Integrated and Robust Storage Assignment:
An E-Grocery Retailing Business Case

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Abstract

In this paper, we deal with a storage assignment problem arising in a fulfilment centre of a major European e-grocery retailer. The centre can be characterised as a hybrid warehouse consisting of a highly efficient and partially automated fast-picking area designed as a pick-and-pass system with multiple stations and a picker-to-parts area. The storage assignment problem considered in this paper comprises the decisions to select the products to be allocated to the fast-picking area, the assignment of the products to picking stations and the determination of a shelf within the assigned station. The objective is to achieve a high level of picking efficiency while respecting station workload balancing and precedence order constraints. We propose to solve this three-level problem using an integrated MILP model. In computational experiments with real-world data, we show that using the proposed integrated approach yields significantly better results than a sequential approach in which the selection of products to be included in the fast-picking area is solved before assigning station and shelf. Furthermore, we provide an extension to the integrated storage assignment model that explicitly accounts for within-week demand variation. In a set of experiments with day-of-week-dependent demands we show that while a storage assignment that is based on average demand figures tends to exhibit a highly imbalanced workload on certain days of the week, the augmented model yields robust storage assignments that are well balanced on each day of the week without compromising the quality of the solutions in terms of picking efficiency.

Keywords: retailing, e-grocery, storage assignment, demand variation, robustness.
1 Introduction

In e-grocery retailing, customers order grocery products online which are then directly delivered at a date and time period determined by the customer. In the recent years, the e-grocery business experienced phases of dynamic growth: For example, following [Hubner et al. (2019)], online sales of food and beverages were expected to grow by 10% to 18% on average per year in Europe, the US and China from 2018 to 2021. This growth was further amplified by the Covid-19 pandemic starting in 2020.

Many of the key players in the e-grocery business are omnichannel grocers having their roots in classical stationary grocery retailing, see [Wollenburg et al. (2018)] for a review of the transition from “brick-and-mortar” to a “brick-and-clicks” grocery retailing and the implications for the underlying logistics networks. When grocers started their e-grocery business, they predominantly resorted to so-called in-store picking where the online orders are picked in existing brick-and-mortar stores located close to the customer from which they are then delivered to the customer. While many companies still resort to this picking strategy in rural regions, it is not well suited to deal with the increasing volume of e-grocery purchases in large cities or metropolitan regions. As a consequence, aiming at increasing the efficiency of the picking process, which according to [Tompkins et al. (2010)] accounts for about 55% of the total warehousing costs, major e-grocery retailers started to establish dedicated warehouses, so-called fulfilment centres or “dark stores”, that are only used for picking e-grocery orders.

The process of warehousing is a key challenge for almost all retailers [Gu et al. (2007)], and a suitable configuration of the warehouse depends on the assortment of the retailer under consideration, characteristics of stock keeping units (SKUs), as well as customer expectations, such as very high service level targets of 97-99% [Ulrich et al. (2021)] and short delivery times, with some retailers even starting to offer same-day delivery. In e-grocery, most retailers offer an assortment of about 12,000 to 15,000 SKUs, some of which have special storage requirements such as the need for refrigeration. At the same time, an average order includes about 30 to 40 order lines. Observe that the need for short delivery times also arises in classical (non-grocery) e-commerce, however, a key difference is the number of lines per customer order: In classical e-commerce, each customer order comprises only a few lines – as reported by, for example, Amazon Germany 1.6. This difference has a huge impact on the design and operation of a
warehouse to the extent that the recent review on warehousing for e-commerce (Boysen et al., 2019) explicitly excludes the e-grocery business.

In this paper, we deal with scientific decision support for a storage assignment problem arising in a fulfilment centre of a major European e-grocery retailer. While the retailer operates various fulfilment centres with different designs and degrees of automation, the one most recently established can be characterised as a hybrid or parallel warehouse in which a part of the assortment is allocated to a traditional picker-to-parts area. At the same time, the other SKUs are allocated to a partially automated area, where boxes sequentially move between stations within a so-called picking loop. At each station, a picker pulls the SKUs for a given customer from a shelve and places them into this box. In more detail, this configuration can be classified as a pick-and-pass system (Jane, 2000; Pan and Wu, 2009).

