Estimator-Based Control for Pure-Feedback Systems With Incomplete Measurements

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ABSTRACT This paper investigates estimator-based control problems for pure-feedback nonlinear systems with incomplete measurements due to the transmission packet losing or sensor saturation. The incomplete measurements can cause the state variables unavailable or distorted, which can degrade the performance of the system. To solve these problems, a state estimator is designed for the data-losing case, based on which two backstepping control methods are developed. The output of the system is subject to a prescribed constraint by using an obstacle Lyapunov function. By solving a linear matrix inequality, the stability conditions of the state estimator and closed-loop system are derived. It is proved that the control scheme can guarantee that all the signals of the closed-loop system are uniformly ultimately bounded in mean square. The effectiveness of the proposed methods is confirmed by simulations.

INDEX TERMS Cyber-physical system, nonlinear system, pure-feedback, controller design, incomplete measurement.

I. INTRODUCTION

In recent years, the control theory is making continuous progress, and the controlled system is becoming more complex. The systems integrating physical processes, computation and networking can be described as cyber-physical systems (CPSs) [1]. Many applications such as smart power grids, smart medical devices and complex physical and chemical processes can be interpreted as CPSs [2]–[6]. Because of the information exchange between subsystems, the control performance of the whole complex system has the potential to be improved. However, due to the complexity of the system, the control of the system has become a challenging problem [7]. For example, the interruption of communication between subsystems, the deterioration of communication parameters or sensor saturation will have a serious impact on the control performance and even the stability of the system [8]–[10]. Hence, many works focus on control problems of CPSs. Authors in [11]–[13] concerned about the controller design of CPSs with packet dropouts. Lu and Yang [14] proposed an input-to-state stabilizing controller for systems under denial of service attacks. Reference [15] investigated the problem of event-triggered control for CPSs in the presence of actuator and sensor attacks. Reference [16] addressed algorithms of state observing for systems that are corrupted by transmission or sensor noise inserted by malicious adversary. References [11], [17], and [18] studied the control problem for systems under the denial of service attacks.

Nonlinearity is an important problem in CPS control, which is more general in the real physical world than the linear description [19]–[22]. In [23], based on the T-S fuzzy model, a state estimator with stochastic incomplete measurements of nonlinear CPSs was designed. Reference [7] concerned with nonlinear CPSs subject to intermittent denial of service attacks. A switching type state estimator was designed and convex design conditions of a backstepping-based controller were derived under the nonlinear approximation by fuzzy functions. However, the nonlinear systems considered by the above works are strict-feedback systems. Pure-feedback systems in non-affine forms are more general in modeling. Many systems, such as chemical systems [24] and aircraft flight systems [25], can be described as pure-feedback systems. Some research focuses on the states estimator and controller design for...
the pure-feedback systems [26]-[29]. The adaptive controller based on fuzzy logic systems is designed for different kinds of pure-feedback nonlinear systems in [26]. Multiinput multi-output (MIMO) pure-feedback nonlinear systems and switched pure-feedback nonlinear systems are concerned in [28] and [29], respectively.

In the field of CPS control, there remain many open problems, such as how to design a state estimator in the absence of measured system information, how to design a Lyapunov function, by which to analyze the stability of the system from the statistical point of view in the case of observation information missing randomly and how to control a pure-feedback CPS with incomplete measurements etc. Inspired by the above considerations, we investigate the problem of estimator based control for nonlinear pure-feedback systems with incomplete measurements such as saturation, packet loss and data absence. The CPSs are described as a pure feedback form. The main contributions of this paper are as follows.

1. The system output is modeled by a random combination of the perfect measurement and defect measurement, and a Luenberger-like nonlinear state estimator is designed for data-losing case with measurement saturation and lost.

2. Based on the state estimator, two backstepping-like controllers are constructed for each one of the normal and abnormal cases. The Lyapunov technique is used to analyze the stability through a random combination of Lyapunov functions for both normal and abnormal cases.

3. Output constraints are taken into account in the pure-feedback nonlinear CPS control problem, and all the signals in the closed-loop system can be kept uniformly ultimately bounded (UUB) in mean square.

The paper is organized as follows. In Section II, a nonlinear pure-feedback system is described and several cases of data transmission are introduced. State estimator design, controller development and stability analysis are shown in Section III. In Section IV, an example of the controller for a nonlinear system is shown, and in Section V we conclude the work of the paper.

