Computational Tools for Supersymmetry Calculations

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I present a brief overview of a variety of computational tools for supersymmetry calculations, including: spectrum generators, cross section and branching fraction calculators, low energy constraints, general purpose event generators, matrix element event generators, SUSY dark matter codes, parameter extraction codes and Les Houches interface tools.

1.1. Introduction

The Standard Model (SM) of particle physics provides an excellent description of almost all physical processes as measured in terrestrial experiments, and is rightly regarded as the crowning achievement of many decades of experimental and theoretical work in elementary particle physics.\(^1\)

As exciting as this is, it is also apparent that the SM cannot account for a wide assortment of astrophysical data, including neutrino oscillations, the matter-anti-matter content of the universe, the presence of dark energy and the presence of dark matter in the universe, and it doesn’t include gravitation. Even before these astrophysical anomalies became evident, it was apparent on theoretical grounds, mainly associated with quadratic divergences in the scalar (Higgs) sector, that the SM was to be regarded as an effective theory valid only at the energy scale of \(\sim 100\) GeV and below. At higher energies, it seemed likely that some new physics must arise, which would be associated with the mechanism for electroweak symmetry breaking.

While a vast array of physics theories beyond the SM have been proposed, the general class of theories including weak scale supersymmetry seem to most naturally solve the theoretical ills of the SM, while at the same time they receive support from a variety of precision experimental

\(^1\)
measurements. Most impressive is the measured values of three SM gauge couplings at the weak scale: when extrapolated to high energies using the renormalization group equations, the gauge couplings very nearly meet at a point under supersymmetric standard model evolution, while they miss badly under SM evolution.

Gauge coupling unification suggests that physics at scales $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV is described by a supersymmetric grand unified theory, and that below $M_{\text{GUT}}$, the correct effective field theory is the Minimal Supersymmetric Standard Model (MSSM), or the MSSM plus gauge singlets (since gauge singlets don’t affect the running of gauge couplings at one loop).

Supersymmetric models predict the existence of a whole new class of matter states at or around the weak scale: the so-called super-partners. Gluinos, charginos, neutralinos, squarks, sleptons plus additional Higgs scalars ($H, A$ and $H^{\pm}$) should all be present in addition to the usual states of matter present in the SM. In order to fully test the hypothesis of weak scale supersymmetry, it seems necessary to actually produce at least some of the superpartners at high energy collider experiments, and to measure many of their properties (mass, spin, coupling strengths and mixing), in order to verify that any new physics signal indeed comes from superpartner production. In addition, the properties of the superpartners will be key to understanding the next level of understanding in the laws of physics, perhaps opening windows to the physics of grand unification and even string theory.

The key link between theoretical musings about various theories of SUSY or other new physics, and the experimental observation of particle tracks and calorimeter depositions in collider detectors is the *event generator program*. Given some theory of new physics, which usually predicts the existence of new matter states or new interactions, the event generator program allows us to compute how such a theory would manifest itself at high energy colliding beam experiments. Thus, event generator programs function as a sort of beacon, showing the way to finding new physics in a vast assortment of collider data.

Searches for new matter states at the CERN LEP2 collider have found no firm new physics signals. We thus conclude that the SM Higgs boson must have mass $m_{H_{\text{SM}}} \gtrsim 114$ GeV, while the charginos of supersymmetry must have mass $m_{\tilde{\chi}_{1}} \gtrsim 103.5$ GeV. The Fermilab Tevatron is probing particle masses such as the gluino up to the 300 GeV level. The CERN LHC is a $pp$ collider which is just now beginning to explore the energy regime
where the SM breaks down, and where new physics ought to lie. LHC is expected to operate at energy scales $\sqrt{s} = 7 - 14$ TeV; this ought to be sufficient to either produce some superpartners, or rule out most particle physics models which include weak scale supersymmetry.

As we enter the LHC era, it is important to review the available calculational tools which are available, that aid in connecting theory to experiment. In this chapter, I present a brief overview of some of the publicly available tools. In Section 1.2, I examine the various

- sparticle mass spectrum calculators,

and the *Les Houche Accord files* which provide a handy interface between these and event generator programs. Sec. 1.3 lists some

- codes which calculate sparticle production rates, decay widths and branching fractions.

Sec. 1.4 reviews

- event generators for SUSY processes, including
  - multi-purpose generators, complete with parton showers, hadronization and underlying events, and
  - more specialized matrix element generators, which tend to focus on specific reactions.

The *Les Houche Event files* allow parton level collider events to be easily passed into general purpose event generators so that showering, hadronization and underlying events can be included. Sec. 1.5 lists

- codes relevant to supersymmetric dark matter calculations,

while Sec. 1.6 examines

- codes designed to extract fundamental theory parameters from sets of experimental measurements.

The supersymmetry parameter analysis (SPA) project seeks to develop a uniform set of conventions which would allow unambiguous extraction of high energy model parameters from various collider measurements of supersymmetric production and decay reactions.

