Hadronization properties of $b$ quarks compared to light quarks in $e^+e^- \rightarrow q\bar{q}$ from 183 to 200 GeV

DELPHI Collaboration

Abstract

The DELPHI detector at LEP has collected 54 pb$^{-1}$ of data at a centre-of-mass energy around 183 GeV during 1997, 158 pb$^{-1}$ around 189 GeV during 1998, and 187 pb$^{-1}$ between 192 and 200 GeV during 1999. These data were used to measure the average charged particle multiplicity in $e^+e^- \rightarrow b\bar{b}$ events, $\langle n \rangle_{b\bar{b}}$, and the difference $\delta_m$ between $\langle n \rangle_{b\bar{b}}$ and the multiplicity, $\langle n \rangle_{l\bar{l}}$, in generic light quark (u,d,s) events:

$$
\begin{align*}
\delta_m(183 \text{ GeV}) &= 4.55 \pm 1.31 \text{(stat)} \pm 0.73 \text{(syst)} \\
\delta_m(189 \text{ GeV}) &= 4.43 \pm 0.85 \text{(stat)} \pm 0.61 \text{(syst)} \\
\delta_m(200 \text{ GeV}) &= 3.39 \pm 0.89 \text{(stat)} \pm 1.01 \text{(syst)} .
\end{align*}
$$

This result is consistent with QCD predictions, while it is inconsistent with calculations assuming that the multiplicity accompanying the decay of a heavy quark is independent of the mass of the quark itself.

(Phys. Lett. B479(2000)118; erratum Phys. Lett. B492(2000)398)
N.I.Zimin, M.Verlato, A.Van Lysebetten
34
39
38
32
45
37
31
30
27
25
14
11
20
19
18
16
13
22

47
52
51
54
50
49
48
47
12
50
49
48
47
12

Department of Physics and Astronomy, Iowa State University, Ames IA 50011-3160, USA

1Department of Physics and Astronomy, Iowa State University, Ames IA 50011-3160, USA
2Physics Department, Univ. Instelling Antwerpen, Universiteitsplein 1, B-2610 Antwerpen, Belgium
and HE, ULB-VUB, Pleinlaan 2, B-1050 Brussels, Belgium
and Faculté des Sciences, Univ. de l’Etat Mons, Av. Maistriau 19, B-7000 Mons, Belgium

3Physics Laboratory, University of Athens, Solonos Str. 104, GR-10680 Athens, Greece

4Department of Physics, University of Bergen, Allégaten 55, NO-5007 Bergen, Norway

5Dipartimento di Fisica, Università di Bologna in INFN, Via Irnerio 40, IT-40126 Bologna, Italy

6Centro Brasileiro de Pesquisas Físicas, rua Xavier Sigaud 150, BR-22290 Rio de Janeiro, Brazil
and Depto. de Física, Pont. Univ. Católica, C.P. 38071 BR-22453 Rio de Janeiro, Brazil

7Comenius University Faculty of Mathematics and Physics, Mlynska Dolina, SK-84215 Bratislava, Slovakia

8Collège de France, Lab. de Physique Corpusculaire, IN2P3-CNRS, FR-75231 Paris Cedex 05, France

9CERN, CH-1211 Geneva 23, Switzerland

10Institut de Recherches Subatomiques, IN2P3 - CNRS/ULP - BP20, FR-67037 Strasbourg Cedex, France

11Now at DESY-Zeuthen, Platanenallee 6, D-15735 Zeuthen, Germany

12Institute of Nuclear Physics, N.C.S.R. Demokritos, P.O. Box 60228, GR-15310 Athens, Greece

13FZU, Inst. of Phys. of the C.A.S. High Energy Physics Division, Na Slovance 2, CZ-180 40, Praha 8, Czech Republic

14Dipartimento di Fisica, Università di Genova e INFN, Via Dodecaneso 33, IT-16146 Genova, Italy

15Institut des Sciences Nucléaires, IN2P3-CNRS, Université de Grenoble 1, FR-38026 Grenoble Cedex, France

16Helsinki Institute of Physics, HIF, P.O. Box 9, FI-00100 Helsinki, Finland

17Joint Institute for Nuclear Research, Dubna, Head Post Office, P.O. Box 79, RU-101 000 Moscow, Russian Federation

18Institut für Experimentelle Kernphysik, Universität Karlsruhe, Postfach 6980, DE-76128 Karlsruhe, Germany

19Institute of Nuclear Physics and University of Mining and Metalurgy, Ul. Kawiory 26a, PL-30055 Krakow, Poland

20Université de Paris-Sud, Lab. de l’Accélérateur Linéaire, IN2P3-CNRS, Bât. 200, FR-91405 Orsay Cedex, France

