Overlap properties and adsorption transition
of two Hamiltonian paths

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We consider a model of two (fully) compact polymer chains, coupled through an attractive interaction. These compact chains are represented by Hamiltonian paths (HP), and the coupling favors the existence of common bonds between the chains. Using a ($n = 0$ component) spin representation for these paths, we show the existence of a phase transition for strong coupling (i.e. at low temperature) towards a “frozen” phase where one chain is completely adsorbed onto the other. By performing a Legendre transform, we obtain the probability distribution of overlaps. The fraction of common bonds between two HP, i.e. their overlap $q$, has both lower ($q_m$) and upper ($q_M$) bounds. This means in particular that two HP with overlap greater than $q_M$ coincide. These results may be of interest in (bio)polymers and in optimization problems.

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I. INTRODUCTION

The study of two coupled identical systems is a familiar topic in Statistical Physics, both in equilibrium [1] and non equilibrium contexts [2]. It is commonly considered in the physics of glassy systems [3,4], where it is a substitute to the time honored conjugate field in the exploration of phase space. In this note, we wish to extend these studies to the case of two compact polymer chains. The coupling between the chains is chosen as an attractive term which favors the existence of common monomers (bonds); in other words, we consider the “adsorption” of a chain onto the other. By a Legendre transform, this model maps onto the overlap probability distribution of the chains.

For simplicity, we will consider fully compact chains. This allows us to use a simple spin representation [5] to describe the chain properties through Hamiltonian paths. Beside its interest in homopolymer physics, this model may yield some insight in the sequence-structure relationship in proteins, or in some related optimization problems such as the travelling salesman. The outline of the paper is as follows: we briefly recall in section II the connection between a single polymer chain and an \( n \)-component spin system, in the limit \( n = 0 \). The extension to the problem of two interacting chains (section III) is then straightforward, and will be solved at a mean field level in section IV. We finally mention some possible consequences of the model.

II. THE SINGLE CHAIN PROBLEM

The connection between polymer physics and spin systems may be presented as follows [6]. Let us consider, on each site \( \vec{r} \) of a \( d \)-dimensional cubic lattice, a spin variable \( \vec{S}_r \), with \( n \) components. The normalization is such that \( \vec{S}_r^2 = n \). Consider the sum:

\[
Z(K) = \int \prod_{\vec{r}} d\mu(\vec{S}_r) \prod_{\{\vec{r},\vec{r}'\}} \left(1 + K\vec{S}_r \cdot \vec{S}_{r'}\right)
\]

where the variable \( K \) denotes the fugacity of a monomer, and the product runs over neighbouring pairs of sites (i.e. bonds) of the lattice. In eq (1), \( d\mu \) is the normalized integration
measure on the \((n-1)\) dimensional sphere of radius \(\sqrt{n}\).

Due to the normalization of \(\vec{S}_r\), it is easy to see that

\[
\int d\mu(\vec{S}_r) \exp(\vec{H}_r \vec{S}_r) = 1 + \frac{\vec{H}_r^2}{2} + O(n)
\]

so that, by taking derivative with respect to \(H^\alpha_r\) \((\alpha = 1, ..., n)\), we obtain:

\[
\int d\mu(\vec{S}_r) 1 = 1; \quad \int d\mu(\vec{S}_r) (S^\alpha_r)^2 = 1
\]

All higher powers of \(\vec{S}_r\) are at least of order \(n\).

Expanding eq (1) in powers of \(K\), and using the previous remark, we see that \(Z(K)\) can be viewed as a sum over all closed loops, each closed loop of length \(l\) contributing a weight \(nK^n\). In the limit \(n \to 0\), only the single connected loops survive, hence the possibility to represent self-avoiding walks (SAW) on the lattice by eq (1). A very useful rewriting of eq (1) is

\[
Z(K) = \int \prod \vec{r} \, d\vec{\phi}_r \, e^{-\frac{1}{4} \sum_{(\vec{r}, \vec{r}')} \phi_{\vec{r}} \Delta_{\vec{r}\vec{r}'} \phi_{\vec{r}'}} \prod \vec{r} \, d\mu(\vec{S}_r) \, \exp(\sqrt{K} \sum_{\vec{r}} \vec{S}_r \vec{\phi}_r)
\]

