In many Chinese cities, traffic streams near bus stops differ from those of bus stops in developed countries. There are usually two lanes at a Chinese curbside stop: a nonmotorized lane and a motorized lane. Bus stops are often located on the nonmotorized lane. When a bus dwells at a curbside stop, nonmotorized vehicles, mainly bicycles, will move to the motorized lane. Thus, the presence of a stopped bus creates a temporary conflict between bicycles and cars, reducing road capacity. A road capacity model based on gap acceptance theory and queuing theory is presented for mixed traffic flow at the curbside stop. Traffic conditions are classified into two types: no stopped bus and presence of stopped bus at the curbside stop. The probabilities of no bus and presence of bus at a stop can be obtained by using the queuing model of bus streams. Under the former condition, the bicycle stream and the car stream have no conflict, and car capacity is not affected by the bicycle stream. Under the latter condition, the conflict between bicycles and cars on the motorized lane leads to an effect on car capacity by the bicycle stream. The effect on car capacity can be derived through gap acceptance theory. Car capacity at the curbside stop is a function of three types of traffic stream—buses, cars, and bicycles—and it may be applicable in traffic analysis and the design of bus stops in other developing Asian cities.

Bust stops are the first point of contact between the passenger and the transit service, and the spacing, location, design, and operation of bus stops significantly influence transit system performance. In the past several decades, traffic planners, designers, and scholars have paid much attention to the location and design of bus stops (1–3). A prominent achievement of this research is a set of guidelines for use in designing and locating bus stops, sponsored by TCRP in the United States (4). The guidelines compile checklists of factors that must be taken into consideration, list the advantages and disadvantages of various bus stop treatments, and discuss the trade-offs among different alternatives. Other researchers focused on the effects of bus stops on traffic flow. For example, Fitzpatrick and Nowlin studied effects of bus stop design on suburban arterial operations (4). Wong et al. analyzed delay at a signal-controlled intersection with a bus stop upstream (5). Koshy and Arasan applied the microscopic traffic simulation model to study the impact of bus stops on the speeds of other vehicles (6). Zhao et al. investigated the capacity drop caused by the combined effects of signalized intersections and bus stops by using a two-lane cellular automata model (7,8). Most existing research of bus stops analyzes only the mixed traffic flow between buses and cars without including nonmotorized vehicles.

As a developing country, China has its own traffic characteristics. A mix of nonmotorized and motorized vehicles is an important traffic type in China. Some surveys show that the nonmotorized vehicle, especially the bicycle, is one of the most widely used traffic tools in Chinese daily travel activity. Typically, there are three types of bus stop in urban areas: curbside stops, bus bays, and bus boarders. The curbside stop is the most common type on minor roadways in many Chinese cities. Figure 1 show the mixed traffic streams at a typical curbside stop. There are two lanes on the urban roadway, the bicycle lane and the motorized lane, and there are three types of traffic stream: bicycle, bus, and car. Bus stops are usually located on the bicycle lane. When a bus dwells at the curbside stop, bicycles move to the motorized lane and go around the stopped bus. Thus, the presence of a stopped bus creates a temporary conflict between bicycles and cars, reducing road capacity. Similar phenomena may be found in other developing Asian countries, for example, Malaysia, Vietnam, and Cambodia.

Because of the special features of mixed traffic, the application of existing traffic models for bus stops, developed by other countries, has not produced a clear effect in Chinese traffic management and control. Some Chinese researchers have realized this, and correlative work is being done. Chen et al. established the link performance function of a roadway section near curbside stops and proposed a statistical (regression analysis) model to fit the relationship between vehicle speed and three types of flow: cars, buses, and bicycles (9). However, statistical models require large numbers of observations from congested flow near bus stops. Such data are difficult to obtain, and thus it is hard to analyze mixed traffic flow capacity on the basis of existing statistical models. Therefore, a new method is needed with which to discuss mixed traffic flow capacity at bus stops.

