1. Introduction

The thermoelectric properties of matter have been drawing the attention of physicists since the first experiments carried out by Seebeck at the beginning of the 19th century. While the properties of bulk materials are already quite well understood [1, 2], the problem of thermoelectricity in confined nanoscale systems still contains issues that need further examination, although these have been intensively researched since the famous publications by Hicks and Dresselhaus [3, 4]. In particular, thermoelectric and spin-thermoelectric properties of strongly correlated quantum dot (QD) systems constitute a field of intensive research [5–17]. It turns out that the understanding of thermoelectric transport properties is not only relevant for possible future applications, but also provides additional information about fundamental interactions and phenomena at the nanoscale. One prominent example is undoubtedly the Kondo effect [18], which in mesoscopic systems has been of great interest for more than two decades [19, 20]. In fact, the Seebeck coefficient for the Kondo quantum dots was not only reliably calculated [21], but also measured [22]. Moreover, the thermopower was also analyzed for double quantum dot (DQD) systems in the isospin Kondo regime, in which the device was shown to work as a minimal thermoelectric generator [23].

In the presence of magnetic field or when the leads are ferromagnetic, the thermoelectric response of the system becomes spin polarized [6, 8]. Spin caloritronic effects of single quantum dots in the Kondo regime have already been studied theoretically [24–26]. Furthermore, the spin-resolved thermoelectricity has also been analyzed in the case of DQD systems, including the weak coupling regime [27] and the case of nonmagnetic leads [28]. In this paper we extend these studies by investigating the spin caloritronic properties of T-shaped double quantum dots strongly coupled to ferromagnetic leads, as sketched schematically in figure 1. Despite the
relative simplicity of the system under consideration, it hosts a variety of interesting many-body phenomena. The screening of subsequent quantum dots gives rise to the two-stage Kondo effect, introducing a cryogenic temperature scale \( T^* \) associated with the second stage of screening [29–32]. On the other hand, the dependence of the Kondo temperature \( T_K \) and \( T^* \) on the DQD level position can lead to Fano-like interference effects [32–38]. These different energy scales, associated with subsequent Kondo screening, can be reflected in the thermoelectric properties of the device [28]. Some aspects of the influence of magnetism on strongly correlated regimes of T-shaped DQDs have also been discussed, mainly in the context of electrical properties, such as linear conductance and current spin polarization. The spin-dependent Fano antiresonance condition in magnetic field [39] and in system with ferromagnetic leads [40] was predicted. Furthermore, the interplay of the two-stage Kondo screening and the ferromagnet-induced exchange field was also studied [41].

The primary goal of the present paper is to analyze the spin caloritronic properties of T-shaped DQD in the case of ferromagnetic contacts. At this point it is worth emphasizing that the presence of ferromagnetic leads is not equivalent to the application of an external magnetic field in the case of DQD with nonmagnetic leads. In fact, we show that the spin caloritronic coefficients are affected, in a very nontrivial manner, by the presence of ferromagnetic correlations. The spin-dependent tunneling results in generation of an effective exchange field, which gives rise to another important energy scale in the problem that conditions the behavior of the spin Seebeck coefficient. We demonstrate that spin polarization of the order of 1% is sufficient to induce a strong spin Seebeck effect in the transport regime where the second stage of screening develops.

Finally, we would like to note that direct observation of the spin Seebeck effect was reported recently in a bulk metallic magnet [42]. In quantum dot systems, experimental evidence remains a challenge. Nevertheless, the closely related spin Peltier effect was observed in a thin metallic layer sandwiched in between two ferromagnets [43]. This setup seems closer to quantum dot geometry; therefore, we believe that our results will, on one hand, stimulate further experimental efforts and, on the other hand, be of assistance in understanding future experimental data.

The paper is organized as follows. In section 2 the model of the device and method used for its solution are explained. The relevant energy scales are outlined in section 3. Main results, concerning the calculated Seebeck and spin Seebeck coefficients are presented and discussed in section 4. Finally, section 5 concludes the paper.

2. Model and methods

The device under consideration consists of two single-level quantum dots in a T-shaped geometry with the first quantum dot (QD1) coupled to external ferromagnetic leads and the second dot (QD2) attached to the first one through the hopping matrix elements \( t \), see figure 1. The system can thus be described by the following two-impurity Anderson Hamiltonian [44],

\[
H = H_{\text{DQD}} + \sum_r H_r + H_{\text{int}}.
\]

The first term corresponds to an isolated DQD and is given by

\[
H_{\text{DQD}} = \sum_{i\sigma} \varepsilon_i n_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_\sigma t(d^\dagger_{i\sigma}d_{i+\sigma} + \text{h.c.}),
\]

where \( n_{i\sigma} = d^\dagger_{i\sigma}d_{i\sigma} \) and \( d^\dagger_{i\sigma} \) creates a spin-\( \sigma \) electron in dot \( i \) with the corresponding energy \( \varepsilon_i \) and \( U \) is the Coulomb interaction parameter in each dot. The ferromagnetic leads are modeled by free-electron Hamiltonian

\[
H_r = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}\sigma} n_{\mathbf{k}\sigma}
\]

(\( r = L \) for left and \( r = R \) for right lead, \( n_{\mathbf{k}\sigma} \) denotes the occupation operator for state characterized by momentum \( \mathbf{k} \), spin \( \sigma \) and lead \( r \), while \( \varepsilon_{\mathbf{k}\sigma} \) is the energy of the corresponding level).

