Anapole nanolasers for mode-locking and ultrafast pulse generation

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Nanophotonics is a rapidly developing field of research with many suggestions for a design of nanoantennas, sensors and miniature metadevices. Despite many proposals for passive nanophotonic devices, the efficient coupling of light to nanoscale optical structures remains a major challenge. In this article, we propose a nanoscale laser based on a tightly confined anapole mode. By harnessing the non-radiating nature of the anapole state, we show how to engineer nanolasers based on InGaAs nanodisks as on-chip sources with unique optical properties. Leveraging on the near-field character of anapole modes, we demonstrate a spontaneously polarized nanolaser able to couple light into waveguide channels with four orders of magnitude intensity than classical nanolasers, as well as the generation of ultrafast (of 100 fs) pulses via spontaneous mode locking of several anapoles. Anapole nanolasers offer an attractive platform for monolithically integrated, silicon photonics sources for advanced and efficient nanoscale circuitry.
In recent years, significant efforts have been devoted to realize nanoscale sources of coherent light aiming at filling the gap between photonics and electronics for both classical and quantum applications. Fundamental advances have been recently reported in semiconductor–dielectric heterojunctions, where nanolasers from direct bandgap materials have been demonstrated. Further progress with these materials requires to address many fundamental challenges, originating when the miniaturization of optical circuitry is pushed to the nanoscale. A first problem comes from the diffraction limit of light that introduces significant radiative losses that are detrimental and severely limit the performance of nanoscale devices. A second issue is represented by the efficient coupling of light from nanophotonics sources, whose emission is strongly nondirectional. All these problems originate from the fact that conventional optical sources couple energy to classical radiating states that are all diffraction limited and difficult to control at the nanoscale.

An intriguing idea is to introduce a nanolaser design concept based on states of matter with unconventional radiation properties that can overcome the aforementioned limitations. States with non-conventional emission of energy have been recently demonstrated in specific types of all-dielectric nanoparticles, realized in both silicon and germanium semiconductors. Under specific conditions, the superposition of internal modes of these nanostructures can generate radiation-less states, called anapoles, that are scattering-free and invisible to the propagation of the electromagnetic fields.

A fundamental question is whether it is possible to employ a non-radiating state of matter, such as an anapole, to engineer a coherent light source based on stimulated light amplification. The possibility to achieve nonlinear amplification of an anapole state is counterintuitive, due to the lack of any optical feedback from a scattering-free state that is totally invisible to a far-field observer. However, if such interaction could be successfully triggered, we would have at disposal a new type of source that is not limited by diffraction or any of the aforementioned problems.

By using rigorous first-principle simulations on the quantum electrodynamics of photons, we show that the light–matter interaction in a nanolaser emitting at the anapole frequency gives rise to a surprisingly stable steady state, where light energy is strongly collected within the anapole and evanescently transmitted in a subwavelength area outside the nanostructure. By leveraging on tunnelling effects and mutual synchronization, we illustrate that such ‘anapole nanolaser’ opens a large manifold of applications, ranging from efficient and polarization controlled energy coupling, to ultrafast pulse generation without the need of external design elements. Anapole nanolasers hold the promise to be an ideal energy source for nanoscale optics in silicon-compatible platforms.

Results

Design of an anapole-based nanolaser. To illustrate the concept of anapole-based nanolaser (see Fig. 1), we consider an all-dielectric resonator composed of a semiconductor nanodisk that is optically pumped in the same scheme of a classical laser, with energy decaying in one - or more - indirect transitions to a final energy state (see Fig. 1b). Unlike a conventional laser, however, in our design the stimulated emission (see Fig. 1b red arrow) amplifies an anapole state, which can be identified by a dip in the scattering cross section of the structure (Fig. 1c, blue line). The anapole mode is generated by the superposition of electric and toroidal dipoles, which produces an electromagnetic mode that is confined in the interior of the nanodisk and does not radiate in the far-field (see Fig. 1d). In the scheme illustrated in Fig. 1, the energy down-converted by stimulated emission at the anapole frequency excites electric and toroidal dipoles that interfere destructively in the far-field. Contrary to lasers based on dark modes, this scheme is not based on resonant states, nor does it require non-radiative energy transitions in order to allow the generation of radiation-less states in purely dielectric materials. In contrast to bound states in the continuum (BIC) lasers recently proposed, the anapole laser does not require extended structures and the radiation-less state is excited with no additional radiating components.

