Low Energy Gauge Unification Theory

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Abstract

Because of the problems arising from the fermion unification in the traditional Grand Unified Theory and the mass hierarchy between the 4-dimensional Planck scale and weak scale, we suggest the low energy gauge unification theory with low high-dimensional Planck scale. We discuss the non-supersymmetric $SU(5)$ model on $M^4 \times S^1/Z_2 \times S^1/Z_2$ and the supersymmetric $SU(5)$ model on $M^4 \times S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2)$. The $SU(5)$ gauge symmetry is broken by the orbifold projection for the zero modes, and the gauge unification is accelerated due to the $SU(5)$ asymmetric light KK states. In our models, we forbid the proton decay, still keep the charge quantization, and automatically solve the fermion mass problem. We also comment on the anomaly cancellation and other possible scenarios for low energy gauge unification.

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1 Introduction

The idea of a Grand Unified Theory (GUT) since its birth \cite{1} has been so attractive that by now GUTs are widely considered as the very good extensions of the Standard Model (SM), especially when we consider the supersymmetry. One impressive success of this idea is that including the radiative corrections, GUTs give the correct weak mixing angle, which is observed in the electroweak (EW) scale experiments. The confusing problem of the $U(1)$ charge quantization is also resolved through embedding the SM hypercharge symmetry into the non-abelian gauge symmetry. Another nice point of GUT is the so-called fermion unification, i. e., the SM fermions of each generation elegantly fit the $\overline{5} + 10$ representation of the $SU(5)$ group, or the $16$ representation of the $SO(10)$ group if we included the right handed neutrinos.

Although these nice points seem quite encouraging to the theoretical physicists who pursue the idea of GUT, fermion unification always gives us problems. In spite of the supporting evidence of the observed $m_\tau \approx m_b$, GUT gives the wrong fermion mass relation $m_e/m_\mu = m_d/m_s$ at 1 GeV scale, which is not supported by experiments. This fermion mass problem may be solved by introducing the Higgs whose dimension is larger than that of the adjoint representation. And the fermion unification naturally leads to the notable proton decay problem. The broken gauge bosons (X and Y in the $SU(5)$ gauge group) can lead to the proton decay through the dimension six operator, and the present experiments pushed them to be at least as massive as $5 \times 10^{15}$ GeV \cite{2}. For the supersymmetric GUTs, this bound may not be harmful because the unification scale is about $2.4 \times 10^{16}$ GeV, however, there exists dimension five proton decay
operator by higgsinos exchange. This raises the famous Higgs doublet-triplet splitting problem. One may soon realize that it’s not easy to solve this doublet-triplet splitting problem without including the complicated interactions and fine-tuning. In short, we do have enough problems from the fermion unification.

Recently, grand unified theories have been revisited in the framework with extra dimensions [4, 5, 6, 7, 8]. Using the orbifold projection, the 5 (or 5)' Higgs fields living in the high dimensional bulk can be naturally splitted into massive triplets and light doublets. Furthermore, it was shown that all the dimension four and five proton decay operators may be completely avoided because of the intrinsic global symmetry [8]. But in this kind of models, the $10^{15}$ GeV compactification scale is naturally involved and the dimension six proton decay operator may give disastrous consequence, so, one has to sacrifice the unification of the first generation fermions. Because of the SUSY flavor problem, the unification of the second generation fermion is also sacrificed. Of course, the wrong prediction of $m_e/m_\mu = m_d/m_s$ is avoided simultaneously [8].

Because of the difficulties in searching for a natural grand unified theory, we might conclude that the fermion unification in the usual form familiar to us always leads to the serious problems. Therefore, we may conjecture that the realistic fundamental theory, which describes the nature, might be the theory with the gauge unification and without the fermion unification. In addition, in the gauge unification theory, we might suppress the proton decay [3]. Then, the scale of gauge unification can be as low as hundreds of TeV if it is possible. This kind of scenario is very interesting if the string scale or high-dimensional Planck scale is at hundreds of TeV scale range, and then, there does not exist the mass hierarchy between the high-dimensional Planck scale and unification scale, although there exists the mass hierarchy between the 4-dimensional Planck scale and weak scale. As a comment, let us briefly discuss how to low the high-dimensional Planck scale. Suppose we have 10-dimensional space-time, the gauge fields live on a 5-brane, the 4-dimensional Planck scale $M_{Pl}$ is related to the 10-dimensional Planck scale $M_*$ and the physical size of the extra dimensions as follows

$$M_{Pl}^2 \sim M_*^8 (R_1 R_2) \tilde{R}^4,$$

where $R_1$ and $R_2$ are the two radii of the extra dimension where gauge fields can propagate, $\tilde{R}$ is the common radius of the extra four space-like extra dimensions. Assuming $M_* \sim 100$ TeV and $R_1, R_2 \sim 10$ TeV, we get $1/\tilde{R} \sim 0.1$/GeV.

Moreover, it is widely believed that even in 4-dimensional theory, the CKM mixing is a direct consequence of the string scale physics. In the approach without fermion unification, we therefore have to ascribe not only the flavor mixing but also all the Yukawa couplings directly to the string scale physics. This idea is further pushed by the observation that involving a TeV scale extra dimension, the accumulating KK excitations may accelerate the gauge coupling running, and low the unification scale, $M_U$, even to TeV scale [3].

