A branch oriented active power loss allocation method for radial distribution networks with distributed generators

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Abstract: The penetration of distributed generation (DG) units in the radial distribution network (RDN) introduces complexity in active power loss allocation (LA) as it leads to reverse current in the network. This current makes power system bidirectional and brings difficulties in the decomposition of cross-terms of power loss equation. To overcome such complications, this paper proposes a new branch oriented LA technique which eliminates the impact of mutual-term mathematically from loss formulation without any assumptions and approximations. It establishes a direct relationship between the subsequent load currents of a branch and its two end voltages in terms of the complex power available at its load ends. The proposed LA scheme is found to be fair as regard to the topology of the RDN. Further, the implementation of DGs may increase/decrease power loss of a system. In order to provide the exact amount of loss reduction benefit to the distributed generator owners (DGOs), a new DG remuneration strategy is developed in this paper which assigns either rewards/penalties to the DGOs after analysing their actual impact towards system loss reduction. The effectiveness of the proposed LA method is investigated against various established LA techniques using two different test systems i.e., 17-bus and 33-bus RDNs.

Keywords: Distributed Generators, Loss Allocation, Power flow, Line loss, Radial distribution network

1. Introduction

The electrical power sector in many countries has been either deregulated or it is in the process of getting deregulated. Several issues are coming up due to this as separate generations, transmissions, and distributions companies are being set up in place of a single state-owned vertically integrated structure, with the main objective, to introduce competition in order to bring down the cost of electricity and to
enhance the service quality. Therefore, it is essential that various wheeling activities should be clearly defined and the corresponding cost must be recovered [1]. The nonlinear relationship between power loss and the injected powers makes loss allocation (LA) process difficult and complicated [2]. In order to make power system more reliable and efficient, distributed generations (DGs) are penetrated at the load ends. Their penetration results reverse current in the network, which again brings difficulties in system LA. Further, the injection of DG power may decrease power loss of a system. From literature, it is observed, some established LA techniques are diverting a part of network loss reduction (NLR) benefit to the consumers’ side as cross-subsidies even if, their impact on radial distribution network (RDN) loss reduction are insignificant. This type of LA is unfair from the DG points of view. Hence, a proper LA scheme is to be formulated which can allocate losses among the network participants with/without DGs, judiciously.

Most of the loss allocation procedures found in literature are observed to be allocating losses in the transmission lines (TLs). Out of these, some LA techniques are improved further for implementation in distribution networks. In [2], a comprehensive analysis has been performed on the characteristics of several loss allocation methodologies. Pro-rata (PR) method [3] assigns losses to the end users as per their power consumptions/injections. Hence, this scheme is unfair to those customers which are placed near to the substation/root bus with equal power consumptions as that of customers connected far away. The problem associated with PR method is not found in MW-mile approach [4] as it allocates losses to the network participants with due consideration to their power consumptions and geographical locations. This approach performs LA by measuring the electrical distance of the load point from the substation bus, and multiplying it with the power rating of the end user under consideration. Still, above two methods execute LA without performing load flow (LF) calculations. Hence, may not be suggested for effective practical implementation. To overcome the drawbacks as stated above, two methods i.e., a Newton-Raphson LF based direct loss coefficient (DLC) technique and a substitution approach of LA were introduced in [5]. DLC scheme is found unsuitable for RDNs due to its high R/X ratio as compared to transmission systems. The total loss allocation of marginal/incremental transmission line (ITL) method [6] extensively used for TLs is always higher than that of the actual loss of the system as obtained from load flow calculation. Thus, it suffers from over-recovery of losses, and for final allocation, a reconciliation procedure is applied for settlement of this extra losses among the end users. Moreover, DLC and ITL techniques are computationally exhausted, as they make use of Hessian and Jacobian matrices, respectively. The
difficulties related to these computational competencies are not observed in the substitution approach [5] where the impact of network participants on system LA is evaluated individually by taking difference between the LA results as obtained in two scenarios i.e. when they are connected and not connected in the RDN. But, this procedure may not be recommended for fair LA as the sum of individual losses of the end-users is different from that of the net LA of the system. The network parameter based Z-bus [7] technique widely used for loss allocation of transmission systems is found to be unsuitable for overhead RDNs due to singularity of Y-bus matrix. Thus, for allocating losses without shunt elements, succinct method (SM) has been developed in [8]. SM establishes a linear relationship between line losses and bus injected power under the condition that network participants will maintain voltage profile within a feasible range as specified. Still, this process faces difficulties when the X/R ratio of a line is more than that of the Q/P ratio available at its receiving end. The LA procedure discussed in [9] performs loss allocation based on the quadratic scheme while method [10] uses a proportional sharing (PS) principle for sharing of losses among their participants. These two approaches may not be suggested for practical implementation as they assign entire loss of the system either to loads or to DG units. The above drawback is not found in the circuit-based branch current decomposition method (BCDM) [11], which is developed from the decomposition of branch current into injected currents and node currents. But, this process is found to be delivering more spatial cross-subsidies to the consumers in the presence of DGs. A tracing based power summation technique (PSMLA) is first introduced for power loss allocation in the distribution network in [12], and is further extended for energy allocation through a statistical analysis of daily load and generation curve in [13]. Savier and Das [14] have first applied a fuzzy-based network reconfiguration technique to find an optimal network and then used a quadratic scheme of LA to test the efficiency of the method before/after reconfiguration using several test systems. To overcome the difficulties associated with the quadratic scheme, they have also employed an exact method of LA based on node voltages and injected currents in [15] to allocate losses in a balanced RDN with/without DGs. But, it remains silent regarding DG remuneration. In [16], a novel power loss allocation scheme has been proposed for unbalanced radial power distribution system. Jahromi et al [17] have presented a three-stepped LA algorithm for allocating losses to both the consumers and distributed generator owners (DGOs). In the first step, buses with more DG capacities than that of their load values are awarded losses; in the next step, load points having more load capacities as compared to DGs are assigned with losses, and lastly, a reconciliation method has been applied to allocate residual losses among the network
participants. In [18], the traditional pro-rata method is modified further for allocating losses at various load levels. This scheme assigns losses to the end users with due consideration to their load demands and physical locations in the network. The technique developed in [19] decomposes the cross-term among the participants as per their contractual power and assigns huge amount of negative losses to the DG connected load points. Sharma and Abhyankar [20] have developed a sequential LA procedure using Shapley value (SSV) and circuit laws to allocate losses in the active RDN. This method removes the time complexity and computational memory burden against conventional Shapley value (CSV) technique because; CSV approach allocates losses by performing permutations of all the entry order of players and possible combinations. In [21], the effectiveness of the developed current summation LA technique has been tested at various load levels and DG capacities where the cross-terms of power loss equation (PLE) have been bifurcated using logarithmic scheme. But, this scheme has certain limitation, that is, the participation factors should be positive and lie within (0-2). However, this may be violated in practical RDNs due to disproportionate load or DG sizes. Kashyap and De [22] presented a PS based LA scheme for selection of the optimal locations and size of the DGs in an active power network. The injected power based LA procedure developed in [23] allocates ‘zero’ losses to all the DG connected load points and penalties to all DG units even if system loss reduces due to DG power injections. Thus, this technique does not provide justice to the DG owners. Kumar et al. [24] have introduced a LA scheme which decomposes the cross-term of PLE using a power factor-based loss allocation factor, and is observed to be operating efficiently at varying power factor scenarios.

