Data Article

Optimal schemes of radial network arch pedestrian bridges: An extensive dataset of solutions under different conditions

Rimantas Belevičius*, Algirdas Juozapaitis, Dainius Rusakevičius, Sigutė Žilėnaitė

Vilnius Gediminas Technical University (VGTU), Saulėtekio al. 11, LT-10223 Vilnius, Lithuania

A R T I C L E   I N F O

Article history:
Received 26 March 2021
Revised 7 May 2021
Accepted 11 May 2021
Available online 15 May 2021

Keywords:
Pedestrian radial network arch bridge
Mass minimization
Topology
Shape
Sizing parameters

A B S T R A C T

This data article provides a series of 220 optimal parameter sets of steel radial network arch pedestrian bridge schemes. Each set includes 10 design parameters for a two-dimensional bridge frame: the arch rise, the number of hangers, their (variable) spread and central angles, and the dimensions of the arch, girder, and hangers. Additionally, solutions are provided for different initial conditions such as constant hanger angles, given ratio values between the arch rise and the bridge span, given hanger diameters, and for the condition of hangers' verticality. These data are related to the research article “Parametric study on mass minimization of radial network arch pedestrian bridges” (Belevičius et al., 2021). In this paper, the bridge scheme was optimized seeking for the minimal bridge mass under different loading cases according to recommendations of Eurocode EN 1991–2. Since the optimization problem is multimodal, and the employed stochastic global optimization algorithms provide different solutions for each run of the algorithm, we render the five best parameters' sets for every problem. In many cases, close objective function values correspond to fairly distinct parameter values. This dataset could be used by bridge designers as an initial design hint by choosing the appropriate parameters set.

DOI of original article: 10.1016/j.engstruct.2021.112182
* Corresponding author.
E-mail address: rimantas.belevicius@vgtu.lt (R. Belevičius).

https://doi.org/10.1016/j.engstruct.2021.112182
Specifications Table

| Subject                  | Engineering                                      |
|--------------------------|--------------------------------------------------|
| Specific subject area    | Civil and Structural Engineering, Computational Mechanics |
| Type of data             | Table                                            |
| How data were acquired   | All the solutions obtained employing the software consisting of three independent original programs: meshing program that from 10 design parameters renders all initial data for finite element (FE) program; FE linear static analysis program, and program that analyses the FE results, evaluates the objective function value, checks the constraints and penalizes the objective function value if needed. The FE program is validated comparing with ANSYS [2] results. As an optimization algorithm, the genetic and particle swarm algorithms from MatLab [3,4] were used. |
| Data format              | Raw Filtered                                     |
| Parameters for data collection | We collected 10 main parameters that fully describe the design of the bridge from the five best solutions for each analyzed problem. The parameters include the arch rise/bridge span ratio, the number of hangers, the hanger spread and central section angles and their variations, and the dimensions of all construction elements. The bridge arch and girder are designed of S355 steel rectangular tube profiles, while the hangers are designed of solid round profiles. |
| Description of data collection | Since every run of the stochastic global optimization algorithm usually ends up with different values of the objective function, each problem was solved several times. The total number of solutions varied depending on the problem's initial conditions and was not less than 50. The calculations were stopped when the median value of the objective function did not change over the last runs of the optimization algorithm. |
| Data source location     | The dataset presented in this paper was collected from Vilnius Gediminas Technical University, Saulėtekio al. 11, LT-10223, Vilnius, Lithuania |
| Data accessibility       | Data: with the article Software: Mendeley data repository, direct URL to archive: https://data.mendeley.com/datasets/ctbgc679/1 |
| Related research article | R. Belevičius, A. Juozapaitis, D. Rusakevičius, Š. Žilenaitė, Parametric Study on Mass Minimization of Radial Network Arch Pedestrian Bridges, Eng. Struct. 2021 (237), DOI: 10.1016/j.engstruct.2021.112182 [1] |

Value of the Data

• This dataset of main topological, shape, and sizing parameters of pedestrian radial network arch bridge has practical importance for the fast designing of pedestrian radial network arch bridges.
• Civil engineers and architects can use this dataset for the designing process. Researchers can use this data to analyze the most influential parameters on the mass of bridges or use the data as a basis for comparison with their findings on optimal schemes of radial network arch bridges.
• Civil engineers and architects can use this data as a hint for their design. All the parameters are provided in continuous forms, and the designer may choose appropriate profiles from the assortment tables and round the values of the angles, rise/span ratio, etc. to the ones more suitable for engineering practice. Since the optimized solution is very sensitive to the smallest parameter changes, after rounding a detailed analysis of the scheme is needed.
1. Data Description

This data article presents optimal values of topological, shape, and sizing parameters of pedestrian radial network arch bridge frame. The nomenclature of topological and shape parameters is shown in Fig. 1.

The arch and the girder are made of S355 steel rectangular tube profiles, while the hangers are designed of solid round profiles. All profiles of rectangular tubes are constant along all length of construction. All hangers have the same diameter. Thus, the sizing parameters include the widths and heights of rectangular tubes of the arch and girder and the diameter of hangers.

Fig. 2 explains how the hanger arrangement is obtained in the case of constant spread and central section angles $\alpha$ and $\beta$. Generally, both angles constantly vary along the bridge span. Going from hanger to hanger in Fig. 1, the spread angle is augmented by $\Delta\alpha$. In case of constantly changing central section angle, it is more convenient to postulate the ratio $\beta_{\text{ini}}$ between initial central section angle and the average angle $\beta_{\text{avg}} = \gamma/n$, where $\gamma = \arcsin\left(\frac{4f^2}{L^2+4f^2}\right)$ is the

![Fig. 1](modified from [1]) Nomenclature of topological and shape parameters: $\alpha$ – hanger spread angle, $\beta$ – central section angle, $\gamma$ – total central angle, $f$ – arch rise, $L$ – bridge span.

![Fig. 2](Generation of the bridge scheme at the constant hanger angles: a) initial scheme of hangers, b) final scheme – obtained by the mirror image.)
total central angle, and \( n \) is the number of sections (the total number of hangers is \( 2n \)). Then the central section angle increment is obtained via 
\[
\Delta \beta = 2 \frac{\gamma - n}{n(n-1)} \beta_{\text{avg}} \beta_{\text{ini}}.
\]

Thus, at \( \beta_{\text{ini}} > 1 \) the hangers’ mesh is denser at the center of the scheme, and vice versa. All the design parameters along with their bound are listed in Table 1.

The bridge scheme was optimized at different initial conditions of parameters.

Tables 2 to 5 present solutions at arbitrary values of all 10 design parameters, constant central section angle \( \beta \), constant spread angle \( \alpha \), and both constant angles \( \beta \) and \( \alpha \). Besides, the total mass of the whole frame \( M \), and the frame elements’ masses \( M_a, M_g, \) and \( M_h \) – masses of an arch, girder, and hangers, all in kg, are provided. The standard deviation and average of solution parameter values through 100 (50) independent numerical experiments also are presented.

Tables 6 to 9 present solutions at four variants of hangers’ angles (arbitrary values of both angles \( \beta \) and \( \alpha \), constant central section angle \( \beta \), constant spread angle \( \alpha \), and both constant angles \( \beta \) and \( \alpha \)) and six constant values of the ratio \( f/L \) (1/2, 1/3, 1/4, 1/5, 1/6, and 1/7 (the arch rise \( f \) is 30, 20, 15, 12, 10, and 8.57, correspondingly)).

Tables 10 to 13 present solutions at four variants of hangers’ angles (arbitrary values of both angles \( \beta \) and \( \alpha \), constant central section angle \( \beta \), constant spread angle \( \alpha \), and both constant angles \( \beta \) and \( \alpha \)), plus four restricted values of hangers’ radius \( r_h \): \( \geq 5, \geq 10, \geq 14, \) and \( \geq 20 \) mm.
Table 3

Five best solutions at $\beta = \text{const.} \ (\beta_{\text{in}} = 1)$.

| Parameters | Five best solutions | Mean | $\sigma$ |
|------------|---------------------|------|---------|
| $f$        | 16.75               | 16.683 | 0.883 |
| $n$        | 33                  | 34    | 6.20 |
| $\alpha$   | 7.87                | 9.01  | 6.33 |
| $\Delta \alpha$ | 1.204        | 1.020 | 2.28 |
| $h_a$      | 0.306               | 0.429 | 0.318 |
| $w_a$      | 0.1156              | 0.0940 | 0.0932 |
| $h_b$      | 0.264               | 0.237 | 0.248 |
| $w_b$      | 0.0871              | 0.0884 | 0.0880 |
| $r_h$      | 0.00542             | 0.00559 | 0.00116 |
| $M$        | 2911                | 2951  | 17.99 |
| $M_g$      | 1499                | 1525  | 16.68 |
| $M_h$      | 788                 | 740   | 97.7 |
| $M_{\text{in}}$ | 624             | 686   | 51.7 |

