Both unitary chiral theories and lattice QCD simulations show that the $DK$ interaction is attractive and can form a bound state, namely, $D_{s0}^+(2317)$. Assuming the validity of the heavy antiquark-diquark symmetry (HADS), the $\Xi_{cc}$ interaction is the same as the $DK$ interaction, which implies the existence of a $\Xi_{cc}\bar{K}$ bound state with a binding energy of 49 – 64 MeV. In this work, we study whether a $\Xi_{cc}\Xi_{cc}\bar{K}$ three-body system binds. The $\Xi_{cc}$ interaction is described by exchanging $\pi$, $\sigma$, $\rho$, and $\omega$ mesons, with the corresponding couplings related to those of the $NN$ interaction via the quark model. We indeed find a $\Xi_{cc}\Xi_{cc}\bar{K}$ bound state, with quantum numbers $J^P = 0^-$, $I = \frac{1}{2}$, $S = 1$ and $C = 4$, and a binding energy of 80 – 118 MeV. It is interesting to note that this system is very similar to the well-known $NN\bar{K}$ system, which has been studied extensively both theoretically and experimentally. Within the same framework, we show the existence of a $NN\bar{K}$ state with a binding energy of 35 – 43 MeV, consistent with the results of other theoretical works and experimental data, which serves as a consistency check on the predicted $\Xi_{cc}\Xi_{cc}\bar{K}$ bound state.

I. INTRODUCTION

In 2007, the LHCb Collaboration reported the observation of a doubly charmed baryon, the $\Xi_{c}^{++}$ [1]. Its quark content is $ccu$, where it is interesting to note that we expect the $cc$ charmed quark pair to be tightly packed together. The theoretical reason behind this is heavy antiquark-diquark symmetry (HADS) [2], a type of heavy-quark symmetry stating that a heavy-quark pair behaves approximately as a heavy antiquark. In practical terms what this means is that the structure of the doubly heavy baryon is the same as the one of a heavy antimeson, i.e. the wave function of the light-quark within the $\Xi_{cc}$ baryon is the same as that in the $\bar{D}^{(*)}$ meson (modulo corrections owing to the finite charm quark mass). Consequently, HADS also implies that many of the findings related to $D^{(*)}$ mesons are likely to apply to $\Xi_{cc}^{(*)}$ baryons. For instance, if there are $DK$ [3, 4] and $DDK$ [7, 9] bound states the same is expected to happen to the $\Xi_{cc}\bar{K}$ and $\Xi_{cc}\Xi_{cc}\bar{K}$ systems. This last system will be the subject of the present manuscript.

The existence of a $DK$ bound state is usually argued on the basis of the experimental location of the $D_{s0}^+(2317)$ [10–12]. The mass of this resonance is excessively low to be accommodated as a $c\bar{s}$ state in the quark-model. Yet from chiral symmetry we expect the $DK$ interaction to be really strong and attractive, leading to the natural explanation that the $D_{s0}^+(2317)$ is a bound state [3, 6]. Indeed the attraction in the $DK$ system has been repeatedly shown to be strong enough to form this state [13–18]. Now, if we consider HADS, then the binding of the $DK$ system implies that the $\Xi_{cc}\bar{K}$ system should bind too, a conclusion which has been pointed out in a series of theoretical works. For instance, Ref. [19] predicts an isoscalar $\Xi_{cc}\bar{K}$ bound state at about 60 ± 20 MeV below threshold. Ref. [20] considers the $\Xi_{cc}\bar{K}$-$\Omega_{cc}\eta$ coupled system, for which binding happens at about 150 MeV below threshold. In Ref. [21] the authors calculated the $\Xi_{cc}\bar{K}$ scattering length to be 2.15 fm, which being positive (and provided that the system is attractive) indicates the existence of a bound state. In Ref. [22], in addition to the next-to-leading order chiral potentials, the $P$-wave excitation between the two heavy quarks was taken into account as a dynamical degree of freedom, a $\Xi_{cc}\bar{K}$ bound state with a binding energy of 50 MeV was predicted.

The bottom-line is that the existence of a $\Xi_{cc}\bar{K}$ bound state is really likely, at least if HADS breaking is not too large. This immediately raises the intriguing question of whether there exists a $\Xi_{cc}\Xi_{cc}\bar{K}$ trimer. The reasons why such a trimer is probable are HADS and the theoretical predictions of a $DDK$ bound state [7, 9, 23]. It is also
interesting to notice that the probable mechanism responsible for the formation of the $DDK$ and $\Xi_{cc}\Xi_{cc}\bar{K}$ trimers is the strong Weinberg-Tomozawa term in the $DK$ and $\Xi_{cc}\bar{K}$ subsystems. This feature is shared with the $NK$ system, which is usually thought to be the most important component of the $\Lambda(1405)$ wave function and which also leads to the formation of $NN\bar{K}$ bound states, as has been extensively studied in both theory \cite{24,25} and experiment \cite{43,44,45,46,47,48,49,50}, with the later supporting the existence of this trimer. Motivated by these facts, in this manuscript we will explore the three-body $\Xi_{cc}\Xi_{cc}\bar{K}$ system, which we conclude to be likely to bind.

