On white holes as particle accelerators

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We analyze scenarios of particle collisions in the metric of a nonextremal black hole that can potentially lead to ultrahigh energy $E_{c.m.}$ in their centre of mass frame. Particle 1 comes from infinity to the black hole horizon while particle 2 emerges from a white hole region. It is shown that unbounded $E_{c.m.}$ require that particle 2 pass close to the bifurcation point. The analogy with collisions inside the horizon is discussed.

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I. INTRODUCTION

Several years ago, an interesting effect was discovered. It turned out that if two particles collide near a rotating extremal black hole, the energy $E_{c.m.}$ in their centre of mass frame can become unbounded [1]. This is called the Bañados-Silk-West (BSW) effect, after the names of its authors. Later on, this effect was generalized to nonextremal horizons [2], generic rotating black holes [3] and even nontotating charged ones [4]. In all these cases it is implied that both particles move towards the horizon as usual for black holes.

Meanwhile, there are also scenarios with head-on collisions when one of particles moves away from the horizon. They were mentioned cursorily in Sec. II G of [5] for the rotating case although the term ”white hole” was not used there. The detailed coherent treatment of this type of scenario was done in [6] where the role of white holes was stressed and it was noticed that unbounded $E_{c.m.}$ appear even for the Schwarzschild metric. As is known, the

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spacetime of an eternal black hole includes inevitably two regions - black hole and white hole ones. In the scenario considered in [6] particle 1 moves towards the future horizon whereas particle 2 approaches the past horizon from the inner white hole region. For nonextremal horizons (which is the subject of the present work) this means in terms of R- and T-regions [7] that particle 2 passes from the expanding T-region to the R one whereas particle 1 moves within our R region as usual. This scenario works for generic particles in contrast to the standard BSW effect where fine tuning between parameters of one particle is required and is valid for generic eternal black - white holes.

In the present work, we analyze this scenario further, describe the main features of relevant trajectories and argue that there exists close similarity between such a scenario and high energy collision inside the horizon.

Some reservations are in order. The existence of white wholes is questionable. In particular, they can be unstable (see Sec. 15 of [8]). However, we can point at least to three factors that support our motivation. (i) Many years ago, an interesting conjecture was pushed forward according to which white holes can act as region retarded in the expansion of surrounded matter in Universe [9]. It is important that the scenario considered there includes, in particular, collision between particles that leave a white hole and those that move outside that corresponds just to our case. (ii) Typically, the structure of spacetime includes alternation of R and T regions. For example, this happens for regular black holes, so-called black universes [10], the motion of self-gravitating shells [12], etc. (iii) Even if (i) and (ii) are not realized in astrophysics in practice, collisions of particles near white holes is an essential ingredient of the theory of high energy collisions. Without this treatment, our understanding of the BSW effect and its modifications would remain incomplete. It is also worth noting that the energetics of white holes was discussed along time ago but in a quite different context [11].

Throughout the paper, we use systems of units in which fundamental constants $G = c = 1$.

II. BASIC EQUATIONS

Let us consider the metric of the eternal hole

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

(1)
where \( f(r_+) = 0 \). For the Schwarzschild metric \( f = 1 - \frac{r_+}{r} \). We consider pure radial motion. Then, for a free particle having the mass \( m \) equations of motion read

\[
\dot{r} = \sigma \sqrt{\varepsilon^2 - f}, \\
\frac{dt}{d\tau} = \frac{\varepsilon}{f}, \\
\frac{dr}{dt} = \sigma \frac{f \sqrt{\varepsilon^2 - f}}{\varepsilon}.
\]

Here, \( \varepsilon = \frac{E}{m}, E \) is the energy, \( \sigma = \pm 1 \) depending on the direction of motion, dot denotes differentiation with respect to the proper time \( \tau \).

Let two particles collide. One can define the energy in the centre of mass in the point of collision according to

\[
E^2_{\text{c.m.}} = -P_{\mu}P^{\mu},
\]

\( P^\mu = m_1u^\mu_1 + m_2u^\mu_2 \), where \( u^\mu \) is the four-velocity. Then,

\[
E^2_{\text{c.m.}} = m_1^2 + m_2^2 + 2m_1m_2\gamma,
\]

\( \gamma = -u_{1\mu}u_2^{\mu} \) is the Lorentz factor of relative motion.

Let particle 1 with \( \sigma = -1 \) and particle 2 with \( \sigma = +1 \) collide at \( r = r_c \). (Hereafter, subscripts ”c” implies that \( r = r_c \).) Then, it follows from the equations of motion that

\[
\gamma = \frac{\varepsilon_1\varepsilon_2 + \sqrt{\varepsilon_1^2 - f_c}\sqrt{\varepsilon_2^2 - f_c}}{f_c},
\]

where \( f_c = f(r_c) \). If collision happens close to the horizon, so \( r_c \to r_+ \), the quantity \( f_c \to 0 \) and we obtain formally diverging expression.

