Abstract

All experimental results concerning possible neutrino oscillations are naturally and simultaneously accounted for in an $E_6$ GUT model. The fermionic mass matrices are dictated by the symmetry breaking and specific radiative corrections and not by the use of “Ansätze” or discrete symmetries.
In a recent paper \[1\], \[2\] (hereafter: “paper I”) we presented in detail an $E_6$ GUT with a very specific set of mass matrices. In particular, the “light” neutrino mass matrix is practically dictated by the quark sector and the scale of the intermediate symmetry breaking. For an intermediate scale of $10^{10} - 10^{12}$ GeV, suggested by the recent values of $\sin^2 \Theta_W$ \[3\], our “solutions” for the neutrino masses and mixing were concentrated exactly around the value, resulting from the latter announced GALLEX experiment \[4\]. Moreover the experimental requirement of large neutrino mixing allowed us to fix the favored breaking chain.

In this letter, we would like to emphasize this fact and use the exact GALLEX results to limit the range of our solutions. This will enable us to fix the hierarchy of the heavy VEV’s and in particular the allowed values of the intermediate breaking scale, which is also the scale of the right-handed (RH) neutrinos.

Our model is based on the following considerations:

1. The superstrong breaking of $E_6$ will be generated by one or several symmetric $\Phi_{351}$. This dictates the direction of the breaking, it must go via $SO(10)$, the only maximal subgroup with singlets in those representations. The further breaking goes then through $SU(5)$ or subgroups of $G_{PS} = SU_C(4) \times SU_L(2) \times SU_R(2)$.

2. The low energy breaking into $SU_C(3) \times U_Q(1)$ will be generated via one $H_{27}$ Higgs representation. In this case all the mass matrices of the standard fermions are proportional to each other on the tree level and can be diagonalized simultaneously

\[
\hat{M}_e^0 = \hat{M}^0 \\
\hat{M}_d^0 = \hat{M}^0 \\
\hat{M}_{\nu, Dir}^0 = a\hat{M}^0 \\
\hat{M}_u^0 = a\hat{M}^0
\]

with

\[
\hat{M}^0 = \text{diag}(\mu_1, \mu_2, \mu_3) \quad \mu_i \in \mathbb{R}. \tag{1}
\]

3. The main requirement of our model is that the one-loop contributions to the mass matrices are dominated by the diagrams of fig. 4. Those diagrams involve in the loop superheavy gauge bosons and fermions but no scalars. This requirement can be justified using arguments of maximal calculability and predictability. Contributions involving scalar loops are less predictable than those involving gauge bosons with known couplings. Only in supersymmetric theories with SUSY broken at a low energy scale, the scalars may play an important role. In those theories, however, the non-renormalization theorems allow one to avoid unwanted couplings in the superpotential, to all orders.
\[ v_{F,f}^0 = O(M_W) \quad v_{F,f} = O(M_X) \]

\[ X^F_f \quad \tilde{X}^F_f \]

\[ f \quad F \quad F^{(c)} \quad f^c \]

Figure 1: One-loop corrections to standard fermion masses containing gauge bosons.

\[ f \quad F \quad F^{(c)} \quad f^c \]

Figure 2: Infinite second order contributions to Higgs self coupling.

This amounts then to fixing by hand the allowed radiative corrections. Our requirement assumes therefore that SUSY is broken at a relatively high scale and that all relevant Yukawa couplings are much smaller than the gauge couplings \(^1\).

Calculability dictates at the same time one more important requirement. It is well known that two-loop corrections to the diagram in fig. 1 diverge (see fig. 2). An obvious way to avoid this problem \(^2\) is to require that the superheavy mass terms are “orthogonal” to the tree level masses of the light fermions, in the family space. In other words, one obtains a calculable theory if in the framework of diagonal tree level mass matrices the radiative contributions are pure off-diagonal. The off-diagonal corrections induced by the

\(^1\)This is well known phenomenologically for the Yukawa couplings of standard fermions except for the top.
diagrams of fig. 1 are given by [5]

\[ \delta M_{F,f} = w_{F,f} M_F \]

\[ w_{F,f} = \frac{3\alpha k_{F,f} \beta_{g,f}^F \beta_{g,f}^F}{4\pi m_{F,f}^2 - \tilde{m}_{F,f}^2} \ln \left( \frac{m_{F,f}}{\tilde{m}_{F,f}} \right)^2. \]  

(3)

\( M_F \) is the mass matrix of the superheavy fermion \( F \), \( m_{F,f} \) and \( \tilde{m}_{F,f} \) are the masses of the gauge bosons \( X_{f}^{F,\pm} \) and \( \tilde{X}_{f}^{F,\pm} \). \( k_{F,f} \) is the gauge boson mixing factor and \( \alpha_{g,f}^{F,\pm}, \beta_{g,f}^{F,\pm} \) are the group theoretical coefficients of the gauge couplings. Since one of the gauge bosons carries a weak isospin \( I_W = 1/2 \), one VEV contributing to the mixing \( k_{F,f} \) is of order \( M_W \):

\[ k_{F,f} = O(\sqrt{v_{F,f} v_{F,f}}) \quad v_{F,f} = O(M_W). \]  

