Rotation in gravitational lenses

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Abstract

Gravitational lensing deflects light. A single lens deflector can only shear images, but cannot induce rotations. Multiple lens planes can induce rotations. Such rotations can be observed in quadruply imaged sources, and can be used to distinguish between two proposed solutions of the flux anomaly problem: substructures in lensing galaxies versus large-scale structure. We predict the expected amount of rotation due to large-scale structure in strong lensing systems, and show how this effect can be measured using $\sim$mas very long baseline interferometry astrometry of quadruple lenses with extended source structures. The magnitude of rotation is around $1^\circ$. The biggest theoretical uncertainty is the power spectrum of dark matter on very small scales. This procedure can potentially be turned around to measure the dark matter power spectrum on very small scales. We list the predicted rms rotation angles for several quadruple lenses with known lens and source redshifts.

Key words: gravitational lensing – galaxies: evolution – galaxies: structure – cosmology: theory – dark matter.

1 INTRODUCTION

Gravitational lensing results from the deflection of light under the gravitational influence of all matter, luminous or otherwise. Its physics is clean, and this effect has allowed the measurement of the distribution of dark matter from galaxy scales (using strong lensing) to the large-scale structure of the Universe (using weak lensing).

Usually, many approximations are made to simplify the calculations. One of these is the Born approximation, where one calculates a small deflection along the unperturbed light path. Some effects, such as image rotation due to multiplane lensing, are not accessible in this approximation. Authors have obtained different results for the magnitude of multiplane weak lensing rotation (Jain, Seljak & White 2000; Cooray & Hu 2002, hereafter CH; Hirata & Seljak 2003, hereafter HS). Schneider (1997) showed that a strong lens plus spatially constant weak lens system is mathematically identical to some other single lens plane system, so only differential rotation at the image positions is observable.

In this paper, we apply the multiple plane lensing calculation to real physical systems: quadruply imaged quasars. We show in this physical example how the rotation effect can be measured, why it is physical and real, estimate its magnitude and show how it can be used to resolve the substructure controversy.

The rotation of images is of current interests in the context of using gravitational lensing to detect substructures predicted by the cold dark matter (CDM) structure formation model. From both semi-analytical studies and numerical simulations, it became clear that hundreds of subhaloes (substructures) are predicted to exist in a Milky Way-type halo (e.g. Kauffmann, White & Guiderdoni 1993; Klypin, Kravtsov & Valenzuela 1999; Moore et al. 1999; Ghigna et al. 2000). In general, about 5–10 per cent of the mass is predicted to be in substructures, with a typical mass spectrum of $n(M)dM \sim M^{-1.8}dM$. If all the substructures form stars, then the predicted number of satellite galaxies exceeds the observed number in a Milky Way-type halo by a large factor. It is now, however, clear that the correspondence between substructures and visible satellite galaxies is not simple (Gao et al. 2004a,b; see also Springel et al. 2001; Diemand, Moore & Stadel 2004; Natarajan & Springer 2004; Nagai & Kravtsov 2005). In particular, if only some substructures house satellite galaxies, then the discrepancy can be alleviated (e.g. Kravtsov et al. 2004). At present, it is not entirely clear whether the internal kinematics of satellite galaxies are consistent with observations (Stoehr et al. 2002; Kazantzidis et al. 2004). Furthermore, the spatial distribution of satellite galaxies in the Milky Way is also somewhat puzzling (Kroupa, Theis & Boily 2005; but see Kang et al. 2005; Liebskind et al. 2005; Zentner et al. 2005).

