Optimal Charging and Discharging Control of Plug-in Hybrid Electric Vehicles in a System-level Power Distribution Network

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Abstract. With the increasing popularity of plug-in hybrid electric vehicles (PHEVs), coordinated charging and discharging of PHEVs has become an important issue in power distribution networks. To this end, we employ a multi-objective optimization model to formulate the problem of coordinated charging and discharging of PHEVs, in which the objectives of load shifting and total costs minimization are both investigated under system’s technical constraints. A hierarchical optimal algorithm combining the iterative water-filling-based algorithm and the consensus-based method is proposed to solve the multi-objective optimization problem. It shows that the proposed algorithm not only enhances stable performance for the power load but also guarantees low-cost satisfaction for the vehicle owners in the system-level power distribution network integrating physical power network with information network together. Numerical simulations are presented to demonstrate the effectiveness of the proposed algorithm.

1. Introduction

Due to low emissions and high efficiency, plug-in hybrid electric vehicles (PHEVs) become increasingly popular. Specifically, a PHEV utilizes combustion engine and Lithium-ion batteries to reduce its fuel consumption [1], thus being more eco-friendly. In addition, PHEVs can offer benefits due to their flexibility in charging and discharging time span and introduce a useful concept called vehicle to grid (V2G) capability [2].

Based on the direction of power flow between PHEVs and grid, existing works can be classified into two categories. One is for charging PHEVs only, known as grid to vehicle (G2V), and the other takes account of both G2V and V2G.

In [3], a decentralized charging control method is proposed to deal with the valley-filling problem, which flattens load demand curve during the night, while satisfying users charging requirements. The authors of [4] point out that users tend to buy electricity at the lowest price to minimize their expenses, but at the same time they do not want their daily driving schedule to be disturbed by some optimal charging strategies. To overcome this challenge, a non-cooperative game approach is applied to maximize individual utility function independently for a group of PHEVs [5], in which the valley filling is achieved and users costs are minimized simultaneously.
Due to the feasibility of V2G mode of discharging based on smart inverters, V2G capability of PHEVs provides some valuable power system services such as regulation, peaking capacity [6]. The authors of [7] model a hierarchical dispatch structure between PHEVs and grid, in which the sincerity degree and the batteries loss are used to determine the priority of scheduling for PHEVs execution.

Considering the popularity of PHEVs in the near future, this paper focuses on a multi-objective optimal scheduling for coordinated charging and discharging of PHEVs in a power distribution system. We propose a hierarchical optimal algorithm to solve the multi-objective optimization problem and verify the effectiveness of the algorithm through numerical simulations. The key technical contributions made in this study can be summarized as follows.

1) A system-level framework of the power distribution network is established for coordinated charging and discharging of PHEVs, where the energy provision plane aims at the power load stability and the energy consumption plane focuses on the total costs minimization.

2) A multi-objective optimization model is employed to formulate coordinated charging and discharging of PHEVs in the power distribution network, in which the stability of power load with a higher priority and the total costs minimization with a lower priority are both optimized under system’s technical constraints.

3) By integrating the iterative water-filling-based algorithm with the consensus-based optimization method, a hierarchical optimal algorithm is proposed to solve the multi-objective optimization problem effectively. It shows that the proposed algorithm not only enables peak shaving and valley filling, but also achieves the global minimization of total costs for vehicle owners.

This paper is organized as follows. System model is presented in Section 2, involving the system-level framework of the power distribution network, the PHEVs charging and discharging model. The problem formulation is given in Section 3. Our main results including problem transformation and the optimal algorithm are presented in Section 4. Numerical simulations are given in Section 5. The conclusion and future work are stated in Section 6.

2. System model

2.1. System-level power distribution system modeling

We consider a system-level framework of a power distribution network for coordinated charging and discharging of PHEVs, as Figure 1 shows. In the energy provision plane, the power grid acts as energy provider and the control center sends control messages based on the electricity load profiles. In the energy consumption plane, there are several energy consumption areas composed of a population of PHEVs and a functional district. Each PHEV equipped with a smart metering device enables to communicate with its neighbours forming a coalition to enlarge their common benefits. Figure 1 illustrates the PHEV charging and discharging scenario in the system-level power distribution network, in which both power flow and signal flow are displayed.