Table 4

Five best solutions at $\alpha = \text{const.} \ (\Delta \alpha = 0)$.

| Parameters | Five best solutions | Mean | $\sigma$ |
|------------|---------------------|------|---------|
| $f$        | 13.88               | 13.73 | 0.533 |
| $n$        | 33                  | 21.7  | 4.79 |
| $\alpha$   | 30.1                | 31.3  | 1.02 |
| $\beta_{\text{in}}$ | 1.289        | 1.334 | 1.037 |
| $h_a$      | 0.275               | 0.383 | 0.076 |
| $w_a$      | 0.1294              | 0.1145 | 0.0168 |
| $h_b$      | 0.294               | 0.285 | 0.0969 |
| $w_b$      | 0.0942              | 0.1005 | 0.0153 |
| $r_h$      | 0.00538             | 0.00116 | 0.000794 |
| $M$        | 3012                | 3159  | 80.9 |
| $M_g$      | 1528                | 1632  | 51.1 |
| $M_h$      | 942                 | 979   | 42.1 |
| $M_{\text{in}}$ | 541             | 547   | 16.6 |

2. Experimental Design, Materials and Methods

To obtain the dataset presented in this paper, numerical experiments were performed employing original software and stochastic global optimization algorithms from MATLAB.

Original software in FORTRAN consists of three independent parts: the first code, meshing program, prepares the whole data set for the FE analysis program from a set of design parameters. The bridge is idealized as a plane frame system. The meshing program evaluates the coordinates of all finite element nodes that are placed at the intersections of structural elements: arch, girder, and hangers. The parts of all structural elements between nodes are idealized as 2-node beam finite elements with 12 degrees of freedom. The hangers that receive only tensile forces and are slender are also idealized as beam finite elements. All connections between finite elements are considered ideal contacts, that is, all connections are perfectly rigid. The program calculates also the stiffness characteristics of all finite elements, evaluates the nodal loads from four loading cases according to the Eurocode EN 1991–2 (Eurocode 1 – Actions on structures – Part 2: Traffic loads on bridges), provides the finite element connectivity information, and imposes the boundary conditions on the degrees of freedom to be excluded. It is assumed that the bridge frame is supported by a hinge at one side and simply supported at the other. Due to
Generally, the tolerance. Following, we consider the structural parameters. For the sake of brevity, we only discuss the algorithm where the parameters are kept constant. Considering the following constraints, one can derive that the thickness of the tube wall is chosen to ensure the local stability of the cross-section. The material of all structural elements is steel S355.

The second original program performs linear static analysis via FE method. The results of the program are validated by comparison with ANSYS. We cannot directly use the ANSYS [2] or more precise analysis types since the bottleneck of the computational procedure is the computation time, and the fast problem-oriented program is an evident necessity.

The third program evaluates the objective function value, i.e. the total mass of the bridge scheme and checks the structural equilibrium constraints, the strength constraints on all structural elements, the stability constraints on the arch elements, and the possible relaxation of any hanger. The relaxation of any hanger conditions a dead penalty. In case of any other constraint violation, the program penalizes the objective function value. For any trespass of the allowable value of \( c_i \), the following penalty on the objective function \( F \) is assigned:

\[
F := F \left( \frac{|c_i - c_{\text{allowable}}|}{c_i} \right) p,
\]

where the penalty factor \( p = 2 \).

The starting module of all software system is MATLAB’s stochastic evolutionary optimization algorithm (EA) requiring no sensitivity information. Two algorithms were employed: the genetic algorithm [3] and particle swarm algorithm [4]. Both EA algorithms provided close results. Initially, the optimization algorithm randomly generates a given number of initial sets of design parameters. Now all three FORTRAN programs, connected to the optimization algorithm as a “black-box”, calculate the objective function value. Based on these results, EA generates the new improved design parameters’ guess. The whole optimization cycle continues until one of the following criteria is reached: the maximum given number of the populations is achieved, or the objective function value does not change over the given number of the last populations or the weighted average of the objective function per given number of generations is less than a given tolerance.

Usually, each run of the program set ends up with different results. Therefore, the optimization is repeated several times until the median value of the objective function does not change. Generally, each problem was run 50–100 times.

### Table 5

Five best solutions at \( \beta, \alpha = \text{const.} \). \( \beta_{\text{ini}} = 1, \Delta \alpha = 0 \).

| Parameters | Five best solutions | 100 solutions |
|------------|---------------------|---------------|
|            | 1   | 2   | 3   | 4   | 5   | Mean | σ   |
| \( f \)    | 13.91 | 13.52 | 13.58 | 13.52 | 14.02 | 13.36 | 0.453 |
| \( n \)    | 36   | 34   | 32   | 32   | 36   | 25   | 5.97 |
| \( \alpha \) | 32.7 | 32.7 | 32.1 | 32.4 | 29.4 | 32.5 | 1.90 |
| \( h\alpha \) | 0.563 | 0.463 | 0.436 | 0.420 | 0.362 | 0.381 | 0.0865 |
| \( w\alpha \) | 0.0822 | 0.0984 | 0.1033 | 0.1058 | 0.1153 | 0.1200 | 0.0158 |
| \( h_k \alpha \) | 0.338 | 0.281 | 0.314 | 0.333 | 0.300 | 0.321 | 0.0721 |
| \( w_k \alpha \) | 0.0857 | 0.0978 | 0.0908 | 0.0876 | 0.0917 | 0.0969 | 0.0124 |
| \( r_h \) | 0.00565 | 0.00582 | 0.00602 | 0.00605 | 0.00583 | 0.00697 | 0.00102 |
| \( M \) | 3231 | 3269 | 3280 | 3282 | 3282 | 3427 | 138.9 |
|      | 1566 | 1616 | 1632 | 1626 | 1618 | 1717 | 88.4 |
|      | 938  | 954  | 949  | 952  | 927  | 1029 | 74.2 |
|      | 726  | 698  | 699  | 704  | 737  | 681  | 43.5 |
### Table 6

Five best solutions at both arbitrary angles $\beta$ and $\alpha$, and given values of the ratio $f/L$.

| Parameters | 1 | 2 | 3 | 4 | 5 | Mean | $\sigma$ |
|------------|---|---|---|---|---|------|---------|
| $f$        | 30 | 30 | 30 | 30 | 30 | 30   | 30      |
|            | 20 | 20 | 20 | 20 | 20 | 20   | 20      |
|            | 15 | 15 | 15 | 15 | 15 | 15   | 15      |
|            | 12 | 12 | 12 | 12 | 12 | 12   | 12      |
|            | 10 | 10 | 10 | 10 | 10 | 10   | 10      |
| $n$        | 28 | 28 | 28 | 28 | 28 | 28   | 28      |
|            | 27 | 27 | 27 | 27 | 27 | 27   | 27      |
|            | 22 | 22 | 22 | 22 | 22 | 22   | 22      |
|            | 20 | 20 | 20 | 20 | 20 | 20   | 20      |
|            | 19 | 19 | 19 | 19 | 19 | 19   | 19      |
| $\alpha$   | 7.8 | 7.8 | 7.8 | 7.8 | 7.8 | 7.8  | 7.8     |
|            | 7.3 | 7.3 | 7.3 | 7.3 | 7.3 | 7.3  | 7.3     |
|            | 8.6 | 8.6 | 8.6 | 8.6 | 8.6 | 8.6  | 8.6     |
| $\Delta \alpha$ | 1.34 | 1.34 | 1.34 | 1.34 | 1.34 | 1.34  | 1.34     |
| $\beta_{\text{m}}$ | 1.47 | 1.47 | 1.47 | 1.47 | 1.47 | 1.47  | 1.47     |
| $h_s$      | 0.472 | 0.472 | 0.472 | 0.472 | 0.472 | 0.472  | 0.472   |
| $w_s$      | 0.1136 | 0.1136 | 0.1136 | 0.1136 | 0.1136 | 0.1136  | 0.1136  |
| $h_g$      | 0.815 | 0.815 | 0.815 | 0.815 | 0.815 | 0.815  | 0.815   |
| $w_g$      | 0.1102 | 0.1102 | 0.1102 | 0.1102 | 0.1102 | 0.1102  | 0.1102  |