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One possible way to detect the (dark) substructures is through the gravitational lensing effect. Simple analytical models in gravitational lenses often fail to reproduce the observed flux ratios (e.g. Kochanek 1991). This discrepancy is commonly referred to as the ‘anomalous flux ratio problem’. This has been proposed as evidence for substructures in the primary lensing galaxies (e.g. Mao & Schneider 1998; Dalal & Kochanek 2002; Metcalf & Zhao 2002). However, as most of the predicted substructures are in the outer part of the lensing galaxies while the lensed images are typically at a projected distance of only a few kpc from the centre, it is unclear whether the predicted amount of substructures in lensing galaxies by CDM is sufficient (Kochanek & Dalal 2004; Mao et al. 2004), so it is important to consider other sources of ‘substructures’. Recently, Metcalf (2005) proposed that substructures along the line of sight can equally explain the discrepancy. A key question naturally arises: how do we know that substructures are from the primary lens or from elsewhere along the line of sight?

In this paper, we examine the rotation of images induced by structures along the line of sight in gravitational lenses (see also Chen et al. 2003). We use a novel power-spectrum approach and consider the fluctuations of surface densities among different images (separated by few tenths to few arcsec) and their effects on the magnifications. Throughout this paper, we adopt the ‘concordance’ Λ CDM cosmology (e.g. Ostriker & Steinhardt 1995; Spergel et al. 2003 and references therein), with a density parameter Ω_m = 0.3, a cosmological constant Ω_Λ = 0.7, a baryon density parameter Ω_b = 0.024 h^{-2} and the power-spectrum normalization σ_8 = 0.9. We adopt a Hubble constant of \( H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

2 LENSING BY LARGE-SCALE STRUCTURE IN THE PRESENCE OF A STRONG LENS

Many multiply imaged gravitational lenses on galaxy scales have been observed. At the time of writing, roughly 100 such systems are known.1 The largest systematic survey, the Jodrell/VLA Astrometric Survey (JVAS) and Cosmic Lens All-Sky Survey (CLASS), has found 22 new galaxy-scale lenses, approximately one-half of which are quadruple lenses (Browne et al. 2003; Myers et al. 2003). Most of these have high-resolution imaging from 0.1 arcsec to mas from Multi-Element Radio-Linked Interferometer Network (MERLIN), Hubble Space Telescope (HST) to very long baseline interferometry (VLBI). Several of these lenses are resolved into multiple components. The system 0128+437 provides a good example (Biggs et al. 2004). Each of the images has been resolved into three subcomponents with VLBI. Such high-resolution images provide an excellent test bed for lensing models.

In addition to the strong lens system which causes the multiple image splitting, all the matter along the line of sight will further deflect the light and contribute to distortions in the image. One such effect is the apparent rotation of images. It can be shown that a single plane lens has a symmetric amplification matrix, which shears but does not rotate images. With multiple planes, image rotation is possible. Observing it may appear non-trivial since it would require prior knowledge of the unlensed image alignment. We will show below (see Section 3) how this can actually be measured if one has a quadruply imaged source with extended source structures.

Several geometric configurations can lead to image rotation. A strong lens has shear and convergence of the order of unity. With sufficiently accurate alignment, a second strong lens could occur along the line of sight. Indeed, such an example has already been seen – the JVAS/CLASS lens system, B2114+022 (Augusto et al. 2001; Chae, Mao & Augusto 2001), has two lensing galaxies at redshifts 0.3157 and 0.5883, respectively, within 2 arcsec of quadruple radio sources.2

However, in general, the expected variation in surface density due to dark matter is small. Integrating the Limber equation, this leads to large-scale structure density variations of the order of a few percent of the critical surface density (e.g. Jain et al. 2000). Therefore, a random lens will typically only have weak lenses in its foreground and background.

The typical splitting angle of a galaxy-scale strong lens is around 1 arcsec. Large-scale structure density fluctuations on such scales are significantly correlated. As shown by Schneider (1997), perfectly correlated weak lensing screens on the scale of image separations do not cause observable rotation. To see the effects of rotation, one must compute the differential weak lensing shear in different images.

