A Field Theory for the Read Operator

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Abstract

We introduce a new field theory for studying quantum Hall systems. The quantum field is a modified version of the bosonic operator introduced by Read. In contrast to Read’s original work we do not work in the lowest Landau level alone, and this leads to a much simpler formalism. We identify an appropriate canonical conjugate field, and write a Hamiltonian that governs the exact dynamics of our bosonic field operators. We describe a Lagrangian formalism, derive the equations of motion for the fields and present a family of mean-field solutions. Finally, we show that these mean field solutions are precisely the Laughlin states. We do not, in this work, address the treatment of fluctuations.
I. INTRODUCTION

Much of our current understanding of the novel behavior in the quantum Hall (QH) regime relies upon microscopic, first quantized, wavefunctions for ideal QH states. The nature of the ordering in these states was first elucidated by Girvin and MacDonald [1] who showed that the Laughlin wavefunctions exhibit a non-trivial form of off-diagonal long range order. More precisely, they showed that there exists a composite operator of the fundamental Fermi fields that obeys Bose statistics and whose off-diagonal density matrix is algebraically long ranged in the Laughlin states [2]. This was an extremely important observation for it opened up the possibility of a Landau-Ginzburg description of the QHE in terms of an order parameter field, thereby bringing it within reach of more systematic computations. Subsequently Zhang, Hansson and Kivelson (ZHK) [3] and Read [4] proceeded to construct such Landau-Ginzburg theories.

The approach taken by ZHK builds directly on the observation of GM and consists of reformulating the dynamics entirely in terms of the bosonic operator. An exact transformation allowed them to rewrite the partition function of two-dimensional electrons placed in an external magnetic field in terms of a set of bosons that continue to see the external magnetic field but also interact among themselves through a statistical or Chern-Simons gauge field that exactly reproduces the Fermi statistics of the original problem. In the new variables a novel mean-field approximation becomes possible, in which the statistical gauge field cancels part of the external magnetic field. In fact at the “magic” fractions in the QHE the cancellation is exact and the bosons are able to condense. So in the bosonic formulation, QH states emerge as condensates of certain composite bosons.

While this mean-field picture is “morally” correct, it isn’t quite right in its details. Recall that we mentioned that GM found only algebraic order for the bosons; that is inconsistent with the true long range order present in the ZHK mean-field state. Fixing this requires that we include corrections from gaussian fluctuations (RPA) [5]. The problem here is that the ZHK field theory takes no advantage of the Landau level structure that the magnetic field
induces; indeed, the mean field wavefunction has extensive content in high Landau levels. Consequently, the Landau level structure has to emerge dynamically and starts to do so only at the RPA level where it can be shown that the asymptotic form of the wavefunction is now of the Laughlin form $^2$.

In contrast, Read was concerned with obtaining a true Landau-Ginzburg description in the sense of writing down classical equations of motion for an order parameter that really has an expectation value. To this end he worked with a bosonic operator that takes advantage of the Landau level structure and whose off-diagonal density matrix is truly long ranged in the Laughlin states. Equivalently, the Laughlin states are ideal condensates of the Read bosons. He then derived a rather complicated effective action that described the dynamics of this operator for small deviations from the Laughlin states that do not violate the constraint of confining the electrons to the lowest Landau level.

While Read’s approach builds in more of the physics of high magnetic fields (i.e. the lowest Landau level constraint) it has proven to be unwieldy for actual calculations. By contrast, the ZHK scheme both in its original form and in a fermionic version $^4$ has proven quite useful, e.g. in work on the phase diagram of QH systems $^8$ and perhaps most notably in the work of Halperin, Lee and Read himself $^9$ on the $\nu = 1/2$ problem. Nevertheless, it suffers from various problems stemming from the uncontrolled inclusion of higher Landau level processes, which brings us to our objective in this paper. (We should note that the Read operator does have the advantage, over its GM/ZHK analog, of being easier to compute in numerical work. This was exploited by Rezayi and Haldane $^1$ in their demonstration that the Read operator fails to condense when the Laughlin states are destabilized by varying the inter-electron interaction, and more recently by Sondhi and Gelfand $^1$ in establishing that the condensation occurs everywhere in the QH phases that derive from the Laughlin states in the presence of disorder.)

