A Four Dimensional Superstring from the Bosonic String with Some Applications.

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A string in four dimensions is constructed by supplementing it with forty four Majorana fermions. The fermions are represented by eleven vectors in the bosonic representation $SO(D - 1, 1)$. The central charge is 26. The fermions are grouped in such a way that the resulting action is world sheet supersymmetric. The energy momentum and current generators satisfy the super-Virasoro algebra. GSO projections are necessary for proving modular invariance. Space-time supersymmetry algebra is deduced and is substantiated for specific modes of zero mass. The symmetry group of the model can descend to the low energy standard model group $SU(3) \times SU_L(2) \times U_Y(1)$ through the Pati-Salam group.

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I. INTRODUCTION

String theory was invented as a sequel to dual resonance models to explain the properties of strongly interacting particles in four dimensions. Assuming a background gravitational field and demanding Weyl invariance, the Einstein equations of general relativity could be deduced. It was believed that about these classical solutions, one can expand and find the quantum corrections. But difficulties arose at the quantum level. Even though the strong interaction amplitude obeyed crossing, it was no longer unitary. There were anomalies and ghosts. Due to these compelling reasons it was necessary for the open string to live in 26 dimensions. At present the most successful theory is a ten dimensional superstring on a Calabi-Yau manifold or an orbifold. However, in order to realise the programme of the string unification of all the four types of interactions, one must eventually arrive at a theory in four flat space-time dimensions, with N=1 supersymmetry and chiral matter fields. Recently one of us has proposed a new type of four dimensional Superstring with these features. Certain questions like equivalence of light cone action to the Superconformal ghost action were raised. This paper is an attempt in elaborating this letter. Considerable research, relating to this paper had also been done by Gates and his collaborators.

A lot of research has been done to construct four dimensional strings, specially in the latter half of the eighties. Antoniadis et al have constructed a four dimensional superstring supplemented by eighteen real fermions in trilinear coupling. The central charge of the construction is 15. Chang and Kumar have discussed the problem with Thirring fermions. Kawai et al have also considered four dimensional models in a different context than the model proposed here. None of these models makes direct contact with the standard model. There are no superconformal ghosts as well. Their contribution of eleven to the central charge is made up by those coming from the Lorentzian fermionic ghosts. This will be shown in section III using light cone gauge, with details in section VIII.

Consistent superstrings as solutions of $D = 26$ bosonic string has been shown to exist by Casper, Engler, Nicolai and Taormina. Quite recently Englert, Hourat and Taormina have extended this work to brane fusion in the bosonic string resulting in a fermionic string. The present work, which has similar purpose, is quite different and novel. We search for supersymmetry within the compactified Nambu-Goto string in 26 dimensions by introducing eleven four-plet of Majorana fermions transforming in the vector representation of the Lorentz Group $SO(3, 1)$ and arrive at a four, not a ten dimensional superstring.

The string, we construct, is essentially, the 26 dimensional ordinary bosonic string in which the bosonic coordinates $X^\mu$ are restricted to be four with $\mu = 0, 1, 2, 3$. We show that an action can be constructed which is supersymmetric.

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All the relevant algebra given in standard text books follow. But subtle differences, which have to be pointed out, are repeated in the paper.

Using vectorial fermions for compactification introduces eleven longitudinal ghost modes. Fortunately there are exactly eleven component subsidiary conditions to eliminate them and make the theory physical. Complication due to so many vectors makes a light cone analysis impracticable. We have to fall back on the covariant formation of superstring theory. The central charge is calculated to be 26.

In section II, we give the details of the world sheet supersymmetric model. The following section III, the local 2D and local 4D supersymmetric actions are discussed. Section IV gives the usual quantization and super-Virasoro algebra is deduced in the section V. Bosonic states are constructed in Section VI. Fadeev- Popov ghosts are introduced and the BRST charge is explicitly given in section VII and VIII. Ramond states have been worked out in section IX. In section X, the mass spectrum of the model and the necessary GSO projections to eliminate the half integral spin states are introduced. In section XI, we show that these projections are necessary to prove the modular invariance of the model. Space-time supersymmetry algebra is satisfied and is shown to exist for the generated zero mass modes in section XII. In section XIII we show how the chain $SO(44) \rightarrow SO(11) \rightarrow SO(6) \times SO(5) \rightarrow SU(4) \times SU(2) \times SU(2)$ from the string theory and then descend to $SU_C(3) \times SU_L(2) \times U_Y(1)$. We calculate that the stringy Pati-Salam group $SU(4) \times SU_L(2) \times SU_R(2)$ breaks at an intermediate mass $M_R \approx 5 \times 10^{14}$ GeV giving the left-handed neutrino a small mass, which has now been observed in the top sector. An action is written down with a curved metric in Section XIV and is world sheet supersymmetric. The vanishing of the one loop $\beta$-function results in the vanishing of the Ricci tensor in the normal one time, three space universe.

The literature on string theory is very vast and exist in most text books on the subject. The references serve only as a guide to elucidate the model.

II. THE MODEL

As stated earlier, the model consists of replacing the 26 vector bosons of the 26-d bosonic string by the 4 bosonic coordinates of the four dimensions and 44 Majorana fermions equivalent to the remaining 22 bosonic co-ordinates. Traditionally, these compactifying fermions are world sheet scalars. The novel feature of this model is that we shall take them as Lorentz vectors in the bosonic representation, $SO(D - 1, 1)$.

We divide the fermions into four groups. They are labelled by the space time indices $\mu = 0, 1, 2, 3$. Each group contains 11 fermions. We begin to search for a four vector which is also an anticommutating Majorana field $\Psi^\mu$ to superpartner $X^\mu$. As a first step, the 11 fermions are divided into two groups, one containing six and the other five fermions. We level one group of real Majorana fields by $\psi^{\mu,j}$ with $j = 1, 2, 3, 4, 5, 6$ and the other by $\phi^{\mu,k}$ with $k = 1, 2, 3, 4, 5 = j - 6$. To differentiate and for convenience, we use $\phi^{\mu,k} = i\bar{\phi}^{\mu,k}$ and scrupulously keep track of the factor ‘$i$’ everywhere; there is an arbitrary phase factor in defining a conjugate to the Majorana spinor. At this stage, we introduce vector like objects $e_\psi$ and $e_\phi$ with eleven components, e.g. for $j = 3$ and $k = 3$ with $\psi$ and $\phi$ as book keeping suffixes,

$$e_\psi^j = (0, 0, 1, 0, 0, 0, 0, 0, 0)$$

and

$$e_\phi^j = (0, 0, 0, 0, 0, 0, 0, 1, 0, 0)$$

with the properties $e_\psi^j e_\psi^{j'} = \delta^j_{j'}$, $e_\phi^j e_\phi^{k'} = \delta^k_{j'}$, and $e_\phi \cdot e_\psi = 0$. We shall frequently use the relation $e_\psi^j e_\psi^{j'} = 6$ and $e_\phi^k e_\phi^{k'} = 5$.

The string action of the model is

$$S = -\frac{1}{2\pi} \int d^2\sigma \left[ \partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^{\mu,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} - i\bar{\phi}^{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k} \right].$$
\( \rho^\alpha \) are the two dimensional Dirac matrices

\[
\rho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},
\]

\( \rho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \)

and obey

\[
\{ \rho^\alpha, \rho^\beta \} = -2\eta^{\alpha\beta}.
\]

Further

\[
\bar{\psi} = \tilde{\psi} \rho^0; \quad \bar{\phi} = \tilde{\phi} \rho^0.
\]

Such an action had also been written down by Gates et al. \[4\].

In general we follow the notations and conventions of reference [10] whenever omitted by us. \( X^\mu(\sigma, \tau) \) are the string coordinates. The fermions \( \psi' \)s and \( \phi' \)'s are decomposed as

\[
\psi = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}, \quad \text{and} \quad \phi = \begin{pmatrix} \phi_- \\ \phi_+ \end{pmatrix}.
\]

The action is found to be invariant under the following infinitesimal transformations

\[
\delta X^\mu = \bar{\epsilon} \left( e^j_\psi \psi^\mu_j + ie^k_\phi \phi^\mu_k \right) = (e^j_\psi \bar{\psi}^\mu_j + ie^k_\phi \bar{\phi}^\mu_k) \epsilon,
\]

\[
\delta \psi^\mu,j = -ie^j_\psi \rho^\alpha \partial^\alpha X^\mu \epsilon, \quad \delta \bar{\psi}^\mu,j = i e^j_\psi \bar{\psi} \rho^\alpha \partial^\alpha X^\mu,
\]

and

\[
\delta \phi^\mu,k = e^k_\phi \rho^\alpha \partial^\alpha X^\mu \epsilon, \quad \delta \bar{\phi}^\mu,k = -e^k_\phi \bar{\phi} \rho^\alpha \partial^\alpha X^\mu.
\]

\( \epsilon \) is an infinitesimally small constant anticommuting Majorana spinor. The commutator of the two supersymmetry transformations gives a spatial translation, namely

\[
[\delta_1, \delta_2] X^\mu = a^\alpha \partial^\alpha X^\mu,
\]

\[
[\delta_1, \delta_2] \psi^\mu,j = a^\alpha \partial^\alpha \psi^\mu,j,
\]

and

\[
[\delta_1, \delta_2] \phi^\mu,k = a^\alpha \partial^\alpha \phi^\mu,k,
\]

where

\[
a^\alpha = 2i \bar{\epsilon}_1 \rho^\alpha \epsilon_2.
\]

