INFINITELY MANY LEAF-WISE INTERSECTIONS ON COTANGENT BUNDLES

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Abstract. If the homology of the free loop space of a closed manifold $B$ is infinite dimensional then generically there exist infinitely many leaf-wise intersection points for fiber-wise star-shaped hypersurfaces in $T^*B$.

1. Introduction

Let $B$ be a closed manifold and $\Sigma \subset T^*B$ be a fiber-wise star-shaped hypersurface with respect to the standard Liouville vector field. $\Sigma$ is foliated by the Reeb flow associated to the Liouville 1-form $\lambda$. We denote by $L_x$ the leaf through $x \in \Sigma$. Let $\psi \in \text{Ham}_c(T^*B)$ be in the space of Hamiltonian diffeomorphisms generated by compactly supported time dependent Hamiltonian functions. Then a leaf-wise intersection is a point $x \in \Sigma$ with the property $\psi(x) \in L_x$. The search for leaf-wise intersections was initiated by Moser in [Mos78] and pursued further in [Ban80, Hof90, EH89, Gin07, Dra08, AF08, Zil08, Gur09, Kan09]. A brief history of the search for leaf-wise intersections is given below.

We call $\Sigma$ non-degenerate if Reeb orbits on $\Sigma$ form a discrete set. A generic $\Sigma$ is non-degenerate, see [CF09, Theorem B.1]. We denote by $L_B$ the free loop space of $B$.

Theorem 1. Let $\dim H_*(L_B) = \infty$. If $\dim B \geq 2$ and $\Sigma$ is non-degenerate then for a generic $\psi \in \text{Ham}_c(T^*B)$ there exist infinitely many leaf-wise intersections.

Remark 1.1.

- To our knowledge all so far known existence results for leaf-wise intersections assert only finite lower bounds. Moreover, all known results make smallness assumptions on either the $C^1$ or Hofer norm of $\psi$.
- The assumption $\dim B \geq 2$ is necessary as the example $B = S^1$ shows.
- If $\pi_1(B)$ is finite then $\dim H_*(L_B) = \infty$ by a theorem of Vigué-Poirrier and Sullivan [VPS76]. If the number of conjugacy classes of $\pi_1(B)$ is infinite then $\dim H_0(L_B) = \infty$. Therefore, the only remaining case is if $\pi_1(B)$ is infinite but the number of conjugacy classes of $\pi_1(B)$ is finite.

1.1. History of the problem and related results. The problem addressed above is a special case of the leaf-wise coisotropic intersection problem. For that let $N \subset (M,\omega)$ be a coisotropic submanifold. Then $N$ is foliated by isotropic leafs, see [MS98, Section 3.3]. The problem asks for a leaf $L$ such that $\phi(L) \cap L \neq \emptyset$ for $\phi \in \text{Ham}_c(M,\omega)$.

The first existence result was obtained by Moser in [Mos78] for simply connected $M$ and $C^1$-small $\phi$. This was later generalized by Banyaga [Ban80] to non-simply connected $M$.
The $C^1$-smallness assumption was replaced by Hofer, Ekeland-Hofer in [Hof90, EH89] for hypersurfaces of restricted contact type in $\mathbb{R}^{2n}$ by a much weaker smallness assumption, namely that the Hofer norm of $\phi$ is smaller than a certain symplectic capacity. Only recently, the result by Ekeland-Hofer was generalized in two different directions. It was extended by Dragnev [Dra08] to so-called “coisotropic submanifolds of contact type in $\mathbb{R}^{2n}$”. Ginzburg [Gin07] generalized from restricted contact type in $\mathbb{R}^{2n}$ to restricted contact type in subcritical Stein manifolds. Moreover, examples by Ginzburg [Gin07] show that the Ekeland-Hofer result is a symplectic rigidity result, namely it becomes wrong for arbitrary hypersurfaces. In [AF08] the authors proved multiplicity results for restricted contact-type hypersurfaces. These were recently generalized by Kang in [Kan09]. Ziltener [Zil08] established multiplicity results in the special case of fibrations. Finally, Gurel [Gur09] obtained existence results for leaf-wise intersections for coisotropic submanifolds of restricted contact type.