21School of Physics and Chemistry, University of Lancaster, Lancaster LA1 4YB, UK

22LIP, IST, FCUL - Av. Elias Garcia, 14-1º, PT-1000 Lisboa Cedex, Portugal

23Department of Physics, P.O. Box 147, Liverpool L69 3BX, UK

24LPNHE, IN2P3-CNRS, Univ. Paris VI et VII, Tour 33 (رده), 4 place Jussieu, FR-75252 Paris Cedex 05, France

25Department of Physics, University of Lund, Sölvegatan 14, SE-223 63 Lund, Sweden

26Université Claude Bernard de Lyon, IPNL, IN2P3-CNRS, FR-69622 Villeurbanne Cedex, France

27Univ. d’Aix - Marseille II - CPP, IN2P3-CNRS, FR-13288 Marseille Cedex 09, France

28Dipartimento di Fisica, Università di Milano e INFN-MILANO, Via Celoria 16, IT-20133 Milan, Italy

29Dipartimento di Fisica, Univ. di Milano-Bicocca and INFN-MILANO, Piazza delle Scienze 2, IT-20126 Milan, Italy

30Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

31IPNP of MFF, Charles Univ., Areal MFF, V Holesovickach 2, CZ-180 00, Praha 8, Czech Republic

32NIKHEF, Postbus 41882, NL-1090 DB Amsterdam, The Netherlands

33National Technical University, Physics Department, Zografou Campus, GR-15773 Athens, Greece

34Physics Department, University of Oslo, Blindern, NO-1000 Oslo 3, Norway

35Dpto. Física, Univ. Oviedo, Avda. Calvo Sotelo s/n, ES-33007 Oviedo, Spain

36Department of Physics, University of Oxford, Keble Road, Oxford OX1 3RH, UK

37Dipartimento di Fisica, Università di Padova e INFN, Via Marzolo 8, IT-35131 Padua, Italy

38Rutherford Appleton Laboratory, Chilton, Didcot OX11 0QX, UK

39Dipartimento di Fisica, Università di Roma II e INFN, Tor Vergata, IT-00133 Rome, Italy

40Dipartimento di Fisica, Università di Roma III e INFN, Via della Vasca Navale 84, IT-00146 Rome, Italy

41DAPNIA/Service de Physique des Particules, CEA-Saclay, FR-91911 Gif-sur-Yvette Cedex, France

42Fisica de Cantabria, Avda. los Castros s/n, ES-39006 Santander, Spain

43Dipartimento di Fisica, Università degli Studi di Roma La Sapienza, Piazzale Aldo Moro 2, IT-00185 Rome, Italy

44Inst. for High Energy Physics, Serpukov P.O. Box 35, Protvino, (Moscow Region), Russian Federation

45J. Stefan Institute, Jamova 39, SI-1000 Ljubljana, Slovenia and Laboratory for Astroparticle Physics, Nova Gorica Polytechnic, Kostanjevitska 16a, SI-5000 Nova Gorica, Slovenia, and Department of Physics, University of Ljubljana, SI-1000 Ljubljana, Slovenia

46Fysikum, Stockholm University, Box 6730, SE-113 85 Stockholm, Sweden

47Dipartimento di Fisica Sperimentale, Università di Torino and INFN, Via P. Giuria 1, IT-10125 Turin, Italy

48Dipartimento di Fisica, Università di Trieste and INFN, Via A. Valerio 2, IT-34127 Trieste, Italy

49Inst. of Física, Universidade de Udine, IT-33100 Udine, Italy

50Univ. Federal do Rio de Janeiro, C.P. 68528 Cidade Univ., Ilha do Fundão BR-21945-970 Rio de Janeiro, Brazil

51IFIC, Valencia-CSIC, and D.F.A.M.N., U. de Valencia, Avda. Dr. Moller 50, ES-46100 Burjasot (Valencia), Spain

52Institut für Hochenergiephysik, Österr. Akad. d. Wissensch., Nikolsdorfergasse 18, AT-1050 Vienna, Austria

53Inst. Nuclear Studies and University of Warsaw, Ul. Hoza 69, PL-00681 Warsaw, Poland

54Fachbereich Physik, University of Wuppertal, Postfach 100 127, DE-42097 Wuppertal, Germany
1 Introduction

The study of the properties of the fragmentation of heavy quarks compared to light quarks offers new insights in perturbative QCD. Particularly important is the difference in charged particle multiplicity between light quark and heavy quark initiated events in $e^+e^-$ annihilations.

In a first approximation one could expect that the multiplicity of hadrons produced in addition to the possible decay products of the primary quark-antiquark is a universal function of the available invariant mass; this would give a difference in charged particle multiplicity between light quark and heavy quark initiated events decreasing with the centre-of-mass energy $E_{cm}$ [1]. QCD predicts, somehow counter-intuitively, that this difference is energy independent; this is motivated by mass effects on the gluon radiation (see [2,3,4] and [5] for a recent review).