(4)

The corrections are in general of the order of magnitude \( \alpha M_W \), depending on the ratios of the superheavy masses in eq. (3). In paper I it is proven that in our breaking scheme \( F = N, \nu^c \) give the leading off-diagonal contributions\( ^6 \) where \( N \) is the \( SO(10) \)-singlet in \( 27_{E_6} \). For group-theoretical reasons the graphs with \( F = N \) contribute to the masses of all standard fermions, whereas those induced by \( F = \nu^c \) are limited to the \( u \)-mass matrix\( ^5 \). Taking these radiative corrections into account we get the following mass matrices:

\[ M_e = \frac{1}{r} (\hat{M}_0 + E) \quad M_d = \hat{M}_0 + pE \]

\[ M_{\nu,Dir} = \frac{1}{r} (a \hat{M}_0 + sE) \quad M_u = a \hat{M}_0 + qE + \Delta. \]  

(5)

The matrices \( E \) and \( \Delta \) represent the contributions induced by \( F = N \) and \( F = \nu^c \) respectively, while their relative strengths are given by \( p, q \) and \( s \). The different renormalization behaviour of the quarks and leptons is taken into consideration by the factor\( ^4 1/r \).

4. The matrix \( \Delta \) is clearly proportional to the RH-neutrino mass matrix \( M_{\nu,R} \):

\[ M_{\nu,R} = \eta \Delta \]

\[ \eta = \frac{1}{w_{\nu e, u}}. \]  

(6)

(7)

\(^2\)This is also true in the case of a superheavy \( SO(10) \)-invariant VEV in the \( 27_{E_6} \)-Higgs–representation. At the first sight this looks dangerous because such a VEV generates large masses for the fermions in \( 10_{SO(10)} \). But for group theoretical reasons contributions from these fermions are restricted to the combinations \( (e, D) \) and \( (d, E) \) (where \( D \) and \( E \) are the charged exotic particles in the \( 10_{SO(10)} \)). In both cases the large VEV \( v \) responsible for gauge boson mixing breaks \( SU(5) \) and consequently leads to a suppression by the large \( SO(10) \) invariant masses of the corresponding gauge bosons (see eq. (3)).

\(^3\)For large top masses this is not a very good approximation, see remark about that latter.
$\eta$ is very large and the see saw mechanism \[\text{[4]}\] is naturally realized. The mass matrix of the light neutrinos is therefore

$$M_{\nu,\text{small}} \simeq \frac{1}{\eta} M_{\nu,\text{Dir}} \Delta^{-1} M_{\nu,\text{Dir}}.$$  \hspace{1cm} (8)

Now, because $N$ and $\nu^c$ are Majorana particles the correction matrices must be symmetrical. They are off-diagonal and in general complex.

We analyzed first the $M_u, M_d, M_e$ matrices of eq. (5) for real entries. In this case, as explained in paper I, there is only one “free” parameter that obeys certain theoretical restrictions. We looked for "solutions" which give the best $\chi^2$ fit to the known masses of the quarks and the charged leptons \[\text{[3]}\] as well as the CKM matrix \[\text{[3]}\]. We found solutions which allow top masses up to $m_t \sim 250$ GeV. This result is not trivial because it is very sensitive to the set of experimental input masses used. Our results were obtained using the by now standard set of masses due to Gasser and Leutwyler \[\text{[3]}\] while only a slightly different set of masses due to Barducci et al. \[\text{[10]}\] allowed for $m_{t,\text{phys}} \lesssim 50$ GeV only.

We showed then in paper I that giving $\Delta$ a phase, one can have CP violation without changing essentially the above results.

By studying different breaking chains of $E_6$ we can limit the freedom in the solutions drastically. We considered different possibilities, in particular:

a) One independent correction matrix, i.e. only one $\Phi_{351}$ used or the superstrong breaking follows the chain $E_6 \rightarrow SO(10) \rightarrow G_1 \rightarrow ... \rightarrow G_n \rightarrow SU_C(3) \times SU_L(2) \times U(1)$ where $G_i \subset G_{PS} = SU_C(4) \times SU_L(2) \times SU_R(2)$.

b) Two independent corrections matrices, i.e. more than one $\Phi_{351}$ used but the superstrong breaking going via $E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU_C(3) \times SU_L(2) \times U(1)$.