The cross-correlation between two image positions separated by an angle Δθ is defined as

\[ r \equiv \xi_s(\Delta \theta)/\xi_s(0), \]

where \( \xi_s(\Delta \theta) \) is the two-dimensional Fourier (Bessel) transform of equation (17) which will be discussed below. The results are shown for four quadruple lenses with known lens and source redshifts, B1422+231 (Patnaik et al. 1992), MG0414+0534 (Hewitt et al. 1992), B1608+656 (Myers et al. 1995) and B2045+265 (Fassnacht et al. 1999). We find a ratio of variances between the difference of two images \( \sigma^2 \) and the individual variances to be

\[ \frac{\sigma^2}{\sigma^2} = 2(1 - r). \]

If the two images are uncorrelated, the difference will have twice the variances of each individual image. Fig. 1 shows the correlation function versus image separation. As can be seen, typically we have \( r > 0.5 \), so the difference mode has a slightly lower variance than each individual image.

We assume that the various images pass through different parts of the strong lens with correspondingly different values of shear and convergence, and also differing weak lens deflectors. We consider a two-plane lens, \( L_1 \) is the strong lens, \( L_2 \) is the weak lens due to large-scale structure, \( O \) is the observer position and \( S \) is the source plane. The geometry is shown in Fig. 2. We consider a quadratic potential on each lens

\[ \sigma \equiv \text{individual variances to be} \]

\[ \Omega \]

\[ \text{structure,} \]

\[ \Omega_1/Lambda_1 \]

\[ (Myers \text{ et al. 1995}) \]

\[ \text{and B2045} \]

\[ \text{known.1} \]

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1 see the CASTLES data base: http://cfa-www.harvard.edu/castles/

2 It is presently unclear whether the radio sources B and C are lensed images.
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Figure 1. The correlation function for convergence ($\kappa$) for different image pairs induced by the large-scale structure; the correlation function at zero lag is the variance in $\kappa$. The boxes indicate the largest and the smallest splitting for four CLASS lenses with known lens and source redshifts. Because splitting angles are small, the $\kappa$ by the large-scale structure at different image positions is strongly correlated. The variance from large-scale structure shear at each image position is the correlation function at zero lag. The covariance for images separated by a lag $\Delta \theta$ can be seen in the plots. Since the covariance (at typical image separations) is of comparable size to the autovariance, which leads to a suppression of the differential shear mode.

Figure 2. Lensing geometry from the observer plane to the source plane. $L_1$ is the plane of the strong lens, while $L_2$ mimics the lensing effects of the large-scale structure. The deflection angles themselves are not observable, but rather the changes in deflection angle. Various quantities are indicated in the figure and used in equations (4)–(7).

plane $\phi = ax_1^2 + 2bx_1x_2 + cx_2^2$, where $(x_1, x_2)$ are the (angular) coordinates in the lens plane. The units for the potential are chosen such that $2\kappa = \nabla^2 \phi$, and the deflection angle is $\hat{\alpha} = \nabla \phi$ due to lens, where we use the same notations as in Schneider, Ehlers & Falco (1992). To distinguish the two lens planes, we will use a prime to denote variables in the $L_2$ plane.

For a general anisotropic lens, the deflection angle is a vector. For our quadratic potential, $\kappa$ and $\gamma_{1,2}$ are constant on the plane:

$$\kappa = \frac{1}{2}(a + c), \quad \gamma_1 = \frac{1}{2}(a - c), \quad \gamma_2 = b.$$  (3)

The quadratic potential is the most general function which leads to constant values of $\kappa$ and $\gamma$. Their constancy on small scales is inferred from the correlation function shown in Fig. 1. In such a potential, the full gravitational lensing effect is straightforward to compute. One can solve the full photon trajectory, which are deflected on the lens planes by the gradient of the potential.

