A QUANTITATIVE INVESTIGATION OF THE POMERON

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Abstract

A comparative investigation of various Pomeron models is carried out through the study of their predicted values of $\sigma_{tot}$, $B$, and $\frac{\alpha_s}{\sigma_{tot}}$ in high energy pp and p$\bar{p}$ scattering. Our results strongly support a picture of the Pomeron in which we have both moderate blackening and expansion of the $p(\bar{p}) - p$ amplitude in impact parameter space as a function of energy in the ISR-SSC domain. In particular, we obtain an excellent reproduction of the data with a hybrid eikonal model which combines the hard Lipatov-like QCD Pomeron with the old fashioned soft Pomeron and Regge terms. Our analysis shows that the additive quark model, at least in the naive form, is not compatible with the data.
At present there are two incongruous descriptions of the Pomeron. One is that based on the conventional phenomenology of soft hadronic scattering and the relevant calculations are carried out [1] within the framework of Reggeon field theory. The soft Pomeron is constructed from multiperipheral hadronic (Regge) exchanges and has $\alpha_P(0) = 1$. This is not compatible with the high energy experimental observation of rising hadronic cross sections. The old fashioned soft Pomeron was, thus replaced [1, 2] by a soft supercritical Pomeron with an intercept $\alpha_P(0) > 0$. This model provides a reasonable description of pp and p$\bar{p}$ scattering in and above the ISR energy range. It’s principle assumption, that the supercritical Pomeron is a simple isolated pole in the complex angular plane, lacks field theoretic justification, and contradicts our experience with the multiperipheral model which fails to produce a pole with $\alpha_P > 1$.

The alternate description of the Pomeron is within the framework of QCD. Lipatov [3] has shown that perturbative QCD, in which a hard Pomeron is built out of multiperipheral high transverse momentum gluon exchanges, predicts a different Pomeron i.e. a series of poles in the complex j plane above unity at $1 < j < 1 + \epsilon$. The resulting Lipatov Pomeron has a more complicated form $s^\alpha/(lns)^\beta$ with $\alpha > 1$. A consequence of a pole with intercept $\alpha(0)$ is that $\sigma_{tot} \propto s^{\alpha(0)-1}$. Hence, both the soft supercritical and the hard QCD Pomeron predict a powerlike rise of the total hadronic cross section. This may continue until the unitarity bound gives rise to screening effects which saturates the growth of the total cross section [4] making $\sigma_{tot} \leq \ln^2 s$, which is compatible with the the Froissart limit [5].

In the following we shall examine a wide class of models of the Pomeron. Our goal is to compare the predictions of these models with the high energy experimental data presently available [6] over the energy range $23 \leq \sqrt{s} \leq 1800$ GeV. The purpose of such a comparative study is to attempt to discriminate between models with different high energy predictions based on data which are apparently [7] below asymptotia.

A quantitative investigation of the various Pomeron models can be readily carried out in the impact parameter space. Our amplitudes are normalized such that $\frac{d\sigma}{dt} = \pi |f(s,t)|^2$ ; $\sigma_{tot} = 4\pi \Im f(s,0)$ where

$$ f(s, t) = \frac{1}{2\pi} \int dB e^{iq \cdot b} a(b, s) \tag{1} $$

and

$$ a(s, b) = \frac{1}{2\pi} \int dqe^{-i\mathbf{q} \cdot \mathbf{b}} f(s, t) \tag{2} $$

Hence we have $\sigma_{tot} = 2 \int dB \Im a(s,b)$ and $\sigma_{el} = \int dB |a(s,b)|^2$. The corresponding forward slope and curvature parameters are

$$ B = \frac{\int dB b^2 a(s, b)}{2 \int dB a(s, b)} \tag{3} $$

$$ C = \frac{1}{32} \frac{\int dB b^4 a(s, b)}{\int dB a(s, b)} - \frac{1}{16} \left| \frac{\int dB b^2 a(s, b)}{\int dB a(s, b)} \right|^2 \tag{4} $$
where \[
\frac{d\sigma}{dt} = \left[ \frac{d\sigma}{dt} \right]_{t=0} e^{(Bt+Ct^2+\ldots)}
\]

