ASTROMETRY AND RADIAL VELOCITIES OF THE PLANET HOST M DWARF GJ 317: NEW TRIGONOMETRIC DISTANCE, METALLICITY, AND UPPER LIMIT TO THE MASS OF GJ 317b

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ABSTRACT

We have obtained precision astrometry of the planet host M dwarf GJ 317 in the framework of the Carnegie Astrometric Planet Search project. The new astrometric measurements give a distance determination of 15.3 pc, 65% further than previous estimates. The resulting absolute magnitudes suggest that it is metal-rich and more massive than previously assumed. This result strengthens the correlation between high metallicity and the presence of gas giants around low-mass stars. At 15.3 pc, the minimal astrometric amplitude for planet candidate GJ 317b is 0.3 mas (edge-on orbit), just below our astrometric sensitivity. However, given the relatively large number of observations and good astrometric precision, a Bayesian Monte Carlo Markov Chain analysis indicates that the mass of planet b has to be smaller than twice the minimum mass with a 99% confidence level, with a most likely value of 2.5 $M_{\text{Jup}}$. Additional radial velocity (RV) measurements obtained with Keck by the Lick-Carnegie Planet search program confirm the presence of an additional very long period planet candidate, with a period of 20 years or more. Even though such an object will imprint a large astrometric wobble on the star, its curvature is yet not evident in the astrometry. Given high metallicity, and the trend indicating that multiple systems are rich in low-mass companions, this system is likely to host additional low-mass planets in its habitable zone that can be readily detected with state-of-the-art optical and near-infrared RV measurements.

Key words: astrometry – planetary systems – stars: individual (GJ 317) – techniques: radial velocities

Online-only material: color figures

1. INTRODUCTION

Astrometric observations complementing radial velocity (RV) measurements provide the only means to measure the dynamical masses of long-period non-transiting exoplanets. The astrometric technique has been largely unsuccessful in the detection of low-mass companions, mainly due to strong systematic effects that are difficult to calibrate through the noise added by Earth’s atmosphere. An example is the planet candidate around the M8.5V star VB 10 (Pravdo & Shaklan 2009), which could not be confirmed by radial velocities (Bean et al. 2010; Anglada-Escudé et al. 2010) nor by further astrometric observations (Lazorenko et al. 2011). Compared to other techniques such as precision Doppler measurements, astrometry cannot be corrected using laboratory standards and relies upon measuring the relative motion of the target star with respect to a number of background and (desirably) slow moving distant sources. Typical FGK planet-hosting dwarfs that should show the largest astrometric wobbles are too bright to be observed simultaneously with background sources, so the perturbing effect of the atmosphere and time-dependent geometric field distortions cannot be easily calibrated in the observations. Some success has been achieved from space. Posterior analysis of Hipparcos astrometry has been used to put upper limits on the masses of detected RV companions, ruling out that most of them are low-mass stars in face-on orbits (Pourbaix & Arenou 2001). A recent reanalysis by Reffert & Quirrenbach (2011) of the residuals to the new Hipparcos solution (van Leeuwen 2010) has been able to confirm the planetary nature of nine objects and a number of brown dwarf candidates by putting upper limits to their masses. In the same study, the planetary signal could be recovered on three systems. Also, precision astrometry of bright stars has been measured using the Fine Guidance Sensors on board the Hubble Space Telescope (e.g., Martioli et al. 2010; McArthur et al. 2010, for the most recent results). Up to now, these are the only astrometric measurements sensitive enough to place actual constraints on the masses of long-period planet candidates and are all obtained by space-based observations.

The Carnegie Astrometric Planet Search is a ground-based program focused on the detection of giant planets around nearby, low-mass stars. The astrometric wobble imprinted by a planet on the central star is inversely proportional to the stellar mass (the smaller the mass, the larger the signal). Also, low-mass stars (M and L dwarfs) are much fainter than typical F, G, and K stars (the most common targets of the RV surveys), simplifying the simultaneous observations of the targets and faint background reference sources. A central part of the project has been the construction of a specialized camera (Boss et al. 2009). CAPSCam uses a Hawaii-2RG HyViSI hybrid array that allows the definition of an arbitrary guide window which can be read out (and reset) rapidly, repeatedly, and independently of the rest of the array. This guide window is centered on our relatively bright target stars, with multiple short exposures avoiding saturation. The rest of the array then integrates for prolonged periods on the background reference grid of fainter stars. This dramatically extends the dynamic range of the composite image. This HyViSI detector is the heart of the CAPScam concept. A full description of the program, instrument performance, and characteristics is given in Boss et al. (2009). The high dynamical range enabled by the guide-window mode permits the observation of stars as bright as magnitude $I = 9$ using integration times as short at 0.2 s on the target object. Even though the best epoch precision
is at the level of 0.4 mas, the long-term accuracy is limited by (yet) unknown systematic effects, that are more pronounced on bright objects. The typical epoch-to-epoch precision at the bright end is about 1 mas. As a complementary part of the planet search program, we follow up a handful of long-period RV planet candidates that should produce a detectable astrometric signal (that is, with a semiamplitude above 0.5 mas). GJ 317 is one such star (\(I \sim 9.32\); Rojo & Ruiz 2003). Johnson et al. (2007, hereinafter JB07) reported the detection of a \(\sim 700\) day period planet candidate with a minimum mass of \(1.2 M_{\text{Jup}}\) and evidence of an additional companion (GJ 317c) with a period of several thousand days. Using that orbital solution and the previously reported distance (\(\sim 9.7\) pc; Jenkins 1963), we estimated a minimum astrometric amplitude (assuming an edge-on orbit) of \(\sim 0.58\) mas for GJ 317b. With an epoch-to-epoch precision of 1 mas, this is a challenging but doable measurement. Assuming a median inclination of 45\(^\circ\), one would expect a semiamplitude of 0.84 mas.

The star has been continuously monitored by the Lick-Carnegie Planet Search group using Keck/HIRES, increasing the number of RV measurements from 18 (JB07) to 38. These additional measurements should better constrain the nature of the long-period planet and permit a search for lower amplitude signals. GJ 317 is a nearby star of astrobiological interest. Its low mass and solar system like configuration (two outer giant planets) make it an outstanding candidate to search for potentially habitable planets with RV observations.

