HOW DOES A QUADRATIC TERM IN THE ENERGY DISPERSION MODIFY THE SINGLE-PARTICLE GREEN’S FUNCTION OF THE TOMONAGA-LUTTINGER MODEL?

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We calculate the effect of a quadratic term in the energy dispersion on the low-energy behavior of the Green’s function of the spinless Tomonaga-Luttinger model (TLM). Assuming that for small wave-vectors $q = k - k_F$ the fermionic excitation energy relative to the Fermi energy is $v_F q + q^2/(2m)$, we explicitly calculate the single-particle Green’s function for finite but small values of $\lambda = q_c/(2k_F)$. Here $k_F$ is the Fermi wave-vector, $q_c$ is the maximal momentum transfered by the interaction, and $v_F = k_F/m$ is the Fermi velocity. Assuming equal forward scattering couplings $g_2 = g_4$, we find that the dominant effect of the quadratic term in the energy dispersion is a renormalization of the anomalous dimension. In particular, at weak coupling the anomalous dimension is $\tilde{\gamma} = \gamma (1 - 2\lambda^2 \gamma)$, where $\gamma$ is the anomalous dimension of the TLM. We also show how to treat the change of the chemical potential due to the interactions within the functional bosonization approach in arbitrary dimensions.

1. Introduction

The exactly solvable Tomonaga-Luttinger model (TLM) has been extremely useful to gain a better understanding of the electron-electron interactions in the normal metallic state of electrons in one spatial dimension ($d = 1$). In this model only the forward scattering part of the electron-electron interaction is retained, which in $d = 1$ can be parameterized by two phenomenological coupling constants $g_2$ and $g_4$ (Ref. 4). The second important feature of the TLM, which is crucial for its exact solubility via bosonization or other methods, is the fact that the energy dispersion of the electrons is exactly linear for all wave-vectors. Because the low-energy and long-wavelength physics is determined by the degrees of freedom in the vicinity of the Fermi points, it is reasonable to expect that the replacement of a general energy dispersion by the linear term in the expansion close to the Fermi points is justified.

The single-particle Green’s function $G(x, t)$ of the TLM in the space-time domain has been calculated by many authors. A detailed discussion of the rather complicated behavior of the Fourier transform $G(k, \omega)$ of $G(x, t)$ in the momentum-frequency domain can be found in Ref. 1. The structure of $G(k, \omega)$ is fundamentally different from the well-known behavior of the Green’s function of a Fermi liquid: There is no quasi-particle peak, but $G(k, \omega)$ contains instead algebraic singularities, which can be interpreted physically in terms of bosonic charge- and spin-excitations...
propagating with different velocities. The absence of a quasiparticle peak implies also that the density of states of the TLM vanishes at the Fermi energy, and that the momentum distribution function exhibits only an algebraic singularity at the Fermi surface, in contrast to the jump discontinuity of a Fermi liquid.

Haldane has emphasized that the above features are not special to the TLM, but are generic to the normal metallic state of interacting electrons in \( d = 1 \). He proposed to call such a state a Luttinger liquid, in contrast to the well-known Fermi liquid, which is the normal metallic state of interacting electrons in \( d = 3 \). In the seminal work, Haldane also emphasized that the various deviations from the idealized TLM can in principle be treated perturbatively within the framework of bosonization, and should only lead to a re-definition of the phenomenological parameters that appear in the Hamiltonian of the TLM, namely the Fermi velocity \( v_F \) and the forward scattering couplings \( g_2 \) and \( g_4 \). For example, in bosonic language the non-linear terms in the expansion of the energy dispersion close to the Fermi points give rise to interactions between the Tomonaga-Luttinger bosons, which can be treated perturbatively within conventional bosonic many-body theory. In Ref. Haldane performed an expansion of the equal-time Green’s function to lowest order in the curvature parameter \( 1/m \), and pointed out that the non-linear terms in the energy dispersion can be expected to modify the anomalous dimension of the TLM. However, the precise value of this renormalization was not obtained in Ref. This will be done in the present work with the help of the functional integral formulation of bosonization.

2. Interacting electrons with dominant forward scattering

In this section we shall briefly describe how the single-particle Green’s function \( G(\mathbf{k}, \omega) \) of interacting fermions with dominant forward scattering can be calculated within the functional bosonization formalism in arbitrary dimensions. In particular, we shall rely on the results of Refs., where a systematic method has been developed to handle the non-linear terms in the expansion of the energy dispersion close to the Fermi surface. It turns out for our calculation it is important to take the renormalization of the chemical potential due to the interaction into account. This renormalization has not been discussed in previous works.

Although we are ultimately interested in \( d = 1 \), all equations in this section are valid in arbitrary \( d \).

2.1. Calculation of the single-particle Green’s function via functional bosonization

For simplicity, we shall ignore the spin degree of freedom. We also assume that the bare energy dispersion is \( \epsilon_k = \frac{k^2}{2m} \), and that the Fourier transform \( f_q \) of the bare interaction between two electrons with momenta \( \mathbf{k} \) and \( \mathbf{k}' \) depends only on the absolute value of the momentum transfer \( \mathbf{k} - \mathbf{k}' \).

It is convenient to consider the Matsubara Green’s function \( G(\mathbf{k}, i\tilde{\omega}_n) \), where
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\[ \tilde{\omega}_n = 2\pi (n + \frac{1}{2})/\beta \] are fermionic Matsubara frequencies. The inverse temperature is denoted by \( \beta \). Suppose that we are interested in \( G(k, i\tilde{\omega}_n) \) at some momentum \( k \) in the vicinity of the Fermi surface. It is then convenient to measure all momenta relative to some reference point \( k^\alpha \) that is close to the Fermi surface, such that the length of \( q = k - k^\alpha \) is small compared with \( |k^\alpha| \). Let us therefore define

\[ G^\alpha(q, i\tilde{\omega}_n) = G(k^\alpha + q, i\tilde{\omega}_n), \tag{1} \]

where the index \( \alpha \) indicates that \( q \) is measured relative to the reference point \( k^\alpha \) in the vicinity of the Fermi surface. The precise choice of \( k^\alpha \) will be discussed shortly.

