Effects of Wall Reflection on the Per-Antenna Power Distribution of ZF-Precoded ULA for Indoor mmWave MU-MIMO Transmissions

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Abstract—Indoor access points (APs) with large-scale antenna arrays are commonly deployed in the vicinity of a wall, where wall reflection (WR) affects the indoor electromagnetic (EM) wave propagation. In this letter, we investigate the effects of WR on the per-antenna power distribution of a transmit uniform linear array (ULA) adopting a zero-forcing (ZF) precoder. A new channel model is constructed to characterise the impact of both the line-of-sight (LOS) path and the WR path on indoor millimetre wave (mmWave) multi-user (MU) multiple-input multiple-output (MIMO) downlink transmissions. Specifically for the dual user equipment (UE) scenario, the ZF precoding matrix is analytically obtained and verified through simulations. The effects of WR on the per-antenna power distribution of the ZF-precoded ULA, in terms of the normalised power distribution and maximum power ratio (MPR), are evaluated through the comparisons between our proposed channel model and the pure LOS channel model. Our analytical and numerical results reveal the impact of AP configurations (the number of antennas and the AP-wall distance), multi-user spatial distribution (the angle of departure (AoD) and length of the LOS path for each user), and wall parameters (permittivity and thickness) on the power distribution across the ZF-precoded ULA. It is found that the effects of WR will exacerbate the uneven power distribution across the ZF-precoded ULA.

Index Terms—Wall reflection, power distribution, ZF precoding, ULA, indoor, mmWave, MU-MIMO, downlink.

I. INTRODUCTION

The combination of multiple-input multiple-output (MIMO) and millimetre wave (mmWave) technologies facilitates massive indoor high-rate wireless applications for future wireless networks by exploiting spatial multiplexing gains even in limited scattering environment [1]–[5]. While the deployment of indoor access point(s) (AP) would typically be optimised to maximise the throughput and/or coverage [6], customers may prefer to deploy the APs in positions that will not lead to inconveniencing usages of a room. A popular solution is to integrate the AP with an interior wall of the building [7]. In this sense, the interactions between the indoor electromagnetic (EM) wave propagation and the interior wall should not be neglected in the deployment of indoor APs [8], [9].

The EM wave bounced off an interior wall would have experienced multiple internal reflections, whose amplitudes and phases are totally changed [10]. The wall reflection (WR) is characterised by the reflection coefficient, which depends on the EM wave’s polarisation, incidence angle, and the wall material’s relative permittivity and thickness [11]. Measurement campaigns in the 28 GHz band show that typical indoor building materials such as clear glass and dry wall usually have strong reflectivity and low penetration [12]. Hence, it is necessary to consider the effects of WR when evaluating indoor wireless performance.

In downlink multi-user (MU) MIMO scenarios, zero-forcing (ZF) precoding has been widely used to suppress inter-user interference. However, the power distribution across a ZF-precoded transmit antenna array has not been sufficiently studied. The power assigned to each transmit antenna affects the efficiency of its corresponding RF power amplifier, and thus influences the power consumption of the RF chains of the AP [13]. It has been shown that the uniform power excitation over all transmit antennas would allow the RF power amplifiers to work with maximum efficiency [14], [15], while significant power variations across different antennas would reduce this efficiency and cause a huge waste of energy. That is why per-antenna power constraints, instead of sum power constraints, need to be well considered for practical precoder design [16].

In this letter, we study the impact of WR on the per-antenna power distribution across the precoded antenna array for indoor mmWave MU-MIMO downlink transmissions. The transmitter is assumed to be equipped with a ZF-precoded uniform linear array (ULA) for analytical tractability. The contribution of this letter is summarised as follows: 1) To capture the effects of WR on the power distribution across the precoded ULA, a new channel model characterising both the line-of-sight (LOS) path and the WR path is proposed for indoor MU-MIMO downlink scenarios, 2) we evaluate the effects of WR on the power distribution across the ZF-precoded ULA that serves multiple user equipment (UEs) simultaneously through the comparisons between our proposed channel model and the pure LOS channel model in [15]. The unevenness in the power distribution across the ZF-precoded ULA becomes more significant under the effects of WR, 3) we analytically derive the entries of the ZF precoding matrix for a dual-UE case, the accuracy of which is verified by simulation results, and 4) our numerical results give insights into how the configuration of transmit antenna array, the spatial distribution of multiple users, the EM and physical properties of the wall, and the mutual coupling influence the power distribution across the ZF-precoded ULA.
II. SYSTEM MODEL