The article is organized as follows: in Sec. II, we explain the $\Xi_{cc}\bar{K}$ and $\Xi_{cc}\Xi_{cc}$ interactions. In Sec. III we construct the three-body wave functions and solve the corresponding Schrödinger equation for the $\Xi_{cc}\Xi_{cc}\bar{K}$ system using the Gaussian Expansion Method (GEM). In Sec. IV we present our predictions and discuss the theoretical uncertainties associated with them. Finally, we summarize our results in Sec. VI.

II. TWO-BODY INTERACTIONS

Here we will explain in detail how to derive the $\Xi_{cc}\bar{K}$ and $\Xi_{cc}\Xi_{cc}$ interactions. For the case of the $\Xi_{cc}\bar{K}$ system the most important part of the interaction is given by the Weinberg-Tomozawa term, which happens to be identical to that of the $DK$ system owing to HADS and chiral symmetry. For the $\Xi_{cc}\Xi_{cc}$ system we will resort to the one boson exchange (OBE) model, where the non-relativistic potential between the two baryons is determined by the exchange of a few light mesons (the pion, the sigma, the rho and the omega). For determining the coupling constants of these light mesons to the doubly-charmed baryons we will resort to HADS and the information we can deduce from the formation of a few light mesons (the pion, the sigma, the rho and the omega). For determining the coupling constants of these light mesons to the doubly-charmed baryons we will resort to HADS and the information we can deduce from the $DD$ two charmed-meson system and the quark model.

A. $0(\frac{1}{2}^-)$ $\Xi_{cc}\bar{K}$ potential

The most important contribution to the $\Xi_{cc}\bar{K}$ interaction is the Weinberg-Tomozawa term between the kaon and the doubly charmed baryon, which in a non-relativistic normalization reads

$$V_{WT}(\bar{q}) = -\frac{C_{WT}(I)}{2f_\pi^2},$$

(1)

with $f_\pi \approx 130$ MeV and $C_{WT}(0) = 2$, $C_{WT}(1) = 0$ for the isoscalar and isovector channels, respectively. This coincides with the standard half-relativistic (relativistic kaon and non-relativistic baryon) normalization $C_{WT}(I)(\omega_K + \omega_K')/2f_\pi^2$.

This potential is exactly the same one as for the $DK$ system as a consequence of two independent facts: the universality of the WT term (and the fact that $D$ and $\Xi_{cc}$ belong to the 3 and 3 representations of SU(3)-flavor) and HADS, which also implies the same interaction in both systems.

The Fourier-transform of the previous potential in coordinate space is

$$V_{WT}(\vec{r}) = -\frac{C_{WT}(I)}{2f_\pi^2} \delta^{(3)}(\vec{r}),$$

(2)

which is singular and requires regularization. For that purpose we will choose a Gaussian regulator of the type

$$V_{WT}(\vec{r}) = -\frac{C_{WT}(I)}{2f_\pi^2} \frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3},$$

(3)

where $R_c$ is a coordinate space cutoff. However the previous expression is still problematic, as the prediction of a bound $\Xi_{cc}\bar{K}$ state and its binding energy depends on the cutoff. If there is an experimentally known bound state, then it is easy to choose the cutoff in order to reproduce that bound state. Though this is not the case for the $\Xi_{cc}\bar{K}$, it happens that the $D_{s0}^*(2317)$ is suspected to be a $DK$ bound state. From this we can set the cutoff in the $DK$ system, which, owing to HADS, should be the same cutoff as in the $\Xi_{cc}\bar{K}$ system.

But there is the more powerful approach of fully renormalizing the $\Xi_{cc}\bar{K} / DK$ interaction, which is what we will do here. For that, we allow the strength of the WT term to vary with the cutoff

$$V(\vec{r}) = C(R_c) \frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3},$$

(4)

where for every value of $R_c$ we determine $C(R_c)$ by reproducing the $D_{s0}^*(2317)$ as a $DK$ bound state. After this we
can predict the binding energy of the $\Xi_{cc}\bar{K}$ system. In addition to this, we can also modify the previous potential with a shorter-range contribution as follows

$$V_{WT}(r) = C(R_c) e^{-r/R_c} + C_S e^{-r/R_S}$$

$$= C_L e^{-r/R_L} + C_S e^{-r/R_S},$$

where we take $R_S < R_c$ and $C_S > 0$ (i.e. repulsive) with $|C_S| > |C(R_c)|$. Notice that we also define the couplings $C'_L$ and $C'_S$, which are equivalent to $C(R_c)$ and $C_S$ but more convenient to use. The purpose of this modification is to take into account the fact that the subleading order corrections to the WT term for the $DK$ interaction are repulsive in nature (with the same thing happening in the $\Xi_{cc}\bar{K}$ system owing to HADS) [14]. For concreteness we will take $R_c = 0.5 - 2.0$ fm and $R_S = 0.1$ fm. For $C'_L$ we will consider two possibilities: $C'_S = 0$ (i.e. we ignore the existence of subleading order corrections) and $C'_S = 1$ GeV (to exaggerate the strength of these corrections). The binding energy we predict for the $\Xi_{cc}\bar{K}$ state lies in the range $B_2 = (50 - 60)$ MeV and are almost independent of $C'_S$. Concrete results for each cutoff can be checked in Table II which we will discuss later on.

