Adaptive Weight Vector Solving Based on the Summation of Partial Eigenbeams in STAP

Ziyi Gong*1,a, Wanlin Wang2, Yinguang He3
1China Academy of Space Technology (Xi’an) Xi’an, China
2China Academy of Space Technology (Xi’an) Xi’an, China
3China Academy of Space Technology (Xi’an) Xi’an, China
*a*zoey7zy@foxmail.com

Abstract. For the side-looking airborne or spaceborne early warning radar, Space-time adaptive processing (STAP) is a well-established technique. The adaptive weight vector can be written in the form of summation that equivalents to the inversion of covariance matrix. In order to reduce the computational complexity of the adaptive weight vector, this paper proposes a modified calculation method based on the weighted sum of partial eigenbeams according to the array model of the radar system. Both the loss of signal-to-interference and noise ratio (LSINR) and adapted patterns are used as performance metrics. Computer simulations verified the effectiveness of the method and its adaptability in traditional full-dimension, fixed-structure reduced dimension STAP.

1. Introduction
Adaptive array signal processing achieves the purpose of detecting useful signals and suppressing interference by processing signals in the space domain. Space-time adaptive processing (STAP) uses not only the spatial information formed by the array, but also the temporal information of pulses at different moments. Therefore, it is an important technique due to the remarkable ability of clutter suppression that facilitates the downstream signal processing tasks suffering from severe clutter such as multi-target detecting [1].

The full-dimension STAP structure was proposed by Brennan and Reed, that is, the STAP optimal processor, but it has strict conditions and a huge amount of calculation, so researchers are trying to find various reduced-dimension STAP algorithms. Among them, the non-data-dependent reduced-dimension STAP performs dimensionality reduction through the structure of the processor [2], usually by using prior knowledge of space-time clutter distribution, choosing different spatial beams and time-domain Doppler channels to construct the reduced-dimension covariance matrix, and then considering the practical situation, select a suitable reduced-dimension structure. This structure-fixed method which does not rely on the received data, has the advantages of simple and easy to implement.

Since most algorithms are based on the inversion of estimated covariance matrix [3], we represent the weight vector from the perspective of eigenbeams. In order to further simplify the calculation of the adaptive weight vector, this paper proposes an adaptive weight vector calculation method based on the partial sum of weighted eigenbeams.
2. Full-Dimension STAP

For a phased array radar with \( N \) array elements, the distance between the array elements is half a wavelength. Each array element receives \( M \) pulses in a coherent processing interval (CPI). The echo data received by the \( l \)-th range gate is expressed as the following \( MN \)-dimension vector

\[
X_l = [x_1^l, x_2^l, \ldots, x_N^l]^T
\]

called space time snapshot. The spatial steering vector and the temporal steering vector are expressed as

\[
s_s(\psi) = [1, \exp(j\psi), \ldots, \exp(j(N-1)\psi)]^T \\
s_\omega(\omega) = [1, \exp(j\omega), \ldots, \exp(j(M-1)\omega)]^T
\]

Where \( \psi \) and \( \omega \) respectively represent the normalized target spatial azimuth frequency and the normalized target Doppler frequency. Correspondingly, the space-time two-dimensional steering vector of the target is the Kronecker product of spatial and temporal steering vector:

\[
s(\psi, \omega) = s_s(\psi) \otimes s_\omega(\omega)
\]

2.1. Clutter power spectrum

Due to the movement of the radar platform, the echo with different arrival angles show different Doppler frequencies, that means the spatial azimuth frequency and the temporal Doppler frequency are coupled. For the antenna array carried by a Side-looking Array (SLA), there is the following relationship between the normalized Doppler frequency and the azimuth frequency of the ground stationary scatterer [4].

\[
\omega = \frac{2vT}{\lambda} \sin \psi
\]

Equation (5) shows that in the absence of ambiguity, the distribution of the received clutter power spectrum on the azimuth sine-Doppler frequency plane is an oblique line. Notice that in the presence of ambiguity, the distribution of the clutter power spectrum presents multiple parallel lines due to folding. The following two spectrum estimation methods are used to present the clutter power spectrum.

![Figure 1](image1.png)

Figure 1 Clutter spectra of SLA

Fig. 1(a) shows the spectrum estimation based on matched filtering is equivalent to Fourier Transform (FT). The clutter power spectrum obtained by this estimator has serious spurious response [5]. Fig. 1(b) shows the pseudo-spectrum estimated by minimum variance distortion response, also known as the Capon spectrum, has the characteristics of high resolution and no redundant sidelobes.

