THE FIRST NEPTUNE ANALOG OR SUPER-EARTH WITH A NEPTUNE-LIKE ORBIT: MOA-2013-BLG-605LB

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ABSTRACT

We present the discovery of the first Neptune analog exoplanet or super-Earth with a Neptune-like orbit, MOA-2013-BLG-605Lb. This planet has a mass similar to that of Neptune or a super-Earth and it orbits at 9 – 14 times the expected position of the snow line, $a_{\text{snow}}$, which is similar to Neptune’s separation of 11 $a_{\text{snow}}$ from the Sun. The planet/host-star mass ratio is $q = (3.6 \pm 0.7) \times 10^{-5}$ and the projected separation normalized by the Einstein radius is $s = 2.39 \pm 0.05$. There are three degenerate physical solutions and two of these are due to a new type of degeneracy in the microlensing parallax parameters, which we designate “the wide degeneracy.” The three models have (i) a Neptune-mass planet with a mass of $M_p = 21.5 \pm 7 M_{\oplus}$ orbiting a low-mass M-dwarf with a mass of $M_\ast = 0.19^{+0.05}_{-0.06} M_\odot$, (ii) a mini-Neptune with $M_p = 7.9^{+1.8}_{-1.2} M_{\oplus}$ orbiting a brown dwarf host with $M_\ast = 0.068^{+0.019}_{-0.018} M_\odot$, and (iii) a super-Earth with $M_p = 3.2^{+0.5}_{-0.3} M_{\oplus}$ orbiting a low-mass brown dwarf host with $M_\ast = 0.025^{+0.005}_{-0.004} M_\odot$, which is slightly favored. The 3D planet–host separations are $4.6^{+2.1}_{-1.2}$ au, $2.1^{+0.2}_{-0.1}$ au, and $0.94^{+0.07}_{-0.06}$ au, which are $8.9^{+10.3}_{-7.7}$, $12^{+5}_{-4}$, or $14^{+11}_{-11}$ times larger than $a_{\text{snow}}$ for these models, respectively. Keck adaptive optics observations confirm that the lens is faint. This discovery suggests that low-mass planets with Neptune-like orbits are common. Therefore processes similar to that one formed Neptune in our own solar system or cold super-Earths may be common in other solar systems.

Key words: Galaxy: bulge – gravitational lensing: micro – planetary systems

1. INTRODUCTION

The formation of the ice giants Uranus and Neptune is not well understood. In the favored core accretion theory, the gas giant planets like Jupiter and Saturn are believed to form through the accumulation of small icy planetesimals into solid cores of about 5–15 $M_\oplus$ in the region beyond the snow line at $a_{\text{snow}} \approx 2.7 (M_\ast/M_\odot)^{1/3}$ (Ida & Lin 2004; Laughlin et al. 2004; Kennedy et al. 2006), where the protoplanetary disk is cold enough for ices (especially water-ice) to condense. However, such a scenario cannot form smaller ice giants like Uranus and Neptune at their current orbital positions, due to the low density

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20 Microlensing Observations in Astrophysics (MOA) Collaboration.
21 Optical Gravitational Lens Experiment (OGLE) Collaboration.
of planetesimals and slow evolution in these orbits (Pollack et al. 1996). One idea is that Uranus and Neptune formed in the Jupiter–Saturn region between ∼5 and ∼17 au, then migrated outwards to their current positions (Fernandez & Ip 1984; Thommes et al. 1999; Helled & Bodenheimer 2014).

The formation of super-Earth exoplanets with a Neptune-like orbit is even less understood. This is partly because we do not even know if they exist, unlike in the case of Neptune. They are also not expected based on the standard core accretion theory for the same reason as the ice giants mentioned above (Ida & Lin 2004). Their formation may be similar or related to the formation of Neptune-like ice giants.

The distribution of such cold ice-giant planets and super-Earths in other solar systems is important for understanding the formation of our own cold ice giants. Also, in our own solar system, the distribution of Kuiper Belt objects (KBOs) is dominated by gravitational interactions with Neptune. Since KBOs hold large amounts of water and other volatiles needed for life, it could be that exo-Neptunes play an important role in the development of life in some exoplanetary systems, whether or not they play this role in our own solar system.

In the 20 years since the first exoplanet discovery (Mayor & Queloz 1995), there have been repeated discoveries of planets that are quite different from those in our own Solar System. However, the detection of planets similar to those in our own Solar System has been more difficult. Only Jupiter analogs have been detected orbiting solar type stars (Wittenmyer et al. 2014), while Jupiter/Saturn (Gaudi et al. 2008; Bennett et al. 2010) and Venus/Earth analogs (Burke et al. 2014; Quintana et al. 2014) have been found orbiting low-mass stars. Very cold, low-mass planets have yet to be explored (see the distribution of known exoplanets as of 2015 October 622 in Figure 1). Cold ice giants like Uranus and Neptune are very difficult to detect with the radial velocity and transit methods owing to their long orbital periods (80–160 years), low orbital velocities, and low transit probabilities. It is even more difficult to detect a super-Earth in such wide orbit. The direct imaging method can detect wide-orbit planets if they are self-luminous, but otherwise they will be far too faint to detect, especially if they are as small as Neptune or a super-Earth.

Recently, low-mass stars (i.e., M-dwarfs) have attracted more interest in exoplanet search programs because of their high detectability of habitable or cold low-mass planets. Kepler’s 150,000 targets contain about 3000 red dwarfs and more than a hundred planetary systems have been found orbiting these stars (Morton & Swift 2014). These results show that smaller planets are more common than larger planets around M-dwarfs, and planets with radii of ∼1.25 R_E are the most common planets in these systems. Dressing & Charbonneau (2013) estimated an occurrence rate of ∼0.5 habitable zone Earth size planets per M-dwarf, and Quintana et al. (2014) found an Earth-radius habitable planet around a ∼0.5 M_☉ M-dwarf. That smaller planets are more common than larger planets around M-dwarfs may be related to the fact that only small mass protoplanetary disks have been found around such low-mass stars (Kennedy & Kenyon 2008). The TRENDS high-contrast imaging survey, in combination with radial velocity measurements, indicates that 6.5% ± 3.0% of M-dwarf stars host one or more massive companions with 1 < m/M_J < 13 and 0 < a < 20 au, however, this survey is not sensitive to cold ice planets (Clanton & Gaudi 2014; Montet et al. 2014).

The gravitational microlensing method is also sensitive to planets around M-dwarfs and even brown dwarfs because it does not rely on the light from the host stars. Microlensing relies upon random alignments between background source stars and foreground lens star–planet systems, and more massive lens stars are only favored by the factor √M, while

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22 http://exoplanet.eu

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Figure 1. The distribution in planetary mass, M_p, vs. the semimajor axis, a (left panel) and a normalized by the snow line (right panel) of exoplanets discovered by various methods. Red circles indicate the microlensing planets. Microlensing planets for which mass measurements have been made are indicated with filled circles. Microlensing planets where the mass has been estimated by a Bayesian analysis are indicated with open circles. The six model solutions for event MOA-2013-BLG-605Lb comprising parallax and the Keplerian orbital motion with the Keplerian prior and the kinematic constraint (M_Σ) are indicated by purple filled circles. Black dots represent the radial velocity planets and blue filled squares are transit planets. Cyan dots are transit planets found by Kepler. Magenta triangles denote planets found via direct imaging. Green open squares denotes planets found via timing measurements. Solar system planets are indicated by their initial. A green vertical dashed line indicates the snow line. All models for MOA-2013-BLG-605Lb are very similar to Neptune, when planet orbit radii are scaled to the snow line (right panel).
smaller masses have shorter timescales which can also bias against detection. So M-dwarf lens stars dominate microlensing events. In contrast to other methods, microlensing is sensitive to low-mass planets down to an Earth-mass (Bennett & Rhie 1996) orbiting beyond the snow line, as shown in Figure 1. Microlensing is therefore complementary to the other planet detection techniques. Statistical analyses of microlensing samples indicate that the planet abundance beyond the snow line is about a factor ~7 larger than the abundance of close-in planets. Neptune-mass planets are more abundant than gas giants around M-dwarfs, and one or more planets per star in total are predicted just beyond the snow line (Gould et al. 2010; Sumi et al. 2010; Cassan et al. 2012).

In about half of the planetary systems found by microlensing, the mass of the host and planets and their projected separation have been measured by microlensing parallax in combination with the finite source effect (Bennett et al. 2008; Gaudi et al. 2008; Muraki et al. 2011; Kains et al. 2013; Tsapras et al. 2014; Udalski et al. 2015) and/or direct detection of the lens flux by high resolution imaging by adaptive optics (AO) (Bennett et al. 2010; Kubas et al. 2012; Batista et al. 2014, 2015) or the Hubble Space Telescope (HST) (Bennett et al. 2006, 2015; Dong et al. 2009a). The probability distribution of physical mass and separations of other events have been estimated using a Bayesian analysis assuming a Galactic model. Among the planetary systems with mass measurements, two of them have very low-mass hosts, less than 0.2\(M_\odot\) and each system has a planetary mass ratio \(q < 0.01\). These two systems are MOA-2007-BLG-192 L \((M_\text{L} = 0.084^{+0.015}_{-0.012} M_\odot, M_\text{p} = 3.2^{+5.2}_{-1.3} M_\oplus)\) (Bennett et al. 2008; Kubas et al. 2012) and MOA-2010-BLG-328 L \((M_\text{L} = 0.11 \pm 0.01 M_\odot, M_\text{p} = 9.2 \pm 2.2 M_\oplus)\) (Furusawa et al. 2013). Neptune analog planets and super-Earths with Neptune-like orbits are still difficult to detect even with microlensing.

Recently, Poleski et al. (2014) found a planet in a Uranus-like orbit with a mass of \(\sim 4 M_\text{Uranus} \) at \(\sim 18\) au around a \(\sim 0.7 M_\odot\) star. This is \(\sim 9\) times the snow line of the host. While their mass estimates are based on a Bayesian analysis and have large uncertainties, their detection demonstrated the ability to detect planets in these orbits with microlensing.

In this paper, we present the detection and the mass measurement of the first Neptune analog MOA-2013-BLG-605Lb via microlensing. We detected the microlensing parallax effect which yielded the mass measurement of the lens system in combination with the finite source effect.

Microlensing parallax can be measured when one observes an event simultaneously from two different locations, either with a telescope on Earth and a space telescope, (Refsdal 1966; Udalski et al. 2015) or with two ground-based telescopes, referred to as terrestrial parallax (Gould et al. 2009). It is known that there is a four-fold degeneracy in these parallax measurements, (Refsdal 1966; Gould 1994). Two elements of this four-fold degeneracy correspond to two different magnitudes of the measured parallax. As a result, the physical parameters of the lens differ between these two degenerate solutions. The other two degenerate solutions in the four-fold degeneracy just arise from a symmetry in the lensing geometry. The physical parameters of the lens are the same between these two degenerate solutions, except the projected velocities which can be used to distinguish among solutions (Calchi Novati et al. 2015a). Most commonly, parallax measurements have been made by observing an event from an accelerated observer; specifically from ground-based observations of an event which is long enough for Earth to move significantly in its orbit around the Sun. This is referred to as orbital parallax (Gould 1992). There is also an analogous four-fold discrete degeneracy for orbital parallax, termed the “jerk parallax” degeneracy and their mirror solutions (Gould 2004; Park et al. 2004). For the binary lens case, there is an approximate degeneracy in the parallax parameters, known as the “ecliptic degeneracy” (Skowron et al. 2011). In this work on event MOA-2013-BLG-605, we report a new type of degeneracy in parallax model solutions, which is specific to widely separated binary lenses. The details of this new degeneracy are presented in Section 4.

