Generation of a Large-scale Magnetic Field in a Convective Full-sphere Cross-helicity Dynamo

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Abstract

We study the effects of the cross-helicity in the full-sphere large-scale mean-field dynamo models of a 0.3 $M_\odot$ star rotating with a period of 10 days. In exploring several dynamo scenarios that stem from magnetic field generation by the cross-helicity effect, we found that the cross-helicity provides the natural generation mechanisms for the large-scale scale axisymmetric and nonaxisymmetric magnetic field. Therefore, the rotating stars with convective envelopes can produce a large-scale magnetic field generated solely due to the turbulent cross-helicity effect (we call it $\gamma$-dynamo). Using mean-field models we compare the properties of the large-scale magnetic field organization that stems from dynamo mechanisms based on the kinetic helicity (associated with the $\alpha^2$ dynamos) and cross-helicity. For the fully convective stars, both generation mechanisms can maintain large-scale dynamos even for the solid body rotation law inside the star. The nonaxisymmetric magnetic configurations become preferable when the cross-helicity and the $\alpha$-effect operate independently of each other. This corresponds to situations with purely $\gamma$ or $\alpha^2$ dynamos. The combination of these scenarios, i.e., the $\gamma \alpha^2$ dynamo, can generate preferably axisymmetric, dipole-like magnetic fields at strengths of several kGs. Thus, we found a new dynamo scenario that is able to generate an axisymmetric magnetic field even in the case of a solid body rotation of the star. We discuss the possible applications of our findings to stellar observations.

Key words: dynamo – magnetic fields – stars: activity – stars: low-mass

1. Introduction

It is widely accepted that the magnetic activity of late-type stars is due to the large-scale hydromagnetic dynamo that results from the actions of differential rotation and cyclic turbulent motions in their convective envelopes (Parker 1979; Krause & Rädler 1980). Solar and stellar observations show that the surface magnetic activity forms a complicated multi-scale structure (Donati & Landstreet 2009; Stenflo 2013). The large-scale organization of the surface magnetic activity on the Sun and other late-type stars could be related to starspots (Berdyugina 2005). Currently, there is no consistent theory that simultaneously explains the large-scale magnetic activity of the Sun and the emergence of sunspots at the solar photosphere. However, these two phenomena can be modeled separately. Moreover, there is no consensus about the details of the origin mechanisms of the large-scale magnetic activity of the Sun and solar-type stars. The models of flux-transport dynamos and the concurrent paradigm of distributed turbulent dynamos are outlined in Charbonneau (2011), Brandenburg & Subramanian (2005), and Pipin (2013). The origin and formations of sunspots and starspots are extensively studied as well (see, e.g., Cheung et al. 2010; Kitiaishvili et al. 2010; Stein & Nordlund 2012; Warnecke et al. 2013).

The results of direct numerical simulations of solar-type stars and M-dwarfs (Brown 2008; Brown et al. 2011; Yadav et al. 2015; Guerrero et al. 2016; Warnecke et al. 2018) show that magnetic field and turbulent convective flows are highly aligned near-surface layers. Generally, it is found that in the regions occupied by the magnetic field, the cross-helicity density $\langle \gamma \rangle = \langle u \cdot b \rangle$ is not zero. Here, $\mathbf{u}$ and $\mathbf{b}$ are the convective velocity and fluctuating magnetic field. Alignment of the velocity and magnetic field is found inside sunspots (Birch 2011). Analysis of the full-disk solar magnetograms shows that, similar to the current helicity (Zhang et al. 2010), the cross-helicity can follow the hemispheric rule (Zhao et al. 2011). In other words, the signs of $\langle \gamma \rangle$ in the north and south hemispheres can be opposite. The existence and origin of the hemispheric rule for the cross-helicity on the Sun are still an open question. The spotiness (or spot-filling factor) and magnetic filling factors of the fast-rotating solar analogs are estimated to be much larger than those for the Sun (Berdyugina 2005). The same is true for the fully convective stars. However, the physical properties of starspots may change with a decrease of the stellar mass (see the above-cited review). The results of stellar magnetic cartography (the so-called ZDI methods) showed that the fast-rotating M-dwarfs demonstrate the strong large-scale dipole (or multi-pole) poloidal magnetic field of strength >1 kG (Morin et al. 2008). Using the solar analog and scaling the sunspot properties to starspots on the M-dwarfs, we could infer that the magnetic fields on those starspots are accompanied by the cross-helicity density magnitude that is observed in sunspots. This leads us to question how the cross-helicity could affect the large-scale dynamo on these objects.

After the work of Krause & Rädler (1980) it was understood that alignment of the turbulent convective velocity and the magnetic field is typical for the saturation stage of the turbulent generation, due to the mean electromotive force (EMF), $\mathbf{E} = (\mathbf{u} \times \mathbf{b})$. This consideration does not account for the effects of cross-helicity that take place in the strongly stratified subsurface layers of the stellar convective envelope. Direct numerical simulations show the directional alignment of the velocity and magnetic field fluctuations in the presence of gradients of either pressure or kinetic energy (Matthaeus et al. 2008). Dynamo scenarios based on the cross-helicity have been suggested in a number of papers (Yoshizawa & Yokoi 1993; Yoshizawa et al. 2000; Yokoi 2013).
In the current framework of dynamo studies, the effects of non-uniform large-scale flows are taken into account only through the differential rotation or so-called $\Omega$-effect ($\nabla \times (\mathbf{U} \times \mathbf{B})$), where $\mathbf{U}$ and $\mathbf{B}$ are the large-scale shear flow and the large-scale magnetic field, respectively. In marked contrast to this differential rotation effect, the non-uniform flow effect has not been considered in the turbulence effects on the mean-field induction represented by the turbulent EMF $\mathbf{E} = \langle \mathbf{u} \times \mathbf{b} \rangle$. This is typically seen in the form of the "anzatz" in which the turbulent EMF is a linear combination of the mean magnetic field and its derivatives:

$$\mathbf{E} = \alpha_{ij} B_i + \beta_{ijk} \partial B_j/\partial x_k + \cdots,$$

which corresponds to a dropping of the mean velocity $\mathbf{U}$ in the fluctuation equations. Because of the Galilean invariance of the fluctuation equations, translational motion can be eliminated from the fluctuation equations. However, strain and rotational motions cannot be eliminated from the fluctuation equations.