In deriving these, the Dirac equations for the spinors have been used. It is interesting to note that we must also choose \( \psi^\mu_j = e_{\psi,j} \Psi^\mu \) and \( \phi^\mu_k = ie_{\phi,k} \Psi^\mu \) leading to the desired

\[
\Psi^\mu = 6 \Psi^\mu - 5 \Psi^\mu = e^j_\psi \psi^\mu_j + ie^k_\phi \phi^\mu_k.
\]
We can recast the infinitesimal transformations as

\[ \delta X^\mu = \bar{\epsilon} \Psi^\mu, \quad (17) \]

\[ \delta \Psi^\mu = -i \rho^\alpha \partial_\alpha X^\mu \epsilon \quad (18) \]

and

\[ [\delta_1, \delta_2] \Psi^\mu = a^\alpha \partial_\alpha \Psi^\mu. \quad (19) \]

The nonvanishing equal time commutator and anticommutators are

\[ [\partial_\pm X^\mu(\sigma, \tau), \partial_\pm X^\nu(\sigma', \tau)] = \pm \frac{\pi}{2} \eta^{\mu\nu} \delta'(\sigma - \sigma') \quad (20) \]

and

\[ \{\psi^\mu_A(\sigma, \tau), \psi^\nu_B(\sigma', \tau)\} = \pi \delta(\sigma - \sigma') \eta^{\mu\nu} \delta_{AB}. \quad (21) \]

Even though

\[ \{\phi^\mu_A(\sigma, \tau), \phi^\nu_B(\tau)\} = \pi \delta(\sigma - \sigma') \eta^{\mu\nu} \delta_{AB}, \quad (22) \]

there are no ghost quanta other than \( \mu = \nu = 0 \) due to a negative creation operator phase.

Also we have,

\[ \{\Psi^\mu_A(\sigma, \tau), \Psi^\nu_B(\sigma', \tau)\} = \pi \delta(\sigma - \sigma') \eta^{\mu\nu} \delta_{AB}. \quad (23) \]

The Noether super-current is

\[ J_{\alpha} = \frac{1}{2} \rho^\beta \rho_\alpha \Psi^\mu \partial_\beta X_\mu. \quad (24) \]

We now follow the standard procedure. The lightcone components of the current and energy momentum tensors are

\[ J_+ = \partial_+ X_\mu \Psi^\mu_+, \quad (25) \]

\[ J_- = \partial_- X_\mu \Psi^\mu_-, \quad (26) \]

\[ T_{++} = \partial_+ X^\mu \partial_+ X_\mu + \frac{i}{2} \psi^\mu_+ j \partial_+ \psi^\mu_+ j + \frac{i}{2} \phi^\mu_+ k \partial_+ \phi^\mu_+ k, \quad (27) \]

and

\[ T_{--} = \partial_- X^\mu \partial_- X_\mu + \frac{i}{2} \psi^\mu_- j \partial_- \psi^\mu_- j + \frac{i}{2} \phi^\mu_- k \partial_- \phi^\mu_- k. \quad (28) \]

where \( \partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma) \).

Next we calculate the following two commutators at equal \( \tau \),

\[ [T_{++}(\sigma), T_{++}(\sigma')] = i \pi \delta'(\sigma - \sigma') \left( T_{++}(\sigma) + T_{++}(\sigma') \right), \quad (29) \]
and
\[
[T_+(\sigma), J_+(\sigma')] = i\pi\delta'(\sigma - \sigma') \left( J_+(\sigma) + \frac{1}{2} J_+(\sigma') \right).
\] (30)

To satisfy the Jacobi Identity
\[
[T_+(\sigma), \{ J_+(\sigma'), J_+(\sigma'') \}] = \{ [T_+(\sigma), J_+(\sigma')], J_+(\sigma'') \} + (\sigma' \leftrightarrow \sigma'').
\] (31)

We use \( \delta(\sigma - \sigma')\delta'(\sigma - \sigma'') + \delta'(\sigma - \sigma')\delta(\sigma - \sigma'') = \delta(\sigma' - \sigma'')\delta'(\sigma - \sigma') \) and verify that
\[
\{ J_+(\sigma), J_+(\sigma') \} = \pi\delta(\sigma - \sigma')T_+(\sigma).
\]

Similarly, the energy momentum tensor equations (27) and (28) can be cast in the form containing \( \Psi \)'s only and the current anticommutators are deduced easily and directly without using the Jacobi Identity. The algebra closes.

We also obtain the component constraints
\[
\partial_\pm X_\mu \psi^\mu_{\pm,j} = \partial_\pm X_\mu \epsilon^j_\psi \Psi^\mu_{\pm} = 0, \ j = 1, 2, \cdots 6
\]
and
\[
\partial_\pm X_\mu \phi^{\mu,k}_{\pm} = \partial_\pm X_\mu \epsilon^{k}_\phi \Psi^\mu_{\pm} = 0, \ k = 1, 2, \cdots 5
\] (35)

These subsidiary postulates enable us to construct null physical states eliminating all negative norm states. It also follows that for the two groups of fermions, the two Lorentz and group invariant constraint equations are
\[
\delta_{\pm} X_\mu e^j_\psi \psi^\mu_{\pm,j} = 0
\] (36)
and
\[
\delta_{\pm} X_\mu e^k_\psi \phi^{\mu,k}_{\pm} = 0
\] (37)

### III. LOCAL TWO DIMENSIONAL AND FOUR DIMENSIONAL SUPERSYMMETRY

In view of equations (16) to (17), \((X^\mu, \Psi^\mu)\) behave like a supersymmetric pair. In addition to this pair, we introduce a ‘zweibein’ \( e_\alpha(\sigma, \tau) \) and its supersymmetric partner \( \chi_\alpha \) into the theory. This \( \chi_\alpha \) is a two dimensional spinor as well as a two dimensional vector. Following reference [9], we deduce the local 2D supersymmetric action.

\[
S = -\frac{1}{2\pi} \int d^2 \sigma \ e \left( h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\mu - i(\bar{\psi}^\mu_{\pm,j} \rho^\alpha \partial_\alpha \psi_{\mu,j} + \bar{\phi}_{\pm,k}^{\mu,k} \rho^\alpha \partial_\alpha \phi_{\mu,k}) \right.
+ \left. 2\bar{\chi}_\alpha \rho^\beta \Psi^\mu_\beta \partial_\beta X^\mu + \frac{1}{2} \bar{\Psi}^\mu_\beta \tilde{\chi}_\alpha \rho^\beta \chi_\alpha \right) \] (38)
which is invariant under local supersymmetric transformations

\[ \delta X^\mu = \bar{\epsilon} \Psi^\mu, \delta \psi^\mu = -i \rho^\alpha \epsilon (\partial_\alpha X^\mu - \bar{\Psi}^\mu \chi^\alpha), \]

and

\[ \delta e_\alpha = -2i \bar{\epsilon} \rho^\alpha \chi^\alpha, \delta \chi^\alpha = \nabla^\alpha \epsilon. \] (39)

The other transformations which leave this action invariant as listed in references [9] and [17]. Variation of the action with respect to \( e_\alpha \) leads to the vanishing of the energy momentum tensor while the variation with respect to \( \chi^\alpha \) gives \( J^\alpha = 0 \). The postulates made in equation (34) are now derived from a gauge principle. However, this action is not invariant under the four dimensional space-time supersymmetry.

In four dimensions, the Green-Schwarz action for \( N = 1 \) local supersymmetry is worked out in references [10] and [18].

\[ S = \frac{1}{2\pi} \int d^2 \sigma \left( \sqrt{g} g^{\alpha \beta} \Pi_\alpha \cdot \Pi_\beta + 2i \epsilon^{\alpha \beta} \partial_\alpha X^\mu \bar{\theta} \Gamma_\mu \partial_\beta \theta \right), \] (40)

where

\[ \Pi^\mu_\alpha = \partial_\alpha X^\mu - i \bar{\theta} \Gamma^\mu \partial_\alpha \theta. \] (41)

and where the \( \theta \) is a genuine space-time fermion fields rather than spacetime vectors. In our case they are \( D = 4 \), four component spinors of \( SO(3,1) \), constructed like the \( \Psi^\mu \). \( \Gamma^\mu \) are the four dimensional Dirac gamma matrices. This action is invariant under global supersymmetry

\[ \delta \theta = \epsilon \]

and

\[ \delta X^\mu = i \bar{\epsilon} \Gamma^\mu \epsilon \] (42)

provided

\[ \Gamma_\mu \Psi_1 \bar{\psi}_2 \Gamma^\mu \Psi_3 = 0 \] (43)

and this is satisfied for our Majorana or Weyl spinors in four dimensions [11].

Consider a supersymmetric parameter \( \kappa^{\alpha \bar{\alpha}} \) where \( \alpha \) represents a two dimensional vector index and ‘\( \alpha \)’ is a spinorial index. It has been shown in detail in reference [10], that the action is invariant under \( \kappa \) transformation wherever global \( \epsilon \) symmetry invariance exists as in four cases listed in reference [9].

The transformations are

\[ \delta \theta = 2i \Gamma \cdot \Pi_\alpha \kappa^{\alpha} \] (44)

and

\[ \delta X^\mu = i \bar{\theta} \Gamma^\mu \delta \theta. \] (45)

Thus the action (40) is obviously spacetime supersymmetric in four dimensions.

