Design and Optimization of VoD schemes with Client Caching in Wireless Multicast Networks

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Abstract—Due to the explosive growth in multimedia traffic, the scalability of video-on-demand (VoD) services is becoming increasingly important. By exploiting the potential cache ability at the client side, the performance of VoD multicast delivery can be improved through video segment pre-caching. In this paper, we address the performance limits of client caching enabled VoD schemes in wireless multicast networks with asynchronous requests. Both reactive and proactive systems are investigated. Specifically, for the reactive system where videos are transmitted on demand, we propose a joint cache allocation and multicast delivery scheme to minimize the average bandwidth consumption under the zero-delay constraint. For the proactive system where videos are periodically broadcasted, a joint design of the cache-bandwidth allocation algorithm and the delivery mechanism is developed to minimize the average waiting time under the total bandwidth constraint. In addition to the full access pattern where clients view videos in their entirety, we further consider the access patterns with random endpoints, fixed-size intervals, and downloading demands, respectively. The impacts of different access patterns on the resource-allocation algorithm and the delivery mechanism are elaborated. Simulation results validate the accuracy of the analytical results and also provide useful insights in designing VoD networks with client caching.

Index Terms—Cache allocation, proactive delivery, reactive delivery, periodic broadcasting, video-on-demand (VoD).

I. INTRODUCTION

The rapid proliferation of smart devices has led to an unprecedented growth in internet traffic. According to Cisco’s most recent report [2], the traffic data of video-on-demand (VoD) services is forecast to grow at a compound annual growth rate of more than 60%. However, the traditional unicast-based delivery mechanism, where a server responds to each client individually, is unlikely to keep pace with the ever-increasing traffic demand. On the other hand, the traffic demand for videos, although massive and ever-increasing, is highly redundant, i.e., the same video is requested multiple times and a small number of videos account for a majority of all requests [3]. Therefore, a promising approach is to deliver these popular videos to multiple clients via multicast.

VoD multicast delivery has attracted significant interest recently. In industry, apart from the broadcasting networks, the evolved multimedia broadcast/multicast service (eMBMS) is introduced in the long term evolution (LTE) networks [4]. In academic, extensive studies have been conducted on the efficient multicast delivery for VoD services [5]–[8]. Among them, one aspect is to provide the reliable and efficient multicast delivery to clients with synchronous requests for the same videos, such as scalable video coding design and cooperative multicast mechanism [5], [6]. Another important issue is to design bandwidth efficient multicast delivery schemes to meet asynchronous requests at different times, including batching, patching, stream merging and periodic broadcasting [7], [8], which is the main focus of this paper.

In addition to the VoD multicast delivery, another important trend is that the cache capacity at client side is increasing rapidly and should be effectively exploited [9], [10]. Therefore, client storage could not only be used as a traditional short-term memory which temporarily buffers ongoing desired video segments at the client request times [11]–[13], but also serve as a long-term memory to pre-cache initial popular video segments ahead of client request times [14]. In this case, the bandwidth consumptions at the server and the network sides are greatly reduced, and also the average client waiting time can be highly saved [15]–[19]. In this paper, we will explore the optimal combination of client caching and multicast delivery for improving the scalability of VoD systems.

A. Related Work

In general, existing VoD multicast schemes for asynchronous requests fall into two transmission modes [20], i.e., reactive and proactive modes. Reactive mode implies that the delivery system is two-way in nature and there exists an uplink channel to report client requests. In this mode, videos are transmitted on demand [11]–[14]. Proactive mode means that the delivery system is only one-way and has no uplink channel to report client demands. In this mode, videos are periodically broadcasted with predefined carouse periods [21]–[23].

For the reactive system, various delivery schemes have been proposed in past decades, including but not limited to batching, patching and merging [11]–[15]. In batching, requests for the same video are delayed for a certain time so that more requests can be served concurrently within one multicast stream [11]. In patching, a client joins a desired ongoing multicast stream, and a unicast/multicast stream is established to patch the missing part [12]. In merging, a client could join several ongoing multicast streams and the patching streams of different clients are merged into one multicast stream [13]. Among these techniques, [13] proves the optimality of merging in terms of the minimum bandwidth requirement. Subsequently, [24] extends this technique to wireless channels based on erasure codes. For multi-video delivery, [25] and [26] propose hybrid transmission mechanisms where popular
videos are periodically broadcasted and less popular videos are served via either grace-patching or unicast. These studies \cite{11}–\cite{13}, \cite{24}–\cite{26} utilize the client storage, however, only for temporary buffering and the potential cache capacity at the client side is not fully exploited. By pre-caching initial video segments into the client storage, \cite{14} adopts the batching delivery mechanism and optimizes the cache allocation to minimize the energy consumption. By buffering an ongoing stream and receiving a multicast patching stream, \cite{15} further proposes the prepopulation assisted batching with multicast patching (PAB-MP). These studies \cite{14}, \cite{15} are based on either batching or patching, and the optimal cache allocation algorithm and the corresponding delivery mechanism for VoD services with client caching are still unknown.

For the proactive system, various periodic broadcasting schemes have been well studied, including skyscraper broadcasting \cite{21}, fast broadcasting \cite{22} and harmonic broadcasting \cite{23}. In all these schemes, videos are divided into a series of segments and each segment is broadcasted periodically on dedicated subchannels. Hu in \cite{27} firstly derives the theoretical lower bandwidth requirement bound for any periodic broadcasting protocols and proposes a greedy equal bandwidth broadcasting (GEBB) scheme that achieves the minimum fixed delay under the bandwidth constraint. Reference \cite{28} further applies the GEBB scheme with fountain codes to wireless systems. However, in \cite{27} and \cite{28}, the client storage is only utilized for temporary buffering and the potential cache capacity at the client side is not effectively exploited. By pre-caching initial video segments at the client side, \cite{16} and \cite{17} develop zero-delay delivery schemes based on polyharmonic broadcasting and GEBB, respectively. Reference \cite{29} investigates the cache-bandwidth allocation and the delivery mechanism for multi-video delivery in digital video broadcasting (DVB) systems. However, the proposed delivery mechanism is designed for video downloading instead of streaming, and the cache-bandwidth allocation is not jointly optimized.

\subsection*{B. Motivation and Contributions}

Despite the aforementioned studies, the following fundamental questions regarding VoD services in reactive and proactive systems with client caching remain unsolved to date.

\begin{itemize}
  \item \textbf{Q1: What are the optimal reactive and proactive multicast delivery mechanisms when the cache capacity at the client side can be exploited? Q2: What is the corresponding optimal resource (e.g., cache and bandwidth) allocation for multi-video delivery?}
  
  In addition, the aforementioned studies rest on the assumption of the full access pattern where clients watch the desired video from the beginning to the end. However, clients might be interested in video intervals rather than full-videos \cite{30}. References \cite{31} and \cite{32} consider interval access patterns with random intervals and fixed-size intervals for VoD services without client caching, respectively. To the best of our knowledge, the impacts of different access patterns on the resource allocation algorithm and the delivery mechanism for VoD services with client caching are also unknown.

In this paper, we attempt to answer the above key questions and provide the performance limits of VoD multicast schemes for both reactive and proactive systems where clients have certain cache capacity. Both full and interval access patterns are investigated. Our main contributions are as follows:

\begin{itemize}
  \item \textbf{Optimal joint cache allocation and multicast delivery scheme for the reactive system:} In Sec. \textbf{III} a joint cache allocation and multicast delivery scheme is developed to minimize the average bandwidth consumption of VoD services in the reactive system under the zero-delay constraint. We first propose a client caching enabled multicast patching (CCE-MP) mechanism which minimizes the average bandwidth consumption given a certain cache allocation. Then we formulate the cache allocation problem under the full access pattern into a convex problem, which can be effectively solved by a water-filling algorithm. This analysis provides a useful insight in choosing the minimum bandwidth-cache resource to meet a certain client request rate.

  \item \textbf{Optimal joint cache-bandwidth allocation and multicast delivery scheme for the proactive system:} In Sec. \textbf{IV} we jointly design the cache-bandwidth allocation algorithm and the multicast delivery mechanism to minimize the average client waiting time for the proactive system under the total bandwidth constraint. Firstly, we propose a client caching enabled GEBB (CCE-GEBB) delivery mechanism and show its optimality in term of the minimum client waiting time given a certain cache-bandwidth allocation. By exploring the structure of the optimal solution, we then simplify the cache-bandwidth allocation problem under the full access pattern to a one-dimensional search of the allocated cache size for the most popular video.

  \item \textbf{Impact of different client access patterns:} We investigate the impacts of different access patterns on the resource allocation algorithm and the multicast delivery mechanism in both reactive and proactive systems. In addition to the content popularity, the optimal scheme also depends on the client request rate and the access pattern in the reactive system. For instance, it is optimal to cache videos evenly for the full access pattern and the interval access pattern with random endpoints under relatively high request rates. Meanwhile, caching simply the most popular videos is optimal for the full access pattern under relatively low client request rates and the fixed-size interval access pattern under all request rates.

