Abstract

Recent evidence of the phenomenal energetics involved in \( \gamma \)-ray burst events strongly suggests that the progenitor must efficiently convert gravitational binding energy into a moderately collimated outflow, possibly in the form of a Poynting jet. We show that an MHD-instability driven dynamo (IDD) operating in a hot accretion disk is capable of generating energetically adequate magnetic flux deposition rates above and below a mildly advective accretion disk structure. The dynamo is driven by the magnetorotational instability (MRI) of a toroidal field in a shear flow and is limited by the buoyancy of ‘horizontal’ flux and by reconnection in the turbulent medium. In the comoving frame of a semi-thick, slim disk setting, the predominant field components reside in surfaces perpendicular to the local meridian and the flux is deposited in funnels that are relatively free of baryons. The efficiency of magnetic energy deposition is estimated to be comparable to the neutrino losses but the strong effective shear induced by the metric on the MRI favors pumping magnetic field energy at low wavenumbers, i.e., field generation at large coherence lengthscales. This, in turn, suggests that an MHD collimation mechanism may deem this process a more viable alternative to neutrino-burst–driven models of \( \gamma \)-ray bursts.

1 Introduction

The combined redshift and fluence measurements of at least five \( \gamma \)-ray burst sources: GRB970508 \( z = .835 \) (Metzger et al. 1997, Kouveliotou et al. 1997), GRB971214 \( z = 3.4 \) (Kulkarni et al. 1998, Kippen et al. 1997), GRB980718 \( z = .966 \) (Djorgovski et al. 1998, Kippen et al. 1998), GRB990123 \( z = 1.6 \) (Kelson et al. 1999, Kippen 1999, Conners et al. 1999), and GRB990510 \( z = 1.619 \) (Vreeswijk et al. 1999, Kippen et al. 1999), plus very large photon energy detections in certain bursts (e.g. Sommer et al. 1994, Hurley et al. 1994, Dingus 1995) and tight size constraints derived from the rapid risetimes of burst triggers (Walker, Schafer & Fenimore 1998) strongly suggest that the release of energy is highly focused by the central engine that propels a \( \gamma \)-ray burst.

In spite of the very large \( \gamma \)-ray energy requirements, \( E_\gamma \simeq \zeta \times 10^{54} \) erg (\( \zeta \equiv \delta \Omega_\nu / 4\pi \)), the efficiency of energy deposition into electromagnetic channels is likely to be very poor, \( E_\gamma / E_{\text{tot}} \equiv \varepsilon \ll [0.01, 0.001] \), if the burst is driven by a neutrino burst in analogy to the processes thought to give birth to supernovae (MacFadyen & Woosley 1998, 1999) or if the burst involves major energy losses to gravitational radiation such as might be the case in compact object merger scenarios (Rasio & Shapiro 1992, 1994, Davies et al. 1994, Janka & Ruffert 1996, 1998; Ruffert & Janka 1998).

Thus, although the monumental observational progress of late at first prompted some workers to suggest that “the physical mechanisms behind \( \gamma \)-ray bursts (were) within reach” (Metzger et al. 1997), the unexpectedly large increase in the energy and/or collimation requirement of any viable \( \gamma \)-ray burst progenitor has deemed such mechanisms at present largely undetermined. Indeed these measurements pose a serious energy budget problem for arguably all gravitational collapse powered theoretical models of \( \gamma \)-ray bursts if the energy release is not moderately collimated \( \zeta \leq 1.0_{-3} \) (see, however, Mészáros, Rees & Wijers 1999).

On the other hand, the angular spreading problem (Fenimore, Epstein & Ho 1996, Fenimore, Mandras & Nayakshin 1996) means that it may be quite implausible for complex millisecond \( \gamma \)-ray variability to be attributed to the interaction of a single fireball with an external medium. The somewhat favored scenario is one in which the variability down to possibly sub-ms timescales portrays flux fluctuations at the emission radius \( R_e \) (Sari & Piran 1997, Kobashi, Piran & Sari 1998) or from flaring within the energy source itself. Fenimore, Ramirez-Ruiz & Wu (1999) assert that the latter must be the case for the brightest burst yet: GRB990123. It is not clear, however, how such radiation could avoid thermalization unless the energy release mechanism yields infinitesimally thin shells (M. Baring, Priv. Comm.).

An attractive solution to the energy budget/collimation problem starts with a directional Poynting-flux dominated outflow (Thompson 1994, Mészáros & Rees 1997) under the premise that such a flow may carry very little baryonic contamination if deposited along a centrifugally (or gravitationally) evacuated funnel such as the angular momentum axis of a black hole-accretion disk system.

The notion that magnetic fields may play an important role in tapping the energy available in accretion disks and in its subsequent re-processing into a high Lorentz factor outflow...
has been surmised by several authors. Attempts have been made at drawing an analog to the standard phenomenology of pulsar electromagnetic emission (e.g., the magnetized differential rotor Katz 1997) and at invoking the Blandford-Znajek mechanism (BZm) (Paczynski 1998) and BZm-like processes at the innermost regions of accretion disks (Ghosh & Abramowicz 1997, Livio, Ogilvie & Pringle 1998).

These estimates of Poynting flux luminosities assume a priori the existence of a nearly equipartition, large-scale ($O(r)$), external magnetic field especially with regards to jet formation processes (Blandford & Payne 1982, Tsinganos & Sauty 1994, Scheuer 1994, Blandford 1994) even if the magnetic collimation mechanism does not explicitly involve significant baryon entrainment (Lynden-Bell 1997, Lynden-Bell & Boily 1994, Appl & Canzemzind 1993). Yet, such an assumption is hard to justify on theoretical grounds. On the one hand, self-generated magnetic fields in thin Keplerian disks are unlikely to attain coherence lengthscales far in excess of the disk’s pressure scale height, $H$, and inverse-cascade estimates in thin disks (Tout and Pringle 1995) indicate that the large scale component is much weaker $O(H/r)$. On the other hand, advection of a large scale fields from the outer disk or stellar mantle is unlikely to permit a substantial steady inflow of material (Ghosh & Abramowicz 1997) and thus preclude the large accretion rate required by the overall energetics.

In addition, jet-launching by large scale magnetic fields is known to possess a large degree of baryon entrainment (see, e.g. Pringle 1993). This ‘problem’ (from the $\gamma$-ray burst standpoint) persists even when the collimation mechanism does not invoke the presence of a baryonic wind (e.g. Lynden-Bell & Boily 1994, Lynden-Bell 1997) because a large scale field connects baryon-free with baryon-loaded regions of the system. Thus, just how the Poynting jet might form is an outstanding theoretical problem posed to remain a subject of controversy and future research for some time.

On the other hand, to our knowledge formal motivation for an external field with the desired strength has yet to be investigated in this setting (with the possible exception of Thompson 1994) who invokes magnetar-like processes, i.e. strong neutrino induced convective instabilities at near nuclear densities. Simple ‘winding-up’ of radial field to produce a toroidal component was invoked by Narayan, Paczyński & Piran (1992) and by Katz (1997) but this mechanism does not replenish the radial field thus failing to operate as a dynamo. Parker ($P$-type) and magnetorotational instabilities ($M$-type or MRI’s for short) are also often invoked without further elaboration on how this processes might operate together under the conditions implied by $\gamma$-ray burst models (see, however, Tout & Pringle 1992, hereafter TP92, for a general, quantitative account of an instability driven dynamo).

We envisioned the accretion disk setting following the standard hyper-accreting black hole model of Popham, Woosley & Fryer (1999, hereafter PWF): $M_{bh} = 3 M_\odot$; $\alpha_{SS} = 0.1$, and $\dot{M} = 0.1 M_\odot$ sec$^{-1}$. These authors find that the onset of photodisintegration and of neutrino cooling at radii $r \leq 70 \left[ GM/c^2 \right]$ yield a mildly advective accretion disk structure ($H_\theta / r \leq 0.4$) with semi-Keplerian rotation rate. The pressure at $r \approx 70$ is dominated by the nucleon gas component and inside this radius, it becomes dominated by radiation at $r \approx 40$. The temperature rises above (1$\gamma$) pair creation threshold at $r \approx 56$. For a viscosity parameter $\alpha_{SS} = 0.1$, a fluid element’s total number of windings is very limited, $N_{wind} \approx 10$, but this increases almost tenfold by reducing $\alpha_{SS}$ by the same factor. The total neutrino luminosity, about 2% of $M c^2$, and annihilation rate efficiency, $\approx 12$%, for this model are probably insufficient to explain moderately bright $\gamma$-ray bursts but the authors remark that magnetohydrodynamic processes might be more efficient at producing a high energy (Poynting) jet.

We elaborate on this suggestion using an instability-driven accretion disk dynamo (IDD) mechanism. Similar versions of this model (Tout & Pringle 1992, hereafter TP92) involving $P$-type and axisymmetric $M$-type (i.e. Balbus-Hawley) instabilities have meet with limited success in explaining dwarf nova quiescence and eruption phenomenology (see Armitage, Livio & Pringle 1996 and references therein). However, invoking Parker’s undulate instability to promote the growth (loss) of vertical (horizontal) field is questionable in turbulent disks where the turbulence is fed by MRI instabilities (Vishniac & Diamond 1992), unless the growth and length scales of $P$-modes could compare favorably to the MRI scales (see §3.3). It is at present unclear what role, if any, $P$ modes may play in the present context where the (presumably) thermal pairs are collisionally well coupled to the nucleon fluid. Following this argument, $P$-modes are assumed to be suppressed thus precluding the meridional field from becoming dynamically significant. By contrast, MRI’s will develop at the largest lengthscales permitted by the local field topology in spite of the turbulent component to the flow/field. The field lines should be, in fact, only mildly stochastic on the largest MRI lengthscales.

