Study on dynamic contact model and force transfer mechanism of hydraulic spherical keyway

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Abstract. In the design of hydraulic structures, keyway are often arranged between adjacent structures to increase their integrity. Spherical keyway is more and more used in the transverse joints of high arch dams. However, the layout of keyway is dense in transverse slot and the simulation workload is huge. Very few reports have been reported on accurately pointing out the working mechanism and force mechanism of transverse joint keyway in high arch dam during earthquake. In this paper, the contact force transmission characteristics of spherical keyway are studied and the contact mechanics equation of spherical keyway is derived. A three-dimensional finite element fine simulation model of spherical keyway is established by using parametric modeling technology, and high performance calculation is carried out by using parallel computing technology. Real-time dynamic simulation of the working behavior of spherical keyway in earthquake reveals the variation of contact position, contact stress and contact mode of spherical keyway in parallel and vertical joints. The research results lay a solid foundation for in-depth seismic response analysis and seismic safety design of high arch dams.

1. Introduction
The structures are usually divided into several pieces by the common construction joint in order to reduce the pouring strength and temperature stress. In the design and construction of large-scale water conservancy projects, which will weaken the integrity of the structure and affect the strength and rigidity of the structure adversely. In order to improve the integrity of the structure, the keyway is often provided on the seam surface to enhance the shear resistance of the seam surface. The hydraulic keyway mainly has triangular, trapezoidal, spherical type and so on. The dam is divided into a number of dam sections along the axis of the dam when designing a high arch dam and the spherical joints between the dam sections are often spherical. Because the horizontal and vertical dams are curved, the cross-section dimensions of the transverse joints are constantly changing, the spherical keyway is densely arranged, and the existence of the keyway greatly changes the force mode and deformation characteristics of the dam. Especially in earthquakes, the interaction between adjacent dam sections becomes very complicated and the deformation of adjacent structures on both sides of the seam surface varies and changes dynamically in real time, resulting in a very complicated contact pattern of the transverse keyway. It is very difficult to simulate the actual working state of the keyway. The measured records of high arch dams subjected to strong earthquakes show that the arch dams have obvious signs of opening and closing in the earthquake engineering. Tu J. [1], Zhang H.Y. et al [2] all considered the influence of transverse joints in the study of seismic performance of arch dams. Chen H.Q. mentioned in the literature [3] that considering only the high dam as the whole structure and ignoring the influence of its longitudinal and transverse joints can not truly reflect the stress state of
the dam in real earthquakes. Therefore, the study of the contact characteristics of the keyway and the actual dynamics of the earthquake process, to understand the transfer mechanism and working mode of the keyway spherical shape, is of great significance for the seismic resistance of the high arch dam.

The existing research on the transverse joint has two kinds of considerations: one is to consider the keyway as a flat seam model of the flat seam, considering the compression resistance and the shear resistance when the seam surface is closed, such as: considering the additional conditions of the keyway mechanics and it is introduced into the contact mechanics constitutive model or the establishment of contact equivalent units [5]–[7]. The second method is to equivalently combine several or dozens of densely arranged keyways into one. Some simple formulas are derived for the contact of equivalent keyway and the shearing effect is analyzed by contact mechanics, which is mostly rectangular and trapezoidal keyways, etc. [8], [9]. For the flat seam model, the slip along the river direction will be significantly reduced during the ground motion due to the shearing ability of the keyway between the rivers[4], which is difficult to reflect the shearing effect of the keyway.

The keyway type of high arch dam in China has been dominated by spherical keyway since the Ertan dam. There is no specific literature on force transmission mechanism and the working mode of the high arch dam spherical keyway. The main reason is that the high arch dam has a large body shape and complex structure, and the calculation workload is very large. The real-time dynamic deformation and contact force transmission modes of the adjacent dam sections of the dam during the current earthquake have not been clearly understood. In order to study the working mechanism of the spherical keyway in the earthquake process, the characteristics of the keyway contact force transmission of the joint surface were studied in detail, and the mechanical equation of the spherical keyway contact was derived. Parametric modeling technology is used to simulate the spherical keyway in terms of shape, size, number, position and working mode, and parallel computing technology is used for high performance calculation. The calculation model and calculation method can dynamically simulate the working behavior of the spherical keyway in earthquake in real time, revealing the working mechanism and force transmission mechanism of the spherical keyway. The research work in this paper lays a solid foundation for in-depth analysis of seismic response of high arch dams considering transverse joints and seismic design.

