Muon-proton Colliders: Leptoquarks and Contact Interactions

Kingman Cheung

Department of Physics, University of California at Davis, Davis, CA 95616

Abstract. The muon-proton (\(\mu p\)) collider is an interesting option of the muon collider. Here we discuss the physics potential of the \(\mu p\) collider; especially, leptoquarks and contact interactions. We calculate the sensitivity reach for the second generation leptoquarks and leptonquons, \(R\)-parity violating squarks, and \(\mu q\) contact interactions for the \(\mu p\) colliders of various energies and luminosities.

INTRODUCTION

Recently, the muon collider has received a lot of attentions [1]. The muon collider of a few hundred GeV center-of-mass energy is considered a Higgs factory [2], where interactions and branching ratios of the Higgs boson can be studied in detail. It is also an excellent place to study the top-quark near the threshold region [3]. Other physics opportunities include precision studies of gauge bosons [3], search for supersymmetry and lepton-number violation, and other new physics. Muon colliders in TeV range should be feasible for studying strong electroweak symmetry breaking [4], lepton-number violation, and search for heavy exotic particles.

The R&D [1,5] of the muon collider is underway. The First Muon Collier (FMC) will have a 200 GeV muon beam on a 200 GeV anti-muon beam, which could possibly be at the Fermilab [5]. With the existing Tevatron proton beam the muon-proton collision becomes a possible option. It would be a 200 GeV \(\otimes\) 1 TeV \(\mu p\) collider, where the first energy is the energy of the muon beam and the second energy is the proton beam energy. The existing lepton-proton collider is the \(ep\) collider at HERA. Lepton-proton colliders have been proved to be successful by the physics results from HERA. In this work, we shall discuss the physics potential of the \(\mu p\) colliders at various energies and luminosities. Other \(\mu p\) colliders that we are considering are 50 GeV \(\otimes\) 1 TeV, 1 TeV \(\otimes\) 1 TeV, and 2 TeV \(\otimes\) 3 TeV. The center-of-mass energies and luminosities of these various designs are summarized in Table 1. The nominal yearly luminosity of the 200 GeV \(\otimes\) 1 TeV \(\mu p\) collider is

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1) Invited talk present at the Fourth International Conference on the Physics Potential and Development of \(\mu^+\mu^-\) Colliders, San Francisco CA, December 1997.
about 13 fb$^{-1}$. Luminosities for other designs are roughly scaled by one quarter power of the muon beam energy and given in Table 1.

**PHYSICS POTENTIAL**

The physics opportunities of $\mu p$ colliders are similar to those of $ep$ colliders, but the sensitivity reach might be very different, which depends on how precise the particles can be identified and measured in $ep$ and $\mu p$ environments. Similar to $ep$ colliders the proton structure functions can be measured to very large $Q^2$ and small $x$ in $\mu p$ colliders of higher energies. At HERA, the $Q^2$ has been measured up to $Q^2 \simeq 10^4$ GeV$^2$ and $x$ down to $x \sim 3 \times 10^{-4}$. At the 200 GeV $\otimes$ 1 TeV $\mu p$ collider the $Q^2$ can be measured up to $10^6$ GeV$^2$. In addition, QCD studies, search for supersymmetry and other exotic particles can be carried out at the $\mu p$ colliders. Here we are particularly interested in the leptoquarks, leptogluons, $R$-parity violating squarks, and the contact interactions that are specific to the muon. The goal here is to estimate the sensitivity reach for these new physics at various energies and luminosities.

**Leptoquarks**

The second generation leptoquarks made up of a muon and a charm or strange quark are particularly interesting at the $\mu p$ collider because they can be directly produced in the $s$-channel processes, e.g.,

$$\mu^\pm c \rightarrow L_{\mu c}^0 .$$  \hspace{1cm} (1)

It is conventional to assume no inter-generational mixing in order to prevent the dangerous flavor-changing neutral currents. The $s$-channel production will give spectacular enhancement in the invariant mass $M$ of the muon and the hadronic final state, or the $x = s/M^2$ distribution.