In this work, we consider an indoor MU-MIMO downlink narrowband system in a LOS environment, as shown in Fig. 1(a). The AP, outfitted with a ULA of $M$ directional antennas, is deployed in parallel with a wall with a distance of $D_1$ m from the wall. The impact of the directional radiation pattern of an AP antenna on the LOS path and the WR path is illustrated in Fig. 1(b). All the $K$ single-antenna UEs locate in the far-field of the AP’s ULA. Due to the random orientation of UEs, the antenna of each UE is assumed as an omnidirectional antenna for analytical tractability. The transmit ULA is assumed to be transparent, i.e., it will not block the WRs to the UEs.

In order to study the effects of WR on the per-antenna power distribution across the ULA, we incorporate the WR path to characterise the EM wave propagation inside a wall. The signal along the WR path experiences multiple reflections inside the wall closest to the AP, named as the considered wall hereafter. Using Friis’ formula, the wireless link between the $m$th antenna and the $k$th UE is deterministically modelled as

$$h_{k,m} \propto G d_{k,\text{LOS}}^{-1} \exp\left(-\beta (d_{k,\text{LOS}} + (m - M/2) D \cos \theta_k)\right) + g \Gamma_k d_{k,\text{WR}}^{-1} \exp\left(-\beta (d_{k,\text{WR}} + (m - M/2) D \cos \varphi_k)\right),$$

where $\propto$ denotes being proportional to, $m \in \{1, 2, \ldots, M\}$, $k \in \{1, 2, \ldots, K\}$, $G$ and $g$ denote the main-lobe gain and side-lobe gain of each antenna, respectively, subject to $G^2 + g^2 = 2$ according to energy conservation law, $d_{k,\text{LOS}}$ and $d_{k,\text{WR}}$ denote the length of the LOS path and the WR path of the $k$th UE, respectively, $\theta_k$ and $\varphi_k$ denote the angle of departure (AOD) of the LOS path and the WR path of the $k$th UE, respectively, $\beta = 2\pi/\lambda$ denotes the wave number with $\lambda$ being the wavelength, $D$ denotes the inter-antenna spacing, and $\Gamma_k$ denotes the $k$th UE’s equivalent reflection coefficient of multiple internal reflections along the WR path and is defined in the following.

Supposing that the considered wall is a homogenous dielectric reflector, the multiple internal reflections inside it are strongly affected by the first-order reflection. In this letter, we mainly focus on the transverse electric (TE) polarisation of the incident EM waves. The proposed approach can be directly applied for the transverse magnetic (TM) polarisation. The first-order reflection coefficient for UE $k$ is given by $[11]$

$$\bar{\Gamma}_k = \frac{\cos \alpha_k - \sqrt{\varepsilon - \sin^2 \alpha_k}}{\cos \alpha_k + \sqrt{\varepsilon - \sin^2 \alpha_k}}$$

with $\alpha_k = (\theta_k - \varphi_k)$ denoting the equivalent incident angle under plane wave assumption and $\varepsilon$ denoting the relative permittivity of the considered wall. Given the thickness of the considered wall as $\zeta$, the equivalent coefficient of multiple internal reflections for UE $k$ is given by

$$\Gamma_k = \frac{1 - \exp(-j2\tau_k)}{1 - \Gamma_k^2 \exp(-j2\tau_k)} \bar{\Gamma}_k,$$

where $\tau_k = \frac{2\pi}{\lambda} \sqrt{\varepsilon - \sin^2 \alpha_k}$.

After precoding at the ULA and propagating through a complex flat-fading channel, the received signals at the UE sides are given by

$$y = HCWs + n,$$

where $y, s, n \in \mathbb{C}^{K \times 1}$ denote the received signal vector, data symbol vector and additive white Gaussian noise vector, respectively, $H \in \mathbb{C}^{M \times K}$ denotes the channel matrix whose elements are given in (1), $W \in \mathbb{C}^{M \times K}$ denotes the precoding matrix, and $C \in \mathbb{C}^{M \times M}$ denotes the mutual coupling matrix whose elements reveal how the radiated power of an antenna is affected by its neighbouring antennas $[17]$.