We design an integrated anapole source starting from standard In0.15Ga0.85As semiconductor that is obtained by combining InAs and GaAs and is a direct bandgap semiconductor that is silicon compatible for monolithic integration. Following the electrical or optical excitation of carriers, the materials sustain stimulated emission of radiation. Both the optical properties and the emission frequency can be tuned by varying on the molar concentration x of InAs in the compound. Figure 2a illustrates the bandgap energy of In0.15Ga0.85As as a function of the Indium molar fraction x. As shown in the figure, the emission of the material can be tuned in a large bandwidth from 873 nm in the near infrared to 3.6 μm in the infrared. In our analysis, we consider a molar fraction x = 0.15, corresponding to an emission wavelength λa = 948 nm. The value x = 0.15 is a standard concentration of InAs that allows monolithic integration directly on silicon, as reported in ref. 18. In addition to that, In0.15Ga0.85As is characterized by low losses in the proximity of the emission frequency λa as illustrated in Supplementary Fig. 1.

Figure 2b illustrates the scattering cross-section Csc of a nanodisk of In0.15Ga0.85As with height 100 nm and different values of diameter d. The figure shows a region where the scattering of the nanodisk is suppressed in all directions (see Fig. 2b dashed line). Such special region, which extends in the visible and in the near infrared for In0.15Ga0.85As nanodisks, is sustained by the presence of non-radiating anapole states that do not possess far-field emission. This is further confirmed by the multipole decomposition of the electromagnetic fields as a function of the incident wavelength for a nanodisk with diameter d = 440 nm and height h = 100 nm (Supplementary Figs 2–5).

Figure 2d presents the scattering cross-section spectrum for a nanodisk with diameter d = 440 nm and height h = 100 nm, whose anapole wavelength coincides with the emission wavelength of the In0.15Ga0.85As semiconductor at 948 nm (red arrow). This configuration constitutes our design geometry as illustrated in Fig. 1c. To maximize the energy trapped inside the nanodisk for the pump beam, the pump wavelength can be placed on a scattering maximum of the nanodisk (see Fig. 2c, green arrow). In the proximity of the anapole frequency, the scattering of the structure is strongly suppressed within a bandwidth of Δλ ≈ 50 nm. This is larger than the amplification bandwidth of the In0.15Ga0.85As semiconductor that is ~20 nm (see Fig. 2d, red solid area) as experimentally measured in refs 18,34. The anapole state is stable against geometrical changes in the structures, as we verified by calculating the scattering cross-section Csc of the structure for varying nanodisk diameter d and height h (see Supplementary Fig. 6). The formation of anapole states dominate at λa = 950 nm in a wide range of geometrical parameters corresponding to 85 nm ≤ h ≤ 105 nm and 435 nm ≤ d ≤ 475 nm that are well within the precision of optical nano fabrication technologies such as electron beam lithography.

In the case of lattice mismatch between the anapole nanodisk and the substrate, we do not expect in general any perturbation in the optical properties of the system, as long as the effect is limited to few lattice sites. However, to reduce lattice-mismatch non-idealities, the InGaAs layer can be grown by using III–V...
Anapole amplification by stimulated emission of radiation. We investigated the process of amplification of anapoles through stimulated emission by the Maxwell–Bloch finite-difference time-domain (MB-FDTD) (see Methods). This technique has proven to furnish extremely realistic results that have been verified against many experiments\textsuperscript{38–41}. In our simulations, we modeled the exact dispersion curve of the In\textsubscript{0.15}Ga\textsubscript{0.85}As (see Supplementary Fig. 1), and we assumed a Lorentzian-shaped gain profile with a dephasing time of $\tau = 40$ ps, whose value has been experimentally measured in refs 18,34.

