Dynamical physically structured data modeling vs. classical time series analysis: A case study related to clinical trial data analysis

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Abstract. At the time series analysis’s core is the fitting of a preselected model—in the most straightforward case a linear combination of basis functions—to experimental data with one of the performance indices, for example, least squares criterion. However, no matter what the type of such a model is and whatever be the fitting performance index, such an approach does not stipulate for reliance on the understanding those dynamical laws of physics dictating the observable data behavior, given their complexity or uncertainty. Our research looks into a question: “What benefits or advantages could mathematical modeling of such laws be capable of giving when included in nature or experimental data analysis?—nowhere more so than in deciding on life-changing events based on time series analysis in medical practice”.

1. Introduction

In time series analysis (TSA) it has become common knowledge that at its core is a procedure of fitting a model to the past experimental data to reveal in them and then use for forecasting of the four main components. The first component being a discernible general tendency to increase, decrease, or stagnate over a long period is called a trend. The second component that is the existence of a repeating pattern over a more extended time, usually two or more years, is called cyclicity. The effect which can be modeled by a periodic component tightly bound to seasonal variations is known as seasonality, the third component of TS. Finally, the absence of any specific and unique character in data behavior caused by unpredictable influences, which are not regular and also do not repeat in a particular pattern, is called irregularity [1, p. 12]. Widely held additive hypothesis considers TS data as a sum of these four components.

In such a case, specialists normally associate a trend with a slowly varying function \( m_t \) and combine cyclicity and seasonality into one periodic attribute \( s_t \). As for irregularity, they view it as a random noise of value \( Y_t \). Thus, the classical TSA implies data decomposition into three summands [2, p. 23]:

\[
X_t = m_t + s_t + Y_t,
\]

where small letters denote deterministic functions of time, and uppercase letters stand for random processes.

In the TSA are many different types (classes) of models to choose. Amongst generic models are static vs. dynamic, deterministic vs. stochastic, linear vs. non-linear types.
Static models have the form of a weighted linear combination of some basis functions whose weighting coefficients are to be adjusted to make the combination as close to data as possible, usually in the sense of least squares. Among the linear dynamic stochastic models one can find Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA), Autoregressive Fractionally Integrated Moving Average (ARFIMA), and Seasonal Autoregressive Integrated Moving Average (SARIMA) models [1, p. 18].

They, except MA, share a common trait: current value $X_t$ depends linearly on several preceding data values $X_{t-k}, k = 1, \ldots, n$; their number $n$ determines the order, or memory size, of the model. Newly developed Artificial Neuron Networks and Support Vector Machines by Vapnik–Chervonenkis [1, pp. 31–41] have set themselves apart as the alternative model types.

A crucial step in the TSA procedure is an appropriate model selection; however, there is no term “appropriate” tentatively identified. In a most general sense, the appropriate model is that it can produce an accurate forecast based on a description of the historical pattern in the data. Selection of such a suitable model can be performed iteratively from a general class of ARIMA models using the Box–Jenkins forecast method [1, pp. 23–24]. The Box-Jenkins methodology for optimal model selection works in three stages: Model Identification, Parameter Estimation, and Diagnosis Checking. Adequacy measures may be different: Variance Accounted For (VAF), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) or others. Judge, whether the model is right or wrong, is possible after the event: the model proved to be effective, viable, relatable, or not.

However, this practice to select the best TS model remains formal as it postulates a model class without emphasis on the dynamical laws of physics (because of their complexity or obscurity) dictating the observable data behavior. With this in mind, our paper raises a question: What benefits or advantages could mathematical modeling of dynamical laws of physics be capable of giving when included in nature or experimental data analysis?—nowhere more so than in deciding on life-changing events based on Time Series Analysis in medical practice.

To set for an answer, we formulate the baseline of the research in Section 2. The baseline clarifies three issues: 1) What is the crux of conventional TSA technology? 2) In the case of nonacceptance of the standard technology, what are the problems to be solved? 3) Do there exist the desired Goal Practically Achieving Tool and efficient algorithms for GPAT usage?

2. Starting positions

2.1. What is the crux of conventional TSA technology?

It is its rigidity typified by over-formalistic postulating a model class without emphasis on those dynamical laws of physics dictating the observable data behavior.