Various LA methods discussed above and available in literature have precisely explained almost all the attributes those are essential for a fair LA. As power loss of a load point is very sensitive to the fluctuation of its node voltage, it is important to maintain voltage profile stable at all the load points; otherwise, it will lead to more system loss. Moreover, the technique used for bifurcation of cross-term of PLE also plays a significant role in system LA. Keeping this in view, this paper proposes a loss allocation method which eliminates the effect of cross-term analytically from the loss formulisation without any assumptions and approximations. Moreover, a new DG remuneration technique is also developed for awarding the exact amount of NLR benefits to the DGOs after analysing their actual contribution towards system loss reduction/enhancement. The novelty of this method can be visualised further from its simplicity and capability of allocating loss among the various network participants as regards to their load levels, DG capacities, power factors, and geographical locations.
In light of the above issues, this paper carries the analysis further. For fair allocation, a new LA method has been formulated in Section-2. Simultaneously, a DG remuneration technique has been developed to provide all the benefits of NLR to the DG owners in Section-2. In Section-3, algorithms relating to the proposed loss allocation and DG remuneration schemes are presented. In order to verify the effectiveness of the proposed approach, the LA results obtained are compared and analysed with various existing methods using two test systems (i.e., 17-bus and 33-bus) in the presence of DG units in Section-4. Finally, conclusive remarks are made in Section-5.

2. Formulation of the proposed branch oriented loss allocation method

This section comprises of two-subsections. The first part describes regarding the formulation of the proposed branch oriented loss allocation (BOLA) method while the second part deals with the development of the proposed DG remuneration technique. The proposed method uses the results of a converged power flow as discussed in [25] for LA analysis. For including DGs in to the computational process, this paper follows PQ-type modeling instead of PV-type because: DG units are normally smaller in size than that of the conventional power generators. Thus, the constant PQ model is sufficient to provide effective load flow solutions for loss calculation of RDNs [26]. Further, DGs installed at load ends of a distribution system are typically not permitted to regulate the voltage; instead, they regulate power and power factor, hence should be modeled as negative PQ-loads [27]. As per Choi and Kim [28] most of the DG units at customer sites are equipped with automatic voltage regulators (AVRs) and operate at constant power output mode. Hence, at this scenario, the voltage output levels of the generator units remain identical to that of system voltages. Therefore, it is advisable to operate the interconnected nodes of DGs as PQ-type in contrast to PV-type. Keeping this in view, the negative load modelling of DG as discussed in [29-30] is preferred for execution of load flow and power loss calculation in this paper. Thus, the entire calculation is carried out by considering DGs as negative constant power loads (PQ-type) throughout the LA procedure.

To make LA computation simple and easier, a unique bus identification scheme is followed in this paper. This scheme is explained by considering a sample 12-bus RDN as shown in Figure 1. In this scheme, the substation/slack bus is numbered as ‘1’, and the subsequent nodes along the main feeder and lateral feeders are indexed in the increasing order. The branches are indexed as one unit less than that of
its receiving end bus numbers which can be verified from Figure-1. In this type of RDN, the total number of buses \( NB \) and branches \( NBR \) are related as:

\[ NBR = NB - 1 \]  

(1)

To avoid conflict between the complex operator \( 'j' \) and branch- \( j \), the branch- \( j \) is represented as branch- \( jj \) in this paper. For easy identification of nodes and faster calculation, several arrays are proposed for storing information relating to the adjacent (Figure 2), subsequent (Figure 3) and previous buses (Figure 2) of the RDN.

An array \( adb[ ] \) of dimension twice that of \( NBR \) is used for storing the adjacent buses of the network. Further, two pointers arrays \( mf[ ] \) and \( mt[ ] \) of dimension equal to \( NB \) are proposed to locate the initial and final memory locations of the adjacent buses relating to a particular node in the network, respectively.

The operations of above arrays are described here through node-4 (i.e. \( a = 4 \)) of the 12 bus RDN. It can be viewed from Figure-1, this node has three adjacent buses i.e. node- 3, 5 and 12 which are stored in the array \( adb[ ] \) within the memory locations 8 to 10. The initial memory location \( s = 8 \) can be fetched from the array \( mf[ ] \) as \( s = mf(a) = mf(4) = 8 \). With this value of \( s \), the first adjacent node ‘3’ can be identified in the array \( adb[ ] \) as \( adb(s) = adb(8) = 3 \) (Figure-2). In the similar manner, the last adjacent node ‘12’ can be accessed using \( mt[ ] \) array as: \( adb(s) = adb(mt(4)) = adb(10) = 12 \). Once initial and final memory locations of the adjacent buses corresponding to a node in the array \( adb[ ] \) are known, other values can be traced within these locations as discussed above. Hence, adjacent buses relating to each node of the RDN can be identified in the similar manner. Moreover, an array \( pb[ ] \) of dimension equal to \( NB \) is used to store the previous bus information corresponding to each nodes of the RDN. The previous bus of node-4 can be fetched from this array as \( pb(a) = pb(4) = 3 \) which can be verified from Figure 2.

Similarly, two other arrays \( nsb[ ] \) and \( sb[ ] \) are proposed to store the information relating to the subsequent buses corresponding to each branch of the RDN. Two pointers arrays, \( mfs[ ] \) and \( mts[ ] \) of dimension equal to \( NB \), are proposed to locate the initial and final memory locations of the subsequent buses relating to a particular branch- \( jj \) in the network, respectively. The subsequent bus information of the considered 12-bus system is provided in Figure 3. The operations of above arrays are explained here through branch-3 (i.e. \( jj = 3 \)) of the 12 bus RDN. It can be viewed from Figure-1, this branch has three numbers of subsequent buses i.e. node- 4, 5 and 12 which are stored in the array \( sb[ ] \) within the memory.
locations 20 to 22. The initial memory location ‘ \( x = 20 \)’ can be fetched from the array \( mfs[] \) as \( x = mfs(jj) = mfs(3) = 20 \). With this value of \( x \), the first subsequent node ‘4’ can be identified in the array \( sb[] \) as \( sb(x) = sb(20) = 4 \). In the similar manner, the last subsequent node ‘12’ can be accessed using \( mts[] \) array as: \( sb(x) = sb\{mts(3)\} = sb(22) = 12 \). Once initial and final memory locations of the subsequent buses corresponding to a branch in the array \( sb[] \) are known, other values can be traced within these locations as discussed above. Thus, subsequent buses relating to each branch of the RDN can be identified in the similar manner. The numbers of subsequent buses corresponding to each branch of the RDN are counted and are stored in the array \( nsb[] \). The formations of these arrays are made using the network data and simple programming techniques in the MATLAB-R2018b environment.