2.2. Optimal weight vector

The optimal weight vector in the traditional full-dimension STAP can be expressed as

\[
w_{\text{opt}} = R^{-1}s(\psi, \omega)
\]

Among them \( s(\psi, \omega) \) is the target space-time steering vector, and is the covariance matrix of noise plus interference (undesired signal). When calculating the weight vector in practical situations, because the covariance matrix is unknown [6], the estimated one needs to be obtained through training data that does not contain the target by a technique, such as Sample Matrix Inversion (SMI) [7]:

\[

\]
3. Reduced-Dimension STAP with Fixed Structure

Using a fixed-structure $MN \times K$ dimensionality reduction transformation matrix $T$, the received snapshot data and target steering vector of a certain distance gate are dimension-reduced.

$$X_r = T^H X$$  \hspace{1cm} (8)

$$s_r = T^H s$$  \hspace{1cm} (9)

From (8), the reduced-dimension covariance matrix is

$$R_r = E(X_rX_r^H) = T^H R T$$  \hspace{1cm} (10)

Accordingly, the reduced-dimension optimal weight is

$$w_r = R_r^{-1}s$$  \hspace{1cm} (11)

4. Eigenbeams of Covariance Matrix

For the covariance matrix $R$, the eigenbeam response refers to the two-dimensional response weighted by the eigenvector of $R$, and the eigenbeam response of the $k$-th eigenvector $e_k$ is \[8\]

$$P_k(\psi, \omega) = \left| e_k^H v(\psi, \omega) \right|^2$$  \hspace{1cm} (13)

According to (6), the optimal weight vector can be written as [9]

$$w_{opt} = R^{-1}s(\psi, \omega) = \sum_{k=1}^{MN} \frac{1}{\lambda_k} e_k^H e_k^H s(\psi, \omega)$$

$$= \frac{1}{\sigma_w^2} s(\psi, \omega) - \frac{1}{\sigma_w^2} s(\psi, \omega) + \sum_{k=1}^{MN} \frac{1}{\lambda_k} e_k^H e_k^H s(\psi, \omega)$$

$$= \frac{1}{\sigma_w^2} \left( s(\psi, \omega) - \sum_{k=1}^{MN} \frac{\lambda_k - \sigma_w^2}{\lambda_k} e_k^H e_k^H s(\psi, \omega) \right)$$  \hspace{1cm} (14)

Analyzing the optimal weight from the perspective of the eigenbeam, the response of the most processor is equivalent to the response of the space-time matched filter minus the sum of the weighted eigenbeam beamformer.

Fig. 2 shows the patterns of several eigenbeams. Fig. 3 (a)(b) and (c)(d) are beam patterns of direct summation and weighted summation of eigenbeams, which shows the sum of all eigenbeams is similar in appearance to the Capon spectrum, and the weighted sum of all eigenbeams is similar in appearance to the clutter FT spectrum. The sum of all eigenbeams appears to be a beam pointing in the direction of the clutter ridge. So by subtracting it, a notch can be formed to suppress the clutter. Part of the eigenbeam sum will be concentrated in the central area of the plane. From (14) the response of the two-dimensional adaptive processor is

$$C_\alpha(\psi, \omega) = \frac{\alpha}{\sigma_w^2} \left[ C_\alpha(\psi, \omega) - \sum_{k=1}^{MN} \frac{\lambda_k - \sigma_w^2}{\lambda_k} [e_k^H s(\psi, \omega)] Q_k(\psi, \omega) \right]$$  \hspace{1cm} (15)

Where $C_\alpha(\psi, \omega)$ is the quiescent beam response, that is, the response matched to the target when there is no external interference but only internal noise. $\lambda_k$ is the eigenvalue of $R$, and $Q_k(\psi, \omega)$ is the corresponding eigenbeam, which is the beam response formed with the eigenvector as the weight. The optimal two-dimensional beamformer is expressed by the weighted sum of the eigenbeams as
When only part of the eigenbeam is used to solve the adaptive weight vector, the effective inverse of the covariance matrix is equivalent to

$$ R_{\text{eff}}^{-1} = I - \sum_{i=1}^{k} \frac{\lambda_i - \lambda_{\text{min}}}{\lambda_i} e_i e_i^H $$

