Matrix representation of an EV’s Sensory and Control System

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Abstract. This article will be the continuation of previously published papers and journal articles. The previous developments have been finished by the suggestion of possible matrix-representation of the sensory and control block diagram system of the examined, fully electric vehicle\(^1\) (EV) [1], [2], [3], [5]. This article will show, how these block diagrams can be represented by some special matrices. The first step in the developments was the creation of the sensory and control map of the system (in this form of expression the “system” is the electrical network of the vehicle). Later, this electrical network, based on functionality, was segmented into the different subsystems, like: EV Control System, Driver Control System, ..., see [2]. Hereupon the subsystems could be represented by the graph of the subsystem and, as it is well known, each graph can be represented as a matrix. The problem arose from the fact, that these subsystems are overlapped with each other. The present article will show, how these overlaps can be handled by a special matrix representation.

1. Introduction and Theoretical Background

At the beginning let me introduce some basic definitions regarding graph theory. In this paper the author will consider the graphs consisting of a set of nodes (vertex - \(v\)) and edges (\(e\)). Symbolically it can be written: \(G=(v,e)\). Based on a created network of points (\(v\)) and lines (\(e\)) (what is basically the graph), -including, but limited-, it can be distinguished as several types of graphs, like: grid-like graph, hierarchical tree graph, polygon type graph, connected- and disconnected graphs, hypergraphs, multigraphs, …etc. [4]. Another aspect concerning graphs is arising from orientation of edges, where we can talk about directed and undirected graphs, moreover regarding weighting of edges, we can talk about weighted and unweighted graphs.

1.1. Mathematical representation of graphs

Each finite graph can be mathematically defined by some type of matrix or some list. The most used matrices are the adjacency matrix and the incidence matrix, and for linked representation is known as an adjacency list [8].

1.1.1. Adjacency matrix

In the case of an adjacency matrix, the elements of the matrix are created from the pairs of vertices (\(v\), (the nodes of the graph). That created matrix will be a square matrix. Usually the adjacency is represented by “1” or “0”. Typically, the diagonal of the matrix contains zeros (if no loop on nodes). In case of an undirected graph, the adjacency matrix is symmetric. The “0” and “1” numbers, of

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\(^1\) The mentioned EV is a NISSAN LEAF Z0, owned by the Óbuda University.
elements of the matrix, come from the convention that each edge, connecting two nodes, adds “1” to the cell in the matrix, resp. “0” where there is no edge between the nodes.

In the case of a directed graph, the direction can be represented 2 ways, see equations (1) and (2). Mathematically it can be defined as follows: Let have a graph \( G=(V, E) \); Where \( V=\{v_i\} \) is the vector of vertices (nodes); \( E=\{e_i\} \) is vector of edges. Let \( A=\{a_{ij}\} \) the adjacency matrix. There exists several definitions for matrix representation of the graphs. In case of equation (1) the matrix contains only “0”-s and “1”-s, because the “incoming” edges are “0” represented, while in equation (2) the \( a_{ij} = -1 \), if the edge \( e_j \) leaves vertex \( v_i \); and \( a_{ij} = 1 \) if it enters vertex \( v_i \), otherwise the element is “0”. This representation is more useful for technical applications. (Sometimes the “+” and “-“ interpretations may be interchanged.)

\[
a_{ij} \begin{cases} 
1 & \text{if } v_i \to e_j \\
0 & \text{otherwise}
\end{cases} 
\]  

(1)

\[
a_{ij} \begin{cases} 
1 & \text{if } v_i \to e_j \\
-1 & \text{if } v_i \leftarrow e_j \\
0 & \text{otherwise}
\end{cases} 
\]  

(2)

In case of weighted graphs, instead of “1”-s, the weight of the appropriate edge is usually used. In practical use, the edges can represent connection wires (electrical engineering, informatics) and the weights should be expected voltage/current values or bit-strings.

1.1.2. Incidence matrix

The most important difference between the adjacency and incidence matrices is, that as long as the adjacency matrix is created from pairs node \( \times \) node, the incidence matrix is encoding the relations between the nodes \( \times \) edges. This will be resulting, that usually the incidence matrix is not a square matrix, rather an \( n \times m \) matrix, where “\( n \)” represents the number of nodes, while “\( m \)” the number of edges. The values of the matrix elements can be, -in particular case-, “1”, if vertex \( v_i \) and edge \( e_j \) are incident, otherwise “0”. For oriented and weighted graphs to be valid, the same rules that were stated previously in case of adjacency/incidence matrices, by adding that: the orientation of the edge is signed by “+” resp. “-“ depending on orientation, (in technical applications the bi-directional transfer will be signed by “\( \pm 1 \)” and in case of weighting, instead of “1” the weights are written also.

