Balancing Adaptability and Non-exploitability in Repeated Games

Anthony DiGiovanni
Ambuj Tewari

Department of Statistics, University of Michigan, Ann Arbor, MI, USA

Abstract

We study the problem of adaptability in repeated games: simultaneously guaranteeing low regret for several classes of opponents. We add the constraint that our algorithm is non-exploitable, in that the opponent lacks an incentive to use an algorithm against which we cannot achieve rewards exceeding some “fair” value. Our solution is an expert algorithm (LAFF), which searches within a set of sub-algorithms that are optimal for each opponent class, and punishes evidence of exploitation by switching to a policy that enforces a fair solution. With benchmarks that depend on the opponent class, we first show that LAFF has sublinear regret uniformly over these classes. Second, we show that LAFF discourages exploitation, because exploitative opponents have linear regret. To our knowledge, this work is the first to provide guarantees for both regret and non-exploitability in multi-agent learning.

1 INTRODUCTION

General-sum repeated games represent interactions between agents aiming to maximize their respective reward functions, with the possibility of compromise over conflicting goals. Despite their simplicity, achieving high rewards in such games is a challenging learning problem due to the complex space of possible opponents. Both the behavior of a given opponent throughout a game, and that opponent’s choice of learning algorithm, may depend on one’s own algorithm. Crandall [2020] argues, based on empirical studies of repeated game tournaments, that a successful agent must achieve two goals. First, it must optimize its actions with respect to its beliefs about the opponent. Second, it should act such that the opponent forms beliefs motivating a response that is beneficial to the agent.

In particular, multi-agent reinforcement learning (MARL) features the following tradeoff: how to adapt to a variety of potential opponents, while also actively shaping other agents’ models of oneself such that they respond with cooperation, rather than exploitation. If an agent commits to a fixed policy to “lead” the other player’s best response [Littman and Stone 2001], it may perform arbitrarily poorly against players that do not converge to such a response. This motivates the design of adaptive algorithms that try to lead, but can retreat to a “Follower” (best response) approach if doing so gives greater rewards [Powers and Shoham 2005; Chakraborty and Stone 2010]. An effective algorithm in this class is S++ [Crandall 2014], which, due to its Follower sub-algorithm, has the drawback that it is exploitable—that is, it rewards agents insisting on unfair bargains (“bully” strategies) [Crandall et al. 2018; Stastny et al. 2021].

A simple motivating example of Follower exploitability is the game of Chicken (Figure 1), between players Row and Column. Suppose Column knows Row will take the apparently optimal action 1 if Column repeats action 2. Column will then want to use the Leader strategy of committing to action 2 to gain the highest reward. Row thus only gets reward 0.25, and if Column has truly committed, an attempt by Row to dissuade this strategy by taking action 2 would give both players reward 0. A cooperative outcome, e.g., alternating between the off-diagonal cells, could be achieved if Row’s learning algorithm were designed to publicly disincentivize commitments to the exploitative Leader strategy.

| 0.5, 0.5 | 0.25, 1 |
|-------|------|
| 1, 0.25 | 0, 0 |

Figure 1: Reward bimatrix for Chicken.

MARL research has largely neglected the latter half of the adaptability vs. non-exploitability tradeoff. Existing algorithms are either evaluated solely by their rewards conditional on given opponents [Powers and Shoham 2005; Crandall 2013], or, when the evaluation criterion does account for the incentives of algorithm selection, the pool of...
Previous MARL algorithms addressing the adaptability half of the tradeoff lack finite-time guarantees on rewards. We aim to provide a theoretically grounded algorithm for repeated games that is both adaptable, by using Leader and Follower sub-algorithms, and non-exploitable. More broadly, this paper addresses a challenge of interest in several areas of machine learning: designing algorithms that account for how the distribution of data the algorithms are applied to may change based on the choice of the algorithms themselves.

Related work Previous algorithms for repeated games have combined Leader and Follower modules, aiming for the following guarantees: worst-case safety, best response to players with bounded memory, and convergence in self-play to Pareto efficiency, i.e., an outcome in which no player can do better without the other doing worse [Powers and Shoham 2004]. Like ours, these algorithms aim for adaptability, but they do not have regret guarantees — the desired properties are only shown to hold asymptotically. Manipulator [Powers and Shoham 2005] achieves these properties by starting with a fixed strategy that maximizes the user’s rewards conditional on the opponent using a best response, and switching to reinforcement learning (RL) with a safety override if that strategy does not yield its target rewards. Related to the self-play guarantee, we prove a more general property of Pareto efficiency against effective RL algorithms (see Section 2.1). Like Manipulator, our approach tests sub-algorithms sequentially. S++ [Crandall 2014] has empirically strong performance on the guarantees above. However, neither of these algorithms guarantee non-exploitability.

Although to our knowledge no previous works have proven non-exploitability in our sense, several algorithms are designed to achieve “fair” Pareto efficiency in self-play without using Follower approaches that would be exploitable. Littman and Stone [2004]’s algorithm for computation of Nash equilibria, like our Leader sub-algorithms, enforces a Pareto efficient outcome by punishing deviations. If an agent played this equilibrium, which satisfies properties of symmetry similar to the outcome our Egalitarian Leader sub-algorithm aims for, it would be non-exploitable. However, committing to this equilibrium precludes learning a best response to fixed strategies that offer higher rewards than the cooperative solution, or exploiting adaptive players, which our Conditional Follower and Bully Leader sub-algorithms achieve, respectively. In two-player bandit problems where the reward bimatrix must be learned, UCRG [Tossou et al. 2020] has near-optimal regret in self-play with respect to the egalitarian bargaining solution (Section 2.2). However, it cannot provably cooperate with agents other than itself, learn best responses, or exploit adaptive players.

Our objectives of adaptability and non-exploitability are inspired by work on learning equilibrium [Brafman and Tenenholtz 2004, Jacq et al. 2020, Clifton and Riché 2021], a solution concept in which players’ learning algorithms are in a Nash equilibrium, beyond merely the equilibrium of an individual game itself. This objective accounts for the dependence of the problems faced by multi-agent learning algorithms on the design of such algorithms.

