Twist-3 light-cone distribution amplitudes of the scalar mesons within the QCD sum rules and their application to the $B \to S$ transition form factors

Hua-Yong Han$^1$, Xing-Gang Wu$^{1,a}$, Hai-Bing Fu$^1$, and Qiong-Lian Zhang$^1$ and Tao Zhong$^2$

1 Department of Physics, Chongqing University, Chongqing 401331, P.R. China
2 Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, P.R. China

Received: 20 April 2013
Published online: 26 June 2013 – © Società Italiana di Fisica / Springer-Verlag 2013
Communicated by Bo-Qiang Ma

Abstract. We investigate the twist-3 light-cone distribution amplitudes (LCDAs) of the scalar mesons $a_0$, $K^*_0$ and $f_0$ within the QCD sum rules. The QCD sum rules are improved by a consistent treatment of the sizable $s$-quark mass effects within the framework of the background field approach. Adopting the valence quark component ($\bar{q}q$) as the dominant structure of the scalar mesons, our estimation for their masses is close to the measured $a_0(1450)$, $K^*_0(1430)$ and $f_0(1710)$. From the sum rules, we obtain the first two non-zero moments of the twist-3 LCDAs $\phi^{2\sigma}_a$: $(\xi^{(4)}_{a,0}) = 0.369 (0.245)$ and $(\xi^{(4)}_{a,1}) = 0.203 (0.093)$; those of the twist-3 LCDAs $\phi^{2\sigma}_{K^*_0}$: $(\xi^{(4)}_{K^*_0,0}) = 0.004 (0.355)$ and $(\xi^{(4)}_{K^*_0,1}) = 0.018 (0.207)$; and those of the twist-3 LCDAs $\phi^{2\sigma}_{f_0}$: $(\xi^{(4)}_{f_0,0}) = 0.335 (0.212)$ and $(\xi^{(4)}_{f_0,1}) = 0.196 (0.088)$, respectively. As an application of those twist-3 LCDAs, we study the $B \to S$ transition form factors by introducing proper chiral currents into the correlator, which is constructed such that the twist-3 LCDAs give dominant contribution and the twist-2 LCDAs make negligible contribution. Our results of the $B \to S$ transition form factors at the large recoil region $q^2 \approx 0$ are consistent with those obtained in the literature, which inversely shows the present twist-3 LCDAs are acceptable.

1 Introduction

Even though lots of works have been done in the literature, the properties of the light scalar mesons are still in ambiguity. In order to get an accurate theoretical prediction on the properties of the scalar mesons and on their applications to high-energy processes, it is very important to provide a good interpretation of their complicated non-perturbative nature.

Among the scalar mesons’ non-perturbative sources, one of the most important thing is their light-cone distribution amplitudes (LCDAs). At present, some pioneering works for both the twist-2 and twist-3 LCDAs of the scalar meson have been done within the QCD sum rules, cf. refs. [1, 2]. According to our experience on the light pseudoscalar twist-3 LCDAs, e.g., the pion and kaon electromagnetic form factors [3, 4] and the $B \to \pi, K$ transition form factors [5, 6], a well-behaved pseudoscalar twist-3 LCDAs in the end-point region can give the conventional power suppressed contributions to the high-energy processes in comparison to those of twist-2 LCDAs. It is interesting to know whether the scalar twist-3 LCDAs also possess such good feature. Moreover, a better understanding of the twist-3 LCDAs is crucial for a reliable estimation. The forthcoming more precise data, e.g., at the large hadronic colliders and the programming super $B$ factories, also requires a more accurate theoretical estimation for the twist-3 contributions.

At present, we will investigate the twist-3 LCDAs of the scalar mesons by incorporating such quark mass effects properly so as to achieve a more accurate theoretical prediction. Within the QCD sum rules, it has been found that the contributions from the quark mass terms (especially those of the $s$-quark) will be comparable to that of the dimension-six operators or even the dimension-four operators, and it can even change the relative importance of the operator expansion series counted by the naive power counting rules, cf. refs. [7–15] for studying the $SU(3)$-breaking effects of the kaon LCDAs, and refs. [16–18] for the cases of the vector twist-3 LCDAs.

For the purpose, we will calculate the Gegenbauer moment of the twist-3 LCDAs for the scalars $a_0$, $K^*_0$ and $f_0$ within the QCD sum rules together with the QCD background field approach. Basic assumption of QCD sum rules is the introduction of non-vanishing vacuum condensates such as the dimension-three quark condensate $\langle \bar{q}q \rangle$. 

---

$^a$ e-mail: wuxg@cqu.edu.cn
the dimension-four gluon condensate \((G^2)^2\), etc. [19]. The QCD background field approach provides a systematic description for those vacuum condensates from the viewpoint of the field theory [20–29]. It assumes that the quark and gluon fields are composed of the background fields and the quantum fluctuations around them. The vacuum expectation values of those background fields describe the non-perturbative effects, while the quantum fluctuations represent the calculable perturbative effects. To take the QCD background field theory as the starting point for the QCD sum rules, it not only shows a distinct physical picture but also greatly simplifies the calculation due to its capability of adopting different gauge conditions for quantum fluctuations and background fields, respectively. Because of the influence from background fields, the quark and gluon propagators shall include a non-perturbative component inevitably, and the quark mass effect can be introduced in a consistent way.

Moreover, the \(B \rightarrow S\) transition form factor within the light-cone sum rules (LCSR) provides a good platform for checking the properties of the scalar LCDAs. In the LCSR approach, a two-point correlation function is introduced and expanded near the light cone \(x^2 = 0\), whose matrix elements are parameterized as LCDAs of increasing twists. By using the conventional currents in the correlator, the form factors will always contain the twist-2 and twist-3 terms simultaneously, which both play important roles for the final LCSDs [1]. Because both the twist-2 and twist-3 LCDAs have their own uncertainties, the entanglement of them makes the estimation largely uncertain. Thus, for the sake of a better accuracy, it is helpful to choose proper chiral currents in the correlators such that either the twist-2 or the twist-3 terms make no contribution to the LCSDs. In ref. [30], a chiral current has been suggested to make the twist-3 terms give zero contribution. At present, we shall introduce another type of chiral current such that the twist-2 terms give no contributions. Furthermore, we will deal with the semileptonic decays \(B \rightarrow S\nu \bar{q}\) and \(B \rightarrow S\bar{\nu}q\), which when compared with the forthcoming data shall be helpful for acquiring valuable information on the twist-3 LCDAs of the scalar particles.

The remaining parts of the paper are organized as follows. In sect. 2, we present the calculation technology for deriving the sum rules for the twist-3 LCDAs of the scalar mesons \(a_0\), \(K^0_s\) and \(f_0\). Then, we present the formulas for the \(B \rightarrow S\) transition form factors. Numerical results and discussions for the scalar mesons’ twist-3 LCDAs and the \(B \rightarrow S\) transition form factors are given in sect. 3. The final section is reserved for a summary. In the appendix, we put the some more subtle points in deriving the sum rules for the twist-3 LCDAs moments.

