Evaluating decay Rates and Asymmetries of $\Lambda_b$ into Light Baryons in LFQM

Zheng-Tao Wei$^1$, Hong-Wei Ke$^2*$ and Xue-Qian Li$^1$

1. School of Physics, Nankai University, Tianjin, 300071, P.R. China,
2. School of Sciences, Tianjin University, Tianjin, 300072, P.R. China

Abstract

In this work we calculate the branching ratios of semi-leptonic and non-leptonic decays of $\Lambda_b$ into light baryons ($p$ and $\Lambda$), as well as the measurable asymmetries which appear in the processes, in the light front quark model (LFQM). In the calculation, we adopt the diquark picture and discuss the justifiability of applying the picture in our case. Our result on the branching ratio of $\Lambda_b \rightarrow \Lambda + J/\psi$ is in good agreement with data. More predictions are made in the same model and the results will be tested in the future experiments which will be conducted at LHCb and even ILC.

PACS numbers: 12.39.Ki, 14.20.Mr

* Corresponding author, email: khw020056@hotmail.com
I. INTRODUCTION

As well known, the $\Lambda_b$ weak decay may give us abundant information about CKM elements, so that it stands as a complementary field to the meson decays. These processes are also good probes for the factorization hypothesis which has been extensively explored for dealing with hadronic transitions [1, 2]. Recently many semi-leptonic and non-leptonic decays of $\Lambda_b$ are observed and measured [3, 4], moreover the LHCb is expected to accumulate a large data-sample of b-hadrons to offer a unique opportunity for studying $\Lambda_b$, thus we would like to investigate the $\Lambda_b$ weak decay more systematically. As for the $\Lambda_b$ decays the key is how to evaluate the form factors which parameterize the hadronic matrix elements. There are many approaches advocated to this aspect [5]. In our previous paper [6] we studied $\Lambda_b$ to $\Lambda_c$ weak decay in the light-front quark model [7] and the results seem to be quite reasonable.

The light-front quark model is a relativistic quark model based on the light-front QCD [7]. The basic ingredient is the hadron light-front wave function which is explicitly Lorentz-invariant. The hadron spin is constructed using the Melosh rotation. The light-front approach has been widely applied to calculate various decay constants and form factors for the meson cases [8, 9, 10, 11, 12].

In our earlier work, we adopted the diquark picture for baryons [6] which especially is well explored and proved to be a good approximation for such processes where the diquarks are not broken during the transition. Indeed, it has been known for a long time that two quarks in a color-antitriplet state attract each other and may form a correlated diquark [13]. The diquark picture of baryons is considered to be appropriate for low momentum-transfer processes [14, 15, 16, 17]. Concretely, under the diquark approximation, $\Lambda_b$ and $\Lambda_c$ are of the one-heavy-quark-one-light-diquark (ud) structure which is analogous to the meson case.

In this paper we will apply these method to $\Lambda_b$ decaying into light hadrons such as proton or $\Lambda$ which is made of three light quarks. These hadrons may also be regarded to possess quark-diquark structure [14].

Some authors [18, 19, 20, 21] calculated the form factors of $\Lambda_b$ decaying into light baryons and the corresponding decay rates. The Ref. [20] explored $\Lambda_b \to p\ell\nu\bar{\nu}$ by using the method of PQCD and they concluded the perturbative analysis is reliable only for $\rho (\equiv \frac{p_{\perp}^2}{M_{\Lambda_b}^2}) > 0.8$. In Ref. [21] the branching ratio of $\Lambda_b \to J/\Psi \Lambda$ in PQCD was evaluated $((1.7 \sim 5.3) \times 10^{-4})$ [18], instead, Cheng used the nonrelativistic quark model to obtain this branching ratio as $1.1 \times 10^{-4}$ which is lower than the experimental value $((4.7 \pm 2.8) \times 10^{-4})$. In a recent study, the authors of [22] used the light-cone sum rules to calculate the $\Lambda_b \to p(\Lambda)$ transition form factors.

In this work, we study the form factors of $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$ in the light-front model with the diquark picture, and then we calculate the rates of $\Lambda_b \to p\pi$, $\Lambda_b \to J/\Psi \Lambda$, as well as several other non-leptonic decays of $\Lambda_b$.

When $\Lambda_b$ decays into light baryons, the energy of the light baryon in the $\Lambda_b$ rest frame is $E = (M_{\Lambda_b}^2 + m^2 - q^2)/(2M_{\Lambda_b})$ which is much larger than its mass $m$ and the hadronic scale $\Lambda_{QCD}$. One important feature of this region is that the light hadrons move nearly along the light cone. It is argued in [23] that the active quark created from b quark by weak interaction
carries most of the energy of the final light baryon. Under the large energy limit (LEET [24]) and heavy quark limit (HQET [25]) we can obtain the relations between $f_3, g_3$ and $f_2, g_2$, which may help to achieve the orders of $f_3, g_3$. We write up these relations in section II, and then derive the form factors $(f_1, f_2, g_1$ and $g_2)$ of $\Lambda_b \rightarrow p$ and $\Lambda_b \rightarrow \Lambda$ in section III. We carry out the numerical computations in section IV. Finally, section V is devoted to discussions by which we will draw our conclusion.

II. FORMULATION

A. The form factors in the large energy limit

The form factors for the weak transition $\Lambda_b \rightarrow H$ where $H$ represents a light baryon (refers to $p, \Lambda$ in this study), are defined in the standard way as

$$M_\mu = \langle H(P', S', S'_z) | \bar{q} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(P, S, S_z) \rangle$$

$$= \bar{u}_H(P', S') \left[ \gamma_\mu f_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M_{\Lambda_b}} f_2(q^2) + \frac{q_\mu}{M_{\Lambda_b}} f_3(q^2) \right] u_{\Lambda_b}(P, S_z)$$

$$- \bar{u}_H(P', S') \left[ \gamma_\mu g_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M_{\Lambda_b}} g_2(q^2) + \frac{q_\mu}{M_{\Lambda_b}} g_3(q^2) \right] \gamma_5 u_{\Lambda_b}(P, S_z),$$

(1)

where $q \equiv P - P'$, $Q$ and $Q'$ denote heavy quark and light quark, $H$ stands as the light baryon, respectively. The above formulation is the most general expression with only constraints of enforcing the Lorentz invariance and parity conservation for strong interaction. There are six form factors $f_i, g_i$ ($i=1,2,3$) in total for the vector and axial vector current $\bar{q} \gamma_\mu (1 - \gamma_5) b$ where the light-quark $q$ denotes $u$ for $p$ and $s$ for $\Lambda$. All the information about the strong interaction is involved in those form factors. Since $S = S' = 1/2$, we will be able to write $| \Lambda_b(P, S, S_z) \rangle$ as $| \Lambda_b(P, S_z) \rangle$ and similarly for $\bar{u}_H(P', S'_z)$ in the following formulations.

Another parametrization in terms of the four-velocities is widely used and is found to be convenient for the heavy-to-heavy transitions, such as $\Lambda_b \rightarrow \Lambda_c$. But for the heavy-to-light transitions at the large recoil region where the energy of final light baryon $H$ is much larger than its mass, it is more convenient to use following formulation. Analogous to heavy quark symmetry in heavy-to-heavy case, there is a large energy symmetry relations for the heavy-to-light at large energy recoil [23]. For the heavy-to-light baryon transition, the symmetry has not been searched up until present. In this subsection, we explore the large-energy symmetry and show that they lead to a simplification of the form factors: the six form factors are reduced to three independent ones.

Let us introduce the velocity $v$ of initial $\Lambda_b$ and a light front unit vector $n$ by

$$v = \frac{P}{M_{\Lambda_b}}, \quad n = \frac{P'}{E},$$

(2)

where $E$ is the energy of $H$. Using these vectors, the amplitude of the weak transition $\Lambda_b \rightarrow H$ is parameterized by

$$M_\mu = \langle H(n, S'_z) | \bar{q} \gamma_\mu (1 - \gamma_5) b | \Lambda_Q(v, S_z) \rangle$$
\[ = \bar{u}_H(n, S_z') [F_1(E)\gamma_\mu + F_2(E)v_\mu + F_3(E)n_\mu] u_{\Lambda_b}(v, S_z) - \bar{u}_H(n, S_z') [G_1(E)\gamma_\mu + G_2(E)v_\mu + G_3(E)n_\mu] \gamma_5 u_{\Lambda_b}(v, S_z). \]  

(3)

Up to leading order in \(1/M_{\Lambda_b}\) the relation between the two parametrization schemes is

\[ f_1 = F_1 + \frac{1}{2}(\frac{F_2}{M_{\Lambda_b}} + \frac{F_3}{E})M_{\Lambda_b}; \quad g_1 = G_1 - \frac{1}{2}(\frac{G_2}{M_{\Lambda_b}} + \frac{G_3}{E})M_{\Lambda_b}; \]
\[ f_2 = \frac{1}{2}(\frac{F_2}{M_{\Lambda_b}} + \frac{F_3}{E})M_{\Lambda_b}; \quad g_2 = \frac{1}{2}(\frac{G_2}{M_{\Lambda_b}} + \frac{G_3}{E})M_{\Lambda_b}; \]
\[ f_3 = \frac{1}{2}(\frac{F_2}{M_{\Lambda_b}} - \frac{F_3}{E})M_{\Lambda_b}; \quad g_3 = \frac{1}{2}(\frac{G_2}{M_{\Lambda_b}} - \frac{G_3}{E})M_{\Lambda_b}. \]  

(4)

where \(M_{\Lambda_b}\) is the mass of \(\Lambda_b\). We have neglected the mass of final light baryon compared to \(M_{\Lambda_b}\).