Unitarity requires \( \text{Im} \ a(s,b) \leq 1 \). In order to satisfy this constraint it is convenient to express \( a(s,b) \) in terms of the complex eikonal function \( \chi(s,b) \) with

\[
a(s,b) = i[1 - e^{i\chi(s,b)}]
\]

where \( \text{Im} \ \chi \geq 0 \). This ensures that unitarity is restored on summing all the eikonal multi-particle exchange amplitudes. At high energy, forward elastic scattering is essentially diffractive and therefore \( \text{Re} \chi \) is very small. We assume that \( \text{Re} \chi \approx 0 \), consequently the amplitude \( a(s,b) \) is purely imaginary and determined by the opaqueness \( \Omega(s,b) = \text{Im} \chi \). This assumption limits our study to the forward direction, where the bulk of the experimental data is present.

We would like to note that considering only the pure imaginary \( \Omega(s,b) \) violates the crossing symmetry of the scattering amplitude. We will describe later how we restore this symmetry, assuming that the real part of the amplitude is small enough. One of our major objectives is to critically examine how much of the observed change in the forward high energy pp and p\overline{p} scattering amplitudes can be associated with blackening as opposed to the expansion of \( \Omega(s,b) \) and \( a(s,b) \) with \( s \). We note that a soft Pomeron trajectory is essentially flat i.e. \( \alpha' \) is either zero or very small. Accordingly, the growth of the cross section associated with the soft Pomeron is attributed mostly to a blackening of the b-space scattering amplitude. This property is common to many other models \[9, 16\]. The b-space characteristics of the Lipatov Pomeron are less clear. In the LLA one expects \[4, 12\] a moderate expansion with energy. We will check whether this result is compatible with data.

Even though we choose an eikonal approach we do not attach any profound theoretical importance to this choice. It is merely a method to restore s-channel unitarity, and it enables us to assess the natural scale of the shadowing corrections. These can manifest themselves either on the hadronic level with a typical scale \( R_h \approx 1 \, \text{fm} \) or on the quark scale with a typical value of \( R_Q \approx 0.1 \, \text{fm} \). We expect the data analysis to provide us with some information on this. We would like to mention here that the eikonal formula (5) allows us to incorporate in our fitting procedure new ideas, such as were discussed in refs. \[12\], \[18\] about the shadowing correction using different assumption about \( \Omega(s,b) \).

To this end we compare several Pomeron models with three experimental quantities, which are particularly sensitive to the above, i.e. \( \sigma_{tot} \), \( B \) and \( \frac{\sigma_{el}}{\sigma_{tot}} \). We note that more than \( \sigma_{el} \) itself, the ratio \( \frac{\sigma_{el}}{\sigma_{tot}} \) is sensitive to the hadron opacity. Over the ISR-Tevatron energy range this ratio was found \[3\] to be in the interval 0.15- 0.25, which is well below the saturation limit of 0.5 corresponding to a fully absorbing black disc \[7\].

The above experimental quantities relate directly to the structure of the inelastic (multiparticle) high energy amplitude. Indeed, \( \sigma_{tot} \) is driven by the inelastic amplitude, \( B \) is the mean radius of the partonic distribution in the impact parameter space and
σ_{el}/σ_{tot} characterizes the hadronic opaqueness which is very sensitive to the scale of the shadowing corrections.

We examine the entire statistics of 74 data points \[6\] of σ_{tot}, B and σ_{el}/σ_{tot} obtained from to pp and p\bar{p} scattering in the ISR-Tevatron energy range. In our handling of the data, we have used the published results as such, without any attempt to discriminate or average between different ISR data sets at the same energy, or re-analyse the raw data as recently advocated \[8\]. In particular, our initial supposition, that a(s,b) is imaginary, prevents us from calculating ρ = \frac{Re f}{Im f} without additional assumptions.

