Statistical properties of twin kHz QPO in neutron star LMXBs

D. H. Wang\textsuperscript{1,2}, L. Chen\textsuperscript{1}, C. M. Zhang\textsuperscript{2,*}, Y. J. Lei\textsuperscript{2}, and J. L. Qu\textsuperscript{3}

\textsuperscript{1} Astronomy Department, Beijing Normal University, Beijing 100875
\textsuperscript{2} National Astronomical Observatories, University of Chinese Academy of Sciences, Beijing 100012, China
\textsuperscript{3} Institute of High Energy Physics, University of Chinese Academy of Sciences, Beijing 100049, China

Key words stars: neutron – X-rays: binaries – accretion, accretion discs

We collect the data of twin kilohertz quasi-periodic oscillations (kHz QPOs) published before 2012 from 26 neutron star (NS) low-mass X-ray binary (LMXB) sources, then we analyze the centroid frequency (\(\nu\)) distribution of twin kHz QPOs (lower frequency \(\nu_1\) and upper frequency \(\nu_2\)) both for Atoll and Z sources. For the data without shift-and-add, we find that Atoll and Z sources show the different distributions of \(\nu_1, \nu_2\) and \(\nu_2/\nu_1\), but the same distribution of \(\Delta \nu\) (difference of twin kHz QPOs), which indicates that twin kHz QPOs may share the common properties of LMXBs and have the same physical origins. The distribution of \(\Delta \nu\) is different from constant value, so is \(\nu_2/\nu_1\) from constant ratio. The weighted mean values and maxima of \(\nu_1\) and \(\nu_2\) in Atoll sources are slightly higher than those in Z sources. We also find that shift-and-add technique can reconstruct the distribution of \(\nu_1\) and \(\Delta \nu\). The K-S test results of \(\nu_1\) and \(\Delta \nu\) between Atoll and Z sources from data with shift-and-add are quite different from those without it, and we think that this may be caused by the selection biases of the sample. We also study the properties of the quality factor (\(Q\)) and the root-mean-squared (rms) amplitude of 4U 0614+09 with the data from the two observational methods, but the errors are too big to make a robust conclusion. The NS spin frequency (\(\nu_s\)) distribution of 28 NS-LMXBs show a bigger mean value (\(\sim 408\) Hz) than that (\(\sim 281\) Hz) of the radio binary millisecond pulsars (MSPs), which may be due to the lack of the spin detections from Z sources (systematically lower than 281 Hz). Furthermore, on the relations between the kHz QPOs and NS spin frequency \(\nu_s\), we find the approximate correlations of the mean values of \(\Delta \nu\) with NS spin and its half, respectively.

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### Table 1  Twin kHz QPOs (without shift-and-add)