In this work we construct an exact quantum field theory for a slightly modified version of the Read operator in the full Hilbert space of the system. It combines the virtues of the ZHK and Read formulations in that, 1) the equations of motion capture the full dynamics
and are not restricted to small fluctuations about the Laughlin states, 2) they are far simpler than if the lowest Landau level constraint were imposed explicitly, and 3) yet the mean field solutions are still precisely the Laughlin states. Our hope is that a calculation at the RPA level in this formulation will yield useful information on mode spectra and corrections to the Laughlin wavefunctions beyond what can be gleaned from a comparable calculation in the ZHK approach. However, we warn the reader that we have not yet solved the problem of actually systematizing such calculations and detail the relevant difficulties in Section V.

In the remainder of the paper we describe the operator algebra for the bosonic field theory, the derivation of the Hamiltonian, a Lagrangian formulation and its mean-field solution and the equivalence of the mean-field solution to the Laughlin wavefunctions. We close with a summary.

II. THE BOSONIC FORMULATION

We consider a set of two-dimensional electrons of mass \( \mu \) and charge \( +e \), placed in a uniform transverse magnetic field of strength \( B \). We take the electrons to be spinless for simplicity and leave unspecified their interaction potential \( V \) as well as the scalar potential \( A_0 \) that represents any uniform and/or random electric fields in the problem. The field theoretic (second quantized) Hamiltonian that describes our system is,

\[
H = \int d^2x \left[ \Psi^\dagger(\vec{x}) \left( -\frac{\hbar^2}{2\mu} \vec{D}^2 + eA_0 \right) \Psi(\vec{x}) \right] + \frac{1}{2} \int d^2x \, d^2x' \, \delta\rho(\vec{x}) V(\vec{x} - \vec{x}') \, \delta\rho(\vec{x}') . \tag{2.1}
\]

Here \( \Psi(\vec{x}) \) is the full electron field operator and not merely its lowest Landau level piece, \( \vec{D} \equiv \vec{\nabla} - i\frac{e}{\hbar c} \vec{A} \) is the covariant derivative inclusive of the vector potential of the uniform field \( (B = \nabla \times \vec{A}(\vec{x})) \) and \( \rho(\vec{x}) \equiv \Psi^\dagger(\vec{x})\Psi(\vec{x}) \) is the density operator whose deviation from its mean value \( \bar{\rho} \) is \( \delta\rho(\vec{x}) \). In addition to specifying \( H \) we need to require that \( \Psi \) is a Fermi field, i.e. it obeys the equal-time anticommutation relations,

\[
\{ \Psi(\vec{x}), \Psi(\vec{x}') \} = \{ \Psi^\dagger(\vec{x}), \Psi^\dagger(\vec{x}') \} = 0
\]

\[
\{ \Psi(\vec{x}), \Psi^\dagger(\vec{x}') \} = \delta^{(2)}(\vec{x} - \vec{x}') . \tag{2.2}
\]
In order to recast this as a bosonic problem we need a set of bosonic operators which we construct in the next section.

\section*{A. Bosonic Operators}

Consider the pair of operators $\Phi(\vec{x})$ and $\Pi(\vec{x})$, defined by

\begin{align}
\Phi(\vec{x}) &\equiv e^{-J(\vec{x})} \Psi(\vec{x}) \\
\Pi(\vec{x}) &\equiv \Psi^\dagger(\vec{x}) e^{J(\vec{x})},
\end{align}

where,

\begin{equation}
J(\vec{x}) \equiv m \int d^2x' \left[ \rho(\vec{x}') \log(z - z') \right] - \frac{|z|^2}{4l^2},
\end{equation}

where $m$ is an odd integer, $z \equiv x_1 + ix_2$ is the complex coordinate on the plane and $l = \sqrt{\frac{\hbar c}{eB}}$ is the magnetic length. Evidently, $\Phi$ and $\Pi$ are not hermitian conjugates as $J$ has both hermitian and anti-hermitian pieces; in fact,

\begin{equation}
\Pi(\vec{x}) = \Phi^\dagger(\vec{x}) e^{J(\vec{x}) + J^\dagger(\vec{x})}.
\end{equation}

Nevertheless, as we now show, they are canonically conjugate Bose fields.

