We present a search for the possible \( I(J^P) = 0(2^+) \) tetraquark state with \( sss\bar{s} \) quark content in quenched improved anisotropic lattice QCD. Using various local and non-local interpolating fields we determine the energies of ground-state and second ground state using variational method. The state is found to be consistent with two-particle scattering state, which is checked to exhibit the expected volume dependence of the spectral weights. In the physical limit, we obtain for the ground state, a mass of \( 2123(33)(58) \) MeV which is higher than the mass of experimentally observed \( f(10) \). The lattice resonance signal obtained in the physical region does not support a localized \( J^P = 2^+ \) tetraquark state in the pion mass region of 300 – 800 MeV. We conclude that the 4q system in question appears as a two-particle scattering state in the quark mass region explored here.

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I. INTRODUCTION

The concept of multi-quark hadrons has received revived interest due to the narrow resonances in the spectrum of states. Recently, several new particles were experimentally discovered and confirmed as the candidates of multi-quark states. These discoveries are expected to reveal new aspects of hadron physics. Among these discoveries, the tetra-quark systems are also interesting in terms of their rich phenomenology, in particular for mesons which still remain a most fascinating subject of research. The 4q states are interesting in terms of the recent experimental discoveries of \( X(3872) \), \( Y(4260) \) and \( D_s(2317) \), which are expected to be tetra-quark candidates.

The Particle Data Group lists 2 tensor mesons with masses in the range 1.9 – 2.2 GeV/c\(^2\) and considers them as well-established. The \( 2^{++} \) candidates, \( f_2(1950) \) and \( f_2(2010) \) are isosinglet. The relevant channels of decay are \( K\bar{K} \) and \( \eta\eta \) for the \( f_2(1950) \) and \( \phi\phi \) and \( K\bar{K} \) for \( f_2(2010) \). Due to their \( K\bar{K} \) decay, one would expect \( f_2(1950) \) and \( f_2(2010) \) are very likely one state; the mass shift could be a measurement error or could be caused by the \( K\bar{K} \) threshold. However, the results for \( f_2(2010) \) favour an intrinsically narrower state, strongly coupled to the \( \phi\phi \) and weakly coupled to the other channel for allowed s-wave decays. Following the recent re-analysis of the BNL data we discuss the state \( f_2(2010) \) as a \( s^2s\bar{s} \) state.

The multi-quark states have been investigated in lattice QCD studied with somewhat mixed results. At the present status of approximations, lattice QCD seems to provide a trustworthy guide into unknown territory in tetra-quark hadron physics. Using the quenched approximation, and discarding quark-antiquark annihilation diagrams, we construct \( s^2s\bar{s} \) sources from multiple operators. Note that we are working in quenched approximation which in principal is unphysical. However, previous lattice results on masses and decay constants turn out to be in good agreement with experimental values. This seems to suggest that it is plausible to use quenched lattice QCD to investigate the mass spectra. We exclude the processes that mix \( qq \) and \( q^2\bar{q}^2 \) and allow the quark masses to vary from small to large values. In the absence of quark annihilation, we do not expect any mixing of \( q^2\bar{q}^2 \) with pure glue. Thus, we can express the \( q^2\bar{q}^2 \) correlation functions in terms of a basis determined by quark exchange diagrams only (ignoring the single, double and annihilation diagrams among Wick’s contractions). Another important question is whether the interpolating operator one uses has a significant overlap with the state in question. To construct an interpolating field which has significant overlap with the 4q system, we adopt the so-called variational method to compute \( 2 \times 2 \) correlation matrix from two different interpolating fields and from its eigenvalues we extract the masses. Thus, assuming that the quenching uncertainties do not effect our conclusions dramatically, we investigate the optimized correlation function and use it to examine lowest-lying tetraquark resonance as \( f_2(s\bar{s}s\bar{s}) \) states in the spectrum of \( 2 \times 2 \) correlation matrix.

II. LATTICE STUDY FOR THE \( f_2(s\bar{s}s\bar{s}) \)

The simplest local interpolators can be written in terms of colour-singlet configuration of a product of colour-neutral meson interpolation fields. We propose a non-\( \phi\phi \) interpolating field to extract the \( f_2(s\bar{s}s\bar{s}) \) tetraquark state. This choice is designed to maximize the possibility to observe attraction between tetraquark constituents at relatively large quark masses. With a \( \phi\phi \) operator it is possible that there is a small amount of
the compact 4$q$ component in the two-body interpolating field since the interpolator may contain a large contamination of $\phi\phi$ scattering states. We adopt the simplest non-$\phi\phi$-type interpolator of the form

$$O_1(x) = \frac{1}{2} \left[ \left( \bar{q}_a(x) (\gamma_\mu)_{\alpha\beta} q^\beta_b(x) \right) \left( \bar{Q}^\dagger_b(x) (\gamma_\mu)_{\lambda\nu} Q^\dagger_a(x) \right) - \left( q \leftrightarrow Q, \bar{q} \leftrightarrow \bar{Q} \right) \right],$$

(1)

with spin $I(J^P) = 0(2^+)$. In the nonrelativistic limit the above non-two state particle can not be decomposed into $\phi\phi$. Thus the 4$q$ state can be singled out as much as possible and the results are less biased by the contamination of two-state scattering states.

