy, provided the Raman lasers are blue-detuned to the excited state. The effects of dissipation manifest in the r.f. spectra as shifted peaks with broadened widths, which serve as a clear experimental signature. Our results, readily observable in current cold-atom experiments, shed light on the fascinating interplay of non-Hermiticity and interaction in few- and many-body open quantum systems.

Introduction:— Non-Hermitian physics have been intensively explored recently in a wide range of experimental systems such as optics, acoustics, and microwave cavities [1, 2]. Whereas most of these studies are limited to single particles, the interplay of non-Hermiticity and interaction, a key element in understanding non-Hermitian many-body quantum systems, has rarely been experimentally investigated. In this context, cold atomic gases provide an ideal platform where such an interplay can be examined in a controllable fashion. This is facilitated by the highly tunable interactions through the Feshbach resonance [3], and, importantly, by the recent implementation of non-Hermiticity via laser-induced one-body [4–6] or two-body [7, 8] dissipation. Based on this latest experimental progress, several very recent theoretical works discussed the impact of interaction on dissipative cold gases of either bosonic [9–12] or fermionic [13, 14] atoms. Given the recent experimental studies of interacting bosons under dissipation [6–8], it is highly desirable to search for other readily accessible non-Hermitian systems (fermionic in particular) where the interplay of interaction and non-Hermiticity leads to non-trivial and experimentally detectable phenomena.

In this work we propose an experimentally feasible scheme where the non-trivial effect of non-Hermitian spin-orbit coupling (SOC) on the molecule formation in a two-component Fermi gas can be probed by the widely used radio-frequency (r.f.) spectroscopy [15]. We start from the two-photon Raman process that has been used to generate SOCs in Hermitian cold-atom systems [16–20]. To make the SOC non-Hermitian, we consider the case where the intermediate excited state is subject to a controllable loss, which, for instance, can be induced by an additional laser, as illustrated in Fig. 1. Such a loss channel makes the SOC strength complex-valued, with tunable real and imaginary parts. Since the SOC does not commute with inter-atomic interactions, the resulting non-Hermitian system is expected to host non-trivial few- and many-body quantum phenomena.

Here we focus on the study of two-body bound states, or molecules, under the non-Hermitian SOC. We find that, in contrast to previously realized imaginary magnetic fields [4, 5], which do not affect the molecular binding of spin-singlet fermions [21], the non-Hermitian SOC has significant impact on the molecule formation. Most remarkably, the presence of dissipation can greatly facilitate the molecule formation, in that it can induce new molecular branches or enhance the bind-
ing energy of existing molecules in a wide parameter regime. This is in distinct contrast to Hermitian systems where the molecule formation is suppressed by one-dimensional SOC [22–24]. Another important feature of these molecules is that their binding energies are generally complex-valued with a negative imaginary part, reflecting their dissipative nature with finite life time. Nevertheless, we show that these molecules can still be experimentally probed via the inverse radio-frequency (r.f.) spectroscopy, provided that the Raman lasers generating the SOC are blue-detuned to the excited state. Signatures of non-Hermiticity manifest themselves as shifted spectral peaks with broadened widths in the r.f. spectrum. Our results pave the way for the experimental exploration of fundamental few-body physics in non-Hermitian fermionic systems, which are indispensable for the understanding of many-body phenomena therein.

**Model:** The single-particle Hamiltonian corresponding to Fig. 1 is \( \langle \hbar = 1 \text{ throughout the paper} \rangle 

\[
H_0 = \int dr \sum_{\sigma,\sigma'} \sum_{q,k} \psi_{\sigma}^\dagger (r) \left[ -\frac{\nabla^2}{2m} + \Omega \delta_{\sigma\sigma'} \right. \\
+ \left. \Omega \sigma x e^{-i\delta q x} + \text{h.c.} \right] \psi_{\sigma'}(r). \tag{1}
\]

