Spin Dynamics of Extrasolar Giant Planets in Planet–Planet Scattering

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Received 2021 March 2; revised 2021 July 29; accepted 2021 August 1; published 2021 October 25

Abstract

Planet–planet scattering best explains the eccentricity distribution of extrasolar giant planets, and past literature showed that the orbits of planets evolve due to planet–planet scattering. This work studies the spin evolution of planets in planet–planet scattering in two-planet systems. Spin can evolve dramatically due to spin–orbit coupling made possible by the evolving spin and orbital precession during the planet–planet scattering phase. The source of torque to planet spin is the stellar torque, and the planet–planet torque contribution is negligible. As a consequence of the evolution of the spin, planets can end up with appreciable obliquities (the angle between a planet’s own orbit normal and spin axis), with the obliquity distribution peaking at about $10^\circ$, and extending to much larger values.

Unified Astronomy Thesaurus concepts: Exoplanets (498); Planetary dynamics (2173)

1. Introduction

Planet–planet scattering is the best model to date for explaining the eccentricity distribution of extrasolar giant planets (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997; Marzari & Weidenschilling 2002; Adams & Laughlin 2003; Chatterjee et al. 2008; Ford & Rasio 2008; Jurić & Tremaine 2008; Raymond et al. 2010; Carrera et al. 2019; Anderson et al. 2020). In the planet–planet scattering scenario, planets are hypothesized to form in closely packed systems. Mutual perturbations trigger orbit crossings and close encounters between planets. Past literature has shown that the orbits of planets (semimajor axis, eccentricity, inclination) change instantaneously during planetary close encounters, before planet collision or ejection. In this paper, we show that as a consequence of planet–planet scattering, the spin of the planets evolves and planets can grow in obliquity. The spin evolution is driven by spin–orbit coupling (due to the torque on the planetary spin from the host star) and the variation of the planet’s orbital inclination in planet–planet scatterings. Obliquity in this work is defined as the angular separation between the spin and orbital axes of a planet. Close encounter is defined as when planets come into each other’s Hill sphere.

The obliquity of a planet may provide clues to its dynamical history. Various mechanisms involving giant impacts and/or secular interactions have been proposed to generate planetary obliquity (e.g., Benz et al. 1989; Dones & Tremaine 1993; Hamilton & Ward 2004; Ward & Hamilton 2004; Rogoszinski & Hamilton 2020; Millholland & Batygin 2019; Millholland & Spalding 2020; Su & Lai 2020; Tremaine 1991). Although there is currently no direct measurement of extrasolar planetary spins and obliquities, constraints can be obtained using high-resolution spectroscopic observations (Snellen et al. 2014; Bryan et al. 2018, 2020), as well as radial velocity signals from directly imaged planets (Vanderburg et al. 2018). High-precision photometry of transiting planets can also help constrain planetary rotations in the future (Seager & Hui 2002; Barnes & Fortney 2003; Carter & Winn 2010; Schwartz et al. 2016).

Recently, Li & Lai (2020) and Li et al. (2020) studied giant planet spin and obliquity produced by planetary mergers. Through analytical calculation and numerical simulations, they showed that planetary collisions lead to rapidly rotating objects and a broad range of obliquities with a characteristic distribution. Two-planet scatterings typically result in planet mergers or ejection of one planet. Li & Lai (2020) focused on the obliquities of the former outcomes, but not the latter. Their simulations (as well as all previous planet scattering simulations) assume the spin of a planet is constant, except when two planets merge. In this paper we show that spin–orbit coupling during the planet–planet scattering phase may change the spin orientation of a planet, and therefore influencing the final planetary obliquities, especially for the ejection remnants.

The next section introduces our method for simulating spin evolution in planet–planet scattering. Because orbital evolution in the planet–planet scattering phase is on timescales shorter than the secular timescale (both during and outside of planetary close encounters), spin evolution requires special treatment beyond the standard secular approaches. Section 4 provides a detailed view the evolution of spin and obliquity. And on the basis of four simulation sets, the section discusses statistical properties of the spin and obliquity of planets in two-planet systems.

