Be X-ray binaries in the SMC as (I) indicators of mass transfer efficiency

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ABSTRACT

Be X-ray binaries (BeXRBs) consist of rapidly rotating Be stars with neutron star companions accreting from the circumstellar emission disk. We compare the observed population of BeXRBs in the Small Magellanic Cloud with simulated populations of BeXRB-like systems produced with the COMPAS population synthesis code. We focus on the apparently higher minimal mass of Be stars in BeXRBs than in the Be population at large. Assuming that BeXRBs experienced only dynamically stable mass transfer, their mass distribution suggests that at least $\sim 30\%$ of the mass donated by the progenitor of the neutron star is typically accreted by the B-star companion. We expect these results to affect predictions for the population of double compact object mergers. A convolution of the simulated BeXRB population with the star formation history of the Small Magellanic Cloud shows that the excess of BeXRBs is most likely explained by this galaxy’s burst of star formation $\sim 40$ Myr ago, rather than by its low metallicity.

1 INTRODUCTION

Be stars are classically defined as main-sequence (MS) stars of spectral type B (e.g. Rivinius et al. 2013; Belczynski & Ziółkowski 2009), although the so called Be phenomenon is recognised to extend from early A to late O spectral types, i.e., masses down to $\sim 3 M_\odot$ (Rivinius et al. 2013 and references therein). The Be phenomenon refers to the presence of Balmer emission lines in the spectrum of a non-supergiant star (Rivinius et al. 2013). Therefore in the Be nomenclature $B$ stands for the most common spectral type and $e$ for the Balmer emission lines. These emission lines trace the presence of a surrounding decretion disk, which is composed of material outflowing from the Be star, and may appear and disappear together with the disk during the star’s life. The decretion disk is strongly linked to high rotational velocities: Be stars are among the most rapid non-compact rotators, with an average velocity of at least $\geq 75\%$ of the Keplerian limit at the equator (Rivinius et al. 2013). Common explanations for such high rotational velocities are initial rotation, evolution during the star’s MS lifetime toward Keplerian velocity (Ekström et al. 2008), and interactions with a companion including mass transfer episodes, tidal locking and mergers (e.g. de Mink et al. 2013).

BeXRBs represent a high fraction of high mass X-ray binaries (HMXRBs); they are composed of a compact object and a Be star (Rappaport & van den Heuvel 1982; van den Heuvel & Rappaport 1987; Reig 2011). The compact object accretes from the decretion disk of the Be star. With the possible exception of the black hole (BH) in MWC 656 (Casares et al. 2014; Munar-Adrover et al. 2014), only slowly rotating neutron stars (NSs)\textsuperscript{1} have been successfully identified as compact objects in BeXRBs (Klus et al. 2014). The accretion onto the NS predominantly occurs close to periastron and is thought to be at the origin of the BeXRB outbursts (Okazaki & Negueruela 2001; Okazaki et al. 2007).

\textsuperscript{1}Two other objects have Be stars which interact with compact objects, but do not show the same phenomenological X-ray behavior as the other BeXRBs. These are PSR B1259-63, which has a fast radio pulsar, and LSI +61 303, the nature of which is debated.

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These X-ray outbursts are observed from binaries in the Milky Way and the Magellanic Clouds (Reig 2011 and references therein). The Small Magellanic Cloud (SMC) is particularly rich in BeXRBs for its mass, with ~70 confirmed systems (Coe & Kirk 2015) compared to ~60–80 in the Galaxy (Reig 2011; Walter et al. 2015; Shao & Li 2014); indeed, all the classified HMXRBs in the SMC except SMC X-1 (Haberl & Sturm 2016) are BeXRBs. As a sub-population of HMXRBs, BeXRBs are characterised by orbital periods in the range of tens to hundreds of days. Be stars in BeXRBs are found at early spectral types (no later than B5 in the Coe & Kirk 2015 SMC catalogue), suggesting masses $\gtrsim 8 M_{\odot}$ (Reig 2011; Shao & Li 2014), significantly higher than for single Be stars.

Here we investigate the origin of BeXRBs, by comparing systems observed in the SMC (Coe & Kirk 2015) against BeXRB-like systems simulated with COMPAS, a rapid binary population synthesis code (Stevenson et al. 2017; Barrett et al. 2018; Vigna-Gómez et al. 2018; Neijssel et al. 2019). We focus on the total number of systems and their mass and orbital period distributions.

Our study suggests that the number of Be stars in interacting binaries is enhanced by accretion-induced spin-up during dynamically stable mass transfer. We conclude that $\gtrsim 30\%$ of the mass lost by the NS-progenitor donor should typically be accreted by the B-star companion to match the observed mass distribution. These findings impact other interacting binary populations, including the formation of double compact objects.

We recap the properties of the BeXRBs reported in the SMC catalogue of Coe & Kirk (2015) in section 2. We briefly describe COMPAS and outline the parameters of our simulations in section 3. We present our findings in section 4 and discuss them in section 5. We conclude in section 6.

2 OBSERVED SAMPLE

We use the 69 SMC BeXRBs listed in the catalogue Coe & Kirk (2015) (CK catalogue hereafter).

The orbital period is reported for 44 of these systems (see figure 1). To estimate the orbital period of the remaining 25 BeXRBs, we use the Corbet relation between the NS spin and orbital periods (Corbet 1984). We fit a linear relationship between the logs of the measured orbital and NS spin periods from the CK catalogue,

$$\log_{10} \left( \frac{P_{\text{orb}}}{\text{days}} \right) = 0.4329 \log_{10} \left( \frac{P_{\text{spin}}}{\text{s}} \right) + 1.043$$

and apply it to the listed BeXRBs with unknown orbital periods.

The CK catalogue lists the eccentricity of only 7 binaries. The eccentricities of BeXRBs have typically been estimated using pulse timing data, with the Doppler shifts on the pulsations at different orbital phases (Coe & Kirk 2015; Townsend et al. 2011). The relatively long period and transient nature of BeXRBs make eccentricity measurements quite challenging.

The uncertain eccentricities also prevent accurate dynamical measurements of the Be star masses. However their spectral type distribution (Antoniou et al. 2009; Maravelias et al. 2014) suggests that their masses do not extend to the low values observed for the general population of other Be stars (down to $\sim 3 M_{\odot}$). While the difference between spectral types of Be stars as a population and Be stars in BeXRBs is clear, the corresponding difference in Be star masses is not straightforward to estimate. Mass estimates from spectral and luminosity types are subject to several uncertainties, concerning both the classification procedures and the physics of the systems. Phenomena such as dust extinction and rotational mixing may bias the inferred properties of the star, and thus its spectral and luminosity classifications. Moreover, the dependence of the star’s spectrum and luminosity on its mass is also partly degenerate with its age and chemical composition. As a conservative estimate for the minimum mass of BeXRBs in the CK catalogue, we use the minimum value of $6 M_{\odot}$ reported in table 4 of Hohle et al. (2010).

In the following, we qualitatively present the main observational selection biases which likely affect the observed population of BeXRBs.

2.1 A qualitative understanding of the selection biases

The bulk of the SMC BeXRB NSs reported in the CK catalogue were found in RXTE scans. These scans typically monitored the SMC on a roughly weekly basis for over a decade, with typical exposure times of $10^4$ seconds, but with continuous observations limited to about 3000 seconds due to Earth occultations that occur on the satellite orbital period of 96 minutes. Orbital periods have been found through a mixture of pulse timing (e.g. Townsend et al. 2011), photometric periodicities (Schuch et al. 2011), and periodicities of the outbursts from the eccentric binaries’ periastron passages.

