Leveraging Evidential Deep Learning Uncertainties with Graph-based Clustering to Detect Anomalies

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Abstract—Understanding and representing traffic patterns are key to detecting anomalies in the maritime domain. To this end, we propose a novel graph-based traffic representation and association scheme to cluster trajectories of vessels using automatic identification system (AIS) data. We utilize the (un)clustered data to train a recurrent neural network (RNN)-based evidential regression model, which can predict a vessel’s trajectory at future timesteps with its corresponding prediction uncertainty. This paper proposes the usage of a deep learning (DL)-based uncertainty estimation in detecting maritime anomalies, such as unusual vessel maneuvering. Furthermore, we utilize the evidential deep learning classifiers to detect unusual turns of vessels and the loss of AIS signal using predicted class probabilities with associated uncertainties. Our experimental results suggest that using graph-based clustered data improves the ability of the DL models to learn the temporal-spatial correlation of data and associated uncertainties. Using different AIS datasets and experiments, we demonstrate that the estimated prediction uncertainty yields fundamental information for the detection of traffic anomalies in the maritime and, possibly in other domains.

Index Terms—Anomaly detection, Evidential deep learning, Regression, Classification, Clustering, Graph, Uncertainty

I. INTRODUCTION

SECURITY is crucial for maritime transportation, as more than 90% of the world-trade is being carried out at sea and is facing significant challenges from natural and unnatural phenomena, such as tough environment, collision, illegal fishing, smuggling, pollution, and piracy [1]. Some of these challenges can be addressed by having a better understanding of maritime traffic patterns in terms of its representation, accurate localization, and its forecasting approaches. Nowadays most ships, excluding small non-passenger ones of less than 300 gross tonnages, are equipped with AIS transponders to provide their static and dynamic information [2]. The AIS transponders use global navigation satellite systems for positioning, with accuracy of up to a few meters [3]. The vessels’ location, navigational status, as well as voyage-related information can be used for collision-avoidance mechanisms, vessel tracking, along with the detection of loss of AIS signal, trajectories, and vessels’ maneuvers.

Despite being equipped with on-board transponders, some vessels meander to engage in suspicious activities. A vessel can intentionally switch-off its AIS transponder, deviate from normal trajectory and turn in an unusual manner. Such situations may arise, for instance, during hijacking of ships by pirates, illegal fishing, rough environment, blockage of canals (e.g., Suez, Panama) amongst others. At the same time, the causes of loss in AIS signals from some vessels at receiving stations could also be unintentional due to limited AIS range, incorrect AIS antenna setup, geographically-induced attenuation or radio interference [4], [5]. Therefore, in addition to correctly representing traffic patterns, it is necessary to detect anomalous (or unusual) vessel trajectories, the loss of signal, unusual turns and other anomalies. In this context, historical and real-time AIS data provide a valuable source of vessel trajectories information to work with in the maritime domain.

Detecting maritime anomalies in real-time has become a key point from operators’ point of view. As there are different kinds of anomalies in maritime domain, they need special attention and mechanisms to be dealt with. The traditional manual surveillance method requires man power, it is therefore often cumbersome and prone to errors. Rule-based approaches with an expert knowledge are simple yet limited by their rigidity [6]. In contrast, statistical and machine learning (ML) algorithms provide promising and complementary solutions to deal with maritime security challenges [7], [8]. Statistical methods, such as Bayesian networks, Kalman filtering, Gaussian processes, or Markov models, have been proven to be very effective in the reconstruction and the prediction of vessels’ trajectories, potentially useful for anomaly detection [9]–[13]. They are generally robust but don’t exploit the short-/long-term inter-dependency of features, e.g., positional and navigation AIS information. Contrarily, the use of ML techniques, such as neural networks, long short-term memory (LSTM), have demonstrated promising results for the anomaly detection problem [14]–[20].

ML-models are often not considered as robust under black swan events (completely unseen scenarios) due to the lack of a model confidence and an uncertainty estimation. Uncertainty estimation could help dealing with critical applications, such as anomaly detection, surveillance, and maritime navigation. We are interested in such applications of ML techniques that include uncertainty characterization. With an evidential deep learning (EDL) for regression [21] and classification [22] approaches, uncertainties associated with data and models could be applied to detect anomalous trajectories and other anomalies. To date and to the authors’ best knowledge, anomaly detection problem has not been approached with ML-based uncertainty estimation in maritime domain using AIS data.

This work leverages model (epistemic) and data (aleatoric)
uncertainties to deal with maritime anomaly detection. In order to detect a set of anomalies using AIS data, we develop clustering, regression and classification models. First, we propose a graph-based traffic clustering and association model to represent maritime traffic patterns. The clustered data may be given as input to an RNN-based EDL regression model to predict the localization and dynamics information of vessels, as well as uncertainties in their predictions. Second, we propose two different EDL classifiers to learn an unusual turn (i.e., fast maneuvers over a short time interval) and AIS on-off switching (OOS) anomalies. This article demonstrates the performance of the EDL regression model for anomalous trajectory detection, and the EDL classification models for the detection of unusual turns and loss of AIS signal (i.e., OOS) over big and small AIS historical datasets collected from the Baltic sea region. The trained EDL classifiers are also shown to be useful in detecting the OOS and the unusual turn anomalies on a test vessel’s trajectories in a validation campaign in another region, namely Bremerhaven (Germany).

The rest of the article is organized as follows. We present a summary of related work in Section II. Section III describes our proposed clustering, regression and classification models useful in detecting various anomalies. Section IV formulates anomalies and presents detection criteria. We evaluate the performance of the models in Section V and conclude the paper in Section VI.