For this fulfilment centre, we consider an integrated storage assignment problem involving three hierarchically related decisions: The first decision is to determine the subset of SKUs to be handled in the picking loop, the second is to determine the station of the picking loop to which a SKU is assigned, and the third one is to determine the shelf within the corresponding station. With the objective of establishing a high picking efficiency, the main goals associated with these decisions are to allocate SKUs with a high picking frequency to the loop, to keep the workload among the stations balanced, and to allocate high-demand SKUs close to the pickers in the stations. In addition, the storage assignment needs to respect aspects such as the need for keeping space between SKUs located next to each other and precedence order constraints to ensure that heavy SKUs do not damage fragile ones in an order box.

As observed by Boysen et al. (2019) and many other authors, demand variation is one of the key challenges to be considered in almost every high-performance retail warehouse. If the demand for SKUs changes due to seasonal impacts or by long-term trends, maintaining a high level of picking efficiency typically requires to adapt the storage assignment by rearranging the storage locations of the SKUs. In case of short-term demand variations such as day-of-week dependent demand for certain SKUs, however, rearranging SKUs is often not possible or not meaningful – as an example, this is the case in the e-grocery fulfilment centre considered in this paper. In order to mitigate the negative impact induced by such short-term demand variations, we propose to aim at determining a robust storage assignment, that is, a storage assignment that performs well for a number of demand scenarios, in particular for day-of-
week-dependent SKU demands. The contributions of this paper can be stated as follows:

First, while there is a considerable amount of work on warehousing in non-grocery e-commerce and for brick-and-mortar retailing, see e.g. the recent reviews by Boysen et al. (2019) and Boysen et al. (2021), our paper is among the first ones dealing with scientific support for tactical decision-making in e-grocery fulfilment centres.

Second, we address a new three-level storage assignment problem occurring in a hybrid warehouse with a pick-and-pass system for fast order picking. For this problem, we propose and solve an integrated MILP formulation. In a set of experiments with real-world data provided by a leading European e-grocery retailer, we show that solving this integrated model is clearly superior to a standard sequential approach in which the selection of SKUs to be included in the fast-pick area is taken before taking zone/station assignment decisions.

Third, in order to cope with day-of-week dependent demand fluctuations, we propose to solve an augmented MILP model that explicitly aims at finding a robust storage assignment for the pick-and-pass system that performs well for each day of the week. Again using real-world data, we show that while a solution determined based on average demands leads to high station imbalances on certain days, the robust solution is balanced on each day of week almost without compromising the storage assignment objective.

The remainder of the paper is structured as follows: In the next section, we introduce the business case and the real-world data set considered in our study. Section 3 provides a review of related literature, followed by Section 4 covering our integrated storage assignment model and computational experiments. In Section 5, we develop a model extension taking into account varying demand patterns with respect to days of week of the SKUs. Finally, we sum up our major findings in the conclusion.

2 Description of the fulfilment warehouse and available SKU data

The e-grocery retailer analysed in this paper mainly operates with a two-stage distribution process. In the first step, SKUs are supplied from national distribution warehouses to local fulfilment centres. These supplies typically take place at each working day between Monday and Saturday. When a supply arrives at a fulfilment centre, all SKUs are stowed in their
allocated shelves. In the second step, customer purchases are served by these fulfilment centres depending on the location of the customer.

In most fulfilment centres, the retailer operates with a traditional picker-to-parts system. To improve operational efficiency, allow for shorter operation times, and increase the number of purchases served within one day, the retailer introduced a higher level of automation within single fulfilment centres. While a fully automated picking process is cost-intensive, in this paper, we consider the case of a partially automated picking loop within a hybrid warehousing system established within one of the retailer’s fulfilment centres. This hybrid system consists of two storage areas, (1) a partially automated picking loop and (2) a traditional picker-to-parts area. While the operational efficiency is higher within the picking loop, its available storage space is limited. As a result, the retailer has to decide which SKUs should be included in the picking loop and which should remain in the picker-to-parts area.

The picking loop consists of eight picking stations, with boxes sequentially visiting the stations. Each box corresponds to one customer purchase, while at each station the picker pulls the SKUs for this purchase from the shelves and places them into the box. Once all SKUs corresponding to a specific customer purchase are placed into the box, it leaves the loop and the purchase is loaded into a vehicle for delivery. Figure 1 provides a schematic sketch of the picking loop.