| Source (22)         | $\nu_1$ (Hz) | $\langle \nu_1 \rangle$ (Hz) | $\nu_2$ (Hz) | $\langle \nu_2 \rangle$ (Hz) | Ref |
|---------------------|-------------|-------------------------------|-------------|-------------------------------|-----|
| Sax J1808.4-3658    | 499 ± 4     | 501 ± 2                       | 685.1 ± 5.1 | 691 ± 4                       | 1   |
| XTE J1807.4-294     | 106 ± 30    | 237 ± 16                      | 337 ± 10    | 437 ± 15                      | 2   |
| Millisecond pulsars (2) |            |                               |             |                               |     |
| 4U 0614+09          | 153.4 ± 5.6 | 651 ± 18                      | 449.4 ± 19.5 | 944 ± 21                      | 3   |
| 4U 1608-52          | 531 ± 17    | 719 ± 48                      | 830.3 ± 5.8 | 918 ± 39                      | 4   |
| 4U 1636-53          | 565.4 ± 5.1 | 839 ± 22                      | 860.4 ± 1.7 | 944 ± 33                      | 5   |
| 4U 1728-34          | 305 ± 8     | 740 ± 20                      | 582 ± 10    | 892 ± 14                      | 6   |
| 4U 1735-44          | 640.5 ± 2.5 | 727 ± 6                       | 981.7 ± 6.7 | 1097 ± 43                     | 7   |
| 4U 1820-30          | 764 ± 6     | 789 ± 5                       | 1055 ± 10   | 1067 ± 4                      | 8   |
| 4U 1915-05          | 223.6 ± 4.7 | 440.6 ± 8.5                   | 513.6 ± 0.2 | 513.8 ± 5.2                   | 9   |
| IGR J17511-3057†    | 72.5 ± 4.9  | 124.9 ± 13.4                  | 179.9 ± 14.9 | 247.7 ± 17.3                  | 10  |
| KS 1731-260         | 898.3 ± 3.3 | 900.7 ± 1.1                   | 1158.6 ± 9.0 | 1173.5 ± 3.0                  | 11  |
| SAX J1750.8-2900    | 936 ± 1     | 936 ± 1                       | 1253 ± 9    | 1253 ± 9                      | 12  |
| XTE J1701-407       | 745 ± 9     | 745 ± 9                       | 1150 ± 7    | 1150 ± 7                      | 13  |
| XTE J123-058        | 847.1 ± 5.5 | 859 ± 6                       | 1102 ± 13   | 1131 ± 8                      | 14  |
| Atoll Sources (12)  |            |                               |             |                               |     |
| Cyg X-1†            | 56.1 ± 1.3  | 82 ± 9                        | 229 ± 18    | 538 ± 26                      | 15  |
| GX 5-1 (1758-250)   | 156 ± 23    | 280 ± 23                      | 478 ± 15    | 671 ± 18                      | 17  |
| GX 17+2 (1813-140)  | 475 ± 7     | 641 ± 24                      | 759.5 ± 5   | 929 ± 24                      | 18  |
| GX 340+0 (1642-455) | 197 ± 70    | 357 ± 22                      | 535 ± 85    | 681 ± 21                      | 19  |
| GX 349+2 (1702-363) | 715 ± 12    | 715 ± 12                      | 985 ± 7     | 985 ± 7                       | 20  |
| Sco X-1 (1617-155)  | 565 ± 4     | 645 ± 9                       | 872 ± 2     | 943 ± 9                       | 21  |
| XTE J1701-462†      | 502.4 ± 23.1 | 629 ± 5                     | 760.8 ± 6.4 | 910 ± 14                      | 22  |

†: The identification of the QPOs is uncertain.
‡: The source shows the properties of both Atoll and Z sources.

1. van Straaten et al. 2005; 2. Wijnands et al. 2003; 3. Linares et al. 2005; 4. Zhang et al. 2006; 5. van Straaten et al. 2000; 6. van Straaten et al. 2002; 7. Altamirano et al. 2008; 8. Wijnands et al. 1997a; 9. Bhattacharyya 2010; 10. Di Salvo et al. 2001; 11. van Straaten et al. 2003; 12. Strohmayer et al. 1999; 13. Migliari et al. 2003; 14. Wijnands et al. 1998a; 15. Ford et al. 1998b; 16. Smale et al. 1997; 17. Boin et al. 2004; 18. Kalamar et al. 2011; 19. Wijnands & van der Klis 1997; 20. Kaaret et al. 2003; 21. Strohmayer et al. 2000; 22. Tomstück et al. 1999; 23. Homan et al. 1999; 24. Boutloukos et al. 2004; 25. Wijnands et al. 1998b; 26. Jonker et al. 2002b; 27. Jonker et al. 1998a; 28. O'Neill et al. 2002; 29. van der Klis et al. 1997; 30. van der Klis et al. 1998; 31. Homan et al. 2007; 32. Sanna et al. 2010.

