Optical detection of a BCS phase transition in a trapped gas of fermionic atoms

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Light scattering from a spin-polarized degenerate Fermi gas of trapped ultracold \(^{6}\text{Li}\) atoms is studied. We find that the scattered light contains information which directly reflects the quantum pair correlation due to the formation of atomic Cooper pairs resulting from a BCS phase transition to a superfluid state. Evidence for pairing can be observed in both the space and time domains.

The realization of Bose-Einstein condensation in trapped atomic gases \cite{6} has generated interest in the atomic physics, quantum optics and condensed matter physics communities. Although the experimental realization of a degenerate atomic Fermi gas has not yet been demonstrated, interest in this subject is increasing \cite{2,3,5}. Of course, the behavior of a degenerate Fermi gas is remarkably different from a degenerate Bose gas. By analogy with the BCS theory of superconductivity in metals, it has been predicted that a degenerate Fermi gas can undergo a BCS phase transition to an atomic superfluid state if the interatomic interaction in the gas is attractive \cite{6}. Experiments to trap and cool \(^{6}\text{Li}\) and \(^{40}\text{K}\) gases into the quantum degenerate regime are underway in several laboratories.

In this paper, we address the question of how to detect the superfluid state after the BCS phase transition. We assume that the Fermi gas has been cooled to near absolute zero, so that all trap levels up to the Fermi energy are filled. An attractive interatomic interaction will cause atoms in the vicinity of the Fermi level to form Cooper pairs, with each pair composed of two quantum correlated atoms behaving as a new composite Bose particle. These bosons automatically undergo Bose-Einstein condensation and form a superfluid. The quantum pair correlation of the Cooper pairs characterizes the superfluid properties of the gas.

A promising experimental approach is to prepare a degenerate gas with atoms in an incoherent mixture of two internal hyperfine state. Such a mixture allows Cooper pairing via an s-wave interaction, and leads to practically attainable BCS-transition temperatures when the scattering length \(a\) is large and negative. This occurs naturally for \(^{6}\text{Li}\) \cite{6}, or can be obtained in the vicinity of a Feshbach resonance for other atoms \cite{2}. We consider here a trapped \(^{6}\text{Li}\) gas in an incoherent mixture of ground states \(|+\rangle = |M_s = 1/2, M_I = 1\rangle\) and \(|-\rangle = |M_s = 1/2, M_I = 0\rangle\) \cite{3}.

The key to observing the superfluid state is to determine the existence of pair correlations. To achieve this goal, we propose to use off-resonance light scattering and Fourier imaging techniques. A laser beam with amplitude \(E_L\), frequency \(\omega_L\), and wave vector \(k\) propagating along the \(z\) direction is used to illuminate the gas. We take the light to be linearly polarized and tuned near resonance between an \(S\) ground state and \(P\) excited state. To avoid incoherent heating of the gas due to spontaneous emission, the magnitude of the laser detuning, \(\delta = \omega_L - \omega_0\), is assumed to be large. In vector quantum field theory \cite{3,4,11}, the atoms in the light field can be described by a four-component atomic field \(\Psi(r) = \psi_+(r) + \psi_{-}(r) + \psi_{+e}(e^+) + \psi_{-e}(e^-)\) with \(\psi_{\pm}\) denoting atoms in the ground-state hyperfine levels \(|\pm\rangle\), and \(\psi_{\pm e}\) in the corresponding excited-state hyperfine levels. For large \(\delta\), the excited-state components can be adiabatically eliminated, yielding a total atomic polarization operator with positive-frequency part \cite{3}

\[\rho_+(r, t) = -i\frac{\varphi}{\hbar\delta} \hat{\rho}(r, t) e^{-i\omega_L t},\]  \hspace{1cm} (1)

where \(\rho_+(r, t) = \psi_+(r, t)\psi_+(r, t) + \psi_{+e}(r, t)\psi_{+e}(r, t)\) denotes the total atomic density operator in the ground state, \(\varphi\) the matrix element of the atomic dipole moment, and \(r\) a location in the gas. Light propagation is determined by the wave equation

\[\nabla^2\rho_+(r, t) - \frac{1}{c^2} \frac{\partial^2 \rho_+(r, t)}{\partial t^2} = \mu_0 \frac{\partial^2 \rho_+(r, t)}{\partial t^2}.\]  \hspace{1cm} (2)