The major defect with this GS action is that the naive covariant quantisation procedure does not work. Only in the light cone gauge, things are much simpler and the theory can be quantised. If one wishes to proceed with a covariant formulation, one has no other choice but to implement the NS-R scheme with GSO projection which is simple and elegant and also equivalent. It can be covariantly quantised and the critical central charge can be calculated. We follow this scheme in the following sections.
IV. QUANTIZATION

As usual the theory is quantized \((\alpha_\mu^\nu = p^\mu)\), with

\[
X^\mu = x^\mu + p^\mu \tau + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos(n\sigma),
\]

or

\[
\partial_\pm X^\mu = \frac{1}{2} \sum_{n=0}^{+\infty} \alpha_n^\mu e^{-in(\tau \pm \sigma)}
\]

(46)

and

\[
[\alpha_\mu, \alpha_\nu^\alpha] = m \delta_{m, -n} \eta^{\mu \nu}.
\]

(47)

We mention in passing that for closed strings

\[
\partial_- X^R_\mu = \sum_{n=0}^{+\infty} \alpha_n^\mu e^{-2in(\sigma - \tau)}
\]

(48)

and

\[
\partial_+ X^L_\mu = \sum_{n=0}^{+\infty} \tilde{\alpha}_n^\mu e^{-2in(\sigma + \tau)}
\]

(49)

The entire algebra can be made applicable to closed string as well.

We first choose the Neveu-Schwarz (NS) boundary condition. Then the mode expansions of the fermions are

\[
\psi_{\mu, j}^{\pm}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_{r, j}^\mu e^{-ir(\tau \pm \sigma)}
\]

(50)

and

\[
\phi_{\mu, k}^{\pm}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in \mathbb{Z} + \frac{1}{2}} b_{r, k}^\mu e^{-ir(\tau \pm \sigma)}
\]

(51)

with real quanta \(b^\prime\), the quantisation is like

\[
\phi_{\mu}^{\pm}(\sigma) = \frac{i}{\sqrt{2}} \sum_{r>0} (b_{r, j}^\mu e^{-ir\sigma} - b_{r, j}^{\mu *} e^{+ir\sigma})
\]

so that \(\{b_{r, j}^\mu, b_{r, j}^{\mu *}\} = \eta^{\mu \nu} \delta_{r, s}\) and similar relations follow. We also have

\[
\Psi_{\mu}^{\pm}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{r \in \mathbb{Z} + \frac{1}{2}} B_{r}^\mu e^{-ir(\tau \pm \sigma)}
\]

(52)

with \(b_{r, j}^\mu = e_{\psi, j} B_{r, j}^\mu, \quad b_{r, k}^{\mu *} = ie_{\phi, k} B_{r, k}^\mu, \quad B_{r}^\mu = e_{\psi} B_{r, j}^\mu + ie_{\phi} b_{r, k}^{\mu *}.\)

(53)

The sum is over all the half-integer modes. The anticommutation relations are

\[
\{b_{r, j}^\mu, b_{r, j}^{\mu *}\} = \eta^{\mu \nu} \delta_{j, j'}, \delta_{r, -s},
\]

(54)

\[
\{b_{r, k}^{\mu}, b_{r, k}^{\mu *}\} = \eta^{\nu \nu} \delta_{k, k'}, \delta_{r, -s}
\]

(55)

and

\[
\{B_{r}^\mu, B_{s}^{\nu}\} = \eta^{\mu \nu} \delta_{r, -s}.
\]

(56)
V. VIRASORO ALGEBRA

Virasoro generators \[12\] are given by the modes of the energy momentum tensor \(T_{++}\) and the Noether current \(J_+\),

\[
L^M_m = \frac{1}{\pi} \int_{-\pi}^{+\pi} \, \text{d} \sigma \, e^{in\sigma} \, T_{++},
\]

(57)

and

\[
G^M_r = \frac{\sqrt{2}}{\pi} \int_{-\pi}^{+\pi} \, \text{d} \sigma \, e^{ir\sigma} \, J_+.
\]

(58)

‘\(M\)’ stands for matter. In terms of creation and annihilation operators

\[
L^M_m = L^{(\alpha)}_m + L^{(b)}_m + L^{(b')}_m,
\]

(59)

where

\[
L^{(\alpha)}_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} : \alpha_{-n} \cdot \alpha_{m+n} :,
\]

(60)

\[
L^{(b)}_m = \frac{1}{2} \sum_{r=-\infty}^{\infty} (r + \frac{1}{2} m) : b_{-r} \cdot b_{m+r} :,
\]

(61)

\[
L^{(b')}_m = \frac{1}{2} \sum_{r=-\infty}^{\infty} (r + \frac{1}{2} m) : b'_{-r} \cdot b'_{m+r} :,
\]

(62)

In each case normal ordering is required. The single dot implies the sum over all qualifying indices. The current generator is

\[
G_r = \sum_{n=\infty} \alpha_n \cdot B_{r+n}.
\]

(63)

Following from the above equations, the Virasoro algebra is

\[
[L^M_m, L^M_n] = (m-n)L^M_{m+n} + A(m) \delta_{m,-n}
\]

(64)

Using the relations

\[
[L^M_m, \alpha^\mu_n] = -n \alpha^\mu_{n+m},
\]

(65)

\[
[L^M_m, B^\mu_n] = -(n + \frac{m}{2})B^\mu_{n+m},
\]

(66)

we get

\[
[L^M_m, G^M_r] = \left( \frac{1}{2} m - r \right) G^M_{m+r}.
\]

(67)

The anticommutator \(\{G^M_r, G^M_s\}\) is obtained by the use of the Jacobi Identity

\[
[\{G^M_r, G^M_s\}, L^M_m] + \{[L^M_m, G^M_r], G^M_s\} + \{[L^M_m, G^M_s], G^M_r\} = 0,
\]

(68)

which implies, consistent with equations (61) and (64),

\[
\{G^M_r, G^M_s\} = 2L^M_{r+s} + B(r)\delta_{r,-s}
\]

(69)
$A(m)$ and $B(r)$ are normal ordering anomalies. Also note that $L^+_m = L_{-m}$ and $G^+_r = G_{-r}$. Taking the vacuum expectation value in the Fock ground state $|0,0\rangle$ with four momentum $p^\mu = 0$ of the commutator $[L_1, L_{-1}]$ and $[L_2, L_{-2}]$, it is easily found that

$$A(m) = \frac{26}{12}(m^3 - m) = \frac{C}{12}(m^3 - m), \quad (70)$$

and using the Jacobi Identity

$$B(r) = \frac{A(2r)}{2r} \quad (71)$$

and

$$B(r) = \frac{26}{3}\left(r^2 - \frac{1}{4}\right) = \frac{C}{3}\left(r^2 - \frac{1}{4}\right) \quad (72)$$

The central charge $C = 26$. This is what is expected. Each bosonic coordinate contribute 1 and each fermionic ones contribute $1/2$, so that the total central charge is +26. The central charge can also be calculated from the leading divergency of the vacuum expectation value of the product of two energy momentum tensors at two world sheet points $x$ and $w$. Since

$$\langle X^\mu(z), X^\nu(w) \rangle \sim \eta^{\mu\nu} \log(z - w)$$

and

$$\langle \psi^{ij}(z), \psi^{ij'}(w) \rangle \sim \eta^{\mu\nu} \delta^{ij'}(z - w)^{-1}$$

and

$$\langle \phi^{jk}(z), \phi^{j'k'}(w) \rangle \sim \eta^{\mu\nu} \delta_{j'k'}(z - w)^{-1},$$

we deduce that, $2\langle T_+(z)T_+(w) \rangle \sim C(z - w)^{-4} + \cdots$ where

$$C = \eta^\mu_\mu + \frac{1}{2}\eta^\mu_j \delta^j_j + \frac{1}{2}\eta^\nu_k \delta^k_k = 26.$$

VI. BOSONIC STATES AND ELIMINATION OF GHOSTS

A physical bosonic state $\Phi$ can be conveniently constructed by operating the generators $L$’s and $G$’s on the vacuum. They satisfy

$$(L^M_0 - 1) | \Phi \rangle = 0, \quad (73)$$

$$L^M_m | \Phi \rangle = 0, \quad m > 0, \quad (74)$$

and

$$G^M_r | \Phi \rangle = 0, \quad r > 0 \quad (75)$$

These physical state conditions enable to exclude the time-like quanta from the physical spectrum. Before doing this rigorously, we first follow the qualitative arguments of Green, Schwarz and Witten [10]. Specialising to a rest frame, we write the physical condition for $L_m$ as

$$\frac{1}{2}p^0 \alpha^0_m | \Phi \rangle + \text{ (terms quadratic in oscillators) | } \Phi \rangle = 0. \quad (76)$$
In this frame, the physical states are generated effectively by the space components of the oscillators only; so that 
\( o^0_m \mid \Phi \rangle = 0 \) following from the constraint that the energy momentum tensor vanishes. Using the condition for the generator \( G_r \),
\[
\left[ G^M_r, o^0_m \right] \mid \Phi \rangle = m \left[ B^0_{m+r} \mid \Phi \rangle = 0. \right.
\]
(77)
Since \( B^0_{m+r} \mid \Phi \rangle = 0, \ B^0_r \mid \Phi \rangle = 0 \) as \( B^0_{m+r} \) anticommutes with \( B^0_r \). Specialising to \( \mu = 0 \), we arrive at the intuitive result from equation (53)
\[
b^{0,j}_r \mid \Phi \rangle = 0
\]
(78)
and
\[
b^{0,k}_r \mid \Phi \rangle = 0.
\]
(79)
Thus the vanishing of the energy-momentym tensor and the current excludes all the time like quanta from the physical space. No negative norm state is expected to show up in the physical spectrum.

