Design and Analysis of Novel Folded Optical Multi-Pass Cell

Gang Cheng¹, Ya-Nan Cao¹,²*, Xing Tian¹, Jia-Jin Chen³ and Jing-Jing Wang⁴

¹State Key Laboratory of Mining Response and Disaster Prevention and Control in Deep Coal Mines, Anhui University of Science and Technology, Huainan, China, ²Institute of Environment-Friendly Materials and Occupational Health of Anhui University of Science and Technology, Wuhu, China, ³Anhui Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Hefei, China, ⁴Institute of Atmospheric Science, Fudan University, Shanghai, China

A novel folded multi-pass cell consisting of three non-coaxial mirrors (spherical mirror or plane mirror) is proposed for laser spectroscopy. Three mirrors of the folded multi-pass cell can arrange in V-shape to form a stable non-coaxial multi-pass cell. Furthermore, in order to research the stability of the multi-pass cell under off-axis mirror’s astigmatism circumstance, an equivalent coaxial multi-pass cell and modified ABCD matrix model for the spot pattern of the folded multi-pass cell is proposed, by which a series of the detailed numerical calculations were implemented to analyze the optical path length of the multi-pass cell. Many spot patterns obtained with a high fill factor improve the utilization efficiency of the surface of the mirror and produce a longer total optical path length. The several typical types of folded multi-pass cells consisting of the different mirrors and base lengths were selected to demonstrate the cell’s self-consistent condition and power for a longer-optical path length. Three effective optical path lengths of 49.6, 97.6 and 173.6 m were obtained, respectively.

Keywords: folded multi-pass cell, non-coaxial mirror, modified ABCD matrix, dense spot pattern, long optical path length

INTRODUCTION

The multi-pass cell (MPC) is regarded as an important part of laser absorption spectroscopy for an effective long optical path length (OPL). In addition, due to its real-time online, high sensitivity, high selectivity non-contracted and non-intrusive advantages, the laser absorption spectroscopy is widely used in the fields of atmosphere, environmental pollution, and industrial process, industrial emissions for trace gas chemical composition analysis, and measurement (CH4, CO2, CO, HCHO, N2O, NH3, etc.) [1–10]. Early White cells, Herriot cells, and Chernin cells [11–13] are still used in a laser-based spectroscopic trace gas sensor due to the advantages of the MPC. The White cells, Herriot cells, and Chernin cells have some drawbacks, such as a relatively large volume, low effective utilization areas of mirrors and complex structure, which limits their application in miniaturization instruments. Considering the advantages of the MPC in laser spectroscopy, the design of miniaturized and weight light MPC will be one of the mainstream development trends in the future. For more compact size and longer OPL, recently, many various MPCs have been reported and developed, in which at least one spherical mirror was replaced with an aspherical mirror. In 2013, BélaTuzson et al. [14, 15] developed a toroidal MPC, which consisted of a single piece of reflective toroidal surface forming a near-concentric cavity in a circular configuration with a volume of merely 40 ml, for this new type of MPC, two effective OPLs of 2.2 and 4.1 m were chosen to demonstrate the cell’s suitability. In 2015, Liu et al. [16] designed a novel compact dense-pattern multi-pass cell (DP-MPC) with a 280 ml sampling volume, whose minimum detectable...
Kong et al. [21] reported a new design method for the multi-pass formed on the end mirrors by numerical simulations. In 2019, aspherical MPC a very rich set of exotic spot patterns can be aspherical MPC, in which the focal length of aspherical mirrors length of 4.2 m and dimensions of 4*4*6 cm³. A limitation of laser absorption spectroscopy originates from the small overlap tend to cause difficulties in multi-pass absorption spectroscopy and limit its sensitivity. The optical interference fringes can be effectively suppressed by wavelength-modulation spectroscopy (WMS) and frequency-modulation spectroscopy (FMS) [23].