To get an idea on how the KK excitations work for our purpose, let us remind the readers in the following way. In 4-dimensional theory, the runnings of the gauge couplings are

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M) + \frac{b_i}{4\pi} \ln \frac{\mu^2}{M^2},$$

where $\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$ refers to $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge groups, $b_i$ is the corresponding beta function. $M$ should be the energy scale not much higher than $m_Z$, for
example $M_{SU(5)}$ in MSSM which is of TeV scale. Because of the big gap from the TeV scale to the 4-dimensional unification scale ($10^{16}$ GeV), changing $M$ around TeV scale is just a tiny change on the boundary conditions and does not affect the precision of gauge coupling unification. Beginning with the initial value of $\alpha_i$ at the EW scale, whether the gauge couplings may unify to be $SU(5)$ symmetric depends on the differences among the beta functions ($b_2 - b_1, b_3 - b_2$). In the SM or MSSM, the gauge and Higgs sectors contribute to the differences of the beta functions while the matter contents do not, because the matter contents are completely $SU(5)$ symmetric. If $N$ copies of gauge and Higgs sectors involved not far from the EW scale, we then have

$$\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu) = \alpha_i^{-1}(M) - \alpha_j^{-1}(M) + N \frac{b_i - b_j}{4\pi} \ln \frac{\mu^2}{M^2}. \quad (3)$$

The unification scale can be lowered in this approach, for instance, $M_U$ can be around tens of TeV if $N = 13$. It’s clear that the precision of gauge coupling unification is of the same level as the original one given by Eq. (2). In the framework of large extra dimension, for example with one extra space-like dimension, we can effectively take the point of view of an 4-dimensional truncated Kaluza-Klein theory and the story is similar but a little bit different. Because of the particle content is accumulated as the energy increased, we have

$$\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu) = \alpha_i^{-1}(M_c) - \alpha_j^{-1}(M_c) + \frac{b_i^0 - b_j^0}{4\pi} \ln \frac{\mu^2}{M_c^2} + \sum_n \frac{b_i^n - b_j^n}{4\pi} \ln \frac{\mu^2}{M_n^2}, \quad (4)$$

where $M_c$ is the compactification scale, $M_n$ is the mass of the KK states, $n > 0$ refers the massive KK levels, $b_i^0$ refers the beta function given by the zero mode. In the simple case $b_i^n - b_j^n = b_i - b_j$, we get

$$\alpha_i^{-1}(\mu) - \alpha_j^{-1}(\mu) = \alpha_i^{-1}(M_c) - \alpha_j^{-1}(M_c) + \frac{b_i^0 - b_j^0}{4\pi} \ln \frac{\mu^2}{M_c^2} + \frac{b_i - b_j}{4\pi} \ln \prod_n \frac{\mu^2}{M_n^2}. \quad (5)$$

This formula then leads to another unification scale $M_U$ which is just one order of magnitude larger than the compactification scale. The gauge coupling unification can be achieved at hundreds of TeV if a 10 TeV compactification scale is involved. If $b_i^0 - b_j^0 = b_i - b_j$, similar to the above example, we have the gauge coupling unification. If $b_i^0 - b_j^0 \neq b_i - b_j$, one may also realise that $b_i^0 - b_j^0$ can not affect too much on the mismatch of the gauge couplings at the scale $M_U$, because the contribution to the gauge coupling running given by the massive KK states is one order of magnitude larger than the zero mode contribution. For example, the SM beta functions are $(b_1, b_2, b_3) = (-41/10, 19/6, 7)$, and the differences among them are $(b_2 - b_1, b_3 - b_2) = (109/15, 23/6)$. As we know, it can not unify the gauge couplings in the 4-dimension and it can not work in the high dimensional models either unless we introduce the extra matter contents, which will be discussed in the section 2. In addition, the MSSM beta functions are $(-33/5, -1, 3)$, and they give the differences $(b_2 - b_1, b_3 - b_2) = (28/5, 4)$. In 5-dimensional theory, this choice can work only if the appropriate particle contents were involved in the high dimensional bulk. In section 3, we will give two scenarios to recover this MSSM differences among the beta functions in the high dimensional supersymmetric model.

In this letter, we will focus on the 6-dimensional space-time or the 5-brane, and try to build the $SU(5)$ models with low energy gauge unification and without fermion unification. We discuss the non-supersymmetric $SU(5)$ model on the space-time $M^4 \times S^1/Z_2 \times S^1/Z_2$ where
there are three pairs of Higgs doublets in the bulk: the supersymmetric \( SU(5) \) model on the space-time \( M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2') \) where there are one pair of Higgs doublets and one pair of hypercharge one singlets on the boundary 4-brane at \( z = \pi R_2/2 \), or one Higgs doublet (\( H_d \)) on the boundary 4-brane at \( z = \pi R_2/2 \). In addition, we forbid the proton decay by putting the matter fields on the suitable 3-branes at the fixed points, and we still have the charge quantization and automatically avoid the fermion mass problem in our models. The gauge unification scale is about hundreds of TeVs, and we would like to point out that in order to avoid the uncertainties from the loop corrections to the masses of the KK states, the supersymmetry breaking soft parameters and Higgs mechanism, we consider the scenario with \( R_1 \sim 10 R_2 \), and then the gauge unification is accelerated by the leading order power one running. We would like to emphasize that for the scenarios with \( R_1 \sim R_2 \), we still have low energy gauge unification, but the gauge unification is accelerated by the subleading order contributions which might be affected by the uncertainties. These uncertainties might be small and definitely deserve further study.

2 Non-Supersymmetric Model

In this section, we would like to discuss the non-supersymmetric \( SU(5) \) model on the space-time \( M^4 \times S^1/(Z_2 \times Z_2) \). The model is given as Model II in Ref. [3], where the gauge unification was not discussed. In this model, the gauge symmetry \( SU(5) \) is broken down to the Standard Model gauge symmetry by orbifold projection, and we only have the zero modes and KK modes of the Standard Model gauge fields and two Higgs doublets on the observable 3-brane at the fixed point. So, we can have the low energy unification, and solve the doublet-triplet splitting problem, the gauge hierarchy problem, and the proton decay problem. Let us review the set-up.

