Note About Hamiltonian Formalism of Modified $F(R)$ Hořava-Lifshitz Gravities and Their Healthy Extension

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ABSTRACT: This note is devoted to the study of Hamiltonian formalism of modified F(R) Hořava-Lifshitz theories of gravity that were proposed recently in arXiv:1001.4102[hep-th]. We also study Hamiltonian formulation of the healthy extended Hořava-Lifshitz gravities and show that these theories have many unusual and interesting properties.

KEYWORDS: Hořava-Lifshitz gravity, F(R) gravity
1. Introduction and Summary

Last year Petr Horava proposed new intriguing approach for the formulation of UV finite quantum theory of gravity \cite{1, 2, 3}. The basic idea of this theory is to modify the UV behavior of the general theory so that the theory is perturbatively renormalizable. However, this modification is only possible on condition when we abandon Lorentz symmetry in the high energy regime: in this context, the Lorentz symmetry is regarded as an approximate symmetry observed only at low energy.

In \cite{4, 11} we introduced version of Hořava-Lifshitz gravity that is related to $F(R)$ theories \footnote{For review and extensive list of references, see \cite{13, 14, 15, 16, 17}.}. This approach was further developed in very interesting paper \cite{8}. It was argued there that such a form of gravity could provide unification of the early time inflation with the late time acceleration. Moreover, the preliminary analysis of the cosmological solution with some promising properties was given there as well.

The goal of this short note is to find the Hamiltonian formulation of modified $F(R)$ Horava-Lifshitz theory. In fact, the Hamiltonian analysis of given theory was already done in \cite{8} but we feel that it deserve to be investigated further. Following \cite{18} we formulate the Hamiltonian formalism for modified $F(R)$ Hořava-Lifshitz gravity and we show that the algebra of constraints is closed for theory that obeys the projectability condition that claims that the lapse function depends on time only $N = N(t)$.

As a counterexample of standard form of well defined Hamiltonian dynamics of modified $F(R)$ theories of gravity that obey the projectability condition we discuss the Hamiltonian analysis of healthy extended Hořava-Lifshitz gravity that was proposed in \cite{5, 6}. Explicitly, since the momentum conjugate to lapse is primary constraint of the theory we find that the preservation of this constraint during the time evolution of the system induces the secondary constraint that has non-zero Poisson bracket with the primary constraint $p_N \approx 0$. In other words, they form the collection of the second class constraints. It is instructive to compare this result with conclusions presented in \cite{9}. It was shown there that the Hořava-Lifshitz gravity without the projectability condition has very peculiar property in the sense that the Hamiltonian constraints are the second class constraints and that
the gravitational Hamiltonian vanishes strongly. However in case of the healthy extended Hořava-Lifshitz gravities we find new and surprising resolution. Explicitly, since $p_N$ and corresponding secondary constraints are the second class constraints their can be explicitly solved. Then we can express $N$ as a function of cannonical variables, at least at principle. Further, the reduced phase space of healthy extended Hořava-Lifshitz theory is spanned by $g_{ij}, p^{ij}$ and there is no gauge freedom related to the time reparameterization of theory since there is not the first class Hamiltonian constraint. Interestingly, this result naturally solves the problem of the closure of the algebra of the Hamiltonian constraints in the Hořava-Lifshitz gravity. Secondly, one can hope that healthy extended Hořava-Lifshitz gravities can provide solution of the problem of time in gravity \(^2\). We hope to return to this interesting problem in future.

The structure of this note is as follows. In the next section (2) we perform the Hamiltonian analysis of modified $F(R)$ Hořava-Lifshitz theory of gravity. In section (3) we perform the Hamiltonian analysis of healthy extended Hořava-Lifshitz gravities and discuss their properties.

