Magnetic coupling in mesoscopic metal/ferromagnet layered systems

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Abstract

We consider a mesoscopic mechanism of the exchange interaction in a system of alternating ferromagnetic/nonmagnetic metallic layers. In the case of small mesoscopic samples the sign and the amplitude of the exchange interaction energy turn out to be random sample-specific quantities. They can be changed by applying an external magnetic field, by attaching to the system superconducting electrodes with different phases of the superconducting order parameter and by changing the chemical potential of electrons in the metal with the help of a gate. In the case of square or cubic geometries of the nonmagnetic layer at low temperature the variance of the exchange energy turns out to be sample size independent.

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The investigation of oscillations of the exchange energy as a function of the nonmagnetic layers’ thickness and the "giant magnetoresistance" in the system of alternating ferromagnetic/nonmagnetic metallic layers has both scientific and technological interest \([1-10]\).

The origin of the exchange interaction between the ferromagnetic (F) layers is the Ruderman- Kittel type interaction: The interaction of itinerant nonmagnetic metal electrons with localized "f" or "d" electrons in the ferromagnets induces a spin polarization in the nonmagnetic metallic (N) layers. This magnetization, in turn, creates the effective interaction between two localized spins in different ferromagnetic layers with the energy

\[
E(\vec{R}_1, \vec{R}_2) = I_{ij}(\vec{R}_1, \vec{R}_2)S_i^1 S_j^2. \tag{1}
\]

Here \(S_i^{1,2}\) are components of localized spins in the F layers and \(\vec{R}_{1,2}\) are their coordinates. We employ the simplest model where conduction "s"-electrons interact with localized "f" or "d" electrons in the F layers via a contact interaction with energy

\[
A \sum_k \delta(\vec{r}_k - \vec{R}) \vec{s}_k \vec{S}. \tag{2}
\]

Here \(\vec{r}_k\) and \(\vec{s}_k\) are coordinates and spins of conduction electrons in the metal which are labeled by the index \(k\) and \(A\) is the interaction constant. In the case where two magnetic ions are embedded in a 3-d nonmagnetic metal Eqs.1,2 lead to a well known form of the exchange energy \([11]\)

\[
I_{ij}(\vec{R}_1, \vec{R}_2) = I_0 \frac{\cos(2p_F |\vec{R}_1 - \vec{R}_2|)}{(p_F |\vec{R}_1 - \vec{R}_2|)^3}. \tag{3}
\]

Here \(I_0 = \frac{9\pi (An)^2}{64 E_F}\) is the interaction energy of adjacent localized spins in ferromagnets, which is of order of ferromagnet’s critical temperature; \(E_F, p_F\) are the Fermi energy and the Fermi momentum, respectively, and \(n = \frac{p_F^3}{3\pi^2}\) is the concentration of electrons. Following to \([6,7,8,9]\) we will use the approximation where the total exchange interaction energy \(\bar{E}\) between the magnetic moments in the F layers is a sum of \(E(\vec{R}_1, \vec{R}_2)\) over coordinates \(\vec{R}_1, \vec{R}_2\) of the localized spins in the ferromagnetic layers.

\[
\bar{E} = \sum_{R_1,R_2} I_{ij}(\vec{R}_1, \vec{R}_2)S_i(1)^1 S_j(2)^2 = \bar{I}_{ij} n_{1i} n_{2j} \tag{4}
\]
Here \( n_{1i} \) and \( n_{2j} \) are components of unit vectors \( \vec{n}_1, \vec{n}_2 \) parallel to magnetizations in the first and the second layers respectively. We assume that the exchange energy inside ferromagnets is large enough and fluctuations of spin magnetization direction along the F layers can be neglected. As a result, in the case of clean N layers \( (L \ll l) \), the value of \( \bar{I}_{ij} \) and the relative orientation of magnetizations of different F layers are oscillating functions of \( L \). \(^{[4-9]}\) Here \( L \) and \( l \) are the thickness and the elastic electron mean free path in the N layer, respectively. The same effects takes place in the system of alternating ferromagnetic/degenerated semiconductor layers.

