Abstract
This paper deals with a numerical study of mixed convection heat transfer in horizontal eccentric annulus. The inner cylinder is supposed hot and rotating, however the outer one is kept cold and motionless. The numerical problem was solved using COMSOL Multiphysics which is based on finite element method. The resolution of the partial differential equations was conducted through an implicit scheme with the use of the damped Newton's method. The present numerical analysis concerns the effect of eccentricity, rotation speed and Rayleigh number on the flow patterns, heat transfer rate, and energy efficiency of the process. It was found that the heat transfer rate increases with the increase of Rayleigh number. In addition, the heat transfer rate drops with the increase of rotation speed. Finally, we have demonstrated that maximum energy efficiency is achieved not only with higher Rayleigh number but also it is maximum with small eccentricity.

Keywords
Annulus, mixed convection, energy efficiency, eccentricity, rotation speed

Introduction
The analysis of heat transfer in cylindrical annulus has been the subject of several old research works. Many configurations were considered in both experimental and numerical studies: concentric motionless annulus, concentric annulus with rotating inner cylinder, eccentric motionless annulus, and eccentric annulus with rotating inner cylinder. The problem of mixed convection heat transfer in cylindrical annulus is still attracting the attention of recent researchers thanks to its importance in many engineering applications such as heat exchangers, centrifugal pumps, wastewater purification systems, etc. Tzeng studied experimentally the heat transfer in cylindrical annulus. The gap between the cylinders was small and only the inner cylinder is rotating. The author carried out a correlation of the heat transfer rate for a large range of rotational Reynolds number (2400 < Re < 45,000). Using an inverse approach, Hsu estimated the viscosity of fluid and the thermal behavior in cylindrical annulus. Shu et al. studied the effect of eccentricity on the flow pattern. Lee studied numerically the case of heat transfer in eccentric annulus. The isotherms, streamlines, and local distributions of Nusselt number were controlled for various rotational Reynolds number (0 < Re < 1120), various Rayleigh numbers, and various eccentricity positions. The authors found that the rotation of the inner...
cylinder causes a decrease of the heat transfer rate. However, when the inner cylinder is motionless, the heat transfer rate increases with Rayleigh number and eccentricity.

The effects of varying boundary conditions and fluid properties were also considerably investigated. Qin et al.\textsuperscript{20} used CO\textsubscript{2} as fluid in the annulus space and studied its impact on the heat transfer characteristics in Taylor Couette flow. Yoo\textsuperscript{21} studied the effect of imposing a constant heat flux on walls for large range of Prandtl number (0.2 < Pr < 1). Hosseini et al.\textsuperscript{22} studied experimentally the heat transfer in an open ended vertical eccentric annulus for different eccentric ratios and different heat fluxes. They demonstrated that the natural convection heat transfer rate increases with the increase of the eccentric ratio from 0 to 0.5. However, for ratios beyond 0.7, the heat transfer decreases and reaches a minimum value with eccentric ratio equal to 1. Ajibade and Bichi\textsuperscript{23,24} investigated the effect of variable fluid properties and thermal radiation. They found that the fluid temperature and velocity decrease with the increase of the conduction rate. In addition, a decrease of the viscosity of the fluid leads to an increase of the velocity in the annulus. The problem of mixed convection concerns also lid driven cavity,\textsuperscript{25,26} porous media, and nanofluid. In fact, Ismael et al.\textsuperscript{27} studied the case of vertically layered fluid-porous medium enclosure with two inner rotating cylinders. The effect of cylinders positions and sizes was then determined for various ranges of Rayleigh number, Darcy number, and rotational speed. Selimefendigil et al.\textsuperscript{28} treated the case of superposed nanofluid and porous layers in square enclosure containing an adiabatic rotating cylinder. They showed that the effect of the rotational speed of the cylinder is more pronounced for large cylinder size.

Recently, the analysis of irreversibility of cylindrical annulus systems through the evaluation of the entropy generation was carried out numerically and experimentally.\textsuperscript{29–36} In particular, Sakly et al.\textsuperscript{35} focused on the effect of geometric and thermodynamic parameters on the entropy generation for the case of unsteady evaporation. They showed that using higher initial temperatures leads to a significant entropy generation. Later, Jarray et al.\textsuperscript{36} studied the combined heat transfer by natural convection and radiation. They used the SNBCK model to solve the radiative part of the numerical problem. They showed that increasing the emissivities of the cylinders causes more radiative entropy generation.

