On Gauge Bosons in the Matrix Model Approach to M Theory

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We discuss the appearance of $E_8 \times E_8$ gauge bosons in Banks, Fischler, Shenker, and Susskind’s zero brane quantum mechanics approach to M theory, compactified on the interval $S^1/Z_2$. The necessary bound states of zero branes are proven to exist by a straightforward application of T-duality and heterotic $Spin(32)/Z_2$-Type I duality. We then study directly the zero brane Hamiltonian in Type $I'$ theory. This Hamiltonian includes couplings between the zero branes and background Dirichlet 8 branes localized at the orientifold planes. We identify states, localized at the orientifold planes, with the requisite gauge boson quantum numbers. An interesting feature is that $E_8$ gauge symmetry relates bound states of different numbers of zero branes.

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1. Introduction

Recently, an intriguing proposal for the microscopic description of M theory was given in [1]. Gravitons propagating in eleven dimensions are bound states of type IIA Dirichlet zero branes [2]. The Ramond-Ramond 1-form charge carried by the zero branes maps to momentum in the 11th dimension. In the infinite momentum frame, the other degrees of freedom of the IIA theory decouple, leaving a large N matrix quantum mechanics describing the interactions of the zero branes [3,1]. Several tests and explorations of this conjecture have been performed in [4,5]. Previous results about maximally supersymmetric quantum mechanics and zero brane dynamics, which play a role in the conjecture of [1], were obtained in [3,7,8,9].

It is of interest to understand matrix theory in more general backgrounds with less supersymmetry. A first step in that direction is to recover the physics of M theory compactified on $S^1/Z_2$, which is believed to govern the behavior of the $E_8 \times E_8$ heterotic string at nonzero coupling [10]. In this description, the $E_8 \times E_8$ gauge bosons propagate in ten dimensions on the ends of the interval $S^1/Z_2$. In this note, we make the simple observation that perturbative T-duality symmetries of string theory together with heterotic/type I duality [2,11] suffice to prove the existence of the bound states of D0 branes which fill out $E_8 \times E_8$ gauge multiplets. We also formulate conditions for the required states in the D0 brane quantum mechanics, and describe states satisfying these conditions for the cases $N = 1$ and $N = 2$.

One approach to the study of nontrivial backgrounds in the matrix theory involves starting with type IIA theory (as in §9 of [1]). One then constructs M theory as the large N quantum mechanics of D0 branes (including all associated BPS degrees of freedom) in the compactified IIA theory. This is the approach that we take in this paper. In section two we review the relevant matrix quantum mechanics which controls the dynamics of zero branes in Type $I'$ theory, as written down in [12]. In section three we present the simple duality argument proving the existence of the required bound states of zero branes to the D8 branes and the orientifold planes. In section four we return to a discussion of the states directly in the quantum mechanics. Section five contains some additional remarks.

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1 One of the most interesting challenges is to understand how this proposal could be implemented in situations where BPS states do not exist.
2. The Type I’ Formulation

We begin with type IIA on the orientifold $S^1/Z_2$, also known as type I’ theory [13]. We will call the $S^1/Z_2$ dimension the tenth dimension, and time the first dimension. In this theory, which is T-dual to type I string theory, we consider the locus on Narain moduli space where 8 D8 branes are at each orientifold plane. This gives a $Spin(16) \times Spin(16)$ gauge symmetry arising from open strings with ends on the 8-branes [14].

So far we are in nine dimensions. As explained in [2], the theory grows a tenth dimension as $\lambda_{I'} \to \infty$. The type I’ coupling, $\lambda_{I'}$, determines the size $R_{11}$ of the eleventh dimension of M theory, as [2]

$$R_{11} = \frac{\lambda_{I}^{2/3}}{I_{I'}}.$$  

As in [1], states with momentum in the eleventh dimension have nontrivial Ramond-Ramond 1-form charge. The $Spin(16) \times Spin(16)$ gauge bosons coming from 8-8 strings do not carry this charge, and do not have momentum in the eleventh dimension.

