Spin-orbit-induced correlations of the local density of states in two-dimensional electron gas

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We study the local density of states (LDOS) of two-dimensional electrons in the presence of spin-orbit (SO) coupling. Although SO coupling has no effect on the average density of states, it manifests itself in the correlations of the LDOS. Namely, the correlation function acquires two satellites centered at energy difference equal to the SO splitting, $2\omega_{\text{SO}}$, of the electron Fermi surface. For a smooth disorder the satellites are well separated from the main peak. Weak Zeeman splitting $\omega_Z \ll \omega_{\text{SO}}$ in a parallel magnetic field causes an anomaly in the shape of the satellites. We consider the effect of SO-induced satellites in the LDOS correlations on the shape of the correlation function of resonant-tunneling conductances at different source-drain biases, which can be measured experimentally. This shape is strongly sensitive to the relation between $\omega_{\text{SO}}$ and $\omega_Z$.

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I. INTRODUCTION.

The density of states, $\nu_0$, in 2D electron gas is energy-independent. It remains energy-independent in a parallel magnetic field, which causes the spin-splitting, $2\omega_Z$, of the electron spectrum. The density of states, corresponding to each spin branch, is equal to $\nu_0/2$. Spin-orbit (SO) coupling also results in the splitting of the electron spectrum into two branches \cite{1}. In this case the density of states in each branch, $\mu = \pm 1$, depends on energy

$$\nu_\mu(\epsilon) = \frac{\nu_0}{2} \left[ 1 + \mu \frac{\omega_{\text{SO}}}{2\epsilon} \right], \quad (1)$$

where $\pm \omega_{\text{SO}}$ is the splitting of a state which had the energy $\epsilon \gg \omega_{\text{SO}}$ in the absence of the SO-coupling. As can be seen from Eq. (1), the net density of states is still identically equal to $\nu_0$. Since the magnitude of the splitting, $2\omega_{\text{SO}}$, at the Fermi level $\epsilon = E_F$ is inversely proportional to the length of the spin rotation, this magnitude represents an important characteristics of the 2D structure, containing the electron gas. The importance of the spin-rotation length was appreciated since 1990, when the proposal for device application of the spin-polarized currents was put forward \cite{2}. This proposal has lately attracted a lot of interest.

If the magnitude of the SO-splitting is large enough, it can be inferred from the beating pattern of the Shubnikov-de Haas oscillations, as was first demonstrated by Dorozhkin and Olshanetskii \cite{3} for Si-based structures, and subsequently \cite{4} by Luo et al. for narrow-gap heterostructures. While for large $\omega_{\text{SO}}$ (several $meV$), as in \cite{4}, the beats of the Shubnikov-de Haas oscillations yield a rather accurate value of SO-splitting, extracting small splittings from the beating pattern is complicated in two regards: (i) very low magnetic fields (with cyclotron energy smaller than $\omega_{\text{SO}}$) and, correspondingly, very low temperatures are required to observe the beatings; (ii) presence of even moderate disorder suppresses the Shubnikov-de Haas oscillations by the Dingle factor, which gets large at low fields.

In the present paper we demonstrate that the disorder can, actually, \textit{reveal} the SO-splitting even if $\omega_{\text{SO}}$ is smaller than the single particle scattering rate, $\tau^{-1}$. In a disordered sample the local density of states (LDOS) fluctuates randomly in space and these fluctuations contain information about SO coupling. Our main point is that this information, which is lost in the average LDOS, is preserved in the correlation function of LDOS at two different energies, $P(\epsilon_1, \epsilon_2)$. This function, whose magnitude
is inversely proportional to the disorder, exhibits peaks at \((\epsilon_1 - \epsilon_2) = \pm 2\omega_{\text{so}}\). Even for \(\omega_{\text{so}} \tau \ll 1\) the peaks are well pronounced when the disorder is smooth, so that the transport relaxation time, \(\tau_{\text{tr}}\), is much longer than \(\tau\). This is illustrated in Fig. 1. Smooth disorder implies that the momentum transfer in a single scattering act is small compared to the Fermi momentum, \(k_F\). In this way, the states with energies \(\epsilon + \omega_{\text{so}}\) and \(\epsilon - \omega_{\text{so}}\) remain correlated after many, \(\sim \tau_{\text{tr}}/\tau\), scattering acts. Therefore, the SO-induced satellites in \(P(\epsilon_1, \epsilon_2)\) are sharp even in the presence of a strong disorder, \(\tau^{-1} > \omega_{\text{so}}\), if the condition \(\omega_{\text{so}} \tau_{\text{tr}} \gg 1\) is met. In terms of underlying physics, the observation that a small SO-splitting can be resolved in a smooth potential is quite analogous to the observation made in Ref. [5] concerning the manifestation of the Landau levels in \(P(\epsilon_1, \epsilon_2)\) in the presence of a strong disorder, when the Landau level structure in the average density of states is completely smeared out [6]. Let us emphasize, though, that in our case (unlike Ref. [5]) the density of states in the absence of disorder is constant, which does not contain \(\omega_{\text{so}}\). Thus, it is only in the presence of disorder that LDOS becomes sensitive to the SO splitting.