(17)
In practical situations, since the covariance matrix is unknown [6], the estimated eigenvalues $\hat{\lambda}_i$ and eigenvectors $\hat{e}_i$ are used instead of $\lambda_i$ and $e_i$ in the formula. Both the inversion of $R$ or the eigen-decomposition requires a lot of computation for full-dimension STAP. When solving eigenvalues and eigenvectors, subspace iterative methods [10] suitable for high-order symmetric matrices is a direct extension of the Power Method, by which the first few larger eigenvalues can be found; the reduced-order Power Method can be used to obtain the eigenvalues from large to small, that is, after obtaining the maximum eigenvalue $\lambda_1$ and its corresponding eigenvector, the compression method can be used to find the order matrix $R_1$ with the largest eigenvalue, so as to convert the second largest eigenvalue $\lambda_2$ into the largest eigenvalue of $R_1$, and proceed accordingly. Combining the weighted sum of some eigenbeams can significantly reduce the computational complexity of solving the full-dimension adaptive weight.

5. Partial Eigenbeams for Adaptive Weight Vector

Take the SLA with $N=18$ elements as an example, the distance between the elements is half the wavelength of the transmitted signal. The number of pulses in a CPI is $M=18$. Thus the degree of freedom (DOF) is $MN$. The platform speed $v=120$m/s, the height $H=6000$m, the pulse repetition frequency $f_r=300$Hz, and the antenna beam points in the direction vertical to the linear array. The target azimuth angle is $0^\circ$, and the Doppler frequency is 20 Hz. In order to compare the performance differences, the loss of signal-to-interference and noise ratio (LSINR) is defined as the ratio of the output signal-to-interference and noise ratio to the two-dimensional matched filter signal-to-noise ratio. On two cases 50 Monte Carlo experiments are performed, and the partial sum of weighted eigenbeams is used to solve the STAP weight vector. The corresponding LSINR and two-dimensional adapted pattern are obtained.

5.1. Full-Dimension Case

When the first $k$ large eigenbeams are used as the weighted sum, As is shown by Fig. 4(a), 12 eigenbeams is not enough to form a complete clutter notch in the two-dimensional adapted pattern. When the number of eigenbeams $k$ is increased to the rank of $R$, as shown in Fig. 4(b), the LSINR is close to optimal, and the LSINR with $5\times$DOF and $1\times$DOF samples is only 0.1838dB and 0.98dB lower than the optimal full-dimension case, respectively. The adapted patterns in Fig. 5 shows with $5\times$DOF samples, when $k$ is 163, it is sufficient to form a complete clutter notch.

As shown in Fig. 4(c)(d), when there are $5\times$DOF samples, increasing $k$, the LSINR decreases slightly until it is about 1.03dB lower than the optimal LSINR. While there are $1\times$DOF samples, increasing $k$, the LSINR drops significantly until it reaches 26dB lower than the optimal case. From Fig. 5(c) to (d), for sufficient samples, although LSINR is maintained, the sidelobe level of the response is increased. Because the smaller the number of SMI samples, the greater the error of small eigenvalues have, as Fig. 6 shows. Using small eigenbeams with rapidly increasing errors for the weighted sum will cause the LSINR to drop dramatically.
Fig. 7 shows that whether with sufficient samples or insufficient samples, there is an interval where the LSINR performance remains close to the optimal level. As long as the number of selected eigenbeams $k$ is in this range, satisfactory LSINR can be guaranteed. Therefore, regardless of the number of samples, it is not necessary to use all the eigenbeams to calculate the weight vector. Abandoning the estimated eigenbeams far from the theoretical ones and using only part of them is enough to form a complete clutter notch and a lower sidelobe of target response.
5.2. Reduced-Dimension Case

The simulation uses a fixed-structure reduced-dimension STAP, taking $K_s=5$ and $K_t=3$ to reduce the $MN$-dimension snapshot vector, and the system DOF becomes 15. There are two jammers located at the azimuth sine of -0.6428 and 0.4226 respectively. Use the Two-Step Nulling (TSN) method to suppress the jamming first [11], and then perform STAP to suppress the clutter; if the TSN method is not used, only the clutter is suppressed. Other parameter settings are the same as in section A.

When only the first 2 or 4 eigenbeams are used, it is already possible to form two vertical notches to suppress the jammings. When only a few large eigenbeams are used, for example $k=4$ in Fig. 8(a), with $5 \times$ DOF samples a complete clutter suppression notch cannot be formed, and the LSINR difference between the sufficient and insufficient samples is small. The one between TSN and no TSN is also small.

Increase $k$, as shown in Fig. 8(b)(c), the LSINR decreases with $1 \times$ DOF samples, and the LSINR without TSN decreases more significantly.

