Surface plasmon in 2D Anderson insulator with interactions

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Abstract

We study the effect of interactions on the zero-temperature a.c. conductivity of 2D Anderson insulator at low frequencies. We show that the enhancement of the real part of conductivity due to the Coulomb correlations in the occupation numbers of localized states results in the change of the sign of imaginary part within a certain frequency range. As a result, the propagation of a surface plasmon in a localized system becomes possible. We analyze the dispersion law of the plasmon for the two cases: unscreened Coulomb interactions and the interactions screened by a gate electrode spaced by some distance from the electron plane.
I. INTRODUCTION

It is believed that in two-dimensions (2D) even small disorder localizes all electron states.\footnote{First hint for the absence of the mobility threshold in 2D came from the calculation of the weak localization correction, $\delta \sigma(\omega)$, to the conductivity at finite frequency $\omega$. It was shown\footnote{that in the limit $k_F l \gg 1$ one has $\delta \sigma(\omega)/\sigma_0 = 1/k_F l \ln |\omega \tau|$, where $k_F$ is the Fermi momentum, $l$ is the mean free path, and $\tau$ is the elastic scattering time; $\sigma_0 = (e^2/h)k_F l$ is the Drude conductivity. The logarithmic singularity in $\delta \sigma(\omega)$ indicates that the zero-temperature ($T = 0$) conductivity is metallic only if $\omega \gg \omega_0$, where} From the finite size, $L$, correction to the zero-frequency conductivity, which is of the order of $1/k_F l \ln(L/l)$, one can estimate the localization length $\xi$ as\footnote{\[ \begin{equation} \xi = l \exp\left(\frac{\pi}{2} k_F l \right). \end{equation} \]} Therefore, starting from metal, one cannot describe the $T = 0$ behavior of the conductivity at frequencies $\omega \sim \omega_0$. The adequate language for this region would be the language of the localized states. Within this language the conductivity originates from the transitions between the localized states induced by an external a.c. field.

Let us briefly remind the corresponding derivation of $\sigma(\omega)$, carried out by Mott\footnote{in the strongly localized regime. The Hamiltonian for strongly localized electrons reads} in the strongly localized regime. The Hamiltonian for strongly localized electrons reads

\[ \mathcal{H} = \sum_i \epsilon_i c_i^\dagger c_i + \sum_{ij} I_{ij}(c_i^\dagger c_j + c_j^\dagger c_i), \] \[ \text{where } \epsilon_i \text{ is the energy of a localized state centered at } \mathbf{r} = \mathbf{r}_i \text{ and } I_{ij} \text{ is the overlap integral which falls off exponentially with distance: } I_{ij} = I_0 \exp(-|\mathbf{r}_i - \mathbf{r}_j|/a), \text{ where } a \text{ is the size of the wave-function of the localized electron. The general expression for the conductivity is given by the Kubo formula} \]
\[ \sigma(\omega) = \frac{ie^2\omega}{\hbar A} \sum_{ij} |\langle i|x|j \rangle|^2 \frac{n_i - n_j}{\omega + \omega_{ij} + i0} \]

where \( \langle i|x|j \rangle \) is the matrix element of \( x \) calculated from the exact eigenstates \( |i \rangle \) and \( |j \rangle \) of the Hamiltonian (3) with energies \( E_i \) and \( E_j \), \( \hbar \omega_{ij} = E_i - E_j \) and \( n_i \) is the occupation number of the state \( |i \rangle \).

The dissipative conductivity, \( \text{Re}\sigma(\omega) \), is determined by the pairs of states with \( \omega_{ij} = \omega \).

At \( \hbar \omega \ll I_0 \) the spatial separation between bare states \( |i \rangle \) and \( |j \rangle \) is much larger than \( a \). Then in the calculation of eigenstates of a resonant pair one should take into account the overlap \( I_{ij} \) within this pair only and neglect the overlap with all the other localized states. This gives

\[ |i \rangle = \frac{\epsilon_i - \epsilon_j}{\Gamma} |i \rangle + \frac{2I_{ij}}{\Gamma} |j \rangle, \quad |j \rangle = \frac{2I_{ij}}{\Gamma} |i \rangle + \frac{\epsilon_j - \epsilon_i}{\Gamma} |j \rangle. \]  

(5)

The corresponding energies are

\[ E_{i,j} = (\epsilon_i + \epsilon_j)/2 \pm \Gamma/2, \quad \Gamma = [(\epsilon_i - \epsilon_j)^2 + 4I_{ij}^2]^{1/2} = \hbar \omega_{ij}. \]  

(6)

Using (3) the matrix element in (4) takes the form

\[ \langle i|x|j \rangle = (x_i - x_j)I_{ij}/\Gamma. \]  

(7)

The contribution to \( \text{Re}\sigma(\omega) \) from pairs with shoulder \( r \) can be presented as

\[ \text{Re}\sigma(\omega, r) = \frac{e^2 \pi x^2 I^2(r)}{\hbar \omega} F(\hbar \omega, r), \]  

(8)

where \( F(\hbar \omega, r) \) is the density of pairs with shoulder \( r \) and excitation energy \( \hbar \omega \) [here \( I(r) = I_0 e^{-r/a} \)]. The density \( F(\hbar \omega, r) \) is determined by the condition that the pair is singly occupied (with energies on the opposite sides from the Fermi level). Then we have

\[ F(\hbar \omega, r) = g^2 \int d\epsilon_1 \int d\epsilon_2 \theta \left( \frac{\hbar \omega}{2} + \frac{\epsilon_1 + \epsilon_2}{2} \right) \theta \left( \frac{\hbar \omega}{2} - \frac{\epsilon_1 + \epsilon_2}{2} \right) \delta \left[ \sqrt{(\epsilon_1 - \epsilon_2)^2 + 4I^2(r)} - \hbar \omega \right] \]  

\[ = \frac{2g^2(\hbar \omega)^2}{\sqrt{(\hbar \omega)^2 - 4I^2(r)}}. \]  

(9)

where \( g \) is the density of states and \( \theta(x) \) is the step-function. Substituting (3) into (8) and integrating over \( r \) we get
\[
\text{Re}\sigma(\omega) = \frac{e^2}{\hbar} (2\pi^2 g^2 \hbar \omega) \int_0^\infty \frac{drr^3 I^2(r)}{\sqrt{(\hbar \omega)^2 - 4 I^2(r)}}.
\]

