Testing dark matter halo properties using self-similarity

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ABSTRACT

We use self-similarity in N-body simulations of scale-free models to test for resolution dependence in the mass function and two-point correlation functions of dark matter halos. We use 1024³ particle simulations performed with the Abacus N-body code and define halos with two different algorithms, “friends of friends” (FOF) and ROCKSTAR. The FOF mass functions show a systematic deviation from self-similarity which is explained by a resolution dependence of the FOF mass assignment previously reported in the literature. Evidence for convergence is observed only starting from halos of several thousand particles, and mass functions are overestimated by as much as 20 – 25% for halos of 50 particles. The mass function of ROCKSTAR halos, on the other hand, shows good convergence from order 50 to 100 particles per halo, with no detectable evidence at the 1% level of any systematic dependence for larger particle numbers. Tests show that the mass unbinding procedure in ROCKSTAR is the key factor in obtaining this much improved resolution. Applying the same analysis to the halo-halo two point correlation function, we find again strong evidence for convergence only for ROCKSTAR halos, at separations sufficiently large so that halos do not overlap. At these separations we can exclude dependence on resolution at the few percent level once halos have of order 50 to 100 particles. At smaller separations results are not converged even at significantly larger particle number, and bigger simulations would be required to establish the resolution required for convergence.

Key words: Cosmological structure formation, gravitational clustering, N-body simulation

1 INTRODUCTION

The approximate description of the cosmological matter field in terms of a decomposition in “halos” has become a central tool of cosmological structure formation (see e.g. Cooray & Sheth (2002)). With the advent of ever more precise observational data, the issue of the precision of this description has become an important practical one. Indeed as theoretical predictions for many observables (e.g. galaxy-galaxy correlation functions) are obtained using construction based on the halo decomposition, their precision relies on that of the latter. The issue of the precision of relevant halo properties, which are obtained uniquely from N-body simulations, is particularly complex as it combines two distinct issues: that of the precision with which mass distribution in the N-body simulation represents the physical limit and that of the halo definition and extraction. Halos are not objects with a unique definition and they are defined in practice by the algorithm adopted to extract them from the N-body simulation. Numerous different ways have been proposed and exploited (see Knebe et al. (2013) for a review). In this article we apply a test of the accuracy with which basic properties of halos, as determined by two different widely used algorithms, can be obtained from cosmological simulations. Specifically we focus on the resolution limits due to the finite particle density used to sample the density field in the N-body method.

The analysis follows very closely that we have reported in a recent paper (Joyce, Garrison & Eisenstein (2020), hereafter P1) in which we have shown how very precise constraints on the convergence to physical values of the two point correlation function (2PCF) of the full matter field can...
be obtained by studying the deviations from self-similarity in its evolution in simulations of a scale-free model. Indeed if such deviations are observed they necessarily imply a dependence of the result on unphysical parameters. In the direct study of the matter field considered in P1, such parameters can only be those introduced by the N-body method, i.e., the box size $L$, initial grid spacing $\Lambda$, force softening $\varepsilon$ and the parameters characterising initial conditions (IC) (as well as the numerical parameters controlling the accuracy of the N-body integration). The halos and their properties as defined by the halo extraction will also necessarily depend at some level on at least some of these unphysical parameters — in particular $\Lambda$ which determines the particle density — and may, depending on the algorithm, introduce other such parameters. Thus by testing for the self-similarity of halo statistics we can simultaneously test for both accuracy limits arising from the underlying field on which the halos are sampled and for accuracy limits arising from the method used to extract the halos.

As the analysis we employ here is completely parallel in its essentials to that presented in detail in P1, we limit ourselves here to a very brief recapitulation of the essential steps with an emphasis on the points which are specific to the halo statistics we consider here. As noted in P1, the self-similarity of scale-free models has been widely recognised since the early development of N-body simulation in cosmology as an instrument to check the reliability of simulation results (e.g. Elstathieu et al. (1988); Colombi, Bouchet & Hernquist (1996)), and such models have been used quite extensively in the study of halo properties (e.g. Navarro, Frenk & White (1997); Cole & Lacey (1996); Knollmann, Power & Knebe (2008); Elahi et al. (2009); Diemer & Kravtsov (2015); Ludlow & Angulo (2017); Diemer & Joyce (2019)), also we refer to P1 for further references to previous literature. The required self-similarity of these models has, however, not been exploited in the way we do here to extract quantitative constraints on resolution.

