Probing sub-eV Dark Matter decays with PTOLEMY

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Abstract. When the Dark Matter mass is below the eV-scale, its cosmological occupation number exceeds the ones of photons from the cosmic microwave background as well as of relic neutrinos. If such Dark Matter decays to pairs of neutrinos, it implies that experiments that seek the detection of the cosmic neutrino background may as well be sensitive to this additional form of “dark radiation”. Here we study the prospects for detection taking into account various options for the forecasted performance of the future PTOLEMY experiment. From a detailed profile likelihood analysis we find that Dark Matter decays with lifetime as large as $10^4$ Gyr or a sub-% Dark Matter fraction decaying today can be discovered. The prospects are facilitated by the distinct spectral event shape that is introduced from galactic and cosmological neutrino dark radiation fluxes. In the process we also clarify the importance of Pauli-blocking in the Dark Matter decay. The scenarios presented in this work can be considered early physics targets in the development of these instruments with relaxed demands on performance and energy resolution.

Keywords: cosmological neutrinos, neutrino experiments, dark matter experiments

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1 Introduction

Besides the cosmic microwave background (CMB), the prediction of the cosmic neutrino background (\(C_\nu\)B) is the second, unequivocal key signature of a hot Big Bang. The Universe must have passed through a stage of billions degrees of Kelvin in order to enable the fusion of light elements from protons and neutrons. At this temperature neutrinos become the main actors balancing the relative abundance of nucleons. However, whereas the measurements of the CMB have elevated Big Bang cosmology to a precision science, the relic radiation of neutrinos from the nucleosynthesis era remains undetected to date.

The observation of relic neutrinos would provide a window into the first second after the Big Bang, and its detection is an important task in cosmology. Today’s observed CMB radiation temperature of \(T = 2.73\) K implies that there are 411 relic photons per cm\(^3\). Assuming a standard cosmological history, it then follows that the average cosmic \(C_\nu\)B number density is 336 cm\(^{-3}\) (see e.g. [1]). Relic neutrinos hence constitute the largest of neutrino fluxes at Earth [2]. However, whereas microwave photons are readily detected, “infrared” neutrinos have exceedingly small cross section, making them literally inert under any ordinary laboratory scheme. Among the various other ideas for detection [3–7], the most prospective way appears to leverage the energy release in the threshold-free electron-neutrino capture reactions of beta-decaying nuclei [8, 9]. Here, the capture on tritium atoms,

\[
\nu_e + T \rightarrow ^3\text{He} + e^- ,
\]

is one of the best candidate reactions. The relatively low \(Q\)-value of the associated super-allowed beta-decay, \(Q_\beta = 18.529(2)\) keV, allows for schemes that achieve the required sub-eV
energy resolution in electron energy, while its half-life $t_{1/2} = 12.32(2)$ yrs implies a sensible experimental timescale where sufficient statistics can be collected.

Because of the minute cross section for neutrino absorption, $\sigma_{\nu}\nu \sim 10^{-44}$ cm$^2$, detecting relic neutrinos takes extremely large amounts of tritium. The PTOLEMY experiment [10, 11] proposes to use 100 g of tritium, which is $10^6$ times more than the best current experiment KATRIN [12] employs. Even with this amount of tritium, the expected detection rate is $\sim 4$ or $\sim 8$ relic neutrinos per year, with a dependence if neutrinos are Dirac or Majorana particles [6, 14]. Both experiments use or plan to use sophisticated schemes that filter electron energies with (sub-)eV precision. Neglecting the recoil of the daughter nucleus, the electron kinetic energy in reaction (1.1) is given by

$$E_e = Q + E_{\nu}. \quad (1.2)$$

Any experiment that aims at detecting the neutrino capture (1.1) in the electron over the common beta background $^3$He $\rightarrow$ e$^- + \bar{\nu}_e$ must hence resolve an amount $E_{\nu}$ at the beta-endpoint energy. For the C$\nu$B— which is guaranteed to be partially composed of massive, non-relativistic neutrinos today — this implies that a successful detection of relic neutrinos is tantamount to measuring neutrino mass.

Given the long road ahead that the efforts in C$\nu$B detection face, it is only just to ask what other kind of physics can be probed with such experiment. For example, it has been proposed that neutrino capture experiments such as PTOLEMY can be used to detect light sterile neutrinos [15], constrain the neutrino lifetime, lepton-asymmetry or thermal history [14], or act as directional Dark Matter (DM) direct detectors [11, 16, 17]. In this work, we consider yet another possibility, namely, the detection of neutrino “dark radiation” (DR). Here, the potentially largest source can be the decay of a fraction $\kappa_{DM}$ of DM of mass $m_{DM}$. To see, if this is prospective at all, we may saturate the cosmological DR flux by assuming the fraction $\kappa_{DM}$ has already decayed (or is currently decaying at an unsuppressed rate) into $N_\nu$ neutrinos — typically $N_\nu = 1, 2$ in simple models — to estimate the ratio of DR to C$\nu$B neutrino absorption,

$$\frac{R_{DR}}{R_{C\nu B}} \sim \frac{N_\nu \kappa_{DM} \Omega_{DM} \rho_{crit}}{m_{DM} n_{C\nu B}} \left(\frac{\sigma c}{\sigma_{\nu\nu}}\right) \sim \mathcal{O}(1) \kappa_{DM} \left(\frac{1 \text{ eV}}{m_{DM}}\right) \eta_{C\nu B}. \quad (1.3)$$

The first factor is the ratio of the cosmological DM number density over the number density $n_{C\nu B}$ of the C$\nu$B. The second factor is the ratio of absorption cross section times the typical velocity of the incoming neutrino; the latter product is to good approximation velocity independent and hence a number close to unity. This rough estimate neglects the flavor and helicity composition of the C$\nu$B as well as additional contributions that may arise from local DM decays, but already demonstrates that if DM with mass below the eV-scale decays, it may be detected in a C$\nu$B experiment [23, 24].

The mass-scale of the decaying DM in (1.3) implies that the only channels of decay into Standard Model (SM) particles are photons and neutrinos. Moreover, when $\kappa_{DM} = \mathcal{O}(1)$ the estimate (1.3) implies that we are to consider bosonic light DM. Fermionic DM

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1Tritium has the life-time $t_{1/2} \sim 12$ years, therefore, the amount of tritium that decays during the experiment will not significantly change, see e.g. [13].

2The possibility of more energetic neutrino DR and its signature in DM direct detection and neutrino experiments was considered in [18–22].

3The total DR flux is independent of DM lifetime $\tau_{DM}$ for $\tau_{DM} \lesssim t_0$, where $t_0$ is the age of the Universe, and for as long as DR remains relativistic [22].
needs to satisfy the Tremaine-Gunn bound $m_{\text{DM}} \gtrsim 300 \text{eV}$ \cite{25–27} and sub-eV fermionic DM cannot comprise its dominant component.\footnote{If $\kappa_{\text{DM}} \ll 1$ the decaying DM can be fermionic for as long it is only light enough to boost its occupation number by $1/\kappa_{\text{DM}}$ to retain an $\mathcal{O}(1)$ number in the estimate (1.3).} There are then several possibilities for a concrete realization of this scenario. A particularly well motivated one is that of a Majoron DM candidate \cite{28}, for which the decay $\text{DM} \to \nu\nu(\bar{\nu}\bar{\nu})$ is a defining feature. The decay rate is linear in DM mass, $\Gamma_{\text{DM} \to \nu\nu} \propto g^2 m_{\text{DM}}$, and the required smallness of the effective parameter $g \sim 10^{-17} \text{eV}/m_{\text{DM}}$ to achieve a cosmological lifetime arises from the global breaking of lepton number at some UV scale $\langle \Phi \rangle$ that provides neutrino with mass, $g \sim m_\nu/\langle \Phi \rangle$. This possibility was considered in \cite{23, 24}.

