A relativistic collisionless shock propagating into an unmagnetized medium leaves behind a strong large-scale magnetic field. This seems to follow from two assumptions: (1) Gamma-ray burst (GRB) afterglows are explained by synchrotron emission of a relativistic shock. (2) The magnetic field cannot exist on microscopic scales only; it would decay by phase-space mixing. Assumption 1 is generally accepted because of an apparent success of the shock synchrotron phenomenological model of GRB afterglows. Assumption 2 is confirmed in this work by a low-dimensional numerical simulation. One may hypothesize that relativistic shock velocities are not essential for the magnetic field generation and that all collisionless shocks propagating into an unmagnetized medium generate strong large-scale magnetic fields. If this hypothesis is true, the first cosmical magnetic fields could have been generated in shocks of the first virialized objects.

Subject headings: magnetic fields — shock waves

1. GAMMA-RAY BURST AFTERTWIGHT PHENOMENOLOGY

It appears that gamma-ray bursts (GRBs) and GRB afterglows can be adequately described by synchrotron emission of relativistic shocks (e.g., Waxman 1997; Sari, Piran, & Narayan 1998; Guetta, Spada, & Waxman 2001). These phenomenological models describe the shock by a set of adjustable parameters. One of these parameters, \( \xi_e \equiv B^2/(8\pi\epsilon) \), is the magnetic energy density divided by the proton energy density \( \epsilon \) downstream of the shock transition. It is thought that \( \xi_e \) is close to unity, say, \( \xi_e \sim 0.1 \). Much smaller values, \( \xi_e \sim 0.001 \), were given by Panaitescu & Kumar (2001), but as we are about to show, even 0.001 is a surprisingly large number.

For the sake of simplicity, consider a mildly relativistic stage of a GRB afterglow, when the Lorentz factor is approximately a few. Assume that unshocked plasma is at a typical interstellar medium temperature, \( \sim 1 \) eV. The unshocked magnetic energy per particle is \( \sim 1 \) eV, because the field has to be confined by the plasma pressure. If hydrodynamics is applicable, then the blast wave is stable (Gruzinov 2000) and there is no magnetic field generation. The magnetic field is amplified only by compression. Then the shocked magnetic energy per particle is less than approximately a few eV, because the shock has a compression factor equal to about a few. But the proton temperature in the shocked plasma is approximately few GeV, giving \( \xi_e \sim 10^{-3} \), which is at least 6 orders of magnitude smaller than required by the synchrotron phenomenological model of GRB afterglow.

In this work, we assume that the synchrotron GRB model is correct. Then the magnetic field should be somehow generated by the shock. For all practical purposes, the unshocked plasma is unmagnetized. The nature of this shock-generated magnetic field is an interesting unsolved problem. Here we try to guess an answer, but we have to acknowledge the speculative nature of our discussion. The right way to solve this problem is by direct numerical simulations of a three-dimensional collisionless (Vlasov) plasma.

2. SHOCK DYNAMO

That collisionless shocks can generate strong, that is, near equipartition, magnetic fields is well known (e.g., Sagdeev 1966). Low-dimensional numerical simulations illustrating this process are also available (e.g., Kazimura et al. 1998; see also the Appendix). The mechanism of the magnetic field generation is the Weibel instability, which can be explained as follows. Within the shock transition region, the distribution function of charged particles is strongly anisotropic. Particles move predominantly perpendicular to the front. These particles might be thought of as straight currents. Currents of the same sign attract each other, leading to bunching of electrons moving in one direction and protons moving in the opposite direction. This current bunching generates magnetic fields.

The length scales and timescales characterizing the instability can be obtained from dimensional analysis. The instability is collisionless—it is described by the Vlasov system of equations (that is, collisionless Boltzmann plus Maxwell). The Vlasov system contains the following dimensional quantities: \( nn, ne \), and \( c \). Here \( n \) is the unshocked plasma density, \( m \) is the proton mass, and \( c \) is the speed of light. We will assume that the shock is mildly relativistic, so it is not characterized by any large or small dimensionless numbers. Then there is only one timescale in the problem—the inverse plasma frequency, \( \omega_p^{-1} \), where \( \omega_p \equiv (4\pi ne^2/m)^{1/2} \). The only length scale is the skin depth, \( \delta \equiv c/\omega_p \).

Now the initial stage of the Weibel instability might be described as follows. The shock width is \( \sim \delta \). Within the shock, a near equipartition magnetic field is generated. The field isotropizes the distribution functions of charged particles. The characteristic length scale of the generated field is \( \sim \delta \).