We describe the observations of, and photometric data for, event MOA-2013-BLG-605 in Sections 2 and 3. The light curve modeling is described in Section 4. In Sections 5 and 6 we present the physical parameters of the lens system and constraints using Keck AO observation. We discuss, in Section 7, the manner in which we might measure the lens mass in the future and we present an overall discussion and our conclusions in Section 8.

2. OBSERVATION

The Microlensing Observations in Astrophysics (MOA; Bond et al. 2001; Sumi et al. 2003) collaboration is carrying out a microlensing survey toward the Galactic bulge from the Mt. John University Observatory in New Zealand. The MOA-II survey (Sumi et al. 2011) is a very high cadence photometric survey of the Galactic bulge with the 1.8 m MOA-II telescope equipped with a 2.2 deg\(^2\) field of view (FOV) CCD camera. The 2013 MOA-II observing strategy called for the six fields \((\sim 13\) deg\(^2\)) with the highest lensing rate to be observed with a 15 minute cadence, while the next six best fields were observed with a 47 minute cadence, and eight additional fields were observed with a 95 minute cadence. Most MOA-II observations use the custom MOA-red wide band filter, which corresponds to the sum of the standard Cousins R- and I-bands. MOA-II issues \(\sim 600\) alerts of microlensing events in real time each year.\(^{23}\)

The Optical Gravitational Lensing Experiment (OGLE; Udalski et al. 2015) also conducts a microlensing survey toward the Galactic bulge with the 1.3 m Warsaw telescope at the Las Campanas Observatory in Chile. The fourth phase of OGLE, OGLE-IV started its high cadence survey observations in 2010 with a 1.4 deg\(^2\) FOV mosaic CCD camera. OGLE observes bulge fields with cadences ranging from one observation every 20 minutes for three central fields to less than one observation every night for the outer bulge fields. Most observations are taken in the standard Kron–Cousin I-band with occasional observations in the Johnson V-band. OGLE-IV issues \(\sim 2000\) microlensing event alerts in real time each year.\(^{24}\)

The microlensing event MOA-2013-BLG-605 was discovered at \((\alpha, \delta)(2000) = (17:58:42.85, -29:23:53.66) [\ell, b] = (1.0583, -2.695^\circ)]\), in MOA field gb9, which is monitored every 15 minutes, and it was announced by the MOA Alert System on 2013 August 30 (HJD\(^{'}\) \equiv HJD \(-2450000 \sim 6535\)). Figure 2 shows the light curve. At the time of its discovery, MOA recognized this event as a possible free-floating planet candidate (Sumi et al. 2011) as the best-fit

\(^{23}\) https://it019909.massey.ac.nz/moa/

\(^{24}\) http://ogle.astrouw.edu.pl/ogle4/ews/ews.html
single lens light curve had an Einstein radius crossing time of $t_E = 0.73 \pm 0.10$ days (See Figure 2). Nearly four weeks later, the OGLE Early Warning System (EWS) (Udalski 2003) detected this event being magnified again with a longer timescale due to the lensing effect of the host star. The OGLE EWS system announced this event as OGLE-2013-BLG-1835 on 2013 September 25 (HJD′ ∼ 6560), as shown in the top panel of Figure 2. The initial short magnification by the planet at HJD′ ∼ 6535 was confirmed by the OGLE survey data. In fact, it should have triggered the OGLE discovery alert but due to an unfortunate deeply hidden bug in the EWS software this did not happen. The later magnification by the host was observed by MOA as well as OGLE.

Follow-up observations of the stellar part of the light curve in the V, I, and H-bands were obtained by the µFUN collaboration using the SMARTS-CTIO 1.3 m telescope. These data were taken mainly to extract the source color. We use the average of these CTIO and OGLE V−I color measurements. CTIO H-band measurements are used to drive H-band source magnitude, which is very important for comparisons to the AO observations (see Section 6).

3. DATA REDUCTION

The MOA images were reduced with MOA’s implementation (Bond et al. 2001) of the difference image analysis (DIA) method (Tomany & Crotts 1996; Alard & Lupton 1998; Alard 2000). In the MOA photometry, we found that there were systematic errors that correlate with the seeing and airmass, as well as the motion, due to differential refraction, of a nearby, possibly unresolved star. There is also a potential systematic error due to the relative proper motion of a nearby star or stars, which we model as linear function in time. We ran a detrending code to measure these effects in the 2011, 2012, and 2014 data, and we removed these trends with additive corrections to the full 2011–2014 data set. (The MOA data from 2006–2010 indicate no significant photometric variations, but they are not included in the light curve analysis.) This detrending procedure improved the fit $\chi^2$ by $\Delta \chi^2 = 0.073$ per data point in the baseline, so it has reduced the systematic photometry errors significantly. This investigation of the systematics is necessary to have confidence in the modeling of the light curve with high order effects in the following section.

The OGLE data were reduced with the OGLE DIA (Woźniak 2000) photometry pipeline (Udalski et al. 2015). In this event, the center of the magnified source star is slightly shifted from the center of the apparent star identified in the reference image, due to blending with one or more unresolved stars. So the OGLE data have been re-reduced with a centroid based difference images, just as the MOA pipeline does (Bond et al. 2001).
Table 1

| Data set     | Telescope     | Filter | \(N_{\text{data}}\) | \(k\)   | \(\epsilon_{\text{min}}\) |
|--------------|---------------|--------|----------------------|--------|--------------------------|
| MOA-Red      | MOA-II 1.8 m  | \(R + I\) | 9675                 | 1.092  | 0.0127                   |
| OGLE-I       | Warsaw 1.3 m  | I      | 5514                 | 3.867  | 0.0109                   |
| OGLE-V       | Warsaw 1.3 m  | V      | 64                   | 1.571  | 0.0                      |
| CTIO-I       | SMARTS-CTIO 1.3 m | I | 15                  | 1.0    | 0.0                      |
| CTIO-V       | SMARTS-CTIO 1.3 m | V | 15                  | 1.0    | 0.0                      |
| CTIO-H       | SMARTS-CTIO 1.3 m | H | 149                 | 1.0    | 0.0                      |

Note. \(N_{\text{data}}\) is the number of data points used in the analysis. \(k\) and \(\epsilon_{\text{min}}\) are the error scaling parameters (see details in the text).

The number of data points used for the light curve modeling are 9675, 5514, and 64 for the MOA-Red, OGLE-I, and OGLE-V passbands, respectively. The photometric error bars provided by the photometry codes give approximate estimates of the absolute photometric uncertainty of each measurement, and we regard them as an accurate representation of the relative uncertainty for each measurement. This is adequate for determining the best light curve model, but in order to determine the uncertainties on the model parameters it is important to have more accurate error bars. We accomplish this with the method presented in Yee et al. (2012). We rescale the errors using the formula, \(\sigma_{i}^{2} = k \cdot \sigma_{i}^{2} + \sigma_{\text{min}}^{2}\), where \(\sigma_{i}\) and \(\sigma_{\text{min}}\) are original and renormalized error bars in magnitudes. The parameters \(k\) and \(\epsilon_{\text{min}}\) are selected so that the cumulative \(\chi^{2}\) distribution sorted by the magnification of the best model is a straight line of slope 1 and \(\chi^{2}/\text{dof} \sim 1\). This procedure yields \(k = 1.092313\) and \(\epsilon_{\text{min}} = 0.012662\) for MOA-Red, \(k = 1.387059\) and \(\epsilon_{\text{min}} = 0.010938\) for OGLE-I, and \(k = 1.571492\) and \(\epsilon_{\text{min}} = 0.0\) for OGLE-V. Note that the changes of the final best-fit model due to this error renormalization are negligible.

CTIO data were reduced by DoPHOT (Schechter et al. 1993), using the point-spread function (PSF)-fitting routine. The number of data points in the CTIO-I, -V, and -H passbands is 15, 15, and 149, respectively. Their error bars are not rescaled, i.e., \(k = 1.0\) and \(\epsilon_{\text{min}} = 0.0\). These CTIO data are not used for light curve modeling, but are used for obtaining the source color in \((V-I)\) and \((I-H)\) in a model-independent way from the linear regression of these light curves by following Dong et al. (2009b) and Calchi Novati et al. (2015b).

Details of the data sets are summarized in Table 1.

4. LIGHT CURVE MODELING

We search for the best-fit models of the standard (static), the parallax, the parallax with the linear orbital motion of the planet, the Keplerian orbital motion with the Keplerian prior, the Galactic kinematic constraint, and the Galactic density prior using the Markov chain Monte Carlo method (MCMC) (Verde et al. 2003). The best-fit models are shown in Tables 2–7 and their physical parameters are in Tables 8–13, respectively (see Section 5).

4.1. Standard (Static) Model

In a point-source point-lens microlensing model, there are three parameters: the time of peak magnification \(t_{p}\), the Einstein radius crossing time \(t_{E}\), and the minimum impact parameter \(\rho_{0}\). The standard binary lens model has four more parameters: the planet–host star mass ratio \(q\), the projected separation normalized by Einstein radius \(s\), the angle of the source trajectory relative to the binary lens axis \(\alpha\), and the ratio of the angular source radius to the angler Einstein radius \(\rho = \theta_{s}/\theta_{E}\). \(\rho\) can only be measured for events that show finite source effects. The measurement of \(\rho\) is important because it allows us to determine the angular Einstein radius \(\theta_{E} = \theta_{s}/\rho\) since the angular source radius, \(\theta_{s}\), can be estimated from its color and extinction-corrected apparent magnitude (Kervella et al. 2004).

We use linear limb-darkening models for the source star using the coefficients, \(u = 0.5863, 0.7585, 0.6327\) for the \(I, V,\) and MOA-Red bands, respectively (Claret 2000). The MOA-Red value is the mean of the \(R\) - and \(I\)-band values. These values were selected from Claret (2000) for a K2 type source star with \(T = 5000\) K, \(\log g = 4.0\) and \(\log [\text{M/H}] = 0\), based on the extinction-corrected, best-fit source \(V-I\) color and brightness (see Section 5).

Initially, the global grid search of the best-fit model was conducted with 9680 fixed grid points across a wide range of three parameters, \(-4.0 < \log q < 0.4, -0.5 < \log s < 0.6,\) and \(0 < \alpha < 2\pi,\) with all other parameters being free. Then the most likely models were refined, allowing all parameters to vary. Using this robust search methodology, we avoid missing any local minimum solutions across the wide range of parameter space. We found that only the model with a wide separation \((s > 1)\) reproduces the observed light curve data. The model corresponds to the source crossing a planetary caustic. Planetary caustics can form far from the primary and any source star that crosses or passes close to such a distant a planetary caustic will impose an signal far from the main microlensing peak. Furthermore, the shape of planetary caustics differ significantly between wide \((s > 1)\) and close \((s < 1)\) configurations in contrast to the close/wide degeneracy for an event crossing a central caustic near the primary. The best-fit standard model parameters are shown in Table 2. The mass ratio of \(q \sim 3 \times 10^{-4}\) and separation of \(s \sim 2.3\) indicate that the companion is a relatively low-mass planet at wide separation.