The results of Yoshizawa & Yokoi (1993) showed that in the presence of the global rotation, the turbulent cross-helicity, defined by the correlation between the velocity and magnetic field fluctuations ($\langle \mathbf{u} \cdot \mathbf{b} \rangle$), should naturally enter the expression of the turbulent EMF as the coupling coefficient for the mean absolute vorticity (rotation and the mean relative vorticity; Yokoi et al. 2016). This suggests that, for a rotating stellar convection zone, the turbulent dynamo mechanism arising from the cross-helicity should be taken into account, as well as the counterparts of the turbulent magnetic diffusivity and turbulent helicity or the so-called $\alpha$-effect.

The cross-helicity may be particularly interesting if considering the dynamo mechanisms operating in fully convective stars. For convenience, we note the physical mechanism behind magnetic field generation via the cross-helicity effect. An illustration of this mechanism is shown in Figure 1(a). We consider a statistically aligned turbulent velocity and magnetic field in the plane that is perpendicular to the rotation axis. The Coriolis force acting on the turbulent motion results in the mean EMF along the rotation axis, $\mathbf{E}^\gamma = \langle \mathbf{u} \times \mathbf{b} \rangle$ (see Figure 1(a)). This EMF can generate the large-scale toroidal magnetic field due to the $\nabla \times \mathbf{E}^\gamma$-term in the induction equation.

An alternative mechanism is given by the $\alpha$-effect, (see Figure 1(b)). In this case, the mean EMF results from the large-scale axial magnetic field and the cyclonic motions in the plane perpendicular to the rotation axis. Note that for a regime of fast rotation, the energy of the turbulent vortices across rotation axis is suppressed. Therefore, the axisymmetric $\alpha^2$-dynamo cannot use the axial magnetic field for the dynamo generation (Rüdiger & Kitchatinov 1993).

Our consideration hints that the cross-helicity effect can generate the axisymmetric magnetic field even in the case of the quasi-2D turbulence that is expected for the fast-rotating M-dwarfs. The differential rotation of the fast-rotating M-dwarfs is rather small (Donati et al. 2008a; Morin et al. 2008). The direct numerical simulations of Browning (2008) show an absence of differential rotation in the magnetic case. Therefore, we can expect that the axisymmetric magnetic field is likely generated by the turbulent mechanisms with no regards to the large-scale shear flow. As mentioned above, the $\alpha$-effect is unlikely to support the axisymmetric dynamo for the case of fast-rotating M-dwarfs.

In this paper, the cross-helicity effects are studied in the full-sphere nonaxisymmetric dynamo. We identify the different dynamo scenarios and study the magnetic properties of the fully convective stars using the nonlinear dynamo models.

2. Basic Equations

The mean-field convective dynamo is governed by the induction equation of the large-scale magnetic field $\mathbf{B}$,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{E} + \mathbf{U} \times \langle \mathbf{B} \rangle),$$

where $\mathbf{E} = \langle \mathbf{u} \times \mathbf{b} \rangle$ is the mean EMF of the turbulent convective velocity, $\mathbf{u}$, and the turbulent magnetic field, $\mathbf{b}$. The mean EMF includes the generation effects due to the helical turbulent flows and the magnetic field, and the generation effect due to the cross-helicity. It also includes the turbulent pumping, and the anisotropic (because of rotation) eddy diffusivity, etc. For convenience, we divide the expression of the mean-EMF into two parts:

$$\mathbf{E} = \mathbf{E}^{\alpha, \beta} + \mathbf{E}^\gamma,$$

where $\mathbf{E}^{\alpha, \beta}$ includes the standard contributions of the mean-EMF, such as the $\alpha$ effect, the turbulent magnetic diffusivity and etc.; $\mathbf{E}^\gamma$ results from the cross-helicity effect (Yokoi 2013, hereafter Y13):

$$\mathbf{E}^\gamma = C_\gamma \frac{\Omega}{\Omega} \langle \mathbf{g} \rangle f_\gamma (\Omega^* \psi_\gamma (\beta),$$

where $\langle \mathbf{g} \rangle = \langle \mathbf{u} \cdot \mathbf{b} \rangle$, $\Omega^* = 4\pi \kappa_c / P^*$ is the Coriolis number, $\tau_c$ is the convective turnover time, and $P^*$ is the period of rotation of a star. We employ a simplified approach to describe the nonlinear feedback due to the Coriolis force and the large-scale magnetic field. A more elaborated framework was presented in Y13. In our study, these effects will be treated in a simplified
way via quenching functions \( f_\psi (\Omega^o) \), and \( \psi_\alpha (\beta) \). The function \( f_\psi (\Omega^o) \) takes into account the nonlinear effect of the Coriolis force in the fast rotation regime. The function \( \psi_\alpha (\beta) \), where \( \beta = \langle |B| \rangle \sqrt{4\pi \mu u^2} \), and \( u' \) is the rms of the convective velocity, describes the magnetic quenching of the cross-helicity dynamo effect. Those quenching functions will be specified later.

\[ \mathcal{E}^{\alpha,n,V} \] includes the other common contributions of the mean-EMF. It is written as follows:

\[ \mathcal{E}^{\alpha,n,V} = \hat{\alpha} \circ (B) + V(r) \times \langle B \rangle - (2\eta_\tau + 2\eta)^{(l)} \nabla \times \langle B \rangle - 2\eta_\tau \frac{\Omega}{\Omega^2} \cdot (\nabla \times \langle B \rangle). \]

(4)

It is convenient to divide the large-scale magnetic field induction vector for axisymmetric and nonaxisymmetric parts as follows:

\[ \langle B \rangle = \hat{B} + \hat{B}, \]

\[ \hat{B} = \hat{b} B + \nabla \times \left( \frac{A \hat{a}}{r \sin \theta} \right), \]

\[ \hat{B} = \nabla \times (\hat{r} T) + \nabla \times \nabla \times (\hat{r} S), \]