Concerning the experimental consequences of SO-induced satellites in \(P(\epsilon_1, \epsilon_2)\), we note that the local density of states governs the probability of electron tunneling from the impurity in the barrier into the 2D gas [7–9]. This fact was recently utilized in the resonant-tunneling spectroscopy [10–14] of the LDOS. More specifically, the correlator of the LDOS determines the behavior of correlation function, \(C(\delta V)\), of the fluctuations of the tunneling conductance with the change, \(\delta V\), of the source-drain bias. From the analysis of \(C(\delta V)\), measured experimentally, the authors [13] were able to extract the quantitative information about the structure of states and their inelastic lifetime in the disordered emitter. In the present paper we demonstrate that the SO-induced correlations in the LDOS give rise to the new characteristic features in \(C(\delta V)\). We also demonstrate that these new features are extremely sensitive to a weak parallel magnetic field.

The paper is organized as follows. In Sect. II we derive the analytic expression for \(P(\epsilon_1 - \epsilon_2)\) in the case of a smooth disorder. In Sect. III we give the qualitative explanation for the ratio between the amplitudes of the main peak in \(P(\epsilon_1 - \epsilon_2)\) and of the SO-induced satellites. In Sect. IV we use the shape of \(P(\epsilon_1 - \epsilon_2)\) to calculate the SO-induced features in the correlator, \(C(\delta V)\), of the tunneling conductances. Concluding remarks are presented in Sect. V.

**II. CALCULATION OF \(P(\epsilon_1 - \epsilon_2)\)**

**A. Hamiltonian and eigenfunctions**

We choose the conventional form, \(\alpha(\hat{\sigma} \times \hat{k})\mathbf{n}\), of the SO-term originating from the confinement potential asymmetry [1]. Here \(\alpha\) is the coupling constant, \(\hat{\sigma}\) is the spin operator, and \(\mathbf{n}\) is the unit vector normal to the two-dimensional plane. In a parallel magnetic field, that induces the Zeeman splitting \(2\omega_Z\), the Hamiltonian of a free electron is

\[
\hat{H} = \frac{\hbar^2 k^2}{2m} + \alpha(\hat{\sigma} \times \hat{k})\mathbf{n} + \omega_{\text{so}}\hat{\sigma}_x,
\]

where \(m\) is the electron mass. The eigenfunctions of the Hamiltonian Eq. (2) are classified according to the chirality, \(\mu = \pm 1\), and form two branches of the spectrum, as illustrated in Fig. 1. In the vicinity of the Fermi surface the spectrum, \(E_\mu(k)\), can be simplified

\[
E_\mu(k) = E_F + \epsilon_\mu(k),
\]

where \(\epsilon_\mu(k)\) is defined as

\[
\epsilon_\mu(k) = \pm \omega_{\text{so}}.
\]
\[ \epsilon_{\mu}(k) = \hbar v_{F}(k - k_{F}) + \mu \Omega(k), \]  

(4)

and

\[ \Omega(k) = \sqrt{\omega_{so}^2 + \omega_{k}^2 + 2\omega_{so}\omega_{k} \sin \phi_{k}}. \]  

(5)

Here \( v_{F} = \hbar k_{F}/m \) is the Fermi velocity, \( \omega_{so} = \alpha k_{F} \), and \( \phi_{k} \) is the azimuthal angle of \( k \). It is convenient to introduce the projection operators \( \hat{\Lambda}_{\mu}(k) \), which are defined through the eigenfunctions

\[ \chi_{\mu}(k) = \frac{1}{2^{1/2}} \begin{pmatrix} 1 \\
-i\mu \exp(i\nu_{k}) \end{pmatrix}, \]

(6)

of the Hamiltonian Eq. (2) as \( \hat{\Lambda}_{\mu}(k) = \chi_{\mu}(k)\chi_{\mu}^{\dagger}(k) \), so that [15]

\[ \hat{\Lambda}_{\mu}(k) = \frac{1}{2} \begin{pmatrix} 1 & i\mu \exp(-i\nu_{k}) \\
-i\mu \exp(i\nu_{k}) & 1 \end{pmatrix} \]

(7)

where the angle \( \nu_{k} \) is related to the azimuthal angle \( \phi_{k} \) as

\[ \tan \nu_{k} = \tan \phi_{k} + \frac{\omega_{k}}{\omega_{so} \cos \phi_{k}}. \]

(8)

In terms of projection operators the Hamiltonian can be presented in a simple form \( \hat{H} = \sum_{\mu} E_{\mu}(k) \hat{\Lambda}_{\mu}(k) \).

B. Correlator of the LDOS in the chirality representation

In the presence of disorder, the LDOS at energy \( \epsilon \) at point \( \mathbf{r} \) is defined as

\[ \nu(\mathbf{r}, \epsilon) = \frac{1}{2\pi i} \text{Tr} \left[ \hat{G}^{A}(\mathbf{r}, \mathbf{r}, \epsilon) - \hat{G}^{R}(\mathbf{r}, \mathbf{r}, \epsilon) \right], \]

(9)

where the advanced and retarded Green’s functions are \( 2 \times 2 \) matrices in the chirality space, and the trace is taken over different chiralities. The main contribution to the LDOS correlator comes from the cross terms \( G^{(R)}G^{(A)} \)

\[ P(\epsilon - \epsilon') = \frac{1}{\nu_{0}^{2}} \langle \delta\nu(\mathbf{r}, \epsilon)\delta\nu(\mathbf{r}, \epsilon') \rangle = \frac{1}{2\pi^{2} \nu_{0}^{2}} \text{Re} \left( \text{Tr} \hat{G}^{R}(\mathbf{r}, \mathbf{r}, \epsilon)\text{Tr} \hat{G}^{A}(\mathbf{r}, \mathbf{r}, \epsilon') \right), \]

