ON SINGLE PARTICLE ENERGIES and NUCLEAR g FACTORS

Shadow Robinson¹ and Larry Zamick²

1. Department of Physics, Millsaps College, Jackson, Mississippi, 39210
2. Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854

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Abstract

If we add one neutron to doubly magic ¹⁰⁰Sn, we can associate the low lying states in ¹⁰¹Sn with single particle states. Thus the J= 5/2 + and J= 7/2 + states are identified as d⁵/₂ and g⁷/₂ states respectively. In ¹⁰¹Sn, these two low lying states are separated by an energy of 0.172 MeV. Currently there is a dispute as to the ordering of these states. We examine how the 2 scenarios—selecting J= 5/2 + as the ground state or J= 7/2 + as the ground state—affect spectra and nuclear g factors of higher mass Sn isotopes in a variety of shell model situations.

1 Introduction

There is disagreement in the literature about the spin assignments of the lowest 2 states in ¹⁰¹Sn, a nucleus which in the simplest picture consists of a doubly magic ¹⁰⁰Sn plus one valence neutron. There is agreement that the splitting of the 2 states is 0.172 MeV, but whether the ground state has angular momentum J= 5/2 + and the excited state J= 7/2 + or vice versa is in dispute. In a theoretical work of Jinag et al. they take the ground state to have J= 7/2 + as per ref. and the excited state J= 5/2 + . These states are associated with the single particle neutron orbitals g⁷/₂ and d⁵/₂. (The effective shell model interaction we will use, sn100pn[5] places the g⁷/₂ below the d⁵/₂ if one uses the default single particle energies.) In this work we will keep an open mind about these 2 scenarios and explore the implications for shell model calculations. We are guided by the observation that the experimental data for the nearby odd A isotopes of Tin prefer the picture of a lower d⁵/₂. In particular, the odd A Tin nuclei show that J= 7/2 + is higher than J= 5/2 + for A=103,105,107 and 109. There is a reversal for A=111 and 113 which we will show is not completely surprising since we are switching from particles to holes.

In ¹¹³Sn, the ground state is J= 1/2, J= 7/2 + is at 77.389 keV while the J= 5/2 + is at 409.83 keV giving us a 332.447 keV splitting with J= 5/2 being the higher state.

2 Calculations.

We will examine the odd A ¹¹³Sn and the even A isotopes of Sn in the g⁷/₂d⁵/₂ neutron model space.

We begin with ¹¹³Sn. The results are shown in Table 1. We first examine this nucleus with no interaction, then with the surface delta interaction (SDI), and finally with the sn100pn interaction[?]. For each interaction we perform the calculation assuming the d⁵/₂ to be the lower shell model orbit (by 0.172 MeV, the splitting in ¹⁰¹Sn), g⁷/₂ to be the lower orbit, and the case where the two orbits are degenerate. For each interaction, we will calculate the J= 7/2 + J= 5/2 splitting. Experimentally, in ¹¹³Sn the J= 7/2 + state is lower than the J= 5/2 state by 0.332 MeV while J= 1/2 + is the ground state.
Table I: The experimental $J = \frac{7}{2}^+ - J = \frac{5}{2}^+$ splitting in the Tin isotopes

| $^{\text{A}}$ | splitting (keV) | $^{\text{SDI}}_{7/2}$ | $^{\text{SDI}}_{5/2}$ | $^{\text{SDI}}_{0}$ | $^{\text{sn100pn}}_{7/2}$ | $^{\text{sn100pn}}_{5/2}$ | $^{\text{sn100pn}}_{0}$ |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 101            | $\pm 172$       | -0.172          | +0.172          | 0               | -0.172          | +0.172          | 0               |
| 103            | 168.0           | -0.008          | +0.055          | +0.033          | 0.244           | 0.283           | 0.279           |
| 105            | 199.7           | -0.024          | 0.046           | 0.005           | 0.258           | 0.347           | 0.305           |
| 107            | 151.2           | +0.015          | -0.001          | 0.014           | 0.264           | 0.246           | 0.246           |
| 109            | 13.38           | 0.003           | -0.037          | -0.025          | 0.212           | 0.103           | 0.177           |
| 111            | -154.48         | 0.031           | -0.094          | -0.011          | -0.038          | -0.284          | -0.164          |
| 113            | -332.447        | -0.008          | -0.352          | -0.018          | -0.656          | -1.0            | -0.829          |

