CAN HIGH ENERGY COSMIC RAYS BE VORTONS?

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A simple model is exhibited in which the remnant density of charged vortons is used to provide candidates for explaining the observed ultra high energy cosmic rays (above $10^{20}$ eV). These vortons would be accelerated in active galaxies and propagated through intergalactic medium with negligible losses of energy. The expected number density of observable events is shown to be consistent with extrapolation of the observations. The spectrum is predicted to be spatially isotropic while its shape is that of an atomic excitation-ionisation, i.e. with a few peaks followed by a continuum; there is also an energy threshold below which no vorton is visible.

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INTRODUCTION

The problem of very high energy cosmic rays \textsuperscript{1} is still an open one: most models based on reasonable astrophysical assumptions seem to indicate a likely maximum value for the energy of any kind of emitted particle at most of $\sim 10^{19}$ eV \textsuperscript{2} and the existence of the microwave background makes it impossible for a proton say to propagate with such energy on scales much larger than a few tens of Mpc; this is the celebrated Greisen-Zatsepin-Kuz'min (GZK) cutoff \textsuperscript{3}. An observation dated October 1991 by the Fly’s Eye detector showed evidence for a cosmic ray at an energy of $(3.2 \pm 0.6) \times 10^{20}$ eV \textsuperscript{4} and a few others reported showers above $10^{20}$ eV \textsuperscript{5} (and well above the GZK cutoff). There does not seem to date to be any completely satisfactory suggestion for explaining these observations (see however Ref. \textsuperscript{6}), and it is the purpose of this paper to propose such a possible explanation that has been overlooked, namely that cosmic rays may consist of (as opposed to originate in) topological defects (as was first proposed in Ref. \textsuperscript{7}).

A fairly common (neither confirmed nor excluded) prediction of particle physics models at energies beyond that of the electroweak scale is the existence of topological defects \textsuperscript{8}. Among these, only cosmic strings seem to be consistent with cosmological constraints (with the possible exception of textures which, as non-localized objects, are not considered here). In particular, strings have been proposed as seeds for galaxy formation as well as sources for the microwave background fluctuations observed by COBE. This requires that the strings appear at the Grand Unified (GUT) scale, a scenario that may have been postponed until this stage).

A second reason to consider such comparatively low energy cosmic strings instead of the more popular GUT strings is that the latter have been shown \textsuperscript{9} to yield a negligible flux of cosmic ray type events as compared to what is currently observed, whatever the mechanism involved. This is mostly based on the idea that, a string being a topologically stable object, emission of very high energy particles must proceed via some removal of the topological stability, which for a GUT string implies either extremely highly energetic phenomena (and we’re back to the previous problem of reaching the required high energies) or relatively rare situations (cusp evaporation, intercommutation and loop collapse). Adding up both cases, the expected flux turns out to be very tiny (roughly $10^{-10}$ times that observed). Our aim here is to show that it is possible in principle to consider high energy rays as cosmic string loops themselves if the scale is not that of GUT but rather much below that scale. This, as we shall see, gives a flux that is comparable with observations.

The paper is organized as follows: in Section \textsuperscript{10}, we recall the basic facts about vortons and how one may expect them to interact with ordinary particles. Then, Section \textsuperscript{11} is devoted to the interaction of a vorton with a proton at rest in order to estimate the order of magnitude of the various phenomena involved in the detection of a $10^{20}$ eV event, including the probability that a vorton interacts with the atmosphere. It is predicted that the typical expected spectrum form should resemble that of an excitation-ionisation spectrum, namely it should consist of lines followed by a continuum. Besides, the interaction probability is found to be quite low (the cross-section is typical of neutrino-hadron interactions at the same energies \textsuperscript{12}, we predict important horizontal showers. In Section \textsuperscript{13}, we present a plausible acceleration mechanism for vortons, which turns out
to be much simpler than most of the standard acceleration mechanisms for protons (Fermi-like mechanisms), without sharing their drawbacks. We then discuss propagation in Section IV where it is shown that no GZK cutoff is to be expected for vortons. It is then argued that this model implies that the most energetic events should be distributed isotropically. Finally we summarize our findings in the conclusion where we also compare the expected properties of these rays with the anticipated detection capabilities of the future Auger observatory.

I. THE PHYSICS OF VORTONS

The objects we shall now consider are vortons [10,11,14], namely loops of cosmic strings endowed with superconducting properties and stabilized by a current. They arise in symmetry breaking theories above the electroweak scale for which the vacuum manifold is not simply connected. In other words, if the high energy vacuum is invariant under the transformation of a symmetry group $G$ and the low energy vacuum under that of the group $H$, the necessary and sufficient condition for the existence of cosmic strings is that the first homotopy group $\pi_1$ of the quotient $G/H$ be nontrivial, $\pi_1(G/H) \neq \{0\}$. A typical example is the scheme $U(1) \rightarrow \{0\}$, which is also the scheme at work in the Landau-Ginzburg model for superconductivity [18], and the strings are the corresponding vortices, much studied experimentally [19]. The vortices appear as infinite or in the form of closed loops which decay via emission of gravitational radiation. The symmetry breaking is achieved thanks to a Higgs field $\Phi$ which in turn can be coupled to other fields, particularly charged ones, which we shall generically write as $\Sigma$. Here we consider only the case where this charge carrier is a scalar particle, noting anyway that because of the formal equivalence between bosons and fermions in the two-dimensional worldsheet generated by the string, one expects similar behaviors for any kind of particle. Besides, as was shown to be sufficient in previous work [22], we shall only consider the coupling of $\Sigma$ with the electromagnetic field as a perturbative effect not modifying much the overall dynamics of the vorton.

