I. INTRODUCTION

Recently, the LHCb Collaboration reported double-charm baryon $\Xi_{cc}^{++}$ in the $\Lambda c\bar{K}^-\pi^+\pi^-$ invariant mass spectrum \cite{1}, which has the mass $3621 \pm 0.72 \pm 0.27 \pm 0.14$ MeV. Obviously, the observation of $\Xi_{cc}^{++}$ plays a crucial role to establish complete baryon family. The announced $\Xi_{cc}^{++}(3621)$ by LHCb may provide input to carry out the research issues around double-charm baryon.

In the past decade, hadronic molecular states were extensively explored with the discovery of charmonium-like $XYZ$ states and $P_c(4380)$ and $P_c(4450)$. The observed charmonium-like $XYZ$ states have stimulated the studies of the interaction between charmed meson and anti-charmed meson, while these reported $P_c$ states make that this study can be extended to interaction between charmed meson and charmed baryon. The observation of double-charm baryon $\Xi_{cc}^{++}(3621)$ inspires our interest in exploring double-charm baryon interacting with a charmed meson. As shown in Fig. 1, investigating the interaction between charmed meson and double-charm baryon is a natural extension of former studies involved in charmed meson interacting with charmed baryon. By this study, we would like to study whether or not there exist possible triple-charm molecular pentaquarks, which can arouse experimentalist’s interest in searching such new type of exotic hadronic molecular states after the LHCb experimental observation of double-charm baryon.

In this work, we mainly focus on the interaction between S-wave double-charm baryon $\Xi_{cc}(3621)$ and S-wave charmed meson ($D$ and $D^*$) and study possible triple-charm molecular pentaquarks. This dynamical interaction can be quantitatively described by one-boson-exchange (OBE) model, which was often adopted to investigate the hadronic molecular states \cite{2-7}. By OBE model, the effective potential of the interaction between S-wave double-charm baryon $\Xi_{cc}(3621)$ and S-wave charmed meson ($D$ and $D^*$) can be extracted, by which we may search for the corresponding molecular states. This information can encourage experimental studies of searching for triple-charm molecular pentaquarks.

![Diagram of the interaction of hadrons and the corresponding connections with charmoium-like $XYZ$ states, $P_c(4380)/P_c(4450)$ and triple-charm molecular pentaquark.](image-url)

An outline of this paper is as follows. In Sec. II, we present the detailed derivation of effective potential related to the interaction between an S-wave $\Xi_{cc}$ baryon and an S-wave charmed meson $D/D^*$, and the corresponding numerical result are presented in III. Finally, we will give a summary in Sec. IV.

II. THE DETAILS OF THE OBTAINED EFFECTIVE POTENTIALS

A. OBE potentials

Our one boson exchange model consists of light meson exchanges $\pi$, $\eta$, $\sigma$, $\rho$, and $\omega$ as shown in Fig. 2. The structure of the interaction Lagrangian is determined by the quantum numbers, and in particular, we remind that $\Xi_{cc}(3621)$ has $J^P = 1/2^+$ \cite{8}. The relevant Lagrangians for three-meson vertices are given by

\[
\mathcal{L}_{D^*D^{*}\sigma} = -2g_{D^*D\sigma}D^*_b\sigma + 2g_{D^*D^i}\cdot D^*_b\sigma, \quad (2.1)
\]

\[
\mathcal{L}_{D^*D^\tau} = -\frac{2g_{D^*D\tau}}{f_\alpha}\epsilon_{\alpha\beta\gamma\lambda}D^\beta_{a\lambda}\partial^\gamma\bar{\psi}_b, \quad (2.2)
\]

\[
\mathcal{L}_{D^*D^\tau\psi} = -\sqrt{2}g_{D^*D\tau}\bar{D}^\tau_{a\lambda}\nabla_{+\lambda} + \sqrt{2}g_{D^*D\tau}\bar{D}^\tau_{a\lambda}\cdot \nabla_{\lambda} - i2\sqrt{2}g_{D^*D^\tau}\bar{D}^\tau_{a\lambda}\left(\partial_{\alpha}\nabla_{\lambda} - \partial_{\lambda}\nabla_{\alpha}\right)_{ba}, \quad (2.3)
\]
which can be constructed by considering the requirement of the heavy quark symmetry and chiral symmetry [9–13].