Table II: Shell Model results for $^{113}$Sn as interaction and lowest orbit are changed. The splitting between the $g_{7/2}$ and $d_{5/2}$ orbits is chosen to be 0.172 MeV to match the $J = \frac{7}{2}^+ - J = \frac{5}{2}^+$ splitting in $^{101}$Sn

| $^\text{A}$ | 7/2 lower | 5/2 lower | 7/2 and 5/2 are degenerate in $^{101}$Sn |
|---------------|-----------|-----------|----------------------------------------|
| No interaction | 5/2 is 0.172 lower | 7/2 is 0.172 lower | they are degenerate |
| SDI | 7/2 is 0.008 lower | 7/2 is 0.352 lower | 7/2 is 0.18 lower |
| sn100pn | 7/2 is 0.657 lower | 7/2 is 1.001 lower | 7/2 is 0.829 lower |

The no interaction cases serves to show how particle hole inversion would naturally invert the order of the two low lying states from their stating point in $^{101}$Sn. The other results show how the relative spacing is affected when realistic matrix elements are included. The particle-hole inversion again serves to lower in $^{113}$Sn the state that was higher in $^{101}$Sn.

How this plays out in nuclei across the low lying Sn isotopes can be seen in Table 3 where we look at results from the even isotopes of Sn ranging from A=102 to 110.

As a note, the input splitting in the sn100pn interaction is 0.320 MeV, chosen to get appropriate results in the $^{132}$Sn region. Changing the splitting to 0.172 MeV as we will do here does not alter any of the conclusions we will draw. We can use the simple formula to calculate the change in energies as one goes from $^{101}$Sn to $^{113}$Sn.

$$\Delta k^{(j)} = -1/(2j+1) \left[ \Sigma (2J+1) \left< ( ( j j c) J [V| ( j j c) J - (-1) (j+jc +J) (jcj )J> \right) \right].$$

The minus sign is for holes.

The results are shown in Table 2.

In Table 3 we show the excitations energy of the lowest $2^+$ of the Sn isotopes. The near constancy of the energies of $J=2^+$ states, as shown in Table 3 has been noted many times before and was a stimulating factor in the development of Talmi’s Generalized Seniority Model[6,7].

Table III: The $J = 2^+$ excitation in the even Tin isotopes

| $^\text{A}$ | Experimental Excitation (keV) | $^{\text{SDI}}_{7/2}$ | $^{\text{SDI}}_{5/2}$ | $^{\text{SDI}}_{0}$ | $^{\text{sn100pn}}_{7/2}$ | $^{\text{sn100pn}}_{5/2}$ | $^{\text{sn100pn}}_{0}$ |
|----------------|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 102            | 887                           | 876             | 924             | 1230            | 1177            | 1270            |
| 104            | 815                           | 790             | 793             | 890             | 858             | 863             |
| 106            | 811                           | 829             | 820             | 933             | 927             | 933             |
| 108            | 819                           | 813             | 810             | 943             | 1045            | 981             |
| 110            | 793                           | 843             | 817             | 1052            | 1098            | 1090            |
| 112            | 826                           | 884             | 1115            | 1072            | 1091            | 1091            |
3 Magnetic g factors.

We can use the work of Yu and Zamick\[8\] to calculate magnetic moments or more precisely nuclear g factors for the even-even Sn isotopes. Here we however used a matrix diagonalization routine.

We can write the g factor as $A g_L + B g_S$. The bare values of $g_L$ and $g_S$ for a neutron are respectively 0 and -3.826. We present results for the SDI iteration in Table 4.

We consider bare values, then $g_L = -0.1 g_S = x^*(-3.826)$ and finally $g_L = x^*+0.1$ $g_S = x^*(-3.826)$ with the renormalization chosen to be $x=0.75$. The negative value of $g_L$ is consistent with meson exchange theory but some have felt the positive value gives a better fit to the data. The data however is sparse.

In Table 4 we examine the case with no splitting between the $g_{7/2}$ and $d_{5/2}$ orbit. In the no split case it was shown in [8] that the g factor is equal to $g_L$ for $^{102}$Sn, which in the bare case is zero. However for $^{104}$Sn and beyond the SDI generates a splitting of the $g_{7/2}$ and $d_{5/2}$ orbits and so one gets non-zero g factors. The behavior is somewhat complex. Focusing on the bare case the g factors which start at zero for $^{102}$Sn become increasingly negative as one goes to 104 and 106 but the turn around and become positive by 110, albeit very small (0.0345). The g factor only becomes substantially positive for $^{112}$Sn (0.3138).

For the case of the SDI interaction with $g_{7/2}$ orbital being the lower one, the g factor in $^{112}$Sn is very small because the splitting $J=\frac{7}{2}^+ - J=\frac{9}{2}^+$ is very small -0.008 MeV as seen in Table 2. If instead we have the $J=\frac{9}{2}$ ground state, the g factor will be larger than the one in Table I, because the energy splitting is larger (-0.352 MeV). In the no split case it is only 0.180 MeV.

