Experimental Implications of Large CP Violation and Final State Interactions in the Search for $D^0 - \bar{D}^0$ Mixing.

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Abstract

We discuss the implications of CP violation as well as final state interaction phases in the experimental search for $D^0 - \bar{D}^0$ mixing. At the present level of sensitivity, these are not yet a significant systematic experimental limitation.

I. INTRODUCTION

As was recently noted by Blaylock, Seiden, and Nir due to final state interaction (FSI) a term proportional to $\Delta M t e^{-\Gamma t}$ may appear in the rate of wrong sign $D$ decays even in the absence of CP violation. Moreover, in some extensions of Standard Model which have large values of both $\Delta M$ and significant CP violation, a similar term may arise. Blaylock et al. have suggested that a value of $\Delta M$ larger than the present experimental limit can be accommodated if one of these previously neglected terms destructively interferes with the other time dependent terms which arise from mixing (proportional to $t^2 e^{-\Gamma t}$) and from doubly Cabibbo suppressed decays (DCSD) (proportional to $e^{-\Gamma t}$). They suggest that this may invalidate the use of existing limits from time dependent mixing studies at fixed target experiments to constrain extensions of the Standard Model.
Below, we give expressions for the time dependence in the general case and then attempt to estimate the maximum size of the terms proportional to $te^{-\Gamma t}$.

II. FORMALISM FOR MIXING

We follow the notation of references [1], [4], [5]. Let the mass eigenstates be $D_S$, $D_L$. Then

$$|D_S> = p|D^0> + q|\bar{D}^0>$$

$$|D_L> = p|D^0> - q|\bar{D}^0>$$

In the limit of no CP violation, $p=q=1/\sqrt{2}$.

Let $\Delta M = M_L - M_S$ and $\Delta \Gamma = \Gamma_L - \Gamma_S$ denote the mass difference and lifetime difference, respectively. Let $A$ denote the amplitude for $<f|H|D^0>$, $B$ the amplitude for $<f|H|\bar{D}^0>$. Let $\lambda = \frac{pA}{qB}$ and $\bar{\lambda} = \frac{qA}{pB}$. The decay rate is then given by

$$\Gamma(D^0(t) \to K^+\pi^-) = \frac{e^{-\Gamma t}}{4} |B|^2 \frac{q}{p} |\lambda|^2 \left\{4|\lambda|^2 + (\Delta M^2 + \frac{\Delta \Gamma^2}{4})t^2 + 2\text{Re}(\lambda)\Delta \Gamma t + 4\text{Im}(\lambda)\Delta Mt\right\}$$

(1)

up to terms of order $t^2$ [1]. The decay rate for the charge conjugate reaction is given by the same expression replacing $\lambda$ with $\bar{\lambda}$, $B$ with $\bar{B}$, and $q/p$ by $p/q$.

$$\Gamma(\bar{D}^0(t) \to K^-\pi^+) = \frac{e^{-\Gamma t}}{4} |\bar{B}|^2 \frac{p}{q} |\bar{\lambda}|^2 \left\{4|\bar{\lambda}|^2 + (\Delta M^2 + \frac{\Delta \Gamma^2}{4})t^2 + 2\text{Re}(\bar{\lambda})\Delta \Gamma t + 4\text{Im}(\bar{\lambda})\Delta Mt\right\}$$

(2)

In the past, it was assumed that the term proportional to $\Delta M \cdot t$ changes sign when averaging over a sample with equal numbers of $D^0$ and $\bar{D}^0$ mesons [6], [7]. This assumption is not correct in general as was noted in Reference [1].

The previous experimental analyses [2], [3], [2] considered the deterioration of the limit in the case when the term proportional to $\Delta \Gamma \cdot t$ interfered destructively with the mixing and
DCSD components. The Standard Model expectation for $\Delta \Gamma$ is many orders of magnitude below the current experimental sensitivity so this interference scenario is very unlikely. In most new physics scenarios which would give $r_{\text{mix}} \sim O(10^{-3})$, $\Delta \Gamma$ is not enhanced whereas values of $\Delta M$ much larger than those expected from the Standard Model are possible. It is also possible to experimentally verify that $\Delta \Gamma$ can be neglected by measuring the $D$ meson lifetime in a CP eigenstate e.g. $D^0 \to K^-K^+$ and comparing to the lifetime in $D^0 \to K^+\pi^+$ [4], [5].

