The Sigma model formulation of the type II string theory in the $AdS_5 \times S_5$ backgrounds with Ramond-Ramond Flux

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Abstract

A sigma model action is constructed for the type II string in the $AdS_5 \times S_5$ back grounds with Ramond-Ramond flux.
1. Introduction

The conjecture of duality between D=4 and $N=4$ super Yang-Mills theory and the type IIB string theory compactified on a five dimensional anti-de Sitter space and a five sphere $AdS_5 \times S_5$ has drawn global attention in recent past [1, 2, 3]. It is not very straightforward to quantize super string with the $AdS_5 \times S_5$ background geometry since there exists self-dual Ramond-Ramond 5-form flux in the Neveu-Schwarz-Ramond (NSR) formulation of string theory. Several attempts are made to quantize string theory in the $AdS_5 \times S_5$ background. The Green-Schwarz formulation of string theory in this background is consistently studied classically [13] and quantization of this is attempted [14]. One faces several technical problems when attempts to covariantly quantize the superstring even in the flat ten dimensional background. Only with the Neveu-Schwarz background, the world-sheet supersymmetry is intact so BRST structure and the Picture-changing operations are smooth for the calculation of physical states. However the Ramond sectors are always represented by the Spin fields. They break the world sheet supersymmetry and give rise to ambiguities of Picture-changing. The spin-spin field correlations give rise to square root cuts and space-time Super symmetry becomes obscure. To illustrate, the space-time supersymmetry generator in the $-\frac{1}{2}$ picture is given by

$$q_\alpha = \int dz e^{-\frac{1}{2} \varphi} S_\alpha$$

(1)

where $\varphi$ comes from the fermionization of the Bosonic ghosts which is usually defined as $\beta = \partial \xi e^{-\varphi}$, $\gamma = \eta e^{\varphi}$ and $S^\alpha$'s are the spin fields of conformal weight $5/8$ constructed from the $\psi^\mu$, the Neveu-Schwarz vector. The supersymmetry (SUSY) algebra is closed on-shell

$$\{q^\alpha_-, q^\beta_\bar{\gamma}\} = \Gamma^{\alpha\beta}_{\mu} \int dz e^{-\frac{1}{2} \varphi} \psi^\mu$$

(2)

where $e^{-\frac{1}{2} \varphi} \psi^\mu$ is not the standard momentum operator. By using a Picture changing operator $P = \{Q, \xi\}$ where $Q$ is the BRST operator, one gets

$$P \int dz e^{-\frac{1}{2} \varphi} \psi^\mu = \int dz \partial x_\mu$$

(3)

the standard momentum operator. Thus one defines one of the generators in the $+\frac{1}{2}$ picture and another in the $-\frac{1}{2}$ picture to get the explicit SUSY

$$\{q^\alpha_+ , q^\beta_\bar{\gamma}\} = \Gamma^{\alpha\beta}_{\mu} \int dz \partial x_\mu$$

(4)

algebra. However it has double the number of required $N = 1$ ten dimensional SUSY generators and it is essential to maintain both the ten dimensional Poincare
invariance and the correct number of SUSY generators. Recently Berenstein and Leigh [6] have an alternative approach to derive sigma model action in the $AdS_5 \times S_5$ background. They take the Ramond operator as $P_{1/2} e^{-\phi S_\alpha}$ where $P_{1/2}$ is the holomorphic square root of the picture changing operator $P$. Then perturbing with Ramond-Ramond vertex operators, around the flat background they are able to generate the $AdS_5 \times S_5$ background by Fischler-Susskind mechanism[7]. The conformal invariance is shown up to two loops.

In the recent past Berkovits and his collaborators [8, 9, 10, 11] chose a novel way to tackle this problem. They redefine the Neveue-Schwarz-Ramond (NSR) variables like Green-Schwarz type of variables such as $\theta^\alpha = e^{1/2 \phi S_\alpha}$ and its canonical conjugate $p_\alpha = e^{-1/2 \phi S_\alpha}$. However $\theta^\alpha$ variables are not all free fields for $\alpha = 1, ..., 16$. There exists OPE among themselves as

$$\theta^\alpha(y)\theta^\beta(z) \rightarrow \frac{1}{(y-z)} \Gamma^{\alpha\beta}_{\mu} e^{\phi} \psi^\mu. \quad (5)$$

