Observations of Mira stars with the IOTA/FLUOR interferometer and comparison with Mira star models

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Abstract

We present K'-band observations of five Mira stars with the IOTA interferometer. The interferograms were obtained with the FLUOR fiber optics beam combiner, which provides high-accuracy visibility measurements in spite of time-variable atmospheric conditions. For the M-type Miras X Oph, R Aql, RU Her, R Ser, and the C-type Mira V CrB we derived the uniform-disk diameters 11.7 mas, 10.9 mas, 8.4 mas, 8.1 mas, and 7.9 mas ($\pm 0.3$ mas), respectively. Simultaneous photometric observations yielded the bolometric fluxes. The derived angular Rosseland radii and the bolometric fluxes allowed the determination of effective temperatures. For instance, the effective temperature of R Aql was determined to be $2970 \pm 110$ K. A linear Rosseland radius for R Aql of $250^{+100}_{-60}$ $R_\odot$ was derived from the angular Rosseland radius of 5.5 mas $\pm 0.2$ mas and the HIPPARCOS parallax of 4.73 mas $\pm 1.19$ mas. The observations were compared with theoretical Mira star models.
of Bessel et al. (1996) and Hofmann et al. (1998). The effective temperatures of the M-type Miras and the linear radius of R Aql indicate fundamental mode pulsation.

Key words: instrumentation: interferometers – stars: AGB and post-AGB – stars: late-type – stars: variables – stars: individual (X Oph, R Aql, RU Her, R Ser, V CrB)

1 Introduction

The resolution of large optical telescopes and interferometers is high enough to resolve the stellar disk of nearby M giants, to reveal photospheric asymmetries and surface structures, and to study the dependence of the diameter on wavelength and variability phase (see e.g. pioneering work by Bonneau & Labeyrie, 1973; Karovska et al., 1991). Theoretical studies (e.g. Watanabe & Kodaira, 1979; Scholz, 1985; Bessel et al., 1989, 1996; Hofmann et al., 1998) show that accurate monochromatic diameter measurements can significantly improve our understanding of M giant atmospheres. With the IOTA (= Infrared-Optical Telescope Array) interferometer, a resolution of ~11.9 mas can be achieved with its largest baseline of 38 m in the K’-band. The IOTA interferometer is located at the Smithsonian Institution’s Whipple Observatory on Mount Hopkins in Arizona. A detailed description of IOTA can be found in Carleton et al. (1994) and Traub et al. (1998). IOTA can be operated in the K-band with the FLUOR (= Fiber Linked Unit for Optical Recombination, Foresto et al. 1997) fiber optics beam combiner. This beam combiner provides high-accuracy visibility measurements in spite of time-variable atmospheric conditions. The single-mode fibers in the beam combiner spatially filter the wavefronts corrugated by atmospheric turbulence (Foresto et al., 1997; Perrin et al., 1998).

2 Observations

The five Miras X Oph, R Aql, RU Her, R Ser, V CrB were observed with the IOTA interferometer on May 16, 17 and 18, 1999. The observations were carried out with the fiber optics beam combiner FLUOR in the K’-band (interference filter with center wavelength / bandwidth of 2.13µm/0.30µm) and

* Based on observations collected at the IOTA/FLUOR interferometer, Whipple Observatory, Mount Hopkins, Arizona.
with 38 m baseline (maximum baseline of the IOTA interferometer). The interferograms were scanned by the delay line during the coherence time of the atmosphere. The OPD length of the scan was \( \sim 100 \mu m \) (OPD = Optical Path Difference). Approximately 100 scans per baseline were recorded. Several reference stars (Table 1) were observed for the calibration of the observations. The calibrated visibilities of the five Miras were obtained with the FLUOR data reduction software package described in Foresto et al. (1997) and Perrin et al. (1998). Preliminary results of these observations have been presented in Hofmann et al. (2000b). Fig. 1 shows the obtained visibility functions of the five Mira stars together with uniform-disk fits. The errors of the derived Mira star diameters are 2.5-3%. In Table 1 the calibrated visibilities and the derived uniform-disk diameters of the five Miras are listed, together with observational parameters (spectral type, variability period \( P \), date of observation, variability phase \( \Phi_{vis} \) in visual light, projected baseline length \( B_p \), calibrated visibilities \( V \), derived uniform-disk diameters \( \Theta_{UD} \), the HIPPARCOS numbers of the reference stars and their uniform disk diameters \( \Theta_{UD,\text{ref}} \)). All of the observed Mira stars have UD diameters smaller than the diffraction-limited resolution \( \lambda/b = 11.9 \) mas of an interferometer with projected baseline \( b = 38 \) m (= largest projected IOTA baseline; \( \lambda = 2.13 \mu m \)). Therefore, all measured visibility points lie at spatial frequencies below the first zero of the UD visibility function of the observed stars (see Fig. 1; a baseline length of 38 m and wavelength of 2.13 \( \mu m \) correspond to a spatial frequency of 85.6 cycles/arcsec). Nevertheless, the UD diameter of the observed stars can be derived from the fit of a model UD visibility function to the measured visibilities.

