Thermal leptogenesis in a TeV scale model for neutrino masses

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Abstract
It is known that the radiative neutrino mass model proposed by Ma could be a consistent framework for dark matter, leptogenesis and suppressed lepton flavor violation if a neutral component of the inert doublet is identified as dark matter and the right-handed neutrinos are of $O(10^7)$ GeV or more. In the same model we explore another scenario such that right-handed neutrinos are in TeV regions and their lightest one is dark matter. It is shown that this scenario requires fine mass degeneracy to generate the appropriate baryon number asymmetry as in the case of resonant leptogenesis. As long as we impose the model to induce the baryon number asymmetry on the basis of thermal leptogenesis, we find that dark matter abundance can not be explained. If this scenario is adopted, the model has to be extended to include some new mechanism to explain it.

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1 Introduction

The explanation of the origin of baryon number asymmetry in the universe is one of unsolved issues remained in the standard model (SM) [1]. Although various baryogenesis scenarios at high energy scales have been proposed, we now know that many of them do not work. The generated baryon number asymmetry there is washed out by a sphaleron process in the electroweak interaction unless the $B - L$ symmetry is violated at some high energy scale [2]. Leptogenesis is a promising scenario since the $B - L$ symmetry is supposed to be violated through Majorana masses of right-handed neutrinos in the seesaw mechanism [3]. As long as the right-handed neutrinos are heavy enough, the sufficient lepton number asymmetry can be produced through the out of thermal equilibrium decay of the lightest right-handed neutrino [4]. This lepton number asymmetry is processed to the baryon number asymmetry due to the sphaleron interaction. However, if we apply this scenario to supersymmetric models, the gravitino problem becomes serious [5]. Since reheating temperature required to escape the gravitino problem is too low to produce sufficiently heavy right-handed neutrinos in the thermal equilibrium, the required lepton number asymmetry could not be generated. A lot of models have been proposed to evade this difficulty [6, 7, 8, 9, 10, 11, 12].

The radiative neutrino mass model proposed by Ma [13, 14, 15, 16] and also its supersymmetric extensions [17] are recognized as the models which can closely relate both small neutrino masses and the origin of dark matter (DM). Within the original Ma model, one can consider a simple scenario which simultaneously explains all baryon number asymmetry, correct dark matter abundance and realistic neutrino masses as discussed in [18]. In that scenario, DM is identified with the lightest neutral component of an inert doublet [19] and the mass of the lightest right-handed neutrino is assumed to be of $O(10^7)$ GeV or more. On the other hand, if the right-handed neutrinos are assumed in TeV regions, the same or worse situation discussed above is found in the model. Although the model seems to have several interesting features, the ordinary thermal leptogenesis seems not to work as the generation mechanism of the lepton number asymmetry as long as DM is identified with the lightest right-handed neutrinos. An obstacle to it is just the lightness of the right-handed neutrinos, while it brings interesting features to the model.

In that case nonthermal leptogenesis could be a consistent scenario for the baryon number asymmetry in this kind of model [20, 21]. However, it is still an interesting
subject to study in what situation the thermal leptogenesis could be applicable in this type of model. For example, resonant leptogenesis might work also in this model if the right-handed neutrino masses are finely degenerate \cite{7,8}. In this scenario, resonant effect caused by the mass degeneracy enhances CP asymmetry in the decay of the right-handed neutrino although light right-handed neutrinos require tiny neutrino Yukawa couplings. On the other hand, the washout brought by lepton number violating processes could be suppressed due to these small neutrino Yukawa couplings. As a result, the sufficient amount of lepton number asymmetry can be generated successfully there. Since the radiative neutrino mass model is characterized by the neutrino mass generation mechanism different from the one considered in the ordinary resonant leptogenesis, a new possibility for the thermal leptogenesis is expected to be found.

In this paper, we propose a scenario for thermal leptogenesis in a nonsupersymmetric radiative neutrino mass model\footnote{A study for TeV scale leptogenesis in a different model can be found in \cite{22}.}. The scenario requires mass degeneracy for some fields, which is realized only in this type of model. It enhances the out of thermal equilibrium decay of a right-handed neutrino and also causes the Boltzmann suppression of washout processes of the lepton number symmetry. These features discriminate the scenario from the ordinary resonant leptogenesis. The scenario is expected to be applicable to a supersymmetric version of the model straightforwardly. In that case the gravitino problem could be escaped.

The paper is organized as follows. In section 2 we briefly address the model and assumptions imposed on the neutrino Yukawa couplings and mass spectrum of new fields added to the SM. After that, we discuss the neutrino oscillation parameters and the CP asymmetry obtained in the decay of a right-handed neutrino. We show that the sufficient amount of baryon number asymmetry could be generated in the model through the study of Boltzmann equations relevant to the lepton number asymmetry. In section 3 we address other phenomenological constraints on the model and discuss whether they could be consistent with the parameters required for the explanation of the baryon number asymmetry. We refer to the required modification for the DM scenario and also supersymmetric extension of the model. Section 4 is devoted to the summary.
2 CP asymmetry in a radiative seesaw model

A model considered here is the radiative neutrino mass model studied in [14, 15]. It is an extension of the standard model (SM) with a scalar doublet $\eta$ and three right-handed neutrinos $N^R_i$. They are assumed to have odd parity of a $Z_2$ symmetry, for which all SM ingredients have even parity. The model is characterized by the following $Z_2$ invariant neutrino Yukawa couplings and scalar potential:

$$-L_y = h_{ij} \bar{N}^R_i \eta^\dagger \ell_j + h^*_{ij} \bar{\ell}_j \eta N^R_i + \frac{1}{2} (M_i \bar{N}^R_i N^c_i + M_i \bar{N}^c_i N^R_i),$$

$$V = m^2_{\eta} \phi^\dagger \phi + m^2_{\eta} \eta^\dagger \eta + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\eta^\dagger \eta)^2 + \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\eta^\dagger \phi) (\phi^\dagger \eta) + \lambda_5 (\phi^\dagger \eta)^2 + h.c.,$$

(1)

where $\ell_L_i$ is a lepton doublet and $\phi$ is an ordinary Higgs doublet. Both Yukawa couplings of charged leptons and masses of the right-handed neutrinos are supposed to be real and flavor diagonal. Since $\eta$ is assumed to have no vacuum expectation value, this $Z_2$ symmetry forbids to generate neutrino masses at tree level. Moreover, since the same $Z_2$ symmetry makes the lightest particle with its odd parity stable, it can be DM.

