Suppression of Complex Hysteretic Resonances in Varying Compliance Vibration of a Ball Bearing

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It is traditionally considered that, due to the Hertzian contact force-deformation relationship, the stiffness of rolling bearings has stiffening characteristics, and gradually researchers find that the supporting characteristics of the system may stiffen, soften, and even coexist from them. The resonant hysteresis affects the stability and safety of the system, and its jumping effect can make an impact on the system. However, the ball bearing contains many nonlinearities such as the Hertzian contact between the rolling elements and raceways, bearing clearance, and time-varying compliances (VC), leading great difficulties to clarify the dynamical mechanism of resonant hysteresis of the system. With the aid of the harmonic balance and alternating frequency/time domain (HB-AFT) method and Floquet theory, this paper will investigate the hysteretic characteristics of the Hertzian contact resonances of a ball bearing system under VC excitations. Moreover, the linearized dynamic bearing stiffness of the system will be presented for assessing the locations of VC resonances, and the nonlinear characteristics of bearing stiffness will also be discussed in depth. Our analysis indicates that the system possesses many types of VC resonances such as the primary, internal, superharmonic, and even combination resonances, and the evolutions of these resonances are presented. Finally, the suppression of resonances and hysteresis of the system will be proposed by adjusting the bearing clearance.

1. Introduction

Rolling-element bearings are characterized by rolling motion of balls or rollers in bearing raceway, often referred to as antifriction bearings in comparison to the sliding motion of regular sleeve bearings [1]. With the advantage of small friction and low-cost maintenance, rolling bearings are the main supporting parts of rotating machinery and machine tools, and also in many cases, the designers use a rolling bearing only because it is easier to select from a manufacturer’s catalog [2]. A major limitation of rolling bearings is that they are subjected to very high alternating and time-varying compliance (VC) oscillating Hertzian stresses at the rolling contacts. Due to the nonlinearities of Hertzian contact and bearing clearance, rolling bearing can bring complex dynamic hysteresis and jumping vibrations to its bearing-rotor systems, which is a basic scientific problem in the field of bearing-rotor dynamics [3–6].

There are two major effects of a health rolling bearing with respect to machine vibration [1]. The first one is the spring characteristics to support a structure, and the second effect occurs because it is an inherent VC excitation source producing time-varying forces. The VC excitation is an inevitable parametrical excitation to generate vibrations to the rotor system, when the rolling elements carry external load moving along the bearing raceway [7]. Vibrations
originating in other parts of the machine can also excite and influence the rolling bearing [2], and in recent years, in-depth studies have been carried out gradually on the dynamic influences on some special operation conditions such as starved lubrication [8, 9] and nonlinear instability [5, 6]. As far as VC vibrations are concerned, Sunnersjö [7] studied dynamic influences on some special operation conditions such originating in other parts of the machine can also excite and influence the rolling bearing [2], and in recent years, in-depth studies have been carried out gradually on the dynamic influences on some special operation conditions such as starved lubrication [8, 9] and nonlinear instability [5, 6]. As far as VC vibrations are concerned, Sunnersjö [7] studied a linear bearing model focusing on the severity of VC vibrations for a cylindrical roller bearing considering the rotor mass and bearing cage speed, which showed that large movements occur at low speed and heavy load. Fukata et al. [10] presented beating and chaotic VC vibrations at critical speeds, which considered a two-degree-of-freedom ball bearing model including the nonlinearities of the Hertzian contact and bearing clearance. Focused on this classical model, the period-doubling and quasiperiodic route to chaos in the VC vibrations has been found by Mevel and Guyader [11, 12], and the effects of the unbalance factors [13], bearing clearance [14, 15], and damping and radial loads [16, 17] have been studied subsequently.

