Meteorite hazard model for a space mission to Mars

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Abstract. Currently, for the world’s space agencies, the robotic exploration of Mars is one of the most important tasks. One of the necessary stages for the implementation of this mission is the development and addition of new information to the State standard “Meteoric substance, spatial distribution model”. Until now, the State Standard has been more detailed in comparison with the American analogue (developed by NASA) and the European one. The standard is a mandatory document in the design of spacecraft. It should be noted that modeling of meteor hazard at a distance from Earth to Mars is a complex problem, since the analysis of the meteor population in near-Earth space does not give a complete picture of the propagation of meteoroids along the Earth-Mars route. Moreover, the further the trajectory of the spacecraft from the Earth’s orbit is, the less the number of near-Earth meteorites becomes. That is, objects that have the same orbital parameters with small bodies crossing the Earth’s orbit. The only way to solve this problem is to build an interpolation regression model, which is based on measurements from the Earth’s surface and observations of space missions. For this purpose, the density of sporadic meteoroids was transformed from the space mission coordinate system to the ground one. This was done in order to analyze meteorite observations by the Mariner 4 and Pioneer 10 spacecrafts. The results of the work made it possible to obtain new data for the spatial distribution of meteoroids on the Earth-Mars path. According to a comparison of our data with the data on the density of space debris in the previous works the most safe for space flights are normalization conditions of distributions of the elements of the orbits of meteoric bodies $P(Z,e,i) < 60$.

1. Introduction

In connection with the planned flights to Mars, there is a problem of ensuring the safety of the functioning of technology and scientific equipment, as well as the life of the crew during a manned flight.

Among the threats to long-term space flight, the meteoroid hazard plays an important role. To predict the meteor hazard, it is necessary to have an idea of the structure of the complex of meteoric bodies in the Solar System. One of the important components of the meteoric complex is a group of single meteoric bodies. The structure of the complex of sporadic meteoric objects (SMO) is determined by the integral flux density and the distribution of heliocentric velocities and radiants of the SMO. At present, structure of the SMO complex in the vicinity of the Earth’s orbit is most reliably known. The SMO complex is described in detail in GOST “Meteoric substance. Spatial distribution model”. The Mars Science Laboratory (MSL) and the third-generation rover Curiosity [1] were
actively working on the surface of Mars. The Mars rover "Perseverance" has begun its observations recently. Our work is aimed at modeling the meteorite hazard during the flight from Earth to Mars [2].

For the creation of a regression interpolation model, data on the zodiacal light [3] and measurements of the meteorite environment of Pioneer 10, Pioneer 11, Mariner 2, Mariner 4, Mars Phoenix, Trace Gas Orbiter and Atmosphere and Volatile EvolutioN (MAVEN) [4, 5] space missions were used. It should be said that the surface of Mars and its moons – Phobos and Deimos – is a source of secondary emissions of meteoroid bodies during their bombardment by meteorites from the asteroid belt [6].

2. Distributions of heliocentric radiants and velocities for heliocentric distances other than 1 au (astronomical unit)

The elongated orbits, close in shape to parabolas, have a significant value of the semi-major axis of the orbit \(a\). Then the value of the eccentricity \((e)\) is close to 1. In this case to reduce the numerical value of \(a\) is introduced the \(Z\) coordinate (reciprocal semi-major axis of the orbit). The orbital elements of particles \(Z = 1/a\) and \(e\), crossing the Earth's orbit, occupy the inner part of the triangular region in the \(Z, e\) coordinate system, bounded by straight lines \(q = 1 = (1 - e)/Z\) and \(Q = 1 = (1 + e)/Z\), where \(q\) is the perihelion distance, \(Q\) is the aphelion distance. For meteors crossing the Earth's orbit, based on the distributions of heliocentric velocities and radiants, it is possible to determine the conditional distributions of the elements of the orbits of meteoric bodies \(P(Z, e, i)\), where \(i\) is orbital inclination angle. Opic E.J. method [7] can be used to determine \(P(Z, e, i)\):

\[
P(Z, e, i) = \frac{1 - e^{3/2}}{q} \times \frac{3 - \frac{1 - e}{q} - 2 \sqrt{q(1 + e) \cos i}}{\sin i \sqrt{2 - \frac{1 - e}{q} q(1 + e)}}.
\]

The flux of SMO, which poses a threat to the structural elements of the spacecraft, depends on the structure of the SMO complex, the trajectory of the spacecraft, and the location of the structural elements on the surface of the spacecraft. Knowledge of the parameters of the spacecraft orbit is necessary to determine the direction from which meteoric bodies approach the spacecraft and, therefore, the possibility of creating an effective surface of the structure (see, for example, [1]).

