Implications of $\eta'$ Coupling In The Chiral Constituent Quark Model

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March 26, 2022

Abstract

Using the latest data pertaining to $\bar{u} - \bar{d}$ asymmetry and the spin polarization functions, detailed implications of the possible values of the coupling strength of the singlet Goldstone boson $\eta'$ have been investigated in the $\chi$CQM with configuration mixing. Using $\Delta u$, $\Delta_3$, $\bar{u} - \bar{d}$ and $\bar{u}/\bar{d}$, the possible ranges of the coupling parameters $a$, $\alpha^2 a$, $\beta^2 a$ and $\zeta^2 a$, representing respectively the probabilities of fluctuations to pions, $K$, $\eta$ and $\eta'$, are found. To further constrain the coupling strength of $\eta'$, detailed fits have been obtained for spin polarization functions, quark distribution functions and baryon octet magnetic moments. The fits clearly establish that a small non-zero value of the coupling of $\eta'$ is preferred over the higher values of $\eta'$ as well as when $\zeta = 0$, the latter implying the absence of $\eta'$ from the dynamics of $\chi$CQM. Our best fit achieves an overall excellent fit to the data, in particular for $\Delta u$, $\Delta d$, $\Delta_8$ as well as the magnetic moments $\mu_n$, $\mu_{\Sigma^-}$, $\mu_{\Sigma^+}$ and $\mu_{\Xi^-}$. The implications of $\eta'$ on the gluon polarization have also been investigated.

The chiral constituent quark model ($\chi$CQM), as formulated by Manohar and Georgi [1], has recently got good deal of attention [2, 3, 4, 5] as it is successful in not only explaining the “proton spin crisis” [6] but is also
able to account for the $\bar{u} - \bar{d}$ asymmetry \cite{7,8,9}, existence of significant strange quark content $\bar{s}$ in the nucleon, baryon magnetic moments \cite{2,3} and hyperon $\beta$–decay parameters etc.. Further, $\chi$CQM with configuration mixing (henceforth to be referred as $\chi$CQM\textsubscript{config}) when coupled with the quark sea polarization and orbital angular momentum (Cheng-Li mechanism \cite{3}) as well as “confinement effects” is able to give an excellent fit \cite{10} to the baryon magnetic moments and a perfect fit for the violation of Coleman Glashow sum rule.

The key to understand the “proton spin problem”, in the $\chi$CQM formalism \cite{3}, is the fluctuation process $q^\pm \to GB + q'^\mp \to (q\bar{q}') + q''\mp$, where GB represents the Goldstone boson and $q\bar{q}' + q'$ constitute the “quark sea” \cite{3,11,5,10}. The effective Lagrangian describing interaction between quarks and a nonet of GBs, consisting of octet and a singlet, can be expressed as

$$\mathcal{L} = g_8\bar{q}\Phi q + g_1\bar{q}\frac{\eta'}{\sqrt{3}} q = g_8\bar{q}\left(\Phi + \frac{\eta'}{\sqrt{3}} I\right) q = g_8\bar{q}\left(\Phi'\right) q,$$

where $\zeta = g_1/g_8$, $g_1$ and $g_8$ are the coupling constants for the singlet and octet GBs, respectively, $I$ is the $3 \times 3$ identity matrix. The GB field $\Phi'$ includes the octet and the singlet GBs. The parameter $a(= |g_8|^2)$ denotes the probability of chiral fluctuation $u(d) \to d(u) + \pi^+(-)$, whereas $\alpha^2a$, $\beta^2a$ and $\zeta^2a$ respectively denote the probabilities of fluctuations $u(d) \to s + K^{-(0)}$, $u(d, s) \to u(d, s) + \eta$, and $u(d, s) \to u(d, s) + \eta'$.

The chiral structure of QCD is known to have intimate connection with the $\eta$ and $\eta'$ dynamics \cite{11}. Recently, in the context of $\chi$CQM, Steven D. Bass \cite{12} has reiterated in detail the deep relationship of the non-perturbative aspects of QCD, including gluon anomaly, and the comparatively large masses of the $\eta$ and $\eta'$ mesons. Similarly, it has been shown earlier by Ohta et al. \cite{13} and recently advocated by Cheng and Li \cite{3} that $\eta'$ could play an important role in the formulation of the $\chi$CQM. On the other hand, it has recently been observed on phenomenological grounds \cite{5} that the new measurement of both the $\bar{u}/\bar{d}$ asymmetry as well as $\bar{u} - \bar{d}$ asymmetry by the NuSea Collaboration \cite{8} may not require substantial contribution of $\eta'$. In this context, it therefore becomes interesting to understand the extent to which the contribution of $\eta'$ is required in the $\chi$CQM thereby giving vital clues to the dynamics of non-perturbative regime of QCD.

