Outage Performance of AF-based Time Division Broadcasting Protocol in the Presence of Co-channel Interference

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Abstract—In this paper, we investigate the outage performance of time division broadcasting (TDBC) protocol in independent but non-identical Rayleigh flat-fading channels, where all nodes are interfered by a finite number of co-channel interferers. We assume that the relay operates in the amplified-and-forward mode. A tight lower bound as well as the asymptotic expression of the outage probability is obtained in closed-form. Through both theoretic analyses and simulation results, we show that the achievable diversity of TDBC protocol is zero in the interference-limited scenario. Moreover, we study the impacts of interference power, number of interferers and relay placement on the outage probability. Finally, the correctness of our analytic results is validated via computer simulations.

I. INTRODUCTION

Recently, two-way relaying (TWR) or bi-directional relaying has emerged as a powerful technique to improve the spectral efficiency of wireless network [1]. A number of relaying protocols have been proposed, such as amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF). For AF relaying, two popular TWR protocols are analog network coding (ANC) [2], which requires two time slots to complete the information exchange between two terminal nodes, and TDBC [3], which needs three time slots. However, TDBC protocol can use the direct link between two terminals even under a half-duplex constraint [3][4], thus can provide higher diversity gain.

Several previous works have investigated the TWR network using TDBC for Rayleigh fading channels, in which relay and terminals are only perturbed by addtive white Gaussian noise (AWGN) [5][7]. The outage performance of AF-based TDBC protocol in Rayleigh fading channels was analyzed in [5][6] and the diversity-multiplexing tradeoff (DMT) was also obtained. In [7], the authors considered relay selection scheme for TDBC protocol and analyzed the outage performance with optimal relay selection. However, signals of terminals (or relay) are often corrupted by co-channel interference (CCI) from other sources that share the same frequency resources in wireless networks [8]. Moreover, for the wireless scenarios with dense frequency reuse, co-channel interference may dominate the AWGN. Therefore, it is necessary to take the effect of CCI into serious consideration in the analysis and design of the practical TDBC protocol. In [9], the performance of ANC protocol corrupted by equal-power interferers was studied, where closed-form expressions of the average bit error rate and outage probability were presented. Outage probability of the cooperative relaying using DF protocol with CCI has been analyzed in [10]. However, for AF-based TDBC protocol, the effect of CCI is still unknown.

In this work, we study the AF-based TDBC protocol where all the nodes (terminals and relay) are interfered by a finite number of co-channel interferers in independent but non-identical Rayleigh flat-fading channels. The system model is described in the next section. In section III, the CDF of the upper bounded SINR is analyzed. Based on the results, a lower bound as well as asymptotic expression of outage probability is obtained. In section IV, the effects of interference power, number of interferers and relay placement on the outage probability are studied.

II. SYSTEM MODEL

We study the TWR network which consists of two terminals and a relay node, as shown in Fig. 1, in which terminal \( T_1 \) and terminal \( T_2 \) exchange statistically independent messages with the help of a relay \( R \). Each node is equipped with a single antenna and operates in the half-duplex mode, that is, a node cannot transmit and receive simultaneously.
The TDBC protocol can be achieved within three time slots, that is, terminal $T_1$ transmits during the first time slot, while $T_2$ and $R$ listen. In time slot 2, $T_2$ transmits while $T_1$ and $R$ listen. It is assumed that both terminals and the relay are interfered by a finite number of co-channel interferers. Denoting $L_R$, $L_1$ and $L_2$ as the total numbers of interferers that affect node $R$, $T_1$ and $T_2$, respectively, the received signals at the relay and $T_i$ during the first two time slots are expressed as

$$y_{Ri} = E_R h_i S_i + E_R \sum_{k=1}^{L_R} d_{R,k} I_{R,k} + n_{Ri},$$

$$y_{ji} = E_j h_0 S_j + E_R \sum_{k=1}^{L_i} d_{T,k} I_{T,k} + n_{ji},$$

respectively, where $i, j \in \{1, 2\}$ and $i \neq j$. $E_i$ and $E_{ji}$ denote the transmit powers of $T_i$ and interferers, respectively. $h_i$, $h_0$ and $h_t$ represent the channel coefficients belonging to the links $T_1 \rightarrow R$, $T_2 \rightarrow R$ and $T_1 \rightarrow T_2$, respectively. The channel reciprocity is assumed. Moreover, $d_{N,k}$ indicates the channel coefficient of link between node $N$ and the $k$th interferer that affects $N$, where $N \in \{T_1, T_2, R\}$. All links are assumed to be independent but non-identical Rayleigh flat-fading. $S_i$ denotes the unit-power symbol transmitted by $T_i$. $I_{N,k}^m$ indicates the unit-power interference signal of $k$th interferer that affects node $N$ during the $m$th time slot, where $N \in \{T_1, T_2, R\}$ and $m \in \{1, 2, 3\}$. Finally, $n_{Ri}/n_{ji}$ denote the AWGN and $n_{Ri}/n_{ji}$ is normally distributed.