(10)

where \( \delta\nu(\mathbf{r}, \epsilon) \) is the deviation of the LDOS from the disorder-free value, \( \nu_{0} \). It is easy to see from Eq. (4) that \( \nu_{0} = m/\pi \hbar^{2} \), and does not depend on energy. To evaluate the correlator Eq. (10) it is convenient to use the chirality representation of the Green functions

\[ \hat{G}^{R,A}(\epsilon, \mathbf{p}) = \sum_{\mu} \frac{\hat{\Lambda}_{\mu}(\mathbf{p})}{\epsilon - \epsilon_{\mu}(\mathbf{p}) \pm i \frac{\tau}{2}} = \sum_{\mu} \hat{\Lambda}_{\mu}(\mathbf{p})G^{R,A}_{\mu}(\epsilon, \mathbf{p}), \]

(11)

where \( \tau \) is the scattering time. As in the absence of the SO-coupling, the LDOS correlator represents the sum of the diffuson and cooperon contributions [16]. Generalized expressions for the corresponding contributions in the chirality representation read

\[ P_{D}(\epsilon - \epsilon') = \frac{1}{2\pi^{2} \nu_{0}^{2}} \text{Re} \int \frac{d\mathbf{q}}{(2\pi)^{2}} \int \frac{d\mathbf{p}}{(2\pi)^{2}} \int \frac{d\mathbf{p}'}{(2\pi)^{2}} \sum_{(\mu, \eta)} \Gamma_{\mu_{1}\eta_{1}}^{\mu_{2}\eta_{2}}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \epsilon - \epsilon') \Phi_{\mu_{1},\eta_{1}}^{*}(\mathbf{p}, \mathbf{p}') \Phi_{\mu_{2},\eta_{2}}(\mathbf{p}, \mathbf{p}') \times \]

\[ \times G^{R}_{\mu_{1}}(\epsilon, \mathbf{p} + \mathbf{q})G_{\eta_{1}}^{R}(\epsilon, \mathbf{p}' + \mathbf{q})G^{A}_{\mu_{2}}(\epsilon', \mathbf{p})G^{A}_{\eta_{2}}(\epsilon', \mathbf{p}'), \]

(12)
domain \mid be presented as \( (17), (18) \). Using Eq. (8), the factor \( \Phi_{\mu_1, \eta_1}(p, p') \Phi^*_{\mu_2, \eta_2}(p, p') \)
\times G^R_{\mu_1}(\epsilon, q - p) G^R_{\eta_1}(\epsilon, q - p') G^A_{\mu_2}(\epsilon', p) G^A_{\eta_2}(\epsilon', p') = P_D(\epsilon' - \epsilon),
(13)
where
\[ \Phi_{\mu, \eta}(p, p') = \frac{1}{2} \{ 1 + \mu \eta \exp (i[\varphi_p - \varphi_{p'}]) \}. \]

The two-particle vertex functions \( \Gamma^\mu_{\mu_2 \eta_2}(p, p', q, \epsilon) \) in Eqs. (12), (13) satisfy the matrix Dyson-type equation
\[ \Gamma^\mu_{\mu_2 \eta_2}(p, p', q, \epsilon) = S(p - p') \Phi_{\mu_1, \eta_1}(p, p') \Phi_{\mu_2, \eta_2}(p, p') + \int \frac{dp_1}{(2\pi)^2} \sum_{\xi_1, \xi_2} K^\mu_{\mu_2 \xi_2}(p, p_1, q, \epsilon) \Gamma^\xi_{\eta_2}(p_1, p', q, \epsilon) \]
(15)
with a kernel
\[ K^\mu_{\mu_2 \xi_2}(p, p_1, q, \epsilon) = \Phi_{\mu_1, \xi_1}(p, p') \Phi_{\mu_2, \xi_2}(p, p') G^R_{\xi_1}(\omega + \epsilon, p_1 + q) G^A_{\xi_2}(\omega, p_1) S(p - p'), \]
(16)
where the function \( S(p - p') \) is the Fourier transform of the correlator of the random potential. Upon integration over \( |p_1| \) Eq. (15) takes the form
\[ \Gamma^\mu_{\mu_2 \eta_2}(p, p', q, \epsilon) = S(p - p') \Phi_{\mu_1, \eta_1}(p, p') \Phi_{\mu_2, \eta_2}(p, p') + \int \frac{dp_1}{(2\pi)^2} \sum_{\xi_1, \xi_2} \tilde{K}^\mu_{\mu_2 \xi_2}(p, p_1, q, \epsilon) \Gamma^\xi_{\eta_2}(p_1, p', q, \epsilon) \]
(17)
where we have introduced the modified kernel \( \tilde{K}^\mu_{\mu_2 \xi_2} \), defined as
\[ \tilde{K}^\mu_{\mu_2 \xi_2}(p, p_1, q, \epsilon) = \left( \frac{m\tau}{2\pi} \right) \frac{\Phi_{\mu_1, \xi_1}(p, p') \Phi_{\mu_2, \xi_2}(p, p') S(p - p')}{1 - i(\omega - \xi_1 \Omega(p_1) + \xi_2 \Omega(p_1 + q))\tau + i\hbar q v_\tau \cos(\phi_p - \phi_q)}. \]
(18)

When the random potential is smooth, the function \( S \) restricts the difference \( p - p' \) within a narrow domain \( |p - p'| \ll k_f \). As we will see below this leads to a drastic simplification of the system Eqs. (17), (18).

