Estimating the cost of constructing and operating a section of a pipeline in the search for its optimal route

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Abstract. The paper considers the calculation of the relative cost of construction and operation of a pipeline section in the search for its optimal route. Comparison of efficiency of application of various options of data representation on unit cost of pipeline route depending on topographical features.

1. Introduction

When designing pipelines, engineers face the task of finding the optimal route, ensuring minimum costs for their construction and further operation [1, 4]. This task is very time consuming, as it requires consideration of many feasible options for passing the route and taking into account a large number of factors affecting the cost of pipeline construction and operation. Currently, the choice of the route is most often based on the expert opinion of the designer.

In [6] an algorithm of searching for the optimum pipeline route in view of topographical features of the area is offered. This algorithm allows one to significantly reduce the time input for design by automating the process of searching for the optimal route, and also to reduce the influence of the human factor. For effective operation of this algorithm, it is required to estimate the cost or weight of the considered pipeline route option most accurately.

2. Methodology

The weight of the line (pipeline section) can be defined as the sum of the weights of all infinitely small segments, of which this section consists. To do this, we integrate the specific weight of each point on the map along the length of the given line:

\[ C = \int_{0}^{L} wdL \]  \hspace{1cm} (1)

where

- \( C \) – total weight of the line in question;
- \( L \) – length of the line in question;
- \( w \) – specific weight of a point on a weight map;
- \( dL \) – length of an elementary segment.
Since, as a result of the work of the route search algorithm, we get a broken line consisting of rectilinear segments, the total weight of this line can be determined as the sum of weights of these segments:

\[ C_i = \int_{L_{i-1}}^{L_i} wdL \]  

(2)

\[ C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \int_{L_{i-1}}^{L_i} wdL \]  

(3)

where  
\( C_i \) – weight of the \( i^{th} \) segment;  
\( L_i \) – length \( i \) of the first segments;  
\( n \) – number of segments.

To determine the cost of the route of the designed pipeline section, it is necessary to draw up a weight map showing the unit cost (specific weight) of the route passing through a particular section of the terrain. The weight map can be presented in vector or raster versions. Each of these options has its own peculiarities for the calculation.

The weight map, represented in a vector form (Figure 1), is divided into several closed areas, for each of which the specific cost of laying the pipeline (specific weight) is indicated.

![Figure 1. Vector map of weights.](image)

In this case, the weight of the pipeline section is determined by summing the products of the weights of the map areas by the lengths of the segments located in these areas:

\[ C = \sum_{i=1}^{n} w_i L_i = \sum_{i=1}^{n} w_i \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \]  

(4)

where  
\( w_i \) – specific weight of the area in which the \( i^{th} \) segment is located;  
\( L_i \) – length of the \( i^{th} \) segment;  
\( x_i, y_i \) – coordinates of the \( i^{th} \) point.

A raster map of weights (Figure 2) is a table for each cell of which the value of the specific cost of pipeline laying (specific weight) is indicated.
Figure 2. Raster map of weights with cells.

The weight of the pipeline section, as in the previous case, can be determined by the formula (4), adding up the products of the specific cell weights by the lengths of the segments located inside these cells. However, in comparison with the calculation of a vector map of weights, this variant requires considerably less processing power to determine the coordinates of the intersection points of the line with the boundaries of the cells of the map.

To ensure that there are no abrupt changes in the specific weight at the borders of the cells of the weight map, it can be specified not for the entire cell, but only for nodes, that is, the intersection points of the grid lines (Figure 3a). In this case, the specific weight of any intermediate point can be found by means of bilinear interpolation (Figure 3b).

Figure 3. Raster map of weights with nodes.

a) general view of the map;
b) finding the specific weight at the intermediate point.
Let us consider one grid cell. The specific weight \( w \) at a point with coordinates \((x, y)\) can be determined by knowing the weights of the node points \( w_1, w_2, w_3, w_4 \) and their coordinates:

\[
\begin{align*}
\frac{w}{w_1} &= \frac{w - w_1}{y - y_1} \quad \frac{w}{w_5} = \frac{w_1}{x - x_1} \quad \frac{w}{w_6} = \frac{w_3}{x - x_1} \\
\end{align*}
\]

where \( w_1, \ldots, w_6 \) – specific weights at points 1 to 6; \( x_1, x_2, y_1, y_2 \) – cell boundary coordinates.

Substituting (6), (7) in (5), we obtain an expression for finding the specific weight at an arbitrary point inside the cell under consideration:

\[
\begin{align*}
\frac{w}{w} &= \frac{w}{w_1} + \left( w_1 - w_2 \right) \frac{x - x_1}{x_2 - x_1} + \left( w_3 - w_1 + \left( w_4 - w_3 - w_2 + w_1 \right) \frac{x - x_1}{x_2 - x_1} \right) \frac{y - y_1}{y_2 - y_1} \quad \frac{w}{w_6} = \frac{w_3}{x - x_1} \\
\end{align*}
\]

We substitute the obtained expression for the specific weight (8) into the initial equation to determine the weight of the segment (2) and transform:

\[
C = \int \left( w_1 + \left( w_2 - w_1 \right) \frac{x - x_1}{x_2 - x_1} + \left( w_3 - w_1 + \left( w_4 - w_3 - w_2 + w_1 \right) \frac{x - x_1}{x_2 - x_1} \right) \frac{y - y_1}{y_2 - y_1} \right) dL
\]

We decompose the elementary segment \( dL \) into horizontal \( dx \) and vertical \( dy \) components (Figure 4).

