Feynman Rules for the Rational Part of the Standard Model One-loop Amplitudes in the ’t Hooft-Veltman $\gamma_5$ Scheme

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ABSTRACT: We study Feynman rules for the rational part $R$ of the Standard Model amplitudes at one-loop level in the ’t Hooft-Veltman $\gamma_5$ scheme. Comparing our results for quantum chromodynamics and electroweak 1-loop amplitudes with that obtained based on the Kreimer-Korner-Schilcher (KKS) $\gamma_5$ scheme, we find the latter result can be recovered when our $\gamma_5$ scheme becomes identical (by setting $g_{5}s = 1$ in our expressions) with the KKS scheme. As an independent check, we also calculate Feynman rules obtained in the KKS scheme, finding our results in complete agreement with formulae presented in the literature. Our results, which are studied in two different $\gamma_5$ schemes, may be useful for clarifying the $\gamma_5$ problem in dimensional regularization. They are helpful to eliminate or find ambiguities arising from different dimensional regularization schemes.

KEYWORDS: NLO, Electroweak interactions, Quantum chromodynamics
1. Introduction

The operation of the Large Hadron Collider (LHC) at CERN has opened a new era in high energy physics. There are many important topics for LHC physics, particularly in hunting Higgs boson(s) and searching for new physics signals beyond the standard model (BSM). However, the existence of large standard model (SM) background makes such studies difficult. In this regard, the next-to-leading order (NLO) corrections to the standard model calculations are necessary and helpful, because these corrections may provide a much improved scale dependence and more reliable estimations in separating SM background from new physics signals. Since the Les Houches workshop in 2005, many NLO corrections in processes with four or more final state particles at the LHC have been successfully performed. They are, e.g., $pp \rightarrow t\bar{t}b\bar{b}+X$, $pp \rightarrow V+3jets$, $pp \rightarrow W^{\pm}+4jets$, $pp \rightarrow VV+2jets$, $pp \rightarrow t\bar{t}+2jets$, $pp(p\bar{p}) \rightarrow W^{\pm}W^{\mp}bb$, $pp \rightarrow b\bar{b}b\bar{b}+X$, and $pp \rightarrow W\gamma+jet$. However, a few SM processes of similar complexity, which are relevant to search for Higgs boson(s) or detection of new particles in physics BSM, are still not available at NLO level, such as $pp \rightarrow t\bar{t}t\bar{t}$. 


Recently, automatic one-loop calculations have become a feasible task after several new and efficient algorithms have been proposed. Some of the most notable methods are the Unitarity\cite{2} based techniques such as the Ossola, Papadopoulos, and Pittau (OPP) reduction method\cite{13}, which reduces the computation of one-loop amplitudes to a problem with a complexity similar to a tree level calculation. Most of the aforementioned processes are computed within this method or its variations. Such approaches rely on the master integrals of one-loop amplitudes that can be simply represented as a linear combination of up to 4-point scalar integrals, i.e., those of boxes, triangles, bubbles and tadpoles. A one-loop amplitude $A$ can be written as

$$A = \sum_i d_i \text{Box}_i + \sum_i c_i \text{Triangle}_i + \sum_i b_i \text{Bubble}_i + \sum_i a_i \text{Tadpole}_i + R.$$  

In the OPP framework, the coefficients $d_i, c_i, b_i, a_i$ and $R_1 (R = R_1 + R_2)$ can be obtained from the cut constructible (CC) part of the amplitude, where the numerators of loop integrals are dealt with in 4 dimensions, and $R_2$ should be derived separately\cite{14}. With the ultraviolet nature of $R_2$\cite{1,15}, we can establish Feynman rules for this part. Here we directly call $R_2$ the rational term $R$ for brevity. In fact, this approach is more efficient than applying the Unitarity reduction method in $d = 4 - 2\varepsilon$ dimensions in practical calculations\cite{11}. Algebraically, even in other methods, such as the conventional Passarino-Veltman reduction procedure\cite{16}, calculations of $R$ are also useful in multi-leg processes\cite{1].

Several previous works studied calculations of the Feynman rules for $R$, and a set of Feynman rules for $R$ in the Standard Model (SM) under one $\gamma_5$ strategy in the ’t Hooft-Feynman gauge, $R_\xi$ gauge, and Unitary gauge has been reported\cite{17,18,19}, and a package $R2SM$ written in FORM is also available\cite{20}. Moreover, some simplifications in extracting rational terms were suggested recently in Ref.\cite{6}. The primary aim of the present paper is to give our result in another $\gamma_5$ regularization scheme, i.e. the ’t Hooft-Veltman $\gamma_5$ scheme. In addition, we would also like to check the results presented in Refs.\cite{17,18}.

This paper is organized as follows. In Section 2, we recall the theory for the rational part $R$, and set our regularization schemes and $\gamma_5$ strategies. The notations and Feynman rules are given in Section 3. Our results in two $\gamma_5$ schemes are expressed in Section 4. Finally, a summary is given in Section 5.

2. The theory of $R$ and regularization schemes

In this section, we will describe the origin of the rational part $R$, and discuss the dimensional regularization scheme (DREG) in two $\gamma_5$ strategies. Due to the divergences in a one-loop calculation, we compute our one-loop integrals in $d = 4 - 2\varepsilon$.
dimensions to regularize the divergences that appear in 4 dimensions. A generic N-point one-loop (sub-) amplitude reads

$$A_N = \int d^d\bar{q} \frac{\tilde{N}(\bar{q})}{D_1 D_2 \ldots D_N}, \quad \bar{D}_k = (\bar{q} + p_k)^2 - (m_k)^2, \quad \bar{q} = q + \tilde{q}, \quad (2.1)$$

where the bar means quantities defined in $d$ dimensions, while the tilde in $d - 4$ dimensions. All quantities of the external particles such as external polarization vectors and external momenta $p_i$ are maintained in 4 dimensions.

In numerator the function $\tilde{N}(\bar{q})$ can be split into a 4-dimensional part $N(q)$ and a $(d - 4)$-dimensional part $\tilde{N}(\tilde{q}^2, \epsilon, q)$, i.e.,

$$\tilde{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, \epsilon, q). \quad (2.2)$$

Here the $\tilde{N}(\tilde{q}^2, \epsilon, q)$ part is related to $R$:

$$R = \int d^d\bar{q} \frac{\tilde{N}(\tilde{q}^2, \epsilon, q)}{D_1 D_2 \ldots D_N}. \quad (2.3)$$

Feynman rules for the $R$ effective vertices can be obtained from all possible one-particle irreducible Green functions, which are enough up to 4 external legs in the SM, accounting for the ultraviolet nature of the rational terms.

The $d$-dimensional momentum $\bar{q}$, the $d$-dimensional metric tensor $\bar{g}_{\mu\nu}$, and the $d$-dimensional Dirac matrices $\bar{\gamma}_\mu$ are all split into a 4-dimensional part and a $-2\epsilon$-dimensional part

$$\bar{q}_\mu = q_\mu + \tilde{q}_\mu, \quad \bar{g}_{\mu\nu} = g_{\mu\nu} + \tilde{g}_{\mu\nu}, \quad \bar{\gamma}_\mu = \gamma_\mu + \tilde{\gamma}_\mu. \quad (2.4)$$

Instead of performing supersymmetry-preserving but technically complicated dimensional reduction (DRED) [21], we use DREG for convenience. Therefore, the $d$-dimensional metric tensor $\tilde{g}_{\mu\nu}$, the $-2\epsilon$-dimensional tensor $\bar{g}_{\mu\nu}$, and the 4-dimensional tensor $g_{\mu\nu}$ satisfy following relations :

$$\tilde{g}_{\mu\nu} g^\nu_\nu = g_{\mu\nu}, \quad \tilde{g}_{\mu\nu} \tilde{g}^\nu_\nu = \bar{g}_{\mu\nu}, \quad \tilde{g}_{\mu\nu} \bar{g}^\nu_\nu = 0, \quad \tilde{g}_{\mu\nu} \tilde{g}^\nu_\nu = \bar{g}_{\mu\nu},$$

$$g_{\mu\nu} g^\nu_\nu = g_{\mu\nu}, \quad \tilde{g}_{\mu\nu} \tilde{g}^\nu_\nu = \bar{g}_{\mu\nu}, \quad \bar{g}^\mu_\mu = d, \quad g^\mu_\mu = 4, \quad \bar{g}^\mu_\mu = -2\epsilon. \quad (2.5)$$

In our regularization we set $d > 4$, so that

$$q^2 \to q^2 + \tilde{q}^2. \quad (2.6)$$

To maintain the advantages of the helicity method for loop calculations, we choose the four dimensional helicity (FDH) [22] and ’t Hooft-Veltman (HV) schemes [23] in the present paper

$$R\big|_{HV} = \int d^d\bar{q} \frac{\tilde{N}(\tilde{q}^2, \epsilon, q)}{D_1 D_2 \ldots D_N},$$

$$R\big|_{FDH} = \int d^d\bar{q} \frac{\tilde{N}(\tilde{q}^2, \epsilon = 0, q)}{D_1 D_2 \ldots D_N}. \quad (2.7)$$
The last comment in this section refers to our \( \gamma_5 \) strategies. We choose two schemes here. One scheme is identical to that used in Refs.\[17, 18\], which was proposed by Kreimer, Körner, and Schilcher \[24\] and we call it the KKS scheme for brevity, and the other one is the ’t Hooft-Veltman scheme \[23, 25, 26\], which gives new results in the present paper. These two schemes are algebraically consistent, at least at one-loop level. In the KKS scheme, we define a unique generator which anti-commutates with all other generators in Clifford algebra, as \( \gamma_5 \). So the anti-commutation relation \( \{ \gamma_5, \bar{\gamma}_\mu \} = 0 \) is maintained. To obtain the anomaly, we give up the cyclic relation in the ordinary Dirac trace, and use a projection relation onto four dimensional subspace

\[
Tr(\gamma_5 \bar{\gamma}_{\mu_1} \cdots \bar{\gamma}_{\mu_2k}) \equiv tr(\mathcal{P}(\gamma_5) \bar{\gamma}_{\mu_1} \cdots \bar{\gamma}_{\mu_2k}), \tag{2.8}
\]

where \( \mathcal{P}(\gamma_5) \equiv \frac{i}{4!} \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \gamma^\mu_1 \gamma^\mu_2 \gamma^\mu_3 \gamma^\mu_4 \) with Lorentz indexes of the totally antisymmetric tensor \( \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \) all in 4 dimensions. To perform calculations for some specific processes unambiguously, we choose a unique ”special vertex”, called the ”reading point”, in all Feynman diagrams. All \( \gamma_5 \)’s are anti-commuted to reach the ”reading point” before performing the \( d \)-dimensional algebra calculation. This treatment generally produces a term that is proportional to the total antisymmetric \( \varepsilon \) tensor with different ”reading points”. The HV scheme defines \( \gamma_5 \equiv \frac{i}{4!} \varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \gamma^\mu_1 \gamma^\mu_2 \gamma^\mu_3 \gamma^\mu_4 \) in \( d \) dimensions. Therefore, the anti-commutation relation is violated by

\[
\{ \gamma_5, \gamma_\mu \} = 0, \quad [\gamma_5, \bar{\gamma}_\mu] = 0. \tag{2.9}
\]

This definition explicitly forbids us to deal with covariant \( \gamma \) algebra and often makes calculations very inconvenient. Nevertheless, it is to date the only known scheme within DREG that has been demonstrated to be consistent in all orders. Actually, the violation of anti-commutation also needs to include an extra finite renormalization by hand in calculations to restore the Ward identities. For instance in the \( W \) boson decays to hadronic partons, the Ward identity is spoiled in the HV \( \gamma_5 \) scheme at one loop level. The \( \mathcal{O}(\alpha_s) \) corrections should include an axial-vector current renormalization

\[
\Gamma^{ren}_{\mu_5} = \left( 1 - \frac{\alpha_s}{2\pi} \frac{C_F}{2} (1 + \lambda_{HV}) \right) \Gamma^{bare}_{\mu_5}, \tag{2.10}
\]

where \( \lambda_{HV} \) is defined in the next section and \( C_F = \frac{N_c^2 - 1}{2N_c} \) is implicitly understood. This non-anticommuting \( \gamma_5 \) scheme was also discussed from the action principles in Ref.\[27\]. Moreover, some issues about dimensional renormalization with \( \gamma_5 \) can also be found in Refs.\[27, 28\].
3. Notations and Feynman rules

In this section, we will briefly describe our notations and tree level Feynman rules that will be used in the next section.

