HARMONIC INDEX OF TOTAL GRAPHS OF SOME GRAPHS

Anandkumar Velusamy
Department of Mathematics, Amrita School of Engineering,
Amrita Vishwa Vidyapeetham, Amrita University, Coimbatore, India

Radha Rajamani Iyer
Department of Mathematics, Amrita School of Engineering,
Amrita Vishwa Vidyapeetham, Amrita University, Coimbatore, India

ABSTRACT
Let \( G = (V, E) \) be a graph with \(|V|\) vertices and \(|E|\) edges and total graph, \( T(G) \)
is obtained from \( G \). In this paper we study the harmonic index of total graph for some
standards graphs, bipartite graph of particular type, regular graph, Grid graph \( G_{m,n} \)
and Complete Binary tree. We also obtained harmonic index of total graphs of bridge
graph of path, cycle and complete graph.

2010 Mathematics Subject Classification: Primary 92E10, Secondary 94C15

Keywords: Total graph, harmonic Index, Grid graph, Binary tree, Bridge graph.

Cite this Article: Anandkumar Velusamy and Radha Rajamani Iyer, Harmonic Index
of Total Graphs of Some Graphs, International Journal of Mechanical Engineering and
Technology 8(8), 2017, pp. 1359–1368.

1. INTRODUCTION
Let \( G(V, E) \) be a simple connected undirected graph with \(|V| = n\) vertices, \(|E| = m\) edges,
\( uv \in E(G) \) if \( u \) and \( v \) are adjacent in \( G \). The Randic index of graph \( G \) is denoted by \( R(G) \)
and it is defined as \( \chi(G) = \sum_{uv \in E(G)}(d_u d_v)^{-1/2} \) [11]. Few related variants of the Randic
index has been introduced in last few years. Sum-connectivity index is one among them introduced by Zhou and Trinajstic in [14,15] and it is defined as \( \chi(G) = \sum_{uv \in E(G)}(d_u + d_v)^{-1/2} \) [11]. General version of sum connectivity index is
\( \chi_{\alpha}(G) = \sum_{uv \in E(G)}(d_u + d_v)^{-\alpha} \) where \( \alpha \) the real number is. It has been found that \( \chi_{\alpha}(G) \)
and \( R(G) \) correlate well between themselves and with \( \pi \)-electron energy of benzenoid
hydrocarbons. In this paper we study the another related variant of the index namely the
Randic harmonic index of a graph \( G \) and is denoted by \( H(G) \) and defined as
Harmonic Index of Total Graphs of Some Graphs

\[ H(G) = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)}. \]

This Index was first appeared in [5]. Zhong found the minimum and maximum values of the harmonic index for simple connected graph and trees, and characterized the corresponding extremal graph. Hanyuan et al., gave some relation between the chromatic number and the harmonic index in 2013.

For every graph \( G \) there exist a graph called the line-graph of \( G \) and is denoted by \( L(G) \), whose points correspond in a one-to-one manner with the lines of \( G \) in such a way that two points in the \( L(G) \) are adjacent if and only if the corresponding lines are adjacent in \( G \). Vertices and edges are elements of a graph \( G \).

For any graph \( G \), total graph, denoted by \( T(G) \), is obtained from \( G \) whose vertex set \( V(T(G)) = V(G) \cup E(G) \) and two vertices are adjacent if and only if the corresponding elements are adjacent or incident in \( G \). Total graph is introduced by Mehdi Behzad and Gary Chartrand in the year 1966. They have presented the characterization of graphs Hamiltonian, total graphs and graph whose total graphs are Eulerian.

**Observation**

If \( v' \) is the point in the total graph of \( G \) corresponding to point \( v \) of \( G \), then the degree of \( v' \), denoted by \( \text{deg } v' \), equals \( 2 \text{deg } v \). Where \( X = v_1v_2 \) is a line of \( G \) and \( X' \) is the corresponding point of \( T(G) \), then \( \text{deg } X' = \text{deg } v_1 + \text{deg } v_2 \).

**2. HARMONIC INDEX OF TOTAL GRAPHS OF SOME GRAPHS**

In this paper Exact values of harmonic index of total graph of standard graphs such as Path, Cycle, Gird Graph, Bipartite graph of a particular type and complete binary tree are obtained.

