Modal Reduction of Synthetic Jet Actuator Based Separation Control with Spectral POD

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Abstract A synthetic jet actuator (SJA) is a zero-net-mass-flux device that imparts fluid momentum and is useful for active flow control (AFC). In many applications, airfoil performance is often limited or degraded by flow separation which is usually associated with loss of lift, increased drag, and kinetic energy losses. Therefore, it is of interest to investigate methods of separation region suppression with the forcing control of SJA. This paper studies the flow behavior of cross flow over an airfoil and how the addition of SJA influences flow characteristics. Using the Spectral Proper Orthogonal Decomposition and LES simulation, flow instabilities in the wake region are analyzed in their different temporal and spatial scales. The objective of this study is to explore the viability of SPOD for separation control and correlating the decomposed flow modes to the aerodynamic performance of airfoil.

Keywords Model Reduction, Aerodynamic Control, Synthetic Jet Actuator

1 Introduction

A synthetic jet actuator transfers momentum to the surroundings by alternately ingesting and expelling fluid through a cavity containing and oscillating diaphragm and has been shown to be a useful AFC device [1,2]. Compared with continuous jets, synthetic jet actuators (SJAs) are low-weight, compact and do not require internal fluid supply lines [3,4]. A schematic diagram of a typical synthetic jet actuator is shown in Fig. 1, which also illustrates the primary structure and shows vortex pairs emanating from a nozzle while the SJA is in operation. An SJA typically has a nozzle or slot connected to a cavity in which a piezoelectric membrane oscillates. By oscillating the diaphragm, the working fluid is alternately ingested and expelled through the nozzle exit, forming a train of discrete vortical structures that impart linear momentum to the flow without net mass injection [5]. The ability of the vortex pairs to overcome the suction velocity during the ingestion stroke depends on its self-induced velocity, which in turn is a function of the vortex strength. The fact that no external fluid source is required combined with the availability of increasingly small vibrating diaphragms, e.g. piezo-electric disks, allows the design of extremely compact devices, even down to MEMS scales [6,7]. In the presence of cross flow, the vortex structure convects downstream, which creates a favorable pressure gradient and momentum exchange for separation suppression.

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Spectral proper orthogonal decomposition (spectral POD, or SPOD) offers a method to identify coherent structures, dominant processes or modes from flow data as done with dynamic mode decomposition (DMD) and other variants of POD [8, 9]. Like POD, SPOD finds an optimal orthogonal basis for the flow data in such a way that the first few modes should capture most of the total energy of variance in the flow data. Combined with the Welch’s method, this leads to both optimality in decomposition and high tolerance to noise. Additionally, it also contains the advantages of DMD in expressing frequencies of the resulting modes, thus it could be applied as substitute of DMD in the study of turbulent flows where most variants of DMD would fail due to noise and stochastic flow behavior. For the statistically stationary flow (no change of mean and variance in time), Towne et al. show that, in fact, SPOD modes represent optimal averaging of an ensemble of DMD modes computed from different realizations of the same flow [10]. The SPOD algorithm in this work is the based on Schmidt et al.’s paper with a kernel based on the cross-spectral density (CSD) tensor and Welch’s method [9], and it was used as model decomposition tool describing the wake region response of airfoil under SJA separation control.

2 Numerical Method and Model

2.1 Description of SPOD Algorithm

As with DMD and snapshot POD, the flow snapshot data, $Q$, sampled at fixed time interval $\Delta t$.

\[
Q = \begin{bmatrix}
q_1^1 & q_1^2 & \ldots & q_1^N \\
q_2^1 & q_2^2 & \ldots & q_2^N \\
\vdots & \vdots & \ddots & \vdots \\
q_M^1 & q_M^2 & \ldots & q_M^N \\
\end{bmatrix},
\]

where $N$ is the total number of temporal snapshots, $M$ is the number of degrees of freedom in one snapshot (number of variables times number of spatial points).
Each snapshot, $Q$, was distributed into blocks along the temporal axis with overlapped segmentation using Welch’s method, following centering (mean subtraction). This produces $N_{blk}$ fast Fourier transform (FFT) blocks each containing $N_{FFT}$ total snapshots and overlap of size $N_{ovl}$ with neighboring blocks.

