The Theory of a Heliospheric Economy

Introduction of preliminary frameworks and quantitative methods for modeling the heliospheric conquest

Submitted in Partial fulfillment of the requirements for the degree of the Bachelor of Arts

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1. Lee:2012gx.
2. gerlach2005profitably.
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Dedication

This thesis is dedicated to the memory of Richard Feynman.
Abstract

Despite more than 50 years of human space exploration, no paper in the field of economics has been published regarding the theory of a space-based economy. The aim of this paper is to develop quantitative techniques to estimate conditions of the human heliospheric expansion. An empirical analysis of current space commercialization and reasoning from first economic principles yields an evolutionary prisoner’s dilemma game on a dynamically scaled heterogeneous Newman-Watts Small World Network to generate a new space. The analysis allows for scalar measurements of behavior, market structures, wealth, and technological prowess, with time measured relative to the system. Four major phases of heliospheric expansion become evident, in which the dynamic of the economic environment drives further exploration. Further research could combine empirical estimations of parameters with computer simulations to prove results to inform long-term business plans or public policy to further incentivize human heliospheric domination.
Chapter 1

Introduction

Despite more than 60 years of man in outer space and libraries of science fiction, there have been no serious attempts within the field of economics to develop the theory behind a space based economy, despite a strong body of work demonstrating the commercial and technical viability of this economy. The purpose of this thesis is to begin the development of a theory of a heliospheric economy. In doing so, we lay out a calculus, the mathematical beginnings of a complete logical set, as a way of enabling calculations and estimations around the heliospheric economy. Our hope is that the empirical analysis, economic framework, and quantitative techniques from this thesis can be developed into a theory of the Heliospheric Economy.

1.1 Literature Review

The amount of research regarding space commercialization is relatively small, especially when restricted to the field of economics. We will begin with the economics
1.1. LITERATURE REVIEW

research regarding space commercialization and related fields, then a brief overview of the various feasibility studies produced, before ending with the development of quantitative techniques within the field.

The first work on space commercialization contends that interplanetary and inter solar trade would fall under the context of international trade theory an argument that Paul Krugman is quick to point out is ignored by other international theorists. Further work regarding a road map for heliospheric domination has been published though this has been criticized as being unfeasible. This brings us to the final papers published in the field by Paul Krugman, both of which are intended to be an open mockery of the preceding research. In the first, Krugman introduces several theories of an interstellar economy based on International Trade Theory He manages to solve the faster-than-light travel paradox by citing a theoretical paper written 9 years into the future. In the year 2030, a further paper will serve to validate all of the contentions of this current paper.

We turn then to fields who have given a serious examination of a profitable space exploration. R Buckminster Fuller first wrote of the Earth as a Spaceship, one that would need to be preserved and replenished with resources in the solar system. Further exploration of harvesting resources from Space is present in his work Critical Path in which his analysis of current trends of civilization in various social contexts shows a clear direction towards space exploration and contends that

1. frankel1975there.
2. krugman2010theory.
3. ohlin1933international; samuelson1961foundations.
4. oneill2000high.
5. Stross:NveALLdi.
6. krugman2010theory.
7. krugman1987impossible.
8. Tarler:2030.
9. fuller1978operating.
10. fuller1982critical.
the wealth available is sufficient to enable all of humanity to become trillionaire’s. Further analysis has argued for profitable space enterprises\textsuperscript{11} surveying the mineral riches available from asteroids\textsuperscript{12} and estimating the energy cost necessary for their extraction and transportation\textsuperscript{13}. Synthesis of this information with classic supply and demand models shows profitability from a microeconomics perspective\textsuperscript{14} and with legal frameworks and theory surrounding space commercialization\textsuperscript{15}. The feasibility of a heliospheric economy is well established, enabling the development of an actual theory of heliospheric economy, a serious approach to what Krugman already wrote\textsuperscript{16}.

The mathematical foundations for analyzing the heliospheric economy are already present. The idea of a transaction cost was first introduced in 1937\textsuperscript{17} and then formalized by Williamson\textsuperscript{18}. Further works establish the relationship between resource costs and transaction costs\textsuperscript{19} and the relationship to labor and endogenous firm specialization\textsuperscript{20}. Many of these theories have been validated empirically using historical data\textsuperscript{21}. Transaction cost theory was first unified with evolutionary game theory by Waneryd\textsuperscript{22} who used an evolutionary prisoner’s dilemma game to analyze transaction costs. Further works formalized many aspects of evolutionary game theory in the context of transaction costs\textsuperscript{23} though the game environment remained relatively primitive. The development of the Newman-Watts Small-World

\textsuperscript{11} gump1990space.
\textsuperscript{12} lewis1997mining.
\textsuperscript{13} badescu2013asteroids.
\textsuperscript{14} gerlach2005profitably.
\textsuperscript{15} Lee:2012gx.
\textsuperscript{16} krugman2010theory.
\textsuperscript{17} ECCA:ECCA386.
\textsuperscript{18} Williamson:1981va.
\textsuperscript{19} Pitelis:1999kp.
\textsuperscript{20} Yang:2006th.
\textsuperscript{21} Broadberry:2005bg.
\textsuperscript{22} Warneryd:1994vf.
\textsuperscript{23} weibull1997evolutionary.
1.2. DEFINITIONS AND SCOPE

Network enabled a more dynamic simulation of agents interacting with transaction costs in evolutionary games, with the first work combining the two exploring a spatial prisoners dilemma game on these networks and then furthered with the evolutionary aspect. Articles published recently have examined the effect of increasingly relaxed assumptions upon the game, with asymmetric payoffs the dynamics of cooperation, the effect of network reciprocity with weighted networks and with heterogeneous networks.

This thesis serves as a progression on using heterogeneous networks combined with using a payoff mechanism based on technology and innovation and using transaction costs as a foundation. By combining this with the economic feasibility that has already been well studied, we hope to develop an excellent understanding of the heliospheric economy.

1.2 Definitions and Scope

The heliosphere is a sphere with its origin at sol, and extending outwards roughly 90 astronomical units, canonically the solar system. Heliospheric economics are any commercial activities taking place within this sphere, which includes all major planets and extends outwards encompassing the Kuiper belt. The reason behind this
1.2. DEFINITIONS AND SCOPE

outer bound limitation is because for humanity to start traveling beyond the solar system represents a significant technological jump beyond our current capabilities. Furthermore, technologists such as R. Buckminster Fuller and Freeman Dyson often describe the harvesting the resources of the heliosphere as a necessary step before expanding outwards. All terrestrial economies are included in this definition, but we will make a further distinction to avoid covering well studied ground. All economic activities occurring in Low Earth Orbit (LEO) or further off the surface of the plan, as well as production activities in the supply chain of these activities will be labeled as heliospheric economic activities. Those that are not included in the definition we will call terrestrial-based economic activities. Heliospheric economics does not study interstellar travel or commercialization, beyond the productive activities required to produce these technologies.

The definition we have placed on the volume of these activities will also create a limitation of the time scales studied by heliospheric economics. A one-way journey to Mars would be anywhere from 150-300 days, and so a return journey would be roughly two years, at current technological levels. This may be one economic transaction, not including production. Then heliospheric economics will study time scales on a different magnitude than current day, where years are the smallest discrete time units that make sense, and even longer term analysis is encouraged. As a result, the scaling of time will be based on logarithmic units, or orders of magnitude, similar to physics. Similarly, very long-term behavior will have to be studied and appropriate quantitative techniques developed to enable this.

A further narrowing of the scope is to focus on a primary based heliospheric economy. This economy will be focused on the harvest, refinement, and transport

35. Fuller1982critical, Fuller1978operating.
36. Dyson1960search.
of resources necessary for manufacturing and production based on Earth. We will assume that the minerals and elements harvested in the heliosphere are either not available on earth, or have become so scarce to render earth side extraction nonviable. A point to draw out is that even manufacturing for space-based activities will be based on Earth, including all machines necessary for extraction, refinement, and logistical support. While additive manufacturing may enable some creation of materials at the mining sites, these will be assumed to not a part of the economy, as they will be consumed almost immediately and drawn from resources either transported from earth or harvested on site.

1.3 Structure of Thesis

This thesis can be divided into four sections in order to make the argument. Section’s 2 and 3 establish the assumptions necessary to build the heliospheric model. Section 2 Assumptions from Empirical Analysis, will use existing empirical research to define the legal framework, historical lessons, economic feasibility and technology for the heliospheric economy. This will generate an incomplete set of assumptions about the economy that will not be sufficient for the heliospheric economy. Section 3 on the other hand, will use a reason-based approach to complete the set of assumptions. This will be based on first principles of economics, as well as speculation from Science-Fiction Literature. In both of these sections, assumptions will be laid out in a very clear format for the reader.

Section’s 4 and 5 will focus on the quantitative techniques and subsequent implications. Section 4 will create an axiomatic system for the model based on the earlier assumptions, use these to create and prove several theorems, along with a proof of the existence of equilibrium conditions. In section 5 these theorems will
1.3. STRUCTURE OF THESIS

have key implications and results explored and placed into an economics context.
Chapter 2

Assumptions from Empirical Analysis

While still relatively small, there is a commercial space economy that exists today, with a legal framework and businesses making a profit. This is the very beginning of the heliospheric economy, and an analysis will yield properties of the heliospheric economy to come. We will trace historical economic development of pioneering spaces, with special emphasis on the Westward expansion of the United States, before looking at existing legal, commercial and technological contexts. While the existing legal framework may inhibit heliospheric exploitation, the commercial space industry demonstrates that space-based industry is not only profitable at current technological levels, but also yields insights into corporate relations in the environment.

2.1 Historical

While there is no record historically of human conquest of the solar system, there is a substantial amount of literature on the expansion of civilizations to extract and
2.1. HISTORICAL

exploit new sources of land and augment the economy. This paper will not provide an exhaustive overview of all of these movements in history, but rather will focus on one, the westward expansion of the United States in the 19th century. We can focus on two aspects of this: what the phases of this expansion were, and how this affected the economy of the United States overall. Answering both of these questions will provide key insights into heliospheric expansion.

The expansion into the western portions of the United States was one that started out small, and as travel became cheaper and more accessible this changed. After the Louisiana purchase in 1803, the first explorers were state funded, the Lewis and Clark expedition, who had basic knowledge in the natural sciences to provide, at the very least, an idea of the natural resources to exploit in the land\cite{Fritz:2004td}. However, traversal of the continent remained an expensive proposition, and so only a few pioneers left the Eastern United States to find opportunity. Despite a relatively high failure rate, many were able to be successful in this new frontier. Some found gold in California, others were able to build farms on the great plains. Given the expense to traverse the rocky mountains, the plethora of pioneers settled down to farm in the Great Plains. Towns began to form, and the railway began to be laid so that farmers could sell their goods in the markets of the Eastern United States\cite{Montgomery:2012wq}. As the railway’s expanded to regions farther and farther afield, the railways enabled even greater waves of pioneers, who no longer had to worry about the risk and expense of using a covered wagon to travel the same distance.

This analysis is by no means even a full analysis of the economic forces driving the Westward Expansion of the United States, but these trends can be translated into the heliospheric economy. In order for an economic expansion to occur,

1. Fritz:2004td.
2. Montgomery:2012wq.
2.1. HISTORICAL

there must first be knowledge of what land is available and the potential for it to create sustenance for those use extract from it. Even though the risk of extraction of those resources remains substantial and a certainty prevents most entrepreneurs from leaving civilization, some do take this risk, and the actualization of their profits removes the uncertainty, though the high cost of travel still prevents all but the most dedicated from becoming pioneers. Those who do pioneer settle at the nearest area where they can become successful, reducing travel costs as much as possible. The need to sell their goods on larger markets drives the establishment of infrastructure, so while future waves of pioneers still settle where the travel cost is the lowest, this is farther and farther afield from the original boundary of the host civilization.

We now turn to the effect this has on innovation and technology, in which we turn to Broadberry and Ghosal, who examine the effect with a mathematical model and compare the productivity of the United States to Britain. The model they utilize is similar in ways to the one we will develop later in the paper, though the lack of generalization means that we must derive it in a different manner. They establish four results:

1. “A network may operate a customised venture efficiently through a group reputation mechanism, while a hierarchy may not.”

2. “A hierarchy may operate a large-scale standardised venture efficiently, but a small-scale standardised venture may be operated efficiently only by a network.”

3. “As venture scale increases, a hierarchy is more likely to operate a standardised venture efficiently, while a network is less likely to operate a customised (or

3. Broadberry:2005bg.
4. Broadberry:2005bg.
4. “A network operating a small-scale customised venture may fail to adapt successfully to an exogenous change in technology that favours large-scale standardised ventures, because network members resist the transition to hierarchy.”

A network is a set of enterprises (or, in this sense, just individual business owners) who can provide a customized service do to their expertise in an area, preferably one in which they are the only one in the economic area with this expertise. The profit they make comes from the lack of competition in the service they offer, not from necessarily offering a lower cost service. A hierarchy is a commercial enterprise that offers a homogeneous product, but one that they can produce at high volume. The profit they make is a more traditional economic profit, one that comes from reducing marginal costs below their competitors, rather than offering a unique product within the market. This distinction is critical to understanding the heliospheric economy, as it offers insight into the optimal operating structure for a given economic environment. This transition was brought on, in part, by the development of the railroads. The railroad is a system that must be operated by a hierarchy, as not knowing which train is on which tracks can cause a significant loss of human life. Forever, the reduction in transportation costs allowed for goods to be sold over a larger range of markets, increasing product homogeneity and forcing competition between businesses that were once in separate markets.

Heliospheric expansion will occur under a completely different societal context, and this cannot be emphasized enough. Broadberry and Ghosal base their model off of macroeconomic data from the 19th century, and so to assume the helio-

5. Broadberry:2005bg.

6. Broadberry:2005bg.
2.2. LEGAL FRAMEWORK

Spheric expansion will marginally have the same context as the United States in 1803 is a folly. What we can generalize is some of the mechanisms that drove this expansion, specifically the interaction between the pioneers and the social environment, a process that substantially changed both. Any attempt to model the heliospheric economy will have to make a similar endeavor, though one in absence of empirical data to fit the model to.