2. Method

The numerical scheme of this work builds on Hong et al. (2015, 2018). The code uses Mercury (Chambers 1999) as a base, and incorporates the shape of planets and spin evolution using a nonsecular equation of motion. The equation of motion for both orbit and spin evolves at every time step. Planets are simulated as oblate and symmetric around their spin axes. The code uses the built in Bulirsch–Stoer algorithm, which produces the best accuracy for close encounters. The faster version—conservative Bulirsch–Stoer—is used, since both the translational acceleration and the torque on spin depend only on position.

2.1. Oblate Planet

The Mercury code is modified to include planet oblateness by modifying the force term generated by the gravitational field around the planet. The gravitational field has a quadruple
moment term $J_2 = 0.0147$, consistent with Jupiter and slightly less than Saturn’s. All simulated planets adopt this oblateness. The $J_2$ moment modifies the spherically symmetric force field around a point mass, to an axisymmetric force field around an extended mass. The code assumes zero reaction time of planet spin to external perturbations, but in reality there might be a time lag due to viscous dissipation for a fluid body (such as a giant planet) to react to a torque. In general, the internal dynamics of a precessing fluid body is rather complicated, potentially involving turbulence and strong dissipation (Papaloizou & Pringle 1982; Barker 2016; Lin & Ogilvie 2017). However, since the internal dissipation conserves angular momentum, we expect that as far as forced precession is concerned, the giant planet effectively behaves as a rigid body to a leading-order approximation. In any case, as in all previous works on planetary obliquities, we will ignore any complications associated with internal fluid motions in this paper.

2.2. Spin Evolution

The spin of an oblate planet evolves under the torque from the central star and other planets. In planetary close encounters, the orbital parameters of a planet change rapidly, and the spin of a planet can receive large torques from another planet, in a shorter period of time than their orbital periods. Therefore, this work adopts an instantaneous equation of motion for the spin, instead of the secular one commonly used.

For an extended mass under the gravitational influence of a point mass, different parts of it receive differing amount of gravitational attraction. The difference in gravitational attraction in different parts of the extended mass causes a torque, leading to spin evolution. The force on a point mass $M$ from an extended mass $m$ is

$$\hat{F} = -\frac{G M m}{r^3}\left\{ \hat{r} - \frac{3J_2}{2} \left( \frac{R}{r} \right)^2 [5(\hat{\rho} \cdot \hat{s})^2 - 1] \hat{\rho} - 2(\hat{\rho} \cdot \hat{s})\hat{s} \right\} + \cdots,$$

where $G$ is the gravitational constant, $r$ the distance between the centers of $m$ and $M$, $\hat{r}$ the unit vector pointing from $m$ to $M$, $R$ the radius of $m$, and $\hat{s}$ the normalized spin vector of $m$ (e.g., Hilton 1991).

The reaction torque per moment of inertia on the extended body is

$$-r \times \frac{d}{dt}(\hat{F}) = \frac{d}{dt}(\Omega \hat{s}),$$

where $\Omega$ is the rotation rate of the planet, which is set to be $2\pi/(10 \text{ hr})$. The rotation period is close to that of Jupiter. For simplicity, the rotation rate is assumed to remain constant throughout the simulations.

Combining the two equations above, we can obtain the instantaneous equation of motion for the spin components of mass $m$:

$$\frac{d}{dt}(\Omega \hat{s}) = \frac{3}{r^3} \frac{G M}{\lambda \Omega} (\hat{r} \cdot \hat{s})(\hat{r} \times \hat{s}),$$

where $\lambda = \frac{I}{m R^2}$ is the normalized moment of inertia. $\lambda$ is set to 0.25, close to that of Jupiter, in all simulations.

The sources of torque on planet spin include the central star and other planets. The implementation of the above equation of motion into Mercury is verified with the past evolution of Mars’s obliquity. The simulation is checked against the result in Ward (1973) using the initial conditions provided in Quinn et al. (1991), with all planets and Pluto included in the simulation. An initial obliquity of 25°18 is applied to Mars. Although his approach was based on secular equations, our result for the obliquity evolution closely resembles Ward (1973) (Figure 1)—this is expected since no close encounters or strong scatterings are involved in the evolution. The integration error limit of the simulations is set at $10^{-12}$ per time step, and the initial time step is 1 day. The integrator automatically adjusts the time step in accordance with the set error limit.