In addition to the strong force, the $\Xi_{cc}\bar{K}$ system also receives contributions from the electromagnetic force if the antikaon happens to be charged (the $\Xi_{cc}$ is always charged, with its particle states being $\Xi_{cc}^+, \Xi_{cc}^{++}$). The two particle components of the $I = 0$ channel in which the $\Xi_{cc}\bar{K}$ interaction is expected to be stronger are

$$|\Xi_{cc}\bar{K}(I = 0)⟩ = \frac{1}{\sqrt{2}} [ |\Xi_{cc}^+\bar{K}^0⟩ + |\Xi_{cc}^{++}\bar{K}^−⟩].$$

For each of these components the Coulomb force reads

$$V^C_{\Xi_{cc}^+\bar{K}^0}(r) = 0,$$

$$V^C_{\Xi_{cc}^{++}\bar{K}^−}(r) = -\frac{2}{r} \alpha,$$

or, equivalently, if we write it with isospin operators

$$V^C_{\Xi_{cc}^+\bar{K}^0}(r) = \frac{\alpha}{r} \left( \frac{\tau_{z,1} + 3}{2} \left( -1 + \tau_{z,2} \right) \right),$$

with $\tau_{z,1}$ and $\tau_{z,2}$ the third component of the isospin operator for the doubly charmed baryon and antikaon, respectively. The problem is that the Coulomb force breaks isospin symmetry, which can be dealt with in two ways. The simplest one is to average the Coulomb force over the particle components of the $I = 0$ state

$$⟨\Xi_{cc}\bar{K}(I = 0) | V^C(r) | \Xi_{cc}\bar{K}(I = 0)⟩ = -\frac{\alpha}{r}.$$  

The other possibility is to consider the $|\Xi_{cc}^+\bar{K}^0⟩$ and $|\Xi_{cc}^{++}\bar{K}^−⟩$ components separately as channels 1 and 2, in which case we can write the full potential as

$$V(r) = V_{WT}(r) \frac{1}{2} \left[ \begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array} \right] - \frac{\alpha}{r} \left[ \begin{array}{cc} 0 & 0 \\ 0 & 2 \end{array} \right].$$

In this work we will choose the first option, the one described by Eq. (11), as this will greatly simplify the formalism required to do the calculations, particularly once we consider the three-body system.

### B. The $\Xi_{cc}\Xi_{cc}$ potential in the one boson exchange model

In the OBE model the interaction between two hadrons is described in terms of the exchange of a series of light mesons, which most commonly include the pion ($\pi$), sigma ($\sigma$), rho ($\rho$) and omega ($\omega$), but sometimes a few more bosons in its more sophisticated incarnations. Indeed the OBE model has provided one of the most quantitatively

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1. HADS is known to be broken at the level of $\Lambda_{QCD}/(m_Q \nu)$ [2], where $\nu$ is the velocity of the heavy quark pair. For a charm quark pair, $m_Q \nu \sim 0.8$ GeV [51], we obtain a breaking of $25 - 40\%$ for HADS. At this moment, there is no concrete experimental information that can help us to estimate the level of the breaking of HADS. Future discovery of the spin 3/2 partner of the $\Xi_{cc}$ should give us a clue, since HADS states

$$m_{\Xi_{cc}} - m_{\Xi_{cc}^0} = \frac{3}{4} (m_{\rho'} - m_{\rho}) \approx 106.5 \text{ MeV.}$$

Lattice QCD studies [52,56] and various models [57,64] suggest a breaking up to 40\%. As a result, the same as Ref. 64, we study the uncertainties due to the breaking of HADS by varying the couplings, $C(R_c)$ and $C_S$, from their central values by 25\%.
successful description of the nuclear forces \cite{66,67} and in principle there is nothing impeding its application to other two-hadron systems. Regarding hadronic molecules, it is interesting to notice that the original speculations about their existence were based on the OBE model \cite{68}, which later has been widely used for predicting or explaining molecular states \cite{69,74}.

The particular version of the OBE model we will use is the one developed in Ref. \cite{75} for the $D\bar{D}$ and $D\bar{D}$ case via HADS. The most important difference of Ref. \cite{75} with previous implementations of the OBE model for heavy hadron molecules is the inclusion of a few of the ideas of the renormalized OBE model of Ref. \cite{76}. In particular we partially renormalize the OBE model, by which we mean the following: the OBE model contains a form factor and a cutoff $\Lambda$, where we determine the cutoff $\Lambda$ from the condition of reproducing a known molecular state. The molecular state chosen is the $X^{(*)}$ (of the ideas of Ref. \cite{75} with previous implementations of the OBE model for heavy hadron molecules is the inclusion of a few

The particular version of the OBE model we will use is the one developed in Ref. \cite{75} for the $D\bar{D}$ system. Instead we will simply indicate how to do it, where the starting point are the definitions of the charmed meson and doubly charmed baryon superfields

$$H_c = \frac{1}{\sqrt{2}} \left[ D + \tilde{D} \right], \quad T_{cc}^{\dagger} = \frac{1}{\sqrt{3}} \tilde{\xi}_{cc}^{\dagger} + \tilde{\xi}_{cc}^{\ast},$$

(13)

which group the $D, D^*$ meson ($\Xi_{cc}, \Xi_{cc}^*$, baryon) fields into a single superfield with good properties with respect to rotations of the heavy quark spin. For implementing HADS there are several possibilities, of which we briefly explain two. One is to group the two superfields $H_c$ and $T_{cc}$ into a new superfield $H_c$ which is invariant under HADS. The other is to simply notice that the previous procedure at the end amount to make the following substitutions in the Lagrangians:

$$\text{Tr} \left[ H_c^4 \mathcal{O} H_c \right] \to T_{cc}^{\dagger} \mathcal{O} T_{cc},$$

(14)

with $\mathcal{O}$ some arbitrary spin operator acting on the superfields. If we do this with the OBE Lagrangian of Ref. \cite{75} we will be able to derive the potentials we will write down below.

