Weak magnetism in chiral quark models

Tommy Ohlsson\textsuperscript{a}, Håkan Snellman\textsuperscript{b}

Theoretical Physics, Department of Physics, Royal Institute of Technology, SE-100 44 Stockholm, Sweden

Received: 29 June 1999

Abstract. We discuss symmetry breaking in the weak magnetism form factors for the semileptonic octet baryon decays. In the chiral quark model, the symmetry breaking can be accounted for in the masses and the quark spin polarizations can take on more general values due to Goldstone boson depolarization. Here we clarify some features of the chiral quark model prediction for the weak magnetism and compare to the corresponding result of the chiral quark soliton model.

1 Introduction

The weak interaction of the baryon octet has been a source of much information concerning the dynamics of the octet baryons. Especially the weak interactions can probe details of the hadronic structure at low energies.

Already early, the conserved vector current (CVC) hypothesis\[1\] extended to flavor SU(3) symmetry made predictions for the so called weak magnetism form factors.

In the quark model (QM), these predictions are easily rederived in the same limit. Introducing explicit SU(3) symmetry breaking through the masses of baryons and quarks, the QM model seems to differ in some respects with the result of the chiral quark soliton model\[3\]. Here we clarify some features of the chiral quark model prediction for the weak magnetism and compare to the corresponding result of the chiral quark soliton model.

2 The weak form factors

The transition matrix element $\mathcal{M}_{B\rightarrow B' l^- \bar{v}_l}$ for the decay $B\rightarrow B' + l^- + \bar{v}_l$ ($q\rightarrow q' + l^- + \bar{v}_l$), is given by

$$\mathcal{M}_{B\rightarrow B' l^- \bar{v}_l} = \frac{G}{\sqrt{2}} V_{qq'} \langle B'(p') | J^\mu_{\text{weak}} | B(p) \rangle L_\mu,$$ (1)

where $G$ is the Fermi coupling constant, $V_{qq'}$ is the $qq'$-element of the Cabibbo–Kobayashi–Maskawa mixing matrix, and $L_\mu$ is the leptonic current.

The hadronic weak current is

$$J^\mu_{\text{weak}} = J^\mu_V - J^\mu_A,$$ (2)

where $J^\mu_V$ is the vector current and $J^\mu_A$ is the axial-vector current.

The matrix element of the vector current in momentum space of the transition $B \rightarrow B' + l^- + \bar{v}_l$ is given by

$$\langle B'(p') | J^\mu_V | B(p) \rangle = \bar{u}(p') \left( f_1(q^2) \gamma^\mu - i \frac{f_2(q^2)}{M_B + M_{B'}} \sigma^{\mu\nu} q_\nu + \frac{f_3(q^2)}{M_B + M_{B'}} q^\mu u(p) \right),$$ (3)

where $M_B$ ($M_{B'}$), $p$ ($p'$), $u(p)$ ($u'(p')$), and $|B(p)\rangle$ ($|B'(p')\rangle$) are the mass, momentum, Dirac spinor, and external baryon state of the initial (final) baryon $B$ ($B'$), respectively, and $q = p - p'$ is the momentum transfer\[3\]. The functions $f_i(q^2)$, where $i = 1, 2, 3$, are the vector current form factors. These form factors are Lorentz scalars and contain information about the hadron dynamics. $f_1$ is the vector form factor, $f_2$ is the induced tensor form factor (or weak magnetism form factor or anomalous magnetic moment form factor), and $f_3$ is the induced scalar form factor.