2.1. Proposed Loss Allocation Scheme

The equivalent current injection (ECI) at a particular node ‘ \( a \)’ (Figure 4) with complex power load \( S_{La} = P_{La} + jQ_{La} \) and node voltage \( V_a \) can be calculated as:

\[
I_{La} = \frac{(P_{La} - jQ_{La})}{V_a^*}, \quad a = 2,3, \ldots, NB
\]  

(2)

The current of any branch- \( jj \) can be computed in terms of its subsequent load currents \( I_{La} \) using the arrays \( mfs[], mts[] \) and \( sb[] \) as:

\[
I(jj) = \sum_{a=sb(mfs(jj))}^{sb(mts(jj))} I_{La}
\]  

(3)

Using Equation 3 and rewriting Equation 2, the current of branch- \( jj \) can be expressed in terms of complex power as:

\[
I(jj) = \sum_{a=sb(mfs(jj))}^{sb(mts(jj))} \frac{P_{La} - jQ_{La}}{V_a^*}
\]  

(4)

Thus, the active power loss of a branch- \( jj \) can be evaluated as:

\[
PLoss(jj) = \text{Re} \left[ (V_s - V_r)^* \cdot I(jj) \right]
\]  

(5)
Where, $V_s$, $V_r$ and $I_{jj}$ are the sending end voltage, receiving end voltage and current of branch ‘$jj$’, respectively.

Substituting Equation 4 in Equation 5, the power loss equation can be expressed as:

$$PLoss(jj) = \text{Real} \left[ (V_s - V_r)^* \cdot \sum_{a = sb(mfs(jj))}^{sb(mts(jj))} \frac{P_{La} - jQ_{La}}{V_a^*} \right]$$  \hfill (6)

Rearranging,

$$PLoss(jj) = \text{Real} \left[ \sum_{a = sb(mfs(jj))}^{sb(mts(jj))} \left( \frac{V_s - V_r}{V_a} \right)^* \cdot \left( P_{La} - jQ_{La} \right) \right]$$  \hfill (7)

Let, $\left( \frac{V_s - V_r}{V_a} \right)^* = X(a) + jY(a)$

The real power loss of branch- $jj$ is then represented as:

$$PLoss(jj) = \sum_{a = sb(mfs(jj))}^{sb(mts(jj))} X(a)P_{La} + Y(a)Q_{La}$$  \hfill (8)

Thus, it can be concluded from Equation 8, power loss of branch- $jj$ can be allocated among the consumers present beyond branch- $jj$. Therefore, the loss share of each subsequent consumer of branch- $jj$ can be computed as:

$$PLoss(jj,a) = X(a)P_{La} + Y(a)Q_{La}$$  \hfill (9)

Total loss allocation to a consumer connected at node- $a$ can be calculated by adding its individual loss shares for each of the branch- $jj$ of the RDN using Equation 9 and is given as:

$$TPLoss(a) = \sum_{jj=1}^{NB-1} PLoss(jj,a)$$  \hfill (10)

Hence, total power loss of the RDN can be evaluated by adding power loss of the individual nodes as:

$$TPLoss = \sum_{a=1}^{NB} TPLoss(a)$$  \hfill (11)
2.2. Proposed DG Remuneration Technique

It is verified from literature, the power loss of a system can be decreased by connecting DG units into the system. Thus, a fair loss allocation method should provide all the benefits of loss reduction to the DGOs as per their individual participation towards NLR. But, it is observed, some LA schemes [11, 17-18] have diverted a part of NLR to the consumers’ side as cross-subsidies even if their impact on NLR is insignificant, which is unfair to the DGOs. However, this drawback is eliminated in the proposed remuneration technique by allocating losses to DG units after analysing their individual contributions towards system loss reduction. The detailed procedure of DG remuneration scheme is presented below. Here, Equation 12 represents total loss of the RDN in the absence of DG units. When DGs are connected, RDN loss is reduced due to flow of reverse current in the network. Hence, at this condition, load flow is carried out considering DGs as negative loads and power loss of the RDN is evaluated using proposed LA formulation which is provided by Equation 13. To find the contributions of DG units towards network loss reduction, the total losses of the RDN obtained at above two scenarios i.e. without and with DGs are subtracted which is shown in Equation 14. Thus, without DGs, the active power loss of the RDN is computed using Equation 11 as:

\[ P_{adg} = \sum_{a=1}^{NB} TPLoss(a) \]  

(12)

Further, the real power loss of the RDN with all DGs is calculated as:

\[ P_{pdg} = \sum_{a=1}^{NB} TPLoss(a) \]  

(13)

Total NLR due to DGs can be evaluated from Equations 12 and 13 as:

\[ P_{dgs} = P_{adg} - P_{pdg} \]  

(14)

The total system loss \( P_{dg}(p) \) without \( (p^{th}) \) DG unit is calculated by connecting the remaining \( (N_{dg} - 1) \) numbers of DGs in the RDN using the power loss Equation 11. Thus, the contribution \( (i.e., In_{dg}(p)) \) of an individual DG unit \( (i.e., p^{th}) \) can be evaluated as:

\[ In_{dg}(p) = P_{dg}(p) - P_{dgs} \]  

(15)

Where, \( In_{dg}(p) \) represents the remuneration allotted to the \( p^{th} \) DG Owner, and \( N_{dg} \) represents the total number of DG connected in the RDN.
\[
T_{\text{In}} = \sum_{p=1}^{N_{\text{dg}}} I_{\text{In}}(p)
\]  

(16)

It is important to note that legacy distribution systems are radial in structure, and there is an obvious variation in node voltages from the source end to tail end. Therefore, there may be a small variation between the values of \( P_{dgs} \) and \( T\text{In}_{\text{dg}} \) at constant power type load due to the variation in node voltages during power flow calculation. However, this variation can be reduced to an insignificant value by injecting numerous DGs into the system, as strategically well located DG units provide adequate voltage support to the system [31]. Thus, keeping this aspect in view, a small difference as expressed in Equation 17 if exists, then, this amount is to be distributed among the DGOs as per their proportional contribution in the exciting remuneration as shown in Equation 18. Hence, the difference in remuneration and its fair allocation is performed as follows:

\[
\text{Diff} (I_{\text{In}}) = P_{dgs} - T\text{In}_{\text{dg}}
\]  

(17)

Thus, the final remuneration provided to the \( p^{th} \) DG owner is:

\[
fI_{\text{In}}(p) = I_{\text{In}}(p) + \left[ \text{Diff} \left( I_{\text{In}} \right) \times \left( \frac{I_{\text{In}}(p)}{T\text{In}_{\text{dg}}} \right) \right]
\]  

(18)

Hence, the final remuneration allocated to all the DG owners is computed as:

\[
fT\text{In}_{\text{dg}} = \sum_{p=1}^{N_{\text{dg}}} fI_{\text{In}}(p)
\]  

(19)

To find the effectiveness of the proposed remuneration technique the total remuneration allocated to DGOs by Equation 19 is compared with the value of \( P_{dgs} \), which is found to be equal in both the case studies: Case-I and II, which can be verified from Section-4.