(continued on next page)
Table 6 (continued)

| Parameters | 1       | 2       | 3       | 4       | 5       | Mean   | σ      |
|------------|---------|---------|---------|---------|---------|--------|--------|
| $r_h$      | 0.00641 | 0.00637 | 0.00627 | 0.00642 | 0.00618 | 0.00767 | 0.001246 |
|            | 0.00585 | 0.00537 | 0.00589 | 0.00582 | 0.00544 | 0.00601 | 0.000485 |
|            | 0.00578 | 0.00611 | 0.00617 | 0.00593 | 0.00597 | 0.00694 | 0.000734 |
|            | 0.00561 | 0.00615 | 0.00659 | 0.00768 | 0.00781 | 0.00802 | 0.000871 |
|            | 0.00793 | 0.00768 | 0.00883 | 0.00871 | 0.00862 | 0.00927 | 0.000916 |
|            | 0.00929 | 0.00958 | 0.00926 | 0.00996 | 0.00997 | 0.01085 | 0.000943 |
| $M$        | 6704    | 6780    | 6940    | 6955    | 6970    | 7891   | 830    |
|            | 2825    | 2833    | 2847    | 2849    | 2851    | 2955   | 103.1  |
|            | 2946    | 2950    | 2956    | 2956    | 2960    | 3046   | 82.3   |
|            | 3244    | 3331    | 3385    | 3387    | 3387    | 3470   | 90.5   |
|            | 3894    | 3903    | 3949    | 3973    | 3994    | 4125   | 118.2  |
|            | 4696    | 4717    | 4721    | 4724    | 4727    | 4860   | 126.1  |
| $M_b$      | 2701    | 2567    | 2792    | 2778    | 2697    | 3243   | 605    |
|            | 1489    | 1470    | 1492    | 1515    | 1499    | 1547   | 66.8   |
|            | 1512    | 1520    | 1546    | 1512    | 1547    | 1571   | 40.4   |
|            | 1702    | 1724    | 1815    | 1767    | 1773    | 1806   | 41.2   |
|            | 2053    | 2096    | 2085    | 2157    | 2059    | 2131   | 70.7   |
|            | 2395    | 2363    | 2422    | 2458    | 2360    | 2476   | 124.6  |
| $M_s$      | 2649    | 2848    | 3007    | 2968    | 2998    | 3326   | 287    |
|            | 682     | 705     | 691     | 684     | 703     | 735    | 46.0   |
|            | 867     | 853     | 853     | 890     | 849     | 906    | 35.2   |
|            | 1047    | 1116    | 1067    | 1118    | 1099    | 1146   | 58.1   |
|            | 1369    | 1321    | 1380    | 1323    | 1465    | 1509   | 118.5  |
|            | 1814    | 1836    | 1839    | 1808    | 1908    | 1918   | 106.2  |
| $M_b$      | 1354    | 1365    | 1140    | 1208    | 1275    | 1321   | 193.8  |
|            | 654     | 659     | 665     | 649     | 649     | 673    | 26.0   |
|            | 567     | 578     | 557     | 554     | 564     | 569    | 41.5   |
|            | 495     | 492     | 503     | 501     | 515     | 517    | 23.3   |
|            | 472     | 486     | 484     | 493     | 470     | 486    | 21.7   |
|            | 488     | 518     | 460     | 458     | 459     | 466    | 26.1   |

Table 7

Five best solutions at $\beta = \text{const.} (\beta_m = 1)$ and given values of the ratio $f/L$.

| Parameters | 1       | 2       | 3       | 4       | 5       | Mean  | σ      |
|------------|---------|---------|---------|---------|---------|-------|--------|
| $f$        | 30      | 30      | 30      | 30      | 30      | 30    | 30     |
|            | 20      | 20      | 20      | 20      | 20      | 20    | 20     |
|            | 15      | 15      | 15      | 15      | 15      | 15    | 15     |
|            | 12      | 12      | 12      | 12      | 12      | 12    | 12     |
|            | 10      | 10      | 10      | 10      | 10      | 10    | 10     |
|            | 8.57    | 8.57    | 8.57    | 8.57    | 8.57    | 8.57  | 8.57   |
| $n$        | 31      | 35      | 28      | 27      | 26      | 22.3  | 5.76   |
|            | 36      | 35      | 29      | 31      | 36      | 30.6  | 4.08   |
|            | 31      | 30      | 33      | 30      | 33      | 26.5  | 3.54   |
|            | 23      | 23      | 26      | 26      | 23      | 17.04 | 4.19   |
|            | 16      | 16      | 15      | 15      | 15      | 11.82 | 1.837  |
|            | 12      | 12      | 10      | 10      | 10      | 8.24  | 1.117  |
| $\alpha$  | 8.66    | 7.38    | 9.46    | 10.34   | 10.24   | 12.08 | 3.27   |
|            | 5.16    | 5.65    | 5.94    | 8.18    | 6.15    | 7.61  | 1.774  |
|            | 17.69   | 17.39   | 18.93   | 18.07   | 20.0    | 21.2  | 2.57   |
|            | 32.4    | 32.3    | 31.7    | 28.7    | 30.9    | 34.1  | 3.66   |
|            | 32.2    | 32.6    | 32.5    | 32.3    | 32.5    | 32.5  | 0.802  |
|            | 34.9    | 34.7    | 35.0    | 35.0    | 35.0    | 35.0  | 0.0537 |

(continued on next page)
Table 7 (continued)

| Parameters | 1   | 2   | 3   | 4   | 5   | Mean |
|------------|-----|-----|-----|-----|-----|------|
| Δα         | 0.845 | 0.810 | 0.906 | 0.878 | 0.945 | 1.039 |
|            | 1.187 | 1.251 | 1.495 | 1.324 | 1.096 | 1.257 |
|            | 0.872 | 0.972 | 0.654 | 0.889 | 0.725 | 0.853 |
|            | 0.426 | 0.469 | 0.418 | 0.561 | 0.653 | 0.628 |
|            | 0.645 | 0.584 | 0.971 | 0.847 | 0.891 | 0.999 |
|            | 1.311 | 1.483 | 1.483 | 1.470 | 1.486 | 1.484 |
| h₀         | 0.495 | 0.465 | 0.557 | 0.510 | 0.519 | 0.555 |
|            | 0.388 | 0.438 | 0.401 | 0.441 | 0.408 | 0.341 |
|            | 0.402 | 0.355 | 0.423 | 0.284 | 0.453 | 0.363 |
|            | 0.381 | 0.356 | 0.405 | 0.213 | 0.274 | 0.425 |
|            | 0.515 | 0.303 | 0.371 | 0.601 | 0.490 | 0.488 |
|            | 0.261 | 0.483 | 0.368 | 0.393 | 0.359 | 0.507 |
| w₀         | 0.1136 | 0.1147 | 0.1067 | 0.1146 | 0.1130 | 0.1202 |
|            | 0.0934 | 0.0837 | 0.0918 | 0.0845 | 0.0914 | 0.1064 |
|            | 0.1025 | 0.1117 | 0.1001 | 0.1268 | 0.0936 | 0.1144 |
|            | 0.1207 | 0.1274 | 0.1182 | 0.1651 | 0.1458 | 0.1202 |
|            | 0.1238 | 0.1620 | 0.1492 | 0.1112 | 0.1278 | 0.1298 |
|            | 0.1999 | 0.1468 | 0.1712 | 0.1638 | 0.1734 | 0.1478 |
| h₀         | 0.949 | 0.904 | 0.952 | 0.962 | 0.964 | 0.900 |
|            | 0.179 | 0.268 | 0.228 | 0.263 | 0.201 | 0.234 |
|            | 0.321 | 0.237 | 0.254 | 0.299 | 0.262 | 0.274 |
|            | 0.410 | 0.284 | 0.179 | 0.211 | 0.253 | 0.314 |
|            | 0.242 | 0.298 | 0.209 | 0.291 | 0.230 | 0.384 |
|            | 0.252 | 0.218 | 0.302 | 0.333 | 0.298 | 0.399 |
| w₉         | 0.1189 | 0.1254 | 0.1197 | 0.1202 | 0.1206 | 0.1341 |
|            | 0.0969 | 0.0806 | 0.0875 | 0.0800 | 0.0897 | 0.0883 |
|            | 0.0815 | 0.0990 | 0.0936 | 0.0869 | 0.0947 | 0.0946 |
|            | 0.0880 | 0.1107 | 0.1378 | 0.1307 | 0.1203 | 0.1114 |
|            | 0.1451 | 0.1390 | 0.1607 | 0.1341 | 0.1524 | 0.1244 |
|            | 0.1745 | 0.1828 | 0.1608 | 0.155 | 0.1624 | 0.1475 |
| rₙ         | 0.00705 | 0.00681 | 0.00744 | 0.00746 | 0.00765 | 0.00832 |
|            | 0.00561 | 0.00570 | 0.00626 | 0.00609 | 0.00572 | 0.00623 |
|            | 0.00570 | 0.00578 | 0.00564 | 0.00587 | 0.00560 | 0.00634 |
|            | 0.00667 | 0.00669 | 0.00632 | 0.00633 | 0.00675 | 0.00800 |
|            | 0.00834 | 0.00835 | 0.00863 | 0.00861 | 0.00861 | 0.00981 |
|            | 0.01009 | 0.01007 | 0.01102 | 0.01101 | 0.01103 | 0.01204 |
| M          | 8053 | 8140 | 8143 | 8161 | 8167 | 8662 |
|            | 3051 | 3061 | 3093 | 3095 | 3098 | 3224 |
|            | 3007 | 3025 | 3045 | 3047 | 3048 | 3138 |
|            | 3404 | 3425 | 3427 | 3437 | 3446 | 3595 |
|            | 4106 | 4110 | 4145 | 4147 | 4159 | 4269 |
|            | 4842 | 4847 | 4862 | 4871 | 4872 | 5247 |
| Mₐ         | 2811 | 2701 | 2884 | 2907 | 2902 | 3267 |
|            | 1480 | 1441 | 1491 | 1464 | 1505 | 1542 |
|            | 1548 | 1559 | 1570 | 1552 | 1536 | 1591 |
|            | 1725 | 1751 | 1762 | 1758 | 1732 | 1828 |
|            | 2194 | 2073 | 2141 | 2202 | 2189 | 2194 |
|            | 2474 | 2514 | 2495 | 2472 | 2498 | 2579 |
| M₉         | 3298 | 3352 | 3334 | 3381 | 3399 | 3578 |
|            | 685 | 725 | 711 | 708 | 671 | 725 |
|            | 847 | 854 | 837 | 865 | 870 | 888 |
|            | 1133 | 1123 | 1112 | 1142 | 1154 | 1199 |
|            | 1438 | 1561 | 1518 | 1466 | 1489 | 1601 |
|            | 1904 | 1865 | 1910 | 1942 | 1915 | 2044 |

