The Force Exerting on Cosmic Bodies in a Quaternionic Field

V. Majerník
Department of Theoretical Physics, Palacký University,
Tř. 17. Listopadu 50, CZ-772 07 Olomouc, Czech Republic
and
Institute of Mathematics, Slovak Academy of Sciences,
Bratislava, Štefánikova 47, Slovak Republic

Abstract

The expression of a time-dependent cosmological constant $\lambda \propto 1/t^2$ is interpreted as the energy density of a special type of the quaternionic field. The Lorenz-like force acting on the moving body in the presence of this quaternionic field is determined. The astronomical and terrestrial effects of this field are presented, and the ways how it can be observably detected is discussed. Finally, a new mechanism of the particle creation and an alternative cosmological scenario in the presence of the cosmic quaternionic field is suggested.

1 Introduction

As is well-known one way to account for the possible cosmic acceleration is the introduction a new type of energy, the so-called quintessence ("dark energy"), a dynamical, spatially inhomogeneous form of energy with negative pressure [1]. A common example is the energy of a slowly evolving scalar field with positive potential energy, similar to the inflation field in the inflation cosmology. The quintessence cosmological scenario (QCDM) is a spatially flat FRW space-time dominated by the radiation at early times, and cold dark matter (CDM) and quintessence (Q) later time. The quintessence is supposed to obey an equation of state of the form

$$p_Qc^{-2} = w_Q \varrho_Q, \quad -1 < w_Q < 0.$$  

(1)

In many models $w_Q$ can vary over time. For the vacuum energy (static cosmological constant), it holds $w_Q = -1$ and $\dot{w}_Q = 0$. The existence of the quintessence, often modelled by a positive non-zero cosmological constant, helps to overcome the cosmological age and tuning problems. The point of view has often been adopted which allows the
quintessence to vary in time, i.e. \( \omega_Q = f(t) \). This means that the corresponding cosmical constant is time-dependent, too. Next, we will consider a cosmological constant \( \lambda \propto 1/t^2 \).

As is well-known the Einstein field equations with a non-zero \( \lambda \) can be rearranged so that their right-hand sides have two terms: the stress-energy tensor of the ordinary matter and an additional stress-energy tensor \( T_{ij}^{(\nu)} \) assigned to \( \lambda \)

\[
T_{ij}^{(\nu)} = \Lambda = \left( \frac{c^4 \lambda}{8\pi G} \right).
\]

A number of authors set phenomenologically \( \Lambda \propto 1/t^2 \) [3-10] (for a review see [2]). Generally, \( \Lambda \) contains in its definition the gravitation constant \( G \) and velocity of light \( c \). The simplest expression for \( \Lambda \propto 1/t^2 \), having the right dimension, and containing \( G \) and \( c \) is

\[
\Lambda = \frac{c^2}{8\pi G t^2},
\]

where \( \kappa \) is a dimensionless constant.

In a very recent article [16], \( \Lambda \) has been interpreted as the field energy of a classical quaternionic field (called \( \Phi \)-field, for short) by written it in the form

\[
\Lambda = \frac{1}{8\pi} \left[ \frac{c}{\sqrt{Gt}} \frac{c}{\sqrt{Gt}} \right] = \frac{\Phi^2}{8\pi} = \frac{c}{\sqrt{Gt}},
\]

where \( \Phi \) is the intensity of a special quaternionic field [12] [13] [11] which is given by the field tensor \( F_{ij} \) \( i, j = 1, 2, 3, 0 \) whose components are defined as \( F_{ij} = 0 \) for \( i \neq j \) and \( F_{11} = F_{22} = F_{33} = -F_{00} = \Phi \). The \( \Phi \)-field belongs to the family of the quaternionic fields (see [12]). The quaternionic field which we consider is given by the field tensor which, in the matrix, has the form

\[
F_{ij} = \begin{pmatrix}
\Phi & 0 & 0 & 0 \\
0 & \Phi & 0 & 0 \\
0 & 0 & \Phi & 0 \\
0 & 0 & 0 & -\Phi
\end{pmatrix}.
\]