I note here that this Chapter is an updated version of the 1997 version by H. Baer and S. Mrenna which appeared in the volume *Perspectives on Supersymmetry*, edited by G. Kane (World Scientific).
1.2. SUSY spectrum calculators

The first step in connecting supersymmetric theory to experiment is to begin with a supersymmetric model, and calculate the expected spectrum of superpartner and Higgs boson masses and couplings. The models we will be focusing on are 4-d supersymmetric quantum field theories with softly broken supersymmetry at the TeV scale. These models might be the low energy effective theory resulting from some even more encompassing theory, such as superstring theory, or a particular SUSY GUT model, or which may invoke some specific mechanism for supersymmetry breaking.

The effective field theory is specified by adopting 1. the gauge symmetry, 2. the (super-) field content and 3. the Lagrangian. In the case of supersymmetric theories, the Lagrangian is derived from the more fundamental superpotential and Kähler potential, and for non-renormalizable models, the gauge kinetic function. The effects of SUSY breaking are encoded in the Lagrangian soft SUSY breaking (SSB) terms. One must also specify the energy scale at which the effective theory and Lagrangian parameters are valid. Since collider experiments will be testing physics at the weak scale $Q \sim 1\text{ TeV}$, while the Lagrangian parameters are frequently specified at much higher scales (e.g. $M_{\text{GUT}}$ or $M_{\text{P}}$), the renormalization group equations (RGEs) must be used to connect the disparate scales in the model.

Once the Lagrangian parameters are known at the weak scale, then the physical (s)particle masses must be identified, often by diagonalizing the relevant mass matrices. Higher order perturbative corrections to the mass eigenstates— at least at 1-loop— are nowadays necessary to gain sufficient accuracy in the predictions.

Numerous researchers have developed private codes to calculate sparticle masses given high scale model inputs. Here, we will focus only on publicly available codes, since these are available to the general user, and are frequently kept up-to-date and user friendly. The first of the publicly available spectrum calculator codes to appear was the ISASUGRA\textsuperscript{7} subprogram of the event generator ISAJET,\textsuperscript{8} in 1994. This was followed by SUSPECT\textsuperscript{9} (1997), SOFTSUSY\textsuperscript{10} (2002) and SPHENO\textsuperscript{11} (2003).

1.2.1. Isasusy, Isasugra and Isajet

ISASUSY is a subprogram of the ISAJET event generator which calculates sparticle mass spectra given a set of 24 SSB input parameters at the weak
scale. The program includes full 1-loop corrections to all sparticle masses. For Higgs masses and couplings, the full 1-loop effective potential is minimized at an optimized scale which accounts for leading 2-loop effects.\textsuperscript{12} Yukawa couplings which are necessary for the loop calculations are evaluated using simple SM running mass expressions.

The Isasugra\textsuperscript{7} program starts with models defined at a much higher mass scale (\textit{e.g.} \(Q = M_{GUT}\)), and calculates the weak scale SUSY parameters via the full set of 2-loop RGEs.\textsuperscript{13} An iterative approach to solving the RGEs is employed, since weak scale threshold corrections which depend on the entire SUSY mass spectrum are included. Once convergence is achieved, then the complete set of 1-loop corrected sparticle and Higgs masses are computed as in Isasusy. Since Isasugra employs full 2-loop running of gauge and Yukawa couplings including threshold corrections—while Isasusy does not—the sparticle masses will differ between Isasusy and Isasugra even for the same weak scale parameter inputs.

A listing of pre-programmed Isasugra models include the following:

- mSUGRA (or CMSSM) model: 4 parameters plus a sign \(m_0, m_{1/2}, A_0, \tan \beta, \text{sign}(\mu)\),
- minimal gauge-mediated SUSY breaking (mGMSB, 4 param’s plus sign plus \(C_{\text{grav}}\)) and non-minimal GMSB,
- non-universal supergravity (19 param’s plus sign)
  - SSB terms can be assigned at any intermediate scale \(M_{\text{weak}} < Q < M_{GUT}\),
  - non-universal Higgs model with weak scale \(\mu\) and \(m_A\) inputs in lieu of \(m_{H_u}^2\) and \(m_{H_d}^2\),
- mSUGRA or NUSUGRA plus right-hand neutrino (RHN),
- minimal and non-minimal anomaly mediation (AMSB),
- mixed moduli-AMSB (mirage mediation),
- hypercharged AMSB.

The related program RGEFLAV computes the complete flavor matrix structure of the SSB terms and Yukawa couplings, including \(CP\)-violating effects.\textsuperscript{14} The webpage is located at \url{http://www.nhn.ou.edu/~isajet/}.

1.2.2. Suspect

SU\textsc{spect}\textsuperscript{9} runs the 2-loop MSSM RGEs to determine weak scale SUSY parameters in the mSUGRA, GMSB and AMSB models, and in the pMSSM (a more general MSSM model). One-loop
sparticle mass corrections are included. Some two loop corrections to Higgs masses are included. The webpage is located at http://www.lpta.univ-montp2.fr/users/kneur/Suspect/.