In this letter, we employ the following empirical model of mutual coupling $[18]$

$$C_{p,q} = \exp\left(-\frac{2d_{p,q}}{\lambda} (\gamma + j\pi)\right), p \neq q \in \{1, 2, \ldots, M\},$$

$$C_{p,p} = 1 - \frac{1}{M} \sum_{p=1}^{M} \sum_{q=1,q\neq p}^{M} C_{p,q},$$

where $C_{p,q}$ and $d_{p,q}$, respectively, denote the mutual coupling coefficient and distance between the $p$th antenna and $q$th antenna, and $\gamma$ is a positive parameter controlling the mutual coupling level.

III. EFFECTS OF WALL REFLECTION ON THE POWER DISTRIBUTION OF ZF-PRECODED ULA

In this section, we investigate how the power is distributed among the $M$ antennas of a ZF-precoded ULA serving $K$ UEs simultaneously. We first apply two metrics to evaluate the power distribution across the transmit precoded ULA for an arbitrary $K$ and then derive the analytical expression of the ZF precoding matrix for a dual-UE case, i.e. $K = 2$.

A. Metrics for Quantifying Power Variation

Considering uncorrelated and unit variance data symbol sequences $s = \sum_{k=1}^{K} w_k s_k$, the transmitted power of the AP’s ULA is given by $E(\|x\|^2) = \sum_{k=1}^{K} \|w_k\|^2$, where $w_k$ and $s_k$ are the $M$-by-$1$ precoding vector and the intended symbol for the $k$th UE, respectively, $E(\cdot)$ denotes the expectation, and $\| \cdot \|_2$ denotes the F-norm. Since the transmitted power is the summation of the F-norm of each UE’s precoding vector, the power distribution across the AP’s ULA can be evaluated based on each UE’s precoding vector. Following $[15]$, we apply the two metrics below to evaluate the power imbalance among the transmit antennas.

The normalised power allocated to the $m$th antenna to serve the $k$th UE is defined as

$$P_{\text{norm}}_{k,m} = \frac{|w_{k,m}|^2}{\|w_k\|^2} = \frac{|w_{k,m}|^2}{\sum_{m=1}^{M} |w_{k,m}|^2},$$

where $w_{k,m}$ is the $m$th element of $w_k$. 

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**Fig. 1.** Indoor MU-MIMO system model for a LOS environment.
Maximum power ratio (MPR) describing the power variation across the $M$ antennas of a transmit ULA is defined as

$$P_k = \frac{\max(P_k^{\text{norm}})}{\min(P_k^{\text{norm}})},$$

where $P_k^{\text{norm}} = (P_{k,1}^{\text{norm}}, \ldots, P_{k,M}^{\text{norm}})$ gives the normalised power levels allocated to the $M$ antennas for serving UE $k$.

### B. Analytical ZF Preceding Matrix for a Dual-UE Case

The array factors (AFs) of the ULA are introduced as

$$\text{AF}_l = \frac{\sin(M\beta D\delta_l/2)}{\sin(\beta D\delta_l/2)},$$

$$\text{AF}_{lk,n} = \frac{\sin(M\beta D\delta_{lk,n}/2)}{\sin(\beta D\delta_{lk,n}/2)},$$

where $k \neq l \in \{1, 2\}$ denotes the UE index, $\delta_l = \cos \theta_l - \cos \phi_l$, $n \in \{3, 4, 5, 6\}$, $\delta_{lk,3} = \cos \theta_k - \cos \theta_l$, $\delta_{lk,4} = \cos \phi_k - \cos \theta_l$, $\delta_{lk,5} = \cos \theta_k - \cos \phi_l$, and $\delta_{lk,6} = \cos \phi_k - \cos \phi_l$. Note that $\text{AF}_{lk,n} = \text{AF}_{kl,n}^*$, in which $\cdot^*$ denotes the complex conjugate.

Given (4), the effective downlink channel model is given by $\hat{H} = HC$. Even though $C$ in (5) is a Toeplitz symmetrical matrix, the closed-form expression of $W$ is still intractable. Fortunately, as stated in [19], the mutual coupling is non-negligible only when $D$ is below 0.2\lambda. In this letter, $D$ is assumed to be half wavelength to guarantee a low spatial correlation [20]. As such, both $C \approx I$ and $\hat{H} \approx H$ are satisfied, especially when $\gamma \geq 4$.