In a previous paper\[3\], we have derived the form factors $f_i \equiv f_i(0)$, where $i = 1, 2, 3$, in the QM. Up to terms linear in the parameters $E \equiv \Delta/\Sigma$ and $\epsilon \equiv \delta/\sigma$, where $\Sigma = M_B + M_{B'}$, $\Delta = M_B - M_{B'}$, $\sigma = m_q + m_{q'}$, and $\delta = m_q - m_{q'}$, these form factors are

$$f_1 = f_1^{\text{QM}},$$ (4)

$$f_2 = \left( \frac{\sigma}{\sigma} G_A - 1 \right) f_1^{\text{QM}},$$ (5)

$$f_3 = \frac{\sigma}{\sigma} (E G_A - \epsilon) f_1^{\text{QM}}.$$ (6)
Here \( G_A \equiv g_1^{QM}/f_1^{QM} \), with \( f_1^{QM} \equiv \langle B'|\lambda_{qq'} \otimes 1|B \rangle \) and \( g_1^{QM} \equiv \langle B'|\lambda_{qq'} \otimes \sigma^z|B \rangle \). The \( \lambda_{qq'} \) is the SU(3) matrix that effectuates the flavor transition and the \( \sigma^z \) operator measures the spin polarizations of the quarks in the baryons.

### 2.1 The weak axial-vector form factors

The weak axial-vector form factors \( G_A \) can be obtained from the SU(6) QM expressed in terms of the parameters \( F \) and \( D \). In the \( \chi \) QM, the \( G_A \)'s are expressed in the quark spin polarizations of the proton, i.e., \( \Delta u, \Delta d, \) and \( \Delta s \). These spin polarizations differ considerably from the ones in the SU(6) QM due to the depolarization of the quark spins by the Goldstone bosons (GBs). The spin polarizations in the \( \chi \) QM are calculated with one GB emission. They are

\[
\Delta u = \frac{a}{3} - \frac{27}{7} a, \\
\Delta d = -\frac{1}{3} - \frac{2}{9} a, \\
\Delta s = -a,
\]

where \( a \) is the parameter which measures the probability of emission of a GB from a quark. Using the relations \( F = \frac{1}{2}(\Delta u - \Delta s) \) and \( D = \frac{1}{2}(\Delta u - 2\Delta d + \Delta s) \), we have

\[
G_A^{np} = \Delta u - \Delta d, \\
G_A^{\Sigma^+ - \Sigma^0} = \frac{1}{2}(\Delta u - \Delta s), \\
g_1^{QM \Sigma^+ A} = \frac{1}{\sqrt{3}}(\Delta u - 2\Delta d + \Delta s), \\
G_A^{\Sigma^- - \Sigma^0} = \Delta d - \Delta s.
\]

for the \( \Delta S = 0 \) decays and

\[
G_A^{\Sigma^- - \Sigma^0} = \Delta d - \Delta s, \\
G_A^{\Sigma^- - \Sigma^0} = \Delta u - \Delta d, \\
G_A^{\Sigma^- - \Sigma^0} = \frac{1}{3}(\Delta u + \Delta d - 2\Delta s), \\
G_A^{\Sigma^- - \Sigma^0} = \frac{1}{3}(2\Delta u - \Delta d - \Delta s), \\
G_A^{\Sigma^- - \Sigma^0} = \Delta u - \Delta d
\]

for the \( \Delta S = 1 \) decays.

The magnetic moments of the octet baryons and the weak axial-vector form factor \( G_A^{np} \) can be used to fit the parameter \( a \) and the quark magnetic moment \( \mu_d \). Using \( \mu_u = -2\mu_q \) and \( \mu_d = 2\mu_d/3 \), we then obtain \( a \approx 0.104 \) and \( \mu_d \approx -1.196 \mu_N \). This gives \( \Delta u \approx 0.90, \Delta d \approx -0.36, \) and \( \Delta s \approx -0.10 \).

In the \( \chi \) QM, the effective quark masses can be determined from the fitted value of \( \mu_d \). The quark masses are then \( m_u^{eff} = m_d^{eff} = m_s^{eff} \approx 260 \) MeV and \( m_s^{eff} = 3m_{u,d}^{eff} \approx 390 \) MeV. In the following, we will use \( m_q \equiv m_q^{eff} \), where \( q = u, d, s \).

The values of the \( G_A^{BB'} \)'s for the \( \chi \) QM are listed in Table [1], where for reference also the axial-vector form factors of the naive QM (NQM) are displayed.