### Table 1

| Star   | spectral type | \( P \) [days] | Date     | \( \Phi_{vis} \) | \( B_p \) [m] | \( V \) | \( \Theta_{UD} \) | ref. stars | \( \Theta_{UD,\text{ref}} \) [mas] |
|--------|---------------|----------------|----------|----------------|----------------|------|-----------------|-------------|-------------------|
| X Oph  | M5e-M9e       | 328            | 99 May 17| 0.71           | 35.47          | 0.2317±0.024 | 11.74±0.30 | 86742       | 4.6±0.5            |
|        |               |                | 99 May 18| 34.75          | 0.2554±0.027   | 98337 | 6.2±0.6        |
|        |               |                | 99 May 18| 34.57          | 0.2279±0.025   | 98438 | 6.1±0.6        |
|        |               |                |          |                | 97278          | 6.8±0.7|
| R Aql  | M5e-M9e       | 284            | 99 May 17| 0.17           | 35.42          | 0.2927±0.027 | 10.90±0.33 | 86742       | 4.6±0.5            |
|        |               |                | 99 May 18| 34.48          | 0.3295±0.031   | 98337 | 6.2±0.6        |
|        |               |                |          |                | 98438          | 6.1±0.6 |
|        |               |                |          |                | 97278          | 6.8±0.7 |
| RU Her | M6e-M9        | 484            | 99 May 17| 0.07           | 37.95          | 0.4768±0.017 | 8.36±0.20  | 71053       | 4.0±0.4            |
|        |               |                |          |                | 37.73          | 0.4769±0.017 | 78159      | 2.8±0.3        |
| R Ser  | M5e-M9e       | 356            | 99 May 18| 0.28           | 35.74          | 0.5467±0.016 | 8.10±0.20  | 61658       | 3.7±0.4            |
|        |               |                |          |                | 75530          | 3.5±0.4  |
|        |               |                |          |                | 85934          | 3.8±0.4  |
| V CrB  | C6,2e(N2e)    | 357            | 99 May 16| 0.07           | 37.78          | 0.5288±0.017 | 7.86±0.24  | 73555       | 2.5±0.3            |
|        |               |                |          |                | 38.02          | 0.5180±0.023 | 81833      | 2.5±0.3        |
2.1 Calibrated visibilities, stellar diameters, errors

The transfer function of the whole instrument at the time of the observation of the object was derived from visibility measurements of several reference stars (2 to 5 different reference stars per object) before and after the object observations. Indirect estimates of the diameter of reference stars had to be used, since only few stellar diameters have been measured up to now. The diameters of the reference stars were derived from the list of angular diameters for giants at K=0 according to Dyck et al. (1996). The accuracy of the estimated diameters of the reference stars is 10%. The transfer function was directly computed as the ratio between the measured fringe contrast and the visibility of the reference star. The visibility of the reference star was derived from its uniform-disk diameter estimated by the method of Dyck et al. (1996). Note, that the errors of the target star diameters are much smaller than 10%. The error $\sigma_{\Theta_{\text{Obj}}}$ of the fitted UD diameter of the target star caused by the uncertainty $\sigma_{\Theta_{\text{Ref}}}$ of the diameter of the reference star is given, according to the Gaussian error propagation law, by

$$
\frac{\sigma_{\Theta_{\text{Obj}}}}{\Theta_{\text{Obj}}} = \left| \frac{V(\Theta_{\text{Obj}})}{V(\Theta_{\text{Ref}})} \right| \cdot \left| \frac{\partial V(\Theta_{\text{Ref}})/\partial \Theta}{\partial V(\Theta_{\text{Obj}})/\partial \Theta} \right| \cdot \frac{\Theta_{\text{Ref}}}{\Theta_{\text{Obj}}} \cdot \frac{\sigma_{\Theta_{\text{Ref}}}}{\Theta_{\text{Ref}}},
$$

where $V(\Theta_{\text{Obj}})$ and $V(\Theta_{\text{Ref}})$ are the visibilities of the target and reference star, respectively. If, for example, one considers R Ser, then $\Theta_{\text{Obj}}=8.10$ mas, $\Theta_{\text{Ref}}=3.81$ mas, $V(\Theta_{\text{Obj}})=0.55$, $V(\Theta_{\text{Ref}})=0.88$, $\partial V(\Theta_{\text{Obj}})/\partial \Theta=0.09$ mas$^{-1}$, and $\partial V(\Theta_{\text{Ref}})/\partial \Theta=0.06$ mas$^{-1}$. According to Eq.(1), the relative error of the UD diameter of the (larger) target star is 2%, while the diameter error of the reference star is 10%.