After performing windowing and FFT, there is a $\hat{Q}_i$ matrix for each frequency. To minimize the computational cost of the simulations, a zero-padding step was done after windowing to increase the frequency resolution of the spectrum.

\[
\hat{Q}_i = \begin{bmatrix} \hat{q}_1^1 & \hat{q}_2^1 & \cdots & \hat{q}_{N_{blk}}^1 \\ \hat{q}_1^2 & \hat{q}_2^2 & \cdots & \hat{q}_{N_{blk}}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{q}_1^M & \hat{q}_2^M & \cdots & \hat{q}_{N_{blk}}^M \end{bmatrix}, \quad \hat{Q}_i = \mathbb{C}^{M \times N_{blk}}, \quad i = 1, 2, \ldots, N_f
\]

The subsequent process is performed on the individual frequencies. Similar to standard POD, SPOD modes and eigenvalues (mode energy) for each frequency were found from the eigenvalue decomposition of the CSD matrix $\hat{C}$.

\[
\hat{C} = \frac{1}{N_{blk} - 1} \hat{Q}_i \hat{Q}_i^H
\]

The SPOD modes $\hat{\Phi}$ and eigenvalues $\hat{\Lambda}$ could be obtained by solving:

\[
\hat{C} W \hat{\Phi} = \hat{\Phi} \hat{\Lambda}
\]

or through the method of snapshots (with a lower computational cost for typical fluid problems with large $M$) by solving:

\[
\hat{Q}_i^H W \hat{Q}_i \Psi = \Psi \hat{\Lambda}, \quad \Phi = \hat{Q}_i \Psi
\]

\[
\hat{\Lambda} = \text{diag}\{\lambda_0, \lambda_1, \ldots, \lambda_{N_{blk}-1}\} \quad \text{(descending order)}
\]

\[
\hat{\Phi} = [\hat{\phi}_0, \hat{\phi}_1, \ldots, \hat{\phi}_{N_{blk}-1}] \quad \text{(corresponding SPOD modes)}
\]

where $W$ is the weighting norm matrix, e.g., the cell volume for non-uniformly sampled cell points. With spatial orthonormality defined as $\Phi^H W \Phi = I$, the spatial modes in $\Phi$ are orthonormal within each frequency, while the spatial modes in different frequencies are not spatially orthogonal but, in a sense, orthogonal in the form of time integral. A graphical representation of the processes mentioned above is shown in Fig 2.

### 2.2 Setup of Flow Control

The flow simulations were been performed using the incompressible LES-WALE model implemented in OpenFoam V1912. The computation domain is a C-type mesh with outer boundary 15 cord lengths away from the airfoil.

The freestream across the airfoil is characterized by the chord length based Reynolds number $Re_c = U_{\infty}c/\nu = 10^5$ with a post stall angle of attack $\alpha = 15^\circ$.

As shown in Fig. 3, the SJA slot is located at $x = 0.5c$ with exit slot width $d = 0.01c$. For the 2D SJA setting, the computation domain is uniform along the spanwise $z$ direction with total width of $0.12c$ and periodic boundaries on both sides to emulate an infinite span.
The outstroke of the synthetic jet has a jet injection angle $\theta$ of 40°. In addition, to reduce the computation cost, the synthetic jets were simulated directly using a sinusoidally oscillating velocity boundary at the slot exit baffles as:

$$U_j = A_0 \sin f_e t$$

The SJAs were actuated with amplitude $A_0$ and excitation frequency $f_e$, corresponding to an excitation Strouhal number $St_e = f_e c/U_\infty$. Moreover, the jet momentum coefficient $C_\mu$ is often used as a parameter in the flow control studies [12, 13]:

$$C_\mu = \frac{T_j}{0.5 \rho_0 U_\infty^2 c}$$

where $U_0$ is the freestream velocity and $c$ is the chord length. $T_j$ is the time-averaged jet momentum per unit span during the outstroke.

$$T_j = \frac{1}{\tau} \rho b \int_0^\tau U_j^2 dt$$
$U_j$ is the phase averaged jet velocity at the exit plane and $b$ is the exit orifice width. $[0, \tau]$ is the outstroke phase duration. For our numerical implementation and the consideration of both injection angle and spanwise jet geometry, the equation above was modified to the form below:

$$T_j = \frac{1}{\tau} \int_0^\tau \text{mag}(U_j) \phi dt$$

where $\phi$ is the flux across the exit plane baffle per unit span and $U_j$ is the spatially averaged jet exit velocity.