Contact nonlinearity is a common nonlinear factor in scientific technology and engineering applications [18, 19]. It has been shown that contact resonances can bring soft resonant hysteresis and jump vibration to the Hertzian contact system with harmonic excitation [20–22]. Kovacic [23] found that constant load has a significant effect on the hysteric characteristics of the resonance response. Sankaravelu et al. [24] early indicated that dynamic hysteresis and jumps exist in VC vibrations of ball bearings. Generally speaking, a high level of VC vibrations is expected whenever the exciting is close to the Hertzian contact resonant frequencies of a ball bearing system. Elsayed et al. [25] illustrated detailed orders of the linear vibration modes for a ball bearing system through finite element modeling analysis. Zhang et al. [5] found that the primary contact resonances of a balanced rigid-rotor ball bearing model can exhibit soft spring behaviors for period-1 VC responses, and these nonlinear vibration modes have been confirmed by experiments subsequently [17]. Besides, period-doubling and chaotic motions can emerge in the VC resonant ranges for a ball bearing system [5, 11, 14]. Dynamic hysteresis as a dangerous bifurcation [26] can bring inside impacts to the system. It has been found that some types of military aircraft had been repaired many times due to the bistability jump faults coming from there aeroengine bearings. Besides, the fatigue life of a rolling contact bearing is a function of the oscillating stress magnitude at the contact [2]. Therefore, the designers must keep in mind to prevent dynamic resonant hysteresis and jumps in bearing-rotor system, which will be addressed in this paper.

As a basic parameter of rolling bearings, bearing clearance has great significance for bearing life, installation, and thermal expansion capacity [1], and it can bring remarkable influence on bearing stiffness [2, 16, 27]. Oswald et al. [28] found that bearing life declines gradually with positive clearance and rapidly with increasing negative clearance, and the life can be maximized for a small negative operating clearance. Yakout et al. [29] presented a statistical analysis to study the correlation between the internal radial clearance and the dynamic characteristics of rolling bearings. In the field of bearing industry, it is generally considered that the adverse vibration and noise behaviors can be reduced by adjusting the ball bearings to zero clearance [30]. Particularly, clearance-free operation is usually desirable for precision or high-speed machinery, such as machine tools and turbines [2].

The motivation of the present paper is to investigate the mechanism of VC contact resonances and their hysteresis characteristics in a ball bearing system, considering control of the main structural parameters of the system such as bearing clearances and damping factors. For doing this, as our previous literature [5], the HB-AFT method will be applied to trace the VC periodic solution branch and its stability characteristics will be analyzed by using Floquet theory.

2. Characteristics of the Ball Bearings

2.1. Governing Equations of VC Motions. A classic 2-degree-of-freedom balanced ball bearing-rotor model (see Figure 1), considering the nonlinearities of Hertzian contact and bearing clearance, is verified to be effective for analysis of radial VC motions of a ball bearing [5, 10, 11, 17], and the following assumptions are required [5]:

1. There is an interference fit between the shaft and inner race of the radial ball bearings, and the outer race is fixed to a rigid support
2. The rotor shaft of the studied ball bearing-rotor system is assumed to be rigid
3. It is under pure rolling condition between the raceway and balls, and the whole system is fault-free
4. The inertia of balls introduces little dynamic effect on the system

The equation of VC motion can be expressed as

\[
m \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + c \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} W \\ 0 \end{bmatrix},
\]

with restoring forces \( F_x \) and \( F_y \)

\[
\begin{bmatrix} F_x \\ F_y \end{bmatrix} = C_b \sum_{i=1}^{N_b} (\delta_iG[\delta_i])^{(3/2)} \begin{bmatrix} \cos \theta_i \\ \sin \theta_i \end{bmatrix},
\]

and

\[
\delta_i = x \cos \theta_i + y \sin \theta_i - \delta_0,
\]

\[
\theta_i = \frac{2\pi(i-1)}{N_b + \Omega t},
\]

where \( m, c, W, \) and \( C_b \) are the equivalent mass, damping factor estimated from [31], constant force, and stiffness coefficient of the bearing system, respectively; \( G[\cdot] \) is a Heaviside function representing the contact conditions [5], obtaining the value 1 (contact case) for positive arguments and the value 0 (noncontact case) for negative or zero arguments; \( \delta_0 \) and \( \theta_i \) are radial deformation of \( i \)th ball (total number of balls, \( N_b \)) and instant angular location; \( 2\delta_0 \) is the
radial internal clearance of the bearing; and $\Omega$ denotes the cage velocity, which is related to the shaft speed $\omega_s$, ball diameter, and pitch diameter of the ball bearing system as

$$\Omega = \frac{\omega_s(1 - D_b/D_h)}{2}. \quad (5)$$

Due to the parametric VC excitation, primary parametric resonance occurs as $\Omega_{vc} = N_b\Omega$ approaches one of the resonant frequencies of equation (8); that is,

$$\Omega_{vc} \in \{\omega_{avg,xx}, \omega_{avg,yy}\}, \quad (6)$$

where $\omega_{avg,xx}$ and $\omega_{avg,yy}$, discussed in the next section, are the averaged first resonant frequencies in the vertical and horizontal radical directions of the system.