### 3 The ratio \( \rho_f \) and the "weak magnetism"

We will now concentrate on the "weak magnetism" form factor \( \rho_f \), which is defined as

\[
\rho_f \equiv \frac{f_2}{f_1}. \tag{19}
\]

Inserting Eqs. (11) and (12) in Eq. (19), we obtain

\[
\rho_f = \frac{\sum a_q}{\sigma} G_A - 1. \tag{20}
\]

Since there are no linear terms in \( E \) and \( \epsilon \) in either \( f_1 \) or \( f_2 \), the formula for \( \rho_f \) is valid up to terms of second order in \( E \) and \( \epsilon \). The quark masses in this and related formulas appear as effective masses, and the parametric dependence of the quark spin polarization \( \Delta q \), where \( q = u, d, s \), on the emission probability \( a \) of GBs incorporates effects of relativistic corrections and other possible dynamical effects on both the magnetic moments and the \( \rho_f \)'s. When these effects are taken into account directly, in terms of a changed structure of the currents, the fits become worse.

The expression (20) for \( \rho_f \) above is closely related to the corresponding formula for the magnetic moments \( \mu_B \) of the octet baryons used in earlier studies. In the same approximation as here, we have

\[
f_1 = Q_B, \tag{21}
\]

\[
f_2 = \sum_{q=u,d,s} \frac{e_q}{2m_q} \Delta q - Q_B \\
\cong \frac{M_B}{M_N} \kappa_B. \tag{22}
\]

where \( e_q \) is the quark charge, \( Q_B = 0, \pm 1 \) is the charge of the baryon, and \( \kappa_B \) is the anomalous magnetic moment of the baryon in nuclear magnetons. It is therefore in principle possible to convert expression (20) above to an expression in terms of the magnetic moments. This will eliminate the parametric model dependence. In the following, we will discuss how this can be done in a way that preserves the absence of terms linear in \( E \) and \( \epsilon \) in Eq. (21).

### 3.1 The weak magnetism and CVC

From SU(3) flavor symmetry the weak magnetism form factors can be related to the magnetic moments of the
nucleons. The result is \[ \rho_f^{np} = (\mu(p) - \mu(n))/\mu_N - 1, \] \[ \rho_f^{\Sigma^-} = (\mu(p) + \frac{1}{2}\mu(n))/\mu_N - 1, \] \[ f_2^{\Sigma^\pm} = -\sqrt{2}\frac{\mu(n)}{\mu_N}, \] \[ \rho_f^{\Sigma^-} = (\mu(p) + 2\mu(n))/\mu_N - 1, \] \[ \rho_f^{\Sigma^-} = (\mu(p) + 2\mu(n))/\mu_N - 1, \] \[ \rho_f^{\Sigma^-} = (\mu(p) - \mu(n))/\mu_N - 1, \] \[ \rho_f^{\Sigma^+} = (\mu(p) + \mu(n))/\mu_N - 1, \] \[ \rho_f^{\Xi^0} = (\mu(p) - \mu(n))/\mu_N - 1. \] (23) (24) (25) (26) (27) (28) (29) (30) (31)

This is called the (extended) CVC hypothesis. In the NQM, using the formula \[ \rho_f^{\Sigma^0} = \frac{2M_N}{2m} (\Delta u - \Delta s) - 1. \] (33)

Using the expressions for \( \mu(\Sigma^-) \) and \( \mu(\Sigma^+) \), we find in this case

\[ \Delta u - \Delta s = \frac{1}{3\mu_d} (\mu(\Sigma^-) - \mu(\Sigma^+)), \] (34)

which leads to

\[ \rho_f^{\Sigma^-} = M_N (\mu(\Sigma^+) - \mu(\Sigma^-)) - 1 \]
\[ = \frac{M_N}{2M_N} (\kappa_{\Sigma^+} - \kappa_{\Sigma^-}). \] (35)

In a similar way, we obtain for the \( \Xi^+ \rightarrow \Xi^0 \) transition

\[ \rho_f^{\Xi^+} = 2M_N (\mu(\Xi^0) - \mu(\Xi^+)) - 1 \]
\[ = \frac{M_N}{2M_N} (\kappa_{\Xi^0} - \kappa_{\Xi^+}). \] (36)