For a better understanding, the theory described before, will be illustrated through a short example, where a simple electric circuit will be converted into the directed graph, what will be mathematically expressed in an adjacent (\( A \)) and incident (\( B \)) matrices, using representation stated in equation (2).
On Fig. 1, the nodes are: 0, 1, 2, 3 and the edges: U, I₁, I₂, I₃, L₁. The notations are set so that the example can be used later for Kirchoff Current Law (KCL) calculations. And now the matrices: the corresponding adjacency matrix \( A = \{a_{i,j}\} \) where the rows (i) and columns (j) are created from the vertices \( v(i,j) \), where \( i,j = \{0,1,2,3\} \).

In case of incidence matrix \( B = \{b(n,m)\} \), the rows \( n = \{0,1,2,3\} \) are created from nodes, and the columns \( m = \{U, I₁, I₂, I₃, L₁\} \) are formed from the edges.

As it can be seen from Fig. 1. and the described matrices, the adjacency matrix “\( A \)” cannot completely describe the system, because it is not clear that nodes “2” and “3” are connected by “I₂” or “I₃”. That is why we must use incidence matrix “\( B \)” for the complete definition of this graph.

From all this, we can conclude, if in the graph two paired nodes are directly connected with more than one edge (parallel edge connections), for mathematical representation the adjacency matrix is insufficient. It is recommended to use an incidence matrix.

2. Graph Creation and Segmentation

In this section the author will refer to his previously published papers and articles, that will develop more to the graphs, see appendix A, and [2]. The system in appendix A can be segmented from several aspects. Based on [3], some hierarchy can be stated, where the higher level is created from modules, the middle level is consisting of units, and the lower level includes the sensors and actuators. The hierarchy is important regarding connections. Between the high and middle level of hierarchy are bi-directional connections, while between the middle and low-level hierarchy are “one-way” connections. The orientation depends on endpoints. In case of actuators the orientation shows outwards from the units, whereas in case of sensors the direction is opposite.

Another aspect of segmentation of the network can be stated based on functionality of the devices. Thus it, is possible to make groups of devices which are systematically connected [2]. Based on this aspect we can distinguish six sub-systems, see appendix A (driver control system, body exterior system, …). The systematization creates the logical blocks of the network, -logical (functional) segmentation.

The physical segmentation can be down based on modularity, where the segments are created based on real modules, e.g.: segment-PDM will be created from devices which are physically
connected to the PDM module, segment-BCM is created similarly, and thus creating a physical segmentation of the system.

The problem of these modularizations are, in both cases, the different modules are overlapping and the created graph will be quite complex, but it shows “the beauty” of the task. The starting point for graph creation has been solved in [3], where the author had to use some simplifications for a more complex representation. In the whole network, 5 modules can be found, which will be numbered as M{1,2,3,4,5}; 10 units U{1,2,3,4,5,6,7,8,9,10}; 20 sensors S{1,...,20} 18 actuators signed by A{1,...,18}, and a Battery signed by B. These can be imagined as nodes of the graph. The edges will be noted by e_{i,j}. The starting figure for this marking can be found in [3].

For better understanding and a clearer graph, the author creates the group of sensors, resp. actuators, belonging to each device (modules and units). The VSP unit will be neglected, because in newer releases (newest version of Leaf Z0) it was no longer installed by the manufacturer. By this way the number of units decrease by one (U{1,...,9}), and two of the actuators are also eliminated (A{1,...,16}), and certainly the number of nodes also, and the edges also will be reduced, see Fig.2.

![Figure 2](image)

**Figure 2.** The simplified graph representation of the Figure A1 in appendix A.

As it can be seen from Fig. 2, the oriented graph has 30 nodes composed from modules (M_{i}), units (U_{i}), sensors (S_{k}) and actuators (A_{l}), and 41 edges (e_{m}). These will create the rows and columns of the matrix.

