On the Energy Measurement of Jets in High-Energy Physics Experiments

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Abstract. As the energies at which the elementary structure of matter is studied increased, the emphasis in scattering experiments has shifted from detecting individual hadrons to fragmenting quarks and gluons, which manifest themselves as particle jets. We investigate and quantify some systematic effects that affect the precision with which the properties of the fragmenting constituents can be determined. We concentrate on the calorimeters that are used to measure the energies, and in particular how the non-compensating nature of a calorimeter affects the energy measurement of different types of quarks and gluons.

1. Introduction
In modern high-energy physics experiments, calorimeters detect the jets which carry primary information about the collisions, and measure their properties. But the calorimeter has a non-trivial problem, its response is different for the hadronic and non-hadronic jet constituents, and energy dependent. At this conference, we present results on how compensating and non-compensating calorimeters respond to different types of quark-jets, and how the charged particle multiplicity and jet energy affect the response of non-compensating calorimeters.

2. Calorimeter Properties
High-energy particles experience the strong interaction with the nuclei in the absorbing calorimeter medium. Typically, $\pi^0$s, $\eta$s, $\pi^\pm$s, kaons, protons and neutrons are produced in such nuclear reactions. Of these products, $\pi^0$s and $\eta$s decay into $\gamma$s and develop em showers. The fraction of the total shower energy carried by this electromagnetic component is defined as the em shower fraction, $f_{em}$. This em shower fraction fluctuates strongly from event-to-event. Figure 1a shows that $<f_{em}>$ increases with the pion energy and depends on the type of absorber material. This energy dependence implies that a non-compensating calorimeter is non-linear for hadron detection. Figure 1b shows the distribution of $f_{em}$ for showers produced by 150 GeV $\pi^-$ in a lead-based hadron calorimeter. The event-to-event fluctuations are large and non-Gaussian.

In contrast, charged pions, kaons, protons, neutrons, and the supplied energy to break up nuclei are classified as the non-electromagnetic component. This binding energy is called “invisible” since it doesn’t contribute to the calorimeter signals. Quantum number conservation leads to an important leading-particle effect in showers induced by baryons and kaons. Figure 2 shows the signal distributions produced by 300 GeV pions and protons. In the pion signal distribution, the high-side tail is populated by events in which a large fraction of the energy
carried by the incoming pion is transferred to a $\pi^0$ in the first nuclear interaction. In proton-induced showers, the leading particle in the first nuclear interaction is always a baryon, $\pi^0$s can only be produced in later stages of the fragmentation process. A similar effect is expected for kaons due to conservation of strangeness. As a result, pions generate larger calorimeter signals than protons (or kaons) of the same energy, and the signal distribution of pions is broader and asymmetric, on average (Figure 2a).

Figure 2. Signal distributions produced by 300 GeV pions and protons in the CMS HF quartz-fiber calorimeter [4].

Figure 3. Energy spectra of the $Z^0$ bosons produced in $pp$ collisions at 8 TeV.

3. The Calorimeter Response

3.1. Jet Simulations

Pythia 8.162 [9] was used for the generation of matrix elements, fragmentation, hadronization and decay. Hadronically decaying $Z^0$ bosons were produced with initial-, final-state parton showering and multi-parton interactions at LEP I ($\sqrt{s} = 91$ GeV) [5], at the Tevatron ($\sqrt{s} = 1.96$ GeV).
Figure 4. Distributions of the fractions of the $Z^0$ energy carried by $\pi^\pm$, $\pi^0$, kaons and $\nu(\bar{\nu})$ in $Z \to u\bar{u}$ processes.

Figure 5. Distributions of the fractions of the $Z^0$ energy carried by $\pi^\pm$, $\pi^0$, kaons and $\nu(\bar{\nu})$ in $Z \to b\bar{b}$ processes.

TeV) [6] and at the LHC ($\sqrt{s} = 8$ TeV) [7]. The purpose of our study is to investigate the parton dependence of the calorimeter response, so we didn’t reconstruct quark and anti-quark jets separately, and considered all final-state particles coming from a $Z^0$ boson. The final-state particles were grouped into 8 categories: (1) $\pi^\pm$s, (2) $\pi^0$s, (3) kaons, (4) baryons, (5) $\gamma$s from sources except $\pi^0$, (6) $e^\pm$, (7) $\mu^\pm$, (8) $\nu$, $\bar{\nu}$. Figure 3 shows that the average energy of $Z^0$ bosons produced at the LHC at $\sqrt{s} = 8$ TeV was 415 GeV. Figures 4 and 5 are simulation results for $Z \to u\bar{u}$ and $Z \to b\bar{b}$ produced in $pp$ collisions at $\sqrt{s} = 8$ TeV, respectively. These are the distributions of the fractions of the $Z^0$ boson energy carried by $\pi^\pm$s, $\pi^0$s, kaons and $\nu(\bar{\nu})$. In the grey boxes, $< n >$ is the average multiplicity and $< E >$ the average fraction of the $Z^0$ boson energy carried by the different jet fragments. The main difference between $Z^0 \to u\bar{u}$ and $Z^0 \to b\bar{b}$ concerns the average energy carried by kaons and neutrinos. The semileptonic decay of $b$-quarks and the strange particles produced in the decay chain of the heavy quarks lead to a substantial increase of kaon and neutrino production.

