Equation of state in the pion condensation phase in the asymmetric nuclear matter using a holographic QCD model

Hiroki Nishihara and Masayasu Harada

1 Department of Physics, Nagoya University, Nagoya 464-8602, Japan

(Dated: July 29, 2014)

We study the asymmetric nuclear matter using a holographic QCD model by introducing a baryonic charge in the infrared boundary. We first show that, in the normal hadron phase, the predicted values of the symmetry energy and it’s slope parameter are comparable with the empirical values. We find that the phase transition from the normal phase to the pion condensation phase is delayed compared with the pure mesonic matter: The critical chemical potential is larger than the pion mass which is obtained for the pure mesonic matter. We also show that, in the pion condensation phase, the pion contribution to the isospin number density increases with the chemical potential, while the baryonic contribution is almost constant. Furthermore, the value of chiral condensation implies that the enhancement of the chiral symmetry breaking occurs in the asymmetric nuclear matter as in the pure mesonic matter. We also give a discussion on how to understand the delay in terms of the 4-dimensional chiral Lagrangian including the rho and omega mesons based on the hidden local symmetry.

PACS numbers: 11.25.Tq, 11.30.Rd, 14.40.Be, 21.65.Cd, 21.65.Mn

I. INTRODUCTION

It is expected that investigation of the hadron physics in extreme conditions will give a clue for our understanding of QCD (Quantum Chromodynamics). In particular studying asymmetric nuclear matter is also important to derive the equation of state inside neutron stars 1, which will give a clue to understand the recently found very heavy neutron star 2, 3.

In the previous work 4, we studied the pion condensation in the pure mesonic matter using a holographic QCD model by introducing the isospin chemical potential as a UV boundary value of the gauge field. We showed that the phase transition from the normal hadron phase to the pion condensation phase is of the second order and the critical value of the isospin chemical potential is equal to the pion mass, consistently with the chiral Lagrangian analysis 5.

In Ref. 4, we studied the $\mu_I$-dependence of the chiral condensate, which generates a massless Nambu-Goldstone boson. Since both $U(1)_V^3$ and $U(1)_V$ are subgroups of the chiral $SU(2)_R \times SU(2)_L$ symmetry, the above structure implies that the chiral symmetry is never restored in the mesonic matter with the isospin chemical potential, and actually the breaking is enhanced in the pion condensation phase. We note that the above properties are obtained in the pure mesonic matter, so that it is interesting to ask whether they are changed by the existence of the nucleon in the matter.

There are several works studying the asymmetric matter. In Refs. 4, 6, the asymmetric nuclear matter was studied by regarding the Reissner-Nordstorm (RN) blackhole charge as the baryon charge in a hard wall holographic QCD model. They studied the transition from the confinement phase to the deconfinement phase and the one from the normal hadron phase to the pion condensation phase. It was shown that the pion condensation phase appears at finite isospin number density with the baryon number density in the confinement phase 6.

In this paper, we adopt a simple way for introducing the baryonic sources: We include a point-like nucleon source at the IR boundary coupling to the iso-triplet vector meson in the hard wall holographic QCD mode as in Ref. 4, and studied the pion condensation in the asymmetric nuclear matter. We will show that the phase transition from the normal hadron phase to the pion condensation phase is delayed in the asymmetric nuclear matter compared with the pure mesonic matter. In other words, the critical chemical potential is larger than the pion mass. On the other hand, the enhancement of the

---

*harada@tkhephys.nagoya-u.ac.jp
*harada@tkhephys.nagoya-u.ac.jp
chiral symmetry breaking still occurs since the chiral condensate $\bar{\sigma}$ keeps increasing with the isospin chemical potential.

This paper is organized as follows: In section II we briefly review the holographic QCD model used in our analysis, and introduce the baryonic charge following Ref. [6]. Section III is devoted to the study of the symmetry energy and the pion mass in the normal hadron phase. In section IV we study the pion condensation phase and obtain the relation between the isospin chemical potential and the isospin number density as well as the chiral condensate. In section V we make an analysis of the pion mass in the normal hadron phase using the four dimensional chiral model based on the hidden local symmetry [10, 11]. We give a summary and discussions in section VI. We also show the equations of motion in appendix A.