2 Calculation technology

2.1 Twist-3 LCDAs of the scalar meson

We adopt the suggestion of the valence quark contents dominant for the scalar mesons \(a_0\), \(K^0_s\) and \(f_0\), which are

\[
\langle S(p)\bar{q}_S(y)q_1(x)|0\rangle = m_s f_S \int_0^1 du e^{i(px+\bar{y}q_1 x)} \phi^*_S(u),
\]

\[
\langle S(p)\bar{q}_S(y)\sigma_{\mu\nu}q_1(x)|0\rangle = -m_s (p_\mu z_\nu - p_\nu z_\mu) f_S \int_0^1 du e^{i(px+\bar{y}q_1 x)} \phi^*_S(u) \frac{\phi^*_S(u)}{6},
\]

where \(z = y - x\), \(m_s\) and \(p\) are mass and momentum of the scalar meson, \(f_S\) is the decay constant of the scalar meson defined by \(\langle S(p)|\bar{q}_S|q_1\rangle = m_s f_S\), \(u\) is the momentum fraction carried by \(q_2\) quark, and \(\bar{u} = 1 - u\). The moments of twist-3 LCDAs are defined as

\[
\langle \xi^n\rangle = \int_0^1 du (2u - 1)^n \phi^*_S(u),
\]

\[
\langle \xi^n\rangle = \int_0^1 du (2u - 1)^n \phi^*_S(u),
\]

which satisfy

\[
\langle 0|\bar{q}_1(0)(iz\cdot \vec{D})^n q_2(0)|S(p)\rangle = m_s f_S (p\cdot z)^n \langle \xi^n\rangle,
\]

\[
\langle 0|\bar{q}_1(0)(iz\cdot \vec{D})^{n+1} \sigma_{\mu\nu}q_2(0)|S(p)\rangle = -i \frac{n+1}{3} m_s f_S (p_\mu z_\nu - p_\nu z_\mu)(p\cdot z)^n \langle \xi^n\rangle.
\]

Here the zeroth moments have been normalized to one, \(\langle \xi^0\rangle = (\xi^0) = 1\).

Figure 1 shows the Feynman diagrams for calculating the LCDAs moments, where the background gluon fields are included in the fermion propagators implicitly and the background quark fields are depicted as crosses.

To study the properties of these LCDAs, one can introduce the following two correlation functions:

\[
i \int d^4x e^{iq\cdot x} \{T\{\bar{q}_1(x)(iz\cdot \vec{D})^n q_2(x), q_2(0)q_1(0)|0\}\} = -i (z\cdot q)^n f^{(n,0)}(q^2),
\]

\[
i \int d^4x e^{iq\cdot x} \{T\{\bar{q}_1(x)\sigma_{\mu\nu}(iz\cdot \vec{D})^{n+1} q_2(x), q_2(0)q_1(0)|0\}\} = i (q_\mu z_\nu - q_\nu z_\mu) \times (z\cdot q)^n f^{(n,0)}(q^2).
\]

Following the standard QCD LCSR within the background field theory [20–22, 24, 28, 29], we can derive the sum rules for the moments of \(\phi^*_S\) and \(\phi^*_S\). For convenience, we present the detailed processes in appendix A.

During the calculation, as has been argued in ref. [8], we should deal with the sizable s-quark mass effects in a more consistent way, which might cause sizable effects comparable to those of the dimension-six condensates. For
the purpose, we adopt the following propagators to do our calculation [8]:

\[ S_F(x, 0) = i \int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot x} \left\{ -\frac{m+\not{q}}{m^2-q^2}+\frac{\gamma^\mu (\not{q}-m)\gamma^\nu}{(m^2-q^2)^2}b_{\mu\nu}+i\left[ 2\frac{\gamma^\mu (\not{q}-m)q^\nu}{(m^2-q^2)^3}\right.ight. \]

\[ +\left. \frac{q^\mu q^\nu}{(m^2-q^2)^2}\right] \right\} , \tag{7} \]

for the quark propagator and

\[ S^{ab}_{\mu\nu}(x, 0) = i \int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot x} \left\{ -\frac{g_{\mu\nu}}{q^2} \delta^{ab} + \cdots \right\} , \tag{8} \]

for the gluon propagator, where \( b_{\mu\nu} = \frac{i}{2} G_{\mu\nu}(0) \) and \( b_{1\nu\mu\rho} = \frac{i}{2} \left[ G_{\nu\mu,\rho}(0) + G_{\mu\nu,\rho}(0) \right] \). Here \( G_{\nu\mu}(x) = g_s T^a G_{\nu\mu}^a(x) \), the gauge-invariant function \( G_{\nu\mu,\rho}(0) = g_s T^a \bar{D}_\rho D^\rho G_{\nu\mu}^a \), \( G_{\nu\mu}(x)|_{x=0} \), and the symbol \( \cdots \) stands for the irrelevant terms that lead to higher-order operators over dimension-six.

The sum rule for the even moments of \( \phi_5^a \) up to dimension-six condensates is

\[ -\frac{1}{3} m_F^2 \bar{ho}_S e^{-m_F^2/M^2} \langle \xi^{2n} \rangle = \frac{3}{4\pi^2} \int_0^1 (2x-1)^{2n} \]

\[ \times \left\{ -(2n+3)x(1-x) + \frac{m_1 m_2 - m_1^2}{2M^2} \right\} M^4 e^{-m_1^2/M^2} dx \]

\[ -\frac{3}{4\pi^2} \int_0^1 (2x-1)^{2n} \left\{ -(2n+3)x(1-x) + \left( 1 + S_s/M^2 \right) \right\} M^4 e^{-S_s/M^2} dx \]

\[ + \frac{1}{M^2} e^{-m_2^2/M^2} dx + \left\{ \langle \xi \rangle q_1 \right\} \left\{ \frac{m_2 + (2n+1)m_1}{2} \right\} + \frac{1}{2M^2} \left\{ \frac{m_1^2 m_2 + 2m_1 m_2^2}{6M^2} \right\} e^{-m_2^2/M^2} + \frac{m_1^2 m_2^2 (2m_1 + m_2)}{2M^4} + \frac{m_1^2 m_2^2}{M^6} e^{-m_2^2/M^2} \]

\[ + \langle g_s \bar{q}_1 \sigma T G q_1 \rangle \left\{ \frac{9(2n-1)m_2 + n(16n-5)m_1}{18M^2} \right\} + \left\{ \frac{12M^4}{9M^2} \right\} e^{-m_2^2/M^2} \]

\[ + \langle g_s^2 \bar{q}_1 q_1 \rangle \left\{ \frac{(8n-3)m_1 + 3m_2 m_1^2}{81M^2} + \frac{m_1^2 m_2^2}{27M^4} \right\} e^{-m_2^2/M^2} \]

\[ + \langle g_s^2 \bar{q}_1 q_1 \rangle \left\{ \frac{4}{9} \left[ \frac{2m_1^2 - m_2^2}{M^2} + \frac{1}{m_2^2} \left( e^{-m_2^2/M^2} - e^{-m_2^2/M^2} \right) \right] \right\} \]

\[ + \langle \bar{q}_1 \leftrightarrow q_2, m_1 \leftrightarrow m_2 \rangle \right\} , \tag{9} \]

where \( m_{12}^2 = m_1^2 x + m_2^2 (1-x) \). The sum rule for the even moments of \( \phi_5^a \) up to dimension-six condensates is

\[ -\frac{1}{3} m_F^2 \bar{ho}_S e^{-m_F^2/M^2} \langle \xi^{2n} \rangle = \frac{3}{4\pi^2} \int_0^1 dx (2x-1)^{2n} M^4 x \]

\[ \times e^{-\frac{m_n^2}{M^2}} \left\{ \frac{m_1^2}{M^2} \right\} M^4 e^{-S_s/M^2} \]

\[ \times \left\{ x(1-x) \left( 1 + S_s/M^2 \right) \right\} M^4 e^{-S_s/M^2} \]

\[ -\langle \alpha_s G^2 \rangle \int_0^1 dx (2x-1)^{2n} \left\{ 1 - \frac{2m_1 m_2}{M^2 x (1-x)} \right\} \]

\[ e^{-\frac{m_n^2}{M^2}} \left\{ \frac{1}{24\pi} \left[ \frac{3m_1^2 + (4n+1) m_1^2}{M^2} + \frac{m_1^2 m_2^2}{M^4} \right] \right\} \]

\[ \times e^{-\frac{m_n^2}{M^2}} \left\{ \frac{m_1}{24\pi} \right\} \int_0^1 dx (2x-1)^{2n} \left\{ 3m_1 + \frac{4n+1) m_1^2}{M^2} + \frac{m_1^2 m_2^2}{M^4} \right\} \]

