Fermion-Condensation Quantum Phase Transition in High-$T_c$ Superconductors

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The effect of a quantum phase transition associated with the appearance of the fermion condensate in an electron liquid on the properties of superconductors is considered. It is shown that the electron system in both superconducting and normal states exhibits characteristic features of a quantum protectorate after the point of this fermion-condensation quantum phase transition. The single-particle spectrum of a superconductor can be represented by two straight lines corresponding to two effective masses $M_{FC}$ and $M_L$. The $M_{FC}$ mass characterizes the spectrum up to the binding energy $E_0$, which is of the order of the superconducting gap in magnitude, and $M_L$ determines the spectrum at higher binding energies. Both effective masses are retained in the normal state; however, $E_0 = 4T$. These results are used to explain the lineshape of single-particle excitations and other remarkable properties of high-$T_c$ superconductors and are in a good agreement with recent experimental data.

Recent experiments using angle-resolved photoemission electron spectroscopy gave rather accurate data on the dispersion of single-particle excitations over a wide range of binding energies. These experiments were carried out with high-$T_c$ superconductors Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ differing in the doping level both at temperatures $T$ below the critical temperature $T_c$ of the decay of superconducting state and at $T_c < T$. It was inferred that the dispersion of quasiparticle excitations $\varepsilon(p)$, where $p$ is momentum, can be described in the energy range (-200—0) meV by two straight lines intersecting at the binding energy $E_0 \simeq (50 - 70)$ meV. This circumstance directly points to the existence of a new energy scale in the self-energy part of quasiparticle excitations at temperatures $T < T_c$ and $T_c < T$. Therefore, new additional constraints can be imposed on the theories that are in principle applicable to the description of properties of high-$T_c$ superconductors. For example, such scale is absent in the theories of normal and marginal Fermi liquids, as well as in the theory based on an idea of the quasiparticle spin–charge separation. The kink in the quasiparticle dispersion law described above could be explained by the interaction of quasiparticles with collective magnetic excitations that was observed in high-$T_c$ superconductors in experiments on inelastic neutron scattering at $T < T_c$ and described in the literature; see, for example, [12, 13]. However, a dispersion kink is also observed at $T_c < T$, when these collective excitations disappear. Moreover, these excitations are successfully described as inelastic neutron scattering from Cooper pairs [14], which is confirmed by other experimental results [15, 16]. With regard to this explanation of the physics of magnetic excitations, it is unlikely that these excitations can significantly affect the single-electron dispersion. Experimental data on single-particle electron spectra of high-$T_c$ superconductors with d-wave symmetry indicate that the perturbation of the superconducting phase and single-particle spectra by phonons, collective states, or impurities is very small. Therefore, this state can be described as a strongly collectivized quantum state or as “quantum protectorate” [11, 12]. From here, it may be inferred that the kink description proposed in [12] is very likely contradictory to the quantum protectorate concept.

In this paper, we show that the dispersion kink can be explained based on the assumption that the electron system of high-$T_c$ superconductor is after the point of the Fermi-condensate quantum phase transition. Thus, the fermion-condensation quantum phase transition (FCQPT) serves as the point separating a normal Fermi liquid from a strongly correlated liquid of a new type that fulfills the quantum protectorate requirements.

Let us start with a brief description of the properties of an electron system with the fermion condensate. Consider a two-dimensional electron liquid in the superconducting state at $T = 0$ on a simple square crystal lattice, which we replace temporarily by a uniform positive charge. Then, the ground state energy $E_{gs}[\kappa(p), n(p)]$ is a functional of the order parameter of the superconducting state $\kappa(p)$ and occupation numbers $n(p)$ and is determined by the known equation

$$E_{gs}[\kappa(p), n(p)] = E[n(p)]$$

$$+ \int V_{pp}(p_1, p_2)\kappa(p_1)\kappa^*(p_2)\frac{dp_1 dp_2}{(2\pi)^2}.$$
The pairing interaction $V_{pp}(\mathbf{p}_1, \mathbf{p}_2)$ is assumed to be weak. The ground-state energy $E[n(\mathbf{p})]$ of the normal Fermi liquid is a functional of occupation numbers $n(\mathbf{p})$, which, at $T = 0$, are related to the order parameter by the simple equation

$$n(\mathbf{p}) = v^2(\mathbf{p}); \quad \kappa(\mathbf{p}) = v(\mathbf{p})\sqrt{1 - v^2(\mathbf{p})}.$$  \hspace{1cm} (2)