A typical observing epoch consists in spending 1 hr of telescope time to obtain 30–50 images on the full \(\sim 6 \times 6\). The integration time of each image varies between 45 and 60 s depending on seeing and environmental conditions. Guide-window exposure time is always set to 0.2 s on GJ 317. Therefore, between 225 and 300 guide-window reads are obtained and co-added to each image of the full field. The field around GJ 317 is rich in background reference stars. The astrometric precision (intranight centroid scatter) is computed by comparing the position of the target star to the reference stars in each of the 30–50 images per epoch. The epoch-to-epoch accuracy is estimated by comparing the observations to the best-fit astrometric model (see Appendix A). For GJ 317 in particular, the typical intranight precision is of the order of 0.5 mas. This precision is illustrated as the error bars in Figure 1. However, the typical epoch-to-epoch accuracy is 0.6 mas in R.A. and 1.2 mas in declination, indicating that a significant source of uncertainty is not random errors but uncalibrated field distortion effects (systematic errors), especially in declination. One epoch (2009 January) was rejected because the camera shutter stopped working, causing severe saturation and charge persistence problems on the target star. The relative astrometric measurements used in the analysis are given in Table 1 and illustrated in Figure 1. The residuals (bottom panels) are obtained by subtracting the best fit to the parallax and proper motion. An overview of the astrometric solution and relevant statistical quantities are given in Section 4. Further details on the astrometric data reduction procedure are given in Appendix A.

The measured motion of GJ 317 is relative to the background stars. As discussed in Appendix A, the reference stars are matched to their predicted positions that, in turn, are also refined on each iteration of the astrometric iterative solution (AIT). As all the stars have the same parallactic motion (except for the amplitude), the average parallactic motion of the reference frame cannot be derived from relative astrometric observations. This translates into a zero-point ambiguity in the measured distance to the target that needs to be corrected. After the final solution is obtained for all the stars in the field, the measured differential parallax of some of the references is used to estimate the zero-point correction as follows. Using the \(B, J, H,\) and \(K\) magnitudes from the NOMAD catalog (Zacharias et al. 2004),

![Figure 1. Top panels: differential astrometry of GJ 317 measured with CAPScam as a function of time. The best fit to the parallax and proper motion is illustrated by the red dashed line. Bottom panels: the residuals after removing the parallax and proper motion. (A color version of this figure is available in the online journal.)](image-url)
photometric distances are obtained to seven stars with all these colors available. The information for all the reference stars is given in Appendix A. Photometric distances to main-sequence stars estimated this way can have uncertainties of up to 20% of their actual values. Note, however, that a star at 330 pc will have a parallax of 3 mas and a corresponding uncertainty in the photometric parallax of only 0.6 mas. In comparison, a star at 50 pc would have a parallax of 20 mas and a corresponding uncertainty of 4 mas. Therefore, we only use stars with nominal photometric distances beyond 300 pc to obtain a more secure determination of the zero-point. The average difference between the photometric parallaxes and the measured ones is the desired offset and amounts to 0.23 ± 0.1 mas for this field in particular. This offset is added to the measured parallax of GJ 317 to provide the final distance determination in Table 2. The fact that the dispersion around the average zero point is small, indicates that this strategy is robust against uncertainties in the photometry and the models. To prove that our procedure is essentially correct, Figure 2 shows the parallaxes of several CAPS program stars compared to the ones published in the literature. We actually found several outliers (not shown here) that typically correspond to previous parallactic measurements done with a small number of epochs and a formal uncertainties larger than 5 mas. These cases and an overview of the CAPS sample will be discussed in a future publication. For the stars shown here, the measurements coincide within the published uncertainties and we find no systematic zero-point offset.

Our measurement of GJ 317’s proper motion is similar to some previous estimates (e.g., Salim & Gould 2003), which is another consistency check of the astrometric calibration procedure (e.g., plate scale and absolute field rotation). However, such proper motion is also relative to the references and requires more assumptions than the parallax zero point to be properly corrected. For galactic kinematic studies, the catalog values of the proper motion given in Salim & Gould (2003, NTT) or Zacharias et al. (2009, UCAC3) should be used instead of our relative measurement (see Table 2). Our relative proper motions in R.A. and decl. agree within 15 and 2 mas yr⁻¹ to the values given in both these catalogs. Even though they differ by more than 1σ (statistical uncertainty), a discrepancy at the level of 15 mas yr⁻¹ is still acceptable due to the unknown zero point in our relative measurements and the uncertainty in the catalogs themselves (random errors but also zonal systematic effects). GJ 317’s proper motion is also given in other catalogs; e.g., Rooser et al. (PPMXL 2010) and Monet et al. (2003, USNO-B1.0). We find that those measurements are unreliable. For example, the values given in PPMXL differ by 560 and 1350 mas yr⁻¹ from our measurement. A similar discrepancy is observed in USNO-B1.0, which is expected given that PPMXL is derived from USNO-B1.0. Given that the field is moderately crowded, we suspect that such mismatch is due to a cross-matching error (two stars with large proper motions are reported within 10'' of the nominal position of GJ 317 in both catalogs). Even though GJ 317 is bright in the NIR, it can be easily confused with background objects at visible wavelengths.

Table 1
Differential Astrometry of GJ 317

| Parameter                  | Value |
|----------------------------|-------|
| μR.A. (mas yr⁻¹)           | -438 ± 5σ |
| μDecl. (mas yr⁻¹)          | 794 ± 5σ |
| Absolute parallax (mas)    | 65.3 ± 0.4σ |
| d (pc)                     | 15.10 ± 0.22σ |
| Calan-ESO spectral type    | M2.5Vb |
| [Fe/H]                     | 0.36 ± 0.2σ |
| T_eff (K)                  | 5150 ± 50σ |

Notes.

- * This work.
- ** Absolute proper motions from the NLTT catalog (Salim & Gould 2003).
- ⋆ Rojo & Ruiz (2003).
- a Mass obtained using absolute magnitudes in J, H, and K; and the Delfosse et al. (2000) calibration.
- d Gizis et al. (2002).

Figure 2. Comparison of the previously published parallaxes to the CAPS program targets with more than 10 epochs. The published parallaxes have been obtained from SIMBAD (http://simbad.u-strasbg.fr/simbad/), and references therein.

Table 2
Properties of GJ 317

| Parameter | Value |
|-----------|-------|
| μR.A. (mas yr⁻¹) | -438 ± 5σ |
| μDecl. (mas yr⁻¹) | 794 ± 5σ |
| Absolute parallax (mas) | 65.3 ± 0.4σ |
| d (pc) | 15.10 ± 0.22σ |
| Calan-ESO spectral type | M2.5Vb |
| [Fe/H] | 0.36 ± 0.2σ |
| T_eff (K) | 5150 ± 50σ |
| Adopted M_⊙ (M_⊙) | 0.42 ± 0.05σ |
| Heliocentric RV (km s⁻¹) | 87.8 ± 1.5σ |
| Heliocentric U/V/W (km s⁻¹) | (-92.7, -52.5, 26.5)σ |
| Galactic U/V/W (km s⁻¹) | (-82.3, 178.3, 33.8)σ |
The measured trigonometric parallax of GJ 317 is 65.3 ± 0.5 mas. This puts the star at a distance of 15.3 ± 0.2 pc compared to the value used by JB07 (110 ± 20 mas or 9.1 ± 1.6 pc) using Jenkins (1963) and Gliese & Jahreiß (1991). The other most recent measurement is 94.2 ± 4.0 mas (van Altena et al. 1995), still off by more than 2σ illustrating that previous parallactic measurements with uncertainties larger than 10 mas have to be taken with caution. Unfortunately, the astrometric amplitude due to an unseen companion is inversely proportional to the distance to the Sun. This effectively suppresses the expected astrometric signal by a factor of 1.7, indicating that previous parallactic measurements with uncertainties larger than 10 mas have to be taken with caution.