![Figure 1. Framework of a system-level power distribution system.](image-url)
2.2. PHEV charging and discharging modeling

Suppose that there are \( n \) PHEVs in a coalition, labelled from \( 1 \) to \( n \) of a set \( V \), and every PHEV has the capability of both G2V and V2G. Let \( k = 0;1;\ldots;N \) denote the time index and \( \varepsilon \ T \) denote the sampling interval. For a PHEV \( i \in V \) over \( N \)-period horizon, the power flow between PHEV \( i \) and grid at time \( k \), denoted by \( x^k \), is restricted as

\[
\dot{i} x^k _{i} \leq \dot{i} x^k _{i} , \tag{1}
\]

where \( \dot{i} x^k _{i} \) (kW) is the maximum charging power and \( \dot{i} x^k _{i} \) (kW) is the maximum discharging power of PHEV \( i \).

The state of charge (SOC) at time \( k \) of PHEV \( i \), denoted by \( s_i(k) \) (%), is the charging level of the battery given by

\[
s_i(k + 1) = s_i(k) + \frac{x^k \ T}{C_{i}^{\text{max}}}, \tag{2}
\]

where \( C_{i}^{\text{max}} \) is the capacity of battery energy of PHEV \( i \) and

\[
\begin{array}{l}
\dot{i} \in < 1 \quad \text{if} \quad x^k \ > 0, \\
\dot{i} > 1 \quad \text{if} \quad x^k \ < 0.
\end{array}
\]

\( \dot{i} \in , \dot{i} \in \ 0;1;1 \) denote the coefficient of charging and discharging of PHEV \( i \). The desire SOC is set by the user, denoted by \( s^i_{r} \), which is the targeted SOC at time \( k = N \) and follows equality constraint:

\[
s_i(0) + \sum_{k=0}^{N-1} \dot{i} (k) x^k = s^i_{r}, \tag{3}
\]

where \( s_i(0) \) is the initial SOC at time \( k = 0 \). Note that \( s^i_{j} \ < \text{min} \text{ } s^i_{j} \text{ } \text{and} \text{ } s^i_{j} \text{ } \text{are the minimum and maximum SOC.}

3. Problem formulation

This work focuses on a multi-objective optimal scheduling for coordinated charging and discharging of PHEVs in the system-level power distribution network. The energy provision plane aims to stabilize the power load by controlling the aggregated power between the PHEVs and grid. On the basis of that, the energy consumption plane aims to achieve the total minimum cost for all users by allotting the aggregated power, while satisfying each user’s requirement for their PHEV to be charged to the required level by the specified time.

A functional area is treated as the basic unit of the energy consumption plane. Divide set \( V \) in the area into two subsets:

\[
V_1 = f \cup \dot{i}, V_2 = f \cup \dot{i}, \tag{4}
\]

where \( f \cup \dot{i} \) is the targeted SOC at the end of the discharging stage. Suppose that the base load demand (non-PHEV) of the area is denoted by \( q^k, k = 0;1;\ldots;N \), which are known to the control center. Let \( k^0 \) denote a time threshold, before and after which the PHEVs only discharge and charge, respectively.

For PHEVs in subset \( V_2 \), they do not discharge during \( 0; k^0 \), and they are only involved in the charging stage after \( k^0 \). For the objective functions with different priorities, if \( f(x) \) has a higher priority than \( g(x) \), we express it as

\[
f(x) > g(x); \tag{4}
\]

The multi-objective optimization problem for coordinated charging and discharging of PHEVs in the area is formulated as follows.
Objectives

\begin{align}
\text{minimize } f(x) &= \sum_{i=1}^{N} \left( \frac{1}{2} k_i^2 x_i^2 + q_i^2 x_i^2 \right) \\
\text{subject to } g(x) &= \sum_{i=1}^{P} \sum_{k=0}^{N-1} \left( s_i x_i - \frac{1}{n} \right) \leq 0; \quad i \in V_1
\end{align}