Note that the Fermi surface is defined as the surface in momentum space where the momentum distribution

\[ n(k) = \frac{1}{\beta} \sum_n G(k, i\tilde{\omega}_n) \tag{2} \]

exhibits some kind of non-analytic behavior. Thus, in general we have to solve the many-body problem in order to know the precise location of the Fermi surface. Of course, if Luttinger’s theorem turns out to be valid in our system, then the volume of the Fermi surface is not changed when the interaction is switched on at constant density. For a spherically symmetric Fermi surface this implies that its radius is precisely identical with the Fermi wave-vector \( k_F \) of the non-interacting system,

\[ k_F = \sqrt{\frac{2m\mu_0}{\beta V}}, \tag{3} \]

where \( \mu_0 \) is the chemical potential of the system without interactions at the same density. In general, however, the precise location of the Fermi surface is a priori not known. In fact, we cannot even exclude the possibility that for some type of interaction the momentum distribution is an analytic function of \( k \), in which case the system does not have a Fermi surface. Note that in the shifted coordinate system the energy dispersion is

\[ \epsilon_{k^\alpha + q} = \epsilon_{k^\alpha} + v^\alpha \cdot q + \frac{q^2}{2m}, \tag{4} \]

where \( v^\alpha = k^\alpha/m \). Assuming that the system is confined to a finite volume \( V \), the real-space imaginary-time Fourier transform of \( G^\alpha(q, i\tilde{\omega}_n) \) is

\[ G^\alpha(r, \tau) = \frac{1}{\beta V} \sum_{q} e^{i(q \cdot r - \tilde{\omega}_n \tau)} G^\alpha(q, i\tilde{\omega}_n), \tag{5} \]

After the usual Hubbard-Stratonovich transformation \( \Box G^\alpha(r, \tau) \) can be written as

\[ G^\alpha(r - r', \tau - \tau') = \langle G^\alpha(r', \tau', \tau') \rangle, \tag{6} \]

where

\[ \left[ -\partial_\tau - v^\alpha \cdot \hat{P}_r - \frac{\hat{P}_r^2}{2m} - \epsilon_{k^\alpha} + \mu - i\phi(r, \tau) \right] G^\alpha(r, \tau, \tau') = \delta(r - r') \delta^\alpha(\tau - \tau'). \tag{7} \]
Here $\hat{P}_r = -i \nabla_r$ is the momentum operator, $\delta^*(\tau) = \frac{1}{\beta} \sum_n e^{i \tilde{\omega}_n \tau}$ is the antiperiodic $\delta$-function, and $\langle \ldots \rangle$ denotes functional averaging with respect to the effective action of the Hubbard-Stratonovich field $\phi(r, \tau)$. The chemical potential of the interacting system is denoted by $\mu$. Let us now choose the reference point $k^\alpha$ such that

$$\epsilon_{k^\alpha} = \mu - \langle i \phi(r, \tau) \rangle .$$

Diagrammatically the term $\langle i \phi(r, \tau) \rangle$ corresponds to the sum of all tadpole diagrams which renormalize the chemical potential, see Fig. 1(a). The leading diagram of this type is just the usual Hartree diagram. We shall include the sum of these diagrams into the definition of the zeroth order Green’s function,

$$G_0^{\alpha}(q, i \tilde{\omega}_n) = \frac{1}{i \tilde{\omega}_n - \epsilon_{k^\alpha} + q + \mu - \langle i \phi(r, \tau) \rangle} = \frac{1}{i \tilde{\omega}_n - \mathbf{v}^\alpha \cdot \mathbf{q} - \frac{q^2}{2m}} .$$

The effective action of the Hubbard-Stratonovich field is given by

$$S_{\text{eff}}\{\phi\} = \frac{\beta V}{2} \sum_{\mathbf{q} n} f^{-1}_{\mathbf{q}} \phi_{-\mathbf{q}, n} \phi_{\mathbf{q}, n} - \text{Tr} \ln[1 - i \hat{G}_0 \hat{\phi}] ,$$

where the trace is over momentum-frequency space, with the infinite matrices $\hat{G}_0$ and $\hat{\phi}$ given by

$$[\hat{G}_0]_{\mathbf{q}, \mathbf{q}' n'} = \delta_{\mathbf{q}, \mathbf{q}'} \delta_{n, n'} G_0^{\alpha}(\mathbf{q}, i \tilde{\omega}_n) ,$$

$$[\hat{\phi}]_{\mathbf{q}, \mathbf{q}' n'} = \phi_{\mathbf{q} - \mathbf{q}', n - n'} .$$

Here

$$\phi_{\mathbf{q}, n} = \frac{1}{\beta V} \int d\mathbf{r} \int_{-\beta/2}^{\beta/2} d\tau e^{-i(\mathbf{q} \cdot \mathbf{r} - \omega_n \tau)} [\phi(\mathbf{r}, \tau) - \langle \phi(\mathbf{r}, \tau) \rangle] ,$$

where $\omega_n = 2\pi n/\beta$ are bosonic Matsubara frequencies. In Eq.(13) we have subtracted the average $\langle \phi(\mathbf{r}, \tau) \rangle$, which by translational invariance is independent of $\mathbf{r}$.
and $\tau$. This subtraction modifies only the zeroth Fourier component $\phi_0 = \phi_{q=0,n=0}$ such that $\langle \phi_0 \rangle = 0$. Note that for non-zero $q$ or $\omega_n$ translational invariance implies $\langle \phi_{q,n} \rangle = 0$.

From Eqs.(11) and (12) it is clear that the Fourier transform $G(\mathbf{r}, \tau)$ of the physical Matsubara Green’s function $G(\mathbf{k}, i\omega_n)$ is simply

$$G(\mathbf{r}, \tau) = e^{i\mathbf{k}_\alpha \cdot \mathbf{r}} G^\alpha(\mathbf{r}, \tau).$$

(14)

If one works with locally linearized energy dispersion the Fermi surface has to be subdivided into several sufficiently small sectors, which are labelled by the discrete index $\alpha$. In this case the right-hand side of Eq.(14) should be summed over all these sectors in order to recover the physical Green’s function. Note that the sectorization would require the introduction of cutoffs in momentum space. However, for our purpose such a subdivision into sectors is really not necessary as long as we restrict ourselves to the calculation of $G(\mathbf{k}_\alpha + \mathbf{q}, \omega)$ for $|\mathbf{q}| \ll |\mathbf{k}_\alpha|$. Of course, without sectorization the curvature of the Fermi surface has to be taken into account.