\end{itemize}

\subsection*{C. Organization}

The remainder of this paper is organized as follows. Section \textbf{II} introduces the system model of the VoD delivery network with client caching. Sections \textbf{III} and \textbf{IV} present the optimal resource allocation and multicast delivery schemes in reactive and proactive systems, respectively. Simulation results are shown in Section \textbf{V} and we conclude in Section \textbf{VI}.

\section*{II. SYSTEM MODEL}

Fig.\textbf{I} shows the simplified logical architecture of the VoD multicast delivery network, which includes a server module, a network module and a client module.
of the videos is denoted by $\{\text{length, bitrate, popularity}\}$, where the popularity is defined as the video access probability. Note that we consider CBR videos in this paper for simplicity, same as [14, 15, 18]. Also, as indicated in [33], the videos in YouTube are encoded into CBR. Moreover, the analysis in this paper can be extended to variable bit rate (VBR) videos since a VBR video can be regarded as a collection of different CBR video chunks and regulated into CBR streams [34].

Denote the length and the bitrate of $V_i$ as $L_i$ and $r_i$, respectively. Since the main purpose of this paper is to reveal the relationship between resource allocation and the content popularity, and also the relationship between length (or bitrate) and popularity of video has not been explicitly discovered so far [18], we assume that all the videos are of equal length $L$ and bitrate $r$ for simplicity, i.e., $L_1 = \ldots = L_M = L$ and $r_1 = \ldots = r_M = r$. Note that the extension to the general case where videos are of different lengths and bitrates is quite straightforward. The popularity distribution vector of the videos is denoted by $\mathbf{p} = [p_1, \ldots, p_M]$, which is assumed to be known apriori (e.g., estimated via some learning procedure [35]), where $\sum_{i=1}^{M} p_i = 1$. In addition, we assume $p_1 \geq p_2 \ldots \geq p_M$, i.e., the popularity rank of $V_i$ is $i$.

### B. Network Module

The transport network is aimed to reliably multicast the desired videos from the server to clients under coverage, which can be guaranteed by forward error correction (FEC) codes at application and physical layers [24, 28]. The corresponding bandwidth efficiency is denoted by $f_B$ (bps/Hz). Depending on whether clients report their demands to the server or not, the network can be divided into the following two types:

1) **Reactive System**: The transport network is reactive delivery if it is a two-way transmission system, and videos are delivered in response to client requests.

2) **Proactive System**: The transport network is proactive delivery if it is one-way and clients have no uplink channel to report their demands. In this case, videos are broadcasted periodically at different carouse periods, e.g., videos with larger popularity are transmitted more frequently. Note that the proactive delivery can be regarded as a special case of the reactive delivery when the gathered request information is not exploited in the reactive system and the server simply broadcasts videos periodically.

### C. Client Module

Each client is equipped with a cache storage and a buffer space. The cache storage is used as a long-term memory to cache video segments ahead of client request times, while the buffer space is a short-term memory which temporarily buffers ongoing desired uncached data at the request times and the buffered data would be released right after consumed. We assume that all clients have the same cache size $C$, where $C \leq M \cdot L$.

The client request events are modeled as a Poisson process with parameter $\lambda_i$ and clients request videos according to the video popularity distribution $\mathbf{p}$. Since Poisson processes remain Poisson processes under merging and splitting, the request arrivals for $V_i$ also follow a Poisson process with $\lambda_i = p_i \cdot \lambda$. When a client demands a certain video, it first checks whether the desired video has been cached, and if so, the client would display the cached part locally while buffering the ongoing uncached part. For the uncached part, the client in the reactive system sends the request to the server and the server will transmit the desired part at a suitable time, while the client in the proactive system needs to wait for the scheduling of the desired part in the periodic broadcasting.

Moreover, different access patterns should be considered since clients might only view a part of a video, e.g., they lose interest and stop watching before the end of the video. In this paper, we consider the following two typical access patterns:

- **Full Access Pattern**: The client views the desired video entirely, i.e., from the beginning to the end.
- **Interval Access Pattern**: Client requests are for video intervals rather than full-videos, e.g., they watch a video from the same beginning but end the watching at random endpoints (i.e., the access pattern with random endpoints [31]), or they watch a video only for fixed-size intervals from random beginnings (i.e., the fixed-size access pattern [32]). In addition, clients may watch interesting video clips until the video is fully saved (i.e., the downloading-demand access pattern [29]). In this paper, we consider the interval access patterns with random endpoints and fixed-size intervals for the reactive system, while the access patterns with random endpoints and downloading demand are investigated for the proactive system.

### III. VoD Delivery in the Reactive System

In this section, we devise a joint cache allocation and multicast delivery scheme to minimize the average bandwidth consumption for VoD services under the zero-delay constraint.

1In addition to client caching, caching within the network (e.g., at the proxy or base stations (BSs)) is also a promising approach. In this paper, we only consider client caching, while BS caching is out of scope here. Note that although BSs with large cache storage can help to save the wired band and the transmission latency from the server to the BSs, it does not actually reduce the wireless link traffic from the BS to the client side, where the wireless capacity is the bottleneck.
Note that lower bandwidth consumption implies more clients can be supported and thus highly scalable. Both full and interval access patterns are investigated.

Since no multicast opportunity exists among different video demands, the bandwidth consumption of each video can be acquired individually. The bandwidth consumption minimization problem can be written in the following general form:

\[
\min_{l,S,d} \sum_{i=1}^{M} b_i(l_i, S_r, d)
\]

s.t. \[\sum_{i=1}^{M} l_i \leq C, \quad 0 \leq l_i \leq L, \forall i \in \{1, \ldots, M\}, \]

where \(b_i(l_i, S_r, d)\) is the average bandwidth consumption of the \(i\)-th video \(V_i\) with cache size \(l_i\) and reactive delivery mechanism \(S_r, d\) under the zero-delay constraint.

We first introduce the optimal multicast delivery mechanism, referred as the client caching enabled multicast patching (CCE-MP) mechanism, consists of the following two operations: a) the server multicasts every desired uncached part at the latest deadline (i.e., at the time of delivery); b) each client starts buffering the desired uncached data from any ongoing multicast stream right after the client request time.

**Proposition 1:** The optimal multicast delivery mechanism, referred as the client caching enabled multicast patching (CCE-MP) mechanism, consists of the following two operations: a) the server multicasts every desired uncached part at the latest deadline (i.e., at the time of delivery); b) each client starts buffering the desired uncached data from any ongoing multicast stream right after the client request time.

**Proof:** See Appendix A.

An example illustrating the basic operations of CCE-MP under the full access pattern is provided in Fig. 2 in the figure, clients A through E intend to view \(V_i\) entirely at different times, i.e., \(t_a\) through \(t_c\). The beginning part with length \(l_i\) has been pre-cached at the client side. For the first request at time \(t_a\) by client A, the server does not respond immediately since client A could enjoy the beginning part locally due to prefix cache. In this case, the latest time to schedule a patching stream \(s_a\) with transmission rate \(r\) should be \(t_c, a = t_a + l_i/r\), right after client A finishes local delivery, where \(r\) is also the slope of orange solid lines in Fig. 2. Meanwhile, client B whose request arrival time is within \([t_a, t_{a,b}]\) is also satisfied by the same multicast stream \(s_a\). When C clicks on the video at time \(t_a\), it immediately buffers the remaining part of the ongoing stream \(s_a\) from video position \(l_c = l_i + r(t_c - t_{a,b})\). Then the server schedules another patching stream \(s_s\) from video position \(l_s\) to position \(l_s\) at time \(t_{s,c}\), where \(t_{s,c} = t_c + l_s/r\) is the latest deadline for client C to receive that part. Similarly, the server schedules \(s_d, s_e\) in response to clients D and E.

### A. Full Access Pattern

For this pattern, we first derive the average bandwidth consumption of the CCE-MP mechanism given a certain cache allocation, and then obtain the optimal cache allocation.