Minimizing the energy $E_{gs}$ in Eq. (1) with respect to occupation numbers and taking into account Eq. (2), we obtain the equation

$$\varepsilon(\mathbf{p}) - \mu = \Delta(\mathbf{p}) - \frac{1 - 2v^2(\mathbf{p})}{2\kappa(\mathbf{p})},$$ \hspace{1cm} (3)

where the single-particle energy $\varepsilon(\mathbf{p})$ is determined by the equation

$$\varepsilon(\mathbf{p}) = \frac{\delta E[n(\mathbf{p})]}{\delta n(\mathbf{p})}.$$ \hspace{1cm} (4)

$\mu$ is chemical potential. The superconducting gap is given by the equation

$$\Delta(\mathbf{p}) = -\int V_{pp}(\mathbf{p}_1, \mathbf{p}_2) \frac{d\mathbf{p}_1}{4\pi^2}.$$ \hspace{1cm} (5)

Let us assume that the interaction $V_{pp} \to 0$. Then, the gap $\Delta(\mathbf{p}) \equiv 0$, and Eq. (3) is reduced to the equation proposed in [15]

$$\varepsilon(\mathbf{p}) - \mu = 0, \text{ if } 0 < n(\mathbf{p}) < 1; \ p_i < p < p_f.$$ \hspace{1cm} (6)

This equation defines a Fermi liquid of a new type for which the order parameter $\kappa(\mathbf{p})$ differs from zero in the $L_{FC}$ range of momenta $p_i \leq p \leq p_f$; the occupation numbers $n(\mathbf{p}) = 1$ and 0 outside the $L_{FC}$ range, as must be in the normal Fermi liquid. It follows from Eq. (6) that the effective mass $M_{FC}$ of quasiparticles in the fermion condensate is infinitely large in the $L_{FC}$ range:

$$\frac{1}{M_{FC}} = \frac{1}{p} \frac{d\varepsilon(p)}{dp} = 0.$$ \hspace{1cm} (7)

The effective mass of normal quasiparticles $M^*_L$ with momenta $p < p_i$ is finite and is defined by the known equations

$$\frac{1}{M^*_L} = \frac{1}{p} \frac{d\varepsilon(p)}{dp} |_{p \leq p_i}.$$ \hspace{1cm} (8)

It follows from Eqs. (7) and (8) that a fermion system with the fermion condensate is broken into two quasiparticle subsystems: the dispersionless part of the single-particle spectrum is occupied by the fermion condensate in the momentum range $L_{FC}$ and is adjoined by the subsystem that is occupied by quasiparticles of finite mass with momenta $p < p_i$. We will assume for simplicity that the fermion condensate occupies a small part of the Fermi sphere $p_f - p_i \ll p_F$, where the Fermi momentum is related by the common equation $p_F = (3\pi^2\rho)^{1/3}$ to the particle density $\rho$. The fermion condensate appears in an electron system at a low density, when the effective electron-electron interaction constant is sufficiently large. In a common electron liquid, this constant is directly proportional to the dimensionless parameter $r_s = 9\pi/4p_Fa_B$, where $a_B$ is the Bohr radius. For simplicity, we will assume that it equals $r_s$. It was shown in [16] that the appearance of the fermion condensate occurs in a system at a certain $r_s = r_{FC} < r_{dw}$ and precedes the appearance of a charge-density wave, which takes place in a two-dimensional electron liquid at $r_{dw} \simeq 6 - 8$. Thus, the FCQPT occurs at $T = 0$ when the parameter $r_s$ attains its critical value $r_{FC}$ and represents a quantum phase transition. At $r_s > r_{FC}$ and $r_s - r_{FC} \ll r_{FC}$, the region $p_f - p_i$ occupied by the fermion condensate is $(p_f - p_i)/p_F \sim r_s - r_{FC}$. This estimate is confirmed by calculations for simple models [21,22].