![Figure 1: Representation of the picking loop.](image)

Figure 2 illustrates the structure of an exemplary station within the picking loop. Each station consists of six racks, four in front of the picker, two outer and two inner, as well as two racks in the back of the picker. The racks in front of the picker contain four shelves with a height of 250 mm each (type 1), while the four shelves within the racks in the back
of the picker have a height of 450 mm (type 2). In total, there are 192 shelves within the whole picking loop, 128 of type 1 and 64 of type 2. The structure of shelves is represented in Figure 10 in the Appendix. Note that some SKUs can be allocated to type 2 shelves only due to their individual height.

To avoid congestion within the picking loop and idle times at some stations, the retailer aims at balancing the processing time and the corresponding workload for pickers between stations. For a given order, the time spent at a station depends on the number of picks, and on the shelf locations of the picked SKUs within the racks. While it is easy to pull SKUs out of a shelf at face level and in front of the picker, the picking process is more time-intensive for SKUs located in the top or bottom shelves of a rack as well as in shelves in the racks in the back of the picker. Therefore, in addition to the decision on allocating SKUs to the picking loop in general, the retailer needs to assign each SKU to a station and to a shelf with respect to the goals mentioned before. In contrast to brick-and-mortar retailing, where the allocation of SKUs to shelves also depends on marketing aspects, the setting of online retailing enables the company with the flexibility to decide on the placement of SKUs based on efficiency only. However, the retailer needs to take into account some additional constraints implicitly considered by customers in brick-and-mortar retailing, such as that large and heavy SKUs need to be placed into the box first in order to mitigate the risk of damaging fragile items.

The data set provided by the e-grocery retailer covers 4,693 different SKUs in total. It provides information on the measurements of each SKU, determining whether the SKU can
be allocated to shelves of type 2 only or also to shelves of type 1. In addition, each SKU is associated with a precedence order rank taking one of the values 1, 2, and 3, where rank 1 corresponds to heavy items which need to be allocated to an early station and rank 3 is used for fragile items. All other SKUs are associated with rank 2. Furthermore, each SKU has a target stock depending on expected customer demand. This target, along with the size of the SKU, determines the space in the shelf to be allocated to the SKU. In addition, the shelf allocation needs to consider handling-related aspects, such as the need to reserve space for a separator if two different SKUs are placed next to each other in a single shelf. As mentioned above, the available space within the loop is not sufficient to store all SKUs in the assortment of the retailer. The decision on which SKUs are included into the picking loop are based on an importance score for each SKU.

![Log importance score](image-a)

![Log number of picks](image-b)

Figure 3: Histograms of the log importance scores and the log number of picks for the SKUs in the assortment of the retailer appropriate for the picking loop.

This importance score, which is defined by the retailer, takes values between 0 and 1, where a higher value corresponds to higher importance of including this SKU into the picking loop. It strongly depends on the number of picks for the specific SKU (correlation coefficient of 0.718), while also additional factors such as the number of units per order and the volume of the SKU affect this score. As the distribution of the importance score has a strong positive skewness, we illustrate the frequency of the logarithm of importance scores for all SKUs in Figure 3(a). The log importance score is approximately normal distributed with mean -7.81
and standard deviation 2.34. This implies that there is only a small number of SKUs having an importance score exceeding 0.1, while the score is fairly small and nearly equal for a large majority of SKUs. Due to the positive skewness of the total number of picks for the SKUs in the assortment of the retailer appropriate to be included in the picking loop, we again show a histogram of the logarithm of the number of picks per SKU in Figure 3 (b). This distribution is again roughly symmetric with a mean log number of picks of about 6.84. For more than 80% of the SKUs, the average number of units per order line is at most 2 (mean 1.70). This confirms prior statements of Boysen et al. (2021) on the characteristics of e-grocery purchases. For some SKUs, however, the average number of units per order line is larger with up to 13.92 units on average (for details see the boxplot in Figure 11 in the Appendix). The target stock for SKUs again varies across the assortment. More than 90% of the SKUs have a target stock of less than 20 units with an average of 9.10 units, implying some flexibility in the assignment due to the limited space needed to assign single SKUs. However, the 1% with the highest target stock have an average of 100.35 and a maximum of 252 units. In addition, the measures of 17.4% of the SKU require an allocation to type 2 shelves only.