The methodological basis in this research is gap acceptance theory (10–13), which is widely used to analyze unsignalized intersections. When a bus dwells at a curbside stop, the conflict between bicycles and cars on the motorized lane is similar to the conflict between vehicles of two different streams at an unsignalized intersection. At a bus stop with mixed traffic flow, especially under congested traffic conditions, the rank of priority from high to low is bus, bicycle, and car, respectively. In addition, Chinese traffic laws regulate that car drivers hold greater responsibility for traffic accidents between nonmotorized and motorized vehicles. Thus, car drivers usually decelerate to give way to nonmotorized vehicles when there is a conflict and car speed is low. Accordingly, the bicycle stream can be regarded as a major stream and the car stream can be a minor
stream during a conflict. Because of lower priority rank, cars have to give way to bicycles on the motorized lane when a bus dwells on the nonmotorized lane. Only when the time interval to the next arriving bicycle is larger than the critical gap has a safe time interval (follow-up time) passed since the departure of the preceding car stream. Car capacity is the function of follow-up times and the availability of headways larger than critical gaps in higher-priority streams (the bicycle stream). Statistical methods are applied to estimate the parameters that describe microscopic properties of the process, such as critical gaps and follow-up times of the car stream, as well as headway distributions of the bicycle stream. Whereas statistical models estimate the effects of road geometry on capacity directly, gap acceptance parameters in stochastic models are adjusted for local conditions.

This paper investigates car capacity at the curbside stop with mixed traffic flow. Mixed traffic flow characteristics on bus stops are analyzed, road capacity based on gap acceptance theory and queuing theory is investigated, and the capacity model is validated by field data in Beijing.

**MIXED TRAFFIC FLOW CHARACTERISTICS AT CURBSIDE STOPS**

**Bus Stream: M/M/k Queuing Model**

Consider a road link near the bus stop as shown in Figure 1. Because of the highest rank of priority, one can assume that the bus stream is not affected by the car and bicycle streams. A sophisticated queuing theory model can be developed on the assumption that the simple bus stream system can be represented by an M/M/k queue. The service counter is the bus stop. The input into the system is formed by the buses approaching from upstream, which are assumed to arrive at random, that is, there are negative exponentially distributed arrival headways with mean $1/\lambda_b$ s (the first M in M/M/k). The dwelling time at the bus stop is the service time, which is also assumed to be independent and negative exponential distributed random variables with mean $1/\mu_b$ s (the second M). Finally, the $k$ in M/M/k stands for $k$ parallel identical servers, that is, the number of existing berths at the bus stop.

Let $N_b$ denote the number of buses in the system that is in equilibrium. According to Medhi [14], for the M/M/k system as a general property, the probability of the empty system is given by

$$P(N_b = 0) = \left( \sum_{j=0}^{k-1} \frac{\rho^j}{j!} + \frac{\rho^k}{k! (1 - \rho)} \right)^{-1}$$ (1)

where $\rho = \lambda_b / \mu_b$ and $\lambda < k\mu$. In the presented bus queuing system, $P(N_b = 0)$ stands for the probability of no stopped bus at the stop. The probability of having at least a stopped bus at the stop is then

$$P(N_b \geq 1) = 1 - P(N_b = 0) = 1 - \left( \sum_{j=0}^{k-1} \frac{\rho^j}{j!} + \frac{\rho^k}{k! (1 - \rho)} \right)^{-1}$$ (2)

In Chinese cities, most curbside stops on minor roadways have two berths. Considering the two-server system, that is, M/M/2, the probability of no stopped bus at the curbside stop is obtained by the equation

$$P(N_b = 0) = \frac{2 - \rho}{2 + \rho} = \frac{2\mu_b - \lambda_b}{2\mu_b + \lambda_b}$$ (3)

**Conflict Streams Between Cars and Bicycles: Gap Acceptance Theory**

When a bus dwells at a stop, there is a conflict between the car stream and the bicycle stream. It is assumed that the volume of the nonmotor (or bicycle) stream (the priority traffic stream) is $q_c$ vehicles/h and the volume of the car stream (nonpriority traffic stream) is $q_b$ (vehicles/h). Bicycles can pass the conflict area without delay. Cars are allowed to enter the conflict area only if the next bicycle is still $t_f$ s away ($t_f$ is the critical gap); otherwise they have to wait. Moreover, cars can enter the conflict area $t_c$ s after the departure of the previous car ($t_c$ is the follow-up time).

When a bus dwells at the curbside stop, the mathematical derivation of the capacity for the car stream, $C(N_b \geq 1)$ (vehicles/h), is as follows. Let $g(t)$, expressed in vehicles/gap, be the number of cars that can enter into a bicycle stream gap of duration $t$. The expected number of these $t$-gaps per hour is $3,600 q_c f(t)$, where, $f(t)$, expressed in gap/vehicle, denotes the statistical density function of the gaps in the bicycle stream. Therefore, the amount of capacity provided by $t$-gaps per hour is $3,600 q_c \cdot f(t) \cdot g(t)$. To get the car capacity, expressed in vehicles/h, integrate over the whole range of bicycle stream gaps:

$$C(N_b \geq 1) = 3,600 q_c \int_0^\infty f(t) \cdot g(t) \, dt$$ (4)

