GRAVITATIONALLY INDUCED SCALAR FIELD FLUCTUATIONS IN THE RADIATION DOMINATED R-W UNIVERSE

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ABSTRACT

It is shown that quantum fluctuations due to a nontrivial gravitational background in the flat radiation dominated universe can play an important cosmological role generating nonvanishing cosmological global charge, e.g. baryon number, asymmetry. The explicit form of the fluctuations at vacuum and at finite temperature is given. Implications for particle physics are discussed.

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1. Introduction

Since the early eighties it has been widely recognized that quantum fluctuations of scalar matter fields may play an important role in cosmology, especially in the context of the inflationary de Sitter epoch [1]. Actually, the case of the scalar field in the de Sitter space has been the most extensively and carefully studied one. The reason is that in the de Sitter space fluctuations of the light fields \((m^2/H^2_I) << 1\), where \(H_I\) is the de Sitter Hubble parameter) grow linearly with time assuming finally a significantly large value of the order of \(H^4_I/m^2\), which singles out the de Sitter universe. It is believed that fluctuations produced at the time of inflation are seen during subsequent stages of the evolution of the universe as energy density inhomogeneities responsible for the formation of the large scale structure. It is also advocated that those fluctuations set initial conditions for the classical evolution of fields in subsequent epochs.

In contrast to the above, it is usually assumed that gravitationally induced scalar field fluctuations in spatially flat radiation dominated (RD) and matter dominated (MD) epochs are irrelevant for particle cosmology. We would like to point out that this assumption is not properly discussed in the literature. On one hand, one observes that in the RD universe the fluctuations (as explained in this letter) decrease in time. On the other hand, they may in principle be large enough to control violation of some symmetries or to alter the evolution of some fields present in field-theoretical models. We want to stress that this problem becomes particularly important in view of the ongoing search for a reliable mechanism for production of the baryon asymmetry in the Universe, the need of the better understanding of the scenarios for late-time phase transitions and discussions of the possible lepton number nonconservation.

In this letter we would like to address explicitly the problem of quantum fluctuations of the massive scalar field during the RD epoch. This epoch covers most of the history of the universe, and the temperature range from, say, \(10^{14}\) GeV down to 10 eV. On that energy scale one can find a lot of interesting phenomena in popular extensions of the standard model such as its supersymmetric version or string inspired models, what, in our opinion, justifies the research reported in this work.

The paper is organized as follows. In Section 1 we set our notation and subsequently evaluate fluctuations of a massive scalar field in the RD flat Robertson-Walker space at vacuum and at finite temperature. In Section 2 we apply our formulae to a general field theoretical model with particular attention paid to two specific examples resem-
bling the Affleck-Dine model [2] and the so called spontaneous baryogenesis scenario [3]. Finally, in the last Section we review our results and present conclusions.

1. Scalar field fluctuations in the RD universe

The RD Universe is the solution to the Einstein’s equations with the energy-momentum tensor in the form $T^\mu_\nu = \text{diag}(-\rho, p, p, p)$. Tracelessness of the $T^\mu_\nu$ implies equation of state for the content of the RD Universe: $\rho = 3p$. In this letter we assume a flat RD space endowed with the metric $g_{\mu\nu} = \text{diag}(-1, a^2(t), a^2(t), a^2(t))$ where $a(t)$ is the RW scale factor given by $a(t) \equiv (u)^{1/2}$ with $u$ defined as $u \equiv t/t_0$, $t_0$ being the beginning of the RD epoch.

We couple a massive scalar field to gravity in the minimal way (note that here the curvature scalar $R$ vanishes identically)

$$S[\phi] = \int d^4x \sqrt{-g} \left(-g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \right).$$ (1)

As usual in this type of analysis we assume that there is no “back-reaction” of the scalar field on the metric, cf. [4]. The equation resulting from eq.(1) is

$$\left(\frac{d^2}{du^2} + \frac{3}{2u} \frac{d}{du} + \frac{k^2 t_0^2}{u} + m^2 t_0^2 \right) \phi_k(u) = 0$$ (2)

where $\phi_k(u)$ is the spatial Fourier transform of the field $\phi(\vec{x}, t)$,

$$\phi(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3 2k} \left( \phi_k(u) e^{i\vec{k}\vec{x}} a^\dagger_k + \text{h.c.} \right)$$ (3)

with $a^\dagger$ and its hermitian conjugate denoting standard creation and anihilation operators respectively.