The coupling between the first dot and the leads is described by the tunneling Hamiltonian

\[
H_{\text{int}} = \sum_{\mathbf{k}\sigma} v_{\mathbf{k}\sigma} (d^\dagger_{1\sigma}c_{\mathbf{k}\sigma} + \text{h.c.}),
\]

where \( c_{\mathbf{k}\sigma} \) is the corresponding annihilation operator and \( v_{\mathbf{k}\sigma} \) denotes the respective tunnel matrix element.

We consider the wide-band limit and assume that only \( s \)-waves couple to the electrodes. This allows us to write the spin-dependent coupling

\[
\Gamma_{r\sigma} = \pi \rho_{\sigma}|v_{\mathbf{k}\sigma}|^2
\]

as a constant (\( \rho_{\sigma} \) denotes the normalized spin-resolved density of states of lead \( r \) at the Fermi level), determined by the leads’ spin polarization \( \rho_{\sigma} \). For parallel configuration of the magnetizations of the leads, one then gets

\[
\Gamma_{r\sigma} = (1 + \sigma p_r)|\Gamma_r|/2, \quad \Gamma_{L\uparrow} \equiv \Gamma_{L\downarrow} \equiv \Gamma_{R\uparrow} \equiv \Gamma_{R\downarrow} = (1 + \sigma p)|\Gamma|/2,
\]

where

\[
p_r = (p_L + p_R)/2
\]

is the effective leads’ spin polarization and we assumed \( \Gamma = \Gamma_L \equiv \Gamma/2 \). We note that in the antiparallel magnetic configuration, for left-right symmetric systems, the couplings become spin independent and the transport properties are similar to those in the nonmagnetic case with a polarization dependent factor. On the other hand, when the system is not symmetric, the behavior is the same as in the case of a parallel magnetic configuration with some new coupling strength and effective spin polarization [45]. Therefore, in the following we will consider only the case of parallel magnetic configuration.

Let \( I \) denote the \( x \)-current (\( x = C \) for charge, \( x = S \) for spin, \( x = Q \) for heat). Using the Boltzmann equation approach and assuming a well-defined Fermi level to be the reference

Figure 1. Schematic of the system. The left (L) and right (R) leads are coupled to the main quantum dot (QD1) via spin-dependent couplings \( \Gamma_{L\sigma} \) and \( \Gamma_{R\sigma} \). The second quantum dot (QD2) is directly coupled only to QD1, with a matrix element \( t \). A small voltage \( V \) (correspondingly spin voltage \( V_\sigma \)) shifts (spin-splits) otherwise equal chemical potentials \( \mu_L = \mu_R = \mu \) symmetrically. There is also a temperature gradient \( \Delta T \) applied symmetrically to the system.
point for energy scale, one can derive the linear-response coefficients connecting currents with voltage $V$ and temperature difference $\Delta T$ [1]

$$
\begin{pmatrix}
L_x \\
I_y \\
I_z
\end{pmatrix} = \sigma \begin{pmatrix}
-\frac{e^2}{2}L_{0x} & \sigma^2e^2L_{0x} & -eL_{1x}/T \\
-\sigma e\frac{h}{2}L_{0x} & -\frac{\sigma^2h}{2}L_{0x} & \sigma L_{1x}/T \\
-\sigma L_{1x}/T & -\sigma eL_{1x}/T & \Delta T
\end{pmatrix},
$$

(2)

where $e$ is the absolute value of electron charge,

$$
L_{\sigma x} = -\frac{1}{h} \int \omega^n \frac{\partial f(\omega)}{\partial \omega} T_\sigma(\omega) d\omega,
$$

(3)

$f(\omega)$ is the Fermi–Dirac distribution function, and $T_\sigma(\omega)$ is the spin-resolved transmission coefficient. Henceforth we will also use notation $L_\sigma = L_{n\uparrow} + L_{n\downarrow}$ and $M_\sigma = L_{n\uparrow} - L_{n\downarrow}$.