Constraints

- Inequality constraints on \( x_i^k \) as

\begin{align}
&\sum_{i=1}^{N} \sum_{k=0}^{N-1} x_i^k = \sum_{i=1}^{N} \sum_{k=0}^{N-1} s_i x_i - \frac{1}{n} \leq 0; \quad i \in V_1
\end{align}

- Equality constraints on \( x_i^k \) as

\begin{align}
\sum_{i=1}^{N} \sum_{k=0}^{N-1} k_i^2 x_i^k = \sum_{i=1}^{N} \sum_{k=0}^{N-1} s_i x_i - \frac{1}{n} = 0; \quad i \in V_1
\end{align}

- Inequality priority constraint on objective functions as

\begin{align}
f(x) > g(x) > h(x);
\end{align}

where \( s_i^k \) and \( t_i^k \) are positive constants used for the total discharging revenue function \( g(x) \) and the total charging cost function \( h(x) \), respectively.

4. Optimal algorithm

4.1. Problem transformation

To solve the multi-objective optimization problem in an efficient manner, we decouple the problem into the single-objective optimization problems based on the objective functions’ priorities with respect to charging and discharging stages of PHEVs.

From the equality constraints (7), we have

\begin{align}
\sum_{i=1}^{N} \sum_{k=0}^{N-1} x_i^k &= \sum_{i=1}^{N} \sum_{k=0}^{N-1} s_i x_i - \frac{1}{n}; \quad i \in V_1
\end{align}

which are equivalent to

\begin{align}
\sum_{i=1}^{N} \sum_{k=0}^{N-1} k_i^2 x_i^k &= \sum_{i=1}^{N} \sum_{k=0}^{N-1} s_i x_i - \frac{1}{n}; \quad i \in V_1
\end{align}

Let \( d^k \) denote the aggregated power between the PHEVs and grid at time \( k \) in the area as:

\begin{align}
d^k &= \sum_{i=1}^{N} x_i^k; \quad 8k = 0; \forall k; \forall i \in V_1
\end{align}

Combining (10) with (11), the multi-objective optimization problem (5)-(8) can be decoupled as follows.
* Optimal Load Shifting Problems with a Higher Priority
1) For the discharging stage:

\[
\min_{k=0} \sum_{i \in V_1} \left( \left(x^k_i - \bar{x}^k_i \right)^2 \right)
\]

s.t. \[
\sum_{i \in V_1} \left( x^k_i - \bar{x}^k_i \right) = P \left( \sum_{i \in \mathcal{I}^1} s_{i} \right) - \sum_{i \in V_2} \left( x^k_i - \bar{x}^k_i \right), \quad k = 0, \ldots, \nu \equiv \mathcal{N} - 1;
\]

\[
X_{\mathcal{I}^1}^{k=0} \left( x^k_i \right)^2 \leq 2 \bar{r}_i^k, \quad \forall i \in \mathcal{I}^1;
\]

\[
0 \leq x^k_i - \bar{x}^k_i \leq 0; 8k = 0; k^{\nu} \equiv 1; i \in V_1;
\]

\[
x^k_i = \sum_{i \in \mathcal{I}^1} s_{i} \max; 8i \in V_1;
\]

\[
x^k_i = d^k; 8k = 0; k^{\nu} \equiv 1;
\]

\[
\sum_{i \in V_1} x^k_i = 0; k^{\nu} \equiv 1;
\]

(12)

