Time-singularity Power Spectral Distribution and Its Application in the Analysis of Radar Sea Clutter

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Abstract. For multifractal time series, singularity power spectrum (SPS) provides the power distribution of fractal subsets with respect to singularity exponent, however fails to trace the time-varying singularity characteristics. Aiming at this shortcoming, this paper has two main objectives. First, it develops SPS method in time-singularity 2D plane, and proposes a novel time-singularity power spectrum distribution (TSPSD) for both deterministic as well as non-stationary random series, thereby emphasizing an intuitive approach to trace the local SPS distribution, derived by the inner dynamics mechanism. Second, it deduces the implementation of TSPSD, based on the self-correlation and SPS algorithm, and particular attention is paid to application of TSPSD in the analysis of radar sea clutter which combines the attractive features of non-stationary and multifractal self-similar processes. Statistical properties as well as tracking capability of TSPSD for radar sea clutter are addressed.

1. Introduction
Fractal and multifractal signal processing are hot topics in current studies [1-2]. At present, the fractal theory mainly focuses in fractal dimension [3], multifractal spectrum (MFS, or singularity spectrum) [4-7] based on differentiability of singular subsets [8-11], which fails to reflect the energy and power measurement of fractal singular subset. Based on power spectrum density function and singularity spectrum, the singularity power spectrum (SPS) [12] is put forward to estimate the power spectrum of fractal subsets with specific singularity exponent. The SPS in the singularity domain is proved to be analogous to power spectral density (PSD) in the Fourier domain, which provides a novel approach of multifractal signal reconstruction based on singularity spectrum and SPS [13]. Further, SPS is generalized into fractional-order SPS [14] to extract the fractional domain singularity spectrum, especially for linear frequency modulation coding signal, commonly used in modern radar system.

In the same time, the fractal theory is widely applied in signal modelling and processing of radar sea clutter. The singularity spectrum analysis of sea clutter is the key technology of the detecting radar for sea target [15], which discovers the dynamics mechanism of the sea surface theoretically. Further, the SPS is also applied in the analysis of sea clutter. The research on radar sea clutter indicates that two sea clutter data can display distinguished singularity power spectrum, in some situation when traditional PSD and MFS fail to distinguish them. Thus, SPS promises alternative characteristic and perhaps new solution for analysis and processing of sea clutter, and further for the target detection and classification in the background of sea clutter.
However, SPS neglects the time information due to the overall statistical analysis and singularity power computation on the fractal subsets. In this paper, a novel time-singularity power spectral distribution (TSPSD) method is proposed to describe the time-varying singularity power spectral (SPS) of multifractal signal, i.e., the power distribution in the time-singularity 2-dimension plane, and thus TSPSD may realize the trace of local SPS distribution, derived by the inner dynamics mechanism.

The following parts are as follows. In section II, we put forward the time-singularity power spectral distribution based on SPS and study the algorithm of TSPSD. In section III, we analyse the radar sea clutter based on TSPSD, compared with the PSD and SPS of sea clutter. In section IV, we conclude and give research prospect.

2. Time-Singularity Power Spectral (TSPSD) Theory and Algorithm

2.1. Singularity Power Spectrum Distribution (SPS)

Let \( X \) be a set of signal, and \( x(t) \in X, 0 \leq t \leq T \) be the continuous signal. The singularity power spectrum of \( x(t) \) can be defined as \([12]\)

\[
P_s(\alpha) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} \left( \frac{|x^\alpha(t)|^2}{T^2} \right) dH(x^\alpha(t)),
\]

where \( x^\alpha(t) \) is the fractal sub-band signal, \( \theta \) is local direction angle [2] of truncation signal \( x_T(t) \), \( dH(x^\alpha(t)) \) denotes differential of Hausdorff measurement [7] of \( x_T(t) \), and \( T(x_T(t)) = \int_{-T/2}^{T/2} \cos \theta dH(x_T(t)) \). Eq. (1) represents the power distribution of fractal subsets with respect to singularity exponent. Based on SPS, a new method of multifractal signal reconstruction was

The SPS method can transform the signal from time domain to the singularity domain, however it fails to contain the time information, in the next section, we will introduce the time dimensional information and transform the time-domain signal into the time-singularity 2-dimension representation and propose the definition of time-singularity power spectral distribution (TSPSD).

2.2. Time-singularity Power Spectrum Distribution (TSPSD)

Referring to the Cohen’s time-frequency analysis, the time-delayed conjugation of fractal signal is chosen as the weigh function, and the self-correlation function is

\[
r_s(t, \tau) = E[x(t + \tau/2)x(t - \tau/2)]
\]

A function \( x(t) \) is said to be in \( C^\alpha \) if there is a polynomial \( P(\alpha) \) satisfying

\[
|r_s(t, u) - P_r(\alpha)| \leq C |u - \tau|^\alpha
\]

for \( u \) sufficiently close to \( \tau \). Then, the local Holder regular degree is \( H(t, \tau) := \sup \{ h : r_s(t, \tau) \in C^h \} \).