Example 1. In the medical practice, four components called in the classical TSA trend (Figure 1), cyclicity (Figure 2) seasonality (Figure 3), and irregularity (Figure 4) are not so important to see when analyzing Human Body Daily Thermometry (HBDT) data. They are visible to a medical man at a glance—whereas, in contrast, it is of much greater importance to get more detailed estimates for the present and, above all, future patient’s condition to timely prevent a harmful course of events. Equally, cyclicity in HBDT data in terms of four phases: (i) Prosperity, (ii) Decline, (iii) Depression, and (iv) Recovery is not as important as that in economic or financial TSA [1].

2.2. In the case of nonacceptance of the standard technology, what are the problems to be solved?

When dynamical models of physics laws can be justifiably found and given a linear state-space representation, the associated Kalman filter recursions are recognized to deliver the best solution,
given all model details are known. An unexpected obstruction of high complexity is here:

- Kalman filter is a determined source goal obtainable only theoretically, but not practically.
- Why? What is the impediment? How to overcome the obstacle?
- Our answer is straightforward, viz.
  (i) Quite apart from the fact that no accessible error criterion exists for any filter, it is critical to construct a Goal Practically Achieving Tool (GPAT) suitable for identifying the optimal filter parameters by a criterion-based self-optimization procedure.
  (ii) If we put the case that such a tool is available, the next pressing need in the filter self-optimizing iterations will be to provide their numerical stability and efficiency.

2.3. Do there exist the desired GPAT and numerically stable and efficient algorithms for it? Yes, they do. Correspondingly to the above two items, we have two solutions:

(i) For the filtering problems, there exists a method of replacing the inaccessible original performance index (OPI), viz. the mean squared error performance index, by an accessible auxiliary performance index (API) whose minimization implies minimization of OPI automatically.
(ii) There exists a class of orthogonalized array computation methods to make Kalman filter— together with the API minimization algorithms—stable and efficient numerically.
These two solutions—in their most general appearance of the problem statement and with all necessary proofs—are available in the recent works [3, 4] and [5].

In Sections 3 to 6, as illustrated by HBDT data analysis, we consider the following four questions: Q-① Are the benchmark HB DT data trustworthy? (Section 3); Q-② What about static Least Squares based sinewave data fitting? (Section 4); Q-③ What about dynamic Kalman filter based data fitting? (Section 5); and Q-④ What if data approximation by the \( n \)-th order power polynomial? (Section 6).

3. Q-①: Is it possible to repose the trust in the data the research is based?

There are known basic terms used for describing chronobiological rhythms such as:

(i) **Acrophase**—Measure of the crest time of rhythm from the cosine curve best fitting the data. It provides the timing of rhythm concerning a defined reference point of time. Local midnight often used for time point for circadian rhythms can be expressed in degrees \( (360^\circ = 1 \text{ period}) \) or time units (hours and minutes for circadian rhythms, days or months for longer rhythms).

(ii) **Midline Estimating Statistic of Rhythm (MESOR)**—The value midway between the highest and lowest values of the (cosine) function best fitting to the data.

(iii) **Amplitude**—One half of the extent of the change in the height of a wave (the difference between the maximum height of the wave and the rhythm-adjusted mean \([\text{MESOR}]\) of the waveform).

Some examples of corresponding medical data are visible in Figures 5 and 6.

![Figure 5](image1.png) **Figure 5.** Simultaneous Gut, Rectal, and Axillary Temperatures [8, 9].

![Figure 6](image2.png) **Figure 6.** Examples of chronobiological rhythms data [8, 10].

Our long-running searches for real human body daily temperature variations have led us to Ulyanovsk Oblast Clinical Hospital. Dr. Prof. Andrey B. Peskov kindly allowed us access to the database of this hospital where he and his colleagues collected data for their research in this area [6]. To make it possible, they invented an original method, device, and firmware for monitoring the temperature of the human body (KMTP-01-MIDA) [7]. As a result, the programmatical access to the database enabled us to create a computer program to display temperature curves in the table format for further study [11]. Ethics Committee of the Ulyanovsk Oblast Clinical Hospital has approved the experimental procedures involving healthy middle-aged subjects. The unique method designed and implemented used the individual sensors pasted on a body at different temperature-sensing points to take the measurements every 5 minutes with an accuracy \( \pm 0.1^\circ \) during the day. The bunch of series realizations entered the database. We selected \( N = 18 \) items to form a set valid in understanding: single age subjects, data registration in time-coordinated fashion, and preferably, no sensor detachments. Averaging results of human body thermometry over the \( N = 18 \) realizations have led us to the notion “normal rhythm.”