Direct computation in the \( \chi \)QM using the formula \[ \rho_f^{\Sigma^0} = \frac{1}{3} (\Delta u - 2\Delta d + \Delta s) (\mu_u - \mu_d). \] (37)

We then obtain

\[ f_2^{\Sigma^0} = -\sqrt{2}(M_A + M_S)\mu(S). \] (38)

In all the cases above, there is an inherent ambiguity in the choice of magnetic moments, since in the SU(6) QM the form factors \( G_A/\sigma \) can be expressed in only two polarization differences, say \( \Delta u - \Delta d \) and \( \Delta u - \Delta s \). In our approximation, the different choices are related by the sum-rule

\[ \mu(p) - \mu(n) + \mu(\Sigma^-) - \mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) = 0, \] (39)

which follows under quite general assumptions on the spin polarizations and the magnetic moments of the quarks, and in particular from the SU(6) QM. This sum-rule is valid to within about 0.5 \( \mu_N \) on the left hand side.

In these cases, \( \sigma = 2m \) and the quark magnetic moments \( \mu_u = -2\mu_d = 1/(3m) \) are related to \( 1/\sigma \) without any symmetry breaking. When we pass to anomalous magnetic moments in Eqs. (32), (33), and (34) it is no longer possible to use Eq. (39), since extra linear terms in \( E \) will then appear.
3.3 The $\Delta S = 1$ cases

The cases with $\Delta S = 1$ are less straightforward, and there is no “natural” way to express the spin polarizations in terms of the magnetic moments, since many different possibilities give the same formal result. To begin with, there is a complication that $\sigma = m + m_s$ in these cases. The crucial factor in the transformation of Eq. (20) into an expression in terms of magnetic moments is, up to normalization, given by an expression of the form

$$A \simeq \frac{1}{\sigma(\mu_d + x\mu_s)},$$

where $x$ is a real parameter. This expression must not contain terms linear in the small quantity $x$. It is easy to see that the condition for this is given by $x = 1$.

Let us study this for the case of the $\Sigma^- \to n$ transition. We have

$$\rho_f^{\Sigma^- n} = \frac{M_\Sigma + m_N}{m + m_s} \frac{\mu(\Sigma^-) - \mu(n)}{\mu_s + 2\mu_d} - 1.$$  

(41)

This can be rewritten as

$$\rho_f^{\Sigma^- n} = (M_\Sigma + m_N)A_1(\mu(n) - \mu(\Sigma^-)) - 1,$$  

(42)

where $A_1 = -1/(m + m_s)(\mu_s + 2\mu_d)$. It is easy to check that this expression is linear in $\epsilon$, since $x = 1/2$. In fact, using $m_s = 3m/2$, we get $A_1 = 9/10$, so the deviation from 1 is 10%.

An alternative way of obtaining the spin polarization for the $\Sigma^- \to n$ transition is to use

$$\mu_{\Sigma n} \equiv \mu(n) + \frac{1}{3} \mu(p) - \mu(\Sigma^-) - \frac{1}{2} \mu(\Sigma^+),$$  

(43)

This gives

$$\rho_f^{\Sigma^- n} = (M_\Sigma + m_N)A_2\mu_{\Sigma n} - 1,$$  

(44)

where

$$A_2 = -\frac{2}{3(m + m_s)(\mu_s + \mu_d)} \simeq 1 + O(\epsilon^2).$$

In fact, for $m_s = 3m/2$, we obtain $A_2 = 24/25$, which is only 4% from 1. In the following, this term will therefore be equal to 1. We can then write

$$\rho_f^{\Sigma^- n} = \frac{M_\Sigma + m_N}{2m_N} \kappa_{\Sigma n},$$

(45)

where $\kappa_{\Sigma n} \equiv \kappa_n + \frac{1}{2} \kappa_p - \kappa_{\Sigma^-} - \frac{1}{2} \kappa_{\Sigma^+}$. Next consider

$$\rho_f^{\Xi^- \Sigma^0} = \frac{M_\Xi + m_\Sigma}{m + m_s}(\Delta u - \Delta d) - 1$$

$$= (\mu(p) - \mu(n)) \frac{1}{\mu_N} - 1 = \kappa_p - \kappa_n,$$  

(46)

where we have used $M_\Xi + M_\Sigma \simeq 3(m + m_s)$. This expression happens to coincide exactly with CVC (see Eq. (28)).

