Diffusion of conserved charges in relativistic heavy ion collisions

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arXiv:1711.08680
Why is Diffusion Important?

![Graph showing high and low baryon density phases in heavy ion collisions.](image)

- **High Baryon Density**
- **Low Baryon Density**

Baryon Density Gradient → **BARYON DIFFUSION**

**HIC**

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Jan Fotakis  
CRC-TR 211 Transport Meeting
The Evolution in (3+1)-Viscous Hydro

Chun Shen et. al. Nucl. Phys. A 967 (2017) 796-799

Hydrodynamical evolution after 7.5 fm

0-5% Au-Au collision at 19.7 GeV

Hadronization on next slide
Why is Diffusion Important

- At Low-Energy Heavy Ion Collisions (e.g. RHIC BES): diffusion could have great impact on dynamical evolution.

Chun Shen et. al. Nucl. Phys. A 967 (2017) 796-799
Diffusion of Conserved Charges in Relativistic Heavy Ion Collisions

- Early dynamical evolution of HIC modeled in Relativistic Dissipative Fluid-Dynamics
- For large evolution times: Navier-Stokes Theory applicable
- One conserved charge (q):

\[ j_q^\mu : \text{Net-charge diffusion current} \]

Particle 4-current:

\[ N_q^\mu = n_0 u^\mu + \kappa_q \nabla^\mu \left( \frac{\mu_q}{T} \right) \]

Net-charge diffusion coefficient

Gradient in thermal potential ~ Gradient in net-charge density
Description of Diffusion

• In multi-component system with **multiple conserved charges**: particles can have any **combination of charges** (e.g. proton: **electric** and **baryon** charge)

• **Net-charge diffusion currents** effect each other

\[
\begin{pmatrix}
\dot{J}_B^\mu \\
\dot{J}_Q^\mu \\
\dot{J}_S^\mu
\end{pmatrix}
= \begin{pmatrix}
\kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\
\kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\
\kappa_{SB} & \kappa_{SQ} & \kappa_{SS}
\end{pmatrix}
\cdot
\begin{pmatrix}
\nabla^\mu \alpha_B \\
\nabla^\mu \alpha_Q \\
\nabla^\mu \alpha_S
\end{pmatrix}
\]

**Off-diagonal coefficients:** Gradients of given charge can **effect diffusion currents of other charges**

**Are the off-diagonal coefficients important?**
The Chapman-Enskog Expansion

- Assume dilute Boltzmann gas with $N_s$ particle species and conserved baryon, strangeness and electric charge close to local equilibrium $\rightarrow$ describe with kinetic theory

$$f^i_k = f^i_{0,k} + \epsilon f^i_{1,k} + O(\epsilon^2)$$

- Neglect non-linear contributions $\rightarrow$ Navier-Stokes limit
The Chapman-Enskog Expansion

- Relativistic Boltzmann equation determines evolution of system

\[ k_i^\mu \partial_\mu f_k^i = - \sum_{j=1}^{N_s} C_{ij} [f_k^i] \]

Chapman-Enskog expansion

\[ \epsilon k_i^\mu \partial_\mu \left( f_{0k}^i + \epsilon f_{0k}^i \right) \approx \epsilon k_i^\mu \partial_\mu f_{0k}^i = -\epsilon \sum_{j=1}^{N_s} C_{ij} [f_{1k}^i] \]

With linearized collision term:

\[ \sum_{j=1}^{N_s} C_{ij} [f_{1k}^i] = \sum_{j=1}^{N_s} \gamma_{ij} \int dK'^{i} dP_i dP_j W_{kk',pp'}^{ij} f_{0k}^i f_{0k'}^j \left( \frac{f_{1k}^i}{f_{0k}^i} + \frac{f_{1k'}^j}{f_{0k'}^j} - \frac{f_{1p}^i}{f_{0p}^i} - \frac{f_{1p'}^i}{f_{0p'}^i} \right) \]

Transition rate: contains (isotropic) cross sections

= information of microscopic interactions
The Chapman-Enskog Expansion

Evaluating derivatives leads to source equation for deviation \( f^{i}_{1k} \)

\[
k_{i}^{\mu} \partial_{\mu} f^{i}_{0k} = - \sum_{j=1}^{N_{s}} C_{ij} [f^{i}_{1k}]
\]