3. Algorithms of the proposed methods

3.1. Algorithm of the proposed loss allocation technique

**STEP-1:** Using network data of the RDN form the arrays \( nsb, sb, mt, mf, adb, mts, mfs, \) and \( pb \).

**STEP-2:** Initialize all branch losses to ‘0’, and all node voltages to ‘1 p.u’ (magnitude) with angle ‘0 radian’.
STEP: 3 The iteration counter is initialized to ‘0’, and tolerance is set to ‘0.0001’.

STEP: 4 ECI of all the load points are evaluated using Equation 2.

STEP: 5 Using mfs, mts, and sb arrays calculate all the branch currents through Equation 3.

STEP: 6 Initialize, \( i = 2 \)

STEP: 7 Update node voltages using forward sweep technique. Identify bus-\( n \) (i.e. previous to bus-\( i \)) and place it in array \( pb(i) \). The voltage of \( i^{th} \) load point is then updated by utilizing the circuit data of \( (i-1)^{th} \) branch and voltage of \( n^{th} \) load point.

STEP: 8 Update, \( i = i + 1 \).

STEP: 9 If \( i = NB \), proceed to step-10, else execute step-7.

STEP: 10 Increase iteration counter by one unit.

STEP: 11 Check the convergence criteria by comparing the newly obtained node voltage values with the previous iteration. If it satisfies, proceed to step-12, else perform step-4.

STEP: 12 Evaluate power loss of all the branches and total power loss of the network.

STEP: 13 Initialize, total loss allocation of each of the load point to zero i.e., \( TPLoss(a) = 0 \), \( a = 2, 3, \ldots, NB \).

STEP: 14 The value of ‘\( i \)’ is initialized to ‘2’.

STEP: 15 The previous load point of node-\( i \) is searched as \( n = pb(i) \) and then \( (V_f - V_i) \) is estimated.

STEP: 16 The subsequent nodes corresponding to \( (i-1)^{th} \) branch are searched from \( sb[ ] \) array, and power loss of this branch is distributed among the subsequent load points using Equation 9.

STEP: 17 Update \( i = i + 1 \) and check, if \( i = NB \), then follow step-18, else proceed to step-15.

STEP: 18 The total loss assigned to each of the end users are evaluated using Equation 10.

3.2. Algorithm of proposed DG remuneration technique

STEP-1: Compute total power loss of the RDN (\( TPLoss \)) in the absence of DG units using Equation 12.

STEP-2: Perform load flow and calculate total power loss of the RDN (\( TPLoss \)) in the presence of all DG units using Equation 13.

STEP-3: Calculate total remuneration (\( P_{dgs} \)) to be provided to the DGOs in (kW) due to loss reduction in the network using Equation 14.
STEP-4: Calculate the reduction in loss by an individual DG unit \((i.e., L_{dg}(p))\) as the difference between the loss allocation of all DGs \((i.e., P_{dgs})\) and \((N_{dg} - 1)\) numbers of DGs \((i.e., P_{dg}(p))\), neglecting the DG unit \((i.e., p^{th} \text{ DG})\) whose remuneration is to be calculated.

STEP-5: Provided remuneration to the \(p^{th}\) DG owner according to Equation 15.

STEP-6: Compute the remuneration to be awarded to each DG units utilizing above two steps.

STEP-7: Final remuneration \((TIn_{dg})\) awarded to the DG owners are evaluated by adding all \(In_{dg}(p)\), where \(p = 1, 2,3,\ldots, N_{dg}\).

STEP-8: Check, is \(Diff (In_{dg}) = 0\), if yes proceed to STEP-9 else, go to STEP-10.

STEP-9: The reward provided to the \(p^{th}\) DG owner is equal to \(In_{dg}(p)\).

STEP-10: Final remuneration awarded to the \(p^{th}\) DGO is calculated using Equation 18.

4. Results and Discussion

In this section, the effectiveness of the proposed BOLA technique has been investigated against various established LA methods using two different test systems \((i.e., 17\text{-bus and 33\text{-bus systems}})\). The detail line data of these two test systems are provided in Table-1. The corresponding load and DG data along with comparative loss allocation results of 17-bus and 33-bus test systems are represented in Table-2 and Table-3, respectively. The load flow and power loss analysis have been carried out by considering DGs as negative loads throughout the computational process. In the LA process, positive values are treated as penalties and negative values as rewards to the participants. All the simulations are performed with a system having following configuration: Intel(R) Core(TM) i3-6006U processor, 2 GHz CPU, 8 GB RAM and Windows-10 operating system.

4.1. Case Study-I (A 17-bus test system)

A 20 KV, 17-bus RDN [20] comprising of 12 load points, 3 DG units and 16 lines is first considered for the implementation of the proposed scheme. The location and size of DG units are finalized as per paper [23]. Thus, three DGs are selected to be placed at nodes 15, 16 and 17 whose details are provided in Table-2. It is observed, without DGs, the total active power loss of the system is 22.74 kW, and it is reduced to 6.64 kW due to injection of DG power into the system. Thus, a total NLR of 16.10 kW occurs due to penetration of DGs into the RDN. The present approach of LA is contemporary and comparable to...
the other established methods (PR [3], Marginal [5], Z-bus [7], SM [8], BCDM [11], Jahromi’s method [17], SSV [20], and Method [23]) as the final allocation of the RDN remains same with/without DGs.

It can be viewed from Table-2; Pro-rata method [3] assigns losses to its network participants as per their load demands. Thus, the consumers of equal ratings connected close to the substation bus (e.g. consumer at node-3) are allocated with an equal amount of losses as that of the consumers placed far away (e.g. consumer at node-9). Moreover, this method assigns penalties to the DGs even if their presence reduces system loss in the network, which is again unfair to the DGOs. Hence, PR approach may not be suggested for fair LA as it assigns losses to the network participants without considering their geographical locations. In contrast, the proposed technique takes care of these issues and provides all the benefits of loss reduction to the DGOs as per their actual contribution towards NLR. The DG connected node-15 is getting the highest remuneration of 6.83 kW as it delivers maximum power into the system. Similarly, the generator at node 17 is rewarded more (5.76 kW) than that of the DG at node-16 (3.52 kW). Thus, a judicious approach of LA and DG remuneration is noticed by the present method.

The ITL based marginal approach [5] provides high volatility and assigns negative losses to the network participants. Instead of giving incentives, it assigns huge losses to DGs, which is undesired. Also, it suffers from loss allocation imbalance between demand and generation sides as observed from Table-2. It can be observed, the DGOs are assigned with high values of positive losses while the consumers at nodes-12, 13 and 15-17 are allocated with large amounts of negative losses. In contrast, the proposed method provides a small incentive to the DG connected nodes due to the reverse current effect, and maximum benefit to the generation side, which is acceptable. Moreover, the marginal method is suffering from computational paradox due to implementation of Jacobian and Hessian matrices in the loss calculation procedure. But, the proposed technique is easy to understand and free from computational complexity.