![FIG. 2: The typical diagram for the Ξ_{cc}D^{(*)} systems with the one-boson-exchange model.](image)

In Refs. [14, 15], the effective Lagrangians for the coupling of S-wave double-charm baryons with light mesons are constructed as

\[
\mathcal{L}_{\Xi_{cc} \to \sigma \Xi_{cc}} = g_{\sigma} \bar{\Xi}_{cc} \sigma \Xi_{cc},
\]

(2.4)

\[
\mathcal{L}_{\Xi_{cc} \to \rho \Pi_{cc}} = g_{\rho} \bar{\Xi}_{cc} \rho \Pi_{cc},
\]

(2.5)

\[
\mathcal{L}_{\Xi_{cc} \to \omega \Omega_{cc}} = h_{\omega} \bar{\Xi}_{cc} \omega \Omega_{cc} + \frac{f_{\omega}}{2M_{\Xi_{cc}}} \bar{\Xi}_{cc} \rho \Pi_{cc} \partial^\nu \partial_\nu \Xi_{cc}.
\]

(2.6)

Here, \( P \) and \( V \) in above Lagrangians correspond to the pseudoscalar and vector mixtures, respectively, i.e.,

\[
P = \left( \begin{array}{c} \pi^- + \frac{\rho^-}{\sqrt{2}} \\ \pi^0 - \frac{\rho^0}{\sqrt{2}} \\ \pi^+ + \frac{\rho^+}{\sqrt{2}} \end{array} \right), \quad V = \left( \begin{array}{c} \rho^- + \frac{\pi^-}{\sqrt{2}} \\ \rho^0 + \frac{\pi^0}{\sqrt{2}} \\ \rho^+ + \frac{\pi^+}{\sqrt{2}} \end{array} \right).
\]

With above effective Lagrangians, one further writes out the scattering amplitudes for the processes \( \Xi_{cc} \to \Xi_{cc}D^{(*)} \) and \( \Xi_{cc}D^{(*)} \to \Xi_{cc}D^{(*)} \) shown in Fig. 2. And then, the effective potential in the momentum space can be related to the scattering amplitude by the Breit approximation, where the general relation is

\[
\mathcal{V}_{\Xi_{cc} \to \Xi_{cc}D^{(*)}}(q) = -\frac{M(h_1 h_2 \to h_3 h_4)}{\sqrt{M_1 M_2 M_3 M_4}}.
\]

(2.7)

where \( M_i \) and \( M_f \) denote the masses of the initial states (\( h_1, h_2 \)) and final states (\( h_3, h_4 \)), respectively. \( M(h_1 h_2 \to h_3 h_4) \) is the scattering amplitude for the \( h_1 h_2 \to h_3 h_4 \) process. Finally, the effective potential in the coordinate space \( \mathcal{V}(r) \) can be extracted by performing Fourier transformation, i.e.,

\[
\mathcal{V}_{\Xi_{cc} \to \Xi_{cc}D^{(*)}}(r) = \int \frac{d^3q}{(2\pi)^3} e^{iq \cdot r} \mathcal{V}_{\Xi_{cc} \to \Xi_{cc}D^{(*)}}(q) \mathcal{F}(q^2, m_{\Xi_{cc}}^2).
\]

(2.8)

In Eq. (2.8), a monopole form factor \( \mathcal{F}(q^2, m_{\Xi_{cc}}^2) = (\Lambda^2 - m_{\Xi_{cc}}^2)/(\Lambda^2 - q^2) \) is introduced at every interaction vertex. It can reflect finite size effect of the hadrons involved in these interactions. Here, \( \Lambda, m_{\Xi_{cc}} \) and \( q \) are the cutoff, mass and four-momentum of the exchanged meson, respectively.