In this work, we build on these previous proposals and, first, study the prospects of an enhanced signature when considering the non-relativistic injection regime, and, second, present a detailed sensitivity study that takes into account the projected performance of PTOLEMY. A Majoron is of course not the only possibility to source low-energy neutrinos. One may equally well consider vector DM particle, associated with a gauged combination of lepton-number and/or baryon minus lepton number, or an eV-scale sterile neutrino with $\kappa_{\text{DM}} < 1$ and enhanced decays to three Standard Model neutrinos through mixing. In this work, we will not go into these various options, but rather choose a phenomenological approach, studying the concrete detectability of decaying sub-eV DM; an exploration of models is left for future work.

The paper is organized as follows: in section 2 we introduce the neutrino DR signal in the neutrino capture reactions with a focus on PTOLEMY. In section 3 we predict the neutrino DR signal for PTOLEMY, considering various cases. In section 4 we forecast the sensitivity of PTOLEMY to detect DR from DM decay, taking into account the backgrounds from beta-decay and relic neutrinos. In section 5 we summarize our results and conclude.

## 2 PTOLEMY neutrino detection rate

The overall capture rate (per tritium atom) is given by a product of neutrino flux $n_\nu v_\nu$ times the capture cross section $\sigma$. Importantly, for as long as the incoming neutrino energy satisfies $E_\nu \ll Q$, the product $\sigma v_\nu$ is a constant, with its only dependence on the helicity composition of the incoming flux \cite{14},

$$n_\nu v_\nu \sigma v_\nu = \left[(1 - v_\nu) n_{\nu h R} + (1 + v_\nu) n_{\nu h L}\right] (\sigma v)_0,$$

with $(\sigma v)_0 \approx 3.7 \cdot 10^{-45} \text{cm}^2$ and where $n_{\nu h L}$ $(n_{\nu h R})$ denote the number density left-helical (right-helical) active neutrinos. This dependence leads to a twice larger event rate when $C\nu B$ neutrinos are Majorana rather than Dirac. For simplicity, in the following we assume that DM decays into equal amounts of right- and left-helical states, which is indeed the case for Majoron decay \cite{24}. By making this assumption, the dependence on helicity composition drops out, simplifying the discussion.

Neutrinos propagate as mass-eigenstates $\nu_i$ and enter as such the detector in an incoherent mixture of flavor states as they have traveled astronomical distances from the source. The reaction to consider is then $\nu_i + T \to ^3\text{He} + e^-$ and the probability of capture is modulated by their electron-flavor content, given by the squared PMNS-matrix element $|U_{ei}|^2$. The total rate of neutrino capture in an experiment with a mass $M_T$ of tritium (PTOLEMY

\text{PTOLEMY neutrino detection rate}
plans to use $M_T = 100$ g [10]) is given by
\[
\Gamma = \frac{M_T}{m_T} \sum_{i=1}^{3} |U_{ei}|^2 \int dE_{\nu_i} \sigma_{\nu_i} \frac{dn_{\nu_i}}{dE_{\nu_i}} \approx \frac{M_T}{m_T} (\sigma v)_{0} \sum_{i=1}^{3} |U_{ei}|^2 n_{\nu,i}.
\] (2.2)

Here, $m_T$ is the mass of one tritium atom, $dn_{\nu,i}/dE_{\nu_i}$ is the energy spectrum of the $i$-th mass eigenstate; $dE_{\nu_i}$ is the total neutrino energy and $v_{\nu,i}$ the associated velocity. For the remainder of this paper, we shall always consider the kinematic regime of low-energy neutrinos where $\sigma v_{\nu,i}$ is a constant, and the local number density $n_{\nu,i}$ alone becomes the figure of merit that informs us about the overall rate. Applied to the $C\nu B$, the number density of relic neutrinos can be written as $n_{\nu,e} = f_{c,i} n_0$, where $n_0 \approx 56$ cm$^{-3}$ is the average number density per neutrino state today, and $f_{c,i}$ is a clustering factor in our Galaxy that ranges from 1 to 1.1 for neutrino masses below 50 meV [29, 30].$^5$ The detection rate of relic neutrinos by PTOLEMY is therefore (cf. [32])
\[
\Gamma_{\text{CNB}} \approx (4 \text{ or } 8) \text{ yr}^{-1} \left( \frac{M_T}{100 \text{ g}} \right).
\] (2.3)

Here we have taken $f_{c,i} = 1$ and used unitarity of the PMNS matrix, $1 = \sum_{i=1}^{3} |U_{ei}|^2$.

The energy spectrum of electrons emitted in the capture is immediately obtained from (2.2) together with (1.2),
\[
\frac{d\Gamma}{dE_e}(E_e) = \frac{M_T}{m_T} (\sigma v)_{0} \sum_{i=1}^{3} |U_{ei}|^2 \frac{dn_{\nu,i}}{dE_{\nu,i}} (E_e - Q).
\] (2.4)

The detectability of a signal depends on its intrinsic shape, the values of neutrino masses and on the energy resolution $\Delta$ of the experiment. We follow [32] and model the latter by a Gaussian with full-width-at-half-maximum (FWHM) given by $\Delta$ to obtain the observed rate,$^5$
\[
\frac{d\tilde{\Gamma}}{dE_e}(E_e) = \frac{1}{\sqrt{2\pi(\Delta/\sqrt{8 \ln 2})}} \int \frac{d\Gamma}{dE_e}(E'_{e}) \exp \left( - \frac{(E'_e - E_e)^2}{2(\Delta/\sqrt{8 \ln 2})^2} \right) dE'_{e}.
\] (2.5)

For the PTOLEMY forecasts below, we shall take $\Delta = 10 \ldots 100$ eV as a representative range covering the optimal to pessimistic range.

3 Neutrino dark radiation from Dark Matter decay

As we have seen in the previous section, when we consider the additional sources of neutrinos with energies $E_{\nu} \ll Q$, the relative local number densities inform us about the absolute rates. The rate of DR-induced capture events is hence related to the $C\nu B$ prediction via,
\[
\Gamma_{\text{DR}} = \frac{n_{\nu,e}^{\text{DR}}}{n_{\nu,e}^{\text{C}\nu B}} \Gamma_{\text{C}\nu B},
\] (3.1)

where $n_{\nu,e}^{\text{C}\nu B} \equiv n_0 \sum_{i=1}^{3} |U_{ei}|^2 f_{c,i}$ is an effective number density of electron neutrinos in the $C\nu B$ and $n_{\nu,e}^{\text{DR}} \equiv \sum_{i=1}^{3} |U_{ei}|^2 n_{\nu,i}^{\text{DR}}$ is an effective number density of electron neutrinos in DR originating from DM decay. $^5$

$^5$The absolute neutrino mass scale is unknown; the best current limit on the sum of neutrino masses is from cosmology, $\sum m_{\nu_i} \leq 0.12$ eV [31]. Therefore, using 50 meV in the estimate of the clustering factor is already close to the limit for an inverted hierarchy, and clustering is almost negligible.
There are then two principal components that source the contributions to $n_{\nu_e}^{\text{DR}}$:

1. *DM decay at cosmological distances* — the averaged neutrino background from DM decays outside of our Galaxy and over the course of history;

2. *DM decay in the Galaxy* — neutrinos from DM decays from the DM halo of our Galaxy at present.

As is turns out, for $\tau_{\text{DM}} \gtrsim t_0$, where $t_0$ is the age of the Universe, these contributions happen to be of almost identical size in total flux. However, the principal difference is that in the former category the neutrinos experience redshifting of their momenta, whereas in the second category, they do not. This has important consequences for the prediction of the event spectra, and in the following we obtain the respective concentration and energy distribution of both DR components.