**Proposition 2.1.** For any graph \( P_n \) of order \( n \geq 4 \), \( H(T(P_n)) = n - \frac{269}{420} \).

**Proposition 2.2.** For any star graph \( S_n \), \( H(T(S_n)) = (n - 1) \left( \frac{2}{n+2} + \frac{3(n+1)}{2(3n-1)} \right) \).

**Theorem 2.3.** Let \( G \) be a graph with \( |V| = n \) and \( |E| = m \) then \( H(T(G)) = \frac{1}{2} H(G) + \frac{2}{d(u) + 2d(v) + d(w)} + 8 \sum_{u \in V(G)} \frac{d(u) + d(v)}{d(u) + d(v)} \cdot \sum_{v \in E(G)} \frac{d(u) + d(v)}{d(u) + d(v)} \cdot \sum_{w \in E(G)} \frac{d(u) + d(v)}{d(u) + d(v)} \).

**Proof.** Total graph of a graph \( G \) has \( E(T(G)) = 3m + \frac{1}{2} \sum_{v \in G} d(v) - m \)

\[ = 2m + \frac{1}{2} \sum_{v \in G} d(v)^2 \]

Each edge belongs any one of the following category.

i. Edges between \( u' \) and \( v' \), where \( u' \) and \( v' \) are vertices corresponding to the vertices \( u \) and \( v \) in \( G \).

ii. Edges between \( X' \) and \( Y' \), where \( X' \) and \( Y' \) are vertices corresponding to the edges \( X \) and \( Y \) in \( G \).

iii. Edges between \( X' \) and \( u' \), where \( X' \) and \( u' \) are vertices corresponding to the edge \( X \) and the vertex \( u \) respectively, \( X = uv \).

http://www.iaeme.com/IJMET/index.asp 1360 editor@iaeme.com
\[ H(T(G)) = \sum_{u'v' \in E(T(G))} \frac{2}{d(u') + d(v')} + \sum_{x'y' \in E(T(G))} \frac{1}{2} \sum_{v \in G} \frac{d(u')} {d(x')} + \frac{2}{d(Y')} + \sum_{x'u' \in E(T(G))} \frac{d(x')} {d(Y')} + \frac{2}{d(u')} \]

\[ = A + B + C. \]

Where,

\[ A = \sum_{u'v' \in E(T(G))} \frac{2}{d(u') + d(v')} = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)} \]

\[ B = \sum_{x'y' \in E(T(G))} \frac{1}{2} \sum_{v \in G} \frac{d(u')} {d(x')} + \frac{2}{d(Y')} = \sum_{xy \in E(L(G))} \frac{d(x) + d(Y) + 4}{d(Y) + d(Y)} \]

\[ C = \sum_{x'u' \in E(T(G))} \frac{2}{d(x') + d(u')} = \sum_{x = uv \in E(G)} \frac{2}{d(u) + 2d(v) + 2d(X) + 2} \]

\[ = \sum_{u \in E(G)} (\frac{2}{d(u) + d(u) + d(v)} + \frac{2}{d(u) + 2d(v) + d(v)}) \]

\[ = \sum_{u \in E(G)} (\frac{2}{3d(u) + d(v)} + \frac{2}{d(u) + 3d(v)}) \]

\[ = \sum_{u \in E(G)} \frac{8d(u) + 8d(v)}{3d(u)^2 + 9d(u)d(v) + d(u)(v) + 3d(v)^2} \]

\[ = \sum_{u \in E(G)} \frac{8(d(u) + d(v))}{3d(u)^2 + 10d(u)d(v) + 3d(v)^2} \]

\[ = 8 \sum_{uv \in E(G)} \left( \frac{d(u) + d(v)}{3d(u) + d(v) + 2d(u)d(v)} \right) \]
From (1), (2) and (3),
\[ H(T(G)) = \frac{1}{2} H(G) \]
\[ + \sum_{uv \in E(G)}^ {M_1(G) - m} \frac{2}{d(u) + 2d(v) + d(w)} + 8 \sum_{uv \in E(G)} \left( \frac{d(u) + d(v)}{3(d(u) + d(v))^2 + 4d(u)d(v)} \right) \]

**Theorem 2.4.** If \( G \) is a regular graph, then \( H(T(G)) = \frac{n(k+2)}{4} \).

**Proof.** If \( G \) is a regular then \( T(G) \) is also regular and it has \( \frac{n}{k+2} \) vertices and degree of all the vertices are the same. Then \( |E(T(G))| = \frac{nk}{2} (k + 2) \).