3. Simulation Setting

This work performs controlled planet–planet scattering experiments with two-planet systems. Four different sets of simulations are listed in Table 1.

The base simulation (Table 2) contains a Sun-like central star, an inner planet with mass $m_1 = 1.5 M_J$ (Jupiter mass), and
an outer planet with mass $m_2 = 1 M_J$. Their radii are equal to that of Jupiter. The inner planet is located at 1 au, and the outer planet at a distance of 2 Hill radii from the inner planet (i.e., $a_2 = a_1 + 2R_H$, with $R_H$ being the Hill radius of the inner planet). Throughout the paper, planets will be called the same name based on their initial position for consistency, although during the scattering phase an inner planet may become more distant than its counterpart. The spacing is set so that close encounters start early in the simulation. The planets are also given small initial eccentricities of 0.002–0.01 to facilitate early close encounters. The inner planet’s orbit normal aligns with the $z$-axis in the Cartesian coordinate system, and the outer planet’s orbit normal is inclined by $3^\circ$ from that of the inner planet. The two planets’ spin axes are aligned with their orbit normal (i.e., obliquity $\theta = 0^\circ$). For any inclination and spin angle presented in the result section, the reference plane is the $xy$ plane, which has a small tilt of about $1^\circ$ from the system’s invariant plane. The planets’ argument of pericenter, longitude of ascending node, and the mean anomaly are randomly distributed from $0^\circ$ to $360^\circ$. The other three simulation sets are constructed by varying one parameter in the base set (Table 2). Simulation set 2 places the inner planet at 0.3 au, simulation set 3 places the inner planet at 5 au, and simulation set 4 is the same as the base except that the spin torque between planets is set to zero, but the stellar torque is retained. The outer planet in sets 2–4 is also located at 2 $R_H$ away from the inner planet. Each simulation is stopped once a planet has been removed from the system, by collision with the star or another planet, or by ejection from the system (semimajor axis >1000 au). Both planets in the system are tracked throughout the simulation.

4. Results

4.1. Spin Dynamics of Planets in Secular Systems

Before presenting our numerical result, we first summarize spin dynamics as numerous previous works on planetary/stellar obliquities are based on secular theory (e.g., Ward 1973; Laskar & Robutel 1993; Ward & Hamilton 2004; Storch et al. 2014; Storch & Lai 2015; Anderson et al. 2016; Millholland & Batygin 2019; Millholland & Spalding 2020; Su & Lai 2020).

In secular systems, when a planet’s own spin and orbital angular momenta are under perturbation, $|\frac{d}{dt}\omega_i|$ and $|\frac{d}{dt}\Omega_i|$ evolve. The obliquity evolution can be categorized into three different regimes. When $|\frac{d}{dt}\omega_i|$ (rate of change of the direction of spin momentum) is much greater than $|\frac{d}{dt}\Omega_i|$ (rate of change of the direction of orbital angular momentum), the spin axis of the planet will follow its own orbit normal, and the planet will retain a constant obliquity. If $|\frac{d}{dt}\omega_i|$ is much smaller than $|\frac{d}{dt}\Omega_i|$, the spin of the planet does not follow its own orbit normal. Instead, it precesses with a constant angular separation around the normal to system’s invariant plane. In two-planet systems, the spin of a planet precesses around the total orbital angular momentum axis of the two planets. When $|\frac{d}{dt}\omega_i| \sim |\frac{d}{dt}\Omega_{ij}|$, resonance crossing may lead to significant spin evolution, and can generate high obliquity planets. Eccentricity evolution in planet–planet scattering can also alter the rate of spin evolution, because it alters $r$-dependent terms in Equation (3). Therefore, eccentricity evolution in planet–planet scattering can also contribute to resonance crossings (e.g., Storch et al. 2014).