Gradient in thermal potential

\[
\sum_{q \in \{B,S,Q\}} f^{i}_{0k} k_{i}^{\mu} \left( \frac{E_{ik} n_{q}}{\epsilon_{0} + P_{0}} - q_{i} \right) \nabla_{\mu} \left( \frac{\mu_{q}}{T} \right) = - \sum_{j=1}^{N_{s}} C_{ij} [f^{i}_{1k}]
\]

L.H.S. of eq. \( \sim \) force term due to gradients in particle density \( \rightarrow \) Navier Stokes currents

Sum over all conserved charges \( \rightarrow \) coupling of diffusion currents
The Chapman-Enskog Expansion

Diffusion currents in kinetic theory:

\[ j_q^\mu = \sum_{i=1}^{N_s} q_i \int dK \; k_i^{(\mu)} f_{1k}^i = \sum_{q'} \kappa_{qq'} \nabla^\mu \left( \frac{\mu q'}{T} \right) \]

We want to calculate THIS

Navier-Stokes limit

In order to do so, we need to solve:

\[ \sum_{q \in \{B,S,Q\}} f_{0k}^i k_i^{\mu} \left( \frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla^\mu \left( \frac{\mu q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij} [f_{1k}^i] \]
The Chapman-Enskog Expansion

\[ \sum_{q \in \{ B, S, Q \}} f^i_{0k} k^\mu_i \left( \frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla_\mu \left( \frac{\mu_q}{T} \right) = - \sum_{j=1}^{N_s} C_{i,j} [f^i_{1k}] \]

Since collision term is linear in \( f^i_{1k} \) the solutions have the general form:

scalar function in energy

\[ f^i_{1k} = \sum_q a^i_q k^\mu_i \nabla_\mu \left( \frac{\mu_q}{T} \right) \]

Expand coefficients in power series in energy:

\[ a^i_q = \sum_{m=0}^{\infty} a^i_{q,m} E^m_{ik} \]
The Chapman-Enskog Expansion

\[
\sum_{q \in \{B, S, Q\}} f_{0k}^i k_i^\mu \left( \frac{E_{ik} n_q}{\epsilon_0 + P_0} - q_i \right) \nabla_\mu \left( \frac{\mu_q}{T} \right) = - \sum_{j=1}^{N_s} C_{ij} [f_{1k}^i]
\]

Truncate series at finite integer M and calculate n-th moment of source equation \( \rightarrow \) set of linear equations for expansion coefficients

Solutions of matrix equation \( \rightarrow \) gives us \( f_{1k}^i \)

\[
\sum_{m=0}^{M} \sum_{j=1}^{N_s} \left( A_{nm}^i \delta^{ij} + C_{nm}^{ij} \right) a_{q,m}^j = b_{q,n}^i
\]

moments of collision term \( \rightarrow \) complicated integrals with information about microscopic interactions

Source term for diffusion
The Chapman-Enskog Expansion

\[ j_q^\mu = \sum_{i=1}^{N_s} q_i \int dK \ k_i^{(\mu)} f_{1k}^i = \sum_{q'} \kappa_{qq'} \nabla^\mu \left( \frac{\mu q'}{T} \right) \]

By comparing both sides we find:

\[ \kappa_{qq'} = \frac{1}{3} \sum_{i=1}^{N_s} q_i \sum_{m=0}^{M} a_{q',m}^i \int dK_i E_{ik}^m (m^2 - E_{ik}^2) f_{0k}^i \]

In our most detailed calculation: \( M = 1 \) and \( N_s = 19 \)
The Relaxation Time Approximation

Calculated for $p\ n\ \bar{p}\ \bar{n}\ K\ \pi$ gas (11 hadron species)

$$\sum_{j=1}^{N_s} C_{ij} [f^i_{1k}] = -\frac{E_{ik}}{\tau} f^{i}_{1k}$$

Relaxation time:

Total baryon density

$$\tau^{-1} = \frac{2}{3} n_{B,\text{tot}} \sigma_0$$

Constant cross section
Results

Hadronic resonance gas...

