THE INVERSE PROBLEMS OF SOME MATHEMATICAL PROGRAMMING PROBLEMS

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Abstract

The non-convex quadratic programming problem and non-monotone linear complementarity problem are NP-complete problems. In this paper we first show that the inverse problem of determining a KKT point of the non-convex quadratic programming is polynomial. We then show that the inverse problems of non-monotone linear complementarity problem are polynomial solvable in some cases, and in another case is NP-hard. Therefore we solve an open question raised by Heuberger on inverse NP-hard problems and prove the CoNP=NP.

Key words: Inverse problem, quadratic programming, Linear complementarity problem, Polynomial algorithm.

Abbreviated title: Inverse problems of mathematical programming problems.

1. Introduction

Since Burton and Toint [3] first introduced inverse shortest path problem, many researchers have contributed to the growing literature on inverse optimization problems. For example, Burton and Toint [3],[4], and Burton, Pulleyblank and Toint [5] have discussed inverse shortest path problem; Huang and Liu [9],[10] have considered inverse linear programming problem and applied it to inverse matching problem and inverse minimum cost flow problem respectively; Zhang, Liu and Ma [19], Sokkalingam, Ahuja and Orlin [18], and Ahuja and Orlin [2] studied inverse minimum spanning tree problem, etc. For a more complete survey on inverse combinatorial optimization problems we refer the reader to Heuberger [7]. Some researchers also did some work on the inverse mathematical programming problems. For example, Huang and Liu [9], and Zhang and Liu [20] studied inverse linear programming problem, Diao and Ding [6] studied inverse convex programming problem, Diao and Ding [6] studied inverse convex programming problem. Most of the inverse problems studied so far are polynomial solvable problems, and most of their inverse problems are polynomial problems too. A natural open question arises: is there a NP-complete problem such that it’s inverse problem is polynomial solvable? This question was proposed in Huang and Liu [9] as an open question for future research. In a survey paper written by Heuberger [7] on inverse combinatorial optimization problems, he proposed three open questions for future research. The first of them is: is there a NP-hard problem such that its inverse problem is polynomial solvable? If so one proves CoNP=NP, a long standing open problem in the theory of computational complexity. Recently, Huang [11] have shown that the inverse Knapsack problem and the inverse problem of integer programming with fixed number of constraints are pseudo polynomial. In this paper, we will further show that the inverse problem of determining the KKT points of a non-convex quadratic programming problem is polynomial solvable; we then show that the inverse problem of determining the optimal solutions of non-monotone linear complementarity problem in some cases are polynomial solvable, in another case is NP-hard.

The theory of inverse optimization can also be viewed as the generalization or complement to the theory of sensitivity analysis and the theory of stability of optimal solutions of optimization. For example, the sensitivity analysis of linear programming (LP) study the range of changes of each
parameter such that the optimal solution will not change if the changes of each parameter is within the range. If the change of one of the parameter is outside the range, then the optimal solution will change. The inverse linear programming [9] study the problem of given any feasible solution \( x^0 \), how to change the parameters such that \( x^0 \) becomes an optimal solution of the LP with the change of the parameters as small as possible under the sense of \( l_p \) norm. Therefore it can be viewed as the generalization of the sensitivity analysis of linear programming. It has been shown in [9] that the inverse problem of linear programming can be solved efficiently.

The sensitivity analysis of NP-complete problems are generally viewed as difficult since there is no polynomial algorithm to solve them. The results in this paper show that for some NP-complete problems, for example the KKT problem of a non-convex quadratic programming and non-monotone LCP problem, the inverse problems of them can be efficiently solved. Therefore it is relatively easy to do the sensitivity analysis for these problems.