The total errors of the calibrated visibilities presented in Table 1 take into account the following errors:

- the statistical error of the raw visibility measurement in each star (e.g. photon and detector noise);
- the errors of the measured gain matrix (link between photometric and interferometric signals, Foresto et al. [1997]);
- the uncertainty on the predicted visibility of the reference star (the error of the estimated diameter of the reference star was assumed to be 10% according to Dyck et al. [1996]);
- the uncertainties of the transfer function at the time of the object measurement (the transfer function is interpolated between measurements of the reference star before and after the object observation; see details in Perrin et al. [1998]).
In addition to the reduction, based on all available reference stars, each target star was also separately reduced with each of the 2-5 different reference stars. All these calibrated visibilities are within the error bars given in Table 1.

The uniform-disk diameters and the diameters corresponding to a model center-to-limb intensity variation (hereafter CLV) were derived from the calibrated visibilities with a $\chi^2$ fit:

$$
\chi^2 = \sum_{i=1}^{N} \frac{|V_i - V(\Theta, u_i)|^2}{\sigma_i^2},
$$  \hspace{1cm} (2)
where \( V_i \) denotes the measured visibility at spatial frequency \( u_i \) with the error \( \sigma_i \), \( V(\Theta, u_i) \) is the visibility of the theoretical CLV (uniform-disk or any model CLV) at frequency \( u_i \) as a function of the disk diameter \( \Theta \), and \( N \) denotes the number of measured visibilities. The diameter error \( \sigma_{\Theta_{\text{Obj}}} \) of the fitted stellar diameter \( \Theta_{\text{Obj}} \) is linked, according to the Gaussian error propagation law, to the total errors \( \sigma_i \) of the derived object visibilities \( V_i \) by

\[
\frac{1}{\sigma^2_{\Theta_{\text{Obj}}}} = \sum_{i=1}^{N} \frac{|a_i|^2}{\sigma_i^2},
\]

where \( a_i \) denotes the derivative \( \partial V(\Theta_{\text{Obj}}, u_i)/\partial \Theta \) of the visibility of the theoretical CLV at the fitted disk diameter \( \Theta_{\text{Obj}} \). The diameter errors listed in Table 1 and used below are derived from the above relation.

### 3 Comparison of the observations with Mira star models

In this section we derive angular diameters from the measured visibilities by fitting different theoretical center-to-limb intensity variations of different Mira star models (Bessel et al. (1996) = BSW96, Hofmann et al. (1998) = HSW98). From these angular diameters and the bolometric fluxes, we derive effective temperatures. For R Aql a HIPPARCOS parallax is available, which allows us to determine linear radii. The comparison of these measured stellar parameters with theoretical ones, indicate whether any of the models is a fair representation of the observed Mira stars. All Mira star models used in this paper are from BSW96 (D and E series) and from HSW98 (P, M and O series). They were developed as possible representations of the prototype Mira variable o Ceti, and hence have periods \( P \) very close to the 332 day period of this star; they differ in pulsation mode, assumed mass \( M \) and assumed luminosity \( L \); and the BSW96 models differ from the (more advanced) HSW98 models with respect to the pulsation modelling technique. The five models represent stars pulsating in the fundamental mode (\( f \); D, P and M models) or in the first-overtone mode (\( o \); E and O models). Table 2 lists the properties of these Mira model series (\( R_p \) = Rosseland radius of the non-pulsating “parent” star of
Table 3
Link between the 22 abscissa values (model-phase combinations m) in Figs. 2 and 3, and the models. The variability phase \( \Phi_{\text{vis}} \) in visual light (\( \Phi_{\text{vis}} \) represents cycle + phase), the Rosseland radius \( R \) and the \( K' \)-band radius \( R_{K'} \) in units of the parent star radius \( R_p \), and the effective temperature \( T_{\text{eff}} \propto (L/R_p^2)^{1/4} \) are also given.

