Infrared Variability due to Magnetic Pressure Driven Jets, Dust Ejection and Quasi-Puffed-Up Inner Rims

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ABSTRACT

The interaction between a YSO stellar magnetic field and its protostellar disc can result in stellar accretional flows and outflows from the inner disc rim. Gas flows with a velocity component perpendicular to disc midplane subject particles to centrifugal acceleration away from the protostar, resulting in particles being catapulted across the face of the disc. The ejected material can produce a “dust fan”, which may be dense enough to mimic the appearance of a “puffed-up” inner disc rim. We derive analytic equations for the time dependent disc toroidal field, the disc magnetic twist, the size of the stable toroidal disc region, the jet speed and the disc region of maximal jet flow speed. We show how the observed infrared variability of the pre-transition disc system LRLL 31 can be modelled by a dust ejecta fan from the inner-most regions of the disc whose height is partially dependent on the jet flow speed. The greater the jet flow speed, the higher is the potential dust fan scale height. An increase in mass accretion onto the star tends to increase the height and optical depth of the dust ejection fan, increasing the amount of 1–8 \( \mu \)m radiation. The subsequent shadow reduces the amount of light falling on the outer disc and decreases the 8–40 \( \mu \)m radiation. A decrease in the accretion rate reverses this scenario, thereby producing the observed “see-saw” infrared variability.

Key words: accretion discs – protoplanetary discs – magnetohydrodynamics – radiative transfer – stars: jets – stars: variables: T Tauri, Herbig Ae/Be

1 INTRODUCTION

Protostars undergo a well-documented, although not fully understood, evolution from a collapsing cloud of gas and dust through to planets orbiting an evolving star (Williams & Cieza 2011). The intervening stages are associated with the formation of a disc of gas and dust surrounding the protostar, where the disc may evolve from a continuous disc to one with an inner hole or an optically thin gap between optically thick inner and outer discs. These latter discs are known as pre-transition and transition discs respectively as they are in a transition phase between a continuous disc to a debris disc from which most of the gas has been removed (Espaillat et al. 2014). The pre-transition disc LRLL 31 is located in the 2-3 Myr old star forming region IC 348 some 315 pc from the Earth. Current observations suggest that LRLL 31 is a G6 star with a rotation period of 3.4 days (Flaherty et al. 2011), a luminosity of 5.0 \( L_\odot \), a radius of about 2.3 \( R_\odot \), a mass of approximately 1.6 \( M_\odot \), and an effective temperature of 5700 K (Pinilla et al. 2014). In addition, observations with the Spitzer Space Telescope show a “see saw” oscillation in the system’s infrared spectrum (Muzerolle et al. 2009). That is, when the flux from the 5 to 8.5 \( \mu \)m range increases then the flux from 8.5 to 40 \( \mu \)m decreases on timescales of weeks (Figure 1).

At wavelengths between 1 and 5 \( \mu \)m, assumed to result from dust emission in the inner disc, a significant change in the LRLL 31 infrared excess can occur daily. As an example, Figure 2 shows the approximate factor of two increase in infrared excess from the LRLL 31 inner disc wall between the 31st of October and the 4th of November 2009. The flux from the inner wall of a disc surrounding a star, \( F_{\text{rim}} \), is approximately given by

\[
F_{\text{rim}} \approx \frac{4\pi R_{\text{rim}} H_{\text{rim}} B_\lambda(T_{\text{rim}})}{d^2} \sin(i),
\]

where \( d \) is the distance between the disc and the observer, \( R_{\text{rim}} \) the distance from the inner disc wall to the centre of the star, \( H_{\text{rim}} \) the height of the inner rim wall measured from the disc midplane to the top of the wall, \( B_\lambda(T_{\text{rim}}) \) the blackbody radiation from the inner disc wall, which is at a
temperature of $r_{\text{rim}}$, and $i$ is the disc inclination angle. In the case of LRLL 31, the disc is thought to be nearly edge on to the observer with $i \sim 90^\circ$ (Flaherty & Muzerolle 2010).

According to Flaherty et al. (2011), analysis of the infrared excess indicates that $r_{\text{rim}}$ and hence $R_{\text{rim}}$ probably remained approximately constant. Thus, to explain the factor of two increase in the infrared excess, equation 1 implies that $H_{\text{rim}}$ increased by around a factor of two during the four days separating the 31st of October and the 4th of November 2009. So again from Flaherty et al. (2011), observations would imply that the covering fraction of the inner disc relative to the central star increased by approximately a factor of five over the course of a month: from $\sim 0.01$ (8th October 2009) to $\sim 0.054$ (8th November 2009), where a possible description of the covering fraction, $f_{\text{cover}}$, is given by the approximate formula:

$$f_{\text{cover}} \approx \frac{2}{\pi} \tan^{-1} \left( \frac{H_{\text{rim}}}{R_{\text{rim}}} \right). \tag{2}$$

Thus, in that one month, the value of $H_{\text{rim}}$ has possibly increased by about a factor of four. At the same time, the deduced mass accretion rate from the disc onto the star also increased by around a factor of four from $\sim 0.4 \times 10^{-8} M_\odot \text{yr}^{-1}$ (8th October 2009) to $\sim 1.6 \times 10^{-8} M_\odot \text{yr}^{-1}$ (8th November 2009) (ibid.).

Other authors have modelled transition disc systems and concluded that parts of the infrared variability might be explained by variation in the height of the inner disc rim, e.g., Juhász et al. (2007) and Sitko et al. (2008). Sitko et al. (2008) also examined disc winds as a possible explanation.

As the inner 0.1 au inner regions of protostellar discs tend to be too small to be resolved with current capabilities (e.g., if 5 mas resolution interferometers such as the Atacama Large Millimetre Array could be used on LRLL 31 then the effective resolution would be $\sim 1.5$ au). Consequently, only indirect data and modelling are available to understand the major physical mechanism that is producing the observed infrared variability in LRLL 31. At least nine separate models have been suggested to explain the deduced variation in scale height of the inner disc. These models range from higher accretion rates that increase the scale height of the inner disc, through to asymmetric, dynamic, warped inner discs, hidden planetary companions, inner disc radial fluctuations and magnetic field effects (Flaherty et al. 2011).

While it is possible that some or all of these explanations may be applicable to particular young stellar systems, in this paper we will attempt to explain the observations as a by-product of the interaction of the stellar magnetic field with the inner disc. Other authors, e.g., Turner et al. (2010), have considered magnetic fields within the disc associated with disc turbulence and the magnetorotational instability. However, they ignored their own jet flow results, so their subsequent deduced variations in the scale height are too small to account for the potential factor of four changes in scale height for LRLL 31.

It is possible that accretional flows via the stellar magnetosphere are sufficiently opaque to produce shadowing on the outer disc. For example, Kulkarni & Romanova (2008) show very interesting numerical examples of such flows, but whether they can produce the observed infrared variability is uncertain as such flows tend to vary on a shorter timescale relative to the observed infrared variability.

Lai & Zhang (2008) modelled the effect of a tilted, rotating stellar magnetic field on the inner region of the disc. They find that waves are produced in the disc, which produces semi-periodic changes in the disc height. They note that such a model may be applicable to neutron star systems, but, to date, infrared variability in LRLL 31 does not seem to be periodic. This may change with more observational data, but at this stage the Lai & Zhang model does not appear to be an applicable mechanism.

The interaction between a stellar magnetosphere and a surrounding accretion disc (Figure 5) produces a significant disc toroidal magnetic field (Matt & Pudritz 2005; Matt et al. 2010) on a probable timescale of hours to days (equation 14 and Appendix A). The toroidal field produces a magnetic pressure (Figure 9 and Appendix B) which may move material away from the disc surface to accrete onto the star or be ejected as an outflow (Zanni & Ferreira 2013). Under certain circumstances (Romanova et al. 2009, 2018), the outflow is produced within a small region at the inner edge.
of the accretion disc (equation 24), a result that is consistent with observations (Lee et al. 2018).

We derive an analytic formula for the jet flow speed near the surface of the inner disc (equation 25). The jet flow speed tends to increase with decreasing distance to the star with the exception of the region near the co-rotation radius where the jet flow turns off (Figure 10).

Numerical simulations for protostellar systems (Zanni & Ferrera 2013; Romanova et al. 2018) and collapsing cloud cores (Price et al. 2012) show that the resulting magneto-hydrodynamic (MHD) jet flows move at an angle relative to the disc midplane. In this study we are interested in the motion of particles that are launched by the jet flows, so for the purposes of establishing a base case scenario, we assume that the jet flow initially moves perpendicular to the disc midplane. By computing the motion of dust grains in the flow, we find that the dust can decouple from the flow and move radially across the face of the disc (Figure 13). A result that is consistent with observations from the Spitzer Space Telescope (Juhász et al. 2012). The resulting dust fan may mimic the appearance of a puffed up inner rim (Figure 14) and possibly account for the observed behaviour of LRL 31 (5). Observations, however, also indicate that the inner edge of the protostellar accretion disc is often within the dust sublimation radius (Carr 2007). This poses a problem for our model, since there may be no dust particles to be entrained in the flow. Again, observations strongly suggest that dust is entrained in outflows from young stars (Petrov et al. 2019). So some dust or macroscopic particles may still be present in the inner disc regions and/or dust condenses in the flow just like dust formation in stellar winds once the flow has moved past the dust sublimation distance (Sedlmayr & Dominik 1995). Either way, our model requires dust to be present at some point in the outflow as it moves away from the inner edge of the accretion disc.