A general solution of the equation (2) is given by confluent hypergeometric functions

$$\phi_k(u) = A_1(k, m) 2ikt_0 e^{-imtu_0} F_1 \left( \frac{1}{4} + \frac{ik^2 t_0}{2m}, \frac{3}{2}, 2imt_0 u \right)$$

$$+ A_2(k, m) \frac{1}{\sqrt{u}} e^{-imtu_0} F_1 \left( \frac{1}{4} + \frac{ik^2 t_0}{2m}, 1/2, 2imt_0 u \right).$$ (4)

The two undetermined coefficients $A_1$ and $A_2$, which may in principle depend on both $m$ and $k$, are not independent if one takes into account quantization condition imposed
on a field $\phi$}

$$[\phi(\vec{x}, t), \partial_t \phi(\vec{y}, t)] = \frac{i}{\sqrt{-g}} \delta^{(3)}(\vec{x} - \vec{y})$$

$$[a_k, a_{k}'^\dagger] = (2\pi)^3 2k \delta^{(3)}(\vec{k} - \vec{k}')$$

(5) (6)

From above equations and decomposition (3) we get a normalization condition

$$\text{Im}(\bar{\phi}_k(u) \partial_t \phi_k(u)) = \frac{k}{\sqrt{-g}}$$

(7)

which translates into the constraint on $A_1$ and $A_2$ (we confine them to be real)

$$A_1(k, m) A_2(k, m) = 1$$

(8)

In most general case there are several ways of fixing both coefficients. One possibility is to use initial conditions set at the timelike surface $t = t_0$ for $\phi$ and $\partial_t \phi$. This is the proper procedure if one knows for example the explicit solution for $\phi$ in the epoch preceding the RD one. We do not assume such a detailed knowledge, hence we use an alternative approach instead. We demand that the “correct” mode functions we choose, which will define our Fock space, should approach at short distances ($k \to \infty$)

the massless positive frequency solution,

$$\phi_k(u) \to \frac{1}{u^{1/2}} e^{2ikt_0 \sqrt{u}}$$

(9)

In this way we obtain the asymptotic behaviour of both coefficients

$$A_{1,2}(k, m) \to 1$$

(10)

Here we assume that $A_1 = A_2 = 1$, what completes the definition of our Fock space.

We set out to calculate the fluctuations of the field $\phi$ i.e. $< 0|\phi^2|0 >$. This quantity is badly divergent and needs renormalization. We define the renormalized fluctuations as the difference between RD and the Minkowski space fluctuations, hence the relevant object to look at is the difference

$$< \phi^2 >_R \equiv < \phi^2 > - < \phi^2 >_M$$

(11)

As we shall see, this definition gives the finite result. It may easily be checked that in terms of Fourier modes $\phi_k(u)$ the renormalized fluctuations (11) are given by the formula

$$< \phi^2 >_R = \int \frac{d^3 k}{(2\pi)^3} \left( \frac{1}{2k} |\phi_k(u)|^2 - \frac{1}{2u^{3/2} \sqrt{k^2/u + m^2}} \right)$$

(12)
Using (4) we can write down the explicit formula

\[
< \phi^2 >_R = \frac{1}{4\pi t^2} \int_0^\infty dy \left[ y \left( \frac{1}{4} + \frac{i y^2}{2 m t}, \frac{1}{2}, 2imt \right) + 2i y \frac{1}{4} \left( \frac{3}{4} + \frac{i y^2}{2 m t}, \frac{3}{2}, 2imt \right) \right]^2 - \frac{y^2}{\sqrt{y^2 + m^2 t^2}}
\]

where \( y = kt/\sqrt{u} \). Unfortunately, the above expression cannot be evaluated in its most general form. However, it is possible to write down the systematic expansion of the mode functions (4) and the integral (12) in terms of \( mt \). Using such an expansion we will be able to discuss reliably fluctuations in the regime of small mass and to control the passage to the massless limit. For the region \( mt > 1 \) we will have to rely on the numerical calculations.

