Model-independent parametrisations of modified gravity have attracted a lot of attention over the past few years; numerous combinations of experiments and observables have been suggested to constrain these parameterisations, and future surveys look very promising. Galaxy Clusters have been mentioned, but not looked at as extensively in the literature as some other probes. Here we look at adding Galaxy Clusters into the mix of observables and examine whether they could improve the constraints on the modified gravity parameters. In particular, we forecast the constraints from combining the Planck CMB spectrum and SZ cluster catalogue and a DES-like Weak Lensing survey. We've found that adding cluster counts improves the constraints obtained from combining CMB and WL data.

1 Introduction

Einstein’s General Relativity (GR) is one of the principal ingredients of modern cosmology. Nonetheless, it is our job as physicists to continue to test even the fundamental pillars of cosmology in order to refine, improve and further justify our model of the universe. Testing GR outside of the solar system can be quite challenging, particularly as the effects of a different theory of gravity could be degenerate with different possible constituents of the universe. This is the case with the current cosmological observations that suggest the presence of dark matter and dark energy. As well as explaining these observations, there are also fundamental physics reasons for considering different theories of gravity: GR is inconsistent with quantum mechanics and the search for "Quantum Gravity" is one of the holy grails of modern physics.

Here, we are interested in testing deviations from GR in a model independent way. There are several advantages to a model independent approach; some alternatives to GR do exist but there is no complete theory of, for example, quantum gravity to draw on. Also, amongst the many options there are no "stand-out" candidates that are universally considered to be strong alternatives. If a model independent approach suggests that the data is inconsistent with concordance cosmology and GR, it will be relatively unambiguous and therefore a strong motivator to develop alternative theories, as well as possibly giving us a clue as to the nature of these theories.

There are studies in the literature on the constraining power of current data and the general conclusion is that the concordance cosmology is consistent with all of the current data, although the data isn’t strongly constraining. Work has also gone into forecasting future experiments and again there is a fair degree of consensus here, namely that future surveys will greatly improve prospects. In this work we will examine the constraints that can be put on model independent modified gravity using the combination of Cosmic Microwave Background anisotropies (CMB) cross-correlated with weak lensing surveys and galaxy cluster counts. For
more details and up to date figures, see [1].

2 Modified Gravity

Our potentials are defined in a flat FRW metric in the Newtonian gauge by \( g_{00} = -1 - 2\Psi(\vec{x}, t), g_{0i} = 0, g_{ij} = a^2\delta_{ij}(1 - 2\Phi(\vec{x}, t)) \). \( \Psi \) is the Newtonian potential and is responsible for the acceleration of massive particles. \( \Phi \) is the curvature potential, which contributes to the acceleration of relativistic particles only.

Several sets of modified gravity parameters (MGPs) have been proposed, see [6] for one of the first papers and [7] for a partial translation table between the different parameterisations. In this work we will use two MGPs, \( \eta \) and \( \mu \), following [8] and implemented in the code MGCAMB, to describe departures from GR. The first, \( \eta \), is the ratio of the two metric potentials, \( \eta = \Psi/\Phi \). This will be approximately unity in GR unless any of the particle species has large anisotropic stress. The second, \( \mu \), is a modification of the poisson equation, and is essentially a time and space dependent Newton’s constant. Fourier expanding the spatial dependence with wavenumbers \( k \) and assuming isotropy, the modification of the Poisson equation is as follows

\[
k^2\Psi(a, k) = -4\pi G a^2 \mu(a, k) \rho(a) \Delta(a, k),
\]

where, \( a \) is the FRW scale factor, \( G \) is Newton’s constant, \( \rho \) is the background density of cold dark matter and \( \Delta \) is the gauge invariant density contrast.

We will assume that GR is valid up to a specified redshift \( z_{mg} = 30 \). Beyond this, we assume that the MGPs transition to a constant value that is different to the GR value. The background expansion history is already constrained to be close to that of a ΛCDM model, we will therefore assume that the modified gravity mimics the expansion history of a standard ΛCDM setup.

3 Observables

3.1 Cluster Counts

Galaxy clusters are some of the largest collapsed structures in the universe. According to the standard ΛCDM cosmology, they typically consist of hot gas bound in a large cold dark matter halo. They are a useful cosmological probe as their size corresponds to scales near the linear to non-linear transition in the underlying dark matter power spectrum. This has several consequences: they probe the tail of the matter perturbation spectrum and are therefore a sensitive probe of growth. In addition, galaxy cluster counts can be predicted accurately from the linear theory matter power spectrum, using semi-analytic formulae or ones calibrated from N-body simulations.