However, to neglect the hyperfine interaction in the mass formulas for the baryons means to discard terms linear in the hyperfine interaction constant, which is generally of the order 50 MeV. This is almost of the same order as the symmetry breaking mass difference $\delta$ between the quark masses. We should therefore not be satisfied with this approximation.

Again, it is possible to use another combination to express the axial-vector coupling constant. This is given by

$$\mu_{\Xi \Sigma} \equiv \mu(\Sigma^+) + \frac{1}{2} \mu(\Sigma^-) - \mu(\Xi^0) - \frac{1}{2} \mu(\Xi^-).$$

(47)

Using this gives

$$\rho_f^{\Xi^- \Sigma^0} = (M_\Xi + m_\Sigma)\mu_{\Xi \Sigma} - 1.$$  

(48)

For later use this can be rewritten as

$$\rho_f^{\Xi^- \Sigma^0} = \frac{M_\Xi + m_\Sigma}{2m_N} (\kappa_{\Sigma^+} + \frac{1}{2} \kappa_{\Sigma^-} - \kappa_{\Xi^0} - \frac{1}{2} \kappa_{\Xi^-}).$$

(49)

Since all $G_A$’s can be expressed in terms of the spin polarization differences $\Delta d - \Delta s$ and $\Delta u - \Delta d$, it is possible to express all other $\Delta S = 1$ transitions in terms of $\mu_{\Sigma n}$ and $\mu_{\Xi \Sigma}$. However, if we want to convert the result from magnetic moments to anomalous magnetic moments, we must also avoid terms that are linear in the mass ratios $E = \Delta / \Sigma$.

Consider therefore next

$$\rho_f^{\Xi^- \Lambda} = \frac{1}{3} \frac{M_\Xi + m_\Lambda}{m + m_s} (\Delta u + \Delta d - 2\Delta s) - 1.$$  

(50)

To avoid terms that are linear in $E$, we must in this case use

$$\mu_{\Xi \Lambda} \equiv \mu(p) + 3\mu(A) - \mu(\Xi^0) - \frac{1}{2} \mu(\Sigma^+) - \frac{5}{2} \mu(\Xi^-).$$

(51)

We then obtain, in the same approximation,

$$\rho_f^{\Xi^- \Lambda} = (M_\Xi + m_\Sigma) \frac{1}{3} \mu_{\Xi \Lambda} - 1 - \frac{M_\Xi + m_\Sigma}{2m_N} \frac{1}{3} \kappa_{\Xi \Lambda} + A_3,$$  

(52)

where $\kappa_{\Xi \Lambda} \equiv \kappa_\Lambda + 3\kappa_\Xi - \kappa_{\Xi^0} - \frac{1}{2} \kappa_{\Sigma^+} - \frac{5}{2} \kappa_{\Xi^-}$ and $A_3 = \frac{1}{9}(M_\Xi + M_\Sigma)(1/(2m_N) - 1/(4m_\Sigma) + 5/(4m_\Xi)) - 1$. It is easy to verify that $A_3 = O(E^2)$ and may therefore be neglected.

Finally, we can express $\rho_f^{\Xi^- p}$ in terms of magnetic moments in the same way starting from

$$\rho_f^{\Xi^- p} = \frac{1}{3} \frac{M_\Xi + m_N}{m + m_s} (2\Delta u - \Delta d - \Delta s) - 1.$$  

(53)

The best way of expressing the form factor uses

$$\mu_{\Xi p} \equiv \mu(n) + \frac{1}{2} \mu(p) + \frac{1}{2} \mu(\Sigma^-) - 3\mu(A) - \mu(\Xi^-)$$

$$= -\frac{3}{2} (2\Delta u - \Delta d - \Delta s)(\mu_d + \mu_s).$$

(54)
Omitting second order mass differences, this gives
\[
\rho_f^{Ap} = (M_A + M_N) \frac{1}{3} \mu_{Ap} - 1 = \frac{M_A + M_N}{2M_N} \frac{1}{3} \kappa_{Ap},
\]
where \(\kappa_{Ap} \equiv \kappa_n + \frac{5}{2} \kappa_p + \frac{1}{2} \kappa_{\Sigma^-} - 3 \kappa_A - \kappa_{\Xi^-}.
\]

