The induced seismicity predicting based on changes in the specific energy of elastic strain of tremor-prone rock layers

Piotr Bańka

Faculty of Mining and Geology, Silesian University of Technology, Akademicka 2, 44-100 Gliwice, Poland

piotr.banka@polsl.pl

Abstract. The paper presents the method of predicting changes of seismic hazards in the runways of the designed excavation workings. The presented method uses correlations between the level of observed induced seismicity and the estimated energy states of the rockmass. It is possible to make predictions of time-space changes in the number and energy expenditure of the tremors accompanying the planned mining workings and the changes in induced seismicity as a function of time. The presented results of the calculations performed for the selected area of the hard coal mine revealed the existence of a correlation between the energy changes, taking place in the rockmass and the observed level of seismicity. These dependencies can be used to predict seismic hazards of the planned works in the same part of deposits.

1. Introduction

The seismic hazard belongs to the most dangerous natural hazards in Polish hard coal mines. In the next few years, this threat will not decrease, despite the possible further reduction of hard coal mining. The persistence of the observed level of danger (or even its increasing) will be caused by the necessity of reaching for residual resources lying at ever greater depths. Mining work carried out in such conditions usually generate high-energy tremors, they are also accompanied by a high rockburst threat. An appropriately early prediction of the seismic hazard on the runway of the designed works allows for the verification of the project in terms of minimizing the risk’s level, as well as for appropriate planning of prevention measures. In the absence of an appropriate network of excavations or the retention of tremor-prone rock layers at a large distance from existing workings, it becomes necessary to use analytical methods to assess the state of danger. These methods, despite the limitations resulting from the necessity to adopt many simplifying assumptions and due to the high variability of strength and deformation parameters of rock layers, enable preliminary, approximate predictions of the potential seismic hazard and rockbursts at the stage of mining works design.

The searching for seismic relationships with geological and mining conditions is mostly empirical. Among the large number of research in this area, it is possible to mention, for example, works from recent years [1 ÷ 4]. The level of recorded seismic activity induced by conducted mining works can be determined by the deformation processes occurring in the rockmass. The possibility of taking into account the indicators describing the deformation processes of rockmass layers and the terrain surface to characterize the induced seismicity level was pointed out in numerous works, including [5 ÷ 10]. In many works, among others [11 ÷ 13], was showed a correlation relationship between the estimated energy changes in the rockmass and the size of the induced seismicity.
During the many years of research conducted at the Institute of Mining of the Silesian University of Technology, was discovered a correlation relationship between the estimated energy changes in the rockmass and the level of induced seismicity. The description of energy changes concerns the elastic strain energy, accumulated in the rockmass as a result of disturbing it, it says nothing about its dissipation and transformation, and consequently does not specify the amount of energy released in the process of damaging a given volume of rocks. In this process, the potential elastic strain energy passes into other types of energy, including the kinetic energy of elastic waves associated with the seismic energy of recorded tremors [14]. In this way, potential elastic strain energy changes translate into the level of induced seismicity. This justifies the searching for relationships between the estimated changes in potential energy and the seismic energy of tremors.

The article discusses the method of using calculated changes of specific energy to determine the magnitude of the number and energy expenditure of tremors; an example of the prediction of the level of seismicity induced in a strongly threatened with tremors area of a hard coal mine is presented.

2. Relationship between calculated changes of the specific energy and the observed seismicity level

The specific energy of elastic strain $\Phi$ [J/m$^3$] is determined by a known Clapeyron equation:

$$\Phi = \frac{1}{2} T_{\sigma} T_{\varepsilon},$$

where:

- $T_{\sigma}$ – tensor of stress state, MPa;
- $T_{\varepsilon}$ – tensor of strain state.

The analytical methods may be used to estimate energy changes taking place in tremor-prone rock layers, e.g. a method based on the solution of the dislocation boundary value problem of the spatial theory of elasticity given by H. Gil [15], determining the distribution of stresses and strains in the half-space around the rectangular void. For this purpose it is also possible to use a much more modern tool, which is the finite difference method (e.g. the FLAC3D program).