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We now denote $D_1$ to be the angular diameter distance from the observer to the first lens, $D_{12}$ is the distance from the first to the second lens, $D_{2s}$ is the distance from the second lens to the source, and so on. We see from Fig. 1 that all the relative deflection angles are very small in units of radians ($\sim 10^{-5}$), so we can expand to first order in the deflection angle (Schneider et al. 1992, chapter 9)

$$
\eta = \frac{D_1}{D_1} \xi_1 - D_{1s} \alpha_1(\xi_1) - D_{2s} \alpha_2(\xi_2), \quad \xi_2 = \frac{D_1}{D_1} \xi_1 - D_{12} \alpha_1(\xi_1).
$$

(4)

where $\eta$, $\xi_1$, and $\xi_2$ denote the position vectors in the source plane and the first and second lens planes, respectively (see Fig. 2); the deflection angles in the first and second planes are given as $\alpha_1$ and $\alpha_2$, respectively.

Following Schneider et al. (1992), we define two reduced deflection angles

$$
\alpha_1 = \frac{D_{1s}}{D_1} \alpha_1, \quad \alpha_2 = \frac{D_{2s}}{D_1} \alpha_2.
$$

(5)

With these, equation (4) can be recast in a very simple form using only angles

$$
y = x_1 - \alpha_1(x_1) - \alpha_2(x_2), \quad x_2 = x_1 - \beta_2 \alpha_2(x_1),
$$

(6)

where $y = \eta/D_1$, $x_1 = \xi_1/D_1$, $x_2 = \xi_2/D_2$ and $\beta_{12} = D_{12} D_2/(D_1 D_2)$.

Lensing shear and magnification can be obtained by studying the change of source position $y$ resulting from a change of apparent angular position $x_1$. In particular, the magnification is given by (Schneider et al. 1992)

$$
A = \frac{\partial y}{\partial x_1} = 1 - U_1 - U_2 + \beta_{12} U_2 U_1,
$$

(7)

where $1$ is a unit matrix, $U_1 = \partial \alpha_1 / \partial x_1$, and $U_2 = \partial \alpha_2 / \partial x_2$.

We are considering the combined effects of a strong and a weak lens. The contribution of the weak lens $U_1$ is small, and we are only interested in its contribution to rotation, so we neglect its linear effect. In the linear (i.e. Born approximation) regime, it is also observationally impossible to distinguish between contributions from the strong and the weak lens. In the product term, we can absorb $\beta_{12}$ into the definition of $U_2$. We define the critical density for the weak lensing large-scale structure to be

$$
\Sigma_{2 crit}^2 \equiv \frac{c^2}{4 \pi G D_{1s} D_{2s}}.
$$

(8)

which is the lensing strength of the large-scale structure as seen by an observer at the strong lens position. This lends a simple observational and computational interpretation of lensing rotation: all distortions visible to an observer at the strong lens position enter linearly into the coupling product of the strong and the weak lens.

An analogous result holds when the weak lensing plane is in front of the strong lens. In our notation, we have

$$
U_1 = \begin{pmatrix} \kappa + \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa - \gamma_1 \end{pmatrix}, \quad U_2 = \begin{pmatrix} \kappa' + \gamma_1' & \gamma_2' \\ \gamma_2' & \kappa' - \gamma_1' \end{pmatrix},
$$

(9)

where $\kappa' = \Sigma_2 / \Sigma_{2 crit}$, and so

$$
A = \begin{pmatrix} \gamma_2 \gamma_2' + (1 - \kappa - \gamma_1)(1 - \kappa' - \gamma_1') & -\gamma_2' (1 - \kappa - \gamma_1) - \gamma_2 (1 - \kappa' + \gamma_1') \\ -\gamma_2(1 - \kappa + \gamma_1) - \gamma_2(1 - \kappa' - \gamma_1') & \gamma_2 \gamma_2' + (1 - \kappa + \gamma_1)(1 - \kappa' + \gamma_1') \end{pmatrix}
$$

$$
\sim \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 + \gamma_2 \gamma_1' \\ -\gamma_2 - \gamma_2 \gamma_1' & 1 + \kappa + \gamma_1 \end{pmatrix}.
$$

(10)

In the last approximate equality, we used the limit that the weak lensing plane (the primed variables) is much smaller than the strong lens plane, but keeping the antisymmetric piece which is relevant for rotations. We define the off-diagonal antisymmetric component as $\omega \equiv \gamma_2 \gamma_1'$. We will discuss the impact of these approximations below.

