Non-Hermitian effects of the intrinsic signs in topologically ordered wavefunctions

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The ground-state wavefunction of double semion model on a two-dimensional hexagonal lattice is given by the superposition of closed domain wall loops weighted by a $\pi$ phase multiplying the number of loops. The presence of these negative signs makes the nature of this state distinguish from the toric code state. To explore the quantum effects of these intrinsic signs, a generic double semion wavefunction with tuning parameters is proposed. In tensor network representation, these phase factors are expressed in terms of auxiliary spins on the dual triangular lattice. The norm of this wavefunction is mapped to the partition function of a two-dimensional triangular Ashkin-Teller model with imaginary magnetic fields and imaginary 3-spin triangular face interactions. Applying the numerical tensor network methods to this parity-time (PT) symmetric statistical model with negative Boltzmann weights, we establish a global phase diagram, where the double semion phase corresponds to the partial order phase of this Ashkin-Teller model. Adjacent to the double semion phase, we find dilute loop phase, dense loop phase with PT symmetry, and dense loop phase with spontaneously broken PT-symmetry. The latter two gapless phases contain non-abelian anyon excitations, characterized by non-unitary conformal field theories.

Introduction.- Topological quantum phases of matter with anyonic excitations has recently attracted considerable attention[1–5]. A prototype topologically ordered state is the toric code model[1], and its ground-state wavefunction is an equal weight superposition of all closed domain-wall loops. A related topologically ordered state is the double semion (DS) model, whose ground-state wavefunction is a superposition of all closed domain-wall loops weighted by a $\pi$ phase multiplying the number of closed loops[3, 6]. Due to the presence of such non-local phase factors, low-energy excitations of the DS phase contain semions with opposite chiralities and bosonic bound states of semions. Actually it has been speculated that the negative signs in the DS wavefunction are intrinsic and cannot be removed by any local transformation[7, 8]. However, the norm of the DS wavefunction is known as same as that of toric code[9, 10]. Therefore, an important open question arises whether there exist novel anyon condensed phases adjacent to the DS phase.

Meanwhile, it has been shown that non-Hermitian and parity-time (PT) symmetric quantum systems provide a useful description of dissipative quantum systems[11–13]. The fundamental property of these systems is that an unusual spontaneous PT-symmetry breaking phase with spectral singularity occurs. In correlated many-body PT-symmetric systems, an exotic universality class of criticality occurs due to the presence of negative/complex Boltzmann weights[14–15]. A well-known example is the Yang-Lee edge singularity[16, 17], where an imaginary magnetic field in the high-temperature Ising model was demonstrated to trigger an exotic phase transition. In this paper, we will establish an intriguing connection between the intrinsic sign problem in the topological wavefunction and the PT-symmetric statistical model with negative Boltzmann weights.

![Phase Diagrams](https://example.com/phase_diagram.png)

**FIG. 1:** (a) The phase diagram of the toric code phase. The Higgs phase and confining phase correspond to electric and magnetic anyon condensed phases, respectively. (b) The phase diagram of the double semion phase. The dense loop A phase is PT-symmetric, while the dense loop B phase spontaneously breaks the PT-symmetry. These two gapless phases are characterized by non-unitary conformal field theories.

To reveal the quantum effects of the intrinsic signs, we propose a generic DS wavefunction with two distinct tuning parameters on a two-dimensional hexagonal lattice. In tensor network representation, the non-local phase factors can be expressed in terms of auxiliary spins on the dual triangular lattice. By integrating out the physical operators, the norm of this wavefunction is mapped to the partition function of a two-
dimensional triangular Ashkin-Teller model with imaginary magnetic fields and imaginary 3-spin triangular face interactions. Using the corner-transfer-matrix renormalization group (CTMRG)\cite{18–21} and variational uniform matrix product state (VUMPS) methods\cite{22–23}, we determine the global phase diagrams of both two topologically ordered phases. Compared to that of the toric code phase (Fig.1a), we find the dilute loop phase with condensed bosonic anyons, dense loop phase with PT symmetry, and dense loop phase with spontaneously broken PT-symmetry in the phase diagram of the DS phase (Fig.1b). The latter two dense loop phases contain non-abelian anyon excitations, described by the non-unitary conformal field theory (CFT) with an effective central charge \( c_\ast = 2 \). The same CFT can arise from one-dimensional Yang-Lee non-abelian anyon systems\cite{25}.