2. Photometry of GJ 317

The photometry of GJ 317 is from the Hipparcos satellite (van Leeuwen 2007) also giving a distance of 15.3 ± 0.2 pc. Therefore, if both stars were of the same spectral type, they should exhibit similar magnitudes. This is clearly not the case (on average, GJ 317 is 1.5 mag fainter in all the bands as given by the Simbad\(^4\) compilation). Rojo & Ruiz (2003) used photometry and low-resolution spectroscopy to obtain a spectral type of M2.5V, but reported a photometric distance estimate of 7 pc, which puts the star even closer. As we show in this section, the updated distance (15.3 pc) and high metallicity seem to solve most of these apparent contradictions.

2.1. Observations: Radial Velocities

The 37 RV measurements presented herein were obtained with the HIRES spectrometer (Vogt et al. 1994) of the Keck I telescope from 2000 January to 2010 March. This adds 18 measurements to those presented in the discovery paper of GJ 317b/c (Johnson et al. 2007). Typical exposure times on this star are 500 s. The Doppler shifts are measured by placing an iodine absorption cell ahead of the spectrometer slit in the converging f/15 beam of the telescope. This gaseous absorption cell superimposes a rich forest of iodine lines on the stellar spectrum, providing a wavelength calibration and proxy for the point-spread function of the spectrometer. For the Keck planet search program, we operate the HIRES spectrometer at a spectral resolving power \( R = 70,000 \) and a wavelength range of 3700–8000 Å, though only the region 5000–6200 Å (with iodine lines) is used in the present Doppler analysis. Doppler shifts from the spectra are determined with the spectral synthesis technique described by Butler et al. (1996). Due to several improvements in our RV extraction pipeline for M dwarfs (see Vogt et al. 2010 for further details), the new RV measurements update those given in Johnson et al. (2007). The HIPPARCOS/Keck has a demonstrated long-term stability better than 3 m s\(^{-1}\) for other stars with similar spectral types (e.g., Vogt et al. 2010). The analysis of the RV data is given in Section 4.1. The differential RV measurements of GJ 317 are given in Table 3 and the average heliocentric RV is given in Table 2. While the astrometry does not show any obvious signal, GJ 317b can be clearly seen in the RV data. The signal of GJ 317c is seen as a long-term quadratic drift in the Doppler measurements (see Section 4.1).

3. GJ 317 IS A METAL-RICH M DWARF

GJ 317 (LHS 2037) was classified as an M3.5V star based on similarities to the high-resolution spectrum of the M3.5V M dwarf GJ 849 (JB07). The assumed distance was 9.1 pc. In JB07, it was noted that the photometry looked anomalous in the sense that the magnitudes did not match those expected for a main-sequence star. The comparison star (GJ 849) has a HIPPARCOS parallax (van Leeuwen 2007) also giving a distance of 9.1 ± 0.2 pc. Therefore, if both stars were of the same spectral type, they should exhibit similar magnitudes. This is clearly not the case (on average, GJ 317 is 1.5 mag fainter in all the bands as given by the Simbad\(^4\) compilation). Rojo & Ruiz (2003) used photometry and low-resolution spectroscopy to obtain a spectral type of M2.5V, but reported a photometric distance estimate of 7 pc, which puts the star even closer. As we show in this section, the updated distance (15.3 pc) and high metallicity seem to solve most of these apparent contradictions.

Given a trigonometric distance, the mass and temperature of GJ 317 can be obtained by fitting the absolute magnitudes to the theoretical models given by Baraffe et al. (1998). Table 4 shows the available photometry of GJ 317 (\( B, V, R, I, J, H, \) and \( K_s \))

\(^4\) http://simbad.u-strasbg.fr/simbad/
Metallicity has an effect on the colors of M dwarfs: a star with super-solar metallicity will have the V-band flux suppressed compared to a star of the same spectral type with solar metallicity. This effect has been used to estimate metallicities of field M dwarfs, so it would not be surprising that the visual magnitudes appear fainter than expected if GJ 317 were, in fact, metal-rich. The best calibrations use the $V - K$ color against the absolute $K$ magnitude to estimate [Fe/H]; Bonfils et al. (2005) or B05, Johnson & Apps (2009) or JA09, and Schlaufman & Laughlin (2010) or SL10. All three calibrations are empirical and are based on measured metallicities of nearby stars with good parallax measurements. Based on the previous distance, JB07 obtained a [Fe/H] = −0.23. This value made GJ 317 the only “metal-poor” M dwarf with detected giant planet candidates. Given the updated $M_K$, all three methods now indicate that the star is indeed metal-rich ([Fe/H] = +0.08 for B05, [Fe/H] = +0.43 using JA09, and [Fe/H] = +0.29 using SL10). Since B05 tends to underestimate metallicities (Johnson & Apps 2009; Rojas-Ayala et al. 2010), we adopt the average of the JA09 and SL10 calibration, i.e., 0.36, as the updated value for [Fe/H].

We now investigate whether or not the color/metallicity relation can explain the apparent extinction in the visible magnitudes when compared to the models or to the mass/luminosity relations. Since the highest metallicity comes from the JA09 calibration, and assuming that $M_K$ is correct, we ask what $V$ magnitude would be required to conclude that the star has [Fe/H] = 0.0 (this is the metallicity assumed by the Baraffe et al. 1998 models). By iterating on the JA09 relation we find that $V = 11.45$ (compared to an observed $V = 11.98$) gives zero metallicity. We can now test whether this value can do a better job fitting the theoretical models. We find that the correction is in the right direction (see Figure 3) and it significantly improves the fit to the $V, J, H$, and $K$ photometry. Also, the revised mass derived from the models is now 0.42 $M_{\odot}$, much closer to the one fitting the $J, H$, and $K$ bands only (0.43 $M_{\odot}$). Moreover, if we apply the mass/luminosity relations in Delfosse et al. (2000) to the “corrected” $V$ magnitude, we obtain $M_V = 0.43 M_{\odot}$ and $M_{V-K} = 0.41 M_{\odot}$, now in perfect agreement with the value obtained from $J, H$, and $K$. A similar behavior would be expected for the $R$ and $I$ bands, but no empirical color metallicity relations have been published on these bands. Delfosse et al. (2000) already mentioned that the mass/luminosity relation using the $V$ color has an excess of dispersion due to the unknown metallicities of M dwarfs. The remarkable agreement in all the quantities recovered after “correcting” $V$ for the effect of metallicity seems to confirm this. As a more definitive proof, it would be desirable to obtain a direct metallicity measurement using the newly developed spectroscopic methods in the near-infrared (e.g., Rojas-Ayala et al. 2010).

