Underwater Acoustic Covert Communication Based on Compressed Sensing

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Abstract. Compressed sensing technology is currently a relatively mature channel estimation technology, and the sparsity of the underwater acoustic channel provides a basis for the application of compressed sensing technology to the underwater acoustic channel. Channel estimation is one of the most commonly used signal reconstruction methods. The signal is reconstructed based on compressed sensing technology to improve the reliability of signal transmission. The key to compressed sensing technology is the SL0 algorithm. Based on the SL0 algorithm, a new objective function is proposed to better approximate the L0 norm. In a very short time, the original signal can be reconstructed accurately and the signal transmission is improved. Finally a simulation experiment is used to compare the proposed objective function with the previously proposed objective function, which proves that the performance of the proposed objective function is better.

1. Introduction
The noise of the marine environment is diverse, mainly natural environmental noise (such as the sound of waves, raindrops.), man-made noise (such as the mechanical noise of ships [1], horn sound.) and the sounds of marine organisms (such as the sounds of dolphins [2]). Man-made noise and the calls of marine organisms exist from time to time. Therefore if animal noise is used as a carrier to transmit concealed information, it is very easy to cause the detector to be alert and cause the transmission of concealed information. However, due to natural environmental noise such as wave sound from time to time, if wave noise is used as a carrier to transmit hidden messages, the attention of the inspector can be reduced to a great extent, so as to realize the covert transmission between the sender and the receiver. Therefore this article intends to use the sound of ocean waves to realize the covert transmission between the two parties in the environment where there is a third-party eavesdropping. This article is to first use the ocean acoustic wave collection device to collect ocean sound waves. The collected ocean sound waves are diverse, and the collected ocean sound waves need to be filtered with the help of filters to obtain the wave sound we want, and then use the DCT technology [3] to embed the message to be transmitted with the pilot signal into the wave sound, and use the wave sound as the carrier of the transmission message, so even if it is transmitted in the channel, it is not easy to cause eavesdropper to doubt the active state of the sender. Bob uses the pilot signal to estimate the channel, and reconstructs the message sent by the sender Alice to the greatest extent reliable transmission.
2. filtering
Ocean noise can be divided into low frequency, intermediate frequency and high frequency noise according to frequency. High-frequency noise is generally the sound of mammals, low-frequency noise is generally the sound of waves, the sound of crustal movement, etc. The sound waves of sea waves are about 5-10Hz (this article assumes that the wave sound is 10Hz). This article intends to use the sound of sea waves as a carrier to transmit hidden information. First, use a deep-sea acoustic wave acquisition device to collect ocean sound waves, pass the collected ocean sound waves through a low-pass filter to filter out high-frequency sound waves and retain low-frequency sound waves; then use direct sequence spread spectrum technology (DSSS) [4] with a high bit rate. The spread spectrum code sequence to spread the sound waves of the ocean waves with a frequency of 5 to 10Hz, and spread them into high-frequency sound waves, and then pass the expanded sound waves through a high-pass filter to filter out low-frequency sound waves. At this time, only the collected sound waves remain the wave sound after spreading; finally the sound wave is de-spread with the same spreading code sequence, and the wave sound wave can be obtained.

3. SL0 algorithm optimization

3.1 Background
Compressed sensing theory [5,6] is a brand-new signal sampling technology, it only needs a few sparse signals to use reconstruction algorithms to achieve accurate reconstruction of the original signal. The sparse reconstruction problem is actually the problem of solving the underdetermined linear equation \( y = P \times x \), where \( y \) is the observation vector, \( x \) is the sparse signal, and \( P \) is the perception matrix of \(( m \ll n )\). When the perception matrix \( P \) satisfies the finite isometric property (RIP), the compressed sensing equation \( y = P \times x \) can be transformed into solving the inverse problem, thereby obtaining an estimate of the original signal. The study found that the underwater acoustic channel is sparse, that is, the signal contains only a few non-zero elements. In the OFDM underwater acoustic system under compressed sensing, the pilot signal is used for channel estimation, and the pilot signal is sent at the transmitting end. The process of transmitting the pilot signal in the channel is the process of linear projection, which is the process of signal compression. The length of the time channel is the length of the original signal \( n \), and the observed value \( m \) is the pilot signal received by the receiving end. The channel model at this time is \( y = P \times x + \omega \), \( P = NF \), where \( y \) is the observation vector, \( x \) is the time domain impulse response of the underwater acoustic channel, \( \omega \) is the data symbol of the subcarrier, \( F \) is the discrete Fourier transform matrix (DFT), \( \omega \) is the environmental noise matrix. When using the pilot signal for channel estimation, the position of the data carrier is set to zero, which \( N \) is expressed as

\[
N(k) = \begin{cases} 
N(k) & k \in N_p \\
0 & k \notin N_p
\end{cases} 
\]

(1)

