An Estimate of the Branching Fraction of $\tau \to \pi\eta'\nu_\tau$

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We calculate the expected branching fraction of the second-class-current decay $\tau \to \pi\eta'\nu_\tau$, motivated by a recent experimental upper-limit determination of this quantity. The largest contribution to the branching fraction is due to the intermediate $a_0(980)$ scalar meson, assuming it is a $\bar{u}d$ state. Smaller contributions arise from $a_0(1450)$, $\rho(770)$, and $\rho(1450)$. Our calculated values are substantially below the experimental upper limit, and are smaller still if the $a_0(980)$ is a four-quark state, as often suggested. Thus, a precise measurement or tight upper limit has the potential to determine the nature of the $a_0(980)$, as well as search for new scalar interactions.

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I. INTRODUCTION

In a recent paper [1], we considered the branching fraction of the isospin-violating decay $\tau \to \pi\eta'\nu_\tau$. We found an expected branching fraction of

$$B \equiv B(\tau \to \pi\eta'\nu_\tau) = (0.3 - 1.0) \times 10^{-5},$$

in rough agreement with a detailed chiral-perturbation-theory calculation [2] and other evaluations [3], which yielded central values in the range

$$B = (1.2 - 1.6) \times 10^{-5}.$$  \hspace{1cm} (2)

The experimental bound on this branching fraction, $B < 1.4 \times 10^{-4}$ [1], was obtained by CLEO with an $e^+e^-$-collision data sample of 3.5 fb$^{-1}$, a fraction of a percent of currently available integrated luminosity. The only related high-luminosity measurement is a stringent $\overline{\text{B}}\overline{\text{B}}$R upper limit on the branching fraction of $\tau \to \pi\eta'\nu_\tau$ [2],

$$B' \equiv B(\tau \to \pi\eta'\nu_\tau) < 7.2 \times 10^{-6} \text{ @90\% CL},$$

obtained with an integrated luminosity of 384 fb$^{-1}$.

The fact that the experimental limit is lower than the results summarized in Eq. (2) raises the question of a possible discrepancy between theory and experiment. Therefore, our goal in this article is to calculate the expected value of $B'$ and compare it to the experimental limit. We adapt the methods used in Ref. [1] to the present case, noting that a chiral-perturbation-theory calculation of this process, as performed for $\tau \to \pi\nu_\tau$ by Neufeld and Rupertsberger [2], would be very useful.

First, we note several similarities and differences between the calculations of $B'$ and $B$:

- The $\bar{u}u + \bar{d}d$ fraction of the wave function which, unlike the $s\bar{s}$ and $gg$ parts, contributes to the decay amplitude, may be smaller for the $\eta'$. While it appears that the magnitude of the $s\bar{s}$ part in relation to that of the light quarks is very similar for both states, the current estimate of the $gg$ fraction of the wave function, $Z_{gg}$, is $|Z_{gg}|^2 = 0.3 \pm 0.2$ [2].

In our calculations we take $Z_{gg} = 0$, as this yields the most conservative limits on $B'$, and since the modification for finite values of $Z_{gg}$ is straightforward.

- Calculations of $B$ in Refs. [1,2,3] rely on extrapolations utilizing intermediate, low-mass $J^{PC} = 1-$ and $0^{++}$ hadrons. Obvious intermediate states for the decay $\tau \to \pi\eta'\nu_\tau$ are the ground-state mesons $\rho(770)$ and $a_0(980)$. In the case of $\tau \to \pi\eta'\nu_\tau$, these are off-shell processes, and the contributions of these resonances are suppressed. On the other hand, we do have now on-shell decays involving the next $1^{--}$ and $0^{++}$ states. These are the $\rho' \equiv \rho(1450)$ and $a_0(1450)$, which contribute to the $P$- and $S$-wave components of the decay, respectively.

