Weighted multiple model adaptive boundary control for a flexible manipulator

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Abstract
In this article, a weighted multiple model adaptive boundary control scheme is proposed for a flexible manipulator with unknown large parameter uncertainties. First, the uncertainties are approximatively covered by a finite number of constant models. Second, based on Euler–Bernoulli beam theory and Hamilton principle, the distributed parameter model of the flexible manipulator is constructed in terms of partial differential equation for each local constant model. Correspondingly, local boundary controllers are designed to control the manipulator movement and suppress its vibration for each partial differential equation model, which are based on Lyapunov stability theory. Then, a novel weighted multiple model adaptive control strategy is developed based on an improved weighting algorithm. The stability of the overall closed-loop system is ensured by virtual equivalent system theory. Finally, numerical simulations are provided to illustrate the feasibility and effectiveness of the proposed control strategy.

Keywords
Flexible manipulator, vibration control, boundary control, weighted multiple model adaptive control, virtual equivalent system

Introduction
Robotic manipulators are widely used in dangerous, heavy, repetitive, and other tasks. Traditional rigid manipulators are realized by heavy materials with strong stiffness.1 They do not have the problem of flexible deformations, so it is relatively easy to obtain good angular position tracking performance. Since a lot of energy is consumed due to slowly moving and large weight, there is a contradiction between

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high speed and high accuracy, which is a challenging problem in the literature. However, flexible manipulators are concerned by engineers and scholars which possess various advantages, such as fast response time, light weight, high efficiency, and low energy consumption. And, they have been widely used in micro-surgical operation,\textsuperscript{1} robots,\textsuperscript{2,3} mechanical industry,\textsuperscript{4} and so on.

Accurate modeling plays an important role in designing a feasible control system. Flexible manipulator is a complicated nonlinear system, whose dynamic model has strongly rigid-flexible coupling characteristic.\textsuperscript{5} In the past decades, there have been many research results of dynamic modeling for flexible manipulators. Generally, bending and shear deflections both exist in each flexible link, which is regarded as Timoshenko beam. Considering that the link length is much larger than the section size, shear deflection can be ignored, then it can be considered as Euler–Bernoulli beam.\textsuperscript{4} Some modeling techniques have been proposed, such as the finite element method, the assumed mode method, and the lumped parameter method.\textsuperscript{4,6} A computationally efficient modeling method using acceleration-based discrete-time transfer matrix is presented for the dynamics analysis of flexible manipulators in Zhang et al.\textsuperscript{6} Besides, the global mode method is proposed to analyze dynamic model in Wei et al.\textsuperscript{7} However, there are two serious issues in the above different discretization modeling techniques. One is that the order of the controller increases with the number of modes, and the other is that the ignored high frequency modes lead to spillover effect in the controller.\textsuperscript{4} To solve these issues, partial differential equation (PDE) model is chosen to design the controller, which can reflect the dynamic characteristics of flexible structure accurately.\textsuperscript{8,9}