The existing experimental tests were not conclusive (see [2] and references therein, [6,7,8,9]). At LEP 2 energies, however, the difference between the QCD prediction and the model ignoring mass effects is large, and the experimental measurement can firmly distinguish between the two hypotheses.

2 Analysis and Results

A description of the DELPHI detector can be found in [10]; its performance is discussed in [11].

Data corresponding to a luminosity of 54 pb$^{-1}$ collected by DELPHI at centre-of-mass (c.m.) energies around 183 GeV during 1997, to 158 pb$^{-1}$ collected around 189 GeV during 1998, and to 187 pb$^{-1}$ collected between 192 and 200 GeV during 1999, were analysed.

The 1999 data were taken at different energies: 25.8 pb$^{-1}$ at 192 GeV, 77.4 pb$^{-1}$ at 196 GeV and 83.8 pb$^{-1}$ at 200 GeV. Each energy was analyzed separately and the results were then combined as described later and attributed to a c.m. energy of 200 GeV.

A preselection of hadronic events was made, requiring at least 10 charged particles with momentum $p$ above 100 MeV/c and less than 1.5 times the beam energy, with an angle $\theta$ with respect to the beam direction between 20° and 160°, a track length of at least 30 cm, a distance of closest approach to the interaction point less than 4 cm in the plane perpendicular to the beam axis and less than $(4/\sin\theta)$ cm along the beam axis, a relative error on the momentum measurement $\Delta p/p < 1$, and a total transverse energy of the charged particles above 0.2$E_{cm}$.

The influence of the detector on the analysis was studied with the full DELPHI simulation program, DELSIM [11]. Events were generated with PYTHIA 5.7 and JETSET 7.4 [12], with parameters tuned to fit LEP1 data from DELPHI [13]. The Parton Shower (PS) model was used. The particles were followed through the detailed geometry of DELPHI giving simulated digitisations in each subdetector. These data were processed with the same reconstruction and analysis programs as the real data.

The hadronic cross-section for $e^+e^-$ interactions above the Z peak is dominated by radiative $q\bar{q}\gamma$ events; the initial state radiated photons (ISR photons) are generally aligned along the beam direction and not detected. In order to compute the hadronic c.m. energy, the procedure described in [14] was used. In this procedure particles are clustered into jets and the effective centre-of-mass energy of the hadronic system, $\sqrt{s'}$, is computed as being the invariant mass of the system recoiling against an ISR photon, possibly unseen.
Events with reconstructed hadronic c.m. energy \( \sqrt{s} \) above 0.9\( E_{cm} \) were used. The selected 1997 (1998, 1999) data sample consisted of 1699 (4583, 4881) hadronic events.

For each year’s data, two samples enriched in (1) \( b-\) events and in (2) \( uds-\) events were selected from the \( b \) tagging variable \( y \) defined as in Ref. \[1\]; this variable represents essentially the probability that none of the tracks in the event comes from a vertex separated from the primary one. To select the samples of the type (2), it was required in addition that the narrow jet broadening \( B_{\text{min}} \) is smaller than 0.065, to reduce the background due to WW and ZZ events. \( B_{\text{min}} \) is defined as follows. The event is separated into two hemispheres \( H_1 \) and \( H_2 \) with respect to the thrust axis, defined by the thrust unit vector \( \hat{t} \). Then, calling \( p_k \) the momentum vector of the \( k \)-th particle,

\[
B_{\text{min}} = \min_{i=1,2} \frac{\sum_{k \in H_i} |\vec{p}_k \times \hat{t}|}{2 \sum_k |\vec{p}_k|}.
\]

The contamination from non-\( q\bar{q} \) events in the samples of type (1) was 7% (8%, 15%), while it was 13% (17%, 20%) in the samples of type (2). After applying the event selection criteria and the cuts to reduce the WW and ZZ background, the purities were approximately 91% (90%, 90%) \( (b- \) events) over the total \( q\bar{q} \) in sample (1), and 79% (79%, 79%) \( (uds- \) events) over the total \( q\bar{q} \) in sample (2). The fractions of \( q \)-type quarks in the \( (i) \)-th sample, \( f_q^{(i)} \), were determined from the simulation. The sample (1) consisted of 103 (326, 416) events; the sample (2) of 590 (1450, 1652) events.

The average charge multiplicity was measured in the samples (1) and (2), after subtracting the background bin-by-bin by means of the simulation. It should be noted that the average multiplicity for a given flavour \( q \) in each sample is equal to \( C_q^{(i)} \times \langle n \rangle_{q\bar{q}} \), with \( C_q^{(i)} \neq 1 \) in general. The factors \( C_q^{(i)} \) account for biases introduced by the application of the \( b \) probability and the jet broadening cuts, as well as for detector effects; these factors were computed by means of the simulation.