Under the large energy limit, the light energetic quark \(q\) is described by the two-component spinor \(\xi = \frac{\gamma^\dagger}{\bar{n}} q\) where \(\bar{n} = 2 - n\) is another light front unit vector and the heavy quark is replaced by \(h_v = e^{i\omega_{0,1,2}(1+\gamma_5)} b\). The weak current \(\bar{q}\Gamma b\) in the full QCD is matched onto the current \(\bar{\xi}\Gamma h_v\) in the effective theory at tree level. For an arbitrary matrix \(\Gamma, \bar{\xi}\Gamma h_v\) has only three independent Dirac structures. One convenient choice is discussed in \([20]\): \(\bar{\xi}h_v, \bar{\xi}\gamma_5 h_v\) and \(\bar{\xi}\gamma_5^\mu h_v\). Thus, we have

\[ \bar{q}\gamma^\mu b = \bar{\xi}\gamma_5^\mu h_v + n^\mu \bar{\xi}h_v, \]
\[ \bar{q}\gamma^\mu_5 b = i\epsilon_{\mu_5}^{\mu} \bar{\xi}\gamma_5^\mu h_v - n^\mu \bar{\xi}\gamma_5 h_v. \]  

(5)

where \(\epsilon_{\mu_5}^{\mu} = \epsilon^{\mu\alpha\beta\gamma} n_\alpha n_\beta\).

The three independent form factors are defined by

\[ \langle H(P', S_z') | \bar{\xi}h_v | \Lambda_b(P, S_z) \rangle = \bar{u}_H(n, S_z) u_{\Lambda_b}(v, S_z) \zeta_0(E), \]
\[ \langle H(P', S_z') | \bar{\xi}\gamma_5 h_v | \Lambda_b(P, S_z) \rangle = \bar{u}_H(n, S_z) \gamma_5 u_{\Lambda_b}(v, S_z) \zeta_5(E), \]
\[ \langle H(P', S_z') | \bar{\xi}\gamma_5^\mu h_v | \Lambda_b(P, S_z) \rangle = \bar{u}_H(n, S_z) \gamma_\mu u_{\Lambda_b}(v, S_z) \zeta_\mu(E). \]  

(6)

Then, we find

\[ F_1 = G_1 = \zeta_\mu(E); \quad F_2 = G_2 = 0; \]
\[ F_3 = \zeta_0(E) - \zeta_\mu(E); \quad G_3 = \zeta_\mu(E) - \zeta_5(E). \]  

(7)

From the above equation, we obtain the relations among the form factors:

\[ f_1 + f_2 = g_1 - g_2; \quad f_2 = -f_3; \quad g_2 = -g_3. \]  

(8)

This is one major result in this work. The \(f_3\) and \(g_3\) are not independent, but related to \(f_2\) and \(g_2\).
B. Vertex function in the light-front approach

In the diquark picture, the heavy baryon $\Lambda_b$ is composed of one heavy quark $b$ and a light diquark $[ud]$. In order to form a color singlet hadron, the diquark $[ud]$ is in a color anti-triplet. Because $\Lambda_b$ is at the ground state, the diquark is a $0^+$ scalar $(s = 0, l = 0)$ and the orbital angular momentum between the diquark and the heavy quark is also zero, i.e. $L = l = 0$. However the situation is complicated for light baryon even thought it is in the ground state. The diquark in light baryon may be a $0^+$ scalar or a $1^-$ vector. Fortunately the diquark is a spectator in the concerned transition and its spin is not affected so that only the scalar diquark can transit into the final baryon and one only needs to consider the scalar diquark structure of the light baryon.

In the light-front approach, the heavy baryon $\Lambda_Q$ composed of only scalar diquark with total momentum $P$ and spin $S = 1/2$ can be written as

$$|\Lambda_Q(P, S, S_z)⟩ = ∫\{d^3p_1\}\{d^3p_2\} 2(2\pi)^3δ^3(\vec{P} - \vec{p}_1 - \vec{p}_2)$$
$$× ∑_{λ_1}Ψ^{S_S_z}(\vec{p}_1, \vec{p}_2, λ_1)C_{αβγ}F^{bc} | Q^α(p_1, λ_1)[q_β q_γ](p_2)⟩, (9)$$

and the light baryon (total momentum $P$, spin $J = 1/2$, composed of $0^+$ scalar diquark and orbital angular momentum $L = 0$) has the similar form,

$$|H(P, S, S_z)⟩ = ∫ d^3p_1d^3p_2 2(2\pi)^3δ^3(\vec{P} - \vec{p}_1 - \vec{p}_2)$$
$$× ∑_{λ_1}Ψ^{S_S_z}(\vec{p}_1, \vec{p}_2, λ_1)C_{αβγ}F^{abc}_L | q_α^β(p_1, λ_1) [q_β q_γ](p_2)⟩, (10)$$

where $Q$, $[q_β q_γ]$ represent heavy quark and diquark respectively and $λ$ denotes the helicity, where $α, β, γ$ and $a, b, c$ are the color and flavor indices, $p_1, p_2$ are the on-mass-shell light-front momenta defined by

$$\vec{p} = (p^+, p_⊥), \quad p_⊥ = (p^1, p^2), \quad p^- = \frac{m^2 + p_⊥^2}{p^+}, \quad (11)$$

and

$$\{d^3p\} ≡ \frac{dp^+ dp_⊥}{2(2\pi)^3}, \quad δ^3(\vec{p}) = δ(p^+)δ^2(p_⊥),$$

$$|Q(p_1, λ_1)[q_β q_γ](p_2)⟩ = b^†_{λ_1}(p_1)a^†(p_2)|0⟩,$$

$$[a(p'), a^†(p)] = 2(2\pi)^3δ^3(\vec{p'} - \vec{p}),$$

$$\{d_{λ'}(p'), d^†_{λ}(p)\} = 2(2\pi)^3δ^3(\vec{p'} - \vec{p})δ_{λ,λ'}.$$ (12)

The coefficient $C_{αβγ}$ is a normalized color factor and $F^{abc}(F^{abc})$ is a normalized flavor coefficient,

$$C_{αβγ}F^{bc}C_{α′β′γ′}F^{b′c′} Q^α(p', λ′_1) [q_β q_γ](p'_2) | Q^α(p, λ_1)[q_β q_γ](p_2)⟩ = 2^2(2\pi)^6δ^3(\vec{p}_1 - \vec{p}_1)δ^3(\vec{p}_2 - \vec{p}_2)δ_{λ_1,λ},$$

$$C_{αβγ}F^{abc}C_{α′β′γ′}F^{a′b′c′} q_α^β(p', λ′_1) [q_β q_γ](p'_2) | q_α^β(p, λ_1)[q_β q_γ](p_2)⟩ = 2^2(2\pi)^6δ^3(\vec{p}_1 - \vec{p}_1)δ^3(\vec{p}_2 - \vec{p}_2)δ_{λ_1,λ}. \quad (13)$$
In order to describe the motion of the constituents, one needs to introduce intrinsic variables \( (x_i, k_{i\perp}) \) with \( i = 1, 2 \) through

\[
\begin{align*}
p_1^+ &= x_1 P^+, & \quad p_2^+ &= x_2 P^+, & \quad x_1 + x_2 &= 1, \\
p_{1\perp} &= x_1 P_\perp + k_{1\perp}, & \quad p_{2\perp} &= x_2 P_\perp + k_{2\perp}, & \quad k_{\perp} &= -k_{1\perp} = k_{2\perp},
\end{align*}
\tag{14}
\]

where \( x_i \)'s are the light-front momentum fractions satisfying \( 0 < x_1, x_2 < 1 \). The variables \( (x_i, k_{i\perp}) \) are independent of the total momentum of the hadron and thus are Lorentz-invariant. The invariant mass square \( M_0^2 \) is defined as

\[
M_0^2 = \frac{k_{1\perp}^2 + m_1^2}{x_1} + \frac{k_{2\perp}^2 + m_2^2}{x_2}.
\tag{15}
\]

