Towards DES/DDES computation of the flow field and heat transfer adopting elliptic relaxation and local correlation based transition approaches

V D’Alessandro\textsuperscript{1}, L Giammichele\textsuperscript{1}, C Lops\textsuperscript{2} and R Ricci\textsuperscript{1}

\textsuperscript{1} Dipartimento di Ingegneria Industriale e Scienze Matematiche
Università Politecnica delle Marche
Via Brecce Bianche 12, 60131 Ancona (AN), Italy
\textsuperscript{2} Dipartimento di Ingegneria e Geologia
Università degli Studi “G. d’Annunzio” di Chieti–Pescara
Viale Pindaro 42, 65127, Pescara (PE), Italy
E-mail: v.dalessandro@univpm.it

Abstract. As Large–Eddy Simulation (LES) shall remain too expensive in the following few decades for the ever increasing number of engineering complexities, researches have shifted much of the attention and effort to hybrid formulations incorporating Reynolds–Averaged Navier–Stokes (RANS) equations and LES in a certain ways. The long term goal of this research activity is to develop accurate hybrid RANS/LES methods. These approaches are particularly appealing in massively separated flows since they offer good prediction performance and they can be considered a good trade–off, in terms of computational resources, if compared with standard LES techniques. In particular with this work we want to deeply investigate Detached–Eddy Simulation (DES) based on: (i) elliptic relaxation turbulence model; (ii) Spalart Allmaras local correlation based transition model (LCTM). Specifically, we analyze $\nu^2$–f DES approach for the convective heat transfer around a sphere at $Re = 5000$ (with $Pr = 0.71$) and we compare these results with the so called delayed version of the model. At the same time we also investigate the LCTM version of the SA model, in the DES context.

1. Introduction
In CFD community is common knowledge that Reynolds–Averaged Navier–Stokes (RANS) equations can completely fail the prediction for a variety of flows due to the modeling of the Reynolds stress term, [1]. Large–Eddy Simulations (LES) could be retained a more reliable approach on which CFD practitioners will rely in the next years. However, this scenario is reasonably quite far since LES of real–life applications requires a very fine grid in the near–wall region, and thus entail a still huge computational cost for general users, [2]. Hybrid RANS/LES approaches are encountering increasing consents by the community because they are a sensible compromise in terms of computational resource requirements and accuracy, [3].
Detached–Eddy Simulation (DES) technique, introduced by Spalart et al. [4], is the most popular hybrid RANS/LES method that operates like RANS in the near–wall regions and like LES in separated flow zones. DES produces good results in several conditions reducing sensibly the computational costs if compared with LES. Moreover DES technique can be easily applied to a wide range of existing RANS models, [5]. Nevertheless it is important to remark that, originally
in 1997, DES was based on Spalart–Allmaras (SA) model, [6]. In the subsequent years DES models have also been introduced into two–equations RANS context. In particular Travin et al. [7] developed a DES technique adopting the SST $k–\omega$ model; the same technique was further modified in various papers too with emphasis to the length scale in the transport equations for turbulence, [8, 9, 10]. Only very recently Jee and Shariff, [11], considered $\bar{v}^2–f$ RANS model as a possible candidate for DES modeling. Mirzaii and Sohankar, [12], developed a $k–\omega–\bar{v}^2–f$ DES technique; and Ashton et al., [13], pursued a DES approach using the $\phi–f$ model. The main advantage about the adoption of elliptic relaxation turbulence models lies in their ability to accurately predict the near–wall effects without specific treatments or expedients. These models are also appealing since they use a length scale based on flow properties, not on grid size like the SA model. In LES mode, elliptic relaxation based DES models solve a transport equation for the Sub–Grid Scales (SGS) kinetic energy that is less empirical than the SGS modified turbulent viscosity, $\tilde{\nu}$, used in the standard SA–DES. The main drawback of these models is that they use four additional equations for turbulence modeling, while other DES techniques are less costly from this point of view.

It is worth mentioning that a critical issue in DES field is not only the turbulence model selection but also the DES formulation itself. Delayed Detached–Eddy Simulation (DDES), [14], and Improved Delayed Detached–Eddy Simulation (IDDES), [15], have been introduced in order to fix some well–known issues of standard DES methods. Indeed, Grid Induced Separation (GIS), Modelled Stress Depletion (MSD) and log–layer mismatch, [16], are overcome with DDES/IDDES.