In the case \( mt < 1 \), the relevant expansion of modes is given by (cf. [5])

\[
\phi_k(u) = \frac{1}{u} \sum_{n=0}^{\infty} \left[ p_n^{(-1/2)}(2imt) j_{n-1}(2y) + ip_n^{(1/2)}(2imt) j_n(2y) \right] \frac{1}{(2y)^{n-1}}
\]

The \( j_n \) is the n-th spherical Bessel function and coefficient \( p_n^{(\mu)} \) can be read from

\[
\sum_{n=0}^{\infty} p_n^{(\mu)}(z) w^n = e^{z/2(\coth(2w) - 1/2w)} \left( \frac{zw}{\sinh(zw)} \right)^{1-\mu}
\]

One easily finds that

\[
|\phi_k(u)|^2 = \frac{1}{u} \left[ 1 + (mt)^2 \left( \frac{\sin^2(2y)}{8y^4} - \frac{1}{2y^2} \right) + O((mt)^4) \right]
\]

On the basis of the expansion we see that \( < \phi^2 > \) is the ultraviolet-finite quantity. It is also infrared finite, since the modes are perfectly regular functions for \( k \to 0 \) (i.e. \( y \to 0 \)).

In the region \( mt < 1 \), with help of the expansion (14), we get the following formula for the leading behaviour:

\[
< \phi^2 > = \frac{m^2}{8\pi^2} \left( -\ln(mt) + (3/2 - \gamma - \ln2) + O((mt)^2) \right)
\]

where \( \gamma \) is the Euler constant. One should note that the fluctuations vanish as \( m \) approaches zero and grow with \( m \) if we keep \( mt \) constant. This agrees with the earlier result for an exactly massless field reported in ref. [6]. The interesting feature of
the formula (17) is its non-analyticity in $m t$ and the appearance of the logarithmic singularity at $t = 0$ which is related to the singularity of the RD Universe at $t = 0$.

In the regime $m t > 1$ the numerical study we have performed shows that the integral in (13), which is solely a function of $m t$, has an oscillating behaviour with the period close to $\pi/m$ and amplitude rising with increasing $m t$. One can easily check that the integral (13) is converging rather quickly, hence in a given interval of the variable $m t$ one can cut-off the integration from above at a numerically determined value $\Lambda$. Given this observation, it is straightforward to find an algebraic approximation for the integral as a function of $m t$. Using the expansion of the Kummer hypergeometric function in terms of the modified Bessel functions of the second kind (cf. [7])

$$1_F^1(a, b, x) = e^{\frac{1}{2}x} \frac{\Gamma(b - a - \frac{1}{2})}{\Gamma(\frac{1}{4} - i x)} \sum_{n=0}^{\infty} \frac{(2b - 2a - 1)_n(b - 2a)_n(b - a - \frac{1}{2} + n)}{n!(b)_n} \times (-1)^n I_{b - a - \frac{1}{2} + n}(\frac{1}{2}x)$$

and expanding the definite integrals in powers of $\Lambda^2/m t$ one obtains in the region $m t > \Lambda^2$ the expression of the form

$$< \phi^2 >_R = \frac{1}{t^2} \sqrt{m t}(a \sin(2mt) + b \cos(2mt) + c + o(\Lambda^2/m t))$$

where the coefficients $a$, $b$, $c$ are

$$a = \frac{1}{(\sqrt{2\pi})^3} \int_{0}^{\Lambda^2} dx e^{-2\pi x} Re(f_1(x)f_2^*(x))$$

$$b = \frac{1}{2(\sqrt{2\pi})^3} \int_{0}^{\Lambda^2} dx e^{-2\pi x} (|f_1(x)|^2 - |f_2(x)|^2)$$