**DS tensor-network wavefunction.** The DS model is defined by the following Hamiltonian which acts on spins \( (\sigma^z = \pm 1) \) living on the edges of a hexagonal lattice\cite{3}.

\[
H_{DS} = -\sum_v A_v - \sum_p B_p.
\]  

Here the star vertex \( A_v \) and the plaquette \( B_p \) shown in Fig.2(a) are given by

\[
A_v = \prod_{k \in E(v)} \sigma_k^z, \quad B_p = \prod_{k \in E(p)} \prod_{j \in E(p)} \langle 1 + \sigma_j^z \rangle / 2,
\]

where \( E(v) \) is the set of edges around a vertex \( v \), \( E(p) \) is the set of inner edges and \( \bar{E}(p) \) is the set of outer edges around a hexagonal \( p \).

**FIG. 2:** (a) The star operator \( A_v \) and plaquette operator \( B_p \) of the DS model. (b) A typical loop configuration where the red circles form \( \sigma^x = -1 \) closed loops. (c) The negative signs can be expressed by a local tensor with three auxiliary Ising (blue) spins, while the red lines denote the domain-walls. (d) The auxiliary spin configuration corresponding to the loop configuration shown in (b).

When all vertex terms \( A_v = +1 \) restrict the Hilbert space of states to the zero-flux subspace, the ground states of the Hamiltonian are stabilized by \( B_p = 1 \) for all plaquettes. Operator \( A_v = -1 \) is associated to a semion or anti-semion excitation, while \( B_p = -1 \) is a bosonic excitation of a semion-antiseion pair. Then the ground state can be written as:

\[
|\Psi_0\rangle = P \prod_p \frac{1 + B_p}{\sqrt{2}} |\uparrow\rangle^\otimes N,
\]

where \( P = (1 + M_y) / \sqrt{2} \), \( |\uparrow\rangle \) is the eigenvector of \( \sigma^z = +1 \), and \( M_y \) is the Wilson line operator that generates \( \sigma^z = -1 \) loop wrapping around the torus in the \( y \) direction. Actually, the loop representation provides a very intuitive way to understand this wavefunction, in which \( \sigma^z = -1 \) and \( \sigma^z = +1 \) states interpret a link of a loop as the presence or absence, shown in Fig.2(b).

Expanding this product, we have the ground state as a superposition of all closed loop configurations with alternative signs\cite{3, 6}.

\[
|\Psi_0\rangle = \sum_{\{L\}} (-1)^{#\{L\}} |\mathcal{L}\rangle,
\]

where \( #\{\mathcal{L}\} \) is the number of closed loops in the loop configuration \( \mathcal{L} \).

It is very useful to express the DS wavefunction in terms of tensor networks\cite{26}. Since the loops among the plaquettes with opposite signs correspond to the domain-walls of local auxiliary Ising spins, which form a triangular dual lattice, as shown in Fig.2(d). Importantly, the non-local signs in the wavefunction can be expressed in terms of these auxiliary Ising spins. When the domain-walls of the Ising paramagnet are oriented with the auxiliary down-spins on the left, a phase factor \( e^{i \alpha} \) is assigned, while the right-turning as the factor \( e^{-i \alpha} \). In the original hexagonal lattice, the difference between the left- and right-turnings for a loop must be six, so we have \( e^{i \alpha} = -1 \). Then the many-body entangled state between the physical spins and auxiliary spins is given by

\[
|\tilde{\Psi}\rangle = \prod_{\langle pqr\rangle} e^{i \phi (s_p + s_q + s_r - 3s_p s_q s_r)} \frac{1 + s_p s_q s_r s_q}{2} |s\rangle |s_{p,q}\rangle,
\]

where \( \langle pqr \rangle \) denotes the nearest-neighbor dual lattice sites, \( \langle pqr \rangle \) stands for the minimal triangular faces. The DS tensor-network wavefunction is obtained by summing over all the auxiliary spins: \(|\Psi_0\rangle = \sum_{\{s\}} \langle s | \tilde{\Psi}\rangle |s\rangle |s_{p,q}\rangle\).