Here $\Gamma^{\alpha\beta}_{\mu}$ are the $16 \times 16$ Pauli matrices in 10 dimensions. It is quite possible to choose a subset of these variables which will be free fields. This can happen when four or six of the dimensions are compactified and one has explicitly $SO(6)$ or $SO(4)$ invariance instead of $SO(9, 1)$ invariance. In Ref.[1] Berkovits et al. have picked four of the free $\theta$ variables and their canonical conjugates to show the explicit SUSY invariance in six dimensions. Then $R^6$ is deformed to $AdS_3 \times S_3$ by adding vertex operator for the Ramond-Ramond three form in the action. The Ramond-Ramond form breaks the $SO(6)$ symmetry to $SO(3) \otimes SO(3)$ which is locally isomorphic to $SO(4)$ and is the remnant of the bosonic symmetry. Thus the Lagrangian describes a supersymmetric sigma model in the group manifold of $SU(2) \times SU(2)$ which can be identified with $AdS_3 \times S_3$ space. The same procedure cannot be extended to ten dimension since it is not possible to find eight of the free fermionic coordinates to describe manifestly ten dimensional supersymmetry. Maximally one can have five free fermionic coordinates so one can have $U(5)$ super Poincare invariance [12].

To avoid these problems we have a new formulation of fermionic coordinates which are linear combination of $S_\alpha$ and $\dot{S}_\dot{\alpha}$ so that we can keep $SO(8) \times SO(2)$ type of invariance. Thus $p_\alpha$ and $\theta_\alpha$ in terms of $8_8$ and $8_C$ of $SO(8)$ are

$$p_\alpha = \left( \sigma^1 S_\alpha + C_{\alpha\dot{\alpha}} \sigma^2 \dot{S}_{\dot{\alpha}} \right) \frac{1}{2} e^{-\phi} \quad (6)$$

$$p_{\dot{\alpha}} = \left( \sigma^1 \dot{S}_{\dot{\alpha}} - C_{\dot{\alpha}\alpha} \sigma^2 S_\alpha \right) \frac{1}{2} e^{-\phi} \quad (7)$$

$$\theta^\alpha = \left( \sigma^1 S_\alpha + C_{\alpha\dot{\alpha}} \sigma^2 \dot{S}_{\dot{\alpha}} \right) \frac{1}{2} e^{1/2 \phi} \quad (8)$$

$$\theta^\dot{\alpha} = \left( \sigma^1 \dot{S}_{\dot{\alpha}} - C_{\dot{\alpha}\alpha} \sigma^2 S_\alpha \right) \frac{1}{2} e^{1/2 \phi} \quad (9)$$
where $\sigma^1$, $\sigma^2$ and $\sigma^3$ are Pauli matrices. Here $C^a\dot{a}$ is symmetric and somewhat like the charge conjugation matrix of $SO(6)$ Clifford algebra where $C\gamma_i^tC^{-1} = -\gamma_i$.

The commutation relations and the operator product expansions (OPE) are given as

$$p^{ij}(z)\theta^b_j(w) = -\frac{1}{2(z-w)}\delta^i_k\delta^a_b$$  \hspace{1cm} (10)

$$p^{\dot{i}\dot{j}}(z)\theta^{\dot{b}}_{\dot{j}}(w) = -\frac{1}{2(z-w)}\delta^{\dot{i}}_k\delta^{\dot{a}}_{\dot{b}}$$  \hspace{1cm} (11)

the other OPE’s are

$$p_ap_b = \theta^a\theta^b = p_{\dot{a}}p_{\dot{b}} = \theta^{\dot{a}}\theta^{\dot{b}} = 0$$  \hspace{1cm} (12)

and also

$$p_a\theta^{\dot{b}} = p_{\dot{a}}\theta^b = 0.$$  \hspace{1cm} (13)

Here $p_a$ and $\theta^a$ are not independent variables since

$$p_a = -i\sigma^3C_{a\dot{a}}\theta^{\dot{a}}e^{-\Phi}.$$  \hspace{1cm} (14)

This gives 8 $p_a$ and 8 $\theta^a$ which are independent variables. This has the correct number of $N = 1$ susy fermionic variables. We exactly follow here the procedure of Berkovits et al. [9] to derive sigma model action with $AdS_5 \times S_5$ as back ground from the flat ten dimensional superspace by perturbing with Ramond-Ramond vertex operator. Our fermionic coordinates are derived from the SUSY generators with $-\frac{1}{2}$ picture. We express the SUSY generators with $+\frac{1}{2}$ picture with these fermionic coordinates only and hence we have to couple to the ghosts. Thus we have only $N = 1$ SUSY in the target space where as we have $N = 2$ SUSY in the world-sheet. In this limitations we cannot write an action with arbitrarily curved background. However this can be done taking another set of superspace variables with $+\frac{1}{2}$ picture and use the harmonic constraints.