In the following, we collect the expressions of the total effective potentials for these discussed \( \Xi_{cc}D \) and \( \Xi_{cc}D^{(*)} \) systems, which include

\[
\mathcal{V}_{\Xi_{cc} \to \Xi_{cc}D}(r) = g_{\sigma} g_{\sigma} \mathcal{G}(I) \mathcal{Y}(\Lambda, m_{\Xi_{cc}}, r) + \frac{1}{2} \mathcal{Y}(\Lambda, m_{\Xi_{cc}}, r),
\]

(2.9)

\[
\mathcal{V}_{\Xi_{cc} \to \Xi_{cc}D^{(*)}}(r) = g_{\rho} g_{\rho} \mathcal{G}(I) \mathcal{Y}(\Lambda, m_{\Xi_{cc}}, r) + \frac{1}{3} \mathcal{Y}(\Lambda, m_{\Xi_{cc}}, r),
\]

(2.10)

Here, \( I \) stands for the isospin for the \( \Xi_{cc}D^{(*)} \) systems. \( \mathcal{G}(I) \) is isospin factor, which is taken as 3/2 for isoscalar systems, and -1/2 for isovector systems. The flavor wave functions \( |I, I_3 \rangle \) involved in this work are

\[
|1, 1 \rangle = |\Xi_{cc}^+ D^{(*)+} \rangle,
\]

(2.11)

\[
|1, 0 \rangle = \frac{1}{\sqrt{2}} \left( |\Xi_{cc}^+ D^{(*)0} - \Xi_{cc}^0 D^{(*)+} \rangle \right),
\]

(2.12)

\[
|1, -1 \rangle = |\Xi_{cc}^0 D^{(*)0} \rangle,
\]

(2.13)

\[
|0, 0 \rangle = \frac{1}{\sqrt{2}} \left( |\Xi_{cc}^+ D^{(*)0} + \Xi_{cc}^0 D^{(*)+} \rangle \right).
\]

(2.14)

In Eqs. (2.9)-(2.10), we define these operators \( \mathcal{O}(r), \mathcal{P}(r), \mathcal{Q}(r), \mathcal{S}(r, a, b) \) and function \( \mathcal{Y}(\Lambda, m, r) \), which can be further expressed as

\[
\mathcal{O}(r) = \frac{1}{r^2} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} r \mathcal{P}(r) = r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \mathcal{Q}(r) = \frac{1}{r} \frac{\partial}{\partial \rho} \mathcal{Q}(r) = \frac{1}{r} \frac{\partial}{\partial r} \rho(r)
\]

\[
\mathcal{Y}(\Lambda, m, r) = \frac{1}{4\pi} \mathcal{F}(e^{-mr} - e^{-\Lambda r}) - \frac{\Lambda^2 - m^2}{8\pi \Lambda} e^{-\Lambda r},
\]

\[
\mathcal{S}(r, a, b) = \mathcal{S}(\rho, a, b) = 3(\rho \cdot a)(\rho \cdot b) - a \cdot b,
\]

respectively.

The study on the deuteron indicates that the S-D mixing related to tensor force is very important \cite{24,25}. In this work, the S-D mixing effect will be also taken into account. Thus, the spin-orbit wave functions $|2S+1 L_J\rangle$ for the $\Xi_{cc}D^{(*)}$ systems involved in S-D mixing effect are

$$
\Xi_{cc}D(1/2^-) = |2S_{1/2}\rangle, \\
\Xi_{cc}D'(1/2^-) = |2S_{1/2}\rangle, \\
\Xi_{cc}D'(3/2^-) = |2S_{3/2}\rangle, \\
\Xi_{cc}D^{(3/2^-)} = |2D_{3/2}\rangle.
$$

In addition, the general expressions of the spin-orbit wave functions for the $\Xi_{cc}D^{(*)}$ systems are constructed as

$$
\Xi_{cc}D : |2S+1 L_J\rangle = \chi_+^{m_s}|Y_{L,m}\rangle, \\
\Xi_{cc}D' : |2S+1 L_J\rangle = \sum_{m,m'} C_{S_{m,s}}^{L,M} C_{S_{m,s}}^{L,M} \chi_+^{m'} |Y_{L,m}\rangle.
$$

Here, $C_{S_{m,s}}^{L,M}$ and $C_{S_{m,s}}^{L,M}$ are the Clebsch-Gordan coefficients. $\chi_+^{m}$ and $Y_{L,m}$ are defined as the polarization vector, spin wave function and spherical harmonics function, respectively.