In scale-free simulations, the initial power spectrum of fluctuations is a power law $P(k) \propto k^n$, where $n$ is a constant and the expansion law is that Einstein de Sitter (i.e. $a(t) \propto t^{2/3}$). One can then infer that, if there is no dependence on any other length or time scales introduced, any statistic must be “self-similar” — i.e. invariant in time if expressed in terms of suitably rescaled space (or mass) variables. For any statistic written as a dimensionless function $F(x_1, x_2 \cdots ; a)$ of the quantities $x_1, x_2 \cdots$ (e.g. separations, angles, masses) it depends on, self-similarity can be expressed simply as

$$F(x_1, x_2 \cdots ; a) = F_0(x_1/X_{1NL}(a), x_2/X_{2NL}(a), \cdots),$$

where each of the $X_{iNL}(a)$ is the temporal dependence of any quantity with the dimensions of $x_i$ inferred from self-similar scaling. The characteristic length scale $R_{NL}$ can be defined by

$$\sigma_{lin}^2(R_{NL}, a) = 1, \quad \sigma_{lin}(R, a) = \sigma_{lin}(\Lambda, a_i)$$

where $\sigma_{lin}(R, a)$ is the linear theory normalized variance of mass in a sphere of radius $R$ at scale-factor $a$. Defining (following the notation of P1) $\sigma_t = \sigma_{lin}(\Lambda, a_i)$ where $a_i$ is the value of the scale factor at the start of the simulation, we can infer that

$$R_{NL}(a) = \Lambda \left( \frac{a}{a_i} \right)^{2/(n+3)}, \quad (3)$$

The characteristic mass scale can then be defined as

$$M_{NL}(a) = \frac{4\pi}{3} \bar{\rho} R_{NL}(a) = 4\pi \frac{3}{3} m_P \left( \frac{a}{a_i} \sigma_t \right)^6/(n+3), \quad (4)$$

where $\bar{\rho}$ is the mean (comoving) mass density and $m_P$ is the mass of a simulation particle.

The two statistics which we consider here are the halo mass function (HMF) and the halo-halo correlation functions. Defining $n(M, a)$ canonically as the number of halos per unit mass interval and per unit comoving volume, we can express conveniently the self-similarity of the HMF as

$$\frac{M_{NL}^2(a)}{\bar{\rho}} n(M, a) = h_0(M/M_{NL}(a)), \quad (5)$$

where $h_0$ is a function. The 2PCF $\xi_{HH}(r, M)$ of halo centres is a function of separation $r$ and of the halo mass $M$. As it is dimensionless, its self-similarity is simply expressed as

$$\xi_{HH}(r, M, a) = \xi_{HH, 0}(r/R_{NL}(a), M/M_{NL}(a)) \quad (6)$$

It is the observed deviations from these scalings that we use here to infer the limitations on accuracy arising in particular from finite particle density.

## 2 SIMULATIONS

As described in P1, we have performed scale-free simulations using the ABACUS cosmological N-body code (Garrison et al. 2018). Background on the ABACUS code and preparation of initial conditions can be found in P1 and references therein. We consider here results for the reference simulation in P1: this is a simulation with $N = 1024^3$ particles of an $n = -2$ scale-free model, with force softening length $\varepsilon = \Lambda/30$. The initial conditions are specified by $\sigma_t = 0.03$, and by a choice of a scale factor fixing corrections applied for discreteness effects at early times.

As discussed in P1 we have chosen values of the numerical parameters controlling the N-body integration — in particular the value of time-stepping parameter, $\eta = 0.15$ — for which numerous tests have shown that the statistics we study here are converged (with respect to these parameters) to a precision well below the percent level. We have, in particular, in P1 reported a comparison with simulations with different values of $\eta$ which confirm this conclusion.

