Nonlinear analysis of axial vibration of five-axis machine tool worktable with double turntable

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Received: 12 August 2021 / Accepted: 11 February 2022 / Published online: 12 March 2022 © The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2022

Abstract
Studies show that the applied torque and load on the worktable of a dual-turntable five-axis machine tool continuously change during the machining process. These variations generate vibration in the worktable along the axial direction, thereby reducing the machining accuracy. In order to improve the machining accuracy of the machine tool and the dynamic characteristics of the worktable, this paper first established the nonlinear dynamic equation of the axial vibration of the machine tool table when the worktable is subjected to torque, swing moment, and load according to the elasticity of the plate and shell, then according to Galerkin, the truncation method solves the dynamic displacement in the axial direction of the worktable, as well as the bifurcation diagram, displacement waveform diagram, phase plane trajectory, and Poincare method solvon diagram of the processing system. Finally, the influence of different parameters, including torque, swing moment, and load on axial vibration of the worktable was analyzed during the tool operation. The obtained results reveal that as the external load changes, the corresponding axial vibration of the worktable is mainly in the state of large periodic motion, where the maximum vibration amplitude reaches 0.09 mm, and the external load has the greatest influence on the axial vibration of the worktable. Moreover, the axial vibration of the worktable is affected by the swing moment. More specifically, in the chaotic state of a small period and small area, the maximum vibration amplitude reaches 0.03 mm, and the swing moment has a negligible effect on the axial vibration of the worktable. The influence of torque and load on the vibration characteristics of the five-axis machine tool table during machining was studied through experiments. The obtained results demonstrate that nonlinear analysis of the table axial vibration of the five-axis machine tool with dual turntables is an effective way to control the stability of the worktable during the processing of the workpiece.

Keywords Five-axis machine tool · Worktable · Axial direction · Nonlinear vibration · Dynamic characteristics

Abbreviations
CNC Numerical control machine
$N_r$ Radial force (N)
$N_\theta$ Circumferential force (N)
$M_r$ Radial bending moment (N·m)
$M_\theta$ Circumferential bending moment (N·m)
$Q_r$ Radial shear force (N)
$q(x, t)$ Unit load produced (N·m$^{-2}$)
c The damping coefficient (N·s·m$^{-1}$)
$\rho$ The density of the worktable material (kg·m$^{-3}$)
h The thickness (mm)
w The displacement of the worktable along the Z-direction (mm)
r The radial length (mm)
u The radial displacement of the worktable (mm)
$\varepsilon_r$ Radial strain components (mm)
$\varepsilon_\theta$ Circumferential strain components (mm)
$E$ Young’s modulus (GPa)
v Poisson’s ratio
$c$ The sum of the inertial force
$D(r)$ The bending stiffness
$\rho h \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} - q(x, t)
\frac{1}{r} \left[ \frac{\partial}{\partial r} \left( r N_r \frac{\partial w}{\partial r} \right) \right]
\frac{\partial Q_r}{\partial r} + \frac{Q_r}{r} - q(x, t)
D(r) = \frac{Eh^3}{12(1-\nu^2)}$
1 Introduction

CNC machining equipment has been widely applied in numerous industries such as energy, transportation, heavy machinery, aerospace, ship manufacturing, and defense. Indeed, it can be considered as an indicator for the manufacturing industry in each country. Recently, many investigations have been conducted on machine tool dynamics. Based on the concepts of the machine tool in the generalized space and stiffness field, Liu and Zhao [1] studied the dynamic characteristics of the machine tool in the generalized space and provided a theoretical basis for optimizing the machining posture of machine tools with more than four axes. Moreover, Mao et al. [2] proposed a high-precision machine tool parameter identification method, which uses the dynamic test data of the entire structure including the node to identify parameters of the dynamic model of the node. Yongsheng and Wentong [3] carried out finite element modeling and modal analysis on the spindle of a heavy-duty machine tool under multiple constraints. They provided a theoretical basis to model the entire machine tool. Luo and Chen [4] proposed a rigid-flexible coupling model of a five-axis CNC system and then studied the influence of the beam structure on the dynamic characteristics of the machine. Moreover, Zhu and Chen [5] proposed a new method to analyze the dynamic characteristics of machine tools based on the unit structure and significantly improved the model through this method. Deng et al. [6] performed dynamic analyses and studied the dynamic response of the machine tool in the generalized space. In this regard, the dynamic flexibility of the spindle front end along the x-, y-, and z-directions within the focus frequency range was used to characterize the overall dynamic characteristics of the machine tool. Baumann et al. [7] analyzed the dynamics of the machine tool based on the machine configuration and processing path and significantly improved the surface quality of the workpiece during the processing. Yin et al. [8] proposed an improved method to identify the dynamic stiffness of nodes and established a machine tool dynamic model considering the dynamic stiffness. Liu (in China) and Altintas (in Canada) [9] performed an improved transfer function-based operational modal analysis and proposed a new model assuming that modal shapes of the spindle components are consistent to monitor variations of the natural frequency and damping ratio of the spindle. Huynh (in Canada) and Altintas (in Canada) [10] proposed a systematic multi-body dynamics modeling of machine tools, considering different structural dynamic characteristics of the machine tool when changing its position in the workspace. With the continuous improvement of machine tool modeling methods, the accuracy of mathematical models has constantly improved.