II. MODEL

In the present analysis, we employ a holographic QCD model given in Refs. [12, 14] for the mesonic part. Then the mesonic action in the five dimensional space is given by

$$ S_5 = S_X + S_{\text{BD}} $$  \hspace{1cm} (II.1)

where

$$ S_X = \int d^4x \int z^m dz \left( \sqrt{g} \text{Tr} \left\{ (DX)^2 - m^2 |X|^2 \right. \right. $$

$$ \left. \left. - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\} \right), $$

$$ S_{\text{BD}} = - \int d^4x \int z^m dz \sqrt{g} \left\{ \lambda_{z_m} |X|^4 - m^2 z_m |X|^2 \right\} \delta (z - z_m) $$  \hspace{1cm} (II.2)

with $m^2 = -3$. The metric is written as

$$ ds^2 = a^2(z) \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) = g_{MN} dx^M dx^N $$  \hspace{1cm} (II.4)

with

$$ a(z) = \frac{1}{z}, $$

$$ \text{a}(z) = \frac{1}{z}, $$  \hspace{1cm} (II.5)

where $z_m$ and $\epsilon$ are the IR-cutoff and UV-cutoff. Here $N$ and $M$ run over 0,1,2,3,5 and $\eta_{\mu\nu}$ is the defined as the Minkovski metric: $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1,-1)$. \(^1\)

The model has the chiral symmetry $U(2)_L \times U(2)_R (=U(1)_L \times U(1)_R \times SU(2)_L \times SU(2)_R)$, under which the fields transform in the following form:

$$ X \rightarrow X' = g_L X g_R^\dagger, $$

$$ L_M \rightarrow L'_M = g_L L_M g_R^\dagger + ig_L \partial_M g_R^\dagger, $$

$$ R_M \rightarrow R'_M = g_R R_M g_L^\dagger + ig_R \partial_M g_L^\dagger $$  \hspace{1cm} (II.6)

with $g_R \in U(2)_R$ and $g_L \in U(2)_L$. The covariant derivative and the field strength are defined as

$$ D_M X = \partial_M X - iL_M X + iX R_M, $$

$$ F^L_{MN} = \partial_M L_N - \partial_N L_M - i[L_M, L_N] $$  \hspace{1cm} (II.9)

and similar for $F^R_{MN}$. These fields are parametrized as

$$ L'_M = \text{Tr} [L_M \sigma^I], \quad R'_M = \text{Tr} [R_M \sigma^I], \quad (II.10) $$

$$ V^I_M = \frac{R'_I L'_M + L'_I R'_M}{2}, \quad A^I_M = \frac{R'_I - L'_I}{2}, $$

$$ X = \frac{1}{2} (S^a \sigma^0 + S^0 \sigma^a) e^{i\pi/2 \sigma^+ + i\eta} $$  \hspace{1cm} (II.11)

where $\sigma^I = (\sigma^0, \sigma^a) = (1, \sigma^a)$ and $\sigma^a$ are the Pauli matrices. In the following analysis we adopt the gauge $L_5 = R_5 = 0$ and the IR boundary condition $F^L_{5M} |_{z_m} = F^R_{5M} |_{z_m} = 0$.

Now, let us include the effects of the nucleon into the model. Here we introduce baryonic sources for the quark number density $\rho_q$ and the baryonic contribution to the isospin number density, denoted by $\rho_I$, through the following term:

$$ S_{\text{int}} = \int d^4x \int z^m dz \left[ V^0_0 \rho_q + V^0_0 \rho_I \right] \delta (z - z_m + \delta z) $$  \hspace{1cm} (II.12)

where $\delta z$ is an infinitesimal length and $V^0_0$ and $V^3_0$ are the gauge fields corresponding to the quark number density and isospin number density. The baryon number density $\rho_B$ is defined as $\rho_B = \rho_q/N_c$. These $\rho_B$ and $\rho_I$ are related with the baryonic condensates as $\rho_B = \langle \bar{\psi} \gamma^0 \bar{\sigma} \psi \rangle$ and $\rho_I = \langle \bar{\psi} \gamma^0 \sigma^a \psi \rangle$. It was assumed that the wave function of the baryon field is expressed as the $\delta$-function which has a peak at a point near the IR boundary [6], which doesn’t modify the IR boundary conditions. Here the gauge corresponding to the vector field is already fixed because a charge is introduced. It should be noted that we assume that the internal energy is not large enough to make a pair creation of a baryon and an anti-baryon. Furthermore, we start from the initial condition where no anti-baryon exists. Then, the present analysis can be applicable only for $|\rho_I| \leq \frac{1}{2} \rho_B$.