It turns out that for interactions that involve only small momentum transfers $|\mathbf{q}| \ll |\mathbf{k}_\alpha|$ it is sufficient to perform the averaging in Eq.(6) within the Gaussian approximation, which amounts to an expansion of the logarithm in Eq.(10) to the second order. Within this approximation the effective action is given by

$$S_{\text{eff}}\{\phi\} \approx i\beta V \rho_0 + \frac{\beta V}{2} \sum_{q,n} f_{\text{RPA}}(\mathbf{q}, i\omega_n) \phi_{-q,-n} \phi_{q,n},$$

(15)

where

$$\rho_0 = \frac{1}{\beta V} \sum_{q,n} G^0_0(\mathbf{q}, i\omega_n),$$

(16)

and

$$f_{\text{RPA}}(\mathbf{q}, i\omega_n) = \frac{f_q}{1 + f_q \Pi_0(\mathbf{q}, i\omega_n)}$$

(17)

is the effective interaction within the random-phase approximation (RPA). The “non-interacting” polarization can be written as

$$\Pi_0(\mathbf{q}, i\omega_n) = -\frac{1}{\beta V} \sum_{q' n'} G^0_0(\mathbf{q}', i\omega_{n'}) G^0_0(\mathbf{q}' + \mathbf{q}, i\omega_{n'+n}).$$

(18)

Let us now focus on the solution of Eq.(15). Making the generalized Schwinger-ansatz

$$\mathcal{G}^\alpha(\mathbf{r}, \mathbf{r}', \tau, \tau') = \mathcal{G}_1^\alpha(\mathbf{r}, \mathbf{r}', \tau, \tau') e^{\Phi^\alpha(\mathbf{r}, \tau) - \Phi^\alpha(\mathbf{r}', \tau')},$$

(19)

we obtain with $\mathbf{k}_\alpha$ given by Eq.(8)

$$\left[ -\partial_{\tau} - v^\alpha \cdot \hat{\mathbf{P}} - \frac{\hat{P}^2}{2m} - \frac{A^\alpha(\mathbf{r}, \tau)}{m} \cdot \hat{\mathbf{P}} \right] \mathcal{G}_1^\alpha(\mathbf{r}, \mathbf{r}', \tau, \tau') + \mathcal{G}_1^\alpha(\mathbf{r}, \mathbf{r'}, \tau, \tau')$$
\[
\times \left\{ \left[ -\partial_\tau - v^\alpha \cdot \hat{P}_r - \frac{\hat{P}_r^2}{2m} \right] \Phi^\alpha (r, \tau) - i[\phi(r, \tau) - \langle \phi(r, \tau) \rangle] - \frac{[A^\alpha(r, \tau)]^2}{2m} \right\} \\
= \delta(r - r') \delta^\alpha(\tau - \tau') \, .
\] (20)

Here
\[ A^\alpha(r, \tau) = \hat{P}_r \Phi^\alpha (r, \tau) \] (21)
is a longitudinal vector potential. Eq.(19) is a gauge transformation. Obviously, there are infinitely many different choices for the gauge factor \( \Phi^\alpha (r, \tau) \). For example, if the bare interaction \( f_q \) is the three-dimensional Coulomb interaction, then \( A^\alpha(r, \tau) \) can be identified with the longitudinal part of the usual vector potential of electromagnetism if we choose \(-\partial_\tau \Phi^\alpha (r, \tau) = i\phi(r, \tau)\). In Ref.\,\,20 it was shown that this choice is useful to resum the leading logarithmic singularities in the perturbative expansion of the Green’s function of two-dimensional disordered electrons that interact with long-range Coulomb forces. To include the renormalization of the chemical potential into the functional bosonization approach, a slight modification of the gauge choice given in Refs.\,\,11,13 is necessary. It is easy to see that Eq.(20) can be satisfied if
\[
\left[ -\partial_\tau - v^\alpha \cdot \hat{P}_r - \frac{\hat{P}_r^2}{2m} - \frac{A^\alpha(r, \tau)}{m} - i\phi_0 - D_0 \right] G_1^\alpha (r, r', \tau, \tau') \\
= \delta(r - r') \delta^\alpha(\tau - \tau') \, .
\] (22)

Here \( \phi_0 \) is the zeroth Fourier component of \( \phi(r, \tau) - \langle \phi(r, \tau) \rangle \) (see Eq.(13)), and \( D_0 \) the average diamagnetic energy,
\[
D_0 = \frac{1}{\beta V} \int dr \int_{-\beta/2}^{\beta/2} d\tau \frac{[\hat{P}_r \Phi^\alpha (r, \tau)]^2}{2m} \, .
\] (24)

By construction the Fourier component corresponding to \( q = 0 \) and \( \omega_n = 0 \) of the right-hand side of Eq.(23) vanishes, so that in the iterative solution of the eikonal equation (23) ambiguities due to vanishing eigenvalues of the differential operator on the left-hand side are avoided. Such an explicit separation of the zeroth Fourier component, which was not discussed in Refs.\,\,11,13, is important to incorporate the renormalization of the chemical potential into functional bosonization.

At this point we solve the eikonal equation (23) to first order in \( \phi(r, \tau) \), which is sufficient to calculate the Debye-Waller factor arising after the averaging from
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the gauge-factor in Eq. (19) to first order in the RPA interaction. The functional averaging of the "prefactor" Green's function is then performed within the self-consistent Born approximation, as described in detail in Ref. 11. The final result for the Green’s function of the interacting many-body system for wave-vectors close to k^α is 11

\[ G(k^\alpha + q, i\omega_n) = \int dr \int_{-\beta/2}^{\beta/2} d\tau e^{-i(q \cdot r - \tilde{\omega}_n \tau)} \tilde{G}^\alpha(r, \tau) e^{Q_1^\alpha(r, \tau)}, \]

with the Debye-Waller factor given by

\[ Q_1^\alpha(r, \tau) = \frac{1}{\beta V} \sum_{qn} f_{RPA}(q, i\omega_n) \left[ 1 - \cos(q \cdot r - \omega_n \tau) \right], \]

and the prefactor Green’s function

\[ \tilde{G}^\alpha(r, \tau) = \frac{1}{\beta V} \sum_{q} e^{i(q \cdot r - \tilde{\omega}_n \tau)} \left[ 1 + Y^\alpha(q, i\tilde{\omega}_n) \right] G_1^\alpha(q, i\tilde{\omega}_n). \]

