Interaction of the pile and surrounding soil during vibration driving

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Abstract. At present, a vibratory method is used for the construction of prefabricated piles and retaining elements of deep excavations. Especially when building in urban development and difficult geological conditions. The research presented in this paper is devoted to the analysis of the interaction of the pile of friction and a two-layer soil base under a vibrating load on the pile head. An analytical solution of the problem is presented in a static and dynamic formulation. The proposed solution takes into account the elastic, viscous and rheological properties of soils. The pile is considered absolutely not compressible. A similar problem is solved by the finite element method using an elastic-viscoplastic model of soils.

1. Introduction

When designing foundations for machines and equipment with dynamic loads, the ground base is considered, in most cases, as weightless and characterized by stiffness coefficients $K_z$, $K_x$ и $K_\phi$. Such a simplified model of the elastic base known as the Winkler model, makes it possible to greatly simplify the solutions of the differential equations of foundation oscillations on a compressible base. Viscous resistance in this solution, as a rule, is not taken into account.

If it is necessary to consider the inelastic resistance of the base, rheological models that take into account the damping properties of the soils are used. The most common are the models of Kelvin-Voigt and Maxwell.

In this paper we consider the problem of oscillation of a pile foundation on a weighted (inertial) foundation taking into account the elastic-viscous properties of the base and friction surfaces along the lateral surfaces of the pile.

The first attempts to consider the adjoining mass of base soils were made by E. Reissner [1], and then by O. Ya. Shekhter [2]. The problem of forced oscillations of a circular die on an elastic, half-space under the influence of a harmonic disturbing force was considered. In the paper [3], Z. G. Ter-Martirosyan considers a buried foundation on a weighty multi-layered foundation.

Taking into account the inertial forces of the base soils leads to the fact that the amplitude of the vibrations of the deep foundation does not go anywhere to infinity, that is, there is no resonance. Moreover, as the Poisson's ratio increases, the energy from the oscillating pile to the ground increases, and, consequently, the damping of the system's oscillations. In addition, taking into account the inertia of the foundation soils leads to aperiodic vibration of the foundation, that is, there is no uniform vibrational motion of the foundation. At the same time, for pile foundations of short length and large width, the influence of inertial properties of the foundation soils is much more pronounced than for piles having a long length and a relatively small area of the sole [4, 5].
Models with “attached mass” are currently used to solve practical problems by various methods, including by equating the volume of soil entering “the active zone of deformation” of the base. When calculating piles of a large cross-section, it is often assumed that the dynamic load is transferred to the ground of the base in the form of a bearing column, a conical (trapezoidal) array, and a half-space [6].

Figure 1. The design scheme of the problem of interaction of a pile with a two-layered base.

2. The static problem
The force acting on the pile head is distributed between the resistance along the side surface of the pile and the resistance at the bottom end [7]:

\[ N = R + T, \]  

(2.1)

where \( N \) is a force on the pile head, kN; \( R \) – resistance under the bottom end of the pile, kN; \( T \) is a resistance along the lateral surface of the pile, kN.

If we accept the telescopic mechanism of interaction between the pile and the surrounding soil (only shear deformations are taken into account in the calculations), the resistance along the lateral surface of the pile is determined by the product of the doubled radius of the pile \( 2a \) (m), the working length of the pile \( l_c \) (m), and shear stresses along the lateral surface \( \tau_a \) (kN/m\(^2\))

\[ T = 2a \pi \tau_a l_c. \]  

(2.2)

Taking into account the dependence (2.2), the pile equilibrium equation (2.1) takes the form

\[ N = 2a \pi \tau_a l_c + R. \]  

(2.3)

The increment of shear strain \( \gamma(r) \) during the driving of the pile is determined by the increment of the settlement \( dS \) and, taking into account the shear modulus of the surrounding pile \( G_1 \), is defined as follows

\[ \gamma(r) = \frac{dS}{dr} = - \frac{\tau(r)}{G_1}. \]  

(2.4)

At the same time, shear stresses along the lateral surface of the pile \( \tau(r) \) can be determined by using the dependencies:
\[ \tau(r) = \tau_a \frac{a}{r}; \quad \tau_a = \frac{T}{2\pi a_l}. \tag{2.5} \]