Since the above DS wavefunction is just the fixed-point wavefunction, it is essential to have a generic DS wavefunction with tuning parameters. When two different magnetic fields \( h'_x \) and \( h'_z \) are present, the DS model with additional Zeeman terms is no longer exactly solved. However, if these additional term are treated perturbably, the ground-state wavefunction is obtained

\[
|\Psi(h'_x, h'_z)\rangle = \left[ 1 + \frac{1}{8} \sum_{\langle pq \rangle} (h'_x \sigma_{p,q}^x + 2 h'_z \sigma_{p,q}^z) \right] |\Psi_0\rangle.
\]
When the wavefunction corrections are expressed as the operator product, we have
\[ |\Psi(h_x, h_z)\rangle = \prod_{(pq)} \left[ \frac{1}{2^s} \left( h_x \sigma^x_{p,q} + h_z \sigma^z_{p,q} \right) \right] |\Psi_0\rangle, \tag{6} \]
which can be regarded as a generic DS wavefunction in the expanded parameter region. Actually, the similar deformation has been used to express the generic toric code wavefunction and Fibonacci quantum-net wavefunction. For convenience, we define \( h_x \equiv h \cos \theta \) and \( h_z \equiv h \sin \theta \). When \( h \rightarrow 1 \), this wavefunction filters out the spin-polarized trivial state. It should be emphasized that this generic wavefunction still has a local parent Hamiltonian. The possible quantum phase transitions of such a Hamiltonian are characterized by the so-called conformal quantum critical points, where all equal-time correlators of local operators are described by two-dimensional CFT.

**Mapping to PT-symmetric statistical models.** To study the possible topological phase transitions of the DS phase, the norm of the wavefunction is considered. By integrating out the physical variables, a double-layer tensor network corresponds to a partition function
\[ \langle \Psi(h, \theta) | \Psi(h, \theta) \rangle = \sum_{(s,t)} P_s P_t \exp[-H(s,t)], \]
and the corresponding statistical model is defined by
\[ H = \sum_{(pq)} \left[ J s_p s_q + t_p t_q \right] + J_4 s_p s_q t_p t_q + J_0 \]
\[ -\frac{i3\alpha}{2} \sum_p (s_p - t_p) + \frac{i3\alpha}{4} \sum_{(pq)} (s_p s_q s_r - t_p t_q t_r), \tag{7} \]
where \( s_p \) and \( t_p \) are the auxiliary Ising spins in the ket and the bra layers, respectively, the 2-spin and 4-spin couplings are given by
\[ J = \frac{1}{4} \log \frac{1 + h^2 - 2h \sin \theta}{1 + h^2 + 2h \sin \theta}, \quad J_4 = \frac{1}{4} \log \frac{4h^2 \cos^2 \theta}{1 + h^2 + 2h^2 \cos 2\theta}, \]
and the operators \( P_s \) and \( P_t \) just contribute the boundary terms of the partition function. This is a two-dimensional triangular Ashkin-Teller model with imaginary magnetic fields and imaginary 3-spin triangular face interactions, whose partition function describes two-coupled O(n = -1) loop models. The imaginary terms of Eq.\( \tag{7} \) just originate from the negative signs in the DS wavefunction. Such a statistical model Eq.\( \tag{7} \) is invariant under the combined operation of parity symmetry \( (s_p \rightarrow -s_p \) and \( t_p \rightarrow -t_p) \) with time-reversal symmetry \( (i \rightarrow -i) \). The PT-symmetry ensures that all eigenvalues of \( H \) are either real or complex conjugate pairs. Since this model has the symmetry \( h_x \rightarrow -h_x \), the parameters of the model are limited to \( 0 \leq h \leq 1 \) and \( -\pi/2 \leq \theta \leq \pi/2 \). This model cannot be solved exactly except for the following two special limits.

When \( h_x = 0 \), we have \( J_4 \rightarrow \infty \), and the Ising spins in two layers are locked together so that the imaginary terms just cancel. The norm of the DS and toric code wavefunctions become the same, and Eq.\( \tag{7} \) is reduced to a single two-dimensional triangular Ising model: \( H = 2J \sum_{(pq)} s_p s_q \) with \( J = \frac{1}{4} \log \frac{1-h_z}{1+h_z} \). The previous studies showed that, for the ferromagnetic coupling \( (h_z > 0) \), a critical point exists at \( h_z = (3/4 - 1)/(3/4 + 1) \), separating the ferromagnetic order (dilute loop) phase from the paramagnetic (partial order) phase, corresponding to the point \( H \) in Fig.1. While for the antiferromagnetic coupling \( (h_z < 0) \), there is no phase transition due to strong spin frustration, up to the multicritical point \( F \) \( (h_z = -1) \).