### 3.1 Neutrino DR from cosmological DM decay

We first consider the cosmological contribution to the local DR number density. Denoting by $\kappa_{\text{DM}} n_{\text{DM},0} = \kappa_{\text{DM}} \Omega_{\text{DM}} \rho_{\text{crit}} / m_{\text{DM}}$ the average number density the decaying DM-component would have today in the limit of infinite lifetime, the number density of the $i$th neutrino mass state can be estimated as

$$n_{\nu_i}^{\text{cosm}} \approx \text{BR}_i \left( 1 - e^{-t_0 / \tau_{\text{DM}}} \right) n_{\nu_i} \kappa_{\text{DM}} n_{\text{DM},0}, \quad \tau_{\text{DM}} \gg t_0,$$

where $\text{BR}_i$ is the branching ratio of DM decay into $i$th neutrino mass state; in the last expression we have exposed the scaling in the limit of long lifetime, $\tau_{\text{DM}} \gg t_0$.

Of course, gravity alone already constrains the lifetime of DM, e.g. from CMB physics. Here, the statement is that either 4% of all of DM could have decayed between recombination and today, or $\kappa_{\text{DM}} / \tau_{\text{DM}} < 6.3 \times 10^{-3} \text{Gyr}$ for lifetimes larger than the age of the Universe. The latter implies that $\tau_{\text{DM}} \gtrsim 12t_0$ if $\kappa_{\text{DM}} = 1$ [33]. For simplicity, we either use $\tau_{\text{DM}} = 10t_0$ with $\kappa_{\text{DM}} = 1$ for the long lifetime regime, or $\kappa_{\text{DM}} = 0.05$ for the short (arbitrary) lifetime regime, even if it implies that we slightly slip into the disfavoured region.

Using eq. (3.2) we obtain the effective number density of cosmological electron neutrinos as,

$$n_{\nu_e}^{\text{cosm}} = \sum_{i=1}^3 |U_{ei}|^2 n_{\nu_i}^{\text{cosm}} \approx 46 \text{ cm}^{-3} \xi \left( \frac{t_0}{\tau_{\text{DM}}} \right) \left( \frac{1 \text{ eV}}{m_{\text{DM}}} \right), \quad \xi = 3N_\nu \sum_{i=1}^3 |U_{ei}|^2 \text{BR}_i,$$

where we have again taken the limit of long lifetime. For equal branching ratios, $\text{BR}_i = 1/3$, the factor $\xi = N_\nu$. Substituting this result into eq. (2.2) the total PTOLEMY detection rate for cosmologically sourced neutrino DR reads

$$\Gamma_{\text{DR}}^{\text{cosm}} \approx 3.2 \text{ yr}^{-1} \xi \left( \frac{M_T}{100 \mu g} \right) \left( \frac{t_0}{\tau_{\text{DM}}} \right) \left( \frac{1 \text{ eV}}{m_{\text{DM}}} \right).$$

We see that for sub-eV DM mass the detection rate of cosmological neutrinos can be higher than for relic neutrinos, see eq. (2.3). The scenario can hence become another target for PTOLEMY. It is, however, beyond the reach of KATRIN which uses $M_T = 100 \mu g$.

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6This estimate neglects the effects of the Universe’s expansion. The exact expression is given in [22] and it makes a 50% downward correction to the estimate for $\tau_{\text{DM}} \gtrsim t_0$. 

To obtain the differential event rate in PTOLEMY, we now turn to the energy spectrum of the cosmological DR component. For simplicity, we shall only consider the case of 2-body decay, DM → ν̅ν or DM → νν. With the initial DM kinetic energy being negligible in comparison to its mass, the neutrinos are initially injected with energy \( m_{DM}/2 \). The associated neutrino momentum then redshifts, and the continuous decay of DM during the cosmic history accumulates to the following spectrum [21],

\[
\frac{dN_{\nu,i}^{\text{cosm}}}{dE_{\nu,i}}(E_{\nu,i}) = \frac{N_{\nu}}{p_{\nu,i}v_{\nu,i}} \frac{BR_i \kappa_{DM} n_{DM,0}}{H(z_{\text{dec}})} e^{-t(z_{\text{dec}})/\tau_{DM}},
\]

(3.5)

where \( z_{\text{dec}} \) is the decay redshift obtained from the redshift of the initial momentum to the momentum at arrival, \( p_{\nu,i} = \sqrt{E_{\nu,i}^2 - m_{\nu,i}^2} \),

\[
z_{\text{dec}} = \sqrt{\frac{m_{DM}^2}{4} - \frac{m_{\nu,i}^2}{2} - 1}.
\]

(3.6)

In the above formula, \( t(z) \) is the cosmic look-back time evaluated at \( z_{\text{dec}} \); for \( \tau_{DM} \gg t_0 \) the exponential factor can be neglected. The energy differential flux itself is given by multiplying (3.5) by the neutrino velocity \( v_{\nu,i} \).

Examples of the signal for different values of neutrino masses and different assumptions about PTOLEMY energy resolutions are shown (together with the galactic component of the signal that is discussed below) in figure 1 as well as figure 4 in appendix B.

### 3.2 Neutrino DR from galactic DM decay

Let us now estimate the contribution to neutrino DR from DM decay inside the Galaxy. For such an estimate it is important to specify the ratio \( v_{\nu}/c \) as it defines the resulting neutrino concentration around us. We will discuss below the two principal cases: neutrinos that escape upon injection and neutrinos that are injected with a speed below the escape speed and are hence retained in the Galaxy.

#### 3.2.1 Escaping neutrinos

If neutrinos are injected at velocities \( v_{\nu} \) that exceed the escape speed \( v_{\text{esc}} \approx 550 \text{ km s}^{-1} \approx 2 \times 10^{-3}c \), they will escape the Galaxy in a time \( t_{\text{esc}} \sim r_\odot/v_{\nu} < 10^{12} \text{ s} \), where \( r_\odot = 8.3 \text{ kpc} \) is the distance from the center of the Galaxy to the Sun. A Galactic contribution with \( v_{\nu} \geq v_{\text{esc}} \) is only present for \( \tau_{DM} \gg t_0 \). Taking the long-lifetime limit, an order of magnitude estimate for the local neutrino number density is hence,

\[
n_{\nu,i}^{\text{gal}} \approx BR_i \frac{t_{\text{esc}}}{\tau_{DM}} N_{\nu} \kappa_{DM} n_{DM,0} \sim 0.4 \frac{c}{v_{\nu}} \left( \frac{BR_i}{1/3} \right) n_{\nu,i}^{\text{cosm}} \left( \tau_{DM} \gg t_0 \right).
\]

(3.7)

For this estimate we used the local DM density \( n_{DM,0} \sim 0.3 \text{ GeV/cm}^3 \) as a representative value. From this estimate we see that the Galactic DR contribution can be comparable with the cosmological one and it can be even larger if the DM mass is such that galactic neutrinos are only semi-relativistic. The Galactic flux that takes into account the DM density profile then reads,

\[
n_{\nu,i}^{\text{gal}} = \frac{BR_i}{v_{\nu,i} \tau_{DM}} r_\odot N_{\nu} \kappa_{DM} n_{DM,0} \langle D \rangle \left( v_{\nu,i} > v_{\text{esc}} \right),
\]

(3.8)
where $\langle D \rangle$ is the whole sky average of the line-of-sight integral as seen from Earth over the galactic DM density distribution (see e.g. [34]),

$$\langle D \rangle = \frac{1}{4\pi} \int Dd\Omega, \quad D = \frac{1}{r_{\odot}\rho_{\odot,\odot}} \int_{\text{l.o.s.}} \rho(r)ds. \quad (3.9)$$

The value of the averaged D-factor normalised in this way describes how much our naive eq. (3.7) underestimates the local concentration of galactic neutrinos. We adopt $\langle D \rangle = 2.19$ obtained from an NFW profile with a mild dependence on other canonical profiles.\footnote{For the NFW profile we use $r_s = 24.4$ kpc, $\rho_s = 0.18$ GeV/cm$^3$; for an Einasto profile with the core radius $r_c = 1$ kpc and $\rho_{\odot,DM} \sim 0.4$ GeV/cm$^3$ one obtains $\langle D \rangle = 2.96$. One may of course entertain the possibility of a core and/or spike at the Galactic center [35–37], amplifying the Galactic DR contribution; the effect is milder than for annihilating DM and we will not go into such possibilities here.}