Only for the sake of accuracy the non-simplified graph contains 54 nodes and 65 edges. Because of the limited boundaries of the paper, the author will mathematically only represent the simplified graph.
3. Mathematical representation and matrix creation

As it can be seen that the graph is oriented. The orientation in the matrix will be represented as follows:

Let me define the incidence matrix by \( B_{30 \times 41} = [b_{ij}] \), where rows are the nodes (vertices) \( v_{ij} \) and the columns are the edges \( e_{ij} \). Then:

\[
\begin{align*}
&b_{i,j} = \\
&\begin{cases} 
1 & v_i \rightarrow e_j \\
-1 & v_i \leftarrow e_j \\
\pm 1 & \text{otherwise}
\end{cases}
\end{align*}
\]

(3)

The grouping of the sensors and actuators is detailed in Table 1. and Table 2.

### Table 1. Group of actuators and associated devices

| groups: | A1 | A2 | A3 | A4 | A5 | A6 | A7 |
|---------|----|----|----|----|----|----|----|
| belonging actuators | A1,2 | A3 | A4,5 | A6-10 | A11-14 | A15 | A16 |

### Table 2. Group of sensors and associated devices

| groups: | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 |
|---------|----|----|----|----|----|----|----|----|
| belonging sensors | S1-3 | S4-9 | S10 | S11 | S12 | S13,14 | S15-19 | S20 |

The simplified incidence matrix of the system, where the first row contains the number of edges, and the first column contains the main control units (M1-3), units (U1-9) and group of the sensors (S1-8) and actuators (A1-7).

Since the graph does not contain multi-edges assigned to the same pair of nodes, it is possible to write up the adjacency matrix \( A_{30 \times 30} = [a_{ij}] \) of the system too. For an adjacency matrix creation, I will use the definition stated in equation (3), because it makes no sense to use the definition stated in (1), since the sensors are connected to the modules representing a real connection between the devices. Only the direction of the data flow is opposite. It makes no sense to represent them by “0”.
And now the appropriate adjacency (\( M_{IA}(S_{5a5}) \)) and incidence (\( M_{II}(S_{5a4}) \)) matrices of the subsystem.

\[
\begin{align*}
M_{IA}(S_{5a5}) &= \begin{pmatrix}
M1 & M4 & U1 & A1 & A2 & e1 & e2 & e8 & e10
\end{pmatrix} \\
M_{II}(S_{5a4}) &= \begin{pmatrix}
M1 & M4 & U1 & A1 & A2 & e1 & e2 & e8 & e10
\end{pmatrix}
\end{align*}
\]

On the other hand, in the case that we want to examine the whole system more precisely, on how the individual elements interact, it is unavoidable to describe the whole matrices.

**Conclusion**

This paper should solve for further research as can be seen in [5], [6], [7]. Furthermore, it showed an example of how the system can be segmented and created an incidence matrix of a simplified electronic system of a given vehicle. Why should this be useful? Usually in real life, instead of the
value of matrix elements (1, 0, ±1), it is possible to write the required voltage level or current on the selected edge (circuit branch). The created matrix can be realized by some FPGA circuit, which can control (check) all of the connections during operation (real-time), and in case of improper data on the selected link some action can be initiated. By this way a “log-file” can be created, that logs the errors, events (by timestamp or stamp, based on traveled kilometers), that have occurred during operation.

Further possible use of the system can be connected to a risk analysis, in the event of a system failure. Here can be examined, how the failure is spreading in the system and how this error will influence the sub-parts of the system.

The nomenclature of abbreviations used in Figure A1., can be found in reference [3]. The abbreviations used in this article can be found in appendix B.

Future Work
In case of mathematical representations of the sub-systems, the layered hyper-matrix should be used, where the connections between the different sub-matrices can be displayed. The theoretical architecture of this hyper-matrix can be seen on Figure 4., where $CM_{(i,j)}$ matrices are the layered sub-matrices of the system.

![Layered hyper-matrix](image)

$$H = \begin{bmatrix}
CM(1, 1)_{1, 1} & \cdots & CM(1, 1)_{1, n} \\
\vdots & \ddots & \vdots \\
CM(1, 1)_{m, 1} & \cdots & CM(1, 1)_{m, n}
\end{bmatrix} \cdots \\
\vdots & \ddots & \vdots \\
CM(I, 1)_{m, n} & \cdots & \begin{bmatrix}
CM(I, 1)_{1, 1} & \cdots & CM(I, 1)_{1, n} \\
\vdots & \ddots & \vdots \\
CM(I, J)_{m, 1} & \cdots & CM(I, J)_{m, n}
\end{bmatrix}
$$

**Figure 4.** Possible solving of the overlapped sub-parts of the system

The question is, how many layers should be formed? Only at the idea level, I could suggest, that so many sub-matrices, as many sub-systems. One sub-matrix contains the elements of the sub-system,
and if any element is connected to more sub-systems; here is reason of using the layer of the matrix. In this case, from Figure 2., it can be seen, that actuator group A1,2 is connected simultaneously to module M1, and M2 too. Here, the first layer of M1 can be created from modules, which are connected only to the M1, and the additional layers would include those elements that are also connected to the other modules.