Because the order parameter of the FCQPT is $\kappa(\mathbf{p})$, the maximum value of the superconducting gap $\Delta_1$ in a system with the fermion condensate $\Delta_1 \sim V_{pp}$, as it follows from Eq. (5). It is pertinent to note that $\kappa(\mathbf{p})$ is determined in this case by the relatively strong particle-hole interaction or by the Landau amplitudes $F_L$. Therefore, the perturbation of the parameter $\kappa(\mathbf{p})$ can be neglected in the first order in $V_{pp}/F_L \ll 1$. It is self-evident that we assume the $L_{FC}$ range to be sufficiently large, so that its perturbation is small compared with the size of this range. Considering that $T_c \sim \Delta_1/2$ in the weak-coupling theory of superconductivity [21,22], we obtain high $T_c$ values for systems with the fermion condensate [13]. At the same time, the single-particle spectrum in the range $L_{FC}$ occupied by the fermion condensate will be disturbed by the interaction $V_{pp}$. This perturbation is quite notable for the effective mass, because the value of $1/M_{FC}$ becomes finite. Simultaneously, the perturbation of the single-particle spectrum at $p < p_i$, as well as the perturbation of the effective mass $M^*_L$, can be neglected.

Let us use Eq. (3) for calculating $M_{FC}$ by differentiating both sides of this equation with respect to momentum $p$ at $p = p_F$

$$\frac{p_F}{M_{FC}} \simeq \frac{\Delta_1}{4\kappa^3(p)} \frac{1}{p_f - p_i} = \frac{2\Delta_1}{p_f - p_i}.$$ \hspace{1cm} (9)

When obtaining Eq. (9), we took into account the facts that $\kappa(p) = 1/2$, at $p = p_F$, the gap $\Delta(\mathbf{p})$ has a maximum at the Fermi surface, and, hence, its derivative there equals zero. The derivative $d\varepsilon(p)/dp$ was calculated with the use of Eq. (2) and the simple estimate $dn(p)/dp \approx -1/(p_f - p_i)$. We may conclude that the electron system with the fermion condensate in the superconducting state is, as before, characterized by two effective masses, and that the single-particle dispersion at $p \sim p_F$ can be approximated by two straight lines. Let us estimate the binding energy $E_0$ at which these lines intersect. Multiplying both sides of Eq. (9) by the
difference \( p_f - p_i \), we obtain
\[
E_0 \simeq \frac{(p_f - p_i)p_F}{M_{FC}^*} \simeq 2\Delta_1. \tag{10}
\]

It follows from this equation that the intersection point of the two straight lines approximating the spectrum does not depend on the difference \( p_f - p_i \), although the effective mass \( M_{FC}^* \) is proportional to this difference. The calculation of \( M_{FC}^* \) at \( T \to T_c \) is completely similar to the preceding; one should only take into account that now \[22\],
\[
v^2(p) = \frac{n(p) - f(p)}{1 - 2f(p)}, \tag{11}
\]
where
\[
f(p) = \frac{1}{1 + \exp(E(p)/T)}; \quad E(p) = \sqrt{(\varepsilon(p) - \mu)^2 + \Delta^2(p)}. \tag{12}
\]

With regard to the facts that the function \( f(p) \) has a maximum at \( p = p_F \) (and its derivative equals zero there) and \( E(p) \ll T \), simple transformations of Eqs. (11) and (12) give
\[
\frac{dv^2(p)}{dp} \simeq -\frac{2T}{E(p)(p_f - p_i)}. \tag{13}
\]

Differentiating both sides of Eq. (3) with respect to momentum and taking into account Eq. (13), we obtain
\[
p_F \simeq \frac{4T}{p_f - p_i}. \tag{14}
\]

It directly follows from Eq. (14) that
\[
E_0 \simeq \frac{(p_f - p_i)p_F}{M_{FC}^*} \simeq 4T. \tag{15}
\]

Considering that \( 2\Delta_1 \simeq T_c \), we conclude by comparing Eqs. (10) and (14) that the effective mass \( M_{FC}^* \) and \( E_0 \) weakly depend on temperature at \( T \leq T_c \).

It follows from the above consideration that the form of the single-particle spectrum \( \varepsilon(p) \) and the order parameter \( \kappa(p) \) are determined by the FCQPT and, therefore, their forms are universal. Actually, the amplitudes \( F_i \) define only the region \( L_{FC} \) occupied by the condensate after the point of the FCQPT. These amplitudes are determined by the properties of the system under consideration, which already include the contribution from impurities, phonons, and other collective excitations. Finally, we may conclude that a system with the fermion condensate is characterized by a universal form of the single-particle spectrum and possesses quantum protectorate features at \( T \leq T_c \).