In tables 5 to 10 we give more detailed results with both the SDI and Sn100pm interactions. We consider all three scenarios - $d_{5/2}$ ground state in $^{103}$Sn, $g_{7/2}$ ground state in $^{101}$Sn and no splitting. We also consider 3 sets of $g_L$ and $g_S$ as shown. Despite the plethora of numbers, some general conclusions can be drawn. Assuming a $J=5/2^+$ for $^{101}$Sn yields, in all cases, larger g factors than if we assume a $J=7/2^+$ ground state. This is understood, as per our discussions above about single particle energies and g factors. In all scenarios the g factor of the $2^+$ state in $^{112}$Sn is larger than the one in $^{110}$Sn. This seems to go against the experimental trend [9,10]. The experimental value of the g factor in $^{112}$Sn is 0.15 with error bar about 0.05 [9]. It would appear we can fit this either with $g_L = -0.1$ $x=0.75$ of the SDI with $J=5/2$ or $g_L = +0.1$ $x=0.75$ of SDI with $J=\frac{9}{2}$ lower. This serves to underline the interconnected nature of interaction details and the use of effective g factors. We will somehow have to have a better understanding of what are the properly renormalized parameters of the magnetic dipole operator. As fundamental calculations lead to a negative effective value of $g_L$ for a neutron this parameterization is appealing, but we cannot say yet that we have a definitive conclusion.

4 Closing remarks

This work emphasizes the importance of values of single particle energies in determining correct g factors in the Tin isotopes. This was also noted in a schematic calculation by Yu and Zamick\[8\] where it was stated that with degenerate single particle energies and a surface delta interaction the g factors in $^{112}$Sn vanished. We here focused on the the $J=\frac{7}{2}^+$, $J=\frac{5}{2}^+$ energy splittings in even-odd Tin isotopes. In $^{101}$Sn we identify these as single particle states $0g_{7/2}$ and $1d_{5/2}$, whilst in $^{113}$Sn these are single hole states. Even in the absence of any interaction the $J=\frac{7}{2}^+, J=\frac{5}{2}^+$ splitting changes as we add neutrons to $^{101}$Sn. Indeed in that case the splitting in $^{113}$Sn is equal and opposite to that in $^{101}$Sn.

We find in all cases the higher the $J=\frac{7}{2}^+$ is above $J=\frac{5}{2}^+$ the larger are the g factors of the $2^+$ states in $^{112}$Sn and $^{110}$Sn. There is an ambiguity in $^{101}$Sn as to whether the $J=\frac{7}{2}^+$ state is above or below the $J=\frac{5}{2}^+$ state. If $J=\frac{5}{2}$ ($d_{5/2}$ orbit) is lower in $^{101}$Sn, then due to particle hole inversion, in $^{113}$Sn the $J=\frac{7}{2}^+$ state will be farther below $J=\frac{9}{2}^+$ than in the scenario where $J=\frac{9}{2}$ ($g_{7/2}$ orbit) is lower. Thus we will in the former scenario ($g_{7/2}$ above $d_{5/2}$) get larger g factors. There is however an ambiguity in this region of what are the best values of the effective g factors $g_L$ and $g_S$. While logically an effective value $g_L = -0.1$ is preferred the claim has been made that a value $g_L = +0.1$ gives a better fit.

3
Table IV: $g$ factors of the Sn isotopes with the SDI interaction

| Sn J=2+ | A     | B     | $g_L^A$ = -0.1 x=0.75 | $g_L^A$ = +0.1 x=0.75 |
|---------|-------|-------|-----------------------|-----------------------|
| 102     | 1.0000| 0.0000| -0.1000               | +0.1000               |
| 104     | 0.9820| 0.0183| -0.0699               | -0.1507               |
| 106     | 0.9757| 0.0244| -0.0923               | -0.1675               |
| 108     | 0.9809| 0.0189| -0.0721               | -0.1523               |
| 110     | 1.0009| -0.0009|0.0345| -0.0750| 0.1268|
| 112     | 1.0826| -0.0826|0.3138| 0.1275| 0.3439|

Table V: $g$ factors in $^{112}$Sn for various scenarios.

| $^{112}$Sn | bare | $g_L^A$ = -0.1 x=0.75 | $g_L^A$ = +0.1 x=0.75 |
|------------|------|-----------------------|-----------------------|
| splitting 0.172 MeV |
| SDI$_{5/2}$ | 0.3771 | 0.1729 | 0.3927 |
| SDI$_{7/2}$ | 0.0110 | -0.0792 | 0.1226 |
| sn100pn$_{5/2}$ | 0.4216 | 0.2052 | 0.4272 |
| sn100pn$_{7/2}$ | 0.4174 | 0.20115 | 0.424 |

We also considered the 2 measured $J=2^+$ $g$ factors $g( ^{112}$Sn) =0.15 [9] and $g( ^{110}$Sn)=0.29 [10]. While one may be able to adjust one’s parameters to fit one of the $g$-factors, in all the cases considered we are not able to fit both values. The calculations give the opposite trend-namely that the $g$ factor of $^{110}$Sn should be smaller than that of $^{112}$Sn. To clarify the whole situation it would be of great help if $g$ factor measurements of other isotopes were were made e.g. $^{108}$Sn and even lighter Sn isotopes.

