Abstract: The existing literature provides various computational models related to the dynamic behavior of strand wire ropes. It starts from the simple longitudinally oscillating beam, to the complex nonlinear multi-body configuration based on helical structural symmetry. The challenge is the prior availability of characteristic parameters for material behavior, structural configuration, and functional capability. Experimental investigation is the main source for evaluation of these characteristics. However, tests have specifically been performed according to each case, minimizing the generalization aspect. This is the main frame of this study. Hereby, the authors propose an ensemble of spectral investigations, applied to a reduced set of experimental tests regarding wire rope dynamics. The research goal consists of wire rope characterization in terms of the flexible and adaptive groups of parameters, related to the conservative and dissipative behaviors. An experimental setup is considered here according to the rope exploitation conditions in order to enable an extension of the method application from the experimental mode to the operational mode. Experiments are conducted based on classical vibration measurement procedures. The analysis is performed using a spectral method ensemble, including discrete Fourier transform, time-frequency joint analysis, and the Prony method. The result show that the proposed assessments can provide suitable information related to a large group of wire rope models.

Keywords: strands wire rope; experimental transitory vibrating regime; stiffness; damping; joint time-frequency analysis; Prony method; matrix pencil method

1. Introduction

Strand wire ropes take part in most existing engineering elements. Wire ropes have a very large area of utilization, ranging from various types of anchorages, traction systems, and industrial applications, to cranes and hoisting systems. Being widely used and providing a very important structural/functional role, strand wire ropes have enabled many actual studies, specifically in terms of being able to accurately evaluate behavioral characteristics under operational conditions. It is also important to predict the extreme charging conditions and the risks in the exploitation of these flexible elements.

The authors of [1] present a co-simulation method based on both multi-body dynamics and finite element analysis. They have developed a dynamic model in the ADAMS/Cable software package related to preliminary evaluation of the time history response of dynamic force in wire rope. Secondly, they have implemented a finite element-based model within ABAQUS FEA software, followed by
static and dynamic computational investigations. The computational results were compared with experimental data, and the results proved accuracy and reliability. Concluding, the authors present the stress distribution and spectrum of wire rope under dynamic loading during the hoisting process and show the most dangerous regions.

The work of [2] contains a model for the dynamic analysis of a load lifting system. The novelty of this analysis is sustained by the method used for solving the model, which is a high-precision direct integration method. Hereby, the author obtained more precise results in the same time with less computing time and high accuracy.

Within [3], Haniszewski proposed a test setup for experimental investigations regarding dynamic phenomena accompanying the lifting process of a load, which allows simultaneously monitoring of a large number of parameters in different regions of the crane system. Based on this test bench, the author developed both experimental and analytical investigations. The same author treated the problem of negative vibrations, which can appear during the use of hoisting systems [4]. Taking into account the various available models for rope and the major importance of rope within the entire hoisting system, he proposed both modified Bouc–Wen and Voigt–Kelvin models. The results were analyzed in the terms of a dynamic factor.

In his paper [5], Argatov presents a refined variant of a discrete mathematical model that is useful in the analysis of a simple helical wire rope. The model details the transverse contraction of a strand due to both the Poisson’s ratio and the local contact deformations. The last aspect was approached as a frictionless unilateral plain strain problem. The author obtained an asymptotic model with constitutive equations in a closed form, being able to accurately predict the deformations of a wire rope strand subjected to both tensile and torsional loads.

A new theoretical method for the analysis of wire ropes simultaneously charged by both tensile and torsional loads was proposed in [6]. The study used the beam assumption and thin rod theory, applied to each wire within the strand structure. A kinematic model was developed on the basis of the hypothesis that is no relative sliding between adjacent wires.

Experiments based on monitoring the longitudinal vibration of a wire rope were developed by Hamilton and presented as part of a study [7]. The author used the principle of a spring-mass oscillator in order to evaluate the parameters of the rope in terms of the elastic and damping properties. The study investigated the transitory vibrating regime of the experimental system containing a mass suspended by a wire rope. The results revealed an acceptable accuracy for wire rope lengths less than 2 m and with differences less than 1% when changing the length.

Simple and reliable theoretical models were presented by Raoof and Davies in their work [8], based on correlations developed by Raoof and Kraincanic. These models are useful to characterize independent wire rope core (IWRC) or fiber core (FC) large-diameter multi-strand wire ropes. The analytical expressions presented by the authors allow the prediction of the axial stiffness with the hypothesis of no-slip or full-slip. These simple models were established based on a very large experimental third-party database which sustains the reliability of the approaches.

The Stomer–Verlet method presented in [9] is very useful in the motion analysis of a multi-body system with wire rope connections between bodies. This method was derived from a discrete Euler–Lagrange (DEL) equation. One main advantage of this approach comes from the fact that the stretching of wire ropes can be mathematically modeled as a constraint for stability. During computational investigations, the DEL equation, with wire ropes as constraints, provides relatively stable solutions.

Within their studies [10,11], Kaczmarczyk and Ostachowicz proposed an interesting distributed parameter model for the dynamic analysis of the behavior of a deep mine hoisting cable. They approached the non-stationary aspects of the problem due to the coupled transversal-longitudinal responses, involving non-linear partial differential equations in a mathematical model. A discrete formulation of this model was obtained using a Rayleigh–Ritz procedure, finally using a non-linear, non-stationary, coupled second-order ODE system applicable for computational-based analyses [10].
The authors treated the problem both in fast and slow time scales in order to investigate the complete response of the system. Equivalent proportional damping was considered for the solving procedure for the stiff problem. The results indicate various transient resonances within non-linear evolutions of the system. An important finding is that small changes in excitation characteristics are able to induce large changes in dynamic response due to the shifting of the resonances [11].