Similarly to the cosmological case, we may obtain effective number density of electron neutrinos

$$n_{\nu_e}^{\text{gal}} = \sum_{i=1}^{3} |U_{ei}|^2 n_{\nu,i}^{\text{gal}} \approx 43 \text{ cm}^{-3} \xi_{\text{DM}} \left( \frac{c}{v_\nu} \right) \left( \frac{10t_0}{\tau_{\text{DM}}} \right) \left( \frac{1 \text{ eV}}{m_{\text{DM}}} \right)^4 \left( \tau_{\text{DM}} \gtrsim t_0 \right), \quad (3.10)$$

where $\xi$ is given by eq. (3.3). Using eq. (2.2) we estimate the PTOLEMY detection rate of galactic DR neutrinos,

$$\Gamma_{\text{gal}} \approx 3.0 \text{ yr}^{-1} \xi_{\text{DM}} \left( \frac{c}{v_\nu} \right) \left( \frac{M_T}{100 \text{ g}} \right) \left( \frac{10t_0}{\tau_{\text{DM}}} \right) \left( \frac{1 \text{ eV}}{m_{\text{DM}}} \right). \quad (3.11)$$

Comparing eqs. (3.4) and (3.11) we see that once the D-factor is taken into account the contributions from Galactic neutrinos and from cosmological neutrinos sourced by DM decay are almost equal in the long-lifetime regime.

We may at this point check whether we stay clear from any suppression factors that arise from Pauli blocking. The maximum occupation number in a Fermi-Dirac gas is attained from

$$n_{\text{max}}^{\nu,\text{gal}} \approx \frac{g_\nu}{(8\pi^3)^{2/3}} \frac{p_\nu^3}{\Delta p_\nu/p_\nu},$$

where $g_\nu = 2$ are the active neutrino degrees of freedom of each massive state. In this section, we consider a kinematic situation where every neutrino, once injected, is on a straight trajectory escaping the Galaxy. In the 2-body decay, a small momentum spread $\Delta p_\nu/p_\nu \sim 10^{-3}$ is inherited from the non-relativistic bound motion of the galactic DM. Integration yields

$$n_{\text{max}}^{\nu,\text{gal}} \approx \frac{g_\nu}{8\pi^3} \frac{p_\nu}{\Delta p_\nu} \left( \frac{\tau_{\text{DM}}}{10^7 \text{ yr}} \right) \left( \frac{m_{\text{DM}}}{1 \text{ eV}} \right)^4 \left( \frac{\tau_{\text{DM}}}{t_0} \right), \quad (3.12)$$

and we have taken the same estimate on the escape time $t_{\text{esc}}$ as above. There is a steep dependence on $m_{\text{DM}}$ and $v_\nu$, and the ratio may drop below unity, signaling that the naive concentration (3.8) is affected by Pauli blocking issues. We take this effect into account by multiplying the relevant rates by a phase-space suppression factor,

$$ps \approx \frac{n_{\text{max}}^{\nu,\text{gal}}}{n_{\nu,\text{gal}}|\text{eq. (3.8)}} \lesssim 1 \quad (3.13)$$

whenever this ratio drops below unity. We note that this kind of blocking is mitigated in a decay with more than 2 final states, as neutrinos then assume a broader distribution in momentum.
Finally, we may find the optimum injection velocity where the concentration saturates \( n_{\nu,\text{gal}}(v_{\nu,\text{optimal}}) = n_{\nu,\text{gal}}'(v_{\nu,\text{optimal}}') \); for this, note that \( n_{\nu,\text{gal}}' \propto v_{\nu}^2 \) whereas \( n_{\nu,\text{gal}} \propto 1/v_{\nu} \) on the account of \( t_{\text{esc}} \). Therefore, the “optimal” velocity is

\[
\frac{v_{\nu,\text{optimal}}}{c} \approx 4 \cdot 10^{-3} \kappa_{\text{DM}}^{1/3} \left( \frac{100}{M_{\nu}} \right)^{2/3} \left( \frac{10t_0}{\tau_{\text{DM}}} \right)^{2/3} \left( \frac{1}{1\text{ eV}} \right)^{1/3} \left( \frac{m_{\text{DM}}}{\text{eV}} \right)^{4/3}. \tag{3.14}
\]

As can be seen, the number can fall below the escape speed. In this case, the estimate is revised; see following section. The maximal galactic event rate for escaping neutrinos reads,

\[
\Gamma_{\text{gal, max}} \approx 750 \text{ yr}^{-1} |U_{e1}|^2 \left( \frac{M_T}{100 \text{ g}} \right)^{2/3} \left( \frac{10t_0}{\tau_{\text{DM}}} \right)^{2/3} \left( \frac{m_{\text{DM}}}{1\text{ eV}} \right)^{1/3}, \text{ for } m_{\text{DM}} \approx 2m_{\nu,\text{lightest}}. \tag{3.15}
\]

This shows that although Pauli-blocking may severely constrain the absolute numbers, in the optimal case the galactic contribution can far exceed the cosmological one.

DR neutrinos from 2-body decay are created with a fixed energy \( E_{\nu} = m_{\text{DM}}/2 \) in the DM rest frame. In the laboratory frame they will have some energy distribution due to random velocities of DM particles with the width \( \Delta E_{\nu}/E_{\nu} \sim v_{\text{DM}}/c \sim 10^{-3} \). This scatter in energy can be neglected compared to the energy resolution of the PTOLEMY detector. Therefore, we may take the Galactic DR neutrino energy distribution as a delta-function,

\[
\frac{d\Gamma_{\nu,i}^{\text{gal}}}{dE_{\nu,i}}(E_{\nu,i}) = n_{\nu,i}^{\text{gal}} \text{ ps} \delta \left( E_{\nu,i} - \frac{m_{\text{DM}}}{2} \right). \tag{3.16}
\]

The Galactic component hence produces a peak at the highest possible energy for neutrino capture signal, \( E_{\nu} = Q + m_{\text{DM}}/2 \), see figure 1. Assuming an optimistic energy resolution of \( \Delta = 10 \text{ meV} \) this peak can be resolved from the cosmological contribution and serves as another signature of the DR scenario.

### 3.2.2 Non-escaping neutrinos

Let us now consider the special case that neutrinos from DM decay are injected at non-relativistic speeds with \( v_{\nu} \ll v_{\text{esc}} \approx 10^{-3}c \). In this case, neutrinos accumulate (and saturate in number) rather than escape. In the 2-body decay benchmark case considered here, it happens when the mass of the lightest neutrino is close to the mass of DM, \( m_{\text{DM}} \approx 2m_{\nu,1} \).

This kinematic arrangement to yield \( v_{\nu} \leq v_{\text{esc}} \) in the decay, requires fine-tuning,

\[
\frac{m_{\text{DM}} - 2m_{\nu}}{m_{\text{DM}}} \lesssim 10^{-6}. \tag{3.17}
\]

Despite the severeness of the condition, it may nevertheless be a natural property in some models for the origin of neutrino masses; see e.g. [38, 39]. On the other hand, if DM were to decay into an \( n \)-body final state with \( n > 2 \), there will always neutrinos with \( v_{\nu} \leq v_{\text{esc}} \). Unless the differential decay rate is strongly IR-biased in the neutrino-energy, the efficiency to inject slow neutrinos is directly proportional to the related phase-space volume, which, again will be a small number. Despite these, at first sight unpalatable circumstances, we shall work out this special case below.
If $v_\nu$ is smaller than the escape velocity from our Galaxy, $v_\nu \ll v_{\text{esc}} \sim 10^{-3}c$, neutrinos rather accumulate than escape. In this case the local neutrino density is\(^8\)

$$n^{\text{gal}}_\nu \approx \frac{t_0}{\tau_{\text{DM}}} N_\nu \kappa_{\text{DM}} n_{\text{DM,}\odot} \times \text{ps},$$  

(3.18)

\(^8\)Gravitationally bound neutrinos can change their helicity when momenta are reversed but spins are not (see e.g. [6]). However, as for such neutrinos $v_\nu \ll 1$, we can neglect each of the velocity-dependent terms in eq. (2.1).
where the escape time is now replaced by the age of the Galaxy, that we have taken as $t_0$ for simplicity. If we neglect the phase space factor \( \text{"ps"} \), eq. (3.18) suggests a concentration in excess of (3.7) by a factor $t_0/t_{\text{esc}}$ which would be enormous.