FIGURE 1 | Transmission of the rays in the folded MPC consisting of three mirrors M₁, M₂, and M₃; the reflection on the M₂, M₃ surface in the x-z plane of a Cartesian coordinate system; R radius of the curvature of the spherical mirror; l₁ l₂, mirror distance; θ inclination angle of the mirror M₂.

OPTICAL SETUP OF THE FOLDED OPTICAL MULTI-PASS CELL

The optical setup for a novel folded optical multi-pass cell is shown in Figure 1. First, the incident ray transmits between two spherical mirrors (M₁ and M₂) and then reflects from the M₂ surface to the spherical mirror (M₃) surface; after that reflects from M₃ to M₂; at the last, the ray is transmitted to M₁ surface to complete a pass count in the structure of the folded MPC.

Compared with the coaxial MPC, the non-coaxial MPC is much more complicated. In order to simplify the analysis of complicated non-coaxial MPC, according to the theory of laser cavity, an equivalent coaxial multi-element MPC is proposed as displayed in Figure 2.

An equivalent cavity consists of two spherical mirrors and a lens that possesses different focal lengths in the meridian plane and sagittal plane due to the astigmatism of the off-axis spherical mirror M₂. In the meridian plane and sagittal plane, the focal length is defined as \( f_{\text{m}} \) = 0.5 \( R \cos (2\theta) \), \( f_{\text{s}} \) = 0.5 \( R \sec (2\theta) \), respectively. If the astigmatism of the off-axis spherical mirror is not considered, a set of beautiful spot patterns on the mirrors cannot be calculated by a numerical calculation model based on the ABCD matrix.

CALCULATION MODEL OF THE SPOT PATTERN OF THE FOLDED OPTICAL MULTI-PASS CELL

In this paper, in consideration of the astigmatism of the off-axis spherical mirror (M₂), the modified 4*4 ABCD matrix describing one complete pass count is proposed, which consists of the standard transmission matrix and the standard reflection matrix of the ray:

\[
M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} d^T & b^T \\ c^T & a^T \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-2}{R} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{-2}{R} & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-2}{R} & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{-2}{R} & 0 & 1 \end{bmatrix}
\]
As follow: the criterion for a stable cell or cavity can be strictly exhibited the spherical mirror mirror as the reference plane, the corresponding eigenvalue judge the stability of the cell is more concise. Taking one MPC is the theoretical study of the self-consistent condition of the FOLDED OPTICAL MPC CONFINEMENT STABILITY OF THE Eqs (4–6).

The criterion for a stable cell or cavity can be strictly exhibited as follow:

\[
\begin{bmatrix}
A - \lambda E & B \\
C & D - \lambda E
\end{bmatrix} = 0
\]

(4)

where \( \lambda \) consists of a pair of real numbers, which means that in a cycle the ray \((x_0, x'_0, y_0, y'_0)\) is at magnification (decrease) of \( \lambda \) times. So, under the self-consistent condition, the ray will diverge (converge) into a point-source of ray in the cavity, which is called an unstable cavity. When \( |A + D| < 2 \), it is concluded that the lambda \( \lambda \) consists of a pair of conjugate complex numbers, whose real part and imaginary part stand for the curvature of the wavefront and the radius of the ray spot, respectively, according to the diagonalization of the complex wavefront. therefore, the optical cavity is stable.

\[
\lambda^2 - (A + D)\lambda + 1 = 0
\]

(5)

\[
\lambda_{1,2} = \frac{A + D}{2} \pm \sqrt{\left(\frac{A + D}{2}\right)^2 - 1}
\]

(6)