To this end note that the only operator appearing in $J(\vec{x})$ is the electron density $\rho(\vec{x}) = \Psi^\dagger(\vec{x}) \Psi(\vec{x})$, which obeys the commutation relation,

\begin{equation}
[\rho(\vec{x}), \Psi(\vec{x}')] = -\Psi(\vec{x}) \delta^2(\vec{x} - \vec{x}').
\end{equation}

Using this one can obtain the following identities:

\begin{align}
e^{-J(\vec{x})} \Psi(\vec{x}') &= (z - z')^m \Psi(\vec{x}') e^{-J(\vec{x})} \\
\Psi^\dagger(\vec{x}') e^{-J(\vec{x})} &= (z - z')^m e^{-J(\vec{x})} \Psi^\dagger(\vec{x}').
\end{align}

It is then straightforward to verify using (2.7) that

\begin{equation}
[\Phi(\vec{x}), \Phi(\vec{x}')] = [\Pi(\vec{x}), \Pi(\vec{x}')] = 0
\end{equation}
Thus, despite the presence of non-unitary factors in their definition in Eq. (2.3), the fields \( \Phi \) and \( \Pi \) form a pair of mutually canonical Bose fields. However, in contrast to standard charged scalar field theories, here \( \Pi \) is not equal to \( \Phi^\dagger \), instead they obey the more complicated relation in Eq. (2.5). This fact, a consequence of the non-unitary transformation in (2.3), has to be borne in mind in doing manipulations with our theory.

Nevertheless, notice that the fermion density \( \rho \), when written in terms of \( \Pi \) and \( \Phi \) still has the standard bosonic form

\[
\rho(\vec{x}) = \Psi^\dagger(\vec{x}) \Psi(\vec{x}) = \Pi(\vec{x}) \Phi(\vec{x}).
\]  

(2.10)

Thus, if \( N \equiv \int d^2x \rho \) is the number operator, then

\[
[N, \Pi(\vec{x})] = \Pi(\vec{x}),
\]  

(2.11)

i.e. the operator \( \Pi(\vec{x}) \) creates one extra composite boson, and the number of composite bosons is the same as the number of the original fermions.

Readers familiar with the work of ZHK will recognize that in the corresponding construction of a Bose field in their approach the operator \( J \) was chosen to contain only the phase of \( (z - z') \), i.e. \( \text{Im} \log(z - z') \) with the consequence that their \( \Pi = \Phi^\dagger \). In Read’s work the creation operator for the bosons differs from our \( \Pi \) only in that he did not include the gaussian factor \( e^{-\frac{|z|^2}{2l^2}} \) in its definition; hence our bosons are essentially the same. The difference between Read’s work and ours arises in that we also construct the canonical conjugate of \( \Pi \) and that, as we show next, allows us to write down a canonical quantum field theory of bosons by changing variables from \( \Psi \) and \( \Psi^\dagger \) to \( \Phi \) and \( \Pi \).

\textbf{B. The Hamiltonian}

Consider the action of the covariant derivative on the electron field. We have,
\[ D\Psi(x) = D(e^{J(x)}\Phi(x)) \]
\[ = (\nabla - ie\frac{\hbar}{c} \vec{A}(x)) \left(e^{J(x)}\Phi(x)\right) \]
\[ = e^{J(x)} \left(\nabla - ie\frac{\hbar}{c} \vec{A}(x) + \nabla J(x)\right) \Phi(x) \]
\[ = e^{J(x)}(D - ie\frac{\hbar}{c} \vec{v}(x)) \Phi(x) \quad (2.12) \]

where,
\[ \vec{v}(x) \equiv \frac{i}{e} \nabla J(x) \quad (2.13) \]