The transport properties can be calculated from Onsager integrals $L_{\sigma x}$ using equation (2). In particular, the electrical and spin conductances are

$$
G \equiv \partial IL_x|_{V_x=0}/\Delta T= e^2L_0,
$$

(4)

$$
G_S \equiv \partial IS_y|_{V_y=0}/\Delta T= -\frac{\hbar}{2}L_0,
$$

(5)

respectively, where $\partial A|_{x=0}$ denotes partial derivative of $A(x,y)$ with respect to $x$, while the condition $y=0$ is fulfilled. Similarly, the heat conductance is given by

$$
\kappa \equiv \partial IL_y|_{V_y=0}/\Delta T= \frac{1}{T} \left( L_2 - \frac{L_0^2}{L_0} \right),
$$

(6)

where the conditions $I_x=0$ and $V_y=0$ in fact determine $V$ as a function of $\Delta T$. In this paper we focus on Seebeck and spin Seebeck coefficients, denoted respectively by $S$ and $S_S$,

$$
S = G^{-1}\partial IS_x|_{V_x=0, I_y=0} = -\frac{1}{eT} \frac{L_1}{L_0},
$$

(7)

$$
S_S = G_S^{-1}\partial IS_y|_{V_y=0} = -\frac{2M_1}{\hbar T} L_0.
$$

(8)

These are related to the (spin) Peltier coefficient $\Pi = \partial k I_y|_{V_y=0, I_x=0}/\Delta T= 0$ ($\Pi_S = \partial I_y|_{V_y=0})$ by $\Pi_S = S_S G_T$. However, we prefer to study $S_S$ instead of $\Pi_S$, because Seebeck coefficient better captures caloric properties at low temperatures. Finally, we can define the (spin) figure of merit

$$
Z_T = S_S G_T \kappa,
$$

(9)

which is a measure of thermodynamic efficiency, and the corresponding power factor

$$
Q_S = S_S^2 G_S,
$$

(10)

which is related to maximal power of the device and the performance under the fixed flow conditions [46].

The transmission coefficient is proportional to the imaginary part of QD1’s retarded Green function, $T_\sigma(\omega) = -\Gamma \text{Im} \langle \langle d_1^\dagger d_1 \rangle^\text{R}(\omega) \rangle$, which we determine with the aid of the numerical renormalization group (NRG) method [47, 48], building the full density matrix from states discarded during the iteration of the NRG procedure [49, 50]. In calculations we use discretization parameter $\Lambda = 2$ and keep 2048 states at each iteration. To perform the computations, we assume flat densities of leads’ states within the cutoff $D = 2U$ and make a transformation to an even–odd basis [51]. This leads us to an effective single-channel formulation of the problem, where for the parallel magnetic configuration the only parameters corresponding to the conduction bands are those related to an effective one, namely $\Gamma$, $p$ and $D \equiv 1$.

The NRG method allows us to obtain reliable results in the whole parameter space of the model, in particular, at finite temperatures. However, NRG forces us to limit our considerations to the linear response regime, where a single Fermi level can be defined for both leads and, thus, the logarithmic discretization, being a key ingredient of the procedure, is well defined [47].

3. Relevant energy scales

The considered device hosts very rich physics at the manifold of energy scales. This can be seen in particular in the temperature dependence of the electrical conductance presented in figure 2. For decoupled second dot (i.e. for $t = 0$), nonmagnetic leads ($p = 0$) and QD1 energy level in the Coulomb valley ($-U \ll \varepsilon_1 \ll 0$), at temperatures below the Kondo temperature $T_K$, the conduction band electrons screen the spin of the electron occupying QD1. This screening results in an additional resonance in the local density of states of the first dot at the Fermi level, which gives rise to an enhancement of the conductance $G$, see the curves for $t = 0$ and $p = 0$ in figures 2(a) and (b). In the Kondo regime $G$ can achieve the unitary limit $G = 2e^2/\hbar$, if the dot is tuned to the point of the particle–hole symmetry (PHS), $\varepsilon_1 = -U/2$, as is done in figure 2(a). The maximal conductance outside the PHS point is slightly smaller, see figure 2(b). For single quantum dots coupled to ferromagnetic leads, the Kondo temperature can be estimated from a scaling approach [52, 53],

$$
T_K \approx \frac{1}{2} \frac{T_U}{2} \exp \left[ \frac{\pi(\varepsilon + U)}{2T_U} \frac{\text{arctanh}(p)}{p} \right].
$$

(11)

Experimentally, the Kondo temperature is typically defined as the temperature at which $G = G_{\text{max}}/2$. For parameters assumed in figure 2(a) in the case of $p = 0$ and $t = 0$, from the temperature dependence of $G$ we find $T_K \approx 0.32T$.

The coupling between quantum dots results in the emergence of another energy scale, $T^*$, which for relatively weak $\varepsilon \ll \Gamma$ is associated with the screening of the second dot’s spin by the continuum formed by QD1 and leads. This screening manifests itself through a decrease of $G$ for temperatures below $T^*$, see the curves for $t = 1/3$, $p = 0$ in figures 2(a) and
is induced in DQD. It is, one can reasonably define, is in general different in each multi-

\[ \Delta \varepsilon_{\text{ex}} \approx \frac{2p\Gamma}{\pi \log \left| \frac{\delta_1}{\delta_1 + U} \right|}. \]  

Figure 2. Linear conductance \( G \) (dashed lines) as a function of temperature \( T \) calculated for \( \delta_2 = -U/2, \Gamma = U/5 \) and (a) \( \delta_1 = -U/2 \), (b) \( \delta_1 = -U/3 \). Solid lines indicate the rescaled and shifted thermal conductance, \( \tilde{\kappa} = L_0^{\text{eff}} \kappa(\alpha T)/\kappa(T) \) with \( \alpha = 2 \) (see text for details). In (a) the curves for \( p = 0 \) and \( p = 0.01 \) (both for \( t = \Gamma/3 \)) are on top of each other.