(10)

For \( \hbar \omega \ll I_0 \) the main contribution to the integral comes from \( r \sim r_\omega = a \ln(2I_0/\hbar \omega) \gg a \) and one obtains

\[
\text{Re}\sigma(\omega) = \sqrt{2\pi^2 e^2} \frac{g^2 a^2 \hbar^2 \omega^2 r_\omega^3}{\hbar}.
\]

(11)

Although we cannot show it explicitly, we argue below that the Mott expression (11) for \( \text{Re}\sigma(\omega) \) is valid also for the Anderson insulator with \( k_F l \gg 1 \). When applying (11) to the Anderson insulator one should replace \( a \) by \( \xi \) from (4) and substitute \( g = m/2\pi\hbar^2 \) (\( m \) is the electron mass). However, the question remains: what is the magnitude of \( I_0 \) in this case? A plausible estimate can be obtained in the spirit of the Thouless picture of localization\[6\] By definition, \( I_0 \) represents the splitting of energy levels of two neighboring localized states (with centers at distance \( \sim \xi \)). The estimate for \( I_0 \) emerges if one equates this splitting to the mean energy spacing for localized states centered within the area \( \sim \xi^2 \), so that \( I_0 \sim 1/g_1 \xi^2 \) (see also Ref. \[5\]).

Note that in 1D case a similar argument leads to \( I_0 \sim 1/g_1 \xi, \) \( g_1 \) being the 1D density of states. Important is that in 1D the Kubo formula can be evaluated exactly\[6\] resulting in the 1D version of the Mott formula, from which one can recover the above estimate for \( I_0 \). Such a mapping was first established by Shklovskii and Efros\[7\].

Using Eq. (10) one can formally evaluate \( \text{Re}\sigma(\omega) \) for \( \hbar \omega > 2I_0 \). In this case the main contribution to the integral comes from \( r \sim a \) and we get

\[
\text{Re}\sigma(\omega) = \frac{3\pi^2 e^2}{4} \frac{(I_0^2 g^2 a^4)}{\hbar}.
\]

(12)

Certainly, the presentation of \( I(r) \) in the form \( I_0 e^{-r/a} \) makes sense only for \( r \gg a \) This means that the numerical coefficient in Eq. (12) is not reliable.

Clearly, at large frequencies \( \omega \gg \omega_0 \) the conductivity of the Anderson insulator should have the Drude form. The fact that Eq. (12), calculated for strongly localized electrons, is also frequency independent allows us to assume that the description of a.c. transport in the
Anderson insulator based on Hamiltonian (3) is accurate within a numerical coefficient. In other words, we assume that despite the complex structure of the electron wave-functions in the Anderson insulator the energy dependence of the matrix elements calculated between these functions is still given by (7).

Eq. (12) provides yet another way to estimate \( I_0 \) in the limit \( k_F l \gg 1 \). Namely, it matches \( \sigma_0 \) if we take \( I_0 \sim (k_F l)^{1/2}/g\xi^2 \). We see that the dependence of \( I_0 \) on \( \xi \) in both estimates is the same; the extra factor \((k_F l)^{1/2}\) presumably can be accounted for the \( \ln(k_F l) \) corrections to the exponent of \( \xi \).

There is also another argument in favor of the above estimate for \( I_0 \). The frequency dependence of \( \text{Re}\sigma(\omega) \) in the Anderson insulator becomes strong for \( \omega \ll \omega_0 \), whereas in the picture of strongly localized electrons the demarcation frequency is \( \omega \sim I_0/\hbar \). Equating \( I_0 \) to \( \hbar\omega_0 \) we get \( I_0 = k_F l/g\xi^2 \), with another extra factor \( k_F l \).

The simplified description of the Anderson insulator based on the Hamiltonian (3) allows one to include into consideration the Coulomb correlations (i.e., the correlations in the occupation numbers of the localized states caused by electron-electron interactions) using the ideas first spelled out in Refs. 8, 7, 9. This is the main goal of the present paper. We study the effect of Coulomb correlations on both \( \text{Re}\sigma(\omega) \) and \( \text{Im}\sigma(\omega) \). The most drastic conclusion we come to is that due to modification of \( \text{Im}\sigma(\omega) \) by the Coulomb correlations a system of localized electrons can support surface plasmons within a certain frequency range. We also show that these plasmons cause an additional structure in the behavior of \( \text{Re}\sigma(\omega) \) at \( \omega > \omega_0 \sim I_0/\hbar \).

The paper is organized as follows. In Section [I] we analyze the polarizability of the localized system in the absence of Coulomb correlations. In Section [II] we introduce the Coulomb correlations and find the dispersion law for the surface plasmons. In Section [III] we study the corrections to the dispersion law due to the resonant scattering of plasmons by pairs of localized states. In Section [IV] we calculate the plasmon contribution to \( \text{Re}\sigma(\omega) \). Section [V] concludes the paper.
II. POLARIZABILITY IN THE ABSENCE OF COULOMB CORRELATIONS

In this section we demonstrate that without Coulomb correlations the 2D Anderson insulator cannot support surface plasmon.

In the framework of the linear response theory the dispersion law of a plasmon, $\omega(q)$, is determined from the condition

$$1 = v(q) \text{Re} P(\omega, q),$$

(13)

where $v(q)$ is the Fourier component of the electron-electron interaction $v(r)$ and $P(\omega, q)$ is the polarization operator. Within a standard approach $P(\omega, q)$ is calculated for non-interacting electrons described by Hamiltonian $\mathcal{H}$:

$$P(\omega, q) = \frac{1}{A} \sum_{ij} |\langle i|e^{iqr}|j \rangle|^2 \frac{n_i - n_j}{\hbar \omega + E_i - E_j + i\delta}.$$

(14)

In the absence of disorder the eigenstates $|i\rangle$ and $|j\rangle$ are the plain waves so that the matrix element in (14) reduces to the delta-function $\delta(i - j - q)$. Then evaluating $P(\omega, q)$ and substituting it into (13) together with 2D Coulomb interaction $v(q) = 2\pi e^2/\kappa q$ ($\kappa$ is the dielectric constant) yields the surface plasmon with the well-known dispersion law

$$\omega(q) = \left[\frac{2\pi ne^2 q}{m\kappa}\right]^{1/2},$$

(15)

where $n$ is the 2D concentration of electrons. The plasmon mode is undamped if $q < \omega/v_F$ where $v_F = (4\pi n)^{1/2}\hbar/m$ is the Fermi velocity (for larger $q$ the Landau damping leads to a finite $\text{Im}P$). The latter condition can be rewritten as $q < 1/2a_B$ where $a_B = \hbar^2 \kappa/me^2$ is the Bohr radius.

