Multi-parameter Fatigue Equivalence Loadings for specification applications

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Abstract

This paper deals with a method to define Equivalent Fatigue Loads (EFL) from in-service load measurement in the case of a structure subjected to multiple variable fatigue loadings. EFL can be used in order to define tests on full scale structures for an experimental validation approach. It can be also used for the evaluation of the severity of the in-service loads, useful step in a reliability approach, in order to optimize the geometry with respect to the fatigue damage. The aim of the paper is to define the mathematical and mechanical concept of the Equivalence Fatigue Method in the case of multi-parameters loading for specification application. This method uses fatigue damage evaluation from statistical in-service measurement data. The specificity of the method presented here is to use a fully multi-parameters equivalence method. Damages is evaluated using life prediction method based on multiaxial criteria. The method is used for rotating structures, applied to a specific family of structure: train wheels. Train wheels are critical safety railway components mainly subjected to random multiaxial fatigue induced by multi-parameters loading. In this paper an approach to evaluate damage from this measurement in order to find the EFL with a damage equivalence optimization is developed.

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1. Introduction

Evaluating damage induced to a structure by a random fatigue loading is difficult and is even harder when there are multiple loadings involved.

In the automotive industry, the fatigue equivalence method is regularly used [18, 3]. This method takes into account the variability of loads to compute the damage in high cycle fatigue in order to evaluate EFL. Customer surveys on car usage are used to identify the loading variability. The main features of this method are:

- In-service loading measurements,
- Analysis of the material fatigue life curve,
- Computation of damage from variable loading in HCF,
- Equivalent Fatigue Loading research.

This method is applied by Bignonnet [2] to railway wheel set axle and has been extended to thermo-mechanical loading in low cycle fatigue [17]. Genet [9] has generalized the method using a multiaxial fatigue criterion (Morel [12]) and multiple loads in the case of proportional loading. All these methods are focused on critical points of the structure. The present paper focuses on finding an accurate EFL for a family of structures.

The first aim of this paper is to use a new method to estimate damage from in-services measurements [14]. The method enables to find Equivalent Fatigue loadings (EFL) for the whole structure through Finite Element computation and Fatigue Equivalence Method. The multiaxial Dang Van criterion is used to evaluate the damage from multiple loadings. The main aim of this method is to find the best Equivalent Fatigue loadings (EFL) for a structure (choice of the complexity of EFL is really important). It can be used to perform a validation test on a bench and can also be used during the design phase to optimize the structure geometry for a specific use. Here, it is applied on a rotating structure: a train wheel.

The first part of the presented work focuses on the loading of the train wheel and the in-services data used in this study. The loading considered is variable and non proportional. The data are assumed to be representative of the life of the structure. The second part develops the model used for the computation of damage. This model takes into account the complex multiaxial stress path using an extended Dang Van criterion. The total damage is computed with a fatigue equivalent stress built using damage accumulation rules. The third part deals with the Equivalent Fatigue Loading research for the entire structure with a genetic algorithm combined with a local optimization. And finally, the last part shows the results of the fatigue equivalence method obtained for several structures and then the family of structures.

2. The railway wheel

2.1. Global fatigue loading

The fatigue loadings used are the global loads applied to train wheels [1] which are the following: Y: Lateral load, Q: Vertical load, I: Press fit (considered as a load).

The variable loads Y and Q are the wheel-rail contact forces. Each revolution of the wheel induce a relevant fatigue cycle under a Y and Q load. The load I is a result of the manufacturing process and accurate statistical information on this variable is not available.

2.2. In-service data

In this study, in-service data obtained from a real train path are used. That large amount of in-service data enables to assume that they are representative of the wheel life. Wheels of the train are equipped with several strain gauges in order to estimate in real time the loads Y and Q.

For each rotation of the wheel the maximum value of each load is extracted. The result of the extraction is given in a load matrix reported in figure 1. The figure represents the number of occurrences for each couple of load. This result represents the equivalent of about 100 km running for a single wheel. These data are chosen to be representative of the whole life of the train wheel. The statistical characteristics of both variables Y and Q are therefore considered as known.
The material used for the wheel is ER7 (a material close to the C45 according to AFNOR designation).

3. Damage evaluation: Finite Element computations and fatigue criterion

3.1. Finite Element computations

In order to evaluate damage by means of a fatigue criterion Elastic Finite Element computations are performed on the wheel under the previous loading conditions.

Constant loads rotating over the structure are applied at each revolution in order to reproduce the evolution of the stresses. An example of mesh is shown in figure 2. We take advantage of the geometrical symmetries of the structure to reduce the number of computations: 1 elastic computation is sufficient to simulate the entire stress path for one rotation to the time-space equivalence.