Note that the flexible structures of flexible manipulators are apt to cause vibrations of links, which even affects the precision of kinematics. Vibration control methods of flexible manipulator mainly includes passive control and active control.\textsuperscript{10} The passive control methods are to increase structural damping by installing high damping material, which are not effective for low frequency vibration. Many scholars tend to study active control strategies. In Benosman and Vey,\textsuperscript{11} a survey has been given on a variety of control methods, including PD control, adaptive control, singular perturbations, and sliding mode control (SMC). Besides, model-based evolution of a hybrid fuzzy adaptive control,\textsuperscript{12} predictive control,\textsuperscript{13} $H_\infty$ disturbance rejection control,\textsuperscript{14} active disturbance rejection control (ADRC),\textsuperscript{15} composite learning control and disturbance observer based control,\textsuperscript{2} second-order proportional integral derivative (PID) terminal SMC,\textsuperscript{16} and SMC based on adaptive neural networks\textsuperscript{17} are developed to restrain the flexible vibration. And neural network control shows a strong ability to deal with uncertainties in complex nonlinear dynamic systems.\textsuperscript{18,19} Note that there are three kinds of active control methods for flexible manipulator.\textsuperscript{20} The first one is to design a classical controller by truncating the dynamic model after discretization, which is known as modal control, such as PD, pole assignment, and adaptive control. The second one is distributed control,\textsuperscript{21,22} which has better effects than the modal control. However, both of them are difficult to realize in reality. Boundary control,\textsuperscript{23–26} as an emerging control method directly based on the PDE model, is to design boundary controllers
acting on one or both terminals of flexible mechanical systems according to the practical application scenes. Boundary control method is easy to construct Lyapunov function, and it does not require distributed controller. In recent years, some boundary control methods are developed, such as sliding mode boundary control, boundary control based on integral-barrier Lyapunov function, and iterative learning algorithm for boundary control. Specifically, spillover problem has been avoided by the boundary control based on the original PDE instead of the truncated model. And the boundary control based on integral-barrier Lyapunov function can effectively suppress vibration with input constraint and parametric uncertainties. Furthermore, the adaptive iterative learning algorithm for boundary control of a flexible manipulator can suppress vibration at a faster rate and restrict the input instability.

As for the previous researches, the control methods deal with the parameter uncertainties of the flexible manipulators based on their robustness or limited self-adaptability, which cannot adapt to the large parameter variations. However, in practical applications, the influences of changeable working environments on the plant cannot be ignored, such as terrestrial, underwater, high and low temperature environments, which will cause large uncertainties of system parameters. However, during the processes of performing some tasks, several system parameters will be changed in a large range, such that the existing controllers cannot guarantee the stability of the system. As for the conventional adaptive control methods, if the initial values of the identification algorithms are far away from the real values of system parameters, they will be restricted by the convergence speed, which makes it difficult to obtain satisfactory transient performance. To solve these issues, an improved weighted multiple model adaptive control (WMMAC) is used to deal with the large uncertainties of system parameters. Benefits from multiple models and controllers in parallel, a more reasonable controller, is used to control the current plant when the system parameters vary largely. Compared with the single controller or other methods, the control performance of the system will be improved obviously. If the parameter estimation of conventional adaptive control can be regard as infinite model identification, multiple model adaptive control can be considered as a finite model identification method, which makes it fast to identify system parameters, especially for dynamic system with large parameter uncertainties. In order to improve the parameter identification speed further, the classical weighting algorithm of WMMAC is replaced by a simple algorithm to reduce the calculation burden. Briefly, the proposed weighted multiple model adaptive boundary control (WMMABC) combined boundary control and WMMAC is designed for the large parameter uncertainties of a flexible manipulator, and the existing other methods cannot solve the problem effectively.

This article focuses on the accurate position tracking and vibration suppression of a flexible manipulator with large uncertainties of system parameters. First, the large variation ranges of system parameters are approximatively divided into several parts with constant parameters. For every part, a local mathematical model for the flexible manipulator is established without complicated process of
mechanical analysis, based on Euler–Bernoulli beam theory and Hamilton principle. Second, a model-based boundary control is designed to ensure that the closed-loop system is stable corresponding to the local model, which works on the motor axis and terminal payload. Then, the WMMABC is developed to solve the issue of large uncertainties through coordinating the local boundary controllers using a novel weighting algorithm. Finally, the overall closed-loop stability is proved based on virtual equivalent system (VES) methodology.

The main contributions of this article are summarized as follows:

1. A WMMABC method is developed to control the flexible manipulator with large uncertainties of system parameters, to the best of our knowledge, which is unrealizable by the existing controllers;
2. By improving the performance index of weighting algorithm, a suitable WMMAC is designed for both position tracking and vibration suppression of the flexible manipulator.

The rest of the article is organized as follows. Section “System description” presents the preliminaries and problem statement. Based on the Lyapunov stability theory, the local boundary controller is designed corresponding to the local constant model in section “Model-based boundary control.” In section “WMMABC,” the WMMAC is introduced into the controller and the stability proof of the overall closed-loop system is given. Then, simulations are carried out for a flexible manipulator with large parameter uncertainties in section “Numerical simulation.” Finally, the conclusion and future works are drawn in section “Conclusion.”