In the case of a strong disorder the eigenstates $i$ and $j$ in (14) are the localized states. The polarization operator (14) can be evaluated in a way similar to that employed in the Introduction for calculation of a.c. conductivity. For small $q$ the matrix element in (14) can be evaluated using (7):

$$\langle i|e^{iqr}|j \rangle = \frac{iqr I(r)}{\Gamma}.$$

(16)
Then the contribution to $P(\omega, q)$ from the pairs with the shoulder $r$ [density $\mathcal{P}(\omega, q, r)$ of the polarization operator] can be presented as

$$
\text{Re}\mathcal{P}(\omega, q, r) = (qr)^2 I^2(r) \int_{2I(r)}^{\infty} d\Gamma \frac{F(\Gamma, r)}{\Gamma (\hbar\omega)^2 - \Gamma^2},
$$

(17)

where $\Gamma$ and $F(\Gamma, r)$ is given by Eqs. (6) and (9), respectively (the integral is understood as principal part). Substituting $F(\Gamma, r)$ into (17) and integrating over $\Gamma$ we obtain

$$
\text{Re}\mathcal{P}(\omega, q, r) = 2g^2 (qr)^2 I^2(r) \int_{2I(r)}^{\infty} d\Gamma \frac{\Gamma}{\sqrt{\Gamma^2 - 4I^2(r)[(\hbar\omega)^2 - \Gamma^2]}}
$$

$$
= -\frac{\pi g^2 (qr)^2 I^2(r)}{\sqrt{4I^2(r) - (\hbar\omega)^2}}, \quad \text{for} \quad \hbar\omega < 2I(r),
$$

$$
= 0, \quad \text{for} \quad \hbar\omega > 2I(r).
$$

(18)

We see that $\text{Re}\mathcal{P}(\omega, q, r)$ is either negative or zero, so that Eq. (13) cannot be satisfied.

In fact, the result (18) is almost obvious. Indeed, the imaginary part of polarization operator density, $\text{Im}\mathcal{P}(\omega, q, r)$, at small $q$ differs from $\text{Re}\sigma(\omega, r)$ in (8) by a factor $e^{2\omega/q^2}$, so it follows from (8) and (9) that

$$
\text{Im}\mathcal{P}(\omega, q, r) = -\frac{\pi g^2 (qr)^2 I^2(r)}{\sqrt{(\hbar\omega)^2 - 4I^2(r)}}, \quad \text{for} \quad \hbar\omega > 2I(r),
$$

$$
= 0, \quad \text{for} \quad \hbar\omega < 2I(r).
$$

(19)

Since $\text{Re}\mathcal{P}(\omega, q, r)$ and $\text{Im}\mathcal{P}(\omega, q, r)$ are connected via the Kramers-Kronig relation, the form (19) immediately follows from (18).

### III. COULOMB CORRELATIONS AND SURFACE PLASMON

In the previous section the polarization operator was evaluated using the pair density (8) which was derived for non-interacting electrons. As it was first pointed out by Efros, interactions modify strongly the density of singly-occupied pairs. The underlying physics is the following. A pair can be singly-occupied even if both energy states reside below the Fermi level. The right condition for the pair to be singly-occupied is that the addition of
a second electron (which interacts with the first one) is energetically unfavorable. Such a Coulomb correlations effectively enhance the density of “soft” pairs (i.e., the pairs with small excitation energy $\Gamma$).

Our goal is to apply the latter argument, which was presented for strongly localized system, to the Anderson insulator with large $\xi$. In order to do so we will adopt two assumptions:

(i) The interactions do not change the localization radius $\xi$.

(ii) The estimate for the overlap integral, $I_0 \sim 1/g\xi^2$, is unchanged in the presence of interactions.

In other words, we assume that switching on the interactions leads to the Coulomb shifts of the eigenenergies but does not affect the wave functions. Note that assumptions (i) and (ii) contradict those made in Refs. 11 and 12, respectively.

As we will see below, the relevant pairs would be those with shoulder $r \sim \xi$. In other words, the relevant transitions shift the position of electron by $\sim \xi$. To establish the form of the density of singly occupied pairs $F(\Gamma, r)$ with such a shoulder one can argue as follows.

An isolated region of a size $\xi$ can be viewed as a small metallic granule. The transfer of an additional electron into this granule leads to the charging energy $U = e^2/2C$, where $C$ is the capacitance of a granule. In other words, the levels in the granule get shifted by an amount $\sim U$. Consider now two neighboring granules and assume that $U \gg I_0$. Due to aforementioned charging effect the highest occupied levels in the two granules typically differ by $\sim U$. Let for concreteness the highest occupied level in the first granule be higher by $U$ than in the second one. Then the sought singly occupied pair with frequency $\Gamma$ can be composed from the top occupied states in the first granule (these states should belong to the energy interval $\Gamma + U$ measured from the highest occupied level) and the empty states in the second granule. Then the density of pairs $F(\Gamma, r)$ (with $r \sim \xi$) can be estimated as $g^2(\Gamma + U)$. With the energy splitting taken into account it can be written as

$$F(\Gamma, r) = \frac{2g^2(\Gamma + U)}{\sqrt{\Gamma^2 - 4I^2(r)}}. \quad (20)$$
Then at \( U = 0 \) we return to (4). Certainly, our consideration, based on artificial arranging the localized states into the granules, provides only the order of magnitude estimate of \( F(\Gamma, r) \). In particular, the numerical coefficient in (20) cannot be found from such a consideration. Our choice of numerical coefficient in (20) provides matching with a similar expression for strongly localized regime.