Neutrino masses are generated through one-loop diagrams. They can be expressed as

$$M^\nu_{ij} = \sum_{k=1}^3 h_{ik} h_{jk} \left[ \frac{\lambda_5 \langle \phi \rangle^2}{8\pi^2 M_k} \frac{M_k^2}{M_k^2 - M_\eta^2} \ln \frac{M_k^2}{M_\eta^2} \right] \equiv \sum_{k=1}^3 h_{ik} h_{jk} \Lambda_k,$$

(2)

where $M_\eta^2 = m^2_{\eta} + (\lambda_3 + \lambda_4) \langle \phi \rangle^2$. In the following study, we consider flavor structure of the neutrino Yukawa couplings such as

$$h^N_{ei} = C \delta_{i2} h_i, \quad h^N_{\mu i} = h^N_{\tau i} \equiv h_i \quad (i = 1, 2); \quad h^N_{e3} = h^N_{\mu 3} = -h^N_{\tau 3} \equiv h_3,$$

(3)

where a constant $C$ is supposed to be real, for simplicity. In this case, the neutrino mass matrix is found to take a simple form as

$$M^\nu = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} h_1^2 \Lambda_1 + \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} h_3^2 \Lambda_3 + \begin{pmatrix} C^2 & C & C \\ C & 1 & 1 \\ C & 1 & 1 \end{pmatrix} h_2^2 \Lambda_2.$$

(4)

If $\lambda_5$ takes a small value such as $O(10^{-10})$, these mass eigenvalues can take suitable values for the explanation of neutrino oscillation data [23] even in the case where both $M_i$ and $M_\eta$ are

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3We assume that $\lambda_5$ and $\langle \phi \rangle$ are real and positive.
$\eta$ are of $O(1)$ TeV and $|h_i| = O(1)$. This is the remarkable feature of this model. The flavor structure defined by eq. (3) is very interesting since it automatically induces the tri-bimaximal MNS matrix for $C = 0$ such as [15]

$$U_{MNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix},$$

(5)

where Majorana phases $\alpha_{1,2}$ are expressed as

$$\alpha_1 = \varphi_3, \quad \alpha_2 = \frac{1}{2} \tan^{-1} \left( \frac{|h_1|^2 \Lambda_1 \sin 2\varphi_1 + |h_2|^2 \Lambda_2 \sin 2\varphi_2}{|h_1|^2 \Lambda_1 \cos 2\varphi_1 + |h_2|^2 \Lambda_2 \cos 2\varphi_2} \right),$$

(6)

by using $\varphi_i = \arg(h_i)$.

We find that the mass eigenvalues should satisfy

$$|h_1|^2 \Lambda_1 + |h_2|^2 \Lambda_2 \simeq \frac{\sqrt{\Delta m_{\text{atm}}^2}}{2}, \quad |h_3|^2 \Lambda_3 \simeq \frac{\sqrt{\Delta m_{\text{sol}}^2}}{3},$$

(7)

where $\Delta m_{\text{atm}}^2$ and $\Delta m_{\text{sol}}^2$ stand for squared mass differences required by the neutrino oscillation data [23]. Recent T2K and Double Chooz data suggest a nonzero value for $\theta_{13}$ [24]. We may include it in this model by introducing a nonzero $C$ as a perturbation for eq. (3). In fact, if we assume that $C|h_2|^2 \Lambda_2 \ll |h_1|^2 \Lambda_1$ and $\frac{e^2}{9} |h_2|^2 \Lambda_2 \ll |h_3|^2 \Lambda_3$ are satisfied, the conditions (7) for the neutrino masses are good approximation even in the case with $C \neq 0$. If the model parameters $\lambda_5, M_\eta$ and $M_{1,2,3}$ are fixed to realize eq. (7), it is obvious that the neutrino oscillation data can be explained successfully in the model.

If we take account of lepton flavor violating processes such as $\mu \to e\gamma$, the experimental bounds might require $M_{1,2} < M_3$ [15]. Since the lightest $Z_2$ odd field is stable, either $N_{R_1}$ or a neutral component of $\eta$ could be a DM candidate. The latter possibility has been considered in a lot of articles [19]. In that case the thermal leptogenesis could give appropriate baryon number asymmetry consistently as long as $M_1 > 10^7$ GeV is satisfied [18]. In the present study we adopt the former possibility and assume the following mass spectrum for the $Z_2$ odd fields:

$$M_1 \lesssim M_\eta \lesssim M_2 < M_3,$$

(8)

Neutrino Yukawa couplings are controlled by the constraints in eq. (7). In Fig. 1 we plot the neutrino Yukawa couplings $|h_{1,3}|$ as a function of $M_{1,3}$ by imposing the neutrino
Fig. 1 Neutrino Yukawa couplings $|h_{1,3}|$ which satisfy the neutrino oscillation data. Each line is plotted as a function of $M_{1,3}$ for typical values of $(\lambda_5, M_\eta)$ which are shown in the figures. A GeV unit is used for the mass scale.

