A Mathematical Study on “Additive Technique” Versus “Branch and Bound Technique” for Solving Binary Programming Problem

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Abstract. A solid body needs adequate supplements from nourishment that we eat each day. Eating pretty much than what our body needs will prompt lack of healthy sustenance (under-nourishment and over-nourishment). In Malaysia, a few reviews have been directed to examine the wholesome status of Malaysians, particularly among youngsters and youths. However there are different methods for taking care of the menu arranging issue and in this paper Binary Programming (BP) is executed. Separately, ”Additive Technique (AT)” and “Branch and Bound Technique (BBT)” are utilized as a part of BP. Both methodologies utilize diverse systems and might yield distinctive ideal arrangements. Along these lines, this study expects to build up a scientific model for eating regimen arranging that meets the essential supplement admission and look at the outcomes yield through additive substance and branch and bound methodologies. The information was gathered from different all inclusive schools and furthermore from the Ministry of Education. The model was illuminated by utilizing the Balas Algorithm through AT and Binary Programming through BBT.

1. Introduction

Schools and institutions give meals over an augmented day and period with a confined spending plan. Research on this issue is advancing keeping in mind the end goal to discover nutritious meals inside the imperatives of the cost of the sustenance [18, 21]. The primary motivation behind the "diet problem" was studied by Stigler in 1945 is to identify issues with human sustenance [6, 17, 18, 21, 24]. This model, as in most operational research models, has been set up on the customary principal suspicion that the leader tries to enhance the traditional
approach and assumption. The issue has kept on being examined by researchers and nutritionists [1, 2, 3, 4, 5, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24]. Hence in this paper, we extended the present information in menu arranging and dieting issues concentrating on Malaysian formulas. We utilized an improvement optimization way to deal with the issue and a model was developed for the utilization of the Ministry of Education, Malaysia. In this paper we used Binary Programming to decide the most nutritious and tasteful meals, while considering the imperatives of the RDA for Malaysian school children 13 to 18 years of age, the cost of the menu, the monetary allowance given by the administration (government) and an assortment necessity. Matlab with the LPSolve programming languages was utilized to take tackle the issue.

2. Branch and Bound Technique
Branch and bound technique (BBT) is an algorithm design for solving binary problem (BP) (e.g 0-1, true-false, yes-no), integer programming problem (IP) and mix integer problem (MIP) or discrete problem (e.g 0.5, 1, 2.5). BBT is commonly used for solving IP where it involves solving multiples LP relaxation by using SIM method and round it up to integer values as a usage in solving the BBT problem. Therefore, the IP problem is much more difficult compared to Linear Programming (LP) problem. A BBT algorithm consists of an organized list or enumeration of possible solutions by means of state space search; the set of candidate solutions is thought of as forming a rooted tree with the full set at the root [8, 9, 20]. The step involves exploring the branches of the Tree Algorithm. This represents the subsets of the solution. Before listing the possible solution which gained from the branch, the solution is checked against the boundaries (upper and lower bound) of the optimal solution. The step is stop when there is no more better solution can be found than the one found so far. Overall the basic idea of BBT is to partition the feasible region into more manageable subdivisions and then to further partition of subdivisions [21, 25, 26, 27].

3. Additive Technique
The Additive Technique (AT) was introduced by Egon Balas in 1965. It is known as the “Additive Algorithm” and proposed to solve a linear programming problem where the variable can only take the value of 0 or 1. AT is a “hand manual” calculation technique and can deal with very limited variables. The process starts by setting all the variables as 0 and uses the BBT technique to find the solution without relying on the linear programming to find the upper bound. The BA does not try to complete the solution, but it tries to search for the cheapest and most feasible solution. Before solving the problem, the standard mathematical form can be written as follows;

(i) the objective function is to

\[
\text{Minimize } Z = \sum_{j=1}^{n} c_j x_j
\]

where it needs to be in the form of a minimization.

(ii) all the \(k\) constraints must be in the form of “\(\leq\)”

\[
\sum_{j=1}^{n} a_{ij} x_j \leq b_i \text{ for all } i=1, 2, \ldots, k
\]
If some of the constraints are in the form “≥”, it must be transformed to
\[
\sum_{j=1}^{n} -a_j X_j \leq -b_i
\]  
(3)
all the constraints in the form “=” must be changed into the following form
\[
\sum_{j=1}^{n} a_j X_j \leq b_i
\]  
(4)
(iii) all the variables \( x_j \) where \( j = 1, 2, 3..n \) must be in the binary unit (0 or 1)

(iv) all the coefficients \( c_j \) in the objective function must be non-negative (positive coefficient). If \( c_j < 0 \) then replace \( x_j \) with \( 1-x_j \). From this transformation, the constant value in the objective function is ignored during the optimization process but will be added back once the final solution is found. For the constant value in the left-hand side (LHS) of the constraint, it must be moved to the right-hand side (RHS) of the constraints.