Given the new distance measurement, we can re-compute its tangential and Galactic three-dimensional velocity to check whether GJ 317 could be a member of a known kinematic group. The $UVW$ velocities in the heliocentric and the Galactic reference frames are given in Table 2. The resulting $U$ and $V$ components are large but not unusual for disk stars. We have integrated the Galactic orbit using a basic potential for the Galaxy (thin disk, thick disk, bulge and halo, see Paczynski 1991) and a fourth-order Runge–Kutta integrator (time steps of 100 years). We find that past and future trajectories lie well within the Galactic disk region. Actually, a large $U$ component

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5 http://ned.ipac.caltech.edu/
Table 5
Orbital Solution for GJ 317b/c from the RV Data Only

|                | GJ 317b (Circular) | GJ 317c (Eccentric) |
|----------------|-------------------|---------------------|
| \( P \) (days) | 692 ± 2           | 71000 ± 1500        | >10^5    |
| \( K \) (m s\(^{-1}\)) | 73.5 ± 2        | 30.5 ± 5            | ~30      |
| \( e \)       | 0.11 ± 0.05       | 0 (fixed)           | 0.81 ± 0.2 |
| \( M_0 \)     | 309 ± 15          | 199^75^25           | 350^p    |
| \( \omega \) (deg) | 340 ± 10     | ⋯                    | 210^a    |
| \( y \) (m s\(^{-1}\)) | 18               | 40^p                |          |

Derived quantities

|                       | \( M \sin i \) (\( M_{\text{Jup}} \)) | \( a \) (AU) | \( \chi^2_{\text{RV}} \) |
|-----------------------|---------------------------------|--------------|-------------------------|
| \( M \sin i \) (\( M_{\text{Jup}} \)) | 1.81 ± 0.05                      | 1.6          | 0.05 ± 1.6              |
| \( a \) (AU)          | 1.148                           | 5.5 AU       | 3.0 ± 1.6               |
| \( \chi^2_{\text{RV}} \) | 210                            | 30.5         | 43                      |

Statistics

|                  | \( N_{\text{RV}} \) | \( \text{rms} \) (m s\(^{-1}\)) | \( \chi^2_{\text{RV}} \) |
|------------------|-------------------|-------------------------------|-------------------------|
| \( N_{\text{RV}} \) | 37                | 37                            | 37                      |
| \( \text{rms} \) (m s\(^{-1}\)) | 8.5               | 7.3                           | 4.36                    |

Notes. All parameters are referred to the initial epoch \( T_0 = 2451550.9925 \). The mass of GJ 317 is assumed to be 0.42 \( M_\odot \).

* The solution for GJ 317b is quite insensitive to the eccentricity of GJ 317c. This solution corresponds to the eccentric fit for GJ 317c given on the rightmost column of this table.

* Unconstrained.

In summary, the previously reported anomalies in the photometry of GJ 317 seem to be related to the previous poor estimate of its distance and its high metal content. The new metallicity determination strengthens the observed correlation of super-solar metallicity and the presence of giant planets around M dwarfs (Rojas-Ayala et al. 2010; Johnson & Apps 2009).

4. ORBITAL ANALYSIS

At the updated distance, our astrometric measurements cannot resolve the wobble of planet b (minimum amplitude ~0.3 mas). Assuming an epoch-to-eclipse precision of 0.9 mas (average between R.A. and decl. precisions) on 18 epochs and Gaussian statistics, an amplitude of 0.30 mas should be detected only with a signal-to-noise ratio (S/N) of 1.4. An amplitude of 0.65 mas should be detected with an S/N of 3 if present. Therefore, at our present precision the amplitude of the signal can only be marginally measured with an S/N between 1 and 4 (depending on the actual orbital inclination). While this is insufficient to obtain an unambiguous detection, our analysis shows that we can actually put tight constraints on the mass of GJ 317b, thus confirming its planetary nature. Because of the reduced sensitivity of the astrometry, the orbital parameters constrained by the RV are not sensitive to the astrometric measurements. Therefore, we first provide a detailed analysis of the RV data and then we discuss the constraints imposed by the astrometric measurements on GJ 317b and GJ 317c.

4.1. RV Analysis

We use the latest stable version of the systemic interface (Meschiari et al. 2009; v1.5.12) to obtain the best fit to the RV data. GJ 317b is clearly seen in the periodogram as a strong peak around 700 days. After subtracting the best orbital fit for GJ 317b (see Table 5), a long-term trend is still apparent in the residuals. Assuming a circular orbit we obtain a period of 7100 days for GJ 317c. Even in the circular orbit case, a broad range of periods and masses are still allowed by the data. For an eccentric solution (\( e_c \approx 0.81 \)), the most likely period falls between 50,000 and 90,000 days (~130–250 years). Compared to the orbital solution of JB07 based on 18 measurements, it is now more clear that a constant slope is insufficient to reproduce the observed RVs and that the curvature of the orbit is clearly detected in the data (see Figure 4). In the case of eccentric orbits, such long-period signals can only be detected if the planet is close to the periastron of its orbit, which might be the case here. In any case, several more years of RV measurements will be required to put a more significant constraint on this orbit.

We find that the orbital solution for GJ 317b remains well constrained irrespective of the orbital fit to GJ 317c. We use a Bayesian Monte Carlo Markov Chain (MCMC) approach to characterize the probability distributions of the free parameters and get a realistic estimate of their uncertainties. The methodology applied to obtain such distributions is described in Appendix B. Using only RV observations, we find that the orbit of planet b is well constrained in a narrow region of the parameter space, while the period and eccentricity of planet c still have broad probability distributions (the 99% confidence level interval allows 20 years < \( P < 240 \) years if the orbit is allowed to be eccentric). These broad distributions are caused because planet c is only detected as a secular quadratic term that can be reproduced by many combinations of parameters (see Gould 2008 for a detailed discussion of the problem). The best-fit orbital solutions for GJ 317b and GJ 317c are shown in Table 5. The uncertainties were obtained using the aforementioned MCMC technique and represent the 68% confidence level intervals around the least-squares solution. Note that, because the mass of the star has been updated, the minimum mass of planet b (1.8 \( M_{\text{Jup}} \)) is larger than the value given by JB07 (1.2 \( M_{\text{Jup}} \)).