- The $\rho$ and $\rho'$ vectors are the quark-model $\bar{u}d$, $S$-wave $1^{--}$ ground state and first radial excitation, respectively. However, the theoretical assignment of the $a_0(980)$ (and, consequently, that of the $a_0(1450)$ as well) is ambiguous, generating the largest uncertainty in both $B$ and $B'$. Conversely, information on these branching fractions can help resolve the longstanding dilemma of the $K\bar{K}$ "threshold" state $a_0(980)$. The significant branching fractions of $a_0(980)$ and $f_0(980)$ decays to $K\bar{K}$, despite the very small phase space, seem inconsistent with these mesons being the ground states of the quark-model scalar nonet, motivating a four-quark ($\bar{u}d\bar{s}s$) interpretation [7]. In this case, the $\bar{u}d$ scalar ground state should most likely be identified with $a_0(1450)$. However, this would make the scalar 190 MeV heavier than the axial vector state $a_1(1260)$, implying a pattern of $L$-$S$ splitting different from what is observed in any other $L = 1$, $qq'$ system. The more appealing possibility, namely, that the two 980-MeV states are indeed just $\bar{u}d$ states, may have been partially resurrected in recent work [8], in which 'tHooft's $\bar{u}d\bar{s}s$s six-quark vertex was utilized to admix the 2- and 4- quark states.
The plan of this note is as follows. As we did in Ref. [1], we discuss separately our estimates of the $P$- and $S$-wave contributions to $B'$. In Sec II we present the more robust results for the $P$-wave part, calculating upper bounds on the contributions of the $\rho$ and $\rho'$ using recently published experimental data involving $\eta'$ and $\tau$ decays. In Sec III we present the less clear-cut estimate of the $S$-wave component. This contribution depends most strongly on whether the $a_0(980)$ is a 4-quark state or the $\bar{u}d$ ground state. In any event, our predictions for $B(\tau \rightarrow \pi \eta' \nu_\tau)$ lie significantly below the BABAR limit [3]. A brief summary and future outlook are given in Sec IV.

II. THE $L=1$ CONTRIBUTION

In Ref. [1], we obtained the $L=1$ contribution to $B$ assuming that it was dominated by the $\rho$, an assumption justified by the large branching fraction $B(\tau \rightarrow \nu_\tau)$. We compared this branching fraction to $B$ using the ratio of coupling constants $g_{\eta \rho \pi}/g_{\rho \pi \pi}$, where $g_{\rho \pi \pi}$ was related to the width of the $\rho$, and $g_{\eta \rho \pi}$ was obtained by analyzing the Dalitz-plot distribution of the decay $\eta \rightarrow \pi^+ \pi^- \pi^0$, taking the scalar contribution to $\eta \rightarrow \pi^+ \pi^- \pi^0$ from $B(\eta \rightarrow \pi^0 \pi^0 \pi^0)$.

This procedure is not directly applicable to $B'$, since there is no experimental information on the Dalitz-plot distribution of the decay $\eta' \rightarrow \pi^+ \pi^- \pi^0$, nor a measurement of $B(\eta' \rightarrow \pi^0 \pi^0 \pi^0)$. Therefore, we make use of the fact that the branching fraction $B(\eta' \rightarrow \pi^+ \pi^- \pi^0)$ depends on the coupling constant $g_{\rho \pi \pi}$, under the conservative assumption that the $\rho^\pm$ states dominate the decay $\eta' \rightarrow \pi^+ \pi^- \pi^0$. This will yield a conservative upper bound on $g_{\eta' \rho \pi}$, from which we obtain an upper bound on the $\rho$ contribution to $\tau \rightarrow \pi \eta' \nu_\tau$. We discuss the likelihood of this assumption and its implications below.

The differential branching fraction of $\eta' \rightarrow \pi^+ \pi^- \pi^0$ as a function of the Dalitz-plot position is given by

$$\frac{d\Gamma(\eta' \rightarrow \pi^+ \pi^- \pi^0)}{\Gamma(\eta')} = \frac{(g_{\eta' \rho \pi}/g_{\rho \pi \pi})^2 Q^2}{384\sqrt{3}\pi^3} m_{\eta'} |\tilde{M}|^2 dX dY,$$  

where

$$Q \equiv m_{\eta'} - 3m_\pi$$

is the kinetic energy in the decay, and

$$X \equiv \sqrt{\frac{3}{Q}} (T_+ - T_-),$$

$$Y \equiv \frac{3}{Q} T_0 - 1$$

are the Dalitz-plot variables, with $T_\epsilon$ being the kinetic energy of the pion with charge $c$. Assuming $\rho$ dominance, we obtain from Eq. (15) of Ref. [1] the reduced matrix element

$$\tilde{M} = -\frac{m_{\eta'}}{m_\rho^2 - \frac{1}{3}m_{\eta'}^2} \frac{Q^2}{1 - \frac{2}{3}r^2(Y^2 + X^2)} (Y^2 - X^2),$$

where

$$r = \frac{m_{\eta'} Q}{m_\rho^2 - \frac{1}{3}m_{\eta'}^2 - m_\pi^2 - i\Gamma_\rho m_\rho} = 1.6 + 0.7i.$$  