The quark number chemical potential $\mu_q$ and the isospin chemical potential $\mu_I$ are introduced as the UV boundary values of the time components of the gauge fields as

$$ V^0_0 |_\epsilon = \mu_q, \quad V^3_0 |_\epsilon = \mu_I $$  \hspace{1cm} (II.13)

The baryon chemical potential is defined as $\mu_B \equiv m_B + N_c \mu_q$ where $m_B$ is the mass of nucleons, $m_B = 939$MeV.
This holographic QCD model involves the following five parameters,
\[ g_5^2, \ z_m, \ m_q, \ \lambda, \ m^2. \]  
(II.16)
To match this model with QCD, the parameter \( g_5^2 \) is adjusted as \[ \frac{1}{g_5^2} = \frac{N_c}{12\pi^2}. \]  
(II.17)
For the physical inputs to determine the parameters, we use the pion mass \( m_\pi = 139.6 \) MeV, the pion decay constant \( f_\pi = 92.4 \) MeV, the \( \rho \) meson mass \( m_\rho = 775.8 \) MeV, and the \( a_0 \) meson mass. As in Ref. \[4\], we use the \( a_0 \) meson mass \( m_{a_0} = 980 \) MeV as a reference value, and see the dependence of our results on the scalar meson mass. The values of the parameters corresponding to \( m_{a_0} = 980 \) MeV are determined as
\[ z_m = 1/(323 \text{ MeV}), \ m_q = 2.29 \text{ MeV}, \ \lambda = 4.4, \ m^2 = 5.39. \]  
(II.18)
As in Ref. \[4\], we assume that the pion condensation phase has the rotational symmetry, \( L_i = R_i = 0 \), and the iso-triplet scalars do not condense, \( S^a = 0 \). Furthermore, we take \( V_0^1 = V_0^2 = 0 \) and \( A_0^3 = \pi^3 = 0 \) which is a set of solutions of the equation of motion (EOM) for these fields. Similarly, the set of the \( \eta = 0 \) and \( A_0^0 = 0 \) satisfies the EOM and we take this solution.

The grandpotential density \( \Omega \) is given from the Lagrangian \( \mathcal{L} \):
\[ \Omega = - \int^z dz \mathcal{L} \]  
(II.19)
where the explicit form of the Lagrangian \( \mathcal{L} \) is shown in Eq. (A.2). One can derive the EOM for the vector field \( V_0^0 \) and \( V_0^3 \) from Eq. (II.19),
\[ \partial_5 \frac{a}{g_5^2} \partial_5 V_0^0 = \rho_q \delta(z - z_m + \delta z), \]
\[ \partial_5 \frac{a}{g_5^2} \partial_5 V_0^3 = \frac{a^3 (S^0)^2}{2} [2 \sin^2 b (V_0 + \theta \sin 2b \sin \zeta)] = \rho_t \delta(z - z_m + \delta z). \]  
(II.20)
By parameterizing \( V_0^0 \) and \( V_0^3 \) as
\[ V_0^0(z) = \mu_q + \varphi^0(z) + g_5^2 \rho_q \int^z_\epsilon d\tilde{z} \tilde{z} \theta(\tilde{z} - z_m + \delta z), \]
\[ V_0^3(z) = \mu_t + \varphi^3(z) + g_5^2 \rho_t \int^z_\epsilon d\tilde{z} \tilde{z} \theta(\tilde{z} - z_m + \delta z) \]  
(II.21)
where \( \theta \) is a step function, Eq. (II.20) is rewritten as
\[ \partial_5 \frac{a}{g_5^2} \partial_5 \varphi^0 = 0, \]
\[ \partial_5 \frac{a}{g_5^2} \partial_5 \varphi^3 = \frac{a^3 (S^0)^2}{2} [2 \sin^2 b (\mu_t + \varphi^3) + \theta \sin 2b \sin \zeta]. \]  
(II.22)
Here the boundary conditions are given by
\[ V_0^{(0,3)} \bigg|_{\epsilon} = \mu_q, \ \partial_5 V_0^{(0,3)} \bigg|_{z_m} = 0 \rightarrow \varphi^{(0,3)} \bigg|_{\epsilon} = 0, \ \partial_5 \varphi^{(0,3)} \bigg|_{z_m} = -g_5^2 z_m \rho_t. \]  
(II.23)

### III. Symmetry Energy and Delay of the Phase Transition

In this section we study the symmetry energy to check whether the present way to introduce the baryonic matter works well in the normal hadron phase by comparing our result with its empirical value. Next, we investigate the dependence of the pion mass on the isospin chemical potential \( \mu_I \) in the normal hadron phase to show the delay of the pion condensation compared with the pure mesonic matter studied in Ref. \[4\]. For studying the hadron phase we set \( b = \theta = 0 \) in the equations of motion in Eqs. (II.20) and (II.22).

