Constraints on $f(R)$ Gravity through the Redshift Space Distortion

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In this paper, a specific family of $f(R)$ models that can produce the ΛCDM background expansion history is constrained by using the currently available geometric and dynamic probes. Our analysis shows that the additional redshift space distortion data set puts very tight constraints on the model parameter $f_{00}$, and we find that $|f_{00}|$ is of $\mathcal{O}(10^{-3})$ order in $1\sigma$ regions. We also discuss the nonlinear matter power spectrum based on different halo fit models.

I. Introduction

In modern cosmology, it is common to realize the late time acceleration of our Universe either by considering an extra energy component namely dark energy or modifying the theory of gravity. However, the two approaches are totally different in nature. Discrimination of dark energy from a modified gravity theory is a crucial issue in theory and cosmic probes. At the background level these are strongly degenerated as both yield the same expansion history of our Universe. But the dynamic evolution of a small perturbation would be different for different gravity theories. Therefore, observations of the large scale structure of our Universe may reveal some clue about the actual scenario.

For the large scale structure, one can only read the correlation of galaxies, the tracers of the distribution of halos. And usually, one uses its Fourier transformation, the galaxy power spectrum $P_g(k)$. To understand the evolution of the matter perturbations even at the linear level, one should assume a relation between the overdensities of galaxy and matter, i.e., $\delta_g = b \delta_m$, where the overdensity for matter $\delta_m$ is well understood in theory. To compare the theory and cosmic observations, the so-called bias factor $b$, which usually depends on scales $k$, should be understood well. This galaxy bias issue limits the use of the matter power spectrum to study the large scale structure formation of Universe. For the nonlinear scale evolution, one still needs a better understanding of the halo model. The study of the nonlinear evolution through $N$-body simulation code with enough resolution and scales is numerically expensive and time-consuming. Thus, it is very difficult to scan model parameter space via the $N$-body simulation technique. In this regard, the so-called HALOFIT model [1] is an alternative plausible choice.

The above argument is based on the observations to the continuity equation for the perturbation evolution. The other best thing is related to the velocity field which comes from the second perturbation equation, the so-called Euler equation. Although peculiar velocities are difficult to be observed directly, if galaxies can be treated as test particles, their peculiar velocities should be directly related to the total matter distribution. Actually, the galaxy maps will be distorted in the line of sight direction by peculiar velocities because of the interpretation of galaxy redshift as its distance. As a result, the overdensities in the redshift and real space are related via

$$\delta_g^R(k) = b \delta_m(k)(1 + \beta \mu^2),$$

where $\beta = f/b$ is distortion factor, $f = d \ln \delta_m/d \ln a$ is the growth function and $\mu = \cos(\theta_k)$, $\theta_k$ being the angle between $\mathbf{k}$ and the line of sight. This is the redshift space distortion or the Kaiser effect [2]. Therefore, the combination $\sigma_8^2 \beta = f \sigma_8$ is independent of galaxy bias in the linear case [3]. The redshift space distortion data are useful to constrain the cosmological parameters space [10]. In this paper, we use the ten $f\sigma_8(z)$ data points as given in Table I for constraining the model parameter space.

| $\#$ | $z$ | $f\sigma_8(z)$ | Survey and Refs |
|------|-----|---------------|-----------------|
| 1    | 0.067 | 0.52 ± 0.06  | 6dFGS (2012) [4] |
| 2    | 0.17  | 0.51 ± 0.06  | 2dFGS (2004) [5] |
| 3    | 0.22  | 0.42 ± 0.07  | WiggleZ (2011) [6] |
| 4    | 0.25  | 0.39 ± 0.05  | SDSS LRG (2011) [7] |
| 5    | 0.37  | 0.43 ± 0.04  | SDSS LRG (2011) [7] |
| 6    | 0.41  | 0.45 ± 0.04  | WiggleZ (2011) [6] |
| 7    | 0.57  | 0.43 ± 0.03  | BOSS CMASS (2012) [8] |
| 8    | 0.60  | 0.43 ± 0.04  | WiggleZ (2011) [6] |
| 9    | 0.78  | 0.38 ± 0.04  | WiggleZ (2011) [6] |
| 10   | 0.80  | 0.47 ± 0.08  | VIPERS (2013) [9] |

TABLE I. The data points of $f\sigma_8(z)$ measured from RSD with the survey references.

