Expanding one-zone model for blazar emission

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ABSTRACT

Context. As a subcategory of active galactic nuclei, blazars are distinguished by their non-thermal variable emission. This emission extends over the whole electromagnetic spectrum and is a consequence of particle acceleration inside their relativistic jets. However, an open question remains regarding the relation of radio emission compared to that of higher frequencies.

Aims. Observations of blazar emission show that the location of radio might be very different from the one where the rest of the spectrum is produced and, thus, often requires separate modeling. We aim to produce both types of emission within the context of a single model.

Methods. We constructed a self-consistent one-zone expanding leptonic model for studying the connection between the radio emission and the emission at higher frequencies. We then applied it to the flaring states of blazars. Assuming an accelerating episode as the source moves down the jet and expands, we numerically studied the electron evolution as they lose energy due to adiabatic expansion and synchrotron or inverse Compton radiation.

Results. We find that high-frequency radiation mimics the electron injection and is mainly produced close to the acceleration site where cooling is strong. In contrast, the radio emission is produced further down the jet when the emitting region has become optically thin with regard to synchrotron self-absorption due to expansion. We briefly present the role of the initial parameters, such as the magnetic field strength, the electron luminosity, and expansion velocity, on the localization of the radio emission site. We show that the expanding one-zone model is inherently different from the non-expanding one and, in addition, it requires more parameters. For example, we apply our approach to the observational data of a Mrk 421 γ-ray and radio flare observed in 2013.

Key words. radiation mechanisms: non-thermal – galaxies: jets – gamma rays: galaxies – radio continuum: galaxies

1. Introduction

Blazars are known as the most extreme subclass of active galactic nuclei (AGN), with their relativistic jets pointing towards the observer. Blazar emission is non-thermal, spreading over the entire electromagnetic spectrum, and it is characterized by rapid variability, high optical polarization, and apparent superluminal motion. One of the characteristic features of this emission is the shape of its spectral energy distribution (SED), which usually has two components: a low-energy component extending from radio to UV and soft X-rays and a high-energy one lying between hard X-rays and TeV γ-rays.

Modeling a blazar’s SED implies the existence of a non-thermal population of particles. In many cases, blazar emission is explained by a relativistic electron population that accelerates and radiates inside a spherical region (for more details, see the following review papers: Padovani et al. 2017; Hovatta & Lindfors 2019; Böttcher 2019; Blandford et al. 2019; Matthews et al. 2020). This is the so-called one-zone leptonic model, which has been applied widely in modeling the blazar SED (e.g., Marscher & Gear 1985; Inoue & Takahara 1996; Mastichiadis & Kirk 1997; Kirk et al. 1998; Kirk & Mastichiadis 1999; Kusunose et al. 2000; Böttcher & Chiang 2002; Tramacere et al. 2011; Asano et al. 2014; Finke 2016; Boula et al. 2019a).

In this approach, the low-frequency component is related to synchrotron radiation while the higher energy one is tied to inverse Compton scattering. Reproducing the observed SED allows us to obtain physical parameter constraints.

The multi-wavelength monitoring programs are a unique way of probing the conditions in blazar jets and unveiling the physical processes responsible for the blazar’s non-thermal variable emission. High-quality data, such as time lags between radio and γ-ray emission, which are associated with significant correlations between these energy bands, contribute to improving the understanding of blazar anatomy (e.g., Pavlidou et al. 2014; Esposito et al. 2015; Hovatta et al. 2015; Angelakis et al. 2016; Tavani et al. 2018; Liodakis et al. 2019).

Despite this multi-year monitoring effort, there is still no consensus about the location of the high-energy (X-rays and γ-rays) emission in blazar jets. In particular, the rapid variability, which is an identifying property of blazars, suggests that the non-thermal emission is typically produced in regions of the jet with a size ≲1 light day (e.g., Aharonian et al. 2007; Aleksic et al. 2011). Although electron synchrotron radiation produced in this region can explain the optical-to-X-ray part of the photon spectrum, in most cases, it cannot account for the jet emission at low frequencies (e.g., ν ≤ 10^{10} Hz); in such compact emitting regions, synchrotron radiation is typically self-absorbed (e.g., van der Laan 1969; Ginzburg & Syrovatskii 1969; Perez-Torres et al. 2001; Kaiser 2006). However, blazar jets are detected up to GHz frequencies having a power-law spectrum (see also Tingay & de Kool 2003; Stawarz et al. 2008; Ghisellini et al. 2017). In order to solve this discrepancy, it has been proposed that the radio emission, in most cases, is not produced in the same region as the γ-ray. The radio-emitting area

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should be larger to be less opaque to synchrotron self-absorption (Marscher 1980, 2014; Marscher & Gear 1985; Ghisellini et al. 1985; Potter & Cotter 2012; Potter 2018; Boula et al. 2019b).

