Study on the optimization of missile take off mass and its influence on aerodynamic parameters

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Abstract. Aiming at the problem of optimal design of take-off quality in the process of missile overall design, on the basis of establishing the optimization model of missile take-off quality, three different optimization algorithms are used to optimize the calculation of missile take-off quality, by comparing the effects of the three algorithms on the takeoff mass optimization and aerodynamic parameters, the models and algorithms for different types of missiles are obtained.

1. Introduction
The overall design of missile is the core part of its development process. The formulation of the parameters in the design stage plays a decisive role in the completion of the follow-up Development Task [1]. In the process of missile overall design, take-off quality design is the key step of quantitative design in the design scheme, and is also the basis of designing missile power system and single structure [2].

With the development of National Defense Science and technology, the performance of weapons is becoming more and more sophisticated. While ensuring the performance of weapons, how to pass the corresponding design at the design stage, it is very important to ensure the attitude and trajectory stability of the missile in the actual flight.

How to optimize the take-off quality at the initial stage of the overall design of the missile can provide data reference support for the determination of the main missile body and aerodynamic parameters. In order to solve these problems, based on the optimization model of missile take-off mass, this paper optimizes the missile take-off mass by using common optimization algorithms, and through comparative analysis, the optimization effects of different algorithms are compared and analyzed to provide effective models and algorithms for the optimization of takeoff mass and aerodynamic parameters for different types of missiles.

2. Calculation method of missile take-off mass

2.1. Numerical statistical estimation method
This algorithm can estimate the initial value of the take-off mass of the missile by investigating the existing missile models and making statistics of the main parameters of these types of missiles. Based on the data analysis, the mathematical relationship between the take-off mass and the main parameters of the missile is obtained.
Assuming that the warhead mass of a newly developed missile has been determined, according to the above methods, the takeoff mass of the missile can be estimated based on the average survey of the warhead and missile [3], as shown in Formula (1):

\[
m_O = \left( \frac{m_O}{m_c} \right)_{\text{average}} \times m_c
\]  

(1)

In formula (1), \( m_O \) is the takeoff mass of the newly developed missile, \( m_c \) is the warhead mass of the newly developed missile, and \( \left( \frac{m_O}{m_c} \right)_{\text{average}} \) is the average value of the reference value of the missile's takeoff mass and warhead mass ratio obtained from the survey.

This estimation method is easy to operate, but it is no longer applicable when the error between the maximum ratio and the minimum ratio exceeds 30% in the statistical data. Moreover, in the process of missile takeoff quality design using this method [3], overall coordination of design parameters should be carried out according to the actual situation, and design indexes should be achieved through multiple adjustments.

2.2. Regression equation estimation method

The basic principle of the regression equation estimation method is similar to the numerical statistical estimation method. By using the sample data of previous statistics and the regression analysis method, the mathematical relationship expression between the takeoff quality and other main missile parameters is obtained. The commonly used methods include least square method and polynomial curve fitting [4]. At the early stage of missile design, based on the completion of the design of the projectile diameter and total length indexes, the takeoff mass of the missile can be obtained according to the previous statistical data, as shown in Formula (2):

\[
m_O = \frac{K_G \times F(L) \times F(D)}{\delta_O}
\]  

(2)

In Equation (2), \( m_O \) is the takeoff mass of the newly developed missile, \( F(L) \) is the empirical function of the missile, \( F(D) \) is the empirical function corresponding to the missile diameter, and \( K_G \) is the relationship coefficient obtained by regression analysis of previous statistical data. The above regression analysis method is applicable to the case where the selected data are relatively appropriate within the statistical data range, and the estimated results have high confidence.

2.3. Mass equation estimation method

Both of the above methods are based on previous design data of similar types of missiles, so as to realize the calculation of takeoff mass pair. If there is no data support, the takeoff mass should be estimated through the design mass equation of the missile.

The takeoff quality of a missile depends on three parts [5], namely, payload, structure and fuel quality. Among them, the payload can be determined by tactical technical requirements and overall scheme analysis, and the structure quality can be estimated by some empirical and semi-empirical formulas. Therefore, the solution of the missile takeoff mass is also the solution of the missile fuel mass (fuel mass ratio) and the determination of the missile mass equation (the mathematical expression of the relationship between the missile takeoff mass, structural characteristics, payload, fuel mass ratio and main design parameters).

Most missiles usually consist of a booster and the main stage of the missile carrying the payload. Therefore, according to the calculation, the takeoff mass [6] can be obtained, as shown in Equation (3):
In Formula (3), \( m_{\text{pm}} \) is the missile payload mass, \( k_{\text{becq}} \) is the missile booster engine structure combustion mass ratio, \( k_{\text{bec}} \) is the booster fuel mass ratio, \( k_{\text{efq}} \) is the combustion mass ratio of the main stage engine, \( k_{\text{scce}} \) is the main engine structure coefficient, \( k_{\text{scp}} \) and is the missile body structure coefficient.

