Supplementary Material

Emergence of cascading dynamics in interacting tipping elements of ecology and climate

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Figure 1: Number of stable fixed points and phase space portraits in a master–slave system with a high positive coupling strength $d_{12} = 0.9 > 0$ depending on the control parameters $c_1$ and $c_2$. The dashed lines represent the intrinsic tipping point of the respective subsystem. The phase space portraits allow to derive the possible critical transitions in the master–slave system. Within the phase space portraits stable fixed points are shown in orange, while unstable fixed points are shown in red. The background color indicates the normalized speed $v = \sqrt{\dot{x}_1^2 + \dot{x}_2^2}/v_{\text{max}}$ going from close to zero (purple) to fast (yellow-green).
Figure 2: Number of stable fixed points of the system consisting of three unidirectionally coupled tipping elements for $c_1 = 0.2 < c_{1\text{crit}}$ depending on the control parameters $c_1$ and $c_2$ and the coupling strengths $d_{21} \leq 0$ and $d_{12} \geq 0$ in a matrix of stability cards. A stability card shows the number of stable fixed points in the $(c_2, c_3)$-space for a specific coupling strength, where a certain number of stable fixed points is associated with a specific color. Note that different areas in the control parameter space with the same color have the same number of stable fixed point but they do not necessarily have the same phase portrait. The dashed lines represent the intrinsic tipping point of the respective subsystem. The position of a stability card in the matrix is determined by the coupling strength.
Figure 3: Number of stable fixed points of the system consisting of three unidirectionally coupled tipping elements for $c_1 = 0.4 > c_{1\text{crit}}$ with $d_{12} = d_{23} = 0.2 > 0$ depending on the control parameters $c_2$ and $c_3$. The dashed lines represent the intrinsic tipping point of the respective subsystem.
Figure 4: Amplified tipping in a system of three unidirectionally coupled tipping elements for an increase of the control parameter $c_2 << c_{2\text{crit}}$ (as indicated by the yellow arrow in the stability card, upper left panel). The central cube shows the flow in the $(x_2, x_3)$-space (in black) as part of the three-dimensional phase space and the remaining stable (in orange) as well as unstable (in red) fixed points for $c_1 = 0.4 > c_{1\text{crit}}$, $c_2 = 0.2$ and $c_3 = 0.0$ with $d_{12} = 0.2 > 0$ and $d_{23} = 0.2 > 0$. An exemplary case of amplified tipping in subsystem $X_2$, where the preceding subsystem $X_1$ is in the tipped state for $c_1 > c_{1\text{crit}}$, is highlighted by the yellow trajectory. Two-dimensional plots arranged around the central cube show the flow in the $(x_2, x_1)$- and $(x_3, x_1)$-space corresponding to the lateral surfaces of the cube.
Figure 5: Amplified tipping in a system of three unidirectionally coupled tipping elements for an increase of the control parameter $c_3 << c_{3\text{crit}}$ (as indicated by the yellow arrow in the stability card, upper left panel). The central cube shows the flow in the $(x_2, x_3)$-space (in black) as part of the three-dimensional phase space and the remaining stable (in orange) as well as unstable (in red) fixed points for $c_1 = 0.4 > c_{1\text{crit}}$, $c_2 = 0.0$ and $c_3 = 0.3$ with $d_{12} = 0.2 > 0$ and $d_{23} = 0.2 > 0$. An exemplary case of amplified tipping in subsystem $X_3$, where the preceding subsystem $X_2$ occupies the tipped state, is highlighted by the yellow trajectory. Two-dimensional plots arranged around the central cube show the flow in the $(x_2, x_1)$- and $(x_3, x_1)$-space corresponding to the lateral surfaces of the cube.