On the Cluster Sunyaev-Zel’dovich Effect and Hubble Constant

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ABSTRACT

This study shows one important effect of preexistent cosmic microwave background temperature fluctuations on the determination of the Hubble constant through Sunyaev-Zel’dovich effect of clusters of galaxies, especially when coupled with the gravitational lensing effect by the same clusters. The effect results in a broad distribution of the apparent Hubble constant. The combination of this effect with other systematic effects such as the Loeb-Refregier Effect seems to provide an explanation for the observationally derived values of the Hubble constant currently available based on the Sunyaev-Zel’dovich effect, if the true value of the Hubble constant is $60 - 80$ km/s/Mpc. It thus becomes possible that the values of the Hubble constant measured by other techniques which generally give a value around $60 - 80$ km/s/Mpc be reconciled with the SZ effect determined values of the Hubble constant, where are systematically lower than others and have a broad distribution.

Subject headings: Cosmology: large-scale structure of Universe – cosmology: cosmic microwave background – cosmology: distance scale – cosmology: gravitational lensing – galaxies: clusters
1. Introduction

The Sunyaev-Zel’dovich (SZ) effect of a cluster of galaxies on the cosmic microwave background (CMB) photons can be used to determine the distance to the cluster hence the Hubble constant ($H_0$), when analysed in conjunction with X-ray observations of the cluster (Cavaliere, Danese, & De Zotti 1977; Gunn 1978; Silk & White 1978; Birkinshaw 1979). For an excellent recent review on this subject and other SZ related topics, see Rephaeli (1995 and references therein). The accuracy of the Hubble constant determination depends upon the accuracy of several assumptions involving both sets of observations (radio and X-ray). Perhaps among the most important are the assumptions of sphericity, isothermality of clusters of galaxies (e.g., Inagaki et al. 1995). In this Letter we point out a completely separate effect on the determination of the Hubble constant due to preexistent, small-amplitude CMB temperature fluctuations before the photons undergo the SZ effect through a cluster. The effect is significantly amplified by the gravitational lensing of the CMB photons by the cluster, because the SZ observational beam size is typically comparable to Einstein radius of the source-lens system. This effect, when coupled with some systematic effects such as the one proposed by Loeb & Refregier (1997) due to the systematic over-removal of background point radio sources in the beam, may provide an explanation for the observed distribution of $H_0$ determined by SZ effect.

2. CMB Fluctuations and $H_0$ Determination Using SZ Effect

The detections of CMB temperature fluctuations on arcminute scales are mostly upper limits or marginal (e.g., Partridge et al. 1997), primarily because of limited sky coverages and instrumental sensitivities. However, there are a number of physical mechanisms suggested, which should generate appreciable fluctuations on the relevant scales. For example, Persi et al. (1995) (see also Scaramella, Cen, & Ostriker 1993) show that SZ
effect due to non-cluster gas can generate $\Delta T/T \sim 10^{-6} - 10^{-5}$ on the arcminute scale, produced naturally by shock heated gas in a network of filaments and sheets during the phase of gravitational collapse of large-scale structure. Loeb (1996) shows that one would expect $\Delta T/T \sim 10^{-6} - 10^{-5}$ due to bremsstrahlung emission from Ly$\alpha$ clouds, given the observed/required meta-galactic ultraviolet radiation field. Depending on the universal ionization history, the Ostriker-Vishniac Effect (Ostriker & Vishniac 1986; Vishniac 1987) could also make appreciable contributions to the CMB fluctuations on the relevant scales (e.g., Persi et al. 1995). To be quantitatively definitive, we will adopt CMB temperature fluctuations observed by Partridge et al. (1997) and assume that they are background CMB temperature fluctuations, meaning that they exist before CMB fluctuations are further induced by hot gas in clusters of galaxies. This is not to say that these fluctuations are primordial; all that we need to assume is that there exists appreciable CMB fluctuations at redshift higher than that of the clusters for which we study their individual SZ effect and then determine the Hubble constant (when coupled with X-ray observations of the clusters).

