Extracting information on black hole horizons

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Received: date / Revised version: date

Abstract. We present some features of Kerr black hole horizons that are replicated on orbits accessible to outside observers. We use the concepts of horizon confinement and replicas to show that outside the outer horizon there exist photon orbits whose frequencies contain information about the inner horizon and that can, in principle, be detected through the emission spectra of black holes. It is shown that such photon orbits exist close to the rotation axis of the Kerr geometry. We argue that these results could be used to recognize and further investigate black holes and their horizons.

Key words. Black hole; Killing horizons; light surfaces; photon orbits.

1 Introduction

The horizon is a paramount feature of black holes (BHs) entering in numerous astrophysical processes and in the understanding of the physics and geometry bordering quantum gravity.

The Event Horizon Telescope (EHT) has produced in 2019 the first BH image, the black hole’s shadow, as a bright ring–like structure with a central dark disk\textsuperscript{1}. More recently, the EHT collaboration has shown the presence of a polarised fraction of light in the M87 galaxy, close to the BH boundary, measuring for the first time light polarisation which is interpreted as a sign of magnetic fields present in\textsuperscript{1}.

In this letter, we study the properties of BH horizons in the Kerr geometry by investigating photon orbits with orbital frequency (relativistic velocity) equal in magnitude to the horizons frequencies. The possible detection of these photons would provide information on the properties of the horizons. Such photon orbits are shown to exist especially in regions close to the BH rotational axis. We interpret the results in the more general framework of metric bundles (MBs), introduced in\textsuperscript{16,17,18,19,20,21,22} and connected to the structures considered in\textsuperscript{9,10,12,18} and\textsuperscript{13,20} and\textsuperscript{13,21,22}.

In\textsuperscript{1} in the wider context of the BHs and naked singularities correspondence, the metric bundles structure was studied in detail, considering metric bundles as curves in a given plane called the extended plane. In that context it was highlighted the possibility that a part of these structures evidences the presence of replicas purposed for the analysis of photon circular motion and the determination of some characteristics of the inner and outer BH horizons particularly close to the rotation axis. Here in Sec.\textsuperscript{2} we provide the analysis and complete classification of the co-rotating and counter-rotating replicas of the horizons of the Kerr metric in the black holes spacetimes, then focusing on the spacial cases of the equatorial plane and the case of extreme Kerr BH. In Sec.\textsuperscript{3} we discuss the results in the bundles frame, interpreting our results through these structures, commenting on the relevance of the results and their phenomenological consequences.

The Kerr geometry

The Kerr geometry is an exact, asymptotically flat, vacuum solution of the Einstein equations, for an axisymmetric, stationary spacetime, with (ADM) mass parameter\textsuperscript{2}$M \geq 0$, rotational parameter (spin—specific angular momentum) $a = J/M \in [0, M]$, where $J$ is the total angular momentum. For $a = 0$, the spacetime is the limiting static and spherically symmetric Schwarzschild geometry. For $a = M$ the solution is known as Kerr extreme BH, for $a > M$ there are naked singularity solutions. For the purposes of this work, it is convenient to represent the Kerr geometry in Boyer-Lindquist coordinates as

\[ ds^2 = -\alpha^2 dt^2 + \frac{\rho \sigma}{\Sigma} (d\phi - \omega_{\text{zamos}} dt)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2, \]

where

\[ \Delta \equiv r^2 - 2Mr + a^2, \quad \Sigma \equiv r^2 + a^2 (1 - \sigma), \quad \rho \equiv (r^2 + a^2)^2 - a^2 \Delta r, \quad \omega_{\text{zamos}} = \frac{2aMr}{\rho}, \]

where $\alpha = \sqrt{\Delta \Sigma / \rho}$ is the lapse function and $\omega_{\text{zamos}} = 2aMr / \rho$ is the frequency of the zero angular momentum observer (ZAMOS).

(1) In the following analysis, to simplify the discussion, when not

\textsuperscript{1} https://eventhorizontelescope.org/.

1 https://eventhorizontelescope.org/.
otherwise specified we use geometrical units with \( r \to r/M \) and \( a \to a/M \).