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appendix A.

Figure A1. The whole electrical system of the mentioned vehicle, see footnote 1, and [2].
appendix B.

Table of groups of modules, units, sensors and actuators, where the different groups are labelled by different colours in the “Nr.” column. The obvious acronyms are not explained.

Table B1. Groups of the simplified units and assigned modules/units/sensors/actuators

| Nr. | Symbol | Acronym          | Meaning                                           |
|-----|--------|------------------|---------------------------------------------------|
| 1   | M1     | PDM              | Power Delivery Module                             |
| 2   | M2     | IPDM E/R         | Intelligent Power Distribution Module, Engine Room|
| 3   | M3     | BCM              | Body Control Module                               |
| 4   | M4     | VCM (VC)         | Vehicle Control Module                            |
| 5   | M5     | COM. METER       | Combination Meter                                 |
| 6   | U1     | Li-Ion Bat. Unit |                                                   |
| 7   | U2     | Cooling Fan Unit |                                                   |
| 8   | U3     | Siren Control    |                                                   |
| 9   | U4     | Air Bag Diagnostic Control |                           |
| 10  | U5     | EPS Control      | Electric Power Steering System                    |
| 11  | U6     | E-d Intelligent | Electrically Driven Intelligent Brake Unit        |
| 12  | U7     | Brake P.S. Backup| Brake Power Supply Backup unit                    |
| 13  | U8     | Low TPMS Warning Unit | TPMS-Tyre Pressure Monitoring System            |
| 14  | U9     | ABS Control      | Anti-lock Braking System                         |
| 15  | A1     | Electric Compressor |                                                |
| 16  | A2     | Traction Motor Inverter |                                    |
| 17  | A3     | Traction Motor   |                                                   |
| 18  | A4     | Wiper Motors     |                                                   |
| 19  | A5     | Door/Mirror Defogger |                                        |
| 20  | A6     | Power Sw Illuminator |                                       |
| 21  | A7     | Lamps: Daytime/ Fog/Room/Luggage |                |
| 22  | A8     | Door Sw.: front/Rear/Tail/Hoods | Door Switch                                      |
| 23  | A9     | iKey warning buzzer | intelligent Key Warning Buzzer                   |
| 24  | A10    | Washer pumps: front/rear |                      |
| 25  | A11    | Charge Connector, Lock actuator |                        |
| 26  | A12    | Parking Actuator  |                                                   |
| 27  | A13    | Electric Water Pump |                                             |
| 28  | A14    | Reverse Lamp Relay |                                               |
| 29  | A15    | Cooling Fun Motors |                                               |
| 30  | A16    | EPS Motor        | Electric Power Steering System Motor             |
| 31  | S1     | NATS Antennas    | Nissan Anti-Theft System                         |
| 32  | S2     | Door/Mirror RC Sw |                                                |
| 33  | S3     | Light Rain Sensor |                                               |
| 34  | S4     | Electric Shift pos. Sensor |                         |
| 35  | S5     | ASCD Steering Sw | Automatic Speed Control Device steering Switch   |
| 36  | S6     | Coolant Temperature Sensor |                               |
| 37  | S7     | Accelerator Pedal Pos. Sensor |                         |
| 38  | S8     | Battery Current Sensor |                                 |
| 39  | S9     | Refrigerant Pressure Sensor |                               |
| 40  | S10    | Stroke Sensor    |                                                   |
| 41  | S11    | Intruder Sensor  |                                                   |
| 42  | S12    | Tyre pressure sensor |                                         |
| 43  | S13    | Satellite Sensor |                                                   |
| 44  | S14    | Seat Belt Pretension Sensing |                               |
| 45  | S15    | Steering Angle Sensor |                                         |
| 46  | S16    | Wheel Speed Sensor |                                               |
| 47  | S17    | Master Cylinder Pressure Sensor |                         |
| 48  | S18    | Yaw/Side/Decel-G. Sensor | Yaw/Side Deceleration Gyro Sensor |
| 49  | S19    | Brake Pedal pos. Sensor |                                      |
| 50  | S20    | Torque Sensor    |                                                   |
| 51  | B      | Battery          | VSP Vehicle Sound for Pedestrians (neglected)     |