For strong lens systems, the shear on $L_1$ is of the order of unity, while the large-structure shear $(\gamma_1', \gamma_2')$ is a few per cent, corresponding to a rotation of the order of a degree. Rotation only results when the principal axes of the two amplification matrices are misaligned. We choose the coordinates such that

$$
U_1 = \begin{pmatrix} \gamma & 0 \\ 0 & -\gamma \end{pmatrix}, \quad U_2 = \begin{pmatrix} \gamma_1' & \gamma_2' \\ \gamma_2' & -\gamma_1' \end{pmatrix}.
$$

(11)

$\gamma_1'$ does not contribute to rotation, so we set it to zero. Their product is a pure antisymmetric matrix

$$
U_2 U_1 = \begin{pmatrix} 0 & \gamma_2 \gamma_2' \\ -\gamma_2 \gamma_2' & 0 \end{pmatrix}.
$$

(12)

When added to a unit matrix, this corresponds to a rotation matrix by an angle $\gamma \gamma_2'$.

In equation (10), we factor the amplification matrix $A = A_{\text{R}} R(\phi)$, as a product of a symmetric matrix and a pure rotation, where the rotation matrix

$$
R = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}.
$$

(13)
We have
\[ \mathbf{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 + \omega \\ -\gamma_2 - \omega & 1 - \kappa + \gamma_1 \end{pmatrix}, \quad (14) \]
where \( \omega \) indicates the importance of rotation. Then, we find
\[ \tan \phi = -\frac{\omega}{1 - \kappa}. \quad (15) \]

The non-rotating amplification matrix \( \mathbf{A} \), has the same convergence as \( \mathbf{A} \) up to \( \mathcal{O}(\omega^2) \), but has its shear components rotated by \( \phi/2 \). We now see the effect of the dropped terms in equation (10). The symmetric off-diagonal terms have no effect on the rotation angle. The terms on the diagonal induce a fractional change of the order of the weak lensing optical depth, unless \( \kappa \) is very close to 1. Thus, the dropping of those terms does not affect the observables.

The different images pass through different parts of the strong lens, each with its own values of shear \( \gamma \). The rotation angle will thus be different for each image. The actual value of the large-scale structure shear depends on the non-linear power spectrum of dark matter at small physical scales (\( \sim \) few kpc scales for galaxy-scale lenses). This is both a function of the primordial power spectrum and its slope, and non-linear gravitational physics. The most accurate models seem to be a combination of \( N \)-body simulations and heuristic models based on stable clustering. Calibrations at larger scales indicate consistency at the better than 20 per cent level on length scales larger than about 100 kpc. In the standard models, the contribution from smaller scales is not dominant, so one would expect forecasts to be good to a factor of 2 (Huffenberger & Seljak 2003).

To forecast the expected rms rotation angle, we compute the expected differential variation in the weak lens screen shear, and apply to equations (12) and (15). Mathematically, shear is a polarization field, which can be described as a trace-free spin-2 tensor field. In two spatial dimensions, any trace-free spin-2 tensor field can be decomposed into two kinematic scalars: the ‘divergence-like’ component, also known as ‘E’ mode which is longitudinal to its Fourier decomposed wave vector, and a pseudo-scalar ‘curl-like’ or ‘B’ mode which is transverse to its wave vector. In weak gravitational lensing, the ‘E’ mode is identical to the convergence field \( \kappa \), and the ‘B’ mode is zero. To compute the statistics of the shear, it thus suffices to calculate the dimensionless variations in projected matter surface density. Its variation is given by the Limber equation. It involves the projection of a three-dimensional non-linear power spectrum
\[ \Delta^2(k, z) \equiv \frac{k^3}{2\pi^2} P(k, z) \quad (16) \]
to a two-dimensional angular power spectrum \( \ell(\ell + 1)C_\ell/2\pi \) (Limber 1954; Kaiser 1992, 1998):
\[ \frac{\ell(\ell + 1)}{2\pi} C_\ell = \frac{\pi}{\ell} \int_0^\ell \Delta^2(\ell/\Delta \chi(z), z) w(z)^2 \chi(z) \frac{d\chi}{dz} \frac{dz}{dz}. \quad (17) \]

