Bimagic Squares of Bimagic Squares and an Open Problem

Inder Jeet Taneja
Departamento de Matemática
Universidade Federal de Santa Catarina
88.040-900 Florianópolis, SC, Brazil.
e-mail: taneja@mtm.ufsc.br
http://www.mtm.ufsc.br/~taneja

Abstract

In this paper we have produced different kinds of bimagic squares based on bimagic squares of order $8 \times 8$, $16 \times 16$, $25 \times 25$, $49 \times 49$, etc. A different technique is applied to produce bimagic square of order $16 \times 16$, $25 \times 25$, $49 \times 49$, etc. The bimagic square of order $8 \times 8$ used is the already known in the literature. The work is neither based on any programming language nor on mathematical results. Just simple combinations are used to produce these bimagic squares. Moreover, in each case we have used consecutive numbers starting from 1.

1 Introduction

In this work we produced different orders of bimagic squares containing bimagic or semi-bimagic squares. Most of the work is based on the bimagic squares of orders $8 \times 8$, $16 \times 16$ and $25 \times 25$. We have also produced new bimagic squares of order $49 \times 49$, $121 \times 121$. The bimagic square of order $8 \times 8$ is obtained long back by Pfeffermann 1891 [5]. The bimagic squares of order $16 \times 16$ and $25 \times 25$ are very much similar to one given in [2] as examples. The difference is that the bimagic square of order $16 \times 16$ appearing in [2] don’t have the property that each sub-block of order $4 \times 4$ as a magic square, while in our case it happens. The bimagic square of order $25 \times 25$ is very much similar to given in [2] but we have applied a little different approach to produce it. The bimagic square of order $49 \times 49$ is long back produced by G. Tarry 1895 [1]. Here also we applied a little different approach. We have produced the bimagic square of order $121 \times 121$ without knowing that is it done before. Based on the approach adopted in this work, we can always produce bimagic squares using squares of prime number such as $13^2 \times 13^2$, $17^2 \times 17^2$, etc.

No programming language is used, just simple combinations are sufficient to produce whole the work. In each case, we have used consecutive numbers starting from 1. Some of these files are available at the authors’ web-site given above.

During construction we observe that in case of orders $k^4 \times k^4$, $k = 4$, 5 and 7, in the previous subgroup $k^3 \times k^3$, $k = 4$, 5 and 7 the bimagic sums has the value in each
case. For more details see the table given at the end as an open problem to prove it mathematically.

Before we proceed, here below are some basic definitions:

(i) A **magic square** is a collection of numbers put as a square matrix, where the sum of element of each row, sum of element of each column and sum of each element of two principal diagonals have the same sum. For simplicity, let us write it as $S_1$.

(ii) **Bimagic square** is a magic square where the sum of square of each element of rows, columns and two principal diagonals are the same. For simplicity, let us write it as $S_2$.

(iii) **Upside down**, i.e., if we rotate it to 180° degree it remains the same.

(iv) **Mirror looking**, i.e., if we put it in front of mirror or see from the other side of the glass, or see on the other side of the paper, it always remains the magic square.

(v) **Universal magic squares**, i.e., magic squares having the property of upside down and mirror looking are considered *universal magic squares*.

A good collection of multimagic squares can be seen in [1]. New upside down and universal magic squares can be seen in Taneja [6]-[11].

## 2 Details

Whole the work we have divided in small parts. In the end we have given magic and bimagic sums in each case.

### 2.1 First Part

In this part we have presented bimagic squares of the following orders:

$16 \times 16, 32 \times 32, 56 \times 56, 64 \times 64, 72 \times 72, 88 \times 88, 96 \times 96, 104 \times 104, 112 \times 112, 128 \times 128, 144 \times 144, 176 \times 176, 208 \times 208,$

$224 \times 224, 256 \times 256, 512 \times 512, 1024 \times 1024, 2048 \times 2048$ and $4096 \times 4096$.