| Model     | \( \Phi_{\text{vis}} \) | \( R/R_p \) | \( R_{K'}/R_p \) | \( T_{\text{eff}}/K \) | m  |
|-----------|--------------------------|--------------|------------------|------------------------|----|
| D27520    | 1+0.0                    | 1.04         | 1.02             | 3020                   | 1  |
| D27760    | 1+0.5                    | 0.91         | 0.90             | 2710                   | 2  |
| D28760    | 2+0.0                    | 1.04         | 1.02             | 3030                   | 3  |
| D28960    | 2+0.5                    | 0.91         | 0.91             | 2690                   | 4  |
| E8300     | 0+0.83                   | 1.16         | 1.14             | 2330                   | 5  |
| E8380     | 1+0.0                    | 1.09         | 1.11             | 2620                   | 6  |
| E8560     | 1+0.21                   | 1.17         | 1.14             | 2610                   | 7  |
| P71800    | 0+0.5                    | 1.20         | 1.03             | 2160                   | 8  |
| P73200    | 1+0.0                    | 1.03         | 0.98             | 3130                   | 9  |
| P73600    | 1+0.5                    | 1.49         | 1.08             | 1930                   | 10 |
| P74200    | 2+0.0                    | 1.04         | 1.12             | 3060                   | 11 |
| P74600    | 2+0.5                    | 1.17         | 1.01             | 2200                   | 12 |
| P75800    | 3+0.0                    | 1.13         | 1.05             | 3060                   | 13 |
| P76200    | 3+0.5                    | 1.13         | 0.95             | 2270                   | 14 |
| P77000    | 4+0.0                    | 1.17         | 1.14             | 2870                   | 15 |
| M96400    | 0+0.5                    | 0.93         | 0.92             | 2310                   | 16 |
| M97600    | 1+0.0                    | 1.19         | 1.15             | 2750                   | 17 |
| M97800    | 1+0.5                    | 0.88         | 0.91             | 2460                   | 18 |
| M98800    | 2+0.0                    | 1.23         | 1.19             | 2650                   | 19 |
| O64210    | 0+0.5                    | 1.12         | 1.08             | 2050                   | 20 |
| O64530    | 0+0.8                    | 0.93         | 0.95             | 2150                   | 21 |
| O64700    | 1+0.0                    | 1.05         | 1.01             | 2310                   | 22 |

The Mira variable [see BSW96/HSW98], i.e. the distance from the parent star’s center, at which the Rosseland optical depth \( \tau_{\text{Ross}} \) equals unity; \( T_{\text{eff}} \propto (L/R_p^2)^{1/4} = \) effective temperature; \( L = \) luminosity). Table 3 provides the link between the 22 abscissa values (model-phase combinations m) in Figs. 2 and 3, and the models, and it additionally lists the variability phase, Rosseland and stellar \( K' \)-band filter radius (definition given below) in units of the parent star radius \( R_p \), and the effective temperature. We compare predictions of these models at different phases and cycles with our measurements.

3.1 Monochromatic radius \( R_{\lambda} \), Rosseland radius \( R \) and stellar filter radius \( R_{f} \)

Monochromatic radius \( R_{\lambda} \): We use the conventional stellar radius definition where the monochromatic radius \( R_{\lambda} \) of a star at wavelength \( \lambda \) is given by the distance from the star’s center at which the optical depth equals unity.
(τ_λ = 1).

**Rosseland radius R**: In analogy, the photospheric stellar radius or Rosseland radius \( R \) is given by the distance from the star’s center at which the Rosseland optical depth equals unity (τ_{Ross} = 1).

**Stellar filter radius \( R_f \) and \( R_{K'} \)**: The stellar filter radius \( R_f \) for filter transmission \( f_λ \) is the intensity and filter weighted radius \( R_f = \int R_λ I_λ f_λ dλ / \int I_λ f_λ dλ \), which we call stellar filter radius \( R_f \) after the definition of Scholz & Takeda (1987). In this equation \( R_λ \) denotes the monochromatic \( τ_λ = 1 \) radius, \( I_λ \) the central intensity spectrum and \( f_λ \) the transmission of the filter. We have calculated the theoretical CLVs of the above-mentioned Mira star models (D, E, P, M and O) at different phases and cycles for the filter K' (center wavelength / bandwidth of 2.13 \( \mu \)m / 0.30 \( \mu \)m) used for the observations. In the near-continuum K'-band window, within which the extinction coefficient varies very little with wavelength in most models, monochromatic radii \( R_λ \) vary very little and are almost identical to the K'-band filter radius \( R_{K'} \).

Note that the Rosseland radius \( R \) is a non-observable quantity based on the Rosseland mean of extinction coefficients, and its correlation with monochromatic (\( R_λ \)) or filter radii (e.g. \( R_{K'} \)) has to be deduced from a model. Often, the K'-band radius and other near-continuum filter radii are close to the Rosseland radius (e.g. HSW98). Some of the models, however, predict wavelength-dependent CLV shapes within the K'-band due to molecular (mainly water) absorption, resulting in a two-component structure (i.e. disk plus tail) of the CLV (Scholz, 2001; Bedding et al., 2001) and contributing to a few noticeable differences between \( R \) and \( R_{K'} \), seen in Table 3.