After removing both signals, there is no conclusive indication of additional planets in the RV data. However, the root mean square (rms) of the residuals (~8 m s\(^{-1}\)) is significantly larger than that expected from the observations of similar stars without planets (rms ~ 3 m s\(^{-1}\), see Section 2.1). Also, the short-period domain (a few days to weeks timescales) is still poorly sampled. Given the abundance of low-mass
objects in multi-planet systems (Howard et al. 2010), additional low-mass companions might be expected to emerge as more RV measurements are obtained.

4.2. Astrometry and Radial Velocities

To perform a joint fit of the astrometry and the RVs, we subtract the best orbital solution for planet c ($e = 0.81$), leaving only the RV signal of GJ 317b in the RV data. The least-squares solution for the combined astrometry and radial velocities is obtained on a grid of fixed period/eccentricities around 690 days (50 test periods between 680 days and 700 days and 20 test eccentricities from 0 to 0.95). All the other parameters are left free. In a last step, the period and the eccentricity are refined starting at the best solution on the grid. The best-fit values (see Figure 5 and Table 6) are then used to initialize a Bayesian MCMC sampler to generate again the a posteriori probability distributions, including now both the RVs and the astrometry in the definition of the likelihood function. The condition equations that the astrometry and the radial velocities have to satisfy together with a brief explanation of the Bayesian MCMC method are given in Appendix B. The same analysis procedure was also used in Anglada-Escudé et al. (2010) to combine astrometric and RV measurements and rule out the existence of the astrometric planet candidate VB10b, at least in moderately eccentric orbits. As in the RV analysis, the steps in the Markov Chain sampler are tuned to accept 15%–30% of the proposed updates, and the first $10^5$ steps are not used in the analysis to avoid oversampling the favored solution. Chains with $5 \times 10^6$ steps are used to generate the numeric realization of the a posteriori distributions for all the parameters. We repeat the process several times and compare the resulting distributions to be sure that the chains are properly converged obtaining good agreement in all the runs.

The best-fit solution (maximum likelihood values) with the corresponding 68% confidence level intervals is given in Table 6. Flat prior distributions have been used in all the cases. Figure 6 shows the final probability distributions for the two parameters that the astrometry can constrain. The fact that the signal is barely detectable bodes ill for the determination of the argument of the node. However, with a 99% confidence level, the inclination is constrained to be greater than 25° (see bottom panel in Figure 6) because a value lower than that would result in a wobble that is not seen. The top panel in Figure 7 illustrates the distributions for the mass and the argument of the node. The distribution for the mass of GJ 317b is clearly non-Gaussian due to the strong lower limit imposed by the RV data on the minimum mass and the more loose constraints imposed by the astrometry on the maximum allowed signal. The bottom panel of Figure 7 shows the histogram of the probability distribution for the mass. Note that an inclination closer to 0 would have two effects: first, the amplitude grows as $1/\sin i$, and second, as the orbit becomes face-on, the amplitude becomes more “two-dimensional.” That is, an edge-on orbit would be seen as a line, while a face-on orbit (inclination close to 0) would show up as a circle increasing its statistical significance. To obtain the range of allowed masses, the obtained

![Figure 5](https://via.placeholder.com/150)

**Figure 5.** Best-fit astrometry + RV joint solution for GJ 317b. Top panels are R.A. and decl. as a function of time. The dashed red line corresponds to the signal of a 4.0 $M_{\text{Jup}}$ mass planet. The dotted red line corresponds to the signal of planet c. The best-fit solution is plotted as a blue line. Bottom panel: the radial velocity data plotted with the best-fit solution after removing the signal of planet c.

(A color version of this figure is available in the online journal.)

| RV Observables | Astrometric Observables |
|----------------|-------------------------|
| $P$ (days)     | $692 \pm 2$             |
| $K$ (m s$^{-1}$) | $75.2 \pm 3.0$          |
| $e$            | $0.11 \pm 0.05$         |
| $M_{\text{op}}$ (deg) | $305 \pm 15$           |
| $\omega$ (deg) | $342 \pm 10$            |

Statistics

| $N_{\text{RV}}$ | $37$ |
|-----------------|------|
| $N_{\text{astro}}$ | $17 \times 2$ |
| rms$_{\text{RV}}$ (m s$^{-1}$) | $7.4$ |
| rms$_{\text{RA}}$ (mas) | $0.70$ |
| rms$_{\text{Decl}}$ (mas) | $1.23$ |
| $T_P$ (Julian date) | $2451656.7^a \pm 35$ |
| $A_{\text{RV}}/N_{\text{RV}}$ | $3.1$ |
| $A_{\text{RA}}/N_{\text{astro}}$ | $1.1$ |
| $A_{\text{Decl}}/N_{\text{astro}}$ | $4.8$ |

Statistics Derived quantities

|                          |                   |
|--------------------------|------------------|
| $P$ (years)              | $1.894 \pm 0.013$|
| $M_{\sin i}$ (M$_{\text{Jup}}$) | $1.8 \pm 0.05$ |
| $a$ (AU)                 | $2.5^{+0.4}_{-0.0}$ |
| $\alpha$ (AU)            | $1.15 \pm 0.05$  |
| Angular separation (mas) | $76$             |

**Table 6**

**Best Orbital Solution for GJ 317 b**

Notes. All parameters are referred to $T_0 = 2451550.9925$.

$^a$ Time of passage through the periastron closer to the initial epoch $T_0$.

$^u$ Unconstrained.
the astrometry (see the residuals in Figure 1), the true mass time, because the period is so long, the eccentricity is poorly constrained. To do this, we removed GJ 317b from the radial velocities and focused on the new measurements put meaningful constraints on the mass of GJ 317b, confirming its planetary nature. Combining astrometric and RV measurements is a complex multi-parametric problem where the final probability distributions for the involved parameters are not necessarily Gaussian. This upper limit has been obtained using a Bayesian MCMC approach which is much better suited than a classical \( \chi^2 \) analysis to put constraints on parameters and obtain confidence intervals. Given that the upper limit to the mass of GJ 317b (\( \sim 3.6 M_{\text{Jup}} \)) is much lower than the planet/brown dwarf boundary, it is the first time that the ground-based astrometric observations have been able to confirm the planetary nature of a substellar companion. The presented radial velocities confirm the presence of an extremely long period planet (period 20 years or more) that is not yet detected in the astrometry.