3 Related work

As discussed in Hübner et al. (2019), establishing efficiency in distribution logistics is one of the most challenging and success-critical tasks for e-grocery retailers. For recent overviews on the specific challenges faced by omni-channel and e-grocery retailers, and the resulting implications on the design and operation of their logistics networks, see e.g. Wollenburg et al. (2018); Hübner et al. (2019) and Rodríguez García et al. (2021). Given these challenges and the increasing relevance of e-grocery retailing, it comes to no surprise that the Operations Research community recently started to develop optimisation-based approaches for supporting decision-making in distribution logistics. To give a few examples, for a fulfilment process based on in-store-picking, Vázquez-Noguerol et al. (2022) propose an optimisation model that allocates customer orders to stores where these orders are picked, and schedules both, the picking as well as the delivery of the order to customers. Dethlefs et al. (2022) consider a setting where the e-grocer operates with both, in-store picking and picking in distribution centres, proposing an approach that integrates the assignment of orders to stores or distribu-
tions centres and the scheduling and routing of deliveries. For a recent overview on the new role of brick-and-mortar stores in omnichannel retailing, see Hubner et al. (2022). However, we are not aware of any paper dealing with optimisation-based decision support for tactical problems such as storage assignment in dedicated e-grocery fulfilment centres. This gap in the research is particularly relevant since, as detailed by Hubner et al. (2019), e-grocery fulfilment is substantially different from fulfilment for non-grocery e-commerce and from warehousing in distribution centres supplying grocery stores. These differences are also reflected in the recent reviews on warehousing for e-commerce by Boysen et al. (2019) and for brick-and-mortar retailing by Boysen et al. (2021), both of which explicitly exclude the case of e-grocery fulfilment warehouses. To illustrate the differences between e-grocery and other e-commerce characteristics and their impact on warehousing, let us consider the number of order lines per transaction. While, based on the observation that customer purchases in e-commerce consists of a few number of order lines only, Boysen et al. (2019) deem the use of a pick-and-pass system as inappropriate for e-commerce. An e-grocery order, in contrast, typically involves dozens of order lines which completely changes this assessment: In fact, a leading European e-grocery retailer decided to use a variant of such a system as part of the fulfilment centre we consider in this paper.

Using this warehouse as underlying business case, we propose an integrated approach for taking the three main storage assignment decisions identified in the review paper by De Koster et al. (2007): allocating SKUs to a specific area of the warehouse (in our case, either to a standard picker-to-parts based area in the warehouse or to the fast-picking area), assigning SKUs to zones of a given area (in our case, stations in the pick-and-pass based picking loop), and the assignment of SKUs to shelves within a given zone (in our case, the picking station). While the exact problem considered here, which, as described above, also involves specific aspects such as order constraints, has not been discussed in the literature so far, we are also not aware of any work that addresses all three storage assignment decisions in an integrated way in other warehousing settings. Next, we will briefly review existing research dealing with taking storage assignment decisions, with an emphasis on pick-and-pass systems considered in this paper.

The highest-level storage assignment decision considered in this paper is which SKUs to assign to the highly efficient picking loop system and which to assign to the picker-to-parts
area of the warehouse. A similar decision arises in the so-called forward reserve allocation problem (see e.g. Hackman et al. 1990 and Walter et al. 2013) which consists in selecting the SKUs to be allocated to the fast-picking area along with the number of units to be allocated of each selected SKU. In that problem, it is assumed that the fast-picking area is re-filled from the reserve area, whereas in our setting each SKU is assigned to a single location in one of the two parts of the warehouse. In contrast to the integrated problem considered in our paper, the forward-reserve allocation problem does not consider the assignment of SKUs to storage locations but only aims at ensuring that the SKUs allocated to the fast-picking area can be assigned to the shelves.

The next decision to be considered in our problem is the assignment of SKUs to zones or stations in the picking loop. As discussed in the review by De Koster et al. (2007), when it comes to assigning SKUs to zones or stations in a pick-and-pass system (also referred to as “progressive zoning”), the most important goal is to balance the workload among the zones. In fact, workload balancing is either part of the objective or the constraint set in most of the works dealing with the optimisation of zone assignment decisions, see e.g. Jane (2000); Jewkes et al. (2004); De Koster and Yu (2008) and Pan et al. (2015). Actually, the positive effect of workload balancing on the performance of pick-and-pass systems was verified in several studies using simulation (Jane 2000; De Koster and Yu 2008; Pan et al. 2015) and approximate models based on queuing theory (Yu and De Koster 2008; Pan et al. 2015; van der Gaast et al. 2020). With the exception of Jane (2000), the majority of the papers dealing with storage assignment in pick-and-pass systems combine the allocation to zones with storage location assignment in shelves. In general, the main goal of this shelf assignment is to determine the shelf locations in a way that SKUs with a high picking frequency have a short picking time. Instead of focusing on picking efficiency, Otto et al. (2017) propose to focus on ergonomic aspects, and, similar to Jewkes et al. (2004), consider an order line system in which the configuration of the zones in terms of zone borders/allocation of rack columns is part of the decision problem (which is not the case in our setting).