2. Hamiltonian Formulation of Modified $F(R)$ Hořava-Lifshitz gravity

Let us consider $D+1$ dimensional manifold $M$ with the coordinates $x^\mu, \mu = 0, \ldots, D$ and where $x^\mu = (t, x), x = (x^1, \ldots, x^D)$. We presume that this space-time is endowed with the metric $\hat{g}_{\mu\nu}(x^\rho)$ with signature $(-, +, \ldots, +)$. Suppose that $M$ can be foliated by a family of space-like surfaces $\Sigma_t$ defined by $t = x^0$. Let $g_{ij}, i, j = 1, \ldots, D$ denotes the metric on $\Sigma_t$ with inverse $g^{ij}$ so that $g_{ij}g^{jk} = \delta^k_i$. We further introduce the operator $\nabla_i$ that is covariant derivative defined with the metric $g_{ij}$. We introduce the future-pointing unit normal vector $n^\mu$ to the surface $\Sigma_t$. In ADM variables we have $n^0 = \sqrt{-\hat{g}^{00}}, n^i = -\hat{g}^{0i}/\sqrt{-\hat{g}^{00}}$. We also define the lapse function $N = 1/\sqrt{-\hat{g}^{00}}$ and the shift function $N^i = -\hat{g}^{0i}/\hat{g}^{00}$. In terms of these variables we write the components of the metric $\hat{g}_{\mu\nu}$ as

$$\hat{g}_{00} = -N^2 + N_i g^{ij} N_j, \quad \hat{g}_{0i} = N_i, \quad \hat{g}_{ij} = g_{ij},$$
$$\hat{g}^{00} = -\frac{1}{N^2}, \quad \hat{g}^{0i} = \frac{N^i}{N^2}, \quad \hat{g}^{ij} = g^{ij} - \frac{N^i N^j}{N^2}.$$  (2.1)

Then it is easy to see that
$$\sqrt{-\det \hat{g}} = N \sqrt{\det g}.$$  (2.2)

We further define the extrinsic derivative
$$K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i).$$  (2.3)

It is well known that the components of the Riemann tensor can be written in terms of ADM variables \(^3\). For example, in case of Riemann curvature we have

$$R = K^{ij} K_{ij} - K^2 + R^{(D)} + \frac{2}{\sqrt{-g}} \partial_i (\sqrt{-g} n^\mu K) - \frac{2}{\sqrt{g N}} \partial_i (\sqrt{g} g^{ij} \partial_j N),$$  (2.4)

\(^2\)For detailed discussion of this problem, see [19].

\(^3\)For review and extensive list of references, see [12].
where $K = K_{ij} g^{ji}$ and where $R^{(D)}$ is Riemann curvature calculated using the metric $g_{ij}$. The new formulation of Hořava-Lifshitz $F(R)$ gravity that was given in [24] is based on the modification of the relation (2.4). In fact, the action introduced there takes the form

$$S_{F(R)} = \int dt d^D x \sqrt{g} N F(\bar{R}) ,$$

where

$$\bar{R} = K_{ij} G^{ijkl} K_{kl} + \frac{2\mu}{\sqrt{-g}} \partial_i (\sqrt{-g} n^\mu K) - \frac{2\mu}{\sqrt{g} N} \partial_i (\sqrt{g} g^{ij} \partial_j N) - E^{ij} G_{ijkl} E^{kl} ,$$

where $\mu$ is constant and where the generalized metric $G^{ijkl}$ is defined as

$$G^{ijkl} = \frac{1}{2} (g^{ik} g^{jl} + g^{il} g^{jk}) - \lambda g^{ij} g^{kl} ,$$

where $\lambda$ is real constant. $E^{ij}$ are defined using the variation of $D$–dimensional action $W(g_{kl})$

$$\sqrt{g} E^{ij} = \frac{\delta W}{\delta g_{ij}} .$$

These objects were introduced in the original work [1]. However we can consider theory when $E_{ij} G^{ijkl} E^{kl}$ is replaced with more general terms that depend on $g_{ij}$ and their covariant derivatives. Further, the action (2.5) is invariant under foliation preserving diffeomorphism

$$t' - t = f(t) , \quad x'^i - x^i = \xi^i(t, x) .$$

Our goal is to perform the detailed Hamiltonian analysis of the theory defined by the action (2.5). In order to do this we introduce two non-dynamical fields $A, B$ and rewrite the action (2.5) into the form