In this paper we present the latest developments of our DES approaches, previously introduced in D’Alessandro et al. [17]. The aim of the work is to test the DDES version of our $\bar{v}^2–f$ DES model, [17], for heat and fluid flow problems; besides we also introduce a DES approach of a Spalart–Allmaras local correlation based transition model proposed and tested by the Authors in [18, 19] for RANS equations. The sphere and the circular cylinder are obvious test cases for a technique with claims over separated flows, [20]. In this paper we have focused our efforts on the flow over the sphere in sub–critical range at $Re = 5000$. Several authors treated this case, thus we have found a sufficient database for the sake of validation. In particular, Li et al. [21] and Dixon et al. [22] performed LES computations of the heat and fluid flow; while Rodriguez et al., [23], conducted Direct Numerical Simulation (DNS) of the flow field. Seidl et al, [24], performed a DNS of a sphere supported at the rear by a stick. Finally, in literature is available an experimental Nusselt number local distribution obtained by Galloway and Sag and reported in Clift et al. textbook, [25].

The paper is organized as follows: in Sec. 2 we present the governing equations of the flow models; Sec. 3 reports a brief description of numerical ingredients used to solve the transport equations. The results are collected in Sec. 4 and, lastly, the conclusions are outlined in Sec. 5.

2. Governing equations

The set of our flow governing equations can be written as

$$
\begin{align*}
\nabla \cdot \mathbf{u} &= 0, \\
\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) + \nabla p - \nabla \cdot ((\nu + \nu_t) (\nabla \mathbf{u} + \nabla \mathbf{u}^T)) &= 0, \tag{1} \\
\frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{u} T) - \nabla \cdot \left( \frac{\nu}{Pr} + \frac{\nu_t}{Pr_t} \right) \nabla T &= 0,
\end{align*}
$$

where $\mathbf{u}$ is the velocity vector, $p = P/\rho$ is the pressure divided by the density and $T$ is temperature; besides $\nu$ is the kinematic viscosity and $Pr$ the Prandtl number. The turbulent Prandtl number, $Pr_t$, is introduced to model the turbulent thermal diffusivity. The turbulent
viscosity, $\nu_t$, needed to take into account the turbulence, is computed according to the specific turbulence models.

2.1. $\overline{v^2}$–f DES model

The $\overline{v^2}$–f DES model here tested was implemented in [17] and it is based on the Lien et al., [26], $\overline{v^2}$–f RANS model. This model has a transport equation for $k$, $\epsilon$ and $\overline{v^2}$. An elliptic relaxation equation for a function $f$ containing time and length scales is also solved. The model switches to LES on the basis of the Spalart et al., [4], criterion related to the flow scales.

The flow model equations read as follows:

$$\frac{\partial k}{\partial t} + \nabla \cdot (\mathbf{u} k) = P - \epsilon_{DES} + \nabla \cdot \left[ (\nu + \nu_t) \nabla k \right],$$

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot (\mathbf{u} \epsilon) = c_{\epsilon 1} \frac{P}{T_{DES}} - \epsilon_{DES} + \nabla \cdot \left[ (\nu + \nu_t) \nabla \epsilon \right],$$

$$\frac{\partial \overline{v^2}}{\partial t} + \nabla \cdot (\mathbf{u} \overline{v^2}) = k_{f_{DES}} - 6 \frac{\overline{v^2}}{k} \epsilon_{DES} + \nabla \cdot \left[ (\nu + \nu_t) \nabla \overline{v^2} \right],$$

$$c_{L}^{2} L_{DES}^{2} \nabla^{2} f - f = 1 \frac{P}{T_{DES}} \left[ (c_{1} - 6) \frac{\overline{v^2}}{k} - \frac{2}{3} (c_{1} - 1) \right] - \frac{P}{k},$$

where the eddy-viscosity is evaluated as: $\nu_t = c_{\mu} \overline{v^2} T_{DES}$. The transition from RANS to the LES is based on the comparison of the length scale $k^{3/2}/\epsilon$ and the grid size, $\Delta$. If $k^{3/2}/\epsilon < C_{DES} \Delta$ the model works in RANS mode with the following conditions:

$$L_{DES} = L_{RANS}, \quad T_{DES} = T_{RANS}, \quad \epsilon_{DES} = \epsilon,$$

where:

$$T_{RANS} = \min \left[ \max \left[ \frac{k}{\epsilon}, c_T \left( \frac{\nu}{\epsilon} \right)^{1/2} \right], \frac{0.6k}{\sqrt{6c_{\mu} \overline{v^2} |\mathbf{D}|}} \right],$$

$$L_{RANS} = \max \left[ \min \left[ \frac{k^{3/2}}{\epsilon}, \frac{k^{3/2}}{\epsilon}, \frac{1}{\sqrt{6c_{\mu} \overline{v^2} |\mathbf{D}|}} \right], c_\eta \left( \frac{\nu^3}{\epsilon} \right)^{1/4} \right];$$

otherwise the model operates in LES mode as follows:

$$L_{DES} = C_{DES} \Delta, \quad T_{DES} = C_{DES} \Delta/\sqrt{k}, \quad \epsilon_{DES} = k^{3/2}/(C_{DES} \Delta).$$