The essential result of this study is that these two breaking chains favor in our model top masses above 100 GeV! Case a) is even more restricted:

$$105 \text{ GeV} \lesssim m_{t,\text{phys}} \lesssim 125 \text{ GeV}.$$  

The neutrino sector plays a special role in the model. Given the superstrong breaking chain, including the corresponding hierarchy of the superheavy VEV’s, the neutrino properties are fixed. This is related to the fact that the mass matrix of the RH neutrinos is explicitly given, in eq. (7), once the quark-sector is solved. In view of the freedom in the quark-sector ($m_t$, phases of the CKM matrix, etc.) we can
predict only the order of magnitude of the neutrino masses. This is the reason why we could use for the renormalization factors between leptons and quarks one representative parameter \( r \), which is fixed by self-consistency arguments in paper I to be \( 2.7 < |r| < 3.3 \). This means actually that for very large top masses the value of \( m_{\nu} \) may be slightly higher than our results. Using the calculations of ref. [11] we expect an additional factor of 2 or so.

In paper I we got the following basic results in the neutrino sector: The chain a) i. e. \( E_6 \rightarrow SO(10) \rightarrow G_{PS} \rightarrow \ldots \) or the usage of only one \( \Phi_{351} \) representation give only small \( \nu \)-mixing \( \lesssim 10^{-3} \). Consequently one needs several \( \Phi_{351} \) representations and the alternative breaking chain b): \( E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU_C(3) \times SU_L(2) \times U(1) \). To have more predictability in last breaking chain we studied the two special cases:

1) \( |H(1,1)| \ll |\lambda_i^2 \phi^i(1,1)| \)

2) \( |H(1,1)| \gg |\lambda_i^2 \phi^i(1,1)| \).

where we used the following definitions: \( H(1,1) \) and \( \phi^i(1,1) \) denote the \( SO(10) \)-invariant VEV’s of \( H_{27} \) and \( \Phi_i^{351} \) respectively and \( \lambda_i \) the effective coupling constant for the mixing between \( H_{27} \) and \( \Phi_i^{351} \). Then the parameters in eq. (5) receive the following simple values:

1) \( p \simeq 1 \quad q \sim s \sim -a \)

2) \( p \simeq 1 \quad q \sim s \sim 1/a \).

This leads to definite predictions. Case 2) is especially interesting because we could show qualitatively in paper I that large \( m_t \) induces large neutrino mixing. This is seen explicitly also in the detailed numerical calculations. The results for \( \nu_e-\nu_\mu \) mixing in the cases 1) and 2) for an intermediate scale of \( \sim 10^{12} \) GeV are given in fig. 3 and fig. 4. They are shown to lie in the parameter range needed for the MSW explanation [12] of the solar neutrino problem. Fig. 3 shows that in case 2) we can get nice MSW solutions for an intermediate scale of \( \sim 10^{10} \) GeV as well. In fig. 3 and fig. 5 we see that our solutions correspond exactly to the two regions allowed by the recently published results of the GALLEX collaboration [14]:

\[
(83 \pm 19 \text{ stat. } \pm 8 \text{ syst.) SNU.} \tag{9}
\]

\footnote{Also fig. 1 corresponds actually to a possible small region in Fig. 1 of the GALLEX paper. In Fig. 5 however, it can be seen that then the depletion of atmospheric neutrinos measured in the Kamiokande experiment cannot be explained simultaneously (shift all solutions \( \sim 3 \) orders of magnitude down to small values in \( \Delta m^2 \)).}
Now we are in the position to determine all solutions of our model compatible with the new GALLEX data. For the cases 1) and 2) the $\nu_\mu$-$\nu_\tau$ mixing of these solutions is shown in fig. 6 and fig. 7 together with the experimental bounds of Kamiokande and Frejus [13]. We find that only in case 2) the depletion of solar neutrinos as well as atmospheric neutrinos can be explained simultaneously. A detailed study of the solutions which obey all those requirements leads to a value of the intermediate scale between $10^{10}$ GeV and $10^{11}$ GeV.
Figure 3: $\nu_e - \nu_\mu$ mixing in the case 1) for an intermediate scale $\sim 10^{12}$ GeV. The curves describe iso-SNU lines for $^{71}$Ga detectors.
Figure 4: $\nu_e - \nu_\mu$ mixing in the case 2) for an intermediate scale $\sim 10^{12}$ GeV. The curves describe iso–SNU lines for $^{71}$Ga detectors.

Figure 5: $\nu_e - \nu_\mu$ mixing in the case 2) for an intermediate scale $\sim 10^{10}$ GeV. The curves describe iso–SNU lines for $^{71}$Ga detectors.
Figure 6: $\nu_\mu-\nu_\tau$ mixing in the case 1) for all solutions compatible with GALLEX data and experimental bounds by Kamiokande and Frejus.

Figure 7: $\nu_\mu-\nu_\tau$ mixing in the case 2) for all solutions compatible with GALLEX data and experimental bounds by Kamiokande and Frejus.
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