Irrespective of the type of picking system, the majority of the articles dealing with storage assignment assume a given demand scenario for which the assignment is optimised. In practice, however, demand is subject to variation: In case of the e-grocery retailer considered in this paper, for example, there are multiple demand variation patterns depending on the day of week
and on the season, and there are long-term trends leading to structural changes in customer buying behaviour (e.g. increased demand for vegan and vegetarian products). Clearly, as noted by Pazour and Carlo (2015), a storage assignment which is optimal for a given demand scenario may be suboptimal for another scenario. As a consequence, there are several papers such as Christofides and Colloff (1973), Chen et al. (2011), or Pazour and Carlo (2015) dealing with rearranging the storage location configuration of warehouses. While such adaptation of the storage assignment is useful in case of long-term demand fluctuations, when it comes to short-term demand variations, such an adaptation is typically not feasible or meaningful from an economical perspective.

In the following exposition, we will use the term robust storage assignment to refer to the problem of determining a storage assignment in presence of short-term variations that cannot be countered by storage reallocations. Research on this topic is relatively scarce and mostly considers warehouse designs that substantially differ from the setting of the business partner considered in this paper: For the case of a unit-load warehouse, Ang et al. (2012) deal with finding a storage allocation policy in presence of varying demands. They show that their policy obtained using robust optimisation significantly outperforms variation-agnostic policies from the literature when it comes to expected performance. For a warehouse storing pallets from a car parts manufacturer, Kofler et al. (2015) discuss a robust storage reallocation strategy, that is, one that is robust to small demand variations, thereby reducing the need for storage reallocations. When it comes to pick-and-pass warehouses, we are not aware of any article dealing with finding a storage assignment that is robust against short-term variations.

4 Integrated three-level storage assignment

In this section, we first propose a MILP model formulation that simultaneously considers the three decisions outlined above: Selection of SKUs to be included in the picking loop, assignment of SKUs to picking stations, and assignment to the shelves in the station. In Subsection 4.2 we present the results from a series of experiments with this model using real-world data.
4.1 Problem description and model formulation

Our problem consists of three main decisions: First, given a set $V$ of SKUs and a hybrid warehouse consisting of a picker-to-parts area with relatively low picking efficiency and a picking loop with a high picking efficiency, we have to decide which SKU to allocate to the picking loop. This allocation is based on a so-called importance score $s_v$ associated with each SKU $v$ which, as described in Section 2, in our case study is provided by the retailer. If we consider only this first decision and aim at maximising the sum of the importance scores, the resulting problem is very similar to the so-called forward-reserve problem reviewed in Section 3. Note, however, that while in the forward-reserve problem it is assumed that the reserve area serves both as a picking area for the SKUs not assigned to the fast-picking zone and as reserve area from which the fast-picking system is restocked, in the problem considered here, each SKU is either assigned to the picking loop or to the picker-to-parts area. Consequently, it is assumed that the (fixed) shelf space taken by an SKU $v$ (characterised by its height $h_v$ and the width $w_v$) is large enough to store all units of the SKU until the next re-supply of the SKU $v$. In this context, observe that $w_v$ is not the width of a unit of a SKU but the width of the target stock of $v$ if it is included in the picking loop; for the problem considered here, $w_v$ is considered as a given and fixed parameter.