1.2.3. **SoftSUSY**

SOFTSUSY\(^{10}\) is a C++ code that calculates 2-loop MSSM RGEs to determine weak scale SUSY parameters in the mSUGRA, mGMSB and mAMSB models, and in the general MSSM. R-parity violating effects are possible. One-loop sparticle mass corrections are included. Some two loop corrections to Higgs masses are included. SOFTSUSY calculates the complete flavor matrix structure of the MSSM soft terms and Yukawa couplings. The webpage is located at http://projects.hepforge.org/softsusy/.

1.2.4. **Spheno**

SPHENO\(^{11}\) is a Fortran 90 code that calculates 2-loop MSSM RGEs to determine weak scale SUSY parameters in the mSUGRA, mGMSB and mAMSB models, and in the general MSSM. One-loop sparticle mass corrections are included. Some two loop corrections to Higgs masses are included. The webpage is located at http://www.physik.uni-wuerzburg.de/~porod/SPheno.html.

1.2.5. **Les Houches Accord (LHA) files**

A standard input/output file under the name of Les Houches Accord (LHA) files has been created. All of the above codes can create LHA output files. The advantage of LHA output files is that various event generator and dark matter codes (see below) can use these as inputs for generating collider events or dark matter observables. The specific form of the LHA files is presented in Ref.\(^{15}\).

In addition, the ISASUGRA and ISASUSY codes output to a special IsaWIG file, which is created expressly for input of sparticle mass spectra and decay branching fractions to the event generator HERWIG.

1.2.6. **Comparison of spectra generator codes**

Several papers have been written comparing the SUSY spectra codes\(^{16}\), although these tend to be all dated material, as the codes are continually being updated and debugged. While many features of these codes
are similar, and so agreement between spectra tends to be good in generic parameter space regions, there are some differences as well. In particular, the codes SuSpect, Softsusy and Spheno all adopt a *sharp cut-off scale* between the MSSM and SM effective theories. Allowance for the sharp cut-off is compensated for by log terms in the 1-loop sparticle and Higgs boson mass corrections. The Isasugra code instead adopts a “tower of effective theories” approach, and incorporates threshold corrections in the 1-loop RGEs. Here, the beta-functions changes each time a sparticle mass threshold is passed over. One loop corrections to non-mixing sparticle masses are implemented at each sparticle’s mass scale, so all logs are minimized. Sparticles that mix have all their SUSY parameters evaluated at the \( M_{SUSY} \equiv \sqrt{m_{\tilde{t}_L}m_{\tilde{t}_R}} \) scale due to a need for consistency amongst the various soft terms that enter the mass matrices. In this way, better accuracy is expected in cases where the sparticle mass spectra is spread across a large energy range, as happens— for instance— in focus point or split SUSY, where scalars are at multi-TeV values or beyond, whereas gauginos can be quite light.

1.3. Sparticle production and decay codes

1.3.1. Production cross sections

The multi-purpose event generators Isajet, Pythia, Herwig, SUSYGen and Sherpa all have a complete set of tree-level SUSY particle production reactions encoded, and can be used to calculate tree-level sparticle production cross sections. In the case of Pythia or Herwig, the LHA files from spectra generators can be used as input to calculate these, or general SUSY parameter inputs are allowed. Isajet does not allow LHA input since it has its own spectra generator. The Isajet code also calculates all sparticle and Higgs boson production reactions for \( e^+e^- \) colliders including variable beam polarization, and bremsstrahlung and beamstrahlung. The Spheno code also calculates lowest order \( e^+e^- \rightarrow SUSY \) cross sections.

The code Prospino has been created to calculate all \( 2 \rightarrow 2 \) supersymmetric production cross sections at hadron colliders at both leading order (LO) and next-to-leading order (NLO) in QCD. The current version of Prospino takes LHA files as its input format.
1.3.2. Decay widths and branching fractions

The programs ISASUSY and ISASUGRA also calculate all sparticle and Higgs boson $1 \rightarrow 2$-body and $1 \rightarrow 3$-body decay widths and branching fractions (BFs). These widths and BFs are output in ISAJET standard output files, and are used internally for event generation. The chargino and neutralino branching fractions are sensitive to the parameter $\tan \beta$ in that at large $\tan \beta$, decays to third generation quarks and leptons are enhanced relative to decays to first/second generation fermions.

The program SUSYHIT$^{20}$ is a relatively new release that combines SUSPECT with the branching fraction codes SDECAY and HDECAY to also generate a table of sparticle and Higgs boson decay widths and branching fractions. Some of the decay modes in SUSYHIT are calculated at NLO in QCD.

The program SPHENO also computes sparticle decay widths and branching fractions.

The branching fractions from ISAJET, SUSYHIT and SPHENO all seem to enjoy excellent agreement with each other. The branching fractions of all these codes can be input to event generators via the LHA input/output files. Early versions of HERWIG took branching fraction inputs from the IsaWIG output files.

Care must be taken in extracting branching fractions for neutralinos and charginos computed internally from PYTHIA in that they may not be valid at large $\tan \beta$ values $\gtrsim 10$, since Yukawa couplings, mixing effects, and decays through intermediate Higgs bosons are neglected.