We consider the 6-dimensional space-time which can be factorized into the product of the ordinary 4-dimensional space-time \( M^4 \) and the orbifold \( S^1/Z_2 \times S^1/Z_2 \). The corresponding coordinates are \( x^\mu, (\mu = 0, 1, 2, 3) \), \( y \equiv x^5 \) and \( z \equiv x^6 \). The radii for the circles along \( y \) direction and \( z \) direction are \( R_1 \) and \( R_2 \), respectively. The orbifold \( S^1/Z_2 \times S^1/Z_2 \) is obtained by \( S^1 \times S^1 \) moduloring the equivalent classes: \( y \sim -y \) and \( z \sim -z \). So, we have four fixed points: \( (y = 0, z = 0), (y = 0, z = \pi R_2), (y = \pi R_1, z = 0) \) and \( (y = \pi R_1, z = \pi R_2) \) that are fixed under two actions, and four fixed lines: \( y = 0, y = \pi R_1, z = 0 \) and \( z = \pi R_2 \) that are fixed under one action. We associate two parity transformations to the fields living in the bulk. For an example, for a field \( \phi(x^\mu, y, z) \) in the fundamental representation we have

\[
\phi(x^\mu, y, z) \rightarrow \phi(x^\mu, -y, z) = \eta^y_0 P \phi(x^\mu, y, z), \quad \phi(x^\mu, y, z) \rightarrow \phi(x^\mu, y, -z) = \eta^z_0 P' \phi(x^\mu, y, z),
\]

where \( \eta^y_0 \) and \( \eta^z_0 \) are \( \pm 1 \).

And we assume the trivial periodic boundary condition for all the fields involved in the model:

\[
\phi(x^\mu, y + 2\pi R_1, z) = \phi(x^\mu, y, z + 2\pi R_2) = \phi(x^\mu, y, z).
\]

Denoting the field with parities \((\pm, \pm)\) under \((P, P')\) by \( \phi_{\pm\pm} \), we obtain the corresponding Fourier expansions [3]: \( \cos \frac{2\pi n}{R_1} \cos \frac{2\pi m}{R_2} \) for \( \phi_{++} \), \( \cos \frac{2\pi n}{R_1} \sin \frac{2\pi m}{R_2} \) for \( \phi_{+-} \), \( \sin \frac{2\pi n}{R_1} \cos \frac{2\pi m}{R_2} \) for \( \phi_{-+} \) and \( \sin \frac{2\pi n}{R_1} \sin \frac{2\pi m}{R_2} \) for \( \phi_{--} \), where \( n \) and \( m \) are non-negative integers. As expected \( \phi_{+-} \) and \( \phi_{--} \) vanish at \( y = 0 \) and \( \pi R_1 \), and \( \phi_{++} \) and \( \phi_{--} \) vanish at \( z = 0 \) and \( \pi R_2 \). The masses of these
Table 1: Parity assignment and masses \( (n \geq 0, m \geq 0) \) of the fields in the SU(5) gauge and Higgs multiplets for the non-supersymmetric model.

| \((P, P')\) | field | mass |
|-------------|-------|------|
| (+, +)      | \(A_\mu^a, H_u^D, H_d^D\) | \(\sqrt{n^2/R_1^2 + m^2/R_2^2}\) |
| (+, -)      | \(A_\mu^a, A_6^a, H_i^T\) | \(\sqrt{n^2/R_1^2 + (m+1)^2/R_2^2}\) |
| (-, +)      | \(A_5^a, A_6^a, H_i^D\) | \(\sqrt{(n+1)^2/R_1^2 + m^2/R_2^2}\) |
| (-, -)      | \(A_\mu^a, H_u^T, H_d^T\) | \(\sqrt{(n+1)^2/R_1^2 + (m+1)^2/R_2^2}\) |

Table 2: The gauge fields, Higgs fields and gauge group on the 3-branes that are located at the fixed points \((y = 0, z = 0)，(y = 0, z = \pi R_2)，(y = \pi R_1, z = 0)\) and \((y = \pi R_1, z = \pi R_2)\), and on the 4-branes that are located at \(y = 0, z = 0, y = \pi R_1\) and \(z = \pi R_2\).

| Brane position | field | gauge group |
|---------------|-------|-------------|
| \((y = 0, z = 0)\) | \(A_\mu^a, H_u^D, H_d^D\) | SU(3) × SU(2) × U(1) |
| \((y = 0, z = \pi R_2)\) | \(A_\mu^a, H_u^D, H_d^D\) | SU(3) × SU(2) × U(1) |
| \((y = \pi R_1, z = 0)\) | \(A_\mu^a, H_u^D, H_d^D\) | SU(3) × SU(2) × U(1) |
| \((y = \pi R_1, z = \pi R_2)\) | \(A_\mu^a, H_u^D, H_d^D\) | SU(3) × SU(2) × U(1) |
| \(y = 0\) or \(y = \pi R_1\) | \(A_\mu^a, A_5^a, A_6^a, H_u^D, H_d^D, H_i^T\) | SU(3) × SU(2) × U(1) |
| \(z = 0\) or \(z = \pi R_2\) | \(A_\mu^a, A_5^a, A_6^a, H_u^D, H_d^D, H_i^D\) | SU(3) × SU(2) × U(1) |

KK modes are given by \(\sqrt{n^2/R_1^2 + m^2/R_2^2}\), \(\sqrt{n^2/R_1^2 + (m+1)^2/R_2^2}\), \(\sqrt{(n+1)^2/R_1^2 + m^2/R_2^2}\) and \(\sqrt{(n+1)^2/R_1^2 + (m+1)^2/R_2^2}\) \((n, m \geq 0)\) for fields of parities \((+, +), (+, -), (-, +)\) and \((-,-)\), respectively.