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The norm of the first term is equal to $-11$ as calculated in reference [10].

Noting that $L^{(b)}_{-1} | 0, p \rangle = L^{(b')}_{-1} | 0, p \rangle = 0$; $L^{(b)}_{-2} = \frac{1}{2} b_{-3/2} b_{-1/2}$ and $L^{(b')}_{-2} = \frac{1}{2} b'_{-3/2} b'_{-1/2}$ the norms of the second and third terms are $\frac{1}{4} (\delta_{\mu\nu} \delta_{jj}) = 6$ and $\frac{1}{4} (\delta_{\mu\nu} \delta_{kk}) = 5$ respectively. The norm of the state given above is $-11 + 6 + 5 = 0$

The ghosts can be eliminated from the tree amplitudes in superstrings in the RNS formulation. The proof given in reference 9, is easily adapted for this string as well. We shall mention certain differences in this superstring. The tree amplitude is

$$A_M = g^{M-2} \langle \phi | V(2) \Delta V(3) \cdots \Delta V(M-1) | \Phi_M \rangle.$$  

$| \phi \rangle$, $V$ and $\Delta$ are suitably chosen to eliminate ghosts. $V$, the vertex operator function should have conformal weight 1. For physical states $(L_0 - 1 | \Phi) = 0$ is the wave equation, so that $\Delta = (L_0 - 1)^{-1}$ is the propagator. This is similar to the $F_1$ picture in the usual superstring theory.

We have to show that

$$\langle \tilde{\chi} | L_n V(2) \Delta V(3) \cdots \Delta (M-1) | \Phi_M \rangle = 0,$$

where $\langle \tilde{\chi} |$ is any state satisfying $(L_0 - 1 + n) | \tilde{\chi} \rangle = 0$.

We need two identities

$$[L_n - L_0 - n + 1] V = V[L_n - L_0 + 1],$$

and

$$[L_n - L_0 + 1] \frac{1}{L_0 - 1} = \frac{1}{L_0 + n - 1} [L_n - L_0 - n + 1].$$

$L_n$ which is in effect the same as $L_n - L_0 - n + 1$, can be pused to the right till the factor at the end. $(L_n - L_0 + 1) | \Phi_M \rangle$ is zero.

The $F_2$ like picture is very illuminating from the point of view of this superstring. As observed already constructing a new state $\tilde{\phi}$ through $\tilde{\phi} = G_{-1/2} \tilde{\Phi}$, we obtain $(L_0 - 1/2) | \tilde{\phi} \rangle = 0$. If we supplement this with a physical state like condition $G_{1/2} | \tilde{\Phi} \rangle = 0$, we deduce that $G_{1/2} | \Phi \rangle = | \Phi \rangle$ and $G_r | \Phi \rangle = G_r | \tilde{\Phi} \rangle = 0$ for $r \geq \frac{3}{2}$. As a consequence, the above tree amplitude of the $F_1$ like picture becomes

$$A_M = g^{M-2} \langle \tilde{\Phi} | G_{1/2} V(2) \tilde{\Delta} V(3) \cdots \tilde{\Delta} V(M-1) | \tilde{\Phi}_M \rangle$$

$$= g^{M-2} \langle \tilde{\Phi} | V(2) \tilde{\Delta} V(3) \cdots \tilde{\Delta} V(M-1) | \tilde{\Phi}_M \rangle,$$

where the $\tilde{\Delta} = (L_0 - \frac{1}{2})^{-1}$ is the new propagator.

Now we have to prove

$$\langle \tilde{\chi} | G_r V(2) \tilde{\Delta} V(3) \cdots \tilde{\Delta} V(M-1) | \tilde{\Phi}_M \rangle = 0,$$

where the state $\langle \tilde{\chi} |$ is such that it is annihilated by $L_0 - \frac{1}{2} + r$. Now we have to move $G_r$ to the right. We introduce an operator $W$ of conformal weight $\frac{1}{2}$ such that $V = [G_r, W]$ and $[G_r, V] = [L_{2r}, W]$. However, the commutator can be replaced by

$$[L_{2r} - L_0 - r + \frac{1}{2}] W - W [L_{2r} - L_0 + \frac{1}{2}] = 0.$$

(87)

Thus $G_r$ is just switched over to the right of $V$. Using

$$G_r \frac{1}{L_0 - \frac{1}{2}} = \frac{1}{L_0 + r - \frac{3}{2}} G_r,$$  

(88)

we can bring past the $V$ and $\tilde{\Delta}$'s till it annihilates against $| \tilde{\Phi}_M \rangle$.

Eventhough, it can be easily shown that the ghosts are eliminated from the trees, it is less obvious for the loops. It is contended that, for the loops, it should be done on a case to case basis.
VII. CONFORMAL GHOSTS

For obtaining a zero central charge so that the anomalies cancel out and natural ghosts are isolated, Faddeev-Popov (FP) ghosts \cite{13} will be introduced. The FP ghost action is

\[ S_{FP} = \frac{1}{\pi} \int (c^+ \partial_- b_{++} + c^- \partial_+ b_{--}) d^2\sigma, \] (89)

where the ghost fields \( b \) and \( c \) satisfy the anticommutator relations

\[ \{b_{++}(\sigma, \tau), c^+ (\sigma', \tau)\} = 2\pi \delta(\sigma - \sigma') \] (90)

and

\[ \{b_{--}(\sigma, \tau), c^- (\sigma', \tau)\} = 2\pi \delta(\sigma - \sigma') \] (91)

and are quantized with the mode expansions

\[ c^\pm = \sum_{-\infty}^{\infty} c_n e^{-in(\tau \pm \sigma)} \] (92)

and

\[ b_{\pm \pm} = \sum_{-\infty}^{\infty} b_n e^{-in(\tau \pm \sigma)} \] (93)

The canonical anticommutator relations for \( c_n \)'s and \( b_n \)'s are

\[ \{c_m, b_n\} = \delta_{m, -n}, \quad \{c_m, c_n\} = \{b_m, b_n\} = 0 \] (94)

Deriving the energy momentum tensor from the action and making the mode expansion, the Virasoro generators for the ghosts (G) are

\[ L_m^G = \sum_{n=-\infty}^{\infty} (m - n) b_{m+n} c_{-n} - a \delta_{m,-n} \] (95)

where \( a \) is the normal ordering constant. These generators satisfy the algebra

\[ [L_m^G, L_n^G] = (m - n) L_{m+n}^G + A^G(m) \delta_{m, -n} \] (96)

The anomaly term is deduced as before and give

\[ A^G(m) = \frac{1}{6} (m - 13m^3) + 2a \] (97)

With \( a = 1 \), this anomaly term becomes

\[ A^G(m) = -\frac{26}{12} (m^3 - m), \quad B^G(r) = -\frac{26}{3} \left( r^2 - \frac{1}{4} \right) \] (98)

The central charge is \(-26\) and cancels the normal order A\((m)\) and B\((r)\) of the\( L \) and G generators. Since \( G^{gh}_r \), the ghost current gauge factor has conformal weight \( \frac{3}{2} \),

\[ [L_m^G, G^{gh}_r] = (m/2 - r) G^{gh}_{m+r} \] (99)
From the Jacobi Identity
\[ \{G_r^{gh}, G_s^{gh}\} = 2L_r^G + \delta_{r,-s}B^G(r). \] (100)

It immediately follows that
\[ G_r^{gh} = L_r^G. \] (101)

In practice, the products of even number of $G_r^{gh}$’s occur in calculations and they can be evaluated in terms of $L_r^G$’s.

The total current generator is
\[ G_r = G_r^M + G_r^{gh}, \] (102)

and we have the anomaly free Super Virasoro algebra,
\[ [L_m, L_n] = (m - n)L_{m+n}, \] (103)
\[ [L_m, G_r] = (m/2 - r)G_{r+m} \] (104)
and
\[ [G_r, G_s] = 2L_{r+s} \] (105)

Thus from the usual conformal field theory we have obtained the algebra of a superconformal field theory. This is the novelty of the proposed string. The BRST [14] charge operator taking the case of constraints (73) is
\[ Q_1 = \sum_{-\infty}^{\infty} L_m^M c_m - \frac{1}{2}\sum_{-\infty}^{\infty}(m - n) : c_{-m} c_{-n} b_{m+n} : -a c_0. \] (106)

and is nilpotent for $a = 1$. This is the open bosonic string.