We note that there is a substantial bias against both detecting wide systems and measuring their orbital periods. Some of the key issues concerning this problem are discussed in (Laycock et al. 2010): the wider systems should be fainter and their periastron passages less frequent. Beyond that, in the pulsation searches, the long pulse period systems are, again, much less likely to be discovered. All the systems with spin periods longer than 500 seconds are indeed discovered by imaging satellites like Chandra and XMM-Newton whose SMC sampling is considerably poorer than that of RXTE (Laycock et al. 2010). Given that the BeXRBs show a strong correlation between orbital period and spin period (Corbet 1984), this, in turn, leads to a bias against the long orbital period systems. This is demonstrated by the 63 pulsars in Haberl & Sturm (2016): 29 of the 32 hosting binaries with shortest NS spin period, have measured orbital periods; while this is the case for only 19 of the binaries hosting the 31 NS with the longest spin periods. A $1.1 \times 10^{-5}$ binomial probability of getting only 19 orbital periods out of 31 objects results from assuming a success rate of 29/32.

Such a bias is to be expected, for a variety of reasons. At long periods, the rates of change of spin frequency due to accretion torques typically seen during outbursts (e.g. Bildsten et al. (1997)) are $\sim 1 - 2$ orders of magnitude larger than those due to Doppler shifts, preventing orbital period measurements via Doppler shifts in pulse timing. Optical spectroscopic monitoring of very long period binaries has typically not been practical due to the challenges in schedul-
ing such observations. We hope that in the new era of large numbers of queue-scheduled telescopes, an efficient management of the observing time will help us filling in the missing orbital periods. Photometric period estimation is notoriously susceptible to incorrect identification due to aperiodic noise (e.g. Press (1978)), and furthermore, the longer period systems are likely to have weaker photometric modulations on the orbital period.

A further penalty impacting the widest BeXRBs concerns the Be decretion disks, which in these cases may never reach the Roche lobe of the NS, or may do so only intermittently, near periastron passages (as is the case for the Galactic system PSR B1259-63, which is a gamma-ray binary near periastron and a radio pulsar binary near apastron – see e.g. Chernyakova et al. 2014), significantly limiting the X-ray emission. Pulsar searches with the Square Kilometer Array likely represent the best path forward to discovering these systems, but in some cases, the Be star lifetimes may exceed the lifetimes of the systems as active pulsars, and in other cases, the pulsar’s opening angle may not be pointed toward Earth, so it is likely that only a statistical sampling of these objects will be obtained with instruments available in the next few decades. Moreover, according to Reig (2011), a significant fraction of the widest Galactic systems are persistent sources, generally characterised by weaker X-ray emission. Similar statistics in the SMC could bias the observed sample of BeXRBs. Finally, instabilities in the Be decretion disks can also impact the long-term detectability of BeXRBs (Rivinius et al. 2013).

3 POPULATION SYNTHESIS CODE: COMPAS

To study the evolution of massive stellar binaries we use the population-synthesis suite of COMPAS (http://compas.science). By rapidly evolving large populations of binaries we can perform statistical studies on the physics of stellar and binary evolution. Similarly to other rapid population-synthesis codes (e.g. Belczynski et al. 2008; Hurley et al. 2002; Eldridge et al. 2017), we rely on analytic approximations of a set of pre-calculated models of single stars and stellar winds to reduce the computational cost (Hurley et al. 2000; Stevenson et al. 2017 and references therein).

We Monte Carlo sample the initial parameters of the binaries. The primary star, which we define to be the initially more massive component of the binary, follows the initial mass function (IMF) of Kroupa (2001), with a minimum mass of 5 M\textsubscript{\odot} and a maximum mass of 150 M\textsubscript{\odot}. We draw the secondary (hereafter the lighter companion at zero-age MS) from a flat mass ratio distribution (Sana et al. 2012). For the binary separation a, we assume a uniform probability density in log a (O’pik 1924), though see Moe & Di Stefano (2017) for updated coupled initial distributions. We set the metallicity to \( Z = 3.5 \times 10^{-3} \sim Z_{\text{SMC}} \) (Davies et al. 2015). For each simulation, we evolve a population of \( 3 \times 10^5 \) binaries.

We generally follow the stellar and binary evolution prescription of Vigna-Gómez et al. (2018), except as highlighted below. Two particularly important sets of choices for BeXRBs are those related to supernovae and mass transfer.

3.1 Supernovae

We determine the properties of the compact object remnant according to the delayed prescriptions of Fryer et al. (2012). The remnant gets a kick from the asymmetric ejection of material. The magnitude of the kick depends on the type of supernova (SN) and the mass of the pre-explosion core, including the amount of stripping during the previous mass transfer episodes (Vigna-Gómez et al. 2018 and references therein). We follow Vigna-Gómez et al. (2018) in drawing the natal kicks from a Maxwellian distribution, whose 1D root mean squared velocity is set to \( \sigma_{1D} = 265 \text{ km/s} \) for core-collapse (CC) SNe and \( \sigma_{1D} = 30 \text{ km/s} \) for electron-capture (EC) and ultra-stripped (US) SNe (Lyne & Lorimer 1994; Hansen & Phinney 1997; Cordes & Chernoff 1998; Arzoumanian et al. 2002b; Pfahl et al. 2002; Podsiadlowski et al. 2004; Schwab et al. 2010; Tauris et al. 2015). We then re-scale the drawn velocity by a “fallback factor”, which depends on the CO core mass of the progenitors. This factor effectively introduces a difference between NS and BH natal kick velocities, with reduced kicks for the latter.

When a star, already stripped of its hydrogen envelope, overflows its Roche lobe, it initiates a further episode of mass transfer. If this interaction removes the entire helium envelope from the donor, the star may experience a SN with reduced natal kicks, an USSN (Pfahl et al. 2002; Podsiadlowski et al. 2004; Tauris et al. 2015). One motivation for this low-kick assumption is that this second episode of mass transfer generally leads to a low core binding energy; this in turn allows for a rapid SN explosion, during which aspherical instabilities that may be responsible for the SN kicks do not have time to develop (Podsiadlowski et al. 2004). Our implementation of USSNe differs from the one adopted in Vigna-Gómez et al. (2018); in the latter case indeed, reduced natal kicks are only applied if the NS progenitors has lost its helium envelope while interacting with a NS. USSNe and mass transfers initiated by stripped stars are active research topics; further investigations are therefore required to determine the conditions for the validity of our assumptions (e.g. stability of such mass transfer episodes and the removal of the entire helium envelope Tauris et al. 2015).

Unlike Vigna-Gómez et al. (2018) and Hurley et al. (2000), our binary evolution model assumes that a star undergoes an ECSN if the mass of its core at the base of the asymptotic giant branch ranges between 1.83 M\textsubscript{\odot} and 2.25 M\textsubscript{\odot} Fryer et al. (2012). Moreover, compared to previous COMPAS versions, we are now allowing stars with carbon-oxygen core masses at SNe above 1.38 M\textsubscript{\odot} to collapse into a neutron star or black hole.

3.2 Mass transfer and specific angular momentum loss

Mass transfer crucially influences the orbital period and component masses. Mass transfer starts when a star overflows its Roche lobe. We determine the dynamical stability of mass transfer by comparing the radial response of the donor star and the response of the Roche lobe radius to mass transfer (see paragraph 2.2.4 of Vigna-Gómez et al. 2018 and section 4.3 below).