2.3 SPOD Setting

After allowing the model time for statistically stationary data to be obtained from the simulation, 1000 snapshots were taken for each case with sampling frequency $f_s = 100$ Hz, which gives a maximum frequency of 50 Hz on the SPOD diagram. Then, these snapshots were distributed over 7 blocks with 400 snapshots and 75% overlap before the application of a Blackman window.

To increase the spectral resolution, the time sequences were padded with zero to achieve a final frequency resolution of 0.1 Hz on the SPOD spectrum, while the exact frequency resolution is $f_s/N_{blk} = 0.25$ Hz.

To display SPOD modes in the following sections, mode values were obtained by multiplying the unit mode vector $\hat{\phi}_i$ and the square root of mode energy $\lambda_i$.

![Fig. 3: 2D geometry and parameters of NACA2412 airfoil with SJA](image)

2.4 Identification of Dominant Coherent Flow Structures with SPOD spectrum

Fig. 6 shows the SPOD spectrum of wake region spanwise vorticity ($\omega_z = \frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}$) for the uncontrolled flow over the airfoil. This leads to SPOD modes optimized in terms of enstrophy, which is more effective in identification of vortical structures than velocity based optimization, especially, when dealing with ambiguous traveling direction of vortices [14, 15]. The mode 0 line represents the eigenvalue $\lambda_0$ or mode energy of the most optimal spatial mode at each frequency, while the following lines indicate the mode energies of the suboptimal orthogonal spatial modes. Since the mode 0 line contains most energy, its corresponding modes are often dominant physical processes indicative of mechanisms of interest, while the subsequent modes are often interpreted mechanically as other orthogonal variations.

There are two methods to extract dominant flow features on the mode 0 line,
– Search for the global peak and subsequent spikes on the spectrum which are indicative of dominant oscillations

– Look for large gaps between the mode 0 line and mode 1 line which means the decomposition behavior is low rank. A low rank decomposition indicate that spatial mode corresponding to first eigenvalue \( \lambda_0 \) are highly dominant and representative in the flow field. This technique is often used in the frequency region showing fast rising or falling trend.

From the power spectral density (PSD) line computed as the summation of all mode energies, for \( St \sim O(1) \) the energy is relative steady. The dominant wake frequency and its second harmonic could be obtained through searching for peaks on the mode 0 line. In the range \( St \sim O(10) \), there is a sharp falling trend, we may also need to look for the large gaps between \( \lambda_0 \) and \( \lambda_1 \) for identification of smaller scale wake, such as wake of attached flow in the later discussion.

3 Comparison with Standard POD in the case of Unexcited Airfoil

SPOD was compared against the standard snapshot POD approach [16, 17], with snapshots taken from 309 \times 141 uniform points within bounding box \([x, y] = [0.96c : 2.5c, -0.2c : 0.5c]\). Fig. 5 shows the two dominant modes extracted from the decomposition and their FFT spectrum. The two POD modes have similar frequency spectra with both having global peaks at 2.9Hz (\( St = 1.93 \)) which also corresponds to the global peak shown in the PSD spectrum shown in Fig. 4. Upon closer examination, these two spatial modes are roughly 90° apart in phase. Together, these two modes could form a reconstruction of the dominant temporal variation of the dominant wake structure.

On the other hand, the SPOD spectrum also gives a global peak at 2.9Hz. Similar to DMD, the leading mode at the peak contains both a real and imaginary part that oscillate at a discrete frequency 90° apart in phase. Thus, the mode shown in Fig. 7 a–b roughly matches the first two modes of snapshot POD.