We need to fill out the predicted $E_8 \times E_8$ gauge group which can propagate in ten dimensions (the dimensions 1-9,11 in our notation). If we take $R_{11}$ finite, the momentum $p_{11}$ of a state in the eleventh dimension is given by

$$p_{11} = \frac{N}{R_{11}} \quad (2.1)$$

where $N$ is the zero brane charge of the state. As $R_{11} \to \infty$, only states with $N/R_{11}$ finite have nonzero $p_{11}$. The adjoint of $E_8$ decomposes as the 120 + 128 of $Spin(16)$. So far we have found a 120 propagating in the dimensions 1-9.

Some interesting aspects of the quantum mechanics of this system were presented in [12] – we follow the notation of that paper with minor modifications. The dynamics of N zero branes near the orientifold plane is (ignoring the 8 D8 branes for now) described by an $SO(N)$ quantum mechanics with 8 supersymmetries. This has coordinates

$$A_{1,10}^{I J}, \quad X_{2,...,9}^{I J}, \quad \text{and} \quad x_{2,...,9}$$

where the $A^{I J}$ are antisymmetric in the $SO(N)$ indices $I$ and $J$ while the $X^{I J}$ are traceless symmetric representations and the $x$s are singlets. The fermionic superpartners are

$$S_a^{I J}, \quad S_\dot{a}^{I J}, \quad \text{and} \quad s_\dot{a}$$

with $S_a$ in the adjoint, $S_\dot{a}$ in the traceless symmetric, and $s_\dot{a}$ a singlet. Here $a$ is an index in the $8_s$ of $SO(8)$ and $\dot{a}$ is an index in the $8_c$. 

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Let us now consider the effects of the 8 D8 branes coincident with the orientifold plane. We will need degrees of freedom coming from the 0-8 strings. Analysis of the worldsheet theory of the 0-8 strings as in §4.2 of [15] reveals that the Neveu-Schwarz sector has vacuum energy $\frac{1}{2}$, so the only massless states are the Ramond-sector states. Therefore, we must include fermions

$$\chi^I_r$$

where \( r \) is an \( SO(16) \) index. Thus we have fermions in the \((16,N)\) representation of \( SO(16) \times SO(N) \), where the \( SO(16) \) lives on the D8 branes.

This system is a modification of that considered in [12] in two respects. Firstly, we are discussing \( SO(N) \) quantum mechanics instead of \( SO(2N) \) (we allow the possibility of unpaired zero branes which are “stuck” to the orientifold plane). Also, we include the vectors of \( SO(N) \) arising from the 0-8 strings.

The Hamiltonian governing the \( SO(N) \) quantum mechanics is

$$H = Tr \left\{ \lambda^I_\nu \left( \frac{1}{2} P^2_i - \frac{1}{2} E^{10}_i \right) + \frac{1}{\lambda^I_\nu} \left( \frac{1}{2} [A_{10}, X_i]^2 - \frac{1}{4} [X_i, X_j]^2 \right) \right. $$

$$+ \left. \frac{i}{2} (-S_a [A_{10}, S_\alpha] - S_\alpha [A_{10}, S_a] + 2X_i \sigma^i_{\alpha\dot{\alpha}} \{S_\alpha, S_{\dot{\alpha}}\} ) \right\} \right. $$

$$+ \chi^I_r A_{10,ij} \chi^J_r + \chi^I_r A_{10,ij} \chi^J_r + \chi^I_r B_{1r} \chi^J_s$$

(2.2)

Here \( E_{10} \) and \( P_i \) are the momenta conjugate to \( A_{10} \) and \( X_i \). As in [12], we have ignored the overall center of mass coordinates \( x_{2,\ldots,9} \). The last term in (2.2) describes the coupling of the spacetime gauge bosons \( B_\mu \) (which come from the 8-8 strings) to the D0 branes.

There are some subtleties with this quantum mechanics system, and we will discuss its analysis in §4. For now let us note that it is not too difficult to motivate the appearance of bound states in the \( \textbf{128} \) (spinor) of \( \text{Spin}(16) \) in this quantum mechanics. Consider a single zero brane in the type \( I' \) theory. It must sit at one of the two orientifold planes (since the \( Z_2 \) symmetry would require zero branes to leave the orientifold plane in pairs) [14]. Quantizing the zero modes of the fermionic open strings between the 0-brane and the 8 D8 branes gives the ground state the quantum numbers of the \( \textbf{128} \). Now suppose there is a bound state consisting of \( 2k + 1 \) D0 branes, i.e. \( 2k \) zero branes bound to the one stuck to the orientifold plane. The \( 2k \) zero branes will on average be separated from the orientifold plane, and hence their fermionic 0-8 strings will not contribute degeneracy to the vacuum.