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Table VI: $g$ factors in $^{110}$Sn for various scenarios.

\begin{tabular}{ccc}
\hline
\text{Sn} & bare & $g_L = -0.1$ x=0.75 \\
\hline
splitting 0.172 MeV & & $g_L = +0.1$ x=0.75 \\
SDI$\frac{5}{2}$ & 0.1748 & 0.026515 & 0.23565 \\
SDI$\frac{7}{2}$ & -0.06585 & -0.14765 & 0.04889 \\
\text{sn100pn}$\frac{5}{2}$ & 0.3516 & 0.1545 & 0.3729 \\
\text{sn100pn}$\frac{7}{2}$ & 0.1139 & -0.01754 & 0.1884 \\
\hline
no splitting & & & \\
SDI & 0.03454 & -0.075 & 0.1268 \\
\text{sn100pn} & 0.26425 & 0.0913 & 0.3051 \\
\hline
\end{tabular}

Table VII: $g$ factors in $^{108}$Sn for various scenarios.

\begin{tabular}{ccc}
\hline
\text{Sn} & bare & $g_L = -0.1$ x=0.75 \\
\hline
splitting 0.172 MeV & & $g_L = +0.1$ x=0.75 \\
SDI$\frac{5}{2}$ & -0.03616 & -0.1262 & 0.07195 \\
SDI$\frac{7}{2}$ & -0.0923 & -0.1668 & -0.028355 \\
\text{sn100pn}$\frac{5}{2}$ & -0.1611 & -0.2166 & -0.0250 \\
\text{sn100pn}$\frac{7}{2}$ & -0.2795 & -0.30235 & -0.11695 \\
\hline
no splitting & & & \\
SDI & -0.07225 & -0.1523 & 0.0439 \\
\text{sn100pn} & -0.2458 & -0.2779 & -0.09075 \\
\hline
\end{tabular}

Table VIII: $g$ factors in $^{106}$Sn for various scenarios.

\begin{tabular}{ccc}
\hline
\text{Sn} & bare & $g_L = -0.1$ x=0.75 \\
\hline
splitting 0.172 MeV & & $g_L = +0.1$ x=0.75 \\
SDI$\frac{5}{2}$ & -0.12265 & -0.18875 & 0.04823 \\
SDI$\frac{7}{2}$ & -0.0734 & -0.1531 & 0.043045 \\
\text{sn100pn}$\frac{5}{2}$ & -0.3673 & -0.36585 & -0.185 \\
\text{sn100pn}$\frac{7}{2}$ & -0.29985 & -0.31705 & -0.13275 \\
\hline
no splitting & & & \\
SDI & -0.0933 & -0.16755 & 0.0276 \\
\text{sn100pn} & -0.32485 & -0.33515 & -0.17015 \\
\hline
\end{tabular}
Table IX: $g$ factors in $^{104}$Sn for various scenarios.

| $^{104}$Sn | bare | $g_L = -0.1 \times 0.75$ | $g_L = +0.1 \times 0.75$ |
|------------|------|--------------------------|--------------------------|
| splitting 0.172 MeV | | | |
| SDI$_{5/2}$ | -0.1634 | -0.21825 | -0.0268 |
| SDI$_{7/2}$ | 0.02912 | -0.0789 | 0.1226 |
| sn100pn$_{5/2}$ | -0.33615 | -0.34335 | -0.1609 |
| sn100pn$_{7/2}$ | -0.2229 | -0.2614 | -0.0730 |
| no splitting | | | |
| SDI | -0.07005 | -0.1507 | 0.04565 |
| sn100pn | -0.28345 | -0.3052 | -0.120 |

Table X: $g$ factors in $^{102}$Sn for various scenarios.

| $^{102}$Sn | bare | $g_L = -0.1 \times 0.75$ | $g_L = +0.1 \times 0.75$ |
|------------|------|--------------------------|--------------------------|
| splitting 0.172 MeV | | | |
| SDI$_{5/2}$ | -0.48105 | -0.4482 | -0.27335 |
| SDI$_{7/2}$ | 0.308 | 0.1230 | 0.339 |
| sn100pn$_{5/2}$ | -0.581 | -0.5205 | -0.3510 |
| sn100pn$_{7/2}$ | 0.3585 | 0.1595 | 0.37825 |
| no splitting | | | |
| SDI | 0 | -0.1 | 0.1 |
| sn100pn | 0.05785 | -0.05805 | 0.1450 |
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