For the foreground large-scale structure, \( z_1 = 0, z_2 = z_1, \Delta \chi(z) = \chi(z) \) where \( z_1 \) is the redshift of the strong lens. For the background large-scale structure, \( z_1 = z_s, z_2 = z_s, \Delta \chi(z) = \chi(z) - \chi(z_s) \), where \( z_s \) is the source redshift. We used the Peacock & Dodds (1996) formulation to obtain the non-linear power from the linear transfer function given by Bardeen et al. (1986). The comoving angular diameter distance is
\[ \chi(z) = c \int_0^z \frac{dz}{H(z)}, \quad (18) \]
where \( H(z) \) is the Hubble constant at redshift \( z \):
\[ H(z) = H_0 \left[ (1 + z)^3 \Omega_m z + 1 - \Omega_m z + 2 \right]^{1/2}. \quad (19) \]

For the comoving angular diameter distance \( \chi \), we used the fitting formula from Pen (1999). In a cosmological context, it is convenient to use comoving angular diameter distances and conformal time, where light rays propagate as they do in an empty universe (White & Hu 2000). The lensing weight is
\[ w(z) = 3 \frac{\Omega_m H_0^2 g(z)}{2} (1 + z), \quad (20) \]
where
\[ g(z) = \frac{[\chi(z) - \chi(z_s)][\chi(z) - \chi(z_s)]}{\chi(z) - \chi(z_s)}. \quad (21) \]

The lensing weighting factor \( g \) corresponds to the distance-weighted terms in equation (8). A similar relation holds for the background lenses.

Table 1 gives the expected rotation angle for the five quadruple lenses in the \( \Lambda \)CDM cosmology. Several simplifying assumptions were made. We attribute all the rotations to the furthest image. The change in strong lens \( \gamma \) between images is taken to be 0.3, which is multiplied by the large-scale structure shear in equation (12). What matters is not the change in the absolute value of \( \gamma \), but the change in each component. We took the variance of the large-scale structure shear to be half of the convergence, which corresponds to the variance in \( \gamma_2 \) in the principal axis frame of the strong lens. In practice, only differences in rotation angles are observable, which depends on the alignment angles of the shear at different image positions. There is a contribution to rotation from the structure in the foreground as well as the background of the lens. The variances were simply added. The ratio of the foreground to background variance is listed in Column 5 in Table 1. With all these caveats, we expect the expected rms image rotation to be good to about a factor of 2, which is comparable to the expected errors on the theoretical lensing power spectrum.
3 MEASURING ROTATION

We only consider the simplest case. The source is made up of three components, \( P^1, P^2 \) and \( P^3 \). We define \( P^3 \) to sit at the origin, \( P^2 \) to sit at \((P^2_x, P^2_y)\) and \( P^1 \) at \((P^1_x, P^1_y)\). We can always choose our coordinate system this way.

The lensed image appears at positions \( A, B, C \) and \( D \) with three components each. We again define the position of the third component to be the origin, and only consider relative distances. The apparent position of \( A^2 \) relative to \( A^1 \) is \( P^2 = D_A A^2 \) and similarly \( P^1 = D_A A^1 \), where the deflection matrix \( D_A \) is the inverse of the amplification matrix \( A \) defined in equation (7) for image \( A \); each image has its own deflection matrix. It will be convenient to concatenate the two position column vectors \( P^1 \) and \( P^2 \) into a \( 2 \times 2 \) matrix \( P \). We can write similar equations for all images, resulting in an apparent 16 equations for 16 unknowns: \( P \) each has four unknowns, and each symmetric amplification matrix has three unknowns. The equations are linear, and homogeneous:

\[
D_A A = P, \quad D_B B = P, \quad D_C C = P, \quad D_D D = P.
\]