The paper is organized as follows: in section 2 we introduce the quadratic programming and show that the inverse problem of finding a KKT point of the non-convex quadratic programming is polynomial solvable. In section 3, we show that the inverse problem of finding an optimal solution of a class of non-monotone linear complementarity problem in some cases are polynomial solvable, in other case is NP-hard. In section 4 we will show that \( \text{CoNP} = \text{NP} \) using the results in section 2. We will give some concluding remarks in section 5.

2. The Inverse Problem of a KKT Point of Non-convex QP

The concept of inverse problem of an optimization problem can be found in [8]. For completion, we state it here again. Given an optimization problem:

\[
\min \{ f(c, x) | x \in D \},
\]

where \( c \in \mathbb{R}^n \) is a parameter vector, \( D \) is the feasible region of \( x \), \( f(c, x) \) is the objective function. Given a feasible solution \( x^0 \) of (1), is there \( \bar{c} \in \mathbb{R}^n \) such that \( x^0 \) is the optimal solution of (1) with \( \bar{c} \) as parameter vector? Formally, let

\[
F(x^0) = \{ \bar{c} \in \mathbb{R}^n | \min \{ f(\bar{c}, x) | x \in D \} = f(\bar{c}, x^0) \},
\]

if \( F(x^0) \neq \emptyset \), define

\[
\min \{ \| c - \bar{c} \| | \bar{c} \in F(x^0) \},
\]

where \( \| . \| \) denotes the norm of the vector, the popular choices for the norms are \( l_1 \), \( l_2 \) and \( l_\infty \). we call (2) the inverse problem of (1).

Now we consider the following general non-convex quadratic programming problem:

\[
\text{(QP)} \quad \begin{array}{rl}
\text{minimize} & Q(x) = \frac{1}{2} x^T Q x + c^T x \\
\text{subject to} & Ax = b, \; x \geq 0,
\end{array}
\]

where \( Q \in \mathbb{R}^{n \times n} \) is an indefinite matrix, \( A \in \mathbb{R}^{m \times n} \) and \( c \in \mathbb{R}^n \). Its dual is:

\[
\text{(QD)} \quad \begin{array}{rl}
\text{maximize} & d(x, y) = b^T y - \frac{1}{2} x^T Q x - c^T x \\
\text{subject to} & A^T y + s - Q x = c, \; x, s \geq 0,
\end{array}
\]
Let \( F(P) = \{ x | Ax = b, x \geq 0 \} \) and \( F(D) = \{ (x, y, s) | A^T y + s - Q x = c, s \geq 0 \} \) be the feasible regions of the (\(QP\)) and (\(QD\)) respectively, then the KKT conditions of the (\(QP\)) – (\(QD\)) are:

\[
\begin{align*}
Ax &= b, \\
A^T y + s - Q x &= c, \\
x^T s &= 0.
\end{align*}
\]

Where \((x, y, s) \in (R^n, R^m, R^n_+)\) is a feasible solution of (\(QP\)). As proved in [15], finding a KKT point of the nonconvex quadratic programming is NP-complete.

Therefore the inverse problem of determining the KKT point of the (\(QP\)) can be stated as follows: given any \(x^0 \in F(P)\), find a matrix \(\bar{Q}\) and a vector \(\bar{c}\) that are closest to \(Q\) and \(c\) such that \(x^0\) is a KKT point of (\(QP\)) with the parameters \(\bar{Q}\) and \(\bar{c}\). Let \(\bar{Q}\) denotes the matrix after changing the parameters in \(Q\), \(\bar{c}\) denotes the vector after changing the parameters in \(c\), then the inverse problem of a KKT point of (\(QP\)) with \(l_2\) norm is:

\[
(IQP) \quad \text{Min} \quad \|Q - \bar{Q}\|_F^2 + \|c - \bar{c}\|^2 \\
\text{s.t.} \quad Ax^0 = b \\
A^T y + s - \bar{Q} x^0 = \bar{c} \\
x^0 T s = 0 \\
s \geq 0.
\]