Computational approaches regarding the behaviors of the wire ropes and cables during intensive and variable dynamic exploitation regimes can be consulted in various research reports [12–14]. Hereby, extensive analyses of the problem regarding cable dynamics during the lifting process have been provided within studies [12,13], where the simulations were based on linear and non-linear, lumped or continuous mass distribution models. In addition, the dynamics of wire ropes, such as in the main hoisting component of technological equipment, have been treated in computational research [14].

Hobbs and Raoof presented a relevant analysis procedure based on an exhaustive literature review inside their research paper [15]. The contact between the wires within the rope strands, the axial stiffness, the hysteresis variability with the charge characteristics, the loaded cable bending problem, and the axial cyclic charging of strand-based wire ropes were clearly presented and extensively discussed within this article. In addition, particular attention was given to the aspects regarding the localized and repeated bending phenomena that frequently occurs close to the cable terminations.

An interesting approach successfully combining computational-based simulation with experimental investigations is provided within [16]. In addition, it has to be mentioned that the second edition of Costello’s study [17] details a comprehensive analysis of wire behavior dynamics and provides useful information for computational approaches based on extensive experimental investigations. In the same area of experimental-based analyses of lifting process dynamics, the research presented in [18] must be mentioned.

Demšić and Raduka [19] proposed an analytical model developed for inhomogeneous boundary conditions which takes into account the system nonlinearities in terms of quadratic and cubic formulations. The authors used both a numerical integration procedure and multiple scales of a perturbation method in order to solve the reduced mathematical model and the parametric oscillations, respectively.

A comprehensive study regarding special cables used at cable-supported bridges was provided within [20]. The authors investigated a wide palette of parameters that are able to characterize the bridge behavior under dynamic loads. Among these, they analyzed the anomalies of damping properties and nonlinear parametric vibration. One has to note the experimental developments presented within this study.

Elata et al. proposed an interesting computational model for the dynamic response analysis of wire ropes with an independent core that are axially charged by a load and a torque [21]. The model fully supposed a double-helix configuration for component wires and assumed a fiber elastic response of individual wires, thus it becomes able to provide the values of the stress at the wire level.

The previous briefly introduction does not have the intention to be exhaustive but has the role of revealing the available literature related to theoretical and applicable research studies into the field of wire ropes. These studies are useful through their gains in dynamics modeling, simulation, and practical implementation. Considering the wide applications of strands wire ropes, this clearly results in the necessity of large and various categories for knowing the characteristic parameters in order to carry out reliable computational analyses. However, the available studies individually treat each type of problem regarding the wire rope dynamic behavior. Operational tests are rather nonexistent. Experimental investigations have been strictly developed for particular cases of simulation models and they do not supply generalizable characteristics. Computational models have plenty of representations, both from the point of view of behavioral schematizations and also solving techniques. Nevertheless, these approaches require many parameters to accurately perform the analysis.

Hereby, the present study presents some useful assessments based on experimental dynamics investigation which are able to provide a flexible and adaptive set of data according to a large category
of numerical models. Flexible means that the post-processing procedure can be extended in respect to the requirements (in terms of parameter type and precision). This is a qualitative approach. Adaptive means that the number of evaluated parameters can be permanently harmonized with the model order. This is a quantitative approach. The main advantages are supplied by the reduced experimental investigations volume, and, in addition, by the capability of operational implementation (where the exploitation regime implies dynamic evolution or the system supports external dynamic perturbation). Thus, the goal of this research is the estimation of wire rope characteristics, closely related to the model complexity, where these parameters will be assumed.

2. Materials and Methods

This research was primarily based on experimental investigations which have developed using a dedicated laboratory setup. Thus, the first stage of analysis features a set of experimental tests developed on strands wire rope samples subjected to both static and dynamic loads. The second stage of analysis features the acquired data post-processing, which were used to evaluate the characteristic parameters derived from the wire rope transitory dynamics.

2.1. Experimental Setup

The experimental laboratory setup used for the first stage of analysis is presented in Figure 1. It consists of a rigid latticed tower that sustains the inspected wire rope sample. The free end of the rope can be charged by a permanent static load and, additionally, by a dynamic excitation. For analysis, a short dynamic pulse was used, in terms of singular acting and strong enough hammer impact [22]. The shock was axially applied downwards of the suspended mass (static load) in a vertical direction, thus, the secondary effects regarding the transversal dynamics of the wire rope were minimized.

Dynamic evolution of the entire ensemble was monitored by a computer-based analysis system that is able to acquire signals from both accelerometers and force transducers (see Figure 1a). The acquisition was assured by the cDAQ-9174© chassis (National Instruments, USA), supplied with NI-USB-9233© (National Instruments, USA) for accelerometers and NI-USB-9237© (National Instruments, USA) for the strain-bridge-based force transducers.

The main parameters regarding the motion of the loading mass hung from the free end of the tested wire rope were monitored through the mass vertical direction oscillations and, respectively, the force within the rope sample (see Figure 1c). These jobs were carried out with the help of a CCP-ICP© 320C34 (PCB, USA) uniaxial accelerometer with a magnetic mounting base and a S9-20kN (HBM, Germany) strain-bridge based traction force transducer.