As the structure remains elastic, a superposition method is used to generate any stress path for a wheel rotation under a combination of loads Y, Q and I. The stress evolution at a point M, at the instant t, due to the loading Y, Q, F can be written as follows:

\[
\sigma(M,Y,Q,F,t) = Y \cdot \sigma_Y(M,t) + Q \cdot \sigma(M,t) + I \cdot \sigma_I(M)
\]

\(\sigma_Y(M,t)\) and \(\sigma_Q(M,t)\) are the stress path (rotation of the load) for respectively \(Y=1\, \text{kN}\) and \(Q=1\, \text{kN}\). \(\sigma_I(M)\) is the stress for \(I=1\, \text{mm}\) interaction between the wheel and the axle at a given point of the structure.

For this analysis the rim and so the contact zone are not studied because they are submitted to fretting fatigue. In the studied part of the wheel, it is verified that the maximum stresses are below the yield stress. This is consistent with a long life fatigue behavior.
3.2. Fatigue life model

3.2.1. Fatigue criterion
To evaluate the damage induced by a given combination of loads, a fatigue equivalent stress $\tau_{DV}$ based on the Dang Van multiaxial high cycle fatigue criterion [7] is used.

$$\max(\tau(t)) + a.p(t) \leq b$$ (2)

$a$ and $b$ are two material parameters. The fatigue stress $\tau_{DV}$ deduced from this criterion is defined as:

$$\tau_{DV} = \max(\tau(t)) + a.p(t)$$ (3)

A graphic representation of the fatigue stress $\tau_{DV}$ is given in figure 3. Other fatigue criteria as proposed by Papadopoulos [13], Sines [16] and Crossland [6] can be used. The used criterion is recommended by standard [4] in the railway industry and applied by authors on railway wheels [15].

3.2.2. Extension of the Fatigue criterion
To evaluate the fatigue life, an extension of the Dang Van criterion to the long life domain is adopted. It consists in the use of a power law (Basquin like model, as introduced by Papadopoulos [13]) relating the number of cycle $N$ to failure to the fatigue stress $\tau_{DV}$ by:

$$N = N_e \cdot \left( \frac{b}{\tau_{DV}} \right)^m$$ (4)

3.2.3. Calibration of the model
The calibration of the materials parameters is performed as follows:

- $a$ is obtained from two different fatigue limits [7];
- $m$ and $b$ are deduced from a Wöhler curve based on the fatigue stress $\tau_{DV}$ (figure 4);
- $N_e$ is a parameter of the method, more precisely defined in section 4.1.

![Fig. 3. Fatigue stress $\tau_{DV}$ calculation exemple](image1)

![Fig. 4. Fatigue life model for ER7 steel](image2)
The fatigue properties for the web are determined from [4], and due to the confidentiality of the data, the values are not given. σ_{mat}, the standard deviation of fatigue strength identified on a \( R = -1 \) torsion only, is also identified. Once all these fatigue parameters are determined we compute the values of the fatigue stress and evaluate the fatigue lifetime.

The fatigue stress \( \tau_{DV} \) does take into account the material properties without the variability of the variable \( \alpha \). This choice is made in order to simplify the method. Other fatigue life models as proposed in [11] can also be used.

### 3.3. Damage accumulation

Palmgren-Miner linear damage accumulation model [8, 10] is used to compute the damage in the structure for its whole life. Using Eq. (4), a cycle \( i \) (single wheel rotation) with a damage variable the damage \( \tau_{DV,i} \), induced a damage \( D_i \) given by:

\[
D_i = \frac{1}{N_i} = \frac{1}{N_e} \cdot \left( \frac{\tau_{DV,i}}{b} \right)^m
\]  

(5)

The damage accumulation is linear:

\[
D = \sum_i D_i = \sum_i \frac{1}{N_i} = \frac{1}{N_e} \cdot \sum_i \left( \frac{\tau_{DV,i}}{b} \right)^m
\]  

(6)

If the sample does not represent the whole life of the wheel, the damage accumulation has to be corrected by a scalar \( k \) (This extrapolation to virtual life is also used in the literature, see [2]):

\[
k = \frac{K_{life}}{K_{sample}}
\]  

(7)

Therefore, a modified expression for the damage is adopted:

\[
D = k \cdot \sum_i D_i = \sum_i \frac{k}{N_i} = \frac{k}{N_e} \cdot \sum_i \left( \frac{\tau_{DV,i}}{b} \right)^m
\]  

(8)

We introduce the equivalent fatigue stress \( \tau_{eq} \) thanks to:

\[
\left( \frac{\tau_{eq}}{b} \right)^m = \frac{k}{N_e} \cdot \sum_i \left( \frac{\tau_{DV,i}}{b} \right)^m \Rightarrow \tau_{eq} = \left( \frac{k}{N_e} \cdot \sum_i \left( \frac{\tau_{DV,i}}{b} \right) \right)^{\frac{1}{m}}
\]  

(9)

This value \( \tau_{eq} \) is obtained for every point and is the severity of the loads and is relative to the local damage \( D \).