Some specialized codes are available for calculating decays modes of the SUSY Higgs bosons. These include FEYNHIGGS,$^{21}$ which calculates MSSM Higgs boson masses at two-loop level, along with branching fractions, CP-SUPERH$^{22}$ which calculates Higgs boson branching fractions including CP-violating parameters, and NHMDECAY,$^{23}$ which calculates Higgs boson masses and branching fractions in the next-to-minimal supersymmetric Standard Model (NMSSM).

1.4. Event generators

Supersymmetric models can be used to calculate sparticle masses and mixings, which in turn allow for a prediction of various sparticle production rates and decay widths into final states containing quarks, leptons, photons, gluons (and LSPs in $R$-parity conserving models). However, quarks
and gluons are never directly measured in any collider detector. Instead, detectors measure tracks of (quasi-)stable charged particles and their momenta as they bend in a magnetic field. They also measure energy deposited in calorimeter cells by hadrons, charged leptons and photons. There is thus a gap between the predictions of supersymmetric models in terms of final states involving quarks, gluons, leptons and photons, with what is actually detected in the experimental apparatus. This gap is bridged by event generator computer programs. Once a collider type and supersymmetric model is specified, the event generator program can produce a complete simulation of the sorts of scattering events that are to be expected. The final state of any simulated scattering event is composed of a listing of electrons, muons, photons and the long-lived hadrons (pions, kaons, nucleons etc.) and their associated 4-vectors that may be measured in a collider experiment.

The underlying idea of SUSY event generator programs is that for a specified collider type ($e^+e^-$, $pp$, $p\bar{p}$, · · ·) and center of mass energy, the event generator will, for any set of model parameters, generate various particle pair production events in the ratio of their production cross sections, and with distributions as given by their differential cross sections. Moreover, the produced sparticles will undergo a (possibly multi-step cascade) decay into a partonic final state, according to branching ratios as fixed by the model. Finally, this partonic final state is converted to one that is comprised of particles that are detected in an experimental apparatus. By generating a large number of “SUSY events” using these computer codes, the user can statistically simulate the various final states that are expected to be produced within the framework of any particular model.

Several general purpose event generator programs that incorporate SUSY are currently available, including Isajet, Pythia, Herwig, SUSYGeN and Sherpa. These include usually just the leading order $2 \rightarrow 2$ SUSY production processes.

In addition, specific $2 \rightarrow n$ ($n \leq 6$) SUSY reactions may be generated by such programs as CompHEP, CalcHEP, MadGraph, SUSY-Grace, Amegic++ and O’Mega. The output of these latter programs can be interfaced with the Pythia or Herwig programs to yield complete scattering event simulations by generating output in the Les Houches Event file (LHE) format. (Isajet generates LHE output, but does not accept LHE files as input, since it includes its own mass and branching fraction generator). Ideally, event generator programs should be flexible enough to enable simulation of SUSY events from a variety of models such as mSUGRA, GMSB, AMSB etc.. This is usually accomplished nowadays by reading in the LHA
model files into the event generators Pythia or Herwig. Since Isajet does its own spectra calculation, it only outputs LHA files, but does not accept them as input.

The simulation of hadron collider scattering events may be broken up into several steps, as illustrated in Fig. 1.1. The steps include:

- the perturbative calculation of the hard scattering subprocess in the parton model, and convolution with parton distribution functions (PDFs),
- inclusion of sparticle cascade decays,
- implementation of perturbative parton showers for initial and final state colored particles, and for other colored particles which may be produced as decay products of heavier objects,
- implementation of a hadronization model which describes the formation of mesons and baryons from quarks and gluons. Also, unstable particles must be decayed to the (quasi-)stable daughters that are ultimately detected in the apparatus, with rates and distributions in accord with their measured or predicted values.
- Finally, the debris from the colored remnants of the initial beams must be modeled to obtain a valid description of physics in the forward regions of the collider detector.

For $e^+e^-$ collider simulations, in addition we have to allow for the possibility of polarized initial beams, and beam-strahlung effects.

1.4.1. Hard scattering

The hard scattering and convolution with PDFs forms the central calculation of event generator programs. The calculations are usually performed at lowest order in perturbation theory, so that the hard scattering is either a $2 \rightarrow 2$ or $2 \rightarrow 1$ scattering process. Matrix element generators are usually used for $2 \rightarrow n$ processes, with $n \geq 3$.

For supersymmetric particle production at a high energy hadron collider such as the LHC, a large number of hard scattering subprocesses are likely to be kinematically accessible. Each subprocess reaction must be convoluted with PDFs so that a total hadronic cross section for each reaction may be determined. The $Q^2$-dependent PDFs commonly used are constructed to be solutions of the Dokshitzer, Gribov, Lipatov, Altarelli, Parisi (DGLAP) QCD evolution equations, which account for multiple collinear emissions of quarks and gluons from the initial state in the leading log approximation.
As $Q^2$ increases, more gluons are radiated, so that the distributions soften for large values of $x$, and correspondingly increase at small $x$ values. Use of a running QCD coupling constant makes the entire calculation valid at leading log level.