where the operator \(\Delta_{\vec{r}\vec{r}'}\) is a lattice \(\Delta\) function \((\Delta_{\vec{r}\vec{r}'} = 1, \text{if } \vec{r} \text{ and } \vec{r}' \text{ are neighbouring sites, and } 0 \text{ otherwise})\). Note that the sum in the exponential term of eq (3) is over the sites of the lattice. Applying the familiar Stratonovich-Hubbard transformation to eq (3), we introduce \(n = 0\) component fields \(\vec{\phi}_r\) and get

\[
Z(K) = \int \prod \vec{r} \, d\vec{\phi}_r \, e^{-\frac{1}{4} \sum_{(\vec{r}, \vec{r}')} \phi_{\vec{r}} \Delta_{\vec{r}\vec{r}'} \phi_{\vec{r}'}} \prod \vec{r} \, d\mu(\vec{S}_r) \, \sqrt{\pi} \exp(\sqrt{K} \sum_{\vec{r}} \vec{S}_r \vec{\phi}_r)
\]

Using eq (2), we get

\[
Z(K) = \int \prod \vec{r} \, d\vec{\phi}_r \, e^{-\frac{1}{4} \sum_{(\vec{r}, \vec{r}')} \phi_{\vec{r}} \Delta_{\vec{r}\vec{r}'} \phi_{\vec{r}'}} \prod \vec{r} \, d\mu(\vec{S}_r) \, \prod \vec{r} \left(1 + \frac{K}{2} \phi_{\vec{r}}^2\right)
\]

So far we have not specified the spatial extension of the SAW: \(Z(K)\) is the grand partition function of the chain, so that the number \(M\) of sites of the lattice is not related to the number \(N\) of bonds of the SAW. We now require that fully compact configurations are the only configurations present in eq (3), i.e. \(M = N\). This full compactness requirement
amounts to keep the term proportional to $nK^N$ in eq (5). The coefficient of this term is simply the number $\mathcal{N}$ of self avoiding fully compact configurations (also called Hamiltonian paths or HP), that is

$$\mathcal{N} = K^{-N} \left( \frac{Z(K)}{n} \right)_{n=0} = \lim_{n \to 0} \frac{1}{n} \int \prod_{\vec{r}} d\vec{\phi} e^{-\frac{1}{2} \sum_{(\vec{r},\vec{r}')} \vec{\phi}_{\vec{r}} \Delta_{\vec{r},\vec{r}'}^{-1} \vec{\phi}_{\vec{r}'} \prod_{\vec{r}} \left( \frac{\vec{\phi}_{\vec{r}}^2}{2} \right)}$$

(6)

Note that $\mathcal{N}$ is simply the canonical partition function for a HP of $N$ bonds. Performing a homogeneous saddle point approximation on $\vec{\phi}$ in eq (6) yields

$$\mathcal{N} = \left( \frac{z}{e} \right)^N$$

(7)

where $z = 2d$ is the coordination number of the lattice. Note that in the context of Hamiltonian paths, an homogeneous solution implies that one deals with periodic boundary conditions.

Equation (6) holds for an arbitrarily connected graph with adjacency matrix $\Delta_{\vec{r},\vec{r}'}$, but the saddle point evaluation is a priori valid only for large enough $z$. Indeed, for the fully connected graph ($z = N$), the number of HP can be directly estimated ($\mathcal{N} = N!/(2N)$), and eq (7) reduces to the Stirling formula.

III. THE COUPLED CHAINS PROBLEM

We now extend the above approach to the case of two interacting self avoiding chains. By “interacting chains”, we mean that the presence of common (or doubly occupied) bonds between the chains is thermodynamically favored, and we are interested in counting the number of configurations of two such HP.

Following eq (1), we denote by $K$ the fugacity of a monomer and consider the sum

$$Z_2(K,\lambda) = \int \prod_{\vec{r}} d\mu(\vec{S}_r) d\mu(\vec{\sigma}_r) \prod_{\{\vec{r},\vec{r}'\}} \left( 1 + K(\vec{S}_r \vec{S}_{\vec{r}'} + \vec{\sigma}_r \vec{\sigma}_{\vec{r}'}) + \lambda K^2 (\vec{S}_r \vec{S}_{\vec{r}'}) (\vec{\sigma}_r \vec{\sigma}_{\vec{r}'}) \right)$$