These states and the others in the \( \textbf{120} \) that we need to fill out the \( \textbf{248} \) of \( E_8 \) propagating in 10d will be exhibited using T-duality in the next section. In particular, we will find that bound states of \textit{even} numbers of D0 branes will comprise the \( \textbf{120} \) with nonzero \( p_{11} \).
3. T-duality and 0 Brane Bound States

In order to establish the existence of the requisite bound states, we can use T-duality and heterotic Spin(32)/Z₂-type I duality to map the problem to a question about perturbative states in the heterotic Spin(32)/Z₂ theory. The type I’ theory is T-dual to the type I theory. The $S^1/Z_2$ of radius $R_{I'}$ in the type I’ theory becomes an $S^1$ of radius $R_I = 1/R_{I'}$ in the type I theory. D0 branes in the type I’ theory map to D1 branes wrapped around the $S^1$ in the type I theory. These in turn correspond to wound perturbative heterotic string states. The relation between the perturbative heterotic $E_8 \times E_8$ and Spin(32)/Z₂ theories on a circle was presented in [16], and most of what follows is an application of that result to the problem at hand.

We are interested in BPS bound states of zero branes, which map to BPS winding states of the Spin(32)/Z₂ heterotic string (with Ramond-Ramond charge mapping to winding number). The left and right moving momentum for heterotic states with momentum number $m$ and winding number $n$ in the presence of Wilson lines are [17]

\[
p_L = (P + An, \frac{m}{2} \frac{A^2 n - A \cdot P}{R_h} - n R_h) \quad (3.1)
\]

\[
p_R = m \frac{A^2 n - A \cdot P}{R_h} + n R_h. \quad (3.2)
\]

Here $P$ is a vector in the internal lattice of the Spin(32)/Z₂ theory, and $R_h$ is the radius of the circle in this theory. A basis for this lattice is

\[
e_i - e_{i+1} \quad i = 1, \ldots, 15
\]

\[
e_{15} + e_{16}
\]

\[
\frac{1}{2} \sum_{i=1}^{16} e_i
\]

where $e_i, i = 1, \ldots, 16$ are unit vectors in $\mathbb{R}^{16}$. The roots of the form $\pm e_i \pm e_j, i,j = 1, \ldots, 16$ have $P^2 = 2$ and give rise to massless gauge bosons. The last entry corresponds to the spinor weight of Spin(32). This weight has length squared 4 and gives rise to massive states in the spinor representation of Spin(32).

We wish to start at the locus in Narain moduli space with Spin(16) × Spin(16) gauge symmetry. This can be achieved in a standard way by a Wilson line

\[
A = \sum_{i=1}^{8} \frac{1}{2} e_i. \quad (3.4)
\]
From (3.1)(3.2), massless perturbative states must satisfy $P \cdot A = 0$. This leaves the following subset of the roots of $Spin(32)/\mathbb{Z}_2$:

$$
\pm e_i \pm e_j \quad i, j = 1, \ldots, 8
$$

$$
\pm e_i \pm e_j \quad i, j = 9, \ldots, 16.
$$

(3.5)

These along with the Cartan generators form the adjoint of $Spin(16) \times Spin(16)$.

States with nonzero $p_{11}$ map to massive BPS states in the heterotic string. These are states where the right-movers are in the ground state [18]. Level matching requires

$$
\frac{p_L^2}{2} - \frac{p_R^2}{2} = 2 - 2N_L.
$$

(3.6)

In order to further specify the relevant states, we must consult the map between heterotic $Spin(32)$ radius $R_h$ and coupling $\lambda_h$ and those of the type $I'$ theory $R_{I'}$, $\lambda_{I'}$:

$$
R_h = \frac{1}{\sqrt{R_{I'} \lambda_{I'}}}, \quad \lambda_h = \frac{R_{I'}}{\lambda_{I'}}.
$$

(3.7)

