CHARACTERIZATION OF KEPLER-91B AND THE INVESTIGATION OF A POTENTIAL TROJAN COMPANION USING EXONEST

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ABSTRACT

Presented here is an independent re-analysis of the Kepler light curve of Kepler-91 (KIC 8219268). Using the EXONEST software package, which provides both Bayesian parameter estimation and Bayesian model testing, we were able to re-confirm the planetary nature of Kepler-91b. In addition to the primary and secondary eclipses of Kepler-91b, a third dimming event appears to occur approximately $60^\circ$ away (in phase) from the secondary eclipse, leading to the hypothesis that a Trojan planet may be located at the L4 or L5 Lagrange points. Here, we present a comprehensive investigation of four possibilities to explain the observed dimming event using all available photometric data from the Kepler Space Telescope, recently obtained radial velocity measurements, and N-body simulations. We find that the photometric model describing Kepler-91b and a Trojan planet is highly favored over the model involving Kepler-91b alone. However, it predicts an unphysically high temperature for the Trojan companion, leading to the conclusion that the extra dimming event is likely a false-positive.

Key words: methods: data analysis -- planets and satellites: detection -- planets and satellites: fundamental parameters -- techniques: photometric

1. INTRODUCTION

Kepler-91 (KIC 8219268) is a red giant branch star in a late stage of stellar evolution. It is orbited by at least one companion (Kepler-91b), which was determined to be of planetary nature by Lillo-Box et al. (2014b). The planetary status has been refuted by two independent groups (Esteves et al. 2013; Sliski & Kipping 2014), but has since been confirmed by radial velocity measurements (Barclay et al. 2015; Lillo-Box et al. 2014a). Kepler-91b previously has been estimated to be a transiting Jupiter-mass planet ($M_p = 0.88^{-0.03}_{+0.17} M_J$) that orbits its red giant host star at one of the shortest known orbital distances of just $a = 2.32^{+0.07}_{-0.22} R_*$, with an orbital period of $T = 6.24650 \pm 0.000082$ days (Batalha et al. 2013; Lillo-Box et al. 2014a, 2014b; Literature values in Table 1). It orbits its host star in a slightly eccentric orbit of $e = 0.066$ inclined by $68.5^\circ$ with respect to the plane of the sky. Being so close to such a giant star ($R_* = 6.30 \pm 0.16 R_\odot$), even at this low inclination the planet is still observed to transit Kepler-91. In addition to the primary transit and secondary eclipse corresponding to Kepler-91b, Lillo-Box et al. (2014b) observed what appears to be a third occultation that occurs approximately $60^\circ$ in orbital phase away from the secondary eclipse. They describe four potential hypotheses explaining this dimming event: (1) a transiting Trojan planet located at either the L4 or L5 Lagrange point, (2) a second more distant planet in a resonant, non-coplanar orbit, (3) the presence of a large transiting exomoon in a resonant orbit around Kepler-91b, (4) instrumental effects associated with the Kepler pipeline or stellar variability.

Previous theoretical studies of exo-Trojans include numerical simulations to investigate the stability of such worlds (Dvorak et al. 2004; Schwarz et al. 2007, 2009), as well as the effect that a Trojan planet would have on the observed transit times of the primary planet (Transit Timing Variations; Ford & Gaudi 2006; Ford & Holman 2007; Madhusudhan & Winn 2009), and a search for Trojans in binary systems based on anomalies in the light curves associated with the correct phase expected from a Trojan (Caton et al. 2001). Despite these studies, the definitive discovery of such a planet remains elusive.

Presented here is a brief discussion of the four possible hypotheses intended to describe the observed flux, and an analysis of approximately 1500 days worth of Kepler observations and published radial velocity measurements of the Kepler-91 system. Three planetary models are applied to the Kepler-91 light curve: two, which model the photometric and radial velocity signals from a single planet, Kepler-91b, in the case of both circular and eccentric orbits, and another that models the photometric and radial velocity signals from Kepler-91b and an exo-Trojan in an eccentric orbit. The analysis was performed using the EXONEST software package (Placek et al. 2014), which relies on Bayesian model selection to statistically test between competing models, thus determining which model best describes a given data set. Since at present, there are no confirmed instances of exo-Trojans, this would be the first photometric evidence of an entirely new class of exoplanet. Furthermore, its existence would provide a unique opportunity to study the effect of stellar flux on two different planets in identical orbits.

1.1. The Trojan Hypothesis

In the restricted three-body problem, there are five equilibrium points known as Lagrange points. The points L1, L2, and L3 lie on the line connecting the primary planet to the
host star, whereas L4 and L5 are positioned 60° in front of, or behind, the primary planet in its own orbit as shown in Figure 1. Any object located at the L4 or L5 Lagrange point will appear stationary (neglecting librations) in a co-rotating reference frame. That is, it will orbit the host star with the same period as that of the primary planet. These orbits at L4 and L5 are stable provided that the host star is greater than 24.96 times the mass of the primary planet, whereas L1, L2, and L3 are unstable saddle points of the gravitational effective potential.

A massive object located at the L4 or L5 Lagrange point will undergo librational motion. The object will display long-period librations about the Lagrange point with period

$$T_{\text{long}} = \frac{T_j}{\sqrt{27/4\mu}},$$
(1)

where $T_j$ is the orbital period of the primary planet, and $\mu = M_p/(M_* + M_p)$, as well as epicyclic librations with period

$$T_{\text{short}} = \frac{T_j}{\sqrt{1 - \frac{27}{8}\mu}}.$$
(2)

The phase difference of 60° between the secondary eclipse of Kepler-91b and the third dimming event observed in the Kepler-91 light curve makes the possibility of a Trojan body very enticing. However, it should be noted that this dimming event would have to be the result of the secondary eclipse of the Trojan—not the primary transit. If it were a primary transit, then the second body would be out of phase from Kepler-91b by 120°, and thus not be a Trojan. The fact that this signal appears in the light curve folded at the orbital period of Kepler-91b, implies that it either occurs at the same period as Kepler-91b, or at an integer value of that period.

### 1.2. The Distant Planet Hypothesis

As mentioned in the previous section, this signal could be the transit of a more distant planet in a non-coplanar, resonant orbit to that of Kepler-91b. The low signal to noise of the raw Kepler light curve of Kepler-91 makes it difficult to simulate such a complex situation, however by considering the light curve folded on the period of Kepler-91b, one may be able to determine the likelihood of such a configuration. If the occultation is due to the presence of a more distant resonant planet, one would expect differences between the occultations occurring at odd, or even integrals of the orbital period of Kepler-91b.