$$c = \frac{1}{2(\sqrt{2\pi})^3} \int_{0}^{\Lambda^2} dx e^{-2\pi x} (|f_1(x)|^2 + |f_2(x)|^2) - \frac{1}{3\sqrt{2\pi}^2} \Lambda^3$$

with functions $f_1$, $f_2$ defined as follows

$$f_1(x) = \Gamma(-\frac{1}{4} - i x) \sum_{n=0}^{\infty} \frac{(1/2 - ix)_n}{n!} \frac{(1/4 - ix + n)(-2ix)_n}{n!} \cos \pi/2(n + 1/4 - ix)$$

$$+ 4\sqrt{x} \Gamma(\frac{1}{4} - ix) \sum_{n=0}^{\infty} \frac{(1/2 - ix)_n}{n!} \frac{(1/4 - ix + n)(-2ix)_n}{n!} \cos \pi/2(n + 3/4 - ix)$$

(23)
\[ f_2(x) = \Gamma\left(-\frac{1}{4} - ix\right) \sum_{n=0}^{\infty} \frac{(-1/2 - 2ix)_n(1/4 - ix + n)(-2ix)_n}{n!(\frac{3}{2})_n} \sin \pi/2(n + 1/4 - ix) + 4\sqrt{x} \Gamma\left(\frac{1}{4} - ix\right) \sum_{n=0}^{\infty} \frac{(1/2 - 2ix)_n(1/4 - ix + n)(-2ix)_n}{n!(\frac{3}{2})_n} \sin \pi/2(n + 3/4 - ix) \]

(24)

where \(a_n \equiv a_{n-1}(a+n-1)\), \(a_0 \equiv 1\). For the reasonable value of \(\Lambda = 3.5\) the least squares fit to numerical data in the interval \((4.0, 17.0)\) gives \(a = -5.9 \times 10^{-4}\), \(b = 4.2 \times 10^{-3}\), \(c = 1.5 \times 10^{-3}\). We note the leading dependence of the \(<\phi^2>_R\) on \(m\) in this range of \(mt\): the fluctuations grow proportionally to the square root of the mass.

One should also note that the fluctuations \(<\phi^2>_R\) as defined here, see (11), (12), are not positive definite. Remarkably, it can easily be shown, that if one perturbs the definitions of the functions \(A_{1,2}(k, m)\) allowing for the appropriate dependence on the ratio \(k^2/m^2\), which amounts to the modification of our Hilbert space, then the regions of an oscillating behaviour with negative values of \(<\phi^2>_R\) are pushed towards increasing values of the argument \(mt\). Since in this letter we do not discuss any specific modification of the Hilbert space beyond the simple and most natural definitions (10) and (11), the non-positiveness of \(<\phi^2>_R\) demands special care when one considers physical applications of the present result, as we do in the next Section. The point is that a physically meaningful quantity which has the interpretation of the dispersion squared should be strictly positive definite. However, we do not rely on the oscillating behaviour of \(<\phi^2>_R\) in the discussion of applications of the present result being only interested in the overall time dependence \(\sqrt{m}/t^{3/2}\). This leading time and mass dependence in the region \(mt > 1\) we believe to be universal, hence we just neglect the scheme-dependent oscillating contributions in what follows.

Up to now we have been calculating curved space vacuum expectation value of \(\phi^2\). However, if we were to take into account that the Universe is “hot”, i.e. it is in fact in a mixed state to which many-particle states may contribute significantly, we should better calculate a thermal average of \(\phi^2\), with finite temperature effects included. Assuming thermal equilibrium of the content of the Universe we have

\[ <\phi^2> |_{T \geq 0} = \int \frac{d^3k}{(2\pi)^32k} |\phi_k|^2 (1 + 2n_k) \]

(25)

where \(\phi_k\) are modes given by (11), (13), and \(n_k\) is the occupation number for the particles
with the comoving momentum $k$. As $n_k$ we take

$$n_k = \frac{1}{\exp(\frac{1}{T} \sqrt{k^2/u + m^2}) - 1} = \frac{1}{\exp(\sqrt{k^2/T_0^2 + m^2/T^2}) - 1}$$