### 2.2. Ball Bearing Nonlinear Stiffness

Bearing stiffness as supporting spring is directly related to load and vibration characteristics of the bearing moving or rotating parts. Due to Hertzian point contact between each rolling element and its raceways of a rolling bearing, the total bearing load $W_i$ versus deformation $\delta_{Wi}$ along the load direction in the static condition exhibits approximately hard (i.e., stiffening) spring characteristics as [3, 4, 31]

$$W_i = \left(1 + 2 \sum_{i=1}^{N_i} \cos^{n+1}\theta_i(t_0)\right)\left(\frac{\delta_{Wi}}{C}\right)^n, \quad (7)$$

where $t$ takes certain $t_0$ in equation (4); $C$ is constant coefficient in connection with bearing structure; $n$ takes 3/2 or 10/9 for ball or roller bearing, representing the Hertzian point contact and line contact, respectively, between the rolling elements and raceways [31]; and then the static bearing stiffness $k_{W_i}$ in the load direction can be expressed as

$$k_{W_i} = \frac{dW_i}{d\delta_{Wi}} = nC \left(1 + 2 \sum_{i=1}^{N_i} \cos^{n+1}\theta_i(t_0)\right)\delta_{W_i}^{n-1} = \frac{nW_i}{\delta_{W_i}}, \quad (8)$$

and the static lateral stiffness (the bearing stiffness in orthogonal direction to the direction of $W_i$) also can be given (see [31]).

Compared with the static bearing stiffness derived in equation (8), it should be noted that the bearing stiffness is time varying and nonlinear coupling with operation conditions such as rotor speed, VC excitation, and system responses, which can make the operating bearing stiffness exhibit complex hard and soft spring characteristics [3], so it is necessary to consider the dynamic bearing stiffness of the system when specializing in hysteretic resonances.

In order to describe the time-varying bearing support characteristics fully, from equation (2), the linearized dynamic bearing stiffness in $x$ and $y$ directions of system (1) can be approximated as follows [11]:

$$\begin{bmatrix} k_{xx}(t) \\ k_{yy}(t) \end{bmatrix} = \begin{bmatrix} \frac{\partial F_x(t)}{\partial x} \\ \frac{\partial F_y(t)}{\partial y} \end{bmatrix}, \quad (9)$$

which is obtained by numerical integration of equation (1).

According to equation (9), dynamic resonant frequencies can be expressed as

$$\begin{bmatrix} \omega_{xx}(t) \\ \omega_{yy}(t) \end{bmatrix} = \begin{bmatrix} \frac{k_{xx}(t)}{m} \\ \frac{k_{yy}(t)}{m} \end{bmatrix}, \quad (10)$$

and then, the equivalent values of dynamic resonant frequencies of the system are averaged as [5]

$$\begin{bmatrix} \omega_{avg,xx} \\ \omega_{avg,yy} \end{bmatrix} = \frac{1}{K_{int}} \sum_{k_{int}=1}^{K_{int}} \begin{bmatrix} \omega_{xx}(t(k_{int})) \\ \omega_{yy}(t(k_{int})) \end{bmatrix}, \quad (11)$$

where $K_{int}$ is the number of numerical integration steps when solving equation (1).

### 3. Methodology

The steady-state periodic response of a linear system is always stable, but it may get unstable in a nonlinear dynamic system. The analysis of periodic motion types and their bifurcation mechanism of a nonlinear system is beneficial to abnormal vibration (e.g., super/subharmonic, combination, chaotic, bistable, and hysteric responses) control and even machine design. The HB-AFT method combined with Floquet stability analysis is an effective way to trace a VC periodic solution branch and its stability characteristics of ball bearing system [5, 6].

Introducing nondimensional time $\tau = \Omega t$ and nondimensional displacements $X = x/\delta_0$ and $Y = y/\delta_0$ to equation (1), then the following nondimensional motion equation is obtained:

$$\frac{d^2}{d\tau^2}X(\tau) = F\left(\frac{d^2}{d\tau^2}X(\tau)X(\tau, \lambda)\right), \quad (12)$$
where $\lambda$ is a control parameter. Take the VC excitation period $T = 2\pi$ of equation (12) as period of the response, $X (r + T)$. Then, $X (r)$ and $F (\cdot)$ can be represented by Fourier series as

$$\begin{bmatrix} X \\ F \end{bmatrix} = \begin{bmatrix} a_0 \\ c_0 \end{bmatrix} + \sum_{k=1}^{K} \begin{bmatrix} a_k \\ c_k \end{bmatrix} \cos (kr) - \begin{bmatrix} b_k \\ d_k \end{bmatrix} \sin (kr).$$