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2. Leaf-wise intersections and Rabinowitz Floer homology

Let $(M, \omega)$ be a symplectic manifold and $f \in C^\infty(M)$ an autonomous Hamiltonian function. Since energy is preserved the hypersurface $\Sigma := f^{-1}(0)$ is invariant under the Hamiltonian flow $\phi_t^f$ of $f$. The Hamiltonian flow $\phi_t^f$ is generated by the Hamiltonian vector filed $X_f$ which is uniquely defined by the equation $\omega(X_f, \cdot ) = df$. If 0 is a regular value of $f$ the hypersurface is a coisotropic submanifold which is foliated by 1-dimensional isotropic leaves, see [MS98, Section 3.3]. If we denote by $L_x$ the leaf through $x \in \Sigma$ we have the equality

$$L_x = \bigcup_{t \in \mathbb{R}} \phi_t^f(x).$$

(2.1)

Given a time-dependent Hamiltonian function $H : [0,1] \times M \rightarrow \mathbb{R}$ with Hamiltonian flow $\phi_t^H$ we are interested in points $x \in \Sigma$ with the property

$$\phi_1^H(x) \in L_x.$$

(2.2)

This notion was introduced and studied by Moser in [Mos78]. Such points are called leaf-wise intersections. For a physical interpretation of leaf-wise intersections it is useful to think of the Hamiltonian $H$ as a perturbation of the conservative Hamiltonian system $\phi_t^f$. More dramatically one can think of $H$ as an earthquake lasting from time $t = 0$ to $t = 1$. Without the earthquake the physical system propagates along a fixed leaf of $\Sigma$. Now we can ask whether the physical system survives the earthquake unharmed. This happens precisely if there exists a leaf-wise intersection. We refer to the article [Mos78] by Moser for further physical applications and examples.

**Definition 2.1.** A leaf-wise intersection $x \in \Sigma$ is called periodic if the leaf $L_x$ is a closed orbit of the flow $\phi_t^f$.

**Definition 2.2.** A pair $\mathfrak{M} = (F, H)$ of Hamiltonian functions $F, H : S^1 \times M \rightarrow \mathbb{R}$ is called a Moser pair if it satisfies

$$F(t, \cdot ) = 0 \quad \forall t \in [\frac{1}{2},1] \quad \text{and} \quad H(t, \cdot ) = 0 \quad \forall t \in [0, \frac{1}{2}],$$

(2.3)
and $F$ is of the form $F(t, x) = \rho(t)f(x)$ for some smooth map $\rho : S^1 \to S^1$ with $\int_0^1 \rho(t)dt = 1$ and $f : M \to \mathbb{R}$.

**Definition 2.3.** We set

$$\mathcal{H} := \{ H \in C^\infty(S^1 \times M) \mid H \text{ has compact support and } H(t, \cdot) = 0 \quad \forall t \in [0, \frac{1}{2}] \}$$

(2.4)

**Remark 2.4.** It’s easy to see that the $\text{Ham}(M, \omega) \equiv \{ \phi^1_H \mid H \in \mathcal{H} \}$, e.g. [AF08].

Let $(M, \omega = -d\lambda)$ be an exact symplectic manifold. Then for a Moser pair $\mathfrak{M} = (F, H)$ the perturbed Rabinowitz action functional is defined by

$$A^{\mathfrak{M}} : \mathcal{L}_M \times \mathbb{R} \to \mathbb{R}$$

$$(v, \eta) \mapsto \int_{S^1} v^*\lambda - \int_0^1 H(t, v)dt - \eta \int_0^1 F(t, v)dt$$

(2.5)

where $\mathcal{L}_M := C^\infty(S^1, M)$. We recall that $\omega(X_F, \cdot) = dF(\cdot)$. Then a critical point $(v, \eta)$ of $A^{\mathfrak{M}}$ is a solution of

$$\left\{ \begin{array}{l}
\partial_t v = \eta X_F(t, v) + X_H(t, v) \\
\int_0^1 F(t, v)dt = 0
\end{array} \right.$$ (2.6)

We observed in [AF08] that critical points of $A^{\mathfrak{M}}$ give rise to leaf-wise intersections.

**Proposition 2.5** ([AF08]). Let $(v, \eta)$ be a critical point of $A^{\mathfrak{M}}$ then $x := v(\frac{1}{2}) \in f^{-1}(0)$ and $\phi^1_H(x) \in L_x$

(2.7)

thus, $x$ is a leaf-wise intersection.

Moreover, the map $\text{Crit} A^{\mathfrak{M}} \to \{ \text{leaf-wise intersections} \}$ is injective unless there exists a periodic leaf-wise intersection (see Definition 2.1).

**Definition 2.6.** A Moser pair $\mathfrak{M} = (F, H)$ is of contact-type if the following four conditions hold.

1. $0$ is a regular value of $f$.
2. $df$ has compact support.
3. The hypersurface $f^{-1}(0)$ is a closed restricted contact type hypersurface of $(M, \lambda)$.
4. The Hamiltonian vector field $X_f$ restricts to the Reeb vector field on $f^{-1}(0)$.

