Estimation of forest lands value by David Ricardo differential rent theory, linear programming model and GIS technologies

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Abstract. Integrated research approach joining the David Ricardo theory of differential rent, linear programming theory and GIS technology used for developing the methodology for differential rent determination and calculations for forest lands in the way similar as to agricultural ones. Special linear programming model was developed and resolved to obtain the rent estimations on the base of information offered by GIS data base on forest plots productivity and location regarding transportation network characteristics. Implementations of forest rent estimations for forest lands value assessments, forest management and economics was discussed. Rent values was calculated as an example for Lisino training and experimental forest of Saint-Petersburg State Forest Technical University. As a result of implementation of rental approach for forest resources assessments the value of forest lands significantly elevates as much as approximately on 5 times. Forest rent as an extra income originated from forest plots natural characteristics suggested to be used as a main source of so-called “long-money” needed for investments in forestry especially in boreal zone with long rotation period.

1. Introduction

An important feature of forest resources is their spatial distribution over the territory, without the formation of deposits, as is the case with such natural resources as oil, gas, coal and others. In addition, forest areas have different productivity, which is manifested in the amount of accumulated wood stock by the maturity age.

According to David Ricardo theory [1] there is two types of differential rent determined as a super profit or extra income resulting from, first, because of high land natural productivity and good location in respect to transportation ways and, second, due to additional investments which elevate the land productivity. Theory of David Ricardo was developed and clearly demonstrated by excellent examples for agricultural lands of different quality and location features. In this paper we would like to apply David Ricardo's theory of differential rent for forest lands. Using special linear programming model [2-4] and GIS data base on forest lands characteristics the methodology for differential rent of first type determination will be presented as well as discussion on use of the rent application in forest management and economics. The article will show that the theory of linear programming, namely the duality theory, applied to a special model makes it possible to calculate rental estimates of forest plots in the form of shadow prices of restrictions on the area of plots of different productivity and location [5-8].
Joining in one approach David Ricardo differential rent theory, linear programming theory and GIS technology provides a significant synergetic research effect in the form of new insight on forest lands value, forest economics and management.

2. Methods and Materials
To determine the differential rent, a special economic and mathematical model has been developed, which allows for the most complete consideration of the location, productivity and quality characteristics of forest plots. The model allows for spatial optimization of the use of forest resources. As a result, the differential rent is determined as a result of solving a special linear programming problem. Rent estimates are calculated on the basis of forest management materials separately for forestry sections that differ in the prevailing species (coniferous, softwood, hardwood) and productivity (high -, medium -, low-productive).

The model has the following form:

\[
\begin{align*}
\sum_{i=1}^{N} g_i \cdot x_i & \rightarrow \text{max} \\
\sum_{i=1}^{N} a_i \cdot x_i & \leq b \\
0 \leq x_i & \leq S_i, i = 1, \ldots, N,
\end{align*}
\]

here, \( x_i \) – unknown variable to be determined - harvested area of compartment \( i, i = 1, \ldots, N \), ha; \( N \) – number of compartments prescribed for harvesting in the current year;

\( a_i \) – wood stock of compartment \( i \) measured in depersonalized cubic meters, m\(^3\)/ha, determined according to the forest management materials as the sum of the wood stocks of the constituent species, taking into account their share in the tree stand composition;

\( S_i \) – area of compartment \( i \), prescribed for harvesting in the current year, ha;

\( g_i = p_i a_i - c_i \) – net income from harvesting of wood on 1 ha of compartment \( i \), RUR/ha,

\( p_i \) – price of 1 depersonalized cubic meter of wood on compartment \( i \), RUR/m\(^3\), it is defined as the average value for species market wood prices, taking into account the species shares in the tree stand composition, size characteristics and merchantable classes;

\( c_i \) – cost of wood harvesting and transportation to agreed point of wood treatment for compartment \( i \), RUR/ha, determined as a sum of harvesting and transportation costs:

\[
\begin{align*}
c_i &= c_i^h + c_i^{tr},
\end{align*}
\]

here, \( c_i^h \) – cost of wood harvesting at the compartment \( i \), which depends on the amount of wood growing stock at the compartment, RUR/ha;

\( c_i^{tr} \) – cost of wood transportation from compartment \( i \) to agreed point of its subsequent processing which depends on transportation distance and number of cubic meters of wood to be transported, RUR/ha.

\( b \) – allowable cutting volume of wood for current year, m\(^3\).

The necessary initial data on the location and productivity of the compartments for calculating rent estimates should be obtained from the forest management database created in the GIS software, and data on market prices based on the analysis of statistics of local forest materials markets.

The objective function of the above model (1) and the restrictions have the following meaning: it is necessary to distribute the annual allowable cutting volume of wood by forest compartments in such a way as to ensure maximum profitability of forest use, while the size of use as a whole should not exceed the annual allowable value and by compartments – their areas.