oscillation data under the assumption $\sin \theta_{13} = 0$. Here $M_2$ and $|h_2|$ are fixed to typical values such as $M_2 = 1.01 M_\eta$ and $|h_2| = 10^{-3.5}$. As long as $|h_1| \gg |h_2|$ is satisfied, we find that the value of $|h_1|$ is not varied by changing the value of $|h_2|$. This figure shows that larger values of $M_\eta$ require larger values of neutrino Yukawa couplings $|h_{1,3}|$ to satisfy the conditions in eq. (7) when $M_{1,3}$ and $\lambda_5$ are fixed. We note that neutrino Yukawa couplings $|h_{1,3}|$ can take values such as $O(10^{-3})$ when we fix $|\lambda_5|$ in suitable ranges. This is favorable for the thermal leptogenesis as seen below.

Now we consider leptogenesis in this model. If we suppose a situation such that $\eta$ has no lepton number and all the right-handed neutrinos decouple at some TeV region, $B - L$ is conserved below this decoupling temperature as in the canonical leptogenesis.\footnote{One may consider the lepton number $L$ defined as $L(\eta) = 1$ and $L(N_{R_i}) = 0$. In this case the lepton number is violated by the $\lambda_5$ term in eq. (1). Since $\lambda_5$ should be small enough for this term to decouple at $T > 100$ GeV \cite{21}, it seems to be difficult to cause sufficient CP asymmetry in the decay of thermal $N_{R_2}$. Thus, we do not consider it here.} Thus, if the excess of the number density of $N_{R_i}$ over the equilibrium value is caused before the freeze-out of the sphaleron interaction, the baryon number asymmetry $n_B$ is expected to be processed from the lepton number asymmetry $n_L(\equiv n_{\ell} - n_{\bar{\ell}})$ generated through the $N_{R_i}$ decay. If we represent the ratio of baryon number asymmetry $n_B$ to an entropy density $s(\equiv \frac{2\pi^2}{45} g_4 T^3)$ as $Y_B$, it is calculated by using the lepton number asymmetry $Y_L(\equiv \frac{n_\ell}{s})$ as

$$Y_B = \frac{8}{23} Y_L(z_{\text{EW}}), \quad (9)$$

where $z_{\text{EW}}$ is related to the sphaleron decoupling temperature $T_{\text{EW}}$ through $z_{\text{EW}} = \frac{M_2}{T_{\text{EW}}}$.\footnote{4}
In eq. (9), we use $B = \frac{8}{23} (B - L)$ which is satisfied also in this model.

Since $N_{R_1}$ is stable due to the $Z_2$ symmetry, leptogenesis based on the decay of the thermal $N_{R_1}$ is not allowed. Thus, the lepton number asymmetry is expected to be produced through the decay of $N_{R_2}$ whose dominant mode is $N_{R_2} \to \ell \alpha \eta^\dagger$. One might expect the enhancement of CP asymmetry in this decay due to the degenerate mass spectrum [5] as in the ordinary resonant leptogenesis [7, 8]. However, we should note that it can not occur here since $N_{R_1}$ is stable. In this model there are also several dangerous lepton number violating processes which wash out the generated lepton number asymmetry. In the Appendix we present the formulas of the reaction density $\gamma$ relevant to the calculation of $Y_L$. The Boltzmann equations for $Y_{N_{R_2}} (\equiv \frac{n_{N_{R_2}}}{s})$ and $Y_L$ are written as [25, 26]

$$
\frac{dY_{N_{R_2}}}{dz} = - \frac{z}{sH(M_2)} \left( \frac{Y_{N_{R_2}}}{Y_{N_{R_2}}^{eq}} - 1 \right) \left[ \gamma_D^{N_2} + \sum_{i=1,3} (\gamma^{(2)}_{N_2 N_i} + \gamma^{(3)}_{N_2 N_i}) \right],
$$

$$
\frac{dY_L}{dz} = \frac{z}{sH(M_2)} \left[ \varepsilon \left( \frac{Y_{N_{R_2}}}{Y_{N_{R_2}}^{eq}} - 1 \right) \gamma_D^{N_2} - \frac{2Y_L}{Y_{\ell}^{eq}} \left( \frac{\gamma_\eta^2}{4} + \gamma_N^{(2)} + \gamma_N^{(3)} \right) \right],
$$

where $H(M_2) = 1.66 g_*^{-1/2} \frac{M_2^2}{m_p^4}$, $Y_{\ell}^{eq} = \frac{45}{\pi^2 g_*}$ and $Y_{N_{R_2}}^{eq}$ stands for the equilibrium value of $Y_{N_{R_2}}$. In these Boltzmann equations we omit several terms whose contributions are negligible compared with others.

The CP asymmetry $\varepsilon$ induced in the $N_{R_2}$ decay comes from the interference between a tree diagram and one-loop vertex or self-energy diagrams as is well known [6]. It can be

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5 Since $N_{R_{1,3}}$ and $\eta$ have sufficient interactions due to the Yukawa interactions with the couplings $h_{1,3}$ and the weak interaction, they are considered to be in the thermal equilibrium during these evolution.