The dynamics of a dynamo process occurring inside an accretion disk are likely to be fairly sensitive to the equilibrium field structure and topology. Because MRI’s are surely present in any highly conductive astrophysical disk, no well defined global field topology is expected (inside the disk). In addition, the field structure could be either diffusive or intermittent, i.e. concentrated in semi-empty flux ropes. In a recent paper (Araya-Góchez 2000a), we have looked into the case of an intermittent field in a static metric. Here we consider both cases in a more general stationary metric.

The field components correspond to azimuthally averaged quantities in the comoving frame, i.e. in the local standard of rest frame. These are consistent with radial field generation from the MRI evolution of a toroidal magnetic field (i.e. non-axisymmetric $M$-type instability) which is, in turn, generated by the sheared radial field; with both components limited by the buoyancy of nearly ‘horizontal’ flux and by

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Here the scale height, $H_\theta = r \Theta_\gamma$, corresponds to an average on spherical surfaces consistent with a not so thin, partially advective disk (see, e.g., Abramowicz, Lanza & Percival 1997).
fast turbulent reconnection of field lines. This model is only weakly sensitive to the details of the turbulent state but it
does depend explicitly on the magnetic Mach number of the
turbulence.

Although MHD disk simulations with vertical stratification
generally fail to support the buoyancy of magnetic fields
(Brandenburg, Nordlund, Stein & Torkelsson 1995; Stone,
Hawley, Gammie & Balbus 1996), these results prove to be
very sensitive to the choice of vertical boundary conditions
and do not account for the effect of radiation/pairs (see §3).

Lastly, we note that the field estimates are not explicitly
sensitive to the precise value of the viscosity parameter, αSS,
but depend rather on local pressure ratios, on the relativistic
generalization of the shear parameter (A\textsuperscript{ort} = Oort's A constant), and on the linear timescale for development of the
non-axisymmetric MRI (which is found to differ from A\textsuperscript{ort}
by a general relativistic correction factor, §3A). Angular
momentum may be transported by the ordered and/or the				
 turbulent components to the field but the relative contributions
to the viscosity are not clear at present.

A review of the slim disk setting in the Kerr geometry and
associated disk thermodynamics is given in [2]. The instability
driven dynamo in a mildly advective disk is laid out in [3]. We then critique and review the assumptions, find steady
state solutions to the dynamo equations and use these to esti-
mate efficiency of the hydromagnetic energy deposition from
field buoyancy in [4].

2 Accretion Disk Setting

2.1 Slim disk in Kerr geometry

The Boyer-Lindquist generalization of the Schwarzschild co-
ordinates (t, r, θ, ϕ) affords the most popular form of the
metric for a rotating Kerr black hole. In normalized geometrical units (c = G = M\textsubscript{bh} = 1), the angular momentum of
the hole is a = J [1/(GM\textsubscript{bh}/c\textsuperscript{2})].

The metric functions of the radial BLF coordinate are
written as relativistic corrections (e.g. Novikov and Thorne
1973) [1]:

\[ A \equiv 1 + a^2/r^2 + 2a^2/r^3, \quad \text{and} \quad D \equiv 1 - 2/r + a^2/r^2, \]

so that, in the equatorial plane, the squared line-element may
be written as:

\[ ds^2 = -\frac{D}{A}dt^2 + r^2A(d\phi - \omega dt)^2 + \frac{1}{D}dr^2, \]

with ω = 2a/Ar\textsuperscript{3} the rate of frame dragging by the hole.

At θ = π/2, the horizon lies at r+ = 1 + \sqrt{1 - a^2} (D\mid_{r=0} = 0)
and no observers with time-like worldlines may keepϕ
constant inside the static limit at r\textsubscript{s} = 2.

The Killing vector fields that belong to the azimuthal and
time symmetries of the metric are deliberately simple in the
BLF: Ψ = (0, 0, 0, 1) and Υ = (1, 0, 0, 0). These may be
used to show that the time and azimuthal components of the
four-velocity, (Ut, Ur, Uθ, Uϕ), are conserved along geodesics.
We denote the latter according to standard notation: Ut =
P\textsubscript{t} ≡ -ξ, and Uϕ = P ϕ ≡ ε.

The unit time-like vector is constructed from the killing
vectors: 1ε \propto Υ + ωΨ. This expression defines “forwards in
time” everywhere in the flow. The rate of frame dragging and
the radius of gyration for circular orbits are quantities de-
duced from inner products of Killing vector fields (Abramowicz,
Chen, Granath & Lasota 1997): ω = -(ΥΨ)/(ΨΨ) and
ev = (ΨΨ)/[Υ + ωΨ], respectively.

A natural frame to treat the local aspects of disk flow is
the locally non-rotating (LNR) frame which is dragged by
the hole at a rate ω with respect to distant observers and
where 1ε = (1, 0, 0, 0) (in our notation \ell denotes the time
component measured in the LNR frame while \ell denotes the
same measured in the LRF).

The flow moves with respect to the LNR frame with az-
imuthal three speed \nu = \dot{x}/\dot{k} (≡ β\phi). This is related to
the angular velocity, Ω, as measured by distant observers by

\[ \nu^2 = v \times (\Omega - \omega) = r \frac{A}{\sqrt{D}} \left( \frac{\dot{x}}{\dot{k}} \right) \approx \frac{\ell}{\gamma} \] . (3)

From the frame that corotates with the fluid, the measured
radial velocity is defined as \beta_r. This choice of frame to mea-
sure the radial velocity dictates that \beta_r \leq 1 everywhere in the
flow (reaching the upper limit at r\textsubscript{s}) and that \beta_r \approx c_s
at the sonic surface (Abramowicz et al. 1997).

The orthonormal thetrad of basis LRF vectors, used to
transform BLF \leftrightarrow LRF, and explicit forms for the four ve-
clocity as function of \beta_r and \ell in the BLF are given in Gammie
and Popham (1998, hereafter GP98). For completeness we
exhibit the contravariant components (i.e. the momenta)
which are easily derivable from the above definitions and from
1\textsubscript{ε}P = -γ:

\[ (U_t, U_r, U_\theta, U_\phi) = \left( -\gamma \frac{\sqrt{D}}{A} - \ell \omega, \frac{\gamma_\phi \beta_\phi}{\sqrt{D}}, 0, \ell \right) \] , (4)

where γ = γ_φγ_r, with γ_φ and γ_r as measured in the frames
indicated above.

By symmetry and for simplicity U\textsuperscript{θ} is assumed to vanish at
the equatorial plane and periodic azimuthal boundary condi-
tions in the “dragged” frame allow for non-axisymmetric
unstable MHD modes to develop while permitting axisym-
metry of the mean flow. Although the equations governing
the flow are greatly simplified by the symmetry conditions, a
potential caveat of this assumption is that low wavenumber

\[ A \equiv 1 + a^2/r^2 + 2a^2/r^3 \]

\[ D \equiv 1 - 2/r + a^2/r^2 \]

\[ ds^2 = -\frac{D}{A}dt^2 + r^2A(d\phi - \omega dt)^2 + \frac{1}{D}dr^2 \]

\[ \omega = 2a/Ar^3 \]
non-axisymmetric unstable azimuthal modes, such as undulate \( P \)-type modes, may violate these conditions if the meanfields become kinematically important.

In accordance with the standard (one-dimensional) slim-disk approach (Abramowicz et al. 1989, Narayan & Yi 1995ab, GP98), the ‘vertical’ average is performed over the meridional and azimuthal coordinates

\[
\int \sqrt{g} \, d\phi d\theta \, f \equiv r^2 \, 2\pi \Theta_{\mu} \, f |_{\theta = \pi/2},
\]

where \( f \) is any dynamical variable, \( \Theta_{\mu} = r^{-1} H_{\theta} \) is the meridional angular pressure scale height of the disk (formally defined below) and where

\[
g \equiv |\text{Det}(g_{\mu\nu})| = (r^2 + a^2 \cos^2 \theta) \, r^2 \sin^2 \theta,
\]
is the determinant of the full metric.