With the pair density (20) we can now easily evaluate \( \text{Re} P(\omega, q, r) \). Calculating the integral over \( \Gamma \) in (17) yields

\[
\text{Re} P(\omega, q, r) = \frac{\pi g^2(qr)^2 I^2(r)}{\sqrt{4I^2(r) - (\hbar \omega)^2}} \left\{ 1 + \frac{2}{\pi} \frac{U}{\hbar \omega} \arctan \left[ \frac{\hbar \omega}{\sqrt{4I^2(r) - (\hbar \omega)^2}} \right] \right\}, \quad \text{for} \quad \hbar \omega < 2I(r),
\]

\[
= \frac{U}{\hbar \omega} \frac{2g^2(qr)^2 I^2(r)}{\sqrt{(\hbar \omega)^2 - 4I^2(r)}} \ln \left[ \frac{(\hbar \omega)^2 - 4I^2(r) + \hbar \omega}{2I(r)} \right], \quad \text{for} \quad \hbar \omega > 2I(r). \tag{21}
\]

The expression (21) for \( \text{Re} P(\omega, q, r) \) can be also obtained, using the Kramers-Kronig relations, from \( \text{Im} P(\omega, q, r) \) which has a simple form

\[
\text{Im} P(\omega, q, r) = -\frac{\pi g^2(qr)^2 I^2(r)}{(\hbar \omega)^2} F(h\omega, r) = -\frac{\pi g^2(qr)^2 I^2(r)}{\sqrt{(\hbar \omega)^2 - 4I^2(r)}} \left( 1 + \frac{U}{\hbar \omega} \right), \quad \text{for} \quad \hbar \omega > 2I(r),
\]

\[
= 0, \quad \text{for} \quad \hbar \omega < 2I(r). \tag{22}
\]

Note that as in (19), \( \text{Im} P(\omega, q, r) \neq 0 \) only for \( \hbar \omega > 2I(r) \).

We see that the enhancement in the density of pairs with small \( \Gamma \) leads to a positive sign of \( \text{Re} P(\omega, q, r) \) for \( \hbar \omega > 2I(r) \). This is our main observation.

In order to obtain \( \text{Re} P(\omega, q) \) one should integrate \( \text{Re} P(\omega, q, r) \) over \( r \). For \( \hbar \omega > 2I_0 \) the main contribution to this integral comes from \( r \sim \xi \) [like in derivation of Eq. (12)] and we get

\[
\text{Re} P(\omega, q) = \frac{3\pi}{4} \frac{q^2 U I_0^2 g^2 \xi^4}{(\hbar \omega)^2} \ln \left( \frac{\hbar \omega}{I_0} \right). \tag{23}
\]

For generality we will assume that there is also a gate at a distance \( d \) from the plane of 2D electrons. In this case the Fourier component of electron-electron interaction has the form

\[
v(q) = \frac{2\pi e^2}{\kappa q} \left( 1 - e^{-2qd} \right). \tag{24}
\]
If the gate is close to the electron plane, that is $q d \ll 1$, we can expand the exponent in (24) and get $v(q) = 4\pi e^2 d/\kappa$. If $d \ll \xi$ then the capacitance $C$ reduces to the capacitance of two disks with area $S = \xi^2$ separated by a distance $d$, so that $C = \kappa S/4\pi d$ and consequently $U = 4\pi e^2 d/\kappa \xi^2$. With these $v(q)$ and $U$, after substituting (23) into the plasmon equation (13), we obtain

$$1 = \frac{3\pi}{4} (q \xi)^2 \left( \frac{U}{\hbar \omega} \right)^2 (I_0 g \xi^2)^2 \ln \left( \frac{\hbar \omega}{I_0} \right).$$

(Certainly, the numerical coefficient in (25) should not be taken seriously. According to the assumption (ii), $I_0 \sim g \xi^2$. Then Eq. (25) yields the following dispersion law for the surface plasmon

$$q(\omega) = \frac{1}{\xi} \left( \frac{\hbar \omega}{U} \right) \ln^{-1/2} \left( \frac{\hbar \omega}{I_0} \right).$$

We see that the dispersion law is close to acoustic.

Let us establish the frequency range for the surface plasmon with dispersion law (26). The validity of expansion (13), $q \xi \ll 1$, implies that $U \gg \hbar \omega \ln^{-1/2} (\hbar \omega / I_0)$. On the other hand, $\hbar \omega > 2I_0$. Then the frequency range for plasmon is $2I_0 \lesssim \hbar \omega \lesssim U$. The necessary condition for this range to be wide is $U \gg I_0$. The ratio $U/I_0 = 4\pi e^2 d/\xi^2 I_0$ with $I_0 \sim 1/g \xi^2$ can be presented as $8\pi^2 d/a_B$, where $a_B = \hbar^2 \kappa / me^2$ is the Bohr radius. Thus, the condition $d \gg a_B$ insures that the plasmon equation (13) has a solution within a wide frequency range. If $d < a_B$, the screening of Coulomb interaction by the gate is strong and the number of soft pairs is not sufficient to change the sign of $\text{Re}P(\omega, q)$.

A similar condition can be obtained from the analysis of $\text{Im}P(\omega, q)$, which can be derived by integration of Eq. (22) over $r$

$$\text{Im}P(\omega, q) = \frac{3\pi^2}{8} \frac{q^2 I_0^2 g^2 \xi^4}{(\hbar \omega)^2} (\hbar \omega + U) \sim \frac{q^2}{(\hbar \omega)^2} (\hbar \omega + U).$$

The origin of $\text{Im}P(\omega, q)$ is the interaction of a plasmon with “resonant” pairs having excitation energy $\omega$. In fact, $\text{Im}P(\omega, q)$ describes the resonant scattering of a plasmon by a pair of localized states. One can introduce a mean free path, $l$, associated with such a scattering
and obtain \( qI \sim \ln^{1/2}(\hbar\omega/I_0)/(1 + \hbar\omega/U) \). Thus, the condition \( 2I_0 \lesssim \hbar\omega \lesssim U \) reduces to the condition \( qI \gg 1 \).