The data we have examined are characterized, in general, by relatively small errors. Consequently, one must carefully evaluate the quality of the fit to this data. In our study we have found that seemingly good reproductions of this data set, yield an unacceptable \[\chi^2/d.f. > 10\]. An additional fitting problem is associated with the fact that most of our data comes from the ISR, with the UA4 and Tevatron providing only 6 high energy points. As a result we had to check, that fits with an acceptable \[\chi^2/d.f.\] also had a reasonable high energy behaviour. We would also like to mention that the detailed high t structure of the elastic cross section is beyond the scope of our investigation. This corresponds to the very fine details of a(b, s) or Ω(b, s) at small b, a region which we do not pretend to investigate.

We have attempted a number of generic forms for the Pomeron, with different degrees of complexity. As we are discussing both pp and p\bar{p} scattering, we have also included in our parametrization, an odd Regge term to account for the p\bar{p} and pp difference. We therefore consider an amplitude of the form

\[a(s, b) = A_P(s, b) \pm A_R(s, b)\]  

Initially, we attempt an orthodox non-unitarized parametrization of the Pomeron. This may possibly be adequate, since the measured data is well below the unitarity limit, and there may be no need to use unitarized expressions in our analysis. At this level we have for the soft Pomeron and Reggeon

\[F_i(s, t) = c_i e^{R_{0i}^2 s^{\alpha_i(t)-1} \sin[\frac{\pi}{2} \alpha_i(t)]}\]  

where \(i = P \text{ or } R\). With linear trajectories \(\alpha_i(t) = \alpha_i(0) + \alpha'_i t\), we can readily take the Fourier transform of Eq.(7) and obtain

\[A_i(s, b) = c_i s^{\alpha_i(0)-1} \frac{1}{|\beta_i|^2} \exp[-\frac{b^2}{4 |\beta_i|^2} R_i^2] \cdot (R_i^2 s^{\frac{\pi}{2} \alpha_i(0) + Z_i} - \frac{\pi}{2} \alpha'_i \cos[\frac{\pi}{2} \alpha_i(0) + Z_i])\]  

where

\[R_i^2 = R_{0i}^2 + \alpha'_i \ln s\]  

\[|\beta_i|^2 = R_{0i}^4 + \frac{\pi^2}{4}\]
Accordingly, $A_i(s,b)$ has four adjustable parameters: $c_i, R^2_{0i}, \alpha_i(0)$ and $\alpha'_i$.

The first model that we have tested has a supercritical soft Pomeron, and we denote it as $a_I(s,b)$. Such a pomeron can be viewed as the consequence of the soft multihadron (multiparton) production at high energies. The best fit parameters and the $\chi^2/d.f.$ for this model are summarized in the upper section of Table I. In our original fit we found that $\alpha'_P \approx 0.3$, $\alpha'_R \approx 1$, and $\alpha_R(0) \approx 0.5$. We have, therefore, fixed these parameters at their traditional values to somewhat improve our $\chi^2/d.f.$ The $\chi^2/d.f. = 2.04$ is misleading. The model, as such, provides a good description of the ISR data at the expense of a very poor reproduction of the UA4 and Tevatron data. This problem was traced to a low fitted value of $\epsilon = \alpha_P - 1$. We have also tested a model in which $\epsilon$ is fixed at 0.085 and denote it as $a_{II}(s,b,\epsilon = 0.085)$. The quality of the fit, presented in the upper section of Table I, is considerably poorer, but yields a fair continuation from the low to the high energy domains.