Let us divide the above bimagic squares in three small groups.

#### 2.1.1 First Small Group

In this subsection we have given the following bimagic squares

$16 \times 16, 64 \times 64, 256 \times 256, 1024 \times 1024$ and $4096 \times 4096$. 
In this group we have the special property that each block of orders 16×16, 64×64, 256×256 and 1024×1024 are also bimagic squares. Also, each block of order 4×4 is a magic square. In case of magic squares of order 256×256 each block of order 64×64 has the same bimagic sum S2. In case of magic squares of order 1024×1024 and 4096×4096 each block of order 256×256 produces a same bimagic sum S2 for 64×64. Using the same procedure we can also calculate the bimagic square of order 128×128. Its values are given in the last section.

2.1.2 Second Small Group

In this subsection we have given the following bimagic squares

\[32 \times 32, 64 \times 64, 96 \times 96, 128 \times 128, 512 \times 512\] and \[2048 \times 2048\].

This group has the special property that each block of orders 8×8, 64×64, 96×96, 128×128, 512×512 and 2048×2048 is either a bimagic or semi-bimagic square but the final group is always a bimagic square. Each block of order 8×8 is always a magic square having the same magic sum S1 in whole the order. The bimagic square of order 160×160 can also be calculated with the same procedure, but we have calculated it in the next part as multiple of 16. If we calculate 128×128 according to first small group i.e., using bimagic square of 16×16, then the next orders 512×512 and 2048×2048 can also be calculated as combinations of bimagic squares of order 16×16. In this case all the bimagic squares of this group goes to first small group except 32×32 and 96×96.

2.1.3 Third Small Group

In this subsection we have given the following bimagic squares

\[56 \times 56, 72 \times 72, 88 \times 88, 104 \times 104, 112 \times 112, 144 \times 144, 176 \times 176, 208 \times 208\] and \[224 \times 224\].

In case of 56×56, 72×72, 88×88 and 104×104, each block of order 8×8 is either a bimagic or semi-bimagic square but the final order is always a bimagic square. Each block of order 8×8 is always a magic square. While, in case of 112×112, 144×144, 176×176, 208×208 and 224×224 each block of order 16×16 is a bimagic square with the property that each block of order 4×4 is a magic magic square.

2.2 Second Part

In this part we have presented bimagic squares of the following orders:

\[40 \times 40, 80 \times 80, 120 \times 120, 160 \times 160, 200 \times 200, 240 \times 240, 400 \times 400, 600 \times 600, 800 \times 800, 960 \times 960, 1000 \times 1000, 1200 \times 1200, 1600 \times 1600, 2000 \times 2000, 2400 \times 2400, 3000 \times 3000, 3200 \times 3200\] and \[4000 \times 4000\].

Let us divide the above bimagic squares in two small groups
2.2.1 First Small Group

In this subsection we have given the following bimagic squares

\[ \begin{align*}
80 \times 80, & \quad 160 \times 160, \quad 240 \times 240, \quad 400 \times 400, \quad 800 \times 800, \quad 960 \times 960, \\
1200 \times 1200, & \quad 1600 \times 1600, \quad 2000 \times 2000, \quad 3200 \times 3200 \text{ and } 4000 \times 4000.
\end{align*} \]

This group we have the special property that each block of orders 16×16 is a bimagic square with each block of order 4×4 as a magic square. In higher cases such as 400×400, each block of order 80×80 is also a bimagic square, etc.

2.2.2 Second Small Group

\[ \begin{align*}
40 \times 40, & \quad 120 \times 120, \quad 200 \times 200, \quad 600 \times 600, \\
1000 \times 1000, & \quad 2400 \times 2400 \text{ and } 3000 \times 3000,
\end{align*} \]

This group has the special property that each block of orders 8×8 is either a bimagic or semi-bimagic but the final group is always a bimagic square. Each block of order 8×8 is always a magic square. In the higher case such as 200×200, each block of order 40×40 is a bimagic or semi-bimagic, etc.

