A Differential Equation Model for the Dynamics of Youth Gambling

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Abstract
Objectives: We examine the dynamics of gambling among young people aged 16–24 years, how prevalence rates of at-risk gambling and problem gambling change as adolescents enter young adulthood, and prevention and control strategies.
Methods: A simple epidemiological model is created using ordinary nonlinear differential equations, and a threshold condition that spreads gambling is identified through stability analysis. We estimate all the model parameters using a longitudinal prevalence study by Winters, Stinchfield, and Botzet to run numerical simulations. Parameters to which the system is most sensitive are isolated using sensitivity analysis.
Results: Problem gambling is endemic among young people, with a steady prevalence of approximately 4–5%. The prevalence of problem gambling is lower in young adults aged 18–24 years than in adolescents aged 16–18 years. At-risk gambling among young adults has increased. The parameters to which the system is most sensitive correspond to primary prevention.
Conclusion: Prevention and control strategies for gambling should involve school education. A mathematical model that includes the effect of early exposure to gambling would be helpful if a longitudinal study can provide data in the future.

1. Introduction

The combination of intensely curious young minds, risk-taking behaviors, sensitivity to peer pressure, increased opportunities for gambling, and the excitement induced by games of chance can be harmful to teenagers and young adults. Considering the effects of excessive gambling on adolescent and young-adult development and the rapid expansion of the legal gambling market, many sociologists and psychologists have studied youth gambling behaviors for the last 20 years or so. However, to the best of our knowledge, no mathematical modeling approach has been used to study the dynamics of youth gambling.

Gambling is betting on an uncertain outcome. Youth gambling occurs in many forms, from simple board games to betting on sports to casino games. Adolescents have little difficulty in accessing games that are...
supposed to be restricted to adults [1]. According to the 2007 Minnesota Student Survey (MSS), the highest underage (≤17 years of age) participation in gambling among Minnesota public school students was observed for lottery gambling [2]. Jacobs reported that lottery play dominates legalized forms of gambling among juveniles in both the United States and Canada [3]. According to Wilber and Potenza, actual rates of participation depend on the accessibility of gambling opportunities and the types of gambling available [4]. Youth gambling involves lower amounts of money and lower frequency, but more strategic forms of gambling than adult gambling. When adolescents become young adults, circumstances change. They are no longer underage gamblers, they have a job, and they have less free time, but they have more money. Does this situation change the rate of prevalence of problem gambling?

Almost all young people try gambling at some time because legalized gambling is well accepted in society and is harmless for most young people, but it begins to cause problems for some individuals as the frequency of their gambling increases. Gambling occurs on a frequency continuum, ranging from experimenting, occasional gambling, and regular gambling, to preoccupation, which has serious adverse consequences [5]. The instruments most commonly used to measure the severity of gambling problems for adults are the South Oaks Gambling Screen (SOGS) [6] and the Diagnostic and Statistical Manual of Mental Disorders, fourth edition (DSM-IV) [7]. Revised versions of these tools for adolescents are the South Oaks Gambling Screen-Revised for Adolescents (SOGS-RA) [8] and the Diagnostic and Statistical Manual of Mental Disorders, fourth edition, Multiple Response - Adapted for Juveniles (DSM-IV-MR-J) [9]. Excessive gambling is identified according to the scores or criteria of each specific instrument. For example, at-risk gambling and problem gambling are defined as a score of 2 or 3 and a score of 4 or above, respectively, on the SOGS-RA/SOGS [10, 11]. Prevalence rates vary depending on the survey methods, instruments, and the geographic areas involved, as well as the source of research funding. Data for New York State indicate that approximately 10% of adolescents are problem gamblers and another 10% are at-risk gamblers [12]. For Nevada, the prevalence estimated for problem gambling and at-risk gambling among adolescents is 2.2% and 9.9%, respectively [13]. Shaffer and Hall estimated that the prevalence of problem gambling among adolescents in the USA and Canada was between 4.4% and 7.4% [14]. Winters et al. used three age categories of 16, 18, and 24 years for Minnesota and found prevalence rates of 2.3%, 4.3%, and 3.9% for problem gambling, and 14.8%, 12.1%, and 21%, respectively, for at-risk gambling [11]. Jacobs used nine US youth gambling surveys from 1989 to 2002 and found an average rate of 3.7% for problem gambling [15]. Welte et al. carried out nationals survey of 2274 young people aged 14–21 years in 2005 and 2007 and calculated a prevalence of 2.1% for problem gambling [16]. LaBrie et al. conducted a large national survey of 10,765 students attending 119 scientifically selected colleges, and found that 2.6% gambled weekly or more frequently [17]. Most of these studies have found that prevalence rates of problem gambling among young people have been stable. However, Winters et al. warned that at-risk gambling increases as adolescents mature to young adulthood [11].