(26)

which is correct for sufficiently large $k$ in view of the choice (9). The (25) is again divergent. However, as usual in finite-temperature calculations, it may be divided into $T = 0$ part and the temperature correction, among which only the former is UV divergent. Hence, we can use the renormalization procedure (11) to get meaningful results even at $T > 0$

$$< \phi^2 >_{\text{renormalized, } T>0} \to < \phi^2 >_R + < \phi^2 >_{R}$$

(27)

where

$$< \phi^2 >_{R} = 2 \int \frac{d^3k}{(2\pi)^3} 2k |\phi_k|^2 \frac{1}{\exp(\sqrt{k^2/T_0^2 + m^2/T^2}) - 1}$$

(28)

This expression may be approximated analytically in two limiting cases: a) $\frac{m}{T} >> 1$, and b) $\frac{m}{T} << 1$. In the case b) one easily gets

$$< \phi^2 >_{R} = \frac{T^2}{12}$$

(29)

exactly as in the flat Minkowski case. In the case a) one can see that in the region which dominates the integral, $k < \frac{mT}{2}$, the second term in (14) is unimportant. Hence we obtain

$$< \phi^2 >_{R} = \frac{\Gamma(5/4)}{\pi^3} \frac{6.64^{3/2}}{(g_*)^{3/4}} \frac{1}{H_i^2} (T_R/T) \frac{4m^{5/2}}{M_p^{5/2}} T^3 e^{-m/T} \cos^2(mt - \frac{3}{8\pi})$$

(30)

which is exponentially suppressed.

2. Implications for particle physics in the expanding Universe

Let us consider a global U(1) symmetry realized in a single complex scalar field model. If $Q$ is the charge of the field $\chi$, the Noether current associated with that symmetry is

$$j^\mu = iQ \{ \bar{\chi} \partial^\mu \chi - \chi \partial^\mu \bar{\chi} \}$$

(31)
(we put $Q=1$ in what follows) and the conservation law for $j^\mu$ in the expanding Universe reads

$$\partial_\mu (a^3(t)j^\mu) = -ia^3(t)\left\{\bar{\chi}\frac{\partial V}{\partial \bar{\chi}} - \chi\frac{\partial V}{\partial \chi}\right\}$$

(32)

One can see that a symmetry is broken once the rhs of (32) is nonvanishing. One can see also that when a symmetry is broken explicitly, the net cosmological charge density gets generated according to the formula

$$a^{-3}\frac{d}{dt}(j^0a^3(t)) \approx -i\left\{\bar{\chi}\frac{\partial V}{\partial \bar{\chi}} - \chi\frac{\partial V}{\partial \chi}\right\}$$

(33)

Let us assume that the term violating the symmetry is

$$\delta V = \frac{\lambda}{2n\Lambda^{2n}}\phi^{2n}$$

(34)

where $\phi \equiv Re(\chi)$ (in this section we assume the absence of derivative couplings, they will be discussed later). Suppose that the initial conditions and the shape of the potential are such that the $Im(\chi)$ and its fluctuations are negligible when compared with $Re(\chi)$ at any time $t$ (this situation may be easily realized in the Affleck-Dine model, cf. [8]). Hence

$$a^{-3}\frac{d}{dt}(j^0a^3(t)) \approx -i\frac{\lambda}{\Lambda^{2n}}\phi^{2n}$$

(35)

We can see that the magnitude of the symmetry violation is proportional to a coupling $\lambda$, inverse powers of some scale $\Lambda$ if $n > 2$, and to some power of the scalar field $\phi$. In this sense one can say that, $\lambda$ and $\Lambda$ being fixed in a given theory, it is the $\phi$ what determines the amount of symmetry breaking. Here the quantum fluctuations of the field $\phi$ come into play. In the quasiclassical picture one can describe the evolution of the quantum field, lets call it $\Phi$, writing it down as the superposition of the quasiclassical field $\phi$ which obeys essentially classical (perhaps perturbatively corrected) equation of motion and quantum fluctuations $\delta\phi$, the dispersion squared of which we identify as $<\phi^2 >_R$. If the potential drives the quasiclassical field to zero, then it may happen that the magnitude of the symmetry breaking term is determined by the dispersion of $\delta\phi$. Using $<\phi^2 >^{2n} = (<\phi^2 >_R)^n$ one gets an estimate

$$a^{-3}\frac{d}{dt}(j^0a^3(t)) = i\frac{\lambda}{\Lambda^{2n}}(<\phi^2 >_R)^n$$

(36)