### Table 2  Twin kHz QPOs (with shift-and-add)

| Source (9)         | $\nu_1$ (Hz) | $\langle \nu_1 \rangle$ (Hz) | $\Delta \nu$ (Hz) | $\langle \Delta \nu \rangle$ (Hz) | Ref |
|-------------------|-------------|-------------------------------|-------------------|----------------------------------|-----|
| Atoll Sources (8) |            |                               |                   |                                 |     |
| 4U 0614+09        | 560.1 ± 2.1 | 644.1 ± 9.7                   | 300.0 ± 11.8      | 320.5 ± 2.3                     | 1   |
| 4U 1608-52        | 473 ± 8.67  | 754 ± 15                      | 225 ± 12          | 305 ± 3                         | 2   |
| 4U 1636-53        | 528.56 ± 16.54 | 841 ± 7                  | 229.86 ± 12.38    | 269 ± 6                         | 3   |
| 4U 1702-43        | 722         | 722                           | 333 ± 5           | 333 ± 5                         | 4   |
| 4U 1705-44        | 776.1 ± 3.9 | 776.1 ± 3.9                   | 298.1 ± 11.1      | 298.1 ± 11.1                    | 5   |
| 4U 1728-34        | 756 ± 8.94  | 746                           | 279 ± 12          | 346 ± 3                         | 6   |
| Aql X-1 (1908+005) | 795.45 ± 0.04 | 798.43 ± 3.73             | 278.1 ± 18.3      | 279.4 ± 1.0                     | 7   |
| IGR J17191-2821   | 681 ± 5.87  | 793 ± 42                      | 315 ± 50          | 349 ± 7                         | 8   |
| Sco X-1 (1617-155)| 531.6 ± 16.6 | 837 ± 6                      | 240.6 ± 4.4       | 293 ± 2                         | 9   |

1. Boutelier et al. 2004; 2. Barret et al. 2005a; 3. Jonker et al. 2000; 4. Méndez et al. 1998b; 5. Di Salvo et al. 2003; 6. Jonker et al. 2000; 7. Barret et al. 2008; 8. Altamirano et al. 2010a; 9. Lin et al. 2011; 10. Méndez & van der Klis 2000.
### Table 3  Twin kHz QPOs (with and without shift-and-add)

| Source (4)† | $\nu_2$ (Hz) | $\langle \nu_2 \rangle$ (Hz) | $\Delta \nu$ | $\langle \Delta \nu \rangle$ |
|-------------|--------------|-----------------|-------------|-----------------|
| Without shift-and-add | | | | |
| 4U 0614+09 | 449.4 ± 19.5 | 1161.8 ± 4.6 | 238.3 ± 6.7 | 316 ± 5 |
| 4U 1608-52 | 830.3 ± 5.8 | 1061.9 ± 6.3 | 277.6 ± 6.4 | 305 ± 5 |
| 4U 1636-53 | 860.4 ± 1.7 | 1194 ± 19 | 249.5 ± 13.0 | 296 ± 5 |
| 4U 1728-34 | 582 ± 10 | 1161 ± 16 | 231 ± 21 | 321 ± 7 |
| Sco X-1 | 872 ± 2 | 1080 ± 3 | 223.1 ± 5.3 | 295 ± 2 |
| With shift-and-add | | | | |
| 4U 0614+09 | 889.4 ± 10.5 | 1144.4 ± 6.8 | 1002 ± 20 | 320.5 ± 2.3 |
| 4U 1608-52 | 799 ± 3 | 1103.9 ± 17.9 | 952 ± 14 | 305 ± 3 |
| 4U 1636-53 | 822.9 ± 2.5 | 1227.7 ± 2.7 | 1078 ± 27 | 269 ± 6 |
| 4U 1728-34 | 925.3 ± 1183.2 | 1085 ± 25 | 279 ± 12 | 346 ± 3 |
| Sco X-1 | 842.1 ± 2.8 | 1142.8 ± 4.4 | 947 ± 12 | 293 ± 2 |

† The reference are the same as Table 1 and 2.