\[ \times e^{-\frac{m_n^2}{M^2}} \left\{ \frac{g_s^2 \bar{q}_1 q_1 \sigma T G q_1 \rangle}{36M^2} \right\} \left\{ \frac{16n+1)m_1 + 6m_2 + m_1^2 m_2^2}{9M^4} \right\} \]

\[ \times e^{-\frac{m_n^2}{M^2}} + \langle g_s^2 \bar{q}_1 q_1 \rangle \left\{ \frac{2m_2^2}{M^2} + \frac{2m_2^2}{M^4} \right\} e^{-m_2^2/M^2} \]

\[ + \langle \bar{q}_1 \leftrightarrow q_2, m_1 \leftrightarrow m_2 \rangle \right\} . \tag{10} \]
The sum rule for the odd moments of $\phi_5^g$ up to dimension-six condensates is

$$-m_{S_{5/2}}^2f_5^2 e^{-m_{S_{5/2}}^2/M^2} (\xi_s^{2n+1}) =$$

$$\frac{3}{4\pi^2} \int_0^1 (2x-1)^{2n+1} \left[ -2(n+2)x(1-x) + \frac{m_1m_2 - m_{12}^2}{M^2} \right] M^4 e^{- \frac{m_{S_{5/2}}^2}{M^2} x} dx$$

$$+ \frac{3}{4\pi^2} \int_0^1 (2x-1)^{2n+1} \left[ -2(n+2)x(1-x) + \frac{2m_1m_2 - m_{12}^2}{M^2} \right] M^4 e^{- \frac{m_{S_{5/2}}^2}{M^2} x} dx$$

$$+ \frac{(n+3)m_2^2 + m_{12}^2}{M^2(1-x)} e^{-\frac{m_{S_{5/2}}^2}{M^2} x} dx$$

$$+ \langle \alpha_s G^2 \rangle \int_0^1 (2x-1)^{2n+1} \left[ \frac{1}{8\pi} - 2(n+1) \right] e^{-\frac{m_{S_{5/2}}^2}{M^2} x} dx$$

$$+ \frac{2m_1m_2 - m_{12}^2}{M^2x(1-x)} e^{-\frac{m_{S_{5/2}}^2}{M^2} x} dx$$

$$+ \frac{2(2n+1)(4n+3)m_1^4 + 6(2n+1)m_1^2m_2 + 3m_2^2}{6M^2}$$

$$+ \frac{m_1^2m_2^2((2n+1)m_1 + m_2)}{2M^4} e^{-\frac{m_{S_{5/2}}^2}{M^2} x}$$

$$+ \langle g_s q_1 \sigma T G q_i \rangle \left[ \frac{36m_2^2 + (2n+1)(16n+3)m_1}{36M^2} \right]$$

$$+ \left[ \frac{(8n+1)m_1 + 3m_2^2}{12M^2} + \frac{m_1m_2^2}{9M^2} \right] e^{-\frac{m_{S_{5/2}}^2}{M^2} x}$$

$$\times e^{-\frac{m_{S_{5/2}}^2}{M^2} x} + 4\alpha_s \langle \bar{q}_1 q_i \rangle \langle \bar{q}_2 q_2 \rangle \frac{2}{9} \left( \frac{e^{-\frac{m_{S_{5/2}}^2}{M^2} x} - 1}{m_2^2} \right)$$

$$\times \left[ e^{-\frac{m_{S_{5/2}}^2}{M^2} x} - e^{-\frac{m_{S_{5/2}}^2}{M^2} x} \right]$$

$$+ 2m_1m_2 \left( \frac{e^{-\frac{m_{S_{5/2}}^2}{M^2} x} - 1}{m_2^2} - \frac{e^{-\frac{m_{S_{5/2}}^2}{M^2} x} - 1}{m_2^2} \right)$$

$$\left[ q_1 \leftrightarrow q_2, m_1 \leftrightarrow m_2 \right] \right\}. \quad (11)$$

The sum rule for the odd moments of $\phi_5^g$ up to dimension-six condensates is

$$\frac{1}{3} \frac{m_2^2}{m_{12}} e^{-\frac{m_{12}^2}{M^2} (\xi_s^{2n+1})} =$$

$$\frac{3}{4\pi^2} \int_0^1 (2x-1)^{2n+1} \times M^4 x(1-x) e^{- \frac{m_{12}^2}{M^2} x} dx$$

$$+ \frac{2m_1m_2 - m_{12}^2}{M^2x(1-x)} e^{-\frac{m_{12}^2}{M^2} x} dx$$

$$+ \frac{(2n+1)(4n+3)m_1^4 + 6(2n+1)m_1^2m_2 + 3m_2^2}{6M^2}$$

$$+ \frac{m_1^2m_2^2((2n+1)m_1 + m_2)}{2M^4} e^{-\frac{m_{12}^2}{M^2} x}$$

$$+ \langle g_s q_1 \sigma T G q_i \rangle \left[ \frac{36m_2^2 + (2n+1)(16n+3)m_1}{36M^2} \right]$$

$$+ \left[ \frac{(8n+1)m_1 + 3m_2^2}{12M^2} + \frac{m_1m_2^2}{9M^2} \right] e^{-\frac{m_{12}^2}{M^2} x}$$

$$\times e^{-\frac{m_{12}^2}{M^2} x} + 4\alpha_s \langle \bar{q}_1 q_i \rangle \langle \bar{q}_2 q_2 \rangle \frac{2}{9} \left( \frac{e^{-\frac{m_{12}^2}{M^2} x} - 1}{m_2^2} \right)$$

$$\times \left[ e^{-\frac{m_{12}^2}{M^2} x} - e^{-\frac{m_{12}^2}{M^2} x} \right]$$

$$+ 2m_1m_2 \left( \frac{e^{-\frac{m_{12}^2}{M^2} x} - 1}{m_2^2} - \frac{e^{-\frac{m_{12}^2}{M^2} x} - 1}{m_2^2} \right)$$

$$\left[ q_1 \leftrightarrow q_2, m_1 \leftrightarrow m_2 \right] \right\}. \quad (12)$$

### 2.2 B → S transition form factors within the light-cone sum rules

The $B \to S$ transition form factors $f_\pm(q^2)$ and $f_T(q^2)$ are defined through the hadronic matrix elements $\langle S(p) | \bar{q}_2 \gamma_\mu \gamma_5 b | B_s(p + q) \rangle$ and $\langle S(p) | \bar{q}_2 \sigma_\mu \gamma_5 q^\nu b | B_s(p + q) \rangle$, which are

$$\langle S(p) | \bar{q}_2 \gamma_\mu \gamma_5 b | B_s(p + q) \rangle = -2ie_{\mu} q^2 - i [f_+(q^2) - f_-(q^2)] q_\mu,$$

$$\langle S(p) | \bar{q}_2 \sigma_\mu \gamma_5 q^\nu b | B_s(p + q) \rangle = [2p_\mu q_\nu - 2q_\nu(p \cdot q)] \times -f_T(q^2) \left( \frac{m_{B_s}^2}{m_{B_s}^2 + m_S^2} \right). \quad (13)$$