Let us here summarize the basic properties of vortons [13] as we expect to observe them. They can be characterized by essentially two integer numbers, namely a topological one, $N$, say, which specifies the winding of the current carrier phase around the loop, and a dynamically conserved number, $Z$, related in the charge-coupled case (to which the present analysis is restricted) with the total amount of electric charge $Q$ that it holds through $Q = Ze$, with $e$ the electron electric charge. Note that both $N$ and $Z$ can be positive or negative, although for $N$ it is just a matter of convention, while in the case of $Z$, the sign is important when there is an external interaction (as in the case of acceleration for instance).

Both these numbers are conserved at a classical level and one expects them to be of the same order of magnitude $Z \sim N \sim 100$ [14]. From these, one can evaluate the total mass-energy of the vorton simply in terms of its characteristic radius $r_{\nu}$, its energy per unit length $U \sim m^2$, and tension $T \sim m^2$ as

$$M_{\nu} = 2\pi r_{\nu}(U+T) \sim m^2r_{\nu},$$

with $m$ the energy scale at which the strings are formed (essentially the string forming Higgs mass). Moreover, we shall assume in what follows that the current carrier mass scale $m_{\sigma}$ is also of order $m$ so that we can keep only one energy scale. Note however that the following analysis can be easily generalized for two different mass scales since in practice it is the current scale that is relevant in most calculations. Then, knowing the angular momentum to be given by $j^2 = UTr_{\nu}/2\pi$ permits to calculate the characteristic vorton circumference as

$$2\pi r_{\nu} = (2\pi)^{1/2}|NZ|^{1/2}(UT)^{-1/4} \sim Z/m$$

with a corresponding mass

$$M_{\nu} \sim Zm.$$ 

Moreover, the mass scale is constrained: depending on various assumptions about the string network evolution and the rate of loop formation as well as the probability that arbitrarily shaped loops end up in vorton states, it can be shown that, if the vortons are stable [12,13], then in order to avoid a cosmological mass excess ($\Omega_{\nu} < 1$), the condition

$$m < 10^9 \text{ GeV},$$

must be satisfied.

Having discussed the basic properties of vortons, let us now turn to a rough evaluation of the typical mass scale expected for $m$ if those vortons were to be seen as cosmic rays, developing air showers. As we shall see in the following section, interaction between a vorton and whatever other particle occurs mainly via inelastic scattering resulting in the extraction of a trapped $\Sigma$ particle. In other words, the current flowing along the string loop can be seen as a bunch of bound states which can be excited provided the interaction energy is large enough. We therefore conclude at the existence of an energy threshold above which the typical expected spectrum should change qualitatively. Besides, and we now come to the second firm prediction of the model, since those particles form bound states, we expect them to show up in the form of a line spectrum, bound states being always quantized; this will be shown on a specific vorton model in section IV.
For the time being, what really matters is the existence of bound state energies the order of magnitude of which we shall now attempt to evaluate.

Let $\Delta E$ be the variation of energy between two energy levels in the vorton, calculated in its rest frame, and $\gamma$ its Lorentz factor in the rest frame of the particle it interacts with (recall we are at the end interested in air showers occurring in the atmosphere where the particles interacting with the vorton, namely essentially quarks composing protons and neutrons, are supposed to be at rest). The energy $\epsilon$ at which the interaction is then seen is obtained by transforming back to the particle’s rest frame,

$$\epsilon \sim \gamma \Delta E.$$  \hspace{1cm} (5)

Denoting by $\tilde{m}$ the particle’s mass and requiring the interaction to actually take place gives

$$\gamma \tilde{m} \sim \Delta E,$$  \hspace{1cm} (6)

so that altogether, Eqs. (5) and (6) imply

$$\gamma^2 \sim \frac{\epsilon}{\tilde{m}}.$$  \hspace{1cm} (7)

from which the characteristic cosmic string scale can be deduced in the following way. From the fact that the angular momentum of the vorton is $J \sim Z^2$, it is seen that the density of states scales like $Z^{-2}$. Therefore, $\Delta E \sim m/Z^2$, so that using (5) and (6) yields

$$m \sim Z^2 \sqrt{\epsilon \tilde{m}}.$$  \hspace{1cm} (8)

We shall now use these relations together with the observations that have been realized on cosmic rays to normalize the energy levels.

Let us apply this evaluation to ultra high energy cosmic rays so that the characteristic observed energy is normalized to $\epsilon \sim 10^{20}$eV. The particles vortons interact with are essentially quarks composing hadrons, so the mass $\tilde{m}$ can be taken to be that of the proton (at this level of approximation, the mass difference between a quark and a proton is negligible). With these numbers in mind, Eq. (8) transforms to $\gamma^2 \sim 10^{11}$ and the cosmic string scale $m \sim 10^{9}$ GeV which, surprisingly enough, turns out to be right at the closure limit given by Eq. (8). It now remains to calculate the effective cross-section between a vorton and a quark in order to know the expected flux and check that it is compatible with the current observational limits.