(8)

where $\vec{\sigma}_r$ has the same properties as $\vec{S}_r$ ($\vec{\sigma}_r^2 = n$), and the parameter $\lambda$ is a measure of the interaction between the chains (see below). The contribution of doubly occupied bonds to eq (8) is proportional to $\lambda^{N_{12}}$, where $N_{12}$ is the number of these bonds (implying $\lambda > 1$).
Following exactly the same steps as in section II, we have

$$Z_2(K, \lambda) = \int \prod_{\vec{r}} d\mu(\vec{S}_r) d\mu(\vec{\sigma}_r) e^{\frac{1}{2} \sum_{(r,r')} \Delta_{r'r'} G_2(\vec{S}, \vec{\sigma})}$$

(9)

with

$$G_2(\vec{S}, \vec{\sigma}) = K(\vec{S}_r \vec{S}_{r'} + \vec{\sigma}_r \vec{\sigma}_{r'}) + (\lambda - 1)K^2(\vec{S}_r \vec{S}_{r'})(\vec{\sigma}_r \vec{\sigma}_{r'})$$

(10)

Performing the Stratonovitch-Hubbard transformation on eqs. (9) and (10) yields

$$Z_2(K, \lambda) = \int \prod_{\vec{r}} d\vec{\phi}_r d\vec{\psi}_r \prod_{\alpha\beta} d\lambda_{\alpha\beta}(r) \ e^{-\frac{1}{2} \sum_{(r,r')} \Delta_{r'r'} A_{r'r'}} \int \prod_{\vec{r}} d\mu(\vec{S}_r) d\mu(\vec{\sigma}_r) e^{\sqrt{K} \sum_{r} B_r}$$

(11)

with

$$A_{r'r'} = \vec{\phi}_r \vec{\phi}_{r'} + \vec{\psi}_r \vec{\psi}_{r'} + \sum_{\alpha\beta} q_{\alpha\beta}(r) q_{\alpha\beta}(r')$$

(12)

and

$$B_r = \vec{S}_r \vec{\phi}_r + \vec{\sigma}_r \vec{\psi}_r + \sqrt{K} \sqrt{\lambda - 1} \sum_{\alpha\beta} q_{\alpha\beta}(r) \phi_{\alpha r} \psi_{\beta r} + K^2 (\lambda - 1) \sum_{\alpha\beta} q_{\alpha\beta}^2 (r)$$

(13)

Note that the previous transformations require two $n$ component fields $\vec{\phi}_r$ and $\vec{\psi}_r$, and an $n \times n$ matrix $q_{\alpha\beta}(r)$, with $n = 0$. One may now perform the integration over the original spin variables $\vec{S}_r$ and $\vec{\sigma}_r$ in eq (11). We obtain

$$Z_2(K, \lambda) = \int \prod_{\vec{r}} d\vec{\phi}_r d\vec{\psi}_r \prod_{\alpha\beta} d\lambda_{\alpha\beta}(r) \ e^{-\frac{1}{2} \sum_{(r,r')} \Delta_{r'r'} A_{r'r'}} \prod_{\vec{r}} C_{r}$$

(14)

where

$$C_{r} = 1 + \frac{K}{2} (\vec{\phi}_r^2 + \vec{\psi}_r^2) + \frac{K^2}{4} \vec{\phi}_r \vec{\psi}_r + \sqrt{K} \sqrt{\lambda - 1} \sum_{\alpha\beta} \phi_{\alpha r} \psi_{\beta r} + \frac{K^2}{2} (\lambda - 1) \sum_{\alpha\beta} q_{\alpha\beta}^2 (r)$$

(15)

So far, we have not specified the spatial extension of the chains. The sum $Z_2(K, \lambda)$ is grand canonical with respect to both singly and doubly occupied bonds. Following the previous section, we now enforce the full compactness constraint for both chains, by keeping the term proportional to $n^2 K^{2N}$ in eq (14). Defining by $Z(\lambda)$, the total number of HP of the two interacting chains, we have
\[ Z(\lambda) = K^{-2N} \left( \frac{Z_2(K, \lambda)}{n^2} \right)_{n=0} \quad (16) \]