The unavoidable residual transitory dynamics of the tower were directly monitored at its upper side, nearby the device that sustains the inspected rope (see Figure 1b). This operation was carried out by two accelerometers (the same type as that mounted on the loading mass), mounted for both the vertical and the horizontal directions of possible motion provided by the free end of the tower. It was assumed that the lateral oscillations of the tower during the tests could be neglected (there were no external excitations applied in that direction and the experimental site does not supply any essential basement displacements).
Figure 1. The laboratory experimental setup: (a) General view with a latticed tower, wire rope sample, static charging device, and compute-based acquisition system; (b) detailed view of the upper side of the tower, with the hung rope device and residual motion monitoring transducers; (c) detailed view of the wire rope charging device with an additional mass applied and transducers for monitoring the forces and oscillations, respectively.

2.2. Tested Wire Rope Sample Characteristics

The strands wire rope considered in this analysis was a regular 6(12+FC)+FC commercial cable, type CA1AA072A© (CABLERO, Romania), 6 mm diameter, based on 6 strands with 12 steel wires of 0.4 mm diameter, designed for a 288-kg maximum service load. The symbol FC within its code means that the rope has a fiber core, both for the strands and for the rope. The unitary length mass is 0.11 kg/m and the theoretical failure load is 9.82 kN.

According to [23] (pp. 40–45), the fill factor $f$ and the rope mass factor $w$ can be used in order to characterize the nominal metallic cross-section area $S$ from the circumscribed area $S_u$ of the cable, and, respectively, the nominal rope length-related mass $m$.

Taking into account the diameters ratio $\delta/d = 0.4/6 = 0.067$, the fill factor $f$ yields a value of 0.32. Thus, the nominal metallic cross-section area yields $S = (\pi/4) \delta f d^2 = 9.05 \text{ mm}^2$. Respectively, with a rope mass factor $w = 0.306$, the nominal rope length-related mass, evaluated with widely accepted empirical expression $m = 0.01 w d^2$, acquires 0.11016 kg/m, which accurately approximates the initially provided value from the producer.

The axial stiffness $E_{wr}$ of a wire rope was evaluated using Hruska’s approach [8]:

$$H = \frac{E_{wr}}{E_{steel}} = \frac{\sum_{j=1}^{m} \left( \sum_{i=1}^{n} S_i \cos^3 \alpha_i \right) \cos^3 \beta_j}{\sum_{j=1}^{m} \left( \sum_{i=1}^{n} S_i / \cos \alpha_i \right) / \cos \beta_j},$$

(1)
where $E_{\text{steel}}$ denotes the steel Young’s modulus, $m$ is the total number of the rope strands, $n$ denotes the total number of wire layers in each strand, including the king wire that is considered the first layer, and $S_i$ is the cross-sectional area of wire within layer $i$ of a strand $j$. The angles $\alpha_i$ and $\beta_j$ respectively denote the lay angle of layer $i$ in strand $j$ and the lay angle of the strand $j$ of the wire rope. Analyzing the denominator of the right-hand-side term in Equation (1), it can obviously be seen that it represents the total net steel area of the normal cross-section of the wire rope. The equation uses the hypotheses of a king wire having $\alpha_i = 0$ and the elliptical shapes of wires and strands within the normal cross-section of the rope.

The parameters in Equation (1) were evaluated based on the schematization depicted in Figure 2a, where $2r_s$ denotes the diameter of the strand circular distribution, $D$ is the wire rope diameter, $L$ is the cable sample length, $h_s$ denotes the strand step, and $\beta$ is the lay angle of the strand. The evaluation of the lay angle of the wires within the ropes, $\alpha_i$, can be obviously obtained based on the same procedure.

The images in Figure 2b contain a region of the cable sample and a sectional view, respectively, and this helps to identify the geometrical parameters of the wire rope sample in terms of the helical step, diameter of circular distribution, and the lay angles of the wires and the strands.

Taking into account that the area $S_i$, and angles $\alpha_i$ and $\beta_j$ have constant values, Equation (1) acquires a simple expression in respect to these angles. For the inspected rope type, the strand diameter is 2 mm, the strands step is 46 mm, and the wires step (within the strand) is 20 mm. Hereby, $H = 0.8152$ and the axial stiffness $E_{\text{axial}} = 1.712 \times 10^5$ MPa (assuming $E_{\text{steel}} = 2.1 \times 10^5$ MPa), which is a value that is framed by the available literature data [23]. In addition, supposing the usual elongation of $\varepsilon \approx 1.7\%$, the stress yield $\sigma \approx 2910$ MPa (also in the range of available values for this kind of small diameter FC steel wire ropes).

Figure 2. The evaluation of the wire rope geometrical parameters: (a) The model of the strands wire rope, used for evaluation of the parameters involved in the lay angle computation of the strand and the rope, respectively [23] (p. 25); (b) detailed images of the wire rope used for the experiments, showing longitudinal and sectional views.
2.3. Post-Processing Techniques

The second stage of analysis featured the numerical investigations of the acquired signals (in terms of forces and accelerations) in order to accurately estimate the main parameters of the dynamic evolution of the experimental ensemble and, finally, to obtain the characteristics of the wire rope sample. It was initially assumed that the characteristics might be required not as simple basic values, but as extended sets of data, being able to accurately approximate the nonlinear dynamics of such a system.