Hence, \( H(T(G)) = \frac{nk}{2} (k + 2) + \sum_{u'v' \in E(T(G))}^ {2(k+2)} \frac{2}{d(u') + d(v')} \)

Here degree of every vertex in \( T(G) \) is \( 2k \).

Therefore,
\[ H(T(G)) = \frac{n(k+2)}{4} \]

**Corollary 2.4.1.** Let \( G_n \) be a cycle with \( n \) vertices then \( H(T(G_n)) = n \).

**Proof** Total graph \( T(G_n) \) is the 4-regular graph of the size \( 4n \)

\[ \therefore H(T(G_n)) = \sum_{u'v' \in E(T(K_n))}^ {2(n+1)} \frac{2}{d(u') + d(v')} \]

\[ = \frac{n(n-1)(n+1)}{2} + \frac{2}{2(n-1) + 2(n-1)} \]

\[ H(T(K_n)) = \frac{n(n+1)}{4} \]

**Theorem 2.5.** Let \( G \) be a bipartite graph with bipartition \( X \) and \( Y \) for all \( i \), \( \deg v_i = \begin{cases} k_1, & \text{if } v_i \in X \\ k_2, & \text{if } v_i \in Y \end{cases} \)

Then, \( H(T(G)) = \frac{mk_1^2 + nk_2^2}{2(k_1+k_2)} + \frac{4(mk_1 + nk_2) + (k_1 + k_2)}{(3k_1 + k_2)(k_1 + 3k_2)} \)

**Proof** Let \( G \) be a bipartite graph with partition \( |X| = m \) and \( |Y| = n \)

Number of edges in \( G \) is \( \frac{mk_1 + nk_2}{2} \)

In the Total graph of \( G \), \( |V(T(G))| = m + n + \frac{mk_1 + nk_2}{2} \)

\[ |E(T(G))| = \frac{mk_1 + nk_2}{2} + \frac{nk_2(k_2 - 1)}{2} + \frac{m(k_1 - 1)}{2} + 2\left( \frac{mk_1 + nk_2}{2} \right) \]

\[ |E(T(G))| = \frac{1}{2}(nk_2(k_2 + 2) + mk_1(k_1 + 2)) \]
Corollary 2.5.1. let $K_{m,n}$ be a complete bipartite graph then $H\left(T(K_{m,n})\right) = \frac{5mn}{6(m+n)^2 + 8mn}$.

Proof. $H\left(T(K_{m,n})\right) = mn \left(\frac{2}{2n+2m} + \frac{2(m+n-2)}{2(m+n+m+n)} + \frac{2}{2n+2m+n} \right)$

\[ = \frac{mn}{n+m} + \frac{mn(m+n-2)}{2(m+n)} + \frac{2mn}{2n+2m+n} + \frac{2mn}{2m+m+n} \]

\[ = \frac{2mn - mn(m+n) - 2mn}{2(m+n)} + \frac{2mn}{3n+m} + \frac{2mn}{3m+n} \]

\[ = \frac{mn}{2} + \frac{2mn}{3n+m} + \frac{2mn}{3m+n} \]

\[ = \frac{5mn}{6(m+n)^2 + 8mn} \]

Theorem 2.6. Let $G$ be a grid graph then

\[ H\left(T(Gm,n)\right) = \frac{1}{16} M_1(G) + \frac{mn}{z} + \frac{99}{260} (m + n) - \left(\frac{41077}{12870}\right) \text{ Where } m, n \geq 5. \]