**Remark 2.7.** If $\Sigma \subset T^*B$ is a fiber-wise star-shaped hypersurface there exists a contact-type Moser pair $\mathfrak{M}$ with $\Sigma = f^{-1}(0)$.

**Definition 2.8.** A Moser pair $\mathfrak{M}$ is called regular if $A^{\mathfrak{M}}$ is Morse.

We recall the following

**Proposition 2.9** ([AF08]). A generic contact-type Moser pair is regular.

For a regular contact-type Moser pair $\mathfrak{M}$ on an exact symplectic manifold which is convex at infinity Rabinowitz Floer homology $RFH_\ast(\mathfrak{M})$ is defined from the chain complex

$$\text{RFC}_k(\mathfrak{M}) := \left\{ \xi = \sum_{\mu_{cz}(c) = k} \xi_c c \mid \# \{ c \in \text{Crit} A^{\mathfrak{M}} \mid \xi_c \neq 0 \in \mathbb{Z}/2, A^{\mathfrak{M}}(c) \geq \kappa \} < \infty \forall \kappa \in \mathbb{R} \right\}$$ (2.8)
where the boundary operator is defined by counting gradient flow lines of $\mathcal{A}^{\mathcal{M}}$ in the sense of Floer homology, see [CF09, AF08] for details. In particular, on cotangent bundles $T^*B$ $RFH_*(\mathcal{M})$ is well-defined.

If the Moser pair is of the form $\mathcal{M} = (F, 0)$ then $\mathcal{A}^{\mathcal{M}}$ is never Morse. But for a generic $F$ the action functional $\mathcal{A}^{\mathcal{M}}$ is Morse-Bott with critical manifold being the disjoint union of constant solutions of the form $(p, 0), p \in f^{-1}(0)$, and a family of circles corresponding to closed characteristics of $\omega$ on $f^{-1}(0)$.

**Definition 2.10.** A Moser pair is called weakly regular if it is of the form just described or if it is regular.

**Remark 2.11.** For weakly regular Moser pairs $RFH_*(\mathcal{M})$ can still be defined by taking the critical points of a Morse function on the critical manifolds as generators, see [CF09] for details.

**Remark 2.12.** We note that if we have two Moser pairs $\mathcal{M}_0 = (F_0, H_0)$ and $\mathcal{M}_1 = (F_1, H_1)$ associated to two fiber-wise star-shaped hypersurfaces $\Sigma_0$ and $\Sigma_1$ then they can be joint through a smooth family of Moser pairs $\mathcal{M}_r = (F_r, H_r)$ such that the corresponding hypersurfaces $\Sigma_r$ remain fiber-wise star-shaped. In particular, each $\mathcal{M}_r$ is a contact-type Moser pair.

Let $\mathcal{M}_r = (F_r, H_r), r \in [0, 1]$ be a smooth family of contact-type Moser pairs. We fix once for all a smooth function $\beta \in C^\infty(\mathbb{R}, [0, 1])$ satisfying $\beta(s) = 0$ for $s \leq 0$, $\beta(s) = 1$ for $s \geq 1$, and $0 \leq \beta' \leq 2$. Then we set

\[ F_s := F^{\beta(s)}, \quad H_s := H^\beta(s), \quad \text{and} \quad \mathcal{M}_s := (F_s, H_s) \quad (2.9) \]

for $s \in \mathbb{R}$. The corresponding $s$-dependent Rabinowitz action functional is

\[ \mathcal{A}_s(v, \eta) := \int_{S^1} v^* \lambda - \int_0^1 H_s(t, v(t)) dt - \eta \int_0^1 F_s(t, v(t)) dt \quad (2.10) \]

It is used to define the standard continuation homomorphisms in Rabinowitz Floer homology, that is, given two weakly regular Moser pairs $\mathcal{M}_0$ and $\mathcal{M}_1$ there exist natural isomorphisms

\[ m^{\mathcal{M}_0}_{\mathcal{M}_1} : RFH_*(\mathcal{M}_0) \longrightarrow RFH_*(\mathcal{M}_1), \quad (2.11) \]

see [AF08] for details.

### 3. Proof of Theorem 1

Let $(B, g)$ be a closed Riemannian manifold and $S_g^*B$ the unit cotangent bundle with respect to $g$. Cutting off the function $\frac{1}{2}(|p|^2 - 1)$ outside a large compact subset of $T^*B$ gives rise to a contact-type Moser pair $\mathcal{M}_0 = (F_0, 0)$ for $S_g^*B$.