II. RELATED WORK

Detecting anomalies is a rather difficult problem as these are rare phenomena and a reference (or ground truth) dataset is generally unavailable. Within the maritime domain, rule-based and clustering methods have been widely utilized to detect anomalies. Rule-based methods are simple and useful in detecting anomalies based on predefined rules of normal and anomalous behaviors of vessels [6], [23]. Output from clustering methods aid the depiction of trajectories into specific traffic patterns. They can help finding abnormality in vessels’ trajectories with respect to so-called normality models or normal patterns. Recently, ML or data-driven models have gained momentum in detecting anomalies, especially those that provide model confidence and data uncertainty. In general, we could divide related work into three sub-tasks involved in anomaly detection, namely: i) traffic data representation using clustering, ii) traffic prediction using statistical and ML methods, and iii) traffic classification. In the following paragraphs, solutions for such three categories are discussed.

Learning and representing motion patterns is a well-known problem that has been investigated across multiple domains [24], [25], including in automotive [26] and maritime domains [27]–[30]. Numerous studies have used unsupervised learning methods, such as k-means, Gaussian mixture model, density-based spatial clustering of applications with noise (DBSCAN), etc., for traffic clustering in maritime domain [27], [29], [31], [32]. The works presented in [27] and [29] utilized the DBSCAN technique and its variants to cluster historical AIS data into route patterns. An Ornstein-Uhlenbeck mean-reverting stochastic process was also applied to represent tracks and detect anomalous ones [33]. Recently, [31] applied a polygonal geographical areas approach to label (cluster) a limited number of routes as input to a trajectory prediction method. A promising graph-based approach was proposed in [32] to represent maritime traffic, whose nodes represent clusters of waypoints, i.e., areas where ships are more likely to change course. The nodes are connected by graph edges corresponding to sea lanes. The above-mentioned clustering methods are sensitive to hyper-parameters. Nevertheless, they are able to represent motion patterns and can be a useful traffic pre-processing step for the prediction task, which we analyze in Sec. V.

Anomaly detection using either the deviation between a predicted trajectory and its reported (measured) one, or considering the probabilistic characteristics of trajectories have also been exploited, using both statistical and ML-based solutions. Statistical methods, such as Kalman filter, particle filter, Bayesian networks or Gaussian processes (GPs) have been used to estimate/predict the vessels’ locations at next time stamp to reconstruct their trajectories and estimate probabilistic characteristics of trajectories to detect anomalies [7], [8], [30], [34]–[37]. ML (or deep learning, DL, as being branded lately) algorithms, such as LSTM and LSTM encoder decoder [38], [39], have been shown to be promising in sequence-to-sequence or time-series prediction in a number of applications, including trajectory prediction tasks [17], [31], [40], [41]. In addition, [16] applied the Variational RNN to learn the probabilistic distribution of trajectories, which help them to detect anomalous trajectories. There have also been efforts towards anomaly classification by means of supervised learning with labeled data. [18] used a random forest algorithm to classify vessels using AIS data streams. [15] proposed a one-class support vector machine-based anomaly detection framework that takes AIS data as well as received signal strength into consideration for analyzing the AIS OOS anomaly. In [19], [20] we showed that neural networks can be used to detect the AIS OOS anomaly with a higher accuracy than other traditional ML algorithms.

Classical ML models generally do not indicate any measure of confidence or uncertainty with their predictions. As the datasets of any application evolve over time and are prone to varying noise levels, it is impossible to use those models in safety-critical applications such as anomaly detection. Thus, it is highly desirable to quantify model and data uncertainty in a trustworthy manner. In this paper, we approach the maritime anomaly detection problem using a novel ML-based learning of model and data uncertainties. To this end, we use a promising EDL regression [21] and classification [22] model. In a nutshell, EDL captures the model evidence (confidence) in support of its prediction. Note that the computationally-intensive, non-parametric GPs [42] and parametric Bayesian neural networks (BNNs) [43], [44] are also solid options for the provision of reliability measures and prediction uncertainty. Orthogonal to GPs and BNNs, EDL enables simultaneous learning of the desired regression/classification task along with data (aleatoric) and model (epistemic) uncertainty estimation, by enforcing evidential priors. Furthermore, an EDL model can be trained to provide uncertainty measures
III. METHODOLOGY FOR ANOMALY DETECTION AND MODELING

Undoubtedly, anomaly detection is a prerequisite for the obtainment of a complete maritime traffic situational awareness. The question resides on how such anomalous patterns are defined. In general, we treat a predictable or acceptable behavior as normal and the rest is classified as anomalies. For example, the acceptable practice to find an anomalous trajectory for a cargo ship is to examine whether the ship deviates from its well-defined maritime route. This method might not be suitable for smaller vessels, e.g., pleasure crafts, fishing vessels, etc. Therefore, we apply EDL techniques to learn data distribution (or patterns) and their underlying uncertainties. There could be different sources of uncertainty in the prediction, when an ML model: i) is trained with in-distribution noisy data, ii) is tested with mismatched or out-of-order distributed data compared to the data used in training, and iii) has wrongly estimated weight and network parameters. The uncertainty caused by the first source can be termed as aleatoric (data) uncertainty, which is irreducible. The second and third sources cause a reducible epistemic (model) uncertainty. For the sake of visualization, Fig. 1 depicts both type of uncertainties with respect to some training data and a model-fit. In other words, aleatoric uncertainty represents the noise distribution of the data (i.e., the reliability of what we measure), while epistemic uncertainty describes the uncertainty over the estimated prediction (i.e., the reliability of what we estimate).

One can illustrate an anomalous behavior of a vessel with respect to its normal trajectories using an example shown in Fig. 2. Here, the anomalous segments clearly deviate from the bulk of trajectories. This anomalous behavior is a combination of an unusual maneuvering and an unusual turn. They can be detected by either i) thresholding the maximum allowed deviation between the posterior ground truth (reported) and the long-term a-priori prediction of a trajectory, which is not trivial for all types of vessels; or ii) quantifying a measure of uncertainty in prediction on successive reported data. With the EDL regression we could estimate the aleatoric or epistemic uncertainty (e.g., for unusual maneuvering), and accept/reject classification of data patterns (e.g., for unusual turn, OOS) based on the obtained epistemic uncertainty.