Where \(\|A\|_f = \sqrt{\sum_{i,j} A_{ij}^2}\) denote the Frobenius norm of the matrix \(A\). Let \(X = Q - \bar{Q}\), \(z = c - \bar{c}\), \(c' = Q x^0 + c\). We define the \(n \times n^2\) block diagonal matrix \(X^0\) as follows:

\[
X^0 = \begin{pmatrix}
x^0 T \\
x^0 T \\
\vdots \\
x^0 T
\end{pmatrix}.
\]

That is, the first \(n\) entries of the first row of \(X^0\) are the row vector of \(x^0\) and rest entries of the first row are 0; the \(n\) to \(2n\) entries of the second row of \(X^0\) are the row vector of \(x^0\) and rest entries are 0; and so on. Let \(A = (a_1, ..., a_n)\) be any \(n \times n\) matrix, where \(a_j\) (\(j = 1, ..., n\)) are columns of \(A\), we define the vector of \(A\) as follows:

\[
vec(A) = (a_1^T, ..., a_n^T)^T.
\]

Then (\(IQP\)) becomes:

\[
(IQP1) \quad \text{Min} \quad \|X\|_F^2 + \|z\|^2 \\
\text{s.t.} \quad A^T y + s + X^0 vec(X^T) + z = c' \\
x^0 T s = 0 \\
s \geq 0.
\]

It is easy to see that the above problem is a convex quadratic optimization problem with \(n^2 + 2n + m\) variables. In order to apply the interior point algorithms for solving (\(IQP1\)), we do some transformations for (\(IQP1\)) first.
We define $w = (\text{vec}(X^T)^T, z^T, y^T)^T$ to be a $\{n^2 + n + m\}$ dimensional vector, $A' = (X^0, I, A^T)$ to be a $n \times \{n^2 + n + m\}$ matrix, $P = \text{diag}(1, ..., 1, 0, ..., 0)$ to be a $\{n^2 + n + m\} \times \{n^2 + n + m\}$ diagonal matrix with first $n^2 + n$ diagonal entries being 1, rest $m$ diagonal entries being 0. Note that $\|X\|_f^2 = \text{vec}(X)^T \text{vec}(X) = \text{vec}(X^T)^T \text{vec}(X)$. Then (IQP1) can be further transformed into:

\[(IQP2) \quad \text{Min} \quad w^T P w \]
\[\text{s.t.} \quad s + A' w = c', \]
\[x^0^T s = 0, \]
\[s \geq 0.\]

Define non-negative vectors $w^+$ and $w^-$ as follows:

$$w^+_i = \begin{cases} w_i, & \text{if } w_i \geq 0; \\ 0, & \text{otherwise;} \end{cases} \quad w^-_i = \begin{cases} -w_i, & \text{if } w_i \leq 0; \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

Let $\bar{w} = (w^T, w^-^T)^T$, $\bar{A} = (A', -A')$, and

$$\bar{P} = \begin{pmatrix} 2P & -2P \\ -2P & 2P \end{pmatrix}.$$ 

Then (IQP2) becomes:

\[(IQP3) \quad \text{Min} \quad \frac{1}{2} \bar{w}^T \bar{P} \bar{w} \]
\[\text{s.t} \quad s + \bar{A} \bar{w} = c', \]
\[x^0^T s = 0, \]
\[s \geq 0, \bar{w} \geq 0.\]

It is easy to show that $\bar{P}$ is positive semidefinite since $P$ is positive semidefinite. Therefore (IQP3) is a standard convex quadratic programming problem. Hence one can use any polynomial interior point algorithm for convex quadratic programming, for example [15], to solve this convex quadratic programming. We conclude this section by following theorem.

**Theorem 2.1** The inverse problem of determining a KKT point of the non-convex quadratic programming (IQP) can be solved in polynomial time.

Note that if the $l_1$ norm is used in (IQP), then it can be transformed into a linear programming problem. We leave it to the reader to try it.