The turbulent kinetic energy $k$ presents a production term which is modelled as $P = 2\nu_t |\mathbf{D}|^2$, with $\mathbf{D}$ the strain rate tensor; on the other hand the production term in the equation for $\overline{v^2}$ is expressed with the following equation:

$$k_{f_{DES}} = \min \left( k_{f}, 5 \frac{\overline{v^2}}{k} \epsilon_{DES} + \frac{2}{3} P \right).$$

Lastly the model constants are the same proposed by Jee and Shariff in [11]:

$$c_{\mu} = 0.22, \quad \sigma_\epsilon = 1.3, \quad c_{c_{1}} = 1.4 \left( 1 + 0.045 \sqrt{k/\overline{v^2}} \right),$$

$$c_{c_{2}} = 1.9, \quad c_1 = 1.4, \quad c_2 = 0.3,$$

$$c_T = 6, \quad c_L = 0.23, \quad c_\eta = 70, \quad C_{DES} = 0.8.$$
2.2. $\vec{v}^2$--$f$ DDES model

The delaying technique that we have introduced adopts the standard shielding function of Spalart et al. [14]. Some model terms are modified as follows:

$$L_{DDES} = f_d C_{DES} \Delta + (1 - f_d) \frac{k^{3/2}}{\epsilon},$$
$$T_{DDES} = L_{DDES} \sqrt{k},$$
$$\epsilon_{DDES} = \frac{k^{3/2}}{L_{DDES}},$$

with

$$f_d = 1 - \tanh \left(8 \frac{\nu_t}{\sqrt{U_{i,j}^2 U_{i,j}^2 k^2 d^2}} \right)^3$$

where $U_{i,j}$ is the velocity gradient, $d$ is the wall distance and $k = 0.41$ is the Karman constant.

2.3. Transitional SA DES model

The turbulent viscosity, $\nu_t$, for transitional SA DES model is computed according to a DES formulation of Spalart–Allmaras LCTM approach introduced by the Authors in [18, 19]. The turbulent viscosity, $\nu_t$, is obtained using the $\tilde{\nu}$ variable as

$$\nu_t = f_{\nu1} \tilde{\nu}. \quad (10)$$

The modified turbulent viscosity $\tilde{\nu}$ is computed solving the following transport equation:

$$\frac{\partial \tilde{\nu}}{\partial t} + \nabla \cdot (\nu \tilde{\nu}) - P_{\tilde{\nu}} + D_{\tilde{\nu}} - \frac{c_{\nu2}}{\sigma} \nabla \cdot \nabla \tilde{\nu} - \frac{1}{\sigma} \nabla \cdot ((\nu + \tilde{\nu}) \nabla \tilde{\nu}) = 0,$$ \quad (11)

the production and destruction terms appearing are defined as follows:

$$P_{\tilde{\nu}} = \gamma_{eff} c_{\nu1} \tilde{S}_{\tilde{\nu}},$$
$$D_{\tilde{\nu}} = \max \left( \min \left( 0, \nu_t \right), 1.0 \right) \left[ c_{w2} f_w \left( \frac{\tilde{\nu}}{d} \right) \right]^2,$$ \quad (12)

note that $\tilde{d}$ is defined as in [4], i.e. $\tilde{d} = \min (d, C_{DES} \Delta)$ with $d$ the distance from the nearest wall. 

The term $\gamma_{eff}$ in eq. 12 is devoted to model the separation–induced transition phenomena and it is defined as follows:

$$\gamma_{eff} = \max \left( \gamma, \gamma_{sep} \right)$$ \quad (13)

with

$$\gamma_{sep} = \min \left( 2.0, \max \left[ 0, \frac{\text{Re}_\nu}{3.235 \text{Re}_{\theta,c}} - 1 \right] F_{reattach}, 2.0 \right) F_{\theta,t},$$ \quad (14)

and

$$F_{reattach} = \exp \left( - \frac{R_T}{20} \right)^4.$$ \quad (15)