The single lens model with a binary source was ruled out as follows. We extracted 88 data points around the planetary anomaly within \(6533.0 < \text{HJD} - \text{E} < 6536.6\) after subtracting the flux contribution from the best-fit single lens model of the primary peak which is fitted without data around the anomaly. We fitted this extracted light curve by the single lens model with a finite source effect. The best-fit \(\chi_{v}^{2}\) is \(\sim 11\) larger than the \(\chi_{v}^{2}\) contributions from the same data points by the planetary models with parallax and orbital motion \(K_{pk}\) (see Section 4.4.3).

Furthermore, the best-fit event timescale of \(t_{E} \sim 2.3\) days is much smaller than \(t_{E} \sim 20\) days of the main peak, while they should be same if a single lens caused both two magnifications.

4.2. Parallax Model with a New Type of Degeneracy

There are higher order effects that require additional parameters. The orbital motion of the Earth can cause the apparent lens–source relative motion to deviate from a constant velocity. This effect is known as the microlensing parallax effect (Gould 1992; Alcock et al. 1995; Smith et al. 2002), and it can be described by the microlensing parallax vector \(\pi_{E} = (\hat{\pi}_{E},, \hat{\pi}_{E})\). The direction of \(\hat{\pi}_{E}\) is the direction of the lens–source relative motion projected on the sky (geocentric proper motion at a fixed time), and the amplitude of the microlensing parallax vector, \(\pi_{E} = au/\hat{r}_{E}\), is the inverse of the
Table 2
Model Parameters. Standard and Parallax-only Models

| Parameter       | Standard | P_+ | P_- | P_++ | P_-- |
|-----------------|----------|-----|-----|------|------|
| \( t_0 \) (HJD) | 6573.056 | 6573.050 | 6573.050 | 6573.051 | 6573.052 |
|                 | 0.008    | 0.009 | 0.009 | 0.009 | 0.010 |
| \( t_E \) (days)| 20.47    | 19.93 | 20.04 | 20.10 | 20.15 |
|                 | 0.13     | 0.32  | 0.29  | 0.32  | 0.28  |
| \( u_0 \) (10^{-2}) | 7.563    | 7.932 | ~7.874 | 7.907 | ~7.890 |
|                 | 0.099    | 0.186 | 0.175 | 0.185 | 0.164 |
| \( q \) (10^{-3}) | 2.762    | 3.460 | 3.431 | 3.530 | 3.611 |
| \( s \)         | 0.105    | 0.211 | 0.204 | 0.202 | 0.215 |
| \( \sigma_0 \) (radian) | 3.0996   | 3.1335 | 3.1534 | 2.9829 | 3.3035 |
| \( \rho \) (10^{-3}) | 0.0004   | 0.0077 | 0.0072 | 0.0084 | 0.0080 |
| \( \alpha E_N \) | 3.37     | 3.88  | 3.85  | 3.91  | 3.95  |
|                 | 0.10     | 0.15  | 0.14  | 0.15  | 0.14  |
| \( \alpha E_E \) | 0.000    | -0.313 | 0.279 | 1.114 | -1.144 |
|                 | 0.000    | 0.076  | 0.071 | 0.086 | 0.082 |
| \( \alpha E \)  | 0.000    | -0.252 | -0.261 | -0.210 | -0.249 |
|                 | 0.000    | 0.107  | 0.100 | 0.104 | 0.104 |
| \( I_s \) (mag) | 0.000    | 0.401  | 0.382 | 1.134 | 1.170 |
|                 | 0.000    | 0.088  | 0.082 | 0.084 | 0.088 |
| \( I_s \) (mag) | 18.167   | 18.117 | 18.125 | 18.120 | 18.123 |
|                 | 0.011    | 0.024  | 0.023 | 0.025 | 0.021 |
| \( H_s \) (mag) | 18.508   | 18.580 | 18.568 | 18.575 | 18.571 |
|                 | 0.015    | 0.037  | 0.034 | 0.038 | 0.032 |
| \( \chi^2 \)   | 15.911   | 15.861 | 15.869 | 15.864 | 15.867 |
| \( \Delta \chi^2_{kp} \) | 0.001    | 0.029  | 0.028 | 0.029 | 0.027 |
| \( \Delta \chi^2_{km} \) | 15251.42 | 15217.49 | 15218.15 | 15216.43 | 15214.27 |
| \( \Delta \chi^2_{gl} \) | 0.000    | 0.000  | 0.000 | 0.000 | 0.000 |
| \( \Delta \chi^2_{g} \) | 0.000    | 0.000  | 0.000 | 0.000 | 0.000 |
| \( \chi^2 \)   | 15251.42 | 15217.49 | 15218.15 | 15216.43 | 15214.27 |
| dof             | 15217    | 15215  | 15215  | 15215  | 15215  |

Note. HJD’ = HJD−2450000. The first subscript of the model P_+ , P_-, and P_++ indicate the models with \( u_0 > 0 \) and \( u_0 < 0 \), respectively. The second subscript indicates the sign of impact parameter to the secondary lens, i.e., "r" and "l" mean the source passes on the same side to the host and planet. The 1σ error is given below each parameter. \( \chi^2 \) is the \( \chi^2 \) from the light curve alone: \( \Delta \chi^2_{kp}, \Delta \chi^2_{km}, \) and \( \Delta \chi^2_{gl} \) are the \( \chi^2 \) penalty due to the Keplerian, kinematic, and Galactic priors. Note that \( \alpha E \) is not a fit parameter.

Einstein radius, projected to the observer plane. Because the Galactic bulge is close to the ecliptic plane, there is an approximate degeneracy in the parallax parameters, known as the “ecliptic degeneracy,” where models with similar parameters but with \( (u_0, \alpha, \pi_{E,N}) = (u_0, \alpha, \pi_{E,N}) \) produce nearly indistinguishable light curves. This corresponds to a reflection of the lens plane with respect to the geometry of Earth’s orbit, (Smith et al. 2003; Skowron et al. 2011).

We found the four degenerate parallax models as shown in Table 2. The light curves of these four models are almost identical to the one shown in Figure 2. The caustics, critical curves, and source trajectory of these models are shown in Figure 3. The “P” scripts indicate models with microlensing parallax. The “r” and “l” subscripts refer to different two-fold degeneracies in the parallax models. The first “r” subscript refers to the sign of the \( u_0 \) parameter, and refers to the “ecliptic degeneracy” mentioned above, and the second “r” subscript refers to a new parallax degeneracy, the wide degeneracy, which is particular to events like this, with a wide separation planet detected through a crossing of the planetary caustic. The light curve measurements indicate the angle, \( \alpha (t_{pcc}) \), between the source trajectory and lens axis at the time of the planetary caustic crossing, \( t_{pcc} \). Due to the reflection symmetry of the lens system, the light curve constrains \( \alpha (t_{pcc}) \) up to a reflection symmetry, as shown in Figure 3. If there were no microlensing parallax, we could use \( \alpha (t_{pcc}) \) to predict the closest approach of the source to the center-of-mass, \( u_0 \), and therefore the peak magnification of the stellar light curve. But, when the microlensing parallax effect is included, the angle \( \alpha \) can vary in time, so that \( \alpha (t_{pcc}) \neq \alpha (t_0) \). For a wide-separation planetary event, like MOA-2013-BLG-605, the light curve basically constrains the microlensing parallax through the three parameters, \( \alpha (t_{pcc}), u_0, \) and \( t_0 \), which is essentially the time of the stellar peak magnification (cf. An & Gould 2001). As a result, the configurations shown in the upper and lower panels of Figure 3 yield nearly identical light curves as shown in Figure 2, even though the source passes in between the two masses in the upper panels and below or above the masses in the bottom two panels. The lower panels imply a larger curvature of the source trajectory and, therefore, a larger microlensing parallax signal. (Note that the model parameters \( \alpha_0 \) and \( S_0 \) given in Tables 2–7 are the \( \alpha \) and \( S \) values at a fixed time \( t_{pcc} = 6573.045 \), following the convention of geocentric microlensing parallax parameters.)
Table 3

| Parameter | P_+L | P_{+L} | P_{+L} | P_{-L} | P_{+L}' | P_{-L}' |
|-----------|-------|--------|--------|--------|--------|--------|
| \( t_0 \) (HJD') | 6573.051 | 6573.051 | 6573.055 | 6573.055 | 6573.055 | 6573.055 |
|             | 0.006  | 0.009  | 0.010  | 0.010  | 0.010  | 0.010  |
| \( t_E \) (days) | 19.95 | 19.90 | 21.33 | 21.24 | 21.25 | 21.22 |
| \( u_0 \) (10^{-2}) | 7.869 | -7.894 | 7.418 | -7.452 | 7.455 | -7.449 |
| \( q \) (10^{-4}) | 3.614 | 3.422 | 4.183 | 4.913 | 2.805 | 2.846 |
| \( s_0 \) | 2.370 | 2.353 | 2.302 | 2.451 | 1.972 | 1.975 |
| \( \alpha_0 \) (radian) | 3.1445 | 3.1444 | 2.9485 | 3.3412 | 2.9614 | 3.3222 |
| \( \rho \) (10^{-3}) | 3.94 | 3.85 | 4.06 | 4.41 | 3.33 | 3.36 |
| \( \pi_{EN} \) | -1.528 | 1.420 | 3.731 | -3.366 | 3.703 | -3.326 |
| \( \pi_{E} \) | 0.194 | 0.226 | 0.213 | 0.340 | 0.283 | 0.318 |
| \( \omega \) (rad yr^{-1}) | -1.145 | 1.087 | 2.459 | -1.974 | 2.550 | -2.117 |
| \( ds/dt \) | -0.277 | -0.403 | -0.935 | 0.434 | -4.104 | -4.099 |
| \( \rho_{E} \) | 0.376 | 0.588 | 0.921 | 0.829 | 1.049 | 0.244 |
| \( I_0 \) (mag) | 18.126 | 18.122 | 18.190 | 18.186 | 18.185 | 18.185 |
| \( I' \) (mag) | 0.025 | 0.017 | 0.021 | 0.023 | 0.024 | 0.023 |
| \( H_0 \) (mag) | 18.566 | 18.572 | 18.476 | 18.481 | 18.482 | 18.482 |
| \( H' \) (mag) | 0.037 | 0.026 | 0.028 | 0.031 | 0.031 | 0.030 |
| \( \chi^2 \) | 15212.02 | 15214.89 | 15204.17 | 15204.36 | 15202.31 | 15202.52 |
| \( \Delta \chi^2_{\text{MOA}} \) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| \( \Delta \chi^2_{\text{OGLE}} \) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| \( \chi^2 \) | 15212.02 | 15214.89 | 15204.17 | 15204.36 | 15202.31 | 15202.52 |
| dof | 15213 | 15213 | 15213 | 15213 | 15213 | 15213 |

Note. Notation is the same as Table 2. The solution \( P_{+L}' \) does not exist with the full Keplerian orbit.

Figure 4 shows the \( \Delta \chi^2 \) distribution of the parallax parameters from the best-fit MCMC models. The best-fit values are compared to those of other models in Figure 6. In late September, the Earth’s acceleration is in the east–west (E–W) direction, so for a typical event, we would expect a better constraint on parallax in the E–W direction, i.e., a smaller error for \( \pi_{E} \). However, in this case, the planetary signal plays a large role in the parallax signal. The angle and timing of caustic entry for a given \( u_0 \) value—which is constrained by the main peak corresponding to the host star—constrain the parallax parameters.

Figure 5 shows the difference in the cumulative \( \chi^2 \) values between the standard and the parallax models as a function of time. We can see that most of the parallax signals come from around the planetary signal in both MOA-Red and OGLE-I as expected.

For all these models, \( q \) and \( s \) are similar to those of the standard model, so the companion is a cold low-mass planet.

