Prediction of Remaining Discharge Time of Battery

Qiming Feng\textsuperscript{1*}, Suping Qian\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, Wuxi Institute of Commerce, Wuxi, Jiangsu, 214153, China

\textsuperscript{2}School of Accounting and Finance, Wuxi Institute of Commerce, Wuxi, Jiangsu, 214153, China

\textsuperscript{*}Corresponding author’s e-mail: feng_qiming@163.com

Abstract. This paper mainly discusses the prediction of discharge time when lead-acid battery discharge at constant current. For one thing, the functional relationship between the discharge depth and voltage under different currents is established, and the remaining discharge time of the battery is calculated according to the mathematical model, too. Moreover, the average relative error is also calculated. For another, the binary function of voltage $U$ with respect to current intensity $I$ and discharge time $t$ is established, and the MRE under different currents is also discussed. The results show that the first method is better than the second.

1. Introduction

Lead acid battery is widely used in industry, military and daily life. Therefore, the study of battery performance has become one of the focuses of research. Lead-acid battery is the most mature battery at present, and the discharge characteristics of lead-acid battery is the main research content\cite{1}. Lead acid battery belongs to the secondary power supply. The chemical reaction between the positive electrode and the negative electrode is reversible. The active substance of the positive electrode is lead dioxide, in the meantime, the active substance of the negative electrode is spongy metal lead, and the electrolyte is dilute sulfuric acid\cite{2}.

In the process of discharge, lead sulphate is produced in both positive and negative active materials, and sulfuric acid is consumed at the same time, which makes the battery polarize in the discharge process\cite{3}. The effect of polarization on the discharge is that the working voltage of the battery decreases, and the degree of polarization varies with the discharge current. Generally, the larger the current is, the more serious the polarization phenomenon is and the more the working voltage drops\cite{4}. The factors that affect the discharge characteristics of lead-acid battery include temperature, battery structure and manufacturing process. In order to simplify the research, the influence is ignored here.

The electrochemical reaction inside the battery is very complex. Therefore, we try to sum up the regularity by a large number of battery discharge tests under the condition of constant current. Therefore, the theoretical model of the discharge characteristics of lead-acid battery can be established by test method.

In the process of discharge of lead-acid battery under the condition of constant current, the voltage monotonically decreases with the discharge time until the set minimum protection voltage ($U_m$ $9V$, in here). The relationship between the voltage and time is called discharge curve. How long the battery can supply power (that is, the remaining discharge time from current voltage to $U_m$) is a question that must be answered in use. The ratio of the remaining capacity of the battery after a period of use to the...
capacity before use is called the battery state of charge. When a fully charged battery is used or placed for a long time, its state of charge will decay.

2. Solution

2.1 Symbol explanation

- $U$ represents voltage
- $I$ represents current at constant current discharge
- $t$ represents discharge time
- $\tau$ represents the time from continuous discharge with constant current to the set minimum protection voltage
- $f$ represents depth of discharge, that is, $f = \frac{t}{\tau}$
- $C$ represents battery discharge capacity
- $N$ represents constants related to battery type

2.2 Analysis of discharge time

When the same batch of batteries is delivered from the factory, discharge tests are carried out with different current to obtain the complete discharge sampling data (these data are called the discharge sampling data for short) [5]. Here, the primary function is used to express the discharge curves. For the capacity characteristics of lead-acid battery, the famous Peukert equation is used

$$I^n \cdot \tau = C$$

(1)

According to the discharge sampling data, we fit the data to obtain $n$. In the figure below, $x$-axis represents log$I$ and $y$-axis represents log $\tau$.

![Figure 1. Relationship between current and discharge time](image)

It is found that the curve is approximately a straight line with a slope of $n = 1.2168$. According to the Peukert equation (1), the value of $C$ can be obtained as follows:

| $I$ (A) | 20  | 30  | 40  | 50  | 60  | 70  | 80  | 90  | 100 |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $C$    | 2402.1 | 2565 | 2557.2 | 2545.4 | 2536.3 | 2526.2 | 2516.8 | 2467 | 2433.5 |

The discharge curves under various currents are made from the discharge sampling data, and the figures are as follows:
From the graph, we can assume that the function between discharge voltage and discharge depth is

\[ U = af^2 + b \]  

(2)

According to the discharge sampling data, the functions under different current can be fitted. The coefficients \( a \) and \( b \) in the functions are shown in the following table:

| \( I(A) \) | 20  | 30  | 40  | 50  | 60  | 70  | 80  | 90  | 100 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( a \)   | -1.245 | -1.207 | -1.212 | -1.157 | -1.134 | -1.112 | -1.092 | -1.07 | -1.058 |
| \( b \)   | 10.48 | 10.46 | 10.47 | 10.42 | 10.39 | 10.33 | 10.29 | 10.26 | 10.23 |