The inner and outer BH horizons are located at
\[
r_{\pm} = M \pm \sqrt{M^{2} - a^{2}},
\]
(2)
and the inner and outer ergospheres in the Kerr geometry are the radii
\[
r_{\pm}^{L} = M \pm \sqrt{M^{2} - a^{2}(1 - \sigma)},
\]
(3)
respectively. We consider null-like circular orbits with rotational frequencies \( \omega_\xi \): \( g(L, L) = 0 \), defined from the Killing vector \( L = \xi^{\phi} + \omega \xi^{\theta} \), in terms of the stationary \( \xi^{\phi} = \partial_{\theta} \) and axisymmetric \( \xi^{\theta} = \partial_{\phi} \) Killing vector fields. The limiting frequencies (or relativistic velocities) \( \omega_\xi \):
\[
\omega_\xi = \frac{2ar \pm \sqrt{\Delta L^{2}}}{\rho},
\]
(4)
are also the limiting frequencies bounding the (time-like) stationary observers four-velocity. In this letter we use the relation \( \omega_\xi(r_\xi) = \omega_\xi^H \), where \( \omega_\xi^H \) are the frequencies of the outer and inner Kerr horizons respectively. The null vector fields \( L^\mu_H = L(r_\xi) = \xi^{\phi} + \omega \xi^{\theta} \) in fact define the horizons of the Kerr BH as Killing horizons (being generators of Killing event horizons). (For \( a = 0 \), the horizon of the Schwarzschild BH is a Killing horizon with respect to the Killing field \( \xi^{\phi} \), consequently the event, apparent, and Killing horizons coincide).

Quantity \( \omega_\xi^H \), expressing the BH rigid rotation, regulates (with the BH surface gravity) the BH thermodynamic laws and The variation of the BH irreducible mass \( \delta M_{\text{irr}} \geq 0 \) constrained by \( \delta M - \delta J \omega_\xi^H \) \( \geq 0 \) for a variation of mass and momentum.

2 Horizon replicas: photon frequencies as horizon frequencies

We study the solutions of \( g(L, L) = 0 \) associated to orbits with radius \( r \), which are different from the horizon radius, but corresponding to a photon orbit with orbital frequency equal in magnitude to the frequency \( \omega_\xi^H \) of the BH horizons, we define these orbits as horizon replicas. We consider the corotating, \( a > 0 \) and the counter-rotating \( a < 0 \) replicas with frequencies that are equal in magnitude to the horizon frequencies. Notice that \( g(L, L)(a, -\omega) = g(L, L)(-a, \omega) = g(L, L)(a, \omega) \).

Replicas therefore connect two null vectors, \( L(r_a, a, \sigma_\xi) \) and \( L(r_p, a, \sigma_\xi) \), where \( r_a \neq r_p \) is an outer or inner Killing horizon and in general \( \sigma_a \neq \sigma_p \). We analyze the two regions \( r < r_a \) (inner region) and \( r > r_a \) (outer region). It turns out that solutions exists on planes close to the BH rotation axis. To represent the solutions, it is convenient to introduce the planes \( (\sigma, \sigma_\xi) \) and spins \( (a, a_\xi) \) that are defined Eqs. (12)[13][14][11] of the Appendix and represented in Fig. (1). We introduce also the replicas \( (r_a, r_p) \) that are the solutions described in Eqs. (2)[9][10]. Moreover, we use the notation \( r_a^L \) and \( r_p^L \) to designate one and two replicas, respectively, which are solutions of the multi-parametric equation for the radius \( r_Q \in [r_a, r_p] \).

The inequality \( \omega_\xi > \omega_\xi^H \) is valid in general, except on the horizons. In the case \( \omega_\xi = -\omega_\xi^H \), there are no solutions. More generally we classify the solutions as follows.