The flow to a single area arbitrarily oriented in space, which is a part of the spacecraft structure, is determined by the ratio:

\[
N = N(m) I,
\]

here \(N(m)\) – flux density of meteoric bodies with a mass greater than \(m\) and \(I\) is flow rate.

If \(m > 10^{-6}\), then \(N(m)\) is calculated by the formula:

\[
N(m) = 4 \times 10^{-14} \Phi(r) m^{-1/2},
\]

\[
I = \iiint H \, d\tau \, d\eta \, dW,
\]

\[
H = f \, P_\epsilon(V) \, P(e, \tau)(W/V)^3 \, \sin^2 \tau \, \cos \eta \, \sin \epsilon,
\]

where \(V, e, \psi\) are the values of the velocity (km/s) and the coordinates of the radiant (degree) of the meteoroid in the heliocentric coordinate system;

\(W, \tau\) and \(\eta\) are the values of the velocity (km/s) and coordinates of the radiant (degree) of the meteoroid in the coordinate system associated with the exposed area;
\( P_e(V) \) – conditional distribution of the velocities of meteoric bodies at a fixed value of \( e \) (s km\(^{-1}\)) in the heliocentric coordinate system;

\( P(e, \psi) \) – two-dimensional distribution of radiant density, degree\(^{-2}\), (probability density of meteoric bodies appearing in the \( e, \psi \) direction in the entire range of velocities and masses) in the heliocentric coordinate system;

\( \Phi(r) \) is a function that determines the change in the flux density of meteoric bodies depending on the distance from the Sun \( r \) between the orbits of the Earth and Mars;

\( f(\tau, \psi) \) is a function that characterizes the obstruction of the area by spacecraft construction elements.

The spherical coordinate system \( e, \psi \) is chosen in such a way that the pole \( e^0 \) coincides with the direction of the velocity vector of a hypothetical body moving at a distance \( R \) from the Sun in a circular orbit. For the Earth’s orbit (\( R = 1 \) au) \( e^0 \) corresponds to the apex of the Earth’s motion, \( e \) is the elongation angle measured from the apex. The angle \( \psi \) is measured from the ecliptic plane (towards the Sun towards – in the direction of north pole of the ecliptic) to the plane passing through the point with the given coordinates \( e, \psi \) and the point \( e^0 = 180^\circ \). Angle range: \( 0^\circ \leq e \leq 180^\circ, 0^\circ \leq \psi \leq 360^\circ \).

The spherical coordinate system \( \tau, \eta \) is associated with the projection of the normal to the unit area under consideration, the direction \( \tau^0 \) and the angle \( \eta \) are measured from the plane in which the points \( e = 0^\circ, e = 180^\circ, \tau = 0^\circ \) lie. It should be considered that the coordinate system \( \tau, \eta \) moves in relation to the coordinate system \( e, \psi \) with the speed and in the direction corresponding to the velocity vector of the spacecraft, therefore, there is a functional dependence between the values \( e, \psi, V \) and \( \tau, \eta, W \).

Due to the established symmetry in the radiant distribution, the values of the function \( P(e, \psi) \) are equal to the values \( P(e, \psi - 180^\circ), P(e, 180^\circ + \psi) \) and \( P(e, 360^\circ - \psi) \). Therefore, in the GOST “Meteoric Substance. Spatial Distribution Model \( P(e, \psi) \) values are given for \( 0^\circ \leq \psi \leq 90^\circ \).

The values of the functions \( P_e(V) \) and \( P(e, \psi) \) are determined from special tables for the heliocentric coordinates \( e, \psi \) and the velocity \( V \) of the meteoroid. Due to the tabular assignment of information about the complex of meteoric bodies, in order to calculate the integral (4), it is necessary to replace it with a triple sum.