The outcome of the previous procedure for the particular case of the $\Xi_{cc} \Xi_{cc}$ system is

$$V_{\text{OBE}} = V_\pi + V_\sigma + V_\rho + V_\omega,$$

(15)

where the contributions from the $\pi, \sigma, \rho$ and $\omega$ read

$$V_\pi(\vec{r}) = \frac{g^2}{6f^2_\pi} \left[ -\frac{1}{9} \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 m_\pi^2 d(m_\pi r, \frac{\Lambda}{m_\pi}) + \frac{1}{9} \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 m_\pi^2 W_Y(m_\pi r, \frac{\Lambda}{m_\pi}) + \frac{1}{9} S_{12}(\vec{r}) m_\pi^2 W_T(m_\pi r, \frac{\Lambda}{m_\pi}) \right],$$

(16)

$$V_\sigma(\vec{r}) = -g_\sigma^2 m_\sigma W_Y(m_\sigma r, \frac{\Lambda}{m_\sigma}),$$

(17)

$$V_\rho(\vec{r}) = \frac{f^2_\rho}{4M^2} \left[ -\frac{2}{27} \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 m_\rho^3 W_Y(m_\rho r, \frac{\Lambda}{m_\rho}) + \frac{2}{27} \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 m_\rho^3 W_T(m_\rho r, \frac{\Lambda}{m_\rho}) - \frac{1}{27} S_{12}(\vec{r}) m_\rho^3 W_T(m_\rho r, \frac{\Lambda}{m_\rho}) \right],$$

(18)

$$V_\omega(\vec{r}) = g_\omega^2 m_\omega W_Y(m_\omega r, \frac{\Lambda}{m_\omega}) + \frac{f^2_\omega}{4M^2} \left[ -\frac{2}{27} \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 m_\omega^3 d(m_\omega r, \frac{\Lambda}{m_\omega}) + \frac{2}{27} \tilde{\sigma}_1 \cdot \tilde{\sigma}_2 m_\omega^3 W_Y(m_\omega r, \frac{\Lambda}{m_\omega}) - \frac{1}{27} S_{12}(\vec{r}) m_\omega^3 W_T(m_\omega r, \frac{\Lambda}{m_\omega}) \right],$$

(19)
where for a monopolar form factor the functions $d$, $W_Y$ and $W_T$ take the form

$$d(x, \lambda) = \frac{(\lambda^2 - 1)^2}{2\lambda} e^{-\lambda x} \frac{1}{4\pi},$$

$$W_Y(x, \lambda) = W_Y(x) - \lambda W_Y(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} e^{-\lambda x} \frac{1}{4\pi},$$

$$W_T(x, \lambda) = W_T(x) - \lambda^3 W_T(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \left(1 + \frac{1}{\lambda x}\right) e^{-\lambda x} \frac{1}{4\pi}.$$  

(20)  

(21)  

(22)

For the coupling constants we follow Ref. [75] and take $g = 0.60$, $g_\sigma = 3.4$, $g_\rho = g_\omega = 2.6$, $f_\rho = f_\omega = g_\omega \kappa_\omega$, $\kappa_\omega = 4.5$ and $M = 1867$ MeV.

Finally we have to include the Coulomb piece, which written in the isospin basis reads

$$V_{_\Xi cc\Xi cc\bar{K}}(r) = \frac{\alpha}{r} \frac{(\tau_+ 1 + 3)}{2} \frac{(\tau_+ 2 + 3)}{2}.$$  

(23)

If we consider the $S = 0$ (singlet) $S$-wave $\Xi_{cc}\Xi_{cc}$ molecule, there are three possible isospin states corresponding to the $\Xi_{cc}^+\Xi_{cc}$, $\Xi_{cc}^+\Xi_{cc}^+$ and $\Xi_{cc}^+\Xi_{cc}^+$ systems. If we consider the $S = 1$ (triplet) case, this corresponds to $\Xi_{cc}^+\Xi_{cc}^+$ and a Coulomb potential

$$V_{_\Xi cc\Xi cc}(r; S = 1, I = 0) = 2 \frac{\alpha}{r}.$$  

(24)

Concrete calculations with the previous parameters indicate that there is no singlet bound state but that a triplet bound state — the charming deuteron — will bind for $\Lambda_S \geq 994$ MeV without Coulomb and $\Lambda_C \geq 1112$ MeV with Coulomb, consistent with the previous prediction of Ref. [77].