Two parameters, $\lambda_{HV}$ and $g5s$, are introduced in our formulae to denote the different DREG and $\gamma_5$ schemes. Here, $\lambda_{HV} = 1(0)$ corresponds to the HV (FDH) regularization scheme, while $g5s = 1(-1)$ corresponds to the KKS (HV) $\gamma_5$ scheme. Our notations are as follows: $L_1 = e, L_2 = \mu, L_3 = \tau, L_4 = \nu_e, L_5 = \nu_\mu, L_6 = \nu_\tau, Q_1 = u, Q_2 = d, Q_3 = s, Q_4 = c, Q_5 = b, Q_6 = t$. In addition, $e_1 = e, e_2 = \mu, e_3 = \tau, \nu_1 = \nu_e, \nu_2 = \nu_\mu, \nu_3 = \nu_\tau, U_1 = u, U_2 = c, U_3 = t, D_1 = d, D_2 = s, D_3 = b$. The vector boson fields are denoted by $A, Z$ and $W^\pm$ with the generic symbol $V$, and the physical scalar Higgs field and the scalar goldstone bosons are written as $H, \phi^0$, and $\phi^\pm$, respectively. Fermions are also generically symbolized by $F$, while the mass, charge, and the third isospin component of $F$ are denoted by $m_F, Q_F,$ and $I_3F$, respectively. $c_w$ and $s_w$ respectively denote sine and cosine of the Weinberg angle. $N_c$ is the number of colors and $V_{u,d,j}$ refers to the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. $\Omega^\pm \equiv \frac{1\pm \gamma_5}{2}$ are the chiral projector operators. Finally, $g_s$ is the coupling constant of strong interaction, while $e$ is the coupling constant of QED.

As we use FeynArts [29] to generate all Feynman amplitudes, our conventions for tree level Feynman rules follow Ref.[30]. Due to the violation of anti-commutation relation in HV $\gamma_5$ scheme, i.e. $\{\gamma_5, \bar{\gamma}_\mu\} \neq 0$, some modifications to conventional Feynman rules are introduced. For coupling to left-handed fields the symmetrically defined vertex $\frac{1}{2}(1 + \gamma_5)\bar{\gamma}_\mu(1 - \gamma_5) = \gamma_\mu(1 - \gamma_5)$, instead of $\bar{\gamma}_\mu(1 - \gamma_5)$, should be used [31]. Especially, the tree level Feynman rules of vertices $F\bar{F}Z$ should be modified as follows

$$\frac{ie}{c_w s_w} (I_3F\bar{\gamma}_\mu \Omega^- - s_w^2 Q_F\bar{\gamma}_\mu) \rightarrow \frac{ie}{c_w s_w} (I_3F\gamma_\mu \Omega^- - s_w^2 Q_F\bar{\gamma}_\mu),$$

(3.1)

and similarly to $F_1\bar{F}_2W^\pm$ vertices.

4. Results

In this section, our results in 't Hooft-Feynman gauge are presented. It is easy to check that if one sets $g5s = 1$, the results in [17, 18] are all recovered.

4.1 Effective vertices in QCD

In this subsection, we give a complete list of all non-vanishing $R$ effective vertices in QCD, where all the internal lines in one-particle irreducible diagrams represent QCD particles.
\[ G^a_\mu \quad \xrightarrow{p} \quad G^b_\nu \quad = \quad \text{Vert}(G^a_\mu, G^b_\nu) \]

(a)

\[ Q^i_l \quad \xrightarrow{p} \quad \bar{Q}^i_m \quad = \quad \text{Vert}(Q^i_l, \bar{Q}^i_m) \]

(b)

**Figure 1:** All possible non-vanishing 2-point vertices in QCD.

### 4.1.1 QCD effective vertices with 2 external legs

All possible non-vanishing 2-point vertices in QCD are shown in Fig. 1.

**Gluon-Gluon vertex**

The corresponding effective vertex is shown in Fig. 1 (a) with the following expression

\[
\text{Vert}(G^a_\mu, G^b_\nu) = \frac{ig_s^2 N_c}{48\pi^2} \delta^{ab} \left[ \frac{p^2}{2} g_{\mu\nu} + \lambda_{HV} \left( g_{\mu\nu} p^2 - p_\mu p_\nu \right) + \sum_Q \frac{p^2 - 6m_Q^2}{N_c} g_{\mu\nu} \right] .
\] (4.1)

**Quark-Quark vertices**

The corresponding diagram is shown in Fig. 1 (b) with the following expression

\[
\text{Vert}(Q^i_l, \bar{Q}^i_m) = \frac{ig_s^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \delta^{ij} \delta_{lm} \left( -\hat{p} + 2m_{Q_i} \right) \lambda_{HV}.
\] (4.2)
4.1.2 QCD effective vertices with 3 external legs

All possible non-vanishing 3-point vertices in QCD are shown in Fig. 2.

**Gluon-Gluon-Gluon vertex**

The corresponding diagram is shown in Fig. 2 (a) with the following expression for $N_f$-flavor quarks

$$\text{Vert}(G^a_{\mu}, G^b_{\nu}, G^c_{\rho}) = -g^3_{s}N_{c}^{2} \left( \frac{7}{4} + \frac{N_{f}}{N_{c}} \right) f^{abc} V_{\mu\nu\rho}(p_{1},p_{2},p_{3}),$$

where

$$V_{\mu\nu\rho}(p_{1},p_{2},p_{3}) = g_{\mu\nu}(p_{2} - p_{1})_{\rho} + g_{\nu\rho}(p_{3} - p_{2})_{\mu} + g_{\rho\mu}(p_{1} - p_{3})_{\nu}.$$
Gluon-Quark-Quark vertex

The corresponding diagram is shown in Fig.2 (b) with the following expression

\[
\text{Vert}(G^a_{\mu}, Q^i_l, \bar{Q}^j_m) = \delta_{lm} \frac{ig^3_s}{16\pi^2} T^a_{ji} \frac{N_c^2 - 1}{2N_c} \gamma_\mu (1 + \lambda_{HV}).
\]  
\(4.5\)

Vector-Quark-Quark vertices

A typical Vector-Quark-Quark vertex is shown in Fig.2 (c) with the following expression

\[
\text{Vert}(V_{\mu}, Q^i_l, \bar{Q}^j_m) = -\frac{g^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \delta^{ij} (1 + \lambda_{HV}) \gamma_\mu \left( v^{VQ_i\bar{Q}_m} + g\tilde{s} a^{VQ_i\bar{Q}_m} \gamma_5 \right).
\]  
\(4.6\)

The actual values of \(V, Q_i, \bar{Q}_m, v^{VQ_i\bar{Q}_m}\) and \(a^{VQ_i\bar{Q}_m}\) are

\[
\begin{align*}
 v^{AQ_iQ_m} &= -ieQ_l \delta_{lm}, \\
 a^{AQ_iQ_m} &= 0, \\
 v^{ZQ_iQ_m} &= ie s_w \left( \frac{I_3Q_l}{2s_w^2} - Q_l \right) \delta_{lm}, \\
 a^{ZQ_iQ_m} &= -ie I_3 Q_l \delta_{lm}, \\
 v^{W-U_iD_m} &= V_{D_m,U_i}^\dagger \frac{ie}{2\sqrt{2}s_w}, \\
 a^{W-U_iD_m} &= -V_{D_m,U_i}^\dagger \frac{ie}{2\sqrt{2}s_w}, \\
 v^{W+D_i\bar{U}_m} &= V_{U_m,D_i} \frac{ie}{2\sqrt{2}s_w}, \\
 a^{W+D_i\bar{U}_m} &= -V_{U_m,D_i} \frac{ie}{2\sqrt{2}s_w}.
\end{align*}
\]  
\(4.7\)

Scalar-Quark-Quark vertices

The generic diagram is shown in Fig.2 (d) with the following expression
\[
\text{Vert}(S, Q^i_l, Q^j_m) = -\frac{g_s^2}{8\pi^2} \left( \frac{N_c^2 - 1}{2N_c} \delta^{ij} (1 + \lambda_{HV}) \right) \left( v^{SQ_l\bar{Q}_m} + g^5 s a^{SQ_l\bar{Q}_m} \gamma_5 \right). \tag{4.8}
\]

The actual values of \(S, Q_l, \bar{Q}_m, v^{SQ_l\bar{Q}_m}\) and \(a^{SQ_l\bar{Q}_m}\) are

\[
\begin{align*}
v^{HQ_l\bar{Q}_m} &= -\delta_{lm} \frac{i e m_{Q_l}}{2m_W s_w}, & a^{HQ_l\bar{Q}_m} &= 0, \\
v^{\phi_0 U_l\bar{U}_m} &= 0, & a^{\phi_0 U_l\bar{U}_m} &= -\delta_{lm} \frac{e m_{U_l}}{2m_W s_w}, \\
v^{\phi_0 D_l\bar{D}_m} &= 0, & a^{\phi_0 D_l\bar{D}_m} &= \delta_{lm} \frac{e m_{D_l}}{2m_W s_w}, \\
v^{\phi^- U_l\bar{D}_m} &= V^a_{D_m, U_l} \frac{i e}{2\sqrt{2}m_W s_w} (m_{U_l} - m_{D_m}), & a^{\phi^- U_l\bar{D}_m} &= V^a_{U_m, D_l} \frac{i e}{2\sqrt{2}m_W s_w} (m_{U_m} + m_{D_l}), \\
v^{\phi^+ D_l\bar{U}_m} &= V^a_{U_m, D_l} \frac{i e}{2\sqrt{2}m_W s_w} (m_{U_m} - m_{D_l}), & a^{\phi^+ D_l\bar{U}_m} &= -V^a_{U_m, D_l} \frac{i e}{2\sqrt{2}m_W s_w} (m_{U_m} + m_{D_l}). \tag{4.9}
\end{align*}
\]

**Vector-Gluon-Gluon vertices**

The generic diagram is shown in Fig.2 (e) with the following expression

\[
\text{Vert}(V^a_{\mu}, G^a_{\nu}, G^b_{\rho}) = -\frac{ig_s^2}{12\pi^2} \delta^{ab} \varepsilon_{\mu \nu \rho (p_1 - p_2)} \sum_l a^{VQ_l\bar{Q}_l}, \tag{4.10}
\]

with all of the expressions of \(a^{VQ_l\bar{Q}_l}\) given in Eq.(4.7).

**Scalar-Gluon-Gluon vertices**

The generic diagram is shown in Fig.2 (f) with the following expression

\[
\text{Vert}(S, G^a_{\mu}, G^b_{\nu}) = \frac{g_s^2}{8\pi^2} \delta^{ab} g_{\mu \nu} \sum_l \left( v^{SQ_l\bar{Q}_l} m_{Q_l} \right), \tag{4.11}
\]

with all of the expressions of \(v^{SQ_l\bar{Q}_l}\) given in Eq.(4.9).
All possible non-vanishing 4-point vertices in QCD are shown in Fig. 3.

**Gluon-Gluon-Gluon-Gluon vertex**

The corresponding diagram is shown in Fig. 3 (a) with the following expression for $N_f$-flavor quarks

$$\text{Vert} \left( G^a_\mu, G^b_\nu, G^c_\rho, G^d_\sigma \right) = \frac{ig_s^4}{48\pi^2} \left( C_1 g_{\mu\nu} g_{\rho\sigma} + C_2 g_{\mu\rho} g_{\nu\sigma} + C_3 g_{\mu\sigma} g_{\nu\rho} \right), \quad (4.12)$$

where

$$C_1 = Tr(\{T^a, T^b\}\{T^c, T^d\}) \left( 5N_c + 2\lambda_{HV}N_c + 6N_f \right)$$
$$- \left( Tr(T^a T^c T^b T^d) + Tr(T^a T^d T^b T^c) \right) \left( 12N_c + 4\lambda_{HV}N_c + 10N_f \right)$$
$$- \left( \delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc} \right),$$

$$C_2 = C_1 (b \leftrightarrow c) = Tr(\{T^a, T^c\}\{T^b, T^d\}) \left( 5N_c + 2\lambda_{HV}N_c + 6N_f \right)$$
$$- \left( Tr(T^a T^b T^c T^d) + Tr(T^a T^d T^c T^b) \right) \left( 12N_c + 4\lambda_{HV}N_c + 10N_f \right)$$
$$- \left( \delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc} \right),$$

$$C_3 = C_1 (b \leftrightarrow d) = Tr(\{T^a, T^d\}\{T^c, T^b\}) \left( 5N_c + 2\lambda_{HV}N_c + 6N_f \right)$$
$$- \left( Tr(T^a T^c T^d T^b) + Tr(T^a T^b T^d T^c) \right) \left( 12N_c + 4\lambda_{HV}N_c + 10N_f \right)$$
$$- \left( \delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc} \right). \quad (4.13)$$
Vector-Vector-Gluon-Gluon vertices

The generic diagram is shown in Fig.3(b) with the following expression

\[ \text{Vert} \left( V_{1\mu}, V_{2\nu}, G^a_{\rho}, G^b_{\sigma} \right) = - \frac{ig^2}{24\pi^2} \delta^{ab} \sum_{l,m} \left[ \left( v^{V_1 Q_l Q_m} v^{V_2 Q_m Q_l} + a^{V_1 Q_l Q_m} a^{V_2 Q_m Q_l} \right) \right. \\
\left. (g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}) + \frac{1 - g5s}{2} 8a^{V_1 Q_l Q_m} a^{V_2 Q_m Q_l} \right] g_{\mu\nu}. \]  

(4.14)

All of the expressions for \( v^{V_1 Q_l Q_m} , a^{V_2 Q_m Q_l} \) are given in Eq.(4.7).