2.2 Legal Framework

One of the primary principles of economics is that a legal framework is necessary for an economy to even operate. Indeed, it is impossible to analyze an economy without a understanding of the legal framework surrounding the economy. It is far outside the scope of this paper to propose what the laws ought to be, so instead we will summarize the current laws governing space. In the context of the heliospheric economy, we need to establish what are the current laws governing commercial and economic activities occurring in outer space. Since there are not countries or nationalities in space, the law surrounding it falls under the jurisdiction of the United Nations. This section will serve as a basic summary of what the law states.

The first treaty on Space by the United Nations, called “The Space Treaty” establishes that the heliosphere is, on principle, not a volume where any national boundaries apply. Signed in 1967, the relatively short treaty makes clear that no nation owns or controls any space outside of earth: “Outer space, including the moon and other celestial bodies, shall be free for exploration and use by all States without discrimination of any kind”. This principle is essential, as analyzing the heliospheric

7. Galloway:1967vq.
economy on nationalistic terms will do us no good. The treaty has been signed and ratified by all current nations who engage in space activities. Another treaty of importance is *A Convention on international liability and a convention on the registration of objects*[^8], which establishes standard liability conventions for if, during the act of spaceflight, another nations property is damaged, the nation that launched the space vehicle is liable. However, this only applies to nations and not commercial entities.

Of crucial importance is that the first treaty mentioned establishes principles only and has no enforcement mechanisms to ensure that states do not abuse its principals[^9]. A fifth treaty was developed, called “The Moon Agreement”, which made it clear that while resources could be exploited and mined in space, there was no national ownership[^10]. This treaty has not been signed by any nation that has activities in space. As a result, the laws governing the heliospheric economy are rather anarchic. There seems to be an agreement that no nation state can annex or own a celestial body, and basic provisions on ensuring that nations communicate with each other to prevent accidents or anything of the like.

As it is, the current legal framework for commercial space enterprises is insufficient to promote a robust economy. The space agreement does not even acknowledge the possibility for commercial space exploration, and the moon agreement, while making mention of it, remains to be ratified and simply establishes a commission to develop relevant principles. Furthermore, the liability convention makes no distinction between air travel and space travel[^11]. The distinction between state actors and a commercial entity is also not clear; there are no mechanisms for two corpo-
2.3. COMMERCIAL ENVIRONMENT

rations from different states to settle liability claims. Finally, the complete lack of a patent system or protection of intellectual property serves as a great inhibition to business in space.

It is not in the scope of this paper to define a new legal system. A significant amount of work has already been done in this area, primarily by the author Ricky Lee. Lee argues that certainly the exploration and prospecting of mineral resources is like to be considered legal, several key issues arise. First of all, no state can claim sovereignty over a celestial body. So while a state may mine a resources, and therefore the commercial entities registered within the state, they make no claim over a celestial body. No current examples exist, but based on the deep sea treaties, it appears that a state may still explore, prospect and exploit resources for scientific resources. Any venture in space to exploit mineral resources must 1. Make all extracted and processed resources available on earth at fair market value and 2. Any monopolistic practices may only occur for a specific activity (laying claim to a specific area to mine) and not in general.

2.3 Commercial Environment

The commercial space industry generates 320 Billion USD in annual revenues, or approximately 0.4% of global GDP. NASA further divides the commercial space industry into 4 sectors that actually produce profit which table breaks down.

12. Lee:2012gx.
13. Lee:2012gx.
14. Lee:2012gx.
15. Association:2014wu.
16. NationalAeronauticsandSpaceAdministration:2014wf.
2.3. COMMERCIAL ENVIRONMENT

| Industry Type                               | Percentage of Revenues |
|---------------------------------------------|------------------------|
| Satellite Servicing                         | 50%                    |
| Space Communications                        | 30%                    |
| Earth Observation                           | 19%                    |
| Commercial Launch and Liquid Rocket Manufacturers | 1%                     |

Table 2.1: Major Space Commercial industries by Revenue

Of special note is that of these three categories, three out of the four, or 99% are tied to specific commercial activities on earth. Only commercial launch and liquid rocket manufacturers may engage in activities that promote commercial activity outside of LEO, but no figures are available on this. The only viable activity it would seem is that which augments the terrestrial economy.

Another note is the interaction between the different types of activities. Earth Observation and Space Communications are the owners of the satellites and deliver the finished product to consumers on earth. However, a space communications company may often transport the data from Earth Observation to the ground. Both Satellite Servicing and Commercial launch are also support industries for the satellite industry. Large defense contractors such as Space X or Lockheed Martin dominate the commercial launch sector, yet this makes up a relatively small proportion of revenues for the overall space industry. No one company has a vertically integrated position in the space sector, and especially outside of commercial launch, no one company has a dominant market position.

We will then look at the satellite industry overall, which is by far the dominant aspect of the space industry. We will classify satellites by two different categories: government vs. private, and functional vs. scientifically focused. Scientifically focused satellites are those focused on Space Science, Research and Development and Remote Sensing, and Meteorology which make up 33% of all satel-
2.3. COMMERCIAL ENVIRONMENT

Figure 2.1: Percentage of Satellites in Low Earth Orbit by Origin

Figure 2.2: Percentage of Satellites in Low Earth Orbit by Use
2.3. COMMERCIAL ENVIRONMENT

In addition, commercial satellites are 65% of the satellites in orbit. Figures ?? and ?? show the proportion of each satellite in orbit. Not only is the private sector far more dominant than the public, but commercial satellites drives the majority of space industry revenues.

It is finally worth noting that the commercial space industry is both high growth and sees a great amount of economic impact. The last 5 years of growth for the satellite industry is depicted in figure z, with an average growth growth rate of 8.67%, growing even during the global recession, creating 209 Billion USD in economic activity in the US economy in 2009 and more than 1 million jobs (1.43% of GDP, and responsible for creating 1 million jobs in a year that lost 4 million).

18. Assocation:2014wu.
19. Assocation:2014wu.
20. Assocation:2014wu.
21. Anonymous:qu8PdXQy.
2.4. Economic Feasibility and Technical Viability

The purpose of this section is to establish what would make a heliospheric economy both in terms of economic prospects and technological feasibility. Based on the previous historical analysis, economic expansion is driven by the prospect of greater land resources, with primary production being the initial mode of production. Unfortunately, the environment outside of earth is extremely hostile to most forms of plant life, rendering the agrarian lifestyle largely nonviable. The one other choice is mineral extraction and production as in initial phase of expansion for the commercial space economy. Economic predictions of this type are well established as are engineering studies, so we will summarize these results in preparation for our heliospheric calculus.

2.4.1 Economic Feasibility

The profitability from heliospheric mineral extraction is largely driven by supply and demand. The mass of the earth is 0.002% of that of the non-solar mass in the solar system\textsuperscript{22}. The distribution of mineral resources of that in the solar system, many are available in relative abundance to that on Earth. The limitation of these elements is what would make the heliospheric economy. As these minerals are depleted, the demand for them will not vary though the decrease in supply will drive prices up.

\textsuperscript{22} The mass of the heliosphere is 1.0014 solar masses, meaning nonsolar mass is 0.0014, and the mass of the earth is $3.0 \cdot 10^{-6}$ solar masses. Already expressed in terms of percentages, then \[ \frac{3.0 \cdot 10^{-6}}{0.0014} \approx 0.002\% \]
2.4. ECONOMIC FEASIBILITY AND TECHNICAL VIABILITY

When the product of price and quantity demanded exceeds the threshold of the cost for mineral extraction, then the heliospheric economy becomes feasible.

There are two scenarios on when specific elements of the periodic table will be depleted, based on (then current) consumption rates from 1974. If consumption were to grow at a 10% annualized rate for all elements, then all resources would be exhausted within 250 years from 1974, 200 years within today. Nickel would be the first to disappear, in approximately 83 years. However, based on average annual growth from 1947 to 1974, the main resources to be concerned about are aluminum and platinum, which would likely be depleted in 197 years and 163 years, respectively. Over the past decade, the annualized growth in consumption has been closed to 5%. The following table shows the percent increase in consumption for platinum from 2000-2013. The clear outlier is the year 2009, the global recession, which when removed yields an annualized growth of closer to 5%. According to the optimists perspective, where growth is assumed to be linear, then we can expect to see a shortage of elements within the next several centuries. This drives the price up for mineral extraction in the heliosphere, creating economic feasibility.

The second scenario is the pessimist one, which assumes an exponential growth in consumption, and that market mechanisms will not necessarily correct for depletion. This may be a more realistic perspective, as the existence of macroeconomic cycles (see 2009 recession) shows us that market corrective mechanisms are not entirely efficient. This yields from the uncertainty of the actual quantity of various elements, for example palladium, a member of the platinum group metals, has a significant amount of reserves in Russia, the quantity of which is considered a state secret. It is entirely possible that prices will not increase as the shortage appears, so

23. Lee:2012gx.
25. Nappi:2013ug.
2.4. ECONOMIC FEASIBILITY AND TECHNICAL VIABILITY

| Mineral      | Resource Base | 0%  | 2%  | 5%  | 10% | Exponential | Average Annual Growth 1947-1974 |
|--------------|---------------|-----|-----|-----|-----|-------------|---------------------------------|
| Aluminum     | 2.00E+18      | 1.66E+11 | 1107 | 468 | 247 | 31          | 9.80%                          |
| Cadmium      | 3.60E+12      | 2.10E+08  | 771  | 332 | 177 |             | 4.70%                          |
| Chromium     | 2.60E+15      | 1.30E+09  | 861  | 368 | 196 | 95          | 5.30%                          |
| Cobalt       | 6.00E+14      | 2.38E+10  | 1009 | 428 | 227 | 111         | 5.80%                          |
| Copper       | 1.50E+15      | 2.16E+08  | 772  | 332 | 177 | 60          | 4.80%                          |
| Gold         | 8.40E+10      | 6.28E+07  | 709  | 307 | 164 | 9           | 2.50%                          |
| Iron         | 1.40E+18      | 2.60E+09  | 898  | 383 | 203 | 93          | 7.70%                          |
| Lead         | 2.90E+14      | 8.35E+07  | 724  | 313 | 164 | 26          | 2.40%                          |
| Magnesium    | 6.72E+17      | 1.32E+11  | 1095 | 463 | 244 |             | 7.70%                          |
| Manganese    | 3.12E+16      | 3.10E+09  | 906  | 386 | 205 | 47          | 6.50%                          |
| Mercury      | 2.10E+12      | 2.24E+08  | 559  | 246 | 133 | 13          | 2.00%                          |
| Nickel       | 2.10E+12      | 3.20E+06  | 881  | 376 | 200 | 53          | 7.30%                          |
| Phosphorus   | 2.88E+16      | 1.90E+09  | 881  | 376 | 200 |             | 7.30%                          |
| Potassium    | 4.08E+17      | 2.21E+10  | 1005 | 427 | 226 |             | 9.00%                          |
| Platinum     | 1.10E+12      | 6.70E+09  | 944  | 402 | 213 | 47          | 9.70%                          |
| Silver       | 1.80E+12      | 1.94E+08  | 766  | 330 | 176 | 13          | 2.20%                          |
| Sulphur      | 9.60E+15      | 2.05E+08  | 769  | 331 | 177 |             | 6.70%                          |
| Tin          | 4.08E+13      | 1.72E+08  | 760  | 327 | 175 | 15          | 2.70%                          |
| Tungsten     | 2.64E+13      | 6.78E+08  | 829  | 355 | 189 | 28          | 3.80%                          |
| Zinc         | 2.20E+15      | 3.99E+11  | 1151 | 486 | 256 | 18          | 4.70%                          |

Table 2.2: Table of estimated reserves of key elements, and years to depletion from 1974 for various consumption growth scenarios, and average annual growth in consumption, 1947-1974.\[24\]

No corporation will be ready to capitalize on palladium extraction immediately when the shortage occurs. In addition, demand for palladium may continue to accelerate, so that the growth rate tends toward an exponential one. Another estimation from 1971\[27\] estimates that at an exponential growth, depletion for most elements would occur within the century, with some elemental depletion occurring within a decade of such an event occurring. Exponential growth has not occurred in the time since 1972.\[27\] meadows1972limits.
2.4. ECONOMIC FEASIBILITY AND TECHNICAL VIABILITY

| Year | Percent Change in Consumption from Previous Years |
|------|--------------------------------------------------|
| 2001 | 9.92%                                            |
| 2002 | 4.07%                                            |
| 2003 | 1.99%                                            |
| 2004 | 0.77%                                            |
| 2005 | 10.17%                                           |
| 2006 | -0.94%                                           |
| 2007 | 4.82%                                            |
| 2008 | -3.39%                                           |
| 2009 | -14.96%                                          |
| 2010 | 16.34%                                           |
| 2011 | 2.40%                                            |
| 2012 | -0.80%                                           |
| 2013 | 4.86%                                            |
| Average | 2.71%                        |
| Variance   | 0.57%                                |
| Average w/o 2009 | 4.18%                        |
| Variance w/o 2009 | 0.31%                                      |

Table 2.3: Table of the percentage change in consumption from the previous year for platinum. Average and variance are indicated with and without the extreme outlier year 2009.

this analysis, though these elements are still depleted at an exponential rate. If an exogenous shock were to occur, then the heliospheric economy would rapidly become viable, assuming that the increase in mineral prices does not keep the cost of space commercialization prohibitive.

In terms of a heliospheric economy, mineral extraction is likely to become economically viable within the short term future. The mechanism to create this is a price differential, the shortage of resources will create the economic incentive to expand outwards into the solar system. This gradient, as a key driver of the heliospheric economy, will indicate the velocity of expansion.
2.4. ECONOMIC FEASIBILITY AND TECHNICAL VIABILITY

2.4.2 Technical Viability

In this section, we will discuss some of the technical limitations of a heliospheric economy. Of course, the ability to land a mission on an asteroid was demonstrated by the Rosetta mission\textsuperscript{28} so we will instead discuss the different difficulties of various types of asteroid missions.