Still, the results of the various monitoring programs raise several questions about the origin of the radio emission of the jets, which in many cases is variable (Hartman et al. 2001; Kharb et al. 2010; Ghirlanda et al. 2011; Arshakian et al. 2012; Myserlis et al. 2018).

Many theoretical studies are based on an idealized model of a conical steady radio jet, while the component that is associated with the variability of the source is related to shock waves that are propagating in the jet (Blandford & Königl 1979; Georganopoulos & Marscher 1998a,b, Katarzyński et al. 2003). Assuming that electrons are accelerated in such shock fronts and are cooled as they move away from it, one gets that the highest-frequency synchrotron component is emitted from a thin layer behind the shock. In contrast, the lower frequency component is produced in a larger volume, simply because the cooling time for higher-energy electrons is less than for the lower-energy ones, Marscher & Gear (1985). Furthermore, this scenario predicts time lags between high-frequency and low-frequency light curves and a specific evolution of the radio spectrum over time. Potter & Cotter (2012), Potter (2018) proposed a model of a ballistic jet with a uniform structure to investigate the relationship between low- and high-energy emission (see also Katarzyński et al. 2003; Hervet et al. 2015; Richter & Spanier 2016; Zacharias et al. 2021).

The goal of this work is to study the relationship between radio and γ-ray activity in blazars and to localize their emission site using a simple framework that allows for a comprehensive search of the parameter space. More specifically, we use a time-dependent numerical code, originally developed to treat the radiative transfer problem in a static spherical geometry (Mastichiadis & Kirk 1995), to compute the emission from an expanding blob of plasma, (see also Zacharias & Wagner 2016; Boula et al. 2019b). This approach allows γ-rays to be produced more efficiently close to the electron acceleration site, whereas radio is produced much later when the blob has expanded enough to become optically thin to synchrotron self-absorption. The numerical code allows us also to study reaccelerating episodes and we show that depending on the distance, this can produce γ-ray flares, which may or may not be accompanied by radio flares.

The paper is structured as follows. In Sect. 2, we present our new one-zone expanding model that considers all the leptonic physical processes. In Sect. 3, we show the dependence on the initial parameters of the transition from the optically thick to the optically thin region due to synchrotron self-absorption. This method guides us to localize the onset of radio emission inside the jet. The initial values of the physical quantities of the source play a vital role in the transition from the optically thick to the optically thin region. In Sect. 4, we present a few characteristic examples for the flaring activity of blazars by assuming electron accelerating episodes and following their evolution. We also apply our model to the flaring episode of Mrk421 that occurred in 2013, explaining the time-lag between γ-ray and radio emission. In Sect. 5, we discuss our results. Finally, in Sect. 6, we give the basic conclusions of our work.

2. One-zone expanding blob model

We consider a blazar with a conical jet whose axis forms an angle of $\theta_{\text{obs}}$ to our line of sight. We assume that due to some accelerating episode, relativistic electrons are injected in a spherical region, which we refer to as the “blob” of an initial radius $R_{0}^{e}$.

The blob moves along the jet axis with a velocity of $u_{\text{jet}} = \beta_{\text{jet}} c \approx c$, corresponding to a Lorentz factor $\Gamma = (1 - \beta_{\text{jet}}^{2})^{-1/2}$ and a Doppler factor of $\delta = [\Gamma (1 - \beta_{\text{jet}} \cos \theta_{\text{obs}})]^{-1}$. At the same time, it expands in a spherically symmetric way with velocity $u_{\text{exp}}$ in its rest frame. Therefore, if we denote $t'$ the time as measured in the rest frame of the blob\(^1\), then for its instantaneous radius, following Boula et al. (2019b), we have:

$$R'(t') = R_{0}^{e} + u_{\text{exp}}(t' - t_{0}'),$$

where $t_{0}'$ is the time of the blob formation. Similarly, the instantaneous distance, $z$, of the blob along the jet at the observer’s frame is given by:

$$z(t) = z_{0} + \beta t c \Gamma (t' - t_{0}'),$$

or

$$z(t) = z_{0} + \beta t c \Gamma [R' - R_{0}^{e}] / u_{\text{exp}},$$

where $z_{0}$ is the blob’s initial distance from the central object. Therefore both quantities, $R'$ and $z$, are functions of the time $t'$.