The absence of the need to use superconformal ghosts may appear puzzling. But we note that the light cone gauge is ghost free. Dropping the helicity suffixes, the light cone vectors
\[ \psi_j^\pm = \frac{1}{\sqrt{2}}(\psi_j^0 \pm \psi_j^3), \quad \phi_k^\pm = \frac{1}{\sqrt{2}}(\phi_k^0 \pm \phi_k^3), \] (107)

have the anticommutators with negative signs.
\[ \{\psi_j^+(\sigma), \psi_j^-(\sigma')\} = -\delta_{j,j'}\pi\delta(\sigma - \sigma'); \{\phi_k^+(\sigma), \phi_k^-(\sigma')\} = -\delta_{k,k'}\pi\delta(\sigma - \sigma'). \] (108)

The total ghost energy momentum tensor comes from the $(0,3)$ coordinates and is given by
\[ T^{gh}(z) = i 2 \left( \psi_j^0 \delta_j^j \psi_j^0 + \psi_j^3 \delta_j^j \psi_j^3 \right) + \frac{i}{2} \left( \phi_k^0 \delta_k^j \phi_k^0 + \phi_k^3 \delta_k^j \phi_k^3 \right). \] (109)

But the correlation function
\[ \langle T^{gh}(z), T^{gh}(\omega) \rangle = \frac{11}{2} \frac{1}{(z - \omega)^2} + \cdots \] (110)

Thus the contribution of these ghosts to the central charge 26 is 11. The remaining charge 15 is the same as for a normal ten dimensional superstring. See appendix for a study of equivalence with FP action. With introduction of FP($\beta, \gamma$) ghosts, the BRST charge changes by
\[ Q'_{BRST} = \sum \gamma_{-r} G_r - \sum \gamma_{-r} \gamma_{-s} b_{r+s}, \] (111)
in the NS sector and similarly for the R sector with integral $r$. This takes care of the current constraints.
VIII. SUPERCONFORMAL GHOSTS

The fermionic light cone part of the action can be written as

\[ S_{L.c}^F = -\frac{i}{\pi} \int d^2 \sigma \bar{\Psi}^+ \rho^a \partial_a \Psi^- \]  \hspace{1cm} (112)

We supplement this Hilbert space by a space where the fermi particles satisfy bose statistics. This is done here by letting \( \bar{\Psi}^+ \) and \( \Psi^- \) depend on Grassman variable \( \bar{\theta} \) and \( \theta \) so that the action

\[ S_{L.c}^F = -\frac{i}{\pi} \int d^2 \sigma \int d\bar{\theta} \int d\theta \bar{\Psi}^+(\bar{\theta}) \rho^a \partial_a \Psi^-(\theta). \]  \hspace{1cm} (113)

Expanding the fields as

\[ \bar{\Psi}(\bar{\theta}) = \cdots + \bar{\theta} \gamma + \cdots \]  \hspace{1cm} (114)

and

\[ +2i \rho^a \Psi^-(\theta) = \cdots + \theta \beta^a + \cdots \]  \hspace{1cm} (115)

We obtain the Superconformal ghost action [10] as

\[ S_{L.c}^F = -\frac{1}{2\pi} \int d^2 \sigma \bar{\gamma} \partial_a \beta^a \]  \hspace{1cm} (116)

The rest is standard. The wave equations are \( \partial \gamma = \partial \beta = 0 \). The energy momentum tensor is

\[ T_{++} = -\frac{1}{4} \gamma \partial_+ \beta - \frac{3}{4} \beta \partial_+ \gamma. \]  \hspace{1cm} (117)

In the quantised form

\[ \gamma(\tau) = \frac{1}{\sqrt{2}} \sum_n \gamma_n(r) e^{-2i\pi n(r)} \]

\[ \beta(\tau) = \frac{1}{\sqrt{2}} \sum_n \beta_n(r) e^{-2i\pi n(r)} \]

for integral \( n \) or half integral \( r \). The nonvanishing relation is \( [\gamma_m, \beta_n] = \delta_{m,-n} \). \( L_{L.c}^M \) which is contained in \( L_n^M \) is now (Ramond)

\[ L_{L.c}^M = \sum_n (n + m/2) : \beta_{m-n} \gamma_n : \]  \hspace{1cm} (118)

In component notations

\[ L_{L.c}^M = \sum_n (n + m/2) \left( \beta_{m-n}^j \gamma_{n,j} + \beta_{m-n}^k \gamma_{n,k} \right). \]  \hspace{1cm} (119)

The conformal dimension of \( \gamma \) is ‘-1/2’ and \( \beta \) is ‘3/2’ as can be deduced from [113],

\[ [L_m^M, \gamma_n] = (-3/2 - n) \gamma_{m+n} \]  \hspace{1cm} (120)

\[ [L_m^M, \beta_n] = (3/2 - n) \beta_{m+n} \]  \hspace{1cm} (121)

The part of the BRST charge which takes care of the current generator constraints[123], for instance, is

\[ Q' = \sum_r G_r \gamma_r - \sum_{rs} \gamma_s \gamma_r b_{s+r}. \]  \hspace{1cm} (122)
Similarly for Ramond sector.
The total BRST charge is then
\[ Q_{BRST} = Q_1 + Q' \] (123)

With \( \{Q_1, Q_1\} = 0 \), it can be directly checked that \( \{Q', Q'\} + 2\{Q_1, Q'\} = 0 \) by using the Virasoro algebra, equation (120) and the following results obtained by Fourier transformation, integration by parts and field equation
\[ \sum_r \sum_s \gamma_s \gamma_r \delta r, -s = \sum_r \sum_s r^2 \gamma_s \gamma_r \delta r, -s = 0. \] (124)

Thus
\[ Q_{BRST} = 0. \] (125)

The nilpotency of \( Q_{BRST} \) proves that the model is unitary and free from anomalies and ghosts.

IX. FERMIONIC STATES

The above deductions can be repeated for Ramond sector \[15\]. We write the main equations. The mode expansion for the fermions are
\[ \psi_{\mu,j}^{\pm}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{\infty} d_{m}^{\mu,j} e^{-im(\tau \pm \sigma)} \] (126)
\[ \phi_{\mu,j}^{\pm}(\sigma, \tau) = \frac{1}{\sqrt{2}} \sum_{-\infty}^{\infty} d_{m}^{\prime \mu,j} e^{-im(\tau \pm \sigma)} \] (127)

The generators of the Virasoro operators are
\[ L_{m}^{(M)} = L_{m}^{(\alpha)} + L_{m}^{(d)} + L_{m}^{(d')} \] (128)
\[ L_{m}^{(d)} = \frac{1}{2} \sum_{n=-\infty}^{\infty} (n + \frac{1}{2} m) : d_{-n} \cdot d_{m+n} : \] (129)
\[ L_{m}^{(d')} = \frac{1}{2} \sum_{n=-\infty}^{\infty} (n + \frac{1}{2} m) : d_{-n} \cdot d_{m+n} ' : \] (130)

and the fermionic current generator is
\[ F_{m}^{M} = \sum_{-\infty}^{\infty} \alpha_{-n} \cdot D_{n+m} \] (131)

The Ramond sector Virasoro algebra is the same as the NS-sector with the replacement of G’s by F’s. It is necessary to define \( L_{0} \) suitably to keep the anomaly equations the same \[10\].

In this Ramond sector, a physical state \(| \Phi \rangle\) should satisfy
\[ F_{n} \mid \Phi \rangle = L_{n} \mid \Phi \rangle = 0 \quad \text{for} \quad n > 0 \] (132)

The normal order anomaly constant in the anticommutators of the Ramond current generators has to be evaluated with care, because the definition of \( F_{0} \) does not have a normal ordering ambiguity. So \( F_{0}^{2} = L_{0} \). Using commutation relation for the \( F \) and the Jacobi Identity we get
\[ \{F_{r}, F_{-r}\} = \frac{2}{r} \{[L_{r}, F_{0}], F_{-r}\} = 2L_{0} + \frac{4}{r} A(r) \]
So

$$B(r) = \frac{4}{r} A(r)$$ (133)

$$B(r) = \frac{C}{3} (r^2 - 1), \quad r \neq 0$$ (134)

A physical state in the fermionic sector satisfies

$$(L_0 - 1) | \Psi \rangle = 0$$ (135)

It follows that

$$(F_0^2 - 1) | \Psi \rangle = (F_0 - 1)(F_0 + 1) | \Psi \rangle = 0$$

The construction of ‘null’ physical states becomes much simpler because all $F_{-m}$ terms can be assigned to $L_{-m}$ terms by the commutation relation $F_{-m} = 2[F_0, L_{-m}]/m$ and $F_0$ has eigenvalues which are roots of eigenvalues of $L_0$ acting on the generic states or states constructed out of the generic states. Thus the zero mass null physical state with $L_0 | \tilde{\chi} \rangle = F_0^2 | \tilde{\chi} \rangle = 0$ is simply

$$| \Psi \rangle = L_{-1} | \tilde{\chi} \rangle$$ (136)

with $L_1 | \Psi \rangle = F_1 | \Psi \rangle = 0$. The next excited state with $(L_0 + 1) | \tilde{\chi} \rangle$ becomes the same as in the bosonic sector. Obtained from the condition $L_1 | \Psi \rangle = 0$,

$$| \Psi \rangle = (L_{-2} + \frac{3}{2} L_{-1}) | \tilde{\chi} \rangle$$

The norm $\langle \Psi | \Psi \rangle = (C - 26)/2$ and vanishes for $C = 26$. It is easy to check that all physical state conditions are satisfied. $F_1 | \Psi \rangle = 2 [L_1, F_0] | \Psi \rangle = 0$ since $L_1 | \Psi \rangle = 0$ and $F_0 | \Psi \rangle = | \Psi \rangle$, $L_2 | \Psi \rangle = F_1 F_1 | \Psi \rangle = 0$ and $F_2 | \Psi \rangle = [L_2, F_0] | \Psi \rangle = 0$. For $C = 26$, there are no negative norm states in the Ramond sector as well.

$$[L_m^G, F_n^{gh}] = \left( \frac{m}{2} - n \right) F_{m+n}^{gh}$$ (137)

and

$$\{ F_n^{gh}, F_m^{gh} \} = 2L_{n+m}^G + B(r) \delta_{n,-m}.$$ (138)

The ghost current anomaly constant is $B^G(r) = -\frac{26}{3} (r^2 - 1)$ from the Jacobi identity and cancels out the $B(r)$ of equation (131).