2) For the charging stage:

\[
\min_{k=k^{\nu}} \sum_{i \in V_1} \left( \left(x^k_i - \bar{x}^k_i \right)^2 \right)
\]

s.t. \[
\sum_{i \in V_1} \left( x^k_i - \bar{x}^k_i \right) = P \left( \sum_{i \in \mathcal{I}^1} s_{i} \right) - \sum_{i \in V_2} \left( x^k_i - \bar{x}^k_i \right), \quad k = 0, \ldots, \nu \equiv \mathcal{N} - 1;
\]

\[
X_{\mathcal{I}^1}^{k=k^{\nu}} \left( x^k_i \right)^2 \leq 2 \bar{r}_i^k, \quad \forall i \in \mathcal{I}^1;
\]

\[
0 \leq x^k_i - \bar{x}^k_i \leq 0; 8k = 0; k^{\nu} \equiv 1; i \in V_1;
\]

\[
x^k_i = \sum_{i \in \mathcal{I}^1} s_{i} \max; 8i \in V_1;
\]

\[
x^k_i = d^k; 8k = 0; k^{\nu} \equiv 1;
\]

\[
\sum_{i \in V_1} x^k_i = 0; k^{\nu} \equiv 1;
\]

(13)

where \( k^{\nu}, s^{\min} \) and \( d^k \) are the optimization variables in (12) and (13).

* Total Costs Minimization Problems with a Lower Priority
1) For the discharging stage:

\[
\max_{k=0} \sum_{i \in \mathcal{I}^1} \left( x^k_i \right)^2
\]

s.t. \[
\sum_{i \in \mathcal{I}^1} x^k_i = P \left( \sum_{i \in \mathcal{I}^1} s_{i} \right); k = 0, \ldots, \nu \equiv \mathcal{N} - 1;
\]

\[
0 \leq x^k_i \leq 0; 8k = 0; k^{\nu} \equiv 1; i \in V_1;
\]

\[
x^k_i = 0; \quad \forall i \in \mathcal{I}^1;
\]

\[
x^k_i = d^k; 8k = 0; k^{\nu} \equiv 1;
\]

\[
\sum_{i \in \mathcal{I}^1} x^k_i = 0; k^{\nu} \equiv 1;
\]

(14)

2) For the charging stage:

\[
\min_{k=k^{\nu}} \sum_{i \in \mathcal{I}^1} \left( x^k_i \right)^2
\]

s.t. \[
\sum_{i \in \mathcal{I}^1} x^k_i = P \left( \sum_{i \in \mathcal{I}^1} s_{i} \right); k = 0, \ldots, \nu \equiv \mathcal{N} - 1;
\]

\[
0 \leq x^k_i \leq 0; 8k = 0; k^{\nu} \equiv 1; i \in V_1;
\]

\[
x^k_i = \sum_{i \in \mathcal{I}^1} s_{i} \max; 8i \in V_1;
\]

\[
x^k_i = d^k; 8k = 0; k^{\nu} \equiv 1;
\]

\[
\sum_{i \in \mathcal{I}^1} x^k_i = 0; k^{\nu} \equiv 1;
\]

(15)

where \( x^k_i \) are the optimization variables of (14) and (15).

4.2. Water-filling-based algorithm for optimal load-shifting problems
In [8], the water-filling algorithm is applied to solve an optimal load shifting problem of using plug-in electric vehicles’ charging and discharging. This paper utilizes this algorithm to solve (12) and (13), which is presented as follows.

Step 1. Setting \( s^{\min} = \frac{1}{2} \) for initialization.

