A CRUCIAL DIPOLE TEST OF THE EXPANSION CENTER UNIVERSE
BASED ON HIGH-Z SCP UNION & UNION2 SUPERNOVAE

ECM paper XV by Luciano Lorenzi
by merging the SAIIt 2011 ECM paper X with the EWASS 2012 ECM paper XII

ABSTRACT

The expansion center Universe (ECU) gives a dipole anisotropy to the Hubble ratio, as \( cz/D = H_0 - a^* \cos \gamma \), at any Hubble depth \( D \). After a long series of successful dipole tests, here is a crucial multiple dipole test at \( z \) bins centred on the mean \( \langle z \rangle \equiv z_0 = 1.0 \), or Hubble depth \( D = c/H_0 \), and based on data from SCP Union & Union2 compilation. Table 5abc lists data of two main samples, with 48 \textit{SCP} SNe Ia and 58 \textit{SCP} SNe Ia respectively. The confirmed dipole anisotropy, shown by 6 primary sample tests and by another 27 from 9 encapsulated \( z \) bins with \( D_L = D \cdot (1 + z) \) assumed and the Hubble Magnitude definition, gives the mean \( \langle a^* \rangle \approx 5.5 \pm 0.3 \) km/s/Mpc as a model independent result, in full accordance with the expansion center model (ECM). That means a maximum \( cz \) range of \( \approx 50000 \) km/s at \( z_0 = 1 \), that is \( c|\Delta z|^{\text{max}} = 2a^*D \) with a decelerating expansion dipole coefficient \( a^*_{\text{ECM}} \approx 5.46 \) H.u. at \( D \approx 4283 \) Mpc. As a complement to the dipole tests, here is a new computation of the relativistic deceleration parameter \( q_0 \), based on the extrapolated total \( M \) spread, that is the deviation of the Hubble Magnitude \( M \) of high-\( z \) SCP Union supernovae at a normal or central redshift \( \langle z \rangle \equiv z_0 = z \ll 1 \) from the absolute magnitude \( M_0 \) at \( z_0 \to 0 \) (cf. parallel paper XVI). A total \( M \) spread according to ECM is derived from 249 high-\( z \) \textit{SCP} SNe listed in paper XVI. In a concordance test with the expansion center model, the obtained new relativistic \( q_0 \gtrsim +2 \) agrees with the value \( q_0 = +2 \) inferred from the ECM paper I eq. (41), when \( R_0 \) is the proper distance at \( t_0 \) of the expansion center from the Galaxy.
1. Introduction

After the construction of the wedge-shaped Hubble diagram of 398 SCP supernovae (Lorenzi 2010), the priority is to check the ECM dipole anisotropy of the Hubble ratio \( cz/D \).

Here a first crucial multiple dipole test has been carried out on the most remote supernovae SNe Ia (a lot of which were observed with the NASA/ESA Hubble Space Telescope), ranging around the mean value \( \langle z \rangle = 1.0 \) and listed in the SCP Union (SCP Union hereafter) compilation by Kowalski et al. (2008) and in the SCP Union2 (SCP Union2 hereafter) by Amanullah et al. (2010).

Such a multiple dipole test is intended to check the expansion center Universe (ECU) described by the fundamental equation (59) of paper I or eq. (1) of paper II, without using the expansion center model (ECM) developed in the ECM paper series and based on the Galaxy radial deceleration coefficient \( a_0 = K_0 R_0 \) of eq. (3) and (9) in paper II, with \( K_0 = \left( \frac{\delta H}{\delta r} \right)_0 = \frac{3H^2}{c^2} \) as resulting in section 4 of paper I and experimentally confirmed in the concluding section of the same paper I. In other words the present contribution confirms the Hubble ratio dipole as a model independent result, with a resulting angular coefficient which has the same value as predicted by the ECM.

The present dipole test includes also the contents of the ECM paper XII, "Evidence for a high deceleration of the relativistic Universe", presented at the European Week of Astronomy and Space Science (EWASS 2012). So paper XV is a combination of paper X with paper XII.

Let us remark that the convention \( M_B \equiv M \) is adopted in this paper XV, while the cited papers I-II-III-IV-V-VI-VII-VIII-IX-X-XI-XII-XIII-XVI are those referenced as Lorenzi 1999a→2012e.

2. The Hubble depth \( D \) from the new Hubble law

The first works on the expansion center Universe (ECU: Lorenzi 1989-91-93) dealt with the trigonometric distance \( r \) as the classic separation between our Milky Way and the nearby galaxies / groups / clusters / superclusters, at depths \( z \lesssim 0.1 \), with luminosity distances \( D_L \equiv r \) assumed. Such an approximation was more suitable in the ECM papers I-II (1999ab), where the light-space \( r \) of galaxies / groups / clusters with \( z \lesssim 0.03 \) was fixed mathematically as

\[
  r = -c(t - t_0)
\]

that is the distance covered by light at a constant speed \( c \) during the whole travel time, from the emission epoch \( t_e = t \) to the present epoch \( t_0 \). Since \( c \) is constant, in eq. (1) \( r \) should correspond to the source distance at the emission epoch. However the "cosmic medium" (CM), with respect to
which light moves at speed $c$, is expanding as does the whole Universe; as a result, the light-space $r$ is larger than the distance at the emission epoch, although its value in light-time represents a measure of that past epoch $t$. Now, while $r$ is unknown, its derivative with respect to the emission time $t$, after putting $dt = \lambda/c$ and $dt_0 = \lambda_0/c$ with $\Delta \lambda = \lambda_0 - \lambda$, becomes the observed and well-known $cz$ (cf. papers V, VIII and IX), as follows:

$$\dot{r} = \frac{dr}{dt} = c \frac{d(t_0 - t)}{dt} = c \frac{\Delta \lambda}{\lambda} = cz$$

(2)

Such $\dot{r} = cz$ enters the ECU new Hubble law (cf. papers I→IX) in Hubble units:

$$\dot{r} = Hr + \Delta H \cdot (r - R \cos \gamma) + R \dot{w} \sin \gamma$$

(3)

Specifically $\gamma$ is the angle between the direction of the huge void center $VC$ ($\alpha_{VC} \approx 9^h; \delta_{VC} \approx +30^\circ$; Bahcall & Soneira 1982), also called expansion center or Big Bang central point (Lorenzi 1989-91-93) distant $R_0$ from the Local Group (LG) at our epoch, and that $\langle \dot{w} \rangle = 0$ (cf. section 7.4 in paper I), can be expressed by an alternative formulation, which easily leads to the Hubble depth $D$. In this case, putting $H_* = H + \Delta H$, eq. (3) may be rewritten in terms of the following sequence:

$$cz = rH_* - R\Delta H \cos \gamma = DH_0 - R\Delta H \cos \gamma \Rightarrow r(H + \Delta H) = DH_0 \Rightarrow$$

$$\Delta H = \Delta H(D) \Rightarrow R\Delta H = a^*(D) \cdot D \Rightarrow c = D \cdot H_0 - a^*(D) \cdot D \cdot \cos \gamma$$

(4)

$$rH_* \equiv DH_0$$

(5)

The identity (6) of the two products shows that here we have two physical quantities, $r$ and $H_0$, and two apparent quantities, $H_*$ and $D$. Consequently, as $H_*$ is the apparent Hubble constant of the observed source at the emission epoch $t$, similarly $D$ results to be the apparent distance of the observed source at the present epoch $t_0$, that is, the Hubble depth $D$ of the ECU Hubble law, according to the formulae reported below. As $H_* > H$, thus $D > D_0$, where $D_0$ represents the proper distance of the observed source at the present epoch $t_0$, while $H$ represents the physical Hubble constant of the observed source at the emission epoch $t$. Based on the canonical terminology, one should distinguish the use of "proper" and "physical" as quantity
adjectives. Indeed the problem is with light-space, which, when considered as the proper distance \( r_{pr} \) referring to the emission epoch \( t \), is shorter than the physical distance \( r \) run by light during the whole travel time. This is the reason for giving the adjective ”physical” to the light-space \( r \), according to the identity (6).