Different aspects should be considered in the dynamics research of ultra-high-precision machine tools. In this regard, numerous investigations have been carried out on the machine tool worktable. Zhao et al. [11] developed a numerical model to analyze the influence of the guideway profile error on the motion accuracy of the hydrostatic worktable. Then, the reaction force of the preloaded thrust ball ring and the hydrostatic circular oil cushion was obtained from the Reynolds equation of the lubricating film. The motion equation of the hydrostatic worktable was deduced, and error contours of the two guide rails were calculated. The obtained results showed that the range of the error contour, speed, and the worktable offset load affect the motion accuracy of the hydrostatic worktable. Liu et al. [12] numerically studied the flow, bearing, and loading capacity of the cycloid hydrostatic oil chamber under different rotational speeds and different boundary conditions. Furthermore, Iwasaki et al. [13] proposed a new method to perform high-precision control of the machine tool table transmission system. The proposed method has a significant performance improvement in the precise contour error control. Reviewing the literature indicates that most investigations on machine tool turntables are focused on large hydrostatic turntables, while few studies have been carried out on small ultra-precision machine tool turntables.

Rotating mechanisms with nonlinear factors are prone to nonlinear vibrations, which adversely affect their performance and service life. Moreover, these mechanisms are closely related to the vibration characteristics and operational stability of the rotor system. Inayat-Hussain and Bifurcations [14] performed numerical simulations and studied the influence of numerous parameters, including gravity, mass ratio, and stiffness ratio on the bifurcation response of an elastic rotor installed in a squeeze film damper with a fixed spring. Qin et al. [15] studied the nonlinear response and bifurcation of a squeeze film damper rotor on an elastic basis and derived the equation of motion. Then a method was proposed to integrate stable solutions. Li et al. [16] established the dynamic model of the rotor blade system. To this end, the axis was modeled as a rotating beam, in which the gyro effect was considered, while the shear deformation was ignored. Moreover, Li et al. [17] established a new dynamic model to analyze the dynamic behavior of the rotor system under the coupling action of nonlinear oil film force, nonlinear sealing force, and disc mass eccentricity. Yang and Zhang [18] studied the nonlinear vibration of an axially moving beam. In this regard, the Galerkin method was applied to truncate the controlled partial differential equations into a set of coupled nonlinear ordinary differential equations. Khadem et al. [19] studied the primary resonance of a simply supported stretched rotating shaft with large amplitude and applied multi-scale methods to complex forms of discrete equations and partial differential...
equations of motion. Samantaray et al. [20] derived the stability condition of the rotor shaft with polynomial nonlinear internal damping. Recently, Luo et al. [21] established a model of a cantilever combined support rotor to investigate the nonlinear vibration phenomena (Pimenov et al. [22, 23]). The mathematical model of elastic displacement in plane milling process is established, and the effects of machining parameters and tool wear on the total elastic displacement of the process system are studied. Accordingly, a theoretical basis was provided to study the nonlinear vibration of the machine tool worktable.

The performed literature survey indicates that nonlinear vibration characteristics and stability of the entire machine tool and the rotating system have been studied comprehensively. However, the lack of nonlinear analysis of the axial vibration of the machine tool table results in non-effective methods to control the table stability. In the present article, an industrial machine tool (VMC-C50, China) was considered as the research object. Dynamic and three-dimensional model of the machine tool table was established first. To this end, table torque, swing moment of the table, and the applied load on the table were considered. Then the Galerkin truncation method was applied to obtain the dynamic displacement in the axial direction of the table, bifurcation diagram, displacement waveform diagram, phase plane trajectory, and Poincaré displacement in the ax of the processing system. Finally, the table torque during the processing was analyzed. It was found that variations of the swing moment and load affect axial vibration of the worktable. This is especially more pronounced in vibration of the double turntable five-axis machine tool where the applied torque, swing moment, and load significantly affect the machine tool worktable, which may result in the axial vibration. Chaotic bifurcation phenomenon of the machine tool is expected to provide technical support for improving the stability of the machine tool table, thereby improving the machining accuracy.

2 Analysis of axial vibration parameters of double rotary machine tool

When the five-axis machine tool operates, the worktable is subjected to the workpiece load and the support load, rotating moment, and swing moment of the bearing. In the present study, a machine tool worktable (VMC-C50, Tuopu, Country) was considered as the research object to model and analyze the nonlinear dynamics of the machine tool worktable.

The circular worktable is regarded as a cylindrical body to analyze the applied forces on the machine tool worktable. Figure 1 indicates that the machine tool worktable is subjected to variable pressure q and the turning moments $T_1$ and $T_2$ rotating about the X- and Z-axes, respectively.

3 Nonlinear dynamic modeling of the axial vibration of the five-axis machine tool worktable

The external force on the machine tool worktable is coupled with the internal stress generated by the worktable. Based on the plate and shell theory, the worktable is split
into infinite units. The force analysis of the units is shown in Fig. 2. It is observed that the unit is subjected to the radial force $N_r$, circumferential force $N_\theta$, radial bending moment $M_r$, circumferential bending moment $M_\theta$, and radial shear force $Q_r$. The angle between the workbench and the rotary shaft is shown in Fig. 3.