For \( r_s(t, \tau) \) and \( H(t, \tau) \), we can calculate the time-varying SPS of \( x(t) \) to trace the power distribution both along with time shaft and singularity axis, i.e.,

\[
P_s(t, \alpha) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} \left( \frac{|r^\alpha_s(t, \tau)|^2}{T^2} \right) dH(r^\alpha_s(t, d\tau))
\]

where \( r^\alpha_s(t, \tau) \) is the fractal sub-band signal of the truncation signal of \( r_s(t, \tau) \), \( \theta_{(\alpha, \tau)} \) is local direction angle of \( r_s(t, \tau) \), \( dH(x^\alpha(r^\alpha_s(t, \tau))) \) denotes differential of Hausdorff measurement of \( r_s(t, \tau) \), and \( T(r_s(t)) = \int_{-T/2}^{T/2} \cos \theta_{(\alpha, \tau)} dH(r_s(t, d\tau)) \).

2.3. Algorithm of TSPSD

Suppose that \( x_n, n = 0, 1, 2, ..., N-1 \) is a one-dimensional series of length \( N \). Define the instantaneous cyclic autocorrelation function of \( x(n) \) as

\[
r_s(k) = x(k)x^*(n+k); n, k = 0, 1, 2, ..., N-1.
\]

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where define $x(k) = \begin{cases} x(k) & k \in [0, N-1] \\ x(k-N), k \in [N, 2N-2] \end{cases}$ to eliminate edge effect, and $k$ is the delayed sample.

The following steps focus on the instantaneous singularity power spectrum based on SPS algorithm at each time $n$. For discrete autocorrelation function $r_n(k)$, the estimation of SPS can refer to the traditional PSD, where the summation of modulus square of the value of the sampling points is described as energy of discrete signal. For $r_n(k)$, calculate $\alpha_n(k)$ of discrete point, and partition singularity subset $r_{\alpha_n}(k, n) = \{(n, r_n(k,n))\}$. Then the energy of fractal subsets can be calculated as

$$P(k, \alpha) \approx \frac{1}{N} \sum_{n} |r_n(k)|^2$$

To obtain the uniform sample of $\alpha$, calculate the singularity exponent $\alpha_n(k)$, assume $\alpha_n(k) \in [\alpha_{\min}, \alpha_{\max}]$, partition the singularity interval uniformly, and the obtained singularity exponent vector is

$$\alpha(m) = [\alpha_{\min}, \alpha_0, \ldots, \alpha_{m-2}, \alpha_{m-1} = \alpha_{\max}] \quad (6)$$

For $\alpha(m)$, assume $\alpha(m) \leq \alpha_n(k) < \alpha(m+1)$, define the discrete fractal sub-band signal or discrete singularity subset as

$$r_{\alpha_n}(k, n) = \{(n, r_n(k)), \alpha_n(k) \in [\alpha(m), \alpha(m+1)]\}. \quad (7)$$

Thus, the TSPS of discrete signal can be written as

$$P(n, \alpha_n) = \sum_k |r_{\alpha_n}(k, n)| / N_{n,\alpha} \quad (8)$$

where $N_{n,\alpha} = \#\{r_{\alpha_n}(k, n)\}$ represents the elemental number of fractal sub-band signal $r_n(k)$. The algorithm flowchart of TSPS is as figure 1.

3. Simulation And Discussion

We analyze the radar sea clutter based on the time-singularity multifractal power spectrum method. The sea surface can be considered as a kind of multifractal, and the angle distribution of the scattering energy contains the spatial fractal character of the multifractal surface after the reciprocity between electromagnetic wave and the fractal surface. There is lots of evidence of the complex multifractal characteristics of the radar ocean clutter. The actual data were obtained from the radio ocean radar.

3.1. The data description of radar sea clutter

The real data of sea radar derived from the lab of anti-jamming of radar, gathered from the sea radar on mobile platform, were analyzed. The detection system is pseudo-random phase modulated CW radar, and the Sea state grade was III level. The height of detection system was above the sea about 10~25m, carrier frequency of detection system was 142MHZ. The echo signal of pseudo-random code phase-modulation signal was received and demodulated by the frequency demodulation and the amplitude demodulation, and the demodulated signal includes amplitude modulated channel and
frequency modulated channel. The demodulated data in frequency channel is sampled with 50 KHz, and two sets of ocean clutter data can be referred to figure 6 in [12].