(7)

where \( \hat{B} \) is the axisymmetric part, and \( B \) is nonaxisymmetric part of the large-scale magnetic field; \( A, B, T, \) and \( S \) are scalar functions; \( \hat{a} \) is the unit vector in the azimuthal direction, and \( \hat{f} \) is the radius vector; \( r \) is the radial distance, and \( \theta \) is the polar angle. The cross-helicity pseudo-scalar is decomposed to the axisymmetric and nonaxisymmetric parts, as well:

\[ \langle \gamma \rangle = \gamma + \tilde{\gamma}. \]

(8)

For the nonaxisymmetric part of the problem we employ the spherical harmonics decomposition, i.e., the scalar functions \( T, S, \) and \( \tilde{\gamma} \) are represented as follows:

\[ T(r, \mu, \phi, t) = \sum \hat{T}_{lm}(r, t) T_{lm}^m \exp (im\phi), \]

\[ S(r, \mu, \phi, t) = \sum \hat{S}_{lm}(r, t) T_{lm}^m \exp (im\phi), \]

\[ \tilde{\gamma}(r, \mu, \phi, t) = \sum \hat{\gamma}_{lm}(r, t) T_{lm}^m \exp (im\phi) \]

(11)

where \( T_{lm}^m \) is the normalized associated Legendre function of degree \( l \geq 1 \) and order \( m \geq 1 \). Note that \( \hat{S}_{l,-m} = \delta_{l,m} \), which is also true for \( \tilde{T}_{l,m} \) and \( \tilde{\gamma}_{l,m} \).

The equations governing the evolution of the axisymmetric part of the magnetic field are as follows:

\[ \frac{\partial B}{\partial t} = \frac{1}{r} \left( \frac{\partial B_{\alpha,n,V}}{\partial r} + \sin \theta \frac{\partial B_{\alpha,n,V}}{\partial \mu} - \sin \theta \frac{\partial (\Omega, A)}{\partial (r, \mu)} \right) - \frac{1}{r} \left( \frac{\partial \mathcal{U}_B}{\partial r} - \sin \theta \frac{\partial \mathcal{U}_B}{\partial \mu} \right) \]

\[ - C_s \sin \theta \left( \frac{\partial}{\partial r} \langle r \gamma \psi_\alpha (\beta) \rangle f_\psi (\Omega^o) \right) \]

\[ - \frac{\partial}{\partial \mu} (\mu \langle r \gamma \psi_\alpha (\beta) \rangle f_\psi (\Omega^o)), \]

(12)

\[ \frac{\partial A}{\partial t} = r \sin \theta \mathcal{E}_{\phi,n,V} - (\mathcal{U} \cdot \nabla) A, \]

(13)

where all the parts of \( \mathcal{E} \), except the cross-helicity effect, are written in symbolic form (see, e.g., Yokoi et al. 2016). Equation (13) does not include a contribution from the cross-helicity. This results in a difference in the cross-helicity dynamo for the axisymmetric and nonaxisymmetric magnetic fields. Another important observation is that the cross-helicity dynamo can contribute to generation of the axisymmetric toroidal magnetic field even when the axisymmetric cross-helicity is zero. It results from condition \( \tilde{\gamma}_\psi_\alpha (\beta) \neq 0 \) for the nonlinear case in the presence of the axisymmetric and nonaxisymmetric magnetic field.

To get the equation for the functions \( S \) and \( T \) we follow the procedure, which is described in detail by Krause & Rädler (1980). For example, to get the equation for the scalar \( S \) we take the scalar product of Equation (1) with \( \hat{f} \), and for the equation governing the scalar function \( T \) we do the same after taking the curl of the Equation (1). Therefore, we will have

\[ \frac{\partial \Delta_0 S}{\partial t} = \hat{f} \cdot \nabla \times (\mathcal{E}^{\alpha,n,V} + \mathcal{U} \times \langle B \rangle), \]

\[ \frac{\partial \Delta_0 T}{\partial t} = \hat{f} \cdot \nabla \times \nabla \times (\mathcal{E}^{\alpha,n,V} + \mathcal{U} \times \langle B \rangle), \]

(14)

(15)

where \((1/r^2)\Delta_0 \) is the Laplace operator on the surface \( r = \text{const.} \)

With the contributions of the cross-helicity, Equations (14) and (15) are rewritten as follows

\[ \frac{\partial \Delta_0 S}{\partial t} = \hat{f} \cdot \nabla \times \left( \mathcal{E}^{\alpha,n,V} + \mathcal{U} \times \langle B \rangle \right) + C_s f_\psi (\Omega^o) \frac{\partial}{\partial \phi} (\gamma \psi_\alpha (\beta)), \]

\[ - \frac{\partial \Delta_0 T}{\partial t} = \hat{f} \cdot \nabla \times \left( \mathcal{E}^{\alpha,n,V} + \mathcal{U} \times \langle B \rangle \right) \]

\[ - C_s f_\psi (\Omega^o) \frac{\Delta_0 (\mu \gamma \psi_\alpha (\beta))}{r} \]

\[ + \frac{C_s}{r} \frac{\partial}{\partial \mu} \left( \sin^2 \theta \frac{\partial}{\partial \mu} r^2 f_\psi (\Omega^o) \psi_\alpha (\beta) \right). \]

(16)

(17)

Some details for deriving Equations (16) and (17) are provided in the Appendix. From these equations, we see that the nonaxisymmetric part of the cross-helicity is coupled with the evolution of the nonaxisymmetric magnetic field. This can provide the dynamo instability of the large-scale nonaxisymmetric magnetic field. In particular, the nonaxisymmetric magnetic field can be generated solely due to the cross-helicity dynamo effect.