A third sample (3) was taken into account by considering the measurement of multiplicity described in \[15\]. This measurement was performed from a sample of 1297 (3444, 3648) hadronic events, with a contamination of 11% (14%, 18%) after applying all the selection criteria; the remaining background mostly comes from the hadronic decay of W and Z pairs. The values \( \langle n \rangle^{(3)} \) shown in Table \[1\] are fully corrected for these backgrounds and for detector effects with their statistical errors; hence the nominal quark flavour ratios appear in the equation (3) below. The systematic errors are reported as the last contribution in Table \[3\].

The measured mean multiplicities together with the event probability cuts and the factors \( f_q^{(i)} \) and \( C_q^{(i)} \) are shown in Table \[1\]. For the 1999 data, the values only at \( \sqrt{s} = 200 \) \( \text{GeV} \) are tabulated.

In each of the three samples, the average multiplicity \( \langle n \rangle \) is a linear combination of the unknowns \( \langle n \rangle_{b\bar{b}} \), \( \langle n \rangle_{l\bar{l}} \) and \( \langle n \rangle_{c\bar{c}} \). One can thus formulate a set of three simultaneous equations to compute these unknowns:

\[
\begin{align*}
\langle n \rangle^{(1)} &= f_b^{(1)} C_b^{(1)} \langle n \rangle_{b\bar{b}} + f_{uds} C_{uds} \langle n \rangle_{l\bar{l}} + f_c^{(1)} C_c^{(1)} \langle n \rangle_{c\bar{c}} , \\
\langle n \rangle^{(2)} &= f_b^{(2)} C_b^{(2)} \langle n \rangle_{b\bar{b}} + f_{uds} C_{uds} \langle n \rangle_{l\bar{l}} + f_c^{(2)} C_c^{(2)} \langle n \rangle_{c\bar{c}} , \\
\langle n \rangle^{(3)} &= f_b^{(3)} \langle n \rangle_{b\bar{b}} + f_{uds} \langle n \rangle_{l\bar{l}} + f_c^{(3)} \langle n \rangle_{c\bar{c}} .
\end{align*}
\]

Solving the above equations gave the following mean charge multiplicities at 183 GeV:

\[
\begin{align*}
\langle n \rangle_{b\bar{b}}(183\text{ GeV}) &= 29.79 \pm 1.11 , \\
\langle n \rangle_{c\bar{c}}(183\text{ GeV}) &= 29.41 \pm 4.05 ,
\end{align*}
\]
Data at 183 GeV

| Sample | b-tag prob. | $f_b^{(i)}$ | $C_b^{(i)}$ | $f_{uds}^{(i)}$ | $C_{uds}^{(i)}$ | $f_c^{(i)}$ | $C_c^{(i)}$ | $\langle n \rangle^{(i)}$ |
|--------|-------------|-------------|-------------|----------------|----------------|-------------|-------------|----------------|
| (1) $P_E < 0.00001$ | 0.914 | 0.921 | 0.017 | 1.24 | 0.069 | 0.903 | 27.43 ± 0.83 |
| (2) $0.2 < P_E < 1.0$ | 0.019 | 0.912 | 0.786 | 0.899 | 0.195 | 0.901 | 23.53 ± 0.33 |
| (3) no cut | 0.162 | – | 0.582 | – | 0.256 | – | 27.05 ± 0.27 |

Data at 189 GeV

| Sample | b-tag prob. | $f_b^{(i)}$ | $C_b^{(i)}$ | $f_{uds}^{(i)}$ | $C_{uds}^{(i)}$ | $f_c^{(i)}$ | $C_c^{(i)}$ | $\langle n \rangle^{(i)}$ |
|--------|-------------|-------------|-------------|----------------|----------------|-------------|-------------|----------------|
| (1) $P_E < 0.00001$ | 0.899 | 0.912 | 0.016 | 1.15 | 0.085 | 0.919 | 27.75 ± 0.48 |
| (2) $0.2 < P_E < 1.0$ | 0.016 | 0.896 | 0.789 | 0.893 | 0.195 | 0.913 | 23.93 ± 0.24 |
| (3) no cut | 0.161 | – | 0.580 | – | 0.259 | – | 27.47 ± 0.18 |