As an extension of studying the influence of the two observational methods on kHz QPOs, we take 4U 0614+09 as an example to analyze the quality factor and rms amplitude. The spin frequency distribution of NS-LMXBs are analyzed, which compared to that of the binary radio millisecond pulsars (MSPs). The whole pictures of kHz QPOs of both Atoll and Z sources are discussed and pointed out.

### 2 Parameter analysis for twin kHz QPOs

We collect the data of twin kHz QPO from 26 NS-LMXB source published before 2012, which includes 2 millisecond pulsars, 16 Atoll sources, 8 Z sources. In some cases, no tables of kHz QPO are provided, and we obtain the data from authors or figures. The identification of the QPOs in IGR J17511-3057 is uncertain (Kalamkar et al., 2011), so we do not use this data when analyzing the distribution of Atoll sources.

According to whether adopting shift-and-add technique, we separate the samples into two groups, the detail information of these sources are shown in Table 1 and 2. Table 1 presents the results without shift-and-add technique, which includes 93 data of Atoll sources and 141 of Z sources. Some of the results are obtained from figures, which are only kept with the integer parts. In case the data has the different up and down errors, we take the bigger one of them as its error, and if the data does not have error, we refer to the mean error of the same source and same observational method as its error. Results from the different authors are usually adopted by the different confidence ranges ($\Delta \chi^2 = 1, \Delta \chi^2 = 2.7$ or both, some authors even do not give the confidence range). For a conservative sense, we take $\Delta \chi^2 = 1$ ($1 \sigma$ single parameter) for all the data to calculate the weighted mean value (with $1 \sigma$ confidence) of $\nu_1$, and $\nu_2$.

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1 The detail information of the sources can be seen in Liu et al., 2007.
Fig. 1 (a)-(d) present the cumulative distribution function curves of $\nu_1$, $\nu_2$, $\Delta\nu$ and $\nu_2/\nu_1$. The data obtained without shift-and-add. The line of $\Delta\nu = 300$ and $\nu_2/\nu_1 = 1.5$ are also plotted in (c) and (d), respectively.

(3.5) (e.g. $\langle \nu_1 \rangle$ and $\langle \nu_2 \rangle$), then we reserve the digit according to the data of that source. Table 2 is similar to Table 1, but without shift-and-add technique, which does not reconstruct the distribution of pairs of frequencies [Abramowicz et al., 2003a; Belloni et al., 2005], and only the frequency difference is meaningful [Jonker et al., 2000]. So, we only show $\nu_1$, $\Delta\nu$ and their weighted mean values (with 1 $\sigma$ confidence, $\langle \nu_1 \rangle$, $\langle \Delta\nu \rangle$), that includes 97 data of Atoll sources and 62 of Z sources. For 4U 1702-43 and 4U 1728-34, there are no errors of $\nu_1$ for all the data, so we only give their mean values. It can be seen from Table 1 and 2 that five sources have the data both with and without shift-and-add. To be clear, we show the ranges of $\nu_2$ and $\Delta\nu$ as well as their mean values of these sources in Table 3.

It is noted that XTE J1701-462 and Cir X-1 are the two special sources. XTE J1701-462 shows the link between “Z-track”, “$\nu$-track” and Atoll behavior in its CCD and HID diagrams [Homan et al., 2010; Lin et al., 2009], and the twin kHz QPOs were detected when this source was in Z phase [Sanna et al., 2010]. Cir X-1 was detected transition between Atoll and Z source [Soleri et al., 2009] and this source was also detected twin kHz QPOs when it showed property much like Z source [Boutloukos et al., 2006, 2008].