**Remark 3.1.** According to a Theorem by Abraham [Abr70] for a generic metric $g$ the Moser pair $\mathcal{M}_0 = (F_0, 0)$ is weakly regular. More precisely, every bumpy metric satisfies this condition.

We recall

**Theorem 3.2.** [CFO09, AS09] For degrees $* \neq 0, 1$

\[ RFH_*(\mathcal{M}_0) \cong \begin{cases} \mathcal{H}_*(\mathcal{L}^B) \\ H^{-*+1}(\mathcal{L}^B) \end{cases} \quad (3.1) \]
Proof of Theorem 1. We fix a fiber-wise star-shaped hypersurface $\Sigma$ and $\psi \in \text{Ham}_{c}(T^*B)$. This gives rise to a Moser pair $\mathcal{M} = (F,H)$. If $\Sigma$ is non-degenerate and $\psi$ is generic the perturbed Rabinowitz action functional $\mathcal{A}^{\mathcal{M}}$ is Morse, see Proposition 2.9. Since $\Sigma$ is fiber-wise star-shaped the Moser pair $\mathcal{M}$ can be joined to $\mathcal{M}_0$ through contact-type Moser pairs, see Remark 2.12. Thus, using the continuation isomorphism

$$m^{\mathcal{M}_0}_{\mathcal{M}}: \text{RFH}_*(\mathcal{M}_0) \longrightarrow \text{RFH}_*(\mathcal{M})$$

we conclude that

$$\text{RFH}_*(\mathcal{M}) \cong \begin{cases} H_*(\mathcal{L}_B) \\
H^{-s+1}(\mathcal{L}_B) \end{cases} \quad (3.3)$$

Since we assume that $\dim H_*(\mathcal{L}_B) = \infty$ we have $\dim \text{RFH}_*(\mathcal{M}) = \infty$ and therefore, the Morse function $\mathcal{A}^{\mathcal{M}}$ has infinitely many critical points. Now, Proposition 2.5 implies that there exist infinitely many leaf-wise intersections or a period leaf-wise intersection. Thus, to prove Theorem 1 we need to exclude the latter for a generic $\psi \in \text{Ham}_{c}(T^*B)$. That is, we need to make sure that for generic $\psi$ the critical points of $\mathcal{A}^{\mathcal{M}}$ do not intersect closed Reeb orbits. This is exactly the content of Theorem 3.3. \qed

We recall that a fiber-wise star-shaped hypersurface $\Sigma$ is called non-degenerate if the set $\mathcal{R}$ of Reeb orbits on $\Sigma$ form a discrete set. A generic $\Sigma$ is non-degenerate, see [CF9, Theorem B.1].

Theorem 3.3. Let $\Sigma = f^{-1}(0) \subset T^*B$ be a non-degenerate star-shaped hypersurface and $\mathcal{M}_0 = (F_0,0)$ be the corresponding weakly regular Moser pair. If $\dim B \geq 2$ then the set

$$\mathcal{H}_\Sigma := \{ H \in \mathcal{H} | A^{(F_0,H)} is Morse and \text{im}(x) \cap \text{im}(y) = \emptyset \ \forall x \in \text{Crit} A^{(F_0,H)}, y \in \mathcal{R} \} \quad (3.4)$$

is generic in $\mathcal{H}$ (see Definition 2.3).

Proof. We set $M := T^*B, \mathcal{L} = W^{1,2}(S^1, M)$, and $\mathcal{H}^k := \{ H \in C^k(S^1 \times M) | H(t,\cdot) = 0 \ \forall t \in [0,\frac{1}{2}] \}$. Furthermore, we define the Banach space bundle $\mathcal{E} \longrightarrow \mathcal{L}$ by $\mathcal{E}_v = L^2(S^1, v^*TM)$. We consider the section $S: \mathcal{L} \times \mathbb{R} \times \mathcal{H}^k \longrightarrow \mathcal{E}^\vee \times \mathbb{R}$ defined by

$$S(v,\eta,H) := dA^{(F_0,H)}(v,\eta) \quad. \quad (3.5)$$

Its vertical differential $DS: T_{(v_0,\eta_0,H)}\mathcal{L} \times \mathbb{R} \times \mathcal{H}^k \longrightarrow \mathcal{E}^\vee_{(v_0,\eta_0,H)}$ at $(v_0,\eta_0,H) \in S^{-1}(0)$ is

$$DS_{(v_0,\eta_0,H)}[(\hat{v},\hat{\eta},\hat{H})] = \mathcal{H}_{(F_0,H)}(v_0,\eta_0)\left[(\hat{v},\hat{\eta},\hat{H}) ; \bullet \right] + \int_0^1 \hat{H}(t,v_0)dt \quad. \quad (3.6)$$

where $\mathcal{H}_{(F_0,H)}$ is the Hessian of $A^{(F_0,H)}$. In [AF08] we proved the following.