6 It is useful to note that the lepton number asymmetry might be considered to be generated through the decay of $\eta$ also. However, since $\eta$ is kept in the thermal equilibrium through various interactions, its out of equilibrium decay does not occur and the lepton number asymmetry is not expected to be produced through its decay.
expressed for the mass spectrum (8) as 

$$
\varepsilon = \frac{1}{16\pi} \left[ \frac{3}{4} + \frac{1}{4} \left( 1 - \frac{M^2}{M^2_r} \right)^2 \right] \sum_{i=1,3} \text{Im} \left[ \left( \sum_{k=e,\mu,\tau} h_{k2}^* h_{ki}^* \right)^2 \right] \sum_{k=e,\mu,\tau} h_{k2}^* h_{k2}^* G \left( \frac{M^2_i}{M^2_r}, \frac{M^2_i}{M^2_r} \right) 
\left[ \frac{4}{2 + C^2} |h_1|^2 G \left( \frac{M^2_i}{M^2_r}, \frac{M^2_i}{M^2_r} \right) \sin 2(\varphi_2 - \varphi_1) \right. 
\left. + \frac{C^2}{2 + C^2} |h_3|^2 G \left( \frac{M^2_i}{M^2_r}, \frac{M^2_i}{M^2_r} \right) \sin 2(\varphi_2 - \varphi_3) \right] 
\equiv \varepsilon_1 \sin 2(\varphi_2 - \varphi_1) + \varepsilon_3 \sin 2(\varphi_2 - \varphi_3). \tag{11}
$$

In this formula $G(x, y)$ is expressed as

$$
G(x, y) = \frac{5}{4} F(x, 0) + \frac{1}{4} F(x, y) + \frac{1}{4} (1 - y)^2 \left[ F(x, 0) + F(x, y) \right], \tag{12}
$$

where $F(x, y)$ is defined by

$$
F(x, y) = \sqrt{x} \left[ 1 - y - (1 + x) \ln \left( \frac{1 - y + x}{x} \right) \right]. \tag{13}
$$

The flavor structure (3) is used in this derivation. The magnitude of $\varepsilon$ is determined by the values of $\varepsilon_{1,3}$ and $\sin 2(\varphi_2 - \varphi_{1,3})$. Since the CP asymmetry induced in the decay of $N_{R_2}$ is required to have sufficient magnitude, $|h_{1,3}|$ should not be largely suppressed. Here we should remind that $|h_{1,3}|$ can take wide range values to generate the appropriate neutrino masses by varying a value of $\lambda_5$ as shown in Fig. 1. Because of this feature of the radiative neutrino mass model, the neutrino Yukawa couplings can take appropriate values for the CP asymmetry even for the TeV scale right-handed neutrinos. Numerical values of $|h_{1,3}|$ and $\varepsilon_{1,3}$ are presented in Table 1 for typical values of model parameters.

It is useful to discuss nature of the favorable parameters before presenting the numerical results of the Boltzmann equations. In order to generate the lepton number asymmetry effectively, a sufficient number of $N_{R_2}$ should be successfully produced as the out of thermal equilibrium states. The lepton number violating processes induced by the light right-handed neutrinos should also be sufficiently suppressed keeping the CP asymmetry $\varepsilon$ the appropriate value. We have to choose parameters which satisfy these simultaneously. As such a promising situation, we consider the one where the inverse
The CP asymmetry parameters $\varepsilon_{1,3}$ and baryon number asymmetry $|Y_B|$ predicted for typical parameters which satisfy the constraints from the neutrino oscillation data. Remaining parameters are fixed as $|h_2| = 10^{-3.5}$, $M_3 = 100 M_\eta$, $\Delta_1 = 10^{-5}$ and $\Delta_2 = 10^{-3}$, where $\Delta_i$ is defined by $\Delta_i = \frac{|M_\eta - M_i|}{M_\eta}$.

In the estimation of $|Y_B|$, the maximum value of $|\sin 2(\varphi_2 - \varphi_1)|$ is assumed. A TeV unit is used for the mass scale.

decay of $\eta$ could be the dominant mode for the lepton number violating processes. Such a case is expected to occur for finely degenerate $M_1$, $M_\eta$ and $M_2$. In the following analysis we assume the degenerate masses such as $\Delta_1 = 10^{-5}$ and $\Delta_2 = 10^{-3}$, where $\Delta_i$ is defined by $\Delta_i = \frac{|M_\eta - M_i|}{M_\eta}$.

In order to confirm this, we study the behavior of the ratio of thermally averaged reaction rate $\langle \Gamma(z) \rangle$ to Hubble parameter $H(z)$ for each relevant processes by assuming the above mentioned degenerate mass spectrum. The thermally averaged reaction rate $\langle \Gamma \rangle$ corresponding to the reaction density $\gamma$ included in eq. (10) are given as

$$\langle \Gamma_{N_D} \rangle = \frac{\gamma_{N_D}}{n_{N_{R_2}}^{\text{eq}}}, \quad \langle \Gamma_{ID}^{\eta} \rangle = \frac{\gamma_{ID}^{\eta}}{n_{\ell}^{\text{eq}}}$$

for the decay of $N_{R_2}$ and the inverse decay of $\eta$, and also

$$\langle \Gamma_{N_{(2,13)}}^{(2,13)} \rangle = \frac{\gamma_{N_{(2,13)}}^{(2,13)}}{n_{\ell}^{\text{eq}}}, \quad \langle \Gamma_{N_2N_{(2,3)}}^{(2,3)} \rangle = \frac{\gamma_{N_2N_{(2,3)}}^{(2,3)}}{n_{N_{R_2}}^{\text{eq}}}$$

for the 2-2 scattering processes given in eqs. (24), (25) and eqs. (26), (27), respectively. If both $\langle \Gamma_{ID}^{\eta} \rangle$ and $\langle \Gamma_{N_{(2,13)}}^{(2,13)} \rangle$ are of $O(1)$ at a neighborhood of $z \sim 1$, we find that the favorable situation for the leptogenesis could be realized for the assumed degenerate masses. In Fig. 2, we plot $\langle \Gamma \rangle$ as a function of $z$ for each process. This figure shows that all the lepton number violating processes could be out of thermal equilibrium at the period $1 < z < 6$. Although these dangerous processes are mediated by rather light right-handed fields, we find that the mass degeneracy could play a crucial role to make them ineffective.
due to the Boltzmann suppression. It could compensate the disfavored effect brought by the lightness of the right-handed neutrinos. It should be noted that the suppression of the washout effect of the lepton number asymmetry in this model is brought as the combined effect of the degenerate masses and the neutrino Yukawa couplings of $O(10^{-3})$.

