A possible new phase of commensurate insulators with disorder: the Mott Glass

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Abstract

A new thermodynamic phase resulting from the competition between a commensurate potential and disorder in interacting fermionic or bosonic systems is predicted. It requires interactions of finite extent. This phase, intermediate between the Mott insulator and the Anderson insulator, is both incompressible and has no gap in the conductivity. The corresponding phase is also predicted for commensurate classical elastic systems in presence of correlated disorder.
The interplay between disorder and interactions gives rise to fascinating problems in condensed matter physics. Although the physics of disordered noninteracting systems is by now well understood, when interactions are included the problem is largely open, with solutions existing for one dimensional systems [1,2] or through approximate scaling [3–5] or mean-field methods [6]. A particularly interesting situation occurs when the non-disordered system possesses a gap. This arises in a large number of systems such as disordered Mott insulators [7–10], systems with external (Peierls or spin-Peierls systems [11]) or internal commensurate potential (ladders or spin ladders [12–15], disordered spin 1 chains [16,17]). This situation is also relevant for classical problems such as elastic systems subjected to both periodic potential and correlated disorder, as encountered e.g. in vortex lattices in superconductors [18].

Although in some cases an infinitesimal disorder can suppress the gap [7] due to Imry-Ma effects, in most cases, a finite amount of disorder is needed to induce gap closure. In the latter case, the complete description of the gap closure and of the physics of the resulting phases is extremely difficult with the usual analytic techniques such as perturbative renormalization group (RG), due to the absence of a weak coupling fixed point. In $d = 1$, where attempts in solving this problem could be made, it was believed [8] that two phases existed: a weak disorder phase where the gap is robust and the system has all the characteristics of the pure gaped insulator, and a strong disorder phase where the gap is totally washed out by disorder and the system is a simple compressible Anderson insulator. However the techniques used so far are either restricted to special points [9] or suffer from serious limitation: the simple perturbative RG of [8] has to be used outside its regime of validity to describe the transition between two strong coupling phases. Furthermore up to now mostly onsite interactions have been studied.

In this Letter we thus reexamine this problem using better suited methods that capture some nonperturbative effects: a variational calculation and a functional renormalization group method. We focus first, for simplicity, on one dimensional interacting spinless fermions in the presence of a commensurate potential [19]. As argued below we expect similar physics to hold for systems with spins. Our main finding is that, in addition to the above mentioned phases, an intermediate phase exists, as shown in Fig. 1.

Quite remarkably although this phase possesses some of the characteristics of an Anderson insulator, namely a non-zero ac conductivity at low frequency, it remains incompressible. Since it is also dominated by disorder, we call it a Mott glass. We discuss here the physical characteristics of this novel phase and its interpretation for fermionic as well as bosonic systems. We argue that such a phase is not specific to $d = 1$ and discuss its generalization to higher dimension both for quantum and classical systems.

Disordered interacting spinless fermions submitted to a commensurate periodic potential are described in one dimension by

$$H = \int dx \left[ -i\hbar v_F (\psi_+^\dagger \partial_x \psi_+ - \psi_-^\dagger \partial_x \psi_-) - g (\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-) + \mu(x) \rho(x) \right] + \int dx_1 dx_2 V(x_1 - x_2) \rho(x_1) \rho(x_2)$$

where $\pm$ denote fermions with momentum close to $\pm k_F$, as is standard in $d = 1$. $V$ is the interaction, $\mu(x)$ an on site random potential and $g$ the strength of the periodic potential opening the gap. In $d = 1$ it is convenient to use a boson representation of the fermion operators [20]. This leads to the action
\[
S/h = \int dx d\tau \left[ \frac{1}{2\pi K} \left[ \frac{1}{v} (\partial_x \phi)^2 + v (\partial_x \phi)^2 \right] - \frac{g}{\pi \alpha h} \cos 2\phi + \frac{\xi(x)}{2\pi \alpha h} e^{i\phi(x)} + c.c. \right]
\]

where \( \alpha \) is a short distance cutoff (of the order of a lattice spacing) and the field \( \phi \) is related to the total density of fermions by \( \rho(x) = -\nabla \phi/\pi \). The interactions are totally absorbed in the Luttinger liquid coefficients \( v \) (the renormalized Fermi velocity) and \( K \), a number that controls the decay of the correlation functions. The noninteracting case \( (V = 0) \) gives \( K = 1 \) and \( v = v_F \). We focus in the following on the repulsive \( (V > 0) \) case for which \( 0 < K < 1 \) depending on the strength and range of \( V \). For weak disorder one can separate in the random potential the Fourier components close to \( q \sim 0 \) (forward scattering) and \( q \sim 2k_F \) (backward scattering). We have retained here, for simplicity, only the backward scattering. Forward scattering, studied in [21], does not lead to qualitative changes in the physics presented here. The backward scattering is modeled by the complex gaussian random variable \( \xi \) such that \( \xi(x)\xi^*(x') = W\delta(x - x') \).