We now consider equations (1), (2) in the following situation. Let

$$\frac{p}{q} = \beta e^{i\phi}$$

$$\frac{A}{B} = \alpha e^{i\delta}$$

and $\alpha^2 = \frac{\Gamma(D^0 \to K^+\pi^-)}{\Gamma(D^0 \to K^-\pi^+)}$. The phase $\phi$ is due to CP violation in the mass matrix. A non-zero value of $\delta$ may arise if the amplitudes $A$ and $B$ have different FSI. Alternatively, if there are complex contributions to one of the amplitudes (e.g. $A$) that are not present in the other (e.g. $B$), this can lead to an overall phase in $\frac{A}{B}$. We have assumed that there is no direct CP violation in the amplitudes (and hence e.g. $\Gamma(D^0 \to K^-\pi^+) = \Gamma(\bar{D}^0 \to K^+\pi^-)$) [8]. In addition, we neglect the small phase in $A/B$ from the CKM matrix, which is approximately $A^2\lambda^4\eta$ in the Wolfenstein parameterization and which lies in the range $(2.3 - 5.3) \times 10^{-4}$.

The decay rate for wrong sign $D^0$ decays to $K^+\pi^-$ is given by

$$\Gamma(D^0(t) \to K^+\pi^-) = \frac{e^{-\Gamma t}}{4}|B|^2 \times$$

$$\frac{1}{\beta^2}(4\alpha^2\beta^2 + (\Delta M^2 + \frac{\Delta \Gamma^2}{4})t^2$$

$$+2\alpha \beta \cos(\phi + \delta)(\Delta \Gamma t) + 4 \sin(\phi + \delta)\alpha \beta \Delta Mt)$$

The corresponding rate for the charge conjugate reaction is obtained by replacing $\phi$ the phase from CP violation with $-\phi$ and by changing $\beta$ to $1/\beta$

$$\Gamma(\bar{D}^0(t) \to K^-\pi^+) = \frac{e^{-\Gamma t}}{4}|\bar{B}|^2 \times$$
\[
\beta^2 \{ 4 \frac{\alpha^2}{\beta^2} + \frac{\Delta M^2 + \Delta \Gamma^2}{4} t^2 \\
+ 2 \frac{\alpha}{\beta} \cos(-\phi + \delta)(\Delta \Gamma t) + 4 \sin(-\phi + \delta) \frac{\alpha}{\beta} \Delta M t \}
\]

In the experimental analyses, the time dependent rate integrated over both types of particles is used:

\[
\Gamma(D^0(t) \to K^+\pi^-) + \Gamma(\bar{D}^0(t) \to K^-\pi^+)
\]

This rate, which will be denoted by \( \Gamma(D^0(t) + \bar{D}^0(t)) \), is given by

\[
\Gamma(D^0(t) + \bar{D}^0(t)) = 2 F(t) \times
\{ 4 \alpha^2 + \frac{1}{2} (\beta^2 + \frac{1}{\beta^2}) (\Delta M^2 + \frac{\Delta \Gamma^2}{4}) t^2 \\
+ \alpha (\beta \cos(-\phi + \delta) + \frac{1}{\beta} \cos(\phi + \delta)) \Delta \Gamma t \\
+ 2 \alpha (\beta \sin(-\phi + \delta) + \frac{1}{\beta} \sin(\phi + \delta)) \Delta M t \}
\]

where \( F(t) = \frac{1}{4} e^{-\Gamma t} |B|^2 \).

### III. EFFECTS OF FSI AND CP VIOLATION

Two scenarios are considered in what follows. First the case of no CP violation but significant final state interactions (FSI) and then the case of both large CP violation and significant final state interaction are examined.

#### A. Effects of FSI

In the first scenario, consider the case of large mixing with \( \Delta M >> \Delta M_{SM} \), the value in the Standard Model. Assume that this does not lead to an enhancement of \( \Delta \Gamma \) i.e. \( \Delta \Gamma_{SM} = \Delta \Gamma << \Delta M \) and allow for non-zero \( \delta \) but no CP violation (\( \phi = 0, \beta = 1 \)). The above equation then reduces to

\[
\Gamma(D^0(t) + \bar{D}^0(t)) = 2 F(t) \{ 4 \alpha^2 + (\Delta M^2 + \frac{\Delta \Gamma^2}{4}) t^2 + 4 \alpha (\sin(\delta)) \Delta M t \}
\]
In order to determine the size of the new term proportional to $\Delta M$ $t$, the values of the phase difference $\delta$ are considered in various models. This will allow an estimate of the additional experimental systematic error that is incurred from ignoring FSI. This phase difference $\delta$ is zero in the limit of exact SU(3) symmetry. The values of $\delta$ from various models are given in Table I. Large values of the phase $\delta$ occur when SU(3) breaking is largest. We use the experimental result from CLEO II for $D^0 \rightarrow K^-\pi^+$ ($\alpha^2 = 0.0077 \pm 0.0025 \pm 0.0025$) and assume that it is entirely due to DCSD. This is found numerically to give the most conservative upper limit on the size of the interference effect.