2 LRL 31

In attempting to understand the behaviour of the LRL 31 inner disc rim, it is helpful to obtain some length scales of the inner LRL 31 disc and star. An important disc length scale is the dust temperature radius of the inner rim, \( R_d \). This is the distance between the inner disc rim and the centre of the protostar required to obtain a dust temperature. It has the approximate formula (Espaillat et al. 2010):

\[
R_d \approx \frac{3(L_\star + L_d)}{16\pi\sigma_{SB} T_d^4} \approx 0.13 \text{ au} \sqrt{(L_\star + L_d)/5L_\odot (T_d/1500 \text{ K})^4} \tag{3}
\]

where \( \sigma_{SB} \) is the Stefan-Boltzmann constant, \( T_d \) is the temperature of the dust, \( L_\star \) is the luminosity of the star (in this case \( 5L_\odot \)) and \( L_d \) is the accretion luminosity given by:

\[
L_d = \frac{G M_* M_{\dot{m}}}{R_*} \left[ 1 - \frac{R_a}{2R_*} \right] \approx 0.15 L_\odot \left( \frac{M_*}{M_\odot} \right) \left( \frac{M_{\dot{m}}}{10^{-8} M_\odot \text{ yr}^{-1}} \right) \left[ 1 - \frac{R_a}{2R_*} \right]. \tag{4}
\]

From equation 4, for LRL 31, \( L_d \approx L_\star \).

Flaherty et al. (2011) suggest an average dust temperature of \( 1800 \text{ K} \) at the inner rim of the LRL 31 disc. This implies that \( R_d \) is approximately equal to 0.09 au.

Another relevant length scale is the truncation radius, \( R_t \), of the inner disc, which is the distance between the star and the inner edge of the disc as a function of mass accretion rate and stellar magnetic field strength. The inner truncation radius is produced by the approximate pressure balance between the infalling accretion disc and the stellar magnetosphere (Ghosh & Lamb 1978), given by:

\[
R_t \approx \left( \frac{4\pi R_*^2 c^6}{\mu_0 M_\odot \sqrt{GM_*}} \right)^{2/7} \approx 0.067 \text{ au} \left( \frac{R_\star/0.15 T_\star}{(M_d/10^{-8} M_\odot \text{ yr}^{-1})(M_\star/0.1 M_\odot)^{1/2}} \right)^{2/7} \tag{5}
\]

where \( R_\star \) is the radius of the star, \( M_d \) the mass accretion rate onto the star, \( M_\star \) the mass of the star, \( \mu_0 \) the permeability of free space, \( G \) the universal gravitational constant, and \( B_\star(R_\star) \) the magnetic field strength at the surface of the star.

For LRL 31, the magnetic field strength at the surface of the star is unknown. However, the average magnetic field strength for protostellar systems is in the kilogauss range (Bouvier et al. 2007). As such, if we set \( B_\star(R_\star) \approx 0.15 T_\star, R_\star \approx 2.3 R_\odot, \) and \( M_\star \approx 1.6 M_\odot \), which implies that a mass accretion rate of \( M_d \approx 0.4 \times 10^{-8} M_\odot \text{ yr}^{-1} \) gives \( R_t \approx 0.13 \) au, while \( M_d \approx 1.6 \times 10^{-8} M_\odot \text{ yr}^{-1} \) gives \( R_t \approx 0.09 \) au. The latter distance is also the deduced dust temperature radius for a mass accretion of \( M_d \approx 1.6 \times 10^{-8} M_\odot \text{ yr}^{-1} \) (Flaherty et al. 2011).

The rotational angular velocity of the star, \( \Omega_\star \), sets the co-rotation radius, which is the distance from the centre of the star where the Keplerian angular velocity, \( \Omega_K(r) \), equals the stellar rotational angular velocity:

\[
\Omega_\star = \Omega_K(R_\co). \tag{6}
\]

with

\[
\Omega_K(r) = \sqrt{\frac{GM_\star}{r^3}}. \tag{7}
\]

\( r \) being the cylindrical radial distance from the star. These equations imply:

\[
R_\co = \left( \frac{GM_\star}{\Omega_\star^2} \right)^{1/3} \approx 0.05 \left( \frac{M_*}{1.6 M_\odot} \right) \left( \frac{P_\star}{3.4 \text{ days}} \right)^{1/3} \text{ au}. \tag{8}
\]

Thus for LRL 31, the co-rotation radius is \( \approx 0.05 \) au from the star. The stellar radius is \( 2.3 R_\odot \approx 0.01 \) au. The distance length scales are summarised in Figure 3, where the inner disc scale heights are calculated from the deduced covering fraction (equation 2). It is of interest to compare the deduced heights of the LRL 31 inner rim with the standard, isothermal scale height, \( h(r) \), of an accretion disc:

\[
h(r) = \frac{2k_B T_g r^3}{G M_\star \dot{m}} \approx 0.0043 \text{ au} \sqrt{(T_g/1000 \text{ K}) (r/0.1 \text{ au})^3} \left( \frac{M_\star}{M_\odot} \right) \left( \frac{\dot{m}}{m_\text{H}} \right) \tag{9}
\]

where \( k_B \) is Boltzmann’s constant, \( T_g \) the gas temperature, \( \dot{m} \) the mass of the hydrogen atom, and \( m_\text{H} \) the mean molecular mass of the gas. This comparison is shown in Figure 4, where the observed inner rim heights for the lower mass accretion rates (data from Flaherty et al. 2011) are smaller, but comparable to the expected isothermal scale height. However, the inner rim heights for the higher mass accretion
rates are significantly higher (over a factor of four in one case) relative to the expected isothermal scale heights, and LRLL 31 has a puffed up inner rim. Such puffed up rims appear to be common in young stellar systems, but a comprehensive explanation for how they are produced has eluded researchers (Vinković 2014). As such, LRLL 31 infrared variability could be linked to the production of puffed up inner disc rims.

In calculating the isothermal scale height, the gas temperature is assumed to be approximately the same as the disc surface temperature, $T_{\text{disc}}(r)$, where the temperature of an optically thick, flat disc subject to stellar radiation (Friedjung 1985; Hartmann 1998) and differential friction in the accretion disc (Frank et al. 2002) is

\[
T_{\text{disc}}(r) \approx \frac{(L_\star + L_\mathrm{w})}{4\pi^2\sigma_{SB}R_\star^2} \left( \frac{1}{r} \right) - \frac{R_\star}{r} \sqrt{1 - \frac{R_\star}{r^2}} + \frac{3GM_\star M_d}{8\pi^2\sigma_{SB}} \left( 1 - \frac{R_\star}{r^2} \right)^{1/4}. \tag{10}
\]

The parameters used for the LRLL 31 calculations are shown in Table 1.

To produce a physical model for puffed-up inner rims, we first consider a model for the interaction between the stellar magnetosphere and the surrounding disc.

### 3 STELLAR MAGNETOSPHERE-DISC INTERACTION

#### 3.1 Magnetic Disc Height and Scale Height

If a co-rotating, stellar magnetosphere interacts with a surrounding disc of gas and dust then a radial disc current, $j_r$, is generated in the disc (Figure 5), where the current has the steady state form (Liffman & Bardou 1999):

\[
j_r = -\sigma_D(r)(\Omega_\star - \Omega_K(r)) B_\star(z) \overline{r}, \tag{11}
\]

where

\[
|B_\star(z)(r)| \approx B_\star(R_\star)(R_\star/r)^3. \tag{12}
\]
with \( \sigma_D \) is the disc electrical conductivity, \( B_{z} \) the \( z \) component of the stellar magnetic field at the midplane of the disc, and \( \hat{r} \) the unit vector in the \( r \) direction.

The radial disc current generates a toroidal magnetic field in the disc with the steady state form \( \left( B_{\phi}, \sigma \right) \) with \( \sigma \) the electrical conductivity of the disc.

\[
B_{\phi}(r, z) = \mu_0 \sigma_D(r) r \Omega(z - \Omega_k(r)) B_z(r, \hat{\phi}),
\]

with \( z \) the perpendicular distance from the midplane of the disc. In equation 13, \( z = 0 \) is located at the midplane of the disc. For this equation, \( z \) has a magnitude that is less than or equal to the height of the disc.