In order to save energy, the enhancement of both heat transfer rate and energy efficiency of heating process becomes an obligation. Some research works that are developed in this context are those of Mazgar et al.\textsuperscript{37} and Ben Abdelmlek et al.\textsuperscript{38} Mazgar et al.\textsuperscript{37} studied numerically the case of horizontal cylinder with partial heating at various locations on the sidewalls. The effects of the location and the size of the heating source on the flow patterns, the dimensionless temperature were delicately analyzed. The authors demonstrated that the best energy efficiency of the heating process is achieved when the heat source is centered on the top of the cylinder. Abdelmlek and Nejma\textsuperscript{38} treated the improvement of the thermal energy efficiency for the case of concentric annulus (i.e. Taylor Couette flow). They proved that the energy efficiency is maximum with low Rayleigh number and low rotational speed.

To our knowledge, the thermal energy efficiency in Taylor-Couette flow as a function of eccentricity and rotational speed has never been investigated until now. Our aim in this paper is then to evaluate the effects of eccentricity, Rayleigh number, and rotational speed on the flow pattern in the annulus and the heat transfer rate. Special attention is given to the effect of mentioned parameters on the thermal energy efficiency of the heating process.

**Mathematical modeling**

**Problem formulation**

The configuration of the physical problem is depicted in Figure 1. It consists of an incompressible flow circulating between two horizontal eccentric cylinders. The inner cylinder of radius $R_{in}$ is hot and rotating with angular velocity $\Omega$. However, the outer one of radius $R_{out}$ is kept cold and motionless. The two cylinders are eccentric: the exterior cylinder is located at the origin of the coordinate system as illustrated in Figure 1, and the inner one is identified by its polar coordinates:

\[
\begin{align*}
\rho_c &= c_R_{out} \\
\Phi_c &= \cdot 
\end{align*}
\]
Assumptions and governing equations

To simplify the numerical analysis, the following assumptions are made in this modeling:

- The length of the cylinders is infinite. So, the flow and the heat transfer are considered bidirectional.
- The flow field is steady, Newtonian, incompressible, and laminar.
- The radiation heat transfer is neglected.
- The Prandtl number is assumed to be equal to 0.71.

According to these assumptions, the non-dimensional governing equations can be described in Cartesian coordinates as follows:

The continuity equation

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$  \hspace{0.5cm} (1)

The momentum equations:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}$$ \hspace{0.5cm} (2)

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + P_r R_o \Theta$$ \hspace{0.5cm} (3)

The energy equation:

$$U \frac{\partial \Theta}{\partial x} + V \frac{\partial \Theta}{\partial y} = \left[ \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \right]$$  \hspace{0.5cm} (4)

Where the non-dimensional variables used in these equations are defined as follows:

$$X = \frac{x}{R_{out}}; Y = \frac{y}{\alpha}; r = \sqrt{X^2 + Y^2}; U = \frac{u R_{out}}{a};$$

$$V = \frac{\nu R_{out}}{\alpha}; W = \frac{\Omega R_{out}^3}{\alpha}; \Theta = \frac{T - T_c}{T_h - T_c}; \Pi = \frac{P R_{out}^2}{\rho a \alpha^2}$$

The local convection heat transfer rate on the inner and the outer cylinders can be expressed in terms of local Nusselt number:

$$Nu_{in, out}(\phi) = 2(1 - f) \left| \frac{\partial \Theta}{\partial r} \right|_{surface}$$  \hspace{0.5cm} (5)

The mean Nusselt number on inner cylinder is calculated averaging equation (5):

$$Nu_{in} = \frac{1}{2\pi} \int_{-\pi}^{\pi} Nu_{in}(\phi) d\phi$$  \hspace{0.5cm} (6)

The mean dimensionless temperature and velocity are calculated respectively as follows:

$$\Theta_a = \frac{1}{\pi(1 - f)^2} \int_{0}^{2\pi} \Theta r dr d\phi$$

$$U_a = \frac{1}{\pi(1 - f)^2} \int_{0}^{2\pi} \sqrt{U^2 + V^2} r dr d\phi$$  \hspace{0.5cm} (7)

In this study, we look for improving the energy efficiency of the heating process which is evaluated through equation (8).

$$\varepsilon = \frac{\Theta_a}{f \cdot Nu_{in}}$$ \hspace{0.5cm} (8)

Numerical procedure and validation

The numerical study was conducted using COMSOL Multiphysics which is based on the finite element method to solve partial differential equations. The resolution of these equations is conducted using an implicit scheme with the use of the damped Newton’s method (damping factor = 0.8) through an adaptive triangular element mesh. The mesh was refined near the wall where the temperature and velocity gradients seem to be important. Then, the grid consists of 21,078 domain elements and 660 boundary elements for the case $e = 0$, $\Phi_c = -\frac{\pi}{6}$, and $f = 0.4$ (Figure 2).