### 3. The Inverse Problem of Linear Complementarity Problem

In this section, we discuss the inverse problem of non-monotone linear complementarity problem. Given $M \in \mathbb{R}^{n \times n}$ and $q \in \mathbb{R}^n$, the linear complementarity problem (LCP) is to find a pair $(x, s) \in \mathbb{R}^n \times \mathbb{R}^n$ such that

$$s = Mx + q, \quad (x, s) \geq 0, \quad \text{and} \quad x^T s = 0.$$
We denote the feasible set of (LCP) as:

\[ F = \{(x, s)|s = Mx + q, \ x, s \geq 0\}. \]

The LCP problem can also be formulated as an optimization problem:

\[
\begin{align*}
(LCP) \quad & \text{Min} & x^T s \\
\text{s.t.} & & s = Mx + q, \ x, s \geq 0.
\end{align*}
\]

If \(M\) (may not be symmetric) is positive semi-definite, then the LCP is called a monotone LCP. Otherwise it is called a non-monotone LCP. Monotone LCPs are “easy” problems since there are polynomial algorithms for solving the monotone LCPs. See, for example, [13] and [14]. If \(M\) is not positive semi-definite, then the LCP becomes a “hard” problem, i.e, NP-complete problem, since it can be transformed into the following non-convex quadratic programming problem:

\[
\begin{align*}
\text{Min} & \quad x^T Mx + q^T x \\
\text{s.t.} & \quad s = Mx + q, \ x, s \geq 0.
\end{align*}
\]

In fact, any feasibility problem of a mixed integer programming problem can be transformed into a LCP problem, see [8]. Therefore non-monotone LCP problems are NP-hard.

To study the inverse problem of non-monotone LCP, we consider three different cases, depends on what information were available to us:

(a): Given \(x^0 \geq 0\), find parameters \(M'\) and \(q'\) and \(s^0 \geq 0\) such that \((x^0, s^0)\) is an optimal solution of the LCP problem with parameters of \(M', q'\) and that \(M', q'\) are closest to \(M, q\).

(b): Given a pair of non-feasible complementary point \((x^0, s^0) \geq 0\) (i.e. \(x^0^T s^0 = 0, s^0 \neq Mx^0 + q\)), find parameters \(M'\) and \(q'\) such that \((x^0, s^0)\) is an optimal solution of the LCP problem with parameters of \(M', q'\) and that \(M', q'\) are closest to \(M, q\).

(c): Given \(s^0 \geq 0\), find parameters \(M'\) and \(q'\) and \(x^0 \geq 0\) such that \((x^0, s^0)\) is an optimal solution of the LCP problem with parameters of \(M', q'\) and that \(M', q'\) are closest to \(M, q\).

We now discuss the case (a) first. The inverse problem of LCP in this case (with \(l_2\) norm) is:

\[
\begin{align*}
(ILCP(a)) \quad & \text{Min} & \|M - M'\|_F^2 + \|q - q'\|^2 \\
\text{s.t.} & \quad s = M'x^0 + q', \\
& & \quad x^0^T s = 0, \ s \geq 0.
\end{align*}
\]

Similar to the discussions in last section, we can transform the \((ILCP(a))\) into a convex programming problem. Let \(X = M - M', z = q - q', \) matrix \(X^0\) be defined as in (3), \(c = Mx^0 + q,\) then the \((ILCP(a))\) becomes:

\[
\begin{align*}
(ILCP(a)1) \quad & \text{Min} & \|X\|_F^2 + \|z\|^2 \\
\text{s.t.} & \quad s + X^0 vec(X^T) + z = c, \\
& & \quad x^0^T s = 0, \ s \geq 0.
\end{align*}
\]