With \( T^{\mu\nu} = pg^{\mu\nu} + \eta u^\mu u^\nu + \Delta_{\text{visc}}^\mu\nu \), the viscous stress-energy tensor, four vertically averaged equations for \( \ell, \beta_r, T, \rho \) and for the angular momentum eigenvalue, \( j \) from particle number, \( (\rho u_{\ell}, \beta) \) tensor, four vertically averaged equations for \( f_a = \rho \) distribution, e.g.

\[
\frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \left( \frac{1}{\rho} \right) \right) + \frac{1}{\rho} \frac{\partial}{\partial \theta} \left( r^2 \sin \theta \frac{\partial}{\partial \theta} \left( \frac{1}{\rho} \right) \right) = 0,
\]
is the radial momentum conservation equation

\[
\gamma_r^2 \frac{d\beta_r}{dr} = - \frac{1}{\rho} \frac{\partial}{\partial r} \left( \frac{1}{\rho} \right) \left( 1 - \frac{\Omega_-}{\Omega_+} \right) + \frac{1}{\rho} \frac{dp}{dr},
\]
where \( p \) is the local total pressure (including the magnetic contribution, e.g. \( \frac{2}{3} \), \( \rho = p + u + p_b \) is the (unnormalized) specific enthalpy and \( \Omega_{\pm} = \pm (r^{3/2} \pm a)^{-1} \) are the prograde and retrograde Keplerian rotation frequencies of circular, planar orbits as measured in the BLF.

PWF find that cooling from neutrino losses and from photodisintegration of He nuclei becomes important as the temperature and density rise. The cooling is calculated explicitly (i.e. by solving the global ADAF). The disk is found to be optically thin to neutrinos for \( M \gtrsim 1.0 \, M_\odot \, \text{sec}^{-1} \), so that neutrino producing processes, pair annihilation and pair capture on nuclei, act to cool the medium at moderate accretion rates.

Where the mass fraction of nucleons \( X_{\text{nuc}} < 1 \), photodisintegration quickly cools off the disk. At moderate accretion rates, \( M \gtrsim 0.1 \, M_\odot \, \text{sec}^{-1} \), photodisintegration is complete at radii \( r \lesssim 70 \, R_\odot \). The nucleon component may then be assumed to behave like an ideal gas; decoupled from the radiation field except to maintain an equilibrium temperature with the thermal bath (no two-temperature model is expected as in the optically thin case).

### 2.2 Fluid component coupling and effective sound speed

Ignoring electron degeneracy pressure, the equation state for the medium has two separate contributions \( p = \rho T + a' T^4 \), where \( a' = \frac{11}{12} a_{\text{rad}} \) accounts for radiation and pairs. The effect of degeneracy in the equation of state is found (by PWF) to be minimal for \( M \lesssim 0.1 \, M_\odot \, \text{sec}^{-1} \). Thus, unless the cooling is significantly enhanced; e.g. if the dynamo constitutes a significant sink of energy from the shear flow, one may cautiously neglect electron degeneracy in the disk.

With the above approximation, the nucleon-gas+radiation medium is expected to posses an effective adiabatic index ranging from that of a pure relativistic radiation-\( e^\pm \) pair gas to that of a non-relativistic, ideal, heavy, nucleon gas \( \Gamma \equiv (\partial p/\partial \rho)_b \in [4/3, 5/3] \). The pressure from an isotropically tangle (i.e. turbulent) magnetic field is included by noting that such field couples well to the (heavy) nucleon-gas component through flux freezing, even though the turbulent field component by itself would be expected to behave like a relativistic gas (e.g. Narayan & Yi 1995b, Quataert & Narayan 1999). The coherent and turbulent component contributions to the pressure are \( p_b^{\text{cohe}} = p_B = B^2/8\pi \), and \( p_b^{\text{turb}} = p_b = b^2/24\pi \) respectively, with \( B_{\text{local}} = B + b \).

Defining \( \beta \equiv p_{\text{rad}}/(p_{\text{gas}} + p_b) \), and assuming a relativistic thermal \( e^\pm \) pair distribution function, the relativistic sound speed is simply (Chandrasekhar 1939, Mihalas & Mihalas 1984)

\[
c_s^2 = \frac{\Gamma}{\beta} \left( \frac{5}{2} + 20\beta + 16\beta^2 \right) \left( \frac{3}{2} + 12\beta(1 + \beta) \right),
\]

Conversely, there is a significant fraction of pairs populating suprathermal energies in a less dense environment, the former would couple to the mean-field component through Parker’s (1965) suprathermal MHD mode thus yielding two natural sound speeds for the medium \( (\cdots \cdots) \).

### 2.3 Vertical structure

In the absence of detailed knowledge on the ‘vertical’ deposition of shear energy, little can be said about the thermal gradient along \( L_b \). The entropy gradient is thus assumed to be covectively stable. Strong stratification, as it occurs in neutron stars where the field has to reach super-equipartition values to break free buoyantly (Thompson 1994), is unlikely to occur in a disk where self-gravity is negligible.

Narayan and Yi (1995a) have pointed out that, at least in the self-similar regime, ADAF’s are well described by “vertically” integrated versions of the fundamental equations as long as the boundary surfaces are spherical in flat geometry. The averaging prescription of Eq. (3) (note a typo in GP98), generalizes this procedure to curved three-space. Because of curvature effects, \( H_\theta \not\equiv r \Theta_{\mu} \not= \int \sqrt{g_{\theta\theta}} \, d\theta \), i.e. the pressure scale-height as calculated in the LRF does not correspond to...
a locally integrated meridional line element. $\mathcal{H}_\theta$ is merely a convenient definition of “height” using the BLF ‘r’ coordinate. This definition breaks down when the disk is very thick, $\Theta_H > 1$, but for $\Theta_H \ll 1$ the correction is small ($\approx 2.5\%$ for $a = .95$). The instability lengthscales in §6 should correspond to locally integrated line elements but, ignoring this small correction, the pressure scale height is used as the standard normalization.

Abramowicz, Lanza and Percival (1997, hereafter ALP) have derived an equation for the pressure scale height which contains only thermodynamic potentials, geodesic flow invariants, and the meridional 4-velocity component and its radial gradient. In the present situation, buoyant instabilities may render the velocity terms non-negligible, but one may still expect the fluid to be subsonic or transonic far from the horizon; effectively “freezing out” close to the horizon (i.e. flowing along $\theta \approx \text{const}$.). In this case, ALP give

$$\mathcal{H}_\theta^2 \approx 2 \frac{\rho_0}{\varrho_0} \frac{r^4}{\mathcal{L}} = \frac{2}{1} \frac{c_s^2}{\Omega z^2}$$

where $\mathcal{L} = \ell^2 - a^2(\ell - 1)$, $c_s$ is the effective soundspeed at the midplane of the disk and $\Omega_z$ is an effective frequency of meridional oscillatory motion (GP98).

### 3 An Instability-driven Dynamo

In spite of being the subject of great theoretical effort since its inception (Sakura & Sunyaev 1973), clear resolution on the question of the origin of anomalous viscosity in accretion disks (Pringle 1981) has only recently occurred. Hawley, Balbus and Winters (1999) conclude that non-magnetic hydodynamic Keplerian accretion is stable (see also Godon and Livio 1999) and that shear-fed MHD turbulence in a fully developed state is capable of explaining anomalous angular momentum transport in accretion disks. (Balbus and Hawley 1991 (BH91), 1992a (BH92a), 1992b, 1996, 1998; Hawley and Balbus 1991, 1996; Hawley, Gammie and Balbus 1995; Brandenburg, Nordlund, Stein & Torkelsson 1995, 1996; Stone, Hawley, Gammie & Balbus 1996; Brandenburg 1998).

MRI induced magnetohydrodynamic turbulence may thus be reasonably proclaimed to be the source of anomalous angular momentum transport in accretion disk systems and the question of whether large scale fields are present must be subjected to their co-existence with the turbulent flow.

Because the IDD is shear-fed, where the disk is advective and gas pressure dominated the low shear rate of an ADAF composed of non-relativistic particles gas starves the dynamo out (recall that when $\Gamma_{\text{effective}} \rightarrow 5/3$, self-similar solutions have nearly spherical accretion $\Omega \rightarrow 0$, Narayan & Yi 1994, Narayan, Mahadevan & Quataert 1998). We thus consider the case where a large-scale field in the outer disk begins to emerge where its influence is dynamically unimportant (e.g., at $r \approx 70$, once photodisintegration is complete).

#### 3.1 On the effect of a radial flow

Although a predominantly toroidal field may be expected a priori (at least on surfaces perpendicular to $\hat{\Omega}_\theta$), the radial flow is non-negligible in the innermost disk. For instance, in the standard model of PWF, the ratio $\mathcal{R}_r/z \equiv (\gamma c_s^2 \beta_r) / \beta_\phi$ which compares the radial and azimuthal three-velocity components as measured in the LNR frame, is small but significant in the range $r \approx [70, 4.5]$. At $r \approx 14$ (where $\Theta_H \approx .4$ is maximum), the sub-Kleinerian $\beta_\phi \approx .26$ and $\mathcal{R} \approx .12$. At $r \approx 4.5$, $\beta_\phi \approx .62$ and $\mathcal{R} \approx .32$ while at $r \approx 3.9$, $\beta_\phi \approx .61$ and $\mathcal{R} \approx .43$ and quickly rising.

We envisioned that the MHD condition of high conductivity in a strongly sheared semi-Keplerian flow produces a mean-field on $\theta = \text{constant}$ surfaces that tracks the meanflow on average (equivalently, one may assume vanishing angular velocity gradients along mean field lines). Thus, since the presence of a small radial field component does not alter the Balbus-Hawley instability (BH91), one may cautiously ignore the effect of a small, passively advected, $B_r$ component also on the (linear) development of non-axisymmetric $\mathcal{M}_5$-type instabilities as calculated form a purely toroidal field (§3.3). Note, however, that these two MRI’s originate from different branches of the MHD wave dispersion relation (the slow and Alfvén branch). This assumption could require re-evaluation when $B_r \approx B_\phi$.

