Abstract

We show that in $D = 4$ AdS, $s \geq 3/2$ partially massless (PM) fermions retain the duality invariances of their flat space massless counterparts. They have tuned ratios $m^2/M^2 \neq 0$ that turn them into sums of effectively massless unconstrained helicity $\pm(s, \cdots, \frac{3}{2})$ excitations, shorn of the lowest (non-dual) helicity $\pm \frac{1}{2}$-rung and -more generally- of succeeding higher rung as well. Each helicity mode is separately duality invariant, like its flat space counterpart.

Keywords: Electromagnetic Duality, Higher spins

1. Introduction

We address, and complete the answer to, the question whether/how free $m = 0$ spin $\geq 1$ systems can retain their known universal ($D = 4$) flat space duality invariance [1] when embedded in (A)dS, rather than flat, backgrounds. Half of the question had actually already been answered in [2], where it was shown that the novel – in dS – PM irreps [3] for $s > 1$ bosons did so (photons always do). The only difference from flat space was, perhaps surprisingly, that rather than having $m = 0$ and just helicities $s$, they now sported a complete range, $\pm(s, \cdots, 1)$ of effectively $m = 0$ helicity excitations, excluding precisely the helicity 0 rung that would have spoiled the duality invariance manifest in each of the higher ones. [Spin 1, being conformally invariant, is a degenerate case, since (A)dS is conformally flat; of course, if studied exactly like its $s > 1$ peers in dS, its duality invariance follows exactly like theirs. Similarly, $s = 3/2$ duality invariance was also exhibited long ago [4] for its “massive”, cosmological SUGRA, version.

For orientation, we recall that the easiest bosonic PM route of [2] uses the dS frame $ds^2 = -dt^2 + e^{2Mt}d\ell^2$, $M^2 \equiv \frac{\Lambda^2}{3}$; there, one first discovers that, in maximal PM, a particular m/M ratio eliminates helicity-0, leaving a sum of unconstrained helicity $\pm(s, \cdots, 1)$ actions. Specifically, for the first non-trivial, $s = 2$ model, the action is that of a transverse-traceless (TT) spatial tensor and a transverse vector ($T_i$); effective masslessness is achieved by the PM tuning of the two mass parameters ($m$, $M$).removing the helicity 0 mode through the residual local scalar gauge invariance of the original action at the PM point, $m^2 = M^2$. However, as we shall see, $m = 0$ models are NOT duality invariant in (A)dS, because their lowest (0 or 1/2) helicities are reinstated there. It was strongly conjectured that the same process (also explicitly performed for $s = 3$) goes through for ALL s: the auxiliary fields, constraints, etc that necessarily decorate the original covariant actions are gone in the final, non-covariant, unconstrained 3+1 form.

Our spinor models also enjoy PM irreps, but in AdS instead of dS. As mentioned, $s = 3/2$ is the basic, and long known, example of a dual invariant $s = 3/2$ tuned system [4]: In order to obtain the cosmological,
necessarily AdS extension of SUGRA, one must add a mass term \( m\bar{\psi}_n \sigma^{mn} \psi_n \) to its massless action, with the tuning \( m \sim \sqrt{-\frac{\Lambda}{3}} = M \). This is exactly equivalent to improving the covariant derivative from \( D_\mu \) to \( D'_\mu = D_\mu + \frac{M}{2} \gamma_\mu \). The effect of this change is to restore the flat space commutativity, \( [\partial_\mu, \partial_\nu] = 0 \rightarrow [D'_\mu, D'_\nu] = 0 \), thereby restoring the flat space invariance of the model under local spinor transformations, now under \( \delta \psi = D'_\mu \alpha(x) \), and so again removing the lowest, here helicity 1/2, excitation \[ \text{[2].} \] Here, the governing variables are the transverse-traceless and \( \gamma_i \)-traceless spinor- spatial tensors \( \psi^{iTT} \). The PM invariance \[ \text{[3]} \] always removes the lowest, here helicity 1/2, leaving an effectively massless (upon, legally, field redefining) array of helicities \( s \in \{s + 1/2, 3/2\} \), each separately duality invariant, but now at the above AdS point.

2. Derivation

For compactness, we will freely use the equations and results of \[ \text{[4]} \]; while that work is ostensibly formulated in dS, it is, as noted there, applicable to our AdS format, the change in sign of \( \Lambda \) corresponds to setting the \( M \) there to \( i\bar{M} \); we will simply keep the \( dS \) notation on the above understanding, rather than wasting space with AdS formalism; we also borrow from \[ \text{[2]} \] it the near-certainty that the procedure and results are uniform for all higher spins: again, while higher spin actions require auxiliary fields and constraint variables, these are all absent from the final unconstrained physical 3+1 actions, here for the gamma-and spatial gradient-transverse, traceless spatial tensor-spinor components.