### III. GAUSSIAN EXPANSION METHOD

Once we have determined all the relevant two-body interactions, we are ready to explore the $\Xi_{cc}\Xi_{cc}\bar{K}$ three-body system. For this we will use the Gaussian Expansion Method (GEM) [78, 79], which is an efficient method to solve few-body systems. The starting point is the Schrödinger equation

$$H \Psi_{JM}^{total} = E \Psi_{JM}^{total},$$  

(25)

with the Hamiltonian

$$\hat{H} = \sum_{i=1}^{3} \frac{p_i^2}{2m_i} - T_{c.m.} + V_{_\Xi cc\bar{K}}(r_1) + V_{_\Xi cc\bar{K}}(r_2) + V_{_\Xi cc\Xi cc}(r_3),$$  

(26)

where $T_{c.m.}$ is the kinetic energy of the center of mass and $V(r)$ is the potential between the two relevant particles. The three possible permutations of the Jacobi coordinates for the $\Xi_{cc}\Xi_{cc}\bar{K}$ system are depicted in Fig.1. The total wave function can be expressed as the sum of the amplitudes of the three re-arrangement channels ($c = 1 - 3$), i.e. the permutations shown in Fig.1 which we write as

$$\Psi_{JM}^{total} = \sum_{c,\alpha} C_{c,\alpha} \Psi_{JM,\alpha}^c (r_c, R_c)$$  

(27)
with \( r_c \) and \( R_c \) the Jacobi coordinates in channel \( c \). As can be appreciated, the wave function is expanded in a series in terms of \( \psi^{cc} \) and the Fermi-Dirac statistics of the two identical \( \Xi \) baryons, which requires

\[
P_{12}\Psi^{\text{total}}_{JM} = -\Psi^{\text{total}}_{JM},
\]

where \( P_{12} \) is the exchange operator of particles 1 and 2. The wave function of each channel has the following form

\[
\Psi_{JM, c}(r_c, R_c) = H_{T,t}^c \otimes [\Phi_{IL,A}(r_c, R_c)]_{JM}
\]

where \( H_{T,t}^c \) is the isospin wave function, and \( \Phi_{IL,A}^c \) the orbital wave function. The isospin wave function in each channel reads as

\[
H_{T,t}^1 = [\eta_1(\Xi_{cc}^1)\eta_1(\tilde{K}^3)]_{t}, \\
H_{T,t}^2 = [\eta_2(\Xi_{cc}^1)\eta_2(\tilde{K}^3)]_{t}, \\
H_{T,t}^3 = [\eta_3(\Xi_{cc}^1)\eta_3(\tilde{K}^3)]_{t}.
\]

The orbital wave function \( \Phi_{IL,A}^c \) is given in terms of the Gaussian basis functions

\[
\Phi_{IL,A}^c(r_c, R_c) = [\phi_{nl,L}^c(r_c)\psi_{N,L}^c(R_c)]_A,
\]

\[
\phi_{nl,m}^c(r_c) = N_{nl}r_c^me^{-\nu_lr_c^2}Y_{lm}(\tilde{r}_c),
\]

\[
\psi_{NLM}^G(R_c) = N_{NL}R_c^l e^{-\lambda_NR_c^2}Y_{LM}(\tilde{R}_c).
\]

Here \( N_{nl}(N_{NL}) \) is the normalization constant of the Gaussian basis and the parameters \( \nu_n \) and \( \lambda_N \) are given by

\[
\nu_n = 1/r_n^2, \quad r_n = r_{\min}a^{n-1} \quad (n = 1, n_{\max}), \\
\lambda_N = 1/R_N^2, \quad R_N = R_{\min}A^{N-1} \quad (N = 1, N_{\max}),
\]

where \( \{n_{\max}, r_{\min}, a \text{ or } r_{\max}\} \) and \( \{N_{\max}, R_{\min}, A \text{ or } R_{\max}\} \) are gaussian basis parameters.

Since the \( \Xi_{cc} \) is a doubly charmed \( \frac{1}{2}^-(\frac{1}{2}^-) \) baryon and \( \tilde{K} \) a \( \frac{1}{2}(0^-) \) meson, considering only \( S \)-wave interactions and the Fermi-Dirac statistics of the two identical \( \Xi_{cc} \) baryons, the quantum numbers of the \( \Xi_{cc}\Xi_{cc}\tilde{K} \) system are \( I(J^P) = \frac{1}{2}(0^-) \). All the configurations of this three-body system are shown in Table I.

As the wave function been constructed, the Schrödinger equation of this system is transformed into a generalized matrix eigenvalue problem by the basis expansion:

\[
[T_{aa'}^{ab} + V_{aa'}^{ab} - EN_{aa'}^{ab} ]C_{b,a'} = 0.
\]

Here, \( T_{aa'}^{ab} \) is the kinetic matrix element, \( V_{aa'}^{ab} \) is the potential matrix element and \( N_{aa'}^{ab} \) is the normalization matrix element. The eigenenergy \( E \) and coefficients are determined by the Rayleigh-Ritz variational principle via the gaussian basis parameters.
IV. THOMAS COLLAPSE IN THE $\Xi_{cc}\Xi_{cc}\bar{K}$ SYSTEM

A problem with the $\Xi_{cc}\Xi_{cc}\bar{K}$ system is that it is Efimov-like. Thus it will be possible for it to show Thomas collapse. In principle this means that the predictions we will make will be cutoff dependent, as the only way to stabilize the energy of the ground state is to include a short-range repulsive three-body force.

