Wandzura-Wilczek approximation for the twist-3 DVCS amplitude

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\section*{Abstract}

We present a derivation of Wandzura-Wilczek (WW) like relations for skewed parton distributions. It is demonstrated for photon-pion scattering that the skewed twist-3 parton distributions contributing to the DVCS amplitude have discontinuities at the points $x = \pm \xi$ in the WW approximation. This may lead to a violation of factorisation for the twist-3 DVCS amplitude with transverse polarization of the virtual photon. We show, however, that the contribution of the divergencies to the scattering of a transversely polarized virtual photon affects DVCS observables only at order $1/Q^2$ and can be neglected at twist-3 accuracy. For the scattering of a longitudinally polarized photon the twist-3 amplitude is free of such divergencies.

\section*{Introduction}

Deeply Virtual Compton Scattering (DVCS) \cite{1, 2} is the cleanest hard process which is sensitive to the Skewed Parton Distributions (SPD) and has been the subject of extensive theoretical investigations for a few years. First experimental data became recently available (see e.g. \cite{3, 4}) and much more data are expected from JLAB, DESY, and CERN in the near future. It was demonstrated\cite{5, 6, 7} that in leading order ($1/Q^0$) the DVCS amplitude factorizes. However, as the typical $Q^2$ are by no means large, studies of the power corrections to the DVCS amplitude are very important.

Recently the DVCS amplitude was computed\cite{8, 9, 10} including the terms of order $O(1/Q)$. The inclusion of such terms is mandatory to conserve the electromagnetic gauge invariance of the DVCS amplitude at this order of $1/Q$. Moreover, they provide the leading contribution to some spin asymmetries. To this order the amplitude depends on a set of new skewed parton distributions. Recently it was shown in ref.\cite{10} that in the so-called Wandzura-Wilczek (WW) approximation these new functions can be expressed in terms of twist-2 SPD’s. WW relations for SPD’s were also discussed in ref.\cite{11}; however the authors
of this paper did not take into account operators which are total derivatives. Obviously these operators are crucial for the description of non-forward matrix elements.

In this note we give a derivation of the WW-like relations which is technically slightly different from the approach used in [10], see the Appendix for details. Then we demonstrate for the case of DVCS on the pion the new SPD’s in WW approximation generically possess discontinuities at the points \( x = \pm \xi \). This leads to a formally divergent expression for the DVCS amplitude, which however contributes to the DVCS observables at the accuracy \( 1/Q^2 \) and hence beyond our accuracy.

We also observed that in the twist-3 DVCS amplitude\( ^\dagger \) with longitudinally polarized virtual photons the divergencies mentioned above are cancelled. We discuss the theoretical implications of this phenomenon and its possible experimental verification.

**DVCS on the pion**

The expression for the DVCS amplitude on the pion as obtained in [8] and reproduced in refs. [9, 10] has the form:

\[
T^{\mu \nu} = -\frac{1}{2P \cdot Q} \int dx \left( \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right) \left[ H_1(x, \xi) \left( -2\xi P^\mu P^\nu - P^\mu Q^\nu - P^\nu Q^\mu \right) + g^{\mu\nu}(P \cdot Q) - \frac{1}{2} P^\mu \Delta^\perp_\nu + \frac{1}{2} P^\nu \Delta^\perp_\mu \right]
\]

\[
- \left[ H_3(x, \xi) + \xi x H_A(x, \xi) \right] \Delta^\perp_\mu \left( 3\xi P^\mu + Q^\mu \right)
\]

\[
- \left[ H_3(x, \xi) - \xi x H_A(x, \xi) \right] \Delta^\perp_\mu \left( \xi P^\nu + Q^\nu \right) \right],
\]

where \( P = (p + p')/2, Q = (q + q')/2, \Delta = q - q' = p' - p = -2\xi P + \Delta_\perp, \) while \( p, p', q, q' \) are the initial and final momenta of pion and photon, respectively. The second and third term contain the new twist-3 contributions to the DVCS amplitude. As \( P \) and \( Q \) are longitudinal and \( \Delta_\perp \) transverse the second corresponds to longitudinally polarized virtual photon and third to transverse polarization.