We first derive the relation between the chemical potential \( \mu_I \) and the isospin number density \( n_I \). For \( b = \theta = 0 \), it is easy to solve the equation of motion (II.22) with the boundary conditions in Eq. (II.23) to have
\[ \varphi^3 = -\frac{g_5^2 \rho_t}{2} z^2. \]  
(III.1)
Substituting this solution into Eq. (II.19), we obtain
\[ \Omega = - \int^z d\tilde{z} \frac{a}{2g_5^2} (\partial_5 V_0^3)^2 = \rho_t V_0^3 \bigg|_{z_m - \delta z} \]
\[ = \frac{g_5^2 z_m}{2} \rho_t^2 - \mu_t \rho_t. \]  
(III.2)
Minimizing the \( \Omega \) for the \( \rho_t \) for a given value of the isospin chemical potential \( \mu_I \) yields the relation between the isospin chemical potential \( \mu_I \) and the isospin density of the asymmetric matter \( \rho_I \):
\[ \rho_I = \frac{2}{g_5^2 z_m} \mu_I. \]  
(III.3)
Here in the normal hadron phase the isospin density \( n_I \) equals to the density \( \rho_I \) because mesons carrying isospin charge do not condense.

Similarly, for the quark number density we also have
\[ \rho_q = \frac{2}{g_5^2 z_m} \mu_q, \]  
(III.4)
which yields the following relation:
\[ \rho_B = \frac{2}{g_5^2 z_m N_c} (\mu_B - m_B). \]  
(III.5)
The energy density of the system is defined as \( \mathcal{E} = \Omega + n_q \mu_q + n_I \mu_I \) at zero temperature and given by
\[ \mathcal{E} = \frac{g_5^2 z_m^2}{4} \rho_I^2 + \frac{g_5^2 z_m^2 N_c^2}{4} \rho_B^2. \]  
(III.6)
Now, the symmetry energy is obtained as \(^2\)

\[
E_{\text{sym}}(\rho_B) \equiv \frac{\partial (E/\rho_B)}{\partial \alpha^2} \bigg|_{\alpha = 0} = \frac{g_{\pi}^2 \gamma}{32 \rho_B} \rho_B
\]

(III.7)

where \(\alpha \equiv \frac{2\mu_1}{\rho_B}\). At the saturation density \(\rho_0 = 0.16 \text{fm}^{-3}\), we can estimate \(E_{\text{sym}}(\rho_0) = 29\text{MeV}\) by using Eq. (II.17) and Eq. (II.18), which is comparable to the empirical value of \(32.3 \pm 1.0\text{MeV}\) \(^\text{[13]}\). In Refs. \(^\text{[16, 17]}\), the value of the parameter \(\gamma\) defined as \(E_{\text{sym}}(\rho_B) = E_{\text{sym}}(\rho_0) \left(\frac{\rho_B}{\rho_0}\right)^\gamma\) is estimated as \(\gamma = 0.55 - 0.69\), which is different from the result of the present analysis, \(\gamma = 1\). We may understand that the deviations of values of \(\gamma\) and \(L\) are caused by the next leading order in the large \(N_c\) expansion. The slope parameter of the symmetry energy is given by

\[
L \equiv 3\rho_0 \left(\frac{\partial E_{\text{sym}}(\rho_B)}{\partial \rho_B}\right) \bigg|_{\rho_B = \rho_0} = \frac{3\rho_0 g_{\pi}^2 \gamma}{16}
\]

(III.8)

and its value is estimated as \(L = 87\text{MeV}\), where its empirical value is known as \(45.2 \pm 10.0\text{MeV}\) \(^\text{[13]}\).

Let us next study the \(\mu_I\)-dependence of the pion mass. The equations of motion for the pion fluctuation up till the quadratic order in the momentum space are given by

\[
\frac{1}{a(aS^0)^2} \partial_0 \left[a(aS^0)^2 \partial_\gamma \pi^\pm\right] + \left\{ E \mp \mu_I \left(1 - \frac{z^2}{z_m^2}\right) \right\} \left[ A_0^\pm \mp \pi^\pm \left(E \mp \mu_I \left(1 - \frac{z^2}{z_m^2}\right)\right)\right] = 0 , \]

\[
\frac{1}{a(aS^0)^2} \partial_0 \left(\frac{a}{g_5^2} \partial_\gamma A_0^\pm\right) - \left[ A_0^\pm + \pi^\pm \left(E \mp \mu_I \left(1 - \frac{z^2}{z_m^2}\right)\right)\right] = 0
\]

(III.9)

where the fields are parameterized as

\[
\pi^\pm = \frac{\pi^1 \pm i\pi^2}{\sqrt{2}} , \quad A_0^\pm = i A_0^1 \pm i A_0^2 \frac{\sqrt{2}}{\sqrt{2}} . \]

(III.10)

\(S^0\) is the solution of Eq. (A.11) and \(E\) is the energy of the static pion.