The Z-bus LA technique [7] assigns negative losses to the customers at DG connected nodes whereas positive losses to the DG owners, which is unfair from the DG point of view. However, the present scheme not only provides benefits only to the DGOs but also to the DG connected consumers. Further, the Z-bus technique may face difficulty for allocating losses in the overhead RDN due to singularity of the admittance matrix, but proposed approach is free from this drawback.

The loss allocation by SM technique [8] is found to be better than PR and marginal methods and very close to the results of the Z-bus method. However, LA by the present method is found to be better than
that of SM technique. Again, the SM technique allocates a huge amount of losses to the generation units even if their penetrations reduce power loss of the RDN, which can be considered as a significant drawback as compared to the proposed method.

The load demand of the consumer at node-17 is more than that of node-15, and they are almost at equal distance from the source node. Hence, the consumer at bus 17 should get more loss as compared to node 15. But, BCDM method [11] allocates less loss to the consumer at node 17, which is unexpected. Further, BCDM awards more remuneration to the DG at node-16 against DG at node-17 even though it injects less power. However, these inconsistencies are not present in the proposed method.

It can be observed, Jahromi’s procedure [17] has become partial for those load buses where DGs are connected. This method allocates zero losses to demands/DGs those are locally injected /consumed by DGs/demands. Thus, the consumers at nodes 15, 16 and 17 are assigned with zero losses because of more generations than demands. Also, some discrepancy is marked in the result of LA between the consumers of nodes 11 and 12 concerning their geographical location and load demand. These difficulties are not found in the proposed LA procedure, and loss allocation between nodes 11 and 12 is observed to be fair.

The loss allocation by a game theory based SSV technique [20] is analogous to the present approach, but it suffers from computational time complexity and memory burden for large electrical networks.

The method discussed in [23] requires a normalization factor for final allocation of losses among the end users whereas proposed method allocates it without reconciliation. The LA results obtained by this approach are found to be very close to that of Jahromi’s method [17], and hence, it also suffers from the same problem of partial LA as discussed in method [17]. Moreover, both methods are doing injustice to the DG owner of bus 17 because; Jahromi’s method allocates zero loss while method [23] assigns a penalty of 0.07 kW to this DG owner. However, the DG owner has been rewarded with the proposed remuneration technique.

Furthermore, in order to verify the performance in terms of their capability to discriminate fair LA, two sets of nodes having equal consumptions but at different geographical locations are identified. At first, a set of two distance nodes 3 and 9 are selected whose difference in LA is calculated and represented in Figure-5. It can be realized; the difference in LA by the present technique is very close to the Z-Bus, SM, BCDM, Jahromi’s method and SSV methods. This difference is zero in PR method as it allocates an equal amount of losses to loads of identical demands. The loss allocation of Method [23] and Marginal methods are more prominent in comparison to the above discussed LA schemes.
Similarly, to test the performance of two close nodes, a set of nodes 5 and 7 are identified and its difference in LA is presented in Figure-6. It can be viewed that the performance of proposed approach is very close to the Z-Bus, SM, BCDM and SSV methods. But, PR scheme assigns an equal amount of losses to both the nodes whereas Marginal method allocates more loss to the consumer at node-5 than that of at node-7, which is undesired. The discrimination of methods [17] and [23] are more as compared to other methods discussed above. However, the proposed method maintains consistency in both the scenarios by allocating an adequate amount of losses among the network participants as regards to their load demands and geographical locations in the RDN.

4.2. Case study-II (A 33-bus system)

A 12.66 kV, 33-bus RDN [32] with total active and reactive power of (3715+ j2300) KVA is considered for analyzing the effectiveness of the proposed method against other established methods. The line data of the test system is presented in Table-1. This test system is modified further by connecting three DGs at nodes 6, 25 and 31 for achieving an optimal RDN. The position and size of the DGs have been selected as per the analytical procedure discussed in [33]. The corresponding load and DG data along with the results of LA are provided in Table-3.

At 100% load level, without DGs, the total real power loss of the 33-bus RDN is found to be 202.67 kW, and it is reduced to 43.42 kW due to the implementation of DGs into the system. Thus, a total loss reduction of 159.25 kW is noticed due to the penetration of DG units into the system. The power injection of DG at node-6 (DG6) is found to be more than that of the other two DGs. In between the other two DGs, the DG at node-25 (DG25) is injecting more power as compared to DG at node-31 (DG31).

It can be observed from table-3, marginal loss coefficient method (MLC) [5] allocates minimum remuneration to DG6 while BCDM provides maximum benefit to DG31, which is unexpected. Modified PR method [18] assigns penalties to DG25 even if there is a total NLR of 159.25 kW in the system due to DG penetration. Further, it provides more remuneration to DG31 against DG25 whereas DG25 is injecting more power as compared to DG31. Other established methods are allocating more incentives to DG31 against DG25, which is unfair. However, a fair allocation with proper DG remuneration is noticed by the proposed method. The present strategy provides maximum benefits to DG6, and out of the other two DGs, it assigns more remuneration to DG25. This remuneration scheme provides all benefits of loss reduction to the DG units after analyzing their exact contribution towards NLR. The proposed technique provides maximum benefits to the DG owners whereas BCDM [11] delivers maximum spatial cross-
subsidies to the consumers. The loss assigned to heavily loaded consumer connected at node-30, is minimum (i.e., 14.63kW) by BCDM method and maximum (i.e., 42.93 kW) by PSMLA method [12]. But, the present method allocates it fairly (i.e., 17.75 kW) with due consideration to the position of nearest DG, load demand and its physical location. Ghaemis’s method [19] allocates large amounts of losses to the heavily loaded buses (bus-7, 8, 14 and 30) as that of PSMLA and Proportional [10] techniques and simultaneously, assigns huge negative losses to the DG connected load points. However, the present method takes care of these issues. The voltage-based LA [18] and MLC [5] offer rewards to the DG owners moderately, whereas proportional method [10] awards more remuneration to the DG owners as compared to the method discussed above. But, the entire benefit is rewarded by the present approach.

Furthermore, to verify the performances of the methods as regards to geographical locations, two sets of nodes of equal demands are identified, whose difference in LA are calculated and presented in Figures-7 and 8 for comparison with the existing techniques. Firstly, a set of two distance nodes 3 and 22 are selected where node-3 is electrically close to the substation bus. Hence, the consumer at node-22 should get more loss than that of the consumer at bus-3. But, it can be verified from Figure-7 that PSMLA, Proportional and Ghaemi’s methods are allocating losses to these consumers in a reverse way. However, proposed method and other remaining established methods as discussed assign it in the proper order. It can also be observed, the discrimination of LA by the present scheme is very close to BCDM, Proportional, modified PR and voltage based LA methods whereas it is very high in MLC.