(13)

First, inserting equation (13) into equation (12), the harmonic balance yields an algebraic relationship

$$g (P, Q, \lambda) = 0,$$

(14)

where

$$\begin{bmatrix} P \\ Q \end{bmatrix}^T = \begin{bmatrix} a_0, a_1, b_1, a_2, b_2, \ldots, a_{K}, b_{K} \\ c_0, c_1, d_1, c_2, d_2, \ldots, c_{K}, d_{K} \end{bmatrix},$$

(15)

are the harmonic terms of $X (r)$ and $F (\cdot)$, respectively.

Second, in order to solve $P$ from equation (14), one needs to formulate $Q$ by $P$, and this can be achieved by the discrete Fourier transforms as

$$X (n) = \text{Real} \left\{ \sum_{k=0}^{K} P_k e^{i(2\pi k n/N)} \right\},$$

(16)

$$Q_k = \frac{\phi}{N} \sum_{n=0}^{N-1} F(n) e^{i(-2\pi k n/N)}.$$

Herein, $P_k = a_k + ib_k$; $N$ is the numbers of samplings in the time domain, and $n = 0, 1, \ldots, N - 1$; $Q_k = c_k + id_k$, and $\phi$ is 1 if $n = 0$; otherwise, $\phi$ is 2; $F (n) = F (X' (n), X (n), \lambda)$ is the value of the restoring force $F (X' (r), X (r), \lambda)$ at the nth discrete time.

Third, $P$ can be obtained from equation (14) by fixed-point iteration method. Besides, the Arc-length method is adopted to bridge over the failure iteration at the turning point due to the singular iterative matrix emerging (see [5]).

Finally, Hsu’s method is applied for Floquet stability analysis of obtained periodic solutions aided as shown in our previous work [32].

4. Results and Discussion

4.1. Specifications. The JIS6306 ball bearing is adopted in this paper, which has been studied commonly during recent years [5, 6, 10–12], and the main parameters of the system studied are given in Table 1. Herein, the damping factor $c$ is estimated by [31]

$$c = (0.25 \text{ to } 2.5) \cdot 10^{-5} \cdot k,$$

(17)

where $k$ is the linearized stiffness of the ball bearing. According to Figure 2 in our paper, we take $k = 6 \times 10^7$, so the damping factor is in range of 150–1500 Ns/m. If doing experimental research, the bearing damping can be measured by experimental modal analysis referring to literature [33].

| Item                      | Value          |
|---------------------------|----------------|
| Contact stiffness $C_b$   | $1.334 \times 10^{10}$ |
| Ball diameter $D_b$ (mm)  | 11.9062        |
| Pitch diameter $D_h$ (mm) | 52.0           |
| Number of balls $N_b$     | 8              |
| Equivalent mass $m$ (kg)  | 20             |
| Damping factor $c$ (Ns/m) | 150–1500       |
| Radial load $W$ (N)       | 196            |

Table 1: Specifications and parameters of JIS6306 ball bearing.

In the following, periodic solutions are traced by HB-AFT method with the cage speed $\Omega$ as the controlled parameter, where a periodic VC motion is called period-$n$ motion if the period of the response is $n$ times the VC parametrically excited period $T_{vc} = 2\pi/\Omega_{vc}$. Herein, the numerical verifications to the HB-AFT tracing results are simulated by the classical explicit Runge-Kutta numerical integration process of literature [34].

4.2. Complex Resonance and Stiffness Characteristics. For $\delta_0 = 6.0 \mu m$ $c = 200$ Ns/m, Figure 3 shows period-1 frequency-response curves of the system, where vertical and horizontal primary resonant amplitudes are traced, respectively. It is clear that soft hysteretic resonance occurs in the vertical direction since the frequency-response curve bends to the left, and the system exhibits coexistence of soft and hard spring characteristics in the horizontal direction, which agrees with previous theoretical and experimental studies [16, 17]. In terms of the bifurcation and stability of the period-1 solution branch considered, subcritical period-doubling bifurcations emerge at $A_1$ and $A_2$, cyclic fold bifurcations occur at turning points $A_3$ and $A_4$, and secondary Hopf bifurcations include quasiperiodic motions at $A_5$ and
Amplitude ($\mu m$)

A_1
A_2
A_5 A_6
A_4
Horizontal amplitude y
Vertical amplitude x

0
5
10
15
140 160 180 200 220 240 260
 Ω (rad/s)

Figure 3: Stable (solid) and unstable (dashed) period-1 frequency-response peak-to-peak curves in (a) the x-direction and (b) the y direction for $\delta_0 = 6.0 \mu m$ and $c = 200$ Ns/m.