Of course, whether this term is really a dominant one or not, it depends on the relative magnitude and time-dependence of $<\phi^2 >_R$ and the classical part of the field. We shall investigate this issue later in this work.
At this point let us discuss the explicit form of $< \phi^2 >_R$ as a function of the temperature. We know that the time dependence for this quantity is given by (17) if $t < \frac{1}{m}$ and by (19) if $t > \frac{1}{m}$. Let us say assume that the RD epoch starts at $t = t_0$. Then $mt \approx \frac{m^2}{H(t_0) H(t_0)}$, where $H(t_0)$ is of the order of the Hubble parameter during inflation, $H(t_0) \approx H_I \approx 10^{14}$ GeV. Assuming then an adiabatic expansion in the RD epoch we get

$$< \phi^2 >_R \approx \frac{m^2}{2} \ln \left( \frac{H_I T^2}{m T^2_R} \right)$$

as long as $T^2 >> T^2 R_{H_I}$ where $T_R$ is the reheating temperature after inflation

$$T_R = \left( \frac{45}{4\pi^3 g_*^*} \right)^{\frac{1}{4}} \min[(H_I M_P)^{\frac{1}{2}}, (\Gamma I M_P)^{\frac{1}{2}}]$$

(here $g_*$ is the number of relativistic degrees of freedom at of reheating and $\Gamma_I$ is the total decay width of the inflaton field – a field which drives the transition from the de Sitter to RD epoch). Let us take for simplicity the case of a “good” reheating which corresponds to $T_R \approx H_I$. This gives the condition

$$T > T_* = (mH_I)^{\frac{1}{2}}$$

If we take $m \leq 10^2$ GeV, $H_I \approx 10^{14}$ GeV then we get $T_* \leq 10^8$ GeV. We note that for a really soft potential with $m \approx 10^{-21}$ eV, one has $T_* \approx 10$ eV which means that in that case the regime where $< \phi^2 >_R$ changes only logarithmically extends over the whole RD epoch. For $T < T_*$ we have instead of (37)

$$< \phi^2 >_R = \frac{O(1) g_*^*}{M_P^4} T^4$$

($g_*$ is the number of relativistic degrees of freedom at temperature T).

Now, let us investigate the evolution of the classical field $\phi$. For simplicity we assume that this evolution is dominated by the mass term in the potential, which is usually a good assumption at least in the perturbative regime. In this case the general solution to the equation of motion is

$$\phi = \frac{1}{(mt)^{\frac{1}{2}}} [C J_{1/4}(mt) + D J_{-1/4}(mt)]$$

If we set initial conditions at $t_0$ such that $z_0 = mt << 1$, then $C = \Gamma(5/4) \phi_0 + \frac{2 \phi_0 z_0}{m}$ and $D = -\frac{4\Gamma(3/4) \partial_0 \phi_0 z_0}{m^{2 + 2/3}}$ where $\phi_0 = \phi(mt_0)$, $\partial_0 \phi_0 = \partial_t \phi(mt_0)$. This gives the $\phi$ at
late times, \( m t > 1 \), in the form

\[
\phi \approx \frac{1}{\sqrt{\pi}} \left( \frac{m t}{2} \right)^{-3/4} \left[ \Gamma(5/4) \phi_0 + \frac{2z_0 \partial_t \phi_0}{m} \right] \cos(mt - 3\pi/8) \tag{42}
\]