The solution to Eq. (2) can be expressed as

\[\rho_+(R, t) = \rho_+(R, t)e^{-i\omega_L t} + \rho_+(R, t)e^{ikz - i\omega_L t},\]  \hspace{1cm} (3)

where \(\rho_+(R, t)\) is the scattered field at position \(R\). For \(R \equiv |R| \gg |r|\), the scattered field has the form \cite{3,11}

\[\rho_+(R, t) = \frac{k^2 e^{ikR}}{R} \int d^3r e^{-ikR} \left[\rho_+(r, t) - \hat{R} \cdot \hat{R} \rho_+(r, t)\hat{R}\right].\]  \hspace{1cm} (4)
where the directional unit vector $\hat{R} = R/R$. From Eqs. (1) and (4), we see that the scattered field depends on the density operator of the gas, so that the averaged spectral intensity of the scattered field received by a photodetector contains the second-order correlation of the atomic field operators $\hat{\rho}$. 

$$\langle \hat{\rho}(r,t)\hat{\rho}(r',t') \rangle \approx \langle \hat{\rho}(r,t)\rangle \langle \hat{\rho}(r',t') \rangle + G(r, r', t, t'),$$  

(5)

where “$\langle \cdot \rangle$” denotes the quantum mechanical expectation value. The first term in Eq. (5), which depends on the total averaged density, describes the contribution to the scattered field by the normal ground-state component. The second term,

$$G(r, r', t, t') \equiv -2\langle \psi_-(r,t)\psi_+(r',t')\rangle\langle \psi_+(r,t)\psi_+(r',t') \rangle,$$  

(6)

gives the quantum pair correlation function arising from the formation of Cooper pairs in the superfluid state.

The contribution of the laser field $E_L$ in Eq. (3) can be removed by imaging the cloud with a dark ground technique, as discussed in Refs. [12]. If a plane located a distance $z_0$ from the atoms is observed in this way, the spectral and spatial intensity distribution measured on the detector will be [13]

$$I(R_{\perp}, \nu) = \int_{-\infty}^{\infty} \int_{-T}^{T} \int d\tau dt \langle E_S^-(R_0, t) \cdot E_S^+(R_0, t + \tau) \rangle,$$  

(7)

where $2T$ is the time interval used for detection, and $R_0 \equiv (R_{\perp}, z_0)$ is a point in the image plane. Equation (4), along with relations (1) and (5), gives the spatial-temporal correlation function of the light field

$$\langle E_S^-(R_0, t) \cdot E_S^+(R_0, t + \tau) \rangle = \frac{9L_l \gamma^2}{16(k \omega_0)^2} \left[ I_1(R_{\perp}, t, \tau) + I_2(R_{\perp}, t, \tau) \right] e^{-i\omega_L \tau},$$  

(8)

where $I_L = E_L^-(R_0) \cdot E_L^+(R_0)$ is the intensity of the incident light and $\gamma$ is the natural linewidth of the transition. The functions $I_1$ and $I_2$ are defined as

$$I_1(R_{\perp}, t, \tau) = \int d^2r_{\perp} d^2r'_{\perp} e^{-i\kappa R_{\perp} \cdot (r_{\perp} - r'_{\perp})/z_0} \langle \hat{\rho}(r_{\perp}, t)\rangle \langle \hat{\rho}(r'_{\perp}, t + \tau) \rangle,$$  

(9)

and

$$I_2(R_{\perp}, t, \tau) = \int d^2\xi e^{-i\kappa R_{\perp} \cdot \xi/z_0} \int d^2r_{\perp} G(r_{\perp}, r_{\perp} - \xi, t, t + \tau),$$  

(10)

where the relative distance between atoms is denoted by $\xi = r_{\perp} - r'_{\perp}$. The function $I_1$ describes the signal from the normal component of the gas and $I_2$ the signal from the Cooper pairs. In general, $I_2$ is much weaker than $I_1$ since the averaged density of atoms in the normal component is far larger than that of the pairs.