For the spin-spin, spin-orbit and tensor force operators in Eq. (2.10), they should be sandwiched by the above spin-orbit wave functions, like $\langle \Xi_{cc}D' | i \sigma \cdot \mathbf{e}_2 \mathbf{e}_4 | \Xi_{cc}D' \rangle$. The obtained relevant numerical matrices are summarized in Table I.

| $\langle \mathbf{L}_2 \cdot \mathbf{e}_2 \mathbf{e}_4 \rangle_{J=1/2}$ | $\langle \mathbf{L}_2 \cdot \mathbf{e}_2 \mathbf{e}_4 \rangle_{J=1/2}$ |
|--------------------------------|--------------------------------|
| $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ |

| $\langle \mathbf{L}_2 \cdot \mathbf{e}_2 \mathbf{e}_4 \rangle_{J=3/2}$ | $\langle \mathbf{L}_2 \cdot \mathbf{e}_2 \mathbf{e}_4 \rangle_{J=3/2}$ |
|--------------------------------|--------------------------------|
| $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ |

| $\langle \mathbf{L}_2 \cdot \mathbf{e}_2 \mathbf{e}_4 \rangle_{J=3/2}$ | $\langle \mathbf{L}_2 \cdot \mathbf{e}_2 \mathbf{e}_4 \rangle_{J=3/2}$ |
|--------------------------------|--------------------------------|
| $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ |

**B. Parameters**

In this work, the parameters are the coupling constants, cutoff $\Lambda$ and masses of the particles. Let us first look at the coupling constants in Eqs. (2.1)-(2.6). For the charmed meson sector, there are four coupling constants $(g_\rho, g_\pi, g_\sigma, g_\omega)$, a pion decay constant $f_\pi = 132$ MeV and $g_\rho = m_\rho/f_\rho = 5.8$. Due to spontaneously broken chiral symmetry \cite{16}, $g_\sigma$ is related to the coupling constant $g$ for the process $D(0^+) \rightarrow D(0^+) + \pi$, $g_\sigma = g/2 \sqrt{6}$, where $g$ is taken as 3.73 in Ref. \cite{13}. In heavy quark symmetry, the coupling constant for $D^* D^* \pi$ interaction is taken the same value of $D^* D^* \pi$ coupling, $g = 0.59$, which is extracted from the decay width of $D^*$ \cite{17}. According to vector meson dominance \cite{17}, $\beta$ is fixed as $\beta = 0.9$. Through a comparison of the form factor between the theoretical calculation from light cone sum rule and lattice QCD, the numerical value for $\lambda$ is determined as 0.56 GeV$^{-1}$ \cite{17}.

With the help of the quark model, coupling constants for the interaction of double-charm baryons and light meson are derived from nucleon-nucleon interaction, i.e.,

$$
\mathcal{L}_N = g_{N\pi\bar{N}N} \bar{N}N + \sqrt{2} g_{N\pi\bar{N}N} \bar{N}N + \frac{f_{NN}}{\sqrt{2}m_N} \bar{N}N + \frac{f_{NN}^*}{\sqrt{2}m_N} \bar{N}N + \frac{f_{NN}}{\sqrt{2}m_N} \bar{N}N + \frac{f_{NN}^*}{\sqrt{2}m_N} \bar{N}N.
$$

We can obtain several relations of coupling constants, i.e.,

$$
g_\sigma = \frac{1}{5} g_{\sigma NN}, \quad g_\rho = \frac{\sqrt{2} m_\rho}{5} g_{\rho NN}, \quad h_\pi = \sqrt{2} g_\rho NN, \quad h_\pi + f_\rho = -\frac{\sqrt{2} m_\rho}{5} (g_{\rho NN} + f_{NN}).
$$

Coupling constants for the nucleon-nucleon interaction are given in Refs. \cite{18-20}. In Table II, we collect numerical values for all the coupling constants.

| $\sigma$ | $\pi/\eta$ | $\rho/\omega$ |
|---------|---------|---------|
| $g_\sigma = 0.76$ | $g_\rho = 4.47$ | $\beta g_\rho = 5.22$, $\lambda g_\rho = 3.25$ |
| $g_{NN} = 5.69$ | $g_{NN} = 13.60$ | $g_{NN} = 0.84$, $f_{NN} = 6.10$ |
| $g_\sigma = -2.82$ | $g_\rho = 14.26$ | $h_\rho = 6.45$, $f_\rho = -28.11$ |