But what happens many deltas downstream (Gruzinov & Waxman 1999)? The standard plasma-physical answer would be as follows: (1) Strong electromagnetic fields occupy a layer several \( \delta \) wide. Within this layer, electromagnetic fields play the role of collisionality, bringing the plasma up the shock adiabatic. Many deltas downstream, there is only shocked plasma and virtually no fields. Electromagnetic fields are weak, because they decay by phase-space mixing (Landau damping). We have confirmed this magnetic field decay scenario, which is the common collisionless shock wisdom, by two-dimensional numerical simulations (see the Appendix).
But, the decaying field scenario is ruled out by astronomical observations, so long as we believe that GRBs and GRB afterglows are synchrotron-emitting shocks. Synchrotron emission of a few skin-deep layers would be negligible.

The two most natural possibilities consistent with the synchrotron GRB model are that (2) strong small-scale fields (scale $\sim \delta$) exist downstream and (3) strong large-scale fields (scale $\sim l$; $l$ is a proper distance from the shock) exist downstream. Now possibility 2 is, in fact, impossible. Magnetic fields do not live on length scales $\sim l$; they decay there on a timescale $\sim \delta$. They can be born there, and then they decay there on a timescale $\sim \omega^{-1}$ due to phase-space mixing. Our numerical simulation confirms that magnetic fields really die out on skin depth scales (see the Appendix), at least in two dimensions. We will assume that the decay of microscopic, skin-deep magnetic fields by the phase-space mixing is a true phenomenon, not an artifact of two dimensions. In three dimensions, the phase-space mixing should be even easier.

Then we are left with possibility 3—strong (but maybe $\xi_0 \sim m_i/m_p$) large-scale (length scale of order the distance from the shock) magnetic fields are somehow generated by a collisionless shock propagating into an initially unmagnetized plasma. We do not understand how this happens. We suspect that particle acceleration is an essential part of the process. The presence of an energetically important population of accelerated particles invalidates the MHD approximation. And one does have to go beyond MHD, because MHD shocks are stable and therefore do not generate magnetic fields.

2 In two dimensions, we see no magnetic field generation on large scales and no particle acceleration.

3 The first magnetic fields

We have suggested that relativistic collisionless shocks generate strong large-scale magnetic fields. However, we do not see how the relativistic effects might be important. We therefore hypothesize that nonrelativistic shocks also generate strong large-scale magnetic fields.

This means, in particular, that the first cosmic magnetic fields could appear together with the first virialized objects. The magnetic fields are generated in virialization shocks, and they are born strong and large-scale. The primordial magnetic field theories are not needed to explain “the seed fields” for a galactic dynamo.

APPENDIX A

WEIBEL INSTABILITY IN 2D3V

Here we show that the magnetic field generated by a 2D3V (two-dimensional in space and three-dimensional in velocity space) Weibel instability decays due to phase-space mixing after a few plasma times. We perform a simplified numerical simulation to illustrate this effect.

We use dimensionless units $e = m = c = 1$. The electromagnetic field is

$$E = (0, 0, -\partial_t A), \quad B = (\partial_x A, -\partial_y A, 0),$$

where the $z$-component of the vector potential is $A = A(x, y, t)$. The absence of the electrostatic field is explained below. Nonrelativistic charges move in a two-dimensional space, $r = (x, y)$, with three-dimensional velocities $(r, v) = (v_x, v_y, u)$. This means that charges are rigid rods elongated along the $z$-coordinate. The equations of motion for positive charges are

$$\dot{r} = v, \quad \dot{v} = u \nabla A, \quad \dot{u} = -\partial_t A - v \cdot \nabla A.$$ 

To simplify our basic system, we will assume that at $t = 0$, and therefore at all later times, each positive charge $(r, v, u)$ is matched by a negative charge $(r, v, -u)$. Under this symmetry, there is no electrostatic component of the field.

We think that the problem of magnetic field generation in shocks will be solved by direct numerical simulations. Our amateur numerical program seems to be sufficient for a two-dimensional case. A carefully designed code should solve the three-dimensional problem. The advocated three-dimensional simulation of a collisionless shock propagating into an unmagnetized medium should be performed regardless of our arguments. This simulation might prove our scenario wrong, but any result should be of great interest, say, one discovers that magnetic fields are not produced. Then, what is the nature of the GRB afterglows?