The invariant mass \( M_0 \) is in general different from the hadron mass \( M \) which satisfies the physical mass-shell condition \( M^2 = P^2 \). This is due to the fact that in the baryon, heavy quark and diquark cannot be on their mass shells simultaneously. We define the internal momenta as

\[
k_i = (k_i^-, k_i^+, k_{i\perp}) = (e_i - k_{iz}, e_i + k_{iz}, k_{i\perp}) = \left( \frac{m_i^2 + k_{i\perp}^2}{x_iM_0}, x_iM_0, k_{i\perp} \right) \tag{16}.
\]

It is easy to obtain

\[
\begin{align*}
M_0 &= e_1 + e_2, \\
e_i &= \frac{x_iM_0}{2} + \frac{m_i^2 + k_{i\perp}^2}{2x_iM_0} = \sqrt{m_i^2 + k_{i\perp}^2}, \\
k_{iz} &= \frac{x_iM_0}{2} - \frac{m_i^2 + k_{i\perp}^2}{2x_iM_0},
\end{align*}
\tag{17}
\]

where \( e_i \) denotes the energy of the \( i \)-th constituent. The momenta \( k_{i\perp} \) and \( k_{iz} \) constitute a momentum vector \( \vec{k}_i = (k_{i\perp}, k_{iz}) \) and correspond to the components in the transverse and \( z \) directions, respectively.

In the momentum space, the function \( \Psi^{SS_i} \) appearing in Eq. (9) is expressed as

\[
\Psi^{SS_i}(\vec{p}_1, \vec{p}_2, \lambda_1) = \left\langle \lambda_1 \left| \mathcal{R}_M^1(x_1, k_{1\perp}, m_1) \right| s_1 \right\rangle \left\langle 00; \frac{1}{2}s_1 \left| \frac{1}{2}S_z \right\rangle \phi(x, k_{\perp}) \right.,
\tag{18}
\]

where \( \phi(x, k_{\perp}) \) is the light-front wave function which describes the momentum distribution of the constituents in the bound state with \( x = x_2, k_{\perp} = k_{2\perp} \); and \( \left\langle 00; \frac{1}{2}s_1 \left| \frac{1}{2}S_z \right\rangle \) is the corresponding Clebsch-Gordan coefficient with total spin of the scalar diquark \( s = s_z = 0; \left\langle \lambda_1 \left| \mathcal{R}_M^1(x_1, k_{1\perp}, m_1) \right| s_1 \right\rangle \) is the well-known Melosh transformation matrix element which transforms the the conventional spin states in the instant form into the light-front helicity eigenstates,

\[
\left\langle \lambda_1 \left| \mathcal{R}_M^1(x_1, k_{1\perp}, m_1) \right| s_1 \right\rangle = \frac{\bar{u}(k_1, \lambda_1)u_D(k_1, s_1)}{2m_1} = \frac{(m_1 + x_1M_0)\delta_{\lambda\lambda s_1} + i\sigma_{\lambda\lambda s_1} \cdot \vec{k}_{1\perp} \times \vec{n}}{\sqrt{(m_1 + x_1M_0)^2 + k_{1\perp}^2}},
\tag{19}
\]
where $u_{(D)}$ denotes a Dirac spinor in the light-front (instant) form and $\vec{n} = (0, 0, 1)$ is a unit vector in the $z$ direction. In practice, it is more convenient to use the covariant form for the Melosh transform matrix \[8, 11\]

$$
\langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \langle 00; \frac{1}{2}s_1 | \frac{1}{2}S_z \rangle = \frac{1}{\sqrt{2(p_1 \cdot \vec{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1) \Gamma u(\vec{P}, S_z),
$$

(20)

where

$$
\Gamma = 1, \quad \bar{P} = p_1 + p_2.
$$

(21)

for the scalar diquark. If the diquark is a vector which is usually supposed to be the case for the $\Sigma_{c(b)}$ baryon, the Melosh transform matrix should be modified (since it is irrelevant to our present work, we omit the corresponding expressions).

The baryon state is normalized as

$$
\langle \Lambda(P', S', S_z') | \Lambda(P, S, S_z) \rangle = 2(2\pi)^3 P^+ \delta^3(\vec{P}' - \vec{P}) \delta_{S'S} \delta_{S_z'S_z},
$$

(22)

the same for $H(P, S, S_z)$.

Thus, the light-front wave function obeys the constraint

$$
\int \frac{dx d^2 k_{\perp}}{2(2\pi^3)} |\phi(x, k_{\perp})|^2 = 1.
$$

(23)

In principle, the wave functions can be obtained by solving the light-front bound state equations. However, it is too hard to calculate them based on the first principle, so that instead, we would like to adopt a phenomenological function, and obviously, a Gaussian form is most preferable,

$$
\phi(x, k_{\perp}) = N \sqrt{\frac{\partial k_{2z}}{\partial x_2}} \exp \left( \frac{-\vec{k}_{\perp}^2}{2\beta^2} \right),
$$

(24)

with

$$
N = 4 \left( \frac{\pi}{\beta^2} \right)^{3/4}, \quad \frac{\partial k_{2z}}{\partial x_2} = \frac{e_1 e_2}{x_1 x_2 M_0}.
$$

(25)

where $\beta$ determines the confinement scale. The phenomenological parameters in the light-front quark model are quark masses and the hadron wave function parameter $\beta$ which should be prior determined before numerical computations can be carried out and we will do the job in the later subsections.

C. $\Lambda_Q \rightarrow H$ weak transitions

Equipped with the light-front quark model description of $| \Lambda_Q(P, S_z) >$ and $| H(P, S_z) >$, we can calculate the weak transition matrix elements

$$
< \Lambda_Q(P', S_z') | \bar{q} \gamma_\mu (1 - \gamma_5) Q | H(P, S_z) >
$$
only the three-momentum rather than the four-momentum is conserved, hence \( \tilde{\psi} \) spinors which make computations more complex. In this framework, at each effective vertex, which are consistent with [18], and 

\[
P = \begin{pmatrix} N & b \\ f & u \end{pmatrix}, \quad x = x', k'_1 = k_1 + x_2 q_1.
\]

where \( N_{IF} \) is a flavor-spin factor of \( I \) (initial particle) decaying into \( F \) (final particle). Following [14], the flavor-spin functions of \( \Lambda_b \), proton and \( \Lambda \) take the forms in the diquark picture

\[
\begin{align*}
\chi_{S^b}^{\Lambda} &= b S_{[u,d]}, \\
\chi_{S}^{p} &= u S_{[u,d]}, \quad \chi_{S}^{V} = [u V_{[u,d]} - \sqrt{2} d V_{[u,d]}/\sqrt{3} \\
\chi_{S}^{A} &= [u S_{[d,s]} - d S_{[u,d]} - 2 s S_{[u,d]}]/\sqrt{6}, \quad \chi_{V}^{A} = [u V_{[d,s]} - d V_{[u,s]}]/\sqrt{2}
\end{align*}
\]

(27)

where \( S \) and \( V \) denote scalar and axial vector diquark. We can get \( N_{\Lambda b p} = \frac{1}{\sqrt{2}}, N_{\Lambda b A} = \frac{1}{\sqrt{2}}, \) which are consistent with [18], and

\[
\Gamma' = \gamma_0 \Gamma \gamma_0 = \Gamma = 1, \\
m_1 = m_b, \quad m'_1 = m_q, \quad m_2 = m_{[u,d]}.
\]

with \( P \) and \( P' \) denoting the momenta of initial and final baryons, \( p_1, p'_1 \) are the momenta of \( b \) and \( c \) quarks, respectively. Because the diquark is a scalar, one does not need to deal with the spinors which make computations more complex. In this framework, at each effective vertex, only the three-momentum rather than the four-momentum is conserved, hence \( \tilde{p}_1 - \tilde{p}'_1 = \tilde{q} \) and \( \tilde{p}_2 = \tilde{p}'_2 \). From \( \tilde{p}_2 = \tilde{p}'_2 \), we have

\[
x' = \frac{P^+}{P^+ + x}, \quad k'_1 = k_1 + x_2 q_1.
\]