Perturbatively both the disorder (in the absence of commensurate potential) and the commensurate potential (in the absence of disorder) are relevant (respectively for \( K < 3/2 \) and \( K < 2 \)). Let us now study this problem using a variational method [22]. We first use replicas to average over disorder in (1). We then approximate the replicated action by the best quadratic action \( S_0 = \frac{1}{2} \int q,\omega_n \phi^a G^{-1}_{ab}(q,\omega_n)\phi^b_{q,\omega_n} \) where the \( G \) are the variational parameters and \( a, b \) replica indices [23]. Skipping the technical details which can be found in [21], we find that the saddle point solution yields

\[
vG_c^{-1}(q,\omega_n) = \frac{1}{\pi K}(\omega_n^2 + v^2 q^2) + m^2 + \Sigma_1(1 - \delta_{n,0}) + I(\omega_n)
\]

where \( G_c = \sum_b G_{ab} \) is the connected Green function. The parameters \( m, \Sigma_1 \) and the function \( I(\omega_n) \) obey closed self consistent equations. The important physical quantities are simply given in term of (2): the static compressibility reads \( \kappa = \lim_{q \to 0} \lim_{\omega \to 0} q^2 G_c(q,\omega) \) while the conductivity is given by the analytical continuation to real frequencies: \( \sigma(\omega) = [\omega_n \lim_{q \to 0} G_c(q,\omega_n)]|_{\omega_n \to \omega + i\delta} \). Note that both quantities are stemming from the same propagator but with different limits \( q \to 0 \) and \( \omega \to 0 \).

Since we expect the physics to be continuous for small enough \( K \) (i.e. repulsive enough interactions), one can gain considerable insight by considering the classical limit \( h \to 0 \), \( K \to 0 \) keeping \( K = K/h \) fixed. In this limit one can solve analytically the saddle point equations and compute \( m, \Sigma_1 \) and \( I(\omega_n) \). The resulting phase diagram is parameterized with two physical lengths (for \( K \to 0 \)): The correlation length (or soliton size) of the pure gapped phase \( d = ((4gK)/(\alpha v))^{-1/2} [24] \) and the localization (or pinning) length \( l_0 = ((\alpha v)^2/(16W K^2))^{1/3} [21]25 \) in the absence of commensurability. We find three phases as shown in Figure [4].

**Mott insulator** [MI]: At weak disorder we find a replica symmetric solution with \( \Sigma_1 = 0 \) but with \( m \neq 0 \) (\( m \) depends on the disorder), \( m \neq 0 \) leads to zero compressibility \( \kappa = 0 \). \( m \) defines the correlation length \( \xi \), with \( \xi^2 = v^2/(\pi K m^2) \), in the presence of both the disorder and the commensurate potential. The effect of disorder is to increase \( \xi \) compared to the pure case, since it reduces the gap created by the commensurate potential. We find that \( \xi \) is given by \( (d/\xi)^2 \exp[\frac{1}{4}(\xi/l_0)^3] = 1 \). We obtain \( \sigma(\omega) = 0 \) if \( \omega \leq \omega_c = m \sqrt{1 + \lambda - 3(3\lambda/2)^{2/3}} \) where \( \lambda = (\xi/l_0)^3 \). The physics of this phase is similar to the simple Mott insulator. However the
gap in the conductivity decreases, when disorder increases, and closes for $\lambda = 2$. For $\lambda > 2$ ($d/l_0 > 0.98$) the RS solution becomes unphysical even though the mass $m$ remains finite at this transition point.

For stronger disorder one must break replica symmetry. In the absence of commensurate potential such a solution describes well the $d = 1$ Anderson insulator [23], in which $\Sigma_1 \neq 0$. Here, however two possibilities arise depending on whether the saddle point allows for $m \neq 0$ or not:

Anderson Glass [AG]: For large disorder compared to the commensurate potential $d/l_0 > 1.58$, $m = 0$ is the only saddle point solution. In this case one recovers exactly the solution of the Anderson insulator with interactions but no commensurate potential. We call it Anderson glass to emphasize that this phase is dominated by disorder (the corresponding phase for bosons, also described by [1], is the Bose glass [1,5]). Within the variational approach, the AG has a finite compressibility (identical to the one of the pure system $\kappa = \pi K/v$) and the conductivity starts as $\sigma(\omega) \sim \omega^2$ showing no gap. Physically this is what is naively expected if the disorder washes out completely the commensurate potential. While the MI and AG were the only two phases accessible by previous techniques [9,8] we find that an intermediate phase exists between them.