Corotating inner horizon replicas with \( \omega_a = \omega_\xi^H \):
\[
r < r_+ : \quad \sqrt{1 - \sigma} \\text{ or } \quad \sqrt{1 + \sigma}.
\]
(5)
\[
r > r_+ : \quad \sigma \in [0, \sigma_{\text{crit}}], \quad (a \in [a_1, 1], \quad r_{a}^{H}):(a \in [a_1, 1], \quad r_{a}^{H}).
\]
(6)

Corotating outer horizon replicas with \( \omega_a = \omega_\xi^H \):
\[
r > r_- : \quad \sqrt{1 - \sigma} \\text{ or } \quad \sqrt{1 + \sigma}.
\]
(7)
\[
r < r_- : \quad \sigma \in [0, \sigma_{\text{crit}}], \quad (a \in [a_1, 1], \quad r_{a}^{H}):(a \in [a_1, 1], \quad r_{a}^{H}).
\]
(8)

Corotating outer horizon replicas with \( \omega_a = \omega_\xi^H \):
\[
r < r_- : \quad \sigma \in [0, \sigma_{\text{crit}}], \quad (a \in [a_1, 1], \quad r_{a}^{H}):(a \in [a_1, 1], \quad r_{a}^{H}).
\]
(9)

Counter-rotating inner horizon replicas with \( \omega_a = -\omega_\xi^H \):
\[
r < r_+ : \quad \sqrt{1 - \sigma} \\text{ or } \quad \sqrt{1 + \sigma}.
\]
(10)
\[
r > r_+ : \quad \sigma \in [0, \sigma_{\text{crit}}], \quad (a \in [a_1, 1], \quad r_{a}^{H}):(a \in [a_1, 1], \quad r_{a}^{H}).
\]
(11)

Counter-rotating outer horizon replicas with \( \omega_a = -\omega_\xi^H \):
\[
r < r_- : \quad \sigma \in [0, \sigma_{\text{crit}}], \quad (a \in [a_1, 1], \quad r_{a}^{H}):(a \in [a_1, 1], \quad r_{a}^{H}).
\]
(12)

Counter-rotating outer horizon replicas with \( \omega_a = -\omega_\xi^H \):
\[
r < r_- : \quad \sigma \in [0, \sigma_{\text{crit}}], \quad (a \in [a_1, 1], \quad r_{a}^{H}):(a \in [a_1, 1], \quad r_{a}^{H}).
\]
(13)
Alternatively,
\[ r > r^+_a : a \in ]0, a_{crit}[: \left( \sigma \in ]0, 1], r^H_0 \right), \]
\[ a = a_{crit} : \left( \sigma \in ]0, 1], r^H_0 \right) : \left( \sigma = 1, r^H_0 \right) \]
\[ a \in ]a_{crit}, 1[: \left( \sigma \in ]0, a_{crit}], r^H_0 \right) : \left( \sigma = \sigma_r, r^H_0 \right) \]
\[ a = 1 : \left( \sigma \in ]0, \sigma_r], r^H_0 \right) : \left( \sigma = \sigma_r, r^H_0 \right), \]
with \( a_{crit} \approx 0.5784M \).

We consider now the particular case of an extreme Kerr BH, \( a = M \), and the situation on the equatorial plane \( \sigma = 1 \). Then, the Extreme Kerr BH:
\[ \omega_+ = \omega_+^H : (r > r_+, \sigma) \in ]0, 1], r^H_0 \right), \]
\[ \omega_- = -\omega_+^H : (r < r_+, \sigma) \in ]0, 1], r^H_0 \right), \]
\[ \omega_+^H : (r > r_+, \sigma) \in ]0, 1], r^H_0 \right) : \left( \sigma = \sigma_r, r^H_0 \right) \]
\[ \omega_+ : (r > r_+, \sigma) \in ]0, 1], r^H_0 \right), \]
\[ \omega_{+} = -\omega_+^H : (r > r_+, \sigma) \in ]0, 1], r^H_0 \right), \]
\[ \omega_- : (r < r_+, \sigma) \in ]0, 1], r^H_0 \right), \]
\[ \omega_+ : \left( \sigma \in ]0, 1], r^H_0 \right) : \left( \sigma = \sigma_r, 1 \right) \]
(18)

(Note the cases \( \omega_{+2} = \omega_+^H \) and \( \omega_- = -\omega_+^H \).)