Many studies agree that gambling is viewed by adolescents as an opportunity to socialize [3, 4, 18–20]. According to Wilber and Potenza, “Peers may introduce others to gambling as a shared social activity. [4]. Peer group gambling, susceptibility to peer pressure, and having peers who gamble, especially peers who gamble excessively, are significant risk factors for excessive gambling [3, 4, 19]. Excessive gambling is accompanied by associated problems, so old friends are replaced by fellow gamblers, bookmakers, and loan sharks [18]. Shaffer and Korn viewed problems associated with gambling as a socially transmitted disease and used the classic public health model for communicable disease [21]. They treated exposure to gambling or activities that promote it as a sequence of social contacts that conceptually act like contagious germs that can lead to adverse health consequences, in this case, problem gambling. Some sociologists have found that certain social phenomena, such as early sexual behavior and juvenile delinquency, are contagious [22–25]. A significant predictor of the occurrence of these phenomena is peer pressure in the sense that the occurrence depends on the number of individuals who are and who might be involved, as well as the frequency, duration, priority, and intensity of association with peers. As a society we have changed our view on gambling because of socially transmitted acceptance of gambling, and allow ourselves to participate in games of chance. Lee and Do studied the dynamics of gambling among older adults using this approach [26].

In the present study, we used a mathematical modeling approach to investigate the dynamics of gambling among young people by creating a simple epidemiological model. We assume that young people are introduced to gambling by a peer, that gambling activities increase when people around them gamble a lot, and that more gambling opportunities are provided. By treating excessive gambling as a socially transmitted disease, environmental peer contagion is expressed using the mass action terms applied in epidemiological models. Our model consists of three classes. To specify the rates at which individuals move from one class to another, the reasons underlying such transitions are discussed. The model seeks to examine the dynamics of the system via stability analysis and a basic reproductive number. The 2005 study by Winters et al. contains a rare longitudinal set of data [27]. We apply our model to
these data to approximate all the model parameters. These parameter values are then applied in a sensitivity analysis using a threshold condition, and numerical simulations are explored. The discussion section focuses on prevention and control strategies. Although research into youth gambling is an active field, this is the first mathematical modeling approach to studying the dynamics of youth gambling.

2. Materials and methods

2.1. Model

Our model focuses on a population of individuals aged 16–24 years who are divided into three classes, \( N \), \( A \), and \( P \). The class specification is based on the group classification of problem severity status used by Winters et al. [27] for which a SOGS-RA/SOGS score is applied. There are 12 identical items in the SOGS-RA and SOGS that assess gambling problem severity. No problem gambling, At-risk gambling, and Problem gambling are defined as scores of 0 or 1, 2 or 3, and 4 or above, respectively, on the 12-item SOGS-RA/SOGS. The No problem gambling class, \( N \), consists of individuals who gamble with no problem at all or who do not gamble. Our At-risk gambling class, \( A \), consists of individuals who have minor symptoms of problem gambling and are at risk of developing excessive gambling. Individuals who belong to the Problem gambling class, \( P \), are addicted to gambling or suffer from frequent or excessive gambling. The problem gambling class includes pathological gambling. We assume that no death occurs from gambling, so the total population \( S = N + A + P \) is a constant.