(29)

with \( x = x_2, x' = x'_2 \). Thus, Eq. [26] is rewritten as

\[
\langle H(P', S'_2) | \tilde{q} \gamma^\mu (1 - \gamma_5) Q | \Lambda_Q(P, S_2) \rangle
\]

\[
= N_{IF} \int \frac{d^2 k_1}{(2\pi)^2} \frac{\phi_H(x', k'_1) \phi_{\Lambda_Q}(x, k_1)}{2 \sqrt{x_1 x'_1 (p_1 \cdot P + m_1 M_0)(p'_1 \cdot P' + m'_1 M'_0)}} \times \bar{u}(\tilde{P}', S'_2)(\tilde{p}'_1 + m'_1) \gamma^\mu (1 - \gamma_5)(\tilde{p}_1 + m_1) u(\tilde{P}, S). \quad (30)
\]

Following [6, 27], we get the the final expressions for the \( \Lambda_Q \rightarrow H \) weak transition form factors

\[
f_1(q^2) = N_{IF} \int \frac{d^2 k_1}{(2\pi)^2} \frac{\phi_H(x', k'_1) \phi_{\Lambda_Q}(x, k_1)}{2 \sqrt{x_1 x'_1 (m_1 + x_1 M_0)^2 + k_2^2} \ [m'_1 + x'_1 M'_0 + m'_1]^2},
\]

\[
g_1(q^2) = N_{IF} \int \frac{d^2 k_1}{(2\pi)^2} \frac{\phi_H(x', k'_1) \phi_{\Lambda_Q}(x, k_1)}{2 \sqrt{x_1 x'_1 (m_1 + x_1 M_0)^2 + k_2^2} \ [m'_1 + x'_1 M'_0 + m'_1]^2},
\]

\[
f_2(q^2) = \frac{N_{IF}}{q'_1} \int \frac{d^2 k_1}{(2\pi)^2} \frac{\phi_H(x', k'_1) \phi_{\Lambda_Q}(x, k_1)}{2 \sqrt{x_1 x'_1 (m_1 + x_1 M_0)^2 + k_2^2} \ [m'_1 + x'_1 M'_0 + m'_1]^2},
\]

\[
M_{\Lambda_Q} = \frac{1}{q'_1} \int \frac{d^2 k_1}{(2\pi)^2} \frac{\phi_H(x', k'_1) \phi_{\Lambda_Q}(x, k_1)}{2 \sqrt{x_1 x'_1 (m_1 + x_1 M_0)^2 + k_2^2} \ [m'_1 + x'_1 M'_0 + m'_1]^2}.
\]

8
\[
\frac{g_2(q^2)}{M_{\Lambda_Q}} = \frac{N_{IF}}{q_{\perp}^2} \int \frac{dxd^2k_{\perp} \phi_H(x', k_{\perp}') \phi_{\Lambda_Q}(x, k_{\perp})[(m_1 + x_1M_0)k'_{\perp} + (m'_1 + x'_1M'_0)k_{\perp}]}{2(2\pi)^3 \sqrt{[(m_1 + x_1M_0)^2 + k^2_{\perp}][k'_{\perp}^2 + (m'_1 + x'_1M'_0)^2 + k'^2_{\perp}]}},
\]

(31)

It is noted that the form factors \(f_3\) and \(g_3\) cannot be extracted in our method because we have imposed the condition \(q^+ = 0\). The fact that the calculated \(f_2\) and \(g_2\) at \(q^2 = 0\) are small compared to \(f_1\) and \(g_1\) and the large energy limit relations \(f_3 = -f_2\) and \(g_3 = -g_2\) show that using the large energy limit relations for \(f_3\) and \(g_3\) does not produce substantial theoretical errors.

III. SEMI-LEPTONIC AND NON-LEPTONIC DECAYS OF TRANSITION \(\Lambda_b \to \) LIGHT HADRONS

In this section, we obtain formulations for the rates of semi-leptonic and non-leptonic processes. In this work, we concern only the exclusive decay modes.

A. Semi-leptonic decays of \(\Lambda_b \to pl\bar{\nu}_l\)

Generally the polarization effects may be important for testing different theoretical models, so that we would pay more attention to the physical consequences brought up by them. The transition amplitude of \(\Lambda_b \to p\) contains several independent helicity components. According to the definitions of the form factors for \(\Lambda_b \to p\) given in Eq. (1), the helicity amplitudes \(H_{i,j}^V\) are related to these form factors through the following expressions [28]

\[
H_{\frac{3}{2},0}^V = \frac{\sqrt{Q^-}}{\sqrt{q^2}} \left((M_{\Lambda_b} + M_{\Lambda_c})f_1 - \frac{q^2}{M_{\Lambda_b}}f_2\right),
\]

\[
H_{\frac{3}{2},1}^V = \sqrt{2Q^-} \left(-f_1 + \frac{M_{\Lambda_b} + M_{\Lambda_c}}{M_{\Lambda_b}}f_2\right),
\]

\[
H_{\frac{1}{2},0}^A = \frac{\sqrt{Q^+}}{\sqrt{q^2}} \left((M_{\Lambda_b} - M_{\Lambda_c})g_1 + \frac{q^2}{M_{\Lambda_b}}g_2\right),
\]

\[
H_{\frac{1}{2},1}^A = \sqrt{2Q^+} \left(-g_1 - \frac{M_{\Lambda_b} - M_{\Lambda_c}}{M_{\Lambda_b}}g_2\right),
\]

(32)

where \(Q_{\pm} = 2(P \cdot P' \pm M_{\Lambda_b}M_p) = 2M_{\Lambda_b}M_p(\omega \pm 1)\).

The helicities of the \(W\)-boson \(\lambda_W\) can be either 0 or 1, corresponding to the longitudinal and transverse polarizations. Following the definitions in literature, we decompose the decay width into a sum of the longitudinal and transverse parts according to the helicity states of the virtual \(W\)-boson. The differential decay rate of \(\Lambda_b \to pl\bar{\nu}_l\) is

\[
\frac{d\Gamma}{d\omega} = \frac{d\Gamma_L}{d\omega} + \frac{d\Gamma_T}{d\omega},
\]

(33)

9
and the longitudinally (L) and transversely (T) polarized rates are respectively \[^28\]

\[
\frac{d\Gamma_L}{d\omega} = \frac{G_F^2 |V_{ub}|^2}{(2\pi)^3} \frac{q^2 p_c M_p}{12M_{\Lambda_b}} \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right],
\]

\[
\frac{d\Gamma_T}{d\omega} = \frac{G_F^2 |V_{ub}|^2}{(2\pi)^3} \frac{q^2 p_c M_p}{12M_{\Lambda_b}} \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right].
\]

(34)

where \(p_c = M_p \sqrt{\omega^2 - 1}\) is the momentum of the proton in the rest frame of \(\Lambda_b\). The relations between \(H_{i,j}\) and \(H^V_{i,j}\) can be found in \[^28\]. Integrating over the solid angle, we obtain the decay rate as

\[
\Gamma = \int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \frac{d\Gamma}{d\omega},
\]

(35)

where the upper bound of the integration \(\omega_{\text{max}} = \frac{1}{2} \left( \frac{M_{\Lambda_b}}{M_p} + \frac{M_p}{M_{\Lambda_b}} \right)\) corresponds to the maximal recoil. In order to compare our results with those in the literatures, we use the variable \(\omega\) in the expression for the differential decay rate.

The polarization of the cascade decay \(\Lambda_b \rightarrow p + W(\rightarrow l\nu)\) is expressed by various asymmetry parameters \[^28, 29\]. Among them, the integrated longitudinal and transverse asymmetries are defined by

\[
a_L = \frac{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right]}{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right]},
\]

\[
a_T = \frac{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]}{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]}.
\]

(36)

The ratio of the longitudinal to transverse decay rates \(R\) is defined by

\[
R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right]}{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]},
\]

(37)

and the longitudinal proton polarization asymmetry \(P_L\) is given as

\[
P_L = \frac{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 \right]}{\int_{\omega_{\text{max}}}^{\omega_{\text{max}}} d\omega \; q^2 p_c \left[ |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]} = \frac{a_T + R a_L}{1 + R}.
\]

(38)

B. Non-leptonic decay of \(\Lambda_b \rightarrow p + M\)

From the theoretical aspects, the non-leptonic decays are much more complicated than the semi-leptonic ones because of the strong interaction. Generally, the present theoretical framework is based on the factorization assumption, where the hadronic matrix element is
factorized into a product of two matrix elements of single currents. One can be written as a decay constant while the other is expressed in terms of a few form factors according to the lorentz structure of the current. For the weak decays of mesons, such factorization approach is verified to work very well for the color-allowed processes and the non-factorizable contributions are negligible.