For all four degenerate solutions, the model parameters of greatest interest are all very similar, except for the microlensing parallax. The ecliptic degeneracy yields nearly identical physical parameters, except that the direction of the lens–source relative motion is different. Potentially, this angle can be measured with follow-up observations (Batista et al. 2015; Bennett et al. 2015). In contrast to the ecliptic degeneracy, the wide degeneracy implies different amplitudes, \( \pi_{E} \), of the microlensing parallax vector, which implies different lens system masses, as discussed in Section 5 below.

Thus, this wide degeneracy presents us with two different classes of physical models, \( P_{\pm+} \) and \( P_{\pm-} \), where the \( P_{\pm} \) models have larger \( \pi_{E} \) implying smaller lens system masses and distance (see Section 5 and Table 8).

These four parallax models are preferred over the standard \( \pi_{E} = 0 \) model in both the MOA-Red and OGLE-I bands by \( (\Delta \chi^2_{\text{MOA}}, \Delta \chi^2_{\text{OGLE}}) = (-21.8, -11.5), (-21.6, -11.1), \)
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Table 4
Model Parameters. Full Keplerian Orbit (K)

| Parameter | $P_{\text{..}}$ K | $P_{\text{..}}$ K | $P_{\text{...}}$ K | $P_{\text{...}}$ K |
|-----------|------------------|------------------|------------------|------------------|
| $t_0$ (HJD$^\dagger$) | 6573.051 | 6573.051 | 6573.054 | 6573.055 |
| $t_0$ (days) | 19.98 | 19.91 | 21.03 | 21.14 |
| $u_0$ (10$^{-2}$) | 7.856 | -7.886 | 7.553 | -7.482 |
| $q$ (10$^{-6}$) | 3.789 | 3.663 | 4.216 | 5.197 |
| $s_0$ | 0.018 | 0.026 | 0.055 | 0.033 |
| $a_0$ (radian) | 3.1474 | 3.1410 | 2.9553 | 3.3528 |
| $\rho$ (10$^{-3}$) | 4.03 | 3.98 | 4.12 | 4.56 |
| $\pi_{\text{EN}}$ | -1.509 | 1.413 | 3.389 | -3.351 |
| $\pi_{\text{ER}}$ | 0.220 | 0.187 | 0.224 | 0.197 |
| $\pi_{\text{E}}$ | 0.112 | -0.016 | 0.033 | -0.114 |
| $\omega$ (rad yr$^{-1}$) | 1.530 | 1.427 | 3.389 | 3.353 |
| $\omega$ (rad yr$^{-1}$) | 0.219 | 0.181 | 0.225 | 0.194 |
| $ds/dt$ (yr$^{-1}$) | 0.304 | 0.293 | 0.241 | 1.431 |
| $dx/dt$ (yr$^{-1}$) | 0.240 | 0.203 | 0.577 | 0.356 |
| $x_s$ | 0.002 | 0.031 | 0.025 | 0.602 |
| $dx/dt$ (yr$^{-1}$) | 0.122 | 0.200 | 0.078 | 0.288 |
| $dE/dt$ | 0.108 | 0.005 | -0.413 | 0.695 |
| $E_0$ (mag) | 0.639 | 0.527 | 0.758 | 0.935 |
| $E_0$ (mag) | 18.127 | 18.123 | 18.171 | 18.180 |
| $E_0$ (mag) | 0.017 | 0.020 | 0.015 | 0.012 |
| $E_0$ (mag) | 18.563 | 18.571 | 18.501 | 18.489 |
| $H_0$ (mag) | 0.026 | 0.030 | 0.020 | 0.016 |
| $H_0$ (mag) | 15.871 | 15.867 | 15.915 | 15.924 |
| $\chi^2_{\text{ke}}$ | 0.023 | 0.026 | 0.022 | 0.200 |
| $\chi^2_{\text{kin}}$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $\chi^2_{\text{gal}}$ | 15212.20 | 15214.92 | 15204.70 | 15204.43 |
| $\chi^2$ | 15212.20 | 15214.92 | 15204.70 | 15204.43 |
| dof | 15211 | 15211 | 15211 | 15211 |

Note. Notation is the same as for Table 2.

The xallarap effect is a light curve distortion caused by the orbital motion of the source star (Griest & Hu 1992; Han & Gould 1997), so it only occurs if the source star has a binary companion (Derue et al. 1999; Alcock et al. 2001). Xallarap can be represented by five additional model parameters. The xallarap vector $\xi_E = (\xi_{E,N}, \xi_{E,E})$ is similar to the parallax vector, $\pi_E$, and represents the direction of the lens–source relative motion. The amplitude of the xallarap vector, $\xi_E = a_\alpha/r_E$, is the semimajor axis of the source’s orbit, $\alpha$, in units of the Einstein radius projected on the source plane, $r_E = \theta_E D_s$. The other xallarap parameters are the direction of the observer relative to the source orbital axis, with vector components R.A.$\xi$ and decl.$\xi$, and the source binary orbital period, $T_\xi$. For an elliptical orbit, two additional parameters are required, the orbital eccentricity, $e$ and time of perihelion, $t_{\text{peri}}$, which we did not consider here as their inclusion did not improve the fit of the model to the data.

We found xallarap models giving only marginally better $\chi^2$ values compared to parallax models for $T_\xi \geq 160$ days and worse values of $\chi^2$ for shorter values of $T_\xi$. This is not surprising as it is known that xallarap effects can mimic parallax effects (Smith et al. 2003; Dong et al. 2009b). Including xallarap yields a slight improvement of $\Delta \chi^2 \sim -5$ for $160 < T_\xi < 200$ days and $\Delta \chi^2 \sim -9$ at $T_\xi > 200$ days. However, these models lead to a xallarap amplitude of $\xi_E \geq 0.26$, which is larger than would be induced by a “normal” main-sequence companion. Here $\xi_E$ is expressed, making use of Kepler’s third law, by

$$\xi_E = \frac{a_\alpha}{r_E} = \frac{1}{T_E} \left( \frac{M_c}{M_\odot} \left( \frac{M_\odot}{M_c + M_\odot} \right) \right)^{1/2} T_\xi^{3/2},$$

These models require a source companion of mass $M_c > 6 M_\odot$ for $T_\xi \geq 160$ days and $M_c > 40 M_\odot$ for $T_\xi \geq 200$ days. Such a heavy object would most likely be a stellar remnant or a black hole—in either case, a rare object and thus an unlikely source companion. For this reason we reject the inclusion of the xallarap in our models.

4.4. Orbital Motion Model

4.4.1. Linear Orbital Motion

The orbital motion of the planet around the host star causes a similar effect as parallax. To a first-order approximation, the orbital motion of the planet is described by two parameters, the rate of change, $\omega = d\alpha/dt$ (rad yr$^{-1}$), of the binary axis angle $\alpha$, and the rate of change $dx/dt$ (yr$^{-1}$), of the projected lens star and planet separation $s$ (Dong et al. 2009b; Batista et al. 2011), as follows,

$$s = s_0 + ds/dt (t - t_{\text{fix}}),$$

$\omega = \alpha_0 + \omega (t - t_{\text{fix}}),$ (2)

where, $s_0$, and $\omega_0$ are instantaneous values of $s$ and $\alpha$ at the time $t_{\text{fix}}$. We required the planet to be bound. That is, the ratio of the projected kinetic energy and potential energy,

$$(KE_{\text{PE}})_{\bot} = \frac{(r_{\text{\bot}}/au)^3}{8\pi^2(M/M_\odot)} \left[ \left( \frac{1}{s dt} \right)^2 + \left( \frac{d\alpha}{dt} \right)^2 \right] \text{years}^2,$$ (3)

which is less than the ratio of kinetic to potential energy (KE/PE) in three dimensions, was required to be less than unity in the MCMC calculations used to determine the model parameter distributions. The four best linear orbital motion models (with scripts “L”) that correspond to each of four parallax models in Table 2, are shown in Table 3. One finds that $\pi_E$ and its uncertainty significantly increased, while $\chi^2$ only improved slightly. This is because of the well known degeneracy between one component of the parallax vector, $\pi_E$, which is
perpendicular to the binary axis and close to \( \pi_{\text{EN}} \) in this case, and the lens orbital rotation on the sky, \( \omega \). As an example, the \( \Delta \chi^2 \) distribution of \( \pi_{\text{EN}} \) and \( \omega \) for the model \( P_{+\sim L} \) is shown in Figure 7.

Note that there are two additional degenerate models \( P_{+\pm L} \) which have smaller \( s_0 \sim 1.97 \) and larger \( ds/dt \sim -4.1 \) yr\(^{-1} \) compared to the other models. Here, the \( s \) of these models are similar to the others, \( s \sim 2.4 \), when the source crosses the planetary caustic. However these models are disfavored with the full Keplerian orbit in the following analysis.

The physical parameters of the lens systems of these models are shown in Table 9 (see details in Section 5). The host stars in these four models have a brown dwarf mass. Note that the \( \text{KE}/\text{PE} \) of these models given in Table 9 are close to unity. The probability of having such a high value is quite low as it requires a very large eccentricity of \( e \sim 1 \), seeing the orbital plane face-on. If the parameters are not well constrained by the light curve, the density distribution of the MCMC chain depends on the prior probability of the fitting parameters in MCMC. Although the linear approximation of the lens motion is good enough in most cases, this parameterization inadvertently assumed the uniform prior on all microlensing fitting parameters, which is not physically justified. We need to use a full Keplerian orbit parameterization to introduce physically justified priors.

### 4.4.2. Full Keplerian Orbit

To take the proper weighting on the orbital parameters, we adopt the full Keplerian parameterization by Skowron et al. (2011). The advantage of the full Keplerian orbit is not only being more accurate and allowing only bound orbital solutions, it also enables us to introduce physically justified priors on the orbital parameters. In addition to the parameters defined above, we introduce the position and velocity along the line of sight, \( s_x \) and...
in units of $r_E$ and $d s_f / d t$ in yr$^{-1}$. Then, the 3D position and velocity of the secondary relative to its host can be described by $(s_0, 0, s_x)$ and $s_0 (\gamma_1, \gamma_2, \gamma_3) = (ds / dt, s_0 \omega, ds_f / dt)$.

We run MCMC fitting using the microlensing parameters with these six instantaneous Cartesian phase-space coordinates, in which we transform the “microlensing” parameters to “Keplerian” parameters, i.e., eccentricity ($e$), time of periapsis ($t_{peri}$), semimajor axis ($a$), and three Euler angles: longitude of the ascending node ($\Omega_{mode}$), inclination ($i$), and argument of periapsis ($\omega_{peri}$). By following Skowron et al. (2011), we assume flat priors on values of eccentricity, time of periapsis, log($a$), and $\omega_{peri}$. Owing to the fact that orbital orientation is random in space, we multiply the prior by $|\sin i|$. We must multiply the Jacobian of the parameter transformation function, $J_{kep} = \left| \frac{\partial (e, a, t_{peri}, \Omega_{mode}, i, \omega_{peri})}{\partial (s_0, \gamma_1, \gamma_2, \gamma_3)} \right|$ (Equation (B6) in Skowron et al. 2011). So we adopt the Keplerian orbit prior of $P_{kep} = j_{kep} |\sin i| a^{-1}$ and added the $\Delta \chi^2$ penalty of $\Delta \chi^2_{kep} = -2 \ln (P_{kep})$.

