Transmission characteristics of magnetic resonance coupling-based multi-load wireless power transmission system

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Abstract: If the magnetic resonance coupling-based wireless power transmission system has multiple receivers, cross-coupling will occur between the transmitter and the receiver, resulting in the variation of transmission characteristics. In this article, the calculation model for mutual inductance between solenoid coils in the same plane has been built to study the transmission characteristics of the system through simulation and testing. The study results show that: the cross-coupling effect between the transmitter and the receiver will cause resonance frequency shift, or even lead to the variation of resonance frequency splitting characteristic; when compared with a single-load system, the multi-load system may increase in total transmission power and efficiency; generally, with the introduction of additional receivers, the receiving power and efficiency of the original loads will decrease; along with the increase in circuit spacing and the decrease in cross-coupling, the transmission characteristics will gradually regress to the state of a single-load system. In this article, the calculation model for mutual inductance is correct and the maximum relative error in these calculating examples is 5.28%.

1 Introduction

Wireless power transmission technology is a kind of technology through which electric equipment gets energy from power source in a non-contact manner, providing possibilities for human-efficiency expressions of the transmission system without relay coil and to discuss the feasibility of improving the system transmission efficiency by using the impedance matching method. In reference [17], resonance frequency shift was observed during test, but no further research was made in theory. The above-mentioned researches have obtained valuable results in such aspects as stability conditions of multi-load transmission, impedance matching method, and variation of equivalent parameters. These results promote the recognition of multi-load energy conversion relationship and the increase in system transmission efficiency. However, to reduce the difficulty in analysing multi-coil and multi-circuit coupling, existing references mainly focus on researching no relay coil and multi-load cases during theoretical modelling. The initial research findings of the Massachusetts Institute of Technology clearly point out the importance of relay coil. References [18–20] also demonstrated the key role of relay coil in improving the system transmission performance. Therefore, the transmission system with relay coils has higher practical value and broader application prospects. In addition, there has been lack of researches on the resonance frequency shift, transmission power, and efficiency variation characteristics. In this paper, a circuit model for double-relay and multi-load system has been built by neglecting the load coupling condition to deduce the analytic calculating formula for mutual inductance of solenoid coils in the same plane. Moreover, the transmission characteristics of the multi-load system and the accuracy of mutual inductance model have been researched through simulation and test.

2 Circuit model for multi-load transmission system

2.1 System model

In the missile and weapon system information cross-linking application, the transmitter was fixed onto the weapon platform, while the receivers were set at the head of the missiles. Before the missiles were loaded into the chamber, the transmitter and receiver coils could form an electromagnetic coupling structure when the missiles were arranged in parallel and successively passed the alternating magnetic field formed by transmitter coils, so that wireless power transmission and operational information set could...
be achieved [21–23]. The coupling structure is shown in the figure below, A is the drive coil, S and D are relay coils, and B is pick-up coil.

As for this application, two receivers are located in the effective coupling area of alternating magnetic field formed by the transmitter in a certain period of time. That is to say, one transmitter supplies wireless power to two receivers at the same time. According to the long/short radial spacing between the two receivers, system modelling could be conducted with/without the consideration of mutual coupling effect of receivers. For this application, the author has built a single-load system model and a multi-load model with consideration of direct coupling of two receivers in reference [21]. However, there is still the lack of circuit model for multi-load system without the consideration of coupling between receivers. This section presents the supplementary research on this model. According to the physical structure in Fig. 1, a circuit model has been built, as shown in Fig. 2. \( R_4 \) is the sum of the internal resistance of drive source and the resistance of drive coil; \( R_{d1} \) and \( R_{d2} \) are the resistances of corresponding coil circuits; \( R_{11} \) and \( R_{12} \) are equivalent resistances of loads; \( M_{ld} \) is the mutual inductance of two adjacent coils; \( L_i \) is the self-inductance of each coil; \( C_i \), \( C_{d1} \), and \( C_{d2} \) are resonant capacitances of corresponding relay coils. In this model, \( M_{ld} \) is equal to 0 when the mutual inductance between relay coils of the two receivers is neglected.