When the mass transfer is dynamically unstable, we assume that the system experiences a common-envelope (CE)
event (for a review see Ivanova et al. 2013). We follow Vigna-Gómez et al. (2018) in assuming that the efficiency parameter for the conversion of orbital energy into unbinding the envelope is $\alpha = 1$. We use fits by Xu & Li (2010), as implemented in StarTrack (Dominik et al. 2012), for the binding energy parameter $\lambda$. Furthermore we assume that the companion star does not accrete during the CE phase.

According to our model, stable mass transfer proceeds on a nuclear or thermal timescale, depending on the evolutionary phase of the donor. The companion may therefore accrete a significant amount of mass during this process. If, however, the accretor is a compact object, we assume that the accretion is Eddington-limited. We denote the ratio of accreted mass $\Delta M_{\text{acc}}$ to mass lost by the donor $\Delta M_{\text{donor}}$ with the efficiency parameter $\beta$:

$$\beta \overset{\text{def}}{=} \frac{\Delta M_{\text{acc}}}{\Delta M_{\text{donor}}}.$$  

When $\beta < 1$, i.e. the mass transfer is non-conservative, mass is lost from the binary system, taking away some orbital angular momentum. In COMPAS, this angular momentum loss is parametrised by

$$\gamma = \frac{h_{\text{loss}}}{h_{\text{binary}}},$$  

the ratio between the specific angular momenta of the ejected material $h_{\text{loss}}$ and the binary $h_{\text{binary}}$. The specific angular momentum (total angular momentum divided by total mass) of a circular binary with mass ratio $q$, total mass $M$ and separation $a$ is $h_{\text{binary}} = q(1+q)^{-2}\sqrt{MGa}$, where $G$ is the universal gravitational constant and we defined the mass ratio $q$ as $q = M_{\text{acc}}/M_{\text{donor}}$.

Below we describe our default model for the fraction of the mass lost by the donor that is captured by the accretor and the specific angular momentum of the material lost from the binary.

### 3.2.1 Default model: $\beta$: THERMAL; $\gamma$: ISO

Our default prescription for the accretion efficiency of mass transfer prior to the current study, motivated by Hurley et al. (2002), estimated the accretion efficiency $\beta$ by comparing the mass loss rate of the donor $M_{\text{donor}}$ to the maximal mass acceptance rate of the accretor $M_{\text{acc,max}}$:

$$\beta = \min\left(1, 10 \times \frac{M_{\text{acc,max}}}{M_{\text{donor}}}\right).$$  

The steady-state mass acceptance rate is set by the time required for the accretor to radiate away the energy carried by the in-falling matter, and is therefore inversely proportional to the accretor’s thermal or Kelvin-Helmholtz timescale:

$$M_{\text{acc,max}} \sim \left(\frac{\varepsilon_{g,\text{acc}}}{L_{\text{acc}}}\right)^{-1} \propto R_{\text{acc}},$$

where $\varepsilon_{g,\text{acc}}$ is the specific gravitational binding energy at the accretor’s surface, $L_{\text{acc}}$ is the accretor’s luminosity and $R_{\text{acc}}$ is its radius at the beginning of mass transfer. In equation 4 we add the factor of ten (Paczynski & Sienkiewicz 1972; Hurley et al. 2002) to approximately account for the possible expansion of the accretor due to the mass transfer. Hereafter we label this prescription for the accretion efficiency parameter $\beta$ as THERMAL.

Alternatively, we can account for the expansion of the accretor up to the point of filling its Roche lobe (assuming the mass transfer to be fully conservative until this point) by using the accretor’s actual Roche lobe radius in place of $R_{\text{acc}}$ in equation (5) and dispensing with the factor of 10 in equation (4). We observe no significant difference between these two approaches for BeXRB predictions. In both simplistic models, we do not account for the change in the accretor luminosity during mass transfer (see, e.g., Kippenhahn & Meyer-Holmeister 1977), using the luminosity at the start of mass transfer for $L_{\text{acc}}$.

In our default model, we assume that the mass lost from the binary during mass transfer instantaneously takes away the angular momentum it had at the surface of the accretor (Hurley et al. 2002; Stevenson et al. 2017). This mode of mass loss is commonly referred to as isotropic re-emission (ISO hereafter in equations and figures) and corresponds to $\gamma_{\text{ISO}} = q^{-1}$.

### 4 RESULTS

In this section we present our findings for the BeXRB formation channels and compare between observed and simulated populations of BeXRBs. We distinguish between simulated intrinsic and simulated SMC populations: the first refers to the binary samples evolved by COMPAS; the second includes its convolution with the star formation rate (SFR) of the SMC and the duration of the BeXRB phase (see equation 6). Figures and tables refer to the latter, and the simulated SMC population is assumed in the text unless stated otherwise.

#### 4.1 Formation channel

We compare observations of BeXRBs in the SMC against the predicted properties of BeXRB-like systems in the synthetic populations of binaries evolved with COMPAS. Unlike some other stellar population synthesis studies (e.g. Belczynski & Ziolkowski 2009; Grudzińska et al. 2015), we avoid a priori cuts on orbital periods based on BeXRB observations. We instead investigate the origin of the observed systems by exploring the key predictions for different formation channels, expanding on the approach proposed for general Be stars by Shao & Li (2014).

According to our simulations, systems that avoid mass transfer can only marginally contribute to the observed sample of BeXRBs. The orbital periods which characterise non-interacting binaries are typically much longer than observations ($\gtrsim 10$ years).

Our simulations also suggest that binaries whose first mass transfer episode was dynamically unstable do not match the observed companion masses in BeXRB.

According to our evolutionary model, dynamical instability is favoured for high mass ratio systems; for the case of BeXRBs, this implies low-mass MS accretors. Moreover we expect minimal accretion during the short-duration CE phase. Therefore, in most NS + MS star systems that evolved through a CE episode, the MS companions are too light to match the observed distribution of Be star masses in BeXRBs (see figure 2). Even when there is a second episode of stable mass transfer following the re-expansion of the stripped primary after the helium main sequence, its
short duration is unlikely to allow for significant accretion onto the Be star. These findings are consistent with the conclusions of Shao & Li (2014), who similarly considered and discarded stable mass transfer following a common envelope phase as a possible formation channel for Be stars in BeXRBs. Moreover, systems experiencing a CE phase are significantly hardened by it and are therefore characterised by orbital periods that are too short (mostly below ∼ 20 days) compared to observations.

This leaves stable mass transfer as the most likely channel for producing BeXRBs, which is consistent with, e.g., Pols et al. (1991); Portegies Zwart (1995); van Bever & Vanbeveren (1997); Shao & Li (2014).

4.1.1 BeXRB-like systems
Motivated by these considerations, we define BeXRB-like systems as binaries:

(i) composed of a NS and a MS star with mass \( M_{\text{MS}} \gtrsim 3 M_\odot \). This mass cut roughly selects stars of B & O spectral types (Belczynski & Ziolkowski 2009; Grudzinska et al. 2015);

(ii) where the MS star is not overflowing its Roche lobe, as this would likely destroy the decretion disk (Panoglou et al. 2016);

(iii) whose secondary accreted during dynamically stable mass transfer from a hydrogen-rich primary (see above).

Mentions of mass transfer in the rest of the paper always refer to Roche lobe overflow from the primary before its collapse into a neutron star. We will indicate these mass transfer episodes preceding the BeXRB stage by the stellar type of the primary at the time of mass transfer, an arrow toward the secondary, and the stellar type of the secondary, e.g., HG → MS for mass transfer from a Hertzsprung gap (HG) primary onto a MS secondary.