The heterotic $E_8 \times E_8$ coupling $\lambda_E$ is related to $R_{I'}$, $\lambda_{I'}$ by

$$
\lambda_E = \frac{R_{I'}^{3/2}}{\lambda_{I'}^{1/2}}.
$$

We see that with $\lambda_E$ held fixed, $R_h \rightarrow 0$ as $\lambda_{I'} \rightarrow \infty$. Therefore from (3.1)(3.2) we see that states that survive in the limit of interest must satisfy

$$
\frac{1}{2}m - \frac{1}{4}A \cdot An - \frac{1}{2}A \cdot P = 0.
$$

(3.8)

Then in order to satisfy (3.6) one must impose either

$$
(P + An)^2 = 2, \quad N_L = 0
$$

(3.9)

or

$$
(P + An)^2 = 0, \quad N_L = 1
$$

(3.10)

The states satisfying (3.6)(3.8)(3.10) have $n$ even and give the gravity multiplet. The vertex operators of the $N_L = 0$ states are

$$
e^{i(p_L + k)X_L}e^{i(p_R + k)X_R}\xi_{\mu}(\partial X_{R}^{\mu} + ik \cdot \psi)_{\mu}.
$$

(3.11)
Here $k$ is the nine-dimensional spacetime momentum, and $X^\mu$ and $\psi^\mu$ are worldsheet bosons and fermions. What are the quantum numbers of the states satisfying (3.6)(3.8)(3.9)? This depends, as anticipated in §2, on whether $n$ is even or odd. The condition is that

$$P + n(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0)$$

(3.12)

have norm squared 2. Let us consider first the case where $P$ is a linear combination of the first two sets of roots, $e_i - e_{i+1}, i = 1, \ldots, 15$ and $e_{15} + e_{16}$ in (3.3). In other words, take first the case where $P$ has integer entries. Also take $n = 2k + 1$ odd. Then in order to make (3.12) square to 2, $P$ must shift it to be of the form

$$P + (2k + 1)A = (\pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0, 0, 0).$$

(3.13)

It is easy to check that the possible $P$s lead to an even number of $+$ signs in (3.13). So we have obtained the spinor $\left(^{128}_1, 1\right)$ of $Spin(16) \times Spin(16)$. What about the $\left(1, ^{128}_1\right)$? This arises in a similar fashion by including in $P$ shifts by the spinor weights $\pm \frac{1}{2}e_1 \pm \frac{1}{2}e_2 \pm \cdots \pm \frac{1}{2}e_{16}$ of $Spin(32)$.

Now if we take $n = 2l$ even, and $P$ to have integer entries, then in order to have (3.12) square to 2 the allowed $P$s must shift it to the form

$$\pm e_i \pm e_j, i, j = 1, \ldots, 8.$$

(3.14)

or

$$\pm e_i \pm e_j, i, j = 9, \ldots, 16.$$

(3.15)

In this way we recover the $\left(120, 1\right) + \left(1, 120\right)$ of $Spin(16) \times Spin(16)$.

Notice that the constraints (3.6)(3.8)(3.9) leave the gravitons and the gauge multiplets that we have just found, i.e. the

$$\left(120, 1\right) + \left(1, 120\right) + \left(128, 1\right) + \left(1, 128\right)$$

of $Spin(16) \times Spin(16)$, as the only surviving states in the limit $R_{11} \sim \lambda^{2/3} \rightarrow \infty$ (along with the gravitons, of course). These states, by the chain of dualities explained above, map to bound states of $n$ D0 branes and 8 D8 branes at the orientifold planes. We have seen that they have the quantum numbers of the requisite $E_8 \times E_8$ gauge bosons. An interesting feature of the states is that part of the $E_8$ adjoint (the 128 of $Spin(16)$) arises from bound states of odd numbers of D0 branes, while the rest (the 120 of $Spin(16)$) comes from bound states of even numbers of D0 branes. So gauge symmetry, like Lorentz symmetry, changes the D0 brane number of the state.

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2 This has been independently noted in a different context in [5].
4. The D0 brane-D8 brane system revisited

In this section we will try to understand the bound states (whose existence we have established in §3) directly in the quantum mechanics system (2.2).