We first show the results with full Keplerian orbit (with scripts “K”) without any priors in Tables 4 and 10. The results are almost the same as the ones with the linear approximation of the orbit. The large eccentricity of $e \sim 1$ seeing the orbital plane face-on ($i \sim 0, 180^\circ$) is as expected from the large (KE/PE) in the linear orbit. The physical parameter of the Keplerian orbits, semimajor axis $a_{kep}$, period $P$, $e$, and $i$, are not well constrained so that they have very large asymmetric error bars in the MCMC in Table 10. Here, when the best-fit is larger or smaller than the 68% confidence interval of the MCMC chains, the upper or lower limit is designated as “+0.0” or “-0.0”, respectively. So the light curve shape itself does not constrain the parameters more than the linear orbit model, except that it ruled out the models with smaller $s_0$ and larger $e$.

| Parameter | M-dwarf | High-mass Brown Dwarf | Low-mass Brown Dwarf |
|-----------|---------|-----------------------|----------------------|
| $i_0$ (HJD$^+$) | 6573.047 | 6573.054 | 6573.050 | 6573.051 | 6573.052 | 6573.054 |
| $t_E$ (days) | 0.009 | 0.009 | 0.009 | 0.008 | 0.009 | 0.010 |
| $u_0$ (10$^{-2}$) | 20.20 | 19.83 | 19.96 | 19.93 | 20.41 | 20.52 |
| $q$ (10$^{-4}$) | 1.63 | 0.23 | 0.10 | 0.24 | 0.32 | 0.28 |
| $s_0$ | 7.979 | -7.960 | 7.890 | -7.905 | 7.804 | -7.749 |
| $s_0$ (radian) | 0.119 | 0.147 | 0.072 | 0.153 | 0.187 | 0.154 |
| $p$ (10$^{-3}$) | 3.526 | 3.124 | 0.131 | 0.158 | 3.812 | 0.054 |
| $\rho$ | 2.388 | 2.344 | 2.394 | 2.382 | 2.396 | 2.407 |
| $\sigma_0$ | 0.018 | 0.012 | 0.011 | 0.021 | 0.015 | 0.014 |
| $\sigma_0$ (rad) | 3.1269 | 3.1568 | 3.1381 | 3.1516 | 2.9719 | 3.3182 |
| $\psi$ | 0.073 | 0.0058 | 0.0027 | 0.0059 | 0.0103 | 0.0102 |
| $\rho (10^{-3})$ | 3.89 | 3.72 | 3.92 | 3.85 | 4.03 | 4.13 |
| $\sigma_0$ | 0.13 | 0.08 | 0.11 | 0.13 | 0.13 | 0.16 |
| $\sigma_0$ (rad) | 0.047 | 0.289 | -0.891 | 0.793 | 2.242 | -2.308 |
| $\sigma_0$ | 0.098 | 0.068 | 0.070 | 0.074 | 0.245 | 0.212 |
| $\omega$ (rad yr$^{-1}$) | 0.302 | 0.040 | -0.552 | 0.512 | 1.074 | -1.065 |
| $d s_f / d t$ (yr$^{-1}$) | 0.036 | 0.030 | 0.065 | 0.069 | 0.162 | 0.141 |
| $s_x$ | 0.047 | -0.304 | 0.002 | 0.001 | -0.001 | 0.049 |
| $s_y$ | 0.076 | 0.185 | 0.057 | 0.083 | 0.035 | 0.119 |
| $d s_f / d t$ (yr$^{-1}$) | 1.917 | 1.078 | 0.149 | 0.003 | 0.023 | 0.243 |
| $s_z$ | 1.126 | 0.732 | 0.370 | 0.019 | 0.212 | 0.564 |
| $H_0$ (mag) | 18.135 | 18.112 | 18.122 | 18.120 | 18.135 | 18.142 |
| $H_0$ (mag) | 18.552 | 18.588 | 18.571 | 18.574 | 18.552 | 18.542 |
| $H_0$ (mag) | 0.020 | 0.028 | 0.011 | 0.029 | 0.036 | 0.029 |
| $H_0$ (mag) | 15.879 | 15.856 | 15.866 | 15.864 | 15.879 | 15.886 |
| $\chi^2_{kep}$ | 152184.9 | 15220.05 | 15215.31 | 15217.48 | 15210.26 | 15207.67 |
| $\Delta \chi^2_{kep}$ | -65.62 | -65.73 | -61.27 | -61.93 | -55.94 | -55.80 |
| $\chi^2_{kep}$ | 13.53 | 5.89 | 7.81 | 4.03 | 1.91 | 1.88 |
| $\Delta \chi^2_{gal}$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $\chi^2$ | 15166.40 | 15160.20 | 15161.85 | 15159.58 | 15156.24 | 15153.75 |
| dof | 15211 | 15211 | 15211 | 15211 | 15211 | 15211 |

Note. Notation is the same as for Table 2.
ds/dt corresponding to $P_{\pm \pm \pm \pm}$. The best-fit parallax vectors are larger than those of the static model as shown in Figure 6. Note that the ratio of 3D kinetic to potential energy (KE/PE) can be calculated in these full Keplerian orbit models, as shown in Table 10, which are also close to unity. This is because the Jacobian of the Keplerian prior, the circular orbits with $e \sim 0$ are preferred contrary to the large eccentricity without the Keplerian prior. This is because the Jacobian $j_{\text{KE}}$ is proportional to $1/(e \sin i)$ as noted by Skowron et al. (2011) and thus smaller values of eccentricity are preferred. Here, technically, the lower limit of eccentricity is set to be $10^{-4}$ to avoid a numerical problem in MCMC as suggested by Skowron et al. (2011).

$\pi_{\text{KE}}$ is reduced by a factor of $1/2 \sim 2/3$ because the circular orbit is preferred by the Keplerian prior. So the lens masses increased, while the hosts are still the high-mass and low-mass brown dwarfs. As for the models $P_{\pm \pm \pm \pm}$, there are other minima with a lower parallax value of $\pi_{\text{HE}} \sim 0.2$ with a similar final $\chi^2$ whose host is a low-mass M-dwarf. This is because $P_{\text{KEP}}$ prefers larger values of $D_t$ by $D_t^\pm$. But $\chi^2$ values from the light curves alone are larger than brown dwarf models. So there seems to be some conflict between the light curve and prior.

### 4.4.3. Stellar Kinematic Constraint

Here, we applied the prior for the Galactic kinematics by following Batista et al. (2011). In Table 11, the projected lens–source relative velocity $\vec{v}_0 = (\vec{v}_t, \vec{v}_s)$ of these $K_p$ models in the Galactic coordinate differs significantly. Those of the M-dwarf models are significantly different from the expected value from the Galactic kinematics as shown in Figure 8. Here we assume a source distance of $D_s = 8$ kpc (Reid 1993; Honma et al. 2012), the proper motion of the Galactic center is $\mu_{GC} = 6.1 \text{ mas yr}^{-1}$ (Backer & Sramek 1999; Reid &
The dispersion of stars in the Galactic coordinates are $\sigma_{\mathrm{disk},l} = 34 \, \mathrm{km \, s}^{-1}$ and $\sigma_{\mathrm{Disk},b} = 18 \, \mathrm{km \, s}^{-1}$ (Binney & Merrifield 1998; Minchev et al. 2013; Sharma et al. 2014). Then the expected average ($\bar{v}_{\ell,\exp}, \bar{v}_{\exp,b}$) and dispersion ($\bar{\sigma}_{\ell,\exp}, \bar{\sigma}_{\exp,b}$) of the lens projected velocity are calculated. The probability of having observed $\bar{v}_{\ell,\exp}$ can be given by

$$P_{\mathrm{kin}} = \exp \left[ -\frac{(\bar{v}_{\ell,\exp} - \bar{v}_{\ell,\exp,b})^2}{2 \bar{\sigma}_{\ell,\exp,b}^2} \right] \exp \left[ -\frac{(\bar{v}_{\exp,b} - \bar{v}_{\exp,b})^2}{2 \bar{\sigma}_{\exp,b}^2} \right].$$

(4)

The $\Delta \chi^2$ penalty of $\Delta \chi^2_{\ell,\exp} = -2 \ln(P_{\ell,\exp})$ is about +16 and +15 for M-dwarf P$_{\ell,\exp}^+ \kappa_p$ and P$_{\ell,\exp}^- \kappa_p$ models, respectively. On the other hand, the penalty is $+9$, $+4$, $+2$, and $+2$ for brown dwarf P$_{\ell,\exp}^+ \kappa_p$, P$_{\ell,\exp}^- \kappa_p \kappa_p$, P$_{\ell,\exp}^+ \kappa_p \kappa_p$, and P$_{\ell,\exp}^- \kappa_p \kappa_p$ models, respectively. So the M-dwarf models are less preferred.

Thus, we conducted MCMC runs by adding the penalty $\Delta \chi^2_{\ell,\exp}$. The results (with scripts “*kpk”) are shown in Tables 6, 12 and Figure 6. The model light curve of P$_{\ell,\exp}^+ \kappa_p^* \kappa_p$ is shown in Figure 2. As expected, $\pi_E$ values for the M-dwarf models increase to ~0.3 to reduce the $\Delta \chi^2$ and the total $\chi^2$ value became similar or larger than the brown dwarf models. In total, low-mass brown dwarf P$_{\ell,\exp}^+ \kappa_p$ models are slightly preferred over other models.

We adopt these $\kappa_p$ models as the main result of this paper because they use the most realistic priors and constraints. See the details in Section 4.5.

### 4.4.4. Galactic Mass Density Prior

Finally, we applied the prior for the Galactic mass density model (Batista et al. 2011; Skowron et al. 2011),

$$P_{\mathrm{gal}} = \nu \left( x, y, z | (\mu) | g(M) | \mathcal{D}_{\ell,\mu}^\ell \right).$$

(5)
which is the microlensing event rate multiplied by the Jacobian of the transformation from microlensing parameters to physical coordinates, \( J_{\text{pal}} = |\partial(D_t, M, \mu)/\partial(t_0, \theta_E, \pi_t)| \). Here \( \nu(x, y, z) \) is the local density of lenses, \( g(M) \) is the mass function, and \( f(\mu) \) is the 2D probability function for a given source–lens relative proper motion, \( \mu = v_i/D_t \) which is set to unity because it is already implemented in \( P_{\text{kin}} \) above. We adopt the Galactic model by Han & Gould (1995) for \( \nu(x, y, z) \) and adopt \( g(M) \propto M^{-1} \) by following Batista et al. (2011).
### Table 12