Because of the degeneracies between dark energy and a modified gravity model at the background level, we mainly focus on a specific family of $f(R)$ models, which produce the ΛCDM background expansion history [11, 13, 14]. Based on this model, the linear and nonlinear matter power spectrum were discussed in Refs. [13, 14], where the model parameter space was also constrained by the SDSS LRG matter power spectrum and the correlation between galaxy and the integrated Sachs-Wolfe effect (gISW). It was reported that CMB+SN+HST+MPK cannot constrain the model parameter space well, while tight constraints were obtained by taking gISW data into account[13]. But these results were obtained at the risk of the galaxy bias issue, i.e., the understanding of
the bias factor $b$ even at the nonlinear scale for the SDSS LRG data sets via the relation $P_g(k) = (1 + Qk^2)/(1 + Ak)P_{lin}(k)$, where $Q$ and $A$ are numbers needed to be calibrated. And it is crucial to calibrate these numbers $Q$ and $A$ for different cosmological models. For the gISW correlation data, we also have the bias parameter problem. However we should avoid the influence coming from a improper bias parameter $b$. The other risk comes from the nonlinear matter power spectrum, which is fitted by the HALOFIT model based on $\Lambda$CDM model through $N$-body simulation. For a modified gravity model, this process should be repeated [14, 15]. Now it was already available for a range of model parameter $|f_{R0}| \lesssim 10^{-4}$, named MGHalofit [15], although it is based on Hu-Sawicki model [16]. It allows us to compare the nonlinear matter power spectrum between theory and cosmic probes, for instance considering the weak lensing, in a suitable range of model parameters. But the galaxy bias issue is still untouche.

Considering above issues, in this paper, we try to use the well-understood linear perturbation to constrain a specific family of $f(R)$ models. One will see that the addition of RSD data sets can tightly constrain the model parameter space.

We arrange this paper as follows. In Section II, we give a brief review of a specific family of $f(R)$ models. The constraint results will be shown in Section III, where we also give a discussion to the nonlinear matter power spectrum based on different halo fits. Section IV carries the concluding remarks.

II. A specific family of $f(R)$ models

The Einstein-Hilbert action in general form for $f(R)$ gravity reads as

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ R + f(R) \right] + \int d^4x \sqrt{-g} L_m, \quad (2)$$

where $L_m$ is the Lagrangian of matter, which will not include the mysterious dark energy as the late time accelerated expansion of our Universe can be realized by the proposed $f(R)$ gravity. For recent reviews for modified gravity theory, see [17–20]. Doing variation with respect to the metric $g_{\mu\nu}$ for the Einstein-Hilbert action, one obtains a generalized Einstein equation which relates the geometry of space-time to the distribution of energy-momentum

$$FR_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \nabla_\mu \nabla_\nu F + g_{\mu\nu} \nabla_\nu \nabla_\nu F = 8\pi GT^{\mu\nu}, \quad (3)$$

where $F = 1 + \frac{\partial f}{\partial R}/R$. It is obvious that the general relativity is recovered when $f(R) = 0$ is chosen. The thorny problem is to determine a form of $f(R)$ which respects the cosmic observations. Here the the cosmological observations include two sides. One is the geometric, i.e., the expansion history of our Universe at the background level. The other is dynamic, i.e., the structure formation history of large scale of Universe via the linear and nonlinear perturbations. The $\Lambda$CDM model is compatible to almost all the cosmic observations at least at the background level. Therefore, an alternative cosmological model should not deviate from the $\Lambda$CDM model too much.