In order to calculate the electron evolution inside the blob, one needs to take into account, as in the standard leptonic model, synchrotron and inverse Compton losses. Furthermore, this case, adiabatic losses should also explicitly be taken into account due to the expansion of the blob. We should also consider the possibility that the magnetic field inside the blob might change over time (or radius) for the same reason, affecting the synchrotron losses. Therefore, we parametrize the magnetic field $B'$ as

$$B'(R') = B_{0}^{e} \left( \frac{R'_{0}^{e}}{R'} \right)^{s},$$

where $R_{0}^{e}$ is the initial value of the magnetic field and $s$ is a free parameter. For the parameter $s$, we have $1 \leq s \leq 2$ from magnetic flux conservation with magnetic-field orientations between the extremes of purely toroidal and purely poloidal fields, respectively.

Similarly, the electron injected power might also be a function of time and we parametrize it as

$$L_{e}^{\text{inj}}(R') = L_{e,0}^{\text{inj}} \left( \frac{R_{0}^{e}}{R'} \right)^{q},$$

where $L_{e,0}^{\text{inj}}$ is the initial electron power and $q$ is a free parameter\(^2\).

We have developed a numerical code, based on Mastichiadis & Kirk (1995), to calculate the evolution of the electron and photon distribution function in an expanding spherical source. This code solves two integro-differential equations, each describing the losses or sinks as well as the injection of relativistic electrons and photons in the emitting region. The kinetic equation for the differential, with respect to energy, electron number, $N_{e}$, is expressed as:

$$\frac{\partial N_{e}(\gamma, R')}{\partial t} + \sum_{i} \frac{\partial}{\partial \gamma} N_{e}(\gamma, R') \frac{\delta_{i}}{t_{\text{esc}}(\gamma, R')} = -Q_{e}(\gamma, R').$$

We use $L_{e}^{\text{inj}}$ to denote the loss terms and $Q_{e}$ to denote the injection terms of relativistic electrons. Also, $\gamma = (1 - u_{e}^{2}/c^{2})^{-1/2}$ the

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\(^1\) Primed symbols refer to the quantities as measured in the co-moving frame.

\(^2\) The free parameters $s$, $q$ are related to the jet dynamics, but in our investigation, we do not restrict them to values from specific models.
The processes of photon-photon absorption and synchrotron self-absorption contribute to the photon sinks as:

\[
\sum \int L_i^\text{pp}(x', R') = L_i^\text{pp}^\text{out}(x', R') + L_i^\text{pp}^\text{in}(x', R'),
\]

where \(L_i^\text{pp}^\text{out}(x', R')\) and \(L_i^\text{pp}^\text{in}(x', R')\) denote the processes of photon-photon absorption and synchrotron self-absorption\(^4\) respectively. The photon source function is given by:

\[
\sum \int Q_i^\text{ph}(x', R') = Q_i^\text{pp}^\text{out}(x', R') + Q_i^\text{pp}^\text{in}(x', R')
\]

where \(Q_i^\text{pp}^\text{out}(x', R')\) and \(Q_i^\text{pp}^\text{in}(x', R')\) are the terms corresponding to synchrotron\(^5\) radiation and to inverse Compton scattering, respectively.

Equation (14) by construction calculates the differential number of photons inside the source when its radius is \(R'\). From here, one can calculate the co-moving spectral luminosity (erg/sec/Hz) of the source using:

\[
L_\nu = \frac{3N_e(x', R')}{\tau_{\text{cross}}(R')} \left(\frac{x'mc^2}{h}\right)
\]

and then calculate the observed flux by imposing the usual Doppler transformations. Therefore, Eq. (14) calculates an instantaneous flux coming from a blob of radius \(R'\).

With regard to checking the numerical scheme presented above, especially as far as the expansion is concerned, the various rates have already been tested, for instance, by Mastichiadis & Kirk (1995) and more recently by Cerruti et al. (2021). Thus, we solve the kinetic equation for the electrons (Eq. (6)) analytically by taking into account only the synchrotron and adiabatic losses and we compare the solution to the results of the code. Figure 1a depicts the electron distribution function in the case of constant electron injection for a decreasing magnetic field. The electron number constantly increases with time as they continuously accumulate inside the source, while the shape of the distribution changes continuously. At early times the distribution is completely cooled due to strong synchrotron losses, however, at later times, adiabatic losses take over and the cooling break moves progressively to higher energies. The analytical and numerical solutions are identical, with the only difference appearing at high energies since we have not included cutoff effects in our analytical solution. Figure 1b depicts the corresponding photon spectrum, which is calculated numerically. The luminosity decreases with time since cooling becomes less efficient at later times while the spectral turnovers correspond to the electron cooling breaks shown in Fig. 1a.