Until now, however, both experimental and theoretical studies of this phenomenon have been restricted to the consideration of the infinite dimensions of both F and N layers and clean N layers. In this article we discuss the opposite case of small sample sizes and disordered N layers where the mesoscopic effects determine both the exchange interaction between the ferromagnetic layers and the conductance of the system. Let us consider, for example, a system of two F layers of sizes \( L_1, L_2, L_3 \) divided by a disordered N layer with \( L > l \) (See Fig.1). In this case the amplitude of the Ruderman-Kittel exchange interaction between a couple of spins located inside different F-layers \( \langle I_{ij}(\vec{R}_1, \vec{R}_2) \rangle \sim \exp(-\frac{|\vec{R}_1 - \vec{R}_2|}{l}) \sim \exp(-\frac{L}{l}) \), averaged over realizations of a random potential (and consequently the total average exchange energy \( \langle \bar{E} \rangle \) between F layers) is exponentially small and can be neglected. \(^{[12]}\) Here brackets \( \langle \rangle \) correspond to averaging over realizations of a random scattering potential. On the other hand, it is well known \(^{[13-16]}\) that the exponential decay of the average \( \langle I_{ij}(\vec{R}_1, \vec{R}_2) \rangle \) is the consequence of randomization of the sign of \( I_{ij}(\vec{R}_1, \vec{R}_2) \) and that the typical amplitude of the interaction \( \sqrt{\langle (I_{ij}(\vec{R}_1, \vec{R}_2))^2 \rangle} \sim |\vec{R}_1 - \vec{R}_2|^{-d} \) decreases with distance in the same way as in the pure case. Here \( d \) is the dimensionality of space. Therefore, in the case \( L \gg l \) the amplitude of the Ruderman-Kittel interaction between the ferromagnetic layers must be controlled by the mesoscopic effects.

In this article we discuss the typical magnitude of the "mesoscopic part" of the exchange energy between the ferromagnetic layers. We show that it is determined by long range correlations between \( I_{ij}(\vec{R}_1, \vec{R}_2) \) and \( I_{k,l}(\vec{R}_3, \vec{R}_4) \) which survive even on distances \( |\vec{R}_1 - \vec{R}_3| \)
|\vec{R}_3|, |\vec{R}_2 - \vec{R}_4| \gg l. As a result, for example, in the case of square or cubic geometries of the N layer and at low temperatures the variance of the exchange energy turns out to be sample size independent.

Let us start with the case where the length $L_2$ of the F-layers is relatively short and one can neglect the fluctuations of the orientation of magnetizations along the F layers. To find the relative angle $\theta(\vec{n}_1, \vec{n}_2)$ between magnetization’s angles in different layers one has to calculate the sign and the amplitude of the quantity $\bar{I}_{ij}$. In the case where metallic region is disordered, $\bar{I}_{ij}$ is a random sample-specific quantity, which can be characterized by its average and moments. One can calculate the correlation function $\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle$ with the help of the standard diagram technique \cite{17} for averaging over realizations of random potential. Here $\delta \bar{I}_{ij} = \bar{I}_{ij} - \langle \bar{I}_{ij} \rangle$. The diagrams which contribute to $\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle$ to lowest order in parameter $\frac{\hbar}{p_Fl} \ll 1$ are shown in Fig.2, where solid lines correspond to electron Green functions in Matsubara representation, dashed lines correspond to the scattering on the random potential and vertexes correspond to the contact magnetic interaction Eq.2. The blocks of the diagrams shown in Fig.2c correspond to the so called Cooperons and Diffusons $\bar{P}_\omega^{c,d}(\vec{r}, \vec{r}')$, which are tensors as functions of electron spin indexes \cite{17}. The diagrams shown in Fig.2a have been considered in \cite{13-16}. They give main contribution to the correlation function $\langle \delta I_{ij}(\vec{R}_1, \vec{R}_2) \delta I_{kl}(\vec{R}_1, \vec{R}_2) \rangle \sim |\vec{R}_1 - \vec{R}_2|^{-d}$. These diagrams correspond, however, to the correlation function $\langle \delta I_{ij}(\vec{R}_1, \vec{R}_2) \delta I_{kl}(\vec{R}_3, \vec{R}_4) \rangle$ which decays exponentially when $|\vec{R}_1 - \vec{R}_3|, |\vec{R}_2 - \vec{R}_4| \gg l$. As a result, for example, in the case $L \sim L_2 \gg L_3 \sim L_1$ the contribution to $\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle$ from these diagrams is of the order of

$$I_0^2(np_F^{-3})^4 \left(\frac{L_1}{L}\right)^2. \quad (5)$$

We assume the density of localized spins in the ferromagnets in of order of $n$.