(b). At the PHS point the conductance drops to 0 as \( G \propto T^2 \) [30], while outside this point some finite conductance remains even in the \( T = 0 \) limit, see figure 2(b). The temperature at which the second stage of screening takes place can be estimated from [30, 37]:

\[ T^* = a T_K e^{-b_{\text{eff}}/k_B T}, \]  

where \( L_0^{\text{eff}} = 4U^2/\pi U^2 - (\delta_1 - \delta_2)^2 \) is the effective exchange interaction between the dots and \( a,b \) are numbers of the order of 1. However, similarly to \( T_K \), we estimate \( T^* \) numerically from the temperature dependence of \( G \), as the temperature at which the conductance drops to half of its maximum value. For parameters assumed in figure 2(a), \( p = 0 \) and \( t = \Gamma/3 \), \( T^* \approx 5.9 \cdot 10^{-4} \).

Unlike in the case of the magnetic field, the influence of the leads’ ferromagnetism on transport properties of the system differs significantly, depending on the presence or lack of particle-hole symmetry in the system. At the PHS point, \( \delta_1 = \delta_2 = -U/2 \), the leads’ spin polarization only slightly modifies \( T_K \) and \( T^* \), see equations (11) and (12), which also causes some minor change in \( G_{\text{max}} \), see figure 2(a). However, outside the PHS point this influence is much more pronounced, as can be seen in figure 2(b). Even relatively low values of spin polarization (see the curve for \( p = 0.01 \)) block the second stage of Kondo screening, while the value of \( p = 0.5 \) is sufficient to suppress the Kondo effect significantly for \( \delta_1 \) considered in the figure. This can be inferred from the strongly reduced maximum value of the conductance and the deviation of the curve corresponding to \( p = 0.5 \) from all the other curves at temperatures of the order of \( T_K \), see figure 2. The suppression of the Kondo effect for large \( p \) is caused by the fact that, in the case of ferromagnetic leads, the renormalization of double quantum dot energy levels due to the hybridization with electrodes becomes spin-dependent, which implies that an effective exchange field \( \Delta \varepsilon_{\text{ex}} \) is induced in DQD. It is worth noticing that \( \Delta \varepsilon_{\text{ex}} \) is in general different in each multi-

\[ \Delta \varepsilon_{\text{ex}} \approx \frac{2p\Gamma}{\pi \log \left| \frac{\delta_1}{\delta_1 + U} \right|}. \]  

The determination of the exchange field in the second dot, denoted by \( \Delta \varepsilon_{\text{ex}}^\text{OQD} \), is a more subtle problem \([40, 54]\). Nevertheless, for \( t \ll \Gamma \), \( \Delta \varepsilon_{\text{ex}}^\text{OQD} \) can be seen as a consequence of coupling between QD2 and the continuum formed by QD1 and the leads. The effective spin-dependent coupling to the second dot \( \Delta \varepsilon_{\text{ex}}^\text{OQD} \) is then proportional to \( t^2/L_s = (1 - \alpha p) L_s \), with \( L_s = (L_{12} + L_{21})/2 \), instead of simply \( L_s \) as in the case of QD1. Note that \( L_s \) is a function of both \( t \) and \( p \), and the effective spin polarization equals \( -p \). Consequently, while the coupling to one of the spin species is larger in the first dot, it can be just opposite in the second dot, which implies that \( \Delta \varepsilon_{\text{ex}}^\text{OQD} \) and \( \Delta \varepsilon_{\text{ex}}^\text{QD1} \) can have different signs \([40]\). Note that a similar situation cannot be reached by applying external magnetic field, which will have the same sign in both quantum dots. By raising the exchange field, detuning from the PHS point by changing either \( \delta_1 \) or \( \delta_2 \) will generally suppress the second stage of the Kondo effect once |\( \Delta \varepsilon_{\text{ex}} \)| \( \gtrsim T^* \) [41]. Moreover, it can also affect the first-stage Kondo effect if |\( \Delta \varepsilon_{\text{ex}} \)| \( \gtrsim T_K \).

In the strong coupling regime and for \( p = 0 \), the modified Wiedemann–Franz law was predicted \([21, 28, 55]\), which states that at \( T < T_K \), \( \mathcal{L} = \kappa(\alpha T)/\kappa(T) \) is a constant, instead of the Lorentz number \( \kappa(T)/\kappa(T) \). The value of this constant equals \( \mathcal{L}_0 = (\pi^2/3)k_B^2e^2 \), while the scale shift \( \alpha \) was estimated to be approximately equal 2. Despite the fact that finite leads’ spin polarization significantly changes the fixed point structure of the renormalization group flow, we found the same behavior in this system, although with slightly worse accuracy. This is illustrated in figure 2, where rescaled and shifted S hall conductance \( \tilde{\kappa}(T) \equiv L_0^{\text{eff}} \kappa(2T)/\kappa(2T) \) is plotted as a function of \( T \) with solid lines. At \( T \lesssim T_K \), all curves overlap to good accuracy with \( G(T) \), which implies that the modified Wiedemann–Franz law also holds in the case of T-shaped DQDs with ferromagnetic contacts.