Here \( \psi_{\sigma} \) is the annihilation field operator for the state \( |\sigma\rangle \), \( q \) is the transferred momentum, and the complex SOC strength \( \Omega \) is given by

\[
\Omega = \frac{\Omega_0}{1 \pm i \tilde{\Gamma}}, \tag{2}
\]

where \( +(-) \) corresponds to the blue (red) single-photon detuning with \( \delta > 0 \) \( (\delta < 0) \), \( \Omega_0 := \eta^2/\delta \) is the Raman-induced SOC strength at zero dissipation \( (\eta \text{ is the single-photon Rabi frequency of either Raman lasers}) \), and \( \Gamma := \Gamma/|\delta| \) is the dimensionless dissipation strength with \( \Gamma \) the decay rate of the intermediate excited state. For a finite dissipation \( \tilde{\Gamma} \), \( \Omega \) is complex and \( H_0 \) features a non-Hermitian SOC term. Note that \( H_0 \) also includes a complex Stark-shift term of strength \( \Omega \) for each spin, which ensures the dissipative nature of the system.

After a U(1) transformation of the field operators, \( \psi_{\sigma}(r) = e^{i\pi x d^{+} \sigma \tilde{\psi}_{\sigma}(r)} \) \( (s_{\pm} = \mp 1) \), \( H_0 \) can be written in momentum space as \( H_0 = \sum_{k} \tilde{\psi}_{k}^\dagger h_{0}(k) \tilde{\psi}_{k} \), with the transformed basis \( \tilde{\psi}_{k} = [\tilde{\psi}_{k\uparrow}, \tilde{\psi}_{k\downarrow}]^T \), and

\[
h_{0}(k) = \frac{1}{2m} (k - q \sigma x e_{\pm})^2 + \Omega \sigma x + \Omega. \tag{3}
\]

The corresponding single-particle dispersion is

\[
\xi_{k\pm} = \epsilon_{k} + E_{q} + \Omega \pm \sqrt{(\epsilon_{k} x / m)^2 + \Omega^2}, \tag{4}
\]

where \( E_{q} = q^2/(2m) \), \( \epsilon_{k} = k^2/(2m) \), and \( \pm \) indicates different helicity branches.

Finally, the s-wave interaction can be written as

\[
U_{\text{int}} = U \sum_{Q,k,k'} \tilde{\psi}_{k\uparrow}^\dagger \tilde{\psi}_{Q-k\uparrow}^\dagger \tilde{\psi}_{Q-k'\downarrow} \tilde{\psi}_{k'\uparrow}, \tag{5}
\]

where the bare interaction \( U \) is related to the s-wave scattering length \( a_{s} \), via \( 1/U = m/(4\pi a_{s}) - 1/V \sum m/k^2 \).

**Single-particle physics:** We first study the impact of dissipation on the single-particle dispersion \( \xi_{k\pm} \). For the rest of the paper and in the figures, we take \( q \) and \( E_{q} \) as the units of momentum and energy, respectively. As shown in Fig. 2(a), Re(\( \xi_{k\pm} \)) are identical for red- and blue-detuned lasers, if both are shifted by the corresponding threshold energy \( \xi_{th} := \text{Re}(\xi_{k-}) \). Here \( k_{m} = (k_{\text{min}}, 0, 0) \), and \( k_{\text{min}} \) is the location of minimum Re(\( \xi_{k-} \)) along \( k_{x} \). For \( \text{Im}(\xi_{k\pm}) \), however, the red- and blue-detuned cases are different [see Fig. 2(b1)(b2)]. In either case, Im(\( \xi_{k\pm} \)) \( < 0 \), indicating a finite life time for single particle.

For our later discussion of molecule formation, it is helpful to highlight several key properties of the single-particle dispersion. First, \( k_{\text{min}} \) is non-monotonic with varying \( \tilde{\Gamma} \); it increases for small \( \tilde{\Gamma} \) and decreases for large \( \tilde{\Gamma} \) [see Figs. 2(c1)(c2)]. Second, the coupling constant \( C \) also varies non-monotonically with \( \tilde{\Gamma} \) [see Fig. 2(d)].
where
\[ C = \langle k_{m_1}^L; k_{m_2}^L | \sum_{kk'} \psi_{kk'}^+ \tilde{\psi}_{kk'}^+ \tilde{\psi}_{kk'} \psi_{kk'}^R | k_{m_1}^R; k_{m_2}^R \rangle, \]
with \([k_{\mu}^R/L]\) denoting the right/left eigenvector satisfying \(h_0|k_{\mu}^R\rangle = \xi_{k\mu}|k_{\mu}^R\rangle\) and \(h_0|k_{\mu}^L\rangle = \xi^*_{k\mu}|k_{\mu}^L\rangle\). Physically, \(C\) is proportional to the s-wave interaction strength between the lowest-energy states in the helicity basis. Furthermore, in Fig. 2(c), we show the energy gap \(G := \text{Re}({\xi_{s+}}) - \text{Re}({\xi_{s-}})\), which decays monotonically with \(\Gamma\).