The main idea of this algorithm is to set all the variables to zero as the objective function has been transformed to a minimization problem. Then, the process continues by assigning each variable at a time. After assigning all the two-possible combinations, two possible outcomes can be determined; (a) an optimal solution; or (b) evidence that there no feasibility could be gained. As explained earlier, the AT is a hand calculation technique with very limited variables; 40 variables to be precise [2, 6, 17]. Therefore, the conversion of hand calculations to computerized calculations will help calculate larger variables [17].

4. Data Collection
There are a few sorts of information expected to construct a menu arranging model. These incorporate the institutionalized cost of every Malaysian menu, the dietary substance for every menu, suggested wholesome day by Recommended Daily Allowance (RDA) which incorporate with upper bound (UB) and lower bound (LB) of every supplement and nutrient for Malaysian boarding school children and the government spending plan for food providers. The monetary allowance is Malaysian Ringgit (RM)15.00 per head each day. There are 11 supplements considered; Vitamins (A, B1, B2 & C), Calcium, Energy, Niacin, Protein, Carbohydrate, Iron and Fat as shown in Table 1. Moreover, 10 sorts of nourishment will be considered in this study; Cereal Based Meal (CBM), Rice Flour Based (RFB), Cereal Flour Based (CFB), Wheat Flour Based (WFB), Seafood and Fish (SF), Meat (MT), Fruit (FT), Vegetable (VT), Beverage (BV) and Miscellaneous (MS) as shown in Table 2. There are 100 of nourishment and beverages to be considered. In light of the information, a binary programming model is created and discussed. In this manner we have 100 variables \((x_1, \ldots, x_{100})\). Each sort of sustenance has its own particular accessible scope of choice as exhibited in Table 4.2. We require 18 dishes from 10 sorts of nourishment for every day.
Table 1. UB and LB of the 11 supplements.

| LB     | Nutrients | UB     |
|--------|-----------|--------|
| 600mg  | Vitamin A | 2800mg |
| 1.1mg  | Vitamin B1| -      |
| 1mg    | Vitamin B2| -      |
| 65mg   | Vitamin C | 1800mg |
| 1000g  | Calcium   | 2500g  |
| 2050kcal| Energy    | 2840kcal|
| 16mg   | Niacin    | 30mg   |
| 54g    | Protein   | -      |
| 180g   | Carbo     | 330g   |
| 15mg   | Iron      | 45mg   |
| 46g    | Fat       | 86g    |

Table 2. Nourishment requirement each day.

| Type of nourishment | Requirement everyday (k) |
|---------------------|--------------------------|
| CBM                 | 2                        |
| RFB                 | 1                        |
| CFB                 | 1                        |
| WFB                 | 1                        |
| SF                  | 1                        |
| MT                  | 1                        |
| FR                  | 2                        |
| VG                  | 2                        |
| BV                  | 6                        |
| MS                  | 1                        |
| **Total Dishes Per Day** | **18**                  |

5. Model Formulation
The primary point of this exploration study is to define a menu arranging model that minimize the budget given by the government to the school cooks, maximizes the variety of food and nutritious necessity relying on the Malaysian RDA prerequisites. Consequently in one day we require 18 dishes that will be reasonably chosen from the 100 dishes that are accessible. In the objective function, we minimize the aggregate cost $Z$, 

Z = \sum_{i=1}^{100} \text{Cost}(x_i) = \sum_{i=1}^{100} w_i x_i \quad (5)

by choosing the dish and giving an acceptable day by day menu. The maximum spending budget gave each day by the government is RM15.00. Along these lines we attempt to limit the cost. The day by day imperatives are,

\[ \text{LB} \leq \sum_{i=1}^{100} \text{Supplements}(x_i) \leq \text{UB} \quad (6) \]

where \( i=1,2,\ldots,11 \), LB and UB is the vector and give an alternate an incentive for every supplement. This is to guarantee that we meet the supplements prerequisites. We have 11 limitations of supplements with lower and upper bound esteems aside from protein, vitamin B1 and B2 as expressed in Table 1. In light of Table 2 we determine the 10 nourishment prerequisites as,

\[ \sum_{i=1}^{10} \text{Type of nourishment} (x_i) = k; \quad (7) \]

where \( i=1,2,\ldots,10 \) with the goal that we can serve 18 dishes for each day. Each of the 100 factors are in binary,

\[ x_i = \{0, 1\} \quad (8) \]

6. Result
The cost is optimal when the lowest optimal value in the objective function is given. It also fulfills all the restrictions and constraints set in the problem. Referring to Table 6.1, there are various types of drinks and foods presented in a 1 day menu. Some of the foods are the same and some are slightly different. However, the major problem here is that the final costs generated by both models are different. The AT gives RM 7.40 while the BBT gives RM 7.30, which is 10 cents cheaper than the AT. However, both results provide a feasible solution. Table 3 shows the nutrient intake for a 1 day menu. The table shows the comparison between the lower and the upper values and the nutrient values that are generated by the programs.