Mott Glass [MG]: For intermediate disorder $0.98 < d/l_0 < 1.58$ a phase with both $\Sigma_1 \neq 0$ and $m \neq 0$ exists. $m$ and $\Sigma_1$ define two characteristic lengths in the intermediate phase, $m^2 + \Sigma_1$ remaining constant in the MI and MG [21]. On the other hand $m$ varies and vanishes (discontinuously within the variational method) at the transition from MG to AG. The MG is thus neither a Mott nor an Anderson insulator. In particular, the optical conductivity has no gap for small frequencies $\sigma(\omega) \sim \omega^2$, while due to $m \neq 0$, the system is incompressible $\kappa = 0$. These properties are shown in Fig. 2.

This result is quite remarkable since by analogy with noninteracting electrons one is tempted to associate a zero compressibility to the absence of available states at the Fermi level and hence to a gap in the conductivity as well. Our solution shows this is not the case, when interactions are turned on the excitations that consists in adding one particle (the important ones for the compressibility) become quite different from the particle hole excitations that dominate the conductivity.

Physical arguments are also in favor of the existence of the Mott Glass, both for systems with or without spins. Let us consider the atomic limit, where the hopping is zero. If the repulsion extends over at least one interparticle distance, leading to small values of $K$, particle hole excitations are lowered in energy by excitonic effects. For example for fermions with spins with both an onsite $U$ and a nearest neighbor $V$ the gap to add one particle is $\Delta = U/2$. On the other hand the minimal particle-hole excitations would be to have the particle and hole on neighboring sites (excitons) and cost $\Delta_{p.h.} = U - V$. When disorder is added the gaps decrease respectively [20] as $\Delta \to \Delta - W$ and $\Delta_{p.h.} \to \Delta_{p.h.} - 2W$. Thus the conductivity gap closes, the compressibility remaining zero [21]. According to this physical picture of the MG, the low frequency behavior of conductivity is dominated by excitons (involving neighboring sites). This is at variance from the AG where the particle and the hole are created on distant sites. This may have consequences on the precise low frequency form of the conductivity such as logarithmic corrections. When hopping is restored, we expect the excitons to dissociate and the MG to disappear above a critical value $K > K^*$. Since finite range is needed for the interactions, in all cases (fermions or bosons) $K^* < 1$. 
In addition we expect $K^* < 1/2$ for fermions with spins. Similar excitonic arguments should also hold in in two- or three-dimensional bosonic and fermionic Mott insulators provided some finite range of interaction is taken into account. In higher dimension, since disorder has a weaker impact on the transport properties, one expects that the important change in the conductivity occurs at the transition between the MI and MG, whereas the compressibility would become non zero only for stronger disorder (transition MG to AG). Numerical investigations would prove valuable. Small gaps facilitate the observation of MG physics (see Fig. 2), making the study of systems already close to a metal–Mott insulator transition particularly interesting.

The properties that we have obtained are thus quite general, depending only on the two gaps closing separately. Given the mapping between a $d$ dimensional quantum problem and a $d+1$ classical one our study also applies to commensurate classical systems in presence of disorder correlated in at least one direction (here the imaginary time $\tau$). \[1\] can be generalized to any dimension to describe a classical elastic system where $\phi$ becomes a displacement field $u$. It applies to systems with internal periodicity, such as a crystal or a charge density wave (with $2\phi = K_0 u$ for reciprocal lattice vector $K_0$) or to non periodic systems such as interfaces. The periodic potential making the system flat while the disorder makes it rough. For these systems a functional RG procedure (FRG) in $d = 4 - \epsilon$ (i.e near $4+1$-dim systems) can be used. For uncorrelated disorder in the absence of an external periodic potential an internally periodic system is described in $d \leq 4$ by a $T = 0$ “Bragg glass” fixed point \[28\]. Adding a periodic potential $\cos(p\phi)$ as a perturbation, a transition was found at $T = 0$ \[29\] between the Bragg glass (for $p > p_c(d)$) (periodic potential irrelevant) and a commensurate phase (for $p < p_c(d)$) (periodic potential relevant). Such a $T = 0$ transition also exists for correlated disorder \[21\].