3.2. The Hadronic Calorimeter Response

The calorimeter response to hadrons, defined as the average calorimeter signal per unit deposited energy and normalized to 1.0 for electrons, can be written as the sum of the em and non-em shower components:

$$ R = < f_{em} > + (1 - < f_{em} >) \frac{h}{e} $$

(1)

The average em shower fraction is described with Grooms’s parameterization (Eq. 2), where $E_0$ is the average energy to produce a pion in the shower development and $k$ is a measure of the average multiplicity in the nuclear reaction. We selected 0.7 GeV and 0.82 for $E_0$ and $k$, respectively [10].

$$ < f_{em}(E) > = 1 - \left[ \frac{E}{E_0} \right]^{(k-1)} $$

(2)

Figure 6 shows the response of the CMS calorimeter system to $\pi^\pm$, $K^\pm$, $p$ and $\bar{p}$ as a function of the energy of the incoming particles [8]. It shows that the hadronic response depends on the
Figure 6. The response of the CMS calorimeter system to different types of particles as a function of energy [8].

energy of the incoming particles and that the response to kaons and protons is smaller than that to pions. The dashed line is the theoretical hadronic response for an $e/h = 2.0$ calorimeter and $k$ is 0.82 (Eqs. 1,2), and agrees well with the $\pi^+$ data. Based on these experimental data, we assume that the response to kaons and baryons is reduced by 20 % compared to that for charged pions.

We determined the response of a calorimeter to a given type of jet as follows. For example, we generated a $Z^0$ boson at the Tevatron and made it decay into an $s\bar{s}$ pair. The average energies of the final state particles were found to be 4.4 GeV ($\pi^\pm$), 4.2 GeV ($\pi^0$), 12.9 GeV (K), 8.7 GeV (baryons), 3.3 GeV (radiative energy $\gamma$s from sources other than $\pi^0$ decay), 2.4 GeV ($e^\pm$), 3.4 GeV ($\mu^\pm$) and 3.6 GeV ($\nu$, $\bar{\nu}$). Next, the calorimeter responses to these fragments were calculated. The final-state particles, $\pi^0$s, $e^\pm$ and $\gamma$s from sources other than $\pi^0$ decay, have a calorimeter response 1. The response to charged pions, kaons and baryons was calculated with Eq. 1 and reduced by 20% in the case of baryons and kaons. In this way, the individual responses of the $e/h = 2.0$ calorimeter to charged pions, kaons and baryons were found to be 0.64, 0.66 and 0.65, respectively. (Anti)neutrinos escape the calorimeter (response 0). We assumed that muons deposit on average 2 GeV in a typical calorimeter. The calorimeter response to muons was in this case thus $2/3.4 = 0.59$. The overall response of this $e/h = 2.0$ calorimeter to $Z^0$s produced at the Tevatron and decaying into $s\bar{s}$ pairs was found by considering the relative contribution of all final state particles. We found $0.64 \times 0.40 \times (\pi^\pm) + 1 \times 0.21 \times (\pi^0) + 0.66 \times 0.26 \times (\text{kaons}) + 0.65 \times 0.11 \times (\text{baryons}) + 1 \times 0.02 \times (\gamma$s from sources other than $\pi^0$ decay) + 1 \times 0.0033 \times (e^\pm) + 0.59 \times 0.00018 \times (\mu^\pm) + 0 \times (\nu, \bar{\nu}) = 0.73$. This method was applied to all hadronic $Z^0$ decay modes at LEP I, the Tevatron and the LHC. In addition, a subset of $Z^0$ bosons with an energy between 0.8 and 1.2 TeV produced in $pp$ collisions at 8 TeV were selected for the highest energy point in this study. We investigated the responses for $e/h = 2$, $e/h = 1.6$ and $e/h = 1$ calorimeter systems.
3.3. Parton dependence

The parton dependence of the response was studied for all hadronic Z\(^0\) decay modes for calorimeters with \(e/h = 1\), \(e/h = 1.6\) and \(e/h = 2.0\), for various Z\(^0\) boson energies. Figure 7 shows the response of a calorimeter with \(e/h = 2.0\) to all hadronic Z decay modes as a function of the Z boson energy.