Scalar-Scalar-Gluon-Gluon vertices

The generic diagram is shown in Fig.3(c) with the following expression

\[ \text{Vert} \left( S_1, S_2, G^a_{\mu}, G^b_{\nu} \right) = \frac{ig^2}{24\pi^2} \delta^{ab} \sum_{l,m} \left[ 3 \left( v^{S_1 Q_l Q_m} v^{S_2 Q_m Q_l} - a^{S_1 Q_l Q_m} a^{S_2 Q_m Q_l} \right) \right. \\
\left. - \frac{1 - g5s}{2} 8a^{S_1 Q_l Q_m} a^{S_2 Q_m Q_l} \right] g_{\mu\nu}. \]  

(4.15)

All of the expressions for \( v^{S_1 Q_l Q_m} , a^{S_2 Q_m Q_l} \) are given in Eq.(4.9).

Vector-Gluon-Gluon-Gluon vertices

The generic diagram is shown in Fig.3(d) with the following expression

\[ \text{Vert} \left( V_{\mu}, G^a_{\nu}, G^b_{\rho}, G^c_{\sigma} \right) = - \frac{g^3}{24\pi^2} \sum_l \left[ Tr(T^a (T^b T^c)) \right] v^{V_1 Q_l} \left( g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} \right) \\
- i9 Tr(T^a [T^b , T^c]) a^{V_1 Q_l} \xi_{\mu\nu\rho\sigma} \right], \]  

(4.16)

with \( \{ A, B \} \equiv AB + BA \) and \( [ A, B ] \equiv AB - BA \). All of the expressions for \( v^{V_1 Q_l} , a^{V_2 Q_l} \) are given in Eq.(4.7).

4.2 Effective vertices in Electroweak part

A complete list of the non-vanishing \( R \) effective vertices in the Electroweak part is given below.
All possible non-vanishing 2-point vertices in Electroweak (EW) are shown in Fig.4.

4.2.1 Electroweak effective vertices with 2 external legs

All possible non-vanishing 2-point vertices in Electroweak (EW) are shown in Fig.4.

**Scalar-Scalar vertices**

The generic effective vertex is shown in Fig.4 (a) with the following expression

\[
\text{Vert} (S_1, S_2) = \frac{i e^2}{16 \pi^2 s_w^2} C^{S_1 S_2}, \quad (4.17)
\]

with the actual value of $S_1$, $S_2$ and $C^{S_1 S_2}$
\[ C^{H\phi^0} = 0, \]
\[ C^{HH} = \frac{1 - 6\lambda_{HV}}{2} \left( 1 + \frac{1}{2c_w^2} \right) m_W^2 - \left( 1 + \frac{1}{2c_w^2} \right) \frac{p^2}{12} + K, \]
\[ C^{\phi^0 \phi^0} = \frac{m_W^2}{4} + \frac{m_H^2}{8c_w^2} + \frac{1 - 4\lambda_{HV}}{4} \left( 1 + \frac{1}{2c_w^2} \right) m_W^2 \]
\[ - \left( 1 + \frac{1}{2c_w^2} \right) \frac{p^2}{12} + g5s K, \]
\[ C^{\phi^- \phi^+} = \frac{m_W^2}{4c_w^2} + \frac{m_H^2}{8} + \frac{(3 - 4\lambda_{HV}) c_w^4 - 2c_w^2 + (\frac{1}{2} - 2\lambda_{HV})}{4c_w^4} m_W^2 \]
\[ - \left( 1 + \frac{1}{2c_w^2} \right) \frac{p^2}{12} + \frac{1}{2m_W^2} \left\{ \frac{1 + g5s}{2} \sum_l \left( m_{\nu_l}^2 \left( m_{\nu_l}^2 - \frac{p^2}{3} \right) \right) \right\} \]
\[ + N_c \sum_{l,m} \left[ V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( \frac{1 + g5s}{2} \right) \left( m_{\nu_l}^2 + m_{D_m}^2 \right) \right] \left( m_{\nu_l}^2 + m_{D_m}^2 - \frac{p^2}{3} \right) \right\}, \]
\[ \text{(4.18)} \]

where

\[ K = \sum_l \left[ \frac{m_{\nu_l}^2}{m_W^2} \left( m_{\nu_l}^2 - \frac{p^2}{6} \right) \right] + N_c \sum_l \left[ \frac{m_{Q_l}^2}{m_W^2} \left( m_{Q_l}^2 - \frac{p^2}{6} \right) \right]. \]
\[ \text{(4.19)} \]

To compare our expressions with that in Ref.\[18\], we use \( m_{\phi^0}^2 = m_Z^2 = \frac{m_W^2}{c_w^2} \) and \( m_{\phi^\pm}^2 = m_W^2 \) in above expressions and fix the masses of neutrinos to be zero.

**Vector-Vector vertices**

The generic effective vertex is shown in Fig.4 (b) with the following expression

\[ \text{Vert} (V_{1\mu}, V_{2\nu}) = -\frac{ie^2}{\pi^2} \left( C_{1\mu}^{V_1V_2} p_{\mu}p_{\nu} + C_{2\mu}^{V_1V_2} g_{\mu\nu} \right), \]
\[ \text{(4.20)} \]

with the actual values of \( V_1, V_2, C_{1\mu}^{V_1V_2} \) and \( C_{2\mu}^{V_1V_2} \)
\[C_{1}^{AA} = -\frac{\lambda_{HV}}{24},\]
\[C_{2}^{AA} = \frac{1}{8} \left[ \frac{1 + 2\lambda_{HV}}{6} p^2 - m_W^2 \right] - \frac{1}{4} \left\{ \sum_{l} \left[ Q_{L_l}^2 \left( m_{L_l}^2 - \frac{p^2}{6} \right) \right] + N_c \sum_{l} \left[ Q_{Q_l}^2 \left( m_{Q_l}^2 - \frac{p^2}{6} \right) \right] \right\},\]
\[C_{1}^{AZ} = \frac{c_w \lambda_{HV}}{24 s_w},\]
\[C_{2}^{AZ} = -\frac{c_w}{8 s_w} \left[ \frac{1 + 2\lambda_{HV}}{6} p^2 - m_W^2 \right] + \frac{1}{4 c_w} \left\{ \sum_{l} \left[ \left( \frac{Q_{L_l} I_{3L_l}}{2 s_w} - Q_{L_l}^2 s_w \right) \left( m_{L_l}^2 - \frac{p^2}{6} \right) \right] + N_c \sum_{l} \left[ \left( \frac{Q_{Q_l} I_{3Q_l}}{2 s_w} - Q_{Q_l}^2 s_w \right) \left( m_{Q_l}^2 - \frac{p^2}{6} \right) \right] \right\},\]
\[C_{1}^{ZZ} = -\frac{c_w^2 \lambda_{HV}}{24 s_w^2},\]
\[C_{2}^{ZZ} = \frac{c_w^2}{8 s_w^2} \left[ \frac{1 + 2\lambda_{HV}}{6} p^2 - m_W^2 \right] + \frac{1}{4 c_w^2} \left\{ \sum_{l} \left[ \left( \frac{Q_{L_l} I_{3L_l}}{2 s_w} - \frac{1 + g 5 s}{2} \frac{I_{3L_l}^2}{2 s_w^2} \right) \left( m_{L_l}^2 - \frac{p^2}{6} \right) \right] + N_c \sum_{l} \left[ \left( \frac{Q_{Q_l} I_{3Q_l}}{2 s_w} - \frac{1 + g 5 s}{2} \frac{I_{3Q_l}^2}{2 s_w^2} - Q_{Q_l}^2 s_w \right) \left( m_{Q_l}^2 - \frac{p^2}{6} \right) \right] \right\} ,\]
\[C_{1}^{W^{-}W^{+}} = -\frac{\lambda_{HV}}{24 s_w^2},\]
\[C_{2}^{W^{-}W^{+}} = \frac{1}{8 s_w^2} \left[ \frac{1 + 2\lambda_{HV}}{6} p^2 - m_W^2 \right] - \frac{1 + g 5 s}{2} \frac{1}{32 s_w^2} \left( \sum_{l} \left( m_{e_l}^2 - \frac{p^2}{3} \right) \right) + N_c \sum_{l,m} \left[ V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( m_{U_l}^2 + m_{D_m}^2 - \frac{p^2}{3} \right) \right] \right\},\]
\[\text{Fermion-Fermion vertices}\]

The generic effective vertex is shown in Fig.4 (c) with the following expression

\[\text{Vert} \left( F_1, \bar{F}_2 \right) = \frac{i e^2}{\pi^2} \left[ \left( C_{-}^{F_1 F_2} \Omega^- + C_{+}^{F_1 F_2} \Omega^+ \right) \bar{p} + C_{0}^{F_1 F_2} \right] \lambda_{HV}, \] (4.22)
with the actual values of \( F_1, \bar{F}_2 \), \( C^{F_1 F_2}_- \), \( C^{F_1 F_2}_+ \) and \( C^{F_1 F_2}_0 \)

\[
\begin{align*}
C^{\bar{U}_i U_m}_- &= \delta_{lm} \frac{Q_{U_i}^2}{16 c_w^2}, \\
C^{\bar{U}_i U_m}_+ &= \delta_{lm} \frac{1}{16} \left[ \frac{1 + g^5 s}{2} \frac{I_{3U_i}^2}{c_w^2 c_w} - \frac{1 + g^5 s}{2} \frac{Q_{U_i} I_{3U_i}}{c_w^2} + \frac{Q_{U_i}^2}{c_w^2} \right. \\
&\left. + \frac{1 + g^5 s}{2} \frac{1}{2 s_w^2} \sum_g \left( V_{U_i, D_g} V_{D_g, U_i}^\dagger \right) \right], \\
C^{\bar{U}_i U_m}_0 &= \delta_{lm} \frac{m_{D_i} Q_{U_i}}{8 c_w^2} \left( Q_{U_i} - \frac{1 + g^5 s}{2} I_{3U_i} \right), \\
C^{\bar{D}_i D_m}_- &= \delta_{lm} \frac{Q_{D_i}^2}{16 c_w^2}, \\
C^{\bar{D}_i D_m}_+ &= \delta_{lm} \frac{1}{16} \left[ \frac{1 + g^5 s}{2} \frac{I_{3D_i}^2}{c_w^2 c_w} - \frac{1 + g^5 s}{2} \frac{Q_{D_i} I_{3D_i}}{c_w^2} + \frac{Q_{D_i}^2}{c_w^2} \right. \\
&\left. + \frac{1 + g^5 s}{2} \frac{1}{2 s_w^2} \sum_g \left( V_{U_i, D_i} V_{D_i, U_i}^\dagger \right) \right], \\
C^{\bar{D}_i D_m}_0 &= \delta_{lm} \frac{m_{D_i} Q_{D_i}}{8 c_w^2} \left( Q_{D_i} - \frac{1 + g^5 s}{2} I_{3D_i} \right),
\end{align*}
\]

\[ (4.23) \]

4.2.2 Electroweak effective vertices with 3 external legs

All possible non-vanishing 3-point vertices in EW are shown in Fig.5.