One can show (Appendix A) that, in principle, the disc toroidal field grows to its steady state value on a timescale, \( \tau_B \), which has the approximate form

\[
\tau_B \approx \frac{\mu_0 \sigma_D(r) h(r)^2}{2} \approx 1.6 \frac{\sigma_D}{10^{-5} \text{ S m}^{-1}} \frac{h}{0.001 \text{ au}}^2 \text{ days}.
\]

In practice, the toroidal field may not reach its steady state value due to field instabilities - for example the inflation of the field that arises when the toroidal field and poloidal field strengths become comparable (Newman et al. 1992; Lovelace et al. 1995).

The interaction between the radial disc current and the generated toroidal field produces a Lorentz compressive \( j \times B \) force on the disc in the \( z \) direction that is directed towards the midplane of the disc. For a disc that is approximately isothermal in the \( z \) direction, this Lorentz compression changes the standard isothermal density profile to (Liffman & Bardou 1999)

\[
\rho(r, z) = \rho_c(r) \exp \left( -\frac{z}{h(r)} \right) - \rho_\infty \left( 1 - \exp \left( -\frac{z}{h_\infty} \right) \right),
\]

with \( \rho_c(r) \) the midplane mass density of the disc gas, \( h(r) \) the standard isothermal scale height, and

\[
\rho_\infty = \mu_0 \sigma_D^2 r^2 B_z^2(r) \frac{\Omega_k(r)}{2 \Omega(z - \Omega_k(r))}.
\]

The first term in equation 15 is the standard isothermal density profile, while the second term introduces the magnetic compression of the disc. The combination of the two terms produces a magnetic disc height due to the sharp cut off in the disc at a distance, \( H_B \), from the midplane of the disc, where

\[
H_B(r) = \frac{h(r)}{2} \ln \left( 1 + \frac{\rho_c(r)}{\rho_\infty} \right).
\]

From equation 16, if \( \sigma_D \to 0 \) and/or \( B_z \to 0 \), then \( \rho_\infty \to 0 \) and equation 15 returns to the standard isothermal disc density profile with \( H_B \to \infty \). The latter property, while physically correct, is mathematically inconvenient and it is useful to define a magnetic scale height, \( h_B \), where \( \rho(r, h_B) = 0 \).
3.2 Magnetic Compression and Disc Conductivity

This magnetic compression effect (obtained, independently, via different derivations by Lovelace et al. (1986); Campbell & Heptinstall (1998) and Liffman & Bardou (1999)) is dependent on the disc conductivity. In Figure 6, we show the decrease in values of $H_B$ and $h_B$ as a function of disc conductivity at the truncation radius of the disc, $R_{\star}$, for the mass accretion rate of $1.6 \times 10^{-8} M_\odot yr^{-1}$ (i.e., for $R_{\star} = 0.09$ au). A magnetically confined disc can suffer significant compression with increasing disc electrical conductivity.

For small disc electrical conductivities, $h_B$ approaches the standard isothermal disc height, $H_B$ is the cut-off of the disc density profile due to magnetic compression, which increases as $\sigma_D \rightarrow 0$.

As discussed in the previous sections, the inner disc of LRLL 31 appears to be “puffed up”, so discussion of magnetised disc compression would appear to be not relevant. However, there is also a wind-up of the toroidal field, (Ap

\section{3.3 Twist and Interaction Region}

From equation 13, the steady state twist of the disc magnetic field, $\gamma_B$, is

\begin{equation}
\gamma_B(r, z) = \frac{B_\phi(r, z)}{B_z(r, z)} = \frac{\mu_0 \sigma_D(r) r z (\Omega_\star - \Omega_K(r))}{10 \text{ days}}.
\end{equation}

An estimate of the maximum twist value as a function of distance from a star is

\begin{equation}
\gamma_{B_{\text{max}}}(r, z) \sim \gamma_B(r, h_B(r)) = \mu_0 \sigma_D(r) h_B(r) (\Omega_\star - \Omega_K(r))
\end{equation}

\begin{equation}
= \mu_0 \sigma_D(r) h_B(r) \Omega_\star \left(1 - \left(\frac{R_{\star}}{r}\right)^{3/2}\right)
\end{equation}

\begin{equation}
= 1.0 \left(\frac{\sigma_D}{10^{-7} \text{Sm}^{-1}}\right) \left(\frac{r}{0.05 \text{ au}}\right) \left(\frac{h_B}{0.001 \text{ au}}\right) \left(\frac{10 \text{ days}}{P_\star}\right).
\end{equation}

From equation 20 we see that the twist at the co-rotation radius, $R_{\star}$, is zero, but for the chosen representative values, it quickly increases as a function of distance $r$ from LRLL 31 to a value much larger than one. Thus, the wound-up toroidal field strength may be orders of magnitude greater than the magnetospheric poloidal field strength. This implies that the stellar dipole field may have expanded, opened and disconnected from the disc (e.g. Uzdensky et al. 2002). Alternatively, wound up, strong toroidal fields may produce collimated jet flows that are perpendicular to the disc midplane (Price et al. 2012).

It is of interest to note the particular region where the twist is possibly stable, i.e. $\gamma_B < 1$. To do this, we set

\begin{equation}
\gamma_{B_{\text{max}}}(r, z) \sim \gamma_B(r, h(r)) = \mu_0 \sigma_D(r) h(r) (\Omega_\star - \Omega_K(r)).
\end{equation}

and let

\begin{equation}
r = R_{\star} + \Delta r,
\end{equation}

where we assume that $\Delta r \ll R_{\star}$. Substituting equation 22 into equation 21 gives

\begin{equation}
\gamma_B(\Delta r, h(r)) = \frac{3 \mu_0 \sigma_D |\Delta r|}{2} \sqrt{\frac{2k_B T}{m}}.
\end{equation}
The interaction of a stellar magnetic field with an accretion disc produces a radial disc current, $B$, that compresses the inner disc and may, simultaneously, produce a magnetic pressure driven outflow with a velocity $v$. Our jet model implicitly assumes that the jet is produced at or near the surface of the disc and that a small toroidal magnetic field would be carried away with the flow. As the jet moves away from the disc, it is likely to expand in the radial direction (Lovelace et al. 1991). The jet speed will also decrease as it moves away from the star.

Setting $\gamma_B = \gamma_c = 1$, where $\gamma_c$ is the ‘critical’ twist (that is the toroidal disc field becomes comparable to the stellar magnetospheric field), gives

$$|\Delta_\gamma| \approx \frac{2\gamma_c}{3\mu_0\sigma_D} \sqrt{\frac{\dot{m}}{2k_B T_g}}$$

$$\approx 1.0 \times 10^{-4} \text{au} \left( \frac{10^{-5} \text{Sm}^{-1}}{\sigma_D} \right) \sqrt{\frac{\dot{m}/m_0}{T_g/1400 \text{ K}}}.
$$

In this case the directions of the magnetic fields and field points opening up at larger distances away from the star. This leads to some interesting disc twist stability regions which are discussed in Matt & Pudritz (2005).

We now suppose that the upper disc atmosphere allows a return current to flow. As discussed in Appendix B, this implies there exist separated layers of peak Pedersen or Hall conductivity that allow trans magnetic field currents to flow. One maximum of conductivity occurs within the disc, approximately at the disc midplane, while the other conductivity maxima occur on the top and bottom disc surfaces. In such a circumstance, the Lorentz force driven by the return currents, $j_{ret}$, and the toroidal fields, $B_\phi$, points away from the disc midplane and so material is forced to move away from the disc (Figure 9).

As derived in Appendix B, the speed, $v_{ex}$, of the outflow produced by the return current at or near the surface of the disc is approximately

$$v_{ex}(r, z_T) = \sqrt{\frac{\mu_0}{\bar{\rho}} \sigma_D(r) r z_0 \Omega_* |1 - (R_c/r)^{3/2}| |B_z(r)|},$$  

where $z_0$ is the distance (or altitude) from the disc midplane to the entrance of the jet flow, $z_T$ is the altitude of the jet flow exit, $\bar{\rho}$ is the average gas mass density within the jet flow, while the $r$ in this case is, approximately, the inner edge of the disc, i.e., $r \approx R_c$. The derivation of equation 25 implicitly assumes that $z_0$ and $z_T$ are $\ll r$. As a consequence,
this is the expression for the jet speed close to the top and bottom surfaces of the inner disc.