Notations: $R^d$ denotes the set of $n$-dimensional real vectors. $P^T$ represents the transpose of the matrix or vector $P$. $\|e\|$ is Euclidean norm of vector $e$. $I$ represents the identity matrix with appropriate dimensions. A positive definite matrix $P$ can be denoted as $P > 0$. $\text{sign}(a)$ is the sign of $a$. $E[x]$ and $D[x]$ denote the expectation and variance of random variable $x$.

II. SYSTEM DESCRIPTION

Consider a class of nonlinear systems in pure-feedback form

$$\begin{align*}
\dot{x}_i &= f_i(\bar{x}_i, x_{j+1}), \quad 1 \leq i \leq n - 1 \\
\dot{x}_n &= f_n(\bar{x}_n, u), \\
y &= v_1 x_1 + v_2 \sigma(x_1) + v_3 \psi(x_1),
\end{align*}$$

(1)

where $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$ is the state vector, $u \in R$, is the control input, and $y \in R$ is the system output. $f_i(\cdot) \in R$, $i = 1, \ldots, n$ denotes a known smooth nonlinear function, $\sigma(\cdot)$ and $\psi(\cdot)$ are saturation function and data-losing function respectively. $v_j, j = 1, 2, 3$, represents the distribution of different scenarios, $v_1 \in \{0, 1\}$, $\sum_{j=1}^{3} v_j = 1$. $v_1, v_2, v_3 = 1$ means the measured value is normal, saturated and lost, respectively.

Let $x_{\mu}$ > 0 be the saturation level of the output measurement, if $|x_1| \leq x_{\mu}$ then $v_1 = 1$ and $v_2, v_3 = 0$. When the measured output is saturated, that is $|x_1| > x_{\mu}$, we can get $v_2 = 1, v_1, v_3 = 0$, and the saturation value is $\sigma(x_1) = \text{sign}(x_1)x_{\mu}$. In the data-losing case, $v_1 = 1$ and $v_2, v_3 = 0$, the received data can be replaced with the previous normal acquisition value. Accordingly, the data-losing function $\psi(\cdot)$ can be expressed as $\psi(x_1) = x_1^*$, where $x_1^*$ denotes the last normal output of the system.

Many cases can cause the loss of observation data, such as sensors for collecting the system output signal being attacked, physical limitation of wireless network or time-division multiplexing of channel. In data-losing case, system output is not available. To cope with the impact of data loss on system performance, the last normal observation value is adopted to replace the losing data rather than discarded and set zero directly as some papers do [7], so as to prevent the occurrence of drastic changes in the observed data.

In order to design virtual controllers and control laws for affine system by using backstepping method, the mean value theorem is adopted [31]. If the nonlinear function $f_i$ in (1) is continuous over $[x_{\mu}^+, x_{\mu}^-]$, then it can be described as

$$f_i(\bar{x}_i, x_{j+1}) = f_i(\bar{x}_i, x_{j+1}^*) + h_i(x_{j+1} - x_{j+1}^*)$$

(2)

where $h_i = [df_i(\bar{x}_i, x_{j+1}^*)/dx_{j+1}]|_{x_{j+1}^*} = x_{\mu}, x_{\mu} = (1 - \rho_i)x_{j+1}^* + \rho_0 x_{j+1}, 0 < \rho_i < 1, i = 1, \ldots, n$, with $x_{\mu+1} = u$.

By setting $x_i^+ = 0$ and substituting (2) into (1), (1) can be rewritten as

$$\begin{align*}
\dot{x}_i &= f_i(\bar{x}_i) + h_i x_{i+1}, 1 \leq i \leq n - 1 \\
\dot{x}_n &= f_n(\bar{x}_n) + h_n u, \\
y &= v_1 x_1 + v_2 \sigma(x_1) + v_3 \psi(x_1),
\end{align*}$$

(3)

The task of this work is to design a states estimator in data-losing case and controllers in normal case and data-losing case for the system (1), which enables all the closed-loop signals of the system (1) to be UUB in mean square and the control error to be within a small boundary.

Note that the $h_i$ in (3), $i = 1, \ldots, n$ can be changed to $h_{i\mu}$ for expressing different parameters in normal case and the data-losing case, where $\mu = 1, 2$, is the subscript denoting the normal case and data-losing case, respectively.

To design the controllers, the following assumptions and definitions are made.