Before we begin, we note that the primary energy and technical difficulty in a space mission is moving into Low Earth Orbit (LEO), requiring an acceleration to a velocity of $8.5\text{km}\cdot\text{s}^{-1}\textsuperscript{29}$ with transfer to other celestial bodies requiring a fraction of that amount. We can expect the development of some sort of station, vastly reducing energies and serving a first point of infrastructure. Since our paper deals exclusively with economies at Low Earth Orbit and beyond, we will assume this is the case. Ranging outwards from Earth, various points for mineral extraction requires increasing amounts of energy and technical knowledge to acquire. The simplest are near-earth asteroids and the moon, both of which posses easy orbital trajectories to establish, and with mineral extraction being relatively energy efficient at these stages, are feasible within current bounds of technology. Outwards to Mars, since no orbital adjustment is required, simply achieving an escape velocity from Earth ($3.7\text{km}\cdot\text{s}^{-1}$) is sufficient to reach the destination. However, due to its high mass compared to the asteroids, a large energy expenditure of around $8\text{km}\cdot\text{s}^{-1}$ is required to return, leaving the total energy expenditure of the trip (not counting transfer to Earth orbit) roughly equivalent to that of achieving escape velocity from Earth, at around ($11.7\text{km}\cdot\text{s}^{-1}$ versus $11.8\text{km}\cdot\text{s}^{-1}$). The main asteroid belt, between Jupiter and Mars, with again similar mineral refinement energies required, and a round trip

\textsuperscript{28} Chang:2014vw.

\textsuperscript{29} Recall that kinetic energy $k_e$ is equal to $k_e = \frac{1}{2}mv^2$
total energy requirement of roughly $6 \text{ km} \cdot \text{s}^{-1}$ is rapidly achievable with current or near future technology. Beyond this point, the energy requirements being to grow rapidly. Harvesting the mineral resources of the celestial bodies around Jupiter requires the ability to generate energy sufficient to escape, requiring acceleration to $59.6 \text{ km} \cdot \text{s}^{-1}$ to escape\footnote{Chartrand:2001ws}. Another issue arises in that the energy required to process an ore, is a function of the purity of the ore as well as the enthalpy (change in heat) required. Throughout the solar system, minerals exist in far greater purity than those available on Earth, but the farther away from the sun, the background temperature of the vacuum becomes even cooler, and the ability to extract energy from sunlight becomes another technical challenge. Other challenges arise as one explores towards the sun. While the energy cost of refinement lowers, the energy cost of returning to Earth increases. The technical challenge of mineral extraction in areas beyond the main belt of asteroids is considerable.

Despite this fact, the actual distribution of mineral resources differs in the

\begin{table}[h]
\centering
\begin{tabular}{|l|c|}
\hline
Transfer & Energy Requirements ($\text{km} \cdot \text{s}^{-1}$) \\
\hline
Surface of the Earth to Low Earth Orbit & 8.5 \\
Surface of the Earth to escape velocity & 11.2 \\
Surface of the Earth to geostationary orbit & 11.8 \\
Low Earth Orbit to escape velocity & 3.2 \\
Low Earth Orbit to Mars transfer orbit & 3.7 \\
Low Earth Orbit to geostationary orbit & 3.5 \\
Low Earth Orbit to highly elliptical Earth orbit & 2.5 \\
Low Earth Orbit to landing on the Moon & 6.3 \\
Low Earth Orbit to typical Near Earth Asteroid & 4.0 \\
Surface of the Moon to Low Earth Orbit (with aerobraking) & 2.4 \\
Typical Near Earth Asteroid to Earth transfer orbit & 1.0 \\
Phobos/Deimos to Low Earth Orbit & 8.0 \\
\hline
\end{tabular}
\caption{Energy Requirements for various missions\footnote{Chartrand:2001ws}}
\end{table}
these three, rough, areas for extraction, meaning that there will still be a drive to overcome these technical challenges. The point of this section was to demonstrate that mineral extraction is a possibility with today’s technology, with further exploitation requiring greater technological advancements.

So we come to the limitation of using empirical analysis and existing research to study human exploration of the heliosphere. We have shown historical factors driving societal conquests of new frontiers, the economic forces driving this expansion as well as the way corporate entities shifted. A basic legal system for space commercialization already is in existence, and while not entirely sufficient, it is beyond what existed in the early days of the Westward expansion of the United States. The market structure of current commercial space endeavors may be limited, but it shows quite clearly the necessity for an initial space based economy to supplement the Earth. In demonstrating the feasibility of the economy and technical viability, we can conclude that the reason why a heliospheric economy has yet to develop is for economic factors. While this establishes boundaries for a heliospheric economy and some of the driving forces, we are still inconclusive on several issues. It is not yet clear how these might develop in an environment entirely different to that on Earth. In the next section, we will answer this question in totality.
Chapter 3

Framework for the Heliospheric Economy

The heliospheric economy, by its very nature, requires a different framework for reasoning about it than conventional economics. The majority of the difference arises from a different treatment of the nature of time within the context of the economy, however this change implies significant changes to other fundamentals of economics, largely surrounding that of fundamental resources and how we consider the economy. While the paradigm of modern economics remains largely the same, the change in our axiomatic consideration of time leads to a principles that, while functionally similar to that of current economic thought, occur in a much different time scale. We will show that when considering longtime, a relational framework is necessitated rather than an atomistic one, and the examination of individual actions falls away. An evolutionary paradigm is the one that best fits these changes, and so require a reconsidering of the fundamental resources in economics: land, capital and labor. Resource allocation and scarcity cease to be the central problem, but a more nuanced consideration of technological innovation and entrepreneurship serves
3.1. TIME

as the underlying principle that ties everything together.

This section serves as a justification for the underlying mathematical structures of the heliospheric economic calculus introduced in the next section, while the previous section establishes empirical first conditions and the specific form that these principles manifest within.

3.1 Time

The development of the heliospheric economy will occur on a time scale greater than what most economists typically use. A one-way journey to Mars may take anywhere from 150-300 days to complete, indicating that it may take five years, from launch to the delivery of minerals to Earth, for a mining expedition in the asteroid belt. This five years represents one economic transaction, where a company produces and transports a product to the market in which it is sold. Compared to a simple economic transaction happening almost instantaneously on Earth in the age of the Internet, and we can already see the difference this might make.

Then a macroeconomic cycle, representing a bundle of economic translations, typically has a period less than 10 years on Earth. A similar analogue in the heliospheric economy may take decades to centuries. Yet we come to the first fundamental difference in our framework; macroeconomic cycles, when averaged out over the long run, tend towards a linear upward trend. The time scale of the heliospheric model will then be unable to look at cyclic periodicity of the macro economy, but rather when this long-term linear trend changes significantly. Perhaps an example is the economic development of the western world after the downfall of the Roman empire. While it is impossible to do a quantitative analysis of world economic output
over that time, humanity say a significant decline in economic stature along with a fragmentation of political structures. The trend then changed, with the steady unification of smaller states into powerful empires with strong economic utilization of resources found all the way around the Earth. In the context of heliospheric economics, two thousand years has one significant event: the reversal of these trends, and the conditions which caused it.

The direct result of this is it allows us to view the heliospheric economy through an evolutionary framework, which while a comprehensive understanding of the Darwinian laws is unavailable in the context of human society, the mathematical relationships are well understood. The importance of the actions of the individual cease to have relevance in this context, where the lifespan of a human being is smaller than the time scales we have determined, so we instead extend corporations, which do not have a biologically limited time span. Even in this case, atomistic actions have very little influence on evolution, unless the corporation controls a significant amount of societal resources. What matters instead is the long term relationships between corporations, especially as the form coalitions and groups large enough to have an impact on the evolutionary environment.

The implications of these principle, that evolution drives the development of the heliospheric economy, are important. First and foremost, the precision of the calculus is changed. We cannot predict exact events of the heliospheric economy, similar to how no one could have predicted the third Reich after the Roman empire until after the fact. The framework we are driving at can identify major trends, phases, and equilibrium points, yet no exact time line can be identified for these occurrences.

Evolutionary is driven by a series of random mutations that affect the abil-
ity of an organism to thrive in an environment, and similarly random events must be well incorporated into our heliospheric economy. The environment then becomes an absolutely essential component to examining the heliosphere, as changes to the environment will produce different optimal operating traits. The key difference between evolutionary biology and evolutionary economics is that humans tend to change the environment, and also are skilled at adapting to new environments. This cyclic relationship is critical to understanding the heliospheric economy, and produces greater accuracy of any model. This is because an optimal set of strategies introduces a long term trend in the economic environment, which humanity then adapts their strategies to retain optimality. This is the mechanism that drives change in the heliospheric economy, and will be expounded upon later in this section.

A final point to make is that relationships between agents drive changes and trends. One person alone does not produce the phenomena of global warming, but the societies they form, as a product of their relationships, do cause anthropic climate change. These relationships are a close analogue to replicator dynamics in evolutionary biology. Relationships change the nature of the individuals, as humans learn from one another. This learning is key, but it is a random process often based on a perceived superiority in the others traits, and relationships themselves are random. Probabilities are a key aspect of the heliospheric economy, and while in the long term they may happen on a predictable basis, the short term offers no insight into the actions of the individual. These relationships are finite, and the chaining of relationships does link every human, or corporation of the world. We can then look at these relationships in a small world network; while everyone is connected, the number of connections between two unrelated individuals can be large. While the hypothesis of six degrees of separation makes sense in this context, the degree of separation can vary widely even today, and may change in evolutionary time scales.
3.2 Fundamental economic resources

Before we expand on the relationships between corporations, it is first important to understand the interaction between them and the environment. There are three essential classes of sustenance for the corporation: land, labour, and capital. Just to clarify what these mean in current economics, land is the raw materials utilized in production, labour is the human power necessary to refine these materials into a marketable product, and capital serves as the machinery and other resources, often measured in currency, required to facilitate production. This analysis will show how the properties of each of these change in the heliospheric context.

3.2.1 Land

Terrestrially, land is a scarce resource, often being the driver of wars between nation states. Indeed, it is finite meaning that at some point it will be exhausted. This is coupled with the fact that the extraction of resources in the land can be destructive, a negative externality that further reduces the amount of resources available. While this is a controversial point in the modern era, especially in regards to the timing of political moves to plan and ensure the long-term sustainability of any economy. Debates rage around the true impact of the negative externalities as well as the time that specific resources will reach the limit of their extraction.

In the heliospheric economy, the magnitude of the resources available creates a significant shift in the consideration of this class of resources. While land still remains finite, it is possible for one or multiple agents to harvest an entire celestial body. Scarcity only materializes in the transportation time to farther celestial resources, rather than those resources themselves. However, while distance
may increase linearly, the volume of resources to be harvested at a given distance, theoretically, grows at a cubic rate\textsuperscript{[1]} Furthermore, the negative externality is removed; complete destruction of a minor asteroid would have very little effect on the environment, in this case the ability of humanity to efficiently survive in the heliosphere. While effects may still remain, one can reason significant different arising from a shift in the mass distribution affecting gravitational trajectories.

Before moving onto other classes of resources, the primary principle to be abstracted from this is that transportation and distribution, not the amount, affect the scarcity of land within space. An increase in transportation technology then removes then increases the amount of resources available and reduces scarcity. In all practicality, the amount of resources is only a function of the technological level.

\subsection*{3.2.2 Labor}

On Earth, the concept of labor is that a corporation pays an amount sufficient to guarantee the survival of the laborer in exchange for the products that are the result of their refinement of materials. The cost of labor to the corporation is the cost to pay for the sustenance of the laborers\textsuperscript{[2]} Labor trades off with capital, as capital becomes more efficient and goes below this living sustenance cost, then capital replaces the labor power traditionally used. Again, controversy does reside around this idea, which could be considered one taken from the Marxist paradigm. However, ensuring the livelihood of its laborers is a necessary condition to ensuring production. The specific conceptualization of knowledge workers, or human capital, is important to

\begin{footnotesize}
\begin{enumerate}
\item The relationship here is that the volume of a sphere, $V = \frac{4}{3}\pi r^3$ grows at a cubic rate compared to the growth of the radius $r$. Removing the $\frac{4}{3}$ term is a valid assumption as the distribution of land in the solar system is not homogeneous
\item While wages are not based solely on this calculation, the concept of a minimum wage is
\end{enumerate}
\end{footnotesize}
In the heliosphere, labor becomes astronomically expensive. The harsh environment of the solar system requires a significant amount of resources just to prevent instantaneous death. Radiation shielding and maintaining a pressurized breathable atmosphere make shelter itself much more costly than that on earth. While water is available in abundance throughout the solar system, a supply must be maintained and food either must be transported, or produced at a cost greater than that of food produced on Earth. These resources all add mass that must be transported to and from a mining site, increasing the cost of fuel to get, and the tendency of extreme acceleration to cause human death also increases travel time. This is not to say that labor will not be present in the heliospheric economy. The is an inherent drive in humanity to see the stars meaning that early stages of development will likely have a higher proportion of labor to equipment. Even ignoring this, human maintenance will be required to keep machinery operational as well as to supervise and make decisions on site, as remote operation still can only occur as fast as the speed of light. That is, a two-way communication takes at least 90 minutes to travel between the Earth and Mars, and using this to directly control machinery without autonomic decision-making capacity could result in disaster.

The point being is that the relative proportion of simple labor compared to capital in production will be small. However, knowledge workers and human capital essentially make up for this difference. Autonomic operation of mineral extraction capital may be developed, but human ingenuity and design of those programs is necessary. For these economies to become efficient, technology must be developed in order to make them efficient. Even as labor on-site scales back, this will be coupled with an increase in the workers producing technological development within a corporation. As a side effect, these knowledge workers can be based on the planet,
meaning that the real cost of them will be far lower and almost ensuring that technology will be a key driver in the heliospheric economy. Using capital to license more advanced technology may serve as a short-term replacement to human ingenuity, but we can be assured the presence of a significant amount of human capital until we can develop artificial sentience. This sort of speculation is beyond that of this paper, but does offer a boundary condition to our reasoning.

### 3.2.3 Capital

Capital is the last resource we are to examine. While capital is often measured in currency, it is important to realize that capital serves as a measure of the potentiality of a production. Currency is easily convertible into the machines and tools required to produce a product terrestrially, and corporations attempt to maintain a strong cash reserve to convert to these productive processes. Terrestrially, conversion of capital into a product is assumed to be efficient and homogeneous across corporations. The consumption of capital generates the energy required for the survival of the corporation. Using capital as a means to compare various corporations implicitly assumes that it can be converted into production of equal capacity and efficiency as any other corporation using capital.

Further examination of this relationship is key to the heliospheric economy. Capital is converted into products, which are then sold to maintain the corporation, similar to how the consumption of food in a living organism is converted into energy. In the heliosphere, this conversion is not nearly as efficient, owing to the large cost of transportation and the low technological level of space-based production techniques. Our assumption that mining will be an initial industry for the economy ensures that
the product is homogeneous, and so competition for reducing costs to mine celestial
dodies is paramount. Capital itself is homogeneous however, ensuring a relatively
boring economy. Competition comes in the place of the conversion of capital into
products; to make a profit a corporation must either ensure that their conversion
is more efficiency (minimize marginal costs) than other corporations in the market,
or by developing a conversion process that is unique and creates a unique product,
such that efficiency in processes is not as necessary as there is no competition.