The form of equation 25 tells us that the jet flow speed goes to zero at \( \rho = \rho_0 \), because the stellar field does not wind up into a toroidal disc field at this point. Trivially, the jet flow speed goes to zero when

\[
\frac{\rho v_{\text{esc}}}{\rho_0} | R + \rho \Omega_{\star} R_{\text{co}} | B_\phi (R_{\text{co}}) \approx 1.45 R_{\text{co}}.
\]

For \( r < R_{\text{co}} \) then the jet speed approaches infinity as \( r \to 0 \). As such the maximum practical jet speed is when the disc is touching the stellar surface

\[
v_{\text{esc}}(R_{\text{co}}) \approx \frac{\rho_0}{\rho} | R_{\text{co}}^2 \Omega_{\star} R_{\text{co}} | B_\phi (R_{\text{co}}) | 1 - (R_{\text{co}}/R_{\text{co}})^{3/2} | B_\phi (R_{\text{co}}) |.
\]

We plot equation 25 as a function of \( r \) in Figure 10, which shows the jet flow speed tends to increase with decreasing distance from the star. Here we have assumed the following parameters: \( \rho_0 = 0.001 \text{ au} \), \( P_{\star} = 3.4 \text{ days} \), \( \sigma_{\text{D}} = 10^{-5} \text{ Sm}^{-1} \), \( \bar{\rho} = 10^{-7} \text{ kgm}^{-3} \), \( \rho_{\star}(R_\star) = 0.15 \text{ T} \), and \( R_{\text{co}} = 0.052 \text{ au} \). These representative values give flow speeds of order 10 to 1000 \text{ kms}^{-1}. For \( r > R_{\text{co}} \) the jet flow speed decreases for decreasing \( r \), while for the region \( R_{\text{co}} < r < R_{\text{co}} \), the jet flow speed increases as \( r \) decreases.

We plot in Figure 11 the jet speed as a ratio of the escape speed, \( v_{\text{esc}} \), where for material in a Keplerian orbit around a star:

\[
v_{\text{esc}}(r) = \sqrt{\frac{GM_{\star}}{r}}.
\]

which is simply the Keplerian speed.

The same parameters are used as for Figure 10, so for the given values of \( \rho_0 \), \( P_{\star} \), \( \sigma_{\text{D}} \), \( \bar{\rho} \), and \( B_{\star} \), the flow in Figure 11 reaches escape speed for the approximate range \( 1.3 < R_{\text{t}}/R_{\text{co}} < 2.1 \). Indeed, it can be shown that for \( r > R_{\text{co}} \), the ratio of the jet speed to the escape speed has a maximum at \( r_{\text{m}} = (7/4)^2/3 R_{\text{co}} \approx 1.59 R_{\text{co}} \). In Figure 11, we also plot the observed values of mass accretion rates in LLRL 31: 0.25, 0.4, 1.2, 1.5 and 1.6\times10^{-8} \text{ Myr}^{-1}. These accretion rates give approximate values of the inner accretion disc radius, \( R_{\text{t}} \), via equation 5. As can be seen from Figure 11, the deduced values of \( R_{\text{t}}/R_{\text{co}} \) from the observed accretion rates move towards the maximum flow speed at 1.59\( R_{\text{co}} \).

In terms of puffed up inner discs, it can be seen from Figure 4, that the maximum inner rim height occurs near 1.7\( R_{\text{co}} \), this is approaching the region where the jet speed relative to the escape speed is a local maximum.

As we will show in the next section, increasing jet ejection speeds forces dust particles to reach higher altitudes and produces regions of raised dust above and below the inner accretion disc that may have the appearance of puffed-up inner rims. So, higher mass accretion rates force the inner rim closer to the star thereby increasing the jet flow speed, which then increases the height of the ejected dust fan and increases the perceived height of the disc inner rim.
from the star (distance in units of the co-rotation radius). In this case, the jet flow exceeds the escape speed for the range $1.3 < R_t/R_{co} < 2.1$. The numbers on the observation points represent the mass accretion rate in units of $10^{-8} \, \text{Myr}^{-1}$. Accretion rates between 1.2 and $1.6 \times 10^{-8} \, \text{Myr}^{-1}$ are in the range where the jet speed is greater than the escape speed. In § 4 we show that jet flows with higher speeds eject dust to higher altitudes, where the resulting dust cloud may have the appearance of a puffed-up inner rim. This may explain the behaviour shown in Figure 4, where increasing mass accretion rates produce higher jet flow speeds which, in turn, produce higher inner disc rims.

4 DUST FANS AND THE CATAPULT EFFECT

In this section we examine the motion of particles that are entrained with a disc outflow or accretional inflow. As mentioned in the introduction, we assume that dust particles are present in the disc at or near the base of the flow, and that dust may condense in the flow in regions where the temperature and gas densities allow dust to nucleate from the gas.

Numerical simulations of MHD outflows show that the outflows tend to leave the disc at an angle which is not perpendicular to the disc midplane (Price et al. 2012; Zanni & Ferreira 2013; Romanova et al. 2018). However, as a base case, we assume that the initial direction of the entrained particles is perpendicular to the disc midplane. This is done to illustrate the potential effect of centrifugal acceleration moving the particles away from the flow direction.

4.1 Particle Ejection Model

In our model, the dust particles are initially in a circular Keplerian orbit at or near the inner truncation radius of the disc. As discussed in § 3.4, we assume that the accretional inflow onto the star and/or the protostellar jet will tend to flow away from the disc in a direction approximately perpendicular to the disc midplane. This will give the particles an initial ‘boost’ velocity that is assumed to be primarily in the $z$ direction. The subsequent motion of the particles is described by the equations given in Appendix C. Although the particles start with a Keplerian azimuthal velocity, the azimuthal velocity of the dust particles as they move above

4.2 Potential Projectile Motion

As an example of the potential projectile paths, Figure 13 shows the generic paths of ten, 1 mm diameter, silicate-like particles ejected with initial vertical speeds, $v_{py}$, of 0.05, 0.15, 0.25 ... 0.95 the local Keplerian orbital speed.

Figure 13(a) shows the case where the initial launch distance from LRLL 31 is assumed to be 0.0464 au, which is 90% of the co-rotation radius $R_{co} \approx 0.0516$ au. The particles are given a stellar co-rotation azimuthal velocity of 148.5 kms$^{-1}$ which is 85% of the local Keplerian orbital speed of 174.9 kms$^{-1}$. The first five particles (1 to 5), with vertical ejection speeds of 8.75, 26.2, 43.7, 61.2 and 78.7 kms$^{-1}$, fall back towards the star. The remaining five particles (ejection speeds: 96.2, 113.7, 131.2, 148.6 and 166 kms$^{-1}$) reach alti-
tudes comparable to the observed puffed up inner rims and fall back to the disc at distances further away from the star.

It is unknown whether stellar outflows or accretional infall can produce such high particle ejection speeds, but there is an obvious direct proportionality between the height of the projectile path above the disc midplane and the magnitude of the vertical ejection speed, $v_z$.

Figure 13(b) shows the case where the initial launch distance from the LRLL 31 is now set to 0.0567 au which is 110% of the co-rotation radius $R_{co}$. The particles are now given a stellar co-rotation azimuthal velocity of 181.5 kms$^{-1}$ or about 114% of the local Keplerian orbital speed of 158.2 kms$^{-1}$. In this case, all the particles (1 through 10) are ejected to larger distances with initial ejection speeds of 7.9, 23.7, 39.5, 55.4, 71.2, 87, 103, 119, 134 and 150 kms$^{-1}$.

In principle, it is relatively easy to obtain dust particle trajectories (Figure 13, Figure C1) that have similar heights to the maximum puffed-up inner rim heights shown in Figures 3 and 4. The height of the dust particle trajectory is simply dependent on the initial $z$ speed of the particle moving away from the disc midplane.

### 4.3 Dust Fan as a Puffed-Up Inner Rim

The particle paths given in Figure 13 are akin to a fan of dust particles emerging from the inner edge of the disc. This idea is displayed schematically in Figure 14. If the number density and size of the ejected dust grains is suitably high and large respectively, then the dust fan can become optically thick and may appear to a distant observer like a “puffed-up” inner rim of the disc.

Bans & Königl (2012) have suggested a similar scenario, where the puffed-up inner rim is produced by dust entrained in a protostellar jetflow from the inner regions of a disc surrounding a young star. Poteet et al. (2011) provided observational evidence that is consistent with the Bans and Königl model when they observed crystalline forsterite dust in the near neighbourhood of the young stellar system HOPS 68, where they suggest this dust was transported from the inner disc regions of HOPS 68 into the surrounding molecular cloud via a jet flow.

The model discussed here is different from the Bans and Königl model as we have the dust initially entrained in an accretion flow onto a star and/or a jet flow emerging from a disc surrounding a star, where the dust subsequently leaves either flow due to centrifugal forces and follows a ballistic path across the face of the disc. Dust in the Bans and Königl model tends to stay entrained in the flow.

Our “catapult” model is consistent with the observational results given in Juházs et al. (2012), who deduced that silicate dust was moving away from the protostar Ex Lup across the face of the disc at radial speeds of about 38 km s$^{-1}$. As discussed in Appendix C, our model can produce such radial speeds for ejected dust particles. For example, Figure C3 shows an extreme case where a radial speed of over 200 km s$^{-1}$ is obtained. It is also possible to obtain an approximate analytic equation that explains how a particular radial speed may be obtained from our particle ejection process (equation C23).