2. Keyway dynamic contact analysis model

2.1 Contact Dynamics Theory

The contact state discrimination method in the dynamic problem is the same as in the static force case, and the form of the dynamic equilibrium equation in the n+1th incremental step is as shown in equation (1):

\[ \mathbf{M} \ddot{\mathbf{u}}_{n+1} + \mathbf{D} \dot{\mathbf{u}}_{n+1} + \mathbf{K} \mathbf{u}_{n+1} = \mathbf{F}_{n+1} + \mathbf{f}_{n} + \Delta \mathbf{f}_{n} \]  

(1)

Where: \( \mathbf{M} \), \( \mathbf{D} \), \( \mathbf{K} \) are structural mass matrix, damping matrix, and stiffness matrix respectively and the damping matrix is assumed by Rayleigh proportional damping[11]. \( \dot{\mathbf{u}}_{n+1}, \mathbf{u}_{n+1}, \mathbf{u}_{n} \) are acceleration, velocity, displacement array respectively. \( \mathbf{F}_{n+1} \) is external load array. \( \mathbf{f}_{n} \) is upper incremental step contact Force array. \( \Delta \mathbf{f}_{n} \) is the contact force increment array added in the incremental step of this level. Equation (1) uses the Newmark method to discretize the dynamic equilibrium equation in time domain as shown in equation (2) [12]:

\[ \mathbf{K} \Delta \mathbf{u}_{n} = \Delta \mathbf{F}_{n} + \Delta \mathbf{f}_{n} \]

\[ \mathbf{K} = (1.0 + \alpha \beta_{1} \Delta t) \mathbf{M} + (\beta_{1} \beta_{2} \Delta t + \beta_{2} \Delta t^{2}) \mathbf{K} \]

\[ \Delta \mathbf{F}_{n} = \mathbf{F}_{n+1} + \mathbf{f}_{n} - \left( \dot{\mathbf{u}}_{n} + \alpha \dot{\mathbf{u}}_{n}^{p} \right) \mathbf{M} - \left( \mathbf{u}_{n}^{p} + \beta \dot{\mathbf{u}}_{n}^{p} \right) \mathbf{K} \]

\[ \mathbf{u}_{n}^{p} = \mathbf{u}_{n} + \Delta \mathbf{u}_{n} + 0.5 \Delta \mathbf{u}_{n}^{2} \]

\[ \dot{\mathbf{u}}_{n}^{p} = \dot{\mathbf{u}}_{n} + \Delta \dot{\mathbf{u}}_{n} \]
Where: \( \mathbf{K} \) is the equivalent stiffness matrix. \( \Delta \mathbf{F}_a \) is the equivalent external load delta array. \( \Delta t \) is the time increment. \( \mathbf{u}_n^p \), \( \dot{\mathbf{u}}_n^p \) are the discrete displacement and velocity array. \( \alpha \), \( \beta \), \( \beta_1 \), \( \beta_2 \) are a constant [13]. After the equation (3) is discretized, the initial contact and the initial separation contact correction are also required by the augmented Lagrangian method [14], [15].

2.2 Spherical keyway contact mechanics model

In this paper, the contact model in ABAQUS is used to introduce contact as a constraint [10]. When considering the initial gap between the transverse joints, due to the discontinuity of the structure, dynamic opening, closing, shearing and contact will occur under the ground motion load. In the dynamic contact process, the contact behavior should satisfy the assumption that the contact master and the slave face are non-intrusive and non-overlapping, and the normal behavior satisfies the relationship between the contact compressive stress and the opening degree \( D \), that is \( p \cdot D = 0 \) \((p > 0, D > 0)\), the tangential behavior satisfies the coulomb friction relationship curve is as shown in equation (3):

\[
\tau = -\mu \cdot p \cdot \text{sgn}(u)
\]  

(3)

Where \( \tau \) is the tangential frictional stress. \( \text{sgn} \) is the sign function that indicates the direction of the frictional stress is opposite to the velocity direction. \( \mu \) is the friction coefficient and \( u \) is the velocity.