Figure 3 summarizes our 3D results for a pumping beam linearly polarized along the $x$ axis and perpendicular to the nanodisk axis. In these simulations we considered a nanodisk of In\textsubscript{0.15}Ga\textsubscript{0.85}As with diameter 440 nm and height 100 nm, exposed to different pumping rates that are measured through the density of carriers $\rho_0$. Figure 3a,b report the intensity and the linewidth of the electromagnetic field of the anapole state amplified inside the nanodisk. The behaviour of the anapole amplitude versus pumping rate (see Fig. 3a,b) is that of a standard laser, with a linear relationship between pumping rate and amplified intensity (see Fig. 3a). As in plasmonic and dielectric nanoparticle lasers\textsuperscript{18,32}, the amplification of anapole states directly follows from stimulated amplification as a balance between field enhancement in the gain material and the system losses, including absorption and near-field energy leakage. The anapole laser also shows an equivalent Schawlow–Townes linewidth (see Fig. 3b) that quickly reaches a stable value of $\sim 2$ nm. This value is in accordance with experiments on lasing emission from InGaAs nanostructures\textsuperscript{18}. The carrier threshold density to lasing, observed in Fig. 3a, depends on the pumping rate $\rho_0$ that is proportional to the population inversion density, or equivalently, to the input pumping rate of the system. By increasing the latter, we increase $\rho_0$ and control the laser threshold condition.

A characteristic time evolution of the $E_x$ field is displayed in Fig. 3a (inset) that shows the reaching of a stable stationary emission state after an initial transient. The corresponding intensity spectrum, reported in Fig. 3b (inset), further confirms that the emission corresponds to the amplification of the anapole state at $\lambda_0 = 948$ nm.

Figure 3c–e illustrates the spatial distribution of the electromagnetic energy of the system at steady state. Here we observe a radically different scenario if compared with a classical nanolaser. The emitted field is in fact totally confined within the nanodisk resonator, with electromagnetic energy decaying exponentially outside the nanodisk region. The lack of far-field emission provides a characteristic signature of the anapole state that is being amplified through stimulate emission, and is the hallmark of a new type of near-field nanolaser that does not radiate in the far field. Inside the nanodisk, we observe that the electromagnetic energy distribution preserves the dipolar shape of the linear anapole state (see Supplementary Fig. 7) that is perpendicular to the polarization of the pumping field $E_p$. We evaluated the correlation between the energy distribution of the anapole state amplified in Fig. 3c–e and that of an ideal anapole in a lossless InGaAs structure, obtaining a correlation amplitude of 96.1%.

We observe that the anapole state is not a resonance of the system, therefore it is not possible to associate intuitive quantities such as quality factors which are employed in the description of resonant laser cavities. Resonant states are in fact observed in the spectral response as Lorentzian energy peaks with full width half maximum equal to $Q_0/\Omega_0$, with $\Omega_0$ being the resonant frequency. An anapole state does not possess a Lorentzian shape and is conversely observed in points of the spectrum where the scattering cross-section vanishes.
This shows that the anapole laser amplifies through stimulated emission an almost ideal radiation-less anapole state. Supplementary Note 1 and Supplementary Figs. 2–5 provide an in-depth analysis of the role of losses in the system.

**Spontaneously polarized nanolasers and photonics couplers.**

The possibility to exploit near-field emission in conjunction with the dipolar nature of the anapole mode opens new pathways to develop spontaneously polarized nanolasers and nanophotonic routers. Polarization control, which is crucial in many linear and nonlinear applications, is quite a challenging task for nanoscale sources and it is in general addressed with the introduction of metals and/or specific geometries. By employing anapole sources we can introduce a new design concept that can be easily integrated into two-dimensional circuits. Figure 4 illustrates our idea, where we consider a nanodisk of In0.15Ga0.85As with 440 nm diameter and 100 nm height placed in proximity of silicon nanowires of width 150 nm and thickness 100 nm. The nanowires are distributed in cross configurations, with the anapole nanolaser placed at the intersection point of the circuit. When the incident beam impinges on the anapole with linear polarization (see Fig. 4a), it excites the corresponding anapole state with localized dipolar field aligned perpendicularly to the pump polarization. This depends on the fact that the energy distribution of the anapole, resulting from the near-field interference between electric and toroidal dipoles, is strongly asymmetric in one spatial direction orthogonal to the input polarization (please see Supplementary Fig. 7). When the anapole is nonlinearly amplified by stimulated emission, the input pump polarization selectively excites the corresponding symmetry direction, leading to different energy...
Figure 4 | Polarization controlled and optical router anapole nanolaser. (a) Working principle of the device based on directional coupling of anapole modes. Light pumping the anapole nanolaser placed in the middle of the circuit amplifies the corresponding anapole mode that tunnels energy in neighboring channels according to the polarization of the input pump. (b) Electromagnetic energy distribution in the nanowire channel at steady state in the case of an $E_x$ polarized pumping beam. (c) Same structure as in (a), but with an In$_{0.15}$Ga$_{0.85}$As spherical nanoparticle replacing the anapole nanolaser (see (e), inset). The nanoparticle has the same radius as the nanodisk and is characterized by a strong resonance at the emission wavelength $\omega_0$. The emission of the nanoparticle is more than one order of magnitude larger than the anapole, as observed by comparing (b, c). However, it does not selectively couple to any channel due to the strongly radiating character of the emission intensity. (d,e) Normalized electromagnetic intensity evolution inside the nanowires. The intensity is normalized to the intensity of the electromagnetic field inside the nanodisk. The intensity coupled in each nanowire channel from the nanospheres (e) is four order of magnitude lower than for the anapole nanolaser (d).