4. Thermopower and spin thermopower

In this section we present and discuss the results on the Seebeck and spin Seebeck coefficients. First, we study their
temperature dependence and then analyze what happens when the degree of the leads’ spin polarization is varied. Finally, we consider the dependence of thermoelectric coefficients on the position of DQD energy levels.

4.1. Temperature dependence

The full temperature dependence of (spin) thermoelectric coefficients is presented in figure 3 for different values of leads’ spin polarization $p$. We cover there a wide class of ferromagnetic materials, starting with the nonmagnetic case and ending with half-metals, for which $p \rightarrow 1$. This figure was calculated for $q_1 = -U/3$ and $q_2 = -U/2$, i.e. outside the PHS point, since for $q_1 = q_2 = -U/2$, the thermopower vanishes due to equal contributions from electron and hole processes. For nonmagnetic systems, the second stage of Kondo effect is mainly responsible for the enhanced transmission for $\omega > 0$. This is visible as a gradual enhancement of $S$ at very low temperatures of the order of $T^2$ [28]. For finite spin polarization, however, a suppression of the second stage of Kondo effect by the exchange field occurs, see figure 2, which suppresses the thermopower peak at $T < T^*$, see figure 3(a). Clearly, leads’ polarization, even as small as $p = 0.01$, is sufficient for the low-temperature peak in $S(T)$ to be strongly suppressed. This is due to the fact that even very low values of $p$ give rise to finite exchange field, see equation (13), which for $p = 0.01$ can already become larger than $T^*$. In a similar spirit, larger values of spin polarization resulting in greater exchange field can affect thermopower behavior at higher temperatures. Interestingly, for $T \approx T_K$ one can then observe a more subtle interplay between the Kondo correlations and the exchange field. For $p < 0.5$, $S(T)$ exhibits a dip with $S(T) < 0$ at $T \approx T_K$, which is characteristic of the (single-stage) Kondo effect [21]. On the other hand, with increasing spin polarization, thermopower changes sign and a positive peak appears instead, see the curves for $p \geq 0.9$ in figure 3(a). This can be explained as follows.

For $T^* < T < T_K$ and $p = 0$, there is a Kondo peak visible in the total transmission coefficient, $T(\omega) = \sum_p T_p(\omega)$, as can be seen in figure 4, which presents the energy dependence of $T(\omega)$ for different values of spin polarization $p$. Because $q_1 > -U/2$, the Kondo peak displays some asymmetry with respect to the Fermi energy ($\omega = 0$). In fact, finite temperature, which is slightly below $T_K$, results in a small shift of the maximum to $\omega > 0$. Because of that, $T(\omega)$ has a finite slope at $\omega = 0$, which is responsible for nonzero thermopower of the device. For $p \neq 0$ the exchange field appears, which grows with increasing $p$. Thus, for sufficiently large spin polarization, $\Delta \varepsilon_{ex}$ can become larger than $T_K$. If this is the case, the Kondo peak becomes suppressed and split by $2\Delta \varepsilon_{ex}$, see figure 4. Moreover, with increasing spin polarization, the levels of DQD become split and the weight of the transmission coefficient becomes shifted to negative energies. This is visible as a gradual enhancement of the negative-$\omega$ Hubbard peak. For very large spin polarization, due to the factors $(1 \pm p)$, the major spin states are mainly responsible for the enhanced transmission for $\omega < 0$. The above-described behavior results in a sign change of the derivative of $T(\omega)$ at $\omega = 0$ with increasing $p$, which gives rise to an enhancement of $S$ at very low temperatures of the order of $T^2$ [28].
to the associated sign change of the Seebeck coefficient visible in figure 3(a).

One could imagine a similar situation for $T \approx T^*$, with a dip in the transmission coefficient corresponding to the second stage of screening being split by $\Delta \epsilon_{\text{ex}}$. However, because $\Delta \epsilon_{\text{ex}}$ becomes larger than $T^*$ at very small values of spin polarization, e.g. at $p = 0.01$ for parameters assumed in figure 3, the difference between $(1 - p)$ and $(1 + p)$ factors in the spin-resolved transmission coefficient is not significant. For this reason the relative depth of dips remains approximately constant and we do not observe a negative peak at $T \approx T^*$ for any value of spin polarization considered in figure 3. However, as presented in section 4.2, a sign change of thermopower in the second stage of screening may occur for $p \approx 0.02$ and is even more pronounced for $\epsilon_1 = -U/4$ instead of $\epsilon_1 = -U/3$.