The non-monotonic behavior of \(\xi_{s+}\) and \(C\) can be understood through perturbation theory in either the small- or large-\(\Gamma\) limit \([25]\). In the large-\(\Gamma\) limit, the SOC strength decays as \(\Omega \sim i\Omega_0/\Gamma\), a direct consequence of the quantum Zeno effect, and Eq. (3) is reduced to the Hamiltonian studied in Ref. \([13]\). An important implication from the non-monotonic behavior is that non-Hermitian SOC would achieve its strongest effect at an intermediate strength \(\Gamma \sim 1\), which we confirm in the molecular calculations below.

**Dissipation-facilitated molecules:**—The experimentally relevant molecular state in our interacting non-Hermitian system is the right eigenvector \(|\Phi^R\rangle\), which satisfies the Lippman-Schwinger equation
\[ |\Phi^R\rangle = G_0 U_{\text{int}} |\Phi^R\rangle. \]
where \(G_0 = (E_2 - H_0 + i0^+)^{-1}\) is the non-interacting Green’s function at the two-body energy \(E_2 = 2\xi_{s+} + E_b\), where \(E_b\) is the two-body binding energy with \(\text{Im}(E_2) = \text{Im}(E_b)\). Since \(U_{\text{int}}\) only acts on the spin-singlet state \(|S = 0\rangle = \frac{|s\uparrow\downarrow\rangle - |s\downarrow\uparrow\rangle}{\sqrt{2}}\), we arrive at the following equation for the two-body state
\[ \frac{1}{U} = \langle S = 0|G_0(0,0)|S = 0\rangle, \]
where the Green’s function can be expanded as
\[ G_0(r, r') = \frac{1}{2} \sum_{k, \mu, \nu} \langle k_{\mu}^L | k_{\mu}^R \rangle \langle -k_{\mu}^L | k_{\mu}^R \rangle (E_2 - \xi_{k\mu} - \xi_{-k\mu}). \]

Different from the Hermitian case, Eq. (8) leads to two coupled equations corresponding to the real and imaginary parts of the equation, which can be solved for the complex binding energy \(E_b\).

Fig. 3 shows typical results of the molecular solution. To demonstrate the effect of dissipation on molecule formation, in Fig. 3(a), we examine the critical coupling \((1/a_s)_c\) to support a two-body bound state as a function of \(\Gamma\), which is obtained by setting \(\text{Re}(E_b) = 0\) in Eq. (8). We can see that apart from a narrow region at very small \(\Gamma\), \((1/a_s)_c\) is greatly reduced with increasing \(\Gamma\), even to negative values when \(\Gamma\) is beyond a certain value [see inset of Fig. 3(a)]. Therefore, molecules can form on the Bardeen-Cooper-Schrieffer side (BCS-side) of the Feshbach resonance, in contrast to the case under one-dimensional Hermitian SOC, where molecules only survive on the Bose-Einstein condensate side (BEC-side) with positive \(a_s\) \([22-24]\). The presence of dissipation-facilitated molecules can be further confirmed in Fig. 3(c). For instance, for a fixed \(1/a_s = 0.5\) (or 0.2), a new molecular branch emerges at \(\Gamma \geq 0.82\) (or \(\geq 1.35\)); for a stronger \(1/a_s = 1\), the molecule binding energy is enhanced by nearly an order of magnitude (from \(-0.29\) to \(-2.24\)) when increasing \(\Gamma\) from 0 to 2. At sufficiently large \(\Gamma\), molecular energy saturates at \(\text{Re}(E_b) = -1/(ma_s^2)\), as quantum Zeno effects become dominant and suppress SOC.