Once the total cross sections are evaluated for all the allowed sub-processes, then reactions may be selected probabilistically (with an assigned weight) using a random number generator. This will yield sparticle events in the ratio predicted by the particular model being simulated.

For sparticle production at $e^+e^-$ colliders, it may also be necessary to
convolute with *electron and positron* PDFs to incorporate bremsstrahlung and beamstrahlung effects. In addition, if beam polarization is used, then each subprocess cross section will depend on beam polarization parameters as well.

### 1.4.2. Parton showers

For reactions occurring at both hadron and lepton colliders, to obtain a realistic portrait of supersymmetric (or Standard Model) events, it is necessary to account for multiple *non-collinear* QCD radiation effects. The evaluation of the cross section using matrix elements for multi-parton final states is prohibitively difficult. Instead, these multiple emissions are approximately included in an event simulation via a parton shower (PS) algorithm. They give rise to effects such as jet broadening, radiation in the forward regions and energy flow into detector regions that are not described by calculations with only a limited number of final state partons.

In leading log approximation (LLA), the cross section for *single* gluon emission from a quark line is given by

\[
\frac{d\sigma}{d^2t} = \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{qq}(z)dz,
\]

where \(\sigma_0\) is the overall hard scattering cross section, \(t\) is the intermediate state virtual quark mass, and \(P_{qq}(z) = \frac{4}{3} \left( \frac{1+z^2}{1-z^2} \right)\) coincides with the Altarelli-Parisi splitting function for \(q' \to qg\) for the fractional momentum of the final quark \(z \equiv \frac{|\vec{p}_q|}{|\vec{p}_{q'}|} < 1\). Interference between various *multiple* gluon emission Feynman graphs, where the gluons are ordered differently, is a subleading effect. Thus, Eq. (1.1) can be applied successively, and gives a factorized probability for each gluon emission. The idea behind the PS algorithm is then to use these approximate emission probabilities (which are exact in the collinear limit), along with exact (non-collinear) kinematics to construct a program which describes multiple non-collinear parton emissions. Notice, however, that the cross section (1.1) is singular as \(t \to 0\) and as \(z \to 1\), i.e. in the regime of collinear and also soft gluon emission. These singularities can be regulated by introducing physically appropriate cut-offs. A cutoff on the value of \(|t|\) of order \(|t_c| \sim 1\) GeV corresponds to the scale below which QCD perturbation theory is no longer valid. A cutoff on \(z\) is also necessary, and physically corresponds to the limit beyond which the gluon is too soft to be resolved.

The PS algorithms available vary in their degree of sophistication. The simplest algorithm was created by Fox and Wolfram in 1979. Their
method was improved to account for interference effects in the angle-ordered algorithm of Marchesini and Webber. In addition, parton emission from heavy particles results in a dead-cone effect, where emissions in the direction of the heavy particle are suppressed. Furthermore, it is possible to include spin correlations in the PS algorithm.

PS algorithms are also applied to the initial state partons. In this case, a backwards shower algorithm is most efficient, which develops the emissions from the hard scattering backwards in time towards the initial state. The backward shower algorithm developed by Sjöstrand makes use of the PDFs evaluated at different energy scales to calculate the initial state parton emission probabilities.

1.4.3. Cascade decays

Not only are there many reactions available via which SUSY particles may be produced at colliders, but once produced, there exist many ways in which superparticles may decay. For the next-to-lightest SUSY particle (NLSP), there may be only one or at most a few ways to decay to the LSP. Thus, for a collider such as LEP or even the Fermilab Tevatron, where only the lightest sparticles will have significant production rates, we might expect that their associated decay patterns will be relatively simple. However, the number of possible final states increases rapidly if squarks and gluinos that can decay into the heavier charginos and neutralinos are accessible, and the book-keeping becomes correspondingly more complicated. Indeed, at the CERN LHC, where the massive strongly interacting sparticles such as gluinos and squarks are expected to be produced at large rates, sparticle cascade decay patterns can be very complex. As a rough estimate, of order $10^3$ subprocess cross sections may be active at LHC energies, with of order 10 decay modes for each sparticle. Naively, this would give of order $10^5$ $2 \rightarrow n$-body subprocesses that would need to be calculated.

Monte Carlo event generators immensely facilitate the analysis of signals from such complex cascade decays, especially in the case where no single decay chain dominates. An event generator can select different cascade decay branches by generating a random number which picks out a particular decay choice, with a weight proportional to the corresponding branching fraction, at each step of the cascade decay. Quarks and gluons produced as the end products of cascade decays will shower off still more quarks and gluons, with probabilities determined by the PS algorithm.

The procedure that we have just described is exact for cascade decays
of spinless particles into two other spinless particles at each step in the cascade. This is because the squared matrix element is just a constant, and there are no spin correlations possible. This is not true in general and in many cases, it can be very important to include the decay matrix element and/or spin correlations in the calculation of cascade decays of sparticles. A general method for incorporating spin correlations based on density matrices has been put forth by Richardson, and incorporated into Herwig.