When $\gamma \geq 4$, the ZF precoding matrix is derived as [21]

$$W = H^H (H^H H^H)^{-1}H^H \left[ \begin{array}{c} H^H \end{array} \right]_{12} \left[ \begin{array}{c} H^H \end{array} \right]_{11},$$

where $\cdot^H$ represents the complex conjugate transpose of a matrix. The four elements of matrix $H^H$ are given by (11)-(14) at the bottom of the next page, where

$$F_l = \exp(j\beta(d_{1,\text{LOS}} - d_{1,\text{WR}})) \cdot \text{AF}_l, l \in \{1, 2\},$$

$$F_{21,3} = \exp(j\beta(d_{1,\text{LOS}} - d_{3,\text{LOS}})) \cdot \text{AF}_{21,3},$$

$$F_{21,4} = \exp(j\beta(d_{1,\text{WR}} - d_{3,\text{LOS}})) \cdot \text{AF}_{21,4},$$

$$F_{21,5} = \exp(j\beta(d_{1,\text{LOS}} - d_{3,\text{WR}})) \cdot \text{AF}_{21,5},$$

$$F_{21,6} = \exp(j\beta(d_{1,\text{WR}} - d_{3,\text{WR}})) \cdot \text{AF}_{21,6}.$$
four-UE case for the number of antennas being 32. Comparing the results obtained using the two channel models, we observe that the effects of WR exacerbates the power variation up to 6 dB across the ZF-precoded ULA, revealing that some antennas are allocated much more power than the others.

Fig. 2(c) and Fig. 2(d) show the impact of the number of antennas on the MPR across the ZF-precoded ULA with or without considering WR for the dual-UE case and the four-UE case, respectively. We can see that the MPRs obtained by our proposed channel model are larger than that obtained by the pure LOS channel model, indicating that the effects of WR exacerbates the uneven power distribution across the ZF-precoded ULA. For the dual-UE case under the pure LOS channel, MPR becomes smaller gradually when the number of antennas increases. However, under our proposed channel, increasing the number of antennas would exacerbate the non-uniform power distribution over the ULA.

Fig. 3(a) illustrates the normalised power distribution of the ZF-precoded ULA for UE 2 under different AP-wall distances for a dual-UE case, where the positions of UE 1 and UE 2 are assumed to be symmetric with respect to the midpoint of the ULA for simplicity. The result of UE 1 is symmetrical to that of UE 2 with respect to the midpoint of ULA. As shown in Fig. 3(a), the variation in the power distribution across the ULA changes greatly with the AP-wall distance, which reveals that the AP-wall distance has to be taken into account carefully.

Fig. 3(b) and Fig. 3(c) present the normalised power distribution of the ZF-precoded ULA for different values of the length (from 1 to 9 m) and the AoD (from 0 to $\pi$) of the LOS path of one user in the dual-UE case, respectively. Fig. 3(b) shows that, as the length of the LOS path increases, the normalised power for serving the UE varies slower within a smaller dynamic range, which is also observed in Fig. 2(a).

In Fig. 3(c), a significant variation appears in the power distribution across the ZF-precoded ULA when $\theta_2$ approaches $\pi/4$. This is ascribed to the location of UE 2 being very close to UE 1 with $\theta_1 = \pi/4$. In this case, the ZF precoder needs to allocate highly different power levels across the ULA to distinguish the two UEs.

From Fig. 3(b)-(c) and Fig. 2(a)-(d), we can see that the power distribution is not sensitive to the length (UE-AP distance) or the AoD of the LOS path under the pure LOS channel model, while under our proposed channel model, different multi-user spatial distributions result in different dynamic ranges of the power distribution over the ULA, which is more evident when the UE is closer to the wall.

In Fig. 4(a)-(c), the two UEs are assumed to be symmetrically located with respect to the midpoint of the ULA for simplicity and only the result of UE 2 is plotted. The result of UE 1 is symmetrical to that of UE 2 with respect to the midpoint of ULA.