For the non-leptonic decays $\Lambda_b^0 \rightarrow p + M$, the effective interaction at the quark level is $b \rightarrow u \bar{q}_1 q_2$. The relevant Hamiltonian is

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{ub} V_{q_1 q_2}^* (c_1 O_1 + c_2 O_2),$$

where $c_i$ denotes the short-distance Wilson coefficient, $V_{ub} (V_{q_1 q_2})$ is the CKM matrix elements, $q_1$ stands for $u$ and $q_2$ for $d$ in the context. Then one needs to evaluate the hadronic matrix elements

$$\langle p M | \mathcal{H}_W | \Lambda_b \rangle = \frac{G_F}{\sqrt{2}} V_{ub} V_{q_1 q_2}^* \sum_{i=1,2} c_i \langle p M | O_i | \Lambda_b \rangle.$$  

(39)

Under the factorization approximation, the hadronic matrix element is reduced to

$$\langle p M | O_i | \Lambda_b \rangle = \langle p | J_\mu | \Lambda_b \rangle \langle M | J'^\mu | 0 \rangle.$$  

(40)

where $J(J')$ is the $V-A$ weak current. The first factor $\langle p | J_\mu | \Lambda_b \rangle$ is parameterized by six form factors as done in Eq. (39). The second factor defines the decay constants as follows

$$\langle P(P) | A_\mu | 0 \rangle = f_P P_\mu,$$

$$\langle S(P) | V_\mu | 0 \rangle = f_S P_\mu,$$

$$\langle V(P, \epsilon^\mu) | V_\mu | 0 \rangle = f_V M_V \epsilon^\mu,$$

$$\langle A(P, \epsilon^\mu) | A_\mu | 0 \rangle = f_A M_A \epsilon^\mu,$$

(42)

where $P(V)$ denotes a pseudoscalar (vector) meson, and $S(A)$ denotes a scalar (axial-vector) meson. In the definitions, we omit a factor $(-i)$ for the pseudoscalar meson decay constant.

In general, the transition amplitude of $\Lambda_b \rightarrow p \pi^-$ can be written as

$$\mathcal{M}(\Lambda_b \rightarrow p P) = \bar{u}_p (A + B \gamma_5) u_{\Lambda_b},$$

$$\mathcal{M}(\Lambda_b \rightarrow p V) = \bar{u}_p \epsilon^\mu [A_1 \gamma_\mu \gamma_5 + A_2 (p_{\Lambda_b})_\mu \gamma_5 + B_1 \gamma_\mu + B_2 (p_{\Lambda_b})_\mu] u_{\Lambda_b},$$

(43)

where $\epsilon^\mu$ is the polarization vector of the final vector or axial-vector mesons. Including the effective Wilson coefficient $a_1 = c_1 + c_2/N_c$, the decay amplitudes under the factorization approximation are

$$A = \lambda f_P (M_{\Lambda_b} - M_{\Lambda_c}) f_1(M^2),$$

$$B = \lambda f_P (M_{\Lambda_b} + M_{\Lambda_c}) g_1(M^2),$$

where $f(P)$ and $f(V)$ denote the form factors.
\[ A_1 = -\lambda f_V M \left[ g_1(M^2) + g_2(M^2) \frac{M_{\Lambda_b} - M_{\Lambda_c}}{M_{\Lambda_b}} \right], \]
\[ A_2 = -2\lambda f_V M \frac{g_2(M^2)}{M_{\Lambda_b}}, \]
\[ B_1 = \lambda f_V M \left[ f_1(M^2) - f_2(M^2) \frac{M_{\Lambda_b} + M_{\Lambda_c}}{M_{\Lambda_b}} \right], \]
\[ B_2 = 2\lambda f_V M \frac{f_2(M^2)}{M_{\Lambda_b}}, \] (44)

where \( \lambda = \frac{G_F}{\sqrt{2}} V_{ub} V_{q_1q_2}^* a_1 \) and \( M \) is the \( \pi \) mass. Replacing \( P, V \) by \( S \) and \( A \) in the above expressions, one can easily obtain similar expressions for scalar and axial-vector mesons.

The decay rates of \( \Lambda_b \to p\pi^- \) and up-down asymmetries are [31]
\[ \Gamma = \frac{p_c}{8\pi} \left[ \frac{(M_{\Lambda_b} + M_p)^2 - M^2}{M_{\Lambda_b}^2} |A|^2 + \frac{(M_{\Lambda_b} - M_p)^2 - M^2}{M_{\Lambda_b}^2} |B|^2 \right], \]
\[ \alpha = -\frac{2\kappa \text{Re}(A^*B)}{|A|^2 + \kappa^2 |B|^2}, \] (45)

where \( p_c \) is the proton momentum in the rest frame of \( \Lambda_b \) and \( \kappa = \frac{p_c}{E_p + M_p} \). For \( \Lambda_b \to \Lambda_c V(A) \) decays, the decay rates and up-down asymmetries are
\[ \Gamma = \frac{p_c(E_p + M_p)}{8\pi M_{\Lambda_b}} \left[ 2 \left(|S|^2 + |P_2|^2\right) + \frac{E^2}{M^2} \left(|S + D|^2 + |P_1|^2\right) \right], \]
\[ \alpha = \frac{4M^2 \text{Re}(S^*P_2) + 2E^2 \text{Re}(S + D)^*P_1}{2M^2 \left(|S|^2 + |P_2|^2\right) + E^2 \left(|S + D|^2 + |P_1|^2\right)}, \] (46)

where \( E \) is the energy of the vector (axial vector) meson, and
\[ S = -A_1, \]
\[ P_1 = -\frac{p_c}{E} \left( \frac{M_{\Lambda_b} + M_p}{E_p + M_p} B_1 + B_2 \right), \]
\[ P_2 = \frac{p_c}{E_p + M_p} B_1, \]
\[ D = -\frac{p_c^2}{E(E_p + M_p)} (A_1 - A_2). \] (47)

C. Non-leptonic decay \( \Lambda_b \to \Lambda + M \)

Theses decays proceed only via the internal W-emission. With the factorization assumption, the amplitude is
\[ A(\Lambda_b \to \Lambda M) = \frac{G_F}{\sqrt{2}} V_{ub} V_{q_1q_2}^* a_2 \langle M | \bar{q}^* \gamma_\mu (1 - \gamma_5) q | 0 \rangle \langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle. \] (48)
TABLE I: Input parameters in LFQM (in units of GeV).

| $m_b$ | $m_s$ | $m_u$ | $m_{[ud]}$ | $\beta_{u,[ud]}$ | $\beta_{b,[ud]}$ | $\beta_{s,[ud]}$ |
|-------|-------|-------|------------|----------------|----------------|----------------|
| 4.4   | 0.45  | 0.3   | 0.5        | 0.3           | 0.4           | 0.3           |

In general, we can use the same formula (Eqs.(42)-(47)) to obtain the decay rates and up-down asymmetries of $\Lambda_b \to \Lambda + M$. Note that: (1) at this time $\lambda$ is replaced by $\sqrt{2} V_{ub} V_{cq_2}^* a_2$, (2) when $q$ and $\bar{q}'$ are $u$ and $\bar{u}$ respectively, the final meson may be $\pi^0$, $\eta$ or $\eta'$. For the decay constants of $\pi^0$, $\eta$ and $\eta'$, we have

$$
\langle \pi^0 | \bar{u} \gamma_\mu \gamma_5 u | 0 \rangle = f_{\pi^0}^u P_\mu,
\langle \eta | \bar{u} \gamma_\mu \gamma_5 u | 0 \rangle = f_{\eta}^u P_\mu,
\langle \eta' | \bar{u} \gamma_\mu \gamma_5 u | 0 \rangle = f_{\eta'}^u P_\mu,
$$

where $f_{\pi^0}^u = f_{\pi}^u \sqrt{2}$, $f_{\eta}^u$ and $f_{\eta'}^u$ can be get form [1].

IV. NUMERICAL RESULTS

In this section we perform the numerical computations of the form factors for $\Lambda_b \to p$ and $\Lambda_b \to \Lambda$, then using them we estimate the rates of $\Lambda_b \to p + l \nu$, $\Lambda_b \to p + M$ and $\Lambda_b \to \Lambda + M$ where $M$ stands as various mesons.

In our calculation, the quark masses of $m_b$ and $m_s$ are taken from [27]; $m_u$ is set to be 0.3 GeV; the mass of diquark $[ud]$, parameters $\beta_{b,[ud]}$, $\beta_{s,[ud]}$ and $\beta_{u,[ud]}$ are chosen from [6, 11, 27]. The baryon masses $M_{\Lambda_b} = 5.624$ GeV, $M_p = 0.938$ GeV, $\Lambda = 1.116$ GeV come from [4]. The input parameters are collected in Table I.