It is clear that one could multiply each solution by a constant and obtain a solution; so one must fix one more parameter. Without loss of generality, one could fix \( P^3_x = 1 \), which just fixes a length scale. If none of the amplifications are known, the solutions are clearly degenerate between a small image that is strongly magnified and a large image that is less magnified. A further degeneracy occurs because we can multiply each equation on the left-hand side by an arbitrary shear matrix. Since we do not know the intrinsic location of the source substructure positions, this is indistinguishable from a constant shear applied to both the lens and the source. This corresponds to a uniform shear plane between the strong lens and the source. For a shear between observer and strong lens, one can similarly symmetrize the deflection matrix by multiplying both the lens and the sources by the inverse shear matrix. This is in accordance with Schneider (1997). Thus, we only measure the differences in shears at the image positions.

Then, we have 16 equations for 15 unknowns, which is overdetermined by one. This allows us to solve for one rotation angle. If one assumes the rotation to be dominated by the most magnified image, say \( A \), we simply take \( D_A \) to be non-symmetric. This allows one to solve for the rotation angle.

In general, however, the large-scale structure shear has two independent components \( \gamma_1 \) and \( \gamma_2 \). To make progress, one can assume them to be Gaussian distributed with standard deviation \( \sigma \). Their one point function is independent, \( P(\gamma_1, \gamma_2) = N(\gamma_1, \sigma)N(\gamma_2, \sigma) \). Equation (12) expresses the off-diagonal components of each deflection matrix in terms of the two large-scale structure shear components. If we fix a value of \( \gamma_1 \), we can solve equation (22) for \( \gamma_2 \), giving us an implicit definition of \( \gamma_2(\gamma_1) \). Then, integrating over all possible values of \( \gamma_1 \) weighted by the probability gives the total likelihood for an assumed \( \sigma \):

\[
L(\sigma) = \int P(\gamma_1, \gamma_2(\gamma_1)) \, d\gamma_1.
\]

One can then solve for the maximum likelihood value of \( \sigma \).

4 DISCUSSION

If one wishes to observe this effect, several challenges must be overcome. Sources must have at least three localizable components, and the position of each must be measured very precisely, to better than 1 per cent of the component separation. Positions are also affected by changes in the lens from one component position to the next. The latter effect is expected to be significantly smaller than rotation because it depends on the change in shear on small scales, while the rotation depends on the large-scale structure shear itself. Basically, the rotation is of the order of the large-scale structure shear. The variation of the large-scale structure shear on source substructure scales is smaller than the shear itself.

We quantify this as follows. Let us assume that the components of the lens have a separation of 10 mas. The model assumed that the shear was constant over the apparent size of the source. The change in shear across the lens is given by \( \xi_z (\Delta \theta = 10 \text{ mas}) \). From Fig. 1, we see that \( \kappa \) has a differential variance of around \( 10^{-4} \) at this separation, corresponding to a per cent change in lensed length scales. This is of comparable magnitude to the rotation effect, which makes it desirable to have more than three components to check. In practice, the actual lensing substructure is suppressed by several factors. Fig. 1 shows the weak lensing correlation in the absence of a strong lens. Lines converge...
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behind the lens, which reduces the corresponding physical scales, and thus the total variance. And only the variation in shear principal axes across three subimage positions affects the rotation, which is suppressed by another dimensionless factor.

It is worth noting that HS proved that the rotation $\omega$ is identical to the B mode of the shear, up to a constant. In this calculation, we only considered the local value of shears, while the E–B decomposition is non-local. We should therefore consider the four entries in the magnification matrix to be independent. In principle, if one had a very high number density of background sources, for example from reionization (Pen 2004), one could also solve for the strong lensing E–B decomposed map (Pen 2000). Such a decomposition also allows one to distinguish between structures in the strong lens plane, and contributions from large-scale structure along the line of sight. The signal-to-noise ratio required for such an exercise is difficult to achieve with current technology. Using quadruply imaged sources allows one to reduce the pairwise noise, and thus solve for the rotational component.