These form factors are the key factors for studying the semileptonic decays $B_s \to Slq_1$ and $B_s \to Sl$. By introducing proper chiral correlators, we obtain the light-cone sum rules for those form factors that depend only on the twist-3 LCDAs of the scalar mesons, and then our twist-3 LCDAs derived in the last subsection apply. More explicitly, we suggest to calculate the following correlators:

$$\Pi_\mu(p,q) = i \int d^4x e^{iqx} \langle S(p) | T \{-\bar{q}_2(x)\gamma_\mu, (1 - \gamma_5) b(x), \bar{b}(0) i(1 + \gamma_5) q_1(0) \} | 0 \rangle,$$

$$\tilde{\Pi}_\mu(p,q) = i \int d^4x e^{iqx} \langle S(p) | T \{-\bar{q}_2(x)\sigma_\mu \gamma_5, (1 + \gamma_5) q^\nu b(x), \bar{b}(0) i(1 + \gamma_5) q_1(0) \} | 0 \rangle.$$


where $q_1$, $q_2$ denotes the light quark field. Following the standard procedures to deal with the correlators which are similar to that of the $B \to$ pseudoscalar transition form factors, cf. refs. [31–35], we can obtain the light-cone sum rules for the form factors $f^{\pm}(q^2)$,

$$f^{\pm}(q^2) = \frac{m_0 + m_{q_1}}{m_B^2} \exp \left( \frac{m_B^2}{M^2} \right) \times \left\{ \int_{0}^{1} \frac{du}{u} \exp \left[ -\frac{m_0^2 + u\bar{q}^2 - \bar{u}q^2}{M^2} \right] \times \left[ m_S f_S(u\phi_S^2(u)) \right] + \frac{1}{3} \phi_B^2(u) + \frac{m_S f_S(6uM^2\phi_S^2(u)(m_0^2 - u\bar{q}^2 + q^2)}{6uM^2} \right\}.

$$

$$= \frac{m_0 + m_{q_1}}{m_B^2} \exp \left( \frac{m_B^2}{M^2} \right) \times \left\{ \int_{0}^{1} \frac{du}{u} \exp \left[ -\frac{m_0^2 + u\bar{q}^2 - \bar{u}q^2}{M^2} \right] \times \left[ m_S f_S(\phi_S^2(u)) \right] - \frac{m_S f_S(6uM^2\phi_S^2(u)(m_0^2 + u\bar{q}^2 - q^2)}{6uM^2} \right\},

$$

$$f_T(q^2) = \frac{(m_0 + m_{q_1})(m_B^2 + m_S^2)}{m_B^2} \exp \left( \frac{m_B^2}{M^2} \right) \times \left\{ \int_{0}^{1} \frac{du}{u} \exp \left[ -\frac{m_0^2 + u\bar{q}^2 - \bar{u}q^2}{M^2} \right] \times \left[ m_S f_S(\phi_S^2(u)) + \frac{m_S f_S(3uM^2\phi_S^2(u)}{3}\right] \right\}.

$$

The mass sum rules are found that the results from both sum rules (9) and (10) are consistent with each other, so we adopt the results from sum rule (9) for a detailed discussion.

3.1 Masses and decay constants for the scalar mesons

Setting $n = 0$ in the sum rules (9) and (10) for the moments of $\phi_S^2$ and $\phi_B^2$, we can further derive the sum rules for the masses of the scalar mesons. The mass sum rules are derived by doing the logarithm of these equations and applying the differential operation $M^4/(\partial M^2)$ over them. It is found that the results from both sum rules (9) and (10) are consistent with each other, so we adopt the results from sum rule (9) for a detailed discussion.

Usually, the threshold parameter $S_n$ (or $S_P$) is taken to be around the squared mass of the scalar’s first excited state. We adopt $S^{\mu}_n = (6.0 \pm 0.5)\text{GeV}^2$ for $a_0$ [1], $S^{\mu}_{n,n} = (5.4 \pm 0.3)\text{GeV}^2$ and $S^{\mu}_{n,n} = (6.5 \pm 0.3)\text{GeV}^2$ for $K_0$ and $f_0$ [2]. According to the SVZ sum rule, the Borel window for the parameter $M$ is determined by the requirement that the dimension-six condensate contribution (SIX) does not exceed 10% and the continuum contribution (CON) is not too large, i.e. less than 30% of the total dispersive integration. In table 1 and fig. 2, the Borel window is obtained by setting SIX < 1% and CON < 30%.

The masses for the scalar mesons are presented in table 1 and their values versus $M^2$ are presented in fig. 2, whose central values are

$$m_{a_0} = 1442\text{MeV}, \quad m_{K_0^*} = 1421\text{MeV}, \quad m_{f_0} = 1634\text{MeV}.$$

These values are close to the physical states $a_0(1450)$, $K_0^*(1430)$ and $f_0(1710)$ [36]. This shows that the valence quark constituent $du$, $us$ and $ss$ are viable choices for studying the properties of $a_0$, $K_0^*$ and $f_0$.

### Table 1. Masses for the scalar mesons. The threshold parameters are taken as $S^{\mu}_{n,n} = (6.0 \pm 0.5)\text{GeV}^2$, $S^{\mu}_{n,n} = (5.4 \pm 0.3)\text{GeV}^2$ and $S^{\mu}_{n,n} = (6.5 \pm 0.3)\text{GeV}^2$.

| Mesons   | $M^2$ (GeV$^2$) | $m$ (GeV) |
|----------|----------------|-----------|
| $a_0$    | 1.037–1.666    | 1.312–1.571 |
| $K_0^*$  | 0.991–1.501    | 1.328–1.514 |
| $f_0$    | 1.284–1.803    | 1.563–1.706 |

3 Numerical results and discussions

We adopt the following input parameters to do our numerical analysis [36–38]:

$$m_u = (2.5 \pm 0.8)\text{MeV},$$

$$m_d = (5.0 \pm 0.8)\text{MeV},$$

$$m_s = (101 \pm 12)\text{MeV},$$

$$\alpha_s(m_c = 1.27\text{GeV}) = 0.39$$

with

$$u_0 = \sqrt{(s_0 - q^2 - p^2)^2 + 4p^4(m_c^2 - q^2)} - (s_0 - q^2 - p^2).$$
Fig. 2. Masses of scalar mesons $a_0$, $K^*_0$ and $f_0$ versus $M^2$ within their Borel windows.

Table 2. Decay constants for the scalar mesons at the energy scale $\mu = 1$ GeV.

| Mesons | $M^2$ (GeV^2) | $f$ (GeV) |
|-------|--------------|-----------|
| $a_0$ | 1.6–2.5      | 0.374–0.377 |
| $K^*_0$ | 1.5–2.2     | 0.357–0.359 |
| $f_0$ | 1.8–2.2      | 0.374–0.378 |

Fig. 3. Decay constants of scalar mesons $a_0$, $K^*_0$ and $f_0$ versus $M^2$ within their Borel windows.

Taking the scalar masses as inputs, we can further calculate the decay constants of the scalar mesons. For the purpose, the Borel window for each meson is redetermined following the same criteria as above, whose values are collected in table 2. The decay constants at the energy scale 1 GeV for the scalar mesons are presented in table 2 and their values versus $M^2$ are presented in fig. 3, whose central values are

$$f_{a_0} = 375 \text{ MeV}, \quad f_{K^*_0} = 358 \text{ MeV}, \quad f_{f_0} = 376 \text{ MeV}.$$  

In the following subsections, all the values of decay constants and moments are given at the scale 1 GeV unless explicitly pointed out.