In time slot 3, $R$ transmits the combined signal to terminals $T_1$ and $T_2$. The combined signal to be transmitted by $R$ can be written as $S_R = A_1 y_{1R} + A_2 y_{2R}$. $A_1$ and $A_2$ denote combining coefficients which can be determined as follows:

$$A_i = \frac{\omega_{ji}}{\omega_1 E_1 |h_1|^2 + \omega_2 E_2 |h_2|^2 + E_1 \sum_{k=1}^{L_R} |d_{R,k}|^2 + 1},$$

where $i \in \{1, 2\}$, $\omega_{ji} \in [0, 1]$ is the power allocation number and $\omega_1 + \omega_2 = 1$. Then the received signal at terminal $T_i$ during the third time slot can be written as

$$y_{Ri} = \sqrt{E_i} h_i S_R + \sqrt{E_i} \sum_{k=1}^{L_i} d_{T,k} I_{T,k} + n_{Ri}$$

$$= \sqrt{E_i} A_1 h_i h_1 S_1 + \sqrt{E_i} A_2 h_2 S_2 + \sqrt{E_i} \sum_{k=1}^{L_i} d_{T,k} I_{T,k}$$

$$+ \sqrt{E_i} A_1 h_i h_1 S_1 + \sqrt{E_i} A_2 h_2 S_2 + \sqrt{E_i} \sum_{k=1}^{L_i} d_{R,k} I_{R,k}$$

$$+ \sqrt{E_i} A_1 h_i h_1 S_1 + \sqrt{E_i} A_2 h_2 S_2 + \sqrt{E_i} \sum_{k=1}^{L_i} d_{R,k} I_{R,k} + n_{Ri},$$

where $n_{Ri} \sim \mathcal{CN}(0, 1)$ is the AWGN and $E_i$ is the transmit power of $R$. In the following, we assume equal power allocation$^2$ between $T_1$, $T_2$ and $R$, i.e., $E_1 = E_2 = E_3 = E$. Since $T_i$ knows its own transmitted symbols, it can cancel the self-interference component in $y_{Ri}$. Therefore, after performing maximal-ratio combining, the instantaneous SINR at terminal $T_i$ can be expressed as in (4), where $i, j \in \{1, 2\}$ and $i \neq j$. By substituting the expressions of $A_1$ and $A_2$ into (4) and performing some manipulations, $\gamma_{T_i}$ can be rewritten as

$$\gamma_{T_i} \approx \gamma_{T_i,D} + \frac{\omega_{ji} |d_{R,k}|^2}{\gamma_{T_i,1} + \gamma_{T_i,2}},$$

where $\gamma_{T_i,1} = \frac{E_i |h_1|^2}{E_i \sum_{k=1}^{L_i} |d_{R,k}|^2 + \omega_i |d_{R,k}|^2 + \omega_2 E_i \sum_{k=1}^{L_i} |d_{T,k}|^2 + \omega_1 + 1}$. Finally, $\gamma_{T_i,2}$ can be rewritten as

$$\gamma_{T_i,2} = \frac{E_i |h_1|^2}{E_i \sum_{k=1}^{L_i} |d_{R,k}|^2 + \omega_i |d_{R,k}|^2 + \omega_2 E_i \sum_{k=1}^{L_i} |d_{T,k}|^2 + \omega_1 + 1}.$$}