Available methods for harmonics investigations include Fast Fourier Transform (FFT) techniques, adaptive filter applications, artificial neural networks, singular value decompositions (SVD), and higher-order spectra (HOSA) [22,24]. Most of these methods are able to work properly for moderate noise levels within analyzed signals and for a narrow domain of frequencies. These methods suppose that the signal only contains harmonics with fixed periodicity intervals. However, real signals might contain (and this is usually the case) inter-harmonics when the periodicity provides variable and very long intervals.

The Prony method, which is intended for use here, assumes that a sampled signal can be re-formulated as a linear combination of exponential functions [25–28]. This approach is not a spectral estimation technique, but it has a close relationship to squares linear prediction algorithms, which are usually used in auto-regressive (AR) or auto-regressive moving average (ARMA) parameter estimation methods. Opposite to AR/ARMA techniques, which try to fit a random model to the second-order data statistics, the Prony method seeks to fit a deterministic exponential-based model to the analyzed signal.

According to the previously mentioned arguments, in order to extract sinusoidal and/or exponential signals from time series data, the Prony method provides a feasible tool. This method allows the decomposition of a complex signal to a sum of exponential functions and transforms this entire computational process into solving a set of linear equations [25–28].

Hereby, assuming a signal \( x(t) \), available through \( n \) samples, where \( x[1], x[2], \ldots, x[n] \), it can be approximated by \( m \) terms [25–28]:

\[
y[i] = \sum_{k=1}^{m} A_k \exp[(b_k + j\omega_k) (i - 1) T_s + j\psi_k],
\]

where \( i = 1, n \), \( A_k \) denotes the amplitude, \( T_s \) denotes the sampling period, \( \alpha_k \) denotes the damping factor, \( \omega_k \) denotes the angular velocity, \( \psi_k \) denotes the initial phase, and \( j^2 = -1 \). This discrete-time expression can be concisely formulated in the form of the following:

\[
y[i] = \sum_{k=1}^{m} h_k z_k^{i-1},
\]

where, based on exponentials, the following can be expressed:

\[
h_k = A_k \exp(j\psi_k), \quad z_k = \exp[(b_k + j\omega_k) T_s].
\]

Considering Equation (3), the estimation problem results from the minimization of the squared error over the \( n \) initial signal values (initial data) as follows [25–28]:

\[
\Delta = \sum_{i=1}^{n} |\varepsilon[i]|^2,
\]

where

\[
\varepsilon[i] = x[i] - y[i] = x[i] - \sum_{k=1}^{m} h_k z_k^{i-1}.
\]
Taking into account this nonlinear problem, we adopted the Prony method, which uses linear equation solutions [26–28]. In fact, this method (also known as a polynomial Prony method), assumes an autoregressive model of order \( p \), which supposes that the value of the sampled signal \( x[i] \) linearly depends on the preceding \( p \) values of \( x \). In order to apply this method, one may use one of the available techniques for solving. Hereby, the original method proposed by Prony (a two-step method which additionally imposes an \( n = 2p \) condition and, in some cases, can become unstable due to ill-conditioned matrix equation solving) was considered. In addition, it can be taken into account the matrix pencil method (MPM), which solves an eigenvalues problem (with the main advantage that is less sensitive to the noises within the signal) [25,28].

In this study, the authors have chosen the MPM. This method starts from two matrices, namely, \( Y_1 \in \mathbb{C}^{m \times n} \) and \( Y_2 \in \mathbb{C}^{m \times n} \). Generally, the set of matrices, with the expressions \( (Y_2 - \lambda Y_1) \) and \( (\lambda \in \mathbb{C}) \), form a matrix pencil. For the case of sampled initial signal \( x \), a rectangular Henkel matrix \( Y \) is considered, with the pencil parameter \( p \), in the following form [25,28]:

\[
Y = \begin{pmatrix}
x[1] & x[2] & \cdots & x[p] & x[p+1] \\
x[2] & x[3] & \cdots & x[p+1] & x[p+2] \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
x[N-p] & x[N-p+1] & \cdots & x[N-1] & x[N]
\end{pmatrix}_{(N-p) \times (p+1)}.
\]  

(7)

The matrix \( Y \) helps to obtain the matrices \( Y_1 \) and \( Y_2 \). Thus, \( Y_1 \) results from \( Y \) without the last column and, respectively, \( Y_2 \) results from \( Y \), but without the first column [25–28]:

\[
Y_1 = \begin{pmatrix}
x[1] & x[2] & \cdots & x[p] \\
x[2] & x[3] & \cdots & x[p+1] \\
\vdots & \vdots & \ddots & \vdots \\
x[N-p] & x[N-p+1] & \cdots & x[N-1]
\end{pmatrix}_{(N-p) \times p}, \\
Y_2 = \begin{pmatrix}
x[2] & \cdots & x[p] & x[p+1] \\
x[3] & \cdots & x[p+1] & x[p+2] \\
\vdots & \vdots & \ddots & \vdots \\
x[N-p] & \cdots & x[N-1] & x[N]
\end{pmatrix}_{(N-p) \times p}.
\]  

(8)

Supposing that \( m \) denotes the real number of poles of the function \( x[i] \), with the conditioning \( m \leq p \leq n-m \), it can be seen that \( z_k \), where \( (k = 1..p) \), are the generalized eigenvalues of the matrix pencil \( (Y_2 - \lambda Y_1) \). Taking into account that matrices \( Y_1 \) and \( Y_2 \) are ill-conditioned, the QZ algorithm is not stable enough to provide adequate results. In this case, it is more accurate to use the following expression:

\[
z_k = eig(Y_1^+ Y_2),
\]  