3. THE FIRST MAGNETIC FIELDS

The two most natural possibilities consistent with the synchrotron GRB model are that (2) strong small-scale fields (scale $\sim \delta$) exist downstream and (3) strong large-scale fields (scale $\sim l$; $l$ is a proper distance from the shock) exist downstream. Now possibility 2 is, in fact, impossible. Magnetic fields do not live on length scales $\sim l$; they decay there on a timescale $\sim \delta$. They can be born there, and then they decay there on a timescale $\sim \omega^{-1}$ due to phase-space mixing. Our numerical simulation confirms that magnetic fields really die out on skin depth scales (see the Appendix), at least in two dimensions. We will assume that the decay of microscopic, skin-deep magnetic fields by the phase-space mixing is a true phenomenon, not an artifact of two dimensions. In three dimensions, the phase-space mixing should be even easier.

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2 In two dimensions, we see no magnetic field generation on large scales and no particle acceleration.

3 The first magnetic fields

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To simplify our basic system, we will assume that at $t = 0$, and therefore at all later times, each positive charge $(r, v, u)$ is matched by a negative charge $(r, v, -u)$. Under this symmetry, there is no electrostatic component of the field.

We think that the problem of magnetic field generation in shocks will be solved by direct numerical simulations. Our amateur numerical program seems to be sufficient for a two-dimensional case. A carefully designed code should solve the three-dimensional problem. The advocated three-dimensional simulation of a collisionless shock propagating into an unmagnetized medium should be performed regardless of our arguments. This simulation might prove our scenario wrong, but any result should be of great interest, say, one discovers that magnetic fields are not produced. Then, what is the nature of the GRB afterglows?

APPENDIX A

WEIBEL INSTABILITY IN 2D3V

Here we show that the magnetic field generated by a 2D3V (two-dimensional in space and three-dimensional in velocity space) Weibel instability decays due to phase-space mixing after a few plasma times. We perform a simplified numerical simulation to illustrate this effect.

We use dimensionless units $e = m = c = 1$. The electromagnetic field is

$$E = (0, 0, -\partial_t A), \quad B = (\partial_x A, -\partial_y A, 0),$$

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To simplify our basic system, we will assume that at $t = 0$, and therefore at all later times, each positive charge $(r, v, u)$ is matched by a negative charge $(r, v, -u)$. Under this symmetry, there is no electrostatic component of the field.

We think that the problem of magnetic field generation in shocks will be solved by direct numerical simulations. Our amateur numerical program seems to be sufficient for a two-dimensional case. A carefully designed code should solve the three-dimensional problem. The advocated three-dimensional simulation of a collisionless shock propagating into an unmagnetized medium should be performed regardless of our arguments. This simulation might prove our scenario wrong, but any result should be of great interest, say, one discovers that magnetic fields are not produced. Then, what is the nature of the GRB afterglows?

3. THE FIRST MAGNETIC FIELDS

Through our simulations, we have seen that relativistic collisionless shocks generate strong large-scale magnetic fields. However, we do not see how the relativistic effects might be important. We therefore hypothesize that nonrelativistic shocks also generate strong large-scale magnetic fields.

This means, in particular, that the first cosmic magnetic fields could appear together with the first virialized objects. The magnetic fields are generated in virialization shocks, and they are born strong and large-scale. The primordial magnetic field theories are not needed to explain “the seed fields” for a galactic dynamo.

APPENDIX A

WEIBEL INSTABILITY IN 2D3V

Here we show that the magnetic field generated by a 2D3V (two-dimensional in space and three-dimensional in velocity space) Weibel instability decays due to phase-space mixing after a few plasma times. We perform a simplified numerical simulation to illustrate this effect.

We use dimensionless units $e = m = c = 1$. The electromagnetic field is

$$E = (0, 0, -\partial_t A), \quad B = (\partial_x A, -\partial_y A, 0),$$

where the $z$-component of the vector potential is $A = A(x, y, t)$. The absence of the electrostatic field is explained below. Nonrelativistic charges move in a two-dimensional space, $r = (x, y)$, with three-dimensional velocities $(r, v) = (v_x, v_y, u)$. This means that charges are rigid rods elongated along the $z$-coordinate. The equations of motion for positive charges are

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We think that the problem of magnetic field generation in shocks will be solved by direct numerical simulations. Our amateur numerical program seems to be sufficient for a two-dimensional case. A carefully designed code should solve the three-dimensional problem. The advocated three-dimensional simulation of a collisionless shock propagating into an unmagnetized medium should be performed regardless of our arguments. This simulation might prove our scenario wrong, but any result should be of great interest, say, one discovers that magnetic fields are not produced. Then, what is the nature of the GRB afterglows?
where \( \langle . . \rangle \) is the average over the unperturbed distribution function.