**X. THE MASS SPECTRUM**

The ghosts are not coupled to the physical states. Therefore the latter must be of the form (up to null state)

$$| \{ n \} p \rangle_M \otimes c_1 | 0 \rangle_G.$$ (139)

$| \{ n \} p \rangle_M$ denotes the occupation numbers and momentum of the physical matter states. The operator $c_1$ lowers the energy of the state by one unit and is necessary for BRST invariance. The ghost excitation is responsible for lowering the ground state energy which produces the shiftable tachyon ($F_2$ picture).

$$(L_0^M - 1) |phys \rangle = 0.$$ (140)
Therefore, the mass shell condition is
\[ \alpha' M^2 = N_B + N_{NS}^F - 1 \quad : \quad NS \] (141)

\[ \alpha' M^2 = N_B + N_R^F - 1 \quad : \quad R \] (142)

where
\[ N_B = \sum_{m=1}^{\infty} \alpha_m \alpha_{-m} \] (143)

and
\[ N_{NS}^F = \sum_{r=1/2}^{\infty} r (b_{-r} \cdot b_r + b'_{-r} \cdot b'_r) \quad : \quad NS \]
\[ N_R^F = \sum_{m=1}^{\infty} m(d_{-m} \cdot d_m + d'_{-m} \cdot d'_m) \quad : \quad R \] (144)

In general, \( \alpha' M^2 = -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots \) in the N. S. sector. In the shifted Hilbert state \( \alpha' M^2 = -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, \cdots \).

Due to the presence of Ramond and Neveu-Schwartz sectors with periodic and anti-periodic boundary conditions, we can effect a GSO projection [17] on the mass spectrum on the NSR model [18]. The desired projection is
\[ G = \frac{1}{2} (1 + (-1)^{F+F'}) \] (145)

where \( F = \sum b_{-r} \cdot b_r \) and \( F' = \sum b'_{-r} \cdot b'_r \). This will eliminate the half integral values from the mass spectrum by choosing \( G=1 \) including the tachyon at \( \alpha M^2 = -\frac{1}{2} \).

XI. MODULAR INVARIANCE

We shall follow the notation of Seiberg and Witten [19] for the spin structures and the result, given by Kaku [18], that the spin structure \( \chi(-\cdot, \tau) \) for a single fermion is given by
\[ \chi(-\cdot, \tau) = q^{-1/24} \text{Tr} q^{2 \sum_n \bar{n}_n} = q^{-1/24} \prod_{n=1}^{\infty} (1 + q^{2n-1}) = \frac{\Theta_3(\tau)}{\eta(\tau)} , \] (146)

where \( \Theta \)'s will be the Jacobi Theta functions [19], \( q = e^{i\pi \tau} \) and \( \eta(\tau)(2\pi) = \Theta_1^{1/3}(\tau) \). \( \eta(\tau) \) is the Dedekind eta function.

In the model there are three groups of oscillators, the four bosons, the twenty four unprimed fermions and twenty primed fermions. There are six constraint equations \( T_{++} = T_{--} = 0 \) and equations (36) and (37). In covariant formulation, the effective number of physical modes is the total number of modes subtracted by the number of invariant constraints on the whole assembly. There are two transverse bosonic and two ‘bosonic’ ghost modes. For the group of twenty \( (SO(5)) \), there are eighteen and for the group of twenty four \( (SO(6)) \), there are twenty two independent fermionic modes. In all there are forty of such fermions.

It is easy to construct the modular invariant partition function for the two physical bosons, namely
\[ \mathcal{P}_B(\tau) = (\text{Im} \tau)^{-2} \eta^{-2}(\tau) \bar{\eta}^{-2}(\tau) , \] (147)

as given in reference [20].
The partition function for the two bosonic ghosts can be calculated from the equations for the Hamiltonian given in reference [10].

\[ \mathcal{P}_{\text{ghost}} = \left| \frac{\Theta_2(\tau)}{\eta(\tau)} \right|^4 \]  

(148)

All that remains is to calculate the partition function of the forty fermions. To obtain a physically viable partition, we observe that the initial internal symmetry of the 44 fermions was SO(44). Then this broke down to four groups of 11 fermions, each group having the internal symmetry SO(11). SO(11) can break to the maximal subgroups as SO(6) × SO(5). We would like to perform the partition of the forty fermions in such a way that the SO(6) × SO(5) symmetry is obvious. The spinor representation of SO(6) has \(2^4 = 8\) space-time fermionic modes. Let \(b^i_{r,k}\) be the anihilation fermion quanta with the SO(6) deive spinor index ‘i’ running from 1 to 8 and the \(k\), a SO(5) vector index running from 1 to 5. The SO(6) × SO(5) invariant Hamiltonian in the NS sector is \(H_{NS}^k = \sum_{r=1}^{5} H_{NS}^k\), with \(H_{NS}^k = \sum_r b^i_{r,k} b^{\dagger}_i b_{r,k} - 8/48\). Further \([H_k, H_{k'}] = 0\), there are five replicas of the group of eight. We may consider five indentical boxes, each box containing the eight fermions. The partition function of the forty fermions will be the partition function of the eight fermions raised to the power of five.

The path integral functions of Seiberg and Witten [19] for the eight fermions in one box is

\[ A((-\), \tau) = (\Theta_3(\tau)/\eta(\tau))^4, \]  

(149)

This is normalised to one. The other three spin structures are related to the above function as given in [19],

\[ A(\(\), \tau) = A((-\), \frac{\tau}{1+\tau}) = -(\Theta_2(\tau)/\eta(\tau))^4, \]  

(150)

and

\[ A((-\), \tau) = A(\(\), \frac{1}{\tau}) = -(\Theta_4(\tau)/\eta(\tau))^4, \]  

(151)

\[ A(\(-\), \tau) = 0. \]  

(152)

The sum of all spin structures is, therefore,

\[ A(\tau) = (\Theta_3(\tau)/\eta(\tau))^4 - (\Theta_2(\tau)/\eta(\tau))^4 - (\Theta_4(\tau)/\eta(\tau))^4 \]

Consider the amplitude

\[ A_N(\tau) = \eta^4(\tau)A(\tau) = \Theta_3^4(\tau) - \Theta_2^4(\tau) - \Theta_4^4(\tau) \]

(153)

Since

\[ \Theta_3(\tau + 1) = \Theta_4(\tau), \quad \Theta_2(\tau + 1) = e^{i\pi/4}\Theta_2(\tau), \quad \Theta_4(\tau + 1) = \Theta_3(\tau), \]

we have

\[ A_N(\tau + 1) = -A_N(\tau) \]

The invariance in \(\tau \rightarrow -1/\tau\) is built in, in the ratios of the theta functions of each structure.

So the modular invariant partition function for all the forty fermions is

\[ \mathcal{P}_F(\tau) = |A(\tau)|^{10} \]

(154)

The total partition function for the model is the product integral

\[ Z = \int \mathcal{P}_B \mathcal{P}_{\text{ghost}} \mathcal{P}_F d\tau \]

We notice that due to the famous Jacobi relation, the \(A(\tau)\), as follows from equation (143) is zero. So the modular invariant partition function \(Z\) vanishes. Thus the necessary condition for space-time supersymmetry is satisfied. At the same time, the constant in the power to which ‘q’ is raised, is still, \(-\frac{27}{24} - \frac{27}{24} - \frac{40}{48} = -1\), confirming the correctness of our calculation.
XI. SUPERSYMMETRY ALGEBRA

One should examine restrictions imposed on the Fock space due to the a supersymmetric algebra. The supersymmetric charge $Q$ should be such as to reproduce equations

$$\delta X^\mu = [X^\mu, \bar{Q} \cdot \epsilon] = \bar{\epsilon} \cdot \Psi^\mu$$

and

$$\delta \Psi^\mu = [\Psi^\mu, \bar{Q} \cdot \epsilon] = -i \rho^\alpha \partial_\alpha X^\mu \epsilon$$

A simple inspection shows that

$$\bar{Q} = -\frac{i}{\pi} \int_0^\pi d\sigma \Psi^\mu \rho^\alpha \rho^0 \partial_\alpha X^\mu$$

leading to

$$Q^\dagger = -\frac{i}{\pi} \int_0^\pi d\sigma \Psi^\mu \rho^\alpha \rho^0 \partial_\alpha X^\mu$$

and

$$Q = \frac{i}{\pi} \int_0^\pi \rho^0 \rho^0 \partial_\alpha X^\mu \Psi^\mu d\sigma$$

By a somewhat lengthy calculation it is deduced that

$$\sum_\alpha \{Q^\dagger_\alpha, Q_\alpha\} = 2H$$

where $H$ is the Hamiltonian of the system. It follows that for the any state $|\Phi_0\rangle$ in the Fock space

$$\sum_\alpha |Q_\alpha \cdot \Phi_0\rangle|^2 = 2 \langle \Phi_0 | H | \Phi_0 \rangle \geq 0$$

It appears essential that the tachyonic states disappear from the physical Fock space as the physical Hamiltonian is a sum of squares of moduli of supersymmetric charges. The ground state is massless. This can be understood as follows, $(L_0 - 1)^{-1}$ being the scalar propagator the scalar tachyonic ground state energy is $(0 | (L_0^NS - 1)^{-1} | 0)$. But there are two fermionic tachyons with same energy. Considering that there is a normal ordering negative sign for the loop fermions, the contribution is $-\langle 0 | (F_0 - 1)^{-1}(F_0 + 1)^{-1} | 0 \rangle = -\langle (L_0 - 1)^{-1} | 0 \rangle$ exactly cancelling the scalar tachyonic energy. This is as it should for supersymmetric theories.