The following closure functions are needed to complete the definition of eq. 11:

$$f_{\nu1} = \frac{\chi^3}{(\chi^3 + c_{\nu1}^3)},$$ \quad (16)
$$g = r + c_{w2} \left( r^{6} - r \right),$$
$$f_w = g \left[ 1 + \frac{c_{w3}^6}{g^{6} + c_{w3}^6} \right]^\frac{1}{2},$$
$$\tilde{S} = [\Omega + \min (0, S - \Omega)] + \frac{\tilde{\nu}}{k^2 d^2} f_{\nu2},$$

$$r = \begin{cases} r_{\max} \quad \frac{\tilde{\nu}}{S k^2 d^2} < 0 \\ \min \left( \frac{\tilde{\nu}}{S k^2 d^2}, r_{\max} \right) \quad \frac{\tilde{\nu}}{S k^2 d^2} \geq 0 \\ \end{cases}.$$
where \( \chi = \tilde{\nu}/\nu \) is the dimensionless turbulent variable, \( \Omega = \sqrt{2} \mathbf{W} : \mathbf{W} \) is the vorticity tensor module, \( S = \sqrt{2D} : \mathbf{D} \) is the strain rate tensor module and \( \tilde{S} \) is a function of both the vorticity magnitude, \( \Omega \), and \( \tilde{\nu} \). \( r_{\text{max}} \) is a positive constant value equal to 10. Finally the following standard closure constants are adopted

\[
\begin{align*}
\sigma &= 2/3, \\
c_{b1} &= 0.1355, \\
c_{b2} &= 0.622, \\
c_{v1} &= 7.1, \\
c_{w1} &= \frac{c_{b1}}{k^2} + \frac{(1 + c_{b2})}{\sigma}, \\
c_{w2} &= 0.3, \\
c_{w3} &= 2, \\
k &= 0.41.
\end{align*}
\] (17)

The transport equations for the transition are:

\[
\begin{align*}
\frac{\partial \gamma}{\partial t} + \nabla \cdot (u\gamma) &= P_\gamma - D_\gamma + \nabla \cdot \left[ \left( \nu + \frac{\nu_t}{\sigma_f} \right) \nabla \gamma \right], \\
\frac{\partial \tilde{R}_{\theta,t}}{\partial t} + \nabla \cdot \left( u\tilde{R}_{\theta,t} \right) &= P_{\theta,t} + \nabla \cdot \left[ \sigma_{\theta,t} \left( \nu + \nu_t \right) \nabla \tilde{R}_{\theta,t} \right].
\end{align*}
\] (20)

The source terms in the \( \gamma \) equation are defined as:

\[
\begin{align*}
P_\gamma &= c_{a1}S \left[ \gamma F_{\text{onset}} \right]^{0.5} \left( 1 - c_{e1}\gamma \right) F_{\text{length}}, \\
D_\gamma &= c_{a2}\Omega_\gamma F_{\text{turb}} \left( c_{e2}\gamma - 1 \right),
\end{align*}
\] (21)

in \( P_\gamma \) the term \( F_{\text{onset}} \) is computed as:

\[
F_{\text{onset}} = \max (F_{\text{onset},2} - F_{\text{onset},3}, 0)
\] (22)

with

\[
\begin{align*}
F_{\text{onset},2} &= \min \left( \max (F_{\text{onset},1}, F_{\text{onset},1}^4), 4 \right), \\
F_{\text{onset},3} &= \max \left( 2 - \left( \frac{R_T}{2.5} \right)^3, 0 \right), \\
F_{\text{onset},1} &= \frac{\text{Re}_\nu}{2.193\text{Re}_{\theta,c}}.
\end{align*}
\] (23)

The terms \( \text{Re}_\nu \) and \( R_T \) are obtained as follows:

\[
\begin{align*}
\text{Re}_\nu &= \frac{Sd^2}{\nu} \\
R_T &= \frac{\nu_t}{\nu}.
\end{align*}
\] (24)

In our approach we redefine the parameter \( R_T \) to avoid the \( \omega \) estimation required in the \( k-\omega \) formulation. In particular the definition based on the viscosity ratio, \( \nu_t/\nu \), enables the SA equation to be adopted. The aspects concerning the terms: \( F_{\text{length}} \) and \( \text{Re}_{\theta,c} \) are described in the next subsection. For the destruction term, \( D_\gamma \), on the other hand, the coefficient \( F_{\text{turb}} \) is defined as:

\[
F_{\text{turb}} = \exp \left( - \frac{R_T}{4} \right)^4.
\] (25)

As for the source terms in the transport equation for \( \text{Re}_{\theta,t}, P_{\theta,t} \), the following equation is adopted:

\[
P_{\theta,t} = \frac{c_{b,t}}{T} \left( \text{Re}_{\theta,t} - \overline{\text{Re}_{\theta,t}} \right) \left( 1 - F_{\theta,t} \right).
\] (26)
F_{\theta,t} is defined as:

\[
F_{\theta,t} = \min \left( \max \left( \exp \left( -\frac{|\mathbf{u}|^2}{375\nu Re_{\theta,t}} \right)^4, 1 - \left( \frac{\gamma - 1/c_{e2}}{1 - 1/c_{e2}} \right)^2 \right), 1.0 \right). \tag{27}
\]

The term \( T \) appearing in the source term of the \( \overline{\theta_{\infty}} \) equation is also defined as follows: \( 500\nu/|\mathbf{u}|^2 \). Finally, the evaluation of \( Re_{\theta,t} \) in eq. 26 is discussed, together with the \( F_{\text{length}} \) coefficient, in the next subsection.