II. CROSS-SECTION VORTON-HADRON

There is no privileged model for cosmic strings, and therefore neither is there for describing vortons. Thus, we can at best evaluate rough orders of magnitude for the cross section we are looking for. There are however various levels of approximation at which a vorton can be described. As mentioned earlier, the characteristic length scale associated with a vorton configuration is expected to be a hundred times larger than its thickness, so the most obvious description of a vorton is that of a classical circle. This is unfortunately of absolutely no use when one is looking for the trapped $\Sigma$ particles along the loop. Hence, as a second level of approximation, we shall consider a vorton to be a torus like configuration in which the field $\Sigma$ feels a confining potential. This is beyond the scope of the present calculation and is left for further investigation. So let us describe a vorton as a sphere of radius $R$ where in fact the mass of $\Sigma$ is a radially dependent function. In order to try and recover the circular geometry, the confined field, expanded as it should in the eigenvectors of angular momentum (spherical harmonics), will be described only by those high values of the total angular momentum (to take into account the fact that $J \gg 1$) as well as its projection along a fixed axis.

The basis for the possible bound states of $\Sigma$ will therefore be given as the set of quantized solutions of the Klein-Gordon equation

$$(\Box + M^2)\Sigma = 0,$$  \hspace{1cm} (9)

where the mass $M = M(r)$. The effective radius $R$ is adjusted in such a way that the geometrical section of the corresponding sphere $\pi R^2$ is that of the equivalent ring if it hits a particle face-on, i.e. $\pi R^2 = \pi [(r_v + r_x)^2 - (r_v - r_x)^2]$, or $R^2 = 4r_v r_x$. In these formula, $r_v$ stands as before for the vorton radius, while $r_x \sim m^{-1}$ is its thickness, i.e. $R \sim Z^1/2 m^{-1}$.

In the particular example which was used to calculate numerically the curve on Fig. 1, the mass function in Eq. (9) is taken as $M(r) = m\Theta(R - r)$, and the trapped field is assumed the separated form $\Sigma(x^a) = u_{\alpha\ell}(r)Y_{\ell M}(\theta, \phi)e^{imX}$, with $Y_{\ell M}(\theta, \phi)$ a spherical harmonic and the radial function provides the quantized energy levels for each value of the angular momentum by imposing equality of the logarithmic derivative of the normalized solutions at $r = R$ (in the simplified model we investigate, those are spherical Bessel and Hankel functions of the first kind). As discussed above, we shall restrict our attention to $M \sim \ell \sim Z^2 \sim 10^4$ in order to take into account, at least partially, the effectively circular (as opposed to spherical) symmetry. For the actual calculations of effective cross-sections, we shall adopt the same normalization convention for all the states, i.e. the bound and free states (including hadron states) are covariantly normalized at $2E$ particles in the volume $V$ for a state of energy $E$ ($\omega_{\alpha\ell M}$ in the case of a bound state).

We now calculate the cross section $\sigma_{\alpha}$ for an incident electromagnetically charged particle, a hadron say for definiteness, in the frame of the loop, to interact inelastically with a vorton. We consider the situation in the rest frame of the loop in which the incident hadron with momentum $p_m = (E_1, p_1)$ hits the $\Sigma$ particle bound state characterized by its angular quantum number $\ell_i$. 

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and bound energy \( \omega_1 \) to yield an outgoing hadron with momentum \( p_{\text{out}} = (E_2, p_2) \) and a final state for the \( \Sigma \) particle which could in principle be either another bound state with quantum number \( \ell_f \) or a free particle state with momentum \( k_f = (\omega_2, k) \); in the notation of the previous section, one has \( \Delta E = \omega_2 - \omega_1 \). In both cases, we assume the decay rates \( d\Gamma_j \) of the resulting products, namely either the excited state or the unstable particle itself, to be much smaller than the energies of the particles so that the cross-section for producing those unstable states effectively factorizes as \( d\sigma = d\sigma_j d\Gamma_j / \Gamma_j \). Integrating over the various decay possibilities (\( \int d\Gamma_j = \Gamma_j \)) shows that it suffices to calculate only the intermediate cross sections \( d\sigma_j \) and sum them up. Note also that because our vorton model is very crude, we cannot expect more than a rough order of magnitude for the cross-section, a detailed quantum examination of the same process being held for another work \cite{23}. This is the reason why we now restrict our attention to the ionisation process, including the possibility of final bound state by considering various energy eigenvalues.

The interaction is assumed to be only electromagnetic. Hence, if \( \psi_{\text{in}} \) and \( \psi_{\text{out}} \) are the wave functions of the hadron under consideration before and after the collision, the hadronic current \( J^h \) we are interested in reads

\[
J^h_\mu \equiv ie(\psi^*_{\text{out}} \partial_\mu \psi_{\text{in}} - (\partial_\mu \psi^*_{\text{out}}) \psi_{\text{in}}),
\]

a similar definition holding for the \( \Sigma \) current \( J^{\Sigma} \) where the “in” state is a bound state, while the “out” state is a free \( \Sigma \) particle. With the electromagnetic field \( A_\mu \), the total Hamiltonian for describing the collision is then

\[
H = J^h_\mu A^\mu J^\Sigma_\beta A^{\beta} + H^{\Sigma} + H^h,
\]

where the self interaction Hamiltonians \( H^{\Sigma} \) and \( H^h \) contain in principle all the information about the internal structure of the interacting particles. In practice, this means that we can work with eigenstates of both these self-interacting Hamiltonians to take into account exactly the vorton structure (this is supposedly done by calculating the bound states) as well as the strong interaction effects binding the quarks in the hadron together; thus, both protons and neutrons can interact with a vorton, a point which is understandable by noting that the vorton being much smaller than the hadron, it interacts essentially with the quarks. Neglecting these corrections of order unity, and working in the Lorentz gauge \( \partial_\mu A^\mu = 0 \), the amplitude for this process is easily calculated semiclassically as (note we do not consider the fermionic degrees of freedom of the hadrons)