Hence,
\[ D^2\Psi = e^{J} (D - ie\frac{\hbar}{c} \vec{v})^2 \Phi \quad (2.14) \]

Inserting this into the starting Hamiltonian (2.1), and using Eqs. (2.3) and (2.10) we get,
\[ H = \int d^2x \left[ \Pi(x) \left(\frac{-\hbar^2}{2\mu} (\nabla - ie\frac{\hbar}{c} \vec{A} + \vec{v})^2 + eA_0\right) \Phi(x) \right] \]
\[ + \frac{1}{2} \int \int d^2x \ d^2x' \delta \rho(\vec{x}) \ V(\vec{x} - \vec{x}') \delta \rho(\vec{x}') \quad (2.15) \]

This Hamiltonian, the auxilliary definitions (2.10) and (2.13) and the commutators (2.8, 2.9) together define a purely bosonic problem that is fully equivalent to our original fermion problem [13].

The vector field \( \vec{v} \) appearing in (2.13) above is constrained in terms of the density by Eq. (2.13), where \( J(\vec{x}) \) is defined in (2.4). Since this \( J(\vec{x}) \) involves more than just the phase of \( (z - z') \), this field \( \vec{v} \) is not the familiar statistical Chern-Simons gauge field \( \vec{a}_{cs} \) used, for instance, in [3]. Because \( J(\vec{x}) \) has real parts, \( \vec{v} \) is a complex vector field. However, we will see now that \( \vec{v} \) is simply related to \( \vec{a}_{cs} \).

In our notation the Chern-Simons field is defined as
\[ \vec{a}_{cs}(\vec{x}) = \frac{-m\hbar c}{e} \nabla_x \int d^2x' \rho(\vec{x}') \ \text{Im} \log(z - z') , \quad (2.16) \]

or equivalently
\[ b \equiv \nabla \times \vec{a}_{cs} = -m\phi_0 \rho \quad (2.17) \]
where $\phi_0 \equiv \frac{hc}{e}$ is the flux quantum. Now, the function $\log z$ obeys the Cauchy-Riemann conditions away from $z = 0$, which can be written as

$$\vec{\nabla}(\text{Re } \log z) = \vec{\nabla}(\text{Im } \log z) \times \hat{k}$$

(2.18)

where $\hat{k}$ is a unit vector perpendicular to the plane. Using this we get,

$$\vec{v}(\vec{x}) = \frac{i\hbar c}{e} \vec{\nabla} J(\vec{x})$$

$$= \frac{i\hbar c}{e} \vec{\nabla}_x m \int d^2 x' \left[ \rho(\vec{x}') \left( \text{Re } \log(z - z') + i \text{Im } \log(z - z') \right) \right] - \frac{|z|^2}{4l^2}$$

$$= \vec{a}_{cs}(\vec{x}) + i \hat{k} \times \vec{a}_{cs}(\vec{x}) - \frac{i\hbar c}{e} \frac{x^2}{2l^2}.$$  

(2.19)

Note that the last term in the above equation is just a c-number term involving the coordinate vector $\vec{x}$. The density dependent operator part of $\vec{v}$ is present entirely through $\vec{a}_{cs}$.

III. THE LAGRANGIAN AND ITS MEAN FIELD SOLUTION

In constructing a Lagrangian formulation it is very useful to implement the constraint (2.17), relating $\vec{a}_{cs}$ to the density $\rho$, by the usual device of introducing a Lagrange multiplier field and recognizing the resulting gauge field action as being the restriction of the Chern-Simons term to transverse vector fields, $\vec{\nabla} \cdot \vec{a}_{cs} = 0$ [3]. Thus the Hamiltonian (2.15) along with the constraint (2.17) will emerge from the following Lagrangian density:

$$\mathcal{L} = \Pi (i\hbar \partial_t - e a_0) \Phi - \frac{e}{2m\phi_0} \epsilon^{\mu\nu\sigma} a_\mu \partial_\nu a_\sigma - H$$

(3.1)

where $H$ is the bosonized Hamiltonian in (2.14), and $a^\mu$ ($\mu = 0, 1, 2$) is the 3-vector $(a_0, \vec{a})$ and we have dropped the subscript on the Chern-Simons gauge field.