**System description**

In this article, a single-link flexible manipulator that moves in the horizontal direction is considered. Some assumptions based on Euler–Bernoulli beam theory are presented as follows, which lay the foundation for the subsequent analysis and design of control scheme.

**Assumption 1.** As the link considered is a prolate thin beam, whose length is much larger than the cross-section size, the deformation in the horizontal direction is much more serious than the axial deformation in general, such that the axial deformation can be ignored and transverse vibration is concerned only.

**Assumption 2.** Compared to the flexible link, the size of tip payload is too small, which is very common in the flexible robotics industry. Thus, the tip payload can be regarded as one particle.

**Assumption 3.** In view of the flexible manipulator material technology, there are constant cross-sectional area and uniform material properties in every part of the link, which can be realized almost perfectly. Even if there are small differences, the impact on the system can be temporarily ignored.
Assumption 4. The clamping device is much less flexible than the link, such that it can be regarded as a central rigid body.

The flexible manipulator system is described in Figure 1. The $XOY$ is the inertia coordinate system and the $xoOy$ is the local rotating reference coordinate system. $EI$ is the uniform flexural rigidity of the beam, $L$ is the length of the beam, $m$ represents the mass of the tip payload, $r$ is the link mass of the unit length, $\tau(t)$ and $I_h$ are the control torque of the manipulator and the moment of inertia. $F(t)$ is the control force of the end actuator, $y(x,t)$ is elastic deformation, and $\theta(t)$ represents the angular position of the hub.

Remark 1. The flexible manipulator parameters are variable with respect to time $t$.

Remark 2. In order to express the analysis process succinctly, the time $t$ is omitted in the function variables, and notations $(\ast)_x = \partial(\ast)/\partial x$ and $(\ast)_t = \partial(\ast)/\partial t$ are used throughout this article.

First, it is assumed that the system parameters are constant at a certain moment, which is to deal with the changes of system parameters by time discretization. Then, the constant parameters can be used to analyze the local mathematical model.

As the elastic deformation of the origin is 0 at any time, that is $y(0) = 0$, and elastic deformation changing rate along the $x$-axis, $y_x(0)$, is also 0. Thus the boundary condition is

$$y(0) = y_x(0) = 0$$ (1)
One point \((x, y(x))\) on the beam in the local rotating reference frame \(xOy\) can be expressed in the inertial frame \(XOY\) as

\[
z(x) = x \theta + y(x)
\]

where \(z(x)\) is manipulator’s offset. Then

\[
\begin{align*}
z(0) &= 0 \\
z_x(0) &= \theta \\
\frac{\partial^p z(x)}{\partial x^n} &= \frac{\partial^p y(x)}{\partial x^n}, \quad n \geq 2
\end{align*}
\]

The Hamilton principle is a modeling method based on the variation of system energy, that is, the sum of the variations for Lagrange function and non-conservative force working is 0 during any time interval. Based on the Hamilton principle, energy equation can be expressed as

\[
\int_{t_1}^{t_2} (\delta E_k - \delta E_p + \delta W_{nc}) \, dt = 0
\]

where \(\delta E_k, \delta E_p\), and \(\delta W_{nc}\) are variations of kinetic energy, potential energy and non-conservative force working, respectively, \(t_1\) and \(t_2\) are two moments, and \(t_1 < t < t_2\) is a operating time interval.