Anomaly detection problem may be solved by quantifying uncertainties with data and a set of three ML models, namely clustering, regression and classification. The clustering models are generally useful in representing vessels’ trajectories into well-defined routes (or sea-lanes). The regression model, on the other hand, forecasts the prospective positions of vessel, as well as estimate model and data uncertainties. Lastly, a classifier could be trained to distinguish between normal and abnormal data patterns, which can be used to detect unusual turn, on-off switching, etc. In this section, we briefly describe the AIS data representation, and present a graph-based clustering method, EDL regression model and EDL classification model for anomaly detection modeling.

A. AIS data representation

AIS messages constitute the primary source of information for the methodology developed in this work. Each vessel’s AIS report may include an identifier, i.e., the maritime mobile service identity (MMSI); kinematic information, i.e., longitude (lon), latitude (lat), speed over ground (sog) and course over ground (cog); a UTC timestamp (time); and other additional information, such as voyage, ship name, ship type, navigation status, etc.

Considering a discrete state-space model, the state vector for the navigation problem for the ith vessel at time index k is formulated as

\[ x_k^i \triangleq [\text{lon}, \text{lat}, \text{cog}, \text{sog}, \text{time}, \text{edge}]^\top, \]  

with edge a feature obtained using the graph-based trajectory association scheme. It is important to note that our anomaly
detection models use different subsets of features in $x_i$, and unlike the kinematic information directly obtained from the AIS data, the edge feature is not known a priori. The notation $X_{k:k+T}^i$ refers to the state of the $i$th vessel between the time periods $k$ and $k+T$, such that

$$X_{k:k+T}^i \triangleq [x_{i,k}, x_{i,k+1}, \cdots, x_{i,k+T-1}], \tag{2}$$

and we use $X^i$ to refer to the complete set of states for the $i$th vessel along the duration of the dataset. In the sequel, the methodology related to trajectory clustering and association, trajectory forecasting and uncertainty estimation, and anomaly classification are detailed.

**B. Trajectory Clustering and Association via Graph-based Representation**

Providing an accurate traffic situation awareness requires addressing which are the usual sea lanes and vessels’ trajectories. Thus, given a real-world historical AIS dataset $D$, we leverage on graph-based clustering to conform the trajectories onto commonly-travelled maritime routes. This process is denoted as graph-based trajectory representation (GTR). Then, we can associate the trajectories from any given dataset via graph-based trajectory association (GTA).

The graph $G$ consists of a set of nodes $V = \{v_j\}_{j=1}^{|V|}$ and a set of edges $E = \{e_j\}_{j=1}^{|E|}$, with $v$ two-dimensional (2D) vectors containing longitude and latitude values, and each $e_j$ containing two distinct nodes in $V$. In this work, the graph’s nodes represent turning points for the vessels’ trajectories, while the edges joining consecutive nodes represent the sea lanes traversed by vessels.

The GTR process, i.e., the estimation of a graph $G$, is summarized in Algorithm 1. First, in Steps 2-4 we reduce the trajectory of each vessel $X^i$ to a set of waypoints $WP^i$ using an iterative Ramer-Douglas-Peucker (RDP) algorithm [15]. $WP^i$ may represent the trajectory’s start, end and turning points, and the parameter $\epsilon$ limits the maximum distance of any intermediate points to their joining segments. The above process is repeated for all vessels’ trajectories and then their associated waypoints are clustered into nodes using the DBSCAN algorithm (Step 5). DBSCAN iteratively searches for the core points in the neighbourhood of the waypoints $WP^i$ within a distance of $\epsilon$, so that there are at least $n_{min}$ number of density-reachable points. We compute the mean of core points in each cluster to get the nodes $V_i$. Lastly, the set of edges are estimated by computing the fraction of times that an edge (sea lane) between two nodes is visited by ships as compared to all visits of the edges of the two nodes. If the fraction is greater than a threshold $\epsilon_{th}$, then the edge is retained (Step 6).

The GTA scheme aims at associating all AIS data from $D$ to the edges of the graph formed by the GTR, as formulated in Algorithm 2. This is performed by computing the perpendicular distances $d_\perp$ from an AIS data of a vessel to all edges in Steps 2-5. The edge with the minimum distance is selected if it lies below a threshold $d_{max}$, otherwise the point is associated to a so-called outlier edge $\emptyset$. Note that this process may assign successive points of a trajectory segment to different edges. Therefore, we compute waypoints $WP^i$ for each vessel’s trajectory $X^i$ in Step 6 using the RDP algorithm. Next, we form a set of edges that was associated (in Step 4) to all intermediate AIS data, i.e., $E_{k_1:k_2}$ between each pair of consecutive waypoints $(x^i_{k_1}, x^i_{k_2}) \in WP^i$ in Steps 7-8. At last, these data points are associated to an edge $e_j$ that has a minimum slope difference from a line formed by joining the waypoints (Step 9). Note that the Steps 7-11 is irrelevant if all points of a segment has been associated the same edge, i.e., $E_{k_1:k_2} = 1$, which could also be an outlier edge.

Our intuition is that a normal trajectory prediction model could be learned more precisely for each labeled trajectory, as compared to an unlabeled trajectory. In addition to AIS data features ($lon, lat, cog, sog$), we could provide the edge label for each AIS data point as an input feature to a trajectory forecasting model, which is described next.

**C. An RNN-EDL Regression Model**

The aim of the RNN-EDL model is to forecast the subsequent $L$ navigation instances for an $i$th vessel, given the last $T$ time instances prior. Thus, we introduce a RNN-EDL regression model

$$f(X_{k:k+T}, W) = X_{k+T|k:k+T+L}, \tag{3}$$

with $X_{k:k+T}$ the input, $X_{k+T+k+T+L}$ the ground truth output, $W$ the model weights optimized during the training process, and $f(\cdot)$ the estimator per se. The model forecasts the estimates $\hat{X}_{k+T|k:k+T+L}$ for $L$ time instances. For simplicity, we will omit the vessel index $i$ henceforth.

