Control of quantum particle dynamics by impulses of magnetic field

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Abstract The problem of control of quantum dynamics particle has been studied. The general equations have obtained for a description of this problem. The criteria for optimal time, reaching the desired point, have been obtained and have analyzed. The different types of spectrum of acting fields using for control, from impulse form of the field to the power form and Gauss distribution force spectrums, have been studied.

Keywords Spin states · Control · Information · Stability · Magnetic field · Quantum dot · Quantum dynamics

1 Introduction

The control problem of different systems from robot manipulators to aeroplanes and space satellites has studied intensively due to very actuality this problem [1–4]. The problems of control and robusting of nonlinear systems are very important too and have been studied in [5–8]. The optimization mathematical approaches have developed to analyze and to control of different natural phenomena as water flow, humidity and so on. For example, for hot regions with lack of water the problem of control of humidity has been studied in [9,10].

Along with the above-mentioned problems, the problem of control of quantum systems now has intensively studied because it is important for quantum calculations [11]. A lot of searches of materials for qubits as bases of element base for quantum calculations have conducted. It has made in several directions, but the most perspective, in our opinion, is the creation of qubits (elements of quantum calculations) on the basis of low-dimensional quantum semiconductor systems with quantum dots. Another important problem for quantum calculations is the development of both the new and adapted under concrete quantum nanostructures algorithms and schemes of the quantum calculations allowing to realize logical schemes and elements. Since 1994 when Shor et al. [12–14] offered effective polynomial algorithm for the solution of the factorization problem on quantum computers, then the intensive development of algorithms of quantum calculations has began. There is the fundamental difference of quantum calculations from classical ones, where the exponential number of operations has necessary for solving factorization problem.

At the present moment, three classes of quantum algorithms were known [15–18]. The first class of algorithms has intended for the solution of number factorization, the second class of algorithms has developed for modeling of the quantum phenomena [19], and the
third class of algorithms has intended for search of object in an unstructured random database [20].

Usually the problems of control and robusting of quantum systems have formulated as following: to make transition from a given initial state of quantum system to a final state with the certain condition. In the case of spin transfer, this condition corresponds to the spin system, governed by impulses of magnetic field. Because the study of above-mentioned problem now is at initial stage, the researches are limited by the formulation of general approaches [21]. But the analytic calculations are important for understanding of physics of processes. This paper devoted to study the spin motion under action of impulse magnetic field by analytic calculations. The criteria for optimal time, reaching the desired point, have been analyzed, and the calculations for the concrete types of control, from impulse of the field to the power and Gauss driving force spectrums, have been made.

2 The equations of motion within a magnetic field with control

As it is well known, the particle motion in a magnetic field has described by the following equation:

$$m \frac{dV}{dt} = \frac{q}{c} [V \times B] + U(t) \frac{q}{c} [V \times h]$$

(1)

where $V$ is the velocity of the particle, $B$ is the constant magnetic field, $m$ is the mass of the particle, $q$ is the charge of the particle, $H(t) = U(t) h$ is the alternating magnetic field, $h$ is the unity vector, $U(t)$ is the absolute value of the control signal.

Considering the equation in components and introducing state vector function $X = (x_1, x_2), X \in R^2$, $U \in R$, we formulate the following model of the controlled system:

$$\dot{X} = (A + U(t) C) X$$

(2)

where the matrices $A$ and $C$ are defined by the following expressions: $A = \begin{pmatrix} 0 & \Omega \\ -\Omega & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix}$, and parameters $\Omega$ and $\omega$ are defined as the following ones: $\Omega = \frac{qB}{mc}$ and $\omega = \frac{qh}{mc}$.