In general, the amplitudes for the $D^0 \rightarrow K^-\pi^+$ and $D^0 \rightarrow K^+\pi^-$ can be written as:

$$A(D^0 \rightarrow K^-\pi^+) = e^{i\delta_3}[(A_1 + C)e^{i(\delta_1 - \delta_3)} + A_3]$$ (5)

$$A(D^0 \rightarrow K^+\pi^-) = -\theta^2 e^{i\delta_3}[(\bar{A}_1 + \bar{C})e^{i(\delta_1 - \delta_3)} + \bar{A}_3]$$ (6)

where $A_1$ and $A_3$ are the quark decay contributions into $I = 1/2$ and $I = 3/2$ final states respectively. $C$ is the W-exchange contribution and $\delta_1$ and $\delta_3$ are the FSI phases. $\bar{A}_i, \bar{C}$ are the corresponding DCSD amplitudes after the CKM factor $-\theta^2$ has been factored out. The phase shifts in a given isospin eigenstate for particles and antiparticles are identical by CPT invariance (which we assume as stated explicitly).

The phase $\delta$ vanishes if two conditions are satisfied: (i) $\delta_1 - \delta_3 = \tilde{\delta}_1 - \tilde{\delta}_3$ and (ii) $A_3/A_1 = \bar{A}_3/\bar{A}_1$. The first condition follows from CPT invariance and the second is satisfied if SU(3) symmetry holds. Hence if SU(3) is an approximate symmetry, the phase $\delta$ should be small. The models used have been tuned to reproduce the observed magnitude of SU(3) breaking in $D$ decays. To obtain more information, we turn to the detailed model fits.

**B. Details of the Models**

In the model of Chau and Cheng,

$$A_1 \approx 0.82, \quad A_3 \approx 0.16, \quad C \approx -0.13$$ (7)
\[ \tilde{A}_1 \approx 1.14, \quad \tilde{A}_3 \approx 0.33, \quad \tilde{C} \approx C \]  \hfill (8)

and

\[ \delta_1 - \delta_3 \approx 90^0, \quad \delta_3 \approx 0. \]  \hfill (9)

Then

\[ A(D^0 \to K^- \pi^+) \approx (0.72)e^{i \cdot 76^0} \]  \hfill (10)

\[ A(D^0 \to K^+ \pi^-) \approx -\theta_c^2(1.01)e^{i \cdot 72^0} \]  \hfill (11)

This yields a phase difference between the two decay modes of \( \delta = 4^0 \). If the W-exchange contribution \( C \) is omitted, the phase difference becomes \( \delta = 5^0 \).

In the model of Buccella et al., one has

\[ A_1 \approx 4.35, \quad A_3 \approx -2.3, \quad C \approx -0.5 \]  \hfill (12)

\[ \tilde{A}_1 \approx 5.2, \quad \tilde{A}_3 \approx -2.3, \quad \tilde{C} \approx -C \]  \hfill (13)

and

\[ \delta_1 - \delta_3 \approx 25^0, \quad \delta_3 \approx 0. \]  \hfill (14)

Then

\[ A(D^0 \to K^- \pi^+) \approx (2)e^{i \cdot 54.3^0} \]  \hfill (15)

\[ A(D^0 \to K^+ \pi^-) \approx -\theta_c^2(3.7)e^{i \cdot 41.2^0} \]  \hfill (16)

leading to a phase difference of \( \delta = 13^0 \). Omitting the W-exchange term gives a slightly smaller value of \( 6^0 \). It should be noted that a \( \delta_1 = 25^0 \) relates to \( \delta_R \) for the \( I = 1/2 \) \( 0^+ \) resonance in the \( K\pi \) channel by
\[
\tan \delta_R = \frac{\Gamma}{2\Delta} = \frac{B \sin \delta_1}{B \cos \delta_1 + (1 - B)}
\]

where \( B = BR(0^+ \rightarrow K\pi) \approx 0.50, \Gamma \approx 200 \text{ MeV}, \Delta = M_R - M_D \approx 70 \text{ MeV} \) and \( \delta_R = (55 - 65)^0 \).