The product $(g_{\eta' \rho \pi}/g_{\rho \pi \pi})^2$ is then found by integrating Eq. (4) over the Dalitz plot. In the $\eta \rightarrow \pi^+ \pi^- \pi^0$ case, we exploited the small value of $r$ to simplify the expression by expanding in $r$. Due to the $O(1)$ value of $r$ for $\eta' \rightarrow \pi^+ \pi^- \pi^0$, we resort to numerical integration, which yields

$$\int |\tilde{M}|^2 dX dY = 2.4.$$  

From this we obtain, using $B(\eta' \rightarrow \pi^+ \pi^- \pi^0) = 3.7 \times 10^{-3}$ [2] and $g_{\rho \pi \pi} = 6.0$ [1],

$$g_{\eta' \rho \pi} < 0.025.$$  

As a cross check, we apply the procedure to the decay $\eta \rightarrow \pi^+ \pi^- \pi^0$, obtaining $g_{\eta \rho \pi} < 0.52$. This value is to be compared to the one obtained from the more precise Dalitz-plot analysis in Ref. [1], $g_{\eta \rho \pi} \approx 0.085$. The factor of 6 ratio between the results reflects the fact that the procedure used here yields but a conservative upper bound, obtained by assuming that the decay $\eta' \rightarrow \pi^+ \pi^- \pi^0$ is dominated by the $\rho^\pm$ resonances. This assumption is manifestly false, as the $\eta' \rightarrow \pi^+ \pi^- \pi^0$ Dalitz-plot distribution is in much better agreement with a flat distribution than with that expected from $\rho^\pm$ dominance [6]. By contrast, in Ref. [1], the value of $g_{\eta \rho \pi}$ obtained from the Dalitz-plot distribution yielded good agreement between the expected and measured values of $B(\eta \rightarrow \pi^+ \pi^- \pi^0)$.

With this point in mind, we proceed to use the upper bound on $g_{\eta' \rho \pi}$ to calculate the upper bound on the $\rho$ contribution to $B(\tau \rightarrow \pi \eta' \nu_\tau)$. We do this by relating $B(\tau \rightarrow \rho(\pi \eta') \nu_\tau)$ to $B(\tau \rightarrow \rho(\pi \eta) \nu_\tau)$ via the ratio of coupling constants and phase-space factors

$$\frac{B(\tau \rightarrow \rho(\pi \eta') \nu_\tau)}{B(\tau \rightarrow \rho(\pi \eta) \nu_\tau)} \approx \frac{(g_{\eta' \rho \pi}/g_{\rho \pi \pi})^2}{V(\tau \rightarrow \rho(\pi \eta') \nu_\tau)} V(\tau \rightarrow \rho(\pi \eta) \nu_\tau),$$  

where $\rho(\pi \eta')$ indicates that the $\rho$ is observed in the $\pi \eta'$ final state, and $V(X)$ is the integral over the Dalitz plot of the three-body decay $X$. The ratio of phase-space integrals is 0.06, with up to 15% variation depending on whether one uses Blatt-Weisskopf and $s$-dependent widths for the $\rho$ and on the choice of angular distribution. Using $B(\tau \rightarrow \rho(\pi \eta) \nu_\tau) = 3.6 \times 10^{-6}$ [1], we obtain

$$B(\tau \rightarrow \rho(\pi \eta') \nu_\tau) < 2 \times 10^{-8},$$  

more than two orders of magnitude below the BABAR upper limit, Eq. [3].