The averaged density and the quantum pair correlation function can be found using vector quantum field theory [9]. In the off-resonant light field, the degenerate Fermi gas is described by the coupled quantum field equations

$$i\hbar \frac{\partial \psi_+}{\partial t} = (H_0 - \mu_+ + V_L - i\hbar \Gamma/2)\psi_+ - \Delta(r)\psi_\perp$$

$$i\hbar \frac{\partial \psi_\perp}{\partial t} = -(H_0 - \mu_- + V_L + i\hbar \Gamma/2)\psi_\perp - \Delta(r)\psi_+,$$  

(11)

where $H_0 = -\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m \omega_r^2 r^2$ is the free Hamiltonian of the trapped Fermi gas, $V_L = \hbar \Omega^2/4\delta$ is the light-induced potential, $\Gamma = \gamma \Omega^2/4\delta^2$ is the rate for spontaneous emission, $\mu_\pm$ are the chemical potentials of the two internal states, and $\Delta(r) = (4\pi \alpha \hbar^2/m)\langle \psi_-(r)\psi_+(r) \rangle$ is the BCS energy gap function [2]. The Rabi frequency of the light field is $\Omega \equiv \langle \hat{\rho} \cdot E_L / \hbar \rangle$. For simplicity we consider the simple case $\mu_+ = \mu_-$ for equal number of atoms in each spin state and introduce the renormalized chemical potential $\mu = \mu_+ - V_L$. Further, a large laser detuning and a weak intensity allow $\Gamma \ll \mu, \Delta$ so that destruction of Cooper pairs by spontaneous emission and interactions involving excited-state atoms [9] can be neglected. Employing an approach similar to that adopted in BCS theory, we approximate the solutions of Eqs. (11) by

$$\psi_\pm(r,t) = \sum_n \left( u_n(r) \hat{b}_{n\pm} e^{-iE_n t/\hbar} \pm v_n(r) \hat{b}_{n\mp}^\dagger e^{iE_n t/\hbar} \right),$$  

(12)
where $\hat{b}_{n\pm}$ are generalized Bogoliubov quasi-particle operators and $E_n$ the excitation energy for the mode indexed by $n$. The superfluid state of the degenerate Fermi gas is characterized by the BCS ground state $|\Phi_{BCS}\rangle$ with the property $\hat{b}_{n\pm}|\Phi_{BCS}\rangle = 0$. From Eqs. (11) with the dissipative terms ignored, the transformation coefficients $\{u_n, v_n\}$ satisfy the celebrated Bogoliubov equations

$$(H_0 - \mu)u_n(r) + \Delta(r)v_n(r) = E_n u_n(r)$$
$$-(H_0 - \mu)v_n(r) + \Delta(r)u_n(r) = E_n v_n(r).$$

(13)

The total averaged density can be expressed as $\langle \hat{\rho}(r, t) \rangle \equiv \langle \Phi_{BCS}|\psi_+^1\psi_+^2 + \psi_-'^1\psi_-'^2|\Phi_{BCS}\rangle = 2 \sum_n |v_n(r)|^2$ and the quantum pair function is

$$G(r, r', t, t') = 2 \sum_{nm} u_n(r)v_n(r')u_m(r)v_m(r)e^{-i(E_n + E_m)(t-t')/\hbar}.$$

(14)