Now, coming back to eqs. (3)(4)(5), here is the formulation for the wedge-shape of the ECU Hubble law, or new Hubble D law (7), with a few specifications:

\[
\begin{align*}
    cz & = [H_0 - a^*(D) \cos \gamma] \cdot D \Rightarrow c |\Delta z|^{\text{max}} = 2a^*D \quad (7) \\
    H_X & = H_0 - a^*(D) \cos \gamma \\
    a^* \neq 0 \quad \text{and} \quad \cos \gamma = 0 \Rightarrow cz = cz_0 = H_0D \quad (9) \\
    D & = cz/H_X = cz_0/H_0 \\
    a^* \equiv 0 \Rightarrow H_X = H_0 \quad (11)
\end{align*}
\]

Hence the new depth law clearly shows the anisotropic behaviour of \( H_X \), whose variability (owing to \( a^* \cos \gamma \)) is responsible for the wedge shape of the Hubble diagram. Furthermore eq. (10) allows us to represent the Hubble depth \( D \) according to the powerful \( h \) convention, as follows:

\[
\begin{align*}
    D & = dh^{-1}Mpc \quad \text{being} \quad d = \frac{cz}{100 \text{ km s}^{-1} \text{Mpc}^{-1}} \quad \text{and} \quad h = \frac{H_X}{100 \text{ km s}^{-1} \text{Mpc}^{-1}} \\
    H_0 & \equiv 70 \text{ km s}^{-1} \text{Mpc}^{-1} \quad \text{and} \quad \langle z \rangle \equiv z_0 = 1.000 \Rightarrow \cos \gamma = 0 \Rightarrow D = \frac{c}{H_0} = 4283 \text{ Mpc} \quad (13)
\end{align*}
\]

3. Two dipole tests on SCP supernovae at \( \langle z \rangle = 1.0 \)

After a long series of successful dipole tests on the nearby Universe (Lorenzi 1991-93-94-99ab-2003b), from historic data sets of about half a century, and that carried out on 53 SCP SNe Ia ranging around \( \langle z \rangle = 0.5 \) (ECM paper VI based on data by Perlmutter at al. 1999 and Knop et al. 2003), here a crucial multiple dipole test at \( z \) bins with \( \langle z \rangle \equiv z_0 = 1.0 \) or Hubble depth \( D(z_0) = c/H_0 = 4283 \text{ Mpc} \) is carried out on SNe Ia data and references from SCP Union compilation (SCP U : Kowalski et al. 2008) and SCP Union2 (SCP U2 : Amanullah et al. 2010), including the results obtained within ”The new wedge-shaped Hubble diagram of 398 SCP supernovae according to the expansion center model” (ECM paper IX: SAIt2010 in Naples).
Assuming directly \( H_0 \equiv 70 \) Hubble units, as the conventional ECM Hubble constant derived from the value \( H_0 = 69.8 \pm 2.8 \) H.u. in 1999 paper II and based on data from Sandage & Tammann (1975), means limiting the fitting of the ECM dipole formula (8) of paper VI to one unknown, through the first eq. (7) rewritten in the form

\[
Y = a^*(D) \cdot (-\cos \gamma) \quad \text{with} \quad Y = \frac{cz}{D} - H_0
\]

(14)

where the supernova Hubble depth \( D \) is computed as the ratio between the cosmological distance \( D_C \) and \( 1 + z \), with \( D_C \equiv D_L \) assumed (cf. papers V-VI-IX), that is by taking the position

\[
D_C = D \cdot (1 + z) \equiv D_L
\]

(15)

and a consequent magnitude formula that we call the Hubble Magnitude (cf. section 3.1 and section 5), while the SNe angular position with respect to the huge void center \( VC \) is expressed by the usual \( \cos \gamma \) formula:

\[
\cos \gamma = \sin \delta_V \sin \delta + \cos \delta_V \cos \delta \cos(\alpha - \alpha_V)
\]

(16)

Table 0 presents the 4 pilot samples from which the useful samples at \( \langle z \rangle = 1.0 \) were extracted. These pilot samples exclude \( z \leq 0.2 \) in order to eliminate the CMB reference (cf. section 3 of paper IX). In particular XVI refers to 249 SNe Ia selected by the SCP Union, those listed in the Appendix of paper IX and lying within the Hubble depth range \( 800 \, Mpc < D < 8000 \, Mpc \). XVII refers to all the 283 SNe Ia at \( z > 0.2 \) of the SCP Union. XVIII refers to 338 SNe Ia selected by the SCP Union2, also at \( z > 0.2 \). XIX refers to all the 359 SNe Ia at \( z > 0.2 \) of the SCP Union2. All the selected data ranging around \( \langle z \rangle = 1.0 \), both from the SCP Union and SCP Union2, have been listed in Table 5abc of section 3.4, as they refer to two main samples, the XVI with 48 \( SCPU \) SNe Ia and the XVIII with 58 \( SCPU2 \) SNe Ia, respectively.

**Table 0**

| Pilot sample | source | N  | \( z \) bin               |
|--------------|--------|----|---------------------------|
| XVI          | Kowalski et al. 2008 (\( SCPU \)) | 249 | 0.200 < \( z \) \leq 1.390 |
| XVII         | Kowalski et al. 2008 (\( SCPU \)) | 283 | 0.200 < \( z \) \leq 1.551 |
| XVIII        | Amanullah et al. 2010 (\( SCPU2 \)) | 338 | 0.200 < \( z \) \leq 1.400 |
| XIX          | Amanullah et al. 2010 (\( SCPU2 \)) | 359 | 0.200 < \( z \) \leq 1.400 |
3.1 1\textsuperscript{st} dipole test: $M = d_0 + d_1 D + d_2 D^2$

The first-type dipole test refers to the pilot samples XVI and XVII of Table 0. Indeed the construction of the new wedge-shaped Hubble diagram, in paper IX, was based on the rigorous coincidence of the SNe Ia Hubble magnitude, as $M = m - 5 \log D_C - 25$, with its computed average trend, $\langle M \rangle = d_2 D^2 + d_1 D + d_0$, according to the adopted relation (24) in paper IX, that is

$$M = m - 5 \log [D \cdot (1 + z)] - 25 \equiv d_2 D^2 + d_1 D + d_0 = \langle M \rangle$$  \hspace{1cm} (17)

We define Hubble Magnitude as the quantity $(m - 5 \log [D \cdot (1 + z)] - 25)$ in H.u., where $D$ is the Hubble depth representing the apparent distance at the present epoch $t_0$ in the Hubble diagram, according to the position (15) and eq. (12).

The introduction of the values $d_0, d_1, d_2, z, m$ in eq. (17) (assuming $d_0 = -18.77; d_1 = -1.421 \cdot H_0/c; d_2 = +0.3589 \cdot H_0^2/c^2$ from paper IX), gave the Hubble depth $D$ through a numerical solution point by point. Hence, after calculating each $\cos \gamma$ according to eq. (16), we obtain $Y$ and a system of equations (14), whose solution by means of the least square method gives the unknown angular coefficient $a^*$. In synthesis the first dipole test may be summarized as follows:

$$[d_0, d_1, d_2, z, m] \Rightarrow D \hspace{1cm} [z, D] \Rightarrow Y \hspace{1cm} \cos \gamma \Rightarrow [Y \Rightarrow (- \cos \gamma)] \Rightarrow a^*$$  \hspace{1cm} (18)

The above procedure is then applied to 10 $z$ bins of the pilot sample XVI ($XVI_{1 \rightarrow 10}$), those having $\langle z \rangle \equiv z_0 = 1.0$, and to the first $z$ bin of the pilot sample XVII ($XVII_1$)(cf. Tables 1ab-2).

3.2 2\textsuperscript{nd} dipole test: $M = M(s_{Min})$

The second-type dipole test has already been applied on the SCP supernovae of paper VI, where the adopted value of the Hubble Magnitude $M_B$ is that minimizing the standard deviation of the unweighted least square fitting. In this case we derive the value of the Hubble depth $D$, to be introduced in eq. (14) for each sample analysed, as follows:

$$[\gamma, z, m, M = M(s_{Min})] \Rightarrow D = \frac{10^{0.2[m - M(s_{Min})]} - 5}{1 + z} \Rightarrow [Y \Rightarrow (- \cos \gamma)] \Rightarrow a^*$$  \hspace{1cm} (19)

Here the test regards all 4 pilot samples XVI-XVII-XVIII-XIX of Table 0; in particular the fitting according to (19) has been carried out both on the 4 primary samples of Table 1a (XVI\textsubscript{1} - XVII\textsubscript{1} - XVIII\textsubscript{1} - XIX\textsubscript{1}: cf. Table 1b), and on a further 18 encapsulated $z$ bins (XVI\textsubscript{2}→10 - XVIII\textsubscript{2}→10: cf. Tables 3-4).
3.3 Solution

All the sample features and results of the above 1st and 2nd dipole test appear in the following Tables 1ab-2-3-4.