#### 3.2 On the use of spherical coordinates

To date, most magnetohydrodynamical stability analyses of accretion disk systems have been performed in the context of the thin disk formulation where the use of cylindrical polar coordinates is customary. In a semi-thick, slim disk setting, the polar $z$ coordinate is preferably replaced by the meridional distance $\approx r \theta$ (2.3).

A fundamental difference in the use of spherical coordinates is that while in a thin disk setting the vertical pressure gradient is balanced by the component of gravity along $\hat{1}_z$, gravity plays no role in balancing pressure gradients along $\hat{1}_\theta$ (ALP). Gravity’s place is taken by the (inertial) centrifugal force which for rotation on spherical shells is $\propto \sin^2 \theta$ (with $\theta = \pi/2 - \phi$). In order to compare this with “gravity along $\hat{1}_z$”, one notes that in the standard thin disk approach $g_z \propto \sin \theta \theta$ and so the fractional difference is of $O(\Theta_H) \approx \sin \theta$. It follows that the gradient in the inertial force along $\hat{1}_0$ is gentler than that of gravity along $\hat{1}_z$ (at least for $\theta < 1$). Note that although this somewhat diminishes the effects of buoyancy and the impact of the inertial force gradient (not

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\* A general relativistic analysis of this issue is complicated by the lack of a precise definition for the centrifugal force in a stationary metric (although such definition exists for a static (Schwarzschild) metric as demonstrated through the formalism of the ‘optical reference geometry’, Abramowicz, Carter & Lasota 1988, Abramowicz 1990, Abramowicz & Prasanna 1990). Note, however, that the meridional hydrostatic equilibrium equation involves an effective frequency of oscillation, Eq 3 (next footnote), while the condition of radial force equilibrium, Eq 4 involves deviations from Keplerian rotation rates instead (Appendix 3).
gravity) on the Parker instability, for $H_0/r \ll 1$ this effect is not severe.

Another difference is that the scale height in spherical coordinates is curvilinear. The scale height enters the general dispersion relation of the Balbus-Hawley (1991) instability by establishing a characteristic lengthscale for the (predominant) vertical–gradient-dependent part of the Brunt–Väisälä frequency (c.f. their Eq [2.8]). This gradient is approximately inversely proportional to the pressure scale height in the horizontal regime independent of the vertical–gradient-dependent part. To simplify the dispersion relation further by assuming rotation on cylindrical shells (i.e. coinciding isobaric and isochoric surfaces), an equivalent assumption in spherical coordinates for a thin disk is rotation on spherical shells and with this, BH91 analysis is straightforwardly carried on to a thicker disk.

Non-axisymmetric $M$-type instabilities of a purely toroidal field only require (§3.7) that $4\Omega A + k_\phi^2 v_{\text{Alf}} < 0$ for non-relativistic Keplerian rotation in the horizontal regime (Foglizzo & Tagger 1994, hereafter FT94). Although the geometry imposes a lower bound on $k_\phi = 1/r$ for nearly Keplerian rotation, the above restriction is independent of scale height and constraints only the toroidal field strength. To first order, general relativistic corrections (§4) do not modify this conclusion. Indeed, magnetorotational instabilities in the horizontal regime are generally insensitive to vertical gradients and the pressure scale height enters only indirectly when the Alfven speed is scaled with respect to the local sound speed of the medium (see the discussion at the end of §3). Lastly, since buoyant motions are commensurate on stratification and since radial pressure gradients force rotation on nearly spherical shells when the shear is strong (i.e. in a mildly advective disk), the $B_\perp$ flux should naturally rise along the $r\dot{\theta}$ local pressure gradient. This is a crucial point that suggests that a fraction of the magnetic flux generated by the dynamo is promptly collimated into the funnels above and below the disk’s surface thus creating favorable conditions for the formation of a Poynting jet.

### 3.3 The dynamo equations

The shear flow generates $B_\phi$ from $B_r$ on the shear timescale $\tau_\delta$. In the absence of a propitious field topology for the development of Parker’s undulate instability (e.g. FT94), the buoyancy of the $B_\perp$ flux, which escapes the disk on a timescale $\tau_B$, limits the growth of $B_\phi$ and $B_r$ and may generate $B_\theta$ albeit inefficiently: The flux buoyancy acts as an interchange instability whereby inhomogeneities along a field domain cross section induce a gradient in the buoyant velocity along the direction of the mean field thus generating a small $B_\theta$ component. Inclusion of this rather uncertain process for meridional field generation adds considerable mathematical complexity to the heuristic equations and will be overlooked in the name of simplicity. Note, however, that although the meridional field is unlikely to attain strengths that could explicitly alter the proposed dynamo equations, its presence does alter the equilibrium turbulent state since the poloidal MRI is more efficient at feeding the turbulence (E. Vishniac, Priv. Comm.). This affects the IDD explicitly through our estimate of the magnetic Mach number for the turbulence ($\S 3.9$).

Non-axisymmetric $M$-type instabilities that develop in a purely toroidal field (FT94, Foglizzo & Tagger 1995 [FT95], Terquem and Papaloizou 1996 [TP96], Papaloizou and Terquem 1997) contribute to both $B_\phi$ and $B_r$ field generation but their growth rates decrease with field strength, c.f. Eq [7], and are likely to be important only for $B_r$ since shear is the primary process for azimuthal field growth. TP96 note that because the toroidal MRI is essentially local in azimuth (at least for small fields) the field need not be entirely toroidal, only local toroidality is required.

Additionally, reconnection of all field components proceeds at a rate that is proportional to the upstream component of the Alfvén velocity, $v_{\text{Alf}} = B/\sqrt{4\pi \rho}$, times the square of the magnetic Mach number of the turbulence (Lazarian & Vishniac 1999) as measured in the LRF.

In Lagrangian, LRF coordinates, the phenomenological two dimensional dynamo equations are self-evident

\[
\begin{align*}
\partial_t B_\phi &= \frac{1}{\tau_\delta} B_r - \frac{1}{\tau_B} B_\phi - \frac{1}{\tau_\delta} B_\phi \\
\partial_t B_r &= \frac{1}{\tau_M} B_\phi - \frac{1}{\tau_B} B_r - \frac{1}{\tau_r} B_r
\end{align*}
\]

(9)

Within the heuristic phenomenology of these equations, when competing gain (loss) timescales differ greatly one may safely drop the weaker terms.

### 3.4 Relativistic scalings

In a classical Keplerian disk, the timescales related to shear, magnetorotational and local vertical oscillations of test particles are all scaled by the local Keplerian rotation frequency (as measured by observers at rest in a global frame). In a relativistic slim disk setting, however, the timescales related to all of these processes are slightly different. The importance of this remark becomes evident when one attempts to generalize the instability scalings (LRF time and length scales) for a slim disk in the Kerr-Newman geometry.

Through Oort’s $A$ parameter (see below), the shear and magnetorotational timescales are intrinsically related to the azimuthal rotation frequency of the disk. For nearly Keplerian prograde rotation the natural scaling is simply $\Omega_+$ (c.f. Eqs [12] & [14]).

On the other hand, the natural frequency of local meridional oscillations of the four velocity GP98\[4\] $\Omega_z$, which is directly involved in determining the local pressure scale height

\[6\] obtained by expanding Carter’s (1968) fourth constant of geodesic motion in the Kerr geometry, $Q$, about $\theta \equiv \pi/2 - \tilde{\theta}$ and taking an affine
\( \mathcal{H}_\Omega \), c.f. Eq 3, differs from the prograde Keplerian rotation rate by a factor \( \mathcal{R} \), where (Riffert & Herold 1995)

\[
\mathcal{R}^2 = \left( \frac{\Omega_\ast}{\Omega_+} \right)^2 \simeq \frac{1 - 4a r^{-3/2} + 3a^2 r^{-2}}{(1 + a r^{-3/2})^2 (1 - 3 r^{-1} + 2 a r^{-3/2})}. \tag{10}
\]

As advertised, the growth rates of \( \mathcal{M} \)-type instabilities are characterized by a relativistic generalization of Oort’s A parameter while for buoyant motions and reconnection timescales (and \( \mathcal{P} \)-type instabilities) the characteristic timescale is (loosely speaking) the meridional ‘Alfvén transit time’: \( \mathcal{H}_\Theta / \nu_{Alf} \). Thus, although the pressure scale height, \( \mathcal{H}_\Omega \), naturally normalizes all meridional lengths, similarly scaled magnetorotational wavenumbers must be corrected by a factor \( \mathcal{R} \) from their classical counterparts

\[
k^\text{Rel}_\mathcal{M} \rightarrow \mathcal{R} k^\text{Class}_\mathcal{M}. \tag{11}
\]

That this is indeed the case may be more easily seen by normalizing either the classical (with \( \Omega = \dot{x}^r / \dot{x}^t \))

\[
4A_\text{Oort} \Omega + (k \cdot v_{Alf})^2 < 0 \tag{12}
\]
or the generalized relativistic stability criterium for shearing instabilities, Eq 3. Because for prograde Keplerian orbits the term driving the radial differential force scales as \( \Omega_+^2 \), normalization of the Alfvén velocity to the local sound speed in the \( (k \cdot v_{Alf})^2 \) term yields the correction factor if one desires to normalize the wavenumber to the disk’s pressure scale height.