The cyclic fold bifurcation and the subcritical bifurcation all can lead to instabilities and dynamic jump to the ball bearing systems. Herein, the period-doubling bifurcations come from one-to-two internal resonances [5, 6], and it is clear that strong couplings between the two degrees of freedom of the system arise in the period-2 response range (see range B-B in Figure 4). At this time, the VC motions exhibit complex hysteresis characteristics of bistability and jumped phenomenon. The most basic motion behavior is the coexistence of period-1 and period-2 motions in B-B range, for example, the bistability characteristics when $\Omega = 230$ rad/s (see Figures 5 and 6).

Meanwhile, as shown in Figure 2, the system possesses two types of time-varying stiffness characteristics, since the dynamic stiffness of the system is related to the response of the system [1, 3]. Therefore, compared to the dynamic stiffness of equation (10), the static stiffness of equation (8) lacks comprehensive perspectives to analyze the stiffness nonlinearity of the system. Moreover, it is worth noting that the coexistence of various stiffness characteristics makes it more difficult to predict and adjust the hysteretic resonances. For example, as shown in Figure 7(a), although the primary resonant hysteresis of period-1 VC motion has been suppressed as the damping coefficient $c$ increasing to 1400 Nm/s, the system still holds hysteretic jumping behaviors due to the nonlinear dynamic stiffness related to the period-2 motions (see Figure 7(b)).

Unstable period 1 solution branch A_5-A_6 in Figure 4 is induced by secondary Hopf bifurcating instability and strongly coupled vibrations produced between the two degrees of freedom of the system in C-C range. Moreover, the complex characteristics of bistabilities and jumps also emerge in this range. For instance, as shown in Figures 8 and 9, quasiperiodic and chaotic motions coexist at $\Omega = 174.3$ rad/s. In terms of the frequency spectrum of the quasiperiodic response considered (see Figure 9(a)), the values of incommensurable dimensionless VC frequencies $p$ and $q$ are about 0.663 and 1.337, respectively, and $(p+q)/2 = 1$, where the value 1 is the dimensionless VC excitation frequency $\Omega_{vc} = N_0\Omega$. The values $p\Omega = 115.561$ rad/s and $q\Omega = 233.039$ rad/s locate near the resonant excitation frequency ranges B-B and D-D, respectively, so it is reasonable to suppose that the case of combination resonance $\omega_p + \omega_q = 2\Omega_{vc}$ occurs in the segment of C-C. It has been found in some pieces of literature [5, 6, 10–16] that the quasiperiodic motion is a typical characteristic of VC vibrations in rolling bearing system, but so far, its physical trigger mechanism is not given. In general, the typical combination resonance of multiple-degree-of-freedom system often can arise in quasiperiodic motions [36–38]. Therefore, it can be considered that the combination resonance may be one of the physical mechanisms of generation of VC quasiperiodic motions. In addition, the existence of combination resonance makes the response characteristics of the system more complicated.

4.3. Control of Resonances and Hysteresis. In certain applications, particularly in high-speed machines or high precision machines, such as machine tools, it is possible to eliminate clearance and introduce small interference (negative clearance) in the bearings, because it increases the bearing stiffness and reduces the run-out and noise due to elastic deformation or clearance [2, 30]. However, the interference should be ensured not to be excessive, because the interference can result in extrarolling contact pressure to reduce the fatigue life of the bearing. Figure 10 shows the change of the resonant amplitude of the two degrees of freedom of the system as bearing clearance $\delta_0$ reduces from 6 $\mu m$ to 0 $\mu m$. Herein, the resonant amplitude in y direction moves to high frequency as $\delta_0$ reduces to 0 $\mu m$, because a
small bearing clearance can stiffen the support of the shaft [2], and it is worth noting that the resonant response is suppressed effectively and hard resonant hysteresis emerges when $\delta_0 = 2 \mu m$. Moreover, the combination and one-to-two internal resonances are eliminated in the condition of clearance-free operation (i.e., $\delta_0 = 0 \mu m$). However, the resonant hysteresis and jumps still exist in the two degrees of freedom of the system.