\[
\langle p_{\text{in}}, \ell_f | p_{\text{out}}, k(\ell_f) \rangle = i \int d^4x g^{\mu\nu} J^h_\mu(x) q^{-2} J^{\Sigma}_\nu(x) \]

where \( q = p_{\text{out}} - p_{\text{in}} \) is the exchanged momentum. At this point, a completely quantum field analysis could be made by expanding the \( \Sigma \) operator in bound and free particle creation and annihilation operators and the hadron field in plane waves and imposing suitable commutation relations (anticommutation to describe properly the hadron states) \cite{23}. We recall that our purpose here is simply to derive a rough order of magnitude estimate for the interaction cross section, which explains why we perform only a semiclassical analysis and neglect the fermionic behaviour of hadrons. Use of Eq. \( (12) \) shall be made later for deriving the details of our oversimplified spherical model, but for the time being, let us derive the expected order of magnitude of the interaction cross-section.

The total cross-section, being a sum over all the confined particles of terms like Eq. \( (12) \) squared is seen to have a factor \( Z^2 e^4 \), with \( e \) the electromagnetic coupling constant. Then, on dimensional grounds, it can be nothing but proportional to \( (\Delta E)^{-2} \). Thus, one ends up with

\[
\sigma_v = \frac{Z^2 e^4}{(\Delta E)^2} \mathcal{F}(\Delta E/m)
\]

\[
\simeq 10^{-32} \left( \frac{10^9 \text{ GeV}}{m} \right)^2 Z_{100}^6 \text{ cm}^2,
\]

where \( Z_{100} \equiv Z/100 \) and the function \( \mathcal{F} \) is a dimensionless quantity that needs a specific model to be evaluated. In the spherical symmetry approximation and for a few energy levels, it is exemplified on Fig. \( 1 \). The line and continuum spectrum characteristic of bound state interactions is clearly shown on the figure, as well as the existence of an energy threshold below which no interaction takes place.

To conclude this section, we consider the probability that a vorton interacts in the way discussed above in the atmosphere. Using the values derived in the previous section for the mass and charge of the vorton shows that the

\[
\text{FIG. 1.} \text{ Expected qualitative form of the cosmic ray spectrum for vortons: it consists of a line spectrum followed by a continuum. This function } \mathcal{F}(\Delta E/m) \text{ [see Eq. \( (13) \)] modulates the cross-section and was calculated using 6 energy levels with } \ell = 10^4, \text{ taking everything numerically into account.}
\]
characteristic ionisation cross section is typical of neutrino interaction at these energies [16]. The probability we are looking for is therefore, using $\rho_{Atm} \sim 10^3 \text{g cm}^{-2}$ for the mean atmospheric depth

$$\alpha_\nu = \sigma_\nu \rho_{Atm}/m_p \sim 10^{-5} - 10^{-4} Z_{100}^6.$$  

(14)

This quantity we shall use later to compute the expected flux of this type of events on earth.

III. ACCELERATION MECHANISM FOR VORTONS

In this section and the following, we turn back to ordinary units in which $c$ and $\hbar$ have their usual values.

Accelerating a particle of charge $Z e$ like a vorton to energies larger than $10^{20}$ eV by means of an electric mechanism requires a potential difference larger than $10^{20}/Z$ Volts. There are basically two classes of astrophysical objects in which such potential differences may be found, namely pulsars and accreting black holes (BH). In fact, potential differences as high as $10^{18}$ Volts are known to exist in the magnetosphere of young pulsars and in that of the accretion disks around massive $(10^7 - 10^8 M_\odot)$ Kerr BH.

Pulsars as sources of high energy vortons can be immediately ruled out since vortons are not expected to be present at the surface of neutron stars. Their mass is so high, as we have just seen, that they would sink toward the center of the neutron star. We shall therefore for now consider the case of accreting BH, supposing those to be the power engine for active galactic nuclei (AGN) radio galaxies and quasars (QSO). The underlying idea is the following: radio jets, $X$ and $\gamma$ rays emissions are due to the acceleration of electrons, positrons and protons by electrostatic fields generated by a Blandford type mechanism [24] near the horizon of the BH. The model is based on the assumption that a fraction $\alpha$ of the total matter is made of vortons [4], which behaves essentially as cold dark matter. Therefore, one can expect their spatial distribution to look like that of ordinary matter. The main idea is then that in an accelerating object, whatever it is, the same fraction $\alpha$ of vortons is accelerated by the same mechanism that works for the protons. However, in reasonable models, the total mass of the vorton is $M_V \sim Z m$ where $Z$ is the charge carried by the vorton and $m$ the scale of symmetry breaking at which the strings were first formed. As shown in Sec. 1, a Lorentz acceleration factor of $10^5 - 10^6$ is actually sufficient to reproduce the data. This means that a very basic acceleration by means of an electric field is enough since at these energies, losses in radiation can be considered negligible. In what follows, we shall justify the above model to show that it encompasses no major difficulties.

It should first be clear, and we want to emphasize that point again, that no exotic mechanism is required for the acceleration (even though the accelerated particles themselves may be considered exotic). Indeed, the acceleration mechanism that produces ultra-relativistic jets in radio galaxies and $X$ and $\gamma$ rays in AGNs and QSOs is enough to accelerate vortons to energies up to $10^{20}$ eV and higher. We want to point out that electrostatic fields are very likely to be responsible for the acceleration of particles in jets of radio galaxies and for the generation of high energy photons (with energies exceeding 1 TeV). Although the origin of such electrostatic fields is not completely understood yet, they may well originate through the quite appealing Blandford–Znajecs mechanism [2]. Let us first describe shortly such a mechanism.