C. Smooth potential; |\epsilon_1 - \epsilon_2| \approx 2\omega_{SO}

In the case of a smooth potential the factors \( \Phi_{\mu, \eta}(p, p') \) for coinciding chiralities, \( \mu = \eta \), differ strongly from those with opposite chiralities, \( \mu = -\eta \). Indeed, when \( |p - p'| \) is small compared to \( k_f \), we have \( \phi_p \approx \phi_{p'} \) and thus \( \varphi_p \approx \varphi_{p'} \), so that \( \Phi_{\mu, \mu} \ll 1 \), whereas \( \Phi_{\mu, \mu} \) is close to unity and can be presented as
\[ \Phi_{\mu, \mu}(p, p') = 1 - \frac{(\varphi_p - \varphi_{p'})^2}{8}. \]
(19)
Using Eq. (8), the factor \( \Phi_{\mu, \mu} \) can be expressed through the angle, \( \phi_{p'} - \phi_p \), between the vectors \( p \) and \( p' \) as follows
\[ \Phi_{\mu, \mu}(p, p') = 1 - \frac{\omega_{SO}^2 (\omega_{SO} + \omega_2 \sin \phi_p)^2}{4 \Omega^4(p)} (\phi_{p'} - \phi_p)^2. \]
(20)
The latter simplification allows to set $\Phi_{\mu,-\mu} = 0$ and $\Phi_{\mu,\mu} = 1$ everywhere except for the kernel of the Dyson equation. As a result, the system Eqs. (17) gets decoupled into closed equations for the elements, $\Gamma_{\eta,\eta}$ with $\mu \neq \eta$, of the matrix $\Gamma_{\mu,\eta}$. The underlying reason for this decoupling is that these elements describe the two-particle motion, in course of which one particle moves within the branch $\mu$ whereas another particle moves within the branch $\eta$. For a smooth potential coupling of these elements to the other branch-nonconserving elements is small in parameter $(1 - \Phi_{\mu,\mu}) \ll 1$.

With the above simplification Eq. (17) takes the form [17]

$$
\Gamma_{\eta,\eta}^{\mu,\mu}(p, p', q, \omega) = S(k_v | \phi_p - \phi_{p'}|) + \int \frac{dp}{(2\pi)^2} \tilde{K}_{\eta,\eta}^{\mu,\mu}(p, p, q, \omega) \Gamma_{\eta,\eta}^{\mu,\mu}(p_1, p', q, \omega),
$$

(21)

while the kernel Eq. (18) simplifies to

$$
\tilde{K}_{\eta,\eta}^{\mu,\mu}(p, p_1, q, \omega) = \left( \frac{mT}{2\pi} \right) \frac{|\Phi_{\mu,\mu}(p, p_1)|^2}{1 - i\tau [\omega - (\mu - \eta) \Omega(p) - ihq_{\eta} \cos(\phi_p - \phi_q)]}.
$$

(22)

Substituting Eq. (22) into Eq. (21) yields an integral equation for the vertex $\Gamma_{\eta,\eta}^{\mu,\mu}$. Note, that this equation differs from the standard equation for the vertex in the absence of the SO-coupling only by the factor $|\Phi_{\mu,\mu}(p, p_1)|^2$ in the integrand, where $\Phi_{\mu,\mu}(p, p_1)$ is defined by Eq. (19). Thus, if the small difference of this factor from unity is neglected, the solution for $\Gamma_{\eta,\eta}^{\mu,\mu}$ would contain a conventional diffusive pole at $\omega = (\mu - \eta) \Omega(p) - iDq^2$, where $D = v_{\eta}^2 \tau_{tr}/2$ is the diffusion coefficient, and

$$
\tau_{tr}^{-1} = \frac{1}{2} \int \frac{dp}{(2\pi)^2} (\phi_k - \phi_p)^2 S(|k - p|) \delta(E_{\mu}(k) - E_{\mu}(p)).
$$

(23)

is the transport relaxation time. Taking into account the small correction originating from the difference $1 - |\Phi_{\mu,\mu}(p, p_1)|^2$ amounts to the imaginary shift of the pole position by $i\tau_{int}^{-1}$, where $\tau_{int}$ is defined as

$$
\tau_{int}^{-1}(k) = \frac{1}{4} \int \frac{dp}{(2\pi)^2} (\varphi_k - \varphi_p)^2 S(|k - p|) \delta(E_{\mu}(k) - E_{\mu}(p)).
$$

(24)

The meaning of $\tau_{int}$ is the interbranch scattering time. Indeed, a general expression for the scattering time between the branches with different chiralities can be written as

$$
\tau_{\mu,-\mu}^{-1}(k) = \frac{1}{2} \int \frac{dp}{(2\pi)^2} \text{Tr} \left( \hat{\Lambda}_{\mu}(k) \hat{\Lambda}_{-\mu}(p) \right) S(|k - p|) \delta(E_{\mu}(k) - E_{-\mu}(p))
$$

$$
= \frac{1}{2} \int \frac{dp}{(2\pi)^2} \left( 1 - |\Phi_{\mu,\mu}(k, p)|^2 \right) S(|k - p|) \delta(E_{\mu}(k) - E_{-\mu}(p)).
$$

(25)

Comparing Eqs. (24) and (25), we see that they differ only by the arguments of the $\delta$-functions. This difference is negligible when $\omega_{so} \ll E_{\varphi}$, and thus we have $\tau_{\mu,-\mu}(k) \approx \tau_{int}(k)$. We also see that the integral Eq. (24) in the expression for $\tau_{int}^{-1}$ differs from the integral in Eq. (23) for $\tau_{tr}^{-1}$ only by the replacement $\varphi_k \rightarrow \phi_k$. Using the relation Eq. (20) between the two angles, we can express $\tau_{int}$ through the transport relaxation time

$$
\tau_{int}(\phi) = 2\tau_{tr} \frac{\omega_{so}^2 + \omega_{so}^2 + 2\omega_{so} \omega_{so} \sin \phi}{\omega_{so}^2 + \omega_{so} \sin \phi^2}.
$$