Here

\[ G_1^\alpha(q, i\tilde{\omega}_n) = \frac{1}{i\tilde{\omega}_n - v^\alpha \cdot q - \frac{q^2}{2m} - \delta \mu_F - \Sigma^\alpha_1(q, i\tilde{\omega}_n)}, \]

\[ \Sigma^\alpha_1(q, i\tilde{\omega}_n) = -\frac{1}{\beta V} \sum_{q' n'} f_{RPA}(q', i\omega_{n'}) G_1^\alpha(q + q', i\omega_{n+n'}) \times \frac{(q q') q'^2}{m^2} + (q q')^2 \frac{m^2}{[i\omega_{n'} - v^\alpha \cdot q' - \frac{q'^2}{2m}] [i\omega_{n'} - v^\alpha \cdot q' + \frac{q'^2}{2m}]}, \]

\[ Y^\alpha(q, i\tilde{\omega}_n) = \frac{1}{\beta V} \sum_{q' n'} f_{RPA}(q', i\omega_{n'}) G_1^\alpha(q + q', i\omega_{n+n'}) \times \frac{q^2 m^2 + 2 q q'}{[i\omega_{n'} - v^\alpha \cdot q' - \frac{q'^2}{2m}] [i\omega_{n'} - v^\alpha \cdot q' + \frac{q'^2}{2m}]}, \]

and

\[ \delta \mu_F = \langle D_0 \rangle = -\frac{1}{\beta V} \sum_{q n} f_{RPA}(q, i\omega_n) \times \frac{q^2 m^2}{[i\omega_n - v^\alpha \cdot q - \frac{q^2}{2m}] [i\omega_n - v^\alpha \cdot q + \frac{q^2}{2m}]} \]

Note that δμF can be identified with the renormalization of the chemical potential arising from the usual "sunrise"-diagram shown in Fig.1(b). To leading order in the bare interaction, this is just the Fock diagram. Note that the Hartree renormalization of the chemical potential has already been taken into account in the definition
of the zeroth order Green’s function by a suitable choice of the reference point \( k^\alpha \), see Eq.(8). This different treatment of the two renormalizations of the chemical potential is justified if we formally treat \( 1/m \) as a small parameter. Obviously, \( \delta \mu_F \propto 1/m \), whereas for the Hartree correction that appears in Eq.(8) we obtain to leading order

\[
\delta \mu_H = \langle i\phi_0 \rangle = \rho_0 f_{\text{RPA}}(0,0) .
\]  

(32)

Keeping in mind that in \( d \) dimensions \( \rho_0 \propto k_F^d \propto m^d \), it is clear that the Hartree renormalization of the chemical potential diverges if we let \( m \to \infty \) holding \( v_F = k_F/m \) constant. This is the reason why this renormalization cannot be treated within a \( 1/m \)-expansion, but should be included in the definition of the zeroth order Green’s function.

It is important to emphasize that the above expansion is quite different from the expansion in powers of \( 1/m \) proposed by Haldane. The advantage of our method is that in the absence of interactions we recover the exact free Green’s function with non-linear energy dispersion, which contains of course infinite orders in \( 1/m \).

In Sec.3 it will become clear that the effective small parameter in our expansion is proportional to \( f_{\text{RPA}}/m \), so that we obtain the exact result in both limits \( f_{\text{RPA}} \to 0 \) or \( 1/m \to 0 \).

### 2.2. Luttinger’s theorem

The Fermi surface of an interacting Fermi system can be defined as the surface in momentum space where the momentum distribution defined in Eq.(2) has some kind of non-analyticity in the zero-temperature limit \( \beta \to \infty \). The Fermi surface is in general not identical with the surface defined by Eq.(8), so that our reference point \( k^\alpha \) is not located precisely on the Fermi surface. Note, however, that according to Luttinger’s theorem the volume enclosed by the Fermi surface is not changed by the interaction, so that the radius of a spherically symmetric Fermi surface is simply \( k_F = \sqrt{2m\mu_0} \). On the other hand, our reference point \( k^\alpha \) is defined by \( \epsilon_{k^\alpha} = \mu - \delta \mu_H \), so that to leading order

\[
|k^\alpha| - k_F = (\mu - \delta \mu_H - \mu_0)/v_F = \delta \mu_F/v_F ,
\]  

(33)

were we have used the fact that to first order in the RPA screened interaction \( \mu = \mu_0 + \delta \mu_H + \delta \mu_F \), see Eqs.(31,32). It is now easy so see that for sufficiently regular interactions the singularity in the momentum distribution occurs at the non-interacting \( k_F \). Substituting Eq.(28) into the definition (8) we obtain

\[
n(k^\alpha + q) = \int d\mathbf{r} e^{-i\mathbf{q} \cdot \mathbf{r}} \tilde{G}^\alpha(\mathbf{r},0) e^{Q_1^\alpha(\mathbf{r},0)}. \]

(34)

Obviously the behavior of this function for small \( \mathbf{q} \) is determined by the long-distance behavior of the prefactor Green’s function \( \tilde{G}^\alpha(\mathbf{r},0) \) and of the Debye-Waller factor \( Q_1^\alpha(\mathbf{r},0) \). In a Fermi liquid \( Q_1^\alpha(\mathbf{r},0) \) approaches a finite constant at large distances, while for a Luttinger liquid the Debye-Waller factor exhibits
logarithmic singularities, which will be discussed in Sec.3.3. The long-distance behavior of \( \tilde{G}^\alpha(r,0) \) is dominated by the singularities of its Fourier transform given in Eqs.\((27)\) and \((28)\). The important point is now that for finite \( \delta \mu \) the singularity occurs at a finite value of \( q \). This is of course due to the fact that we have not measured the momenta relative to the true Fermi surface. It is therefore convenient to shift the momentum \( q \) in Eq.\((28)\) by setting

\[ q = q' - \delta k^\alpha, \tag{35} \]

where \( \delta k^\alpha \) should be chosen such that it cancels the chemical potential shift in Eq.\((28)\). For our spherical Fermi surface this means that

\[ \delta k^\alpha = k^\alpha - k_F \frac{k^\alpha}{|k^\alpha|} = \frac{\delta \mu}{v_F} \frac{k^\alpha}{|k^\alpha|}. \tag{36} \]