### 3.4 The weak magnetism in the chiral quark soliton model

The weak magnetic form factors have also been calculated in the \(\chi\)QSM. The result, after normalization in our convention, is given in Eq. (56) as
\[
\rho_f^{pp} = \kappa_p - \kappa_n,
\]
\[
\rho_f^{\Sigma^- \Sigma^0} = \frac{M_\Sigma}{2M_N} (\kappa_{\Sigma^+} - \kappa_{\Sigma^-}),
\]
\[
f_2^{\Sigma^+ A} = -\sqrt{2} \frac{M_\Sigma + M_A}{2M_N} \frac{\mu_A}{\mu_N},
\]
\[
\rho_f^{\Sigma^- n} = \frac{M_\Sigma + M_N}{2M_N} \times (\kappa_n + \frac{1}{2} \kappa_p - \kappa_{\Sigma^-} - \frac{1}{2} \kappa_{\Sigma^+}),
\]
\[
\rho_f^{\Xi^- \Xi^0} = \frac{M_\Xi + M_\Sigma}{2M_N} \times (\kappa_{\Xi^+} + \frac{1}{2} \kappa_{\Sigma^-} - \kappa_{\Xi^0} - \frac{1}{2} \kappa_{\Xi^-}),
\]
\[
\rho_f^{\Xi^- A} = \frac{M_\Xi + M_A}{2M_N} \times (\kappa_p + 3 \kappa_A - \kappa_{\Xi^0} - \frac{3}{2} \kappa_{\Sigma^+} - \frac{5}{2} \kappa_{\Xi^-}),
\]
\[
\rho_f^{Ap} = \frac{M_A + M_N}{2M_N} \times (\kappa_n + \frac{5}{2} \kappa_p + \frac{1}{2} \kappa_{\Sigma^-} - 3 \kappa_A - \kappa_{\Xi^-}).
\]

We have here neglected the possible change in the transition from the anomalous magnetic moments to the full magnetic moments that might be related to the change in normalization. By this, we mean that in the above expressions, \(\kappa_B = \mu_B/\mu_N - Q_B M_N/M_B\), and the normalization, of course, is of relevance when the symmetry is broken.

However, our understanding is that, apart from a factor of \(M_B/(M_B + M_B')\), these differences in our normalization are of the order \(O(m^2)\) or \(O(m_N/N_c)\), i.e. in terms that are anyhow neglected in the above formulas, and the corresponding terms of second order or higher in the mass ratios that are neglected in our calculations.

From Section 3 of the paper, it should be clear then, that the method of expressing \(\rho_f\) that avoids introducing linear terms in the symmetry breaking masses, in general produces expressions that coincide with those above. This shows that the \(\chi\)QM and the \(\chi\)QSM give the same results when linear terms in the symmetry breaking are eliminated.

### 4 Discussion

The particular choice of combinations of magnetic moments, that enables one to express the \(\rho_f\)'s in term of anomalous magnetic moments, are enforced from the cancelation of linear terms in both the quark and baryonic mass differences. The analysis presented here shows that when this is done the \(\chi\)QM and the \(\chi\)QSM give the same results. The earlier noticed numerical differences are related to the difficulty to reproduce the octet baryonic magnetic moments in the \(\chi\)QM without symmetry breaking in the spin polarizations. This can be seen e.g. in the case of \(\Xi^0\) and \(\Xi^-\).

It is of course possible to stop and be satisfied at the level where the \(\rho_f\)'s are expressed in terms of magnetic moments. Then, since all \(G_A\)'s can be expressed in terms of only two spin polarization differences, there are several equivalent relations for the \(\rho_f\)'s related to sum rules for the \(G_A\)'s. On top of that, there is in this case also the possibility to use the sum-rules for the magnetic moments to find alternative ways to express the \(\rho_f\)'s.