Then, it is assumed that constant $t_f$ and $t_c$ values, constant traffic volumes for each traffic stream, and exponential distribution for bicycle stream headways, that is, the statistical density function of bicycle stream headways, can be given by

$$f(t) = q_b e^{-\lambda_b t} \quad t > 0$$ (5)

In addition, assume a stepwise constant function for $g(t)$:

$$g(t) = \sum_{i=0}^\infty i \cdot p_i(t)$$ (6)

where $p_i(t)$ denotes the probability that $i$ cars enter a gap in the bicycle stream of duration $t$. It can be expressed by

$$p_i(t) = \begin{cases} 1 & \text{for } t_f + (i-1) \cdot t_s \leq t < t_f + i \cdot t_s \\ 0 & \text{elsewhere} \end{cases}$$ (7)
With a combination of Equations 4, 5, and 6, car capacity when a bus dwells at the stop can be evaluated by elementary probability theory methods.

\[
C(N_b \geq 1) = 3.600 \sum_{i=0}^{\infty} i \cdot \frac{e^{-\frac{\rho}{\mu_b}} \cdot \left(1 - e^{-\frac{\rho}{\mu_b}}\right)^i}{1 - e^{-\frac{\rho}{\mu_b}}} 
\]

where \( \rho = \frac{\lambda_b}{\mu_b} \) denotes the volume of bicycle stream, \( \frac{1}{\mu_b} \) is the mean of bus dwelling time, \( \lambda_b \) is the arrival rate of bus at the stop, \( t_f \) is the critical gap and follow-up time when cars enter into the conflict area between bicycle stream and car stream, and \( C(N_b = 0) \) denotes the total car capacity when a bus dwell at the stop, which can be obtained by using Equation 8. In addition, \( C(N_b \geq 1) \) and \( C(N_b = 0) \) are all expressed in vehicles per hour.

\[
C_i = \frac{3.600}{t_f} \int_{0}^{\infty} e^{-\frac{\rho}{\mu_b}} \cdot \frac{1}{1 - e^{-\frac{\rho}{\mu_b}}} \cdot \frac{1}{1 - e^{-\frac{\rho}{\mu_b}}} 
\]

where \( t_f \) is the follow-up time of the car stream and \( C(N_b \geq 1) \) denotes car capacity when a bus dwell at the stop, which can be obtained by using Equation 8. In addition, \( C_i, C(N_b = 0), \) and \( C(N_b \geq 1) \) are all expressed in vehicles per hour.

By combining Equations 1, 2, 8, 10, and 11, one can obtain a car capacity model at the curbside stop. Car capacity at the stop is the function of the three types of traffic stream, buses, cars, and bicycles, which can be given by

\[
C_i = \left[ \frac{3.600}{t_f} \int_{0}^{\infty} e^{-\frac{\rho}{\mu_b}} \cdot \frac{1}{1 - e^{-\frac{\rho}{\mu_b}}} \cdot \frac{1}{1 - e^{-\frac{\rho}{\mu_b}}} \right] \cdot 3.600 q_e \cdot e^{-\frac{\rho}{\mu_b}} 
\]

\[
C = \frac{3.600}{t_f} \int_{0}^{\infty} e^{-\frac{\rho}{\mu_b}} \cdot \frac{1}{1 - e^{-\frac{\rho}{\mu_b}}} \cdot \frac{1}{1 - e^{-\frac{\rho}{\mu_b}}} 
\]

where

\[
k = k \text{ berths at the curbside stop}, \quad \rho = \frac{\lambda_b}{\mu_b} \quad \frac{1}{\mu_b} = \text{mean of bus arrival headways}, \quad \frac{1}{\mu_b} = \text{mean of bus dwelling time}, \quad \lambda_b = \text{volume of bicycle stream,} \quad t_c, t_f = \text{critical gap and follow-up time when cars enter into the conflict area between bicycle stream and car stream.} \quad \lambda_b, \frac{1}{\mu_b} = \text{are expressed in seconds, and} q_e = \text{expressed in vehicles per second.} \quad \text{For the special curbside stop with two berths, according to Equations 3, 8, 10, and 11, car capacity can be obtained as}
\]

\[
C_i = \frac{2 \mu_b - \lambda_b}{2 \mu_b + \lambda_b} \cdot \frac{3.600}{t_f} + \frac{2 \lambda_b}{2 \mu_b + \lambda_b} \cdot 3.600 q_e \cdot e^{-\frac{\rho}{\mu_b}} 
\]

\[
C = \frac{2 \mu_b - \lambda_b}{2 \mu_b + \lambda_b} \cdot \frac{3.600}{t_f} + \frac{2 \lambda_b}{2 \mu_b + \lambda_b} \cdot 3.600 q_e \cdot e^{-\frac{\rho}{\mu_b}} 
\]