Since the discussed hadrons are not point-like particles, we should introduce the monopole form factor in each interactive vertex. For cutoff $\Lambda$, we recall a cutoff relation on charge root-mean-square radius of the interactive hadron, $\Lambda_1/\Lambda_2 = \sqrt{<r^2>/<r^2>}$, which is based on

$$
<r^2> = \frac{-6}{f(r^2, m_r^2)} \frac{\partial f(r^2, m_r^2)}{\partial q^2} \bigg|_{q^2 = 0} \approx 6 \Lambda_2^2.
$$

In Ref. \cite{21}, cutoff $\Lambda$ for the charm and bottom mesons are estimated by

$$
\Lambda_D = 1.35 \Lambda_N, \quad \Lambda_B = 1.29 \Lambda_N,
$$

where $\Lambda_N, \Lambda_D$ and $\Lambda_B$ correspond to the cutoffs for nucleon, $D$ and $B$ mesons, respectively. To fix the cutoff of nucleon $\Lambda_N$,
we would like to adopt the OBE model to reproduce the binding energy of deuteron. According to the effective Lagrangian in Eq. (2.15), the OBE effective potential for nucleon-nucleon system with \( I(J^P) = 0(1^+ \) is

\[
V(r) = - \frac{g_{\sigma NN}^2}{4m_N^2} \sigma_1 \cdot \sigma_2 Y(\Lambda, m_{\sigma}, r) \\
+ \frac{g_{\sigma NN}^2}{2m_N^2} \left[ \sigma_1 \cdot \sigma_2 O(r) + S(\hat{r}, \sigma_1, \sigma_2) \right] \\
\times \left[ - Y(\Lambda, m_{\pi}, r) + \frac{1}{9} Y(\Lambda, m_{\eta}, r) \right] \\
+ \frac{g_{\pi NN}^2}{2m_N^2} \left[ \sigma_1 \cdot \sigma_2 + \frac{f_{\rho NN}^2}{2m_N^2} \sigma_1 \cdot \sigma_2 O(r) \right] \\
+ \frac{g_{\eta NN}^2}{2m_N^2} (\sigma_1 + \sigma_2) \cdot LQ(r) \\
- \frac{g_{\sigma NN}^2 + 2g_{\rho NN}^2 f_{\rho NN}^2 - 2f_{\rho NN}^2}{12m_N^2} \sigma_1 \cdot \sigma_2 O(r) \\
- \frac{g_{\eta NN}^2 - 8g_{\rho NN}^2 f_{\rho NN}^2 + f_{\rho NN}^2}{12m_N^2} S(\hat{r}, \sigma_1, \sigma_2) \right] \\
\times \left[ - 3Y(\Lambda, m_{\rho}, r) + Y(\Lambda, m_{\omega}, r) \right].
\]

(2.19)

Under considered the S-D mixing effect, operators for spin-spin, spin-orbit and tensor force interactions in Eq. (2.19) are replaced by

\[
\langle \sigma_1 \cdot \sigma_2 \rangle \mapsto \begin{pmatrix}
0 \\
0
\end{pmatrix}, \quad \langle S(\hat{r}, \sigma_1, \sigma_2) \rangle \mapsto \begin{pmatrix}
0 & \sqrt{8} \\
\sqrt{8} & -2
\end{pmatrix},
\]

\[
\langle (\sigma_1 + \sigma_2) \cdot L \rangle \mapsto \begin{pmatrix}
0 & 0 \\
0 & -6
\end{pmatrix},
\]

(2.20)

respectively.

By solving the coupled channel Schrödinger equation, we can reproduce deuteron binding energy \( E = -2.23 \) MeV and obtain the corresponding root-mean-square radius \( r_{\text{RMS}} = 3.74 \) fm, when \( \Lambda_0 \) is taken as 0.862 GeV. Thus, by the cutoff relations in Eq. (2.18), \( \Lambda_0 \) is estimated as 1.164 GeV.

Additionally, the masses and spin-parity quantum numbers for all involved hadrons are listed in Table III.