The third model we have tested is that based on the Lipatov-like hard Pomeron, where we set

$$A_{L1}(s,b) = \frac{a_1 s^{a_2}}{(lns)^{a_3}} \exp\left[-\frac{b^2}{R^2_{L1}(s)}\right]$$

with

$$R^2_{L1}(s) = a_4 + a_5 (lns)^{a_6}$$

The $a_i$ denote six adjustable parameters. The parametrization considered is

$$a_{III}(s,b) = A_{L1}(s,b) \pm A_R(s,b)$$

This parametrization assumes the soft contribution to be entirely associated with the Regge sector whereas the hard contribution is given by the Lipatov-like Pomeron driven by hard multihadron (multiparton) production.

The form we have chosen for $A_{L1}$ is very similar in its energy dependence to the form suggested by Bourrely, Soffer and Wu [9]. The two models are, however, fundamentally different. In the Bourrely, Soffer, Wu model, $A(s,b)$ has a fixed $b$ distribution independent of $s$. Accordingly, the rise in $\sigma_{tot}$ is due only to the blackening of $A(s,b)$. The Lipatov 1 amplitude that we have examined, e.g. Eq. (10), has a $b$ distribution which is energy dependent, and hence $R^2_{L1}$ is $s$ dependent, see Eq. (11).

Our best fit with the corresponding $\chi^2/d.f.$ is summarized in the upper section of Table I. As can be seen, the hard Lipatov-like Pomeron provides an excellent fit, which is considerably better than the soft Pomeron descriptions. However, it is important to note that neither of these models are suitable parametrizations at very high energies as the power dependence of $s$ will eventually violate unitarity. This is particularly true for the Lipatov $A_{L1}(s,b)$ where we have $A_{L1}(s,b=0) > 1$ for $\sqrt{s} > 62$ TeV.
To overcome this problem we have also examined the above three Pomeron models in an eikonal parametrization, where we set

\[ \Omega_I(s, b) = A_P(s, b) \pm A_R(s, b) \]  
\[ \Omega_{II}(s, b, \epsilon = 0.085) = A_P(s, b, \epsilon = 0.085) \pm A_R(s, b) \]  
\[ \Omega_{III}(s, b) = A_L(s, b) \pm A_R(s, b) \] (13)

Our motivation for utilizing the eikonal expansion rests, not only on the need to unitarize our amplitudes and study the shadowing effects, but also on the knowledge that the eikonal description provides a natural explanation for the changing logarithmic slope of \( \frac{d\sigma}{dt} \) with \( t \). Some prudence must be exercised at this stage. Our investigation aims at understanding the general characteristics of the Pomeron. As is well known, neither of Eqs.(13 - 15) can reproduce the dip structure at ISR and its slow change to a shoulder as observed by UA4. However, these are exceedingly small phenomena where \( \frac{d\sigma}{dt} \) at the second peak is \( 10^{-5} - 10^{-6} \) smaller than \( \frac{d\sigma}{dt} \) in the forward direction. Accordingly, when viewed in \( b \)-space it corresponds to a very subtle substructure of \( \Omega(b, s) \) at small \( b \), which is completely outside the scope of our investigation.

Our best fits and the corresponding \( \chi^2/d.f. \) are presented in the middle section of Table I. We again note, that the \( \Omega_I \) fit agrees well with the ISR data, but fails to reproduce the high energy data, due to the the low value of \( \epsilon \) in this fit. The quality of \( \Omega_{II} \) fit is considerably worse than the one obtained without eikonalization. We have therefore attempted a hybrid model

\[ \Omega_{IV}(s, b) = A_L(s, b) + A_P(s, b) \pm A_R(s, b) \] (16)

Namely, we associate the increase of \( \sigma_{tot} \) with the hard QCD Lipatov-like Pomeron which is superimposed on a non-leading soft Pomeron with \( \alpha_P(0) = 1 \), plus a Regge contribution. As can be seen from the middle section of Table I, we obtain an excellent fit with \( \chi^2/d.f. = 0.96 \). This is demonstrated in Figs. 1-3 where we show the hybrid \( \Omega_{IV} \) fit compared to the relevant experimental data. The energy scale was chosen so as to include our predictions for LHC and SCC. The quality of our fit is even more impressive if we consider the fact that some of the ISR data points, at the same energy differ by more than one standard deviation. The one data point which is not in agreement with the fit is the \( \frac{\sigma_{el}}{\sigma_{tot}} \) value as measured by UA4. We call attention to the fact that an alternative analysis of the UA4 raw data \[ \text{[8]} \] yields a ratio which is compatible with our fit.