(26)

With shifted diffusion pole, the final expression for the vertex $\Gamma_{\eta,\eta}^{\mu,\mu}$ takes the form

$$
\Gamma_{\eta,\eta}^{\mu,\mu}(p, p', q, \omega) = \frac{iS(k_v | \phi_p - \phi_{p'}|)}{\omega - (\mu - \eta) \Omega(p) + iDq^2 + i\tau_{int}^{-1}(p)}.
$$

(27)
Substituting this form into Eq. (12) yields the following expression for the diffusion contribution to the correlator of LDOS

$$ P_D(\epsilon) = \frac{\tau}{16\pi^3} \sum_{\mu \neq \eta} \Re \int_0^{1/v_F \tau_r} dq \int_0^{2\pi} d\phi_P \int_0^{2\pi} d\phi_P' \frac{S(k_x |\phi_P - \phi_P'|)}{2\pi} - i [\epsilon - (\mu - \eta) \Omega(p)] + Dq^2 + \tau_{int}^{-1}(p). \quad (28) $$

Two integrations (over q and over $\phi_{P'}$) in Eq. (28) can be readily performed resulting in the following expression for SO-induced satellites $P_D(\epsilon)$

$$ P_D(\epsilon) = \frac{1}{4\pi E_F \tau_{tr}} \sum_{\mu \neq \eta} L_{\mu,\eta}(\epsilon), \quad (29) $$

where the functions $L_{\mu,\eta}(\epsilon)$ are the following azimuthal averages

$$ L_{\mu,\eta}(\epsilon) = \int_0^{2\pi} d\phi \ln \left[ (\epsilon - (\mu - \eta) \Omega(\phi))^2 \tau_{tr}^2 + \frac{\tau_{tr}^2}{\tau_{int}^{-1}(\phi)} \right]. \quad (30) $$

Since $L_{\mu,\eta}(\epsilon) = L_{\eta,\mu}(-\epsilon)$ the cooperon contribution, $P_C(\epsilon)$, to the LDOS correlator coincides with $P_D(\epsilon)$, as it should be expected on general grounds. Thus, Eqs. (29) and (30) constitute our final result. The functions $L_{\mu,\eta}(\epsilon)$, which determine the energy dependence of the correlator, exhibit a singular dependence on a weak magnetic field, $\omega_z \ll \omega_{SO}$ for $\mu = -\eta$, as demonstrated in the next subsection.

### D. Shape of the satellites

As it follows from Eq. (26), in a weak magnetic field we have $\tau_{int}(\phi) \approx 2\tau_{tr} = const(\phi)$. Then for the satellites, centered at $\epsilon = \pm 2\omega_{SO}$ we can identify the $\phi$-dependence of the integrand in Eq. (30). This dependence comes from $\Omega(\phi)$, defined by Eq. (5). For $\omega_z \ll \omega_{SO}$, we have $\Omega(\phi) \approx \omega_{SO} + \omega_z \sin \phi$. Upon substituting this form into Eq. (30), the integration can be performed analytically with the use of the following identity

$$ \frac{1}{2\pi} \int_0^{2\pi} d\phi \ln (a + b \sin \phi)^2 = \left\{ \begin{array}{ll} \ln \left[ a + \sqrt{a^2 - b^2} \right]^2 & \text{for } |a| > |b|, \\ 2 \ln |b| & \text{for } |a| < |b|, \end{array} \right. \quad (31) $$

yielding

$$ P(\epsilon) = -\left( \frac{1}{\pi E_F \tau_{tr}} \right) \left\{ \begin{array}{ll} \frac{\ln |\epsilon - 2\omega_{SO}|}{\tau_{tr}} & \text{for } |\epsilon - 2\omega_{SO}| > \omega_z, \\ \frac{\ln \omega_z \tau_{tr}}{\tau_{tr}} & \text{for } |\epsilon - 2\omega_{SO}| \leq \omega_z. \end{array} \right. \quad (32) $$

The satellites of the correlator $P(\epsilon)$, calculated numerically without the assumption $\omega_z \ll \omega_{SO}$, are shown in Fig. 2. We see that the plateau in the shape of the satellites is very well pronounced, but slightly tilted. The reason for this tilt is the effect of the “background” originating from the main peak.

### E. Smooth potential; $|\epsilon_1 - \epsilon_2| \ll \omega_{SO}$

In this subsection we solve the system of Eqs. (17) with kernels defined by Eq. (18)) for small energy difference $|\epsilon_1 - \epsilon_2| \ll \omega_{SO}$. Firstly, we point out that the vertex functions $\Gamma_{\mu,\mu}^{\mu,\mu}$ and $\Gamma_{\mu,\mu}^{-\mu,\mu}$ are relevant, since in Eq. (18) $\xi_1$ must be equal to $\xi_2$. At the first glance, in equation (17) for $\Gamma_{\mu,\mu}^{\mu,\mu}$, only
the term in the r.h.s. containing the same $\Gamma_{\mu,\mu}^{-}$ should be kept. Indeed, the coupling of this term to $\Gamma_{\mu,\mu}^{+}$ is determined by the kernel $\tilde{K}_{\mu,\mu}^{-}$, which, as it is seen from Eq. (18) contains a square of the small parameter $\Phi_{\mu,\mu}^{-}$. However, unlike the case of the satellites, the “feedback” becomes important for small $|\epsilon_1 - \epsilon_2|$. Namely, the equation (17) for $\Gamma_{\mu,\mu}^{+}$ contains in the r.h.s. the coupling to $\Gamma_{\mu,\mu}^{-}$ with the same small coefficient $\propto \Phi_{\mu,\mu}^{2}$. Specifics of the small energy difference, as compared to satellites, is that both $\Gamma_{\mu,\mu}^{+}$ and $\Gamma_{\mu,\mu}^{-}$ are resonant in this case. Thus Eq. (17) reduces to the following system of coupled equations