Fig. 4(a) and Fig. 4(b) present the normalised power distribution across the ZF-precoded ULA for different values of the real part of wall permittivity (from 1 to 9) and wall thickness (from 0.1 to 0.3 m) for the dual-UE case, respectively. We can see from Fig. 4(a) that the non-uniform power distribution across the ZF-precoded ULA will be exaggerated with the increase of the real part of wall permittivity. Differences as large as 8 dB are observed between the ends of the ULA. In Fig. 4(b), the power distribution does not change with the wall thickness. This is because the wavelength at 28 GHz, i.e. 1 cm, is not comparable to the typical wall thickness.

Fig. 4(c) plots the normalised power distribution across the ZF-precoded ULA for different values of $\gamma$. We can see that the power distribution across the ZF-precoded ULA changes slightly with $\gamma$ ranging from 1.5 to 4.5, corresponding to the mutual coupling level between two adjacent antennas from

\[
\begin{align*}
[H^H]_{11} &= M(G^2d^{-2}_{1,\text{LOS}} + g^2d^{-2}_{1,\text{WR}}|\Gamma_1|^2) + 2Ggd^{-1}_{1,\text{WR}}d^{-1}_{1,\text{LOS}}\text{Re}(\Gamma_1 F_1) \quad (11) \\
[H^H]_{22} &= M(G^2d^{-2}_{2,\text{LOS}} + g^2d^{-2}_{2,\text{WR}}|\Gamma_2|^2) + 2Ggd^{-1}_{2,\text{WR}}d^{-1}_{2,\text{LOS}}\text{Re}(\Gamma_2 F_2) \quad (12) \\
[H^H]_{12} &= Gd^{-1}_{2,\text{LOS}}d^{-1}_{1,\text{LOS}}F_{1,23} + Ggd^{-1}_{2,\text{LOS}}d^{-1}_{1,\text{WR}}\Gamma_1 F_{1,24} + Ggd^{-1}_{1,\text{LOS}}d^{-1}_{2,\text{WR}}\Gamma_2 F_{2,12,5} + g^2d^{-1}_{1,\text{WR}}d^{-1}_{2,\text{WR}}\Gamma_1 \Gamma_2 F_{12,6} \quad (13) \\
[H^H]_{21} &= Gd^{-1}_{2,\text{LOS}}d^{-1}_{1,\text{LOS}}F_{2,21,3} + Ggd^{-1}_{2,\text{LOS}}d^{-1}_{1,\text{WR}}\Gamma_1 F_{2,21,4} + Ggd^{-1}_{1,\text{LOS}}d^{-1}_{2,\text{WR}}\Gamma_2 F_{2,21,5} + g^2d^{-1}_{1,\text{WR}}d^{-1}_{2,\text{WR}}\Gamma_1 \Gamma_2 F_{21,6} \quad (14)
\end{align*}
\]
γ -13.03 dB to -39.09 dB [18]. However, for a very small value of γ, e.g., γ = 0.5 corresponding to the mutual coupling level between two adjacent antennas of -4.34 dB, the mutual coupling has to be considered while analysing the power distribution over the AP’s ULA.

Fig. 4. The impact of wall permittivity, thickness, and mutual coupling on the normalised power distribution across the ZF-precoded ULA, where d1,LOS = d2,LOS = 2 m, θ1 = π/2, θ2 = 2π/5. M = 16, D1 = 0.05 m.

V. CONCLUSION AND FUTURE WORKS

In this letter, we have built an indoor MU-MIMO downlink channel model that involves both the LOS path and the WR path. The expression of the ZF precoding matrix for the dual-UE case was analytically derived. For the scenario where the AP is deployed close to a wall, we compared the power distribution across the ULA under our channel model and that under the pure LOS channel model, in terms of the normalised power distribution and MPR. Numerical results have shown that the power distribution becomes more uneven across the ZF-precoded ULA under the effects of WR. Even though the proposed numerical approach has shed light on how WR impacts the per-antenna power distribution, a comprehensive measurement campaign is still required to verify the results of this work.

The following research will be considered in future works: 1) Experimental validation of the proposed approach; 2) Consideration of actual antenna patterns; 3) The impact of an arbitrary arrangement of the AP; 4) The effects of WR on the power distribution across an MMSE/MRT-precoded massive MIMO antenna array; 5) Indoor rich scattering environment; and 6) MmWave wideband wireless system.

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