The amplitude depends on new SPD’s defined as\( ^\dagger \)

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x(P_n)} \langle p' | \bar{\psi} \left( -\frac{\lambda}{2} n \right) \gamma^\mu \psi \left( \frac{\lambda}{2} n \right) | p \rangle = P^\mu H_1(x, \xi) + \Delta^\perp_\mu H_3(x, \xi),
\]

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x(P_n)} \langle p' | \bar{\psi} \left( -\frac{\lambda}{2} n \right) \gamma^\mu \gamma_5 \psi \left( \frac{\lambda}{2} n \right) | p \rangle = i\varepsilon_{\mu\alpha\beta\delta} \Delta^\alpha P^\beta n^\delta H_A(x, \xi).
\]

Here the light-cone vector \( n \) is normalized as \( n \cdot P = 1 \)

Recently in ref. [10] it was suggested to use the Wandzura-Wilczek approximation to express functions like \( H_3 \) in terms of twist-2 function \( H_1 \). This approximation gives the kinematical part of twist 3, while the genuine, dynamical, higher twist is neglected. If

\( ^* \) We use the notion of 'twist-3 amplitude' as synonymous to 'amplitude of order 1/Q'

\( ^\dagger \) Note that the function \( H_A \) in ref. [8] is defined with opposite sign
one adopts the operator definition of twist, this approximation, because of neglecting the
operators containing the gluon field strength, corresponds to twist 2. In the Appendix we
give alternative derivation of such relations and in the next section we show that the WW
approximation generically leads to formally divergent expression for the DVCS amplitude.
We also specify the twist-3 helicity amplitudes which are free of such divergencies.

Discontinuities of the twist-3 distributions in WW approximation

In this section we study the properties of the twist-3 distributions in the WW approximation.
We shall see that generically they possess discontinuities at the points \( x = \pm \xi \). We shall
illustrate this by the example of \( H_3(x, \xi) \).

Using the general expression (A.11) discussed in the Appendix it is easy to obtain the
function \( H_3(x, \xi) \) in WW approximation. It can be expressed in terms of the twist-2 SPD
\( H_1(x, \xi) \) as follows (c.f. [10]):

\[
H^{\text{WW}}_3(x, \xi) = \frac{1}{4}\left\{ \theta(x > \xi) \int_{x}^{1} \frac{du}{u - \xi} \left[ \frac{\partial H_1(u, \xi)}{\partial \xi} - \frac{\partial H_1(u, \xi)}{\partial u} \right] + \theta(x < \xi) \int_{-1}^{x} \frac{du}{u - \xi} \left[ \frac{\partial H_1(u, \xi)}{\partial \xi} + \frac{\partial H_1(u, \xi)}{\partial u} \right] \right\}
\]

\[
= -\theta(x < \xi) \int_{x}^{1} \frac{du}{u + \xi} \left[ \frac{\partial H_1(u, \xi)}{\partial \xi} - \frac{\partial H_1(u, \xi)}{\partial u} \right] + \theta(x > -\xi) \int_{-1}^{x} \frac{du}{u + \xi} \left[ \frac{\partial H_1(u, \xi)}{\partial \xi} + \frac{\partial H_1(u, \xi)}{\partial u} \right].
\]

Note that in the limit \( \xi = 0 \) this reduces to an expression, which is similar to the standard
WW approximation for the transverse spin structure function \( g_T \). Note that the specific form
of (4) guarantees the correct symmetry properties of \( H_3 \) [8], which is a sensitive check of our
result. Another interesting limit to discuss is \( \xi = 1 \). In this limit our expression is similar
to the corresponding relation for the light cone amplitude for vector mesons (which is the
closest case worked out in literature, namely by Ball and Braun [17]). The relation (4) also
allow to generalize the WW relation for vector mesons distribution amplitudes obtained in
[17] to the case of a meson of arbitrary spin. This can be done with help of crossing relations
between pion SPD’s and distribution amplitudes of resonances [14].

From eq. (4) one can derive the following relations for the Mellin moments of \( H_1 \) and
\( H_3 \):

\[
\int_{-1}^{1} dx H_3(x, \xi) = -\frac{1}{2} \frac{d}{d \xi} \int_{-1}^{1} dx H_1(x, \xi) = 0,
\]

where the last equation follows from the polynomiality condition for SPDs [2]. This relation
is an analog of the Burkhardt-Cottingham sum rule, it is valid beyond WW approximation.
This particular moment, like the other even moments, corresponds to the non-singlet part
of the SPDs, while the contribution of the singlet part is zero for the trivial reason that
the pion SPD antisymmetric in \( x \). Another interesting relation, which is an analog of the
Efremov-Leader-Teryaev sum rule \cite{16}, can be obtained for the second Mellin moments of the SPD’s, corresponding to the singlet part, appearing in DVCS:

\[
\int_{-1}^{1} dxx H_3(x, \xi) = -\frac{1}{4} \frac{d}{d\xi} \int_{-1}^{1} dxx H_1(x, \xi) = \frac{1}{2} \xi M_2, \quad (6)
\]

where \(M_2\) is a momentum fraction carried by the quark in the pion. To obtain the last equality we used the generalized momentum sum rule for SPDs at \(\Delta^2 = 0\) as derived in \cite{15}. Let us stress that this relation is valid beyond the WW approximation. Its generalizations to the nucleon case was recently discussed in \cite{9}.