The diagrams Fig.2b give much smaller contribution to $< \delta I_{ij}(\vec{R}_1, \vec{R}_2) \delta I_{kl}(\vec{R}_1, \vec{R}_2) >$, but they describe long range correlations $\langle \delta I_{ij}(\vec{R}_1, \vec{R}_2) \delta I_{ij}(\vec{R}_3, \vec{R}_4) \rangle \sim R^{-4(d-1)}$ when $|\vec{R}_1 - \vec{R}_3| \sim |\vec{R}_2 - \vec{R}_4| = R \gg l$. As a result, it is these diagrams that determine the of the correlation function of the interlayer exchange energy $\langle \bar{I}_{ij} \bar{I}_{kl} \rangle$ at $L \gg l$. The qualitative
explanation of the origin of the correlation is as follows. The mesoscopic fluctuations of the exchange energy $\delta I_{ij}(\vec{R}_1, \vec{R}_2)$ result from the interference of random probability amplitudes of diffusion paths between the points $\vec{R}_1$ and $\vec{R}_2$. Among these paths there are some which visit points $\vec{R}_3$ and $\vec{R}_4$ (An example is the line ”a” in Fig.1a). This leads to the mentioned above correlation between $I_{ij}(\vec{R}_1, \vec{R}_2)$ and $I_{kl}(\vec{R}_3, \vec{R}_4)$. As a result, we have:

$$\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle = \frac{2}{\pi} I_0^2 E_F^2 T \sum_m \omega \int d\vec{R}_1 d\vec{R}_2 d\vec{R}_3 d\vec{R}_4$$

$$(\hat{\sigma}_i \dot{P}_\omega^c(\vec{R}_1, \vec{R}_2) \hat{\sigma}_k \dot{P}_\omega^c(\vec{R}_2, \vec{R}_3) \hat{\sigma}_j \dot{P}_\omega^c(\vec{R}_3, \vec{R}_4) \hat{\sigma}_l \dot{P}_\omega^c(\vec{R}_4, \vec{R}_1) + \hat{\sigma}_i \dot{P}_\omega^d(\vec{R}_1, \vec{R}_2) \hat{\sigma}_j \dot{P}_\omega^d(\vec{R}_2, \vec{R}_3) \hat{\sigma}_k \dot{P}_\omega^d(\vec{R}_3, \vec{R}_4) \hat{\sigma}_l \dot{P}_\omega^d(\vec{R}_4, \vec{R}_1)). \quad (6)$$

Here $\omega = \pi (2m + 1) T$ is the Matsubara frequency, $m$ is an integer number $T$ is the temperature and $\hat{\sigma}_i$ are spin operators. Integrations over $\vec{R}_1$, $\vec{R}_3$ and $\vec{R}_2$, $\vec{R}_4$ in Eq.6 is performed over volumes of the first and the second ferromagnetic layers respectively. Results of calculation of Eq.6 depend on the ratio between the lengths $L, L_2, L_T = \sqrt{\frac{D}{T}}$, $L_{so} = \sqrt{D \tau_{so}}$ and on boundary conditions for Cooperons and Diffusons. Here $L_{so}, \tau_{so}$ are the spin-orbit relaxation length and time, respectively, and $D$ is the electron diffusion coefficient in the N layer. In the case of the ”open” geometry of the N layer shown in Fig.1a and $L_T, L_{so} \gg L > l; L, L_2 \gg L_1, L_3$, we have

$$< \delta \bar{I}_{ij} \delta \bar{I}_{kl} > = \frac{5 \cdot 2^\frac{3}{2} \zeta(\frac{5}{2})}{3^2 \pi^2} X \frac{I_0^2}{(p_F L_1)^4} \delta_{ij} \delta_{kl} \quad (7)$$