While the secular equations do not strictly apply to our problem, they can be used to provide a qualitative understanding of how a planet may attain a significant obliquity during the scattering process. Consider planet $m_1$ on an approximately circular orbit (with semimajor axis $a_1$) while planet $m_2$ on an eccentric orbit (with $a_2$ and $e_2$) interact with $m_1$ near its pericenter (thus $a_2(1 - e_2) \approx a_1$). The spin axis of $m_1$ precesses around its orbital axis $\hat{L}_1$ (driven by the torque from the host star) at a rate given by

$$\omega_i = \frac{3k_2 M_s}{2\lambda} \left( \frac{R_1}{a_1} \right)^3 \Omega_1,$$

where $k_q = J_2(Gm_1/R_1^3)^{1/2}$ (with $R_1$ and $\Omega_1$ the radius and rotation rate of $m_1$). On the other hand, the orbital axis $\hat{L}_i$ precesses around the total angular momentum axis $\hat{L}_1 + \hat{L}_2$ at the rate

$$\omega_l \approx \frac{3m_2}{4M_s} \left( \frac{a_1}{a_2} \right)^3 \frac{m_1}{(1 - e_2)^{3/2}},$$

where $m_1$ is the mean motion of $m_1$. Equation (5) assumes $a_2(1 - e_2) \gg a_1$ (e.g., Liu et al. 2014); when this is not satisfied, the actual $\omega_l$ can be larger by a factor of a few. The ratio of the two precession frequencies is

$$\eta = \frac{\omega_l}{\omega_i} \approx \frac{3.8\lambda}{k_q} \left( \frac{\rho_1}{g \text{ cm}^{-3}} \right) \left( \frac{M_n}{M_{\odot}} \right)^{-3/2} \left( \frac{m_2}{M_J} \right) \left( \frac{P_1}{10 \text{ hr}} \right) \times \left[ \frac{a_1}{a_2(1 - e_2)} \right]^{3/2} \left( \frac{1 - e_2}{1 + e_2} \right)^{3/2},$$

where $P_1 = 2\pi/\Omega_1$ and $\rho_1$ are the rotation period and mean density of of the planet, and $\lambda \sim k_q$ for giant planets.

During planet–planet scatterings that lead to planet ejection, $a_2$ and $e_2$ increase (while $a_2(1 - e_2)$ remains close to $a_1$) as a result of energy exchange between $m_1$ and $m_2$, in a process analogous to diffusion (see Pu & Lai 2021). Thus the ratio $\eta$ can pass through ~1 in a dynamical way. This “transient” resonance crossing may potentially give rise to appreciable obliquity excitation of the planet.

4.2. Spin Dynamics in Planet–Planet Scattering

In planet–planet scattering, the spin of a planet evolves under torques from the host star and other planets in the system. Because of the short timescale evolution of planetary orbits, and very short duration of planetary close encounters, we use the nonlinear equation for the planet’s spin evolution (Equation (1)). Here we discuss how planets can experience significant spin evolution and gain high obliquity in planet–planet scattering. During the planet–planet scattering phase, close planetary encounters, as well as strong mutual perturbation outside of planetary close encounters, can change the planet’s orbital elements by large amounts over short periods, which in turn can change $|\frac{d}{dt}\omega_i|$ and $|\frac{d}{dt}\Omega_i|$. This can drive the system close to a state where $|\frac{d}{dt}\omega_i| \sim |\frac{d}{dt}\Omega_{ij}|$, and thus lead to large variations in the obliquity due to stellar torques.

Figures 2 and 3 show an example of the spin and orbit evolution of a two-planet system in the base simulation set for the first two million years. Figure 3 shows typical orbital evolution of planets during the planet–planet scattering phase. The spin of the planets evolves chaotically. The outer planet’s spin axis evolves beyond $80^\circ$ from its initial orientation at 2
The inner planet’s spin evolution is relatively stable. Figure 2 also shows the corresponding evolution of inclination and obliquity. The dramatic spin evolution of the outer planet leads to its high obliquity ($\text{max} = 90^\circ$), under a relatively modest inclination. The inner planet experiences very little spin evolution, with a maximum displacement of $1^\circ$. The contrast in the scale of spin evolution between the outer and inner planets can be explained by considering the three regimes laid out in the last subsection. When the planet’s orbital precession rate $|\frac{d\hat{L}}{dt}|$ is close to $|\frac{d\hat{s}}{dt}|$, resonance effects can cause the spin to significantly evolve. In the top panel of Figure 4, the ratio between $|\frac{d\hat{L}}{dt}|$ and $|\frac{d\hat{s}}{dt}|$ for the outer planet lingers around 1 very frequently ($\log_{10}\sqrt{|\frac{d\hat{L}}{dt}|/|\frac{d\hat{s}}{dt}|} \sim 0$), so its spin evolves significantly. $|\frac{d\hat{L}}{dt}|$ and $|\frac{d\hat{s}}{dt}|$ have chances to be close to each other because in planet–planet scattering, both can evolve significantly, as shown in the middle and bottom panels of Figure 4.