The equatorial plane \( \sigma = 1 \):

In [15] it was discussed the existence of two corotating orbits \( r^H_0 \) such that \( \omega_+ (r^H_0) = \omega_+^H \) respectively, and \( r^H_0 > r_+ < r^H_0 \) (almost everywhere but at \( a_{0} = 0 \) for the static spacetime and \( a_0 = M \) for the extreme Kerr BH). More in general here we find,
\[ \omega_+ = \omega_+^H : (r > r_+, \sigma) \in ]0, 1], r^H_0 \right), \]
\[ \omega_- = -\omega_+^H : (r < r_+, \sigma) \in ]0, 1], r^H_0 \right), \]
\[ \omega_+ : (r > r_+, \sigma) \in ]0, 1], r^H_0 \right) : \left( \sigma = \sigma_r, 1 \right) \]
(19)

Inner horizon counter-rotating replicas with \( \omega = -\omega_+^H \):
\[ \omega = -\omega_+^H : (r \in ]0, r_+], a \in ]0, 1], \sigma \in ]0, 1], r^H_0 \right) \]
\[ r > r^+_a : (\sigma \in ]0, a_{crit}], r^H_0 \right) : \left( \sigma = \sigma_r, r^H_0 \right) \]
\[ a \in ]a_{crit}, 1[: (\sigma \in ]0, a_{crit}], r^H_0 \right) : \left( \sigma = \sigma_r, r^H_0 \right) \]
(23)

Inner horizon corotating replicas with \( \omega = \omega_+^H \):
\[ \omega = \omega_+^H : (r > r_+, \sigma) \in ]0, a_{crit}] : \left( a = a_r, r^H_0 \right) \]
\[ r \in ]0, r_+], \sigma \in ]a_{crit}, 1]: (a_r, r^H_0) : (a \in ]0, a_{crit}], r^H_0 \right) \]
\[ a = 1, \sigma = 1, r^H_0 \right) : \left( \sigma = 1, a \in ]0, 1], r^H_0 \right) \]
(24)

Outer horizon corotating replicas with \( \omega = +\omega_+^H \):
\[ \omega = +\omega_+^H : (r \in ]0, r_+], a \in ]0, 1], \sigma \in ]0, 1], r^H_0 \right) \]
\[ r > r^+_a : (\sigma \in ]0, a_{crit}], r^H_0 \right) : \left( \sigma = \sigma_r, r^H_0 \right) \]
\[ a \in ]a_{crit}, 1[: (\sigma \in ]0, a_{crit}], r^H_0 \right) : \left( \sigma = a_r, r^H_0 \right) \]
\[ \omega = +\omega_+^H : (r \in ]0, r_+], \sigma \in ]a_{crit}, 1] \]
\[ r > r^+_a : (\sigma \in ]0, 1], a \in ]0, 1], (a = 1, \sigma \in ]a_{crit}, 1] \]
\[ r > r_+ : (a \in ]0, 1], \sigma \in ]0, a_{crit}] : (a = 1, \sigma \in ]a_{crit}, 1] \]
(25)

Examples of replicas are shown in Fig. (1). To clarify this concept in Fig. (2), we show a set of replicas of the inner and outer horizons. A study of the asymptotic region is illustrated also in Fig. (1). As clear from Figs (1), there are two extreme values of \( \theta \) for the existence of the inner horizon replicas in the outer region. A further interesting aspect is the asymptotic behavior (for large \( r/M \)), where the outer horizon and the curves representing the inner horizon replicas close and approach the BH poles (i.e., \( \theta = \{0, \pi\} \), that is, for larger \( r \) and small \( \sigma \)). For \( r \to +\infty \) there is \( \omega_+ = 0 \). This means that at a fixed angle \( \sigma \), the inner horizon replica approaches the outer horizon replica. Similarly, at a fixed radius \( r/M \) and for values of \( \theta \) approaching the BH poles, there are two replicas for two horizon frequencies, respectively. We note that for small \( \sigma \) and large \( r \) the two curves, inner and outer horizon replicas, get closer.