The validity of the proposed numerical code was verified by comparing our numerical results for the case of concentric cylinders displayed in Figure 3 (i.e. $e = 0$) with the results of Desrayaud et al.\textsuperscript{39} and those of Kuehn and Goldstein\textsuperscript{8} for different Rayleigh number. The comparison is resumed in Table 1. Moreover, we have determined analytically the ortho-radial velocity in steady state when $f = 0.5$ and $W = 10^3$. To simplify the analytical resolution, we considered the azimuthal symmetry and we supposed that the gravity forces were neglected. Then, the ortho-radial velocity is expressed as follows:
The evolution of the ratio $u_t/U_0$ in the annular space determined analytically is compared with the evolution estimated using COMSOL Multiphysics in Figure 4. The agreement is found to be good, and shows that our numerical code can correctly predict the mixed convection in eccentric cylinders.

### Results and discussions

**Local distributions for different eccentricity angles $\Phi_e$**

Figure 5 illustrates local distributions of dimensionless temperature, dimensionless velocity, gauge pressure, and local Nusselt numbers for $f=0.4$, $W=10^3$, $Ra=10^4$.

\[
 u_t = \frac{\Omega}{R^2_{ex} - R^2_{in}} \left( \frac{R^2_{ex}R^2_{in}}{r} - R^2_{in}r \right) \tag{9}
\]

The evolution of the ratio $u_t/U_0$ in the annular space determined analytically is compared with the evolution estimated using COMSOL Multiphysics in Figure 4. The agreement is found to be good, and shows that our numerical code can correctly predict the mixed convection in eccentric cylinders.

### Table 1. Validation of the numerical model.

| Ra   | $Nu_{in}^{8}$ | $Nu_{in}^{39}$ | $Nu_{in}$ present work |
|------|---------------|----------------|------------------------|
| $10^3$ | 1.081         | 1.109          | 1.081                  |
| $10^4$ | 2.010         | 2.004          | 1.985                  |
| $5.10^4$ | 3.024         | 3.031          | 2.978                  |

Figure 4. Validation with analytical model.

rise in $Nu_{out}$ values in this area. Similar behavior is noted for the other eccentricity angles.

**Local distributions for different inner cylinder rotation speed**

The effect of the inner cylinder rotation speed on the local distribution of dimensionless temperature and local Nusselt number is showed in Figures 6 to 9. Note that the effect is displayed for the four eccentric angles $\Phi_e$ treated in this study.

For $W = 0$, the two cylinders are stationary. The movement of fluid in the annular space is then governed by the buoyancy forces. Indeed, the flow follows two counter-rotating cells. The hot fluid near the inner cylinder rises upward under the effect of its density difference following an ascending thermal plume. The temperature of the fluid is practically equal to the temperature of the hot cylinder. The heat transfer rate with the inner cylinder is then constant except for the part of the thermal plume where the values of $Nu_{in}$ drop because of the low temperature gradient in this part. When the hot fluid reaches the top of the outer cylinder, it is divided into two counter-rotating cells. The temperature gradient is then very large in the upper part of the annular space, which explains the rise in $Nu_{out}$ values in this region. The same observations are also found with $\Phi_e = 180^\circ$ displayed in Figure 8.
Given the eccentricity of the inner cylinder, the left rotating cell is dominant (similarly, the right cell dominates in the case of $\Phi_c = 180^\circ$). Thus, the heat transfer rate with the outer cylinder is very low due to the temperature gradient drop. As for the right cell (the left cell for $\Phi_c = 180^\circ$), fluid movement is difficult in the restriction zone. This allows lower heat exchange rate with the hot cylinder. The lower part of the annular eccentric space is characterized by stagnant fluid temperature, causing almost no heat transfer with the outer cylinder.

By passing to $W = 10^3$, a competition between the buoyancy forces and the viscous forces begins. The heat is then transferred by forced convection. The heat transfer with the hot cylinder is then fair. Also, we notice that the left cell (the right cell for $\Phi_c = 180^\circ$) decreases in size, and its temperature is stagnant and is equal to the cold temperature. Thus, the heat transfer with the cold cylinder is zero in this zone. However, the heat exchange rate improves in the restriction zone (to the right of the hot cylinder for $\Phi_c = 0^\circ$) where fluid circulation is difficult and the temperature gradient with the outer cylinder is improved.