For the inner planet, $|\frac{d\hat{L}}{dt}|/|\frac{d\hat{s}}{dt}|$ stays much higher than 1 for the most part, so its spin evolves very little in the example depicted in Figure 2. However, the inner (remnant) planet does have a chance to attain larger obliquities, as revealed in our ensemble of simulations (see Section 4.3).
4.3. Statistics

This section discusses the results of planet–planet scattering in systems with two giant planets in four simulation sets (Table 1). The statistics use only simulations with one surviving planet after the other planet is ejected via planet–planet scattering (23.26% of all simulations; see Table 1), which is around 5500 simulations in each set. In two-planet systems, the instability boundary is very narrow. If both planets survive, its very likely they have none or very little interactions, so it is not very interesting to study them for our purpose. Simulations with planet–star (2.71% of all simulations) or planet–planet collisions (73.61% of all simulations; see Table 1) are excluded from the statistics, as obliquity evolution for the collision of fluid bodies is beyond the scope of this work (see Li & Lai 2020; Li et al. 2020). Simulations with energy error \( \frac{E-E_0}{E} \) greater than \( O(10^{-4}) \) are also excluded (0.36% of all simulations), which is an adequate threshold for multiplanet systems (Barnes & Quinn 2004; Raymond et al. 2010). A negligible fraction of systems (0.03% of all simulations) that do not experience planet destruction in \( 10^8 \) yr are also excluded. Figures 5 and 6 show the distribution of the surviving planet’s final spin orientation and obliquity for simulation sets 1–3. The spin inclination is measured relative to the z-axis, which is aligned with the inner planet’s initial spin and orbit normal. The final spin and obliquity distribution is quite broad compared with the orbital inclination distribution (Figure 7). Overall, the final spin and obliquity distributions (Table 3) are broader than the inclination distributions, with 19.51% planets’ obliquity beyond 40°, and 8.77% of the surviving planets are on retrograde obliquity (i.e., the rotation is in reversed direction relative to the orbit). The inclination distribution only has 0.12% planets beyond 40°. The rate of highly oblique planets (>40°) are 17.86%, 31.98%, and 10.43% in sets 1, 2, and 3. There are 5.07% surviving planets on retrograde obliquity in the base set, 21.46% in set 2, and 3.21% in set 3. The distribution of obliquity for set 3 is more clustered at lower regions compared to the other sets. From the rate of high and retrograde obliquity in set 2 (the inner planet ends up at 0.19 au after the outer planet is ejected), we may expect a high rate of highly oblique planets close to the central star, and some oblique planets further out (Figures 5 and 6). A moderate peak in Figure 6 for close-in planets in sets 1 and 2 between obliquities 170°–180° implies that Venus-like obliquity is possible. The simulation results imply that planets closer to the star receive more torque from it and become more evolved in obliquity.

The final spin distributions of the three simulation sets all fit best with a log-normal distribution (Figure 8):

\[
f(s, \sigma) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{1}{2} \left( \frac{\ln(s) - \mu}{\sigma} \right)^2},
\]

\( \sigma \) is the standard deviation, and \( \mu \) the mean of the log-normal distribution. We do not have an explanation for the log-normal distribution, but we find it provides a good fit—this is a finding from our numerical experiments. Table 4 shows the statistics of the log-normal fit.