The turnover rate in the population, which is the rate at which individuals enter and leave the system, is modeled using a per capita rate \( \mu \). All transition rates are per capita. Gambling disorders occur on a continuum. An individual in class \( N \) is introduced to gambling as a social activity by peers and engages in occasional gambling, but as the urge to gamble and the intensity of gambling increase, the individual transitions to class \( A \). We model this using a peer pressure rate \( \beta \), which is directly related to the proportion of at-risk gamblers and problem gamblers already existing among the peer population, \( (A + P)/S \). Members of class \( A \) may move back to class \( N \) by cutting down the frequency and intensity of their gambling, which is measured using a parameter \( \eta \), or may develop excessive gambling. The latter step could be induced by peer influence, measured in terms of the proportion of peers in the Problem gambling class, \( P/S \), multiplied by a proportionality constant \( \alpha \), or by some other factors such as neurobiology, psychology, parental gambling, and/or delinquent behavior patterns [19, 21], modeled using \( \gamma \). Therefore, the overall per capita problem gambling rate is \( \alpha P/S + \gamma \). There are also two transition directions from class \( P \). Problem gamblers may recover naturally [28] or with professional help. When a problem gambler recovers to a gambler with no problems, a transition to \( N \) occurs; this is modeled using \( \psi \). As gambling problem severity is reduced, a problem gambler transitions to \( A \); we model this using a constant per capita rate \( \theta \). Since addiction to gambling is hard to eliminate, a gambler may relapse back to class \( P \) or may recover fully to move to class \( N \) before leaving the population.

Figure 1 summarizes the model in schematic form. We can now write a system of three differential equations and the governing compartmental model as follows:

\[
\begin{align*}
\frac{dN}{dt} &= \mu S - \beta N A/P + \eta A + \psi P - \mu N \\
\frac{dA}{dt} &= \beta N A/S + \theta P - \alpha A P/S - (\gamma + \eta + \mu) A \\
\frac{dP}{dt} &= \alpha A P/S + \gamma A - (\theta + \psi + \mu) P
\end{align*}
\]

\( S = N + A + P \).

To rescale the system so that the model is independent of the population size \( S \), we choose the dimensionless variables \( n = nS \), \( a = aS \), and \( p = pS \) and obtain the model

\[
\begin{align*}
\dot{n} &= \mu - \beta n(a + p) + \eta a + \psi p - \mu n \\
\dot{a} &= \beta n(a + p) + \theta p - \alpha a p - (\gamma + \eta + \mu) a \\
\dot{p} &= \alpha a p + \gamma a - (\theta + \psi + \mu) p
\end{align*}
\]

\( l = n + a + p \).

To simplify these expressions, we denote the rates of all outflows from \( A \) and \( P \) by \( \omega \) and \( \sigma \), respectively:

\( \omega = \gamma + \eta + \mu, \quad \sigma = \theta + \psi + \mu \).

The model can be reduced to two differential equations for subsequent analysis since the total population is constant.

2.2. Parameter values

To establish model projections, we need to estimate values for the model parameters. Winters et al. described developmental pathways for the severity of youth problem gambling [27] using a three-wave longitudinal data set [11]. The study was conducted with more than 900 families, and 305 participants completed all three SOGS-RA/SOGS assessments in 1990 (T1), 1992 (T2), and 1997–1998 (T3) at mean ages of 16.0,
17.6, and 23.8 years, respectively. Although SOGS-RA was used at T1 and T2, and SOGS at T3, the researchers chose 12 items that are identical in SOGS-RA and SOGS. They identified developmental gambling groups as N (no gambling problem), A (at-risk gambling), and P (problem gambling), and expressed pathways of gambling severity over the three time periods. For example, APN describes a pathway of at-risk gambling at T1, problem gambling at T2, and no gambling problem at T3. There are 27 combinations over three waves. The data provide new incidence and prevalence rates. The measures were for gambling in the previous year, so young people aged 16, 18, and 24 on average were asked about their gambling activities in the previous year. Using these data, we approximate two sets of values for the model parameters, one set for adolescents aged 16–18 years and the other for young adults aged 18–24 years.