2.3 Third Part

In this section we have presented bimagic squares of following orders:

\[ \begin{align*}
25 \times 25, & \quad 125 \times 125 \text{ and } 625 \times 625
\end{align*} \]

Here each block of order 5×5 is a magic square with the same sum S1. Each block of orders 25×25 is a bimagic square. In case of 625×625, each block of order 125×125 is also a bimagic square with the same sum S2.

2.4 Forth Part

In this section we have presented bimagic squares of the following orders:

\[ \begin{align*}
49 \times 49, & \quad 343 \times 343 \text{ and } 2401 \times 2401.
\end{align*} \]

Here each block of order 7×7 is a magic square with the same sum S1. Each block of order 49×49 is a bimagic square. Moreover, in case of 2401×2401 each block of order 343×343 has the same bimagic sum S2.

2.5 Fifth Part

In this section we have presented bimagic squares of the following orders:

\[ \begin{align*}
121 \times 121, & \quad 1331 \times 1331.
\end{align*} \]

Here each block of order 11×11 is a magic square with the same sum S1. Each block of order 121×121 is a bimagic square. The next order 14641×14641 multiple of 11 with 1331 is not calculated because of higher values.
3 Semi-Bimagic Squares

Following the same approach applied to obtain bimagic squares in the above sections, we can still have semi-bimagic squares of orders $24 \times 24$ and $48 \times 48$ with the property that in case of $24 \times 24$, each block of order $8 \times 8$ is a magic square with three blocks of order $8 \times 8$ as bimagic and six blocks as semi-bimagic. In case of $48 \times 48$, each block of order $4 \times 4$ is a magic square. Here we have 9 blocks of bimagic squares of order $16 \times 16$. For the numerical values of these semi-bimagic squares see the last section.

4 Bimagic Squares of Magic Squares

Tarry-Cazalas [3] in 1934 (see [1]) gave a bimagic square of order $9 \times 9$. Following the idea of Tarry-Cazalas and the approach adopted above, we obtained here bimagic squares of orders:

$$81 \times 81 \text{ and } 729 \times 729.$$ 

These two bimagic squares have the property that each block of order $9 \times 9$ is just a magic square, rather than bimagic as in the other cases studied above. Also the sum of all 9 member in each block of order $3 \times 3$ has the same sum as of $S_{1,9,9}$. If we follow the idea of Pfeffermann (see [1]) of bimagic square of order $9 \times 9$ and use our approach, we are unable to get bimagic squares of orders $81 \times 81$ and $729 \times 729$. In whole the work, this is the only case, where we don’t have subgroups of bimagic squares.

We can still have bimagic squares of orders $81 \times 81$ and $729 \times 729$ considering just magic square of order $9 \times 9$. In this situation, we get bimagic squares with the property that each small groups of orders $3 \times 3$ and $9 \times 9$ are semi-magic squares finally giving bimagic squares of orders $81 \times 81$ and $729 \times 729$. For details see the files given in authors’ site [13].

5 Open Problem

The bimagic square of order $16 \times 16$ already known in the literature don’t have the property that each block of order $4 \times 4$ as a magic square. This we have done in this work. We have produced bimagic squares of order $25 \times 25$, $49 \times 49$, $121 \times 121$, etc. in a different approach than the one already known in the literature. Interesting the approach adopted lead us the following property:
We observe from the above table that in the first sub-group in each case the $S_2$ is same, i.e., $k^4 \rightarrow k^3 \rightarrow k^2 \rightarrow k^1$, $k = 4$, 5 and 7. Now the question is to prove it mathematically? Moreover, if we go to higher order, for example in case of 1024×1024, the bimagic squares of order 64×64 has the same sum $S_2$ in each group of 256×256, while the bimagic sums $S_2$ for 256×256 give different values.