Let us first translate the BPS condition for the mass $M$

$$M = p_R = \frac{\tilde{m}}{2R_h} + nR_h$$

(4.1)

to the type $I'$ theory. By making use of the map (3.4) and making a Weyl rescaling of the metric to type $I'$ string frame, this becomes

$$M = \frac{\tilde{m}}{2} R_{I'} + \frac{n}{\lambda_{I'}}.$$  

(4.2)

Here $\frac{1}{2} \tilde{m} = \frac{1}{2}m - \frac{1}{4}A \cdot A_n \cdot \frac{1}{2}A \cdot P$. So the condition (3.8) that $\tilde{m} = 0$ translates to the statement that states surviving in the limit $\lambda_{I'}, R_{I'} \to \infty$ do not involve open strings stretched across the interval $S^1/\mathbb{Z}_2$. The Lorentz quantum numbers also arise in a simple way from the BPS condition, as in the analysis of the graviton Lorentz quantum numbers in [1]. We start with 8 supercharges; a state killed by half the supersymmetries then has degeneracy $2^4 = 16$. This is the correct counting for a 10d $N = 1$ vector multiplet.

The challenge is to understand how the surviving states constitute precisely the gauge bosons of interest (as well as the graviton). On the heterotic side of the duality discussed in §3, there is an extra constraint (3.6). This arises from the level-matching constraint $L_0 - \bar{L}_0 = 0$, where $L_0$, $\bar{L}_0$ are the left and right-moving Hamiltonians on the string worldsheet.

The D0 brane quantum mechanics system has the following structure. If we ignore for now the coupling to the spacetime gauge fields $B^\mu$, the Hamiltonian (2.2) takes the form

$$H = H_0 + \delta H$$

(4.3)

where $H_0$ is the part of the Hamiltonian involving only the 0-0 strings [12], and

$$\delta H = \chi_r^I A^1_{I,J} \chi_r^J + \chi_r^I A^{10}_{I,J} \chi_r^J.$$ 

(4.4)

Here $A^1$ is the $SO(n)$ gauge field, and $A^{10}$ is its bosonic superpartner which describes the motion of the pair of zero branes away from the orientifold plane. The supersymmetry variation of the two terms in $\delta H$ cancel, so we have

$$[Q_a, \delta H] = 0.$$ 

(4.5)
We can take the gauge $A_1 = 0$ as in $[12]$, keeping in mind that its supersymmetry transformation is involved in ensuring (4.3). As explained in $[12]$, there are operators (supercharges) $Q_a$ which satisfy

$$\{Q_a, Q_b\} = \delta_{ab} H_0.$$  \tag{4.6}

Now the fermions $\chi^I_r$ do not transform under supersymmetry, and hence do not appear in the supercharges, so

$$\delta H \neq \{Q_a, Q_b\} \zeta^{ab}$$  \tag{4.7}

for any constant $\zeta^{ab}$. This structure is similar to the analogous problem in the T-dual setup. There one is interested in worldsheet properties of the D string. On its worldsheet, there are left-moving fermions coming from the 1-9 strings $[11]$. The supersymmetry is right-moving, so that

$$\{\bar{Q}_+, Q_+\} = \bar{L}_0 = H + P.$$  \tag{4.8}

Here $H = L_0 + \bar{L}_0$ is the Hamiltonian, and $P = L_0 - \bar{L}_0$ is the momentum.

We are interested in the quantum numbers of the BPS states of this system. For simplicity, let us begin with $n$ small. First recall the case $n = 1$. Then, as discussed in §2, the D0 brane is stuck at the orientifold plane, where 8 D8 branes also lie. In terms of the formalism (2.2), this means that the Hamiltonian does not depend on the fermions $\chi$ at all. Quantizing the $\chi$ zero modes then gives precisely the spinor of $Spin(16)$, the $128$.

Now let us move on to the case $n = 2$. Here one has two D0 branes, which can move off the orientifold plane as a mirror pair. Let us define

$$\chi_r = \frac{1}{\sqrt{2}}(\chi^1_r + i\chi^2_r)$$

$$\bar{\chi}_r = \frac{1}{\sqrt{2}}(\chi^1_r - i\chi^2_r)$$  \tag{4.9}

The couplings in $\delta H$ reduce to (in $A_1 = 0$ gauge)

$$\delta H_{n=2} = A_{12}^{10} \sum_{r=1}^{16} \bar{\chi}_r \chi_r.$$  \tag{4.10}

The canonical anticommutator involving the $\chi$s is

$$\{\bar{\chi}_r, \chi_s\} = \delta_{r,s}.$$  \tag{4.11}

Now take the vacuum $|0\rangle$ to satisfy

$$\chi_r |0\rangle = 0.$$  \tag{4.12}
Then

$$\delta H |0\rangle = 0. \quad (4.13)$$

We expect that an analogue of the GSO projection (a $\mathbb{Z}_2$ discrete gauge symmetry) will remove states created by an odd number of $\bar{\chi}$s.