In the general case, all coefficients in Equation (2) depend on the Coriolis number \( \Omega^c \equiv 4\pi \tau_c / P^c \), where \( \tau_c \) is the convective turnover time and the \( P^c \) is the period of rotation of a star. Also, the magnetic feedback on the generation and transport effects in the \( \mathcal{E} \) should be taken into account. For the case of \( \Omega^c \gtrsim 1 \), the \( \alpha \)-effect tensor can be represented as follows (Rüdiger & Kitchatinov 1993; Pipin 2008):

\[ \alpha_{ij} \approx c_\mu u' |A^{(o)}| \cos \theta \psi_\alpha (\beta) f_s^{(a)} (\Omega^o) \left\{ \delta_{ij} - \frac{\Omega_i \Omega_j}{\Omega^2} \right\}, \]

(18)

where \( A^{(o)} = \nabla \log p \) is the gradient of the mean density. Although \( f_s^{(a)} (\Omega^o) \rightarrow \pi/2 \) when \( \Omega^o \gtrsim 1 \), we will keep the
dependence on the Coriolis number for the nonlinear solution. For the case \( \Omega^* \gg 1 \), the magnitude of the kinetic part of the \( \alpha \) effect is given by

\[
\alpha_\alpha = \frac{\pi}{2} c_\alpha u_t |\mathbf{A}^\alpha|.
\]

(19)

The magnetic quenching function of the kinetic part of the \( \alpha \) effect is defined by

\[
\psi_\alpha = \frac{5}{128 \beta^3} \left( 16 \beta^2 - 3 - 3(4\beta^2 - 1) \arctan \left( \frac{2\beta}{3} \right) \right),
\]

(20)

where \( \beta = \left( \langle |\mathbf{B}|^2 \right)^{1/2} \). For the cross-helicity dynamo effect we assume that \( \psi_r = \psi_\alpha \). The given assumption is because of a lack of an appropriate mean-field alternative for the magnetic quenching of the cross-helicity dynamo effect. The theory of the cross-helicity effect, which was reviewed by Yokoi (2013), includes a detailed description of dynamical coupling of the turbulent cross-helicity evolution with the large-scale magnetic field and other quantities like the turbulent kinetic and magnetic energy, the kinetic and magnetic helicity, etc. However, implementation of that approach in the stellar dynamo models is a complicated task that we leave for future studies. Also, for the sake of simplicity, we skip the magnetic quenching, due to the magnetic helicity conservation (cf. Pipin et al. 2013).

The turbulent pumping of the mean field contains the sum of the contributions due to the mean density gradient (see, Pipin 2008, hereafter P08) and the mean-field magnetic buoyancy (Kitchatinov & Pipin 1993),

\[
V^{(p)} \times \langle \mathbf{B} \rangle = 3 \eta_\tau f_1^{(a)}(\Omega^*) \left( \frac{\langle \mathbf{B} \rangle \cdot \mathbf{A}^\alpha}{\Omega^2} \right) \Omega \times \langle \mathbf{B} \rangle
- \left( \frac{\langle \mathbf{B} \rangle \cdot \Omega}{\Omega^*} \right) (\Omega \times \mathbf{A}^\alpha)
+ \frac{\alpha_{\text{MLT}} u^t}{\Gamma_1} \beta^* K(\beta^*) \mathbf{g} \times \langle \mathbf{B} \rangle,
\]

(21)

where \( \alpha_{\text{MLT}} \) is the parameter of the mixing length theory, \( \Gamma_1 \) is the adiabatic exponent, the function \( K(\beta^*) \) is defined as in Kitchatinov & Pipin (1993), and \( \mathbf{g} \) is the unit vector in the radial direction. When \( \Omega^* \gg 1 \), we have \( f_1^{(a)}(\Omega^*) \rightarrow \pi / (8\Omega^*) \).

The function of the Coriolis number \( f_1^{(a)}(\Omega^*) \) is given in P08. The dependence of the Coriolis number \( f_1^{(a)}(\Omega^*) \) on the Coriolis number is as follows

\[
\eta_\tau = \frac{\eta_0}{\Omega^2} \left( 1 - \frac{\arctan \Omega^*}{\Omega^*} \right)
\]

\[
\eta_\tau = \frac{3 \eta_0}{4 \Omega^2} \left( \Omega^2 + 3 \right) \arctan \Omega^* - 3 \Omega^* - 3
\]

where the eddy magnetic diffusivity coefficient is defined as \( \eta_0 = \nu_0 / \text{Pm}_\tau \), with \( \nu_0 = u^t / 3 \) being the eddy viscosity.

The quenching of the cross-helicity dynamo for the fast-rotating case has not previously been studied. We will assume that the cross-helicity effect is quenched in the same way as the turbulent magnetic diffusivity coefficients, i.e., we put

\[
f_1(\Omega^*) = \frac{1}{\pi} \left( \frac{\arctan \Omega^*}{1 + \Omega^*} \right).
\]

(22)

Equation (22) affects the amplitude of the cross-helicity effect in the large-scale dynamo. Similar to the magnetic quenching of the cross-helicity generation effect (see the comment below Equation (20)), the given assumption is because of a lack of an appropriate mean-field alternative.

The evolution of the cross-helicity is governed by the conservation law

\[
\frac{\partial \langle \gamma \rangle}{\partial t} = \frac{1}{3\eta_\tau} \left( \langle \mathbf{B} \rangle \cdot \nabla \right) \mathbf{B} - 2 \mathbf{E} \cdot \mathbf{\Omega} + \eta_0 \Delta \langle \gamma \rangle.
\]

(23)

Under stellar conditions, the typical spatial scale of the density stratification is much less than the spatial scale of the mean magnetic field. Thus, the first term in Equation (23) dominates the second one. Neither rotation-induced anisotropy of the \( \alpha \)-effect, the eddy diffusivity, nor the pumping contribute to the cross-helicity generation. Substituting the general expression of the mean-EMF into Equation (23) we get

\[
\frac{\partial \langle \gamma \rangle}{\partial t} = \frac{\eta_0}{\tau_\gamma} \left( \langle \mathbf{B} \rangle \cdot \mathbf{A}^\alpha \right) + 2 \gamma_\tau \mathbf{\Omega} \cdot (\nabla \times \langle \mathbf{B} \rangle)
- \frac{\Omega_{\text{MLT}} u^t}{\Gamma_1} \beta^2 K(\beta^*) \sin \theta \langle \mathbf{B} \rangle + \eta_0 \Delta \langle \gamma \rangle.
\]

(24)

For the numerical solution, we reduce the equations to a dimensionless form. The radial distance is measured in units of solar radius, as expected for stellar astrophysics. Thus, we will have the following set of parameters: \( \Omega^* \), is the rotation rate of the star, \( \nu_0 \) is the magnitude of the eddy viscosity, and \( \text{Pm}_\tau \) is the magnetic Prandtl number.