Figure 2, shows the binned time series for Kepler-91b, folded at the accepted orbital period of Kepler-91b (solid black curve), and twice that of Kepler-91b (solid gray and dashed black curves). One can clearly see differences between the third dimming event occurring during the first and second halves of the double-period. While there appears to still be an occultation occurring, there is an odd feature toward one side of the dimming. During the first half of the double-period, there is an increase in the flux while during the second half of the double-period there is a decrease occurring at the same orbital phase, which can be seen more clearly in Figure 2(B). There are however significant odd/even effects for the primary eclipse of Kepler-91b as well, implying that this hypothesis may not be the correct explanation. These odd/even effects will be discussed in more detail in Section 3.5.

### 1.3. The Exomoon Hypothesis

A sizable exomoon in a resonant orbit may also be the cause of this third dimming event. The moon would need to appear to transit Kepler-91b 60° in phase before or after its secondary eclipse of the host star. In addition, this exomoon would need to have an orbital period that is the same as, or an integer multiple of that associated with Kepler-91b. This exomoon hypothesis and the stability of such a configuration is discussed in detail in Section 3.4.

### 1.4. Instrumental Effects and Stellar Variability

The Kepler science pipeline component Presearch Data Conditioning (PDC) identifies and removes systematic errors that are highly correlated across the ensemble of target stars on each CCD readout channel. It also identifies outliers and transients associated with radiation damage to the CCDs at the pixel level, called sudden pixel sensitivity dropouts. Global instrumental signatures are identified via singular value decomposition (SVD) of the quiet stars on each CCD readout channel. PDC-MAP applies a Bayesian maximum a posteriori (MAP) approach (Smith et al. 2012; Stumpe et al. 2014) to constrain the fit coefficients for the SVD-identified co-trending.

### Table 1

| Stellar Parameters From Literature | | | | | | |
|---|---|---|---|---|---|---|
| Mass, ($M_\odot$) | $1.31 \pm 0.10$ | | | | | |
| Radius, ($R_\odot$) | $6.30 \pm 0.16$ | | | | | |
| Effective Temperature, (K) | $4550 \pm 75$ | | | | | |
| Surface Gravity, log $g_*$, (c.g.s.) | $2.953 \pm 0.007$ | | | | | |
| Gravity Darkening Exponent | 0.733 | | | | | |
| Lin. Limb Darkening Coeff. | 0.549 | | | | | |
| Quad. Limb Darkening Coeff. ($a_1$, $a_2$) | (0.69, 0.05) | | | | | |
| Kepler Magnitude | 12.495 | | | | | |

| Estimated Planetary Parameters | | | | | | |
|---|---|---|---|---|---|---|
| Mass, ($M_1$) | $1.09 \pm 0.20$ | | | | | |
| Radius, ($R_1$) | $1.384 \pm 0.054$ | | | | | |
| Density, ($\rho_1$) | $0.33 \pm 0.05$ | | | | | |
| Period, (day) | $6.24650 \pm 0.000082$ | | | | | |
| Inclination, (deg) | 68.5 \( \pm \) 0.6 | | | | | |
| Eccentricity | 0.066 \( \pm \) 0.003 | | | | | |
| $a/R_*$ | 2.32 \( \pm \) 0.17 | | | | | |
| Equilibrium Temperature, (K) | 2460 \( \pm \) 120 | | | | | |

Figure 1. Schematic of Lagrange points in the restricted three-body problem. The L4 and L5 Lagrange points are located at the same orbital distance from the star, but lag or lead the primary planet by 60°.
basis vectors that are then projected out of each stellar light curve. This data reduction step can coincidentally remove stellar variations correlated with the instrumental signatures, especially on timescales longer than 20 days and can in some cases, introduce systematic signatures into individual light curves. However, it is highly unlikely that PDC-MAP introduced consistent brightening and dimming events at the period of Kepler-91b, as such signatures are not evident in the co-trending basis vectors which are formulated for each individual observing segment or quarter. Moreover, the transits and secondary occultation of Kepler-91b and the deep secondary occultation of the candidate Trojan planet are readily identifiable in the simple aperture photometry (PA-SAP) produced by the Kepler science pipeline, in addition to the PDC-MAP light curves analyzed in this paper (Jenkins et al. 2010; Twicken et al. 2010).

Lillo-Box et al. (2014b) were able to exclude stellar oscillations as the source of the extra dimming by performing a zero-order cleaning of the solar-like oscillations of the star using a high-pass filter in Fourier space.

2. METHODS

For this analysis, we utilized the EXONEST software package (Knuth et al. 2012; Placek et al. 2013, 2014), which employs a Bayesian inference engine capable of employing Nested Sampling (Skilling 2006) and MultiNest (Feroz et al. 2009, 2011, 2013), as well as Metropolis–Hastings Markov chain Monte Carlo (Metropolis et al. 1953) and Simulated Annealing (Otten & van Ginneken 1989). The Multi-Nest engine was employed for this study for its ability to efficiently explore complicated parameter spaces.

The EXONEST software package provides the ability to perform both Bayesian parameter estimation and model testing. That is, given a photometric model that describes a hypothetical planetary system, EXONEST allows one to obtain model parameter estimates as well as an estimate of the Bayesian evidence, which is used to statistically test one model against another. A model found to have an overwhelmingly large corresponding evidence value compared to the others is then said to be favored over the other model(s) describing the particular data set (Sivia & Skilling 2006; Knuth et al. 2015). The measure of one model’s favorability over another is quantified by the Bayes’ factor, which is the ratio of the model evidences. Nested Sampling and Multi-Nest both compute log-evidences, so the Bayes’ factor is calculated as the exponential of the difference between the log-evidences.