The other type of interpolating field is one in which quarks and anti-quarks are coupled into a set of diquark and antidiquark, respectively and has the form

$$O_2(x) = \epsilon_{abc} \left[ \bar{q}_b CT Q_c \right] \epsilon_{ade} \left[ \bar{q}_d CT \bar{Q}_e \right].$$

(2)

Accounting for both colour and flavour antisymmetry, possible $\Gamma$'s are restricted within $\gamma_5$ and $\gamma_i$. For $\Gamma = \gamma_5\gamma_i$ ($i = 1, 2, 3$), the above diquark operator transforms like $J^P = 1^-$. For concreteness, we simulate the flavour combination $[ss]$ and $[\bar{s}s]$.

To extract energies $E_n$ of $s^2\bar{s}^2$ we compute the $2 \times 2$ correlation matrix

$$C_{ij}(t) = \sum_x \text{tr} \left[ \langle (O_i(x), t) \bar{O}_j(0, 0) \rangle \right] U,$$

(3)

where the trace sum over the Dirac space, and the subscripts $f$ and $U$ denote fermionic average and gauge field ensemble average, respectively. Following [12, 16, 28] we solve the eigenvalue equation

$$C(t_0) v_k(t_0) = \lambda_k(t_0) v_k(t_0)$$

(4)

to determine the eigenvectors $v_k(t_0)$. We use these eigenvectors to project the correlation matrices to the space corresponding to the $n$ largest eigenvalues $\lambda_n(t_0)$

$$C_{ij}^n(t) = \langle v_i, C(t) v_j \rangle, \quad i, j = 1, \ldots, n$$

(5)

and solve the generalised eigenvalue problem equation for the projected correlation matrix $C_{ij}^n$. The resulting large-time dependence of the eigenvalues $\lambda_n(t)$ allows a determination of ground and excited-state energies. The mass can be extracted by a hyperbolic-cosine fit to $\lambda_n(t)$ for the range of $t$ in which effective mass

$$M_{eff}(t) = \ln \left[ \frac{\lambda(t)}{\lambda(t+1)} \right]$$

(6)

attains a plateau. In order to show the existence or absence of the signature of tetraquark resonance on lattice, we establish lowest and as well as the second-lowest energy levels for our 4$q$ system.

Using a tadpole-improved anisotropic gluon action [29], we generate quenched configurations on two lattice volumes $16^3 \times 64$ and $16^3 \times 80$ (with periodic boundary conditions in all directions). After discarding the initial sweeps, a total of 200 configurations are accumulated for measurements at $\beta = 4.0$ Quark propagators are computed by using a tadpole-improved clover quark action on the anisotropic lattice [30]. All the coefficients in the action are evaluated from tree-level tadpole improvement.

The bare mass of the strange quark is determined by extracting the mass of the vector meson $M_\phi$. At $m_q a_t = 0.066$, we obtain $\kappa_t = 0.2404$, which produces a mass for the $\phi$ of 1.237(2) in lattice units. Using the mass of the nucleon in the chiral limit, we find that the ratio $M_\phi/M_N$ at the chiral limit is 1.059 ± 0.014, which is in good agreement with the physical ratio of 1.087. This verifies that the strange bare quark mass of 0.07 used is very close (within 3%) to the physical strange quark mass. The quark propagators are then computed at seven values of the hopping parameter $k_t$ which cover the strange quark mass region of $m_s < m_q < 2m_s$, i.e., $a_t m_q = 0.07, 0.075, 0.08, 0.09, 0.105, 0.115, 0.12$. Inspired by the good agreement of the ratio with the experimental value, the scale was set alternatively by $M_\phi/M_N$. Using the experimental value 938 MeV for the nucleon mass, the spacing of our lattice is $a_s = 0.473(2)$ fm.

### III. RESULTS AND DISCUSSION

Figs. [I] illustrate the two lowest energy levels extracted by fitting the effective masses over appropriate $t$ intervals. The ground state eigenvalues show a conventional time-dependence near $t \approx T/2$ and hence the mass can be accurately extracted using Eq. [II]. We choose one "best fit" which is insensitive to the fit range, has high confidence level and reasonable statistical errors. We then confirm this by looking at the plateau region of the correlator. Statistical errors of masses are estimated by the jackknife method and the goodness of the fit is gauged by the $\chi^2/\text{N}_{\text{DF}}$, chosen according to criteria that $\chi^2/\text{N}_{\text{DF}}$ is preferably close to 1.0.