Scalar-Fermion-Fermion vertices

The generic effective vertex is shown in Fig.5 (a) with the following expression

\[ \text{Scalar-Fermion-Fermion vertices} \]

\[ \text{The generic effective vertex is shown in Fig.5 (a) with the following expression} \]
\[
\text{Vert}(S, F_1, \bar{F}_2) = \text{Vert}(S, F_1, \bar{F}_2) = \text{Vert}(V_\mu, S_1, S_2)
\]

\[
\text{Vert}(V_\mu, F_1, \bar{F}_2) = \text{Vert}(V_\mu, F_1, \bar{F}_2) = \text{Vert}(S, V_\mu, V_2^\nu)
\]

\[
\text{Vert}(S_1, S_2, S_3) = \text{Vert}(S_1, S_2, S_3) = \text{Vert}(V_\nu, V_2^\alpha, V_3^\beta)
\]

\[
\text{Vert}(G_\mu^I, Q_1^I, \bar{Q}_m^I) = \text{Vert}(G_\mu^I, Q_1^I, \bar{Q}_m^I)
\]

**Figure 5:** All possible non-vanishing 3-point vertices in EW.

\[
\text{Vert} (S, F_1, \bar{F}_2) = \frac{e^3}{\pi^2} \left( C_{-}^{SF_1F_2} \Omega^- + C_{+}^{SF_1F_2} \Omega^+ \right), \quad (4.24)
\]

With its actual values of $S, F_1, \bar{F}_2, C_{-}^{SF_1F_2}$ and $C_{+}^{SF_1F_2}$
\[ C_{-H U \bar{u}_m} = \delta_{lm} \frac{im_{U_i}}{8m_W s_w} \left[ \frac{(5 + 3g5s + 8\lambda_{HV}) Q_{U_i}^2}{16c_w^2} + \frac{1 + g5s}{2} \frac{1}{16s_w^2} \right] \]
\[ + \sum_g \left( V_{U_i, D_g} V_{D_g, U_i}^\dagger \right) + \frac{1 + g5s}{2} \frac{I_{3U_i}}{c_w^2} \left( \frac{I_{3U_i}}{8s_w^2} - \frac{(1 + \lambda_{HV}) Q_{U_i}}{2} \right) \]
\[ + \frac{1}{16m_W^2 s_w^2} \sum_g \left( m_{D_g}^2 V_{U_i, D_g} V_{D_g, U_i}^\dagger \right) \],

\[ C_{+H U \bar{u}_m} = C_{-H U \bar{u}_m}, \]

\[ C_{-H D_i \bar{d}_m} = \delta_{lm} \frac{im_{D_i}}{8m_W s_w} \left[ \frac{(1 + 3g5s + 4\lambda_{HV}) Q_{D_i}^2}{8c_w^2} + \frac{1 + g5s}{2} \frac{1}{16s_w^2} \right] \]
\[ + \sum_g \left( V_{D_i, D_g} V_{D_g, D_i}^\dagger \right) + \frac{1 + g5s}{2} \frac{I_{3D_i}}{c_w^2} \left( \frac{I_{3D_i}}{8s_w^2} - \frac{(1 + \lambda_{HV}) Q_{D_i}}{2} \right) \]
\[ + \frac{1}{16m_W^2 s_w^2} \sum_g \left( m_{U_i}^2 V_{U_i, D_i} V_{D_i, U_i}^\dagger \right) \],

\[ C_{+H D_i \bar{d}_m} = C_{-H D_i \bar{d}_m}, \]

\[ C_{-H L \bar{l}_m} = \delta_{lm} \frac{im_{L_i}}{8m_W s_w} \left[ \frac{(3 + g5s + 4\lambda_{HV}) Q_{L_i}^2}{8c_w^2} + \frac{1 + g5s}{2} \frac{1}{16s_w^2} \right] \]
\[ + \frac{1 + g5s}{2} \frac{I_{3L_i}}{c_w^2} \left( \frac{I_{3L_i}}{8s_w^2} - \frac{(1 + \lambda_{HV}) Q_{L_i}}{2} \right) \],

\[ C_{+H L \bar{l}_m} = C_{-H L \bar{l}_m}, \]

\[ C_{-\phi U_i \bar{u}_m} = -g5s \delta_{lm} \frac{m_{U_i}}{4m_W s_w} \left[ \frac{(5 + 3g5s + 8\lambda_{HV}) Q_{U_i}^2}{16c_w^2} + \frac{1 + g5s}{2} \frac{1}{32s_w^2} \right] \]
\[ + \sum_g \left( V_{U_i, D_g} V_{D_g, U_i}^\dagger \right) + \frac{1 + g5s}{2} \frac{I_{3U_i}}{c_w^2} \left( \frac{1}{32s_w^2} - \frac{(1 + \lambda_{HV}) Q_{U_i} I_{3U_i}}{2} \right) \]
\[ - \frac{1}{16m_W^2 s_w^2} \sum_g \left( m_{D_g}^2 I_{3D_g} V_{U_i, D_g} V_{D_g, U_i}^\dagger \right) \],

\[ C_{+\phi U_i \bar{u}_m} = -C_{-\phi U_i \bar{u}_m}, \]
\[ C_{\phi D_i D_m} = -g_5 s \delta_{lm} \frac{m_{D_i}}{4 m_W s_w} \left[ \frac{(1 + 3g_5 s + 4\lambda_{HV}) Q_{D_i}^2 I_{3D_i}}{8 c_w^2} - \frac{1 + g_5 s}{2} \frac{1}{32 s_w^2} \right] \]

\[ \sum_g \left( V_{U_g, D_i} V_{D_i, U_g}^\dagger \right) + \frac{1 + g_5 s}{2} \frac{I_{3D_i}}{c_w^2} \left( \frac{1}{32 s_w^2} - \frac{(1 + \lambda_{HV}) Q_{D_i} I_{3D_i}}{2} \right) \]

\[ - \frac{1}{16 m_W^2 s_w^2} \sum_g \left( m_{U_g}^2 I_{3U_g} V_{U_g, D_i} V_{D_i, U_g}^\dagger \right) \],

\[ C_{\phi L_i L_m} = C_{\phi D_i D_m} \],

\[ C_{\phi^\dagger L_i L_m} = -C_{\phi L_i L_m} \],

\[ C_{\phi^\dagger U_i D_m} = - \frac{1 + g_5 s}{2} \frac{3}{4 \sqrt{2} m_W s_w} \left[ \frac{-1}{16 c_w^2} (1 + 8 (1 + \lambda_{HV}) Q_{U_i} Q_{D_m}) - \frac{3}{32 s_w^2} \right] \]

\[ - \frac{m_{U_i}^2}{16 m_W^2 s_w^2} + \frac{I_{3U_i}}{16 c_w^2} (1 + 8 (1 + \lambda_{HV}) Q_{D_m}) \right] + \frac{1 - g_5 s}{2} \frac{3}{4 \sqrt{2} m_W s_w} \]

\[ \left[ \frac{-1}{16 c_w^2} (1 + 8 (1 + \lambda_{HV}) Q_{U_i} Q_{D_m}) - \frac{m_{D_m}^2}{16 m_W^2 s_w^2} \frac{3 m_{D_m} - m_{U_i}}{48 m_{U_i} c_w^2} \right] \],

\[ C_{\phi^\dagger U_i D_m} = \left[ \frac{-1}{16 c_w^2} (1 + 8 (1 + \lambda_{HV}) Q_{U_i} Q_{D_m}) - \frac{3}{32 s_w^2} \right] \]

\[ - \frac{m_{D_m}^2}{16 m_W^2 s_w^2} + \frac{I_{3D_m}}{16 c_w^2} (1 + 8 (1 + \lambda_{HV}) Q_{U_i}) \] + \frac{1 - g_5 s}{2} \frac{3}{4 \sqrt{2} m_W s_w} \]

\[ \left[ \frac{-1}{16 c_w^2} (1 + 8 (1 + \lambda_{HV}) Q_{U_i} Q_{D_m}) - \frac{m_{U_i}^2}{16 m_W^2 s_w^2} \frac{5 m_{D_m} - 3 m_{U_i}}{48 m_{D_m} c_w^2} \right] \],

\[ C_{\phi^\dagger U_i \bar{U}_m} = - \left( C_{\phi^\dagger U_m \bar{D}_i} \right)^* \],

\[ C_{\phi^\dagger \nu \bar{e}_m} = - \delta_{lm} \frac{m_{e_l}}{4 \sqrt{2} m_W s_w} \left[ \frac{Q_{e_l}}{16 c_w^2} - \frac{1 + g_5 s}{2} \frac{3}{32 s_w^2} \right] \]

\[ + \frac{1 + g_5 s}{2} \frac{I_{3e_l}}{16 c_w^2} (1 + 8 (1 + \lambda_{HV}) Q_{e_l}) \],

\[ C_{\phi^\dagger \nu \bar{e}_m} = 0 \],

\[ C_{\phi^\dagger e_l \bar{\nu}_m} = - \left( C_{\phi^\dagger \nu \bar{e}_l} \right)^* \].

(4.25)

**Vector-Fermion-Fermion vertices**

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The generic effective vertex is shown in Fig. 5 (b) with the following expression

\[
\text{Vert} (V_{\mu}, F_1, F_2) = \frac{ie^3}{\pi^2} \left( C_+^{\nu F_1 F_2} \Omega^+ + C_-^{\nu F_1 F_2} \Omega^- \right) \gamma_\mu, \tag{4.26}
\]

with the actual values of \( V, F_1, F_2, C_+^{\nu F_1 F_2} \) and \( C_-^{\nu F_1 F_2} \)

\[
C_{-}^{U_i C_m} = \delta_{l m} \left[ \frac{1 + \lambda_{HV}}{4c_w} Q_{U_i}^3 + \frac{m_{U_i}^2}{8s_w m_W^2} \left( \frac{1}{2} \sum g \left( V_{U_i, D_\delta} V_{D_\delta, U_i} Q_{D_\delta} \right) \right) + \frac{Q_{U_i}}{4} + Q_{U_i} I_{3U_i}^2 \right],
\]

\[
C_{+}^{U_i C_m} = \delta_{l m} \left[ \frac{1 + \lambda_{HV}}{4c_w} Q_{U_i}^3 - \frac{2 + (1 + g5s) \lambda_{HV}}{4c_w} Q_{U_i}^2 I_{3U_i} + \frac{2 + (1 + g5s) \lambda_{HV}}{8s_w^2 m_W^2} Q_{U_i} I_{3U_i}^2 + \frac{1}{4s_w^2} \sum g \left( V_{U_i, D_\delta} V_{D_\delta, U_i} m_{D_\delta}^2 Q_{D_\delta} \right) \right],
\]

\[
C_{-}^{A D_i D_m} = \delta_{l m} \left[ \frac{1 + \lambda_{HV}}{4c_w} Q_{D_i}^3 + \frac{m_{D_i}^2}{8s_w m_W^2} \left( \frac{1}{2} \sum g \left( V_{U_g, D_i} V_{D_i, U_g} Q_{U_g} \right) \right) \right],
\]

\[
C_{+}^{A D_i D_m} = \delta_{l m} \left[ \frac{1 + \lambda_{HV}}{4c_w} Q_{D_i}^3 - \frac{2 + (1 + g5s) \lambda_{HV}}{4c_w} Q_{D_i}^2 I_{3D_i} + \frac{2 + (1 + g5s) \lambda_{HV}}{8s_w^2 m_W^2} Q_{D_i} I_{3D_i}^2 + \frac{1}{4s_w^2} \sum g \left( V_{U_g, D_i} V_{D_i, U_g} m_{U_g}^2 Q_{U_g} \right) \right],
\]