Proof. The total graph of grid graph has $|V| = 3mn - m - n$ and $|E| = \frac{1}{2} M_1(G) + 4mn - 2(m + n)$. The degree pairs of the number of edges 8, 8, 4, 16, 8, 4(m + 2n - 13), 4(3m + 3n - 16), 2(m + n - 4), 4, 8(m + n - 5), $\frac{1}{2} M_1(G)$ + $4mn - 26(m + n)$ + 84 in the $T(Gm,n)$ are

(4,5), (4,6), (5,5), (5,6), (5,7), (6,6), (6,7), (6,8), (7,7), (7,8), (8,8) respectively.

\[ H\left(T(Gm,n)\right) = 8 \left(\frac{2}{4+5}\right) + 8 \left(\frac{2}{4+6}\right) + 4 \left(\frac{2}{5+5}\right) + 16 \left(\frac{2}{5+6}\right) + 8 \left(\frac{2}{5+7}\right) + 4(2m + 2n - 13) \left(\frac{2}{6+6}\right) + 2(3m + 3n - 16) \left(\frac{2}{6+7}\right) + 2(m + n - 4) \left(\frac{2}{6+8}\right) + 4 \left(\frac{2}{7+7}\right) + 8(m + n - 5) \left(\frac{2}{7+8}\right) + \frac{1}{2} M_1(G) + 4mn - 26(m + n) + 84 \left(\frac{2}{8+8}\right) \]
Harmonic Index of Total Graphs of Some Graphs

\[ H(T(Gm, n)) = \frac{1}{16} M1(G) + \frac{mn}{2} + \sum_{u'v' \in (T(G))} \frac{2}{d(u') + d(v')} \]

\[ = 2^t \left( \frac{2}{2 + 4} + 2^t \left( \frac{2}{2 + 6} + 2^{t-1} \left( \frac{2}{4 + 4} + \frac{2}{4 + 5} + (2^{t+1} + 2) \left( \frac{2}{4 + 6} \right) \right) \right) \right) \]

\[ + \left( \frac{2}{5 + 5} \right) + 6 \left( \frac{2}{5 + 6} \right) + (9.2^{l-1} - 22) \left( \frac{2}{6 + 6} \right) \]

\[ = 2^t \left( \frac{2}{6} + 2^t \left( \frac{2}{8} + 2^{t-1} \left( \frac{2}{8} + 2 \left( \frac{2}{9} \right) + 2^{t+1} \left( \frac{2}{10} + 2 \left( \frac{2}{10} + 6 \left( \frac{2}{11} \right) \right) \right) \right) \right) \]

\[ + 9.2^{t-1} \left( \frac{2}{12} \right) - 22 \left( \frac{2}{12} \right) \]

\[ = 2^t \left( \frac{2}{6} + \frac{2}{8} + \frac{2}{16} + \frac{4}{10} + \frac{18}{24} \right) + 4 \left( \frac{1}{9} + \frac{1}{10} + \frac{1}{20} + \frac{3}{11} + \frac{11}{12} \right) \]

\[ = 2^{t+1} \left( \frac{40 + 30 + 15 + 48 + 90}{240} \right) - \frac{758}{495} \]

\[ \therefore H(T(T)) = 2^t \left( \frac{223}{120} \right) - \frac{758}{495}. \]

3. HARMONIC INDEX OF TOTAL GRAPH OF BRIDGE GRAPHS

Let \( (G)_{i=1}^d \) be a set of finite pairwise disjoint molecular graphs with \( v_i \in V \{G_i\} \). The bridge graph \( B(G_1, G_2, \ldots, G_d) = B(G_1, G_2, \ldots, G_d; v_1, v_2, \ldots, v_d) \) of \( \{G\}_{i=1}^d \) with respect to the vertices \( \{v_i\}_{i=1}^d \) is the graph obtained from the graphs \( G_1, G_2, \ldots, G_d \) by connecting the vertices \( v_i \) and \( v_{i+1} \) using an edge for all \( i = 1, 2, \ldots, d - 1 \). In this section we derive formula to obtain the harmonic index of total graphs of the bridge graphs when main part of the bridge graphs are isomorphic to path\( (Pn) \), cycle\( (Cn) \), complete graph\( (Kn) \). Here we denote
$G_d(H, v) = B(H, \ldots, H, v, \ldots, v)$ for special situations of bridge molecular graph. In theorems we consider the graphs only $d \geq 5$ and $n \geq 4$. 