Spin correlation effects are especially important for precision measurements at $e^+e^-$ linear colliders. While retaining spin correlations may be less crucial in many situations at a hadron collider, this is not always the case. For instance, relativistic $\tau^-$ leptons produced from $W$ decay are always left-handed, while those produced from a charged Higgs decay are always right-handed. Likewise, the polarization of the taus from $\tilde{\tau}_1$ decays depends on the stau mixing angle. Since the undetectable energy carried off by $\nu_{\tau}$ from tau decay depends sensitively on the parent tau helicity, it is necessary to include effects of tau polarization in any consideration involving the energy of “tau jets”. By evaluating the mean polarization of taus in any particular process, these effects can be incorporated, at least on average, into event generator programs such as Isajet. Of course, such a procedure would not include correlations between decay products of two taus produced in the same reaction.

Another aspect is to include appropriately the complete 3-body decay matrix elements. While some programs merely use a flat phase space distribution, Isajet and Herwig include pre-programmed exact decay matrix elements.

1.4.4. Models of hadronization

Once sparticles have been produced and have decayed through their cascades, and parton showers have been evolved up to the point where the partons have virtuality smaller than $\sim 1$ GeV$^2$, the partons must be converted to hadrons. This is a non-perturbative process, and one must appeal to phenomenological models for its description. The independent hadronization (IH) model of Field and Feynman is the simplest such model to implement. In this picture, a new quark anti-quark pair $q_1\bar{q}_1$ can be created in the color field of the parent quark $q_0$. Then the $q_0\bar{q}_1$ pair can turn into a meson with a longitudinal momentum fraction described by a phenomenological function, with the remainder of the longitudinal momentum carried
by the quark $q_1$. This process is repeated by the creation of a $q_2\bar{q}_2$ pair in the color field of $q_1$, and so on down the line to $q_n\bar{q}_n$. A host of mesons are thus produced, and decayed to the quasi-stable $\pi$, $K$, $\cdots$ mesons according to their experimental properties. The final residual quark $q_n$ will have very little energy, and can be discarded without significantly affecting jet physics. Finally, a small transverse momentum can be added according to a pre-assigned Gaussian probability distribution to obtain a better description of the data. Quark fragmentation into baryons is also possible by creation of diquark pairs in its color field, and can be incorporated. The IH scheme, with many parameters tuned to fit the data, will thus describe the bulk features of hadronization needed for event simulation programs.

The string model of hadronization developed at Lund$^{34}$ is a more sophisticated model than IH, which treats hadron production as a universal process independent of the environment of the fragmenting quark. In the string model, a produced quark-antiquark pair is assumed to be connected by a color flux tube or string. As the quark-antiquark pair moves apart, more and more energy is stored in the string until it is energetically favorable for the string to break, creating a new quark-antiquark pair. Gluons are regarded as kinks in the string. The string model correctly accounts for color flow in the hadronization process, as opposed to the IH model. In $e^+e^- \rightarrow q\bar{q}g$ (3-jet) events, the string model predicts fewer produced hadrons in the regions between jets than the IH model, in accord with observation.

A third scheme for hadronization is known as the cluster hadronization model.$^{35}$ In this case, color flow is still accounted for, but quarks and antiquarks that are nearby in phase space will form a cluster, and will hadronize according to preassigned probabilities. This model avoids non-locality problems associated with the string hadronization model, where quarks and antiquarks separated by spacelike distances can affect the hadronization process.

### 1.4.5. Beam remnants

Finally, at a hadron collider the colored remnants of the nucleon that did not participate in the hard scattering must be accounted for. These beam remnant effects produce additional energy flow, especially in the far forward regions of the detector. A variety of approaches are available to describe these non-perturbative processes, including models involving Pomeron exchange and multiple scatterings. In addition, the beam remnants must
be hadronized as well, and appear to require a different parametrization from “minimum bias” events where there are only beam jets but no hard scattering.

1.4.6. **Multi-purpose event generators**

Publicly available event generators for SUSY processes include,

- **ISAJET**: (H. Baer, F. Paige, S. Protopopescu and X. Tata),
  [http://www.nhn.ou.edu/~isajet/](http://www.nhn.ou.edu/~isajet/)

- **PYTHIA**: (T. Sjöstrand, L. Lönnblad and S. Mrenna),
  [http://www.thep.lu.se/~torbjorn/Pythia.html](http://www.thep.lu.se/~torbjorn/Pythia.html)

- **HERWIG**: (G. Corcella, I. G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M. Seymour and B. R. Webber),
  [http://hepwww.rl.ac.uk/theory/seymour/herwig/](http://hepwww.rl.ac.uk/theory/seymour/herwig/)

- **SUSYGEN**: (N. Ghodbane, S. Katsanevas, P. Morawitz and E. Perez),
  [http://lyoinfo.in2p3.fr/susygen/susygen3.html](http://lyoinfo.in2p3.fr/susygen/susygen3.html)

- **SHERPA**: (T. Gleisberg, S. Höche, F. Krauss, M. Schönherr, S. Schumann, F. Siegert and j. Winter)
  [http://projects.hepforge.org/sherpa/dokuwiki/doku.php](http://projects.hepforge.org/sherpa/dokuwiki/doku.php)

The event generator program **ISAJET** was originally developed in the late 1970’s to describe scattering events at the ill-fated ISABELLE pp collider at Brookhaven National Laboratory. It was developed by F. Paige and S. Protopopescu to generate SM and beyond scattering events at hadron and $e^+e^-$ colliders. H. Baer and X. Tata, in collaboration with Paige and Protopopescu, developed **ISAJET** to give a realistic portrayal of SUSY scattering events. **ISAJET** uses the IH model for hadronization, and the original Fox-Wolfram (Sjöstrand) PS shower algorithm for final state (initial state) parton showers. It includes an $n$-cut Pomeron model to describe beam-jet evolution.