\[
C_{-}^{A e\bar{e} m} = \delta_{l m} \left[ \frac{1 + \lambda_{HV}}{4c_w} Q_{e_l}^3 + \frac{m_{e_l}^2}{8s_W^2 s_w^2} \left( \frac{Q_{e_l}}{4} + Q_{e_l} I_{3e_l}^2 \right) \right],
\]
\[C_{Ae_l^m}^{A_{\ell m}} = \delta_{lm} \left\{ \frac{1}{4} \left[ 1 + \lambda_{HV} Q_{e_l}^3 - \frac{2 + (1 + g^5 s) \lambda_{HV} Q_{e_l}^2 I_{3 e_l}}{4 c_w^2} + \frac{2 + (1 + g^5 s) \lambda_{HV} Q_{e_l}^2 I_{3 e_l}}{8 s_w^2 c_w^2} \left( 1 + \frac{4 I_{3 e_l}^2}{8 m_W^2} c_w^2 \right) - \frac{1 + g^5 s}{2} \right] \right\},
\]
\[C_{Ae_l^m}^{A_{\ell m}} = 0,
\]
\[C_{ZU_l^m}^{Z_{\ell m}} = \delta_{lm} \left\{ \frac{1}{8 c_w} \left\{ \frac{(1 + \lambda_{HV}) s_w}{2 c_w^2} Q_{U_l}^3 + \frac{m_{U_l}^2}{8 s_w m_W^2} \left[ \sum_g \left( V_{U_l, D_g} V_{D_g, U_l}^\dagger \right) \frac{Q_{D_g} - \frac{1 + g^5 s}{2} I_{3D_g}}{2} \right] - \frac{1 - g^5 s}{2} \right\} \right\},
\]
\[C_{ZU_l^m}^{Z_{\ell m}} = \delta_{lm} \left\{ \frac{1}{8 c_w} \left\{ \frac{(1 + \lambda_{HV}) s_w}{2 c_w^2} Q_{U_l}^3 - \frac{(2 + (1 + g^5 s) \lambda_{HV}) (4 s_w^2 + 1 + g^5 s)}{8 c_w^2 s_w} \right\} \right\},
\]
\[C_{ZU_l^m}^{Z_{\ell m}} = \left\{ \frac{Q_{U_l}^2 I_{3 U_l} + (2 + g^5 s) \frac{2}{4 s_w c_w^2} Q_{U_l} I_{3 U_l}^2}{2} - \frac{1 + g^5 s}{2} \right\} \frac{1}{2} \frac{m_{U_l}^2}{8 m_W^2} s_w^3 + \frac{1}{2} \frac{1}{4 m_W^2} \sum_g \left( V_{U_l, D_g} V_{D_g, U_l}^\dagger \right) \frac{Q_{D_g} - \frac{1 + g^5 s}{2} I_{3D_g}}{2} \frac{m_{D_g}^2}{s_w^2} \right\} \right\},
\]
\[C_{ZD_l^m}^{Z_{\ell m}} = \delta_{lm} \left\{ \frac{1}{8 c_w} \left\{ \frac{(1 + \lambda_{HV}) s_w}{2 c_w^2} Q_{D_l}^3 + \frac{m_{D_l}^2}{8 s_w m_W^2} \left[ \sum_g \left( V_{U_g, D_l} V_{D_l, U_g}^\dagger \right) \frac{Q_{U_g} - \frac{1 + g^5 s}{2} I_{3U_g}}{2} \right] - \frac{1 - g^5 s}{2} \frac{1 + \lambda_{HV}}{2} \right\} \right\},
\]
\[ C_{± \bar{D}_1 \bar{D}_m} = \delta_{\bar{D}_m \bar{D}_1} \frac{1}{8c_w} \left\{ \frac{(1 + \lambda_{HV}) s_w}{2c_w^2} Q^3_{\bar{D}_1} - \frac{(2 + (1 + g5s) \lambda_{HV}) (4s_w^2 + 1 + g5s)}{8c_w^2 s_w} \right\} \]

\[ Q^2_{\bar{D}_1} I_{3\bar{D}_1} + (2 + g5s) \frac{2 + (1 + g5s) \lambda_{HV}}{4s_w c_w^2} Q_{\bar{D}_1} I^2_{3\bar{D}_1} \]

\[ - \frac{1 + g5s}{2} \frac{(1 + \lambda_{HV}) I^3_{3\bar{D}_1}}{2s_w c_w^2} - \frac{1 - g5s}{2} \frac{m_{\bar{D}_1}^2 I_{3\bar{D}_1}}{8m^2_W s_w^3} + \frac{1}{2s_w} \left[ \frac{1}{4m^2_W} \right] \]

\[ \sum_g \left( V_{U_g, D_1} V^\dagger_{D_1, U_g} m^2_{U_g} \left( Q_{U_g} - \frac{1 - g5s}{2} \frac{I_{3U_g}}{s_w^2} \right) \right) \]

\[ + \frac{1 + 4I^2_{3\bar{D}_1} m_{\bar{D}_1}^2 Q_{\bar{D}_1}}{8m^2_W} + \frac{2 + (1 + g5s) \lambda_{HV}}{4} \]

\[ \sum_g \left( V_{U_g, D_1} V^\dagger_{D_1, U_g} \left( Q_{U_g} + \frac{1 + g5s}{2} \frac{c_w^2 - I_{3U_g}}{s_w^2} \right) \right) \}

\[ C_{±\bar{e}_i \bar{e}_m} = \delta_{\bar{e}_m \bar{e}_i} \frac{1}{8c_w} \left\{ \frac{(1 + \lambda_{HV}) s_w}{2c_w^2} Q^3_{\bar{e}_i} + \frac{m^2_{\bar{e}_i}}{4s_w m^2_W} \left[ \frac{-1 + g5s}{2} \frac{1}{4s_w^2} + \frac{1}{2} \right] \left( Q_{\bar{e}_i} - \frac{1 + g5s}{2} \frac{I_{3\bar{e}_i}}{s_w^2} \right) \right\} \]

\[ \left\{ \frac{1 - g5s}{2} \frac{(1 + \lambda_{HV}) I^3_{3\bar{e}_i}}{s_w^3 c_w^2} - \frac{1 + g5s}{2} \frac{m^2_{\bar{e}_i} I_{3\bar{e}_i}}{4m^2_W s_w^3} \right\} \]

\[ \left[ \frac{1}{2s_w} \left[ \frac{m^2_{\bar{e}_i} Q_{\bar{e}_i} (1 + 4I^2_{3\bar{e}_i})}{4m^2_W} + \frac{1 + g5s}{2} \frac{(1 + \lambda_{HV}) (c_w^2 - I_{3\bar{e}_i})}{s_w^2} \right] \right] \]

\[ C_{±\bar{\nu}_m} = 0, \]

\[ C_{±\bar{\nu}_m} = \delta_{\bar{\nu}_m \bar{\nu}_i} \frac{1}{16c_w} \left\{ \frac{-1 + g5s}{2} \frac{(1 + \lambda_{HV}) I^3_{3\bar{\nu}_i}}{s_w^3 c_w^2} + \frac{1}{2s_w} \left[ \frac{m^2_{\bar{\nu}_i} Q_{\bar{\nu}_i}}{2m^2_W} \right] \frac{1 + g5s}{2} \frac{I_{3\bar{\nu}_i}}{s_w} \right\} \]

\[ \left\{ \left( 1 + \frac{1 + g5s}{2} \lambda_{HV} \right) \left( Q_{\bar{\nu}_i} - \frac{1 + g5s}{2} \frac{c_w^2 + I_{3\bar{\nu}_i}}{s_w^2} \right) \right\} \]

\[ - \frac{1 - g5s}{2} \frac{I_{3\bar{\nu}_i} m^2_{\bar{\nu}_i}}{4m^2_W s_w^3} \}

\[ - 21 - \]
\[ C_{W^- U_i D_m}^{m} = -\frac{1 - g s^5}{2} \frac{V^\dagger_{Dm,U_i}}{16 \sqrt{2 s_w}} (1 + \lambda_{H V}) \frac{Q_{U_i} Q_{D_m}}{c_w^2}, \]
\[ C_{W^- U_i D_m}^{+} = \frac{1 + g s^5}{2} \frac{V^\dagger_{Dm,U_i}}{16 \sqrt{2 s_w}} (1 + \lambda_{H V}) \left[ \frac{Q_{D_m} I_3 U_i + Q_{D_m} I_3 D_m - Q_{U_i} Q_{D_m}}{c_w^2} - \frac{1}{s_w^2} + \frac{1}{4s_w^2 c_w^2} \right], \]
\[ C_{W^+ D_i U_m}^{m} = \left( C_{W^- U_m D_i}^{m} \right)^*, \]
\[ C_{W^- \nu_l e_m}^{m} = 0, \]
\[ C_{W^- \nu_l e_m}^{+} = \frac{1 + g s^5}{2} \frac{\delta_{lm}}{16 \sqrt{2 s_w}} \left[ \frac{Q_{\nu_l} I_3 \nu_l}{c_w^2} - \frac{1}{s_w^2} + \frac{1}{4s_w^2 c_w^2} \right] (1 + \lambda_{H V}), \]
\[ C_{W^+ e_l \nu_m}^{m} = \left( C_{W^- \nu_m e_l}^{m} \right)^*. \]

(4.27)

Scalar-Scalar-Scalar vertices

The generic effective vertex is shown in Fig.5 (c) with the following expression

\[ \text{Vert} (S_1, S_2, S_3) = \frac{i e^3}{\pi^2} C_{S_1 S_2 S_3}, \]

with its actual values of \( S_1, S_2, S_3 \) and \( C_{S_1 S_2 S_3} \)

\[ C^{HH \phi^0} = 0, \quad C^{\phi^0 \phi^0 \phi^0} = 0, \quad C^{\phi^0 \phi^- \phi^+} = 0, \]
\[ C^{HHH} = \frac{3}{32 s_w^2} \left[ \frac{1 - 4 \lambda_{H V}}{2} m_W + \frac{1}{m_W^2} \left( \sum_l m_{e_l}^4 + N_c \sum_l m_{Q_l}^4 \right) \right. \]
\[ \left. + \left( 1 + \frac{1}{2 c_w^2} \right) \frac{m_W^2}{4 m_W} + \left( 1 - 4 \lambda_{H V} \right) m_W \right], \]
\[ C^{H \phi^0 \phi^0} = \frac{1}{8 s_w^2} \left[ \frac{1 - 4 \lambda_{H V}}{8} m_W + \frac{2 g s^5 - 1}{4 m_W^3} \left( \sum_l m_{e_l}^4 + N_c \sum_l m_{Q_l}^4 \right) \right. \]
\[ \left. + \left( 1 + \frac{1}{2 c_w^2} \right) \frac{m_W^2}{4 m_W} + \left( 1 - 4 \lambda_{H V} \right) m_W \right]. \]
\[ C^{H\phi^{-}\phi^{+}} = \frac{1}{32s_w^3} \left\{ \frac{1 - 4\lambda_{HV}}{4} m_W \left( 3 + \frac{s_w^2(1 + c_w^2)}{c_w^4} \right) + \frac{(1 + 2c_w^2)m_H^2}{8c_w^2m_W} \right\} \]
\[ + \frac{1}{m_W^3} \left\{ \frac{1 + g5s}{2} \sum_l m_{e_l}^4 + N_c \sum_{l,m} \left[ V_{U_l,D_m} V_{D_m,U_l}^\dagger \mathrm{m}_U \left( m_{U_l}^2 + m_{D_m}^2 - \frac{1 - g5s}{2} 2m_{U_l}m_{D_m} \right) \right] \right\} \]
\[ = \left( \frac{1 + g5s}{2} \left( m_{U_l}^4 + m_{D_m}^4 \right) - \frac{1 - g5s}{2} 2m_{U_l}m_{D_m} \right) \left( m_{U_l}^2 + m_{U_l}m_{D_m} + m_{D_m}^2 \right) \} \right\} \} . \] (4.29)

Vector-Scalar-Scalar vertices

The generic effective vertex is shown in Fig.5 (d) with the following expression

\[ \text{Vert} (V_{\mu}, S_1, S_2) = \frac{e^3}{\pi^2} C^{V S_1 S_2}_{\mu} , \] (4.30)

with the actual values of \( V, S_1, S_2 \) and \( C^{V S_1 S_2}_{\mu} \)

\[ C^{AHH}_{\mu} = 0, \quad C^{ZHH}_{\mu} = 0, \quad C^{A\phi^0\phi^0}_{\mu} = 0, \quad C^{Z\phi^0\phi^0}_{\mu} = 0, \]
\[ C^{A\phi^0H}_{\mu} = \frac{5}{192s_w^2} (p_1 - p_2)_\mu , \]
\[ C^{Z\phi^0H}_{\mu} = -\frac{1}{96s_w^2c_w} \left[ \frac{1 + 2c_w^2 + 20c_w^4}{8s_w^2c_w^2} \right] + \left( \frac{1}{s_w^2m_W^2} \sum_l m_{e_l}^2 + N_c \sum_l m_{Q_l}^2 \right) \] \[ (p_1 - p_2)_\mu = \frac{1 - g5s}{2} \frac{3}{96s_wc_w} \left[ \frac{1}{s_w^2m_W^2} \left( \sum_l m_{e_l}^2 + N_c \sum_l m_{Q_l}^2 \right) \right] (p_2)_\mu , \]
\[ C^{A\phi^+\phi^-}_{\mu} = g5s \frac{i}{48s_w^2} \left[ \frac{1 + 12c_w^2}{8s_w^2c_w^2} + \frac{3 + g5s}{4m_W^2} \sum_l (m_{e_l}^2) + N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( m_{U_l}^2 + m_{D_m}^2 \right) \right) \right] \] \[ \left( \frac{1 - g5s}{2} 2m_{U_l}m_{D_m} \right) \right) \} (p_1 - p_2)_\mu \]
\[ C^{Z\phi^+\phi^-}_{\mu} = g5s \frac{i}{48s_wc_w} \left\{ \frac{1 - 24c_w^4}{16c_w^2s_w^2} + \frac{3 + g5s}{4m_W^2} \sum_l \left( m_{e_l}^2 \left( 1 - \frac{1}{2s_w^2} \right) \right) \right\} \] \[ + N_c \sum_{l,m} \left( \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( m_{U_l}^2 + m_{D_m}^2 \right) - \frac{1 - g5s}{2} 2m_{U_l}m_{D_m} \right) \right) \] \[ \left( 1 - \frac{1}{2s_w^2} \right) \right\} \} (p_1 - p_2)_\mu ; \]
Scalar-Vector-Vector vertices