To compare our results with observations, we infer the properties of our simulated SMC population of BeXRB-like systems by convolving the lifetime of modelled systems with the SMC SFR history (see figure 3). The distribution of orbital periods \( P_{\text{orb}} \), and companion masses \( M_{\text{MS}} \) of currently observable BeXRBs is given by

\[
\frac{dN}{dP_{\text{orb}}dM_{\text{MS}}} = \int \frac{dN}{dM_{\text{SR}}(-t)} \frac{dN}{dM_{\text{SR}}dP_{\text{orb}}dM_{\text{MS}}(t)} dt ,
\]

where the first term in the integrand is the SMC star formation rate history (measured at time \( t \) before the observing time 0), and the second term is the number of BeXRBs per unit orbital period per unit companion mass per unit star-forming mass at time \( t \) after star formation, as simulated by COMPAS. \( dN/dM_{\text{SR}}dP_{\text{orb}}dM_{\text{MS}}(t) \) can be also thought as the efficiency function for converting a mass \( M_{\text{SR}} \) of star formation into a number density of BeXRBs per unit orbital period per unit mass of the MS (Be) star, with the system still present after a time \( t \) since its formation. In practice, this integral is evaluated through a Monte Carlo approximation using our synthetic population. We estimate statistical uncertainties from this integration to be of the same order as the ones due to the imperfectly known SMC SFR history as reported in figure 16 of Rubele et al. 2015. One \( \sigma \) uncertainties are indicated with shaded areas in figures 2 and 1.

Similarly to Pols et al. (1991), we find that all selected binaries have experienced mass transfer initiated by the primary after hydrogen exhaustion (case B mass transfer). In particular, according to our default model, the primary of BeXRB-like systems always overflows its Roche lobe at the HG stage (hydrogen shell burning with contracting core, HG → MS). More evolved donors are ruled out by our stability criteria, which favour dynamically unstable mass transfer for giants that develop a deep convective envelope (Soberman et al. 1997).

About \( \lesssim 10\% \) of our simulated intrinsic population of BeXRBs start mass transfer while the primary is burning hydrogen in its core (case A mass transfer), at the MS stage (MS → MS). However, after re-weighting by the SMC SFR history to obtain the currently observable population (according to equation 6), this fraction becomes negligible (\(< 2\% \)). Systems initiating stable mass transfer at early stages are likely quite massive binaries. This penalises them as they both quickly pass through the BeXRB phase and miss the peak of the SMC SFR (current BeXRBs having experienced MS → MS mass transfer were born after the peak of SFR).

A considerable fraction of binaries in our simulated BeXRB populations (in our default model: \( \sim 20\% \) of the intrinsic population and \( \sim 50\% \) of the SMC one) experience a second mass transfer episode after the primary has been stripped by its hydrogen envelope (case BB mass transfer). This mass transfer episode, which in the case of BeXRB formation always involves a MS accretor, happens during the primary’s helium Hertzsprung gap (HeHG) stage, after the primary completes its helium main sequence evolution (HeHG → MS). In these cases, we assume that the primary becomes an ultra-stripped star and if it explodes in a SN, it experiences lower natal kicks (USSN). Together with the typically low orbital separations, these reduced kicks make the disruption of the binary during a SN very unlikely. Therefore, these systems are prominent in the simulated intrinsic population of BeXRBs. Systems experiencing case BB mass transfer are also favoured by re-weighting by the SMC SFR according to equation 6. Their progenitors are characterised by relatively low mass ratios \( q_{\text{HG}} = M_{\text{acc}}/M_{\text{donor}} \lesssim 0.4 \), which is one of the reasons why they are brought to close separations after the initial mass transfer episode that strips hydrogen off the primary. This difference between the masses of donors and accretors yields lighter MS (Be) companion stars (due to both their initially lower masses and generally less efficient accretion). In turn this entails a longer duration of the BeXRB phase and larger formation time, which matches the time since the peak of the SMC SFR. However, the predicted orbital period and MS star mass distributions depend only very weakly on the details of case BB mass transfer. This is mostly because of the short time scales and the relatively small amount of mass loss/exchange.

In figure 4 we show our preferred BeXRB formation channel, indicating the fraction of the simulated SMC population of binaries undergoing multiple mass transfer episodes.
4.2 Comparisons: Be star mass and orbital period distributions

We compared observations to our simulated SMC population of BeXRBs in figures 5, 2 and 1 and in table 1. According to our default SMC model (see figures 5 and 2), most of the detected BeXRBs should have a MS companion $M_{\text{MS}} < 6 M_\odot$. The low value at which the distribution peaks clearly contradicts observations (see, e.g., table 4 in Hohle et al. 2010), according to which Be stars in BeXRBs should be appreciably more massive than general Be stars. Because the synthetic BeXRBs with low-mass companions do not cluster toward high orbital separations, selection effects are unlikely to resolve this incompatibility.

As shown in figure 5, the main determining factor for the companion mass distribution is the accretion efficiency $\beta$ during the HG $\rightarrow$ MS mass transfer. Lower $\beta_{\text{HG}}$ values reduce accretion of the primary hydrogen envelope onto the secondary, leaving the MS mass of initially light stars almost unchanged. Such binaries containing initially low-mass MS stars dominate our simulated SMC population, because they are favoured by both the IMF and the re-weighting for the lifetime of the BeXRB stage.

Moreover, the orbital period distribution from our default model (dashed blue line in figure 1) does not match observations well. The predicted overabundance at long orbital periods can be explained by observational selection effects (see section 2.1). However, the predicted but unobserved BeXRBs with short orbital periods indicate a failure of this model.

The predicted peak at short orbital periods is mostly due to the population of systems experiencing USSNe (see e.g. figure 6). As mentioned above, these binaries are very likely to survive the SN and are characterised by small separations and low masses. In our default model, this population is only partially suppressed by the stability criteria (Podsiadlowski et al. 1992), which allow all systems with $q_{\text{HG}} \gtrsim 0.23$ to engage in stable HG $\rightarrow$ MS mass transfer.

4.2.1 Variations from the default model

The inconsistencies between observed and predicted BeXRB populations motivated us to investigate some of the assumptions on which our default model relies. We focus on the accretion efficiency $\beta$ (see equation 2) and the specific angular momentum loss $\gamma$ (see equation 3). When we plot MS mass and orbital period distributions, we label our variations showing the assumed models for accretion efficiency and specific angular momentum loss. We also explicitly highlight our default model (see section 3.2.1) and preferred variation. A model that assumes fixed accretion efficiencies ($\beta_{\text{HG}} = 0.5$, with fully conservative case $A$ and case BB mass transfer) and isotropic re-emission of non-accreted material from the surface of the accretor is our preferred model, as justified below (this model is also identified as $\beta$: HG0.5MS1 BB1; $\gamma$: ISO).

In figure 2, we show the impact of the accretion efficiency during HG $\beta_{\text{HG}}$, by varying it from the THERMAL prescription to different fixed values: 0.5, 0.75, 1. Because our results only very weakly depend on MS $\rightarrow$ MS and HeHG $\rightarrow$ MS mass transfers, they are always assumed to be conservative. This is shown by the label $\beta$: MS1 and $\beta$: BB1. For these variations we keep the isotropic re-emission model for the specific angular momentum loss. Figure 2 shows that, as expected, applying increasingly higher accretion efficiencies during the HG $\rightarrow$ MS mass transfer moves the mass distribution toward increasingly higher MS star masses, to better match the observed range. However, we also notice that the area below the curve, which represents the total number of BeXRB-like systems in our simulated population, shrinks for progressively more conservative mass transfer (see also table 1).