Contributions We propose an algorithm (LAFF) that, to our knowledge, is the first proven to have both strong performance against different classes of players in repeated games and a guarantee of non-exploitability, formalized in Section 2.2. Specifically, these classes consist of stationary algorithms (“Bounded Memory”), unpredictable adversaries (“Adversarial”), and adaptive RL agents (“Follower”). LAFF’s modular design allows for extensions to a broader variety of opponent classes in future work. We propose regret metrics appropriate for games against followers, based on the goal of Pareto efficiency. Our method of proof of adaptability and non-exploitability is novel, applying “optimistic” principles at two levels. First, LAFF starts with the sub-algorithm (or expert) that would give the highest expected rewards if the opponent were in that expert’s target class (“potential”), then proceeds through experts in descending order of potential. Second, LAFF chooses whether to switch experts by comparing the potential of the active expert with its empirical average reward plus a slack term, which decreases with the time for which the expert is used. For non-exploitability and regret against followers, we use the properties of an enforceable bargaining solution (see Section 2.2) to upper-bound the other player’s rewards.

2 PRELIMINARIES

We study a special class of Markov games: repeated games with a bounded memory state representation [Powers and Shoham 2005] and public randomization.

2.1 SETUP AND OPPONENT CLASSIFICATION

Consider a repeated game over \( T \) time steps, defined for players \( i = 1, 2 \) by action spaces \( \mathcal{A}^{(i)} \), reward matrices \( \mathbf{R}^{(i)} \), and a fixed player memory length \( K \in \mathbb{N} \). Here, all \( \mathbf{R}^{(i)}(a^{(1)}, a^{(2)}) \in [0, 1] \) are known by both players. At time \( t \) the following random variables are drawn: \( S_t \) for state, \( \mathcal{A}^{(i)} \) for actions, and \( \mathbf{R}^{(i)}(\mathcal{A}^{(1)}, \mathcal{A}^{(2)}) \) for rewards. A state space \( \mathcal{S} := (\mathcal{A}^{(1)})^K \times (\mathcal{A}^{(2)})^K \times \{0, 1\}^{2K+2} \), and transition probabilities \( \mathcal{P}(s'|s, a^{(1)}, a^{(2)}) \) between states, are induced by two features: (1) the tuple of both players’ last \( K \) actions, and (2) the tuple of the last \( K \) and current outcome of a randomization signal, for each player. (See Section 2.1.2 of [Mailath and Samuelson 2006].) Thus, players condition their actions on their memory of the last \( K \) time steps, and a signal that permits correlated action choices.
Formally, let $(w^{(1)}, w^{(2)}) \in [0, 1]^2$ be weights chosen by the \( \text{resp.} \) players at time \( t \) and draw \( X_t \sim \text{Unif}(0, 1) \) independent of all other random variables in the game. Then, letting \( y_t^{(i)} \) be the realized value of \( Y_t^{(i)} := [X_t < w_t^{(i)}, \text{the second feature at time} \ t \ is \ (y_t^{(1)} - y_t^{(2)})] \). This allows the players to correlate actions through the public signal \( X_t \), even if one player unilaterally generates the signal. For instance, in Chicken (Figure 1), players could flip a fair coin \( w_t^{(1)} = w_t^{(2)} = 0.5 \) at each time step and play the pair of actions leading to the top-right cell when it comes up heads, otherwise play the bottom-left cell. In this framework, at each time step each player has a choice of both a weight \( w_t^{(i)} \) and policy \( \pi_t^{(i)} : S \rightarrow \Delta(A^{(i)}) \), a mapping from states to distributions over actions.

Given a fixed policy of player 2, a repeated game is a Markov decision process (MDP) given by \((S, A^{(1)}, r, p)\) as follows. Let \( a_t^{(i)}(s) \) be the last action of player \( i \) that defines state \( s \). Here, \( r : S \times A^{(1)} \rightarrow [0, 1] \) is \( r(s, a) = R^{(1)}(a(s), a_t^{(2)}(s)) \), and \( p : S \times A^{(1)} \times S \rightarrow [0, 1] \) is \( p(s' | s, a) = \sum_a \pi(a(s'))(a_t^{(2)}(s)) \). A policy is called Markov if it is conditioned only on the current state.

The problem faced by our learner, player 1, depends on which of the following classes player 2’s algorithm is in:

1. **Bounded Memory**: (i) Player 2 uses a constant \( w^{(2)} \), reported at the start of the game; (ii) \( \pi^{(2)} \) is Markov and does not depend on time or player 1’s signals \( w_t^{(1)} \) or \( y_t^{(1)} \); and (iii) for all \( s, a^{(2)}(s) > 0 \).

2. **Adversarial**: Player 2 selects actions according to any arbitrary distribution, which may depend on the history of play and on player 1’s policy at each time step.

3. **Follower**: A Follower learns a best response when player 1 is “eventually stationary” (formalizing the follower concept in [Littman and Stone, 2001]), and when the value of that best response meets player 2’s standard of fairness. For some fairness threshold \( V^{(2)} \geq 0 \) (depending on the game), player 2’s algorithm has the following properties. Suppose that after time \( T_0 \), player 1 always plays a Bounded Memory algorithm (without condition 3), which induces an MDP of finite diameter \( D \) where player 2’s optimal average reward is at least \( V^{(2)} \). Then with probability at least \( 1 - \delta \), player 2’s regret up to time \( T \) (see Section 2.3) is bounded by \( C_1T_0 + C_2D(SAT \log(T/\delta))^{3/2} \) for constants \( C_1, C_2 \).

A repeated game against a Bounded Memory player is equivalent to a communicating MDP [Puterman, 1994]. A Follower formalizes an agent that models our agent as an MDP (Leader), and the regret bound in our definition is of a standard form for RL algorithms [Wei et al., 2020]. Many MARL algorithms take this approach at least partially [Powers and Shoham, 2005] and construct corresponding experts, we use several concepts from bargaining theory. We also illustrate these concepts in the game of Chicken from the first introduction (Example 2.1). Define the security values \( \mu^{(i)}_S := \max_{\nu_1, \nu_2} \min_{v_1, v_2} \mathbf{v}_1^\top R^{(i)} \mathbf{v}_2 \), i.e., the rewards that each player can guarantee regardless of their opponent’s actions, with player 1’s maximin strategy as \( \mathbf{v}_1^{(i)} = \arg \max_{\nu_1, \nu_2} \min_{v_1, v_2} \mathbf{v}_1^\top R^{(i)} \mathbf{v}_2 \). Let \( \mathcal{G} := \{ (R^{(1)}(i, j), R^{(2)}(i, j)) \mid i \in A^{(1)}, j \in A^{(2)} \} \), the set of reward.