Analytical methods allow to calculate energetic changes without differentiating mechanical parameters and geometry of considered rock layers. These methods are very numerically effective, allow calculation of energy changes in any time intervals and areas, so that instantaneous or local extremes are not omitted – large energy changes associated with configuration and time of operation. This is very important because the values of potential energy undergo strong changes in the period when the rockmass is covered by the influence of the progressing exploitation front. However, these methods require a strong idealization of the rock medium (continuous, homogeneous, isotropic, linear elastic center). Therefore, a typical comparative approach is required (prediction precedence by computational tests for the appropriate range of the already performed operation) to determine the values of parameters characterizing the rockmass, at which the forecast results will be the closest to reality.

Work [16] analyzed the possibility of increasing the accuracy of the description of energy changes occurring in the rockmass by using the finite difference method, which does not require such a strong idealization of the center. Numerical methods allow using practically any complex description of geomechanical processes occurring in the rockmass. However, there are two major limitations in taking full advantage of these opportunities. The first of them, which is gradually losing its importance, is the limited computing power of computers. The second obstacle is the ignorance of the exact geological structure of the rockmass and the geomechanical parameters of the rocks, often causing that the large possibilities of numerical methods are not fully used. As shown by the results of the test calculations [16], with the appropriate selection of parameters of the analytical method, it is possible to obtain comparable results of calculations of energy changes to those obtained by the numerical method.
The nature of changes in the value of specific energy and induced seismicity are similar. By way of example, figure 1 shows an example of the formation of seismicity induced by mining works carried out in the area of the impact of the stopped operation edge (a), as well as the remains of the deposit left in the adjacent seam (b). There were used data on induced seismicity published in the paper [17]. Value of seismic activity E/W was calculated by dividing the sum of registered during one day energy of tremors and coal output (in the same period). The graphs also show changes in specific energy values calculated using the analytical method and the FLAC3D program.

![Figure 1](image_url)

Figure 1. Changes of the values of seismicity and specific energy during the working of longwall under the mining edge (a) and the residue (b) in the overlying seam.

Tremors induced by mining exploitation occur both on the runways and in the goafs of the operating longwalls (during loading and unloading of the deformed rock layer). Therefore, it is reasonable to compare the magnitude of seismicity induced both with calculated increases and decreases in specific energy. The dissipation of accumulated energy is different when the rocks are not damage, and different when the rocks have already been damaged (as a result of the impact of previously conducted mining works). Considering the observations above, the adopted regression models include the calculated increases (decreases) of specific energy with the separation of the taking place before the hypothetical damage of the rock (W effort rate <1) and after it (W effort rate ≥ 1). In order to determine whether rocks have been damaged, the Hoek-Brown generalized condition is used [18]. Energy changes taking place in one or several rock layers are taken into account.

The method of regression analysis was used for searching the relationships between energy states of the rockmass and the level of induced seismicity. This method allows to approximate the functional dependence with another, simpler function, such as a polynomial that contains the appropriate variables and approximates the function sought in certain limited intervals [19]. Space-time changes in seismicity and the formation of these changes over time were analysed.

2.1. Model of seismic changes in a space-time profile

In this model, the number of tremors per surface unit or the energy expenditure of tremors per unit of surface within the adopted time interval determines the expression:

\[ E_{\text{cal} \ l} = a_0 + \sum_{k=1}^{Nl} \sum_{j=1}^{4} a_{kj} f_{jkl} + \epsilon \quad \text{for} \quad l = 1 \ldots Np \quad (2) \]

where:
- \( E_{\text{cal} \ l} \) - considered seismic rate calculated in point \( l \),
- \( Ni \) - number of tremor-prone rock layers,
- \( Np \) - number of calculation points,
\( a_0, a_{kj} \) - parameters of the regression model,