Lens Physical Parameters. Keplerian Prior + Kinematic Constraint (Kpk)

| Parameter | M-dwarf | High-mass Dwarf | Low-mass Dwarf |
|-----------|---------|----------------|---------------|
| \( \theta_E \) (mas) | \( P_{-K_{pk}} \) | \( P_{+K_{pk}} \) | \( P_{+K_{pk}} \) | \( P_{+K_{pk}} \) |
| \( \rho_{g_{pk}} \) (mas yr\(^{-1}\)) | 8.48 ± 0.63 | 9.03 ± 0.63 | 8.51 ± 0.61 | 8.68 ± 0.65 | 8.11 ± 0.61 | 7.86 ± 0.61 |
| \( \rho_{het,N} \) (mas yr\(^{-1}\)) | 3.16 ± 0.69 | 8.26 ± 0.59 | -8.45 ± 0.59 | 8.19 ± 0.63 | 7.44 ± 0.61 | -8.48 ± 0.61 |
| \( \rho_{het,E} \) (mas yr\(^{-1}\)) | -8.65 ± 0.63 | -3.69 ± 0.24 | -3.14 ± 0.17 | -2.72 ± 0.15 | -2.20 ± 0.03 | -2.47 ± 0.05 |
| \( D_h \) (kpc) | 3.59 ± 0.30 | 3.55 ± 0.20 | 1.76 ± 0.15 | 1.97 ± 0.12 | 0.86 ± 0.08 | 0.85 ± 0.08 |
| \( M_9 \) (M\(_\odot\)) | 0.185 ± 0.034 | 0.198 ± 0.065 | 0.066 ± 0.006 | 0.073 ± 0.004 | 0.025 ± 0.004 | 0.024 ± 0.003 |
| \( M_{P_{gal}} \) | 21.77 ± 5.31 | 20.56 ± 6.66 | 7.47 ± 0.87 | 8.25 ± 0.67 | 3.23 ± 0.29 | 3.24 ± 0.38 |
| \( a_{2.5} \) (au) | 4.13 ± 0.48 | 4.18 ± 0.96 | 2.01 ± 0.14 | 2.22 ± 0.12 | 0.95 ± 0.08 | 0.93 ± 0.08 |
| \( a_{2.5}/a_{snow} \) | 8.25 ± 1.09 | 7.83 ± 0.20 | 11.82 ± 0.44 | 11.25 ± 0.72 | 13.90 ± 0.42 | 14.34 ± 0.75 |
| \( v_{exp} \) (au) | 5.06 ± 0.59 | 5.12 ± 1.17 | 2.46 ± 0.17 | 2.72 ± 0.15 | 1.17 ± 0.09 | 1.14 ± 0.09 |
| \( a_{exp}/a_{snow} \) | 10.11 ± 1.55 | 9.59 ± 2.20 | 14.49 ± 0.86 | 13.78 ± 0.88 | 17.02 ± 1.10 | 17.57 ± 0.89 |
| \( a_{2.5}/a_{snow} \) | 4.62 ± 0.71 | 4.60 ± 2.46 | 2.01 ± 0.10 | 2.22 ± 0.14 | 0.95 ± 0.03 | 0.93 ± 0.06 |
| \( P \) (years) | 9.23 ± 10.23 | 8.63 ± 7.94 | 11.83 ± 0.08 | 11.26 ± 0.93 | 13.90 ± 0.58 | 14.42 ± 1.00 |
| \( \gamma_0 \) (km s\(^{-1}\)) | 23.0 ± 4.5 | 22.2 ± 4.9 | 11.4 ± 0.1 | 12.2 ± 0.7 | 14.1 ± 0.1 | 16.3 ± 0.1 |
| \( \gamma_0 \) (km s\(^{-1}\)) | -92.4 ± 26.5 | 177.3 ± 53.3 | -85.5 ± 5.7 | 82.8 ± 8.6 | 37.2 ± 3.6 | -27.7 ± 2.9 |
| \( \gamma_{tot} \) (K) | 263.4 ± 44.8 | 233.3 ± 36.5 | -9.2 ± 3.6 | 86.1 ± 5.3 | 33.2 ± 2.0 | -2.6 ± 1.7 |
| \( \gamma_{tot} \) (K) | 25.5 | 27.9 | 13.1 | 14.1 | 7.1 | 7.1 |
| \( \gamma_{tot} \) (K) | 0.439 | 0.856 | 0.498 | 0.498 | 0.497 | 0.480 |
| \( \gamma_{tot} \) (K) | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 | 0.500 |

**Note.** Notation is the same as for Table 10.

The results of the MCMC runs adding a penalty of \( \Delta \chi_2^2 = -2 \ln(P_{gal}) \) are shown (with scripts “Kpk”) in Tables 7, 13 and Figure 6. The light curves, caustics, critical curves, and source trajectory of the models comprising both parallax and planetary orbital motion with various different priors are almost the same as that of the parallax-only models as shown in Figures 2 and 3. Overall, the lens masses slightly increased relative to that of the Kpk models because this P\(_{gal}\) prefers larger \( D_h \) i.e., smaller \( \pi_E \).

In addition to above six models, there are two more minima for P\(_{+\pm 0.05Kpk}\) with a lower parallax value of \( \pi_E \sim 0.8 \) with similar final \( \chi^2 \) values. This is also because the prior P\(_{gal}\) prefers larger \( D_h \) and smaller \( \pi_E \) values. These solutions happen to have similar \( \theta_E \) values to those of high-mass brown dwarf P\(_{+\pm 0.05Kpk}\) models, hence the similar physical parameters.

As for the P\(_{+\pm 0.05Kpk}\) models, there are two other minima which each have a much smaller parallax value of \( \pi_E \sim 0.035 \) and a smaller final \( \chi^2 = 15107 \). This is because their large values of \( D_h = 7 \) kpc are preferred by P\(_{gal}\). These solutions have a very heavy host mass of \( M_9 \sim 1.7M_\odot \), which would be quite rare. In addition, these solutions have a value of \( \chi^2 \sim 15224 \) which is larger than any other model with parallax, which conflicts with the preference of P\(_{gal}\). Furthermore, these models have very bright B-band source magnitudes, \( H_i = 15.816 \pm 0.017 \) and \( H_i = 15.824 \pm 0.018 \) for P\(_{+\pm 0.05Kpk}\) and P\(_{+\pm 0.05Kpk}\), respectively. These are too bright compared to the Keck AO measurement of the target, \( H = 15.90 \pm 0.02 \), by 3\( \sigma \) (see Section 6). If we assume that the host is a main-sequence star, then the total brightness of the source plus lens is expected to be brighter and ruled out by Keck measurements by more than 4\( \sigma \). For these reasons we do not consider these solutions to be real, and they are not listed among the other solutions in the tables.

#### 4.5. Model Selection

We have presented a large number of models with various high order effects and priors. Here we summarize which of these models are preferred over the others. As discussed in Section 4.2, the parallax signal looks qualitatively real because the signal comes from the theoretically expected part of the light curve and it is consistent in both the MOA and OGLE-I data sets. The \( \chi^2 \) improvement by the parallax-only models over the standard model are \( \Delta \chi^2 = -33.3 \sim -37.1 \) with two additional parameters, which is equivalent to a confidence level of \( 5.4 \sim 5.8\sigma \) and is formally significant.

Furthermore, we compared the models by using the common statistical criterion, Akaike’s Information Criterion (AIC), AIC = \( \chi^2 + 2n_{param} \), and the Bayesian information criterion (BIC), BIC = \( \chi^2 + n_{param} \ln(N_{data}) \) (Burnham & Anderson 2002), which includes a penalty discouraging an over-fitting. The smaller the AIC and/or BIC values are the better model is. Here we adopt the number of data points \( N_{data} = 2913 \) during the event at 6450 < HJD' < 6620 for BIC because the baseline data outside this range do not constrain the parallax signal.

The differences in these criteria between the parallax-only models and standard model are \( \Delta \text{AIC} = -29 \sim -33 \) and \( \Delta \text{BIC} = -17 \sim -21 \). Thus the parallax-only models are better than the standard model. The parallax models with a linear orbit (L) are also better than the standard model by \( \Delta \text{AIC} = -28 \sim -41 \) and \( \Delta \text{BIC} = -5 \sim -17 \). The parallax models with full Keplerian orbits with two more parameters are better than the standard in AIC but not in BIC.

The \( \Delta \chi^2 \) of the parallax models with a linear orbit (L) relative to the parallax-only models are only marginal, \( -3 \sim -12 \) with two additional parameters. The differences in these criteria are \( \Delta \text{AIC} = +0.7 \sim -8 \) and \( \Delta \text{BIC} = +13 \sim +4 \), and...
### Table 13

Lens Physical Parameters. Keplerian Prior + Kinematic + Galactic Density Prior (Kpkg)

| Parameter          | M-dwarf | High-mass Brown Dwarf | Low-mass Brown Dwarf |
|--------------------|---------|-----------------------|----------------------|
| $\theta_0$ (mas)   | $0.482 \pm 0.033$ | $0.498 \pm 0.034$ | $0.487 \pm 0.035$ | $0.487 \pm 0.034$ | $0.485 \pm 0.036$ | $0.493 \pm 0.036$ | $0.469 \pm 0.035$ | $0.461 \pm 0.035$ |
| $v_{\text{los}}$ (mas yr$^{-1}$) | $8.53 \pm 0.60$ | $9.01 \pm 0.64$ | $8.66 \pm 0.65$ | $8.72 \pm 0.63$ | $8.65 \pm 0.67$ | $8.74 \pm 0.66$ | $8.17 \pm 0.63$ | $7.96 \pm 0.62$ |
| $v_{\text{hel}}$ (mas yr$^{-1}$) | $1.71 \pm 0.13$ | $8.51 \pm 0.61$ | $-8.47 \pm 0.61$ | $8.28 \pm 0.62$ | $8.22 \pm 0.66$ | $-8.67 \pm 0.63$ | $7.52 \pm 0.63$ | $-8.56 \pm 0.62$ |
| $\mu$ (mas yr$^{-1}$) | $-8.59 \pm 0.59$ | $-2.96 \pm 0.19$ | $-3.42 \pm 0.20$ | $-2.53 \pm 0.14$ | $-2.47 \pm 0.14$ | $-3.17 \pm 0.18$ | $-2.16 \pm 0.03$ | $-2.53 \pm 0.06$ |
| $D_1$ (kpc) | $3.74 \pm 0.68$ | $3.70 \pm 0.49$ | $1.93 \pm 0.38$ | $2.00 \pm 0.28$ | $1.95 \pm 0.16$ | $1.81 \pm 0.38$ | $0.86 \pm 0.10$ | $0.87 \pm 0.07$ |
| $M_\text{b}$ ($M_\odot$) | $0.201 \pm 0.053$ | $0.210 \pm 0.033$ | $0.074 \pm 0.022$ | $0.077 \pm 0.009$ | $0.074 \pm 0.007$ | $0.070 \pm 0.003$ | $0.026 \pm 0.005$ | $0.026 \pm 0.003$ |
| $M_p$ ($M_\odot$) | $23.54 \pm 6.29$ | $21.96 \pm 3.84$ | $8.45 \pm 2.26$ | $8.66 \pm 1.07$ | $8.35 \pm 0.75$ | $7.67 \pm 0.40$ | $3.27 \pm 0.43$ | $3.39 \pm 0.29$ |
| $\alpha_\text{L}$ (au) | $4.31 \pm 0.65$ | $4.33 \pm 0.54$ | $2.22 \pm 0.44$ | $2.31 \pm 0.19$ | $2.25 \pm 0.14$ | $2.08 \pm 0.47$ | $0.97 \pm 0.02$ | $0.97 \pm 0.08$ |
| $\alpha_\text{E}$ (au) | $7.95 \pm 1.41$ | $7.64 \pm 1.47$ | $11.17 \pm 0.82$ | $11.08 \pm 1.25$ | $11.22 \pm 0.93$ | $11.09 \pm 0.71$ | $13.73 \pm 0.25$ | $13.99 \pm 0.60$ |
| $e_{\text{exp}}$ (au) | $0.52 \pm 0.88$ | $0.53 \pm 0.67$ | $2.72 \pm 0.54$ | $2.83 \pm 0.23$ | $2.75 \pm 0.17$ | $2.55 \pm 0.58$ | $1.19 \pm 0.10$ | $1.18 \pm 0.05$ |
| $P$ (years) | $9.73 \pm 1.72$ | $9.36 \pm 1.80$ | $13.68 \pm 1.01$ | $13.57 \pm 1.53$ | $13.74 \pm 1.14$ | $13.58 \pm 0.87$ | $16.81 \pm 1.54$ | $17.13 \pm 1.62$ |
| $\gamma$ | $21.8 \pm 22.4$ | $22.8 \pm 25.3$ | $14.2 \pm 2.0$ | $12.7 \pm 2.9$ | $12.4 \pm 6.1$ | $14.1 \pm 2.8$ | $13.74 \pm 7.07$ | $14.16 \pm 4.96$ |
| $\gamma_0$ (km s$^{-1}$) | $-83.2 \pm 37.0$ | $210.0 \pm 51.7$ | $-99.4 \pm 15.3$ | $88.7 \pm 13.8$ | $86.0 \pm 5.8$ | $-91.0 \pm 11.1$ | $37.8 \pm 1.7$ | $-29.2 \pm 2.5$ |
| $T_{\text{rot}}$ (K) | $290.9 \pm 60.9$ | $234.5 \pm 33.2$ | $-8.3 \pm 1.0$ | $88.6 \pm 8.4$ | $84.9 \pm 3.5$ | $-10.7 \pm 7.0$ | $33.5 \pm 1.0$ | $-2.8 \pm 1.5$ |
| $K$ | $27.8 \pm 4.5$ | $28.9 \pm 6.1$ | $15.4 \pm 3.4$ | $15.4 \pm 3.4$ | $14.7 \pm 3.4$ | $13.5 \pm 3.4$ | $8.0 \pm 2.0$ | $7.5 \pm 2.0$ |
| $KE/PE$ | $0.464$ | $0.081$ | $0.430$ | $0.477$ | $0.175$ | $0.196$ | $0.499$ | $0.470$ |
| $KE/PE$ | $0.500$ | $0.500$ | $0.500$ | $0.500$ | $0.500$ | $0.500$ | $0.500$ | $0.500$ |