Corresponding to a certain voltage, the average relative error between the discharged time obtained by the model and the sampled discharged time is \( MRE \). The accuracy of using the discharge curve to predict the residual capacity and discharge time depends on the quality of the discharge curve in the low voltage section. However, the sampling points in the low voltage section are relatively sparse. Based on this fact, the 231 voltage sample points shall be extracted at the maximum interval of not more than 0.005V from \( U_m \). And the \( MRE \) is calculated from these voltage sample points, which is,

\[ MRE = \frac{1}{231} \sum_{i=1}^{231} \frac{|T_i - t_i|}{t_i} \]  

(3)

For any given current, the corresponding estimated voltage can be calculated, and the \( MRE \) is as follows:

| \( I(A) \) | 20  | 30  | 40  | 50  | 60  | 70  | 80  | 90  | 100 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( MRE \) (\%) | 0.42 | 0.40 | 0.15 | 0.21 | 0.26 | 0.20 | 0.14 | 0.20 | 0.19 |

For the discharge of new battery, it is assumed that the current is 30A, 40A, 50A, 60A and 70A respectively. By substituting \( u = 9.8 \)V into the model (2), we can get the discharge depth \( f \) corresponding to the battery discharging to 9.8V. Meanwhile, according to Peukert equation (1) and discharge capacity \( C \) corresponding to different current, the time \( \tau \) from discharge to minimum protection voltage can be calculated.

So the remaining discharge time of the battery can be obtained by using the discharge depth \( f \) and discharge time \( \tau \), that is, the remaining discharge time

\[ T = \tau - t \]

\[ = \tau - f \tau. \]

The results are as follows:
Table 4. The remaining discharge time

| I (A) | 30  | 40  | 50  | 60  | 70  |
|-------|-----|-----|-----|-----|-----|
| t (min) | 1815 | 1280 | 958 | 757 | 577 |
| \(\tau_1\) (min) | 2446.6 | 1724 | 1314 | 1052.6 | 872.57 |
| \(\tau_2\) (min) | 2421.1 | 1706 | 1300.4 | 1041.6 | 863.5 |
| \(\tau_2 - t\) (min) | 606.1 | 426 | 342.4 | 384.6 | 277.9 |

2.3 Analysis of MRE

According to Peukert equation (1), the model (2), and the discharge depth \(f\), the binary function of voltage \(U\) with respect to current \(I\) and discharge time \(t\) can be established, that is

\[ U(I, t) = \frac{a}{C^2} \cdot I^{2n} \cdot t^2 + b \]  

Here, \(C\) takes the value in table 1. By using the data corresponding to 231 voltage sample points, the function can be obtained, that is

\[ U(I, t) = -1.5722 \times 10^{-7} \cdot I^{2n} \cdot t^2 + 10.2906 \]  

Here, \(1.2h \leq t \leq 63h\), \(20A \leq I \leq 100A\).

By using the model (5), the MRE can be calculated under different current, as shown in the following table:

| I (A) | 20  | 30  | 40  | 50  | 60  | 70  | 80  | 90  | 100 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| MRE (%) | 11.55 | 6.8 | 5.19 | 3.74 | 6.13 | 3.94 | 2.87 | 2.50 | 4.64 |

Substituting \(I = 55A\) into the function(5), the discharge curve at this time can be made, as shown in the figure below.

Figure 4. The discharge curve(x-axis represents time and y-axis represents voltage)

3. Conclusion

Through the discussion of MRE in this paper, we can see that the accuracy of the first method is better than the second method. We can select more data to fit, and improve the model to achieve better results. Peukert equation is used to analyse the relationship between the discharge depth and the discharge voltage of lead-acid battery, which is simple in calculation and significant in process.

Acknowledgments

This paper is supported by the Famous Teacher Project of Wuxi Institute of Commerce.

References

[1] Liu, G.L. (2008) Introduction to Lead-acid Battery Technology. Mechanical Industry Press, Beijing. pp.246-247.
[2] Zhu, S.R. (1998) Battery Handbook. Tianjin University Press, Tianjin. pp. 1-47.
[3] Liu, G.L. (1990) Peukert Equation Research. Power Technology, 1990 (1): 13-18.

[4] Lu, G.Q., etc. (2001) Principle and Manufacture of lead battery. National Defence Industry Press, Beijing. pp.31-57.

[5] National College Students Mathematical Modelling Competition of Higher Education Social Cup, http://www.mcm.edu.cn.