It should be noted that the distribution of radiants and velocities were obtained from ground-based observations of meteors. As a result, they provide information on meteors in that part of the SMO complex that can cross the Earth’s orbit. These are meteoric bodies with perihelion distances less than or equal to 1 au, and aphelion distances greater than or equal to 1 au. Such meteoroids make up a fairly small part of the entire SMO complex. In order to make up for the lack of information about the part of the complex that is inaccessible for observation from the Earth, one can act in several ways.

One way is to assume that the distributions of heliocentric radiants and SMO velocities at any heliocentric distance are the same as in the vicinity of the Earth’s orbit, i.e. correspond to the near-Earth model of the SMO complex. Obviously, this method abstracts from additional information about meteoric bodies that do not cross the Earth’s orbit.

3. Extrapolation of orbital elements distributions \( P(Z, e, i) \)

Another way is that conditional distributions of eccentricities and reverse semi-major axes (\( Z = 1/a \)) are determined from the model distributions of heliocentric radiants and SMO velocities. Here the criterion is the value of the inclination of the meteoroid orbit. Based on the extrapolation of these model distributions approximated by spline functions, the distributions of orbital elements at other (other than 1 au) heliocentric distances are determined. After that, from these extrapolated distributions of orbital elements, the distribution of heliocentric radiants and SMO velocities at a given heliocentric distance is determined.

The planet’s circular orbit of a radius \( R \) can be crossed by celestial bodies that have \( q \leq R \) and \( Q \geq R \).
Thus,
\[
\frac{(1 - e)}{Z} \leq R \leq \frac{(1 + e)}{Z}.
\] (6)

Figure 1 shows the areas of penetration of meteoric bodies into the orbits of the Earth and Mars. The Orbital Element Distributions (OED) of meteoric bodies are found analytically: using the Jacobian of the functional transformation of the absolute value of heliocentric velocity and the coordinates of the heliocentric radiant to the desired SMO orbital elements. Moreover, a transformation occurs from the flux density to the number of particles crossing the sphere with a radius of 1 au and moving in orbits with given elements [2].

OED \(P(a, e, i))\) is determined based on the distribution of heliocentric velocities and radiant density \(P(V, E, \Psi)\) using the following relation:

\[
P(Z, e, i) = f \sin E \frac{P(V, E, \Psi) D(V, E, \Psi)}{D(a, e, i)},
\] (7)

here

\[
f = a^{3/2} \cos \sum \sin i,
\] (8)

[4], and

\[
\cos \sum = \sin E \cos \Psi.
\] (9)

![Figure 1. Penetration areas of small celestial bodies](image)

The Jacobian of the functional transformation is as follows:
\[
J = \frac{D(a,e,i)}{D(V,E,\Psi)} = \begin{vmatrix}
\frac{\partial a}{\partial V} & \frac{\partial a}{\partial E} & \frac{\partial a}{\partial \Psi} \\
\frac{\partial e}{\partial V} & \frac{\partial e}{\partial E} & \frac{\partial e}{\partial \Psi} \\
\frac{\partial i}{\partial V} & \frac{\partial i}{\partial E} & \frac{\partial i}{\partial \Psi}
\end{vmatrix}.
\]  

(10)

From the functional dependences of the components of the heliocentric velocity vector on the orbital elements, one can obtain [3] that

\[
j = 2 \nu^3 a/(\cos \Psi \sin^2 E).
\]

(11)

The Jacobian

\[
(J_{22})^{-1} = 2 \nu^3 \cos \Psi \sin^2 E/\{a \ e\}
\]

(12)
transforms the distribution \(P(V,E,\Psi)\) to the corresponding OED.

