Topics in Light Hadron Mass Spectrum in Quenched QCD *

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Several topics concerning the light hadron spectrum are discussed. Flavor symmetry breaking effects and the problem of quenched chiral logarithm are examined with pion mass data for the Kogut-Susskind quark action, and light meson decay constants for the Wilson action calculated with non-perturbative renormalization constants are discussed. Results for quark masses are also given both for the Kogut-Susskind and Wilson actions.

1. Introduction

In this report we discuss several issues concerning the light hadron mass spectrum in quenched QCD with the Kogut-Susskind (KS) and Wilson quark actions. For the KS action we use data generated in our $B_K$ measurements in the range $\beta = 5.7 - 6.4$. The Wilson data come from $f_B$ and $B_K$ measurements at $\beta = 5.9 - 6.3$. We refer to refs. \cite{1,2,3} for simulation details, concentrating here on physics issues.

2. Flavor breaking with the KS action

Spectral quantities for the KS action are expected to have flavor breaking errors of $O(a^2)$. For the pion spectrum in physical units this means that

\begin{align*}
m^2_{\pi(NG)} &= B_m a + O(m_q^2) \\
m^2_{\pi(non-NG)} &= \Lambda^4 a^2 + B_m a + O(m_q^2)
\end{align*}

to leading order in $a$, where $\Lambda$ denotes the symmetry-breaking scale, NG means the Nambu-Goldstone channel $\gamma_5 \otimes \xi_5$ and non-NG any other channels. An interesting implication is that the mass splitting $\Delta m_\pi = m_{\pi(NG)} - m_{\pi(non-NG)}$ should vanish linearly $\Delta m_\pi = O(a)$ at $m_q = 0$, while the behavior should change to $\Delta m_\pi = O(a^2)$ for fixed and finite values of $m_q$ satisfying $B_m a \gg \Lambda^4 a^2$. These predictions are confirmed in our data as shown in fig. \ref{fig:1}, where we take $\pi(\gamma_5 \gamma_4 \otimes \xi_5 \xi_4)$ for the non-Nambu-Goldstone pion. This conforms the result of a similar test made previously \cite{4}.

3. Quenched chiral logarithm

The Nambu-Goldstone pion mass for the KS action offers a good testing ground for the predictions of quenched chiral perturbation theory \cite{5}. We fit our data for $m_{\pi(\gamma_5 \otimes \xi_5)}$ for equal and unequal quark masses to the form (fit-1),

\begin{align*}
(m_\pi a)^2 &= A(m_1 + m_2) a \left[1 - \delta \left\{ \log \frac{2A m_1}{\Lambda} \\
&+ \frac{m_2}{m_2 - m_1} \log \frac{m_2}{m_1} \right\} \right] \\
&+ C_4(m_1 + m_2)^2 a^2 + C_5 m_1 m_2 a^2.
\end{align*}
Figure 1. $\Delta m = m_\pi(\gamma_5 \gamma_\tau \otimes \xi_5) - m_\pi(\gamma_5 \otimes \xi_5)$ as a function of $a$. Lines are linear ($m_q = 0$) or quadratic ($m_q = 50$ MeV) fits.

Figure 2. Parameter $\delta$ extracted from the Nambu-Goldstone pion as a function of $m_\rho a$. See text for details.

The quadratic terms are added since fits with only the logarithm term become untenable as $\beta$ increases beyond $\beta = 6$. Fits are also made in which the quark masses inside the logarithm are corrected by a constant shift to take into account the non-Nambu-Goldstone nature of $\eta'$ in the KS formalism (fit-2).

We find both types of fits to be acceptable, and the resulting $\delta$ reasonably stable as a function of lattice spacing as shown in fig. 2. In this sense our data are consistent with the presence of the quenched chiral logarithm. The magnitude of $\delta$, $\delta = 0.05 - 0.07$ for fit-1 and $\delta = 0.08 - 0.1$ for fit-2, however, is a factor 2–3 smaller than is expected in the real world $\delta \approx 0.2$.