Consider a magnetized rotating neutron star. Its surface is supposed to be a perfect conductor, so that in the rest frame of the surface of the neutron star, the tangential component $E_\theta$ of the electrostatic field must vanish, a condition which, when written in the inertial frame takes the form

$$E_\theta - (\frac{\Omega \times B}{c} R_\ast)_\theta = 0,$$

(15)

where $\Omega$, $B$ and $R_\ast$ are respectively the angular velocity, surface magnetic field and radius of the neutron star. This gives very energetic electric field lines along which charged particles can be easily accelerated.

An analogous mechanism for generating electric field lines works for Kerr BHs, with the unfortunate difference that a bare BH cannot have a magnetic field, the latter being supplied by the accretion disk via a dynamo mechanism. The surface of the BH behaves like a rotating conductor with angular velocity $\Omega = \dot{a} c/M$, with $\dot{a}$ and $M$ the Kerr solution parameters somehow identifiable with the total mass $(M)$ and the angular momentum (for $0 < \dot{a} < 1$) of the source. Now because of the boundary conditions on the surface of the BH, an electrostatic
field is created which yields a potential difference given by [25]

\[ \Delta V = \frac{\hat{a} B}{M} D^2, \]  

(16)

where \( B \) is the poloidal component of the magnetic field generated by the accretion disk and \( D \) the typical length scale of the electrostatic field, \( D \approx M \), i.e., \( \Delta V \approx \hat{a} BM \) (see Fig. 3).

The magnetic field is estimated to be \( 10^4 \) gauss for the most active galaxies. In fact, under the hypothesis that the magnetic energy density is in equipartition equilibrium with the thermal energy of the disk, one finds

\[ B = \left( \frac{L_e}{c} \right)^{1/2} \frac{c^2}{GM} \approx 4 \times 10^4 M_8^{-1/2} \text{ gauss}, \]

(17)

where \( L_e \) is the Eddington luminosity

\[ L_e = \frac{4\pi G c M_p}{\sigma_T} \approx 4 \times 10^{46} M_8 \text{ erg s}^{-1}, \]

(18)

\( m_p \) being the mass of the proton, \( \sigma_T \) the Thompson cross-section [24] and \( M_8 = (M/10^8 M_\odot) \) being the BH mass in units of \( 10^8 \) solar masses. Taking into account Eq. (16), we then end up with a potential difference given by

\[ \Delta V \approx 1.7 \times 10^{20} \hat{a} M_8^{1/2} \text{ Volts}, \]

(19)

or, equivalently

\[ \Delta V = \sqrt{\frac{L}{c}} \hat{a} \approx 5 \times 10^{19} \hat{a} L_{45} \text{ Volts}, \]

(20)

\( L_{45} \) being the luminosity in units of \( 10^{45} \text{ erg sec}^{-1} \) [25]. Estimates of \( B \) [Eq. (17)] and \( \Delta V \) [Eq. (16)] are very rough and depend crucially on the conversion efficiency of gravitational energy of the disk into electromagnetic energy. As already stated, the parameter \( \hat{a} \) is less than unity and is related with the angular momentum \( J \) of the BH through

\[ J = \frac{\hat{a} c G M^2}{\delta}, \]

(21)

where \( M \) is now expressed in ordinary units. For old AGNs and QSOs, \( \hat{a} \) can be larger than 0.9.

So high a potential difference is very efficient to accelerate particles and can thus be the source for the electromagnetic energy of AGNs, QSOs and radio galaxies. Similar powerful sources of electromagnetic energy can be found in the magnetosphere of the accretion disks and by radiation accelerated winds from the disk. Interested readers will find details and useful references in the review article [27].

Hot spots in radio galaxy jets are also good candidates as acceleration regions of very high energy particles. Shock waves in the jets of radio galaxies are the main mechanism allowing acceleration of protons at energies \( \approx 10^{20} \text{ eV} \), while protons in the near zone of the BH cannot be accelerated at so high energies because of the losses due to the unfavorable environment (synchrotron and Compton losses) [26]. Vortons on the other hand do not suffer from this drawback since because of their high mass, they have a Lorentz factor which is at most of the order of \( 10^6 \) as we have seen above.

The synchrotron energy loss per unit length is

\[ \frac{dE}{dx} = -\frac{2}{3} \frac{(Ze)^2}{\rho^2} \gamma^4, \]

(22)

with \( \rho \) the curvature radius of the vorton trajectory along the magnetic field lines. If we take \( \rho \) equal to the Schwarzchild radius \( r_s = 2GM/c^2 \) of the BH, we obtain the total energy loss \( \delta E \) as

\[ \delta E = \frac{(Ze)^2}{3GM} \gamma^4 \]

\[ = 5.2 \times 10^{-33} Z^2 A \gamma^4 \text{ erg} \]

(23)

where \( A \) is the acceleration path expressed in Schwarzchild units. For \( Z = 100 \) and \( \gamma = 10^6 \), the synchrotron losses are \( \approx 10^{-4} \text{ erg} \) (with \( A = 1 \)), a very small quantity indeed with respect to the required final energy of \( 10^8 \text{ erg} \).