Assumption 1: Giving a function $f_i, i = 1, \ldots, n$, there exist known constants $m_i$, such that $\forall X_1, X_2 \in R^n$, the inequality below holds:

$$\|f_i(X_1) - f_i(X_2)\| \leq m_i \|X_1 - X_2\|.$$

(4)
Assumption 2: The $h_{ei}$ is assumed to be positive, $i = 1, \cdots, n$, and satisfies

$$0 < s < h_{ei} < S < \infty$$

where $s$ and $S$ represent constants and $s$ is assumed to be known.

### III. MAIN RESULTS

Due to the physical limitations of devices and the problems from the cyber side, the output of system (1) may be affected by saturation and packet losing. No matter saturation or packet losing, the current observation value cannot be used, therefore these two situations can be combined into one case, which can be named as data-losing case. In this paper, for the data-losing case, the current observation value is replaced by the previous normal value. Next we will discuss the design of a state estimator for the data-losing case and system controllers for two cases, the normal case and the data-losing case.

### A. CONTROLLER DESIGN IN NORMAL CASE

In the normal case, the system states can be observed, based on which, the virtual control law and control law will be developed using backstepping method. The following coordinate transformation is introduced:

$$Z_{11} = x_1 = y, \quad Z_{ii} = x_i - \alpha_{i(i-1)},$$

where $i = 2, \ldots, n$, and $\alpha_{i(i-1)}$ is the virtual control signal. The $\alpha_{i(i-1)}$ will be designed below. From (3) and (6), one can get

$$\dot{Z}_{11} = f_1(x_1) + h_{11}x_2,$$

$$\dot{Z}_{ii} = f_i(x_i) + h_{1i}x_{i+1} - \dot{\alpha}_{i(i-1)}.$$  

Step 1: Choose the Lyapunov function as $V_{11} = \frac{1}{2} \tan^2 (\frac{\pi Z_{11}}{2\tau})$, where $\tau$ is a performance function with $\tau = (\tau_0 - \tau_\infty) e^{-\frac{t}{\tau_\infty}} + \tau_\infty$. Using the Lyapunov function defined in this step, the system output $Z_{11}$ can be limited to the range of output constraint function $\tau$. Differentiating $V_{11}$ yields

$$\dot{V}_{11} = \lambda \left( \frac{z_{11} - z_{11}\tan \frac{\pi z_{11}}{2\tau}}{\tau} \right)$$

$$= \lambda \left( f_1(x_1) + h_{11}x_2 - \frac{z_{11}}{\tau} \right)$$

$$= \lambda \left( f_1(x_1) - \frac{z_{11}}{\tau} + h_{11}Z_{12} + h_{11}\alpha_{11} \right).$$  

where $\lambda = \pi \tan (\frac{\pi Z_{11}}{2\tau}) / 2\tau \cos^2 (\frac{\pi Z_{11}}{2\tau})$. Defining $w = f_1(x_1) - \frac{z_{11}}{\tau}$, it can be obtained from Assumption 2 and Young’s inequality that

$$\lambda w \leq \frac{1}{2} \pi^2 \lambda w^2 + \frac{1}{2\tau}$$

Design the virtual control function $\alpha_{11}(x_1, \tau)$ as:

$$\alpha_{11} = -c_{11}\tau \tan (\frac{\pi Z_{11}}{2\tau}) \cos^2 (\frac{\pi Z_{11}}{2\tau}) - \frac{\lambda w^2}{2},$$

where $c_{11}$ is a positive constant. From Assumption 2 and Young’s inequality, one has

$$\lambda h_{11}\alpha_{11} \leq -\frac{1}{2} \pi^2 \lambda S^2 + \frac{1}{2\tau} \pi Z_{11}.$$  

Then (8) becomes

$$\dot{V}_{11} \leq -\frac{c_{11}\pi}{2} \tan^2 \left( \frac{\pi Z_{11}}{2\tau} \right) + \frac{1}{2\tau} \pi Z_{11}.$$  

Step 2: Choose the Lyapunov function as $V_{12} = \frac{1}{2} Z_{12}^2 + V_{11}$. Differentiating $V_{12}$ along with (7) and (10) yields

$$\dot{V}_{12} = Z_{12} (\dot{x}_2 - \dot{\alpha}_{11}) + \dot{V}_{11}$$

$$= Z_{12} (f_2(x_2) + h_{12}(Z_{13} + \alpha_{12}) - \dot{\alpha}_{11}) + \dot{V}_{11}$$

$$= Z_{12} (f_2(x_2) + h_{12}\alpha_{12} + \lambda h_{11} - \dot{\alpha}_{11}) + h_{12}Z_{12}\dot{Z}_{13}$$

$$= -\frac{c_{11}\pi}{2} \tan^2 \left( \frac{\pi Z_{11}}{2\tau} \right) + \frac{1}{2\tau} \pi Z_{11}.$$  