With the help of expression (4) we can easily compute the behaviour of \(H_3\) near the points \(x = \pm \xi\). Let us consider the difference of left and right limits of the function \(H_3\) at \(x \to \pm \xi\), the result is:

\[
\lim_{\delta \to 0} \left\{ H_3^{WW}(\xi + \delta, \xi) - H_3^{WW}(\xi - \delta, \xi) \right\} = -\frac{1}{4} v p \int_{-1}^{1} du \left[ \frac{\partial H_1(u, \xi)}{\partial \xi} + \frac{\partial H_1(u, \xi)}{\partial u} \right], \quad (7)
\]

\[
\lim_{\delta \to 0} \left\{ H_3^{WW}(-\xi + \delta, \xi) - H_3^{WW}(-\xi - \delta, \xi) \right\} = -\frac{1}{4} v p \int_{-1}^{1} du \left[ \frac{\partial H_1(u, \xi)}{\partial \xi} - \frac{\partial H_1(u, \xi)}{\partial u} \right], \quad (8)
\]

where \(v p \int \) denotes a principle value integral (\textit{valeur principal}). From these expressions we see that for a wide class of functional forms for \(H_1\) the corresponding function \(H_3\) exhibits discontinuities at the points \(x = \pm \xi\). Even for smooth functions \(H_1\) the discontinuities are non-zero. To clarify this point, it is instructive to integrate by part, imposing the natural condition \(H_1(1, \xi) = H_1(-1, \xi) = 0\). One gets:

\[
\lim_{\delta \to 0} \left\{ H_3^{WW}(\xi + \delta, \xi) - H_3^{WW}(\xi - \delta, \xi) \right\} = -\frac{1}{4} \frac{d}{d\xi} v p \int_{-1}^{1} du H_1(u, \xi) \frac{1}{u - \xi}, \quad (9)
\]

\[
\lim_{\delta \to 0} \left\{ H_3^{WW}(-\xi + \delta, \xi) - H_3^{WW}(-\xi - \delta, \xi) \right\} = -\frac{1}{4} \frac{d}{d\xi} v p \int_{-1}^{1} du H_1(u, \xi) \frac{1}{u + \xi}. \quad (10)
\]

The sum of the two jumps which arises in the convolution integral for the amplitude can be expressed in terms of the real part of the twist-2 DVCS amplitude as follows:

\[
\lim_{\delta \to 0} \left\{ H_3^{WW}(\xi + \delta, \xi) - H_3^{WW}(\xi - \delta, \xi) + H_3^{WW}(-\xi + \delta, \xi) - H_3^{WW}(-\xi - \delta, \xi) \right\}
\]

\[
= -\frac{1}{4} \frac{d}{d\xi} v p \int_{-1}^{1} du H_1(u, \xi) \left\{ \frac{1}{u + \xi} + \frac{1}{u - \xi} \right\}. \quad (11)
\]

The latter expression is nothing else than the derivative of the real part of twist two DVCS amplitude. As DVCS is an observable physical process (and as already been observed) the integral on the right hand side of (11) cannot be zero and because the physics has to show a non-trivial dependence on the skewedness parameter it cannot be independent of \(\xi\). This shows that the r.h.s. of (11) cannot vanish and therefore \(H_3\) really has to show discontinuities.