The mechanism of dissipation-facilitated molecules is closely related to the single-particle physics discussed previously, mainly in two aspects: the enhanced low-energy coupling constant [Fig. 2(d)] and the reduced energy gap \(G\) [Fig. 2(e)] under dissipation. These factors make the pairwise scattering of fermions much easier in the low-energy subspace, giving rise to enhanced molecule formation. Moreover, such enhancement is most dramatic at intermediate dissipation, where \(\Gamma\) is neither too small nor too large. This, again, is consistent with the non-monotonic behavior we show in Fig. 2.

Here we emphasize that the type of Raman detuning (red- or blue-detuned), which determines the sign of \(\Omega_0\), does not alter the results of \((1/a_s)_c\) and \(\text{Re}(E_b)\), but it
Re\((E_b)\) as shown in Fig. 3(b)(d). For both red- and blue-detuned Raman lasers, we have \(\text{Im}(E_b) < 0\), suggesting that the molecules are all dissipative and have finite life time. In the following, we discuss how to detect these molecules in experiment.

**Detection:** We now turn to the experimental detection of dissipation-facilitated molecules using r.f. spectroscopy. Consider a third hyperfine state \(|1\rangle\), which has no interaction or SOC with \(|\uparrow\rangle\) and \(|\downarrow\rangle\), but can be coupled to \(|\downarrow\rangle\) by a r.f. field with the Hamiltonian

\[
H_{rf} = \Omega_{rf} \int dx \left( e^{-i\omega t + iqx} \hat{\psi}_\uparrow(x,t) \hat{\psi}_\downarrow(x,t) + H.c. \right). \tag{10}
\]

We focus on two different types of commonly used r.f. measurements in cold atoms: the direct r.f. spectroscopy, where the r.f. field breaks down molecules and transfers atoms from the \(|\downarrow\rangle\) state to an empty \(|1\rangle\) state; and the inverse r.f. spectroscopy, where atoms are initialized in \(|1\rangle\) and are transferred to \(|\downarrow\rangle\) state by the r.f. field to form the molecular state. Both types of measurements have been successfully implemented in cold Fermi gases under Hermitian SOC [19, 26].

Applying the linear response theory to non-Hermitian systems and by incorporating the effect of molecule states [25], we obtain the transition rate

\[
R_d(\omega) = -\sum_{\lambda=\pm} \text{Im} \left[ \frac{-k^2_k |\psi_k^R| (\Phi^L_\lambda |\psi_k^R| - k^R_k)}{\omega + (E_2 - \xi - k\lambda - \epsilon_{k+qe_e} + i0^+)} \right]
\]

for the direct r.f. process, and

\[
R_i(\omega) = -\sum_{\lambda=\pm} n_F(\epsilon_{k+qe_e}) \times \text{Im} \left[ \frac{-k^2_k |\psi_k^R| (\Phi^L_\lambda |\psi_k^R| - k^R_k)}{\omega - (E_2 - \xi - k\lambda - \epsilon_{k+qe_e} + i0^+)} \right]
\]

for the inverse r.f. process.

In the Hermitian case with \(\Gamma = 0\), Eqs. (11)(12) can be reduced to the familiar Fermi’s golden rule of the corresponding r.f. process in Hermitian SOC systems [25]. The Fermi’s golden rule guarantees a positive-definite transition rate \(R(\omega) > 0\) [26, 27]. However, for a non-Hermitian system with a finite \(\Gamma\), due to the complex eigen-energies and eigenstates involved in Eqs. (11)(12), the Fermi’s golden rule breaks down in general. A crucial question is whether the transfer rate can still be positive to allow the experimental detection of molecules?

By noting that the molecules undergo a dissociation (association) process in the direct (inverse) r.f. spectroscopy, we recognize that important insights can be gained by comparing the lifetime of the molecule \(\tau_m\) (as inferred from its imaginary energy) and that of unbound fermions \(\tau_f\). Specifically, there are two scenarios:

(i) In the direct r.f. spectroscopy, the molecule is formed before the r.f. excitation. It follows that, to ensure a positive transition rate, the molecule should be relatively more stable compared to unbound fermions after the r.f. excitation, i.e., \(\tau_m > \tau_f\). Mathematically, this is equivalent to requiring \(-\text{Im}(E_b) < -\text{Im}(\xi_k\lambda)\) [28], such that the denominator in the summand of Eq. (11) has a positive imaginary part.