Renaming again \( q' \rightarrow q \), we obtain

\[ \tilde{G}^\alpha(r,0) = e^{-i\delta k^\alpha \cdot r} G_2^\alpha(r,0), \tag{37} \]

with

\[ G_2^\alpha(r,0) = \frac{1}{\beta V} \sum_{q^\alpha} e^{iq \cdot r} \times \frac{1 + Y^\alpha(q, i\omega_n)}{i\omega_n - (v^\alpha - \frac{4k^\alpha}{m}) \cdot q - \frac{q^2}{2m} - \Sigma_1^\alpha(q, i\omega_n)}. \tag{38} \]

Since we retain only terms to first order in \( f_{\text{RPA}} \), we have ignored the shift \( \delta k^\alpha \) in the arguments of \( \Sigma_1^\alpha \) and \( Y^\alpha \). This is justified, because according to Eqs.\((31,36)\) the shift \( \delta k^\alpha \) is already of order \( f_{\text{RPA}} \). The momentum distribution can then be written as

\[ n(k^\alpha - \delta k^\alpha + q) = \int d_r e^{-i(q-r) \cdot r} G_2^\alpha(r,0) e^{Q_1^\alpha(r,0)}. \tag{39} \]

The important point is now that for a conventional Fermi liquid the right-hand side of Eq.\((39)\) is non-analytic at \( q = 0 \). Because by construction \( |k^\alpha - \delta k^\alpha| = k_F = \sqrt{2m \mu_0} \), the momentum distribution is non-analytic at the non-interacting Fermi surface, in agreement with Luttinger’s theorem. In Sec.3 we shall explicitly evaluate the above expression in one dimension, and verify that even in this case Luttinger’s theorem remains valid.

The reader may find our discussion in this section rather pedantic. However, as will become evident in Sec.3, for the calculation of the effect of the quadratic term in the energy dispersion on the spectral function of the TLM it is crucial to take the shift of the chemical potential due to the interaction into account. Of course, if the location of the true Fermi surface is known a priori (this is the case, for example, in systems where Luttinger’s theorem is valid and the Fermi surface is known to be spherically symmetric), it seems more natural to choose a point on the true Fermi surface as the reference point for the linearization of the energy dispersion.
However, for general interactions and energy dispersions the location of the Fermi surface is not known a priori. In this case the general method outlined above can be used to calculate the true Fermi surface perturbatively. In the following section we shall explicitly demonstrate how this method works in one dimension.

3. Non-linear energy dispersion in one dimension

In $d = 1$ there are only two Fermi points, which can be labelled by $\alpha = \pm$. The zeroth order Green’s function defined in Eq.(9) can then be written as

$$G^0_\alpha(q, i\tilde{\omega}_n) = \frac{1}{i\tilde{\omega}_n - \alpha \tilde{v}_F q -\frac{q^2}{2m}}. \quad (40)$$

The reference points $k^\alpha$ and the renormalized Fermi velocity $\tilde{v}_F$ are (see Eq.(8))

$$k^\alpha = \alpha \sqrt{2m(\mu_0 + \delta\mu_F)}, \quad (41)$$
$$\tilde{v}_F = \sqrt{2(\mu_0 + \delta\mu_F)/m}. \quad (42)$$

It is important to emphasize that according to the generalized Luttinger theorem even in $d = 1$ the volume of the Fermi surface is invariant as we switch on the interaction at constant density. Hence, the Fermi wave-vector $k_F$ is still given in terms of the chemical potential of the non-interacting system, $k_F = \sqrt{2m\mu_0}$. Therefore the velocity $\tilde{v}_F = |k^\alpha|/m$ is not identical with the bare Fermi velocity $v_F = k_F/m$.

In this section we shall explicitly evaluate the expressions of Sec.2 in one dimension for a bare interaction of the form

$$f_q = f_0 \Theta(q_c - |q|), \quad q_c \ll k_F. \quad (43)$$

Note that this corresponds to the TLM with forward scattering interactions $g_2(q) = g_4(q) = f_q$. In one dimension the polarization function in Eq.(18) can for $V \to \infty$ and $\beta \to \infty$ be written as

$$\Pi_0(q, i\omega) = \frac{m}{2\pi q} \ln \left[ \frac{(\tilde{v}_F q + \frac{q^2}{2m}) + \omega^2}{(\tilde{v}_F q - \frac{q^2}{2m}) + \omega^2} \right]. \quad (44)$$

In the limit $1/m \to 0$ this reduces to

$$\Pi_0^{lin}(q, i\omega) = \frac{1}{\pi v_F} \frac{(v_F q)^2}{(v_F q)^2 + \omega^2}, \quad 1/m \to 0. \quad (45)$$

For finite $m$ the RPA interaction can be written as

$$f_{RPA}(q, i\omega) = \frac{\pi \tilde{v}_F \tilde{F}_q}{1 + \tilde{F}_q P(\frac{\omega}{2m \tilde{v}_F} \frac{\omega}{\tilde{v}_F q})}, \quad (46)$$

where we have introduced the dimensionless interaction

$$\tilde{F}_q = \tilde{F}_0 \Theta(q_c - |q|), \quad \tilde{F}_0 = \frac{f_0}{\pi \tilde{v}_F}, \quad (47)$$

In one dimension the polarization function in Eq.(18) can for $V \to \infty$ and $\beta \to \infty$ be written as

$$\Pi_0(q, i\omega) = \frac{m}{2\pi q} \ln \left[ \frac{(\tilde{v}_F q + \frac{q^2}{2m}) + \omega^2}{(\tilde{v}_F q - \frac{q^2}{2m}) + \omega^2} \right]. \quad (44)$$

In the limit $1/m \to 0$ this reduces to

$$\Pi_0^{lin}(q, i\omega) = \frac{1}{\pi v_F} \frac{(v_F q)^2}{(v_F q)^2 + \omega^2}, \quad 1/m \to 0. \quad (45)$$

For finite $m$ the RPA interaction can be written as

$$f_{RPA}(q, i\omega) = \frac{\pi \tilde{v}_F \tilde{F}_q}{1 + \tilde{F}_q P(\frac{\omega}{2m \tilde{v}_F} \frac{\omega}{\tilde{v}_F q})}, \quad (46)$$

where we have introduced the dimensionless interaction

$$\tilde{F}_q = \tilde{F}_0 \Theta(q_c - |q|), \quad \tilde{F}_0 = \frac{f_0}{\pi \tilde{v}_F}, \quad (47)$$

and the dimensionless polarization
\[
P(k, iu) = \frac{1}{4k} \ln \left[ \frac{(1 + k)^2 + u^2}{(1 - k)^2 + u^2} \right]. \tag{48}
\]