However, there is an essential subtlety here, not discussed in [5], [6]. The effect under consideration involves not one horizon as usual in the BSW effect but two different horizons - the future (black hole) horizon and the past (white hole) one. In such a situation a new problem arises that remained irrelevant for collisions near a black hole horizon only. To gain the effect, one should guarantee, first of all, that collision does occur near the horizon. Otherwise, either particle approaches its own horizon in different points of the spacetime diagram (see Fig. 1) and no near-horizon collision happens. It is possible somewhere far from the horizon but this case is uninteresting since the gamma factor \( \gamma \) remains modest.
To elucidate the essence of matter, it makes sense to introduce the Kruskal coordinates that cover all spacetime including the black and white hole regions. Then, according to the standard formulas, we use in the region \( r > r_+ \) (our R region) coordinates

\[
U = -\exp(-\kappa u), \quad V = \exp(\kappa v),
\]

\[
u = t - r^*, \quad v = t + r^*,
\]

where the so-called tortoise coordinate equals

\[
r^* = \int r \frac{dr}{f},
\]

\( \kappa \) is the surface gravity. From \((8) - (10)\) useful relation follow:

\[
UV = \exp(2\kappa r^*),
\]

\[
\frac{V}{|U|} = \exp(2\kappa t).
\]

Near the horizon,

\[
r^* \approx \frac{1}{2\kappa} \ln \frac{r - r_+}{r_+} + C,
\]

where \( C \) is a constant,

\[
f \approx 2\kappa r_+ UV.
\]

For the Schwarzschild metric, with the constant of integration chosen properly, one finds exact expressions

\[
r^* = r + r_+ \ln \frac{r - r_+}{r_+},
\]
\[
\kappa = \frac{1}{2r_+},
\]

\[
f = \frac{r_+}{r} \exp\left(-\frac{r}{r_+}\right)UV.
\]

Then, the metric takes the form

\[
ds^2 = -FdUdV + r^2(d\theta^2 + \sin^2 \theta d\phi^2),
\]

\[
F = f \frac{du}{dU} \frac{dv}{dV} = -\frac{f}{UV\kappa^2}.
\]

We must have in the point of collision

\[
U_1 = U_2, V_1 = V_2.
\]

In terms of the \( t \) coordinate,

\[
t_1(r_c) = t_2(r_c).
\]

Here, according to (1)

\[
t_1(r) = \varepsilon_1 \int_r^{r_1} \frac{dr}{f\sqrt{\varepsilon_1^2 - f}},
\]

where \( r_1 \) is the starting point of motion of particle 1, so \( t_1(r_1) = 0 \). In a similar way,

\[
t_2(r) = t_1(r_c) - \varepsilon_2 \int_r^{r_c} \frac{dr}{f\sqrt{\varepsilon_2^2 - f}}
\]

When particle 2 crossed the past horizon, \( r \to r_+ \) and \( t_2 \to -\infty \) that signals about failure of the original coordinate system (1). However, the proper time \( \tau \) stays finite, so particles 1 and 2 can meet in the point \( r_c \).

The choice of the constant of integration in (23) ensures that (20) is satisfied. If particle 1 comes from infinity, \( \varepsilon \geq 1 \). It terms of the Kruskal coordinates, the equations of motion follow from (2) - (4) and (8) - (10). They read

\[
\frac{dU}{dr} = -\frac{\sigma \kappa U}{\sqrt{\varepsilon^2 - f(\varepsilon + \sigma\sqrt{\varepsilon^2 - f})}},
\]

\[
\frac{dV}{dr} = \frac{\kappa V(\sqrt{\varepsilon^2 - f} + \sigma \varepsilon)}{f\sqrt{\varepsilon^2 - f}}.
\]
III. KINEMATICS OF COLLISION

In the \((U, V)\) coordinate system, particle 2 crosses the white hole horizon in the point \((U_2, 0)\). Particle 1 would cross the black hole horizon in the point \((0, V_1)\), unless the collision happened. In general, their trajectories intersect in the intermediate point with \(|U_c| = O(1)\), \(V_c = O(1)\) this gives a modest \(\gamma\). To gain large \(E_{\text{c.m.}}\), we must arrange collision very nearly to the horizon, where \(f_c\) is small, so \(\gamma\) is big according to \((7)\). As in the present work we are interested in the effects near the white hole horizon \(V = 0\), we require

\[V_c \ll 1.\] (26)

This entails consequences for the properties of a trajectory of each particle.

A. Particle 1

By assumption, particle 1 started its motion at \(t_1 = 0\). We can choose, say, that for \(t < 0\) it remained in the state of the rest, \(r = r_1 = \text{const.}\). Then, \(t > 0\) on its further trajectory. It is seen from \([12]\) that \(|U_c| < V_c\). This means that collision could not happen near a generic point of the white horizon where \(U = O(1)\), \(V = 0\) since this would have been in contradiction with \([12]\) and \((26)\). As now both \(|U_c| \ll 1\) and \(V_c \ll 1\), the collision happens near the bifurcation point \(V = 0 = U\).