\( N_t \) - number of unit time intervals,

\( \varepsilon \) - random component,

\[
f_{1kl} = \sum_{i=1}^{N_t} \Delta \Phi_{l(W<1)}^k \]

- the sum of instantaneous increases of specific energy calculated in point \( l \) for the \( k \)-th rock layer, when the effort rate \( W < 1 \),

\[
f_{2kl} = \sum_{i=1}^{N_t} \Delta \Phi_{l(W<1)}^k \]

- the sum of instantaneous decreases in specific energy calculated in point \( l \) for \( k \)-th rock layer, when the effort rate \( W < 1 \),

\[
f_{3kl} = \sum_{i=1}^{N_t} \Delta \Phi_{l(W \geq 1)}^k \]

- the sum of instantaneous increases of specific energy calculated in point \( l \) for the \( k \)-th rock layer, when the effort rate \( W \geq 1 \),

\[
f_{4kl} = \sum_{i=1}^{N_t} \Delta \Phi_{l(W \geq 1)}^k \]

- the sum of instantaneous decreases in specific energy calculated in point \( l \) for the \( k \)-th rock layer, when the effort rate \( W \geq 1 \).

2.2. Model of changes in seismicity over time

In this model, the number of tremors per unit of time or the sum of seismic energy of tremors in a unit of time is determined by the expression:

\[
E_{cal\ i} = a_0 + \sum_{k=1}^{N_l} \sum_{j=1}^{4} a_{kj} g_{jkl} + \varepsilon \quad for \quad l = 1 \ldots N_t
\]

where:

\( E_{cal\ i} \) - considered seismic rate calculated in the \( i \)-th time interval,

\[
g_{1ki} = \sum_{l=1}^{N_p} \Delta \Phi_{l(W<1)}^k
\]

- calculated in the \( i \)-th time interval, for the \( k \)-th rock layer of the sum of instantaneous increases of specific energy, when the effort rate \( W < 1 \),

\[
g_{2ki} = \sum_{l=1}^{N_p} \Delta \Phi_{l(W<1)}^k
\]

- calculated in the \( i \)-th time interval, for the \( k \)-th rock layer, the sum of instantaneous decreases in specific energy, when the effort rate \( W < 1 \),

\[
g_{3ki} = \sum_{l=1}^{N_p} \Delta \Phi_{l(W \geq 1)}^k
\]

- calculated in the \( i \)-th time interval, for the \( k \)-th rock layer of the sum of instantaneous increases of specific energy, when the effort rate \( W \geq 1 \),

\[
g_{4ki} = \sum_{l=1}^{N_p} \Delta \Phi_{l(W \geq 1)}^k
\]

- calculated in the \( i \)-th time interval, for the \( k \)-th rock layer of the sum of instantaneous decreases in specific energy, when the effort rate \( W \geq 1 \).

Other markings – as above.

In addition, the following limitations have been accepted:

\[ a_{kj} \geq 0 \quad for \quad k = 1 \ldots N_l, j = 1 \ldots 4 \] (4)

To determine the parameters of the model (minimization of the objective function - L1 or L2 norm of the vector of differences calculated and observed values of the number or the energy expenditure of tremors) the evolutionary difference algorithm [20] is used.

In the conducted research the following indices characterizing the level of seismicity of exploitation regions were adopted. During the analysis of space-time distributions:

- number of tremors per surface unit \([1/(10^4 \text{ m}^2)]\),
- energy expenditure of tremors per unit of volume \([J/\text{m}^3]\).
In turn, when analysing the distributions of recorded seismicity over time, the following indices were calculated:

- number of tremors registered in a unit time interval, e.g. 7 days [1/week],
- total energy of tremors that occurred in the analysed time period, e.g. 7 days [J/week].