It is evident that some of the parameters are rough estimates in the absence of precise data; in addition, the size of our data set is small. Hence, we are mainly interested in examining how closely our model activity agrees with the empirical study, and in suggesting control strategies.

2.2.1. Adolescents aged 16–18 years

To calculate the entrance and removal rate $\mu$, we estimate the average time an individual remained in the study group between T1 and T2. The baseline sample consisted of 910 families who had at least one individual aged 15 to 18 years in residence at T1. However, 208 did not consent to participation (208/910 ≈ 22.9%). A further 170 of the 702 participants at T1 did not participate at T2, representing 24.2%. Averaging yields a dropout rate of approximately 23.6%, which means that at least three-quarters of the participants remained in the group for 2 years, although it is unknown how long those who dropped out (about 25%) remained in the group before they left. We therefore use an estimate of 1.75 years ($0.75*2 + 0.25*1 = 1.75$), $1/\mu = 1.75$, yielding $\mu = 1/1.75$ year$^{-1}$.

None of the problem gamblers had recovered by T2, which provides $\psi = 0$.

A total of 25 of the 47 at-risk gamblers at T1 had become no problem gamblers by T2, yielding $\eta \approx (25/47)/2 = 0.266$ year$^{-1}$.

There were 47 at-risk gamblers and seven problem gamblers among 305 participants at T1, giving a prevalence of $(47 + 7)/305 \approx 17.7\%$ for A and P in the population at T1. Some 13 new at-risk gamblers developed over 2 years among 251 individuals with no gambling problem at all. Ten individuals with no problem at all at T1 had become problem gamblers by T2. According to the model assumption of a frequency continuum for gambling, we presumed that such individuals would be at-risk gamblers first before they became problem gamblers. Although we know that it took less than 2 years for those 10 individuals to become at-risk gamblers and then problem gamblers, the duration of their stay in the $A$ or $N$ class is unknown. Hence, we consider the change in proportion of no problem gambling at all over 2 years, and suppose $n' = \frac{251}{305} - \frac{251}{305}/2 \approx 0.0033$. By rewriting Eq. (1) as $\beta = \frac{-0.0033 + \mu(1 - n) + \eta a + \psi p}{(n(a + p))}$, with the above values for $\mu$, $\psi$, $n = 251/305$, $a = 47/305$, and $p = 7/305$, we obtain $\beta \approx 0.953$ year$^{-1}$. This indicates that the 10 individuals stayed in class $N$ for approximately 3 or 4 months (0.28 years) on average. Furthermore, we obtain a per capita rate of increase in the incidence of $(\frac{13/251}{2} \text{ years} + \frac{10/251}{0.28} \text{ years})$ year$^{-1}$, and divide this by the 17.7% prevalence of at-risk and problem gamblers in the population at T1 to obtain an approximation of $\beta$ as above.

The average per capita problem gambling rate is $ap + \gamma$. Some 12 new problem gamblers were observed at T2: two of these were at-risk gamblers at T1; the rest correspond to the 10 individuals discussed above, giving a proportion of at-risk gamblers who became problem gamblers before leaving the population of $(ap + \gamma)/(\mu + \eta + ap + \gamma) = 12/57$. Using the values of $\mu$ and $\eta$ above, $ap + \gamma \approx 0.223$. Since there are no data available for estimating how many individuals became problem gamblers because of peer pressure and how many did so because of factors other than peer pressure, we assume that $\alpha$ is one order of magnitude greater than $\gamma$, and estimate $\gamma = 0.18$ years$^{-1}$ and $\alpha = 1.88$ year$^{-1}$ with $p = 0.023$.