For the transition model, the following closure constants are adopted:

\[
\begin{align*}
c_{a1} &= 2.0, \quad c_{a2} = 0.06, \quad c_{e1} = 1.0, \tag{28} \\
c_{e2} &= 50, \quad c_{\theta,t} = 0.03, \quad \sigma_f = 1.0, \tag{29} \\
\sigma_{\theta,t} &= 2.0. \tag{30}
\end{align*}
\]

2.4. Empirical correlations in the SA DES transitional model

The transition Reynolds number, \( Re_{\theta,t} \), is modelled in our work through a correlation developed by Menter et al. [27]:

\[
Re_{\theta,t} = \left\{ \begin{array}{ll}
(1173.51 - 589.428 \cdot Tu + 0.2196/Tu^2) F(\lambda_{\theta}) & Tu \leq 1.3 \\
331.5 (Tu - 0.5668)^{-0.671} F(\lambda_{\theta}) & Tu > 1.3
\end{array} \right., \tag{31}
\]

\[
F(\lambda_{\theta}) = \left\{ \begin{array}{ll}
1 + [12.986\lambda_{\theta} + 123.66\lambda_{\theta}^2 + 405.689\lambda_{\theta}^3] \exp \left( -\frac{(Tu^{1.5})^{1.5}}{\lambda_{\theta}} \right) & \lambda_{\theta} \leq 0 \\
1 + 0.275 [1 - \exp (-35\lambda_{\theta})] \exp \left( -\frac{Tu^{1.5}}{\lambda_{\theta}} \right) & \lambda_{\theta} > 0
\end{array} \right. \tag{32}
\]

Note that \( Re_{\theta,t} \) is computed by iterating on the value of \( \theta_t \), since \( Re_{\theta,t} \) is a function of \( \theta_t \) itself because of the presence of \( \lambda_{\theta} = (\theta^2/\nu) \, d|\mathbf{u}|/ds \). It is also really important to note that the correlations in eq. 31, 32 contain the turbulence intensity \( Tu \). In our approach we adopt the techniques introduced in [28, 29] to handle this contribution. Specifically, we establish that \( Tu = Tu_{\infty} \) for all the points of the flow field. Differently, other turbulence models such as \( k-\omega \) can compute directly \( Tu \).

\( Re_{\theta,c} \), which appears in eq. 23, is the critical Reynolds number where the intermittency, \( \gamma \), starts to increase in the boundary layer. This typically occurs upstream from the transition Reynolds number, \( Re_{\theta,t} \). This element is introduced to model the turbulence start in the boundary layer and when appreciable turbulence levels are reached. As regards \( F_{\text{length}} \), appearing in the production term of the \( \gamma \) transport equation, is modelled through a correlation that controls the length of the transition region. For \( Re_{\theta,c} \) and \( F_{\text{length}} \) we use the correlations introduced by Malan et al. [30]:

\[
Re_{\theta,c} = \min \left( 0.615\overline{Re_{\theta,t}} + 61.5, \overline{Re_{\theta,t}} \right), \tag{33}
\]

\[
F_{\text{length}} = \min \left( \exp \left( 7.168 - 0.01173\overline{Re_{\theta,t}} \right) + 0.5, 300 \right). \tag{34}
\]

2.5. Boundary conditions

In the \( \overline{\nu^2} - f \) model we have adopted the same set of conditions proposed in [11]. However a different treatment has been devised for \( \epsilon \) wall condition. Indeed, \( \epsilon_w = \nu \frac{\partial^2 k}{\partial y^2} \) was used by Jeje and Shariff, [11], here we use the formulation proposed by Chien, [31], for RANS equations. This kind of condition is obtained introducing the Taylor expansions of velocity components close to the wall in the \( \epsilon \) definition; it reads as:

\[
\epsilon_w = \nu \frac{2k}{y^2}. \tag{35}
\]
The approach was already successfully tested for the flow past a cylinder at \( Re = 3900 \) in [17]. The main advantage of Chien wall boundary condition is that it does not require the second derivative evaluation for \( k \), thus it can be easily implemented retaining the spatial accuracy.