Moreover, the consumers at nodes 9 and 10 are having the same load demands and are at electrically close to each other. It can be verified from Figure- 8 that the present approach, BCDM, modified PR and Voltage based LA [18] methods treat these consumers identically with better loss allocation as compared to PSMLA, Proportional and Ghaemi’s methods. However, the discrimination of LA between the customers of the same ratings placed close to each other is better in the proposed method, while it is poor in MLC technique. Thus, it is verified that the present LA procedure is allocating losses with due consideration to the network topology of the RDN. Again, in order to investigate the LA results as regards to power factor (p.f.), three heavily loaded end users connected at nodes- 30, 31 and 32 are identified. The p.f. of a consumer at node-30 (p.f., 0.32 lagging) is found to be very poor than that of consumers at nodes- 31 and 32 (p.f., 0.9 lagging). It is noticed, the load point with poor p.f. has been assigned with more loss as compared to the other two consumers, which is fair and justified. Further, the efficiency of
the proposed method is also tested at larger networks using two test systems i.e., 69-bus RDN [23] and 136-bus RDN [34], providing a total loss allocation of 225 kW and 320.36 kW at the base case with a computational time of 0.0193 second and 0.1943 second, respectively. In light of the above discussions, the proposed LA strategy is found to be more suitable and efficient for practical implementation in comparison to other methods discussed.

Since, there are more and more renewable energy sources in distribution networks, the output of DGs are actually time-varying and fair amount of uncertainties are also involved, which cannot be regarded as constant loads as considered in this paper due to the lack of scope of the paper. However, the LA approach proposed in this paper can be effectively applied as a building block in the scenario-based approach to capture the uncertainties involved with the DGs and loads.

5. Conclusion

In this paper, a new active power loss allocation method is developed by eliminating the impact of cross-term mathematically from the loss formulation. For LA with DGs, the DGs are modeled as negative power injections and are included in the LA process. Moreover, it is not involved in any over-recovery of losses, so, normalization of the allocated losses is not required. The effectiveness of the proposed technique is verified on two test systems in the presence of DGs. The results obtained are promising as compared to other established methods. It is investigated some of the established methods as discussed (PR [3], Marginal [5], Z-Bus [7], SM [8], Jahromi’s [17], Modified PR [18] and Method [23]) assign penalties to the DGOs, even if their penetration has reduced system loss, which is unfair. Also, some existing techniques are diverting a part of NLR as cross-subsidies towards the consumer side even though their impact on NLR is insignificant. However, the proposed approach allocates all the benefits of NLR to the DGOs. Hence, it is more accurate and may be suggested for practical implementation.

Nomenclature

| Symbol | Description                          | Symbol | Description                          |
|--------|--------------------------------------|--------|--------------------------------------|
| $P$    | Real Power in (kW)                   | $NB$   | Total number of nodes in the RDN     |
| $Q$    | Reactive Power in (kVAR)             | $NBR$  | Total number of branches in the RDN  |
| $R$    | Resistance in (Ω)                    | $X$    | Reactance in (Ω)                     |
| $a$    | Node number                          | $jj$   | Branch Number                        |
| $V_s$  | Sending end voltage of branch- $jj$  | $V_r$  | Receiving end voltage of branch- $jj$|
| $V_a$  | Voltage of node- $a$                 | $I(jj)$| Current of branch- $jj$              |
| Symbol | Description |
|--------|-------------|
| adb[]  | Array used to store adjacent buses relating to each node of the RDN |
| sb[]   | Array used to store subsequent buses relating to each branch of the RDN |
| mf[]   | Array used to store initial memory location of the adjacent buses relating to each node of the RDN in array adb[] |
| mt[]   | Array used to store final memory location of the adjacent buses relating to each node of the RDN in array adb[] |
| mfs[]  | Array used to store initial memory location of the subsequent buses relating to each branch of the RDN in array sb[] |
| mts[]  | Array used to store final memory location of the subsequent buses relating to each branch of the RDN in array sb[] |
| pb[]   | Array used to store previous buses relating to each node of the RDN |
| nsb[]  | Array used to store information of total subsequent buses relating to each branch of the RDN |
| s      | Represents the memory location in array adb[] |
| x      | Represents the memory location in array sb[] |
| $S_{La}$ | Net Complex power connected at node-a |
| $Q_{La}$ | Net Reactive power connected at node-a |
| $P_{La}$ | Load current at node-a |
| $P_{Loss}(jj)$ | Power loss of branch- jj |
| $P^*(jj, a)$ | Power loss of branch- jj, which is assigned to node- a |
| $P_{Loss}(a)$ | Total Power loss assigned to the participant of node- a |
| $TP_{Loss}(a)$ | Total Power loss of the RDN |
| $P_{pdg}(p)$ | Power loss of the RDN without $p^{th}$ DG unit |
| $P_{dgs}(p)$ | Power loss reduction due to DGs |
| $In_{dg}(p)$ | Contribution of $p^{th}$ DG unit towards system loss reduction |
| $N_{dg}$ | Total number of DG units connected in the RDN |
| $Diff(In_{dg})$ | Difference between actual and calculated value of DG remuneration |
| $TIn_{dg}$ | Calculated value of DG remuneration |
| $fIn_{dg}(p)$ | Final remuneration awarded to the $p^{th}$ DG owner |
| $fTIn_{dg}$ | Final remuneration awarded to the DG owners |
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\[ S_{La} = P_{La} + jQ_{La} \]

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Table 1. Detail line data of 17 and 33-bus test systems