The average density and pair function can be calculated by self-consistently solving Eqs. (13). In the normal degenerate ground state, energy levels below the Fermi level $E_F$ are occupied, while those above are empty. The effect of interatomic interactions is to cause scattering between nearby energy levels, which creates an energy shell near $E_F$ where normally unoccupied states in the normal ground state acquire an amplitude to be occupied, and states below $E_F$ have some amplitude to be unoccupied. The stronger the interatomic interaction is, the wider the energy shell and the more atoms are available to form Cooper pairs. Physically, the coefficients $u_n$ and $v_n$ in Eqs. (13) determine the amplitudes for atoms to be scattered into the pair states. To evaluate these amplitudes, we expand the coefficients as $u_n = \sum_q u_{nq}\phi_q$ and $v_n = \sum_q v_{nq}\phi_q$, in terms of the eigenstates $\phi_q$ of the single-atom Hamiltonian $H_0$. In principle the sum over $q$ in the coefficients should extend from zero to infinity. However, we should note that in the BCS theory, Equation (13) is a direct result of the Born approximation by replacing the realistic non-local quantum pair function is degenerate ground state, energy levels below the Fermi level $E_F$, and the more atoms are available to form Cooper pairs. Physically, the coefficients $u_n$ and $v_n$ in Eqs. (13) determine the amplitudes for atoms to be scattered into the pair states. To evaluate these amplitudes, we expand the coefficients as $u_n = \sum_q u_{nq}\phi_q$ and $v_n = \sum_q v_{nq}\phi_q$, in terms of the eigenstates $\phi_q$ of the single-atom Hamiltonian $H_0$. In principle the sum over $q$ in the coefficients should extend from zero to infinity. However, we should note that in the BCS theory, Equation (13) is a direct result of the Born approximation by replacing the realistic non-local interatomic interaction $V(\vec{r})$ by a local contact potential $V(\vec{r}) = 4\pi\hbar^2 a\delta(\vec{r})/m$. We know that the Born approximation is only valid for low-energy scattering. The invalidity of the approximation in high-energy scattering regime produces an ultra-violet divergence in the BCS theory. In the case of superconductivity, the ultra-violet divergence naturally vanishes by considering the fact that the phonon-exchange induced interaction between electrons can be cut-off in the Debye frequency. However, in the case of degenerate Fermi gas of atoms, to avoid the ultra-violet divergence, an exact theory for superfluid phase transition must take the realistic shape of the exact non-local triplet potential into account. Recently two independent approaches to remove the ultra-violet divergence in the BCS theory of degenerate Fermi gas of atoms have been proposed [20]. One is to renormalize the interaction potential in terms of the Lippman-Schwinger equation [21] and the other is to employ the more exact pseudo-potential approximation [22]. However for a first guess, the Born approximation provides a simple and reasonable way to evaluate the gap energy and the pair correlation if an appropriate momentum cut-off is introduced to remove the ultra-violet divergence. Now the question is how to choose a physically valid momentum cut-off $\hbar k_c$. To determine the cut-off range, we must use the fact that Born approximation only gives the correct evaluation in the low-energy scattering regime with $k|a| < 1$. Hence the validity of the present theory based on Born approximation requires a cut-off $k_c < |a|^{-1}$. For $^6$Li atom, this is in the order of Fermi wave number $k_F$. With such a cut-off, we numerically evaluate the energy gap, the total averaged atomic density and the quantum pair function.

To be concrete, we assume that $N = 2 \times 10^3$ $^6$Li atoms in each spin state are confined by a magnetic trap with an oscillation frequency $\omega = 2\pi \times 150$ Hz. With these values, $\hbar k_c \approx 100 \hbar \omega \approx 740$ nK, and the peak value of the energy gap is $\Delta(0) \approx 5\hbar \omega = 36$ nK. For a degenerate Fermi gas in a harmonic trap, the characteristic size of the average density is given by the Fermi radius $r_F = [2E_F/\hbar^2\omega^2]^{1/2} \approx 48 \mu m$ [14], while the length scale of the pair correlation function is $r_c \sim k_F^{-1}$, where $k_F = (2mE_F/\hbar^2)^{1/2} \approx 2\pi \times 6800$ cm$^{-1}$ is the Fermi wavenumber. The numerical result for the correlation function is shown in Fig. 2, along with the spatial variation of the energy gap.

We need emphasize that in the homogeneous gas, the correlation length (pair size) at zero temperature is defined in terms of the so-called coherence length $\xi_c = \hbar v_F/\Delta(0)$. The coherence length determines the region where the pair function extends $\xi_c$. However within the region, the pair function still contains shorter oscillation structure which has the scale $r_c$. In fact from our numerical result for the trapped gas, we see that the pair function indeed varies with such a length scale. Now we will explain how such a scale can be observed by optical imaging.

Assuming that a plane at $z_0 = 2$ cm is imaged with unit magnification and with a transition wavelength $\lambda = 670$ nm, the image size is $z_0/k_F \sim 0.09$ mm for the normal component and $z_0/k_r \sim 1.9$ cm for the pair component, differing by a factor of $2E_F/\hbar \omega$. The calculated images for a gas below and above the critical temperature for the BCS phase transition are shown in Fig. 3(a) and (b), respectively. It is seen that when the transition occurs, a spatially broadened image appears. The physical situation is depicted in Fig. 3, where the small-scale structure induced by pairing causes light to scatter at a larger angle than that scattering from the cloud itself.