Although both SPOD and POD identify similar dominant flow structures, there POD mode spectra are continuous, and there is no way to control the suppression of frequency components [18]. Unwanted spectral components could lead to spectral leakage and contamination of suboptimal modes, which limits the exploration of subdominant flow structures of different temporal or spatial scales. For example, SPOD could generate the description of 2nd harmonic of the leading mode (Fig. 7 c–d), whereas standard POD failed to generate a meaningful representations of the flow structure in the subsequent modes. On the contrary, SPOD has the advantage of frequency discretization from DMD, leads to both optimality from POD and the separation of temporal or spatial scales within flow structures. In addition, analogous to the estimation of the PSD, it is robust within the confidence bounds of the Welch’s method [19]. Therefore, to a certain degree, SPOD allows comparison among different cases of SJA excitation in the flowing discussion.
Fig. 4: Power Spectral Density of Cross-Stream Velocity @ [x=1.5c, y=0.12c]. Dash line labels @[2.9, 5.8]Hz.

Fig. 5: The leading two spatial modes generated from POD and their FFT Time Spectra
Fig. 6: SPOD spectrum ($\omega_z$) for unexcited airfoil wake region. Mode 0 and Mode 1 correspond to the largest and second largest eigenvalues (mode energies) for each frequency. Green lines correspond to the subsequent eigenvalues. PSD line represent the summation of all eigenvalues. Dash line labels @[2.9, 5.8]Hz.

(a) Mode 0 Real @2.9Hz [Base Harmonic]  
(b) Mode 0 Imaginary @2.9Hz [Base Harmonic]  
(c) Mode 0 Real @5.8Hz [2nd Harmonic]  
(d) Mode 0 Imaginary @5.8Hz [2nd Harmonic]

Fig. 7: Dominant wake spatial modes corresponding to the dash line labels
4 Airfoil Excited with Synthetic Jet Actuator

The SJA was operated at various momentum coefficients at 0.286%, 0.601% and 0.927% respectively. The generated SPOD spectra and modes are shown in Fig. 8–14 and Table 1 summarizes the global peak features and aerodynamic performances. Additionally, flow profiles for these cases are provided in the Appendix.

Using the global peak in the spectrum as the separated flow instability, the frequency of the separated wake $f_{ws}$ is 2.9Hz ($St=1.93$) for unexcited flow. At the lowest SJA intensity $C_{\mu}=0.286$% and $St=8$, there is a rightward shift of the global peak and an increase of $f_{ws}$ to 3.4Hz ($St=2.27$). Fig. 8a shows this leads to narrower spacing between the counter rotating vortices in the wake train. From classical scaling, the dominant frequency associated with the wake should be proportional to freestream velocity and inversely proportional to the width of the wake [20]. With the decreased width of the separation bubble after excitation, the frequency of wake structure is expected to increase.

Further increasing $C_{\mu}$ shows a clear effect of the SJA excitation observed and the suppression of mode energies of the global peaks at $f_{ws}$, while $f_{ws}$ remains relatively constant after the initial SJA intensity. The suppression effect grows with increased $C_{\mu}$, as shown in Fig. 8 a–c, with weakening of the magnitude of separated wake structure. Since these large scale wake modes are associated with the separated flow, their mode energies have strong correlation with the size of separation zone. A similar representation of such suppression effect of separated wake was also seen by Goodfellow et al. [12] where it was obtained through the PSDs at a monitor point velocity behind the trailing edge of airfoil.

Another feature of SJA separation suppression discovered in the previous experimental study was the enhancement of wake of the attached flow that originates from the trailing edge. On the SPOD spectrum, this corresponds to the $\lambda_0$ extrusions on the downward slope around $St = 10$, which are organized and sharpened with increase of $C_{\mu}$.