The expression for the WW-part \(H^{WW}_A(x, \xi)\) of the function \(H_A(x, \xi)\) is slightly different:

\[
H^{WW}_A(x, \xi) = \frac{1}{4\xi} \left\{ \theta(x > \xi) \int_{\xi}^{1} du \frac{\partial H_1(u, \xi)}{\partial u} + \xi \frac{\partial H_1(u, \xi)}{\partial \xi} \right\},
\]
\[ -\theta(x < \xi) \int_{x - 1}^{x} \frac{du}{u - \xi} \left[ u \frac{\partial H_1(u, \xi)}{\partial u} + \xi \frac{\partial H_1(u, \xi)}{\partial \xi} \right] \]
\[ -\theta(x > -\xi) \int_{x}^{1} \frac{du}{u + \xi} \left[ u \frac{\partial H_1(u, \xi)}{\partial u} + \xi \frac{\partial H_1(u, \xi)}{\partial \xi} \right] \]
\[ + \theta(x < -\xi) \int_{-1}^{x} \frac{du}{u + \xi} \left[ u \frac{\partial H_1(u, \xi)}{\partial u} + \xi \frac{\partial H_1(u, \xi)}{\partial \xi} \right] \]  \hspace{1cm} (12)

It can be checked that the function \( H_{WW}^A(x, \xi) \) also possesses discontinuities at the points \( x = \pm \xi \). The expression for these discontinuities has the form:

\[ \lim_{\delta \to 0} \left\{ H_{WW}^A(\pm \xi + \delta, \xi) - H_{WW}^A(\pm \xi - \delta, \xi) \right\} = \pm \frac{1}{4} \frac{d}{d\xi} \int_{-1}^{1} \frac{du}{u \mp \xi} H_1(u, \xi) \]  \hspace{1cm} (13)

Note that the difference of these discontinuities (which is the combination entering the amplitude \( \Pi \)) coincides with \( \Pi \). The presence of discontinuities in the functions \( H_3 \) and \( H_A \) exactly at the points \( x = \pm \xi \) leads to a formally divergent result for the amplitude at the order \( O(1/Q) \), see expression eq. \( \Pi \). This indicates a violation of factorisation in order \( O(1/Q) \) for the DVCS amplitude in WW approximation. The considered violation disappears for virtual final photon, as the pole of the quark propagator no more occurs at the point \( x = \xi \). Thus the production of Drell-Yan pairs does not pose any problems. Experimental studies comparing real and virtual photon production thus provide an excellent opportunity to check factorisation.

It is important to realize that the formal divergencies of the amplitude in the WW approximation are canceled for certain combinations of helicity amplitudes because the jumps in \( H_3 \) and \( H_A \) are related to each other, see eqs. \( \Pi \). Since the divergencies occur only at the point \( x = \pm \xi \) it is sufficient that the jumps cancel at this specific value to save factorisability. Specifically there is no problem for the amplitudes with longitudinal polarization of the virtual photon, because the corresponding amplitude is proportional to the following combination of the functions \( H_3 \) and \( H_A \):

\[ \varepsilon_{\mu L}^{\mu} T_{\mu \nu} \propto \Delta_{\perp, \nu} \int dx \left( \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi - i\epsilon} \right) \left[ H_3(x, \xi) + \frac{\xi}{x} H_A(x, \xi) \right]. \]  \hspace{1cm} (14)

Thus the discontinuities generate divergencies only for the twist-3 DVCS amplitude with transversely polarized virtual photon. The contribution of the "problematic" part of the twist-3 DVCS amplitude (corresponding to scattering of the transversely polarized virtual photons) to the differential cross section is suppressed after the contraction with the polarization vector of the emitted real photon by two powers of the hard scale, i.e. by \( 1/Q^2 \), relative to the leading order result and do not contribute to observables in order \( O(1/Q) \). The DVCS differential cross section to this accuracy gets contributions only from the longitudinal part of the twist-3 amplitude which is free from divergencies.

The physical reason of these peculiarities for the transverse photon case is rather similar to the origin of the famous Callan-Gross relation. The dangerous pole of the quark propagator, which in combination with jumps lead to the violation of factorisation, corresponds to an on-shell quark, which may absorb only transverse photons. We therefore expect that this situation should persist also in the case of nucleon and deuteron targets. This would be interesting to check by an explicit calculations.
The considered relationship between twist-2 and twist-3 terms can also be discussed with regard to electromagnetic gauge invariance. The leading twist-2 term is gauge invariant only if one neglects the transverse component of the momentum transfer $\Delta_\perp$. In $O(\Delta_\perp)$ accuracy it requires the consideration of a quark-gluon diagram, whose contribution by use of the equation of motions is expressed through the twist-2 SPD $H_1$ and the new twist three SPDs $H_3, H_A$. While the $H_1$ term combines with the leading result to a gauge-invariant expression (see two first lines in eq. (1)), as anticipated in ref. [13], the other terms provide the additional contribution which violates factorisation in the WW approximation for transverse polarization of the virtual photon.