To clearly illustrate the point, we will first adopt a set of numbers and make a few simplifying assumptions:

1) the true Hubble constant is $H_{0,true} = 65\text{km/s/Mpc}$;

2) X-ray observations of clusters and the interpretations of them are error-free;

3) the clusters are spherical with a singular isothermal profile for the total mass with one-dimensional velocity dispersion of $1021 \text{ km/s}$ (corresponding to a gravitational lensing bending angle of $30^\circ$), and a true SZ effect of $(\Delta T/T)_{\text{SZ}} = 2.0 \times 10^{-4}$ (the observed arcs by gravitational lensing of rich clusters have sizes comparable to $30^\circ$ and the observed SZ effect in clusters is about $(1 - 6) \times 10^{-4}$ with a mean about $2.0 \times 10^{-4}$ [see Rephaeli 1995 for a summary]);
4) the profile of the observational beam is, for the simplicity of calculation, assumed to be a two-dimensional top-hat with a radius of $60^\circ$ and the beam is precisely centered on each cluster center;

5) we adopt the CMB fluctuations around arcminute scales from Partridge et al. (1997), adapted as columns 1 and 2 in Table 1, and assume that the fluctuations are Gaussian. Note that, when the CMB maps are generated, we follow that the scales indicated in Table 1 are FWHM’s but with a Gaussian profile (i.e., FWHM = $2.35\sigma_g$, where $\sigma_g$ is the radius of the Gaussian window). This is not to be confused with the SZ observation’s beam shape, which is assumed to be a top-hat for the convenience of calculation (assumption 4, above).

The rms fluctuation of the CMB map just due to pre-SZ CMB fluctuations alone, as indicated by the third column in Table 1, on a top-hat circle of radius $60^\circ$ is $6.8 \times 10^{-6}$. The distribution of fluctuations on a top-hat circle of radius $60^\circ$ is shown as heavy histogram in Panel (a) of Figure 1 for a sample of 1000 random beams. The light dashed curve is the Gaussian fit with the same variance and zero mean. Since the Hubble constant determined from SZ effect is proportional to $(\Delta T/T)^{-2}_{SZ}$ (Cavaliere, Danese, & De Zotti 1977; Gunn 1978; Silk & White 1978; Birkinshaw 1979), we immediately obtain a distribution of $H_0$, given the distribution of $\Delta T/T$. Thus, the light dashed curve in Panel (a) of Figure 1, $f(\Delta T/T)$, can be translated into a distribution of $H_0$, which is shown as the dashed curve in Figure 2, $g(H_0)$. The short vertical bar at $H_0 = 65$ km/s/Mpc indicates the (adopted) true value of the Hubble constant. Note that, although the light dashed curve in Panel (a) is Gaussian, the dashed curve in Figure 2 is not Gaussian because the translation from $f(\Delta T/T)$ to $g(H_0)$ is nonlinear.

Next, we examine how gravitational lensing of CMB photons by clusters alter the CMB fluctuations on the arcminute scale around the clusters. In order to do this, we first generate synthetic CMB maps which agree with observations on $6^\circ$ – $80^\circ$ scales tabulated
in Table 1. We find that a two-dimensional power-law power spectrum of index $-0.42$ (i.e., $P_k \propto k^{-0.42}$) produces a satisfactory fit to observations, as indicated by the third column in Table 1. We normalize the fluctuations by requiring $(\Delta T/T)_{\text{syn}} = (\Delta T/T)_{\text{obs}} = 1.2 \times 10^{-5}$ at FWHM$= 60^\circ$.