For what concerns \( \gamma - \text{Re}_{\theta} \)–\( \text{SA} \) model standard boundary conditions are adopted for \( \nu : \nu_\infty = 3\nu \) at the free stream and \( \nu = 0 \) at the wall, while the boundary condition for \( \gamma \) at the wall is zero normal flux. At the inlet, the value of \( \gamma \) is 1. The boundary condition for \( \text{Re}_{\theta} \) at the wall is zero flux, while at the inlet \( \text{Re}_{\theta} \) is calculated from the specific empirical correlation based on the inlet turbulence intensity.

3. Numerical solution

The computations performed in this work are obtained by means of OpenFOAM v.2.3.0. In particular, we have used a modified version of pisoFoam, which is the unsteady solver for incompressible flows available in the OpenFOAM official releases, including the solution of the energy equation. The solver uses the well established PISO algorithm, [32], for pressure–velocity decoupling and it is based on a colocated finite volume approach; the Rhie–Chow correction is adopted to remove oscillations in the solutions, [33].

All diffusive terms and pressure gradient were approximated with a second-order accuracy while a second-order implicit Euler method (BDF–2, [34]) was used for time integration. The time-step size was selected in order to obtain \( \text{Co} \simeq 0.2 \) for accuracy reasons.

For all computations the high–resolution NVD Gamma scheme, [35], was selected for the convective terms; a second order central scheme was adopted for the momentum equation.

3.1. Computational domain

In this paper we have built two different structured hexaedral cells grids having respectively \( n_c \simeq 3.6 \cdot 10^6 \) and \( n_c = 4.5 \cdot 10^5 \) cells.

The domain is a cylinder of radius equal to 20 times the sphere one, \( D \). A distance between the sphere surface and the inflow boundary of 14 \( D \) was used for both grids; in the wake region the domains develop for 30 \( D \) behind the sphere surface. Lastly, the height of cells next to the walls is fixed to: \( \Delta r/D = 2 \cdot 10^{-4} \). Finer grid computations were run on MARCONI–A2 system hosted by CINECA using 256 CPU–cores, while coarse case was run on Linux Cluster, with sixteen Intel Xeon E5–2603v3 based nodes for a total of 192 CPU cores operating at 1.6 GHz.

4. Results

This section illustrates our numerical solutions for the forced convective heat transfer over a sphere at \( Re = 5000 \). A fixed wall temperature condition on the sphere surface was imposed as in [21, 22]. The models presented in Sec. 2 were tested and compared with the standard \( \text{SA–DES} \) approach.

For this case the attached boundary layer is laminar. Moreover the flow experiences a laminar separation thus, as also noted by Rodriguez et al. [23], the separated shear layer is Kelvin–Helmholtz unstable and large vortex rings which finally break up to feed a turbulent wake, [24].

The angular distribution of the mean pressure coefficient, \( c_p = 2 (\langle p \rangle - p_\infty) / \rho u_\infty^2 \), is plotted in Fig. 1(a). In the same figure \( c_p \) is compared with Rodriguez et al., [23]. A good agreement is achieved. The angular position of the pressure minimum is well captured, being placed at an azimuth angle, \( \varphi \), of 72° which is the same of Rodriguez et al., while Seidl et al. [24] found \( \varphi = 71° \) on a sphere with a rear support. The predicted local Nusselt number distribution, \( \text{Nu}_\varphi = -[D/(T_w - T_\infty)] \partial_n T_{|w} \), is presented in Fig. 1(b) and it is compared with data from literature [21, 22, 25]. \( \nu^2 \cdot f \) based techniques and \( \text{SA–DES} \) converge to very close solutions; a
Table 1. Mean drag and Nusselt number

| Case                        | \( \langle C_D \rangle \) | \( \langle Nu \rangle \) |
|-----------------------------|-----------------|-----------------|
| \( v^2 f \) DES             | 0.386           | 38.967          |
| \( v^2 f \) DDES            | 0.392           | 38.965          |
| \( \gamma - \text{Re}_{\theta,t} \) SA DES fine | 0.393           | 39.375          |
| \( \gamma - \text{Re}_{\theta,t} \) SA DES coarse | 0.392           | 37.264          |
| SA DES                      | 0.396           | 39.239          |
| LES Dixon et al. [22]       | 0.409           | 42.912          |
| LES Li et al. [21]          | 0.409           | 46.310          |

different trend can be noted for \( \gamma - \text{Re}_{\theta,t} \)–SA but, globally, current results are in overall satisfactory agreement with the ones presented in literature. This is hold true for our finer grid results, while coarse grid results, obtained only for \( \gamma - \text{Re}_{\theta,t} \)–SA, completely deviate from other solutions for \( \varphi > 90^\circ \). The average Nusselt number, \( \langle Nu \rangle \), results clearly reflects Fig. 1(b) evidence. Indeed,

\( \text{Nu}_\varphi \) is almost under predicted if compared with literature thus, as expected, \( \langle Nu \rangle \) undergoes a similar trend, see Tab. 1. On the other hand, looking at the same table, the average drag coefficient, \( \langle C_D \rangle = 2 \langle D \rangle / \rho u^2_\infty A_{\text{ref}} \), is only slightly underestimated by our techniques.

