Thermal radiation and chemical reaction effects on mixed bioconvection of nanoliquid in a horizontal channel along with microorganisms

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Abstract. The effects of thermal radiation and chemical reaction on the mixed bioconvection nanofluid flow with gyrotactic microorganisms along horizontal channel are analysed. The homotopy analysis method (HAM) is assigned to find the analytic solution of ordinary non-linear differential equations using appropriate similarity variables. The physical interpretation of the required solutions is exhibited through plots. It is observed that the temperature of the fluid increases when rising the values of thermal radiation and thermophoresis parameter. Also the solute concentration and the mass transfer rate are highly influenced by the chemical reaction parameter.

Keywords: Nanofluid, Bioconvection, Microorganisms, Thermal Radiation, Chemical reaction, Horizontal channel.

1. Introduction

Recently, many researchers have been focusing on nanofluid since it extremely increases the heat transfer properties of the base liquid[1-3]. Matin et al. [4] analysed the forced convection stream with chemical reaction of first order through a porous channel on the wall. Halim et al. [5] discussed active and passive controls of the Williamson nanofluid stagnation-pointflow on a stretching/shrinking surface. They established that the skinfriction risesin both surfaces as the value of Williamson parameter raises.

The nanofluid with gyrotactic microorganisms has an enormous number of applications in microbial fuel cell technology and medical filtration device technologies. The motion of themicroorganisms cannot be affected by the motion of nanoparticles since the motion of nanoparticles can be decided by Brownian motion and thermophoresis. The density of microorganismssare little more than water and on average swim upwards. When the upper surface is excessively dense by the presence of the collection of microorganisms, it is destabilized. Then it leadsto a crumbling of microorganisms and the production of macroscopic convection. Gyrotacticmicroorganisms was examined by Pedley et al.[6] and Bees et al.[7]. The nanofluid containinggyrotactic microorganisms with different effects are discussed by [8].

The combined effects of thermal radiation and chemical reaction gives tremendous applications in the chemicalindustries. Eswaramoorthi et al.[9] discussed the radiation andchemical reaction concepts on viscoelastic boundary layer flow through a stretching surfacealong various effects. They observed that the increase irradiation parameterincreases heat transferrate and chemical reaction parameter enhances mass transfer rate. Karthikeyan et al.[10] and Niranjan et al. [11] examined the chemical reaction and radiation effects of a stagnation-pointflow with various factors and environment. Various authors contribute to this type of research through different flow [12, 13].

The aim of this work is to analyse thermal radiation and chemical reaction effects based onthe model given by Xu et al.[14]. The analytic solution is obtained by the method of homotopyanalysis using similarity transformations. This paper is structured as follows: section 2
contain the mathematical formulation, the result and discussion is given in section 3 and the concluding remarks are carried out in section 4.

2. Mathematical Modelling

Consider the combined bio-convection of nanofluid in a horizontal channel of length \(2L\) filled along gyrotactic microorganisms as shown in the Figure 1. The axes of the horizontal and vertical directions to the channel walls are along the axes of \(x\) and \(y\), respectively. Let the velocity of upper and lower walls are \(u = ax\). Let \(T_1\) and \(T_2\) are the constant temperatures lie on the top and the bottom of the walls. Suppose \(N_1\) and \(N_2\) are constant densities of the movable microorganisms. Two different solutal concentrations are taken on the walls. Let us consider on the upper wall, the nanoparticle volume fraction satisfies the passively controlled model, but on the lower wall, it is constant. The equations of continuity, momentum, heat energy, nanoliquid concentration, solutal concentration and the microorganisms density are:

\[
\begin{align*}
\nabla \cdot \mathbf{v} &= 0, \\
\rho (\nabla \cdot \mathbf{v}) &= -\nabla p + \mu \nabla ^2 \mathbf{v}, \\
\mathbf{v} \cdot \nabla T &= \alpha \nabla ^2 T + \tau \left[ D_p \nabla \mathbf{T} \cdot \nabla C + \frac{\partial C}{\partial t_1} \nabla T \cdot \nabla T \right] - \frac{1}{\rho c_p} \nabla q_r, \\
(\mathbf{v} \cdot \nabla) C &= D_p \nabla ^2 C + \frac{\partial C}{\partial t_1} \nabla ^2 T, \\
(\mathbf{v} \cdot \nabla) \Gamma &= D_\Gamma \nabla ^2 \Gamma - k_1 (\Gamma - \Gamma_0), \\
\Delta j &= 0
\end{align*}
\]