3. Correlations between number and energy expenditure of tremors and energy changes

The analysis of the impact of energy changes occurring in the rockmass on the level of induced seismicity was carried out for the area of works that took place in conditions of high risk of tremors and rockbursts in one of the hard coal mines located in Upper Silesia Coal Basin, Poland. The exploitation was carried out in the areas of two longwalls 002 and 003 in the upper layer of seam 504, the thickness of which varies from 2.8 m to 7.0 m (the height of the longwall was up to 3.4 m). The depth of the deposit is about 850 m.

In the area of the presented longwall excavations, there are exploitation edges in the above-declining seams, including 413 at a distance of about 270 m, 416 at a distance of about 160 m, 418 at a distance of about 130 m and 502 at a distance of about 70 m from seam 504.

Mining work in longwalls 002 and 003 induced 5000 tremors with a total energy of about $4 \times 10^7$ J. The strongest phenomenon occurred during operating the longwall 002 and reached the energy of $6 \times 10^6$ J. There were registered 3816 tremors with energy of $10^2$ J, 964 with energy of $10^3$ J, 194 of energy of the order of $10^4$ J. There occurred 26 high energy phenomena, including 20 with the energy of $10^5$ J and 6 with energy of the order of $10^6$ J.

The period of conducting the works was divided into two sub-periods:

- 04.2009 ÷ 12.2012 - using data on registered seismicity in this time interval and calculated values of indicators characterizing energy changes, estimation of regression model parameters was carried out,
- 05.2014 ÷ 05.2015 - for which, using the determined regression equations, calculations of changes in the number and density of energy of tremors were performed. The predicted changes in seismicity were compared with those observed in the course of works in the longwall 003.

3.1. Description of changes in seismicity in the space-time profile

3.1.1. Analysis of energy density of tremors. The results of calculations of the regression model parameters (2) based on the data observed during the operating longwall 002 are given in table 1. The applied model includes a constant $a_0$. It is worth to note, that very similar results were obtained for a model that does not contain this constant [21].

| Layer number | Depth [m] | $a_{i1}$ | $a_{i2}$ | $a_{i3}$ | $a_{i4}$ |
|--------------|-----------|----------|----------|----------|----------|
| 1            | 400       | 122.386  | 0        | 0        | 32.193   |
| 2            | 500       | 97.636   | 11.5     | 4.923    | 18.195   |
| 3            | 750       | 10.984   | 31.148   | 0.042    | 9.279    |
| 4            | 830       | 8.814    | 0        | 0        | 1.228    |

The determined regression equation enabled relatively accurate reproduction of changes in the energy density value of tremors observed on longwall runway 002. Figure 2a shows the observed, whereas in figure 2b, there is presented the energy density distribution calculated from the determined equation prepared for the longwall operating period 002. The increased values of the energy expenditure of tremors within the range of the impact of the edges produced in the seam 416, and also
in the seam 502 are visible. Unlike the regionalization, the extreme values of the observed energy expenditure of tremors could not be recovered. The value of the mean square error is 1.93 J/m³, the value of the linear correlation coefficient is 0.8 - the coefficient of determination is 0.64, i.e. 64% of the observed variation of the energy density of tremors is explained by the determined equation.

![Figure 2](image)

**Figure 2.** Distributions of observed (a) and calculated (b) energy density of tremors [J/m³] during the working of longwall 002.

The regression equation (2) has been used to predict changes in energy density of tremors on longwall 003. The result of the energy expenditure prediction for tremors made for the second stage of works, shown in figure 3b, was compared with the observed distribution of tremors energy density, which is shown in figure 3a.

![Figure 3](image)

**Figure 3.** Distributions of observed (a) and predicted (b) energy density of tremors [J/m³] during the working of longwall 003.

The estimated model allowed a relatively accurate prediction of the induced seismicity on longwall 003. The value of the mean square error is 2.1 J/m³, the value of the linear correlation coefficient between the observed and predicted values is 0.7 (value of the determination coefficient of about 0.5). In particular, the most endangered area, which was the final section of the longwall, was aptly identified. It is important that the results indicated the possibility of a higher value of the maximum energy expenditure of tremors than that observed during the guiding of the longwall 002. However, it was not large enough to suggest on this basis the earlier stop of the longwall run. In general, the prediction for the majority of area of longwall 003 was a bit over-heated. This fact can be explained,
except the inevitable mistakes resulting from the simplicity of the model used, also with a very wide range of rockbursts’s prevention.