![Figure 13](image)

**Figure 13.** Trajectories for 1 mm diameter, silicate-like particles that are ejected perpendicular to the midplane. (a) Ten particles are ejected from $r = 0.9R_{co} = 0.0564$ au. Particle 1 to 5 have vertical ejection speeds of 8.75, 26.2, 43.7, 61.2 and 78.7 kms$^{-1}$, respectively, and these particles subsequently fall back towards the star. Particles 6 to 10 have vertical ejection speeds of 98.2, 113.7, 131.2, 148.6 and 166 kms$^{-1}$, respectively, and they reach altitudes comparable to the observed puffed up inner rims before they fall back towards the disc midplane away from the star. (b) Ten particles are ejected from $r = 1.1R_{co} = 0.0567$ au with speeds ranging from 7.9 to 150 kms$^{-1}$. For this case all the particles subsequently move away from the star.

### 5 DUST FAN SEDS

It is useful to examine whether a dust fan/puffed up inner disc is a feasible explanation for the “see saw” oscillation in the mid-infrared spectrum of LLRL 31 (Figure 1). To study this idea, we use the Monte Carlo radiative transfer code Hochunk3D (Whitney et al. 2003a,b, 2013) to simulate a protostellar system with a puffed-up inner rim.

In a later paper, we hope to simulate a bipolar outflow with a full dust fan. However, as a start we use a schematic replication of the dust fan effect by assuming that the dust is ejected in a thin channel or thin wall located at the inner edge of the disc, where the channel is perpendicular to the midplane of the disc, which is a slight modification of Figure 14.
Hochunk3D has an inbuilt model of a protostellar disc and makes provision for puffed inner rim walls through an additive scale height. This is added to the isothermal scale height:

\[ h_{\text{rim}}(r) = h_{\text{fid}} \left( H_{\text{rim}} \exp \left( \frac{r - R_t}{L_{\text{rim}}} \right) \right), \tag{29} \]

where \( h_{\text{rim}} \) is the puffed-rim scale factor, \( h_{\text{fid}} \) is the fiducial scale height (i.e. the isothermal scale height at radius \( r \)), and \( L_{\text{rim}} \) is the radial scale length for the puffed inner rim. In our case, we have \( R_t = 0.086 \text{ au}, \) \( h_{\text{fid}} = 0.00137 \text{ au} \) and \( L_{\text{rim}} = 0.01 \text{ au}. \) The first two values are representative for a LLRL 31 high accretion scenario, while the latter value is a tentative minimum thickness for a puffed up inner rim based on the simulations shown in Figure 13. These parameters and equation 29 provide us with a channel-like, puffed-up inner rim, as shown in Figure 15.

Figure 15 shows the density cross section of our model protostellar disc that surrounds LLRL 31. The inner disc is shown in Figure 15(a), while Figure 15(b) shows the gap in the disc between 1 and 15 au. The mass density ranges from \( 10^{-8.3} \text{ g cm}^{-3} \) to \( 10^{-20.3} \text{ g cm}^{-3}. \)

To a first approximation, the puffed up inner rim may be produced by a gas flow with a speed given by the parameterised form of equation C13

\[ v_{\text{g}} = \frac{M_\odot}{4\pi R_t \rho g \Delta} \left( \frac{M_\odot}{10^{-8} M_\odot} \right)^{y^{-1}} \left( \frac{R_t/0.1 \text{ au}}{\rho g/10^{-7} \text{ kgm}^{-3}} \right) \left( \frac{\Delta/0.01 \text{ au}}{0.00137} \right). \tag{30} \]

To compute the relevant spectral energy distribution, we consider a range of inner rim heights, \( H_{\text{rim}} \) from 1 to 3.6 scale heights, where we note that the higher rim heights correspond to higher outflow ejection speeds. The resulting SEDs are shown in Figure 16, a puffed up inner rim has a greater radiative flux in the 5 to 8 \( \mu \text{m} \) range and a lower flux in the 8 to 40 \( \mu \text{m} \) region relative to smaller inner rims.

The results in Figure 16 are qualitatively similar to those shown in Figure 1. The pivot point in both figures is around 8 \( \mu \text{m}. \) The correspondence between model and observations is close, but not exact since we wish only to demonstrate that a puffed rim, as inspired by our dust fan model, can approximately reproduce observations.

In our model, the changing value of \( H_{\text{rim}} \) would be directly due to the changing speed of the jet flow generated at the inner disc rim. As discussed in § 3.4 there are two local maxima for the jet flow speed: a maximum when the inner disc rim is touching the stellar surface and another maximum, relative to the escape speed, when the inner disc rim
is at the approximate distance of 1.59\(R_\odot\) from the centre of the star.

As the accretion rate changes and the inner rim approaches the 1.59\(R_\odot\) point then the jet flow speed increases. As a consequence, as is shown in § 4, ejected dust particles can reach higher altitudes after the particles are catapulted from the flow and subsequently move radially away from the star across the face of the accretion disc. It is the increase in the jet flow speed that produces the increase in the height of the dust fan and the consequent, perceived increase in the height of the puffed-up inner rim. We show more detailed radiative transfer modelling of LRLL 31 in Bryan et al. (2019).

6 CONCLUSIONS

In this study, we have derived a model for the mid-infrared variability of the young stellar system LLRL 31, which displays a decrease in the 8 to 40 \(\mu m\) flux when there is an increase in the 1 to 8 \(\mu m\) flux and vice versa (Figure 1). We have concluded, as have other authors, that this variability is primarily due to the perceived change in the rim height of the inner disc surrounding the central star.

When the inner rim height is perceived to increase, the inner wall is heated by stellar radiation and there is an increase in the 1 to 8 \(\mu m\) flux. A puffed inner rim also produces a shadow that obscures the outer disc, thereby resulting in the decrease in the 8 to 40 \(\mu m\) flux. Similarly, the opposite occurs when the perceived inner disc rim decreases in height. We say, “perceived inner disc”, because this deduced change in height may not actually be occurring to the disc itself, but may be produced by an optically thick fan of dust that is ejected from the disc due to the accretion of dust and gas onto the star.

As accreting gas in the disc moves towards the star, there is an interaction between the poloidal, approximately dipole, stellar magnetic field and the disc. The resulting toroidal disc field can produce an outflow such that gas and dust is ejected with a component of the flow that is perpendicular to the disc midplane. As this dusty gas moves away from the disc midplane, the dust may centrifugally decouple from the gas flow and move on a ballistic trajectory across the face of the disc. We suggest that the resulting inner disc dust fan may produce a shadow over the outer disc and provide the distant perception of a “puffed-up” inner rim. The dust may be resident in the inner disc rim and/or it may have condensed in the outflow.

This model has allowed us to derive a number of analytic formulae: the speed of the jet flow produced from a toroidal magnetic disc field at or near the surface on an accretion disc (equation 25), the time dependent disc toroidal field (equation A15), the disc magnetic twist (equation 19), the size of the disc region where the magnetic twist is likely to be stable (equation 24), the distance from the star for the maximal jet flow speed (equations 26 and 27) plus the radial speeds of particles ejected from the jet flow (equations C20 and C23).

This theoretical work indicates that the major timescale for this process is the magnetic diffusion time scale of the inner disc (equation 14) due to the wind up of the stellar magnetosphere into a disc toroidal field. This timescale is dependent on the conductivity of the inner disc, but plausible conductivity values suggest a timescale of days, which is consistent with observations (Figure 2).

As such, puffed-up inner rims may be symptomatic of the magnetic interaction between a star and surrounding accretion disc. They are also indicative of the radial transport of processed dust from the inner regions of an accretion disc to the outer regions. Such a result is consistent with the Stardust mission results, where the dust obtained from Comet Wild 2 had been exposed to temperatures greater than 1000 K (Brownlee 2014). The ballistic radial transport of dust has also been observed via the Spitzer Space Telescope (Poteet et al. 2011; Juhász et al. 2012). Puffed-up inner rims and the subsequent radial transport of dust may be an intimately intertwined process that is applicable to many young stellar systems including the some of the very first radial transport processes in the early Solar System.