### 4. Equivalence method

#### 4.1. Equivalent loading

The value of the fatigue stress \( \tau_{DV} \) an the values of the in-service damage \( D_i \) can be calculated for any values of the loads parameters at every point of the structure. In the specific case of the train wheel, the parameters are the values of the rotating loads \( Q \) and \( Y \).

Loading parameter vector is the following for the in-service data (\( N_e \) is the number of cycles in the in-service data):

\[
\lambda = \begin{bmatrix} Q_1 \ldots Q_i \ldots Q_{N_e} \\ Y_1 \ldots Y_i \ldots Y_{N_e} \end{bmatrix} \quad \text{or} \quad \lambda_i = \begin{bmatrix} Q_i \\ Y_i \end{bmatrix}
\]  

(10)
As shown before, for one cycle and at any point $M$ of the structure we can calculate $\tau_{DV}(\lambda, M)$. Furthermore, we can compute for every cycle: $D_i(\lambda, i, M)$. And finally, the damage from the life of the structure is:

$$ D(M) = \sum_i D_i(\lambda, i, M) $$

(11)

The equivalent loading is the following, chosen for $N_e$ cycles:

$$ \lambda_{eq} = \left\{ \frac{Q_{eq,1} \cdots Q_{eq,i} \cdots Q_{eq,N_e}}{Y_{eq,1} \cdots Y_{eq,i} \cdots Y_{eq,N_e}} \right\} \text{ or } \lambda_{eq,i} = \left\{ \frac{Q_{eq,i}}{Y_{eq,i}} \right\} $$

(12)

The damage induced by the equivalent loading is the following:

$$ D_{eq}(M) = \sum_i D_{eq,i}(\lambda_{eq,i}, M) $$

(13)

Strict fatigue equivalence is obtained when, for all the points $M$ of the structure, $D_{eq}(M) = D(M)$.

4.2. Finding best equivalent loading

Obviously, we will choose an equivalent load as simple as possible, and the strict equality between the damages will not be obtained for all point. For a given type of Equivalent Load, we look for the best one. The best equivalent $\lambda_{eq}$ is the following:

$$ \lambda_{eq} = \ar g m i n_{\lambda_{eq}} \left[ \sum_M \frac{D(M)}{\sum_i D_{eq,i}(\lambda_{eq,i}, M)} - 1 \right]^2 $$

(14)

In order to obtain a good equivalence with the Equivalent Fatigue Loading, we can try more complex EFL. For instance the first EFL can be a single value of the loads parameters, so one block repeated $N_e$ times. In order to have a better equivalence it is important to try more blocks. The effect of the number of blocks is studied in [14].

The function minimized in equation 14 is the mean error over the structure between the damage induced by the loading during the life of the structure and the damages induced by the Equivalent Fatigue Loading. Other indicators are used as the maximum damage and the minimum damage over the structure.

In order to compute with equation 14 an optimization scheme is needed. In this paper both a genetic algorithm and a local algorithm are used. The genetic algorithm is used first in order to find best convex sector of the domain and then the local algorithm finds the EFL in this particular sector at a lower cost. This algorithm is derived from [5].

Stopping criteria are built in order to stop the two algorithms in a relevant computing time. These criteria are not discussed in this paper.

4.1. The family of structure

In this part 3 wheels are studied in order to compare the values of the EFL for a given in-service data set. The generic geometry of a train wheel is introduced in Appendix A. The 2D meshes of the 3 studied wheels are shown on figure 5.

Considering the family of structures is equivalent to considering the 3 wheels as one structure.
4.2. Computation of $\tau_{eq}$

The first step of the method is to evaluate the severity of the structure considering the presented in service data set. The value of the fatigue stress $\tau_{eq}$ is computed for every point of these 3 structures. $\tau_{eq}$ is used to find the hot point of the structure because it takes into account the variability of the loading and the damage induced by each cycle. The results of the computations are shown on a 2D mesh of the web of the structures (Fig 6). Because the considered wheels are not used on the same vehicles, the values can’t be compared one to another. Figure 7 illustrates the computation of $\tau_{eq}$ for the critical point of the wheel A.