3 Discussion

The replica analysis of Sec. (2) evidences how every photon circular frequency \( \omega \) can be read as a BH horizon frequency, and in this sense every photon circular orbit can be read as an horizon replica. Replicas can therefore be grouped in structures, known as Metric Killing bundles (MBs) containing all (and only) the horizon replicas of every Kerr geometry, for a given (photon) frequency \( \omega \), called bundle characteristic frequency. Therefore MBs also connect measures in different spacetimes, BH geometries and BH and naked singularities, grouping photon orbits in different geometries, characterized by equal value of the photon orbital frequency \( \omega \). (Replicas in NSs were interpreted in [15] as “horizons remnants” and, more generally, appear connected to the “pre-horizon regime” introduced in [69]). The connection between replicas and MBs appear evident by the MBs definition: MBs with characteristic frequency \( \omega \) are solutions of the condition \( a = g(\mathcal{L}, \mathcal{L}) = 0 \) for the fixed, constant \( \omega \). Therefore we can read the results of Sec. (2) in terms of bundles properties.

MBs can be represented as curves on a plane, known as extended plane, tangent to the curve \( a_0 = \sqrt{(2M - r)} \), representing all the BHs horizons. The horizon curve \( a_0 > 0 \) is a function of the horizons radius \( r/M \) in the extended plane, for the inner BH horizons, where \( r \in [0, M] \) (with \( \omega = +\omega_+^H \geq 1/2 \), or the outer BH horizon, for \( r \in [M, 2M] \) (with \( \omega = -\omega_+^H \in [0, 1/2] \)).

Introducing the quantity \( \mathcal{A} \equiv a \sqrt{\mathcal{R}} \), the extended plane of the Kerr geometry is the plane \( \mathcal{A}/M - r/M, \alpha/\alpha - M/rM \) in the special case of the equatorial plane. (Note that the horizon curve is obviously independent of the polar angle \( \theta \)). Using the variable \( \mathcal{A} \) it is possible to connect points in different planes \( \sigma \), as done in Sec. (2). The significance of the MBs with respect to the Kerr horizons lies in the fact that, per construction, each bundle is a curve tangent to the horizon curve in the extended plane. Consequently MB set must necessarily contain at least one BH geometry in
functions of the plane $\sigma_v$. The limiting planes (blue sphere is the outer horizon $r_H$, $r_H \in [0, M]$ is an inner horizon, and $r_H \in [M, 2M]$ is an outer horizon. The gray curves denote the ergosurfaces $r^\pm$. Inner and outer horizons correspond to vertical lines, with $r_H = M$ for the extreme case $a = M$. The inner horizon $r_H = 0.16M$ corresponds to $a = 0.542586M$, $r_H = r_{crit}$ corresponds to $a_{crit} = 0.5784M$, $r_H = 2/5M$ to $a = 0.8M$, and $r_H = M$ to $a = M$. The limiting planes ($\sigma_{\parallel}, \sigma_{\perp}$) are depicted as functions of the dimensionless spin $a/M$. Alternatively, the plots show the limiting spins $(a_{\parallel}, a_{\perp})$ as functions of the plane $\sigma$—Eqs (52, 33, 54, 51)]. The values $a_{crit} = 0.5784M$, $\sigma_{\parallel} = 2\left(8 - 3\sqrt{7}\right) = 0.125492$, and $\sigma_{\perp} = 2\left(2 - \sqrt{3}\right) \approx 0.536$, are shown explicitly.