Moreover, $q(x, t)$ is the unit load produced by the workpiece with a mass $m$ on the worktable. Then the balance equation of the worktable was established, where $c$ is the damping coefficient, $\rho$ is the density of the worktable material, and $h$ is the thickness. The sum of the inertial force $\rho h \frac{\partial^2 w}{\partial t^2}$, the damping force $c \frac{\partial w}{\partial t}$, and the elastic restoring force $\frac{\partial Q_t}{\partial r} + \frac{Q_t}{r} - \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( r N_\theta \frac{\partial w}{\partial r} \right) \right]$ on the worktable is combined with the external load $-q(x, t)$ to reach the following expression for the equilibrium state:

$$\rho h \frac{\partial^2 w}{\partial t^2} + c \frac{\partial w}{\partial t} + \frac{\partial Q_t}{\partial r} + \frac{Q_t}{r} - \frac{1}{r} \left[ \frac{\partial}{\partial r} \left( r N_\theta \frac{\partial w}{\partial r} \right) \right] = -q(x, t) \quad (1)$$

According to the von Carmen displacement equation, the circumferential and radial displacement and strain equations of the circular table are in the form below:

$$\varepsilon_r = \frac{\partial u}{\partial r} + \frac{1}{2} \left( \frac{\partial w}{\partial r} \right)^2$$

$$\varepsilon_\theta = \frac{u}{r} \quad (3)$$

When solving a plane problem by stress, it is necessary to derive a supplementary equation reflecting stress originating from geometric and physical conditions. Therefore, the compatibility equation of strain and displacement is established as a supplementary equation for solving the stress function. Based on the displacement strain Eqs. (2) and (3), the compatibility equation can be expressed as the following:

$$\frac{\partial (r \varepsilon_\theta)}{\partial r} + \frac{1}{2} \left( \frac{\partial w}{\partial r} \right)^2 - \varepsilon_r = 0 \quad (4)$$

where $w$ is the displacement of the worktable along the $Z$-direction, $r$ is the radial length, $u$ is the radial displacement of the worktable, and $\varepsilon_r$ and $\varepsilon_\theta$ are the radial and circumferential strain components, respectively. Moreover, $E$ and $\nu$ denote Young’s modulus and Poisson’s ratio, respectively. Similarly, the balanced equation of the moment of the circular worktable can be expressed as the following:

$$\frac{\partial M_r}{\partial r} + \frac{M_r - M_\theta}{r} + Q_t = 0 \quad (5)$$
where \( M_r = -D(r) \left( \frac{\partial^2 w}{\partial r^2} + \frac{v}{r} \frac{\partial w}{\partial r} \right) - M_1 \) and \( M_\theta = -D(r) \frac{\partial N_\theta}{\partial r} + \frac{N_r - N_\theta}{r} = 0 \) \( (9) \)

Moreover, \( D(r) = \frac{E h^3}{12(1-v^2)} \) is the bending stiffness.

Substituting Eqs. (7), (8) and (9) into Eq. (4) results in the following expression for the compatibility equation:

\[
\frac{\partial}{\partial r} \left[ r \left( \frac{\partial (r N_r)}{\partial r} - v N_r \right) \right] - \left( \frac{\partial (r N_r)}{\partial r} - v N_r \right) - \left( N_r - v \frac{\partial (r N_r)}{\partial r} \right) + \frac{E h}{2} \left( \frac{\partial w}{\partial r} \right)^2 = 0
\]

\( (10) \)

\( M_1 = T_1 \sin \varphi \) where \( \varphi \) is the angle between each point of the worktable and the swing axis, \( M_2 = T_2. N_r \) and \( N_\theta \) are the radial and circumferential force, and \( M_r \) and \( M_\theta \) denote the radial and circumferential bending moment, respectively.

Substituting Eq. (5) into Eq. (1) yields the following expression:

\[
\frac{\partial}{\partial \bar{r}} \left[ \frac{1}{\bar{r}} D(\bar{r}) \left( \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{v}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} \right) - M_1 - D(\bar{r}) \left( \frac{1}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} + \frac{v}{\bar{r}} \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} \right) \right] + M_2 + \frac{\partial}{\partial \bar{r}} \left[ D(\bar{r}) \left( \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{v}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} \right) + M_1 \right]
\]

\[
\frac{1}{\bar{r}} \left[ \frac{1}{\bar{r}} D(r) \left( \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{v}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} \right) - M_1 - D(r) \left( \frac{1}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} + \frac{v}{\bar{r}} \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} \right) \right] + M_2 + \frac{\partial}{\partial \bar{r}} \left[ D(\bar{r}) \left( \frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{v}{\bar{r}} \frac{\partial \bar{w}}{\partial \bar{r}} \right) + M_1 \right]
\]

\[
\frac{\partial}{\partial \bar{r}} \left[ \frac{1}{\bar{r}} \left( \frac{\partial (\bar{r} N_r)}{\partial \bar{r}} - v \bar{N}_r \right) \right] - \left( \frac{\partial (\bar{r} N_r)}{\partial \bar{r}} - v \bar{N}_r \right) - \left( \bar{N}_r - v \frac{\partial (\bar{r} N_r)}{\partial \bar{r}} \right) + \frac{E h}{2} \left( \frac{\partial \bar{w}}{\partial \bar{r}} \right)^2 = 0
\]

\( (11) \)

\( (12) \)

Radial and circumferential stress–strain equations of the worktable can be expressed as follows:

\[
\varepsilon_r = \frac{1}{E} \left( \frac{N_r}{h} - \frac{N_\theta}{h} \right)
\]

\( \varepsilon_\theta = \frac{1}{E} \left( \frac{N_\theta}{h} - \frac{N_r}{h} \right) \)

\( (7) \)

\( (8) \)