### 3.2 Derived angular stellar K'-band radius \( R_{aK'}^\alpha \) (\( \alpha = \text{angular}, \ K' = \text{K'}-\text{band}, \ m = \text{model-phase combination} \)) and angular Rosseland radius \( R_{a}^\alpha \) of the observed Miras

The derived angular stellar K'-band radii \( R_{aK'}^\alpha \) (corresponding to the model-phase combinations \( m \); Table 3) were determined by least-squares fits between the measured Mira star visibilities (Fig. 1) and the visibilities of the corresponding model CLVs (of the models described in Table 3).

Additionally, the angular Rosseland radii \( R_{a}^\alpha \) were derived from the obtained stellar K'-band radii \( R_{K'}^\alpha \) and the theoretical ratios between \( R/R_p \) and \( R_{K'}/R_p \) given in Table 3. In the following subsections, we apply CLVs predicted from all five models at phases both near our observations and, for comparison, also at other phases.
Fig. 2. Comparison of measured R Aql radii and theoretical model radii: (left) linear Rosseland radii $R_m$ and (right) linear stellar K′-band radii $R_{K′,m}$ for all 22 model-phase combinations m. Measured linear radii derived from models with phases close to our observations (= filled squares) and far from our observations (open squares) are shown. The theoretical model radii are plotted with open circles. Table 3 gives the link between the abscissa values (model-phase combinations m) and the models and their phases.

### 3.3 Linear radii

We have derived linear stellar K′-band radii $R_{K′,m}$ and linear Rosseland radii $R_m$ of R Aql from the measured angular stellar K′-band radii $R^a_{K′,m}$ and Rosseland radii $R^a_m$ by using the R Aql HIPPARCOS parallax of 4.73±1.19 mas (ESA, 1997; Van Leeuwen et al., 1997). The HIPPARCOS parallaxes of the other four observed Miras have too large errors to reliably estimate stellar linear radii. Fig. 2 shows the obtained linear Rosseland radii $R_m$ and stellar K′-band radii $R_{K′,m}$ of R Aql for all model-phase combinations m. The theoretical Rosseland radii of the D, M and P fundamental mode model series at all available near-maximum phases are close (i.e. within the error bars) to the measured Rosseland radii of R Aql. The theoretical Rosseland radii of the first-overtone models E and O are clearly too large compared with measured Rosseland radii. The same conclusions are also valid for the linear stellar filter radii $R_{K′}$ (Fig. 2).

Table 4 lists measured linear Rosseland radii of R Aql derived from the well-fitting near-maximum fundamental mode models D, P and M and the corresponding theoretical ones. The model-averaged measured linear Rosseland radii are averages over those cycles of the model where the measured and theoretical linear Rosseland radius values are within the error bar (see Fig. 2).
Fig. 3. Comparison of measured effective temperatures of the 5 observed Mira stars and the model effective temperatures (see text). The measured effective temperatures are derived from the angular Rosseland radii determined by least-square fits between the measured visibilities and the visibilities of the corresponding theoretical CLVs given by the models of BSW96 & HSW98. The measured $T_{\text{eff}}$ values are given with $1\sigma$-error bars. Measured $T_{\text{eff}}$ values derived from models at phases close to our observations (= filled squares) and far from our observations (open squares) are shown. The theoretical model effective temperatures are plotted with open circles. Table 3 shows the link between the abscissa values and the models and their phases.
Table 4
Measured linear Rosseland radii of R Aql for the well-fitting near-maximum fundamental mode models D, P and M and a comparison with theory. The model-averaged measured linear Rosseland radii are averages over those cycles (at phases close to our observation) of the model where the measured and theoretical linear Rosseland radius values are within the error bar. For comparison, the linear uniform disk radius of R Aql is $248^{+93}_{-56} R_{\odot}$.

| model | measured linear | theoretical linear |
|-------|----------------|-------------------|
|       | Rosseland radius ($R_{\odot}$) | Rosseland radius ($R_{\odot}$) |
| D     | $259^{+99}_{-59}$ | 246               |
| P     | $246^{+100}_{-60}$ | 263               |
| M     | $252^{+97}_{-58}$ | 315               |