Here we give our numerical results for the generated baryon number asymmetry in the present model. In Fig. 3, we show the evolution of $Y_{N_{R2}}$, $\Delta_{N_{R2}} (\equiv |Y_{N_{R2}} - Y_{N_{R2}}^{eq}|)$ and $|Y_L|$ which are obtained by solving the Boltzmann equations (10) for the cases (b) and (d) listed in Table 1. In this calculation we assume that $\sin 2(\varphi_2 - \varphi_1)$ takes a maximum value, for simplicity. The figure shows that $|Y_L|$ reaches a constant value before the sphaleron decoupling if we consider the sphaleron decoupling temperature as $T_{EW} \sim 140$ GeV. This temperature $T_{EW}$ corresponds to the Higgs mass such as 125 GeV [27], which could be considered as the promising one from the recent ATLAS and CMS data. The obtained $Y_B$ through this analysis is also listed in Table 1 for each case. Their order is correct but the values are a little bit smaller than the one expected from the big bang nucleosynthesis. However, we should remind that they are obtained for the very simple flavor structure of neutrino Yukawa couplings. The model could also give nonzero values for $\sin \theta_{13}$ if $C \neq 0$ is assumed. The predicted value of $|\sin \theta_{13}|$ for each case is also listed in Table 1. These results could be changed by modifying the flavor structure of neutrino Yukawa couplings. If we adopt other type of flavor structure which can be consistent with the neutrino oscillation data, desired values for both $Y_B$ and $|\sin \theta_{13}|$ might be derived within this framework. Although this study is an interesting subject, it is beyond the scope of this paper.
This leptogenesis scenario is characterized by the requirement that the masses of $N_{R_1,2}$ and $\eta$ should be finely degenerate. This situation may be considered similar to the resonant leptogenesis. However, the required mass degeneracy is much milder and also has different nature compared with the one of the resonant leptogenesis where the required mass degeneracy among the right-handed neutrinos is smaller than $10^{-10}$ [7]. We note that this difference is caused by the neutrino mass generation mechanism, especially, the existence of the small coupling $\lambda_5$. Since the same Yukawa couplings contribute to both the generation of the lepton number asymmetry and its washout, some extra suppression for the washout processes is required to keep the lepton number asymmetry in the favorable range. In the present model, this effect could be brought through the mass degeneracy of the relevant fields which suppress both the relevant decay and also the lepton number violating scattering processes. As long as the reheating temperature satisfies $T > M_2$, the $N_{R_2}$ could be in the thermal equilibrium. Thus, we find that rather low reheating temperature such as $10^5$ GeV could make this leptogenesis scenario applicable.

3 Phenomenological constrains

We have seen that the suitable lepton number asymmetry could be thermally generated in this radiative neutrino mass model for a rather low reheating temperature. In this section we study this possibility further by taking account of other phenomenological constraints in a quantitative way.
First, we start to address a problem relevant to the CP phase, which eventually appears when we consider the leptogenesis. In general, the interactions introduced to generate the neutrino masses could also contribute to the electric dipole moment of an electron (EDME) through one-loop diagrams if they violate the CP invariance. However, as long as we confine the additional interaction for leptons to the one given in eq. (1), we find that the EDME is not induced at one-loop level but can be induced through a two-loop diagram with internal $W^\pm$ lines. Even if the Majorana phases $\alpha_{1,2}$ given in eq. (6) are assumed to take a maximum value, the EDME ($d_e/e$) induced by such a diagram is roughly estimated to be $O(G_F^2 m_e^3)$. It is much smaller than the present experimental upper bound [28]. Thus, the constraint from the EDME does not contradict with the present leptogenesis scenario.

Second, the lepton flavor violating processes such as $\ell_i \rightarrow \ell_j \gamma$ are induced through one loop diagrams similar to the ones for the neutrino masses. However, since they are irrelevant to $\lambda_5$ unlike the neutrino masses, large contributions could be generated from them. In fact, the relevant branching ratio can be expressed as [15]

$$\begin{align*}
\text{Br}(\mu \rightarrow e \gamma) &\simeq \frac{3\alpha}{64\pi(G_F M_\eta^2)^2} \left[ C|h_2|^2 F_2 \left( \frac{M_2^2}{M_\eta^2} \right) + |h_3|^2 F_2 \left( \frac{M_3^2}{M_\eta^2} \right) \right]^2, \\
\text{Br}(\tau \rightarrow \mu \gamma) &\simeq \frac{0.51\alpha}{64\pi(G_F M_\eta^2)^2} \left[ (|h_1|^2 + |h_2|^2) F_2 \left( \frac{M_1^2}{M_\eta^2} \right) - |h_3|^2 F_2 \left( \frac{M_3^2}{M_\eta^2} \right) \right]^2,
\end{align*}$$

(16)

where $F_2(r)$ is defined as

$$F_2(r) = \frac{1 - 6r + 3r^2 + 2r^3 - 6r^2 \ln r}{6(1 - r)^4}. \quad (17)$$

In this derivation we use $M_1 \sim M_2$. It is known that these processes severely constrain the model if the neutrino Yukawa couplings take values of $O(1)$ which are convenient for the explanation of the relic abundance of the lightest right-handed neutrino. However, the neutrino Yukawa couplings have rather small values such as $O(10^{-3})$ in the cases of Table 1, eq. (16) gives negligibly small values compared with the present experimental upper bounds [29].