3
The normal signal is produced by coherent scattering and is therefore proportional to \((2N)^2\), as can be verified by reference to Eq. (9). The pair signal, however, arises from spontaneous Raman scattering between pairs above and below the energy gap, and is found using Eq. (10) to be proportional to the number of pairs \(N_p\). This number is determined by the number of atoms in an energy shell of width \(\Delta\) centered on \(E_F\), so \(N_p \approx 3N\Delta/E_F\). For the parameters given above, \(N_p \approx 3 \times 10^4\) and the ratio of the peak signal intensities is \(I_2(0)/I_1(0) \approx 2 \times 10^{-7}\). It is difficult to experimentally measure a signal with such a large dynamic range, but the pair signal can be revealed by using a nearly opaque spatial filter to attenuate the normal signal. If the diameter of the filter is chosen to be approximately equal to the spatial dimension of the normal signal image, it will affect only the central region of the pair signal, and both contributions can be observed with the same intensity scale.

Finally, we calculate the scattered light spectrum. For the normal degenerate ground state, a single spectral line is obtained at the frequency of the incident light. For the superfluid state, the spectrum exhibits a double-peaked structure as shown in Fig. [4]. The coherent peak is from scattering by the normal component. The frequency shift of the sideband line is approximately twice the gap energy, confirming that the sideband is due to Raman scattering by pairs. The long oscillating tail of the sideband is due to modulated broadening from the center of mass motion of atoms at the trap frequency. Hence, the presence of the shifted peak provides another effective method to detect the BCS phase transition and can be used to directly determine the gap energy.

The theory presented here was simplified by the neglect of spontaneous emission, permitting, for example, the assumption that \(\Delta\) remains constant during probing. However, the pair signal depends on breaking pairs by incoherent spontaneous Raman scattering, and thus requires spontaneous emission. The theory is therefore valid only in the weak-signal limit, where \(\Gamma \ll T^{-1}\). Larger signals could be obtained experimentally by allowing \(\Gamma \sim T^{-1}\), but quantitative interpretation would then be more difficult.

In conclusion, we have studied off-resonance light scattering by a trapped degenerate Fermi gas. The results show that both spatial imaging and the scattered light spectrum give clear signatures for the BCS phase transition to a gaseous superfluid state.

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FIG. 1. The normalized spatial distributions of the equal-time quantum pair correlation function, with the distance $\xi$ between atoms scaled by the Fermi wavenumber $k_F$. The dashed curve is the correlation due to Cooper pairs with center of mass located at the trap center and the solid curve is the average contribution of all Cooper pairs. The inset shows the spatial dependence of the energy gap, normalized to the center of the trap and with position scaled by the Fermi radius $r_F$. 
FIG. 2. The spatial image measured a distance $z_0$ from the atoms, with (a) the trapped Fermi gas in the normal degenerate ground state and (b) in the superfluid state after the BCS phase transition. In both images, the central peak is clipped and actually extends by a factor of $\sim 10^6$ above the axes shown. The radial size of the normal component is approximately $z_0/k_{rF}$, while the pair component is larger, extending to $z_0/k_{rc}$. 
FIG. 3. Schematic of the imaging technique. The white area represents the incident probe laser, the dark gray the light coherently scattered by the cloud, and the light gray the light scattered by Cooper pairs. The small length scale of the pair structure scatters light at a relatively large angle, so by measuring the intensity in the far field, the components can be distinguished. Dark ground imaging techniques can be used to eliminate the contribution of the probe laser itself.
FIG. 4. The normalized scattered light spectrum of the scattered field. The frequency shift $\Delta \nu$ is scaled by the trap frequency $\omega$. For the solid curve, the gap energy $\Delta \approx 5 \hbar \omega = 36 \text{ nK}$, and for the dotted curve $\Delta = 72 \text{ nK}$. A spatial filter with a transmission of $\sim 10^{-4}$ is used to reduce the strong signal from the normal component to the same level as that from the pair component.