The nucleon mass and pion-nucleon sigma term from a chiral analysis of $N_f = 2$ lattice QCD world data

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Abstract. We investigate the pion-mass dependence of the nucleon mass within the covariant $SU(2)$ baryon chiral perturbation theory up to order $p^4$ with and without explicit $\Delta(1232)$ degrees of freedom. We fit lattice QCD data from several collaborations for 2 and 2+1 flavor ensembles. Here, we emphasize our $N_f = 2$ study where the inclusion the $\Delta(1232)$ contributions stabilizes the fits. We correct for finite volume and spacing effects, set independently the lattice QCD scale by a Sommer-scale of $r_0 = 0.493(23)\text{fm}$ and also include one $\sigma_{\pi N}$ IQCD data point at $M_\pi \approx 290\text{MeV}$. We obtain low-energy constants of natural size which are compatible with the rather linear pion-mass dependence observed in lattice QCD. We report a value of $\sigma_{\pi N} = 41(5)(4)\text{MeV}$ for the 2 flavor case and $\sigma_{\pi N} = 52(3)(8)\text{MeV}$ for 2+1 flavors.

1 Introduction

The current quark mass dependence of observables from lattice QCD simulations (lQCD) for unphysical values provides an additional perspective on QCD itself. Many simulations with two dynamical degenerated light quarks ($N_f = 2$) or two degenerated light and one heavy quark ($N_f = 2+1$) are now available with light quark masses spanning from unphysical heavy values down to nearly physical ones. To investigate this extensive data set the (baryon) chiral perturbation theory ($B\chi$PT) represents a prominent tool. By matching $B\chi$PT to IQCD data for unphysical quark masses, low-energy constants (LECs) can be extracted, which are then used for predictions at the physical point.

We performed such a matching for the quark-mass ($m_u = m_d = \overline{m}$) dependence of the nucleon mass $M_N(\overline{m})$ by separately fitting to $N_f = 2$ and 2+1 IQCD data. We concentrate here on our $N_f = 2$ results and refer to [1] for more details on the 2+1 ones.

An important derivative of $M_N(\overline{m})$ is given by the Hellmann-Feynman theorem:

$$\overline{m} \frac{\partial}{\partial \overline{m}} M_N(\overline{m}) = \sigma_{\pi N} = \overline{m} \langle N[\bar{u}u + \bar{d}d]N \rangle ,$$

which relates $M_N(\overline{m})$ to the so-called $\sigma_{\pi N}$ term. The $\sigma_{\pi N}$ term can be seen as a measure of the contribution from the explicit chiral symmetry breaking to the nucleon mass. Since the $\sigma_{\pi N}$ can also be defined by the nucleon scalar form factor at zero four-momentum transfer squared, it can also...
be isolated from $\pi N$-scattering data. In this sense, the quark-mass dependence of the nucleon mass relates lQCD, $\beta_N$PT and experiment.

2 Nucleon mass and covariant baryon chiral perturbation theory

The chiral structure of the nucleon mass is parametrized by the covariant $\beta_N$PT up to order $p^4$ as:

$$M_N^{(4)}(\bar{M}_\pi^2) = M_0 - c_14M_\pi^4 + \frac{1}{2}\pi M_\pi^4 + \frac{c_1}{8\pi^2}M_\pi^4 \ln \frac{M_\pi^2}{M_0^2} + \Sigma_{\text{loops}}^{(3) + (4)}(\bar{M}_\pi^2) + O(p^5).$$

(2)

where $M_\pi^2 \sim \bar{m}$ and $f_\pi$ are the pion mass and pion decay constant. The loop-contributions $\Sigma_{\text{loops}}^{(3) + (4)}$ can also contain explicit $\Delta(1232)$ contributions. We refer to Ref. [1] for all the details. We fit the nucleon chiral-limit mass $M_0$ and the two LECs $c_1$ and $\sigma$ to lQCD data and obtain through Eq. (1) a $\sigma_{\pi N}$ value. Explicitly, we use the $N_f = 2$ lQCD data in its dimensionless form from $(r_0M_\pi, r_0M_N)$, with $r_0$ being the Sommer-scale, and minimize the function:

$$\chi^2 = \sum_i \left[ \frac{\bar{M}_N^{(4)}(M_\pi^2) + \Sigma_N^{(4)}(M_\pi^2, L) + \bar{c}_i\bar{a}^2 - d_i(M_\pi^2, L)}{\sigma_i} \right]^2,$$

(3)

with $\bar{M}_N^{(4)} = r_0M_N^{(4)}$, $\bar{M}_\pi^2 = (r_0M_\pi)^2$, $\Sigma_N^{(4)} = r_0\Sigma_N^{(4)}$.

(4)

where $d_i(M_\pi^2, L)$ are the data points with uncertainties $\sigma_i$ for a lattice of size $L$ and spacing $a$. The terms $\bar{c}_i\bar{a}^2$ parametrize finite spacing effects for each lQCD action separately. The self-energy $\Sigma_N^{(4)}$ contains also finite volume corrections. Furthermore, we use the physical nucleon mass to determine the Sommer-scale $r_0$ recursively and self-consistently inside the fit. In our $SU(2)$ fits to $N_f = 2 + 1$ data we assume that the strange quark contributions are integrated out and absorbed into the LECs.