A. Form factor

In LFQM, the calculation of form factors is performed in the frame $q^+ = 0$ with $q^2 = -q_{\perp}^2 \leq 0$, only the values of the form factors in the space-like region can be obtained. The advantage of this choice is that the so-called Z-graph contribution arising from the non-valence quarks vanishes. In order to obtain the physical form factors, an extrapolation from the space-like region to the time-like region is required. Following [27], the form factors in the space-like region can be parameterized in a three-parameter form as

$$
F(q^2) = \frac{F(0)}{\left(1 - \frac{q^2}{M_{\Lambda_b}^2}\right) \left[1 - a \left(\frac{q^2}{M_{\Lambda_b}^2}\right) + b \left(\frac{q^2}{M_{\Lambda_b}^2}\right)^2\right]},
$$

where $F$ represents the form factor $f_{1,2}$ and $g_{1,2}$. The parameters $a$, $b$ and $F(0)$ are fixed by performing a three-parameter fit to the form factors in the space-like region which were
TABLE II: The $\Lambda_b \to p$ form factors in the three-parameter form.

| $F$ | $F(0)$ | $a$ | $b$  |
|-----|--------|-----|-----|
| $f_1$ | 0.1131 | 1.70 | 1.60 |
| $f_2$ | -0.0356 | 2.50 | 2.57 |
| $g_1$ | 0.1112 | 1.65 | 1.60 |
| $g_2$ | -0.0097 | 2.80 | 2.70 |

TABLE III: The $\Lambda_b \to \Lambda$ form factors in the three-parameter form.

| $F$ | $F(0)$ | $a$ | $b$  |
|-----|--------|-----|-----|
| $f_1$ | 0.1081 | 1.70 | 1.60 |
| $f_2$ | -0.0311 | 2.50 | 2.50 |
| $g_1$ | 0.1065 | 1.70 | 1.40 |
| $g_2$ | -0.0064 | 2.70 | 2.70 |

obtained in previous sections. We then use these parameters to determine the physical form factors in the time-like region. The fitted values of $a$, $b$ and $F(0)$ for different form factors $f_{1,2}$ and $g_{1,2}$ are given in Table II and III. The $q^2$ dependence of the form factors is plotted in Fig. 1.

From Fig. 1 we can see that there is only a tiny difference between $f_1$ and $g_1$, i.e. they are close to each other. $g_2$ is small comparing to $f_1$ and $g_1$. This is the same as the conclusion of [6, 32]. But the difference between $f_2$ and $g_2$ increases as $q^2$ increases. This will break the

![FIG. 1: (a) Form factors $f_1$ and $g_1$ of $\Lambda_b \to p$. (b) Form factors $f_2$ and $g_2$ of $\Lambda_b \to p.$](image-url)
TABLE IV: The $\Lambda_b \to p$ form factors in the the form of Eq.(51).

| $F$ | $r_1$ | $r_2$ | $M_{fit}$ |
|-----|-------|-------|-----------|
| $f_1$ | -0.183 | 0.295 | 6.8       |
| $f_2$ | -0.176 | 0.287 | 6.8       |
| $g_1$ | 0.080  | -0.114| 6.8       |
| $g_2$ | 0.023  | -0.032| 6.8       |

TABLE V: The branching ratios and polarization asymmetries of $\Lambda_b \to pl\bar{\nu}_l$.

| $BR$ | $a_L$ | $a_T$ | $R$ | $P_L$ |
|------|-------|-------|-----|-------|
| $2.54 \times 10^{-4}$ | -0.99 | -0.96 | 1.11 | -0.97 |

large energy limit relation $f_1 + f_2 = g_1 - g_2$ proposed in the Section II A.

Our method of smooth extrapolation of from factors from space- to time-like momentum regions is by no means an analytical continuation in the rigorously mathematical sense but an extension, although it is used in many phenomenological analysis. In [33], the authors suggest to write the form factor as a dispersion relation in $q^2$ with a lowest-lying pole plus a contribution from multiparticle states. We follow this scheme and use a parametrization method adopted in [34]

$$F(q^2) = \frac{r_1}{1 - \frac{q^2}{M_{fit}^2}} + \frac{r_2}{1 - \left(\frac{q^2}{M_{fit}^2}\right)^2}, \quad (51)$$

The parameters $r_1$, $r_2$ and $M_{fit}$ are fixed in the space-like regions for the transition of $\Lambda_b \to p$. The results are presented in Table IV. We also plot the form factors in the new parametrization method in Fig. I for a comparison. From Fig. I we can find there is a little difference between the form factors fitted by the above two methods. In particular, the $f_1$ and $g_1$ in the two methods are nearly the same. The difference of $f_2$ and $g_2$ in the methods increases when $q^2$ increases, but due to smallness of their values, they will not produce substantial errors to our predictions.

B. Semi-leptonic decay of $\Lambda_b \to p + l\bar{\nu}_l$

With the form factors given in the above subsection, we are able to calculate the branching ratio and various asymmetries of $\Lambda_b \to p l\bar{\nu}_l$ decay. Table V presents our numerical predictions. The ratio of longitudinal to transverse rates $R > 1$ implies that the longitudinal polarization dominates.
TABLE VI: Meson decay constants $f$ (in units of MeV) \cite{11, 18}.

| meson | $\pi$ | $\rho$ | $K$ | $K^*$ | $D$ | $D^*$ | $D_s$ | $D_s^*$ | $a_1$ | $J/\psi$ |
|-------|------|-------|-----|------|-----|------|------|--------|------|----------|
| $f$   | 131  | 216   | 160 | 210  | 200 | 220  | 230  | 230    | 203  | 395      |

TABLE VII: Branching ratio and up-down asymmetry for non-leptonic decay $\Lambda^0_b \rightarrow p + M$.

| Branching ratios | Up-down asymmetries | Exp |
|------------------|----------------------|-----|
| $\Lambda^0_b \rightarrow p + \pi^-$ | $3.15 \times 10^{-6}$ | -1  | $(3.5 \pm 0.6(stat) \pm 0.9(syst)) \times 10^{-6}$ |
| $\Lambda^0_b \rightarrow p + \rho$ | $6.12 \times 10^{-6}$ | -0.873 | - |
| $\Lambda^0_b \rightarrow p + a_1$ | $4.08 \times 10^{-6}$ | -0.741 | - |
| $\Lambda^0_b \rightarrow p + D^-$ | $5.75 \times 10^{-7}$ | -0.998 | - |
| $\Lambda^0_b \rightarrow p + D^{*-}$ | $6.05 \times 10^{-7}$ | -0.546 | - |
| $\Lambda^0_b \rightarrow p + D_s$ | $1.36 \times 10^{-5}$ | -0.897 | - |
| $\Lambda^0_b \rightarrow p + D_s^*$ | $6.70 \times 10^{-6}$ | -0.514 | - |
| $\Lambda^0_b \rightarrow p + K$ | $2.58 \times 10^{-7}$ | -1  | $(5.8 \pm 0.8(stat) \pm 1.5(syst)) \times 10^{-6}$ |
| $\Lambda^0_b \rightarrow p + K^*$ | $3.21 \times 10^{-7}$ | -0.850 | - |

C. Non-leptonic decays of $\Lambda_b \rightarrow p + M$ and $\Lambda_b \rightarrow \Lambda + M$

The non-leptonic decays $\Lambda_b \rightarrow p(\Lambda) + M$ in the factorization approach have been studied in the previous section. Now, we present our numerical predictions on the decay rates and relevant measurable quantities. The CKM matrix elements take the values \cite{4}

$$
V_{ud} = 0.97377, \quad V_{us} = 0.2257, \quad V_{cd} = 0.230, \\
V_{cs} = 0.957, \quad V_{cb} = 0.0416, \quad V_{ub} = 0.00413,
$$

and the effective Wilson coefficient $a_1 = 1$ \cite{27}, $a_2 = 0.23$ \cite{18}. The meson decay constants are shown in Table VI.

The predictions for $\Lambda^0_b \rightarrow p + M$ are provided in Table VII. The Table VIII demonstrates a comparison of our result with other approaches and experimental data for $\Lambda^0_b \rightarrow \Lambda J/\psi$. In the Table IX we give predictions on the rates of $\Lambda^0_b \rightarrow \Lambda$+meson.