Fig. 1. Upper line panels and bottom left panel. Replicas analysis for five different spin values. Plots show the horizons $r_+$ frequencies $\omega_H$ (red curve), $-\omega_H$ (dashed red curve), and $-\omega_H$ (dashed black curve) on the plane $(0, r/M)$ for different values of the BH horizon radius $r_H$. Here, the BH spin is $a_+ = \sqrt{r_H(2 - r_H)}$, $r_H \in [0, M]$ is an inner horizon, and $r_H \in [M, 2M]$ is an outer horizon. The gray curves denote the ergosurfaces $r^\pm$. Inner and outer horizons correspond to vertical lines, with $r_H = M$ for the extreme case $a = M$. The inner horizon $r_H = 0.16M$ corresponds to $a = 0.542586M$, $r_H = r_{crit}$ corresponds to $a_{crit} = 0.5784M$, $r_H = 2/5M$ to $a = 0.8M$, and $r_H = M$ to $a = M$. The limiting planes ($\sigma_{\parallel}, \sigma_{\perp}$) are depicted as functions of the dimensionless spin $a/M$. Alternatively, the plots show the limiting spins $(a_{\parallel}, a_{\perp})$ as functions of the plane $\sigma$—Eqs (52, 33, 54, 51)]. The values $a_{crit} = 0.5784M$, $\sigma_{\parallel} = 2\left(8 - 3\sqrt{7}\right) = 0.125492$, and $\sigma_{\perp} = 2\left(2 - \sqrt{3}\right) \approx 0.536$, are shown explicitly.

Fig. 2. Examples of horizon replicas are shown for fixed planes $\sigma = \sin^2 \theta$ and spins $a/M$. The green sphere represents the inner horizon $r_-$, the blue sphere is the outer horizon $r_+$, the light gray surfaces are the inner and outer ergosurfaces $r^\pm$. Orange orbits are replicas of the outer horizon frequency $\omega_H$, orange dashed curves are replicas of $-\omega_H$, purple curves are replicas of $\omega_H$, and purple dashed curves are replicas of $-\omega_H$. In left and right panels, the coordinates are $\{x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta\}$. The singularity corresponds to the point $r = 0$ at $x = 0$, $y = 0$, and $z = 0$. In the central panel, the coordinates are $\{x = \sqrt{a^2 + r^2} \sin \theta \cos \phi, y = \sqrt{a^2 + r^2} \sin \theta \sin \phi, z = r \cos \theta\}$. The singularity is the central disk of radius $a$. The spins are selected in agreement with the analysis of Fig. 1. Here $a_{crit} = 0.5784M$.

