Construction of nested pattern matrix with large girth for PDMA systems

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In this letter, a new method is proposed to construct the pattern matrix (PM) with both nested structure and large girth for pattern division multiple access (PDMA) system. The proposed method contains two steps. Firstly, by performing the Kronecker product on a cycle-free base-PM with overload factor \( \beta > 1 \), an expanded-PM with overload factor \( \beta^2 \) is obtained. Then, puncturing the specific columns with high-weight of that expanded-PM, the derived nested PM achieves large girth and low row-weight, which lead to low error propagation and low computational complexity under message passing-based detector. Simulation results verified the above idea.

Introduction: For sparse code spreading based non-orthogonal multiple access system, such as pattern division multiple access (PDMA) [1], the design of the pattern matrix (PM) is a key issue since it plays a crucial role in achieving a trade-off between the bit error rate (BER) performance and the detection complexity under standard message passing algorithm (MPA) [2]. Although in fading channel, this type of detector is less sensitive to the channel correlation than it is in the additive white Gaussian noise (AWGN) case, a well-designed PM is also valuable in terms of the convergence speed and computational complexity as shown in this work. In [3], an empirical method is proposed to design PMs for PDMA system according to the diversity order principle, and obtained two PMs, for 150% and 200% overload cases, respectively, which are also shown in Figure 1 for the convenience of readers. The overload factor \( \beta \) of PDMA system is defined as the ratio of the number of users \( K \) to that of resource elements (REs) \( N \), that is, \( \beta = K/N \). The number of non-zero elements in each column of the PM is defined as the column-weight of that column. Similarly, the number of non-zero elements in each row of the PM is defined as row-weight. Assume that the total transmit power of each user is \( P \), when the \( i \)-th column of the PM is \( d_{vi} \), then the transmit power of user-\( i \) at each RE with non-zero element is set to \( P/d_{vi} \). In this work, only a case is considered that all users transmit with the same total power \( P \), which is the worst situation in the perspective of multi-user detection. It needs to be pointed out that potential error propagation exists for that expanded-PM, the derived nested PM achieves large girth and low row-weight, which lead to low error propagation and low computational complexity under message passing-based detector. Simulation results verified the above idea.

Fig. 1: PMs of PDMA system in [3]. \( S_{2,3} \) for 150% overload. \( S_{4,8} \) for 200% overload

The factor graph description of the construction procedure. Factor graph of base-PM. Factor graph of \( B^2 \). Factor graph of \( S_{4,8} \).

where \( \circ \) is the Hadamard product. \( H \in \mathbb{C}^{N \times N} \) is the channel matrix, where each entry \( h_{ij} \) denotes the channel fading coefficient of user-\( i \) over the \( j \)-th RE. \( S_{i,K} \) is the pattern matrix. \( n \in \mathbb{C}^{N \times 1} \) is the complex baseband AWGN with \( n \sim CN(0, \sigma_n^2 I_N) \).

Proposed two steps method to construct PMs: It is not difficult to find that \( S_{2,3} \) cycle-free but the PM \( S_{4,8} \) in [3] girth 4, which means potential error propagation exists for \( S_{4,8} \) during the first two iterations of the MPA-based detection according to [2]. In this work, we proposed a new method to construct PMs with nested structure and large girth. The proposed method contains two steps and is summarized as follows:

1. Step 1: Extend the base-PM by performing Kronecker product operation
2. Step 2: Puncture the variable nodes with high column-weight

The first step is to extend the base-PM to an expanded-PM by Kronecker product. As doing so once, the overload factor increases to \( \beta > 1 \) times. For example, let \( B \) denote the base-PM, and \( B = S_{2,3} \) with overload factor \( \beta = K/N = 3/2 = 1.5 \). The base PM is extended \( N \) times as follows:

\[
B^N \triangleq B^{0:N} = B \otimes B \otimes \cdots \otimes B, \quad (2)
\]

where \( \otimes \) denotes the Kronecker product. For example, \( B^2 = B^{0:2} = B \otimes B \), and is expressed in as Figure 2.

An intuitive understanding of this expansion operation is given by factor graph [2]. As illustrated in Figure 3(a), the factor graph of the base-PM \( B = S_{2,3} \) is cycle-free and has three variable nodes and two function nodes \( x_1, x_2, x_3 \) and \( y_1, y_2 \). When variable node \( x_i \) and function node \( y_j \) is connected by an edge, \( E(i,j) = 1 \), and vice verse. When the Kronecker product operation is performed, the variable nodes and function nodes are duplicated as follows:

\[
\begin{align*}
x_1 &\rightarrow (x_1, x_1, x_2, x_3), \\
x_2 &\rightarrow (x_2, x_2, x_2, x_2), \\
x_3 &\rightarrow (x_3, x_3, x_3, x_3), \\
y_1 &\rightarrow (y_1, y_2, y_1, y_2), \\
y_2 &\rightarrow (y_2, y_2, y_2).
\end{align*}
\]

The edge connection is depicted by

\[
E((i, k), (j, l)) = \begin{cases} 1 & \text{if } E(i, j) = 1 \text{ and } E(k, l) = 1 \\ 0 & \text{otherwise}. \end{cases}
\]