The impedance mapping analysis method is used to map the pick-up coil circuit to the previous circuit successively [21]. The mapping process is given in Fig. 3.

In Fig. 3a:

\[
R'_{dd1} = \frac{(oM_{ld})^2R_{dd1}}{R_{dd1} + X_{dd1}'}, \quad R''_{dd1} = \frac{(oM_{ld})^2R_{dd1}''}{R_{dd1}'' + X_{dd1}''}
\]

In Fig. 3b:

\[
L'_{dd1} = \frac{M_{dd1}'}{(R_{dd1} + X_{dd1}')}C_{dd1}, \quad L''_{dd1} = \frac{M_{dd1}''}{(R_{dd1}'' + X_{dd1}'')}C_{dd1}'
\]

In Fig. 3c:

\[
I_{in} = \frac{V_{in}}{\sqrt{(R_{dd1} + R''_{dd1})^2 + \omega^2L''_{dd1} - (1/\omega^2C''_{dd1})^2}}
\]

\[
a = \arctg \frac{\omega(L_{dd1} + L''_{dd1}) - (1/\omega^2C''_{dd1})}{R_{dd1} + R''_{dd1}}
\]
As a result, the powers absorbed by the two loads are as follows:

\[ P_I = 0.5I_{in}R_{i'''} \]

\[ P_M = 0.5I_{in}R_{i''''} \]

The total transmission efficiency of the system can be expressed as the sum of transmission efficiency \( \eta = I_{in}R_{i''''}/V_{in}\cos\alpha \) and \( \eta_i = I_{in}R_{i''''}/V_{in}\cos\alpha \) of load \( R_{i1} \) and load \( R_{i2} \):

\[ \eta = \eta_i + \eta \]

In combination of the author's research findings [21], complete circuit models for magnetic resonance coupling-based wireless power transmission system have been built for double-relay transmission system, including a single-load system model and the multi-load system models with/without the consideration of the coupling effect between receivers.

### 2.2 Simulation analysis of transmission characteristics

A set of coil parameters were taken to conduct simulation analysis in five cases. During analysis, the system transmission rules under strong and weak coupling effect of multiple loads were analysed by fixing the transmitter, receiver 1, and the coupling parameter between the transmitter and the receiver, introducing receiver 2, changing the coupling parameters between receiver 2 and the transmitter and between receiver 2 and receiver 1, and simulating the distance between two receivers. Refer to Table 1 for main parameters. In Cases 1–4, the situation that two receivers are in the same plane was simulated, as shown in Fig. 1. In Case 5, strong coupling effect between receiver 2 and the transmitter and between receiver 2 and receiver 1 was simulated, just like the situation that they are close to the axis of the original transmission channel. Case 1: only receiver 1 exists. Case 2: the two receivers are close to each other and in a strong coupling state, with \( M_{dd} = 0.18 \mu H \) and \( M_{dd} = 0.02 \mu H \). Case 3: The two receivers are a little far away from each other and in a weak coupling state, with \( M_{dd} = 0.08 \mu H \) and \( M_{dd} = 0.01 \mu H \). Case 4: The two receivers are rather far away from each other and their mutual coupling effect could be neglected; in addition, the coupling effect between receiver 2 and the transmitter is weak, with \( M_{dd} = 0 \) and \( M_{dd} = 0.002 \mu H \). Case 5: \( M_{dd} = M_{dd} = 0.18 \mu H \).