$$
\Gamma_{\mu,\mu}(p, p', q, \omega) = S(k_v |\phi_p - \phi_{p'}|) + \int \frac{d\phi_{p_1}}{(2\pi)^2} \tilde{K}_{\mu,\mu}(p, p_1, q, \omega) \Gamma_{\mu,\mu}(p_1, p', q, \omega)
$$

$$
\Gamma_{\mu,\mu}^{-}(p, p', q, \omega) = S(k_v |\phi_p - \phi_{p'}|) |\Phi_{\mu,\mu}(p, p_1)|^2 + \int \frac{d\phi_{p_1}}{(2\pi)^2} \tilde{K}_{\mu,\mu}^{-}(p, p_1, q, \omega) \Gamma_{\mu,\mu}^{-}(p_1, p', q, \omega)
$$

Solution of the system (33), (34) yields [20]

$$
\Gamma_{\mu,\mu}(p, p', q, \omega) = \frac{i}{2\tau} S(k_v |\phi_p - \phi_{p'}| \left\{ \frac{1}{\omega + iDq^2 + i\tau_f^{-1}(p)} + \frac{1}{\omega + iDq^2 + 2i\tau_{int}^{-1}(p)} \right\}
$$

$$
\Gamma_{\mu,\mu}^{-}(p, p', q, \omega) = \frac{i}{2\tau} S(k_v |\phi_p - \phi_{p'}| \left\{ \frac{1}{\omega + iDq^2 + i\tau_f^{-1}(p)} - \frac{1}{\omega + iDq^2 + 2i\tau_{int}^{-1}(p)} \right\}
$$

where the diffusion pole in the first term is cut by the inelastic time, $\tau_f$. Substituting $\Gamma_{\mu,\mu}^{+}$ and $\Gamma_{\mu,\mu}^{-}$ into the general expression Eq. (12) and keeping only the terms with coefficients $\Phi_{\mu,\mu} \approx 1$, we arrive at the final result

$$
P(\epsilon) = - \left( \frac{1}{2\pi E_F \tau_{tr}} \right) \left[ \ln \left( \epsilon^2 \tau_{tr}^2 + \frac{\tau^2_{tr}}{\tau_f} \right) + \ln \left( \epsilon^2 \tau_{tr}^2 + 1 \right) \right].
$$

Note that each term in the sum (37) is comprised of two (equal) contributions from $\Gamma_{\mu,\mu}^{1,1}$ and $\Gamma_{\mu,\mu}^{-1,-1}$. In the derivation of Eq. (37), as well as of Eq. (29), we used $(v_s \tau_{tr})^{-1}$ as the upper cutoff in the integrals over $q$. In principle, there is also a contribution from the ballistic region, up to $q \approx k_v$. This contribution does not affect the correlation function of the tunneling conductance, studied in Sec. IV.

### III. DISCUSSION

The general case of an arbitrary SO-splitting would require a lengthy calculation, involving $4 \times 4$ diffusion and cooperon matrices [18,19]. However, in the most interesting case of a strong splitting, when $\omega_{iso} \tau_{int} \gg 1$, a considerable simplification occurs: the LDOS correlation function, $P(\epsilon)$, at large enough $\epsilon \gg \tau_{int}^{-1}$ can be expressed in terms of that in the absence of splitting, $P(\epsilon, \omega_{iso} = 0) \equiv P_0(\epsilon)$. The splitting energy $2\omega_{iso}$ enters only through the argument of $P_0$ and the low-energy cutoff $h/\tau_{int}$, which assumes the role of dephasing time.
To establish the relation between \( P \) and \( P_0 \), we notice [20] that the states with the same energy, \( E \), have two different values of the wave number, \( \hbar k_{\pm} = \sqrt{2mE} \pm \omega_{\text{so}}/v_p \), depending on the branch (see Fig. 1). Similarly, states at energy \( E' = E + \epsilon \) have wave numbers \( \hbar k'_{\pm} = \sqrt{2mE} + (\epsilon \pm \omega_{\text{so}})/v_p \). Thus, we can identify four contributions to \( P(\omega) \), coming from correlating each of the \( k_{\pm} \) states with either of \( k'_{\pm} \) states. Since, for \( \omega_{\text{so}} \tau_{\text{int}} \gg 1 \), the degree of correlation between a pair of states depends only on the difference of the wave numbers, all four contributions have the same functional form. It is given by the function \( P_0(\epsilon) \) with the corresponding arguments, which can be either \( \hbar v_p(k'_{\pm} - k_{\pm}) = \epsilon \) or \( \hbar v_p(k'_{\pm} - k_{\pm}) = \epsilon + 2\omega_{\text{so}} \), or \( \hbar v_p(k'_{-} - k_{+}) = \epsilon - 2\omega_{\text{so}} \). It must be emphasized that, since separate branches cannot be resolved beyond the time \( \tau_{\text{int}} \), the correlator \( P_0 \) for \( |\epsilon| \approx 2\omega_{\text{so}} \) must be computed with a low-energy cutoff \( \hbar/\tau_{\text{int}} \). Hence the condition \( \epsilon \tau_{\text{int}} \gg 1 \), adopted in this qualitative consideration, insures that the SO-induced satellites in \( P(\epsilon) \) do not overlap with the main peak. Combining the four contributions, we obtain

\[
P(\epsilon) = \frac{1}{4} [2P_0(\epsilon) + P_0(\epsilon + 2\omega_{\text{so}}) + P_0(\epsilon - 2\omega_{\text{so}})] .
\]  