where \(\mathbf{v} = (u, v)\) being the velocity components of nanoliquid in the Cartesian coordinates, \(p\) and \(T\) be the pressure and heat energy, \(C\) is the nanoparticle volume fraction, \(\Gamma\) - the solutal concentration, \(\rho\), \(\mu\) and \(\alpha\) are respectively density, viscosity and thermal diffusivity of the nanofluid and \(c_p\) is the specific heat.

By Rosseland approximation, the radiative heat flux is given by \(q_r = -\frac{4\sigma^*}{3k^*} \frac{dT}{dy}\), where \(\sigma^*\) be the Stefan-Boltzman constant, \(k^*\) is the coefficient of absorption. Suppose the temperature difference inside the flow is \(T^4\), it can be expressed by using truncated Taylor’s series at \(T_0\) as \(T^4 \approx 4T_0^3(T - 3T_0^4)\). The ratio of the efficient heat energy measure of the nanoparticle to the basic liquid is represented by \(\tau = (\rho c_p)_p/(\rho c_p)_f\). \(D_p\) and \(D_\Gamma\) are called the two diffusion coefficients. The flux of microorganisms is \(j\) with respect to the convection of fluid, self-motivated swimming, and scattering, which is given by \(j = N \mathbf{v} + N \hat{\mathbf{v}} - D_t \nabla \mathbf{N}\) here \(N\) is the microorganisms motile density, the nanofluid cell swimming average directional velocity with the microorganisms diffusivity \(D_t\) is given by \(\hat{\mathbf{v}} = (\hat{u}, \hat{v})\) where \(\hat{u} = (bW_c/C_0) \frac{ac}{dx}, \hat{v} = (bW_c/C_0) \frac{dc}{dy}\), the chemotaxis constant is \(b\) and the maximum speed of swimming cell is \(W_c\).

For a horizontal 2D channel flow problem, equation(2) can be written by

\[
\begin{align*}
\frac{\partial u_1}{\partial x} + v \frac{\partial u_1}{\partial y} &= -\frac{1}{p_f} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial ^2 u_1}{\partial x^2} + \frac{\partial ^2 u_1}{\partial y^2} \right)
\end{align*}
\]
by using the following transformation, the previous equations are simplified

$$\zeta = \frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y} = -\nabla^2 \psi,$$

where $$u_1 = \frac{\partial \psi}{\partial y}$$ and $$v_1 = \frac{\partial \psi}{\partial x}$$.

From the above equations, the expanded governing equations (1)-(6) are given by

$$u_1 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_1}{\partial y} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} \right),$$

and

$$u_1 \zeta_x + v_1 \zeta_y = \nu (\zeta_{xx} + \zeta_{yy}).$$

$$u_1 T_x + v_1 T_y = \alpha \left( T_{xx} + T_{yy} \right) - \frac{1}{\rho C_p} \left( q_x + q_y \right) + \tau \left[ D_b \left( C_x T_x + C_y T_y \right) + \frac{D_T}{T_0} \left( T_x^2 + T_y^2 \right) \right],$$

$$u_1 C_x + v_1 C_y = D_b \left( C_{xx} + C_{yy} \right) + \frac{D_T}{T_0} \left( T_{xx} + T_{yy} \right),$$

$$u_1 \Gamma_x + v_1 \Gamma_y = D \left( \Gamma_{xx} + \Gamma_{yy} \right) - k_i (\Gamma - \Gamma_0),$$

and

$$u_1 N_x + v_1 N_y = D_b N_{yy}.$$  

The conditions at wall and centre of the channel are as follows:

$$u_1 = ax, \quad v_1 = 0, \quad T = T_2, \quad D_b \frac{dC}{dy} + \frac{D_T}{T_2} \frac{dT}{dy} = 0, \quad \Gamma = \Gamma_1, \quad N = N_2 \quad \text{as} \quad y = L;$$

$$u_1 = 0, \quad v_1 = 0 \quad \text{as} \quad y = 0; \quad u_1 = ax, \quad v_1 = 0, \quad T = T_1, \quad C = C_1, \quad \Gamma = \Gamma_2, \quad N = N_1 \quad \text{as} \quad y = -L.$$