Let \(w^T = (vec(X^0)^T, z^T)\) be a \(n^2 + n\) dimensional vector, \(A' = (X^0, I)\) be a \(n \times \{n^2 + n\}\) matrix, then \((ILCP(a)1)\) becomes:
(ILCP(a)2) \[ \begin{align*} \text{Min} & \quad w^T w \\ \text{s.t.} & \quad s + A'w = c, \\ & \quad x^0^T s = 0, \\ & \quad s \geq 0. \end{align*} \]

Define non-negative vectors \( w^+ \) and \( w^- \) as in (4), and \( \bar{w} = (w^T, w^-)^T \), \( \bar{A} = (A', -A') \), then (ILCP(a)2) becomes:

(ILCP(a)3) \[ \begin{align*} \text{Min} & \quad \bar{w}^T \bar{w} \\ \text{s.t} & \quad s + \bar{A}\bar{w} = c, \\ & \quad x^0^T s = 0, \\ & \quad s \geq 0, \ \bar{w} \geq 0. \end{align*} \]

It is easy to see that (ILCP(a)3) is a convex quadratic programming problem, therefore it can be solved by any polynomial interior point algorithms for quadratic programming. See, for example, [14]. Hence we have the following conclusion.

**Theorem 3.1.** Given any \( x^0 \geq 0 \), the inverse problems of (LCP) in case (a), i.e., ILCP(a) can be solved in polynomial time.

For the case (b), the inverse problem (with \( l_2 \) norm) becomes:

(ILCP(b)) \[ \begin{align*} \text{Min} & \quad \|M - M'\|^2_j + \|q - q'\|^2 \\ \text{s.t.} & \quad s^0 = M'x^0 + q', \\ & \quad x'^^T s^0 = 0. \end{align*} \]

Similarly, let \( X = M - M' \), \( z = q - q' \), matrix \( X^0 \) be defined as in (), vector \( c = Mx^0 + q - s^0 \), \( w^T = (\text{vec}(X^T)^T, z^T) \), non-negative vectors \( w^+ \) and \( w^- \) be defined as in (4), and \( \bar{w} = (w^+ T, w^- T)T \), \( A' = (X^0, I) \) and \( A = (A', -A') \), then the (ILCP(b)) becomes:

(ILCP(b)1) \[ \begin{align*} \text{Min} & \quad \bar{w}^T \bar{w} \\ \text{s.t} & \quad \bar{A}\bar{w} = c, \\ & \quad \bar{w} \geq 0. \end{align*} \]

It is also easy to see that (ILCP(b)1) is a convex quadratic programming problem, therefore it can also be solved by any polynomial interior point algorithms for quadratic programming. Hence we have the following theorem.

**Theorem 3.2.** Given any complementary pair \( (x^0, s^0) \geq 0 \), the inverse problem of (LCP) in case (b), i.e., ILCP(b), can be solved in polynomial time.

For the case (c), the inverse problem with \( l_1 \) norm becomes:

(ILCP(c)) \[ \begin{align*} \text{Min} & \quad \|M - M'\| + \|q - q'\| \\ \text{s.t.} & \quad s^0 = M'x + q', \\ & \quad x^T s^0 = 0, \end{align*} \]
Let \( Y = M - M', z = q - q', c = q - s^0 \), then (ILCP\((c)\)) becomes:

\[
(\text{ILCP\((c)\))1) \quad \text{Min} \quad \sum_{i,j=1}^{n} |Y_{ij}| + \sum_{i=1}^{n} |z_i| \\
\text{s.t.} \quad Yx - Mx + z = c, \\
\quad x^Ts^0 = 0, \\
\quad x \geq 0.
\]

Let \( Y_j \) denotes the jth column of the matrix \( Y \) \((j = 1, ... , n)\), then \( Y_j^T \) is the jth row of the matrix \( Y \) \((j = 1, ... , n)\). Therefore the constraints

\[ Yx - Mx + z = c \]

in (ILCP\((c)1\)) can be written in \( n \) quadratic constraints:

\[ x^T(Y^T)_j - x^T(M^T)_j + z_j = c_j, \quad j = 1, ..., n. \]