### III. Outage Probability Analysis

#### A. Lower Bound of the Exact Outage Probability

In this section, the outage probability of the AF-based TDBC protocol in the presence of CCI is studied. For brevity of analysis and without loss of generality, we focus on the outage probability at terminal $T_1$ in the rest of this work. By definition, the outage event occurs when the mutual information at $T_1$ falls below the target rate $R_i$, or equivalently, the output SINR at $T_1$ is below the target SINR $\varphi$. Therefore, the outage probability at terminal $T_1$ can be written as

$$P_{OUT}^T(R_i) = \Pr (I_{T_1} < R_i) = F_{\gamma_{T_1}}(\varphi),$$

where $I_{T_1} = \frac{1}{\lambda} \log (1 + \gamma_{T_1})$ indicates the mutual information at terminal $T_1$, and $\varphi = 2^{R_1} - 1$. $F_\varphi(y)$ represents the CDF of random variable (RV) $\varphi$. However, it is very difficult to obtain the exact expression of $F_{\gamma_{T_1}}(\varphi)$ in closed-form. To circumvent this obstacle, we introduce a tight upper bound on the received SINR at $T_1$ by employing a widely used inequality, i.e., $\nu_1 \nu_2 / (\nu_1 + \nu_2) \leq \min \{\nu_1, \nu_2\}$, where $\nu_1$ and $\nu_2$ are positive numbers, then we shall have

$$\gamma_{T_1} \leq \gamma_{UB} = \gamma_{T_1,D} + \min \{\gamma_{T_1,1}, \gamma_{T_1,2}\}.\quad (8)$$

Next, we will determine the CDF of the upper bounded SINR. For convenience of analysis, letting $X \triangleq E_i |h_1|^2$, $Y \triangleq E_i |h_2|^2$, $Z \triangleq E_i |h_0|^2$, $S \triangleq E_i \sum_{k=1}^{L_i} |d_{R,k}|^2$ and $T \triangleq E_i \sum_{k=1}^{L_i} |d_{T,k}|^2$. Note that $X$, $Y$ and $Z$ are exponential RVs with means $E_{\Omega_1}$, $E_{\Omega_2}$ and $E_{\Omega_0}$, respectively, where $\Omega_i$ indicates the variance of $h_i$, $i \in \{0, 1, 2\}$. Letting $\gamma_m = \min \{\gamma_{T_1,1}, \gamma_{T_1,2}\}$, then with the help of total probability theorem, the CDF of $\gamma_m$ conditioned on $S$ and $T$ can be written as in (9), where $F_{\gamma_m | (S,T)}(x)$ is the CDF of $\gamma_m$.

$^1$As in [6], here it is assumed that $R$ knows the channel gains of links $T_1 \rightarrow R$ and $T_2 \rightarrow R$, and the total interference power (instantaneous) at $R$. Moreover, it is assumed that $T_1$ knows the channel gains of links $T_1 \rightarrow R$, $T_2 \rightarrow R$, $T_1 \rightarrow T_2$ as well as the total interference powers (instantaneous) at $R$ and $T_1$.

$^2$Similar as in [6] and [10], the assumption of equal power allocation dose not make the analysis in this work lose generality because the variances of the channel coefficients between $T_1$, $T_2$ and $R$ may be different.
\[
\gamma_T = \frac{E|h_0|^2}{E\sum_{k=1}^{L_T} |d_{T,k}|^2 + 1} + \frac{A_T^2 E^2|h_1|^2|h_2|^2}{E\sum_{k=1}^{L_T} |d_{R,k}|^2 + 1} + E\sum_{k=1}^{L_T} |d_{T,k}|^2 + 1,
\]

(4)

\[
F_{\gamma_T|\gamma_1(S,T)}(\phi) = 1 - \Pr(\gamma_{T,D} > \phi | S,T) - \Pr(\gamma_{T,D} < \phi, \gamma_m > \phi - \gamma_{T,D} | S,T)
= 1 - \int_{\phi}^{\infty} f_{\gamma_{T,D}|S,T}(r) \, dr - \int_{0}^{\phi} \left(1 - F_{\gamma_{m,1}|S,T}(r)\right) f_{\gamma_{T,0}|S,T}(r) \, dr
\]

(9)

conditioned on \(S\) and \(T\) which can be written as

\[
F_{\gamma_{m,1}|S,T}(x) = 1 - \prod_{i=1}^{2} \left(1 - F_{\gamma_{m,1,1}|S,T}(x)\right)
= 1 - \exp\left(-\frac{T+1}{E\Omega_0} x\right) \exp\left(-\frac{S + \omega_1 T + 1 + \omega_1 x}{\omega_2 E\Omega_2}\right)
\]