There is, however, a subtlety in this procedure which does not arise in ZHK’s construction and has to do with the gauge invariance of the resulting action and hence the freedom to pick gauges different from transverse gauge. First, note that the action is manifestly invariant with respect to gauge changes of the external field, i.e. the transformations,
\[
\vec{A} \to \vec{A} - \frac{\hbar c}{e} \vec{\nabla} \Lambda(\vec{x}, t)
\]
\[
A_0 \to A_0 + \frac{\hbar}{e} \partial_0 \Lambda(\vec{x}, t)
\]
\[
\Phi \to e^{-i \Lambda(\vec{x}, t)} \Phi
\]
\[
\Pi \to e^{+i \Lambda(\vec{x}, t)} \Pi .
\] (3.2)

However, the invariance with respect to gauge changes of the Chern-Simons field is more restricted. Gauge transformations of the form (3.2) with \((A_0, \vec{A})\) replaced by \((a_0, \vec{a})\) leave the action invariant only if \(\Lambda(\vec{x}, t)\) is independent of \(\vec{x}\), i.e. if they do not involve the spatial gauge field at all. In addition, there is a class of modified gauge transformations for the spatial components that do not involve the temporal component of the gauge field and have the following form. Let \(f(z) = u(\vec{x}) + i w(\vec{x})\) be an analytic function of \(z\). Then the action is invariant under,

\[
\vec{a} \to \vec{a} - \frac{\hbar c}{e} \vec{\nabla} w(\vec{x})
\]
\[
\Phi \to e^{-f(z)} \Phi
\]
\[
\Pi \to e^{+f(z)} \Pi .
\] (3.3)

(The variation of the gauge field implies that \(J \to J + f\) which ensures that the constraint Eq. (2.5) is preserved.)

As a consequence, the Chern-Simons field here is not really a gauge field and by the Chern-Simons term we necessarily mean its restriction to transverse gauge \[14\]. This is not a practical issue in this paper or in the computations we have in mind for they are typically carried out in transverse gauge anyway. Nevertheless, the implications of this feature of our theory, in particular the significance of the modified gauge invariance (3.3) and its possible connection to work on \(W_\infty\) algebras \[15\], remain a subject for future work.

The field equations arising from the Lagrangian density (3.1) are a “non-linear Schrödinger equation” \[16\],

\[
(i\hbar \partial_t - e(a_0 + A_0))\Phi(\vec{x}) = -\frac{\hbar^2}{2\mu} \left[ \vec{\nabla} - \frac{ie}{\hbar c} \left( \vec{A} + \vec{a} + i \vec{k} \times \vec{a} - \frac{i\hbar c}{e} \frac{\vec{x}}{2l^2} \right) \right]^2 \Phi(\vec{x})
\]
+ \left( \int d^2x' V(\vec{x} - \vec{x}') \delta \rho(\vec{x}') \right) \Phi(\vec{x}) , \quad (3.4)

along with the modified Chern-Simons field-current identities,

\[ \nabla \times \vec{a} = -m_{\phi_0} \Pi \Phi \quad (3.5) \]

\[ \hat{k} \times \left( -\partial_0 \vec{a} - \vec{\nabla} a_0 \right) = \frac{m_{\phi_0}}{c} \left( \vec{j} - i \hat{k} \times \vec{j} \right) \quad (3.6) \]

where,

\[ \vec{j} = \frac{\hbar}{2\mu_i} \left[ \Pi (\vec{D} \Phi) - (\vec{D} \Pi) \Phi \right] \quad (3.7) \]

\[ \vec{D} \equiv \vec{\nabla} - \frac{ic}{\hbar c} \left( \vec{A} + \vec{a} + i \hat{k} \times \vec{a} - \frac{i\hbar c}{e} \frac{\vec{x}}{2l^2} \right) \quad (3.8) \]

Although this current \( \vec{j} \) does not look manifestly hermitian, it is in fact just the usual hermitian electron current operator, as can be verified by rewriting it in terms of the Fermi fields.