The models discussed above predict

\[
\frac{BR(D^0 \rightarrow K^+\pi^-)}{BR(D^0 \rightarrow K^-\pi^+)} = (2.3 - 3.4) \tan^4 \theta_c
\]

which is compatible with the CLEO II measurement. There are also other models for \( D \) decays in which a value for the phase difference \( \delta \) can be extracted [14]. Since it is difficult to assign errors to these predictions, we regard \( 0^0 - 13^0 \) as a reasonable range for \( \delta \). In order to explore the range of \( \delta \) in the models, we have calculated the value of \( \delta \) omitting the W-exchange term. This corresponds to a dramatic change in the parameters of the models.

| TABLE I. Values of \( \delta \) in various phenomenological models of \( D \) meson decay. |
|-----------------|-----------------|
| Exact SU(3) limit [18] | \( 0^0 \) |
| Chau and Cheng [12] | \( 4^0 \) |
| Chau and Cheng (no W-exchange) [12] | \( 5^0 \) |
| Buccella et al. | \( 13^0 \) |
| Buccella et al. (no W-exchange) | \( 6^0 \) |

C. Summary of the Interference Effect from FSI

To summarize, the phenomenological models which have been tuned to agree with the observed branching fractions and the fits to the \( D \) meson data give \( \delta \) in the range of \( 5^0 - 13^0 \).

To evaluate the possible experimental consequences, consider the case of maximal destructive interference (\( \phi = 0, \beta = 1 \)), with \( \delta = 13^0 \). We allow a one standard deviation variation on \( R_{DCSD} = \alpha^2 \) from the CLEO II measurement in order to obtain an upper limit on the effect.
of the interference term. We set \( r_{\text{mix}} \), the ratio of integrated rates for mixed events relative to unmixsed events, to the E691 upper bound \[13\]. The contributions of the mixing term, the DCSD term, and the term proportional to \( \Delta M t \) are shown in Figure 1. These time dependent searches are most sensitive to excess events from mixing for \( t > 0.22 \text{ ps} = \frac{\tau_{D^0}}{2} \), where the combinatorial backgrounds are manageable and where the mixing term is expected to peak. In addition, there is no loss in efficiency for the mixing component when this cut is imposed. An upper limit of \( t < 4.0 \text{ ps} \) is also imposed. The change in the observed event yield for various values of \( R_{\text{DCSD}} \) and maximal destructive interference are given in Table II. These were calculated for the scenario with maximal destructive interference and \( \delta = 13^\circ \). We also give the change in the observed event yield for \( t > 2\tau_{D^0} \) (this is the region where mixing peaks and the experiments are most sensitive) in Table III. This change is at most 10-15\% and is well within the experimental systematic error assigned by the E691 and E791 experiments to their limits.

**TABLE II.** The change in wrong sign event yield for \( t > 0.22 \text{ ps} \) with maximal destructive interference, \( r_{\text{mix}} = 0.37\% \), and \( \delta = 13^\circ \).

| \( R_{\text{DCSD}} \) | \( \Delta \text{ Yield (\%)} \) |
|------------------------|-----------------|
| 0.0052 | 10\% |
| 0.0077 | 8\% |
| 0.0102 | 1\% |

**TABLE III.** The change in wrong sign event yield for \( t > 0.88 \text{ ps} (2\tau_{D^0}) \) with maximal destructive interference, \( r_{\text{mix}} = 0.37\% \), and \( \delta = 13^\circ \).

| \( R_{\text{DCSD}} \) | \( \Delta \text{ Yield (\%)} \) |
|------------------------|-----------------|
| 0.0052 | 12\% |
| 0.0077 | 10\% |
| 0.0102 | 9\% |
IV. EFFECTS OF CP VIOLATION