Specifically, the main features of the 4 primary samples at \( \langle z \rangle = 1.0 \) are listed in Table 1a, with the following data: Sample ordinal number; number N of supernovae of the sample; sample z bin; mean \( \langle z \rangle \) of the z bin; unweighted mathematical mean \( \langle m_B^{\text{max}} \rangle \) of the sample SNe magnitudes; mean \( \langle \cos \gamma \rangle \) of the sample SNe \( \cos \gamma \).

### Table 1a

| Sample | N   | \( z \) bin       | \( \langle z \rangle \) | \( \langle m_B^{\text{max}} \rangle \) | \( \langle \cos \gamma \rangle \) |
|--------|-----|-------------------|-------------------------|----------------------------------------|----------------------------------|
| XVI    | 48  | \( 0.830 \leq z \leq 1.390 \) | 1.001                   | 24.84                                   | +0.29                            |
| XVII   | 64  | \( 0.811 \leq z \leq 1.400 \) | 0.992                   | 24.81                                   | +0.27                            |
| XVIII  | 58  | \( 0.812 \leq z \leq 1.400 \) | 0.995                   | 24.82                                   | +0.24                            |
| XIX    | 62  | \( 0.812 \leq z \leq 1.400 \) | 0.996                   | 24.80                                   | +0.27                            |

The dipole test on the above 4 primary samples of Table 1a was made possible thanks to data and references taken from the SCP papers.

The fitting results are shown in Table 1b, where two 1st type primary sample tests and four 2nd type primary sample tests are presented numerically. In particular the 8 columns of Table 1b present the following data: Test identification name (TID); sample ordinal number; number N of the sample supernovae; the fitting standard deviation \( s \) in H.u. that results either from the Hubble Magnitude values provided by the function \( M(D) \) according to the paper IX parameters or as the minimum value of the standard deviation \( s_{\text{Min}} \), corresponding to the listed Hubble Magnitude \( M(s_{\text{Min}}) \); the function \( M(D) \) or the \( M \) value which minimizes the standard deviation \( s \) in the dipole least square fitting; average Hubble depth \( \langle D \rangle \) of the sample SNe whose individual \( D \) come from the previous \( M \) value; the resulting angular coefficient \( a^* \) of eq. (14) with its standard deviation; the correlated maximum \( cz \) range of the fitted sample, obtained by \( c|\Delta z|^{\text{max}} = 2a^*D \) with \( D = 4283 \, \text{Mpc} \) as the adopted Hubble depth at the central redshift \( z_0 = 1.000 \).
Table 1b

| TID | Sample | N  | s          | M            | ⟨D⟩  | a*            | c|Δz|_{max} |
|-----|--------|----|------------|--------------|------|---------------|-----------|
| A1  | XVI1   | 48 | 10.374     | d₀ + d₁D + d₂D² | 4419 | 6.0 ± 2.5     | ≈ 51000   |
| B1  | XVI1   | 48 | s_{Min} = 7.2108 | M(s_{Min}) = −19.89 | 4477 | 5.5 ± 1.7     | ≈ 47000   |
| C1  | XVII1  | 64 | 18.654     | d₀ + d₁D + d₂D² | 4427 | 4.8 ± 3.8     | ≈ 41000   |
| D1  | XVII1  | 64 | s_{Min} = 12.779 | M(s_{Min}) = −19.95 | 4614 | 5.9 ± 2.6     | ≈ 51000   |
| E1  | XVIII1 | 58 | s_{Min} = 7.4327 | M(s_{Min}) = −19.87 | 4420 | 5.1 ± 1.7     | ≈ 44000   |
| F1  | XIX1   | 62 | s_{Min} = 7.7170 | M(s_{Min}) = −19.89 | 4432 | 4.3 ± 1.7     | ≈ 37000   |

Table 1b demonstrates the full success of this dipole test at ⟨z⟩ = 1.00. Clearly the lower standard deviation s indicate B1 and E1 as being the best test fittings. It is remarkable that the mathematical mean of the 6 angular coefficients a* coming from the 6 primary tests of Table 1b gives ⟨a*⟩ = 5.3 ± 0.3 H.u. as a model independent result, in full accordance with the value a^{ECM}_* = 5.46 predicted by the expansion center model (cf. section 4). Consequently the maximum cz range of the resulting dipole nears a value of about 50000 km s⁻¹, as the last column of c|Δz|_{max} certifies.

A further remark on Table 1b must be made about the scattering between the resulting mean ⟨D⟩ and the value of the Hubble depth D = c/H₀ at z₀ = 1. Such a scattering is clearly tied to the solution in the 5th column, whose M values in modulus result to be sensibly greater (at minimum ∼ 0.05 magnitudes) than both M = ⟨m_B^{max}⟩ − 5 log(2c/H₀) − 25 ≅ −19.82 after introducing ⟨m_B^{max}⟩ = 24.84 and M = d₀ + d₁D + d₂D² = −19.83 being D = 4283 Mpc. Its entity, as ΔD = ⟨D⟩ − D with ΔD ≈ 0.04, may be due to some small systematic perturbation or distortion effect (cf. section 3.1 of the parallel paper XVI).

The following Tables 2-3-4 present another 27 sample tests, divided into 3 groups of 9 encapsulated z bins; each group comes from the primary sample tests, A1, B1, E1 of Table 1b. Therefore the column contents maintain the same headings as Table 1. However Table 2 lacks the M column because of its variable value as M(D).
Table 2

| TID | Sample | N  | \(z\) bin | \(\langle z \rangle\) | \(s\)  | \(a^*\)  | \(c|\Delta z|^{\text{max}}\) |
|-----|--------|----|------------|----------------|------|--------|----------------|
| A2  | XVI\(_2\) | 45 | 0.833 \(\leq z \leq 1.37\) | 1.000 | 10.625 | 6.3 \(\pm 2.6\) | \(\approx 54000\) |
| A3  | XVI\(_3\) | 42 | 0.84 \(\leq z \leq 1.35\) | 0.999 | 10.703 | 7.1 \(\pm 2.7\) | \(\approx 61000\) |
| A4  | XVI\(_4\) | 39 | 0.85 \(\leq z \leq 1.31\) | 0.998 | 11.046 | 7.5 \(\pm 2.9\) | \(\approx 64000\) |
| A5  | XVI\(_5\) | 36 | 0.86 \(\leq z \leq 1.30\) | 0.998 | 11.443 | 7.1 \(\pm 3.2\) | \(\approx 61000\) |
| A6  | XVI\(_6\) | 33 | 0.87 \(\leq z \leq 1.27\) | 0.997 | 11.895 | 7.1 \(\pm 3.3\) | \(\approx 61000\) |
| A7  | XVI\(_7\) | 30 | 0.88 \(\leq z \leq 1.23\) | 0.996 | 11.791 | 6.3 \(\pm 3.3\) | \(\approx 54000\) |
| A8  | XVI\(_8\) | 27 | 0.90 \(\leq z \leq 1.20\) | 0.996 | 11.502 | 4.3 \(\pm 3.5\) | \(\approx 37000\) |
| A9  | XVI\(_9\) | 24 | 0.92 \(\leq z \leq 1.15\) | 0.995 | 11.474 | 2.0 \(\pm 3.7\) | \(\approx 17000\) |
| A10 | XVI\(_{10}\) | 21 | 0.93 \(\leq z \leq 1.13\) | 0.985 | 10.880 | 1.7 \(\pm 3.8\) | \(\approx 15000\) |

Table 3

| TID | Sample | N  | \(z\) bin | \(\langle z \rangle\) | \(s_{\text{Min}}\) | \(M(s_{\text{Min}})\) | \(a^*\)  | \(c|\Delta z|^{\text{max}}\) |
|-----|--------|----|------------|----------------|-----------|----------------|--------|----------------|
| B2  | XVI\(_2\) | 45 | 0.833 \(\leq z \leq 1.37\) | 1.000 | 7.385 | -19.89 | 5.7 \(\pm 1.8\) | \(\approx 49000\) |
| B3  | XVI\(_3\) | 42 | 0.84 \(\leq z \leq 1.35\) | 0.999 | 7.609 | -19.88 | 5.6 \(\pm 1.9\) | \(\approx 48000\) |
| B4  | XVI\(_4\) | 39 | 0.85 \(\leq z \leq 1.31\) | 0.998 | 7.756 | -19.89 | 6.5 \(\pm 2.1\) | \(\approx 56000\) |
| B5  | XVI\(_5\) | 36 | 0.86 \(\leq z \leq 1.30\) | 0.998 | 7.859 | -19.89 | 5.8 \(\pm 2.2\) | \(\approx 50000\) |
| B6  | XVI\(_6\) | 33 | 0.87 \(\leq z \leq 1.27\) | 0.997 | 8.179 | -19.89 | 5.7 \(\pm 2.3\) | \(\approx 49000\) |
| B7  | XVI\(_7\) | 30 | 0.88 \(\leq z \leq 1.23\) | 0.996 | 7.729 | -19.91 | 5.6 \(\pm 2.2\) | \(\approx 48000\) |
| B8  | XVI\(_8\) | 27 | 0.90 \(\leq z \leq 1.20\) | 0.996 | 7.395 | -19.92 | 4.2 \(\pm 2.2\) | \(\approx 36000\) |
| B9  | XVI\(_9\) | 24 | 0.92 \(\leq z \leq 1.15\) | 0.995 | 7.428 | -19.93 | 3.4 \(\pm 2.4\) | \(\approx 29000\) |
| B10 | XVI\(_{10}\) | 21 | 0.93 \(\leq z \leq 1.13\) | 0.985 | 6.786 | -19.93 | 4.0 \(\pm 2.4\) | \(\approx 34000\) |
Table 4