The boundary conditions are as follows. The cross-helicity and magnetic field are put to zero at the inner boundary, which is close to the center of the star. For the top, we use the vacuum boundary conditions for the magnetic field. Formulation of the boundary condition for the cross-helicity at the top is still an open issue; we place the radial derivative to zero at the top. The impact of the cross-helicity boundary condition on the large-scale dynamo can be considered in a future work.

2.1. The Possible Dynamo Scenarios

The possible dynamo scenarios depend upon the magnetic field generation mechanisms, such as the \( \alpha \)-effect, the so-called \( \Omega \)-effect (associating with the differential rotation) and the cross-helicity (denoted as the \( \gamma \)-effect). Following the conventions of the dynamo theory (Krause & Rädler 1980), we can identify the following scenarios: \( \alpha^2 \), \( \alpha^2 \Omega \), \( \gamma^2 \), \( \gamma^2 \Omega \), \( \alpha^2 \gamma^2 \), and \( \alpha \gamma^2 \Omega \). More scenarios can be found in Krause & Rädler (1980). From the point of view of this study, the scenarios of \( \gamma^2 \), \( \alpha \gamma^2 \), and \( \alpha \gamma^2 \Omega \) are of particular interest. All of them depend on the cross-helicity generation governed by Equation (24).

Conceivable scenarios for the cross-helicity dynamos depend on the physical mechanisms of cross-helicity production. In the simplest scenario, the cross-helicity is generated from the axial current, e.g., the term \( 2 \eta_\tau \mathbf{\Omega} \cdot (\nabla \times \langle \mathbf{B} \rangle) \) on the RHS of Equation (24). If we consider the axisymmetric magnetic field, the \( \gamma^2 \) dynamo can generate the toroidal magnetic field from the cross-helicity effect. The poloidal field is decoupled from the system of the dynamo equation and it can have only a decaying solution. This scenario was discussed previously by...
Yoshizawa & Yokoi (1993) and Yokoi (2013) for the dynamo in accretion disks. In a stellar convection zone, the cross-helicity generation due to density stratification can be one of the relevant mechanisms, especially at the near-surface layers (Rüdiger et al. 2011). We assume this will be the primary mechanism for cross-helicity generation. This effect is accounted for by the first term in the cross-helicity evolution Equation (24). With regards to the density stratification, all the dynamo equations are coupled and there is a possibility a $\gamma^2$ dynamo. In this case, only the nonaxisymmetric modes can be unstable in the linear solution because the mean EMF $\mathcal{E} = -C_y \gamma^2 \tau \Omega + ...$ has no contribution in the equation for the axisymmetric poloidal magnetic field (associated with the potential $A$). In what follows we discuss $\gamma^2$, $a^2 \gamma^2$, and $a^2 \gamma^2 \Omega$ scenarios based on the cross-helicity production effect, which comes from the first term of Equation (24).

3. Results

3.1. The Eigenvalue Problem

For the linear eigenvalue problem we consider the simplified set of equations. We assume that the eddy magnetic diffusivity, $\eta_0 = \nu_0 / \text{Pm}_T$ with $\nu_0 = 5 \cdot 10^{10} \text{cm}^2 \text{s}^{-1}$, is constant over radial distance. This corresponds to a set of parameters in our model for the 0.3 $M_\odot$ star rotating with a period of 10 days. The density gradient scale has a sharp variation in the upper part of the star and it is nearly constant in depth. It was found that it is important to keep the spatial variations of the $\mathcal{A}^{(p)}$ for the cross-helicity evolution equation. For the eigenvalue problem we introduce a new variable, $\xi = R_\odot \mathcal{A}^{(p)}$, and employ the adiabatic profile of the density variation scale,

$$\xi(r) = \frac{1}{2} \frac{R_r R_\odot}{r (R_\odot - r)}.$$  \hspace{1cm} (25)

The parameter $\xi$ is nearly uniform in the bulk of the star, having $\xi \approx 10$, and it rapidly falls to $\xi \approx -500$ toward the surface. For the sake of simplicity, we put the constant $\xi = -50$ in the pumping terms. The amplitude of the $\alpha$-effect will be determined by the parameter $C_\alpha = \alpha_0 R_\odot / \nu_0$. An additional parameter is the ratio $C_\tau = R_\odot^2 / (\nu_0 \tau_c)$. It determines the generation of the cross-helicity. Therefore, in the linear problem, the reduced expression of the mean-EMF is

$$\dot{\mathcal{E}}_i = C_\alpha \text{Pm}_T^{-1} \{ \langle B \rangle - \frac{\Omega (\langle B \rangle \cdot \langle B \rangle)}{\Omega^2} \} + C_\tau \dot{\gamma},$$

$$+ \text{Pm}_T^{-1} \{ \frac{(\Omega \cdot \xi) \Omega \cdot \langle B \rangle}{\Omega^2} - (\Omega \cdot \langle B \rangle) (\Omega \cdot \xi) \}.$$

$$- \text{Pm}_T^{-1} (1 + 2a) \nabla \times \langle B \rangle + 2a \frac{\Omega}{\Omega^2} (\Omega \cdot (\nabla \times \langle B \rangle)).$$  \hspace{1cm} (26)

where $a = \eta_\tau / \eta_T$. In what follows we assume that $a = 1$. Also, in the linear problem the parameter $C_\alpha = C \tau_\odot \tau_c$ in Equation (26) absorbs the Coriolis number dependence. The hat sign in Equation (26) means that the mean EMF was scaled about $\nu_0$. The cross-helicity is governed by the equation

$$\frac{\partial \dot{\gamma}}{\partial t} = C_\tau \text{Pm}_T^{-1} \{ \langle B \rangle \cdot \xi \} + \text{Pm}_T^{-1} \Delta \dot{\gamma}.$$  \hspace{1cm} (27)

Our purpose is to investigate the eigenvalue solution of Equations (1), (26), and (27) for the set of parameters $C_\alpha$, $C_\tau$, and $C_T$. The effect of the differential rotation can be controlled by the angular velocity of the star and the distribution of the differential rotation. We use $C_T = 100$ because the typical diffusive timescale is of the order of 100 years and $\tau_c \approx 1$ year for this star. For the external layers of the star, the $C_T$ is much larger. We consider the profile of the differential rotation from our previous paper (Pipin 2017). It is illustrated in Figure 2.