EXONEST currently models four physical mechanisms in addition to transits and secondary eclipses that affect the photometric signal obtained from an exoplanetary system. The first is the reflection of starlight off of the atmosphere or surface of the planet (Seager et al. 2000; Jenkins & Doyle 2003; Seager 2010; Perryman 2011; Placek et al. 2014), which is given by

\[
\frac{F_R(t)}{F_*} = \frac{A_g R_p^2}{2 r(t)^3} (1 + \cos \theta(t))
\]  

where \(A_g\) is the geometric albedo, \(R_p\) is the planetary radius, \(r(t)\) is the star–planet separation distance, and \(\theta(t)\) is the planetary phase angle. The second is the thermally emitted light from both the day- and night-sides of the planet (Charbonneau et al. 2005; Borucki et al. 2009; Placek et al. 2014), given by

\[
\frac{F_D(t)}{F_*} = \frac{1}{2} \frac{B(T_d)}{B(T_{eff})} \left( \frac{R_p}{R_*} \right)^2 (1 + \cos \theta(t))
\]  

and

\[
\frac{F_N(t)}{F_*} = \frac{1}{2} \frac{B(T_n)}{B(T_{eff})} \left( \frac{R_p}{R_*} \right)^2 (1 + \cos \theta(t) - \pi),
\]  

where \(T_d\) and \(T_n\) are the day- and night-side temperatures of the planet, \(R_*\) is the radius of the host star, and \(B(T)\) is the observed thermal radiation in the Kepler bandpass from a blackbody emitter at temperature \(T\). Both of these effects induce variations in the observed light as the planet orbits its host star and it goes through its phases (New, Crescent, Quarter, Full). The third effect is Doppler beaming (or Boosting), which is a relativistic effect that causes an increase in

\[
\frac{F_D(t)}{F_*} = \frac{1}{2} \frac{B(T_d)}{B(T_{eff})} \left( \frac{R_p}{R_*} \right)^2 (1 + \cos \theta(t))
\]  

and

\[
\frac{F_N(t)}{F_*} = \frac{1}{2} \frac{B(T_n)}{B(T_{eff})} \left( \frac{R_p}{R_*} \right)^2 (1 + \cos \theta(t) - \pi),
\]
in observed flux when the host star is moving toward an observer, and a decrease when the star recedes from the observer (Loeb & Gaudi 2003; Rybicki & Lightman 2008; Placek et al. 2014). This induces variations in the observed light curve since stars with planets around them orbit the common center of mass of the system, and thus periodically move toward and away from an observer. The relative flux due to Doppler beaming is found by

\[
\frac{F_E(t)}{F_s} = 1 + 4 \beta_s(t) \tag{6}
\]

where \(\beta_s\) is the stellar radial velocity along the observer’s line of sight. The fourth effect accounts for ellipsoidal variation, which are due to the gravitational tidal warping of the stellar surface due to the proximity of a massive planet (Loeb & Gaudi 2003; Faigler & Mazeh 2011; Shporer et al. 2011; Esteves et al. 2013; Placek et al. 2014). A massive planet will warp the stellar surface into an ellipsoid whose semi-axis will follow the planet throughout its orbit. This results in observed flux variations at 1/2 the orbital period (twice the frequency) due to both the changing cross-sectional area of the star and an accompanying gravity darkening at the sub-planetary point and at the corresponding antipodal point. This is approximated as

\[
\frac{F_E(t)}{F_s} = \frac{M_p}{M_*} \left(\frac{R_*}{r(t)}\right)^3 \sin^2 i \cos 2\theta(t) \tag{7}
\]

where \(M_p\) and \(M_*\) are the planetary and stellar masses, \(i\) is the orbital inclination, and \(\alpha\) is related to the linear limb-darkening coefficient \(u\), and the gravity darkening coefficient \(g\) by

\[
\alpha = \frac{0.15(15 + u)(1 + g)}{3 - u}. \tag{8}
\]

In addition to modeling the photometric effects in the case of both circular and eccentric orbits for Kepler-91b alone, a two-planet model was created, which assumed that the orbit of a second body is of the same orbital period \(T\), eccentricity \(e\), inclination \(i\), and argument of periastron \(\omega\) as that of Kepler-91b, but offset by an orbital phase of \(\Delta\phi\).

Kepler data from quarters 1–16 were used to analyze Kepler-91, which spans approximately 1550 days, and includes 63,214 data points. The following stellar properties were also assumed: \(u_1 = 0.05\), \(u_2 = 0.05\), \(u = 0.549\), \(g = 0.733\) (Claret & Bloemen 2011; Esteves et al. 2013). Here, the parameters \(u\) and \(g\) represent the linear limb-darkening coefficient and the gravity darkening coefficient respectively. Similarly, the parameters \(u_1\) and \(u_2\) represent the quadratic limb-darkening coefficients. The data were also folded on the accepted value of the orbital period, which assumes that both objects have an orbital period of \(T = 6.24650\) days.

2.1. Prior Probabilities and The Likelihood Function

The Bayesian inference engine takes as inputs the prior probabilities for each model parameter and a likelihood function describing the expected noise distribution.

The prior probability distribution of a particular model parameter quantifies the knowledge one has about that parameter before analyzing the data. For this study, each model parameter is assigned an uninformative uniform prior probability over a reasonable range as shown in Table 2. Since stellar characteristics such as mass, radius, and effective temperature have all been estimated previously, we assign a Gaussian prior (Table 2) for these parameters defined by the published values in Table 1.

The likelihood function depends on the particular forward model and the expected nature of the noise. In many situations one may assume that the noise is Gaussian distributed, however Kepler-91 displays a significant amount of correlated (red) noise likely induced by stellar oscillations. In order to accommodate the presence of correlated noise, we employ a nearest-neighbor approach introduced by Sivia & Skilling (2006) where the strength of correlations among nearest neighbors is described by the parameter \(\epsilon\), which varies from \([-1, 1]\). Taking these nearest neighbor correlations into account one can obtain a log-likelihood function of the form (Sivia & Skilling 2006)

\[
\log L = -\frac{N}{2} \log(2\pi\sigma^2) - \frac{(N - 1)}{2} \log(1 - \epsilon^2) + \frac{Q}{2(1 - \epsilon^2)} \tag{9}
\]

where \(N\) is the number of data points, \(\sigma^2\) the noise variance, and \(Q\) is related to the sum of the squared residuals, \(\chi^2\), by

\[
Q = \chi^2 + \epsilon \left( \chi^2 - \phi \right) - 2\psi. \tag{10}
\]

Here, \(\phi\) is defined as the sum of the first and last squared residuals, and \(\psi\) is the sum of the nearest neighbor residuals, which can be calculated using

\[
\psi = \sum_{i=1}^{N-1} R_i R_{i+1}. \tag{11}
\]
Note. In addition to parameter estimates, $\chi^2$, which is the sum of the squared residuals, and $\log \, L_{\text{max}}$, which represents the maximum of the likelihood function—the probability of observing a data set given a model set of model parameter values. The lower the $\chi^2$ value, the better the fit whereas the higher the $\log \, L$ the better the fit. For this particular set of photometric models, the two-planet model has the largest $\log \, Z$ making it the most favored model from a bayesian model selection standpoint. The two-planet model also has the lowest $\chi^2$, and highest $\log \, L_{\text{max}}$ values making it a better fit compared to the two-one-planet models. The nearest-neighbor correlated noise log-likelihood function described in Section 2.1 was used for each of these simulations and the correlation strength $\varepsilon$ was consistently estimated to be $0.365 \pm 0.004$ for each model.