Again, when we are sourcing fermionic DR from a bosonic parent with $\kappa_{\text{DM}} = O(1)$, we need to include the restriction on phase space density by the factor \( \text{"ps"} \). To estimate its importance, let us consider for the parent DM phase space distribution function a non-truncated Maxwellian, for concreteness again using the local value $n_{\text{DM}}$,⊙:

$$f_{\text{DM}}(|p|) = \kappa_{\text{DM}} n_{\text{DM},\odot} \left( \frac{1}{2\pi m_{\text{DM}}^2 \sigma^2} \right)^{3/2} \exp \left( -\frac{p^2}{2\sigma^2 m_{\text{DM}}^2} \right),$$

such that $\int d^3p f_{\text{DM}} = \kappa_{\text{DM}} n_{\text{DM},\odot}$; $\sigma$ is the one-dimensional velocity dispersion. The maximum value is attained for $|p| = 0$ which we denote by $f_{\text{DM}}^{\text{max}} = f_{\text{DM}}(0)$. In turn, the precise distribution function of the created non-relativistic neutrinos is difficult to know, because the neutrino spends significant time in the Galaxy and is subject to the same thermalization processes as DM. However, if we are to consider a strictly non-relativistic injection with $v_\nu \ll v_{\text{esc}}$, which is possible in the finely-tuned 2-body decay, it is not unreasonable to assume that the DM phase space density is largely inherited. Irrespective of the detailed functional form $f_\nu(|p|)$, however, we may particularly expect that $\tilde{f}_{\text{max}}^{\nu} \approx f_{\text{max}}^{\text{DM}}$ holds well, if Pauli-blocking can be neglected.

As mentioned in the previous section, Fermi-Dirac statistics tells us that the maximum phase space density is $f_\nu^{\text{max}} = f_\nu / (8\pi^3)$, and Pauli-blocking in the decay needs to be taken into account, whenever this density becomes saturated. Therefore, we may evaluate the “in-medium” phase space suppression factor from the ratio

$$\text{ps} \approx \frac{f_\nu^{\text{max}}}{f_{\text{DM}}^{\text{max}}} = \frac{g_\nu (2\pi \sigma^2)^{3/2} m_{\text{DM}}^4}{8\pi^3 \kappa_{\text{DM}} \rho_{\text{DM},\odot}} \approx 4 \times 10^{-6} \frac{g_\nu}{\kappa_{\text{DM}}} \left( \frac{m_{\text{DM}}}{\text{eV}} \right)^4.$$

On the right hand side we have used $\sigma = v_c / \sqrt{2}$ where $v_c \approx 220 \text{ km/s}$ is the circular velocity of the solar system. This is a punishing factor and implies that the enhancement in the local concentration is at best moderate for $m_{\text{DM}} \approx 1 \text{ eV}$, and even turns into a suppression factor for lower DM mass when neutrinos are not evacuated from the galaxy like in the relativistic case above. The arguments above are similar in the spirit that underlie the ones leading to the Gunn-Tremaine bound [25] and essentially a manifestation of Liouville’s theorem; see also [26] and the recent update in [27].

Since the solar mass splitting is $\sqrt{\Delta m^2_\odot} \approx 10^{-2} \text{ eV}$, the degeneracy condition (3.17) can only hold for one of the three neutrino mass eigenstates. If the degeneracy holds for the lightest of neutrino states, $\nu_1$, it remains the only kinematically allowed decay channel and $v_\nu \lesssim v_{\text{esc}}$ is guaranteed. If the degeneracy is with a heavier state, then the branching ratio into the “slow channel” will be suppressed by a model-dependent factor $\sim (v_\nu/c)^n$, $n \geq 1$ as a lighter final state is available. In our numerical results, we will assume for simplicity that the fine-tuning happens for the lightest neutrino in which case the branching fraction of DM decay into the lightest neutrino mass state $i = 1$ is equal to one.

### 4 PTOLEMY sensitivity

We now proceed to forecast the sensitivity to neutrino DR on the concrete example of PTOLEMY. The canonical event shape for $C\nu B$ detection is a large beta-background until
the endpoint energy that needs to get filtered in order to detect the small capture signals that are offset by a small amount given by their neutrino masses. In the current context, both constitute backgrounds to a DR search. However, as we argued above, the DR signal can extend in energy up to \( \sim 1 \) eV above the endpoint, into an essentially background free region.

To treat both cases simultaneously, we use a binned profile likelihood and simulate the experiment with assumed 1 yr and 5 yr exposures and a target mass of 100 g by generating Monte Carlo mock representations. We consider the neutrino-induced capture events from sources \( \alpha = \beta, C \nu B \), and DR together with their associated energy spectra \( d\Gamma_{\alpha}(E_e, \theta)/dE_e \) that are obtained by folding the theoretical rates with an Gaussian energy resolution \( \Delta \) according to (2.5). The model parameters that enter these predictions are \( \tau_{DM} \) and \( m_{DM} \) for DR with \( \theta = \tau_{DM} \); for \( \alpha = \beta, C \nu B \) \( \theta \) there are no fit parameters and \( \theta \) is null. The likelihood function under the hypothesis H for fixed neutrino mass hierarchy (NO, IO), fixed DM mass \( m_{DM} \) and absolute neutrino mass scale given by \( m_{\nu_1} \) reads,

\[
\mathcal{L}(\hat{\theta}|H) = \prod_{i=1}^{N_{\text{bin}}} \frac{e^{-\varepsilon} \sum_{\alpha} \mu_{\alpha}^i(\hat{\theta})}{N_{\text{obs}}^i} \left[ \sum_{\alpha} \mu_{\alpha}^i(\hat{\theta}) \right]^{N_{\text{obs}}^i}. \tag{4.1}
\]

For our analysis, we divide the signal region in \( E_e - Q \) from \(-25, -75, -150\) meV (for \( \Delta = 10, 50, 100\) meV, respectively) to 300 meV into \( N_{\text{bin}} = 100 \) equidistant bins and from 0.3 eV to 30 eV into \( N_{\text{bin}} = 50 \) logarithmic bins. The reason for such division is owed to computational efficiency, since at higher energies we enter the background-free region. The expected number of events in each bin \( i \) of source \( \alpha \) is denoted by \( \mu_{\alpha}^i \) and \( N_{\alpha}^i \) is the associated random number of observed events in each bin that is drawn from a Poisson distribution; \( N_{\text{obs}}^i = \sum_{\alpha} N_{\alpha}^i \).

A discovery of a DR signal in presence of backgrounds then amounts to a rejection of the background-only hypothesis \( H_0 \) for sources \( \alpha = \beta, C \nu B \). Here, the negative log-likelihood then serves as test statistic for the hypothesis test, \( q = -2 \ln \mathcal{L}(\hat{\theta}|H_0)/\mathcal{L}(\hat{\theta}|H_1) \), where \( \hat{\theta} \) maximizes the likelihood under the background-only hypothesis \( H_0: \tau_{DM} \to \infty \) and \( \hat{\theta} \) maximizes \( \mathcal{L} \) for signal plus background, \( H_1: \tau_{DM} \neq 0 \). A distribution in \( q \) under \( H_1 \) is obtained by generating \( 10^3 \) mock data-sets for each combination of \( (\tau_{DM}, m_{DM}) \), until the entire parameter space is scanned; in turn, the distribution in \( q \) with mock-data generated under \( H_0 \) we verified that it follows a \( \chi^2 \) distribution with one degree of freedom as per Wilk’s theorem [40]. The significance distributions are then given by \( Z = \sqrt{q} \). The discovery criterion at 3\( \sigma \) significance implies that \( H_0 \) is rejected with 99.865\% probability (\( p \)-value \( p_0 = 0.00135 \)). For a chosen confidence level of 90\% we require that a given experiment has a 90\% probability to detect at least a signal with 3\( \sigma \) significance. Hence, this leads to a detection of the signal, if 90\% of the mock-data sets generated under \( H_1 \) lie above the discovery criterion \( Z \geq 3\sigma \), where \( H_1 \) is accepted and where at the same time \( H_0 \) is rejected; see [22, 41] for further details on this procedure.