In a linear solution, all the partial dynamo modes are decoupled. We restrict our discussion to a few partial modes of the large-scale magnetic field, including the axisymmetric modes $S_0$ and $A_0$ and the nonaxisymmetric modes $S_1$ and $A_1$. We follow the convention suggested by Krause & Rädler (1980): the letter “S” means the mode symmetrical about the equator and the letter “A” is for the antisymmetric mode.

With no regards to the cross-helicity dynamo effect, e.g., in the cases $C_T = 0$ or $C_\alpha = 0$, the large-scale dynamo instability is provided by the $a^2$ or $a^4 \Omega$ scenarios. For the $a^2$ scenario, the critical $C_\alpha$ does not depend on $\text{Pm}_T$. Also, in this scenario, the nonaxisymmetric modes are preferable, having thresholds at $C_{\alpha}^{(\text{s})} \approx 37$ for the $A_1$ mode and at $C_{\alpha}^{(\text{s})} \approx 42$ for the $S_1$ mode and for $a = 0$. The thresholds are about half a factor higher for $a = 1$ than for $a = 0$. This means that the additional diffusive mixing of the large-scale magnetic field quenches the efficiency of the dynamo mechanisms. In the $a^2 \Omega$ dynamo the instability...
depends largely on the parameter $Pm_T$, because this parameter controls the efficiency of the large-scale magnetic field stretching by the differential rotation. For $Pm_T = 20$, the axisymmetric modes are initially unstable, having thresholds around $C_{m(\gamma)} = 10$.

Magnetic field generation by the cross-helicity effect adds new parameters to the study. Results are shown in Figures 3(a) and (b). We restrict the study by fixing the $C_{m}$ parameter below the dynamo thresholds of the $\alpha^2$ and $\alpha^2\Omega$ dynamo regimes. We use $C_{m} = 10$ and the anisotropy parameter $a = 0$, and study the dynamo instability against the parameter $C_{b}$ for the variable magnetic Prandtl number $Pm_T$. Figure 3(a) shows results for the pure turbulent dynamo scenarios, i.e., the differential rotation is disregarded. It is found that the mode A1 keeps the least dynamo thresholds. Also, we found the $\gamma^2$ scenario has smaller dynamo instability thresholds for modes A1 and S1 than the $\alpha^2\gamma^2$ scenario. Therefore, we can conclude that the magnetic field generation by the concurrent cross-helicity and $\alpha^2$ dynamo reduces the efficiency of both dynamo mechanisms. The results of the linear analysis tells us that without the differential rotation, the nonaxisymmetric dynamo solution is preferable. We can conclude that during the linear stage of the dynamo process the $\gamma^2$ scenario does not give any preference for the axisymmetric magnetic field generation. Moreover, as was anticipated from the dynamo equations, the $\gamma^2$ scenario provides additional mechanisms for nonaxisymmetric magnetic field generation.

Figure 3(b) shows that, accounting for the differential rotation effect, i.e., considering the $\alpha^2\Omega\gamma^2$ dynamo scenario, the instability thresholds for all the modes are close to each other when increasing $Pm_T$. Also, the efficiency of the axisymmetric dynamo instability increases with increasing $Pm_T$. Within the studied parameter range of $Pm_T$, the nonaxisymmetric mode A1 remains the most unstable. We also studied instability for the spatially non-uniform density stratification parameter $\xi$. In this case the dynamo instability thresholds are about a factor of magnitude larger than those in the case of constant $\xi$. However, the order of the instability thresholds among the different partial dynamo modes remains the same as is shown in Figures 3(a) and (b).

3.2. The Nonlinear Solution

For the nonlinear solution, we employ the model that keeps the spatial dependence of the turbulent parameters provided by the MESA code and solution of the heat transport problem (see Pipin 2017). Using the results of the eigenvalue problem, we bear in mind that the parameters $C_b$ and $C_{\gamma}$ in Equations (26) and (27) absorb the dependence upon the parameter $\tau_c$. It is found that the Coriolis number parameter $\Omega^b = 4\pi \tau_c / P^\star$, where $P^\star = 10$ days, varies from about 1 near the surface to 200 in the depths of the star. This means that the critical threshold $C_{\gamma} = 2 C_b/\Omega^b \ll 1$ $\sim$ 0.01 for the $\alpha^2\Omega\gamma^2$ dynamo if $Pm_T = 3$. We use this $Pm_T$ in all models below. Three different models will be considered. Parameters of the models are listed in Table 1. The model M1 represents the $\gamma^2$ scenario, the model M2 represents the $\alpha^2\gamma^2$ scenario and M3 stands for $\alpha^2\gamma^2$. In the latter case, we disregard the contribution of the axisymmetric cross-helicity in the mean-EMF. This imitates the situation when the mean cross-helicity has no hemispheric sign rule. The nonlinear combination of the nonaxisymmetric magnetic field and cross-helicity can produce the axisymmetric magnetic field. The model M3 was introduced to study this situation. In this paper, we consider models with solid body rotation.

Table 1

| Model | Scenario | $C_b$ | $C_{\gamma}$ |
|-------|----------|------|-------------|
| M1    | $\gamma^2$ | 0.01 | ...         |
| M2    | $\alpha^2\gamma^2$ | 0.01 | 0.03        |
| M3    | $\alpha^2\gamma^2$ | 0.01 | 0.03        |

Regime $\gamma^2$ provides the simplest scenario of the cross-helicity dynamo. Contrary to the $\alpha^2$ dynamo, it excites only in the nonaxisymmetric regime. The $\gamma^2$ works only in the nonaxisymmetric regime. In this scenario, evolutions of the axisymmetric components of the toroidal and poloidal magnetic fields are decoupled. Figures 4 and 5 show evolutions of the partial modes in the model M1, as well as snapshots of the magnetic field and the cross-helicity distributions at the stationary stage of evolution. It is seen that the axisymmetric mode of the toroidal magnetic field evolves non-monotonically, showing growth at the beginning and eventually decaying. In the nonlinear case, the cross-helicity that is produced by the nonaxisymmetric magnetic field may contribute to the generation of the axisymmetric toroidal magnetic field, because in general $\langle \gamma \psi_\alpha(\beta) \rangle \neq 0$ (see Equation (12)). Therefore, the nonaxisymmetric cross-helicity affects the generation of the axisymmetric toroidal magnetic field. However, evolution of the axisymmetric poloidal magnetic field is decoupled from the toroidal magnetic field and the axisymmetric cross-helicity.
Therefore, there is no true axisymmetric dynamo in this case. The axisymmetric field starts to decay when parts of the product \(\gamma \psi, (\beta)\) get synchronized. Both the nonaxisymmetric cross-helicity density \(\gamma\) and magnetic field can be represented by the equatorial dipole, which changes orientation while rotating around the axis of stellar rotation. This phenomenon is known as the nonaxisymmetric dynamo waves.