We choose the following parity assignments for \(P\) and \(P'\):

\[
P = \text{diag}\{-1, -1, -1, 1, 1\}, \quad P' = \text{diag}\{-1, -1, -1, 1, 1\}.
\]  

Upon the parity transformation, the gauge generators of SU(5) group are separated into two classes: the generators of the SM gauge group \(T^a\), and the other broken gauge generators \(T^\hat{a}\), and \(T^\rho\) and \(T^\hat{\rho}\) satisfy the following equations

\[
P'T^aP = P^\rho T^\hat{a}P = T^a, \quad P'T^\hat{a}P = P^\rho T^\rho P = -T^\hat{a}.
\]

Now let us discuss gauge coupling unification. Notice that even if \(R_1 \sim R_2\) involved, it is still possible to get the gauge coupling unification, because the orbifold projection makes parts of the fields disappear in some of the KK states and then the relative running can accelerated.
However in the case of $R_1 \sim R_2$, the dominating radiative corrections to the gauge couplings are basically $SU(5)$ symmetric because of the $SU(5)$ bulk theory, and the relative running is given by the subleading order contributions which depend largely on the differences of the beta functions and detailed masses of different fields. If including the loop corrections to the masses of the KK states, the Higgs mechanism to break EW symmetry (and the supersymmetry breaking soft parameters in the supersymmetric case), it might be possible that the precision on the gauge coupling unification would be changed a lot. Therefore, we consider the scenario with $R_1 \sim 10R_2$ and make the relative running to be at leading order in the whole radiative corrections. If the relative running is at the leading order as the form of the last term in Eq. (5), it’s clear that uncertainties in the masses of the KK states can not change the precision of the gauge coupling unification although they may affect the scale a little bit at which the gauge coupling unification is reached.

At the energy scale $1/R_1 < \mu < 1/R_2$, from the model described above we would have a 5-dimensional effective theory with the SM gauge group in the 5-dimensional bulk and at each KK level, there are states coming from $A^a_i$ and $A^i_b$ fields. To achieve gauge coupling unification we add three pairs of two Higgs (5 and 5) in the 6-dimensional bulk. We may use the orbifold projection to eliminate the zero modes of two pairs of them from the 4-dimensional effective theory. For one pair of them, denoting as $H_u$ and $H_d$, the parity transformations are given by Eq. (8) and (8), and $\eta^u = +1$ and $\eta^d = +1$. For the other two pairs, denoting as $H_i$ where $i = 1, 2, 3, 4$, we choose $\eta^i = -1$ and $\eta^{i*} = +1$. The parities and mass spectrum of the gauge fields and Higgs fields are given in the Table 1. And in Table 2, we present the gauge fields, Higgs fields and gauge group on the 3-branes that are located at the fixed points ($y = 0, z = 0$), ($y = 0, z = \pi R_2$), ($y = \pi R_1, z = 0$) and ($y = \pi R_1, z = \pi R_2$), and on the 4-branes that are located at $y = 0, z = 0, y = \pi R_1$ and $z = \pi R_2$.

Table 1, we obtain that the triplet Higgs do not have zero and light KK modes while the doublet Higgs do have light KK modes. In short, in the effective 5-dimensional theory, we have the SM gauge fields, adjoint scalar fields $A^a_i$, lepton-quark scalar $A^i_a$ with hypercharge $5/6$ and three pairs of two Higgs doublets. For each KK level below the scale $1/R_2$, ($b_1, b_2, b_3$) = ($-\frac{43}{30}, \frac{11}{2}, \frac{21}{2}$), ($b_2 - b_1, b_3 - b_2$) = (104,15,14/3) and indeed it can lead to the gauge coupling unification. We assume the energy scale where the gauge coupling unified is the cutoff scale, $\Lambda$, of the theory. For example, assuming $1/R_2 = 6/R_1$, we find that the gauge coupling unification can be achieved at $\Lambda \approx 190$ TeV for $1/R_1 = 10$ TeV. Running above the energy scale $1/R_2$ can also reduce the discrepancies among the gauge couplings considerably because although the contributions from those KK states with masses $\sqrt{n^2/R_1^2 + m^2/R_2^2}$ in which $m > 0$ are basically $SU(5)$ symmetric, there exist the dominant contributions to the relative gauge couplings running from the KK states with mass $n/R_1$, i. e., $m = 0$. The above example with $\Lambda R_1 \approx 19$ and $\Lambda R_2 \approx 3$ confirms our expectations, that is the $SU(5)$ asymmetric contributions are indeed at the leading order.

From Table 2, we know that on the 3-brane at any one of four fixed points, there exist only the SM gauge fields and one pair of Higgs doublets $H_u$ and $H_d$. We can then put the SM fermions on one of the 3-branes, for example, the 3-brane at ($y = 0, z = 0$). This theory is completely anomaly-free, and there are no operators which can lead to the proton decay, as discussed in the Ref. (3). We can couple the Higgs doublets, $H_u$ and $H_d$, to the SM fermions, like the Model II of the two Higgs doublet Model. Thus, the charge quantization is achieved
from the consistent condition if we included the Yukawa couplings on the observable 3-brane because the \( U(1) \) charge has to be balanced in the Yukawa couplings. And together with the four anomaly-free conditions, we can determine the \( U(1) \) charges of the SM fermions.

In short, the physical picture in our model is that above the low compactification scale \( (1/R_1) \), \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge fields and three pairs of Higgs doublets propagate in the 5-dimensional space-time, and the light KK states with masses \( n/R_1 \) accelerate the gauge coupling unification. Above the high compactification scale \( (1/R_2) \), we should have the \( SU(5) \) gauge theory, and the gauge coupling unification is achieved at the cutoff scale. So, at least two extra dimensions in this kind of models should be involved.

### 3 Supersymmetric Model

In this section, we would like to discuss the low energy supersymmetric \( SU(5) \) model, which is the supersymmetric extension of the model in the last subsection.

One of the two possibilities is that we consider the 6-dimensional \( N = 1 \) supersymmetric theory. In terms of the 4-dimensional language, this theory has \( N = 2 \) supersymmetry and the gauge superfield is described by a vector superfield \( V \) and a chiral superfield \( \Phi \). The scalar component of \( \Phi \) can be written as \( A_5 + i A_6 \). However, it is not hard for one to figure out that we can not construct the 6-dimensional \( N = 1 \) supersymmetric \( SU(5) \) model, which is the supersymmetric generalization of the model in the last subsection, because the discrete symmetry should act on \( A_5 \) and \( A_6 \) simultaneously and \( R_1 = R_2 \) is required. Thus, we have to consider the 6-dimensional \( N = 2 \) supersymmetric theory.