We define these types of capital and their corresponding conversion mech-

anisms as ordinary capital and space capital. Ordinary capital is identical to current
conceptions of capital and hence its name. Many corporations produce a homoge-
neous product, so profit maximization comes in the form of reducing marginal costs.
This capital conversion is then a function of the mastery of existing technologies.
This capital corresponds directly with regular labor.

Space capital, on the other hand, is based on the development of processes
to extract and harvest raw materials that are unique to the market. The intent is
to be the first to the market, rather than better than everyone else on the market.
Space capital is then the product of the knowledge workers, or of human capital.
The previous work on the scarcity of land indicates that space capital is far more
profitable than regular capital in the heliosphere. An incremental increase in space
capital results in a cubic increase in profits, whereas a linear increase in regular
capital results in a linear increase in profits.

We have mentioned the environment several times in this section while
intentionally failing to give it more meaning. The environment contains a full allo-
cation of land, readily converted into profit, through labour and capital. However,
labour and capital must be spent to achieve this, so while technological advance-
ment can change the physical time to reach land, the capacity to acquire labor and capital during this time is very important. So the wage level and interest rate serve as an impedance in the environment; the greater they are the more expensive it is to extract from land, and slowing down the time it takes for corporations to conduct economic transactions. This increases the length of the relations between corporations, creating greater economic fulfillment within them, but decreasing the possibility of these connections. This serves to model inflation and its effect on the heliosphere quite well, as well as the rate of learning. The aggregate effect of the corporations then produces an environmental change, to which they change their strategies, which facilitates another environmental change. This dynamic is what ensures the continual progression of the heliospheric development, and it is easy to identify this effect in contemporary economies.

At this point, the relationship between land, labour, and capital is an important one to expound upon. Land has an increasing abundance with the distance from Earth. At a fixed distance, there is a finite and identifiable scarcity of resources, which is where regular capital and labour take effect. To compete with corporations at an identical distance, marginal costs must be reduced, which is brought about by creating efficiency in the consumption of regular labor and capital. However, the ability to move operations to a greater distance from earth increases the abundance of resources and lowers the number of competitors present in the market. This shift, however, requires human capital and space capital to be made. Regular capital and labor are based on the mastery of the heliospheric economy at its current development, and space and human capital are an advancement of the conquest of heliospheric space. Reconciling these two different processes is the focus of technology in the next section.
3.2.4 Currency, Accounting, and The Economic Transaction

We are going to use a similar assumption to previous works from engineers in regards to the currency used in a heliospheric economy, which is to consider energy as the underlying metric for value within a heliospheric economy. This makes sense in several fashions, first of all with the long-time that we have defined, inflation becomes very difficult to model, especially considering that many currency revaluations may occur in one time period. While currency may undergo inflation, the first law of thermodynamics states that neither energy nor matter may be created or destroyed. We can consider the actual resource allocation for a company to be the potential energy, where currency represents energy and physical resources are matter. The process of production is converting that energy into matter, and the process of selling a product is conversion of that matter into energy once again. While the variations in transactions can vary considerably, over the long run this is a very appropriate way at modelling heliospheric economies. Similar to how the heisenberg uncertainty principle prevents total measurement of particles, the actions of these particles over time and in sufficiently large quantities was enough to derive classical physics. We are ultimately doing something very similar.

As a result, the efficiency of each one of these processes is paramount. Keep in mind that ultimate efficiency of conversion is given by

\[ E = mc^2 \]

where \( c \) is the speed of light. Inflation occurs as less efficient processes become more efficient; the total loss of matter and energy in the conversion is less, making the

3. Lee:2012gx; gerlach2005profitably; Sanchez:2011fv.
3.2. FUNDAMENTAL ECONOMIC RESOURCES

amount available greater. However the total amount of resources introduced into the system has not changed, just wastage is greatly reduced. Costs can be defined in two parts, the transportation as well as mining and refinement. For transportation, we are given the the formula

\[ F_t = U_l + U_r \]

where \( U_l \) is the energy impulse required for launch and \( U_r \) is the energy for a rendezvous trajectory. Table 2 gives the breakdown for these energies. Remember that \( 2F_t \) is the total transportation cost for a full mining mission. In addition, there are the mining and refinement costs, \( F_r \) given by

\[ F_r = \frac{E_0}{g \cdot \eta_1} + \frac{\Delta H}{\eta_2} \]

where \( E_0 \) is the energy for ore beneficiation, \( g \) is the grade of the ore, \( \eta_1 \) is the efficiency of ore beneficiation, \( \delta H \) is the enthalpy of smelting and \( \eta_2 \) is the efficiency of the process. Finally, we have

\[ C = 2F_t + F_r \quad (3.1) \]

Ultimately, the presents a mass of ore that has been refined and transported back to Earth for selling. We will consider the revenue to be

\[ R = \frac{1}{2}mv^2 \]

where \( v \) is the velocity of currency at that time (in reality, a function of supply and demand, and harking back to previous economic approximations of inflation\[4\] As a

4. soter2001near; Sanchez:2011fv.
5. Johnson:2001.
6. Fleming:1962be.
result, if we consider the price vector \( \vec{\pi} = \langle \pi, \pi \rangle^T \) and \( v = D_{\vec{\pi}} \), then we can see how a price differential between heliospheric we can identify how prices affect the relative revenue and then profitability of a space-based enterprise.

3.3 Technology

At this point, it should be apparent that the central problem of the heliospheric economy is not the scarcity and allocation of resources, but rather the scarcity and allocation of technology. Before examining the role this plays on the heliospheric economy, we must give a rigorous examination of what technology is.

Technology serves a role as the set of traits in the evolutionary approach, a qualitative collection of knowledge that in some way enhances the production process. However, technology is not developed atomistically, it is rather the combination and advancement of a vast set of technologies that preceded it. The invention of the wheel was a necessary precursor to the automobile, an invention with vast importance but many other technologies involved in its conception. The idea of a precursor and a successor technology is vital, a wheel is primitive compared to that of the automobile. So we can arrange these technologies according to levels, corresponding to how many precursor technologies are between it and a base technology. However, not every technology can count every technology at a lower level as a predecessor. We can then group these into technological branches, technologies that all share a common predecessor, but technologies in different branches share no common technology. This necessitates the conception of a zero technology, the technology that is the predecessor to all technologies. We assume the existence of this technology to make

7. : Earth.
: Heliospheric
the mathematics work, but refining the zero technology is a process best left to philosophers.

In addition to creating new, more advanced technologies, technologies at the same level can be combined into other technologies as well, even with unrelated technologies. Taking the concept of the automobile, and combining it with a cannon creates the tank, a concept very different from the car but the product of two other technologies. It may be tempting to say that the car was the predecessor to the tank, but the tank is not a more advanced functionality of the car, but a different one altogether. The tank can be considered as an advancement to horse drawn artillery however, and conceptualizing a world where the tank was invent before the car is possible, as the car is an advancement of the horse drawn carriage. In actuality, the automobile serves as the advancement to horse drawn carriages. Distinguishing that horse drawn mobility, is the shared predecessor between the tank and the car. This succeeded to the internal combustion engine, so three distinct technologies all share a common predecessor. The car and the tank are mastery of the internal combustion engine, and the internal combustion engine is the advancement of horse-based locomotion.

We then define three different types of technologies for each level. An innovative technology is defined by having the most number of connections both to technologies within its level, and to creating a very diverse array of successor technologies. An example of an innovative technology is the computer. This is in contrast with a fringe technology, technologies with a relatively low amount of connection and which span a far smaller amount of successor technologies. These technologies are important still but do not factor much into the advancement of technological progress. Anything that is not an innovation or a fringe technology is just a regular technology, with a median amount of connections. Due to patent system, entire tech-
nological levels eventually become part of the public domain, but new advancements of technology remain proprietary for some time to come. Even if a new technology is developed that is currently at this level, it can be derived from existing technologies with expired patents, essentially categorizing it as being in the public domain as well. However, the development of technology by a corporation remains private, creating a fragmented set of advanced technologies from branches of technologies in the public domain. This creates a series of fragmented technological sets, where companies may have only a small portion of the technological level they reside in. This heterogeneity of technology forces interactions between corporations. The decision to license another corporation’s technology may be essential for the productive process involving that corporation’s already developed technology. The combinatorial aspects of technology also means that a full collaboration of technology may allow for the mastery of technologies of levels not in the public domain and involving branches the two initial corporations had not yet developed. These two interactions, licensing and collaboration, while being a crude categorization of corporation technological sharing, serve as an excellent starting point for their examination over long periods of time. In truth, collaboration has analogues to the mating process to share genes in evolutionary biology, and licensing is similar to organisms sharing resources with one another without the exchange of genetic material. Both are absolutely essential to the propagation of a species, but while the first advances it, the second only maintains short-term survival.

The technological space itself is infinite and with no existing maximum, and so as technology advances the fragmentation of any one possessor of it becomes greater. In today’s age, oftentimes disciplinary collaborations are required to produce new academic truth, which serves as an excellent example of the process we are describing. Since corporations have no biologically unlimited lifespan and near
3.3. TECHNOLOGY

perfect record keeping, the capacity to store and retain knowledge is far greater, but producing it is still a human process. Technology is what enables the profitability through the consumption of labour and capital resources, and so corporations with larger technological sets will have a greater profitability in all situations. Technological mastery makes the basic components of manufacturing more efficient, regular capital and labour, but human capital drives technological advancement to produce the space capital to harvest more distant resources.

Technology is the new scarcity, as it removes the scarcity of land and ensures greater profit-making potential. Corporations still strive to maximize profit, but the process of technological advancement versus mastery differentiates the type of profit they can produce. Technological mastery ensures market saturation and drives down marginal costs, but as this process occurs the marginal profit becomes greatly diminished. This drives technological advancement, which while opening up new products, will eventually see a return to technological mastery to drive down marginal costs as others expand into this new space. No matter the situation, expanding the technological set is what enables greater profits.

So concludes our framework for examining the heliospheric economy. This framework is what serves as the basis for the heliospheric calculus in the next section, which creates a mathematical basis for working through the framework. While we now have a framework for examining the human conquest of the heliosphere in evolutionary time, keep in mind that this process is still fundamentally a random one, with different assumptions regarding economic resources, but it must be based on starting conditions surrounding the economy. Where possible, this section attempts to avoid them, except where necessary as enabled by Lee’s work. The derivation of the calculus following is one more routed in application, combining the empirical analysis in the preceding section to assign mathematical structures to framework.
3.3. TECHNOLOGY

They also establish initial conditions, which the interaction between the agent and the environment necessitates. The calculus can easily be adjusted to include more nuanced assumptions, developed from this framework. However, in reading the next section, it is THIS framework that drives the mathematical relations, with the empirical analysis creating the objects which have less importance.
In this section, we present the derivation of a mathematical calculus which allows for the determination of several properties of the heliospheric economy. That is, the quantitative methods developed below can fit a wide variety of assumptions about the heliospheric economy, not just the ones this paper makes. While our process is an axiomatic one in nature, we make no claims regarding the completeness about the set of axioms chosen, and only go so far to show that our functions are well defined, but do not prove any significant theorems from the calculus.

A calculus is the logical subset of a first-order logical system, a set of rules corresponding to each of the constants, variables, or functions for the system. Further exploration of this set, $T^*$, yields the theorems within the system. The scope of this paper is simply to define these rules and explore these properties, without attempts to complete the logical system as a whole. This is why we call the paper a calculus.

The derivation presented here is meant to give a ready an idea of the
basic concepts necessary for using the calculus, rather than presenting a rigorous derivation. These are presented in full detail in Appendix ??.

The calculus itself was designed with the intent of several properties. It balances using mathematical structures to model more complex and nuanced assumptions, with calculations that are easily made with current computing power. The structures can be easily modified by changing the variables of various parameters, or be expanded for as many dimensions as necessary. In addition, probabilistic parameters allow for a great many of scenarios to be simulated. The focus of the calculation is upon relationships between companies, and this makes macroeconomic variables calculated with dynamic interactions between the agents and the environments. However, the calculus sacrifices precision for this flexibility. Time is only introduced in relation to the model, so no time measurements can be ascertained. The derivation operates on assumptions that the nature of human interactions or economies will not change, and in ignorance of possibly disruptive exogenous events. While the model operates on time scales of centuries and can handle a moderate deviation from current behavior, but if an extreme deviation occurs then the calculus breaks. No attempts at boundary examination are made.

The calculus is based on an evolutionary game theory model, where companies use a set of technologies, or traits, that enable them to more efficiently utilize the resources they have, capital and labor. They are attracted towards potential profit, which can accelerate them towards technological advancement or mastery. Companies compete in this game to get a larger technology set, as this augments their economic capability. The game is essentially a prisoners dilemma game, where a defect is the licensing of a technology and cooperation involves sharing technology sets. We use line integrals to measure properties of their technology sets which affects the size of their wealth allocations. They play the game in a small world network, in
4.1. OVERVIEW OF THE QUANTITATIVE METHODS

which the interest rate and wage rate affect the length of the connections between them. Interest rate and wage rate also affect the distance between technologies, and hence the amount of time it takes for a company to endogenously grow their technology. The connections in the technology set then approximate a company’s internal investments, while in the small-world network this represents connections with external companies. To measure the aggregate economy, one only has to sum up those connections. After every turn in the game, each company’s wealth allocation is modified to reflect the technologies they achieved. Along with a mechanism to reflect learned behaviors, the tendency of the environment to change according to agent actions, and several probabilistic mechanism create a robust and varied evolutionary equilibrium, where established Evolutionary Stable Strategies (ESS) dominate and the system in equilibrium until the environment changes enough such that an invader strategy becomes more profitable, establishing a new dynamic equilibrium. These equilibria enable the easy determination of probabilistic scenarios.