6 Numerical Values

Here below are the numerical values in each case. Some files of these bimagic squares can be downloaded at authors' web-site [13]:

- **Bimagic square of order 16×16**
  
  \[
  S_{1_{4\times4}} := \frac{1}{4} S_{1_{16\times16}} = 514
  
  S_{1_{16\times16}} := 2056
  
  S_{2_{16\times16}} := 351576
  
  - **Semi-Bimagic square of order 24×24**
  
  \[
  S_{1_{8\times8}} := \frac{1}{4} S_{1_{24\times24}} = 2308
  
  S_{1_{24\times24}} := 6924
  
  S_{2_{24\times24}} := 2661124 - \text{(rows and columns)}
  
  S_{2_{24\times24}} := 2654292 - \text{(diagonal - 1)}
  
  S_{2_{24\times24}} := 2714116 - \text{(diagonal - 2)}
  
  - **Bimagic square of order 25×25**
  
  \[
  S_{1_{5\times5}} := \frac{1}{5} S_{1_{25\times25}} = 1565
  
  S_{1_{25\times25}} := 7825
  
  S_{2_{25\times25}} := 3263025
  
  6
• Bimagic square of order $32 \times 32$

\[
S_{1_{8 \times 8}} := \frac{1}{7} S_{1_{32 \times 32}} = 4100 \\
S_{1_{32 \times 32}} := 16400 \\
S_{2_{32 \times 32}} := 1120100
\]

• Bimagic square of order $40 \times 40$

\[
S_{1_{8 \times 8}} := \frac{1}{9} S_{1_{40 \times 40}} = 6404 \\
S_{1_{40 \times 40}} := 32020 \\
S_{2_{40 \times 40}} := 32165340
\]

• Semi-Bimagic square of order $48 \times 48$

\[
S_{1_{4 \times 4}} := \frac{1}{7} S_{1_{16 \times 16}} = \frac{1}{12} S_{1_{48 \times 48}} = 4610 \\
S_{1_{48 \times 48}} := 55320 \\
S_{2_{48 \times 48}} := 84989960 - \text{(rows and columns)} \\
S_{2_{48 \times 48}} := 84990120 - \text{(diagonal - 1)} \\
S_{2_{48 \times 48}} := 85358600 - \text{(diagonal - 2)}
\]

• Bimagic square of order $49 \times 49$

\[
S_{1_{7 \times 7}} := \frac{1}{7} S_{1_{49 \times 49}} = 8407 \\
S_{1_{49 \times 49}} := 58849 \\
S_{2_{49 \times 49}} := 94217249
\]

• Bimagic square of order $56 \times 56$

\[
S_{1_{8 \times 8}} := \frac{1}{7} S_{1_{56 \times 56}} = 12548 \\
S_{1_{56 \times 56}} := 87836 \\
S_{2_{56 \times 56}} := 183665076
\]

• Bimagic square of order $64 \times 64$

\[
S_{1_{8 \times 8}} := \frac{1}{8} S_{1_{64 \times 64}} = 16338 \\
\text{or} \\
S_{1_{4 \times 4}} := \frac{1}{7} S_{1_{16 \times 16}} = \frac{1}{16} S_{1_{64 \times 64}} = 8194 \\
S_{1_{64 \times 64}} := 131104 \\
S_{2_{64 \times 64}} := 358045024
\]

• Bimagic square of order $72 \times 72$

\[
S_{1_{8 \times 8}} := \frac{1}{9} S_{1_{72 \times 72}} = 20740 \\
S_{1_{72 \times 72}} := 186660 \\
S_{2_{72 \times 72}} := 645159180
\]
• Bimagic square of order $80 \times 80$

$$S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{20} S_{180 \times 80} = 12802$$
$$S_{180 \times 80} := 256040$$
$$S_{280 \times 80} := 1092522680$$