The balanced equation of radial membrane force and circumferential membrane force is as follows:
When space is integrated by the Galerkin displacement equation, only the differential function over time remains. Subsequently, dynamic equation of the worktable’s derivative with respect to time can be obtained in the form below:

\[
\ddot{A}(\tau) + c_1 \dot{A}(\tau) + c_2 A(\tau)^3 + c_3 A(\tau) + c_4 = 0
\]

where \( \tau = \omega t \), and coefficients \( c_1, c_2, c_3, \) and \( c_4 \) are defined as follows:

\[
c_1 = \frac{h c \sin^3(\pi \tau)}{2 m_0 \omega^3 c_1}
\]

\[
c_2 = \frac{18 h c}{\rho c \omega^3 c_1} \left( -24 r^2 \pi^6 + 12 r^2 \pi^4 \sin 2(\pi \tau) + 24 r^2 \pi^4 \sin 2(\pi \tau) - 12 r^2 \pi^3 \sin 2(\pi \tau) \sin 2(\pi \tau) 
  - 6r^2 \pi^4 \sin 4(\pi \tau) + 3r^2 \pi^3 \sin 2(\pi \tau) \sin 4(\pi \tau) + \frac{18 h c E_0}{\rho c \omega^3 c_1} (-24 r^2 \pi^5 + 24 r^2 \pi^4 - 2(\pi \tau) 
  - 6r^2 \pi^4 \sin 4(\pi \tau) + 12r^2 \pi^4 \sin 2(\pi \tau) - 12r^2 \pi^3 \sin 2(\pi \tau) \sin 2(\pi \tau) + 3r^2 \pi^3 \sin 4(\pi \tau) \sin 2(\pi \tau))
\]

\[
c_3 = \frac{h c}{2 m_0} (1 - \cos(\pi \tau)) - \frac{r \pi^5 D_{12}}{\rho c \omega^3} - \frac{r \pi^5}{3 m_0 (1 - \gamma)^2} \left( - \frac{E_0}{16 r^5 \pi^3} - 6r^6 \sin 2(\pi \tau) + 16r^6 \pi^2 \sin 2(\pi \tau) + 6r^5 \cos 2(\pi \tau) (2r \pi + r \sin 2(\pi \tau)) - 6r^6 \sin 2(\pi \tau) + 4r^6 \pi^2 \sin 2(\pi \tau) 
  + 12r^6 \pi^2 \sin 2(\pi \tau) - 12r^6 \pi^3 \sin 2(\pi \tau) \sin 2(\pi \tau) + 12r^6 \pi^3 \sin 2(\pi \tau) \sin 2(\pi \tau) + 2r \pi - 4r \pi \cos 2(\pi \tau) + r \sin 2(\pi \tau))
\]

\[
c_4 = 19.6 m (r \pi - r \sin [2\pi \tau])^2 + 22 M_1 r \sin [2\pi \tau] + 18 M_2 r \sin [2\pi \tau]
\]

4 Numerical simulation analysis of the axial vibration of the five-axis machine tool turntable

In the present study, the CNC five-axis machine tool worktable is considered as the research object. Supposedly, the worktable is an ideal cylinder made of HT200. The thickness and radius of the worktable are \( h = 0.05 m \) and \( a = 0.3 m \), respectively. Moreover, density, elastic modulus, and Poisson’s ratio of the material are \( \rho_1 = 7840 kg/m^3 \), \( E = 150 GPa \), and \( \nu = 0.25 \), respectively. The initial displacement and the vibration velocity are \( x(0) = 0 \) and \( \dot{x}(0) = 0 m/s \). Substituting the size and physical parameters into Eq. (14), the bifurcation diagram, vibration displacement diagram, phase plane trajectory diagram, and Poincaré cross-section diagram of the worktable axial vibration system can be obtained.

![Fig. 4 Bifurcation diagram of system vibration with a varying rotating torque](image-url)

4.1 The influence of the rotating torque on the worktable

When the swing moment of the machine tool worktable is \( M_1 = 0 N\cdot m \) and the mass of the workpiece on the worktable is \( m = 30 kg \), the size of the turning moment \( M_1 \) changes the size of the turning moment of the machine tool worktable.
and varies between $0\text{ N} \cdot \text{m} \sim 800\text{ N} \cdot \text{m}$. Figure 4 shows the distribution of the bifurcation of the system against $M_1$. It is observed that chaos and period of the axial vibration system increases as the applied torque increases. When the magnitude of the worktable torque $M_1$ varies between $0\text{ N} \cdot \text{m} \sim 300\text{ N} \cdot \text{m}$, the worktable axial vibration system is in a single-cycle motion state, and the maximum vibration amplitude is less than $0.02\text{ mm}$. When the worktable torque $M_1$ approaches $300\text{ N} \cdot \text{m}$, the worktable axial vibration system enters a bifurcation motion state. When the magnitude of the worktable torque $M_1$ changes between $300\text{ N} \cdot \text{m} \sim 310\text{ N} \cdot \text{m}$, the system is in a two-period motion state, and the maximum vibration amplitude is less than $0.03\text{ mm}$. When the torque $M_1$ of the worktable varies within the range $310\text{ N} \cdot \text{m} \sim 350\text{ N} \cdot \text{m}$, the corresponding axial vibration system enters a large-area chaotic motion, and the maximum vibration amplitude is less than $0.08\text{ mm}$. On the other hand, when the worktable torque $M_1$ reaches $350\text{ N} \cdot \text{m}$, the worktable axial vibration system enters a bifurcation motion. As $M_1$ further increases and varies within the range $350\text{ N} \cdot \text{m} \sim 400\text{ N} \cdot \text{m}$, the system enters a four-period movement state. Moreover, the worktable axial vibration system enters a large-area chaotic motion when $M_1$ varies in the range $400\text{ N} \cdot \text{m} \sim 480\text{ N} \cdot \text{m}$, and the maximum vibration amplitude is less than $0.08\text{ mm}$. When $M_1$ reaches $480\text{ N} \cdot \text{m}$, the worktable axial vibration system enters a single-cycle motion state. When the size of the worktable torque $M_1$ varies in the range $480\text{ N} \cdot \text{m} \sim 570\text{ N} \cdot \text{m}$, the system enters a single-cycle motion state, and the maximum vibration amplitude is less than $0.06\text{ mm}$. When the worktable torque $M_1$ reaches $570\text{ N} \cdot \text{m}$, the worktable axial vibration system enters a bifurcation motion state. As the worktable torque $M_1$ further increases and varies in the range $570\text{ N} \cdot \text{m} \sim 600\text{ N} \cdot \text{m}$, the system enters a two-period motion state, where the maximum vibration amplitude is less than $0.07\text{ mm}$. When the worktable torque $M_1$ nears $600\text{ N} \cdot \text{m}$, the system enters a four-period motion state, where the maximum vibration amplitude is less than $0.08\text{ mm}$. Based on the performed numerical simulations, the vibration displacement diagram, phase plane trajectory diagram, and Poincaré cross-section diagram of the worktable axial vibration system are shown in Figs. 5, 6 and 7 under different torques.