3. Missile take-off quality optimization model and aerodynamic parameters

3.1. Missile take-off quality optimization model

In this paper, the mass equation estimation method described in the first section is adopted to estimate the takeoff mass of the missile. The takeoff mass of the missile can be divided into the takeoff mass of the first stage booster and the main stage of the missile carrying payload at the second stage. The specific expression is as follows:

\[
m_{O} = m_{1} + m_{2}
\]  

(4)

The motion equation of the missile in the vertical plane is as follows:

\[
\begin{align*}
    m \frac{dv}{dt} &= P_{2} - \frac{1}{2} \rho v^{2} c_{s} S_{2} - mg \sin \theta \\
    \frac{dx}{dt} &= v \\
    \theta &= \theta(t) \\
    m &= m_{2} - \int_{0}^{t} m_{d} dt = m_{2} - m_{p}(t)
\end{align*}
\]

(5)

In view of the errors caused by the original models "assume the missile to have equal acceleration motion" and Simpson integral method for the approximate calculation of definite integral, the corresponding optimization calculation of fuel mass ratio in the endurance stage can be carried out.

3.2. Effect of take-off quality on aerodynamic parameters

After obtaining the optimization result of takeoff quality, the corresponding aerodynamic parameter calculation process is shown in Figure 1.
4. Calculation and analysis

Through the above research, this paper adopts genetic algorithm, particle swarm optimization and quadratic programming algorithm to optimize the design of the missile takeoff mass. The calculation results of the missile takeoff mass before and after optimization are shown in Table 1.

Table 1. Comparison of optimal calculation results of missile take-off mass

| parameter       | Original model take off mass | Quadratic programming | Genetic algorithm | Particle swarm algorithm |
|-----------------|-------------------------------|-----------------------|-------------------|-------------------------|
|                 | Optimized quality | Change ratio | Optimized quality | Change ratio | Optimized quality | Change ratio |
| Take off quality| 337.81kg         | 331.01kg     | 2.01%             | 325.11kg       | 3.76%             | 323.67kg       | 4.19%      |
| First mass      | 151.89kg         | 150.96kg     | 0.61%             | 145.63kg       | 4.12%             | 144.32kg       | 4.98%      |
| Secondary mass  | 185.92kg         | 180.05kg     | 3.15%             | 179.48kg       | 3.46%             | 179.35kg       | 3.53%      |

It can be seen from the results in Table 1 that the optimization ratio of the quadratic programming algorithm is not obvious, but the optimization ratio of the genetic algorithm and the example group algorithm is relatively obvious. In order to observe the convergence rate of each algorithm more intuitively, two test functions in the unimodal function are used to test the convergence rate of the three optimization algorithms, and the test results are compared and analyzed. The expressions of the two unimodal functions are shown in Equations (6) and (7).

\[ F(x) = \sum_{i=1}^{n} x_i^2 \]  
\[ \text{(6)} \]
The test results of the three algorithms using function \( F(x) \) are shown in Figure 2, and the test results of the three algorithms using function \( G(x) \) are shown in Figure 3.

\[
G(x) = \sum_{i=1}^{n-1} [100(x_i + 1 - x_i^2)^2 + (x_i - 1)^2] 
\]  
(7)

The test results of the three algorithms using function \( F(x) \) are shown in Figure 2, and the test results of the three algorithms using function \( G(x) \) are shown in Figure 3.

![Figure 2. F(x) Comparison of the convergence speed test results of the three algorithms](image)

![Figure 3. G(x) Comparison of the convergence speed test results of the three algorithms](image)

It can be seen from the analysis results in FIG. 2 and FIG. 3 that the convergence speed of pSO is obviously better than the other two algorithms. In order to compare the degree of influence of the optimization results obtained by the three algorithms on the aerodynamic parameters of the missile, the lift-drag ratio coefficient was obtained under the condition that the missile velocity was 1.5mA and the attack angle was 2° by following the calculation process as shown in Figure 1. The specific statistical results are shown in Table 2. It can be seen from the statistical results in the table that, on the premise of completing the takeoff quality optimization, the three algorithms can better ensure the aerodynamic parameters' index satisfaction, among which the index results obtained by the particle swarm optimization algorithm are the closest to the design index.
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| parameter       | Design indicators | Quadratic programming | Genetic algorithm | Particle swarm algorithm |
|-----------------|-------------------|-----------------------|-------------------|-------------------------|
|                 |                   | Optimization index    | Error rate        | Optimization index      | Error rate |
|                 |                   |                       |                   |                         |            |
|                 |                   |                       |                   |                         |            |
|                 |                   |                       |                   |                         |            |

5. Summary
Missile in the course of the development of the overall design is the core part, in view of the overall design process of missile off the quality of the optimization design problem, this paper first introduces the method for estimating the quality of several common take-off, design the missile off quality optimization model, and will take off between quality optimization and aerodynamic parameters calculation process has carried on the detailed elaboration, on this basis, using quadratic programming, genetic algorithm and particle swarm optimization (pso) on the quality of the missile off optimized calculation, calculation result by comparing three kinds of algorithm of missile quality optimization and aerodynamic parameters affect the degree of taking off, It is concluded that particle swarm optimization can not only obtain the optimal calculation results, but also ensure the satisfaction of aerodynamic parameters, which provides reference for the subsequent optimization of takeoff quality and aerodynamic parameters of different types of missiles.

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