The temperature dependence of the power factor corresponding to the Seebeck coefficient shown in figure 3(a) is presented in figure 3(b). It exhibits local maxima for temperatures corresponding to peaks (or dips) visible in $S$, including a small peak at $T \approx U$, associated with thermally excited hole-like (due to $\epsilon_1 < 0$) transport. The inset in figure 3(b) displays the thermoelectric figure of merit $ZT$ as a function of temperature. It also exhibits all the peaks of $S(T)$, however, the contributions at intermediate temperatures, $T^* < T < T_K$, are somewhat suppressed by quite large heat conductance; see in particular the curve for $p = 0.01$ in figure 3(b).

We now move to the discussion of spin thermoelectric properties of the considered device. A large conventional thermopower present at $T \approx T^*$ gives a hope that breaking the spin-reversal symmetry by finite lead spin polarization will generate a considerable spin thermopower. However, generation of $S_3$ at such low temperatures is a matter of a delicate compromise. This is because while the spin Seebeck coefficient can be generally enhanced by increasing spin polarization, the second stage of screening and, consequently, the conventional thermopower become strongly suppressed if $p$ is too large, as already explained in the discussion of figure 3(a). Nevertheless, as can be inferred from figure 3(c), there are such values of spin polarization for which the symmetry is sufficiently broken and a maximum in $S_3(T)$ appears, see the curves for $p = 0.001$ and $p = 0.01$. We note that $\hbar/2 \cdot S_3^{\text{max}} < e S_3^{\text{max}}$ for $p = 0.001$ ($S_3^{\text{max}}$ denotes the maximal value of $S_3(T)$ for a given value of $p$), while for $p = 0.01$ the opposite inequality holds. Indeed, in the latter case the spin thermopower exceeds the conventional thermopower.

With increasing the degree of spin polarization of the leads, the maximum in $S_3$ moves to larger temperatures and for $p \geq 0.5$ the spin Seebeck coefficient exhibits a peak at $T \approx T_K$, see figure 3(c). Contrary to the case of conventional Seebeck coefficient, the peak at $T \approx T_K$ has the same sign as the low-temperature peak for small spin polarization. This can be surprising, because the peak in $S(T)$ changes sign when $T$ increases from $T \approx T^*$ to $T \approx T_K$, see figure 3(a). However, in the case of spin Seebeck coefficient one needs to keep in mind that, for assumed parameters, the exchange field in QD1 is opposite to the exchange field in QD2, which compensates for this effect.

One can be surprised that $p$ of the order of one percent is sufficient to induce a significant spin Seebeck coefficient. However, it is advisable to recall, that $S_3$ is in fact a ratio of a small spin bias $V_S$ and a small temperature gradient $\Delta T$. Thus, even the largest value of $S_3$, despite its fundamental aspects, does not guarantee a practical importance of the result, if the corresponding power factor $Q_S$ is too small. For this reason in figure 3(d) we show the temperature dependence of $Q_S$. It can be clearly seen that the peak in $Q_S(T)$ corresponding to $p = 0.001$ is a few times smaller than the peak corresponding to $p = 0.01$. This remains in agreement with intuition, that a lead spin polarization with degree much smaller than 1% cannot induce a significant spin current (although this very small spin current can still be much larger than the corresponding charge current). On the other hand, the peak of $Q_S(T)$

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Figure 5. Thermopower (a), spin thermopower (b) and the corresponding power factors, (c) and (d), plotted as a function of spin polarization $p$ for different values of the level position of the first quantum dot, as indicated in the figure. The other parameters are the same as in figure 3 with $T = 10^{-4}T$. 
for \( p = 0.01 \) is larger than the peaks corresponding to stronger lead polarization. This implies that the thermoelectric performance of the considered device is best in the regime of the second stage of screening. Nevertheless, the corresponding spin-thermoelectric figure of merit \( Z_s \) which is plotted in the inset of figure 3(d), is not too spectacular, with values only slightly exceeding 0.05 \( \cdot (2e/h) \).

4.2. Dependence on the leads’ spin polarization

In this section we analyze how the spin thermoelectric properties depend on the magnitude of the exchange field, focusing on the second stage of the Kondo effect. We thus assume the same parameters as in the previous section and set \( T = 10^{-4} \), which is of the order of \( T_c \), and study the dependence on spin polarization for different values of QD1 level position. According to equation (13), the exchange field is linear in \( p \) and also in \( q_1 \) near the PHS point, since \( \log(a/(a + U)) \approx 4(a + U/2)/U \). As can be seen in figure 5(a), which displays the dependence of \( S \) on \( p \), the Seebeck coefficient in the regime of small spin polarization is a nonincreasing function of \( p \) (note the logarithmic scale for \( p \) in the plot). At sufficiently low spin polarization, \( S \) retains its value for a nonmagnetic system. However, for any \( q_1 \) there is some critical value of \( p \), which we denote \( p_c \), above which the Seebeck coefficient becomes suppressed. This critical polarization decreases monotonically with increasing detuning from the PHS point, and is related to some critical value of the exchange field, \( \Delta s_{ex} \approx T^* \), overcoming the second stage of the Kondo effect. As can be seen in figure 5(a), the height of \( S(p) \) maximum depends on \( q_1 \) in a nonmonotonic manner. This suggests a nontrivial dependence of \( S \) on \( q_1 \), which is explained in section 4.3.