From the SPOD spectrum of the highest excitation level $C_{\mu}=0.927$%, the small scale modes associated with the attached flow are described mainly by the two signature frequencies of attached flow $f_{wa}$ on the downward slope at 14.8Hz and 17.3Hz (Fig. 10 a-b). Coincidentally, the frequency difference between these two modes corresponds to the newly emerging large scale spike at 2.5Hz (Fig. 9 d). As a result, these two modes alternate between in phase and out of phase periodically to form periodic dispersion of wake of attached flow, which is coupled with the newly emerging large scale instability at 2.5Hz.

The temporal reconstruction of periodic dispersion through summation of the two SPOD modes is shown in Fig. 11. The same reconstruction logic for DMD mode was applied to these complex SPOD modes, with the contribution of each mode at time $t$ to the field defined as [21]:

$$V_i(t) = \sqrt{\lambda_i} \left( Re\{\hat{\phi}_i\} \cos(2\pi ft) + Im\{\hat{\phi}_i\} \cos(2\pi ft + \pi/2) \right)$$

Another case of SJA excitation was simulated with $St_e=2$, which is close to the frequency of the initial separated wake instability. However, unlike the SJA excitation from previous cases at higher frequency, this low frequency excitation is not time invariant relative the large scale flow structures of separated flow. On the SPOD spectrum, rather than suppression of global peak in $St \sim O(1)$, this leads to organization of large spikes at the harmonics of $St_e$, with suppression of side lobes near initial global peak. Even through lift and flow reattachment were improved due to the excitation, as shown in Table 1 and mean flow profile, the corresponding spike modes (Fig. 14) are signs of unsteady reattachment.

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Fig. 8: SPOD spectrum ($\omega_z$) for 2D SJA excited airfoil wake region ($St_e = 8$)
Modal Reduction of SJA Separation Control with SPOD

| $St_e$ | $C_\mu$ | $\text{max}[U_j]$ | Global Peak $f_{gp}/f_{ws}$ | $\lambda_{gp}$ | $C_l$ | $C_d$ | L/D  |
|--------|---------|-------------------|----------------------------|---------------|------|------|------|
| -      | -       | 2.9Hz             | $2.765 \times 10^5$        | 1.241         | 0.0781 | 0.0781 | 15.88 |
| 8      | 0.286%  | 1m/s              | 3.4Hz                      | $3.869 \times 10^5$ | 1.271 | 0.0777 | 16.35 |
| 8      | 0.601%  | 1.45m/s           | 3.6Hz                      | $2.030 \times 10^5$ | 1.304 | 0.0773 | 16.87 |
| 8      | 0.927%  | 1.8m/s            | 3.5Hz                      | $1.409 \times 10^5$ | 1.332 | 0.0770 | 17.30 |
| 2      | 0.927%  | 1.8m/s            | 3.0Hz                      | $4.274 \times 10^5$ | 1.354 | 0.0794 | 17.04 |

Table 1: SPOD Global Peak Features and Force Coefficients (2D SJA)

(a) $C_\mu = 0.286\%$ @ 3.4Hz
(b) $C_\mu = 0.601\%$ @ 3.6Hz
(c) $C_\mu = 0.927\%$ @ 3.5Hz
(d) $C_\mu = 0.927\%$ @ 2.5Hz

Fig. 9: Large Scale Dominant Wake Modes for excitation frequency $St_e = 8$

(a) $C_\mu = 0.923\%$ @ 14.8Hz
(b) $C_\mu = 0.923\%$ @ 17.3Hz

Fig. 10: Small Scale Wake Modes of Attached Flow near $St \sim O(10)$
Fig. 11: Temporal Reconstruction of Reattached Wake from Two Dominant Modes over Dispersion Period $T_0 = 1/2.5Hz$

(a) $t = 0T_0$
(b) $t = 0.25T_0$
(c) $t = 0.5T_0$
(d) $t = 0.75T_0$

Fig. 12: Aerodynamic Performance and Global Peak Mode Energy vs $C_\mu (St_e = 8)$

(a) $C_L$ and $L/D$
(b) $\lambda_{pp}$
and organized vortex shedding, which agrees with the periodic wake circulation observed by Goodfellow et al with modulated excitation at the frequency of initial separated instability [12]. When compared with the case at $St_e = 8$ with same $C_{\mu}$, this lead to higher drag which could be attributed to the larger momentum deficit due to the unsuppressed large scale circulations [22].