3.1.2. Analysis of the number of tremors. The results of the calculation of the regression model parameters based on the data observed in the first period - during the operating of the longwall 002 are given in table 2.

Table 2. Results of computation of regression model (2) parameters – taking into account tremors registered during the working of longwall 002, analysing the number of tremors per $10^4$ m$^2$.

| Layer number | Depth [m] | $a_{ij}$ | $a_{2j}$ | $a_{3j}$ | $a_{4j}$ |
|--------------|-----------|----------|----------|----------|----------|
| 1            | 400       | 382.395  | 0.000    | 0.000    | 209.807  |
| 2            | 500       | 298.478  | 458.351  | 16.391   | 316.949  |
| 3            | 750       | 130.647  | 399.711  | 0.000    | 261.143  |
| 4            | 830       | 368.690  | 0.000    | 0.000    | 0.000    |

Analogically, as in the previous case, the determined regression equation made it possible to reproduce the changes in the number of tremors that occurred on longwall 002 in a correct way. Figure 4a presents the observed, while in figure 4b, the distribution of number of tremors calculated for the period of 002 operating longwall. The value of the mean square error is $24.1 \times 10^{-4}$ m$^2$, the value of the linear correlation coefficient is 0.8 - the determination coefficient is 0.58, i.e. 58% of the observed variation in the number of tremors is explained by the determined equation.

Figure 4. Distributions of observed (a) and calculated (b) number of tremors [$x10^{-4}$ m$^2$] during the working of longwall 002.

The determined regression equation was used to make a prediction of changes in the number of tremors on longwall runway 003. The result of the seismic prediction prepared for the second stage of works, shown in figure 5b, was compared with the observed distribution of number of tremors, which is shown in figure 5a.

Similarly to the analysis of the energy expenditure of tremors, the determined equation was able to accurately indicate the most endangered region, which was the final section of the longwall 003. The value of the mean square error is $46.1 \times 10^{-4}$ m$^2$, the value of the linear correlation coefficient between observed and predicted values is 0.6 (value of the determination coefficient of about 0.4).
3.2. Description of changes in seismicity over time

In some cases, it may be useful to use the (3) model, which allows description of induced seismicity changes in time. The use of this model is particularly appropriate in cases where data of registered tremors come from areas that are difficult to observe and therefore the location of tremors epicenters can be loaded with significant errors. During the estimation of parameters of equation (3) errors of the location of occurring tremors do not affect the value of regression coefficients. Similarly, as during the analysis of space-time changes in seismicity, the results of calculations for two indicators characterizing the level of observed seismic hazard were given: logarithm of the sum of tremors’ energy that occurred in 7 days (table 3) and the number of tremors that occurred in the same, 7-date period (table 4).

![Figure 5](image)

**Figure 5.** Distributions of observed (a) and predicted (b) number of tremors \([x10^4 \ m^2]\) during the working of longwall 003.

**Table 3.** Results of computation of regression model (3) parameters – taking into account tremors registered during the working of longwall 002, analysed the total energy of tremors which occurred in 7-days periods of time \([J/7 \ days]\).

| Layer number | Depth [m] | Values of the regression model parameters; \(a_0 = 4.560616\) |
|--------------|-----------|--------------------------------------------------|
| 1            | 400       | \(a_0 = 4.560616\) |
| 2            | 500       | \(a_0 = 0\) |
| 3            | 750       | \(a_0 = 0\) |
| 4            | 830       | \(a_0 = 0\) |

**Table 4.** Results of computation of regression model (3) parameters – taking into account tremors registered during the working of longwall 002, analysed the number of tremors which occurred in 7-days periods of time \([1/7 \ days]\).