4.1 Overview of the Quantitative Methods

The point of this calculus is to easily calculate the properties of vectors \( \vec{H} \). The vector space \( \mathbb{H} \), the human vector space, is unusual in that it is the product of several other spaces

\[
\mathbb{H} = T \otimes A \otimes S \otimes W
\]  

(4.1)

Where \( T \) is the technological space, \( A \) is the Agent space, \( S \) is the strategy space, and \( W \) is the wealth space. For a generic \( \vec{H} \in \mathbb{H} \), this represents the trajectory for a time \( t \) for human behavior, wealth and technological prowess. As such we will be able to calculate the following quantities:
4.1. OVERVIEW OF THE QUANTITATIVE METHODS

- $\nabla \vec{H}$ is an indication of the equilibrium of the system.
- $\kappa(\vec{H})$ is the rate towards the next evolutionary change.
- $\partial \vec{H}$ is the trajectory for market structures, strategic behavior and technology for the variable taken.
- $\int \int \int \vec{H} d\mathbb{H}$ is the economic potential of the heliospheric economy, as well as the total wealth accumulated.
- $\int C \vec{H} d\mathbb{W}, A$ represents the actual GDP of the heliospheric economy.
- $\int C \vec{H} d\mathbb{W}$ represents capital or labour spending within the economy.
- $\frac{\partial}{\partial K} \frac{d}{dT} \int C \vec{H} d\mathbb{W}$ is the change in interest rates.
- $\frac{\partial}{\partial L} \frac{d}{dT} \int C \vec{H} d\mathbb{W}$ is the change in interest rates.
- $\frac{\partial}{\partial S} \frac{d}{dT} \int C \vec{H} d\mathbb{W}$ is the change in strategy over time.
- $\frac{\partial}{\partial A} \frac{d}{dT} \int C \vec{H} d\mathbb{W}$ represents the change in market strategies.
- $\kappa(T)$ is the momentum of the technological space.

The rest of this section will go through the derivation of this vector space $\mathbb{H}$. For $A, S$ and $T$, these structures are not vector spaces themselves, but through various tensor products with the wealth space we are able to construct $\mathbb{H}$. A very careful note however, time is actually a factor of all these spaces and is endogenous to the system we are examining. In no way is it continuous. Once these spaces have been constructed, the final construction of $\mathbb{H}$ will be done as a proof. The intent of the derivation is readability, and so the underlying mathematics has largely been shifted to the appendices.
4.2 Wealth Space

The wealth space is by far the simplest space we will construct. There are two independent dimensions within the wealth space, capital $K$ and labour $L$, where the basis of the space is

$$\{(k,0), (0,l)\}$$

where $k,l \in \mathbb{R}$ are the unit costs of capital and labour. Then, we can assign coordinates to anything in the wealth space, simply by $W \mapsto \mathbb{R}^2$. While capital and labour are supposed to be uniquely positive quantities, when $k,l \in \mathbb{R}^+$ this represents a revenue, and when $k,l \in \mathbb{R}^-$ then this represents a cost. Notice the distinct absence of the time variable $t$. Since $W \simeq \mathbb{R}^2$, it is a full vector space. The importance of this space cannot be understated, as the tensor product with many spaces grants a continuous or quasi-continuous distance metric.

To factor in a time variable and expand $W$ to $\mathbb{R}^3$, we maintain the current axis, but relabel them to $k_r$ and $l_r$, which are ordinary labor and capital. This is meant to be capital and labor used for production at a time $t$, with existing productive capacity. $l_s$, human capital, and $k_s$, space capital, are capital for future production. These independent of regular capital and labour, so we set $t = \sqrt{k_s^2 + l_s^2}$ and set the basis to be the vectors

$$\{(k_r,0,0), (0,l_r,0), (0,0,t)\}$$

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4.3 Technological Space

The technological space is a partially ordered set of all technologies, with the ordering relation \( \leq \). This is an equivalence relation, and \( \tau \leq \tau' \) when \( \tau \) is a predecessor to \( \tau' \). Then the dimensions of \( T \) are two, \((A, M)\) where \( A \) is the time axis and \( M \perp A \) \( \tau' \) is more advanced, and so we say that \( H(\tau') - H(\tau) = t \) where \( t \) is a unit of time.

Time is a blanket variable, incorporating risk factors and other costs. In addition, we establish a zero technology \( \tau_\perp \) such that

\[
\tau_\perp = \bigvee_{i=1}^{\infty} \tau_i
\]

(4.4)

This is necessary for establishing algebraic operations with \( T \) but not for calculation.

We establish the following relations where are useful:

\[
\tau \sim \tau' \iff H(\tau) = H(\tau')
\]

(4.5)

\[
\tau \equiv \tau' \iff \tau \land \tau' \neq \tau_\perp
\]

(4.6)

\[
\tau \not\equiv \tau' \iff \tau \land \tau' = \tau_\perp
\]

(4.7)

We can construct equivalence classes, labeled \([\tau]_b\) which represent related technologies. If we say \([\tau]_{b\pm t}\) this represents the number of technological levels to form the basis, if the \( \pm t \) is left out you can assume that \( H([\tau]_b) = 2 \) We suggest the following geometry for technologies at a fixed height, contained in figures 4.1 and 4.2 which gives an easy to calculate side length \( s \).

As for the geometry with respect to time \( t \), we define the interconnectivity between
4.3. TECHNOLOGICAL SPACE

Figure 4.1: Geometry on the $\text{Tr}(\tau)$ plane for a technology set at fixed height $h$.

Figure 4.2: Geometry on the $lw$ plane for a technology set at fixed height $h + 1$

Which allows for the following geometries to be assigned, with the center point being $H(\tau) = t$ and the peripheraries being $H(\tau) = t + 1$. The technological space, $\mathcal{T}$ is intended to be a continuous meet semi-lattice, with a Scott-topology. By utilizing its

1. Please remember that there is no time distance as of yet. All this indicates a is different time
4.3. TECHNOLOGICAL SPACE

Figure 4.3: $W_4$

Figure 4.4: $W_5$

Figure 4.5: $W_7$
4.3. TECHNOLOGICAL SPACE

compliment, we form a Lawson-topology which gives sufficient continuity to combine it with the wealth space.

\[ T \otimes W = \{ m = \sqrt{k_r^2 + l_r^2}; a = \sqrt{k_s^2 + l_s^2} \} \] (4.12)

This graph now ensures that the unit distance for technologies at similar heights is now defined by their capital cost and their labor cost to develop. Our variable time \( t \) remains endogenous but is now a function of \( w \) and \( l \), and incorporates an inflationary factor. We are going to now make the distinction of capital and labour into four types, \( k_r \) and \( l_r \) represent ordinary capital and labour, \( k_s \) and \( l_s \) represent space capital and space labour, which are the capital necessary to create new types of production. From now on, \( W \) dim 3, so that integrating with respect to \( W \) includes the time. We say that

\[
\begin{align*}
t &= \sqrt{k_s^2 + l_s^2} \\
s &= \sqrt{k_r^2 + l_r^2}
\end{align*}
\] (4.13) (4.14)

The chain function \( C(\tau, \tau') \) which was the least number of \( \leq \) between the two, now has a distance ascribed to it. To avoid confusion, the function \( D : T \mapsto T \) now represents the cost, such that

\[
D(\tau, \tau') = \int_C C(\tau, \tau')dW
\] (4.15)

Recall that negative capital and labor are costs, so this is actually a retrospective process. We define one more function, \( \| \tau \| \). This is based of the region

\[
\Pi_r = \{ \tau' \in T : \tau \in [\tau]_{k=1} \text{ and } \tau \leq \tau' \}
\] (4.16)
4.4. AGENT SPACE

Or the $\tau$ on the same level with the same basis in the previous level as $\tau$, and the next successors $\tau$. Then

$$\|\tau\| = \int \int \int dW$$  \hspace{1cm} (4.17)

Finally, we define the inner product of a two technology’s to be the distance between them and is defined

$$\|\tau \cdot \tau'\| = \frac{[\tau] \cup [\tau']}{[\tau] \cap [\tau']}$$  \hspace{1cm} (4.18)

4.4 Agent Space

An agent $A$, is a function

$$A : (W \times T) \mapsto (W \times \mathcal{I})$$  \hspace{1cm} (4.19)

where $\mathcal{I} \subset T$. $\mathcal{I}$ is just an initial, randomly generated set. We find a single chain $C(\mathcal{I})$ such that there exists a connection between every technology. The endowment, $\bar{w} \in \mathbb{R}^4$ is given by

$$\begin{pmatrix}
k_1^r \\
k_1^s \\
l_1^r \\
l_1^s
\end{pmatrix} = \begin{pmatrix}
k_0^r \\
k_0^s \\
l_0^r \\
l_0^s
\end{pmatrix} + \begin{pmatrix}
D_{k_r}(\mathcal{I}) \\
D_{k_s}(\mathcal{I}) \\
D_{l_r}(\mathcal{I}) \\
D_{l_s}(T\mathcal{I})
\end{pmatrix}$$

Where $k^0$, $l^0$ are initial values, and the distance metric is still the line integral, just with respect to whatever variable we are measuring. We further calculate a value $A$,
4.4. AGENT SPACE

the advancement metric as

\[ A = \frac{\iint_C \mathcal{T} \, d\mathcal{W}}{\iiint_C \mathcal{T} \, d\mathcal{W}} \]  

(4.20)

The mastery metric is calculated by

\[ M = \sum_{H=1}^{n} \int_C \mathcal{T} \, d\mathcal{W} \]  

(4.21)

We then compute another metric, which refers to the potential of an agent as

\[ \|A\| = M(k_r + l_r) + A^3(k_s + l_s) \]  

(4.22)

The growth potential is going to be represented as

\[ |A| = \|A\| - D(\mathcal{T}) \]  

(4.23)

We define the technological difference between two companies, \( \forall \tau \in \mathcal{T}, \tau' \in \mathcal{T}' \) then

\[ \|A \cdot A'\| = \sum_{\tau \in \mathcal{T} \cup \mathcal{T}'} \frac{||\tau \cup \tau'||}{||\tau \cap \tau'||} \]  

(4.24)

This represents essentially working capital for the agent. For growth from \( \mathcal{T}_0 \) to \( \mathcal{T}_1 \) such that there is a \( \tau_1 \in \mathcal{T}_1 \setminus \mathcal{T}_0 \) then \( D(\tau_1, \mathcal{T}_0) < |A| \). This is the function by which \( A \) grows.

However, \( \mathcal{T} \) has two dimensions, \( (A, M) \), and so we construct the vectors of growth,
4.4. AGENT SPACE

or $\vec{G} = \langle A, M \rangle$ such that

\[
\vec{A} = \left\langle \frac{\|A\| \tau_a}{D(\tau_a^2)}, 0 \right\rangle \tag{4.25}
\]

\[
\vec{M} = \left\langle 0, \frac{\|A\| \tau_m}{D(\tau_m^2)} \cdot |A| \right\rangle \tag{4.26}
\]

\[
\vec{G} = (\vec{A} \times \vec{M}) \cdot |A| \tag{4.27}
\]

The size of the agent space is limited to $n$ agents, and is a one dimensional lattice, such that $A_{i-1} \to A_i \to A_{i+1} \to \cdots \to A_{i-2} \to A_{i-1}$ and which forms a closed world network, with, again, side length $s = \sqrt{w^2 + l^2}$. We chose $j$ agents randomly, our hub agents, $[A]_h$ and connect them with a set of agents $A \notin A_h$ of any distance. Figure [??] depicts a generic example of such a network. The distance metric is given by:

\[
D(A_i, A_j \neq i) = \int_C C(A_i, A_j) d\mathbb{W} \tag{4.29}
\]

Which allows us to define a neighborhood $\Omega_A$ where

\[
\Omega_{A_i} = \{ A_j \in \mathbb{A} : D(A_i, A_j) < |A| \} \tag{4.30}
\]

Then we construct the final endowment, $\vec{w}_2$ is created by $\vec{w}_2 = \vec{w}_1 + \sum_{\Omega_A} D(A, \Omega_A)$. To finalize our construction of this space, we create several other quantities,

\[
\deg(A) = \frac{\sum_{i=1}^{n} \|A_i\| \deg(A_i)}{\int_C C(A)d\mathbb{W}} \tag{4.31}
\]
4.4. AGENT SPACE

Figure 4.6: A Newman-Watts Small World with \( n = 20 \) agents, such that each agent is connected to \( k = 2 \) neighbors, and with \( j = 3 \) hub agents with 3 connections each.

Which is the average degree of the space. Then the heterogeneity of the system is measured by

\[
\text{deg}(A) = \frac{1}{n} \sum_{\text{deg}(A)} \text{deg}(A)^2 |\text{deg}(A)| - \overline{\text{deg}(A)}^2
\]  

(4.32)

We end this section with three very important functions, the interest rate function, the wage rate function, and the GDP function.

\[
GDP = \int_C C(A) d\mathbb{W} + \int_C C(\tau \in A) d\mathbb{W}
\]  

(4.33)
4.5. STRATEGY SPACE

Which means that

$$\Delta GDP = \frac{GDP_{t+1} - GDP_t}{GDP_t}$$  \hspace{1cm} (4.34)

Which we then define $r$ the growth in the interest rate, and $w$, the growth in the wage rate as

$$r = \frac{\partial}{\partial K} \Delta GDP$$  \hspace{1cm} (4.35)

$$w = \frac{\partial}{\partial L} \Delta GDP$$  \hspace{1cm} (4.36)

4.5 Strategy Space

The strategy space, $S$ is again a simple one, composed of two vectors

$$\vec{C} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \vec{D} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The product $A \otimes S$ gives each agent $A \in A$ a unique strategy function, $S : A \mapsto S$ that is given by

$$S(A) = \frac{|\vec{A}|}{|\vec{G}|} \vec{C} + \frac{|\vec{A}|}{|\vec{G}|} \vec{D}$$  \hspace{1cm} (4.37)

However, due to the discreteness of these strategies, the decision is made by

$$\begin{cases} \vec{C} & \text{if } |\vec{C}| > |\vec{D}| \\ \vec{D} & \text{if } |\vec{D}| > |\vec{C}| \end{cases}$$  \hspace{1cm} (4.38)
4.6 Final Construction and Measurement

The final construction from this model comes from the game $\Gamma(\Lambda, S, P)$. The above are already well defined, so we will define the payoff space as

$$P = T \otimes S$$  \hspace{1cm} (4.39)