**Note.** Notation is the same as for Table 10.
this is worse for the full Keplerian orbit with two more parameters. Thus, the inclusion of the orbital motion is not justified by these criteria.

However, the reason that we must introduce the orbital motion is not to improve the goodness of the fit, but to avoid the bias of the value and the underestimation of the uncertainty in parallax parameters due to the known degeneracy between the parallax and the lens orbital motion as shown in Section 4.4.1.

Furthermore, even though the full Keplerian orbit does not improve the goodness of the fit with two additional parameters, its incorporation has the following benefits. As Skowron et al. (2011) noted, in addition to being more accurate, the advantage of the full Keplerian orbit is to avoid all unbound orbital solutions (with eccentricity $>1$) and to enable the introduction of priors on the values of orbital parameters directly into MCMC calculations. If the uncertainty of the parameters are relatively large like in this event, the density distribution of the MCMC chain depends on the prior probability of the fitting parameters in MCMC. However, the linear orbital motion parameterization inadvertently assumed the uniform prior on all microlensing fitting parameters, which is not physically justified. On the other hand, the full Keplerian orbit parameterization enables us to properly weigh the MCMC chains with physically justifiﬁed priors.

We conducted the modeling with three different sets of relatively realistic priors including the Keplerian prior, i.e., $K_p$, $K_{pk}$, and $K_{pkg}$. We think that the model $K_{pk}$ is more realistic than $K_p$ as the galactic kinematics constraint is applied. The models with the Galactic mass density prior $K_{pkg}$ may also be useful, but we do not know if the assumption that the distribution of the planetary systems is uniform throughout the Galaxy is valid. Thus we adopt the $K_{pk}$ models as the main result of this paper.

It is important to note that the results of all these models $K_p$, $K_{pk}$, and $K_{pkg}$, are basically the same within their errors, thus
our main conclusion does not depend on the choice of these priors.

Among the models in $K_{\text{pk}}$, the low-mass brown dwarf models, i.e., $P_{++}$, $P_{++}$, and $P_{++}$, are slightly preferred by both $\chi^2$ and $H_s$ (see Section 6). But we accept all the models equally as possible solutions.

5. LENS PROPERTIES

The lens physical parameters can be derived for this event because we could measure both the parallax and finite source effects in the light curve.

The OGLE-IV calibrated color–magnitude diagram in a $2' \times 2'$ region around the event is shown in Figure 9. Figure 9 also shows the center of the red clump giants (RCGs) $(V - I)_{\text{RC,obs}} = (2.047, 15.73) \pm (0.002, 0.04)$ and the model-independent OGLE $V - I$ source color found by linear regression and the best-fit source $I$ magnitude of the model $P_{++}$. $(V - I)_{s,0} = (1.985, 18.13) \pm (0.008, 0.02)$. The $I_s$ for other models are almost the same, as shown in Tables 2–7.

Assuming the source suffers the same extinction and reddening as the RCGs and using the expected extinction-free RCG centroid $(V - I)_{\text{RC,0}} = (1.06, 14.39) \pm (0.06, 0.04)$ at this position (Bensby et al. 2013; Nataf et al. 2013), we estimated the extinction-free color and magnitude of the source as $(V - I)_{s,0} = (1.00, 16.80) \pm (0.06, 0.06)$. This color measurement is consistent with the independent measurement of $(V - I)_{s,0} = 1.02 \pm 0.06$ by the CTIO telescope (see Section 3). We use the average of OGLE and CTIO colors, $(V - I)_{s,0} = 1.01 \pm 0.06$ in the following analyses. Here the errors in $(V - I)_{s,0}$ are dominated by the error in $(V - I)_{\text{RC,0}}$.

These values are consistent with the source being a K2 subgiant (Bessell & Brett 1988).

Following Fukui et al. (2015), we estimated the source angular radius, $\theta_s$, by using the relation between the limb-darkened stellar angular diameter, $\theta_{\text{LD}}$, $(V - I)$, and $I$ given by Equation (4) of Fukui et al. (2015). This relation is derived from a subset of the interferometrically measured stellar radii in Boyajian et al. (2014), in which the dispersion of the relation is ~2% using only stars with $3900 < T_{\text{eff}} < 7000$ K to improve the fit for FGK stars. This yields the source radius of $\theta_s = \theta_{\text{LD}}/2 = 1.84 \pm 0.12 \mu\text{as}$.

The spectrum of the source was taken by the UVES spectrograph on the Very Large Telescope at a time when the source was still magnified as a Target of Opportunity observation. Reductions were carried out with the UVES pipeline (Ballester et al. 2000). The observation and data analysis have been performed in the same manor as Bensby et al. (2011, 2013). This gives the source effective temperature, $T_{\text{eff}} = 4854 \pm 66$ K, the gravity, $\log g = 3.30 \pm 0.14$, and the metallicity, $[\text{Fe}/\text{H}] = -0.17 \pm 0.09$ (T. Bensby et al. 2016, in preparation). By using these values and the relation by Casagrande et al. (2010), we derive the extinction-free source colors, $(V - I)_{s,0,\text{spec}} = 1.036 \pm 0.047$ and $(V - H)_{s,0,\text{spec}} = 2.244 \pm 0.078$. Thus we obtain $(I - H)_{s,0,\text{spec}} = 1.208 \pm 0.091$. The extinction-free $H$-band source magnitude is given as $H_{s,0} = I_{s,0} - (I - H)_{s,0,\text{spec}} = 15.59 \pm 0.11$.

The expected extinction-free $(I - H)_{s,0}$ from the measured $(V - I)_{s,0}$ are $(I - H)_{s,0} = 1.188 \pm 0.082$ and $(I - H)_{s,0} = 1.119 \pm 0.074$ using the stellar color–color relation of Bessell & Brett (1988) and Kenyon & Hartmann (1995), respectively. These are roughly consistent with $(I - H)_{s,0,\text{spec}}$. By a linear regression of the OGLE-$I$-band light curve and the CTIO $H$-band light curve (see Section 3), which are calibrated to the 2MASS scale, we obtain the source $(I - H)$ color as,

$$(I - H)_{s,0} = 2.256 \pm 0.016. \tag{6}$$

Correcting the extinction by using the measured extinction and reddening $(E(V - I), A_I)$ by RCGs and the extinction law of Chen et al. (2013), we obtain $(I - H)_{s,0,\text{OGLE:CTIO}} = 1.259 \pm 0.071$, which is also consistent with $(V - H)_{s,0,\text{spec}}$.

Then we obtain $\theta_s = 1.90 \pm 0.11 \mu\text{as}$ by using the relation between $\theta_{\text{LD}}$, $H_{s,0}$, $(V - H)_{s,0}$, and $[\text{Fe}/\text{H}]$, given by Equation (9) of Fukui et al. (2015), which is also driven in the same way as Equation (4) of Fukui et al. (2015) but with the metallicity term. Note that $H$ in the relation is in the Johnson magnitude system. Thus the observed $H$-band source magnitude, which is in the 2MASS system, is converted to the Johnson system by following Fukui et al. (2015). This is consistent with the above value. The average of the above values is,

$$\theta_s = 1.87 \pm 0.12 \mu\text{as}, \tag{7}$$

where we adopt the larger error from the estimate with $(V - I, I)$, conservatively. This value is about the median of those from the other models and differences with them are less than 2%, thus we adopt this value for all models in the following analysis.

We also tested the traditional method as follows. Following Yoo et al. (2004), the dereddened source color and brightness $(V - K, K_{s,0} = (2.2, 15.6)$ are estimated using the observed $(V - I, I_{s,0})$ and the color–brightness relation of Kenyon & Hartmann (1995). By using this $(V - K, K_{s,0}$ relation and the empirical color/brightness-radius relation of Kervella et al. (2004), we
estimated the source angular radius, \( \theta_s = 1.85 \pm 0.16 \mu \text{as} \),
where the error includes uncertainties in the color conversion
and the color/brightness-radius relations. This is consistent
with the above value.

The physical parameters of all models are listed in Tables 8–13. The physical properties of three models with realistic priors and constraints, i.e., \( K_{p} \), \( K_{pk} \), and \( K_{pkg} \), are basically same within the error bars. In the following analysis, we focus on the model \( K_{pk} \).

Here we summarize the physical properties of the lens system by showing the average values of various models in Table 12 for clarity. The averages are taken without any weighting by their error bars. The uncertainties are given by the maximum and minimum values of \( 1\sigma \) upper and lower limits of all (or a group of) models. The angular Einstein radii, and geocentric lens–source relative proper motion \( \mu_{geo} \) which are independent of the parallax values, are estimated, respectively, as follows,

\[
\theta_E = \frac{\theta_s}{\rho} = 0.48 \pm 0.06 \text{ mas},
\]

\[
\mu_{geo} = \frac{\theta_E}{r_E} = 8.4 \pm 1.2 \text{ mas yr}^{-1}.
\]  

This \( \mu_{geo} \) is consistent with the typical value for disk lenses of \( \mu \sim 5–10 \text{ mas yr}^{-1} \) (Han & Gould 1995).