If, as in our numerical simulation, at the initial time all velocities are parallel to \( z \), equation (A6) gives the instability growth rate,

\[
-\omega^2 \approx -\frac{\langle u^2 \rangle k^2}{1 + k^2/\Omega^2},
\]

This means that the fastest growing modes are small-scale, with wavelength \( \lambda \approx 1/\Omega \). The characteristic growth rate is \( \sim u\Omega \).

### A2. NONLINEAR THEORY

As the instability develops, the magnetic field turns the particles’ trajectories, and a nonzero velocity dispersion in the \((x, y)\)-plane appears. This stabilizes the short wavelengths first, because in the first nonvanishing order in \( v \), equation (A6) gives, for \( k \gg \Omega \),

\[
-\omega^2 \approx \Omega^2 \langle u^2 \rangle - \frac{3}{2} \frac{\langle u^2 v^2 \rangle}{\langle u^2 \rangle} k^2.
\]

One expects a maximal conversion of kinetic into magnetic energy at about the time when one Larmor circle in the generated magnetic field has been completed by a typical particle. This should occur at length scales somewhat larger than \( 1/\Omega \), because the generated field has to be somewhat subequipartition, that is, less than \( \Omega u \).

Further evolution is governed by conservation of the mean squared potential. As follows from translation invariance along \( z \), the generalized momentum, \( p = u(t) + A[x(t), y(t)] \), is conserved for each particle. In particular, \( I = \sum p^2/\mathcal{N} \), where the sum is over all \( N \) particles, is conserved; \( I \) has dimensions of the squared potential, that is, \( B^2L^2 \), where \( B \) is the magnetic field and \( L \) is the length. One may then suggest the following scenario for the magnetic field evolution. The length scale \( L \) grows, and \( I \) is conserved; therefore, the magnetic energy \( \langle B^2 \rangle \) decreases as \( I/L^2 \). To determine \( L \) as a function of the evolution time \( t \), we assume that \( L/t \) is close to the Alfvén velocity, which is proportional to \( B \). It follows that \( \langle B^2 \rangle \approx t^{-1} \).

### A3. NUMERICAL SIMULATIONS

In our simulations, we integrated the particle trajectories (eq. [A2]) and solved the electrodynamics equations \( \partial_t A = \)
\(-E, \partial_t E = -\nabla^2 A - j\) on a grid. Space was a periodic square of size 1. We used the simplest straightforward numerical algorithm. With a 100 \times 100 grid for an electromagnetic field, we needed \(N = 2,000,000\) particles to compensate for the discreteness effects. For \(\Omega = 100\), with the time step of 0.0005, energy is conserved to 0.3\%, and the mean squared potential (see § A2) is conserved to 3\%.

Initial conditions are as follows. The electromagnetic field is zero. Particles are at random spatial positions, with \(v = 0\) and \(u\) randomly chosen from an interval \((-0.2, 0.2)\). This velocity might seem to be too high for a nonrelativistic approximation to apply, since in our units the speed of light is 1. However, as we have checked, even the fastest particle in our run does not get faster than about 0.5. Results are shown in Figures 1–3.

At \(t = 1\), 16\% of energy is converted into magnetic energy. The mean magnetic field is \(B \sim 3\). Since velocity is \(v \sim 0.1\), the corresponding Larmor radius is \(\sim 0.03\), close to the length scale seen in Figure 1. As seen from Figure 2, the length scale of the field grows. As we have said, this is accompanied by the magnetic energy decay. As seen from Figure 3, magnetic energy is indeed described by the \(t^{-1}\) law.

REFERENCES

Gruzinov, A. 2000, preprint (astro-ph/0012364)
Gruzinov, A., & Waxman, E. 1999, ApJ, 511, 852
Guetta, D., Spada, M., & Waxman, E. 2001, ApJ, 557, 399
Kazimura, Y., Sakai, J. I., Neubert, T., & Bulanov, S. V. 1998, ApJ, 498, L183
Lifshitz, E. M., & Pitaevskii, L. P. 1981, Physical Kinetics (New York: Pergamon)
Panaitescu, A., & Kumar, P. 2001, ApJ, 554, 667
Sagdeev, R. Z. 1966, Rev. Plasma Phys., 4, 23
Sari, R., Piran, T., & Narayan, R. 1998, ApJ, 497, L17
Waxman, E. 1997, ApJ, 485, L5