### 2.2 BACKGROUND ON BARGAINING THEORY

To define appropriate optimality criteria for these opponent classes and construct corresponding experts, we use several concepts from bargaining theory. We also illustrate these concepts in the game of Chicken from the introduction (Example 2.1). Define the security values \( \mu^{(i)}_S := \max_{\nu_1, \nu_2} \min_{v_1, v_2} \mathbf{v}_1^\top R^{(i)} \mathbf{v}_2 \), i.e., the rewards that each player can guarantee regardless of their opponent’s actions, with player 1’s maximin strategy as \( \mathbf{v}_1^{(i)} = \arg \max_{\nu_1, \nu_2} \min_{v_1, v_2} \mathbf{v}_1^\top R^{(i)} \mathbf{v}_2 \). Let \( \mathcal{G} := \{ (R^{(1)}(i, j), R^{(2)}(i, j)) \mid i \in A^{(1)}, j \in A^{(2)} \} \), the set of reward.
The metric of regret, which we aim to minimize, varies according to the player pair (i, j). The optimal strategy for player 1, given by \( \mu_1 = \arg\max \{u_1, u_2\} \), maximizes the expected utility for player 1 at each step. The EBS is given by \( \mu_2 = \arg\max \{u_1, u_2\} \), which is the optimal strategy for player 2.

As shown in Definition 2.1, the EBS can be enforced using an algorithm \( \mathcal{B} \), which is guaranteed to achieve the optimal value \( \mu_2 \). The regret of player 1 against player 2 is given by \( \mu_1 - \mu_2 \), which is minimized by ensuring that \( \mu_1 \) is the maximizer of the joint utility function for player 1.

The regret of player 2 against player 1 is given by \( \mu_2 - \mu_1 \), which is minimized by ensuring that \( \mu_2 \) is the minimizer of the joint utility function for player 2.

The EBS is a benchmark for evaluating the performance of algorithms in multi-agent learning. The regret of an algorithm \( \mathcal{B} \) against a benchmark \( \mu_2 \) is given by \( \mu_1 - \mu_2 \), which is minimized by ensuring that \( \mu_1 \) is the maximizer of the joint utility function for player 1.

**Adversarial** Against an Adversarial player, an appropriate benchmark is the greatest expected value that player 1 can guarantee, no matter player 2’s actions. This is player 1’s security value: \( \mu(\mathcal{B}) = \mu_2 \). Note the distinction from external regret used in adversarial bandits and MDPs. While the problem is trivial if player 2 is known to be Adversarial, since one can always play the maximin strategy, our challenge is to maintain low Adversarial regret without losing guarantees on other regret measures. This corresponds to safety in multi-agent learning [Powers and Shoham 2004].

**Follower** The concept of regret against a Follower is more complex. Player 2’s sequence of policies can vary significantly based on player 1’s actions. Evaluating our algorithm by the maximum average reward in hindsight would have to account for this counterfactual dependence [Cran, dall 2014]. However, by considering enforceability, we can define benchmarks by lower bounds on this maximum, constrained by the Follower’s fairness value \( V^{(2)} \). We consider two cases depending on \( V^{(2)} \), focusing on the extremes where the Follower either accepts nothing less than the EBS or accepts any enforceable bargain. In principle, our framework could be extended for other \( V^{(2)} \) values.

First, the EBS is Pareto efficient, meaning we cannot achieve a security value of \( \mu_2 \) without player 2 receiving less than \( \mu_2 \). When the EBS can be enforced with a fixed policy, \( \mu_2 \) is thus an appropriate benchmark if the fairness threshold \( V^{(2)} \) is player 2’s part of the EBS pair. The EBS is not always enforceable for finite \( K \), however. In this case, the enforceable version of the EBS is the maximizer of the objective \( f(u_1, u_2) = \min\{u_1 - \mu_2\} \) in \( U(\epsilon) \) for some \( \epsilon > 0 \). For this first case, we therefore consider \( V^{(2)} = \mu_2 \), where player 2 follows conditionally. If \( U(\epsilon) \) is empty, \( \mu_2 = \mu_2 \). We set \( \mu(\mathcal{B}) = \mu_2 \).

The second case is \( V^{(2)} = 0 \), i.e., player 2 follows unconditionally. Here, we compute the maximizer over \( U(\epsilon) \) of \( f(u_1, u_2) = u_1 \). Let \( \mu_2(\mu_1) = \mu_2 \) be the solution to this optimization problem. The Convex Polytope. Then, the state-independent optimal average reward is given by \( \mu_{ex}(1) := \max_{\epsilon \in \Pi} \lim_{t \to \infty} \frac{1}{t} \mathbb{E}_{\epsilon}(\sum_{i=0}^{t} R^{(1)}_i | S_0) \). Here, \( \mu_1 = \mu_{ex}(1) \).

**Bounded Memory** By condition 3 for Bounded Memory, player 2 induces a communicating MDP. Let \( \Pi \) be the set of time-independent deterministic Markov policies. Then the state-independent optimal average reward is given by \( \mu_{ex}(1) := \max_{\epsilon \in \Pi} \lim_{t \to \infty} \frac{1}{t} \mathbb{E}_{\epsilon}(\sum_{i=0}^{t} R^{(1)}_i | S_0) \). Here, \( \mu_1 = \mu_{ex}(1) \).

**2.3 OBJECTIVES**

The metric of regret, which we aim to minimize, varies based on the class of player 2 our algorithm faces. For a player 2 algorithm \( \mathcal{B} \), regret with respect to a benchmark \( \mu_2(\mathcal{B}) \) is \( R(T) := T \mu_2(\mathcal{B}) - \sum_{t=1}^{T} R^{(1)}_t \).