### Table 3. Moments from the scalar density sum rules.

| Mesons | $\langle \xi_s \rangle$ | $M^2$ (GeV^2) |
|-------|----------------------|--------------|
| $a_0$ | $\langle \xi_{s,a_0}^2 \rangle = (0.367-0.371)$ | 2.0–2.5 |
|       | $\langle \xi_{s,a_0}^4 \rangle = (0.24-0.25)$ | 2.0–2.6 |
| $K^*_0$ | $\langle \xi_{s,K^*_0}^2 \rangle = (0.33-0.44) \times 10^{-2}$ | 2.0–3.5 |
|       | $\langle \xi_{s,K^*_0}^4 \rangle = (0.352-0.358)$ | 1.8–2.3 |
| $f_0$ | $\langle \xi_{s,f_0}^2 \rangle = (0.331-0.339)$ | 1.8–2.2 |
|       | $\langle \xi_{s,f_0}^4 \rangle = (0.204-0.220)$ | 1.8–2.2 |

### Table 4. Moments from the tensor sum rules.

| Mesons | $\langle \xi_s \rangle$ | $M^2$ (GeV^2) |
|-------|----------------------|--------------|
| $a_0$ | $\langle \xi_{s,a_0}^2 \rangle = (0.203-0.204)$ | 2.0–2.5 |
|       | $\langle \xi_{s,a_0}^4 \rangle = (0.092-0.094)$ | 2.0–2.5 |
| $K^*_0$ | $\langle \xi_{s,K^*_0}^2 \rangle = (0.99-2.54) \times 10^{-2}$ | 1.8–7.8 |
|       | $\langle \xi_{s,K^*_0}^4 \rangle = (0.206-0.208)$ | 1.8–2.3 |
| $f_0$ | $\langle \xi_{s,f_0}^2 \rangle = (0.192-0.199)$ | 1.5–2.2 |
|       | $\langle \xi_{s,f_0}^4 \rangle = (0.085-0.091)$ | 1.8–2.2 |

### 3.2 Moments for the scalar mesons

By using the masses and decay constants for the scalar mesons, we further calculate the moments of these mesons from the sum rules (9)–(12). Considering the conservation of charge parity and isospin symmetry, the odd moments for $a_0$ and $f_0$ meson twist-3 distribution amplitudes should vanish. While taking $SU(3)$ symmetry-breaking effects into account, the odd moments for $K^*_0$ meson are non-zero. Therefore, the moments we aim to derive are the second and fourth moments for the $a_0$ and $f_0$ meson, and the first and second moments for the $K^*_0$ meson. Similar to the way to get masses and decay constants, we first find the stable Borel window for the sum rules of each moments. The choice of Borel parameters for each meson, together with the moments for the scalar mesons, are collected in tables 3 and 4. In tables 3 and 4, the Borel windows are determined by setting $\text{SIX} < 5\%$ and $\text{CON} < 30\%$. The moments for the scalar mesons versus $M^2$ are presented in figs. 4–6. It is noted that the moments within their Borel windows are very stable and their central values are

$$\langle \xi_{s,a_0}^{2(4)} \rangle = 0.369 (0.245), \quad \langle \xi_{s,a_0}^{4(4)} \rangle = 0.203 (0.093),$$  
$$\langle \xi_{s,K^*_0}^{1(2)} \rangle = 0.004 (0.355), \quad \langle \xi_{s,K^*_0}^{1(2)} \rangle = 0.018 (0.207),$$  
$$\langle \xi_{s,f_0}^{2(4)} \rangle = 0.335 (0.212), \quad \langle \xi_{s,f_0}^{4(4)} \rangle = 0.196 (0.088),$$  

It is noted that these moments are different from ref. [2], especially for the moments of $K^*_0$ and $f_0$. By using the same input parameters, we can obtain consistent moments for $a_0$. Then, the differences are mainly caused by the different treatment of the mass terms (the s-quark mass terms) involved in the hard part calculation\(^1\). Similar to

\(^1\) Since the current s-quark mass is of the order of $\Lambda_{QCD}$, it is reasonable to assume that those terms will have sizable effects.
the kaonic case [8], it is found that those $m_s$ terms do provide sizable contributions. Thus, they should be treated consistently with those of the higher-dimensional matrix elements. By taking all the mass terms consistently into consideration, more reliable masses and decay constants, and hence more accurate moments can be obtained.

3.3 A discussion on the scalar’s three-particle twist-3 LCDAs

For each scalar meson $S$, there are three twist-3 distribution amplitudes $\phi^1_S$, $\phi^2_S$ and $\phi_{3S}$. To be of useful reference, we make a discussion on the three-particle twist-3 LCDA, which, similar to the pseudoscalar and the vector cases [18, 43, 44], is defined as

$$\langle 0 | \bar{q}_2(x) \sigma_{\mu\nu} g G_{\alpha\beta} (-v x) q_1 (-x) | S(q) \rangle = i f_{3S} \left[ q_\alpha (q_\mu \delta_{\nu\beta} - q_\nu \delta_{\mu\beta}) - (\alpha \leftrightarrow \beta) \right] \times \int D\alpha_1 e^{i p x (-\alpha_1 + \alpha_2 + \alpha_3)} \phi_{3S}(\alpha_1),$$

(24)

where $D\alpha_1 = d\alpha_1 d\alpha_2 d\alpha_3 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 1)$. Defining

$$R_1 = \frac{1}{m_{a_0}} \frac{f_{3a_0}}{f_{a_0}}, \quad R_2 = \frac{1}{m_{K^*_0}} \frac{f_{3K^*_0}}{f_{K^*_0}}, \quad R_3 = \frac{1}{m_{f_0}} \frac{f_{3f_0}}{f_{f_0}},$$

the kaonic case [8], it is found that those $m_s$ terms do provide sizable contributions. Thus, they should be treated consistently with those of the higher-dimensional matrix elements. By taking all the mass terms consistently into consideration, more reliable masses and decay constants, and hence more accurate moments can be obtained.

Fig. 4. Moments of the scalar meson $a_0$ versus $M^2$ within their Borel windows.

Fig. 5. Moments of the scalar meson $K^*_0$ versus $M^2$ within their Borel windows.

Fig. 6. Moments of the scalar meson $f_0$ versus $M^2$ within their Borel windows.
These equations show that all $R_{\tau}$ are less than 1%, so in the usual calculations, one can safely neglect the scalar meson’s three-particle twist-3 LCDA in comparison to the two-particle twist-3 LCDAs. This is different from the pion and the kaon, whose similar ratios $R_{\tau} (R_K) \sim 5$% [45]; then for the pionic or kaonic processes, one may need to take $\phi_{3\pi}$ or $\phi_{3K}$ into consideration.

### 3.4 Scalar meson distribution amplitudes

The scalar mesons’ twist-3 LCDAs can be expanded into a series of Gegenbauer polynomials [41,42],

\[
\phi_S^a (u, \mu) = 1 + \sum_{m=1}^{\infty} a_m (\mu) C_m^{3/2} (2u - 1),
\]

where $C_m^{3/2} (2u - 1)$ are Gegenbauer polynomials. The Gegenbauer moments $a_m, b_m$ can be related to moments $\langle x^m \rangle$ and $\langle x^m \rangle$ defined in eqs. (3) and (4). By using the orthogonality of Gegenbauer polynomials,

\[
\int_0^1 du C_n^{3/2} (2u - 1) C_m^{3/2} (2u - 1) = \frac{1}{2n + 1} \delta_{mn},
\]

we obtain the usual Gegenbauer moments,

\[
a_1 = 3 \langle \xi \rangle, \quad a_2 = \frac{5}{2} (3 \langle \xi^2 \rangle - 1),
\]

\[
a_4 = \frac{9}{8} (35 \langle \xi^4 \rangle - 30 \langle \xi^2 \rangle + 3),
\]

\[
b_1 = \frac{5}{3} \langle \xi \rangle, \quad b_2 = \frac{7}{12} (5 \langle \xi^2 \rangle - 1),
\]

\[
b_4 = \frac{11}{24} (21 \langle \xi^4 \rangle - 14 \langle \xi^2 \rangle + 1).
\]

Using the moments $\langle x^m \rangle$ and $\langle x^m \rangle$ derived in the last subsection, one can obtain the Gegenbauer moments $a_m$ and $b_m$ by using the relations (38) and (39). For convenience, we present the first two non-zero ones for each meson at two different energy scales in tables 5 and 6, where

### Table 5. Gegenbauer moments for $\phi_S^a$ at two energy scales, 1 GeV and 2.4 GeV, where $S$ stands for the scalar meson $a_0, k_0^*$ or $f_0$, respectively.

