Effects of artificial square nanorods array to square bulk superconducting plate

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Abstract. It is known that the superconducting properties are improved by adding nanorods to a superconductor. As increasing nanorods into superconductor, the pinning force becomes stronger. Using the molecular dynamics method, we investigate vortex motions in a superconductor with a nanorod array. We obtain trajectories of vortices and standard deviation of vortex positions. We find peculiar temperature dependence of vortex motion.

1. Introduction
Vortices in a superconductor tend to make lattice structures at low temperature. However, the lattice structures are broken by external sources, such as temperature and electric current. For stronger disturbance, vortices move more freely. It is known that impurities in a superconductor can pin vortices, then, vortex motion is restricted. If there are regular pinning sites which match with vortex lattice, vortex lattice structures become harder against disturbances. Then superconductor shows higher critical current density $J_c$.

Recently, nanorods are much studied as artificial impurities. Especially, it is known that as adding straight nanorods parallel to an external magnetic field in the superconductor, the pinning force becomes strong, and the critical current density becomes high [1-4]. So, we simulate how the vortices move in a superconducting plate including nanorods.

In this study, using the molecular dynamics method, we investigate the motion of vortices in a superconducting plate including nanorods. We obtain standard deviations ($\sigma$) of vortex position, which depend on temperature.

2. Method
In the molecular dynamics method, we consider following equations of motion of vortices,

$$\eta \frac{dr}{dt} = f_{\text{nanorod}} + f_{v} + f_{f},$$

(1)
where $\mathbf{r}_i$ is a position of an $i$-th vortex, and $\eta$ is the viscosity. $f_{\text{nanorod}}$ is a pinning force of nanorods, which is given as,

$$f_{\text{pinning}} = \sum_j \frac{1}{p_{ij}} \frac{f_{pj}}{r_{pj}^{p_{ij}-1}} \left( \sum_{i=1}^{2n} \left( \mathbf{r} - \mathbf{r}_j \right) \cdot \mathbf{n}_j \right)^{p_{ij}}.$$  

(2)

Here $f_p$ and $r_p$ are strength and range of the pinning force, and $\mathbf{r}_j$ is the position of $i$-th nanorod. Also $p_{ij}$ is a power of distance in the pinning potential, and $\mathbf{n}_j$ is a normal vector of the polygon. Second term $f_{vi}$ is a sum of vortex-vortex interaction forces from other vortices.

$$f_{vi} = \sum_j f_{vij},$$  

(3)

$$f_{vij} = f_0 \sum_j K_1 \left( \frac{r_i}{\lambda} \right) \left( \mathbf{r}_i - \mathbf{r}_j \right) .$$  

(4)

$$f_0 = \frac{\Phi_0^2}{8\pi^2 \lambda^2} .$$  

(5)

where $\lambda$ is the penetration depth, $f_0$ is strength of the vortex-vortex interaction force, $\Phi_0$ is the quantum flux ($\Phi_0 = \frac{eh}{2\pi}$), and $K_1(r)$ is the MacDonald function. Therefore, the force depends on temperature $T$ through temperature dependence of $\lambda = \lambda(T)$. The MacDonald function decay exponentially for $r$ greater than $\lambda$, and for computational efficiency, we can cut off the interaction at $15\lambda$ safely. The last term is the thermal fluctuation force, which satisfies following equation,

$$\left\langle f_{t1}(t_1) \cdot f_{t2}(t_2) \right\rangle = 2\eta k_B T \delta(t_1-t_2).$$  

(6)

here $k_B$ is the Boltzmann constant. Temperature dependence of vortex motion comes from the thermal fluctuation $f_f$ and also the vortex-vortex interaction $[5]$.  

3. Results

We consider a bulk $(20\lambda_0 \times 20\lambda_0)$ square superconducting plate with impurities under the periodic boundary condition, and the superconducting plate including square 400 nanorods. Shapes of nanorods

![Figure 1](image-url)  

**Figure 1.** $20\lambda_0 \times 20\lambda_0$ square superconducting plate including 400 nanorods
are square and they form a square lattice as shown in figure 1. In this configuration, current around a vortex may be deformed, and then, the vortex-vortex interaction may be modified from equation (3). However, in this study, we ignore this effect. We will discuss this effect in section 4. In the simulation, first, vortices move at higher temperature, then, temperature decreases and vortices move until equilibrium is reached. Then we investigate vortex motion using standard deviation of vortex positions.