The local distributions of temperature and Nusselt number for the case of $\Phi_c = 90^\circ$ are depicted in Figure 7 for different rotation speeds of the hot cylinder. The movement of the fluid follows the rotation of the cylinder. The heat is then transferred by forced convection. The heat transfer with the hot cylinder is then fair. Also, we notice that the left cell (the right cell for $\Phi_c = 180^\circ$) decreases in size, and its temperature is stagnant and is equal to the cold temperature. Thus, the heat transfer with the cold cylinder is zero in this zone. However, the heat exchange rate improves in the restriction zone (to the right of the hot cylinder for $\Phi_c = 0^\circ$) where fluid circulation is difficult and the temperature gradient with the outer cylinder is improved.

Figure 5. Local distributions of: (a) dimensionless temperature, (b) dimensionless velocity, (c) gauge pressure, and (d) Nusselt number for $f = 0.4$, $e = 0.3$, $W = 10^3$, $Ra = 10^4$, and different $\Phi_c$. 

$Khaoula$ and $Fayçal$
cylinder. We notice that the heat transfer takes place completely in the upper part of the annular space. Indeed, the buoyancy forces cause the thermal plume to direct vertically above the hot cylinder. On meeting the outer cylinder, the fluid is divided into two cells and flows down the annular space with a temperature almost equal to the cold temperature. This prevents heat exchange in the lower part and explains the low values of $Nu_{\text{out}}$. Finally, the cold fluid returns to the hot cylinder where it gains a large amount of heat. By applying a speed of rotation to the inner cylinder, we observe a deflection of the thermal plume to the left under the effect of the resultant of viscous forces and those of buoyancy. That causes in return a deviation of the values of $Nu_{\text{in}}$. By further increasing the speed of rotation of the hot cylinder, viscous forces monopolize the movement of the fluid in the annulus. An equitable distribution of heat transfer is then noticed with the hot cylinder. The maximum heat transfer with the outer cylinder is in the right part above the hot cylinder, where the temperature gradient is quite high.

For $\Phi_c = 270^\circ$, the local distributions of temperature and Nusselt numbers are illustrated in Figure 9 for different rotation speeds of the inner cylinder. The movement of the fluid is symmetrical when the two cylinders are stationary. Indeed, under the effect of the difference in density, the hot fluid rises upwards from the annular space following a vertical thermal plume. The heat transfer with the inner cylinder then weakens at this area, but it is maximum at the outer cylinder. On the other hand, in the lower part of the annular space, the fluid gains a significant amount of heat from the inner cylinder since the temperature gradient is maximum. By applying a speed of rotation to the hot cylinder: $W = 10^2$, we observe a deflection of the thermal plume due to the mixed convection. Finally, passing to $W = 10^3$, viscous forces govern the movement of the fluid. A series of rings are then formed around the hot cylinder, and the upper part of the annulus remains stationary and slows the rate of heat transfer.

### Local distributions for different Rayleigh number

The effect of Rayleigh number on local distributions of temperature and local Nusselt numbers is displayed in Figures 10 to 13 for the four eccentric angles studied in this work. Figure 10 shows the local distributions for the case of $\Phi_c = 0^\circ$. As it can be seen, for $Ra = 10^3$, we notice the presence of two contrarotating cells. The first
one is dominant and localized around the hot cylinder indicating that the transfer mode is that by conduction. The heat transfer rate is practically zero on the left cell for $Ra = 10^3$. We notice that the convective cell increases in size as the Rayleigh number increases, promoting heat exchange. This is why $Nu_{out}$ improves with increasing Rayleigh. We also note the appearance of a third rotating cell when we increase Rayleigh to $Ra = 10^5$. The latter boosts heat transfer with the outer cylinder and explains the improved $Nu_{out}$ on the left side.

Local distributions for the case $\Phi_c = 90^\circ$ are shown in Figure 11. The series of rings developed around the hot cylinder tighten when increasing the Rayleigh number, indicating that the transfer mode changes from a purely conductive transfer to a mixture of conductive and convective transfer. The secondary cell changes focus and its size decreases with $Ra = 10^4$. In addition, the transfer rate with the inner cylinder improves with increasing Rayleigh as the convection increases. On the other hand, the transfer with the outer cylinder weakens and undergoes a deformation with $Ra = 10^5$ given the deformation of the dominant cell.