A basic conclusion that can be drawn from our analysis, with or without eikonalization, is that \( \sigma_{tot} \) growth is associated with both the expansion and the blackening of \( a(s, b) \) and \( \Omega(s, b) \) with \( s \). In the hybrid model we have

\[ \frac{\Omega(\sqrt{s}=1800, b=0)}{\Omega(\sqrt{s}=23, b=0)} = 1.50 \]  
\[ \frac{R^2(\sqrt{s}=1800)}{R^2(\sqrt{s}=23)} = 1.48. \]  
An identical result was obtained by Chou and Yang \[ \text{[11]} \] who found, in the framework of their model, the need to change both the normalization and the radius of their \( \Omega(s, b) \) as a function of \( s \).
The results of the Lipatov 1 sector of the hybrid model can be readily compared with some theoretical expectations [4, 12] in which

\[ \Omega(s, b) = \sigma_0 s^{\omega_0} \exp \left( -\frac{b^2}{R^2(s)} \right) \]  

with

\[ R^2(s) = R^2_0 + 4\alpha_{\text{eff}}' \sqrt{\ln s} \]  

In the above one should distinguish between parameters that are determined as a direct consequence of perturbative QCD, and those which depend on the boundary conditions and the LLA used by the authors of references [4, 12]. On the qualitative level our fit supports the Lipatov energy dependence

\[ s^a_2 (\ln s)^a_3 \]  

with both \( a_2 \) and \( a_3 \) positive as expected. We also obtain \( R^2_{L1}(s) \propto \sqrt{\ln s} \), in agreement with references [4, 12]. On the other hand, we see that the actual numerical values of our fitted \( a_2, a_3, a_4 \) and \( a_5 \) differ from the numbers calculated by references [4, 12]. This difference can be traced to the fact that the above calculations were made in the LLA with specific boundary conditions. Both may require some modifications. We note that Lipatov 1 in the low energy limit predicts \( B \) values which are considerably smaller than the experimental data. This is compensated, however, by the non leading soft sector, producing an overall reproduction of the data. This is not surprising, as the hybrid model suggests a smooth, rather than an abrupt (threshold), continuation from soft to hard physics. Below the black limit, Lipatov 1 produces a \((\ln s)^{\frac{3}{2}}\) dependence of \( \sigma_{\text{tot}} \) on \( s \). This is less than the \((\ln s)^2\) allowed by the Froissart limit [3]. In order to investigate this point further, we have also examined an alternative Lipatov 2 parametrization in which \( R^2_{L1} \) of Eq. (11) is replaced by

\[ R^2_{L2} = a_4 + a_5 \sqrt{\ln s} + a_6 \ln s \]  

and we have

\[ \Omega_V(s, b) = A_{L2}(s, b) + A_P(s, b) \pm A_R(s, b) \]  

This model has an \((\ln s)^2\) dependence built in, provided \( a_6 > 0 \). Our best fit with this form is \( \chi^2 = 1.02 \), is also given in the middle sector of Table I. We find \( a_6 = 0.10 \) whereas \( a_5 = 0.789 \), rather close to the LLA estimates.