Similarly, it can be obtained that

$$Z_{12} h_{11} \leq \frac{s_{12} Z_{12}^2 \lambda^2 h_{11}^2}{2S^2} + \frac{S^2}{2s}$$

$$= \frac{s_{12}^2 \lambda^2}{2} + \frac{S^2}{2s}$$  

$$Z_{12} (f_2(x_2) - \dot{\alpha}_{11}) \leq -\frac{s_{12}^2 (f_2(x_2) - \dot{\alpha}_{11})^2}{2}$$

$$+ \frac{1}{2\tau}.$$  

Select the virtual control law $\alpha_{12}(x_2, \dot{\alpha}_{11})$ as

$$\alpha_{12} = -c_{12} Z_{12} - \frac{Z_{12}^2 \lambda^2}{2} - \frac{Z_{12} (f_2(x_2) - \dot{\alpha}_{11})^2}{2},$$

where $c_{12}$ is a positive constant. It can be obtained that

$$Z_{12} h_{12} \alpha_{12} \leq -c_{12} sZ_{12}^2$$

$$- \frac{s_{12}^2 \lambda^2}{2}$$

$$- \frac{Z_{12}^2 s (f_2(x_2) - \dot{\alpha}_{11})^2}{2},$$

which results in

$$\dot{V}_{12} \leq -c_{12} sZ_{12}^2 + \frac{S^2}{2s} + \frac{1}{s} + h_{12}Z_{12}\dot{Z}_{13}$$

$$- \frac{c_{11}\pi}{2} \tan^2 \left( \frac{\pi Z_{11}}{2\tau} \right).$$

Step $i, \ i = 3, \ldots, n$: Choose the Lyapunov function as $V_{ii} = \frac{1}{2} Z_{ii}^2 + V_{i(i-1)}$, with

$$\dot{V}_{i(i-1)} \leq -\sum_{j=2}^{i-1} c_{ij}s_{ij}Z_{ij}^2 + \frac{(i - 2)S^2}{2s}$$

$$+ \frac{i - 1}{2s} + h_{(i-1)}Z_{(i-1)i}$$

$$- \frac{c_{11}\pi}{2} \tan^2 \left( \frac{\pi Z_{11}}{2\tau} \right).$$
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(32) and (33) into (31), one has

\[ V_{20} = e^T Pe, \]

where \( P \) is a positive definite matrix to be designed. The derivative of \( V_{20} \) is

\[ \dot{V}_{20} = 2e^T P(-Ke + \Delta F + K\Delta e). \]

Using Assumption 1, one has \( \|\Delta F\| = \|f_i(x_i) - f_i(\hat{x}_i)\| \leq m_i \|e\| \). Due to \( M = \max_i[m_i^2] \), the following inequalities hold

\[ 2e^T P\Delta F \leq e^T PP e + \Delta F^T \Delta F \leq e^T PP e + Me^T e \] (32)

Using Young’s inequality, we can get that

\[ 2e^T PK \Delta e \leq \frac{1}{\eta^2} e^T PPK^T Pe + \eta^2 H^2. \] (33)

Substituting (32) and (33) into (31), one has

\[ \dot{V}_{20} \leq e^T \left( -2PK + PP + \frac{1}{\eta^2} PKK^T P + MI \right) e + \eta^2 H^2. \] (34)

Based on the estimator in the data-losing case designed above, an output-based control law will be developed using backstepping method, and similar to the normal case, the coordinate transformation in the data-losing case is defined as

\[ Z_{21} = \hat{x}_i, \quad Z_{2i} = \hat{x}_i - a_{2i(i-1)}, \quad i = 2, \ldots, n. \] (35)

where \( a_{2i(i-1)} \) is the virtual control law.

