The Attractive Hubbard Model in 2D: Is it capable of describing a pseudogap and preformed pairs?

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Deviations from Fermi liquid behavior are well documented in the normal state of the cuprate superconductors, and some of these differences seem to be related to pre-transitional features appearing at temperatures above $T_c$. The observation of a pseudogap, e.g. in ARPES experiments, is a familiar example of this physics. One potential explanation for this behavior involves preformed pairs with finite lifetimes existing in the normal state above $T_c$. In this way two characteristic temperatures can be established. A higher one $T^*$ at which pairs begin to form and the actual $T_c$ at which a phase-coherent superconducting phase is established.

In order to test these ideas we have investigated the negative U Hubbard model in two dimensions in the fully self-consistent ladder approximation at low electron densities. In the non self-consistent version of this theory the system always shows an instability towards Bose-condensation of infinite lifetime pairs. In contrast to this, pairs obtain a finite lifetime due to pair-pair interaction and the sharp two-particle bound state is strongly lifetime broadened when self-consistency is applied. A quasi-particle scattering rate which varies linearly with temperature is also found.

The fully self-consistent calculation we were able to perform using a $\mathbf{k}$-averaged approximation in which the self-energy loses its $\mathbf{k}$-dispersion due to a $\mathbf{k}$-average. This approximation is found to preserve the essential physics.

Keywords: negative-U Hubbard model, two-particle bound states, pseudogap, non Fermi-liquid properties

74.20 Mn 74.25.-q 74.25.Fy 74.25.Nf 74.72.-h 74.20-z

I. INTRODUCTION:

In three dimensions (3D) the negative U Hubbard model has two well understood borderline cases: One is the weak coupling limit $U \ll t$ (t is the transfer). In this case the model shows superconductivity of mean-field, BCS type with a large coherence length of the cooper pairs. The second well understood case is the limit of strong coupling and low densities $U \gg t$, $n \approx 0$. In this case pairs of electrons which now form composite Bosons condense into a two particle bound state and the Fermi surface is destroyed.

In 2D however the situation is different. Here it has been shown by Schmitt-Rink et al. that for any coupling strength the Fermi surface is lost and Bose condensation (at $T=0$) takes place. The calculation that was used in was a T-matrix approximation (taking into account ladder diagrams in the particle–particle channel) in its non self-consistent, non conserving form which is only valid in the zero density limit, $n \to 0$.

The motivation to study a 2D system with low but finite densities and intermediate coupling strength ($U \approx$ bandwidth $W$) comes from the high-T$_c$ cuprates. In these systems the normal state transport is governed by 2D CuO$_2$ planes, the quasiparticle density is low but finite (one still finds a Fermi surface) and the coherence length of a cooper pair is small ($\xi \approx 3-4$ lattice constants). However the negative U Hubbard model in its simplest form shows only s-wave pairing. But we believe that full understanding of the simple s-wave problem is a necessary condition to extend the calculations to more complicated models. We are confident that many parts of the physics developed here for the s-wave case will survive when more complicated d-wave pairing is considered.

II. CALCULATION:

When trying to expand the known results for the zero density limit towards finite densities the main additional interaction which has to be taken into account is the interaction between pairs. The lowest order interaction term would be the exchange of two electrons between two pairs. Such interactions are included into the equations by extending the non selfconsistent work to a fully selfconsistent as has been discussed in e.g. . In order to perform such a fully selfconsistent calculation we use dynamical mean field theory which in our particular problem becomes not only exact in infinite dimensions but also for the limit of large correlations ($U > t$) which is in our particular problem even more important.

In this case the $\mathbf{k}$ independent self-energy (we denote the $\mathbf{k}$-average with over-lined quantities)

$$\Sigma(i\omega_n) = \frac{1}{\beta} \sum_m \Gamma(i\omega_m + i\omega_n) G(i\omega_m)$$

(1)
results from a $\mathbf{K}$ independent vertex function $\Gamma(i\Omega_n)$. The vertex function now consists of two different parts: The one particle continuum and the bound state. In the large $U$ limit the $\mathbf{K}$-dispersion of the bound state vanishes and therefore the $\mathbf{K}$ averaged vertex function will already be a good approximation in two dimensions. To calculate this vertex function from the susceptibility we apply another approximation

$$\Gamma(i\Omega) \approx \frac{U^2 \chi(i\Omega_n)}{1 - U \chi(i\Omega_n)}$$

(2)

By doing this the next term which is neglected is of the order of the mean square deviation of the susceptibility, $\chi^2 - \chi^2$ as is discussed in detail in [5]. To get the full set of equations to solve selfconsistently we further need the equation for $\chi$

$$\chi(\mathbf{K}, i\Omega_n) = -\frac{1}{N\beta} \sum_{m, \mathbf{k}} G(\mathbf{K} - \mathbf{k}, i\Omega_n - i\omega_m) G(\mathbf{k}, i\omega_m)$$

(3)

and for the one particle Green function:

$$G(\mathbf{k}, i\omega_n) = (G^0(\mathbf{k}, i\omega_n)^{-1} - \Sigma(i\omega_n))^{-1}$$

(4)

When solving these equations non selfconsistently we reproduce the results of [5]. That means the Fermi surface is lost at low temperatures and we get Bose condensation into the two–particle bound state.

### III. RESULTS AND DISCUSSION

When doing a fully selfconsistent calculation the situation changes drastically; the infinite lifetime bound state gets strongly lifetime broadened and merges with the one particle continuum. We further regain a Fermi surface [3]. In Fig.1 we have plotted the dispersion of the one particle density of states $\Lambda(k, \omega)$ where the self-energy was obtained via a fully selfconsistent calculation on a quasi-2D system with a constant initial density of states. We have chosen several $k$-points along the $(1,1)$ direction. The correlation strongly broadens and renormalizes the one particle peak but at the Fermi energy we obtain a clear quasiparticle peak. We therefore find no pseudo gap from our calculation.

![Fig. 1. For a density of $n = 0.3$, an attractive correlation of $U = -8t$, and a temperature of $k_B T = 0.1t$, the $k$-dispersion of the one particle spectral function is shown. The self-energy was obtained by a fully selfconsistent T-matrix calculation and the $k$-points are chosen along the (1,1) direction. Especially at the Fermi energy we obtain a quasiparticle peak whereas the incoherent broadening comes mainly into play away from $k_F$. The energy units along the axis are chosen in units of half the bandwidth $\Delta$.](image)

From our results it seems to follow that at low but finite quasiparticle densities the intuitive picture of pairs of electrons which condense as composite Bosons breaks down. In the following we discuss some arguments which support this result: When thinking in such a Bose picture the effective hopping of pairs is given by second order perturbation theory. The kinetic energy of such a pair is therefore given by an effective hopping of $t^2$. But only in the zero density limit the interaction between such Bosons can be considered to be small. At finite densities the dominant interaction is given by the fact that a pair has a smaller number of virtual hopping processes due to the existence of other pairs. Such interaction is therefore caused by the Pauli principle, is repulsive can not be neglected in comparison with the kinetic energy of the pair. Such kind of discussion is well known in nuclear physics [3]. When one therefore maps the Hamiltonian with strongly interacting Fermions onto Bosons one ends up, at finite density, with strongly interacting Bosons which does not solve the problem.

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