The Lagrangian of the second generation leptoquark with the muon and charm and strange quarks is given by

$$\mathcal{L} = \lambda_{\mu q}^0 \bar{q} \mu L_{\mu q}^0 + \lambda_{\mu q}^1 \bar{q} \gamma_\mu \mu L_{\mu q}^{1\mu} + h.c. ,$$ \hspace{1cm} (2)

| $\sqrt{s}$ (GeV) | $\mathcal{L}$ (fb$^{-1}$) |
|------------------|------------------|
| 30 GeV $\otimes$ 820 GeV | 314 | 0.1 |
| 50 GeV $\otimes$ 1 TeV | 447 | 2 |
| 200 GeV $\otimes$ 1 TeV | 894 | 13 |
| 1 TeV $\otimes$ 1 TeV | 2000 | 110 |
| 2 TeV $\otimes$ 3 TeV | 4899 | 280 |
where \( q = c, s \) and the superscripts \((0, 1)\) on the leptoquark field denote the scalar and the vector leptoquarks, respectively. The production cross section of the leptoquark at the \( \mu p \) collider is given by

\[
\sigma = \frac{\pi \lambda^2}{4s} q(x, Q^2) \times (J + 1),
\]

where \( J \) is the spin of the leptoquark and \( q(x, Q^2) \) is the parton luminosity.

**Leptogluons**

A leptogluon has a spin of either \( 1/2 \) or \( 3/2 \), a lepton quantum number (in this case it is the muon), and a color quantum number (the same as gluon.) The interaction Lagrangian for a spin \( 1/2 \) leptogluon is given by

\[
\mathcal{L} = g_s \frac{M_{L\mu g}}{2\Lambda^2} \bar{L} \sigma^{\mu\nu} L \sigma_{\mu
u} \delta_{ab} + \text{h.c.,}
\]

where \( \Lambda \) is the scale that determines the strength of the interaction. The decay width of the leptogluon into a muon and a gluon is given by

\[
\Gamma(L_{\mu g} \rightarrow \mu g) = \frac{\alpha_s M_{L\mu g}^5}{2\Lambda^4}.
\]

The leptogluon can be produced in \( s \)-channel in a \( \mu p \) collider and the production cross section is given by

\[
\sigma = \frac{4\pi^2\alpha_s}{s} \left( \frac{M_{L\mu g}^2}{\Lambda^2} \right)^2 g(x, Q^2),
\]

where \( g(x, Q^2) \) is the gluon luminosity.

**\( R \)-parity Violating Squarks**

\( R \)-parity is in general assumed in supersymmetry, but there is no theoretical reasons why \( R \)-parity should conserve. \( R \)-parity violation is included by introducing additional terms in the superpotential:

\[
\mathcal{W}_R = \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k + \mu_i L_i H_u,
\]

where \( L, \bar{E}, Q, \bar{U}, \bar{D}, H_u \) are superfields. The relevant term in the superpotential for the direct production of the \( R \)-parity violating squark at the \( \mu p \) collider is \( \lambda'_{ijk} L_i Q_j \bar{D}_k \). The corresponding Lagrangian is
\[ \mathcal{L}_{LQ_jD_k} = \lambda'_{ijk} \left[ \bar{e}_{iL} d_{kR} u_{jL} + \bar{u}_{jL} d_{kR} e_{iL} + \bar{d}_{kR} (\bar{e}_{iL})^c u_{jL} \right. \\
\left. - \bar{\nu}_{iL} d_{kR} \nu_{jL} - \bar{\nu}_{jL} d_{kR} \nu_{iL} - \bar{d}_{kR} (\bar{\nu}_{iL})^c d_{jL} \right] + h.c. \]  

(8)

where \( i, j, k \) are the family indices, and \( c \) denotes the charge conjugate. The \( R \)-parity violating squarks can be considered special scalar leptoquarks.

The cross section for \( \mu^+p \rightarrow \tilde{t}_L \rightarrow \mu^+X \) is given by

\[ \sigma_{\tilde{t}L} = \frac{\pi |\lambda'_{231}|^2}{4s} d \left( \frac{m^2_{\tilde{t}_L}}{s}, Q^2 = m^2_{\tilde{t}_L} \right), \]

(9)

where \( d \) is the down-quark luminosity. The above formula can be easily changed to represent the production of other squarks with the corresponding subscripts in \( \lambda' \) and the parton luminosity. If kinematically allowed the leptoquarks, leptogluons, and the \( R \)-parity violating squarks are produced in \( s \)-channel and thus give rise to spectacular enhancement in a single bin of the invariant mass \( M \) distribution or the \( x \) distribution.