(9)

where \( Y_1^+ \) denotes the Moore–Penrose pseudo-inverse matrix of \( Y_1 \). In such conditions, the damping \( b_k \) and the frequency \( f_k = 2\pi \omega_k \) results from the eigenvalues of \( z_k \) [25,28].

\[
b_k = \frac{\ln|z_k|}{T_s}, \quad f_k = \frac{1}{2\pi T_s} \arctan \left( \frac{\text{Im}(z_k)}{\text{Re}(z_k)} \right).
\]  

(10)

Finally, by solving the system [25,28]

\[
Z_{p \times p} h_{p \times 1} = x_{p \times 1},
\]  

(11)

the amplitude \( A_k \) and the phase \( \psi_k \) [25,28] may be found:

\[
A_k = |h_k|, \quad \psi_k = \arctan \left( \frac{\text{Im}(h_k)}{\text{Re}(h_k)} \right).
\]  

(12)
3. Results

The experimental investigations considered many tests based on five wire rope samples with the characteristics presented in the previous chapter of this paper. The number of rope samples was initially stipulated by the research theme. Within this process, the variables were (1) the static loading mass, (2) the application mode of the initial shock, and (3) the acquisition procedure in terms of analog data filtering, taking into account or not prior sampling of the signals. Based on the initial evaluation of the results, the authors adopted two significant cases for presentation and discussion within this paper.

Hereby, it was considered that the static loading masses should be 19 and 8 kg, respectively. The same rope sample was used, with an effective length $L = 2.200 \text{ m}$ between the hook devices. It was assumed that the analog filtered signal (low-pass filter with $f_{\text{cut}} = 400 \text{ Hz}$) was provided by the force transducer. For both cases, multiple tests were performed. Two situations for each case were adopted for graphical presentation.

The raw acceleration signals are presented in Figure 3, acquired onto the loading mass and nearby the upper hanging point (in both the vertical and horizontal directions). Each set of diagrams contain the three timed signals and the corresponding FFT spectral magnitude, respectively.

The signals provided by the force transducer (raw digital and analog filtered signals) are depicted in Figure 4, where each group of diagrams represent both the timed evolution and spectral composition (in terms of the FFT magnitude).

The diagrams in Figure 5 present the results of a joint time-frequency analysis [22] of the absolute motion of the loading mass in terms of acceleration. The exclusive motion of the mass results through elimination of the parasitic motions, recorded on the upper hanging device, from the signal directly acquired at the loading point (see Figure 1). The absolute acceleration of the mass, as a function of time, is presented in Figure 6, together with its spectral composition. Obviously, the diagrams in Figure 6 are related to each testing situation. The essential magnitudes were evaluated and red circles were marked on the spectral diagrams in order to identify the minimum set of eigenfrequencies in the system evolution. The evaluation of these points was performed with the help of a computational routine which identifies the peaks within the signal [22] that simultaneously satisfy the following conditions: Minimum height raised above 10% of maximum magnitude and prominence greater than 0.03 (value adopted based on the initial analysis of all signals spectra). The procedure only takes into account the first five peaks, because the analysis of high frequency range does not meet the purposes of this research. These aspects have to be mentioned because of the various numbers of marked peaks on the spectra graphs in Figure 6.
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Figure 3. The raw signals of acceleration in terms of both the timed evolution and the spectral magnitude: (a) First situation within the case of $m = 8$ kg; (b) second situation within the case of $m = 8$ kg; (c) first situation within the case of $m = 19$ kg; (d) second situation within case of $m = 19$ kg.
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Figure 4. The raw signals from the force transducer in terms of the timed evolution and spectral magnitude: (a) First situation within the case of \(m = 8\) kg; (b) second situation within the case of \(m = 8\) kg; (c) first situation within the case of \(m = 19\) kg; (d) second situation within the case of \(m = 19\) kg.

Figure 5. Joint time-frequency analysis of the absolute motion of the loading mass in terms of the acceleration signal: (a) First situation within the case of \(m = 8\) kg; (b) second situation within the case of \(m = 8\) kg; (c) first situation within the case of \(m = 19\) kg; (d) second situation within the case of \(m = 19\) kg.
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Figure 6. Timed evolution and related spectral magnitude of the absolute motion in terms of acceleration, recorded at the loading mass: (a) First situation within the case of $m = 8$ kg; (b) second situation within the case of $m = 8$ kg; (c) first situation within the case of $m = 19$ kg; (d) second situation within the case of $m = 19$ kg. Note: Red circles on the right-side diagrams denote the maximum peaks satisfying the imposed conditions (see text for details).
The authors have evaluated the transfer function, taking into account the force signal, as the input to the system, and the loading mass acceleration signal, as the output of the system [22, 24, 29]. In order to facilitate a comparative analysis between excitation, the transfer function, and system response, Figure 7 simultaneously presents the three spectra (in terms of the FFT magnitude). The four groups of diagrams in Figure 7 correspond to each considered test.

![Transfer Function Diagrams](image)

**Figure 7.** Transfer function of the tested ensemble, comparatively presented with the input and output spectra (magnitudes): (a) First situation within the case of \( m = 8 \, \text{kg} \); (b) second situation within the case of \( m = 8 \, \text{kg} \); (c) first situation within the case of \( m = 19 \, \text{kg} \); (d) second situation within the case of \( m = 19 \, \text{kg} \).