Here $X$ is a factor, which is of the order of unity when $L \sim L_2 \leq L_T$ and $\zeta(x)$ is the Zeta function. In different limiting cases we have:

$$X = \begin{cases} 
\left( \frac{L_2}{L} \right)^4 & \text{if } L_T > L_2 > L \\
\left( \frac{L_1 L_2}{L^4} \right) \delta_{ij} \delta_{kl} & \text{if } L_2 > L_T > L 
\end{cases} \quad (8)$$

In the case $L > L_T$ the expression for $X$ acquires an additional exponentially small factor $\exp(-\frac{L}{L_T})$. In the case $L_{so} > L$ the minimum of the exchange energy corresponds to parallel or antiparallel orientation of the layer’s magnetizations ($\theta$ equals zero or $\pi$). In the opposite limit $L_{so} \ll L$ we get the same formula as Eq.8 but without the factor $\delta_{ij} \delta_{kl}$.
This means that the exchange interaction between the F layers is of the Dzialoshinski-Moria type and a minimum of the exchange energy corresponds to a sample specific angle \( \theta(\vec{n}_1, \vec{n}_2) \) distributed randomly in the interval \((0, \pi)\). It is interesting that in the case \( L \sim L_2 < L_T \) Eq.7 turns out to be independent of \( L \). Let us note that the diagrams shown in Fig.2a give \( \sqrt{\langle (\delta \bar{I}_{ij})^2 \rangle} \sim L^{-1} \).

While deriving the results presented above we neglected the sensitivity of the boundary conditions for Cooperons and Diffusons to the change of magnetization directions in F-layers. In the case of the open sample geometry Fig.1a this is correct, provided \( v_F / L_1 \ll 1 \). To get an estimate for \( \langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle \) in the opposite limit one has to substitute the factor \( A(L_1p_F) \) in Eq.8 for \( E_F \). For example, in the case \( L_T > L \sim L_2 > L_{so} \) we have

\[
\langle \delta \bar{I}_{ij} \delta \bar{I}_{kl} \rangle \sim E_F^2(p_Fl)^{-2} \sim \frac{\hbar}{\tau}.
\]

(9)

Here \( \tau \) is the elastic mean free path in the metal. We would like to stress that the origin of both Eq.7 and Eq.9 is the existence of the long range correlation of signs if the quantities \( I_{ij}(\vec{R}_1, \vec{R}_2) \) and \( I_{kl}(\vec{R}_3, \vec{R}_4) \) on distances much larger than \( l \).

In the case when the length of the ferromagnetic layers \( L_2 \) is long enough one should take into account the random fluctuations of the direction of magnetization along the layers. A solution of this problem is beyond the scope of this paper.

As it is usual in the physics of mesoscopic metals \(^{19,20}\), the external magnetic field changes the electron interference pattern and, thereby, \( \theta(\vec{n}_1, \vec{n}_2) \) turns out to be a random sample-specific oscillating function of the magnetic field \( H \). In the case of the open geometry of the N layer shown in Fig.1a for \( L_2 \sim L \) the characteristic period of these fluctuations is \(^{19,20}\) \( \delta H_1 \sim \frac{\Phi_0}{L} \). Here \( \Phi_0 \) is the flux quantum. There is also another characteristic magnetic field in the system which corresponds to the interaction energy between the external magnetic field and magnetic moments of the F layers which is of order of the exchange energy between the layers: \( \delta H_2 \sim \frac{\bar{I}_{ij}}{\mu_B p_F L_1 L_2 L_3} \sim \frac{\hbar}{\tau} \frac{1}{\mu_B p_F L_1 L_3} \). Here \( \mu \) is the Bohr magneton. At large enough \( L \), \( \delta H_2 >> \delta H_1 \) and therefore the exchange energy \( \bar{I}_{ij} \) will be a random sample-specific oscillating function of the magnetic field in the situation where the interaction
energy between the magnetic field and the ferromagnetic moment is still negligible. As a result, the relative orientation of magnetizations of the F-layers, characterized by $\theta(\vec{n}_1, \vec{n}_2)$, will be a random function of the magnetic field. In the opposite limit, when $\delta H_2 < \delta H_1$, $\theta(\vec{n}_1, \vec{n}_2)$ monotonically decreases with the magnetic field. Even in this case it is possible to see the random oscillations of $\theta(\vec{n}_1, \vec{n}_2)$ near the magnetic field corresponding to spin-flop transitions.