Now consider the contribution of CP violation. Let $\beta = 1 - \epsilon$ and $\frac{1}{\beta} = 1 + \epsilon$. We assume $\epsilon$ is small compared to 1 and retain only terms linear in $\epsilon$; this is justified in the SM and even more so when $\Delta M$ is enhanced and $\Delta \Gamma/\Delta M << 1$. We allow the phase $\phi$ to be arbitrary. With these definitions and $\Delta \Gamma << \Delta M$, the expression for $\Gamma(D^0(t) + \bar{D}^0(t))$ now becomes:

$$\Gamma(D^0(t) + \bar{D}^0(t)) = 2F(t) \times \left\{ 4\alpha^2 + (\Delta M^2 + \frac{\Delta \Gamma^2}{4})t^2 \right\}$$
$$+ \alpha(\cos(-\phi + \delta) + \cos(\phi + \delta))\Delta \Gamma t + \alpha\epsilon(\cos(\phi + \delta) - \cos(-\phi + \delta))\Delta \Gamma t$$
$$+ 2\alpha(\sin(-\phi + \delta) + \sin(\phi + \delta))\Delta M t + 2\alpha\epsilon(\sin(-\phi + \delta) - \sin(\phi + \delta))\Delta M t$$

The quantity $\epsilon$ is assumed to be small as in Ref [1], however, the CP violating phase $\phi$ can be large as is the case for certain extensions of the Standard Model. The quantity $\epsilon$ for $D$ mixing is given by [17]

$$\epsilon \approx \frac{-2\text{ Im}(\frac{M_{12}^*\Gamma_{12}}{2})}{\frac{1}{2}\Delta M^2 + \frac{\Delta \Gamma^2}{4}}$$

in the Standard Model and is already small ($\epsilon < O(2\%)$) [17].

In new physics scenarios with $\Delta \Gamma_{SM} = \Delta \Gamma << \Delta M$,

$$\tan \phi \approx \text{ Im}(\frac{M_{12}}{\Delta M})$$

For non standard models with $\text{ Im}(M_{12})/\Delta M$ of order unity, $\tan \phi$ may be large (O(1)). By contrast,

$$\epsilon \approx 2(\frac{\Delta \Gamma}{\Delta M})\text{ Im}(M_{12}) \approx 2(\frac{\Delta \Gamma}{\Delta M}) << 1$$

The crucial point is that $\epsilon$ is proportional to $1/\Delta M$ and is highly suppressed if $\tan \phi$ is of order unity and $\Delta M$ is enhanced. It is important to note that while $\tan(\phi)$ can be much
larger than the Standard Model expectation $\epsilon$ will be even smaller than the value in the Standard Model for new physics scenarios in which $\Delta M$ is enhanced.

The total wrong sign rate can then be reduced to

$$\Gamma(D^0(t) + \bar{D}^0(t)) = 2F(t) \times$$

$$\{4\alpha^2 + (\Delta M^2 + \frac{\Delta \Gamma^2}{4})t^2$$

$$+2\alpha(\cos(\phi) \cos(\delta))\Delta \Gamma \ t - 2\alpha\epsilon(\sin(\phi) \sin(\delta))\Delta \Gamma \ t$$

$$+4\alpha\epsilon(\cos(\delta) \sin(\phi))\Delta M \ t + 4\alpha(\sin(\delta) \cos(\phi))\Delta M \ t\}$$

With $\epsilon$ as given above and $\Delta \Gamma << \Delta M$, the expression for $\Gamma(D^0(t) + \bar{D}^0(t))$ becomes

$$\Gamma(D^0(t) + \bar{D}^0(t)) = 2F(t) \times$$

$$\{4\alpha^2 + (\Delta M^2)t^2$$

$$+4\alpha(\sin(\delta) \cos(\phi))\Delta M \ t\}$$

Hence, the term due to CP violation is too small to be observable when $\Delta M$ and $\text{Im}(M_{12})$ are enhanced.

As experimental sensitivity improves and become sensitive to mixing at the level $r_{\text{mix}} < 10^{-4}$, it is possible that better sensitivity to $D^0 - \bar{D}^0$ mixing can be achieved by fitting the time distribution of $\Gamma(D^0 \to K^+\pi^-) - \Gamma(D^0 \to K^-\pi^+)$. This rate, which will henceforth be denoted $\Gamma(D^0(t) - \bar{D}^0(t))$, is given by

$$\Gamma(D^0(t) - \bar{D}^0(t)) = 2F(t) \times$$

$$\{2\alpha\epsilon(\cos(\phi) \cos(\delta))\Delta \Gamma \ t - 2\alpha(\sin(\phi) \sin(\delta))\Delta \Gamma \ t$$

$$+4\alpha(\cos(\delta) \sin(\phi))\Delta M \ t + 4\alpha\epsilon(\sin(\delta) \cos(\phi))\Delta M \ t\}$$

