Translation process of mathematics representation: From graphics to symbols and vice versa

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Abstract. The objective of this research is to examine the process of translation of mathematical representation from symbols to graphics and vice versa. Translation of representation refers to a basic skill to develop a concept, understanding, and to promote students’ mathematical thinking. This qualitative descriptive research employed task-assigning and task-based interview method. It is revealed that the students can perform every process of the translation of representation from symbol to graphic and vice versa. Translation among mathematical representations comprises four steps, i.e. unpacking the source, preliminary coordination, constructing the target, and determining equivalence. The process of translation of mathematical representations between graphics and symbols can be done without involving another form of representations. The students follow each process that is relatively similar, even though the construction of the translation output is somewhat different.

1. Introduction

Representation is among the important standard in mathematics subject due to its contribution in enhancing student’s understanding [1, 2]. NCTM proposes five standard processes that a student must possess, namely problem-solving, reasoning and verification, communication, connection and representation [1]. The use of representation can develop and deepen the students’ understanding of mathematics concepts. The representation allows the students to create a connection between the concepts and communicate their thoughts through representation.

Representation, generally, is an approach of an individual to express their ideas and concepts [1, 2]. This term is described as a complex, crucial concept that corresponds to a particular object. It also functions to represent, to symbolize, and to provide the information regarding the object [3]. Representation is the way the students externalize and present their works [4-6]. Representation in learning functions to develop and optimize student’s thinking; this is a process of construction and abstraction of mathematical insight [7]. In addition to producing new configuration and construction, thinking process to grasp the concept, operation, and mathematical correlations of a configuration is the manifestation of the output of representation [7]. This process signifies representation takes two steps, i.e. internal and external process [2, 8, 9].

Internal and external representation influences each other; it determines the way a person expresses or illustrate mathematical concepts [10-12]. During the thinking process to solve a mathematical problem, one needs the internal representation before manifesting the idea through the external representation. This present study emphasizes the external representation of a student. Goldin and
Shteingold categorize this representation into three forms, namely formal notation form (algebra symbol), visual-spatial (graphic, table, etc.) and verbal form (written or spoken) [8].

The standard for the mastery on representation in learning comprises the capability to successfully select, implement, and perform translation of mathematical representation in solving a particular problem [1]. The key to understand, communicate, and perform mathematical operations correlate to performing translation among the representation of graphic, symbol and verbal. Furthermore, translation enables the teacher to monitor the student’s understanding of a concept [13]. In other words, comprehending the process of translation among the representation is crucial. One is urged to comprehend the way the student translates, and identify their mistakes during that process. The information regarding such a problem is, without question, essential to boost the learning quality.

Translation is a process of constructing a mapped mathematical representation to another representation [14]. Similarly, translation can be defined as a cognitive process of transforming information from a representation form to another [15]. One of the examples is explaining a representation of a symbol in the form of a graphic. Two terms are used in translation between mathematical representations, namely source and target representation. The core of translation is to transform a representation into another form; this process demands ability in defining, identifying, manipulating and constructing.

There are four steps of translation among mathematical representations, i.e. unpacking the source, preliminary coordination, constructing the target, and determining equivalence [13, 15-17]. Furthermore, their study denotes that every student performs the same four steps of translation, but they implement different strategies in translating certain mathematical representations. The differences are due to the difference in the student’s ability.

This present study discusses the process of translation of mathematical representation from symbol to graphic and vice versa. The representation of the symbol, in this context, refers to the mathematical model, statement, or algebra symbol that serves to reveal the student’s concepts on mathematics. Representation of graphic is a picture or graphic that is used in explaining the concept of mathematics of the students. With that being said, the translation between the representation is from symbol to graphic and from graphic to symbol. The indicators of the translation of mathematical representation are described in Table 1.

| Translation Steps          | Indicator                                                        |
|----------------------------|------------------------------------------------------------------|
| Unpacking The Source       | Mentioning information contained in the task                     |
|                            | Mentioning the representation given                              |
| Preliminary Coordination   | Determining formula, strategy and initial step to form the       |
|                            | representation as requested                                      |
| Constructing the Target    | Forming the requested representation as problem-solving to the   |
|                            | given task                                                       |
| Determining Equivalence    | Summing up the result of the representation according to the     |
|                            | problem given                                                    |

A study finds out that most of the students can follow every step in translating the verbal representation to the graphics [7]. Such an outcome is also relevant to this study as the objective of both research is to investigate the translation process; the difference is only on the representation form that is used by the students. This research focuses on the representation of graphic and symbol while put more emphasize on verbal and graphic representation.

Another study compares the mistakes between students who have graduated and those who are in the final year of their study. The mistakes vary among the students, and these are common mistakes [18-20].

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In the light of the above discussion, this present study aims at discussing the translation of mathematical representation from graphic to the symbol and vice versa. It is expected that this research serves as a reference to evaluate the process of designing an appropriate learning material.

2. Method
This descriptive research employed a qualitative approach to investigate a particular issue. Its focus is to identify and describe the translation between mathematical representations, from graphics to symbols and vice versa. The data were from the written and spoken form; these were obtained from the students as the research participants. Furthermore, the students were assigned a translation of mathematical representation task to generate descriptive data. They also participated in a task-based interview to gain other information.