So, the answer to Question ① is affirmative.
4. **Q-2**: Is the non-dynamical LS-based solution a reasonable one for the real HBDT data processing?

Let us have a set of experiments and suppose their results $y_k$ behave or operate during testing like a harmonic oscillator:

$$ y_k \approx \theta^* + A_n \sin(k \omega_n \tau + \phi_n) = \theta^* + a_n \sin(k \omega_n \tau) + b_n \cos(k \omega_n \tau) \triangleq \tilde{y}_k,$$

$v_k \triangleq y_k - \tilde{y}_k: \quad \tilde{y}_k \triangleq \theta^* + A_n \sin(k \omega_n \tau) + b_n \cos(k \omega_n \tau)$,

$k = 1, 2, \ldots, K > 3; \quad a_n = A_n \cos \phi_n, \quad b_n = A_n \sin \phi_n.$

Formally, data vector $y \triangleq [y_1 \mid \cdots \mid y_K]^T$ obtains the model representation $y = H x + v$ with

$$ H = \begin{bmatrix} 1 & \sin(1 \omega_n \tau) & \cos(1 \omega_n \tau) \\ 1 & \sin(2 \omega_n \tau) & \cos(2 \omega_n \tau) \\ \vdots & \vdots & \vdots \\ 1 & \sin(K \omega_n \tau) & \cos(K \omega_n \tau) \end{bmatrix}, $$

vector $x \triangleq [\theta^* \mid a_n \mid b_n]^T$ of three unknowns, and model error $K$-vector $v$. This model is linear in $x$ since $T_n = 24 \text{ hours} = 1440 \text{ minutes}$ and $\omega_n \triangleq 2 \pi / T_n$ are the known values, $\tau = 5 \text{ minutes}$. Model functions

$$ \tilde{y}_k \triangleq \theta^* + A_n \sin(k \omega_n \tau + \phi_n) = \theta^* + a_n \sin(k \omega_n \tau) + b_n \cos(k \omega_n \tau),$$

$k = 1, 2, \ldots, K$, when they best fit the data $y$, ought to use the LS optimal parameters $\hat{x}$. This $\hat{x}$ satisfies the so-called normal equations:

$$ (H^T H) \hat{x} = H^T y. $$

- While experimenting, we selected the axillary temperatures measured and considered valid for $N = 18$ subjects and computed the averaging sequence of temperatures to substitute it as vector $y$ in the above normal equations.
- By this way, we found $\hat{x} = [34.78^\circ \text{C} \mid 0.8141 \mid -0.0661]^T$. This LS solution suggests that $\theta^* \approx 34.78^\circ \text{C}$, $a_n \approx 0.81$, $b_n \approx -0.06$, and $A_n \approx 0.81$.

The following Figure 7 shows $y_k$ blue circles obtained from real data by averaging over $N = 18$ individual HBDT data, vs. LS modeled measurements

$$ \tilde{y}_k \triangleq h_k^T \hat{x} = \hat{\theta}^* + \hat{a}_n \sin(k \omega_n \tau) + \hat{b}_n \cos(k \omega_n \tau),$$

(red line crossed on the figure) where $h_k^T$ is the $k$-th row of matrix $H$, viz.

$$ h_k^T = \begin{bmatrix} 1 & \sin(k \omega_n \tau) & \cos(k \omega_n \tau) \end{bmatrix}. $$

Here, we can conclude the following:

1. Although one can substitute the sequential computing (Kalman-like) procedure for the above simultaneous solution of the normal equations, it does not alter the significant WEAKNESS of the method (cf. ❺ and ❻ below).
2. It lays in the fact that this method completely ignores the dynamics law that is to dictate the random component in observable data.
3. The non-dynamical nature of this LS-based method makes it not a useful tool for real HBDT data processing.
Figure 7. Results of static Least Squares based sinewave fitting of the averaged real data time series.

This method is of utility to medical staff only for estimating the MESOR which is $\theta^*$, and amplitude $A_n = \sqrt{a_n^2 + b_n^2}$ of the sinusoidal wave, as they officially defined in medical practice, and its phase $\phi_n = \arctan(b_n/a_n)$ through the estimates of parameters $a_n = A_n \cos(\phi_n)$ and $b_n = A_n \sin(\phi_n)$.