Effects of Individual Traffic Stream on Car Capacity

Differences in the arrival rate of the bus stream affect car capacity at the curbside stop, as shown in Figure 2. Here, the critical gap was 2.5 s, the follow-up time was 2 s, the mean of dwell times was 15 s, and bicycle stream headways followed the negative exponential distribution. At the same bicycle flow rate, the probability of no stopped bus at the stop decreases with the increasing arrival rates of bus stream. This reduces the car capacity. In addition, Figure 2 shows...
that the conflict increases with the increasing bicycle flow rate, which causes a decrease in car capacity near bus stops. Similarly, the dwell times of buses at the stop affect car capacity. As Figure 3 displays, the increasing dwell time of buses increases the probability of the conflict between bicycles and cars, reducing road capacity. Figures 2 and 3 indicate that at different buses’ arrival rates or dwell times, the differences in car capacity increase with the increasing bicycle flow rate.

In addition, both the critical gaps and the follow-up times affect car capacity near the curbside stop. Figure 4 gives the effect of critical gaps on car capacity. At the same bicycle flow rate, the queuing delay time of the car stream increases with the increasing critical gap. This reduces car capacity. Under conditions of light bicycle traffic and up to 2,000 vehicles/h, the difference of car capacities at different critical gaps increases with the increasing bicycle flow. When the bicycle stream is higher than 2,000 vehicles/h, on the contrary, the difference of car capacities decreases with the increase of bicycle flow. This is because at high bicycle flow rates, fewer cars can pass the conflict area of bus stops with the increasing critical gaps when a bus dwells at the stop (see Figure 5). A comparison of Figures 2, 3, and 4 indicates that, at low bicycle flow rates, the effect of critical gaps on car capacity is larger than that of bus arrival rates as well as that of bus dwell times on car capacity.

COMPARISON WITH FIELD MEASUREMENTS

Field data collected at bus stops in Beijing were used to calibrate the new capacity model at curbside stops with the interaction between transit (bus stops), bicycle flow, and car flow. The survey areas were
urban locations in the north of Beijing. After panel observations, the bus stop near the Xue-Yuan-Lu expressway was chosen. Figure 6 shows the surveyed roadway section. Data on spot flows, speeds, densities, and vehicle classifications were collected in the summer and fall of 2007 in one direction for 15 min and were categorized into a group.

### Estimation of Critical Gap Parameters

The two critical gap parameters that need to be estimated are the follow-up time $t_f$ and the critical gap $t_c$. The follow-up time in this paper is the mean headway between queued cars that move through the stopped bus during the longer gaps in the bicycle stream. A probabilistic approach can be used to estimate the follow-up time. Consider the example of two bicycles passing by a stopped bus at 2.0 and 10.0 s. If there is a queue of, say, four cars that want to pass the curbside stop, and if they depart at 2.4, 4.8, 7.3, and 9.9, then the headways between cars are 4.8 − 2.4, 7.3 − 4.8, and 9.9 − 7.3. The average headway between four cars is 2.5 s. This process is repeated for a number of larger bicycle gaps, and an overall average headway between the queued cars is estimated. This average headway is the follow-up time. If a car was not in a queue, then the preceding headway would not be included.

The estimation of critical gaps from observed traffic flow patterns is one of the most difficult tasks in empirical traffic engineering science. The difficulty in estimation of the critical gap is that it cannot be directly measured. All that is known is that a driver’s individual critical gap is greater than the largest gap rejected and shorter than the accepted gap for that driver. If the accepted gap is shorter than the largest rejected gap, then the driver is considered to be inattentive. These data are changed to a value just below the accepted gap. Numerous techniques have been proposed. Brilon et al. compared some important methods and found that the maximum likelihood procedure and Hewitt’s method gave the best performance for a wide range of minor stream and major stream flows (15). Therefore, the maximum likelihood method is used here to estimate the critical gap of cars at the curbside stop.

### Average Delay and Comparison with Measured Delays

To calculate the average delay, a classical approach may be used. The formula, which is contained in the *Highway Capacity Manual* as Equation 17-38, may be applied to calculate the average delay for a car stream (2).

Figure 7 compares the measured and calculated delays at the proposed approach. In the proposed approach in this paper, the mean percent error is 6.9% and the mean absolute percentage error is 10.4%. Thus, the proposed model at bus stops with mixed traffic flow is desirable.