Both \( \Omega_{IV} \) and \( \Omega_V \) models produce a curvature \( C \) which changes sign at \( \sqrt{s} \approx 1200 \) GeV and becomes more negative with increasing \( s \). This result which is compatible with other models, is just another indication that the data examined is far from asymptotia. We note that the non-eikonalized models predict very small \( C \) values over the entire range \( 23 \leq \sqrt{s} \leq 1800 \) GeV. Unfortunately, a direct experimental measurement
of C at \( t = 0 \) is not possible, and the extrapolation of C measured at some finite \( t \) to \( t=0 \), is not reliable.

As a check of the parametrization obtained in fit \( \Omega_{IV} \) (where the parameters were determined from fitting data in the energy range \( 23 \leq \sqrt{s} \leq 1800 \) GeV), we have also evaluated the predictions of the model for \( \sigma_{tot}, \frac{\sigma_{el}}{\sigma_{tot}} \), and B for lower energies i.e. \( 5 \leq \sqrt{s} \leq 23 \) GeV. The results are in very good agreement with the experimental data are shown in Figs. 1-4.

As a further check on our assumption of taking \( \text{Re} f(s,t) \approx 0 \) for small values of \( t \), we evaluate the differential cross-section obtained from the contribution of \( \text{Im} f(s,t) \) only. From the equality \( \text{Im} f(s,0) = \frac{\sigma_{tot}}{4\pi} \), we take

\[
\frac{d\sigma}{dt}_{t=0} \approx \frac{\sigma_{tot}^2}{16\pi}
\]

so that

\[
\frac{d\sigma}{dt} \approx \frac{1}{16\pi} \sigma_{tot}^2 \cdot \exp(Bt + Ct^2)
\]

for evaluating the differential cross-section at small \( t \) values. In Fig. 5 we show how our best parametrization describes the small \( t \) - behaviour of the differential elastic cross section. We interpret this figure as support of the approximations used, and to show how close the physical picture we extract from this fitting, is to reality.

We now discuss the real part of the scattering amplitude which is intimately connected with the crossing symmetry. Our basic assumption was that the real part is small. We now check this assumption and calculate the real part for our parametrizations. To do this, we separate the different contributions of positive and negative signature in the whole amplitude \( f(s, t) \) (see Eq. (1)). In our case, this is easy to do, since the negative signature contribution is due only to reggeon exchange. Finally

\[
f(s, t) = f_+(s, t) \pm f_-(s, t)
\]

for \( pp \) and \( p\bar{p} \) respectively.

1. The real part of the negative signature contribution is given by the usual formula, namely

\[
\frac{\text{Re} f_-(s, t)}{\text{Im} f_-(s, t)} = \tan\left(\frac{\pi}{2} \alpha_{\text{Reggeon}}(t)\right).
\]

2. For the positive signature we calculated \( \text{Re} f_+(s, t) \) as follows

\[
\frac{\text{Re} f_+(s, t = 0)}{\text{Im} f_+(s, t = 0)} = -\cot\left(\frac{\pi}{2} \alpha_{\text{eff}}^+_p(t = 0)\right).
\]

where \( \alpha_{\text{eff}}^+_p(t = 0) \) was calculated by fitting \( f_+(s, t = 0) \) as a power \( s^{\alpha_{\text{eff}}^+_p(t = 0)} \) in the wide range of energy. Fig. 6 shows the result of the above calculation for our the best
fit and illustrates that \( \rho = \frac{Re f(s,t=0)}{Im f(s,t=0)} \) is small enough to support the assumption that one can neglect the real part of the amplitude for small values of momentum transfer squared. One can also see that we get a good description of the experimental data that have not been included in the fitting procedure, except of the UA4 data point at \( \sqrt{s} = 546 \text{ GeV} \), which has been a subject of much debate [8].