Hence (ILCP\((c)1\)) is NP-hard since an optimization problem with linear objective function and one non-convex quadratic constraint is a NP-complete problem. See for example, [11]. Now we can make following conclusion:

**Theorem 3.3.** Given any \( s^0 \geq 0 \), the inverse problem of (LCP) in case (c) is NP-hard.

### 4. The Proof of CoNP=NP

As pointed out by Heuberger in [7] as one of the open problems: if one can find a NP-hard optimization problem such that its inverse problem can be solved in polynomial time, then one proves CoNP=NP, a long standing open problem in computational complexity. But there he only gave a brief discussion, no formal proof. In this section we give a formal proof of CoNP=NP using the results obtained in above sections. Let us first review some of the useful concepts and results related to CoNP. We will follow the notations in [17].

**Definition 4.1:** (See P. 384 of [17]) Problem \( \bar{A} \) is the complement of Problem \( A \) if the set of strings with symbols in \( \sum \) that are encodings of yes instances of \( \bar{A} \) are exactly those that are not encodings of yes instances of \( A \).

**Definition 4.2:** (P. 385 of [17]) The class CoNP is the class of all problems that are complements of problems in NP.

**Theorem 4.3:** (P. 385 of [17]) If the complement of an NP-complete problem is in NP, then NP=CoNP.

Now let us consider the following decision problem:

**P1** Given an instance of \((Q, c, A, b)\), is there a feasible solution \( x^0 \) such that it is a KKT point of \((QP)\)?

It is well known that \( P1 \) is NP-complete [16]. By Definition 4.1 the complement of \( P1 \) is:
All the feasible solutions of \((QP)\) are not the KKT points of \((QP)\), or equivalently \((QP)\) has no KKT point.

Regarding the \(P2\) we have the following theorem.

**Theorem 4.4:** \(P2\) is in \(NP\).

**Proof:** Given any feasible solution \(x^0\) of \((QP)\), we can check whether it is a KKT point of \((QP)\) in polynomial time by solving the inverse problem of \((QP)\), i.e., \((IQP)\). \(x^0\) is not a KKT point of \((QP)\) if and only if \(\bar{Q} \neq Q\) or \(\bar{c} \neq c\). Since \((QP)\) have many feasible solutions (more than exponential number), so \(P2\) is in \(NP\).

Using theorem 4.3 and theorem 4.4 we have the following main result of the section.

**Theorem 4.5** \(\text{CoNP}=\text{NP}\).

5. Conclusions

In this paper we have shown that the inverse problem of determining the KKT point of the non-convex quadratic programming is polynomial solvable. We also showed that the inverse problems of determining the optimal solutions of the non-monotone linear complementarity problem in cases (a) and (a) are polynomial, in case (c) is \(NP\)-hard. We showed that the inverse problem of determining a KKT point of the non-convex \((QP)\) with \(l_2\) norm is equivalent to solve a convex quadratic programming, therefore can be solved by polynomial interior point algorithms. We then showed that the inverse problems of determining an optimal solution of the non-monotone linear complementarity problem with \(l_2\) norm in cases (a) and (b) are also equivalent to solve a convex quadratic programming problem, hence can also be solved by polynomial interior point algorithms. In case (c), we showed that the inverse problem of non-monotone LCP with \(l_1\) norm is equivalent to solve an optimization problem with linear objective function and \(n\) non-convex quadratic constraints, therefore is \(NP\)-hard. The results in this paper solved an open question raised by Heuberger in [7] on whether there exists an \(NP\)-hard problem such that its inverse problem is polynomial solvable. If so one proves \(\text{CoNP}=\text{NP}\), a long standing open problem in the theory of computational complexity.

It is also interesting to observe that finding the KKT points of a general quadratic programming problem and solving the non-convex linear complementary problem are the only \(NP\)-complete optimization problems we have found so far whose inverse problems are polynomial solvable. Are there any other \(NP\)-complete optimization problems whose inverse problems are polynomial solvable? The main difficulty is that there are no optimality conditions for most of the \(NP\)-complete optimization problems.