These equations have a simple mean field solution \[ \] for the situation where the external electric potential \( A_0 \) is absent and the uniform magnetic field \( B = \nabla \times \vec{A} \) is chosen so that the filling fraction is

\[ \nu \equiv \frac{\bar{\rho}_{\phi_0}}{B} = \frac{1}{m} , \quad (3.9) \]

where \( m \) is the odd integer in the fermion to boson transformation function \( J \) defined in (2.4). The solution describes a homogeneous state and is given by the fields,

\[ \Phi(\vec{x}) = \Pi(\vec{x}) = \sqrt{\rho} \]

\[ \bar{a}_{\rho}(\vec{x}) = \frac{m_{\bar{\rho}\phi_0}}{2} (\vec{x} \times \hat{k}) \]

\[ a_0 = 0 . \quad (3.10) \]

As the boson field has a uniform phase and non-vanishing amplitude everywhere, this solution describes an ideal condensate of the composite bosons \[ \].
In order to verify that this is indeed a solution, we begin by noting that density $\rho = \Pi \Phi$ equals its mean value $\bar{\rho}$ everywhere. It follows that a constant $\Phi$ solves (3.4) provided the gauge fields that enters the covariant derivatives vanish. For the temporal gauge field this is trivially true. For the spatial gauge field we note that,

$$\vec{a} \bar{\rho}(\vec{x}) = -\frac{1}{2}m\phi_0 \hat{k} \times \vec{x}$$

$$= -\frac{1}{2}B \hat{k} \times \vec{x}$$

$$= -\vec{A},$$

(3.11)

and hence the combination $\vec{a} + \vec{A}$ vanishes. This condition for picking out uniform states, that the Chern-Simons field at mean density cancels the external field $\vec{A}$, is already known from [3]. But the statistical gauge field appearing in the covariant derivative in our Lagrangian and field equation (3.1 and 3.4) is not just $\vec{A} + \vec{a}_{cs}$. It also contains imaginary pieces. However, we also have the additional the result that

$$\hat{k} \times \vec{a} \bar{\rho} = \frac{B}{2} \vec{x}$$

$$= \frac{ch \vec{x}}{e 2l^2}$$

(3.12)

This last equality tells us that the extra imaginary pieces of the statistical gauge field, i.e. the third and fourth terms in Eq. (2.19), also cancel one another. Altogether, we have, for $\rho = \bar{\rho}$,

$$\vec{A} + \vec{a} \bar{\rho} + i \hat{k} \times \vec{a} \bar{\rho} - i \frac{ch \vec{x}}{e 2l^2} = 0$$

(3.13)

and hence the forms (3.10) satisfy Eq. (3.4).

It is also straightforward to verify that the forms (3.10) solve the field-current identity (3.5). Finally, readers concerned about the consistency of the solutions for $\Phi$ and $\Pi$ should note that for our solutions $J + J^\dagger = 0$. 

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IV. THE MEAN FIELD WAVEFUNCTION

We now show that the mean field solution of the last section is, in the first quantized fermionic representation, exactly the Laughlin state. To see this note from Eq. (2.11) that in our bosonized formulation, an N-particle state is obtained by the action of N powers of $\Pi$ on the vacuum. Hence the translationally invariant mean field state, where all the bosons have condensed into the $k = 0$ mode, has the (arbitrarily normalized) form,

$$|N\rangle_{MF} = \frac{1}{N!} \left[ \int d^2x \Pi(\vec{x}) \right]^N |O\rangle$$

(4.1)

where $|O\rangle$ is the no particle (vacuum) state, and $N, V \to \infty$ with $\frac{N}{V} = \bar{\rho}$. The first quantized electron wavefunction associated with this state is

$$\psi_{MF}(\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_N) = \langle O | \Psi(\vec{y}_1) \ldots \Psi(\vec{y}_N) |N\rangle_{MF}$$

$$= \frac{1}{N!} \langle O | e^{J(\vec{y}_1)} \Phi(\vec{y}_1) e^{J(\vec{y}_2)} \Phi(\vec{y}_2) \ldots e^{J(\vec{y}_N)} \Phi(\vec{y}_N)$$

$$\times \int d^2x_1 \Pi(\vec{x}_1) \int d^2x_2 \Pi(\vec{x}_2) \ldots \int d^2x_N \Pi(\vec{x}_N) |O\rangle \ .$$