In section 2. we describe the world-sheet and the space-time supersymmetry of our action. In section 3. we show how to deform $R^{10}$ to $AdS_5 \times S_5$ by adding the vertex operator of the Ramond-Ramond five form in the action. We show that the remnant bosonic symmetry is $SU(4) \times SU(4)$ which indicates that the target space to be $SU(4|4)$ super group manifold. In section 4. we conclude very briefly and the gamma matrices are explicitly given in the appendix.
2. The Space time Supersymmetry formulation

In the $-\frac{1}{2}$ picture we define our supersymmetric generator as

$$q^\alpha_- = \oint dz j^\alpha_- = \int dz p_\alpha$$

where

$$p_\alpha = \begin{pmatrix} p_a \\ p_\dot{a} \end{pmatrix}$$

From the OPEs (c.f. (7)–(12)) we can show that

$$Tr \left( \{q^\alpha_-, q^\beta_-\} \right) = 0,$$

where $Tr$ is over the $\sigma$ matrices. Applying the picture changing operator $\{Q, \xi\}$ on $j^\alpha_-$ we get

$$j^\alpha_+ = b \eta e^{2\varphi} j^\alpha_- + \bar{j}^\alpha_+$$

where $\bar{j}_+ = (\bar{j}_+^a \bar{j}_+^\dot{a})$ and

$$\bar{j}^\alpha_+ = \left( \Gamma^\mu_{ab} \theta^b \partial x_\mu + \Gamma^\mu_{\dot{a}b} \bar{\theta}^b \right) \partial x_\mu$$

and

$$\bar{j}^\dot{a}_+ = \left( \Gamma^\mu_{\dot{a}b} \bar{\theta}^b + \Gamma^\mu_{\dot{a}\dot{b}} \bar{\theta}^\dot{b} \right) \partial x_\mu.$$

This gives OPE

$$\{j^\alpha_- (z), j^\beta_+ (w)\} = \frac{1}{2(z-w)} \delta^i_k \Gamma^\mu_\alpha \partial x_\mu$$

which is the correct covariant off-shell space time supersymmetric commutation relation.

The action is

$$S = \frac{1}{2\pi} \int \left[ \partial x_\mu \partial x^\mu + S_{\text{ghost}} + p_\dot{a} \bar{\theta}^\dot{a} + \hat{p}_\dot{a} \bar{\theta}^\dot{a} \\
+ \lambda^a \left( p_a - i \sigma^a C_{a\dot{a}} e^{-\varphi} \theta^\dot{a} \right) + \hat{\lambda}^\dot{a} \left( \hat{p}_\dot{a} + i \sigma^\dot{a} C^a_{\dot{a}} e^{-\varphi} \theta^a \right) \\
+ \hat{\lambda}^a \left( p_a - i \sigma^a C^a_{\dot{a}} e^{-\varphi} \bar{\theta}^\dot{a} \right) + \lambda^\dot{a} \left( \hat{p}_a + i \sigma^a C^a_{\dot{a}} e^{-\varphi} \bar{\theta}^a \right) \right]$$

where

$$S_{\text{ghost}} = S_{bc} + S_{\beta\gamma} + S_{ge}.$$
Here $S_{ge}$ denotes the extra non-interacting ghosts to make total central charge zero. The right moving fields are all denoted with a hat. Here $\lambda^\alpha$'s are all Lagrange multiplier to use the constraint of eq.(14). We use here all the ghosts in the bosonized form as $b = e^{-i\sigma}$ and $c = e^{i\sigma}$ and $\eta = e^{ik}$ and $\xi = e^{-ik}$.