(ii) In the inverse r.f. spectroscopy, the molecule is formed after the r.f. excitation, such that the condition \(\tau_m < \tau_f\) is required to guarantee a positive transition rate. This is equivalent to \(-\text{Im}(E_b) > -\text{Im}(\xi_k\lambda)\), such that the denominator in the summand of Eq. (12) again has a positive imaginary part.

In the present case, the comparison of \(\text{Im}(E_b)\) and \(\text{Im}(\xi_k\lambda)\) suggests that the requirement in (i) can hardly be satisfied (for either red- or blue-detuned Raman lasers), whereas that in (ii) can be met over a considerable parameter range, under the condition that the Raman laser is blue-detuned. Indeed, the full numerical calculation of \(R(\omega)\) from Eqs. (11)(12) confirms the feasibility of achieving a positive transition rate using the inverse r.f. spectroscopy with blue-detuned Raman lasers. This is demonstrated in Figs. 4(a1)(a2), where \(R_i\) stays positive over a large frequency range, especially around the peak position \(\tilde{\omega}_p\). Note that we have used a shifted frequency \(\tilde{\omega} = \omega - 2\xi_{th} + \xi_q\) in Fig. 4, to better compare the peak location \(\tilde{\omega}_p\) of the r.f. spectrum against the real binding energy \(\text{Re}(E_b)\).

In Fig. 4, we also find that the evolution of \(R_i(\tilde{\omega})\) with different \(\Gamma\) can be classified into two regimes. When \(\Gamma\) is small [Fig. 4(a1)], increasing \(\Gamma\) leads to a broadening of

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**FIG. 4.** (Color online) Detection of molecules in the inverse r.f. spectroscopy under blue-detuned Raman lasers. (a1)(a2) Transition rate \(R_i\) at different \(\Gamma\). We use the shifted frequency \(\tilde{\omega}\) (see main text for definition) as x-axis, where the locations of \(\text{Re}(E_b)\) for each case are marked with arrows of the same color. (b)(c) The peak position \(\tilde{\omega}_p\) and the FWHM of r.f. spectrum as functions of \(\Gamma\). In (b) we also show \(\text{Re}(E_b)\) to guide the eye. In all plots we take \(\Omega_0 = 2\) and \(1/a_s = 1\).
enhanced by dissipation. On the other hand, when $\tilde{\Gamma}$ is sufficiently large [Fig. 4(a2)], further increase in $\tilde{\Gamma}$ leads to a narrower peak width in $R_i$, with $\tilde{\omega}_p$ moving to the right (less negative). This is the regime where the molecule is less affected by non-Hermitian SOC due to the quantum Zeno effect. In Figs. 4(b)(c), we extract the peak position $\tilde{\omega}_p$ and the full width at half magnitude (FWHM) of the r.f. spectrum, both of which show non-monotonic evolutions with increasing $\tilde{\Gamma}$. In particular, we confirm that the evolution of $\tilde{\omega}_p$ indeed follows the same trend as in $\text{Re}(E_b)$ [Fig. 4(b)]. We therefore conclude that the r.f. spectrum is experimentally detectable and captures the key properties of dissipation-facilitated molecules.

Summary and outlook:— In summary, we have proposed a realistic experimental scheme to explore the interplay of interaction and non-Hermitian SOC. We demonstrate the existence of dissipation-facilitated molecules, and show that they can be detected using inverse r.f. spectroscopy. Our results serve as a guide for the experimental observation of non-trivial effects of non-Hermiticity in interacting Fermi gases, which has so far eluded experimental efforts. Future studies would include the dissipation-induced Fermi superfluidity in the corresponding many-body systems, as well as other exotic few- and many-body quantum states where non-Hermiticity plays a key role.

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Supplemental Materials

In this Supplemental Materials, we provide more details on the analysis of non-monotonicity in single-particle properties, the derivation of transition rate in r.f. measurements, as well as on the breakdown of conventional Fermi’s golden rule under non-Hermiticity.

Perturbative expansion

The non-monotonic behavior of $k_{\text{min}}$ and $C$ can be understood through perturbation theory in either the small- or large-$\Gamma$ limit.