1) **Average Bandwidth Consumption of CCE-MP:** Similar to the optimal merging without client caching in [13], the average bandwidth consumption of CCE-MP can be derived by splitting a video into arbitrary small portions and obtaining the average bandwidth consumption of each portion individually. As illustrated in Fig. 3, we take the transmission of a small portion \(dx\) at an arbitrary length offset \(x\) of the \(i\)-th video for example, where \(l_i \leq x \leq L\). Let \(t_i^s\) be the time interval between the previous transmission of \(dx\) and the following first video request. Let \(T_i^s\) denote the time interval between two successive transmissions of \(dx\). We then have \(T_i^s = t_i^s + x/r\) according to operation (a) in Proposition 1 since the transmission of \(dx\) is triggered by the first request and scheduled until the display reaches position \(x\). Meanwhile, the following requests can all be satisfied by the same transmission of \(dx\) according to operation (b). It can be verified that the transmission of \(dx\) follows a renewal process. Let \(S(t)\) denote the total data amount for delivering \(dx\) from time 0 to \(t\). Due to the property of the renewal process, the average bandwidth consumption for delivering \(dx\) is

\[
\hat{b}_i^F = \frac{1}{S(t)} \lim_{t \to \infty} \frac{\text{d}S(t)}{\text{d}t} = \frac{dx}{f_B(T_i^s)},
\]

where \(E(T_i^s)\) denotes the expectation of \(T_i^s\). Therefore, the average bandwidth consumption of \(V_i\) can be written as

\[
b_i^F = \int_{l_i}^{L} \frac{1}{f_B(x + x/r)} dx = \frac{r}{f_B} \left( \frac{L - l_i}{l_i + x} + 1 \right).
\]

**Remark 1:** When \(l_i = 0\), we have \(b_i^F = \frac{r}{f_B} \ln \left( \frac{L}{l_i} + 1 \right)\). In this case, CCE-MP reduces to the optimal merging without client caching in [13]. Compared to \(l_i = 0\), (4) indicates the following two benefits of client caching: a) local cache gain incurred by the pre-cached part; b) multicast gain by allowing the server to delay the delivery due to local cache and serve a batch of requests via a single multicast stream.

**Remark 2:** When \(l_i \gg l_c\), we have \(E(T_i^s) = \frac{x}{r}\) and \(b_i^F = \frac{1}{f_B} \ln \left( \frac{L}{l_i} + 1 \right)\). In this case, CCE-MP reduces to the proactive

![Image](76x637 to 86x666)

**Fig. 2.** Example of CCE-MP under the full access pattern.

![Image](Image 76x684 to 148x731)

**Fig. 3.** Illustration of the derivation of the average bandwidth consumption.
delivery mechanism which periodically broadcasts the video, i.e., portion $dx$ at offset $x$ is broadcasted every $\frac{1}{2}$ time units, where $l_i \leq x \leq L$. Therefore, $\frac{r}{f_B} \ln(\frac{L}{x})$ is also the minimum average bandwidth consumption of $V_i$ with cache size $l_i$ under the zero-delay constraint in the proactive system.

Let $b_{i,batch}$ denote the average bandwidth consumption of $V_i$ with cache size $l_i$ under the batching method. In the batching method [14], multiple client requests for the same video that arrive within a batching window (i.e., the local displaying period due to prefix cache) are grouped and served via a single multicast transmission. According to [14], we have

$$b_{i,batch} = \frac{r}{f_B} \frac{L - l_i}{l_i + \frac{r}{f_B}} + \frac{l_i}{2},$$

decreasing with the increase of cache size $l_i$. When $l_i = 0$, we have $b_{i,batch} = \lambda_i L / f_B$ and the batching method reduces to serving each client request via unicasting. We have the following lemma.

**Lemma 1**: Compared to the batching method, the bandwidth saving of CCE-MP becomes smaller with larger cache size, i.e., $b_{i,batch} - b_{i,cache}^{FA}$ decreases with increasing cache size $l_i$.

**Proof**: See Appendix B.

2) **Cache Allocation**: The cache allocation problem is

$$\min_i \sum_{i=1}^{M} \frac{r}{f_B} \ln \left( \frac{L - l_i}{l_i + \frac{r}{f_B}} + 1 \right)$$

s.t. **(2)**, **(3)**.

**Lemma 2**: (Water-filling Algorithm) The optimal cache allocation is

$$l_i = \min \left( \beta r - \frac{1}{\lambda_i}, L \right), \quad \text{for } i \in \{1, 2, \ldots, M\},$$

where $x^+ = \max(x, 0)$ and $\beta$ can be effectively solved by the bisection method under the storage constraint.

**Proof**: See Appendix C.

As illustrated in Fig. 4, the water-filling algorithm allocates larger cache sizes to videos with larger popularity. For instance, the videos with larger popularity below the water level are cached to reach either the entire video length $L$ (i.e., $V_1$ and $V_2$) or the water level (i.e., $V_3$ to $V_6$). Meanwhile, the videos with smaller popularity above the water level, i.e., $V_7$ to $V_{10}$, have no cache allocated.

### B. Interval Access Pattern with Random Endpoints

Here we consider the interval access pattern where clients watch a video from the same beginning to random endpoints. For simplicity, the endpoints are uniformly distributed.

1) **Average Bandwidth Consumption of CCE-MP**: Due to the uniform distribution of the endpoints, the probability that a client finishes watching the video before portion $dx$ at offset $x$ is $x/L$. Hence the client request rate for $dx$ of $V_i$ is also a Poisson process with parameter $\frac{(L-x)L_i}{L}$, yielding $E(T_i^x) = \frac{L}{(L-x)L_i} + \frac{x}{r}$. Let $\eta_i = \frac{L}{L-x} + \frac{x}{2}$, the minimum average bandwidth consumption of $V_i$ with cache size $l_i$ is

$$b_{i,RE} = \frac{1}{f_B} \int \frac{l_i}{(L-x)L_i} + \frac{x}{r} \, dx = \left( \frac{r}{2f_B} + \frac{L}{4\eta_i f_B} \right) \left( \ln \left( \frac{L}{2} + \eta_i \right) - \ln \left( l_i + \eta_i - \frac{L}{2} \right) \right) + \left( \frac{r}{2f_B} - \frac{Lr}{4\eta_i f_B} \right) \left( \ln \left( \eta_i - \frac{L}{2} \right) - \ln \left( \eta_i + L - l_i \right) \right).$$

**Remark 3**: When $\lambda_i \gg \frac{1}{\eta_i}$, we have $\eta_i = \frac{L}{2}$ and $b_{i,RE} = \frac{1}{f_B} \ln(\frac{L}{\eta_i}) = \frac{L}{2}$. In this case, CCE-MP also reduces to the proactive delivery mechanism which broadcasts the video periodically, and the bandwidth consumption is the same as that of the full access pattern.

2) **Cache Allocation**: Since $\frac{\partial^2 b_{i,RE}}{\partial l_i^2} = \frac{r}{f_B (rL + L_i L)} \left( -2 \frac{L}{(rL + L_i L)^2} \right) \geq 0$, the cache allocation problem under this pattern is also convex, yielding the following lemma.

**Lemma 3**: The optimal cache allocation under the interval access pattern with uniformly distributed endpoints is

$$l_i = \min \left( \frac{\beta r + L}{2} \left( -\frac{(rL - \frac{L}{2})^2}{4\lambda_i} + \frac{Lr}{\lambda_i} \right), L \right)$$

for $i \in \{1, \ldots, M\}$, where $\beta$ can be effectively obtained by the bisection method.

**Proof**: The proof is similar to **Lemma 2**.

### C. Interval Access Pattern with Fixed-size Intervals

In addition to the interval access pattern with random endpoints, we also consider the fixed-size interval access pattern with random beginnings proposed in [12] i.e., each request is for a segment of duration $D$ starting from a random point, and videos are cyclic, which means access may proceed past the end of a video by cycling to the beginning of it.

1) **Average Bandwidth Consumption of CCE-MP**: The average bandwidth consumption under the fixed-size interval access pattern is derived based on the following proposition.

**Proposition 2**: (Ref. [12], Sec. 3.1) The mean interval for delivering $dx$ of the $i$-th video is

$$E[T_i^x] = \sqrt{\frac{\pi L}{2r\lambda_i}} \text{erf} \left( \sqrt{\frac{r\lambda_i}{2L}} \right) + \frac{L}{Dr\lambda_i} \exp \left( -\frac{D^2 r\lambda_i}{2L} \right),$$

where $\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t \exp(-s^2) \, ds$ denotes the error function.

2) Since the general case with random size intervals is too complex to be analyzed [12], we select the fixed-size interval access pattern proposed in [12] to reveal the cache allocation for the access pattern with random start points.
Note that $E[T^x_i]$ is irrelevant to video position $x$ and we can drop the upper index of $T^x_i$ as $T_i$. This is due to the fact that all parts of the video are of equal importance for the fixed-size interval access with cyclic display. We then have

$$p^fS_i = \int_{l_i}^{L} \frac{dx}{fBE(T_i)} = \frac{L - l_i}{fBE(T_i)}$$  \hspace{1cm} (11)$$

2) **Cache Allocation**: The cache allocation problem is

$$\min \sum_{i=1}^{M} \frac{L - l_i}{fBE(T_i)}$$  \hspace{1cm} (12)

s.t. \hspace{0.5cm} (2), (3)

and we have the following lemma.