The generic effective vertex is shown in Fig. 5 (e) with the following expression

\[
\text{Vert} (S, V_1, V_2) = \frac{ie^3}{\pi^2} C^{SV_1 V_2} g_{\mu \nu},
\]  

with the actual values of \( S, V_1, V_2 \) and \( C^{SV_1 V_2} \).
$C^{\phi AA} = 0, \quad C^{\phi AZ} = 0, \quad C^{\phi ZZ} = 0, \quad C^{\phi W^- W^+} = 0,$

$C^{HAA} = -\frac{1}{8s_w} \left[ \frac{1}{m_W} \left( \sum_l \left( m_L^2 Q_{L_l}^2 + N_c \sum_l \left( m_Q^2 Q_{Q_l}^2 \right) \right) + m_W \right) \right]$, 

$C^{HAZ} = \frac{1}{8c_w} \left\{ \frac{1}{m_W} \left[ \sum_l \left( m_L^2 Q_{L_l} \left( \frac{I_{3L_l}}{2s_w^2} - Q_{L_l} \right) \right) \right] + N_c \sum_l \left( m_Q^2 \frac{I_{3Q_l}}{2s_w^2} - Q_{Q_l} \right) \right\} + \frac{m_W^2 (1 + 2c_w^2)}{4s_w^2}$, 

$C^{HZZ} = \frac{1}{8} \left\{ \frac{1}{m_W c_w} \left[ \sum_l \left( m_L^2 \left( \frac{Q_{L_l} I_{3L_l}}{s_w} - Q_{L_l} s_w - \frac{1 + g\bar{5} s}{2} \frac{I_{3L_l}^2}{s_w^3} \right) \right) \right] + N_c \sum_l \left( m_Q^2 \left( \frac{Q_{Q_l} I_{3L_l}}{s_w} - Q_{Q_l} s_w - \frac{1 + g\bar{5} s}{2} \frac{I_{3Q_l}^2}{s_w^3} \right) \right) \right\} + \frac{m_W (s_w^2 - 2)}{2s_w^3}$, 

$C^{HW^- W^+} = -\frac{1}{8s_w^3} \left\{ \frac{1 + g\bar{5} s}{2} \frac{1}{4m_W} \left[ \sum_l m_{L_l}^2 \right] + N_c \sum_{l,m} \left( V_{U_l D_m} \left( m_{U_l}^2 + m_{D_m}^2 \right) \right) \right\} + m_W$, 

$C^{\phi^- AW^+} = C^{\phi^- W^- A} = \frac{K}{32s_w^3}$, 

$C^{\phi^- ZW^+} = C^{\phi^- W^- Z} = \frac{K}{32s_w c_w}$, \hspace{1cm} (4.33) 

where

$K = m_W + \frac{N_c}{m_W} \sum_{l,m} \left( V_{U_l D_m} \left( m_{U_l}^2 + m_{D_m}^2 \right) \right)$. \hspace{1cm} (4.34) 

**Vector-Vector-Vector vertices**

The generic effective vertex is shown in Fig.5 (f) with the following expression

$$\text{Vert} \left( V_{1\mu}, V_{2\nu}, V_{3\rho} \right) = \frac{i e^3}{\pi^2} C_{\mu\nu\rho}^{V_1 V_2 V_3},$$ \hspace{1cm} (4.35)
with the actual values of $V_1$, $V_2$, $V_3$, and $C_{\mu\nu\rho}^{V_1V_2V_3}$

$$C_{\mu\nu\rho}^{AAA} = C_{\mu\nu\rho}^{AAZ} = C_{\mu\nu\rho}^{AZZ} = C_{\mu\nu\rho}^{ZZZ} = 0,$$

$$C_{\mu\nu\rho}^{AW+W^-} = KV_{\mu\nu\rho} + \frac{1 + 3g5s}{2} \frac{i}{288s_w^2} \left[ N_c \sum_{l,m=1}^{3} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger - 9 \right) \right]$$

$$\left( \varepsilon_{(p_2-p_3)\mu\nu\rho} + \frac{1 - g5s}{2} 3iV_{2\mu\nu\rho} \right) + \frac{1 - g5s}{2} \frac{1}{8s_w^2} V_{2\mu\nu\rho},$$

$$C_{\mu\nu\rho}^{ZW+W^-} = -\frac{c_w}{s_w} \left( K - \frac{1 - g5s}{2} \frac{1}{4s_w^2} \right) V_{1\mu\nu\rho} + \frac{1 - g5s}{2} \frac{1}{8c_w s_w} V_{3\mu\nu\rho}$$

$$+ \frac{1 + 3g5s}{2} \frac{1}{288c_w s_w^3} \left( N_c \sum_{l,m=1}^{3} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger - 9 \right) \right)$$

$$\left( is_w^2 \varepsilon_{(p_2-p_3)\mu\nu\rho} - \frac{1 - g5s}{2} 3 \left( -2V_{1\mu\nu\rho} + s_w^2 V_{2\mu\nu\rho} \right) \right), \quad (4.36)$$

where

$$K = \frac{7 + 4\lambda_{HV}}{96s_w^2} + \frac{1}{48s_w^2} \left( 3 + N_c \sum_{l,m=1}^{3} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \right) \right),$$

$$V_{1\mu\nu\rho} = g_{\mu\rho} (p_2 - p_1)_\rho + g_{\nu\rho} (p_3 - p_2)_\mu + g_{\rho\nu} (p_1 - p_3)_\nu,$$

$$V_{2\mu\nu} = 3g_{\mu\rho} (p_1)_\rho + g_{\nu\rho} (p_2 - p_3)_\mu - 3g_{\rho\nu} (p_1)_\nu,$$

$$V_{3\mu\nu} = g_{\mu\rho} (p_2 - p_3)_\rho + g_{\nu\rho} (p_2 - p_3)_\nu - g_{\rho\nu} (p_2 - p_3)_\mu. \quad (4.37)$$

Gluon-Quark-Quark vertices

In previous subsection, we have listed all mixed $R$ QCD/EW vertices with internal QCD particles. The remaining non-vanishing mixed $R$ QCD/EW vertices with EW internal particles are those Gluon-Quark-Quark vertices. Its generic diagram is shown in Fig.5 (g) with the following expression

$$\text{Vert} \left( G_{\mu}^a, Q_i^l, \bar{Q}_m^j \right) = \frac{ig_s^2}{\pi^2} T_{ji}^a \left( C_{Q_i^l Q_m^j}^{Q_i^l Q_m^j} \Omega^- + C_{Q_i^l Q_m^j}^{Q_i^l Q_m^j} \Omega^+ \right) \gamma_{\mu}. \quad (4.38)$$

The actual values of $Q_l$, $\bar{Q}_m$, $C_{Q_l Q_m}^{Q_l Q_m}$ and $C_{Q_l Q_m}^{Q_l Q_m}$ are
\[ C_{U_i U_m} = \delta_{lm} \frac{1}{16} \left[ (1 + \lambda_{HV}) \frac{Q^2_{U_i}}{c_w^2} + \frac{m^2_{U_i}}{2 s_w^2 m_W^2} \left( \frac{1}{2} \sum_g \left( V_{U_i, D_g} V_{D_g, U_i}^\dagger \right) \right) + \frac{1}{4} + I_{3U_i} \right] \],

\[ C_{U_i U_m} = \delta_{lm} \frac{1}{16} \left[ (1 + \lambda_{HV}) \frac{Q^2_{U_i}}{c_w^2} + \frac{2 + (1 + g5s) \lambda_{HV}}{2 c_w^2} \left( \frac{I_{3U_i}^2}{s_w^2} - 2 Q_{U_i} I_{3U_i} \right) + \frac{2 + (1 + g5s) \lambda_{HV}}{4 s_w^2} \sum_g \left( V_{U_i, D_g} V_{D_g, U_i}^\dagger \right) \right],

\[ C_{D_l D_m} = \delta_{lm} \frac{1}{16} \left[ (1 + \lambda_{HV}) \frac{Q^2_{D_l}}{c_w^2} + \frac{m^2_{D_l}}{2 s_w^2 m_W^2} \left( \frac{1}{2} \sum_g \left( V_{U_g, D_l} V_{D_l, U_g}^\dagger \right) \right) + \frac{1}{4} + I_{3D_l} \right] \],

\[ C_{D_l D_m} = \delta_{lm} \frac{1}{16} \left[ (1 + \lambda_{HV}) \frac{Q^2_{D_l}}{c_w^2} + \frac{2 + (1 + g5s) \lambda_{HV}}{2 c_w^2} \left( \frac{I_{3D_l}^2}{s_w^2} - 2 Q_{D_l} I_{3D_l} \right) + \frac{2 + (1 + g5s) \lambda_{HV}}{4 s_w^2} \sum_g \left( V_{U_g, D_l} V_{D_l, U_g}^\dagger \right) \right] . \]

### 4.2.3 Electroweak effective vertices with 4 external legs

In this part, we list all possible non-vanishing 4-point vertices in EW, which are shown in Fig. 6.

**Scalar-Scalar-Scalar-Scalar vertices**

The generic effective vertex is shown in Fig. 6 (a) with the following expression

\[ \text{Vert} (S_1, S_2, S_3, S_4) = \frac{i e^4}{\pi^2} C^{S_1 S_2 S_3 S_4} \],

with the actual values of \( S_1, S_2, S_3, S_4 \) and \( C^{S_1 S_2 S_3 S_4} \).
Figure 6: All possible non-vanishing 4-point vertices in EW.

\[ C^{HHH0} = C^{0000} = C^{0000} = 0, \]
\[ C^{HHH} = C^{000} = \frac{1}{64s_w^4} K_1, \]
\[ C^{HH00} = \frac{1}{192s_w^4} \left( K_1 - \frac{1 - g5s}{2} \frac{4}{m_W^4} \left( \sum_l m_{e_l}^4 + N_c \sum_l m_{q_l}^4 \right) \right), \]
\[ C^{HH-0} = \frac{1}{64s_w^4} \left\{ K_2 - \frac{1 - g5s}{2} \frac{2N_c}{3m_W^4} \sum_{l,m} \left[ V_{Ul,Dm} V_{Dm,Ul}^\dagger m_{Ul} m_{Dm} \left( m_{Ul}^2 + m_{Ul} m_{Dm} + m_{Dm}^2 \right) \right] \right\}, \]
\[ C^{\phi^0 \phi^0 \phi^+} = \frac{1}{64 s_w^4} \left\{ K_2 + \frac{1 - g s}{2} \frac{2N_c}{3m_W^2} \sum_{l,m} \left[ V_{U_l, D_m} V_{D_m, U_l}^\dagger m_{U_l} m_{D_m} \left( m_{U_l}^2 - m_{U_l} m_{D_m} + m_{D_m}^2 \right) \right] \right\}, \]

\[ C^{\phi^- \phi^0 \phi^+} = \frac{1}{32 s_w^4} K_3, \quad (4.41) \]

where

\[
K_1 = \frac{1}{m_W^2} \left[ \frac{5}{m_W^2} \left( \sum_g m_{e_1}^4 + N_c \sum_g m_{q_1}^4 \right) + \frac{3m_H^2}{2} \left( 1 + \frac{1}{2c_w^2} \right) \right] \\
+ \frac{1 - 12 \lambda_{HV}}{2} \left( 1 + \frac{1}{2c_w^2} \right),
\]

\[
K_2 = \frac{1}{m_W^2} \left[ \frac{9 + g s}{6m_W^2} \left( \sum_l m_{e_1}^4 + N_c \sum_{l,m} V_{U_l, D_m} V_{D_m, U_l}^\dagger \left( m_{U_l}^4 + m_{D_m}^4 \right) \right) \right] \\
+ \frac{m^2_H}{2} \left( 1 + \frac{1}{2c_w^2} \right) \left( 1 + \frac{1 - 12 \lambda_{HV}}{4} \left( 1 + \frac{s_w^2}{3c_w^2} \left( 1 + \frac{1}{c_w^2} \right) \right) \right),
\]

\[
K_3 = \frac{1}{m_W^2} \left[ \frac{9 + g s}{6m_W^2} \left( \sum_l m_{e_1}^4 + N_c \sum_{l,j,k} V_{U_l, D_j} V_{D_j, U_k} V_{U_k, D_i} V_{D_i, U_l}^\dagger \left( \left( m_{U_k}^2 m_{D_k}^2 + m_{D_i}^2 m_{D_i}^2 + \frac{1 - g s}{4} m_{U_k} m_{U_k} m_{D_i} m_{D_i} \right) \right) \right) \right] \\
+ \frac{m^2_H}{2} \left( 1 + \frac{1}{2c_w^2} \right) \left( 1 + 12 \lambda_{HV} \left( \frac{1 + s_w^4}{4} + \frac{1}{6} \left( s_w^2 + \frac{2s_w^6}{c_w^2} \right) + \frac{s_w^8}{12c_w^4} \right) \right) \quad (4.42)
\]