Vector $\hat{x} \triangleq [\hat{\theta}^* \, | \, \hat{a}_n \, | \, \hat{b}_n]^T$ delivers the estimates necessary for the above item ❹.

Vector $\hat{x} \triangleq [\hat{\theta}^* \, | \, \hat{a}_n \, | \, \hat{b}_n]^T$ induces launching information for dynamic Kalman filter-based data fitting, viz. $\theta^*$, and initial values $x_1(0) = b_n$, $x_2(0) = a_n \omega_n$.

5. Q-❸: Is it good, bad, or indifferent to implement the dynamic Kalman filter based data fitting?

The individual sensors pasted on a body show their behavior in terms of unpredictable external exposure modeled as a combination of deterministic and stochastic noise components. The deterministic component represents the main circadian rhythm of body temperature, whereas the stochastic component describes the statistical measurement errors. For this deterministic component, a reasonable approximation is a dynamic model described by the van der Pol equation:

$$\ddot{x}(t) + \epsilon \omega_n \left( \frac{4}{A_n^2} x(t)^2 - 1 \right) \dot{x}(t) + \omega_n^2 x(t) = 0$$

where $\omega_n$ is the oscillation rate, $A_n$ is the limit cycle amplitude, and $\epsilon$ is a deterministic parameter controlling the shape of the periodic component.

A small value of $\epsilon$ leads to the signal $x(t)$ to have a sinusoidal shape. This type of model is often used for real-time modeling processes in Biomedicine, Engineering, and Chemistry because these processes often show self-sustaining oscillations in which energy is fed into small oscillations and removed from large swings.

Our strategy in this section is as follows.
First, letting $\varepsilon$ in the van der Pol equation converge to zero, we get the harmonic oscillator having constant phase and amplitude.

Second, we design the Kalman filter whose job is to process a new portion of measurement data as it becomes available.

Third, we develop software to make the filter adaptable to remove parameters’ uncertainty.

Finally, we test the performance of the software using real-life data.

Constructing and further identifying an adequate mathematical model able to reflect the inner dynamics of the object under study is the task of primary importance. This task is in the class of inverse problems related to clinical diagnosis and having a general biological or medical focus.

The first Linear Discrete-Time Dynamic (LDTD) model for circadian human body temperature dynamics has appeared in [12]. The API method for their parameter identification has proved out in [13]. Using [13], we model the thermometry data $y(t_k)$ as composed of the cyclical and irregular components, $y(t_k) = x_1(t_k) + x_3(t_k)$, in the following state equations:

$$
\begin{align*}
\dot{x}_1(t) &= x_2(t)dt, \\
\dot{x}_2(t) &= -\omega_n^2 x_1(t)dt, \\
\dot{x}_3(t) &= -(1/T)(x_3(t) - \theta^*)dt + \sigma \sqrt{2/T} d\beta(t), \\
\end{align*}
$$

with some initial values $x_1(0) = b_n$, $x_2(0) = a_n \omega_n$, and $x_3(0) = 0$ of the state variables, where $\omega_n = 2\pi/T_n$ rad/min; $T_n = 24$ h; $\{\cdot\}_n$ reads $\{\cdot\}_{\text{natural}}$; $x_3(t)$ is the Ornstein–Uhlenbeck random process. Its parameters $T > 0$ and $\sigma > 0$ are unknown, and so we deem them to identify. $\beta(t)$ is a standard (zero mean and unit diffusion) Wiener process with $\lim_{t_0 \to -\infty} \beta(t_0) = 0$ (a.s.).

Parameter $\theta^*$ is the daily mean of body temperature (MESOR); $a_n = A_n \cos(\phi_n)$ and $b_n = A_n \sin(\phi_n)$ are parameters for amplitude $A_n = \sqrt{a_n^2 + b_n^2}$ and phase $\phi_n$ of the sinusoidal wave $x_1(t_k) = A_n \sin(\omega_n t_k + \phi_n) = a_n \sin(k \omega_n \tau) + b_n \cos(k \omega_n \tau)$ that models the cyclical component of the body temperature at time $t_k = k \tau$ with $\tau$ being a sampling interval; $x_1(t_k)$ solves the first two state equations.