From Fig. 2 to Fig. 4 we present mean streamwise velocity and its fluctuations in the weak region averaged on \( \sim 1000 D/u_\infty \) time units. Note that the origin of the reference frame is the center of the sphere and the \( x \)-axis is parallel to undisturbed flow direction. Our flow statistics are directly compared with DNS data of Rodriguez et al. [23]. A good agreement between finer grid results and DNS data was obtained. Coarser grid does not produce reliable results and for this reason is further considered in the following discussions. Major discrepancies between DES/DDES results are observed in the streamwise velocity fluctuations for \( x/D = 1 \). These differences are mainly due to the different ability of the considered turbulence models in the prediction of laminar–to–turbulent transition in the wake region. This effect is also evidenced in
Fig. 5 where we represent the vortical structures by $Q$–isosurfaces. The $Q$–criterion defines eddy structures as regions with positive second invariant of the velocity gradient tensor. In particular for $Q > 0$ the vorticity prevails over the strain, i.e. the strength of rotation is greater than the strain–rate. Therefore, in Fig. 5 it is easy to observe that unorganized flow structures appear in different wake locations for the different flow models. At $x/D = 2 \frac{\nu^2}{f}$ DES/DDES models seems to be in better agreement with reference data, differently at $x/D = 3$ SA methods perform better than the other ones. As the regards the mean streamwise velocity we can note that at $x/D = 1$ all models produce very close results, see Fig. 2. For $x/D > 1$ it is easy to observe as SA based techniques under predict the velocity deficit, see Fig. 3 and Fig. 4. Hence we retain the DES/DDES techniques based on elliptic relaxation techniques are the best choice for this kind of test case.

![Figure 2](image-url)

Figure 2. First and second order statistics of the streamwise velocity at different positions in the wake, $x/D = 1$.

4.1. Discussion

The proposed benchmark allowed to test the considered DES models as LES drawing a rigorous comparison with literature data. Indeed, we have to remember that DES models have been developed to work in RANS mode in the wall region. Moreover standard RANS models have been thought for fully turbulent flows and calibrated with turbulence data. The flow features here considered reveal critical issues for standard DES models since the turbulent transition occurs in the wake region. We have also to take into account that standard RANS models are also able to predict a laminar–to–turbulent transition although the rigorous mathematical reasons of this phenomenon are not still known [36]. However the quality of the results can be often questionable. For this reason we have decided to introduce $\gamma$–$\tilde{Re}_\theta$–SA model in the DES context. Moreover with this work it is our intention to investigate the impact of DDES shielding function in the $\nu^2$–$f$ DES context. The results show that the proposed models are sufficiently reliable in LES mode especially $\nu^2$–$f$ DDES since $\gamma$–$\tilde{Re}_\theta$–SA does not perform appropriately in the prediction of the velocity deficit. As regards the RANS behaviour is very clear that all the models converge to the same solution prior the flow separation; differently in the separated flow
Figure 3. First and second order statistics of the streamwise velocity at different positions in the wake, $x/D = 2$.

(a) $\bar{u}/u_\infty$
(b) $\bar{u}'^2/u_\infty^2$

Figure 4. First and second order statistics of the streamwise velocity at different positions in the wake, $x/D = 3$.

(a) $\bar{u}/u_\infty$
(b) $\bar{u}'^2/u_\infty^2$
region all the models converge to the same solution with the exception of $\gamma^{\tilde{\text{Re}}_{\theta, t}}$-SA. However, on the basis of the presented results is not possible to evince a clear improvement of the wall behaviour using a similar approach.

5. Conclusion
In this paper we have presented the preliminary benchmarking of two DES models, i.e. $v^2-f$ DDES and $\gamma^{\tilde{\text{Re}}_{\theta, t}}$-SA DES. Forced convective heat transfer over a sphere at $\text{Re} = 5000$ at fixed wall temperature was considered. The proposed benchmark is critical due to the flow features. Both RANS and LES modes were assessed. The results put in evidence a good behaviour when LES is active for $v^2-f$ DDES model, while SA based models are not optimal in the velocity
deficit prediction. On the other hand in RANS region we have noted that all the models, with the exception of $\gamma - \text{Re}_\theta$-SA, converge to the same solution prior the flow separation. We want also to highlight that, from $\gamma - \text{Re}_\theta$-SA DES seminal results, is not possible to evince a clear improvement of the wall behaviour using the proposed approach. Future work will be devoted to the assessment of the proposed models for internal flow with the possibility to consider their recalibration.