### 3.5 Linear shear

We choose to model the shear flow in a linear shearing sheet approximation (Goldreich & Lynden-Bell 1965) noting the well known limitations of such an approach, i.e. ignoring global curvature (Ogilvie and Pringle 1996, TP96) and nonlinear shear effects. The influence of shear is thus parameterized by a relativistic generalization of Oort’s first constant, \( \tau_\text{r}^{-1} = 2A_{\text{Class}} = d_{\text{in}, r} \Omega = \frac{3}{2} \Gamma_{\text{Kep}}, \) for a Keplerian disk.

A relativistic expression for Oort’s A value that contains the radial gradient of \( \ell \) and \( \beta_r \) implicitly is (Appendix 4.1)

\[
\sigma_{r \varphi}(r, \beta_r, \Omega) = \mathcal{A} \Omega^2 \left\{ \frac{1}{2} \frac{d \Omega}{d \ln r} \right\} = A_{\text{Oort}}. \tag{13}
\]

This result differs subtly from the one in Novikov and Thorne (1974) where \( \gamma \) goes to \( \gamma_{\varphi} \) for a thin disk with negligible radial flow.

For nearly prograde/retrargrade Keplerian orbits, from Eq 13, one finds

\[
\left| \frac{1}{\tau_{\text{r} \text{Rel}}^2} \right| \simeq \frac{3}{2} \mathcal{A} \frac{a^2}{1 + a r^{-3/2}} \equiv -2A_{\text{Oort}}. \tag{14}
\]

derivative

\[
U_{\beta}^{-1} \left\{ Q = U_{\beta}^2 + \cos^2 \theta \left( \frac{\ell^2}{\sin^2 \theta} - a^2 (c^2 - 1) \right) \right\}
\]

Shear-fed instabilities, on the other hand, pertain to growth rates slightly different from \( A_{\text{Oort}} \) due to relativistic effects\(^7\).

Shear also forces the radial wavenumber of perturbations to evolve according to

\[
k_r(t) = k_r^0 - 2A \mathcal{R} \varphi t. \tag{15}
\]

However, we shall concern ourselves only with the horizontal regime of magnetorotational instabilities, \( k_r / k_\theta \rightarrow 0 \), noting that this constraint yields the highest growth rate from both the Balbus-Hawley (BH92a) instability (3.6) and toroidal, non-axisymmetric \( \mathcal{M} \)-modes (\( \mathcal{M} \)).

### 3.6 The poloidal (Balbus-Hawley) MRI

Classically, the axisymmetric magnetorotational instability (Balbus & Hawley 1991) only requires that \( \partial_t \Omega < 0 \) and that the \( B_\psi \) field be “weak” in the sense \( \nu_{Alf}^2 < (1/\pi) |\mathcal{A}| \mathcal{H} \) which is derived by imposing an upper limit (2H) on the ‘vertical’ wavelength in the Keplerian thin disk limit (note that the radial field component is irrelevant in the horizontal regime: \( k_r / k_\theta \rightarrow 0 \) BH92a). On the other hand, the classical instability criterion (Chandrasekhar 1961 and Appendix 3.6), \( 4A_\Omega + k_\theta \nu_{Alf}^2 < 0 \), sets a upper limit on \( k_{\max} = \sqrt{3 \Omega / \nu_{Alf}^2} \) beyond which the energy cost of bending field lines on small scales suppresses the instability. The fastest growth occurs for \( k_\theta = \Omega / \nu_{Alf}^2 \) at a rate equals \( A_{\text{Class}} \) as long as \( k_\theta \gg \pi / \mathcal{H}_\Omega \)

\[= \pi \sqrt{\frac{1}{2} \times \Omega / c_s} \]

Although a fully general relativistic stability analysis of MHD disk systems in the Kerr geometry is beyond the scope of this paper, in Appendix 4 we derive an approximate (see footnote 6) relativistic timescale, corrected for the form of the “radial differential force” in the Kerr geometry. For nearly Keplerian orbits, this procedure leads to replacement of the term driving the (classical) radial differential force \( d_{\text{in}, r} \Omega^2 \)

\[4 \Omega^2 + A_{\text{Class}} \]

by \( 4 \Omega^2 + A_{\text{Class}} \). For prograde orbits; using Eqs 13 & 23 to estimate the timescale and Eq 11 to normalize the optimal wavenumber, one finds

\[
\left| \frac{1}{\tau_{\text{r} \text{Rel}}^2} \right|_{\max} = 3 \mathcal{A}^2 \frac{4}{\gamma^2} \Omega_+ \left( = -A_{\text{Class}} \right) \tag{16}
\]

(compare this with Eq 12), and

\[
\mathcal{H}_\Theta k_{\text{in}} = \sqrt{\frac{2}{\mathcal{A}} \frac{c_s}{\Omega_+}} \nu_{Alf} \approx \sqrt{\frac{2}{\Gamma} \frac{c_s}{\nu_{Alf}}} \mathcal{H}_\Theta. \tag{17}
\]

We identified the r.h.s of Eq 16 with the maximum growth rate of shear-fed instabilities (such as \( \mathcal{M} \)-type) in relativistic prograde Keplerian Kerr disks as measured in the LRF of the fluid:

\[
\left| \frac{1}{\tau_{\text{r} \text{Rel}}^2} \right|_{\max} = -A_{\text{Class}} \]

\( \gamma \)

The relativistic shearing and magnetorotational timescales are derived by modifying only the relativistic analog to the classical “radial differential force” (FT94), and assuming that, both, the centrifugal and the coriolis forces act as they do classically, c.f. Eqs 27 and footnote and discussion at the end of Appendix 4.
3.7 The non-axisymmetric toroidal MRI

Although BH92b originally found that the most unstable magnetorotational modes correspond to axisymmetric modes evolving in a poloidal field configuration (through destabilization of Alfvén modes), several authors (FT94, TP96, Matsumoto & Tajima 1995) find similar growth rates for non-axisymmetric modes in a purely toroidal field configuration (albeit at larger poloidal wavenumbers, Balbus 1998). Note that in hyper-accreting γ-ray burst black holes, non-axisymmetric modes may be strongly excited if these feed the angular momentum loss through an associated gravitational wave emission.

This instability corresponds to destabilization of the slow MHD mode of wave propagation (FT94, TP96) as is also the case for Parker’s undulate instability. FT94 (see also TP96, Shu 1974) remark that the inertial effect of rotation enters through the coriolis force term, slightly twisting flux tubes of displaced fluid elements in the azimuthal direction, and through the radial differential force.

If the ratio \( k_r/k_\theta \ll 1 \) (\( \Rightarrow \) ‘horizontal’ regime), meridionally localized disturbances are more prone to the radial differential force. If this force is strong enough to overcome the magnetic tension this results in the non-axisymmetric magnetorotational instability (FT95). Radially localized perturbations \( k_\theta/k_r \ll 1 \) (\( \Rightarrow \) ‘asymptotic’ limit, c.f. Eq [13]), on the other hand, would naturally evolve into Parker unstable bellowations (FT95). Radially localized perturbations \( k_\theta/k_r \ll 1 \) (\( \Rightarrow \) ‘asymptotic’ limit, c.f. Eq [13]), on the other hand, would naturally evolve into Parker unstable (3.8).

We use the Kerr geometry generalization of the radial differential force, \( \hat{A} \) and define \( \alpha_\varphi \equiv \hat{P}_{B_\varphi}/(\hat{p}_{gas} + \hat{p}_b) \) and a rescaled \( \tau_M \) and \( A_{\alpha_\varphi} < 0 \)

\[
\hat{\tau}_M \equiv \frac{\tau_M}{\Omega_+}, \quad \hat{A} \equiv \frac{A_{\alpha_\varphi}}{\Omega_+},
\]

(16)

to adopt the results in FT95 as approximate relativativistic generalizations of the toroidal MRI scales.

For prograde, nearly Keplerian rotation

\[
|A_{\alpha_\varphi}|^2 \hat{\tau}_M^2 = 
\]

\[
\frac{1}{2} \left\{ \sqrt{(1 + 2\alpha_\varphi)(1 + 2\alpha_\varphi(1 + \hat{A})) + 1 + \alpha_\varphi(2 + \hat{A})} \right\}
\]

(17)

and

\[
H_\Omega^2 k_M^2 = -\frac{2}{\Gamma} \left( \frac{c_s}{v_{\alpha_\varphi}^{eff}} \right)^2 \times (2\hat{A} + (1 + \alpha_\varphi)(\hat{\tau}_M^{-2} - 2))
\]

\[
= -\frac{2}{\Gamma} \left( \frac{c_s}{v_{\alpha_\varphi}^{eff}} \right)^2 \times \hat{\delta}
\]

(18)

for the non-axisymmetric MRI modes of most vigorous growth.