Among the recurrent networks, the LSTM encoder-decoder is known for its superior performance at temporal prediction [19] and it constitutes the learning algorithm for our

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**Algorithm 1** Graph-based Traffic Representation (GTR)

1: **Input**: Dataset $D$, parameters: $\epsilon, \bar{\epsilon}, n_{min}, \epsilon_{th}$
2: for each $i$ vessel in $D$ do
3: \hspace{1em} $WP^i \leftarrow$ RDP($X^i, \epsilon$)
4: end for
5: Nodes $V \leftarrow$ DBSCAN($WP, \epsilon, n_{min}$)
6: Edges $E$ with $#\ of\ visits > \epsilon_{th}$
7: **Return**: $G(V, E)$ for data association

**Algorithm 2** Graph-based Traffic Association (GTA)

1: **Input**: Dataset $D$, graph $G$, parameters $d_{max}, \epsilon$
2: for each $i$ vessel in $D$ do
3: \hspace{1em} for each $k$ time in $X^i$ do
4: \hspace{2em} $x^i_{k}[\text{edge}] = \{j, \text{if} \ min_{1 \leq j \leq |E|} d_\perp(e_j, x^i_k) \leq d_{max}\}$ \hspace{1em} \hspace{1em} otherwise \hspace{1em} \hspace{1em} $\emptyset$
5: \hspace{1em} end for
6: \hspace{1em} $WP^i \leftarrow$ RDP($X^i, \epsilon$)
7: for each $(k_1, k_2)$ time pair in $WP^i$ do
8: \hspace{1em} $E_{k_1:k_2} \leftarrow \{x^i_{k}[\text{edge}]\}$
9: \hspace{1em} $x^i_{k}[\text{edge}] \leftarrow j, \min_{1 \leq j \leq |E|_i} |\nabla e_j - \nabla (x^i_{k_1} , x^i_{k_2})|$
10: end for
11: end for
12: **Return**: $x^i_{k}[\text{edge}], \forall i, k \in D$
can be extracted from this data are very limited and hence the evidence for a certain prediction (even when given as mean and variance) is also limited. As a result, the higher order distribution, which is parameterized by this evidence, represents the uncertainty as well.

Based on the evidential modelling, the probability of a prediction $x_k$ is given by

$$ p(x_k|m) = \int_{\theta} p(x_k|\theta)p(\theta|m)d\theta. \quad (4) $$

For our approach, we follow [21] and model the aleatoric uncertainty by a normal distribution with unknown mean and variance, i.e. $\theta = (\mu, \sigma^2)$ and use its conjugate prior, i.e., Normal Inverse-Gamma (NIG), as a fitting distribution for representing the epistemic uncertainty. The NIG distribution is parameterized by four scalar parameters, i.e.

$$ m = (\hat{x}, v, \alpha, \beta) \quad \text{with} \quad \hat{x} \in \mathbb{R}, v > 0, \alpha > 1, \beta > 0. \quad (5) $$

$$ \mu \sim \mathcal{N}(\hat{x}, \sigma^2/v) \quad \text{and} \quad \sigma^2 \sim \Gamma^{-1}(\alpha, \beta) \quad (6) $$

where $\Gamma^{-1}$ is the inverse gamma distribution.

For the probability of a prediction we have to consider the marginal likelihood $p(x_k|m)$, which is given as the Student’s-t

$$ p(x_k|m) = St(x_k; \hat{x}_k, \beta(1 + v)/v\alpha, 2\alpha). \quad (7) $$

Please refer to [21] and their supplementary materials for further details and the derivations.

During the training process the network parameters $W$ are optimized in order to minimize the negative log-likelihood

$$ -\log p \left( \int_{\theta} p(x_k|\theta)p(\theta|m)d\theta \right). \quad (8) $$

Given a $k$th input sequence $X_{k:k+T}$, the loss function $L_k$ for a ground-truth output sequence $X_{k+T:k+T+L}$, the predicted parameters $m^d_k$ and an estimated mean $\hat{x}^d_k$ can be computed as [21]

$$ L_k(W) = L_k^{NLL}(W) + \lambda L_k^{R}(W) $$

$$ = \sum_{j=k+T}^{k+T+L-1} \sum_{d=1}^{n} -\log p(x^d_j|m^d_k) + \lambda |x^d_j - \hat{x}^d_j|2\sigma^2_j + \alpha^d_j, \quad (9) $$

where the first loss component is used for minimizing the negative log-likelihood (NLL) of an observation $x_k$ and the second one is an evidence regularizer that motivates to collect less evidence when the predictions are false with a scalar $\lambda$.

The RNN-EDL model is trained to output four evidential distribution parameters for each output $x^d_k$, $d = 1, 2, \ldots, n$, in $X_k$, i.e., $m^d_k = (\hat{x}^d_k, v^d_k, \alpha^d_k, \beta^d_k)$ for an $L$-length output sequence $X_{k+T:k+T+L}$. Given a NIG distribution and following [21], we can compute the next timestep prediction at $j = k+T$ for each feature $d = 1, 2, \ldots, n$, given an input sequence $X_{k:k+T}$, as $\hat{x}^d_j = \mathbb{E}[\mu^d_j]$, the aleatoric uncertainty as $\sigma^2_j \times \text{var}[\mu^d_j]$, and the epistemic uncertainty by

$$ \text{var}[\mu^d_j] = \frac{\beta^d_j}{v^d_j(\alpha^d_j - 1)}. \quad (10) $$
by setting a threshold on acceptable uncertainty the classification of data patterns (e.g., for unusual turn, OOS) These classifiers are trained with different inputs and networks unusual turn detection and other for the AIS OOS detection.