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REFERENCES

[1] Ahuja, R.K. and J.B. Orlin, Combinatorial algorithms for inverse network flow problems, Working Paper, Sloan School of Management, MIT, Cambridge, MA, 1998.
[2] Ahuja, R.K. and J.B. Orlin, A fast algorithm for the inverse spanning tree problem, J. of Algorithms, 34, 177-193(2000).
[3] Burton, D. and Ph.L. Toint, On an instance of the inverse shortest paths problem, Mathematical Programming, 53, 45-61(1992).
[4] Burton, D. and Ph.L. Toint, On the use of an inverse shortest paths algorithm for recovering linearly correlated costs, Mathematical Programming, 63, 1-22(1994).
[5] Burton, D., B. Pulleyblank and Ph. L. Toint, The inverse shortest paths problem with upper bounds on shortest paths costs, In Network Optimization, edited by P. Pardalos, D.W. Hearn and W.H. Hager, Lecture notes in Economics and Mathematical Systems, Volumn 450, pp. 156-171(1997).
[6] Diao, Z. and M. Ding, Models and Algorithms for inverse convex quadratic programming problem, Chinese J. of OR, Vol. 4(4)(2000), 88-94.
[7] Heuberger, C., Inverse combinatorial optimization: A survey on problems, methods, and results, J. of Combinatorial Optimization, 8(3), 329-361, 2003.
[8] Horst, R., P.M. Pardalos and N.V. Thoai, Introduction to Global Optimization, Kluwer Academic Publisher, 1995.
[9] Huang, S. and Z. Liu, On the inverse problem of linear programming and its application to minimum weight perfect k-matching, European Journal of Operational Research, 112, 421-426(1999).
[10] Huang, S. and Z. Liu, On the inverse minimum cost flow problem, Advances in Operations Research and Systems Engineering, World Publishing, Co., 30-37, 1998.
[11] Huang, S., Inverse Problems of some NP-complete Problems, Lecture Notes in Computer Sciences, LNCS 3521, N. Megiddo, Y. Xu and B. Zhu (eds),, Springer-Verlag, Berlin, Heidelberg, 422-429, 2005.
[12] Garry, M. R. and D. S. Johnson, COMPUTERS AND INTRACTABILITY, A Guide to the Theory of NP-Complete, W. H. Freeman and Company, 1979.
[13] Ji, J., F. A. Potra and S. Huang, Predictor-corrector method for linear complementarity problems with polynomial complexity and superlinear convergence, J. of Optimization Theory and Applications, 84, 187-199, 1995.
[14] Kojima, M., S. Mizuno and A. Yoshise, A polynomial-time algorithm for a class of linear complementarity problems, Math. Programming, 44, 1-26, 1989.
[15] Monteiro, R. D. C. and I. Adler, Interior path following primal-dual algorithms: Part II: Convex quadratic programming, Math. Programming, 44, 43-66, 1989.
[16] Murty, K. G. and S. N. Kabadi, Some NP-complete problems in quadratic and nonlinear programming, Math. Programming, 39: 117-129(1987).
[17] Papadimitriou, C.H. and K. Steiglitz, Combinatorial Optimization: Algorithms and Complexity, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1982.
[18] Sokkalingam, P.T., R.K. Ahuja and J.B Orlin, Solving inverse spanning tree problems through network flow techniques, Operations Research, 47, 291-298(1999).
[19] Zhang, J., Z., Liu and Z., Ma, On the inverse problem of minimum spanning tree with partition constraints, Mathematical Methods of Operations Research, 44, 171-188(1996).
[20] Zhang, J., Z., Liu, Calculating some inverse linear programming problems, J. Comput. Appl. Math., 72(1996), 261-273.