To form this construction, we note that $T$ can be partitioned into many separate boolean algebras, which form a complete ring. Hence we can form a ring $\langle T, \land, \lor \rangle$, where the $T \in A$ are the elements of this operation. They are defined as such, for $\tau_i \in T$, such that $|T| = n$ and $\tau_j \in T'$, and $|T'| = m$

$$\land : T \cup (\bigwedge_{i,j=1}^{\max(n,m)} \tau_i, \tau_j \setminus T)$$  \hspace{1cm} (4.40)

$$\lor : T \cup (\bigvee_{i,j=1}^{\max(n,m)} \tau_i, \tau_j \setminus T)$$  \hspace{1cm} (4.41)

Then we form the $T_{2 \times 2}$ matrix as $S_i \times S_j$ and form

$$\begin{bmatrix}
\vec{(C)}_i & \vec{(D)}_i \\
\vec{(C)}_j & T_i \lor T_j \\
\vec{(D)}_j & T_i \land T_j - \\
\end{bmatrix}$$
A second matrix $W_{2 \times 2}$ is given by
\[
\begin{bmatrix}
(C)_i & (D)_i \\
(C)_j & 0 & |A| - |D| \\
(D)_j & |A| + |\bar{C}| & 0
\end{bmatrix}
\]

Then
\[
\Pi_{2 \times 2} = T_{2 \times 2} \oplus W_{2 \times 2}
\]  \hspace{1cm} (4.43)

The final payout to the player is given by
\[
\Pi = \sum_{j \in \Omega} s_i^T \Pi s_j
\]  \hspace{1cm} (4.44)

Each player attempts to $\max_{T,W} \Pi$, this gives them greater economic capacity, and greater ability to make profits. However, the strategy function must be modified one more time.

\[
S(A) = |\bar{A}| \sum_{j \in \Omega_A} \deg(A) |A| \frac{|\bar{C}|}{|\bar{G}|} \sum_{j \in \Omega_A} \deg(A_j) \cdot |A_j| \frac{|\bar{C}|}{|\bar{G}|} + |\bar{A}| \sum_{j \in \Omega_A} \deg(A) |A| \frac{|\bar{D}|}{|\bar{G}|} \sum_{j \in \Omega_A} \deg(A_j) \cdot |A_j| \frac{|\bar{D}|}{|\bar{G}|}(4.45)
\]

Each player has a learning function $L(A)$, which is defined as the following

\[
\frac{\sum_{\Omega(A)} \|T_A \cdot T_{\Omega(A)}\|}{\sum_{\Omega(A)} \|A\| \|\Omega(A)\|}(4.46)
\]

This defines the scalar multiple for which the probability of which a player learns is decided. For each time in this period, then

\[
GDP = \sum_{i=1}^{n} \Pi_i + \sum_{i=1}^{n} |\bar{G}_i|\]  \hspace{1cm} (4.47)
4.6. FINAL CONSTRUCTION AND MEASUREMENT

Then we define

\[ r = \frac{D_k(\Gamma_{t+1}) - D_k(\Gamma_t)}{D_k(\Gamma_t)} \]  \hspace{1cm} (4.48)\\
\[ w = \frac{D_l(\Gamma_{t+1}) - D_l(\Gamma_t)}{D_l(\Gamma_t)} \]  \hspace{1cm} (4.49)

We utilize a standard IS-LM model, with the Earth economy being the foreign economy where the inputs \( IS - LM(GDP, r_t, w_t) = \rho, r_{t+1}, w_{t+1} \), where \( \rho \) represents the inflationary index. We then scale the inputs of the basis \( W \) by

\[
\delta r \cdot \begin{pmatrix}
k_r \\
0 \\
0
\end{pmatrix} \
\hspace{1cm} (4.50)\\
\delta w \cdot \begin{pmatrix}
l_r \\
0
\end{pmatrix} \
\hspace{1cm} (4.51)\\
\rho \cdot \begin{pmatrix}
0 \\
0 \\
\sqrt{w_s^2 + l_s^2}
\end{pmatrix} \
\hspace{1cm} (4.52)\\
\hspace{1cm} (4.53)

The vector space \( \mathbb{H} \), as we so long promised, is going to be the linear map \( h \in \mathcal{L}(\Gamma_t, \Gamma_{t+1}) \) such that

\[ H = \bigcup_{t=1}^{\infty} h(\Gamma_t) \]  \hspace{1cm} (4.54)

A key aspect of the model is not just measuring the economic output, but also ideas and resources are circulating throughout the space. We use percolation to measure
this. In a Newmann-Watts small-world network, the percolation is given by

\[ \phi = \frac{(1 - \text{deg}(A))^n}{2\text{deg}(A)} \]  

(4.55)

This is a probabilistic measure, and so we define \( \rho = \begin{cases} 1 & \phi \geq 0.5 \\ 0 & \phi < 0.5 \end{cases} \). If \( \rho = 1 \), this means that the network is still connected, and so we can continue as before. If \( \rho = 0 \) then the network has split, meaning there are now two (or more) separate Newmann-Watts networks, each without any interaction. This can be considered as heliospheric nations, and each plays their own evolutionary game against one another. We measure the GDP separately for each one.
4.6. FINAL CONSTRUCTION AND MEASUREMENT

Theorem 4.6.0.1 For any $\vec{H} \in \mathbb{H}$, the following quantities can be calculated:

\[
\begin{align*}
\nabla \vec{H} & \\
\kappa(\vec{H}) & \\
\partial \vec{H} & \\
\int\int\int \vec{H} d\mathbb{H} & \\
\int\int \vec{H} d\mathbb{W}, \mathbb{A} & \\
\int \vec{H} d\mathbb{W} & \\
\frac{\partial}{\partial K} \frac{d}{dT} \int \vec{H} d\mathbb{W} & \\
\frac{\partial}{\partial L} \frac{d}{dT} \int \vec{H} d\mathbb{W} & \\
\frac{\partial}{\partial S} \frac{d}{dT} \int \vec{H} d\mathbb{A} & \\
\frac{\partial}{\partial \mathbb{A}} \frac{d}{dT} \int \vec{H} d\mathbb{W} & \\
\kappa(\mathbb{T}) & \\
\end{align*}
\]
Chapter 5

Lessons from the Model

5.1 Phases of Heliospheric Expansion

Since this is an evolutionary game, we are able to determine when the model is in evolutionary stability. This occurs when $\forall S_j \in A, \Pi(S_i, S(A)) \geq \Pi(S_j \neq i, S(A))$. This also happens when $\eta = \frac{\|W_{t+1}\|\beta_3}{\|W_{t+1}\|} - \frac{\|W_t\|\beta_3}{\|W_t\|} = 0$, or the rate of change of learning is zero. However, changes in the environment, brought about changes in $T(T)$ and $w(t)$ and $r(t)$ enable different mutant strategies to become dominant and moves the system to a different set of dominant market strategies. WE break these into four rough phases

**Phase I**: Initial Space Commercialization

**Phase II**: Technology Pioneering

**Phase III**: Near Earth Homogenization
5.1. PHASES OF HELIOSPHERIC EXPANSION

**Phase IV:** Heliospheric Segmentation and Specialization

We will then explore the dominant strategies, market strategies, and the environmental changes that denote each phase.

5.1.1 Initial Space Commercialization

Phase I is the current heliospheric era that we are in now. In this age, the ideal profit structure for space commercialization is one that is based on a terrestrially based model, where $\frac{\|\tau_{A}\|}{\|\tau_{M}\|} < 1$, where satellite technologies are a service used by those on Earth. The large amount of actual satellite providers implies that for that level of technology, satellite technologies are relatively mature; that is every company’s technological allocation includes the whole set of technology related technologies. For these satellite companies, $M(\mathcal{T}) > A(\mathcal{T})$, meaning that profits come from the efficiency of basis capital and labour. On the other hand, the other set of rocketry technology, which can be viewed as independent of that of satellite technology $\mathcal{\wedge} \tau_s, \tau_r = \tau_\perp$. Companies with $\tau_r$ are heavily specialized technology, such that each company may poses a disjoint set of top technologies for rocketry. In addition, $M(\mathcal{T}) < A(\mathcal{T})$ so these companies compete on technology rather than price. The dominant strategy of the satellite companies is to defect, where a satellite company purchases rockets to lift their satellites into space with no retainment of the technology. On the other hand, since $\Pi(S_c) = \Pi(S_D)$ for the commercial space companies (advances in satellite manufacture do little to aid their productive capacity, meaning that, for $\tau_i$ being a rocket technology, and $\tau_j$ representing a satellite technology $\tau_i \not= \tau_j \rightarrow \tau_i \cdot \tau_j = \tau_i$) The equilibrium present is obvious. Satellite companies compete on price, providing a relatively homogeneous product where the ability to provide a lower cost service directly results in higher demand. Then, while
5.1. PHASES OF HELIOSPHERIC EXPANSION

internal technological development still occurs, they are entirely on the horizontal level. The profit incentive fails to exist to promote technological advancement rather than mastery. This, combined with the predominance of a defect strategies, limits the growth of the vertices and hence any real growth in space GDP.

What governs this is that $\|A\| < 1$, the attraction to where companies will tend toward mastery rather than advance. Since $\kappa \propto \frac{\Pi(a)}{\Pi(b)}$, this will increase as the price differential does. If as Lee predicts, the relative scarcity of terrestrial arguments drives the price of these up, then $\|rA\| > 1$, which means that companies will tend towards the advancement of technology, and $\frac{\partial S}{\partial C} > 0$ and $\frac{\partial S}{\partial C} < 0$, giving us $\nabla S = (+C, -D)$ As a result, a new set of agents would be introduced into the system, striving towards technological advancement. With increasing wealth allocations in comparison to the existing companies, then the process of learning and replicator dynamics ensures that most space commercializers occur a low-growth niche, adapt the strategies of the space pioneers, or simply cease to exist. A new set of agents comes to dominate the environment.

5.1.2 Technological Pioneering

The above changes ensure a shift in the evolutionary dominant strategies. Whereas before, companies competed based on price, now technology is what creates greater profits. As a result, to maximize their wealth, the technological pioneers will seek to expand their technological allocation as fast as possible. The apparent technological space at this level is much larger, so initially companies will only see advancements in small subsets of their technological set. The resulting shift is $\Pi(S_c) > \Pi(S_D)$ leading to cooperation as the dominant strategy. While each firm may only have a very specialized form of the technological set, their collaboration will form coali-
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.1: Technological Equilibrium for Phase 1
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.2: Strategic Equilibrium for Phase 1
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.3: Total Equilibrium for Phase 1
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.4: Heliospheric Domination at the first phase
tions, shifting the market structure to a set of companies with specialization, but their coalition can form a vertically and horizontally integrated structure, in which agents are parts of large coalitions. As a result, the Newton Watts small world network develops a small amount of clusters, with every company in them considered a neighbor, with each coalition competing against one another.

While in the early days, many of the profit making ventures will serve as feasibility studies, of which the large profit venture will further attract more companies with a lower risk profile, eventually the rapid chaos of the game becomes one that is much slower. The coalitions will quickly gain a technological mastery at the current technological level, with the profit potential of the next greater level continuously drawing them in. Eventually, with the mastery of each technological level occurring, the divergence of the technological set will increase the time it takes to develop new technologies.

Then, once again, there is a shift between $\vec{A}$ and $\vec{M}$, as technological levels gradually increase then it will become, once again, more economical to focus on $\rho_m$. The profit potential from operating efficiently on the margins begins to overtake that of the profit potential for those with better technology. This is aided by the fact that once mining technology reaches a certain level, then it is remarkably similar to that on earth. Given the homogeneity of this product, technological gains that increase the efficiency of how one mines it (mastery) rather than increasing the ability to mine it (advancement) yields far greater profitability gains. Geographically, this would happen at or around the area between Earth and Jupiter, which requires the simplest mining techniques. The abundance of resources to harvest provides no near term limitation of profit potentiality. A further effect of this is that the increased efficiency of capital and labour usage helps to drive down interest rates, bringing down costs overall for those participating in the space economy. This provides an
5.1. PHASES OF HELIOSPHERIC EXPANSION

opportunity for enterprises on earth with massive amounts of capital to effectively expand into space. The overall master of technology ensures that employing a defect strategy (licensing of technology) ensures that one corporation can license an entire technological level. In effect, much of this licensing would come from agents that have yet to join a larger coalition; their absorption will mark the end of the technological pioneering era. While some of the fastest developments in heliospheric domination do occur in this era, the exponential increase of resources and cost of technological advancement yields the transition into phase III. Sometime during this era, the GDP of space will surpass that of earth. The combination of rapid technological advancement as well as an increased number of interactions beyond space bound companies ensures that the production of the space economy will surpass Earth itself. During this era, however, the increased amount of resources harvested will grow faster than the GDP itself, ensuring that real interest and wage rates stay relatively constant. The heliospheric economy will then have transitioned to a large foreign economy compared to that of earth, necessitating the development of central bank institutions as well as providing greater stability to the fabric of the the Newman-Watts network. This stability reduces the overall volatility of the model, making the evolutionary stability in succeeding stages a much harder force to overcome.

5.1.3 Near Earth Homogenization

Once the heliospheric expansion has reached the main asteroid point between Mars and Jupiter, we can expect a long-term expanded equilibrium period. After this point, extraction of minerals becomes far more technically involved, and the distribution of resources becomes much more sparse, with the technological advancements required to harvest them requiring a much greater degree of specialization. The momentum of companies in the technology shifts considerably towards mastery
of current technologies rather than simple advancement. This is aided greatly by the sheer magnitude of resources available in the main asteroid belt, so the relative scarcity will remain low for a considerable amount of time, and the size of the corporations operating in this space will yield considerable resistance to evolutionary change. With the clustering of corporations and the amount of capital considerably greater, geospatial segmentation will begin to form. In order to reduce the amount of
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.6: Strategic Equilibrium for Phase 2
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Figure 5.7: Total Equilibrium for Phase 2
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Figure 5.8: Heliospheric Domination at the second phase
5.1. PHASES OF HELIOSPHERIC EXPANSION

risk inherently involved in space-commercialization, purchasing sectors of the asteroid belt and extracting them over a period of time will keep relative risk lower, thus ensuring a greater ability to acquire capital through banks to fund these operations. A necessary condition for this eventuality is the operation of a functional legal system and government to maintain these lands claims, as well as banks and a central bank to loan capital. During this period, we can expect the development of the first, fully operational space-based government in order to manage this commercialization and operation. To promote greater heliospheric conquest, an infrastructure system will also be developed, starting with transportation networks based around space stations, which will likely have government representatives at them. While corporations likely developed their own transportation infrastructure in the transition from Phase 2 to Phase 3, this will be integrated or centralized around these space stations, forming regional hubs with major transportation networks between them. Costs then become greatly reduced for companies who are in one of these geospatial hubs, reducing the transactional costs for them working together, and forming oligopolistic or monopolistic market structures. As the process continues, there will be less and less incentive for technological advancement, and this is why we call this phase of the economy Near Earth Homogenization.