One interesting parameter that is able to characterize the system behavior under dynamic excitations is the dynamic rigidity [30]. This parameter relates to the perturbing force with the system response (in terms of displacement) and provides information regarding the dynamic evolution of this ratio in respect to the perturbation frequency. The spectra of dynamic rigidity, according to the four considered cases, are presented in Figure 8.

Additionally, in the second stage of analysis, the Prony method was used in order to estimate an extended range of spectral parameters, such as the amplitude, frequencies, damping, and phase. Practical implementation of this procedure uses MPM. During the analysis, different values for the pencil parameter were tested, finally adopting the value of 512. This value assures an adequate ratio between the additional parameters and computational resource requirements.

Hereby, the diagrams in Figure 9 show the amplitude, damping factor, and damping ratio respectively, in respect to the frequency. These diagrams were grouped according to the four presented experimental cases. Supposing that the Prony algorithm directly provided amplitudes and damping factors, the damping ratio was subsequently evaluated based on the available data from the Prony/MPM implementations. In addition, an equivalent value of the damping ratio was estimated using the hypothesis of the same energy values supplied for both the initial system (simulated by the finite sum of exponential functions) and for an equivalent single-degree-of-freedom (SDoF) system with a natural frequency identical to the dominant maximum peaks satisfying the imposed conditions (see text for details).
of exponential functions) and for an equivalent single-degree-of-freedom (SDoF) system with a natural frequency identical to the dominant frequency within the signal spectra [22,30–32]. Taking into account the five maximum dominant spectral peaks previously evaluated, this results in the five maximum values of the equivalent damping ratio. These equivalent values are depicted in the damping ratio graphs within Figure 9 with dashed red lines.

Supposing that each frequency provided by the Prony/MPM algorithms can be considered the natural frequency of a SDoF linear model, with a correspondent amplitude, damping, and phase (also available from the same algorithm), it becomes simple to evaluate a theoretical response of a multi-degree-of-freedom (MDoF) linear model as a finite sum of all components supplied by the Prony method.

These cumulative responses have a graphical representation, such as a function of two variables, e.g., the perturbation and the natural frequencies (see Figures 10a, 11a, 12a and 13a). The correlated overlapping spectral diagrams, following the SDoF natural frequency parameter, are presented in Figures 10b, 11b, 12b, and 13b. The global response of the equivalent MDoF model was depicted using a blue continuous thick line on each diagram within Figure 10, Figure 11, Figure 12, Figure 13 according to the assumed tests/cases for presentation.

![Dynamic Rigidity Spectra](image)

**Figure 8.** Dynamic rigidity spectra: (a) First situation within the case of \(m = 8\) kg; (b) second situation within the case of \(m = 8\) kg; (c) first situation within the case of \(m = 19\) kg; (d) second situation within the case of \(m = 19\) kg.
Taking into account the five maximum dominant spectral peaks previously evaluated, this results in the five maximum values of the equivalent damping ratio. These equivalent values are depicted in the damping ratio graphs within Figure 9 with dashed red lines.

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Figure 8. Dynamic rigidity spectra: (a) First situation within the case of \( m = 8 \) kg; (b) second situation within the case of \( m = 8 \) kg; (c) first situation within the case of \( m = 19 \) kg; (d) second situation within the case of \( m = 19 \) kg.

Figure 9. Amplitudes, damping factor, and damping ratio provided by the Prony method as functions of the modal frequencies (according to the first 512 terms of signal decomposition): (a) First situation within the case of \( m = 8 \) kg; (b) second situation within the case of \( m = 8 \) kg; (c) first situation within the case of \( m = 19 \) kg; (d) second situation within the case of \( m = 19 \) kg. Red dashed lines within the damping ratio diagrams denote an equivalent damping ratio (see text for details).

Figure 10. Double-spectra evolution of the first 512 terms in the exponential function decomposition according to the Prony method with the first test within the case of \( m = 8 \) kg: (a) Behavior of each component in respect to the perturbation frequency in the range of interest; (b) overlapped spectral diagrams. The blue continuous thick lines on the graphs denote the response spectra of the linear system, assuming the available terms.
Figure 10. Double-spectra evolution of the first 512 terms in the exponential function decomposition according to the Prony method with the first test within the case of $m = 8$ kg: (a) Behavior of each component in respect to the perturbation frequency in the range of interest; (b) overlapped spectral diagrams. The blue continuous thick lines on the graphs denote the response spectra of the linear system, assuming the available terms.

Figure 11. Double-spectra evolution of the first 512 terms in the exponential function decomposition according to the Prony method for the second test within the case of $m = 8$ kg: (a) Behavior of each component in respect to the perturbation frequency in the range of interest; (b) overlapped spectral diagrams. The blue continuous thick lines on graphs denote the response spectra of the linear system, assuming the available terms.

Figure 12. Double-spectra evolution of the first 512 terms in the exponential function decomposition according to the Prony method for the first test within the case of $m = 19$ kg: (a) Behavior of each component in respect to the perturbation frequency in the range of interest; (b) overlapped spectral diagrams. The blue continuous thick lines on graphs denote the response spectra of the linear system, assuming the available terms.
Figure 13. Double-spectra evolution of the first 512 terms in the exponential function decomposition according to the Prony method for the second test within the case of \( m = 19 \) kg: (a) Behavior of each component in respect to the perturbation frequency in the range of interest; (b) overlapped spectral diagrams. The blue continuous thick lines on graphs denote the response spectra of a cumulative linear system, assuming the available terms.