**Lemma 4**: The optimal cache allocation under the fixed-size interval access pattern is

$$l_i = \begin{cases} 
L & \text{if } 1 \leq i \leq k - 1 \\
C \mod L & \text{if } i = k \\
0 & \text{if } k + 1 \leq i \leq M
\end{cases}$$  \hspace{1cm} (13)

where $k = \lceil C/L \rceil + 1$.

**Proof**: Cache allocation problem (12) is equivalent to the problem

$$\max \sum_{i=1}^{M} \frac{l_i}{fBE(T_i)}$$  \hspace{1cm} (14)

s.t. \hspace{0.5cm} (2), (3)

which belongs to fractional knapsack problems, where the knapsack capacity is $C$ and the value of caching the unit size of $V_i$ would be $\frac{1}{fBE(T_i)}$. The optimal solution for the fractional knapsack problem is the greedy algorithm, which chooses the videos with the highest $\frac{1}{fBE(T_i)}$ values and caches them up to the full length $L$ until the knapsack capacity $C$ is used up [36, 37]. Since $E(T_i)$ decreases with larger $\lambda_i$ and the videos are ranked in the descending order of the popularity, the greedy algorithm then reduces to cache the most popular videos up to the full length $L$ until the cache storage capacity $C$ is used up, i.e., the optimal cache allocation is Eq. (13). Thus the proof is completed.

**Remark 4**: The optimal cache allocation here is independent of the total client request rate. Therefore, under any request rate, the optimal cache allocation algorithm for the fixed-size interval access pattern is to cache the most popular videos only, termed as **Popular-Cache**.

### D. Extreme Case Analyses

Two special cases of client request rates are investigated to provide further insight into the impact of different access patterns on the cache allocation algorithms.

1) $\lambda \rightarrow 0$: When $\lambda$ is relatively small, no multicast opportunity exists even among client requests for the same video. In this case, the server satisfies each request via unicast.

- **Full access pattern**: The average bandwidth consumption of $V_i$ under the unicast-based transmission is $\lambda_i(L - l_i)/fB$. The cache allocation problem becomes

$$\min \sum_{i=1}^{M} \frac{\lambda_i(L - l_i)}{fB}$$  \hspace{1cm} (15)

s.t. \hspace{0.5cm} (2), (3),

which requires maximizing $\sum_{i=1}^{M} \lambda_i l_i / fB$, reducing to a fractional knapsack problem. Similar to problem (12), the optimal solution is **Popular-Cache**.

- **Interval access pattern with random endpoints**: The average bandwidth consumption of $V_i$ is $\frac{1}{fB} \int_{l_i}^{L} \frac{(L - l_i) \lambda_i}{fE(T_i)} dx = \frac{(l_i - L)^2 \lambda_i}{2lf_fB}$, and the cache allocation problem becomes

$$\min \sum_{i=1}^{M} \frac{(l_i - L)^2 \lambda_i}{2lf_fB}$$  \hspace{1cm} (16)

s.t. \hspace{0.5cm} (2), (3),

which is a convex problem. The optimal solution is $l_i = (L - \frac{i}{M})^+$ for $i \in \{1, \ldots, M\}$ rather than Popular-Cache, where $\beta$ can be solved by the bisection method.

- **Fixed-size interval access pattern**: Popular-Cache is optimal under any client request rate.

2) $\lambda \rightarrow +\infty$: When $\lambda$ is relatively large, the cache allocation algorithms for different access patterns are as follows.

- **Full access pattern**: The average bandwidth consumption of $V_i$ becomes $\frac{1}{fB} \ln(l_i)$, and CCE-MP reduces to the proactive delivery mechanism which broadcasts the video periodically. We then have

$$\min \sum_{i=1}^{M} \frac{r}{fB} \ln \left( \frac{l_i}{l_i} \right)$$  \hspace{1cm} (17)

s.t. \hspace{0.5cm} (2), (3),

which is also a convex problem. The optimal solution is to evenly allocate the cache capacity among all videos, i.e., $l_1 = \ldots = l_M = \frac{C}{M}$. We term it as **Even-Cache**. The total bandwidth consumption in this case is $\frac{M}{fB} \ln \left( \frac{M}{C} \right)$, served as the “upper bound” of the optimal scheme under any client request rate.

- **Interval access pattern with random endpoints**: The bandwidth consumption of $V_i$ is $\frac{r}{fB} \ln(l_i)$, the same as that of the full access pattern. Therefore, Even-Cache is optimal.

- **Fixed-size interval access pattern**: Based on Lemma 4 the optimal cache allocation for this pattern is Popular-Cache under any client request rate. Therefore, different from the bandwidth patterns with the same beginning where Even-Cache is optimal, Popular-Cache is optimal for this pattern when request rate is relatively large. Note that when request rate becomes relatively large, the bandwidth consumption of $V_i$ under both the full access pattern and the interval access pattern with random endpoints become the same constant value $\frac{r}{fB} \ln(l_i)$ eventually, yielding the Even-Cache allocation among all videos. However, this is not the case for the fixed-size interval access pattern. When $\lambda$ becomes relatively large, we have $E(T_i) = \sqrt{\frac{2M}{\lambda_i}}$. 

```python
def even_cache_allocation(l, M, C):
    return C / M

def popular_cache_allocation(l, M, C, l_max):
    return l

def even_cache_upper_bound(M, C):
    return (M / fB) * ln(M / C)

```

```python
import math

# Example usage
M = 10
C = 100
l_i = [i for i in range(1, M + 1)]

l_even = [even_cache_allocation(l_i, M, C) for l_i in l_i]
l_popular = [popular_cache_allocation(l_i, M, C, l_max) for l_i in l_i]
l_upper_bound = [even_cache_upper_bound(M, C) for l_i in l_i]

```
based on Eq. (10), and the bandwidth consumption of $V_i$ is $b_i^d = \frac{l_i - l_i^r}{f_{\text{nn}}} \sqrt{\frac{2d_i}{n_i}}$, still increasing with larger $d_i$ value. Meanwhile, a larger popularity for a video implies a larger request rate. Therefore, a video with larger popularity should have larger cache storage size. Since it belongs to a fractional knapsack problem according to the proof part of Lemma 4, Popular-Cache is optimal here.

IV. VoD Delivery in the Proactive System

Instead of minimizing the bandwidth consumption of VoD services under the zero-delay constraint for the proactive system, we jointly design the cache-bandwidth allocation and the multicast delivery to minimize the average client waiting time under the total bandwidth constraint. Note that for limited bandwidth-cache resource, the waiting time performance might not be guaranteed for each client. In this case, the typical performance metric is to minimize the blocking probability if client requests are blocked when their waiting times exceed their waiting tolerance, or to minimize the average waiting time. In this section, same as [18], [25], [29], we focus on the average waiting time minimization problem. Both full and interval access patterns are considered.

Let $B$ denote the total bandwidth and $b = [b_1, \ldots, b_M]$ denote the bandwidth allocation of each video, where $b_i$ is the allocated bandwidth for broadcasting the $i$-th video periodically. The bandwidth constraint can be then written as $\sum_{i=1}^{M} b_i \leq B$. Let $S_{pd}$ and $d_i(b_i, l_i, S_{pd})$ denote the adopted proactive delivery mechanism and the corresponding waiting time for the $i$-th video with allocated bandwidth $b_i$ and cache size $l_i$, respectively. The average waiting time minimization problem can be then written in the following general form:

$$\min_{b, l, S_{pd}} \sum_{i=1}^{M} p_i d_i(b_i, l_i, S_{pd})$$

s.t. \[
\left\{ \begin{array}{l}
\sum_{i=1}^{M} b_i \leq B, \sum_{i=1}^{M} l_i \leq C, \\
b_i \geq 0, 0 \leq l_i \leq L, \forall i \in \{1, \ldots, M\}.
\end{array} \right.
\]

To minimize the average client waiting time, we first introduce the optimal delivery mechanism given a certain cache-bandwidth allocation, and then develop the corresponding optimal cache-bandwidth allocation algorithm. To begin with, we consider the traditional full access pattern.

A. Full Access Pattern

For this pattern, the greedy equal bandwidth broadcasting (GEBB) mechanism is optimal in the proactive system without client caching [27]. In GEBB, the bandwidth for a certain video is equally divided into several subchannels and the video is also divided into different segments. Within each

This problem reduces to Problem 17 with the minimum bandwidth consumption $\frac{M b_i}{\max(\frac{M b_i}{L}, l_i)}$. In this case, the cache capacity is evenly allocated among all videos, and portion of data at offset $x$ is broadcasted every $\frac{1}{r}$ time units, where $0 \leq x \leq L$.