When \( h = 1 \) but \( \theta \neq \pm\pi/2 \), the inter-layer coupling \( J_4 = 0 \) and the model Hamiltonian of the decoupled layer is reduced to
\[ H_s = J \sum_{(pq)} s_p s_q - \frac{i3\alpha}{2} \sum_p s_p + \frac{i3\alpha}{4} \sum_{(pq)} s_p s_q s_r, \tag{8} \]
where \( J = \frac{1}{4} \log \frac{1+ \sin \theta}{1+ \sin \theta} \). The partition function of this model corresponds to a single-layer \( O(-1) \) loop model:
\[ Z_s \propto \sum_{\{\ell\}} e^{-2J|\ell|(-1)^\#|\ell|}, \tag{9} \]
where \( |\ell| \) is the combined length of the closed loops. By mapping to the triangular Potts model, two exactly solvable points were found. The first one \( c = \frac{1}{2} + \sqrt{3} \) is a critical point \( \theta \) (Fig.1b) separating the gapped dilute loop phase from gapless dense loop phase, characterized by a non-unitary CFT with a central charge \( c = -3/5 \). The other one \( c = -\sqrt{2} - 3 \) represents a stable fixed-point \( E \) (Fig.1b) of the dense loop phase, which is described by the non-unitary CFT with a central charge \( c = -7 \). The same CFT also arises from a ferromagnetic Yang-Lee non-abelian anyon chain, which is a particular Galois conjugate of the unitary \( SU(2)_3 \) CFT. The other one \( c = -\sqrt{2} - 3 \) represents a stable fixed-point \( E \) (Fig.1b) of the dense loop phase, which is described by the non-unitary CFT with a central charge \( c = -7 \). The same CFT also arises from a ferromagnetic Yang-Lee non-abelian anyon chain, which is a particular Galois conjugate of the unitary \( SU(2)_3 \) CFT. The previous studies had found that the scaling dimensions of primary fields of this CFT are given by \( \Delta = -\frac{2}{3}, -2, 0, 1/3, \) and an effective central charge is defined by \( c_s = c - 12\Delta_{\text{min}} = 1 \).

With the model Hamiltonian Eq.\( \tag{8} \), we performed numerical calculations. Since the convergence of the CTMRG tensor-network methods is very slow in the presence of negative scaling dimensions, we resort to exact diagonalization on a finite-size lattice. To our surprise, another critical point \( D \) (Fig.1b) is discovered, dividing the dense loop phase into the PT-symmetric phase with real free-energy density and spontaneously broken PT-
symmetry phase with a complex one. The same numerical behavior is obtained at the exactly solvable point \( A \) as the dense loop phase is approached from the dilute loop phase. The detailed discussions are given in Supplementary Materials.

Tensor-network numerical calculations.- To establish the global phase diagram for the DS wavefunction, we will apply the numerical VUMPS method \cite{15, 21} to the PT-symmetric statistical model Eq.(7) with negative Boltzmann weights. As shown in Fig.3, we first perform tensor contractions by combining every two nearest-neighbor triangle faces into one unit of a large square lattice, and integrating out the physical indices leads to the partition function as a double-layer tensor network,

\[
Z = t \text{Tr} \left( \bigotimes_{\text{vertex}} Q \right) = \text{Tr}(T^{h_c}),
\]

where “tTr” denotes the tensor contraction over all auxiliary indices, \( Q \) is the double tensor on the large square lattice, \( T \) is the column-to-column transfer matrix operator, and \( L_x \) is the number of columns. Note that \( T \) is an one-dimensional non-Hermitian operator. In order to determine the phase boundaries, we need to calculate the correlation length, whose divergent peaks give rise to the position of the continuous phase transitions.

