Improving Comb Aliasing Rejection Using Sharpening of Modified Comb

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Abstract. This paper presents an efficient method to improve the comb aliasing rejection in a comb decimation filter without increasing its passband droop. This problem is important since aliasing and comb passband droop may deteriorate the decimated signal. We propose here to apply sharpening of the modified comb in the second stage of a two-stage comb structure. The modified comb is obtained by decreasing the middle coefficient of the impulse response of the cascade of two combs by 1/2. The sharpening polynomial with the first order tangencies is used here. As a result, the comb folding bands, where the aliasing occur, become wider and with an increased attenuation in comparison with the original comb filter. However, this improvement in the folding bands did not result in an increased passband droop. The compensator from literature is used to further decrease passband droop. The method is illustrated with examples and compared with the original comb and the methods proposed in literature for increasing aliasing rejection.

1. Introduction

This paper treats the problem of the increasing aliasing rejection in comb decimation filters. It is well known that the comb filter is the most popular decimation filter, due to its simplicity, since all its coefficients are equal to unity [1].

The system function of the comb filter is given as:

\[ H(z) = \frac{1-z^{-M}}{M(1-z^{-1})}, \]  

where \( M \) is the decimation factor.

Comb filter attenuates aliasing which occurs in the folding bands, (bands around the comb zeros). However, this attenuation is not enough in many applications, and should be increased [2].

The simplest method to increase the aliasing rejection is to use the cascade of \( K \) combs (1). \( K \) is called the order of the comb filter. The system function of the comb filter of order \( K \) is given as:

\[ H(z)^K = \left[ \frac{1-z^{-M}}{M(1-z^{-1})} \right]^K. \] 

The increase of \( K \) increases aliasing rejection, and also increases the comb passband droop, which may deteriorate the decimated signal. This problem is illustrated in Figure 1, where the overall comb magnitude responses, and the passband zooms, for two values of \( K \), are presented.
Different methods have been proposed to improve comb aliasing rejection. The rotation of comb zeros, requiring two multipliers, was proposed in [3-4]. Additionally, a recursive term is introduced with the perfect cancellation of poles and zeros on the unit circle for the case, in which the coefficients are presented in an infinite precision. However, when the coefficients are presented in a finite precision, the cancellation of poles and zeros can be lost, leading to instability.

Different methods have been proposed to get the improvement of the aliasing rejection without introducing multipliers and possible instability. The authors in [5-7] used combs of different lengths, while the authors in [8-10] introduced additional zeros into comb folding bands using simple multiplierless filters. The sharpening technique using Kaiser sharpening polynomial was introduced in [11]. Recently, different sharpening polynomials were proposed in [12-14], and the designs of the corresponding compensators to compensate for the passband droop, were also elaborated.

As a difference to works in [11-14], where the sharpening of comb filters was applied, we consider here the sharpening of a modified comb at the second stage. As a result, we get an improved aliasing rejection and a decreased passband droop in comparison with the equivalent comb. In that way, a simple comb compensator can be used to compensate for the passband droop.

The rest of the paper is organized as follows. Next section briefly presents the modified comb and the sharpening technique. The proposed filter is introduced in Section 3, and compared with the equivalent comb filter. The passband compensation is also addressed. The comparisons of the
proposed method with the methods recently proposed in literature are presented in Section 4. Finally, the conclusions are given in Section 5.

2. Modified comb and sharpened comb

In this section we present brief review of the modified comb and the sharpened comb from literature.

2.1. Modified comb [8]

The modified comb was introduced in [8-9]. This filter is obtained by subtracting the \( \frac{1}{2} \) from the middle coefficient of the impulse response of the cascade of two comb filters. The system function of the proposed comb filter can be presented using the modified comb as [8]:

\[
G(z) = H^{K-2}(z)H_m(z),
\]

where \( K \) is the order of the equivalent comb filter and the modified comb is equal to,

\[
H_m(z) = \left[ \frac{1-z^{-M}}{M(1-z^{-1})} \right]^2 - z^{-M} \left( M - \frac{1}{2} \right).
\]

Next example illustrates the benefit of the filter (3).