Finally, we examine the consistency of the scenario with the DM relic abundance. Since $N_{R_1}$ is the lightest $Z_2$ odd particle, it is stable to be DM. Thus, its relic abundance should satisfy $\Omega_{N_{R_1}} h^2 = 0.11$ which is obtained from observations of the WMAP [30]. Here we remind that $N_{R_1}$ interacts with other fields only through the neutrino Yukawa coupling.
Although it is required to have values of $O(1)$ to realize the favored value of $\Omega_{N_{R_1}} h^2$ as shown in the previous work [14, 15], the above study suggests that it should be much smaller to explain both the neutrino oscillation data and the baryon number asymmetry in the universe. The relic abundance of $N_{R_1}$ is so large to overclose the universe in the present scenario.

This problem might be solved by introducing some interaction which makes $N_{R_1}$ unstable but induces no other substantial effect. As such a simple example, we may consider the gravity induced $Z_2$ violating interaction similar to the Weinberg operator [31] such as

$$\mathcal{L}_V = \frac{f_i}{M_{pl}} (\ell_i^T \phi^*) (\eta^\dagger \ell_i) + \text{h.c.}. \quad (18)$$

This interaction brings three body decay $N_{R_1} \rightarrow \bar{\ell} \ell \ell$ for $N_{R_1}$. In this case the lifetime of $N_{R_1}$ is estimated as

$$\tau_{N_{R_1}} \simeq 3 \times 10^{16} \left( \frac{1 \text{ TeV}}{M_1} \right) \left( \frac{10^{-3}}{|h_1|} \right)^2 \text{sec} \quad (19)$$

for $f_i = O(1)$. Since this lifetime is shorter than the age of universe, $N_{R_1}$ can not be DM and then we need to introduce a new DM candidate. If we consider the embedding of the thermal leptogenesis in this radiative neutrino mass model in the simple way, the model seems to lose the close relation between the neutrino masses and the existence of DM generally. We order several comments on this point.

As long as we confine our consideration to the thermal leptogenesis in this radiative neutrino mass model, we can only explain two of three experimental results which suggest physics beyond the SM, that is, the neutrino oscillation data and the DM relic abundance as discussed in [14, 15, 16], or the neutrino oscillation data and the baryon number asymmetry as studied here. In the former case, we need to find some new generation mechanism of the baryon number asymmetry. The promising scenario is nonthermal leptogenesis, which has been discussed in [20, 21].

In the latter case, we need to modify the model so as to include some new scenario for DM. We discuss two scenarios here. The first one is to introduce additional interaction which contribute to the pair annihilation of $N_{R_1}$. As such an example, one might consider an flavor blind abelian gauge interaction at TeV regions, which induces $s$-channel pair

\footnote{Since $M_1$ are degenerate with $M_2$ and $M_\eta$, the coannihilation of $N_{R_1}$ with $N_{R_2}$ and $\eta$ should be taken into account. However, even in that case the neutrino Yukawa couplings $h_{1,2}$ are too small to reduce its relic abundance sufficiently.}
annihilation of $N_{R_1}$. This type of model has been discussed in [15]. Unfortunately, this extension seems to make the thermal leptogenesis useless, since the same interaction also contributes the pair annihilation of $N_{R_2}$ which keeps $N_{R_2}$ in the thermal equilibrium until larger $z$. Another promising solution for this issue is to introduce an interaction $y_i S N^c_{R_i} N_{R_i}$ with a $Z_2$ even scalar field $S$, which has been studied in other context in [16]. If Yukawa couplings satisfy $|y_1| \gg |y_{2,3}|$, this could mainly contribute to the pair annihilation of $N_{R_1}$ through s-channel exchange of $S$. It could reduce the relic abundance of $N_{R_1}$ substantially due to the resonance as long as the mass of $S$ satisfies the condition $m^2_S \simeq 4M^2_1$. Detailed study of this issue will be given elsewhere.

The second one is to construct a supersymmetric version of the present model. It might be done straightforwardly following the prescription shown in [17]. Thermal leptogenesis is expected to be formulated along the similar line to the present model. An interesting point in this extension is that the artificial hierarchy assumed for various couplings and masses might be derived on the basis of symmetry. In fact, if we introduce an anomalous U(1) symmetry at high energy regions, the required hierarchical structure of the couplings and masses might also be generated via its spontaneous breaking as discussed in [17]. This extension may also open a new possibility for DM. Since $N_{R_1}$ can be unstable there, the lightest neutralino could be dominant component of DM. In this extended model, we might have a supersymmetric model with no gravitino problem since the required reheating temperature could be $10^5$ GeV as shown here. These extensions of the model might give a simultaneous explanation within the thermal leptogenesis framework for the three crucial problems in the SM, that is, the baryon number asymmetry in the universe, the small neutrino masses consistent with the neutrino oscillation data, and the DM relic abundance.

4 Summary

The radiative neutrino mass model has been originally proposed as the model which could give the consistent explanation for the neutrino masses and mixings and also the relic abundance of DM on the basis of TeV scale physics. If we assume the simple flavor structure for the neutrino Yukawa couplings in this model, the tri-bimaximal neutrino mixing and the suppression of lepton flavor violating processes such as $\mu \to e\gamma$ can be
easily derived. However, unfortunately, it is not so easy to embed the thermal leptogenesis in this model because of the lightness of right-handed neutrinos. In this paper we have studied under what condition thermal leptogenesis could be applicable in the model with the right-handed neutrinos of $O(1)$ TeV mass for the explanation of the baryon number asymmetry in the universe. Our result is that the finely degenerate mass spectrum of $Z_2$ odd fields could allow the model to generate the sufficient baryon number asymmetry by thermal leptogenesis, although the masses of right-handed neutrinos are not huge but of $O(1)$ TeV. Since the relevant right-handed neutrino is light, the reheating temperature could be less than $10^5$ GeV. This suggests that the supersymmetric extension of the model could escape the notorious gravitino problem.