Fig. 4 shows the receiver power and efficiency-based transmission characteristic curves in five cases. According to this figure, the following conclusions could be drawn: the addition of receiver 2 may greatly affect the transmission performance of receiver 1, not only causing variation in receiving power or efficiency of \( R_{i1} \), but also leading to resonance frequency shift; strong coupling effect between the two receivers may cause the variation of splitting characteristics of power- and efficiency-based frequency response curves, such as the curve in Case 2; in general, the introduction of an additional receiver may lead to the decrease in power and efficiency of the original receiver, such as those in Cases 2 and 3, and the reason for this mainly lies in that the resonance state of the original system is disturbed and partial energy is absorbed after the addition of receiver 2; in Case 5, the power increase of the original load \( R_{i1} \) and the rise-up of total transmission power and efficiency of the system were observed with the addition of receiver 2, and the reason for this is that receiver 2 is located near the axis of the original transmission system, forming a strong coupling effect with both the transmitter and receiver 1 and serving as the relay coil of \( R_{i1} \); along with the decrease in mutual inductance of receiver 2 and the original circuit, cross-coupling effect becomes weaker and its impact on the transmission performance of the original system also becomes less, as shown in the diagrams of Case 2 to Case 4, until the impact is negligible, at which moment the transmission characteristics after the addition of receiver 2 are similar to those when only receiver 1 exists, that is the curve in Case 1 almost coincides with that in Case 4.

### 3 Calculation of mutual inductance between solenoid coils in the same plane

#### 3.1 Mutual inductance model

According to the analysis results in the previous section, cross-coupling effect between the transmitter and the receiver has great impact on the transmission characteristics of the multi-load system, and different values of mutual inductance between relay coils cause different variation in transmission characteristics. For these reasons, the mutual inductance between coils is a key parameter affecting the system transmission characteristics. Therefore, it is necessary to build a mathematical model for mutual inductance and to deduce its analytical expression. Two mutual inductance models exist in the coupling structure given in Fig. 1. One model involves the transmitter and receiver coils, which are parallel in terms of axis but are not coaxial. The other one involves two receiver coils, which are solenoid coils in the same plane. For the first model structure, references [24–27] present analytic calculating formulas based on different principles and provide available solutions for theoretical calculation even though the effect of conductor spacing change between different turns in the two coils is neglected [24, 25] or the formulas become complicated due to the introduction of special functions [26, 27]. As a result of limited space of this paper, the structure is not deduced again. Reference [28] provides fairly comprehensive calculation formulas for mutual inductance between coils in various structures. Generally, specific tables or curves need to be looked up for determination of some parameters and then these parameters can be used for continuous calculation with such formulas. These formulas deliver poor calculation accuracy and are limited in utilisation convenience. For the second model structure, this section deduces the calculation formula for mutual inductance between two solenoid coils in the same plane according to the Biot–Savart law.

As for the coupling structure given in Fig. 1, the load coils are solenoid coils in the same plane, as shown in Fig. 5. To make the research conclusions be more generic, assume that the number of turns of these two coils is \( N_1 \) and \( N_2 \), respectively, the radii are \( r_1 \) and \( r_2 \), respectively, the distance between coil centres is \( d_{12} \), the diameter of conductor is \( d \), and all coil turns are wound tightly.

First, consideration is given to the mutual inductance \( M_{dd} \) between two single-turn coils in the same plane. Current elements in these single-turn coils are \( dI_1 \) and \( dI_2 \), respectively. As the magnitude of current does not affect mutual inductance, such current elements could be simplified into vector elements \( dl_1 \) and \( dl_2 \).

#### Table 1 Simulation parameters

| Parameter | Value |
|-----------|-------|
| \( L_a, L_{d1}, L_{d2} \) | 0.90 \( \mu H \) |
| \( R_a \) | 0.20 \( \Omega \) |
| \( L_a, L_{d1}, L_{d2} \) | 2.85 \( \mu H \) |
| \( R_a, R_{d1}, R_{d2} \) | 0.40 \( \Omega \) |
| \( C_a, C_{d1}, C_{d2} \) | 8.89 nF |
| \( R_L, R_{i1}, R_{i2} \) | 50 \( \Omega \) |
| \( M_{ds}, M_{ds1}, M_{ds2} \) | 1.12 \( \mu H \) |
| \( M_{dd1}, M_{dd2} \) | 0.11 \( \mu H \) |
| \( V_a \) | sinusoidal V |

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The included angles between vector elements ($d_1$ and $d_2$) and axes ($x$ and $x'$) of the coil coordinate system are denoted as $\alpha$ and $\beta$, the corresponding radii of these vector elements are $d\alpha$ and $d\beta$, and the distance between these vector elements is denoted as $d_0$, as shown in Fig. 6.