A similar calculation can be performed for the Compton losses; let us first compute the Thompson cross section \( \sigma_T^c \) of a vorton:

\[ \sigma_T^c = 8\pi \frac{(Ze)^4}{3M_v^2 c^4} \]

\[ = 1.6 \times 10^{-45} Z^2 \left( \frac{10^9 m_p}{m} \right)^2 \text{ cm}^2 \]

(24)

(recall \( M_v = Zm \)). It is now easy to compute the mean free path of a vorton in the photon bath around a BH. For a given luminosity \( L \), the photon density \( n_{ph} \) reads

\[ n_{ph} \approx \frac{L}{4\pi R^2 c h \nu} \]

(25)

where \( R \) is the radius of the source and \( \nu \) is a typical frequency of the radiated spectrum. Again fixing \( R \) to be the Schwarzchild radius of the BH, we have

\[ n_{ph} = 2 \times 10^{15} L_{45} M_8^{-2} \left( \frac{1 \text{ keV}}{E_{ph}} \right) \text{ photons/cm}^3 \]

(26)

so that the vorton mean free path \( \lambda_v = (n_{ph} \sigma_T^c)^{-1} \) is

\[ \lambda_v = 2 \times 10^{29} Z^{-2} \left( \frac{m}{10^9 m_p} \right)^2 M_8^2 \left( \frac{E_{ph}}{1 \text{ keV}} \right) L_{45}^{-1} \text{ cm} \]

(27)

which is much greater than the typical acceleration length \( \ell_s \approx 3 \times 10^{13} M_8 \) cm.
In brief, the acceleration of vortons in the hot spots of radio galaxies is more favorable than the same mechanism applied to protons because synchrotron and Compton losses scale respectively like

\[ Z^2 \left( \frac{m_p}{M_V} \right)^4 \simeq 10^{-22} Z_{100}^{-2} \text{ (synchrotron)} \]

and

\[ Z^4 \left( \frac{m_p}{M_V} \right)^4 \simeq 10^{-18} Z_{100}^0 \text{ (Compton)} \]

In what follows, we postulate that vortons that are uniformly distributed in the accretion disk around a BH can reach the region of high electrostatic field. Taking into account Eq. (13), we see that they can gain energies of

\[ E_V = 10^{20} \hat{a} \lambda_{45} \text{ eV}, \tag{28} \]

so that even active galaxies with luminosities as low as \(10^{43} \text{ erg/s}\) can accelerate vorton to \(10^{20} \text{ eV}\) if the angular momentum of the BH is large enough that \(\hat{a} \sim 1\).

As already stated, acceleration at these energies of vortons can happen also in the radio galaxy hot spots. The possibility of accelerating them near the central BH means that under the hypothesis that monopolar electrostatic and rotating BH is the power engine of all active galaxies \([27]\), Seyfert I, II, QSO and AGN are potential vorton acceleration sites, thereby tremendously increasing the number of sites as compared to the proton case since there is no GZK cutoff in their case, as we shall now see.

**IV. ENERGY CONSIDERATIONS AND PROPAGATION.**

The flux of cosmic rays with energy higher than \(10^{20} \text{ eV}\) is estimated of the order of \(10^{-20} \text{ cm}^{-2} \text{ s}^{-1} \text{ ster}^{-1}\), i.e. a flux \(10^{-19} \text{ cm}^{-2} \text{ s}^{-1}\), so the energy flux of particles having energies \(E = E_{20} \times 10^{20} \text{ eV}\) is

\[ \Phi = 10^{-22} E_{20} \text{ erg cm}^{-2} \text{ s}^{-1}. \tag{29} \]

If \(P_v\) is the probability that a vorton interacts with the atmosphere and if all the particles with energies greater than \(10^{20} \text{ eV}\) are vortons, then the actual flux (as opposed to the observed one) is

\[ \Phi = 10^{-6} E_{20} P_5 \text{ erg cm}^{-2} \text{ s}^{-1}, \tag{30} \]

with \(P_5 = P_v / 10^{-5}\) (see Sec. I). In order to achieve this flux, extragalactic sources up to a distance \(D\) must supply a power \(W\)

\[ W = 10^{47} E_{20} P_5 \left( \frac{100 \text{ Mpc}}{D} \right)^2 \text{ erg s}^{-1}. \tag{31} \]

Assuming \(10^5\) potential sources (Sy I, I, QSO) \([28]\) means that each source must supply a power

\[ W = 10^{44} E_{20} P_5 \left( \frac{1 \text{ Gpc}}{D} \right)^2 \text{ erg s}^{-1}, \tag{32} \]

power too uncomfortably close to the averaged power radiated by active galaxies. Note the neutrino hypothesis suffers even more of the same drawback.

Until now, we have assumed the typical mean free path of a \(10^{20} \text{ eV}\) vorton to be much larger than the Hubble radius, with the meaning that a vorton can reach us, even if emitted at large redshifts. We shall now prove that point by estimating the energy losses the vorton has to experience on his way.

We start with synchrotron radiation due to intergalactic magnetic field \(B\) since this is supposed to be the most efficient mechanism. The energy loss per unit time is

\[ \frac{dE}{dt} = 2 \left( \frac{Ze}{E} \right)^4 E^{2} c \gamma B^2, \tag{33} \]

in a mean magnetic field \(B = (B^2)^{1/2}\) if the Larmor radius of the vorton is less than the correlation length of the magnetic field \(B\). The typical slow down timescale for the vorton is thus given by

\[ t_v = \frac{3}{2} \frac{M^4 c^7}{E_20 (B^2)(Ze)^2} \]

\[ = 4.6 \times 10^{59} \left( \frac{B}{10^{-8} \text{ gauss}} \right)^2 E_{20}^{-1} \left( \frac{m_{10^9 m_p}}{M^4} \right)^4 \text{ s} \tag{34} \]

which is clearly sufficiently large that there is not even a cosmological cutoff for vortons.