In the absence of gate \([qd \gg 1]\) one should take \( U = e^2/\kappa\xi \), so that \( U/I_0 \sim \xi/a_B \gg 1 \). Then after a simple algebra we obtain

\[
q(\omega) = \frac{1}{\xi} \left( \frac{\hbar\omega}{U} \right)^2 \ln^{-1} \left( \frac{\hbar\omega}{I_0} \right).
\]

(28)

It can be shown that the above analysis of the validity applies in this case as well and leads to the same frequency range \( 2I_0 \lesssim \hbar\omega \lesssim U \). Within this range we again have \( q\xi \ll 1 \).

Thus, one should use Eq. (26) for \( qd < 1 \) and Eq. (28) for \( qd > 1 \). On the other hand, the magnitude of \( U \) depends on the ratio \( d/\xi \). Note that for \( d > \xi \) one still can have \( qd < 1 \). In this case the dispersion law is given by Eq. (26) with \( U = e^2/\kappa\xi \).

**IV. RENORMALIZATION OF THE PLASMON DISPERSION LAW**

In the previous section, when calculating the polarization operator, we took into account the Coulomb correlations within a pair, but neglected the effect of polarization of surrounding pairs on a given pair. On the other hand, by averaging of the polarization operator \([14]\) over frequencies of pairs \( \Gamma \) and their shoulders \( r \) we have effectively replaced the localized system by a medium. The average polarization of this medium gave rise to a plasmon mode. Within this procedure the “feedback” from surrounding pairs reduces to the interaction of a given pair with plasmons. In the present section we study the renormalization of the plasmon spectrum due to this effect.

Generally, the plasmon excitation is defined as a pole in the density-density correlation function, \( \Pi(\omega, q, q') \), which is related to the polarization operator \( P(\omega, q, q') \) by the Dyson equation

\[
\Pi(\omega, q, q') = P(\omega, q, q') + \int \frac{dq_1}{(2\pi)^2} P(\omega, q, q_1) v(q_1) \Pi(\omega, q_1, q').
\]

(29)

Before averaging, both \( P(\omega, q, q') \) and \( \Pi(\omega, q, q') \) depend on two momentum variables \( q_1 \).
and \(q'\). The approximation we made above reduces to replacing of \(P(\omega, q, q')\) by its average \(P(\omega, q)\), so that the solution of (29) takes the form
\[
\Pi(\omega, q) = \frac{P(\omega, q)}{1 - v(q)P(\omega, q)}.
\]
(30)

Then the pole of \(\Pi(\omega, q)\) is determined by the plasmon equation (13).

As a next step, we took for \(P(\omega, q)\) its expression (14) for non-interacting electrons, which represents a sum of polarizations of pairs,
\[
P(\omega, q) = \frac{1}{A} \sum_{ij} |\langle i| e^{iqr}| j \rangle|^2 P_{ij}(\omega), \quad P_{ij}(\omega) = \frac{n_i - n_j}{\hbar\omega + E_i - E_j + i0},
\]
(31)
and performed the summation neglecting correlations between the pairs but with the Coulomb correlations within a pair included.

The renormalized \(P_{ij}(\omega)\) for a given pair can be obtained from the following procedure. The function \(\Pi(\omega, q)\) has a diagrammatic presentation in a form of a series of bubbles (\(ij\)), corresponding to \(P_{ij}\), connected by the Coulomb interaction lines (see Fig. 1a). First, we arrange into a single block the sum over all combinations of bubbles which appear between two bubbles (\(ij\)) (see Fig. 1b). Then we replace this block by its average, so that the result can be presented as two bubbles (\(ij\)) connected by a plasmon propagator \(\Pi(\omega, q)\) (see Fig. 1b) (there is also an extra factor \(v(q)\) in each vertex). The renormalized bubble (\(ij\)) can be then obtained by summing up the series, consisting from the bubbles (\(ij\)), connected by plasmon lines (see Fig. 1c). The resulting expression for \(P_{ij}(\omega)\) reads
\[
P_{ij}(\omega) \equiv \frac{P_{ij}(\omega)}{1 - P_{ij}(\omega)R_{ij}(\omega)},
\]
(32)
with
\[
R_{ij}(\omega) = \int \frac{d\mathbf{q}}{(2\pi)^2} |\langle i| e^{iqr}| j \rangle|^2 v^2(q)\Pi(\omega, q).
\]
(33)

Finally, replacing \(P_{ij}(\omega)\) in (31) by \(P_{ij}(\omega)\) we obtain
\[
P(\omega, q) = \frac{1}{A} \sum_{ij} |\langle i| e^{iqr}| j \rangle|^2 \frac{n_i - n_j}{\hbar\omega + E_i - E_j - (n_i - n_j)R_{ij}(\omega)}.
\]
(34)
The equations (30), (33), and (34) form a closed system which determines \( \Pi(\omega, q) \) and, correspondingly, the renormalized dispersion law of a plasmon in a self-consistent way. The approximation made in order to get the closed system [replacement of a block by a sought function \( \Pi(\omega, q) \)] is known as the effective-medium approximation.

The analysis of the system (30,33,34) reveals that the renormalization of the plasmon dispersion law is weak. Namely, the appearance of the term \( R_{ij}(\omega) \) in the denominator of (34) has the physical meaning that a pair \( (ij) \) acquires a finite life-time \( \tau_{ij} \) due to the interaction with plasmons. We will show that this life-time is long, i.e., \( 1/\tau_{ij} \ll \omega_{ij} \). It can be readily seen that the difference between the renormalized polarization \( \text{Re} P(\omega, q) \) and \( \text{Re} P(\omega, q) \) originates from resonant pairs with \( (\omega - \omega_{ij}) \sim 1/\tau_{ij} \) and \( r \sim \xi \) (note that the main term, \( \text{Re} P(\omega, q) \), is determined by the entire interval \( \omega_{ij} \sim \omega \)). If we neglect the dependence of the matrix element in (34) on \( \omega_{ij} \) then the renormalization correction to \( \text{Re} P(\omega, q) \) would be identically zero. A finite correction results from a slight asymmetry of the matrix element within the narrow interval \( (\omega - \omega_{ij}) \sim 1/\tau_{ij} \). Then the relative magnitude of the correction is of the order of \( 1/\omega \tau_{ij} \) with \( \tau_{ij} \) calculated for a pair with \( \omega_{ij} = \omega \). Expecting \( \tau_{ij} \) to be long, we can calculate it by substituting the nonrenormalized dispersion law of a plasmon into Eq. (33). Performing the integration we obtain

\[
R_{ij}(\omega) = \frac{\hbar}{\tau_{ij}} \sim \hbar \omega \left( \frac{\hbar \omega}{U} \right) \left( \frac{I_0}{U} \right)^2 \ln^2 \left( \frac{\hbar \omega}{I_0} \right), \quad \text{with gate,}
\]

\[
\sim \hbar \omega \left( \frac{\hbar \omega}{U} \right)^3 \left( \frac{I_0}{U} \right)^2 \ln^{-3} \left( \frac{\hbar \omega}{I_0} \right), \quad \text{without gate. (35)}
\]