According to the Biot–Savart law, the mutual inductance ($M_0$) between two single-turn coils in the same plane could be expressed as:

$$M_0 = \mu_0 \frac{4\pi}{\lambda} \oint_{l_1} \oint_{l_2} \frac{dl_1 dl_2}{d_0}(8)$$

It can be known from Fig. 6 that the geometrical relationship of single-turn coils in the same plane is $d_1 = r_1 d\alpha l_1$, $d_2 = r_2 d\beta l_2$, and $l_1, l_2 = \cos(\beta - \alpha)$, where $l_1 = l_1 \| l_1 \|$ and $l_2 = l_2 \| l_2 \|$ are the unit vectors of vectors $dl_1$ and $dl_2$, respectively.

Then:

$$dl_1dl_2 = r_1 d\alpha dl_1 \cos(\beta - \alpha)\quad (9)$$

The distance ($d_0$) between vector elements $dl_1$ and $dl_2$ could be expressed as:

$$d_0 = \sqrt{(r_1' - r_1')^2 + d_{12}'^2}\quad (10)$$

where $r_1' = r_1 \cos \alpha$, $r_2' = r_2 \cos \beta$, $d_{12}' = d_{12} - r_1 \sin \alpha + r_2 \sin \beta$. The above expressions are for two single-turn coils in the same plane. As for multi-turn solenoid coils, the $i$th turn of coil 1 and the $j$th turn of coil 2 are not always in the same plane. Therefore, expression (10) is required to be modified. The distance between vector elements of any two parallel coils ($d_0'$) could be expressed as:

$$d_0' = \sqrt{(r_1' - r_2')^2 + d_{12}'^2 + i - j d_2}\quad (11)$$

Therefore, the calculation formula of mutual inductance between the $i$th turn of coil 1 and the $j$th turn of coil 2 is as follows:

$$M_{ij} = \mu_0 \frac{4\pi}{\lambda} \oint_{l_1} \oint_{l_2} \frac{dl_1 dl_2}{d_0'}\quad (12)$$

The following calculation formula of mutual inductance between any two solenoid coils in the same plane could be deduced according to the superposition principle:
3.2 Verification of model accuracy

Two groups of coils were wound to verify the accuracy of calculation formula of mutual inductance. The diameters of all conductors were 0.69 mm. In Group 1, the coil diameter was 76.69 mm and four turns of coils were tightly wound. In Group 2, the coil diameter was 46.69 mm and ten turns of coils were tightly wound. The comparison between theoretical and experimental values is shown in the figure below. In the two groups of experiment, the maximum relative errors between theoretical and experimental values are 4.49 and 5.28%, respectively, indicating that the analytic calculating formula mentioned herein is highly accurate (Fig. 7).

4 Experimental analysis of transmission characteristics

To verify the correctness of theoretical analysis, two groups of experimental systems were established. Refer to Fig. 8a for schematic diagram of experiment arrangement, in which two receivers were in the same plane. Group 1: both the diameters of transmitter and receiver coils are 76.69 mm and these coils are formed by arranging and winding multiple turns of copper conductors with a diameter of 0.69 mm, including two turns for coils A and B and four turns for coils S and D; in addition, the distance between outer edges of coils S and D is 55 mm, with \( C_{d1} = C_{d2} = 8.89 \text{ nF} \), as shown in Fig. 8b. Group 2: the diameter of transmitting coil is 113 mm, the diameters of all receiver coils are 51 mm, and these coils are formed by arranging and winding multiple turns of copper conductors with a diameter of 0.9 mm, including two turns for coils A and B and five turns for coils S and D; in addition, the distance between outer edges of coils S and D is 35 mm, with \( C_{s1} = 1 \text{ nF} \) and \( C_{d1} = C_{d2} = 2.82 \text{ nF} \). The peak sinusoidal signal (1 V) produced by the signal generator was input into coil A. The resistance of all loads used in the experiment was 50 Ω. Power test was conducted using TCP312 current probe and MDO3014 oscilloscope.