For completeness, the Kolmogorov–Smirnov (K-S) test shows that the final eccentricity distributions between the three simulation sets are similar, with K-S statistics 0.08–0.10 and \( p \)-values 0.96–1.0 (Figure 9). The low value of K-S statistics implies the distributions are similar, and the \( p \)-value is above our set 95% confidence level, so one cannot reject the null hypothesis that the distributions are the same. This result agrees with previous findings (Li et al. 2020). For the total number of close encounters, close-in planets in set 2 experience more than the other 2 sets, and set 1 and set 3 have nearly identical distributions as shown by the K-S test. We have also found that swapping the masses of the inner planet with the outer planet does not produce a significant difference in the distributions of final obliquities and spin inclinations, as tested for simulation set 3.

Simulation set 4 has the same initial setting as the base set, but torques between planets are turned off. Figure 10 shows the
The histograms of final spin inclinations for the base set and set 4. The distributions have a K-S statistic of 0.023 and p-value of 1.00. The statistics cannot reject the null hypothesis that the two distributions are the same, and the statistical evidence is very strong that planet–planet torque is very insignificant. As shown in Figure 11, we can see that the stellar torque is greater than planet–planet torque most of the time. Although the torque between planets is very large during close encounters (at the first blue peak ∼20 yr, the torque from the inner planet is ∼10^5 greater than the stellar torque), due to their short duration and small mass of planets relative to the star, statistically the summed effect of planet–planet torque is not very important. In the simulation shown in Figure 11, the summed torque on the outer planet from the star is 3 × 10^4 greater than from the inner planet in the first 2 million years of simulation time, so planet–planet torque is rather insignificant compared to stellar torque. The results show that planet–planet torque does not statistically affect simulation results with regards to spin or obliquity (Lee et al. 2007), and helps conclude that the torque from the central star is the main driver for the spin and obliquity evolution of the planets.

### Table 4

| Set  | μ   | Std-dev | K-S Statistics | p-value |
|------|-----|---------|----------------|---------|
| 1 + 4 (1 au) | 22.55 | 0.84 | 0.047 | 0.99 |
| 2 (0.3 au) | 19.71 | 0.63 | 0.079 | 0.95 |
| 3 (5 au) | 14.40 | 0.90 | 0.085 | 0.84 |

Figure 8. Log-normal fit of final spin inclination of sets 1–4. Set 4 is mixed into the base set in the 1 au plot, as their distributions are statistically identical. The fit parameters are given in Table 4.

Figure 9. Same as Figure 5, but for the distribution of the final eccentricity of planets.

Figure 10. The distributions of final spin inclination of base set and set 4. The K-S statistic between the two distributions is 0.023 with a p-value of 1.0.

5. Summary and Discussion

We have carried out numerical simulations of scatterings between two giant planets, keeping track of the spin evolution of each planet due to nonsecular spin–orbit coupling. Our results show that planet–planet scattering is a viable mechanism for tilting planets, and provides some probability of producing planets with appreciable obliquities, including retrograde rotations, especially for planets that are close-in during the scattering phase. Spin–orbit coupling can be triggered by the orbital evolution during the planet–planet scattering phase, and leads to chaotic evolution of spin inclination and obliquity. The final spin distribution of the remaining planet in an initially two-planet system fits best with log-normal distributions. The main driver for the spin evolution of planets is the central star, accompanied by inclination and eccentricity evolution from planet–planet scattering. Planetary collision is outside the scope of this work (see Li & Lai 2020; Li et al. 2020), but it may be capable of producing highly oblique planets.

Overall, the results presented in this paper suggest a new mechanism for generating modest planet obliquities even without giant impacts, planet–planet collisions, or long-term secular processes. This mechanism takes place during the dynamical (scattering) phase of a planetary system. There is evidence that both our solar system and exoplanetary systems...
have experienced such a dynamical phase. Therefore, it is possible that the mechanism studied in this paper contributes to the observed obliquities of giant planets, including those detected in extrasolar systems (e.g., Bryan et al. 2020; see also Jennings & Chiang 2021). However, as our dynamical obliquity-generation process is stochastic, we can only predict the distribution of the final obliquities, rather than the obliquity for a specific system.

While our paper was under review, we became aware of the work by Gongjie Li (Li 2021), who studied a similar obliquity-generation mechanism in three-planet scatterings.

The work was supported by the JWST Project through grant NNX17AL71A. D.L. is supported in part by NASA grant 80NSSC19K0444.

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