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REFERENCES

Adams F. C., Gregory S. G., 2012, ApJ, 744, 55
Bans A., Königl A., 2012, ApJ, 758, 100

Figure 16. Resulting spectral energy distributions (SEDs) for a protostellar disc that is modelled on LRLL 31 with varying inner rim height. The SEDs show similar behaviour to observations as displayed in Figure 1, where the higher puffed up inner rims produce more flux in the 5 to 8 \(\mu m\) range and less flux in the 8 to 40 \(\mu m\) range. The SED pivot point is around 8 \(\mu m\). The disc is inclined at an angle of 75\(^\circ\) with an assumed extinction of \(A_V = 8.4\). The inset displays the heights of the puffed-up inner rim in terms of natural scale height, \(h\), at the inner rim.
APPENDIX A: TOROIDAL FIELD GROWTH

As illustrated in Figure 5, we make the plausible, first-order approximation that the dipole component of the stellar magnetic field co-rotates with the star and this magnetic field interacts with the surrounding accretion disc. As the stellar magnetic field moves over the accretion disc, it will generate a toroidal field in the disc. To obtain a timescale for the development of the disc toroidal field and the subsequent changes in the inner disc, we require the induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \quad (A1)$$

where $\mathbf{B}$ is the magnetic vector field, $t$ the time and $\eta$ is the magnetic diffusivity with

$$\eta = \frac{1}{\mu_0 \sigma D} \quad (A2)$$

The assumption of co-rotation with the star of the stellar magnetic field implies that at the disc surface, the speed of the stellar field, $v_\phi$ is

$$v_\phi = \tau \Omega \tilde{\phi} \quad (A3)$$

where $\tilde{\phi}$ is the unit vector in the cylindrical coordinate azimuthal direction. The azimuthal velocity of the disc relative to the co-rotating stellar field, $v_{DB}$ is

$$v_{DB}(r) = (v_\phi(r) - r \Omega \tilde{r}) \tilde{\phi} \quad (A4)$$

where $v_\phi(r)$ is the Keplerian azimuthal velocity ($v_\phi(r) = \tau \Omega \tilde{r}$). In equation A1,

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = \frac{\partial}{\partial z} \left( \frac{(v_\phi(r) - r \Omega \tilde{r}) B_z}{\tau} \right) \tilde{\phi} \quad (A5)$$

with $B \approx B_z$ at or near the midplane of the accretion disc. Thus, assuming axisymmetry

$$\nabla \times (\mathbf{v} \times \mathbf{B}) = \frac{\partial}{\partial z} \left( \frac{(v_\phi(r) - r \Omega \tilde{r}) B_z}{\tau} \right) \tilde{\phi} \quad (A6)$$

For this analysis, we are assuming that $\mathbf{B} = (0, B_\phi, B_z)$ and that the change in $B_\phi$ as a function of $r$ is small relative to the change of $B_\phi$ in $z$. So

$$\nabla^2 B_\phi = \frac{\partial^2 B_\phi}{\partial z^2} = \frac{B_\phi}{r^2} \quad (A7)$$

and equation A1 becomes

$$\frac{\partial B_\phi}{\partial t} \approx \frac{\partial}{\partial z} \left( \frac{(v_\phi(r) - r \Omega \tilde{r}) B_z}{\tau} \right) + \eta \frac{\partial^2 B_\phi}{\partial z^2} - \frac{B_\phi}{r^2} \quad (A8)$$

Integrating the components of equation A8 with respect to $z$ gives

$$\int_0^h B_\phi dz = \tilde{B}_\phi h, \quad (A9)$$

with $\tilde{B}_\phi$ the height averaged value of $B_\phi$.

$$\int_0^h \frac{\partial B_\phi}{\partial t} dz = h \frac{\partial \tilde{B}_\phi}{\partial t} \quad (A10)$$
where we have assumed that $\partial h/\partial t$ can be neglected.

\[
\int_0^h \frac{\partial \varphi}{\partial z} dz = (\varphi_B(h) - \varphi_B(0)) B_z(r),
\]
(A11)

\[
\int_0^h \frac{\partial^2 \varphi}{\partial z^2} dz = \frac{\partial B_\phi}{\partial z} \bigg|_0^h - \frac{\partial B_\phi}{\partial z} \bigg|_0^h \approx \frac{2}{h} B_\phi,
\]
(A12)

where we have set $\partial B_\phi/\partial z|_b = 0$ as a boundary condition as we should expect that the generated toroidal field will decrease with increasing $z$ as one moves away from the disc surface ($z > h$). The boundary condition $\partial B_\phi/\partial z|_0 = (2/h) B_\phi$, is a semi-plausible ansatz.

Finally,

\[
\frac{\eta h}{r^2} B_\phi dz = \frac{\eta h B_\phi}{r^2},
\]
(A13)

where we neglect the latter term as $(h/r) \ll 1$.

Putting all this together, our height averaged form of equation A8 is

\[
\frac{\partial \bar{B}_\phi}{\partial t} + \frac{2\eta}{h^2} \bar{B}_\phi \approx \frac{(\varphi_B(h) - \varphi_B(0)) B_z(r)}{h},
\]
(A14)

which, for constant $r$, is a first order differential equation with constant coefficients and has the solution

\[
\bar{B}_\phi(r) \approx \bar{B}_\phi(0) e^{-\frac{2\eta}{h} r} + \frac{h}{2\eta} (\varphi_B(h) - \varphi_B(0)) B_z(r) \left(1 - e^{-\frac{2\eta}{h} r}\right).
\]
(A15)

Here the e-folding time scale for the build-up in the toroidal field has the expected dimensional form:

\[
\tau_B = \frac{h(r)^2}{2\eta} = \frac{\mu_0 \sigma_T h(r)^2}{2}.
\]
(A16)

So as $t \to \infty$, we have the steady state form

\[
\bar{B}_\phi(r) \approx \frac{h \mu_0 \sigma_T (r)}{2} (\varphi_B(h) - \varphi_B(0)) B_z(r).
\]
(A17)

By assumption the corona at the surface of the disc is corotating with the star:

\[
\varphi_B(0) \approx 0,
\]
(A18)

while at the midplane of the disc

\[
\varphi_B(0) \approx r \Omega_k(r) - r \Omega_\star.
\]
(A19)

So

\[
\bar{B}_\phi(r) \approx \frac{h \mu_0 \sigma_T (r)}{2} (\Omega_\star - \Omega_k(r)) B_z(r),
\]
(A20)

which is the height-averaged integral $\left(\frac{1}{h} \int_0^h B_\phi(r, z) dz\right)$ of equation 13.

**APPENDIX B: MAGNETIC PRESSURE DRIVEN FLOW**

**B1 MAGNETIC FIELD**

In Figure 5, we show a radial current generated by the relative motion of the stellar magnetic field and the disc. The assumed direction of the stellar field and the directions of rotation of the disc and star produce radial disc current flows that are within the disc and flow towards the star. However, for the current to exist then there must be a return current, otherwise, charge separation would occur in the disc and the current would shut down. We thereby assume that the disc surface is also conductive and there is a return current along the disc surface to complete the circuit. For this to occur, there must exist separated layers of peak Pedersen or Hall conductivity that allow trans magnetic field currents to flow. One maximum of conductivity occurs within the disc, while the other conductivity maxima occur on the separate disc surfaces.

Such altitude dependent, multiple conductivity maxima do not occur in the Earth’s ionosphere, but have been observed in the upper atmosphere of Titan (Rosenqvist et al. 2009). Such a phenomenon may also occur in the inner discs around young stars, where the inner disc is interacting with a stellar magnetosphere.

The inner disc current shown in Figure 5, interacts with the wrapped-up disc toroidal field to compress the inner disc (Figure 8). Conversely, because the surface current flows in the opposite direction then its interaction with the disc toroidal field pushes material away from the disc surface (Figure 9). The general current flows and magnetic fields are shown in Figure B1.

This figure is somewhat busy and complex, but we first concentrate on the current flows. The internal disc current density $j_\text{int}$ flows through the disc towards the star, it then flows up the stellar magnetosphere as a total current, $I_\text{ret}$, and out across the upper surface of the disc with a return current density $I_\text{ret}$. Finally, it flows back to the disc along the stellar magnetosphere, which we represent via the current $I_\text{ret}$. This jet acceleration region is assumed to take up only a small area of the inner disc starting from the inner truncation radius, $R_\text{t}$, to a slightly larger radius of $R_\text{t} + \Delta r$. The base of the jet acceleration region is located at height $z_0$, which is on or near the surface of the disc. The top of the acceleration jet occurs at $\gamma r t$, where $\gamma r t$ is also located in the upper regions of the disc. So $z_0 < \gamma r t < r$. The magnetospheric current, $I_\text{M}$, generates a toroidal magnetic field $B_\phi$ in the acceleration region. The radial return Pedersen (or, possibly, Hall current), $I_\text{ret}$, bleeds off the magnetospheric current and interacts with the toroidal field to produce a Lorentz force that pushes the disc gas away from the disc plane, thereby producing the jet flow.

The toroidal magnetic field within the acceleration region is given by Ampere’s Law:

\[
\nabla \times B = \mu_0 j.
\]
(B1)

Integrating the right hand side of equation B1 over the area element $d\mathbf{a}$ shown in Figure B1 gives

\[
\int_A \mu_0 j \cdot d\mathbf{a} = -2\pi \mu_0 r \int_{z_0}^{\gamma r t} j_\text{ret}(z) dz = -2\mu_0 \sigma_T j_\text{ret}(z) (z - z_0),
\]
(B2)

where the final equality is derived from the Mean Value Theorem with $z_0 \in [z_0, z]$.