We show simulation results on 400 vortices in a square superconducting plate with 400 nanorods that form a square lattice. So, numbers of vortices and nanorods match. The number of vortices may correspond to the external magnetic field. In order to see how vortices move, we use standard deviation $\sigma$ of a vortex position, which is defined as

$$\sigma_i^2 = \left\langle \left( r_i - \langle r_i \rangle \right)^2 \right\rangle,$$

where the average is a time average. In figure 2, we show temperature dependence of averaged standard deviation $\sigma$ for $0.9 < T/T_c < 0.99$, where $T_c$ is the critical temperature. Here $\sigma$ is normalized by an average vortex distance $l_N$, where $l_N = \sqrt{20\lambda_0 \times 20\lambda_0}/N$ for $N$ vortices. $\sigma$ becomes smaller for larger $f_{\text{nanorod}}$. From figure 2 we can see that 400 nanorods affect the vortex motion, and vortices are less movable for larger $f_{\text{nanorod}}$. But at $f_{\text{nanorod}}/f_0 = 0.008$, vortices are more moveable comparing with the motion at $f_{\text{nanorod}}/f_0 = 0.006, 0.007$. This behavior might come from balance of $f_{\text{vi}}$ and $f_{\text{nanorod}}$. Also, we can see that at $T/T_c = 0.92$, and 0.94, the vortex motion becomes relatively less movable. This behavior also might come from balance between temperature dependent $f_{\text{vi}}$ and $f_{\text{nanorod}}$.  

![Figure 2](image_url)
Above $T/T_c = 0.97$, $\sigma$ with nanorod increases rather rapidly. So, the vortex lattice melting transition may occur around $T/T_c = 0.97$.

We can see the trajectory of vortices from following figures 3-6. Figure 3 shows trajectories in the superconducting plate without nanorods. The trajectories of vortices are gradually extended for higher temperature. At $T/T_c = 0.9$, vortices form a lattice, and then the vortex lattice melts at $T/T_c = 0.99$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{The vortex trajectories at $T/T_c$ is from 0.9 to 0.99 in the superconducting plate without nanorods.}
\end{figure}

Figures 4-6 shows vortex trajectories in the superconducting plate with 400 nanorods for $f_{\text{nanorod}}/f_0 = 0.013, 0.027$, and 0.004. We can see that the stronger $f_{\text{nanorod}}$, the smaller movement of vortices. In order to clarify the influence of nanorods on the vortex motion, we count nanorods through which vortices pass. We denote it the number of nanorods without passed vortex and show this number on upper-left corner in each figure. It becomes the largest at $T/T_c = 0.99$ for each Figure. At $T/T_c = 0.99$, the number of nanorods without passed vortex becomes maximum in all figures. They are 19 in figure 4, 40 in figure 5, and 53 in figure 6. When the temperature is close to transition temperature, we may expect that all of nanorods could not trap vortices. But if the thermal fluctuation force is not strong compare to the pinning force, vortices can remain inside of the nanorod. This is because, close to the $T_c$, the thermal fluctuation becomes large and also vortex-vortex interaction becomes long-range and weak due to longer $\lambda (T)$. We discuss that the reason is that as the vortex-vortex interaction force becomes small at high temperature referencing equations (4) and (5), the vortex-vortex distance becomes short, some vortices exist between nanorod and another, and the number of nanorods consequently increases. At lower temperature, vortex motions are small, thermal fluctuation force becomes weak and the vortex-vortex interaction force becomes short range but strong due to short $\lambda (T)$. Then, the number of nanorods without passed vortex becomes small.
Figure 4. The vortex trajectories at $T/T_c$ is from 0.9 to 0.99 in the superconducting plate including 400 nanorods when $f_{\text{nanorod}}/f_0 = 0.013$.

Figure 5. The vortex trajectories at $T/T_c$ is from 0.9 to 0.99 in the superconducting plate including 400 nanorods when $f_{\text{nanorod}}/f_0 = 0.027$. 
Summary

We have investigated vortex motion in a square bulk superconducting plate with 400 nanorods, using the molecular dynamics method. We have found that at $T/T_c = 0.92, 0.94$, vortices become less movable. Moreover, the number of nanorods through which vortices pass increases with increasing temperature.

In this study, we only considered square nanorods and assume the vortex-vortex interaction is not affected by nanorods. Even so, the square lattice of vortices becomes stable in our simulation. We think this is because the repulsive interaction between the pair of vortices in nearest neighbor square nanorods is effectively large comparing with that in other nanorod lattices, such as a triangular lattice. However, the vortex-vortex interaction, equation (4), comes from the circular current around the vortex. This current will be modified by nanorods, because of broken of the circular symmetry around the vortex. So, if we take into account of this modification of the current, the vortex-vortex interaction may have the four-fold symmetry around the nanorod-trapped vortex. Then we expect more stability of the square lattice of vortices.

Including this modified vortex-vortex interaction is a future problem. Also, we will investigate the vortex number dependence of vortex motion with square nanorods. In experiments, circular nanorods are usually used for pinning vortices. We expect weak stability of square lattice of vortices in the square lattice of circular nanorods, because of the repulsive interaction may have circular symmetry. But we expect stability of the vortex lattice does not much dependent on the lattice structure for circular nanorods comparing with square nanorods. These points will be investigated in the future.

References

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Figure 6. The vortex trajectories at $T/T_c$ is from 0.9 to 0.99 in the superconducting plate including 400 nanorods when $f_{\text{nanorod}}/f_0 = 0.04$. 
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