Figure 2 shows an example when all processes are taken into account in the numerical code. It depicts the time evolution of the SED when the source constantly expands, while the magnetic field strength and the electron injection luminosity decrease with time, that is, \(q = s = 1\). The addition of all processes complicates the evolution over the case of Fig. 1, however, some basic features remain the same. The photon luminosity decreases with time, represented by the color gradient in the logarithmic scale. The SED has two characteristics bumps:

\(^4\) In our code we use the full expression in the calculation of synchrotron self-absorption in contrast to the \(\delta\)-function approach that was used in Mastichiadis & Kirk (1995).

\(^5\) Here we use the full expression in our calculations in contrast to the \(\delta\)-function approach that was used in Mastichiadis & Kirk (1995).
Fig. 1. Electron and photon distribution. (a) Plot of the electron energy distribution at different time-steps for synchrotron and adiabatic losses only. The values of the parameters are $R'_0 = 10^{15}$ cm, $L_{\text{inj}}' = 10^{42}$ erg s$^{-1}$, $u_{\text{exp}} = 0.1$ c, $B'_0 = 100$ G, $\gamma_{\text{min}} = 1$, $\gamma_{\text{max}} = 10^6$, $p = 2$, and $\delta = 10$, while the profile of magnetic field strength decreases along $B' \propto R'^{-2}$ and the electron luminosity is constant in time. Full lines represent analytical solutions, while dashed lines the corresponding numerical ones. (b) Numerical results of the photon SED for the time instances of the depicted electrons.

Fig. 2. Time evolution of the SED when all the physical processes are taken into account. The parameters have the values: $R'_0 = 10^{15}$ cm, $B_0 = 3.16$ G, $L_{\text{inj}}' = 10^{43}$ erg s$^{-1}$, $t'_{\text{esc},e} = R'/c$, $u_{\text{exp}} = 0.1$ c, $\gamma_{\text{min}} = 1$, $\gamma_{\text{max}} = 10^4$, $p = 2$, and $\delta = 10$. The magnetic field and electron luminosity profiles are $\propto R'^{-1}$. The colorbar refers to time, the time unit is given in days in the co-moving frame and it is on a logarithmic scale.

the low-frequency component is produced by synchrotron radiation while the higher one is produced due to Synchrotron Self-Compton (SSC). The synchrotron frequency peak is given by the expression $\nu_{\text{syn}}' \propto B'(R')\gamma_{\text{max}}^3$, so as the magnetic field strength decreases, the peak shifts to lower energies. The same behavior is shown by the Compton peak, which depends both on the
energy of synchrotron photons and on $\gamma_{\text{max}}$ through SSC mechanism. Also, the SSC luminosity decreases quadratically compared to the synchrotron one. Moreover, the spectrum shows a break at radio frequencies due to synchrotron self-absorption, which shifts to lower energies as time increases and the source expands. The slope of the spectrum below the synchrotron self-absorption frequency has the characteristic value of $\nu F_{\nu} \propto \nu^{7/2}$. At all frequencies, the spectral flux peaks at the beginning of the expansion, apart from the radio band which shows a peak at later times. The produced luminosity at these frequencies increases with time, reaching a peak at the transition from the optically thin to the optically thick region, and then decreases. We expand on this issue in the next section.

### 3. Localization of the onset of radio emission

Very-long-baseline interferometry (VLBI) studies have provided a great deal of information about the structure of the jets and, specifically, about pc-scale jets. As mentioned in the previous section, we can detect radio emission mainly from the regions where the source is optically thin at radio frequencies. In this section, we investigate the role of the expansion velocity on the transition from the optically thin to the optically thick region in the case of an expanding blob (Fig. 3).

In this case, the frequency below which the synchrotron radiation is absorbed can be derived by the condition for the optical depth $\tau_{\text{abs}} = \alpha'_{\text{abs}} R' \sim 1$, where $\alpha'_{\text{abs}} (t')$ is the absorption coefficient, for instance, in Eq. (6.53) from Rybicki & Lightman (1979). The synchrotron self-absorption frequency $\nu'_{\text{ssa}}$ depends on the specifics of the electron distribution and on the magnetic field. In our treatment, both of these depend on the radius of the blob and therefore on the co-moving time (Eqs. (4) and (5)).