| Layer number | Depth [m] | Values of the regression model parameters; \(a_0 = 0\) |
|--------------|-----------|--------------------------------------------------|
| 1            | 400       | \(a_0 = 0\) |
| 2            | 500       | \(a_0 = 0\) |
| 3            | 750       | \(a_0 = 0\) |
| 4            | 830       | \(a_0 = 0\) |
It is noteworthy that the energy processes occurring in the second tremor-prone layer, lying at a depth of 500 m, do not affect the observed changes in time-induced seismicity. The results of the reproduction of the observed number of tremors and their total energy in the weekly time intervals, for the longwall operating period 002 (dashed line) and the predictions of these indicators for the longwall operating period 003 (continuous line) are shown in figure 6. In addition, in these drawings a distribution of changes in observed seismicity over time is reported (thick, continuous line).

![Figure 6. Distributions of observed, computed and predicted logarithms of total energy of tremors [J/7 days] (a) and number of tremors [1/7 days] (b) registered in 7-days periods of time.](image)

As a result of the calculations made for the seismicity described by the logarithms of the sum of tremors energy in unit time intervals (figure 6a), the following estimates of regression accuracy were obtained: for longwall guide period 002 (estimation of model parameters) the mean square error value is 0.33 J/7 days, in turn the value of the linear correlation coefficient between observed and predicted values is 0.8 (value of the coefficient of determination about 0.6); for the second period, operating longwall 003, i.e. for the time interval in which the prediction of changes in seismicity was made, the mean square error value is 0.34 J/7 days, while the value of the linear correlation coefficient between observed and predicted values is 0.7 (value of the determination coefficient about 0.5).

If the level of induced seismicity was characterized by specifying the number of phenomena that occurred in a unit time interval (figure 6b), the estimation of the regression accuracy was as follows: for longwall operating period 002 the mean square error value is 37 tremors/7 days and the value of the linear correlation coefficient is 0.6 (value of the coefficient of determination about 0.4); for the period of prediction of changes in seismicity, the mean square error value is 35 tremors/7 days, while the value of the linear correlation coefficient is 0.5 (the value of the determination coefficient is about 0.3). Despite the low values of the coefficient of determination and the big values of the mean square error, it is worth noting that the period of the greatest seismic threat accompanying the works run in the longwall 003 and its level was accurately predicted.

4. Summary
The calculations carried out for many regions allowed to determine the existence of correlative relationships between parameters characterizing the course of energy processes occurring in tremor-prone rock layers and the level of induced seismicity. These dependencies can be used to assess the level of seismicity on runs of designed works in the same area, with accuracy sufficient for mining practice.

The results of previous studies indicate that the most accurate results of prognostic calculations should be expected during works at a small distance from tremor-prone layers and within the range of the impact of near exploited edges. In the case of hard rock layers or past exploitation at large
distances from the selected seam, the inference regarding the size of seismicity on the planned longwall runs is subject to a greater error. The presented method may be a supplement to the methods used to assess the status of a seismic hazard. This method can be used primarily in those areas where the seismicity level is determined by the configuration of edges and remains in the exploited seam as well as in neighboring seams, and to a lesser extent, the variable geological structure of the rock mass. The method is local, it means that prediction with the use of the determined regression equation should be performed only for the region from which the data used to estimate the model parameters came from.