The effective mass is found to be stable using different values of $t$ in Eq. [II], which suggests that the ground state in question is correctly projected. Suppressing any data point which has error larger than its mean value, the possible plateau is seen in the region $5 \leq t \leq 12$ with reasonable errors, where the single-state dominance is expected to be achieved. Fitting the effective mass in the window $t = 6 \text{–} 11$ is found to optimize the $\chi^2/\text{N}_{\text{DF}}$. To avoid the clutter in Fig. [I] we do not show the points at larger $t$ values which have larger error bars, and have little or no influence on the fits. The best fit curve to the 4$q$ data has $\chi^2/\text{N}_{\text{DF}} = 0.87$. The results for the masses corresponding to the various values of the hopping parameter $\kappa_t$ are tabulated in Table [II].

To interpret the ground state in terms of signatures of a lattice resonance, we look at three possible scenarios. First, we extract the mass splitting between the tetraquark $0(2^+)$ and the noninteracting $\phi + \phi$ two-
lattice units, for various values of $\kappa$. Table I: The masses of the 4q, Kaon and $\phi$ states, in the lattice units, for various values of $\kappa$.

| $\kappa$ | $M_{4q}$ | $M_{4q}^L$ | $M_K$ | $M_\phi$ |
|----------|----------|------------|-------|---------|
| 0.2410   | 2.187(4) | 2.223(6)   | 0.579(2) | 1.029(3) |
| 0.2420   | 2.055(7) | 2.083(9)   | 0.507(5) | 0.973(6) |
| 0.2435   | 1.951(13)| 1.973(11)  | 0.475(8) | 0.926(11)|
| 0.2440   | 1.812(19)| 1.825(21)  | 0.440(5) | 0.860(15)|
| 0.2450   | 1.724(24)| 1.751(37)  | 0.417(13)| 0.822(28)|
| 0.2455   | 1.655(27)| 1.673(44)  | 0.401(14)| 0.787(17)|
| 0.2462   | 1.591(23)| 1.606(49)  | 0.383(19)| 0.759(24)|

FIG. 1: Effective mass of the $I(J) = 0(2^+)$ colour-singlet lowest-lying ground state. The data correspond to $m_\pi \simeq 361$ MeV (triangles) and 824 MeV (circles).

It is interesting to note that our results are consistent with quenched one-loop results despite the fact that the disconnected contributions were not inserted in our calculation. This implies that disconnected correlator has very small or negligible contribution than the connected one at several time separations.

To confirm or discard the signature observed in Fig. 2, we examine the second scenario, i.e., the volume dependence of the spectral weight of these states. Theoretically, if the state is a genuine resonance, then its spectral weight should be almost constant for any lattices with the same lattice spacing. On the other hand, if it is a two-particle scattering state, then its spectral weight has an explicit $1/V^2$ dependence[22]. In the following, we shall use the ratio of the spectral weights on two spatial volumes $16^3$ and $20^3$ to discriminate whether the hadronic state in question is a resonance or a scattering state.

FIG. 2: The energy shift of the lowest $I(J^P) = 0(2^+)$ state as a function of $m_\pi L$. The solid and dashed lines correspond quenched one-loop chiral perturbation results for $\delta = 0.12$ and 0.15, respectively.

We see clearly that the masses derived for the tetraquark state are consistently higher than the lowest two-particle state. The mass difference is over 100 MeV at small quark masses and weakly dependent on $m_\pi L$. The positive mass difference observed in this range of pion mass suggests that the observed signal is unlikely to be a tetraquark. We also notice that our data are reasonably consistent with one-loop quenched perturbation results[31] for $m_\pi L \geq 4.3$ for $\delta = 0.12$ and 0.15.

1 Since we are using the quenched approximation, the extraction of energy shift in a finite box using full QCD one-loop chiral perturbation theory is not applicable.
be verified by analyzing the 4q potential of the tetraquark system. We do not intend to pursue such an analysis here since this is not the focus of our present study. The spectral weight ratio of the first excited state turns out to be consistent with 1.95, confirming our speculation that it is two-particle scattering state. The two states are reasonably well separated compared to the decay width of \( f(2010) \).

Finally, the mass differences extracted can be extrapolated to the physical limit, which is the next important issue \cite{33}. Since quenched spectroscopy is quite reliable for mass ratio of stable particles, it is physically even more motivated to extrapolate mass ratios rather than masses or mass differences. This allows for the cancellation of systematic errors since the hadron states are generated from the same gauge field configurations and hence systematic errors are strongly correlated. We use a set of data points with smallest \( m_s^2 \) to capture the chiral log behaviour. Fig. 4 collects and displays the resulting mass ratios, illustrated in Table II extrapolated to the physical limit using linear and quadratic fits in \( m_s^2 \). The difference between these two extrapolations gives some information about systematic uncertainties in the extrapolated quantities. Performing such extrapolations to mass ratios, we adopt the choice which shows the smoothest scaling behaviour for the final value, and use others to estimate the systematic errors.

