ISOLATED PHOTONS WITHOUT FRAGMENTATION CONTRIBUTION

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I illustrate how to define an isolated-photon cross section which is independent of the parton-to-photon fragmentation contribution, it is infrared safe to all orders in perturbative QCD, and it is exclusive in the kinematical variables of the photon and of the accompanying jets. This isolation prescription is applicable to any kind of polarized or unpolarized hard collisions. It can also be used without any modifications in the case of two or more isolated photons in the final state. I present results for one-photon production in hadronic collisions, and I compare them with existing results obtained in the framework of the conventional cone approach.

1 Introduction

The production of photons in hard collisions is a valuable tool to study the interactions of the elementary constituents of the nuclear matter. Some of the intricacies related to the dependence upon long-distance effects, which heavily influence the study of single-inclusive hadron production, are avoided. In hadronic collisions, a much smaller number of partonic subprocesses is involved in photon production with respect to jet production. Besides allowing a relatively clean test of the theoretical predictions, especially in $e^+e^-$ collisions, photon production has proven to be very important in pinning down the gluon density in the proton in an intermediate $x$ range, thus providing complementary information to that from DIS.

It is customary to ascribe the production of photons in hard collisions to two different mechanisms. In the direct process, the photon enters the partonic hard scattering, characterized by a large energy scale. In the fragmentation process, a QCD parton (quark or gluon) fragments non-perturbatively into a photon, at a scale of the order of the typical hadronic mass. In the latter case, all the unknowns of the fragmentation mechanism are collected into two functions (the quark-to-photon and gluon-to-photon fragmentation functions) which, although universal, must be determined by comparison with the data, and are not calculable in perturbative QCD. Loosely speaking, one can experimentally define direct photons as those which are well isolated from the final state hadrons, and photons produced via fragmentation as those which lie inside hadronic jets. However, from the theoretical point of view neither the direct photon cross section (resulting from all the Feynman diagrams with a photon leg) nor the fragmented photon cross section (obtained by convoluting the QCD parton cross section with a bare parton-to-photon fragmentation function) are separately well defined, being divergent order-by-order in perturbation theory; it is only their sum which is divergence-free and can play the role of a physical observable.

In high-energy collisions, the study of photon production is complicated by the background due to hadrons decaying into photons (mainly, $\pi^0 \rightarrow \gamma\gamma$). It is well known that the signal-to-background ratio is enhanced by applying the so-called isolation condition: the tagged photon is required to be far away from any energetic hadron. To be consistent with experimental measurements, the theoretical predictions must implement the isolation condition as well. Regardless of the specific isolation prescription, in perturbative QCD it is not possible to separate sharply the photon from the partons; in fact, this would constrain the phase space of soft gluons, thus spoiling the cancellation of infrared divergences which is crucial in order to get a sensible cross section. Two methods have been devised to tackle this problem. In the cone approach, a cone is drawn around the photon axis; if only a small hadronic energy (compared to the photon energy) is found inside the cone, the partons accompanying the photon are clustered with a given jet-finding algorithm. In the democratic approach, the photon is treated as a parton as far as the jet-finding algorithm is concerned. At the end of the clustering procedure, the configuration corresponds to an isolated photon event only if the ratio of the hadronic energy found inside the jet containing the photon over the total energy of the jet itself is smaller than a fixed amount, usually of the order of 10%.

In both the cone and the democratic approaches, the fragmentation mechanism does contribute to the cross section, although to a lesser extent with respect to the
case of non-isolated photon production. This is inconvenient if one aims to study the underlying dynamics, because of the large uncertainties introduced by the very poorly known fragmentation functions.

In this talk, based on ref. [1], I will show that it is possible to modify the cone approach in order to get a cross section which only depends upon the direct process. I argue that this prescription is infrared safe at any order in perturbative QCD. The definition of the isolated-photon cross section is given in section 2. Section 3 presents phenomenological applications for the case of isolated-photon production at hadronic colliders. The conclusions are reported in section 4.