Thus, the OED is as follows:

\[
P(Z,e,i) = P(V,E,\Psi) e a^{5/2} \sin i /\{2 \nu^3\}
\]

(13)

**Table 1.** Distributions \(P(Z,e,i)\) depending on \(Z\) and \(e\) for the orbital inclination angle \(i = 5^\circ\)

| \(Z\) \(\backslash e\) | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.45 |
|---|---|---|---|---|---|---|---|---|
| 0,35 | 753,24 | 699,16 | 604,30 | |
| 0,45 | 419,30 | 816,53 | 563,74 | 455,35 |
| 0,55 | 100,92 | 351,19 | 615,76 | 428,31 | 306,41 |
| 0,60 | 102,04 | 407,98 | 96,50 | 283,09 | 487,00 | 292,89 | 157,46 |
| 0,65 | 17,83 | 90,60 | 342,14 | 92,07 | 214,99 | 322,23 | 157,46 | 48,38 |
| 0,70 | 20,42 | 79,16 | 276,30 | 87,64 | 146,88 | 157,46 | 48,38 | 72,61 |
| 0,75 | 23,01 | 67,71 | 210,46 | 83,21 | 78,78 | 26,17 | 39,28 | 39,28 |
| 0,80 | 25,61 | 56,27 | 114,62 | 78,78 | 26,17 | 26,17 | 39,28 | 13,98 |
| 0,85 | 28,20 | 44,83 | 78,78 | 61,99 | 26,17 | 39,28 | 39,28 | 13,98 |
| 0,90 | 30,80 | 33,39 | 26,27 | 12,21 | 12,21 | 18,33 | 7,14 | 7,14 |
| 0,95 | 33,39 | 25,54 | 26,27 | 12,21 | 18,33 | 7,14 | 7,14 |
| 1,00 | 31,60 | 25,54 | 29,22 | 12,21 | 18,33 | 7,14 | 7,14 |
| 1,05 | 31,60 | 25,54 | 29,22 | 12,21 | 18,33 | 7,14 | 7,14 |
| 1,10 | 12,29 | 9,40 | 9,67 | 5,00 | 7,50 | 7,50 | 3,25 | 3,25 |
| 1,15 | 12,29 | 9,40 | 9,67 | 5,00 | 7,50 | 7,50 | 3,25 | 3,25 |
| 1,20 | 1,81 | 1,81 | 2,72 | 1,31 |
| 1,25 | 0,57 | 0,86 | 0,86 |
| 1,30 | 1,06 | 0,57 | 0,86 |
| 1,35 | 1,06 | 0,57 |
| 1,40 | 1,06 | 0,57 |
Table 1 shows an example of the distribution of the orbital elements $Z$ and $e$ for the orbital inclination angle $i = 5^\circ$. Bold and italic fonts show part of the distributions obtained by extrapolating the distributions related to the area available for observations from the Earth.

As a result, two-dimensional distributions of orbital elements are modeled for different inclination angles. The reverse recalculation of the distributions $P(Z, e, i)$ gives the distributions of heliocentric velocities and radiants $P(V, E, \Psi)$ for a heliocentric distance greater than 1 au.

4. Summary and conclusions

Space missions have shown, based on measurements of three scanners mounted on a spacecraft platform, that there is an exponential increase in the SMO flux density on the trajectory from Earth to Mars orbit [8]. The density increases 2 times for bodies weighing more than 6-10 g and 6 times for meteoroids in the mass range 10-12 g [9]. The flux density sharply decreases to normal outside the orbit of Mars [10]. For the first time, this hypothesis and theoretical studies were carried out at St. Petersburg State University, and our studies have confirmed these conclusions. When analyzing the height of the geopotential of the northern hemisphere, the hypothesis of the existence of geostrophic winds was confirmed [11]. It is known that tidal temperatures have a significant impact on the predicted movement of geostrophic winds, and the determination of altitude characteristics is a rather difficult problem [12]. Therefore, the development of appropriate methods for accounting for these processes is a new and promising task [13]. As a result, a study of near-Earth space, including the lower atmosphere, lower upper mesosphere, lower thermosphere and ionosphere, was carried out using radiophysical methods, and the parameters of the influence of the dynamics of the lower and middle atmosphere on the propagation of radio waves were determined [14].

The results of the work can be used in projects for the preparation of space missions to the bodies of the solar system [15]. Из аналогичного сопоставления плотности метеорного вещества с плотностью космического мусора [16] можно сделать вывод, что безопасным пределом являются значения $P(Z, e, i) < 60$.

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