Rewrite the above DPS model (1) with vector-matrix notations and call it **3dCRPM = 3-dimension Continuous Real-valued Physically-structured Model**:

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix}_t =
\begin{bmatrix}
0 & 1 & 0 \\
-\omega_n^2 & 0 & 0 \\
0 & 0 & -\lambda
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}_t +
\begin{bmatrix}
0 \\
0 \\
\eta
\end{bmatrix} u_t +
\begin{bmatrix}
0 \\
0 \\
\eta
\end{bmatrix} \dot{w}_t, \\
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix} =
\begin{bmatrix}
b_n \\
a_n \omega_n \\
0
\end{bmatrix} x_t + v_t, \ t \in [0, \infty);
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}_t =
\begin{bmatrix}
b_n \\
a_n \omega_n \\
0
\end{bmatrix} ; \ \eta \triangleq \sigma \sqrt{2\lambda}, \ \lambda \triangleq 1/T, \ u_t \triangleq \theta^*.
$$

Perform one of equivalent transformation of basis, viz. $x^* = T_1^{-1} x$ with $T_1^{-1} = \frac{1}{2}
\begin{bmatrix}
1 & -\omega_n^{-1} & 0 \\
0 & 0 & 2
\end{bmatrix}$

to change to **3dCRCM = 3-dimension Continuous Real-valued Canonical (Jordan) Model**. This
3dCRCM marked by * follows:

\[
\begin{bmatrix}
  \dot{x}_{1}^* \\
  \dot{x}_{2}^* \\
  \dot{x}_{3}^*
\end{bmatrix}
= \begin{bmatrix}
  0 & -\omega_n & 0 \\
  \omega_n & 0 & 0 \\
  0 & 0 & -\lambda
\end{bmatrix}
\begin{bmatrix}
  x_{1}^* \\
  x_{2}^* \\
  x_{3}^*
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  0 \\
  \lambda
\end{bmatrix}
u_t + \begin{bmatrix}
  0 \\
  0 \\
  \eta
\end{bmatrix}\hat{w}_t,
\]

\[y_t = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x_t^* + v_t, \quad \begin{bmatrix}
  x_{1}^* \\
  x_{2}^* \\
  x_{3}^*
\end{bmatrix}
= (1/2)\begin{bmatrix}
  b_n + a_n \\
  b_n - a_n
\end{bmatrix}, \quad \eta \triangleq \sigma \sqrt{2\lambda}, \quad \lambda \triangleq 1/T, \quad u_t \triangleq \theta^*.
\]

It corresponds to the state equations of the dynamical physically structured (DPS) model (1) as a starting point to answer the question: What about dynamic Kalman filter based data fitting? Is it right to refer to it as a substitute for or complement to the conventional TSA analysis?

To begin from this starting point, we do a transition to discrete time to obtain 3dDRCM, the 3-dimension Discrete Real-valued Canonical (Jordan) Model:

\[
x_k^* = \begin{bmatrix}
  \cos \omega_n \tau & -\sin \omega_n \tau & 0 \\
  \sin \omega_n \tau & \cos \omega_n \tau & 0 \\
  0 & 0 & d
\end{bmatrix}x_{k-1}^* + \begin{bmatrix}
  0 \\
  0 \\
  1 - d
\end{bmatrix}u_k + \begin{bmatrix}
  0 \\
  0 \\
  \sigma \sqrt{1 - d^2}
\end{bmatrix}w_{d,k},
\]

\[y_k = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x_k^* + v_k, \quad k = 1, 2, \ldots, K,
\]

\[
\begin{bmatrix}
  x_{1}^* \\
  x_{2}^* \\
  x_{3}^*
\end{bmatrix}
= (1/2)\begin{bmatrix}
  b_n + a_n \\
  b_n - a_n
\end{bmatrix}, \quad \eta \triangleq \sigma \sqrt{2\lambda}, \quad \lambda \triangleq 1/T, \quad d \triangleq e^{-\lambda \tau}, \quad u_k \triangleq \theta^*.
\]

Discrete-time white Gaussian noises (DGWNs) \(w_{d,k} \sim \mathcal{N}(0,Q)\) and \(v_k \sim \mathcal{N}(0,R)\) are mutually independent variables. We hold \(Q\) and \(R\) unknown and the “true” value \(\theta = [\lambda \mid \sigma]^T\) of vector parameter \(\theta \triangleq [\lambda \mid \sigma]^T\) to be identified from the available data \(Y_K^T = \{y_1, y_2, \ldots, y_K\}\) where \(K = 288\) is the number of daily measurements registered at 5-minute intervals \((\tau = 5)\).

