Progress in constraining axion and non-Newtonian gravity from the Casimir effect

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We consider recent progress in constraining the axion-nucleon coupling constants and the Yukawa-type corrections to Newtonian gravity from experiments on measuring the Casimir interaction. After a brief review of previously obtained constraints, we concentrate on the new Casimir-less experiment, which allows to strengthen the known results up to factors of 60 and 1000 for the axion-like particles and Yukawa-type corrections, respectively. We also discuss possibilities allowing to further strengthen the constraints on axion-nucleon coupling constants, and propose a new experiment aiming to achieve this goal.

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1. Introduction

Both axions and non-Newtonian gravity have a long history. The Yukawa- and power-type corrections to the Newton law of gravitation follow from the exchange of light massive or massless elementary particles, respectively, between atoms of two macrobodies (see the detailed review in Ref. [1]. The same corrections to Newtonian gravity were introduced in extra-dimensional models with a low-energy compactification scale [2,3]. The light pseudoscalar particle axion was predicted in quantum chromodynamics as a consequence of broken Peccei-Quinn symmetry [4]. The latter was postulated in order to avoid strong PC violation and large electric dipole moment of a neutron which are excluded by the experimental data. Axions play an important role in astrophysics and cosmology because they are considered as possible constituents of dark matter [5,6].

Strong constraints on the corrections to Newton gravitational law have been obtained from the astronomical observations, geophysical experiments and from the experiments of Eötvös and Cavendish type [7,8]. In the beginning of the 21st century several new Eötvös- and Cavendish-type experiments were performed in order to
strengthen these constraints in the interaction region below several hundreds micrometers (see Ref. 11 for a review). However, no signature of deviations from the Newtonian gravity have been observed.

During the last few decades a lot of experiments on the search of axions were performed using their interactions with photons, electrons and nucleons (see Refs. 9 and 12–14 for a review). In addition to the laboratory experiments, many constraints on the axion parameters were obtained from astrophysics and cosmology, e.g., from stellar cooling, neutrino data of supernova SN 1987A, and attempts to observe axions emitted by the Sun or belonging to the dark matter. As a result, the originally introduced axions were constrained to a very narrow band in the parameter space and many different models of the so-called axion-like particles were proposed. The most of these models can be attributed to one of the two sets of models of QCD (or hadronic) axions or Grand Unified Theory (GUT) axions.

It has long been known that strong constraints on the Yukawa-type and power-type corrections to Newtonian gravitational law can be obtained from measurements of the van der Waals and Casimir force. Precise measurements of the Casimir interaction performed during the last few years allowed for significant strengthening of previously known constraints on the Yukawa-type corrections to Newton’s law in the interaction range below one micrometer (see Refs. 24 and 25 for a review). Recently it was shown that the same measurements place strong constraints on the coupling constants of the GUT axions to nucleons in the wide range of axion masses (see Ref. 30 for a review).

In this paper we describe the most modern constraints on the axion-nucleon coupling constants and Yukawa-type corrections to Newton’s law obtained from the Casimir effect. In Sec. 2 we consider the effective potentials following from different elementary processes and explain what kind of constraints could be obtained from each of them. We also attract reader’s attention to one unresolved problem. In Sec. 3 we briefly review the constraints on the coupling constants of the GUT axions to nucleons following from measurements of the thermal Casimir-Polder force. In Sec. 4 the same is done regarding the constraints following from measurements of the gradient of the Casimir force, Casimir pressure and lateral Casimir force. Sections 5 and 6 are devoted to the most strong constraints on the Yukawa-type corrections to Newton’s law and axion-nucleon coupling constants obtained from the new Casimir-less experiment. Section 7 contains our conclusions and discussion including the proposal of a pioneer Casimir experiment exploiting the polarized test bodies.

We use the system of units in which $\hbar = c = 1$.