3.1. Chemical potential

Let us first consider the Fock-renormalization of the chemical potential given in Eq.(31), which turns out to be finite even in \(d = 1\). Introducing the dimensionless variables \( p = q/q_c, u = \omega/(\tilde{v}_F q) \) and the small parameter
\[
\tilde{\lambda} = \frac{q_c}{2m\tilde{v}_F}, \tag{49}
\]
we obtain from Eq.(31)
\[
\delta \mu_F = \langle D_0 \rangle = -\frac{q_c^2}{8\pi m} \int_{-1}^{1} dp |p| \int_{-\infty}^{\infty} du \frac{\tilde{F}_0 A(\tilde{\lambda}p, iu)}{1 + \tilde{F}_0 P(\tilde{\lambda}p, iu)}, \tag{50}
\]
where
\[
A(k, iu) = \frac{1}{[iu - \alpha - k][iu - \alpha + k]}. \tag{51}
\]
For small \( \tilde{\lambda} \) we may expand the functions \( P(\tilde{\lambda}p, iu) \) and \( A(\tilde{\lambda}p, iu) \) in powers of \( \tilde{\lambda} \). To leading order we obtain
\[
P(\tilde{\lambda}p, iu) \approx \frac{1}{1 + u^2}, \tag{52}
\]
\[
A(\tilde{\lambda}p, iu) \approx \frac{1}{(iu - \alpha)^2}. \tag{53}
\]
Recall that \( \alpha = \pm 1 \) labels the two Fermi points. The integrations in Eq.(50) are then easily performed, and we obtain
\[
\delta \mu_F = \frac{q_c^2}{4m} \tilde{\gamma} = \frac{\tilde{v}_F q_c \tilde{\lambda}}{2} \tilde{\gamma}, \tag{54}
\]
where
\[
\tilde{\gamma} = \frac{\tilde{F}_0^2}{2\sqrt{1 + \tilde{F}_0(\sqrt{1 + \tilde{F}_0} + 1)^2}} \tag{55}
\]
will be identified below with the anomalous dimension of our model. Using \( \rho_0 = m\tilde{v}_F/\pi \) the Hartree renormalization of the chemical potential can be written as
\[
\delta \mu_H = \langle i\phi_0 \rangle = m\tilde{v}_F^2 \frac{\tilde{F}_0}{1 + \tilde{F}_0}, \tag{56}
\]
so that the total shift of the chemical potential is
\[
\delta \mu = \delta \mu_H + \delta \mu_F = m\tilde{v}_F^2 \frac{\tilde{F}_0}{1 + \tilde{F}_0} + \frac{q_c^2}{4m} \tilde{\gamma}. \tag{57}
\]
Note that the Hartree renormalization is divergent in the Tomonaga-Luttinger limit $m \to \infty$ with $k_F/m = \text{const}$. This is the reason why it is necessary to include this renormalization into the definition of the zeroth order Green’s function. Note also that by construction the prefactor Green’s function (38) depends on the bare Fermi velocity $v_F = k_F/m$. This follows directly from Eqs.(36,42,54), which imply in $d = 1$ for the renormalized Fermi velocity

$$\tilde{v}_F = v_F[1 + \lambda^2 \gamma],$$

and hence for the relevant velocity that appears in the prefactor Green’s function

$$v^\alpha - \frac{\delta k^\alpha}{m} = \alpha \tilde{v}_F - \alpha \frac{\delta \mu_F}{\tilde{v}_F} = \alpha v_F.$$  

Here

$$\lambda = \frac{q_c}{2 m v_F}$$

is defined in terms of the bare Fermi velocity. Obviously

$$\tilde{\lambda} = \lambda + O(\lambda^3),$$

so that to leading order in an expansion in powers of $\lambda$ we may replace $\tilde{\lambda} \to \lambda$.

### 3.2. Prefactor Green’s function

Next, we calculate the functions $\Sigma^\alpha(q, i\omega)$ and $Y^\alpha(q, i\omega)$ defined in Eqs.(29) and (30) for $V \to \infty$ and $\beta \to \infty$. For small values of $q$ and $\omega$ and to leading order in $f_{\text{RPA}}$ it is sufficient to replace $G_1^\alpha \to G_0^\alpha$ on the right-hand sides of Eqs.(29) and (30). This neglect of self-consistency is justified a posteriori: because the result of the lowest order Born-approximation is finite and small, we presume that the self-consistent Born-approximation is not necessary. Introducing the same dimensionless variables as above, we obtain

$$\Sigma^\alpha(q, i\omega) = -\tilde{v}_F q \frac{\tilde{\gamma}}{\pi} \int_{-1}^{1} dp |p| \int_{-\infty}^{\infty} du \frac{\tilde{F}_0 A(\tilde{\lambda} p, iu)}{1 + \tilde{F}_0 P(\tilde{\lambda} p, iu)}$$

$$\times \frac{p + \frac{q_c}{\tilde{v}_F}}{G_0^{-1} + p[iu - \alpha - \lambda (2 \frac{q_c}{\tilde{v}_F} + p)]},$$

$$Y^\alpha(q, i\omega) = \frac{\tilde{\lambda}}{\pi} \int_{-1}^{1} dp \int_{-\infty}^{\infty} du \frac{\tilde{F}_0 A(\tilde{\lambda} p, iu)}{1 + \tilde{F}_0 P(\tilde{\lambda} p, iu)}$$

$$\times \frac{|p| + 2 \text{sgn}(p) \frac{q_c}{\tilde{v}_F}}{G_0^{-1} + p[iu - \alpha - \lambda (2 \frac{q_c}{\tilde{v}_F} + p)]},$$

where

$$\tilde{G}_0^{-1} = \frac{i\omega + \alpha v_F q - \frac{q^2}{2m}}{\tilde{v}_F q_c}. $$
To make further progress, we expand Eqs. (62) and (63) in powers of $\tilde{\lambda}$. To leading order, we may simply set $\tilde{\lambda} = 0$ in the integrands and replace $\tilde{v}_F \to v_F$, $\tilde{\lambda} \to \lambda$. The integrations can then be performed analytically, but the result is rather complicated and not very illuminating. For our purpose, we only need the leading terms for small $q$ and $\omega$, which are