| TID | Sample | N   | z bin | \langle z \rangle | s_{Min} | M(s_{Min}) | a^* | c|\Delta z|^{\text{max}} |
|-----|--------|-----|-------|------------------|--------|-----------|-----|----------------|
| E2  | XVIII2 | 54  | 0.817 \leq z \leq 1.39 | 0.998  | 6.787  | -19.88 | 4.6 \pm 1.6 | \approx 39000 |
| E3  | XVIII3 | 50  | 0.823 \leq z \leq 1.38 | 1.001  | 6.478  | -19.90 | 5.9 \pm 1.6 | \approx 51000 |
| E4  | XVIII4 | 46  | 0.833 \leq z \leq 1.33 | 0.993  | 6.616  | -19.89 | 6.0 \pm 1.7 | \approx 51000 |
| E5  | XVIII5 | 42  | 0.845 \leq z \leq 1.33 | 1.007  | 6.689  | -19.90 | 7.1 \pm 1.8 | \approx 61000 |
| E6  | XVIII6 | 38  | 0.860 \leq z \leq 1.30  | 1.000  | 6.574  | -19.91 | 6.8 \pm 1.9 | \approx 58000 |
| E7  | XVIII7 | 34  | 0.873 \leq z \leq 1.27  | 1.003  | 6.740  | -19.93 | 7.3 \pm 2.0 | \approx 63000 |
| E8  | XVIII8 | 30  | 0.885 \leq z \leq 1.20  | 0.995  | 5.646  | -19.95 | 5.7 \pm 1.8 | \approx 49000 |
| E9  | XVIII9 | 26  | 0.920 \leq z \leq 1.15  | 0.998  | 5.770  | -19.95 | 4.8 \pm 2.0 | \approx 41000 |
| E10 | XVIII10| 22  | 0.930 \leq z \leq 1.12  | 0.983  | 5.078  | -19.95 | 4.9 \pm 1.9 | \approx 42000 |

Again, the mathematical mean of the angular coefficient \(a^*\) of these 27 further dipole tests is 
\[
\langle a^* \rangle = 5.52 \pm 0.30 \text{ H.u., while all 33 } a^* \text{ values listed in Tables 1b-2-3-4 give the mean}
\]
\[
\langle a^* \rangle = 5.47 \pm 0.25 \text{ km s}^{-1}\text{ Mpc}^{-1}
\]

as the final result of this model independent dipole test.

Indeed the resulting value 5.47 of \(\langle a^* \rangle\) coincides perfectly with the value \(a^*_{ECM} = 5.46\) furnished by the expansion center model at the central redshift \(z_0 = 1\) (cf. the next section 4.1). Consequently, the new Hubble D law of eq. (7) gives a corresponding mean for the maximum \(cz\) range of the SNe Ia cz dipole at \(D = 4283 \text{ Mpc}\), that is

\[
\langle c|\Delta z|^{\text{max}} \rangle = 2D\langle a^* \rangle = 46900 \pm 2200 \text{ km s}^{-1}
\]
3.4 Tabled data set of 2 SCP SNe Ia samples at $\langle z \rangle = 1.00$

Tables 5a-5b-5c list a basic data set of this crucial dipole test of the expansion center Universe; this base set, which concerns two primary samples of Table 1a, XVI and XVIII - of 48 SCP U SNe Ia and 58 SCP U2 SNe Ia respectively - , lists all the data needed to check ten 1st type dipole tests and twenty 2nd type dipole tests. We must point out that the last column of Tables 5a and 5b list the $Y$ values of the Test A1 supernovae, as $Y_{A1} = c\bar{z} - H_0$ where $c\bar{z}$ and $D$ appear in Table 3abcdefghi of the previous ECM paper IX, while Table 5c refers only to a further 12 SNe Ia data, those necessary to complete the 2nd type Test E1→E10. These listed values of $Y_{A1}$ are also useful for checking the other 9 dipole tests, corresponding to 9 encapsulated $z$ bins and called A2-A3-A4-A5-A6-A7-A8-A9-A10 in Table 2. Of course the $Y$ values of the B and E series do not appear in Table 5 because their values change according to the variable $M(s_{Min})$ solution of each corresponding 2nd type dipole test. Specifically, the columns of Table 5a and Table 5b report in order the following data: Supernova name according to the reference number of column 4th; right ascension $\alpha$ and declination $\delta$ as given in the reference in column 4; footnote reference number of the work in which the previous tabled values of right ascension (R.A.) and declination (Decl.) appear; redshift $z$ of supernova or host galaxy as listed in the SCP papers, but rounded off to the third decimal place as the CMB reference likely affects the value for about 0.001 on average; supernova magnitudes $m_{SCP U}$ and $m_{SCP U2}$ as $m_{B}^{\text{max}}$ values listed in the SCP Union compilation (SCP U : Kowalski et al. 2008) and in SCP Union2 (SCP U2 : Amanullah et al. 2010); $-\cos \gamma$ value of the supernova according to eq. (16) computed using the supernova astronomical coordinates ($\alpha, \delta$) listed on the Internet (ref. 1: Harvard-IAU 2003) or in the reference papers cited in the footnote to Table 5c; $Y_{A1}$ value as above explained.

The dipole diagrams referring to the listed data in Table 5abc, those of the Tests A1-B1-E1 of Table 1b, are reported in Figures 1-2-3, respectively. These 3 primary sample tests are graphically presented by as many dipole plots of $Y = c\bar{z} - H_0$ against the ($-\cos \gamma$) value of each corresponding supernova. In the cartesian plane $(x, y)$ of Figures 1-2-3 the resulting fitting equations, as $y = f(x)$, are included together with the value of the coefficient of determination $R^2$. Note that the mean of the 3 $a^*$, as $\langle a^* \rangle = 5.5 \pm 0.3$ H.u., coincides with the mean (20).
| Name | R.A. | Decl. | ref. | $z_{SCP}$ | $m_{SCP}$ | $m_{SCP}^2$ | $- \cos \gamma$ | $Y_{A1}$ |
|------|------|-------|------|-----------|----------|-----------|----------------|---------|
| 1997ck | 16 53.0 | +35 04 | 1 | 0.970 | 24.72 | 24.69 | -0.0482 | +1.7 |
| 1997ap | 13 47.2 | +02 24 | 1 | 0.830 | 24.34 | 24.31 | -0.2912 | +1.2 |
| 1999fm | 02 30.6 | +01 10 | 1 | 0.950 | 24.30 | 24.22 | +0.1006 | +21.7 |
| 1999fk | 02 28.9 | +01 16 | 1 | 1.057 | 24.77 | 24.74 | +0.1061 | +10.4 |
| 2002aa | 07 48.8 | +10 18 | 1 | 0.946 | 24.60 | 24.69 | -0.0482 | +1.7 |
| 2002x | 08 48.5 | +02 24 | 1 | 0.830 | 24.34 | 24.31 | -0.2912 | +1.2 |
| 2002w | 08 47.9 | +01 16 | 1 | 1.057 | 24.77 | 24.74 | +0.1061 | +10.4 |
| 2001kd | 07 50.5 | +10 21 | 1 | 0.936 | 24.96 | 24.82 | -0.9007 | +4.5 |
| 2001jm | 04 39.2 | -01 33 | 1 | 0.978 | 24.50 | 24.44 | -0.3511 | +14.3 |
| 2001jh | 02 29.0 | +00 21 | 1 | 0.859 | 24.73 | 24.68 | -0.9684 | -12.7 |
| 2001hu | 07 50.6 | +09 58 | 1 | 0.882 | 24.91 | 24.71 | -0.9007 | -16.5 |
| 2001hs | 04 39.4 | -01 33 | 1 | 0.833 | 24.26 | 24.22 | -0.3503 | +5.5 |
| 2001fs | 04 39.5 | -01 28 | 1 | 0.874 | 25.12 | 25.07 | -0.3514 | -24.1 |
| 1997ek | 04 56.2 | -03 41 | 1 | 0.860 | 24.48 | 24.47 | -0.3875 | -1.3 |
| 04Eag | 12:37:20.75 | +62:13:41.50 | 2 | 1.020 | 24.97 | 24.93 | -0.6777 | -4.0 |
| 04Gre | 03:32:21.49 | -27:46:58.30 | 2 | 1.140 | 24.73 | 24.75 | +0.1253 | +24.5 |
| 04Man | 12:36:34.81 | +62:15:49.06 | 2 | 0.854 | 24.53 | 24.55 | -0.6786 | -4.4 |
| 04Mcg | 03:32:10.02 | -27:49:49.98 | 2 | 1.370 | 25.73 | 25.64 | +0.1263 | -2.4 |
| 04Omb | 03:32:25.34 | -27:45:03.01 | 2 | 0.975 | 24.88 | 24.88 | +0.1248 | -5.1 |
| 04Pat | 12:38:09.00 | +62:18:47.24 | 2 | 0.970 | 25.02 | 24.99 | -0.6762 | -11.5 |
| 04Sas | 12:36:54.11 | +62:08:22.76 | 2 | 1.390 | 25.82 | 26.00 | -0.6786 | -4.4 |
| 05Fer | 12:36:25.10 | +62:15:23.84 | 2 | 1.020 | 24.83 | 24.80 | -0.6789 | +2.6 |
| 05Gab | 12:36:13.83 | +62:12:07.56 | 2 | 1.120 | 25.07 | 25.09 | -0.6794 | +2.0 |
| 05Lan | 12:36:56.72 | +62:12:53.33 | 2 | 1.230 | 26.02 | 26.05 | -0.6783 | -22.4 |
Table 5b