The total kinetic energy of the system is produced by the motions of the joint, link and tip payload, which is

\[
E_k = \frac{1}{2} I_h \dot{\theta}^2 + \frac{1}{2} \int_0^L \rho \dot{z}^2(x) \, dx + \frac{1}{2} m \dot{z}^2(L)
\]

The potential energy caused by elastic deformation is

\[
E_p = \frac{1}{2} \int_0^L EI_y^2(x) \, dx
\]

The system non-conservative force working is

\[
W_{nc} = \tau \theta + Fz(L)
\]

Using the methods of variation and integration by parts, the Hamilton equation is
\[ - \int_{t_1}^{t_2} \left[ \rho \ddot{z}(x) + EIz_{xxxx}(x) \right] \delta z(x) dx dt \\
- \int_{t_1}^{t_2} [I_h \ddot{z}_x(0) = EIz_{xx}(0) - \tau] \delta z_x(0) dt \\
- \int_{t_1}^{t_2} [m \ddot{z}(L) = EIz_{xx}(L) - F] \delta z(L) dt \\
- \int_{t_1}^{t_2} [EIz_{xx}(L)] \delta z_x(L) dt = 0 \]  

In equation (8), \( \delta z(x) \), \( \delta z_x(0) \), \( \delta z(L) \), and \( \delta z_x(L) \) are independent variables, such that the four parts on the left side are linear independent. It can be obtained that every term is 0. Therefore, the PDE dynamic model 34 is as follows

\[
\begin{cases}
\rho \ddot{z}(x) = - EIz_{xxxx}(x) \\
\tau = I_h \ddot{z}_x(0) - EIz_{xx}(0) \\
F = m \ddot{z}(L) - EIz_{xx}(L) \\
z_{xx}(L) = 0
\end{cases}
\]  

where \( \ddot{z}(x) = x \ddot{\theta} + \ddot{y}(x) \) and \( \ddot{z}(L) = L \ddot{\theta} + \ddot{y}(L) \).

It is worth noting that the dynamic model parameters, \( EI \), \( m \), \( I_h \), and so on, are variable with respect to time. Generally, there are three cases about the plant, which are the unknown time-invariable parameters, constant parameters after finite jumping, and infinite jumping parameters.

**Model-based boundary control**

As mentioned in the previous section, the dynamic model parameters are unknown or variable with respect to time in a large range. However, the changes of parameters cannot be infinitely fast in reality, which makes them be regarded as constant values at a certain moment. Even if there are some uncertainty in a small range, the stability of the closed-loop system can be ensured by designing appropriate control parameters. For these reasons, it can be assumed that the model parameters are constant at every certain moment, which lays the foundation for designing conventional boundary controller for the local model after time discretization.

By designing Lyapunov function, PD boundary control law is developed to suppress flexible vibration of the manipulator, that are \( y(x, t) \to 0 \), \( \dot{y}(x, t) \to 0 \), which is relatively simple to construct multiple model adaptive controller in the next step. Similarly, for expressing the analysis process simply, the time \( t \) of the time-varying parameters is omitted in the following contents.
The boundary control law can be designed as

\[
\begin{align*}
\tau &= - k_p (\theta - \theta_d) - k_d (\dot{\theta} - \dot{\theta}_d) \\
F &= - k u_a + m \ddot{z}_{xxx}(L)
\end{align*}
\] (10)

where \( u_a = \dot{z}(L) - z_{xxx}(L) \), \( k_p \), and \( k_d \) are proportional and differential coefficients, respectively, \( \theta_d \) is the target angle, \( k \) is vibration suppression correlation coefficient. The boundary controller works on the motor axis and terminal payload respectively by \( \tau \) and \( F \).

**Remark 3.** The required signals \( \theta \) and \( z(L) \) in equation (10) can be measured by the rotary encoder and strain gage installed on the motor rotor and clamping device, respectively. Their partial derivatives can be calculated by the difference algorithm.

**Remark 4.** \( \tau \) is the output torque of the motor, and \( F \) can be obtained by a flap device or jet thrust attitude adjustment device at the end of the manipulator.