Note that when the correlation strength, $\varepsilon = 0$, the log-likelihood function reduces to a Gaussian distribution of the form

$$
\log \, L = -\frac{N}{2} \log (2\pi \sigma^2) - \frac{\chi^2}{2\sigma^2}.
$$

For this study, both the correlated noise likelihood (9) and a Gaussian likelihood (12) are used to analyze the Kepler-91b time series.

3. RESULTS

This section summarizes our results from modeling the Kepler photometry from quarters 1–16 and the published radial velocity measurements of Kepler-91b (Barclay et al. 2015; Lillo-Box et al. 2014a). In addition, a short study of the stability of the hypothesized Trojan is presented using parameter values predicted by our analysis of the Kepler data.

3.1. From Photometry

Results from our analyses are summarized in Table 3. The parameter estimates obtained from the one-planet model (Kepler-91b only) indicate that Kepler-91b is a hot-Jupiter with a mass of $M_p = 0.91 \pm 0.06 M_J$ and radius of $R_p = 1.39 \pm 0.02 R_J$ consistent with results from Lillo-Box et al. (2014b). The dayside temperature was found to be $T_d = 2441.7 \pm 250.7$ K. However, this may be significantly underestimated due to the mid-eclipse brightening event, which can be seen in Figure 6 at orbital phase of $\sim 0.52$. This day-side temperature is consistent with the equilibrium temperature calculated by Lillo-Box et al. (2014b) assuming all incident thermal flux is re-radiated back into space ($T_{\text{eq}} = 2460_{-40}^{+120}$ K). Based on the model log-evidences, the eccentric orbit model is significantly favored over the circular orbit model by a Bayes’ factor of $\sim \exp(13) \approx 440,000$. However, the derived orbital eccentricity was lower than that of Lillo-Box et al. (2014b) at $\varepsilon = 0.028 \pm 0.004$.

In the case of the two-planet model, which considers a photometric signal from both Kepler-91b and the hypothesized Trojan, the log-evidence was such that the two-planet eccentric orbit model is favored by a Bayes’ factor of $\sim \exp(16) \approx 9$ million over the single planet eccentric orbit model. The improvement in fitting is also demonstrated by the increased maximum likelihood of the solution and lower $\chi^2$ (see Table 3). The predicted composite photometric signal from both objects is displayed in Figure 3. Note that the third occultation that occurs around orbital phase 0.7 is modeled by EXONEST as the secondary eclipse of the Trojan, as predicted, as well as a fourth occultation around an orbital phase of 1.2, which represents the transit of the Trojan. Parameter estimates associated with Kepler-91b do not change significantly by adding another planet to the model. The mass $M_p = 0.99 \pm 0.07 M_J$, radius $R_p = 1.38 \pm 0.02 R_J$, albedo $A_p = 0.39 \pm 0.17$, and dayside temperature $T_d = 2513.2 \pm 317.9$ K are all within $1 \sigma$ of the estimates from the one-planet model. Parameter estimates from the two-planet eccentric model suggest that the hypothesized Trojan has a mass of $M_p = 0.025 \pm 0.019 M_J$ or $M_p = 8.89 \pm 6.04 M_{J\oplus}$, and radius of $R_p = 0.28 \pm 0.05 R_{J\oplus}$ giving it a mean density of $\bar{\rho} = 1.98 \pm 0.77$ g/cm$^3$, which implies that, if this is a planet, it could potentially be classified as sub-Neptunian. This estimate of the density is consistent with results from Rogers (2015), who showed using the known populations of sub-Neptune planets that planets with radii greater than 1.6 $R_{J\oplus}$ are most likely not rocky, but gaseous. In addition, the presence of a Trojan body is not only supported by the log-evidence calculations, but also by the model prediction for the planet-Trojan phase difference, which is $\Delta \phi = 65^\circ \pm 1^\circ$.

This model also predicted very high day and night-side brightness temperatures of $T_d = 5184.6 \pm 531.5$ K and $T_n = 2372.6 \pm 1283.6$ K, respectively, which will be further discussed in Section 4. As for the primary planet, the two-
Figure 3. Kepler-91b plus Trojan fit for Kepler-91. Dark gray data points represent the time series binned and averaged at the Kepler cadence, and the black line is the model fit. Notice that the third occultation, occurring at an orbital phase 0.7, has been modeled by EXONEST as a secondary eclipse as one would have expected from a Trojan. Also, notice that the primary for the Trojan is much smaller in depth than the secondary, implying either a reduced transit depth due to the system’s geometry or extremely hot day and night-sides.

Figure 4. Two-dimensional representations of the posterior parameter estimates associated with Kepler-91b in the case of the two-planet model. Of note, is the poorly constrained geometric albedo, which is caused by the degeneracy between the thermal emission and reflected light effects. This degeneracy is further illustrated in the $T_d$ vs. $A_g$ plot where there is a slight banana-shaped ridge and causes large error bars on their respective parameter estimates.
planet model predicts Kepler-91b to have day and night-side temperatures that are within $1\sigma$ of each other ($T_d = 2513.2 \pm 317.9$ K and $T_n = 2871.8 \pm 183.6$), which is not unexpected for short-period hot Jupiters around red giant stars (Spiegel & Madhusudhan 2012).

This model also predicted an unexpectedly high albedo for the Trojan $A_g = 0.49 \pm 0.28$, which is likely due to the strong degeneracy between the albedo $A_g$ and the day-side temperature $T_d$. As indicated by the large uncertainties, there is simply not enough information in the data for EXONEST to disentangle the reflected and emitted components and to well-determine the geometric albedo of the hypothetical Trojan.

Two-dimensional representations of the posterior probability of the model parameters are illustrated in Figures 4 and 5. Notice the degeneracy between $T_d$ and $A_g$ in Figure 4. A planet can have a higher day-side temperature and lower geometric albedo and output the same flux as a highly reflective planet (high geometric albedo) with a low day-side temperature. One can also see that many of the parameters associated with the Trojan exhibit broad ridge-like structures, which explain our inability to precisely estimate them.

Based on the estimate of the Jovian mass obtained from the two-planet model, the expected long- and short-period librations of the Trojan can be calculated to be $T_{long} = 90.32 \pm 4.80$ days, and $T_{short} = 6.254 \pm 0.001$ days, respectively. The short period epicyclic librations have approximately the same period as the orbital period of the two planets around the host star. The long period librations were searched for using a Lomb–Scargle periodogram, however no pronounced periodicities were discovered.