The resulting discovery potentials are shown in the left (right) panel of figure 2 for normal (inverted) neutrino mass hierarchy; we additionally take the lightest neutrino as massless, \( m_{\nu_1} = 0 \) (\( m_{\nu_3} = 0 \)). We assume that DR is sourced from \( X \to \nu \bar{\nu} \), i.e. \( N_{\nu} = 1 \) with a decaying DM fraction of 100\%, \( \kappa_{DM} = 1 \). The blue and green sets of lines are associated with exposures of 100 g yr and 500 g yr, respectively. The solid, dashed, and dash-dotted lines correspond to progressively worsening assumptions on the energy resolution, \( \Delta = 10, 50 \) and 100 meV, respectively. The thin vertical lines show the kinematic thresholds for the decays into the heavier neutrino mass eigenstates \( m_{\nu_2, 3} \). The gray shaded region shows the
Figure 2. Discovery reach of PTOLEMY as a function of progenitor mass $m_{\text{DM}}$ and lifetime in units of the age of the Universe, $\tau_{\text{DM}}/t_0$. An exposure of 100 g yr (blue lines) and 500 g yr (green lines) has been assumed for various projected performances on the electron energy resolution $\Delta$ as labeled, with 10 eV (100 eV) being the optimal (most conservative) case. All of DM is assumed to be decaying $\kappa_{\text{DM}} = 1$. The mass of the lightest neutrino is $m_{\nu_1} = 0$. In the left (right) panel the mass of the lightest neutrino is $m_{\nu_1} = 0$ ($m_{\nu_3} = 0$) and a normal (inverted) hierarchy is assumed.

Figure 3. Sensitivity of PTOLEMY to the decaying fraction of DM, $\kappa_{\text{DM}}$, as a function of DM mass; same labeling as in figure 2.

cosmological limit on the decaying cold DM lifetime, $\tau_{\text{DM}} \gtrsim 35 t_0$ [42]; see also [33, 43, 44]. Finally, the thin green line in the left panel is obtained when Pauli-blocking is neglected.

Both panels establish the sensitivity to the maximum DM lifetime, directly related to the minimum detectable DR flux (a 3$\sigma$ significance in the general presence of the $C\nu B$
background. In the high mass region $m_{\text{DM}} \gtrsim 100$ meV, the discovery potential is almost independent on the neutrino mass hierarchy and a general $1/m_{\text{DM}}$ scaling can be seen. At around 100 meV progenitor mass, this trend is broken by the presence of the CνB peak generated by the heaviest neutrino, $m_{\nu_3} = 50$ meV ($m_{\nu_{1,2}} \approx 50$ meV) assuming normal (inverted) hierarchy. In the left panel, the sensitivity for $\Delta = 10$ meV reaches its optimum at $m_{\text{DM}} \sim 50$ meV, whereas in the inverted hierarchy scenario (right panel) the lightest neutrino has a smaller contribution to the DR signal due to the smaller squared PMNS-matrix element $|U_{e3}|^2$. Therefore, smaller lifetimes, i.e. a larger DR flux, are necessary to discover the signal in comparison to the right panel. For $m_{\text{DM}} \lesssim 40$ meV, the continuous tritium beta background starts playing a role, suppressing the lifetime reach in both panels. However, this is eventually counterbalanced by the growing decaying DM number density with $1/m_{\text{DM}}$ and the sensitivity is again improved for diminishing DM mass. However, only for the NO the lines extend above the cosmological limit.

We conclude that a discovery of decaying DM with $m_{\text{DM}} \gtrsim 100$ meV is possible with rather relaxed assumptions on energy resolution. For lighter DM mass, an optimum energy resolution is critical to suppress the bleeding of the beta background into the signal region, and decaying DM with neutrino final states is discoverable in the normal ordering across the entire conceivable mass range.

Finally, we may consider the possibility that a fraction $\kappa_{\text{DM}}$ of DM decays with arbitrary lifetime and ask for the sensitivity of PTOLEMY to $\kappa_{\text{DM}}$. For this, we saturate the flux by choosing an optimal lifetime, $\tau_{\text{DM}} = t_0$, so that the fraction $\kappa_{\text{DM}}$ decays today with an unsuppressed rate. The model parameters that enter in the likelihood in eq. (4.1) are now $\kappa_{\text{DM}}$ and $m_{\text{DM}}$ for DR with $\bar{\theta} = \kappa_{\text{DM}}$. Figure 3 presents the $3\sigma$ discovery sensitivity to $\kappa_{\text{DM}}$ as a function of progenitor mass as above. As expected, the discovery potentials in the $(\kappa_{\text{DM}}, m_{\text{DM}})$-plane exhibit an inverse behaviour with respect to the contours in the $(\tau_{\text{DM}}, m_{\text{DM}})$-plane in figure 2. The discoverable region is hence affected by the same limiting factors as were discussed above. We conclude that with an exposure of 100 g yr (500 g yr) PTOLEMY is capable to detect a decaying fraction of $\sim 1\%$ ($\sim 0.1\%$) with an optimal energy resolution of $\Delta = 10$ meV. For the pessimistic case $\Delta = 100$ meV it takes the larger of assumed exposures to compete with cosmological limits with a mild prospect to detect DR originating from a decay with progenitor mass $0.1 \lesssim m_{\text{DM}}/\text{eV} \lesssim 2$.

5 Conclusions

PTOLEMY is a visionary and ambitious experiment. Its main science goal — the detection of relic neutrinos — would mark a resounding success for a key prediction of hot Big Bang cosmology, but will require significant breakthroughs in experimental technology. When entering such unexplored areas we are not safe from unexpected difficulties and obstacles. In this work we demonstrate through a detailed profile likelihood study that even before PTOLEMY reaches the level of performance (first of all, energy resolution and statistics) it can potentially detect a signal from new physics that can accede the SM relic neutrino signal, namely, the detection of neutrino DR. Such DR may be sourced by the decay of (a component of) DM with sub-eV mass. The potential signal in PTOLEMY can then be classified as follows:

- In the most generic case (see figure 1) DM decays into a 2-body neutrino final state which results in an additional peak located at $E_{\text{peak}} = Q + m_{\text{DM}}/2$. The number of events in this signal may be equal or larger than in the signal from the CνB for the
DM mass range $2m_{\nu_1} \leq m_{DM} \lesssim 1\, \text{eV}$. This peak comes from DM decay in our Galaxy. Additionally, extragalactic DM decays give rise to a second component of the signal, similar in overall magnitude but with events almost equally distributed in electron energy between $Q$ and $Q + m_{DM}/2$.

- There is a special case when neutrinos are efficiently released at semi- or non-relativistic velocities, either in a suitably arranged decay with more than two final states, or when $m_{DM}/2 \simeq m_{\nu_1}$ in the 2-body decay. The local concentration of neutrinos can then be enhanced by Galactic DM decays, reaching a maximum when the injection velocity is in the vicinity of the Milky Way’s escape speed.