The Lyapunov function is presented as

\[
V(t) = \frac{1}{2} \int_0^L \rho \dot{z}^2(x) dx + \frac{1}{2} EI \int_0^L y_{xx}^2(x) dx + \frac{1}{2} \dot{z} \dot{\dot{z}} + \frac{1}{2} k_p e^2 + \frac{1}{2} m u_a^2 + \alpha \rho \int_0^L x \dot{z}(x) \dot{z}(x) dx + \alpha I_k \dot{e}^2
\] (11)

where \( e = \theta - \theta_d \), \( \dot{e} = \dot{\theta} - \dot{\theta}_d \), and \( \alpha \) is a positive real number.

In equation (11), the first two terms are link kinetic energy and potential energy, which are designed for suppressing the flexible deformation and bending variation rate. The third and fourth terms are control error index, and the others are auxiliary terms. Furthermore, combined with equation (2), it can be easily obtained that \( z e(x) = x e + y(x) \), \( z e_\dot{x}(x) = e + y_\dot{x}(x) \), and \( z e_{xx}(x) = y_{xx}(x) \).

Since the next analysis process is roughly similar to Jiang et al.,\textsuperscript{34} except the consideration of the infinite dimensional disturbance, which can make it simple to construct multiple model adaptive controller and reduce the calculation burden. In order to ensure the analysis process can be derived, some constraint conditions should be satisfied, which can be known easily and is omitted in this section. Furthermore, derivative of Lyapunov function is satisfied the inequality \( \dot{V}(t) \leq - \lambda V(t) \), where \( \lambda \) is a positive real number. Then, it concludes that

\[
V(t) \leq V(0) e^{-\lambda t}
\] (12)

where \( V(0) \) is bounded. Obviously, when time tends to infinity, the Lyapunov function tends to 0, which is exponential convergence.
Remark 5. In this section, the large uncertainties are divided into several parts with constant parameter values. The boundary controller and Lyapunov stability analysis are based on the local dynamic model, which can only ensure the local stability of the controlled plant, instead of the overall closed-loop stability.

WMMABC

In this study, the main problem is that the dynamic model parameters, $EI$, $m$, $I_h$, and so on, are variable with respect to time, including the unknown time-invariable parameters, constant parameters after finite jumping, and infinite jumping parameters. Multiple model adaptive control can be regard as a kind of discretization modeling techniques, by which the variation ranges of system parameters are divided into several parts. For every part, the model parameters are regarded as constant values and the corresponding boundary controller has been designed well in the last section. With the variation of system parameters, a weighting algorithm can fuse local boundary controllers to produce appropriate control signals for the current situation.

Remark 6. On one hand, because the designed local controllers own certain robustness, and they are fused by the weighting algorithm, the small uncertainty can be allowed between the local model and the plant. On the other hand, an adaptive model can be added to enhance the effective coverage of the real plant, which can improve the control performance further.

In this section, an improved WMMAC is designed as follows. First, model set containing finite models is built to cover and approach the real system plant. Next, local boundary controllers for the models are designed, which constitute the controller set. Finally, the weighting algorithm is used to generate appropriate response signals based on the system performance index.

Model set and controller set

According to the parameter uncertainty ranges of the controlled plant, several local models are established to cover them, forming the model set $\Omega$: $\Omega = \{M_i|i = 1, 2, 3, \ldots, n\}$, where $M_i$ is the local model and $n$ is the quantity of local models. They lay the foundation for the design of the controllers.

Thus, in the model set, the difference of the local models is the system parameters. And local model with constant parameters has been analyzed in section “System description.” The quantity and specific parameters of local models are shown in the numerical simulation.

Then, the boundary controller can be designed for each model in the model set $\Omega$. Correspondingly, the local controllers construct the controller set $C$: $C = \{C_i|i = 1, 2, 3, \ldots, n\}$, which are PD boundary controllers and designed to control the local models. The local stability analysis based on Lyapunov function has been explained in section “Model-based boundary control.” Meanwhile, the
designed parameters of each boundary controller corresponding to the local model are presented in the numerical simulation.

**Remark 7.** The issue of model set and controller set construction in multiple model adaptive control is complex. In order to cover and approach the real plant reasonably, the running data samples of the system can be clustered to build the model set, based on the Vinnicombe distance or others. For conciseness, the model set in the numerical simulation section will be directly assumed to be known, which can reasonably cover the plant, so that the effectiveness of the proposed strategy can be shown clearly.