5. DISCUSSION

We have obtained precision astrometric measurements of the M dwarf GJ 317 and combined these with new radial velocities. Even though the signal is not fully resolved by the astrometry, the new measurements put meaningful constraints on the mass of GJ 317b, confirming its planetary nature. Combining astrometric and RV measurements is a complex multi-parametric problem where the final probability distributions for the involved parameters are not necessarily Gaussian. This upper limit has been obtained using a Bayesian MCMC approach which is much better suited than a classical \( \chi^2 \) analysis to put constraints on parameters and obtain confidence intervals. Given that the upper limit to the mass of GJ 317b (\( \sim 3.6 M_{\text{Jup}} \)) is much lower than the planet/brown dwarf boundary, it is the first time that the ground-based astrometric observations have been able to confirm the planetary nature of a substellar companion. The presented radial velocities confirm the presence of an extremely long period planet (period 20 years or more) that is not yet detected in the astrometry.

Other RV surveys (e.g., Zechmeister et al. 2009; Johnson et al. 2007) have found a low occurrence rate of moderate-to-short-period gas giants around M dwarfs, with the resonant pair GJ 876b/c the only remarkable exception (Rivera et al. 2010).
No hot Jupiters ($P < 30$ days) have been reported around a low-mass star. A handful of gas giants with periods longer than 30 days have been found around a few early type M dwarfs (e.g., GJ 179b, GJ 832b, GJ 849b, HIP 79431b, HIP 57050b; see the exoplanet encyclopedia for an up-to-date list). This seems to indicate that M dwarfs have trouble forming and/or keeping giant planets in tight orbits.

According to recent studies, the frequency of M dwarfs hosting gas giants seems rather low compared to more massive solar-type stars (e.g., Endl et al. 2006; Johnson et al. 2010). This is an expected consequence of the core-accretion model for giant planet formation (Laughlin et al. 2004; Kennedy & Kenyon 2008; Alibert et al. 2011). Also, the new distance measurement indicates that GJ 317 is metal-rich. With GJ 317 now in the club, all the M dwarfs with reported giant planets are metal-rich (Rojas-Ayala et al. 2010; Johnson & Apps 2009). The competing mechanism model for planet formation (Boss 1997, disk instability model) should not be very sensitive to the metallicity of the host star, so this has also been suggested as evidence in favor the core-accretion scenario (e.g., see Ida & Lin 2004; Mordasini et al. 2009). However, core-accretion is still too slow to form long-period gas giant planets around low-mass stars, except for exceptionally long-lived disks (Ida & Lin 2005). Also, core-accretion fails to reproduce the overabundance of planets in close-in orbits around all stellar types found by Kepler/NASA (Borucki et al. 2011). On the other hand, disk instability is able to form gas giants around M dwarfs with short lifetime disks (Boss 2006, 2011). Given these two issues (timescale problem and incorrect prediction of the observed planet distributions), we find it more natural to invoke the disk instability mechanism to explain the formation of GJ 317c, and possibly GJ 317b as well. Given that the number of nearby M dwarfs surveyed for planets is still small, more detections in a statistically larger sample are required to put real constraints on the metallicity–gas giant connection. This is one of the long-term goals of the Carnegie Astrometric Planet Search project.

GJ 317 is one of the faintest targets in the Lick-Carnegie planet search program, and precision radial velocities in the optical are limited by photon noise and require intensive use of large aperture telescopes (Keck/HIRES) to reach the few m s$^{-1}$ precision level. Since it is a cool star ($T < 4000$ K), most of its flux is in the near-infrared; so it will be an excellent target for precision RV measurements when the new generation of near-infrared spectrographs comes online (Bean et al. 2010; Figueira et al. 2010; Anglada-Escudé & Plavchan 2011). Also, the star lies in the optimal magnitude range $(12 < V < 15)$ for the Gaia/ESA space astrometry mission (Lindegren 2010) and the PRIMA/Very Large Telescope interferometer (Koehler et al. 2010). Both instruments will have a single measurement precision better than 0.1 mas and the orbit of GJ 317b (minimal semiamplitude of 0.3 mas) will be clearly resolved once an orbital period (∼700 days) is covered. Since the astrometric signal of the long-period planet GJ 317c should be evident in a few more years of CAPScam observations as a quadratic term, we will continue monitoring this star at lower cadence to measure its orbital inclination and mass.

Finally, we highlight some unique features of GJ 317. It is a relatively bright, low-mass star with super-solar metallicity, so it seems an ideal target to look for low-mass terrestrial planets in its habitable zone. Recent studies (Mayor et al. 2009; Howard et al. 2010) indicate that 30% of the dwarf stars host planets in the super-Earth mass range. This fraction seems to be even higher in multi-planetary systems. Such planets in the habitable zone around GJ 317 should have orbital periods of a few weeks and RV amplitudes of several m s$^{-1}$. The fact that the system contains a pair of long-period giants might have helped to deliver volatile compounds to the inner orbits as seemed to have happened in the early solar system (Crida 2009). On the other hand, significantly eccentric orbits are possible for both outer planets, which might be a problem for the long-term stability of the inner orbits (Chambers & Cassen 1997).
Finally, the outermost giant planet might be detectable by direct imaging in the near future (e.g., Lagrange et al. 2010 found a substellar companion to $\beta$ Pic at a similar angular separation). An image of this planet combined with the astrometry and the RV would provide a model-independent measurement of its mass. In summary, in a few years GJ 317 might become one of the better characterized planetary systems beyond our own.

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Facilities: Du Pont (CAPScam), Keck:1 (HIRES)

APPENDIX A

THE ATPa PIPELINE: ASTROMETRIC EXTRACTION, CALIBRATION, AND SOLUTION

The reduction of the astrometric observations is done using the ATPa software, specifically developed for the CAPS project. Further details on the various steps of the astrometric data reduction are given in Boss et al. (2009). The astrometric processing done by ATPa can be outlined as follows. First, the position of all the stars in each image is extracted and mapped to a tangent plane to the sky centered at the nominal coordinates of the target star at the first epoch of observation. For each image, a subset of reference stars are matched to their predicted position to adjust the telescope pointing, plate scale, rotation, and fit for a geometric field distortion. These predicted positions are initialized using the stars extracted from a high-quality image. Using this image-by-image solution, the position of each star in the field is mapped to the frame defined by the reference stars. Then, all the measured positions in a night (or epoch) are averaged to obtain the epoch position and its uncertainty of each star in the field. Finally, all the epochs are fitted to a linear astrometric model (position, proper motion, and parallax) and the reference frame is refined by selecting those stars showing the smallest residuals. This process is iterated a few times until convergence is reached, that is, when the rms of the residuals of the reference stars does not change significantly (e.g., less than 0.05 mas). We call this process the AIT. The result is a catalog of positions, parallaxes, and proper motions for all the stars in the field.