\[
\sum_{j=1}^{n} \exp(\tilde{y}_j) / \sum_{j=1}^{K} \exp(\tilde{y}_j)
\]

In doing so it does not take the magnitude of the logits into account and hence fails to represent the certainty of the predicted probability vector. Therefore, authors in \cite{22} came up with an idea to replace the softmax activation with any other continuous operator, for example ReLU\((x) = \max(x, 0)\), and consider \(\alpha = f(\Theta, X_{k:T}) + 1\) (i.e., output+1) as parameters of a multivariate Dirichlet distribution \(\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_K)\). An architecture of EDL classifier is depicted in Fig. 4. Notice that unlike the evidential regressor, which models outputs with the continuous Gaussian distribution, the evidential classifier utilizes the categorical Dirichlet’s distribution, with a sum of squares loss function \(|y_c - \hat{p}_c|^2_2\). Once the network learns the parameters \(\alpha\), its mean, can be taken as an estimate of the class probabilities, shown in Eq. (11). The epistemic uncertainty on the prediction is computed as the inverse of total evidence or Dirichlet strength \(S = \sum_{c=1}^{K} \alpha_c\), for \(K\) output classes in \cite{22}.

\[
\hat{p}_c = \alpha_c / S, c = 1, 2, \ldots, K
\]

\[
u = K / S
\]

In this paper we train two EDL classifiers: one for the unusual turn detection and other for the AIS OOS detection. These classifiers are trained with different inputs and networks required for the learning purposes. The prediction uncertainty in Eq. (12) together with (11) can be used to accept or reject the classification of data patterns (e.g., for unusual turn, OOS) by setting a threshold on acceptable uncertainty \(\nu_{th}\). In the next section, we describe how we detect different anomalies using the proposed models.

D. EDL Classifiers for Anomalous Pattern (OOS and Turn) Detection

A standard neural network outputs \(\tilde{y} = f(\Theta, X_{k:T}) = a_{n+1}(W_{n+1, \Theta} W_n \ldots a_1(W_1 \cdot X_{k:k+T})\ldots)\) with the number of hidden layers \(n\), activation functions \(a_j(\cdot), j = 1, 2, \ldots, n\), and network parameters \(\Theta = [W_{1:n+1}, b]\), where \(W\) is the weight vector of the connections between two layers, and \(b\) is a bias vector. For classification, generally a softmax activation is used to convert the predicted values \((\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_K \in \mathbb{R})\) of the output layer to class probabilities \(p_c = \text{softmax}(\tilde{y}_c) = \exp(\tilde{y}_c) / \sum_{j=1}^{K} \exp(\tilde{y}_j)\). The epistemic uncertainty on the prediction is computed as the inverse of total evidence or Dirichlet strength \(S = \sum_{c=1}^{K} \alpha_c\), for \(K\) output classes in \cite{12}.

\[
\hat{p}_c = \alpha_c / S, c = 1, 2, \ldots, K
\]

\[
u = K / S
\]

IV. Anomaly Definitions and Detection Criteria

The following subsections describes three anomalies and presents the detection criteria.

A. Anomalous Trajectory (AT)

In order to detect anomalous trajectories, each trajectory is divided into smaller segments. Let us assume that a trajectory segment contains \(N + T - 1\) consecutive AIS messages covered by a sliding window of length \(T\) with a step size of one. This results into \(N\) sequences per segment. To simplify the evaluation of uncertainties, we keep the output sequence length as \(L = 1\).

A vessel’s trajectory is termed as anomalous if it contains one or more anomalous segments. Note that high epistemic uncertainty may represent anomalous trajectory. However, different output features per AIS data are predicted with different uncertainties and it might not be a good idea to analyze a given track with respect to the uncertainty-threshold of other normal tracks. Thus, we define a trajectory segment as anomalous if the predicted sequences of the segment have an abrupt transition in their epistemic uncertainties. Precisely, a contiguous segment \(X_{0:}\) \(N+T\) with \(N\) sequences is termed as anomalous if the minimum segment uncertainty over the normalized uncertainties obtained over all the predicted outputs is smaller than a threshold \(\Theta_{AT}\), i.e.,

\[
\min_d \left[ \min_j (\text{var}[\mu_d^j]) / \max_j (\text{var}[\mu_d^j]) \right] < \Theta_{AT}, \tag{13}
\]

where Eq. (13) detects some abrupt transitions in the output uncertainties \(\text{var}[\mu_d^j]\) over \(j = T, T+1, \ldots, N+T-1\) sequences for each feature \(d = 1, 2, \ldots, n\). It selects the feature \(d\) and output sequence \(j\) with the minimum normalized uncertainties. If this value is below \(\Theta_{AT}\), then the segment is considered as anomalous. The normalization of the uncertainties of all \(N\) predicted output sequences helps in two ways. First, it makes a fair comparison of uncertainties on all features. Second, the normalization of a set of values representing a larger transition leads to a wider range of scaled values, from closer to 0 to an upper limit of 1. Thus, setting a maximum threshold \(\Theta_{AT}\) in Eq. (13) helps us in detecting the larger transitions in the uncertainties, which leads to the detection of anomalous trajectories.