These estimates are in fact subject to the condition

\[-\hat{A} < 1 + (2\alpha + 1)^{-1}.\]

In a stronger effective shear flow the optimal wavenumber in Eq [18] becomes imaginary and the toroidal field is unstable to a faster radial interchange (\( k_r = 0 \)) instability according to the Rayleigh criterion (T. Fogliizzo, Priv. Comm.). Interestingly, this destabilization process is mathematically similar to the one that occurs when suprathermal particles push Parker instability into interchange-like mode (FT94) for \( p_{\text{suprathermal}}/p_{\text{gas}} > 3 + 4\alpha \) (see below).

To the IDD, this means that when condition [19] is not meet, the toroidal MRI does not grow the radial field nor does it provide structure nor field reversals in the linear analysis. Moreover, note that when the wavenumber becomes very small, the local analysis is out of its intended regime and global curvature effects become important. The IDD is thus inadequate in this limit. Fortunately, condition [19] is only meet at most within \( r \leq 5 \left[ GM/c^2 \right] \) for a non-rotating black hole and it moves inward for a rotating black hole (§4.2).

3.8 On the buoyancy induced by radiation and \( e^{\pm} \) pairs

One may physically expect that \( e^{\pm} \) pairs would enhance the buoyancy of magnetic flux if such a lighter fluid component coupled to the ‘other’ light relativistic fluid of the medium, i.e. the magnetic field. Yet, further elaboration on such a conjecture requires some understanding of the dynamics of quasi-equilibrium pair plasmas in magnetic fields well in excess of the Q.E.D. field scale (\( B_Q = m^2/e(c^3/h) \simeq 44. \) TG), a problem that has yet to be explored.

For instance, under this circumstance pairs are created (annihilated) mainly via 1 γ decay (production) with strongly asymmetrical energy profiles for each member of the pair (Daughterly & Harding 1983, Harding 1986). It is thus conceivable that at very high disk temperatures the pairs may have considerable energy density in a non-thermal distribution of suprathermal particles. This being the case, Parker’s suprathermal mode of hydromagnetic wave propagation (which strictly speaking applies in the collisionless regime) will enhance Parker’s undulate instability possibly pushing the latter into an exchange-like mode (FT94) with a normalized growth rate \( (\omega_{\text{Growth}}/\Omega)^2 \propto (\alpha_\perp - \beta)/(1 + 2\alpha_\perp) \), where \( \alpha_\perp \) is calculated from the \( B_\perp \) field, i.e. including \( B_\gamma \) in contrast to Eqs [17] & [18].

On the other hand, (thermal) transrelativistic pairs are well coupled to the nucleon component even when coulomb drag is low (due to the neutronization of the material) and ambipolar diffusion of the field-pair component through neutron matter is dismal (although the turbulence may help the latter). Thus, the most likely mechanism for fast horizontal flux escape would have to involve nucleon density deficits inside flux ropes or, in the case of a diffuse field, turbulent diffusion.
A plausible mechanism that promotes baryon unloading from field lines is turbulent pumping (Vishniac 1995a) by the MRI which must favorably compete with turbulent diffusion of matter back onto flux ropes. Under this assumption, the stretch, twist and fold of field lines by (enthalpy-weighted) sub-Alfvénic turbulence augments the field energy density and releases matter from field lines that would otherwise be “frozen-in”. For the marginal case of Alfvénic turbulence, nearly empty $B_\perp$ flux ropes in a gas pressure dominated disk acquire a drag limited buoyant velocity $v_b \propto (v_{\parallel \text{Alf}}^2)^2/c_s$. Moreover, assuming efficient diffusion of radiation and $e^\pm$ pairs into the flux tubes, in this picture the buoyancy loss rate, $v_b/\mathcal{H}_\theta$, is enhanced by a factor $\lesssim p/p_{\text{gas}}$ (Vishniac 1995b).

On the other hand, in a diffuse field configuration the local value of the turbulent helicity implies migration of the field to flux poor regions of the accretion disk (E. Vishniac, Priv. Comm.). Phenomenologically, this is also born in numerical simulations with vertical stratification (Brandenburg et al. 1995, Stone et al. 1996). Since the main contribution to the field generation occurs in regions of highest pressure, i.e. the disk midplane, this argument yields a systematic meridional motion of the field (up to a gradient in the diffusion coefficient)

$$v_b = \frac{2}{\Gamma} \frac{v_{\text{turb}} \tau_{\text{cor}}}{\mathcal{H}_\theta} \simeq \sqrt{\frac{2}{\Gamma} \left( \frac{v_{\parallel \text{Alf}}}{c_s} \right)^2} \left( \frac{\tau_{\text{cor}}}{\Omega_+^{-1}} \right) M_\text{B}^2$$

(20)

where $\tau_{\text{cor}} \simeq \tau_M$ is the correlation timescale of the turbulence and $M_\text{B} \equiv v_{\text{turb}}/v_{\text{Alf}}$ is its magnetic Mach number on the largest eddy scale.

The estimate for the field loss rate from the disk depends on whether or not the radiation/pairs enhance the buoyancy loss rate. In the context of a diffuse field configuration (which is on more solid grounds), this enhancement is only a conjecture. With this in mind, using Eq. [21] and defining $\xi = p_{\text{gas}}/p$, we write the characteristic time as a meridional Alfvén ‘transit time’ (admittedly a misleading nomenclature given that Alfvén waves move along field lines)

$$\tau_\theta(\mathbf{B}_\phi, \mathbf{B}_r) = \frac{\mathcal{H}_\theta}{v_b} = \eta \frac{\mathcal{H}_\theta}{v_{\parallel \text{Alf}}^2} = \eta' \left( \frac{c_s}{v_{\parallel \text{Alf}}} \right) \frac{1}{\Omega_+}$$

(21)

where

$$\eta' = \sqrt{\frac{2}{\Gamma} \left( \frac{1}{\eta} \right)^2} \xi^{-2} \left( \frac{c_s}{v_{\parallel \text{Alf}}} \right) \left( \frac{\Omega_+^{-1}}{\tau_{\text{cor}}} \right) M_\text{B}^{-2}$$

(22)

### 3.9 Turbulent reconnection

It is well known that in spite of generally predicted low reconnection rates (Sweet 1958, Parker 1979), well studied systems such as the solar corona and chromosphere indicate that reconnection is fast once it begins (Dere 1996, Innes et al. 1997), essentially occurring at an order of magnitude below the Alfvén speed.

Following this observation, Lazarian & Vishniac (1999) show that in the Goldreich & Sridhar (1997) model of strong MHD turbulence, reconnection in a weakly stochastic field occurs at a fraction of the Alfvén speed which equals the square of the magnetic Mach number of the turbulence $M_\text{B}^2$.

A plausible mechanism that promotes baryon unloading is the radial, azimuthal field reversal lengthscale, $X_\varphi$. Note that the meridional length scales associate with the poloidal MRI are not relevant since the meridional field is assumed to be weak (albeit quickly unstable).

It has been argued (TP92) that the shear process generally reduces radial length scales until these supply the fundamental reconnection channel. This is indeed the case when any of the coherent field pumping process is slow when compared to shear, $\tau_s \gg \tau_\varphi \equiv \Omega_+^{-1}$; but this is never the case when the MRI is the culprit (even for the slower manifestations of the instability). One may thus considerably simplify the calculation by ignoring this aspect of shear in the reconnection process.

On the other hand, following TP92, we obtain an estimate of the radial, azimuthal field reversal lengthscale, $X_\varphi$ by noting that the time evolution of wavenumbers implied...
by Eq (13) during one MRI timescale couples the azimuthal lengthscale to the radial lengthscale, i.e., 
\[ l_y^c = \frac{(\tau_M/\tau_s) \times l_r^c}. \]
Thus
\[ Y_r = \frac{\pi}{k_M} \quad \text{and} \quad X_\phi \equiv \left( \frac{\tau_s}{\tau_M} \right) \times \frac{\pi}{k_M}. \quad (23) \]

3.10 Equilibrium Solutions

In general, Eqs (9) are implicitly complex due to the non-linearities introduced by the field dependence of gain timescales, Eqs (14, 17), the buoyancy loss rate Eq (21), and the reconnection times which involve the perpendicular timescales, Eqs (14, 17), the buoyancy loss rate Eq (21), and the non-linearities introduced by the field dependence of gain

Scaling wavenumbers to the inverse pressure scale height \( k \rightarrow k/\hbar \), and defining

\[ \frac{\Gamma}{2} \varepsilon_r \equiv \frac{Y_r}{\hbar} = \frac{\pi}{k_M} \quad \text{and} \quad \frac{\Gamma}{2} \varepsilon_\phi \equiv \frac{X_\phi}{\hbar} = \frac{\dot{A}}{A'} \frac{\pi}{k_M}. \quad (24) \]

and

\[ \dot{\Lambda} = \frac{\tau_M^{-1}}{\Omega_r} \quad \text{and} \quad 2\dot{\Lambda} = \frac{\tau_s^{-1}}{\Omega_r}, \]

(c.f. Eq (10), although these are positive definite), a set of normalized dynamo equations for prograde orbits follows

\[ \partial_t B_\phi' = 2\dot{\Lambda} B_\phi' - \frac{1}{\eta} B_r' B_\perp - \frac{1}{\varepsilon_\phi} (B_\phi')^2, \quad (25) \]

\[ \partial_t B_r' = \dot{\Lambda} B_\phi' - \frac{1}{\eta} B_r' B_\perp - \frac{1}{\varepsilon_r} (B_r')^2, \quad (26) \]

where the fields are in velocity units and normalized to the soundspeed \( (B' = B \times \sqrt{\pi r c_s}) \) and the time is normalized to the inverse of the prograde Keplerian frequency \( \dot{t}' = \Omega_r t' \).