**Vector-Vector-Vector-Vector vertices**

The generic effective vertex is shown in Fig. 8 (b) with the following expression

\[
\text{Vert} \left( V_{1\mu}, V_{2\nu}, V_{3\rho}, V_{4\sigma} \right) = \frac{ie^4}{2} \left( C_1^{V_1 V_2 V_3 V_4} g_{\mu \nu} g_{\rho \sigma} + C_2^{V_1 V_2 V_3 V_4} g_{\mu \rho} g_{\nu \sigma} \right) \\
+ C_3^{V_1 V_2 V_3 V_4} g_{\mu \sigma} g_{\nu \rho} + C_0^{V_1 V_2 V_3 V_4} \epsilon_{\mu \nu \rho \sigma}, \quad (4.43)\]

with the actual values of \( V_1, V_2, V_3, V_4, C_1^{V_1 V_2 V_3 V_4}, C_2^{V_1 V_2 V_3 V_4}, C_3^{V_1 V_2 V_3 V_4} \) and \( C_0^{V_1 V_2 V_3 V_4} \).
\[ C_{1}^{AAAA} = \frac{1}{12} \left( -1 + \sum_{l} Q_{L_{l}}^4 + N_{c} \sum_{l} Q_{Q_{l}}^4 \right), \]

\[ C_{2}^{AAAA} = C_{3}^{AAAA} = C_{1}^{AAAA}, \]

\[ C_{0}^{AAAA} = 0, \]

\[ C_{1}^{AAAZ} = \frac{1}{12} \left[ \frac{s_{w}}{s_{w}} + \sum_{l} \left( \frac{s_{w}}{c_{w}} Q_{L_{l}}^4 - \frac{1}{2 s_{w} c_{w}} Q_{L_{l}}^2 I_{3 L_{l}} \right) \right. \]

\[ + N_{c} \sum_{l} \left( \frac{s_{w}}{c_{w}} Q_{Q_{l}}^4 - \frac{1}{2 s_{w} c_{w}} Q_{Q_{l}}^3 I_{3 Q_{l}} \right), \]

\[ C_{2}^{AAAZ} = C_{3}^{AAAZ} = C_{1}^{AAAZ}, \]

\[ C_{0}^{AAAZ} = 0, \]

\[ C_{1}^{AAZZ} = \frac{1}{12} \left[ \frac{c_{w}}{s_{w}^2} + \frac{1}{2} \sum_{l} \left( \frac{s_{w}}{c_{w}^2} Q_{L_{l}}^4 + \left( \frac{s_{w}}{c_{w}} Q_{L_{l}}^2 - \frac{1}{s_{w} c_{w}} Q_{L_{l}} I_{3 L_{l}} \right)^2 \right) \right. \]

\[ + N_{c} \frac{1}{2} \sum_{l} \left( \frac{s_{w}}{c_{w}} Q_{Q_{l}}^4 + \left( \frac{s_{w}}{c_{w}} Q_{Q_{l}}^2 - \frac{1}{s_{w} c_{w}} Q_{Q_{l}} I_{3 Q_{l}} \right)^2 \right) \]

\[ + \frac{1 - g_{5S}}{2} 2 \left( \sum_{l} \frac{1}{s_{w} c_{w}} Q_{L_{l}}^2 I_{3 L_{l}} - N_{c} \sum_{l} \frac{1}{s_{w} c_{w}} Q_{Q_{l}}^2 I_{3 Q_{l}} \right), \]

\[ C_{2}^{AAZZ} = \frac{1}{12} \left[ \frac{c_{w}}{s_{w}^2} + \frac{1}{2} \sum_{l} \left( \frac{s_{w}}{c_{w}^2} Q_{L_{l}}^4 + \left( \frac{s_{w}}{c_{w}} Q_{L_{l}}^2 - \frac{1}{s_{w} c_{w}} Q_{L_{l}} I_{3 L_{l}} \right)^2 \right) \right. \]

\[ + N_{c} \frac{1}{2} \sum_{l} \left( \frac{s_{w}}{c_{w}} Q_{Q_{l}}^4 + \left( \frac{s_{w}}{c_{w}} Q_{Q_{l}}^2 - \frac{1}{s_{w} c_{w}} Q_{Q_{l}} I_{3 Q_{l}} \right)^2 \right), \]

\[ C_{3}^{AAZZ} = C_{2}^{AAZZ}, \]

\[ C_{0}^{AAZZ} = 0, \]

\[ C_{1}^{AZZZ} = \frac{1}{12} \left[ \frac{c_{w}^3}{s_{w}^3} + \sum_{l} \left( \frac{s_{w}^3}{c_{w}^3} Q_{L_{l}}^4 - \frac{3 s_{w}}{2 c_{w}^3} Q_{L_{l}}^2 I_{3 L_{l}} + \frac{5 - 2 g_{5S}}{2 s_{w} c_{w}} Q_{L_{l}}^2 I_{3 L_{l}} \right) \right. \]

\[ + \frac{2 - g_{5S}}{2 s_{w} c_{w}^3} Q_{L_{l}} I_{3 L_{l}}^3 \right) + N_{c} \sum_{l} \left( \frac{s_{w}^3}{c_{w}^3} Q_{Q_{l}}^4 - \frac{3 s_{w}}{2 c_{w}^3} Q_{Q_{l}}^3 I_{3 Q_{l}} \right. \]

\[ + \frac{5 - 2 g_{5S}}{2 s_{w} c_{w}^3} Q_{Q_{l}}^2 I_{3 Q_{l}}^2 - \frac{2 - g_{5S}}{2 s_{w} c_{w}^3} Q_{Q_{l}} I_{3 Q_{l}}^3 \right), \]

\[ C_{2}^{AZZZ} = C_{3}^{AZZZ} = C_{1}^{AZZZ}, \]

\[ C_{0}^{AZZZ} = 0, \]
\[ C_{zzzz} = \frac{1}{12} \left[ -\frac{c_w^4}{s_w^4} + \sum_{l} \left( \frac{s_w^4}{c_w^4} Q_{L_l}^4 - \frac{2s_w^2}{c_w^2} Q_{L_l}^3 I_{3L_l} + \frac{5 - 2g5s}{c_w^4} Q_{L_l}^2 I_{3L_l}^2 \right) \right. \\
- \frac{4 - 2g5s}{s_w^2 c_w^4} Q_{L_l}^3 I_{3L_l} + \frac{5 - 3g5s}{4s_w^4 c_w^4} I_{3L_l}^3 \left. \right) + N_c \sum_{l} \left( \frac{s_w^4}{c_w^4} Q_{Q_l}^4 - \frac{2s_w^2}{c_w^2} Q_{Q_l}^3 I_{3Q_l} + \frac{5 - 2g5s}{c_w^4} Q_{Q_l}^2 I_{3Q_l}^2 \right) \\
- \frac{4 - 2g5s}{s_w^2 c_w^4} Q_{Q_l}^3 I_{3Q_l} + \frac{5 - 3g5s}{4s_w^4 c_w^4} I_{3Q_l}^3 \right] , \\
C_{zzzz}^A = C_{zzzz}^A , \\
C_{zzzz}^0 = 0 , \\
C_{1}^{AAW-W^+} = \frac{1}{16s_w^2} \left[ \frac{10 + 4\lambda_{HV}}{3} + 3 \frac{7 - g5s}{6} + \frac{(51 - g5s) N_c}{54} \sum_{l,m} V_{U_l,D_m} V_{D_m,U_l}^\dagger \right] , \\
C_{2}^{AAW-W^+} = -\frac{1}{16s_w^2} \left[ \frac{7 + 2\lambda_{HV}}{3} + 1 + \frac{11N_c}{27} \sum_{l,m} V_{U_l,D_m} V_{D_m,U_l}^\dagger \right] , \\
C_{3}^{AAW-W^+} = C_{2}^{AAW-W^+} , \\
C_{0}^{AAW-W^+} = \frac{1 + g5s}{2} \frac{i}{48s_w^2} \left( N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \right) - 9 \right) , \\
C_{1}^{AZW-W^+} = \frac{1}{16s_w c_w} \left[ \frac{10 + 4\lambda_{HV}}{3} + 3 \left( \frac{7 - g5s}{6} - \frac{10 + g5s}{12s_w^2} \right) \right. \\
+ N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( \frac{51 - g5s}{54} - \frac{10 + g5s}{12s_w^2} \right) V_{U_l,D_m} V_{D_m,U_l}^\dagger \right) \left. \right] , \\
C_{2}^{AZW-W^+} = \frac{1}{16s_w c_w} \left[ \frac{7 + 2\lambda_{HV}}{3} + 3 \left( \frac{4 + g5s}{12s_w^2} - \frac{1}{3} \right) \right. \\
+ N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( \frac{4 + g5s}{12s_w^2} - \frac{11}{27} \right) \right) \left. \right] , \\
C_{3}^{AZW-W^+} = C_{2}^{AZW-W^+} , \\
C_{0}^{AZW-W^+} = \frac{i}{192c_w s_w^3} \left( \frac{1 + g5s}{2} \left( 4s_w^2 - 3 \right) + \frac{1 - g5s}{2} \right) \left( N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \right) - 9 \right) , \\
C_{1}^{ZZW-W^+} = \frac{5 + 2\lambda_{HV}}{24s_w^4} + \frac{1}{16c_w^2} \left[ 3 \left( \frac{7 - g5s}{6} - \frac{10 + g5s}{6s_w^2} + \frac{23 - g5s}{24s_w^4} \right) \right. \\
+ N_c \left( \frac{51 - g5s}{54} - \frac{10 + g5s}{6s_w^2} + \frac{23 - g5s}{24s_w^4} \right) \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \right) \left. \right] \right] , \\
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\[
C_{zzw-w^+} = -\left(\frac{7 + 2\lambda_{HV}}{48s_w^4}\right)c_w^2 + \frac{1}{16c_w^2} \left\{ 3\left(\frac{1}{3} + \frac{4 + g5s}{6s_w^2} - \frac{9 + g5s}{24s_w^4}\right) + N_c \left( -\frac{11}{27} + \frac{4 + g5s}{6s_w^2} - \frac{9 + g5s}{24s_w^4}\right) \sum_{l,m} V_{U_i,D_m} V_{D_m,U_l}^\dagger \right\},
\]

\[
C_{zzw-w^+}^3 = C_{zzw-w^+}^2, \\
C_{zzw-w^+}^0 = -i\left(12\left(1 + g5s\right) + \left(g5s - 1\right)c_w^2\right) \frac{1152s_w^2c_w^2}{\sum_{l,m} \left( V_{U_i,D_m} V_{D_m,U_l}^\dagger \right) - 9},
\]

\[
C_{w-w^+w^+}^1 = \frac{1}{16s_w^4} \left( \frac{3 + 2\lambda_{HV}}{3} + \frac{3}{12} \frac{7 - g5s}{12} \right) \\
+ \frac{(7 - g5s)N_c}{12} \sum_{i,j,k,m} \left( V_{U_i,D_j} V_{D_j,U_k}^\dagger V_{U_k,D_m} V_{D_m,U_i}^\dagger \right)
\]

\[
C_{w-w^+w^+}^2 = -\frac{1}{8s_w^4} \left( \frac{7 + 2\lambda_{HV}}{3} + \frac{9 + g5s}{24} \right) \\
+ \frac{(9 + g5s)N_c}{24} \sum_{i,j,k,m} \left( V_{U_i,D_j} V_{D_j,U_k}^\dagger V_{U_k,D_m} V_{D_m,U_i}^\dagger \right)
\]

\[
C_{w-w^+w^+}^3 = C_{w-w^+w^+}^1, \\
C_{w-w^+w^+}^0 = 0. \quad (4.44)
\]