To ensure controlled finite volume effects and an acceptable chiral convergence of our $\beta_N$PT results, we restrict our fits to $M_N$-data points fulfilling $M_\pi L > 3.8$ and $r_0M_\pi < 1.11$. Additionally, the QCDSF collaboration obtained one direct $\sigma_{\pi N}$ data point at $M_\pi \approx 290$ MeV [2] with which we also perform simultaneous fits to nucleon mass data and that $\sigma_{\pi N}(290)$ point.

3 Results and conclusions

We summarize some of our results of Ref. [1]. Figure 1 shows our fits to the lQCD data from the $N_f = 2$ ensembles of the BGR, ETMC, Mainz and QCDSF collaborations [2-5]. We fitted the data without and with explicit $\Delta(1232)$ contributions, left and right figures respectively. The dashed and solid lines correspond to the ex-/inclusion of the one $\sigma_{\pi N}(290)$ point, respectively. Our fits yield $2 < \chi^2/d.o.f. < 3$ reflecting that some of the data are marginally consistent.

By including the one $\sigma_{\pi N}(290)$ point in the $\Delta(1232)$-less fit we reduce the uncertainties, although, the shape of the pion-mass dependence changes noticeably. This is also seen in the obtained $\sigma_{\pi N}$ value at the physical point, which changes from $\sigma_{\pi N} = 62(13)$ MeV to $41(3)$ MeV. The situation is different when we include the explicit $\Delta(1232)$ contributions in our fit formula. For this case, the inclusion of the one $\sigma_{\pi N}(290)$ point does not change the pion-mass dependence much. Differences are mainly visible at higher pion masses and the $\sigma_{\pi N}$ term at the physical point turns out to be $\sigma_{\pi N} = 41(3)$ MeV for both cases. We conclude that for the present data situation the inclusion of $\Delta(1232)$ contributions stabilizes the fits and that the reported (only one) $\sigma_{\pi N}(290)$ point is more compatible with the $\beta_N$PT with $\Delta(1232)$ rather than with the $\Delta(1232)$-less one. It will be interesting to see if further direct lQCD calculations of the $\sigma_{\pi N}$ at unphysical pion-masses will confirm this conclusion.
The left panel of Fig. 1 shows the pion-mass dependence of the $\sigma_{\pi N}$ term obtained from our $B\chi$PT fits with $\Delta(1232)$ contributions, solid line, and without, dashed-line. As discussed above, the changes of the slope of the dashed line could indicate that the $\Delta(1232)$-less $B\chi$PT has problems to account for the $\sigma_{\pi N}(290)$ point. As for our final $\sigma_{\pi N}$ value at the physical point for the $N_f = 2$ fits we quote:

$$\sigma_{\pi N}^{N_f=2} = 41(5)(4) \text{ MeV},$$

which corresponds to our result for the $B\chi$PT with $\Delta(1232)$ contributions. For the determination of the uncertainties we refer to Ref. [1].

The right panel of Fig. 2 compares our $B\chi$PT results from fits to $N_f = 2$ and $N_f = 2 + 1$ data. Differences are within the errorbars of the input data but translate into a $\sim 11$ MeV difference in the $\sigma_{\pi N}$ value. We obtain for our $N_f = 2 + 1$ fits the higher value of

$$\sigma_{\pi N}^{N_f=2+1} = 52(3)(8) \text{ MeV}.$$

Both values are compatible within the uncertainties, however, with the present data we cannot unambiguously determine the origin of the 11 MeV difference of the central values. As a first point, we see that the $N_f = 2$ data does not constrain the small pion-mass region much. This results in an up to $\sim 13\%$ smaller $c_1$ value which determines solely the $\sigma_{\pi N}$ at leading order. As a second point, the $\sigma_{\pi N}(290)$ data point brought in the $N_f = 2$ case the $\Delta(1232)$ and $\Delta(1232)$-less results together. Such low-$M_{\pi}$ $\sigma_{\pi N}$ data points are not available for the $N_f = 2 + 1$ case and our fits yield by $9 \sim 11$ MeV different $\sigma_{\pi N}$ results for the two cases. The above $N_f = 2 + 1$ value is the average of our $\Delta(1232)$ and $\Delta(1232)$-less results. Furthermore, since the strange quark is treated differently in the two IQCD data sets, one could expect that part of the difference comes also from this fact. Note also that the latest value extracted from pure $\pi$-$N$ scattering data yields $\sigma_{\pi N} = 59(7) \text{ MeV}$ [6].

In summary, we fitted IQCD data for $N_f = 2$ and $N_f = 2 + 1$ ensembles by a $SU(2)$ $B\chi$PT formula up to $p^4$ with and without $\Delta(1232)$ degrees of freedom. Even though the present data set is extensive, we observed systematic uncertainties stemming mostly from the distribution of the data points. New data for the following cases would further reduce these systematic effects: a) more $N_f = 2$ data points.
in the low-$M_\pi$ region, b) even one direct calculation of the $\sigma_{\pi N}$ at $M_\pi < 300$ MeV for the $N_f = 2 + 1$ case.

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