Technically, the extraction of the term proportional to \( K^2 \) in \( Z_2(K, \lambda) \), amounts to keep on each site \( \vec{r} \) the terms proportional to \( K^2 \) in eqs.(14) and (15). We may therefore write

\[ Z(\lambda) = \lim_{n \to 0} \frac{1}{n^2} \int \prod \phi_r \phi_{r'} \prod q_{\alpha\beta}(r) e^{-\frac{1}{2} \sum (r, r') A_{r'r'}^{\alpha\beta}} \prod \mathcal{D}_r \quad (17) \]

where \( A_{r'r'} \) is given in eq (12) and

\[ \mathcal{D}_r = \frac{\phi_r^2 \psi_r^2}{4} + \sqrt{\lambda - 1} \sum_{\alpha\beta} \phi_r^{\alpha} q_{\alpha\beta}(r) \psi_r^{\beta} + \frac{1}{2} (\lambda - 1) \sum_{\alpha\beta} q_{\alpha\beta}(r)^2 \quad (18) \]

Note that \( Z(\lambda) \) is still grand canonical with respect to the number \( N_{12} \) of common bonds between the two HP, since, as mentionned above, the term with \( N_{12} \) common bonds in \( Z(\lambda) \) yields a contribution proportional to \( \lambda^{N_{12}} \).

One may also interpret eq (14) as giving the partition function of two HP of \( N \) bonds, with an attractive interaction energy \( \varepsilon \) favoring common bonds, at temperature \( T \), that is

\[ Z(\varepsilon) = \sum_{(HP_1, HP_2)} e^{\beta\varepsilon N_{12}} \quad (19) \]

with \( \beta = \frac{1}{T} \). The identity between eqs.(14) and (19), i.e between \( Z(\lambda) \) and \( Z(\varepsilon) \), yields the familiar result \( \lambda = e^{\beta\varepsilon} \). Moreover, equation (19) allows us to derive bounds for the partition function \( Z(\varepsilon) \) (or \( Z(\lambda) \)), namely

\[ N e^{\beta\varepsilon N} \leq Z(\varepsilon) \leq N^2 \quad (20) \]

where \( N \) is given in eq (7). From now on, we will set \( \varepsilon = 1 \).

**IV. SADDLE POINT APPROXIMATION AND PHASE TRANSITIONS**

Since an exact evaluation of eqs (17,18) seems to be out of reach, we will use a saddle point approximation with respect to the variables \( \phi_r, \psi_r, \) and \( q_{\alpha\beta}(r) \). This saddle point will be further restricted to be space independent (\( \phi_r = \phi, \psi_r = \psi, q_{\alpha\beta}(r) = q_{\alpha\beta} \)), again implying periodic boundary conditions. Setting \( Z(\lambda) = e^{-N\omega(\lambda)} \), we get
\[ \omega(\lambda) = \text{Min}_{(\phi^\alpha, \psi^\beta, q_{\alpha\beta})} \left( \frac{1}{4d} (\overline{\phi}^2 + \overline{\psi}^2 + \sum_{\alpha\beta} q_{\alpha\beta}^2) - \log D \right) \] (21)

where

\[ D = \frac{\overline{\phi}^2 \overline{\psi}^2}{4} + \sqrt{\lambda - 1} \sum_{\alpha\beta} \phi^\alpha q_{\alpha\beta} \psi^\beta + \frac{1}{2} (\lambda - 1) \sum_{\alpha\beta} q_{\alpha\beta}^2 \] (22)

The saddle point equations are easily solved, by introducing the quantities \( \Phi = \overline{\phi}^2 \), \( \Psi = \overline{\psi}^2 \), \( R = \sum_{\alpha\beta} \phi^\alpha q_{\alpha\beta} \psi^\beta \), and \( Q = \sum_{\alpha\beta} q_{\alpha\beta}^2 \). Denoting the saddle point values with a subscript 0, we obtain

\[ \frac{\Phi_0}{2d} = \frac{\frac{1}{2} \Phi_0^2 + \sqrt{\lambda - 1} R_0}{D_0} \] (23)

\[ \frac{R_0}{2d} = \frac{(\lambda - 1) R_0 + \sqrt{\lambda - 1} \Phi_0^2}{D_0} \] (24)

\[ \frac{Q_0}{2d} = \frac{(\lambda - 1) Q_0 + \sqrt{\lambda - 1} R_0}{D_0} \] (25)

together with \( \Phi_0 = \Psi_0 \). In eqs. (23-27), the denominator is given by

\[ D_0 = \frac{\Phi_0^2}{4} + \sqrt{\lambda - 1} R_0 + \frac{(\lambda - 1)}{2} Q_0 \] (26)

One then gets

\[ \omega_0(\lambda) = \frac{\Phi_0}{2d} + \frac{Q_0}{4d} - \log D_0 \] (27)

At this point, one has to resort to a numerical solution of the saddle point equations. Note that the possible solutions obey simple equalities or inequalities, such as: \( \Phi_0, Q_0 > 0 \), \( \Phi_0 + Q_0 = 4d \), \( R_0^2 = Q_0 \Phi_0^2 \).