We also note that our calculation did not use the Born approximation. We modelled each lens plane by a quadratic potential as inferred from the $\kappa$ correlation function, and computed the full deflection trajectories. Recently, calculations of rotations by large-scale structure at different lens planes resulted in different answers, depending on whether calculations were done with light rays (HS) or with shears on light bundles (CH). In the CH calculation, ray bundles were propagated along unperturbed trajectories, but distortions were accumulated. The distortions are relative deflections of neighbouring rays, which accumulate rotations through the same effect as discussed in this paper. This allows CH to see rotation without considering the explicit deflection of the centres of each ray. In terms of a full Taylor expansion of light rays along unperturbed paths, this includes some of the effects beyond the Born approximation. HS included all leading order corrections to the Born approximation, and thus differed in their answer. In our case, the angular scales involved are very small, and we can assume the rotation to be dominated by the common large-scale structure shear at different image positions. The CH and HS approaches do not differ in our model since the potential is taken to be quadratic, and all second derivatives are constant.

Schneider (1997) constructs an equivalent single plane strong lens from a strong lens plus constant weak shear. He showed that such a uniform large-scale structure shear is not observable in image rotations. In comparison, our paper focuses on the (observable) image rotations caused by the difference in shears (i.e. the non-uniform component) among different images.

For the rotations observable, our model assumes a source plane consisting of three compact components with unknown positions. If all three components are quadruply lensed, there are a total of 12 angular positions, i.e. we have 24 observables. We subtract 8 degrees of freedom for the global lens model because we do not know the macroscopic lens deflection angle. This leaves 16 constraints. Then, using equation (22) we can solve for the relative positions in the source plane modulo a scaling (due to the mass sheet degeneracy), which is three in number, as well as the values of the amplification matrix at each image position, which is four sets of three numbers, plus one rotation, for a total of 16 parameters. This suggests that small-scale structure in a single lens is observationally distinguishable from weak shear at a different redshift.

We have made several approximations, which will affect the results at some level. We used the difference in shears at the two furthest image positions. In the analysis, we then assumed that three of the images had no rotation, and only the furthest accounted for the multiplane rotation. Reality is more complex, and all images have differential weak lensing shear, and thus some level of differential rotation. Simulations are needed to quantify this simplifying assumption.

In this paper, we have concentrated on the rotation induced by the weak large-scale structure. However, for the cases where one has multiple lenses along the line of sight, the rotation can be more significant. For the best-fitting model in Chae, Mao & Augusto (2001), we find that the two Jacobian matrices are given by

$$ J = \begin{bmatrix} 0.176 & -0.093 \\ -0.258 & -0.595 \end{bmatrix}, \begin{bmatrix} 0.888 & -0.174 \\ -1.48 & 0.481 \end{bmatrix}. $$

(24)

for images A and D, corresponding to magnifications $-7.78$ and $2.49$, respectively. Clearly, for the more highly magnified image A, the Jacobian is highly asymmetric and the rotation is quite significant.

5 CONCLUSIONS

We have computed the expected rotation from uncorrelated foreground and background large-scale structure in strong lensing systems. We have shown that this effect is in principle observable with precise VLBI imaging of quadruply imaged lens systems. The rotation is physical and observable in the example of a source plane consisting of three point sources. If observed, it can unambiguously determine the shear amalyly problem is caused by substructure on the lens plane, or by uncorrelated structures along the line of sight.

In addition, it demonstrates how one can extract information about small-scale dark matter structure along the line of sight to a lens. A measurement of rotation would measure the variance of $\kappa$ from large-scale structure at the smallest scales. This affects the scatter in supernovae lensing effects, and may have potential to measure the primordial power on small scales, and possibly a tilt or running spectral index.

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