![Figure 7](image)

**Figure 7.** The response of the non-compensation calorimeter with \(e/h = 2\) to all hadronic Z decay modes as a function of the Z boson energy.

![Figure 8](image)

**Figure 8.** The response of the non-compensating calorimeter with \(e/h = 2\) to jets as a function of jet energy.

The response increases as a function of energy because of the increased em component. The response to \(s\) jets is smaller than that to \(u, d\) jets by about 3% because kaons in \(s\) jets carry a larger fraction of the shower energy than those in \(u\) and \(d\) jets, and kaon showers have a reduced \(< f_{em} >\) value. Also, the response to \(c, b\) jets is smaller than that to light quark jets due to the combination of strangeness conservation and \(\mu, \nu\) leakage. All response curves in Figure 8 were parameterized as follows:

\[
R_p = 1 - \left[ \frac{E}{p_0} \right]^{(p_1 - 1)}
\]  

(3)

The coefficients \(p_0\) and \(p_1\) for different types of quark jets and different calorimeters are listed in Table 1. Figure 9 shows the \(e/h\) dependence of the calorimeter response to different types of quark jets. The response of the compensating calorimeter to \(s\) jets is same as that to \(u, d\) jets because the calorimeter response to kaons is 1, and the different kaon contents for \(u, d\) and \(s\) jets don’t affect the calorimeter response. The response to \(c, b\) jets is smaller because of semi-leptonic decays, in which particles escape detections. For the non-compensating calorimeters, the response to \(s\) jets is smaller than that to \(u, d\) jets because of the reduced em energy fraction of kaon showers. In addition, for \(c\) and \(b\) jets, a combination of the kaon content and semi-leptonic decay leads to a smaller calorimeter response than for the light quark jets.
Table 1. The values of the coefficients $p_0$ and $p_1$ that parameterize the calorimeter response to different types of quark jets as a function of jet energy for the calorimeters with $e/h = 1.6$ and $2.0$ (Eq. 3).

\[
\begin{array}{cccccc}
\text{Parameter} & u & d & s & c & b \\
p_0 \text{ (GeV)} & 0.0033 & 0.0031 & 0.0031 & 0.0004 & 0.00004 \\
p_1 & 0.836 & 0.837 & 0.845 & 0.877 & 0.910 \\
\end{array}
\]

Table 1. The values of the coefficients $p_0$ and $p_1$ that parameterize the calorimeter response to different types of quark jets as a function of jet energy for the calorimeters with $e/h = 1.6$ and $2.0$ (Eq. 3).

\[
\begin{array}{cccccc}
\text{Parameter} & u & d & s & c & b \\
p_0 \text{ (GeV)} & 0.0168 & 0.0185 & 0.0178 & 0.0056 & 0.00115 \\
p_1 & 0.838 & 0.836 & 0.846 & 0.869 & 0.898 \\
\end{array}
\]

Figure 9. The responses of the three $e/h$ values of the calorimeters to the different types of quark jets.

Figure 10. The response of a calorimeter with $e/h = 2.0$ to jets as a function of the charged-particle multiplicity of jets.

3.4. Multiplicity dependence

To investigate the charged particle multiplicity dependence of the calorimeter response, $u$, $d$ quark jets in a calorimeter with a $e/h = 2.0$ calorimeter were used. The effects were studied for $Z^0$s produced either at LEP I or at the LHC ($\sqrt{s} = 8$ TeV). In the latter case, a sample of $Z^0$s with energies between 300 and 400 GeV was selected. Figure 10 shows the response to these jets as a function of the charged-particle multiplicity. The response to LHC jets is larger than to 45 GeV jets. Also, high multiplicity jets have smaller response than low multiplicity jets because of the lower average energy of the charged fragments, and the corresponding smaller em shower fraction. The effects of non-compensation on the calorimeter response thus depend on both the charged particle multiplicity and the jet energy.

4. Conclusions

The calorimeter response to jets depends on the type of fragmenting parton, the final-state multiplicity, the jet energy and the $e/h$ value of the calorimeter. The effects of jet energy and final-state multiplicity are eliminated if $e/h = 1$. If $e/h \neq 1$, then the response increases with
the jet energy and decreases with increasing final state multiplicity. The response to gluon jets is also reduced compared to u and d jets of the same energy since gluon jets have a larger final state multiplicity than quark jets. If $c/h \neq 1$, then the response depends on the relative contribution of baryons and strange mesons to the final state, because the response to these jet fragments is reduced w.r.t. the response to charged pions. The response to c and b quark jets is reduced because of the semi-leptonic decay processes.

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