There are several reasons which explain the drop in the predicted number of BeXRB-like systems for more conservative mass transfer. First of all, the more massive the MS secondary stars become, the shorter is the duration of the BeXRB stage. Secondly, more conservative mass transfer leads to larger typical separations, so the binaries are more likely to be disrupted by the SN kicks. Larger separations after the HG $\rightarrow$ MS mass transfer also imply fewer binaries engaging case BB mass transfers. In turn, this yields to fewer primaries experiencing the reduced kicks of the USSNe and so fewer systems surviving the SN explosion. Finally, assuming the isotropic re-emission model for the specific angular momentum loss, the response of the Roche lobe to mass transfer favours more unstable interactions for higher $\beta$ values. The amount of specific angular momentum loss during mass transfer depends on the location from where the matter leaves the system. In our default model, we assume isotropic re-emission from the surface of the accretor. Other common scenarios consider matter leaving from the surface of the donor, fast or Jeans mode ($\gamma$: JEANS heareafter), or a circumbinary ring ($\gamma$: CIRC heareafter). In our settings, the semimajor axis of the circumbinary ring $a_{\text{ring}}$ is fixed to twice the binary’s semimajor axis $a$ (Artymowicz & Lubow 1994) (see figure A1).

In figure 2 we also report a variation of the default model in which we change the prescription for the angular momentum loss to the circumbinary ring mode ($\beta = \text{THERMAL}; \gamma = \text{CIRC}$). Our results confirm the conclusions of Portegies Zwart (1995), who first suggested high angular momentum loss to explain the Be mass distribution in BeXRBs. Portegies Zwart (1995) suggested matter leaving from the second Lagrangian point or, similarly, with a specific angular momentum 6 times higher than the binary’s one ($\gamma = 6$). This last case also qualitatively resembles our results for $\gamma = \text{CIRC}$. By assuming:

$$\gamma_{\text{CIRC}} = \frac{(1+q)^2}{q} \sqrt{\frac{a_{\text{ring}}}{a}} = \sqrt{\frac{(1+q)^2}{q}}$$

we are indeed considerably increasing the angular momentum lost by our simulated SMC binaries, particularly disfavouring low mass ratio systems, which are now very likely to merge. Consequently, the MS secondaries’ mass in the surviving binaries matches well the observed mass range of Be stars in BeXRBs (see figure 2). However, as shown in

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2 Stable mass transfer initiated by MS stars is commonly accepted to be close to conservative (Schneider et al. 2015); it is similarly so also in our default model (see equation 4). The accretion efficiency during case BB mass transfer has been less studied; we decide to fix its value to 1 to be conservative in our conclusions concerning HG $\rightarrow$ MS mass transfer.
The expected number of BeXRB-like systems drops considerably below the 69 observed BeXRBs listed in the CK catalogue. This is due to the increased number of mergers and our stability criteria, which in the circumbinary ring mode disfavour stable HG \( \rightarrow \) MS mass transfer, especially if these are highly non-conservative, as is often the case for our default THERMAL \( \beta \) model (see figure 5). Moreover the expected orbital period distribution of the simulated BeXRB population in this model peaks at a few days, in disagreement with observations. Therefore according to our results, high angular momentum losses are unlikely to explain the high masses of MS stars in BeXRBs. These considerations also strongly disfavour more extreme angular momentum losses, as would be the case for matter leaving the binary from L2 or L3.

In figure 1, we show the orbital period distributions of our simulated SMC populations of BeXRBs, assuming constant accretion efficiencies \( \beta : \) HG0.5 MS1 BB1 and different angular momentum models. This assumption for the accretion efficiencies allows the masses of the MS companion at the BeXRB stage to be consistent with the observed range and to depend only slightly on the angular momentum loss models (see figure A3). For comparison, we also plot the orbital period distribution for fully conservative mass transfer (black line) and for our default model (dashed blue line).

In purple we show the orbital period distribution under the assumption of isotropic re-emission from the surface of the accretor (our preferred model). For the reasons mentioned above, this predicts fewer BeXRBs than the default model and shows a drastic decrease in the number of systems with orbital periods below \( \sim 30 \) days.

The orange curve shows that the simulated SMC BeXRB population shrinks and moves toward shorter separations when assuming high angular momentum losses (see equation 7).

In yellow, we plot the orbital period distribution for the BeXRB population obtained under the assumption that material lost from the system during non-conservative mass transfer does so from the surface of the donor. Under this condition, \( \gamma_{\text{Jeans}} = q \). During the first mass transfer episode this value is always less than one, meaning the binary increases its specific angular momentum by losing matter and so widens. Contrary to isotropic re-emission from the accretor surface, the more unequal the component masses, the less angular momentum is assumed to leave the system. This scenario is therefore appealing for moving short period systems toward higher separations. However, according to our stability criteria, this combination of \( \beta \) and \( \gamma \) values allows binaries with mass ratio as low as \( \sim 0.14 \) at the onset of mass transfer to engage in stable HG \( \rightarrow \) MS mass transfer. Most of these low \( q \) systems end up contributing to the short orbital period tail after their primaries experience an USSN, leading to an orbital period peak around \( \sim 20 \) days, too low to match the observations.

In table 1 we report the number of BeXRB-like systems in our simulations with orbital period below the maximum reported in the CK catalogue (\( \sim 520 \) days) as a function of the different accretion efficiency (columns) and angular momentum loss (rows) models.

### 4.3 Stability criteria: impact on synthetic BeXRB populations

According to our binary evolution models, HG \( \rightarrow \) MS is the most relevant mass transfer episode for the formation of BeXRBs (see section 4.1). In COMPAS, the stability criterion is based on the stellar type of the donor (HG) and on the response of the Roche lobe, which in turn depends on the values of \( \beta \) and \( \gamma \) of the specific system. In our default model, our assumptions determine a critical mass ratio of \( \sim 0.23 \), compared e.g. to the Claeyss et al. (2014) value of 0.25: only systems with mass ratio above this value can experience stable HG \( \rightarrow \) MS mass transfer and so become BeXRB-like systems (see table B1 for typical critical mass ratios assuming different \( \beta \) and \( \gamma \) models).

The stability of mass transfer is, however, highly uncertain. In particular, when rapid accretion causes the accretor to swell and over-fill its Roche lobe in, if the envelope is co-rotating, the orbit of the binary may shrink due to angular momentum loss from the system without leading to a classical CE spiral-in. After some time, when the donor under-fills its Roche lobe, a stable mass transfer phase begins (see figures 3–5 in Podsiadlowski et al. 1992). This idea was originally applied to stars with convective envelopes. However Pols (1994) and Wellstein et al. (2001) have shown that a similar behaviour is also expected for most massive HG donors, if the mass ratio is less than \( \sim 0.7 \) (see, e.g., figure 5 in Pols 1994). These systems are all expected to form a contact phase with at least a temporary CE in which they lose a lot of angular momentum and may therefore merge. HG stars may nonetheless be able to avoid a runaway plunge-in, because of their radiative envelopes. The details of this process are not well understood and it is therefore possible that some of the systems that we assume can successfully reach a BeXRB stage, should instead have merged. Figure 7 shows the systems that would survive this mass transfer phase under more stringent conditions on the mass ratio. It indicates that, in order to match the observed number of SMC BeXRBs, only a small fraction of binaries can merge prematurely through this channel.