2. Effective Potentials

We begin with an exchange of one light scalar particle of mass $M$ between two particles with masses $m_1$ and $m_2$ located at the points $r_1$ and $r_2$. It is common
knowledge that this process results in the spin-independent Yukawa-type effective potential. It is conventional to parametrize it as a correction to Newton’s law

\[ V(|r_{12}|) = -\frac{G m_1 m_2}{|r_{12}|} \left( 1 + \alpha e^{-|r_{12}|/\lambda} \right), \tag{1} \]

where \( r_{12} = r_1 - r_2 \), \( G \) is the gravitational constant, \( \alpha \) is a dimensionless interaction constant, and \( \lambda = 1/M \) is the Compton wavelength of light scalar particle (i.e., the interaction range). If the Yukawa-type correction originates from extra-dimensional models (see Sec. 1), the quantity \( \lambda \) characterizes the size of a compact manifold formed by the extra dimensions. \[ \lambda \]

If the scalar particle is massless, \( \lambda \rightarrow \infty \) and we return from (1) to the potential which is inverse proportional to separation similar to the Newtonian potential. There are also power-type effective potentials with higher powers which are usually parametrized as corrections to Newtonian gravity

\[ V_n(|r_{12}|) = -\frac{G m_1 m_2}{|r_{12}|} \left[ 1 + \Lambda_n \left( \frac{r_0}{|r_{12}|} \right)^{n-1} \right], \tag{2} \]

where \( n = 1, 2, 3, \ldots \), \( \Lambda_n \) is a dimensionless interaction constant, and \( r_0 = 10^{-15} \text{ m} \) is chosen to preserve the correct dimension of energy at different \( n \). The effective potentials of power type arise due to an exchange of an even number of massless pseudoscalar particles, such as axions, and also due to exchanges of two neutrinos, two goldstinos, or other massless fermions.

The Casimir effect was used to constrain the axion-nucleon coupling constants. Because of this, here we consider the effective potentials arising due to the exchange of axions between protons and neutrons belonging to two different test bodies. The interaction of GUT axion-like particles \( a \) with nucleons \( \psi \) is described by the pseudoscalar Lagrangian density

\[ \mathcal{L}_{ps} = -i g_{ak} \bar{\psi} \gamma_5 \psi a, \tag{3} \]

where \( g_{ak} \) is the dimensionless coupling constant of an axion to a proton (\( k = p \)) or a neutron (\( k = n \)), and \( \gamma_a \) with \( a = 0, 1, 2, 3, 5 \) are the Dirac matrices. The interaction of QCD axions (which are pseudo-Nambu-Goldstone bosons) with nucleons is described by the pseudovector Lagrangian density

\[ \mathcal{L}_{pv} = \frac{g_{ak}}{2m_a} \bar{\psi} \gamma_5 \gamma_\mu \psi \partial^\mu a, \tag{4} \]

where \( m_a \) is the axion mass and the effective interaction constant \( g_{ak}/(2m_a) \) is dimensional. Note that the mass and interaction constant of the GUT axions are the independent parameters, whereas the mass and interaction constant of the QCD axions are connected by some relationship.

It is interesting that on a tree level both Lagrangian densities \( (3) \) and \( (4) \) lead to one and the same action after an integration by parts. As a result, both \( (3) \) and \( (4) \) lead to common effective potential due to the exchange of either one GUT or

\[ \text{progress in constraining axion and non-Newtonian gravity from the Casimir effect} \]
one QCD axion between two nucleons:

\[
V_{kl}(r_{12}; \sigma_1, \sigma_2) = \frac{g_{ak} g_{al}}{16\pi m_k m_l} \left[ (\sigma_1 \cdot n)(\sigma_2 \cdot n) \left( \frac{m_a^2}{|r_{12}|} + \frac{3 m_a}{|r_{12}|^2} + \frac{3}{|r_{12}|^3} \right) \right. \\
- \left. (\sigma_1 \cdot \sigma_2) \left( \frac{m_a}{|r_{12}|^2} + \frac{1}{|r_{12}|^3} \right) \right] \exp^{-m_a |r_{12}|},
\]

(5)

where \( n = r_{12}/|r_{12}| \) is the unit vector, and \( m_k, m_l \) and \( \sigma_1, \sigma_2 \) are the nucleon masses and spins, respectively. This potential, however, depends on the nucleon spins. The resulting interaction between two unpolarized test bodies, used in already performed experiments on measuring the Casimir force, averages to zero. Therefore, these experiments are not suitable for constraining the axion-nucleon interaction caused by the exchange of one axion.