The 6-dimensional \( N = 2 \) supersymmetric theory is anomaly-free in the bulk. In terms of 4-dimensional language, the 6-dimensional \( N = 2 \) supersymmetric theory corresponds to the 4-dimensional \( N = 4 \) supersymmetric theory. The gauge superfield can be decomposed to be one vector superfield, \( V \), and three chiral superfields, \( \Sigma_5, \Sigma_6 \) and \( \Phi \). In the Wess-Zumino gauge and 4-dimensional \( N = 1 \) language, the bulk action is

\[
S = \int d^6x \left\{ \int d^2\theta \left( \frac{1}{4kg^2} W^\alpha W_\alpha + \frac{1}{kg^2} \left( \Phi \partial_\mu \Sigma_6 - \Phi \partial_\mu \Sigma_5 - \frac{1}{\sqrt{2}} \Phi [\Sigma_5, \Sigma_6] \right) \right) + \text{h.c.} \right\} + \int d^4\theta \frac{1}{kg^2} \text{Tr} \left\{ \sum_{i=5}^{6} \left( (\sqrt{2} \partial_i + \Sigma_i^\dagger) e^{-V} (-\sqrt{2} \partial_i + \Sigma_i) e^V + \partial_i e^{-V} \partial_i e^V \right) + \Phi^+ e^{-V} \Phi e^V \right\}.
\]

We consider the space-time \( M^4 \times S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2) \), where the orbifold \( S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2) \) is defined by \( S^1 \times S^1 \) moduloing the equivalent classes

\[
P_y : y \sim -y, \quad P'_y : y' \sim -y',
\]
\[
P_z : z \sim -z, \quad P'_z : z' \sim -z',
\]

where \( y' = y - \pi R_1/2 \) and \( z' = z - \pi R_2/2 \). The physical space is in the region: \( 0 \leq y \leq \pi R_1/2 \) and \( 0 \leq z \leq \pi R_2/2 \). The detail set-up and discussions can be found in Ref. [7].

From the action Eq. (10), we obtain that under \( P_y \) and \( P'_y \), the vector multiplets transform as:

\[
V(x^\mu, -y, z) = P_y V(x^\mu, y, z)(P_y)^{-1}, \quad V(x^\mu, -y', z) = P'_y V(x^\mu, y', z)(P'_y)^{-1}.
\]

7
Table 3: Parity assignment and masses \((n \geq 0, m \geq 0)\) for the vector multiplet in the supersymmetric \(SU(5)\) model on \(M^4 \times S^1/(Z_2 \times Z'_2) \times S^1/(Z_2 \times Z'_2)\). And we include the Higgs superfields \((H, H^c)\) on the fixed 4-brane at \(z = \pi R_2/2\).

\[
\begin{array}{|c|c|c|}
\hline
(P^u, P^u', P^s, P^{s'}) & \text{field} & \text{mass} \\
\hline
(+, +, +, +) & V_\mu^a & \sqrt{(2n)^2/R_1^2 + (2m)^2/R_2^2} \\
(+, -, +, -) & V_{\mu}^\alpha & \sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2} \\
(-, -, +, +) & \Sigma_5^a & \sqrt{(2n+2)^2/R_1^2 + (2m)^2/R_2^2} \\
(-, +, +, -) & \Sigma_5^\alpha & \sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2} \\
(+, +, -, -) & \Sigma_6^a & \sqrt{(2n)^2/R_1^2 + (2m+2)^2/R_2^2} \\
(+, -, -, +) & \Sigma_6^\alpha & \sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2} \\
(-, -, -, -) & \Phi^a & \sqrt{(2n+2)^2/R_1^2 + (2m+2)^2/R_2^2} \\
(-, +, -, +) & \Phi^\alpha & \sqrt{(2n+1)^2/R_1^2 + (2m+1)^2/R_2^2} \\
(P^y = +, P^{y'} = +) & H & 2n/R_1 \\
(P^y = -, P^{y'} = -) & H^c & (2n+2)/R_1 \\
\hline
\end{array}
\]

\[
\Sigma_5(x^\mu, -y, z) = -P_y \Sigma_5(x^\mu, y, z)(P_y)^{-1}, \quad \Sigma_5(x^\mu, -y', z) = -P_y' \Sigma_5(x^\mu, y', z)(P_y')^{-1},
\]

\[
\Sigma_6(x^\mu, -y, z) = P_y \Sigma_6(x^\mu, y, z)(P_y)^{-1}, \quad \Sigma_6(x^\mu, -y', z) = P_y' \Sigma_6(x^\mu, y', z)(P_y')^{-1},
\]

\[
\Phi(x^\mu, -y, z) = -P_y \Phi(x^\mu, y, z)(P_y)^{-1}, \quad \Phi(x^\mu, -y', z) = -P_y' \Phi(x^\mu, y', z)(P_y')^{-1}.
\]

For \(P_z\) and \(P_z'\), the vector multiplet transformations are similar to those under \(P_y\) and \(P_y'\), i.e., we just make the following transformation on subscripts: \(y \leftrightarrow z\) and \(5 \leftrightarrow 6\).

We choose the following representations for \((P_y, P_y', P_z, P_z')\):

\[
P_y = (+1, +1, +1, +1, +1), \quad P_y' = (+1, +1, +1, -1, -1),
\]

\[
P_z = (+1, +1, +1, +1, +1), \quad P_z' = (+1, +1, +1, -1, -1).
\]

The KK mode expansions for the bulk fields can be found in Ref. [8]. The parity assignment and particle spectrum for the gauge fields and the Higgs fields on the 4-brane at \(z = \pi R_2/2\) are given in Table 3, and the gauge superfields, the number of 4-dimensional supersymmetry and gauge group on the 3-brane or 4-brane are shown in Table 4.