**Figure 4**: Numerical bifurcation diagram of (a) $X(\tau)$ and (b) $Y(\tau)$ when $\Omega$ is sweeping up (blue dots) and down (magenta dots) for $\delta_0 = 6.0 \mu m$ and (c) $c = 200 \text{Ns/m}$.

**Figure 5**: Time series and phase orbits of coexisting period-1 (black line) and period-2 (blue line) motions for $\delta_0 = 6.0 \mu m$ and $c = 200 \text{Ns/m}$ when $\Omega = 230 \text{rad/s}$.

**Figure 6**: Power spectra of $X(\tau)$ of (a) period-1 and (b) period-2 motions of $X(\tau)$ for $\delta_0 = 6.0 \mu m$ and $c = 200 \text{Ns/m}$ when $\Omega = 230 \text{rad/s}$. 
One can see in Figure 11(a) that the soft hysteresis in vertical resonant range gradually eliminates as the bearing clearance $\delta_0$ reduces from 0.2 $\mu$m to $-0.6$ $\mu$m, and then the vertical resonant amplitude gets larger as $\delta_0$ continues to reduce from $-0.6$ $\mu$m to $-1.2$ $\mu$m (see Figure 11)(b). This difference is due to the variation of the stiffness $c$.

Figure 7: Influence of damping on (a) period-1 frequency-response peak-to-peak curves in the $x$-direction and (b) numerical bifurcation diagram of $X(\tau)$ when $\Omega$ is sweeping up (blue dots) and down (magenta dots) for $\delta_0 = 6.0$ $\mu$m and $c = 1400$ Ns/m. Herein, the arrows express the jumping locations and directions of the responses, and solid and dashed lines refer to stable and unstable responses, respectively.

Figure 8: Time series $X(\tau)$ and their Poincare sections of coexisting quasiperiodic (black line or dots) and chaos (blue line dots) motions for $\delta_0 = 6.0$ $\mu$m and $c = 200$ Ns/m when $\Omega = 174.3$ rad/s.

Figure 9: Power spectra of $X(\tau)$ of (a) quasiperiodic and (b) chaos motions of $X(\tau)$ for $\delta_0 = 6.0$ $\mu$m and $c = 200$ Ns/m when $\Omega = 174.3$ rad/s.
characteristics. As the bearing clearance decreases to $-0.8\,\mu m$, $-1.0\,\mu m$, and $-1.2\,\mu m$, the stiffness in the horizontal direction is close to the stiffness in the vertical direction, which induces strong coupling resonance between the two degrees of freedom of the system. For example, when $\delta_0$ takes $-1.2\,\mu m$, the two degrees of freedom are remarkably coupled in horizontal and vertical resonant ranges (see Figure 12(a)), respectively, and combination resonance again arises between these two resonant ranges (see $E_1-E_2$ branch in Figure 12(b)). In other words, the vibration amplitudes, bearing dynamic loads, and even the bearing life can be optimized for a proper small negative operating clearance, which agrees well with the results of literature [28].

In terms of hysteresis characteristics considered, due to the effect of clearance nonlinearity elimination as $\delta_0$ is negative, at this moment, the Hertzian contact nonlinearity of the system plays a dominant role. Therefore, as illustrated in Figure 12(a), the resonant amplitudes nearly all bend to left as the Hertzian contact deformation exhibits a stiffening nonlinearity relationship. In addition, compared with resonant responses for positive bearing clearance cases shown in Figure 10, for $\delta_0 = -1.2\,\mu m$, the resonant excitation frequency is higher in the horizontal direction than in the vertical direction (see Figure 12(a)), which agrees with the equivalent dynamic resonant frequencies predicted by equation (11) (see Figure 13(a)), and Figure 13(b) illustrates that selection of proper bearing clearance can suppress the hysteresis in the vertical direction effectively resonance and make the system operate in a quasilinear stiffness interval.