**Scalar-Scalar-Vector-Vector vertices**

The generic effective vertex is shown in Fig. 8 (c) with the following expression

\[
\text{Vert} \left( S_1, S_2, V_1\mu, V_2\nu \right) = \frac{ie^4}{\pi^2} C^{S_1S_2V_1V_2} g_{\mu\nu}, \quad (4.45)
\]

with the actual values of $S_1$, $S_2$, $V_1$, $V_2$ and $C^{S_1S_2V_1V_2}$
\[ C^{H\phi^0 AA} = C^{H\phi^0 AZ} = C^{H\phi^0 ZZ} = C^{H\phi^0 W^+ W^-} = 0, \]
\[ C^{HHAA} = \frac{1}{16s_w^2} \left[ \frac{1}{12} - \frac{1}{m_W^2} \left( \sum_l \left( Q_L^2 m_{L_l}^2 + N_c \sum_l \left( Q_{Q_l}^2 m_{Q_l}^2 \right) \right) \right) \right], \]
\[ C^{\phi^0\phi^0 AA} = \frac{1}{16s_w^2} \left[ \frac{1}{12} - \frac{7 - 4g_5s}{3m_W^2} \left( \sum_l \left( Q_L^2 m_{L_l}^2 + N_c \sum_l \left( Q_{Q_l}^2 m_{Q_l}^2 \right) \right) \right) \right], \]
\[ C^{HHAZ} = \frac{1}{16s_w^2} \left[ \frac{4 + s_w^2}{12s_w^2 c_w} + \frac{1}{m_W^2 c_w} \left( \sum_l \left( Q_L^2 m_{L_l}^2 \left( \frac{I_{3L_l}}{2s_w^2} - Q_{L_l} \right) \right) \right) \right] + N_c \sum_l \left( Q_{Q_l}^2 m_{Q_l}^2 \left( \frac{I_{3Q_l}}{2s_w^2} - Q_{Q_l} \right) \right), \]
\[ C^{\phi^0\phi^0 AZ} = \frac{1}{16s_w^2} \left[ \frac{4 + s_w^2}{12s_w^2 c_w} + \frac{7 - 4g_5s}{3m_W^2 c_w} \left( \sum_l \left( Q_L^2 m_{L_l}^2 \left( \frac{I_{3L_l}}{2s_w^2} - Q_{L_l} \right) \right) \right) \right] + N_c \sum_l \left( Q_{Q_l}^2 m_{Q_l}^2 \left( \frac{I_{3Q_l}}{2s_w^2} - Q_{Q_l} \right) \right), \]
\[ C^{HHZZ} = -\frac{1}{16c_w^2} \left[ \frac{1 + 2c_w^2 + 40c_w^4}{48s_w^2 c_w} \right] \left[ \sum_l \left( m_{L_l}^2 \left( Q_{L_l}^2 + \frac{(7 + g_5s) I_{3L_l}^2}{64s_w^4} - \frac{Q_{L_l} I_{3L_l}}{s_w^2} \right) \right) \right] + N_c \sum_l \left( m_{Q_l}^2 \left( Q_{Q_l}^2 + \frac{(7 + g_5s) I_{3Q_l}^2}{64s_w^4} - \frac{Q_{Q_l} I_{3Q_l}}{s_w^2} \right) \right), \]
\[ C^{\phi^0\phi^0 ZZ} = -\frac{1}{16c_w^2} \left[ \frac{1 + 2c_w^2 + 40c_w^4}{48s_w^2 c_w} \right] \left[ \sum_l \left( m_{L_l}^2 \left( Q_{L_l}^2 + \frac{(71 + 17g_5s) I_{3L_l}^2}{664s_w^4} - \frac{Q_{L_l} I_{3L_l}}{s_w^2} \right) \right) \right] + N_c \sum_l \left( m_{Q_l}^2 \left( Q_{Q_l}^2 + \frac{(71 + 17g_5s) I_{3Q_l}^2}{664s_w^4} - \frac{Q_{Q_l} I_{3Q_l}}{s_w^2} \right) \right), \]
\[ C^{HHW^- W^+} = -\frac{1}{48s_w^2} \left[ \frac{1 + 38c_w^2}{16c_w^2} \right] \left[ \sum_l m_{e_l}^2 + N_c \sum_{l,m} \left( V_{U_l D_m} V_{D_m U_l}^{\dagger} \left( m_{U_l}^2 + m_{D_m}^2 \right) \right) \right]. \]
\[
C^{\phi \phi W^- W^+} = -\frac{1}{48s_w^4} \left[ \frac{1 + 38c_w^2}{16c_w^2} + \frac{11 - 3g5s}{8m_W^2} \left( \sum_l m_{e_l}^2 \right) \\
+ N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( m_{U_l}^2 + m_{D_m}^2 - \frac{1 - g5s}{2} \frac{4}{l} m_{U_l} m_{D_m} \right) \right) \right],
\]

\[
CH^{\phi^+ AW^-} = C^{H^+ \phi^- AW^+} = \frac{1}{24s_w^3} \left( 1 + \frac{22c_w^2}{32c_w^2} + K_1 \right),
\]

\[
C^{\phi^+ AW^-} = -\frac{i}{24s_w^3} \left( 1 + \frac{22c_w^2}{32c_w^2} + K_2 \right),
\]

\[
C^{\phi^- AW^+} = -C^{\phi^+ AW^-},
\]

\[
CH^{\phi^+ ZW^-} = C^{H^+ \phi^- ZW^+} = \frac{s_w}{c_w} C^{H^+ AW^-},
\]

\[
C^{\phi^+ ZW^-} = -\frac{i}{24s_w^2 c_w} \left( 1 + \frac{22c_w^2}{32c_w^2} + K_3 \right),
\]

\[
C^{\phi^- ZW^+} = -C^{\phi^+ ZW^-},
\]

\[
C^{\phi^- \phi^+ AA} = -\frac{1}{12s_w^2} \left[ \frac{1 + 21c_w^2}{16c_w^2} + \frac{1}{m_W^2} \left( \frac{9 - g5s}{8} \sum_l m_{e_l}^2 \right) \\
+ \frac{5N_c}{6} \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( m_{U_l}^2 + m_{D_m}^2 + \frac{1 - g5s}{2} \frac{(m_{U_l} + m_{D_m})^2}{30} \right) \right) \right],
\]

\[
C^{\phi^- \phi^+ AZ} = \frac{1}{12s_w c_w} \left\{ \frac{42c_w^4 - 10c_w^2 - 1}{32s_w^2 c_w} \\
- \frac{1}{m_W^2} \left[ \sum_l \left( m_{e_l}^2 Q_{e_l} \left( \frac{9 - g5s}{8} \frac{Q_{e_l}}{s_w^2} + \frac{5I_3}{8} \right) \right) \right) \\
+ N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( m_{U_l}^2 \left( \frac{5}{6} - \frac{I_3}{m_{D_m}^2 s_w^2} \right) Q_{D_m} - \frac{5}{8} Q_{U_l} \right) \right) \\
+ m_{D_m}^2 \left( \frac{5}{6} - \frac{I_3}{s_w^2} \left( Q_{U_l} - \frac{5}{8} Q_{D_m} \right) \right) \\
+ \frac{1 - g5s}{2} \frac{1}{36} (m_{U_l} + m_{D_m})^2 \right) \right\},
\]
\[ C^{\phi^- \phi^+ ZZ} = \frac{1}{12c_w^2} \left\{ \frac{-1 + 2c_w^2 + 44c_w^4 - 84c_w^6}{64s_w^2c_w^2} - \frac{1}{m_W^2} \left[ \sum_{l} \left( m_{\ell_l}^2 \left( \frac{9 - 2g5s}{8} \right) Q_{\ell_l}^2 \right) \right] \right\}, \]

\[ + \frac{5Q_{\ell_l}^2}{4s_w^2} \left( \frac{I_3_{\ell_l}^2}{s_w^2} \right) \] + \left( m_{U_l}^2 \left( \frac{5}{6} - \frac{I_{3_{D_m}}^2}{s_w^2} \right) \right) + N_c \sum_{l,m} \left[ V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( \frac{2Q_{D_m} - \frac{5}{4}Q_{U_l}}{s_w^2} \right) \right] \]

\[ + m_{D_m}^2 \left( \frac{5}{6} - \frac{I_{3_{D_m}}^2}{s_w^2} \right) \left( 2Q_{D_m} - \frac{5}{4}Q_{D_m} \right) + \left( \frac{1 - g5s}{2} \right) \left( \frac{m_{U_l}m_{U_{l_k}} + m_{U_l}m_{D_j} + m_{U_l}m_{D_i} + m_{U_k}m_{D_j}}{8s_w^4} \right) \] \}

\[ C^{\phi^- \phi^+ W^+ W^-} = -\frac{1}{48s_w^2} \left\{ \frac{38c_w^2 + 1}{16c_w^2} + \frac{1}{m_W^2} \left[ \left( \frac{9 - 2g5s}{8} \right) \sum_{l} m_{\ell_l}^2 \right] \right\}, \]

\[ + \frac{N_c}{4} \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \right) \left[ \left( \frac{2m_{U_l}^2 + 2m_{U_{l_k}}^2 + 2m_{D_j} + 2m_{D_i}^2}{3} \right) \right] \]

\[ + \left( \frac{1 - g5s}{2} \right) \left( \frac{m_{U_l}m_{U_{l_k}} + m_{U_l}m_{D_j} + m_{U_l}m_{D_i} + m_{U_k}m_{D_j} + m_{U_l}m_{D_i} + m_{D_j}m_{D_i}}{8s_w^4} \right) \] \}

\[ \text{where} \]

\[ K_1 = \frac{1}{8m_W^2} \left[ \sum_l m_{\ell_l}^2 + N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( 3m_{D_m}^2 + 2m_{U_l}^2 \right) \right) \right], \]

\[ K_2 = \frac{1}{8m_W^2} \left[ \left( -1 + 2g5s \right) \sum_l m_{\ell_l}^2 \right] \]

\[ + N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( \frac{7 + 2g5s}{3} m_{D_m}^2 + \frac{2 + 4g5s}{3} m_{U_l}^2 \right) \right), \]

\[ K_3 = \frac{1}{8m_W^2} \left[ \left( -1 + 2g5s \right) \sum_l m_{\ell_l}^2 + \frac{1 - g5s}{2} \frac{1}{s_w^2} \sum_l m_{\ell_l}^2 \right] \]

\[ + N_c \sum_{l,m} \left( V_{U_l,D_m} V_{D_m,U_l}^\dagger \left( \frac{7 + 2g5s}{3} m_{D_m}^2 + \frac{2 + 4g5s}{3} m_{U_l}^2 \right) \right) \]

\[ + \left( \frac{1 - g5s}{2} \right) \left( \frac{m_{U_l}^2 + m_{D_i}^2}{8s_w^4} \right) \] \}

\[ \text{All possible non-vanishing } R \text{ effective vertices in SM are listed above. Our results are the same as those in Refs. [17, 18] in the same } \gamma_5 \text{ scheme. In addition, all terms} \]
proportional to $\varepsilon$ evidently vanish in the formulae after including all fermions in SM. This is guaranteed by the cancelation of gauge anomaly in SM.

5. Summary

In summary, we have studied Feynman rules for the rational part $R$ of the Standard Model amplitudes at one-loop level in the ’t Hooft-Veltman $\gamma_5$ scheme. Comparing Feynman rules obtained in this scheme for quantum chromodynamics and electroweak 1-loop amplitudes with that obtained in another $\gamma_5$ scheme (the KKS scheme) in Refs.[17, 18], we find the latter result can be recovered after setting $g5s = 1$ in our results. As an independent check, we also calculated Feynman rules in the KKS scheme, and found results completely in agreement with the analytical expressions given in their updated version, as mentioned in Ref.[33].

With these rational terms, one can simplify fermion chains or Dirac traces in 4 dimensions at the amplitude level, resulting in more convenient general calculations for multi-leg processes, compared with directly solving $d$ dimension Dirac algebras. To clarify some existing ambiguities in DREG, we give our results in FDH and HV DREG, as well as in KKS and HV $\gamma_5$ schemes. To be more reliable, our results are checked and found to be in agreement with previously published works (see Refs.[17, 18] and Ref.[33]) in the KKS $\gamma_5$ scheme. In particular, the expressions for $R$ in the HV $\gamma_5$ scheme are given in this paper to meet the needs of some specific NLO calculations.

As the HV $\gamma_5$ scheme is the only rigorously proved consistent regime to all orders, our results in this $\gamma_5$ scheme are surely useful and necessary in clarifying the $\gamma_5$ problem in DREG. The only dimensional regularization dependent terms (except scaleless bubbles) in one-loop calculations are rational terms $R$. These Feynman rules are also helpful in eliminating or finding the uncertainties arising from freedoms in DREG. For instance, in NLO QCD corrections to the $W$ boson hadronic decays, the rational terms $R$ in HV and KKS $\gamma_5$ schemes are different (see Eq.(4.6)), being right-handed in HV scheme while left-handed in KKS scheme. These differences are the only differences after including virtual parts, counter terms, and real radiations in these two $\gamma_5$ schemes. Actually, the violation of the anti-commutation relation between $\gamma_5$ and $\gamma_\mu$ in HV spoils the Ward identity, and a finite renormalization is needed in this scheme to obtain the same expressions in HV and KKS (see the details in Ref.[32]).

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