More than in figure 5(a) we can also notice a small sign change of \( S(p) \) for \( q_1 = -U/4 \) and \( q_1 = -U/3 \). This is in fact a consequence of the same phenomenon as that responsible for the sign change of \( S(T) \) for \( T \approx T_c \) described in section 4.1. The main difference is that the dip in the transmission coefficient, corresponding to the second stage of the Kondo screening, more easily gets smeared, than split. For this reason, the negative peak of \( S \) is rather small and develops only in a narrow range of parameters, see figure 5(a).

We also note that very close to the PHS point, one can observe a large peak in \( S(p) \), see the curve for \( q_1 = -0.499U \) in figure 5(a). This result, however, may be considered somewhat artificial. According to equation (7), \( S \) is proportional to the ratio of \( L_3 \) and \( L_0 \). Exactly at the PHS point, \( L_3 \) is always 0 while \( L_0 \) decreases with temperature as \( T^5 \). Moreover, \( L_0 \) is a symmetric function of the detuning from the PHS point, while \( L_3 \) is an anti-symmetric function. Thus, for \( |q_1 - U/2| \gtrsim T \), when \( L_0 \) and \( L_3 \) are set by detuning, \( S \) may reach really large values. The role of spin polarization is here to split the transmission coefficient dip and cause \( T(\omega) \) to possess a finite slope at \( \omega = 0 \), which additionally enhances \( L_1 \). However, despite large value of \( S \), the system does not conduct in this regime (neither heat, nor current, nor spin), so the result is not really physically interesting. This is confirmed by the values of \( Q \), which are presented in figure 5(c). While for \( q_1 > -0.49U \), the peaks of \( S(p) \) correspond to the peaks of \( Q(p) \), this is not the case for \( q_1 = -0.499U \); \( Q \) is not enhanced in the regime of large \( p \).

The dependence of the spin Seebeck coefficient \( S_s \) on \( p \) is presented in figure 5(b). It significantly differs from the \( p \)-dependence of \( S \), since \( S_s \) vanishes for \( p = 0 \). In fact, \( S_s(p) \) exhibits a peak, whose position varies with \( q_1 \) by a few orders of magnitude. The height of the peak increases when \( q_1 \) approaches the PHS point. The existence of this peak is a consequence of a balance between the exchange field and the second stage of screening. If \( p \) is small enough, spin-reversal symmetry is approximately preserved and \( S_s \approx 0 \). On the other hand, large values of spin polarization result in strong exchange field, which destroys the second stage of the Kondo effect, and thus decrease the spin caloritronic effects.

![Figure 6. Spin thermopower (a), the corresponding power factor (b) and the spin-thermoelectric figure of merit (c) as a function of \( q_1 \) for different values of hopping between the two leads, as indicated. The parameters are the same as in figure 3 with \( T = 10^{-4} \) and \( p = 0.5 \). The inset shows peaks of the spin thermopower at lower \( T = 10^{-1} \).](image-url)
The exchange field can also be changed by tuning $\varepsilon_1$, which allows for moving the peak in $S_S(p)$ to the desired range of $p$. The flexibility of the device upon this kind of tuning is reduced by the power factor corresponding to spin thermoelectric effects, $Q_S$, which is shown in figure 5(d). $Q_S$ as a function of $p$ exhibits peaks corresponding to those present in $S_S(p)$ for all values of $\varepsilon_1$ considered. The height of these peaks is the largest for $U_0 = 0.49 \Delta / T_0$ and drops significantly for $U < 0.011 \Delta / T_0$. This is associated with the suppression of the conductance already discussed in the case of conventional Seebeck coefficient. Moreover, as can be seen in figure 5, for a finite value of spin polarization, there exists such a value of $\varepsilon_1$ for which large peaks in $S_S(p)$ and $Q_S(p)$ occur.

4.3. Dependence on the position of QD1 energy level

As follows from figure 5, the dependence of $S_S$ on $\varepsilon_1$ for large $p$ is quite sharp. This is related to Fano-like interference, which occurs between transport paths through a weakly coupled molecular state of DQD that is a resonant one and another, strongly coupled state serving as the background [33, 36–38].

To shed more light on this behavior, in figure 6 we now plot the full $\varepsilon_1$-dependence of $S_S$ for fixed $p = 0.5$ and the other parameters the same as in figure 5. In this figure we also study the influence of different hopping between the dots $t$, which strongly affects the formation of molecular states in DQD and, thus, strongly influences the interference effects.

