State estimation of packet loss network based on dimension compression of measurement information

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Abstract. With packet loss, this paper proposes a method to compress the dimension of measurement information obtained by the sensor which is distributed in the wireless sensor network with single sensor and single communication link. Based on the minimum mean square error criterion, the state estimation method under the condition of dimension compression of measurement information is designed by using the improved Kalman filter when measurement noise and system noise are correlated. And the convergence condition of the estimation algorithm is analyzed. The simulation platform of matlab is used to verify the proposed estimation method. It is proved that the estimation accuracy of the measured physical quantity can be improved by using the dimension compression of measurement information estimation method, so as to improve the reliability of the whole network system.

1. Introduction
With the development of wireless technology, the related application of wireless sensor network (WSN) becomes more and more extensive, which requires the deployment of more sensors in the application field. The function of sensor is estimating the monitored physical quantity in WSN. However, the estimated physical quantity cannot be directly measured by sensors with the influence of noise. In order to estimate the physical quantity, we can get it from the measurement value polluted by noise using the correlation estimation algorithm. State estimation algorithms including Kalman filter, Particle filter, Wiener filter and so on are mainly used to estimate the variables changing with time. In application, the measured values obtained by sensors in WSN are constantly changing. Therefore, this paper mainly designs the estimation algorithm based on the Kalman filter under the linear minimum mean square error criterion. Kalman filter algorithm can realize the optimal estimation of a single sensor to the physical quantity through recursive filter from a series of measured values.

End-to-end data transmission which is reliable becomes a very significant problem in WSN. However, many factors cause the unreliability of wireless network data transmission. Mainly reflected in: the amount of information collected by sensors is huge, the bandwidth of communication link is limited, and it is easy to generate network congestion, which lead to the loss of data packets transmitted by sensors. Wireless communication technology which can be affected by the external environment easily always produces data packet loss.

Scholars have done a lot of research on packet loss. In [1], the lost packets were compensated by prediction. In [2], the method of zero input was used for packet loss compensation. The algorithm mentioned above effectively reduces the influence of lossy packet on the performance of the estimator. It is also very significant to analyze the estimation stability in the case of packet loss. In [3], the authors considered a discrete-time linear system with Bernoulli distribution with packet loss, and obtained the condition of divergence of the expectation estimated error covariance. [4] proposed a
scheme to combine the current measurements and the lost measured values obtained by the sensor. In [5], it was proposed to transfer the Kalman estimation obtained by the sensor instead of the original measurement values, and the condition of system stability was obtained. Most of the above studies are focused on Bernoulli packet loss.

In [6], a linear coding scheme was proposed, the measurement values in a finite length were linearly combined. In this scheme, the dimension of transmission measurement information is consistent with the original measurement output. In this paper, we innovate on the basis of linear coding, and compress the dimension of measurement information to scalar. The task of this paper is to give the compression scheme of measurement information dimension and the corresponding state estimation algorithm. By compressing the dimension of the measurement information obtained by the sensor, the dimension of transmission data can be reduced and the bandwidth of communication link can be saved. The condition of covariance convergence can be obtained by analyzing the convergence of the proposed algorithm with packet loss. Simulation results show that the proposed algorithm can resist packet loss and improve the reliability of the network system.

2. Problem description

Assume that a sensor estimates the state of a target physical quantity, the state equation of the system is

$$x_{k+1} = Ax_k + w_k$$  \hspace{1cm} (1)

among them, $x_k \in \mathbb{R}^{nx1}$ is the state variable at time $k$, $A \in \mathbb{R}^{nxn}$ is the system matrix, the initial state of the system is $x_0$ and its distribution is $x_0 \sim N(\bar{x}_0, P_0)$, in which the initial state estimation error covariance is $P_0 \geq 0$. $w_k \in \mathbb{R}^{nx1}$ is Gaussian white noise, and its distribution is $w_k \sim N(0, W)$ and $W \geq 0$.

The measurement equation of the sensor is

$$y_k = Cx_k + v_k$$  \hspace{1cm} (2)

among them, $y_k \in \mathbb{R}^{mx1}$ is the measurement output of the sensor, $C \in \mathbb{R}^{mxn}$ is the measurement matrix, the measurement noise $v_k \in \mathbb{R}^{mx1}$ is Gaussian white noise, its distribution is $v_k \sim N(0, V)$. It is assumed that $x_0, w_k, v_k$ are uncorrelated and $(A, C)$ is observable.