Another way to change the relative orientations of the F-layers is demonstrated in Fig.1b. Namely, $\theta(\vec{n}_1, \vec{n}_2)$ is a random sample specific function of the order parameter phase difference $(\chi_1 - \chi_2)$ in superconductors $S_1$ and $S_2$ in Fig.1b. The reason for this is that some diffusive paths connecting points 1 and 2 in Fig.1b can visit superconductors (line "b" in Fig.1b) and the corresponding amplitude of probability to travel along these paths acquire the additional phase $(\chi_1 - \chi_2)$ \[20\]. Another consequence of the phase dependence of the exchange energy is that the critical Josephson current of the device shown in Fig.1b depends on the angle $\theta$ between magnetizations of F layers. At last, a change of electrical or chemical potentials in the metal (or in the degenerate semiconductor) also lead to the oscillations of $\bar{I}_{ij}$. In the case $L \sim L_2$ characteristic period of the oscillations as a function of the chemical potential $\mu$ is of the order of $\delta \mu \sim E_c = \frac{D}{L^2}$. \[19\] In the case when the degenerate semiconductor is a part of a MOSFET one can change $\mu$ by changing the voltage on the gate.

The resistance of the system considered above is a random oscillating function of the external magnetic field. There are two mechanisms which cause these oscillations. 1. The usual mechanism for mesoscopic metallic samples: \[19,20\] amplitudes of probability to travel along diffusion paths acquire additional random phases, proportional to the magnetic field. The characteristic period of the oscillations is of the order of $\delta H_1$. 2. The magnetic field induced change of $\theta(\vec{n}_1, \vec{n}_2)$ which corresponds to a change of the boundary conditions for electrons in nonmagnetic metal. As a result the rotation of the ferromagnetic layer’s magnetizations leads to mesoscopic fluctuations of the resistance of the sample. The characteristic period of the oscillations correspond to $(\theta(H) - \theta(0)) \sim 1$. 

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The calculations presented above do not take into account the dipole interaction between magnetic moments in different F-layers. This interaction decays with the distance between the layers $L$ and can be much less than the considered above mesoscopic part of the exchange energy, which is independent of $L$ if $L \ll L_T$.

In conclusion, we would like to mention that the considered above mechanism can determine random anisotropy of the exchange interaction in bulk disordered ferromagnets. Let us introduce an exchange field

$$\tilde{H}_{\text{ex}} = \frac{1}{v} \int_v \tilde{H}_{\text{ex}} d\vec{r}. \quad (10)$$

averaged over a volume $v = L_0^3$, $l < L_{so} < L_0 < L_T$. Here $\tilde{H}_{\text{ex}}$ is the local exchange field acting on localized spins in ferromagnet. The correlation function of this field can be calculated with the help of the diagram Fig.2b.

$$\langle \tilde{H}_{\text{ex}}(\vec{r}) \tilde{H}_{\text{exj}}(\vec{r}') \rangle = \frac{I_p^2}{\mu^2 (p_F l)^2 (p_F L_0)^2} \frac{1}{f(|\vec{r} - \vec{r}'|)} \quad (11)$$

Here $f(r)$ is the function which is equal to unity at $r \ll L_0$, decays as $f(r) \sim (\frac{L_0}{r})^6$ at $L_0 < r < L_T$ and exponentially decays when $r > L_T$. The above-mentioned long range correlation between $\delta I_{ij}(\vec{R}_1, \vec{R}_2)$ and $\delta I_{kl}(\vec{R}_3, \vec{R}_4)$ manifests itself in the factor $(p_F L_0)^{-2}$ in Eq.11. An assumption about short range correlations would lead to an expression which is proportional to $(p_F L_0)^{-3}$.

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FIGURES

Fig.1. Schematic pictures of the ferromagnet-nonmagnet layered systems.

Fig.2 Diagrams for the correlation function $\langle \delta \tilde{I}_{ij} \delta \tilde{I}_{kl} \rangle$. 