Based on Table 4, each method gives a different nutrient intake but both methods fulfill the nutrient and food group requirements. It shows that the AT picked a higher nutrient intake in certain areas compared to the BBT. The AT generated a higher nutrient intake for energy, fat, niacin, vitamin C, protein, and vitamin B2 compared to the BBT. However, both nutrient intakes generated by AT and BBT are feasible. Therefore, it can be concluded in this case that the BBT provide an optimal and feasible solution, and the AT also gives a near optimal solution.
### Table 3. Extended BP results for a 1 day menu. Cost: RM 7.30.

| Amount | AT         | Food Groups | BBT                              | Amount |
|--------|------------|-------------|----------------------------------|--------|
| 1      | Milk, cow, fresh | Beverages   | Coffee powder, instant Orange flavoured drink, powder | 1      |
| 1      | Orange flavoured drink, powder |            | Plain water                        | 1      |
| 2      | Plain water   |            | Milk, UHT, low-fat, recombined Syrup rose | 1      |
| 1      | Milk, UHT, low-fat, recombined Syrup rose | Beverages | Coffee powder, instant Orange flavoured drink, powder | 1      |
| 1      | Cookies, peanut [B] | Cereal Flour Based | Cookies, peanut | 1      |
| 1      | Kuih buah Melaka [M] | Rice Flour Based | Kuih buah Melaka | 1      |
| 1      | Rice, fried [D] | Cereal Meal Based | Rice, fried | 1      |
| 1      | Rice, cooked [L] | Cereal Meal Based | Rice, cooked | 1      |
| 1      | Beef, fried [D] | Meat Dishes | Beef, fried | 1      |
| 1      | Asam gelugor, pucuk [L] | Vegetables | Bean, string Fern shoots | 1      |
| 1      | Cabbage, common [D] | Vegetables | Bean, string Fern shoots | 1      |
| 1      | Banana [L] | Fruits | Banana | 1      |
| 1      | Guava [D] | Fruits | Guava | 1      |
| 1      | Cucur udang [E] | Wheat Flour Based | Kuih kapit | 1      |
| 1      | Black pomfret, fried [L] | Fish and Seafood Dishes | Shrimp, small, cooked in chilli | 1      |
| 1      | Bingka ubi kayu [S] | Miscellaneous | Pengat keledek, gula merah | 1      |
| 18     | Total Food per day |             | RM7.40 | 18     |
|        | Total Cost |             | RM7.30 |        |
Table 4. Comparison of nutrient intake between the AT and BBT.

|          | Lower Bound | AT    | BBT    | Upper Bound |
|----------|-------------|-------|--------|-------------|
| Energy   | 2050        | 2209  | 2187   | 2840        |
| Carbohydrate | 180        | 300.2 | 328.9  | 330         |
| Fat      | 46          | 67.8  | 64.1   | 86          |
| Calcium  | 1000        | 1003  | 1015   | 2500        |
| Niacin   | 16          | 18.5  | 16.1   | 30          |
| Iron     | 15          | 37.91 | 41.8   | 45          |
| Vitamin A| 600         | 1430  | 1576   | 2800        |
| Vitamin C| 65          | 267.1 | 245.6  | 1800        |
| Protein  | 54          | 90    | 74.3   | -           |
| Vitamin B1| 1.1        | 1.21  | 1.48   | -           |
| Vitamin B2| 1          | 2.67  | 2.09   | -           |
| Price    | 7.40        | 7.30  | -      | -           |

7. Conclusion
The researcher have delivered an appropriate menu arrange that can be utilized as a guide for the administration of the school. The model was tackled utilizing Matlab with LPSolve. AT only focuses on the optimal Z value and is less concerned with the variables. Once it reaches the optimal value, while satisfying all the constraints, it will stop the process. AT would not look for any further improvements on the variables or constraints. Table 3 showed that both methods yield slightly different optimal Z values (by 10 cent) and different optimal selected variables. This is probably because of the repetition of plain water affecting the solution with a wider range of variables. This is to check the performance of both algorithms when a few elements (variables) are changed. The possible explanations that can be made regarding the AT solution are: (i) it involves many variables which make it more complex. The largest problem Balas ever handled consisted of 40 variables and 22 constraints. Byrne & Proll in 1969 [17] dealt with only 33 variables and 25 constraints, and (ii) there is a possibility that manual calculations can give slightly different results when converted to a computerized technique.

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