For the \(\rho_f\)'s the existing experimental data is given in Table 3.

Let us consider this table.

The CVC values listed are in a way half experimental results, since they use the measured values of the anomalous magnetic moments for the nucleons as input data to calculate these values.

Since the magnetic moments of the quarks are fitted to the magnetic moments of the proton and neutron, the SU(6) QM results should coincide with CVC in the absence of symmetry breaking and are not listed.

All values obtained for the \(\rho_f\)'s in the \(\chi\)QM lie within the experimental errors, where experimental data exist. (The experimental results have large errors, though.)

In one case, that of neutron decay, we can see that \(\rho_f(\chi\text{QM}) \approx \rho_f(\text{CVC})\). For the other decays, the \(\rho_f\)'s of the \(\chi\)QM incorporate effects of vector current non-conservation due to the mass differences between the iso-multiplets as well as depolarization of the spin due to GB emission.

All calculated values for the \(\chi\)QM have the same sign as the CVC values and they are also close in magnitude. The numerical results cannot be expected to be much better than within 10%. Already isospin is violated to a few percent.

For comparison, we have in two cases calculated the \(\rho_f\)'s obtained by neglecting the hyperfine interaction, since it is rather small. The results are

\[
\rho_f^{\Xi^- \Sigma^0} = (\mu(p) - \mu(n))/\mu_N - 1 \simeq 3.71
\]

and

\[
\rho_f^{\Xi^- A} = (\mu(\Sigma^+) + \mu(\Xi^0) - \mu(\Xi^-) - \mu(\Sigma^-))/(3\mu_N) - 1
\]

\[
= 0.01 \pm 0.02,
\]

which both are very close to the values using Eqs. (49) and (62), respectively.

### 5 Summary and conclusions

We have studied the baryonic weak magnetism form factors in detail in the spirit of the \(\chi\)QM and compared the
results with the $\chi$QSM. The comparison shows that the results are in good agreement, and that the differences are of the order of reliability of the results in all cases. This might indicate that the main part of the symmetry breaking is accounted for in these formulas. The numerical results are presented in Tables 1 and 2.

The present investigation has used the SU(3) symmetric coupling in the $\chi$QSM and the static approximation for the quarks. A natural improvement would be to incorporate lowest order non-static effects and further SU(3) symmetry breaking effects [5,6], to obtain better agreement with experimental data. In particular, we expect that this would lead to a closer agreement with the $\rho_f$ ratios obtained from direct application of Eq. (2), since symmetry breaking can better account for the octet baryon magnetic moments [2]. SU(3) symmetry breaking also leads to better agreement for $g_A^{np}$ [12,13].

Acknowledgments. This work was supported by the Swedish Natural Science Research Council (NFR), Contract No. F- AA/FU03281-312. Support for this work was also provided by the Engineer Ernst Johnson Foundation (T.O.).