Since both ratios, \( \hbar \omega / U \) and \( I_0 / U \) are small, the correction to the dispersion law, \( \delta q(\omega)/q(\omega) \sim 1/\omega \tau_{ij} \), is negligible.

In the next section we will see that the corresponding renormalization of \( \text{Re} \sigma(\omega) \) is much larger than the renormalization of the dispersion law.
V. RENORMALIZATION OF THE REAL PART OF THE CONDUCTIVITY

As we have seen in the previous section, the renormalization of the polarization operator results in an appearance of $i\hbar/\tau_{ij}$ in the denominator of Eq. (34). Correspondingly, the renormalized expression (4) for $\text{Re} \sigma(\omega)$ can now be rewritten as

$$\text{Re} \sigma(\omega) = e^2 \omega \hbar A \sum_{ij} \frac{\left[ I_{ij}(x_i - x_j) \right]^2}{\hbar \omega_{ij}} \frac{1}{\omega + \omega_{ij} + i/\tau_{ij}},$$

(36)

where the summation is performed over the singly occupied pairs $ij$ and in this way the Coulomb correlations are taken into account. In the limit $\tau_{ij} \to \infty$ and for $\omega > 2I_0$ one should use the pair density $F(\Gamma, r)$, given by Eq. (20), in order to perform the summation. The result is determined by resonant pairs with $\omega = \omega_{ij} = \Gamma/\hbar$:

$$\text{Re} \sigma(\omega) = \frac{3\pi^2 e^2}{4} \frac{\hbar}{\bar{h}} (I_0 g \xi^2)^2 \left(1 + \frac{U}{\hbar \omega}\right) = \sigma_0 \left(1 + \frac{U}{\hbar \omega}\right).$$

(37)

We see that within the frequency interval $2I_0 < \hbar \omega < U$ the real part of the conductivity exceeds the Drude value due to Coulomb correlations.

When calculating the correction to $\text{Re} \sigma(\omega)$ caused by the finite value of $\tau_{ij}$ it is important to realize that $\hbar/\tau_{ij}$ is maximal for soft pairs with small $\omega_{ij}$. This is because the matrix element $\langle i | e^{iqr} | j \rangle$ is proportional to $1/\omega_{ij}$. In the previous Section this was not important since the correction to $\text{Re} P(\omega, q)$ came from the resonant pairs only. Here, however, we have $\text{Im}(\omega + \omega_{ij} + i/\tau_{ij})^{-1} \propto 1/\tau_{ij} \propto 1/\omega_{ij}^2$, so that the soft pairs give the main contribution to the correction $\delta \text{Re} \sigma(\omega)$. Assuming $\omega_{ij} \ll \omega$ we can present this correction in the form

$$\text{Re} \sigma(\omega) = e^2 \frac{\hbar}{\hbar \omega A} \sum_{ij} \left[ \frac{I_{ij}(x_i - x_j)}{\hbar \omega_{ij}} \right]^2 \int \frac{d\mathbf{q}}{(2\pi)^2} q^2 v^2(q) \text{Im} \Pi(\omega, q),$$

(38)

where we have substituted $\hbar/\tau_{ij} = R_{ij}$ from Eq. (33). The sum over pairs is again evaluated with the pair density (20)

$$\frac{1}{A} \sum_{ij} \left[ \frac{I_{ij}(x_i - x_j)}{\hbar \omega_{ij}} \right]^2 = \frac{1}{A} \int d\mathbf{r} r^4 I^4(r) \int_{2I(r)}^{\infty} d\Gamma F(\Gamma, r) \frac{F(\Gamma, r)}{\Gamma^4}.$$  

(39)
The main contribution to the integral over $\Gamma$ comes from the lower limit $\Gamma \sim I(r)$. Then the integral over $r$ is again determined by $r \sim \xi$, so that the sum (39) appears to be $\sim I_0 \xi^6 g^2 U$.

As in the previous section, the value of the integral in (39) takes different values in the presence and in the absence of a gate. Finally we obtain

$$\frac{\delta \text{Re}\sigma(\omega)}{\sigma_0} \sim \left(\frac{\hbar \omega}{U}\right)^5 \left(\frac{\hbar \omega}{I_0}\right) \ln^{-3} \left(\frac{\hbar \omega}{I_0}\right),$$

with gate,

$$\sim \left(\frac{\hbar \omega}{U}\right)^3 \left(\frac{\hbar \omega}{I_0}\right) \ln^{-2} \left(\frac{\hbar \omega}{I_0}\right),$$

without gate. (40)

Comparing (35) to (40) we see that both corrections are of the same order $(I_0/U)^2$ at $\hbar \omega \sim I_0$.

However the correction to Re$\sigma$ is much bigger at $\hbar \omega \gg I_0$. This reveals a new mechanism of absorption of a.c. field: by resonant excitation of plasmons. More precisely, the field polarizes the soft pairs (with $\omega_{ij} \sim I_0/\hbar$) and the induced polarization excites the plasmon waves. Therefore, the energy of a.c. field is effectively absorbed by plasmons. The rapid increase of $\delta \sigma$ with $\omega$ is caused by the number (phase volume) of plasmons which absorb the field.