Therefore, following the assumptions above, the expression of synchrotron self-absorption for our numerical model takes the form:

$$
\nu'_{\text{ssa}}(R') = \left[ k_1 k_2(p) \frac{\nu_0^{(0)} R_0^{\gamma/2}}{\rho_{\text{exp}}} \frac{1}{R'^2} \left( \frac{R_0}{R'} \right)^{\alpha_{\text{abs}}/2} \right]^{2/3} 
\times \left\{ \begin{array}{ll}
\left[ \nu_0 \ln \left( \frac{\nu_0}{\nu_{\text{ssa}}'} \right) \right]^{2/3} & \text{for } q = 1 \\
\left[ \frac{k_e}{1-q} \left( \frac{k_e}{\nu_{\text{ssa}}'} \right)^{q-1} - 1 \right]^{2/3} & \text{for } q \neq 1
\end{array} \right.
$$

where

$$
k_1 = \frac{3 \sqrt{3} q^3}{32 \pi^2 m_e}
$$

and

$$
k_2(p) = \left( \frac{3 q}{2 \pi m_e c^5} \right)^{2/3} \Gamma \left( \frac{3p + 2}{12} \right) \Gamma \left( \frac{3p + 22}{12} \right) \sin \frac{\pi q}{3} \alpha,
$$

where $\Gamma(y)$ is the Gamma function. The luminosity at the synchrotron self-absorption frequency becomes, for instance, as per Stawarz et al. (2008):

$$
[v' L'_{\nu}]_{\text{syn-ssa}} = \frac{c \sigma_T}{12 \pi m_e c^2} \frac{B^2(R')}{\nu_{\text{exp}}} \int_{\nu_{\text{max}}}^{\nu_{\text{min}}} \nu^{2} \nu' \mathrm{d} \nu' \left( \frac{4 \pi m_e c v'_{\text{ssa}}(R')}{3 e B(R')} \right)^{2/3}.
$$

where $\sigma_T$ is the Thomson cross-section.

As Eqs. (19) and (20) show, both the synchrotron self-absorption frequency and its corresponding luminosity depend on the initial values and profiles of the key parameters. As we show below, for all physical combinations of these, the source starts self-absorbed and at some point, it becomes transparent. Practically speaking, the distance $z_{\text{ssa}}(v')$ at which this occurs marks the onset of radio emission at frequency $v'$ and corresponds to the maximum of the lightcurve at that frequency, which can be found from Eq. (2) at a time $t'_{\text{ssa}}(v')$ defined by $\tau_{\text{ssa}}(t'_{\text{ssa}}(v')) = 1$. Therefore for $z > z_{\text{ssa}}(v')$ the source is optically thin.

An example of the above can be seen in Fig. 4, whichdepicts the radio lightcurve at 95 GHz as a function of time (as measured on Earth) for three different values of the injected electron luminosity in $\log L'_{\nu}^{(0)}$ (erg/sec). As we mentioned in Fig. 2, the radio luminosity increases initially with time, it reaches a peak during the transition from the optically thick to the optically thin regime and then decreases. The transition occurs later and is brighter for higher electron luminosities because the number of electrons increases with luminosity. This result is in qualitative agreement with Eqs. (19) and (20). As it turns out, the slopes of the lightcurves depend on the profile of the magnetic field strength (e.g., for $B' \propto R^{-2}$ the decay is faster in comparison with $B' \propto R^{-1}$) and of the electron luminosity.

Figure 5 depicts the luminosity at 95 GHz as a function of $z$ when all parameters are kept constant except for the expansion velocity (Fig. 5a) and the initial value of the magnetic field (Fig. 5b). Figure 5a shows that as the velocity increases, the transition from the optically thick to the optically thin regime occurs at smaller distances, a result that is expected from Eq. (19).

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6 Here, we neglect energy losses and electron escape from the source.

7 When we use the symbols $F_{\nu}$, $L_{\nu}$ where $i$ is a specific photon energy/frequency, we present the flux/luminosity for this specific energy/frequency, respectively.
Qualitatively this is to be expected as all the physical quantities change faster; therefore the transition occurs closer to the central engine.

Figure 5b that examines the effects of the blob’s initial magnetic field $B_0'$ on the transition distance $z_{\text{trans}}$, shows that the transition to the optically thin regime occurs at higher distances as $B_0'$ increases, while the luminosity during the transition also increases. It is interesting to note that transition occurs around the parsec scale for the most commonly assumed parameters. Finally, we should also mention that we find a very similar behavior if we were to increase, instead of $B_0'$, the initial electron luminosity.