The total mass and distance of the lens system can be given by \( M = \theta_E/(\kappa_{\sigma}) \) and \( D_1 = (\pi_{\sigma}/\theta_E + \pi_s) \), where \( \kappa = 4G/ (c^2 \sigma) = 8.144 \text{ mas} M_1^{-1} \), \( \pi_s = au/D_1 \), and \( D_1 \sim 8 \text{ kpc} \) is the distance to the source (Dong et al. 2009a). Thus these quantities depend on the parallax parameter and we have three groups of solutions in the models, i.e., small \( \pi_E \sim 0.3 \) (\( P_{\pm \pm} K_{pk} \)), medium \( \pi_E \sim 0.8 \) (\( P_{\pm} K_{pk} \) and \( P_{\pm \pm} K_{pk} \)), and large \( \pi_E \sim 2 \) (\( P_{\pm \pm} K_{pk} \)). The distance to the system, \( D_h \), the mass of the host, \( M_h \), and planet, \( M_p \), and their projected separation, \( a_{\perp} \), of these solutions are,

\[
D_1 = 3.6^{+0.6}_{-0.8} \text{ kpc}, 1.8^{+0.4}_{-0.2} \text{ kpc}, \text{ or } 0.85^{+0.13}_{-0.08} \text{ kpc},
\]

\[
M_h = \frac{M}{1 + q} = 0.19^{+0.05}_{-0.06} M_\odot, 0.068^{+0.019}_{-0.011} M_\odot, \text{ or } 0.025^{+0.005}_{-0.004} M_\odot,
\]
The semimajor axes \( a_{\text{kep}} \) normalized by the snow line, 
\[
a_{\text{snow}} = 2.7 \left( \frac{M_\star}{M_\odot} \right),
\]
are
\[
\frac{a_{\text{kep}}}{a_{\text{snow}}} = 8.9^{+10.5}_{-1.4}, \quad 12^{+7}_{-1}, \quad \text{or} \quad 14^{+11}_{-11}.
\]  

The effective temperature of the planet at the time of its formation based on the host mass and host–planet separation are also given in the tables.

The small parallax models suggest that the planet has a mass similar to Neptune (17\(M_\oplus\)) orbiting a very low-mass M-dwarf in the Galactic disk. The planet is very cold as the estimated separation is \(8.9^{+10.5}_{-1.4}\) times larger than the snow line. This is comparable to Neptune’s semimajor axis, i.e., 11 times larger than the Sun’s snow line. This interpretation of the planetary signal for MOA-2013-BLG-605Lb, therefore, suggests the planet is a Neptune analog.

The medium parallax models correspond to a miniature Neptune (or large-mass “super-Earth”) orbiting a high-mass brown dwarf host. The planet is even colder as the planetary orbit radius is \(12^{+7}_{-1}\) times larger than the snow line, which is also similar to that of Neptune.

The large parallax models correspond to a three-times Earth-mass planet orbiting a low-mass brown dwarf host. The planet is colder because the planetary orbit radius is \(14^{+11}_{-11}\) times larger than the snow line.

These solutions of \(K_{pk}\) are compared to the planets found by other methods in Figure 1. As one can see in the right-hand panel of Figure 1, in either group of models, this planet is the coldest low-mass planet ever found and it is very similar to Neptune.
We observed with the NIRC2 instrument mounted on KECK-II the microlensing target MOA-2013-BLG-0605 on 2015 July 26. We used the wide camera, giving a pixel scale of 0.04 arcsec and a FOV of 40 arcsec. We adopted a five-position dithering pattern, and performed 30 exposures of 10 s each. We performed dark subtraction and flatfielding in the standard manner for an IR detector. We then stacked the frames using Swarp (Bertin et al. 2002), without subtracting the background. The final image is shown in Figure 10.

For absolute calibration, we used images from the VVV survey performed with the VISTA 4 m telescope at Paranal (Minniti et al. 2010). We extracted a 3 arcmin JHK band image centered on the target. We computed a PSF model using PSFEX software (Bertin & Arnouts 1996), and measured fluxes on the frames using SExtractor with this PSF model. We cross identified the stars from the field with 2MASS catalogs. We selected 300 stars that are bright but not saturated on VVV, and derived the photometric zero points with an accuracy of 0.004 mag. We then used the VVV catalog to perform the astrometric calibration of the KECK frame.

We then measured the fluxes using SExtractor as described in (Batista et al. 2014). We cross identified 39 stars in both the VVV and KECK images. We excluded the stars saturated on KECK, and derive the zero point of KECK photometry. In the non-AO PSF, there are two stars, the source with $H = 15.90 \pm 0.02$ and a blend at $\sim 0.3$ arcsec to the south with $H = 17.01 \pm 0.03$. Here, these two stars are blended in the OGLE reference image and the cataloged centroid is between them. The actual source position during the magnification on the OGLE difference image was precisely measured as shown by the red cross in Figure 10. This clearly resolved the source and showed that the blend measured in the fitting process is not the lens.

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solutions from the parallax measurements that the host is a low-mass M-dwarf or a brown dwarf mentioned in Section 5. The low-mass brown dwarf models are slightly preferred. The $H_s$ of the standard model is consistent with the Keck results, but we concluded that the parallax models are better as discussed in Section 4.5.

As mentioned in Section 4.4.4, there are two minima with a much smaller parallax value of $\pi_E \sim 0.035$ for the $P_{\pm 2}$ models. In addition to the rarity of their heavy host mass of $\sim 1.7M_\odot$, which might be a stellar remnant, their source magnitudes $H_s = 15.824 \pm 0.018$ and $H_s = 15.816 \pm 0.017$ are $2.9\sigma$ and $3.2\sigma$ brighter than the Keck measurement. So these models are not likely real. If their host is a main-sequence star, then the total brightness of the source plus lens is ruled out by the Keck measurements by more than $4\sigma$.

7. FUTURE MASS MEASUREMENT

Let us consider the prospects for resolving the degeneracy and characterizing the host and the planet. In the first epoch of Keck AO observations, we could not detect any excess light, which confirmed the lens is faint. If the second epoch is taken by HST or AO observations, then we may directly detect the host (or possibly its companion). We can then measure the lens mass and distance or place a stronger upper limit on the lens mass.

In Tables 8–13, the geocentric proper motions are reported as $\mu_{\text{geo}} = \theta_E / \pi_E = 8 \sim 9$ mas yr$^{-1}$. The heliocentric proper motion is given by (Janczak et al. 2010),

$$\mu_{\text{hel}} = \mu_{\text{geo}} + v_{\odot, \text{rel}} \frac{\pi_{\text{rel}}}{\text{au}},$$

where $v_{\odot, \text{rel}} = (v_{\odot, \text{rel}, N}, v_{\odot, \text{rel}, E}) = (-2.96, -8.24)$ km s$^{-1}$ is the velocity of the Earth projected on the plane of the sky at the peak of the event. The estimated $\mu_{\text{hel}} = (\mu_{\text{hel}, N}, \mu_{\text{hel}, E})$ of each model is shown in Tables 8–13, and they are about $8 \sim 9$ mas yr$^{-1}$. Hence it is clear that the lens will be separately resolved by HST or AO observations in 5–10 years’ time given a diffraction limit of 50 mas. Or, if we do not see any luminous object, then the lens is a sub-stellar object.

Not only the value but also the direction of expected relative proper motion would help us to know if it was the lens or just an ambient star when we detect such star at 80 mas from the source ten years later.

8. DISCUSSION AND CONCLUSION

There are three physical planetary solutions for the MOA-2013-BLG-605L system. One comprises a Neptune-mass planet at a wide separation from a very low-mass M-dwarf host star, having a very similar temperature as Neptune when the planet was formed. The second solution comprises a mini-Neptune around a high-mass brown dwarf which was even colder than Neptune when it was formed. The third one is a super-Earth around a low-mass brown dwarf.

These degenerate solutions may be resolved by future high resolution imaging of the lens by the HST or ground-based telescopes using AOs, after waiting a period of time for the positions of the lens and the source to diverge. We may detect an M-dwarf lens host star, but we do not expect to detect a brown dwarf host star by such direct imaging.

In either case, the host is one of the three least massive main-sequence stars orbited by a planet for which the planet’s mass
was measured and for which the planet–host mass ratio is \( q < 0.01 \). The other low host mass, low planet mass systems are MOA-2007-BLG-192L \((M_h = 0.084^{+0.009}_{-0.012} M_{\odot}, M_p = 3.2^{+5.3}_{-1.1} M_{\odot}, r_p = 0.66^{+0.21}_{-0.32} \) au) (Bennett et al. 2008; Kubas et al. 2012) and MOA-2010-BLG-328L \((M_h = 0.11 \pm 0.01 M_{\odot}, M_p = 9.2 \pm 2.2 M_{\odot}, r_p = 0.92 \pm 0.16 \) au) (Furusawa et al. 2013).

These planets found around very low-mass \( (\sim 0.1 M_{\odot}) \) hosts have relatively small masses themselves, ranging from super-Earth to Neptune mass. In contrast, a roughly equal number of giant planets and planets with Neptune-mass or less have been found across the whole mass range of host stars. This may imply that the formation of gas giants is more difficult around very low-mass stars compared to average K-M dwarf stars with masses of \( \sim 0.5 M_{\odot} \), which is the typical host star for microlensing planets. This is somewhat as predicted by the core accretion model of planetary formation, but this work provides the first observational evidence supporting this prediction.

This could be the first exoplanet around a brown dwarf with a mass measurement having a planetary mass ratio \( q < 0.03 \). There are three brown dwarf binaries where one of the components is in the planetary mass regime ( Choi et al. 2013; Han et al. 2013). However, their mass ratios are large \( q \gtrsim 0.08 \), suggesting that their formation may be considered more akin to binary formation than planetary formation.

The separation of the planet is very wide, \( 8.9^{+10.5}_{-14.7}, 12^{+11}_{-7} \), or \( 14^{+11}_{-7} \) times larger than the snow line of \( \sim 0.5 (M/0.2 M_{\odot}) \) au, \( \sim 0.2 (M/0.07 M_{\odot}) \), or \( \sim 0.08 (M/0.03 M_{\odot}) \) au, respectively, as seen in Figure 1. The effective temperature of the planet when it was formed, based on the host mass and the planet–host separation, is \( \sim 26, \sim 13, \) or \( \sim 7 \) K, the coldest planet found to date apart from those planets found by the direct imaging method, which can currently only find heavy gas giant planets of more than a few Jupiter masses. The effective temperatures of these heavy gas giants are a few hundreds of K or higher due to their internal heat. In either interpretation, planet MOA-2013-BLG-605Lb is orbiting around one of the least massive objects found to date at a very wide separation. The planet is the coldest exoplanet discovered so far. This is the first observed example of a Neptune-like exoplanet in terms of mass and temperature or a super-Earth with Neptune-like temperature, which are important factors in any planetary formation theory.

The probability of detecting such wide separation low-mass planets is very low, even using microlensing. The probability of a source crossing the planetary caustic is proportional to the size of the planetary caustic, \( w_c \sim 4q^{1/2}s^{-2} \sim 0.01 \) (Han 2006), divided by half the circle with radius of separation \( s \), i.e., \( P \sim 4/\pi q^{1/2}s^{-3} \sim 1 \times 10^{-3} \) \((s = 2.4) \). It is an order of magnitude smaller than planets at \( s \sim 1 \), where \( \sim 10 \) planets with Neptune-mass or less have been found by microlensing. This may imply that such low-mass planets with masses less than that of Neptune at \( a \sim 10 d_{\text{snow}} \) are as common as low-mass planets at a few times the distance of the snow line (Gould et al. 2010; Sumi et al. 2010; Cassan et al. 2012).

This conclusion may challenge the standard core accretion model and other formation models ( Ida & Lin 2004) which predict few low-mass planets with Neptune-like orbits at \( >10 d_{\text{snow}} \). More accurate measurements of the abundance and distribution of such low-mass ice planets are very important in the study of the formation of Neptune and in the study of planet formation mechanisms in general. The microlensing exoplanet search by NASA’s \( \text{WFIRST} \) satellite is expected to detect hundreds of low-mass planets with Neptune-like orbits and will constrain further planetary formation models.

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