By using the successive similarity transformations

$$\psi(x, y) = ax F_1(\eta), \quad \eta = \frac{y}{L}, \quad \theta(\eta) = \frac{T - T_0}{T_2 - T_0}, \quad \phi(\eta) = \frac{C - C_0}{C_0}, \quad \gamma(\eta) = \frac{\Gamma - \Gamma_0}{\Gamma_2 - \Gamma_0}, \quad S(\eta) = \frac{N}{N_2},$$

in Eqs. (7)-(12), continuity equation (7) is obviously true and the simplified form of other equations are given by

$$f_1''' + Re(F_1 fis - f_1' f_1) = 0,$$

$$\left[ 1 + \frac{4}{3} Rd \right] \dot{\phi} = (RePr) f_1 \theta + N b \phi \dot{\phi} + N t \phi^2 = 0,$$

$$\phi' + \frac{N t}{Nb} \dot{\theta} + (ReLe) f_1 \phi = 0,$$

$$\gamma' + ReSc \dot{f}_1 \gamma + CrSc \dot{\gamma} = 0,$$

$$s' - Pe_b \phi (s' + s \phi) + (ReSc) f_1 s = 0,$$  

along with the following conditions at boundary

$$\theta = \delta_0, \quad \phi = \delta_1, \quad \gamma = \delta_2, \quad s = \delta_3 \quad \text{as} \quad \eta = -1: \quad f_1 = 0, \quad \dot{f}_1 = 0 \quad \text{as} \quad \eta = 0;$$

$$f_1 = 0, \quad f_1' = 1, \quad \theta = 1, \quad N b \phi + N t \dot{\theta} = 0, \quad \gamma = 1, \quad s = 1 \quad \text{as} \quad \eta = +1.$$

The dimensionless numbers are given by

$$Re = \frac{a L^2}{v}, \quad Pr = \frac{v}{\alpha}, \quad Nb = \frac{\pi D_b C_0}{\alpha}, \quad Nt = \tau \left( \frac{D_{T}}{T_0} \right) \frac{T_2 - T_0}{\alpha}, \quad Le = \frac{a}{D_{b}}, \quad Pe_b = \frac{bW_c}{D_n}, \quad Sc = \frac{v}{D_{n}}.$$  

3
\[ Cr = \frac{I^2 k_C}{v}, \quad Rd = \frac{4\sigma T_3^3}{k' k}, \quad Sc_C = \frac{\nu}{D}, \quad \delta_y = \frac{T_1-T_0}{T_2-T_0}, \quad \delta_y = \frac{C_1-C_0}{C_0}, \quad \delta_y = \frac{T_1-T_0}{T_2-T_0}, \quad \delta_y = \frac{N_1}{N_2}. \]

In convection flow problems, very important to aware the amount of heat transfer and mass transfer between the solid wall and the liquid. To calculate the heat and mass transfer rates over the channel, it is essential to give description of the local Nusselt number and the local Sherwood number along the boundary. The definitions of the local Nusselt number and local Sherwood number are listed by

\[ Nu = \frac{q_w x}{k(T_2-T_0)}, \quad \text{where the heat flux } q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} - \frac{4\sigma^*}{3k'} \left( \frac{\partial T^4}{\partial y} \right)_{y=0}, \]

\[ Sh = \frac{j_w x}{D b C_0}, \quad \text{where the mass flux } j_w = -D b \left( \frac{\partial C}{\partial y} \right)_{y=0}. \]

From the above equations, the dimensionless form of the Nusselt and the Sherwood numbers of this model are given by \( Nu = -\left[ 1 + \frac{4}{3} Rd \right] \theta'(\eta) \) and \( Sh = -\gamma'(\eta) \) at \( \eta = 0 \).