Two of seven problem gamblers at T1 moved back to the at-risk class at T2, which is approximately $\theta = (2/7)/2$ year$^{-1}$. 

![Schematic diagram of the model](image-url)
We use these parameter values to compute $R_0$. Proposition 2 in the Appendix provides $R_0 \approx 1.208$.

2.2.2. Young adults aged 18–24 years

Some 305 of the 350 target participants were successfully contacted at T3, yielding a dropout rate of 12.9%. Therefore, we estimate that at least 87% of the participants remained in the study group, and take 5.3 years as the average time for which an individual remained in the group, yielding $\mu = \frac{1}{5.3}$ year$^{-1}$.

Four of 16 problem gamblers at T2 recovered and moved to class N over approximately 6 years (from 1992 to 1997–1998), so $\psi = (4/16)/6 \approx 0.042$ year$^{-1}$.

Twenty of 36 at-risk gamblers at T2 had become no problem gamblers by T3, yielding $\eta = (20/36)/6 \approx 0.093$ year$^{-1}$.

We approximate $\beta = 0.406$ year$^{-1}$: 45 new at-risk gamblers developed over 6 years among 253 individuals in class N. Three individuals with no gambling problem at all at T1 had become problem gamblers by T3. Using the analysis carried out to find $\beta$ for adolescents, we assume that these three individuals remained in class $A$ for 3 or 4 months (0.3 years). There were 36 at-risk gamblers and 16 problem gamblers at T2, which represents a prevalence of $(36 + 16)/305 \approx 17\%$ of at-risk or problem gambling in the population at T2. Dividing the per capita rate of increase in incidence of $(45/253)/6 + (3/253)/0.3$ by 0.17 provides an approximation of $\beta$. Since the population of $N$ changed from 253 at T2 to 229 at T3, the average change in the rate of $\beta$ is $((229/305) - (253/305))/6 \approx -0.013$. Solving $n' = -0.013$ for $\beta$ using Eq. (1) gives $\beta \approx 0.412$, which is very close to our approximation.

Two of 36 at-risk gamblers at T2 had become problem gamblers by T3, and three individuals with no problem gambling at all at T2 (the same three individuals as discussed in the paragraph above) had become problem gamblers by T3. Hence, the proportion of at-risk gamblers who become problem gamblers before leaving the population is $(\alpha p + \gamma)/((\mu + \eta + \alpha p + \gamma) = 5/39$, which provides $\alpha p + \gamma \approx 0.041$ year$^{-1}$ with the values of $\mu$ and $\eta$. Since there are no data available for measuring $\gamma$ and $\alpha$, we consider that as young adults begin their careers, gambling is more affected by factors other than peer pressure, such as job stress, money requirements, and exposure to heavy gambling, than as an adolescent [11, 29]. Hence, we assume that $\gamma$ and $\alpha$ are of the same order, that is, $\alpha p + \gamma \approx \gamma$, and take $\gamma = 0.04$ year$^{-1}$ and $\alpha p = 0.001$. With $\mu = 16/305$, we obtain $\alpha \approx 0.019$ year$^{-1}$.

Five of 16 problem gamblers at T2 had reduced the frequency and intensity of their gambling and moved to the at-risk class by T3, providing $\theta = (5/16)/6 \approx 0.052$ year$^{-1}$.

These parameter values provide $R_0 \approx 1.466$.

3. Results

Since $R_0 > 1$ for both adolescents and young adults, there is an endemic equilibrium according to Proposition 3 in the Appendix, that is, a gambling problem is endemic among young people.