We now turn to the description of the system at \( T > T_c \), which is given by the equation of the Fermi-liquid theory \[1\],

\[
\frac{\delta(F - \mu N)}{\delta n(p, T)} = \varepsilon(p, T) - \mu(T) - T \ln \frac{1 - n(p, T)}{n(p, T)} = 0. \tag{16}
\]

Here, \( F \) is free energy, which, as well as energy \( E \), is a functional of occupation numbers \( n(p, T) \). The occupation numbers now depend on momentum and temperature, and the quasiparticle energy \( \varepsilon(p, T) \) is defined by Eq. (4). Assuming that \( T_c = 0 \) and \( T \to 0 \) in Eq. (16) and that the occupation numbers differ from zero and unity in the range \( L_{FC} \), we obtain that the term \( T \ln(...) \to 0 \), and Eq. (16) is reduced to Eq. (6) for the fermion condensate \[13\]. If the interaction \( V_{pp} = 0 \), the FCQPT is absent at any finite temperature. Actually, as shown above, the order parameter \( \kappa(p) \) after the point of the FCQPT differs from zero in the region \( L_{FC} \), and the gap \( \Delta(p) \equiv 0 \). From here, it is clear that the critical temperature of this transition equals zero. However, a trace of this quantum phase transition persists in its radical effect on the properties of the system up to temperatures \( T \ll T_f \), where \( T_f \) is a temperature at which the effect of this phase transition disappears. For example, the system entropy can be taken as such a property, resulting in the estimate \[21\],
\[
\frac{T_f}{\varepsilon_F} \simeq \frac{p_F^2 - p_i^2}{p_F} \simeq \frac{\Omega_{FC}}{\Omega_F}, \tag{17}
\]

where \( \Omega_{FC} \sim (p_f - p_i)p_F \) is the volume occupied by the fermion condensate, \( \Omega_F \) is the volume of the Fermi sphere, and \( \varepsilon_F \) is the Fermi energy. Taking into account that the occupation numbers at \( T \ll T_f \) are defined by Eq. (6) and \( n(p, T) = n(p) \), we obtain from Eq. (16)
\[
\varepsilon(p, T) - \mu(T) = T \ln \frac{1 - n(p)}{n(p)} \simeq T \frac{1 - 2n(p)}{n(p)} \big|_{p \approx p_F}. \tag{18}
\]

Differentiating both sides of Eq. (18) with respect to momentum \( p \) and using the estimate \( dn(p)/dp \simeq -1/(p_f - p_i) \), we obtain the approximate value for the effective mass
\[
\frac{p_F}{M_{FC}^*} \simeq \frac{4T}{p_f - p_i} \big|_{T \ll T_f}. \tag{19}
\]

Multiplying both sides of Eq. (19) by the difference \( p_f - p_i \), we obtain for the parameter
\[
E_0 \simeq 4T. \tag{20}
\]

Equations (19) and (20) indicate that the mass \( M_{FC}^* \) and the energy \( E_0 \) start to depend on temperature at \( T_c \leq T \ll T_f \). However, this dependence is very weak at \( T \simeq T_c \), as is evident from a comparison of Eqs. (14), (15), (19), and (20). We may conclude that the system under consideration still possesses quantum protectorate...
features at these temperatures, because the spectrum of the system is determined by solutions of Eq. (6) and temperature. It is evident from Eqs. (18), (19), and (20) that this spectrum has a universal character and is weakly affected by phonons, collective states, etc.

We now turn to the description of the experimental data [1–3] using the results presented above. We return to the consideration of an electron system on a square lattice. Experimental studies showed that the Fermi surface in the case of the \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) metal has a shape of an approximately regular circle with the center at the point \((\pi, \pi)\) of the Brillouin zone filled with hole states [24]. A Van Hove singularity is located in the vicinity of the \((\pi, 0)\) point, and an almost dispersionless section of the spectrum is observed in this region (see, for example, [23]). This allows the suggestion to be made that the fermion condensate is disposed in the vicinity of this point [24]. The straight line YT, which is known as the line of zeros of the Brillouin zone, passes through the points \((\pi, \pi) - (0, 0)\) at an angle of \(\pi/4\) to the straight line \(YM\) passing through the points \((\pi, \pi) - (\pi, 0)\). The density of states attains a minimum at the point of intersection of the YT line and the Fermi surface. The single-particle spectrum was measured along the lines parallel to YT and YM, from the line of zeros to the \(YM\) line. As a result, it was shown that the parameter \(E_0\) is constant for a given sample; that is, it does not depend on the angle \(\phi\), reckoned from the line of zeros to \(YM\). The angle (kink) between the straight line characterizing the part of the spectrum with the binding energy lower than \(E_0\) and the straight line related to the spectrum with the binding energy higher than \(E_0\) grows with increasing \(\phi\) and with decreasing doping level [23]. This general pattern is retained at \(T > T_c\) [4].