References
[1] Abdul-Wahed M K, Al Heib M and Senfaute G 2006 Mining-induced seismicity: seismic measurement using multiplet approach and numerical modeling International Journal of Coal Geol 66/1 pp 137–47
[2] Babenko E V 2010 Prediction of mining induced seismicity around moving longwall Transactions of UkrNDMI NAN Ukraine 7 pp 76–85
[3] Kudela J, Kurnik S and Rusinek J 2009 Ocena wpływu koncentracji wydobycia na poziom zagrożenia sejsmicznego w partii VII pokładu 209, KW SA KWK „Piast” Prace Naukowe GIG 4/2 pp 169–79
[4] Majcherczyk T and Olechowski S 2011 Parametry frontu eksploatacyjnego a zagrożenie sejsmiczne w trzonie W1 Kopalni Kamiennego „Rydułtowy-Anna” Górnictwo i Środowisko 4/2 pp 232–42
[5] Białek J, Drężła B and Jaworski A 1992 Próba ustalenia zależności funkcyjnych pomiędzy przebiegiem deformacji górotworu w czasie a energią sejsmiczną dla warunków KWK Rydultowy Publ. Inst. Geophys. Pol. Acad. Sc. M-16 pp 279–87
[6] Bańka P 2004 Determination of potential level of rockburst hazard based on the result of analytical predictions of stress distributions and the level of induced seismicity Acta Geodynamicza et Geomaterialia IRSM AS CR 1 pp 19–26
[7] Głowacka E 1991 Ocena zagrożenia sejsmicznego górotworu z uwzględnieniem przebiegu eksploatacji Thesis Instytut Geofizyki PAN Warszawa
[8] Goszcz A 1988 Wpływ gradientu prędkości obniżania się powierzchni pod wpływem robót górniczych na stan zagrożenia wstrząsami górniczymi Zeszyty Naukowe AGH seria Górnictwo 141 pp 63–9
[9] Jarmużek J, Siewierski S and Wilczyński W 1987 Wpływ powierzchni wybranego złoża oraz deformacji terenu na aktywność sejsmiczną górotworu na przykładzie ZG Rudna Zeszyty Naukowe AGH seria Górnictwo 129 pp 67–73
[10] Waniór J 1992 Metoda prognozowania wstrząsów i tąpnięć w oparciu o wyniki pomiarów geodezyjnych (Częstochowa: PTPNOZ)
[11] Bańka P and Jaworski A 2007 Analitycznie symulowane zmiany stanów naprężeniowo-energetycznych górotworu w kontekście potencjalnego zagrożenia tąpiami robót górniczych Mechanizacja i automatyzacja w górnictwie 9 pp 17–27
[12] Białek J, Bańka P and Jaworski A 1999 Analityczne prognozowanie zmian energii sprężystej warstw skalnych Zeszyty Naukowe Politechniki Śląskiej s. Górnictwo 239 pp 93–102
[13] Jaworski A 2004 Description of induced seismicity level basing on analytically calculated changes of elastic energy in rock layers subjected to deformation Acta Geodynamicza Et Geomaterialia 1 pp 7–17
[14] Salamon M D G 1984 Energy Considerations in Rock Mechanics: Fundamental Results IS.Afr.Inst.Min.Metal. 84/8 pp 951–1099
[15] Gil H 1991 The Theory of Strata Mechanics (Warszawa: PWN)
[16] Bańka P 2013 Modelowanie zmian sejsmiczności indukowanej na podstawie szacowanych stanów energetycznych górotworu (Gliwice: Wydawnictwo Politechniki Śląskiej)
[17] Gerlach Z 1990 Empiryczne modele przewidywania stanu zwiększonego zagrożenia tąpiami
w oparciu o wyniki sejsmologii górniczej Thesis AGH Kraków

[18] Hoek E, Carranza-Torres C and Corkum B 2002 Hoek-Brown Failure Criterion – 2002 edition Proc. NARMS-TAC Conference Toronto pp 267–73.

[19] Draper N R and Smith H 1966 Applied Regression Analysis (New York: Wiley)

[20] Storn R and Price K 1997 Differential Evolution – A simple and efficient heuristic for global optimization over continuous spaces Journal of Global Optimization 11 pp 341–59

[21] Bańka P, Chmiela A, Menendez Fernandez M, Fernandez Muniz Z and Sanchez A B 2016 Predicting changes in induced seismicity on the basis of estimated rock mass energy states Int. J. Rock. Mech. Min. Sci. 95 pp 79–86