Now let us describe each step in more detail. The centroids of the stars are measured by binning the stellar profile in the X- and Y-directions on a box of 12 pixels ($\sim 2''$) around the pixel with maximum flux. This one-dimensional profile is then precisely centroided using a function called Tukey’s biweight function (see p. 697 in Press et al. 1992). Many simulations and tests with different profiles showed that this approach provided the best centroid accuracy and robustness (see Boss et al. 2009 for further details). The flux is also measured on a circular aperture of 10 pixels around the obtained centroid and the sky obtained from a ring of 15–20 pixels is then subtracted. As a result, each image produces a list of star positions and fluxes. A preliminary centroid accuracy for a given night is empirically obtained using a rough reference frame consisting of the best 10 stars in the field in terms of centroid nominal precision. Finally, the nominal optical distortion of CAPScam is used to map the pixel positions to a local tangent planet to the sky. This nominal field distortion was obtained in 2011 April by observing a moderately crowded field on a rectangular grid of 16 positions spaced by 40'' (northeast tip of the rectangle centered at 16:00:11.41, −40:12:42.2). This method of measuring the nominal optical distortion is described in Anderson (2007), and references therein.

To initialize the astrometric iterative solution, the extracted positions of one image are used as a template to generate an initial catalog. In this first iteration, all the images are matched to this catalog using a linear distortion model. This is, assuming that the predicted local tangent plane position of the ith reference star is $(u_i, v_i)$, the geometric calibration consists in finding the coefficients that satisfy

$$ u_i = a_0 + a_1 x_i + a_2 y_i, $$

$$ v_i = b_0 + b_1 x_i + b_2 y_i, $$

where $x_i$ and $y_i$ are the extracted positions of the references obtained in the one night processing step after applying the nominal CAPScam distortion. The same transformation is then applied to all the stars in the field. The nightly averaged $u$ and $v$ of each star is obtained (this is what we call an astrometric epoch measurement) and the intranight standard deviation divided by the square root of the number of observations is used as the associated epoch uncertainty (i.e., uncertainties in Table 1). Finally, all the epochs are used to update the astrometric solution for each star by fitting their motion to

$$ u(t) = u_0 + \mu_u (t - t_0) - \Pi p_u(t) + \Delta_u, $$

$$ v(t) = v_0 + \mu_v (t - t_0) - \Pi p_v(t) + \Delta_v, $$

where $\mu_u$ and $\mu_v$ are the proper motion in the direction of increasing R.A. and decl., respectively (in mas yr$^{-1}$), $\Pi$ is the parallax in mas, $u_0$ and $v_0$ are constant offsets (in mas) that provide the star position on the local plane at the reference epoch $t_0$ assuming zero parallax. The numbers $p_u$ and $p_v$ are the so-called parallax factors and correspond to the parallactic apparent motion projected on the direction of increasing R.A. and decl. at the observing instant $t$. To ensure maximal precision, the parallax factors are derived using the position of the geocenter from the DE405 JPL Ephemeris of the solar system. Note that the five free parameters in this equation ($u_0$, $v_0$, $\mu_u$, $\mu_v$, and $\Pi$) are linear. Therefore, the corresponding system of normal equations can be efficiently solved in a single least-squares step. The perspective acceleration for GJ 317 (Dravins et al. 1999) is negligible given the relatively short time baseline of this data.
set (∼0.042 mas over two years) and is not included in the processing. A simple derivation of this astrometric model and additional second-order terms are outlined in Anglada-Escudé & Debes (2010). $\Delta_\alpha$ and $\Delta_\delta$ contain possible perturbations to the baseline astrometric model but they are not adjusted during the AIT. For the target star, $\Delta_\alpha$ and $\Delta_\delta$ include the astrometric Keplerian motion following the prescriptions outlined in Appendix B.2. Such functions implicitly depend on the Keplerian parameters in a complicate nonlinear fashion, specially when combining astrometric and RV observations.

Because this field is particularly rich in stars, our software is able to automatically select 11 very stable references within 120′ of the target. By stable we mean that the rms of the epoch-to-epoch residuals (rms) is able to automatically select 11 very stable references within 0.042 mas. In a second iteration these 11 references are used to re-compute the field distortion of each image with respect to the updated version of the catalog. This time, a second-order distortion correction (six coefficients are adjusted on each axis) is fitted to each image. The zero-point corrections is the average of the references changes by less than 0.05 mas with respect to the previous iteration. For this field in particular, only five iterations were required to reach convergence. The astrometric solution for the reference stars and their catalog information (Zacharias et al. 2004) are given in Tables 7 and 8, and their distribution on the field of view of CAPScam is illustrated in Figure 9.

### APPENDIX B

**BAYESIAN MONTE CARLO MARKOV CHAINS**

Bayesian statistics allow computation of the probability distributions of the free parameters constrained by a set of observations. The general strategy we use in this work is based on the methods described in detail in Ford (2005), to which we strongly encourage readers to refer. To obtain the parameter distributions compatible with the data we have to obtain the likelihood function which, in this case, is just the product of the probability distributions of all the available observations. That is, assuming that the uncertainties in the measurements follow a Gaussian distribution, the likelihood function $L$ reads

$$ L = \kappa \exp \left(-\frac{1}{2} \chi^2[\hat{\alpha}] \right), $$

(\text{B1})

where $\chi^2$ is the classic definition of the weighted least-squares statistic, $x_{\text{obs}}$ can be any kind of observations (e.g., an RV measurement, an astrometric offset, an instant of transit), $\sigma_k$ is the uncertainty on such measurement, $x_{\text{model}}[\hat{\alpha}]$ are the predictions of the model to be tested, $\hat{\alpha}$ is a vector containing the free parameters to be investigated, and $\kappa$ is a normalization constant. This $L$ multiplied by the prior distributions of the
follow a Gaussian distribution with 0 mean and \( \sigma = J_k \), where each \( J_k \) should be of the order of the expected uncertainty on the \( k \)th parameter. If these \( J_k \) are too small, the MCMC will require many jumps to sample the region of interest. If \( J_k \) is too large, the MCMC will rarely accept updates and the parameter space will be poorly sampled again. To optimize convergence, Ford (2005) and other authors found that these \( J_k \) need to be tuned so only 15%–30% of the proposed jumps are accepted. To perform this optimization, we initialize each \( J_k \) using the formal uncertainty of each parameter obtained from the least-squares solver and run \( 10^5 \) MCMC steps. If one parameter has an acceptance rate higher than 30%, we multiply \( J_k \) by 1.5. If the acceptance ratio is lower than 15%, we divide the corresponding \( J_k \) by 1.5. This process is iterated until all the parameters have acceptance rates between 15% and 30%. The precise values of these \( J_k \) are not critical as long as the MCMC converges to the equilibrium distribution. This can be tested by running different chains and checking that the obtained \( P[\hat{a}] \) are compatible with each other. Given a multidimensional parameter space, one needs many steps to properly sample \( P[\hat{a}] \). This method is computationally expensive and takes a long time to converge if the Markov Chain is not initialized close to the favored solution. As a general rule, we initialize the Markov Chains within three-standard deviations as obtained from the least-squares solver on the optimal solution. Once an MCMC is ready and tuned, we typically run it over \( 5 \times 10^5 \) steps rejecting the first \( 10^5 \) to avoid oversampling the initial least-squares solution. The resulting list of parameters is saved in a file for further processing (e.g., marginalization, confidence level estimates).