| Name  | R.A.     | Decl.     | ref. | $z_{SCP}$ | $m_{SCP}$ | $m_{SCP2}$ | $- \cos \gamma$ | $Y_{A1}$ |
|-------|----------|-----------|------|----------|-----------|------------|----------------|----------|
| 05Red | 12:37:01.70 | +62:12:23.98 | 2    | 1.190    | 25.76     | 25.63      | -0.6782        | -18.0    |
| 05Spo | 12:37:06.53 | +62:15:11.70 | 2    | 0.839    | 24.20     | 24.15      | -0.6799        | +9.5     |
| 05Str | 12:36:20.63 | +62:10:50.58 | 2    | 1.010    | 25.03     | 24.92      | -0.6793        | -7.7     |
| 2002dd | 12:36.9   | +62:13    | 1    | 0.950    | 24.66     | 24.61      | -0.6784        | +2.0     |
| 2002fw | 03:32.6   | -27:47    | 1    | 1.300    | 25.65     | 25.65      | +0.1244        | -5.3     |
| 2002hp | 03:32.4   | -27:46    | 1    | 1.305    | 25.41     | 25.51      | +0.1250        | +6.1     |
| 2002ki | 12:37.5   | +62:21    | 1    | 1.140    | 25.35     | 25.37      | -0.6770        | -7.8     |
| 2003az | 12:37.3   | +62:19    | 1    | 1.265    | 25.68     | 25.73      | -0.6774        | -9.5     |
| 2003dy | 12:37.2   | +62:11    | 1    | 1.340    | 25.77     | 25.70      | -0.6780        | -6.6     |
| 2003eq | 12:37.8   | +62:14    | 1    | 0.840    | 24.35     | 24.34      | -0.6770        | +2.1     |
| 04D4bk | 22:15:07.681 | -18:03:36.79 | 3    | 0.840    | 24.31     | 24.32      | +0.9345        | +4.1     |
| 04D3nr | 14:22:38.526 | +52:11:15.06 | 3    | 0.960    | 24.54     | 24.56      | -0.4827        | +9.5     |
| 04D3ki | 14:19:34.598 | +52:17:32.61 | 3    | 0.930    | 24.87     | 24.69      | -0.4885        | -9.8     |
| 04D3cp | 14:20:23.954 | +52:49:15.45 | 3    | 0.830    | 24.24     | 24.11      | -0.4884        | +6.1     |
| 04D4dw | 22:16:44.667 | -17:50:02.38 | 3    | 0.961    | 24.57     | 24.55      | +0.9317        | +8.1     |
| 04D3lp | 14:19:50.911 | +52:30:11.88 | 3    | 0.983    | 24.93     | 24.97      | -0.4886        | -6.4     |
| 03D4cy | 22:13:40.441 | -17:40:54.12 | 3    | 0.927    | 24.72     | 24.55      | +0.9347        | -3.7     |
| 03D1ew | 02:24:14.079 | -04:39:56.93 | 3    | 0.868    | 24.37     | 24.31      | +0.1748        | +5.1     |
| 04D3dd | 14:17:48.411 | +52:28:14.57 | 3    | 1.010    | 25.12     | 24.88      | -0.4931        | -11.4    |
| 03D4di | 22:14:10.249 | -17:30:24.18 | 3    | 0.905    | 24.29     | 24.24      | +0.9335        | +15.0    |
| 03D4cx | 22:14:33.754 | -17:35:15.35 | 3    | 0.949    | 24.50     | 24.47      | +0.9333        | +10.1    |
| 04D3ml | 14:16:39.095 | +53:05:35.89 | 3    | 0.950    | 24.55     | 24.51      | -0.4976        | +7.6     |
| 04D3gx | 14:20:13.666 | +52:16:58.33 | 3    | 0.910    | 24.71     | 24.67      | -0.4870        | -5.3     |
| 03D1cm | 02:24:55.294 | -04:23:03.61 | 3    | 0.870    | 24.46     | 24.54      | +0.1699        | +1.0     |
Table 5c

| Name   | R.A.   | Decl.  | ref. | \( z_{SCP} \) | \( m_{SCP/2} \) | \(- \cos \gamma\) |
|--------|--------|--------|------|---------------|----------------|------------------|
| 2001jf | 02 28.1| +00 27 | 1    | 0.815         | 24.83          | +0.1162          |
| 2001hy | 08 49.8| +44 15 | 1    | 0.812         | 24.86          | -0.9686          |
| 04D3lu | 14:21:08.009 | +52:58:29.74 | 1 | 0.822 | 24.34 | -0.4872 |
| 04D3nc | 14:16:18.224 | +52:16:26.09 | 3 | 0.817 | 24.24 | -0.4959 |
| 03D4cn | 22:16:34.600 | -17:16:13.55 | 3 | 0.818 | 24.63 | +0.9297 |
| 1999fj | 02 28.4 | +00 39 | 1 | 0.816 | 24.17 | +0.1134 |
| 2002fx | 03 32.1 | -27 45 | 1 | 1.400 | 25.65 | +0.1258 |
| 2003aj | 03 32.7 | -27 55 | 1 | 1.307 | 26.97 | +0.1253 |
| 2003XX | 12:37:29.00 | +62:11:27.8 | 4 | 0.935 | 24.48 | -0.6776 |
| 2001cw | 15 23.1 | +29 40 | 1 | 0.953 | 24.71 | -0.1718 |
| 2001gn | 14 02.0 | +05 05 | 1 | 1.124 | 25.37 | -0.2603 |
| 2001hb | 13 57.2 | +4 20 | 1 | 1.030 | 24.79 | -0.2715 |

R.A.&Decl. references in Table 5abc:

1 Harvard-IAU, [http://cfa-www.harvard.edu/iau/lists/Supernovae.html](http://cfa-www.harvard.edu/iau/lists/Supernovae.html)
2 Riess, A.G. et al. (2007) as referenced by Kowalski, M. et al. (2008)
3 Astier, P. et al. (2006) as referenced by Kowalski, M. et al. (2008)
4 Riess, A.G. et al. (2004) as referenced by Kowalski, M. et al. (2008)
4. The expansion center model reconfirmed