Some admissible Fock space states are

**NS eigenstates**

$$\prod_{n,\mu} \prod_{m,\nu} \{\alpha_\mu^n\} \{B^\nu_{-m}\} \ | 0\rangle$$

**R eigenstates**

$$\prod_{n,\mu} \prod_{m,0} \{\alpha_\mu^n\} \{D^\nu_{-m}\} \ | 0\rangle u$$

$u$ is a spinor. GSO projection is implied for the N-S eigenstates. Let us construct the zero mass modes. The tachyonic vacuum will be denoted by $|0\rangle$ and the zero mass ground state by $|\phi_0\rangle$.

We start with the supergravity multiplet. The ground state

$$B^\mu_{-1/2} B^\nu_{-1/2} | 0\rangle \epsilon_{\mu\nu}$$
has zero mass. Due to the physical state conditions $G_{1/2} | \phi_0 \rangle = 0$

$$p^\mu \epsilon_{\mu \nu} = p^\nu \epsilon_{\mu \nu} = 0$$  \hspace{1cm} (163)

It describes a massless antisymmetric tensor $A_{\mu \nu} = 1/2(\epsilon_{\mu \nu} - \epsilon_{\nu \mu})$, which turns out to be a pseudoscalar, a massless scalar $\epsilon_{\mu \nu}$ of spin 0 and a massless symmetric terms of spin 2: $\epsilon_{\mu \nu} = 1/2(\epsilon_{\mu \nu} + \epsilon_{\nu \mu})$, which is traceless.

The other zero mass spinorial states are

$$\alpha_{-1}^\mu | 0 \rangle u_1^\mu$$  \hspace{1cm} (164)

$$D_{-1}^\mu | 0 \rangle u_2^\mu$$  \hspace{1cm} (165)

$u_1^\mu, u_2^\mu$ are spinor four vectors and are distinguished by $[10]$

$$\gamma_5 u_1^\mu = u_1^\mu$$  \hspace{1cm} (166)

$$\gamma_5 u_2^\mu = -u_2^\mu$$  \hspace{1cm} (167)

We shall consider them together as a four component spin vector $u_\mu$. The condition $F_0 | \phi_0 \rangle = 0$, $F_1 | \phi_0 \rangle = 0$, $L_1 | \phi_0 \rangle = 0$ lead to the condition

$$\gamma \cdot p u_\mu = p^\mu u_\mu = \gamma^\mu u_\mu = 0$$  \hspace{1cm} (168)

This state contains not only a spin $3/2$ but also a spin $1/2$ state. They can be projected out. The details have been given by GSO in reference [17].

We now count the number of physical degrees of freedom:

Graviton 2 degrees of freedom: $\rho_\mu^a$

Dilaton, $\epsilon_{\mu \nu}$, 1

Antisymmetric tensor 1

Spin 3/2 2 $u_\mu$

Spin 1/2 2

The numbers of the fermions and the bosons are equal. They can be grouped together as the gravitational ($\rho_\mu^a, u_\mu$) and the matter ($A, B, u$) multiplets.

The massless ground state vector is represented by

$$\alpha_{-1}^\mu | 0 \rangle \epsilon_\mu(p)$$  \hspace{1cm} (169)

Here, because of the $L_0$ condition, $p^2 \epsilon_\mu = 0$: The constraint $L_1 | \phi_0 \rangle = 0$ gives the Lorentz condition $p \cdot \epsilon = 0$. The external photon polarisation vector can be subjected to an on shell gauge transformation $\epsilon_\mu(p) \rightarrow \epsilon_\mu(p) + \lambda p_\mu$. Therefore the state

$$p_\mu \alpha_{-1}^\mu | 0 \rangle \lambda = L_{-1} | 0 \rangle \lambda$$  \hspace{1cm} (170)

decouples from the physical system. There are only two degrees of freedom left. However, from the Ramond sector we have the spinor

$$p_\mu \alpha_{-1}^\mu | 0 \rangle u(p) = F_{-1} | 0 \rangle u(p)$$  \hspace{1cm} (171)

with $\gamma \cdot p u(p) = 0$ from the physical state condition. Further, as already noted, $\gamma_5 u(p) = u(p)$. So the member of the fermionic degrees of freedom is again two, just like the vector boson. Thus for all the zero mass states the bosonic and the fermionic degrees of freedom are equal. Before passing on to the next section, we mention that space time supersymmetry has been examined in a standard like model in reference [23].
XIII. APPROACH TO STANDARD LIKE MODEL

In this section we start by constructing the gauge and matter fields from the creation operators of the NS and R sectors.

Writing $b^\mu$ for the creation operator $b^{-1/2}_{\mu}$ of the NS sector, the massless gauge bosons $A^{\mu\nu}_{ij}$ can be obtained from the traceless field strength tensor of an adjoint representation,

$$F_{\mu\nu,ij} = \epsilon_{\mu\nu\lambda\sigma} (b^\lambda_j b^\sigma_i - b^\lambda_i b^\sigma_j) | 0 \rangle$$

$$= \partial_\mu A_{\nu,ij} - \partial_\nu A_{\mu,ij} + g (A_{\nu,ik} A_{\mu,kj} - A_{\nu,ik} A_{\mu,kj})$$  \hspace{1cm} (172)

When $i, j$ run from 1 to 11, we have the 55 of $SO(11)$; with $i, j$ running from 1 to 6 or 1 to 5, we get the 15 of $SO(6)$ or the 10 of $SO(5)$ respectively. Thus the $SO(6) \otimes SO(5)$ symmetry of action(3) is also the gauge symmetry.

To construct the massless fermionic modes of the Ramond sector, we equate the creation operator, when $d_{-1} = M_k = (d_k + d_1^k)$ for $k = 1, \cdots 5$; $\Gamma_k = (d_{k-5} - d_{k-5}^k)/i$ for $k = 6, \cdots 10$ and for $k = 11$, $\Gamma_k = (d_{11} + d_{11}^k)$. These $\Gamma$’s obey the anticommutator relation

$$\{ \Gamma_k, \Gamma_l \} = 2\delta_{k,l}$$  \hspace{1cm} (173)

and also

$$\Gamma_{11} = \Gamma_1 \cdots \Gamma_{10}$$  \hspace{1cm} (174)

Each $\Gamma$ is a $2^5 \times 2^5$ matrix and the $SO(11)$ spinors are 32 dimensional. The massless spinors are

$$\Psi_\alpha = \Gamma^k_{\alpha\beta} | 0 \rangle u^k_{\beta}, \hspace{0.5cm} \alpha, \beta = 1, \cdots 32$$  \hspace{1cm} (175)

The spinors decompose as follows: when $SO(11) \to SO(6) \times SO(5)$; 32 = $(4, 4) + (\bar{4}, 4)$ and when $SO(11) \to SU(4) \times SU(2) \times SU(2)$, 32 = $(4, 2, 1) + (\bar{4}, 2, 1) + (4, 2, 1) + (\bar{4}, 2, 1)$. Finally in the descent to the standard model

$$SO(11) \to SU(3) \times SU_L(2) \times U_Y(1)$$  \hspace{1cm} (176)

32 = $(3, 2, 1/3) + (3, 1, 4/3) + (3, 1, -2/3) + (1, 2, -1) + (1, 1, -2) + \epsilon_L$, and $\nu_R$. $\nu_R$ is an addition to the usual standard model. Thus we show that the fermionic spectrum are derivable as string excitations directly in this model. One of the main motivation of constructing this superstring is to show that the internal symmetry group makes a direct contact with the standard model which explains all available experimental data with a high degree of accuracy. As already noted, the internal symmetry group was $SO(44)$. We divided these fermions in groups of eleven where each group was characterised by a space-time index $\mu = 0, 1, 2, 3$. All the four groups are similar, but not identical. The group with $\mu = 0$ is different and physically absent. The other three groups of eleven, $\mu = 1, 2, 3$ are all identical.

$SO(6)$ and $SO(5)$ are the maximal subgroups of $SO(11)$ 22. Similarly $SU(3)$ and $U(1)$, two $SU(2)$s are the maximal subgroups of $SO(6) \equiv SU(4)$ and $SO(5)$ respectively. So by the use of Wilson lines, the gauge group $SO(11)$ can break to the Pati Salam group 24, $SU(4) \times SU_L(2) \times SU_R(2)$ and then descent to $SU(3) \times U(1) \times SU(2) \times SU(2)$ without breaking supersymmetry. Thus the string is directly related to low energy groups accessible to phenomenology. This is an interesting feature of the model. Departing from stringiness, let us record briefly some consequences of phenomenology which are not found in literature. The most convinient scheme of descending to the standard model is

$$SO(11) \longrightarrow SO(6) \times SO(5)$$

$$\downarrow M_X$$

$$SU(4) \times SU_L(2) \times SU_R(2)$$

$$\downarrow M_R$$

$$\times$$

$$\downarrow M_S$$

$$SU_C(3) \times SU_L(2) \times U_Y(1)$$  \hspace{1cm} (177)
Such a scheme and similar ones have been extensively studied \cite{25}. Invoking charge quantization \cite{25}, $SU(4)$ may be broken to $UB-L(1) \times SUc(3)$ and subsequently $UB-L(1)$ may squeeze with $SUr(2)$ to yield $Uy(1)$. Unification mass is $M_X = M_{GUT}$, the left-right symmetry breaks at $M_R$ and supersymmetry is broken at $M_S = M_{SUSY}$. The renormalisation equations for the evolution of the coupling constants are easily written down \cite{26}.