For the SKUs to be allocated to the picking loop, the second decision to be considered is the assignment of the SKU $v$ to one station $k_v$ from the set $K$ of stations which we assume is ordered and indexed by integers ($K = \{1, \ldots, |K|\}$). This station assignment, which can be viewed as a zone assignment in a pick-and-pass system, needs to consider two main aspects: First, the assignment has to respect a set of precedence order constraints. As described above, we assume that each SKU is associated with a precedence rank $o_v \in O$, with $O = \{1, \ldots, |O|\}$. In order to avoid damaging of SKUs, the station assignment decision needs to make sure that for each pair of two SKUs $v$ and $v'$ with $o_v \leq o_{v'}$, $v$ is assigned to the same or an earlier station as $v'$, that is, $k_v \leq k_{v'}$. In addition to respecting these precedence order constraints, the workload among the stations in terms of number of picks should be balanced. Here, following the requirement of our business partner, we assume that there is a given threshold $\delta$ denoting the maximum permitted relative deviation of the workload $z_k$ of a station $k$ from the

1Note that among all SKUs in the assortment of the retailer, $V$ only comprises those that can be handled by the picking loop – as an example, $V$ does not include any SKUs that need to be refrigerated.
average value over all stations in terms of picking operations per day. This average workload can be calculated as $z = \frac{1}{|K|} \sum_{k \in K} z_k$.

The third storage assignment decision is the assignment of SKUs to shelves within the stations in a way that SKUs with a high number of picks are stored in shelves that are fast and easy to reach by the picker. It is assumed that each SKU is assigned to a single shelve. We denote the set of shelves at a given station $k$ with $R_k$ and the set of all shelves in the picking loop with $R$, that is $R = \bigcup_{k \in K} R_k$. Each shelf $r \in R$ has a height $h_r$ and a width $w_r$, and an SKU $v$ can only be assigned to a shelf $r$ if $h_v \leq h_r$ and $w_v \leq w_r$, giving rise to the definition of a set $R_v \subseteq R$ of shelves that can fit SKU $v$. Note that in addition to this, we can further restrict the set $R_v$ based on the precedence rank $o_v$: As an example, if all SKUs with rank 1 fit into the first two stations, this implies that an SKU $v$ with $o_v = 1$ cannot be assigned to a shelf in a station $k > 2$ without either leaving shelf space empty in the first two stations or violating the precedence constraints. If more than one SKU is stored in a shelf, there needs to be a minimum distance $g$ between each two SKUs stored next to each other. In addition, a shelf $r$ is associated with a distance $d_r$ from the picker at the corresponding station $k_r$. We assume that every pick is carried out separately, and, following the definition used by the management of the e-grocery retailer, we define the picking efficiency of a shelf $r$ as the inverse of the distance $d_r$, that is, as $\frac{1}{d_r}$. Using this definition, the goal in this subproblem is to allocate SKUs with a large number of picks in shelves exhibiting a high picking efficiency. Accordingly, in this subproblem we maximise the average efficiency per pick.

Since the three storage assignment decisions described above are highly interdependent, we aim at considering them simultaneously in an integrated problem. Among these three decisions, only the first and the third one are associated with an objective function, namely maximising the sum of the importance scores of the SKUs selected for the picking loop and maximising the picking efficiency of the shelf allocation. In the integrated problem, we combine these two objectives in form of a linear combination by introducing a weight $\alpha$ to be multiplied with the second part of the objective.

Next, we propose a MILP formulation for the integrated problem. The main decision variables in this formulation are the binary variables $x_{v,r}$. $x_{v,r}$ takes the value 1 if SKU $v$ is assigned to shelf $r$ and 0 otherwise. The values of these variables determine the values of the second set of variables considered in the model, namely the variables $z_k$ denoting the
workload in terms of total number of picks assigned to station $k$. Furthermore, we introduce
the integer variable $y_o$ representing the last station (that is, the station with the highest
index $k$) to which a SKU with precedence rank $o$ is assigned. Given these variables and
the parameters introduced above, we are now ready to present the MILP formulation of the
integrated problem:

$$\begin{align*}
\text{max} & \sum_{v \in V} \sum_{r \in R^o} s_v x_{v,r} + \alpha \cdot \frac{1}{\sum_{v \in V} \sum_{r \in R^o} \frac{1}{d_r} p_v x_{v,r}} \\
\sum_{r \in R^v} x_{v,r} & \leq 1 \quad \forall \ v \in V \\
k_r x_{v,r} & \leq y_o \quad \forall o \in O, v \in V^o, r \in R^v \\
k_r x_{v,r} & \geq y_{o-1} \quad \forall o \in O \setminus 1, v \in V^o, j \in R^v \\
z_k &= \sum_{v \in V} \sum_{r \in R^v} p_v x_{v,r} \quad \forall k \in K \\
z_k &\leq (1 + \delta) \cdot \frac{1}{|K|} \sum_{k \in K} z_k \quad \forall k \in K \\
z_k &\geq (1 - \delta) \cdot \frac{1}{|K|} \sum_{k \in K} z_k \quad \forall k \in K \\
w_r &\geq \sum_{v \in V^r} (w_v + g) \cdot x_{ij} - g \quad \forall \ r \in R \\
x_{v,r} &\in \{0, 1\} \quad \forall \ v \in V, r \in R^v \\
y_o &\in \{1, \ldots, |K|\} \quad \forall \ o \in O
\end{align*}$$
The objective function consists of a weighted combination of the two parts mentioned above: Part I corresponds to the maximisation of the total importance score, while part II represents the maximisation of the average efficiency per pick. By adjusting the parameter $\alpha$, the relative importance of the two objectives can be adjusted by the decision maker.

Constraint set (1) ensures that each SKU is assigned to at most one shelf in the picking loop. The Constraints (2) and (3) enforce the precedence order constraints: Constraint set (2) imposes that $y_o$ is at least as big as the maximum shelf index $k$ assigned to any SKU with order rank $o$, and (3) ensures that all SKUs with a precedence rank $o$ other than 1 are assigned to a station $k \geq y_{o-1}$, that is, to a station corresponding to the last station containing a SKU with the next smaller rank or to a station later in the loop. The Constraints (4)−(6) enforce balanced workload among the stations. Constraint set (4) is used to determine the value of the auxiliary variables $z_v$ representing the total number of picking operations allocated to station $k$. Using this variable, Constraints (5) and (6) ensure that the workload allocated to each station respects the maximum permitted relative deviation from the average workload among all stations. Constraints (7) ensure that the total width of the SKUs assigned to a shelf $r$ plus the required gaps between each pair of SKUs in a shelf does not exceed the width $w_r$ of the shelf. Finally, Constraints (8) and (9) enforce the domains of the variables $x_{v,r}$ and $y_o$.

4.2 Computational experiments

In this section, we present the results from a number of experiments with the model presented above, using real-world data from the e-grocery retailer considered in this paper. In a first set of experiments, we explore the solution behaviour with respect to convergence of the duality gap, i.e. the relative difference between a solution found by the optimiser and a lower bound, over time. In addition, we discuss the impact of the weighting factor $\alpha$ on the values of the two parts of the objective function, given that we allow for a fixed relative deviation $\delta$ in the number of picks between stations. This allows us to derive an interval of reasonable values for the weighting factor $\alpha$. Furthermore, we consider the effect of the allowed deviation $\delta$ between stations on the structure of the solutions. Finally, we also compare our integrated three-level storage assignment approach to a sequential proceeding, i.e. solving the area allocation
problem (akin to the forward reserve problem) and the assignment of the selected SKUs to shelves consecutively. All experiments were conducted with the Gurobi optimiser version 9.0.2 on a computer with 16 GB RAM and a 11th Gen Intel(R) Core™ 94i7 Processor 2.30 GHz.

Experiments on the runtime

In a first analysis, we use an exemplary weighting factor $\alpha = 250$ and allow for a deviation of picks between stations of $\delta = 1\%$. Figure 4 shows (a) the objective value and (b) the gap to the lower bound after a given runtime of up to 12 hours. As we refer to the solution found if a gap of 0.5% is reached in most of our analyses, we additionally emphasise the resulting values at this point of time by the red dotted lines. In this exemplary setting, we reach the intended gap after a runtime of about 90 minutes. As we cover a tactical problem of the retailer, that is not solved regularly, even longer runtimes could be allowed. However, our results show that the progress in further reduction of the gap is slow, as even an additional hour of runtime reduces the gap by 0.01 additional percentage points from 0.48% to 0.47% only.

![Runtime vs. Objective Value](image1)

![Runtime vs. Gap](image2)

Figure 4: Representation of the objective value and gap to the lower bound in percent depending on the runtime in minutes of up to 12 hours using $\alpha = 250$ and $\delta = 1\%$. The red solid line corresponds to a gap of 0.5%.