The event generator **PYTHIA** was developed mainly by T. Sjöstrand in the early 1980s to implement the Lund string model for event generation. **PYTHIA** uses the FW virtuality-ordered shower model, but with an angle-ordered veto. S. Mrenna contributed the inclusion of SUSY processes in **PYTHIA**.

The event generator **HERWIG** was developed in the mid-1980s to describe scattering events with angle-ordered parton showers, which accounted for interference effects neglected in the FW shower approach. **HERWIG** imple-
ments a cluster hadronization model. \texttt{HERWIG} is notable in that it includes sparticle production and decay spin correlations using density matrix techniques.$^{32}$

The program \texttt{SUSYGen} was developed by S. Katsanevas and P. Morawitz to generate $e^+e^- \rightarrow SUSY$ events for the LEP experiments. \texttt{SUSYGen} interfaces with \texttt{Pythia} for hadronization and showering. \texttt{SUSYGen} has since been upgraded to also generate events for hadron colliders.

The program \texttt{Sherpa} was developed as a new generation event generator in the $C++$ language. It calculates subprocess reactions using \texttt{Amegic++}. It includes its own shower and cluster hadronization routines.

### 1.4.7. Matrix element generators

For generating various $2 \rightarrow n$ scattering reactions using complete matrix elements, a number of automated tree-level codes are available.

The code \texttt{CompHEP} by E. Boos \textit{et al.}$^{36}$ is designed to take one directly from a Lagrangian to distributions. Feynman rules can be calculated using the \texttt{LanHEP} code,$^{37}$ and then \texttt{CompHEP} will generate the squared matrix element by constructing the squared amplitude, taking traces, and storing the output as subroutines. \texttt{CompHEP} also includes code for doing the phase space integration, convolution with PDFs, and after integration, numerical output, or output in terms of histograms.

The code \texttt{CalcHEP}$^{38}$ by Pukhov, Belyaev and Christensen is very similar to \texttt{CompHEP}, and was in fact created as a spin-off by some of the original authors of \texttt{CompHEP}.

The code \texttt{MadGraph/MadEvent} was developed by Stelzer and Long and others.$^{39}$ It allows the user to input initial and final state particles, and then generates all Feynman diagrams along with a subroutine which evaluates the scattering amplitude as a complex number using the \texttt{Helas} helicity amplitude subroutines developed by Hagiwara and Murayama.$^{40}$ Since \texttt{MadGraph} directly evaluates the amplitude, and not amplitude squared, computational sampling of the squared matrix element should be faster than programs which evaluate traces over gamma matrices. The latest versions of \texttt{MadGraph}, updated to \texttt{MadEvent}, will convolute with PDFs and perform phase space integration and evaluate distributions as well. A number of models for BSM physics, including the MSSM, are available in \texttt{MadGraph/MadEvent}.
The program O’Mega by Ohl, Reuter and Schwinn, also generates tree-level SM and MSSM amplitudes, and can work in concert with the Whizard program for event generation.\textsuperscript{41,42}

The program SUSY-Grace by Tanaka, Kuroda, Kaneko, Jimbo and Kon also generates SM and MSSM amplitudes, and generates scattering events in association with the Grappa program.\textsuperscript{43,44}

1.4.8. Les Houches Event (LHE) files
A Les Houches Event (LHE) file format has been proposed\textsuperscript{45} which allows for a simple communication between parton level event generators and all purpose generators such as Pythia and Herwig. This is particularly useful when matrix element generators like CalcHEP or MadGraph are used, but the user needs a complete event output including parton showers, hadronization and underlying event simulation.

The LHE file is an ascii file which includes lines pertaining to the generator initialization. It then follows with a listing of partons (particle ID code), their associated 4-vectors and color flow information. The generators Pythia and Herwig then can read in these files, to add on showering, hadronization and underlying event. A sample SUSY event in LHE format is listed below. It lists a reaction with $sg \rightarrow \tilde{s}\tilde{g}$, with $\tilde{s} \rightarrow s\chi^0_1$ and $\tilde{g} \rightarrow \tilde{d}\bar{d}$ and then $\tilde{d} \rightarrow d\chi^0_1$. After listing a line of event characteristics, the event listing follows. The first column corresponds to particle ID, 2nd column to stability of particle, 3rd and 4th columns list the source of the particle, 5th and 6th columns relate to color flow, and 7th column is the $x$-component of the energy-momentum four-vector. The four-vector listing has been truncated to fit on the page.