(continued on next page)
Table 7 (continued)

| Parameters | 1   | 2   | 3   | 4   | 5   | Mean  | σ    |
|------------|-----|-----|-----|-----|-----|-------|------|
| $M_b$      | 1943| 2087| 1925| 1873| 1866| 1816  | 173.6|
|            | 886 | 895 | 891 | 923 | 923 | 956   | 48.1 |
|            | 611 | 612 | 638 | 630 | 642 | 658   | 34.3 |
|            | 546 | 551 | 553 | 538 | 560 | 568   | 30.1 |
|            | 474 | 476 | 486 | 478 | 481 | 475   | 7.10 |
|            | 465 | 467 | 457 | 456 | 458 | 434   | 14.0 |

Table 8

Five best solutions at $\alpha = \text{const.} \ (\Delta \alpha = 0)$ and given values of the ratio $f/L$.

| Parameters | 1   | 2   | 3   | 4   | 5   | Mean  | σ    |
|------------|-----|-----|-----|-----|-----|-------|------|
| $f$        | 30  | 30  | 30  | 30  | 30  | 31.4  | 3.63 |
|            | 20  | 20  | 20  | 20  | 20  | 30.3  | 1.922|
|            | 15  | 15  | 15  | 15  | 15  | 30.8  | 1.071|
|            | 12  | 12  | 12  | 12  | 12  | 34.4  | 1.995|
|            | 10  | 10  | 10  | 10  | 10  | 39.0  | 2.18 |
|            | 8.57| 8.57| 8.57| 8.57| 8.57| 41.4  | 1.566|
| $n$        | 39  | 43  | 39  | 33  | 43  | 31.4  | 3.63 |
|            | 10  | 11  | 13  | 9   | 8   | 14.76 | 5.82 |
|            | 33  | 33  | 28  | 26  | 27  | 21.9  | 4.64 |
|            | 29  | 28  | 26  | 28  | 24  | 20.8  | 4.88 |
|            | 23  | 22  | 22  | 19  | 19  | 15.3  | 3.30 |
|            | 16  | 15  | 15  | 15  | 15  | 10.9  | 2.39 |
| $\alpha$   | 19.83| 19.96| 19.93| 20.1| 19.33| 20.8  | 0.590|
|            | 31.5 | 30.9 | 30.1 | 32.2 | 32.9 | 30.3  | 1.922|
|            | 28.7 | 30.6 | 29.1 | 29.6 | 30.4 | 30.8  | 1.071|
|            | 34.6 | 32.5 | 34.9 | 33.6 | 35.0 | 34.4  | 1.995|
|            | 36.2 | 35.2 | 36.8 | 38.0 | 39.0 | 39.0  | 2.18 |
|            | 39.4 | 39.7 | 40.4 | 40.4 | 40.1 | 41.4  | 1.566|
| $\beta_{ei}$| 0.994| 0.759| 0.880| 1.101| 0.808| 0.817  | 0.1907|
|            | 1.5  | 1.451| 1.484| 1.5  | 1.5  | 1.423  | 0.0963|
|            | 1.374| 1.377| 1.360| 1.357| 1.353| 1.345  | 0.0342|
|            | 1.282| 1.217| 1.165| 1.214| 1.242| 1.221  | 0.0830|
|            | 1.195| 1.266| 1.258| 1.277| 1.109| 1.185  | 0.1075|
|            | 0.980| 1.138| 1.139| 1.138| 0.895| 1.101  | 0.1014|
| $h_a$      | 0.235| 0.234| 0.237| 0.287| 0.229| 0.309  | 0.0713|
|            | 0.655| 0.645| 0.569| 0.657| 0.709| 0.495  | 0.1233|
|            | 0.1919| 0.510| 0.282| 0.380| 0.319| 0.393  | 0.0910|
|            | 0.368| 0.486| 0.387| 0.581| 0.325| 0.389  | 0.0927|
|            | 0.331| 0.295| 0.399| 0.422| 0.423| 0.474  | 0.1280|
|            | 0.320| 0.403| 0.306| 0.421| 0.483| 0.478  | 0.1060|
| $w_a$      | 0.1855| 0.1797| 0.1855| 0.1798| 0.1775| 0.1817  | 0.0103|
|            | 0.0821| 0.0813| 0.0877| 0.0903| 0.0919| 0.1070  | 0.0184|
|            | 0.1501| 0.0831| 0.1268| 0.1072| 0.1171| 0.1080  | 0.0170|
|            | 0.1203| 0.1031| 0.1187| 0.0894| 0.1316| 0.1225  | 0.0168|
|            | 0.1512| 0.1613| 0.1361| 0.1333| 0.1324| 0.1281  | 0.0205|
|            | 0.1753| 0.1572| 0.1785| 0.1548| 0.1499| 0.1465  | 0.0207|
| $h_g$      | 1.017| 1.019| 1.013| 1.015| 1.013| 1.012  | 0.0389|
|            | 0.662| 0.680| 0.698| 0.589| 0.527| 0.705  | 0.0987|
|            | 0.366| 0.359| 0.352| 0.351| 0.349| 0.324  | 0.0392|
|            | 0.308| 0.226| 0.289| 0.214| 0.269| 0.306  | 0.0744|
|            | 0.232| 0.179| 0.248| 0.227| 0.264| 0.329  | 0.0923|
|            | 0.290| 0.279| 0.273| 0.256| 0.237| 0.403  | 0.1146|

(continued on next page)
Table 8 (continued)