**Example 2.1.** In Chicken (Figure 1), both players’ security value is 0.25, guaranteed by playing action 1. The EBS for player 1 is found by solving the optimization problem from Section 3.2.4 of Tossou et al. [2020] under the constraint in Definition 1. A similar procedure, applied to the objective of maximizing only player 1’s reward, gives the Bully solution for the second Leader expert. We provide details on these solutions in the Appendix.
guarantee on an algorithm’s performance against its best response. It is unclear how to characterize the best response to an algorithm capable of adapting to several opponent classes. Given this, we focus on a tractable and practically relevant subproblem: guaranteeing that the best response to our algorithm is not a “bully” in the sense discussed in the introduction, which is the most common exploitative strategy in MARL literature [Powers and Shoham 2005; Littman and Stone 2001; Press and Dyson 2012; Littman and Stone 2005]. Even this weaker guarantee is absent from previous work, and we show numerically in Section 5 that this suffices for our algorithm to be in learning equilibrium with itself (see Section 1) in a pool of top-performing algorithms.

Definition 2. Let player 2 be Bounded Memory, and \( \mu_{M}^{(1)} \) and \( \mu_{M}^{(2)} \) be the expected rewards for players 1 and 2 when player 1 uses \( v^{(1)} \) and player 2 uses \( \pi^{(2)} \). An algorithm \( \mathcal{A} \) is \((V^{(1)}, \eta_{c})\)-non-exploitable if, whenever \( \mu_{\ast}^{(1)} < V^{(1)} - \eta_{c} \) and \( \mu_{\ast}^{(2)} > \mu_{\ast}^{(2)} \) for all \( c > 0 \) player 2’s regret with respect to \( \mu_{\ast}^{(2)} + c \) against \( \mathcal{A} \) is \( O(T) \).

Our algorithm is exploitable if player 2 can profit (do better than \( \mu_{\ast}^{(2)} \)) from a policy against which we cannot achieve close to some value corresponding to a standard of fairness. The hyperparameter \( V^{(1)} \) tunes the tradeoff between exploitability and flexibility to various opponents. Player 2 does not profit from exploitation if it incur linear regret.

Example 2.2. In Chicken (Figure 1), let \( V^{(1)} = 0.625 \) (i.e., the EBS), and consider the following strategies: a) always play action 2, b) always play the opponent’s last action, and c) play the best response to the empirical distribution of the opponent’s past actions. Strategy (a) is exploitative Bounded Memory. Thus, we argue that an effective algorithm should avoid playing the “best response” of action 1, instead discouraging the use of this strategy by, e.g., consistently playing the EBS (see Egalitarian Leader in the next section). Strategy (b) is also Bounded Memory, but not exploitative since one can achieve at least \( V^{(1)} \) against this player on average. Our algorithm should therefore learn the best response to (b). Strategy (c) is a Follower with \( V^{(2)} = 0 \), thus our algorithm should converge to consistently playing action 2 against (c), achieving the Bully value.

3 LEAD AND FOLLOW FAIRLY (LAFF)

We apply an expert algorithm to a set of experts designed for our target classes. Expert algorithms use an active expert to choose an action at a given time, and switch active experts based on their relative performance [Crandall 2014]. LAFF switches experts sequentially, going to the next expert in a predefined sequence only if the rewards obtained by its active expert fall short of the current target value. Some of the experts are also designed to guarantee non-exploitability.

![Algorithmic components](image)

Figure 2: Algorithmic components (white) of LAFF’s experts (gray). An arrow from one node to another means the former is used in computation of the output by the latter.

3.1 DESCRIPTION OF EXPERTS

LAFF uses an active expert for an epoch of length \( H \) before checking whether to switch. Let \( \tau \) be the time elapsed since LAFF started using the current instance of the active expert (at time \( t_{i} + 1 \)), and define \( \tau_{1, \tau} := \frac{1}{\tau} \sum_{i=t_{i}+1}^{i+\tau} R_{i}^{(1)} \) and \( \tau_{2, \tau} := \frac{1}{\tau} \sum_{i=t_{i}+1}^{i+\tau} R_{i}^{(2)} \). See Figure 2 for a summary of algorithmic elements that these experts depend on.

Conditional Follower (\( \phi_{F} \)) Recall the benchmarks \( \mu_{B,E}^{(1)} \), \( \mu_{E}^{(1)} \), and \( \mu_{S}^{(1)} \) from Section 2.3. To handle cases where \( \mu_{B,E}^{(1)} \) against a Bounded Memory player 2 lies between these values, LAFF uses  \( \phi_{F} \) multiple times in the sequence (called “instances”). This expert starts off equivalent to Optimistic Q-learning [Wei et al. 2020], whose regret bound (in an MDP with \( S \) states and \( A \) actions) with probability at least \( 1 - \delta \) is \( RQ(\tau, \delta) = O((SA\log(S))^{1/3}T^{2/3}) \). After each subepoch of length \( H^{1/2} \), if \( \tau_{1, \tau} < V^{(1)} - RQ(\tau, \delta/T) \), this expert switches to the Egalitarian Leader \( \phi_{E} \) (below) for as long as any instance of \( \phi_{F} \) is used. Otherwise, it uses Optimistic Q-learning for the next subepoch.

Conditional Maximin (\( \phi_{M} \)) Initially, \( \phi_{M} \) uses the policy \( \pi^{(1)}(\cdot|s) = v^{(1)} \) for all \( s \). Let \( \eta_{m} > 0 \) be a slack variable, chosen based on the class of Adversarial players considered in Theorem 1. After each subepoch, if \( \tau_{1, \tau} > \mu_{B,E}^{(2)} - \eta_{m} + \sqrt{\log(T/\delta)/2\tau_{1, \tau}} \), this expert switches to the Egalitarian Leader \( \phi_{E} \) for the rest of the game. Otherwise, it uses \( v^{(1)} \) for the next subepoch.