As for the blocking probability minimization problem in the proactive system, the result is trivial, i.e., it is optimal to evenly allocate cache and bandwidth among the most popular videos such that the waiting times for these videos just reach the tolerance.

\begin{align*}
\text{Fig. 5. Illustration of the CCE-GEBB delivery mechanism for } V_i \text{ with allocated cache size } l_i \text{ and broadcast bandwidth } b_i, \text{ where the grey blocks are the buffered data after the client request time.}
\end{align*}
When $e^{\frac{L}{r_i}b_i}l_i = L$, the first segment $S_i$ has been buffered right after the end of local cache is displayed (i.e., $w_i = l_i/r_i$), yielding zero-delay for the $i$-th video. Note that there is no need to waste extra cache-bandwidth resource achieving $e^{\frac{L}{r_i}b_i}l_i > L$ while maintaining the same viewing experience, then we have

$$d_i^{FA} = \frac{L - e^{\frac{L}{r_i}b_i}l_i}{r(e^{\frac{L}{r_i}b_i} - 1)} \quad \text{s.t.} \quad e^{\frac{L}{r_i}b_i}l_i \leq L. \quad (25)$$

**Proposition 3:** CCE-GEBB is the optimal proactive delivery mechanism to minimize the waiting time for the $i$-th video with bandwidth $b_i$ and cache size $l_i$.

**Proof:** See Appendix D.

2) **Cache-bandwidth Allocation:** Given the developed CCE-GEBB delivery mechanism, the cache-bandwidth allocation problem becomes

$$\min_{b, l} \sum_{i=1}^{M} p_i \left( \frac{L - e^{\frac{L}{r_i}b_i}l_i}{r(e^{\frac{L}{r_i}b_i} - 1)} \right) \quad \text{s.t.} \quad \sum_{i=1}^{M} b_i \leq B, \sum_{i=1}^{M} l_i \leq C, \quad b_i \geq 0, 0 \leq l_i \leq L e^{\frac{L}{r_i}b_i}, \forall i \in \{1, \ldots, M\}. \quad (26)$$

We first introduce the following proposition about the structure of the optimal solution.

**Proposition 4:** If $V_k$ has non-zero cache allocated and experiences non-zero delay, i.e., $0 < l_k < L e^{\frac{L}{r_i}b_i}$, then

a) $V_1$ to $V_k$ with larger popularity experience zero-delay while the remaining $M - k$ videos with smaller popularity have non-zero delay and no cache storage allocated.

b) For the first $k - 1$ videos with zero-delay, the storage and bandwidth are evenly allocated, i.e., $b_1 = \ldots = b_{k-1}$ and $l_1 = \ldots = l_{k-1}$.

**Proof:** See Appendix E.

Based on the structure of the optimal allocation in **Proposition 4**, we then have the following lemma to solve the cache-bandwidth allocation problem.

**Lemma 5:** For the full access pattern, the cache-bandwidth allocation problem of $2M$ variables can be simplified to a one-dimensional search of the first cache size $l_1$

$$(l_1^*, b_1^*) = \arg \max_{l_1} \phi(l_1),$$

where $l_1 \in [L/M, \min(L/C)]$ and $\phi(l_1)$ is the average waiting time in terms of $l_1$ when the cache allocation is

$$l_i = \begin{cases} \left\lfloor \frac{C}{i} \right\rfloor + 1 & \text{if } 0 \leq i \leq k - 1, \\ C \mod l & \text{if } i = k, \\ 0 & \text{if } k + 1 \leq i \leq M, \end{cases} \quad \text{if } l_1 < C,$$

in which $k = \left\lfloor \frac{L}{C} \right\rfloor + 1$ is the threshold video number which has cache storage allocated.

If $l_1 < C$, the bandwidth allocation becomes

$$b_i = \begin{cases} \frac{L_l}{r} \ln \frac{L_l}{l_i} & \text{if } 1 \leq i \leq k - 1, \\ \frac{L_l}{r} \ln \left( \frac{2p_i\beta L(l_i)}{\sqrt{\pi p_i\beta L(l_i)^2 + 4p_i\beta (L - l_i)}} \right) & \text{if } k \leq i \leq M, \end{cases}$$

where $\beta$ meets the bandwidth constraint $\sum_{i=1}^{M} b_i = B$.

If $l_1 = C \leq L$, the cache capacity is allocated to the first video and we have, for $i = 1, 2, \ldots, M$,

$$b_i = \frac{r}{r} \ln \left( \frac{2 + p_i \beta (L - l_i) + \sqrt{p_i \beta (L - l_i)^2 + 4p_i \beta (L - l_i)}}{2} \right) \quad (31)$$

**Proof:** See Appendix F.

If and only if the total bandwidth meets $B \geq \frac{M}{ln(L)}$, zero-delay can be achieved for all videos, which is consistent with the extreme case analysis in Sec. III-D.

B. Interval Access Pattern with Random Endpoints

For the interval access pattern with uniformly distributed endpoints, we first introduce the proactive delivery mechanism and then derive the optimal cache-bandwidth allocation.

1) **CCE-GEBB Delivery Mechanism:** Clients who finish the watching of the $i$-th video before position $l_i$ experience zero-delay, and the corresponding probability is $l_i/L$ due to the uniform distribution of endpoints. For the remaining clients interested in the uncached part, the waiting time is $d_i^{FA}$ as indicated in Fig. [5]. Since CCE-GEBB minimizes $d_i^{FA}$, it also achieves the minimum waiting time under this access pattern. Based on CCE-GEBB, the average waiting time of $V_i$ is

$$d_i^{RE} = \frac{L - l_i}{L} - d_i^{FA} = \frac{L - l_i}{L} \left( \frac{L - l_i}{L(r(e^{\frac{L}{r_i}b_i} - 1))} - \frac{l_i}{r} \right). \quad (32)$$

2) **Cache-bandwidth Allocation:** Let $\chi_i = 1/e^{\frac{L}{r_i}b_i}$ for $i \in \{1, \ldots, M\}$, and note that the cache-bandwidth allocation problem can be rewritten as

$$\min_{x, l} \sum_{i=1}^{M} p_i \left( L - l_i \right) \frac{L - x_i}{rL} \left( \frac{L - l_i}{x_i - 1} - L \right) \quad (33)$$

s.t.

$$\sum_{i=1}^{M} \ln \frac{L_l}{l_i} \leq B \ln \frac{L_l}{l_i}, \sum_{i=1}^{M} l_i \leq C, \quad 0 < x_i \leq 1, 0 \leq l_i \leq L x_i, \forall i \in \{1, \ldots, M\}.$$ 

**Proposition 5:** Problem (33) is convex.

**Proof:** The hessian matrix of $d_i^{RE}$ becomes

$$H \left[ \frac{\partial^2 d_i^{RE}}{\partial x_i \partial x_j} \quad \frac{\partial^2 d_i^{RE}}{\partial x_i \partial l_i} \quad \frac{\partial^2 d_i^{RE}}{\partial l_i \partial l_i} \right] = \frac{2}{rL(1 - x_i)} \left[ \frac{L - l_i}{x_i - 1} \quad \frac{L - l_i}{x_i - 1} \quad \frac{l_i}{x_i - 1} \right] \geq 0,$$

thus $d_i^{RE}$ is convex in $x_i$ and $l_i$, and the objective function is convex along with convex constraints. Hence, the problem is convex and can be solved by the interior-point method.

Similar to **Proposition 4**, we also have the following statement for this access pattern.

**Proposition 6:** If $V_k$ experiences zero-delay, then

a) $V_1$ to $V_{k-1}$ experience zero-delay.

b) For the videos with zero-delay, the cache size and the bandwidth are evenly allocated.
C. Access Pattern with Downloading Demand

In this subsection, we consider the downloading-demand access pattern where each client selectively watches interested video clips until the desired video is fully saved. In this case, the client waiting time reduces to the downloading time.

1) CCE-GEBB Delivery Mechanism: Under this access pattern, no part in a video has a higher timing priority than others in the transmission and each bit of the video should be sent at the same frequency. In this case, the total number of subchannels in CCE-GEBB becomes 1 (i.e., \( n = 1 \) in Fig. 5), and CCE-GEBB reduces to the traditional broadcast carousel where the uncached data of \( V_i \) is cyclically transmitted via one subchannel with bandwidth \( b_i \). The downloading time of \( V_i \) with bandwidth \( b_i \) and cache size \( l_i \) is given by

\[
d_{i}^{DD} = \frac{L - l_i}{f_b b_i}
\]

2) Cache-bandwidth Allocation: The resource allocation problem becomes

\[
\min_{b, l} \sum_{i=1}^{M} p_i \frac{L - l_i}{f_b b_i}
\]

s.t. \[\text{(19)}\].

And we have the following proposition.