Each sensor estimates the measured physical quantity from the measurement value polluted by noise, in this process, the sensor is the estimator. Due to network congestion and other factors, data packets will be lost, which affects the overall estimation performance of the network. Suppose the packet loss process is Bernoulli process, $\gamma_k$ indicates whether the packet is lost or not, $\gamma_k = 1$ indicates that the packet is successfully transferred, otherwise, $\gamma_k = 0$ indicates that the packet is lost.

The condition of the research object of the system is as follows:

**Condition 1.** The maximum eigenvalue amplitude of system matrix is greater than 1.

**Remark.** If the maximum eigenvalue amplitude of system matrix is not greater than 1, even if all the measurement information of the sensor is lost, the state estimation of the whole system is still stable. It is not meaningful to study the estimation performance of this system.

In general, the more dimension of the measurement information the sensor gets, the more communication link resources it consumes. Therefore, in order to reduce the influence of packet loss and estimate effectively, the algorithm in this paper must be designed to ensure that the dimension of measurement information after preprocessing is smaller than the raw measurement information, so that the dimension is a scalar. Different from the standard Kalman filter, the algorithm proposed in this paper preprocesses the measurement information at the sensor end: set the buffer space with the length as $L$, combine the measurement values before time $k$ and at the time $k$, multiply them by the
weighted vector coefficient to compress the dimension, and then transmit them to the estimator through the communication link. The block diagram of state estimation of sensor is shown in figure 1.

![Figure 1. Block diagram of state estimation under compression of measurement information dimension](image)

### 3. Design of dimension compression method and state estimation algorithm

#### 3.1. Measurement information dimension compression scheme

This section mainly proposes a method to compress the measurement information dimension, that is, to combine the measurement values, and the weighting coefficient is the vector \( \mathbf{k} \). The specific combination scheme is: the weighting coefficient is

\[
\mathbf{k} = \begin{bmatrix}
\mathbf{k}_1 \\
\mathbf{k}_2 \\
\vdots \\
\mathbf{k}_L
\end{bmatrix}
\]

where \( \mathbf{k}_l \in \mathbb{R}^m \), \( l = 1 \cdots L \). The output value of the combined measurement is

\[
z_k = H_k \mathbf{y}_k = \begin{bmatrix}
H_{k1} \mathbf{y}_k \\
H_{k2} \mathbf{y}_k \\
\vdots \\
H_{kL} \mathbf{y}_k
\end{bmatrix}
\]

where \( \mathbf{y}_k = \begin{bmatrix}
\mathbf{y}_{k-L+1}^T \\
\mathbf{y}_{k-L+2}^T \\
\vdots \\
\mathbf{y}_k^T
\end{bmatrix} \) and \( H_k = \begin{bmatrix}
H_{k1} \\
H_{k2} \\
\vdots \\
H_{kL}
\end{bmatrix} \) is a \( (L-1) \times m \) matrix. According to the literature [6], the period of coefficient vector \( \{H_k : k \in \mathbb{N}\} \) is \( \tau \geq n \) (\( H_k = H_{k+\tau} \)), which \( H_0, H_1, \cdots \) is randomly generated by a continuous distribution.

#### 3.2. State estimation algorithm

After the measurement information is compressed using the dimension compression scheme, it can be seen from the definition of \( z_k \) that the noise of \( z_k \) is correlated with those involved in \( z_{k-1}, z_{k-2}, \cdots, z_{k-L+1} \). Then, the standard Kalman filter algorithm is no longer applicable. Therefore, in this section, the Kalman filter algorithm which the system noise and measurement noise is related is used to obtain the estimator. The state equation and measurement equation is established.