Egalitarian Leader (\( \phi_{E} \)) If there is no enforceable EBS, let \( \phi_{E} \equiv v^{(1)} \). Otherwise, let the EBS action pairs be denoted \( (a_{E}^{(1)}(y), a_{E}^{(2)}(y)) \) for \( y = 0,1 \), and the weight on the first action pair be \( \alpha_{E} \). While \( e \)-enforceability requires that a punishment of length \( K \) is sufficient to make a reward pair player 2’s best response, this length may not be necessary. We therefore consider the least harsh punishment (if any) needed to enforce the EBS, that is, the value \( K' \leq K \) satisfying \( K' = \max\left\{ 0, \frac{\tau_{1}(a_{E}^{(1)}(0), a_{E}^{(2)}(0), a_{E}^{(1)}(1), a_{E}^{(2)}(1)) + \epsilon}{\mu_{E}^{(2)} - \eta_{m}} \right\} \).

Let \( v^{(1)} := \arg\min_{v_{1}} \max_{a_{1}} V_{1}^{(2)} R_{2}^{(2)} v_{2} \), player 1’s punishment strategy. Recall that policies in our framework are
conditioned on binary signals \(Y^{(i)}\), whose distributions are determined by players’ reported weights \(w^{(i)}_k\). Then, for the first \(K'\) time steps, with the realized value \(y^{(1)}_t\) of the signal given by \(w^{(1)}_k = \alpha_k\) for all \(t\), \(\phi_E\) plays \(a^{(1)}_E(y^{(1)}_t)\). (This ensures that, if LAFF switches to \(\phi_E\) mid-game, player 2 is not punished for having played actions other than the EBS before LAFF started signaling enforcement of the EBS.) Afterwards, \(\phi_E\) uses the following stationary policy. If, for any of the past \(K'\) timesteps, player 2 has played \(A_t^{(2)} \neq a^{(1)}_E(y^{(2)}_t)\) — i.e., deviated from the EBS — the distribution over actions for that state is \(v^{(1)}_F\). Otherwise, \(a^{(1)}_E(y^{(1)}_t)\) is played.

**Bully Leader (\(\phi_B\))** This expert is defined like \(\phi_E\), but using the Bully solution from Section 2.2 (maximizing the selfish objective). If there is no enforceable solution, given by \((a^{(1)}_B(y), a^{(2)}_B(y))\) for \(y = 0, 1\) and \(\alpha_B\), let \(\phi_B \equiv v^{(1)}_M\). Otherwise, define \(\phi_B\) just as \(\phi_E\) for this solution.

### 3.2 ALGORITHM

We design the selection of experts by LAFF (Algorithm 1) such that, for any of our target classes, LAFF eventually commits to the optimal expert against player 2 in a sequence \(\{\phi_j\}_j\). Over an epoch, the active expert is executed, and we update this expert’s average rewards since it was made active (line 5). Afterwards, LAFF switches to the next expert in the schedule if and only if it rejects the hypothesis that the current expert’s expected value exceeds its corresponding target \(\mu_j\) (line 7). The false positive rate of this hypothesis test is controlled by a function \(B\), which decreases with \(\sqrt{T}\). We define \(B\) in the proof of Lemma 1 (see Appendix). Because \(\mu^{(1)}_{B,e} > \mu^{(1)}_{E,e} \geq \mu^{(1)}_S\), and the optimal reward \(\mu^{(1)}_e\) against a Bounded Memory player may be greater than \(\mu^{(1)}_{B,e}\) or in between these values, \(\{\phi_j\}_j\) prioritizes the order of experts based on the optimal average reward they could achieve against the corresponding player 2 class (line 1).

### 4 ANALYSIS

We will now show that LAFF meets our key criteria of adaptability and non-exploitability. See Appendix for proofs of lemmas and the detailed proof of Theorem 1. Lemma 1 shows that with high probability player 2’s rewards against \(\phi_E\) are not much greater than the EBS (thus non-exploitability is feasible), and player 1’s rewards against a Follower are near the target when the correct Leader is used.

**Lemma 1. (Reward Bounds When LAFF Leads)** If player 1 uses \(\phi_E\) over a sequence of length \(\tau + K'\) starting at time \(t^* + 1\), then with probability at least \(1 - \frac{3\delta}{T}\):

\[
\sum_{t = t^* + 1}^{t^* + K' + \tau} R^{(2)}_t \leq K' + 1 + \tau \mu^{(2)}_{E,e} + 3\sqrt{\frac{3}{2} \tau \log\left(\frac{T}{\delta}\right)}.
\]

If player 2 is a Follower with \(V^{(2)} = 0\), and player 1 uses \(\phi_E\), then with probability at least \(1 - \frac{5\delta}{T}\), we have \(V^{(1)}_{i,\tau} \geq \mu^{(1)}_{B,e} - V^{(2)}(\tau)\). If \(V^{(2)} = \mu^{(1)}_{E,e}\) or player 1 uses \(\phi_e\), then with probability at least \(1 - \frac{5\delta}{T}\), we have \(V^{(1)}_{i,\tau} \geq \mu^{(1)}_{E,e} - V^{(2)}(\tau)\).

**Lemma 2. (False Positive and Negative Control of Exploitation Test)** Consider a sequence of \(k\) epochs each of length \(H\). Let \(m^{(1)}_F\) or \(m^{(1)}_M\) be, respectively, the index of the subepoch within this sequence at the start of which \(\phi_F\) or \(\phi_M\) switches to punish with \(\phi_E\), if at all (if not, let \(m^{(1)}_F = m^{(1)}_M = \infty\)). Let \(\eta_m = \frac{2kH}{H/2+\delta/T} + \sqrt{\frac{25k^2\log(20)}{c_0H}}\).\(^\dagger\) Let \(\eta_m \geq \frac{\log(T/\delta)}{2(H/2-\delta)} + \sqrt{\frac{64c_1\log(20)}{2c_1H} - \frac{64c_1\log(20)}{2c_1H} + \frac{64c_1\log(20)}{2c_1H} + \frac{64c_1\log(20)}{2c_1H}}\), where \(\lambda\) and \(N_\delta\) are constants with respect to time defined in Lemma 4 (see Appendix).