The critical mass ratio not only determines how many simulated systems can become BeXRB-like, but also their distribution in MS masses and orbital periods (e.g. Shao & Li 2014; Portegies Zwart & Verbunt 1996). The smaller the critical mass ratio, the more systems experience stable mass transfer. Then systems with low \( q \) values may dominate the distributions of the simulated BeXRB population, as they are characterised by long-lived low mass MS companions, as the minimal primary mass is limited by the requirement of generating a NS (see e.g. Jeans in figures 1 and A3). A very low mass ratio threshold for stability can therefore predict many short orbital period systems; the contrary happens for very high thresholds, which can drastically reduce the number of systems with orbital periods below \( \sim 100 \) days. This is shown in figure 7, which also indicates that the typical critical mass ratio for HG \( \rightarrow \) MS stability should range between 0.2 – 0.3 for our treatment of USSNe.

In figure 6 we show the orbital period distribution of BeXRB-like systems obtained by assuming a different stability criterion, this time based on the temperature of the donor, instead of its stellar type. In this model, when a post-MS star fills its Roche lobe we approximate the response of
the stellar radius to mass loss with a constant logarithmic derivative if its temperature is above \( \log_{10}(T/K) = 3.73 \) (i.e. when, according to Belczynski et al. 2008, the star has a radiative envelope), and with condensed polytrope models (Soberman et al. 1997) otherwise. The stability of mass transfer is then determined by comparing the stellar response with the Roche lobe response. This variation for determining the stellar response allows early core helium burning stars to engage in stable mass transfer onto a MS star and so to contribute to our synthetic population of BeXRBs. The simulated SMC population of BeXRB-like systems in figure 6 has been evolved by extending the preferred model \((\beta : HG0.5\text{MS1BB1}; \gamma : ISO)\) to a constant \(\beta = 0.5\) for stable mass transfer from core helium burning donors. We find the results obtained under this assumption to be in good agreement with observations.

5 DISCUSSION

5.1 Accretion efficiency during stable mass transfer

The comparison between observations and our synthetic population of BeXRB-like systems suggests that our default model underestimates the accretion efficiency during HG → MS mass transfer. Indeed the simplified fully conservative and half conservative variations allow the simulated Be-star mass distribution to match the observed range. According to our study, fully conservative mass transfer is however disfavoured by the low predicted number of synthetic SMC BeXRB-like systems (see also Shao & Li (2014)). Higher angular momentum losses can also explain the Be mass distribution, but they are unable to reproduce the high number of BeXRBs observed in the SMC.

Our findings on accretion efficiency during HG → MS mass transfer are supported by other studies of massive X-ray binaries (e.g., Shao & Li 2014; Kaper et al. 1995). On the other hand, Wolf-Rayet stars in binaries with a MS companion strongly suggest highly non-conservative mass transfers (Shao & Li 2016; Petrovic et al. 2005). Interestingly, de Mink et al. (2007) point out a weak correlation between orbital period and the efficiency of mass accretion, possibly due to tides and rotationally limited accretion. However, as pointed out by Shao & Li (2014), rotationally limited accretion entails highly non-conservative mass transfer, which would leave most of the MS star masses in BeXRB-like systems below \(6 M_\odot\), in contradiction with observations (see figure 5).

In our default model, very low accretion efficiencies occur if, at the beginning of the mass transfer episode, the donor mass loss timescale and the timescale of the thermal response of the accretor differ by almost two orders of magnitude (as shown by equation 4), i.e., if their evolutionary stage is very different. The need to reduce the rate of occurrence of these low accretion-efficiency events in order to match observations could indicate that such mass transfer events lead to dynamical instability when the accretor overflows its Roche lobe (e.g., Narain & Sugimoto 1976; Dewi et al. 2006; Ivanova et al. 2013). However, a more conservative stable mass transfer than predicted by the simplistic default model is likely required to reproduce both the observed number of systems and the Be-star mass distribution.

5.2 Metallicty or age as the driver of the SMC BeXRB excess?

According to our preferred formation channel, the recent peak of the SMC SFR (∼20–40 Myrs ago) matches the typical time necessary to form BeXRB-like systems in our simulations, as shown in figure 3. Similar formation times have been observationally confirmed for HMXRBs (Williams et al. 2013, 2018). This, together with the strong observational selection effects which impact X-ray detections in the Milky Way, might explain why the number of detected BeXRBs is so similar in the SMC and the Galaxy, despite the very different masses and current SFRs (see also Antoniou et al. 2019, 2010).

In the general framework of the Be phenomenon, metallicity has also been proven to be a crucial parameter (Martayan et al. 2007). In the case of BeXRBs, our results suggest that all stable HG → MS mass transfer may generate a Be star. Assuming this is true, i.e. assuming for simplicity that all MS stars in BeXRB-like systems are Be stars, the effect of metallicity reduces to its impact on binary evolution. Because of the metallicity dependence of HG expansion, high metallicities support short period BeXRB-like systems, whose primary has experienced an USSN, while low metallicities favour wide binaries.

To test how much our synthetic BeXRB population is sensitive to this parameter, we repeated the same analyses assuming either half the SMC metallicity or the Galactic metallicity. The overall change in predicted number of BeXRB-like systems is \(\lesssim 15\%\) assuming fully conservative mass transfer and \(\lesssim 25\%\) for our default model. According to our default model higher metallicity results in a larger population of synthetic BeXRBs. Conversely, the highest numbers of BeXRB-like systems are reached at the lowest metallicities if assuming fully conservative mass transfer.

5.3 What can the absence of BeXRBs with short orbital period imply?

Most of our simulated SMC BeXRB populations over-predict the number of systems at short orbital periods. As mentioned above, this overabundance is closely connected to the USSNe. According to our model, USSNe are characterised by low NS natal kicks and short separations, and are therefore unlikely to disrupt the binary (assuming \(\sigma_{1D} = 30\text{ km s}^{-1}\), the binary survival rate is \(\sim 98.5\%\)). We can reconcile observations with predictions with two different approaches. Firstly, we can change our assumptions and so suppress this sub-population. In the following we list three possible examples:

- We could change the stability criteria, allowing only systems with higher mass ratio to experience stable mass transfer (see e.g. \(q_{\text{HG}} > 0.3\) of figure 7), assume a different initial mass ratio distribution or different stellar response to mass loss.
- Alternatively we could assign higher natal kick velocities to NSs born through USSNe. This, however, may contradict observations (Schwab et al. 2010; Beniamini & Piran 2016; Brisken et al. 2002).
- Another possibility invokes more conservative mass transfer. As discussed in section 4.2.1, the contribution of

\[ \beta \rightarrow HG_0.\text{MS1BB1; ISO} \]

\[ \beta = 0.5 \]
binaries whose primary has experienced a USSNe, decreases when assuming higher accretion efficiency.

However, the overall number of predicted BeXRBs provides an additional constraint (see section 5.5); for example, setting \( q_{HG} > 0.3 \) without further modifications lowers the overall number of BeXRB-like systems below the 69 observations reported in the CK catalogue.

Additional selection effects may also explain the absence of short orbital period BeXRBs in the observed sample. MS stars in binaries with orbital period below a week may be unable to create a decretion disk because of tidal interactions (Panoglou et al. 2016). Tidal locking can become relevant at periods \( \lesssim 20 \) days, and could slow down the rotation of MS stars and so avoid the formation of the decretion disk.

### 5.4 Impact of accretion efficiency on double compact objects (DCOs)

Adjusting mass transfer prescriptions to match the Be star mass distribution in BeXRBs affects the rate of formation DCOs. Here we discuss the impact of the various prescriptions on the formation of DCOs at the metallicity of the SMC.