The spin-independent effective potential is obtained from the exchange of two GUT axions between two nucleons. The respective calculation is straightforward because the quantum field theory with the Lagrangian density (3) is renormalizable. The result is:

\[
V_{kl}(r_{12}) = -\frac{g_{ak} g_{al}}{32\pi^3 m_k m_l} m_a |r_{12}| K_1(2m_a |r_{12}|),
\]

(6)

where \( K_1(z) \) is the modified Bessel function of the second kind, and it is assumed that \( |r_{12}| \gg 1/m_k, l \). In the limiting case \( m_a \to 0 \) one obtains from (6) an attractive power-type potential

\[
V_{kl}(r_{12}) = -\frac{g_{ak} g_{al}}{64\pi^3 m_k m_l} \frac{1}{|r_{12}|^3},
\]

(7)

i.e., the same result as follows from the exchange of two massless pseudoscalar particles between two fermions.

The exchange of two QCD axions between two nucleons is a much more complicated process. The point is that the quantum field theory with the Lagrangian density (4) is not renormalizable. According to our knowledge, the effective potential due to exchange of two massive QCD axions between two fermions is not yet obtained. As to the case of two massless pseudoscalar particles, the Lagrangian density leads to

\[
V_{kl}(r_{12}) = \frac{3g_{ak}^2 g_{al}^2}{128\pi^3 m_k^2 m_l^2} \frac{1}{|r_{12}|^5},
\]

(8)

which is, surprisingly, a repulsive potential, as opposed to the effective potentials (6) and (7) which are the attractive ones.

This problem invites further investigation.

3. Constraints on Axion-Like Particles from Measuring the Casimir-Polder Force

In this and next sections we briefly review the constraints on axion-nucleon coupling constants obtained from several laboratory experiments. This is done under the
natural assumption that \( g_{an} = g_{ap} \). Before dealing with the Casimir-Polder force, we mention the constraints following from the magnetometer measurements and several gravitational experiments.

The magnetometer using spin-polarized K and \(^3\)He atoms was used to obtain the constraints on \( g_{an} \) and \( m_a \) shown by the line 1 in Fig. 1. Note that for all lines in Fig. 1 the axion mass and the axion-nucleon coupling constant are considered as independent parameters. The regions below the lines are allowed and the regions above the lines are prohibited by the results or respective experiment.

The gravitational experiments of Eötvös and Cavendish types have long been used for constraining the axion-nucleon coupling constants for the GUT axions. In doing so, the additional force due to two-axion exchange was calculated by the additive summation of internucleonic potentials over the volumes of the test bodies taking into account their isotopic composition. The same method was later applied to experiments on measuring the Casimir interaction. The dashed lines 2 and 3 in Fig. 1 show the constraints obtained from the Cavendish-type experiments and from the Eötvös-type experiment, respectively. The solid line 4 indicates the most modern gravitational constraints on the GUT axion obtained from the recent Cavendish-type experiment. As is seen in Fig. 1, the line 4 provides rather strong constraints on the masses and coupling constants of GUT axions in the mass interval centered at \( m_a = 10 \mu\text{eV} \). With increasing axion mass, the strength of these constraints quickly decreases.