**Proposition 7:** The optimal cache allocation for this access pattern is Popular-Cache, i.e.,

\[
l_i = \begin{cases} 
L & \text{if } i \leq k, \\
C \mod L & \text{if } i = k, \\
0 & \text{if } i > k,
\end{cases}
\]

where \( k = [C/L] + 1 \). The corresponding bandwidth allocation is

\[
b_i = \frac{B \sqrt{p_i (L - l_i)}}{M} \sum_{j=1}^{M} p_j (L - l_j), \quad \text{for } i = 1, 2, \ldots, M.
\]

**Proof:** See Appendix G.

V. PERFORMANCE EVALUATION

In this section, simulations are provided to validate the performance gain of the proposed schemes in both reactive and proactive systems. The default parameter settings are shown in Table I.

| Parameter | Description | Value |
|-----------|-------------|-------|
| \( M \)  | Number of videos | 200   |
| \( \alpha \) | Zipf parameter | 0.8   |
| \( L \)  | Video length  | 150 MB (10 minutes) |
| \( r \)  | Video bitrate | 2 Mbps |
| \( f_b \) | Bandwidth efficiency | 4 bps/Hz |
| \( C \)  | Cache size    | 3000 MB |
| \( A \)  | Client request rate | 0.5 s⁻¹ |

Table I: Default parameter settings

The downloading-demand access pattern is not studied for the reactive system in Sec. III since the considered zero-delay constraint is not practical for this pattern. Instead, this pattern can be investigated for the reactive system given a maximum downloading time constraint.

Fig. 6. Impact of request rate \( \lambda \) on the average bandwidth consumptions under the full access pattern.

Table II In our simulation, the number of videos in the library is taken as 200. Each video is of bitrate 2 Mbps and duration 10 minutes [13]. The popularity of each video is distributed according to a Zipf law of parameter \( \alpha \) [8], where \( \alpha \) governs the skewness of the popularity. The popularity is uniform over videos for \( \alpha = 0 \), and becomes more skewed as \( \alpha \) grows. We select \( \alpha = 0.8 \) as the default value [15], where 47% client requests concentrate on the 10% popular videos. The client cache size is 3000 MB (2.93 GB), which is reasonable for smart devices with increasing cache storage size (e.g., 16 GB).

Table II illustrates the evaluated schemes adopted in the simulation. In Batch [14], multiple client requests for the same video that arrive within a batching window (i.e., the local displaying period due to prefix cache) are grouped and served via a single multicast transmission. In the PAB-MP scheme [15], in addition to the prepopulation assisted batching, clients can join an ongoing multicast stream and multicast patching streams are scheduled to patch the missing parts.

A. Reactive System

For the reactive system, the impacts of the client request rate, the Zipf parameter, the cache size and the number of videos on the average bandwidth consumptions of different schemes are illustrated in Figs. 6, 7, 8 and 9 respectively. In addition, the impact of the access pattern is shown in Fig. 10.

**Impact of client request rate:** The impact of client request rate \( \lambda \) on the average bandwidth consumption under the full access pattern is presented in Fig. 6, where R-optimal (simu., 2 MB) and R-optimal (simu., 2 Mbit) stand for the practical case that video chunks are of 2 MB (75 chunks) and 2 Mbit (600 chunks), respectively, rather than the arbitrary small size in R-optimal (theo.). The smaller the video chunk size, the smaller the performance degregation compared to R-optimal (theo.). Note that the simulation result of R-optimal (simu., 2 Mbit) achieves nearly the same performance with R-optimal (theo.), hence the chunk video size 2 Mbit is adopted in the following simulations for the reactive system. In addition, the performances of Batch, PAB-MP, R-popularCache and the
TABLE II
ILLUSTRATION OF DIFFERENT SCHEMES

| Scheme      | System | Delivery Mechanism | Cache Allocation | Bandwidth Allocation |
|-------------|--------|--------------------|------------------|----------------------|
| R-optimal   | reactive | CCE-MP            | optimal          | —                    |
| R-popularCache | reactive | CCE-MP          | Popular-Cache    | —                    |
| R-evenCache | reactive | CCE-MP            | Even-Cache       | —                    |
| Batch [14]  | reactive | Batch             | [14]             | —                    |
| PAB-MP [15] | reactive | PAB-MP            | [15]             | —                    |
| P-optimal   | proactive | CCE-GEBB         | optimal          | optimal              |
| P-popularCache | proactive | CCE-GEBB     | Popular-Cache    | optimal              |
| P-evenCache | proactive | CCE-GEBB         | Even-Cache       | optimal              |
| P-even      | proactive | CCE-GEBB         | Even-Cache       | evenly allocated     |
| P-noStorage | proactive | GEBB             | —                | optimal              |

proposed R-optimal are nearly the same under relatively low request rates (e.g., $\lambda = 0.01$). Since there is little chance to merge multiple client requests at that low request rate, the server responds to almost all client requests via unicast. In this case, it is optimal to simply cache the most popular videos. As $\lambda$ increases, by buffering one ongoing stream and later receiving a corresponding multicast patching stream, PAB-MP outperforms Batch where clients join no ongoing streams. However, both Batch and PAB-MP suffer significant performance losses compared to R-optimal since R-optimal utilizes every desired part of ongoing streams. For instance, up to 47% (or 28%) bandwidth saving can be achieved by R-optimal compared to Batch (or PAB-MP) at $\lambda = 2$ (or 7), and 223% (or 62%) more requests can be supported by R-optimal compared to Batch (or PAB-MP) at bandwidth consumption 160 MHz. Moreover, R-evenCache has almost the same performance with R-optimal when $\lambda \geq 7$, which coincides with the extreme case analysis in Sec. III-D, i.e., it is optimal to evenly allocate the cache capacity among all videos under relatively high request rates.

Impact of Zipf parameter: Fig. 7 illustrates the average bandwidth consumptions of various schemes vs. $\alpha$ under the full access pattern. When $\alpha = 0$, the popularity is uniformly distributed and R-optimal reduces to R-evenCache. As $\alpha$ increases, more requests concentrate on the first few videos, resulting in less bandwidth consumption for all schemes. Note that R-evenCache, which employs CCE-MP, is even worse than Batch for $\alpha > 1.2$ since the adopted Even-Cache ignores the popularity property. In addition, R-popularCache performs nearly the same as R-optimal for that the first few cached videos dominate most requests for large $\alpha$.

Impact of cache size: As shown in Fig. 8 the average bandwidth consumptions of all schemes decrease with increasing cache size since a larger cache size provides larger local-cache and multicast gains. Note that R-popularCache outperforms R-evenCache under the settings $\lambda = 0.5$ (low request rate) and $\alpha = 0.8$ (highly skewed popularity). Compared with Batch, R-optimal saves 25% bandwidth consumption at the same cache size 0.1ML, and reduces 52% cache consumption while achieving the same bandwidth consumption. As the cache size increases, the performance gap between R-optimal and Batch becomes smaller, which coincides with Lemma 1. Moreover, R-optimal in Fig. 8 indicates the minimum cache-bandwidth resource required for supporting a certain client request rate, e.g., (0.1ML, 55 MHz) and (0.2ML, 44 MHz), which can be used as a guideline for VoD services with client caching.

Impact of the total number of videos: The impact of the number of videos on the bandwidth consumptions of
different schemes is illustrated in Fig. 9 where the Unicast-popularCache scheme caches only the most popular videos and serves the remaining video requests via unicasting. With the increase of the number of videos, the gap between Unicast-popularCache and Batch become relatively small. The reason is that the cache size and the video popularity for each video becomes smaller with larger $M$, and fewer client requests are batched for the videos with smaller cache size and smaller video popularity through a single transmission. In this case, the performance of Batch would reduce to that of Unicast-noStorage. However, PAB-MP and R-optimal still have notable bandwidth saving compared to Batch even with $M = 4000$, since both schemes utilizes the patching method to exploit the ongoing streams. When $M = 4000$, the cache storage can only cache 0.5% of the total videos. However, Unicast-popularCache still saves 21.6% bandwidth consumption compared to Unicast-noStorage. The reason is that the most popular 0.5 percent of total videos accounts for 21.6% of the total requests when $M = 4000$, showing the effectiveness of client caching.

Impact of different access patterns: Fig. 10 illustrates the average bandwidth consumptions of different access patterns vs. the client request rate, where “FA”, “RE” and “FS” represent the full access pattern, the interval access pattern with random endpoints and the fixed-size interval access pattern, respectively. Under relatively low client request rates (e.g., $\lambda = 0.1$), client requests could not be merged and the server responds by unicast. Therefore, R-popularCache is optimal under “FA” while it suffers a performance loss under “RE”. As $\lambda$ increases, the average bandwidth consumption of “FS” is even larger than that of “FA” since more multicast opportunities could be exploited by access patterns with the same beginning. Meanwhile, the performance gap of R-optimal between “FA” and “RE” becomes smaller, and both patterns approach the “upper bound” with increasing $\lambda$. Furthermore, R-popularCache is optimal under “FS” for all request rates. Therefore, the numerical results are consistent with the theoretical analyses in Sec. III.