| Parameters | 1 | 2 | 3 | 4 | 5 | Mean | σ |
|------------|---|---|---|---|---|------|---|
| $W_g$      | 0.1271 | 0.1275 | 0.1273 | 0.1269 | 0.1268 | 0.1297 | 0.00764 |
|            | 0.0836 | 0.0865 | 0.0872 | 0.0898 | 0.0869 | 0.0982 | 0.01271 |
|            | 0.0800 | 0.0817 | 0.0816 | 0.0806 | 0.0838 | 0.0898 | 0.00911 |
|            | 0.1026 | 0.1196 | 0.1060 | 0.1243 | 0.1108 | 0.1068 | 0.01404 |
|            | 0.1392 | 0.1549 | 0.1357 | 0.1416 | 0.1345 | 0.1251 | 0.01656 |
|            | 0.1592 | 0.1597 | 0.1641 | 0.1637 | 0.1624 | 0.1403 | 0.01964 |
| $r_h$      | 0.00605 | 0.00628 | 0.00633 | 0.00652 | 0.00659 | 0.00724 | 0.00505 |
|            | 0.01003 | 0.00952 | 0.00886 | 0.01042 | 0.01115 | 0.00876 | 0.001534 |
|            | 0.00535 | 0.00542 | 0.00580 | 0.00605 | 0.00595 | 0.00677 | 0.000754 |
|            | 0.00598 | 0.00602 | 0.00621 | 0.00604 | 0.00653 | 0.00720 | 0.000891 |
|            | 0.00698 | 0.00719 | 0.00719 | 0.00781 | 0.00761 | 0.00875 | 0.000980 |
|            | 0.00883 | 0.00941 | 0.00959 | 0.00959 | 0.00899 | 0.01071 | 0.001056 |
| $M$        | 8490 | 8601 | 8615 | 8721 | 8729 | 9159 | 346 |
|            | 4454 | 4529 | 4553 | 4614 | 4621 | 5012 | 366 |
|            | 3010 | 3015 | 3030 | 3042 | 3051 | 3135 | 69.2 |
|            | 3281 | 3300 | 3312 | 3312 | 3313 | 3386 | 70.3 |
|            | 3804 | 3807 | 3829 | 3879 | 3903 | 4006 | 93.6 |
|            | 4659 | 4683 | 4696 | 4699 | 4709 | 4825 | 112.1 |
| $M_a$      | 3126 | 2976 | 3138 | 3372 | 2892 | 3570 | 332 |
|            | 2001 | 1953 | 1903 | 2229 | 2432 | 2082 | 376 |
|            | 1518 | 1482 | 1545 | 1564 | 1525 | 1583 | 60.0 |
|            | 1670 | 1736 | 1710 | 1718 | 1705 | 1747 | 38.3 |
|            | 2008 | 2023 | 2013 | 2050 | 2034 | 2079 | 59.4 |
|            | 2348 | 2389 | 2333 | 2418 | 2579 | 2443 | 92.7 |
| $M_g$      | 3778 | 3798 | 3773 | 3769 | 3759 | 3843 | 116.7 |
|            | 1620 | 1723 | 1780 | 1583 | 1383 | 2045 | 349 |
|            | 923  | 933  | 915  | 901  | 937  | 954  | 34.2 |
|            | 1085 | 1059 | 1080 | 1079 | 1084 | 1114 | 38.4 |
|            | 1322 | 1317 | 1336 | 1337 | 1375 | 1431 | 80.8 |
|            | 1834 | 1794 | 1839 | 1757 | 1657 | 1918 | 111.9 |
| $M_h$      | 1586 | 1825 | 1704 | 1580 | 2078 | 1747 | 158.3 |
|            | 833  | 853  | 871  | 802  | 805  | 885  | 81.3 |
|            | 568  | 600  | 569  | 577  | 589  | 598  | 16.4 |
|            | 525  | 505  | 522  | 516  | 524  | 525  | 37.6 |
|            | 474  | 467  | 480  | 493  | 494  | 495  | 28.2 |
|            | 477  | 499  | 524  | 524  | 473  | 464  | 21.8 |

Table 9
Five best solutions at $\beta$, $\alpha = \text{const.}$ ($\beta_m = 1$, $\Delta \alpha = 0$) and given values of the ratio $f/L$.

| Parameters | 1 | 2 | 3 | 4 | 5 | Mean | σ |
|------------|---|---|---|---|---|------|---|
| $f$        | 30 | 30 | 30 | 30 | 30 | 30   | 30 |
|            | 20 | 20 | 20 | 20 | 20 | 20   | 20 |
|            | 15 | 15 | 15 | 15 | 15 | 15   | 15 |
|            | 12 | 12 | 12 | 12 | 12 | 12   | 12 |
|            | 10 | 10 | 10 | 10 | 10 | 10   | 10 |
|            | 8.57 | 8.57 | 8.57 | 8.57 | 8.57 | 8.57 | 8.57 |
| $n$        | 35 | 35 | 30 | 30 | 30 | 22.2 | 4.37 |
|            | 30 | 11 | 11 | 11 | 13 | 18.20 | 7.34 |
|            | 34 | 37 | 34 | 34 | 37 | 23.3 | 8.28 |
|            | 38 | 36 | 37 | 37 | 33 | 24.0 | 6.08 |
|            | 23 | 22 | 22 | 22 | 22 | 17.80 | 3.00 |
|            | 15 | 14 | 14 | 14 | 14 | 12.38 | 2.25 |

(continued on next page)
| Parameters | 1     | 2     | 3     | 4     | 5     | Mean | σ   |
|-----------|-------|-------|-------|-------|-------|------|-----|
| α         | 19.53 | 19.53 | 20.7  | 20.6  | 20.6  | 21.9 | 3   |
|           | 32.9  | 32.3  | 32.4  | 32.4  | 32.0  | 31.1 | 3   |
|           | 31.0  | 30.6  | 31.1  | 31.3  | 31.1  | 32.8 | 3   |
|           | 34.7  | 33.7  | 32.4  | 30.6  | 36.6  | 36.3 | 3   |
|           | 37.4  | 38.8  | 38.4  | 39.5  | 38.7  | 40.0 | 2   |
|           | 39.7  | 40.0  | 39.6  | 39.6  | 40.1  | 40.7 | 2   |
| h_a       | 0.249 | 0.267 | 0.284 | 0.286 | 0.295 | 0.500 | 0  |
|           | 0.665 | 0.567 | 0.559 | 0.546 | 0.602 | 0.456 | 0  |
|           | 0.380 | 0.518 | 0.429 | 0.491 | 0.378 | 0.399 | 0  |
|           | 0.365 | 0.325 | 0.330 | 0.287 | 0.460 | 0.384 | 0  |
|           | 0.270 | 0.287 | 0.392 | 0.397 | 0.427 | 0.423 | 0  |
|           | 0.514 | 0.414 | 0.347 | 0.342 | 0.512 | 0.471 | 0  |
| w_a       | 0.1977| 0.1826| 0.1930| 0.1912| 0.1865| 0.1649| 0  |
|           | 0.0848| 0.0962| 0.0979| 0.1006| 0.0856| 0.1097| 0  |
|           | 0.1043| 0.0858| 0.0966| 0.0886| 0.1049| 0.1106| 0  |
|           | 0.1207| 0.1296| 0.1305| 0.1405| 0.1025| 0.1251| 0  |
|           | 0.1643| 0.1578| 0.1383| 0.1341| 0.1322| 0.1361| 0  |
|           | 0.1380| 0.1534| 0.1710| 0.1715| 0.1378| 0.1489| 0  |
| h_g       | 1.013 | 1.004 | 1.018 | 1.018 | 1.016 | 1.006 | 0  |
|           | 0.786 | 0.782 | 0.758 | 0.755 | 0.826 | 0.835 | 0  |
|           | 0.381 | 0.362 | 0.365 | 0.374 | 0.384 | 0.375 | 0  |
|           | 0.230 | 0.205 | 0.201 | 0.233 | 0.437 | 0.324 | 0  |
|           | 0.248 | 0.306 | 0.225 | 0.320 | 0.210 | 0.331 | 0  |
|           | 0.326 | 0.331 | 0.277 | 0.286 | 0.337 | 0.354 | 0  |
| w_g       | 0.1266| 0.1280| 0.1273| 0.1273| 0.1279| 0.1297| 0  |
|           | 0.0985| 0.0977| 0.1033| 0.1040| 0.1032| 0.1104| 0  |
|           | 0.0804| 0.0815| 0.0831| 0.0810| 0.0807| 0.0863| 0  |
|           | 0.1188| 0.1266| 0.1279| 0.1196| 0.0818| 0.1051| 0  |
|           | 0.1430| 0.1294| 0.1452| 0.1229| 0.1477| 0.1279| 0  |
|           | 0.1471| 0.1497| 0.1634| 0.1625| 0.1452| 0.1481| 0  |
| r_b       | 0.00616| 0.00683| 0.00667| 0.00677| 0.00692| 0.00812| 0  |
|           | 0.01088| 0.01075| 0.01076| 0.01076| 0.01007| 0.00907| 0  |
|           | 0.00582| 0.00612| 0.00589| 0.00611| 0.00590| 0.00778| 0  |
|           | 0.00544| 0.00535| 0.00552| 0.00551| 0.00574| 0.00683| 0  |
|           | 0.00675| 0.00693| 0.00702| 0.00716| 0.00709| 0.00797| 0  |
|           | 0.00916| 0.00910| 0.00925| 0.00916| 0.00921| 0.00997| 0  |
| M         | 8764 | 8883 | 8985 | 8989 | 9025 | 9757 | 3  |
|           | 5417 | 5488 | 5585 | 5621 | 5689 | 5985 | 2  |
|           | 3352 | 3354 | 3361 | 3363 | 3367 | 3545 | 1  |
|           | 3319 | 3323 | 3350 | 3352 | 3352 | 3485 | 1  |
|           | 3879 | 3882 | 3893 | 3893 | 3901 | 4030 | 1  |
|           | 4687 | 4690 | 4694 | 4694 | 4696 | 4801 | 1  |
| M_g       | 3534 | 3296 | 3697 | 3660 | 3608 | 4322 | 4  |
|           | 2101 | 2105 | 2121 | 2145 | 1943 | 2004 | 3  |
|           | 1513 | 1558 | 1522 | 1542 | 1517 | 1639 | 3  |
|           | 1668 | 1673 | 1706 | 1703 | 1647 | 1765 | 3  |
|           | 1959 | 1929 | 2026 | 1970 | 2047 | 2044 | 3  |
|           | 2451 | 2362 | 2396 | 2379 | 2438 | 2456 | 3  |
| M_s       | 3748 | 3766 | 3789 | 3787 | 3804 | 3820 | 4  |
|           | 2264 | 2233 | 2309 | 2320 | 2493 | 2713 | 4  |
|           | 960  | 936  | 964  | 954  | 970  | 1022 | 3  |
|           | 1063 | 1075 | 1078 | 1084 | 1100 | 1135 | 3  |
|           | 1433 | 1449 | 1375 | 1403 | 1350 | 1472 | 3  |
|           | 1786 | 1849 | 1841 | 1866 | 1801 | 1880 | 3  |