As mentioned above, this model describes various physical problems, in particular a particle motion in a magnetic field and spin precession in a magnetic field. From the control theory point of view, the formulated problem belongs to the degenerate class of the optimal control problems because there are additional invariants—the integrals of motion in accordance with the law of conservation of energy:

$$X_1^2 + X_2^2 = \text{const}, \quad (m = \text{const})$$

(3)

The control problem of particle motion in a magnetic field has formulated as the following: to find the time to reach a given point depending on the control parameters—the amplitude and the period of control pulse, as well as the spectrum of external impact. The functional, describing the reaching of the given point in space and solving this problem, has defined:

$$L(T_f) = 1 - \left( (x_1(T_f), x_1^f) + (x_2(T_f), x_2^f) \right)$$

(4)

Here $X^f = (x_1^f, x_2^f)$ is final state vector function, $X(T_f) = (x_1, x_2)$ is current state vector function. The optimal time to reach the given point has defined by fulfilling the condition of functional extreme (minimum).

2.1 The motion of particles within a magnetic field under external impacts

Let’s consider the circular particle motion using the complex variable form. Thus, a complex variable $z = x_1 + i x_2 = |z| e^{-i \phi}$ has introduced; then, the system of the motion equations has transformed to the following simple form:

$$\frac{d |z|}{dt} = 0, \quad \frac{d \phi}{dt} = \Omega + U(t)$$

(5)

The first equation corresponds to the law of energy conservation (the conservation of amplitude). The second equation describes the phase change while circular motion under the control impact. Thus, the circular motion with the defined radius is completely determined by its phase values. The change of the phase is determined by the external fields in turn.

According to Eq. (5), the general solution has such a form:

$$\phi(t) = \Omega t + \omega \int U(t) dt$$

(6)
3 Control of particle motion

According to the above conditions, formulated for optimal control, to define the optimal time for reaching the given point we need to build the functional (4). Let’s construct it.

Consider some current point on the circle.
\[ z = x_1 + i x_2 = |z| e^{-i\varphi}, \]
where the phase is defined by Eq. (5). The final point on the circle has chosen, for example, as the angle equals to 45°:
\[ z^* = x_1^* + i x_2^* = |z| e^{-i\pi/4} \]

Then the functional to be found takes this form:
\[ L (T_f) = 1 - \frac{y^2}{2} \left( 1 + \sin \left( 2 \left( \omega t + \int U (t) dt \right) \right) \right) \]

Accordingly the condition of the functional minimum is defined by the condition for phase:
\[ \varphi (T_F) = \omega T_F + \int_0^{T_F} U (t) dt = 2\pi \]

Thus, the time for optimal reaching the given point at the \( T^1 \) trajectory is defined by the functional Eq. (4).

4 The dependence of particles phase on the control amplitude, duration and specter

In the absence of external impact \( U (t) \) regular standard solution with the initial angular velocity \( \Omega \) arises:

\[
\begin{align*}
V_x (t) &= V_0 \cos (\Omega t + \varphi) \\
V_y (t) &= V_0 \sin (\Omega t + \varphi)
\end{align*}
\]

Then the time for reaching is defined by the statement:
\[ T_F = \Delta \varphi / \omega = \pi / 4\omega \]

These solutions are known as Trunk solutions in mathematics [21].

The dependence of the phase of particle circular motion on the parameters of control has analyzed, and the time for reaching has calculated below.

(1) In case of pulse external impact the control function takes the form:
\[ U (t) = U_0 \theta (t - \varepsilon) - \theta (t) \text{ or} \]
\[ U (t) = U_0 \sum_{n} (t - n \tau) - \theta (t). \]

Then time \( T_F \) has defined by the expression:
\[ T_F = \frac{2\pi - U_0 \tau}{\omega} = \frac{2\pi}{\omega} - \tau \left( \frac{\omega_0}{\omega} \right) \]

The value of the time for reaching position decreases in comparison with the case of impact absence due to the impact amplitude \( \omega_0 \). When the condition fulfilled:
\[ \omega_0 \tau = 2\pi \]

the reaching is immediate: \( T_F = 0 \)

(2) In case of exponentially temporal dependence of impacts the control function takes this form:
\[ U (t) = U_0 e^{-\alpha t} \]

Consider the case of adiabatic field; the time to reach the desired point increases:

(a) \( \alpha T_F << 1 \), then \( T_F = \frac{\pi}{4} / \omega_0 + \alpha \)

(b) \( \alpha T_F >> 1 \), then \( T_F = \frac{\pi}{4} - \frac{U_0}{\omega} \)