Data at 200 GeV

| Sample | b-tag prob. | $f_b^{(i)}$ | $C_b^{(i)}$ | $f_{uds}^{(i)}$ | $C_{uds}^{(i)}$ | $f_c^{(i)}$ | $C_c^{(i)}$ | $\langle n \rangle^{(i)}$ |
|--------|-------------|-------------|-------------|----------------|----------------|-------------|-------------|----------------|
| (1) $P_E < 0.00001$ | 0.880 | 0.928 | 0.026 | 1.11 | 0.094 | 0.881 | 27.31 ± 0.71 |
| (2) $0.2 < P_E < 1.0$ | 0.017 | 0.867 | 0.785 | 0.900 | 0.199 | 0.921 | 23.64 ± 0.37 |
| (3) no cut | 0.159 | – | 0.579 | – | 0.262 | – | 27.52 ± 0.29 |

Table 1: Mean multiplicities, $\langle n \rangle$, in three event samples of different flavour content, $f_q$, and correction factors $C_q$. The errors quoted on $\langle n \rangle$ are statistical only. The last dataset contains only the data at 200 GeV from 1999.

$$\langle n \rangle_{\bar{b}b}(183 \text{ GeV}) = 25.25 \pm 1.35,$$

$$\delta_{bl}(183 \text{ GeV}) = 4.55 \pm 1.31,$$

with correlation coefficient of $-0.45$ between $\langle n \rangle_{\bar{b}b}$ and $\langle n \rangle_{\bar{t}t}$, and at 189 GeV:

$$\langle n \rangle_{\bar{b}b}(189 \text{ GeV}) = 30.53 \pm 0.70,$$

$$\langle n \rangle_{c\bar{c}}(189 \text{ GeV}) = 28.63 \pm 2.81,$$

$$\langle n \rangle_{\bar{t}t}(189 \text{ GeV}) = 26.10 \pm 0.97,$$

$$\delta_{bl}(189 \text{ GeV}) = 4.43 \pm 0.85,$$

with correlation coefficient of $-0.52$ between $\langle n \rangle_{\bar{b}b}$ and $\langle n \rangle_{\bar{t}t}$.

From the 1999 data, the results obtained for each energy are tabulated in Table 1. The values were scaled to 200 GeV using JETSET and then a weighted average was calculated using the inverse of the square of the statistical error as weight. One obtains

$$\langle n \rangle_{\bar{b}b}(200 \text{ GeV}) = 29.38 \pm 0.65,$$

$$\langle n \rangle_{c\bar{c}}(200 \text{ GeV}) = 29.89 \pm 2.92,$$

$$\langle n \rangle_{\bar{t}t}(200 \text{ GeV}) = 25.99 \pm 1.03,$$

$$\delta_{bl}(200 \text{ GeV}) = 3.39 \pm 0.89,$$

with average correlation coefficient of $-0.52$ between $\langle n \rangle_{\bar{b}b}$ and $\langle n \rangle_{\bar{t}t}$. The difference between the average of the values rescaled to 200 GeV and the average of the values without the scaling was added in quadrature to the final systematic error. This difference is anyway small (0.16 units for $\langle n \rangle_{\bar{b}b}$ and less than 0.01 units for $\delta_{bl}$).

The relatively large uncertainty of the measured mean multiplicities for charm stems from the inability of the $P_E$ variable to extract a $c$-enriched sample of events.

It should be noted that the transition between particle and detector level measurements in equations (1) and (2) is done via multiplicative factors $C$ applied to the mean value
The validity of this procedure requires that the simulation used to compute the $C$ values reproduces the real data well; the $\chi^2$/DF at centre of mass energies of 183, 189 and 200 GeV are respectively 0.81, 1.17 and 0.67 for the sample (1) and 0.93, 1.44 and 1.36 for the sample (2).

The analysis was repeated with different cuts applied to the $b$-tag probability, $P_E$, and the results for the $\delta_{bl}$ were found to be quite stable (see Figure 1). A systematic error was evaluated as half of the difference between the greatest and the smallest multiplicity values obtained from varying the cut on $P_E$ from $0.5 \times 10^{-5}$ to $1.5 \times 10^{-5}$.

The uncertainty due to the event selection in sample (2) was investigated by repeating the analysis after variation of the narrow jet broadening cut, from 0.05 to 0.08. Half of the differences between the greatest and the smallest multiplicities were added in quadrature to the systematic error previously calculated. The propagated systematic error in the total multiplicity in equation (3) from [15] was also added in quadrature to the systematic error. Uncertainties arising from the modelling of short-lived particles in the simulation were considered. The main physics sources of these uncertainties come from the assumed lifetime of B-hadrons ($\tau_B = 1.564 \pm 0.014$ ps) [16], and the $D^+$, $D^0$ lifetimes and production rates [16]. Also a variation in the modelling of the $b$ fragmentation was investigated, by allowing the average fractional energy of a $B$ hadron to vary by 1.5%. The same relative uncertainty was assumed as in [6].