Figure 14 gives the power spectra of the dynamic displacement parameter (DDP) [7]: $\text{DDP} = x_{\text{peak-to-peak}} + y_{\text{peak-to-peak}}$, where $x_{\text{peak-to-peak}} = x(t)_{\text{max}} - x(t)_{\text{min}}$ and $y_{\text{peak-to-peak}} = y(t)_{\text{max}} - y(t)_{\text{min}}$. In order to reflect the hysteresis characteristics in this figure, we get the DDP value when the controlled parameter $\Omega$ runs up and down by the way in literature [6]. In general, as shown in Figure 14, the system contains many types of VC resonant
characteristics, such as primary resonances $R_1$ and $R_2$ in vertical and horizontal directions, respectively, and also superharmonic resonances $R_3$ and $R_4$ are aroused in these two directions. Moreover, the forms of $\omega_p + \omega_q = 2\Omega_{vc}$ and $4\cdot\omega_p + \omega_q = 8\Omega_{vc}$ combination resonances labeled as $R_5$ and $R_6$, respectively, exist for this system. Figure 15 shows that equation (11) can provide a good prediction of typical resonance locations, where $\omega_p = \omega_{avg.yy}$ and $\omega_q = \omega_{avg.xx}$, so it is a valuable way for analysis of VC resonances for the ball bearing systems.
5. Conclusions