| Sending end node | Receiving end node | Branch No. | Resistance of the branch (in pu) | Reactance of the branch (in pu) | Charging Capacitance (pu) | Sending end node | Receiving end node | Branch No. | Resistance of the branch (in Ω) | Reactance of the branch (in Ω) |
|------------------|--------------------|------------|----------------------------------|---------------------------------|---------------------------|------------------|--------------------|------------|------------------------------|-------------------------------|
| 1                | 2                  | 1          | 0.0025                           | 0.0026                          | 0.03                      | 1                | 2                  | 1          | 0.0922                       | 0.047                         |
| 2                | 3                  | 2          | 0.0008                           | 0.0008                          | 0.02                      | 2                | 3                  | 2          | 0.493                        | 0.2511                        |
| 3                | 4                  | 3          | 0.0007                           | 0.0007                          | 0.02                      | 3                | 4                  | 3          | 0.366                        | 0.1864                        |
| 2                | 5                  | 4          | 0.0007                           | 0.0007                          | 0                         | 4                | 5                  | 4          | 0.3811                       | 0.1941                        |
| 5                | 6                  | 5          | 0.002                            | 0.0021                          | 0.02                      | 5                | 6                  | 5          | 0.819                        | 0.707                         |
| 6                | 7                  | 6          | 0.0009                           | 0.0009                          | 0.01                      | 6                | 7                  | 6          | 0.1872                       | 0.6188                        |
| 7                | 8                  | 7          | 0.0017                           | 0.0017                          | 0.01                      | 7                | 8                  | 7          | 0.7114                       | 0.2351                        |
| 5                | 9                  | 8          | 0.0021                           | 0.0022                          | 0.02                      | 8                | 9                  | 9          | 1.03                         | 0.74                          |
| 6                | 10                 | 9          | 0.0001                           | 0.0001                          | 0                         | 9                | 10                 | 9          | 1.044                        | 0.74                          |
| 10               | 11                 | 10         | 0.0006                           | 0.0006                          | 0                         | 10               | 11                 | 10         | 0.1966                       | 0.065                         |
| 10               | 12                 | 11         | 0.0018                           | 0.0018                          | 0                         | 11               | 12                 | 11         | 0.3744                       | 0.1238                        |
| 12               | 13                 | 12         | 0.0003                           | 0.0003                          | 0                         | 12               | 13                 | 12         | 1.468                        | 1.155                         |
| 12               | 14                 | 13         | 0.0011                           | 0.0011                          | 0                         | 13               | 14                 | 13         | 0.5416                       | 0.7129                        |
| 14               | 15                 | 14         | 0.0011                           | 0.0011                          | 0                         | 14               | 15                 | 14         | 0.591                        | 0.526                         |
| 15               | 16                 | 15         | 0.0001                           | 0.0001                          | 0                         | 15               | 16                 | 15         | 0.7463                       | 0.545                         |
| 14               | 17                 | 16         | 0.0007                           | 0.0007                          | 0                         | 16               | 17                 | 16         | 1.289                        | 1.721                         |
| -                | -                  | -          | -                                | -                              | -                         | -                | -                  | -          | -                            | -                             |
| -                | -                  | -          | -                                | -                              | -                         | 17               | 18                 | 17         | 0.732                        | 0.574                         |
| -                | -                  | -          | -                                | -                              | -                         | 2                | 19                 | 18         | 0.164                        | 0.1565                        |
| -                | -                  | -          | -                                | -                              | -                         | 19               | 20                 | 19         | 1.5042                       | 1.3554                        |
| -                | -                  | -          | -                                | -                              | -                         | 20               | 21                 | 20         | 0.4095                       | 0.4784                        |
| -                | -                  | -          | -                                | -                              | -                         | 21               | 22                 | 21         | 0.7089                       | 0.9373                        |
| -                | -                  | -          | -                                | -                              | -                         | 3                | 23                 | 22         | 0.4512                       | 0.3083                        |
| -                | -                  | -          | -                                | -                              | -                         | 23               | 24                 | 23         | 0.898                        | 0.7091                        |
| -                | -                  | -          | -                                | -                              | -                         | 24               | 25                 | 24         | 0.896                        | 0.7011                        |
| -                | -                  | -          | -                                | -                              | -                         | 26               | 27                 | 26         | 0.2842                       | 0.1447                        |
| -                | -                  | -          | -                                | -                              | -                         | 27               | 28                 | 27         | 1.059                        | 0.9337                        |
| -                | -                  | -          | -                                | -                              | -                         | 28               | 29                 | 28         | 0.8042                       | 0.7006                        |
| -                | -                  | -          | -                                | -                              | -                         | 29               | 30                 | 29         | 0.5075                       | 0.2585                        |
| -                | -                  | -          | -                                | -                              | -                         | 30               | 31                 | 30         | 0.9744                       | 0.963                         |
| -                | -                  | -          | -                                | -                              | -                         | 31               | 32                 | 31         | 0.3105                       | 0.3619                        |
| -                | -                  | -          | -                                | -                              | -                         | 32               | 33                 | 32         | 0.341                        | 0.5302                        |
Table 2. Load/DG data and Loss Allocation results of a 17-bus test system

| Bus No. | Load/Generation at Nodes | Loss Allocation at various nodes by different methods |
|---------|--------------------------|-----------------------------------------------------|
|         | P (kW) | Q (kVAR) | Proposed Method | PR Method [3] | Marginal Method [5] | Z-Bus Method [7] | SM Method [8] | BCDM Method [11] | Jahromi's method [17] | SSV Method [20] | Method [23] |
| 1       | 0.00   | 0.00     | 0.00            | 0.00          | 0.00              | 0.00             | 0.00             | 0.00             | 0.00             | 0.00            | 0.00          |
| 2       | 0.00   | 0.00     | 0.00            | 0.00          | 0.00              | 0.00             | 0.00             | 0.00             | 0.00             | 0.00            | 0.00          |
| 3       | 89.00  | 50.00    | 0.36            | 0.16          | 0.34              | 0.22             | 0.22             | 0.33             | 0.09             | 0.33            | 0.08          |
| 4       | 111.00 | 63.00    | 0.46            | 0.2           | 0.49              | 0.29             | 0.29             | 0.43             | 0.18             | 0.42            | 0.16          |
| 5       | 140.00 | 80.00    | 0.67            | 0.25          | 0.56              | 0.43             | 0.44             | 0.62             | 0.26             | 0.60            | 0.27          |
| 6       | 0.00   | 0.00     | 0.00            | 0.00          | 0.00              | 0.00             | 0.00             | 0.00             | 0.00             | 0.00            | 0.00          |
| 7       | 141.00 | 80.00    | 1.02            | 0.25          | 0.52              | 0.78             | 0.79             | 0.96             | 0.87             | 0.93            | 0.81          |
| 8       | 338.00 | 192.00   | 2.61            | 0.6           | 2.08              | 2.12             | 2.15             | 2.56             | 3.39             | 2.49            | 3.24          |
| 9       | 89.00  | 50.00    | 0.44            | 0.16          | 0.49              | 0.30             | 0.3               | 0.41             | 0.16             | 0.40            | 0.18          |
| 10      | 0.00   | 0.00     | 0.00            | 0.00          | 0.00              | 0.00             | 0.00             | 0.00             | 0.00             | 0.00            | 0.00          |
| 11      | 152.00 | 86.00    | 1.06            | 0.27          | 0.37              | 0.77             | 0.79             | 0.97             | 1.24             | 0.94            | 1.13          |
| 12      | 266.00 | 151.00   | 1.71            | 0.49          | -0.30             | 1.37             | 1.42             | 1.70             | 0.39             | 1.67            | 0.42          |
| 13      | 10.00  | 5.00     | 0.07            | 0.02          | -0.03             | 0.05             | 0.05             | 0.06             | 0.01             | 0.06            | 0.01          |
| 14      | 0.00   | 0.00     | 0.00            | 0.00          | 0.00              | 0.00             | 0.00             | 0.00             | 0.00             | 0.00            | 0.00          |
| 15      | 205.00 | 116.00   | -0.68           | 0.37          | -1.98             | -0.25            | -0.15            | 1.05             | 0.00             | 1.16            | 0.00          |
| 16      | 72.00  | 41.00    | -0.89           | 0.13          | -0.80             | -0.09            | -0.06            | 0.39             | 0.00             | 0.40            | 0.00          |
| 17      | 241.00 | 137.00   | -0.19           | 0.43          | -2.14             | -0.51            | -0.18            | 0.72             | 0.00             | 1.42            | 0.00          |
| DG15    | 300.00 | 145.29   | -6.83           | 1.31          | 2.79              | 0.37             | 0.22             | -1.53            | 0.03             | -1.62           | 0.16          |
| DG16    | 200.00 | 96.86    | -3.52           | 0.87          | 2.10              | 0.24             | 0.17             | -1.08            | 0.02             | -1.08           | 0.11          |
| DG17    | 260.00 | 125.92   | -5.76           | 1.13          | 2.15              | 0.55             | 0.19             | -0.78            | 0.00             | -1.48           | 0.07          |
|         | Total Loss | 6.64 | 6.64 | 6.64 | 6.64 | 6.64 | 6.64 | 6.64 | 6.64 | 6.64 | 6.64 | 6.64 |