**Figure 1** Total graph of Bridge graph

**Proposition 3.1.** Let $P_n$ a graph with $n$ vertices then the total graph of the bridge graph $G_d(P_n, v)$ denoted by $T(G_d(P_n, v))$ has $2nd - 1$ vertices and $3nd + 4d - 7$ edges.

**Proposition 3.2.** Let $C_n$ a graph with $n$ vertices then the total graph of the bridge graph $G_d(C_n, v)$ denoted by $T(G_d(C_n, v))$ has $2dn + d - 1$ vertices and $4dn + 8d - 9$ edges.

**Proposition 3.3.** Let $K_n$ a graph with $n$ vertices then the total graph of the bridge graph $G_d(K_n, v)$ denoted by $T(G_d(K_n, v))$ has $\frac{1}{2}(dn^2 + dn + 2d - 2)$ vertices and $\frac{1}{2}(dn^3 + 3dn + 4d - 4n - 6)$ edges.

**Theorem 3.4.** Let $P_n$ be a path with $n$ vertices. Then $H \left( T(G_d(P_n, v)) \right) = \left( \frac{3dn}{4} + \frac{226d}{3465} - \frac{7121}{13860} \right)$.

**Proof.** Edge set of the $T(G_d(P_n, v))$ can be categorized as follows:

| Edge category | No.of Edges |
|---------------|-------------|
| $E_{23}$      | $d$         |
| $E_{24}$      | $d$         |
| $E_{34}$      | $(d + 2)$   |
| $E_{45}$      | $(d - 2)$   |
| $E_{44}$      | $3d(n - 3) + 6$ |
| $E_{45}$      | $d + 2$     |
| $E_{56}$      | $d$         |
| $E_{55}$      | 2           |
Harmonic Index of Total Graphs of Some Graphs

Table 1 Edges in $T(G_d(P_n, v))$

| Edge category | No.of Edges |
|---------------|-------------|
| $E_{44}$      | $4d - 9d$   |
| $E_{45}$      | $8$         |
| $E_{46}$      | $4(d - 1)$  |
| $E_{48}$      | $2(d - 2)$  |
| $E_{55}$      | $2$         |
| $E_{56}$      | $4$         |
| $E_{66}$      | $(d - 2)$   |
| $E_{67}$      | $6$         |
| $E_{68}$      | $2(3d - 7)$ |
| $E_{88}$      | $(4d - 13)$ |

$$H\left(T(G_d(P_n, v))\right) = d \left(\frac{2}{5}\right) + d \left(\frac{2}{6}\right) + (d + 2) \left(\frac{2}{7}\right) + (d - 2) \left(\frac{2}{8}\right) + \left(3d(n - 3)\right) \left(\frac{2}{8}\right) + (d + 2) \left(\frac{2}{9}\right) + d \left(\frac{2}{10}\right) + 2 \left(\frac{2}{11}\right) + (3d - 4) \left(\frac{2}{11}\right) + (4d - 13) \left(\frac{2}{12}\right)$$

$$= 2d \left(\frac{9194}{6930} - 1\right) - 2 \left(\frac{7121}{27720}\right) + \frac{3dn}{4}$$

$$= \left(\frac{3dn}{4} + \frac{2264d}{3465} - \frac{7121}{13860}\right).$$

Theorem 3.5. Let $C_n$ be a cycle with $n$ vertices. Then $H\left(T(G_d(C_n, v))\right) = \left(\frac{3dn}{4} + \frac{2264d}{3465} - \frac{7121}{13860}\right)$.