Table 5
Observational data and measured effective temperatures. For each star the effective temperature according to its UD diameter is listed. For R Aql and RU Her measured $T_{\text{eff}}$ values are given which were derived from the Rosseland radii of those model-phase combinations (at phases close to our observation), where the theoretical $T_{\text{eff}}$ value and the measured one are within the $2\sigma$-error bar (see Fig. 3). The indices D, P, M refer to the pulsation models.

| Star  | Date   | $\Phi_{\text{vis}}$ | $K$ | $F_{\text{bol}}$ | $T_{\text{eff},\text{UD}}$ | $T_{\text{eff},\text{D}}$ | $T_{\text{eff},\text{P}}$ | $T_{\text{eff},\text{M}}$ |
|-------|--------|---------------------|-----|-----------------|-----------------|-----------------|-----------------|-----------------|
|       | 1999   | [mag]               | [mag] | $[10^{-8} \text{erg/cm}^2 \text{s}]$ | [K] | [K] | [K] | [K] |
| X Oph | May 27 | 0.71                | -0.83 | 287.7±28.8 | 2810±90 | - | - | - |
| R Aql | May 28 | 0.17                | -0.86 | 305.6±30.6 | 2960±100 | 2900±90 | 3010±120 | - |
| RU Her| May 21 | 0.07                | -0.11 | 153.2±15.3 | 2850±80 | - | 2970±80 | 2840±80 |
| R Ser | May 21 | 0.28                | 0.02  | 130.3±13.1 | 2780±80 | - | - | - |
| V CrB | May 27 | 0.07                | 0.96  | 59.9±6.0 | 2320±70 | - | - | - |

3.4 Effective temperature

Effective temperatures of each observed Mira star were derived from its angular Rosseland radii $R_{\text{m}}^a$ and its bolometric flux using the relation

$$T_{\text{eff}} = 2341 \text{ K} \times (F_{\text{bol}}/\phi^2)^{1/4}$$

(4)

where $F_{\text{bol}}$ is the apparent bolometric flux in units of $10^{-8} \text{erg/cm}^2 \text{s}^{-1}$ and $\phi = 2 \times R_{\text{m}}^a$ is the angular Rosseland diameter in mas. The bolometric fluxes were derived from optical ($VR$), near-infrared ($JHKLM$) and mid-infrared (e.g. IRAS data) photometry (see Appendix A). Interstellar extinction corrections were applied which, however, affect the fluxes only mildly (see Table A.1). The near-infrared photometry was carried out with the 1.25 m telescope at the Crimean station of the Sternberg Astronomical Institute in Moscow 12 days after our visibility observations. The V band photometry was taken from the AAVSO data base (Mattei, 2001) at the time of our observations. The R band data were derived from observations with the 1.25 m telescope at the
Crimean station of the Sternberg Astronomical Institute and the AAVSO data base. The conventional approximation for calculating bolometric fluxes is to use a black body function to interpolate between photometric observations in the near-infrared (JHKLM). However, applying only this black body as a description of the whole spectral energy distribution, yields bolometric fluxes being in most cases significantly larger than those based also on additional photometry in V and mid-infrared. In Appendix A, the procedure for deriving bolometric fluxes used in this paper is described in detail. The errors of the JHKLM photometry do not exceed 0.03 magnitudes, whereas the errors of the V and R photometry are approx. 0.6 magnitudes, which affect the bolometric fluxes. The errors of the bolometric fluxes were determined by integrating the SED derived from the photometric data at the upper and lower boundary of the error bar. The derived bolometric fluxes have errors of approx. 10%.

Fig. 3 shows a comparison of the measured and theoretical effective temperatures.

(a) For the two M stars X Oph and R Ser, the phases of the models do not fit the phases at observations (X Oph: only the E model at phase 0.83 and the O model at phase 0.80 are close to the phase 0.71 at observation; R Ser: only the E model at phase 1.21 is close to the phase 0.28 at observation).

(b) For the C-type Mira V CrB, the information drawn from Fig. 3 should be considered with due caution, because the K′-band continuum is slightly contaminated by molecular lines in both M and C stars, and because even the pure-continuum CLV might be influenced by the different structures of M- and C-type atmospheres. The good description by the high-mass, first-overtone O model may be a chance match.

(c) For the remaining stars, R Aql and RU Her, all five models are available at phases close to the observations (phase 0.17 for R Aql and 0.07 for RU Her). Note, however, that RU Her has an appreciably longer period than the o Ceti period adopted for all model series. For R Aql and RU Her, phase- and model-dependent effective temperatures could be derived, which are listed in Table 5.