Generally, the appearance of the jumps just at the points $x = |\xi|$ is not unnatural from the physical point of view, as this is just a transition point between the regions, where SPD has quite different physical meaning [5], accommodated, in particular, in the two-component model of SPDs [15].

As final remark we note that preliminary estimates of the function $H_3$ at a low normalization point in the instanton model give a function without discontinuities and hence indicates that the WW approximation for SPD’s is not valid within this specific model.

Conclusions

We demonstrated by explicit calculations that Wandzura-Wilczek like relations for SPD’s entering the description of the DVCS amplitude at order $O(1/Q)$ lead to a violation of factorisation for the twist-3 DVCS amplitude with transversely polarized virtual photons. However, one can easily see that the dangerous divergencies do not contribute to DVCS observables at the order $1/Q$ but at the order $1/Q^2$. Therefore these divergencies affect only twist-4 corrections which are beyond the scope of the present paper. One cannot exclude that the kinematical contributions of twist 4 will cancel the considered divergencies. This promising opportunity is suggested by the paper [20], in which a part of the $1/Q^2$ term is identified which makes the contribution of the jumps equal to zero after the contraction with the real photon polarization vector is performed.

The divergencies in the amplitude are related to the fact that the additional functions obtained with the help of WW-like relations generically contain discontinuities at the points $x = \pm \xi$ what in turn leads to formally divergent results for the DVCS amplitude with a transversely polarized photon. However the divergencies contribute to the observables only at the accuracy $1/Q^2$ and can be neglected at twist-3 accuracy. We observed that the divergencies are canceled in the twist-3 DVCS amplitude with a longitudinally polarized virtual photons. Such a cancelation of the divergencies in the longitudinal twist-3 amplitude implies that the DVCS observables up to accuracy $O(1/Q)$ can be estimated using the WW approximation, at least at leading order in $\alpha_s$. The case of DVCS off the nucleon will be studied elsewhere.

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were independently found by A. Radyushkin and C. Weiss [20]. We are grateful to them for the discussions. The work of N.K. was supported by the DFG, project No. 920585. O.V.T. was partially supported by RFFI grant 00-02-16696. N.K. and O.V.T. are thankful to Klaus Goeke for invitation to Bochum University where the idea to write these notes has appeared.

APPENDIX: Derivation of the WW relations

Here we briefly describe the method used to derive the WW relations for skewed parton distributions. Our approach is very close to approach used in [10]. The difference lies only in technical details. We have also been informed that similar results have been obtained independently in [20].

Our starting point is the following equations derived in [19]:

\[
\bar{\psi}(x)\gamma_\mu \psi(-x) = \int_0^1 dt \frac{\partial}{\partial x_\mu} \bar{\psi}(tx)\gamma_\mu \psi(-tx) - i \epsilon_{\mu \nu \alpha \beta} \int_0^1 dt \, t x' D^\alpha \left[ \bar{\psi}(tx) \gamma^\beta \gamma^5 \psi(-tx) \right] - \int_0^1 dt \int_{-t}^t dv \bar{\psi}(tx) \left\{ v i g \tilde{G}_{\mu \nu}(vx) - t \gamma_5 g \tilde{G}_{\mu \nu}(vx) \right\} x' \gamma^\nu \gamma^5 \psi(-tx) \tag{A.1}
\]

and

\[
\bar{\psi}(x)\gamma_\mu \gamma_5 \psi(-x) = \int_0^1 dt \frac{\partial}{\partial x_\mu} \bar{\psi}(tx)\gamma_\mu \gamma_5 \psi(-tx) - i \epsilon_{\mu \nu \alpha \beta} \int_0^1 dt \, t x' D^\alpha \left[ \bar{\psi}(tx) \gamma^\beta \psi(-tx) \right] - \int_0^1 dt \int_{-t}^t dv \bar{\psi}(tx) \left\{ t g \tilde{G}_{\mu \nu}(vx) + v \gamma_5 i g \tilde{G}_{\mu \nu}(vx) \right\} x' \gamma^\nu \psi(-tx) \tag{A.2}
\]

where we do not write out explicitly all the path-ordered gauge factors, and \(D_\alpha\) denotes the total derivative defined as :

\[
D_\alpha \left\{ \bar{\psi}(tx) \Gamma[t x, -tx] \psi(-tx) \right\} \equiv \frac{\partial}{\partial y^\alpha} \left\{ \bar{\psi}(tx + y) \Gamma[t x + y, -tx + y] \psi(-tx + y) \right\} \bigg|_{y \to 0}, \tag{A.3}
\]

with generic Dirac matrix structure \(\Gamma\) and \([x, y] = \text{Pexp}[i g \int_0^1 dt (x - y)_\mu \alpha^\mu(t x + (1-t)y)]\). Note that in the matrix elements the total derivative can be easily converted to the momentum transfer:

\[
\langle p' | D_\mu \bar{\psi}(tx) \Gamma[t x, -tx] \psi(-tx) | p \rangle = i (p' - p)_\mu \langle p' | \bar{\psi}(tx) \Gamma[t x, -tx] \psi(-tx) | p \rangle \tag{A.4}
\]