$$S_{F(R)} = \int dt d^D x \sqrt{g} N (B(\bar{R} - A) + F(A)) .$$

It is easy to see that solving the equation of motion with respect to $A, B$ this action reduces into (2.4). On the other hand when we perform integration by parts we obtain the action in the form

$$S_{F(R)} = \int dt d^D x \left( \sqrt{g} N B (K_{ij} G^{ijkl} K_{kl} - E^{ij} G_{ijkl} E^{kl} - A) + \sqrt{g} N F(A) - 2\mu \sqrt{g} (\partial_i B - N^i \partial_i B) K + 2\mu \partial_i B \sqrt{g} g^{ij} \partial_j N \right) ,$$

where we ignored the boundary terms. From this form of the action we clearly see that $B$ is now dynamical field. In fact, from the action (2.11) we find the conjugate momenta

$$p_N = \frac{\delta S_{F(R)}}{\delta \partial_t N} \approx 0 , \quad p_i = \frac{\delta S_{F(R)}}{\delta \partial_t N^i} \approx 0 , \quad p_A = \frac{\delta S_{F(R)}}{\delta \partial_t A} \approx 0 ,$$

$$\dot{p}^{ij} = \frac{\delta S_{F(R)}}{\delta \partial_t g_{ij}} = \sqrt{g} \left( B G^{ijkl} K_{kl} - \frac{2\mu g^{ij}}{N} (\partial_i B - N^i \partial_i B) \right) ,$$

$$\pi = \frac{\delta S_{F(R)}}{\delta \partial_t B} = -2\mu \sqrt{g} K .$$

(2.12)
The first line in (2.12) implies that $p_N, p_i$ are primary constraints of the theory. On the other hand the relations on the second and third line in (2.12) can be inverted so that

$$\partial_t B - N^i \partial_i B = -\frac{N}{2\mu D \sqrt{g}} \left( \frac{1}{2\mu} B (1 - \lambda D) \pi + p^{ij} g_{ji} \right),$$

$$K_{ij} = \frac{1}{B \sqrt{g}} G_{ijkl} \left( \frac{p^{kl} - \frac{1}{D} g^{kl}}{D \sqrt{g}} \left( \frac{1}{2\mu} B (1 - \lambda D) \pi + p^{kl} g_{ik} \right) \right),$$

(2.13)

where we used the fact that

$$g_{ij} G_{ijkl} = (1 - \lambda D) g^{kl}.$$  

(2.14)

Using these results it is straightforward exercise to find corresponding Hamiltonian

$$H = \int d^D x (N \mathcal{H}_T + N^i \mathcal{H}_i + v^A p_A + v^N p_N + v^i p_i),$$

(2.15)

where

$$\mathcal{H}_T = \frac{1}{B \sqrt{g}} p^{ij} G_{ijkl} p_{kl} - \frac{1}{D \mu \sqrt{g}} (1 - \lambda D)^2 \pi p^{ij} g_{ji} - \frac{1}{BD \sqrt{g}} (1 - \lambda D) (\pi_{ij} g_{ji})^2 +$$

$$+ \frac{1}{\sqrt{g}} \left( \frac{(1 - \lambda D)^2 B}{4D \mu^2} \right) ((1 - \lambda D)^2 - 2) \pi^2 +$$

$$+ \sqrt{g} B (E^{ij} G_{ijkl} E^{kl} + A) - \sqrt{g} F(A) + 2\mu \partial_i [\partial_j B \sqrt{g} g^{ij}],$$

$$\mathcal{H}_i = -2 g_{ik} \nabla_j p_{kj} + \pi \partial_i B,$$

(2.16)

and where we included the primary constraints $p_N \approx 0, p_i \approx 0, p_A \approx 0$. Note that as opposite to the Hamiltonian analysis presented in [8] we find that $B$ is dynamical field. Further, the consistency of the primary constraints with the time evolution of the system implies following secondary constraints

$$\partial_t p_N(x) = \{p_N(x), H\} = -\mathcal{H}_T(x) \approx 0,$$

$$\partial_t p_i(x) = \{p_i(x), H\} = -\mathcal{H}_i(x) \approx 0,$$

$$\partial_t p_A(x) = \{p_A(x), H\} = -\sqrt{g} N (B - F'(A))(x) \equiv -\sqrt{g} N G_A(x) \approx 0.$$