(continued on next page)
### Table 9 (continued)

| Parameters | 1   | 2   | 3   | 4   | 5   | Mean | σ  |
|------------|-----|-----|-----|-----|-----|------|----|
| $M_h$      | 1482| 1822| 1498| 1542| 1613| 1614 | 161.0 |
|            | 1052| 1150| 1155| 1157| 1253| 1268 | 131.1 |
|            | 879 | 860 | 875 | 866 | 880 | 884  | 38.2 |
|            | 588 | 575 | 567 | 566 | 605 | 586  | 21.9 |
|            | 488 | 504 | 491 | 520 | 504 | 514  | 26.0 |
|            | 449 | 478 | 457 | 448 | 457 | 465  | 24.4 |

### Table 10

Five best solutions at both arbitrary angles $\beta$ and $\alpha$, and given values of $r_s$.

| Parameters | 1   | 2   | 3   | 4   | 5   | Mean |
|------------|-----|-----|-----|-----|-----|------|
| $f$        | 17.49| 16.80| 17.03| 16.27| 16.39| 14.70 |
|            | 13.85| 13.75| 13.70| 13.62| 14.74| 14.17 |
|            | 13.92| 14.61| 13.24| 13.68| 13.74| 13.21 |
|            | 12.29| 13.20| 12.36| 12.38| 12.46| 12.41 |
| $n$        | 35   | 37   | 22   | 25   | 31   | 18.79 |
|            | 11   | 12   | 12   | 12   | 11   | 12    |
|            | 9    | 10   | 9    | 10   | 10   | 10.36 |
|            | 7    | 8    | 7    | 7    | 8    | 8.02  |
| $\alpha$   | 10.96| 13.44| 13.69| 17.43| 12.78| 22.7  |
|            | 28.0 | 25.4 | 27.4 | 25.9 | 24.6 | 27.9  |
|            | 32.2 | 28.7 | 34.3 | 29.4 | 31.2 | 29.3  |
|            | 33.2 | 30.9 | 33.4 | 33.1 | 31.2 | 30.9  |
| $\Delta\alpha$ | 0.947| 0.752| 1.280| 0.833| 0.957| 0.931 |
|            | 1.096| 1.109| 0.935| 1.085| 1.398| 1.058 |
|            | 1.370| 1.032| 1.435| 1.128| 0.946| 1.226 |
|            | 1.447| 1.394| 1.201| 1.431| 0.891| 1.270 |
| $\beta_{ini}$ | 1.309| 1.342| 1.272| 1.286| 1.258| 1.272 |
|            | 1.354| 1.354| 1.325| 1.348| 1.377| 1.305 |
|            | 1.278| 1.382| 1.260| 1.395| 1.367| 1.250 |
|            | 1.309| 1.304| 1.313| 1.310| 1.389| 1.252 |
| $h_g$      | 0.286| 0.327| 0.470| 0.348| 0.245| 0.386 |
|            | 0.582| 0.500| 0.572| 0.476| 0.539| 0.496 |
|            | 0.641| 0.562| 0.622| 0.522| 0.568| 0.494 |
|            | 0.634| 0.639| 0.675| 0.676| 0.694| 0.576 |
| $w_s$      | 0.1155| 0.1083| 0.0866| 0.1076| 0.1318| 0.1119 |
|            | 0.0844| 0.0961| 0.0865| 0.1007| 0.0894| 0.0988 |
|            | 0.0826| 0.0889| 0.0854| 0.0969| 0.0881| 0.1094 |
|            | 0.0982| 0.0893| 0.0934| 0.0930| 0.0873| 0.1093 |
| $h_g$      | 0.225| 0.1536| 0.280| 0.292| 0.1909| 0.288 |
|            | 0.438| 0.410| 0.429| 0.418| 0.369| 0.334 |
|            | 0.463| 0.442| 0.401| 0.483| 0.353| 0.386 |
|            | 0.554| 0.561| 0.493| 0.446| 0.546| 0.474 |
| $w_s$      | 0.0926| 0.1113| 0.0836| 0.0834| 0.1053| 0.0965 |
|            | 0.0800| 0.0828| 0.0801| 0.0814| 0.0911| 0.0991 |
|            | 0.0800| 0.0821| 0.0937| 0.0800| 0.1032| 0.1003 |
|            | 0.0850| 0.0801| 0.0959| 0.1049| 0.0890| 0.1044 |
| $r_s \geq 5$ | 0.00515| 0.00511| 0.00649| 0.00609| 0.00550| 0.00738 |
| $\geq 10$ | 0.01| 0.01| 0.01| 0.01| 0.01| 0.01003 |
| $\geq 14$ | 0.014| 0.014| 0.014| 0.014| 0.014| 0.01401 |
| $\geq 20$ | 0.02| 0.02| 0.02| 0.02| 0.02| 0.02 |

(continued on next page)
Table 10 (continued)

| Parameters | 1     | 2     | 3     | 4     | 5     | Mean | σ    |
|------------|-------|-------|-------|-------|-------|------|------|
| \( M \)    | 2801  | 2806  | 2874  | 2878  | 2884  | 3123 | 133.4|
|            | 3319  | 3359  | 3365  | 3371  | 3371  | 3477 | 104.2|
|            | 3899  | 3935  | 3947  | 3951  | 3972  | 4192 | 186.7|
|            | 4832  | 4885  | 4890  | 4918  | 4937  | 5218 | 228  |
| \( M_a \)  | 1446  | 1455  | 1502  | 1499  | 1512  | 1621 | 67.2 |
|            | 1658  | 1684  | 1675  | 1701  | 1683  | 1716 | 76.4 |
|            | 1766  | 1726  | 1762  | 1759  | 1700  | 1890 | 131.7|
|            | 2064  | 1898  | 2066  | 2060  | 1969  | 2120 | 183.3|
| \( M_g \)  | 757   | 753   | 785   | 810   | 800   | 943  | 80.5 |
|            | 1075  | 1056  | 1057  | 1053  | 1085  | 1090 | 72.5 |
|            | 1127  | 1114  | 1200  | 1169  | 1216  | 1234 | 115.3|
|            | 1410  | 1334  | 1463  | 1494  | 1466  | 1534 | 181.8|
| \( M_s \)  | 598   | 598   | 587   | 569   | 573   | 559  | 22.4 |
|            | 585   | 619   | 634   | 617   | 604   | 671  | 88.1 |
|            | 1005  | 1095  | 985   | 1022  | 1056  | 1068 | 124.5|
|            | 1357  | 1652  | 1360  | 1365  | 1502  | 1565 | 250  |

Table 11

Five best solutions at \( \beta = \text{const.} (\beta_{\text{ini}} = 1) \) and given values of \( r_h \).  