The tunability of the formation of an anapole state in the final anapole state (see Fig. 4a). The electromagnetic field distribution of the anapole couples only into the channels that overlap with the anapole mode, routing light according to the polarization of the input field. Figure 4b reports a volumetric mapping of the spatial distribution of the electromagnetic energy at steady state and illustrate this dynamics. The near-field characteristics of the anapole allows electromagnetic energy to be efficiently coupled inside nanowire channels even of such a small size. To evaluate this functionality more quantitatively, we considered the same scenario with a standard nanolaser, characterized by a classical spherical nanoparticle with diameter $d = 448 \text{ nm}$ and resonance at $948 \text{ nm}$ (Fig. 4e, inset). Figure 4c illustrates the outcome. Due to the strongly radiating nature of the components amplified in the nanoparticle, light scatters almost everywhere without coupling in the nanowire channels. By measuring at the light energy in the nanowire channels, we observe that with an anapole nanolaser we have four orders of magnitude more energy inside the channel with respect to the standard nanoparticle laser (see Fig. 4d,e). Such remarkable result depends on the fact that an anapole nanolaser enables by design near-field energy coupling, a mechanism that is extremely efficient in transferring energy in neighbouring structures. In Supplementary Fig. 8 we considered in detail the comparison with another integrated lasing structure, namely a nanodisk with the same size of the anapole laser but now set to emit at $1,125 \text{ nm}$. At this wavelength, the structure shows a strongly dipolar energy emission that well couples with the fundamental mode of the nanowire channels. In this case the energy couples inside all channels. The energy transferred in the waveguides is 15 times lower that the one observed with the anapole, showing the superior performances of the anapole laser.

The tunability of the formation of an anapole state in the In$_{0.15}$Ga$_{0.85}$As material (see Fig. 2) and the compact geometry of the anapole near-field nanolaser allows a large design flexibility, where multiple anapoles can be optically pumped within hundreds of nm, feeding light energy into complex nanoscale circuits. In this scenario, even the use of a scanning near-field optical microscope nanopip a plasmonic nanoantenna is very challenging, as the macroscopic size of the coupling structure does not easily allow the excitation of selective channels nearby in space.

Anapole mode locking and on-chip ultrafast pulse generation. The evanescent emission of energy from anapole nanolasers opens to new concepts for ultrafast pulse generation that results from a mechanism of mode locking that is based on the near-field synchronization of different anapoles. In photonics, synchronization phenomena that are usually exploited for the generation of optical pulses require expensive setup with specific design elements, such as Q-switching components, saturable absorbers and optical modulators, that cannot be miniaturized to the nanoscale.

Recent experiments reported the observation of mutual synchronization in space of laser sources emitting at the same frequency$^{30–52}$ that can reconfigure the spatial distribution of the emitted energy in the far field. These results open the question of whether it is possible to exploit the mutual synchronization in time of different anapole lasers, designing a new process of mode locking that generates pulses without the need of external design elements than the laser itself.