the case of corotating frequency $\omega a > 0$. For instance, the special bundle with characteristic frequency $\omega = 1/2$, refers to the extreme Kerr BH spacetime as its tangent point to the horizon curve is $(r = M, a = M)$ with $\omega = \omega_H = 1/2$ and, on the equatorial plane, where the extended plane is $a - r$, the metric bundle set contains the extreme Kerr BH only and NSs. In this sense, remarkably, the horizon curve in the extended plane is generated as envelope surface of all the MBs curves. The characteristic frequency of the bundles is therefore in particular the frequency of the BH horizon defined by the tangency point, hence each other point of the MB curve is its horizon replicas, and all the geometries of the curves are connected by this property. Regarding the representation of counter-rotating horizons replicas described in Eqs (11, 12, 13, 14, 16, 20, 21, 23, 24, 25) in the extended plane, we note that, by using the symmetries of the $\mathcal{L}$ tensor and the Kerr metric tensor discussed in Sec. (2), we can represent counter-rotating replicas in a compact and immediate way by extending the extended plane into the negative region, with $\mathcal{A} < 0$ (equivalently $a < 0$). In this representation, the bundle characteristic frequency is always positive, and the curves extend, mostly continuously, in the negative section of the extended plane, at $a < 0$, grouping corotating replicas of the horizon frequency in the positive sector $a > 0$, and horizon counter-rotating replicas in the extension $a < 0$. In this extension, the horizon curve is a circle of radius $M$ centered on the point $(a = 0, r = M)$. Per construction the region $(a^2 < a^2_r$ or $r \in [r_-, r_+])$, bounded by the horizon curve, is inaccessible for the bundles which are not defined in this region. In this context, the inaccessibility of some frequencies for the observer (as for a part of the inner horizon curve frequencies), as concept of horizon confinement highlighted in Sec. (2), can be expressed in the MBs frame as “local causal ball”, that is a entirely con-
sunglight frequency, $\omega$ is the characteristic frequency of the horizon curve, defining the characteristic frequency of the bundle as horizon frequency. However, the bundle has also an origin, corresponding to the point $r_0 = r_\infty(a_0)$, which is repeated on the bundle with that characteristic frequency and tangent to the inner or outer horizon curve in the extended plane. The horizon frequency is the characteristic frequency equal to the outer horizon frequencies, which are located in the inner region of the extended plane. A limiting special case is represented by the equatorial plane, $\sigma = 0$, where the extended plane is $a = r$ and the bundles origin is $d_0 = 1/\omega$. This relevant case has been analyzed in Eqs. (19, 20, 21, 22). We evidenced how MBs with BH origins are tangent to the inner horizon (for any plane $\sigma$), however for frequencies $\omega = \omega_{\infty} \geq 1$ with $a_0 \in [0, 0.8M]$ and tangent radius $r_0 \in [0, 2/5M]$, MBs are confined in the region of the extended plane upper-bounded by the inner horizon. Thus, more generally, for large values of $\sigma \in [0, 1]$, and particularly on the equatorial plane MBs with BH origin spin $d_0$ are confined in the inner region of the extended plane. Consequently, on these planes $\sigma$, all the frequencies $\omega \geq 1$ defining these bundles are confined and cannot be found in the outer region, i.e. $r > r_\infty$. Therefore, the confinement of the MBs tangent to the inner horizons can be overcome, as noted in Sec. (2), by considering that the zero-quantity $g(\mathcal{L}, \mathcal{L})$ depends explicitly on the plane $\sigma > 0$ (and not from the bundle origin $d_0$ only). For each point of the horizons curve, it is possible to find a replica for a plane $\sigma$ bounded in $\sigma < \sigma_{\text{crit}}$. This implies that close to the rotational axis, $\sigma \ll 1$, for a spacetime spin $a$ it is possible to find an inner horizon replica in the outer region $r > r_\infty$. Consequently, bundles can extract “information” on the inner horizons frequencies near the rotation axis and, therefore, in this sense, the inner region is not entirely confined. Another significant aspect of the horizon confinement concerns the intriguing possibility of extracting information from counter-rotating orbits, while from the observational view-point, we established that the rotational axis of a Kerr BH may contain important information about the singularity and horizons.

**Acknowledgements**

This work was partially supported by UNAM-DGAPA-PAPIIT, Grant No. 114520, Conacyt-Mexico, Grant No. A1-S-31269, and by the Ministry of Education and Science of Kazakhstan, Grant No. BR05236730 and AP05133630.
A Solutions

\[ r_n : \sum_{i=0}^{8} \alpha_i r^i = 0, \quad \text{where} \]
\[ m_0 = a^i(\sigma - 1)^2 \left[ a^i\sigma^2 + 8(a^2 - 2)\sigma + 16 \right]; \]
\[ m_1 = -4a^i(\sigma - 1) \left[ a^i\sigma^2 + 8(a^2 + 8)\sigma - 4 \right] - 8\sigma(\sigma + 1) - 16; \]
\[ n_2 = 2 \left[ a^2(\sigma - 2)\sigma^2 + 2a^i\sigma^2(\sigma - 2)\sigma + 4\sigma + 6\right] + \\
8a^i\sigma(\sigma(\sigma + 6) - 5) + 2 + 32a^2(\sigma + 1)^2; \]
\[ n_3 = -4 \left[ a^i(\sigma - 2)\sigma^2 + 16a^2(\sigma^2 + 1) + 4a^i\sigma^2(\sigma - 4)\sigma + 2\right]; \]
\[ n_4 = a^2 \left[ \sigma \left( a^i\sigma^2(\sigma - 2)\sigma + 4\sigma^4(3 - 2\sigma) + 32\sigma - 48 \right) + 16 \right]; \]
\[ n_5 = 4a^i \left[ a^2\sigma^2(\sigma - 1) - 8 \right]; \]
\[ n_6 = -2\sigma \left[ a^2(\sigma - 2)\sigma - 4\sigma^2 + 8 \right]; \quad n_7 = 0 \]
\[ n_8 = a^2\sigma^2 \]