To study the variation of the $\chi$CQM parameters and the role of the coupling strength of $\eta'$ in obtaining the fit, one needs to formulate the experimentally measurable quantities having implications for these parameters. The spin structure of a nucleon is defined as \cite{3,4,5,10} $\hat{B} \equiv \langle B|N|B \rangle$, where $|B\rangle$ is the nucleon wavefunction and $N$ is the number operator giving
the number of $q^\pm$ quarks. The contribution to the proton spin in $\chi$CQM$_{\text{config}}$ is given by the spin polarizations defined as $\Delta q = q^+ - q^-$. After formulating the spin polarizations of various quarks, we consider several measured quantities which are expressed in terms of the above mentioned spin polarization functions. The flavor non-singlet components $\Delta_3 = \Delta u - \Delta d$ and $\Delta_8 = \Delta u + \Delta d - 2\Delta s$, usually calculated in the $\chi$CQM, are obtained from the neutron $\beta-$decay and the weak decays of hyperons. The flavor non-singlet component $\Delta_3$ is related to the well known Bjorken sum rule [14]. Another quantity which is usually evaluated is the flavor singlet component $\Delta \Sigma = \frac{1}{2}(\Delta u + \Delta d + \Delta s)$, in the $\Delta s = 0$ limit, this reduces to the Ellis-Jaffe sum rule [15]. Apart from the above mentioned spin polarization we have also considered the quark distribution functions which have implications for $\zeta$ as well as for other $\chi$CQM parameters. For example, the antiquark flavor contents of the “quark sea”, the deviation of Gottfried sum rule [9], related to the $\bar{u}(x)$ and $\bar{d}(x)$ quark distributions, $\bar{u}/\bar{d}$ and the fractions of the quark content defined as $f_q = \frac{q^+ + \bar{q}}{\sum_q(q+\bar{q})}$.

With a view to phenomenologically estimating the coupling strength of the singlet Goldstone boson $\eta'$ in $\chi$CQM$_{\text{config}}$, we have carried out a detailed analysis using the latest data regarding $\bar{u} - \bar{d}$ asymmetry, the spin polarization functions and the baryon octet magnetic moments. As a first step of the analysis, we have found from broad considerations the required ranges of these parameters using the data pertaining to $\Delta u$, $\Delta_3$, $\bar{u} - \bar{d}$, $\bar{u}/\bar{d}$ etc.. After obtaining the ranges, analysis has been carried out corresponding to four different sets of the $\chi$CQM parameters within the ranges. In the first case, the pion fluctuation parameter $a$ is taken as 0.1, whereas $\Delta u$, $\Delta_3$, $\bar{u} - \bar{d}$, $\bar{u}/\bar{d}$ etc. are fitted by treating the other three parameters to be free. This analysis yields $|\zeta| = 0.65$, $\alpha = 0.4$ and $\beta = 0.7$ and is referred to as Case I. A similar analysis has also been also been carried out by taking $a = 0.1$, $|\zeta| = 0.70$, $\alpha = 0.4$ and $\beta = 0.6$ and is referred to as Case II. Our best fit (Case IV) is obtained by varying $a$, $\zeta$ and $\alpha$, the parameter $\beta$ is taken to be equal to $\alpha$ and the best fit values of the parameters are $a = 0.13$, $|\zeta| = 0.10$, $\alpha = \beta = 0.45$. We have also carried out a fit where there is no contribution of the singlet GB ($\zeta = 0$) and $a$, $\alpha$ as well as $\beta$ are treated free, yielding $a = 0.14$, $\alpha = 0.4$ and $\beta = 0.2$ and referred to as Case III.

In Table I, we have presented the results of our fits mentioned above. A comparison of all the fits clearly shows that our best fit is not only better than other fits carried out here but also provides an excellent overall fit to
the data particularly in the case of $\Delta u, \Delta d, \Delta s, \mu_n, \mu_{\Sigma^-}, \mu_{\Sigma^+}, \mu_{\Xi^-}$ and $\mu_{\Xi^+}$. It needs to be mentioned that $\Delta s$ cannot be fitted for "higher values" of $|\zeta|$ even after scanning the entire parameter space for $a, \alpha$ and $\beta$, suggesting that only the lower values of $|\zeta|$ are compatible with data. In Table II, we have presented the results corresponding to quark distribution functions. In this case also the fit for the lower values of $|\zeta|$ is better as compared to the higher values. It is interesting to observe that even for a small deviation in the value of $\zeta$, $f_s$ gets affected significantly, therefore a measurement of $f_s$ would give a very strong signal about the coupling strength of $\eta'$ in the $\chi$CQM. It may be mentioned that our conclusion regarding the small but non-zero value of $|\zeta|$ being preferred over $\zeta = 0$ is not only in agreement with the theoretical considerations based on the arguments of Cheng and Li [3] and those of S. Bass [12]. It seems that the phenomenological analyses of spin polarization functions, quark distribution functions and baryon octet magnetic moments, strongly suggest a small but non-zero value of $|\zeta|$ within the dynamics of chiral constituent quark model, suggesting an important role for $\eta'$ in the non-perturbative regime of QCD.