This fact is also called cosmological model degeneracy. Thus, to discriminate one model from the other, reliable cosmic observations are demanded to break this degeneracy. The large scale structure formation information of Universe is promising to break the possible degeneracy because the structure formation history may differ significantly in models having same expansion history at the background level. Following this, one can detect a possible deviation from general relativity or rule out an alternative model. While constructing a model which predicts the expansion history as of $\Lambda$CDM model, one can compare the expansion rate and its time variation in the two models. A form of $f(R)$ having the expansion history as of $\Lambda$CDM model is [11, 12]

$$f(R) = -2\Lambda - \sigma \left( \frac{\Lambda}{R - 4\Lambda} \right)^{p_s - 1} \mathcal{F}_1 \left[ q_s, p_s - 1; r_s, -\frac{\Lambda}{R - 4\Lambda} \right], \quad (4)$$

where $\Lambda$ is the cosmological constant and $\sigma$ is a constant parameter

$$\sigma = \frac{D}{p_s - 1} \left( \frac{\Omega_m}{\Omega_L} \right)^{p_s} 3\Omega_m \Lambda H_0^2, \quad (5)$$

and $\mathcal{F}_1[a; b; c; z]$ is the Gaussian hypergeometric function and the indices are given by

$$q_s = 1 + \frac{\sqrt{3}}{12}, \quad r_s = 1 + \frac{\sqrt{3}}{6}, \quad p_s = \frac{5 + \sqrt{3}}{12}. \quad (6)$$

This family of $f(R)$ models is specified by the only extra model parameter $D$ as a comparison to the $\Lambda$CDM model. This extra model parameter $D$ relates to the current value $B_0$ of the Compton wavelength

$$B = \frac{dR}{dH} \frac{H}{1 + f_R d\ln a d\ln a}, \quad (7)$$

via

$$B_0 = \frac{2Dp_s}{(\Omega_m)^2 \left[ 1 + D_2 F_1 \left[ q_s, p_s, r_s; -\frac{\Omega_L}{\Omega_m} \right] \right]} \times \left\{ \Omega_m \Lambda^2 F_1 \left[ \left. \begin{array}{c} q_s + 1, p_s + 1; r_s + 1; -\frac{\Omega_L}{\Omega_m} \end{array} \right| \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \r
find out the sensitive dependence on the values of model parameter $f_{R0}$ (or $D, B_0$). For future using the MGHaloFit to fit the nonlinear matter power spectrum, in this paper, we will take $f_{R0}$ as a free model parameter, then $D$ and $B_0$ are derived model parameters via the Eq. (8) and Eq. (9) respectively. In Figure 1, the dependence to the model parameter $f_{R0}$ was shown, where the curves from the top to the bottom but the last one are plotted for the values of $f_{R0}: -10^{-2}, -10^{-3}, -10^{-4}, -10^{-5}, -10^{-6}$. The last one is for ΛCDM model. Here the other relevant cosmological parameters are fixed to their mean values obtained in Planck 2013 [21]. With this observations, one can expect to obtain a tight constraint to the model parameter $f_{R0}$ when RSD data points are included. If the values of $f_{R0}$ can be confined to a range less than $10^{-4}$ through the linear perturbation, one can safely use the MGHaloFit to obtain the nonlinear matter power spectrum prepared for comparison to the weak gradational lensing probes.

But before using the fσ8(z) to constrain the model parameter space, we should check the potential risk of the dependence of fσ8(z) to the scale k for a modified gravity theory, here it is the specific family of f(R) models. To do that, we plotted the evolution of fσ8(z) with respect to k at different redshift $z \in [0, 1.0]$ in Figure 2. For example, please see the bottom curve for $z = 0$, the deviation to constant value amounts to 2% which is much less than the error bars. Based on this fact, it would be safe to use the RSD data points to constrain this specific family of f(R) models. In the worst (maybe the best) cases where a strong dependence on the scale k appears for a specific f(R) model, one can see strong evidence beyond the standard ΛCDM model or rule out this kind of model from the cosmic probes which seems independence to the scale k for fσ8(z).