The presence of correlated noise was verified by the algorithm, which predicted a nearest-neighbor correlation strength, $\epsilon$, of approximately $0.365 \pm 0.004$ for all models. It is certainly possible that the Jovian has set up oscillations in the host star. However, for these oscillations to manifest themselves as a pulse wave with a pulsewidth $\tau$ that mimics the Trojan’s secondary eclipse, the star would need to be oscillating according to something of the form

$$f(t) = C_1 + C_2 \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin \left( \frac{n\pi t}{T} \right) \cos \left( \frac{2n\pi t}{T} \right)$$

where $T$ is the period of the Jovian and $C_1$ and $C_2$ are constants. It is highly unlikely that some resonance excited by a driving
oscillation at period $T$ would result in an overall stellar oscillation consisting of frequency components with amplitudes precisely modulated by a quantity $\tau$ (appearing in the functional form above) that is in agreement with the transit/eclipse duration of the Jovian, which relies on the viewpoint-dependent impact parameter.

A Gaussian likelihood that neglects correlations among the observed data was also used in the analysis. With the Gaussian likelihood, the Jovian+Trojan model was again favored by a Bayes’ factor of $\sim\exp(8) \approx 3000$ over the other two photometric models. The parameter values associated with the Jovian and Trojan were also similar, not deviating more than 1 $\sigma$ from the estimates obtained using the correlated noise likelihood function (in Table 3). Comparing log-evidences, the correlated noise likelihood ($\log Z = 411.937.4 \pm 0.9$) is favored over the Gaussian ($\log Z = 407.408 \pm 0.8$) by a Bayes’ factor of $\sim\exp(4500)$, indicating that the correlated noise model provides a significantly better description of the observed data.

### 3.2. From Radial Velocity

Bayesian model testing was also performed on the published radial velocity measurements (Barclay et al. 2015; Lillo-Box et al. 2014a) in order to determine if a model that includes the radial velocity signal from a Trojan planet would be favored over a model that neglects a Trojan. Assumed quantities for these simulations include only the stellar mass ($M_s = 1.31 \pm 0.10$ $M_\odot$), and orbital period ($T = 6.24650$ days) as opposed to the analysis performed by Lillo-Box et al. (2014a), which assumed the eccentricity to be that which was estimated from their analysis of photometry. Based on model log-evidences, the eccentric one-planet model was slightly favored over the circular model by a Bayes’ factor of $\sim\exp(1.5) \approx 4.5$ in agreement with the photometric results. The eccentric one-planet model predicts an eccentricity of $e = 0.041 \pm 0.028$, which is also in agreement (within uncertainty) with results presented in the previous section as well as the photometric analysis performed by Lillo-Box et al. (2014b). This model also predicts a minimum planetary mass associated with Kepler-91b of $M_p \sin i = 0.86 \pm 0.09M_\oplus$, which along with the orbital inclination obtained from photometry, implies a true mass of $M_p = 0.92 \pm 0.10M_\oplus$, which is in agreement with the results from photometry. These two models were then tested against an eccentric two-planet model that assumes the second planet to be in the same orbit as the Jovian, but trailing it by a phase angle $\Delta \phi$. This model was neither favored nor rejected by the eccentric one-planet model. However, it is interesting that adding an additional two parameters to describe the hypothetical Trojan did not significantly change the log-evidence. The one-planet eccentric model yielded a higher log-evidence, with a difference in log-evidence between the two models being $\sim\exp(1.53)$. The fits for all three of these radial velocity models are shown in Figure 6. It should also be noted that attempting to fit two radial velocity signals with the same period will result in significant degeneracies as can be seen in the semi-amplitudes estimated in Table 4. The predicted semi-amplitudes correspond to uncertainties in $M_p \sin i$ that are approximately 50% of the mean value. However, this does not affect ones ability to perform Bayesian model testing. The two planet model also predicted a phase difference of $\Delta \phi = 79^\circ 06 \pm 37^\circ 8$. While the uncertainty here is large, this does not rule out the expected phase difference for a Trojan of $60^\circ$. These radial velocity measurements neither confirm or reject the Trojan hypothesis. In addition to comparing log-evidences, another common way to compare different models is to look at the $\chi^2$ value, which represents the sum of the squared residuals. Having the lowest $\chi^2$, the two-planet model represents a better fit to the observed data. In a similar fashion, one may compare the log-likelihoods of a set of models, which describes the probability of observing a particular data set given a set of model parameters. The higher the log-likelihood, the better the fit to the data. The two-planet model clearly has the highest maximum log-likelihood, but is not favored in log-evidence because the fit was not improved sufficiently to overcome the penalty of adding additional parameters to the model, which is taken into account when calculating the log-evidence.

### 3.3. Stability of Trojan Configuration

In order to test the Trojan planet hypothesis further, a preliminary stability study was performed. One hundred (100) orbital configurations were generated by sampling the posterior obtained from the Kepler photometry. Initial conditions for each orbital configuration were determined by sampling from the posterior distributions obtained in the photometric two-planet simulations for $M_\star$, $R_\star$, $M_{\text{tr}}$, $T_{\text{short}}$, $e$, $a/R_\star$, and $\Delta \phi$ with the means and standard deviations being those that are listed in Table 3 for those that correspond to the photometric model parameters, and Table 1 for those which are previously published values. Each orbital configuration was tested for stability using the following cuts. The semimajor axes of the Jovian and hypothetical Trojan orbits cannot increase or decrease by more than 30% of the original value, the eccentricity of the orbits for either object cannot exceed one ($e \neq 1$), which would indicate a parabolic or hyperbolic trajectory, and the maximum allowed change in energy of the system was $10^{-4}$ (Barnes & Quinn 2004). The equations of motion for the three-body problem were integrated up to 50,000 days, which corresponds to approximately 8000 orbits of the Jovian and Trojan. All orbital configurations for the Trojan were found to be stable over this time period. Furthermore, the results predict librational periods of $T_{\text{hang}} = 80.36 \pm 11.21$ days and $T_{\text{short}} = 5.71 \pm 0.83$ days, which both agree with the theoretical values predicted by Equations (1) and (2). The relatively large uncertainties on these librational periods are likely due to the sampling of the initial conditions from the posterior. In the future longer integration times will be necessary to solidify the stability of such an orbital configuration.

### 3.4. Stability of Exomoon Configuration

A similar stability study was performed to investigate the exomoon hypothesis. Each parameter was again sampled from the posterior obtained from the photometric simulations. This excludes the mass and semimajor axis of the hypothesized exomoon since they were not modeled in the light curve. Due to the fact that the occultations associated with the hypothesized third body are observed in the light curve folded on the period of the Jovian, it would be reasonable to assume that the two bodies are in resonance. Therefore, the period of the exomoon was assumed to be $T_m = nT_J$, where $n$ ranges from $n = [0.1, 3]$ in increments of 0.1 and $T_J = 6.2465$ days. For each value of $n$, 100 orbital configurations were generated.