Refer to Table 2 for the experimental results. The experimental results show that: the addition of load \( R_{L2} \) may cause resonance frequency shift of the original transmission system; the closer the two receivers are, the more significant the cross-coupling effect will be and the frequency shift will also increase; with the experiment conditions of Group 1, where \( d_{d1} = 0 \) and \( d_{12} = 80 \text{ mm} \), resonance frequency splitting of \( R_{L1} \) was observed; with the increase in the distance between receiver 2 and the transmitter and between receiver 2 and receiver 1, the cross-coupling effect becomes weaker and the transmission characteristics of \( R_{L1} \) also gradually resume to the original state (only \( R_{L1} \) is provided), and it can be foreseen that the state of \( R_{L1} \) may be fully resumed provided that the distance between receiver 2 and the original system is long enough; under general experimental conditions, the addition of load \( R_{L2} \) may cause the decrease in total transmission power and efficiency of \( R_{L1} \) and the system, however, rise-up of total transmission efficiency was also observed, such as the experimental results of Group 2, where \( d_{d1} = 22.8 \text{ mm} \) and \( d_{12} = 75.6 \text{ mm} \).

In addition to the above-mentioned experimental results, one more experiment was also carried out for Group 1, in which the distance between outer edges of coils S and D1 was adjusted to 70 mm, receiver 2 was set in the transmitter and receiver channel to make the transmitter, receiver 2, and receiver 1 in the same axis, and the distance between outer edges of coils S and D2 was adjusted to 31 mm. Under such experimental conditions, the power of \( R_{L1} \) was increased from 2.50 to 2.75 mW and its resonance frequency was shifted from 1.015 to 1.033 MHz, when compared with the conditions without receiver 2. In this situation, receiver 2 which was set in the original power transmission channel served as an additional relay coil of \( R_{L1} \), showing consistency with the research conclusions of references [18–20]. That is to say, transmission performance of the original system can be increased by adding relay coils. Although this operating state is a little different from the coupling structure (as shown in Fig. 1) of the research objects in this paper, it is a transmission form in theoretical analysis. These experimental results verify the correctness of the conclusions from theoretical analysis.

5 Conclusions

In this paper, researches were conducted on the transmission characteristics of magnetic resonance coupling-based multi-load wireless power transmission system. A mutual inductance coupling circuit model for multi-load transmission system was built and the calculation formula of mutual inductance between any two solenoid coils in the same plane was deduced. In the end, the correctness of theoretical analysis was verified by experiments.
The research results show that: the calculation formula of mutual inductance proposed in this paper is correct and of high calculation accuracy, and the maximum relative error in the calculating examples is 0.28%; cross-coupling effect between transmitter and receiver of a multi-load system has great impact on the transmission characteristics of the system, resulting in resonance frequency shift and variation of splitting characteristics of power- and efficiency-based frequency response curves, and this impact will reduce along with the decrease in cross-coupling effect; in general, the introduction of an additional receiver into the transmission system always leads to the decrease in power and efficiency of the original receiver; however, the coupling effect will become weak if the distance between transmitter and receiver of the original system is long, and the power of the original load may increase if the additional receiver is set in the original transmission channel. These research conclusions enrich the magnetic resonance coupling-based wireless power transmission theory and have reference significance in the design of multi-load wireless transmission systems.

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7 References

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