It is clear that at the energy scale \(\mu < 1/R_1\) and \(1/R_2\), we get the 4-dimensional \(N = 1\) effective theory described by the zero mode of \(V^\alpha\). Similar to the reasons pointed out in section 2, we assume that \(10/R_1 \sim 1/R_2\). From the parity assignment in the Table 3, at the energy scale \(1/R_1 < \mu < 1/R_2\) we get a 5-dimensional \(N = 1\) effective theory described by the zero modes of \(V^\alpha\) and \(\Sigma_5^a\) along the \(z\) direction. In order to avoid the proton decay, we put the quark fields \((Q, U\) and \(D\)) on the 3-brane at \((y = \pi R_1/2, z = \pi R_2/2)\), and the lepton and
Table 4: For the supersymmetric model $SU(5)$ on $M^4 \times S^1 / (Z_2 \times Z_2') \times S^1 / (Z_2 \times Z_2')$, the gauge superfields, the number of 4-dimensional supersymmetry and gauge symmetry on the 3-brane which is located at the fixed point $(y = 0, z = 0)$, $(y = 0, z = \pi R_2/2)$, $(y = \pi R_1/2, z = 0)$, and $(y = \pi R_1/2, z = \pi R_2/2)$, or on the 4-brane which is located at the fixed line $y = 0$, $z = 0$, $y = \pi R_1/2$, $z = \pi R_2/2$. We also include the quark fields on the 3-brane, considering the right-handed up-type quark $U$ and down-type quark $D$ that are on the 3-brane at $(y = \pi R_1/2, z = \pi R_2/2)$, the lepton doublet $L$, right handed lepton $E$ and neutrino $N$ that are on the 3-brane at $(y = 0, z = \pi R_2/2)$.

| Brane Position       | Fields               | SUSY | Gauge Symmetry |
|----------------------|----------------------|------|----------------|
| $(0, 0)$             | $V^A_\mu$            | $N = 1$ | $SU(5)$       |
| $(0, \pi R_2/2)$     | $V^a_\mu$, $\Sigma^a_6$, $L$, $E$, $N$ | $N=1$ | $SU(3) \times SU(2) \times U(1)$ |
| $(\pi R_1/2, 0)$     | $V^a_\mu$, $\Sigma^a_5$ | $N=1$ | $SU(3) \times SU(2) \times U(1)$ |
| $(\pi R_1/2, \pi R_2/2)$ | $V^a_\mu$, $\Phi^a$, $Q$, $U$, $D$ | $N=1$ | $SU(3) \times SU(2) \times U(1)$ |
| $y = 0$              | $V^A_\mu$, $\Sigma^A_6$ | $N=2$ | $SU(5)$       |
| $z = 0$              | $V^A_\mu$, $\Sigma^A_5$ | $N=2$ | $SU(5)$       |
| $y = \pi R_1/2$      | $V^a_\mu$, $\Sigma^a_5$, $\Sigma^a_6$, $\Phi^a$ | $N=2$ | $SU(3) \times SU(2) \times U(1)$ |
| $z = \pi R_2/2$      | $V^a_\mu$, $\Sigma^a_5$, $\Sigma^a_6$, $\Phi^a$ | $N=2$ | $SU(3) \times SU(2) \times U(1)$ |

neutrino fields ($L$, $E$, $N$) on the 3-brane at $(y = 0, z = \pi R_2/2)$. Let us discuss the anomaly cancellation. The $SU(2)$ is a safe Lie algebra. Because we put all the quark fields on the 3-brane at $(y = \pi R_1/2, z = \pi R_2/2)$, there are no $SU(3)$ anomaly. So, the possible anomaly must involve at least one $U(1)$. In addition, the anomaly localized on the 3-brane at $(y = \pi R_1/2, z = \pi R_2/2)$ and the anomaly localized on the 3-brane at $(y = 0, z = \pi R_2/2)$ have the opposite sign and same magnitude, and there are two ways to cancel the anomaly:

1) Similar to the discussions on 5-dimensional orbifold [1], we introduce the following Chen-Simons term on the 4-brane (covering space-time $M^4 \times S^1 \times (z = \pi R_2/2)$) at $z = \pi R_2/2$ \[ L_{CS} = -\frac{1}{4} \beta \frac{1}{128 \pi^2} \int_{M^4 \times S^1 \times S^1} d^4x dy dz \delta(z - \pi R_2/2) \theta(y) \epsilon_{MNOPQ} A^M F^{NO} F^{PQ}, \] (18)

where $\beta$ is the constant to be adjusted to cancel the anomaly, $\theta(y) = +1$ for $y \in (0, \pi R_1/2) \cup (\pi R_1, 3\pi R_1/2)$, and $\theta(y) = -1$ for $y \in (\pi R_1/2, \pi R_1) \cup (3\pi R_1/2, 2\pi R_1)$.

2) We introduce the following topological term on the covering space-time $M^4 \times S^1 \times S^1$ [2]

\[ L = -\frac{1}{8} \beta \frac{1}{128 \pi^2} \int_{M^4 \times S^1 \times S^1} d^4x dy dz \epsilon_{LMNOPQ} \partial^L \theta(y, z) A^M F^{NO} F^{PQ}, \] (19)

where $\theta(y, z)$ satisfies the equation

\[ [\partial_y, \partial_z] \theta = 2\pi (\delta(y) - \delta(y - \pi R_1/2) + \delta(y - 3\pi R_1/2) - \delta(y - 2\pi R_1)) \]

\[ ^1\text{For simplicity, we discuss the pure } U(1) \text{ case and the discussions for the other cases are similar.} \]
\[
\times (\delta(z - \pi R_2/2) + \delta(z - 3\pi R_2/2)) \,.
\]  