3. Results and analysis

The set of non-linear differential equations (13)-(17) are evaluated by using HAM technique subject to the values of parameters \( h_f = h_g = -1.5, \quad h_b = h_r = -0.2, \quad h_t = -0.5 \). The fixed values of other parameters are taken as \( Pr = Sc = Le = Pe_b = Sc_e = Rd = Cr = 1, \quad Nb = Nt = 0.5, \quad \delta_y = \delta_y = 0.5, \quad \delta_y = 1 \). The Radiation parameter \( (Rd) \) and Chemical reaction parameter \( (Cr) \) lie between 0 to 2, \( -20 \leq Re \leq 20 \) for Reynolds number, Brownian motion and Thermophoresis parameters lie between 0 to 5.

The profile of temperature, nanoparticle volume fraction and heat transfer rate under the radiation are displayed in Fig.2(a-c). Obviously, the radiation stimulate the surface temperature and reduce concentration of the liquid. Hence, the thermal energy of the nanoliquid enhances when growing the values of \( Rd \). Microorganisms density enhances with \( Rd \) the lower part of the channel and it reduces in the upper part. For the negative values of \( Re \), the energy transport reduces but it enhances when the values of \( Re \) are positive.

**Figure 2(a).** The temperature for different entries of \( Rd \) with \( Re = \pm 20, \quad Cr = 1 \).

**Figure 2(b).** The density of microorganisms for various entries of \( Rd \) with \( Re = \pm 20, \quad Cr = 1 \).
Figure 2(c). Nusselt number versus Re including \( Nb = Nt = 0.5, Cr = 1 \).

Figure 3(a). The solute concentration for various entries of \( Cr \) including \( Re = \pm 1 \), \( Nb = Nt = 0.5, Rd = 1 \).

Fig.3(a-b) discuss the impact of chemical reaction for the solutal concentration and mass transfer rate of the nanoliquid. It is established that the solute concentration is more pronounced with the rise of \( Cr \). The mass transfer rate declines for the negative values of \( Cr \) and it get enhances for the positive values of \( Cr \). It is observed from the Fig.4(a) and 4(b) that the influence of Brownian motion parameteron nanoparticle concentration and density of microorganisms. The increasing values of \( Nb \) enhances nanoparticle concentration near the upper wall. The opposite trend is discovered on growing the values of \( Nb \) in the upper and lower portions of channel, respectively.

Figure 3(b). Sherwood number versus Re including \( Nb = Nt = 0.5, Rd = 1 \).

Figure 4(a). The nanoparticle concentration for various entries of \( Nb \) including \( Re = \pm 10 \), \( Cr = Rd = 1, Nt = 0.5 \).

Figure 4(b). The density of microorganisms for various entries of \( Nb \) with \( Nt = 0.5, Cr = Rd = 1, Re = \pm 10 \).

Figure 5(a). The temperature for different entries of \( Nt \) with \( Re = \pm 20 \), \( Nb = 0.5, Cr = Rd = 1 \).
Figure 5(b). The nanoparticle concentration for various entries of $N_t$ including $Re = \pm 10$, $Cr = Rd = 1, Nb = 0.5$.

Figure 5(c). The microorganisms density for various entries of $N_t$ with $Re = \pm 10$, $Cr = Rd = 1, Nb = 0.5$.

Figure 5(d). Nusselt number versus $Re$ including $Nb = 1, Cr = Rd = 1$.

Fig.5(a-d) displayed the thermophoresis parameter effect on temperature, concentration of nanoparticle and microorganisms and the heat transport inside the channel. It is established that the temperature inside the channel raises when $N_t$ reduces. Growing the values of $N_t$, the concentration of the nanofluid diminishes. It is detected that $S(\eta)$ enhances with $N_t$ in the lower portion of the channel and the reverse trend is observed in the upper portion. The local Nusselt number behaves in the same manner with respect to $Re$ as like thermal radiation.