According to our default (preferred) model, the formation rate of binary black holes (BBHs) merging within the age of the Universe per unit star forming mass is \( \sim 3.5(2.1) \times 10^{-5} M_{\odot}^{-1} \). Our default model also yields the highest formation rate of neutron star – black hole binaries (NSBHs) merging within the age of the Universe \( \sim 1.5 \times 10^{-5} M_{\odot}^{-1} \). This drops by a factor of \( \sim 1/3 \) for our preferred model. In general the contribution of BeXRB-like systems to the population of NSBH binaries merging within the age of the Universe strongly depends on the prescription for angular momentum loss. It is around \( \sim 20\% \) (\( \sim 24\% \) for our default model and \( \sim 18\% \) for our preferred one) if we assume isotropic re-emission of the ejected material from the surface of the accretor.

The formation rate of binary neutron stars (BNSs) is sensitive to the assumed accretion efficiency. It is equal to \( \sim 1.7(1.2) \times 10^{-5} M_{\odot}^{-1} \) per unit star forming mass for our default (preferred) model and rises to \( \sim 2.5 \times 10^{-5} M_{\odot}^{-1} \) for fully conservative mass transfer. The percentage of BNSs merging within the age of the Universe varies from \( \sim 65\% \) for the default model to \( \sim 83\% \) for the preferred model to \( \sim 93\% \) when non-accreted material is assumed to carry away the specific angular momentum of a circumbinary ring. Our models also demonstrate that a significant fraction of the BNS population (generally \( \gtrsim 1/3 \)) experienced a BeXRB-like phase\(^3\). In particular, in our preferred model, almost all BNSs (\( \sim 96\% \) of the overall population and \( \sim 98\% \) among those merging within the age of the Universe) experienced a BeXRB-like phase. These results are consistent with Vigna-Gómez et al. (2018), where the authors study NS binaries in the Galaxy and conclude that the main formation channel begins with stable mass transfer from a HG donor onto a MS secondary.

Carefully quantifying the impact of our variations on the DCO merger rates for gravitational-wave observations would require an integration over the cosmic star formation history (see e.g. Neijssel et al. 2019), which is beyond the scope of this paper.

### 5.5 On the predicted number of BeXRB-like systems

The low yield of our simulations suggests that the majority of B stars in binary systems with a NS are Be stars. The enhanced ratio of Be stars to B stars in these interacting systems is consistent with the hypothesis that the Be phenomenon originates from accretion, or is boosted by it (de Mink et al. 2013). On the other hand, the challenge of producing more BeXRB-like systems in simulations suggests that the existing census of BeXRBs in the SMC is already close to complete, with negligible selection effects at orbital periods below a year.

There are, however, several uncertainties/complications which may affect our estimated number of BeXRB-like systems.

- In COMPAS, we currently do not account for rotation. This may impact our estimate of BeXRB-like systems, as we are not accounting for rotational mixing and the consequence additional hydrogen available for nuclear burning at the MS stage. The associated change in the lifetime of the MS stars would change the duration of the BeXRB stage and the number of our synthetic SMC BeXRBs.
- Another uncertainty, which might affect the predicted numbers of simulated BeXRBs, is the magnitude of SN kicks. There is ongoing debate about the frequency of low natal kicks in the observed pulsar population (e.g., Hobbs et al. 2005; Arzoumanian et al. 2002a; Verbunt et al. 2017; Bray & Eldridge 2016, 2018). For example, Podsiałowski et al. (2004) argued that low-mass iron core collapse SNe might experience lower kicks compared to typical CCSNe.
- As shown in section 4, the stability criteria for mass transfer may play a key role. Some of the mass transfer episodes designated as dynamically unstable in our simulations could, in fact, be stable and produce BeXRBs. However, increasing the number of systems undergoing stable HG → MS mass transfer with low mass ratio \( q \) would considerably increase the unobserved population of BeXRB-like systems at short orbital period, assuming the primaries experience low-kick USSNe which allow such binaries to survive.
- Conversely, some of the systems undergoing a CE event before the first SN may contribute to the observed population of BeXRBs if:
  - a significant amount of mass is accreted during dynamically unstable mass transfer, which appears unlikely (MacLeod & Ramirez-Ruiz 2015; De et al. 2019);
  - not much mass needs to be accreted to spin up the MS stars (Packet 1981) (or the Be phenomenon is not linked to accretion), so CE events are sufficient to produce Be stars — however, to avoid BeXRBs with low-mass companions that are not present in the observational sample,
CE events with low-mass companions would lead to mergers; or
- the systems which experience a subsequent dynamically stable mass transfer episode after a CE event acquire a significant amount of mass, despite the short duration of this mass transfer episode and the significant difference in the thermal timescales of the donor and the accretor.

- The significant scatter of the Corbet relation could shift some of the 25 BeXRBs with unknown orbital properties in the CK catalogue to orbital periods larger than the maximum measured one of \( \sim 520 \) days. This seems reasonable given some of their relatively large spin periods (3 have spin periods above the maximum of the 44 systems with known orbital periods and 9 have it above the one corresponding to \( P_{\text{orb}} = 520 \) days according to the fit of equation (1)). This would reduce the number of observed systems with \( P_{\text{orb}} < 520 \) days that we are trying to match.
- The number of simulated SMC BeXRBs could be slightly increased by considering the contribution of post-MS stars to the Be star population. Indeed the CK lists luminosity classifications that range between type II and type V. However, we only consider MS stars as the luminosity classification may not be indicative of the actual photometric magnitudes (see e.g. McBride et al. 2008), due to both difficulties of the measurements involved and the heterogeneity of the adopted methods.

Finally we should also consider the significant uncertainties in the total number of observed BeXRBs in the SMC. Maravelias et al. (2019) and Haberl & Sturm (2016) argue that the SMC contains about \( \sim 120 \) HMXRBs, of which almost all are supposed to contain a Be star. Only about half of those (the 69 reported in the CK catalogue) show X-ray pulsations, clearly identifying NSs as the accreting compact object in the binary. The remaining systems may contain BHs/white dwarfs or NSs, with unknown, but most likely large (\( \gtrsim 100 \) s, Haberl & Sturm 2016) spin periods. Further observations are needed to measure the fraction of HMXRBs not exhibiting X-ray pulsations that contain NSs. If this fraction is high, or if an appreciable population of NS + MS systems with no X-ray emission is discovered, our models may need further revision.

6 CONCLUSIONS

Our models strongly suggest that observed BeXRBs experienced one or more dynamically stable mass transfer episodes initiated by the progenitor of the NS.

We find that stable mass transfer with accretion efficiency \( \gtrsim 0.3 \) matches the observed properties of BeXRBs in the SMC. A similar result was presented by Shao & Li (2014). This points to an increased incidence of Be stars in interacting binaries through spin-up generated by mass accretion.

We show that a deeper understanding of selection effects and of the occurrence rate of Be vs B stars in binary systems with a NS can improve inference on the critical mass ratio for stable mass transfer, accretion efficiency and angular momentum loss during HG \( \rightarrow \) MS mass transfer and/or USSN kicks.