Equation (III.9) together with the boundary conditions, \(\pi^\gamma |_{\gamma = 0} = 0\), yield the value of the \(E\) as the eigenvalue. The lowest value of the eigenvalue \(E\) is identified with the pion mass.

Figure II shows the \(\mu_I\) dependence of the pion mass in the normal hadron phase. The \(\pi^+\) mass drawn by the red curve increases with the isospin chemical potential. The \(\pi^-\) mass by the green curve, on the other hand, decreases and reaches zero at \(\mu_I = 235\text{MeV}\), which implies that the \(\pi^-\) condenses and that the transition to the pion condensation phase occurs. We would like to stress that the \(\pi^-\) mass here decreases more slowly than the one obtained in the pure mesonic matter shown by the blue curve. One can easily see that the critical value of the isospin chemical potential for the phase transition is larger than the pion mass for the pure mesonic case. This is due to the existence of the baryons in the matter, which can also be understood by an analysis of the chiral Lagrangian based on the Hidden Local Symmetry as shown in section V.

---

\(^2\) Note that this definition of symmetry energy is different from the one used in Ref. \(^\text{[3]}\).
From the Lagrangian Eq. (A.2), the equations of motion are obtained as
\[
\partial_5 \left( -a^3 \partial_5 S^0 \right) + a^3 S^0 \left( \partial_5 b \right)^2 - 3a^5 S^0 - a^3 S^0 \left[ \sin^2 b \left( \varphi^3 + \mu_I \right)^2 + \theta \sin 2b \sin \zeta \left( \varphi^3 + \mu_I \right) + \theta^2 - \theta^2 \sin^2 b \sin^2 \zeta \right] = 0 ,
\]
\[
\partial_5 \left( -a^3 \left( S^0 \right)^2 \partial_5 b \right) - \frac{a^3 \left( S^0 \right)^2}{2} \left[ \sin 2b \left\{ \left( \varphi^3 + \mu_I \right)^2 - \theta^2 \sin^2 \zeta \right\} + 2\theta \cos 2b \sin \zeta \left( \varphi^3 + \mu_I \right) \right] = 0 ,
\]
\[
\partial_5 \left( \frac{a}{g^2} \partial_5 \theta \right) - \frac{a}{g^2} \partial_5 \zeta = 0 ,
\]
\[
\partial_5 \left( \frac{a}{g^2} \partial_5 \varphi^3 \right) - \frac{a^3 \left( S^0 \right)^2}{2} \left[ \theta \sin 2b \cos \zeta \left( \varphi^3 + \mu_I \right) - \theta^2 \sin^2 b \sin 2\zeta \right] = 0 ,
\]
\[
\partial_5 \left( \frac{a}{g^2} \partial_5 \varphi^3 \right) - \frac{a^3 \left( S^0 \right)^2}{2} \left[ 2 \sin^2 b \left( \varphi^3 + \mu_I \right) + \theta \sin 2b \sin \zeta \right] = 0 .
\]

These differential equations are solved with the boundary conditions listed in Table I. One can derive
\[
\frac{\partial \Omega}{\partial \rho_I} = 0 .
\]

In Fig. 2 we show the resultant relation between the isospin density and the isospin chemical potential obtained by solving Eq. (IV.1). For \(\mu_I < 235 \text{MeV}\) there is no pion condensation, so that the isospin number density increases linearly with the chemical potential following Eq. (IV.2) as drawn by the red curve in Fig. 2. At \(\mu_I = 235 \text{MeV}\) the phase transition occurs from the normal hadron phase to the pion condensation phase. This critical chemical potential \(\mu_I^c = 235 \text{MeV}\) is consistent with the one determined from the pion mass shown in Fig. 1 but the value is larger than the critical value for the pure mesonic matter, for which \(\mu_I = m_\pi \) as seen by the green curve in Fig. 2. This delay of the phase transition is due to the existence of the baryons, which can also be understood by an analysis of the four dimensional chiral Lagrangian as shown in the next section.
in Fig. 2 while the baryonic contribution by the blue curve is almost constant: $\rho_I \sim 0.2 \text{ fm}^{-3}$. As a result the mesonic contribution dominates the isospin number density. This implies that the energy provided by the isospin chemical potential is mostly used for generating the pion condensation rather than converting the neutron into proton.