As shown in this paper, unfortunately, the close relation between the neutrino masses and the existence of DM is lost if we try to embed the thermal leptogenesis in the model. The neutrino Yukawa couplings required by the thermal leptogenesis are too small to reduce the relic abundance of the DM candidate $N_{R_1}$. We need some extension of the model to resolve this problem by introducing suitable interaction for $N_{R_1}$ which makes $N_{R_1}$ unstable or contributes to the $N_{R_1}$ annihilation. Anyway, an interesting point is that the extension discussed in this paper could give a closely correlated explanation for recently clarified phenomena relevant to physics beyond the SM. They seem to deserve further study.

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8 We should remind that the original Ma model can be consistent with the thermal leptogenesis if the the lightest neutral component of the inert doublet $\eta$ is identified as DM [18]. However, the right-handed neutrino should be of $(10^7)$ GeV or more in that scenario.
Appendix

In this appendix we give the formulas of the reaction density of the relevant processes. Since $N_{R1,3}$ and $\eta$ are considered to be in the thermal equilibrium due to the Yukawa interactions and other interactions, we need to study the Boltzmann equations only for the number density of $N_{R2}$ and the lepton number asymmetry as discussed in the text. For the processes relevant to their evolution, we can refer to the reaction density given in [26]. However, interaction terms of $\eta$ and $N_{R2}$ are restricted by the $Z_2$ symmetry as shown in eq. (1). It causes large difference from the ordinary seesaw leptogenesis. We need to modify them by taking account of the features of the present model such that $\eta$ has a large mass comparable with the one of $N_{R1,2}$ and the neutrino Yukawa couplings have the flavor structure given in eq. (3). In order to give the expression for the reaction density of the relevant processes, we introduce dimensionless variables

$$x = \frac{s}{M_2^2}, \quad a_j = \frac{M_j^2}{M_2^2}, \quad a_\eta = \frac{M_\eta^2}{M_2^2},$$

where $s$ is the squared center of mass energy.

The reaction density for the decay of $N_{R2}$ and $\eta$ can be expressed as

$$\gamma_D^{N_2} = \frac{|h_2|^2(2 + C^2)}{8\pi^3} M_2^4 (1 - a_\eta)^2 \frac{K_1(z)}{z},$$
$$\gamma_D^\eta = \frac{|h_1|^2}{8\pi^3 a_\eta^{3/2}} M_\eta^4 \left(1 - a_1 a_\eta \right)^2 \frac{K_1(\sqrt{a_\eta z})}{z},$$

where $K_1(z)$ is the modified Bessel function of the second kind. The reaction density for the scattering processes are expressed as

$$\gamma(ab \rightarrow ij) = \frac{T}{64\pi^4} \int_{s_{\text{min}}}^\infty ds \hat{\sigma}(s) \sqrt{s} K_1 \left(\frac{\sqrt{s}}{T}\right),$$

where $s_{\text{min}} = \max[(m_a + m_b)^2, (m_i + m_j)^2]$ and $\hat{\sigma}(s)$ is the reduced cross section. In order to express the reduced cross section for the scattering processes relevant to eq. (20), we define the quantities as

$$\frac{1}{D_1(x)} = \frac{1}{x - a_1}, \quad \frac{1}{D_3(x)} = \frac{x - a_3}{(x - a_3)^2 + a_3 c_3}, \quad c_3 = \frac{9a_3}{16\pi^2} |h_3|^4,$$
$$\lambda_{ij} = \left[ x - (\sqrt{a_i} + \sqrt{a_j})^2 \right] \left[ x - (\sqrt{a_i} - \sqrt{a_j})^2 \right],$$
$$L_{ij} = \ln \left[ \frac{x - a_i - a_j + 2a_\eta + \sqrt{\lambda_{ij}}}{x - a_i - a_j + 2a_\eta - \sqrt{\lambda_{ij}}} \right],$$
$$L'_{ij} = \ln \left[ \frac{\sqrt{x}(x - a_i - a_j - 2a_\eta) + \sqrt{\lambda_{ij}(x - 4a_\eta)}}{\sqrt{x}(x - a_i - a_j - 2a_\eta) - \sqrt{\lambda_{ij}(x - 4a_\eta)}} \right].$$

(23)
The relevant reduced cross section are summarized as follows.

As the lepton number violating scattering processes induced through the \( N_{R_1,3} \) exchange, we have

\[
\hat{\sigma}^{(2)}_N(x) = \frac{1}{2\pi} \left[ 4|h_1|^4 \frac{(x - a_\eta)^2 a_3}{x^3} \left\{ \frac{x^2}{xa_1 - a_\eta^2} + \frac{2x}{D_1(x)} + \frac{(x - a_\eta)^2}{2D_1(x)^2} \right\} \right. \\
- \left\{ \frac{x^2}{(x - a_\eta)^2} \left( 1 + \frac{2(x + a_1) - 4a_\eta}{D_1(x)} \right) \ln \left( \frac{x(x + a_1 - 2a_\eta)}{xa_1 - a_\eta^2} \right) \right\} \\
+ \left\{ 9|h_3|^4 \frac{(x - a_\eta)^2 a_3}{x^3} \left\{ \frac{x^2}{xa_3 - a_\eta^2} + \frac{2x}{D_3(x)} + \frac{(x - a_\eta)^2}{2D_3(x)^2} \right\} \right. \\
- \left\{ \frac{x^2}{(x - a_\eta)^2} \left( 1 + \frac{2(x + a_3) - 4a_\eta}{D_3(x)} \right) \ln \left( \frac{x(x + a_3 - 2a_\eta)}{xa_3 - a_\eta^2} \right) \right\} \right] (24)
\]