### B.1. Radial Velocities Only

In what follows, all equations used to describe the Keplerian motion and its observables are based on the expressions given in Wright & Howard (2009). When only radial velocities are used, the \( \chi^2 \) function required by Equation (B1) reads

\[
\chi^2_P[\hat{a}] = \sum_{k=1}^{N_r} \left( \frac{\text{RV}_k - \text{RV}_{\text{model}}[t_k; \hat{a}]}{\sigma_k} \right)^2, \quad (B3)
\]

where \( \text{RV}_k \) are the heliocentric RVs, \( t_k \) are the heliocentric instants of observation, and \( \sigma_k \) are the associated uncertainties. The predicted radial velocities by the model \( \text{RV}_{\text{model}} \) are given by

\[
\text{RV}_{\text{model}}[t; \alpha] = \gamma + K \cos(\omega + \nu(t)) + e \cos \omega, \quad (B4)
\]

where \( \nu(t) \) is the so-called true anomaly and is obtained as a function of time using the relations

\[
\tan \frac{\nu(t)}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E(t)}{2}, \quad (B5)
\]

\[
E(t) - e \sin E(t) = \frac{2\pi}{P} - M_0. \quad (B6)
\]

\( E \) is the so-called eccentric anomaly and is obtained by solving numerically the implicit Kepler Equation (B6).\(^9\) The constant \( K \) is the RV semiamplitude and relates to the physical parameters of the system as follows:

\[
K^3 = \frac{2\pi G}{P(1-e^2)^{3/2}} \frac{m_1^3 \sin^3 i}{(M_*+m_p)^2}, \quad (B7)
\]

\(^9\) [http://mathworld.wolfram.com/KeplersEquation.html](http://mathworld.wolfram.com/KeplersEquation.html)
where $G$ is the gravitational constant in MKS units. The physical free parameters $\hat{\alpha}$ in the condition Equation (B4) to be solved are as follows.

1. $\gamma$: systematic RV of the star (or RV offset) in m s$^{-1}$.
2. $m \sin i$: planet mass times the orbital inclination (or minimum mass) in kg.
3. $P$: orbital period in seconds.
4. $e$: orbital eccentricity (from 0 to 1 for bound orbits).
5. $\omega$: argument of the periastron in radians.
6. $M_0$: initial mean anomaly in radians.

When only dealing with RV measurements, the least-squares solvers and the Bayesian MCMC directly optimize the parameters listed above.

B.2. Astrometry and Radial Velocities

When two-dimensional astrometric measurements $u$, $v$ and radial velocities are available, the $\chi^2$ function required by Equation (B1) reads

$$
\chi^2 = \sum_k \left( \frac{\text{RV}_{\text{obs}} - \text{RV}_{\text{model}}[t_k \hat{\alpha}]}{\sigma_k} \right)^2
$$

(B8)

$$
+ \sum_s \left( \frac{u_s - u_{\text{model}}[t_s \hat{\alpha}]}{\sigma_s} \right)^2 + \sum_s \left( \frac{v_s - v_{\text{model}}[t_s \hat{\alpha}]}{\sigma_s} \right)^2,
$$

(B9)

where $\text{RV}_{\text{model}}[t_k \hat{\alpha}]$ is given in Equation (B4). The astrometric condition equations in the local tangent plane coordinates read

$$
u[t; \hat{\alpha}] = u_0 + \mu_a (t - t_0) - \Pi p_a(t) + \Delta_a(t)\quad (B10)
$$

$$
\nu[t; \hat{\alpha}] = v_0 + \mu_\delta (t - t_0) - \Pi p_\delta(t) + \Delta_\delta(t),\quad (B11)
$$

where all the linear parameters $u_0$, $v_0$, $\mu_a$, $\mu_\delta$, and $\Pi$ are already described in Appendix A. The Keplerian parameters are included in $\Delta_a$ and $\Delta_\delta$ as

$$
\Delta_a(t) = B X(t) + G Y(t)\quad (B12)
$$

$$
\Delta_\delta(t) = A X(t) + F Y(t)\quad (B13)
$$

$$
A = a(\cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i)\quad (B14)
$$

$$
B = a(\sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i)\quad (B15)
$$

$$
F = a(- \cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i)\quad (B16)
$$

$$
G = a(- \sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i)\quad (B17)
$$

where $A$, $B$, $F$, and $G$ are the so-called Thiele Innes constants. $a$ is the orbital semimajor axis and is related to the other parameters as

$$
\left( \frac{a}{\text{AU}} \right)^3 = \frac{(\Pi/1000)^3 (m/M_\odot)^3}{(M_\odot + m)/M_\odot^2} (P/\text{yr})^2,\quad (B18)
$$

where $M_\odot$ is the mass of the sun in kg and yr is a year in seconds. All the time dependence in the astrometric motion in Equations (B12) and (B13) are in $X(t)$ and $Y(t)$. $X$ and $Y$ represent the Keplerian motion of the star on a coordinate system coplanar with the orbital plane with the $X$-axis pointing to the orbital periastron. $X$ and $Y$ depend on time through the eccentric anomaly $E$ as

$$
X(t) = \cos E(t) - e\quad (B19)
$$

$$
Y(t) = \sqrt{1 - e^2} \sin E(t)\quad (B20)
$$

and $E$ has to be solved as for the radial velocities using Equation (B6). The free parameters $\hat{\alpha}$ for the combined astrometric and RV measurements are as follows.

1. $\gamma$: systematic RV of the star, (or RV offset) in m s$^{-1}$.
2. $m$: planet mass in kg.
3. $P$: orbital period in seconds.
4. $e$: orbital eccentricity (from 0 to 1 for bound orbits).
5. $i$: orbital inclination with respect to the plane of the sky in radians ($0$ corresponds to a face-on orbit).
6. $\Omega$: argument of the node in radians (orientation of the orbit on the sky with respect to the local north. Positive from north to east).
7. $\omega$: argument of the periastron in radians. This is the angle between the node and the periastron of the system as measured on the orbital plane.
8. $M_0$: initial mean anomaly in radians.
9. $u_0$: offset in R.A. in mas.
10. $v_0$: offset in decl. in mas.
11. $\mu_\alpha$: proper motion in R.A. in mas yr$^{-1}$.
12. $\mu_\delta$: proper motion in decl. in mas yr$^{-1}$.
13. $\Pi$: parallax in mas.

A more detailed description of these parameters can be found elsewhere. This prescription is based on the definitions given by Wright & Howard (2009).

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