- Use 19 different, massive species $\pi^0,\pi^\pm, K^\pm, K^0, p, \bar{p}, n, \bar{n}, \Sigma^0,\Sigma^\pm, \Lambda, \bar{\Lambda}$
- Isotropic cross sections
- Use PDG data
- Other cross sections: GiBUU, UrQMD or constant
Results

Simplified (conformal) QGP model...

- Use 7 massless species \( u, \bar{u}, d, \bar{d}, s, \bar{s}, g \)

- Simplified approach: Fix shear viscosity to express isotropic cross section in terms of temperature

\[
\frac{\eta}{s} = \frac{1}{4\pi} \quad \Rightarrow \quad \sigma_{tot} = \frac{0.716}{T^2}
\]

Calculate diffusion coefficients for the hadron gas for \( T < 160 \text{ MeV} \) and for higher temperatures in the simplified QGP model \( \rightarrow \) phase transition area is **NOT** covered by our calculations
The diffusion matrix

$$\begin{pmatrix}
j_B^\mu \\
j_Q^\mu \\
j_S^\mu \\
\end{pmatrix} = \begin{pmatrix}
k_{BB} & k_{BQ} & k_{BS} \\
k_{QB} & k_{QQ} & k_{QS} \\
k_{SB} & k_{SQ} & k_{SS} \\
\end{pmatrix} \cdot \begin{pmatrix}
\nabla^\mu \alpha_B \\
\nabla^\mu \alpha_Q \\
\nabla^\mu \alpha_S \\
\end{pmatrix}$$

Diffusion matrix is symmetric! → Onsager Theorem holds
Baryon current

\[
\begin{pmatrix}
\dot{j}_B^\mu \\
\dot{j}_Q^\mu \\
\dot{j}_S^\mu
\end{pmatrix} =
\begin{pmatrix}
\kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\
\kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\
\kappa_{SB} & \kappa_{SQ} & \kappa_{SS}
\end{pmatrix} \cdot
\begin{pmatrix}
\nabla^\mu \alpha_B \\
\nabla^\mu \alpha_Q \\
\nabla^\mu \alpha_S
\end{pmatrix}
\]

- Largest contribution
- Nearly constant at \( \mu_B = 600 \text{ MeV} \)
- So far only used coefficient

- Much smaller than others
- QGP-part vanishes at \( \mu_B = 0 \)
- Strong \( \mu_B \) dependence

- Negative contribution!
- Similar strength as \( \kappa_{BB} \)
- Could drastically reduce baryon current
Electric current

\[
\begin{align*}
\mathbf{j}^{\mu} &= \kappa_{QB} \nabla^{\mu} \alpha_B + \kappa_{QQ} \nabla^{\mu} \alpha_Q + \kappa_{QS} \nabla^{\mu} \alpha_S
\end{align*}
\]

- Smaller than others
- QGP-part vanishes at \( \mu_B = 0 \)
- Strong \( \mu_B \) dependence

- \( \mu_B = 0 \) same as electric conductivity
- Only decreasing behavior in \( T \)

QGP: strongest contribution
Strangeness current

$$j_S^\mu = \kappa_{SB} \nabla^\mu \alpha_B + \kappa_{SQ} \nabla^\mu \alpha_Q + \kappa_{SS} \nabla^\mu \alpha_S$$

1. Negative contribution
2. Could also drastically reduce strange currents
3. 1 Magnitude smaller than \(\kappa_{SS}\)
4. Charged Kaons contribute to electric currents (see \(\kappa_{QQ}\))
5. By far most important contribution
Conclusion

- First calculation of complete diffusion matrix of baryon, electric and strangeness charges in Navier-Stokes limit with first order Chapman-Enskog expansion
- Classical hadron gas with realistic isotropic cross sections and simple conformal QGP model were used

- HRG: dependence of coefficients on temperature and baryo-chemical potential
- Strong coupling of all gradients to (almost) all currents → large off-diagonal coefficients
- Suggestion: Off-diagonal terms should not be neglected!
- Can be used in (hydro) models
Outlook

• Calculation scheme can be used to calculate other Navier-Stokes coefficients

• Investigate effects in viscous hydro simulations → Observables?

• Compare to other models: SMASH? BAMPS? lQCD?