| Meson | $\mu = 1$ GeV | | $\mu = 2.4$ GeV | |
|-------|---------------|---|---------------|---|
|       | $a_1$ | $a_2$ | $a_4$ | $a_1$ | $a_2$ | $a_4$ |
| $a_0$ | 0 | 0.302-0.323 | 0.491-0.944 | 0 | 0.225-0.241 | 0.307-0.591 |
| $k_0^*$ | 0.0109-0.0143 | 0.163-0.211 | - | 0.0095-0.0125 | 0.122-0.157 | - |
| $f_0$ | 0 | -0.022-0.049 | 0.140-0.892 | 0 | -0.016-0.036 | 0.088-0.56 |

### Table 6. Gegenbauer moments for $\phi_S^a$ at two energy scales, 1 GeV and 2.4 GeV, where $S$ stands for the scalar meson $a_0, k_0^*$ or $f_0$, respectively.

| Meson | $\mu = 1$ GeV | | $\mu = 2.4$ GeV | |
|-------|---------------|---|---------------|---|
|       | $b_1$ | $b_2$ | $b_4$ | $b_1$ | $b_2$ | $b_4$ |
| $a_0$ | 0 | 0.011-0.013 | 0.050-0.073 | 0 | 0.008-0.009 | 0.031-0.045 |
| $k_0^*$ | 0.0216-0.0468 | 0.019-0.028 | - | 0.017-0.037 | 0.013-0.029 | - |
| $f_0$ | 0 | -0.0291-0.0014 | 0.030-0.106 | 0 | -0.0206-0.0010 | 0.018-0.065 |
the Gegenbauer moments at $\mu = 2.4$ GeV are obtained by using the evolution equations.

The LCDAs $\phi_5^b(u)$ and $\phi_5^u(u)$ at $\mu = 1$ GeV are presented in figs. 7 and 8. As a comparison we also present the results of ref. [2] in figs. 7 and 8.

3.5 Properties of the $B \to S$ transition form factors

In doing the numerical calculation we adopt [1, 36, 46]: $m_{s0} = 5.279$ GeV, $m_{B_s} = 5.368$ GeV, $f_{B_s} = (0.19 \pm 0.02)$ GeV, $f_{B_s} = (0.23 \pm 0.02)$ GeV, $m_b = (4.68 \pm 0.03)$ GeV. As for the energy scale of the $B \to S$ transitions, we adopt $\mu_b = \sqrt{m_{B_s}^2 - m_s^2} \sim 2.4$ GeV. To calculate the form factors one has to evolve the parameters to the scale $\mu_b$, which can be derived by using the renormalization group equations (A.1) and (A.2). We take the threshold parameter $S^{B_0}_0 = S^{B_0}_0 = (33 \pm 1)$ GeV$^2$ [30]. The Borel window is determined by requiring both the contributions from higher excited resonances and continuum states be less than 30%, which results in $10$ GeV$^2 \leq M^2 \leq 15$ GeV$^2$. As shown in fig. 9, the sum rules for the form factors at the large recoil region vary slightly within such Borel window.

We present the $B \to S$ form factors at the large recoil point $q^2 = 0$ in table 7, where the theoretical uncertainties are around 20%, which can be derived by varying the Borel parameter $M$, the threshold values $S^{B_0}_0$ and $S^{B_0}_3$, the $b$ quark mass, the decay constants of the scalar mesons, and the Gegenbauer moments for the twist-3 LCDAs of the scalar mesons, within their reasonable regions. As a comparison, we also put several typical estimations from the SR, the LCSR and the pQCD approaches [30, 46–51], in table 7. The form factors $f_+(q^2)$ for various scalar mesons $a_0^+(1450), K_0^+(1430), K_0^{*+}(1430)$ and $f_0(1500)$ rise slightly with the increment of $q^2$, as can be shown explicitly in fig. 10.

3.6 Two semi-leptonic channels $B \to Sl\bar{\nu}$ and $B \to Sl\bar{\nu}$

Next, as an application of the $B \to S$ transition form factors, we discuss the properties of the two semi-leptonic channels, $B \to Sl\bar{\nu}$ and $B \to Sl\bar{\nu}$, whose differential decay widths can be expressed as

$$
\frac{d\Gamma}{dq^2}(B(s) \to Sl\bar{\nu}) = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_{B}^2} \frac{q^2 - m_d^2}{m_B^2} \frac{1}{4q^2} \left[ (m_B^2 - m_S^2 + q^2)^2 - m_S^2 (m_B^2 + 2q^2) \right] \\
\times \left[ (q^2 - m_B^2)^2 (q^2 - m_B^2 - m_S^2)^2 - 4m_B^2 (q^2 - m_B^2 - m_S^2)^2 f_+^2 (q^2) \right] \\
+ 3m_d^2 (m_B^2 - m_S^2)^2 \left( f_+(q^2) + \frac{q^2}{m_B^2 - m_S^2} f_-(q^2) \right)^2, \quad (40)
$$
The dashed, the dotted and the dash-dotted lines are for $\bar{B}_0 \to a_0^+(1540)$, $\bar{B}_0 \to K_0^+(1430)$, $\bar{B}_0^0 \to K_0^+(1430)$ and $\bar{B}_0^0 \to f_0(1500)$, respectively. The input parameters are taken as their central values.

Table 7. Numerical results for the $B \to S$ transition form factors at the large recoil point $q^2 = 0$, where $S$ stands for the scalar mesons $a_0^+$, $f_0$, $K_0^{++}$, and $K_0^+$, respectively.

| Processes          | Methods      | $f_+$ | $f_-$ | $f_T$ |
|--------------------|--------------|-------|-------|-------|
| $B^0 \to a_0^+(1540)$ | This work    | 0.44 ± 0.04 | 0.26 ± 0.04 | 0.43 ± 0.04 |
|                   | LCSR [30]    | 0.53 | -0.53 | -       |
|                   | pQCD [47]    | 0.68 | -0.92 |        |
|                   | CLF [48]     | 0.26 | -       |        |
| $\bar{B}_0^\prime \to f_0(1500)$ | This work    | 0.39 ± 0.04 | 0.25 ± 0.04 | 0.41 ± 0.04 |
|                   | LCSR [30]    | 0.41 | -0.41 | 0.59   |
|                   | pQCD [47]    | 0.60 | -       | 0.82   |
| $\bar{B}_0^\prime \to K_0^+(1430)$ | This work    | 0.45 ± 0.04 | 0.28 ± 0.04 | 0.46 ± 0.04 |
|                   | LCSR [30]    | 0.49 | -0.41 | 0.60   |
|                   | SR [49]      | 0.40 | -0.49 | 0.69   |
|                   | pQCD [47]    | 0.31 | -0.31 | -0.26  |
|                   | CLF [48]     | 0.26 | -       | -0.78  |
|                   | LFCQM [51]   | -0.26 | 0.21 | -0.34 |

and

$$\frac{df}{dq^2}(B(\gamma) \to Sll) = \frac{G_F^2 |V_{tb}V_{ts}|^2 m_t^2 \alpha_{em}}{1536 \pi^3 s} \times \left( 1 - \frac{4r_t}{s} \right)^{1/2} \left[ \left( 1 + \frac{2r_t}{s} \right) \phi_{Sl}^2 \alpha_S + \phi_{Sl}^1 r_t \delta_S \right]. \quad (41)$$

Fig. 9. Prediction of $f_+(q^2 = 0)$ within the LCSR approach with chiral currents versus the Borel parameter $M$. The solid, the dashed, and the dash-dotted lines are for $B_0 \to a_0^+(1540)$, $\bar{B}_0 \to K_0^+(1430)$, $\bar{B}_0^0 \to K_0^{++}(1430)$ and $\bar{B}_0^0 \to f_0(1500)$, respectively. The input parameters are taken as their central values.