B. Proactive system

For the proactive system, we aim to minimize the average client waiting time under the total bandwidth constraint. The impacts of the cache size, the Zipf parameter and the access pattern are described as follows, where the evaluated total bandwidth is 130 MHz.

Impact of cache size: The impact of the cache size on the average waiting times of different schemes under the full access pattern is illustrated in Fig. 11 where P-popularCache caches only the most popular videos while P-evenCache and P-even evenly cache the prefixes of all videos. As indicated in Table II the bandwidth in P-evenCache is optimally allocated given the Even-Cache allocation while it is still evenly allocated in P-even. “CCE-EBB, n=16” (or n=64) stands for the practical scenario where each video is transmitted over 16 (or 64) subchannels rather than infinite subchannels. Note that the more subchannels are allocated for each video, the less performance degradation is obtained compared to the infinite case, e.g., “CCE-EBB, n=64” achieves nearly
Optimal. In addition, P-evenCache and P-even perform worse than P-popularCache under small cache size settings (e.g., $C < 0.075ML$) while P-evenCache achieves nearly optimal under large cache size settings (e.g., $C \geq 0.25ML$). This is due to the fact that the bandwidth and the cache capacity are evenly allocated among videos with zero-delay, and a larger cache size yields more videos with zero-delay. Compared with P-popularCache (or R-evenCache), the proposed R-optimal reduces 59% (or 58%) average waiting time at the same cache size 0.2ML (or 0.1ML). Moreover, Fig. 14 provides useful insights in choosing the appropriate cache size to meet the average waiting time constraint, e.g., R-optimal with cache size 0.2ML saves 88% cache consumption compared to P-popularCache while meeting the same average waiting time constraint (i.e., 25 s). Similar results can also be expected for the interval access pattern with random endpoints.

Fig. 12 illustrates the impact of the cache size on the average waiting time under the downloading-demand access pattern. This pattern is CCE-GEBB with $n = 1$ (i.e., the traditional broadcast carousel), and the cache allocation is to cache the most popular videos only. When $C/(ML) = 0.4$, the average waiting time of the optimal scheme is still 67 seconds, while it is zero under the full access pattern in Fig. 11.

Impact of Zipf parameter: Fig. 13 illustrates the average waiting times of different access patterns vs. the Zipf parameter $\alpha$, where “FA”, “RE” and “DD” represent the full access pattern, the interval access pattern with random endpoints and the access pattern with downloading demand, respectively. As $\alpha$ increases, the performance gap between “FA” and “RE” becomes negligible since the optimal schemes under both access patterns aim to provide zero-delay for videos with larger popularity. Meanwhile, the performance gap between “DD” and other two access patterns becomes much smaller with increasing $\alpha$. This is due to the fact that Popular-Cache is optimal for “DD”, and most client requests concentrate on the popular contents already cached at client side for large $\alpha$.

Impact of different access patterns: The cache-bandwidth allocations under “FA”, “RE” and “DD” are illustrated in Figs. 14(a), 14(b) and 14(c), respectively. For popular videos with zero-delay under “FA” and “RE” (i.e., $V_1$ to $V_4$ in Fig. 14(a) and $V_5$ to $V_6$ in Fig. 14(b)), the cache and the bandwidth are evenly allocated. Meanwhile, the videos with zero-delay are entirely cached under “DD” (i.e., $V_1$ in Fig. 14(c)). For “FA”, at most one video (i.e., $V_5$ in Fig. 14(a)) with non-zero delay has cache allocated due to the greedy property of the solution for fractional knapsack problems, while several videos (i.e., $V_4$ to $V_6$ in Fig. 14(b)) with non-zero delay could have cache allocated under “RE”. In addition, Popular-Cache is optimal under “DD” in Fig. 14(c). Therefore, the numerical results are consistent with the theoretical analyses in Sec. IV.

VI. CONCLUSION

This paper has investigated the optimal joint resource allocation and multicast delivery schemes for VoD services in reactive and proactive systems with client caching. Both full and interval access patterns have been considered. For
the reactive system, we have developed a joint cache allocation and multicast delivery scheme to minimize the average bandwidth consumption under the zero-delay constraint. We observe that in addition to the video popularity, the cache allocation algorithm also relies on the client request rate and the access pattern. For the proactive system, we have jointly designed the cache-bandwidth allocation algorithm and the CCE-GEBB delivery mechanism to minimize the average waiting time under the total bandwidth constraint. Note that CCE-GEBB with infinite subchannels is optimal for both the full access pattern and the interval access pattern with random endpoints, and the cache capacity is evenly allocated among videos with zero-delay. Meanwhile, CCE-GEBB with only one subchannel is optimal for the access pattern with downloading demand, in which case the optimal cache allocation is to cache the most popular contents entirely. These results can be used as a guideline for the VoD network with client caching, e.g., the required minimum bandwidth-cache resource under a certain client request rate in the reactive system, or under a certain average waiting time constraint in the proactive system.

APPENDIX A: PROOF OF PROPOSITION

Assuming by contradiction that a part of a desired video is multicast before the time of display, then the average bandwidth consumption could be increased since the following requests for the same part before that time are not benefited by the transmission and more data needs to be sent. Also if a client does not start buffering the desired uncached data from an ongoing stream right after the request time, then the useful data in the ongoing stream might not be fully utilized, resulting in extra data transmission for that client. Therefore, CCE-MP is optimal to minimize the average bandwidth consumption given a certain cache allocation under any access pattern.

APPENDIX B: PROOF OF LEMMA

The bandwidth consumption of \( V_i \) under CCE-MP can be rewritten as

\[
b_i^{FA} = \frac{r}{f_B} \ln \left( \frac{L - l_i}{l_i + \frac{l_i}{f_B}} + 1 \right) = \frac{r}{f_B} \ln \left( \frac{f_B b_i^{batch}}{r} + 1 \right).\tag{38}
\]

We then have

\[
\frac{\partial b_i^{batch} - b_i^{FA}}{\partial b_i^{batch}} = \frac{\ln \left( \frac{f_B b_i^{batch}}{r} + 1 \right) - \frac{f_B b_i^{batch}}{r} b_i^{batch} \ln \left( \frac{f_B b_i^{batch}}{r} + 1 \right)}{(b_i^{batch})^2}.\tag{39}
\]

Denote \( g(t) = \ln(t + 1) - \frac{t}{t + 1} \) where \( t \geq 0 \), we then have \( g'(t) = \frac{1}{(t + 1)^2} \geq 0 \). Therefore, \( g(t) \geq g(0) = 0 \) and we have

\[
\frac{\partial b_i^{batch} - b_i^{FA}}{\partial b_i^{batch}} = \frac{r}{f_B} \frac{g\left( \frac{f_B b_i^{batch}}{r} \right)}{\left( \frac{f_B b_i^{batch}}{r} \right)^2} \geq 0.\tag{40}
\]

Therefore, \( \frac{b_i^{batch} - b_i^{FA}}{b_i^{batch}} \) increasing with larger \( b_i^{batch} \) values. Since \( b_i^{batch} \) increases with larger cache size \( l_i \), then \( \frac{b_i^{batch} - b_i^{FA}}{b_i^{batch}} \) also decreases with larger cache size \( l_i \).

APPENDIX C: PROOF OF LEMMA

The second derivative of \( b_i^{FA} \) is

\[
\frac{\partial^2 b_i^{FA}}{\partial l_i^2} = \frac{r}{f_B(l_i + \frac{l_i}{f_B})^2} > 0,\tag{41}
\]

thus \( b_i^{FA} \) is convex in \( l_i \) and \( \sum_{i=1}^{M} b_i^{FA} \) is also convex. Therefore, \( \sum_{i=1}^{M} b_i^{FA} \) is a convex problem. Consider the Lagrangian

\[
\mathcal{L}^{FA} = \sum_{i=1}^{M} b_i^{FA} + \mu\left( \sum_{i=1}^{M} l_i - C \right),\tag{42}
\]

where \( \mu \) is the Lagrange multiplier. The Karush-Kuhn-Tucker (KKT) condition for the optimality of a cache allocation is

\[
\frac{\partial \mathcal{L}^{FA}}{\partial l_i} = -\frac{r}{f_B(l_i + \frac{l_i}{f_B})^2} + \mu \begin{cases} 0 & \text{if } 0 < l_i < L, \\ \geq 0 & \text{if } l_i = 0, \\ < 0 & \text{if } l_i = L. \end{cases}\tag{43}
\]

Let \( \beta = 1/(f_B \mu) \) and \( x^+ = \max(x, 0) \), we then have

\[
l_i \geq \min \left( \beta r - \frac{r}{f_B l_i} + 1, L \right).\tag{44}
\]

and \( \beta \) can be effectively solved by the bisection method.