The effect of a variation of 1% in the fraction $R_b$ of $b\bar{b}$ events and of 3% in the fraction $R_c$ of $c\bar{c}$ events was found to be negligible. Since the multiplicity difference, $\delta_{bl}$, was found to be independent of energy within errors, the effect of the modelling of the initial state radiation is also expected to be negligible.

The contributions to the systematic error are summarized in Table 3.

The final mean values of the event multiplicity in $b$ events are $\langle n \rangle_{b\bar{b}}(183 \text{ GeV}) = 29.79 \pm 1.11(\text{stat}) \pm 0.28(\text{syst})$, $\langle n \rangle_{b\bar{b}}(189 \text{ GeV}) = 30.53 \pm 0.70(\text{stat}) \pm 0.34(\text{syst})$, and $\langle n \rangle_{b\bar{b}}(200 \text{ GeV}) = 29.38 \pm 0.65(\text{stat}) \pm 0.50(\text{syst})$. The multiplicity difference between

| $E_{cm}$ | $\langle n \rangle_{b\bar{b}}$ | $\langle n \rangle_{c\bar{c}}$ | $\langle n \rangle_{u\bar{u}}$ | $\delta_{bl}$ |
|----------|-----------------|----------------|-----------------|-----------|
| 192 GeV  | $27.57 \pm 1.56$ | $30.63 \pm 7.70$ | $25.54 \pm 2.75$ | $2.03 \pm 2.36$ |
| 196 GeV  | $29.58 \pm 0.97$ | $26.75 \pm 4.45$ | $27.12 \pm 1.58$ | $2.46 \pm 1.37$ |
| 200 GeV  | $29.55 \pm 1.06$ | $32.42 \pm 4.43$ | $24.75 \pm 1.54$ | $4.79 \pm 1.34$ |

Table 2: Multiplicities measured for each energy during 1999.

Table 3: Contributions to the systematic errors on $\langle n \rangle_{b\bar{b}}$ and $\delta_{bl}$. 
\( b\bar{b} \) and light quark-antiquark events measured at the different energies is:

\[
\begin{align*}
\delta_{bb}(183 \text{ GeV}) &= 4.55 \pm 1.31 (\text{stat}) \pm 0.73 (\text{syst}), \\
\delta_{bb}(189 \text{ GeV}) &= 4.43 \pm 0.85 (\text{stat}) \pm 0.61 (\text{syst}), \\
\delta_{bb}(200 \text{ GeV}) &= 3.39 \pm 0.89 (\text{stat}) \pm 1.01 (\text{syst}).
\end{align*}
\]

These values include the products of \( K_0^0 \) and \( \Lambda \) decays. The uncertainties on the modelling of the detector largely cancel out in the difference.

Our results on \( \delta_{bb} \) are plotted in Figure 2 and compared with previous results in the literature.

3 Comparison with Models and QCD Predictions

Flavour-Independent Fragmentation — In a model in which the hadronization is independent of the mass of the quarks, one can assume that the non-leading multiplicity in an event, i.e., the light quark multiplicity which accompanies the decay products of the primary hadrons, is governed by the effective energy available to the fragmentation system following the production of the primary hadrons \([1]\). One can thus write:

\[
\delta_{bb}(E_{cm}) = 2 \langle n_{B}^{\text{decay}} \rangle + \int_0^1 dx_B f_{E_{cm}}(x_B) \int_0^1 dx_B f_{E_{cm}}(x_B) \, n_{\bar{q}} \left( 1 - \frac{x_B + x_B}{2} \right) E_{cm}
\]

where \( \langle n_{B}^{\text{decay}} \rangle \) is the average number of charged particles coming from the decay of a \( B \) hadron, \( x_B \) is the fraction of the beam energy taken by the \( B \) hadron, and \( f_{E_{cm}}(x_B) \) is the \( b \) fragmentation function.

We assumed \( 2 \langle n_{B}^{\text{decay}} \rangle = 11.0 \pm 0.2 \) \([2]\), consistent with the average \( \langle n_{B}^{\text{decay}} \rangle = 5.7 \pm 0.3 \) measured at LEP \([7]\). For \( f_{E_{cm}}(x_B) \), we assumed a Peterson function with hardness parameter \( \epsilon_p = 0.0047 \pm 0.0010 \) \([10]\), evolving with energy as in \([12]\) to take into account the effects of scaling violations. The value of \( n_{\bar{q}}(E) \) was computed from the fit to a perturbative QCD formula \([18]\) including the resummation of leading (LLA) and next-to-leading (NLLA) corrections, which reproduces well the measured charged multiplicities \([15]\), with appropriate corrections to remove the effect of heavy quarks \([19]\) and leading particles.