Fig. 10. Form factors $f_+(q^2)$ within the LCSR approach with chiral currents. The solid, the dashed, and the dash-dotted lines are for $\bar{B}_0 \to a_0^+(1540)$, $B_0 \to K_0^+(1430)$, $\bar{B}_0^0 \to K_0^{++}(1430)$ and $\bar{B}_0^0 \to f_0(1500)$, respectively. Here the Borel parameter $M^2 = 12 GeV^2$ within the LCSR approach.

where $m_l$ denotes the mass of a final-state lepton, and

$s = q^2/m_B^2$, \hspace{0.5cm} r_l = m_l^2/m_B^2$, \hspace{0.5cm} r_S = m_S^2/m_B^2$. \hspace{0.5cm} (42)

$$\varphi_S = (1 - r_S)^2 - 2s (1 + r_S) + s^2, \hspace{0.5cm} \alpha_S = \left| C_q^s f_+(q^2) - 2 C_T f_T(q^2) \right|^2 + \left| C_{10} f_+(q^2) \right|^2, \hspace{0.5cm} (43)$$

$$\delta_S = 6 |C_{10}|^2 \left\{ (2 (1 + r_S) - s) |f_+(q^2)|^2 + (1 - r_S) 2 Re[f_+(q^2) f^*_-(q^2)] + s |f_-(q^2)|^2 \right\}. \hspace{0.5cm} (44)$$

Except for the $B \to S$ transition form factors, we adopt the same input parameters as those of ref. [30]. Our results for the curves for the differential decay rates are presented in figs. 11 and 12, which have a different arising trend versus the increment of $q^2$ in comparison to that of ref. [30]. In ref. [30], the authors also adopt the chiral currents in calculating the $B \to S$ transition form factors. However their chiral currents are different from ours, where only the twist-2 scalar LCDAs are retained and the twist-3 scalar LCDAs make no contributions. Those two different QCD sum rules can be taken as a cross-check of the chiral LCSR approach and be confirmed by the future experiments.

4 Summary

The masses, the decay constants and the twist-3 LCDAs of the scalar mesons $a_0$, $K_0^+$ and $f_0$ have been investigated in this work by using the QCD sum rules together with the QCD background field theory. Our present estimations are improved by a consistent treatment of the mass effects and a RG improved treatment of the input parameters. It is found that the $m_t$ terms provide sizable contributions, which should be treated consistently with those of higher-dimensional matrix elements. While, by taking all the mass terms consistently into consideration, more reliable masses and decay constants, and hence more accurate moments, can be obtained.
As for $a_0$ meson, the second and fourth moments of the twist-3 LCDAs $\phi_{s,a}^{s,a}$ are

$$\langle \xi^{(2)}_{s,a} \rangle = 0.369 (0.245) \quad \text{and} \quad \langle \xi^{(4)}_{s,a} \rangle = 0.203 (0.093),$$

whose uncertainties are about 10%; as for the $K_1^*$ meson, the first and second moments of the twist-3 LCDAs $\phi_{K_1^*}^{s,a}$ are

$$\langle \xi^{(1)}_{s,K_1^*} \rangle = 0.004 (0.355) \quad \text{and} \quad \langle \xi^{(2)}_{s,K_1^*} \rangle = 0.018 (0.207),$$

whose uncertainties are about 10%–15%; as for the case of the $f_0$ meson, the second and fourth moments of the twist-3 LCDAs $\phi_{f_0}^{s,a}$ are

$$\langle \xi^{(2)}_{s,f_0} \rangle = 0.335 (0.212) \quad \text{and} \quad \langle \xi^{(4)}_{s,f_0} \rangle = 0.196 (0.088),$$

whose uncertainties are within 15%.

As an application of these twist-3 LCDAs, we have studied the $B \to S$ transition form factors and their corresponding decay rates. For the purpose, the chiral currents are adopted in our LCSR calculation of $B \to S$ transition form factors, in which the twist-3 LCDAs make dominant contributions. Our results for the transition form factors at the large recoil point $q^2 \approx 0$ are consistent with those obtained in the literature; while, the arising trends for the form factors versus $q^2$, and hence the differential decay widths for the $B \to S$ semileptonic decays, are somewhat different. This can be checked by a future experimental data.

Our results for the twist-3 LCDAs are useful for pQCD calculation. Basing on the Gegenbauer moments of the twist-3 LCDAs, if further taking a proper transverse momentum dependence, e.g., the BHL-transverse momentum dependence [52, 53], we can construct a better-behaved scalar wave function in which the end-point singularity can be effectively suppressed. For example, following the similar idea in constructing the pseudo-scalar meson’s twist-3 WF model, it is natural to assume that one can obtain reasonable twist-3 contributions to the scalar-meson-involved high-energy processes.

HYH thanks Y.M. Wang and Y.J. Sun for helpful discussions. This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant No.CDJS11100002 and No.CQDXXL-2012-Z002, by Natural Science Foundation of China under Grant No.11075225 and No.11275280, and by the Program for New Century Excellent Talents in University under Grant NO.NCET-10-0882.

Appendix A. Details for deriving the twist-3 LCDAs

The LCDAs of the scalar mesons are normalized as $\int_0^1 du \times \phi_{s}^{s} (u, \mu) = \int_0^1 du \phi_{s}^{s} (u, \mu) = 1$. The initial energy scale for the bound state and the non-perturbative matrix elements are set as the values of the Borel parameter $M$. Their values at any other scale $\mu$ can be obtained from
the RG equations, for example, the decay constant, the quark mass and the condensates are related by [39, 40]

\[
\begin{align*}
  f_S(M) &= f_S(\mu) \left( \frac{\alpha_s(\mu)}{\alpha_s(M)} \right)^{4/b}, \\
  m_q(M) &= m_q(\mu) \left( \frac{\alpha_s(\mu)}{\alpha_s(M)} \right)^{-4/b}, \\
  \langle \bar{q}q \rangle_M &= \langle \bar{q}q \rangle_0 \left( \frac{\alpha_s(\mu)}{\alpha_s(M)} \right)^{4/b}, \\
  \langle g_s q^2 \rangle_M &= \langle g_s q^2 \rangle_0 \left( \frac{\alpha_s(\mu)}{\alpha_s(M)} \right)^{-2/3b}, \\
  \langle \alpha_s G^2 \rangle_M &= \langle \alpha_s G^2 \rangle_0, 
\end{align*}
\]

(1.1)

where \( b = (33 - 2n_f)/3 \) with \( n_f \) the active flavor number. And the RG equations for Gegenbaum moments are [54, 55]

\[
\begin{align*}
  \langle a_n(\mu) \rangle &= \langle a_n(\mu_0) \rangle \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{-\gamma_n^S/b}, \\
  \langle b_n(\mu) \rangle &= \langle b_n(\mu_0) \rangle \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{-\gamma_n^T/b},
\end{align*}
\]

(2.2)

where the one-loop anomalous dimensions are

\[
\begin{align*}
  \gamma_n^S &= C_F \left( 1 - \frac{8}{(n+1)(n+2)} + 4 \sum_{j=2}^{n+1} \frac{1}{j} \right), \\
  \gamma_n^T &= C_F \left( 1 + 4 \sum_{j=2}^{n+2} \frac{1}{j} \right).
\end{align*}
\]

(3.3)

with \( C_F = 4/3 \).