Then, suppose player 2 is Bounded Memory, and \(\phi_E\) is used. If \(\mu^{(1)}_e < V^{(1)}(1) - \eta_m\), then with probability at least \(1 - \delta\), \(m^{(1)}_F \leq \frac{[H^2/2] - \delta}{T}\). If \(\mu^{(1)}_e \geq V^{(1)}(1)\), then with probability at most \(kH^2/2\delta\), \(m^{(1)}_F \leq \infty\). If \(\phi_F\) is used, and \(\mu^{(2)}_{E,e} > \mu^{(2)}_{E,e}\), then with probability at least \(1 - \delta\), \(m^{(1)}_M \leq \frac{[H^2/2] - \delta}{T}\).

Suppose player 2 is Adversarial, with a sequence of action distributions \(\{\pi^{(2)}_i\}\) such that, for any \(M \geq H/2 - K\) and \(i, \frac{1}{M} \sum_{t=i+1}^{i+M} R^{(2)}_t \leq \mu^{(2)}_{E,e} - \eta_m\). Then, if \(\phi_F\) is used, and \(\mu^{(2)}_{E,e} \leq \mu^{(2)}_{E,e}\), then with probability at most \(kH^2/2\delta\), \(m^{(1)}_M \leq \infty\).

Our main result, Theorem 1, claims that 1) against each of our target classes, LAFF achieves a regret bound of the...
with probability at least $\frac{1}{5}$, wards are sufficiently high that the condition for switching whose target class matches the opponent (thus guaranteeing we exclude from our target class some subset of opponents. Then, we need only show that once LAFF reaches the expert for each opponent class, we need to show

Proof Sketch. That is, we cannot guarantee low Adversarial regret against its target opponent class, by, respectively: the definitions of $\mu_{E,e}$ and $\mu_{E,\epsilon}$, or

Let $\eta_m$ and $\eta_e$ satisfy the conditions of Lemma 2. Then, with probability at least $1 - 5\delta$, LAFF satisfies:

$$
\max \mathcal{R}(T) = O(\mathcal{R}_Q(T, \delta/T)).
$$

Further, with probability at least $1 - 6\delta$, LAFF is $(\nu^{(1)}, \eta_e)$-non-exploitable when there exists an enforceable EBS.

If there is no enforceable EBS, $\mu_{E,e}^{(2)} = \mu_{E,\epsilon}^{(2)}$ and so we cannot guarantee player 2 does worse than $\mu_{E,\epsilon}^{(2)}$ in expectation. The class of Adversarial players for which Theorem 1 holds is technically restrictive. However, non-exploitability requires that for each strategy (expert) used by our algorithm that could be exploited, including Conditional Maximin, we exclude from our target class some subset of opponents. That is, we cannot guarantee low Adversarial regret against players who receive more than the EBS value against maximin, because such players may exploit us.

Proof Sketch. For each opponent class, we need to show that with high probability LAFF does not lock in to a suboptimal expert for that class. If LAFF locks in to an expert for which the corresponding target value $\mu_j$ is greater than the opponent’s benchmark $\mu(\mathfrak{B})$, this implies LAFF consistently receives rewards such that “regret” with respect to $\mu_j$ grows like $\mathcal{R}_Q$, by design of $B(\tau)$. But since the benchmark is less than $\mu_j$, the true regret is also bounded as desired.

We therefore only need to consider the cases of $\mu_j \leq \mu(\mathfrak{B})$. First, we know that each expert achieves at most $\mathcal{R}_Q$ regret against its target opponent class, by, respectively: the definitions of $\mathcal{R}_Q$ (for non-exploitotive Bounded Memory) and maximin (for Adversarial), and Lemma 1 (for Followers). Lemma 2 ensures with high probability that $\phi_F$ and $\phi_M$ do not switch to $\phi_E$ when not exploited, so they inherit the desired regret bounds.

Then, we need only show that once LAFF reaches the expert whose target class matches the opponent (thus guaranteeing low regret using that expert), with high probability LAFF does not switch. But if using the corresponding expert gives LAFF low regret with respect to $\mu(\mathfrak{B})$, then its rewards are sufficiently high that the condition for switching experts (line 7 of Algorithm 1) never holds. The first claim of the theorem follows.

To show non-exploitability, suppose LAFF locks in to the first instance of $\phi_F$. By Lemma 2, $\phi_F$ detects evidence of exploitation sufficiently early that the remaining time left in the game is linear in $T$. After detecting exploitation, $\phi_F$ plays the same policy as $\phi_E$. But by Lemma 3 against this policy player 2 cannot guarantee an average reward greater than $\mu_{E,\epsilon}^{(2)}$ plus a term that vanishes at a rate $T^{1/4}$. The second claim of the theorem follows for the other possible locked-in experts as well by considering two facts. First, whenever $\phi_E$ or $\phi_M$ is used, Lemma 2 again bounds player 2’s rewards, since by Pareto efficiency of the EBS player 2’s rewards from the Bully solution cannot exceed $\mu_{E,\epsilon}^{(2)}$. Second, if LAFF reaches $\phi_M$, again Lemma 2 ensures sufficiently fast detection of exploitation with high probability.

5 NUMERICAL EXPERIMENTS

Code for the experiments in this section is available on Github. We evaluate LAFF by three empirical metrics. First, we find LAFF’s empirical regret against one algorithm from each target class. Second, LAFF and a set of top-performing repeated games algorithms compete in a round-robin tournament. For each algorithm, we find its rewards against its best response algorithm in this set, and check if it is in a learning equilibrium by applying a Nash equilibrium solver [Knight and Campbell 2018] to the matrices of empirical rewards for algorithm pairs. These criteria evaluate exploitability: more exploitable algorithms have lower rewards against algorithms that optimize against them, and an exploitable algorithm cannot be in equilibrium with itself unless the fairness threshold $\nu^{(1)}$ is low. Finally, we perform a replicator dynamic simulation [Crandall et al. 2018]. Each generation, the algorithms’ fitness values are computed as averages of the round-robin scores weighted by the distribution of the population of algorithms. Then, the population distribution is updated in proportion to fitness. This evaluates how well a given algorithm performs when the distribution of its opponents is determined by those algorithms’ own performance. Exploitability is thus implicitly penalized by accounting for opponents’ incentives. Details on the implementation of these experiments are in the Appendix. We set $\nu^{(1)} = \mu_{E,\epsilon}^{(2)}$.