### Contact Interactions

The effective four-fermion contact interactions can arise from fermion compositeness or exchanges of heavy particles like heavy \( Z' \), heavy leptoquarks, or other exotic particles. The conventional effective Lagrangian of \( llqq \) (\( l = e, \mu \)) contact interactions has the form [6]

\[ L_{NC} = \sum_q \left[ \eta_{LL} \left( \bar{l}_L \gamma_{\mu} l_L \right) \left( \bar{q}_L \gamma^\mu q_L \right) + \eta_{RR} \left( \bar{l}_R \gamma_{\mu} l_R \right) \left( \bar{q}_R \gamma^\mu q_R \right) \right. \\
+ \eta_{LR} \left( \bar{l}_L \gamma_{\mu} l_L \right) \left( \bar{q}_R \gamma^\mu q_R \right) + \eta_{RL} \left( \bar{l}_R \gamma_{\mu} l_R \right) \left( \bar{q}_L \gamma^\mu q_L \right) \right], \]

(10)

where the eight independent coefficients \( \eta_{\alpha\beta}^{lu} \) and \( \eta_{\alpha\beta}^{ld} \) have dimension \((\text{TeV})^{-2}\) and are conventionally expressed as \( \eta_{\alpha\beta}^{lu} = \epsilon g^2 / \Lambda_{iq}^2 \) with a fixed \( g^2 = 4\pi \).

We introduce the reduced amplitudes \( M_{\alpha\beta}^{\mu q} \), where the subscripts label the chiralities of the initial lepton (\( \alpha \)) and quark (\( \beta \)). The SM tree level reduced amplitude for \( \mu q \rightarrow \mu q \) is

\[ M_{\alpha\beta}^{\mu q}(\hat{t}) = -\frac{e^2 Q_q}{\hat{t}} + \frac{e^2}{\sin^2 \theta_w \cos^2 \theta_w} \frac{g_{\alpha}^{f} g_{\beta}^{f}}{t - m_Z^2}, \quad \alpha, \beta = L, R \]

(11)

where \( \hat{t} = -Q^2 \) is the Mandelstam variable, \( g_{\alpha}^{f} = T_{3f} - \sin^2 \theta_w Q_f \) and \( g_{\beta}^{f} = -\sin^2 \theta_w Q_f \), and \( T_{3f} \) and \( Q_f \) are, respectively, the third component of the SU(2) isospin and the electric charge of the fermion \( f \) in units of the proton charge, and \( e^2 = 4\pi \alpha_{em} \). The new physics contributions to the reduced amplitudes \( M_{\alpha\beta} \) from the \( \mu\mu qq \) contact interactions of Eq. (10) are
$$\Delta M_{\alpha\beta} = \eta^{iq}_{\alpha\beta}, \quad \alpha, \beta = L, R.$$  \hspace{1cm} (12)

The differential cross section are given by

$$\frac{d\sigma(\mu^+p)}{dx \; dy} = \frac{sx}{16\pi} \left\{ u(x, Q^2) \left[ |M_{\mu u}^{LL}|^2 + |M_{\mu u}^{LR}|^2 + (1 - y)^2 \left( |M_{\mu u}^{LL}|^2 + |M_{\mu u}^{RR}|^2 \right) \right] + d(x, Q^2) \left[ |M_{\mu u}^{RL}|^2 + |M_{\mu u}^{RR}|^2 + (1 - y)^2 \left( |M_{\mu u}^{LL}|^2 + |M_{\mu u}^{RL}|^2 \right) \right] \right\} \hspace{1cm} (13)$$

$$\frac{d\sigma(\mu^-p)}{dx \; dy} = \frac{sx}{16\pi} \left\{ u(x, Q^2) \left[ |M_{\mu u}^{LL}|^2 + |M_{\mu u}^{RR}|^2 + (1 - y)^2 \left( |M_{\mu u}^{LL}|^2 + |M_{\mu u}^{RL}|^2 \right) \right] + d(x, Q^2) \left[ |M_{\mu u}^{RL}|^2 + |M_{\mu u}^{RR}|^2 + (1 - y)^2 \left( |M_{\mu u}^{LL}|^2 + |M_{\mu u}^{RL}|^2 \right) \right] \right\} \hspace{1cm} (14)$$

The above contact interactions do not enhance the cross section in a single bin of the invariant mass distribution like the light leptoquarks do, instead, contact interactions enhance the cross section at the large $Q^2$ tail.