Next, we evaluate the contribution of the on-shell $\rho'$. One expects that this state, being a radial excitation and hence having a node in its wave-function, couples to the ground-state particles $\eta$ and $\pi$ more weakly than
the $\rho$. We hypothesize that this $\rho'$ suppression mechanism works equally strongly for the final states $\pi\eta'$ and $\pi\pi$, leading to an equality of the ratios of the squared matrix elements

\[
\frac{\mathcal{B}(\tau \to \rho' (\pi\eta')_{\mu\nu})}{\mathcal{B}(\tau \to \rho(\pi\eta')_{\mu\nu})} = \frac{V(\tau \to \rho' (\pi\eta')_{\mu\nu})}{V(\tau \to \rho(\pi\eta')_{\mu\nu})} \approx \frac{\mathcal{B}(\tau \to \rho' (\pi\eta')_{\mu\nu})}{\mathcal{B}(\tau \to \rho(\pi\eta')_{\mu\nu})},
\]

(13)

The relevant phase-space integral ratios are

\[
\frac{V(\tau \to \rho(\pi\eta')_{\mu\nu})}{V(\tau \to \rho' (\pi\eta')_{\mu\nu})} \approx 0.06,
\]

(14)

\[
\frac{V(\tau \to \rho(\pi\eta')_{\mu\nu})}{V(\tau \to \rho' (\pi\eta')_{\mu\nu})} \approx 2.5.
\]

We use the upper bound of Eq. (12) and the central value plus one standard deviation of the recent Belle result \[10\]

\[
\sqrt{\frac{\mathcal{B}(\tau \to \rho' (\pi\eta')_{\mu\nu})}{\mathcal{B}(\tau \to \rho(\pi\eta')_{\mu\nu})}} = 0.15 \pm 0.05 \pm 0.15 -0.04
\]

(15)

to obtain the conservative upper limit

\[
\mathcal{B}(\tau \to \rho' (\pi\eta')_{\mu\nu}) < 8 \times 10^{-8}.
\]

(16)

We note that this is an upper bound both due to the way we use Eq. (15) and since Eq. (12) is an upper bound.

### III. THE $L = 0$ CONTRIBUTION

Calculating the $L = 0$ contributions to $\mathcal{B}'$ is not as straightforward as the $L = 1$ case, where one can make use of the dominant $\rho$ coupling to the leptonic vector current. Therefore, is important to evaluate the scalar component using different methods, as has been done for the $\tau \to \pi\eta\tau$ decay \[4, 7, 8\]. It should be noted that these calculations are performed under the assumption that the relevant scalar resonances are $\bar{u}d$ states. The coupling of a 4-quark state to the $\bar{u}d$ scalar current is "Zweig-Rule" suppressed, making it significantly smaller than the predictions.

Here we perform a more detailed version of the calculation used in Ref. [1]. We begin with the ratio of branching fractions

\[
\frac{\mathcal{R}^{\bar{u}}_{a_1}}{\mathcal{R}^{\bar{u}}_{a_1}} = \frac{\mathcal{B}(\tau \to a_0 \nu_{\tau})}{\mathcal{B}(\tau \to a_1 \nu_{\tau})} = \frac{p_{a_0}}{p_{a_1}} \times \frac{|\langle a_0 | V_{u\mu} | 0 \rangle |^2}{|\langle a_1 | V_{u\mu} | 0 \rangle |^2} \times \frac{\langle \bar{a}_0 | J^\mu_{\tau} | \xbar{\psi}_a \rangle |^2}{\langle \bar{a}_1 | J^\mu_{\tau} | \xbar{\psi}_a \rangle |^2}.
\]

(17)

where $a_0$ stands for either $a_0(980)$ or $a_0(1450)$, $a_1$ is the $a_1(1260)$, $p_X$ is the $\tau$-rest-frame momentum of the products of the decay $\tau \to X \nu_{\tau}$, $V_{u\mu} \equiv \bar{\psi}_u (x) \gamma_\mu \psi_d (x)$ is the hadronic vector current, $A_{u\mu} \equiv \bar{\psi}_u (x) \gamma_\mu \gamma_5 \psi_d (x)$ is the hadronic axial vector current, and $J^\mu_{\tau} \equiv \bar{x}_{\tau} (x) \gamma^\mu (1 - \gamma_5) x_{\tau} (x)$ is the leptonic current. The calculation of the leptonic parts of this ratio is well defined, while all the uncertainty in the hadronic parts comes down to a single parameter $\xi$, which shall be defined shortly. With this in mind, we can take the $a_0$ matrix element to be

\[
\langle a_0 | V_{u\mu} | 0 \rangle = f_0 \frac{q_\mu}{m_{a_0}} \langle a_0 | S_h | 0 \rangle,
\]