Actually to show the existence of the Thomas collapse in this system we will use the Efimov effect as a proxy. The Efimov effect refers to the appearance of a geometric spectrum in the three-body system when a few of the interacting particles are in the unitary limit, i.e. their scattering lengths diverge. This is complementary to the Thomas collapse: reducing the range of the interaction is equivalent to a relative increase of the scattering length when expressed in units of the range. The presence of the Efimov effect can be deduced from the Faddeev equations for a contact-range potential, which we will not derive in detail here but can be consulted in Refs. [80–82]. Instead we will simply use the results derived in other works: if we consider a three-body system of the type $AAB$, where $A$ and $B$ are two different species of particles and the $AB$ interaction is resonant, the condition for having the Efimov effect is

$$\lambda_{\alpha} = \frac{\sin 2\alpha}{2\alpha} \leq \lambda,$$

with $\lambda$ a geometric factor depending on the characteristics of the $AB$ interaction and quantum numbers of the system, and $\alpha$ an angle given by

$$\alpha = \arcsin \left( \frac{1}{1 + \frac{m_B}{m_A}} \right).$$

For the $\Xi_{cc}\Xi_{cc}\bar{K}$ system we have that $m_A = m(\Xi_{cc})$ and $m_B = m(K)$, which gives $\lambda_{\alpha} \approx 0.389$. The factor $\lambda$ is given in this case by the condition that the $\Xi_{cc}\bar{K}$ interaction is strong in the isospin $I = 0$ channel, and can be calculated from the matrix element of the isospin wave functions from the two different permutations of the doubly charmed baryons:

$$\lambda = \langle H_{c=1}^{1/2,0}|H_{c=2}^{5/2,0}\rangle = \frac{1}{2}.$$

From this we have that $\lambda \geq \lambda_{\alpha}$: the conclusion is that for the $\Xi_{cc}\Xi_{cc}\bar{K}$ system the Efimov effect can indeed happen. But of course, owing to the fact that the $\Xi_{cc}\bar{K}$ system is far from the unitary limit (i.e. they do not form a shallow bound state), what we expect instead is Thomas collapse. For comparison purposes, in the $DDK$ and $NN\bar{K}$ systems we have $\lambda_{\alpha} \approx 0.531$ and 0.693, respectively, from which we deduce that there is no Thomas collapse in these two cases.

V. PREDICTIONS

Once we have determined the required two-body inputs, the calculation of prospective $\Xi_{cc}\Xi_{cc}\bar{K}$ trimer predictions is straightforward. For this we will use the GEM, which we have already explained in Section III. As already discussed, the $\Xi_{cc}\Xi_{cc}\bar{K}$ three-body system suffers from Thomas collapse, i.e. if the range of the involved two-body $\Xi_{cc}\bar{K}$ interaction would be reduced to zero, the three-body system would collapse. Of course in the real world this does not happen because the range the $\Xi_{cc}\bar{K}$ interaction is finite, but this collapse will manifest itself as a strong dependence on the cutoff $R_c$ we have chosen to regularize the potential. Finally, we notice that the $\Xi_{cc}\Xi_{cc}\bar{K}$ system is completely analogous to the $NN\bar{K}$ one: the doubly-charmed baryons and the nucleons belong to the same irreducible representation of the spin and isospin and are therefore interchangeable modulo two difference, one being the masses and the other being the strength of the WT term with the antikaon. For this reason we will also present calculations of the $NN\bar{K}$ trimer.

A. The $\frac{1}{2}(0^-)$ $\Xi_{cc}\Xi_{cc}\bar{K}$ system

We now present here the $\Xi_{cc}\bar{K}$ dimer and $\Xi_{cc}\Xi_{cc}\bar{K}$ trimer predictions. We begin with the dimer, as it is the basic building block for the calculation of the trimer binding energy. As already explained the $\Xi_{cc}\bar{K}$ interaction is given by a contact-range potential the strength of which can be related to that of the $DK$ interaction by means of HADS. For the form of the potential we use Eq. (6), where we let the cutoff float in the $R_c = 0.5 - 2.0$ fm window but set $R_c = 0.1$ fm for the second cutoff we use to model short-range repulsion. We determine the coupling $C(R_c)$ from the condition of reproducing the $D_{s0}^{'+}(2317)$ as a $DK$ bound state with a binding energy of 45 MeV, where the resulting potential is shown in Fig. 2. From this potential and HADS, we predict the $\Xi_{cc}\bar{K}$ dimer to have a binding energy of
49 − 64 MeV in the absence of a repulsive core, and 49 − 63 MeV if there is a repulsive core with $C_S = 1000$ MeV, from which we deduce that the influence of a repulsive core is negligible. A more detailed compilation of the binding energies of the dimer for different cutoffs can be found in Table II. We note that the binding energy of the $\Xi_{cc}\bar{K}$ bound state is larger than that of the $\bar{D}K$ bound state because the $\Xi_{cc}$ baryon is heavier than the $D$ meson.

Now for the $\Xi_{cc}\Xi_{cc}\bar{K}$ system, besides the $\Xi_{cc}\bar{K}$ interaction explained in the previous paragraph, we also need the $\Xi_{cc}\Xi_{cc}$ potential. For this we use the OBE potential described in Eqs. (15–19). With this we arrive at the results we show in Table III.