The OPE’s of free fields are
\[
x^\mu(z)x^\nu(w) = \eta^{\mu\nu} \log(z - w)^2, \quad \kappa(z)\kappa(w) = \sigma(z)\sigma(w) = -\log(z - w) \quad (24)
\]

The energy momentum tensor is
\[
T = \frac{1}{2} \partial x_\mu \partial x^\mu + p_\alpha \partial \theta^\alpha + T_{\text{ghost}}. \quad (25)
\]

To express the super Virasore generator we need the expression of $\psi$ in terms of the superspace variable $\theta$. Thus we first express spin fields as
\[
S^a = (1 + i)\left[\sigma_1^1 \theta^a e^{-\frac{i}{2} \phi} + C^a \sigma^2 p_a e^{\frac{1}{2} \phi}\right] \quad (26)
\]
and
\[
S^\dot{a} = (1 - i)\left[\sigma_1^1 \dot{\theta}^\dot{a} e^{-\frac{i}{2} \phi} + C^\dot{a} \sigma^2 \dot{p}_\dot{a} e^{\frac{1}{2} \phi}\right]. \quad (27)
\]

We define here
\[
A^a = e^{\frac{1}{2} \phi} S^a \\
B^a = e^{-\frac{3}{2} \phi} S^a \\
C^\dot{a} = e^{\frac{1}{2} \phi} S^\dot{a} \\
D^\dot{a} = e^{-\frac{3}{2} \phi} S^\dot{a}
\]
so that we can denote $\psi$ as
\[
\psi_+ = \Gamma_{ab} A^a B^b e^{\phi} \\
\psi_- = \Gamma_{\dot{a}\dot{b}} C^\dot{a} D^\dot{b} e^{\phi} \\
\psi_i = \Gamma_{\dot{a}b} A^\dot{a} C^b e^{\phi}. \quad (29)
\]

The super Virasore generator for the critical $N = 1$ system is
\[
G = \psi_\mu \partial x^\mu. \quad (30)
\]

which can be expressed in terms of new world sheet variables. The BRST operator is
\[
Q_{\text{BRST}} = \frac{1}{2\pi} \int dz \left[cT + ib\partial c + \frac{1}{2} \partial \varphi \partial \varphi + \partial^2 \varphi + i\eta \partial \xi + i\eta e^{\varphi} G + ib\eta \partial \eta e^{2\varphi}\right] \quad (31)
\]

We can have a twisted $N = 2$ super Virasore algebra by taking $G^+ = J_{\text{BRST}}$ and $G^- = b$. 5
3. The Sigma model formulation

We deform the flat ten dimensional space-time to $AdS_5 \times S^5$ by adding a vertex operator $V_H$ depicting Ramond-Ramond flux. Let’s denote a self dual five form as $iH_{01234} = H_{56789}$. The vertex operator due to this will break $SO(10)$ symmetry to $SO(5) \times SO(5)$ rotational symmetry. We set this to be a constant

$$iH_{01234} = H_{56789} = 2g$$

where $g$ is a small parameter measuring the strength of the coupling. This is taken to be a constant so that it can maintain the translational symmetry of the $R^{10}$. The vertex operator in the $-\frac{1}{2}$ picture is $V_{H^-} = q^{-}\hat{q}^{-}$ and in the $+\frac{1}{2}$ picture $V_{H^+} = q^{+}\hat{q}^{+}$. We take here the most general one as the linear combination of both of them.

$$V_H = g \left[ q^+ \hat{q}^- + q^- \hat{q}^+ \right]$$

$$= g \int \left[ p_\alpha \hat{p}_\alpha + (p_\alpha e^\phi - \Gamma_{\alpha\beta}^{\mu} \theta^\beta \partial x_\mu) \left( \hat{p}_\alpha e^\phi - \Gamma_{\alpha\beta}^{\mu} \hat{\theta}^\beta \bar{\partial} x_\mu \right) \right]$$

where $\phi = i(\kappa - \sigma) + 2\varphi$. After a scaling $p_\alpha = g^{-\frac{1}{2}} p_\alpha$, $\theta = g^{-\frac{1}{2}} \theta$ and $e^\phi = -ge^\phi$ one gets

$$V_H = \int \left[ p_\alpha \hat{p}_\alpha - g^2 \left( p_\alpha e^\phi + \Gamma_{\alpha\beta}^{\mu} \theta^\beta \partial x_\mu \right) \left( \hat{p}_\alpha e^\phi + \Gamma_{\alpha\beta}^{\mu} \hat{\theta}^\beta \bar{\partial} x_\mu \right) \right]$$