ACKNOWLEDGMENTS
H.D. would like to thank DST, Government of India and the organizers of SPIN2006, Kyoto University, for financial support.

References
[1] S. Weinberg, Physica A 96, 327 (1979); A. Manohar and H. Georgi, Nucl. Phys. B 234, 189 (1984).
[2] E.J. Eichten, I. Hinchcliffe and C. Quigg, Phys. Rev. D 45, 2269 (1992).
[3] T.P. Cheng and Ling Fong Li, Phys. Rev. D 57, 344 (1998); ibid., Phys. Rev. Lett. 80, 2789 (1998).

[4] X. Song, Phys. Rev. D 57, 4114 (1998).

[5] J. Linde, T. Ohlsson and Hakan Snellman, Phys. Rev. D 57, 452 (1998).

[6] EMC Collaboration, J. Ashman et al., Phys. Lett. 206B, 364 (1988); SMC Collaboration, P. Adams et al., Phys. Rev. D 56, 5330 (1997).

[7] New Muon Collaboration, M. Arneodo et al., Phys. Rev. D 50, R1 (1994).

[8] E866/NuSea Collaboration, R. S. Towell et al., ibid. 64, 052002 (2001).

[9] K. Gottfried, Phys. Rev. Lett. 18, 1174 (1967).

[10] H. Dahiya and M. Gupta, Phys. Rev. D 64, 014013 (2001); 66, 051501 (2002); 67, 114015 (2003).

[11] S. Weinberg, Phys. Rev. D 11, 3583 (1975); G ’t hooft, Phys. Rev. Lett. 37, 8 (1976); M. Ida, Prog. Theor. Phys. 61, 618, 1784 (1979); Y. Hosotani, Prog. Theor. Phys. 61, 1452 (1979).

[12] Steven D. Bass, Phys. Lett. 463B, 286 (1999); Nucl. Phys. Proc. Suppl. 105, 56 (2002).

[13] K. Kawarabayashi and N. Ohta, Nucl. Phys. B 175, 477 (1980); Prog. Theor. Phys. 66, 1789 (1981); H. Hata, T. Kugo and N. Ohta, Nucl. Phys. B 178, 527 (1981).

[14] J.D. Bjorken, Phys. Rev. 148, 1467 (1966); Phys. Rev. D 1, 1376 (1970).

[15] J. Ellis and R.L. Jaffe, Phys. Rev. D 9, 1444 (1974); ibid. 10, 1669 (1974).

[16] W.-M. Yao et al., J. Phys. G 33, 1 (2006).

[17] A.O. Bazarko et al., Z. Phys C 65, 189 (1995).
Table 1: The calculated values of the spin polarization functions and baryon octet magnetic moments for different cases. The value of the mixing angle $\phi$ is taken to be 20°.
| Parameter | Data | $\chi$CQM |
|-----------|------|-----------|
|           | Case I | Case II | Case III | Case IV |
| $|\zeta| = 0.65$ | $|\zeta| = 0.70$ | $\zeta = 0$ | $|\zeta| = 0.10$ |
| $a = 0.1$  | $a = 0.1$ | $a = 0.14$ | $a = 0.13$ |
| $\alpha = 0.4$ | $\alpha = 0.4$ | $\alpha = 0.4$ | $\alpha = 0.45$ |
| $\beta = 0.7$ | $\beta = 0.6$ | $\beta = 0.2$ | $\beta = 0.45$ |
| $\bar{u}$ | $0.168$ | $0.167$ | $0.250$ | $0.233$ |
| $\bar{d}$ | $0.288$ | $0.293$ | $0.366$ | $0.350$ |
| $\bar{s}$ | $0.108$ | $0.104$ | $0.07$ | $0.07$ |
| $\bar{u} - \bar{d}$ | $-0.118 \pm 0.015$ | $-0.120$ | $-0.127$ | $-0.116$ | $-0.117$ |
| $\bar{u}/\bar{d}$ | $0.67 \pm 0.06$ | $0.58$ | $0.57$ | $0.68$ | $0.67$ |
| $I_G$ | $0.254 \pm 0.005$ | $0.253$ | $0.248$ | $0.255$ | $0.255$ |
| $f_u$ | $0.655$ | $0.654$ | $0.677$ | $0.675$ |
| $f_d$ | $0.442$ | $0.445$ | $0.470$ | $0.466$ |
| $f_s$ | $0.10 \pm 0.06$ | $0.061$ | $0.058$ | $0.039$ | $0.039$ |
| $f_3$ | $0.213$ | $0.209$ | $0.207$ | $0.209$ |
| $f_8$ | $0.975$ | $0.982$ | $1.07$ | $1.06$ |
| $f_3/f_8$ | $0.21 \pm 0.05$ | $0.22$ | $0.21$ | $0.19$ | $0.20$ |

Table 2: The calculated values of the quark flavor distribution functions for different cases.