For $\Gamma \ll 1$, the dissipation can be treated as perturbation. Up to the lowest order, we have $\Omega = \Omega_0(1 + i\Gamma)$, whose imaginary part, $\text{Im}(\Omega) \propto \Gamma$, leads to the expressions $k_{\text{min}} = q\sqrt{1 - s^2(1 - 3\Gamma^2)}$ and $C = 1 - s^2(1 - 3\Gamma^2)$, with $s := \Omega_0/(2E_q) \ll 1$. Thus, both $k_{\text{min}}$ and $\omega_{\text{min}}$ increase quadratically with $\Gamma$. In the opposite limit $\Gamma \gg 1$, however, the SOC strength decays as $\Omega \sim \Omega_0/\Gamma$, and the $\Omega$ term in (3) behaves just like an imaginary magnetic field. In this limit, we treat $\Omega$ as perturbation, and get $k_{\text{min}} = q\sqrt{1 + s^2/\Gamma^2}$ and $C = 1 + s^2/\Gamma^2$, both decreasing with larger $\Gamma$.

The distinct trends of $k_{\text{min}}$ and $C$ in different limits of $\Gamma$ explain the non-monotonicity shown in Fig. 2 of the main text. We also note that behavior in the large $\Gamma$ limit can be understood through the quantum Zeno effects, which effectively suppress the off-diagonal non-Hermitian SOC term.

Transition rate in radio-frequency spectroscopy

Within the linear response theory, the transition rate, which serves as the experimental signal, can be related to the spin-flip correlation function in the frequency space

$$D(i\omega) = \frac{1}{\beta} \sum_k \sum_n G_{\lambda\lambda}(k, i\omega_n)G_{11}(k + qe_x, i\omega_n \pm i\omega),$$

(S1)

where the sign $(\pm)$ corresponds to the direct (inverse) r.f. process, $\omega$ and $\omega_n$ are both fermionic Matsubara frequencies, $G_{11}(k, i\omega_n) = (i\omega_n - \epsilon_k)^{-1}$ is the Green’s function for state $|1\rangle$, and $G_{\lambda\lambda}$ is the Green’s function for state $|\lambda\rangle$, which, in view of the molecular state, can be written as

$$G_{\lambda\lambda}(k, i\omega_n) = \sum_{\lambda=\pm} \frac{\langle -k_{\lambda}^R | \bar{\psi}_{k_{\lambda}} | \Phi^R \rangle \langle \Phi^L | \bar{\psi}_{k_{\lambda}}^\dagger | -k_{\lambda}^L \rangle}{i\omega_n - (E_2 - \xi_{-k\lambda})}.$$  

(S2)

Here $|\Phi^R\rangle$ ($|\Phi^L\rangle$) is the right (left) eigenvector for the molecular state with eigenenergy $E_2$ ($E_2^*$). The transition rate is then given by $R(\omega) = -\text{Im}D(i\omega \rightarrow \omega + i0^+)$, which can be simplified to Eq.11 for the direct r.f. process and to Eq.12 for the inverse r.f. process.

Fermi’s golden rule modified by non-Hermiticity

In the Hermitian limit with $\Gamma = 0$, Eqs. (11)(12) are reduced to the familiar Fermi’s golden rule of the corresponding r.f. process in spin-orbit-coupled Hermitian systems, with experimentally detectable positive transfer rate $R(\omega) > 0$. Specifically, in the Hermitian limit, $|k_{\lambda}^R\rangle = |k_{\lambda}^L\rangle \equiv |k_{\lambda}\rangle$, $|\Phi^R\rangle = |\Phi^L\rangle \equiv |\Phi\rangle$, and all energies are real. It follows that Eqs. (11)(12) exactly reproduce the formula of Fermi’s golden rule under according r.f. process. For instance, the direct r.f. spectrum is reduced from Eq. (11) to

$$R_d(\omega) = \pi \sum_{\lambda=\pm} |\langle -k_{\lambda}^R | \bar{\psi}_{k_{\lambda}} | \Phi \rangle|^2 \delta(\omega + E_2 - \xi_{-k\lambda} - \epsilon_{k + qe_x}).$$

(S3)

The Fermi’s golden rule results in a positive-definite transition rate. However, for the non-Hermitian case here, both the numerator and denominator in the summand of Eqs. (11)(12) can be complex. The conventional Fermi’s golden rule breaks down, and the resulting $R(\omega)$ is significantly modified and not positive-definite anymore.