(2.17)

Since $\{p_A(x), G_A(y)\} = F''(A) \delta(x-y)$ we see that $\{p_A, G_A\}$ are the second class constraints and hence can be explicitly solved. The solving the first one we set $p_A$ strongly zero while solving the second one we find $F'(A) = B$. If we presume that $F'$ is invertible we can express $A$ as a function of $B$ so that $A = \Psi(B)$ for some function $\Psi$. Finally, since $\{p^{ij}, p_A\} = \{g_{ij}, p_A\} = 0$ we see that the Dirac brackets between canonical variables coincide with Poisson brackets.

Let us consider the smeared form of the spatial diffeomorphism generator

$$T_S = \int d^D x \xi^i \mathcal{H}_i.$$

(2.18)
It is easy to see that this generates the spatial diffeomorphism since

\[
\{ T_S, B(x) \} = -\xi^i(x) \partial_i B(x) , \\
\{ T_S, \pi(x) \} = -\xi^i(x) \partial_i \pi(x) - \partial_i \xi^i(x) \pi(x) , \\
\{ T_S, g_{ij}(x) \} = -\xi^k(x) \partial_k g_{ij}(x) - g_{jk}(x) \partial_i \xi^k(x) - g_{ik}(x) \partial_j \xi^k(x) , \\
\{ T_S, p^{ij}(x) \} = -\partial_k p^{ij}(x) \xi^k(x) - p^{kj}(x) \partial_i \xi^k(x) + p^{jk}(x) \partial_i \xi^k(x) + p^{ik}(x) \partial_j \xi^k(x) .
\]

(2.19)

Using the Poisson bracket between \( T_S \) and \( B \) we find

\[
\{ T_S, A(B(x)) \} = \frac{\delta A(x)}{\delta B(x)} \{ T_S, B(x) \} = - \frac{\delta A(x)}{\delta B(x)} \xi^k(x) \partial_k B(x) = - \xi^k(x) \partial_k A(x) .
\]

(2.20)

Then we find following Poisson bracket

\[
\{ T_S, \mathcal{H}_T(x) \} = -\xi^k(x) \partial_k \mathcal{H}_T(x) - \mathcal{H}_T(x) \partial_k \xi^k(x)
\]

(2.21)

that implies

\[
\{ T_S(\xi), T_T(f) \} = \int d^Dx (\partial_k f \xi^k) \mathcal{H}_T = T_T(\partial_k f \xi^k) .
\]

(2.22)

Note that the right side in the expression above vanishes for constant \( f \).

Finally we calculate the Poisson bracket of \( T_T(f), T_T(g) \). Clearly the calculations of the Poisson bracket \( \{ \mathcal{H}_T(x), \mathcal{H}_T(y) \} \) will be as intricate as the calculation of the Poisson bracket between these constraints in standard Hořava-Lifshitz gravity. The structure of these brackets was analyzed in [7, 9] with the outline that \( \mathcal{H}_T \) are the second class constraints with unclear physical meaning of this theory. On the other hand it is possible to find consistent physical theory (at least on the classical level) in case when we impose the projectability condition that claims that \( N = N(t) \). Then the local primary constraint \( p_N(x) \approx 0 \) is replaced with the global one \( p_N \approx 0 \) and its preservation during the time evolution of the system implies the global constraint 4

\[
T = \int d^Dx \mathcal{H}_T(x) \approx 0 .
\]

(2.23)

Then we find that the Hamiltonian is the linear combination of the first class constraints

\[
H = v^N p_N + v^i p_i + N T + T_S(N_i) .
\]

(2.24)

Finally using the fact that

\[
\{ T_S(\xi), T_S(\eta) \} = T_S(\xi^i \partial_i \eta^k - \eta^i \partial_i \xi^k)
\]

(2.25)