During the machining process of the workpiece on a dual-turntable five-axis machine tool, the machine worktable undergoes chaotic bifurcation originating from the continuous variation of the applied torque. It is worth noting that these bifurcations are unfavorable for the worktable stability so it is rotated through Fig. 4 and then the driving torque with small amplitude and less cycle is selected as the turning torque of the machine tool to the worktable during processing. For example, when the driving torque varies within the range of $0\text{ N} \cdot \text{m} \sim 300\text{ N} \cdot \text{m}$, the axial vibration system of the worktable is in a single-cycle motion state and the maximum vibration amplitude is $0.02\text{ mm}$.
The influence of the swing moment on the worktable

When the turning torque of the worktable is $M_1 = 0 \text{N} \cdot \text{m}$ and the mass of the workpiece is $m = 0 \text{kg}$, the swing moment $M_2$ varies within the range $0 \text{N} \cdot \text{m} \sim 1500 \text{N} \cdot \text{m}$. Under these circumstances, the swing moment of the machine tool worktable changes, and variations of the system bifurcation $M_2$ can be obtained. The obtained results in this regard are shown in Fig. 8. It is observed that when the swing moment $M_2$ of the worktable varies within the range of $0 \text{N} \cdot \text{m} \sim 300 \text{N} \cdot \text{m}$, the corresponding system vibration of the worktable has chaotic motions, where the maximum vibration amplitude is less than 0.03 mm. When the swing moment $M_2$ reaches $300 \text{N} \cdot \text{m}$, the axial vibration system of the worktable transfers from chaotic motions to small periodic motions. Moreover, when the swing moment $M_2$ changes between $300 \text{N} \cdot \text{m} \sim 600 \text{N} \cdot \text{m}$, the system is still in the region with small periodic motions, where the maximum vibration amplitude is less than 0.02 mm. As the swing moment further increases and varies in range $600 \text{N} \cdot \text{m} \sim 700 \text{N} \cdot \text{m}$, the axial vibration system of the worktable enters a small-area chaotic state, where the maximum vibration amplitude increases to 0.03 mm. Furthermore, when the swing moment $M_2$ of the worktable reaches $700 \text{N} \cdot \text{m}$, the axial vibration system of the worktable transfers from chaotic motions to a region with small periodic motions. When $M_2$ of varies between $700 \text{N} \cdot \text{m} \sim 950 \text{N} \cdot \text{m}$, the system is in a state of small periodic motion. When the swing moment $M_2$ of the worktable approaches $950 \text{N} \cdot \text{m}$, the corresponding axial vibration system enters a state of chaotic motion, where the maximum vibration amplitude is less than 0.03 mm. When the swing moment of the worktable varies from $950 \text{N} \cdot \text{m} \sim 1050 \text{N} \cdot \text{m}$, the axial vibration system enters a small period of motion, where the maximum vibration amplitude is less than 0.01 mm. When the swing moment $M_2$ of the worktable reaches $1050 \text{N} \cdot \text{m}$, the system enters a state of small periodic motion. When $M_2$ varies from $1050 \text{N} \cdot \text{m} \sim 1200 \text{N} \cdot \text{m}$, the axial vibration system of the worktable is in a state of large-area chaotic motion. Moreover, when $M_2$ varies within the range $1200 \text{N} \cdot \text{m} \sim 1500 \text{N} \cdot \text{m}$, the system is in a state of small periodic motion.

The obtained results indicate that as the swing moment of the machine tool worktable increases continuously, the axial vibration system of the worktable enters chaos and small periods alternately. This is unfavorable to worktable stability. In this regard, the vibration bifurcation of the system with different swing moments is presented in Fig. 8.
is found that the driving torque with a small amplitude and less period is selected as the swing torque of the machine tool. For example, when the driving torque varies within the range $950 \text{N} \cdot \text{m} \sim 1050 \text{N} \cdot \text{m}$, the axial vibration system is in a small periodic motion state and the maximum vibration amplitude is 0.01 mm.