Note that with correction $\delta \sigma(\omega)$ the total conductivity Re$\sigma(\omega)$ exhibits a rather complicated behavior. For $\hbar \omega < 2I_0$ the conductivity increases with $\omega$. Then it passes through a maximum at $\omega \sim I_0/\hbar$ and falls off with $\omega$ according to Eq. (37). However at $\hbar \omega \sim U(I_0/U)^{1/4}$ (with gate) and $\hbar \omega \sim U(I_0/U)^{1/6}$ (without gate) we have $\delta \text{Re}\sigma(\omega) \sim \text{Re}\sigma(\omega)$ and the conductivity starts rising again. On the other hand, the expression for $\delta \text{Re}\sigma$ was derived assuming that it is small. Therefore in the region $\delta \text{Re}\sigma > \text{Re}\sigma(\omega)$ the renormalization of $\sigma(\omega)$ by plasmons is strong. In this case one cannot calculate $\tau_{ij}$ using the bare polarization operator. The full analysis of the system (30, 33, 34) in this frequency range is out of the scope of the present paper.

VI. CONCLUSION

In the present paper we argue that the wave of electric field can propagate along the surface of the 2D Anderson insulator. The field originates from the density fluctuations of
localized electrons. One should distinguish this wave from the usual plasmon in an ideal 2D gas with the dispersion law given by Eq. (13): (i) the derivation of (13) implies that \( \omega \gg 1/\tau \) while we predict the existence of a plasmon at much lower frequencies \( \omega \gtrsim I_0/\hbar \). The minimal frequencies \( 1/\tau \) and \( I_0/\hbar \) differ by a factor \( \exp(\pi k_Fl) \); (ii) the plasmon (13) results from the solution of Eq. (13) with polarization operator calculated for free electrons. Within this approximation Eq. (13) has no solutions for localized electrons. The solution appears only if one takes into account the Coulomb correlations in the occupation numbers of the localized states.

The obvious consequence of the existence of a plasmon excitation is that localized electrons “feel” a fluctuating electric field \( \mathcal{E}(\omega, q) \) with the spectral density

\[
\langle |\mathcal{E}(\omega, q)|^2 \rangle = \frac{\kappa^2}{2\pi e^2} \frac{q^2v^2(q)\text{Im}P(\omega, q)}{[1 - v(q)\text{Re}P(\omega, q)]^2 + [v(q)\text{Im}P(\omega, q)]^2},
\]

where \( P(\omega, q) \) is the polarization operator (34). The spectral density (41) has a peak at \( \omega = \omega(q) \) corresponding to the dispersion law of a plasmon, which is different with and without gate.

The basic assumption of our theory is that in the presence of interactions the Anderson insulator is characterized by two energy scales: \( I_0 \) - the spacing between energy levels in the area \( \xi^2 \) and the charging energy \( U \) which is the Coulomb interaction of two localized electrons separated by a distance \( \sim \xi \). This energy modifies the pair density \( F(\omega, r) \). Such a modification results in an enhancement\(^7\text{-}9\)\(^,\)\(^14\) of the dissipative conductivity \( \text{Re}\sigma(\omega) \). If we denote as \( \rho(\omega) \) the matrix element of \( r \) calculated for a pair with frequency \( \omega \) then

\[
\text{Re}\sigma(\omega) \propto \omega \rho^2(\omega)F(\omega, r_\omega).
\]

As it was discussed in the Introduction, it is plausible to assume that for the Anderson insulator in the absence of interactions \( \rho(\omega) \) behaves as \( \rho(\omega) \sim r_\omega \) for \( \omega \lesssim I_0/\hbar \) and \( \rho(\omega) \sim \xi I_0/\omega \) for \( \omega \gtrsim I_0/\hbar \). Since the pair density in the absence of interactions is proportional to \( \omega \) we reproduce Eqs. (11) and (12):

\[
\text{Re}\sigma(\omega) \propto \omega^2, \quad \text{for} \quad \omega \lesssim I_0/\hbar,
\]

(43a)
\[
\text{Re}\sigma(\omega) = const, \quad \text{for } \omega \gtrsim I_0/\hbar,
\] (43b)

Where we neglected a logarithmic factor in (43a). We have assumed that the frequency dependence of \(\rho(\omega)\) remains the same in the presence of interactions. At the same time, \(F(\omega, \xi)\) is changed drastically by interactions: \(F(\omega, \xi) = \text{const} \text{ for } \omega \lesssim U/\hbar\) and \(F(\omega, \xi) \propto \omega\) for \(\omega \gtrsim U/\hbar\). As a result, we get the following frequency dependence of \(\text{Re}\sigma(\omega)\) in the presence of interactions [see also (37)]:

\[
\begin{align*}
\text{Re}\sigma(\omega) \propto \omega, & \quad \text{for } \omega \lesssim I_0/\hbar, \\
\text{Re}\sigma(\omega) \propto 1/\omega, & \quad \text{for } I_0/\hbar \lesssim \omega \lesssim U/\hbar, \\
\text{Re}\sigma(\omega) = \text{const}, & \quad \text{for } \omega \gtrsim U/\hbar,
\end{align*}
\] (44a, b, c)

This simple analysis shows that \(\text{Re}\sigma(\omega)\) exhibits a maximum at \(\omega \sim I_0/\hbar\) (see also the end of the previous section). In fact, this maximum is intimately related to the existence of a plasmon. Indeed, for Eq. (13) to have solutions we need a positive sign of \(\text{Re}\mathbf{P}(\omega, q)\), which is equivalent to a positive \(\text{Im}\sigma(\omega)\). At the same time, \(\text{Im}\sigma(\omega)\) can be obtained from \(\text{Re}\sigma(\omega)\) using the Kramers-Kronig relation. In order to trace how the modification of \(\text{Re}\sigma(\omega)\) by Coulomb correlations leads to the change of sign of \(\text{Im}\sigma(\omega)\), we can interpolate the frequency dependence of \(\rho(\omega)\) as \(\rho(\omega) \propto [(\hbar\omega)^2 + 4I_0^2]^{-1/2}\) and the frequency dependence of \(F(\omega, \xi)\) as \(F(\omega, \xi) \propto (\hbar\omega + U)\). Then we have

\[
\text{Re}G(\omega) = \frac{\hbar\omega(\hbar\omega + U)}{(\hbar\omega)^2 + 4I_0^2}, \quad \omega > 0,
\] (45)

where \(G = \frac{\sigma}{e^2/\hbar}\) is the conductance. It can be easily seen that Eq. (45) reproduces correctly all the limiting cases (44) and (44) both with and without charging effect. The imaginary part of the conductance calculated from (45) has a simple form:

\[
\text{Im}G(\omega) = -\frac{\hbar\omega[2I_0 - (2U/\pi) \ln(\hbar\omega/2I_0)]}{(\hbar\omega)^2 + 4I_0^2}, \quad \omega > 0,
\] (46)