Step 1: Define Lyapunov function \( V_{21} = \frac{1}{2} Z_{21}^2 \) with

\[ \dot{V}_{21} = Z_{21}(f_i(\hat{x}_i) + h_2(Z_{22} + \alpha_{21}) + k_1 e'_i). \] (36)
where \( c_2 \) is a positive constant. Substituting (37), (38) and (39) into (36), one has

\[
\dot{V}_{2i} \leq -sZ_{2i}^2 + i - 1 + h_{2i}Z_{2i-1}Z_{2i},
\]

and its derivative can be expressed as

\[
\dot{V}_{2i} = Z_{2i}(f_1(\hat{x}_1) + h_2\alpha_2 + k_1\epsilon'_1 + Z_{2i-1}h_{2i-1})
\]

Design the virtual control law \( \alpha_{21} \) as

\[
\alpha_{21} = -c_{21}Z_{21} - \frac{Z_{21}^2 (f_1(\hat{x}_1) + k_1\epsilon'_1)^2}{2},
\]

where \( c_{21} \) is a positive constant. Letting \( \xi_1 = 1, \mu = E[e] \), and all the \( s_i \) are positive definite matrices, the stability of the closed-loop system (1) can be established.

**Proof:** The expectation of the Lyapunov function V = \( \xi_1 V_1 + \xi_2 V_2 \) can be expressed as

\[
E[V] = \xi_1 E[V_1] + \xi_2 E[V_2]
\]

where \( \xi_1, \xi_2 \) are the probabilities of the normal case and data-losing case respectively, \( \xi_1 + \xi_2 = 1, \mu = E[e] \), and all the \( s_i \) are positive definite matrices.

**C. STABILITY ANALYSIS**

Combining the analysis results (24) and (45) of the Lyapunov function, the stability of the closed-loop system (1) can be established.

**Theorem 1:** Consider system (1) with the normal and data-losing information transmission cases. Under Assumptions 1, 2, and 3, if there exist a set of positive definite matrices \( P, K \), and positive constants \( c_1, c_2, j = 1, \ldots, n \), such that \( \Lambda < -aP, c_{11} \geq a/s\pi, c_{1j} > a/2, j = 2, \ldots, n, \) and \( c_{2j} > a/2, j = 1, \ldots, n \), where \( a \) is a positive constant, then the proposed control scheme with the states estimator (25), control laws (10), (16), (21), (39), and (43) can ensure that all the signals in the closed-loop system remain UUB in mean square.

**Proof:** The expectation of the Lyapunov function V = \( \xi_1 V_1 + \xi_2 V_2 \) can be expressed as

\[
E[V] = \xi_1 E[V_1] + \xi_2 E[V_2]
\]

where \( \xi_1, \xi_2 \) are the probabilities of the normal case and data-losing case respectively, \( \xi_1 + \xi_2 = 1, \mu = E[e] \), and all the \( s_i \) are positive definite matrices.
The inequality $\Lambda < -aP$ is the precondition of system stability, which, by using Schur complement and defining $H = PK$, can be guaranteed by the following linear matrix inequality (LMI):

$$
\begin{bmatrix}
(MI - 2H) & P & \frac{1}{2}H \\
P & -I & \eta \\
\frac{1}{2}H^T & \eta & 0 - I
\end{bmatrix} < -aP. \tag{51}
$$

**IV. SIMULATION RESULTS**

The effectiveness of the proposed control method is validated in this section. Consider a strict-feedback nonlinear system governed by the following equations:

\[
\begin{align*}
\dot{x}_1 &= x_1^2 + 0.2x_2 \\
\dot{x}_2 &= x_1^2x_2 + u + 0.2u^3
\end{align*}
\tag{52}
\]

where $x_1$ and $x_2$ are the state variables, $u$ is the actual control input. The initial conditions are selected to be $[x_1(0), x_2(0)]^T = [0.3, 0.2]^T$. The simulation time is set to $t \in [0, 10s]$, and the occurrence time of the data-losing case is set to $t \in [2s, 4s] \cup [5s, 7s]$. The virtual control law, control input and the state estimator are calculated and listed as follows:

For the normal case:

\[
\begin{align*}
\alpha_{11} &= -c_{11} \tau \tan \left( \frac{\pi Z_{11}}{2\tau} \right) \cos^2 \left( \frac{\pi Z_{11}}{2\tau} \right) \\
&\quad - \frac{\lambda (x_1^2 - x_1 \dot{x}_1)^2}{2} \\
u_1 &= -c_{12} Z_{12} - \frac{Z_{12} \lambda x_1^2}{2} - \frac{1}{2} Z_{12} (x_1^2 x_2 - \dot{\alpha}_{11})^2 \\
\dot{\alpha}_{11} &= -c_{11} \tau \cos(2\gamma) \dot{y} - c_{11} \tau \tan(\gamma) \cos^2(\gamma) \\
&\quad - \lambda \left( x_1^2 - \frac{x_1 \dot{x}_1}{2} \right) 2x_1 \dot{x}_1 \\
&\quad + \lambda \left( x_1^2 - \frac{x_1 \dot{x}_1}{2} \right) \left( \dot{x}_1 \dot{x}_1 + x_1 \ddot{x}_1 \right) \\
&\quad + \frac{1}{2} \left( x_1^2 + k_1 e_1' \right)^2 \\
\dot{x}_1 &= x_1^2 x_2 + u + 0.2u^3 + k_2 \left( x_2' - \dot{x}_2 \right). \tag{53}
\end{align*}
\]

where $\gamma = \frac{\pi x_1}{2\tau}$, $\dot{y} = \frac{\pi (x_1 - x_1 \dot{x}_1)}{2\tau^2}$, $\ddot{x}_1 = -a(\tau_0 - \tau_\infty)e^{-at}$ and $\ddot{\alpha}_1 = a^2(\tau_0 - \tau_\infty)e^{-at}$.

For the data-losing case:

\[
\begin{align*}
\alpha_{21} &= -c_{21} \hat{x}_1 - \frac{1}{2} \frac{Z_{21} (\hat{x}_1^2 + k_1 e_1')}{2} \\
u_2 &= -c_{22} Z_{22} - \frac{Z_{22} \lambda x_1^2}{2} - \frac{1}{2} Z_{22} (x_1^2 \dot{x}_2 + k_2 e_2' - \dot{\alpha}_{21})^2 \\
\dot{\alpha}_{21} &= -c_{21} \hat{x}_1 - \frac{1}{2} \frac{Z_{21} (\hat{x}_1^2 + k_1 e_1')}{2} \\
&\quad + \hat{x}_1 \left( \hat{x}_1^2 + k_1 e_1' \right) + \left( 2x_1 \dot{x}_1 - k_1 \dot{x}_1 \right) \\
\dot{\hat{x}}_1 &= \hat{x}_1^2 + 0.2 \ddot{x}_2 + k_1 \left( x_1' - \dot{x}_1 \right) \\
\dot{\hat{x}}_2 &= \hat{x}_1^2 \dot{x}_2 + u_2 + 0.2u_2^3 + k_2 \left( x_2' - \ddot{x}_2 \right). \tag{54}
\end{align*}
\]

According to the parameter constraints described in Theorem 1 and (51), the parameters in (53) and (54) are chosen as $c_{11} = c_{12} = 0.5$, $c_{21} = c_{22} = 1.5$, $k_1 = 10$, $k_2 = 2$, $\tau_0 = 0.5$, $\tau_\infty = 0.1$, $a = 2$. The simulation results are shown in Figs. 1-6. Figs 1, 2 and 3 show the states $x_1$, $x_2$ and control input $u$ in the normal case, respectively. For the data-losing case, the saturation value of the system output $x_1$ is set to 0.15, and the measured value is lost at $t \in [2s, 4s] \cup [5s, 7s]$. Figs. 4 and 5 show that the estimation states $\hat{x}_1$ and $\hat{x}_2$ follow states $x_1$ and $x_2$ respectively. Fig. 6 gives the control law $u$ in the data-losing case. The saturation value 0.15 for the output $x_1$ is lower than the initial value 0.3, hence, the output estimation value $\hat{x}_1$ in Fig. 4 does not
The stability of the closed-loop CPSs has been analyzed based on the Lyapunov theory. The effectiveness of the proposed scheme has been confirmed by simulation results. An estimator has been designed to obtain the state in data-losing case. It can be observed from Figs. 1 and 4 that the output exceeds the saturation value and the stability of the system is still maintained. Because the constraints are introduced for the output, it can be observed from Figs. 1 and 4 that the system output does not exceed the constraints $\tau$. It can be seen from the simulation results that the proposed method results in a stable closed-loop nonlinear system, although the system performance is degraded during the period of data losses.

V. CONCLUSIONS

An estimator-based control method has been designed for a class of nonlinear pure-feedback CPSs with incomplete measurements due to the packet losing and sensors saturation in this paper. An estimator has been designed to obtain the system states under the data-losing case, based on which, two controllers have been developed for the normal case and the data-losing case. The stability of the closed-loop CPSs has been analyzed based on the Lyapunov theory. The effectiveness of the proposed scheme has been confirmed by simulation results.

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