Now we consider the constraints on \( g_{an} \) and \( m_a \) of the GUT axions obtained from measurements of the thermal Casimir-Polder force between \(^87\)Rb atoms belonging to a Bose-Einstein condensate cloud and an amorphous SiO\(_2\) plate. In this dynamic experiment, the condensate cloud was placed in a magnetic trap near

![Fig. 1](image_url)
a plate, and the dipole oscillations in it with the natural frequency $\omega_{0z}$ and some constant amplitude were excited in the direction $z$ perpendicular to the plate. The distance $d$ between the center of mass of a cloud and a plate varied from 6.88 to 11 $\mu$m, i.e., in the thermal regime. The Casimir-Polder force, acting between $^{87}$Rb atoms and a plate, leads to a shift of the oscillation frequency $\omega_{0z}$ to some value $\omega_z$, and the relative frequency shift

$$\gamma_z = \frac{|\omega_{0z} - \omega_z|}{\omega_{0z}}$$

was measured as a function of separation $d$. By solving the oscillator problem under the influence of external (Casimir-Polder) force, the quantity $\gamma_z$ was also calculated. For this purpose, the Casimir-Polder force between $^{87}$Rb atoms and a plate was calculated using the Lifshitz theory of dispersion forces with subsequent averaging over the condensate cloud.

The comparison of experiment with theory has led to a conclusion that the calculation results are in a good agreement with the data in the limits of the experimental errors $\Delta \gamma_z(d)$ determined at the 67% confidence level. This conclusion was obtained by omitting the role of free charge carriers in the SiO$_2$ plate which are in fact present at any nonzero temperature. As was shown later, the account of charge carriers in the plate material results in an exclusion of the theoretical predictions of the Lifshitz theory by the data (see the discussion in Refs. 22 and 23).

The two-axion exchange between protons and neutrons belonging to $^{87}$Rb atoms and to SiO$_2$ plate results in some additional force. The respective additional frequency shift $\gamma_{z,\text{add}}$ was calculated by the additive summation of potentials with subsequent averaging over the condensate cloud. This additional shift was not observed and, thus, it should be constrained in the limits of the experimental error

$$\gamma_{z,\text{add}}(d) \leq \Delta \gamma_z(d).$$

The constraints on the axion-nucleon coupling constant following from Eq. (10) are shown by the dashed line 5 in Fig. 1. As is seen in the figure, the strength of these constraints decreases with increasing axion mass, but it is stronger than that of line 4 in the region of axion masses centered at $m_a = 10^{-2}$ eV.

Several other experiments on measuring the Casimir interaction used for constraining the parameters of the GUT axions are considered in the next section.

4. Constraints on Axion-Like Particles from Various Measurements of the Casimir Interaction

The most precise measurements of the Casimir interaction were performed by means of an atomic force microscope and micromechanical oscillator in the configuration of a metal-coated sphere and a metal-coated plate. Here, we concentrate on the case of Au coatings and do not consider other metals, such as Al, Cu or Ni, because they result in weaker constraints.
4.1. Gradient of the Casimir force

The gradient of the Casimir force between Au-coated surfaces of a hollow glass sphere and a sapphire plate was measured\textsuperscript{48, 49} by means of dynamic atomic force microscope over the separation region from 235 to 500 nm. Below the Au-coating both test bodies were covered with various additional material layers. This does not influence the Casimir force but should be taken into account in calculation of the additional force due to two-axion exchange. The measured gradients $F'_{\text{C}}(d)$ were compared with theoretical predictions of the Lifshitz theory and found to be in a good agreement under a condition that the relaxation properties of conduction electrons are omitted in computations\textsuperscript{48} Note that the measurement data of all precise experiments on the Casimir force agree with theory only when the relaxation properties of conduction electrons are omitted (for metals) or the free charge carriers are disregarded (for dielectrics; see Sec. 3). The two experiments which disagree with this observation are shown to be erroneous\textsuperscript{50–52}

The gradient of the additional force $F'_{\text{add}}(d)$ due to two-axion exchange acting between a sphere and a plate was calculated\textsuperscript{27} using Eq. (6) with account of the layer structure of these test bodies and their isotopic composition\textsuperscript{1} In the limits of the measurement error $\Delta F'_{\text{C}}(d)$ (which is used here at the 67% confidence level) the theoretical predictions for the gradient of the Casimir force have been confirmed. Thus, one arrives at

$$F'_{\text{add}}(d) \leq \Delta F'_{\text{C}}(d). \quad (11)$$

The constraints on the axion-nucleon coupling constants following from (11) are presented by the dashed line 6 in Fig. 1 as the function of an axion mass. The comparison of the lines 6 and 5 shows that the constraints obtained from measurements of the gradient of the Casimir force are stronger than those obtained from measurements of the Casimir-Polder force up to a factor of 170.