(10)

and \(f_{\gamma_{T,0}|T}(x)\) is the PDF of \(\gamma_{T,D}\) conditioned on \(T\) which can be expressed as

\[
f_{\gamma_{T,0}|T}(x) = \frac{T+1}{E\Omega_0} \exp\left(-\frac{T+1}{E\Omega_0} x\right).
\]

(11)

Then the CDF of \(\gamma_{T,D}^B\) can be obtained by averaging the conditioned CDF with respect to the PDFs of \(S\) and \(T\), i.e.,

\[
F_{\gamma_{T,D}^B}(\phi) = \int_{0}^{\infty} \int_{0}^{\infty} f_S(s) f_T(t) F_{\gamma_{T,D}^B|S,T}(\phi) \, ds \, dt
= 1 - \int_{0}^{\infty} \int_{0}^{\infty} f_T(t) \left(1 - F_{\gamma_{T,0}|T}(r)\right) dr \, dt
\]

\[
- \int_{0}^{\infty} \int_{0}^{\infty} f_S(s) \left(1 - F_{\gamma_{T,0}|T}(r)\right) dr \, ds,
\]

(12)

where \(f_S(s)\) and \(f_T(t)\) are the PDFs of RVs \(S\) and \(T\), respectively. Note that \(T\) is the sum of a finite number of exponential RVs with different means. Hence with the help of [12], the PDF of \(T\) can be written as

\[
f_T(t) = \sum_{k=1}^{L_T} \frac{\rho_k}{E_{\rho_k}} \exp\left(-\frac{t}{E_{\rho_k}}\right),\]

where \(\rho_k\) is the variance of \(d_{T,k}\) and \(p_k = \prod_{j=1,j\neq k}^{L_T} \frac{1}{\rho_{j,k}}\) for \(L_T \geq 2\) and \(p_k = \frac{1}{\rho_{1,k}}\) for \(L_T = 1\). Substituting the expressions of \(f_T(t)\) and \(f_{\gamma_{T,0}|T}(r)\) into (12), we can obtain

\[
P_1(\phi) = \exp\left(-\frac{\phi}{E\Omega_0}\right) \sum_j p_j E\Omega_0 / E_{\rho_{1,j}} + \frac{1}{\rho_{1,j}}.
\]

Moreover, denoting \(\rho_{R,k}\) as the variance of \(d_{R,k}\), the PDF of \(S\) can be given by

\[
f_S(s) = \sum_{k=1}^{L_R} \frac{1}{E_{\rho_{R,k}}} \exp\left(-\frac{\rho_{R,k}}{E_{\rho_{R,k}}}s\right),\]

where \(q_k = \prod_{j=1,j\neq k}^{L_R} \frac{1}{\rho_{R,k}-\rho_{R,j}}\) for \(L_R \geq 2\) and \(q_k = \frac{1}{\rho_{R,k}}\) for \(L_R = 1\). By substituting the PDFs of \(S\) and \(T\) into (12) and using the results of Appendix, the third term in the right-hand side of (12) (denoted by \(P_2(\phi)\)) can be evaluated as

\[
P_2(\phi) = \frac{\omega_2}{E_1^2} \exp\left(-\frac{\phi}{E\Omega_1}\right) \sum_j \sum_k \frac{1}{\rho_{j,k}} p_j q_k
\]

\[
\times \left(\frac{E\Omega_1 \lambda_1}{\phi + \varphi + E\Omega_1 / E_{\rho_{1,j}}} - \frac{E\Omega_1 \lambda_1}{\phi + E\Omega_1 / E_{\rho_{1,j}}} \exp\left(-\frac{\phi}{E}\right) \right)
\]

\[
\left(1 + \frac{E\Omega_0}{\phi + E\Omega_0 / E_{\rho_{1,j}}} \right) \Theta_k(\phi) + \left(1 + \frac{E\Omega_0}{\phi + E\Omega_0 / E_{\rho_{1,j}}} \right) E\Omega_1 / E_{\rho_{1,j}} \right) \Theta_k(\phi),
\]