A few comments are in order at this point. First, the 3-body binding energy of the $\Xi_{cc}\Xi_{cc}\bar{K}$ system increases as the cutoff $R_c$ decreases. The origin of this cutoff dependence lies in the fact that the $\Xi_{cc}\Xi_{cc}\bar{K}$ system is susceptible to Thomas collapse, i.e. if we were to reduce the cutoff $R_c$ to zero, the binding energy of the system will diverge: $B_3 \to \infty$. For the cutoff range we have chosen, i.e. $R_c = 0.5 − 2.0$ fm, the binding energy of the trimer varies between 80 − 118 MeV. Indeed it can be appreciated that the possibility of Thomas collapse in the $\Xi_{cc}\Xi_{cc}\bar{K}$ system translates into a considerable cutoff dependence of the results, yet the conclusion that the system binds is solid, as it happens even for really soft cutoffs like $R_c = 2.0$ fm. Second, if $R_c$ is large enough (1.5 − 2.0 fm), a second bound state appears. This additional bound state is expected to be a cutoff artifact: the cutoffs for which it appears are relatively soft, definitely larger than the size of the hadrons we are considering here. Be it as it may, the bottom-line is that the existence of the $\Xi_{cc}\Xi_{cc}\bar{K}$ bound state is rather robust.

The uncertainties of our predictions as listed in Table III come from two different sources. One is violation of HADS, which is not a perfectly preserved symmetry, but instead it is expected to be broken at the 25% level. This

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3 The Coulomb interaction is found to affect the binding energy only by several MeV, and therefore has been neglected in this work.
will affect the two two-body potentials on which the calculation of the $\Xi_{cc}\Xi_{cc}\bar{K}$ trimer relies: the $\Xi_{cc}\bar{K}$ and the $\Xi_{cc}\Xi_{cc}$ potentials. In both cases we are inferring the strength of the potential from HADS and the corresponding potential for the $DK$ and $DD$ systems, which means that the mentioned 25% relative uncertainty applies. In addition the individual couplings of the $\Xi_{cc}\Xi_{cc}$ potential inherit the same type of uncertainties as in the $DD$ potential, as discussed in Ref. 76. The origin of these uncertainties lies in the problem of determining the value of the different coupling constants in the OBE model, where we simply assume them to compound into a single uncertainty of about 30% in the value of the OBE potential. These two sources of uncertainty — HADS and the OBE couplings — are then summed in quadrature (as we expect them to be independent error sources)

$$\text{Error} = \sqrt{\text{Error}(\text{WT})^2 + \text{Error}(\text{OBE})^2}$$ (39)

where Error(WT) is calculated by scaling the $\Xi_{cc}\bar{K}$ potential by a factor of 0.75 – 1.25, while keeping the OBE $\Xi_{cc}\Xi_{cc}$ potential unchanged. Similarly, Error(OBE) is calculated by scaling the OBE $\Xi_{cc}\Xi_{cc}$ potential with a factor of 0.7 – 1.3, while keeping the $\Xi_{cc}\bar{K}$ potential unchanged. We find that the influence on the binding energy form the Error(WT) is about a few tens of MeV and that from Error(OBE) is less than 1 MeV. Therefore, we conclude that the dominant uncertainty in the binding energy of the trimer is the $\Xi_{cc}\bar{K}$ interaction.

B. The $NN\bar{K}$ system as an analogue of the $\Xi_{cc}\Xi_{cc}\bar{K}$ system

As previously mentioned, the charmed meson $D$ and doubly charmed baryon $\Xi_{cc}$ can be seen as an analogue to the nucleon: while the heavy quarks act as spectators, the light-quark within these heavy hadrons belong to the same spin and isospin irreducible representations as the nucleon (see, e.g., 77). Thus it is natural to see the $DDK$ and $\Xi_{cc}\Xi_{cc}\bar{K}$ systems as a heavy counterpart of the $NN\bar{K}$ system.

Furthermore, the origin of the same $\bar{K}N$ interaction is the WT term which is also responsible for the binding of the $DK$ and $\Xi_{cc}\bar{K}$ systems. Here there is a difference though: the nucleon belongs to the 8 representation of the $SU(3)$-flavor group, while the $D$ and $\Xi_{cc}$ heavy hadrons to the 3 and 3 ones. This means that the strength of the WT term is not the same for $\bar{K}N$ as it is for $DK$ and $\Xi_{cc}\bar{K}$ (in fact, for $\bar{K}N$ it is more attractive). Using the WT term or the chiral potential as kernel to the Lippmann-Schwinger or Bethe-Salpeter equations, one can describe the $\Lambda(1405)$ as a $\bar{K}N$ bound state 83,90. In addition, the $NN\bar{K}$ system has been studied rather extensively and it seems that all the approaches lead to the conclusion that it should bind 26,38,41,42, while only differing in minor details. If this were not enough, all the experiments performed so far support the existence of such a state 43–45, 47–49, again only differing in minor details.

Following the logic with which we studied the $\Xi_{cc}\Xi_{cc}\bar{K}$ system, here we calculate the binding energy of the $NN\bar{K}$ bound state using the $\bar{K}N$ and $NN$ potentials as input. For fixing the strength of the $\bar{K}N$ interaction we simply reproduce the location of the $\Lambda(1405)$ with the potential of Eq. (9). For the $NN$ interaction we use the OBE model. The binding energy of the $NN\bar{K}$ trimer is listed in Table III for different cutoffs. In the same table we also show the values of the coupling that reproduces the $\Lambda(1405)$ pole as a $\bar{K}N$ bound state with a binding energy $B_2 = 29.4\text{ MeV}$. The binding energy of the $NN\bar{K}$ trimer ranges from $35 – 43\text{ MeV}$, where the cutoff dependence is relatively weak in comparison to the $\Xi_{cc}\Xi_{cc}\bar{K}$ system. The reason is that the $NN\bar{K}$ does not suffer from Thomas collapse as a consequence that the mass ratio between the nucleon and the kaon is not large enough as to trigger this effect. We notice that the inclusion of the $NN\bar{K}$ system in the present manuscript should be viewed mostly as a consistency check of the $\Xi_{cc}\Xi_{cc}\bar{K}$ calculation: there is a large literature of calculations of the $NN\bar{K}$ system that are far more sophisticated that the one presented here.