Proof. Edge set of the $T(G_d(C_n, v))$ can be categorized as follows:

| Edge category | No.of Edges |
|---------------|-------------|
| $E_{44}$      | $4dn - 9d$  |
| $E_{45}$      | $8$         |
| $E_{46}$      | $4(d - 1)$  |
| $E_{48}$      | $2(d - 2)$  |
| $E_{55}$      | $2$         |
| $E_{56}$      | $4$         |
| $E_{66}$      | $(d - 2)$   |
| $E_{67}$      | $6$         |
| $E_{68}$      | $2(3d - 7)$ |
| $E_{88}$      | $(4d - 13)$ |

$$H\left(T(G_d(C_n, v))\right) = (4dn - 9d) \left(\frac{2}{8}\right) + 8 \left(\frac{2}{9}\right) + 4(d - 1) \left(\frac{2}{10}\right) + 2 \left(\frac{2}{11}\right) + 4 \left(\frac{2}{11}\right) + 4 \left(\frac{2}{12}\right) + (d - 2) \left(\frac{2}{12}\right) + 6 \left(\frac{2}{13}\right) + (6d - 14) \left(\frac{2}{14}\right) + 4 \left(\frac{2}{15}\right) + (4d - 13) \left(\frac{2}{16}\right)$$

$$= \left(\frac{3dn}{4} + \frac{2264d}{3465} - \frac{7121}{13860}\right).$$

Theorem 3.6. Let $k_n$ be a path with $n$ vertices then $H\left(T(G_d(k_n, v))\right) = \left(\frac{d}{4}\right) (n^2 - 2n) + \left(\frac{d}{4n-3}\right) (n - 1)^2 + \left(\frac{1}{2n-1}\right) (dn^2 - 2dn + 3n - n^2 + d - 2) + \frac{1}{4n} (dn^2 - dn - 2n^2 + 6n - 4) + \left(\frac{4}{4n-1}\right) (n - 1) + \left(\frac{1}{2(n+1)}\right) (3dn - 3d - 8n + 10) + \left(\frac{4n}{4n+1}\right) + \left(\frac{1}{2(n+1)}\right) (4d - 13) + \left(\frac{8}{4n+3}\right).$

Proof. The total number of edges in $T(G_d(k_n, v))$ is $\frac{1}{2} (d n^3 + 3dn + 4d - 4n - 6)$ and edge set of $T(G_d(k_n, v))$ can be categorized as follows:
Anandkumar Velusamy and Radha Rajamani Iyer

| Edge category | No.of Edges |
|---------------|-------------|
| $E_{(2(n-1))(2(n-1))}$ | $\frac{d}{2} (n^3 - 3n^2 + 2n)$ |
| $E_{(2(n-1))(2(n-1))}$ | $2(n-1)^2$ |
| $E_{(2(n-1))(2n)}$ | $(d - 2)(n-1)^2 + 2(n-1)$ |
| $E_{(2(n-1))(2(n+1))}$ | $(d - 2)(n-1)$ |
| $E_{(2(n-1))(2(n-1))}$ | $(n-1)(n-2)$ |
| $E_{(2(n-1))(2n)}$ | $2(n-1)$ |
| $E_{(2(n-1))(2(n+1))}$ | $2(n-1)$ |
| $E_{(2n)(2n)}$ | $\frac{1}{2}(dn^2 - 3dn + 2d - 2n^2 + 6n - 4)$ |
| $E_{(2n)(2(n+1))}$ | $3dn - 3d - 8n + 10$ |
| $E_{(2n)(2n+1)}$ | $2n$ |
| $E_{(2(n+1))(2(n+1))}$ | $4d - 3$ |
| $E_{(2(n+1))(2(n+1))}$ | $4$ |

Table 3 Edges in $T(G_d(K_n, v))$

$$H \left( T(G_d(K_n, v)) \right) = \left( \frac{d}{4} \right) (n^2 - 2n) + \left( \frac{4}{4n-3} \right) (n-1)^2 + \left( \frac{1}{2n-1} \right) (dn^2 - 2dn + 3n - n^2 + d - 2) + \frac{1}{4n} (dn^2 - dn - 2n^2 + 6n - 4) + \left( \frac{4}{4n-1} \right) (n-1) + \left( \frac{1}{2n+1} \right) (3dn - 3d - 8n + 10) + \left( \frac{4n}{4n+1} \right) + \left( \frac{1}{2(n+1)} \right) (4d - 13) + \left( \frac{8}{4n+3} \right).$$

4. CONCLUSION

In this paper we determined many results of harmonic index of total graph of many standard graphs and total graph of bridge graph of some important graphs. Further we study the harmonic index of some important molecular structure.

ACKNOWLEDGEMENT

The authors are thankful to the anonymous referees for their useful comments.