For an exomoon with mass $M_m = 0.1 M_\oplus$, there were zero stable orbits for each value of $n$. In the case of the 1:10 resonance, every configuration resulted in the moon crashing
Table 4
Model Parameters for All Three Models Applied to the Kepler-91 Radial Velocity Measurements

| Parameter | Two-planet Model | One-planet Model Eccentric | One-planet Model Circular |
|-----------|------------------|-----------------------------|---------------------------|
|           | Kepler-91b       | Trojan Candidate            | Kepler-91b                | Kepler-91b                |
| $K$ (m s$^{-1}$) | 50.94 ± 27.98     | 58.77 ± 26.56               | 83.67 ± 7.77              | 82.28 ± 11.57             |
| $\omega$ (rad)  | 3.02 ± 1.69       |    ...                      | 2.82 ± 2.23              |    ...                    |
| $e$         | 0.047 ± 0.028     |    ...                      | 0.041 ± 0.028            |    ...                    |
| $M_p \sin i$ (M$_J$) | 0.48 ± 0.23     | 0.61 ± 0.28                 | 0.86 ± 0.09              | 0.86 ± 0.12               |
| $\Delta$ (deg) |    ...            | 79.06 ± 37.8               |    ...                   |    ...                    |
| $\gamma$ (m s$^{-1}$) | −31.00 ± 0.04 |    ...                      | −62.010 ± 0.006          | −62.011 ± 0.008           |
| $\log L$     | −241.53 ± 0.26    | −240.31 ± 0.35              | −241.50 ± 0.36           | −241.50 ± 0.36            |
| $\chi^2$     | $1.5732 \times 10^5$ | $1.5769 \times 10^5$ | $1.6622 \times 10^5$ | $1.6622 \times 10^5$ |
| $\log L_{max}$ | −224.6390         | −225.6953                   | −226.6576                | −226.6576                 |

Note. In addition to parameter estimates, included is the $\chi^2$ value, which is the sum of the squared residuals, and $\log L_{max}$ value, which represents the maximum of the likelihood function—the probability of observing a data set given a model set of model parameter values. The lower the $\chi^2$ value, the better the fit whereas the higher the $\log L$ the better the fit. For this particular set of radial velocity models, the eccentric one-planet model has the largest log $L$ making it the most favored one-planet model from a bayesian model selection standpoint. Notice that the two-planet model also has the lowest $\chi^2$, and highest $\log L_{max}$ values making it a better fit compared to the two one-planet models.

Table 5
Model Testing Results on odd/Even Subsets of the 16 Quarters of Kepler Observations

| Subset     | $\log Z_{two} - \log Z_{one}$ | $\chi^2_{two}$ (ppm) | $\chi^2_{one}$ (ppm) | $\log L_{max,two}$ | $\log L_{max,one}$ |
|------------|-------------------------------|-----------------------|-----------------------|---------------------|---------------------|
| Even—All Data | +11.5                        | 4559.10               | 4562.24               | 2.04710e05         | 2.04693e05         |
| Odd—All Data  | −1.5                         | 4634.87               | 4634.59               | 2.07267e05         | 2.07258e05         |
| Even—First Half | +13.9                      | 2289.05               | 2293.96               | 1.02945e05         | 1.02924e05         |
| Odd—First Half | −6.9                      | 2339.86               | 2339.45               | 1.02903e05         | 1.02905e05         |
| Even—Second Half | +3.2                      | 2304.50               | 2307.24               | 1.04567e05         | 1.04557e05         |
| Odd—Second Half | −0.6                      | 2242.90               | 2242.19               | 1.01635e05         | 1.01633e05         |

Note. Positive values indicate that the two-planet model was favored over the one-planet model. The two-planet model is favored in the even periods, and the odd periods favor the one-planet model. Also listed are the $\chi^2$ values for each simulation. Notice the odd/even differences are evident in both log-evidence and $\chi^2$ values.

into the Jovian. For the 1:5 resonance, 78% of the configurations resulted in the Moon crashing into the Jovian, and 22% resulted in the moon being expelled into a more distant orbit around the host star. The remaining configurations all resulted in the moon being shot out to more distant orbits around the host star, likely due to the fact that it is semimajor axis was larger than the radius of the Hill sphere of the Jovian, which is approximated by

$$r_H \approx a(1 - e) \left( \frac{M_{Jov}}{3M_*} \right)^{1/3}. \quad (13)$$

A moon that is outside of the Hill sphere of a planet, is not gravitationally bound to that planet. Based on the posterior estimates from photometry, the radius of the Hill sphere in the case of Kepler-91b is $r_H = 0.0041 \pm 0.0004$ AU. This would translate to orbital periods for the moon of $T_m = 0.51 \pm 0.08 T_{jov}$, which verifies the unstable behavior observed in the numerical simulations.

3.5. Odd–Even Effects and Systematic Noise

As shown in Figure 2, there are odd/even effects that appear in the light curve folded at twice the orbital period of the Jovian. These appear to occur halfway through the transit of the Jovian and apparent secondary of the Trojan (see Figure 2) and when binned at the Kepler cadence and folded on the accepted period of the Jovian apparently disappear. To test whether or not these are significant in affecting the model testing results, we tested the one-planet model against the two-planet model using six different subsets of the entire time-series. The subsets were defined as follows: the even/odd periods of the Jovian over the all quarters, the even/odd periods of the Jovian over the first eight quarters, and the even/odd periods of the Jovian over the last eight quarters for a total of six subsets. Results are displayed in Table 5.