For all the wheels, a critical zone is observed at the bottom of the web on the external face. Other critical zones can be observed depending on the curvature of the web.
4.3. Choice of the EFL

In order to define the EFL, 3 types of cycle are defined according to the 3 loading configurations:

- Straight track $Q_{eq}^1 > 0$;
- Curved track $Q_{eq}^2 > 0$ and $Y_{eq}^2 > 0$;
- Switch point $Q_{eq}^3 > 0$ and $Y_{eq}^3 < 0$.

A two bloc EFL is considered, with a first block in curve configuration and a second block on switch point configuration with half of $N_e$ the cycles in each configuration:

$$\text{For } i \in [1, N_e \cdot 0.5], \lambda_{eq,i} = \left\{ \frac{Q_{eq}^2}{Y_{eq}^2} \right\} \text{ and for } i \in [N_e \cdot 0.5 + 1, N_e], \lambda_{eq,i} = \left\{ \frac{Q_{eq}^3}{Y_{eq}^3} \right\}$$

4.4. Equivalent loading computation

4.4.1. EFL for different wheels

The equivalence method is performed on the skin of Wheel A. After 20 steps of the genetic algorithm and 87 steps of the local algorithm the values of the equivalent and the equivalence indicators are on Tab. 2.

Figure 7 shows the discrepancies of the structure. Both the maximum and the mean discrepancy over the structure are low and thus the EFL is validated.

| Table 1 EFL computations results |
|----------------------------------|
| Structure | $Q_{eq}^1$ | $Y_{eq}^1$ | $Q_{eq}^2$ | $Y_{eq}^2$ | meanErr | maxErr |
| Wheel A   | 0.71       | 0.43       | 0.69       | -0.05      | 9.50 · 10^{-2} | 9.95 · 10^{-3} |
| Wheel B   | 0.70       | 0.43       | 0.69       | -0.05      | 2.06 · 10^{-1} | 1.18 · 10^{-2} |
| Wheel C   | 0.71       | 0.42       | 0.69       | -0.05      | 1.15 · 10^{-1} | 1.37 · 10^{-2} |
| Wheel Family | 0.71 | 0.43 | 0.69 | -0.05 | 1.21 · 10^{-3} | 3.91 · 10^{-3} |

Fig. 7. (a) Mean damage discrepancies and (b) Loads values during the optimization; (c) Computation of $\tau_{eq}$

The equivalence method is also performed on the skin of Wheel B and C. The equivalence indicators are on Tab. 2. These equivalents are also validated. Discrepancies are plotted on Fig. 8.
4.4.2. EFL for the family of structure

The EFL computed for the different wheels are really close. In this method, the use of the family is relevant. The equivalence method has to be performed on the skin of the 3 wheels in order to obtain a relevant EFL for the family of the structure.

The equivalence method is performed on the skin of the wheels. After 20 steps of the genetic algorithm and 102 steps of the local algorithm the values of the equivalent and the equivalence indicators are on Tab. 2.

The evolutions of the mean damage discrepancy over the skin of the structure, and the values of the EFL parameters at each iteration of the optimization scheme, are shown on figure. Both the maximum and the mean discrepancies over the structure are low and thus the EFL is validated. Furthermore, the parameters of this EFL are almost the same as the parameters obtained for the 3 different wheels.

![Fig. 8. Damage discrepancies on the wheels; (a) Wheel A; (b) Wheel B; (c) Wheel C](image)

5. Conclusion

This paper gives a method to define multi-parameters Equivalent Fatigue Loadings relevant for industrial structures subjected to multi-input and multiaxial variable fatigue loadings.

This paper focuses on rotating structures but the adaptation to another type of structure is of course possible. The main lines of the approach are the following:

1. The method requires large in-service data in order to take into account the variability of the loads during the whole life of the structure.
2. A simple analysis of the fatigue strength is required (Classical fatigue tests must be performed) to compute damage (using in this paper an extended Dang Van Criterion).
3. A fatigue equivalence method is used to compute local equivalent fatigue stress taking into account the variable loads. This variable is a local measurement of the damage.
4. A mathematical method to find Equivalent Fatigue Loading is described. This method uses a genetic algorithm in order to compute an accurate EFL for the whole structure.

The method was applied to several structures and it showed the need of finding the good loading parameters to describe the EFL. The results show that the EFL that were calculated for different structures were really close to each other. Thus computing an EFL for the family of structure is efficient and reasonable is this case.

Moreover the method only requires classical computational resources. This method to evaluate EFL can be used in design process but also in validation.
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