The next possibility is the set of states

$$|\mathbf{120}\rangle = \bar{\chi}_r \bar{\chi}_s |0\rangle. \quad (4.14)$$

This state transforms in the $\mathbf{120}$ of $SO(16)$. It corresponds to a configuration of two 0-8 strings. It satisfies

$$\delta H |\mathbf{120}\rangle = 2A_{12}^{10} |\mathbf{120}\rangle. \quad (4.15)$$

So far we have been treating $A^{10}$ as a background field. We must also take into account the fields $X_{2,\ldots,9}^{IJ}, S_a^I$ which transform in the traceless symmetric representation of $SO(2)$. As pointed out in [12], there is a branch classically where $X^{IJ} \neq 0$ and $A^{10} = 0$. We can now begin to develop a microscopic description of how the gauge bosons are bound to the ends of the world. From (4.10), we see that $H_0$ kills the BPS states we are interested in (which have the quantum numbers of Lorentz vectors). This means that in order for a state to be simultaneously BPS and an eigenstate of the full Hamiltonian, it must also be an eigenstate of $\delta H$. For our states, which satisfy (4.15), this requires that

$$A_{12}^{10} |\mathbf{120}\rangle = 0. \quad (4.16)$$

As $X^{IJ} \to \infty$, we can analyze the system semiclassically and indeed the wavefunction asymptotes to an eigenvector of $A_{12}^{10}$ with eigenvalue zero.

One puzzling feature of this picture is that there would seem to be an $SO(16)$-invariant state, the vacuum $|0\rangle$, which is also stuck to the wall for $X^{IJ} \to \infty$. One difference between this state and the $|\mathbf{120}\rangle$ is that $|0\rangle$ is not constrained by the analogue of (4.16), since it satisfies (4.13). So this state is not required by BPS to lie at the end of the world. One possibility is that its wavefunction is generally supported away from the wall, where the appropriate zero-brane Hamiltonian has enhanced supersymmetry (it becomes the maximally supersymmetric Hamiltonian of [7,8]) and these states become gravitons.

We have already recovered states with the expected quantum numbers. What about the states created by more (even) powers of $\bar{\chi}$? These states are not protected from decay into the states we have already found. It would be interesting to study these decay
processes directly, using the couplings in the Hamiltonian between the spacetime gauge field $B^\mu$ and the quantum-mechanical coordinates $X^{IJ}$, $\chi$, and $A_{IJ}^{10}$.

It is difficult to prove directly the existence of bound states at threshold with larger $n$. Assuming that such bound states exist the analysis given here explains how the quantum numbers of the 128 and the 120 arise. The indirect derivation using duality in the previous section ensures that the requisite states exist.

5. Comments

The existence of a matrix model formulation of the $E_8 \times E_8$ heterotic string theory (M theory on $S^1/Z_2$) requires the existence of bound states of zero branes to D8 branes at orientifold planes in the Type I' theory. The derivation provided in §3 constitutes a proof of their existence assuming only T-duality and heterotic $Spin(32)/Z_2$-type I duality. In this sense proving the existence of the gauge bosons with $p_{11} \neq 0$ in the matrix theory is easier than the analogous problem for the gravitons discussed in [1] (although Sen has given strong arguments for the existence of the relevant zero brane bound states in that context as well [19]). This is because the gauge bosons propagate only in ten dimensions.

In [1] D0 brane bound state scattering was studied and shown to reproduce low-energy graviton exchange. This involves exchange of closed string states, which is an annulus computation in the open string channel. Scattering of gauge bosons, on the other hand, involves exchange of open string states (the 8-8 strings). This is simply a tree-level diagram encoding the scattering of two 0-8 strings (including the effects of nonzero D0 brane velocity) by exchange of an 8-8 string.