**Example 1:** We consider \( M=14 \) and \( K=4 \). Figure 2 compares magnitude responses of the equivalent comb filter of the order \( K \), and the filter (3).

Note that filter (3) has wider folding bands than the equivalent comb filter, thus providing better aliasing attenuation.

![Figure 2. Magnitude responses of filter comb and filter given in [8].](image)

2.2. Sharpened comb [11]

We will use the sharpening polynomial \( \text{Sh} \{ x \} = 3x^2 - 2x^3 \), introduced in [11] where \( x \) is a comb filter of order \( K \), and \( \text{Sh} \{ x \} \) means sharpening of \( x \).

The sharpening polynomial improves the comb magnitude characteristic in both, passband and stopband, as shown in next example.

**Example 2:** We choose \( M=15 \) and \( K=1 \). Figure 3 contrasts the magnitude responses of comb and the sharpened comb.

The zoom in the passband is also shown in order to demonstrate that the sharpening improves also the passband of the original comb.
3. Proposed filter

In this section we first present improving of aliasing rejection. Next we present the improving the passband characteristic of the proposed filter.

3.1. Improving aliasing rejection

We propose to sharpen the modified comb at the second stage instead of to sharpen comb itself. The system function of the proposed filter is given as:

\[ H_{sh}(z) = H_1^K(z)H_2^{K-3}(z^{M_1})Sh\{H_{zm}(z^{M_1})\}, \]  

(5)

where \( H_1(z) \) and \( H_2(z) \) are the system functions of combs at first and the second stages, respectively, and \( H_{zm}(z) \) is the system function of the modified comb \( H_2(z) \).

Note that the system functions of all filters at the second stage are expanded by \( M_1 \) since the overall system function is presented at high input rate.

The method is illustrated in the following example.

Example 3: We consider \( M=16 \) and \( K=4 \). Figure 4 contrasts magnitude responses of the equivalent comb and the proposed filter.

Note that the proposed filter has wider folding bands, thus providing a better aliasing rejection than the equivalent comb filter of order \( K \). Additionally, the proposed filter exhibits lesser passband droop.
a. Overall magnitude responses.

b. Passband zoom.

Figure 4. Magnitude responses of the proposed sharpened filter and the equivalent comb filter.

3.2. Improving passband

In order to decrease the passband droop in the proposed sharpened filter (5), we applied the compensator $C(z)$ at low rate. We propose here to adapt the compensator, proposed for the comb compensation in [15]. The magnitude characteristic of this compensator is given as:

$$C(e^{j\omega}) = \left[1 + A\sin^4\left(\frac{\omega M}{2}\right)\right] \times \left[1 + A\sin^2\left(\frac{\omega M}{2}\right)\right],$$  

(6)

where the parameters $A$ and $B$ are obtained using MATLAB simulation, using the similar procedure as described in [15] for a comb filter. Instead of comb filter used in [15], we use here the sharpened modified comb (5).

Table I shows the obtained results for parameters $A$ and $B$, and the absolute values of the maximum passband deviations $\delta$ in dBs, of the compensated filter for a given values of $K$. The passband edge is equal to $\omega_p=\pi/5$.

It is worth to mention that the parameters in Table 1 do not depend on the decimation factor $M$. The total number of compensator adders is $N_a=9$, for all cases in Table 1.

Using (5) we get the system function of the proposed sharpened compensated filter as,

$$H_p(z) = H_{sh}(z)C(z^{M}),$$  

(7)

where $H_{sh}(z)$ is given in (5) and $C(z)$ is the system function of the compensator.
Observe that the system function of the compensator $C(z)$ is expanded by $M$ since the compensator works at low rate, i.e. after the decimation by $M$.