References

1. R.P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958)
2. B.H. Kellett, Phys. Rev. D 10, 2269 (1974)
3. J.F. Donoghue and B.R. Holstein, Phys. Rev. D 25, 206 (1982)
4. A. Manohar and H. Georgi, Nucl. Phys. B234, 189 (1984)
5. E.J. Eichten, I. Hirschfile, and C. Quigg, Phys. Rev. D 45, 2269 (1992)
6. T.P. Cheng and L.-F. Li, Phys. Rev. Lett. 74, 2872 (1995)
7. H.-C. Kim, M.V. Polyakov, M. Praszalowicz, and K. Gohe, Int. J. Mod. Phys. E 57, 299 (1998)
8. T. Ohlsson and H. Snellman, Eur. Phys. J. C 6, 285 (1999)
9. J.F. Donoghue, E. Golowich, and B.R. Holstein, Dynamics of the Standard Model (Cambridge, Cambridge 1992)
10. P. Renton, Electroweak Interactions: An Introduction to the Physics of Quarks & Leptons (Cambridge, Cambridge 1990)
11. F.E. Close, The Nucleon Spin Crisis Bible, RAL-93-034, 1993 (unpublished); F.E. Close and R.G. Roberts, Phys. Lett. B 316, 165 (1993)
12. J. Linde, T. Ohlsson, and H. Snellman, Phys. Rev. D 57, 452 (1998)
13. K. Dannbom, L Ya. Glozman, C. Helminen, and D.O. Riska, Nucl. Phys. A616, 555 (1997)
14. H.J. Weber and K. Bodoor, Int. J. Mod. Phys. E 5, 603 (1997)
15. T.P. Cheng and L.-F. Li, Phys. Rev. D 57, 344 (1998)
16. X. Song, J.S. McCarthy, and H.J. Weber, Phys. Rev. D 55, 2624 (1997)
17. Particle Data Group, C. Caso et al., Review of Particle Physics, Eur. Phys. J. C 3, 1 (1998)
18. M. Bourquin et al., Z. Phys. C 12, 307 (1982); 21, 1 (1983); 21, 17 (1983); 21, 27 (1983)
19. J.-M. Gaillard and G. Saugave, Ann. Rev. Nucl. Part. Sci. 34, 351 (1984)

| Table 1. Weak axial-vector form factors, $G_A^{BB'}$. |
|---------------------------------------------|
| The values in the NQM column are the SU(6) values for the weak axial-vector form factors and the values in the $\chi$QSM column are obtained from the quark spin polarizations. $g_{1QMSigma}^+\Lambda$ are given instead of $G_A^{\Sigma^+\Lambda}$, since $f_1^{QMSigma-A} = 0$. The experimental values for $G_A^{BB'}$ have been obtained from Ref. [7], except for the $g_{1QMSigma}^+\Lambda$ and $G_A^{\Sigma^+\Lambda}$ values, which are CERN WA2 [8,9] results from branching ratio measurements. |

| Quantity          | Experimental value | NQM       | $\chi$QSM |
|-------------------|--------------------|-----------|------------|
| $G_A^{BB'}$       | $1.2670 \pm 0.0035$| $\frac{5}{4}$ | $1.26$  |
| $G_A^{BB'}\chi^0$ | $-\frac{3}{4}$   | $0.50$    |            |
| $G_A^{BB'}\chi^+\Lambda$ | $0.589 \pm 0.016$ | $\sqrt{\frac{3}{4}}$ | $0.62$  |
| $g_{1QMSigma-A}$  | $-\frac{1}{4}$   | $-0.25$   |            |

| Table 2. The ratios $\rho_f^{BB'} \equiv \frac{\rho_f^{BB'}}{f_1^{BB'}}$. The experimental values have been obtained from Ref. [10] (see also Ref. [19]). $f_1^{\Sigma^+\Lambda}$ are given instead of $\rho_f^{\Sigma^+\Lambda}$, since $f_1^{\Sigma^+\Lambda} = 0$. |

| Quantity          | Experimental value | CVC       | $\chi$QSM |
|-------------------|--------------------|-----------|------------|
| $\rho_f^{BB'}$    | $3.71 \pm 0.00$    | $3.71$    | $3.53$    |
| $\rho_f^{BB'}\chi^0$ | $-0.84$    | $1.31$    | $1.30$    |
| $f_1^{\Sigma^+\Lambda}$ | $3.52 \pm 3.52$ | $2.34$    | $2.73$    |
| $f_1^{\Sigma^+\Lambda}$ | $-2.34$    | $2.72$    | $2.80$    |
| $\rho_f^{BB'}$    | $-2.03 \pm 2.27$  | $-2.03 \pm 2.27$ | $-2.03 \pm 2.27$ |
| $\rho_f^{BB'}\chi^0$ | $-0.44 \pm 0.46$ | $-0.12 \pm 0.06$ | $-0.44 \pm 0.46$ |
| $\rho_f^{BB'}\chi^+\Lambda$ | $2.43 \pm 1.49$ | $1.79$    | $1.38$    |
| $\rho_f^{BB'}\chi^0\Lambda$ | $-3.71$    | $3.83$    | $-3.71$    |