| Parameters | 1    | 2    | 3    | 4    | 5    | Mean | σ    |
|------------|------|------|------|------|------|------|------|
| \( f \)    | 16.75| 16.81| 16.43| 15.62| 16.75| 14.68| 0.883|
|            | 14.89| 15.22| 14.79| 14.29| 15.85| 14.32| 0.848|
|            | 14.57| 14.99| 14.96| 15.30| 14.41| 13.48| 0.946|
|            | 12.89| 12.15| 12.86| 12.56| 12.73| 12.13| 0.758|
| \( n \)    | 33   | 34   | 37   | 34   | 30   | 20.4 | 6.20 |
|            | 10   | 11   | 11   | 10   | 11   | 12.37| 1.851|
|            | 9    | 10   | 10   | 10   | 9    | 10.01| 1.267|
|            | 6.133| 7    | 7    | 6    | 7    | 7.35 | 0.682|
| \( \alpha \)| 7.87 | 9.01 | 13.19| 12.04| 7.03 | 22.8 | 6.33 |
|            | 32.3 | 30.6 | 32.0 | 34.1 | 28.9 | 28.1 | 4.29 |
|            | 34.1 | 31.6 | 32.1 | 30.7 | 32.1 | 29.9 | 3.42 |
|            | 34.6 | 43.2 | 36.8 | 35.6 | 36.3 | 37.2 | 2.97 |
| \( \Delta \alpha \)| 1.204| 1.020| 0.790| 0.951| 1.272| 1.144| 0.211|
|            | 1.193| 1.193| 1.087| 1.088| 1.251| 1.233| 0.180|
|            | 0.922| 1.390| 1.199| 1.409| 1.407| 1.214| 0.201|
|            | 0.847| 0.749| 1.003| 0.538| 1.465| 1.016| 0.300|
| \( h_a \)| 0.306 | 0.429 | 0.420 | 0.257 | 0.332 | 0.410 | 0.0922 |
|            | 0.594 | 0.558 | 0.559 | 0.565 | 0.534 | 0.448 | 0.0825 |
|            | 0.598 | 0.570 | 0.563 | 0.556 | 0.632 | 0.486 | 0.0793 |
|            | 0.720 | 0.658 | 0.675 | 0.719 | 0.666 | 0.594 | 0.0710 |
| \( w_a \)| 0.1156 | 0.0940 | 0.0944 | 0.1316 | 0.1132 | 0.1098 | 0.0147 |
|            | 0.0814 | 0.0877 | 0.0882 | 0.0881 | 0.0916 | 0.1121 | 0.0183 |
|            | 0.0855 | 0.0853 | 0.0861 | 0.0866 | 0.0846 | 0.1140 | 0.0170 |
|            | 0.0905 | 0.0913 | 0.0905 | 0.0913 | 0.0919 | 0.1061 | 0.0138 |
| \( h_g \)| 0.264 | 0.237 | 0.290 | 0.252 | 0.248 | 0.305 | 0.0650 |
|            | 0.299 | 0.394 | 0.413 | 0.363 | 0.383 | 0.337 | 0.0673 |
|            | 0.370 | 0.319 | 0.302 | 0.286 | 0.418 | 0.384 | 0.0881 |
|            | 0.645 | 0.435 | 0.593 | 0.541 | 0.535 | 0.475 | 0.0863 |

(continued on next page)
### Table 11 (continued)

| Parameters | Five best solutions | 100 solutions |
|------------|---------------------|---------------|
|            | Mean | σ |
| **w_h**    |      |    |
| 0.0871     | 0.0884 | 0.0815 | 0.0924 | 0.0880 | 0.0940 |
| 0.1071     | 0.0822 | 0.0805 | 0.0946 | 0.0816 | 0.0995 |
| 0.0900     | 0.0976 | 0.1029 | 0.1060 | 0.0894 | 0.1036 |
| 0.0806     | 0.0952 | 0.0800 | 0.0972 | 0.0880 | 0.1030 |
| r_h ≥ 5    |      |    |
| ≥ 10       | 0.00542 | 0.00559 | 0.00531 | 0.00544 | 0.00586 | 0.00728 |
| ≥ 14       | 0.01005 | 0.01 | 0.01 | 0.01 | 0.01002 | 6.3E-05 |
| ≥ 20       | 0.014 | 0.014 | 0.014 | 0.014 | 0.01401 | 4.3E-05 |
| M          |      |    |
| 2911       | 2951 | 2954 | 2965 | 2968 | 3246 |
| 3452       | 3473 | 3477 | 3486 | 3500 | 3691 |
| 3983       | 4055 | 4081 | 4094 | 4108 | 4365 |
| 4962       | 4979 | 5025 | 5033 | 5035 | 5226 |
| M_a        |      |    |
| 1499       | 1525 | 1495 | 1536 | 1555 | 1668 |
| 1651       | 1709 | 1708 | 1708 | 1748 | 1825 |
| 1743       | 1677 | 1680 | 1681 | 1805 | 1965 |
| 2127       | 1963 | 2009 | 2134 | 2013 | 2105 |
| M_g        |      |    |
| 788        | 740 | 782 | 818 | 761 | 955 |
| 1120       | 1014 | 1031 | 1118 | 982 | 1104 |
| 1071       | 1050 | 1074 | 1071 | 1174 | 1277 |
| 1519       | 1307 | 1398 | 1609 | 1421 | 1515 |
| M_h        |      |    |
| 624        | 686 | 676 | 611 | 652 | 623 |
| 682        | 751 | 738 | 660 | 770 | 762 |
| 1170       | 1327 | 1327 | 1342 | 1129 | 1122 |
| 1316       | 1709 | 1617 | 1291 | 1602 | 1607 |

### Table 12

Five best solutions at α = const. (Δα = 0) and given values of r_h.

| Parameters | Five best solutions | 100 solutions |
|------------|---------------------|---------------|
|            | Mean | σ |
| **f**      |      |    |
| 13.88      | 14.39 | 14.49 | 14.46 | 14.28 | 13.73 | 0.533 |
| 15.08      | 14.70 | 14.93 | 14.09 | 14.63 | 13.39 | 0.742 |
| 14.24      | 14.34 | 12.34 | 14.09 | 13.10 | 12.73 | 0.877 |
| 11.82      | 12.34 | 13.31 | 12.91 | 12.37 | 12.22 | 0.864 |
| **n**      |      |    |
| 33         | 30 | 30 | 24 | 26 | 21.7 | 4.79 |
| 11         | 12 | 12 | 11 | 13 | 13.52 | 1.568 |
| 10         | 10 | 10 | 10 | 10 | 10.58 | 1.012 |
| 8          | 8 | 8 | 8 | 8 | 8.57 | 1.153 |
| **α**      |      |    |
| 30.07      | 28.99 | 30.16 | 30.28 | 29.88 | 31.26 | 1.102 |
| 30.91      | 32.45 | 33.21 | 33.67 | 33.92 | 32.31 | 1.415 |
| 33.58      | 33.69 | 34.64 | 34.39 | 33.13 | 32.83 | 1.342 |
| 33.54      | 32.41 | 30.29 | 31.10 | 34.72 | 33.13 | 1.244 |
| **β_in**   |      |    |
| 1.289      | 1.334 | 1.328 | 1.285 | 1.312 | 1.297 | 0.0372 |
| 1.500      | 1.451 | 1.365 | 1.387 | 1.462 | 1.359 | 0.0580 |
| 1.447      | 1.431 | 1.341 | 1.412 | 1.401 | 1.353 | 0.0839 |
| 1.425      | 1.421 | 1.432 | 1.428 | 1.327 | 1.296 | 0.1236 |
| **h^n**    |      |    |
| 0.275      | 0.280 | 0.357 | 0.379 | 0.249 | 0.383 | 0.0876 |
| 0.566      | 0.512 | 0.583 | 0.573 | 0.479 | 0.429 | 0.0827 |
| 0.608      | 0.601 | 0.559 | 0.550 | 0.596 | 0.499 | 0.0879 |
| 0.644      | 0.597 | 0.637 | 0.584 | 0.619 | 0.539 | 0.0825 |

(continued on next page)
Table 12 (continued)

| Parameters | 1     | 2     | 3     | 4     | 5     | Mean  | σ       |
|------------|-------|-------|-------|-------|-------|-------|---------|
| $w_a$      | 0.1294| 0.1295| 0.1110| 0.1086| 0.1379| 0.1145| 0.0168  |
|            | 0.0877| 0.0954| 0.0831| 0.0878| 0.0947| 0.1143| 0.0175  |
|            | 0.0853| 0.0852| 0.0949| 0.0928| 0.0909| 0.1097| 0.0182  |
|            | 0.0924| 0.0978| 0.0918| 0.1006| 0.0980| 0.1166| 0.0173  |
| $h_g$      | 0.294 | 0.301 | 0.275 | 0.331 | 0.347 | 0.285 | 0.0696  |
|            | 0.380 | 0.401 | 0.236 | 0.350 | 0.232 | 0.336 | 0.0794  |
|            | 0.437 | 0.349 | 0.476 | 0.355 | 0.449 | 0.414 | 0.0801  |
|            | 0.568 | 0.537 | 0.552 | 0.575 | 0.587 | 0.441 | 0.1035  |
| $w_g$      | 0.0942| 0.0905| 0.0982| 0.0856| 0.0848| 0.1005| 0.0153  |
|            | 0.0876| 0.0800| 0.1158| 0.0970| 0.1207| 0.0993| 0.0166  |
|            | 0.0826| 0.1000| 0.0800| 0.0983| 0.0869| 0.0959| 0.0150  |
|            | 0.0901| 0.0926| 0.0871| 0.0846| 0.0801| 0.1095| 0.0212  |
| $r_n \geq 5$ | 0.00538 | 0.00563 | 0.00565 | 0.00629 | 0.00605 | 0.00683 | 0.000794 |
| $\geq 10$  | 0.01  | 0.01  | 0.01  | 0.01  | 0.01  | 0.01  | 3.4E-06  |
| $\geq 14$  | 0.014 | 0.014 | 0.014 | 0.014 | 0.014 | 0.01402 | 0.000112 |
| $\geq 20$  | 0.02  | 0.02  | 0.02  | 0.02  | 0.02  | 0.0200 | 9.7E-05  |
| $M$        | 3012  | 3027  | 3051  | 3053  | 3061  | 3159  | 80.9    |
|            | 3409  | 3420  | 3440  | 3458  | 3474  | 3586  | 96.5    |
|            | 3954  | 4000  | 4004  | 4014  | 4018  | 4168  | 142.8   |
|            | 4901  | 4941  | 4964  | 4968  | 4971  | 5314  | 213     |
| $M_h$      | 1528  | 1564  | 1540  | 1568  | 1564  | 1632  | 51.1    |
|            | 1725  | 1733  | 1663  | 1719  | 1621  | 1778  | 63.1    |
|            | 1756  | 1740  | 1784  | 1765  | 1818  | 1893  | 100.3   |
|            | 1942  | 1954  | 1955  | 1997  | 2021  | 2164  | 191.1   |
| $M_g$      | 942   | 915   | 944   | 921   | 946   | 979   | 42.1    |
|            | 1060  | 997   | 1046  | 1120  | 1093  | 1091  | 53.6    |
|            | 1112  | 1158  | 1154  | 1152  | 1205  | 1244  | 95.6    |
|            | 1538  | 1513  | 1446  | 1447  | 1387  | 1518  | 140.3   |
| $M_b$      | 541   | 548   | 567   | 563   | 550   | 547   | 16.6    |
|            | 624   | 690   | 731   | 619   | 761   | 717   | 82.2    |
|            | 1086  | 1103  | 1066  | 1097  | 994   | 1031  | 97.2    |
|            | 1422  | 1474  | 1563  | 1524  | 1562  | 1633  | 208     |