CONFINEMENT STABILITY OF THE FOLDED OPTICAL MPC

The theoretical study of the self-consistent condition of the MPC is the first step before designing any optical MPC (choice of the curvature radius, base length of the cell). Based on laser cavity theory, the optical cavity’s eigenvalue method used to judge the stability of the cell is more concise. Taking one mirror as the reference plane, the corresponding eigenvalue lambda \( \lambda \) of the ABCD matrix of the equivalent MPC in the meridian plane or the sagittal plane is determined by Eqs (4–6). The criterion for a stable cell or cavity can be strictly exhibited as follow:

\[
\begin{bmatrix}
A - \lambda E & B \\
C & D - \lambda E
\end{bmatrix} = 0
\]

(4)

When \( |A + D| > 2 \), it is found that the lambda \( \lambda \) consists of a pair of real numbers, which means that in a cycle the ray \((x_0, x'_0, y_0, y'_0)\) is at magnification (decrease) of \( \lambda \) times. So, under the self-consistent condition, the ray will diverge (converge) into a point-source of ray in the cavity, which is called an unstable cavity. When \( |A + D| < 2 \), it is concluded that the lambda \( \lambda \) consists of a pair of conjugate complex numbers, whose real part and imaginary part stand for the curvature of the wavefront and the radius of the ray spot, respectively, according to the diagonalization of the complex wavefront. therefore, the optical cavity is stable.

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\lambda_{1,2} = \frac{A + D}{2} \pm \sqrt{\left(\frac{A + D}{2}\right)^2 - 1}
\]

(6)

OPTICAL ARRANGEMENT OF THE FOLDED OPTICAL MULTI-PASS CELL

The optical arrangement of the folded MPC involving three identical spherical mirrors is shown in Figure 3. As a requirement to get a long OPL, three high reflectivity dielectric-coating spherical mirrors (reflectivity of 99.993% at the wavelength of 1.573 µm) are employed to form a folded MPC. The three identical spherical mirrors have a diameter of (50.8 mm) and a radius of (1,000 mm), which are non-coaxial arrangements, and the arm’s base lengths \( L_1 \) and \( L_2 \) are 150 mm, 200 mm, respectively. The spherical mirror \( M_1 \) and \( M_3 \) are 90° angles to the optical axis respectively, which is equivalent to \( \theta \) with a value of 10° as shown in Figure 1. Around the MPC, there...
are gas-in and gas-out holes for gas change. The spherical mirror M₁ possesses ray-in and ray-out windows as displayed in the mechanical structure drawing Figure 3.

RESULTS AND DISCUSSION

Three typical folded MPC are selected to show, whose eigenvalue \( \lambda \) is the conjugate complex numbers with value of \((-0.28 \pm 0.9600i, -0.28 \pm 0.9600i), (-0.8487 \pm 0.5289i, -0.8377 \pm 0.5461i), (-0.7524 \pm 0.6588i, -0.7393 \pm 0.6734i)\), respectively, which means that those folded MPCs selected to show is stable. On the base of the stable cavity, by adjusting incident ray’s parameters such as initial incident angle, initial entry location on M₁, mirrors separated distance \( (L₁, L₂) \) or distance \( (L₁, L₂) \) between mirrors, some satisfying number of the reflections on \( x-y \) plane of the mirrors can be obtained. Therefore, it is demonstrated that an original folded MPC is powerful to create a long OPL. In the calculation model, with a fixed inclination angle \( \theta \) (10°) of the spherical mirror M₂, the longest OPL is the spot pattern of Figure 6 with a value of 173.6 m. In fact, a ratio of the total OPL to the volume can better reflect the space utilization of the

| Pattern | \((x₀, y₀, z₀)\) (mm) | \((x\,', y\,', z\,)\) | \(L₁\) (mm) | \(L₂\) (mm) | \(R\) (mm) | OPL (m) | V (cm³) | RLV (cm⁻²) |
|---------|------------------|-----------------|------------|------------|----------|--------|--------|----------|
| Figure 4 | (0, 16, -199.5) | (-0.08, 0.008, 2.1) | 2\(^{102}\) | 2\(^{102}\) | 103 | ∞ | 103 | 49.6 | 950 | 5 |
| Figure 5 | (16, 16, 209) | (0.007, 0.007, 2.1) | 2\(^{102}\) | 2\(^{102}\) | 103 | 103 | 103 | 97.6 | 950 | 10 |
| Figure 6 | (15, 16, -155) | (0.008, 0.007, 1.5) | 1.5\(^{102}\) | 2\(^{102}\) | 103 | 103 | 103 | 173.6 | 780 | 22 |