**SENSITIVITY REACH**

The 95% sensitivity of the contact interaction scale that can be reached by $\mu p$ colliders at various center-of-mass energies and luminosities are performed in the following. We use the $x$-$y$ distribution to investigate the sensitivity to the new contact interactions. We divide the $x$-$y$ plane into a grid with $0.05 < x < 0.95$ and $0.05 < y < 0.95$ and 0.1 interval in both $x$ and $y$ directions. We calculate the number of events predicted by the standard model in each bin, call it $n_{i}^{sm}$. We use an overall efficiency of 0.8. We assume that the observed number of events is given by the standard model. We vary one $n_{\alpha\beta}^{iq}$ at a time while keeping others zero and

| TABLE 2. | The 95% sensitivity of the $\Lambda^{iq}_{\alpha\beta}$, ($\alpha, \beta = L, R$; $q = u, d$) that can be reached at the various $\mu^+p$ colliders, by assuming that what will be observed is given by the SM prediction. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\sqrt{s}$ (GeV) | 30GeV $\otimes$ 820GeV | 50GeV $\otimes$ 1TeV | 200GeV $\otimes$ 1TeV | 1TeV $\otimes$ 1TeV | 2TeV $\otimes$ 3TeV |
| $\mathcal{L}$ (fb$^{-1}$) | 314 | 447 | 894 | 2000 | 4899 |
| $\Lambda^{uu}_{LL}$ | + | 2.4 | 4.0 | 8.9 | 24 |
| $\Lambda^{dd}_{LL}$ | 1.9 | 3.4 | 6.5 | 17 |
| $\Lambda^{uu}_{LR}$ | 1.2 | 2.1 | 5.6 | 14 |
| $\Lambda^{dd}_{LR}$ | 0.8 | 1.5 | 3.7 | 9.9 |
| $\Lambda^{uu}_{RL}$ | 1.1 | 2.1 | 5.6 | 10 |
| $\Lambda^{dd}_{RL}$ | 1.2 | 2.2 | 5.6 | 14 |
| $\Lambda^{uu}_{RR}$ | 0.8 | 1.5 | 3.7 | 9.9 |
| $\Lambda^{dd}_{RR}$ | 0.8 | 1.5 | 3.7 | 9.9 |
calculate the predicted number of events in each bin, call it \( n^\text{th}_i \). We then calculate the \( \chi^2 \) using

\[
\chi^2 = \sum_i 2(n^\text{th}_i - n^\text{sm}_i) + 2n^\text{sm}_i \log \left( \frac{n^\text{sm}_i}{n^\text{th}_i} \right),
\]

where the sum is over all \( 9 \times 9 \) bins. We know that for a larger \( \eta \) we will obtain a larger \( \chi^2 \), which means that it is really different from the standard model beyond statistical fluctuation. Here we have 80 degree of freedom, and so for a 95\% CL we set \( \chi^2 = 102 \). We then repeat for another \( \eta \).

The sensitivity reach of \( \Lambda_{\mu q}^{\alpha \beta} \) is tabulated in Table 2. The sensitivity reach depends on the sign of the contact term. The maximum reach of \( \Lambda \) at each center-of-mass energy roughly scales as \( \Lambda \sim 10 \sqrt{s} \). The effect of luminosity on \( \Lambda \) is rather small: \( \Lambda \) only scales as the 1/4th power of the luminosity.

To estimate the sensitivity reach for \( R \)-parity violating squarks we start with \( \lambda'_{231} \) for the top squark and the down quark luminosity. We assume the enhancement in cross section is in the mass bin of \( 0.9 \, m_{\tilde{t}_L} < m < 1.1 \, m_{\tilde{t}_L} \). We calculate the number of events predicted by the standard model in this bin, call it \( n^\text{sm}_i \). Again, we use an overall efficiency of 0.8. Then we use the poisson statistics to estimate the \( n^\text{th}_i \) that \( n^\text{sm}_i \) can fluctuate to at the 95\% CL:

\[
\sum_{n=0}^{n^\text{th}_i} \frac{(n^\text{sm}_i)^n e^{-n^\text{sm}_i}}{n!} > 0.95
\]