The heterotic string arises in a straightforward way from duality as well in this formalism. The heterotic string with zero winding number maps to the D string in the type I theory. Upon T-duality to type I', this becomes a D2 brane stretched between the orientifold planes. It is interesting to think about obtaining the heterotic string directly in the matrix model by using an analogue of the wrapped membrane of [1]. In the heterotic theory one expects to only obtain membranes stretched between the two orientifold planes. Recall that in [1] the membrane extended in the $i,j$ directions is obtained by finding (infinite) matrices with nonzero $Tr([X_i,X_j])$. This means that, in the notation of §2, one expects

$$Tr([X_i,A_{10}])$$
to be the only nonzero trace in the $N \to \infty$ limit. This follows directly from the symmetry properties of the matrices: The $X_i$ are symmetric while $A_{10}$ is antisymmetric, so even in the infinite $N$ limit only commutators with one factor of $A_{10}$ can have nonzero trace.\footnote{This was pointed out to us by O. Aharony.}

The quantum mechanics of the system (2.2) has a very rich structure which we have only begun to explore. There are at least two features which deserve further exploration: the change in the zero brane number under $E_8$ gauge transformations, and the importance of the branch $X^{IJ} \neq 0$ in producing states bound to the orientifold plane. There are also interesting resonances \cite{12}, as in the maximally supersymmetric case \cite{7,8,9}. Decays involving the background spacetime gauge fields would be interesting to study. In any case it is clear that applying the prescription of \cite{1} to the heterotic/type I/type $I'$ string yields an intriguing microscopic picture of its physics.

Acknowledgments

We are grateful to O. Aharony, T. Banks, W. Fischler, J. Maldacena, S. Shenker, and L. Susskind for useful discussions. This work was supported in part by DOE DE-FG02-96ER40559.
References

[1] T. Banks, W. Fischler, S. Shenker, and L. Susskind, hep-th/9610043.
[2] E. Witten, Nucl. Phys. B443 (1995) 85, hep-th/9503124.
[3] E. Witten, Nucl. Phys. B460 (1996) 335, hep-th/9510135.
[4] M. Berkooz and M. Douglas, hep-th/9610236
  V. Periwal, hep-th/9611103
  L. Susskind, hep-th/9611163
  O. Ganor, S. Ramgoolam, and W. Taylor, hep-th/961202
  O. Aharony and M. Berkooz, hep-th/9611215
  G. Lifschytz and S. Mathur, hep-th/9612087
  N. Ishibashi, H. Kawai, Y. Kitazawa, and A. Tsuchiya, hep-th/9612115
  M. Li, hep-th/9612144.
[5] M. Douglas, hep-th/9612126.
[6] M. Claudson and M.B. Halpern, Nucl. Phys. B250 (1985) 689;
  M. Baake, P. Reinicke, and V. Rittenberg, J. Math. Phys. 26 (1985) 1070;
  R. Flume, Ann. Phys. 164 (1985) 189.
[7] U. Danielsson, G. Ferretti, and B. Sundborg, hep-th/9603081.
[8] D. Kabat and P. Pouliot, Phys. Rev. Lett. 77 (1996) 1004, hep-th/9603127.
[9] M. Douglas, D. Kabat, P. Pouliot, and S. Shenker, hep-th/9608024.
[10] P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506, hep-th/9510209.
[11] J. Polchinski and E. Witten, Nucl. Phys. B460 (1996) 525, hep-th/9510168.
[12] U. Danielsson and G. Ferretti, hep-th/9610082.
[13] J. Dai, R. Leigh, and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073.
[14] J. Polchinski, S. Chaudhuri, and C. Johnson, hep-th/9602052.
[15] J. Polchinski, hep-th/9611050.
[16] P. Ginsparg, Phys. Rev. D35 (1987) 648.
[17] K. Narain, M. Sarmadi, and E. Witten, Nucl. Phys. B279 (1987) 369.
[18] A. Dabholkar and J. Harvey, Phys. Rev. Lett. 63 (1989) 478.
[19] A. Sen, Phys. Rev. D54 (1996) 2964, hep-th/9510229
  A. Sen, Mod. Phys. Lett. A11 (1996) 827, hep-th/9512203.