4.2. Casimir pressure

The Casimir pressure between two Au-coated parallel plates was determined dynamically by means of micromechanical oscillator in the configuration of a sapphire sphere and a silicon plate\textsuperscript{53, 54} Both the test bodies were coated by the layers of Cr prior to a Au coating. The indirectly measured Casimir pressure was compared with the Lifshitz theory and found to be in a very good agreement in the limits of the experimental errors $\Delta P_{\text{C}}(d)$. The latter were determined at a 95% confidence level, but recalculated to a 67% confidence level in order to obtain the constraints comparable with those discussed above. The comparison was made with omitted relaxation properties of conduction electrons.

The additional pressure $P_{\text{add}}(d)$ due to two-axion exchange between nucleons of the test bodies taking into account their layer structure was calculated (see Ref. 28 for details). The constraints on the axion-nucleon coupling constants and masses
were obtained from the inequality

\[ |P_{\text{add}}(d)| \leq \Delta P_C(d). \]

(12)

In Fig. 1 these constraints are shown by the solid line 7. As is seen in the figure, they significantly improve the constraints of lines 5 and 6. Specifically, at \( m_a = 1 \) eV the constraints of line 7 are by a factor of 3.2 stronger than those of line 6.

4.3. Lateral Casimir force

It has been known that if the test bodies are corrugated and there is some phase shift \( \varphi \neq 0 \) between corrugations, then the Casimir force acts not only perpendicular to their surfaces, but also along them \( 22, 23 \). This is the so-called lateral Casimir force. The lateral Casimir force between two Au-coated surfaces of a sphere and a plate covered with the longitudinal sinusoidal corrugations was first observed in Refs. \( 55 \) and \( 56 \) and found to be in qualitative agreement with theory based on the proximity force approximation. The precise measurements of the lateral Casimir force in similar configuration (but with shorter corrugation period equal to \( \Lambda = 574.4 \) nm), as a function of the phase shift, were performed over the separation region from 120 to 190 nm between the mean levels of corrugations \( 57, 58 \). The lateral Casimir force was independently computed using the exact theory, which generalizes the Lifshitz theory for the case of arbitrary shaped boundary surfaces of the test bodies. The comparison between experiment and theory demonstrated a good agreement in the limits of the experimental error of force measurements \( \Delta F_C^{\text{lat}}(d) \) determined at a 95% confidence level \( 57, 58 \).

In Ref. 29 the effective potential (6) was used to calculate the additional force, \( F_{\text{add}}^{\text{lat}}(d) \), which arises between the corrugated test bodies due to two-axion exchange. This was done taking into account the material properties of a plate (a hard epoxy coated with a layer of Au) and a sphere (a polystyrene core coated with a layer of Cr and then with a layer of Au). The maximum magnitude of the additional force at fixed separation is achieved at the phase shift \( \varphi_0 = \pi/2 \). Taking into account that no deviations were observed from theoretical predictions, the constraints on the parameters of GUT axions were found from the inequality

\[ \max |F_{\text{add}}^{\text{lat}}(d)| \leq \Delta F_C^{\text{lat}}(d), \]

(13)

where the experimental error was recalculated to the 67% confidence level. The obtained constraints are shown by the solid line 8 in Fig. 1 in the mass range from 1 to 20 eV. For \( m_a \geq 8 \) eV they are stronger than the constraints obtained by means of micromechanical oscillator (the line 7 in Fig. 1).