(4.2)

Now we can use the identity,

$$\Phi(\vec{y}_1) e^{J(\vec{y}_2)} = e^{J(\vec{y}_2)} \Phi(\vec{y}_1) (z_1 - z_2)^m ,$$

(4.3)

to move all the factors of $e^J$ to the left, which yields

$$\psi_{MF}(\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_N)$$

$$= \frac{1}{N!} \prod_{i<j} (z_i - z_j)^m \langle O | e^{\sum_i J(\vec{y}_i)} \Phi(\vec{y}_1) \ldots \Phi(\vec{y}_N)$$

$$\times \int d^2x_1 \Pi(\vec{x}_1) \int d^2x_2 \Pi(\vec{x}_2) \ldots \int d^2x_N \Pi(\vec{x}_N) |O\rangle \ .$$

(4.4)

Next we use Wick’s theorem for the product of the $\Phi$’s and $\Pi$’s and note that the former annihilate the vacuum to obtain

$$\psi_{MF}(\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_N) = \prod_{i<j} (z_i - z_j)^m \langle O | e^{\sum_i J(\vec{y}_i)} |O\rangle \ .$$

(4.5)

Finally, as the vacuum has no particles, only the gaussian factor in $J$ contributes and we have the result,
\[
\psi_{MF}(\vec{y}_1, \vec{y}_2, \ldots, \vec{y}_N) = \prod_{i<j} (z_i - z_j)^m e^{\sum_i \frac{|z_i|^2}{4l^2}}.
\] (4.6)

Thus the mean field state directly yields the complete Laughlin wavefunction. This is in contrast to the bosonized theory of ZHK, where the mean field state contains only the correct phase of the Laughlin wavefunction and not its zeroes or the gaussian factor; as already mentioned those can be obtained correctly only upon including fluctuations.

**V. SUMMARY AND PROSPECTS**

In this paper we have set up a bosonic field theory for the quantum Hall effect using an operator algebra based on Read’s operator. The field theory admits mean-field states at the fractions \( \nu = 1/m \) that are ideal condensates in the Bose language and correspond exactly to the Laughlin states in terms of the electrons. In order to treat fluctuations about these states and to calculate the spectra of the various collective modes it is necessary to perturb about the mean field Hamiltonian. In our formulation, the mean-field Hamiltonian has the simple form,

\[
H_{MF} = -\frac{\hbar^2}{2\mu} \int d^2 x \Pi(\vec{x}) \nabla^2 \Phi(\vec{x})
\] (5.1)

and hence its eigenstates are all known exactly. Nevertheless, \( H_{MF} \) is non-hermitian and hence states with different energies are not necessarily orthogonal. (The full Hamiltonian is perfectly hermitian; however the mean-field theory dictates that we decompose it as the sum of two non-hermitian pieces.) This requires then, that the perturbation theory explicitly take account of the non-orthogonality and that we possess tractable expressions for the overlaps between different states \[19\]. We expect to discuss progress on this problem elsewhere \[20\].

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[13] The constraint Eq. (2.5) relating $\Pi$ and $\Phi$ does not need to be enforced separately; it is implicit in the equations of motion for the two fields.

[14] This conclusion follows as $w(\vec{x}, t)$ is harmonic and hence does not allow transformations between field configurations with different divergences.
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[16] There is also a related equation for Π.

[17] We use the term “mean-field solution” loosely. Evidently, what we present here are solutions to the classical equations of motion.

[18] The off-diagonal correlation function is Π(\vec{x}) \Phi(\vec{x}') and it is long ranged in the mean field state.

[19] Similar issues arise in attempting to treat the \( \nu = 1/2 \) problem in a purely lowest Landau level approach (F. D. M. Haldane, private communication).

[20] R. Rajaraman and S. L. Sondhi (work in progress).