4. Conclusion

In this paper, mixed bioconvection of nanofluid with microorganisms across horizontal channel bearing thermal radiation, chemical reaction, Brownian motion and thermophoresis parameters is illustrated in this study. The influences of these parameters are given by the virtue of graphs. The influences of different physical parameters in heat and mass transfer inside the channel are also discussed by using graphs. The main results are listed as below:

- The radiation and thermophoresis parameter increases the temperature in the channel.
- The nanoparticle concentration enhances on rising the Brownian motion parameter, whereas it diminishes on growing the values of thermophoresis parameter.
- The chemical reaction regulates the solutal concentration and the mass transfer rate.
- The heat transfer rate reduces with respect to the radiation and thermophoresis parameter.

References
[1] Xu H and Pop I 2012 Fully developed mixed convection flow in a vertical channel filled with nanofluids Int. Comm. Heat and Mass Trans.39 1086–1092
[2] Sivasankaran S, Aasaithambi T and Rajan S 2014 Natural convection of nanofluids in a cavity
with linearly varying wall temperature\textit{Maejo Int. J. Sci. and Tech.}\textbf{238}149–162

[3] Ramly N A, Sivasankaran S and Noor N F M 2016 Numerical Solution of Cheng-Minkowycz Natural Convection Nanofluid Flow with Zero Flux \textit{AIP Conf. Proce.}\textbf{1750} 030020-1-8

[4] Matin M H and Pop I 2013 Forced convection heat and mass transfer flow of a nanofluid through a porous channel with a first order chemical reaction on the wall \textit{Int. Com. in Heat and Mass Trans.}\textbf{46} 134–141

[5] Halim N A, Sivasankaran S and Noor N F M 2017 Active and passive controls of the Williamson stagnation nanofluid flow over a stretching/shrinking surface \textit{Natural Comp. Appl.}\textbf{28 (Suppl 1)} S1023–S1033

[6] Pedley T J, Hill N A and Kessler J O 1988 The growth of bioconvection patterns in a uniform suspension of gyrotactic micro-organisms \textit{J. Fluid Mech.}\textbf{195} 223–237

[7] Bees M A and Hill N A 1999 Non-linear bioconvection in a deep suspension of gyrotactic swimming micro-organisms \textit{J. Mathe. Bio.}\textbf{38} 135–168

[8] Raees A, Xu H and Liao S J 2015 Unsteady mixed nano-bioconvection flow in a horizontal channel with its upper plate expanding or contracting \textit{Int. J. Heat and Mass Tran.}\textbf{86} 174–182

[9] Eswaramoorthi S, Bhuvaneswari M, Sivasankaran S and Rajan S 2016 Soret and Dufour effects on viscoelastic boundary layer flow over a stretching surface with convective boundary condition with radiation and chemical reaction \textit{Sci. IranicaB}\textbf{23(6)} 2575-2586

[10] Karthikeyan S, Bhuvaneswari M, Sivasankaran S and Rajan S 2016 Soret and Dufour Effects on MHD Mixed Convection Heat and Mass Transfer of a Stagnation Point Flow towards a Vertical Plate in a Porous Medium with Chemical Reaction Radiation and Heat Generation \textit{J. App. Fluid Mech.}\textbf{9(3)} 1447-1455

[11] Niranjan H, Sivasankaran S, Bhuvaneswari M and Siri Z 2015 Effects of chemical reaction on MHD mixed convection stagnation point flow toward a vertical plate in a porous medium with radiation and heat generation \textit{J. Phy. Conf. Series}\textbf{662} 012014-1-9

[12] Rahman M M and Eltayeb I A 2013 Radiative heat transfer in a hydromagnetic nanofluid pasta non-linear stretching surface with convective boundary condition \textit{Meccanica}\textbf{48} 601–615

[13] Eswaramoorthi S, Bhuvaneswari M, Sivasankaran S, Niranjan H and Rajan S 2017 Effect of partial slip and chemical reaction on convection of a viscoelastic fluid over a stretching surface with Cattaneo-Christov heat flux model \textit{J. Phy. Conf. Series: Materials Sci. and Engg.}\textbf{263} 062009-1-9

[14] Xu H and Pop L 2014 Fully developed mixed convection flow in a horizontal channel filled by a nanofluid containing both nanoparticles and gyrotactic microorganisms \textit{Euro. J. Mech. B/Fluids}\textbf{46} 37 - 45