We ran numerical simulations using the parameter values for adolescents and the initial condition $(N,A,P) = (251,47,7)$, which is the data at T1. Figure 2a and b reveal an increase of problem gambling since T1 and a decrease in at-risk gambling around T2, respectively. Both of these model behaviors agree with the data, but with slower decreasing and increasing rates. The endemic prevalence of problem gambling among adolescents is approximately 5.8% according to the profile in Figure 2c. Data from other studies range between 2% and 7%. Figure 2c reveals the local stability of the endemic equilibrium solution, that is, any solution that starts close enough to the endemic equilibrium point approaches it, although we could not confirm this analytically.

Using the data at T2, $(N,A,P) = (253,36,16)$ as an initial condition, and the parameter values for young adults, it is clear from Figure 3a that the number of at-risk gamblers sharply increased, in strong agreement with data reported by Winters et al. [11, 27]. Figure 3b shows that the prevalence of problem gambling in young adults is approximately 4%, which is currently the national average.

Our model behavior parallels the common notion that problem gambling is more prevalent among adolescents than among adults [4, 30]. Sociologists explain this phenomenon in terms of the shifting focus of young adults to issues such as getting a real job, succeeding at college, and marriage, among others.

Our model confirms the following: (1) at-risk gambling and problem gambling are endemic among young people; (2) the prevalence of problem gambling among young people is stable; (3) prevalence rates of problem gambling are 5.8% and 4% for adolescents and young adults, respectively, which are similar to the national average; (4) the prevalence of problem gambling among young adults is lower than that among adolescents, which parallels the common notion that problem gambling is more prevalent among adolescents than among adults [4, 30]; and (5) at-risk gambling among young adults has increased.

4. Discussion

The increase in at-risk gamblers is translated to an increase in the value of the model parameter $\beta$, to which the system is most sensitive. To see how a small perturbation to $q$ affects the threshold condition $R$, we define an index of the sensitivity of $R$ to $q$ as $|S_q| = |\partial R/\partial q|/(q/R)$. Our threshold condition is $R_0$. The parameter with the highest sensitivity index for $R_0$ among all the parameters is the one to which our system
is most sensitive. Using the parameter values estimated in the previous section, we find that the highest is $|S_b|$ and the second highest is $|S_h|$. Therefore, reducing the value of $\beta$ is the best way to decrease the value of $R_0$. Working on reducing $\beta$ and increasing $\eta$ corresponds to primary prevention; likewise, increasing the values of $\theta$ and $\psi$ and decreasing the value of $\gamma$ are related to secondary prevention, harm reduction, and treatment, which are the terms used by Shaffer and Korn [21]. The sensitivity analysis explains mathematically why the former is more important than the latter. In practice, primary prevention costs less and is more effective than the alternatives. There is ample evidence that severe gambling among adolescents results in increased delinquency, poor familial interaction, and poor coping skills when facing demanding situations [31, 32], which represents a social problem as well as a public health problem. However, when primary prevention fails, the alternatives must be considered.