The solution of the expansion center model (ECM) within the very nearby Universe, using the 83 individual galaxies listed by Sandage & Tammann in their paper V (S&T 1975) - at \( z < 0.02 \) with \( \langle z \rangle \equiv z_0 = 0.0066 \) or \( D(z_0) = 28.3 \text{ Mpc} \) and single redshifts \( z \) corrected only for the Sun’s motion in the Local Group through the standard vector of 300 \( \text{km s}^{-1} \) towards \( l = 90^0; b = 0^0 \), gives the values

\[
H_0 = 70 \pm 3 \text{ km s}^{-1} \text{Mpc}^{-1} \quad a_0 \cong 12.7 \text{ km s}^{-1} \text{Mpc}^{-1} \quad R_0 = 260 \pm 22 \text{ Mpc} \tag{22}
\]

as resulting from the Hubble ratio eq. (3) of the ECM paper II (Lorenzi 1999b), here rewritten as the formulation for the wedge-shape of the new Hubble law, that is

\[
ct = [H_0 - a_{\text{ECM}}^* (D) \cdot \cos \gamma] \cdot D \tag{23}
\]

with

\[
ct = \dot{r} \quad a_{\text{ECM}}^* = a_0 (1 - x)^{\frac{3}{2}} / (1 + x) \quad D = \frac{ct}{3H_0} \left( \frac{1 + x}{1 - x} \right) \quad x = \frac{3H_0 r}{c} \quad a_0 = \frac{3H_0^2 R_0}{c} \tag{24}
\]

and

\[
\cos \gamma \equiv 0 \Rightarrow z_0 = \frac{x}{3} \left( \frac{1 + x}{1 - x} \right) \tag{25}
\]

4.1 A few ECM values at \( z_0 = 1 \)

The direct extension and application of the previous formulae to the Deep Universe with its central redshift \( z_0 = 1 \) leads to the approximate ECM values, as follows:

\[
z_0 = 1 \Rightarrow x^2 + 4x - 3 = 0 \Rightarrow x = 0.6457513 \Rightarrow r \cong 922 \text{ Mpc} \tag{26}
\]

\[
z_0 = 1 \Rightarrow D = \frac{c}{H_0} \Rightarrow D \cong 4283 \text{ Mpc} \tag{27}
\]

\[
z_0 = 1 \Rightarrow a_{\text{ECM}}^*(x) = a_{\text{ECM}}^*(D) \cong 5.46 \text{ km s}^{-1} \text{Mpc}^{-1} \tag{28}
\]

\[
z_0 = 1 \Rightarrow c|\Delta z|_{\text{max}} = 2a_{\text{ECM}}^* \cdot D \cong 46770 \text{ km s}^{-1} \tag{29}
\]

In addition, recalling the formulae (cf. papers I-III)

\[
R = R_0 (1 - x)^{\frac{3}{2}} \quad t = t_0 (1 - x) \quad t_0 = \frac{1}{3H_0} \tag{30}
\]

we obtain the Galaxy distance \( R \) from the expansion center at the epoch \( t \) of \( z_0 = 1 \) :

\[
z_0 = 1 \Rightarrow R = 184 \pm 16 \text{ Mpc} \text{ at the epoch } t \approx 1.7 \times 10^9 \text{ years} \tag{31}
\]
4.2 The ECM decelerating expansion

In conclusion the whole of the results, collected without and within the ECM in about a quarter of a century, give strong observational evidence for an expansion center Universe, radially decelerated towards the huge void center $\text{VC}$ ($\alpha_{\text{VC}} \approx 9^h; \delta_{\text{VC}} \approx +30^\circ$: Bahcall & Soneira 1982), in full accordance with what is described within the 1999 ECM paper II. That scientific evidence coming from historic and recent observational data sets shows the physical consistency of a decelerating expansion dipole, detectable at any Hubble depth $D$, both in the nearby and deep Universe, with a resulting angular coefficient $a^*$ or $a_0$ of the linear fitting of the Hubble ratio $cz/D$ plotted versus $-\cos \gamma$ or $-X$, respectively.

The angular coefficient $a_0$, which was called Galaxy radial deceleration coefficient (cf. paper II), is represented below through the multiple formulation in Hubble units as follows

$$a_0 = K_0R_0 = R_0 \left( \frac{\delta H}{\delta r} \right)_{r=0} = \frac{3H_0^2R_0}{c} = \frac{-3c}{2} \left( \frac{\delta^2 R_{MW}}{\delta r^2} \right)_{r=0} \text{ km s}^{-1} \text{ Mpc}^{-1} \tag{32}$$

Here the Galaxy radial deceleration towards the expansion center $\text{VC}$, in c.g.s. units, results to be

$$\left( \frac{\delta^2 R_{MW}}{\delta t^2} \right)_{t=t_0} = -2H_0^2R_0 \approx -8.2 \times 10^{-9} \text{ cm s}^{-2} \tag{33}$$

corresponding to the value of the cosmic matter density $\rho_0$ at our epoch $t_0$, given by the paper VII formula (21), that is

$$\rho_0 = \frac{3H_0^2}{8\pi G_0} = 2.3^{+0.7}_{-0.5} \times 10^{-28} \text{ g cm}^{-3} \tag{34}$$

where $n = V_R/\dot{V}_{ECM} = 24.8^{+4.7}_{-3.9}$, while the Galaxy radial and transversal velocities, after fixing the cosmic rotation $\dot{\vartheta}_0 = y_0H_0$ with the resulting $y_0 = 3.2^{+0.4}_{-0.3}$, take the values

$$\dot{R}_0 = H_0R_0 \approx 1.8 \times 10^9 \text{ cm s}^{-1} \quad R_0\dot{\vartheta}_0 \approx 6 \times 10^9 \text{ cm s}^{-1} \tag{35}$$

respectively (cf. formulae and numerical values in the ECM paper VII).

4.3 Measuring the deceleration parameter $q_0$

The above sections 4.1 - 4.2, following the previous crucial dipole test that has been made possible thanks to the Supernova Cosmology Project (Perlmutter et. al. 1999 - Knop et al. 2003 - Kowalski et al. 2008 - Amanullah et al. 2010), undertake another confirmation of the expansion center model (ECM), specifically remarking the strong physical evidence for a high cosmic deceleration towards the expansion center, even of the very nearby Universe, where the calculation of the
decelerating expansion dipole coefficient $a^* \equiv a_0$, the controversial value of $H_0$ and the Galaxy radial run $R_0$ follow easily from historic distance measures, such as those made over three decades from the 1950s by Hubble’s fellow scientist, Prof. Allan Sandage (Wikipedia 2011).

On the other hand, the very different context of relativistic observational cosmology includes the problem of measuring the \textbf{deceleration parameter} $q_0$, whose computed values of the past century seemed to span a positive range, according to the analysis of the Hubble diagram of rich clusters (e.g.: Sandage 1972 : $q_0 \cong +1.0 \pm 0.5$ - Rowan-Robinson 1996 : $q_0 \cong +1.6 \pm 0.4$). But in the first decade of the century the measurement and confirmation of a resulting negative value of the deceleration parameter ($\langle q_0 \rangle \approx -1$ : John 2004), based on the analysis of SCP high-$z$ SNe data within relativistic cosmology, led to the discovery of the accelerating expansion through the observation of distant supernovae.

However, it is possible to show that even the relativistic Universe of the supernovae Ia is undergoing a highly decelerating expansion, that means finding a high positive value of $q_0$, in accordance with the expansion center model. These are the contents of the ECM paper XII, presented at the meeting EWASS 2012 and here attached to the main contents of paper X, as section 5 of the new paper XV.

\section{Evidence for a high deceleration of the relativistic Universe}

A new calculation of the relativistic deceleration parameter $q_0$ is the topic of the present section, following the APPENDIX - November 2011 - "Introduction to the Hubble Magnitude and a new relativistic $q_0$" to the ECM paper X (SAIt2011 in Palermo).

\subsection{Formulation of the SNe Ia absolute magnitude}

Regarding the supernovae of the SCP Union compilation (Kowalski et al. 2008), let us proceed through a few statements which refer to a $z$-bin normal point of the redshift $z$, the corresponding intrinsic luminosity $L$ and the apparent magnitude $m$, within a classical model-independent cosmology. While the $z$-bin normal redshift $\langle z \rangle$, like the normal apparent magnitude $\langle m \rangle$, is an observed quantity $O$, and $z_0$, like $m_0 = m(z_0)$, is a calculated central value $C$ (cf. papers IX-XVI), here $O - C = 0$ is assumed to hold for a \textbf{normalized-central supernova Ia}; that means the following:

\begin{equation}
    z = \langle z \rangle \equiv z_0
\end{equation}
\[ L \equiv L(z_0) = \alpha L_0 \quad \text{with} \quad \alpha = \alpha(z_0) \geq 1 \]  
(37)

\[ z_0 \to 0 \Rightarrow L = L_0 \]  
(38)

\[ m = \langle m \rangle \equiv m_0 = -2.5 \log \frac{L}{4\pi d_L^2} + \text{const.} = -2.5 \log(\alpha L_0) + 5 \log D_L + 30 + C \]  
(39)

with \( C = \text{const.} + 2.5 \log(4\pi) \) and \( d_L = D_L \times 10^6 \equiv d_L(z_0) \) parsecs.