VI. SUMMARY

In this work we have investigated the $\Xi_{cc}\Xi_{cc}\bar{K}$ system. We have reached the conclusion that it likely binds. This is mostly a consequence of HADS, a type of heavy-quark symmetry that relates the $\Xi_{cc}\bar{K}$ interaction to the $DK$ one. From this symmetry and the fact that previous theoretical explorations point out to the possibility of bound $DDK$ 71,8 and $DDDK$ clusters 9, the natural expectation is that the $\Xi_{cc}\Xi_{cc}\bar{K}$ system binds too. Concrete calculations within the GEM framework indicate that (i) the two-body $\Xi_{cc}\bar{K}$ system has a binding energy $B_2 = 49 – 64\text{ MeV}$ (ii) while for the three-body $\Xi_{cc}\Xi_{cc}\bar{K}$ system the binding energy is $B_3 = 80 – 118\text{ MeV}$.

In the case of the two-body calculation, from HADS we expect the $\Xi_{cc}\bar{K}$ potential to be identical to the $DK$ one, where for the later case the strength of the potential can be completely determined from the hypothesis that the $D_{s0}^*(2317)$ is a $DK$ bound state. At the level of approximation we are considering, this interaction is given by the WT term, though we additionally considered the possibility of a short-range repulsive core in the $\Xi_{cc}\bar{K}$ potential to mimic
TABLE III. Parameters ($C S$, $C(R_c)$ in MeV, and $R_s$, $R_c$ in fm) of the $N\bar{K}$ potential and the binding energies ($B_2$ and $B_3$ in MeV) of the $N\bar{K}$ and $I(J^P) = \frac{1}{2} (0^-)$ $N\bar{K}\bar{K}$ bound states. The parameters are determined by reproducing the $\Lambda(1405)$ with the binding energy 29.4 MeV with respect to the $N\bar{K}$ threshold.

| $C(S)$ | $R_c$ | $B_2(N\bar{K})$ | $B_3(N\bar{K})$ |
|--------|-------|-----------------|-----------------|
| $C(S) = 0$ | $R_c = 0.1$ | 29.4 | 35.2 |
| $-925.9$ | $0.5$ | 29.4 | 39.3 |
| $-316.4$ | $1.0$ | 29.4 | 41.8 |
| $-132.6$ | $2.0$ | 29.4 | 42.5 |
| $-89.2$ | $3.0$ | 29.4 | 42.5 |
| $C(S) = 1000$ | $R_c = 0.1$ | 29.4 | 35.4 |
| $-946.6$ | $0.5$ | 29.4 | 39.4 |
| $-319.8$ | $1.0$ | 29.4 | 41.8 |
| $-133.2$ | $2.0$ | 29.4 | 42.5 |
| $-89.4$ | $3.0$ | 29.4 | 42.5 |

the effect of subleading corrections. The uncertainty in the two-body binding energy comes mostly from violations of HADS, which we take to be as large as 25% percent.

For the three-body $\Xi_{cc}\Xi_{cc}\bar{K}$ trimer the uncertainties are definitely larger because besides HADS we also have a sizable cutoff dependence: this trimer is in principle susceptible to Thomas collapse, i.e. if the range of the $\Xi_{cc}\bar{K}$ interaction were to be taken to zero the trimer binding energy would diverge, hence the cutoff dependence. This problem can be effectively circumvented by the fact that the size of the $\Xi_{cc}$ or $\bar{K}$ hadrons is finite or, more elegantly, by the inclusion of a repulsive short-range three-body force that will stabilize the results. Here we opt for the finite-cutoff solution, owing to its simplicity but also to the fact that the strength of a prospective three-body force should be determined from the data. Be it as it may, we consider the existence of a relatively compact $\Xi_{cc}\Xi_{cc}\bar{K}$ trimer as a robust conclusion. Besides, we can deduce the existence of additional trimers from the other heavy-quark symmetries. For instance, from heavy-flavor symmetry there should be $\Xi_{bc}\Xi_{bc}\bar{K}$ and $\Xi_{bb}\Xi_{bb}\bar{K}$ trimers.

Finally we applied our framework to study the $N\bar{K}$ and $NN\bar{K}$ systems. The motivation is a very simple analogy between the charmed mesons $D$, the doubly charmed baryons $\Xi_{cc}$ and the nucleon $N$, the three of which belong to the same representation of light-quark spin and isospin. From this analogy the only differences between few nucleons and few doubly-charmed baryons systems are the specific details of the potential, i.e. the coupling constants and masses. If we determine the $N\bar{K}$ interaction from the condition that the $\Lambda(1405)$ is a $N\bar{K}$ bound state, we reach the conclusion that the binding energy of the $NN\bar{K}$ system is $B_3 = 35 - 43$ MeV with respect to the three-body mass threshold, which is consistent with previous studies and experiments.

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