At-risk young gamblers are potential problem gamblers when circumstances or opportunities for increased gambling are encountered. Although the study by Winters et al. could not confirm that early gambling exposure and involvement would trigger serious gambling in adulthood [11], more recent studies have proved that gambling experience at an early age or early childhood impulsivity predicts problem gambling in the teenage years or adulthood [15, 33, 34]. This evidence leads to a debate on abstinence versus harm reduction as an approach [35] and discussions on lessening youth gambling problems by reducing impulsivity [34]. A future study should include a mathematical model that considers the effect of early exposure and a longitudinal study that can provide data. Other behaviors connected to problem gambling are poor school performance, substance abuse, and parental gambling history [36], among others. Investigators from Harvard Medical School and the Harvard School of Public Health found that according to a national survey of more than 10,000 college students, the most distinctive differences between gamblers and non-gamblers are being a male and watching television for more than 3 hours a day [17]. In summary, researchers agree on the risk factors for problem gambling.
Schools should address the danger of problem gambling [17,32,37]. School counselors should routinely survey questions related to gambling. According to Gupta and colleagues, teaching of active coping strategies to high-risk young individuals is a very effective approach and should be “a substantial part of school-based prevention initiatives” [32]. Shaffer et al. found that all schools investigated had alcohol policies, but fewer than a quarter of them had gambling policies [37]. Educators and staff in schools should be trained to engage in the early identification for prevention efforts to succeed. Many young people are unaware of the seriousness of addiction to gambling. Schools should educate their students about the risks associated with gambling in the way that substance abuse is often discussed. Young people have been exposed to lotto-playing by their parents and television commercials for Powerball. Many adolescents do not relate lotto to gambling and misunderstand the age limit of lotto-playing [1] because of the absence of any education on the danger of serious gambling. A particular approach suggested by Stinchfield is to develop and evaluate prevention programs and messages designed for specific groups of young people, ranging from information to intensive prevention efforts [38]. This is echoed by Wilber and Potenza, who emphasized the importance of evidence-based strategies and an improved understanding of the impact of various levels of specific types of gambling [4].

Various reports confirm that underage gambling, including casino access, is easy. An experienced individual may easily propagate the excitement of risk-taking to friends. The most vulnerable period for peer pressure is adolescence; underage gambling often occurs as a group activity. Prevention of underage gambling should be stressed. Recommendations include strengthening of regulations, enforcement of such regulations, and evaluation of their efficacy [4,38].

To reduce harm and treat young people who have already developed problem gambling, help channels and services should be available. First, many such individuals may not recognize that their gambling is problematic. Second, some young people may lack the maturity to admit that they are problem gamblers. In both situations, continuous monitoring of gambling activity is helpful. A parent, teacher, community counselor, or friend may intervene to provide resources. Stinchfield suggested a 24-hour telephone helpline, assessment, referral, and treatment services as possible interventions [38]. Even when young people present for treatment, different approaches are required, depending on the patient’s maturity and developmental level [18]. For example, considering peer influence on adolescents, a Gambling Anonymous program could pair a problem gambler with a recovered youth who matches the characteristics of individuals and problems associated with gambling. Since few adolescents seek treatment, research has been very limited. In particular, pharmacotherapies for pathological gambling in young people have come from studies conducted for adults, so questions on efficacy and tolerability remain [4,38]. Further systematic and evidence-based research is necessary to develop treatments for gambling disorders.

Appendix.

There is only one possible end state for this model, the gambling problem free equilibrium \( (n,a,p) = (1,0,0) \), which is the most desirable state. If individuals leave the age group faster than they become at-risk gamblers, no problem gambling state will ever arise in the population. We state this mathematically in the following proposition.

**Proposition 1.** If \( \beta < \mu \), the gambling problem free equilibrium is globally stable, that is, all solutions approach \((1,0,0)\).

**Proof.** We define the Lyapunov function \( V = a + p \) and denote \((1,0,0)\) by \( x^* \). Then \( V(x^*) = 0 \) and \( V'(x^*) > 0 \) for all \( x \neq x^* \). It remains to show that \( dV/\ dt < 0 \) for \( x \neq x^* \) if \( \beta < \mu \).

\[
V' = a' + p' = \beta n (a + p) - (\eta + \mu) a - (\psi + \mu) p \\
\leq \beta (a + p) - \mu (a + p),
\]

which is negative if \( \beta < \mu \). This implies that the best strategies that control problem gambling reduce \( \beta \) to this level. Although \( \beta < \mu \) is too ambitious to achieve, we show later that reducing \( \beta \) is still the best strategy for reducing the prevalence of at-risk gambling and problem gambling. We proceed to find the basic reproductive number, \( R_0 \). In epidemiological models, \( R_0 \) is interpreted as the average number of secondary cases caused by a typical single infected individual. Hence, the disease spreads if \( R_0 > 1 \) and dies out if \( R_0 < 1 \). A sociological term for this is the tipping point, which is the point at which a stable system changes to an unstable one or vice versa, which is a threshold condition.