The prediction of the model in which the hadronization is independent of the quark mass is plotted in Figure 2. The reason for the drop with collision energy is that the heavy quark system carries away a large fraction of the available energy, approximately (i.e., neglecting scaling violations) linear with \( \sqrt{s} \), while the multiplicity growth with \( \sqrt{s} \) is less than linear. There are several variations of this model in the literature, leading to slightly different predictions (see \([7]\) and references therein). The result from substituting in Eq. \([7]\) \( n_{\bar{q}} \left( 1 - \frac{x_B + x_B}{2} \right) E_{cm} \) with \( n_{\bar{q}} \left( E_{cm} \sqrt{(1 - x_B)(1 - x_B)} \right) \) as in \([7]\), or approximating the Peterson fragmentation function with a Dirac delta function at \( \langle x_B \rangle \), are within the errors. Also by using for \( n_{\bar{q}} \) the expression in \([7]\) one stays within the band in Figure 2. The prediction as plotted in Figure 2 agrees with the one calculated in \([5]\).

QCD Calculation — The large mass of the \( b \) quark, in comparison to the scale of the strong interaction, \( \Lambda \simeq 0.2 \) GeV, results in a natural cut off for the emission of gluon bremsstrahlung. Furthermore, where the c.m. energy greatly exceeds the scale of the \( b \) quark mass, the inclusive spectrum of heavy quark production is expected to be well described by perturbative QCD in the Modified Leading Logarithmic Approximation (MLLA, \([20]\)).
The value of $\delta_{bl}$ has been calculated in perturbative QCD[2,3]:

$$\delta_{bl} = 2\langle n_B^{(\text{decay})} \rangle - \langle n_{l\bar{l}} \rangle (\sqrt{s} = e^{1/2}m_{b}) + O(\alpha_s(m_{b}))\langle n_{l\bar{l}} \rangle (\sqrt{s} = m_{b}) .$$  \hspace{1cm} (8)

The reason for the appearance of the $e^{1/2}$ factor in the above expression is discussed in detail in[3]. The calculation of the actual value of $\delta_{bl}$ in [2] on the basis of the first two terms in (8) gives a value of $5.5 \pm 0.8$. A different calculation of $\delta_{bl}$ gives $3.68$ [3]. These two calculations assume $m_{b} = 5 \text{ GeV}/c^2$ and $m_{b} = 4.8 \text{ GeV}/c^2$ respectively, and difference parametrizations for the function $\langle n_{l\bar{l}} \rangle (\sqrt{s})$. The dependence of the perturbative part in Eq. (8) on $m_{b}$ is such that moving the $m_{b}$ value from $5 \text{ GeV}/c^2$ to $4 \text{ GeV}/c^2$ induces a change of $+0.6$ units of multiplicity.

The difference of the results in [2] and in [3] demonstrates the importance of the contribution proportional to $\alpha_s(m_{b})$. A less restrictive condition is the calculation of upper limits: an upper limit $\delta_{bl} < 4.1$ is given in [3], based on the maximization of the nonperturbative term; $\delta_{bl} < 4$ is obtained from phenomenological arguments in Ref. [4].

Although the presence of the last term in the equation limits the accuracy in the calculation of $\delta_{bl}$, QCD tells that $\delta_{bl}$ is fairly independent of $E_{cm}$. In this article the average of the experimental values of $\delta_{bl}$ up to $m_{Z}$ included, $\langle \delta_{bl} \rangle = 2.96 \pm 0.20$ (dominated by the LEP 1 data), is taken as the high energy prediction from QCD. The accuracy of the measurement at the $Z$ is thus used to constrain the theoretical prediction.

Our measurement of $\delta_{bl}$, as seen in Figure 2, is consistent with the prediction of energy independence based on perturbative QCD, and more than three standard deviations larger than predicted by the naive model presented in the beginning of this section.

4 Conclusions

The difference $\delta_{bl}$ between the average charged particle multiplicity $\langle n \rangle_{bl}$ in $e^+e^- \rightarrow b\bar{b}$ events and the multiplicity in generic light quark $l = u, d, s$ events has been measured at centre-of-mass energies of 183, 189 and 200 GeV:

$$\begin{align*}
\delta_{bl}(183 \text{ GeV}) &= 4.55 \pm 1.31(\text{stat}) \pm 0.73(\text{syst}) \\
\delta_{bl}(189 \text{ GeV}) &= 4.43 \pm 0.85(\text{stat}) \pm 0.61(\text{syst}) \\
\delta_{bl}(200 \text{ GeV}) &= 3.39 \pm 0.89(\text{stat}) \pm 1.01(\text{syst}) .
\end{align*}$$

This difference is in agreement with QCD predictions, while it is inconsistent with calculations assuming that the multiplicity accompanying the decay of a heavy quark is independent of the mass of the quark itself.