To understand Eq. (13), consider a trajectory segment with \(N + T - 1 = 39\) consecutive AIS data, where there are \(N = 30\) sequences, each with length \(T = 10\) data. The regression model iteratively predicts 30 corresponding output sequences, each with a predicted sequence of length \(L = 1\). Let us assume that the epistemic uncertainties for the features \(lon\) (\(d = 1\)) and \(lat\) (\(d = 2\)) using Eq. (10) and for 30 consecutive predictions are \(\text{var}[\mu_d^j] = [0.01, 0.02, \ldots, 0.30, 0.31]\), \(d = 1, 2; j = 10, 11, \ldots, 39\). The normalized values of uncertainties over all predicted sequences \(j\) are simply \(\text{var}[\mu_d^j] / 0.30, \text{var}[\mu_d^{j^2}] / 0.31\). The minimum over all predictions \(j\) would result in 0.01/0.30 and 0.02/0.31 for the \(lon\) and \(lat\), respectively. Under these settings, the trajectory segment is detected as anomalous if the threshold \(\Theta_{AT}\) is set larger than 1/30 (i.e., the minimum over all considered features).
B. Unusual Turn (UT) Anomaly

We are interested in developing an EDL classifier that learns whether a vessel took a turn of more (or less) than a threshold $\Theta_{UT}$, given an AIS data sequence of length $T$. To this end, we first compute the change in the course of a vessel using the $\text{cog} \in [0^\circ, 360^\circ]$ values from the data sequence $X_{0:T}$ by Eq. (14).

$$\theta = \begin{cases} \max(M) - \min(M) + \max(L) - \min(L), & M, L \neq \emptyset \\ \max(X_{0:T}[\text{cog}]) - \min(X_{0:T}[\text{cog}]), & \text{otherwise} \end{cases}$$ (14)

The second line on the right hand side of Eq. (14) computes the course change by the difference between the maximum and the minimum reported $\text{cog}$ values. However, the condition overlooks a scenario when a vessel turns across true north, i.e., from $359^\circ$ to $0^\circ$ or the other way. The first line in Eq. (14) captures this scenario by formulating two sets of $\text{cog}$ values reported within $[270^\circ, 360^\circ)$ using $M = \{x_k[\text{cog}] \in X_{0:T} | x_k[\text{cog}] \geq 270^\circ, k = 0, \ldots, T - 1\}$ and within $[0^\circ, 90^\circ]$ using $L = \{x_k[\text{cog}] \in X_{0:T} | x_k[\text{cog}] \leq 90^\circ, k = 0, \ldots, T - 1\}$. The sum of the changes in course across the true north then correctly computes the vessels’ turn using the sample $X_{0:T}$.

A sequence $X_{0:T}$ is termed as anomalous if the turn is larger than the threshold i.e., $\theta_{X_{0:T}} > \Theta_{UT}$. It is important to mention that Eq. (14) is used to label input data sequences into the normal and the anomaly categories, which are used for the training of the EDL unusual turn classifier. A predicted class is accepted or rejected based on its associated uncertainty.

We consider a flexible approach to define a trajectory segment for the unusual turn detection, as the duration of a vessel’s turn is unknown. The segment of a track $X_{k_1:k_2}$ with reported times $k_1 < k_2$ is termed as an unusual turn anomaly if all of its samples are anomalous and the previous $X_{k_1-1}$ and the following $X_{k_2+1}$ samples are normal. This helps in the minimization of the number of reported anomalies and the frequency that the maritime authority are to be alerted, as we noticed during a project validation campaign (results are reported in Sec. V).

C. AIS OOS Anomaly

In order to detect an OOS anomaly due to loss of AIS signal, the EDL classifier model needs to learn the time difference between two consecutive AIS data. A sample $X_{k:k+2}$ is termed as an AIS OOS anomaly if the time difference between two consecutive AIS data reported in the sample is greater than a threshold $\Theta_{OOS}$, as shown in Eq. (15). We either accept or reject the predicted class of a given input sample depending on whether its associated uncertainty $u$ (see Eq. (12)) is lower or higher than a threshold $u_{th}$, respectively.

$$x_{k+1}[\text{time}] - x_k[\text{time}] > \Theta_{OOS}$$ (15)

V. EXPERIMENTAL RESULTS

In this section the proposed anomaly detection methods are evaluated, namely clustering, regression and classification on various real-world AIS data. To this end, we describe the data preprocessing step, experimental settings, and present results from various scenarios in the following subsections.

A. Experimental Setup

**Dataset Preparation:** The two AIS datasets evaluated were provided by: i) the German waterways and shipping administration (WSV) for January 2016, and ii) AISHUB for January 2020. Henceforth, they are referred to as $D_1$ and $D_2$, respectively. The data is filtered in the region of interest (ROI) with a rectangular bounding box from $(11.5^\circ, 54.2^\circ)$ to $(12.5^\circ, 54.5^\circ)$. AIS messages that reported speed over ground within 30 knots, and navigation status as 0 (i.e., underway using engine) are considered. Additionally, the ship type is stored (e.g., passenger: 60-69, cargo: 70-79, tanker: 80-89) for all vessels within the ROI. After these preprocessing steps, $D_1$ and $D_2$ contain $\approx 1.2$ and $0.1$ million AIS positions, respectively. Figure 5 depicts trajectories of vessels that have been extracted from the datasets. Notice how the trajectories in $D_2$ (shown in right) have much higher number of missing data segments than $D_1$ (shown in left). The reason is that $D_1$
Fig. 6: Left: Figure depicts the nodes and edges (in black) as created by our graph-based clustering scheme maritime traffic data (in blue). Right: Figure shows the refined nodes and edges (in black) of a graph, and the corresponding data traffic association.

is satellite-based, and data in \( D_2 \) were collected by AIS base stations.

**Models and Parameters**

The graph-based clustering (GTR) process in Algorithm 1 uses \( \epsilon = 1000 \) in the RDP function (Step 3), and \( \varepsilon = 20 \) and \( n_{\text{min}} = 1500 \) in the DBSCAN method (Step 5). A threshold of \( \varepsilon_{\text{th}} = 0.3 \) is used for the edge selection (Step 6). The data association (GTA) process in Algorithm 2 uses a distance threshold of 7 km (Step 4), and \( \epsilon = 500 \) in the RDP function (Step 6).

We use different network architectures and data feature sets for the regressor and classifiers. The RNN-EDL regressor (see Fig. 3) has an encoder and a decoder with one LSTM hidden layer, each with 128 neurons. The scalar constant \( \lambda \) in the loss function is set to 0.01. Moreover, each AIS data is represented by feature set \( x_k = [\text{lon, lat, cog, sog, edge}]^\top \). The edge feature is omitted if the model is not graph-based.