(14)

where \(\Phi_0 = \frac{1}{\varphi + \omega_1 / \omega_2} \neq 0, \Phi_b = \frac{1}{\varphi + \omega_1 / \omega_2} = \frac{\lambda_2}{\lambda_1}, \lambda_2 = \left(\frac{1}{\omega_2} + \frac{1}{\omega_1 + \frac{1}{\omega_2}}\right)^{-1}\) and \(\lambda_1 = \left(\frac{1}{\omega_2} + \frac{1}{\omega_1 + \frac{1}{\omega_2}}\right)^{-1}\) and \(\beta_{j,k} = \frac{E\Omega_1 (1 + \omega_1 / \omega_2 \Phi_b)}{E\Omega_1 (1 + \omega_1 / \omega_2 \Phi_b)}\). Then we will test the convergence of the infinite series involved in the expressions of \(\Theta_k(\phi)\) and \(\Xi_j(\phi)\). Defining

\[
\Delta_l(\eta_1, \eta_2, \eta_3, \eta_4) = \frac{\eta_1^l \eta_4^{(l+1)}}{(\eta_2 \phi + \eta_1 + \phi \eta_4)},
\]

(18)

where \(0 < \eta_1 \leq \eta_2 \leq \eta_3 > 0\). It can be shown that \(\Theta_k(\phi) = \Delta_l(\frac{1}{E\Omega_1 / E_{\rho_{1,j}}} + \frac{\phi}{E}, \phi)\) \((\Phi_b > 0)\) and \(\Xi_j(\phi) = \exp\left(-\frac{\phi}{E}\right) \Delta_l(\frac{\phi}{E} + \frac{\eta_1}{\eta_2 \phi + \eta_1 + \phi \eta_4}, \phi_0 - \frac{\phi_0}{E})\) \((\Phi_b < 0 < \Phi_a)\), thus it is sufficient to prove that the infinite series \(\sum_{l=1}^{\infty} \Delta_l(\eta_1, \eta_2, \eta_3, \eta_4)\) is convergent. Using [13, 3.381.1], it can be shown that

\[
\lim_{l \to \infty} \sqrt{l} \Delta_l(\eta_1, \eta_2, \eta_3, \eta_4) = \lim_{l \to \infty} \sqrt{l} \left(\frac{\eta_1}{\eta_2 \phi + \eta_1 + \phi \eta_4}\right) \int_{0}^{\phi} r^l \exp\left(-\eta_4 r\right) dr
\]

\[
\leq \frac{\eta_1}{\eta_2 \phi + \eta_1 + \phi \eta_4} < 1
\]

(19)

Note that the series expression of \(\Theta_k(\phi)\) \((\Phi_b > 0)\) is also valid for the case of \(\Phi_b < 0\). However, we present another closed-form expression without infinite series to facilitate the computation of \(\Theta_k(\phi)\) when \(\Phi_b < 0\). Similarly, it can be seen the series expression of \(\Xi_j(\phi)\) \((\Phi_b < 0)\) in (16) is always valid except for the case \((\Phi_a = 0 \text{ or } \Phi_b = 0)\).
By the root test \[14\], it can be seen that the infinite series in (15) and (16) are always convergent when \( \varphi < \infty \).

Finally, the lower bound of the outage probability for the AF-based TDBC protocol in the presence of CCI can be derived by substituting (13) and (14) (or (17)) in to (12), i.e.,

\[
P_T^\text{OUT-LB} (R_t) = F_{\gamma_{T1}} (\varphi) = 1 - P_1 (\varphi) - P_2 (\varphi). \quad (20)
\]

B. Asymptotic Analysis

To offer an intuitive observation into the effect of CCI on the outage performance, we develop asymptotic analysis on the outage probability based on the analysis of subsection A. According to \[15\] - \[16\], the asymptotic expression can be derived by performing McLaurin series expansion to \( F_{\gamma_{T1}} (\varphi) \) and taking only the first two order terms. Wherein the McLaurin series expansion of \( \Theta_k (\varphi) \) can be given by

\[
\Theta_k (\varphi) = \Theta_k (0) + \Theta_k^{(1)} (0) \varphi + \frac{\Theta_k^{(2)} (0)}{2} \varphi^2 + O (\varphi^2), \quad (21)
\]

where \( O (\delta) \) indicates the higher order term of \( \delta \) and \( \Theta_k^{(n)} (0) \) \((n=1,2)\) can be determined by \[13, 0.410\]:

\[
\Theta_k^{(1)} (0) = \left[ g (\varphi, r) \right] \bigg|_{r = \varphi = 0}
\]

\[
\Theta_k^{(2)} (0) = \left[ \frac{dg (\varphi, r)}{d\varphi} \right] \bigg|_{r = \varphi = 0} + \frac{d}{d\varphi} \left( g (\varphi, r) \right) \bigg|_{r = \varphi = 0},
\]