6. Acknowledgements
We acknowledge the CINECA Award N. HP10CIZMYW YEAR 2017 under the ISCRA initiative, for providing high-performance computing resources and support.

References
[1] Pope S 2000 Turbulent flows (Cambridge University Press)
[2] Sagaut P and Deck S 2009 Philos T Roy Soc A 367 2849–2860
[3] Chaouat B 2017 Flow Turbul Combust 99 279–327
[4] Spalart P, Jou W, Strelets M and Allmaras S 1997 1st AFOSR Int. Conf. on DNS/LES ed Liu C and Liu Z (Ruston, LA)
[5] Bunge, U and Mockett, C and Thiele, F 2007 Aerosp Sci and Technol 11 376 – 385 ISSN 1270-9638
[6] Spalart P and Allmaras S 1994 Rech Aérospatiale 1 5–21
[7] Travin A, Shur M, Strelets M and Spalart P 2002 Proceedings of EUROMECH Colloquium. pp 239–254
[8] Yan J, Mockett C and Thiele F 2005 Flow Turbul Combust 74 85–102
[9] Davidson L 2006 European Conference on Computational Fluid Dynamics (Netherlands)
[10] Menter F, Kuntz M and Langtry R 2003 Proceedings of Turbulence Heat and Mass Transfer 4
[11] Jee K and Shariff K 2014 Int J Heat Fluid Fl 46 84–101
[12] Mirzaei M and Sohankar A 2015 Aerosp Sci Technol 43 199 – 212
[13] Ashton N, Revell A, Prosser R and Uribe J 2013 AIAA J 51 513–518
[14] Spalart P, Deck S, Shur M, Squires K, Strelets M and Travin A 2006 Theor Comp Fluid Dyn 20 181–195
[15] Shur M, Spalart P, Strelets M and Travin A 2008 Int J Heat Fluid Fl 29 1638–1649
[16] Spalart P 2009 Annu Rev Fluid Mech 41 181–202
[17] D’Alessandro V, Montelpare S and Ricci R 2016 Comput Fluids 136 152–169
[18] D’Alessandro V, Montelpare S, Ricci R and Zoppi A 2017 Energy 130 402–419
[19] D’Alessandro V, Garbuglia F, Montelpare S and Zoppi A 2017 J Phys: Conf Ser 923
[20] Costantinescu G and Squires K 2004 Physics of Fluids 16 1449–1465
[21] Li S, Yang J and Wang Q 2017 Appl Therm Eng 121 810 – 819 ISSN 1359-4311
[22] Dixon A, Taskin M, Nijemeisland M and Stitt E 2011 Comput Chem Eng 35 1171 – 1185 ISSN 0098-1354
[23] Rodriguez I, Lehmkühl I, Borrell R, Oliva A and Pérez-Segarra C 2010 V European Conference on Computational Fluid Dynamics ed Pereira J and Sequeira A (Lisbon)
[24] Seidl, V and Muzafferija, S and Peric, M 1998 Appl Sci Res 59 379 – 394 ISSN 1270-9638
[25] Clift, R, Grace J R and Weber M E 1978 Bubbles drops and particles (New York: Academic Press)
[26] Lien F, Kalitzin G and Durbin P 1998 RANS modeling for compressible and transitional flows Proceedings of the Summer Program Center for Turbulence Research
[27] Menter F, Langtry R, Likki S, Suzen Y, Huang P and Volker S 2006 Journal of Turbomachinery 128 413–422
[28] Medida S and Baeder J 2011 American Helicopter Society 67th Annual Forum
[29] Medida S 2014 Correlation-based Transition Modeling for External Aerodynamic Flows Ph.D. thesis University of Maryland
[30] Malan P, Suluksna K and Juntasaro E 2009 47th AIAA Aerospace Sciences Meeting (Orlando, FL)
[31] Chien K 1982 AIAA Journal 20 33–38
[32] Issa R 1986 J Comput Phys 62 40–65
[33] Ferziger J and Peric M 1999 Computational Methods for Fluid Dynamics (Springer)
[34] Geurts B 2004 Elements of Direct and Large-Eddy Simulation (R.T. Edwards)
[35] Jasak H, Weller H and Gosman A 1999 Intl J Num Meth Fl 31 431–449
[36] Durbin P and Petterson Reif B 2010 Statistical Theory and Modeling for Turbulent Flows, 2nd Edition (Wiley)