There is a number of avenues to explore further in our proposal. First of all, concrete models of sub-eV DM should be explored and how they embed themselves into the bigger scheme of things, such as relic density generation; such program has already started in [23, 24]. Is it possible to find well-motivated or natural cases where the non-relativistic injection boosts the detection prospects? On the signal side, we may quantitatively address the question to what degree it is possible to discriminate between early ($\tau_{DM} \lesssim t_0$) and late ($\tau_{DM} \gtrsim t_0$) decays by virtue of the Galactic peak. In summary, there is a scientific case for relic neutrino searches such as PTOLEMY that is connected to another pressing topic in modern physics, namely, the quest in understanding the most basic properties of DM, such as its lifetime and mass-scale. A detection of DR in a future $C\nu B$ experiment may shed light on these questions.

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A A solar neutrino basin?

Another intense local source of neutrinos is the Sun. Here, one may first wonder if the most prominent of fluxes, the $pp$-flux may constitute a background for PTOLEMY. However, it is easy to see that the falling statistical beta spectrum with decreasing energy yields a small flux, e.g. at $E_\nu = 1\, \text{eV}$ it is $dF_{pp}/dE_\nu \simeq 10^{-2}\, \text{cm}^{-2}\, \text{s}^{-1}\, \text{keV}^{-1}$. Overall, the spectrum translates into a minute local concentration $n_{\nu,pp} \sim 10^{-16}$ of $pp$-neutrinos below 1 eV energy. In fact, the low energy region is largely dominated by the flux from plasmon decay [45]. From figure 2 in [46] one finds a differential flux value $dF/dE = 10 \,\text{cm}^{-2} \,\text{s}^{-1} \,\text{keV}^{-1}$ at a neutrino energy of $1\, \text{eV}$, still falling significantly short for an interesting instantaneous concentration.

The above arguments neglect neutrino mass. Neutrinos produced in the Sun may, however, also be gravitationally trapped within the solar system because of their finite masses. Such possibility has recently been proposed in [47] as an amplification scheme for probing light new physics that may be produced in Sun. We may take a quick estimate to demonstrate, that for neutrinos this mechanism is negligible to obtain a reasonable neutrino concentration at Earth.

Let the flux of neutrinos that reaches Earth but do not escape the solar system be $\Delta F$. On dimensional grounds, the number density of neutrinos at Earth is then of order,

$$n_{\nu,\text{solar}} \sim \frac{\Delta F t_\odot}{r_E}, \quad (A.1)$$

- 14 –
where $t_\odot \approx 4.5 \times 10^9$ yr is the age of the solar system and $r_E = 1$ AU is the distance between Sun and Earth. To estimate the neutrino flux we may take the above quoted value of 10 cm$^{-2}$ s$^{-1}$ keV$^{-1}$ at 1 eV energy from [46], hence overestimating the relevant non-relativistic portion at even lower energy. Neutrinos that reach Earth but do not escape the solar system have a narrow energy distribution with a width

$$\Delta E = \frac{GM_\odot m_\nu}{r_E} \approx 10^{-8} \text{ eV} \left( \frac{m_\nu}{1 \text{ eV}} \right).$$

(A.2)

We may then limit the associated flux of such neutrinos from above,

$$\Delta F \lesssim \frac{dF}{dE}(E = 1 \text{ eV}) \Delta E \approx 10^{-10} \text{ cm}^{-2} \text{ s}^{-1} \left( \frac{m_\nu}{1 \text{ eV}} \right)$$

(A.3)

Substituting this value into (A.1) we arrive at

$$n_{\nu,\text{solar}} \sim 10^{-6} \text{ cm}^{-3}.$$  

(A.4)

This number is already overestimation of the trapped neutrino density at Earth and it is eleven orders of magnitude smaller than for relic neutrinos. This means that solar neutrinos do not constitute a background for relic neutrino searches.

B Inverse ordering

In this appendix figure 4 presents the pendant to figure 1 for an inverted neutrino mass ordering with two heavier states split by the smaller solar mass difference.

C Discovery potential for $m_{\nu_1} = 50$ meV ($m_{\nu_3} = 50$ meV)

In figures 5, 6 we present the discovery potentials for $m_{\nu_1} = 50$ meV ($m_{\nu_3} = 50$ meV) for normal (inverted) mass hierarchy. In this case, the minimum allowed DM mass is 100 meV and the tritium beta background does not play a role. Hence, the C$\nu$B is the only background that enters in the analysis and only alters the limits around $m_{\text{DM}} = 100$ meV compared to the figures 2, 3.

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9 It appears that finite neutrino masses were not taken into account in the numerical results of [46], but their inclusion would render the neutrino flux even smaller.
Figure 4. Examples of the DR neutrino signals from DM decay with $m_{DM} = 1, 0.5$ and 0.2 eV and inverse neutrino mass ordering (IO) with $m_{\nu_1} = 0$ (left panel) and 50 meV (right panel). The DM lifetime is taken as $\tau_{DM} = 10^9$ and the detector energy resolution is assumed to be $\Delta = 10$ meV, 50 meV, and 100 meV from top to bottom.
Figure 5. Reach of PTOLEMY at $3\sigma$ significance as a function of progenitor mass $m_{DM}$ and lifetime in units of the age of the Universe, $\tau_{DM}/t_0$. An exposure of 100 g yr (blue lines) and 500 g yr (green lines) has been assumed for various projected performances on the electron energy resolution $\Delta$ as labeled, with 10 eV (100 eV) being the optimal (most conservative) case. All of DM is assumed to be decaying $\kappa_{DM} = 1$. In the left (right) panel the mass of the lightest neutrino is $m_{\nu_1} = 50$ meV ($m_{\nu_3} = 50$ meV) and a normal (inverted) hierarchy is assumed.

Figure 6. Reach of PTOLEMY at $3\sigma$ significance to the decaying fraction of DM, $\kappa_{DM}$, as a function of DM mass. The mass of the lightest neutrino is $m_{\nu_1} = 50$ meV ($m_{\nu_3} = 50$ meV); a normal (inverted) hierarchy is assumed.
References

[1] V.A. Rubakov and D.S. Gorbunov, *Introduction to the Theory of the Early Universe: Hot big bang theory*, World Scientific, Singapore, (2017).

[2] E. Vitagliano, I. Tamborra and G. Raffelt, *Grand Unified Neutrino Spectrum at Earth: Sources and Spectral Components*, Rev. Mod. Phys. 92 (2020) 045006 [arXiv:1910.11878] [inSPIRE].

[3] L. Stodolsky, *Speculations on Detection of the Neutrino Sea*, Phys. Rev. Lett. 34 (1975) 110 [Erratum ibid. 34 (1975) 508] [inSPIRE].

[4] B.F. Shvartsman, V.B. Braginsky, S.S. Gershtein, Y.B. Zeldovich and M.Y. Khlopov, *Possibility of detecting relict massive neutrinos*, JETP Lett. 36 (1982) 277 [Pisma Zh. Eksp. Teor. Fiz. 36 (1982) 224] [inSPIRE].

[5] P. Langacker, J.P. Leveille and J. Sheiman, *On the Detection of Cosmological Neutrinos by Coherent Scattering*, Phys. Rev. D 27 (1983) 1228 [inSPIRE].

[6] G. Duda, G. Gelmini and S. Nussinov, *Expected signals in relic neutrino detectors*, Phys. Rev. D 64 (2001) 122001 [hep-ph/0107027] [inSPIRE].

[7] V. Domcke and M. Spinrath, *Detection prospects for the Cosmic Neutrino Background using laser interferometers*, JCAP 06 (2017) 055 [arXiv:1703.08629] [inSPIRE].

[8] J.m. Irvine and R. Humphreys, *Neutrino masses and the cosmic neutrino background*, J. Phys. G 9 (1983) 847 [inSPIRE].

[9] A.G. Cocco, G. Mangano and M. Messina, *Probing low energy neutrinos with neutrino capture on beta decaying nuclei*, JCAP 06 (2007) 015 [hep-ph/0703075] [inSPIRE].

[10] S. Betts et al., *Development of a Relic Neutrino Detection Experiment at PTOLEMY: Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield*, in Community Summer Study 2013: Snowmass on the Mississippi, (2013) [arXiv:1307.4738] [inSPIRE].