Positive differences in the log-evidences indicate that the two-planet model is favored over the one-planet, whereas negative differences indicate that the one-planet model is favored. Clearly the odd/even effects are influencing the model testing results, since the two-planet model is favored over the one-planet model in the even periods, but not in the odd periods. This is also apparent in the model fits as the minimum chi-squared values (bolded in Table 5) alternate between even and odd periods. Since the libration period is estimated to be on the order of the orbital period of the Jovian, it is unlikely that the libration of a Trojan companion could explain this result. It is very possible that there is an unmodeled stellar effect or star–planet interaction that is responsible for these odd/even differences. The maximum log-likelihoods for each model are also listed in Table 5. Unlike the chi-squared values, these do not alternate between odd and even periods likely due to the fact that there are free parameters affecting the likelihood function. Namely the standard deviation of the noise, $\sigma$, and the correlation coefficient, $\epsilon$. The one-planet model has fewer degrees of freedom compared to the two-planet model. As such there may be features in the data that the model cannot fit, and
Figure 6. Model fits to the radial velocity measurements of Kepler-91 (Lillo-Box et al. 2014a). The three models displayed are the one-planet in eccentric (solid line with dots; log $Z = -240.31 \pm 0.35$, $\chi^2 = 1.5769 \times 10^3$, log $L_{\text{max}} = -225.6953$) and circular (solid line; $log Z = -241.50 \pm 0.36$, $\chi^2 = 1.6622 \times 10^3$, log $L_{\text{max}} = -226.6576$) orbits, as well as the two-planet eccentric model (dashed; log $Z = -241.53 \pm 0.26$, $\chi^2 = 1.5732 \times 10^3$, log $L_{\text{max}} = -224.6390$) that attempts to describe the observed radial velocity measurements by including the jovian and a candidate Trojan planet. The bottom window of the plot displays the corresponding residuals for each model.

Thus treats as noise by altering the estimated parameters that correspond to the likelihood function. Since there is suspected to be a significant amount of systematic noise in the light curve of Kepler-91b, these extra dimming events may be caused by some unknown systematic effect(s). If the signal in the phase-folded light curve is truly from a Trojan companion, one should only rarely see similar dimming events (in depth and duration), when the light curve is folded on other random periods.

A transit detection routine was created to search for such eclipses and determine their frequency at various periods. First, the signal from the jovian was subtracted from the light curve using the best-fit model from the photometric analysis in Section 3.1. Then, the residuals corresponding to the intervals over which the jovian was in transit and when it was in secondary eclipse were removed. These residuals were then folded on random periods ranging from [0, 100] days. Detections were deemed to have occurred if the data points dipped below the mean of the entire data set, at which time the area under the curve would be computed. A larger transit would correspond to a smaller, more negative, area under the curve. If any detections corresponded to a greater area under the curve than the apparent Trojan signal ($2.2 \times 10^{-18}$), it would be discarded. In addition, if any events were found to have durations inconsistent with the apparent Trojan signal they would also be discarded. The apparent Trojan eclipse has a duration of approximately 0.459 days. Events would be deemed detections if they had durations of 0.459 ± 0.05 days. This process was repeated for 10,000 random periods. Figure 7 displays six examples of the detected dimming events as well as the apparent Trojan signal. Each window displays only part of the phase-folded residual as these detections often have durations much smaller than the phase as shown in Figure 8, dimming events similar to that associated with the hypothetical Trojan were more readily discovered at longer periods. While only four were detected around the orbital period of the jovian, they were nevertheless detected making the systematic noise a serious candidate to explain the extra dimming.

4. DISCUSSION

Presented here is a re-analysis of the Kepler-91 system. The results from the one-planet model for both photometry and radial velocity measurements re-confirm the planetary nature of the companion, Kepler-91b, as a hot-Jupiter with characteristics similar to those estimated by Lillo-Box et al. (2014a). Based on the model log-evidence calculations, Kepler-91b is considerably more likely to be in an eccentric orbit, rather than circular, which is consistent with previous results. The estimate of the day-side temperature is consistent with the equilibrium temperature of the planet, although a possible confounding effect is that of the mid-eclipse brightening event that occurs during the secondary eclipse of Kepler-91b. The predicted albedo is also consistent for short-period hot-Jupiters (Angerhausen et al. 2014).

The Trojan planet hypothesis was tested by applying a two-planet model (Planet+Trojan, eccentric) to the Kepler photometric data that aims to describe Kepler-91b and a hypothetical Trojan planet. This two-planet model was significantly favored over the one-planet model by a Bayes’ factor of $\sim \exp(9) \approx 8000$, but was neither ruled out (nor verified) by the RV data obtained by Lillo-Box et al. (2014a). When performing Bayesian model testing, there are two important factors: the ratio of the prior probabilities of the models and the ratio of the evidences of the data. When the prior probabilities of the models are assumed to be equal one can focus on the ratio of the evidences, or equivalently, the difference between their logarithms (the log odds ratio; Knuth et al. 2015). In the analysis presented here, we focused on the differences between the logarithms of the evidence, which implicitly assumes that the prior probabilities of the models are equal. However, this assumption is questionable since our experience to date does not lead us to truly believe that it is as likely for Kepler-91 to
have a Jovian planet as it is for Kepler-91 to have a Jovian planet with a Trojan companion. The log evidences must be compared with this in mind. Statistical evidence for the existence of Trojan planets has been uncovered (Hippke & Angerhausen 2015). However, given that at present there are about 5000 exoplanet candidates and a Trojan companion to a planet has yet to be found, one could crudely estimate the ratio of probabilities to be something on the order of $1/5000$ against the Jovian+Trojan model, which corresponds to a log probability of about 6.9. Thus one would not be confident in such assertions unless the log evidence in support of a Trojan was proportionately higher. The fact that the log-evidence differences supporting a Trojan is on the order of 16 (see Section 3.1) yields an overall support with a probability on the order of $\exp(16 - 6.9) \approx 9000$.

Based on the model log-evidences for the single-planet case, we re-confirm the planetary nature of Kepler-91b with an estimated minimum mass of $M_p \sin i = 0.86 \pm 0.09 M_J$, which

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**Figure 7.** Six examples of the randomized folding routine including a zoomed in view of the apparent Trojan signal (top). The gray data points represent the Kepler-91b residuals folded on the period listed above each window binned at the *Kepler* cadence. Detected dimming events are labeled by black dots. These events are all approximately the same duration (0.495 days), however the durations appear different in each window as they are folded at different periods. For longer periods, the duration will appear to decrease.
corresponds to a true mass of $M_p = 0.92 \pm 0.10 M_J$ given the
orbital inclination of $i = 69.98 \pm 0.18$ determined using the
photometric data. This estimate of the mass of Kepler-91b is
consistent with the analysis of photometric data performed
in Section 3.1, along with the results from Lillo-Box et al.
(2014a). The eccentricity of the orbit was found to be
e = 0.041 \pm 0.028, which also agrees with the photometric
analysis within uncertainty. Applying the two-planet (Jovian +
Trojan) model did not confirm or reject the Trojan hypothesis
based on model log-evidences, however the two-planet model
did have the lowest $\chi^2$ value, and highest log-likelihood
making it a better fit to the data than the other models despite
the Bayesian evidence being slightly lower than the one-planet
eccentric model.