By an exact (“true”) value, we will always mean a parameter value of the Best Linear Approximation (BLA) to the unknown behavior of a system. The BLA denotes the linear system that fits best the available data. It is known to be the Kalman filter, the optimal one. So, to identify \(\theta = [\lambda \mid \sigma]^T\) of the unknown \(\theta \triangleq [\lambda \mid \sigma]^T\) is, in sober fact, to find the optimal KF parameter.

It explains why we need an instrument (called GPAT in subsections 2.2 and 2.3) for self-optimization of the adaptive Kalman filter (AKF)—in other words, identification of the Optimal Kalman Filter (OKF): \([\text{AKF} \rightarrow \text{OKF}]\). Our solution for that looks as follows.

**Phase 1**

**Experiment 1:**
- Identify \(\hat{\theta} = [\hat{\lambda} \mid \hat{\sigma}]^T\) for 3dDRCM (3) on a series realization-by-realization basis using the available data \([11]\) for \(N = 18\) selected subjects.
- In so doing, we use our API method \([13]\) with minimizing the API by the gradient Steepest Descent Method \([14]\), viz. the MATLAB-function \(\text{fminunc}\).
- By averaging the separate estimated results \(\hat{\theta}_n\), we obtain \(\hat{\theta} = \sum_{n=1}^{N} \hat{\theta}_n\).
- As the final result, we have got the averaged estimated values: \(\hat{\lambda} = 0.0168\) and \(\hat{\sigma} = 1.9252\).

**Experiment 2:**
Figure 8. Actual data (blue circles) vs. the data modeled by 3dDRCM (yellow check marks). Both graphs after averaging over $N = 18$ realizations.

Figure 9. Actual data (blue circles) vs. the estimates generated by the Kalman filter predictively for a day ahead based on model 3dDRCM (full black line).

- Substitute the averaged estimated values $\bar{\lambda} = 0.0168$ and $\bar{\sigma} = 1.9252$ for $\lambda$ and $\sigma$ in 3dDRCM (3) to generate on this basis the $N = 18$ instances of an artificially created type of data; we can look at them as the modeled data of HBDT.

- We aim to average them and compare the result with the real HBDT data averaged over the same $N = 18$ instances from the database to prove or disprove eligibility of using the developed 3dDRCM with that estimated parameter $\bar{\theta} = [\bar{\lambda} | \bar{\sigma}]$.

- The result of this study is in Figure 8.

**FINDINGS:**

(i) Mean values of actual and modeled data averaged over the measurement time interval are, correspondingly, $m_{\text{actual}} = 34.78$ and $m_{\text{modeled}} = 34.73$.

(ii) These values are very close together.

(iii) The decreased mean level of HBDT stems from the fact that some instances of HBDT curves contained abnormal (decreased) values due to tablet-like sensor detachment from the subjects’ common integuments.

(iv) At a glance, one may conclude that the modeled data more or less recreate the look of actual (real) data. Therefore, the developed model 3dDRCM (3) even if meant to be a simple one, can be accepted as able to “catch” the primary thermal homeostasis mechanism’s characters rather than merely to approximate data.

(v) We have chosen the estimated values $\bar{\lambda} = 0.0168$ and $\bar{\sigma} = 1.9252$ for the “true” $\bar{\lambda} = 0.0168$ and $\bar{\sigma} = 1.9252$ on the basis of this experiment.

This project milestone involves an experimental examination of the developed optimal estimator functionality.

**Phase ❷ Experiment ③:**

- We need to know if the estimator can correctly determine a randomly selected patient’s thermal state and predict it for the next day based on measurements of the previous day.
• For this doing, we substitute the found values \( \dot{\lambda} \) and \( \dot{\sigma} \) into the Kalman filter equations that correspond to the developed model 3dDRCM and will predict the randomly selected patient’s thermal state for the next day.
• The result of this study is in Figure 9.

**Findings:**

(i) From Figure 9, it is apparent that our method provides a means for getting estimates of satisfactory quality in the task of body temperature controlling and, possibly, predicting progress for a day ahead.