Rent estimates of forest compartments can be obtained in the form of so called shadow prices of the area constraints of the linear programming model (1) described above, which are the solutions of
the dual linear programming model obtained simultaneously with the solution of the original one [9].

The dual problem has the following form:

\[
\begin{align*}
\text{min} & \quad w \cdot b + \sum_{i=1}^{N} u_i \cdot S_i \\
\text{subject to} & \quad w \cdot a_i + u_i \geq g_i, i = 1, \ldots, N \\
& \quad w \geq 0 \\
& \quad u_i \geq 0, i = 1, \ldots, N.
\end{align*}
\] (3)

Here, \(w, u_i\) – shadow prices of the constraints of direct linear programming model (1) that make sense of the rent estimates of wood cut down within allowable volume – \(w\), RUR/m³ and forest compartments – \(u_i\), RUR/ha.

The below analysis of the direct and dual linear programming models decisions will recover the rent sense of shadow prices for direct model constraints which totally coincides with differential rent of first type determination done in David Ricardo theory.

Let \(x^*, w^*, u^*_i\) – optimal decisions of direct and dual linear programming models (1) and (3) respectively, if so then according to linear programming theory two following cases are possible:

1) \(x^*_n < S_n\), that is, the compartment \(n\) is not used completely, then for such plots the equation is fulfilled, which indicates that the differential rent on such plots is not appears:

\[
u^*_n = g_n - w^* \cdot a_n = 0
\] (4)

From this equation \(w^*\) may be determined:

\[
w^* = \frac{g_n}{a_n}.
\] (5)

2) \(x^*_i = S_i\), then:

\[
u^*_i = g_i - w^* \cdot a_i = g_i - \frac{g_n}{a_n} \cdot a_i = \left(\frac{g_i}{a_i} - \frac{g_n}{a_n}\right) \cdot a_i,
\] (6)

here, \(u^*_i\) – is a rent estimate for forest compartment \(i\).

The further analysis allows us to establish that the rent estimate \(u^*_i\) includes two rental components: rent by location in respect to transportation ways and rent by productivity, both of which should be taken into account when organizing the rational use of forest resources.

Let the quantitative productivity of two forest compartments be the same, i.e. \(a_i = a_n = a\), then:

\[
u^*_i = g_i - g_n = p_i \cdot a_i - c_i - p_n \cdot a_n + c_n = a \cdot (p_i - p_n) + (c_n - c_i).
\] (7)

In this case, the rent assessment of the compartment is equal to the sum of its qualitative assessment, determined by the difference in the prices of the cubic meter and the difference in the costs of harvesting and transporting of wood on this forest compartment and the worthiest one. The difference in the wood price will optionally occur in the case of difference in the size and quality characteristics of the wood stock, such as the species composition of the stand and the yield of merchantable wood.

In the cost structure, there are two components associated with the cost of cutting down the wood stock and transport costs (2). If the wood stock on both compartments is equal harvesting cost also will be the same, if, in addition, the prices of the cubic meter of wood are equal, and we get:
That is, the rent estimate of the compartment will be equal to the difference in transport costs, which exactly corresponds to the classical definition of the differential rent by the location of the plot.

If the costs of harvesting and transporting of wood are the same in the analyzed and worthiest forest compartment, i.e. $c_i = c_n = c$, then the rental estimate has the following form:

$$u^*_i = a_i \cdot (p_i - p_n) + c \cdot \left( \frac{a_i}{a_n} - 1 \right)$$

(9)

The first term in this formula characterizes, as before, the quality of the wood stock on the plot $i$ in comparison with $n$, the second term will be greater than zero, only if $a_i > a_n$, i.e., with greater productivity of the plot in a quantitative sense (in terms of the number of cubic meters per hectare). If the prices of the cubic meter of this plot and the closing one are equal, then only the quantitative component will remain in the rent assessment, which also corresponds to the classical definition of the differential rent on the productivity of the forest plot.

Thus, in the general case, the values $u^*_i$ are the sum of the rent for the location and productivity of the forest compartment and they can be determined as a result of solving a special linear programming problem.

3. Results and Discussion

Rent estimates for the forest compartments of the exploitable part of the Lisinsky training and experimental forest were calculated. The determination of rent estimates for forest compartments is reduced to the solution of a special linear programming problem, which can be done using special software, for example MS Excel software package.