(38)

Obviously, the above general consideration, applies to the both diffuson and cooperon contributions to the LDOS correlator, \( P \). This consideration clarifies why the height of the central peak in \( P(\epsilon) \) is twice the height of each of the SO satellites. The factor 2 in the first term of Eq. (38) appears because the argument \( \epsilon \) occurs twice out of the four possibilities mentioned above.

IV. MANIFESTATION IN THE TUNNELING SPECTROSCOPY

In the tunneling-spectroscopy experiments [21,22,10,11,23,24,12–14] the measured quantity is a resonant-tunneling current, \( I_{sd} \), through a localized impurity state as a function of the source-drain bias, \( V_{sd} \). Such an impurity plays the role of a “spectrometer” since its energy position, \( \epsilon_0 \), changes with \( V_{sd} \). Within a narrow range, \( \delta V \), of \( V_{sd} \) this change is linear, i.e., \( \Delta = \epsilon_0(V + \delta V) - \epsilon_0(V) \approx \beta \delta V \), where \( \beta \) is the structure-specific parameter. Experimentally [21,22,10,11,23,24,12–14], measurements are performed in the plateau regime when the temperature is much lower than \( V_{sd} \), so that the value of resonant-tunneling current

\[
I(V_{sd}) = \frac{e}{\hbar} \int_{-\infty}^{\infty} d\epsilon \frac{\gamma_l(\epsilon)\gamma_r(\epsilon)}{[\epsilon - \epsilon_0(V_{sd})]^2 + [\gamma_l(\epsilon) + \gamma_r(\epsilon)]^2/4}
\]  

(39)

is temperature independent. In Eq. (39), \( \gamma_l \) and \( \gamma_r \) stand for the tunneling widths, associated with the escape from the impurity into the emitter and collector, respectively. These widths are proportional to the LDOS in the electrodes. Typically, the widths \( \gamma_l \) and \( \gamma_r \) differ strongly, \( \gamma_r \gg \gamma_l \). In addition, the energy dependence of \( \gamma_r \) is weak, so that it can be considered as a constant. As a result, it is a tunneling coupling to collector that dominates the width of the Lorentzian in Eq. (39). With regard to the weak fluctuating dependence of current, \( I \), on \( V_{sd} \) in the plateau regime, it is exclusively due to the energy dependence of \( \gamma_l \), which, in turn, originates from the fluctuations of the LDOS.

Quantitative analysis of the experimental data is performed (see e.g., Ref. [13]) by plotting the correlator \( C(\delta V) = \langle \delta g(V + \delta V)\delta g(V) \rangle \) of the fluctuations of the differential conductance \( g = dI_{sd}/dV \) around its average value, \( \langle g \rangle \), as a function of \( \delta V \) for different values of \( V = V_{sd} \). The expression for \( C(\delta V) \) in terms of the LDOS correlator \( P(\epsilon - \epsilon') \) is obtained (see [9,25]) by, first, calculating the the correlator of current fluctuations \( \langle \delta I(V_{sd})\delta I(V_{sd}') \rangle \) and then taking derivatives with respect to \( V_{sd} \) and \( V_{sd}' \). Using the fact that a convolution of two lorentzians is also a lorentzian, the final expression for \( C(\delta V) \) can be written in the form
\[ C(\delta V) = \langle \delta g(V_a + \delta V)\delta g(V_a) \rangle = -\left(\frac{\gamma}{2}\right)^2 \frac{\partial^2}{\partial \Delta^2} \left\{ \gamma_r \int \frac{d\omega |P(\omega) + P(-\omega)|}{(\omega - \Delta)^2 + \gamma_r^2} \right\}, \] (40)

If, following Ref. [9], we substitute the conventional form \( P(\omega) \propto \ln(\omega^2 \tau_f^2 + 1) \) into Eq. (40), where \( \tau_f \) is the “floating up” time of a hole created as a result of the tunneling act, then Eq. (40) yields \( C(\delta V) = F_2(\delta V/V_c) \), where the dimensionless function \( F_2(x) \) is defined as

\[ F_2(x) = \frac{1 - x^2}{(1 + x^2)^2}, \] (41)

and the characteristic value, \( V_c \), is given by \( V_c = \beta^{-1} (\gamma_r + \hbar/\tau_f) \). In the presence of the SO-coupling the correlator \( P(\omega) \) has three peaks, centered at \( \omega = 0 \) and \( \omega = \pm 2\omega_{so} \). Without Zeeman splitting all three peaks have the same shape, which allows to express the correlator, Eq. (40), in terms of the function \( F_2 \) as follows

\[ C(\delta V) = \frac{1}{4} F_2 \left( \frac{\delta V}{V_c} \right) + \frac{1}{4} F_2 \left( \frac{\delta V}{V_c'} \right) + \frac{1}{4} F_2 \left[ \frac{\beta \delta V - 2\omega_{so}}{\beta V_c'} \right] + \frac{1}{4} F_2 \left[ \frac{\beta \delta V + 2\omega_{so}}{\beta V_c'} \right], \] (42)