(d) For the smaller stars RU Her and R Ser, extremely large measured effective temperatures are seen in Fig. 3 at model-phase combination m=11 (near-maximum model P74200). The P74200 model predicts a very pronounced two-component, disk-plus-tail CLV structure, as mentioned above, which essentially distorts the central maximum of the visibility of the tail-free curve at smaller spatial frequencies. For the small disks of RU Her and R Ser, the large measured visibilities (V = 0.48 and 0.55, respectively) lie in the inner portion of the central maximum, which should be mostly affected by a two-component CLV structure. Obviously, such a pronounced structure is not
present in these stars. For the larger disks of X Oph and R Aql, the smaller values of $V = 0.23$ and 0.29, respectively, belong to higher spatial frequencies at which the influence of a CLV tail, if it were present, would not be very conspicuous. Observations with several baselines would be necessary for detecting tail-generated distortions of the visibility curve.

(e) Interestingly, the visibility fit of V CrB whose disk size is similar to those of RU Her and R Ser would be compatible with a distinct two-component brightness distribution, but no model studies of limb-darkening of C-type Miras in the K (K′) bandpass are available at the present.

The four M stars X Oph, R Aql, RU Her and R Ser clearly do not fit the first-overtone models E and O. For the two M stars, R Aql and RU Her, observed at near-maximum phase, however, the measured and theoretical $T_{\text{eff}}$ values are within the 2$\sigma$-error bar for some cycles of the fundamental mode models D, P and M (R Aql: all cycles of the D model and 3 of 4 cycles of the P model; RU Her: 1 of 4 cycles of the P model and 1 of 2 cycles of the M model). Table 5 lists the measured bolometric flux, the $T_{\text{eff}}$ values derived from the measured UD diameter, and for the two M stars R Aql and RU Her measured $T_{\text{eff}}$ values for selected models close to the phase at observation. These model-averaged $T_{\text{eff}}$ values were obtained by averaging only those values where the theoretical $T_{\text{eff}}$ values and the measured ones are within the 2$\sigma$-error bar. The $T_{\text{eff}}$ values derived from the UD diameters are in good agreement with the model-derived effective temperatures.

4 Discussion

We derived angular uniform-disk diameters $\Theta_{\text{UD}}$ of five Mira stars (Table 1) from K′-band visibility measurements with the IOTA interferometer and the FLUOR beam combiner at 38 m baseline. Using simultaneously observed bolometric fluxes and the measured uniform-disk diameters we obtained $T_{\text{eff,UD}}$ values given in Table 5.

Previous interferometric K-band observations of some of our target stars (R Aql, X Oph, R Ser) were carried out by Van Belle et al. (1996) at similar phases. Their derived uniform-disk diameters (R Aql: $\Phi_{\text{vis}} = 0.90$: 10.76±0.61 mas, X Oph: $\Phi_{\text{vis}} = 0.75$: 12.30±0.66 mas, R Ser: $\Phi_{\text{vis}} = 0.32$: 8.56±0.58 mas) are in good agreement with our observations. Their effective temperatures, derived from measured angular Rosseland radii (R Aql: 3189±147 K, X Oph: 3041±160 K, R Ser: 2804±144 K), are also in agreement with our results.

The comparison of the observations with Mira star models, with respect to the effective temperature, suggests that the four observed M Miras can approxi-
mately be represented by the fundamental mode models D, P or M, whereas
the overtone models are much too cool. For the two M stars R Aql and RU Her,
phase- and model-dependent effective temperatures could be derived, which
are listed in Table 5. These effective temperatures are within the error bars
of the \( T_{\text{eff}} \) values obtained with the measured uniform disk diameter. Any
more accurate model interpretation of our four M-type stars would require
an extension of the parameter range (Table 2) and refining the phase spacing
(Table 3) of available Mira model grids. A quantitative study of V CrB can
only be given on the basis of C-type Mira models which are not yet available.

For R Aql, a useful HIPPARCOS parallax (4.73±1.19 mas) is available and
it is therefore possible to compare measured linear Rosseland and stellar K′-
band radii with the theoretical radii of the BSW96 and HSW98 models. The
measured radii were derived by fitting theoretical (BSW96, HSW98) center-to-limb intensity variations to the visibility data. In Table 4, the measured linear
Rosseland radii derived from the well fitting near-maximum fundamental mode
models D, P and M are listed. From the measured linear Rosseland radii of R
Aql the pulsation mode could not be determined because of the large parallax
error.

The comparison suggests that R Aql can well be represented by the funda-
mental mode D or P model. Note, however, that observations in more filters
than just one continuum filter and more baselines may be necessary for safely
distinguishing a well-fitting model from an accidental match (cf. Hofmann et
al. 2000a). Furthermore, in order to discover the tail structure predicted by
some models, future observations should cover more baselines.

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Strasbourg.