### III. Results and Discussion

In this section, we show the constraint results to the specific family of f(R) models from the geometric and dynamic probes. For the geometric one, we will use the supernova Ia data from SDSS-II/SNLS3 joint light-curve analysis [22], the baryon acoustic oscillation $D_L(0.106) = 456 \pm 27$ [Mpc] from 6dF Galaxy Redshift Survey [23]; $D_L(0.35)/r_s = 8.88 \pm 0.17$ from SDSS DR7 data [24]; $D_L(0.57)/r_s = 13.62 \pm 0.22$ from BOSS DR9 data [25], the present Hubble parameter $H_0 = 73.8 \pm 2.4$ [km s$^{-1}$]Mpc$^{-1}$] from HST [26], and the full information of CMB recently released by Planck 2013 (which include the high-l TT likelihood (CAMSpec) up to a maximum multipole number of $l_{max} = 2500$ from $l = 50$, the low-l TT likelihood (lowl) up to $l = 49$) [27] with the addition of the low-l TE, EE, BB likelihood up to $l = 32$ from WMAP9.

For the dynamic one, we use the RSD data which was already listed in Table I. For using the growth function, the values of $f = d\ln \Delta_m/d\ln a$ at different values $a$ and $k$ are stored in a two dimensional table, where $\Delta_m = \delta_m + 3H(1 + \omega_m)\eta_m/k^2$ is overdensity for matter.

We perform a global fitting on the Computing Cluster for Cosmos by using the publicly available package CosmoMC [28] in the following model parameter space

$$P = (\Omega_b h^2, \Omega_c h^2, 100\theta_{MC}, \tau, n_s, \ln(10^10 A_s), f_{R0}). \quad (10)$$

their priors are shown in the second column of Table II. The running was stopped when the Gelman & Rubin $R - 1$ parameter $R - 1 \sim 0.02$ was arrived; that guarantees the accurate
TABLE II. The mean and best fit values with 1σ errors for the interested and derived cosmological parameters, where the Planck 2013, WMAP9, BAO, SN, HST and RSD data sets were used.

| Parameters | Priors | Mean with errors | Best fit |
|------------|--------|------------------|----------|
| $\Omega_{\text{b}} h^2$ | $[0.005, 0.1]$ | $0.02243^{+0.00026}_{-0.00028}$ | $0.02249$ |
| $\Omega_{\text{m}} h^2$ | $[0.01, 0.99]$ | $0.1163^{+0.0011}_{-0.0012}$ | $0.1162$ |
| $100\Omega_{\text{m}}$ | $[0.5, 10]$ | $1.0417^{+0.0006}_{-0.0006}$ | $1.04203$ |
| $\tau$ | $[0.01, 0.81]$ | $0.078^{+0.051}_{-0.041}$ | $0.076$ |
| ln(10^{10}A_s) | $[27.4, 7]$ | $3.054^{+0.049}_{-0.049}$ | $3.048$ |
| $n_s$ | $[0.9, 1.1]$ | $0.9685^{+0.0057}_{-0.0057}$ | $0.9676$ |
| $f_{R0} \times 10^{-6}$ | $[-100, 0]$ | $-5.93^{+3.10}_{-3.10}$ | $-1.2$ |

$H_0$ | ... | $68.99^{+0.25}_{-0.25}$ | 69.19 |
$\Omega_{\text{b}}$ | ... | $0.0707^{+0.0083}_{-0.0083}$ | $0.7089$ |
$\Omega_{\text{m}}$ | ... | $0.2930^{+0.0083}_{-0.0083}$ | $0.2912$ |
$\sigma_8$ | ... | $0.830^{+0.020}_{-0.020}$ | $0.811$ |
$\tau$ | ... | $9.82^{+2.04}_{-2.04}$ | $9.69$ |
Age/Gyr | ... | $13.752^{+0.000009}_{-0.000009}$ | $13.735$ |
$D$ | ... | $-0.00000101^{+0.00000007}_{-0.00000004}$ | $-0.00000020$ |
$log(B_0)$ | ... | $-4.61^{+1.23}_{-1.23}$ | $-5.19$ |

FIG. 3. The 1D marginalized distribution and 2D contours for interested model parameters with 68% C.L., 95% C.L. by using the Planck 2013, WMAP9, BAO, BAO, JLA, HST and RSD data sets.