### 4.3 The influence of the applied load on the worktable

When the swing moment of the machine tool worktable is set to $M_2=0 \text{N} \cdot \text{m}$, the turning moment is $M_1=0 \text{N} \cdot \text{m}$ and the mass of the workpiece on the worktable varies in the range $0 \sim 80 \text{kg}$, the bifurcation diagram of the system is shown in Fig. 12. It is observed that the worktable axial vibration system mainly appears alternately with four cycles and chaos. When the load-bearing mass of the worktable changes from $0 \sim 25 \text{ kg}$, the system is in a four-cycle motion state with the maximum vibration amplitude less than 0.06 mm. When $m$ reaches 25 kg, the axial vibration system of the worktable transfers from the four-period region to a chaotic state. Furthermore, when the load-bearing mass $m$ varies within the range $25 \sim 50 \text{ kg}$, the axial vibration system is in a state of large-area chaotic motion, where the maximum amplitude of vibration is less than 0.08 mm. As $m$ further increases and reaches 50 kg, the system enters a multi-period movement state. When the load-bearing mass of the worktable varies from $50 \sim 67 \text{ kg}$, the axial vibration system is in a multi-period motion state, where the maximum vibration amplitude is less than 0.06 mm. When the load-bearing mass of the worktable reaches 67 kg, the axial vibration system transfers from the multi-period region to a large-area chaotic state. Moreover, when $m$ varies within the range $67 \sim 74 \text{ kg}$, the system has large-area chaotic motions, where the maximum vibration amplitude is less than 0.09 mm. When the worktable bearing mass $m$ reaches 74 kg, the worktable axial vibration system enters a multi-period motion state. When $m$ changes between $50 \sim 67 \text{ kg}$, the axial vibration system is in a multi-period motion state, where the maximum vibration amplitude is less than 0.05 mm. Based on the performed numerical simulations, the vibration displacement diagram, phase plane trajectory diagram, and Poincaré cross-section diagram are presented in Figs. 13, 14 and 15 under different load-bearing masses.

The obtained results show that as the machine tool worktable changes continuously, the worktable axial vibration system alternates between a 4-period zone and a large-area chaotic zone. It should be indicated that this variation is unfavorable for worktable stability. In this regard, the system with dynamic torque change is presented in Fig. 12. In the vibration bifurcation diagram, the swing angle of the A-axis is adjusted to change the positive pressure of the workpiece on the worktable and obtain a stable load area, select the load-interval with small amplitude and less cycle as the load stable area of the worktable during processing, such as the workpiece load at $0 \sim 25 \text{ kg}$. It is concluded that the axial vibration system of the lower worktable is in a 4-cycle motion state and the maximum vibration amplitude is 0.06 mm.
5 Experimental verification of the axial vibration of the five-axis machine tool worktable

Analyzing the influence of different parameters, including the rotating torque, turning torque, and applied load on the worktable on the chaotic bifurcation and axial vibration of the worktable during machining reveals that chaotic vibration is one of the key factors affecting the processing quality. In this section, an axial vibration experiment of a machine tool table (Tuopu VMC-C50, Country) is applied to verify the vibration state of the table when the table is subjected to changes in torque and load. In the experiment, the worktable is made of gray cast iron HT200 with a radius and thickness of 300 mm and 50 mm, respectively. A high-precision power analyzer (FLUKE Norma 5000, Country) is used to directly measure the output torque of the A/C axis. During the torque measurement, the worktable moves uniformly and uses vibration generated by the self-excited force of the machine tool to measure axial vibrations. An acceleration sensor (Dytran 356A25, China) is installed on the outermost part of the worktable to gather acceleration data. Moreover, a data acquisition and analysis system (Donghua DH5922, China) is used to adopt signals. The configuration of the test system is shown in Fig. 16. The structure of the test system is shown in Figs. 17 and 18. During the experiments, worktable torque $M_1$ and worktable swing torque $M_2$ are set to 220N · m ~ 360N · m and 20N · m, respectively. Moreover, the workpiece mass $m$ varies in the range 0 ~ 40 kg.

The experiment adopts the method of controlled variables, which controls one of the three parameters of turntable torque, turntable swing moment, and workpiece quality as a fixed value, and the other two parameters are used as variables for the experiment. And under the condition that the three parameters are not changed, 30 experiments are carried out. The experimental results conform to the normal distribution, and the value within 90% of the confidence level is taken as the confidence interval through the function

$$ z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} $$

When the machine tool table moves at a constant speed of 30rpm, the swing moment is $M_2=0$N · m and the mass of the workpiece on the worktable is $m=30$ kg, the worktable torque varies from 50N · m ~ 300N · m, and the calculated maximum acceleration values are compared with the experiment. The comparison results in Table 1 show that the comparison result shows that the error between the experimental result and the calculated result is 12.3%. In the axial vibration experiment, the swing moment of the first group of worktables is set to $M_2=0$N · m and the torque $M_1$ and the quality of the workpiece $m$ Suppose the table torque $M_1$ is 10N · m at the interval of 400N · m ~ 470N · m, received by the worktable are changed. The quality of the workpiece is $m$ at the interval of 0 ~ 40 kg to 10 kg. In the second set of the experimental workbench, torque $M_1$ is set to $M_1=0$N · m, while the pendulum moment $M_2$ and the workpiece mass $m$ change. In this regard, the workbench pendulum moment 10N · m in the 10 kg interval is 20N · m, the workpiece mass $m$ is 10 kg at the 0 ~ 40 kg interval. Accelerations are measured twice and the highest value is taken as the result. Measured axial vibration acceleration of the worktable is presented in Tables 2 and 3. Accordingly, distributions of table torque-workpiece mass and table load-vibration acceleration are presented in Figs. 19 and 20, respectively.