To confirm the existence of the MG phase it is necessary to study the phase where the periodic potential is relevant (since $p = 2 < p_c(d)$ here). Since this goes beyond the perturbative FRG analysis, we consider the toy model where the cosine is replaced by a quadratic term (a good approximation when the periodic potential is relevant) defined by the energy:

\[
\frac{H}{T} = \frac{1}{T} \int d^dxd\tau \left[ \frac{1}{2}(c(\nabla u)^2 + c_{44}(\partial_{\tau} u)^2 + m^2 u^2) + V(x, u(x, \tau)) \right]
\]  

(3)

where $\tau$ is the coordinate along which disorder is correlated (e.g. the magnetic field for vortices with columnar defects), $T$ is the classical temperature ($\hbar$ for the quantum problem) and the gaussian disorder has a correlator $\overline{V(x, u)V(x', u')} = \delta^d(x-x') R(u - u')$ (for a periodic problem, $R(u)$ is itself periodic). $c$ and $c_{44}$ are the elastic moduli, analogous to $1/K$ for the quantum problem \[1\]. This $d+1$ dimensional model can be studied perturbatively (for small $m$ and disorder). For $m = 0$ and $T = 0$, both for uncorrelated \[30\] and correlated disorder \[31\] a cusp-like nonanalyticity in the renormalized disorder $R(u)$ develops beyond the Larkin length $R_c$ (corresponding to metastability and barriers in the dynamics) \[32\]. Remarkably, we find that this feature persists even when $m > 0$, while one usually expects that a mass smoothes out singularities. This can be seen from the RG equation for $\Delta(u) = -R''(u)$:

\[
\partial_t \Delta(u) = \epsilon \Delta(u) + \tilde{T}_t \Delta''(u) + f_t(\Delta''(u)(\Delta(0) - \Delta(u))) - \Delta'(u)^2
\]

(4)
with $f_l = \frac{1}{8\pi^2}(1 + \mu e^2)^{-2}$, $\mu = m^2 a^2$ and at zero temperature $T_l = 0$. Integrating the closed equation for $\Delta''(0)$ one finds that the cusp persists (i.e. $-\Delta''_{l=+\infty}(0) = +\infty$) provided that $R_c < R^*_c(\mu)$ (with $R^*_c(\mu) \sim 1/\sqrt{\mu}$ for small $\mu$) while it is washed out $(-\Delta''_{l=+\infty}(0) < +\infty)$ for weaker disorder. Our FRG study shows that this $T = 0$ transition in the renormalized disorder, exists both for correlated and uncorrelated disorder. For uncorrelated disorder our findings are of interest for the question of the existence of an intermediate “glassy flat phase”. In that case, however, no sharp signature of this transition exists in two point correlation functions and a physical order parameter remains to be found, which makes the existence of such a phase still controversial [33, 34]. On the contrary for correlated disorder the transition seen in the FRG has much stronger physical consequences. Because of the lack of rotational invariance (in $(x, \tau))$ the existence of the cusp and the transition directly affects two point correlation functions. Indeed, the tilt modulus $c_{44}$ renormalizes as $\partial_l c_{44} = -f(l)\Delta''(0)c_{44}$. Integrating at $T = 0$ one finds that $c_{44}(l = +\infty)$ is finite for $R_c > R^*_c(\mu)$ but that it is infinite for $R_c < R^*_c(\mu)$. Furthermore we find (see [21] for details) that for correlated disorder a small temperature ($\tilde{T}_l > 0$) does not affect the transition (the phase where $c_{44}(l = +\infty) = \infty$ survives) whereas for uncorrelated disorder the effective temperature goes to a constant ($\sim \mu^{1-\epsilon/2}$) washing out the cusp. Thus the toy model for correlated disorder exhibits at low temperature, within the FRG, a transition between two phases. The first one is identified with the MI, where the mass (commensurability) destroys the metastability and restores isotropy in $x, \tau$ at large scale. The second one is the MG, which is glassy with metastable states despite the presence of the mass. It shares some properties with the AG such as $c_{44} = \infty$. This implies non analyticity of the Green’s functions in frequency. The AG itself corresponds to the phase where the periodic potential is irrelevant ($m = 0$). This provides strong evidence for the intermediate phase proposed in this paper, which, besides electrons, should be obtained in classical commensurate elastic systems with disorder.
REFERENCES