for \( \ell_\alpha \eta \rightarrow \bar{\ell}_\beta \eta \) and also

\[
\hat{\sigma}^{(13)}_N(x) = \frac{1}{2\pi} \left[ 4|h_1|^4 \left\{ \frac{a_1 x(x - 4a_\eta)}{a_1 x + (a_1 - a_\eta)^2} \right\} \right. \\
+ \left\{ \frac{a_3 x(x - 4a_\eta)}{a_3 x + (a_3 - a_\eta)^2} \right\} \\
+ \left\{ 9|h_3|^4 \frac{a_3 x(x - 4a_\eta)}{a_3 x + (a_3 - a_\eta)^2} \right\} \ln \left( \frac{a_1 x + (a_1 - a_\eta)^2}{x + 2a_1 - 2a_\eta} \right) \right] (25)
\]

for \( \ell_\alpha \ell_\beta \rightarrow \eta \eta \). Here we note that cross terms are cancelled because of the assumed flavor structure [3]. Although there are other lepton number violating processes \( N_{R_i}N_{R_j} \rightarrow \ell_\alpha \ell_\beta \) induced through the \( \eta \) exchange, they can be safely neglected in the analysis due to the additional suppression caused by the smallness of \( \lambda_5 \).

As the lepton number conserving scattering processes which contribute to determine the number density of \( N_{R_2} \), we have

\[
\hat{\sigma}^{(2)}_{N_1N_2}(x) = \frac{1}{4\pi} \left[ 2(2 + C^2)|h_1|^2|h_2|^2 \sqrt{\lambda_{12}} x \left( 1 + \frac{(a_1 - a_\eta)(a_2 - a_\eta)}{(a_1 - a_\eta)(a_2 - a_\eta) + xa_\eta} \right) \right. \\
+ \left\{ a_1 + a_2 - 2a_\eta \frac{L_{12}}{x} \right\} - 4\text{Re}[\{h_1^*h_2\}] \frac{2\sqrt{\lambda_{12}^2}}{x - a_1 - a_2 + 2a_\eta}, \right. \\
\hat{\sigma}^{(2)}_{N_3N_2}(x) = \frac{1}{4\pi} \left[ 3(2 + C^2)|h_3|^2|h_2|^2 \sqrt{\lambda_{32}} x \left( 1 + \frac{(a_3 - a_\eta)(a_2 - a_\eta)}{(a_3 - a_\eta)(a_2 - a_\eta) + xa_\eta} \right) \right. \\
+ \left\{ a_3 + a_2 - 2a_\eta \frac{L_{32}}{x} \right\} - C^2\text{Re}[\{h_3^*h_2\}] \frac{2\sqrt{\lambda_{32}^2}}{x - a_3 - a_2 + 2a_\eta} \right] (26)
\]
for $N_{R_1}N_{R_2} \to \ell_\alpha \bar{\ell}_\beta$ which are induced through the $\eta$ exchange and also

\begin{align*}
\hat{\sigma}_{N_1N_2}^{(3)}(x) &= \frac{1}{\pi} \left| |h_1|^2 |h_2|^2 \right| \left\{ \frac{(x - 4a_\eta)}{x^{1/2}} \sqrt{\lambda_{12}} \left( \frac{\sqrt{\lambda_{12}}}{x} \right) - 2 \right. \\
&\quad + \frac{4a_\eta(a_1 - a_2)^2}{(a_\eta - a_1)(a_\eta - a_2)x + (a_1 - a_2)^2a_\eta} + \left( 1 - 2\frac{a_\eta}{x} \right) L'_{12} \right\} \\
&\quad - \text{Re}[\langle h_1^* h_2^2 \rangle \left( \frac{\sqrt{\lambda_{12}}}{x} + \frac{2(a_2^2 - a_1 a_2) L'_{12}}{(x^2 - 4 x a_\eta)^{1/2}(x - a_1 - a_2 - 2a_\eta)} \right)], \\
\hat{\sigma}_{N_3N_2}^{(3)}(x) &= \frac{C^2}{4\pi} \left[ |h_3|^2 |h_2|^2 \right] \left\{ \frac{(x - 4a_\eta)}{x^{1/2}} \sqrt{\lambda_{32}} \left( \frac{\sqrt{\lambda_{32}}}{x} \right) - 2 \right. \\
&\quad + \frac{4a_\eta(a_3 - a_2)^2}{(a_\eta - a_3)(a_\eta - a_2)x + (a_3 - a_2)^2a_\eta} + \left( 1 - 2\frac{a_\eta}{x} \right) L'_{12} \right\} \\
&\quad - \text{Re}[\langle h_3^* h_2^2 \rangle \left( \frac{\sqrt{\lambda_{32}}}{x} + \frac{2(a_2^2 - a_3 a_2) L'_{32}}{(x^2 - 4 x a_\eta)^{1/2}(x - a_3 - a_2 - 2a_\eta)} \right)] \quad (27)
\end{align*}

for $N_{R_1}N_{R_2} \to \eta\eta^\dagger$ which are induced through the $\ell_\alpha$ exchange. It may be useful to note that the cross terms in these reduced cross sections become zero if the maximum CP phases are assumed as $\sin 2(\varphi_2 - \varphi_{1,3}) = 1$. 

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