Step 2. Using the decentralized water-filling-based algorithm [8] to solve the problem:
Denote the optimal solution of (16) by \((d_{k}^{in})^{\ast}\). After convergence, we obtain a time stamp \(k_{in}\) such that
\[
d_{k}^{in} = 0;8k < k_{in}; \text{ and } d_{k}^{in} > 0;8k, k_{in};
\]
Denote the water level of \(k = k_{in}\) by \(s_{in}\) such that
\[
s_{in} = d_{k}^{in} + q_{k}^{in};
\]
Step 3. Using the decentralized water-filling-based algorithm [8] to solve the problem:
\[
\min_{k=0} s_{i} \cdot d_{k}^{i} + q_{k}^{i};
\]
\[
\sum_{i \in Y_{1}} s_{i} \cdot d_{k}^{i} = 0;8k = 0;6\cdot N; i = 1;
\]
\[
\sum_{i \in Y_{2}} s_{i} \cdot C_{i}^{max};
\]
Denote the optimal solution of (19) by \((d_{k}^{out})^{\ast}\). After convergence, we obtain a time stamp \(k_{out}\) such that
\[
d_{k}^{out} < 0;8k < k_{out}; \text{ and } d_{k}^{out} = 0;8k < k_{out};
\]
Denote the water level of \(k = k_{out}\) by \(s_{out}\) such that
\[
s_{out} = d_{k}^{out} + q_{k}^{out};
\]
Step 4: Comparing two water levels \(s_{in}\) and \(s_{out}\).
1) If \(s_{in} < s_{out}\), then \((d_{k}^{i})^{\ast} = (d_{k}^{out})^{\ast} = k_{out}^{\ast}, k_{in}^{\ast} = k_{out}^{\ast}\) and \(s_{min} = s_{i}\);
2) If \(s_{in} > s_{out}\), then we set a new auxiliary variable \(s_{\pm} = (s_{i} + s_{i})/2\) and \(s^{\pm} = \pm\) Running Step 2 and 3 again and updating \(s_{\pm}\) and \(s_{\pm}\) by
\[
\begin{align*}
\hat{s} = \pm s_{\pm} = s_{\pm} \text{ if } s_{in} < s_{out}; \\
\hat{s} = s_{\pm} = s_{\pm} \text{ if } s_{in} > s_{out}; \\
\hat{s} = \pm (s_{\pm} + s_{\pm})/2:
\end{align*}
\]
Step 5: Rerunning step 1 to step 4 until \(s_{in} = s_{out}\), then \((d_{k}^{i})^{\ast} = (d_{k}^{out})^{\ast} = (d_{k}^{in})^{\ast}, k_{y} = k_{out}^{\ast}\) and \(s_{min} = s_{i}\).
After getting the solution of (12) and (13), we present the consensus-based optimization method to solve (14) and (15) subsequently.

4.3. Consensus-Based Method for Total Costs Minimization Problems
The consensus-based optimization method is proposed in our previous work [9], in which the global minimization of dynamic resource allocation problem is achieved using this method. This paper applies this method to PHEVs discharging and charging scenario solving (14) and (15), which is interpreted as follows.
Step 1. Initializing the variables to be updated through the equation (7) in [9].
Step 2. Let each PHEV \(i\) at time \(k\) have its own copy of the Lagrange multiplier to satisfy the aggregated power equality constraints in (14) and (15), and updating the Lagrange multiplier such that
all Lagrange multipliers reach consensus at the optimal value according to the consensus-based iteration (8a) in [9];

Step 3. Let each PHEV\(i\) have an alternating Lagrange multiplier to satisfy the target SOC equality constraints in (14) and (15), and updating the alternating Lagrange multiplier according to the consensus-based iteration (8b) in [9];

Step 4. Mapping the estimated power state of PHEV\(i\) at time \(k\) into the interval \([-¹x_i;¹x_i]\) according to the nonlinear projection (8c) in [9];

Step 5. Due to the nonlinear projection, the estimated power state may not be a feasible solution for (14) and (15). To overcome this issue, let each PHEV\(i\) at time \(k\) associate with a surplus variable to temporarily store the resulting deviation. Then updating the surplus variables according to the consensus-based iteration (8d) in [9], in which the surplus variables can be averaged with its neighbours and converge to zero.

Step 6. Rerunning step 2 to step 5 until all variables converge to the optimal value.

5. Simulation

In this section, the case of high-level PHEV penetration with 10 homogenous PHEVs is utilized to demonstrate the effectiveness of the proposed hierarchical algorithm. In this example, the starting time of all PHEVs is assumed to be 1 and the ending time is assumed to be 84, corresponding to 19:00 to 07:00 (next day). The sampling interval \(T = 8.5714\) min. We set \(s = 20\%\) and other PHEVs parameters are given in Table 1. The simulation results are shown in Figure 2 and Figure 3.