(18)

where $f_0$ is an isospin-violation suppression factor, and $S_h \equiv \bar{\psi}_a (x) \psi_d (x)$ is the scalar current operator. The weak vector current is conserved up to the difference between the $u$- and $d$-quark masses, plus a smaller electromagnetic part that we neglect. Therefore,

\[
\partial^\mu V_{h\mu} \approx (m_d - m_u) S_h.
\]

(19)

Using this relation in Eq. (18) yields

\[
f_0 = \frac{m_d - m_u}{m_{a_0}}.
\]

(20)

We use the fact that both the $a_0$ and the $a_1$ are $P$-wave states to relate the axial and scalar decay constants

\[
\langle a_1 | A_{\mu} | 0 \rangle = \xi \langle a_0 | S_{\xi} | 0 \rangle.
\]

(21)

We note that this is reminiscent of applying $SU(6)$ \[11\] or, in this case, just $SU(4)$ \[12\] flavor-spin symmetry to the $(L = 0)$ 15-plet plus singlet containing the $\pi$, $\rho$, $\eta$, and $\omega$, or the $(L = 1)$ states $a_0$, $a_1$, $f_0$, and $h_1$.

Naively, one expects $\xi$ in Eq. (21) to be of order unity. However, this parameter incorporates all the hadronic uncertainty in our procedure. With Eqs. (18)-(21), Eq. (17) becomes, after spin averaging and index contraction,

\[
\mathcal{R}^{\bar{u}}_{a_1} = |\xi|^2 \frac{p_{a_0}}{p_{a_1}} \left( \frac{m_d - m_u}{m_{a_0}} \right)^2 \times \frac{m_k^2 - m_{a_1}^2}{1 + 2 (m_\tau/m_{a_1})^2}.
\]

(22)

This yields the branching fractions

\[
\mathcal{B}(\tau \to a_0(980) \nu_{\tau}) = 1.6 \times 10^{-6} |\xi|^2,
\]

\[
\mathcal{B}(\tau \to a_0(1450) \nu_{\tau}) = 6.4 \times 10^{-8} |\xi|^2,
\]

(23)

where, as in Ref. [1], we chose the mass difference of the two light quarks to be 4 MeV \[13\] and, assuming that the $\tau \to 3 \pi \nu_{\tau}$ decay is dominated by the $a_0$, we took $\mathcal{B}(\tau \to a_1 \nu_{\tau}) = 0.18$. We compare $\mathcal{B}(\tau \to a_0(980) \nu_{\tau})$ of Eq. (23) with the value $\mathcal{B} = 1.2 \times 10^{-5}$, obtained from the more elaborate calculation of Ref. [8]. Minus the $\rho$ contribution to $\mathcal{B}$, which is $3.6 \times 10^{-6}$ \[1\]. This yields $|\xi|^2 \approx 5$, from which we conclude

\[
\mathcal{B}(\tau \to a_0(1450) \nu_{\tau}) \approx 3 \times 10^{-7}.
\]

(24)

The $a_0(1450)$ contribution to $\tau \to \pi\eta' \nu_{\tau}$ depends also on the branching fraction $\mathcal{B}(a_0(1450) \to \pi\eta')$, regarding
which there is only partial information. However, from the branching-fraction measurements that have been made \cite{13}, it is clear that \( B(a_0(1450) \to \pi \eta') < 0.3 \). Hence

\[
B(\tau \to a_0(1450)_{(\pi \eta')} \nu_\tau) < 1 \times 10^{-7}. \tag{25}
\]

If the \( a_0(1450) \) is a radial excitation, which is the case if the \( a_0(980) \) is the \( \bar{u}d \) ground state, then \( B(\tau \to a_0(1450)_{(\pi \eta')} \nu_\tau) \) should be suppressed by an additional wave-function overlap factor.

Next, we look at the contribution of the \( a_0(980) \) to \( \tau \to \pi \eta' \nu_\tau \), which can be extracted from the relation

\[
\frac{B(\tau \to \nu a_0(980)_{(\pi \eta'})}{B(\tau \to \nu a_0(980)_{(\pi \eta)})} = \frac{V(\tau \to \nu a_0(980)_{(\pi \eta')})}{V(\tau \to \nu a_0(980)_{(\pi \eta)})} R_{\eta}^{\eta'}, \tag{26}
\]

where

\[
R_{\eta}^{\eta'} = \left| \frac{\mathcal{M}(a_0(980) \to \pi \eta')}{\mathcal{M}(a_0(980) \to \pi \eta)} \right|^2 \tag{27}
\]