Since the 6-dimensional bulk theory is basically 4-dimensional \( N = 4 \) supersymmetric theory, we can not put the Higgs fields in the whole bulk. On the other hand, we can put the Higgs doublets on the boundary 4-brane at \( z = \pi R_2/2 \), which preserves the 4-dimensional \( N = 2 \) supersymmetry. The 5-dimensional Higgs superfield can be described by two 4-dimensional chiral Higgs superfields, \( H \) and \( H^c \). The orbifold can project out one of them on the boundary 3-brane, and makes the 4-dimensional effective theory to be a chiral \( N = 1 \) supersymmetric theory. As noted in the last section, in order to have the gauge coupling unification, we should be careful on how to put the Higgs fields in high dimension. One might expect that putting one pair of Higgs doublets on the 4-brane at \( z = \pi R_2/2 \) might work. However, noticing that at each KK level with mass \((2n + 2)/R_1\), we have four chiral Higgs doublet fields, \( V^a \) and \( \Sigma_0^a \). Then, the beta functions are \((b_1, b_2, b_3) = (-6/5, 2, 6)\) and \((b_2 - b_1, b_3 - b_2) = (16/5, 4)\). Comparing with the MSSM beta function difference \((b_2 - b_1, b_3 - b_2) = (28/5, 4)\), we find that the above suggestion is impossible to unify the gauge couplings. To rescue from this problem, we may put two extra singlets with hypercharge 1 on the 4-brane at \( z = \pi R_2/2 \), and the beta functions become \((b_1, b_2, b_3) = (-18/5, 2, 6)\). It just recovers what we have in the MSSM, i.e., \((b_2 - b_1, b_3 - b_2) = (18/5, 3)\), and can work as expected. For example, assuming \( 1/R_2 = 20/R_1 \) and \( 1/R_1 = 10 \) TeV, we can achieve the gauge coupling unification at around 530 TeV with the MSSM threshold scale \( M_{SUSY} \) varying 200 – 1000 GeV. Just as the case in 4-dimension, the uncertainty of the boundary conditions given by the supersymmetry threshold gives only small corrections.

Another choice is that we put only one Higgs doublet on the 4-brane at \( z = \pi R_2/2 \), which may be identified as 4-dimensional \( H_d \) and its mirror partner \( H_d^c \). The chiral superfield \( H_u \) can be put on the 3-brane at \((y = \pi R_1/2, z = \pi R_2/2)\). The beta function is \((b_1, b_2, b_3) = (-3/5, 3, 6)\) and the difference among them is \((b_2 - b_1, b_3 - b_2) = (18/5, 3)\). As observed in [3], this choice can lead to the gauge coupling unification. Assuming \( 1/R_2 = 20/R_1 \) and \( 1/R_1 = 10 \) TeV, we obtain that the unification scale \( M_U \) is about 770 TeV with \( M_{SUSY} \) varying from 200 – 1000 GeV. Similar to above, the anomaly from the Higgs field \( H_d \) and \( H_d^c \) on the 4-brane can be cancelled by introducing the suitable Chen-Simons terms on the 4-brane at \( z = \pi R_2/2 \) or the topological term in the bulk. However, the anomaly from the chiral superfield \( H_u \) can not be cancelled unless we introduce another Higgs doublet, say \( \tilde{H}_u \), on the 3-brane at \((y = \pi R_1/2, z = \pi R_2/2)\) or \((y = 0, z = \pi R_2/2)\). Of course, we have to make \( \tilde{H}_u \) massive and let it disappear in the effective 4-dimensional theory below the compactification scale.

On the 3-brane at \((y = \pi R_1/2, z = \pi R_2/2)\), the field \( \Phi^a (3, 2, -\frac{5}{6}) \) does not vanish. So, we have the localized superpotential \( H_u D^a \Phi^a \) on the 3-brane at \((y = \pi R_1/2, z = \pi R_2/2)\). Since the \( U(1) \) charge has to be balanced on the 3-brane localized Yukawa superpotential and we must have the anomaly cancellation, we then have the charge quantization from the consistent conditions of these models.

### 4 Discussions and Conclusion

First, as noted before, we concentrate on the scenarios with \( 1/R_2 \sim 10/R_1 \) in order to suppress the possible uncertainties on the gauge coupling unification. For the case with \( 1/R_2 \sim 1/R_1 \), the dominating radiative corrections given by the massive KK excitations are basically \( SU(5) \) symmetric and the relative runnings are given by the subleading contributions which might
Figure 1: $b_3 - b_2$ versus $b_2 - b_1$. The allowed region by gauge coupling unification for the supersymmetric models is in between the two lines with slopes 1.19 and 1.60. Similar results hold for the non-supersymmetric model.

suffer the uncertainties from the loop corrections to the masses of the KK states, the supersymmetry breaking soft parameters and the Higgs mechanism. However, these uncertainties deserve further and careful study. Because if these uncertainties are indeed small, we can construct many new models with low energy gauge unification, for example, the 6-dimensional $N = 1$ or $N = 2$ supersymmetric $SU(5)$ model on the space-time $M^4 \times T^2/Z_6$, and the 6-dimensional $N = 1$ or $N = 2$ supersymmetric $SU(5)$, $SU(6)$, $SO(10)$ and $E_6$ models on the space-time $M^4 \times D^2$ or $M^4 \times A^2$ where $D^2$ is the disc and $A^2$ is the annulus.