Proposition 3.4. The operator $DS_{(v_0,\eta_0,H)}$ is surjective for $(v_0,\eta_0,H) \in S^{-1}(0)$. In fact, $DS_{(v_0,\eta_0,H)}$ is surjective when restricted to the space

$$\mathcal{V} := \{(\hat{v},\hat{\eta},\hat{H}) \in T_{(v_0,\eta_0,H)}\mathcal{L} \times \mathbb{R} \times \mathcal{H}^k | \hat{v}(\frac{1}{2}) = 0 \} \quad. \quad (3.7)$$

Thus, by the implicit function theorem the universal moduli space

$$\mathcal{M} := S^{-1}(0) \quad. \quad (3.8)$$

is a smooth Banach manifold. We consider the projection $\Pi: \mathcal{M} \longrightarrow \mathcal{H}^k$. Then $A^{(F_0,H)}$ is Morse if and only if $H$ is a regular value of $\Pi$, which by the theorem of Sard-Smale form a generic set (for $k$ large enough). Moreover, the Morse condition is $C^k$-open. Thus, for functions in an open and dense subset of $\mathcal{H}^k$ the functional $A^{(F_0,H)}$ is Morse.
Next we define the evaluation map
\[ \text{ev} : \mathcal{M} \rightarrow \Sigma \]
\[ (v_0, \eta_0, H) \mapsto v_0(\frac{1}{2}) \quad (3.9) \]
From Proposition 3.4 together with Lemma 3.5 below it follows that the evaluation map
\[ \text{ev}_H := \text{ev}(\cdot, \cdot, H) : \text{Crit} \mathcal{A}(F_0, H) \rightarrow \Sigma \]
is a submersion for a generic choice of H. Thus, the preimage of the one dimensional set \( \mathcal{R}^\tau := \{ \text{Reeb orbits with period } \leq \tau \} \) under \( \text{ev}_H \) doesn’t intersect \( \text{Crit} \mathcal{A}(F_0, H) \) using that \( \dim T^* B \geq 4 \). Therefore, the set
\[ \mathcal{H}_\Sigma^n := \{ H \in \mathcal{H}^n \mid \mathcal{A}(F_0, H) \text{ is Morse and } \text{im}(x) \cap \text{im}(y) = \emptyset \ \forall x \in \text{Crit} \mathcal{A}(F_0, H), y \in \mathcal{R}^n \} \quad (3.10) \]
is generic in \( \mathcal{H} \) for all \( n \in \mathbb{N} \). Now, the set \( \mathcal{H}_\Sigma^n \) is a countable intersection of the sets \( \mathcal{H}_\Sigma^n \), \( n \in \mathbb{N} \). This proves the assertion of Theorem 3.3. \( \square \)

We learned the following Lemma from Dietmar Salamon.

**Lemma 3.5.** Let \( \mathcal{E} \rightarrow B \) be a Banach bundle and \( s : B \rightarrow \mathcal{E} \) a smooth section. Moreover, let \( \phi : B \rightarrow N \) be a smooth map into the Banach manifold \( N \). We fix a point \( x \in s^{-1}(0) \subset B \) and set \( K := \ker d\phi(x) \subset T_x B \) and assume the following two conditions.

1. The vertical differential \( Ds|_K : K \rightarrow \mathcal{E}_x \) is surjective.
2. \( d\phi(x) : T_x B \rightarrow T_{\phi(x)} N \) is surjective.

Then \( d\phi(x)|_{\ker Ds(x)} : \ker Ds(x) \rightarrow T_{\phi(x)} N \) is surjective.

For convenience we provide a proof here.

**Proof.** We fix \( \xi \in T_{\phi(x)} N \). Condition (2) implies that there exists \( \eta \in T_{\phi(x)} B \) satisfying \( d\phi(x)\eta = \xi \). Condition (1) implies that there exists \( \zeta \in K \subset T_x B \) satisfying \( Ds(x)\zeta = Ds(x)\eta \). We set \( \tau := \eta - \zeta \) and compute
\[ Ds(x)\tau = Ds(x)\eta - Ds(x)\zeta = 0 \quad (3.11) \]
thus, \( \tau \in \ker Ds(x) \). Moreover,
\[ d\phi(x)\tau = d\phi(x)\eta - d\phi(x)\zeta = d\phi(x)\eta = \xi \quad (3.12) \]
proving the Lemma. \( \square \)

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