REFERENCES

[1] G.H. Fath-Tabar, A.Hamzeh, and S.Hossein-Zadeh, $GA_2$ index of some graph operation, Filo-mat, 24(1):21-28, 2010.
[2] V.Anandkumar, Radha Rajamani Iyer, Zagreb Indices of the Total Graphs of Graphs, Journal of Applied Sciences Research 12(3)(2016)59-64.
[3] V.Anandkumar and Iyer, R. R., on the hyper-zagreb index of some operations on graphs, International Journal of Pure and Applied Mathematics, vol. 112 (2017) 239-252.
Harmonic Index of Total Graphs of Some Graphs

[4] Xinli Xu, Relationships between harmonic index and other topological indices, applied mathematical Sciences, 6(41):2013-2018, 2012.

[5] Mojgan Mogharrab, Ivan Gutman, Bridge Graphs and Their Topological Indices, MATCH Commun. Math. Comput. Chem. 69 (2013) 579-587.

[6] Toufik Mansour, Matthias Schork, The vertex PI index and Szeged index of bridge graphs, Discrete Applied Mathematics 157 (2009) 16001606.

[7] Wei Gao, Muhammad Kamran Siddiqui, Muhammad Imran, Muhammad Kamran Jamil, Mohammad Reza Farahani, Forgotten topological index of chemical structure in drugs, Saudi Pharmaceutical Journal (2016) 24, 258264

[8] Ismael G.Yero and Juan A.Rodriguez-Velazquez, On the Randic index of corono product graph, International Scholarly Research notices, 2011.

[9] M.Randic, on characterization of molecular branching, J.Am.Chem.Soc.97 (1975)6609-6615.

[10] L.Pogliani, from molecular connectivity indices to semi empirical connectivity terms: recent trends in graph theoretical descriptors, Chem.Rev.100 (2000)3827-3858.

[11] I. Gutman, B.Furtula(Eds.), Recent Results in the Theory of Randic Index, Univ.Kragujevac,2008.

[12] B.Zhou, N.Trinajstic, on general sum-connectivity index, J.Math. Chem.47(2010)210-218.

[13] B.Lucic, N.Trinajstic, B.Zhou, Comparison between the sum-connectivity index and product-connectivity index for benzenoid hydrocarbons,Chem.Phys.Lett.475(2009)146-148.

[14] Z.Du,B.Zhou, N.Trinajstic, On the general sum-connectivity index of trees, appl.Math.Lett.24(2011)402-405.

[15] Lingping Zhong, The harmonic index for graphs, applied Mathematics Letters 25(2012)561-566.

[16] Hanyuan Deng, S.Balachandran, S.K.Ayyaswamy, Y.B.VenkataKrishnan, On the harmonic index and the chromatic number of a graph, Discrete applied Mathematics 161(2013)2740-2744.

[17] Z.Du, B.Zhou, On sum-connectivity index of bicyclic graphs, Bull. Math. Sci. Soc. (2)35(1)(2012)101-117.

[18] Z.Du, B.Zhou, N.Trinajstic, minimum sum-connectivity indices of trees and unicyclic graphs of a given matching number, J.Math. Chem.47 (2010)842-855.

[19] R.Xing, B.Zhou, N.Trinajstic, sum-connectivity index of molecular trees,J.Math. Chem.48 (2010)583-591.

[20] L.Zhong, The harmonic index for graphs, appl. Math. Lett.25 (2012)561-566.

[21] B.Shwetha Shetty, V.Lokesha and P.S.Ranjini, on the harmonic index of graph operations,Transactions on combinatorics,4(4)(2015),5-14.

[22] Tabitha Agnes Mangam and Joseph Varghese Kureethara. Diametral Paths in Total Graphs of Complete Graphs, Complete Bipartite Graphs and Wheels. International Journal of Civil Engineering and Technology, 8(5), 2017, pp. 1212–1219

[23] Rajesh A V, Bindu S J.and Rekha T., Reduction of Harmonic Distortion in BLDC Drive using Bl-Buck Boost Converter BLDC Drive. International Journal of Electrical Engineering & Technology, 7(5), 2016, pp. 79–88