2.1 Analysis for the centroid frequency

For the data without shift-and-add, the statistical results of kHz QPO frequency are as follows: For Atoll (Z) sources, the weighted mean values (with 1 $\sigma$ confidence) of $\nu_1$, $\nu_2$, $\Delta\nu$ and $\nu_2/\nu_1$ are $744 \pm 10$ Hz, $937 \pm 12$ Hz, $303 \pm 3$ Hz and $1.42 \pm 0.01$ ($537 \pm 16$ Hz, $886 \pm 11$ Hz, $302 \pm 2$ Hz and $1.47 \pm 0.01$). The maxima of $\nu_1$ and $\nu_2$ in Atoll sources are $936$ Hz and $1253$ Hz (SAX J1750.8-2900) while the corresponding maxima in Z sources are $852$ Hz and $1081$ Hz (Sco X-1), respectively. For the data without shift-and-add, we take the mean errors of Atoll sources as the errors of 4U 1702-43 and 4U 1728-34, then obtain the results (with 1 $\sigma$ confidence): for Atoll (Z) sources, the mean of $\nu_1$ and $\Delta\nu$ are $791 \pm 7$ Hz and $311 \pm 3$ Hz ($837 \pm 6$ Hz and $293 \pm 2$ Hz), respectively. Table 3 shows that most of the maxima and mean values of $\nu_2$ of data with shift-and-add are bigger than those without shift-and-add. But we are not sure whether it results from the observational methods. We adopt K-S test to compare the centroid frequency distributions of kHz QPO.

\[ \text{We get } 571 \pm 16 \text{ Hz when calculating the weighted mean value of } \nu_2, \text{ in which only two data point below this value. Considering that the weights of this two data may be over estimated, we replace them with mean weight of Atoll sources and the recalculated result is } 937 \pm 12 \text{ Hz.} \]
Fig. 2  (a)-(b) present the cumulative distribution function curves of $\nu_1$ and $\Delta \nu$, in which the data obtained with shift-and-add.  (c)-(d) present the results of 4U 0614+09, where Yes-B means data with shift-and-add and from Boutelier et al. 2009. No-v means data without shift-and-add and from van Straaten et al. 2000. (e)-(f) are similar to (c)-(d), but data from Méndez & van der Klis 2000 and van der Klis et al. 1997, respectively.
between Atoll and Z sources as shown in Table 4, where the significance level \( \alpha = 0.05 \) for all the tests (the two groups data have the different distributions if the test p-value is less than \( \alpha \)). The results obtained from data with and without shift-and-add are quite different: for the data without shift-and-add, K-S test indicates that \( \nu_1, \nu_2, \Delta \nu \) of Atoll sources have the different distributions from those of Z sources, but \( \Delta \nu \) show the consistent distribution between the two types of sources (see Table 4 and Fig. 2). On the contrary, K-S test from the data with shift-and-add show that the two types of sources have the consistent distribution of \( \nu_1, \nu_2, \Delta \nu \) (see Table 4 and Fig. 2 (a)-(b)). If we take the data of the two observational methods together, K-S test shows that \( \nu_1, \nu_2, \Delta \nu \) are all different from those of Z sources. It is obviously to see that the p-value of K-S test of \( \Delta \nu \) changes sharply when adding the data with shift-and-add into the data without it (see Table 4). In order to find the reason that causes the different results, we select two sources: 4U 0614+09 (van Straaten et al. 2000, 27 data; Boutelier et al. 2009, 24 data) and Sco X-1 (van der Klis et al. 1997, 39 data; Méndez & van der Klis 2000, 49 data), in which there exist the data with and without shift-and-add. The K-S test of these two sources show the consistent distributions of \( \nu_1, \Delta \nu \) between the two different methods (see Table 4 and Fig. 2 (c)-(f)). When we compare the distributions of data from the different methods with all the data, we find that the distributions of \( \nu_1, \Delta \nu \) from the two methods are consistent in Atoll sources, but inconsistent in Z sources (see Table 4). There is only one Z source (Sco X-1) with shift-and-add, which has the relatively bigger \( \nu_1 \) and smaller \( \Delta \nu \) than other Z sources (see Table 2 and Fig. 2 (a)-(b)). So, this selection bias, as well as the abundant data of Sco X-1, cannot reconstruct the frequency distribution of all Z sources, which may cause the different results. We also show the line \( \Delta \nu = \text{constant} \) (300 Hz) and \( \nu_2/\nu_1 = 1.5 \) in Figure 2 (c)-(d) respectively. K-S test indicates that \( \Delta \nu \) is far from a constant distribution, so is \( \nu_2/\nu_1 \) (see also Table 4).