**Performance index and weighting algorithm**

In order to illustrate the weighting algorithm\(^ {35} \) conveniently, a concise block diagram of the WMMAC system is shown in Figure 2.

As shown in Figure 2, based on the each output \( u_i(t) \) of local controller \( C_i \) and the corresponding weight \( p_i(t) \), the global control signal \( u(t) \) is obtained

\[
  u(t) = \sum_{i=1}^{n} p_i(t) u_i(t) \quad (13)
\]

In order to reduce the calculation burden and relax the convergence conditions, a weighting algorithm based on model output error performance index is adopted. In view of the fact that most actual system are controlled by computers, the weights can only be calculated at the sampling time. To the best of our knowledge, there are two methods to deal with the discontinuity of control signals. One is that the system model can be controlled by discrete signals after discretization, and the other is that a zero-order holder can be used to obtain continuous weights. For succinct expression, the weighting algorithm is given in discrete form, so weights are \( p_i(k) \), weighting calculation indexes are \( l_i(k), i = 1, 2, \ldots, n \). Their initial values are set as follows

![Figure 2. Simplified block diagram of a WMMAC system.](image-url)
\[ p_i(0) = l_i(0) = \frac{1}{n}, i = 1, 2, \ldots, n \]  

(14)

The error performance index is

\[ l'_i(k) = \varepsilon + \frac{1}{k} \left\{ \beta \left[ e^2_i(k) + y^2_i(k) \right] + \gamma \sum_{j=1}^{k-1} \left[ e^2_i(j) + y^2_i(j) \right] \right\} \]  

(15)

where \( \varepsilon \) is a very small positive value for avoiding incalculable situations from happening, \( \beta \) and \( \gamma \) reflect the weight between the present and the historical moment, and \( \beta + \gamma = 1 \).

Then, the weights can be obtained by

\[ l'_\text{min}(k) = \min l'_i(k) \]  

(16)

\[ l_i(k) = \begin{cases} l_i(k-1), & H = 1 \\ l_i(k-1)H^{\text{ceil}}(\frac{1}{1-H}), & H < 1 \end{cases} \]  

(17)

where \( H = l'_\text{min}(k)/l'_i(k) \), and \( \text{ceil}(x) \) is the ceiling function that generates the smallest integer not less than \( x \)

\[ p_i(k) = \frac{l_i(k)}{\sum_{i=1}^{n} l_i(k)} \]  

(18)

Remark 8. In contrast to the weighting algorithm in Zhang,\textsuperscript{36} the error performance index in this article is more reasonable and efficient, which can gradually forget the historical accumulation effect and consider both the movement and vibration suppression of the flexible manipulator.

In addition, all the limit operations in this article are in the sense of probability 1. Thus, there is a weighting calculation situation that should be stated, which is that if the weight of one model is 1, the others are 0. When the system parameters are changed, the weights of the unselected model should be recalculated, which is not attainable according to equation (17). Therefore, the threshold value can be designed for limiting the weights to be 0.

**Stability analysis**

On the basis of weighting algorithm convergence and VES methodology,\textsuperscript{37} this section shows the stability analysis of the overall closed-loop system, in which the local controllers are designed by PD boundary control.

Primarily, it should be explained that the stability of WMMAC means the boundedness of its input–output signals and the convergence of its performance index to the local control systems.\textsuperscript{35}

**Theorem.** If a WMMABC system has the following properties:
1. Variation range of system parameters can be approximated by model set $\Omega$, and the approximation error is bounded;

2. The local boundary control law in equation (10) is well defined, corresponding to the local model, whose closed-loop system is stable;

3. For a slow time-varying system, among any models switching, there is a model $M_j \in \Omega$ closest to the current plant in the following sense with probability $1$:

$$P_{Tl+1} + k_r = T_l + 1 - \min \left\{ S_j \right\}_{i \neq j}, i \neq j, T_l, l = 0, 1, 2, \ldots$$

where $d$ is the system delay, $S_j$ is a constant, $S_i$ is a constant or infinity and $S_j < S_i$, $i \neq j$, $T_l, l = 0, 1, 2, \ldots$ is the jumping time sequence for uncertain parameters. Then the overall closed-loop system is stable.