In a steady state, these equations must satisfy

\[ B_r' = \frac{B_r'}{2\Lambda} \left[ M_0^B \right]^2 \times \left\{ \frac{2\sqrt{3}}{\xi A} B_\perp^2 + \frac{\sqrt{3}}{\pi} \frac{\dot{\Lambda}}{\Lambda} \delta \right\}, \quad (27) \]

\[ B_\phi' = \frac{B_\phi'}{\Lambda} \left[ M_0^B \right]^2 \times \left\{ \frac{2\sqrt{3}}{\xi A} B_\perp^2 + \frac{\sqrt{3}}{\pi} \frac{B_r'}{B_\phi'} \delta \right\}, \quad (28) \]

which we solve using a multidimensional Newton-Raphson method (e.g., Numerical Recipes in C, Press et al. 1988).

4 A toy calculation

4.1 Self-critique

The instability driven dynamo depends self-consistently on the relativistic shear, \( 2\Lambda' \), and MRI, \( \Lambda \), timescales, on the coherent field pumping lengthscale supplied by the MRI, \( \delta \), and on the magnetic Mach number of the turbulence, \( M_0^B \).

A phenomenological scaling for the buoyant velocity in the turbulent medium, Eq (20), follows from the limiting cases of nearly empty flux tubes and turbulent diffusion of a spread field (and by analogy to other buoyant instabilities such as the Parker instability).

An open issue is whether a radiation pressure dominated environment helps the buoyancy of the field and what this means for the magnetic energy deposition rate, particularly in the context of a \( \gamma \)-ray burst. If the emerging magnetic flux (in ropes) is relatively baryon-free, this helps its escape from the disk. On the other hand, the meridional diffusion of \( B_\perp \) flux in a semi-thick disk setting also tends to separate the field from the (heavier) baryon component, aiding in the escape of flux+radiation and pairs.

In order to address the overall energetics and to gain a qualitative feel of the postulated hydromagnetic energy conversion process, we have constructed a toy model which adopts the pressure ratios from the standard model of PWF and (non–self-consistently) assesses the energy loss from the buoyancy of the field. Evidently, a realistic model should account for the back-reaction of the field on the flow which in the context of an accretion disk means primarily the angular transport implied by the -\( r \phi_\perp \) component of the magnetic stress (note that in the notation of the previous section, \( \sigma_{SS}^{mag} = B_r' B_\phi' \)). In the absence of other significant sources of ‘anomalous’ viscosity, a self-consistent hydromagnetic accretion disk model is possible in principle when our equations are solved with a compatible set of relativistic hydrodynamic equations (e.g. GP98, Abramowicz et al. 1997) under the assumption that the angular momentum is transported by the largest magnetic eddies.

The radial flow (3.1) is passively accounted for by carrying out the analysis in the comoving frame and assuming that the linear timescale of the non-axisymmetric MRI faithfully reflects the growth rate of the radial field (an assumption that is not free of criticism). On the less optimistic side, the radial interchange instability (3.7) of the \( B_\perp \) flux in a strong effective shear field shuts the dynamo off at the innermost section of the disk where most energy could be extracted. This instability is already present when \( 1 > \Lambda < 1 + (2\alpha - 1)^{-1} \) (T. Foglizzo, Priv. Comm.) but with a slower growth rate than the MRI. Thus, even in the absence of substantial meridional flux (Park & Vishniac 1994), radial buoyancy may change the picture we have portrayed hereby (on the brighter side, the radial interchange opposes the advection of the magnetic field). In this regard, note that condition (19) depends only on the toroidal field strength when \( k_\phi \rightarrow 0 \) but we cap the latter at \( k_\phi = 1/\varpi \).

4.2 The magnetic energy deposition rate

The local magnetic energy flux (from buoyancy) is obtained from the part of the time derivative of the energy density \( \mathcal{H}_\theta f_B \) which is lost to buoyant motions

\[ \mathcal{H}_\theta f_B = -\frac{1}{4\pi} \left\{ B_r \dot{B}_r + B_\phi \dot{B}_\phi \right\} = \left[ \frac{c^3}{GM} \right] r^{-3/2} \frac{\Gamma p}{\xi A} \frac{\dot{M}^B}{B_\perp^4} \times \left[ M_0^B \right]^2 \times B_\perp^4 \quad (29) \]
where \( \dot{B}_r \) and \( \dot{B}_\phi \) equal the second terms in Eqs \([25 \& 26]\).

Adopting the results from the standard model of PWF (c.f. \$3 \& 3.1), this form for the specific power output is used in four estimates of the steady magnetic flux deposition rate for \( r \in [4.5, 40] \); non-rotating black hole with \((h_{\text{Buoy}}=a.00)\), and without enhanced buoyancy \((h_{\text{Buoy}}=a.00)\), and nearly maximally rotating black hole, \( a = .95 \), with \((h_{\text{Buoy}}=a.95)\), and without \((h_{\text{Buoy}}=a.95)\) enhanced buoyancy. Curiously, because the fields are higher in the low buoyancy scenario, these yield the highest output rates (in erg sec\(^{-1}\)): 8.1\(_{+51}\) \((h_{\text{Buoy}}=a.00)\) and \(1.0\_{+52}\) \((h_{\text{Buoy}}=a.95)\) vs \(5.1\_{+51}\) \((h_{\text{Buoy}}=a.00)\) and \(6.6\_{+51}\) \((h_{\text{Buoy}}=a.95)\).

The ‘half-luminosity’ radius displays some interesting behavior as well: for \((h_{\text{Buoy}}=a.00)\), the IDD becomes operational (c.f. radial interchange instability) at \( r_{\text{min}} \approx 5.3 \) and \( r_\text{c} \approx 10.3 \), while for \((h_{\text{Buoy}}=a.00)\), \( r_{\text{min}} \approx 5.9 \) and \( r_\text{c} \approx 9.2 \), because of the higher stationary fields in the low buoyancy case: \([B_\phi', B_r'] = [.72, .80] \) vs \([.42, .48]\) at \( r_{\text{min}} \), respectively.

For nearly maximally rotating holes, the IDD operates down to smaller radii because frame dragging yields a slower effective shear: \( r_{\text{min}} \approx 5.4, B_\phi' = .93 \) and \( r_\text{c} \approx 8.5 \) for \((h_{\text{Buoy}}=a.95)\) and \( r_{\text{min}} \approx 4.6, B_\phi' = .57 \) and \( r_\text{c} \approx 8.5 \) for \((h_{\text{Buoy}}=a.95)\).

The implied \( \alpha_{\text{SS}} \)-parameter at the minimum operational radius is highest for maximally rotating, low buoyancy disks \( \approx .80 \), and smallest for non-rotating, highly buoyant disks \( \approx .21 \). This magnetic stress decreases quickly at larger radii: for low buoyancy, \( \alpha_{\text{SS}} \approx .39 \), and \( .31 \) at \( r = 10 \), and 20, respectively; while for high buoyancy, \( \alpha_{\text{SS}} \approx .16 \), and \( .17 \) at \( r = 10 \), and 20.

### 4.3 Poynting jets and \( \gamma \)-ray burst engines

We have constructed a reasonable set of heuristic two dimensional dynamo equations for magnetic field components in the comoving frame under the premise of negligible generation of meridional field. These equations approximately account for field generation by the shear flow and by the non-axisymmetric MRI in the Kerr-Newman geometry. Two local processes are invoked as field loss terms: turbulent flux buoyancy (or vertical diffusion) and turbulent reconnection. Self-consistent equilibrium solutions to these equations are found in proximity to equipartition field values by adopting pressure ratios and the scale height from the standard model of PWF. Although, one must not take these estimates at face value until a self-consistent calculation is displayed, the total hydromagnetic energy deposition rate from buoyancy is found to be at least comparable to the neutrino luminosity in that scenario, \( L_\nu \approx 3.5\_{+51} \) erg sec\(^{-1}\).

The magnetic field is removed from the disk interior and its ultimate fate depends on the details of the disk corona dynamics which I do not address here. At \( \theta \sim \Theta_\text{H} \), some of this flux can be expected to undergo an inverse cascade to form larger coherent field structures (Tout and Pringle 1995). This large scale field is the relevant one with regards to Poynting jet production. However, ignoring curvature effects, the field generated by the non-axisymmetric MRI in a strong effective shear flow, c.f. \([3.1]\), pushes the optimal wavenumber to its lower bound which implies pumping of coherent field structures with a lengthscale of \( \mathcal{O}(\pi \omega) \sim \pi r \). In other words, the coherence lengthscale of the toroidal MRI in a strongly sheared rotor is large. Thus, in principle all the energy deposited at the innermost radii may go into a Poynting flux. In the low buoyancy, high spin case, our estimate indicates that this may be as much as 10% of the total output rate, without invoking inverse cascade arguments. Invoking the Blandford-Znajek mechanism can only increase the above estimate. If fully self-consistent models yield similar results, this could render the proposed hydromagnetic energy conversion mechanism a more viable alternative to neutrino-burst driven models of \( \gamma \)-ray bursts.