where \( g (\varphi, r) = \left( \frac{\varphi}{E\Omega_2} + \frac{\omega_2 E\Omega_2}{E_1 \rho_{R,k}} \right)^{-l+1} \exp \left( -\frac{\varphi E}{E_1} r \right) \). Moreover, the McLaurin series expansion of \( \Xi_j (\varphi) \) can be calculated using the similar method as in the above. Finally, the asymptotic expression of \( F_{\gamma_{T1}} (\varphi) \) can be written as

\[
F_{\gamma_{T1}} (\varphi) \approx \left( \frac{E_1}{E} \right)^2 \varphi^2 \frac{\varphi^2}{2 \Omega_0} \left[ \sum_j \sum_k \frac{p_j q_k \rho_{R,k}^2}{\omega_2 E\Omega_2} \left( \frac{\rho_{1,j} + \rho_{1,j}^2}{E_1} \right) \right] \sum_j \frac{p_j}{E_1^r} \left( \frac{\rho_{1,j}}{E_1^r} + \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \left( \frac{\rho_{1,j}}{E_1^r} + \frac{2 \rho_{1,j}}{\lambda_2} \right). \quad (23)
\]

Through the asymptotic expression, it can be seen that when the ratio of useful power to interference power is constant, the AF-based TDBC protocol dose not achieve any diversity.

IV. SIMULATION RESULTS

In this section, we provide the simulation results to verify our theoretical analyses on the outage probability. It is assumed that \( T_1, T_2 \) and \( R \) are located in a straight line and \( R \) is between \( T_1 \) and \( T_2 \). The distance between two terminals is normalized to 1 and the path loss exponent is set to 4 [17], thus the variances of \( h_0, h_1 \) and \( h_2 \) can be computed as \( \Omega_0 = 1, \Omega_1 = D_1^{-4} \) and \( \Omega_2 = (1 - D_1)^{-4} \), respectively, where \( D_1 \) indicates the normalized distance between \( T_1 \) and \( R \). The normalized distances between node \( N \) and the interferers that interfere \( N \) are assumed to be evenly distributed on the interval \((\alpha_1, \alpha_2) = (1, 1.5)\), where \( N \in \{T_1, T_2, R\} \). Hence, \( \rho_{R,k} \) and \( \rho_{i,k} \) can be determined by \( \rho_{R,k} = (\alpha_1 + (k - 1)(\alpha_2 - \alpha_1)/(L_R - 1))^{-4} \) and \( \rho_{i,k} = (\alpha_1 + (k - 1)(\alpha_2 - \alpha_1)/(L_R - 1))^{-4} \).

In Fig. 2, the outage performance at \( T_1 \) is presented as a function of the transmit power \( E \), where \( E/E_1 \) is fixed.
expected. Finally, a fine agreement between the analytic results and simulations can be observed from the figures.

Fig. 2: Outage performance at $T_1$ with fixed $E/E_t$, where $D_1 = 0.5$, $\omega_1 = 0.5$ and $R_t = 1$ bit/s/Hz.

Fig. 3: Outage performance at $T_1$ versus $E/E_t$, where $E_t = 5$dB, $D_1 = 0.5$, $\omega_1 = 0.5$ and $R_t = 1$ bit/s/Hz.

at 30dB. The outage performance of the scenario without interference is also presented as a benchmark. It is seen that the outage performance degrades as the numbers of interferers increase. Furthermore, the slope of outage probability curves are steep in the low SNR region ($E < 15$dB). This is because the power of AWGN dominates the interference power. While a performance floor can be observed in the high SNR region due to the dominant role of interference power. This phenomenon also indicates that the achievable diversity order of the TDCC protocol in the interference-limited scenario is zero. Fig. 3 studies the outage performance at $T_1$ against $E/E_t$. From Fig. 3, it can be seen that the outage probability increases as the numbers of interferers as well as $E_t/E$ increase as expected. Finally, a fine agreement between the analytic results and simulations can be observed from the figures.

Fig. 4 investigates the effect of relay placement on the outage performance, where numbers of interferers that affect $T_1$ and $R$ may different. Without loss of generality, we set $L_1 = 1$ and let $L_R$ increase from 1, 5, 10 to 15. Then we examine the outage performance at $T_1$ as a function of $D_1$. It is seen from the figure that the optimal relay placement moves toward $T_2$ as the number of interferers (and total interference power) that affects the relay increases. This is because the AF operation adopted by the relay. From equation (4), we can see that, for given $(\alpha_1, \alpha_2)$, when $L_R$ increases, to decrease the amplified interference (i.e., the term $\left(A_1^2 + A_2^2\right) |h_1|^2 (E_t \sum_{k=1}^{L_R} d_{R,k}^2 + 1)$), the relay should move toward $T_2$ to decrease $|h_1|^2$.