Figure 3 shows the dependence of the equation of state on the scalar meson mass. The value of parameter $\lambda$ is determined from the mass of the $a_0$ meson, where $\lambda = 1.0, 4.4$ and 100 correspond to the $m_{a_0} = 610, 980$ and 1210 MeV, respectively. We find that the critical value of the isospin chemical potential is independent of $\lambda$ and the behavior of the equation of state is not sensitive to the value of $\lambda$.

As we stated in the introduction, the existence of the isospin chemical potential $\mu_I$ explicitly breaks the chiral symmetry group $SU(2)_R \times SU(2)_L$ down to its subgroup $U(1)^{(3)}_R \times U(1)^{(3)}_L = U(1)^{(3)}_V \times U(1)^{(3)}_A$, where the superscript $(3)$ implies that the generator $T_3$ of $SU(2)$ is used for the $U(1)$ as $\exp[i\theta L T_3] \in U(1)^{(3)}_V$. For studying the order parameters for the phase transition, we define the following $\pi$-condensate and the “$\sigma$”-condensate [4]:

$$\langle \pi^0 \rangle \equiv \frac{1}{2} \text{Tr} \left[ i \sigma^a a \left( \frac{\partial}{\partial \tau} \frac{X}{z} \right) \right]_{\epsilon} = \langle \bar{q} \gamma_5 \sigma^a q \rangle \ ,$$

$$\langle \sigma \rangle \equiv \frac{1}{2} \text{Tr} \left[ a \left( \frac{\partial}{\partial \tau} \frac{X}{z} \right) \right]_{\epsilon} = \langle \bar{q} q \rangle \ . \quad (IV.7)$$

We plot the “$\sigma$”-condensate and the $\pi$-condensate obtained by the present analysis in Fig. 4 together with those condensates for the pure mesonic matter. This figure shows that the present behavior is quite similar to the previous one except the difference of the phase transition point: In the normal hadron phase the “$\sigma$”-condensate exists, which leads to the break down of the $U(1)^{(3)}_A$ symmetry, but $\pi$-condensate is zero. At the phase transition point, the $\pi$-condensate appears, which spontaneously breaks the $U(1)^{(3)}_V$ symmetry, while the “$\sigma$”-condensate starts to decrease very rapidly. For large $\mu_I$, the “$\sigma$”-condensate is almost zero while the $\pi$-condensate keeps increasing.

We next show the chiral circle in Fig. 5. The red solid curve shows that the behavior for the nuclear matter is quite similar to the one for the pure mesonic matter shown by the green dotted line. Although the “$\sigma$”-condensate decreases and the $\pi$-condensate increases, the chiral condensate defined by

$$\hat{\sigma} \equiv \sqrt{\langle \sigma \rangle^2 + \langle \pi^0 \rangle^2} \quad (IV.8)$$

stays constant until about 150 MeV above the critical chemical potential. In the large $\mu_I$ region, the chiral condensate $\hat{\sigma}$ grows very rapidly. This implies that the enhancement of the chiral symmetry breaking occurs in the asymmetric nuclear matter, similarly to the one in the pure mesonic matter as shown in Ref. [4].
V. AN ANALYSIS BY THE CHIRAL LAGRANGIAN BASED ON THE HIDDEN LOCAL SYMMETRY

In this section, we show that the delay of the phase transition to the pion condensation phase is understood as the baryonic matter effect in the framework of the four dimensional chiral Lagrangian including the ρ meson based on the hidden local symmetry (HLS) [10, 11].

The mesonic part of the HLS Lagrangian is given by

\[ \mathcal{L} = F^2 \pi \left[ \alpha_{\mu} \partial^\mu \hat{\alpha} \right] + a F^2 \pi \left[ \alpha_{\mu} \partial^\mu \hat{\alpha} \right] 
+ \frac{F^2}{4} \pi \left[ \xi_L \xi^L_R + \xi_R \xi^L_L \right] - \frac{1}{2g^2} \pi \left[ V_{\mu \nu} V^{\mu \nu} \right] \]

where \( \chi \) is an external field which has the expectation value corresponding to the pion mass, \( \langle \chi \rangle = m_\pi \). The \( \hat{\alpha}_{\mu} \) and \( \hat{\alpha}_{\mu} \) are defined as

\[ \hat{\alpha}_{\mu} = \frac{D_{\mu} \xi_L - \xi_L D_{\mu}}{2t} \]

where \( \xi_{L,R} \) is the fields, including pion, \( V_\mu \) is the gauge field including the rho and omega mesons and the covariant derivative of these fields are

\[ D_{\mu} \xi_L = \partial_{\mu} \xi_L - i V_{\mu} \xi_L + i \xi_L \mathcal{L}_\mu \]
\[ D_{\mu} \xi_R = \partial_{\mu} \xi_R - i V_{\mu} \xi_R + i \xi_R \mathcal{L}_\mu \]

The baryon and isospin chemical potentials, \( \mu_B \) and \( \mu_I \), are introduced as the expectation value of the time component of the external gauge fields: \( \langle L_0 \rangle = \langle R_0 \rangle = \frac{\mu_B \sigma^0 + \mu_I \sigma^3}{2} \).