The average radial return current $j_\text{ret}$ is given by the definition

\[
j_\text{ret}(r) = \frac{1}{\gamma r t - z_0} \int_{z_0}^{\gamma r t} j_\text{ret}(z) dz = \frac{I_\text{ret}}{2\pi r (\gamma r t - z_0)},
\]
(B3)

so we can write

\[
\int_A \mu_0 j \cdot d\mathbf{a} = -\mu_0 I_\text{ret} j_\text{ret}(r) (z - z_0),
\]
(B4)
The magnitude of the total Lorentz force, $F_l$, generated in the acceleration region is

$$F_l = \int_{r_1}^{r_0} dr \int_{z_0}^{z_T} dz \int_0^{2\pi} r f_I d\theta \approx \frac{\mu_0 I_M}{4\pi} \ln \frac{r_0}{r_1} \left(2 - \frac{r}{I_M}\right),$$

(B12)

where we note that in deriving equation B12, we have made the approximation that

$$\int_{r_1}^{r_0} dr \int_{z_0}^{z_T} dz j_{ret}(z) \mu_0 I_M \left(1 - \frac{r}{I_M}\right) \approx \int_{r_0}^{r_0} dr j_{ret} \mu_0 I_M \int_{z_0}^{z_T} dz \left(1 - \frac{r}{I_M} \frac{z - z_0}{z_T - z_0}\right).$$

(B13)

If this approximation is true, then the total force from the acceleration region is independent of the $z$ behaviour of $j_{ret}$.

For the case where $I_r = I_M$ then equation B12 has the form

$$F_l \approx \frac{\mu_0 I_M^2}{4\pi} \ln \frac{r_0}{r_1}. \quad \text{(B14)}$$

So the total driving force is dependent on the central current flow and the radial size of the propulsion region.

### B3 APPROXIMATE JET SPEED

It would be useful to obtain an approximate equation for the flow speed of this magnetic jet system. An intuitive idea of how this system behaves can be obtained by solving a simplified momentum equation by ignoring gravity and pressure gradient:

$$\rho \frac{dv}{dz} = \frac{p}{2} \frac{d(v^2)}{dz} = f(r, z) \quad \text{(B15)}$$

$$\Rightarrow \int_{z_0}^{z_T} \rho \frac{d(v^2)}{dz} dz = \frac{p(z_{c2})}{2} \int_{z_0}^{z_T} \frac{d(v^2)}{dz} dz = \frac{p(z_{c2})}{2} (v^2(z_T) - v^2(z_0)), \quad \text{(B16)}$$

$$\int_{z_0}^{z_T} f(r, z) dz = \frac{j_{ret}(z_{c3}) \mu_0 I_M}{2\pi r} (z_T - z_0) \left(1 - \frac{r}{I_M}\right), \quad \text{(B17)}$$

where we have used the Mean Value Theorem with $z_{c2}$ and $z_{c3} \in [z_0, z_T]$.

Putting all this together gives:

$$v^2(r, z_T) \approx v^2(r, z_0) + \frac{j_{ret}(z_{c3}) \mu_0 I_M}{\rho(z_{c3}) \pi r} (z_T - z_0) \left(1 - \frac{r}{I_M}\right). \quad \text{(B18)}$$

Making the approximations that

$$v^2(r, z_0) \ll v^2(r, z_T), \quad \text{(B19)}$$

$$j_{ret}(r, z_{c3}) \approx j_{ret} = \frac{l_t}{2\pi r (z_T - z_0)}, \quad \text{(B20)}$$

$$\rho(z_{c3}) \approx \rho = \frac{\int_{z_0}^{z_T} \rho dz}{z_T - z_0}, \quad \text{(B21)}$$

then equation B18 becomes

$$v^2(r, z_T) \approx \frac{\mu_0 l_t I_M}{2\pi r^2 \rho} \left(1 - \frac{r}{I_M}\right). \quad \text{(B22)}$$

If we suppose that all the magnetospheric current between the star and the disc is converted into radial current,
i.e., $I_t = I_M$ then the exit speed of the gas from the acceleration region is

$$v(r, z_t) = \frac{\sqrt{\frac{20}{\rho}} \frac{I_M}{2\pi r}}{\sqrt{\mu_0}} \frac{|B_0(r, z_0)|}{|B_0(r)|} = \frac{\sqrt{\frac{20}{\rho}} \frac{I_0(r_0)}{2\pi r_0}}{\sqrt{\mu_0}} \frac{|B_0(r)|}{|B_0(r)|}.$$

where we have used equations B6 and 13 to obtain the right hand side of equation B23. It is useful to rewrite equation B23 as

$$v(r, z_t) \approx \sqrt{\frac{20}{\rho}} \frac{I_0(r_0)}{2\pi r_0} \rho \Omega_{*}(r_0) - \Omega_{*}(r) |B_z(r)|.$$

We note that the above derivation has ignored gravity as we have implicitly assumed that the jet propulsion occurs at or near the disc surface and that $z \ll r$. This assumption may be incorrect, but if jet flows are produced from or near the disc surface then the observed outflow speed will not be the same as the value given by equation B23 as the jet flow will have had to overcome the gravitational potential of the star. To obtain an approximate value for the final flow speed, one can use a Bernoulli-like equation which includes gravity and angular velocity, e.g., equation 47 of Liffman & Siora (1997).

APPENDIX C: PARTICLE MOTION

C1 EQUATIONS OF MOTION

Suppose that dust particles are, initially, in a circular Keplerian orbit at or near the inner truncation radius of the disc. As discussed in the § 3.4, the accretional inflow and/or the protostellar jet flow gives the particles an initial ‘boost’ velocity that is assumed to be primarily in the $z$ direction as this is perpendicular to the disc midplane. If we assume that the self-gravity of the disc is negligible compared to the gravity of the protostar then the equations of motion for a particle in the cylindrical coordinate $r, \phi$ and $z$ directions are:

$$m_d \ddot{p}_d = m_d \rho_d \dot{p}_d^2 - \frac{GM_* m_d \rho_d}{(r_p^2 + c_p^2)^{3/2}} \frac{C_D}{2} \rho_p \pi a_d^2 \rho_g \dot{v}_g \cdot \hat{r},$$

$$m_d \ddot{\phi}_d + 2m_d \dot{\phi}_d \dot{p}_d = -\frac{C_D}{2} \rho_p \pi a_d^2 \rho_g \dot{v}_g \cdot \hat{\phi},$$

$$m_d \ddot{z}_d = -\frac{GM_* m_d \dot{z}_d}{(r_p^2 + c_p^2)^{3/2}} \frac{C_D}{2} \rho_p \pi a_d^2 \rho_g \dot{v}_g \cdot \hat{z},$$

where $r_p, \phi_d$ and $z_d$ are the cylindrical coordinates of the dust particle, $\rho_d$ the average mass density of the gas, $C_D$ the drag coefficient for the interaction between the gas and the dust particle, and $v_g$ and $\dot{v}_g$ are the gas flow velocity and dust velocity, respectively, with $v_{g} = v_{p} - \dot{v}_g$. Symbols with a caret and tilde are unit vectors, while $m_d$ is the mass of an individual, approximately spherical, dust grain, so

$$m_d = 4 \pi a_d^3 \rho_d.$$

with $a_d$ the average dust grain radius and $\rho_d$ the average mass density of the dust grain. The azimuthal gas speed is $\dot{v}_g \approx r \Omega_{*}$, as we assume that the gas flow arises at the inner truncation radius and the gas flow is coupled to the protostellar magnetosphere, which is, to a first approximation, co-rotating with the protostar.

We can normalise the above equations by setting $r_p = r_p/r_0$, $z_d = z_d/r_0$ and $t = t_0$, where $r_0$ is the initial value of $r_p$ for the particle and $P_0$ is the orbital period of an object with a circular orbit of radius $r_0$.

$$P_0 = 2\pi \sqrt{\frac{r_0^3}{GM_*}}.$$ (C5)

Dropping the primes on the main non-dimensional variables, the equations of motion become:

$$\dot{r}_p = r_p \ddot{r}_p - \frac{4\pi^2 r}{(r_p^2 + c_p^2)^{3/2}} - \frac{3C_D \rho_p \pi a_d^2 \rho_g \ddot{v}_g}{8 \Delta u_d} \rho_g \cdot \hat{r},$$

$$\dot{\phi}_p = -2r_p \ddot{\phi}_p - \frac{3C_D \rho_p \pi a_d^2 \rho_g \ddot{v}_g}{8 \Delta u_d} \rho_g \cdot \hat{\phi},$$

$$\dot{z}_p = -\frac{4\pi^2 r}{(r_p^2 + c_p^2)^{3/2}} - \frac{3C_D \rho_p \pi a_d^2 \rho_g \ddot{v}_g}{8 \Delta u_d} \rho_g \cdot \hat{z},$$

where $v'_{g l} = (P_0/\Delta u_g) v_{g l}$, $v'_{g z} = (P_0/\Delta u_g) v_{g z}$, and $\Omega_{*} = P_0 \Omega_{*}$.