TABLE VIII: Branching ratio and up-down asymmetry for non-leptonic decay $\Lambda^0_b \rightarrow \Lambda J/\psi$ within different theoretical approaches and data from experiment.

| Br($\times 10^{-4}$) | This work | \cite{21} | \cite{31} | \cite{35} | \cite{36} | Exp.[4] |
|----------------------|-------------|-----------|-----------|-----------|-----------|---------|
|                     | 3.94        | 1.65 ~ 5.27 | 1.6       | 2.55      | 6.037     | 4.7 ± 2.8 |
| $\alpha$            | -0.204      | -0.17 ~ -0.14 | -0.1    | -0.208   | -0.18     | -       |

From Table VIII we can find that there are some differences among the predictions by
TABLE IX: Branching ratio and up-down asymmetry for non-leptonic decay $\Lambda_b^0 \rightarrow \Lambda + M$ with different theoretical approaches.

| Decay Mode       | Branching Ratio (x 10^−8) | Up-down Asymmetry |
|------------------|---------------------------|-------------------|
| $\Lambda_b^0 \rightarrow \Lambda + \pi^0$ | 7.49                       | -1                |
| $\Lambda_b^0 \rightarrow \Lambda + \eta$ | 5.46                       | -1                |
| $\Lambda_b^0 \rightarrow \Lambda + \eta'$ | 2.29                       | -1                |
| $\Lambda_b^0 \rightarrow \Lambda + D^0$ | 4.54 x 10^−5             | -0.998            |
| $\Lambda_b^0 \rightarrow \Lambda + D^{0*}$ | 4.78 x 10^−5             | -0.551            |
| $\Lambda_b^0 \rightarrow \Lambda + \bar{D}^0$ | 8.76 x 10^−6             | -0.998            |
| $\Lambda_b^0 \rightarrow \Lambda + D^{0*}$ | 5.08 x 10^−6             | -0.551            |

Various theoretical approaches. In our calculation, the $f_1(m_{J/\psi}^2), g_1(m_{J/\psi}^2)$ is nearly equal, however $g_1(m_{J/\psi}^2)$ is bigger than $f_1(m_{J/\psi}^2)$ in [18, 31].

V. CONCLUSION

In this work, we carefully investigate the processes where a heavy baryon decays into a light baryon plus a lepton pair (semi-leptonic decay) or a meson (non-leptonic decay) in terms of the light-front-quark model (LFQM). Besides the regular input parameters such as the quark masses and well measured decay constants of various mesons, there is only one free parameter to be determined, that is $\beta$ in the light front wavefunction. In our earlier work [6], by fitting the data of the semi-leptonic decays $\Lambda_b \rightarrow \Lambda_c + l + \bar{\nu}$, we obtained the values of $\beta_{b[ud]}$. Similarly, we fix the values $\beta_{u[ud]}$ for proton and $\beta_{s[ud]}$ for $\Lambda$.

Our numerical results are shown in corresponding tables and some measurable quantities such as the up-down asymmetries are also evaluated. A clear comparison of our prediction on the decay rate of $\Lambda_b \rightarrow \Lambda + J/\psi$ with the results predicted by other models and as well as the experimental data is also explicitly presented. One can notice that our result for $\Lambda_b \rightarrow \Lambda + J/\psi$ is 3.94 x 10^−4 which is in good agreement with the data. The success is not too surprising even though the model we adopt is much simplified. Definitely this value obtained in this work is closer to the central value of measurement than the previous evaluations, but since there is a large uncertainty in the data, one still cannot justify which model is more preferable than others because within two standard deviations, all the numerical results achieved with all the approaches listed in the table are consistent with data. The asymmetry parameter which may be important for determining the applicability of the adopted model, is estimated as −0.204, which is generally consistent with that obtained in other models and approaches. Of course the details, especially the branching ratios will be further tested by the more accurate experiments in the future.

Besides the semi-leptonic decays, we also estimate the branching ratios of several non-leptonic decay modes which are listed in Table VIII. Recently the CDF collaboration [37] has measured the branching ratios of $\Lambda_b \rightarrow p + \pi^−$ and $\Lambda_b \rightarrow p + K^−$ as $BR(\Lambda_b \rightarrow p + \pi^−) =$
\((3.5\pm 0.6(stat)\pm 0.9(syst))\times 10^{-6}\) and \(BR(\Lambda_b \to p+K^-) = (5.8\pm 0.8(stat)\pm 1.5(syst))\times 10^{-6}\). Our prediction on \(BR(\Lambda_b \to p+\pi^-)\) is consistent with the measurement of the CDF within one standard deviation, but for \(BR(\Lambda_b \to p+K^-)\) our value is \(2.58 \times 10^{-7}\), one order smaller than the data of the CDF collaboration. Following the literature, in our calculation, we employ the factorization scheme where the emitted pseudoscalar meson (\(\pi\) or \(K\)) is factorized out and described by the common-accepted form factor \(b\) case \(\Lambda\) anomalously larger than \(BR\) suppression of order \(\alpha\) and \(V\) be the loop where top quark is the intermediate fermion. In the case \(\Lambda_b \to p+K^-\), the CKM entry would be \(58\) \(\approx C\) \(\pi\) \(\sin \theta_C \approx 0.22\), thus comparing with \(\Lambda_b \to p+\pi^-\), the amplitude of the process \(\Lambda_b \to p+K^-\) is suppressed by a factor \(\frac{f_K}{f_\pi}\sin \theta_C \sim 0.27\). Thus besides a small difference between the final phase spaces of the two reactions, one can roughly estimate that \(BR(\Lambda_b \to p+K^-)/BR(\Lambda_b \to p+\pi^-) \sim 0.07\), and this estimate is consistent with our numerical results. Therefore the smallness of \(BR(\Lambda_b \to p+K^-)\) seems reasonable. However the data of CDF show completely different results that \(BR(\Lambda_b \to p+K^-)\) is anomalously larger than \(BR(\Lambda_b \to p+\pi^-)\).

In fact, in our calculations on the non-leptonic decays, we only consider the contributions from the tree diagrams and neglect the penguin-loop effects. For the mode of \(\Lambda_b \to p+\pi^-\), the penguin contribution can be safely neglected compared to the tree level. However, for the mode of \(\Lambda_b \to p+K^-\), the tree level contribution is suppressed by the Cabibbo-Kabayashi-Maskawa entry \(V_{ub}V_{us}^\ast\) while for the penguin diagram, the main contribution comes from the loop where top quark is the intermediate fermion. In the case, the CKM entry would be \(V_{ub}V_{ts}^\ast\) which is almost two orders larger than \(V_{ub}V_{us}^\ast\). Thus even though there is a loop suppression of order \(\alpha_s/4\pi\), it is compensated by the much larger CKM entry. This situation was discussed in \([38]\) where the authors used the pQCD method to carry out the calculations. In fact, we make a rough estimation of the contribution from the top-penguin, and the result is almost five times larger than the contribution from the tree diagram given above.

However, from another aspect, when the penguin diagram is taken into account, the factorization is dubious. That is why we do not include the loop contributions in this present work, but will make a detailed discussion in our coming paper.

Actually, even including the penguin contribution, the theoretically estimated branching ratio of \(BR(\Lambda_b \to p+K^-)\) is still below the data and obviously smaller than that of \(BR(\Lambda_b \to p+\pi^-)\). If this measurement is valid and approved by further experiments, it would be a new anomaly which may hint an unknown mechanism which dominates the transition or new physics beyond the standard model\(^1\) and it is also consistent with the result of \([38]\).

The good agreement of our results on the semileptonic decays of \(\Lambda_b\) to light baryon and several non-leptonic decay modes with data indicates the following points.

First, the diquark picture: as we know, two quarks in a color-anti-triplet attract each

---

\(^1\) We thank Dr. D. Tonelli for bringing our attention to the new measurements of the CDF collaboration on \(BR(\Lambda_b \to p+\pi^-)\) and \(BR(\Lambda_b \to p+K^-)\).
other and constitute a Cooper-pair-like subject of spin 0 or 1. However, until now, many theorists still doubt the justifiability of the diquark picture. It is true that even though the diquark structure was raised almost as early as the birth of the quark model, its validity or reasonability of application is still in sharp dispute. In fact, it should be rigorously testified by experiments. We have argued that for some processes, the diquark picture may be more applicable than in the others. Actually, in our case, we can convince ourselves that the picture should apply. As aforementioned, diquark is only a spectator in the transitions which we concern in this work, therefore its inner structure may not affect the numerical results much. Secondly, the produced baryon is very relativistic, i.e. very close to the light-cone, generally the details of the inner structure of the spectator diquark may not be important, this interpretation is somehow similar to the parton picture which was conceived out by Feynman and Bjorken long time ago. Namely at very high energy collisions, the interaction among partons can be ignored at the leading order, thus in our case the interaction between the quark which undergoes a transition, and the spectator diquark should be weak and negligible. Third is that the small effects caused by the inner structure of the diquark may be partly included in the parameter $\beta$ of the light-front wavefunction. The agreement with data indicates that the diquark picture and the light-front quark model indeed apply in the analysis of the heavy baryon transiting into a light one.