### III. NUMERICAL RESULTS

Before producing the numerical calculation, we firstly perform a qualitative analysis of the properties of the OBE effective potentials for the \( \Xi_{cc}D^{(*)} \) systems. In general, the pseudoscalar meson \( \pi \), scalar meson \( \sigma \) and vector meson \( \rho/\omega \) exchanges provide long-range, intermediate-range and short-range contribution for the interaction between S-wave \( \Xi_{cc} \) and S-wave charmed meson, respectively.

For the \( \Xi_{cc}D \) system, there do not exist the \( \pi \) and \( \eta \) exchanges since the \( \Delta \Delta \sigma/\Delta \Delta \pi \) interactions are forbidden by symmetry. The \( \sigma \) and \( \omega \) exchanges provide attractive forces [23], while the effective potentials from the \( \rho/\omega \) exchange are repulsive and attractive for the isovector and isoscalar systems, respectively, and have relation \( V_{\text{isoscalar}} = -3V_{\text{isovector}} \). Thus, the isoscalar \( \Xi_{cc}D \) system can more easily form bound state than the isovector \( \Xi_{cc}D \) system.

For the \( \Xi_{cc}D^{*} \) system, besides considering the intermediate-range and short-range interactions from the \( \sigma/\rho/\omega \) exchanges, the \( \pi \) exchange and S-D wave mixing effect are also included in our calculation. As showed in Eq. (2.10), the spin-spin interaction, spin-orbit interaction and tensor force are involved in the total OBE effective potentials for the \( \Xi_{cc}D^{*} \) system. According to the experience of studying on deuteron [24, 25], the long-range force from pion exchange, tensor force and S-D mixing effect are very important when proton and neutron form a loosely bound deuteron. Compared with the \( \Xi_{cc}D \) system, more bound solutions for the \( \Xi_{cc}D^{*} \) system can be found.

In the following, we present the concrete numerical results with the effective potentials in Eqs. (2.9)-(2.10) when cutoff \( \Lambda_0 \) is taken as 1.164 GeV. We find some interesting results:

1. For the \( \Xi_{cc}D \) system, an isoscalar state can be a molecular candidate, which has binding energy \( E = -0.22 \) MeV and root-mean-square radius \( r_{\text{RMS}} = 5.10 \) fm. In Fig. 3, the \( r \) dependence of its radial wave function is shown, which reflects the \( \Xi_{cc}D(0[1/2^-]) \) state to be a loosely bound triple-charm molecular state.

2. For the \( \Xi_{cc}D^{*} \) system, there is only one good molecular candidate, the \( \Xi_{cc}D^{*} \) state with \( I(J^P) = 0(3/2^-) \). Its binding energy is -18.71 MeV and root-mean-square radius is 0.85 fm. In comparison with the radial wave functions showed in Fig. 3, we find that its dominant channel is \( \Xi_{cc} D^{*} [d^4 S_{1/2}] \).

Since the mass of \( \Xi_{cc}D \) system is very close to the \( \Xi_{cc}D^{*} \) system, and the pion exchange is also allowed for the process \( \Xi_{cc}D^{*} \rightarrow \Xi_{cc}D \), we further consider the coupled channel effect for the \( \Xi_{cc}D/\Xi_{cc}D^{*} \) system. With the same parameters input, we find that the binding energy for the \( \Xi_{cc}D/\Xi_{cc}D^{*} \) system with \( 1/2^- \) is \( E = -3.05 \) MeV, and the dominant channel is \( \Xi_{cc}D^{*} [d^4 S_{1/2}] \) with probability around 99 percent. Thus, the coupled channel effect plays a rather minor role for the \( \Xi_{cc}D/\Xi_{cc}D^{*} \) system.

| Hadrons | \( I(J^P) \) | Mass (MeV) |
|---------|------------|---------|
| \( \sigma \) | 0\(^+(0^+)\) | 600 |
| \( \eta \) | 0\(^+(0^+)\) | 547.85 |
| \( \omega \) | 0\(^-(1^-)\) | 782.65 |
| \( D \) | 1/2(0^-) | 1867.21 |
| \( B \) | 1/2(0^-) | 5279.42 |
| \( \Xi_{cc} \) | 1/2(1/2^-) | 3621.4 |

| Hadrons | \( I(J^P) \) | Mass (MeV) |
|---------|------------|---------|
| \( \pi \) | 1\(^-(0^-)\) | 137.27 |
| \( \rho \) | 1\(^-(1^-)\) | 775.49 |
| \( N \) | 1/2(1/2^+) | 938.27 |
| \( D' \) | 1/2(1^-) | 2008.56 |
| \( B' \) | 1/2(1^-) | 5325.2 |