• Bimagic square of order $81 \times 81$

$$S_{9 \times 9} := \frac{1}{9} S_{81 \times 81} = 29529$$
$$S_{81 \times 81} := 442416$$
$$S_{281 \times 81} := 1162527201$$

• Bimagic square of order $88 \times 88$

$$S_{8 \times 8} = \frac{1}{11} S_{88 \times 88} = 30980$$
$$S_{88 \times 88} := 340780$$
$$S_{288 \times 88} := 175947140$$

• Bimagic square of order $96 \times 96$

$$S_{8 \times 8} = \frac{1}{12} S_{96 \times 96} = 36868$$
$$S_{96 \times 96} := 442416$$
$$S_{2120 \times 120} := 2718351376$$

• Bimagic square of order $104 \times 104$

$$S_{8 \times 8} = \frac{1}{13} S_{104 \times 104} = 43268$$
$$S_{104 \times 104} := 562484$$
$$S_{2104 \times 104} := 4056072124$$

• Bimagic square of order $112 \times 112$

$$S_{4 \times 4} := \frac{1}{32} S_{16 \times 16} = \frac{1}{28} S_{1112 \times 112} = 25090$$
$$S_{1112 \times 112} := 702520$$
$$S_{2112 \times 112} := 5875174760$$

• Bimagic square of order $120 \times 120$

$$S_{8 \times 8} := \frac{1}{10} S_{120 \times 120} = 57604$$
$$S_{120 \times 120} := 864060$$
$$S_{2120 \times 120} := 8295264020$$

• Bimagic square of order $121 \times 121$

$$S_{11 \times 11} := \frac{1}{11} S_{121 \times 121} = 80531$$
$$S_{121 \times 121} := 885841$$
$S_{121 \times 121} := 8646694001$

- **Bimagic square of order $125 \times 125$**
  
  $S_{1_{5 \times 5}} := \frac{1}{3} S_{1_{25 \times 25}} = \frac{1}{25} S_{1_{125 \times 125}} = 39065$
  
  $S_{1_{125 \times 125}} := 976625$
  
  $S_{2_{125 \times 125}} := 10173502625$

- **Bimagic square of order $128 \times 128$**

  $S_{1_{4 \times 4}} := \frac{1}{4} S_{1_{16 \times 16}} = \frac{1}{16} S_{1_{128 \times 128}} = 65540$

  or

  $S_{1_{128 \times 128}} := 1048640$
  
  $S_{2_{128 \times 128}} := 11454294720$

- **Bimagic square of order $144 \times 144$**

  $S_{1_{4 \times 4}} := \frac{1}{4} S_{1_{16 \times 16}} = \frac{1}{16} S_{1_{144 \times 144}} = 41474$

  $S_{1_{144 \times 144}} := 1493064$
  
  $S_{2_{144 \times 144}} := 20640614424$

- **Bimagic square of order $160 \times 160$**

  $S_{1_{4 \times 4}} := \frac{1}{4} S_{1_{16 \times 16}} = \frac{1}{16} S_{1_{160 \times 160}} = 51202$

  $S_{1_{160 \times 160}} := 2048080$
  
  $S_{2_{160 \times 160}} := 34954581360$

- **Bimagic square of order $176 \times 176$**

  $S_{1_{4 \times 4}} := \frac{1}{4} S_{1_{16 \times 16}} = \frac{1}{16} S_{1_{176 \times 176}} = 61954$

  $S_{1_{176 \times 176}} := 2725976$
  
  $S_{2_{176 \times 176}} := 56294130376$

- **Bimagic square of order $200 \times 200$**

  $S_{1_{8 \times 8}} := \frac{1}{3} S_{1_{40 \times 40}} = \frac{1}{25} S_{1_{200 \times 200}} = 160004$

  $S_{1_{200 \times 200}} := 4000100$
  
  $S_{2_{200 \times 200}} := 106670666700$

- **Bimagic square of order $208 \times 208$**

  $S_{1_{4 \times 4}} := \frac{1}{4} S_{1_{16 \times 16}} = \frac{1}{16} S_{1_{208 \times 208}} = 86530$