Define \( \eta_k \) as \( \eta_k = (H_k \mathbf{y}_{k-L+1})^T (H_k \mathbf{y}_{k-L+2})^T (H_k \mathbf{y}_{k-L+1})^T \), we can get a new equation

\[
\eta_{k+1} = F_k \eta_k + G_k \mathbf{y}_k
\]

where

\[
F_k = \begin{bmatrix}
0 & J_k \\
0 & 0
\end{bmatrix} \in \mathbb{R}^{(L-1)\times(L-1)}, \quad G_k = \begin{bmatrix}
0 \\
H_{(k+1)(L-1)}
\end{bmatrix} \in \mathbb{R}^{(L-1)\times m}
\]

\[
J_k = \begin{bmatrix}
H_{(k+1)1}^T & H_{k2}^T & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & H_{(k+1)(L-2)}^T H_{kL}^T
\end{bmatrix}
\]

\( J_k \) is a \( (L-2) \times (L-2) \) dimensional matrix, and \( \mathbf{y}_k \) is \( (L-1) \times 1 \). Especially, when \( L = 2 \), \( F_k = 0 \).
Define \( u_k = [x_k^T \eta_k^T]^T \in \mathbb{R}^{(L-1)+n} \), we can get the improved state equation and measurement equation for measurement information \( z_k \) as follows

\[
\begin{align*}
 u_{k+1} &= \Phi_k u_k + \sigma_k \\
 z_k &= h_k u_k + \rho_k
\end{align*}
\] (5)

with \( \Phi_k = \begin{bmatrix} A & 0 \\ G_k C & F_k \end{bmatrix} \), \( \sigma_k = \begin{bmatrix} w_k \\ G_k V_k \end{bmatrix} \), \( \rho_k = H_{kl} V_k \), \( h_k = [H_{kl} \times C \ 1 \ \cdots \ 1] \in \mathbb{R}^{(L-1)+n} \),

\[
S_k = \mathbb{E}[(\sigma_k \rho_j)]^T \delta_{kj} = \begin{bmatrix} G_k V_k^T \\ 0 \\ G_k V G_k^T \end{bmatrix}, \quad R_k = \mathbb{E}[\rho_k \rho_j] = H_{kl} V H_{kl}^T, \quad \overline{Q}_k = \mathbb{E}[\sigma_k \sigma_j] = \begin{bmatrix} W & 0 \\ 0 & G_k V G_k^T \end{bmatrix}.
\]

Based on the above variables, the state estimator and the corresponding error covariance are obtained by using the improved Kalman filter which can be expressed by

\[
\hat{u}_{k+1|k} = \Phi_k \hat{u}_{k|k-1} - \gamma_k (\Phi_k \Sigma_{k-1|k-1} h_k^T + S_k) (h_k \Sigma_{k-1|k-1} h_k^T + R_k)^{-1}(z_k - h_k \hat{u}_{k|k-1})
\] (7)

\[
\Sigma_{k+1|k} = \Phi_k \Sigma_{k|k-1} \Phi_k^T - \gamma_k \Sigma_{k|k-1} + \gamma_k (\Phi_k \Sigma_{k-1|k-1} h_k^T + S_k) (h_k \Sigma_{k-1|k-1} h_k^T + R_k)^{-1} (\Phi_k \Sigma_{k-1|k-1} h_k^T + S_k)^T \gamma_k
\] (8)

3.3 Convergence analysis of estimation algorithm under packet loss

Consider a stochastic time-varying system

\[
\begin{align*}
 x_{k+1} &= A x_k + w_k \\
 y_k &= C_k x_k + v_k
\end{align*}
\] (9)

(10)

The difference between this system and system (1) (2) is that the measurement matrix \( C_k \) is time-varying. Set \( m = n - \text{rank}(C_k) + 1 \).

**Definition 2.** With any \( \tau \geq m \), \( k \geq \tau - 1 \), if for any \( 1 \leq i_1 < i_2 < \cdots < i_{m-1} < \tau \), the matrix

\[
O(k, k-i_1, k-i_2, \ldots, k-i_{m-1}) = \begin{bmatrix} C_k \\ C_{k-i_1} A^{-1} \\ \vdots \\ C_{k-i_{m-1}} A^{-m+1} \end{bmatrix}
\] (11)

is fully column rank. Then \( (A, \{C_k : k \in \mathbb{N}\}) \) is strongly observable and the period is \( \tau \).

**Lemma 4** Based on Condition 1, if the period of the coding vector \( \{H_k : k \in \mathbb{N}\} \) is \( \tau \geq n \) (\( H_k = H_{k+1} \)), and all of \( H_0, H_1, \ldots, H_{\tau-1} \) are generated by a continuous distribution, then the probability of the strongly observable system \( (A, \{C_k : k \in \mathbb{N}\}) \) with period \( \tau \) is 1.