From previous analysis we have learnt that for \( T < T_* \) the \( \langle \phi^2 \rangle_R \) falls off as \( \sqrt{m t} t^2 \), hence we conclude that \( \frac{\langle \phi^2 \rangle_R}{\phi^2} |_{T > T_*} \sim \frac{m^2}{[\Gamma(5/4)\phi_0 + \frac{2z_0 \partial_t \phi_0}{m}]^2} \) which does not depend on time (and temperature). This implies that if the initial conditions happen to make this ratio large, then the fluctuations dominate over the classical part of the field. Moreover, in general the field \( \phi \) has some additional couplings to light particles, which facilitate decay of the field \( \phi \) with the decay width \( \Gamma_\phi \). This changes the behaviour of the classical field \( \phi \), namely \( \phi^2 \rightarrow \exp(-\Gamma t) \phi^2 \). Actually, as pointed out by several authors in the context of the Affleck-Dine mechanism (which corresponds to our toy model when \( n=2 \)) the \( \Gamma_\phi \) should be large in order to avoid an unobservable excess of the net charge produced during symmetry violation [8]. We want to stress that in such a case, the \( \langle \phi^2 \rangle_R \), decaying accordingly to the power law, dominates the divergence of the Noether current and the net cosmological charge density even at the late times.

Let us check whether at \( T > T_* \) the \( \langle \phi^2 \rangle_R \) may be significantly large. From (37) and (42) we obtain, averaging over oscillations,

\[
\frac{\langle \phi^2 \rangle_R}{\phi^2} |_{T > T_*} \approx \frac{2m^2}{(\Gamma(5/4)\phi_0 + \frac{2z_0 \partial_t \phi_0}{m})^2} \tag{43}
\]

That means that fluctuations are important as long as the condition

\[
\phi_0 + \frac{z_0 \partial_t \phi_0}{m} < m \tag{44}
\]

is fulfilled. One can see that even if \( m << H_I \) and \( \phi_0 \sim H_I \), which happens to be the case if the initial conditions at \( t = t_0 \) are produced by large quantum fluctuations in the preceding de Sitter epoch [8], the condition (44) may be fulfilled provided that \( \partial_t \phi_0 \) is sufficiently large and negative.

Finally, let us consider models where a massive scalar \( \phi \) is derivatively coupled to other particle species. This situation corresponds for instance to models possessing pseudogoldstone bosons with nonvanishing masses. The relevant scenario is similar to that of the “spontaneous baryogenesis” described in ref. [5]. If a Lagrangian has a coupling of the form \( L_\phi = -\frac{1}{4} \phi \partial_\mu \phi \partial^\mu \) (\( f \) being some, presumably large, mass scale) where \( \partial_\mu \phi \partial^\mu \) is a divergence of a current corresponding to some explicitly broken symmetry, the
baryon number symmetry for instance. Then, as we have shown, there are fluctuations in the field $\phi$ with dispersion $\sqrt{<\phi^2>_R}$. We may represent them as the effective term in the Lagrangian

$$L_{\delta\phi} = -\frac{1}{f} \sqrt{<\phi^2>_R} \partial_\mu j^\mu$$  \hspace{1cm} (45)$$

Up to the total divergence (45) is equivalent to

$$L_{\delta\phi} = \frac{1}{f} \partial_0 \sqrt{<\phi^2>_R} j^0$$  \hspace{1cm} (46)$$

This produces an effective chemical potential $\mu = -\frac{1}{f} \partial_0 \sqrt{<\phi^2>_R}$ for the charge density $j^0$ which means a nonzero cosmological charge density generated in thermal equilibrium. Explicitly, cf. [9],

$$j^0 \approx -\frac{1}{f} \partial_t \sqrt{<\phi^2>_R} T^2$$  \hspace{1cm} (47)$$

or charge to entropy ratio

$$j^0/s \approx -\frac{1}{fg_* T} \partial_t \sqrt{<\phi^2>_R}$$  \hspace{1cm} (48)$$

where $g_*$ is the number of relativistic degrees of freedom at temperature $T$. The above estimate gives in the case of our toy model

$$j^0/s \approx \frac{m}{4g_* T} \frac{1}{t\sqrt{\ln(1/mt)}}$$  \hspace{1cm} (49)$$