$$\Sigma_1^\alpha(q, i\omega) = \delta v_1^\alpha q + O(q^2, \omega q, \omega^2) + O(\lambda^3), \quad (65)$$

$$\delta v_1^\alpha = -\alpha v_F \lambda^2 \frac{2}{\sqrt{1 + F_0}} \gamma, \quad (66)$$

where $\gamma$ is the anomalous dimension of the TLM, which is obtained from Eq. (55) by replacing $\tilde{F}_0 \to f_0/\pi v_F$. For the function $Y_1^\alpha(q, i\omega)$ we obtain to leading order

$$Y_1^\alpha(q, i\omega) = \lambda \left[ c_1 \frac{i\omega}{v_F q} + c_2 \frac{q}{q_c} \right] \ln \left( \frac{q_c}{q} \right) + O(\omega, q), \quad (67)$$

where $c_1$ and $c_2$ are numerical constants that depend on $F_0$.

### 3.3. Debye-Waller factor

Finally, consider the Debye-Waller factor defined in Eq. (26). Introducing again the dimensionless integration variables $p = q/q_c$ and $u = \omega/(\tilde{v}_F q)$, we obtain in $d = 1$

$$Q_1^\alpha(x, \tau) = \frac{1}{4\pi} \int_{-1}^1 dp \int_{-\infty}^\infty du \frac{\tilde{F}_0 A(\tilde{\lambda} p,iu)}{1 + \tilde{F}_0 P(\tilde{\lambda} p, iu)} \times \left[ 1 - \cos(p(\tilde{x} - u\tilde{\tau})) \right], \quad (68)$$

where $\tilde{x} = q x$ and $\tilde{\tau} = \tilde{v}_F q_c \tau$. Since we are only interested in the leading behavior of $Q_1^\alpha(x, \tau)$ for large $x$ or $\tau$, we may set $\tilde{\lambda} = 0$ in the integrand of Eq. (68). This is easily seen from the fact that the factor of $1/|p|$ in the integrand is responsible for a logarithmic growth of $Q_1^\alpha(x, \tau)$ for large $x$ or $\tau$. If we now expand the integrand of Eq. (68) in powers of $\tilde{\lambda}$, we obtain additional powers of $p$, so that the resulting contributions to $Q_1^\alpha(x, \tau)$ are bounded and hence can be neglected as far as the leading logarithmic behavior is concerned. In the limit $|\tau| \gg (\tilde{v}_F q_c)^{-1}$ and $|x| \gg q_c^{-1}$, the leading term is

$$Q_1^\alpha(x, \tau) \sim \frac{\tilde{\gamma}}{2} \ln \left[ \frac{q_c^{-2}}{x^2 + (v_c \tau)^2} \right] + \ln \left[ \frac{x + i\alpha \tilde{v}_F \tau}{x + i\alpha v_c \tau} \right], \quad (69)$$

where

$$v_c = \tilde{v}_F \sqrt{1 + \tilde{F}_0} \quad (70)$$

is the velocity of collective charge oscillations and $\tilde{\gamma}$ is the renormalized anomalous dimension, see Eq. (57). Eq. (69) is the well-known result of the TLM, but with renormalized parameters $\tilde{\gamma}, v_c$ and $\tilde{v}_F$. A more rigorous way to obtain Eq. (69) is to first perform the $u$-integration in Eq. (68) without expanding the integrand in powers of $\tilde{\lambda}$. Then, in the same limit as above, namely for large $x$ and $\tau$, we can also perform
the $\rho$-integration analytically, which leads precisely to the presented result. Note that Eq. (69) implicitly depends on our small parameter $\lambda$, because the renormalized parameters $\tilde{\gamma}$, $v_c$, and $\tilde{v}_F$ are $\lambda$-dependent. Hence, Eq. (69) goes beyond a simple expansion in powers of $\lambda$.

### 3.4. Spectral function

Combining all the results of the previous subsections, we finally obtain for the Matsubara Green’s function for wave-vectors $q$ close to the two Fermi points $\pm k_F = \pm \sqrt{2m\mu_0}$,

$$G(\alpha k_F + q, i\omega) = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} d\tau e^{-i(qx - \omega\tau)} G_2^\alpha(x, \tau) \times \frac{x + i\alpha \tilde{v}_F \tau}{x + i\alpha v_c \tau} e^{-\tilde{\gamma}/2},$$

with

$$G_2^\alpha(x, \tau) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} \int_{-\infty}^{\infty} d\omega e^{i(qx - \omega\tau)} \frac{1 + Y^\alpha(q, i\omega)}{i\omega - \alpha v_1 q \left[1 + \lambda \frac{v_1}{v_1 q}\right]}.$$  

Here

$$v_1 = v_F \left[1 - \lambda^2 \frac{2}{\sqrt{1 + F_0^2}}\right] + O(\lambda^3),$$

where we have used Eqs. (59, 66). Because the anomalous dimension in Eq. (69) depends on the effective dimensionless coupling $\tilde{F}_0 = f_0/(\pi \tilde{v}_F)$, which involves the renormalized Fermi velocity $\tilde{v}_F$, we obtain a small correction to the anomalous dimension $\gamma$ of the TLM. Keeping in mind that our result for the renormalization of the chemical potential is based on a weak coupling expansion, we obtain from Eq. (69) in the Tomonaga-Luttinger limit $\lambda \to 0$ and for small $\tilde{F}_0$,

$$\gamma = \frac{f_0^2}{8(\pi v_F)^2}.$$  

Hence, for finite $\lambda$ the effective anomalous dimension is

$$\tilde{\gamma} = \frac{\gamma}{1 + 2\lambda^2 \gamma} \approx \gamma [1 - 2\lambda^2 \gamma + O(\lambda^3)].$$

This is the leading correction to the anomalous dimension of the TLM due to the quadratic term of the energy dispersion at weak coupling. We expect that the non-Gaussian corrections to the Debye-Waller factor do not change the correction term in Eq. (75): simple power counting shows that these non-Gaussian corrections do not contain any infrared divergences, so that they cannot give rise to logarithmically growing corrections to the Debye-Waller factor, which are necessary to renormalize the anomalous dimension.