Two students with the highest mathematical skills were selected as the representative of other students with the same capacity. The reason for selecting these participants is to explore their translation process; the teachers can also benefit from the data as it serves as the benchmark of the students with different mathematical ability. The students were assigned a mathematical problem on straight line equations to translate the mathematical representation from graphic to a symbol. On the question is a straight line drawn on a Cartesian diagram with two coordinates from the line. The students were asked to solve the equation where its output is in the representation of the symbol. Moreover, the same mathematical problem also assigned to the similar model of the question to identify the process of translation of representation from symbol to graphic. The difference is that the test contains two straight lines and the students were asked to draw a straight line based on the equation in one Cartesian diagram.

3. Results and Discussion

3.1. Translation of mathematical representation from graphic to a symbol
The results of the translation of two research participants show several differences. The performance of Subject 1 and Subject 2 to translate the mathematical representation from graphic to the symbol are given in Figure 1 and Figure 2 respectively.

![Figure 1. Translation of Subject 1 from graphics to symbol](image1)

![Figure 2. Translation of Subject 2 from graphics to symbol](image2)
In Figure 1 and Figure 2, the way the students select coordinate 1 and 2 by which it results in a difference of the value \((x_1, y_1)\) and \((x_2, y_2)\). Despite this, however, the process yields the same outcome. The processes of translation of mathematical representation the students from graphic to the symbol are explained as follows.

3.1.1. **Unpacking the source.** The result of Subject 1 reveals that the subject wrote the coordinates given on the picture without explicitly write the word “given”. According to the interview, Subject 1 can identify the given representation and the representation that is asked. This participant explains that the problem is to find out the equations of a line from a picture or graphic whose two coordinates are already identified. Participant 2 or S2 also successfully identifies the given representation and the representation that is asked. On the answer sheet, this participant seems to proceed to the process without constructing the first step directly.

3.1.2. **Preliminary coordination.** This step is not present on the answer sheets of both students. The interview signifies that the students apply the formula of straight line equations as the problems provide a picture of line with two coordinates.

3.1.3. **Constructing the target.** The reconstruction of the representation from graphic to symbol between two subjects is different despite that the students apply the same formula. This difference is because the way the students select coordinate 1 and 2 differently. The interview reveals that both students explain the detail of the process of solving the math problem.

3.1.4. **Determining equivalence.** The students do not conclude the output of translation on their answer sheets. It is, however, reflected by the way they clarify the answer. Both Subject 1 and Subject 2 explain that they investigate the appropriateness of the answer and the test by substituting the given coordinates to the obtained equation.

Although the processes are not explicitly shown on the answer sheet, it is revealed that the students follow all four steps of the translation of mathematical representations from graphic to symbol. All four steps of translation are successfully identified from the interview result. Furthermore, there is no repetitive process in the student’s translation. All four steps that the students follow are similar. However, there is a difference in the step of constructing the target since the identification of the information on the given representation varies depending on each student.

3.2. **Translation of mathematical representation from symbol to graphic.** The results of the translation from symbol to graphics of Subject 1 is described in Figure 3.

![Figure 3. Translation of Subject 1 from symbol to graphics](image)
Meanwhile, results of the translation of mathematical representation from symbol to graphics of Subject 2 is described in Figure 4.

![Figure 4. Translation of mathematical representation of Subject 2 from symbol to graphic](image)

This step is not present on the answer sheets of both students; this is particularly on the first and the last step. The process of each translation step of the two students is discussed as follows.

3.2.1. **Unpacking the source.** This step is not present on the answer sheets of both students. Nevertheless, the students can explain and identify each information from the given test.

3.2.2. **Preliminary coordination.** Both Subject 1 and Subject 2 share the same step before constructing the test given in the form of two equations and translate the representation into a straight line graphic on a coordinate axis. They begin the step by finding out the two coordinates for each equation. Subject 1 argues that “identifying the coordinates should be done first as a line is formed by two coordinates minimally”. Subject 1 explains how to spot the coordinates of the two equations by substituting $x$ or $y$ with “0”. This is to ease the process of finding out the coordinates that intercept with the line equation.

3.2.3. **Constructing the target.** This step is not presented clearly since the output of the problem-solving is in the form of a picture. The interview reports that the students draw the two lines on a Cartesian plane, meaning that the process that the students perform is relatively the same. Following the process of pointing out the coordinates of each equation is the students begin drawing on the
Cartesian plane, marking the coordinates that have been discovered and further joining the coordinates of each equation.

3.2.4. Determining equivalence. This step is not explicitly presented on the answer sheets of the students; it is rather identified from the interview. Both students sum up their answer and check it to clarify whether the answer suits the problem given. This clarification is done by identifying the intercept of the two coordinates and ensure whether the result is appropriate to the intercept obtained from the calculation.

Although the processes are not explicitly shown on the answer sheet, it is revealed that the students follow all four steps of the translation of mathematical representations from graphic to symbol. All four steps of translation are successfully identified from the interview result; the students also perform the same step of translation. Furthermore, there is no repetitive process in the student’s translation.

4. Conclusion
Even though some of the processes are not presented during working on the equation, it is revealed that the students can perform every process of the translation of representation from symbol to graphic and vice versa. Translation among mathematical representations comprises four steps, i.e. unpacking the source, preliminary coordination, constructing the target, and determining equivalence. The process of translation of mathematical representations between graphics and symbols can be done without involving another form of representations. The students follow each process that is relatively similar, even though the construction of the translation output is somewhat different.

5. Acknowledgement
The authors thank the Universitas Negeri Surabaya for giving us the opportunity and facilitating us in completing this research. Thanks also to the Head of Mathematics Education Department who helped us in carrying out this research. Lastly, authors also thank the school and mathematics teacher at the MTs Al Huda Gorontalo who had provided the opportunity for researchers to complete this research.

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