The key, Dirac, equation satisfied by these amplitudes is given by Eq. (14) there:

\[
\gamma^0 \partial_0 \psi^{iTT} + e^{-MT} \Phi \psi^{iTT} + \left[ m + (2-s)\gamma^0 M \right] \psi^{iTT} = 0
\]

Clearly, \[ \text{[1]} \] differs from its flat space, massless, counterpart in two basic respects: it contains a “mass” term, \( \sim aM\gamma^0 + b m \), as well as the factor \( e^{-MT} \) in the spatial derivative term. The latter is essentially an irrelevant numerical coefficient in the Hamiltonian for spatial duality transformation purposes, also present and harmless for bosons, as explained in \[ \text{[2]} \]. To remove the offending “mass” terms, consider for concreteness irrelevant numerical coefficient in the Hamiltonian for spatial duality transformation purposes, also present and harmless for bosons, as explained in \[ \text{[2]} \]. To remove the offending “mass” terms, consider for concreteness

\[ s = \frac{5}{2} \]

where we face -\( \frac{M}{2}\gamma^0 + m = \frac{M}{2}\gamma^0 + (m - \gamma^0 M) \). The parenthesis vanishes at the PM point, because \( \gamma^0 \) is diagonal with \( \pm i \) entries, provided we add, beyond \( m^2 + M^2 = 0 \), the fermionic requirement that the upper/lower components of \( \psi \) obey its respective roots \( \pm im + M = 0 \). The remaining, \( M/2\gamma^0 \), term is simply removed by rescaling \( \psi \) by \( \exp(M/t) \), to leave the sum of flat space pure helicity \( > 1/2 \) actions (modulo the irrelevant \( e^{-MT} \) term in the Hamiltonian). While one might worry that any amount of M-dependence can be removed this way, the process here is really an artifact of the AdS gauge choice: we had proceeded in conformal AdS gauge,

\[
ds^2 = (MT)^{-2}(-dT^2 + dt^2),
\]

from the start, we would have found the fully flat form of (1),

\[
\left[ \gamma^0 \frac{\partial}{\partial T} + \Phi \right] \psi^{iTT} = 0,
\]

since PM actions are all conformally invariant (indeed, that is their special virtue). We can also recover \[ \text{[3]} \] from the, final, massless \[ \text{[1]} \],

\[
[\exp(M/t)\gamma^0 \partial_0 + \Phi] \psi^{iTT} = 0,
\]

by performing the (trivial) gauge transformation from our \( t \rightarrow \) to the \( T \rightarrow \) frame \[ \text{[2]} \]. But \[ \text{[3]} \] is just the flat space, E-B, form given in \[ \text{[4]} \] namely

\[
\gamma^0 E + B = 0.
\]

This is both manifestly \( (E \leftrightarrow B) \) rotation invariant, and a time-local canonical transformation, in terms of the underlying canonical pair, as detailed in \[ \text{[1]} \] for all spins. Indeed, the same, natural, conformal frame

\[ ^1 \text{Different values of the } s \text{ can be reduced to this case by performing the field the redefinition } \psi^{iTT} \rightarrow e^{(s-5/2)MT} \psi^{iTT}. \]
could have been used for the bosonic case [2] directly, or also reached by transforming to T-frame there, to remove the \exp (MT) factor in the corresponding t-frame Hamiltonian there, starting from its PM form,

\[ \mathcal{L}_{\text{boson}} = p^a \dot{q}_a - \exp(-MT) \frac{1}{2} [p^2 + q^2] \]  

where the summed index a runs over all helicities \(>0\) (or \(>1\) etc., in the various other PM levels discussed below).

We remark finally that for spins \(>3/2\), there exist different PM levels, each exciting more lower helicities, until only helicity \(\pm s\) is left. Each of these occurs at different \(m/M\) ratios, and each is duality invariant by tuning \(M\) to remove the \(m\) term in the corresponding Dirac equation, then rescaling the spinor-tensor amplitude to remove whatever \(M\)–dependence remains. In this sense there is in fact a much larger set of dual-invariant PM–levels for any \(s\), than the unique \(m = 0\) one in flat space.

3. Summary

We have shown that all \(s \geq 3/2\) PM free fermionic models in suitably PM tuned AdS are duality-invariant under the same transformations as in flat space, separately for each effectively massless helicity \((>1/2)\) component. Together with the existing—essentially identical bosonic PM duality invariances in dS [2], this establishes the maximal curved spacetime generalization of flat space free higher spin field duality invariances.

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References

[1] S. Deser and D. Seminara, Phys. Lett. B 607, 317 (2005) [hep-th/0411169].
[2] S. Deser and A. Waldron, Phys. Rev. D 87, 087702 (2013) arXiv:1301.2238 [hep-th].
[3] S. Deser and A. Waldron, Phys. Lett. B 513, 137 (2001) [hep-th/0105181].
[4] S. Deser, J. H. Kay and K. S. Stelle, Phys. Rev. D 16, 2448 (1977).
[5] S. Deser and B. Zumino, Phys. Lett. B 62, 335 (1976).
[6] S. Deser and A. Waldron, Phys. Lett. B 508, 347 (2001) [hep-th/0103255].