P: Active power  Q: Reactive power  DG: Distributed generator  PR: Pro-rata method  SM: Succinct method  BCDM: Branch current decomposition method  SSV: Sequential shapley value method
Table 3. Load/DG data and Loss Allocation results of a 33-bus test system

| Bus No. | P (kW) | Q (kVAR) | Load/Generation at Nodes | Loss Allocation at various nodes by different Loss Allocation methods |
|---------|--------|----------|--------------------------|---------------------------------------------------------------------|
|         |        |          | Proposed Method | MLC Method [5] | BCDM Method [11] | PSMLA Method [12] | Ghaemi's Method [19] | Branch oriented PR method [18] | Voltage based LA [18] | Proportional Method [10] |
| 1       | 0      | 0        | 0.07          | 0.03           | 0.05            | 0.2              | 0.17              | 0.07                          | 0.07                      | 0.19                                           |
| 2       | 100    | 60       | 0.24          | 0.1            | 0.12            | 0.81             | 0.73              | 0.23                          | 0.22                      | 0.83                                           |
| 3       | 120    | 80       | 0.53          | 0.17           | 0.26            | 2.49             | 1.91              | 0.5                           | 0.51                      | 2.17                                           |
| 4       | 60     | 30       | 0.26          | 0.08           | 0.07            | 0.77             | 0.82              | 0.24                          | 0.25                      | 0.93                                           |
| 5       | 60     | 20       | -12.7         | 0.06           | -0.05           | 1.05             | -53.66            | 0.24                          | 0.27                      | 1.28                                           |
| 6       | 200    | 100      | 1.6           | 0.41           | 0.35            | 11.06            | 10.08             | 1.48                          | 1.59                      | 8.14                                           |
| 7       | 200    | 100      | 2.6           | 0.97           | 1.34            | 12.73            | 10.08             | 2.48                          | 2.58                      | 9.53                                           |
| 8       | 60     | 20       | 1.05          | 0.45           | 0.61            | 1.56             | 1.91              | 1.01                          | 1.04                      | 1.87                                           |
| 9       | 60     | 20       | 1.37          | 0.62           | 0.91            | 1.81             | 2.15              | 1.33                          | 1.38                      | 2.15                                           |
| 10      | 45     | 30       | 1.15          | 0.55           | 0.94            | 1.26             | 1.71              | 1.12                          | 1.14                      | 1.7                                            |
| 11      | 60     | 35       | 1.6           | 0.77           | 1.28            | 2.31             | 2.64              | 1.56                          | 1.59                      | 2.64                                           |
| 12      | 60     | 35       | 1.92          | 0.96           | 1.62            | 2.59             | 2.91              | 1.87                          | 1.91                      | 2.96                                           |
| 13      | 120    | 80       | 4.06          | 2.14           | 3.65            | 10.43            | 8.65              | 3.94                          | 4.04                      | 8.79                                           |
| 14      | 60     | 10       | 2.01          | 0.94           | 1.35            | 2.13             | 2.39              | 1.96                          | 2.01                      | 2.47                                           |
| 15      | 60     | 20       | 2.11          | 1.02           | 1.61            | 2.36             | 2.68              | 2.06                          | 2.11                      | 2.78                                           |
| 16      | 60     | 20       | 2.21          | 1.07           | 1.7             | 2.44             | 2.75              | 2.15                          | 2.21                      | 2.86                                           |
| 17      | 60     | 40       | 3.38          | 1.69           | 2.79            | 6.02             | 4.97              | 3.28                          | 3.37                      | 5.66                                           |
| 18      | 90     | 40       | 0.1           | 1.66           | 0.08            | 0.19             | 0.17              | 0.1                           | 0.11                      | 0.19                                           |
| 19      | 90     | 40       | 0.38          | 1.82           | 0.36            | 0.46             | 0.41              | 0.38                          | 0.37                      | 0.47                                           |
| 20      | 90     | 40       | 0.43          | 1.85           | 0.41            | 0.51             | 0.46              | 0.43                          | 0.42                      | 0.52                                           |
| 21      | 90     | 40       | 0.47          | 1.87           | 0.45            | 0.56             | 0.49              | 0.47                          | 0.47                      | 0.56                                           |
| 22      | 90     | 50       | 0.29          | 1.98           | 0.2             | 0.93             | 0.85              | 0.26                          | 0.26                      | 0.96                                           |
| 23      | 420    | 200      | 1.26          | 9.04           | 1.07            | 11.2             | 6.72              | 1.04                          | 1.03                      | 7.64                                           |
| 24      | 420    | 200      | -0.15         | 8.42           | 0.07            | 11.4             | -4.94             | -0.30                         | 0.02                      | 7.28                                           |
| 25      | 60     | 25       | 0.43          | 1.22           | 0.03            | 1.15             | 1.21              | 0.39                          | 0.42                      | 1.39                                           |
| 26      | 60     | 25       | 0.55          | 1.28           | 0.11            | 1.19             | 1.26              | 0.52                          | 0.79                      | 1.45                                           |
| 27      | 60     | 20       | 1.02          | 1.44           | 0.29            | 1.24             | 1.33              | 0.98                          | 1.21                      | 1.57                                           |
| 28      | 120    | 70       | 3.45          | 3.48           | 1.54            | 6.38             | 4.83              | 3.36                          | 3.41                      | 5.59                                           |
| 29      | 200    | 600      | 17.75         | 16.55          | 14.63           | 42.93            | 30.68             | 17.64                         | 21.74                     | 33.92                                          |
| 30      | 150    | 70       | 4.63          | 4.3            | 1.97            | 9.42             | -12.34            | 4.5                           | 4.67                      | 7.57                                           |
| 31      | 210    | 100      | -2.84         | 6.13           | 2.68            | 16.34            | 10.27             | 6.5                           | 6.6                       | 12                                             |
| 32      | 60     | 40       | 2.19          | 1.91           | 1.07            | 1.86             | 1.99              | 2.15                          | 2.29                      | 2.3                                            |
| DG6     | 2043.954 | 989.932 | -120.33       | -2.37          | -0.05           | -69.26           | -54.77             | -11.76                        | -13.76                    | -62.54                                          |
| DG25    | 695.869 | 521.901 | -21.44        | -15.75         | -0.04           | -19.84           | -11.55             | 0.55                          | -1.25                     | -12.87                                          |
| DG31    | 520.808 | 0        | -17.48        | -13.55         | -0.16           | -35.4            | -18.82             | -9.42                         | -11.77                    | -21.67                                          |

Total Loss: 43.42 43.42 43.42 43.42 43.42 43.42 43.42 43.42

P: Active power  Q: Reactive power  DG: Distributed generator  MLC: Marginal loss coefficient method  BCDM: Branch current decomposition method  PSMLA: Power summation technique  PR: Pro-rata method  LA: Loss allocation
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