4. Simulating flaring episodes in blazars

Blazars are characterized by their variability across the electromagnetic spectrum on time scales from minutes to years. Every flaring episode has its characteristic signature, a fact that makes its modeling a challenge. There are many studies in the literature (e.g., Camenzind & Krockenberger 1992; Wagner & Witzel 1995; Bhatta et al. 2013; Marscher 2014) that are based on the flare’s dependence on the accretion disk and jet plasma variations. Here, we use a simplified way to simulate flaring activity and we present two approaches: radio flares produced due to the expansion of the blob and flares, at all frequencies, produced as a result of particle reaccelerating episodes. In this section, we focus not only on radio flares, but we also examine higher frequencies.

4.1. Flares by energetic blobs

The continuous injection of a series of expanding blobs could create emissions at all values of $z$. If these blobs had similar initial conditions, then the superposition of their emission at all distances could create a steady-state, quiescent jet (e.g., Potter 2018; Bolla et al. 2019b). Supposing that one of the blobs has different initial properties than the rest (which make the steady-state emission), this leads to a disturbance in the emission that will propagate with the particular blob down the jet. In the case where the initial parameters of this disturbance are such that it could lead to higher emission than that of the surrounding blobs, then a flaring episode will ensue and it is possible to calculate the time-dependent emission from it.

According to our analysis in Sect. 2, for conditions where both the electron injection luminosity and the magnetic field drop outwards (i.e., $q > 0$ and $s > 0$), the high-energy photons show a peak very early, close to the base of the jet – see Fig. 2. This occurs because the choice of the parameters makes
radiation losses more important at the beginning of the expansion. At radio frequencies, however, the peak of the corresponding lightcurve occurs at much larger distances when the blob has reached the region of the transition from the optically thick to the optically thin regime – see Sect. 3. The details of the shape of the radio flare will depend on many parameters, such as the magnetic field strength and the electrons’ power profile on the distance. These parameters, along with the expansion velocity, determine in principle the time-lags between higher frequencies which show a maximum essentially at the base of the jet and the radio frequencies which have their maximum much further out (see also Boula et al. 2019b).

The above can be encoded in Fig. 6, which depicts the distance where the peak of the radio flare occurs as a function of the initial magnetic field value for two magnetic field profiles and two initial electron injection luminosities. The size of the circles scales with the peak radio luminosity at 95 GHz, while the color encodings shows the peak of γ-ray luminosity between 100 MeV and 10 GeV – we note that the γ-ray peak is not contemporaneous to the peak of the radio flare as it has occurred at earlier times, soon after the injection of the energetic blob.

Several conclusions can be drawn from Fig. 6. Radio flares tend to be more energetic and occur at larger distances along the jet when $B_0'$ is higher (results already drawn from Fig. 5b). Furthermore, an anti-correlation between the peak luminosities in γ-ray and radio is found. This is to be expected because for the same injected electron luminosity, higher values of $B_0'$ tend to suppress the γ-ray which are produced by the SSC process. Furthermore, the source becomes optically thin at smaller distances when the magnetic field strength and electron power decrease faster with radius. Finally, when the initial power of the injected electrons becomes higher, then all quantities increase but in a slower way than the linear (compare Figs. 6a and b).

$^8$ In the present analysis, high-energy photons are produced by synchrotron self-Compton; thus, these results refer mainly to BL Lac objects, where the external photon fields are negligible.

4.2. Flares by reaccelerating episodes

An alternative way to produce a flare is to consider a reaccelerating episode in an expanding blob when this is at a certain distance, $z_*$, from the origin. As in the previous sections, we have not included an acceleration mechanism; instead, we approach this case by increasing the injected power of the relativistic electrons through a Lorentzian function in time:

$$Q_\ell(\gamma, t) = q_\ell(t)\gamma^{-p} \left(1 + \frac{\alpha w^2}{4(R' - t')^2 + w^2}\right)$$

$$= q_\ell 0 \left(\frac{R}{R'}\right)^q \gamma^{-p} \left(1 + \frac{\alpha w^2}{4(R' - t')^2 + w^2}\right),$$

where $\alpha$ is the value at maximum, $w$' the width of the injection, and $t'$ the time of the maximum that corresponds to $z_*$.

The Lorentzian form of the injection is partly motivated by the shape of the pulses that are observed (see also Sikora et al. 2001; Wang & Kusunose 2002).