The drag coefficient, $C_D$, is given by

$$C_D(s) = \frac{2}{3\pi} \sqrt{\frac{\pi T_p}{\bar{T}}} + \frac{2 \bar{x}^2 + 1}{\pi \bar{x}^3} \exp(-\bar{x}^2) + \frac{4 \bar{x}^4 + 4 \bar{x}^2 - 1}{2 \bar{x}^4} \operatorname{erf}(s),$$

(Hayes & Probstin 1959; Probstin 1968) where $T_p$ is the temperature of the particle, erf the error function, $\exp$ the exponential function, and $s$ is the thermal Mach number:

$$s = |\bar{v}_g|/\bar{u}_T,$$ (C10)

with the thermal gas speed:

$$v_T = \sqrt{2 \bar{k}_B T_g/m}.$$ (C11)

To compute the velocity and mass density of the gas flow, there are at least two scenarios that could be considered: accretional mass flow from the disc onto the star and/or an outflow that is ejecting material from the disc. Of course, it is possible that outflows and accretional inflows are manifestations of the same phenomena. As such, we will consider the case of accretional flow onto the star.

At the truncation radius, the infalling gas and dust will initially tend to flow along the stellar field lines with a gas velocity, $v_g$, in the $z$ direction in an (assumed) axisymmetric channel of initial width $\Delta$ (Figure 12). Several authors have developed detailed and elegant flow models for the velocity and density of the infalling gas, e.g., Adams & Gregory (2012) and references therein. However, for our purposes, we have adopted the standard boundary layer value for $\Delta$, where the stellar magnetosphere at $R_0$ replaces the surface of a compact object (e.g., equation (6.10) in Frank et al. (2002)):

$$\Delta = h(R_0)^2 / R_0 = 2 \times 10^{-5} au (h/0.001 au)^2 (R_0/0.05 au).$$ (C12)

So, we can write for the mass flow rate in the channel by using the conservation of mass

$$M_\Delta = 2\pi R_0 \Delta \rho_g v_g.$$ (C13)
Combining equations C12 and C13 gives

$$\dot{r}_p = \frac{M_a}{4\pi h(R_0)^2 v_g}.$$  \hfill (C14)

### C2 NUMERICAL RESULTS FOR LRL1 31 & ANALYTIC TESTS

This system of equations can be solved via standard techniques and as an example, we assumed that the particles and accretional gas flow initially started at or near the midplane (\(z = 0\)) of the inner edge of the disc, i.e., at the truncation radius (\(R_0 \approx 0.09\) au), with a corresponding mass accretion rate of \(M_a \approx 1.6 \times 10^{-8} M_{\odot} \text{yr}^{-1}\). The resulting width of the channel was \(\Delta = 5000 \text{ km} \approx 3.3 \times 10^{-5} \text{ au}\) and the dust particles were placed at the inner edge of the gas flow, so they had to travel through the entire width of the accretional flow before they could escape the flow. We set the particle drag to zero, once the dust particles left the initial gas flow. The speed of the \(z\) component of the initial gas flow, \(v_{gz}\), was set as a free parameter, which, in turn, determined the mass density of the gas flow via equation C14. The other gas velocity components were: \(v_p = 0\) and \(v_{\phi g} = r \Omega_*\). The results for different particle ejection speeds are shown in Figure C1.

The dust particle parameters used were \(a_d = 0.5\mu m\), \(\rho_d = 3 \text{ g cm}^{-3}\), with the initial velocity components: \(v_{pe} = 0\), \(v_{\phi e} = v_k(R_0)\), (the Keplerian speed at \(R_0\)) and \(v_{re} = v_{\phi e}\). As the launching, radial distance \(R_0 \approx 0.09\) au > \(R_{co} \approx 0.05\) au then \(v_{\phi e} \approx R_0 \Omega_* \approx 290 \text{ km s}^{-1}\), which is around 2.2 times the Keplerian speed at \(R_0\). Therefore, the dust particles are subject to radial acceleration away from the star due to centrifugal force derived from the gas flow.

To check that the numerical solver was working correctly, we cross checked our numerical results with some available analytic solutions for the particle paths. For example, when the particle is free of the gas flow then equation C2 has the form

$$r_p \dot{\phi}_p + 2r_p \dot{\phi}_p = 0$$  \hfill (C15)

This equation has the solution

$$r_p^2 \dot{\phi}_p = \text{constant} = \ell$$  \hfill (C16)

i.e., the specific angular momentum of the particle, \(\ell\), when it is not subject to a torque, is a constant. It follows that

$$v_{\phi p} = r_p \dot{\phi}_p = \frac{\ell_0 \phi_0}{r_p} = \frac{\ell_0}{r_p},$$  \hfill (C17)

where, in this case, \(r_0 = R_0\) and \(\phi_0 = \Omega_*\). Note that even though the dust particle started with Keplerian azimuthal velocity, gas drag accelerated the particle to the co-rotational azimuthal gas velocity (Figure C2). The numerical calculation reproduced the values from the analytic solution: equation C17 with \(\phi_0 = \Omega_*\).

Other, approximate, analytic solutions are available when one considers the radial equation of motion, equation C1, without gas drag:

$$\ddot{r}_p = \frac{d}{dr} \left( r_p v_{pe}^2 - \frac{GM_* r_p}{(r_p^2 + z^2)^{3/2}} \right) = \frac{\ell_0^2}{r_0^3} - \frac{GM_* r_p}{(r_p^2 + z^2)^{3/2}}.$$  \hfill (C18)

Suppose the particle is given a significant boost velocity in the \(z\) direction such that \(z \to \infty\) (or \(z\) becomes comparable to \(r\)) then

$$\dot{r}_p \approx \frac{\ell_0^2}{r_0^3} > 0,$$  \hfill (C19)

and the particle starts to accelerate in the radial direction with the subsequent radial speed

$$v_{pr} \approx \ell_0 \sqrt{\frac{1}{r_0^2} - \frac{1}{r_p^2}}.$$  \hfill (C20)

So, in this scenario, the particle increases in radial speed and we have

$$v_{pe} \to \ell_0 r_0^2 \approx \eta_p \phi_0 \approx \sqrt{\frac{GM_*}{r_0}} \to \infty,$$  \hfill (C21)

It could be argued that setting \(z \to \infty\) is slightly unrealistic.
The particle azimuthal velocity, $v_p \phi$, as a function of distance from the LRLL 31 protostar. The dust particle has an initial azimuthal velocity equal to the Keplerian speed of 127 km s$^{-1}$, but is quickly spun up by the mass flow from the accretion disc at the inner truncation radius, $R_t$, to the stellar corotational speed, $\Omega^\star$, of 280 km s$^{-1}$. Once the particle is free from the gas flow, in an assumed gas drag free environment, its angular momentum is constant and its azimuthal velocity decreases as $r_p^{-1}$, as predicted from equation C17. This can be seen from the azimuthal velocities at 0.1 and 1 au, which decrease from 245 km s$^{-1}$ to 24.5 km s$^{-1}$.

Let us assume that the boost in the $z$ direction is small and that $z \ll r$. For such a case, another approximate analytic solution is available when one considers the radial equation of motion, equation C1, without dust drag:

$$\ddot{r}_p = v_p \frac{d v_p}{d r} = r_p \frac{d^2 \phi}{d r^2} - \frac{GM^\star}{(r_p^2 + z_p^2)^{3/2}} \approx \frac{r_p^2 \phi_0^2}{r_p^2} - \frac{GM^\star}{r_p^2},$$

(C22)

where we have used equation C17 and assumed $z \ll r$. Equation C22 has the solution

$$v_{pr} = \sqrt{\frac{r_0^2 \phi_0^2}{r_p^2} \left(1 - \left(\frac{r_0}{r_p}\right)^2\right) - \frac{2GM^\star}{r_0} \left(1 - \frac{r_0}{r_p}\right)}.$$

(C23)

When $r_p \to \infty$ then

$$v_{pr} \to \bar{v}_{pr} \approx \sqrt{\frac{2GM^\star}{r_0}}.$$

(C24)

In Figure C3, we compare these analytic equations with the numerical solutions, where it can be seen that there is very little difference between the numerical solution of equation C1 for the radial velocity of a particle ejected from the inner region of the LRLL 31 accretion disc relative to an analytic approximation given by equation C23. For this case, the expected asymptotic radial speed for large $r_p$, as obtained from equation C24 is $v_{pr} \approx 215$ km s$^{-1}$.

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