An alternative method for examining the various suggested forms for the Pomeron is based on the additive quark model [14]. In such an approach we introduce two scales for the hadron: its size with a radius of \( R_h \approx 1 \text{ fm} \), and the size of a constituent quark with \( R_Q \approx 0.1 - 0.2 \text{ fm} \). Based on this slightly naive picture, we can then follow Chou and Yang [10] and write the scattering amplitude in the form

\[
f(s,t) = 9G^2(t)f_{QQ}(s,t)
\]

where \( G^2(t) = \left( \frac{\nu^2}{\nu^2 - t} \right)^2 \) with \( \nu^2 = 0.71 \text{ GeV}^2 \), and \( f_{QQ} \) denotes the quark-quark scattering amplitude. We now apply our assumptions for the Pomeron and Regge structures to the QQ amplitude.

We assume a simplified static model [2, 4] where the QQ interaction is observed by static spectator quarks. In such a model, we define \( a^{QQ}(s,b) \) and \( \Omega^{QQ}(s,b) \) in analogy to Eq.(5), obtaining \( \sigma^{QQ}, \sigma_{el}^{QQ}, B_{QQ} \) and \( C_{QQ} \). The hadronic properties are derived from:

\[
\sigma_{tot} = 9\sigma_{tot}^{QQ}
\]

\[
B = B_{QQ} + \frac{8}{\nu^2}
\]

\[
C = C_{QQ} + \frac{4}{\nu^4}
\]

From the relation

\[
\sigma_{el} = \frac{\sigma_{tot}^2}{16\pi} \int dt e^{Bt+Ct^2}
\]

we can calculate the ratio \( \frac{\sigma_{el}}{\sigma_{tot}} \). We have examined this model in detail, utilizing the various parametrizations shown before, and were unable to fit the data. We consistently obtained \( \frac{\sigma_{el}}{\sigma_{tot}} > 10 \), which is indicative of our inability to reconcile, \( \sigma_{tot} \), the slope B, and the curvature parameter C in such a model. It has been suggested [4, 15], that the QQ amplitudes approach the black disc limit. To test this, we have also examined a parametrization in which

\[
a^{QQ}(s,b) = (1 - \frac{a_1}{s^{a_2}}) \cdot [\theta[a_3\ln s + a_4 - b]] +
\]

\[3 \text{ The knowledge of } Im f(s,t) \text{ and } \rho \text{ enables us to calculate the real part of the amplitude at any } t \text{ utilizing } [13]
\]

\[
Re f(s,t) = \rho \frac{d}{dt}(Im f(s,t)).
\]

The above expression completes our restoration of the crossing symmetry in the case when \( \rho \) is small enough.
\[ \theta[b - a_3 \ln s - a_4] \cdot \exp[-\frac{(b - a_3 \ln s - a_4)^2}{a_5}] \pm a^{QQ}_R(s, b) \]  

(31)

Here again, we obtained a \( \chi^2_{d.f.} > 10 \). Our consistent inability to obtain a reasonable fit, is a strong indication that the additive quark model, at least in the naive form we have studied, is not amenable to our problem.

Finally, we have also examined the possibility that the increase of \( \sigma_{\text{tot}} \) with increasing \( s \) is connected to the onset of minijets produced by parton-parton collisions at energies above the ISR region [16]. In such a model the b-space transform of \( G^2(t) \) is normalized by the hard parton-parton cross sections to which we also add a soft background. We disregard the fact that the various authors of reference [16] actually calculated an inclusive rather than a total cross section, and examine the generic form

\[ \Omega_{\text{VI}}(s, b) = [a_1 s^{a_2} / (\ln s)^{a_3} + a_4 + \frac{a_5}{\sqrt{s}} \frac{\nu^5 b^3}{96\pi}] K_3(\nu b) \]  

(32)

It is obvious that in this type of model, the increase in \( \sigma_{\text{tot}} \) with \( s \) is due to an increase of \( \Omega \) through blackening rather than expansion, which will only start when \( a(s, b = 0) = 1 \). Our results for this parametrization are summarized at the bottom of Table I. The \( \chi^2_{d.f.} = 3.7 \) we have obtained implies that such a model cannot accurately account for the observed data. We therefore disagree with the conclusions of Block, Halzen and Margolis [7] who have normalized such a model to the Tevatron data in order to make predictions for energies in the LHC and SSC domain. Our analysis, which shows both a blackening and expansion of \( a(s, b) \) and \( \Omega(s, b) \), excludes the possibility that the onset of minijets is the exclusive reason for rise of \( \sigma_{\text{tot}} \) with energy.