In Fig. 2 we have plotted \(\text{Re}G(\omega)\) and \(\text{Im}G(\omega)\) for different ratios \(U/2I_0\). We see that in the absence of Coulomb correlations \((U = 0)\), \(\text{Im}G(\omega)\) is strictly negative (in our calculation in
Section it turns to zero for $\omega > I_0/\hbar$). However, at finite $U$ the change of sign occurs at $\omega = (2I_0/\hbar) \exp(\pi I_0/U)$. For $U \gg I_0$ this frequency is just $2I_0/\hbar$.

As a final remark, let us outline the difference between our approach and that of Ref. [10]. In Ref. [10] the authors addressed electron-electron interactions in the strongly localized regime when the localization radius is much smaller than the interpair separation. In this case singly-occupied pairs can be considered as point-like dipoles. The dipole moment $p_k$ induced by an external field $E_0 e^{-i\omega t}$ can be written as

$$p_k^\mu = \alpha_k^{\mu\nu}[E_0^\nu + \sum_{l \neq k} E_k^{(l)\nu}], \quad (47)$$

where $E_k^{(l)\nu}$ is a component $\nu$ of the electric field caused by polarization of a dipole $l$ which acts on the dipole $k$ (summation over repeating indices $\nu$ is implied). The polarizability $\alpha_k^{\mu\nu}$ of a dipole $k$ has the form

$$\alpha_k^{\mu\nu} = \frac{2e^2}{\hbar} \rho_k^\mu \rho_k^\nu \omega_k = \frac{e^2}{\hbar} \rho_k^\mu \rho_k^\nu \sum_{ij} P_{ij}, \quad (48)$$

where $\rho_k^\mu$ and $\omega_k$ are correspondingly the matrix element and the frequency of the dipole $k$. Here $P_{ij}$ is given by Eq. (31) with the states $i$ and $j$ making up the dipole $k$. To keep the discussion simple we will assume that the polarizability is isotropic, i.e., $\alpha_k^{\mu\nu} = \alpha_k \delta_{\mu\nu}$. The field $E_k^{(l)\nu}$ can, in turn, be expressed through $p_k$ as

$$E_k^{(l)\mu} = -\frac{\partial}{\partial R^\mu} \frac{p_l^\nu R}{R^3} \bigg|_{R=R_k-R_l}, \quad (49)$$

where $R_k$ is the position of the dipole $k$.

Upon substituting (15) into (17) we obtain an infinite system of linear equations. It can be easily shown that iterating this system leads to the renormalization of polarizabilities of dipoles $\alpha_k$

$$\alpha_k = \frac{e^2}{\hbar} \rho_k^2 \sum_{lj} \frac{P_{lj}}{1 - P_{lj} \Sigma_k} = \frac{2e^2}{\hbar} \rho_k^2 (\omega_k + \Sigma_k) \rho_k^\mu (\omega_k + \Sigma_k)^2, \quad (50a)$$

$$\Sigma_k = -\sum_l \alpha_l \left( \frac{\partial}{\partial R^\mu} \frac{R^\nu}{R^3} \right)_{R=R_k-R_l} \left( \frac{\partial}{\partial R^\nu} \frac{R^\mu}{R^3} \right)_{R=R_k-R_l} + \cdots. \quad (50b)$$

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Here $\Sigma_k$ is the self-energy. Fleishman and Anderson\textsuperscript{16} analyzed this self-energy using the arguments similar to those put forward by Anderson\textsuperscript{17} when he demonstrated the existence of localization transition for eigenfunctions of the Schrödinger equation with disorder. They argued that $\text{Im}\Sigma$ takes a finite value with non-zero probability. This means that in the absence of an external field the system

$$p_k + \alpha_k \nabla \sum_{l \neq k} \frac{p_l R}{R^3} \bigg|_{R=R_k-R_l} = 0,$$

has delocalized solutions.\textsuperscript{18} In other words, by analogy to the Schrödinger equation in the tight-binding approximation, the eigenstates $\{p_k\}$ of the system (51) extend throughout the entire volume.

Fleishman and Anderson\textsuperscript{16} considered a three-dimensional system and neglected the Coulomb correlations. Note that if one rewrites the system (51) in the momentum representation and averages it by factorizing the average of the product $\alpha p$ (mean-field) then one arrives at the plasmon equation (13). As it was demonstrated in Section II, this equation has no propagating solutions in the absence of Coulomb correlations. Our central point is that with the Coulomb correlations the delocalized solution exists even at the mean-field level. This solution, that is surface plasmon, is specific for the two-dimensional system and is characterized by the dispersion law $\omega(q)$.

An interesting question that could be addressed within the same approach is how the Coulomb correlations modify the a.c. Hall conductivity $\sigma_{xy}(\omega)$ of the Anderson insulator.\textsuperscript{19}

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FIGURES

FIG. 1. (a) Diagrammatic presentation of the density-density correlation function $\Pi(\omega, q)$; bubble $(ij)$ stands for $P_{ij}(\omega)$ while dashed line corresponds to the Coulomb interaction $v(q)$. Each vertex contains matrix element $\langle i|e^{iqr}|j \rangle$. (b) The block connecting two bubbles $(ij)$ is replaced by a plasmon line. (c) The renormalization of the bubble $(ij)$ caused by interaction with plasmons.

FIG. 2. The real (a) and imaginary (b) parts of conductance is plotted as a function of dimensionless frequency $\hbar\omega/2I_0$ for $U/2I_0 = 0$ (solid line), $U/2I_0 = 3$ (long-dashed line), and $U/2I_0 = 8$ (dashed line).
\[ \Pi = \quad + \quad + \quad + \quad \cdots \cdots \] 

(a)

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(b)

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(c)

**Fig. 1**
Fig. 2