(3) In the case of Gaussian impact \( U (t) = U_0 e^{-\frac{\beta^2}{2} t^2} \) the control function takes this form:
(b) \( \beta T_F \gg 1 \), then \( \omega T_F + \frac{U_0 \sqrt{\beta}}{\beta} = \frac{\pi}{4} \) and

\[
T_F = \frac{\pi}{4} - \frac{U_0 \sqrt{\beta}}{\omega} \tag{17}
\]

(4) In case of power distribution of the action the function takes this form:

\[
U(t) = U_0 \left( \frac{t}{\tau} \right)^\gamma, \quad \gamma > 0.
\]

\[
\frac{T_F}{\tau} U(t) dt = \frac{U_0}{\gamma + 1} \left( \frac{t}{\tau} \right)^{\gamma+1} = \frac{1}{\gamma + 1} \frac{U_0}{\tau^\gamma} T_F^{\gamma+1}
\]

\[
\frac{\pi}{4} = \frac{U_0}{(\gamma + 1) \tau^\gamma} T_F^{\gamma+1} + \omega T_F,
\]

\[
(\omega_0 \tau) \left( \frac{T_F}{\tau} \right)^{1-|\gamma|} + \omega T_F = \frac{\pi}{4}, \quad -\infty < \gamma \leq -1.
\]

Consider three cases:

(a) \(-\infty < \gamma < -1\); then, we have the equation for optimal time:

\[
\omega_0 \tau \left( \frac{T_F}{\tau} \right)^{1-|\gamma|} + \omega T_F = \frac{\pi}{4}, \tag{19}
\]

and then, in case of strong impacts \(-\omega_0 \tau \left( \frac{T_F}{\tau} \right)^{1-|\gamma|} << \omega T_F\) the following expression has obtained:

\[
T_F = \frac{\pi}{4\omega} \tag{20}
\]

In the opposite case of weak impacts

\[
-\omega_0 \tau \left( \frac{T_F}{\tau} \right)^{1-|\gamma|} \gg \omega T_F,
\]

the another formula has followed:

\[
\frac{T_F}{\tau} = \left( \frac{\pi}{4\omega_0 \tau} \right)^{\frac{1}{|\gamma|-1}} \tag{21}
\]

The dependence on the parameters of signal spectrum of reaching time has increased.

(b) \(-1 < \gamma < 0\); the following equation has obtained:

\[
(\omega_0 \tau) \left( \frac{T_F}{\tau} \right)^{\gamma+1} + \omega T_F = \frac{\pi}{4}. \tag{22}
\]

At small values, the time to reach the point does not depend on the specter of time distribution \( T_F = \frac{\pi}{4\omega} \).

When \( T_F \to \infty \)

\[
T_F = \left( \frac{\pi}{4\omega_0 \tau} \right)^{\frac{1}{|\gamma|-1}} \tag{23}
\]

At large values, the time to reach the point has defined by the specter of distribution.

(c) \( \gamma > 0 \),

and then \( (T_F)^{\gamma+1} + \omega T_F = \frac{\pi}{4} \) that yields

\[
T_F = \tau \left( \frac{\pi}{4\omega_0 \tau} \right)^{\frac{1}{\gamma+1}} \tag{24}
\]

At short times, the impact time to reach the point is not sensitive to the specter and then \( T_F = \frac{\pi}{4\omega} \). At large times, strong impact time to reach the point has defined by the type of the specter.

5 Conclusion

Thus, the particle motion by the impact of external fields has been studied. The possibility to control the motion and reaching the given point under different types of external impact has been investigated. The dependence of time to reach the given point at the trajectory on the spectrum of the magnetic field has been established. For weak impact of fields that usually corresponds to the case of small times, the optimal time of reaching the final position has an universal value, which corresponded to trunk fundamental solutions. At the case of large times, the optimal time depends on the parameters of magnetic field: form and amplitude of the field, spectrum of fields and so on. This dependence shows that there is a possibility using different form and amplitude, duration and spectrum of fields to control of optimal time and as result to manage this process. These results have been applied for the problem of control of spin systems and further for problem of quantum calculations.

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