\( \Phi \) is the only field variable in it. \( F_{ij} \) is a symmetric field tensor with the components \( F_{ii} = \Phi \) \( i = 1, 2, 3 \), \( F_{ii} = -\Phi \) \( i = 0 \), and \( F_{ij} = 0 \) \( i \neq j \). It is easily to show that \( \Phi \) is transformed as a scalar under Lorentz transformation [16]. The field equations of the \( \Phi \)-field in the differential are

\[
\nabla \Phi = k \vec{J} \quad i = 1, 2, 3 \quad \text{and} \quad -\frac{1}{c} \frac{\partial \Phi}{\partial t} = k_0 J_0, \quad i = 0,
\]

where

\[
k = \frac{4\pi \sqrt{G}}{c} \quad \text{and} \quad k_0 = 8\pi \sqrt{G}.
\]
These equations are first-order differential equations whose solution can be found given the source terms. Assuming the spatial homogeneity of the $\Phi$-field it becomes independent of spatial coordinates therefore it holds $J_1 = J_2 = J_3 = 0$. The source of the $\Phi$-field is its own mass density associated with the field energy density, i.e. $\Phi^2/8\pi c^2$, therefore, it holds $J_0 = k_0 \Phi^2/8\pi c^2$. $J_0$ is dependent only on time. The energy density associated with the field is

$$E_\Phi = \frac{\Phi^2}{8\pi}.$$ 

Since the current 4-vector in the everywhere local rest frame has only one non-zero component, $J_0$, Eqs.(3) become

$$\nabla \Phi = 0$$

$$\frac{1}{c} \frac{d\Phi}{dt} = \frac{4\pi \sqrt{G}\Phi^2}{8\pi c^2} = \frac{\sqrt{G}\Phi^2}{2c^2}.$$ 

whose solution is

$$\Phi(t) = \frac{c}{\sqrt{G}(t + t_0)},$$

where $t_0$ is the integration constant given by the boundary condition. $\Lambda \propto 1/t^2$ has been considered by several authors with different physical motivations, e.g. Lau [6] adopted the Dirac large-number hypothesis of variable $G$, Kendo and Fukui [3] and others operated in the context of a modified Brans-Dicke theory etc.

In analogy with the electromagnetic field, the quaternionic field acts on the moving ”charged” objects with the Lorenz-like force. In [16] we supposed that the $\Phi$-field interacts with all form of energy and matter and the coupling constant $k$ is from the dimensional reason equal to $\sqrt{G}$. The ”charge” of the $\Phi$-field for a point mass $m_0$ is $\sqrt{G}m_0$. Since the momentum of a moving particle is $p_i = m_0v_i$, $i = 1, 2, 3, 0$, its current is given as $J_i = \sqrt{G}m_0v_i = \sqrt{G}p_i$. For the Lorenz-like force acting on this particle in the $\Phi$-field we get

$$F_i = c^{-1}\sqrt{G}m\Phi v_i = c^{-1}\sqrt{G}\Phi p_i.$$ 

In what follows we shall study the possible effect of the $\Phi$-field on the moving bodies in solar and galactic conditions on the large time scale.

2 Force of the $\Phi$-field exerted on the moving bodies

It is to be expected that the cosmical $\Phi$-field manifests itself in the present-day solar and galactic physical conditions only: (i) at the large mass concentrations, (ii) at the large velocities of massive objects (iii) during the large time and space scales. For the sake of simplicity, we confine ourselves to the non-relativistic case, i.e. we suppose that $m =$ const. and $v \ll c$. Then Eq.(6) turns out to be

$$m\dot{v} = c^{-1}\sqrt{G}\Phi mv.$$ 

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Below, we present three possible effects of the quaternionic field in the solar and galactic conditions:

(i) The increase of the velocity of the moving bodies in the $\Phi$-field. Since $c^{-1}\sqrt{G}\Phi = 1/t$ we get a simple differential equation $\dot{v} = \beta v$ where $\beta = 1/t$ the solution of which is $v = Ct$. A free moving object in the quaternionic field is accelerated by a constant acceleration $C$. This acceleration is due to the immense smallness of $\beta \approx 1/10^{18}$ in the present-day extremely small. As is well-known for a given time instant the Hubble constant $H$ is equal in the whole Universe. Supposing $\beta|_0 = H$, the solution of equation $\dot{v} = Hv$ becomes $v = Hr + C$, where $C$ is an integration constant. Setting $C = 0$ we get a Hubble-like law $v = Hr$.