When 12 and 14 eigenbeams are used in Fig. 8(c)(d), although the adapted pattern forms a complete clutter notch, the sidelobe of the target response is increased. It can be seen from the two-dimensional adapted pattern that the use of partial eigenbeams and solving the dimension reduction weight vector requires a trade-off between the performance of the clutter notch and the target response main-sidelobe ratio. Fig. 9 also shows that with the addition of small eigenbeams, the difference in the impact of the number of samples on the LSINR performance is widened. It is because the error of small eigenvalues increases sharply.
6. Conclusion

Giving the appearances of direct sum and weighted sum of eigenbeams on a two-dimensional plane, the full-dimension STAP optimal weight vector is represented by eigenbeams and applied to SMI, and partial eigenbeams are used to solve the adaptive weight vector. The simulation results show that when calculating the weight vector, no matter the covariance matrix is full-dimensional or reduced-dimensional, only part of its eigenbeams are required to participate in the weighted sum to obtain satisfactory LSINR. Specifically, in the full-dimension case, if the sample number is large enough, only the first 11% to 15% of the eigenbeams are needed to achieve a near-optimal LSINR; in the reduced-dimension case with a fixed structure, the number of eigenbeams is much smaller than the full-dimension case. Hence forming the complete clutter notch requires a larger part of eigenbeams than the full-dimension case. In the presence of blanket jamming, although a few eigenbeams are not enough to form a complete clutter suppression notch, they can form a counterpart for jamming. As the number of eigenbeams increases, it is possible to provide clutter and jamming notches closer to the optimal, but the sidelobes of the target response increase and the LSINR performance decreases.

Figure 8 LSINR and adapted patterns with 5×DOF samples, k eigenbeams
(a) $k=4$, (b) $k=9$, (c) $k=12$, (d) $k=14$

Figure 9 Theoretical and estimated eigenspectra (reduced-dimension)

References

[1] Y. Zhou, X. Chen, Y. Li, L. Wang, B. Jiang, and D. Fang, "A fast STAP method using persymmetry covariance matrix estimation for clutter suppression in airborne MIMO radar," EURASIP Journal on Advances in Signal Processing, vol. 2019, no. 1, 12/01 2019.

[2] G. Joseph, Space-Time Adaptive Processing for Radar, 2 ed. Boston: Artech, 2014.

[3] E. Makhoul, S. V. Baumgartner, M. Jäger, and A. Broquetas, "Multichannel SAR-GMTI in Maritime Scenarios With F-SAR and TerraSAR-X Sensors," IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing, vol. 8, no. 11, pp. 5052-5067, 2015.

[4] J. Ward, "Space-time adaptive processing for airborne radar," in IEE Colloquium on Space-Time Adaptive Processing, London, 1998.

[5] M. Dimitris G, I. Vinay K, and K. Stephen M, Statistical and adaptive signal processing: spectral estimation, signal modeling, adaptive filtering. Norwood: Artech, 2003.
[6] W. Zhang, R. An, N. He, Z. He, and H. Li, "Reduced Dimension STAP Based on Sparse Recovery in Heterogeneous Clutter Environments," IEEE Transactions on Aerospace and Electronic Systems, vol. 56, no. 1, pp. 785-795, 2020.

[7] H. Xu, Z. Yang, S. He, M. Tian, G. Liao, and Y. Sun, "A generalized sample weighting method in heterogeneous environment for space-time adaptive processing," Digital Signal Processing, vol. 72, pp. 147-159, 2018/01/01/ 2018.

[8] S. Burintramart, T. K. Sarkar, Y. Zhang, and M. C. Wicks, "Performance comparison between statistical-based and direct data domain STAPs," Digital Signal Processing, vol. 17, no. 4, pp. 737-755, 2007/07/01/ 2007.

[9] X. Wang, E. Aboutanios, and M. G. Amin, "Slow radar target detection in heterogeneous clutter using thinned space-time adaptive processing," IET Radar, Sonar & Navigation, vol. 10, no. 4, pp. 726-734, 2016.

[10] T. Zhang, G. H. Golub, and K. H. Law, "Subspace iterative methods for eigenvalue problems," Linear Algebra and its Applications, vol. 294, no. 1, pp. 239-258, 1999/06/15/ 1999.

[11] M. Witter, "Analysis of two-step nulling of RF interference and ground clutter with PAMIR," in EUSAR 2012; 9th European Conference on Synthetic Aperture Radar, 2012, pp. 259-262.