The EDL classifiers are trained with the same order of magnitude of normal and anomalous data samples, by randomly deleting some excessive normal samples in dataset \( D_1 \) and \( D_2 \). All models are trained using an Adam optimiser.

**Evaluation Metrics:** In the case of anomalous trajectory detection, there is no reference dataset for anomalous tracks \([16]\), a quantitative analysis in terms of accuracy or a false alarm rate is therefore not feasible. In contrast, we can evaluate the EDL classifiers for detecting OOS and UT anomalies using the following standard metrics: the accuracy measure, confusion matrix, false positive or negative rate. Nevertheless, since anomalies are subjective in nature, each detected anomaly must undergo an inspection by data (here, AIS) experts before raising any alarm.

**B. Graph-based Clustering**

Before showing the effectiveness of the GTR and GTA algorithms in trajectory prediction and uncertainty estimation, we present the results on how a graph can represent vessel movement patterns using dataset \( D_1 \). Here, we consider only passenger, cargo and tanker vessels, as they follow well-defined trajectories. Fig. 6 (left) depicts the original graph in black that is formed by data plotted in blue. Notice that the nodes are either at the borders or at waypoints, and the shown bidirectional edges very well represent vessel motion patterns. However, we can also see that the GTR algorithm places a node around \((12.1^\circ, 54.3^\circ)\) without any further edge connection, misrepresenting underlying vessel movements.

Note that, since the parameters of RDP and DBSCAN are sensitive to the dataset in question, the computed graphs might be inadequate to represent all of the important sea lanes. This is particularly true in scenarios where the AIS-range is limited. There may be trajectories that are recorded in-between out-of-range source(s) or destination(s). Therefore, an expert knowledge that might improve the traffic representation, for example, by manually adding a node/edge to account for such limitation. Thus, as an offline process we refined the graph by

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1The source code is at www.github.com/sansastra/uncertainty_edl_graph.
Fig. 7: The plots show the CDFs on positional error between prediction and ground truth for with and without (W/O) graph-based models, as well as for different unseen vessels of ship types \([0, 60)\) and \([60, 90)\) in the dataset \(D_1\). Left: Results from the RNN-EDL regression model. Right: Results from the RNN model without the EDL layer.

Fig. 8: Left: Figure shows the ground truth and the corresponding predicted trajectories for the left, normal, and right trajectory segments by the graph-based RNN-EDL model. Right: Figure shows the epistemic uncertainties estimated on successive predictions of output data features (\(\text{lon}, \text{lat}, \text{cog}, \text{and} \ \text{sog}\)) for the left, normal, and right trajectory segments by the graph-based RNN-EDL model.

Deletion of the node and connecting two edges, as shown in Fig. 6 (right). The latter additionally shows the association of data points to one of the edges in a color-coded fashion where a so-called outlier edge is represented in grey. Notice that the majority of movements show acceptable assignment to edges. Only some segments of trajectories have been wrongly (though subjective) assigned owing to their complex manoeuvres.

**Impact of Graph-based clustering:** Clustering the trajectories has a huge impact on the accuracy of trajectory prediction as well as on the corresponding uncertainty estimation. Fig. 6 (left) illustrates this observation using a cumulative distribution/density function (CDF), which is obtained on the Euclidean distances (or positional errors) between ground truth and predicted positions on trajectories of ships in the dataset \(D_1\). Aside from the superior performance of the graph-based prediction model, we can also notice that it is challenging to predict trajectories of ships in range \([0, 60)\), as compared to passenger, cargo and tanker vessels \((60, 90))\). More importantly, the RNN-EDL model without graph-based clustering predicts more than 80% of target positions with deviation by at least 30 km from the ground-truth data. The reason is that the model fails to fit targets with the Gaussian distribution, as the maritime data is dispersed. The clustered data, on the other hand, simplifies the model’s ability to learn the motion pattern and fit the distribution. To further investigate the impact of the EDL layer, the latter was removed from the RNN-EDL model. The trained RNN model is then used for the prediction of trajectories, and to plot CDFs of positional errors in Fig. 7 (right). We observe that the prediction error significantly decreased for the RNN model without the graph-based clustering. Nevertheless, RNN-EDL with graph in Fig. 7 (left) outperforms others. In the next subsection, we show results of only the RNN-EDL model with graph based clustering.

**C. Anomalous Trajectories Detection**

We evaluate three trajectory segments: a normal track and two anomalous tracks in the left and right to get an insight on
the trajectory prediction and associated epistemic uncertainties in Fig. 8. Note that the left and right tracks are synthetic data based on recorded data from a real-world (or normal) track in the middle by changing the longitudes and latitudes. We can see from Fig. 8 (left) that the ground truth data closely follows the predicted ones. Given an input sequence, the model is able to correctly forecast the vessel’s position at a future time step not only for the normal track but also for the unseen (left and right) trajectories, which would not be the case with the other models.

In order to test the efficacy of the regression model’s learning ability in uncertainty estimation, Fig. 8 (right) depicts the epistemic uncertainties of all three segments based on the data features. We observe that the left and right tracks show an abrupt transition (or jump) in uncertainty with respect to the normal for all features. Also, note that despite having similar cog and sog values in all three segments, uncertainties in cog clearly show the transitions in opposite direction for the left and right tracks, as the vessel’s movement was in an opposite direction. This shows that the model learnt the correlation among features well. It is also important to mention that when we trained the RNN-EDL regressor model with only the Rostock-Gedser dataset (see trajectories in Fig. 2), the relative change in the uncertainties is even higher (not shown for sake of brevity), as it is easier for the model to learn the data distribution of trajectories between two ports accurately. However, we are interested in developing a single global model to detect all anomalous trajectories in the ROI.