This paper focuses on the characteristics of Hertzian contact resonances and dynamic hysteresis of a ball bearing system. The HB-AFT method and Floquet theory are applied to trace the periodic response behaviors of the system. It is found that the system contains many types of resonances such as primary, combination, and superharmonic resonances, which makes the response characteristics of the system more complicated, and the linearized dynamic bearing stiffness can be used to assess the locations of these resonances. It should be emphasized that combination resonance can bring quasiperiodic motions to the system. Moreover, the coexistence of two types of time-varying stiffness characteristics is demonstrated. It is presented that bearing clearance has significant effects on the dynamic characteristics of ball bearing system, including the bearing stiffness, resonant amplitudes, and their hysteretic behaviors. Our analysis indicates that the hysteretic resonances can be suppressed effectively by adjusting the bearing clearances around the clearance-free operations, and this method may be beneficial for the engineering study and inspection of resonant vibration control and even bearing life optimization of rolling bearing systems. In the future, the intrinsic correlations between typical resonances, Hertzian contact nonlinearity, bearing clearance, and even asymmetric stiffness characteristics of the system should be further clarified.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] T. A. Harris, Rolling Bearing Analysis, John Wiley & Sons, Hoboken, NJ, USA, 4th edition, 2001.
[2] A. Harnoy, Bearing Design in Machinery: Engineering Tribology and Lubrication, Marcel Dekker, New York, NY, USA, 2002.
[3] F. F. Ehrich, Handbook of Rotordynamics, McGraw-Hill, New York, NY, USA, 1992.
[4] M. L. Adams, Rotating Machinery Vibration: From Analysis to Troubleshooting, Taylor & Francis, Oxfordshire, UK, 2009.
[5] Z. Zhang, Y. Chen, and Q. Cao, “Bifurcations and hysteresis of varying compliance vibrations in the primary parametric resonance for a ball bearing,” Journal of Sound and Vibration, vol. 350, pp. 171–184, 2015.
[6] Z. Y. Zhang, X. T. Rui, R. Yang, and Y. S. Chen, “Control of period-doubling and chaos in varying compliance resonances for a ball bearing,” ASME Journal of Applied Mechanics, vol. 87, p. 2020, Article ID 021005.
[7] C. S. Sunnersjö, “Varying compliance vibrations of rolling bearings,” Journal of Sound and Vibration, vol. 58, no. 3, pp. 363–373, 1978.
[8] H. T. Shi, X. T. Bai, K. Zhang et al., “Influence of uneven loading condition on the sound radiation of starved lubricated full ceramic ball bearings,” Journal of Sound and Vibration, vol. 461, Article ID 114910, 2019.
[9] H. T. Shi and X. T. Bai, “Model-based uneven loading condition monitoring of full ceramic ball bearings in starved lubrication,” Mechanical Systems and Signal Processing, vol. 139, Article ID 106583, 2020.
[10] S. Fukata, E. H. Gao, T. Kondou, T. Ayabe, and H. Tamura, “On the radial vibration of ball bearings: computer simulation,” Bulletin of JSME, vol. 28, no. 239, pp. 899–904, 1985.
[11] B. Mevel and J. L. Guyader, “Routes to chaos in ball bearings,” Journal of Sound and Vibration, vol. 162, no. 3, pp. 471–487, 1993.
[12] B. Mevel and J. L. Guyader, “Experiments on routes to chaos in ball bearings,” Journal of Sound and Vibration, vol. 318, no. 3, pp. 549–564, 2008.
[13] M. Tiwari, K. Gupta, and O. Prakash, “Dynamic response of an unbalanced rotor supported on ball bearings,” Journal of Sound and Vibration, vol. 238, no. 5, pp. 757–779, 2000.
[14] M. Tiwari, K. Gupta, and O. Prakash, “Effect of radial internal clearance of a ball bearing on the dynamics of a balanced horizontal rotor,” Journal of Sound and Vibration, vol. 238, no. 5, pp. 723–736, 2000.
[15] C.-q. Bai, Q.-y. Xu, and X.-l. Zhang, “Nonlinear stability of balanced rotor due to effect of ball bearing internal clearance,” Applied Mathematics and Mechanics, vol. 27, no. 2, pp. 175–186, 2006.
[16] Z. Zhang, Y. Chen, and Z. Li, “Influencing factors of the dynamic hysteresis in varying compliance vibrations of a ball bearing,” Science China Technological Sciences, vol. 58, no. 5, pp. 775–782, 2015.
[17] Y. L. Jin, R. Yang, L. Hou, Y. S. Chen, and Z. Y. Zhang, “Experiments and numerical results for varying compliance contact resonance in a rigid rotor-ball bearing system,” ASME Journal of Tribology, vol. 139, Article ID 041103, 2017.
[18] R. A. Ibrahim, Vibo-impact Dynamics: Modelling, Mapping and Applications, Springer-Verlag, Berlin, Germany, 2009.
[19] C. F. Li, Z. X. Zhang, Q. Y. Yang, and P. Y. Li, “Experiments on the geometrically nonlinear vibration of thin-walled cylindrical shells with points supported boundary condition,” Journal of Sound and Vibration, vol. 473, Article ID 115226, 2020.
[20] R. Nayak, “Contact vibrations,” Journal of Sound and Vibration, vol. 22, no. 3, pp. 297–322, 1991.
[21] D. P. Hess and A. Soom, “Normal vibrations and friction under harmonic loads: Part 1-hertzian contacts,” Journal of Tribology, vol. 113, no. 1, pp. 80–86, 1991.
[22] E. Rigaud and J. Perret-Liaudet, “Experiments and numerical results on non-linear vibrations of an impacting hertzian contact. Part 1: harmonic excitation,” Journal of Sound and Vibration, vol. 265, no. 2, pp. 289–307, 2003.
[23] I. Kovacic, “Forced vibrations of oscillators with a purely nonlinear power-form restoring force,” Journal of Sound and Vibration, vol. 330, no. 17, pp. 4313–4327, 2011.
[24] A. Sankaravelu, S. T. Noah, and C. P. Burger, “Bifurcation and chaos in ball bearings,” in Proceedings of ASME Nonlinear and Stochastic Dynamics, Chicago, USA, August 1994.
[25] E. S. Elsayed, A. M. Elkhatib, and M. Yakout, “Vibration modal analysis of rolling element bearing,” in Proceedings of 4th International Conference on Integrity, Reliability and Failure, Funchal, Madeira, June 2013.
[26] J. M. T. Thompson, H. B. Stewart, and Y. Ueda, “Safe, explosive, and dangerous bifurcations in dissipative dynamical systems,” Physical Review E, vol. 49, no. 2, pp. 1019–1027, 1994.
[27] S. H. Ghafari, E. M. Abdel-Rahman, F. Golnaraghi, and F. Ismail, “Vibrations of balanced fault-free ball bearings,” Journal of Sound and Vibration, vol. 329, no. 9, pp. 1332–1347, 2010.
[28] F. B. Oswald, E. V. Zaretsky, and J. V. Poplawski, “Effect of internal clearance on load distribution and life of radially loaded ball and roller bearings,” Tribology Transactions, vol. 55, no. 2, pp. 245–265, 2012.
[29] M. Yakout, M. G. A. Nassef, and S. Backar, “Effect of clearances in rolling element bearings on their dynamic performance, quality and operating life,” Journal of Mechanical Science and Technology, vol. 33, no. 5, pp. 2037–2042, 2019.
[30] Schaeffler Technologies AG & Co KG, The Design of Rolling Bearing Mountings: Design Examples Covering Machines, Vehicles and Equipment, Fag Bearing company Ltd., Schweinfurt, Germany, 2012.
[31] K. Erwin, Dynamics of Rotor and Foundation, Springer-Verlag, Berlin, Germany, 1993.