V. CONCLUSIONS

In this paper, we study the effect of CCI on the AF-based TDCC protocol. Lower bound of the outage probability is derived and is shown to provide a good match with the simulation results. Meanwhile, a simpler asymptotic expression of outage probability is also provided. We show through both analytic and simulation results that the achievable diversity of the TDCC protocol in the interference limited scenario is zero. Moreover, we investigate the effect of relay placement on the outage probability and show that when only consider the outage performance at one terminal (e.g. $T_1$), as the number of interferers that interferes the relay increases, the optimal relay placement needs to move toward $T_2$ in order to obtain the optimal outage probability at $T_1$.

APPENDIX

Substituting the PDFs of $S$ and $T$ into (12) and interchanging the integration order, we can obtain

$$P_2(\varphi) = \frac{1}{E}\text{exp}\left(-\left(\frac{1}{\Omega_1} + \frac{\omega_1}{\Omega_2}\right)\frac{\varphi}{E}\right) \sum_k \sum_j \frac{p_j q_k}{E \Omega_0}$$

$$\int_0^\varphi \exp\left(\frac{-\Phi_k}{E}\right) \int_0^\infty \exp\left(-\left[\frac{\varphi - r}{\omega_2\Omega_2} \frac{1}{E_1 \rho_{R,k}} + \frac{1}{E_I \rho_1,j}\right]s\right) ds dr \times (t + 1) dt dr.$$

Case 1 ($\Phi_0 \neq 0$): In this case, solving the integrals with respect to $s$ and $t$, we can yield

$$P_2(\varphi) = \frac{1}{E}\text{exp}\left(-\left(\frac{1}{\Omega_1} + \frac{\omega_1 + 1}{2\Omega_2}\right)\frac{\varphi}{E}\right) \times \sum_k \sum_j \frac{E\Omega_1}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{\omega_2}{\omega_1} + \frac{\omega_2}{\omega_1} + \frac{\omega_2}{\omega_1}} \exp\left(-\frac{\Phi_k}{E}\right)$$

$$\times \left[\frac{E\Omega_2}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} - \frac{E\Omega_2}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} \frac{\varphi}{\omega_2} \exp\left(-\frac{\Phi_k}{E}\right) \right]$$

$$\times \left[\frac{1}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} \frac{\varphi}{\omega_2} \exp\left(-\frac{\Phi_k}{E}\right) \right]$$

$$\times \left[\frac{1}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} \frac{\varphi}{\omega_2} \exp\left(-\frac{\Phi_k}{E}\right) \right]$$

$$\times \left[\frac{1}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} \frac{\varphi}{\omega_2} \exp\left(-\frac{\Phi_k}{E}\right) \right]$$

$$\times \left[\frac{1}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} \frac{\varphi}{\omega_2} \exp\left(-\frac{\Phi_k}{E}\right) \right]$$

$$\times \left[\frac{1}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} \frac{\varphi}{\omega_2} \exp\left(-\frac{\Phi_k}{E}\right) \right]$$

$$\times \left[\frac{1}{\varphi + \frac{E\Omega_0}{\omega_2 + \frac{1}{\omega_2 + \frac{1}{\omega_1}}}} \frac{\varphi}{\omega_2} \exp\left(-\frac{\Phi_k}{E}\right) \right]$$

where $\Theta_k(\varphi)$ and $\Xi_j(\varphi)$ are expressed as

$$\Theta_k(\varphi) = \int_0^\varphi \frac{1}{E\Omega_0} \exp\left(-\frac{\Phi_k}{E}\right) dr$$

$$\Xi_j(\varphi) = \int_0^\varphi \frac{1}{E\Omega_0} \exp\left(-\frac{\Phi_k}{E}\right) dr.$$ (26)
When $\Phi_b > 0$, to solve the integral $\Theta_k(\varphi)$, we apply Taylor series expansion \( \frac{\varphi}{E_{\text{TX}}} + \frac{\omega_2 E_{\Omega_k}}{E_{\text{I} \text{PR},k}} \) = $E\Omega_2 \sum_{l=0}^{\infty} \varphi^l L_l / (\varphi + \omega_2 E \Omega_2 / E_{\text{I} \text{PR},k})^{l+1}$. Then based on [13, 3.381.1], the integral can be solved into