Note that the normal Casimir force between sinusoidally corrugated bodies, acting perpendicular to the surfaces, has also been measured \( 59, 60 \). This experiment does not lead to stronger constraints on an axion, but is used in the next section when discussing the Yukawa-type corrections to Newtonian gravity.
5. New Casimir-Less Experiment and Stronger Constraints on Non-Newtonian Gravity

First we briefly list the strongest constraints on the parameters $\alpha$ and $\lambda$ of the effective potential (1) obtained from measurements of the Casimir interaction [the strongest present constraints on the constants of power-type potentials (2) follow from the Eötvos- and Cavendish-type experiments [23, 25].

The Yukawa-type interaction energy between the test bodies in experiments on measuring the Casimir interaction is obtained by the integration of the effective potential (4) over the volumes of the test bodies. In doing so, the Newtonian contribution to the results at submicrometer separations turns out to be negligibly small, as compared to the errors in force measurements, and can be neglected. The most strong constraints on $\alpha$ and $\lambda$ in the micrometer and submicrometer interaction range are summarized in Fig. 2. The line 1 was obtained from measurements of the lateral Casimir force between corrugated surfaces [57, 58]. This experiment was discussed in the previous section. It leads to the strongest constraints in the interaction region from $\lambda = 1.6$ to 11.6 nm. The constraints on $\alpha$, $\lambda$ found from measurements of the normal Casimir force between corrugated surfaces at the angle of 2.4° between corrugations [59, 60] are shown by the line 2. These constraints are the strongest in the interaction region from 11.6 to 17.2 nm. The line 3 indicates the constraints found from indirect measurement of the Casimir pressure by means of micromechanical oscillator (see Sec. 4.2). Until very recently they were the strongest ones over the wider interaction region from 17.2 to 89 nm.

At larger $\lambda$ strong constraints on $\alpha$ have been obtained from measurements of another type, the so-called Casimir-less experiment [62]. In this approach the Casimir force was nullified by using the difference force measurement, and the Yukawa-type force was restricted by the force sensitivity of the measurement device (see below for the improved version of such kind experiment). The constraint found in this way [64] are shown by the line 4 in Fig. 2. They extend from $\lambda = 89$ nm to $\lambda = 891$ nm. Finally, the line 5 demonstrates the constraints obtained from measurements of the Casimir force between Au-coated surfaces of a plate and a spherical lens of centimeter-size radius. These constraints are the most strong in the interval $0.891 \mu m < \lambda < 3.16 \mu m$. At $\lambda \geq 3.16 \mu m$ the strongest constraints on $\alpha$ shown by the dashed line 6 follow from the Cavendish-type experiments [44, 64, 65].

Very recently, new and significantly improved Casimir-less experiment has been performed [66]. In this experiment, a differential force between a Au-coated sphere and either a Au sector or a Si sector of the structured disc deposited on a Si substrate and coated with overlayers of Cr and Au was measured. In such away, the contribution of the Casimir force between two Au surfaces to the differential force was nullified. Then the measurement result was determined solely by the difference in the forces due to the exchange of some hypothetical particles when the sphere is above Au or Si sectors. The differential Yukawa-type force $F_{\text{add, Yu}}^{\text{Au, Au}}(d) - F_{\text{add, Yu}}^{\text{Au, Si}}(d)$ was calculated with account of material composition of the test bodies. Taking
into account that no statistically meaningful signal was observed, the constraints on \( \alpha \) and \( \lambda \) were obtained from an inequality:

\[
|F_{\text{add, Au, Yu}}^{\text{Au}}(d) - F_{\text{add, Si, Yu}}^{\text{Si}}(d)| \leq \Delta F(d). \tag{14}
\]

Here, the minimum detectable force \( \Delta F(d) \) varied between 0.1 and 0.2 fN at different separation distances.