FIGURE 4 | Spot patterns simulated by optical software, (A–C) Spot patterns on spherical mirrors M₁, M₂, and M₃ of three different MPCs, respectively. The initial ray parameters for the spot pattern are listed in Table 1. Pass counts \( n = 61 \) times.

FIGURE 5 | Spot patterns simulated by optical software, (A–C) Spot patterns on spherical mirrors M₁, M₂, and M₃ of three different MPCs, respectively. The initial ray parameters for the spot pattern are listed in Table 1. Pass counts \( n = 122 \) times.
According to the ratio of OPL to V (RLV) in Table 1, the most efficient space utilization is the spot pattern with a value of 22 in Figure 6. Those extraordinary spot patterns result mainly from the astigmatism of the off-axis spherical mirror M₂, which plays an important role in the spot pattern evolution caused by the off-axis spherical mirror’s astigmatism.
When the important ray’s parameter such as the incident location \( (x_0, y_0, z_0) \), incident vector \( (x', y', z') \), mirrors spacing \( (L_1, L_2) \), and curvature radius \( (R) \) is depicted in Table 1 of the spherical mirrors, each spot pattern in Figures 4–6 can evolve more spot patterns to increase the spot density or down. In Figures 4, 6 the pass counts are 61 times, 122 times, and 248 times, respectively. It is found that the number of the reflection of the ray on the \( M_2 \) is twice as much as that on the \( M_1, M_3 \) mirror surface due to the folded MPC consisting of two subs-resonators. In Figures 4–6 the OPL of the folded MPC is 49.6, 97.6 and 173.6 m under different ray parameters shown in Table 1.

In order to validate the calculation model based on the modified ABCD matrix, a comparison of the blue spot patterns in Figures 7A–9A simulated by optical software with the red spot patterns Figures 7B–9B from the modified ABCD matrix model shows that the consistency between the calculated and simulated spot patterns demonstrates the validity of the modified 4*4 ABCD matrix with the astigmatism of the off-axis mirror. It is found that the slight difference between Figures 9A,B mainly results from the ABCD matrix’s paraxial approximation error which is amplified in hundreds of cycles. For non-paraxial rays, the paraxial approximation error will make the spot pattern seriously distorted in multiple cycles.

**CONCLUSION**

In conclusion, a new folded MPC and a modified ABCD matrix with astigmatism are proposed. The spot patterns from the calculation model and optical software show that the new folded MPC is powerful to the long OPL with rich spot patterns, which are very same as the cylindrical and astigmatic MPC. For the new MPC, the excellent ratio of the total OPL to the volume can realize a highly sensitive and compact gas sensor with a low cost due to the use of a common spherical mirror and plane mirror to form two subs-MPCs. Compact or portable MPCs such as the one displayed on paper have many uses in various fields such as climate change, atmospheric monitoring, and medical diagnostics. Further topics of interest include the manufacture and testing of the folded MPC, as well as the investigation of laser beam spot interference and thermal stability of the folded MPC of this type.

**DATA AVAILABILITY STATEMENT**

The original contributions presented in the study are included in the article-supplementary material, further inquiries can be directed to the corresponding author.

**AUTHOR CONTRIBUTIONS**

GC, Y-NC, XT, J-JC, and J-JW contributed to the writing of the manuscript and to the interpretation of results. GC: Conceptualization, Writing-Original draft preparation. Y-NC and XT: Software, Validation. J-JC and J-JW: Writing-Reviewing and Editing.

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