Since the spherical keyway have the same shear resistance in all directions, their shear behavior is similar to that direction of the river. A schematic view of a cut surface of a keyway passing through a center plane parallel to the shear direction, as shown in Fig. 1. The radius of spherical keyway is \( R \), the heightof keyway is \( h \), and the inter-groove gap is \( d \). There are three contact states during the keyway motion: 1) When \( d \leq 0 \), the spherical keyway is completely closed; 2) When \( d > R \cdot (1 - \cos \beta) \), the spherical keyway is completely separated from the corresponding spherical groove, and the spherical keyway does not cause contact behavior; 3) When \( 0 \leq d \leq R \cdot (1 - \cos \beta) \), the spherical keyway may undergo rotation in the spherical core surface in this state, and the contact portion of the black thick line as shown in Fig. 1 may be brought into contact.

In the third state, the contact problem of the spherical keyway can be simplified into the point-to-contact problem in the black thick line region, and the pitch of the point pairs in the tangent plane is divided into a normal pitch \( d_n \) and a tangential distance \( d_u \), \( d_v \). The variation of the normal and tangential spacing during the dynamic contact behavior is \( \Delta d_n \), \( \Delta d_u \), \( \Delta d_v \) respectively. From the equation (4), the normal contact forces \( f_n \) and tangential contact forces \( f_u \), \( f_v \) are obtained. Contact generation produces three states during the movement, namely: 1) When opening degree \( d + \Delta d > 0 \), contact is open state; 2) When opening degree \( d + \Delta d = 0 \), contact is closed state; 3) When opening degree \( d + \Delta d = 0 \), \( \sqrt{f_u^2 + f_v^2} = \mu \cdot f_n \), contact is slip state.

The contact between the contact planes of the respective tangent planes with the spacing parallel to the tangent plane is \( -R < n < R \). The total contact force of the keyway is obtained and the shear resistance of the keyway is as shown in the equation (4):

\[
F_i = \int_{-R}^{R} \left( f_u^i + f_v^i + f_n^i \right) dn
\]  

(4)

Where: the summation symbol \( \sum \) represents the sum of the contact forces of all pairs of contact segments and the contact force superscript \( y \) represents the projection of the contact force \( f \) in the tangent plane against the shear direction \( Y \) direction. \( dn \) indicating integration in the direction of the spacing.

2.3 Finite element parametric modeling and contact algorithm

In this paper, the parametric modeling technology is used to establish the refined keyway geometry model. Based on the above-mentioned keyway contact mechanics model, the contact surface discrete
method of face-to-face discrete method is used to simulate the contact behavior of the spherical keyway and the simulation finite element model is established accordingly.

Parametric modeling of the keyway geometry model bases on Solidworks software which is not only one of the BIM modeling software, but also the most installed modeling software today. Through the automatic dimension association of the model part sketch, the model can be parametrically modeled to quickly build the model part, and the complex structure can be modeled by reassembly. Using the professional meshing software Hypermesh to mesh the keyway geometry model completed in Solidworks, select different 2D meshing and 3D mesh projection methods, and use its built-in grid inspection tool to evaluate and optimize the quality of meshing, in order to reduce the probability of error when solving subsequent convergence problems, the full hexahedral refined mesh of the overall model can be realized, and the overall flow chart is shown in Fig. 2.

In general, the calculation accuracy of the surface-to-surface discrete method is higher than that of the point-to-surface discrete method, and the selection of the master-slave surface has little effect on the calculation result compared with the point-to-surface discrete method. Therefore, the error of the point-to-surface discrete method can be reduced in the ground motion analysis. The friction slip formula adopts the finite sliding method, which mainly uses the path-based tracking algorithm to consider the relative displacement path of the nodes in the incremental process, which is beneficial to the calculation and convergence problem when the incremental displacement is large. In order to simulate the real situation, the law is inaccessible. The contact normal behavior adopts the hard contact behavior, and the tangential direction adopts the Coulomb friction law, that is, the frictional coefficient is expressed by the friction coefficient.

3. Force transfer mechanism of spherical keyway subjected to static load

3.1. Establishment of a parametric model

In this paper, two adjacent concrete columns with spherical keyways arranged between the joints are studied, and the contact behavior under static load and earthquake action reveals the working behavior of the spherical keyway. The size of column P₁ (left) and P₂ (right) are both 5×6×50 m, and spherical keyways are arranged in the adjacent faces. The column and coordinate system are as shown in Fig. 3. The size of a single spherical keyway is shown in Fig. 4. The initial gap of the column is 2 cm, and the number of spherical keyways is 6×50, a total of 300. The above parametric modeling technique is used to carry out the fine division of the column mesh, and the three-dimensional finite element model is shown in Fig. 5. The contact was subjected to the contact mechanics model described above with a friction coefficient \( \mu = 0.7 \) [2]. Both columns are made of concrete C25 material with a density of 2400 kg/m\(^3\), an elastic modulus of 28GPa and a Poisson's ratio of 0.167. For the sake of simplicity, the static load is taken as follows: a horizontal line load \( F \) along the X-axis direction is applied to the left side of the top of the column \( P₁ \), \( F = 500 \) kN/m.
3.2 Force transfer mechanism of spherical keyway