It is expressed as the set of the positions of all sub-carrier pilots \( N(k), k=(1,2,...,n) \) and the matrix form of \( N(k) \) is

\[
N(k) = \begin{bmatrix} 
N(1) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & N(n)
\end{bmatrix} 
\]

(2)

3.2 New objective function
The algorithm has two key issues. One is the selection of a smooth continuous function that approximates the L0 norm (the number of non-zero elements in the vector), and the other is to modify the Newton method based on the selected objective function to achieve more efficient and accurate signal reconstruction. The objective function has been proved to have many options, choose compound trigonometric function to approximate L0 norm [7], choose hyperbolic tangent function to approximate L0 norm [8], based on modified hyperbolic tangent function [9], choose approximate
hyperbolic tangent function to approximate L0 norm [10]. In order to improve the reconstruction performance and get closer to the L0 norm, this paper proposes an optimization function of the approximate hyperbolic tangent function as the objective function of the approximate L0 norm [7], and its function expression is

$$f_x(x_i) = \frac{e^{\frac{x_i}{\sigma}} - 1}{e^{\frac{x_i}{\sigma}} + 1}$$  \hspace{1cm} (3)$$

Among them

$$\lim_{\sigma \to 0} f_x = \begin{cases} 0, & x_i = 0, \\ 1, & x_i \neq 0 \end{cases}$$

represents an element of the random sparse signal $x$ and $\sigma$ represents the control parameter of the function. A smaller $\sigma$, can make the objective function closer to the L0 norm.

Definition $F_{\sigma}(X) = \sum_{i} f_x(x_i)$, so the L0 norm can be approximately expressed as $\|X\|_0 = \lim_{\sigma \to 0} F_{\sigma}(X)$, and the solution of this equation can be expressed as

$$\begin{cases} \min & F_{\sigma}(X) \\ s.t. & y = PX + \omega \end{cases}$$  \hspace{1cm} (4)$$

In the expression (4), the magnitude of $\sigma$ determines the degree of approximation of the function, but 0 cannot be obtained, so a decreasing sequence $[\sigma_0, \sigma_1, \sigma_2, ..., \sigma_\infty]$ can be used to gradually verify the optimal solution $\sigma \to 0$ under different values $\sigma_j$. This article is set $\sigma = 0.1$, the result of comparing the objective function of the text with other functions is shown in the figure below.

![Figure 1: Distribution diagram of each objective function when $\sigma=0.1$](image)

It can be seen from Figure 1 that the objective function proposed in this paper is significantly better than other functions in terms of internal "steepness" $X = [-0.5, 0.5]$, so the performance of the approximate L0 norm is better than other functions. It can be seen from the literature [7] that the approximate degree of approximation of the approximate hyperbolic tangent function is better than that of the compound trigonometric function, the hyperbolic tangent function and the Gaussian function, so it is only necessary to prove that the function proposed in this paper is better than the approximate hyperbolic tangent function. The function expression of the approximate hyperbolic tangent function is

$$\rho_x(x_i) = \frac{e^{\frac{x_i}{\sigma}} - 1}{e^{\frac{x_i}{\sigma}} + 1}$$  \hspace{1cm} (5)$$

might as well make

$$\rho_x(x_i) f_x(x_i) - \rho_x(x_i) = \frac{e^{\frac{x_i}{\sigma}} - 1}{e^{\frac{x_i}{\sigma}} + 1} \frac{\sigma}{e^{\frac{x_i}{\sigma}} + 1}$$  \hspace{1cm} (6)$$

prove that for any $\sigma \geq 0$, $x_i \in [-1, 1]$ have
\( g(x_j) \geq 0 \) constant establishment. Because \( g(-x_j) = g(x_j) \), So only discuss the situation, When \( x_j = 0 \), \( g(x) = 0 \) Satisfy the meaning of the question, when \( x_i \in (0, +1) \),

\[
g(x) = \frac{\|f_s(x) - \rho_\sigma(x)\|^2}{\|e\|^2}
\]

(7)

Simplify and get

\[
g(x) = \begin{cases}
\frac{\|f_s(x) - \rho_\sigma(x)\|^2}{\|e\|^2} & \text{if } x_i \geq 0 \\
\frac{\|f_s(x) - \rho_\sigma(x)\|^2}{\|e\|^2} & \text{if } x_i < 0
\end{cases}
\]

(8)

Because \( \frac{\|f_s(x) - \rho_\sigma(x)\|^2}{\|e\|^2} > 0 \), So only need to discuss the negative and positive of \( e^{-\sigma^2} \), Because \( x_i \in (0, +1] \) there is \( x_i^2 \geq x_i \) always at time, so no matter \( \sigma \) what value it takes, there are \( e^{-\sigma^2} \geq e^{-\sigma^2} \) both \( x_i \in (0, +1] \) inside and \( g(x) \geq 0 \), due to the symmetry of even functions, \( x_i \in [-1, 1] \), have \( f_s(x) \geq \rho_\sigma(x) \), so the approximation degree of the approximate L0 norm of the objective function proposed in this paper is better than other functions.