TABLE III: Masses and spin-parity of the hadrons involved in our study [22].
To summarize, two possible triple-charm molecular pentaquarks are predicted, which are the \(\Xi_{cc}D\) state with \(I(J^P) = 0(1/2^-)\) and the \(\Xi_{cc}D^*\) state with \(I(J^P) = 0(3/2^-)\). Our results also verify that the intermediate-range and short-range forces from \(\sigma\), \(\rho\), and \(\omega\) exchanges are helpful in generating a bound state as suggested in Ref. [23]. The allowed strong decay models for these two possible triple-charm molecular pentaquarks are \(\Omega_{cc}\sigma\), \(\Omega_{cc}\rho\), \(\Omega_{cc}\omega\). However, we need to indicate that the \(\Omega_{cc}\) baryon is still missing in experiment, which is a big challenge to search for these predicted triple-charm molecular pentaquarks via these allowed decay channels.

For the \(\Xi_{cc}D^*/\Xi_{cc}B^*\) systems, the properties of \(\pi\) exchange and \(\omega\) exchanges are very different from those for the \(\Xi_{cc}D^*/\Xi_{cc}B^*\) systems. Our result shows that the isoscalar \(\Xi_{cc}D^*\) and \(\Xi_{cc}B^*\) systems with \(J^P = 1/2^-\) can be the possible molecular candidates. Moreover, the \(\Xi_{cc}D^*\) molecule with \(I(J^P) = 0(1/2^-)\) is loosely bound states. Possible two-body strong decay channels for the \(\Xi_{cc}D^*/\Xi_{cc}B^*\) molecular states with \(I(J^P) = 0(1/2^-)\) include \(\Lambda_c(1/2^+)+\eta_c/B_c(0^-),\ \Lambda_c(1/2^+)+J/\psi/B_c^*(1^-)\), respectively.

### IV. Summary

The observation of double-charm baryon [1] provides us a good opportunity to study possible triple-charm molecular pentaquarks composed by an S-wave double-charm baryon and an S-wave charmed meson. In Ref. [27], F. K. Guo et al. once predicted the possible triple-heavy pentaquarks composed of a heavy meson and a doubly-heavy baryon by exploring the consequences of heavy flavour, heavy quark spin and heavy antiquark-diquark symmetries for hadronic molecules within an effective field theory framework. In this work, we explore the interaction between an S-wave double charm baryon \(\Xi_{cc}\) and an S-wave charmed meson \(D/D^*\) by adopting the OBE model, and further predict the existence of possible triple-charm molecular pentaquark, which include the \(\Xi_{cc}D\) state with \(I(J^P) = 0(1/2^-)\) and the \(\Xi_{cc}D^*\) state with \(I(J^P) = 0(3/2^-)\).

In addition, the derived formula of the effective potentials for the \(\Xi_{cc}D^*/\Xi_{cc}B^*\) system can be extended to study the \(\Xi_{cc}B/\Xi_{cc}B^*\) systems. Two possible molecular candidates are predicted, which are the \(\Xi_{cc}B\) state with \(I(J^P) = 0(1/2^-)\) and \(\Xi_{cc}B^*\) state with \(I(J^P) = 0(3/2^-)\). In addition, as a byproduct,
we further extend our investigation to the $\Xi_{cc}\bar{D}(\sim)/\Xi_{cc}\bar{B}(\sim)$ systems. Our results indicate that there can exist two molecular pentaquarks, the $\Xi_{cc}\bar{D}$ and $\Xi_{cc}\bar{B}$ states with $I(J^{P}) = 0(1/2^{-})$.

This information of these predicted triple-charm molecular pentaquarks and their partners may stimulate further interest in searching for them in near future. In the past 14 years, abundant observations of charmonium-like XYZ states has made the study of multiquark states become a hot issue of hadron physics. With the running of accelerator facilities such as KEK and LHCb, we have good reasons to believe that experimentalist will bring us more surprises. We also expect that these prediction of triple-charm pentaquarks come true. Obviously, more theoretical and experimental effort are needed.

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