Our set of competitors to LAFF consists of Bounded Memory (Bully, Forgiving Generalized Tit-for-Tat or FTFT), Follower (M-Qubed, Q-Learning, Fictitious Play), and expert (Manipulator, S++) algorithms. See Appendix for details and sources. We chose these algorithms because, first, they performed well in a repeated games tournament [Crandall et al. 2018], and second, they cover our opponent classes. S++ and Manipulator do not fall cleanly into any of those classes, but they are the closest comparisons in previous lit-
Unconditional Follower (Q-Learning)        Conditional Follower (LAFF)         Bounded Memory (FTFT)

Unconditional Follower (Q-Learning)        Conditional Follower (LAFF)         Bounded Memory (FTFT)

Unconditional Follower (Q-Learning)        Conditional Follower (LAFF)         Bounded Memory (FTFT)

Unconditional Follower (Q-Learning)        Conditional Follower (LAFF)         Bounded Memory (FTFT)

Figure 3: The first four plots show LAFF’s average regret, in each of 11 games detailed in the Appendix, for the following opponents: Unconditional Follower (Q-Learning), Conditional Follower (LAFF), Bounded Memory (FTFT), Adversarial (Manipulator). The last plot shows the regret of an Exploitative (Bully) algorithm against LAFF.

error to LAFF, since they adapt to a variety of opponents by switching between Leader and Follower experts.

To ensure sufficient diversity of test games, we choose games based on the taxonomy of Figure 1 in Bruns [2010]. Six game families are categorized by the structures of their Nash equilibria. We use two games from each family, one with symmetric rewards and one with asymmetric, except Cyclic, which has no symmetric games (see Appendix).

Regret Bounds  Figure 3 shows LAFF’s regret, averaged over 50 trials, in games against an algorithm from each target class, and the regret of an exploitative Bounded Memory algorithm against LAFF. We chose Manipulator as “Adversarial” because it does not play the EBS and is not a pure Leader or Follower. However, in the symmetric Unfair game, the empirical rewards indicate that Manipulator attempts to exploit LAFF, so LAFF punishes Manipulator at the expense of the Adversarial regret guarantee. From the plot evaluating player 2’s regret, we also exclude four games where player 2’s Bully solution equals the EBS, since in these cases $\mu^{(1)} \geq V^{(1)}$ (player 1 is not exploited by playing the optimal policy). In most games, LAFF’s regret eventually plateaus, while the exploitative player has linear regret, showing that LAFF is non-exploitable. In three games, LAFF has linear regret against an Unconditional Follower and non-exploitative Bounded Memory player. This may be due to the practical difficulty of choosing hyperparameters for tests used to decide when to switch to the next expert; these tests depend on some unknown quantities, so for our experiments, we tuned $B(\tau)$ on a training set of four games that are not included in the set of 11 games for these results (see Appendix). Longer time horizons may be required for the conditions on $\eta_e$ in Lemma 2 to hold. We used a horizon of $T = 2 \cdot 10^5$ to be on the same approximate scale as experiments in other works on repeated games [Crandall and Goodrich, 2010, Littman and Stone, 2005, Crandall, 2014].

Figure 4: Replicator dynamic results, where the bold curves are average population shares and shaded regions are plus and minus one standard deviation.

Round Robin  Table 1 shows the average rewards of each algorithm pair across the 11 games and 50 trials, which provide an empirical bimatrix for the learning game, i.e., a meta-game in which users choose algorithms to deploy across different repeated games. An algorithm’s reward against its best response (highlighted in blue) measures how much it bullies when possible and avoids exploitation.
Table 1: Rewards of algorithm pairs, averaged over games and trials (pure learning equilibria in are highlighted in bold text, and each algorithm’s reward against its best response is in blue)

|        | S++  | Manipulator | M-Qubed | Bully      | Q-Learning | LAFF  | FTFT  | FP   |
|--------|------|-------------|---------|------------|------------|-------|-------|------|
| S++    | 0.75, 0.76 | 0.73, 0.80 | 0.73, 0.81 | 0.65, 0.77 | 0.82, 0.76 | 0.71, 0.8 | 0.70, 0.68 | 0.72, 0.55 |
| Manipulator | 0.87, 0.68 | 0.76, 0.71 | 0.77, 0.65 | 0.65, 0.77 | 0.89, 0.67 | 0.70, 0.65 | 0.71, 0.60 | 0.76, 0.55 |
| M-Qubed | 0.88, 0.68 | 0.68, 0.68 | 0.80, 0.74 | 0.65, 0.80 | 0.79, 0.75 | 0.76, 0.73 | 0.78, 0.65 | 0.62, 0.56 |
| Bully   | 0.86, 0.61 | 0.83, 0.60 | 0.85, 0.61 | 0.48, 0.44 | 0.91, 0.63 | 0.61, 0.49 | 0.72, 0.55 | 0.76, 0.56 |
| Q-Learning | 0.82, 0.77 | 0.73, 0.83 | 0.79, 0.67 | 0.68, 0.85 | 0.83, 0.74 | 0.71, 0.84 | 0.81, 0.67 | 0.64, 0.56 |
| LAFF    | 0.87, 0.65 | 0.71, 0.66 | 0.74, 0.72 | 0.55, 0.61 | 0.90, 0.66 | 0.77, 0.74 | 0.80, 0.70 | 0.75, 0.57 |
| FTFT    | 0.64, 0.70 | 0.49, 0.71 | 0.59, 0.76 | 0.60, 0.71 | 0.59, 0.78 | 0.61, 0.78 | 0.80, 0.75 | 0.46, 0.72 |
| FP      | 0.70, 0.73 | 0.66, 0.74 | 0.66, 0.55 | 0.63, 0.73 | 0.69, 0.57 | 0.61, 0.71 | 0.71, 0.60 | 0.68, 0.55 |

Both as player 1 and player 2, LAFF is second by this metric, behind Bully. We also highlight the pure strategy Nash equilibria of this learning game (in bold), noting that LAFF is in a learning equilibrium with itself. Unfortunately, the pairing in which Q-Learning follows Bully is also an equilibrium. Thus there is an equilibrium selection problem, e.g., both users might choose Bully and receive very low rewards. However, in practice it may be easier for users to coordinate on both using LAFF, because there is no conflict over choosing which side is the Leader (Bully) versus the Follower (Q-Learning).