4. Discussion

A comparative analysis of the raw acceleration signals reveals dominant frequencies within the spectra. A pertinent evaluation of these values must take into account the force signal spectra. Thus, the spectral peaks in the range of interest becomes quite visible around 10 Hz and in the domain of 90–400 Hz. The loading mass acceleration spectra presents a rather wide plate evolution from 10 to 1000 Hz, containing a maximum of three essential peaks, comparative with the other two acceleration signals, which provide many of distinctive peaks (see Figure 3). This qualitative difference is due to the initial shock directly applied to the mass and, respectively, to the dissipative and elastic characteristics of the rope mounted between the two points of acceleration monitoring. The spectral force diagrams, which are even as noisy as the acceleration diagrams, provide two clear main peaks around the values of 10 and 200 Hz, respectively (see Figure 4). An interesting signal is that acquired nearby the upper hanging device in the horizontal direction, because this preponderantly contains the effects due to the support tower bending motion. Hereby, the spectrum of this signal contains a very clear maximum in the range of 9.25 to 10.75 Hz, depending on the test case, which represents the first modal frequency of the tower. This preliminary analysis offers the primary qualitative findings, which are useful for the next evaluations.

The effective (absolute) acceleration of the loading mass is able to characterize the dynamics of the wire rope used to suspend the mass oscillator. This signal was obtained from the initial acceleration recorded on the mass, through elimination of the perturbing signals due to additional unavoidable motion of the support tower. Timed evolutions of these signals were firstly evaluated using a joint time-frequency algorithm and the results are depicted in Figure 5. This investigation helps to correctly identify the time-frequency areas within the effective mass acceleration signal, showing the transitory regimes of each dominant frequency in terms of both the initial time and during the timed evolution. The results show the two relevant spectral components supplied by the support structure (low frequency area) and, respectively, by the rope–mass-vibrating ensemble (with most relevant values centered around 200 Hz).

Following the previous qualitative results, the acquired signals were quantitatively analyzed using, firstly, a classical FFT algorithm. The magnitude diagrams were computationally scanned for the five maximum peaks (this value was settled after the preliminary analysis that had indicated the average number of peaks provided by the entire test set).

As it can be seen in Figure 6, the peaks were grouped around 10 and 200 Hz. For the cases/situations within this paper, the frequency values of marked peaks in Figure 6 are presented in Table 1. For the first case \( (m = 8 \text{kg}) \), the scanning procedure identified five values in each situation. For the second
case, the procedure yielded only three values, where all attended the domains of interest. Additionally, identified components at 2.5, 6.75, 220, and 302 Hz appeared in a certain singular situation. However, following the graphs, it can be clearly seen that these dominant frequencies were also included by all signals, but with various amplitudes and prominences, thus that they were not constantly pointed out by the algorithm. Low values (2.5 and 6.75 Hz) result from the cable transversal parasitic oscillations and high values (220 and 302 Hz) results from the dynamics of the hooking devices.

Table 1. Peaks frequencies related to the diagrams in Figure 8.

| Case/Situation of Analysis | Frequency (Hz)   |
|---------------------------|-----------------|
| \(m = 8\) kg/situation I  | 2.499938, 6.749831, 9.99975, 11.24972, **190.9952** |
| \(m = 8\) kg/situation II | 9.99975, 11.24972, **188.7453**, 219.9945, 301.9925 |
| \(m = 19\) kg/situation I | 9.249769, 10.74973, **186.7453** |
| \(m = 19\) kg/situation II | 9.249769, 10.74973, **189.4953** |

Force and acceleration can be assumed as the input and the output, respectively, of the rope–load ensemble. Hereby, the transfer function, in terms of driving point function, becomes easily estimated for each inspected case (see Figure 7). The effective acceleration of loading mass (Figure 6), double integrated during the analysis time, was considered in order to obtain the displacement signal. The driving point transfer function represents the system admittance viewed through the input point, thus it is able to provide useful information related to the evolution of dynamic rigidity with respect to the excitation frequency (Figure 8). Targeted frequencies around 190 Hz were pointed out in the dynamic rigidity graphs, but low frequencies (9 to 11 Hz) were not present because this parameter exclusively characterizes the wire rope dynamics.

Starting from the hypothesis that the product \(kL = ES = \text{const.}\), where \(k\) denotes the rigidity, \(L\) is the wire rope sample effective length, \(E\) is the rope Young’s modulus, and \(S\) denotes the effective sectional area of the rope \([7,14,30]\), it can be seen that the sample rigidity features a constant value for all assumed tests because the same cable sample with a constant length was used. According to initial evaluation of the rope parameters (see the values within Section 2.2), the rigidity was \(7.04 \times 10^5\) Nm\(^{-1}\). The experimental results provided rather different values for the cable rigidity because of dynamic effects (in fact, we discussed the dynamic rigidity). Thus, the values of this parameter acquired the following values: \(9.8 \times 10^5\) Nm\(^{-1}\) for the case of \(m = 8\) kg and \(24 \times 10^5\) Nm\(^{-1}\) for the case of \(m = 19\) kg.