2 Isolation prescription

I will now sketch the main ideas which allow to define an isolated-photon cross section that does not depend upon the fragmentation contribution. The key observation is that the fragmentation mechanism in QCD is a purely collinear phenomenon; therefore, in order to cancel the contribution of the fragmentation functions, it is sufficient to veto all the kinematical configurations where a parton is collinear to the photon. However, this must be accomplished without spoiling the cancellation of the infrared singularities due to soft gluon emission. These two conditions are seemingly incompatible: indeed, the latter amounts to the requirement that no region of forbidden radiation be present in the phase space, which is exactly what is needed in order for the first condition to be fulfilled. Therefore, since the cancellation of singularities is mandatory to get an infrared-safe cross section, one has to relax the first condition: instead of vetoing the collinear configurations, we can try to suppress them. Indeed, this can be achieved in the following way (I restrict for the moment to the case of $e^+ e^-$ collisions). A cone of (fixed) half-angle $\delta_0$ is drawn around the photon axis. Then, for all $\delta \leq \delta_0$, the total amount of hadronic energy $E_{\text{tot}}(\delta)$ found inside the cone of half-angle $\delta$ drawn around the photon axis is required to fulfill the following condition

$$E_{\text{tot}}(\delta) \leq K \delta^2,$$

(1)

where $K$ is some energy scale (the form $K \delta^2$ is chosen for illustrative purposes; it will be generalized in the following). According to eq. (1), a soft gluon can be arbitrarily close to the photon, and the cancellation of infrared poles is not spoiled. On the other hand, eq. (1) implies that the energy of a parton emitted exactly collinear to the photon must vanish. Therefore, fragmentation process does not contribute to the cross section, being restricted to the zero-measure set $z = 1$. Consistently, the quark-photon collinear singularities in the direct part are also cancelled, by effect of the damping associated with the energy of the quark getting soft.

The isolation prescription given above can now be refined and extended to any kind of hard collisions. To this purpose, I consider the class of scattering events whose final state contains a set of hadrons, labelled by the index $i$, with four-momenta $k_i$, and a hard photon with four-momentum $k_\gamma$. After fixing the parameter $\delta_0$, which defines the isolation cone, the following procedure (isolation cuts) is applied.

1. For each $i$, evaluate the angular distance $R_{i\gamma}$ between $i$ and the photon. The angular distance is defined, in the case of $e^+ e^-$ collisions, to be

$$R_{i\gamma} = \delta_{i\gamma},$$

(2)

where $\delta_{i\gamma}$ is the angle between the three-momenta of $i$ and $\gamma$. In the case of hadronic collisions I define instead

$$R_{i\gamma} = \sqrt{(\eta_i - \eta_\gamma)^2 + (\varphi_i - \varphi_\gamma)^2},$$

(3)

where $\eta$ and $\varphi$ are the pseudorapidity and azimuthal angle respectively.

2. Reject the event unless the following condition is fulfilled

$$\sum_i E_i \theta(\delta - R_{i\gamma}) \leq \mathcal{X}(\delta) \text{ for all } \delta \leq \delta_0,$$

(4)

where $E_i$ is the energy of hadron $i$ and, due to $\theta(\delta - R_{i\gamma})$, the sum gets contribution only from those hadrons whose angular distance from the photon is smaller than or equal to $\delta$. The function $\mathcal{X}$, which plays the rôle of $K\delta^2$ in eq. (1), is fixed and will be given in the following. The function $\mathcal{X}$ must vanish when its argument tends to zero, $\mathcal{X}(\delta) \rightarrow 0$ for $\delta \rightarrow 0$. At hadron colliders, the transverse energy $E_{\text{tr}}$ must be used instead of $E_i$.

3. Apply a jet-finding algorithm to the hadrons of the event (therefore, the photon is excluded). This will result in a set of $m + m'$ bunches of well-collimated hadrons, which I denote as candidate jets. $m$ ($m'$) is the number of candidate jets which lie outside (inside) the isolation cone, in the sense of the angular distance defined by eqs. (2) or (3).

4. Apply any other additional cuts to the photon and to the $m$ candidate jets which lie outside the cone (for example, the cut over the minimum observable (transverse) energy of the jets must be applied here).

An event which is not rejected when the isolation cuts are applied is by definition an isolated-photon plus
m-jet event. The key point in the above procedure is step 2: hadrons are allowed inside the isolation cone, provided that eq. (5) is fulfilled. This in turn implies the possibility for a candidate jet to be inside the isolation cone. It would not make much sense to define a cross section exclusive in the variables of such a jet, which cannot be too hard. For this reason, in the physical observable that I define here, the jets which accompany the photon are the candidate jets outside the isolation cone which also pass the cuts of step 4. The resulting cross section is therefore totally exclusive in the variables of these jets and of the photon, and inclusive in the variables of the hadrons found inside the isolation cone.