In the limit that $\Delta \Gamma << \Delta M$ and $\phi$ is large, this reduces to

$$\Gamma(D^0(t) - \bar{D}^0(t)) \approx 2F(t) \{4\alpha(\cos(\delta) \sin(\phi))\Delta M \ t + 4\alpha\epsilon(\sin(\delta) \sin(\phi))\Delta M \ t\}$$

or neglecting the small term proportional to $\epsilon \sin(\delta)$,
\[ \Gamma(D^0(t) - \bar{D}^0(t)) \approx 2F(t)[4\alpha(\cos(\delta) \sin(\phi))\Delta M \, t] \] (26)

Note that in this case, the long lived tail of DCSD does not contribute to the signal. In addition, as noted by Wolfenstein [18], for small values of \(\Delta M\), the term proportional to \(\Delta M \, t\) will be larger than the term in \(\Gamma(D^0(t) + \bar{D}^0(t))\) which is proportional to \((\Delta M \, t)^2\). This feature is illustrated in Figs. 2 (a), 2 (b).

V. CONCLUSIONS

The formalism presented here must be modified for the case of multibody modes such as \(D^0 \rightarrow K^+\pi^0\) or \(D^0 \rightarrow K^+\pi^-\pi^+\pi^-\). For these other modes, an additional complication is that the value of the final state phase difference, \(\delta\), may be different from the value in the case of \(D^0 \rightarrow K^+\pi^- / \bar{D}^0 \rightarrow K^+\pi^-\) and is not guaranteed to be small. It should also be remembered that limits on \(D^0 - \bar{D}^0\) mixing from studies of semileptonic decays do not have the complications from DCSD and other hadronic effects discussed here.

At the present level of sensitivity and with reasonable (though model dependent) values for the phase difference \(\delta\), the \(\Delta M \, t\) term which arises from FSI does not dramatically change the observed event yield for experiments which study the time dependence of mixing and is not yet a significant systematic experimental limitation. We suggest that future experiments determine systematic errors on their limits by using an upper limit on the phase difference \(\delta\).

The contribution from the corresponding term proportional to \(\Delta M \, t\) due to CP violation which arises in extensions of Standard Model is highly suppressed. This term is not observable at the present level of experimental sensitivity. However, as emphasized by Liu [4] and by Wolfenstein [18], this term should not be neglected as experimental examination of the \(D^0(t) - \bar{D}^0(t)\) distribution may allow more sensitive searches for \(D^0 - \bar{D}^0\) mixing in the future if the CP violating phase is large.

This work was supported in part by the United States Department of Energy under grant
DE-FG 03-94ER40833 and by Tokkuri Tei. We thank G. Burdman, E. Golowich, J. Hewett, D. Kaplan, T. Liu, Y. Nir, and M. Witherell for useful and enjoyable discussions.
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[15] The ratio of integrated rates for mixed and unmixed events is

$$\frac{(\Delta M/\Gamma)^2 + (\Delta \Gamma/\Gamma)^2/4}{2 + (\Delta M/\Gamma)^2}.$$  

In the limit that the width difference is small, $r_{mix} = 1/2(\Delta M/\Gamma)^2$.

[16] For small values, $\epsilon$ is related to the usual parameter $\epsilon_D$ by $\epsilon \approx -2\text{Re}(\epsilon_D)$.

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FIG. 1. The case of maximal destructive interference. The time dependence of DCSD (open circles), mixing (dotted), interference (dash-dotted) mixing and DCSD without interference (solid points), mixing and DCSD with interference (dashed) are shown. For this plot, the mixing rate is taken to be $\Delta M/\Gamma = 0.086$ ($r_{mix} = 0.37\%$) and the DCSD rate is taken to be the central value of the branching fraction for $D^0 \rightarrow K^+\pi^-$ determined by CLEO II (i.e. $R_{DCSD} = 0.0077$). Other scenarios are discussed in the text.
FIG. 2. The time dependence for wrong sign events for the case of small mixing ($\Delta M/\Gamma = 0.02$) for (a) $\Gamma(D^0(t) + \bar{D}^0(t))$, where the dotted component is the FSI interference term and the dashed component is the usual mixing term (both scaled up by a factor of two in order to be visible), and for (b) $\Gamma(D^0(t) - \bar{D}^0(t))$. Note that in (b) there is no background to mixing from DCSD.