Here we introduce the following terms including the baryons explicitly:

\[ \hat{N} \gamma^\mu D_{\mu} \hat{N} + G \hat{N} \gamma^\mu \hat{\alpha}_{\mu} \hat{N} \]

where \( \hat{N} \) is the baryon field and \( D_{\mu} \) is a covariant derivative defined as \( D_{\mu} \hat{N} = (\partial_{\mu} - i V_{\mu}) \hat{N} \). We replace the bilinear baryon fields by the mean field as

\[ \left( V^3_0 + G \hat{\alpha}_{\mu} \right) \rho_I + \left( V^0_0 + G \hat{\alpha}_{\mu} \right) \rho_B \]

where \( \hat{\alpha}_{\mu} \) is the time component of the neutral rho meson and \( V^3_0 \) is the time component of the omega meson.

Taking the unitary gauge of the HLS and integrating out the rho and omega mesons and assuming the rotational symmetry, we obtain the following effective Lagrangian for the pion coupling to the baryonic sources:

\[ \mathcal{L} = F^2 \pi \left[ \alpha_{\mu} \partial^\mu \hat{\alpha} \right] + a F^2 \pi \left[ \alpha_{\mu} \partial^\mu \hat{\alpha} \right] 
+ \frac{F^2}{4} \pi \left[ \xi_L \xi^L_R + \xi_R \xi^L_L \right] - \frac{1}{2a^2} \pi \left[ \rho_B \right] - \frac{1}{2a^3} \pi \left[ \rho_B \right] \]

where \( a' = \frac{a}{a - 1} \) and \( \alpha_{\mu} = \alpha_{\mu} + V_\mu \). Existence of the terms in the last line of Eq. (V.5) causes the deviation from the result obtained by the \( O(p^2) \) chiral Lagrangian without the baryonic sources, which delays the transition to the pion condensation comparing to the holographic QCD model (the red curve). The dotted black line corresponds to the case for the pure pion matter, \( a' = 0 \). This figure shows that the point at which the curve reaches zero depends on the value of \( a' \). The critical value of the isospin chemical potential for \( 0 < a' < 1 \) is larger than the pion mass, which implies that delay of the transition is understood by using a model based on the HLS with the baryonic sources.

VI. A SUMMARY AND DISCUSSIONS

We introduced a baryonic source at the IR boundary coupling to the iso-triplet vector meson in the hard wall holographic QCD mode, and studied the pion condensation in the asymmetric nuclear matter. We showed that the phase transition from the normal matter to the pion condensation phase is delayed in the asymmetric nuclear matter compared with the pure mesonic matter. Furthermore, our result shows that the meson contribution to the isospin number density increases with the chemical potential, while the baryon contribution stays constant. This implies that the chiral symmetry breaking is enhanced in the asymmetric nuclear matter as in the pure mesonic matter.

We show the phase diagram obtained from the preset analysis in Fig. 7, where the blue and red area express
the hadron phase and the pion condensation phase respectively. The phase transition is of the second order. There are uncolored area in which our analysis is not applicable because of $|\rho_I| \leq \frac{1}{2} \rho_B$. On the $\mu_I$ axis, $\mu_B = 0$, the phase transition to the pion condensation occurs at which the isospin chemical potential is equal to the pion mass as shown in Ref. [4]. On the other hand, at $\mu_B \neq 0$, done by the present analysis, the critical point of the transition is delayed compared with at $\mu_B = 0$.

Fig. 7: Phase diagram: $\mu_B$ vs. $\mu_I$. The blue and red area express the hadron phase and the pion condensation phase respectively. Our analysis done in this paper is not applicable in the uncolored area.

In the present analysis, we put the baryonic charge at the IR boundary. In more general case, the charge is spread into the bulk by the gauge interaction. Furthermore, the coupling of the baryon to the scalar mesons is not included. Such effects could be included by the holographic mean field approach [18, 19], which is left for future publication.

As we mentioned in the introduction, References [2, 3] studied the asymmetric matter in the hard wall holographic QCD model. Our results for the meson mass splitting and the symmetry energy are comparable to their results.