Next, Eq. (13) is used to detect anomalous trajectories with thresholds $\Theta_{AT} = 0.7$, and 0.4, and segment size $N = 30$. Figures 9 and 10 show all the anomalous segments of each track in red for two sets of vessels in $D_1$ and $D_2$, respectively. Obviously, decreasing the threshold decreases the number of detected anomalous segments, as the threshold is an upper bound in Eq. (13) and a lower threshold value only detects larger transitions. The passenger, cargo and tanker ships’ trajectories (left plots) show lower number of anomalous segments than other ship types shown in the right plots. We can say that the model is able to detect some unusual maneuvering while missing out on some tracks that seem to show anomalous behavior. At the same time, some detected anomaly segments do not seem to be anomalous after a careful
look at their tracks. Nevertheless, we observe that having a common threshold for all ships is not the best solution. Instead, we can set a lower threshold for passenger, cargo, and tanker vessels, and a higher threshold for other vessel types. Another possible solution could be to find uncertainty thresholds of normal tracks, though computationally intensive and subjective in nature, in smaller grids of the ROI and compare them with the estimated uncertainty on any trajectory under evaluation.

D. Unusual Turn and OOS Detection

For an unusual turn detection, we choose a threshold $\Theta_{UT} = 30^\circ$, considering that larger vessels generally do not take more than $30^\circ$ of turn within a short interval $T \times \tau = 3$ minutes, unless constrained by their geographical locations (which is not considered here). Thus, we evaluate the model accuracy only for passenger, cargo and tanker vessels. On the other hand, we set a threshold $\Theta_{OOS} = 3$ minutes for the OOS anomaly detection (Eq. 15).

In Fig. 11 we show the accuracy of the EDL classifiers against the uncertainty threshold for all samples as well as for only anomalous samples in the datasets. We accept the classification of a sample into a class if it has the highest probability and the prediction uncertainty is below a threshold $u_{th}$. In other words, the sample’s classification by the model is rejected if the epistemic uncertainty is above a threshold. The following observations are made. Firstly, increasing the threshold improves the accuracy, which suggests that the models fit well to the Dirichlet categorical distribution. Secondly, the UT model classifies most of the samples with higher uncertainty ($> 0.2$) as compared to the OOS classifier ($< 0.1$), and it achieves 90% accuracy if the uncertainty threshold $u_{th} = 0.4$. Thirdly, the UT classifier not only misclassifies anomalous samples as normal, but also the other way round, which can be seen from the accuracy obtained on all samples and only the anomalous samples. Finally, the OOS classifier achieves an overall accuracy of $\approx 99\%$ on all samples for uncertainty thresholds greater than 0.1. The accuracy of the EDL models for UT and OOS detection on only anomalous samples are lower at uncertainty thresholds below 0.4, compared to the accuracy computed on all samples. The reason is that the models produce larger uncertainties on their predictions. This suggests that the models find it relatively hard to learn
Fig. 11: Left and right figures show the accuracy w.r.t. uncertainty threshold by the AIS OOS classifier and the unusual turn (UT) classifier evaluating all samples and only the anomalous samples in test datasets $D_1$ and $D_2$, respectively.

Fig. 12: The left plot shows the unusual turn ($>30^\circ$) segments (red) detected on a test vessel’s trajectory (black). The right plot shows the OOS (red) detected on another test vessel moving from bottom to up direction (black).

anomalous samples than the normal samples.

Unlike the RNN-EDL regressor model, the trained EDL classifiers on ROI data could be used to detect UT and OOS anomalies on AIS data beyond ROI. We, therefore, evaluate the EDL classifiers on two additional scenarios: one for the dump cleaning (UT), and other for the AIS blackout (OOS) exercises that was performed by a test vessel in a validation campaign of a project in July 2020 in the Bremerhaven region in Germany. The uncertainty threshold $u_{th}$ is set to 0.4. Fig. 12 (left) shows all the unusual turn segments detected, and the right plot shows the detected AIS blackout event in red in the test vessel’s trajectories. We observe that an unusual turn segment on the upper left region is undetected in the left plot. Nevertheless, the classifiers are able to detect all other UT and OOS anomalies well.

VI. CONCLUDING REMARKS

A novel graph-based clustering and evidential deep learning approaches has been proposed to detect maritime anomalies using AIS data. First, we proposed a graph-based AIS track representation and association scheme. We showed that the clustered data is more suitable for learning the AIS trajectories by the EDL model than the unclustered data. Furthermore, we evaluated three different anomalies: anomalous trajectory, unusual turn and the loss of AIS signal, using the EDL regressor and classifiers using the epistemic uncertainties in prediction. The anomalous tracks are detected by evaluating jumps in evidential uncertainties on trajectory segment prediction. Similarly, the unusual turn anomaly and the loss of AIS signal are detected by taking into account the class probabilities and prediction uncertainty. In summary, we make the following observations based on the proposed models and analysis presented in the paper.

- A graph-based clustering is essential for the RNN-EDL time-series prediction model in order to learn distribution of maritime trajectories. In contrast to an RNN model without the EDL layer, the prediction performance of the RNN-EDL model decreases significantly when trained
without the graph-based clustered data.

- The epistemic uncertainties obtained on prediction by the EDL regressor and classifier models are very useful in anomaly detection. Typically, a lower threshold for passenger, cargo and tankers vessels and higher for other ship types can be set to detect anomalous tracks. Additionally, large vessels taking usually fixed routes show lower number of anomalous track segments than other vessels, for example, fishing.

This work opens deeper research activities within the field of leveraging deep learning uncertainties in maritime anomaly detection. Moreover, the work might be useful in detecting anomalies in other domains of transportation. Lastly, in the absence of reference anomalous routes, expert knowledge will always be required before taking any action upon detection of anomalies.

ACKNOWLEDGMENT

We would like to thank Dr. Frank Heymann for providing valuable comments that improved the paper. This work has been partially funded by the German federal ministry for economic affairs and energy project IntelliMar, and the German federal ministry of education and research project European Maritime Safety III.

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