Our results can be interpreted in two ways. They first describe the (thermal) properties of two HP coupled via eq (19): \( \log Z(\lambda) = -N \omega(\lambda) \) is then, up to a temperature factor, the free energy (with \( \lambda = e^\beta \)). On the other hand, a Legendre transformation with respect to \( \log \lambda \), gives information on the number \( N_{12} \) of common bonds between the HP, i.e. on their overlap properties. For convenience, we define \( N_{12} = Nq \), and
\[ \Theta(q) = \oint \frac{d\lambda}{2i\pi \lambda} Z(\lambda)\lambda^{-N_q} \quad (28) \]

A saddle point evaluation of eq (28) gives

\[ q = -\frac{\partial \omega(\lambda)}{\partial \log \lambda} \quad (29) \]

Using eqs (23-27), we obtain

\[ q_0 = \frac{\lambda}{\lambda - 1} \frac{Q_0}{4d} \quad (30) \]

We now present our results along both ways.

A. Thermal properties of the coupled Hamiltonian paths

Unless otherwise specified, our results are given for \( d = 3 \), and the main parameter of this section is the temperature \( T (T = \frac{1}{\log \lambda}) \). The saddle point equations (23-27) yield the free energy \( f_0(T) = T\omega_0(e^T) \) as a function of \( T \) (see Figure 1).

FIG. 1. The free energy \( f \) as a function of temperature. The saddle point result \( f_0 \) (full line) crosses the free energy \( \gamma \) of the fully adsorbed phase (hatched line) at point A \( (T = T_3) \).

Solving numerically the above equations, we get the following results

1) \( T_1 = \infty \)
In this case, we find a finite fraction of common bonds \( q_0^{(1)} = \frac{1}{d} = \frac{1}{3} \). This value corresponds to a random choice of a common bond among the \( d = 3 \) lattice dimensions, and is the smallest possible value \( q_m \) of the overlap.

2) \( T_2 = \frac{1}{\log(d+1)} \simeq 0.7213 \)

One then has \( \Phi_0 = Q_0 = 2d = 6 \). This point is in some sense a disorder point \([9]\) where the values of \( \Phi_0 \) (linked to the entropy of a single HP) and \( Q_0 \) (linked to the overlap of the two HP) cross. It corresponds to an overlap \( q_0^{(2)} = \frac{d+1}{2d} = \frac{2}{3} \).

3) \( T_3 \simeq 0.5846 \): the complete adsorption transition.

At this point, the two HP system “freezes” into a single HP; one may also say that one chain gets fully adsorbed onto the other. Of course, the entropy does not vanish below \( T_3 \), but becomes equal to the entropy of a single HP, see eq \((7)\). Note that this solution is not a saddle point solution. It must nevertheless be taken into account, since its free energy per monomer which reads

\[
\gamma(T) = -1 - T \log\left(\frac{\tilde{z}}{e}\right)
\]

is an upper bound (see eq \((20)\)). The transition temperature \( T_3 \) is thus defined by \( \gamma(T_3) = f_0(T_3) \) (see Figure \([\text{I}]\)\). The mechanism of this first order freezing transition is analogous to the one studied in ref. \([10]\), in the context of polymer crystallization. Note that \( q \) jumps from \( q_0^{(3)} \simeq 0.7572 \) to \( q = 1 \) across the transition. From a thermodynamic point of view, we have thus found that the overlap fraction of two HP has a maximum value \( q_M \simeq 0.7572 \), beyond which the two HP coincide.

Note that, as the space dimension \( d \) increases, the adsorption temperature \( T_3 \) decreases. In particular, the transition does not occur for the fully connected graph \((d \to \infty)\).