Second, we learned from the discussions in sections 2 and 3 that basically we only need check $(b_2 - b_1, b_3 - b_2)$ of the 5-dimensional KK excitations to examine whether the particle content can lead to the gauge coupling unification. In particular, as shown in the section 2 checking in this way is also reliable even in the 6-dimensional setup if $1/R_2 \sim 10/R_1$ involved. Because of the orbifold projection, parts of the $SU(5)$ representations disappear in the KK states of masses $(n + 1)/R_1$ or $2(n + 1)/R_1$ and this makes the power one contributions survive from leading power two $SU(5)$ symmetric contributions (actually the power two contributions vanish in the supersymmetric model because of the $N = 4$ supersymmetry). If $1/R_2 \sim 10/R_1$ (or $\Lambda R_1 \sim 10\Lambda R_2$) involved, the $SU(5)$ asymmetric power one contribution is then at the leading order which can suppress the possible uncertainties on the gauge coupling unification. In this framework, we have several scenarios that do have low energy unification: (I) the non-supersymmetric $SU(5)$ scenario with three pairs of bulk Higgs doublets where the differences among the beta functions are $(b_2 - b_1, b_3 - b_2) = (104/15, 14/3)$; (II) the supersymmetric $SU(5)$ scenario with one pair of Higgs doublets and one pair of hypercharge one singlets on the boundary 4-brane at $z = \pi R_2/2$, where the differences among the beta functions are $(b_2 - b_1, b_3 - b_2) = (28/5, 4)$; (III) the supersymmetric $SU(5)$ scenario with one Higgs doublet on the boundary 4-brane at $z = \pi R_2/2$, where the differences among the beta functions are $(b_2 - b_1, b_3 - b_2) = (18/5, 3)$. We remind the readers that $b_2 - b_1$ and $b_3 - b_2$ satisfy a roughly linear dependence, i.e., the larger $b_3 - b_2$, the larger $b_2 - b_1$ is needed. By the way, we would like to emphasize that two natural scenarios do not have the gauge unification, and they are:
(1) for the non-supersymmetric case, $SU(5)$ gauge fields and one pair of Higgs doublets (or one Higgs doublet) in the bulk; (2) for the supersymmetric case, $SU(5)$ gauge fields in the bulk and one pair of Higgs doublets on the boundary 4-brane at $z = \pi R_2/2$.

This phenomenon is not strange to us in view of the fact that the dominating one-loop running equation is actually linearly dependent on the beta functions, as can be seen in Eq.s (2), (3), (4) and (5). To achieve the gauge coupling unification, we should adjust the particle content so that the more rapid the relative running between $\alpha_3^{-1}$ and $\alpha_2^{-1}$, the more rapid the relative running between $\alpha_2^{-1}$ and $\alpha_1^{-1}$, as we need to obtain the gauge unification. If one of them becomes slower, so does the other to get the gauge coupling unification. In the Figure 1, we give a plot on the correlated region for $b_2 - b_1$ and $b_3 - b_2$. We may understand the slope by realizing that

$$\frac{\alpha^{-1}_1(m_Z) - \alpha^{-1}_2(m_Z)}{\alpha^{-1}_2(m_Z) - \alpha^{-1}_3(m_Z)} \approx 1.38. \tag{21}$$

Using Eq. (2) or (5), we can translate this number to be the required ratio $r = (b_2 - b_1)/(b_3 - b_2)$. Taking the values of $\alpha_i$’s at 10 TeV instead and assuming the $M_{SUSY}$ varies in the range $200 - 1000$ GeV, we can get the required ratio, $r$. Further assuming that the mismatch at the $M_U$ scale, $\delta = (\alpha_3^{-1} - \alpha_2^{-1})/\alpha_2^{-1}$, is less than 5%, and the unified $\alpha^{-1}$ is smaller than 50, we obtain the region which is allowed by the gauge unification: $1.19 < r < 1.60$. (Without supersymmetry, the bound is roughly $1.15 < r < 1.52$ at 10 TeV scale and $1.20 < r < 1.56$ at 1 TeV scale). The two lines in the Fig. 1 correspond to these two bounds. The above successful scenarios have the following ratios: (I) $r = (104/15)/(14/3) \approx 1.49$; (II) $r = 28/5/4 = 1.4$; (II) $r = (18/5)/3 = 1.2$. And these three $r$ are plotted as three points in the Fig. 1.

One interesting question still remains, which is whether we can find a natural and simple scenario in high dimension which can unify the gauge couplings without adding the exotic particles or extra Higgs by hand. According to our experiences, this is a really hard question.

Furthermore, there exists another kind of scenarios that might deserve further study. We can use the Higgs mechanism to break the GUT gauge symmetry and assume that the GUT breaking scale is much larger than the scale of $1/R_1$ and $1/R_2$. And we can use the orbifold projection to forbid the proton decay operators, and use the KK states to accelerate the gauge unification. Of course, the uncertainties on the gauge unification are avoided. The non-supersymmetric models works similarly, and we might discuss any GUT models on the space-time $M^4 \times [S^1/(Z_2 \times Z_2')]^n$, or $M^4 \times D^2$, or $M^4 \times A^2$, for example SO(10) and $E_6$. However, it seems to us that we might not construct the supersymmetric extensions of this kind of models, the key points are: (1) the Higgs, which break the gauge symmetry, can not break the parity, i. e., the neutral Higgs which has VEV must transform trivially under the discrete symmetry which acts on the extra space manifold; (2) the Higgs on the boundary brane can not give the large masses (compare to $1/R$) to the bulk fields by Higgs mechanism, although they might change the boundary conditions for the bulk fields [13]; (3) the 4-dimensional $N = 2$ hypermultiplets only have gauge interactions [4].

In short, because of the problems arising from the fermion unification in the traditional Grand Unified Theory and the mass hierarchy between the 4-dimensional Planck scale and weak scale, we suggest the low energy gauge unification theory with low high-dimensional Planck scale. We discuss the non-supersymmetric $SU(5)$ model on the space-time $M^4 \times S^1/(Z_2 \times Z_2)$ where there are three pairs of Higgs doublets in the bulk; the supersymmetric $SU(5)$ model on the space-time $M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2)$ where there are one pair of Higgs doublets.
and one pair of hypercharge one singlets on the boundary 4-brane at \( z = \pi R_2 / 2 \), or one Higgs doublet \((H_d)\) on the boundary 4-brane at \( z = \pi R_2 / 2 \). The \( SU(5) \) gauge symmetry is broken by the orbifold projection for the zero modes, and the gauge unification is accelerated due to the \( SU(5) \) asymmetric light KK states. In our models, we forbid the proton decay, still keep the charge quantization, and automatically solve the fermion mass problem. We also comment on the anomaly cancellation and the other possible scenarios for low energy gauge unification.

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