is the square of the ratio between the relevant hadronic-decay matrix elements. We assume that \( R_{\eta}^{\eta'} \) equals the corresponding ratio of \( a_0(1450) \)-decay matrix elements, and is hence obtained from

\[
R_{\eta}^{\eta'} \approx \frac{B(a_0(1450) \to \pi \eta')}{B(a_0(1450) \to \pi \eta)} \times \frac{p_{\eta'}}{p_\eta}, \tag{28}
\]

where \( p_X \) is the \( a_0(1450) \)-rest-frame momentum of the products of the decay \( a_0(1450) \to \pi X \). Given the \( \sim 50\% \) error \cite{13} on the ratio of branching fractions appearing in Eq. \( \tag{28} \) and the uncertainty on the \( a_0(1450) \) width, \( R_{\eta}^{\eta'} \) comes out in the range \([0.25, 1.25]\). The ratio of the phase-space integrals in Eq. \( \tag{29} \) is 0.06, with some dependence on what one takes for the \( a_0(980) \) width. Using the range for \( B \) from Eq. \( \tag{2} \), we obtain

\[
B(\tau \to a_0(980)_{(\pi \eta')} \nu_\tau) \approx [0.2 \text{ to } 1.2] \times 10^{-6}. \tag{29}
\]

**IV. CONCLUSIONS**

Combining Eqs. \( \tag{12}, \tag{13}, \tag{25}, \) and \( \tag{29} \), we obtain the branching fraction limit

\[
B(\tau \to \pi \eta' \nu_\tau) < 1.4 \times 10^{-6}, \tag{30}
\]

in no conflict with the experimental upper limit, Eq. \( \tag{33} \), which is about five times greater. Our result is dominated by the \( a_0(980) \) contribution, assuming it is a \( \bar{u}d \) state.

The experimental limit was obtained with only a third of the currently available BaBar and Belle data sets, and with the \( \eta \) reconstructed only in the \( \gamma \gamma \) final state. Therefore, an improvement in the limit can be expected from the current generation of \( B \) factories, but probably not to the level of Eq. \( \tag{30} \). By contrast, a Super \( B \) factory \cite{14}, with two orders of magnitude more luminosity, will be able to use \( B \) and \( B' \) to investigate the nature of the \( a_0(980) \) and to search for new interactions mediated by heavy scalars \cite{1}.

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\[\text{[1] S. Nussinov and A. Soffer, Phys. Rev. D 78, 033006 (2008) [arXiv:0806.3022 [hep-ph]].}\]
\[\text{[2] H. Neufeld and H. Rupertsherberger, Z. Phys. C 68, 91 (1995).}\]
\[\text{[3] A. Pich, Phys. Lett. B 196, 561 (1987); S. Tisserant and T. N. Truong, Phys. Lett. B 115, 264 (1982);}\]
\[\text{[4] J. E. Bartelt et al. [CLEO Collaboration], Phys. Rev. Lett. 76, 4119 (1996).}\]
\[\text{[5] B. Aubert et al. [BaBar Collaboration], Phys. Rev. D 77, 112002 (2008) [arXiv:0803.0772 [hep-ex]].}\]
\[\text{[6] R. Escirbano, arXiv:0807.4201 [hep-ph]; F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B 648, 267 (2007) [arXiv:hep-ex/0612029].}\]
\[\text{[7] R. L. Jaffe, Phys. Rev. D 15, 267 (1977).}\]
\[\text{[8] G. 't Hooft, G. Isidori, L. Maiani, A. D. Polosa and V. Riquer, arXiv:0801.2288 [hep-ph].}\]
\[\text{[9] P. Naik et al. [CLEO Collaboration], Phys. Rev. Lett. 102, 061801 (2009) [arXiv:0809.2587 [hep-ex]].}\]
\[\text{[10] M. Fujikawa et al. [Belle Collaboration], Phys. Rev. D 78, 072006 (2008) [arXiv:0805.3773 [hep-ex]].}\]
\[\text{[11] F. Gursey and L. A. Radicati, Phys. Rev. Lett. 13, 173 (1964).}\]
\[\text{[12] E. Wigner, Phys. Rev. 51, 106 (1937).}\]
\[\text{[13] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).}\]
\[\text{[14] M. Bona et al., arXiv:0709.0451 [hep-ex].}\]