A Bolometric fluxes

The bolometric fluxes are determined by integrating the spectral energy dis-
tributions (SEDs). The present \( JHKLM \) photometry were complemented by
coeval V data of the AAVSO (Mattei, 2001). Inspection of \( BVRJ \) photom-
etry taken one pulsational cycle later, nearly at the same phase, shows good
agreement with the AAVSO data. From this data the \( R \) fluxes at epoch and
cycle of the present observations were interpolated. Table A.1 presents the \( V \)
(AAVSO), R (interpolated) and JHKLM magnitudes. In the long-wavelength regime the respective IRAS measurements were taken into account, and in the case of R Aql also 8-30 µm photometry of Epchtein et al. (1980) taken at almost the same phase as our data (Φ = 0.11). Interstellar extinction corrections, A_V, are given by Whitelock & Feast (2000) and were considered by adopting the method of Savage & Mathis (1979) with A_V = 3.1E(B − V). However, extinction is generally small (see Table A.1) and affects the fluxes only mildly.

Table A.1
Pulsational phase, Φ_v, photometric data in the V (0.55 µm, AAVSO), R (0.71 µm, interpolated), J (1.25 µm), H (1.62 µm), K (2.20 µm), L (3.50 µm), and M (4.80 µm) band (in mag), adopted extinction A_V (in mag), total bolometric flux, F_{bol}, based on a three-component fit to the VRJ, JHKLM and mid-infrared data, flux difference δF_{bol} of a SED fit based only on JHKLM photometry, and flux contribution ∆F_{dust} due to dust emission.

| Star   | Φ_v  | V   | R   | J   | H   | K   | L   | M   | A_V | F_{bol} | δF_{bol} | ∆F_{dust} |
|--------|------|-----|-----|-----|-----|-----|-----|-----|-----|--------|----------|-----------|
| X Oph  | 0.71 | 8.30| 5.43| 0.55| -0.32| -0.83| -1.49| -1.25| 0.18 | 287.7 | +26.2%   | 2.4%      |
| R Aql  | 0.17 | 7.90| 5.01| 0.47| -0.40| -0.86| -1.40| -1.32| 0.23 | 305.6 | +29.5%   | 1.5%      |
| RU Her | 0.07 | 7.70| 5.12| 1.28| 0.53 | -0.11| -0.82| -0.66| 0.04 | 153.2 | +12.9%   | 1.8%      |
| R Ser  | 0.28 | 9.90| 5.55| 1.26| 0.45 | 0.02 | -0.51| -0.30| 0.04 | 130.3 | +40.7%   | 2.7%      |
| V CrB  | 0.07 | 8.10| 5.73| 2.79| 1.68 | 0.96 | -0.10| -0.13| 0.03 | 59.9  | -6.0%    | 2.4%      |

The JHKLM photometry can be well fitted with a black body. For the fitting procedure the Levenberg-Marquardt method (see Press et al. 1992) was applied. However, these black body fits are not well suited to represent the optical and mid-infrared data (see Fig. A.1). In the optical regime the SED can be significantly depressed compared to a black body due to absorption bands (e.g. of TiO; see Scholz & Takeda 1987). For example, all oxygen-rich Mira stars of the present sample show V magnitudes well below that of a black body. To find a fair representation of the optical flux, the VRJ photometry was fitted separately, applying the Levenberg-Marquardt method. Fit functions resembling black bodies, but showing larger spectral indices, proved to give good matches to the observational data and smooth transitions to the near-infrared photometry.

On the other hand, longwards of 10 µm dust emission may give additional, albeit small, flux contributions. For instance, the IRAS data of R Aql does not follow the Rayleigh-Jeans curve indicating the existence of possibly significant dust emission. However, we found the respective flux contributions due to dust emission to be less than 3% (Table A.1), and therefore to be of only minor importance.

Table A.1 presents the photometric data in the V (0.55 µm, AAVSO), R (0.71 µm, interpolated), J (1.25 µm), H (1.62 µm), K (2.20 µm), L (3.50 µm), and M (4.80 µm) band, the pulsational phase at date of observation, the adopted extinction, and the total bolometric flux. The flux difference of a
Fig. A.1. Spectral energy distributions of R Aql and R Ser. Circles refer to AAVSO data and present photometry (see Tab. A.1), squares to the photometry of Epchtein et al. (1980), and asterisks to IRAS (1985) data. The solid line represents the black-body fit to the $JHKLM$ data, and the thick dashed line refers to the overall fit.

SED fit based only on the $JHKLM$ photometry and the flux contribution due to dust emission are given as well. Fig. A.1 illustrates photometric data and various fits for R Aql and R Ser.

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