Figure 12 displays the case of $\Phi_c = 180^\circ$. The movement of fluid similarly follows two counter-rotating cells. A dominant cell around the hot cylinder is in the form of consecutive, equidistant rings. The distance between these rings decreases with increasing Rayleigh, as the convection subsides. The secondary cell is located to the right of the hot cylinder. The effect of increasing the Rayleigh number on the heat transfer rate is very small from $Ra = 10^3$ to $Ra = 10^4$. On the other hand, the effect is remarkable with $Ra = 10^5$, where the secondary cell increases in size. The temperature gradient also improves on this cell causing an improvement in heat transfer with the inner cylinder. In addition, a deformation of the $Nu_{out}$ curve at the lower right part of the annulus can be explained by the low temperature gradient in this area. An improvement of $Nu_{out}$ is also noted to the right of the hot cylinder, more precisely in the restricted zone where the circulation of the fluid is difficult.

Local distributions for the case $\Phi_c = 270^\circ$ are shown in Figure 13, the effect of increasing the Rayleigh number is remarkable with $Ra = 10^5$. Indeed, the cell developed above the hot cylinder increases in size and becomes the dominant cell. It covers more than three quarters of the annular space with $Ra = 10^5$. The hot fluid collects in the lower left part of the annulus and causes a decrease in heat transfer with the inner cylinder.
Effect of eccentricity position

Figure 14 illustrates the evolution of dimensionless temperature as a function of the eccentricity angle $\Phi_c$, for different radial coordinates $e$. As it can be seen, the dimensionless temperature decreases by increasing $e$. This is because of the increase in the size of the dead cell; which has very low velocity (i.e. the left cell for $\Phi_c = 0^\circ$). The dead cell slows down the heat transfer. In addition, one notices that for higher $e$, the dimensionless temperature is more sensitive to the azimuthal coordinates $\Phi_c$. Figure 15 depicts the effect of $\Phi_c$ on the local Nusselt number $N_u_{in}$ for different radial positions $e$. We note that the effect of azimuth is accentuated when increasing $e$. The heat transfer rate with the hot cylinder is minimal for an azimuthal angle $\Phi_c = 0^\circ$. It is remarkable that the heating process is more efficient with small $e$ whatever the azimuth. In addition, it should be noted that for high radial positions $e$, the heating process is more reliable with lower azimuthal angles.

Effect of the rotation speed of the inner cylinder

The evolution of dimensionless temperature as a function of the azimuthal coordinates is illustrated in Figure 17 for different rotational speeds $W$. We observe a sinusoidal shape, the amplitude of which decreases as the speed of rotation increases. Finally the curve becomes practically constant for $W = 10^5$. In addition, the fluid reaches a maximum temperature for $W = 0$ and $W = 10^2$ with an azimuthal angle $\Phi_c = 270^\circ$. Regarding the heat transfer rate with the inner cylinder, we can see in Figure 18 the evolution of the Nusselt number $N_u_{in}$ as a function of the $\Phi_c$ for different rotational speeds. Note that for $0 < W < 10^3$, the shape of $N_u_{in}$ is sinusoidal, the amplitude of which decreases by increasing the speed of rotation of the inner cylinder. For $W = 10^4$, the transfer rate increases and it is almost constant whatever the azimuthal angle $\Phi_c$.

Figure 19 illustrates the effect of rotational speed on the energy efficiency of the heating process. A rotational speed $W = 10^3$ seems to be the best speed which provides the highest energy efficiency. For this same speed, it is also noted that the energy efficiency of the
process is maximum for an azimuthal position of $30^\circ < \Phi_c < 60^\circ$. However, a very high rotational speed $W = 10^4$ causes a decrease in the energy efficiency of the system for the different angles of eccentricity.

**Effect of Rayleigh number**

The change in the dimensionless temperature of the fluid as a function of the $\Phi_c$ angle is illustrated in Figure 20 for different Rayleigh numbers. We note for the values $Ra = 10^3$ and $Ra = 10^4$, the dimensionless temperature is very insensitive to the variation of the $\Phi_c$ angle. On the other hand, for $Ra = 10^5$, it is very dependent on the $\Phi_c$ angle, and its evolution is sinusoidal. A maximum value of the fluid is reached with $Ra = 10^5$, and an angle $300^\circ < \Phi_c < 330^\circ$.