Fig. 6. shows the calculation results of the contact stress on the joint surface. The results show that the left columnar joint keyway only has the top row of keyway contacts, which is located in the bottom area of the row of keyways. The contact area has stress concentration, and the maximum values of contact normal stress and tangential stress are 17.28 MPa, 11.76 MPa respectively. None of the remaining rows of keyways are in contact state. It can be seen from the contact mode and the contact stress that a columnar body with a spherical keyway surface with equal spacing and initial clearance is likely to have only a small amount of keyway contact under static load, and these keyways may be shear damaged due to excessive loading. Therefore, the working characteristics of the keyway and the force transmission mode are closely related to the arrangement and shape of the keyway, especially the loading of the structure which further illustrates the importance of simulation analysis.

4. Force transfer mechanism and working mode of spherical keyway during earthquake

4.1 Earthquake input

In order to investigate the working mode of the spherical keyway under dynamic load, some changes were made to the above static model. The density of the P1 column is set to be twice that of the original static example, which is 4800 kg/m³. The purpose is to investigate the seismic response caused by the two columns with different vibration frequencies during the earthquake. The dynamic elastic modulus of the two columns is taken as 1.5 times of the static condition according to the seismic specification, and the values are all 42 GPa, and the remaining material parameters remain unchanged. The artificial seismic waves generated by the design response spectrum are input from the X-direction and the Z-direction at the bottom of the column. The time history curve of the seismic wave is shown in Fig. 7. The total calculation time is 20s and the step length is 0.01s.

4.2 Z-direction earthquake

It is mainly considered that the two columnar bodies are displaced due to the inconsistent vibration frequency, resulting in different Z-direction vibration magnitudes. When solving the natural vibration frequency of the system, the bottom of the column is fully constrained, and the X-direction displacement of the external x-direction surface of the column is constrained. The natural vibration frequency is solved and the Rayleigh damping value of the system is calculated. The calculation
results show that when the ground motion is close to 12 seconds, the deformation of the top of the column is large, and the two columns are dynamically changed during the Z-direction earthquake. Fig. 8. shows three states of contact normal stress change within 0.04 s between the 11.53 s and 11.57 s of the P1 column joint surface. The contact behavior of the spherical keyway occurs in the top region of the column. At 11.53s, the keyway at the upper right corner of the seam surface is the most stressed, and the remaining keyways are successively decreased. The keyway in the upper left corner is not in contact, indicating that there is a clockwise twisting tendency in the XZ plane of the P1 column and the amount of twisting is different, referring to Fig. 8(a). At the end of the 11.55s, the two rows of keyways are in contact with each other, and the contact stress of the same row of keyways is relatively uniform, indicating that the top Z displacement of the two columns is equivalent, and the torsion in the XZ plane of the column is small, referring to Fig. 8 (b). At 11.57s, the maximum value of the contact keyway contact normal stress is transferred to the upper left corner. And the rest keyway is successively decremented while the keyway on the top right is not in contact, and the contact state is somewhat opposite to that at 11.53s, referring to Fig. 8 (c). It can be seen that when the two column with different vibration frequencies vibrate laterally, due to the vibration displacement is not synchronized, and the column will also have a certain torsional deformation under the shearing action of the keyway. Therefore, it can also reflect the mutual shearing effect of the arch dam along the river to the adjacent dam section during the earthquake, which enhances the integrity of the dam to some extent.