Our simulations also suggest that currently observed BeXRBs were born during the peak of SFR in the SMC, at least partially explaining the observed abundance of BeXRBs. This is supported by the predicted age of interacting systems at the NS+MS-star stage. In terms of binary evolution, metallicity does not appear to play a major role in explaining the abundance of BeXRBs in the SMC. Our models also suggest that metallicity may not be crucial even in terms of Be vs B star occurrence in BeXRBs. We find that there is not much room for a population of non-interacting NS + MS systems (which could represent the fraction of NS + MS binaries containing a normal B star): the sample of observed BeXRBs in the SMC must be close to complete, at least for relatively close systems. Only then can our models produce enough BeXRBs to be consistent with observations.

The requirements placed on mass transfer stability and accretion efficiency by BeXRB observations impact models of other massive binaries, including predictions for DCO mergers.

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Table 1. The table reports the number of BeXRBs with orbital period under 520 days (longest period listed in the CK catalogue), predicted in the SMC according to our synthetic populations of binaries. According to the CK catalogue there are at least 69 BeXRBs in the SMC, of which at the very least 44 (all systems with observed orbital periods) have period below 520 days. The rows correspond to different modes of angular momentum loss, while the columns represent different accretion efficiency models.

| ISO | HG0.5 MS1 BB1 | CONSERVATIVE |
|-----|--------------|--------------|
| 190 ± 20 | 85 ± 10 | 45 ± 5 |
| 18 ± 2 | 40 ± 5 | as above |
| 130 ± 10 | 130 ± 5 | as above |

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Figure 1. Orbital period distribution of our simulated SMC BeXRB-like systems. Different colours correspond to different mass transfer models; shading indicates $1\sigma$ uncertainties. For this plot we focus on simulations where the model for how conservative mass transfer is ($\beta$) is fixed to HG0.5MS1BB1, which seems to better predict Be masses and the number of BeXRBs. For reference, we also plot in dashed light-blue and in black the results obtained for the default and the fully conservative mass-transfer models. In dark grey we show the distribution of the 44 observed orbital periods reported in the CK catalogue. In light grey we show the orbital period distribution obtained by applying the Corbet relation fit from equation (1) to the 25 systems in the CK catalogue which only have measured spin periods.

Figure 2. Mass distribution of the MS stars in our simulated SMC BeXRB-like systems. Different colours correspond to different mass transfer models; shading indicates $1\sigma$ uncertainties. We focus on $\gamma$: ISO as this better predicts the orbital distribution of BeXRBs; we however also report a single case where $\beta$: THERMAL and $\gamma$: CIRC, to show how increasing the angular momentum loss leads to heavier MS stars, despite the relatively low accretion efficiency. In blue and dark green we show our default and preferred models. In grey we show the region of observed Be star masses; the light grey corresponds to a conservative estimate of this range, based on the Be spectral type distribution (Hohle et al. 2010); in darker grey we highlight the range where we expect the bulk of the masses (see text for explanation).

Figure 3. In teal, we plot the Star Formation Rate as a function of time in Myrs, as reported in Rubele et al. 2015 (y-axis on the right). In violet we show the formation time distribution for BeXRB-like systems in our default simulation, weighted by the duration of the BeXRB-like phase and normalised to 1. The formation time distribution peaks around the time of the recent SMC starburst.

Figure 4. Schematic representation of our preferred BeXRB formation channel. All the mass transfer episodes are stable. The symbols represent the evolutionary stages of each star in the binary: main-sequence (MS); Hertzsprung gap (HG); Helium main-sequence (HeMS); Helium Hertzsprung gap (HeHG) and neutron star (NS). After the mass transfer episode from a Hertzsprung-gap primary, the accreting secondary spins up and becomes a Be star in our model.
Figure 5. Simulated main-sequence companion (Be star) mass $M_{MS}$ versus the efficiency of mass transfer for a Hertzsprung-gap donor $\beta_{HG}$. The colour bar shows the intrinsic probability, for BeXRB-like systems, of obtaining a specific combination of $M_{MS}$ and $\beta$ according to our default model. As in figure 2, the grey area shows the range of observed Be star masses, according to our conservative interpretation (Hohle et al. 2010).

Figure 6. Orbital period distribution of BeXRB-like systems assuming the alternative stability criterion based on temperature, described in section 4.3. Shaded grey areas follow the description of figure 1. Our synthetic BeXRB-like systems are divided into two sub-populations, based on the stripping level of the NS progenitor: the dotted line marks binaries with ultra-stripped (US) primaries, which therefore undergo USSNe; while the dashed line shows the BeXRB-like systems whose primary at the time of SN has been stripped of its hydrogen envelope only (SS).

Figure 7. The effect of different critical mass ratios (in different shades of blue) on the predicted SMC orbital period distribution and total number of BeXRBs. We focus on mass transfers from HG primaries, as this is the crucial episode in our preferred BeXRB formation channel. We show results for the preferred model: constant accretion efficiencies ($\beta : HG0.5 MS1 BB1$) and isotropic re-emission from the surface of the accretor ($\gamma : ISO$). Our default stability criteria correspond to mass transfer with $q_{HG} > 0.23$ being typically stable. The meaning of shaded areas is the same as in figure 1.
Figure A1. A schematic representation of the most commonly used angular momentum loss models and their underlying assumptions in terms of where mass is lost from. In this plot, acc is an abbreviation for accretor. The colours of the matter leaving the systems for each specific angular momentum prescription match the colours used to represent results in figures 1 and A3.

Figure A2. Orbital period distribution of the simulated BeXRB-like systems corresponding to the MS mass distributions reported in figure 2. The meaning of shaded areas is the same as in figure 1.

APPENDIX A: ORBITAL PERIOD AND MASS DISTRIBUTION

In figure A1, we show a schematic representation of the three different scenarios assumed for the angular momentum loss during mass transfer. From left to right we show the case of matter leaving the binary from: the surface of the accretor (isotropic re-emission, also our default and preferred angular momentum loss mode); a circumbinary ring with semimajor axis twice that of the binary (circumbinary ring); and the surface of the donor (fast or Jeans mode). The colours match those used in figures 1 and A3. We show the MS mass (figure A3) and BeXRB orbital period (figure A2) distributions corresponding to the mass transfer models explored in figures 1 and 2.

Table B1. Critical mass ratios for mass transfer initiated by HG stars, depending on the assumed accretion efficiency $\beta$ and specific angular momentum loss $\gamma$. In our model, the stability threshold is determined for each binary by comparing the responses of the donor radius and the Roche lobe size to mass transfer. According to our default model, the donor response to mass loss depends on its stellar type and is fixed to a logarithmic derivative of 6.5 for a HG star. The response of the Roche lobe to mass loss is determined by the specific angular momentum lost, the accretion efficiency and the mass ratio of each binary, so the table lists typical values. In the table we show that, assuming isotropic re-emission, the critical mass ratio can vary between 0.22 and 0.26 depending on the choice of $\beta$. These values can be compared with the fixed mass ratio of 0.25 adopted in (Claeys et al. 2014), who also assumed isotropic re-emission from the surface of the accretor.

APPENDIX B: STABILITY

In table B1 we report the critical mass ratio for mass transfer initiated by a HG star. According to our model such a donor has a response to mass transfer characterised by $\zeta^* = 6.5$, where $\zeta^*$ is defined in terms of the star’s radius $R^*$ and its mass $m$ as $\zeta^* \equiv d \ln (R^*)/d \ln (m)$. $\zeta^*$ is then compared to $\zeta_{\text{RL}}$, which represents the response of the size of the Roche lobe to mass transfer. $\zeta_{\text{RL}}$ depends on the assumed accretion efficiency $\beta$, angular momentum loss $\gamma$ and mass ratio $q$ of the binary. When $\zeta^* \geq \zeta_{\text{RL}}$ the mass transfer is stable.