Figure 5a shows the design principle of our idea, where we consider a linear array of anapole nanolasers with slightly different radii. Due to the radius variation, each anapole possesses a different emission frequency $\omega_n = \omega_0 + n \cdot \Delta \omega$ that lies in the 20 nm amplification bandwidth of the In$_{0.15}$Ga$_{0.85}$As semiconductor (see Fig. 2b). The collective emission from the anapole...
The anapole chain, in this scheme, behaves as a system of weakly coupled, nonlinear oscillators. Nonlinearity is here provided by the presence of the amplifying optical semiconductor. Weak coupling, conversely, arises from the spatial overlapping of the anapole emitted fields (see Fig. 3). Each anapole behaves initially as an independent oscillator at the frequency $\omega_n$ and phase $\phi_n$ as illustrated in Fig. 5b. During the time evolution of the system, the frequencies $\omega_n$ and phases $\phi_n$ interact and are allowed to interact via nearest-neighbour coupling. To understand the dynamical outcomes of these interactions, we developed a simplified model of light–matter interaction by generalizing the theory of coupled laser systems \(^5\) to the case of multiple anapole oscillators. For a realistic anapole laser with nonzero $x$, the frequency of oscillation of each electric field becomes influenced by the carrier population $N_n$. On a timescale smaller than $1/\mu$, no effect is practically observed and the anapole network behaves as a classical Kuramoto model. Conversely, for times larger than $1/\mu$ the synchronization conditions between the anapole oscillators change, with some anapoles getting detuned from the others. Supplementary Figure 9c,d illustrates an example, obtained for $x = 1$ in an ensemble of 10 anapoles.

Synchronization events in the anapole chain open the interesting possibility to control the time dynamics of the laser at the ultrafast scale. When a synchronization event takes place, in fact, the total emission intensity $I(t)$ acquires the temporal profile of a train of pulses, expressed by the superposition of the

$$I(t) = \sum_n A_n e^{i\phi_n(t)} |E_n(t)|^2.$$
harmonics that becomes synchronized:

$$I(t) \propto \left| A_0 \sum_n e^{i\omega_n t + n\delta\omega t} \right|^2 = A_0^2 \left| \frac{\sin(N_n \omega t)}{\sin(\omega t)} \right|^2. \quad (2)$$

The period of the train is $T = 2\pi/\delta\omega$ and the individual pulse width is $\Delta = 2\pi/N_0 \delta\omega$. These quantities depend on the number $N$ and frequency spacing $\delta\omega$ of the anapoles that become synchronized. Following equation (1), the synchronization process is controlled by the pumping rate $P$ and coupling coefficients $C_{mn}$ as shown in Supplementary Fig. 9.

To validate this theoretical picture for a realistic anapole laser, we performed first-principle simulations on the anapole chain. Figure 5d–g shows the lasing behaviour of the anapole array. Figure 5d shows a snapshot of a 3D volume mapping of the energy distribution inside the nanodisk. The pulse profile, as shown in (b), is characteristic of a system with a large number of synchronized frequencies. The intensity in (a) is normalized to the intensity of the electromagnetic field inside the nanodisk.

![Figure 6](image)

Figure 6 | Ultrafast pulse engineering by tailoring the anapole geometry. (a,b) Modified anapole array configuration where the nanoparticles from the symmetric structure in Fig. 5 have been shifted of $6^\circ$ along $y$. (b) Shows a section of the electromagnetic energy steady-state distribution. (c,e) Steady-state electromagnetic intensity inside the waveguide, characterized by the formation of 145 fs long pulses repeating within a 1.65 ps period. The pulse profile, as shown in d, is characteristic of a system with a large number of synchronized frequencies. The intensity in (a) is normalized to the intensity of the electromagnetic field inside the nanodisk.

The dynamical model described by equation (1) of the anapole chain suggests that the number of anapole that becomes synchronized can be changed by acting on the coupling matrix elements $C_{mn}$. This observation is stimulated by the similitude of equation (1) and the dynamics of oscillatory neural networks\textsuperscript{55}. Simple calculations on equation (1) show that in the case where the connection matrix is suitably engineered to possess real eigenvalues $\varepsilon$ with orthogonal eigenvectors, the populations converge to the same value $N_n(t) = -\varepsilon$ and all oscillators tend to synchronize their phases. This case is numerically illustrated in Supplementary Fig. 9e,f.