The Jovian + Trojan model provides similar estimates of the
Kepler-91b parameters again supporting the idea that the planet is
a hot Jupiter with mass $M_p = 0.99 \pm 0.07 M_J$, radius
$R_p = 1.38 \pm 0.02 R_J$, albedo $A_p = 0.39 \pm 0.17$ and day-
and night-side temperatures of $2513.2 \pm 317.9$ K and $2871.8 \pm
183.6$ K, respectively. The day- and night-side temperatures of
Kepler-91b in this model are equal to within uncertainty, and
consistent with the expected equilibrium temperature of the
planet suggesting that it may have high winds that equilibrate
its day and night sides. Results also suggest that a Trojan would be
sub-Neptunian, with a radius $R_p = 2.91 \pm 0.56 R_J$, and with a
mass of $M_p = 8.89 \pm 6.04 M_J$. The mass was not well estimated
in this case because of the apparent lack of Doppler boosting and
ellipsoidal variations associated with this object. The day and
night-side temperatures were determined to be a scorching
$T_d = 5184.6 \pm 531.5$ K and $T_n = 2372.6 \pm 1283.6$ K, respec-
tively. While the uncertainty in these estimates are such that
the temperatures are not conclusive, it is possible, according
to the model, that the day-side of the planet may be greater
than or approximately equal to the effective temperature of the
host star, which has been determined to be 4550 \pm 75 K (Lillo-
Box et al., 2014b). The two-planet model fit illustrated in Figure 6
shows the comparatively small depth of the Trojan transit to the

affected by the mid-transit and mid-eclipse brightening events,
which are neither modeled nor well-understood. In addition,
there are other known effects, which are not presently accounted
for by EXONEST that could result in increased temperature
estimates. These include the extra illumination of the side of the
planet opposite the star due to the planet’s proximity (see for example,
Figure 10 in Lillo-Box et al. 2014b) in addition to the high
inclusion of the orbit, which would allow part of the
day-side of the planet to be visible from Earth during transit.
Also, a highly inclined and slightly eccentric orbit has a high
probability to produce a grazing transit (see for example
WASP-67; Hellier et al. 2012). However, using posterior
estimates of the planetary radius and inclination (Table 3) along
with the published values of the stellar radius and semimajor axis
(Table 1), one can calculate the estimated impact parameter,
b = 5.06 \pm 0.19 R_*$. Figure 9 displays the sky-projected distance
between the center of the host star and the center of the
planet, which shows that Kepler-91b is most likely not in a
grazing orbit.

In addition, if a Trojan companion were present, both the
Jovian and Trojan would undergo librations as discussed in
Sections 1.1 and 3.3. Our simulations, based on the current
model parameter estimates, show that the orbital phase of the
Jovian planet would not significantly vary, whereas the
Trojan’s phase would vary. One would expect that this would
result in a smearing of the Trojan transit in the phase-folded
light curve. To properly model the system, $N$-body fitting
routines should be employed rather than the Keplerian fitting
used here. While the libration periods obtained from our
current model parameter estimates do not seem to be able to
explain the observed even/odd period effects, a carefully
modeling of the planet–planet interactions could account for
additional unusual features observed in the light curve, such as
the mideclipse brightening events.

Given the available data and the models employed, it is not
yet possible to come to a conclusion as to the presence of a
Trojan partner to Kepler-91b. In favor of the Trojan hypothesis
is the fact that the Bayesian evidence of the Jovian+Trojan

Figure 8. Histogram of the detected dimming events in the light curve of
Kepler-91b folded at random periods. Notice that the events are more abundant
at periods of approximately 70 days and very rare near the orbital period of the
Jovian (6.2465 days).

Figure 9. Sky-projected separation between the centers of the planet and star.
The black arc represents the stellar limb ($R_* = 6.30 \pm 0.16 R_J$) and the
shaded regions represent the $\pm 1 \sigma$ values for the stellar radius. The planet
is depicted as the small black circle and the vertical line through the planet
represents the $\pm 1 \sigma$ values for the impact parameter ($b = 5.06 \pm 0.19 R_*$),
and the dotted line is the sky-projected orbital path of the planet.

$\sum_{i=1}^{n} (y_i - \mu)^2$
model is \( \exp(16) \) times greater than the Jovian model. This hypothesis is still highly probable if one considers a reasonable prior probability reflecting the fact that a Trojan planet has never been observed in the set of 5000 or so exoplanet candidates.\(^5\) It is also remarkable that the model selected the appropriate relative phase for the Trojan companion. While the probability of this occurring is not overwhelming, it is on the order of 1/36. However, this correct positioning of a Trojan occurred at the expense of having the secondary transit be only during the Jovian eclipses, but also during hypothetical Trojan eclipses may be mimicking a Trojan-like signal. At this stage, given the available data and the models employed, it is impossible to say anything definitive concerning the presence of a Trojan companion.

5. SUMMARY AND OUTLOOK

We were able to confirm the planetary nature of Kepler-91b using the EXONEST code for photometric analysis of its optical lightcurve obtained with the Kepler Space telescope and radial velocity analysis on data taken from the literature. We find that introducing an additional object to the system can explain the extra dimming in the lightcurve: an EXONEST photometric model including an object in the same orbit as Kepler-91b, but shifted by \( \sim 60^\circ \), produces a significantly higher Bayesian evidence than a model without this sub-Neptunian Trojan companion (see Section 3.1). This model to describe the light curve is not ruled out by radial velocity, and the orbit seems to be stable over the course of 50,000 days, or 8000 cycles. On the other hand, this model also predicted an unphysical day-side temperature and a secondary eclipse depth greater than that of the transit, which would imply that either there are unknown heating mechanisms for such an object, or that it is a false-positive, which is far more likely.

We are able to exclude previously suggested alternative explanations such as the presence of a Moon, a resonant outer planet or instrumental effect for the observed dimmings caused by the hypothetical Trojan (see Sections 1.2–1.4). However, we detect a significant odd/even effect in the phase-folded light curve, and similar dimming events in the light curve folded on randomized periods, both of which could mimic the hypothetical Trojan signal.

The Kepler-91 system is currently of great interest since it is a short-period hot-Jupiter orbiting a star in its red giant stage. If this Trojan planet is confirmed it would not only be the first detection of an exo-Trojan planet, but also would provide the unique opportunity to study two different worlds that are in identical stellar environments thus promising to provide insights into star–planet interactions. Despite this comprehensive investigation of the potential Trojan companion, given the data at hand we cannot conclude that it is the cause of the extra dimming events observed in the Kepler light curve.

\(^5\) Trojans have however been observed in our solar system at the Lagrange points of Venus, Earth, Mars, Jupiter, Uranus, and Neptune. In addition, Saturn’s moon’s Tethys and Dione each have two Trojan moons.

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