The moments of the LCDAs for the scalar mesons are calculated under the background field approach. For the purpose, we adopt the following two correlation functions:

\[
\begin{align*}
  i \int d^4 x e^{i q x} \langle 0 | T \{ \bar{q}_1(x) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right) q_2(x), \bar{q}_2(0) q_1(0) \} | 0 \rangle = \langle \bar{q}q \rangle_0 \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{n+1} (z \cdot q)^n I_z^{(n,0)}(q^2), \\
  i \int d^4 x e^{i q x} \langle 0 | T \{ \bar{q}_1(x) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right) \sigma_{\mu\nu} \bar{q}_2(x), \bar{q}_2(0) q_1(0) \} | 0 \rangle = \langle \bar{q}q \rangle_0 \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{n+1} (z \cdot q)^n f_\mu^{(n,0)}(q^2).
\end{align*}
\]

(4.4)

The product expansion can be expressed as

\[
\begin{align*}
  I_z^n(q^2)_{QCD} &= \frac{3(n+1)}{4\pi^2} \int_0^1 dx (2x-1)^n \{ [q^2 x(x-1) + m_1^2] \}
  \times \left( 1 - \ln \frac{q^2 x(x-1) + m_1^2}{\mu^2} \right) \\
  + \left( -n \right)[q^2 x(x-1) + m_1^2] \left( \frac{3}{2} - \ln \frac{q^2 x(x-1) + m_1^2}{\mu^2} \right) \\
  + \left( \alpha_s G^2 \right) \frac{1}{8\pi} \int_0^1 dx x(x-1)(2x-1)^n \left( - \frac{n+1}{q^2 x(x-1) + m_1^2} \\
  + \left\{ \frac{2m_1^2 - 2m_1^2}{q^2 x(x-1) + m_1^2} \right\} \\
  + [2m_1^2 + (n+1)m_1] \right\}
\end{align*}
\]

(5.5)

where \( m_1^2 = m^2 + x^2 - 1 \).
On the other hand, the correlation functions (A.5) and (A.6) can be derived by inserting a complete set of quantum states $\Sigma|n\rangle\langle n|$ in the physical region,

\[
\text{Im} I_\sigma^n(q^2)^{\text{had}} = -\pi\delta(q^2 - m_\Sigma^2) m_\Sigma^2 f_\Sigma^n(\xi^n) \\
+ \frac{3}{4\pi} \int_0^1 dx (2x - 1)^n \\
\times \left\{ (n+3)q^2 x(x-1) + (n+2)m_\Sigma^2 + m_1 m_2 \right\} \\
\times \theta(q^2 - s_\sigma),
\]

\[
\text{Im} I_\sigma^n(q^2)^{\text{had}} = -\pi\delta(q^2 - m_\Sigma^2) \frac{n+1}{3} m_\Sigma^2 f_\Sigma^n(\xi^n) \\
+ \frac{3(n+1)}{4\pi} \int_0^1 dx (2x - 1)^n \\
\times \left\{ q^2 x(x-1) + m_\Sigma^2 \right\} \theta(q^2 - s_\sigma).
\]

In deriving the above equations, we have implicitly adopted the quark-hadron duality. These two expressions of correlators (A.5) and (A.6) can be matched through the dispersion relation

\[
\frac{1}{\pi} \int ds \frac{\text{Im} I(s)^{\text{had}}}{s - q^2} = I(q^2)^{\text{QCD}}.
\]

One can apply the Borel transformation to both sides to suppress the unknown higher-dimensional condensates and the continuum contributions as much as possible. For an arbitrary $q^2$ polynomials, the Borel transformation implies [56]

\[
\mathcal{B}_{M^2} = \lim_{q^2 \to -n^2, n=1} \frac{(-q^2)^{(n+1)}}{n!} \left( \frac{d}{dq^2} \right)^n,
\]

where $M$ is the Borel parameter.

Applying the Borel transformation to the dispersion relation, it results in

\[
\frac{1}{\pi} \int ds e^{-s/M^2} \text{Im} I(s)^{\text{had}} = \mathcal{B}_{M^2} I(q^2)^{\text{QCD}},
\]

which finally leads to the sum rules (9, 11, 10, 12) adopted in the body of the text.

References

1. H.Y. Cheng, C.K. Chua, K.C. Yang, Phys. Rev. D 73, 014017 (2006).
2. C.D. Liu, Y.M. Wang, H. Zou, Phys. Rev. D 75, 056001 (2007).
3. T. Huang, X.G. Wu, Phys. Rev. D 70, 093013 (2004).
4. X.G. Wu, T. Huang, JHEP 04, 043 (2008).
5. T. Huang, X.G. Wu, Phys. Rev. D 71, 034018 (2005).
6. X.G. Wu, T. Huang, Z.Y. Fang, Eur. Phys. J. C 52, 561 (2007).
7. T. Zhong, X.G. Wu, J.W. Zhang, Y.Q. Tang, Z.Y. Fang, Phys. Rev. D 83, 036002 (2011).
8. T. Zhong, X.G. Wu, H.Y. Han, Q.L. Liao, H.B. Fu, Z.Y. Fang, Commun. Theor. Phys. 58, 261 (2012).
9. K.G. Chetyrkin, A. Khodjamirian, A.A. Pivovarov, Phys. Lett. B 661, 250 (2008).
10. P. Ball, V.M. Braun, A. Lenz, JHEP 05, 004 (2006).
11. P. Ball, JHEP 01, 010 (1999).
12. P. Ball, G.W. Jones, JHEP 03, 069 (2007).
13. A. Khodjamirian, Th. Mannel, M. Melcher, Phys. Rev. D 70, 094002 (2004).
14. P. Ball, R. Zwicky, Phys. Lett. B 633, 289 (2006).
15. V.M. Braun, A. Lenz, Phys. Rev. D 70, 074020 (2004).
16. P. Ball, V.M. Braun, Y. Koike, K. Tanaka, Nucl. Phys. B 529, 523 (1998).
17. I.E. Halperin, Phys. Rev. D 57, 1680 (1998).
18. P. Ball, V.M. Braun, Nucl. Phys. B 543, 201 (1999).
19. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147, 385 (1979).
20. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Fortschr. Phys. 32, 585 (1984).
21. W. Hubschmid, S. Mallik, Nucl. Phys. B 207, 29 (1982).
22. J. Govaerts, F. de Viron, X.D. Xiang, Phys. Rev. D 73, 014017 (2006).
23. J. Govaerts, F. de Viron, D. Gusbin, J. Weyers, Phys. Lett. B 128, 262 (1983).
24. J. Govaerts, F. de Viron, D. Gusbin, J. Weyers, Nucl. Phys. B 248, 1 (1984).
25. J. Ambjorn, R.J. Hughes, Ann. Phys. 145, 340 (1983).
26. J. Ambjorn, R.J. Hughes, Nucl. Phys. B 217, 336 (1983).
27. W. Hubschmid, S. Mallik, Nucl. Phys. B 207, 29 (1982).
28. J. Govaerts, F. de Viron, X.D. Xiang, Phys. Rev. D 73, 014017 (2006).
49. M.Z. Yang, Phys. Rev. D 73, 034027 (2006).
50. T.M. Aliev, K. Azizi, M. Savci, Phys. Rev. D 76, 074017 (2007).
51. C.H. Chen, C.Q. Geng, C.C. Lih, C.C. Liu, Phys. Rev. D 75, 074010 (2007).
52. S.J. Brodsky, T. Huang, G.P. Lepage, in Particles and Fields-2, Proceedings of the Banff Summer Institute, Banff, Alberta, 1981, edited by A.Z. Capri, A.N. Kamal (Plenum, New York, 1983) p. 143.
53. T. Huang, in Proceedings of the XXth International Conference on High Energy Physics, Madison, Wisconsin, 1980, edited by L. Durand, L.G. Pondrom, AIP Conference Proceedings, Vol. 69 (AIP, New York, 1981) p. 1000.
54. D.J. Gross, F. Wilczek, Phys. Rev. D 9, 980 (1974).
55. M. Shifman, M.I. Vysotsky, Nucl. Phys. B 186, 475 (1981).
56. P. Colangelo, A. Khodjamirian, At the Frontier of Particle Physics Handbook of QCD, edited by M. Shifman (World Scientific, 2001).