for $T > T_*$ and

$$j^0/s \approx \frac{O(10^{-2})}{g_* T} \frac{1}{t^{7/4}m^{1/4}}$$  \hspace{1cm} (50)$$

for $T < T_*$. One can see that both expressions fall off as time elapses, and that the decrease at $T < T_*$ is faster than at $T > T_*$, essentially $\sim T^{5/2}$ below $T_*$ and $\sim T$ above. If there is no phase transition in the model before the end of RD epoch, then the final charge to entropy ratio produced will be equal to (49) or (50) taken at the “decoupling” temperature $T_D$. This is the temperature at which symmetry violating interactions fall off from equilibrium or the one which corresponds to the end of RD stage, when the shape of the fluctuations changes qualitatively i.e. at $T_D \approx T_f$ close to 10 eV. As previously, the numerical values predicted depend on various details of a model under investigation. For example, let us take $T_D = 10$ eV and $g_* = 100$. Then if we require the charge-to-entropy ratio to be equal to $10^{-10}$, as it should be for the baryonic charge, then we get the condition $m = f^4 \times 10^{-77}$GeV, which gives
\[ m = 10^{-17}\text{GeV for } f = 10^{15}\text{GeV and } m = 1\text{GeV for } f = 10^{19}\text{GeV}. \]

3. Conclusions

In this letter we have found explicit expressions for a massive scalar field fluctuations in the flat radiation dominated universe. It turns out that in the region of small \( mt \), i.e. shortly after the beginning of the RD epoch or for very light fields, the fluctuations decrease with time only logarithmically and are proportional to the square of the mass of the field in question. For large \( mt \), i.e. very late or for a heavy field, the time dependence is stronger, \( 1/t^{3/2} \), but the mass dependence becomes weaker – proportional to the square root of \( m \). As far as finite temperatures are concerned, we have concluded that the “radiation-dominated” background modifies Minkowski space results rather weakly. At low temperatures, i.e. at large ratios \( m/T \), the thermal contribution is exponentially suppressed the suppression becoming stronger as the temperature decreases with time. At high temperatures the result coincides essentially with that of Minkowski space. In general, fluctuations vanish when one takes the limit \( m \to 0 \).

Given all that we argue that the fluctuations may still play a significant role in particle physics models, which has been illustrated in the second part of the work. Within the family of models we discuss, the case when our parameter \( n \) equals 2 corresponds precisely to the Affleck-Dine model, and the higher \( n \) terms are often encountered in the important class of string inspired models. Hence we conclude that fluctuations we have described constitute the phenomenon which is relevant in a very general situation when some cosmological charge density, first of all the baryonic charge density, is supposed to be generated during the radiation dominated epoch. We also note that although the inflationary scenario is widely accepted, our results do not rely on the existence of the de Sitter epoch preceding the RD stage in the early Universe.

In conclusion, we have demonstrated that quantum fluctuations due to a nontrivial gravitational background during radiation dominated epoch in the evolution of the Universe do in fact exist and may have observable consequences for cosmology of realistic particle models.
References

[1] J.M.Bardeen,P.J.Steinhardt,M.S.Turner, Phys. Rev. D28, 679 (1983);
    A.D.Linde, Phys. Lett. 116B, 335 (1982).

[2] I.Affleck,M.Dine, Nucl. Phys. B249, 361 (1985).

[3] A.Cohen,D.Kaplan, Phys. Lett. 199B, 251 (1987).

[4] N.Birrel,P.C.Davies, Quantum Fields in Curved Space, Cambridge University Press, Cambridge 1984.

[5] H.Buchholz, Die Konfluent Hypergeometrische Funktion, Springer-Verlag, Berlin 1953.

[6] C.Pathinayake,L.H.Ford, Phys. Rev. D37, 2099 (1988).

[7] M.Abramowitz,I.Stegun, Handbook of Mathematical Functions, Dover Publications, Inc., New York.

[8] A.D.Dolgov, Yukawa Institute preprint YITP/K-940, 1991.

[9] E.W.Kolb,M.S.Turner, The Early Universe, Addison-Wesley Publishing Company, 1990.