Figure 19 reveals that the chaotic vibration of the worktable torque $M_1$ appears at 400N · m ~ 470N · m. Table 2 indicates that when the worktable torque is greater than 420N · m, the vibration acceleration of the system decreases with the same load. Meanwhile, when the worktable torque varies in the range of 420N · m ~ 440N · m, vibration acceleration decreases rapidly. Finally, a weak cycle phenomenon appears in the system as the load increases.

Different acceleration values obtained from the two experiments in Fig. 20 show that the swing moment $M_2$ of the worktable has chaotic vibration at 220N · m ~ 360N · m, and the fastest vibration acceleration appears when the swing moment of 240N · m ~ 280N · m.
Fig. 12 Vibration bifurcation diagram with load variation
Fig. 13  Diagrams of the vibration displacement, phase plane, and Poincaré cross-section at $m = 13$ kg

Fig. 14  Diagrams of vibration displacement, phase plane, and Poincaré cross-section at $m = 26$ kg

Fig. 15  Diagrams of vibration displacement, phase plane, and Poincaré cross-section at $m = 70$ kg

Fig. 16  Configuration of the test system
It is observed that when the swing moment of the worktable is constant and the applied load of the worktable is less than 30 kg, the vibration acceleration of the system slowly increases with the increase of the applied torque. Furthermore, when the applied load exceeds 40 kg, the vibration acceleration decreases slowly. This is a kind of random vibration phenomenon whose amplitude is within a certain range, and there is uncertainty.

| Worktable torque $M_1$ | 50 N·m | 100 N·m | 150 N·m | 200 N·m | 250 N·m | 300 N·m |
|------------------------|--------|--------|--------|--------|--------|--------|
| Experimental value $a(m/s^2)$ | 0.98 | 0.99 | 0.95 | 0.94 | 0.96 | 0.93 |
| Theoretical calculation value $a(m/s^2)$ | 0.86 | 0.85 | 0.83 | 0.84 | 0.86 | 0.83 |

Fig. 17 Vibration experiment site of the worktable

Fig. 18 Test site of machine tool A/C axis output torque
Table 2  Torque-workbench and load-vibration acceleration data

| First experiment |  |  |  |  |  |  |  |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 400              | 1.37             | 1.53             | 1.50             | 1.13             | 1.24             | 400              | 1.67             | 1.56             | 1.40             | 1.65             | 1.24             |
| 410              | 1.42             | 1.63             | 2.23             | 1.57             | 1.86             | 410              | 1.31             | 1.15             | 2.23             | 1.15             | 1.19             |
| 420              | 1.53             | 2.25             | 1.41             | 2.05             | 2.31             | 420              | 1.57             | 1.66             | 1.75             | 2.35             | 2.61             |
| 430              | 1.24             | 1.01             | 2.37             | 1.85             | 1.05             | 430              | 1.01             | 1.91             | 2.54             | 1.48             | 1.55             |
| 440              | 1.11             | 1.93             | 1.19             | 2.03             | 1.05             | 440              | 1.32             | 1.43             | 1.16             | 2.16             | 1.61             |
| 450              | 1.23             | 1.02             | 1.16             | 1.05             | 1.92             | 450              | 2.23             | 1.12             | 1.35             | 1.55             | 1.92             |
| 460              | 1.35             | 1.07             | 1.24             | 0.97             | 2.14             | 460              | 1.15             | 1.37             | 1.62             | 0.53             | 2.61             |
| 470              | 1.22             | 1.40             | 1.32             | 1.35             | 1.08             | 470              | 1.26             | 1.21             | 1.51             | 1.76             | 1.18             |

Second experiment

| 400 | 1.67 | 1.56 | 1.40 | 1.65 | 1.24 |
| 410 | 1.31 | 1.15 | 2.23 | 1.15 | 1.19 |
| 420 | 1.57 | 1.66 | 1.75 | 2.35 | 2.61 |
| 430 | 1.01 | 1.91 | 2.54 | 1.48 | 1.55 |
| 440 | 1.32 | 1.43 | 1.16 | 2.16 | 1.61 |
| 450 | 2.23 | 1.12 | 1.35 | 1.55 | 1.92 |
| 460 | 1.15 | 1.37 | 1.62 | 0.53 | 2.61 |
| 470 | 1.26 | 1.21 | 1.51 | 1.76 | 1.18 |

Table 3  Swing-workbench and load-vibration acceleration data

| First experiment |  |  |  |  |  |  |  |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| 200              | 1.53             | 1.94             | 1.46             | 1.64             | 1.45             | 400              | 200              | 1.56             | 1.15             | 1.35             | 1.78             |
| 220              | 1.31             | 1.43             | 2.09             | 1.24             | 2.05             | 410              | 220              | 1.51             | 1.63             | 2.84             | 1.56             |
| 240              | 1.46             | 2.04             | 1.65             | 1.45             | 1.68             | 420              | 240              | 1.31             | 1.15             | 2.78             | 1.96             |
| 260              | 1.14             | 1.42             | 0.95             | 1.96             | 1.35             | 430              | 260              | 1.26             | 0.68             | 0.97             | 1.13             |
| 280              | 1.45             | 0.99             | 0.99             | 0.89             | 1.38             | 440              | 280              | 1.74             | 0.15             | 0.99             | 1.32             |
| 300              | 1.31             | 1.16             | 1.64             | 0.93             | 1.54             | 450              | 300              | 1.31             | 0.96             | 1.87             | 0.96             |
| 320              | 1.49             | 1.58             | 1.25             | 1.46             | 1.63             | 460              | 320              | 1.81             | 1.85             | 1.63             | 1.53             |
| 340              | 1.43             | 1.49             | 1.63             | 1.35             | 1.54             | 470              | 340              | 1.93             | 1.51             | 1.87             | 1.02             |