This assumption allows us to examine the impact of our free parameters, especially the expansion velocity, on the shape of the flares and to study the interplay between radiative and adiabatic losses.

Figure 7 shows the role of $t_{\text{exp}}$ on the pulse shape. Here, all parameters remain constant between cases except of $t_{\text{exp}}$. The light curves depict the behavior of γ-ray in a reaccelerating episode which occurs at a certain $t'$ that is the same for all cases. This means that episodes with smaller velocities produce flares closer to the origin, where both the magnetic field strength and the electron power are larger. This explains the fact that the γ-ray luminosity is decreasing with increasing $t_{\text{exp}}$. At the same time, low expansion velocities produce more symmetrical flares because the cooling timescale is $t_{\text{cool}} \ll t_{\text{dyn}} = t_{\text{esc}} = cB_0/(\gamma_0^2 M)$, where $t_{\text{dyn}}$ is the dynamical timescale. On the other hand, fast expansion has a large impact on the cooling timescale because the magnetic field drops substantially as the reaccelerating episode occurs at larger distances and, as a result, the flare appears both asymmetric and extended.
4.3. Application to the flare of Mrk421

Mrk421 is one of the most well-studied blazars. In April 2013, the source was in intense activity recorded from X-rays to TeV energies. Following this detection the Combined Array for Research in Millimeter-Wave Astronomy (CARMA) observations started. According to Hovatta et al. (2015) the activity of the source in the γ-ray started in March 2013, while the light curve reached its peak on April 14, 2013. The photon flux almost quadrupled within 30 days, while the total activity of the event was about 100 days. In radio frequencies, the intense activity started in April and reached its maximum flux in June with an increase of about a factor of 1.5. In radio frequencies, the source was recorded at 15 GHz and 95 GHz with a slight lag of the two peaks and a complete correlation between them. However, in the data analysis, it appeared that there was also a correlation between γ-ray\(^9\) and radio.

Motivated by this analysis, we model the observational data as recorded in the 2013 Mrk 421 flaring activity. Our approach to studying this particular flare is to apply the method described earlier in this paper. An energetic plasma blob is produced at a distance \(z_0 = 0.1\) pc from the origin and moves along its axis. The γ-ray (Fig. 10a) are produced by a particle reaccelerating episode at \(z_\gamma = 0.6\) pc through the SSC process. At this distance, the jet is opaque to radio emission due to the synchrotron self-absorption. However, as the plasma blob expands, it becomes optically thin and the radio waves at 95 GHz escape at \(z_{\text{esc}}(95\,\text{GHz}) \approx 1.1\) pc from the origin and this transition is recorded as a flare at the specific frequency (see Fig. 10b). The total electron energy required to power this flare is \(\sim 10^{49}\) erg. More blobs could explain the rest activity of Mrk421. Despite the fact that the fit is not perfect, it should be compared to a similar fit presented in Fig. 4 of Hovatta et al. (2015), which was obtained using the non-expanding version, Mastichiadis & Kirk (1995) of the present code.

\(^9\) Data by Fermi γ-ray telescope.
The code, by construction, does not provide a steady-state solution but rather a series of snapshots of the photon spectrum (total injected power, upper and lower energy cutoffs, and slope in the case of a power-law distribution). Despite the fact that there are ways that to make relations between at least some of these parameters physically (see, e.g., Boula et al. 2019b), for the purpose of the present paper, we used a more generic approach and we have left them as free. We note that we do not vary the shape of the electron distribution (high and low energy cutoffs plus a power-law slope) with the distance because this would have added more free parameters or, alternatively, it would have required a detailed acceleration scheme that we have not included in the present version of the code.

As a first example, we apply the expanding code to treat the radio emission of blazars. This is usually problematic for the non-expanding one-zone models since they provide fits that require a small size for the source and, thus, a high electron number density. This leads, in turn, to an intense synchrotron self-absorption of the radio photons and one requires a different region to produce them (e.g., Marscher 1980; Ghisellini et al. 1985; Marscher & Gear 1985). With the present code, we show that while high-energy photons can be produced close to the base of the jet where the bulk of the injection power takes place, radio photons of some frequency appear when the radius of the blob is such that the optical depth at that particular frequency drops below unity. This occurs at much larger distances from the origin, where the blob has opened up and both the electron density and the magnetic field have dropped. We find that this distance depends on many parameters, but the rule of thumb is that it appears further away from the origin for higher initial values of the electron luminosity and magnetic field strength. If therefore, both of these quantities drop outwards, it is expected that high-energy radiation will be produced very close to the site of the blob formation, while radio photons appear much further away.}