2.2 Analysis for the quality factor and the rms amplitude

As an extension of studying the influence of the two observational methods on kHz QPOs, We test the abrupt drop phenomena in \( Q_1 \) vs. \( \nu_1 \) and \( \text{rms}_S \) vs. \( \nu_1 \) plots. We select the source 4U 0614+09 (van Straaten et al. 2000 and Boutelier et al. 2009; see also Table 1 and 2) as a sample, where the data are obtained by the techniques with and without shift-and-add. We show results in Figure 3, where the drop in \( Q_1 \) vs. \( \nu_1 \) and \( \text{rms}_S \) vs. \( \nu_1 \) plots can be seen if combining the two group data together, but the errors are too big to make a robust conclusion.

3 Spin analysis in NS-LMXBs

In the samples, 28 sources have the inferred NS spins from the periodic or nearly periodic X-ray oscillations (Boutloukos & Lamb, 2008), where 9 sources are the accretion-powered millisecond pulsars, 22 sources are nuclear-powered millisecond pulsars, 3 sources are intermittent accretion-powered oscillations pulsars (Lamb et al., 2009), and 4 sources have been detected both accretion-powered and nuclear-powered oscillations and 2 sources have been detected both intermittent accretion-powered and nuclear-powered oscillations. The detail information of NS spins are listed in Table 5, where one can see that all Z sources have not yet been detected the inferred NS spins. The spin of XTE J1739-285 (1122 Hz) has not yet been confirmed (see Kaaret et al. 2007), so we neglect this data when analyzing the result. Figure 4 shows the NS spin frequency cumulative distribution function (CDF) curves of LMXBs, and the range is from 95 Hz to 619 Hz with the mean value 408 Hz. Considering that NS in LMXB
undersgoes the spin-up process and will form a radio millisecond pulsar (Bhattacharya & van den Heuvel, 1991), we also analyze the NS spin frequency of 136 binary radio millisecond (> 50 Hz or < 20 ms) pulsars, with the spin range of 52–716 Hz (1.4–19.4 ms) and the mean value of 281 Hz. The CDF of which is also shown in Figure 4, where we notice that the NS spin frequencies of LMXBs and binary radio MSPs share the similar range. The mean value of these two types of NS spins is shown in Table 4, indicating that they share the different distributions. The mean value of NS spin in NS-LMXBs is bigger than that in binary radio MSPs.

Van der Klis et al. 1997 and Méndez & van der Klis 2000. With vs. without shift-and-add. atoll vs. Z (with shift-and-add)

Table 4  K-S test result ($\alpha = 0.05$)