The general method to obtain Wandzura-Wilczek relations is to take the matrix elements in the LHS and RHS (A.1) and (A.2) and insert their parametrisation. This provides us with a system of equations for the twist three functions like \(H_3\) and \(H_A\). Such a method was effectively also used for the \(\rho\)-meson distribution amplitudes, see [19] [18]. In the case of skewed distributions it is more convenient to solve the system of equations (A.1), (A.2) at the operator level and then take the matrix elements. Operator solution means that one has to express non-symmetrical operator \(\bar{\psi}(x)\gamma_\mu \gamma_5 \psi(-x)\) through the two point symmetrical operators \(\bar{\psi}(x)\gamma_\mu \gamma_5 \psi(-x)\) and three point quark-gluon operators.
Consider as an example the solution for the vector operator. Substituting (A.2) into (A.1) we obtain an equation which contains only one non-symmetrical vector operator:

\[
u \bar{\psi}(ux)\gamma_\mu \psi(-ux) = [(xD)^2 - x^2(D)^2] \int_0^u dt \int_0^u dt' \bar{\psi}(tx)\gamma_\mu \psi(-tx) + \frac{\partial}{\partial x_\mu} \int_0^u dt \bar{\psi}(tx)\bar{\psi}(tx)\gamma_5 \psi(-tx) \int_0^u dt' \bar{\psi}(tx)\gamma_5 \psi(-tx) + \ldots
\]

where ellipses stands for the contributions of the three point quark-gluon operators. It is convenient to rewrite this equation in the compact form:

\[
f(u) = k^2 \int_0^u (u - t) f(t) + \varphi(u)
\]

where we introduced the following notations:

\[
f(u) = u \bar{\psi}(ux)\gamma_\mu \psi(-ux), \quad f(0) = 0,
\]

\[
k = \sqrt{(xD)^2 - x^2(D)^2}
\]

and \(\varphi(u)\) denotes the known terms in the RHS of the (A.3). Our aim is to solve this integral equation and find \(f(u)\). It it easy to see that (A.3) can be reduced to the simple second order differential equation:

\[
f''(u) = k^2 f(u) + \varphi''(u),
\]

with the boundary conditions:

\[
f(0) = 0, \quad f'(0) = \frac{\partial}{\partial x_\mu} \bar{\psi}(0)\bar{\psi}(0)
\]

The solution is

\[
f(t) = e^{-tk} \int_0^t du e^{2uk} \left[ f'(0) + \int_0^u d\alpha e^{-\alpha k} \varphi''(\alpha) \right]
\]

Substituting the definitions (A.7) we obtain an explicit expression. We then simplify the full answer by neglecting the square of the momentum transfer. This corresponds to the formal substitutions \(D^2 = 0, \quad k = xD\). Then after simple manipulations we obtain the following operator equation:

\[
\bar{\psi}(x)\gamma_\mu \psi(-x) = \frac{1}{2} \int_0^1 d\alpha \left\{ \alpha D_\mu \left( e^{-\alpha(xD)} - e^{\alpha(xD)} \right) + \left( e^{-\alpha(xD)} + e^{\alpha(xD)} \right) \frac{\partial}{\partial x_\mu} \right\} \bar{\psi}(\alpha x)\bar{\psi}(\alpha x) + \int_0^1 du e^{(2u-1)(xD)} \int_0^u d\alpha e^{-\alpha(xD)} \frac{\partial}{\partial x_k} \bar{\psi}(\alpha x)\gamma_5 \psi(-\alpha x) + \ldots
\]

where \(\bar{\alpha} = 1 - \alpha\) and ellipses are contributions of the three point quark-gluon operators which we do not write explicitly for the sake of simplicity. This operator relation can be used for the derivation of various WW relations, because in the RHS of the eq. (A.11) only two-point quark operators of twist-2 contribute in this case. Also the general solution (A.10) can be applied for all-order summation of the kinematical power suppressed terms in the DVCS amplitude.
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