Table 1. Simulation parameters of ten PHEVs.

| Variables | PHEV No. | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|-----------|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| \(k_i^{in} - k_i^{out}\) (kW) |          | 3.5   | 6.0   | 6.2   | 7.2   | 9.5   | 8.6   | 8.5   | 7.5   | 6.5   | 5.9   |
| \(C_i^{max}\) (kWh) |          | 60.54 | 68.26 | 70.64 | 63.95 | 72.32 | 70.93 | 65.93 | 69.29 | 68.15 | 59.41 |
| \(r_i^{in}\) (%) |          | 95.53 | 90.34 | 93.80 | 86.56 | 89.64 | 95.07 | 93.71 | 95.55 | 92.21 | 85.39 |
| \(l_i^{out}\) (%) |          | 112.85| 108.82| 110.10| 115.70| 107.93| 111.77| 111.66| 109.18| 108.34| 108.53|
| \(s_i(0)\) (%) |          | 54.31 | 44.14 | 56.61 | 45.11 | 58.31 | 46.14 | 36.21 | 45.21 | 45.35 | 38.14 |

![Figure 2. Optimal load shifting with ten PHEVs](image1)

In Figure 2, one can see that the regulated total demand curve turns into a straight line under the coordinated charging and discharging scheduling. By contrast, the coordinated charging scheduling achieves the valley filling only. Figure 2 shows that the coordinated charging and discharging scheduling contributes to the power load stability better. In Figure 3, without coordinated charging scheduling, each PHEV charges its battery with maximum power as soon as it arrives. Compared with
the three modes of PHEV operation, Figure 3 demonstrates that the coordination of charging and discharging scheduling achieves the lowest daily energy payments for the users, which can reduce the total costs of the users and encourage them to provide load-shifting service for the power grid.

6. Conclusion and future work
This paper focuses on a multi-objective optimal scheduling for coordinated charging and discharging of PHEVs in the system-level power distribution network. The stability of power load with a higher priority and the total costs minimization with a lower priority are both optimized under system’s technical constraints. To solve the multi-objective optimization problem, a hierarchical optimal algorithm is proposed, in which the iterative water-filling-based algorithm and the consensus-based optimization method play a pivotal role. The simulation results demonstrate the effectiveness of the proposed algorithm. Future work will consider a more practical case in the power distribution network, where PHEVs arrive and leave randomly based on preferences and convenience of the vehicle owners. This practical situation may be solved by the proposed algorithm with a moving horizon approach.

7. References
[1] Fathabadi H 2018 Plug-in hybrid electric vehicles: replacing internal combustion engine with clean and renewable energy based auxiliary power sources IEEE Trans. Power Electron. 33 9611
[2] Hashmi A and Gul M T 2018 Integrating E-vehicle into the power system by the execution of vehicle-to-grid (V2G) terminology-A review Int. Conf. Eng. Emerg. Technol. 12 1
[3] Mou Y, Xing H, Lin Z and Fu M 2018 Decentralized optimal demand-side management for PHEV charging in a smart grid IEEE Trans. Smart Grid. 6 726
[4] Hu Z, Zhan K, Zhang H and Song Y 2018 Pricing mechanisms design for guiding electric vehicle charging to fill load valley Appl. Energy. 178 155
[5] Tajeddini M A and Kebriaei H 2019 A mean-field game method for decentralized charging coordination of a large population of plug-in electric vehicles IEEE Syst. J. 13 854
[6] Kaiser A, Nguyen A, Pham R, Granados M and Le H T 2018 Efficient Interfacing Electric Vehicles with Grid using Bi-directional Smart Inverter IEEE Transp. Electrif. Conf. Expo. 1 714
[7] López M A, De La Torre S, Martín S and Aguado J A 2015 Demand-side management in smart grid operation considering electric vehicles load shifting and vehicle-to-grid support Int. J. Electr. Power Energy Syst. 64 689
[8] Xing H, Fu M, Lin Z and Mou Y 2016 Decentralized optimal scheduling for charging and discharging of plug-in electric vehicles in smart grids IEEE Trans. Power Syst. 31 4118
[9] Li W, Lin Z, Xu Y, Zhang J, Song S, Wang W and G. Yan 2017 An expanded distributed algorithm for dynamic resource allocation over strongly connected topologies IEEE Int. Conf. Control Sci. Syst. Eng. 20 500

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