In order to get a sigma model type of action we rescale all the fields $(x, p_\alpha, \theta^\alpha, e^\phi, \hat{p}_\alpha, \hat{\theta}^\alpha, \ldots)$ to $g^{-1}(x, p_\alpha, \theta^\alpha, e^\phi, \hat{p}_\alpha, \hat{\theta}^\alpha, \ldots)$ to get

$$S = g^{-2} \int \left[ \partial x \bar{\partial} x + p_\alpha \bar{\partial} \theta^\alpha + \hat{p}_\alpha \bar{\partial} \hat{\theta}^\alpha + p_\alpha \hat{p}_\alpha + \lambda^\alpha p_\alpha + \check{\lambda}^\alpha \hat{p}_\alpha + \left( p_\alpha e^\phi + \Gamma_{\alpha\beta}^{\mu} \theta^\beta \partial x_\mu \right) \left( \hat{p}_\alpha e^\phi + \Gamma_{\alpha\beta}^{\mu} \hat{\theta}^\beta \bar{\partial} x_\mu \right) \right.$$ 

$$+ i \sigma^3 C_{\alpha\beta} \left\{ e^{-\varphi} \left( \lambda^\beta \theta^\alpha - \lambda^\alpha \theta^\beta \right) + e^{\hat{\varphi}} \left( \check{\lambda}^\beta \hat{\theta}^\alpha - \check{\lambda}^\alpha \hat{\theta}^\beta \right) \right\}$$

$$= g^{-2} \int \left[ \partial x \bar{\partial} x + p_\alpha \bar{\partial} \theta^\alpha + \hat{p}_\alpha \bar{\partial} \hat{\theta}^\alpha \right.$$ 

$$+ \bar{\lambda}^\alpha p_\alpha + \lambda^\alpha \hat{p}_\alpha + \left( p_\alpha e^\phi + \Gamma_{\alpha\beta}^{\mu} \theta^\beta \partial x_\mu \right) \left( \hat{p}_\alpha e^\phi + \Gamma_{\alpha\beta}^{\mu} \hat{\theta}^\beta \bar{\partial} x_\mu \right) \right.$$ 

$$+ i \sigma^3 C_{\alpha\beta} \left\{ e^{-\varphi} \left( \lambda^\beta \theta^\alpha - \lambda^\alpha \theta^\beta \right) + e^{\hat{\varphi}} \left( \check{\lambda}^\beta \hat{\theta}^\alpha - \check{\lambda}^\alpha \hat{\theta}^\beta \right) \right\}$$

Here $e^\phi$ and $e^{\hat{\varphi}}$ are zero weight conformal fields. To start with we set them zero and separately consider them perturbatively since they scale like the perturbative parameter $g$. Here $p_\alpha$ and $\hat{p}_\alpha$ are like auxiliary variables so we integrate these variables by using their equation of motion which gives

$$\bar{\partial} \theta^\alpha + \lambda^\alpha = -\hat{p}_\alpha$$

and

$$\bar{\partial} \hat{\theta}^\alpha + \check{\lambda}^\alpha = -p_\alpha.$$
Substituting these we get
\[ S = g^{-2} \int \left[ \partial x \bar{\partial} x + \lambda^\alpha \dot{\lambda}^\alpha - \partial \theta^\alpha \bar{\partial} \dot{\theta}^\alpha \ight. \\
+ \Gamma^\mu_{\alpha\beta} \Gamma^\nu_{\alpha\gamma} \theta^\beta \dot{\theta}^\gamma \partial x_\mu \bar{\partial} x_\nu \\
\left. + i \sigma^3 C^{\alpha\alpha} \left\{ e^{-\varphi} \left( \lambda^\dot{a} \theta^\dot{a} - \lambda^a \dot{\theta}^a \right) + e^{\dot{\varphi}} \left( \dot{\lambda}^\dot{a} \dot{\theta}^{} - \dot{\lambda}^a \dot{\theta}^a \right) \right\} \right] \] (38)