In order to be definite, I choose
\[ \mathcal{X}(\delta) = E_\gamma \epsilon_\gamma \left( \frac{1 - \cos \delta}{1 - \cos \delta_0} \right)^n, \tag{5} \]
where \( E_\gamma \) is the photon energy (in the case of hadron collisions, \( E_\gamma \)) must be replaced by the transverse energy of the photon, \( E_{\gamma T} \), and \( \epsilon_\gamma \), and \( n \) are positive numbers of order one. As will be discussed in the following, the choice of the value of these parameters is arbitrary to a very large extent. The fact that \( n > 0 \) guarantees that
\[ \lim_{\delta \to 0} \mathcal{X}(\delta) = 0. \tag{6} \]
Furthermore, we have
\[ \mathcal{X}(\delta) \neq 0 \quad \text{if} \quad \delta \neq 0. \tag{7} \]

The information contained in eqs. (5) and (6) are sufficient to investigate the infrared properties of the isolated-photon observables. I remind the reader that in QCD any jet cross section is easily written in terms of measurement functions. Given a \( N \)-parton configuration \( \{k_i\}_i=1^N \), the application of a jet-finding algorithm results in a set of \( M \) jets with momenta \( \{q_a\}_a=1^M \). This can be formally expressed by the measurement function
\[ \mathcal{S}_N \left( \{q_a\}_a=1^M; \{k_i\}_i=1^N \right), \tag{8} \]
which embeds the definition of the jet four-momenta in terms of the parton four-momenta. It has been shown that, at next-to-leading order and for an arbitrary type of collisions, the infrared-safeness requirement on the cross section can be formulated in terms of conditions relating the measurement functions \( \mathcal{S}_N \) for different \( N \) (see for example refs. [4,5]). These conditions can be extended without any difficulties to higher perturbative orders. Here, I stress that the measurement function in eq. (8) implements an infrared-safe jet cross section definition, which I will apply to the partons accompanying the photon in a candidate isolated-photon event. By labeling the partons in such a way that
\[ R_{i\gamma} \geq R_{j\gamma} \quad \text{if} \quad i > j, \tag{9} \]
I define
\[ \mathcal{S}_{\gamma,N} \left( \{k_{\gamma a}\}_a=1^M; \{k_i\}_i=1^N \right) = \mathcal{S}_N \left( \{q_a\}_a=1^M; \{k_i\}_i=1^N \right) \times \prod_{i=1}^N \mathcal{I}_i, \tag{10} \]
\[ \mathcal{I}_i = \theta \left( \mathcal{X} \left( \min \{R_{i\gamma}, \delta_0\} \right) - \sum_{j=1}^{i-1} E_j \theta (\delta_0 - R_{j\gamma}) \right). \tag{11} \]

It is easy to understand that eq. (11) is equivalent to the isolation cuts described above. In particular, the quantity \( \prod_{i=1}^N \mathcal{I}_i \) is equivalent to step 2. Therefore, \( \mathcal{S}_{\gamma,N} \) is the measurement function relevant for the isolated-photon plus jets cross section: it vanishes when applied to those parton configurations where the photon is non-isolated. From the point of view of the proof of the infrared safeness of the cross section, the case of isolated photon plus jets is almost identical to the case of jet production. In particular, the various functions \( \mathcal{S}_{\gamma,N} \) with different \( N \) must fulfill a given set of conditions. This issue has been discussed in details in ref. [4]. It is straightforward to see that eq. (11) indeed fulfills the requirements given in ref. [4] for this to hold, the properties given in eqs. (5) and (6) are crucial. The isolation condition presented in this paper therefore induces an isolated-photon cross section which is formally infrared safe to all orders in perturbative QCD. Notice that eqs. (5) and (6) are the only conditions that the function \( \mathcal{X} \) must fulfill in order for the isolation prescription to define an infrared-safe cross section. In other words, the specific form of the function \( \mathcal{X} \) is not important in what discussed here, provided that eqs. (5) and (6) are satisfied.

However, a word of caution is necessary. Although the cross section is formally infrared safe, it has to be stressed that the isolation cuts have an impact on the local subtraction of singularities. It is therefore conceivable that, at some order in perturbation theory, the isolation condition may result into a divergent cross section. To the best of my knowledge, no proof has been given that a (formally) infrared-safe cross section is also free to all orders in perturbation theory of the divergences possibly induced by the isolation cuts. This issue is discussed in some details in ref. [4]. In that paper, it is shown that the isolation prescription given above defines a divergent-free cross section at least at next-to-leading order in QCD, for hadron-hadron, photon-hadron and e+e− collisions. It is also argued that no problem should arise at higher orders in perturbation theory.