To describe these experimental data, we assume the following model: the volume of the fermion condensate \(\Omega_{FC}\) depends on the angle \(\phi\), \(\Omega_{FC}(\phi) \sim (p_f(\phi) - p_i(\phi))p_F\), increases with increasing \(\phi\) and attains a maximum at \((\pi, 0)\). Along with that, \(r_s\) grows with decreasing doping level and exceeds the critical value \(r_{FC}\) in the optimal doping region. Note that the values of \(r_s\) corresponding to the optimal doping level are close to \(r_{FC}\) [15, 25, 26], whereas strong fluctuations of the charge density or charge-density waves are observed in undoped samples [24]. From here, we may conclude that the formation of the fermion condensate in copper oxides is a quite determinate process stemming from the general properties of a low-density electron liquid.

It follows from Eq. (15) that the energy \(E_0\) does not depend on the angle \(\phi\) at \(T \leq T_c\). It also follows from Eq. (14) that the kink increases with increasing \(\phi\), because the effective mass linearly depends on the difference \(p_f(\phi) - p_i(\phi)\). Comparing Eqs. (9), (10), (14), and (15), one can conclude that these properties weakly depend on temperature at \(T \leq T_c\). Equations (19) and (20) demonstrate that this behavior persists at \(T_c \leq T\); however, a temperature dependence appears. According to experimental data, \(E_0 \simeq (50 - 70)\) meV [33], which is in agreement with Eqs. (10) and (20), because \(E_0 \simeq 2\Delta_1\) in these materials. The volume of the phase condensate \(\Omega_{FC}\) in our model grows with increasing \(r_s\), and the mass \(M_{FC}\) correspondingly increases, as evident from Eqs. (9) and (14). Hence, the dispersion kink in the single-particle spectrum must increase with decreasing doping level. Because \(E_0 \simeq 2\Delta_1\), the kink point must shift towards higher binding energies as the doping level decreases. All these results are in a good agreement with experimental data [23].

The lineshape of a single-particle excitation is another important characteristic property of this excitation that can be measured experimentally. The lineshape \(L(q, \omega)\) is a function of two variables. Measurements carried out at a fixed binding energy \(\omega = \omega_0\), where \(\omega_0\) is the energy of the single-particle excitation under study, determine the lineshape \(L(q, \omega = \omega_0)\), as a function of momentum \(q\). As shown above, the effective mass \(M_{FC}\) is finite if \(T > 0\), or \(\Delta \neq 0\). Therefore, the system behaves as a normal liquid characterized by a certain effective mass at energies \(\omega < 4T\) (or \(\omega < 2\Delta_1\), if \(T < T_c\)). Quasiparticles with energies of the order of temperature will be involved in rescattering processes, which determine the width of the single-particle excitation. As follows from Eq. (20), these are precisely the quasiparticles with mass \(M_{FC}\), which leads to the width of the order of \(T\) [23]. It was this behavior that was observed in experiments on measuring the lineshape at a fixed energy, when well-defined quasiparticles at the Fermi level were found even in the region of the \((\pi, 0)\) point [3]. The lineshape can be determined differently as a function of energy \(\omega\) at a fixed momentum \(q\). At small \(\omega\), the line will have a characteristic maximum and width as well as in the case of fixed energy \(\omega\). At energies \(\omega \geq 4T\) (or \(\omega \geq 2\Delta_1\), if \(T < T_c\)), quasiparticles of mass \(M_{FC}\) will come into play, which will lead to a growth of the function that determines the lineshape. Thus, this line will have a characteristic shape: a maximum, then a minimum, and then again a flat maximum. This result is in a qualitative agreement with the experiment [23, 24]. On the other hand, one may follow the procedure suggested in [8], using the Kramers-Krönig transformation to construct the imaginary part of the self-energy starting with the real one. As we have seen above, the real part of the self-energy in systems with FC can be represented by two straight lines intersecting near the point \(E_0\) and characterized by the two effective masses. As a result, the lineshape \(L(q = q_0, \omega)\) of the quasiparticle peak as a function of the binding energy \(\omega\) possesses a complex peak-dip-hump structure [3] directly defined by the existence of the two effective masses \(M_1^{\text{FC}}\) and \(M_2^{\text{FC}}\) [23].

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