The first 2-tuple \((i, k)\) in \( E((i, k), (j, l)) \) denotes the subscript of variable node of \( B^2 \) while the second 2-tuple \((j, l)\) denotes the subscript of function node. The Tanner graph of expanded-PM \( B^2 \) is shown in Figure 3(b). It is shown that the expanded-graph has the similar structure.
The complexity of standard message passing-based PDMA receiver is $O(\lambda^m)$, where $d_f$ is the maximum row-weight of the PM. Take $S_{4,8}$ for example compared with the baseline $S_{4,6}$ with maximum $d_f = 4$, its computational complexity will be reduced by a half since the maximum $d_f$ for PM $S_{4,8}$ is 3. Furthermore, all row-weight of $S_{4,8}$ are equal and all REs have about the same decoding complexity, which are beneficial for hardware implementation with pipeline. It is also observed that the proposed PMs result in lower peak to average power ratio.

**Conclusion:** A description of the novel method was given to construct nested PM for PDMA systems. The proposed PMs with large girth and low row-weight implied the high convergence speed and low computational complexity. Numerical simulation results show that both schemes with four inner iterations and five outer iterations are sufficient to obtain satisfactory performance. The proposed PM scheme was practical, with low complexity and suitable for PDMA systems.

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With the base-PM. The second step is intended to puncture the expanded-PM to avoid short cycles and to prohibit the unexpected growth of maximum row-weight. As shown in Figure 2, the last column of PM $B^3$ with weight 4 is punctured, which is emphasized with dashed box. There are two reasons for this operation. On one hand, as shown in Figure 3(b), the variable node $s_{1,3}$ introduces 4 length-4 cycles in the factor graph, which is unexpected in PM designation. On the other hand, by puncturing this weight-4 column, all the row-weight of $B^3$ are the same and equal to 3. As a result, the derived PM is obtained, denoted as $S_{3,8}$ with $K = 8$ and $N = 4$, which is also shown as a submatrix of $S_{20}$ in Figure 5. The overload factor of $S_{3,8}$ is $\beta = 2$ and the girth is $g = 8$ as shown in bold line in Figure 3(c). Figure 4 is the PM $S_{2,8}$ with 8 rows and 27 columns. All weight-4 columns and the weight-8 column emphasized with dashed box are punctured for the same reasons as aforementioned. The derived PM, denoted as $S_{20}$, shown in Figure 5, has overload factor $\beta = 2.5$ and the girth is also $g = 8$.

**Nested pattern matrix:** For $S_{20}$ in Figure 5, the first two rows and the first three columns are the same as PM $S_{2,1}$ while the first four rows and eight columns are PM $S_{4,8}$. We call this nested pattern matrix in this designation.

**Simulation results:** The bit error ratio (BER) performance comparison between the proposed PM and the existing ones in [3] are given in this section. Rate $R_s = 0.5$ low-density parity check (LDPC) code with length $N_s = 2304$ bits is employed [4]. Independent identical distributed (i.i.d) fading channel is used for simplicity and perfect channel state information is known at the receiver but not at the transmitter. Binary phase shift keying (BPSK) modulation is adopted with mapping 0 → +1 and 1 → −1. Let $In_{iter}$ denote the number of iterations for MPA detector, and $Out_{iter}$ denotes the number of iterations between MPA detector and LDPC decoders. In all simulations, LDPC decoders perform 30 standard belief propagation iterations. Figure 6 shows the average BER performance of the PM $S_{4,8}$ with $In_{iter} = 2, 3, 4, 5, 10$ as $Out_{iter} = 5$ is fixed. It can be found that the MPA detector converges within 4 inner iterations. Further increase the number of inner iterations does not improve the performance. Since the girth of the proposed PM $S_{4,8}$ is 8, we conclude that $In_{iter} = 4$ is sufficient for the MPA-based detector to converge. Figure 7 shows the BER performance simulation results for both the proposed PM $S_{4,8}$ and PM $S_{4,6}$ with $Out_{iter} = 1, 2, 3, 4, 5, 10, 30$ under the fixed $In_{iter} = 4$. According to the simulation results, for both the proposed PM $S_{4,8}$ and PM $S_{4,6}$ cases, when $In_{iter} = 4$, the performance gap between $Out_{iter} = 10$ case and $Out_{iter} = 30$ case is marginal. The performance loss of $Out_{iter} = 5$ compared with the case $Out_{iter} = 10$ is only 0.1 dB. We conclude that $Out_{iter} = 5$ is sufficient and reasonable for both cases, and the performance loss is negligible.

**Figures:**

- **Fig. 4** The PM of $B^3 = B^3 \odot B \odot B$. The columns with dash box will be punctured to obtain $S_{20}$.
- **Fig. 5** The PM $S_{3,8}$ is nested in $S_{4,8}$, and $S_{3,8}$ is nested in $S_{20}$.
- **Fig. 6** BER performance of PM $S_{4,8}$ with different $In_{iter}$ iterations as $Out_{iter} = 5$ is fixed.
- **Fig. 7** BER performance comparison between $S_{4,8}$ and $S_{4,6}$ with different $Out_{iter}$ iterations as $In_{iter} = 4$ is fixed.
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