The most general supersymmetric transformations of various fields generated by
\[ v_\pm^\alpha \dot{q}_\pm^a + \hat{v}_\pm^\alpha \dot{\hat{q}}_\pm^a \] are
\[ \delta \theta^\alpha = v_+^\alpha + v_+^\alpha e^\varphi + i \sigma^3 v_+^\beta \hat{F}_{\beta \alpha} + \hat{v}_+^\beta (\Gamma^\mu)^{\alpha\beta} x_\mu \] (39)

and
\[ \delta x^\mu = v_+^\beta (\Gamma^\mu)^{\beta\alpha} \theta^\alpha + \hat{v}_+^\beta (\Gamma^\mu)^{\beta\dot{\alpha}} \dot{\theta}^\dot{\alpha} \] (40)

where
\[ \hat{F}_{\beta \alpha} = \left[ (\Gamma^\nu)^{\dot{\alpha}\dot{\alpha}} C^{\alpha\alpha} \delta_{\beta\alpha} + (\Gamma^\nu)^{\beta\dot{\alpha}} C^{\alpha\alpha} \delta_{\dot{\beta}\dot{\alpha}} \right] F_\mu \] (41)

and
\[ F_\mu = \int \frac{e^\varphi}{(z - w)^{32}} \partial x_\mu \] (42)

If we drop the term of \( e^\varphi \) and \( v_-^\alpha \) which is like the translation in the superspace we observe that \( x, \theta, \dot{\theta} \) transform under susy on to themselves and we get a rotation in the super space. To verify the number of bosonic generators we take
\[ Tr \{ \delta^2 x_\mu \} = v_+^\alpha \left[ (\Gamma^\nu)^{\alpha\beta} \hat{F}_\nu^{\beta\gamma} - \hat{F}_\nu^{\beta\gamma} (\Gamma^\nu)^{\beta\dot{\gamma}} \right] \dot{\theta}_+^\gamma x_\nu = R_{\mu\nu} x_\nu \] (43)

Here \( R_{\mu\nu} \) is the pure rotation of the bosonic coordinate \( x_\mu \) which has 30 independent parameters.[4]

1 In the \( SO(8) \) decompositions there are three antisymmetric \( \gamma \) and five symmetric \( \gamma \) matrices (c.f. Appendix). Thus rotational parameters are
\[ v^a (\gamma_i - \gamma_i^t) \hat{v}^a \]
\[ v^\dot{a} (\gamma_i - \gamma_i^t) \hat{v}^\dot{a} \]
give 6 parameters. Similarly
\[ v^a (\gamma_i \gamma_j^t - \gamma_j^t \gamma_i) \hat{v}^a \]
\[ v^\dot{a} (\gamma_i \gamma_j^t - \gamma_j^t \gamma_i) \hat{v}^\dot{a} \]
give 18 parameters and
\[ v^a (\gamma_i \gamma_8^t - \gamma_i^t \gamma_8) \hat{v}^a \]
\[ v^\dot{a} (\gamma_i \gamma_8^t - \gamma_i^t \gamma_8) \hat{v}^\dot{a} \]
give 6 more elements. This gives total 30 matrix elements.
Now if we integrate over $\lambda$ and $\hat{\lambda}$ and neglecting the terms which couples to $b\eta$ ghosts we get

$$S = g^{-2} \int \left[ \partial x \bar{\partial} x - \bar{\partial} \theta^a \partial \bar{\theta}^a + \Gamma^\mu_{\alpha\beta} \Gamma^\nu_{\alpha\gamma} \theta^\beta \bar{\theta}^\gamma \partial x_\mu \bar{\partial} x_\nu + e^{\phi + \tilde{\phi}} \theta^a \tilde{\Gamma}^\alpha + x^\mu x^\nu \bar{\partial} x_\mu \bar{\partial} x_\nu \right] \quad (44)$$

Now we treat

$$\theta^a_q = \begin{pmatrix} \theta^a \\ \hat{\theta}^a \\ \theta^\dagger \\ \hat{\theta}^\dagger \end{pmatrix} \quad (45)$$

as a quartet of the continuous symmetry of $SU(4)$ which is clearly seen if take the second variation of $\theta^a$ due to supersymmetry. Besides here is a symmetry for $z \Leftrightarrow \bar{z}$

$$\theta^a_q = \epsilon_{qp} \theta^a_p \quad (46)$$

where $\epsilon_{qp}$ is an antisymmetric $4 \times 4$ matrix $i\sigma^1 \times \sigma^2$. So without the $b\eta$ ghost coupling we get the action

$$S = g^{-2} \int \left[ \partial_i x^\mu \partial_i x_\mu - \epsilon_{qp} \partial_i \theta^a_p \partial_i \theta^a_q + x^\mu x^\nu \partial_i x_\mu \partial_i x_\nu + e^{\phi + \tilde{\phi}} (\theta_1^a \theta_4^a - \theta_3^a \theta_2^a) \right] \quad (47)$$