### CONCLUSION

Road capacity at the curbside stop with mixed traffic flow is investigated on the basis of gap acceptance theory and queuing theory. Two conditions of bus stops are considered: that there is a stopped bus or there is not. If there is no bus at the stop, the car stream is not affected by the bicycle stream and its capacity equals the capacity on an uninterrupted roadway. While the bus is at the stop, its presence on a nonmotorized lane causes conflict between bicycles and cars on the motorized lane because bicycles change lanes. In this instance, car capacity near bus stops can be obtained by using gap acceptance theory, which is widely used on unsignalized intersections. Combining two conditions, car capacity at the curbside stop is the function of three types of traffic stream, buses, cars, and bicycles. The analysis shows that both the bus stream and the bicycle stream have significant effects on car capacity. The results may be applicable in the traffic analysis and design of bus stops in other developing Asian cities.

Although this study gives valuable insight into road capacity at a bus stop with mixed traffic flow, further research work is suggested. First, it is assumed here that bicycles have absolute priority over cars when a bus dwells at the stop. However, the minor-stream vehicles could affect the major-stream vehicles (16). Field observations show that the limited priority mechanism does take place at bus stops. Study of a new road capacity model that assumes limited priority, a particular example of shared priority, for car and bicycle streams, is
needed. Second, a delay model on roadways near bus stops with mixed flow can be estimated on the basis of queuing theory. In addition, the plan and design problems of bus stops with mixed traffic flow should be further researched.

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REFERENCES

1. Levinson, H. S., and K. R. St. Jacques. Bus Capacity Revisited. In Transportation Research Record 1618, TRB, National Research Council, Washington, D.C., 1998, pp. 189–199.
2. Highway Capacity Manual. TRB, National Research Council, Washington, D.C., 2000.
3. Fitzpatrick, K., K. Hall, D. Perdinson, and L. Nowlin. TCRP Report 19: Guidelines for the Location and Design of Bus Stops. TRB, National Research Council, Washington, D.C., 1996.
4. Fitzpatrick, K., and R. L. Nowlin. Effects of Bus Stop Design on Suburban Arterial Operations. In Transportation Research Record 1571, TRB, National Research Council, Washington, D.C., 1997, pp. 31–41.
5. Wong, S. C., H. Yang, W. S. A. Yeung, S. L. Cheuk, and M. K. Lo. Delay at Signal-Controlled Intersection with Bus Stop Upstream. Journal of Transportation Engineering, Vol. 124, No. 3, 1998, pp. 229–234.
6. Koshy, R. Z., and V. A. Arasan. Influence of Bus Stops on Flow Characteristics of Mixed Traffic. Journal of Transportation Engineering, Vol. 131, No. 8, 2005, pp. 640–643.
7. Zhao, X. M., Z. Y. Gao, and B. Jia. The Capacity Drop Caused by the Combined Effect of the Intersection and the Bus Stop in a CA Model. Physica A, Vol. 385, 2007, pp. 645–658.
8. Zhao, X. M., Z. Y. Gao, and K. P. Li. The Capacity of Two Neighbour Intersections Considering the Influence of the Bus Stop. Physica A, Vol. 387, 2008, pp. 4649–4656.
9. Chen, X. W., W. Wang, J. Y. Wang, and F. L. Chen. Link Performance Functions Based on the Traffic Management and Control (in Chinese). Journal of Highway and Transportation Research and Development, Vol. 17, No. 3, 2000, pp. 40–42.
10. Pollatschek, M. A., A. Polus, and M. Livneh. A Decision Model for Gap Acceptance and Capacity at Intersections. Transportation Research A, Vol. 36, 2002, pp. 649–663.
11. Brilon, W., and N. Wu. Capacity at Unsignalized Two-Stage Priority Intersections. Transportation Research A, Vol. 33, 1999, pp. 275–289.
12. Tanyel, S., and N. Yayla. A Discussion on the Parameters of Cowan M3 Distribution for Turkey. Transportation Research A, Vol. 37, 2003, pp. 129–143.
13. Gartner, N. H., C. Messer, and A. K. Rathi. Monograph on Traffic Flow Theory: A State of the Art Report. FHWA, U.S. Department of Transportation, 1996.
14. Medhi, J. Stochastic Models in Queueing Theory. Academic Press, New York, 1991.
15. Brilon, W., R. Koenig, and R. J. Troutbeck. Useful Estimation Procedures for Critical Gaps. Transportation Research A, Vol. 33, 1999, pp. 161–186.
16. Troutbeck, R. J., and S. Kako. Limited Priority Merge at Unsignalized Intersections. Transportation Research A, Vol. 33, 1999, pp. 291–304.

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