Because our result for the vertex function $Y^\alpha(q, i\omega)$ in Eq. (67) vanishes in the limit $q, \omega \to 0$, it does not modify the qualitative behavior of the spectral function for
small $\lambda$ (Ref.22). To leading order, we may therefore set $Y^\alpha(q,i\omega) \to 0$. Restricting
ourselves to the limit of large $x$ and $\tau$ and neglecting the term of order $q^2$ in the
denominator of Eq.(72), we obtain

$$G^\alpha_2(x,\tau) = \frac{1}{2\pi i x + i\alpha v_1 \tau}, \quad |x|, |\tau| \to \infty. \quad (76)$$

The imaginary-time Green’s function in Eq.(71) becomes then

$$G(\alpha k_F + q, i\omega) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} d\tau e^{-i(qx - \omega \tau)} \times \frac{1}{x + i\alpha v_c \tau} \left[ \frac{q_c^{-2}}{x^2 + (v_c \tau)^2} \right]^{\tilde{\gamma}/2} \frac{x + i\alpha \tilde{v}_F \tau}{x + i\alpha v_1 \tau}. \quad (77)$$

Note that for finite $\lambda$ and $F_0$ all three velocities $v_1$, $\tilde{v}_F$ and $v_c$ are different. Only
for linearized energy dispersion (i.e. for the TLM) $\tilde{v}_F = v_1$, so that the last factor
in Eq.(77) is exactly unity. For finite $\lambda$, however, the velocity degeneracy is broken,
which has rather spectacular consequences for the spectral function: there appears
an additional algebraic singularity in the spectral function, which is not present in
the TLM. The fact that even a small breaking of the velocity degeneracy in a
special model of interacting fermions in $d = 1$ can lead to new singularities in the
spectral function has recently been pointed out by Ho and Coleman.23 Because
in our model the curvature of the energy band implies a velocity dispersion, it is
not unreasonable to expect that the curvature leads to new features in the spectral
function, which are not present in the TLM. However, at this point we cannot
exclude the possibility that this new feature, whose appearance depends in a rather
subtle way on a small renormalization of effective velocities, is completely washed
out by non-Gaussian corrections or by the subleading corrections to the Debye-
Waller factor, which have been neglected in our leading order calculation. Moreover,
this singularity has a very small weight (which can be shown22 to vanishes as $\lambda^2$ for
$\lambda \to 0$), so that it is not of any practical importance.

For these reasons we shall replace the last factor of Eq.(77) by unity in the
following analysis. Note, however, that the charge velocity $v_c$ and the anomalous
dimension $\tilde{\gamma}$ are defined in terms of the renormalized velocity $\tilde{v}_F = v_F(1 + \lambda^2 \tilde{\gamma})$, which is slightly larger than the bare Fermi velocity $v_F = k_F/m$. To calculate the
spectral function

$$A^\alpha(q,\omega) = -\frac{1}{\pi} \text{Im} G(\alpha k_F + q, \omega + i0^+) \quad (78)$$

from Eq.(77) with $\tilde{v}_F = v_1$, it is convenient to first consider the real time Green’s function

$$G^\alpha_\sigma(x,t) = i\Theta(t)G^\alpha(x, i\tau \to -t + i0). \quad (79)$$

The spectral function for $\omega > 0$ is then given by

$$G^\alpha_\sigma(q, \omega) = 2\pi \Theta(\omega)A^\alpha(q, \omega), \quad (80)$$
where
\[ G_\alpha(q, \omega) = i \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx e^{-i(qx - \omega t)} G_\alpha^\ast(x, t). \] (81)

We obtain
\[
A^\alpha(q, \omega) = \left( \frac{1}{2vcq_c} \right) \tilde{\gamma} \frac{\tilde{\gamma}}{2\Gamma^2(1 + \frac{\tilde{\gamma}}{2})} \times \Theta(\omega - |v_c q|)(\omega + \alpha v_c q)^{\frac{\tilde{\gamma}}{2}}(\omega - \alpha v_c q)^{\frac{\tilde{\gamma}}{2} - 1}. \] (82)

Note that \( \tilde{\gamma} \) can be expanded in powers of \( \lambda \), but a perturbative calculation of \( A^\alpha(q, \omega) \) in powers of \( \lambda \) would lead to unphysical logarithmic singularities. To illustrate this point, consider the momentum distribution. From Eq. (34) it is easy to show that
\[ n(\alpha k_F + q) = \frac{1}{2} - C \text{sgn}(q) \left| \frac{q}{q_c} \right|^{\tilde{\gamma}}, \] (83)
where \( C \) is a numerical constant. Using \( \tilde{\gamma} = \gamma - 2\lambda^2 \gamma^2 \), an expansion of the second term in powers of \( \lambda \) yields
\[ \left| \frac{q}{q_c} \right|^{\tilde{\gamma}} = \left| \frac{q}{q_c} \right|^{\gamma} \left[ 1 + 2\lambda^2 \gamma \ln \left| \frac{q_c}{q} \right| + O(\lambda^4) \right]. \] (84)

Thus, a naive expansion of \( n(\alpha k_F + q) \) in powers of \( \lambda \) would generate logarithmic terms, which become arbitrary large for sufficiently small \( q \). Our method effectively resums all these corrections.

4. Conclusion

In this work we have studied how in one dimension the quadratic term in the expansion of the energy dispersion close to the Fermi points modifies the spectral function of the TLM. The most important effect of the non-linearity is a renormalization of the anomalous dimension, which we have explicitly calculated to leading order in the small dimensionless parameter \( \lambda = q_c/(2mv_F) \). Note that this implies that correlation functions can in general not be expanded in powers of the \( \lambda \), because they exhibit algebraic singularities. This leads to a non-analytic \( \lambda \)-dependence. We have also pointed out that the non-linear terms in the energy dispersion might give rise to new features in the spectral function, which are not present in the spectral function of the TLM. However, we cannot exclude the possibility that these non-universal features are washed out by higher order corrections which have been neglected in our calculation.

For a proper treatment of the quadratic term in the energy dispersion it is crucial to take the renormalization of the chemical potential due to the interaction into account. We have shown how to include this effect into the functional bosonization approach in arbitrary dimension, emphasizing that the ”tadpole” diagrams describing a Hartree-renormalization of the chemical potential have to be treated exactly.
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