A similar conclusion holds for the energy losses due to the microwave background. Note however the essential difference in this case between protons and vortons: if protons were pointlike particles with no internal quark structure, they would be able to propagate almost freely in the microwave background, even at these energies, and they decay dominantly because the binding energy between quarks and gluons is much less than the proton mass. Vortons, on the other hand, not only have energy levels which are much higher than protons, but also they propagate with a relatively low velocity (\(\gamma \sim 10^6\)). Therefore, vortons, once accelerated can arrive on earth with undegraded energy: supposing they were accelerated at the time of formation of the galaxies (\(z \sim 1\)) \([29]\), they would by now have lost a mere factor of two because of cosmological redshift. The interesting point in that observation is that for \(z \sim 1\), the ratio between active and normal galaxies is about 0.1 \([29, 30]\). Keeping that in mind, we can now calculate the high energy vorton density under the hypothesis that vortons were accelerated at the birth of galaxies.

If one assumes that all the unexplained cosmic rays with energy larger that \(10^{20} \text{ eV}\) are made of vortons, their density will be
\[ n_v = \frac{4\pi \Phi_v}{c \mathcal{P}_v} = 4.2 \times 10^{-25} \frac{\Phi_{20}}{\mathcal{P}_5} \text{ V} \cdot \text{cm}^{-1}, \] (35)

where \( \Phi_v \) (and \( \Phi_{20} = \Phi_v / 10^{-20} \text{ cm}^{-2} \text{s}^{-1} \text{ster}^{-1} \)) is the high energy cosmic ray flux.

The total number of vortons \( N_v \) in a sphere of radius corresponding to \( z = 1 \) is

\[ N_v = 3 \times 10^{60} h^{-3} \frac{\Phi_{20}}{\mathcal{P}_5}, \] (36)

and their energy is \( 10^{20}(1 + z)N_v \) eV = \( 6 \times 10^{68} h^{-3} \Phi_{20} / \mathcal{P}_5 \) ergs, where \( h \) is the Hubble constant \( H_0 \) in units of 75 km s\(^{-1}\) Mpc\(^{-1}\). Such an amount of energy can be released by a number of active galaxies \( N_G \)

\[ N_G = \frac{2 \times 10^9}{\varepsilon} h^{-3} \frac{\Phi_{20}}{\mathcal{P}_5} T_9^{-1} L_{43}, \] (37)

with \( T_9 \) the duty time of the galaxies in millions of years and \( \varepsilon \) the ratio between the power used to accelerate vortons and the electromagnetic luminosity; note this latter parameter is not constrained and can in fact exceed unity.

The baryonic mass in the sphere with \( z = 1 \) is \( M_b = 5.8 \times 10^{54} h^{-1} \text{ g} \), so that the total number of galaxies is

\[ N_G \approx 3 \times 10^{60} h^{-1} M_{10}^{-1}, \] (38)

where \( M_{10} \) is the galaxy mass in \( 10^{10} \) solar masses. From Refs. [22][30], we know the ratio between active and normal galaxies to be of order 10%, so the required efficiency is, expressed as a function of solar masses accreted by the BH per year \( M \),

\[ \varepsilon = 10^{-4} h^{-2} \frac{\Phi_{20}}{\mathcal{P}_5} \mathcal{P}_5 M_{10} \left( \frac{M}{M_\odot} \right)^{-1}. \] (39)

Finally, let us compute the required number of vortons present in the universe in the form of dark matter. Let \( \alpha \) be the ratio between the total vorton and baryon masses in the universe. The ratio \( \eta \) between vorton and baryon number densities is

\[ \eta = \frac{\alpha}{M_v \varepsilon}, \] (40)

and assuming the baryon density to be 10% of the critical density \( n_b = 6.6 \times 10^{-7} \text{ h}^2 \text{ baryons/cm}^3 \) we finally obtain

\[ \alpha = 6.4 \times 10^{-4} \frac{\Phi_{20}}{\mathcal{P}_5} Z_{100} \left( \frac{m}{10^9 m_p} \right). \] (41)

This result is understandable in two ways. Either all vortons present in AGNs are indeed accelerated, in which case vortons are, according to this calculation, expected to represent roughly less than a thousandth of the matter in the universe, or, conversely, vortons do fill the universe so that the actual value of \( \alpha \) should exceed unity. In the latter case, our calculation reveals that either the interaction probability is lower than what we evaluated above, or only a small fraction of vortons get accelerated. In any case, it should be clear that our model provides a very high energy cosmic ray flux which can be made to agree with observations.

**CONCLUSIONS**

We have exhibited a model for explaining extremely high energy cosmic rays that have recently been observed. More of these events are expected to be observed in the near future by the Auger Observatory [17]. We propose that they are bound states of very massive particles in vortons, i.e. loops of superconducting cosmic strings stabilized by a current, that are are freed by inelastic collision with atmospheric hadrons. The model, in its simplest form, has only one free parameter, namely the energy scale at which the current sets up in cosmic strings. This could easily be extended to take into account a possible difference between this mass scale and the string energy scale itself [14]. In this particular framework however, the mass scale is constrained by requiring that vortons do not overfill the universe, a bound that is almost saturated by the demand that high energy cosmic rays are made of vortons. This interesting coincidence could imply, if verified, that a non negligible fraction of the dark matter in the universe consists of vortons.