**Proof.** First, the convergence of the weighting algorithm is also ensured and the specific process is omitted, which can be obviously obtained following the proof in Zhang.36 Second, based on the Lyapunov function in equation (11), it can be proved that the local control law makes the corresponding closed-loop system stable obviously. That is, when $t$ tends to infinity, $\theta$ tends to $\theta_d$ and $y(x)$ tends to 0 for all $x \in [0, L]$.

Then, the VES of a time-varying WMMAC system, as shown in Figure 3, can be decomposed into two subsystems,35,37 see Figures 4 and 5, where $M_j \in \Omega$ is one of the models covering variation range of system parameters and $\Delta u(t) = u(t) - u_i(t)$.

Considering that parameter jumps cannot be infinitely fast in reality, the system parameters can be regarded as constant values at a certain working point. So, the decomposed subsystem 1 in Figure 4 is equivalent to the time-invariant system in the input–output sense, whose stability can be proved according to Zhang.35 Then, based on switching system theory,38 the subsystem 1 of VES is overall closed-loop stable, that are $y'(t) \rightarrow y_r(t)$ and $\theta'(t) \rightarrow \theta_d(t)$. In addition, the subsystem 2 is a stable deterministic system and the approximation errors $e(t)$ and $y_e(t)$ are bounded, whose stability is obvious.

![Figure 3. VES of a time-varying WMMAC system.](image-url)
Furthermore, based on proof by contradiction and the Cauchy–Schwarz inequality, $(\int f(*)g(*)d(*))^2 \leq \int f^2(*)d(*) \int g^2(*)d(*)$, where $f(*)$ and $g(*)$ are two functions, conclusions can be drawn that the overall closed-loop system is stable, that are $y(t) \rightarrow y_r(t)$, $\theta(t) \rightarrow \theta_d(t)$.

That completes the stability proof of the WMMABC system.

**Numerical simulation**

The effectiveness of the proposed WMMABC is illustrated by numerical simulation in MATLAB. In order to realize the simulation of PDE model without losing the flexible characteristics of the manipulator, the finite difference approach is adopted and the space–time steps are as small as possible, whose detailed values based on debugging experience are time step $\Delta t = 5 \times 10^{-4}$ s and manipulator space step $\Delta x = 0.01$ m.

In general, the boundary control has certain robustness, and the parameter variations of the flexible manipulator can be obtained through the clustering of sample data and theoretical analysis. Therefore, the parameter uncertainties can be effectively covered by a finite number of representative constant local models. For simplicity, there are two possible models to cover the parameter uncertainties, whose parameters have been obtained. Without losing the authenticity of the simulation system, the specific model parameters are given and adjusted based on the parameters in Jiang et al.\textsuperscript{34} The parameters of model 1 are $EI_1 = 3$, $\rho_1 = 0.2$, $m_1 = 0.1$, and $I_{h1} = 0.1$, and the parameters of model 2 are $EI_2 = 0.5$, $\rho_2 = 0.2$, $m_2 = 0.5$, and $I_{h2} = 1$. According to condition 2 in the Theorem, two corresponding
boundary control laws \( \tau(t) \) and \( F(t) \) can be designed as equation (10), which respectively act on two terminals of the flexible manipulator. The local controllers are designed based on Lyapunov’s direct method, whose parameter values are obtained by trial and error combined with the constraints for the controllers in simulation debugging. The parameters of controller 1 are \( k_{p1} = 13, k_{d1} = 4, \) and \( k_1 = 2, \) and the parameters of controller 2 are \( k_{p2} = 50, k_{d2} = 30, \) and \( k_2 = 20. \) It can be assumed that the manipulator parameters are changed from model 1 to model 2 at the seventh second. It is obvious that the system simulation meets the conditions of the Theorem, which is easy to be satisfied in practice without losing authenticity.