Most of our results will be given in terms of dimensionless quantities. As we will focus primarily on resolution effects associated with the finite particle density, the associated bounds on mass will be given in units of the particle mass. As time variable we use, as in P1, $\log_2(a/a_0)$, where the reference $a_0$ is defined by

$$\sigma_{lin}(\Lambda, a_0) = 0.56, \quad (7)$$

which is a simple estimate for the time at which the first non-linear structures appear in the simulation ($0.56 \approx \delta_c/3$ where $\delta_c$ is the estimated threshold linear density fluctuation for virialization). We take our first output at $a = a_0$, and as in P1, subsequent outputs at scale-factors $\{a_1, a_2 \cdots a_P\}$ with equal logarithmic spacing chosen as

$$\frac{M_{NL}(a_{i+1})}{M_{NL}(a_i)} = \sqrt{2}, \quad (8)$$

i.e. for every two intervals the theoretical non-linear mass

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scale grows by a factor of 2. The last output corresponds to \( P = 37 \), at which time the largest halos contain of order a million particles\(^1\).

### 2.1 Halo Extraction

For each of our 38 outputs, we have constructed two halos catalogs using the Friends of Friends (FOF) algorithm (see Knebe et al. (2013) for references) and the ROCKSTAR code (Behroozi, Wechsler & Wu 2013).

FOF uses a single parameter \( b \) (linking length) to link particles separated by a distance smaller than \( b \). The most common choice for its value is \( b = 0.2 \) which corresponds to a theoretical threshold density of \( \sim 80 \) times the mean matter density (More et al. 2011). Here we use \( b = 0.138 \), corresponding to a critical density of approximately 240 times the mean matter density. Our generated catalog retains only halos of at least 25 particles.

ROCKSTAR (Robust Overdensity Calculation using K-Space Topologically Adaptive Refinement) (Behroozi, Wechsler & Wu 2013) uses a sophisticated algorithm that is based on initial FOF groups in phase space that are refined using spherical overdensity and phase space proximity criteria. We have run with the default parameters of the publicly available code, except that we have set the parameter fixing the minimal size of the initial FOF groups taken as seeds to 25. The output catalogs, which contain halos down to two particles, are expected thus to be incomplete when halos have less than 25 particles. ROCKSTAR provides numerous outputs corresponding to different mass definitions, and also a labelling identifying “parent” halos from sub-halos (and sub-sub-halos etc.). We will focus in what follows on the default output catalog, which include all halos assigned their estimated gravitationally bound mass inside the virial radius. We will discuss briefly below the importance of this specific choice for our results.

### 3 Halo Mass Function

As we have discussed, if the HMF of extracted halos corresponds to its physical (continuum) limit it should be self-similar. This means that the measured value of \( M^2_{\text{NL}} n(M,a) \), where \( n(M,a) \) is the usual HMF, should be invariant as a function of time, when plotted as a function \( M/M_{\text{NL}} \). Figures 1 and 2 show the corresponding plots for the FOF and ROCKSTAR halos, respectively. For clarity we show in each figure only a subsample of 7 of our 38 available outputs.

These figures show that the self-similar scaling appears to apply to a very good approximation to both FOF and ROCKSTAR catalogs. To be more quantitative requires us to assess what the converged answer actually is. To do this, we follow the method defined in P1 and consider the rescaled HMF at fixed values of \( M/M_{\text{NL}} \) as a function of \( \log_2(a/a_0) \). This corresponds to a projection of Figures 1 and 2 along the \( x \)-axis. The results are shown in Figure 3 for the FOF halos, and in Figure 4 for the ROCKSTAR halos. Each panel shows the measured value of the rescaled mass function for all 38 snapshots for a finite bin of halo mass \( M/M_{\text{NL}} \). The bins have an equal logarithmic spacing with a width of approximately 40% their central value, and the subsample of bins shown in the figures are equally spaced across the full range of \( M/M_{\text{NL}} \) sampled by the simulation. To facilitate comparison, the \( y \)-axis in all panels of the two figures has been chosen to have the same logarithmic range, with \( \log_{\text{max}}/\log_{\text{min}} = 2 \). For the ROCKSTAR halos each plot also has a small subplot showing the fractional change \( \Delta Y/Y \) of \( Y = M^2_{\text{NL}} n(M,a) \) between consecutive snapshots i.e. at each \( a_i \) we plot \( \Delta Y/Y = (Y_i/Y_{i-1}) - 1 \).