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### A Relativistic Shearing Instabilities

In a local kinematic analysis of the Hill (1878) equations for a “sticky disk”, Balbus & Hawley (1992a) have shown that the equations of motion for a (corotating) Lagrangian displacement vector \( \xi \) (with \( 1_x \rightarrow 1_x \), and \( 1_y \rightarrow 1_y \))

\[
\dot{\xi}_{xx} - 2\Omega\dot{\xi}_{xy} = -\{4\Omega_{Oort} + \kappa_x\}\xi_x
\]

\[
\dot{\xi}_{yy} + 2\Omega\dot{\xi}_{yx} = -\kappa_y \xi_y
\]

yield a dispersion relation identical to the one derived from an MHD stability analysis of axisymmetric perturbations of a purely poloidal disk field configuration in the limit \( k_r/k_\theta \rightarrow 0 \), provided that

\[
\kappa_x = \kappa_y = k_\theta^2 v_{A,\text{Alf}}^2.
\]

Notably, although both terms in the curly brackets of Eqs \([31]\) are stabilizing, their combined influence is destabilizing (Balbus & Hawley 1996) because of the spatial coupling.

Foglizzo & Tagger (1995) find an identical set of equations for the magnetorotational instability of a purely toroidal field in the horizontal regime, provided that the “spring con-
stated” are replaced by
\[ \mathcal{K}_x = k_x^2 \nu_{\text{Alt}, \phi}^2 \quad \text{and} \quad \mathcal{K}_y = k_y^2 \left( \frac{v_{\text{Alt}, \phi}^2}{c_x^2 + v_{\text{Alt}, \phi}^2} \right). \]

This is not surprising given the generality of the magnetorotational destabilization process (BH92a).

In either case, the dispersion relation is of the form
\[ \tau^4 - b \tau^2 + c = 0, \]
with \( b = 4(\Omega^2 + \Lambda) + (\mathcal{K}_x + \mathcal{K}_y) \), and \( c = \mathcal{K}_y(4\Lambda + \mathcal{K}_x) \), where we have set \( d_{\text{in}, r} \Omega^2 = 4\Lambda \) (recall \( \Lambda_{\text{Oort}} = \frac{1}{2} d_{\text{in}, r} \Omega \)). This dispersion relation yields a generalized classical instability criterium (i.e., \( \tau^2 < 0 \))
\[ 4 \Lambda_{\text{Oort}} \Omega + (k \cdot \nu_{\text{Alt}})^2 < 0 \quad (31) \]
for magnetorotational instabilities in either poloidal or toroidal magnetic field topologies, in the horizontal regime \( k_r/k_\theta \rightarrow 0 \). Because in most cases of interest to us \( \Lambda_{\text{Oort}} < 0 \), the first term in Eq \( 31 \) drives the destabilization process.

Classically, this term \( 4\Lambda \equiv d_{\text{in}, r} \Omega^2 \) may be derived by considering the unbalance of inertial forces induced by deviations from Keplerian rotation of radially displaced fluid elements, i.e. induced by the imbalance of centrifugal vs. radial gravity (FT95). Alternatively, in the Hill equations (which follow from Galilean transformations of the Newtonian equations of motion) the equivalent expression, which contains Oort’s A constant implicitly, may be obtained by expanding the two factors in \( -r \Omega^2 \) about the equilibrium point and keeping only linear terms in \( \delta r \).

In the general theory of relativity, on the other hand, the effect of gravity is geometrically included in the equation of motion and the meaning of force is intrinsically different from that in the Newtonian theory even if the acceleration is measured in a frame instantaneously at rest (Abramowicz & Prasanna 1990, Abramowicz 1990). Still, in the LRF a procedure similar to the radial differential force analysis (c.f. Eq \( 35-37 \) of FT95) may be used to derive an approximate relativistic analog to Eq \( 31 \).

Although geodesic Keplerian flow does not ‘feel’ gravity, deviations from relativistic Keplerian rotation unsettle a flow in otherwise radial equilibrium (ignoring radial pressure gradients). The equation that describes this behavior was first derived by Lasota (1994, see also Abramowicz et al. 1997 and GP98)
\[ \frac{1}{2} \dot{D} \beta^2 = - \frac{\gamma^2}{\gamma} \frac{\mathcal{A}}{D} \frac{1}{r^2} \left( 1 - \frac{\Omega}{\Omega_+} \right) \left( 1 - \frac{\Omega}{\Omega_-} \right). \quad (32) \]
Recall that \( \beta \) is the radial speed as measured by an observer at rest in the corotating (CRF) frame.

Noting that the radial speed as measured in the LRF by definition equals zero (albeit instantaneously) we re-write the l.h.s. of Eq \( 32 \) in terms of the radial gradient of this \( v_r \)
\[ \frac{1}{2} \dot{D} \beta^2 = \frac{1}{2} \gamma r^{-4} \frac{D}{r} \nu^2. \quad (33) \]

Since the gravitational potential is implicit in the geometry, the r.h.s. of Eq \( 32 \) may be interpreted as a Newtonian acceleration measured in the local rest frame, aka \( p_r = -\partial_r \mathcal{H} = -\partial_r T \) for a “free” particle (note that the LRF is the only frame where, loosely speaking, it is safe to treat a ‘force’ as a having its Newtonian meaning).

Next, we expand the two terms in parenthesis using the definitions for the prograde and retrograde Keplerian frequencies in BLF coordinates \( \Omega_{\pm} = (r^{3/2} \pm a)^{-1} \) and Taylor expand \( \Omega^2 \) around either Keplerian frequency
\[ \left( 1 - \frac{\Omega}{\Omega_+} \right) \left( 1 - \frac{\Omega}{\Omega_-} \right) = \left\{ 1 - 2\Omega - \Omega^2 (r^3 - a^2) \right\} \equiv \left( \frac{1}{\Omega_+ \Omega_-} \pm \frac{a}{\Omega_+} \right) \delta \Omega^2 |_{\text{eq}}(34) \]
where the difference \( \delta \Omega^2 \) is taken with respect to (frame-dragged) relativistic Keplerian rotation rates.

Lastly, putting together Eqs \( 12, 33, 32 \) to construct the radial differential force, defining the relativistic shear rate in the LRF
\[ \Lambda^2_{\text{Oort}} = A_r^2 \left\{ \frac{1}{D} \frac{d \Omega}{d \ln r} \right\} \quad (35) \]
(recall this expression contains the radial gradients of \( \ell \) and \( \beta \), implicitly); and noting that all other quantities in Eq \( 31 \) are locally defined values, the desired relativistic stability criterium obtains
\[ -4 \left( \frac{1}{D} \right) \frac{1}{r^3} \left( \frac{1}{\Omega_+ \Omega_-} \pm \frac{a}{\Omega_+} \right) A^2_{\text{Oort}} \Omega_{\pm} + (k \cdot \nu_{\text{Alt}})^2 < 0. \quad (36) \]
(recall \( \Omega_{\pm} < 0 \)).

The timescale derived from the relativistic modification to the radial differential force follows straightforwardly by replacing the expression in the curly bracket of Eqs \( 35 \) by its relativistic equivalent in the l.h.s. of Ineq \( 36 \), and following the analysis of BH92a thereon. In the limit \( k_r/k_\theta \rightarrow 0 \) (the horizontal regime) this timescale should be identified with the maximum growth rate for (magnetic) shearing instabilities
\[ \left| \frac{1}{D} \right|_{\text{max}} = - \gamma^2 \left\{ \frac{A}{D} \right\} \frac{1}{r^3} \left\{ r^3 - a^2 + a(r^2 \pm a) \right\}. \quad (37) \]
This expression reduces to the correct limit when \( a \rightarrow 0 \), with the shear force reversal at \( r = 3 \) embodied by the expression for \( A^2_{\text{Rel}} \), as remarked by GP98 (see also, Anderson & Lemos 1988, Abramowicz & Prasanna 1990).

For Keplerian, prograde orbits; using Eqs \( 12, 37 \) one finds
\[ - \Lambda^2_{\text{Rel}} = \left| \frac{1}{D} \right|_{\text{max}} = \frac{3}{4} \gamma^2 \left\{ \frac{A}{D} \right\} \Omega_+, \quad (38) \]
while for retrograde disks, the strength of the instability is somewhat diminished from its classical growth rate.

In the horizontal regime, one may expect the maximum growth rate of shear-fed instabilities (above) to differ from the relativistic shear rate (compare Eqs \([14 & 12]\)) because the former corresponds (classically) to oblique local fluid excursions with \(\xi^x = -\xi^y\) (see \(\S 2.4\) of BH92a) while the gradient of the velocity field (i.e. the shear) is maximized for excursions perpendicular to \(L\). Since the local tetrad of basis LRF vectors is rotated by the geometry (c.f. \(\S 2.1\)), it is not surprising that new metric corrections are involved in the growth rate for magnetorotational instabilities. While this argument does not prove the validity of Eq \([37]\) (which is, after all, an approximate result), it does motivate our finding.

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