The dependence of $S_S$ on $\varepsilon_1$ calculated at $T = 10^{-4} \Gamma$, i.e. for temperature corresponding to that used in figure 5, is shown as an inset to figure 6(a). However, in this case for the considered range of $\varepsilon_1$ the spin-thermoelectric power factor $Q_S$ is quite small, as explained in the previous section (not shown in the plot). Moreover, it becomes even more suppressed with increasing $t$. For this reason, the main results shown in figure 6 are calculated at larger temperature, $T = 10^{-3} \Gamma$, which is of the order of $T^*$ for $t = \Gamma / 3$. At this temperature, outside the PHS point, the conductance is not yet fully suppressed due to the second stage of Kondo effect, and $Q_S$ values are larger, as can be seen in figure 6(b).

At first sight, one can immediately notice a striking qualitative similarity between the curves shown in figure 6(a) and those in the inset. A closer look, however, reveals some differences. First of all, the two plots have different scales for the horizontal axis. It turns out that the sharp interference peaks get broadened with increasing temperature. Moreover, the width of those peaks scales approximately linearly with $T$, while the maximal value of $S_S$ is rather independent of temperature. We also note that $S_S$ is anti-symmetric around the PHS point, which is caused by the corresponding sign change of the exchange field $\Delta \varepsilon_{ex}$ around this point.

The spin-thermoelectric power factor as a function of $\varepsilon_1$ is shown in figure 6(b). One can clearly see that $Q_S$ is optimized for $t = \Gamma / 3$ and only in this case reaches considerable values. For smaller values of hopping $t$, the temperature considered in figure 6(b) is above $T^*$ and $S_S$ is not enhanced. On the other hand, for larger hoppings, $T \ll T^*$ and the conductance is generally blocked by the second stage of screening. Since $Q_S$ must be sufficiently large for any measurement to be possible,
one should not overestimate the meaning of large spin-thermolectric figure of merit. With this in mind, let us analyze figure 6(c), which presents $ZST$ as a function of $S_3$.

As can be seen in the figure, $ZST$ exhibits maxima for those values of $S_3$ for which $|S_3|$ has peaks. Due to the square dependence of $ZST$ on $S_3$, see equation (9), the differences in peak heights are now more prominent than in the case of $S_3$. The influences of thermal and electrical conductance compensate each other. The maximal $ZST$ equals $0.5 \cdot 2e/\hbar$, which is quite large, see figure 6(c). However, it occurs for strong $t$, for which $Qs$ is rather low and the measurement is hardly possible. On the other hand, for $t = \Gamma/3$, corresponding to reasonably large $Qs$, maximal $ZST$ remains of the order of $0.1 \cdot 2e/\hbar$.

Finally, to make the analysis of (spin) thermoelectric properties of our magnetic device complete, in figure 7 we present the thermopowers and the corresponding power factors as a function of temperature and QD1 energy level. One can see that both $S$ and $S_3$ change sign in the PHS point. However, $S$ as a function of $T$ exhibits more sign changes than $S_3(T)$. The regimes of large Seebeck and spin Seebeck coefficients can be clearly identified in the figure. While for $S$ and $Q$ the largest values are obtained at relatively high $T$ and large detunings from the PHS point, $S_3$ and $Q_3$ are maximized for temperatures of the order of $T^*$ and close to (but not at) the PHS point.

5. Conclusions

We have analyzed the thermoelectric and spin-thermoelectric properties of the DQD in a T-shaped configuration, coupled to two leads magnetized in parallel. The calculations were performed in the linear response regime with the aid of the NRG and we focused on the parameter regime where the system exhibits the two-stage Kondo effect. We determined the full temperature dependence of the (spin) Seebeck coefficient, together with the corresponding power factor and figure of merit. We also studied the dependence of the spin caloritronic properties on the degree of spin polarization of the leads, dot level detuning and the strength of hopping between the dots. It was demonstrated that the thermal conductance fulfills the modified Wiedemann–Franz law found previously for nonmagnetic systems. In addition, we showed that the spin Seebeck coefficient can be strongly enhanced in the regime corresponding to the second stage of the Kondo effect. This enhancement is very sensitive to the value of lead spin polarization. Moreover, it can be tuned by changing the DQD parameters, such as level position and hopping between the dots. We also showed that in order to keep the power factor at an experimentally relevant level, one needs to set the temperature of the order of $T^*$. Since $T^*$ strongly depends on $t$, this effect can be tuned by changing the hopping between the dots and the temperature.

We would also like to emphasize that the spin thermoelectric properties of the considered device are very sensitive to the spin polarization of the leads, and even small values of $p$ (of the order of 1%) can induce large spin Seebeck effect. Such a value of spin polarization may be a consequence of current-induced spin accumulation even for very small driving currents. It can also occur in the case of the anti-parallel configuration of lead magnetization for two asymmetrically coupled electrodes. Then, even very small coupling asymmetry changes the effective spin polarization from 0 to a finite value of $p = (\Gamma_D - \Gamma_L)/\Gamma$. It therefore seems quite realistic to expect $p \gtrsim 0.01$ in an experiment, which will cause the conventional Seebeck effect to be strongly suppressed (compared to the nonmagnetic case) and the spin Seebeck effect to be present and even possibly strong. All this implies that the effects studied in this paper may also be relevant for a system in which one would not expect them to appear.

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