Moreover, since we employ the factorization scheme to deal with the non-leptonic decays, we find that to some modes, it works well, but to some modes where loop contributions may dominate or just are comparable to the tree contributions, the scenario encounters serious challenges.

We further investigate the measurable polarization asymmetries. Because the information on the polarization asymmetries may be more sensitive to the model adopted in the theoretical calculations than the decay width, accurate measurements would discriminate various models and indicate how to improve the details of the models.

Moreover, we also predict the rates and asymmetries of several similar modes of $\Lambda_b$ non-leptonic decays in the same model, and the results are listed in Table IIIV of last section. The numbers will be tested in the future.

Fortunately, the high luminosity at LHCb can provide large database on $\Lambda_b$ and moreover, with great improvements of experimental facility and detection technique, we expect that more and more accurate measurements will be carried out in the near future and theorists will be able to further testify, improve, or even negate our present models. Indeed, the baryons are much more complicated than mesons, but careful studies on the processes where baryons are involved would be very beneficial for getting better insight into the hadron structure and underlying principles, especially the non-perturbative QCD effects including the factorization and plausibility of the diquark picture. The LHCb will be an ideal place to do the job.
Acknowledgements:

The work is supported by the National Natural Science Foundation of China and the Special Grant for the PH.D program of the Chinese Education Ministry, one of us (Ke) would like to thank the Tianjin University for financial support.

[1] A. Ali, G. Kramer and C. Lü, Phys. Rev. D 58, 094009 (1998).
[2] Y. Chen, H. Cheng, B. Tseng, K. Yang, Phys. Rev. D 60, 094014 (1999); Z. Wei, arXiv:hep-ph/0310173; C. Chen, C. Geng, Z. Wei, Eur. Phys. J. C 46, 367 (2006).
[3] J. Abdallah et al., DELPHI Collaboration, Phys. Lett. B 585, 63-84 (2004).
[4] W. Yao et al., Particle Data Group, J. Phys. G 33, 1 (2006).
[5] C. Albertus, E. Hernandez and J. Nieves, Phys. Rev. D 71, 014012 (2005); H. Shih, S. Lee and H. Li, Phys. Rev. D 61, 114002 (2000); M. Huang, H. Jin, J. Körner and C. Liu, Phys. Lett. B 629, 27 (2005); M. Pervin, W. Roberts and S. Capstick, Phys. Rev. C 72, 035201 (2005); D. Ebert, R. Faustov and V. Galkin, Phys. Rev. D 73, 094002 (2006); F. Cardarelli and S. Simula, Phys. Rev. D 60, 074018 (1999).
[6] H. Ke, X. Li and Z. Wei, Phys. Rev. D 77, 014020 (2008).
[7] M. Terent’ev, Sov. J. Nucl. Phys. 24, 106 (1976); V. Berestetsky and M. Terent’ev, ibid. 24, 547 (1976); 25, 347 (1977); P. Chung, F. Coester, and W. Polyzou, Phys. Lett. B 205, 545 (1988).
[8] W. Jaus, Phys. Rev. D 41, 3394 (1990); W. Jaus, Phys. Rev. D 44, 2851 (1991); W. Jaus, Phys. Rev. D 53, 1349(1996); D 54, 5904(E)(1996); 60, 054026 (1999).
[9] C. Ji, P. Chung and S. Cotanch, Phys. Rev. D 45, 4214 (1992).
[10] H. Cheng, C. Cheung and C. Hwang, Phys. Rev. D 55, 1559 (1997).
[11] H. Cheng, C. Chua and C. Hwang, Phys. Rev. D 69, 074025 (2004).
[12] C. Hwang and Z. Wei, J. Phys. G 34, 687 (2007); C. D. Lu, W. Wang and Z. T. Wei, Phys. Rev. D 76, 014013 (2007) [arXiv:hep-ph/0701265]; Z. T. Wei, H. W. Ke and X. F. Yang, Phys. Rev. D 80, 015022 (2009) [arXiv:0905.3069 [hep-ph]]; H. W. Ke, X. Q. Li and Z. T. Wei, arXiv:0907.5465 [hep-ph].
[13] H. Dosch, M. Jamin and B. Stech, Z. Phys. C 42, 167 (1989).
[14] P. Kroll, B. Quadder and W. Schweiger, Nucl. Phys. B 316, 373 (1989); P. Ball, H.G. Dosch, Z. Phys. C 51, 445 1991; J. Körner and P. Kroll, Phys. Lett. B 293, 201 (1992); R. Jakob, P. Kroll, M. Schurmann and W. Schweiger, Z. Phys. A 347, 109 (1993); J. Bolz, P. Kroll and J. Körner, Z. Phys. A 350, 145 (1994); J. Körner and P. Kroll, Z. Phys. C 57, 383 (1993).
[15] F. Wilczek, arXiv: hep-ph/0409168
[16] D. Ebert, R. Faustov and V. Galkin, Phys. Rev. D 73, 094002 (2006); X. Guo and T. Muta, Phys. Rev. D 54, 4629 (1996); X. Guo, A. Thomas and A. Williams, ibid D 59, 116007 (1999); Y. Yu, H. Ke, Y. Ding, X. Guo, H. Jin, X. Li, P. Shen and G. Wang, Commun. Theor. Phys. 46, 1031 (2006); Y. Yu, H. Ke, Y. Ding, X. Guo, H. Jin, X. Li, P. Shen and G. Wang, arXiv:
[17] B. Ma, D. Qing and I. Schmidt, Phys. Rev. C 66, 048201 (2002).
[18] H. Cheng, and B. Tseng, Phys. Rev. D 53, 1457 (1996).
[19] C. S. Huang, C. F. Qiao and H. G. Yan, Phys. Lett. B 437, 403 (1998) [arXiv:hep-ph/9805452].
[20] H. Shih, S. Lee and H. Li, Phys. Rev. D 59, 094014 (1999).
[21] C. Chou, H. Shih, S. Lee and H. Li, Phys. Rev. D 65, 074030 (2002).
[22] Y. M. Wang, Y. L. Shen and C. D. Lu, [arXiv:0907.4008 hep-ph].
[23] J. Charles, A. Yauonace, L. Oliver, O. Pöne and J. Raynal, Phys. Rev. D 60, 014001 (1999).
[24] M. Dugan and B. Grinstein, Phys. Lett. B255, 583 (1991).
[25] N. Isgur and M. Wise, Phys. Lett. B232, 113 (1989); N. Isgur and M. Wise, Nucl.Phys. B 348, 276 (1991); H. Georgi, ibid B348, 293 (1991).
[26] C.W. Bauer, S. Fleming, D. Pirjol and I.W. Stewart, Phys. Rev. D 63 (2001) 114020.
[27] H. Cheng, C. Chua and C. Hwang, Phys. Rev. D 70, 034007 (2004).
[28] J. Körner and M. Krämer, Phys. Lett. B 275, 495 (1992); P. Bialas, J. Körner, M. Krämer, and K. Zalewski, Z. Phys. C 57, 115 (1993); J. Körner, M. Krämer and D. Pirjol, Prog. Part. Nucl. Phys. 33, 787 (1994).
[29] D. Ebert, R. Faustov and V. Galkin, Phys. Rev. D 73, 094002 (2006).
[30] J. Körner and M. Krämer, Z. Phys. C 55, 659 (1992).
[31] H. Cheng, Phys. Rev. D 56, 2799 (1997).
[32] P. Guo, H. Ke, X. Li, C. Lü and Y. Wang, Phys. Rev. D 75, 054017 (2007).
[33] D. Becirevic and A. B. Kaidalov, Phys. Lett. B 478, 417 (2000) [arXiv:hep-ph/9904490].
[34] P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005) [arXiv:hep-ph/0406232]; ibid D71, 014029 (2005) [arXiv:hep-ph/0412079].
[35] R. Mohanta, A. Giri, M. Khanna, M. Ishida, S. Ishida and M. Oda, Prog. Theor. Phys. 101, 959 (1999).
[36] Fayyazuddin and Riazuddin, Phys. Rev. D 58, 014016 (1998).
[37] T. Aaltonen, et al., CDF Collaboration, Phys. Rev. Lett. 103, 031801 (2009).
[38] C. Lü, Y. Wang, H. Zou, A. Alia and G. Kramer, Phys. Rev. D 80, 034001 (2009) [arXiv:0906.1479[hep-ph]].
[39] Z. Wei, H. Ke and X. Li, in preparation.