(ii) The increase of the kinetic energy of the moving bodies in the $\Phi$-field. The gain of kinetic energy of a moving body per time unit in the quaternionic field if $(f_i \parallel v_i)$ is

\[
\frac{dE}{dt} = F_i v_i = c^{-1}\sqrt{G}\beta m v^2 = 2\sqrt{G}c^{-1}\Phi E_{\text{kin}} = 2\beta E_{\text{kin}}. \quad (9)
\]

Again, the increase of the kinetic energy of a moving object is extremely small. However, for a rapid rotating dense body it may represent a considerable value. For example, a pulsar rotating around its axis with the angular velocity $\omega$ having the moment of insertion $I$. Its kinetic energy is $E_{\text{kin}} \approx I \omega^2$ and its change in the quaternionic field is $dE_{\text{kin}}/dt \approx \beta I \omega^2 \approx 10^{32}$ which is a value only of some orders of magnitude smaller than the energy output of a pulsar [20].

(iii) The change of the kinetic parameters of the gravitationally bounded moving bodies. This can be best demonstrated by describing the motion of the Earth around the Sun taking into account Eq.(7). It holds

\[
\vec{F}_1 + \vec{F}_2 = -\frac{GM_{\odot} m_{\oplus}}{r^2} \vec{r}, \quad (10)
\]

where $\vec{F}_1 = m_{\oplus} \vec{r}$ and $\vec{F}_2 = m_{\oplus} \beta \vec{r}$. Inserting $\vec{F}_1$ and $\vec{F}_2$ in Eq.(10) we have

\[
\frac{d}{dt}(\beta \vec{r}) = -\beta \frac{GM_{\odot}}{r^2} \vec{r}
\]

from which it follows

\[
\beta r^2 \dot{\phi} = \text{const.} = h \quad (11)
\]

The reciprocal radius $u$ satisfies the equation

\[
\frac{d^2u}{d\phi} + u = \beta^2 \frac{GM_{\odot}}{h^2},
\]

the general solution of which is

\[
u(\phi) = \frac{C}{r^2} + c_2 \cos(\phi) - c_1 \sin(\phi)
\]
Setting $c_1 = c_2 = 0$, i.e. supposing that the orbit is circle we get

$$r \sim K t^2$$

(12)

Hence, the distance of the Earth and the Sun varies with time like

$$r \sim \frac{1}{GM_\odot \beta^2} \sim \frac{1}{\beta^2} \sim t^2.$$  

(13)

According to Eq.(13) the distance between the Earth and the Sun is increasing direct proportional to the square of time. There have been several atomic time measurements of the period of the Moon orbiting around the Earth. A description of the work of some independent research groups can be found in Van Flander’s article [18]. We simply point that after subtracting the gravitational perturbative (tidal) effects, Van Flanders gives

$$\frac{\dot{P}}{P} = \frac{\dot{n}}{n} = (3.2 \pm 1).10^{-10}/yr,$$

where $n = 2\pi/P$ is the angular velocity. Using Eq.(11) we get comparable value

$$|\dot{n}/n| \approx 4.10^{10}/yr.$$  

However, given the complexity of the data analysis, we must certainly await further confirmation by different, independent test before concluding whether the $\Phi$-field really affects the motion of cosmic bodies. Nevertheless, it can be asserted that at present, there exists no evidence against the influence of the $\Phi$-field on the moving bodies at the level of the supposed present-day intensity of the $\Phi$-field. Due to large value of the cosmic time the present-day effects of the $\Phi$-field lei on the limit of the observability. However, they had, probably, strong influence in the early universe. For example, the strong $\Phi$-field can destabilize the large rotating mass concentration (e.g. quasars) forming from them the present-day galaxies. We note that the enlargement of distance of the Earth and the Sun is also suggested by the large numbers hypothesis presented by Dirac in 1937 (for detail see [17]). In this hypotheses Dirac supposed that $G \propto 1/t$. The astrophysical and geological consequences of this hypothesis are discussed in details in [19][15].

3 The creation of particle in the $\Phi$-field

Another interesting effect of the cosmic $\Phi$-field is the possibility of the creation of real particles from the virtual ones. Particle creation in nonstationary strong fields is well-known phenomenon studied intensively in seventies (see, e.g.,[14]). There are several proposed ways for the creation of real particles from the virtual ones in the very strong and nonstationary gravitation field. We propose here a new mechanism of the creation of real particles from the virtual ones in the presence of the $\Phi$-field.
We note that our further consideration on the creation of matter from the vacuum quantum excitation are done by a semiclassical way, although we realize that they should be performed in terms of an adequate theory of quantum gravity. However, as is well-known, when constructing a quantum theory of gravity one meets conceptual and technical problems. The usual concepts of field quantization cannot be simple applied to gravity because standard field quantization (e.g., eld field) is normally done in flat spacetime. It is impossible to separate the field equations and the background curved space because the field equations determine the curvature of spacetime. Moreover, the classical quantized field equations are linear and that of gravitation are non-linear and weak-field linearized gravitation field is not renormalizable. There is even no exact criterion on which time and space scales one has necessary to apply quantum laws for gravitation field. Therefore, we take the inequality $|A| \leq h$, where $A$ is the classical action, as a criterion for a possible application of quantum physics in gravity.