4) In the present mean field like description, one may continue the high temperature branch of the free energy \( f_0(T) \) beyond the full adsorption point, implying the existence of a metastable state (see the branch AB of Figure \([\text{I}]\)). In this respect, a particular temperature may be defined, where the entropy \( s_0(T) = -\frac{\partial f_0(T)}{\partial T} \) equals that of a single HP:

\[
s_0(T_4) = \log\left(\frac{\tilde{z}}{e}\right)
\]
leading to $T_4 \simeq 0.4350$, and in turn to $q_0^{(4)} \simeq 0.9627$. Furthermore, one may argue that this metastable branch is defined up to the point where the (metastable) overlap parameter $q$ reaches the value one. If this is correct, we find from the saddle point equation that this occurs for $T = T_5 \simeq 0.3898$, with a positive entropy $s_0(T_5) \simeq 0.498$. We do not have a clear understanding of the metastable branch, and more work is needed on this point.

**B. Overlaps and Legendre Transform**

As previously mentionned, our results can also be interpreted in terms of the overlap properties of the two HP. A convenient function to characterize these properties is the overlap probability distribution $P(q)$ defined as

$$P(q) = \frac{1}{N^2} \sum_{(HP_1, HP_2)} \delta(N_{12} - Nq)$$

where $N$ is given in eq (7) and $N_{12}$ is the number of common bonds of the HP. Eqs.(28) and (33) imply that

$$P(q) = \frac{1}{N^2} \Theta(q)$$

From a strictly thermodynamic point of view, we have the result that the overlap probability distribution $P(q)$ is defined only for $q_m < q < q_M$, and for $q = 1$. The existence of a metastable branch beyond $q_M$ is not easy to interpret: the saddle point evaluation (29) of the Legendre transform (28) then becomes ill-defined, since $\lambda(q)$ becomes a multivalued function.

For $q_m < q < q_M$, the relation between $\lambda$ (i.e. temperature) and $q$ (i.e. overlap) can be inverted through eq (29). There is thus a one to one correspondance between overlap and temperature in the region $1 < \lambda < \lambda_3 = e^{\frac{1}{T_3}} \simeq 5.532$. The thermal properties of the coupled system imply that, for $1 < \lambda < \lambda_3$, we have $P(q) = \delta(q_0 - q)$, where $q_0$ is given by eq (30). On the other hand, for $\lambda > \lambda_3$, we have $P(q) = \delta(1 - q)$.

Another quantity of interest is the entropy, considered as a function of the overlap $q$. It is given by
Performing the saddle point evaluation on $\lambda$ in eq (28), we get

$$s_0(q) = -\omega_0(\lambda) - q \log \lambda$$

(36)

where $\lambda = \lambda(q)$ is given by eq (30). The phase transition for $q = q_M$ can be interpreted (see Figure 2) as a Maxwell construction since the results of the previous section can be rewritten as

$$s_0(q_M) - s_0(1) = (1 - q_M) \log \lambda$$

(37)

FIG. 2. The entropy $s(q)$ as a function of the overlap $q$. The full line denotes the saddle point result $s_0(q)$; the hatched line is the result of the Maxwell construction with the fully adsorbed state. Note that AB corresponds to the metastable branch of Figure 4.

Beyond the homogeneous saddle point approximation, our results in the region $q_M < q < 1$ are coherent with a phase coexistence picture between a fraction $x$ of the phase $q = q_M$ and a fraction $1 - x$ of the fully adsorbed phase $q = 1$. Such a phase coexistence will give rise to an effective overlap $q_M < q_{eff} = xq_M + (1 - x) < 1.$
V. CONCLUSION

We have considered the overlap and/or thermal properties of two coupled Hamiltonian paths (HP), in a homogenous saddle point approximation. We have found (for $d = 3$) a phase transition at finite coupling, between an entropy dominated phase and a completely adsorbed phase, where the two HP have overlap $q = 1$. This phase transition disappears when $d \to \infty$. Our results may be relevant in a proteic context (NMR, structure alignment). If the number of constraints exceeds a certain threshold, the existence of $q_M$ suggests that a single spatial structure may survive. For longer polymers, the existence of a first order transition, raises many questions (metastability, glass transition,..). Finally, we remark that the coupled chains system of this paper undergoes a phase transition at a finite value of the coupling constant $\lambda$, whereas in spin glasses, the two replica system undergoes a (spin glass) transition when the coupling constant vanishes [34].

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