**ACKNOWLEDGEMENTS**

The authors would like to thank Kenji Fukushima for stimulating discussion on the symmetry structure. This work was supported in part by Grant-in-Aid for Scientific Research on Innovative Areas (No. 2104) “Quest on New Hadrons with Variety of Flavors” from MEXT, and the JSPS Grant-in-Aid for Scientific Research (S) No. 22224003, (c) No. 24540266.

**Appendix A: Equations of motion**

At the vacuum, non zero value of $S^0$ brakes the chiral symmetry to the vector part of its symmetry. The isosinglet scalar field $S^0$ satisfies the following equation of motion (EOM) and the boundary conditions:

$$
\partial_5 a^2 \partial_5 S^0 + 3a^5 S^0 = 0 ,
$$

$$
m_q = \left. \frac{S^0}{z} \right|_c ,
$$

$$
\left[ \partial_5 S^0 + \frac{S^0}{2z_m} \left( \lambda \left( S^0 \right)^2 - 2m^2 \right) \right]_{z_m} = 0 \quad (A.1)
$$

where the $m_q$ corresponds to the explicit braking of the chiral symmetry due to the current quark mass.

Using the assumptions given in section II and the variables parameterized in Eqs. (II.12) and (II.13), the Lagrangian $\mathcal{L}$ is written as

$$
\mathcal{L} = \frac{a^3}{2} \left[ - \left( \partial_5 S^0 \right)^2 - \left( S^0 \right)^2 \left( \partial_5 b \right)^2 \right] + \frac{3a^5}{2} \left( S^0 \right)^2 + \frac{a^3}{2} \left[ \sin^2 b \left( V_0^3 \right)^2 + \theta \sin 2b \sin \zeta V_0^3 + \theta^2 - \theta^2 \sin^2 b \sin^2 \zeta \right] + \frac{a}{2a^3} \left[ \left( \partial_5 V_0^3 \right)^2 + \left( \partial_5 V^3 \right)^2 + \left( \partial_5 \theta \right)^2 + \theta^2 \left( \partial_5 \zeta \right)^2 \right] + \rho q V_0^3 \delta(z - z_m + \delta z) + \rho I V_0^3 \delta(z - z_m + \delta z) \quad (A.2)
$$

where

$$
e^{i z x^a \sigma^a} \equiv \cos b + i \sin b \sigma^3 ,
$$

$$
A_0^a = (\theta \cos \zeta, \theta \sin \zeta, 0) . \quad (A.3)
$$

For convenience, we fixed $\pi^2 = 0$ by using the isospin symmetry $U(1)_I$ which is the subgroup of $U(1)_L^3 \times U(1)_R^3$. 


[1] See, e.g. J. M. Lattimer and M. Prakash, Phys. Rept. 442 (2007) 109-165 and references therein.
[2] P. Demorest, T. Pennucci, S. Ransom, M. Roberts and J. Hessels, Nature 467, 1081 (2010).
[3] J. Antoniadis, P. C. C. Freire, N. Wex, T. M. Tauris, R. S. Lynch, M. H. van Kerkwijk, M. Kramer and C. Bassa et al., Science 340, 6131 (2013).
[4] H. Nishihara and M. Harada, Phys. Rev. D 89, 076001 (2014).
[5] D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 86, 592 (2001).
[6] K. -Y. Kim, S. -J. Sin and I. Zahed, JHEP 0801, 002 (2008).
[7] K. Jo, B. -H. Lee, C. Park and S. -J. Sin, JHEP 1006, 022 (2010).
[8] C. Park, Phys. Lett. B 708, 324 (2012).
[9] B. -H. Lee, S. Mamedov, S. Nam and C. Park, JHEP 1308, 045 (2013).
[10] M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. 164, 217 (1988).
[11] M. Harada and K. Yamawaki, Phys. Rept. 381, 1 (2003).
[12] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95, 261602 (2005).
[13] L. Da Rold and A. Pomarol, Nucl. Phys. B 721, 79 (2005).
[14] L. Da Rold and A. Pomarol, JHEP 0601, 157 (2006).
[15] Z. Zhang and L. -W. Chen, Phys. Lett. B 726, 234 (2013).
[16] L. -W. Chen, C. M. Ko and B. -A. Li, Phys. Rev. Lett. 94, 032701 (2005).
[17] D. V. Shetty, S. J. Yennello and G. A. Souliotis, Phys. Rev. C 76, 024606 (2007) [Erratum-ibid. C 76, 039902 (2007)].
[18] M. Harada, S. Nakamura and S. Takemoto, Phys. Rev. D 86, 021901 (2012).
[19] B. -R. He and M. Harada, Phys. Rev. D 88, no. 9, 095007 (2013).