4Clearly this constraint takes the same form as \( T_T(f) \) for constant \( f = 1 \).
and also the equation (2.22) when we impose the condition \( f = 1 \) we find that the constraints
\[ T \approx 0 , \quad T_S(\xi) \approx 0 \]
are consistent with the time evolution of the system since
\[
\begin{align*}
\partial_t T &= \{T, H\} \approx 0 , \\
\partial_t T_S(\xi) &= \{T_S(\xi), H\} \approx 0 .
\end{align*}
\]

\[ (2.26) \]

Let us conclude our results. We derived the Hamiltonian formulation of modified \( F(R) \) Hořava-Lifshitz gravity and argued that this is a consistent theory when the projectability condition is imposed. Observe that the requirement of the consistency of the constraints with the time evolution implies the secondary constraints only which is different from analysis presented in [8]. Explicitly, it was argued there the consistency of the constraints with the time evolution of the system could lead to the the possibility of the generation of tertiary constraints or constraints of higher order until the closure of constraints is established.

### 3. Hamiltonian Dynamics of Healthy Extended Hořava-Lifshitz Gravity

The healthy extended of Hořava-Lifshitz theory was proposed in [5] in order to improve some pathological properties of the Hořava-Lifshitz gravity without projectability condition. Explicitly, the healthy extended Hořava-Lifshitz gravity is the version the Hořava-Lifshitz theory without projectability and without detailed balance condition imposed that contains additional vector \( a_i \) constructed from the lapse function \( N(t, x) \) as
\[
a_i = \frac{\partial_i N}{N} \quad (3.1)
\]
Note that under foliation preserving diffeomorphism where \( N'(t', x') = N(t, x)(1 - \dot{f}(t)) \) we find that \( a_i \) transforms as
\[
a'_i(t', x') = a_i(t, x) - a_j(t, x)\partial_i \xi^j(t, x) . \quad (3.2)
\]
Let us now consider the healthy extension of modified \( F(R) \) Hořava-Lifshitz theory of gravity defined by the action
\[
S = \int dt d^Dx \sqrt{g}N(B(\tilde{R} - V(g_{ij}, a_i) - A) + F(A)) , \quad (3.3)
\]
where \( V(g, a) \) is an additional potential term that depends on \( a_i \) and on \( g_{ij} \). Performing the same analysis as in previous section we find the Hamiltonian in the form
\[
\begin{align*}
H &= \int d^Dx \left( N(H_T + B\sqrt{g}V) + N^i \mathcal{H}_i + \\
&\quad + v^i p_i + v^N p_N + v^A p_A \right) ,
\end{align*}
\]

\[ (3.4) \]
where $\mathcal{H}_T$ and $\mathcal{H}_i$ are the same as in case of modified $F(R)$ Hořava-Lifshitz gravity. The crucial point of the Hamiltonian analysis of the healthy extended Hořava-Lifshitz gravity is that the condition of the preservation of the primary constraint $p_N \approx 0$ implies following secondary one

$$
\partial_t p_N(x) = \{p_N(x), H\} = -(\mathcal{H}_T(x) + B \sqrt{g} V(x)) +
\frac{1}{N} \partial_i \left( NB \frac{\delta V}{\delta a_i} \right)(x) \equiv -\dot{\mathcal{H}}_T(x) \approx 0
$$

(3.5)

using

$$
\left\{ p_N(x), \int d^D y NB \sqrt{g} V(g, a) \right\} = -B \sqrt{g} V(x) + \frac{1}{N} \partial_i \left( NB \sqrt{g} \frac{\delta V}{\delta a_i} \right)(x)
$$

(3.6)

The general analysis of the constraint systems implies that the total Hamiltonian is the sum of the original Hamiltonian and all constraints so that the Hamiltonian takes the form

$$
H = \int d^D x (N\mathcal{H}_T + \sqrt{g} BV) + N\mathcal{H}_i + v_T \dot{\mathcal{H}}_T + v^N p_N + v^i p_i \right).
$$