| Parameter | p-value |
|-----------|---------|
| Atoll vs. Z (without shift-and-add) | |
| $\nu_1$ | $6.1 \times 10^{-8}$ |
| $\nu_2$ | $5.8 \times 10^{-11}$ |
| $\Delta \nu$ | $7.4 \times 10^{-2}$ |
| $\nu_2 / \nu_1$ | $4.3 \times 10^{-5}$ |
| Atoll vs. Z (with shift-and-add) | |
| $\nu_1$ | $4.1 \times 10^{-1}$ |
| $\Delta \nu$ | $1.0 \times 10^{-6}$ |
| Atoll vs. Z (with and without shift-and-add) | |
| $\nu_1$ | $1.4 \times 10^{-8}$ |
| $\nu_2$ | $1.4 \times 10^{-12}$ |
| $\Delta \nu$ | $7.4 \times 10^{-6}$ |
| $\nu_2 / \nu_1$ | $8.4 \times 10^{-5}$ |
| With vs. without shift-and-add (4U 0614+09$^3$) | |
| $\nu_1$ | $1.2 \times 10^{-1}$ |
| $\Delta \nu$ | $6.2 \times 10^{-1}$ |
| With vs. without shift-and-add (Sco X-1$^3$) | |
| $\nu_1$ | $3.6 \times 10^{-1}$ |
| $\Delta \nu$ | $3.6 \times 10^{-1}$ |
| With vs. without shift-and-add (Atoll) | |
| $\nu_1$ | $1.8 \times 10^{-1}$ |
| $\Delta \nu$ | $5.0 \times 10^{-1}$ |
| With vs. without shift-and-add (Z) | |
| $\nu_1$ | $1.9 \times 10^{-8}$ |
| $\Delta \nu$ | $4.1 \times 10^{-2}$ |
| $\Delta \nu$ (Atoll, without shift-and-add) vs. 300 Hz | $1.4 \times 10^{-14}$ |
| $\Delta \nu$ (Z, without shift-and-add) vs. 300 Hz | $7.9 \times 10^{-18}$ |
| $\nu_2 / \nu_1$ (Atoll, without shift-and-add) vs. 1.5 | $1.2 \times 10^{-15}$ |
| $\nu_2 / \nu_1$ (Z, without shift-and-add) vs. 1.5 | $9.2 \times 10^{-20}$ |
| NS spin in LMXBs vs. in Binary radio MSPs | $1.2 \times 10^{-3}$ |

$^3$ The data comes from Australia Telescope National Facility (ATNF) pulsar catalog.

In the samples, 12 sources have both the detected twin kHz QPOs and NS spins. We try to analyze the relation between peak separations of twin kHz QPOs and their NS spin frequencies. Figure 5 shows $\Delta \nu$ vs. $\nu_2$ plot (both for data with and without shift-and-add). It can be seen that there are two approximate clusters in the figure, one relates to the line of $\Delta \nu = \nu_2$ while the other relates to $\Delta \nu = 0.5 \nu_2$. These correlations are not obvious because of the large spread of the data.

4 Discussions and Conclusions

From the kHz QPO data published before 2012, the following statistical results are obtained below:

(1). The K-S test results from data without shift-and-add technique show the inconsistency of $\nu_1$, $\nu_2$, $\nu_2 / \nu_1$ distributions between Atoll and Z sources, which may result from...
Table 5  NS spins in LMXBs

| Source (28)        | ν_s (Hz) | Type | Ref   |
|--------------------|----------|------|-------|
| HETE J1900.1-2455  | 377      | I    | 1:8   |
| IGR J00291+5934    | 598      | A    | 1     |
| NGC 6440 X-2       | 206      | AN   | 2     |
| SAX J1808.4-3658   | 401      | AN   | 1     |
| XTE J0929-314      | 185      | A    | 1     |
| XTE J1739-285      | 1122     | N    | 3     |
| XTE J1751-305      | 435      | A    | 1     |
| XTE J1807.4-294    | 191      | A    | 1     |
| XTE J1814-338      | 314      | AN   | 1     |
| HETE J1900.1-2455  | 377      | I    | 1:8   |
| IGR J00291+5934    | 598      | A    | 1     |
| NGC 6440 X-2       | 206      | AN   | 2     |
| SAX J1808.4-3658   | 401      | AN   | 1     |
| XTE J0929-314      | 185      | A    | 1     |
| XTE J1739-285      | 1122     | N    | 3     |
| XTE J1751-305      | 435      | A    | 1     |
| XTE J1807.4-294    | 191      | A    | 1     |
| XTE J1814-338      | 314      | AN   | 1     |