Given that the range of \( M/M_{\text{NL}} \) is constant in each plot, and that \( M_{\text{NL}} \) expressed in units of particle mass increases monotonically as a function of time, the \( x \)-axis of these plots

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\(^1\) Although we do not use redshift \( z \) here, it may be instructive to note that if we define it so that \( z = 0 \) at our final time, the starting red-shift of our simulation is \( z_i \approx 157 \), while \( a = a_0 \) corresponds to \( z \approx 7.5 \).
can be labelled also by the monotonically increasing number of particles per halo. Thus each plot effectively shows the measured mass function as a function of increasing resolution. The dashed vertical lines in each plot indicate a resolution corresponding to 50 and 5000 particles respectively (for the geometric centre of the bin of $M_{NL}$). At any intermediate time the number of particles can easily be read off given our chosen snapshot spacing Eq. (8). There are thus no halos in each plot until the time at which the particle number per halo reaches the minimal number of particles in halos included in the catalog (25 for FOF, 2 for Rockstar), and it is only in the last bin, corresponding to the largest mass at given $M_{NL}$, that there are halos at the first output time (i.e. at $a = a_0$ defined by Eq. (7)).

While in a given plot the number of particles per halo increases monotonically from left to right, the average number of halos contributing to the measure of the HMF in the corresponding bin decreases monotonically: this number is proportional to the simulation volume in units of the characteristic volume $R_{NL}$, so, in the approximation that $M_{NL}(M, a)$ is constant, it is proportional to $1/M_{NL}$. Thus we expect the effects of sparseness of sampling in the finite bins to manifest itself as increasing noise in the measured signal at later times. This is indeed what we see in Figures 3 and 4, for the larger values of $M/M_{NL}$ (for which the number of halos per logarithmic interval of mass decreases strongly). Conversely we see that most of our plots are clearly not dominated by such sampling noise, despite our use of rather narrow mass bins. We can thus clearly identify systematic dependencies on resolution alone.

### 3.1 FOF halos

We see in Figure 3 that once the time is reached at which the mass range in the bin is unaffected by the intrinsic lower limit on the number of particles in the catalog, the rescaled HMF apparently decreases monotonically as the number of particles per halo increases, until it becomes noisy due to sparseness effects at the later times. There is marginal evidence, at best, for convergence towards a resolution independent value in a few of the mass bins. In these cases the apparent flattening out of the curves occurs at several thousand particles. For halos of $50 - 100$ particles, the in-
ferred converged value is then systematically overestimated by $20 - 25\%$.

This observed monotonic decrease in the region where the mass function is well measured has an obvious interpretation in terms of resolution dependence of the mass measured by the FOF algorithm. Indeed studies in the literature (Warren et al. 2006; More et al. 2011) have shown, by using the FOF algorithm on idealized isolated halos, that it systematically overestimates mass because of finite size effects that lead to percolation into sub-critical density regions. As the underlying rescaled mass function is decreasing functions of mass, halos at a given measured FOF mass are then overestimated at lower resolution. Comparing to Warren et al. (2006) and More et al. (2011) we find agreement in the order of magnitude of the effect. Our method has the considerable advantage of allowing its precise "in situ" quantification in a cosmological simulation.

3.2 Rockstar halos

For ROCKSTAR halos, Figure 4 shows that the rescaled mass function is much more self-similar than for FOF. In all but the first bin, the data clearly shows convergence in a range of
scale-factors: the visually apparent plateau in each case corresponds, as can be seen in the lower panel, to fluctuations of the derivative which appear to be roughly symmetric about zero. Down to the intrinsic limit on precision corresponding to these fluctuations (of order 2% here), we thus conclude that there is convergence to a resolution independent physical value of the mass function.

We observe further that the point of onset of this convergence, right across the several decades of $M/M_{NL}$, corresponds to a number of particles per halo situated around 50 particles, and at most 100. For the larger mass bins we see both the growth in the fluctuations due to finite sampling and also strong systematic deviations from the converged plateau. The latter are a direct manifestation of the finite box size: as shown in P1 using the analysis of the matter correlation function, by $\log_2(a/a_0) \approx 2.5$ the finite size of the simulation box begins to give rise to strong systematic deviations from self-similarity even at non-linear scales.