We denote $\alpha_i = g_i^2/4\pi$ where $g_i$ is the constant related to the $i^{th}$ group, $\alpha_{\Sigma} = g_{\Sigma}^2/4\pi$ where $g_{\Sigma}$ is the coupling constant at the GUT energy and $t_{XY} = \frac{1}{2\pi} \log e M_X/M_Y$. The lowest order evolution equations are

$$
\alpha_3^{-1}(M_Z) = \alpha_G^{-1} + b_3 t_{SZ} + b_{3s} t_{RS} + b_{4s} t_{XR},
$$

$$
\alpha_2^{-1}(M_Z) = \alpha_G^{-1} + b_2 t_{SZ} + b_{2s} t_{RS} + b_{2s} t_{XR},
$$

$$
\alpha_1^{-1}(M_Z) = \alpha_G^{-1} + b_1 t_{SZ} + b_{1s} t_{RS} + (\frac{2}{5} b_{4s} + \frac{3}{5} b_{2s}) t_{XR}.
$$

(178)

(179)

(180)

$b_i$ and $b_{is}$ are the well known non-susy and susy coefficients of the $\beta$-function respectively. The experimental values at $M_Z = 91.18$ GeV are taken to be \cite{25}

$$
\alpha_1^{-1} = 59.036, \alpha_2^{-1} = 29.656, \alpha_3^{-1} = 7.69
$$

(181)

To these, we add the expected string unification value

$$
M_X = M_{GUT} = M_{string} = g_{\Sigma}(5 \times 10^{17})\text{GeV}
$$

(182)

We have four unknown quantities to calculate from the four known values.

Notice that the quantities, $b_1 - 3/5b_2 = 6$, $b_{1s} - 3/5b_{2s} = 6$, $b_3 = -7$, $b_{3s} = -3$ and $b_{4s} = -6$ are independent of the required number of Higgs doublets. So we rewrite the above three equations as

$$
\alpha_1^{-1} - 3/5\alpha_2^{-1} - 2/5\alpha_3^{-1} = 8.8t_{SZ} + 7.2t_{RS}
$$

(183)

$$
\alpha_1^{-1} - 3/5\alpha_2^{-1} = 2/5\alpha_G^{-1} + 6t_{SZ} + 6t_{RS} - 2.4t_{XR}
$$

(184)

$$
\alpha_3^{-1} = \alpha_G^{-1} - 7t_{SZ} - 3t_{RS} - 6t_{XR}
$$

(185)

The solutions are $M_{SUSY} = 5 \times 10^9$ GeV, $M_R = 5 \times 10^{14}$ GeV, $M_X = 2.87 \times 10^{17}$ GeV and $g_{\Sigma}^2 = 0.566$. With the value of $M_R$ found here, the mass of the left-handed tau neutrino \cite{29} is calculated to be about $\frac{1}{25}$ eV. following references \cite{25} and \cite{28}. We have used $m_{top}(M_R) \approx 140$ GeV in the formula for the neutrino mass $m_{\nu\tau}$,

$$
m_{\nu\tau} = -\frac{m_{top}^2}{M_R}
$$

(186)

**XIV. CURVED METRIC AND EINSTEIN EQUATION**

We shall work in the orthonormal gauge where the zwebian is constant and the gravitino field vanishes. The background gravitational field will be denoted by $g_{\mu\nu}(x)$. The supersymmetric action is guessed to be \cite{30}

$$S = -\frac{1}{2} \int d^2\sigma \left\{ \left[ \partial_\alpha X^\mu \partial_\alpha X^\nu + i\psi^{\mu-j} \rho^\sigma \left( \partial_\alpha \psi^\nu_j + \Gamma^\nu_{\lambda\sigma} \partial_\lambda X^j \psi^\sigma \right) \right] \right.$$

$$
+ i\phi^{\mu-k} \rho^\alpha \left( \partial_\alpha \phi^\nu_k + \Gamma^\nu_{\lambda\alpha} \partial_\lambda X^k \phi^\sigma \right) \right\} g_{\mu\nu}(X)
$$

$$
- \frac{1}{6} R_{\mu\nu\lambda\sigma}(X) \left\{ \psi^{\mu-j} \psi^j \psi^{\nu-j} \phi_{\sigma} + \phi^{\mu-k} \phi^k \phi^{\nu-k} \phi_{\sigma} \right\}
$$

(187)
where the metric connection, the Christoffel symbol is

\[ \Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} \left( \partial_\lambda g_{\sigma\nu} + \partial_\nu g_{\sigma\lambda} - \partial_\sigma g_{\lambda\nu} \right) \]  

and the Riemann Tensor is

\[ R^\mu_{\nu\lambda\sigma} = \partial_\sigma \Gamma^\mu_{\nu\lambda} - \partial_\lambda \Gamma^\mu_{\nu\sigma} + \Gamma^\mu_{\nu\lambda} \Gamma^\nu_{\sigma\rho} - \Gamma^\mu_{\nu\rho} \Gamma^\rho_{\lambda\sigma} \]  

Indeed, we find by lengthy and tedious calculation that the above action is invariant under the following global supersymmetric transformations

\[ \delta X^\mu = \bar{\epsilon} \left( \epsilon^j \psi_j^\mu + i \epsilon^k \phi_k^\mu \right) \]

\[ \delta \psi_{\mu,j} = -i \epsilon_j^\rho \rho^\alpha \partial_\alpha X^\mu \epsilon - \Gamma^\mu_{\nu\lambda} (\bar{\epsilon} \psi_{\nu,j}^\lambda) \psi_j^\lambda \]

\[ \delta \phi_{\mu,k} = \epsilon_k^\rho \rho^\alpha \partial_\alpha X^\mu \epsilon - i \Gamma^\mu_{\nu\lambda} (\bar{\epsilon} \phi_{\nu,k}^\lambda) \phi_k^\lambda \]

We proceed to calculate the one-loop beta function or equivalently the one loop counter term by expanding the curved metric round \( X^\mu = X_0^\mu \). Replacing \( X^\mu \to X_0^\mu + X^\mu \), we have the perturbative expansion

\[ g_{\mu\nu}(X^\rho) = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\nu\lambda\kappa}(X_0^\mu) X^\lambda X^\kappa \]

Eventually the lowest order counter term is obtained by contracting two of the \( X^\mu \)'s that appear in the action equation. The quadratically divergent integrals that arise due to contraction of terms like \( \langle \partial_\lambda X^\mu \partial_\nu X^\rho \rangle \) are discarded in dimensional regularisation. The logarithmic divergence in \( 2 + \epsilon \) dimension is

\[ \langle X^\lambda(\sigma) X^k(\sigma') \rangle_{\sigma \to \sigma'} = \pi \eta^{\lambda k} \int \frac{dk^{2+\epsilon}}{(2\pi)^{2+\epsilon}} \frac{1}{k^2} \sim \frac{\eta^{\lambda k}}{2\epsilon} \]  

The infinite term in the one loop effective action is, thus,

\[ \Delta S = -\frac{1}{12\pi\epsilon} \int d^2\sigma (\partial_\lambda X^\mu \partial_\nu X^\nu + i \psi_{\mu,j}^\rho \rho^\alpha \partial_\alpha \psi_j^\nu + i \phi_{\mu,k}^\rho \rho^\alpha \partial_\alpha \phi_k^\nu) R_{\mu\nu}(X^\rho) \]

\( R_{\mu\nu}(X^\rho) \) is the Ricci tensor. Vanishing of this term which is needed for a finite theory or the vanishing of the one loop \( \beta \)-function, namely

\[ R_{\mu\nu}(X^\rho) = 0 \]

are the familiar Einstein equations of General Relativity in vacuum.

\[ \text{XV. CONCLUSION} \]

It is remarkable that we have been able to discuss physics from the Planck scale to the Kamiokonda neutrino scale within the same framework. The starting point has been a Nambu-Goto string in four dimensions to which forty four Majorana fermions in four groups have been added. The resulting string has an action which is world sheet supersymmetric. Super-Virasoro algebra for the energymomentum tensor and current generators is established. Conformal ghosts are introduced whose contributions cancel the anomalies. BRST charge is explicitly constructed. Since the expectation value of the Hamiltonian is deduced to be the sum of squares of the supersymmetric charges, the tachyons should be absent. Thus we have been able to show that the 26\( D \) ordinary string behaves like a 4\( D \) superstring. This paper is to elaborate the material contained in [31].
By the use of Wilson lines, the gauge symmetry $SO(11)$ can break to Pati Salam group and $SU_C(3) \times SU_L(2) \times SU_R(2) \times U(1)$ preserving supersymmetry. The left right symmetry and supersymmetry are broken at intermediate mass scales. By the usual see-saw mechanism, the left handed neutrino develops a small mass of about $\frac{1}{25}$ ev. Finally the descent is complete at $SU_C(3) \times SU_L(2) \times U_Y(1)$. There is no gap left between $M_{GUT}$ and $M_{string}$ by choice.

A global supersymmetric action in orthonormal gauge has been constructed in a gravitational background field. The vanishing of the one loop divergent contribution to the action require that the Ricci tensor vanishes which are the Einsteins equations of general relativity in vacuum.

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