The resulting rent estimates have the dimension of RUR/ha and are specific, so for the entire object of calculation (for example, forest compartment), the total rent is determined by the following formula as the sum of products of the rent estimate of the compartments on its area designated for harvest:

$$R = \sum_{i=1}^{N} u^*_i \cdot x^*_i$$

(10)

The results of the calculations are presented in table 1. From table 1 it may be learned that total amount of annual rent $R$ for Lisino training and experimental forest is as much as 27 904 771.8 RUR, meanwhile mean forest rent per 1 hectare is equal to 158 459.8 RUR. Also table 1 data demonstrate that for 3 compartments forest rent is absent because forest on these areas not harvested at all (compartments 7 and 13) or only partly (compartment 8).

One of the most important applications of rental calculations is estimation of leasing rate for harvesting companies per cubic meter of harvested wood. To calculate the leasing rate $A$ per unit of wood resource on the basis of the differential rent, its value is determined as a ratio of total amount of annual forest rent and annual allowable cutting volume of wood:

$$A = \frac{R}{b},$$

here, $A$ – leasing rate per 1 cubic meter, RUR/m$^3$, $b$ - annual allowable cutting volume of wood, m$^3$.

Allowable cutting level of wood was established as much as 52500 m$^3$/year, if so

$$A = \frac{R}{b} = \frac{27904771.8}{52500} = 531.6 \text{ RUR/m}^3$$
This calculated on the base of differential rent approach leasing rate exceeds the local regional average by about 5 times.

Table 1. The results of rent estimates calculations for forest compartments.

| Compartment number | Area, hectares | Rent per 1 ha, RUR/ha | Rent per compartment, RUR |
|---------------------|----------------|------------------------|--------------------------|
|                     | Total          | Harvested              | Not harvested            |                           |
| 1                   | 18.0           | 18.0                   | 0.0                      | 30560.3                  | 5500084.8                |
| 2                   | 36.0           | 36.0                   | 0.0                      | 122246.2                 | 4400863.3               |
| 3                   | 13.0           | 13.0                   | 0.0                      | 206243.7                 | 2681168.7               |
| 4                   | 12.0           | 12.0                   | 0.0                      | 136295.0                 | 1635540.3              |
| 5                   | 11.0           | 11.0                   | 0.0                      | 94812.6                  | 1042938.7              |
| 6                   | 10.0           | 10.0                   | 0.0                      | 241687.2                 | 2416872.4              |
| 7                   | 10.0           | 0.0                    | 10.0                     | 0.0                      | 0.0                     |
| 8                   | 11.0           | 9.5                    | 1.5                      | 0.0                      | 0.0                     |
| 9                   | 9.5            | 9.5                    | 0.0                      | 304906.3                 | 2896609.7              |
| 10                  | 9.5            | 9.5                    | 0.0                      | 289326.8                 | 2748604.5              |
| 11                  | 10.0           | 0.0                    | 10.0                     | 0.0                      | 0.0                     |
| 12                  | 18.0           | 18.0                   | 0.0                      | 254560.5                 | 4582089.3              |
| 13                  | 8.1            | 0.0                    | 8.1                      | 0.0                      | 0.0                     |
| Sum or mean         | 176.1          | 146.5                  | 29.6                     | 158459.8                 | 27904771.8             |

It is interesting to note that the share of the calculated rent in the total income from the use of forests (80 834 296.3 RUR/year) is 34.5%, and the share of the calculated leasing rate in the price of an impersonal cubic meter of wood (1991.0 RUR/m³) is 26.7%, these indicates that currently forest resources are leased by private harvesting companies at a very low price.

The total rent for some forest area may be used for calculations of this forest area (land) sell price P by using well known formula for capitalization of annual amount of rent:

\[ P = \frac{R}{r}, \]

here, \( r \) – is rate of interest. As a result of implementation of rental approach for forest resources assessments the value of forest lands will significantly elevates. For Russian Federation this fact may be important because of possible forest lands privatization in the future which if so should be done on the base of forest lands real price.

4. Conclusion

Extrapolation of David Ricardo differential rent theory firstly developed for agricultural lands on forestry lands together with mathematical modeling and GIS technologies giving precise information on forest resources characteristics regarding its location and productivity seems to be perspective as a useful approach for forests value assessments especially taken into account the need for environment protection under current global problems such as possible climate changes.

The introduction of forest rent assessments in the practice of the organization of forestry and forest management [10-14] will contribute to:

- increasing the overall level of its profitability,
- equalization of economic conditions of management of loggers and other forest users,
- equal involvement in the economic turnover of all areas (allotments) of the forest fund,
- prevention of predatory exploitation of well-located and productive forest areas (allotments) of the forest fund.
The ability of forest resources use to offer an extra income in the form of differential forest rent should be taken into account when organizing market-based rational forestry. Forestry in boreal zone with long rotation periods, as long as 100 years in average, needs for long-term investments in forestry and require a special, proprietary source of financing so called “long money”, such a non-budgetary, non-inflationary source of funds may be rent payments [15].

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