[1] T. Giamarchi and H. J. Schulz, Phys. Rev. B 37, 325 (1988), and ref. therein.
[2] M. V. Feigelman and V. M. Vinokur, Sol. State Comm. 45, 603 (1983).
[3] A. M. Finkelstein, Z. Phys. B 56, 189 (1984).
[4] For a review see: D. Belitz and T. R. Kirkpatrick, Rev. Mod. Phys. 66 261 (1994).
[5] M. P. A. Fisher, P. B. Weichman, G. Grinstein, and D. S. Fisher, Phys. Rev. B 40, 546 (1989).
[6] V. Dobrosavljevic and G. Kotliar, Phys. Rev. Lett. 78, 3943 (1997).
[7] R. Shankar, Int. J. Mod. Phys. B 4, 2371 (1990).
[8] S. Fujimoto and N. Kawakami, Phys. Rev. B 54, R11018 (1996).
[9] M. Mori and H. Fukuyama, J. Phys. Soc. Jpn. 65, 3604 (1996).
[10] M. A. Paalanen, J. E. Graebner, R. N. Bhatt, and S. Sachdev, Phys. Rev. Lett. 61, 597 (1988).
[11] B. Grenier et al., Phys. Rev. B 57, 3444 (1998).
[12] E. Orignac and T. Giamarchi, Phys. Rev. B 56 7167 (1997); 57 5812 (1998).
[13] M. Azuma and al., Phys. Rev. B 55, 8658 (1997).
[14] A. S. Carter et al., Phys. Rev. Lett. 77, 1378 (1996).
[15] S. Fujimoto and N. Kawakami, Phys. Rev. B 56, 9360 (1997).
[16] C. Monthus, O. Golinelli and T. Jolicoeur, cond-mat 9705231.
[17] N. Kawakami and S. Fujimoto, Phys. Rev. B 52, 6189 (1995).
[18] G. Blatter et al., Rev. Mod. Phys. 66, 1125 (1994).
[19] In one dimension, the Mott transition can be viewed as caused by a $4k_F$ periodic potential (see e.g. [24]). However to study the simpler case of spinless fermions (while retaining the main features of the fermions with spins), we use instead a $2k_F$ periodic potential which allows to study gap formation while avoiding the destruction of the gap by infinitesimal disorder that occurs in the latter case [7].
[20] F. D. M. Haldane, J. Phys. C 14, 2585 (1981).
[21] E. Orignac, T. Giamarchi and P. Le Doussal, to be published.
[22] M. Mezard and G. Parisi, J. de Phys. I (Paris) 4, 809 (1991); E. I. Shakhnovich and A. M. Gutin, J. Phys. A 22, 1647 (1989).
[23] T. Giamarchi and P. Le Doussal, Phys. Rev. B 53, 15206 (1996).
[24] T. Giamarchi, Physica B 230-232, 975 (1997).
[25] H. Fukuyama and P. A. Lee, Phys. Rev. B 17, 535 (1978).
[26] For small bounded disorder the gap of the Mott insulator phase is robust.
[27] Note that in the disordered $d = 1$ Wigner crystal (without a commensurate potential) the compressibility is $\kappa = \lim_{q \to 0} q^2 \log(1/q) = 0$ whereas the optical conductivity has no gap at low frequencies. The arguments given here show that similar physics occurs in the presence of a commensurate potential and interactions of short range.
[28] T. Giamarchi and P. Le Doussal, Phys. Rev. B 52, 1242 (1995).
[29] T. Emig and T. Nattermann, Phys. Rev. Lett. 79, 5090 (1997).
[30] D. S. Fisher, Phys. Rev. Lett. 56, 1964 (1986).
[31] L. Balents, Europhys. Lett. 24, 489 (1993).
[32] Note that at $T = 0$ the $\omega_n = 0$ mode (i.e $u(x, \tau)$ constant in $\tau$) decouples and identifies exactly with the problem of uncorrelated disorder in $d$ dimension.
[33] J. P. Bouchaud and A. Georges, Phys. Rev. Lett. 68, 3908 (1992).
[34] T. Emig and T. Nattermann, preprint cond-mat 9808318.
FIG. 1. The three phases as defined in the text in the disorder $W$, commensurate potential $g$ plane for a fixed value of the interactions. Both MI and MG are incompressible. Both MG and AG have no gap in the conductivity.

FIG. 2. Conductivity in the MI (solid line) and MG phases (dashed line). Insert: density $n$ versus the chemical potential $\mu$. 