![Figure 8. P1 column local contact normal stress contours(Pa).](image)

4.3 X-direction earthquake

Fig. 9. shows the contact surface normal stress of the joint keyway of the column P1 under the X-direction earthquake between 11.66s and 11.68s. At 11.66s, the seam contact behavior occurs in the upper part of the column, with approximately 4 rows of keyways in contact. The top keyway is most stressed, decreasing from top to bottom, as shown in Fig. 9 (a). At 11.67s, the entire seam surface is almost completely contacted except for the top row and the bottom row of keyways. At this time, the shear resistance of the seam surface is very strong, as shown in Fig. 9 (b). At 11.68s, all the keyways of the seam surface are not in contact. It can be seen that the shearing capacity is the worst at this time, and the adjacent columns have no effect on each other, as shown in Fig. 9 (c). Seismic analysis of the X-direction shows that the columns will be completely closed and fully open during the earthquake. The adjacent columns will have a reciprocating interaction in the X-direction earthquake, but the time interval is very short, which is only 0.02s. It can be extended that the high arch dam may open and close between the dam sections during strong earthquakes, but the time interval is short and may not cause serious damage.
5. Conclusion
In this paper, the contact force characteristics of spherical keyway are studied and the mechanical equation of spherical keyway contact is derived. Using parametric modeling technology, a three-dimensional finite element simulation model of spherical keyway is established, and parallel computing technology is used for high performance calculation. Real-time dynamic simulation of the working behavior of the spherical keyway during earthquakes reveals the working mechanism and force transmission mechanism of the spherical keyway. The main conclusions are as follows:

1. The position, stress and mode of contact between keyway are constantly changing during the earthquake.
2. The contact state of the spherical keyway is highly correlated with the direction of seismic action. Earthquakes in parallel with the seam surface mainly cause keyway contact at the top region of the seam surface and may cause torsion of adjacent structures. An earthquake perpendicular to the seam surface, which may cause repeated opening and closing of the seam surface.
3. Due to the high-frequency characteristics of the earthquake, the time interval between repeated opening and closing changes or reciprocating torsion of adjacent structures is very short, in this case, the columnar body of 50 m height is only about 0.01 s to 0.02 s. The research results of the spherical keyway contact model and the force transmission mechanism can be extended to the seismic analysis of the high arch dam.
4. The research results of the spherical keyway contact model and the force transmission mechanism can be extended to the seismic analysis of the high arch dam.

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References
[1] Tu J., Chen H.Q., et al. (2001)Study on the simulation scheme of transverse joint in nonlinear seismic response analysis of high arch dam. J. Hyd. Eng., (02):18-25.
[2] Zhang H.Y., Zhang L.J. (2011)Study on seismic behavior of high arch dam with transverse joints based on virtual crack model. J. Hyd. Eng., 30(06):148-152+165.
[3] Chen H.Q. (2017)Challenges in seismic design of high concrete dams. Hydropower and Pumped Storage, 3(02): 1-13+49.
[4] Chen D.H., Bao T.F., et al. (2018)Dynamic analysis of a double curved arch dam considering the influence of transverse joints. Hydropower Energy Science, 36(04): 61-64.
[5] Zhang L.J, Fu Z.X, et al. (1997)Study on the influence of cross construction joints on the integrity of concrete structures. J. Hyd. Eng., (03): 21-26+64.
[6] Zhang L.J. (2005)Mechanical model of hydraulic structure contact problem and its application in the Three Gorges Project. Hohai University.
[7] Zhang C.H., Pan J.W. (2009) Influence of seismic input mechanisms and radiation damping on arch dam response. Soil Dynamics & Earthquake Engineering, 29(9): 1282-1293.

[8] Zhang D. (2007) Analysis of the influence of arch dam transverse joint and keyway form on seismic response of dam. Dalian University of Technology.

[9] Du C.B., Jiang S.Y. (2010) Analysis of the influence of arch dam transverse joint keyway on seismic response of arch dam. J. Hyd. Eng., 29(05):1-5+21.

[10] Li X.Y. (2013) Study on the influence of seam setting on seismic performance of arch dam. Northwest A&F University.

[11] Li X.J., Lu W. (2009) Explicit Finite Element Analysis of Seismic Responses of Underground Powerhouse Caverns of Hydroelectric Power Stations. J. Hyd. Eng., 28(05):41-46.

[12] Zhao L.H., Li T.C., et al. (2007) A dynamic contact model for nonlinear seismic response of high arch dam with transverse joints. J. Hyd. Eng., (04):91-95.

[13] Katona MC, Zienkiewicz OC. (1985) A unified set of single step algorithms part 3: The beta-m method, a generalization of the Newmark scheme. International Journal for Numerical Methods in Engineering, 21(7): 1345-1359.

[14] Jiang Y.S., Su C. (2010) Numerical analysis method of engineering contact problems. J. Hyd. Eng., 36(04): 75-78+94.

[15] Chen B.K., Li R.F. (1996) Some Notes on Finite Element Method for Impact-Dynamic Contact Problem. CHN J. Com. Mec., (02):248-252.