$$\Theta_k(\varphi) = \sum_{l=0}^{\infty} \frac{E\Omega_2}{(\varphi + \omega_2 E \Omega_2 / E_{\text{I} \text{PR},k})^{l+1}} \left( \frac{\Phi_b}{E} \right)^{-l+1} \phi \left( l + 1, \frac{\varphi \Phi_b}{E} \right).$$  \hspace{1cm} (27)

When $\Phi_b < 0$, $\Theta_k(\varphi)$ can be solved by replacing $(\varphi - r)$ with $t$, and then using the integral result reported in [13, 3.352.1],

$$\Theta_k(\varphi) = \exp \left( -\Phi_b \frac{t}{E} \right) \int_0^\varphi \frac{1}{E\Omega_2 + \omega_2 \frac{E \Omega_2}{E_{\text{I} \text{PR},k}}} \exp \left( \frac{\Phi_b}{E} t \right) dt$$

$$= E\Omega_2 \exp \left( -\Phi_b \frac{\varphi}{E} + \omega_2 E \Omega_2 \frac{E_{\text{I} \text{PR},k}}{E} \right) \times \left( \frac{\Phi_b}{E} \right)^{-1} \phi \left( l + 1, \frac{\varphi \Phi_b}{E} \right).$$  \hspace{1cm} (28)

On the other hand, using [13, 3.352.1] on $\Xi_j(\varphi)$ when $\Phi_a > \Phi_b > 0$, we can yield $\Xi_j(\varphi) = \frac{\Phi_b}{E \Phi_a} \exp \left( \frac{\Phi_b}{E} \right) (\exp (\frac{\Phi_b}{E}) - \exp (-\frac{\Phi_b}{E}))$. To solve the integral $\Xi_j(\varphi)$ when $\Phi_b < 0 < \Phi_a$, similar approach as in the case of $\Phi_b > 0$ for $\Theta_k(\varphi)$ can be used, and we obtain

$$\Xi_j(\varphi) = \exp \left( -\Phi_b \frac{\varphi}{E} \right) \int_0^\varphi \frac{1}{E\Omega_2 + \omega_2 \frac{E \Omega_2}{E_{\text{I} \text{PR},k}}} \exp \left( \frac{\Phi_b}{E} t \right) dt$$

$$= \exp \left( -\Phi_b \frac{\varphi}{E} \right) \sum_{l=0}^{\infty} \frac{E \Omega_2 \Omega_l}{(\varphi + E \Omega_2 / E_{\text{I} \text{PR},k})^{l+1}} \times \left( \frac{\Phi_b}{E} \right)^{-l+1} \phi \left( l + 1, -\frac{\varphi \Phi_b}{E} \right).$$  \hspace{1cm} (29)

Using the similar method as in the case of $\Phi_b < 0$ for $\Theta_k(\varphi)$, it can be shown that, when $\Phi_b < \Phi_a < 0$,

$$\Xi_j(\varphi) = \frac{E}{\Phi_a} \exp \left( -\Phi_b \frac{\varphi}{E} \right) \phi \left( \varphi + \frac{E \Omega_2 / E_{\text{I} \text{PR},k}}{\Omega_0 \Phi_a} \right) \times \left( \frac{\Phi_b}{E} \right) \phi \left( \varphi + \frac{E \Omega_2 / E_{\text{I} \text{PR},k}}{\Omega_0 \Phi_a} \right).$$  \hspace{1cm} (30)

Moreover, it is very easy to verify that $\Theta_k(\varphi)$ and $\Xi_j(\varphi)$ can be expressed as in (15) and (16) when $\Phi_b = 0$, thus the derivations details for this case are omitted.

Case2 ($\Phi_a = 0$): In this case, it is easy to verify by using the equation $\Phi_a = 0$ that the sum of the first term and third term in bracket {} of (25) equals to zero, thus $P_2(\varphi)$ can be simplified to (17).

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Fig. 4: Outage performance at $T_1$ versus relay placement, $E = 30$ dB, $E_1 = 10$ dB, $\omega_1 = 0.5$, $R_t = 1$ bit/s/Hz and $L_1 = 1$.

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