Given the previous calculations (2.4, 2.5), the settlement on the lateral surface of the pile \( S(r) \) is calculated using the following formula:

\[ S(r) = \int_a^r \gamma(r) \, dr = -\frac{\tau_a a^2}{G_1} \int_a^r \frac{1}{r} \, dr = -\frac{\tau_a a}{G_1} \ln(r) + C_1. \tag{2.6} \]

To determine the integration constant \( C_1 \), let us set the boundary conditions for which the radius of the pile influence zone corresponds to \( r = b \) and the vertical displacement on the model boundary does not exist \( S(r) = 0 \). The influence zone \( b \), in accordance with the current understanding of the interaction between the pile and the surrounding soil, is \( b = 6a \). However, if necessary, another meaning can be adopted. Under these boundary conditions, the integration constant takes the form

\[ C_1 = \frac{\tau_a a}{G_1} \ln(b). \tag{2.7} \]

Finally, taking into account formulas (2.6) and (2.7), the vertical displacement of the pile along the lateral surface is determined by the dependence

\[ S(r) = \frac{\tau_a a}{G_1} \ln \left( \frac{b}{r} \right), \tag{2.8} \]

at the same time, at the “pile – soil” contact \( (r = a) \), the vertical displacement is determined

\[ S_a = \frac{\tau_a a}{G_1} \ln \left( \frac{b}{a} \right). \tag{2.9} \]

The displacement of the lower end of the pile, taking into account the Poisson ratio \( \nu_2 \) and the modulus of deformation of the underlying layer of soil \( E_2 \), can be found from the well-known Schleicher formula:

\[ S_i = \frac{\omega_0 N 2a (1 - \nu_2^2)}{E_2}, \tag{2.10} \]

in which the coefficient \( \omega_0 \) depends on the shape of the cross-section of the pile.

If in the expression (2.10) we take the coefficient \( \omega_0 \) responsible for the square shape of the pile section equal to 0.8, and replace the modulus of deformation of the underlying soil by the shear modulus, we obtain the following dependence:

\[ S_i = 0.8 \frac{N (1 - \nu_2^2)}{a G_2}. \tag{2.11} \]

Given the infinite stiffness of the pile material, we can equate the displacement along the lateral surface of the pile and along the bottom end

\[ S_r = S_i. \tag{2.12} \]

From equation (2.10) and (2.11), a dependence can be obtained to determine the force \( T \) acting on the lateral surface of the pile:
\[
T = 2\pi al\omega_0 N \left(\frac{1-\nu_2}{G_1}\frac{b}{a}\right) G_2.
\] (2.13)

Based on the description of the interaction between the pile and the surrounding soil under static action, we turn to the solution of the dynamic problem.

3. The dynamic problem

It is known \([4, 5]\) that in the case of a weightless base, the dynamic balance of a pile with one degree of freedom under the action of an oscillatory force can be determined by a differential equation:

\[
M_c \frac{d^2z}{dt^2} + \eta_2 \frac{dz}{dt} + k_2z + T = N + \Delta N \cdot \sin(\omega t),
\] (3.1)

where \(z\) - vertical displacement of the pile (m); \(M_c\) - pile weight (kN); \(T\) - resistance along the lateral surface of the pile (kN), determined by the equation (2.12). In addition, the mechanical parameters of the soil under the sole of the pile are used in the equation: elastic \(k_2\) (kN/m\(^2\)) and viscous \(\eta_2\) (s\(\cdot\)kN/m\(^2\)) coefficients. Components \(\eta_2 \frac{dz}{dt}\) and \(k_2z\) correspond to the damping and elastic parts of the base reaction. \(\omega\) is the angular velocity of oscillations of the pile, connected with the frequency of oscillations \(f\) (s\(^{-1}\)) by the formula \(\omega = 2\pi f\).

![Figure 2. Calculation schemes for oscillating the pile taking into account the elastic-viscous properties of the soil under the bottom end of the pile.](image)

In this paper, we propose an approximate scheme for calculating the pile oscillation on a weighted basis taking into account the friction of the lateral surface of the foundation with the soil, and also the elastic-viscous properties of the soil. The ground base is represented as a thick layer of finite thickness. In this case, the layer with mass \(m_2\) will oscillate independently and, consequently, a system with one degree of freedom will be obtained \([8, 9, 10]\).
For a mathematical description of the oscillation of such a system, the determination of the stiffness coefficient of the underlying layer $k_2$, mean static $N$ and dynamic $\Delta N \sin(\omega t)$ loads.