[11] PTOLEMY collaboration, *PTOLEMY: A Proposal for Thermal Relic Detection of Massive Neutrinos and Directional Detection of MeV Dark Matter*, arXiv:1808.01892 [inSPIRE].

[12] KATRIN collaboration, *KATRIN: A next generation tritium beta decay experiment with sub-eV sensitivity for the electron neutrino mass. Letter of intent*, hep-ex/0109033 [inSPIRE].

[13] Y.F. Li and Z.-z. Xing, *Possible Capture of keV Sterile Neutrino Dark Matter on Radioactive $\beta$-decaying Nuclei*, Phys. Lett. B 695 (2011) 205 [arXiv:1009.5870] [inSPIRE].

[14] A.J. Long, C. Lunardini and E. Sabancilar, *Detecting non-relativistic cosmic neutrinos by capture on tritium: phenomenology and physics potential*, JCAP 08 (2014) 038 [arXiv:1405.7654] [inSPIRE].

[15] Y.F. Li, Z.-z. Xing and S. Luo, *Direct Detection of the Cosmic Neutrino Background Including Light Sterile Neutrinos*, Phys. Lett. B 692 (2010) 261 [arXiv:1007.0914] [inSPIRE].

[16] Y. Hochberg, Y. Kahn, M. Lisanti, C.G. Tully and K.M. Zurek, *Directional detection of dark matter with two-dimensional targets*, Phys. Lett. B 772 (2017) 239 [arXiv:1606.08849] [inSPIRE].

[17] J.A. Dror, G. Elor and R. Megehee, *Absorption of Fermionic Dark Matter by Nuclear Targets*, JHEP 02 (2020) 134 [arXiv:1908.10861] [inSPIRE].

[18] S. Palomares-Ruiz, *Model-Independent Bound on the Dark Matter Lifetime*, Phys. Lett. B 665 (2008) 50 [arXiv:0712.1937] [inSPIRE].

[19] S. Palomares-Ruiz and S. Pascoli, *Testing MeV dark matter with neutrino detectors*, Phys. Rev. D 77 (2008) 025025 [arXiv:0710.5420] [inSPIRE].
[20] C. Garcia-Cely and J. Heeck, Neutrino Lines from Majoron Dark Matter, *JHEP* 05 (2017) 102 [arXiv:1701.07209] [nSPIRE].

[21] Y. Cui, M. Pospelov and J. Pradler, Signatures of Dark Radiation in Neutrino and Dark Matter Detectors, *Phys. Rev. D* 97 (2018) 103004 [arXiv:1711.04531] [nSPIRE].

[22] M. Nikolic, S. Kulkarni and J. Pradler, The neutrino-floor in the presence of dark radiation, arXiv:2008.13557 [nSPIRE].

[23] D. McKeen, Cosmic neutrino background search experiments as decaying dark matter detectors, *Phys. Rev. D* 100 (2019) 015028 [arXiv:1812.08178] [nSPIRE].

[24] Z. Chacko, P. Du and M. Geller, Detecting a Secondary Cosmic Neutrino Background from Majoron Decays in Neutrino Capture Experiments, *Phys. Rev. D* 100 (2019) 015050 [arXiv:1812.11154] [nSPIRE].

[25] S. Tremaine and J.E. Gunn, Dynamical Role of Light Neutral Leptons in Cosmology, *Phys. Rev. Lett.* 42 (1979) 407 [nSPIRE].

[26] A. Boyarsky, O. Ruchayskiy and D. Iakubovskyi, A lower bound on the mass of Dark Matter particles, *JCAP* 03 (2009) 005 [arXiv:0808.3902] [nSPIRE].

[27] J. Alvey et al., New constraints on the mass of fermionic dark matter from dwarf spheroidal galaxies, *Mon. Not. Roy. Astron. Soc.* 501 (2021) 1188 [arXiv:2010.03572] [nSPIRE].

[28] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, Are There Real Goldstone Bosons Associated with Broken Lepton Number?, *Phys. Lett. B* 98 (1981) 265 [nSPIRE].

[29] A. Ringwald and Y.Y.Y. Wong, Gravitational clustering of relic neutrinos and implications for their detection, *JCAP* 12 (2004) 005 [hep-ph/0408241] [nSPIRE].

[30] K. Akita, S. Hurwitz and M. Yamaguchi, Precise Capture Rates of Cosmic Neutrinos and Their Implications on Cosmology, arXiv:2010.04454 [nSPIRE].

[31] *Planck* collaboration, Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* 641 (2020) A6 [arXiv:1807.06209] [nSPIRE].

[32] *PTOLEMY* collaboration, Neutrino physics with the PTOLEMY project: active neutrino properties and the light sterile case, *JCAP* 07 (2019) 047 [arXiv:1902.05508] [nSPIRE].

[33] V. Poulin, P.D. Serpico and J. Lesgourgues, A fresh look at linear cosmological constraints on a decaying dark matter component, *JCAP* 08 (2016) 036 [arXiv:1606.02073] [nSPIRE].

[34] M. Cirelli et al., PPPC 4 DM ID: A Poor Particle Physicist Cookbook for Dark Matter Indirect Detection, *JCAP* 03 (2011) 051 [Erratum ibid. 10 (2012) E01] [arXiv:1012.4515] [nSPIRE].

[35] P. Gondolo and J. Silk, Dark matter annihilation at the galactic center, *Phys. Rev. Lett.* 83 (1999) 1719 [astro-ph/9906391] [nSPIRE].

[36] P. Ullio, H. Zhao and M. Kamionkowski, A dark matter spike at the galactic center?, *Phys. Rev. D* 64 (2001) 043504 [astro-ph/0101481] [nSPIRE].

[37] A.A. Dutton and A.V. Macciò, Cold dark matter haloes in the Planck era: evolution of structural parameters for Einasto and NFW profiles, *Mon. Not. Roy. Astron. Soc.* 441 (2014) 3359 [arXiv:1402.7073] [nSPIRE].

[38] G. Dvali, S. Folkerts and A. Franca, How neutrino protects the axion, *Phys. Rev. D* 89 (2014) 105025 [arXiv:1312.7273] [nSPIRE].

[39] G. Dvali and L. Fucik, Small neutrino masses from gravitational θ-term, *Phys. Rev. D* 93 (2016) 113002 [arXiv:1602.03191] [nSPIRE].

[40] S.S. Wilks, The Large-Sample Distribution of the Likelihood Ratio for Testing Composite Hypotheses, *Annals Math. Statist.* 9 (1938) 60 [nSPIRE].
[41] J. Billard, L. Strigari and E. Figueroa-Feliciano, *Implication of neutrino backgrounds on the reach of next generation dark matter direct detection experiments*, Phys. Rev. D 89 (2014) 023524 [arXiv:1307.5458] [INSPIRE].

[42] DES collaboration, *Constraints on Decaying Dark Matter with DES-Y1 and external data*, arXiv:2011.04606 [INSPIRE].

[43] K. Enqvist, S. Nadathur, T. Sekiguchi and T. Takahashi, *Decaying dark matter and the tension in $\sigma_8$*, JCAP 09 (2015) 067 [arXiv:1505.05511] [INSPIRE].

[44] A. Nygaard, T. Tram and S. Hannestad, *Updated constraints on decaying cold dark matter*, arXiv:2011.01632 [INSPIRE].

[45] G. Raffelt, *Stars as laboratories for fundamental physics: The astrophysics of neutrinos, axions, and other weakly interacting particles*, Chicago University Press, Chicago, U.S.A. (1996) [INSPIRE].

[46] E. Vitagliano, J. Redondo and G. Raffelt, *Solar neutrino flux at keV energies*, JCAP 12 (2017) 010 [arXiv:1708.02248] [INSPIRE].

[47] K. Van Tilburg, *Stellar Basins of Gravitationally Bound Particles*, arXiv:2006.12431 [INSPIRE].