The absolute magnitude has by convention the formulation

\[ M_{\alpha} = -2.5 \log \frac{L}{4\pi \cdot 10^2} + \text{const.} = -2.5 \log(\alpha L_0) + 5 + C \]  
(40)

So we have

\[ M_{\alpha} = m - 5 \log D_L - 25 = M_0 - 2.5 \log \alpha \]  
(41)

being

\[ M_0 = -2.5 \log L_0 + 5 + C \]  
(42)

Hence the difference between the SNe Ia intrinsic absolute magnitude \( M_{\alpha} \) and the hypothetical absolute magnitude \( M_0 \) of the same source located at 10 parsecs at our epoch \( t_0 \) can be defined as the \( M_{\alpha} \) spread, by the formula

\[ M_{\alpha} - M_0 = -2.5 \log \alpha \]  
(43)

### 5.2 Relativistic Magnitude \( M_R(z_0) \)

Once taken into account eq. (41) as the classical formulation of the absolute magnitude in cosmology, the first problem is to give a correct formula to the luminosity distance \( D_L \). In relativistic cosmology we have

\[ D_L = D_0 \cdot (1 + z_0) \]  
(44)

where \( D_0 = D_{pr}(t_0) \) represents the proper distance in Megaparsecs at the present epoch \( t = t_0 \) of a supernova Ia, with redshift \( z = \langle z \rangle \equiv z_0 \), according to the eqs. (36)(37)(38)(39). As the relativistic formula of \( D_0 \) (cf. Coles & Lucchin 1995) is the following

\[ D_0 = \frac{cz_0}{H_0} \left[ 1 - \frac{z_0}{2} (1 + q_0) + ... \right] \]  
(45)

we obtain

\[ D_L = \frac{cz_0}{H_0} \left[ 1 + \frac{z_0}{2} (1 - q_0) + ... \right] \]  
(46)
where $c$ is the velocity of light in $km/s$, $H_0 = H(t_0)$ the Hubble constant at $t = t_0$ in $km/s/Mpc$ and $q_0$ a dimensionless deceleration parameter, that can be written without the relativistic scale factor or expansion parameter, as follows

$$q_0 = -\frac{\ddot{D}_{pr}(t_0) \cdot D_{pr}(t_0)}{\dot{D}_{pr}^2(t_0)} = -\frac{\dot{D}_0 D_0}{D_0^2}$$ (47)

after recalling the relativistic Hubble law

$$\dot{D}_{pr}(t_0) = H_0 D_0$$ (48)

and expanding the proper distance $D_{pr}(t)$ at the epoch $t$ in a power-series:

$$D_{pr}(t) = D_0 \cdot \left[ 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2(t - t_0)^2 + ... \right]$$ (49)

The introduction of $D_L$ eq. (46) into $M_\alpha(z_0)$ eq. (41) leads to the formulation of the relativistic absolute magnitude $M_R(z_0)$, that is

$$M_\alpha(z_0) \equiv M_R(z_0) = m_0 - 25 + 5 \log H_0 - 5 \log cz_0 - 1.0857 \cdot (1 - q_0) \cdot z_0 + ...$$ (50)

It is important to remark that, owing to the assumed constancy of the SNe Ia mean intrinsic luminosity, at least for $z_0 \ll 1$ or $\alpha(z_0) = 1$, the previous relativistic magnitude $M_R(z_0)$ of a supernova Ia, with $z = \langle z \rangle \equiv z_0$ and $m = \langle m \rangle \equiv m_0$ assumed, must be practically coinciding with the relativistic absolute magnitude $M_0$ of a hypothetical supernova Ia with a redshift $z_0 \to 0$. Therefore with $z_0 \ll 1$, for instance at $z_0 = 0.001$ and within the observational limits, we can write

$$M_0 = m_0 - 25 + 5 \log H_0 - 5 \log cz_0 - 1.0857 \cdot (1 - q_0) \cdot z_0$$ (51)

At the same time, if we assume the constancy of $M_R(z_0)$ as $M_0$ at any $z_0$ according to eq. (50), the observation of many distant SNe Ia with different $z_0$ allows us to compute the deceleration parameter $q_0$. A negative value of $q_0$ implies the accelerating expansion of the relativistic Universe, as discovered by the Nobel scientists Saul Perlmutter, Brian Schmidt and Adam Riess (2011).

### 5.3 Hubble Magnitude $M(z_0)$ and its total spread

The relativistic Hubble law (48) can be replaced by the alternative law

$$cz_0 = H_0 D$$ (52)
where now $D = \frac{cz_0}{H_0}$ is an apparent distance, which differs from the proper distance $D_0 = D_{pr}(t_0)$ according to eq. (45). $D$ is the Hubble depth of a central point $z_0$ (cf. papers IX-XVI). Consequently, as in eq. (44), here a different luminosity distance $D_C$ (that is the ECM cosmological distance) based on the Hubble depth $D$ can be formulated as follows

$$D_C = D \cdot (1 + z_0) = \frac{cz_0}{H_0}(1 + z_0)$$ (53)

At this point, the introduction of $D_L = D_C$ in eq. (41) leads to the mathematical definition of the Hubble Magnitude $M(z_0)$ of a supernova Ia, with $z = \langle z \rangle \equiv z_0$ and $m = \langle m \rangle \equiv m_0$ assumed:

$$M(z_0) = m_0 - 5 \log [D \cdot (1 + z_0)] - 25$$ (54)

or

$$M(z_0) = m_0 - 25 + 5 \log H_0 - 5 \log cz_0 - 5 \log (1 + z_0)$$ (55)

One can remark that the previous eq. (53) of $D_C$ results to be practically coinciding with the relativistic $D_L$ eq. (46) at the second order (cf. Attilio Ferrari 2011), that is at $z_0 \ll 1$, after introducing the obtained value $q_0 \approx -1$ ($\langle q_0 \rangle \approx -0.77$: John 2004) from the SNe Ia observational cosmology; as a consequence, also the eq. (54) or (55) of the Hubble Magnitude $M(z_0)$ results to coincide with the relativistic formulation of the absolute magnitude $M_\alpha(z_0) \equiv M_R(z_0)$ at $z_0 \ll 1$ (cf. eq. (41) and (50)). At the same time such a $q_0 \approx -1$ is the result of the assumed constancy of $M_R(z_0)$ as $M_0$ at any $z_0$, that means $\alpha = 1$ assumed in eq. (41). So the relativistic cosmology implies that the Hubble Magnitude $M(z_0)$ at $z_0 \ll 1$ coincides with the relativistic absolute magnitude $M_0$ of eq. (51) at $z_0 \to 0$.

Hence the relativistic result can be summarized through the following two statements:

$$M_R(z_0) \equiv M_0 \text{ at any } z_0 \Rightarrow q_0 \approx -1$$ (56)

$$q_0 = -1 \text{ at } z_0 \ll 1 \Rightarrow M(z_0) = M_R(z_0) = M_0 \Rightarrow M(z_0) - M_0 = 0$$ (57)

The previous $M(z_0) - M_0$ is the total $M$ spread, that is the deviation of the Hubble Magnitude $M(z_0)$, of a supernova Ia with redshift $z = \langle z \rangle \equiv z_0$, from the $M_0$ value corresponding to the same $z_0$. The below eq. (58) gives the relativistic expression of that total $M(z_0)$ spread:

$$M(z_0) - M_0 = -5 \log(1 + z_0) + 1.0857 \cdot (1 - q_0) \cdot z_0 + ...$$ (58)

It is remarkable that the previous eq. (58) was obtained from the difference between eq. (55) and the $M_0$ eq. (51), with the consequent elimination both of $H_0$ and $m_0$. 