\[ r_n : \sum_{i=0}^{8} \alpha_i r^i = 0 \]
\[ z_0 = a^2(\sigma - 1)^2 \left[ a^2\sigma^2 + 8(a^2 - 2)\sigma + 16 \right]; \]
\[ z_1 = -2a^2(\sigma - 1) \left[ a^2\sigma^2 - 4 \right] \sigma(\sigma(\sigma + 1) - 4) + 4; \]
\[ z_2 = a^2 \left[ \sigma \left( a^i\sigma^2(\sigma - 3)\sigma - 1\sigma + 4\sigma^4(4 - 3\sigma) + 16(\sigma - 2) \right) + 16 \right]; \]
\[ z_3 = 4a^2 \left[ a^2 - 2 \right]^2\sigma^2; \]
\[ z_4 = \sigma \left[ a^2\sigma^2(3 - 2\sigma) + 4a^2(\sigma + 2) - 16 \right]; \quad z_5 = 2a^2\sigma^2; \quad z_6 = a^2\sigma^2. \]

\[ a_i = \sum_{i=0}^{10} e_i d_i = 0, \quad \text{where} \]
\[ c_0 = -6912(\sigma - 1)^2\sigma; \]
\[ c_1 = 16(\sigma - 1)[\sigma(\sigma(9\sigma + 447) - 312) - 16]; \]
\[ c_2 = 8\sigma(\sigma(\sigma(1045 + 411\sigma)(\sigma - 1228) + 588) + 16); \]
\[ c_3 = \sigma^{2} \left[ \sigma(\sigma(201\sigma + 1160) - 2784) + 1408 \right] + 16; \]
\[ c_4 = -2(\sigma - 1)\sigma^4(3\sigma(5\sigma - 28) + 68); \quad c_{10} = (\sigma - 1)^2\sigma^2; \]
\[ \sigma_0 : \sum_{i=0}^{8} c_i \sigma^i = 0 \]
\[ x_0 = 256\sigma^2; \quad x_1 = 128(\sigma^2 + 37a^2 - 54); \]
\[ x_2 = 16(\sigma^2 + 294a^2 - 759a^2 + 864); \]
\[ x_3 = 32(44a^2 - 307a^2 + 175a^2 - 216); \]
\[ x_4 = 8\sigma(17a^2 - 348a^2 + 1045a^2 + 194); \]
\[ x_5 = -8a^2(38a^2 + 145a^2 + 41); \]
\[ x_6 = a^6(a^2 + 198a^2 + 201); \quad x_7 = -2a^6(a^2 + 15); \quad x_8 = a^{10}; \]
$a_i : \sum_{i=0}^{18} l_i a_i^i, \quad i \equiv 0, \quad (34)$

\[ l_0 = -110592 \sigma (\sigma + 1)^5; \]
\[ l_1 = 256 [\sigma (\sigma (\sigma (\sigma (313 \sigma + 2748) + 3606) + 9508) + 1128) + 16]; \]
\[ l_2 = 256 [\sigma (\sigma (\sigma (\sigma (2041 \sigma + 6499) + 6741) + 961) - 10310) + 972) - 824) - 32]; \]
\[ l_3 = 32 [\sigma (\sigma (\sigma (\sigma (283 \sigma - 28506) + 64139) - 163688) + 147576) - 13168) + 960) + 128]; \]
\[ l_4 = 256 [\sigma (\sigma (\sigma (\sigma (919 \sigma + 44056) - 215280) + 436672) - 631360) + 533248)] + 20992) + 256]; \]
\[ l_5 = 2 \sigma [\sigma (\sigma (\sigma (\sigma (102 - 19 \sigma) + 116) - 18304) + 49312) - 44864) + 12544) + 1152]; \]
\[ l_6 = 2 \sigma [\sigma (\sigma (\sigma (\sigma (147712 - 631360) + 533248)] + 20992) + 256); \]
\[ l_7 = -2 (\sigma - 2) (\sigma - 1) \sigma^5 [\sigma (\sigma + 36) - 36]; \]
\[ l_8 = (\sigma - 1)^5 \sigma^8. \]

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