The scenario of the \(\alpha \gamma^2\) dynamo has possible axisymmetric magnetic field generation. Figures 6 and 7 show the evolution of the partial modes in model M2, as well as snapshots of the magnetic field and the cross-helicity distributions at the stationary stage of evolution. We use the output of model M1 as an initial condition for model M2. Panels (a) and (b) of Figure 6 show that the axisymmetric toroidal magnetic field started to grow at the beginning phase, where it shows some oscillations. The dynamo solution reaches a stage with a constant dipole-like distribution at the end of the simulation. A similar behavior is demonstrated by the cross-helicity evolution. The cross-helicity has opposite signs in the northern and southern hemispheres. The polar magnetic field in model M2 reaches a magnitude of 2kG. At the end of the simulations, the model maintains a substantial nonaxisymmetric magnetic field. It has a strength that is one order of magnitude less than the axisymmetric magnetic field. Snapshots of the magnetic field and cross-helicity distributions show that these nonaxisymmetric components concentrate in the near equatorial regions. The field lines of the magnetic field distribution show that the overall configuration of the magnetic field is dipole-like both inside and outside the star.

Using the output of model M2 we conduct an additional run in which we neglect magnetic field generation by cross-helicity effects. This returns the dynamo model to the \(\alpha^2\) scenario. Similar to Pipin (2017) we find the nonaxisymmetric magnetic field at the end of the run. Also, we made additional runs with the decreased \(C_\alpha\). For the given parameter \(C_\alpha\) it was found that the model remains axisymmetric magnetic even in the case when \(C_\alpha\) is a factor of 2 less than it is in model M2.

The interesting question is if the axisymmetric dynamo can be sustained by the cross-helicity generation effect when the spatially averaged cross-helicity is zero. Model M3 illustrates...
this scenario. In this model, we disregard the contribution of the axisymmetric cross-helicity in the mean-EMF by neglecting the contributions of the axisymmetric magnetic field in the equation of the cross-helicity evolution. Results are shown in Figures 8 and 9. Results show that contrary to the pure $\gamma^2$ scenario, the axisymmetric magnetic field is generated. This means that the azimuthally averaged cross-helicity dynamo effect is not zero, $\langle \gamma \psi_r (\beta) \rangle \neq 0$, because the terms of the product $\langle \gamma \psi_r (\beta) \rangle$ are not synchronized in the azimuthal direction. It is caused by the nonlinear generation of the nonaxisymmetric magnetic field both by the $\gamma^2$ and the $\alpha^2$ mechanisms. We see that the strength of the axisymmetric and nonaxisymmetric toroidal magnetic field is the same by order of magnitude. The axisymmetric magnetic field shows the solar-like time–latitude evolution of the toroidal magnetic field inside the star. The radial magnetic field at the surface shows the dominant nonaxisymmetric magnetic field and the nonaxisymmetric distribution of the cross-helicity. During the nonlinear evolution the pattern of the magnetic field distribution shown in Figure 9(b) moves about the axis of rotation, representing the azimuthal dynamo wave. Also, it weakly oscillates around the perpendicular axis, which corresponds to the axis of the equatorial dipole. For this reason, the model can show the nearly axisymmetric configuration of the polar magnetic field during the minima and maxima of the axisymmetric magnetic field cycle. The frequency of the nonaxisymmetric $m = 1$ mode is twice that of the axisymmetric one.

4. Discussion and Conclusions

The physical origin of this cross-helicity effect lies in the combination of the local angular-momentum conservation in a
rotational motion and the presence of the velocity-magnetic field correlation (Yokoi 2013). Unlike the $\Omega$ effect, the cross-helicity effect does not depend on the particular configuration of the differential rotation. Provided that a finite turbulent cross-helicity exists, the cross-helicity effect should work in the presence of the absolute vorticity (rotation and relative vorticity). This means that we can expect the cross-helicity dynamo mechanism to work even in the case in which the differential rotation is negligibly small. How and how much cross-helicity exists in turbulence is another problem. In our models, the turbulent cross-helicity is generated by means of the large-scale magnetic field and density stratification. This generation mechanism was analytically found in a number of papers (Yokoi 1999, 2013; Pipin et al. 2011; Rüdiger et al. 2011) using the mean-field magnetohydrodynamics framework. Our results show that this turbulent cross-helicity generation effect results in a number of new dynamo scenarios.

It was shown that for the solid body rotation regime there are three possible dynamo scenarios: the $\gamma^2$-dynamo (pure cross-helicity dynamo), the $\alpha^2\gamma^2$-dynamo, and its modification—the $\alpha^2\gamma^2$ dynamo. The latter is operating from the purely nonaxisymmetric cross-helicity distribution. In the nonlinear case, both the $\gamma^2$ and the $\alpha^2$ dynamo scenarios sustain only the nonaxisymmetric magnetic field. For the $\gamma^2$ scenario the evolutions of axisymmetric components of the magnetic field are decoupled. Therefore, this regime cannot sustain the axisymmetric magnetic field against decay.