To quantify these observations a little more precisely we plot in Figure 5, for all 40 bins of $M/M_{NL}$, from which those shown in Figure 4 are sampled, the converged range expressed in terms of the particle number per halo (evaluated at the geometric centre of the bin). The range is determined here as the largest contiguous set of snapshots in which $\Delta Y/Y < 0.03$, corresponding to the green-dashed lines in the sub-panels of Figure 4, and in which $\Delta Y$ changes sign at least once. As anticipated, we see that the lower cut-off on particle number per halo we determined fluctuates in a range around 50 particles up to at most 100 particles. The upper cut-offs in the plot quantify limits on the accurate determination of the HMF arising from the finite size of the simulation box. The increasing upper cut-off at smaller $M/M_{NL}$ reflect the fact that in any such bin the largest mass which can be potentially resolved is $(M/M_{NL}) \times M_{NL}(a_f)$ where $a_f$ is the scale-factor at the stopping time of the simulation. This maximal mass thus grows linearly with $M/M_{NL}$ until, at some value of $M/M_{NL}$, it reaches a value at which the sparseness of halos in the finite volume starts make the measure of the very HMF noisy. The maximal value is then fixed as a function of time and thus decreases approximately as a function of $M/M_{NL}$, as observed.

The chosen threshold value of $\Delta Y/Y$, inferred from the level of residual fluctuations in the subplots in Figure 4, gives a measure of the precision with which the mass function, in the chosen binning, can be determined. Varying the size of our bins we observe that this precision level changes, reaching a minimal level for most bins of well under one percent when we divide the full range of $M/M_{NL}$ into only a couple bins. Irrespective of these binnings we find that we do not find any evidence for systematic evolution of the mass function with resolution when the halos contain more than of order 100 particles. On the other hand this statement becomes weaker as the residual fluctuations decrease well below the 1% per cent level because the bins themselves become very broad.

### 3.3 Comparison with other Rockstar catalogs

Our analysis clearly indicates that it is the resolution dependence of mass assignment in the FOF algorithm that causes the poor accuracy in the determination of the HMF. As ROCKSTAR does so much better, the question obviously arises as to why there is such a great difference, given that ROCKSTAR is built itself on an initial FOF selection. It is evident that a potentially important difference can arise from the mass unbinding performed by ROCKSTAR, which gives the final mass assigned to the halos in the default output catalog we have analysed here. As ROCKSTAR provides also output catalogs including both bound and unbound mass it is easy to test whether this is the case. Figure 6 shows a comparison between convergence plots, like those in Figures 3 and 4, but for a single chosen bin of rescaled mass $M/M_{NL} \in [0.20, 0.25]$. The left panel compares the catalog we have analysed above (bound mass only) with a catalog in which the same halos are assigned their unbound mass. The central panel compares the same two catalogs but now including only the parent halos. The right panel shows the catalogs of subhalos only, with and without bound mass. From these plots we see that it is indeed clearly the mass unbinding which corrects for the resolution dependence in the FOF masses. However they also show that the mass unbinding essentially affects only the population of subhalos, and thus a selection of the parent halos only, using the unbound mass, leads also to good convergence of the HMF. We have compared in detail our quantitative results for the parent only catalogs with those we obtained above and found them to be essentially unchanged. We will report a fuller analysis and in particular an analysis of the subhalo catalogs elsewhere.
4 CORRELATION OF HALO CENTERS

We now consider the correlation properties of the clustering as characterised by $\xi_{HH}$, the two-point correlation function (2PCF) of halo centers. As discussed in the introduction, because $\xi_{HH}$ is in principal a function of both mass and separation, to test for self-similarity we need to consider its value in bins of both rescaled mass and separation. Thus we divide the halos in bins of $M/M_{NL}$ and compare the measured 2PCF of the halos in a given bin, as a function of time, in bins of rescaled separation $r/R_{NL}$. As in P1 we have used the code Corrfunc (Sinha & Garrison 2020) to compute the 2PCFs.

We do not present here results for the FoF halos, be-
caused, as can be anticipated, the non-similarity of the HMF we have seen leads to a strong breaking of self-similarity also in the halo 2PCF. Like for the HMF we find only at most some weak marginal convergence in $\xi_{HH}$, and our conclusion is that this finder is unsuitable for precision measurement of a physical clustering signal in halos, unless a lower cut-off at at least several thousand particles in used. To determine these bounds reliably would require considerably larger simulations than ours.

For ROCKSTAR halos our results are exemplified by those for the case displayed in Figure 7. As in our convergence analysis of the mass function, we use the catalog containing all halos (i.e. both parent halos and subhalos) labelled by their bound virial mass. Figure 7 shows, for a single chosen bin, $M/M_{NL} \in [1.485, 2.169]$, and in a similar format as used for the ROCKSTAR mass function, the measured $\xi_{HH}$; each plot in Figure 7 shows its evolution as a function of time for a bin of rescaled separations $r/R_{NL}$ (corresponding to the centers of the same bins as in Figure 7).