**Lemma 5** Based on Condition 1, if the period of the coding vector \( \{H_k : k \in \mathbb{N}\} \) is \( \tau \geq n \) (\( H_k = H_{k+1} \)), and all of \( H_0, H_1, \ldots, H_{\tau-1} \) are generated by a continuous distribution, if the number of packets received by the sensor in the time period \( [(j-1)\tau, j\tau) \) is at least \( m \), then there will exist \( \beta > 0 \) which is independent of \( P_0 \) such that \( P_{\tau|j \tau} < \beta^j \).

**Theorem 6** Based on Condition 1, if the period of the coding vector \( \{H_k : k \in \mathbb{N}\} \) is \( \tau \geq n \) (\( H_k = H_{k+1} \)), and all of \( H_0, H_1, \ldots, H_{\tau-1} \) are generated by a continuous distribution, the convergence condition of the state estimator with probability 1 is as follows:

\[
(1 - p) \|A\|_\max^2 \langle P(\tau, n) \rangle^{1/\tau} < 1
\] (12)
where \( P(\tau, n) = \sum_{i=0}^{\infty} \left( \frac{\tau}{i!} \right) \left( \frac{P}{1-P} \right)^i \geq 1, \) and \( \left( \frac{\tau}{i!} \right) \) is a combination of \( i \) choosing from \( \tau \).

**Proof.** To get optimal estimator, the convergence analysis of the state estimation algorithm is to find out that the condition of \( \limsup_{k \to \infty} E[P_{k|k}] < \infty \). The period \( \tau \) of \( H_k \) is limited which will result in \( \limsup_{k \to \infty} E[P_{k|k}] < \infty \) is equal to \( \limsup_{k \to \infty} E[P_{k|k}] < \infty \). Therefore, in order to get (12), we only need to prove that under the condition of theorem 6, \( \limsup_{k \to \infty} E[P_{k|k}] < \infty \) is established with the probability 1.

For any \( j = 0, 1, \cdots k \), define the event \( \Omega_{j,k}^n \) as "in time period \([k-(k-1)\tau, k\tau)\), \([j-k+1] \) the number of packets received is less than \( n \), while the number of packets received in \(([j-k+1], j\tau)\) is greater than or equal to \( n \)." The probability of \( \Omega_{j,k}^n \) is \( p_{j,k}^n \). Especially, \( \Omega_{0,k}^n \) means that less than \( n \) packets are received in the time period \([k-(k-1)\tau, k\tau)\), \([k-(k-1)\tau, (k-1)\tau)\), \cdots \([-1], [0, \tau)\).

Let \( \bar{z} = [CA^{n-1}, CA^{n-2}, \ldots, C]^T \). According to lemma 4, \((A, H_k \bar{z}; k \in N)\) is a strong observable system with period \( \tau \). This satisfies the condition of lemma 5, from which we can get \( E[P_{k|k}] < B \). Since the divergence rate of the estimation error covariance is not more than \( \lambda_{\max}(A)^2 \left( \lambda_{\max}(A) \text{ refers to the maximum eigenvalue of the matrix} \right) \). Using the above results, we can further obtain that there is a constant \( \varepsilon > 0 \) such that

\[
E[P_{k|k}] = \sum_{j=0}^{k} E[P_{k|k} \Omega_{j,k}^n] p_{j,k}^n < \varepsilon \sum_{j=0}^{k} \lambda_{\max}(A)^2 (k-j+1)^{\tau} E[P_{k|k} \Omega_{j,k}^n] p_{j,k}^n \\
< \varepsilon \beta \lambda_{\max}(A)^2 \sum_{j=0}^{k} \lambda_{\max}(A)^2 (k-j+1)^{\tau} p_{j,k}^n I \]

The probability of \( \Omega_{j,k}^n \) is

\[
p_{j,k}^n = \sum_{i=0}^{\infty} \left( \frac{\tau}{i!} \right) \left( \frac{P}{1-P} \right)^i \left( 1 - \sum_{i=0}^{\infty} \left( \frac{\tau}{i!} \right) \left( \frac{P}{1-P} \right)^i \right) \\
= ((1-P)^\tau P(\tau, n))^{(k-j)} (1-(1-P)^\tau P(\tau, n)) \]

\[
p_{j,k}^n \text{ is substituted into } E[P_{k|k}] \text{, then we can get} \\
E[P_{k|k}] < \varepsilon \beta \lambda_{\max}(A)^2 \sum_{j=0}^{k} ((1-P)^\tau P(\tau, n))^{(k-j)} (1-(1-P)^\tau P(\tau, n)) \]

Thus, the convergence condition of state estimation can be determined as

\[
(1-P)\beta \lambda_{\max}(A)^2 (P(\tau, n))^{1/\tau} < 1. \]

4. Simulation

According to the convergence condition (12) of state estimation, the relationship between the boundary value of packet loss rate and the period of weighted vector coefficient can be obtained as shown in figure 2.