The secondary stage of analysis was related to the dissipative characteristics provided by the experiments. The dissipation of the wire rope-loading mass ensemble could be globally characterized through an overall damping factor, evaluated based on timed effective acceleration diagrams (see Figure 6). The simplest evaluation procedure consists of an exponential function-based interpolation of the magnitude of the Hilbert Transform applied to the inspected function \([22,31]\). This method, such as the classical procedure of the logarithm decrement, is affected by certain operational errors regarding the initial parameters, the time length of the processed signal, and the nonlinearities within the system evolution. However, it was adopted because it avoids the uncertainty of the maximum point picking procedure.

The results according to the Hilbert Transform-based method are presented in Table 2. It has to be noted that there is a relative wide spread of the values, which may produce certain difficulties in the selection of the parametrical values for some MDoF computational models.

In order to accurately simulate the behavior of a real ensemble, an equivalent linear MDoF theoretical system is suitable for use. The dissipative characteristics of such models require supplementary parameters. Thus, an additional computational procedure was adopted, based on Prony approximation and MPM, which were briefly presented in Section 2.3. The results of these analyses were comparatively presented in Figure 9, and the MDoF system approximations were, respectively, depicted in Figures 10–13. The evaluation had supposed the value of 512 for the pencil parameter. Thus, 256 serviceable values were yielded, respectively, for the amplitude, damping, and phase parameters,
in respect to the frequency. These values can be taken into account for any linear computational model intended to simulate the dynamic behavior of a wire rope.

Within some practical situations, a computational approximation is often required with the simplest models, such as the SDoF schematization [16,33]. An equivalent value of the dissipative characteristic is required for these cases. Using an algorithm that supposes a SDoF system, enabling the same energy with the initial MDoF system, it becomes easy to evaluate the equivalent damping parameter (in terms of the damping ratio) [22,30–32]. Successively, we assumed the natural frequencies for the SDoF system (see values in Table 1) and the equivalent damping ratios were computed. In addition, the damping factors were also evaluated. The results are given in Table 3. It has to be noted that there are consistent values within each testing case, but a wide spread between the cases, which shows the rope dissipation dependence with respect to the loading conditions.

Table 2. Overall damping factors related to the diagrams in Figure 8.

| Case/Situation of Analysis | Overall Damping Factor (s⁻¹) |
|----------------------------|-------------------------------|
| m = 8 kg/sit. I            | 24.5810                      |
| m = 8 kg/sit. II           | 23.8685                      |
| m = 19 kg/sit. I           | 12.7252                      |
| m = 19 kg/sit. II          | 14.2641                      |

Table 3. Equivalent damping factors and ratios related to the diagrams in Figure 8.

| Case/Situation of Analysis | Equivalent SDoF Damping Ratio (-) | Equivalent SDoF Damping Factor (s⁻¹) |
|----------------------------|-----------------------------------|-------------------------------------|
| m = 8 kg/sit. I            | 0.0312, 0.0312, 0.0312, 0.0312, 0.0192 | 0.4908, 1.3233, 1.9589, 2.2047, 23.0028 |
| m = 8 kg/sit. II           | 0.0311, 0.0312, 0.0186, 0.0174, 0.0193 | 1.9567, 2.2024, 22.0632, 24.1166, 36.6487 |
| m = 19 kg/sit. I           | 0.0312, 0.0312, 0.0072              | 1.8117, 2.1062, 8.3916               |
| m = 19 kg/sit. II          | 0.0312, 0.0312, 0.0077              | 1.8152, 2.1102, 9.1820               |

Comparative analysis of values in Tables 2 and 3 reveals certain differences between the damping factors for the two evaluative algorithms. Basic justification of this fact results from the appreciative qualitative character of the first proposed algorithm. However, this technique was presented because it is easily implemented and requires reduced computing resources (comparative to the second technique). Regarding the differences between the values of the two cases according to the second algorithm (significant values in Table 3 are denoted in boldface), the damping factor depends on the excitation. Thus, in Table 4, the damping coefficient was exclusively provided as that evaluated for the significant values. The average values of this parameter were 360.528 and 333.906 Nsm⁻¹, which presents a 2.1% and 4.7% maximum deviation related to the computed values.

Table 4. Equivalent damping coefficients related to the data in Table 3.

| Case/Situation of Analysis | Equivalent SDoF Damping Coefficient (Nsm⁻¹) |
|----------------------------|---------------------------------------------|
| m = 8 kg/sit. I            | 368.048                                     |
| m = 8 kg/sit. II           | 353.008                                     |
| m = 19 kg/sit. I           | 318.896                                     |
| m = 19 kg/sit. II          | 348.916                                     |
5. Conclusions

The discussion section contains findings related to both the conservative and dissipative aspects within dynamic behaviors of strands wire ropes. It was shown that the methods used for investigation of dynamic data are able to provide suitable information, depending on the numerical model requirement. The findings within this study were systematically presented, starting with a SDoF model (completely characterized by the global damping and stiffness) and progressing to complex MDoF approaches (which usually require a variable range frequency, depending on the dissipations and rigidities). Hereby, the presented assessments have demonstrated the flexibility and adaptability of the proposed techniques and, additionally, with the advantage of reduced experimental investigations. This study only includes laboratory tests based on an experimental setup, according to the initial research theme. This limitation does not restrict the applicability of the findings, as per the study objective. However, the operational mode of application was not completely proven. This research sustains the feasibility of the proposed post-processing assessments. Future developments will be focused on operational test applications using the same analysis technique but instead applied to operationally acquired signals. Thus, it will be able to provide suitable arguments for enlarging the utilization range of the authors’ assessments.

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