  $S_{1_{208 \times 208}} := 4499560$
  
  $S_{2_{208 \times 208}} := 129780809080$
• Bimagic square of order $224 \times 224$

\[
S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{82} S_{224 \times 224} = 100354
\]

\[
S_{224 \times 224} := 5619824
\]

\[
S_{224 \times 224} := 187988732624
\]

• Bimagic square of order $240 \times 240$

\[
S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{60} S_{240 \times 240} = 115202
\]

\[
S_{240 \times 240} := 6912120
\]

\[
S_{240 \times 240} := 265427712040
\]

• Bimagic square of order $256 \times 256$

\[
S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{16} S_{64 \times 64} = \frac{1}{64} S_{256 \times 256} = 131074
\]

\[
S_{64 \times 64} := 2097184
\]

\[
S_{256 \times 256} := 91628066144
\]

\[
S_{256 \times 256} := 8388736
\]

\[
S_{256 \times 256} := 366512264576
\]

• Bimagic square of order $343 \times 343$

\[
S_{7 \times 7} := \frac{1}{7} S_{149 \times 49} = \frac{1}{49} S_{343 \times 343} = 41775
\]

\[
S_{343 \times 343} := 20176975
\]

\[
S_{343 \times 343} := 1582540680175
\]

• Bimagic square of order $400 \times 400$

\[
S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{20} S_{80 \times 80} = \frac{1}{105} S_{400 \times 400} = 320002
\]

\[
S_{400 \times 400} := 32000200
\]

\[
S_{400 \times 400} := 3413365333400
\]

• Bimagic square of order $512 \times 512$

\[
S_{8 \times 8} := \frac{1}{8} S_{32 \times 32} = \frac{1}{64} S_{512 \times 512} = 1048580
\]

\[
\text{or}
\]

\[
S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{32} S_{128 \times 128} = \frac{1}{128} S_{512 \times 512} = 524290
\]

\[
S_{512 \times 512} := 67109120
\]

\[
S_{512 \times 512} := 11728191138560
\]

• Bimagic square of order $600 \times 600$

\[
S_{8 \times 8} := \frac{1}{15} S_{120 \times 120} = \frac{1}{75} S_{600 \times 600} = 1440004
\]

\[
S_{600 \times 600} := 1018000300
\]

\[
S_{600 \times 600} := 25920108000100
\]
• Bimagic square of order 625×625

\[ \begin{align*}
S_{1,5 \times 5} & := \frac{1}{5} S_{1,25 \times 25} = \frac{1}{25} S_{1,125 \times 125} = \frac{1}{125} S_{1,625 \times 625} = 976565 \\
S_{1,125 \times 125} & := 24414125 \\
S_{2,125 \times 125} & := 6357853190125 \\
S_{1,625 \times 625} & := 1220706625 \\
S_{2,625 \times 625} & := 31789265950625
\end{align*} \]

• Bimagic square of order 729×729

\[ \begin{align*}
S_{1,9 \times 9} & := \frac{1}{9} S_{1,729 \times 729} = 2391489 \\
S_{1,729 \times 729} & := 193710609 \\
S_{2,729 \times 729} & := 68630571075249
\end{align*} \]

• Bimagic square of order 800×800

\[ \begin{align*}
S_{1,4 \times 4} & := \frac{1}{16} S_{1,160 \times 160} = \frac{1}{60} S_{1,800 \times 800} = 5120008 \\
S_{1,800 \times 800} & := 256000400 \\
S_{2,800 \times 800} & := 109226922666800
\end{align*} \]

• Bimagic square of order 960×960

\[ \begin{align*}
S_{1,4 \times 4} & := \frac{1}{4} S_{1,16 \times 16} = \frac{1}{64} S_{1,240 \times 240} = \frac{1}{240} S_{1,960 \times 960} = 1843202 \\
S_{1,960 \times 960} & := 442368480 \\
S_{2,960 \times 960} & := 271791341568160
\end{align*} \]