where \( V_c' = V_c + \beta^{-1} \hbar/\tau_{int} = \beta^{-1} (\gamma_r + \hbar/\tau_f + \hbar/\tau_{int}) \), \( V_c'' = V_c + 2\beta^{-1} \hbar/\tau_{int} = \beta^{-1} (\gamma_r + \hbar/\tau_f + 2\hbar/\tau_{int}) \), and \( \tau_{int} \) is the intersubband scattering time defined by Eq. (26). It is seen from Eq. (42) that the SO peaks in \( P(\omega) \) give rise to the satellites in \( C(\delta V) \) at \( \delta V = 2\beta^{-1}\omega_{so} \). Thus, the SO-coupling has a noticeable effect on the correlator of the tunnel conductances when \( 2\beta^{-1}\omega_{so} \geq V_c' \). In Fig. 3 the correlator \( C \) is plotted vs. \( \delta V/V_c' \) for three different values of the dimensionless SO-splitting \( \omega_{so}' = \omega_{so}/\beta V_c' \). It is seen, that at \( \omega_{so}' = 0.5 \) the correlator \( C(\delta V/V_c') \) develops a characteristic “shoulder”, while at \( \omega_{so}' = 0.7 \) it exhibits a well-developed additional maximum. In the experiments [11–14] the characteristic width of the impurity level, \( \gamma_r \), was \( \lesssim 1K \). This suggests that the SO satellites should be well resolved in tunneling spectroscopy of the narrow-gap semiconductors, where \( \omega_{so} \) is of the order of several \( meV \). Concerning the GaAs-based structures, the SO splitting there is much smaller (of the order of \( 1K \)), and depends strongly on the details of the confinement potential. Therefore, the observation of the additional peaks in tunneling spectroscopy would require a higher “spectrometer resolution”. The characteristic signature of the SO-related features in the correlator \( C(\delta V) \) would be a strong sensitivity of the correlator to a weak parallel magnetic field. In Fig. 4 we plot \( C(\delta V/V_c'') \) for the same values \( \omega_{so}' = 0.5 \) and \( \omega_{so}' = 0.7 \) as in Fig. 3 and for different ratios \( \omega_{so}'/\omega_{so} \). The curves were calculated using Eqs. (29), (37) for \( P(\omega) \). We see that both the “shoulder” at \( \omega_{so}' = 0.5 \) and additional maximum at \( \omega_{so}' = 0.7 \) do not disappear with increasing \( \omega_{so} \), but rather get shifted down. Remarkably, the effect of the parallel field is noticeable already at very small \( \omega_{so}'/\omega_{so} \sim 0.1 \).

V. CONCLUSIONS

In the present paper we have demonstrated that the intrinsic SO-splitting of the electron spectrum in the absence of disorder manifests itself in a disorder-induced effect - mesoscopic fluctuations of the local density of states. This observation suggests that the splitting, \( 2\omega_{so} \), must show up in the resonant tunneling spectroscopy in the form of additional peaks in the correlator \( C(\delta V) \). The fact that the positions of these peaks do not depend on the disorder offers a possibility to measure experimentally the magnitude of the splitting. Note that, unlike the magnetotransport experiments, where the SO-related features are quickly washed out with increasing temperature [26], the resonant tunneling current in the plateau regime depends on temperature rather weakly [12]. As it was
demonstrated in Sec.II, the correlator, $P$, of the LDOS develops a plateau in a weak parallel magnetic field $\omega_Z \ll \omega_{SO}$. This singular behavior manifests itself in the anomalous sensitivity of the correlator of the tunneling conductances to $\omega_Z$, as illustrated in Fig. 4.

VI. ACKNOWLEDGEMENTS

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FIG. 1. Schematic illustration of the processes responsible for SO-induced peaks in LDOS correlator at $\epsilon = \pm 2\omega_{\text{SO}}$.

FIG. 2. Correlator of the LDOS, $P(\epsilon_1, \epsilon_2)$, calculated from Eqs. (29), (37) for $\omega_{\text{SO}}\tau_r = 10$ and $\tau_f/\tau_r = 10$, is plotted versus dimensionless energy $\epsilon/\omega_{\text{SO}}$. The numbers near the lines show the value of $\omega_{\text{SO}}/\omega_{\text{so}}$. For convenience different curves are shifted along the vertical axis.
FIG. 3. The correlator $C(\delta V) = \langle \delta g(V) \delta g(V + \delta V) \rangle$ of differential tunneling conductances is plotted from Eq. (42) versus dimensionless voltage difference $\delta V/V'_c$ for different values of the SO-splitting: $\omega_{\text{SO}} = 0$ (solid line); $\omega_{\text{SO}} = 0.5V'_c$ (dotted line); $\omega_{\text{SO}} = 0.7V'_c$ (dashed line). For simplicity, we have assumed $\gamma_r \gtrsim \hbar/\tau_{\text{int}}$, so that $V'_c \approx V_c \approx V''_c$.

FIG. 4. Evolution of the SO-related features in the correlator $C(\delta V)$ with parallel magnetic field is shown for same $\omega_{\text{SO}} = 0.5V'_c$ (a) and $\omega_{\text{SO}} = 0.7V'_c$ (b) as in Fig. 3. Solid lines $\omega_2 = 0$; dashed lines $\omega_2 = 0.1\omega_{\text{SO}}$; dotted lines $\omega_2 = 0.2\omega_{\text{SO}}$. 