4. Light Quark Masses

Continuum extrapolations of light quark masses have been recently discussed by several authors [6,7]. In fig. 3(a) we plot our results for $m = (m_u + m_d)/2$ in the $\overline{MS}$ scheme at $\mu = 2$ GeV for the Wilson (solid circles) and KS (bursts) actions. Results for the strange quark mass determined from $m_\phi$ are shown in fig. 3(b) (estimates from $m_K$ are not shown, being essentially $m_s \approx 200$). Our results are obtained by a 2-loop running of tadpole-improved $\overline{MS}$ masses at the scale $1/a$ calculated with $\alpha_V(1/a)$. The lattice spacing is fixed by $m_\rho$.

For the Wilson action our results show a weaker $a$ dependence compared with those of other groups, albeit errors are large and the lattice size used is rather small ($L a \approx 1.9$ fm). As a result, the agreement of Wilson and KS results after continuum extrapolation is less apparent (cf. ref. [6]). We note, however, that the one-loop renormalization factor for the KS action has a large value of almost two in the range of our data. Higher order corrections might well bring the results for the KS action into agreement with those of the Wilson action.

5. Meson decay constants for the Wilson action

Determination of hadronic matrix elements often suffers from uncertainties in renormalization factors. An exceptional case is the vector meson decay constant $f_V^1$ for which the renormalization constant $Z_V$ for the local vector current can be precisely determined non-perturbatively with the use of the conserved current [11]. For our Wilson runs at $\beta = 5.9, 6.1$ and $6.3$, we obtain $Z_V = 0.532(4), 0.595(3)$ and $0.640(4)$, respectively.

In fig. 4 we plot $f_V^{-1}$ as a function of $(m_\pi/m_\rho)^2$ for the three $\beta$ values using the non-perturbative result $Z_V = (1 - 0.820 \alpha_V(1/a))/8K_c$. We find an almost perfect scaling with the non-perturbative estimate of $Z_V$ as was noted previously [9], while the perturbative $Z$ factor gives results significantly varying with $\beta$.

Making a linear extrapolation of the non-
perturbative results in $m_\rho a$, we obtain $f_\rho^{-1} = 0.270(25)$ and $f_\phi^{-1} = 0.239(10)$ in the continuum limit. These values are in a good agreement with the experiment $f_\rho^{-1} = 0.287(7)$ and $f_\phi^{-1} = 0.234(3)$. For comparison use of perturbative $Z_V$ leads to the results $f_\rho^{-1} = 0.255(27)$ and $f_\phi^{-1} = 0.217(12)$ which are smaller than the experimental values by one standard deviation.

For the axial vector current, a non-perturbative estimate of $Z_A$ using chiral Ward identities gives $Z_A = 0.741(9), 0.751(11)$ and $0.822(19)$ for $\beta=5.9, 6.1$ and $6.3$ [3]. Using these results and data from ref. [2] we obtain in the continuum limit $f_\pi = 132(19)$ MeV and $f_K = 154(21)$ MeV. Estimating $Z_A$ by tadpole-improved perturbation theory, we find $f_\pi = 122(20)$ MeV and $f_K = 144(18)$ MeV. Both methods give results consistent with experiment within errors.

REFERENCES
1. JLQCD Collaboration (presented by S. Aoki), this volume.
2. JLQCD Collaboration (presented by S. Hashimoto), this volume.
3. JLQCD Collaboration (presented by Y. Kumamashi), this volume.
4. S. R. Sharpe et al., Nucl. Phys. B (Proc. Suppl.) 26 (1992) 197.
5. S. R. Sharpe, Phys. Rev. D41 (1990) 3233; ibid. D46 (1992) 3146; C. Bernard and M. Golterman, ibid. D46 (1992) 853.
6. For a review, see P. Mackenzie, this volume.
7. R. Gupta and T. Bhattacharya, Los Alamos preprint LAUR-96-1840 [hep-lat/9605039].
8. F. Butler, H. Chen, J. Sexton, A. Vaccarino and D. Weingarten, Nucl. Phys. B430 (1994) 179.
9. QCDPAX Collaboration (Y. Iwasaki et al.), Phys. Rev. D53 (1996) 6443.
10. T. Bhattacharya, R. Gupta, G. Kilcup and S. Sharpe, Phys. Rev. D53 (1996) 6486.
11. L. Maiani and G. Martinelli, Phys. Lett. B178 (1986) 265.