Our theoretical predictions for the number of clusters in redshift bins will be compared to predicted SZ catalogues for a number of future observational stages. The SZ effect [9] is a nearly redshift independent tracer of clusters that is due to the rescattering of CMB photons by hot intracluster gas. The observational limits on SZ observations are, in principle, determined simply by resolution and sky coverage.

3.2 CMB

With the release of Planck satellite [10] results only a few years away we are entering an era where observations of the CMB total intensity spectrum will have reached the sample variance limit throughout scales where primary effects dominate the signal. The sensitivity to MGPs in the CMB spectrum is restricted to the largest scales and the main signal that will arise on these scales is the ISW effect. This is sourced as the Universe transitions into a dark energy
dominated model and the potential starts to decay. The effect can be described by the integral of the time-derivative of the sum of metric potentials along the line of sight.

3.3 Weak lensing

The third observable we will use is the convergence power spectrum from weak lensing surveys. Weak lensing is a relatively new cosmological tool and is a measure of the small distortions of background galaxies caused by gravitational lensing by large scale structure \cite{11,12}. Distortions of individual background galaxies are virtually impossible to measure due to the intrinsic ellipticity of galaxies. However, statistical results averaging over large numbers of galaxies are now routinely reported.

For our initial weak lensing survey, we consider a DES-like survey. DES\cite{13} (Dark Energy Survey) is a ground based survey that is scheduled to begin observations in 2011. It will survey 5000 sq deg over 5 years and aims to constrain dark energy with 4 probes: supernovae, BAO, galaxy clusters and weak lensing, the latter being the probe we are interested in here.

4 Forecasts

In this Section we carry out forecasts for two future observational ‘Stages’. For weak lensing and cluster counts we will assume two distinct observational stages corresponding to short and long term development of survey sizes and accuracies. This is unnecessary for the CMB as the data from Planck over the range of interest will be cosmic variance limited and therefore essentially as good as theoretically possible.

Stage 1 corresponds to a Planck-like SZ survey and a DES-like weak lensing survey. DES will be carried out on the Cerro Tololo Inter-American Observatory in the Chilean Andes and should start taking data in late 2011. The stage 2 weak lensing survey is based on the LSST\cite{14}, due to begin taking data in 2020. The stage 2 SZ survey corresponds to a Planck-like survey, but with a better flux resolution, allowing smaller mass clusters to be detected.

Our forecasts will be based on Fisher matrix\cite{15,16} estimates of errors in a subset of parameters comprising the MGPs $\eta$ and $\mu$ and two parameters from the standard model that are expected to be most correlated with them, namely, the total matter density $\Omega_m$ and the primordial amplitude of scalar curvature perturbations $A$.

5 Results

| Parameter   | CMB and WL cross-correlation | Clusters added | Cluster counts self calibrated |
|-------------|-----------------------------|---------------|-------------------------------|
| $\Omega_m$  | 0.00104                     | 0.00104       | 0.00104                       |
| $\log (10^{10} A)$ | 0.00215                 | 0.00214       | 0.00214                       |
| $\eta$      | 0.0300                      | 0.00548       | 0.0150                        |
| $\mu$       | 0.00835                     | 0.00234       | 0.00451                       |

After cross-correlating the CMB with weak lensing, the constraints on the MGPs are quite good. This is due to the complementarity between the data sets; with the CMB providing strong constraints on the standard parameters, any degeneracies between the standard parameters and the MGPs in the weak lensing data are broken. However, since both the CMB and weak lensing rely on the sum of the two potentials, there is still a degeneracy between the MGPs that is affecting the constraints. This is where the galaxy cluster counts are useful, as only $\Psi$ is relevant and hence only $\mu$ contributes. Thus, the data from the cluster counts breaks...
the degeneracy between the MGPs from the CMB and Weak lensing, creating a much tighter constraint as shown in figure 1. There are some uncertainties associated with cluster counts and these are also shown in figure 1. Although marginalising over these uncertainties reduces the impact of clusters, clusters still add to the constraining power of the CMB and Weak lensing. The constraints on the parameters for the first stage of experiments are shown in table 1. In addition, figure 2 shows how galaxy clusters are still a worthwhile addition to cross correlated CMB and weak lensing measurements for the longer term survey.

6 Conclusion

Over the next 5-10 years, deviations from GR should be well constrained, and the concordance cosmology will either be more secure or may even have undergone a paradigm shift. If the latter is the case, then the results from the model independent tests could be crucial in helping to find a new theory of gravity.

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