3 Photon production in hadron-hadron collisions

In this section, I will present next-to-leading order predictions for isolated-photon production at hadron colliders,
adopting the isolation prescription discussed in section 2. Eqs. (10) and (11) can be straightforwardly used to construct a Monte Carlo program as described in ref. 9. The resulting code outputs the kinematical variables of the partons and of the photon plus a suitable weight. The isolation condition and the jet-finding algorithm are implemented at the very last step of the computation.

It has been known since a long time that next-to-leading order corrections are necessary in order to sensibly compare data with theoretical predictions. In particular, it has been shown in ref. 10 that a consistent next-to-leading order treatment of both the direct and the fragmentation part in the conventional isolation prescription allows to reasonably describe the data on isolated-photon production at the Tevatron. However, there are indications that pure QCD can not describe both the Tevatron and the SpS data. It would be interesting to understand whether this discrepancy is related to the particular isolation cuts chosen; this can be done by comparing the theoretical and experimental results obtained by imposing a different isolation criterion, like the one presented in ref. 3 and in this talk. No data are presently available which correspond to the cuts discussed in section 2. However, as a preliminary step it is mandatory to study the perturbative stability of the resulting cross section, the size of radiative corrections (by comparing the leading order result with the next-to-leading order result) and the dependence upon the renormalization ($\mu_R$) and factorization ($\mu_F$) scales. As far as the isolation condition is concerned, I used the function $X$ given in eq. (5), fixing $\epsilon = 1$ and varying the parameters $n$ and $\delta_0$ in the ranges $0.5 \leq n \leq 4$ and $0.3 \leq \delta_0 \leq 1$ respectively. Other functional forms for $X$ have been adopted as well, with results comparable to those obtained with eq. (5). As expected from the general arguments given in ref. 3. As expected from the general arguments given in ref. 3, a reasonable choice for the parameters (which minimizes the scale dependence of both the single-inclusive and double-differential observables in the largest possible range) requires $n \approx 1$ and $\delta_0$ much larger than the half-angle of the isolation cone used in the conventional cone prescription. To compare with results in the literature, I therefore choose $n = 1$ and $\delta_0 = 1$. The scale dependence of the photon $p_T$ spectrum presented in refs. 10, 11 is obtained by setting $\mu_R = \mu_F$, and amounts to a variation of about 10% with respect to the default curve. Remarkably, this is also the scale dependence which one obtains by adopting the isolation prescription given here. It has to be stressed that, at the Tevatron energy, the $\mu_R = \mu_F$ scale dependence of the Born result is smaller than that of the next-to-leading order result. However, this is not the indication of a failure of the perturbative expansion as it might seem. Indeed, this fact arises from an incidental cancellation between the effects due to the $\mu_R$ and $\mu_F$ variations. If the two scales are varied independently, and the results are eventually combined, it turns out that the next-to-leading order results is (although mildly) more stable than the leading order one, as can be seen from fig. 1. The $\mu_F$ dependence is sizably reduced when going from leading to next-to-leading order, while the $\mu_R$ dependence stays almost the same: this is due to
the isolation cuts, which perturb the cancellation of the soft-gluon effects and therefore have an impact on the renormalization scale dependence. If \( \mu_R \) and \( \mu_F \) are varied independently, the overall scale dependence amounts to a variation of about 20% with respect to the default result. We can therefore conclude that the isolation prescription given here induces a reasonably stable cross section in perturbation theory; the results are comparable to those obtained with the conventional cone isolation prescription.

Since the next-to-leading order predictions are perturbatively stable, the possibility can be considered of using high-energy isolated-photon data to extract the gluon density at smaller \( x \) with respect to the average \( x \) obtained at fixed target experiments. This issue is investigated in fig. 2. From the figure, we see that the \( qg \) channel contribution is dominant, as is customary also with other isolation prescriptions. However, the span induced by varying the parton densities is smaller than the uncertainty due to scale dependence. Therefore, it appears that isolated-photon data at Tevatron can not be used to severely constrain the gluon density in the proton.

4 Conclusions

I presented a definition for the isolation of a photon from surrounding hadrons which is based on a modified cone approach. The resulting cross section does not get any contribution from the uncalculable parton-to-photon fragmentation functions; still, it is infrared safe to all orders in perturbative QCD and thus defines a physical observable. The isolation prescription can be applied to any kind of hard (polarized or unpolarized) scattering process, as well as to the case of several isolated photons in the final state. I presented phenomenological results for one-photon plus jets observables in hadron-hadron collisions at high energy. The next-to-leading order cross section displays a good perturbative stability. The scale dependence is of the same order of that which is obtained with conventional isolation prescriptions.

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