(3.7)

where $v_T$ is Lagrange multiplier related to the new constraint $\dot{\mathcal{H}}_T$. Observe that as opposite to the case of canonical gravity or standard Hořava-Lifshitz theory $N$ does not appear as Lagrange multiplier in the Hamiltonian (3.7). This is the first indication of the slightly unusual behavior of this theory. In order to investigate the properties of given theory further we introduce the smeared form of the Hamiltonian constraint $\mathbf{T}_T(f) = \int d^D x f(x) \mathcal{H}_T(x)$. Then we find

$$
\{p_N, \mathbf{T}_T(f)\} = \frac{1}{N} f \partial_i \left( B \sqrt{g} \frac{\delta V}{\delta a_i} \right) +
\frac{1}{N} \partial_i \left( \frac{f}{N} \right) B \sqrt{g} \frac{\delta^2 V}{\delta a_i \delta a_j} + \partial_j \left( \frac{f}{N} \right) B \sqrt{g} \frac{\delta^2 V}{\delta a_i \delta a_j}.
$$

(3.8)

Since the Hamiltonian can be written as

$$
H = \int d^D x (N\mathcal{H}_T + \sqrt{g} BV) + v^N p_N + v^i p_i + \mathbf{T}_T(v_T) + \mathbf{T}_S(N^i)
$$

(3.9)

we find that the time derivative of $p_N$ is equal to

$$
\partial_t p_N = \{p_N, H\} \approx \frac{1}{N} v_T \partial_i \left( B \sqrt{g} \frac{\delta V}{\delta a_i} \right) +
\frac{1}{N} \partial_i \left( \frac{v_T}{N} \right) B \sqrt{g} \frac{\delta^2 V}{\delta a_i \delta a_j} + \partial_j \left( \frac{v_T}{N} \right) B \sqrt{g} \frac{\delta^2 V}{\delta a_i \delta a_j}.
$$

(3.10)
In principle this equation can be solved for $v_T$ so that it is determined by the dynamical variables. In other words, $p_N$ and $\tilde{\mathcal{H}}_T$ form the second class constraints and consequently there is no gauge freedom related to the constraint $\tilde{\mathcal{H}}_T$. However this fact has very interesting consequences for the structure of the theory. Explicitly, since $p_N(x), \tilde{\mathcal{H}}_T(x)$ are second class constraints they can be explicitly solved. The solution of the first one is $p_N(x) = 0$ strongly. On the other hand we suggest that the constraint $\tilde{\mathcal{H}}_T(x) = 0$ can be solved for $a_i = \frac{\partial N}{N}$ and hence $N$ can be expressed as a function of dynamical variables $g_{ij}, p^{ij}$

$$N = \Phi(g_{ij}, p^{ij}) \quad (3.11)$$

Further, since the Poisson brackets between $g_{ij}, p^{ij}$ and $p_N$ vanish we find that the Dirac brackets between canonical variables $g_{ij}, p^{ij}$ that span the reduced phase space of the theory coincide with the Poisson brackets. Finally, using (3.11 in (3.9) we find that the Hamiltonian on the reduced phase space takes the form

$$H = \int d^Dx(\Phi(\mathcal{H}_T + \sqrt{g}BV(\Phi)) + v^i p_i) + + T_S(N^i) \quad (3.12)$$

We see that this Hamiltonian contains generator of the spatial diffeomorphism that is the first class constraint. The presence of this constraint is a consequence of the fact that this theory is invariant under spatial diffeomorphism. Observe that there is no gauge freedom related to time reparameterization. This result suggests that even if the structure of the healthy extended Hořava-Lifshitz gravity is completely different from general relativity it has the potential that it can solve the long standing problem of time in general relativity. It would be very interesting to study this theory further for some examples of the potential $V$ that allow to find $N$ as a function of canonical variables and hence find Hamiltonian on reduced phase space. We hope to return to this problem in future.

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5I would like to thank to Diego Blas, Oriol Pujolas and Sergey Sibiryakov for suggesting me this interpretation.

6By the problem of time in General Relativity (GR) one means that GR is a completely parametrised system. That is, there is no natural notion of time due to the diffeomorphism invariance of the theory and therefore the canonical Hamiltonian which generates time reparametrisations vanishes. In fact, instead of a Hamiltonian there are an infinite number of spatial diffeomorphism and Hamiltonian constraints respectively, of which the canonical Hamiltonian is a linear combination, which generate infinitesimal spacetime diffeomorphisms.
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