An interesting new effect was considered. It is found that, in general, it is possible to generate some axisymmetric magnetic field in a nonlinear regime if the spatial variations of the nonaxisymmetric distributions of the cross-helicity and magnetic energy are not synchronized in the azimuthal direction. This allows generation of the axisymmetric toroidal magnetic field due to the cross-helicity dynamo effect, because for the axisymmetric part of $\langle \gamma \psi_r(\beta) \rangle$ we have

$$\langle \gamma \psi_r(\beta) \rangle = \gamma \psi_r(\beta) + \gamma \psi_r(\beta) = 0,$$  

where the first term on the RHS is zero in the $\gamma^2$ regime. The second term of the RHS of this equation is not necessarily zero. The results for model M1 show that in the $\gamma^2$ scenario the $\gamma$ and $\psi_r(\beta)$ become synchronized. This prevents axisymmetric toroidal magnetic field generation. On the other hand, if there is an additional mechanism of nonaxisymmetric magnetic field generation, e.g., the $\alpha^2$ dynamo, then the axisymmetric dynamo can be excited even if the axisymmetric part of the cross-helicity is zero. This was demonstrated by model M3 using the $\alpha^2\gamma^2$ scenario. There, the effect of the axisymmetric cross-helicity generation was disregarded. The given mechanism shows a mixture of the nonaxisymmetric and axisymmetric modes, with the equatorial dipole-like mode being dominant.

One of the most important findings of our work is that the axisymmetric magnetic field can be generated through cross-
helicity generation and $\alpha^2$; in this case we employ the standard formulation of the mean EMF suggested by Yoshizawa et al. (2000). The dynamo mechanism operates with regards to the axisymmetric and nonaxisymmetric cross-helicity generation. In this case, the strong axisymmetric dipole-like magnetic field is generated. Unlike the nonlinear $\alpha^2\Omega$ regimes (cf. Pipin 2017), this scenario produces the constant in a timed magnetic field configuration that is antisymmetric about equator cross-helicity, toroidal, and radial magnetic field distributions. Our scenario was demonstrated for the solid body regime. This means that it can be realized on the fast-rotating M-dwarfs with periods of rotation of about 1 day, which often show only a small amount of differential rotation (Donati et al. 2008a). Moreover, direct numerical simulations, e.g., Browning (2008), show suppression of the differential rotation in nonaxisymmetric regimes. For solid body rotation, the $\alpha^2$ dynamo produces the nonaxisymmetric magnetic field (Chabrier & Küker 2006; Elstner & Rüdiger 2007). This is because the $\alpha$-effect cannot use the component of the large-scale magnetic field along rotation for generation of the axial EMF, and this results from the anisotropic $\alpha$-effect in the case of a high Coriolis number (see Equation 18 and Rüdiger & Kitchatinov 1993). The cross-helicity can generate the poloidal EMF in this case (Yokoi 2013). This enables generation of the axisymmetric magnetic field by the $\alpha^2\gamma^2$-dynamo. The magnetic field configuration produced in our model of the $\alpha^2\gamma^2$-dynamo is very similar to those which found in fast-rotating M-dwarfs, e.g., V374 Peg and YZ CMi (Donati et al. 2008a, 2008b; Donati & Landstreet 2009).

This paper demonstrates the possibility of a cross-helicity dynamo in a rather simplified model. A number of simplifications were made. They should be relaxed in future studies. For example, for further application, there is a need for a better understanding of the cross-helicity generation effects in the case of fast rotation and a strong magnetic field. Also, the efficiency of the magnetic field generation by the cross-helicity should be compared with the standard dynamo mechanisms in a more realistic setup of the dynamo model, such as, for instance, with regards to the dynamo effect of the differential rotation and magnetic feedback on the global flows. Direct numerical simulations indicate that even a small amount of differential rotation, which is observed in fast-rotating stars, can maintain the axisymmetric $\alpha^2\Omega$ dynamo, e.g., Dobler et al. (2006) and Yadav et al. (2015). We think that reconstruction of the results of direct numerical simulations with the help of mean-field models (e.g., Schrinner 2011) and with regards to the cross-helicity effects, can reveal the potential feasibility of the cross-helicity dynamo in the interiors of fully convective stars. In our paper we employ a rather simplified approach to modeling magnetic field generation by cross-helicity effects for fast-rotating regimes. We hope that future work might shed more light about the usability of magnetic field generation through the cross-helicity effects in stellar dynamos.

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Appendix

To derive evolution equations for the nonaxisymmetric parts of the magnetic field, we use the approach suggested by Krause & Rädler (1980) and some useful identities (more of them can be found in their book). For any scalar functions $T$ and $S$ and radius vector $\hat{r}$ we have:

$$\nabla \times (\hat{r} T) = -\hat{r} \times \nabla T$$

$$\nabla \times \nabla \times (\hat{r} S) = \nabla \frac{\partial S}{\partial r} - r \Delta S,$$

$$\hat{r} \cdot \nabla \times \nabla \times (\hat{r} S) = -\Delta_0 S = -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial S}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 S}{\partial \phi^2}.$$

(29)

To derive Equation (14), we substitute $B = \hat{\phi} B + \nabla \times \left( \frac{\hat{\alpha} \phi}{r \sin \theta} \right) + \nabla \times (\hat{r} T) + \nabla \times \nabla \times (\hat{r} S)$ into the LHS of Equation (1) while taking into account nonaxisymmetry and Equation (29), and we get $\frac{\partial \phi B}{\partial \theta} = -\frac{\partial_0 \Delta S}{\partial \theta}$. For the cross-helicity contribution to the RHS of that equation we have:

$$\hat{r} \cdot \nabla \times (E') = C_\gamma \hat{r} \cdot \nabla \times \left( \frac{\Omega}{\Omega} \langle \gamma \rangle f_\gamma (\Omega^0) \psi_\gamma (\beta) \right)$$

$$= C_\gamma \hat{r} \cdot \nabla \times \left( \frac{\hat{r}}{r} \mu - \hat{\theta} \sin \theta \langle \gamma \rangle f_\gamma (\Omega^0) \psi_\gamma (\beta) \right)$$

$$= C_\gamma \nabla \cdot (\hat{r} \times \hat{\theta}) \sin \theta \langle \gamma \rangle f_\gamma (\Omega^0) \psi_\gamma (\beta)$$

$$= C_\gamma \frac{\partial}{\partial \phi} \langle \gamma \rangle f_\gamma (\Omega^0) \psi_\gamma (\beta),$$

where $\hat{\theta}$ is the unit vector along the polar angle coordinate.

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