We summarize with the following conclusions and remarks;

1) Our parametrization denoted by \( \Omega_{\text{IV}} \) provides a very good description of the experimental data over the energy range \( 5 \leq \sqrt{s} \leq 1800 \text{ GeV} \).
2) Regardless of details, the data in the \( 23 \leq \sqrt{s} \leq 1800 \text{ GeV} \) range shows both moderate blackening and expansion of \( a(s, b) \).
3) The data is compatible with a smooth transition from a soft to a hard Pomeron contribution which can account for the rise of \( \sigma_{\text{tot}} \) with \( s \). We find that the onset of hard minijet production cannot exclusively account for the rise of \( \sigma_{\text{tot}} \). Our analysis provides strong evidence that the Lipatov Pomeron is the proper way to describe the rise in \( \sigma_{\text{tot}} \) which is due to hard processes, such as minijet production.
4) The additive quark model, at least in it’s naive form, fails completely to provide a simultaneous fit to the data.

5) The various models we have discussed produce somewhat different values of \( \sigma_{\text{tot}} \), \( B \) and \( \frac{\sigma_{\text{tot}}}{\sigma_{\text{II}}} \) at \( \sqrt{s} = 40 \text{ TeV} \). These are summarized in Table II. We note that our predictions for \( \sigma_{\text{tot}} \) are higher than the 121 mb predicted by reference [7] and the 125 mb predicted by reference [8]. It is interesting to check our asymptotic predictions at

\footnote{After the completion of this work we learnt of a more sophisticated approach to the quark model (see refs. [17] [18]) that takes into account the interaction between more than one quark from each hadron.}
the Planck scale \( s \approx 10^{38} \text{ GeV}^2 \). We observe that, asymptotically, the various models compared in Table II have reached the black limit where \( \frac{\sigma}{\sigma_{\text{tot}}} = 0.5 \), and have converged to the same limiting cross section \( \sigma_{\text{tot}} = 1010 \text{ mb} \). If we compare this value with the Froissart bound

\[
\sigma_{\text{tot}} = \frac{\pi}{\mu^2} \ln^2 \frac{s}{s_0}
\]

we find that \( \mu \approx 3 \text{ GeV} \) for \( \sqrt{s_0} < 23 \text{ GeV} \).

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Table 1: Values of parameter for different parameterizations of the Pomeron.
Figure captions

Figure 1  $\sigma_{tot}$ in mb vs $\sqrt{s}$ in GeV. The measured cross sections are from [6]. The dashed (solid) line are the predictions for pp ($p\bar{p}$) for fit $\Omega_{IV}$.

Figure 2  $B$, the measured and predicted nuclear slope parameter in units of $(GeV)^{-2}$ vs $\sqrt{s}$ in GeV. Data from [6]. Dashed (solid) curve as in Fig. 1.

Figure 3  The ratio $\frac{\sigma_{el}}{\sigma_{tot}}$ vs. $\sqrt{s}$ in GeV. Data from [6]. Dashed (solid) curve as in Fig. 1.

Figure 4  The predicted value of the curvature parameter $C$ in units of $(GeV)^{-4}$ vs. $\sqrt{s}$ in GeV. Dashed (solid) curve as in Fig. 1.

Figure 5  Elastic differential $\bar{p}p$ cross section at small -$t$, at representative energies. Data from [6].

Figure 6  Ratio of real to imaginary parts of forward elastic amplitude vs. $\sqrt{s}$ in GeV. Data from [6]. Dashed (solid) curve as in Fig. 1.