We should also emphasize that adiabatic losses can become dominant as the blob expands and this has severe consequences
on the radiative efficiency of the electrons; in particular, at large distances from the origin, even freshly injected electrons become inefficient emitters. This can be seen mostly at higher frequencies, even during the duration of a flare. Generally speaking, we find that more asymmetric flares imply high expansion velocities since parameters such as the magnetic field can vary a great distance from the origin, even freshly injected electrons become effective emitters. This can be seen mostly at higher frequencies, even during the duration of a flare. Therefore, it is conceivable that, for instance, a $\gamma$-ray flare starts when electrons are dominated by radiative losses and ends when they are in the adiabatic phase\textsuperscript{11}. In the non-expanding case, such asymmetries can be achieved only by fine-tuning the electron escape timescale from the source. However, we should mention that we have not considered the light travel time effects between photons emitted from the back and the front side of the blob, which are also expected to play a role in the flare shape, Zacharias & Schlickeiser (2013).

As a second example, we have added an injection of relativistic electrons in the form of a Lorentzian, which occurs on top of the power-law decay of electron power that we used in the previous example. We use this approach in order to study the relation of the produced $\gamma$-ray and radio flares. Even this simple case shows clearly that one can detect both $\gamma$-ray and radio in the case when the reaccelerating episode occurs in a region that is optically thin to radio frequencies. If the episode occurs further in, only $\gamma$-ray (and other high-frequencies) flares are detected but not radio flares\textsuperscript{12}. While it is possible for one to make a parametric study, here, we have refrained from doing so due to the large number of parameters involved, which would have obscured our results. However, as we have shown in Boula et al. (2019b) that a wide range of time lags is expected when both $\gamma$-ray and radio flares appear. As a final example, we have shown a fit to a particular radio and $\gamma$-ray flare of Mrk 421 that occurred in 2013, described in greater detail in Hovatta et al. (2015).

While this code is best suited to follow the radiative patterns of accelerating episodes as they move down the jet, it does not provide the SED for a global jet. We could envisage a steady-state jet spectrum by a superposition of the spectra of blobs created at a constant rate with the same initial conditions that emit continuously as they travel along the jet Boula et al. (2019b) but here we do not deal with this option. It ought to be mentioned that there are other numerical approaches to calculate the produced photons in the case of blazar jets (e.g., Potter & Cotter 2012; Tramacere et al. 2011; Zacharias et al. 2021). Specifically, Potter (2018) presented similar results using a different geometry, however, for the emitting region.

Concluding we can say that the expanding one-zone leptonic model presented in this paper is inherently different from the usual non-expanding one. While the latter has fewer free parameters and can reach a steady-state, the former provides perhaps a more realistic prescription for the blob evolution as it takes into account expansion, fully fledged adiabatic losses and the light-crossing time of a varying source. However, this happens at the expense of more free parameters that could partly obscure the physical mechanisms at the source – this is particularly true in the case where a flaring episode is simulated. In the present paper, we show only a few characteristic examples emphasizing the connection between radio and $\gamma$-ray frequencies and we have drawn some general conclusions. We believe that other conclusions can be drawn only by fitting data on a source-to-source basis using both temporal and spectral information.

6. Conclusions

In this paper, we present a new numerical code that describes a one-zone leptonic expanding model and we use it to investigate some general trends of the multwavelength activity of

\textsuperscript{11} Electron physical escape might also play a role; see the analysis presented in Wang & Kusunose (2002).

\textsuperscript{12} On the other hand, if we suppose that the high-frequencies peak luminosities are below the steady-state emission, it is possible only the radio to appear above the steady-state and be detected as a flare.
blazars. More precisely we assume a spherical blob of plasma that expands and propagates down along the blazar jet’s axis. We self-consistently solve the time-dependent integro-differential kinetic equations that describe the evolution of particles and photons by taking the leptonic physical processes into account and by assuming that all physical quantities are related to the comoving time. By applying this code to blazar jets we are able to:

- localize the distance of radio emission onset depending on the fundamental physical quantities of the source, such as the magnetic field, the electron luminosity and the initial source’s radius at the base of the jet.
- connect radio and γ-ray flares by assuming electron accelerating episodes.

This numerical tool can be used for a comprehensive study of flaring activity, as in the case of blazars, both for BL Lac objects and Flat Spectrum Radio Quasars (where we consider external photon fields). It may also be applied to compact jetted sources, such as active galactic nuclei, in general, as well as micro-quasars, tidal disruption events, and gamma-ray bursts.

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