• Bimagic square of order 1000×1000

\[ \begin{align*}
S_{1,8 \times 8} & := \frac{1}{8} S_{1,40 \times 40} = \frac{1}{25} S_{1,200 \times 200} = \frac{1}{125} S_{1,1000 \times 1000} = 4000004 \\
S_{1,1000 \times 1000} & := 500000500 \\
S_{2,1000 \times 1000} & := 333333833333500
\end{align*} \]

• Bimagic square of order 1024×1024

\[ \begin{align*}
S_{1,4 \times 4} & := \frac{1}{16} S_{1,16 \times 16} = \frac{1}{64} S_{1,64 \times 64} = \frac{1}{256} S_{1,256 \times 256} = \frac{1}{125} S_{1,1024 \times 1024} = 2097154 \\
S_{1,1024 \times 1024} & := 536871424 \\
S_{2,1024 \times 1024} & := 375300505818624
\end{align*} \]

• Bimagic square of order 1200×1200

\[ \begin{align*}
S_{1,4 \times 4} & := \frac{1}{16} S_{1,16 \times 16} = \frac{1}{64} S_{1,240 \times 240} = \frac{1}{256} S_{1,1200 \times 1200} = 2880002 \\
S_{1,1200 \times 1200} & := 864000600 \\
S_{2,1200 \times 1200} & := 829440864000200
\end{align*} \]

• Bimagic square of order 1331×1331
$S_{11 \times 11} := \frac{1}{11} S_{121 \times 121} = \frac{1}{121} S_{1331 \times 1331} = 9743591$

$S_{1331 \times 1331} := 1178974511$

$S_{2 \times 1331} := 1392417235445951$

**Bimagic square of order $1600 \times 1600$**

$S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{20} S_{180 \times 80} = \frac{1}{100} S_{1400 \times 400} = \frac{1}{400} S_{1600 \times 1600} = 5120002$

$S_{1600 \times 1600} := 2048000800$

$S_{2 \times 1600} := 3495255381333600$

**Bimagic square of order $2000 \times 2000$**

$S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{20} S_{180 \times 80} = \frac{1}{100} S_{1400 \times 400} = \frac{1}{400} S_{2000 \times 2000} = 8000002$

$S_{2 \times 2000} := 4000001000$

$S_{2 \times 2000} := 106667066667000$

**Bimagic square of order $2048 \times 2048$**

$S_{18 \times 8} := \frac{1}{8} S_{132 \times 32} = \frac{1}{64} S_{1512 \times 512} = \frac{1}{256} S_{12048 \times 2048} = 16777220$

or

$S_{14 \times 4} := \frac{1}{4} S_{16 \times 16} = \frac{1}{32} S_{1128 \times 128} = \frac{1}{128} S_{1512 \times 512} = \frac{1}{512} S_{2048 \times 2048} = 8388610$

$S_{2 \times 2048} := 4294968320$

$S_{2 \times 2048} := 12009603301288960$

**Bimagic square of order $2400 \times 2400$**

$S_{18 \times 8} := \frac{1}{8} S_{1120 \times 120} = \frac{1}{60} S_{1600 \times 600} = \frac{1}{300} S_{2400 \times 2400} = 23040004$

$S_{2 \times 2400} := 6912001200$

$S_{2 \times 2400} := 26542086912000400$

**Bimagic square of order $2401 \times 2401$**

$S_{17 \times 7} := \frac{1}{7} S_{149 \times 49} = \frac{1}{49} S_{1343 \times 343} = \frac{1}{343} S_{12401 \times 2401} = 20176807$