\begin{verbatim}<event>
10 2160 1.00000 0.768145E+06 ...
 3  -1  0  0  101  0  0.000000E+00 ...
21  -1  0  0  102  101  0.000000E+00 ...
2000003  2  1  2  103  0  0.220402E+03 ...
1000021  2  1  2  102  103  -0.220402E+03 ...
1000022  1  3  0  0  0  0.778961E+02 ...
 3  1  3  0  103  0  0.142506E+03 ...
2000002  2  4  0  102  0  -0.185972E+03 ...
 2  1  4  0  0  103  -0.344299E+02 ...
1000022  1  7  0  0  0  0.800434E+02 ...
 2  1  7  0  102  0  -0.266016E+03 ...
</event>
\end{verbatim}
1.5. Dark matter codes

In response to the increasing precision of data corresponding to the density of dark matter in the universe, several public codes have been developed which evaluate key astrophysical observables in supersymmetric (and other) models.

1.5.1. DarkSUSY

The DarkSUSY code, developed by Gondolo et al.,\textsuperscript{46} evaluates the relic density of neutralino dark matter in SUSY models. DarkSUSY computes all relevant neutralino annihilation and co-annihilation processes, and solves the Boltzmann equation to output the current density of neutralino CDM. It accepts input files from Isajet/Isasugra or from LHA input files. DarkSUSY also calculates: spin-independent and spin-dependent neutralino-nucleon scattering rates (direct WIMP detection), and indirect neutralino detection rates, such as: muon flux from neutralino annihilation in the core of earth or sun, flux of $\gamma$ rays, $\bar{p}s$, $e^+s$ and $\bar{d}s$ from neutralino annihilation in the galactic core or halo. The halo annihilation rates all depend on an assumed form for the galactic dark matter density profile.

1.5.2. Micromegas

Micromegas was developed by Belanger et al.,\textsuperscript{47} and also evaluates the neutralino relic density due to all annihilation and co-annihilation processes. It also computes the WIMP relic density for a variety of other non-SUSY models. It also outputs neutralino direct and indirect detection rates, $b \rightarrow s\gamma$ branching fraction, $(g-2)_\mu^{SUSY}$, $BF(B_s \rightarrow \mu^+\mu^-)$ and the thermally averaged neutralino annihilation cross section, which is key input to neutralino halo annihilation calculations.

1.5.3. Isatools

Isatools is part of the ISAJET package. It includes a subroutine IsaReD\textsuperscript{48} to evaluate the neutralino relic density, the direct neutralino detection rates via spin-independent and spin-dependent scattering, the $b \rightarrow s\gamma$ branching fraction, $(g-2)_\mu^{SUSY}$, $BF(B_s \rightarrow \mu^+\mu^-)$ and the thermally averaged neutralino annihilation cross section, which is key input to neutralino halo annihilation calculations.
1.6. Parameter fitting codes

If supersymmetry is indeed discovered at the Tevatron, LHC and/or a linear $e^+e^-$ collider, then an exciting task will be to make precision measurements of all sparticle masses, spins, couplings and mixings. Once these are known, then, if the MSSM is indeed the correct effective theory all the way from $M_{\text{weak}}$ to $M_{\text{GUT}}$, it is possible to map out the GUT scale values of the soft SUSY breaking parameters. Once these are known, important information will be gained which will allow for the construction of SUSY models at or beyond the GUT scale. Two such codes are available which accomplish this task: \textit{Sfitter}\textsuperscript{49} and \textit{Fittino}\textsuperscript{50}.

1.7. SPA convention

The supersymmetry parameter analysis (SPA) project\textsuperscript{51} is an attempt to achieve co-ordination between the various sparticle mass generation codes, event generators, relic density codes, and parameter fitting codes, with a goal in mind to determine the fundamental SUSY Lagrangian. In the SPA convention, all programs should input/ouput SUSY parameters in the $\overline{\text{DR}}$ scheme at the $Q = 1$ TeV scale. Once this benchmark is set, then all remaining calculations may proceed from this common agreed upon point.

1.8. Summary

In the past decade, there has been an explosion of interest in supersymmetry phenomenology. This is exhibited in part by the corresponding development of numerous computational tools to aid in supersymmetry calculations for expected collider events and for dark matter observables. Supersymmetry has certainly been an enduring theme in high energy physics. Hopefully, at the dawn of the LHC era, we are on the verge of actual discovery of supersymmetry. In this case, many of these tools for SUSY will be put to good hard use, as the community analyzes the upcoming collider data.

We expect that new tools for SUSY will emerge, which will be more focused on the new matter states that might appear. As an example, if SUSY is discovered, then the MSSM (or perhaps NMSSM) may become the new SM, and radiative corrections will have to be calculated for any remaining production and decay reactions, and in a form suitable for embedding in event generator programs. The clues we find pertaining to dark matter will impact on all astrophysical codes. In addition, new tools should also
emerge that facilitate model building, as the clues we expect to emerge from the data point the way to a new paradigm in physics beyond the Standard Model.

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