The last term is due to the constraint that originally $\theta^a$ and $\hat{\theta}^a$ are not all independent and there exists OPE among them. That is if explicitly calculated it halves the 8 components of $\theta^a$ so that we have total 16 components of fermions instead of the present one with 32 components. This action is a perfect sigma model action with a Lagrangian of the form of $G_{IJ} \partial_i \Phi^I \partial_i \Phi^J$ where $G_{IJ}$ is the metric of the $SU(4|4)$ and $\Phi$ are the coordinates on the group manifold.

4. Conclusion

In this article we present the preliminary version of our big endeavour and there are many more things which deserve further investigations. Our construction of fermionic variables enable us to make the theory covariant. We get clearly two sets of fermionic variables those are non-interacting. We never face the complications of quantizing the theory with local fermionic symmetry with mixed first class and second class constraints which usually plagues the covariant quantization problem. The coupling of zero weight ghost field $e^\phi$ and $\tilde{e}^{\tilde{\phi}}$ is neglected here in order to make the action look simple. Our next step is to take this in to account. The bosonic rotations of $x_\mu$ has 30 elements (c.f. eqn.(43)) and the four copies of fermionic coordinates (c.f.eqn (42)) indicate that the target space is a homogeneous space of $SU(4|4)$ super group manifolds. To verify our model we can make a semi classical analysis as has been done by de Boer et al. \[15\] for the $AdS_3$ and compare the
correlation functions of $<T(x)T(y)>_{\text{worldsheet}}$ with that of the target space on the boundary. There is no problem in finding the dilaton -dilaton correlation function and compare with the results of Maldacena conjecture. We are not repeating these things here. The whole purpose of our exercise is to extend this things to higher genus Riemann surface and take all the ghost couplings which we keep it for the further investigation.

5. Appendix

\[
\Gamma^0 = \begin{pmatrix} 1_8 & 0 \\ 0 & 1_8 \end{pmatrix}, \quad \bar{\Gamma}^0 = \begin{pmatrix} -1_8 & 0 \\ 0 & -1_8 \end{pmatrix},
\]
\[
\Gamma^i = \begin{pmatrix} 0 & \gamma^i_{\dot{a}a} \\ \tilde{\gamma}^i_{\dot{a}a} & 0 \end{pmatrix}, \quad \bar{\Gamma}^i = \begin{pmatrix} 0 & \gamma^i_{\dot{a}a} \\ \tilde{\gamma}^i_{\dot{a}a} & 0 \end{pmatrix},
\]
\[
\Gamma^9 = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix}, \quad \bar{\Gamma}^9 = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix}.
\]

(48)

Here $\gamma^i_{\dot{a}a}, \tilde{\gamma}^i_{\dot{a}a} = -(\gamma^i_{\dot{a}a})^T$ are $SO(8)$ $\gamma$-matrices for $i = 1, \ldots, 7$ and $\gamma^8 = 1_8$ which obey

\[
\gamma^i \tilde{\gamma}^j + \gamma^j \tilde{\gamma}^i = 2\delta^{ij}1_8,
\]

and $i, a, \dot{a} = 1, \ldots, 8$. More explicitly

\[
\gamma_1 = -i\sigma^2 \times \sigma^2 \times \sigma^2
\]
\[
\gamma_2 = i\sigma^2 \times \sigma^3 \times 1
\]
\[
\gamma_3 = i\sigma^2 \times \sigma^1 \times 1
\]
\[
\gamma_4 = -i\sigma^2 \times \sigma^2 \times \sigma^1
\]
\[
\gamma_5 = i\sigma^1 \times 1 \times 1
\]
\[
\gamma_6 = i\sigma^3 \times 1 \times 1
\]
\[
\gamma_7 = -i\sigma^2 \times \sigma^2 \times \sigma^3
\]
\[
\gamma_8 = 1 \times 1 \times 1
\]

(50)

Similarly

\[
C = \sigma^2 \times 1 \times \sigma^2.
\]

(51)
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