In Fig. 2, the constraints following from the inequality \( \text{(14)} \) are shown by the line 7, which reproduces the original line in Fig. 4 of Ref. 66. As is seen in Fig. 2, the new constraints are the strongest ones over a wide interaction region extending from 40 nm to 8 \( \mu \)m. Thus, the new constraints of Ref. 66 significantly, up to a factor of \( 10^3 \), strengthen the results of several previous experiments, which is a major progress in the field.

6. Constraining Axion from the Casimir-Less Experiment

The new Casimir-less experiment allows to strengthen constraints not only on the non-Newtonian gravity, but on the axion-nucleon coupling constants as well. The differential force due to two-axion exchange was calculated using Eq. \( \text{(6)} \) with account of material and isotopic composition of the test bodies. Note that in fact the disc used consisted of the alternating concentric strips of Au and Si rather than sectors, but this does not influence the values of the additional differential force.

The constraints on \( g_{\text{cn}} \) and \( m_a \) were found from the inequality:

\[
|F_{\text{add, Au}}^{\text{Au}}(d) - F_{\text{add, Si}}^{\text{Si}}(d)| \leq \Delta F(d), \tag{15}
\]

Fig. 2. Constraints on the Yukawa-type corrections to Newton’s law of gravitation obtained from measurements of the lateral and normal Casimir force between corrugated surfaces (line 1 and 2, respectively), of the Casimir pressure (line 3), from the old Casimir-less experiment (line 4), from measurements of the Casimir force between a plate and a spherical lens (line 5), from the Cavendish-type experiment (line 6), and from the new Casimir-less experiment (line 7) are shown as functions of the interaction range. The regions above each line are prohibited, and below each line are allowed.
which means that no signal due to any hypothetical force was registered. Note that if both attractive forces ($F_{\text{add}}$ due to two-axion exchange and $F_{\text{add,Yu}}$ due to exchange of one scalar particle) exist in nature and contribute to the measured differential force, the constraints imposed on each of them by the data would be even stronger than those obtained in Refs. 66 and 67 from the inequalities (14) and (15), respectively. In Fig. 1 the constraints on the axion-nucleon interaction of GUT axions following from Eq. (15) are shown by the line 9. As is seen in Fig. 1, the constraints of line 9 significantly improve the previously known constraints of lines 4 and 7 in the wide region of axion masses from $1.7 \times 10^{-3}$ to 0.9 eV. The largest strengthening by a factor of 60 holds for $m_a = 4.9 \times 10^{-3}$ eV. The obtained strengthening is not as strong as in the case of Yukawa-type corrections to Newtonian gravity. This is explained by the fact that for axions it was necessary to use the process of two-axion exchange, whereas for gravitation the exchange of one scalar particle was considered. We note also that the constraints of this and previous sections obtained from the Casimir-less experiment are free of any problem in theoretical understanding of the Casimir force discussed in Secs. 3 and 4.

7. Conclusions and Discussion

In the foregoing, we have summarized the most recent results on constraining the axion-nucleon coupling constants and the Yukawa-type corrections to Newtonian gravity from the Casimir effect. It was shown that the major progress was achieved very recently in both these fields and even more can be expected in near future with increasing precision in measurements of the Casimir interaction.

It is pertinent to note, however, that all the constraints on axion-nucleon coupling constants following from measurements of the Casimir interaction, gravitational and Casimir-less experiments were obtained by using the Lagrangian density (3) and the effective potential (6) due to an exchange of two axions. Thus, all these constraints are valid for only the GUT axion-like particles. There is an unresolved problem what is the effective potential due to exchange of two QCD axions between two nucleons. If this problem were solved, the already performed experiments on measuring the Casimir interaction could be used for constraining the parameters of QCD axions.

As another prospective opportunity we propose an experiment on measuring the Casimir interaction between spin-polarized (magnetized) test bodies. In this case the simplest process of one-axion exchange between the two nucleons and the effective potential (9), valid for both the QCD and GUT axions, can be used because it leads to a nonzero result. The constraints obtained in such a way would be not only significantly stronger, but much more universal by being applicable to all kinds of axions and axion-like particles.
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