B. Particle 2

Let us consider particle 2 moving from a white hole. It has \(\sigma = +1\). We want to arrange collision near the past horizon, so \(r_c - r_+\) is small. Then, we have

\[U_c \approx U_+ + \left(\frac{dU}{dr}\right)_+ (r_c - r_+) \approx U_+ - \frac{\kappa U_+}{2\varepsilon^2_2} (r_c - r_+) \approx U_+(1 - \frac{f_c}{4\varepsilon^2_2}),\] (27)

where \(U_+ = U(r_+)\) and we used the fact that near the horizon

\[f(r) \approx 2\kappa(r - r_+).\] (28)

For any finite \(\varepsilon_2\), this gives a small correction to \(U_+\), so \(U_c \approx U_+\). Thus both the point where particle 2 intersects the horizon and the point of collision are situated near the bifurcation point.
For completeness, we will also discuss the case when both $\varepsilon_2$ and $f_c$ are small and have the same order,

$$\varepsilon_2 \sim f_c,$$

so the Taylor expansion (27) does not work. In the near-horizon region eq. (24) for particle 2 gives us

$$\frac{d \ln |U|}{dr} \approx -\frac{\kappa}{\sqrt{\varepsilon_2^2 - 2\kappa(r - r_+)}} \frac{1}{\sqrt{\varepsilon_2^2 - 2\kappa(r - r_+) + \varepsilon_2}},$$

where we took into account (28). It is convenient to use parametrization $\varepsilon_2^2 = 2\kappa(r_0 - r_c)$. Here, $r_0$ is close to $r_c$ which, in turn, is close to $r_+$. Collision should occur before particle 2 reaches the turning point, otherwise $\sigma$ would change the sign and the head-on collision would not occur. Therefore, $r_c \leq \frac{r_+ + r_0}{2}$.

Then, after integration with the boundary condition $U(r_c) = U_c$, one finds that

$$U \approx U_c \left(\sqrt{\delta + \sqrt{\delta - x}}\right),$$

where $\delta = \frac{r_0 - r_c}{r_+}$, $x = \frac{r - r_+}{r_+}$ and $\alpha = \frac{r_c - r_+}{r_+} = x(r_c)$. To have radicals nonnegative, we require $\alpha \leq \delta$. Thus

$$U_+ \approx U_c \frac{2}{1 + \sqrt{1 - \frac{\alpha}{\delta}}}. \quad (32)$$

We see that $U_+$ and $U_c$ have the same order, so both of them are small according to the explanation given above. Thus in both cases $\varepsilon_2 \gg f_c$ and $\varepsilon_2 \sim f_c \ll 1$ collision occurs near the bifurcation point. See Fig. 2, where the scale is magnified to show the whole picture distinctly.

It is worth noting that if $\varepsilon_2 \sim \sqrt{f_c}$, $\varepsilon_1 = O(1)$, $\gamma = O(f_c^{-1/2})$, so the growth of $E_{c.m.}$ is more slow than in the case $\varepsilon_2 \gg \sqrt{f_c}$ when $\gamma = O(f_c^{-1})$. If both particles have energies $\varepsilon_1 \sim \varepsilon_2 \sim \sqrt{f_c}$, $\gamma = O(1)$ and the effect of high energy collision is absent.

Formally, there exists one more case when particle 2 falls back into a black hole. However, collisions near the black hole horizon were already discussed intensively in literature and it is known that without a fine-tuned (critical) particle high $E_{c.m.}$ are impossible [1] - [3]. Therefore, we do not consider this case.

IV. DISCUSSION AND CONCLUSIONS

If two particle moving along the line in opposite directions collide in the flat spacetime, one can arrange collision in any given point adjusting an initial position of, say, particle 2 to
its energy. However, we saw that for head-on collision near the white-black hole horizons the situation is different. The particle emerging from a white hole region should pass close to the bifurcation point, although not through the bifurcation point itself since otherwise it would come into the contracting T region instead of our R one. And, this is true irrespective of the energy of particle 2. Collision of both particles 1 and 2 also happens near the bifurcation point.

It is instructive to compare the results with those found for collisions inside black (white) holes since in both cases one is faced with the existence of two branches of the horizon. At first, it was claimed in [13] that collisions near the inner nonextremal horizon lead to the unbounded growth of $E_{c.m.}$ in a manner similar to collisions near the event horizon of rotating black holes [1]. Later on, this result was refuted [14] because of impossibility to arrange collision kinematically since each particle approaches to its horizon. The similar conclusion was made in the end of Sec. 3 in [15]. However, more careful treatment showed that the effect of high $E_{c.m.}$ can be saved [16] - [18] if particles pass very close to the bifurcation point. We see that similar situation happens in the present case although the number of suitable scenarios actually reduced to one. This is because kinematics of the problem is more restricted (particle 1 moves in the R region only, particle 2 moves from a white hole to the R region).

Thus white holes in combination with the black ones can serve as accelerators of particles
to ultra-high energies. In contrast to the standard BSW effect \[1\], no fine-tuning of parameters is required but, instead, there is a kinematic restriction. This is necessary if we want to achieve unbounded $E_{c.m.}$.

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