Now we are ready to formulate the *Results and Benefits* of our approach.

**RESULTS = ① + ②:**

① We have invented a method of replacing the inaccessible original performance index (OPI) viz. the mean squared error performance index, by an accessible auxiliary performance index (API) whose minimization implies minimization of OPI automatically thereby enabling the criterion-based filter adaptation.

② We have developed novel orthogonalized array computation methods to make Kalman filter—together with the API minimization algorithms—stable numerically therethrough emphasizing their application in a rough environment.

**BENEFITS = ① + ② + ③:**

① Results of our research exposed in this section by mathematical constructions of an adequate mathematical model able to reflect the inner dynamics of the object under study, viz. the circadian human body temperature dynamics, affirm the applicability of this innovative approach.

② We have developed an instrument for self-optimization of the adaptive Kalman filter. It is the novel tool for identification of the optimal Kalman filter intended for different applications admitting a linear state-space representation for many processes under study.

③ It is right to refer to the Kalman filter based data fitting as a reasonable complement to the conventional TSA analysis.

6. **Q-④: Is it reasonable to approximate the real HBDT data by a power \( n \)-order polynomial?**

In the Least Squares method, there are more ways to approximate HBDT data than those shown in Figure 7.

Let us examine the approximation of HBDT data by a power \( n \)-order polynomial. Given the available data set \( Y_1^K = \{y_1, y_2, \ldots, y_K\} \) where \( K = 288 \) is the number of daily measurements registered at 5-minute intervals, find an \( n \)-order power polynomial

\[
p^{(n)}(x) = p_1 x^n + p_2 x^{n-1} + \ldots + p_n x + p_{n+1}
\]

whose coefficients solve the multi-dimensional optimization problem

\[
\min_{p_1, p_2, \ldots, p_{n+1}} \sum_{i=1}^{K} \left( p^{(n)}(x_i) - y_i \right)^2.
\]

The order \( n \) must be less than number \( K = 288 \) for the polynomial to be unique and in the interests of smoothing.

Our experiment strategy follows.
Figure 10. Real HBDT data (blue circles) and the simulated ones based on a power polynomial of the 30-th order (full line).

- Conduct computation experiments on the same HBDT data as before.
- Seek polynomial coefficients with the MATLAB-function polyfit.
- Determine the polynomial order from the condition for a minimum of the residual norm

\[ R_{\text{norm}} = \sqrt{\min_{p_1, p_2, \ldots, p_{n+1}} \sum_{i=1}^{K} (p^{(n)}(x_i) - y_i)^2}. \]

Numerical results shown in Figure 10 are indicative of the following FINDINGS:
(i) The simulated data approximate the past real-world data very good.
(ii) This remark is going to lead to the conclusion that this method allegedly is worthwhile for use in modeling the circadian human body temperature variability.
(iii) In very deed, it is an incorrect conclusion.
(iv) By increasing the polynomial order, it is possible to work for passing the polynomial through all points of the real-world data on the plot.
(v) In so doing, however, dies out the overarching objective: To enucleate characteristic trends, general patterns or hidden physics mechanisms inherent to the phenomenon in and of itself, rather than a member function taken from the database individually.

So, it is not reasonable to approximate the real HBDT data by a power \( n \)-order polynomial. Most certainly, this method is not worthwhile for use in modeling the circadian human body temperature variability.

7. Conclusions
What is the best practical outcome we have achieved?
We have tested the dynamical Kalman filter based solution in comparison with classical TSA methods. This solution has had a reality check by contrast with the traditional TSA methods (sinewave and power-polynomial fitting) at the same HBDT data to learn the limits of methods’ abilities. Our generalized findings from the numerical experiments follow.
Inclusion of the dynamical physically structured model of the natural mechanism generating real data into the TSA enables to “catch” all the mechanism’s native characters rather than merely to approximate data.

This approach leads us immediately to the Kalman recursions allowing a unified and practical approach to prediction and estimation for all processes having a state-space representation.

Although this paper presents a case study related to medical practice, uses of dynamical physically structured modeling—in the heightened sense of the word—are many and varied. It is a multidisciplinary endeavor located at the intersection of the fields of Mathematics and Physics such as Computer Optics & Nanophotonics [15], Image processing & Remote Sensing of the Earth [16], and others.

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