**Table 1.** Compensator parameters and values of the absolute deviation $\delta$ in dBs

| $K$ | $A$ | $B$ | $\delta$ in dBs |
|-----|-----|-----|-----------------|
| 3   | $2^{-3}$ | $2^{-4}$ | 0.026            |
| 4   | $2^{-3}$ | $2^{-2}$ | 0.037            |
| 5   | $2^0$     | $2^{-2}$ | 0.035            |
| 6   | $2^0$     | $2^{-1}$ | 0.028            |

Next example illustrates the compensation of the proposed filter from Example 3, ($M=16$, $K=4$).

**Example 4:** The compensator parameters are obtained from Table 1, for $K=4$. The magnitude responses of the proposed sharpened, and the proposed compensated sharpened filters are shown in Figure 5. The passband zoom is also shown to present the passband improvement. The passband deviation in the compensated filter is equal to 0.027 dBs. Note that the compensator does not affect the aliasing rejection.

**Figure 5.** Magnitude responses of proposed sharpened and sharpened compensated filters.
4. Comparisons
In this section we present the comparison of the proposed method, with the method in [8] and the sharpening methods recently proposed in literature.

4.1 Comparison with method in [8]
Since the proposed method is the modification of the method in [8], our first comparison is with the method in [8]. We will compare only alias rejection since in method [8] is not presented the compensation.

We chose $M=20$, $M_1=4$, $M_2=5$, and $K=5$. The magnitude responses are compared in Figure 6. We can observe that the proposed method provides much better aliasing reduction.

4.2. Comparison with method in [13]
The compensator design for different sharpening polynomials was proposed in [13]. The obtained results are shown in table for $M=32$. For a sake of comparison, we consider the sharpening polynomial $p(x)=2^8x^2-2^5x^4+x^6$, where $x$ is a comb filter of order 1. In the proposed design we have $M_1=4$, $M_2=8$, and $K=4$. The compensator parameters are obtained from Table 1.

The magnitude responses are shown in Figure 7. We can observe that the proposed method provides better magnitude characteristic in both, the passband and the stopband. Additionally, the proposed method has lesser complexity since the sharpening is performed at lower rate.

4.3. Comparison with method in [14]
A two-stage structure, in which the sharpening is applied at the second stage, was proposed in [14]. The coefficients of the sharpening polynomial are obtained using linear programming optimization. The example of design, presented in table in [14] for $M=32$, is used here for the comparison. Like in [14], in the proposed method $M_1=4$, $M_2=8$. In the proposed method, $K=3$ and the parameters of compensator are obtained from Table 1 for $K=3$.

The magnitude responses are contrasted in Figure 8. The proposed method has better aliasing rejection, while method in [14] has slightly better passband compensation.

Moreover, the advantage of the proposed method is the design simplicity, since the design parameters do not depend on $M$, and the compensator parameters are given in table for any value of $M$. 
5. Conclusions

This paper presents a simple method to increase aliasing rejection and improve passband characteristic in the comb decimation filter. The proposed filter has two stages decimated by $M_1$ and $M_2$, respectively, where the overall decimation factor is equal to $M = M_1 \cdot M_2$.

In the first stage is the comb filter, while in the second stage is the cascade of comb filters, and the sharpened modified comb filter.

The sharpening methods recently proposed in literature, require new parameter calculations for each new value of $M$. In contrast, in the method proposed here, the parameters of the proposed sharpened filter do not depend on $M$.

Similarly, the compensator parameters depend only on the comb parameter $K$, and are provided in table. The presented examples show that the proposed method provides better aliasing rejection and better or similar passband characteristics in comparison with methods from literature. The complexity is decreased by realizing the sharpening at the second stage. In future work we plan to design a new compensator in order to get a better compensation in the passband.
a. Overall magnitude responses.

b. Passband zoom.

**Figure 8.** Comparison with method in [14].

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