Figure 2. The PSD and SPS analysis of Radar ocean clutter. (a) PSD analysis; (b) SPS analysis.

Figure 3. The contour of TSPSD of real radar ocean clutter (a) for dataset 1; (b) for dataset 2

Figure 4. The TSPSD of real sea clutter at certain time point n=50, n=150, and n=400; (a) for dataset 1; (b) for dataset 2.

Figure 2(a) shows power spectrum density of radar ocean clutter and figure 2(b) shows simulation comparison of their singularity power spectrum (SPS). It can be seen that SPS distribution gives power distribution of radar ocean clutter along with singularity exponent. Besides, there exist visible differences between the SPS of two groups ocean clutter, and the SPS of data 2 has a wider interval of singularity exponent than data 1. Obviously, the SPS distribution provides more fractal characteristics than the traditional power spectrum distribution. [14]

Figure 3 show simulation results of TSPSD of sea clutter, and figure 4 display the instantaneous singularity power spectrum at n = 50, 150, and 400. From figure 4, we can find that for dataset 1, at n = 50 the singularity interval of SPS lies in [0.3, 0.78], and the power peak appears at \( \alpha = 0.36 \), while at n = 150 the singularity interval of SPS lies in [0.35, 0.85], and the peak appears in the interval [0.4, 0.43]. For dataset 2, there also exit great difference in the peak and singularity interval of TSPSD, and especially, at singularity \( \alpha = 0.75 \), the power of \( P_2 (400,0.75) \) reaches the peak and is more than -10dB, while \( P_2 (50,0.75), P_2 (150,0.75) \) are less than -30dB.
3.2. The IPIX Radar Datasets
This section adopts the TSPSD to deal with the IPIX radar Datasets. The measurement was made using the McMaster IPIX radar at the east coast of Canada. In this section, we adopt the #54(19931111_163625_starea.cdf) data file, which contain data sets of 14 range bins, each range bin contains 131072 data. The detailed IPIX radar parameters can be seen on the website http://soma.ece.mcmaster.ca. The range bin 8 where the target is the strongest is labeled as primary target bin and the neighboring range bins 7,9,10 where the target may also be visible labeled as secondary target bins.

Figure 5. The TSPSD of real IPIX radar dataset with 1024 sampling points (a) for pure sea clutter range bin 2; (b) primary target bin 8.

Figure 6. The contour of TSPSD of real IPIX radar dataset (a) for bin 2 (pure sea clutter range bin); (b) for bin 8 (primary target bin)

Figure 5 shows the TSPSD of real IPIX radar dataset with and without target, and Figure 6 presents the contour of figure 5. From figure 5, we can see that the TSPSD of pure sea clutter distributes in the range of $\alpha \in [0,0.45]$ with center singularity exponent $\alpha_c = 0.3$, while primary target bin in $\alpha \in [0.1,0.95]$ with $\alpha_c = 0.5$. The total power of primary target bin is larger than that of pure sea clutter. From figure 6, we can find that the TSPSD of both target bin and the sea clutter bins fluctuate along with the varying of time points, and the fluctuation of the latter is more violent than the former. We infer that the fluctuation of TSPSD of primary target bin is mainly caused by the interaction between the target and sea surface, and also influenced by the radar system and signal waveform. The underlying mechanism related to the fluctuation of TSPSD remains to further study and explain. Even so, above parameters, such as range and center of singularity exponents and the singularity power spectrum can be used to detect the radar target within the sea clutter.

According to the simulation analysis, we can deduce that (1) The TSPSD gives power distribution of sea clutter with respect to the time axis and the singularity exponent, i.e. the time-varying fractal power of instantaneous self-correlation signal. (2) There are violent fluctuations in the TSPSD of radar ocean clutter, which indicates that the TSPSD of radar ocean clutter is time variant and non-stationary in the singularity domain. It conforms to dynamics characteristic of real sea clutter. (3) There exist remarkable differences between the TSPSD of sea clutter with and without target, reflected in the fluctuation and statistical parameter of TSPSD.
4. Conclusion
In this paper, we proposed the conception and the mathematical expression of time singularity power spectrum distribution (TSPSD) and apply it to analysis of radar sea clutter. The singularity power spectrum of multifractal signal in the time-singularity 2-dimension plane provides richer information about power distribution. Also, we studied the approximate algorithm of TSPSD, based on algorithm of SPS of discrete signal. Simulation indicates that TSPSD can display more singular detail features, and be expected to provide richer characteristics for signal detection than SPS and MFS method, in background of sea clutters. Further study will focus on TSPSD of LFM radar echo signal and the possible application of TSPSD in the radar target detection and recognition.

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