Table 13
Five best solutions at $\beta$, $\alpha = \text{const.}$ ($\beta_{\text{ini}} = 1$, $\Delta \alpha = 0$) and given values of $r_n$.

| Parameters | 1     | 2     | 3     | 4     | 5     | Mean  | σ       |
|------------|-------|-------|-------|-------|-------|-------|---------|
| $f$        | 13.91 | 13.52 | 13.58 | 13.52 | 14.02 | 13.36 | 0.453   |
|            | 15.14 | 15.49 | 14.74 | 15.00 | 14.67 | 13.99 | 1.008   |
|            | 15.02 | 14.98 | 14.90 | 14.30 | 13.71 | 13.03 | 0.772   |
|            | 13.75 | 13.28 | 12.73 | 12.86 | 12.60 | 12.36 | 0.927   |
| $n$        | 36    | 34    | 32    | 32    | 36    | 25    | 5.97    |
|            | 12    | 11    | 12    | 11    | 11    | 12.84 | 1.633   |
|            | 9     | 9     | 8     | 8     | 8     | 9.78  | 1.314   |
|            | 6     | 7     | 7     | 7     | 8     | 7.87  | 0.892   |
| $\alpha$   | 32.7  | 32.7  | 32.1  | 32.4  | 29.4  | 32.5  | 1.190   |
|            | 34.8  | 34.5  | 34.9  | 34.5  | 35.0  | 33.2  | 1.508   |
|            | 34.3  | 34.4  | 34.6  | 34.7  | 31.0  | 32.2  | 1.254   |
|            | 33.7  | 33.4  | 34.7  | 34.1  | 33.5  | 32.9  | 1.072   |

(continued on next page)
Table 13 (continued)

| Parameters | Five best solutions | 100 solutions |
|------------|---------------------|---------------|
|            | 1                   | 2             | 3             | 4             | 5             | Mean | σ          |
| $h_a$      | 0.563               | 0.463         | 0.436         | 0.420         | 0.362         | 0.381 | 0.0865     |
|            | 0.516               | 0.512         | 0.569         | 0.618         | 0.545         | 0.439 | 0.0907     |
|            | 0.611               | 0.555         | 0.549         | 0.636         | 0.678         | 0.508 | 0.0896     |
|            | 0.699               | 0.682         | 0.639         | 0.604         | 0.590         | 0.547 | 0.0808     |
| $w_a$      | 0.0822              | 0.0984        | 0.1033        | 0.1058        | 0.1153        | 0.1200 | 0.0158     |
|            | 0.0934              | 0.0948        | 0.0896        | 0.0835        | 0.0944        | 0.1179 | 0.0210     |
|            | 0.0887              | 0.0980        | 0.0988        | 0.0903        | 0.0890        | 0.1151 | 0.0172     |
|            | 0.0933              | 0.0918        | 0.0993        | 0.1050        | 0.1037        | 0.1159 | 0.0165     |
| $h_g$      | 0.338               | 0.281         | 0.314         | 0.333         | 0.300         | 0.321 | 0.0721     |
|            | 0.360               | 0.337         | 0.362         | 0.427         | 0.400         | 0.342 | 0.0741     |
|            | 0.431               | 0.444         | 0.412         | 0.541         | 0.580         | 0.407 | 0.1015     |
|            | 0.568               | 0.579         | 0.590         | 0.574         | 0.554         | 0.524 | 0.0950     |
| $w_g$      | 0.0857              | 0.0978        | 0.0908        | 0.0876        | 0.0917        | 0.0969 | 0.0124     |
|            | 0.0897              | 0.0941        | 0.0906        | 0.0826        | 0.0855        | 0.1030 | 0.0174     |
|            | 0.0819              | 0.0804        | 0.0856        | 0.0800        | 0.0812        | 0.1081 | 0.0218     |
|            | 0.0854              | 0.0832        | 0.0855        | 0.0863        | 0.0894        | 0.1047 | 0.0173     |
| $r_h$      | 0.00565             | 0.00582       | 0.00602       | 0.00605       | 0.00583       | 0.00697 | 0.00102    |
| $\geq 5$  | 0.01                | 0.01022       | 0.01          | 0.01023       | 0.0102        | 0.01008 | 0.00018    |
| $\geq 10$ | 0.014               | 0.014         | 0.014         | 0.014         | 0.014         | 0.014 | 1.1E-05    |
| $\geq 14$ | 0.02                | 0.02          | 0.02          | 0.02          | 0.02          | 0.02001 | 3.7E-05    |
| $M$        | 3231                | 3269          | 3280          | 3282          | 3282          | 3427   | 138.9       |
|            | 3609                | 3619          | 3643          | 3646          | 3649          | 3853   | 145.8       |
|            | 4144                | 4202          | 4204          | 4226          | 4289          | 4489   | 134.6       |
|            | 5010                | 5062          | 5126          | 5155          | 5260          | 5472   | 190.1       |
| $M_a$      | 1566                | 1616          | 1632          | 1626          | 1618          | 1717   | 88.4        |
|            | 1713                | 1740          | 1764          | 1763          | 1800          | 1890   | 97.3        |
|            | 1865                | 1920          | 1916          | 1947          | 2009          | 2045   | 63.1        |
|            | 2173                | 2072          | 2117          | 2155          | 2074          | 2176   | 109.1       |
| $M_g$      | 938                 | 954           | 949           | 952           | 927           | 1029   | 74.2        |
|            | 1044                | 1047          | 1061          | 1090          | 1076          | 1160   | 89.7        |
|            | 1088                | 1093          | 1103          | 1290          | 1393          | 1400   | 126.5       |
|            | 1449                | 1430          | 1499          | 1480          | 1494          | 1683   | 235         |
| $M_h$      | 726                 | 698           | 699           | 704           | 737           | 681    | 43.5        |
|            | 852                 | 831           | 818           | 793           | 773           | 803    | 79.1        |
|            | 1192                | 1189          | 1185          | 989           | 887           | 1044   | 123.7       |
|            | 1387                | 1560          | 1510          | 1519          | 1692          | 1613   | 219         |
CRediT Author Statement

Rimantas Belevičius: Conceptualization, Methodology, Software, Supervision; Algirdas Juozapaitis: Formal analysis, Writing – Original draft preparation; Dainius Rusakevičius: Investigations, Validation, Visualization; Sigutė Žilėnaitė: Investigations, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships which have, or could be perceived to have, influenced the work reported in this article.

Acknowledgments

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors

References

[1] R. Belevičius, A. Juozapaitis, D. Rusakevičius, S. Žilėnaitė, Parametric study on mass minimization of radial network arch pedestrian bridges, Eng. Struct. (237) (2021), doi:10.1016/j.engstruct.2021.112182.

[2] ANSYS. https://ansys.com 2021 (accessed 16 March 2021).

[3] K. Deep, K.P. Singh, M.L. Kansal, C. Mohan, A real coded genetic algorithm for solving integer and mixed integer optimization problems, Appl. Math. Comput. 212 (2009) 505–518, doi:10.1016/j.amc.2009.02.044.

[4] E. Mezura-Montes, C.A. Coello Coello, Constraint-handling in nature-inspired numerical optimization: past, present and future, Swarm Evolut. Comput. 1 (2011) 173–194, doi:10.1016/j.swevo.2011.10.001.