**Replicator Dynamic** On average over 1000 runs, LAFF converges to 100% of the population in the pool of algorithms (Figure 4), based on fitness computed as the minimum of an algorithm’s average reward over the set of games when playing as player 1 versus player 2. This metric matches the motivation for the EBS; algorithm users will not know a priori which of the two “sides” of the game they will be in. Thus, they may prefer their algorithm to cooperate with itself (maximize an egalitarian objective), instead of bullying its copy in hopes of being on the side of the bully.

6 DISCUSSION

When choosing algorithms for multi-agent interactions, users will have to trade off robustness to the variety of possible algorithms they might face, with avoiding providing other users incentives to exploit them [Stastny et al., 2021]. We have presented an algorithm for repeated games that balances these desiderata. Both properties can facilitate cooperation between learning agents, while still allowing them to accept generous offers. If LAFF faces an agent who “follows” fair, Pareto efficient bargaining proposals, the Egalitarian Leader leads them to a mutual benefit over their security values. If the other agent’s fairness standard is different, the Conditional Follower can follow this alternative proposal using RL if it is not exploitative; otherwise, the exploitation penalty encourages the other player to be more cooperative. Against exploitable agents, the Bully Leader can benefit from a more self-interested bargain. Finally, if the other player is unwilling to cooperate at all but is not exploitative, Conditional Maximin ensures safety. In future work, more experts can be added based on agent classes that we have neglected. For example, while LAFF includes Leader experts only for the extreme cases in which player 2 has a high or minimal fairness standard, one could add Leaders for other bargaining solutions.

The biggest limitations of our approach are restrictive assumptions required for our non-exploitability criterion, and the strictness of this criterion. The margin $\eta_e$ is small only for sufficiently large time horizons, hence the linear regret in some of our experiments. Though LAFF successfully punishes players against whom it receives less than fair rewards, this is only strategically necessary when such players benefit from playing this way (genuine “exploitation”). It may not be practically necessary to modify the experts to not punish when the opponent also does worse, because an opponent would not have an incentive to lead with a Pareto inefficient policy. Finally, we note that our approach is not intended to provide the optimal balance of the adaptability-exploitability tradeoff; in particular, keeping a fixed fairness threshold may not be ideal if it prevents an algorithm from cooperating with algorithms that follow other intuitively “fair” standards [Stastny et al., 2021].

Author Contributions

Both authors conceived and carried out the research project jointly. A.D. wrote the paper and code for numerical experiments. A.T. helped edit the paper.

Acknowledgements

A.D. acknowledges the support of a grant from the Center on Long-Term Risk Fund.
References

Ronen I. Brafman and Moshe Tennenholtz. Efficient learning equilibrium. *Artificial Intelligence*, 159:27–47, 2004.

Bryan Bruns. Navigating the topology of 2x2 games: an introductory note on payoff families, normalization, and natural order. 2010. URL https://arxiv.org/abs/1010.4727.

Doran Chakraborty and Peter Stone. Convergence, targeted optimality and safety in multiagent learning. *Proceedings of the Twenty-seventh International Conference on Machine Learning (ICML)*, 2010.

Jesse Clifton and Maxime Riché. Towards cooperation in learning games, 2021.

Jacob W. Crandall. Towards minimizing disappointment in repeated games. *Journal of Artificial Intelligence Research*, 49:111–142, 2014.

Jacob W. Crandall. When autonomous agents model other agents: An appeal for altered judgment coupled with mouths, ears, and a little more tape. *Artificial Intelligence*, 2020.

Jacob W. Crandall and Michael A. Goodrich. Learning to compete, coordinate, and cooperate in repeated games using reinforcement learning. *Machine Learning*, 82:281–314, 2010.

Aristide C.Y. Tossou, Christos Dimitrakakis, Jaroslaw Rzepecki, and Katja Hofmann. A novel individually rational objective in multi-agent multi-armed bandits: Algorithms and regret bounds. *Proc. of the 19th International Conference on Autonomous Agents and Multiagent Systems*, 2020.

Vincent Knight and James Campbell. Nashpy: A Python library for the computation of Nash equilibria. *Journal of Open Source Software*, 3(30):904, 2018. doi: 10.21105/joss.00904. URL https://doi.org/10.21105/joss.00904.

Michael Littman and Peter Stone. Implicit negotiation in repeated games. *Revised Papers from the 8th International Workshop on Intelligent Agents VIII*, 2001.

George Mailath and Larry Samuelson. *Repeated Games and Reputations: Long-Run Relationships*. Oxford University Press, 2006.

Shie Mannor and John N. Tsitsiklis. On the empirical state-action frequencies in Markov decision processes under general policies. *Mathematics of Operations Research*, 30:545–561, 2005.

Rob Powers and Yoav Shoham. New criteria and a new algorithm for learning in multi-agent systems. *Neural Information Processing Systems*, 2004.

Rob Powers and Yoav Shoham. Learning against opponents with bounded memory. *Proceedings of the 19th International Joint Conference on Artificial Intelligence*, 2005.

William H. Press and Freeman J. Dyson. Iterated prisoner’s dilemma contains strategies that dominate any evolutionary opponent. *Proceedings of the National Academy of Sciences*, 109(26):10409–10413, 2012. ISSN 0027-8424. doi: 10.1073/pnas.1206569109. URL https://www.pnas.org/content/109/26/10409.

Martin L. Puterman. *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley & Sons, Inc., New York, NY, USA, 1994.

Julian Stastny, Maxime Riché, Alexander Lyzhov, Johannes Treutlein, Allan Dafoe, and Jesse Clifton. Normative disagreement as a challenge for cooperative AI, 2021.

Chen-Yu Wei, Mehdi Jafarnia-Jahromi, Haipeng Luo, Hiteshi Sharma, and Rahul Jain. Model-free reinforcement learning in infinite-horizon average-reward Markov decision processes. volume 119 of *Proceedings of the 37th International Conference on Machine Learning*, 2020.