In addition, the initial conditions of variables are all set to be 0, and the desired positions are \( u_{d1} = 0, 0.5/C2, 0 \) for \( t < 2 \), \( u_{d2} = 0.5, 7/C2, 3 \) for \( t > 2 \), and \( u_{d3} = 0.5, 7/C2, 10 \).

The simulation results are presented in Figures 6–9. As shown in Figure 6, the angle positions of the flexible manipulator asymptotically tend to the target positions within 2.5 s along with the changes of target positions, and the angle speed is relatively smooth without oscillations frequently. Even when the system plant parameters jump largely at the seventh second, the controller can also ensure the angle position tracking accurately. Figure 7 shows the flexible deformations of each point on the flexible manipulator changes as time goes on. Since the target angle position changes at the 3.5th and 7th seconds, which is a square wave function as shown in Figure 6, the flexible deformations are difficult to avoid. Fortunately, the manipulator is quickly stabilized and the flexible deformations are effectively suppressed within 3.0 s. Especially when the system parameters change largely, the control system can effectively suppress the flexible deformation without obvious chattering, which mainly benefits from the implementation of WMMAC. The control inputs demonstrated in Figure 8 are restrained with \([-10 \text{ N}, 20 \text{ N}]\), which shows that the control system just requires a small amount of energy. The weights of models are
shown in Figure 9, whose switching process takes about 20 steps in the simulation when the plant parameters jump.

Remark 9. According to the above simulation results, especially those shown in Figures 6 and 7, the position tracking and vibration suppression are effective and stable, which are due to the satisfaction of the Theorem conditions, that are the stabilizing and tracking of the local boundary control strategy and the convergence of weighting algorithm. In summary, the Theorem effectively ensures the overall closed-loop system stability and provides theoretical guidance for the design of the controller. At the same time, the effective results of system simulation further verify the reliability of the Theorem.

Figure 7. Flexible deformations of the manipulator with the WMMABC.

Figure 8. Weighted multiple model adaptive boundary control inputs.
In addition, to illustrate the improvement of control performance, the vibration suppression effect using conventional boundary control is given in Figure 10. As shown in Figures 6 and 7, it is obvious that the joint motor $\tau$ and tip controller $F$ can work well to regulate angle position and suppress vibration, even when the manipulator parameters changed significantly at the seventh second. However, the conventional boundary controller fail to suppress vibration for this situation, shown in Figure 10. Intuitively, when the system parameters jump largely, the conventional boundary controller may fail to meet the control requirements due to its limited robustness or self-adaptability. But, the proposed WMMABC can effectively coordinate multiple controllers in parallel to ensure reasonable control.

Figure 9. Weights for the local controllers of WMMABC system.

Figure 10. Flexible deformations of the manipulator with conventional boundary control.
signals. Thus, the control performance of the system can be improved. To sum up, simulation results show that it is necessary to develop WMMABC strategy for the angle tracking and vibration suppression of flexible manipulators with large parameter uncertainties.

Conclusion

This article focuses on position control and vibration suppression issues of a flexible manipulator subjected to the large parameter uncertainties. To our knowledge, the proposed WMMABC based on improved weighting algorithm is first designed for the flexible manipulator, and the stability of the overall closed-loop system is ensured by Lyapunov stability theory and VES theory. The simulation results illustrate the effectiveness of the proposed control strategy. The main advantages of the proposed method are that (1) it can deal with the large parameter uncertainties, which cannot be accomplished by the existing other controllers, and (2) the performance index of weighting algorithm considers both angular position and flexible deformation, which can avoid system response chattering or instability caused by incorrect operations of weight calculation. However, the design process of the control method is complicated, especially the construction of model set. Therefore, the future works will be focused on construction and dynamic optimization of a reasonable precise model set, the position control and vibration suppression for a flexible manipulator with input and output constraints, and the practical experiments.

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