3.3 The concrete realization of PNSL0 algorithm

The SL0 algorithm uses the Gaussian function as the objective function to approximate the L0 norm, converts the problem of solving the minimum L0 norm into a convex optimization problem, and solves the problem through the gradient descent method. However, the local solution of the gradient descent method is a global solution only when the objective function is a convex function, and in the two-dimensional case, affected by the condition number of the Hesse matrix, "saw tooth phenomenon" is prone to appear, which will cause the algorithm to slow down. It also leads to a decrease in the accuracy of the estimation. In order to avoid the above problems, this paper will use the modified Newton's method based on the objective function proposed in this paper to solve the problem to improve the convergence speed of the algorithm and the accuracy of the estimation.

Knowing that there is noise in the underwater acoustic channel, the form of the L0 norm can be expressed as:

\[
\min_{x \in \mathbb{R}^n} \|x\|_0 = \min_{x \in \mathbb{R}^n} \sum_{i=1}^{n} I_i(x)
\]

(9)

\( I_i(x) = \begin{cases} 1 & \text{if } x_i \neq 0 \\ 0 & \text{if } x_i = 0 \end{cases} \)

First, calculate the gradient descent direction of the objective function in this article \( d_i = -\nabla f_s(x) \), which is expressed as:

\[
\nabla f_s(x) = \left[ \frac{\partial f_s(x)}{\partial x_1}, \frac{\partial f_s(x)}{\partial x_2}, \ldots, \frac{\partial f_s(x)}{\partial x_n} \right]
\]

(9)

Newton's \( d_2 \) direction is expressed as:

\[
d_2 = -\nabla^2 f_s(x)^{-1} \nabla f_s(x)
\]

the Hesse matrix is expressed as
\[ V^T f_x(x) = \begin{pmatrix} \frac{\partial f_x(x)}{\partial x_1} & 0 & \cdots & 0 \\ 0 & \frac{\partial f_x(x)}{\partial x_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial f_x(x)}{\partial x_n} \end{pmatrix} \] 

(10)

Where diagonal elements

\[ \frac{\partial f_x(x)}{\partial x_i} = \frac{v^2}{\sigma^2} + \frac{5}{\alpha \sigma} \left( 10 \frac{v^2}{\sigma^2} + \sigma^2 \right) \frac{m_i^2}{e^m}, \]

which is the diagonal element of the Hesse matrix. Obviously, the diagonal element is greater than 0 and it is always true, then the Hesse matrix is a positive definite matrix. Finally, the corrected Newton direction can be obtained

\[ d_I = -G^T V f_x(x) = \begin{pmatrix} \frac{m_1^2}{e^m} \\ \frac{m_2^2}{e^m} \\ \vdots \\ \frac{m_n^2}{e^m} \end{pmatrix} \]

(11)

\[ 3.4 \text{ Simulation estimation} \]

In order to verify the performance of the PNSL0 algorithm proposed in this paper, we use the BELLHOP model to simulate the dilute hydrophobic acoustic channel [11]. The BELLHOP model [12-14] uses the Gaussian beam tracking method to calculate the sound field in a horizontally non-uniform environment. Verify the performance of the PNSL0 algorithm and compare it with the mean square error (MSE) of MSL0, ARESL0, MRESL0 and MARESL0 under different signal-to-noise ratio (SNR) environments. The expression of the mean square error is

\[ \text{MSE} = E \left[ \left\| x - \hat{x} \right\|^2 \right] \]

(13)

According to formula (13), the smaller the value of MSE, the better the performance of the algorithm. In this experiment, the range of the signal-to-noise ratio SNR is set to, and the result is shown in the figure below.
As can be seen from the figure above, when the signal-to-noise ratio is the same, the MSE value of PNSL0 is the smallest, and its performance is better than several other algorithms. Under the condition of low signal-to-noise ratio, the algorithm proposed in this paper has strong reconstruction and fast calculation speed. Robustness is good.

4. Conclusion
In response to the need for hidden transmission of underwater information, this article starts with the best and most common sound waves in the marine environment, and considers embedding information in the sound of ocean waves, so it is not easy for a third party to discover the transmission of messages. However, there are many kinds of sound waves in the ocean. Filtering technology is used to extract the wave sound from the collected sound waves, and then DCT technology is used to embed the message sequence to be sent into the wave sound, and OFDM technology is used to send the message sequence to the receiver. And then reconstruct the received message sequence based on compressed sensing technology. The new objective function proposed in this paper improves the accuracy of the reconstructed message, thereby improving the reliable transmission of the entire system message; finally the Bellhop simulation experiment is used to simulate the underwater acoustic channel, and the algorithm proposed in the text and the algorithm proposed in the reference are used. By comparison, it is concluded that the performance of the algorithm in this article is better.

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