Acknowledgements

We are grateful to Jorge Dias de Deus, Vladimir Petrov, Alexander Kisselev, Valery Khoze and Torbjörn Sjöstrand for useful discussions. We are greatly indebted to our technical collaborators, to the members of the CERN-SL Division for the excellent performance of the LEP collider, and to the funding agencies for their support in building and operating the DELPHI detector. We acknowledge in particular the support of Austrian Federal Ministry of Science and Traffics, GZ 616.364/2-III/2a/98, FNRS-FWO, Belgium, FINEP, CNPq, CAPES, FUJB and FAPERJ, Brazil, Czech Ministry of Industry and Trade, GA CR 202/96/0450 and GA AVCR A1010521,
Danish Natural Research Council,
Commission of the European Communities (DG XII),
Direction des Sciences de la Matière, CEA, France,
Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, Germany,
General Secretariat for Research and Technology, Greece,
National Science Foundation (NWO) and Foundation for Research on Matter (FOM),
The Netherlands,
Norwegian Research Council,
State Committee for Scientific Research, Poland, 2P03B06015, 2P03B1116 and SPUB/P03/178/98,
JNICT–Junta Nacional de Investigação Científica e Tecnológica, Portugal,
Vedecka grantova agentura MS SR, Slovakia, Nr. 95/5195/134,
Ministry of Science and Technology of the Republic of Slovenia,
CICYT, Spain, AEN96–1661 and AEN96-1681,
The Swedish Natural Science Research Council,
Particle Physics and Astronomy Research Council, UK,
Department of Energy, USA, DE–FG02–94ER40817.
References

[1] A. Kisselev, V. Petrov and O. Yushchenko, Z. Phys. C41 (1988) 521.
[2] B.A. Schumm, Y.L. Dokshitzer, V.A. Khoze and D.S. Koetke, Phys. Rev. Lett. 69 (1992) 3025.
[3] V.A. Petrov and A.V. Kisselev, IHEP 94-83 and Z. Phys. C66 (1995) 453.
[4] J. Dias de Deus, Phys. Lett. B355 (1995) 539.
[5] V.A. Khoze and W. Ochs, Int. J. Mod. Phys. A12 (1997) 2949.
[6] DELPHI Coll., P. Abreu et al., Phys. Lett. B347 (1995) 447.
[7] OPAL Coll., R. Akers et al., Phys. Lett. B352 (1995) 176.
[8] SLD Coll., K. Abe et al., Phys. Lett. B386 (1996) 475.
[9] TOPAZ Coll., K. Nagai et al., Phys. Lett. B278 (1992) 506.
[10] DELPHI Coll., P. Abreu et al., Nucl. Instr. Meth. A303 (1991) 233.
[11] DELPHI Coll., P. Abreu et al., Nucl. Instr. Meth. A378 (1996) 57.
[12] T. Sjöstrand, Comp. Phys. Comm. 82 (1994) 74.
[13] DELPHI Coll., P. Abreu et al., Z. Phys. C77 (1996) 11.
[14] P. Abreu et al., Nucl. Instr. Methods A427 (1999) 487.
[15] DELPHI Coll., P. Abreu et al., “Charged and Identified Particles from the Hadronic Decay of W Bosons and in $e^+e^- \to q\bar{q}$ from 130 to 200 GeV”, CERN-EP-2000-023 submitted to Eur. Phys. J. C.
[16] Particle Data Group, Eur. Phys. J. C3 (1998) 1.
[17] A. De Angelis, “Properties of the $Z \to b\bar{b}$ events”, Proc. XXIV Symposium on Multiparticle dynamics (Vietri 1994), p. 359;
“Light and Heavy Flavour Production at LEP 1 and LEP 1.5”, Proc. XIX Workshop on High Energy Physics and Field Theory (Protvino, June 1996), p. 80;
Nucl. Phys. B79 (1999) 475.
[18] B.R. Webber, Phys. Lett. B143 (1984) 501 and references therein.
[19] A. De Angelis, Proc. EPS-HEP Conference, Bruxelles 1995, p.63.
[20] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, “Basics of Perturbative QCD”, Ed. J. Trân Thanh Vân, Editions Frontières, Gif-sur-Yvette, France, 1991.
Figure 1: Stability of $\delta_{bl} = \langle n \rangle_{bb} - \langle n \rangle_{ll}$ with respect to variations of the cut on the b-tagging variable, $y$. Notice that the errors in the plot are correlated (see text). The arrow indicates the value used in the analyses.
Figure 2: The present measurement of $\delta_{bt}$ compared to previous measurements as a function of the centre-of-mass energy, to the QCD prediction (taken as the average of the values up to the Z included, see the text), and to the expectation from flavour-independent fragmentation. The inner error bars represent the statistical error; the full bars show the sum in quadrature of the statistical and systematic errors.