The inclusion of RSD data set leads to very tight constraints on the model parameter $f_{R0} = -5.93^{+3.10}_{-3.10} \times 10^{-6}$ at 68% C.L. (see Table II). In Figure 4, we show the sensitive dependence of $f\sigma_8(z)$ on the model parameter $f_{R0}$, where one can see that the larger values of $f_{R0}$ predict the larger values of $f\sigma_8(z)$. Actually, it is already seen from the linear matter power spectrum as shown in Figure 1. This is one of the main finding of this work. When this model parameter is well constrained on the linear scale, much time can be saved in $N$-body simulation by specifying the values obtained from the linear matter power spectrum.

FIG. 4. The effects to $f\sigma_8(z)$ for different values of $f_{R0} = -10^{-2}, -10^{-3}, -10^{-4}, -10^{-5}, -2.29 \times 10^{-6}$ from the top to the bottom, where the values of other relevant cosmological model parameters were fixed to their best fit values listed in Table II.

Now let us move to the discussion of the nonlinear matter power spectrum at redshift $z = 0$ obtained from HALOFIT, MGHalofit and the fitting formula given PPFFit in Ref. [14] where the other relevant cosmological model parameters were fixed to their best fitting values as obtained above in Table II. This comparison can provide clues to the difference of matter power spectrum at the nonlinear scales. We show the linear and nonlinear matter power spectrum corrected by different fitting in Figure 5.

The top curve in Figure 5 shows the relative difference of linear matter power spectrum between $f(R)$ and $\Lambda$CDM. This difference is due to the modification of the Newtonian constant $G$, that has been understood very well in Ref. [14]. The second curve in Figure 5 on the right side from the top was obtained from the standard Halofit formula. That predicts relative large nonlinear matter power spectrum. The third curve in Figure 5 on the right side from the top shows the relative difference of the nonleianr matter power spectrum corrected by GR nonlinear power spectrum from standard Halofit model via the PPFFit [14]

$$P(k, z) = \frac{P_{\text{non-GR}}(k, z) + (C_{ni1}k^\alpha + C_{ni2})\Sigma^2(k, z)P_{GR}(k, z)}{1 + (C_{ni1}k^\alpha + C_{ni2})\Sigma^2(k, z)}$$

(11)

where $P_{GR}$ is the power spectrum in $\Lambda$CDM model and $\Sigma^2(k, z)$ is given by

$$\Sigma^2(k, z) = \left[\frac{k^3}{2\pi^2}P_{\text{lin}}(k, z)\right]^{1/3}.$$

(12)
therefore, the linear matter power spectrum in a $f(R)$ gravity corrected by the standard HALOFIT model is taken as a substitute. One can also find this kind of correction in Ref. [29, 30]. Here the values of $C_{a1} = 0.02349462$, $C_{a2} = 0.4634951$ and $\alpha = 2.251794$ were adopted for the case of $f_{R0} = -10^{-4}$ at the redshift $z = 0$, because the values of $C_{a1}$, $C_{a2}$ and $\alpha$ for the case of $f_{R0} = -10^{-6}$ at the redshift $z = 0$ are still unavailable now. Therefore, we should keep in mind that different values of $C_{a1}$, $C_{a2}$ and $\alpha$ will change the shape and amplitude of the matter power spectrum at the nonlinear scale. To understand the changes, we plotted the nonlinear matter power spectrum with combination of different values of $C_{a1}$, $C_{a2}$ and $\alpha$ in Figure 6. The larger values of $C_{a1}$ will decrease the matter spectrum at the region larger than $k > 1h$/Mpc. The larger values of $C_{a2}$ will increase the matter spectrum at the region larger than $k > 0.2h$/Mpc. The larger values of $\alpha$ will decrease the matter spectrum at the region larger than $k > 1h$/Mpc. Then choosing a combination carefully, a corrected power spectrum can mimic the evolution of the nonlinear matter power spectrum fitted from the MGHaloifit model or HALOFIT model in $\Lambda$CDM model at a given redshift, say $z = 0$. However, it is still hard to model the dependence of the model parameter $C_{a1}$, $C_{a2}$ and $\alpha$ to $f_{R0}$ at different redshifts [14]. The MGHaloifit works in the range $[f_{R0}] = [10^{-6}, 10^{-4}]$ and $z \leq 1$, it is free from this kind of difficulties. As a comparison to the naive HALOFIT and GR correction model, MGHaloifit predicts relative small deviation to the $\Lambda$CDM model based on HALOFIT. One should worry about the suitability of MGHaloifit for this specific family of $f(R)$ models, because MGHaloifit is obtained based on Hu-Sawicki model [29], but for this tiny $[f_{R0}] \sim 10^{-6}$, it is difficult to detect a model not only because of the accuracy of the fitting formula but also because of the complicated astrophysical systematics on such scales [15]. Based on these points, MGHaloifit would be a better choice.