This stage can be characterized as one that greatly reduces the volatility of the heliospheric calculus that we have characterized. The time intervals for any changes will be greatly increased, and evolutionary progress may take a hiatus. Since our focus is on the long future, much of this era remains relatively uninteresting to the heliospheric model, though using state action to introduce a greater degree of heterogeneity will cause several exogenous shocks to the equilibrium; however the dominant strategy of consolidate then defect will remain the same and our heliospheric economy remains in an equilibrium. At this point, there is also very little
5.1. PHASES OF HELIOSPHERIC EXPANSION

chance of regression to an all-terrestrial civilization. There are several factors that might eventually shift this equilibrium, though which one occurs is beyond the scope of this paper. The first is the continual scarcity problem, at some point in the future the cost of continuing to search for and extract resources in the main asteroid belt may become large enough to promote companies to seek advancement again. Another is that the barrier to entry to the main belt asteroids will become extremely high; once this reaches a certain point, new agents will find the cost of advancing their technology to be more economical. If the regulatory costs of the heliospheric government become too large, either through taxes, break-ups of perpetual monopolies, or just simple space regulations, again costs will go up. In any sense, what will create a new evolutionary equilibrium will be that, through whatever mechanism, the attraction of profit from further expansion will exceed the ability to make profit as is. The process of the transition to Phase IV will be slower, especially compared to previously rapid changes, but eventually the environment will shift more and more corporations towards the outer bounds of the solar system.

5.1.4 Heliospheric Segmentation and Specialization

The only places for expansion at this point will be the far inner solar system, the space closes to the sun, and the outer solar system, everything from the Gas Giants to the Oort belt. The technologies required to succeed in these areas will be extremely different; dealing with the extreme conditions very close to Sol, to a diverse set of challenges that await in the outer solar system. Transportation costs in the outer solar system will be much higher, though the potential profitability much greater, compared to the far inner solar system with lower profit potentials but lower transportation costs. The result of this is a massive divergence of the technological
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.9: Technological Equilibrium for Phase 3
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.10: Strategic Equilibrium for Phase 3
Figure 5.11: Total Equilibrium for Phase 3
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Figure 5.12: Heliospheric Domination at the third phase
sets between companies operating in either sphere, branches that will be virtually irreconcilable. Collaborative processes will be such that two companies choosing to cooperate that are from different sides of the solar system will see no new technologies of importance added to their technological sets, and the distance between the technological sets will render most licensing unlikely. The only technology unifying the two will be that developed for main asteroid belt extraction, technology that is far into the public domain.

Between these two portions of the solar system, we are likely to see homogenization and mastery similar to that which happened in the main asteroid belt. The heliospheric calculus is undefined beyond this point, and it is difficult to speculate much further. It is very possible that the gulf in the technology sets will be beyond unification, forcing a split in the Newton-Watts network, meaning that connections may not necessarily exist between agents operating in the inner versus outer solar system. This may imply a war between factions, especially if resources in one part of the system runs out before another. Or, if the technology set has advanced sufficiently that the innovation of inter solar travel has occurred, this may signal a new wave of human expansion that this paper will not attempt to explore. This marks the last era of heliospheric conquest, a venerable human achievement. The following table is a summary of the properties of various heliospheric phases:

| Phase | $\nabla T$ | $\frac{\partial W}{\partial k, \partial l_t}$ | $\frac{\partial W}{\partial k, \partial l_c}$ | $\nabla S$ | $\nabla \Gamma$ | $\frac{d}{dt}$ $\deg(A)$ |
|-------|-----------|---------------------------------|---------------------------------|--------|--------|----------------|
| I     | $\leftrightarrow$ | 0 | + | D | 0 | 0 |
| II    | $\uparrow$ | + | + | C | + | 0 |
| III   | $\leftrightarrow$ | 0 | + | D | - | + |
| IV    | $\uparrow$ | + | - | C | $\leftrightarrow$ | - |

Table 5.1: Summary of the Phases of Heliospheric Expansion
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Figure 5.13: Technological Equilibrium for Phase 4
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.14: Strategic Equilibrium for Phase 4
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.15: Total Equilibrium for Phase 4
5.1. PHASES OF HELIOSPHERIC EXPANSION

Figure 5.16: Heliospheric Domination at the fourth phase
5.2 Inhibitions to Heliospheric Expansion

We will briefly touch upon what could inhibit the heliospheric expansion. The primary driver of the heliospheric economy is the technological set. We use the abstract variable \( t = h \) within this set, but we should note that \( t \) implicitly includes the risk within heliospheric expansion. Without a substantial and well-defined set of laws defining the heliospheric economy, this risk remains high, and is arguably why heliospheric expansion is not occurring today. We assume that the existence of one corporation having a highly profitable mining expedition is sufficient to make this risk seem negligible; however if such an expedition were to happen and to fail, especially due to a systematic flaw such as the UN Space treaties, then this could greatly inhibit or prevent space exploration. Scarcity is what will drive space exploration, but if resources become too scarce to drive space exploration, such as if an essential element is used to the extreme and prevents the manufacture of heavy lift rockets, say, or something else altogether, then heliospheric conquest may never happen.

We can extend this problem throughout the phases. If a war or other exogenous shock were to remove a significant amount of resources, then it is possible that the entire system could collapse. As corporate entities would no longer to sustain themselves with commercial space enterprises, a recessionary strategy may be the most relevant. One major limitation of the model is that it allows for no possibility for regression, this is unrefined because it is impossible to reconstruct a complete technology tree from scattered sources. The exogenous shock we are discussing would have to be massive to cause such a regression, on the magnitude of or near an extinction level event. A significant proportion of the mass of harvestable resources would have to be destroyed or rendered unusable.
5.2. INHIBITIONS TO HELIOSPHERIC EXPANSION

Another potentiality is that the constant interaction between agents drives the evolutionary dynamic, meaning that consolidation of these agents into a very small amount would cause heliospheric expansion to grind to a halt. If one corporation dominates the entirety of the space sector, this would be a likelihood. While this occurs regularly in Earth-based economies, typically the state forcibly removes market dominance from the corporate entity. The mechanism for this may be rendered inoperable for the heliospheric economy. Indeed, the only mechanism for enforcement would be an embargo of heliospheric minerals, a move that may seriously hamper the terrestrial economy. The destruction that could be caused by a heliospheric attack on Earth is incalculable and relatively cheap; constant acceleration of even a small mass from the Main Asteroid belt would impose kinetic energy equal to or greater than the atomic bomb on a small part of the planets surface. As always, the possibility of endogenous extinction remains a constant in the scale of human evolution. If an economic system were to develop that is not reliant on competition, which is certainly a possibility when discussing the long-future, then again heliospheric conquest of our type will grind to a halt. This system may still promote heliospheric expansion through a different mechanism, simply that it will not have the same properties or characteristics of the one we described.

A final possibility is that of human nature, while probability may push humanity towards the expansion, we expect a somewhat heterogeneous distribution of different behaviors. This assumption about the probability distribution may not be entirely accurate, and even if so, the system may itself create a scenario. These scenarios are unlikely, especially since modern civilization promotes the autonomy of behaviors, this may still occur. Removal of the feedback system would create an even greater tendency for this; that is the macro economy must be a byproduct of the individual agents operating within it, no matter how small their contribution.
Without this feedback, then only the most extreme of competitive strategies will survive. The system is one that may be resistant to most exogenous events, but even the most determined of individuals can have an effect on evolution.
Chapter 6

Concluding Remarks

In this paper, we attempt to combine empirical analysis with mathematical reasoning to generate a framework for examining the heliospheric economy. After considering historical, legal, economic, and feasibility aspects of the current commercial space enterprises and their development, we examine the effect of using a very long time frame at examining economics, and show that an evolutionary paradigm is more appropriate than conventional economic models, and then show that the differences to land, labor, and capital are best unified with a careful consideration of technology. The resulting calculus allows us to predict properties of the different phases of heliospheric expansion, as well as inhibitors to the conquest of the solar system. Where the model sacrifices precision for accuracy, we can discern these properties, but no exact time line nor specific determination of events that disrupt the evolutionary equilibria.

The calculus is developed in a purely theoretical space, and so using empirical results to validate or reject the hypotheses of the model beckon as a possible area
for future research. The first step would be refining the technological space by using experimental techniques to estimate the geometry of the actual human idea space. We suggest utilizing either the patent system or academic citations to ascertain how preceding ideas and innovations are incorporated into new theories and technologies, and determining boundaries for the difference between peripheral, base, and innovative technologies, and then using econometric techniques to define the basis for which technologies grow with respect to time and various economic conditions. Furthermore, by tracking how memes go viral on Facebook, Twitter, or other social media platforms would give an indication of geometry of the idea space for a fixed time. These parameters could modify the topology of the tech space we defined. If new topological structures are utilized, we note that the entirety of the space must be well defined, meet the standards for being a continuous semi-meet lattice, such that every technology must be related. If a complete technological space can be defined for a period in history, such as for the United States of America in the years 1800-1900, historical results can be used to provide model validation.

Beyond refining the geometry of the technology space, a computer simulation of the calculus would be able to determine more precise mathematical results. In combining the data provided by Ricky Lee\textsuperscript{1}a computer simulation with randomly allocated technology sets, wealth allocations, and behavioral parameters and environmental variables would be able to simulate various scenarios of the heliospheric economy and examine any intersections. This would yield a probabilistic estimation of the likelihood of various serious events happening the heliospheric economy, as well as better definitions of the boundary conditions. Public policy and corporate strategy could then be created to aspire to optimal outcomes for either the state or the corporate entity involved.

1. Lee:2012gx.
A second area of general research is generated from the refinement of the model itself. It was far beyond the scope of this paper to ensure a logically consistent mathematical theory, only to lay out essential axioms, definitions, and quantitative techniques to examine the heliospheric economy. Further exploration of the theorems, lemmas, and corollaries might yield a mathematically rich theory for modelling the development of human society; a stark contrast in that evolution of humanity largely takes place in the sharing of ideas, a far more complex area of investigation that most biologically based evolutionary models. In combination with the experiments suggested above, a mathematical calculus for examining human society can be determined, albeit one that only works for sufficiently large periods of time. Furthermore, if completeness and compactness can be shown, translation into a second-order logic may yield important theoretic results for the field of Quantum Game Theory, along with practical applications for the real-time regulation and maintenance of the heliospheric economy in order to produce optimal results for the successive evolution of humanity.
Chapter 7

Appendix

7.1 Appendix 1: Summary of Variables

7.1.1 Spaces

| Space | Description             | Properties                                      |
|-------|-------------------------|-------------------------------------------------|
| T     | Technological Space     | Continuous semi-meet lattice with a Lawson topology. |
| W     | Wealth Space            | Vector Space                                    |
| A     | Agent Space             | Dynamically scaled Heterogenous                 |
| S     | Strategy Space          | Newman-Watts Small World Network                |
|       |                         | Unit simplex for $k$ strategies                 |

Table 7.1: Summary of Spaces
### 7.1.2 Variables

| Variable | Meaning                                      | Domain       |
|----------|----------------------------------------------|--------------|
| \( \tau \) | A technology                                 | \( \mathbb{T} \) |
| \( \tau_b \) | Base technology                              | \( \mathbb{T} \) |
| \( \tau_p \) | Peripheral technology                         | \( \mathbb{T} \) |
| \( \tau_c \) | Innovative technology                         | \( \mathbb{T} \) |
| \( \tau_\perp \) | Zero Technology                               | \( \mathbb{T} \) |
| \( \Pi_\tau \) | Neighborhood of technology                    | \( \Pi_\tau \subset \mathbb{T} \) |
| \( K \) | Capital                                       | \( \mathbb{R}^+ \) |
| \( L \) | Labour                                        | \( \mathbb{R}^+ \) |
| \( k_r \) | Ordinary capital for production               | \( k_r \in \mathbb{R}^+ \) |
| \( k_s \) | Space capital for production                  | \( k_s \in \mathbb{R}^+ \) |
| \( l_r \) | Labour for production                         | \( l_r \in \mathbb{R}^+ \) |
| \( l_s \) | Human capital for production                  | \( l_s \in \mathbb{R}^+ \) |
| \( A_i \) | An agent                                      | \( A_i \in \mathbb{A} \) |
| \( \Omega_{A_i} \) | The Agent Neighborhood                         | \( \Omega_{A_i} \subset \mathbb{A} \) |
| \( [A]_h \) | Hub Agents                                    | \( [A]_h \subset \mathbb{A} \) |
| \( n \) | Number of Agents                              | \( n \in \mathbb{N} \) |
| \( j \) | Number of Hub Agents                           | \( j \in \mathbb{N} \) |
| \( S_c = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) | Cooperation Strategy                          | \( S_c \in \mathbb{S} \) |
| \( S_d = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \) | Defect Strategy                               | \( S_d \in \mathbb{S} \) |

Table 7.2: Summary of Variables
### 7.1.3 Functions and Operations

| Function | Calculation | Explanation |
|----------|-------------|-------------|
| $\|\tau, \cdot \tau'\|$ | $\|\tau\| \cdot \|\tau'\|$ | How related each technology is |
| $A(A)$ | $\int \int \int C T dW$ | Agents level of technological advancement |
| $M(A)$ | $\sum_n \int_C \int \int \int dW$ | Agents level of technological mastery |
| $G = \langle \vec{M}, \vec{A} \rangle$ | $\|\vec{G}\| \cdot \vec{A} \times \vec{M}$ | Growth direction of an agent |
| $\|A\|$ | $M(k_r + l_r) + A^3(k_s + l_s)$ | Profit potential of an agent |
| $C(\tau')$ | $\int_C D(\tau, \tau') dW$ | Cost to grow to a new technology |
| $R(\tau')$ | $\int \int \int D(\tau, \tau') dW \Pi(\tau')$ | Revenue potential of a new new technology |
| $S(A)$ | $\sum_j s_j T \Pi, s_j$ | Payout for an agent |
| $\Pi(A)$ | $\sum_{\Omega(A)} |T_{A} T_{\Omega(A)}|$ | Total learning for an agent |

Table 7.3: Summary of Functions and Operators
### Measurements

| Measurement | Description |
|-------------|-------------|
| $\nabla \vec{H}$ | is an indication of the equilibrium of the system. |
| $\kappa(\vec{H})$ | is the rate towards the next evolutionary change. |
| $\partial \vec{H}$ | is the trajectory for market structures strategic behavior and technology for the variable taken. |
| $\iiint H d\vec{H}$ | is the economic potential of the heliospheric economy as well as the total wealth accumulated. |
| $\iint C \vec{H}dW\Lambda$ | represents the actual GDP of the heliospheric economy. |
| $\int C \vec{H}dW$ | represents capital or labor spending within the economy. |
| $\frac{\partial}{\partial K} \int C \vec{H}d\Lambda$ | is the change in interest rates. |
| $\frac{\partial}{\partial L} \int C \vec{H}d\Lambda$ | is the change in interest rates. |
| $\frac{\partial}{\partial S} \int C \vec{H}d\Lambda$ | is the change in strategy over time. |
| $\frac{\partial}{\partial A} \int C \vec{H}d\Lambda$ | represents the change in market strategies. |
| $\kappa(T)$ | is the momentum of the technological space. |

Table 7.4: Summary of Measurements
7.2 Appendix 2: Mathematical Proofs

The technology space is the only novel proof that we have developed, and is based on the work from two sources, of which the footnote is included for the individual proofs. Proofs for the agent space and for all games have already been done and can easily be verified with existing literature.