| TABLE 3. 95\% sensitivity on \( \lambda'_{231} \) for a few choices of \( m_{\tilde{t}_L} \) at various \( \mu^+ p \) colliders. |
|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \sqrt{s} \) (GeV) | 30GeV \( \otimes \) 820GeV | 50GeV \( \otimes \) 1TeV | 200GeV \( \otimes \) 1TeV | 1TeV \( \otimes \) 1TeV | 2TeV \( \otimes \) 3TeV |
| \( \mathcal{L} (\text{fb}^{-1}) \) | 314 | 447 | 894 | 2000 | 4899 |
| \( m_{\tilde{t}_L} \) (GeV) | 0.1 | 2 | 13 | 110 | 280 |
| 200 | 0.014 | 0.0045 | 0.0025 | 0.0015 | 0.0010 |
| 300 | - | 0.0094 | 0.0032 | 0.0018 | 0.0014 |
| 400 | - | 0.095 | 0.0041 | 0.0021 | 0.0017 |
| 500 | - | - | 0.0056 | 0.0024 | 0.0019 |
| 600 | - | - | 0.0086 | 0.0027 | 0.0021 |
| 700 | - | - | - | 0.016 | 0.0030 |
| 800 | - | - | - | 0.045 | 0.0033 |
| 900 | - | - | - | - | 0.0037 |
| 1000 | - | - | - | - | 0.0043 |
| 1500 | - | - | - | - | 0.0053 |
| 2000 | - | - | - | - | 0.0043 |
| 2500 | - | - | - | - | 0.0056 |
| 3000 | - | - | - | - | 0.0078 |
| 3500 | - | - | - | - | 0.013 |
| 4000 | - | - | - | - | 0.024 |
| 4500 | - | - | - | - | 0.081 |
where $n^{th}$ is the first $n$ that the above inequality is satisfied. Once the $n^{th}$ is obtained the $\lambda_{231}'$ can be obtained using Eq. (9).

The sensitivity reach of $\lambda_{231}'$ is tabulated in Table 3. We have also calculated the sensitivity reach of $\lambda_{232}'$ using the strange quark luminosity. We found that the reach is typically worse than that of $\lambda_{231}'$: for small $m_{\tilde{t}}$ the reach is about a factor of two worse while for large $m_{\tilde{t}}$ the reach can be ten times worse. This is because the strange quark luminosity is rather large at small $x$ but very small at large $x$.

The results for the second generation leptoquarks and leptogluons are summarized in Table 4 [7]. Here only a simple criteria is defined for the discovery of the leptoquarks and leptogluons. Assuming no background and requiring five signal events for the discovery the sensitivity reach is at 99% CL.

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**TABLE 4.** 99% sensitivity reach on the coupling $\lambda$ for the second generation leptoquarks and the new physics scale $\Lambda$ for leptogluon via the resonance production $\mu\bar{p} \rightarrow L$.

| $\sqrt{s}$ (GeV) | $30\text{GeV} \otimes 820\text{GeV}$ | $50\text{GeV} \otimes 1\text{TeV}$ | $200\text{GeV} \otimes 1\text{TeV}$ | $2\text{TeV} \otimes 4\text{TeV}$ |
|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| $\mathcal{L}(fb^{-1})$ | 314 | 447 | 894 | 5657 |
| $M^0_{\mu c}$ | 200 GeV | 300 GeV | 500 GeV | 1500 GeV |
| $\lambda^0_{\mu c}$ | 0.089 | 0.043 | 0.010 | 0.0014 |
| $M^0_{\mu s}$ | 200 GeV | 300 GeV | 500 GeV | 1500 GeV |
| $\lambda^0_{\mu s}$ | 0.068 | 0.034 | 0.0080 | 0.0011 |
| $M^1_{\mu c}$ | 200 GeV | 300 GeV | 500 GeV | 1500 GeV |
| $\lambda^1_{\mu c}$ | 0.063 | 0.031 | 0.0072 | 0.0010 |
| $M^1_{\mu s}$ | 200 GeV | 300 GeV | 500 GeV | 1500 GeV |
| $\lambda^1_{\mu s}$ | 0.048 | 0.024 | 0.0055 | 0.0008 |
| $M_{\mu g}$ | 200 GeV | 300 GeV | 500 GeV | 1500 GeV |
| $\Lambda_{\mu g}$ (TeV) | 20 | 49 | 190 | 1700 |