Atoll Sources (13)

| Source (28)    | ν_s (Hz) | Type | Ref   |
|----------------|----------|------|-------|
| 4U 0614+09     | 415      | N    | 4     |
| 4U 1608-52     | 619      | N    | 1     |
| 4U 1636-53     | 581      | N    | 1     |
| 4U 1702-43     | 330      | N    | 1     |
| 4U 1728-34     | 363      | N    | 1     |
| 4U 1915-05     | 270      | N    | 1     |
| A 1744-361     | 530      | N    | 1     |
| Aql X-1 (1908+005) | 550  | IN   | 1:9   |
| IGR J17191-282 | 294      | N    | 1     |
| IGR J17511-3057| 245      | AN   | 5     |
| KS 1731-260    | 524      | N    | 1     |
| SAX J1750.8-2900 | 601  | N    | 1     |
| XB 1254-690    | 95       | N    | 6     |

Other Sources (6)

| Source (28) | ν_s (Hz) | Type | Ref   |
|-------------|----------|------|-------|
| EXO 0748-676 | 522      | N    | 7     |
| GS 1826-238  | 611      | N    | 1     |
| MXB 1659-298 | 567      | N    | 1     |
| MXB 1743-29  | 589      | N    | 1     |
| SAX J1748.9-2021 | 442  | IN   | 1;10  |
| SWIFT J1756.9-2508 | 182 | A    | 1     |

†: NS spin frequency has not yet been confirmed;
A: accretion-powered millisecond pulsar;
N: nuclear-powered millisecond pulsar;
I: intermittent accretion-powered oscillations pulsar;
1. Reference in Boutloukos & Lamb 2008;
2. Altamirano et al. 2010c;
3. Kaaret et al. 2007;
4. Strohmayer et al. 2008a;
5. Altamirano et al. 2010b;
6. Bhattacharyya 2007;
7. Galloway et al. 2010;
8. Galloway et al. 2007;
9. Casella et al. 2008;
10. Gavriil et al. 2007, Altamirano et al. 2008a, Patruno 2008.

the different properties of the two types of sources. The result also show the consistency of ∆ν distributions, which indicates the twin kHz QPO of Atoll and Z sources to be the same physical origins. It can be seen from Figure 3 and Table 3 that the distribution of ∆ν is quite different from the prediction of beat model of twin kHz QPOs, so is ν2/ν1 different from the constant ratio (3:2). The similar conclusion is also noticed by Belloni et al. (2005).

(2). From the results of data without shift-and-add, the weighted mean values of ν1 and ∆ν of Atoll sources of low luminosity are a little higher than those of Z sources of high luminosity. The maximum of ν2 in Atoll (Z) sources is 1253 Hz (1081 Hz) (see Table 1), which is the same order as the Keplerian orbital frequency near the NS surface (for the NS with radius 15 km and mass 1.4 M⊙, see Zhang 2004), so the kHz QPOs of Atoll sources could occur, usually, closer to the NS surface than those of Z sources.

(3). For 4U 0614+09 and Sco X-1, the K-S test result shows the ν1 and ∆ν distributions of the different observational methods are consistent, which may imply that shift-and-add technique can reconstruct the distribution of ν1 and ∆ν. The K-S test results of ν1 and ∆ν between Atoll and Z sources from data with shift-and-add are quite different from those without it, and we think that there is only one Z source (Sco X-1) with shift-and-add, which has the relatively bigger ν1 and smaller ∆ν than those of the other Z sources. So, this selection bias, as well as the abundant data of Sco X-1, may cause the different results.

(4). We test the abrupt drop phenomenon in Q1 vs. ν1 and rms1 vs. ν1 plots with data from different observational methods. We find that 4U 0614+09 shows the drop in Q1 vs. ν1 and rms1 vs. ν1 plots if combining the data with and without shift-and-add. But the errors are too big to make a certain conclusion.

(5). Considering that NS in LMXB undergoes the spin-up process and will form a radio millisecond pulsar (Bhattacharya & van den Heuvel 1991), NS spins in two systems may have a correlation, so we compare the NS spin frequency distribution in LMXBs with that in the binary radio MSPs (see Fig 4). The range of NS spin frequencies in LMXBs (95-619 Hz) is similar to...
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