The evolution of $Nu_a$ as a function of the $\Phi_c$ angle is illustrated in Figure 21 for different Rayleigh values. The sinusoidal shape is very small in amplitude for $Ra = 10^3$, $Ra = 10^4$, and it becomes accentuated by increasing $Ra$. A maximum heat transfer rate is noted for $Ra = 10^5$ and $\Phi_c = 240^\circ$.

Figure 22 illustrates the evolution of energy efficiency as a function of $\Phi_c$ for different Rayleigh numbers. The best energy efficiency of the studied process is reached with a Rayleigh number $Ra = 10^5$ and $\Phi_c = 60^\circ$. Additionally, the intersection between the sinusoidal curves indicates that there are other configurations that maximize the energy efficiency of the heating process. For example, for low Rayleigh numbers ($Ra = 10^3$), the eccentricity angles $120^\circ < \Phi_c$
Figure 15. Effect of eccentricity on average Nusselt number $Nu_{in}$.

Figure 16. Effect of eccentricity on energy efficiency.

Figure 17. Effect of rotation speed on dimensionless temperature.

Figure 18. Effect of rotation speed on average Nusselt number $Nu_{in}$.

Figure 19. Effect of rotation speed on energy efficiency.

Figure 20. Effect of Rayleigh number on dimensionless temperature.
guarantee maximum energy efficiency of the process.

Conclusion
In this work, mixed convection heat transfer in eccentric annulus was numerically investigated. The inner cylinder is hot and rotational, however the outer one is cold and motionless. The flow pattern in the form of streamlines and isotherms was explored for different Rayleigh numbers, rotational speeds and eccentricity positions. The effects of mentioned parameters on local distribution of Nusselt number and heat transfer rate were also involved. It was found that the heat transfer rate improves with the increase of both Rayleigh number Ra and eccentricity e. However, it decreases considerably with the increase of the speed of inner cylinder rotation. In addition, we have demonstrated that when the inner cylinder is motionless \((W = 0)\), the heat transfer is mainly conducted by natural convection. However, when increasing the rotational speeds \((W = 10^3)\), the convection heat transfer becomes mixed. With higher rotational speeds \((W = 10^4)\), the heat is mostly exchanged by forced convection. Concerning the energy efficiency, we have demonstrated that it increases well with the rotation speed, but it decreases and reaches a constant value with very high speeds. In addition, small eccentricities lead to more efficient heating process. Finally, we have found that the energy efficiency of the studied process achieves its maximum not only with higher Rayleigh number, but also with lower Rayleigh number if the azimuth angle is \(120^\circ < \Phi_c < 330^\circ\). Future work will focus on the effect of radiation on the heat transfer process in concentric and eccentric annulus. It would be interesting to improve the efficiency and overall performance of the process when heat transfer by radiation is considered.

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**Appendix**

**Notations**

| Symbol | Description                  |
|--------|------------------------------|
| $e$    | eccentricity ratio          |
| $f$    | radius ratio                |
| $g$    | acceleration due to gravity (m s$^{-2}$) |
| $Nu$   | Nusselt number              |
| $P$    | pressure (Pa)               |
| $Pr$   | Prandtl number              |
| $R$    | cylinder radius (m)         |
| $Ra$   | Rayleigh number             |
| $(r,\phi)$ | cylindrical coordinate system |
| $T$    | temperature (K)             |
| $\Delta T$ | temperature difference, $T_h - T_c$ (K) |
| $(U,V)$ | dimensionless velocity components |
| $(u,v)$ | velocity components (m s$^{-1}$) |
| $W$    | dimensionless angular velocity |
| $(X,Y)$ | dimensionless Cartesian coordinates |
| $(x,y)$ | Cartesian coordinates        |

**Greek symbols**

| Symbol | Description                  |
|--------|------------------------------|
| $\alpha$ | thermal diffusivity (m$^2$s$^{-1}$) |
| $\beta$ | thermal expansion coefficient (K$^{-1}$) |
| $\varepsilon$ | energy efficiency |
| $\Theta$ | dimensionless temperature |
| $\nu$ | kinematic viscosity (m$^2$s$^{-1}$) |
| $\Pi$ | dimensionless pressure |
| $\rho$ | density (kg m$^{-3}$) |
| $(\rho_c,\Phi_c)$ | polar coordinates of inner cylinder |
| $\Omega$ | angular velocity (rad s$^{-1}$) |

**Subscripts**

| Symbol | Description |
|--------|-------------|
| $a$    | average     |
| $c$    | cold        |
| $h$    | hot         |
| int    | inner       |
| out    | outer       |