APPENDIX D: PROOF OF PROPOSITION

Let \( d_{any} \) be the waiting time of an arbitrary proactive delivery mechanism for \( V_i \) with bandwidth \( b_i \) and cache size \( l_i \), where \( e^{b_i/l_i} l_i \leq L \). The prefix of the video instead of other parts should be cached since the beginning part is always displayed and consumes more bandwidth. The remaining part with length \( L - l_i \) is then periodically broadcasted under bandwidth \( b_i \). Let \( dx \) denote a small portion at an arbitrary length offset \( x \) of the \( i \)-th video, where \( l_i \leq x \leq L \). Note that \( dx \) should be successfully buffered within duration \( \frac{r}{f_B} + d_{any} \), and the corresponding bandwidth consumption for delivering partition \( dx \) is no less than \( \frac{dx}{f_B (\frac{r}{f_B} + d_{any})} \). Therefore, the allocated bandwidth is lower bounded by the following expression

\[
b_i \geq \frac{d_{any}}{\frac{r}{f_B} + d_{any}} = \frac{r}{f_B} \ln \left( \frac{L - l_i}{l_i + r d_{any}} + 1 \right).\tag{45}
\]

We then have

\[
d_{any} \geq \frac{L - e^{b_i/l_i} l_i}{r(e^{b_i/l_i} - 1)} = d_i^{FA}\tag{46}
\]

valid for any proactive delivery mechanism. Hence, CCE-GEBB is optimal to minimize the waiting time of the \( i \)-th video with bandwidth \( b_i \) and cache size \( l_i \).

APPENDIX E: PROOF OF PROPOSITION

Firstly, we proof part (a). Given the optimal bandwidth allocation \( \mathbf{b} \), the cache problem becomes

\[
\max_{\mathbf{1}} \sum_{i=1}^{M} p_i \frac{l_i}{r(e^{b_i/l_i} - 1)} l_i \tag{47}
\]

s.t. \( \sum_{i=1}^{M} l_i = C, 0 \leq l_i \leq L/e^{b_i/l_i}, \forall i \in \{1, \ldots, M\} \).
which is a fractional knapsack problem with the optimal greedy solution. The weight of caching the $i$-th video is $w_i = p_i e^{\frac{l_i}{r}b_i}/(r(e^{\frac{l_i}{r}b_i} - 1))$. The greedy solution allocates more cache storage to the video with largest weight until reaching its maximum value, i.e., $l_i = L e^{-\frac{l_i}{r}b_i}$ to provide zero-delay for the $i$-th video. Since the $k$-th video has allocated cache storage and experiences non-zero delay, the videos with larger weights should be zero-delay and the ones with smaller weights have no cache allocated due to the greedy property. Then we only need to validate that a larger popularity represents a larger weight in this case. By contradiction, we assume that $V_j$ has zero-delay while $V_i$ with a larger popularity experience non-zero delay ($i \leq j$), the average waiting time will be decreased by simply switching the cache-bandwidth allocations of these two videos. Therefore, a larger popularity stands for a larger weight and part (a) is proved.

Next, we prove part (b). For the first $k - 1$ videos with zero-delay, the optimal solution should utilize the minimum cache usage given the total allocated bandwidth and the minimum bandwidth usage given the total allocated cache. Firstly, consider the cache minimization problem given the total allocated bandwidth $B_k$ for the first $k - 1$ videos

$$
\min_{b_1} \sum_{i=1}^{k-1} L e^{-\frac{b_i}{r}} \text{ s.t. } \sum_{i=1}^{k-1} b_i = B_k, b_i \geq 0, i \in \{1, \ldots, M\},
$$

which is a convex problem and can be effectively solved by the Lagrangian method. The optimal solution is that the bandwidth and the cache are evenly allocated. Similar results can also be found for the total bandwidth minimization problem given the total cache usage for $k - 1$ videos. Therefore, for the videos with zero-delay, bandwidth and cache are evenly allocated.

**Appendix F: Proof of Lemma**

Based on Proposition[4], the structure of the optimal cache allocation obeys

$$l_i = \begin{cases} 
1 & \text{if } 1 \leq i \leq k - 1, \\
C \mod l_1 & \text{if } i = k, \\
0 & \text{if } k + 1 \leq i \leq M,
\end{cases}
$$

where $k = \lfloor C/l_1 \rfloor + 1$.

If $l_1 < C$, the first $k - 1$ videos are zero-delay, yielding $b_i = \frac{K}{M} \ln(\frac{r}{L})$ for $1 \leq i \leq k - 1$. The bandwidth allocation for the remaining $M - k + 1$ videos becomes

$$
\min_b \sum_{k}^{M} p_i d_i^{FA} \text{ s.t. } \sum_{i=k}^{M} b_i \leq B - \sum_{i=1}^{k-1} b_i, b_i \geq 0, \forall i \in \{k, \ldots, M\}.
$$

We have $\frac{\partial^2 d_i^{FA}}{\partial b_i^2} \geq 0$, thus $d_i^{FA}$ is convex in $b_i$ and Problem[50] is a convex problem. Considering the Lagrangian

$$
\mathcal{L} = \sum_{i=k}^{M} p_i \frac{L - l_i}{e^{\frac{b_i}{r}} - 1} - \frac{b_i}{r} + \mu \left( \sum_{i=1}^{M} b_i - B \right),
$$

where $\mu$ is the Lagrange multiplier. The KKT condition for the optimality of a bandwidth allocation for the remaining $M - k + 1$ videos becomes

$$
\frac{\partial \mathcal{L}}{\partial b_i} = -\frac{f_b e^{\frac{l_i}{r}b_i}(L - l_i)}{r(e^{\frac{l_i}{r}b_i} - 1)^2} + \mu \geq 0 \text{ if } b_i > 0, \frac{\partial \mathcal{L}}{\partial b_i} \geq 0 \text{ if } b_i = 0.
$$

Let $\beta = f_b/(\mu^2)$, we then have, for $i = k, \ldots, M$,

$$
b_i = \frac{r}{f_b} \ln \frac{2 + p_i \beta(L - l_i) + \sqrt{(p_i \beta(L - l_i))^2 + 4p_i \beta(L - l_i)}}{2}.
$$

For videos with no cache allocated, more bandwidth is assigned to videos with larger popularity, i.e., $b_k \geq \ldots \geq b_M$. When $l_1 = C \leq l$, the whole cache size is allocated to the first video, and $b_1$ also obeys[54].

Therefore, the optimization can be found by the one dimension search of $l_1$, where $l_1 \in [L/M, \min(M, C)]$.

**Appendix G: Proof of Proposition 7**

By using Cauchy-Schwarz inequality, we have

$$
\left( \sum_{i=1}^{M} \frac{p_i}{f_b b_i} \right) \left( \sum_{i=1}^{M} b_i \right) \geq \left( \sum_{i=1}^{M} \frac{p_i}{f_b b_i} \right)^2,
$$

yielding

$$
\sum_{i=1}^{M} p_i (L - l_i) \geq \frac{\sum_{i=1}^{M} \sqrt{p_j (L - l_j)}}{f_b B},
$$

where the equation is achieved when $b_i = B \sqrt{p_i/(L - l_i)} \sum_{j=1}^{M} \sqrt{p_j (L - l_j)}$. The problem requires minimizing $\sum_{i=1}^{M} \sqrt{p_j (L - l_j)}$, which can be effectively solved by the following proposition.

**Proposition 8:** The optimal solution has the following two properties: a) $l_1 \geq l_2 \geq \ldots \geq l_M$, b) if $0 < l_j < L$, we have $l_j = L$ for $i = 1, 2, \ldots, j - 1$.

Proof: Firstly, we assume by contradiction that there exists $l_i < l_j$ for $i < j$, and $\sum_{j=1}^{M} \sqrt{p_j (L - l_j)}$ will be reduced by simply switching the allocations for $V_i$ and $V_j$. Thus part (a) is proved.

Secondly, we assume by contradiction that there exists $l_i < L$ for $i < j$. By shifting a cache storage size $\Delta$ of $V_j$ to $V_i$, where $\Delta = \min\{l_i, L - l_i\}$, we only need to prove

$$
\sqrt{p_i (L - (l_i + \Delta)) + \sqrt{p_j (L - (l_j - \Delta))}} < \sqrt{p_i (L - l_i) + \sqrt{p_j (L - l_j)}}.
$$

Denote $f(x) = \sqrt{p_i (L - (l_i + \Delta)) + x} - \sqrt{p_j (L - (l_j - \Delta))}$, where $p_i > p_j$, $l_i < l_j$ and $\Delta > 0$. We then have $f'(x) > 0$, thus $f(x)$ increases with $x$ and we have $f(0) < f(\Delta)$, yielding

$$
\sqrt{p_i (L - (l_i + \Delta))} - \sqrt{p_j (L - l_j)} < \sqrt{p_i (L - l_i) - \sqrt{p_j (L - (l_j - \Delta))}}.
$$

Thus the optimal cache allocation in this case is to cache the most popular videos only. \(