$S_{1343 \times 343} := 141237649$

$S_{2 \times 343} := 3799632717121140$

$S_{2 \times 2401} := 6920644801$

$S_{2 \times 2401} := 26597429019848001$

**Bimagic square of order $3000 \times 3000$**

$S_{18 \times 8} := \frac{1}{8} S_{1120 \times 120} = \frac{1}{70} S_{1600 \times 600} = \frac{1}{370} S_{13000 \times 3000} = 36000004$

$S_{2 \times 3000} := 13500001500$

$S_{2 \times 3000} := 8100013500000500$

**Bimagic square of order $3200 \times 3200$**
\begin{align*}
S_{1_{4\times4}} &: = \frac{1}{10} S_{1_{160\times160}} = \frac{1}{50} S_{1_{800\times800}} = 1280002 \\
S_{1_{3200\times3200}} &: = 256000400 \\
S_{2_{3200\times3200}} &: = 10922692266800
\end{align*}

- Bimagic square of order $4000 \times 4000$

\begin{align*}
S_{1_{4\times4}} &: = \frac{1}{10} S_{1_{160\times160}} = \frac{1}{50} S_{1_{800\times800}} = \frac{1}{250} S_{1_{4000\times4000}} = 12800008 \\
S_{1_{4000\times4000}} &: = 32000002000 \\
S_{2_{4000\times4000}} &: = 34133365333334000
\end{align*}

- Bimagic square of order $4096 \times 4096$

\begin{align*}
S_{1_{4\times4}} &: = \frac{1}{10} S_{1_{16\times16}} = \frac{1}{16} S_{1_{64\times64}} = \frac{1}{64} S_{1_{256\times256}} = \frac{1}{256} S_{1_{1024\times1024}} = \frac{1}{1024} S_{1_{4096\times4096}} = 33554434 \\
S_{1_{4096\times4096}} &: = 34359740416 \\
S_{2_{4096\times4096}} &: = 384307202562021376
\end{align*}

References

[1] C. Boyer - Multimagic Squares, http://www.multimagie.com.

[2] H. Derksen, C. Eggermont and A. Essen, Multimagic Squares, The American Mathematical Monthly, Vol. 114, N 8, October 2007, pages 703-713. Also in http://arxiv.org/abs/math.CO/0504083

[3] Général E. Cazalas. Carrés magiques au degré $n$. Séries numérales de G. Tarry. Avec un aperçu historique et une bibliographie des figures magiques - 1934.

[4] H. Heinz - Magic Squares, Magic Stars and Other Patterns http://www.magic-squares.net.

[5] G. Pfeffermann, Les Tablettes du Chercheur, Journal des Jeux d’Esprit et de Combinaisons, (fortnightly magazine) issues of 1891 Paris.

[6] I.J. Taneja - DIGITAL ERA: Magic Squares and 8th May 2010 (08.05.2010), http://arxiv.org/abs/1005.1384.

[7] I.J. Taneja - Universal Bimagic Squares and the day 10th October 2010 (10.10.10), http://arxiv.org/abs/1010.2083.

[8] I.J. Taneja - DIGITAL ERA: Universal Bimagic Squares, http://arxiv.org/abs/1010.2541.

[9] I.J. TANEJA, Upside Down Numerical Equation, Bimagic Squares, and the day September 11, http://arxiv.org/abs/1010.4186.
[10] I.J. TANEJA, Equivalent Versions of "Khajuraho" and "Lo-Shu" Magic Squares and the day 1st October 2010 (01.10.2010), http://arxiv.org/abs/1011.0451.

[11] I.J. TANEJA, DIGITAL ERA: Four Digit Magic Squares and the day May 11, 2010 (11.05.2010) - To appear in Journal of Combinatorics, Information and System Sciences.

[12] I.J. TANEJA, Upside down Magic, Bimagic, Palindromic Squares and Pythagoras Theorem on a Palindromic Day - 11.02.2011, http://arxiv.org/abs/1102.2394v2.

[13] I.J. TANEJA, Bimagic Squares Files - http://www.mtm.ufsc.br/~taneja.