Let us here summarize our findings concerning the model. First, as mentioned earlier, it essentially has only one free parameter, namely the mass scale at which the superconducting current sets up in the core of the cosmic strings. As it turns out, demanding these objects to be candidates for the few \( 10^{20} \text{eV} \) events through interaction with atmospheric protons completely fixes this parameter to \( 10^{9} \text{ GeV} \). There doesn't seem to be any way out of this prediction, which renders the model very falsifiable.

Another point worth mentioning is that the vorton-proton (or neutron) cross section is quite weak, giving, at these energies, a total interaction probability between \( 10^{-5} \) and \( 10^{-4} \) with the earth atmosphere. Note again that this probability depends on nothing but the fixed energy scale, and that, in order to fit the observed data, it implies a high energy vorton flux whose numerical value is also therefore determined.

No high energy cosmic ray model is complete without specifying acceleration and propagation processes. In our case, acceleration is performed very simply by kicking the vortons (at least those which have been ionized somehow beforehand) with the high electrostatic fields that are expected to be present in AGNs. The required quantity of energy turns out to exist in these objects and because of the relatively low amount of acceleration required (\( \gamma \lesssim 10^6 \)), losses are negligible and the vortons can escape the acceleration zone. This is, to the best of our knowledge, the most efficient mean of extracting \( 10^{20} \text{eV} \) in a single particle out of any astrophysical object.

Similarly to the fact that energy losses are negligible in the region of acceleration, and for the same reason, the propagation in intergalactic medium is done almost without any collision, thus making this medium effectively transparent to very high energy vortons. As a result, there is no reason for a cosmological (GZK) cutoff and vortons can come from as far as a redshift of a few.
This is satisfactory because the ratio of active to normal galaxies increases as one goes farther away, and the final vorton flux we end up with thanks to this fact is actually quite large and indeed requires only a very small fraction of the total mass density in the universe to consist in vortons (of the order of $10^{-4}$) if all vortons are ionized (and therefore accelerated), or, if one believes the standard vorton predictions [3] that for a mass scale of $10^{19}$ GeV they should be the dominant part of the dark matter, then one just needs the same tiny fraction of vorton to be accelerated. The absence of any cutoff also means we predict a spatial isotropy together with a correlation with active galaxies.

Interacting with the atmosphere with a very low probability (and then presenting a threshold followed by a line spectrum, presumably unobservable unfortunately), vortons are here predicted to give rise to mostly horizontal air showers, just like the neutrinos. Let us stress however that until now, it has not been possible to find any astrophysical mechanism that would give neutrinos such high energies. Hence, although our vortons are indeed hypothetical in the sense that we don’t know what is the actual theory that describes physics beyond the electroweak scale at which we expect to find them, it should however be clear that they do not, apart in their existence itself, imply any new mechanism of any kind.

Finally it is worth pointing out that the acceleration mechanism we have proposed is just one amongst a few other acceleration mechanisms already proposed in the same context such as the Fermi mechanism. Vortons can be accelerated as well across the shocks of the jets of the radio galaxies. We prefer however the direct acceleration mechanism because of its high efficiency and therefore the energy budget constraints can be more easily fulfilled. Note that this is not the case for the neutrinos: in fact, as already stated, $10^{20}$ eV neutrinos have interaction cross sections (with hadrons) that are roughly equivalent to vorton’s. Consequently, it is highly questionable whether the energetic constraints required to accelerate the parent charged particle of the neutrinos in the lobes of radio galaxies can be satisfied.

To end up this conclusion, let us compare our expectations with those of the Auger Observatory [17], a project specifically designed to accumulate more statistics on cosmic rays with energies in excess of $10^{19}$ eV. The Auger Observatory will have an energy resolution less than 20% which is presumably going to be insufficient to actually allow definite conclusion about the line features. In spite of this difficulty, it is however not impossible that part of the line signal might be found using correlation function techniques after a power-law subtraction will have been made in the data: no significant feature should be left by almost all the competing candidates. Another characteristic of the Auger Observatory concerns its ability to identify the primary of an air shower through accurate measurement of the shower maximum ($\Delta X_{\text{max}} \sim 20$ g-cm$^{-2}$). In a bound state model such as the one we propose here, the line part of the spectrum should be initiated by radiative decay of the excited vortons, hence the primary should be a photon. Then once the continuum is reached, some other particle (the $\Sigma$ in our model) plays the part of the primary, initiating a shower whose maximum is expected somewhere else. It is not clear yet, because very model dependent, whether the Auger Observatory accuracy in this measurement will be sufficient, but it is also a firm prediction of any bound state model. Finally, according to our calculation, the interaction probability with the atmosphere is very low, with the result that many horizontal showers are expected. Contrary to most other cosmic ray detectors, the Auger Observatory will be very efficient to detect those, being also understandable as a neutrino detector with an acceptance of 10 km$^3$·sr water equivalent for horizontal air showers at $10^{19}$ eV [17]. Finally, although the angular resolution of the two detectors (giant array and air fluorescence) in the Auger Observatory should be of the order of less than 2°, some events will be observed in hybrid mode by both with a resolution of 0.3°, a precision that is expected to be enough to conclude on the isotropy of the cosmic ray sources.

All these facts lead to the conclusion that our model has a very high potential for being either confirmed or ruled out shortly after the Auger Observatory will be started.

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