![Figure 5](image-url)  
**FIG. 5.** The linear and nonlinear matter power spectrum at redshift $z = 0$ for a specific family of $f(R)$ model with HALOFIT, MGHaloifit and GR correction PPFit [14] and that for $\Lambda$CDM model in GR, where the values of other relevant cosmological model parameters were fixed to their best fit values listed in Table II.

![Figure 6](image-url)  
**FIG. 6.** The nonlinear matter power spectrum from PPFit at redshift $z = 0$ for combinations of different values of $C_{a1}$, $C_{a2}$ and $\alpha$, where the values of other relevant cosmological model parameters were fixed to their best fit values listed in Table II.

**IV. Conclusion**

In this paper, a specific family of $f(R)$ models which can produce the $\Lambda$CDM background expansion history has been tightly constrained with an addition of the redshift space distortion data $f\sigma_8(z)$ combining the other cosmic observations.
which include SN, BAO, CMB and HST. The constraint results show that the values of $f_{R0}$ should be of $O(10^{-6})$ order in $1\sigma$ regions. This tight constraint is obtained mainly due to fact that the linear matter power spectrum in this $f(R)$ theory is much sensitive to the values of $f_{R0}$ as shown in Figure 1 and Figure 4.

We have analyzed the nonlinear matter power spectrum at redshift $z = 0$ in the specific family of $f(R)$ models using three fitting methods for the best fit values of model parameters. The first one is the standard HALOFIT model, where the matter power spectrum deviates from the $\Lambda$CDM about 20% at the nonlinear scales. The second is the PPFfit method which is based on the HALOFIT model with a correction from the $\Lambda$CDM model nonlinear power spectrum. At a fixed redshift say $z = 0$, in principle, by carefully choosing values of model parameters $C_{a1}, C_{a2}$ and $\alpha$, almost the same nonlinear matter power spectrum as of $\Lambda$CDM model can be produced, but it is hard to model the dependence of $C_{a1}, C_{a2}$ and $\alpha$ on $f_{R0}$ at different redshifts. The third one is MGHalofit model, although it is modeled based on Hu-Sawicki model and works in the range of $|f_{R0}| \in [10^{-6}, 10^{-4}]$ and $z \leq 1$, the resultant nonlinear matter power spectrum can almost mimic the $\Lambda$CDM model with very small deviation from $\Lambda$CDM model. Also the dependence to the parameter $f_{R0}$ is well modeled. Based on this point, MGHalofit would be a better choice, although it is based on the analysis of Hu-Sawicki model. With the very small values of $f_{R0}$, it is difficult to detect a model not only because of the accuracy of the fitting formula but also the complicated astrophysical systematics on such scales [15].

Acknowledgments

The authors thank Dr. Bin Hu and Dr. Jun-Qing Xia at ICTP for helpful discussions. The author thanks Dr. Gong-Bo Zhao and Dr. Yuting Wang for hospitality and useful discussions in NAOC. This work is supported in part by NSFC under the Grants No. 11275035 and "the Fundamental Research Funds for the Central Universities” under the Grants No. DUT13LK01 and the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (No. Y4KF101CJ1).

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