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5.4 Computation of the deceleration parameter $q_0$

From eq. (58), only for $z_0 \ll 1$, the formula for the deceleration parameter $q_0$ becomes

$$q_0 \cong +1 - 4.605 \times z_0^{-1} \times \log(1 + z_0) - 0.921 \times z_0^{-1} \times [M(z_0) - M_0]$$

Eq. (59), that is eq. A19 of the APPENDIX - November 2011 - to the ECM paper X, appears to be the solution to an apparent paradox in relativistic cosmology. In fact, when $M(z_0) = M_0$ at $z = z_0 \ll 1$ is assumed, eq. (59) still gives the negative value $q_0 \approx -1$ in accordance with the Deep Universe analysis carried out by the Nobel scientists Saul Perlmutter, Brian Schmidt and Adam Riess (2011), while the total $M$ spreads which can be inferred from the SCP Union data give $q_0$ a positive value, according to the preliminary results obtained in the cited Appendix of paper X. On this occasion, the "absolute magnitude analysis of the SCP Union supernovae" of the parallel paper XVI makes it possible to calculate and extrapolate a few cubic fittings of the SNe Hubble Magnitude $M$ versus $z = \langle z \rangle \equiv z_0$ and the correlated spreads $[M - M_0]$ at $z_0 \ll 1$, with $\langle M \rangle \equiv M(z_0) \equiv M$ assumed, according to the equation

$$M = A_0 + A_1 z_0 + A_2 z_0^2 + A_3 z_0^3$$

where $A_0 \equiv M_0$. Let us remark that the extrapolated trends of the normal or central Hubble Magnitude $M$ of the supernovae Ia at low central redshifts $z_0 \equiv \langle z \rangle \ll 1$ have a sharp negative variation, which clearly contrasts with the almost constant trend due to a relativistic $q_0 \approx -1$.

In particular the fitting solutions, (41)(55)(59)(70) of paper XVI and graphically represented by the fit lines of the Appendix Figures 14-22-24-29 of the same parallel paper, give four total $M$ spreads, whose numerical values with the resulting $q_0$ from eq. (59) are collected below in Table 6, Table 7, Table 8 and Table 9, respectively. In particular the values of Table 9 refer to the final $M$ solution (70) of paper XVI, with $M_0 = -17.9$.

| Table 6 | Table 7 | Table 8 | Table 9 |
|---------|---------|---------|---------|
| $z_0$   | $M - M_0$ | $q_0$   | $M - M_0$ | $q_0$   | $M - M_0$ | $q_0$   |
| 0.001   | -0.00411 | +2.79   | -0.00335 | +2.09   | -0.00319 | +1.93   | -0.00426 | +2.92   |
| 0.01    | -0.04085 | +2.77   | -0.03325 | +2.07   | -0.03161 | +1.92   | -0.04229 | +2.91   |
| 0.1     | -0.3807  | +2.60   | -0.3096  | +1.95   | -0.2929  | +1.79   | -0.3946  | +2.73   |

Clearly, the main result here presented is $q_0 \gtrsim +2$, which shows relativistic cosmology expressing a positive and high value of the deceleration parameter $q_0$. 

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5.5 The relativistic $q_0$ from 249 high-$z$ SNe Ia according to ECM

More rigorously, within the ECM context, the cubic fitting (60) for the calculation of the total $M$ spread to introduce into eq. (59) should be applied exclusively to individual points of central Hubble Magnitude, that is to say only $M(z_0)$ values based on central $m_0$ and $z_0$ in eq. (55) which refer to individual supernovae with $\cos \gamma \approx 0$ (cf. eqs. 18 in paper IX). Alternatively, on the grounds of the dipole analysis carried out on 249 high-$z$ SCP Union supernovae according to the expansion center model, one can fit 249 Hubble Magnitudes $M_z$ plotted against the ECM Hubble depth $D_z$ (cf. Table 3abcdefg of papers XI-XVI), as shown in the diagram of Figure 4, which replicates Figure 6 of paper XVI. In this case, after applying

$$D_z = cz/H_X = cz_0/H_0 = D$$ (61)

with $H_0 = 70 \text{ km/s/Mpc}$ assumed (cf. paper I and II), the cubic line (60) fitting the 249 individual points $M_z$ becomes

$$M_z(z_0) = A_0 + A_1 z_0 + A_2 z_0^2 + A_3 z_0^3 = d_0 + d_1 D + d_2 D^2 + d_3 D^3 = M_z(D)$$ (62)

being

$$A_0 = d_0 \quad A_1 = d_1 c/H_0 \quad A_2 = d_2 c^2/H_0^2 \quad A_3 = d_3 c^3/H_0^3$$ (63)

The solution, from the automatic fitting based on the Hubble depth $D = D_z$, gives

$$d_0 = -18.15 \quad d_1 = -9.05 E - 04 \quad d_2 = +1.69 E - 07 \quad d_3 = -1.15 E - 11$$ (64)

where $R^2 = 0.382$ is the value of the coefficient of determination.

The above cubic solution, reported as $y = f(x)$ in the Figure 4 area, together with the value of $R^2$, gives the total $M$ spread a more reliable extrapolated value at $z_0 = 0.001$ or $D = 4.28 \text{ Mpc}$, the following

$$[M(z_0) - M_0] = [M_z(z_0) - A_0] = -0.00387 \quad \text{at} \ z_0 = 0.001$$

whose introduction in the formula (59) leads to a high value of the relativistic $q_0$, that is

$$q_0 = +2.57$$ (65)
5.6 Concordance test on Galaxy radial deceleration

The section "Verso una nuova cosmogonia della concordanza" of the ECM paper VIII, "Steps towards the expansion center cosmology", represents the first step of a search for some points of contact between the ECM and relativistic cosmology. Here a new concordance test may take into account the calculation of the Galaxy radial deceleration in paper VII, based on the main ECM motion equations. In particular eq. (41) of paper I, that is

\[ \ddot{R}_{MW_{t=t_0}} = -2H_0^2(t_0) \cdot R_{(cm)}(t_0) \] (66)

becomes perfectly equivalent to the relativistic

\[ \ddot{D}_0 = -q_0 H_0^2 D_0 \] (67)

that follows from the previous eqs. (47)(48), when \( R_0 = D_0 \) is considered to be the proper distance at \( t_0 \) of the expansion center from the Galaxy. In this case the expansion center model gives the relativistic deceleration parameter \( q_0 \) the value of +2.

Now the calculation in c.g.s. units of the relativistic deceleration \( \ddot{D}_0 \) of eq. (67), with \( q_0 = +2 \) or \( q_0 = +3 \) applied respectively, after adopting the ECM values \( H_0 = 69.8 \pm 2.8 \ km \ s^{-1} Mpc^{-1} \) based on data by Sandage & Tammann (1975) and \( D_0 \approx 260 \ Mpc \) as the Galaxy distance from the huge void center (Bahcall & Soneira 1982) (cf. papers I-II-VII and author 1991), leads to the values listed below, in Table 10.

| \( H_0 \)   | \( D_0 \)   | \( \ddot{D}_0(q_0 = +2) \) | \( \ddot{D}_0(q_0 = +3) \) |
|-----------|-----------|-----------------|-----------------|
| \((2.263 \pm 0.091) \times 10^{-18} \ s^{-1}\) | \( \approx 8.0 \times 10^{26} cm \) | \(-0.8 \times 10^{-8} cm/s^2\) | \(-1.2 \times 10^{-8} cm/s^2\) |

The results reported in Table 10 can be summarized through a single Galaxy radial deceleration \( \ddot{R}_0 \equiv \ddot{D}_0 \), according to the following order of magnitude:

\[ \ddot{R}_0 \approx -10^{-8} cm/s^2 \] (68)
6. Conclusion

In conclusion the above multiple dipole test on SCP SNe at \( \langle z \rangle = 1.0 \) represents a last crucial proof of the expansion center Universe. Moreover the expansion center model and the involved Galaxy deceleration are here fully reconfirmed at Hubble depths of the Deep Universe. That called for a further ECM dipole analysis on all 249 High-\( z \) SCP Union supernovae, those here referred to as pilot sample XVI in Table 0 and listed in paper IX as usable supernovae with \( z > 0.2 \) of the SCP Union compilation (Kowalski et al.2008). A dipole and absolute magnitude analysis of these \( SCPU \) SNe Ia within the expansion center model has been carried out in the parallel paper XVI, with important consequences for observational and relativistic cosmology. In particular a new positive value of the relativistic parameter \( q_0 \) comes out from the extrapolated behaviour of supernovae Ia at redshift \( z = \langle z \rangle \equiv z_0 = 0.001 \). In other words even the expansion of the relativistic Universe is shown to be decelerating, through a data analysis referring to the nearby instead of the deep Universe. At the same time such an analysis follows from extrapolated fittings of high-\( z \) SNe Ia \( M \) normal or central points, which have the advantage of being negligibly affected by the perturbation due to cosmic rotation (cf. parallel paper XVI).

In the author’s view, the previous results represent the overcoming of the relativistic paradox of the accelerating cosmic expansion and its so-called dark energy. Consequently here relativistic cosmology says Einstein was right (Einstein-de Sitter 1932) when he rejected the cosmological constant, whose introduction was the greatest blunder of his life, as he remarked much later, according to George Gamow (1970).
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