In order to verify the estimation performance of the estimation algorithm (7) and (8) with packet loss, firstly, the period of weighting vector coefficient is taken as \( d = 20 \). As shown in figure 2, the average packet arrival rate should meet \( p \geq 0.65 \). Take three values of the average packet arrival rate
for simulation analysis, which are $p_1 = 0.65$, $p_2 = 0.75$, $p_3 = 0.85$. A comparison is made between the estimation algorithm proposed in this paper and the intermittent Kalman filter algorithm proposed in [7] with 1000 Monte Carlo experiments. By comparing the trace of estimation error covariance, we can get the estimation effect under three packet loss values as shown in figure 3, figure 4 and figure 5.

The convergence condition of the algorithm in [7] is $1 < \frac{(1-p)\lambda_{\text{max}}(A)|^4}{\lambda_{\text{max}}(P(20,2))^{1/20}}$ with packet loss. Compared with (12), when $p_1 = 0.65$, the condition of convergence of the intermittent Kalman filter is not satisfied as $1 > \frac{(1-p)\lambda_{\text{max}}(A)|^4}{\lambda_{\text{max}}(P(20,2))^{1/20}} = 1.7719 > 1$. Similarly, when $p_2 = 0.75$, $1 > \frac{(1-p)\lambda_{\text{max}}(A)|^4}{\lambda_{\text{max}}(P(20,2))^{1/20}} = 2.6561 > 1$ while formula (12) is established $1 < \frac{(1-p)\lambda_{\text{max}}(A)|^4}{\lambda_{\text{max}}(P(20,2))^{1/20}} = 0.6909 < 1$. From figure 3 and figure 4, when the vertical axis is viewed in logarithmic coordinates, the trace of the state estimation error covariance of the intermittent measurement Kalman filter without the measurement information dimension compression is far greater than that of the dimension compression.

When $p_3 = 0.85$, the convergence condition of the intermittent Kalman filter algorithm is satisfied, that is, $1 < \frac{(1-p)\lambda_{\text{max}}(A)|^4}{\lambda_{\text{max}}(P(20,2))^{1/20}} = 0.7594 < 1$. The convergence condition of the estimation method proposed in this paper is also satisfied, that is, $1 < \frac{(1-p)\lambda_{\text{max}}(A)|^4}{\lambda_{\text{max}}(P(20,2))^{1/20}} = 0.4277 < 1$. It can be seen from figure 5 that the estimation error covariance of the two estimation methods is very close, and the estimation effect is almost the same.
Figure 5. Error covariance comparison under the average packet arrival rate of 0.85

The above analysis shows that under the packet loss rate satisfying the convergence condition, the performance of the measurement information dimension compression state estimation algorithm proposed in this paper is better than that of the intermittent Kalman filter algorithm without any processing, which can improve the estimation accuracy. When the packet loss rate is very small (the Kalman filter of intermittent measurement remains stable), the estimation effect under the condition of dimension compression of measurement information is similar to that of the Kalman filter algorithm of intermittent measurement. Therefore, the measurement information dimension compression estimation method can improve the reliability of the network.

5. Conclusions
The higher the dimension of the measurement information obtained by the sensor, the more communication link resources are needed for data transmission. Due to the limited channel bandwidth, it is easy to cause the loss of measurement data packets, which has an influence on the reliability of the sensor network. In order to reduce the channel burden and the impact of packet loss, this paper proposes a dimension compression method to compress the measurement information into a scalar. Then, the state estimation algorithm is designed, the convergence of the state estimation algorithm is analyzed, and the convergence condition is obtained. Under three different packet loss rates, simulations are carried out to verify the validity of the state estimation method under the measurement information dimension compression method which meets the convergence condition, and it is showed that the proposed algorithm can improve the reliability of network data transmission.

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