According to quantum theory, the vacuum contains many virtual particle-anti-particle pairs whose lifetime $\Delta t$ is bounded by the uncertainty relation $\Delta E \Delta t > h$ [21]. The proposed mechanism for the particle creation in the $\Phi$-eld is based on the force relation (6). During the lifetime of the virtual particles the Lorentz-like force (6) acts on them and so they gain energy. To estimate this energy we use simple heuristic arguments. As is well-known, any virtual particle can only exist within limited lifetime and its kinetics is bounded to the uncertainty relation $\Delta p \Delta x > h$. Therefore, the momentum of a virtual particle $p$ is approximately given as $p \approx h \Delta x^{-1}$. If we insert this momentum into Eq.(6) and multiply it by $\Delta x$, then the energy of virtual particle $\Delta E$, gained from the ambient $\Phi$-eld during its lifetime, is

$$F \Delta x = \Delta E = \sqrt{G \Phi(t)} \frac{h}{c}.$$  \hspace{1cm} (15)

When the $\Phi$-eld is sufficiently strong then it can supply enough energy to the virtual particles during their lifetime and so spontaneously create real particles from the virtual pairs. The energy necessary for a particle to be created is equal to $m_v c^2$ ($m_v$ is the rest mass of the real particle). At least, this energy must be supplied from the ambient $\Phi$-eld to a virtual particle during its lifetime. Inserting $\Phi$ into Eq. (15), we have

$$\Delta E \approx \frac{h}{(t + t_0)},$$

Two cases may occur: (i) If $m_v c^2 < \Delta E$, then the energy supplied from the $\Phi$-eld is sufficient for creating real particles of mass $m_v$ and, eventually, gives them an additional kinetic energy. (ii) If $m_v > c^2 \Delta E$, then the supplied energy is not sufficient for creating the real particles of mass $m_v$ but only the energy excitations in vacuum. The additional kinetic energy of the created particles, when $\Delta E > m_v c^2$, is

$$E_{kin} = \Delta E - m_v c^2 = \frac{h}{(t + t_0)} - m_v c^2.$$
As is well-known in Friedmann’s cosmology the scale parameter \( R \) varies in time according to the equation

\[
\ddot{R} = \frac{4\pi G}{3} \left[ -\rho - 3p + 2\Lambda \right] R.
\]  

(18)

If we assume that at the very beginning of the cosmical evolution no ordinary matter was presented only the energy of the cosmic \( \Phi \)-field then we have

\[
\ddot{R} \approx 2\Lambda R.
\]

The solution to this equation describes the exponential expansion of the early universe

\[
R(t) \approx A \exp(2\Lambda t).
\]

This inflationary phase of the cosmological evolution is supposed to stop when an massive creation of mass particles in the \( \Phi \)-field began by means of mechanism described above. The proposed cosmological evolution started with purely field-dominated epoch during which the inflation took place, after which a massive creation of particles began. During the time interval \((\approx 0, 10^{-20})\), the masses of the created particles lie in the range from \(10^{-5}\) to \(10^{-27}\) g. Their kinetic energy was \( E_{\text{kin}} = [h/(t + t_0)] - m_0 c^2\). \( E_{\text{kin}} \) of the created nucleons has reached values up to \(10^{-5}\) erg, which corresponds to the temperature of \(10^{21}\) K. Today, energies of the virtual pairs, gained during their lifetime, are immense small, therefore, they represent only a certain local energy excitations of the vacuum.

4 Conclusion

From what has been said so far it follows:
(i) The \( \Phi \)-field exerts a force on the moving cosmic bodies which is given by Eq.(6).
(ii) This force has negligible effects on moving bodies in the present time but might be important in the early stage of cosmological evolution.
(iii) A creation of real particle occurs in the \( \Phi \)-field which is proportional to intensity of the \( \Phi \)-field.

Summing up, we can state that if the cosmic quaternionic field does exist then it affects the kinetic parameters of moving bodies, causes the creation of real particle from virtual one and enormously enlarges the temperature of the early universe.

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