![FIG. 3: (a) The DS tensor-network wavefunction. The large blue circles denote the auxiliary spins, while small circles are the physical spins, and the yellow dashed box shows the coarse graining unit. (b) The local tensor \( Q \) on the large square lattice and \( Q \) is the double tensor. (c) The one-dimensional column-to-column transfer matrix operator \( T \) of the double-layer partition function.](image)

When the imaginary terms in Eq.(7) are absent, the VUMPS method \cite{22, 23, 24} is useful and the phase diagram for the toric code phase is calculated as a comparison. As displayed in Fig.1a, we find that the toric code phase is enclosed by the Higgs and confining phases, corresponding to the electric and magnetic anyon condensed phases, respectively. Both condensed phases are gapped, and between them there is a critical line \( IJ \) which is described by Kosterlitz-Thouless theory \cite{29}. The critical lines \( JH \) and \( JKF \) are described by the two-dimensional Ising transition, and the tricritical point \( J \) is exactly the critical point of the 4-state Potts model \cite{28}. The detailed results are included in the Supplementary Material.

To determine the phase diagram of the DS phase, we show the correlation length \( \xi \) as a function of \( h_x \) along the cut \( h_z = 0.5 \) in Fig. 4a. As \( h_z \) decreases from unity, a sharp peak first appears at \( h_z \approx 0.271 \), indicating a phase transition from the dilute loop phase to the DS phase. The peak positions are nearly the same for bond dimensions \( D = 40, 50, 60, 70 \). When \( h_z \) further decreases, a hump appears around \( h_z \approx -0.470 \) and gradually becomes a peak with increasing bond dimension. After this hump, the model enters the dense loop phase with PT-symmetry. Inside this phase, we calculate the entanglement entropy at the point \( (h_x = 0.5, h_z = -0.8) \). By fitting with the scaling relation \cite{43} \( S = (c_x / 6) \log \xi + S_0 \), we estimate an effective central charge \( c_x \approx 2.00 \) as shown in Fig.4(b). Because the stable fixed-point \( E \) of the dense loop phase is contained in this region, we can make a conclusion that the PT-symmetric dense loop phase is characterized by the non-unitary CFT with the central charge \( c = -7 \times 2 \), consistent with our numerical results.

Moreover, along the \( h_x \)-axis we show the correlation length \( \xi \) with the bond dimensions \( D = 50, 60, 70 \) in Fig.4(c). As \( h_x \) increases from small values, a peak appears around \( h_x \approx 0.785 \), which is enhanced by larger bond dimensions. However, the CTMRG calculations for \( h_x > 0.785 \) converge extremely slow, and we found the largest eigenvalue of the transfer matrix operator becomes complex. So the resulting dense loop phase spontaneously breaks the PT-symmetry \cite{11, 15}. The boundaries of the new phase are determined by the real-to-complex condition, so the dense loop phase is divided into the dense loop A and B phases, consistent with the results discussed in the limit of \( h = 1 \). Inside the dense
loop B phase, the entanglement entropy is calculated at the point \((h_x = 0.85, h_y = 0)\). Fitting with the scaling relation yields the effective central charge \(c_e \approx 2.02\) shown in Fig.4d. Although a numerical difficulty is encountered, we can conclude that the dense loop B phase is still described by the non-unitary CFT with the effective central charge \(c_e = 2\).

With these numerical results, the global phase diagram can be fully established as shown in Fig.1b. We also determine that both BC and CF are the continuous phase transitions with an effective central charge \(c_e \approx 1\), and the phase transition along AB line shares the same non-unitary CFT as the critical point A. However, the nature of the dense loop B phase and its phase transitions to its neighbor phases have not been completely resolved, and we hope to develop new numerical methods that are able to deal with them in the future.

Conclusions.- We have considered a generic DS phase to explore the quantum effects of the negative signs in its ground-state wavefunction. In tensor network representation, the norm of this wavefunction has been mapped to the partition function of a two-dimensional triangular Ashkin-Teller model with imaginary magnetic fields and imaginary 3-spin triangular face interactions. Adjacent to the DS topological phase, we found a gapped dilute loop phase and two different dense loop phases, whose low-energy excitations contain non-unitary, non-abelian anyons. Our study has confirmed that the negative signs in the topological wavefunction are intrinsic and deeply connected to negative Boltzmann weights of a PT-symmetric statistical model, shedding new light on the understanding of topological ordered phases of matter.

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