7.2.1 Technology Space

Definition 7.2.1.1 Let $\mathbb{T}$ be the set with elements $\tau \in \mathbb{T}$ representing technologies, with the partial ordering $\leq$. $\tau \leq \tau'$ simply means that $\tau$ is the predecessor to $\tau'$. Furthermore

$$\tau \sim \tau' \iff \tau \leq \tau' \text{ and } \tau' \leq \tau$$

and that

$$\text{Tr}(\tau) = \{\tau' \in \mathbb{T} : \tau' \sim \tau\}.$$ 

In addition, a chain, notated as $C(\tau, \tau')$, is:

$$C(\tau_i, \tau \neq i) : \tau_i \leq \tau_{i+1} \leq \cdots \leq \tau_{j-1} \leq \tau_j$$

The following are the aximomatic assumptions that we will make about the technology set. These should be assumed to be true:

$$\forall \tau, \tau' \in \mathbb{T}, \exists C(\tau, \tau')$$

(A1)

1. weibull1997evolutionary; newman1999renormalization; Newman:1999dw.
7.2. APPENDIX 2: MATHEMATICAL PROOFS

\[\Delta : \text{Tr}(\tau) \mapsto \text{Tr}(\tau)' \text{ such that } \Delta \in \mathcal{P}_{m \geq 2}(\mathbb{T})\]  
(A2)

\[V(\text{Tr}(\tau)) \simeq \prod_{k=0}^{h} D_{k,6}\]  
(A3)

Where \(V(\text{Tr}(\tau))\) is the vertex set of a given trace, and \(D_{k,6}\) is the dihedral group.

**Proposition 7.2.1.2** Let \(\mathbb{T}\) be the technology set. If \(T^n \subset \mathbb{T}\) is such that

\[T^n = \{\tau \in \mathbb{T} : \tau \leq \text{Tr}_n(\mathbb{T})\}\]

Then \(\langle T^n, \Delta, \cup \rangle\) forms a boolean ring, where, \(\Delta\) is the set difference operation

\[\delta : A \cup B \setminus A \cap B\]

**Proof.** By [A1] \(\forall \tau, \tau' \in \text{Tr}_n \mathbb{T}, \exists C(\tau, \tau')\), which implies \(\exists \tau_{\perp}\) where

\[\tau_{\perp} = \bigwedge_{\text{Tr}_n} \tau\]

or that

\[\forall \tau \in T^n, \tau_{\perp} < \tau\]
Then consider the upset, $\tau \uparrow = \{ \tau' \in T : \tau > \tau' \}$.

\[
\begin{align*}
[\tau_\bot] \uparrow &= T \\
[\text{Tr}_n] \uparrow &= T \\
[T] \uparrow &= \tau_\bot[\text{Tr}_n] \uparrow = \tau_\bot
\end{align*}
\]

Then $[\text{Tr}_n] \uparrow \cap [\tau_\bot] \uparrow = T^n$ is a complete boolean lattice. We can verify this

\[
\begin{align*}
\bigwedge_{\tau \in T^n} [\text{Tr}_n] \uparrow &= T^n \\
\bigwedge_{\tau \in T^n} [\tau_\bot] \uparrow &= T^n \\
\bigvee_{\tau \in T^n} [\text{Tr}_n] \uparrow &= \tau_\bot \bigvee_{\tau \in T^n} [\tau_\bot] \uparrow = \tau_\bot
\end{align*}
\]

**Property 1** The additive identity is $\tau_\bot$, as

\[
[\tau \delta \tau_\bot = [\tau] \cup [\tau_\bot] \setminus [\tau] \cap [\tau_\bot] \cup [\tau] \cap [\tau_\bot] \cap [\tau_\bot] = [\tau] \cup \emptyset = [\tau] \cap \emptyset = [\tau] \setminus \emptyset = \tau
\]

2. davey2002introduction.
3. gierz2003continuous.
Property 2 The multiplicative identity is $T^n$.

$$\tau \cdot T^n = [\tau] \cap [T^n]$$
$$= [\tau]$$
$$= \tau$$

Property 3 All $\tau \in T^n$ are idempotent.

$$\tau^2 = [\tau] \cap [\tau]$$
$$= [\tau_\perp] \cap [\tau_\perp]$$
$$= \tau_\perp$$

Property 4 The left distributive law holds.

$$\tau_i \cdot (\tau_j + \tau_k) = [\tau_j] \Delta [\tau_k]$$
$$= [\tau_j] \cup [\tau_k]$$
$$= (\tau_j \cap \tau_k)$$
$$= (\tau_i \cdot \tau_j) + (\tau_i \cdot \tau_k)$$

Property 5 Multiplication is associative

$$\tau_i \cdot (\tau_j \cdot \tau_k) = (\tau_i \cdot \tau_j) \cdot \tau_k$$
$$= [\tau_i] \cdot [\tau_j]$$
$$= [\tau_i] \cdot [\tau_j] \cdot [\tau_k]$$
7.2. APPENDIX 2: MATHEMATICAL PROOFS

By A3 all vertices are described by the dihedral groups \( D_{k,6} \), then each technology class is isomorphic to a finitely generated abelian group.\[4 \]

\[ \begin{align*}
\tau_c &= D_0 \simeq \mathbb{Z}_1 \\
\tau_b &= D_6 \simeq \mathbb{Z}_6 \\
\tau_p &= D_{12} \simeq \mathbb{Z}_{12}
\end{align*} \]

(7.1) (7.2) (7.3)

Since all of these are finitely generated abelian groups, each element can generate an entire group.

By A2 we let the change function \( \Delta : Tr(\tau) \mapsto Tr(\tau') \) be the piecewise defined function as such

\[ \Delta(\tau) = \begin{cases} 
D_3 & \text{if } \tau_p \\
D_4 & \text{if } \tau_b \\
D_6 & \text{if } \tau_c
\end{cases} \] \quad \text{(P1)}

Since each element is a finitely generated abelian group, then the ring contains additive commutative subgroups. Since the common factor is 6, then \( \mathbb{T} \) forms a finitely generated abelian group of the form \( \mathbb{T} \simeq \mathbb{Z}_m \) where \( m = 2^{2h-3}3^{h-1}h \).

The right distributive law holds, since the additive subgroups are commutative.

Since \( \mathbb{T}^n \) contains a commutative additive subgroup (the finitely generated abelian groups), is closed, contains a multiplicative identity and operations, and the right and left distributive laws hold, and multiplication is associative, \( \mathbb{T}^n \) is an abelian subgroup. \( \cdot \)

**Proposition 7.2.1.3** \( \mathbb{T} = \bigcup_{i=1}^{N} \mathbb{T}^i \) is a boolean space as well. **Proof.** By the previous result we have already shown that \( \mathbb{T}^n \) is a boolean algebra. We perform induction on

4. mikhalev2002concise.
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the set $S = \{ n \in \mathbb{N} : T^n \text{ is a boolean algebra} \}$. Assume that $T^n$ is a boolean algebra.
Consider that $T^n \subset T^{n+1}$. Moresover, $T^n \in [\operatorname{tr}_{n+1}]$. Since, $\forall \tau, \tau' \in \operatorname{Tr}_{n+1}$ Then $\tau \land \tau' = \tau'' \in T^n$. Then, if $T^n$ is closed, it is $T^{n+1}$ is closed as well. $\tau_\bot$ still remains the additive identity, and $T^{n+1} \cdot \tau = \tau$. Then $T^{n+1}$ is a boolean algebra.
Consider $T^{n+2}$. Then $T^n, T^{n+1} \subset T^{n+2}$. The same arguments apply for closure, as well as the additive identity, and the new multiplicative identity. Hence $T^{n+2}$.
Therefore, by the principle of strong induction, $\bigcup_{i=1}^n T^i = \mathbb{T}$ is an algebraic space. ■

**Proposition 7.2.1.4** $\mathbb{T}$ is a quasicontinuous domain with a Lawson Topology

**Proof.** We define

$$\text{Pred}(\tau) = \{ \downarrow \tau : \downarrow \tau \text{ is finite}, \downarrow \tau \ll \tau \}$$

Since $\text{Pred}(\tau)$ is directed downwards towards $\tau_\bot$, and $\forall \tau' \not\leq \tau$, then $\exists \downarrow \in \text{Pred}(\tau)$ with $\tau' \not\downarrow \tau$

By the demonstration of the properties of each equivalence class of $\tau$, this is true $\forall \tau \in \mathbb{T}$

Therefore, $\mathbb{T}$ is a quasicontinuous domain. Since $\forall \downarrow \exists \uparrow [\tau]$ then the duality principle holds, so $\mathbb{T}$ has a lawson topology. ■

**Proposition 7.2.1.5** $\mathbb{T} \otimes \mathbb{W}$ is a vector space in $\mathbb{R}^3$

**Proof.** By $A^i \tau_c, \tau_b, \tau_p$ can be formed into the adjacency matrix $\operatorname{Tr}(\tau)$ where

$$A = \begin{bmatrix}
[\tau_c : \tau_c] & [\tau_b : \tau_c] & [\tau_p : \tau_c] \\
[\tau_c : \tau_b] & [\tau_b : \tau_b] & [\tau_p : \tau_b] \\
[\tau_c : \tau_p] & [\tau_b : \tau_p] & [\tau_p : \tau_p]
\end{bmatrix}$$

5. gierz2003continuous.
6. xu2013topological.
such that \[ \tau_c : \tau_b \] = \[ \tau_b : \tau_c \]^T and \[ \tau_c : \tau_p \] = \[ \tau_p : \tau_c \] = \emptyset

\[
[\tau_c : \tau_c] = [1] \quad (P2)
\]

\[ \dim[\tau_b : \tau_b] = 6 \times 6 \tau_{j,k} = 1 \iff |j - k| = 1 \quad (P3) \]

\[ \dim[\tau_p : \tau_p] = 12 \times 12 \tau_{j,k} = 1 \iff |j - k| = 1 \quad (P4) \]

\[ \dim[\tau_b : \tau_p] = 6 \times 12 \tau_{j,k} = 1 \iff |j - k| \leq 4 \quad (P5) \]

We then assign the linear map \( \Delta \in \mathcal{L}(\text{Tr}(\tau), \tau(\tau')) \)

\[
[\tau_c : \tau_c'] = [1] \quad (P6)
\]

\[
[\tau_c : \tau_{ij}] = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (P7)
\]

\[ [\tau_b : \tau_{ij}] = \tau_{j,k} = 1 \iff |j - k| \leq 4 \quad (P8) \]

\[ [\tau_p : \tau_{ij}] = \tau_{j,k} = 1 \iff |j - k| \leq 3 \quad (P9) \]
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Then we can assign a norm to \( T^7 \) by solving for each \( \tau \) in a three dimensional matrix. Directly adjacent \( \tau, \tau' \) have \( \|\tau, \tau'\| = u_{\text{Tr}} \), and for unadjacent, it is \( \|\tau_i, \tau^n\| = \sum_i \|\tau_i, \tau_{i+1}\| \). We should note that for each \( h + 1 \) then

\[
u_{h+1} = \sqrt{2}u_h. \tag{P10}\]

As a result, \( u_h = \sqrt{5} \) for \( h = 1 \).

Then to compute \( T \otimes W \), we set

\[
M(T \otimes W) = \left( \begin{array}{c} \sqrt{l_2^2 + k_2^2} = u_{\text{Tr}} \\ \sqrt{l_2^2 + k_2^2} = u_h \end{array} \right) \tag{7.4}
\]

In addition

\[
M(\tau + \tau') = M([\tau]^{\|}) + M([\tau']^{\|}) \tag{P11}
\]

\[
M(\tau + \tau') = M([\tau]^{\|}) + M([\tau']^{\|}) \tag{P12}
\]

\[
M(\tau \cdot \tau') = M(\tau) \wedge M(\tau') \tag{P13}
\]

Since \( T, W \) were closed spaces, then \( T \otimes W \) is also closed. The operations of \( T \) continue to apply towards the space \( T \otimes W \).

However, we define the inner product

\[
\tau \cdot \tau' = \frac{|[\tau]^{\|} \cup [\tau']^{\|}|}{|[\tau]^{\|} \cap [\tau']^{\|}|} \tag{D1}
\]

7. mikhalev2002concise.
In addition, simple technology addition is

\[ \tau + \tau' = \text{Coor}_W(\tau) + \text{Coor}_W(\tau') \]  

(D2)

We define scalar multiplication of a technology as

\[ \lambda \tau = \lambda \text{Coor}_W(\tau) \]  

(D3)

Then with a norm, an inner product, scalars in \( \mathbb{R} \), closure, vector addition with our operations for the \( T \), we conclude that \( T \otimes W \) is a vector space.

**Proposition 7.2.1.6** All properties of calculus in \( \mathbb{R}^n \) apply to \( T \otimes W \), so long as care is taken to acknowledge that any functions are not defined at \( \tau \), and the commutivity of partial derivatives does not apply. *Proof.* \( T \otimes W \) is a continuous vector space.