For each synthetic CMB map of size $12.8' \times 12.8'$, with pixel size of $6'' \times 6''$, we center the cluster at the optical axis connecting the observer and the center of the source plane map. For simplicity we assume that we live in an $\Omega = 1$ universe, the sources (CMB photons) are placed at a distance of $D_s = 6000h^{-1}\text{Mpc}$ (i.e., $z_s >> 1$) from the observer, and the lens (the cluster) is at a distance of $D_l = 1000h^{-1}\text{Mpc}$ (i.e, $z_l \sim 0.44$) from the observer (note that only the combination $(D_s - D_l)/D_s$ enters our calculation of ray tracing; see below). Each CMB source pixel of size $6'' \times 6''$ is divided into smaller (square) sub-pixels. The number of sub-pixels ($N_{\text{sub}}$) for each such division is adaptive depending on the amplification ($\mu$) of the coarse pixels (for an optimal efficiency of calculation). The photons of sub-pixels are ray traced through the cluster potential and collected in the image plane with pixels of exactly the same size ($6'' \times 6''$) as that of the coarse pixels in the source plane. The accuracy of the ray tracing method is measured as follows. We keep increasing the number of sub-pixels until the fluctuations on a top-hat of radius $60''$ of the CMB map in the image plane is less than $5 \times 10^{-7}$ for a source plane map with zero fluctuations on all scales; note that an ideal method should give zero fluctuations in the image plane map in this case. This is sufficiently accurate since the real fluctuations are on the order of $1.0 \times 10^{-5}$. We find that $N_{\text{sub}}(\mu) = 4096\mu^6$ is required to give such a satisfactory accuracy.

We ray trace 1000 independent maps and the final distribution of $\Delta T/T$ on the top-hat beam of radius $60''$, $F(\Delta T/T)$, is shown as heavy solid histogram in Panel (b) of Figure 1. The variance of $F(\Delta T/T)$ is $1.3 \times 10^{-5}$ compared to $6.8 \times 10^{-6}$, the variance of $f(\Delta T/T)$, the distribution prior to the gravitational lensing effect by the clusters. The light dashed
curve in the same panel is the Gaussian fit with the same variance ($1.3 \times 10^{-5}$).

We can translate the light dashed curve in Panel (b) of Figure 1 into a distribution of $H_0$, shown as the dotted curve in Figure 2. It is worthwhile to consider other effects on the determination of SZ effect of clusters. Loeb & Refregier (1997; LR effect) point out a systematic effect due to gravitational lensing of discrete background radio sources by the clusters, which causes an over-removal of unresolved radio background thus an underestimate of $\Delta T/T$ of the true SZ effect of the cluster (i.e., a more negative value of $\Delta T/T$ than the true value) and subsequently an underestimate of $H_0$. They find an underestimate of $\Delta T/T$ by approximately 5%. We here include the LR effect. We combine the light dashed curve in Panel (b) of Figure 1 with the Lb effect (assuming to be 5% underestimate of $\Delta T/T$) and the resulting distribution of the apparent $H_0$ is shown as the solid curve in Figure 2. Also shown as long-dashed vertical bar is the apparent Hubble constant when only LR effect is taken into account. We see that the lensing-coupled pre-SZ CMB fluctuations produce a fairly broad distribution (with a FWHM of about 18 km/s/Mpc). Even if the distribution of $\Delta T/T$ is symmetric, that of $H_0$ is not. We find that the average and median values of $H_0$ for the three cases shown in Figure 2 (dashed, dotted and solid curves) are (65.23, 65.00)km/sec/Mpc, (65.87, 65.00)km/sec/Mpc and (59.20, 58.50)km/sec/Mpc, respectively.

It is probably still too early to make solid statistical comparisons between this model and observations due to two obvious reasons. First, the model is perhaps over-simplified. Second, the observed sample is still too small [a current total of nine $H_0$ measurements from SZ effect, see Table 2 of Rephaeli (1995)]. For the convenience of the reader, we list the values of the nine available $H_0$ values from SZ measurements (all in units of km/s/Mpc): $24 \pm 11$, $32^{+19}_{-15}$, $38^{+18}_{-16}$, $40 \pm 9$, $57^{+61}_{-39}$, $65 \pm 25$, $74^{+29}_{-24}$, $76^{+22}_{-19}$, $82^{+35}_{-22}$. Nevertheless it is quite encouraging that the effect considered here provides a reasonable explanation for the
broadness of the observed distribution of $H_0$.