Second experiment

| 400 | 200 | 1.56 | 1.15 | 1.35 | 1.78 |
| 410 | 220 | 1.51 | 1.63 | 2.84 | 1.56 |
| 420 | 240 | 1.31 | 1.15 | 2.78 | 1.96 |
| 430 | 260 | 1.26 | 0.68 | 0.97 | 1.13 |
| 440 | 280 | 1.74 | 0.15 | 0.99 | 1.32 |
| 450 | 300 | 1.31 | 0.96 | 1.87 | 0.96 |
| 460 | 320 | 1.81 | 1.85 | 1.63 | 1.53 |
| 470 | 340 | 1.93 | 1.51 | 1.87 | 1.02 |

Fig. 19  Distribution of the torque-table against load-vibration acceleration

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Conclusion

1. A dynamic model of the axial vibration of the worktable is established, and the nonlinear dynamic equation of the axial vibration of the machine tool worktable is derived considering the elasticity of the plate and shell. The established model is studied in cases with different torques, swing moments, and loads. Then, the Galerkin truncation method is applied to calculate the dynamic displacement in the axial direction of the worktable and obtain the bifurcation diagram, displacement waveform diagram, phase plane trajectory, and Poincaré cross-section diagram of the processing system. The obtained results show that in the machining process on a dual-table five-axis machine tool, the machine tool table is affected by the applied torque, swing moment, and load, and chaotic bifurcation appears in the axial vibration system. It is found that when the swing moment of the machine tool table and the mass of the workpiece are $M_2=0\text{N} \cdot \text{m}$ and $m=30\text{ kg}$, respectively, the corresponding table torque is $50\text{N} \cdot \text{m} \sim 300\text{N} \cdot \text{m}$. Relative error between the maximum theoretical and the experimental values is 12.3%.

2. In the VMC-C50 machine tool as an example, the performed theoretical analysis indicates that when the load and swing moments are constant, the axial vibration system of the worktable is in a single-cycle motion state and the vibration amplitude is $0.02\text{ mm}$, which is the smallest amplitude under the driving torque of $0\text{N} \cdot \text{m} \sim 300\text{N} \cdot \text{m}$. When the applied load and torque are constant, the machine tool worktable is affected by the swing moment. Consequently, the worktable axial vibration alternately appears in a state of chaos and small cycles, and the worktable axial vibration is under the driving torque of $950\text{N} \cdot \text{m} \sim 1050\text{N} \cdot \text{m}$. In this case, the vibration amplitude is $0.01\text{ mm}$, which is the lowest vibration in this state. When the torque and swing moment are constant, the machine tool table is affected by the load, and the main motion of the table axial vibration appears alternately in the 4-period zone and the large-area chaotic zone. Adjust the swing angle of the A-axis to change. The positive pressure of the workpiece results in a stable load area. The load ranges with small amplitudes and few cycles can be selected as the load stable area of the worktable during processing. Under the workpiece load at $0 \sim 25\text{kg}$, the axial vibration of the worktable is in a 4-period movement state. Meanwhile, the vibration amplitude is $0.06\text{ mm}$.

3. The performed experiments reveal that when the worktable torque approaches $400\text{N} \cdot \text{m} \sim 470\text{N} \cdot \text{m}$, chaotic vibration occurs. When the worktable torque exceeds $420\text{N} \cdot \text{m}$, as the load increases, the axial vibration acceleration of the system decreases. When the moment varies in the range $420\text{N} \cdot \text{m} \sim 440\text{N} \cdot \text{m}$, the axial vibration decreases rapidly. When the worktable torque is constant and the load increases, a weak periodic phenomenon appears in the system. Chaotic vibration of the worktable swing occurs at $220\text{N} \cdot \text{m} \sim 360\text{N} \cdot \text{m}$, and the axial vibration acceleration decreases when the swing moment approaches $M_2$. When the swing moment of the worktable is constant, the axial vibration acceleration of the system slowly increases as the torque increases. When the worktable load is less than $30\text{ kg}$, the vibration acceleration slowly decreases when the worktable load is greater than $40\text{kg}$.

4. The research results provide a new design analysis method for the dynamic analysis of the table of the
five-axis machine tool with two tables. It is possible to accurately find the conditions and range of the chaotic phenomenon during processing and avoid the chaotic phenomenon through reasonable control. The nonlinear analysis of the table axial vibration of the five-axis machine tool with double turntable can improve the stability of the machine table and control the machining accuracy and provide technical support.

Funding  Project supported by the National Natural Science Foundation of China (51772010S009).

Availability of data and material  The data is transparent.

Declarations

Ethics approval  Yes.

Consent to participate  Yes, the authors consent to participate.

Consent to publish  Yes, the authors consent to publish.

Conflicts of interest  The authors declare no competing interests.

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