Fig. 13: SPOD spectrum ($\omega_z$) for 2D SJA excited airfoil wake region, ($St_e = 2$)

(a) $C_{\mu} = 0.927\%$ @ [3.0Hz, 6.0Hz]
(b) $C_{\mu} = 0.286\%$ @ 3.0Hz
(c) $C_{\mu} = 0.601\%$ @ 6.0Hz

Fig. 14: Large Scale Dominant Wake Modes for excitation frequency $St_e = 2$  

5 Conclusions

SPOD was employed as a supplement of traditional monitor point based spectral analysis. It offers a method to analyze the characteristics of the near wake region of separated flow
globally, with identification of dominant wake frequencies and their corresponding spatial structures. As a result, the selection of monitor point, which often troubled by the sensitivity of insensitivity of flow structures, could be avoid in the experiment, provided with global flow visualization tools. Specific spatial structures associated with a particular frequency could be isolated through FFT, with robust estimation of the corresponding mode energy through the Welch’s method.

When SPOD was compared against snapshot POD for an uncontrolled airfoil, both methods successfully isolated the dominant wake frequency of separated flow that was verified by monitor point PSD behind the airfoil. However, only SPOD could achieve separation of scales and identification of subdominant wake structures, such as the harmonic of the dominant wake and wake of attached flow.

After increasing $C_\mu$ beyond a certain threshold at a moderate frequency $St_e = 8$, the peak mode energy of separated wake was found to decrease with a corresponding increase of aerodynamic performance and distinct peaks corresponding to the attached wake. These findings agree with the PSD results of SJA excitation at moderate excitation frequency above the dominant wake frequency. As well, the relationship between the newly emerging large scale flow instability under high excitation and periodic dispersion of wake of attached flow was discovered through the temporal reconstruction from the signature wake modes. On the other hand, the SJA excitation at a frequency close to the initial $f_{ws}$ leads to organization of coherent wake structures at the harmonics of $St_e$.

**Declarations**

**Availability of supporting data**

The datasets used and analysed during the current study are available from the corresponding author on reasonable request.

**Competing interests**

The authors declare that they have no competing interests.

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**Authors’ contributions**

XS implemented the model, checked the results, wrote the first draft and revised the manuscript. PS was responsible for funding and revision of the manuscript.
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6 Appendix

6.1 Validation of Result for Unexcited Airfoil

The simulation result obtained from LES-WALE simulation for unexcited airfoil was compared against the Xfoil result with critical amplification factor $N_{cr} = 5$. With Xfoil, the transition to turbulent flow was predicted through $e^2$ method, and this $N_{cr}$ corresponds to turbulence intensity ($Tu$) of $0.371\%$ through the relationship $N_{cr} = -8.43 - 2.4 \times \ln(Tu\% / 100\%)$ [23].

![Graph of Cp vs x/c of LES-WALE mean and Xfoil prediction](image)
Table 2: Comparison of Aerodynamic performance indicators

| Source       | $C_l$ | $C_d$ | L/D | Separation Point $x_{sep}$ |
|--------------|-------|-------|-----|---------------------------|
| LES-WALE     | 1.241 | 0.0781| 15.88| 0.60c                     |
| XFOIL $N_r = 5$ | 1.189 | 0.0768| 15.49| 0.60c                     |

6.2 Velocity Profiles for Flow around Airfoil

Fig. 16: Mean Velocity Profiles ($St_r = 8$). Contours @ $U_x = [0, -0.05, -0.1]$ m/s
Fig. 17: Q Criterion Isosurface ($St_e = 8$). Color Map $U_x$

(a) Mean Velocity
(b) $C_\mu = 0.286$
(c) $C_\mu = 0.601$
(d) $C_\mu = 0.927$

Fig. 18: Flow Profiles for $St_e = 2$, $C_\mu = 0.927%$. 

(a) Q Criterion Isosurface. Color Map $U_x$
(b) Mean Velocity