3. Conclusions

We show that the background CMB fluctuations, especially when they are coupled with the gravitational lensing effect by clusters of galaxies, have one important effect on the determination of the Hubble constant through Sunyaev-Zel’dovich effect of the clusters (for the adopted set of characteristic numbers for the cluster, which seem fairly realistic compared to those of real clusters of interest): a broad distribution of the apparent Hubble constant is produced with a FWHM about 30% of the apparent mean value. The combination of this effect with other systematic effects such as the Loeb-Refregier Effect seems to provide a reasonable explanation for the observationally derived values of the Hubble constant currently available, if the true value of the Hubble constant is $\sim 65$ km/s/Mpc. Thus, it becomes possible that the values of $H_0$ measured by other techniques which generally give a value around $60-80$ km/s/Mpc [e.g., $73 \pm 10$ km/s/Mpc ($1\sigma$) from Freedman, Madore, & Kennicutt 1997 based on HST observations of Cepheids; $64 \pm 6$ km/s/Mpc ($1\sigma$) from Riess, Press, & Kirshner 1996 based on type Ia supernova multicolor light-curve shapes; $64 \pm 13$ km/s/Mpc (95% confidence level) from Kundic et al. 1997 based on gravitational lensing time delay measurements; $70 \pm 5$ km/s/Mpc ($1\sigma$) from Giovanelli 1997 using I-band Tully-Fisher relation; $81 \pm 6$ km/s/Mpc ($1\sigma$) from Tonry 1997 using surface brightness fluctuations] be reconciled with the SZ effect determined values of $H_0$.

It may be possible, at least in principle, that one can use a large sample of SZ measured Hubble constant to infer the fluctuations of the CMB at the relevant scales, when the Hubble constant is independently measured to high accuracy by methods such as that using detached eclipsing binaries (Paczynski 1997).
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Fig. 1.— Panel (a) shows the rms fluctuation of the CMB map on a top-hat circle of radius 60" before the gravitational lensing effect by the clusters is taken into account (solid histogram). The light dashed curve is the Gaussian fit with a variance of $6.8 \times 10^{-6}$ and zero mean. Panel (b) shows the rms fluctuation of the CMB map on a top-hat circle of radius 60" after the gravitational lensing effect by the clusters is taken into account (solid histogram). The light dashed curve in the same panel is the Gaussian fit with the same variance ($1.3 \times 10^{-5}$).

Fig. 2.— shows the distribution of $H_0$. Note that the true Hubble constant is assumed to be 65km/s/Mpc, as shown as a vertical solid line at the bottom x-axis. The dashed and dotted curves indicate the distributions of the apparent Hubble constant due to CMB fluctuations without and with gravitational lensing effect, respectively. The solid curve combines the CMB fluctuations and gravitational lensing effect with the Loeb-Refregier Effect (assuming to be 5% underestimate of $\Delta T/T$ due to Loeb-Refregier Effect). Also shown as long-dashed vertical bar is the apparent Hubble constant when only Loeb-Refregier Effect is taken into account.
Table 1. CMB fluctuations on arcminute scales

| FWHM | $(\Delta T/T)_{obs} (10^{-5})$ | $(\Delta T/T)_{sim} (10^{-5})$ |
|------|-------------------------------|----------------------------------|
| 6"   | 7.4 ± 8.1                     | 6.9                              |
| 10"  | 5.0 ± 5.1                     | 5.1                              |
| 18"  | 3.3 ± 3.1                     | 3.2                              |
| 30"  | 2.6 ± 1.9                     | 2.1                              |
| 60"  | 1.2 ± 1.4                     | 1.2                              |
| 80"  | 1.5 ± 1.5                     | 0.98                             |
Figure 1

(a) without lensing

(b) with lensing

$\Delta T / T$ (60°) vs. $f(\Delta T / T)$ and $F(\Delta T / T)$