Let us express the length of an elementary segment through its projections onto the horizontal and vertical axes:
\[ dL = \frac{dx}{\cos \alpha} = \frac{dy}{\sin \alpha} \]

\[
\cos \alpha = \frac{(x_i - x_{i-1})}{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}}; \quad \sin \alpha = \frac{(y_i - y_{i-1})}{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}}
\]

\[ dL = \frac{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}}{(x_i - x_{i-1})} \, dx = \frac{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}}{(y_i - y_{i-1})} \, dy \quad (10) \]

We write the equation of the straight line in a general form and substitute the coordinates of the beginning and end of the segment:

\[
y = ax + b
\]

\[
\begin{cases}
  y_{i-1} = ax_{i-1} + b \\
  y_i = ax_i + b
\end{cases}
\]

\[
a = \frac{y_i - y_{i-1}}{x_i - x_{i-1}}, \quad b = y_i - \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \, x_i
\]

\[
\begin{cases}
  y = \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \, x + y_i - \frac{y_i - y_{i-1}}{x_i - x_{i-1}} \, x_i, \quad \text{if } x_{i-1} \neq x_i \\
  x = x_i, \quad \text{if } x_{i-1} = x_i
\end{cases} \quad (11)
\]

Substituting expressions (10), (11) in (9), we can obtain a formula for determining the weight of a segment inside one cell of the map. If the segment is not vertical, that is, \( x_{i-1} \neq x_i \), then:

\[
C = \frac{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}}{(x_i - x_{i-1})} \left( w_1 \int_{x_{i-1}}^{x_i} dx + \frac{w_2 - w_1}{x_2 - x_1} \int_{x_{i-1}}^{x_i} (x - x_1) dx \\
+ \frac{w_3 - w_1}{y_2 - y_1} \int_{x_{i-1}}^{x_i} (ax + b - y_1) dx + \frac{w_4 - w_3 - w_2 + w_1}{(x_2 - x_1)(y_2 - y_1)} \int_{x_{i-1}}^{x_i} (x - x_1)(ax + b - y_1) dx \right)
\]

\[
C_i = \frac{\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}}{(x_i - x_{i-1})} \left( w_1 \int_{x_{i-1}}^{x_i} dx + \frac{w_2 - w_1}{x_2 - x_1} \int_{x_{i-1}}^{x_i} (x - x_1) dx \\
+ \frac{w_3 - w_1}{y_2 - y_1} \int_{x_{i-1}}^{x_i} (ax + b - y_1) dx \\
+ \frac{w_4 - w_3 - w_2 + w_1}{(x_2 - x_1)(y_2 - y_1)} \int_{x_{i-1}}^{x_i} (ax^2 + (b - y_1 - ax_1)x + (y_1 - b)x_1) dx \right) \quad (12)
\]

After integrating, we obtain:
\[ C_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \left( w_1 + \frac{w_2 - w_1}{x_2 - x_1} \left( \frac{x_i + x_{i-1}}{2} - x_1 \right) \right. \]
\[ + \frac{w_3 - w_4}{y_2 - y_1} \left( \frac{a(x_i + x_{i-1})}{2} - (b - y_1) \right) \]
\[ + \frac{w_4 - w_3 - w_2 + w_1}{(x_2 - x_1)(y_2 - y_1)} \left( \frac{a(x_i^2 + x_i x_{i-1} + x_{i-1}^2)}{3} + \frac{(b - y_1 - ax_1)(x_i + x_{i-1})}{2} \right) \]
\[ + (y_1 - b)x_1 \left. \right) \quad (13) \]

If the segment is vertical, that is \(x_{i-1} = x_i\), then:

\[ C = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \int \left( w_1 + (w_2 - w_1) \frac{x - x_1}{x_2 - x_1} \right. \]
\[ + \left( w_3 - w_1 + (w_4 - w_3 - w_2 + w_1) \frac{x_i - x_{i-1}}{x_2 - x_1} \right) \frac{x - x_1}{x_2 - x_1} \frac{y - y_1}{y_2 - y_1} \]
\[ d y \]
\[ C_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \left( w_1 + (w_2 - w_1) \frac{x_i - x_1}{x_2 - x_1} \right. \]
\[ + \left( w_3 - w_1 + (w_4 - w_3 - w_2 + w_1) \frac{x_i - x_{i-1}}{x_2 - x_1} \right) \frac{y_i}{y_2 - y_1} \int \left( y - y_1 \right) d y \quad (14) \]

After integrating, we obtain:

\[ C_i = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \left( w_1 + (w_2 - w_1) \frac{x_i - x_1}{x_2 - x_1} \right. \]
\[ + \left( w_3 - w_1 + (w_4 - w_3 - w_2 + w_1) \frac{x_i - x_{i-1}}{x_2 - x_1} \right) \frac{y_i}{y_2 - y_1} \left( y_i + y_{i-1} - y_1 \right) \quad (15) \]

Formulas (13), (15) allow calculating the weight of a pipeline section using a variant of a map of weights with smooth transitions between nodes. However, this option requires more processing power than the option with the weight of the cells of the map.

3. Conclusion

Thus, the use of a vector map of weights allows the most accurate estimate of the cost of the route of the pipeline section, but because of the difficulty in determining the coordinates of the intersection of the segments of the route with the boundaries of the map areas, it requires more computational resources and time for making calculations. Moreover, the time spent directly depends on the area of the considered section of the map (on the number and complexity of the selected areas).

The use of raster variants of the weight map significantly reduces the time spent on calculations, but gives a less accurate result due to the error in determining the boundaries of map areas with different specific weights. When the resolution of the map increases, the error decreases, but the memory input and the calculation time of the pipeline route cost increase.
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