To solve this problem, we used computer software Mathcad, which is a system of computer algebra from the class of computer-aided design systems, focused on the preparation of interactive documents with calculations and visual accompaniment. The present work uses the version of Mathcad 15. To solve the differential equation, the built-in function “Odesolve” is used to solve fourth-order Runge-Kutta algorithm for solving differential equations, described in the majority of special literature on computational methods [11].

The results of the solution of the differential equation (3.1) are presented in the form of a graph of the dependence of the vertical displacements of the pile on the time of the dynamic action (Figure 3). We note that the solution of Eq. (3.1) is satisfied taking into account the rheological properties of the soils. When solving a problem, we use the completion of hardening and softening based on the dependencies

$$k_2 = k_0 \cdot e^{\beta t},$$

$$\eta_2 = \eta_0 \cdot e^{\alpha t},$$

where $k_0$ and $\eta_0$ are the initial values of the stiffness and viscosity coefficients, respectively; $\alpha$ and $\beta$ parameters of hardening and softening of the soil under the lower end of the pile. Values of rheological coefficients $\alpha$ and $\beta$ can be determined on the basis of special laboratory tests of soils in stabilometers at different rates of application of vertical load (kinematic regime). Examples of the determination of these coefficients can be found in [3, 11 - 13].

Options piles have been taken for the calculations: pile weight of 19.44 kN, diameter of 0.3 m, under the influence of a perturbing impulsive force $\Delta N \sin(\omega t)$. It can be seen that in this case there is no sinusoidal oscillation and a residual pile settlement. The maximum amplitude of pile oscillations and the damping rate of oscillations are due primarily to the rheological properties of the soil under the lower end of the pile. In addition, the mass of the pile affects the stabilization of vibrations, in other words - its geometric parameters.

![Figure 3. The curve of the oscillations of the “pile-base” system constructed on the basis of the solution of equation (3.1) with the help of the MathCAD PC taking into account hardening and friction of the soils along the lateral surface of the pile.](image)

Next, we will consider an identical problem of interaction between a pile and surrounding soil using the finite element method (FEM). To implement the FEM calculations, the geotechnical software PLAXIS BV, the Netherlands, was used. The problem was solved in the formulation of 2D. Geometric dimensions, physical and mechanical properties of the soil and pile material were assumed to be identical to the previously considered analytical solution. As the calculation model of soil was
used elastic-plastic model of Coulomb - Mohr. The viscous resistance was determined by means of the coefficients of damping of the soils along the Rayleigh.

The resulting solution in the form of a graph of the dependence of the vertical movements of the pile stem on the time of dynamic action is shown in Figure 4. Despite the fact that the absolute values of the obtained pile displacements do not coincide with the analytical solution, one can speak of qualitative convergence of calculations. Based on such a comparative analysis, it can be concluded that the Coulomb - Mohr model, widely used in FEM calculations, can be used to predict the interaction of piles and surrounding soil during vibrational immersion or the operation of vibratory equipment on the pile head.

4. Conclusions
Before we summarize, we note that in the framework of such a small volume article it is impossible to consider all aspects of the study. A detailed analysis of each of the aspects mentioned will necessarily be presented in subsequent surveys. The most general conclusions can be formulated as follows.

1. The proposed design scheme (Figure 1) with sufficient accuracy describes the oscillations of a weighty pile on an elastically viscous soil base, taking into account the resistance along the lateral surface of the pile due to frictional forces on the contact "pile – surrounding soil" [14, 15].

2. Examples of solving problems on oscillations of the "pile-base" system based on the proposed model and their comparison with the results of FEM on PLAXIS 2D PC showed their satisfactory qualitative convergence. In the numerical solution, an elastoplastic Mohr - Coulomb model with Rayleigh damping was used [16]. Thanks to the widespread use of this model, it is possible to solve an engineering problem without using laborious analytical solutions.

3. Allowance for the friction along the lateral surface of the pile on the ground in the model considered (Figure 1) has a significant effect on the characteristics of the oscillation of the system "pile-ground", including the amplitude and the accumulation of the residual sediment piles.

5. Acknowledgments
This work was financially supported by Ministry of Education and Science of the Russian Federation (1.4984.2017/6.7). All tests were carried out using research equipment of The Head Regional Shared Research Facilities of the Moscow State University of Civil Engineering (RFMEFI59317X0006).
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