**Proposition 2.** The basic reproductive number is

\[
R_0 = \frac{\beta (\theta + \psi + \mu) + \beta \gamma + \gamma \theta}{(\gamma + \eta + \mu)(\theta + \psi + \mu)}.
\]

If \( R_0 < 1 \), the gambling free equilibrium is locally asymptotically stable, that is, solutions that start close enough approach \((1,0,0)\).

**Proof.** The Jacobian of \( n \), \( a \), and \( p \) evaluated at the gambling problem free equilibrium \((1,0,0)\) is

\[
J = \begin{bmatrix}
-\mu & -\beta + \eta & -\beta + \psi \\
0 & \beta - (\gamma + \eta + \mu) & \beta + \theta \\
0 & \gamma & -(\theta + \psi + \mu)
\end{bmatrix}.
\]

We use the notations \( \omega \) and \( \sigma \) defined in \((5)\). All eigenvalues are negative if \( \beta - \omega - \sigma < 0 \) and
Note that if $\beta - \omega - \sigma < 0$. Therefore, we define
\[ R_0 = \frac{\beta \sigma + \beta \gamma + \gamma \theta}{\omega \sigma}. \] (7)

Note that if $R_0 < 1$, inequality (6) holds, which completes the proof. To interpret the tipping point $R_0$, we rewrite (7) as
\[ R_0 = \frac{\beta + \beta \gamma + \gamma \theta}{\omega \sigma}. \]

The first term, $\beta/\omega$, expresses the notion that individuals enter class $A$ at a rate of $\beta$ and leave after an average time of $1/\omega$. The second term, $(\beta/\omega)(\gamma/\sigma)$, is the proportion of at-risk individuals who become problem gamblers. Finally, $(\gamma/\sigma)(\beta/\omega)$ is the proportion of problem gamblers who move back to the At-risk gambling class. Therefore, if the sum of these quantities is less than 1, at-risk gambling and problem gambling will be controlled as long as there is no sudden huge entry of at-risk gamblers and problem gamblers into the population, that is, the population is locally stable. We now discuss whether we have an endemic equilibrium if $R_0 > 1$. Note that $R_0$ is independent of $\sigma$, so we investigate the role of $\beta$ in the following.

**Proposition 3.** If $R_0 > 1$, we have either one or three endemic equilibria. If $R_0 < 1$, we have either none or two endemic equilibria. If $\alpha = 0$, that is, at-risk gamblers become problem gamblers only for reasons other than peer pressure, there is only one endemic equilibrium if $R_0 > 1$, and none otherwise. Multiple endemic equilibria, if any, exist only for a positive bounded $\alpha$.

**Proof.** We use (4) to replace $n$ by $n = 1 - a - p$, and substitute this in Eq. (9), multiply by $-\sigma$ and divide by $\alpha$. Then we obtain a linear function,$$a' = g(a) = -\beta(\sigma^2 + 2\sigma\gamma + \gamma^2)\gamma + d_0,$$where$$d_0 = \sigma(\gamma\beta + \beta + \gamma\theta - \sigma^2).$$

The slope of $g$ is always negative and$$g(0) = d_0 > 0 \iff R_0 > 1.$$Hence, if at-risk gamblers become problem gamblers because of factors other than problem gamblers, then there is only one endemic equilibrium if $R_0 > 1$, and none otherwise. Suppose that $\alpha$ is arbitrarily large. Then we see that $p^* = 0$ from (8), which implies that $p^* = 0$, as seen in Eq. (2), and thus there is no endemic equilibrium solution. Therefore, no endemic equilibria exist if $\alpha$ is arbitrarily large.

**Conflicts of interest**

All authors declare no conflicts of interest.

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