The data at smallest five quark masses behave almost linearly in \( m_s^2 \) and both the linear and quadratic fits essentially gave the identical results. The contributions from the uncertainties due to chiral logarithms in the physical limit are seen to be significantly less dominant. The mass difference \( \Delta M \) is \( \sim 100 \) MeV at the smaller quark masses, and weakly dependent on \( m_s^2 \). The signature of repulsion at quark masses near the physical regime would imply no evidence of the resonance in the \( J = 2 \) channel. If this mass difference from two-\( \phi \) threshold can be explained by the two-\( \phi \) interaction, then the \( s^2s^2 \) state can be regarded as a two-\( \phi \) scattering state.

To verify whether analysis at relatively large quark masses would affect the manifestation of the \( J = 2 \) state and aid to confirm the indication of a resonance, we allow the quark mass to be \( m_q > 2m_s \) so that the threshold for the decay \( q^2q^2 \rightarrow (q\bar{q})(q\bar{q}) \) is elevated. The heavy quark mass suppresses relativistic effects, which complicates the interpretation of light-quark states. The resulting extracted mass ratios are shown in Fig. 5 and tabulated in Table III.

The behaviour observed for the mass differences between the \( J = 2 \) and the two-particle states, at large quark masses, implies that at larger quark masses, the data appear above the two-\( \phi \) threshold by \( \sim 95 \) MeV and remains constant in magnitude as the physical regime is approached. This trend continues in the physical limit where the masses exhibit the opposite behaviour to that which would be expected in the presence of binding. Again, the positive mass difference could be a signature of repulsion in this channel. This suggests that instead of a bound state, we appear to be seeing a scattering state.
in $J = 2$ channel. Since the mass difference between the reported experimental $s^2s^2$ mass and the physical $2m_\phi$ continuum is $\sim 20$ MeV, the observed signal is too heavy to be identified with the empirical $f_2(1950)$ or $f_2(2010)$.

Using the physical kaon mass, $M_K = 503(5)$ MeV, we obtain a mass estimates of $2123(33)(58)$ MeV and $2137(39)(64)$ for the $s^2s^2$ tetraquark ground state and the second ground-state, respectively. In each case, the first error is statistical, and second one is our estimate of combined systematic uncertainty including those coming from chiral extrapolation and quenching effects. Note that we cannot estimate the discretization error since we have only one lattice spacing to work with. Given the fact that the ratio does not show any scaling violations, we could also quote the value of this quantity on our finest lattice, which has the smallest error. Nevertheless, order 2% errors on the finally quoted values are mostly due to the chiral extrapolations. The quenching errors might be the largest source of uncertainty. Note however, that in the case of mass ratios of stable hadrons, this is not expected to be very important. It has been shown \cite{34} that with an appropriate definition of scale, the mass ratios of stable hadrons are described correctly by the quenched approximation on the $1 - 2\%$ level. To this end we also calculated the pseudoscalar to vector meson ratio $R_{SP}$ and pseudoscalar to nucleon mass ratio $R_{SN}$ and found that in the physical limit these ratios differ about 1% from their corresponding experimental values. So we quote our quenching errors to be less than two percent.

IV. SUMMARY AND CONCLUSION

We presented the results of our investigation on the tetraquark systems in improved anisotropic lattice QCD in the quenched approximation. The mass of $J = 2$ state was computed using field operators, which are motivated by the non-$\pi\pi$ and diquark structure. In the quenched approximation, our results suggest that our interpolators have sufficient overlap with $f_2(s\bar{s}s\bar{s})$ to allow a successful correlation matrix analysis and produced the evidence that the mass of the lowest-lying state only agrees marginally with the mass of $f(2010)$. In the region of pion mass which we are able to access, we saw no evidence of attraction that could be associated with the existence of a resonance in $J = 2$ channel. Since our estimated value for the mass of $f_2(s\bar{s}s\bar{s})$ is close to its experimental value, we suspect that might be the $f(2010)$ resonance captured by our optimized correlator. However, on the other hand, our spectral weight ratio for two different lattice volumes deviates from one (the essential criterion for resonance) with large errors for small quark masses, observed state exhibits the expected volume dependence in the spectral weight for two particles in a box. The ground-state is found to be consistent with scattering state. Our estimated values serve as predictions of lattice QCD in quenched approximation. Indeed, our simulation does not include dynamical quarks, the final conclusions will have to wait till both disconnected correlators and annihilation contributions are incorporated.

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