The criteria used to obtain these regions are analogous to those used for Figure 5, and are illustrated by the sets of green lines in the panels if Figure 7). Note that this plot is for the bins which satisfy the convergence criterion, and thus excludes the first two bins in Figure 7 in which $\xi_{HH}$ appears to be systematically resolution dependent up to a limit of at least several hundred particles per halo.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure8.png}
\caption{Mass resolution limits for halo 2PCF. For halos in the same mass bin as Figure 7, $M/M_{NL} \in [1.485, 2.169]$, the range (expressed in particle number per halo) in which convergence is observed, as a function of rescaled separations $r/R_{NL}$ (corresponding to the centers of the same bins as in Figure 7). The criteria used to obtain these regions are analogous to those used for Figure 5, and are illustrated by the sets of green lines in the panels if Figure 7). Note that this plot is for the bins which satisfy the convergence criterion, and thus excludes the first two bins in Figure 7 in which $\xi_{HH}$ appears to be systematically resolution dependent up to a limit of at least several hundred particles per halo.}
\end{figure}

We proceed in the same manner as above for the mass function to state these results more precisely. Figure 8 shows for halos with mass in the bin $M/M_{NL} \in [1.485, 2.169]$ the range of particle number per halo as a function of $r/R_{NL}$ in which the simple convergence criterion illustrated by the red dashed lines in Figure 7 is satisfied. The absence of a bound at small distances corresponds to the observed lack of convergence, and we see the trend for convergence to improve strongly above a few times the typical virial radius in the mass bin.

\section{5 CONCLUSION}
Extending the methods applied in P1 to the full matter 2PCF, we have shown that self-similarity in scale-free cosmological N-body simulations provides a stringent test for the convergence to the physical (continuum) limit also of the statistical properties of halos extracted from such simulations. For halos selected using the simple FOF algorithm, our analysis identifies and allows the precise quantification of a resolution dependence of the assigned mass which has been anticipated in previous work (Warren et al. 2006; More et al. 2011) through study of isolated idealized halos. The ROCKSTAR algorithm, on the other hand, shows for the halo mass functions good convergence to a resolution independent value starting from of order 50 to 100 particles, and no measurable dependence on resolution for greater particle number at the level of precision we can probe (conservat-
For the halo-halo 2PCF (for ROCKSTAR) we have found very similar results, albeit at a lower level of precision (conservatively, or order several %) and at separations above that at which contributing pairs can overlap. For smaller separations at which halos overlap we see, on the other hand, clear evidence for strong resolution dependence of the measured correlation amplitude up to at least several hundreds of particles per halo.

Our results (like those in P1) have been derived for the $n = -2$ scale-free model and (by construction) cannot be directly applied to physical cosmological models, which are not scale-free. The essential general conclusions we have drawn are, nevertheless, stated in a model independent form, and even our more quantitative results can naively be extended to any model by simply identifying the relevant parameters (here, $M_{NL}$ and $R_{NL}$) with their appropriate (redshift dependent) values. Such a naive extrapolation assumes, however, that the results we have found do not depend on the difference between the $n = -2$ power spectrum and the physical one, are also unaffected by deviations from EdS cosmology. For what concerns the resolution effects we have quantified it appears very reasonable to assume that this is the case: these effects in principle, as we argued, arise from the halo finder and would not be expected to depend sensitively on the exact nature of the underlying density field on which they are used. Nevertheless in future work we will probe directly the model dependence of these results by performing the same analysis for different scale-free models. We will also explore further the dependence of our inferred constraints on other relevant parameters — box size, force smoothing, parameters controlling initial conditions — which we have not discussed at length here. Our results also show that it would also be potentially very instructive to perform significantly larger simulations of these models. As illustrated in particular by our results for the halo 2PCF at very small scales, by further reducing sampling noise, we should be able to determine the resolution necessary to obtain physically converged results at these scales rather than just diagnose that resolution is poor as we have done here. Indeed similar considerations are very likely to be relevant to the construction of HOD models e.g. correlations between parent halos and subhalos or between halos in different mass ranges.

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