In the anapole chain, coupling coefficients can be altered by acting on the geometric topology of the array. A thorough analysis on this topic goes beyond the scope of this article, and is deferred to a future specialized work. We will here consider a simple and illustrative case, where we shift by $6^\circ$ the anapole array along the direction $y$ (see Fig. 6a). In this configuration the emission dynamics exhibits the appearance of a larger number of synchronized frequencies in the spectrum (see Fig. 6e), and the stable emission of a pulse train of period 1.65 ps (see Fig. 6c) and pulse duration 145 fs (see Fig. 6d). Figure 6b shows a section along the $z$ plane of the energy distribution in the structure, and well illustrates the mechanism of formation of the energy pulses inside the waveguide channel by the complex interactions among the different anapoles.

By comparing the emission spectra of Figs 5g and 6e we observe that the dynamics of mode locking in the anapole nanolasers involve the selective locking of different spectral frequencies that vary in number and position according to the spatial configuration considered (linear in Fig. 5 versus shifted array in Fig. 6). This result opens interesting possibilities to use complex anapole arrays as sources for future optical integrated neurocomputers\textsuperscript{55}.

The mode-locking dynamics of Figs 5 and 6 is a characteristic feature of anapoles that is not observed in conventional nanoparticle lasers. Supplementary Fig. 10 reports the results for nanoparticles with diameter 448 nm, arranged in the same geometry of Fig. 5. In this case there is no pulse generation: the
collective emission does not mutually synchronize and very little energy is coupled into the nanowire.

**Methods**

**Scattering calculations.** All scattering efficiencies for 3D structures have been computed by means of the freely available package DDSCAT. The interaction between the FDTD electromagnetic fields \( \mathbf{E} \) and \( \mathbf{H} \) and the atoms composing the amplifier is expressed by the polarization response of the material \( \mathbf{P} = \mathbf{P}_{\text{lin}} + \mathbf{P}_{\text{non}} \) that is decomposed into a linear \( \mathbf{P}_{\text{lin}} \) and nonlinear contribution \( \mathbf{P}_{\text{non}} \). The latter is related to the atomic transitions in the amplifier by the following set of MB equations:

\[
\begin{align*}
\frac{\partial \mathbf{E}}{\partial t} &= -\frac{i}{\hbar} \mathbf{H} - \frac{1}{\mu} \nabla \times \mathbf{E}, \\
\frac{\partial \mathbf{H}}{\partial t} &= \frac{i}{\hbar} \mathbf{E} + \left( \mathbf{P}_{\text{lin}} + \mathbf{P}_{\text{non}} \right), \quad \left\{ \begin{array}{l}
\mathbf{P}_{\text{lin}} = \text{eq} \mathbf{N}_l \left[ \mathbf{S}_l + \mathbf{S}_d + \mathbf{S}_a \right], \\
\mathbf{S}_d = \sum \mathbf{P}_{\text{lin}} - \frac{1}{\gamma} \left( \mathbf{S}_l - \mathbf{S}_d \right), \\
\mathbf{P}_{\text{non}}(t) = e\gamma t \int (t' - t') \mathbf{E}(t') dt'
\end{array} \right. \\
\end{align*}
\]

where \( \hbar \) is the quantum displacement length, \( \gamma \) is the electric charge, \( \mathbf{N}_l \) represents the density of polarizable atoms in the excited state and \( \gamma \) are the atomic decaying constant rates. In equation (3), \( \mathbf{S}_d \) is the dipole component of the Bloch coherence vector \( \mathbf{S} \) that groups different complex-valued elements of the density matrix \( \rho_{\text{non}} \) into a real-valued vector state. A detailed derivation of equation (3), including the expressions for the coupling \( \Gamma_m \) and Hamiltonian \( \mathbf{H} \) components can be found in refs 57,58. As discussed in ref. 41, our MB implementation explicitly considers a realistic source of quantum noise, while the density of excited atoms \( \mathbf{N}_l \) plays the role of an external pumping rate. The optical properties of InGaAs were taken from published experimental data. The dephasing time for the active levels of InGaAs is set as \( \tau = 40 \) fs, following experimental measurements available in the literature. Material dispersion was included in the FDTD algorithm by employing a representation based on a multipolar expansion that correctly represents the media response at visible wavelengths.

**Data Availability**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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**Author contributions**
J.S.T.G. and A.E.M. carried out scattering calculations. J.S.T.G. performed first-principle parallel simulations. Y.S.K. and A.F. supervised the research. All authors contributed to the analysis of data. All authors contributed equally to manuscript preparation.

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