Effect of the objective weight $\alpha$

As introduced above, the objective function consists of two parts, where the first (I) covers the sum of importance scores of SKUs allocated to the picking loop area and the second (II) controls for the picking efficiency in the picking loop. In fact, the absolute value of part I is larger by approximately factor 180. Therefore, we consider the impact of different values for
Figure 5: Values of part I (importance score) and part II (picking efficiency) of the objective function depending on the weighting factor $\alpha$.

the weighting factor $\alpha$ on our results in a set of experiments. We limit the relative deviation of picks between stations for these experiments to $\delta = 1\%$, while we terminate the optimisation when either a gap of 0.5% or a predefined time limit of 15 minutes is reached. Figure 5 gives an overview on the values for both parts of the objective function depending on $\alpha$. The left part of the figure illustrates that the sum of importance scores of the SKUs assigned to the picking loop takes the highest values for $\alpha < 400$. At the same time, part II of the objective function strongly increases until $\alpha = 100$ and remains almost in the same interval afterwards. Figure 6 gives additional insights in the structure of the solutions by showing the number of picks for a given distance between the picker and the shelf for $\alpha \in \{0, 1, 75, 250, 400\}$. We exclude SKUs with a height exceeding 250 mm from these plots as they can be allocated to type II shelves only and would bias the findings discussed here. While Figure 6 indicates no clear pattern for $\alpha = 0$ (i.e. a setting where the distance between the picker and the corresponding shelf does not affect the objective value), starting from $\alpha = 75$ SKUs with a high number of picks are allocated to shelves close to the picker, with results changing only slightly for larger values of $\alpha$. However, there is still a small number of outliers in each figure. For example, given the allocation for $\alpha = 75$, there are some SKUs with a high number of picks which are still allocated to shelves with a distance of 950 mm, 1900 mm, and 2850 mm, respectively. These specific SKUs have a high width and, therefore, the model favours the allocation of more but smaller SKUs with a high number of picks over these SKUs to shelves close to the picker. To conclude, $\alpha$ should be determined within the interval $[100, 400]$. For our ongoing analyses we fix $\alpha = 250$. 
Figure 6: Allocation of SKUs and corresponding picks to shelves with given distance to the picker for different values of $\alpha$ and $\delta = 1\%$. Note that the figure is limited to SKUs with a height of up to 250 mm.

**Effect of the workload balancing parameter $\delta$**

In the following, we additionally analyse the effect of the allowed deviation of picks between stations $\delta$ on the runtime until reaching a gap of 0.5\% as well as the resulting objective value. Again choosing $\alpha = 250$, we vary the permitted relative deviation between stations in the set $\delta \in \{1\%, 1.5\%, 2\%, 2.5\%, 3\%\}$. Table [I] first shows the number of SKUs included in the picking loop as well as the corresponding total importance score of theses SKUs, i.e. part I of the objective function. We find that both values vary only slightly without a clear pattern depending on the allowed deviation $\delta$. The next two columns report the picking efficiency in the picking loop, i.e. part II of the objective function as well as the total objective value. Except for $\delta = 2\%$, the increased flexibility due to a higher level of deviation allowed enables a slight increase in the picking efficiency and also in the total objective value. In total, we can increase the objective value by 0.07\% when allowing for a maximum relative deviation of 3\% instead of 1\% only. Still, the structure of the solution varies. In particular, for $\delta = 1.5\%$ more SKUs are allocated to the picking loop, although the total importance score decreases. At the same time, the computation time until reaching a gap of 0.5\% is much higher than for all other settings. From a managerial point of view, these results suggest to focus on limiting
the allowed deviation, such that the workload between stations is balanced and the risk of congestion is small, while the objective value accounting for the importance of SKUs allocated to the picking loop as well as picking efficiency reduces only slightly.

| $\delta$ | number of SKUs | objective part I | objective part II | objective value | maximum relative deviation | runtime | gap |
|----------|----------------|------------------|-------------------|-----------------|--------------------------|---------|-----|
| 1.0%     | 1497           | 26.56            | 0.1864            | 73.165          | 0.98                     | 6181s   | 0.48% |
| 1.5%     | 1507           | 26.54            | 0.1866            | 73.183          | 1.43                     | 30337s  | 0.46% |
| 2.0%     | 1511           | 26.55            | 0.1864            | 73.136          | 1.97                     | 5830s   | 0.52% |
| 2.5%     | 1503           | 26.56            | 0.1866            | 73.214          | 2.38                     | 8313s   | 0.41% |
| 3.0%     | 1497           | 26.54            | 0.1867            | 73.216          | 2.64                     | 4488s   | 0.41% |

Table 1: Summary statistics on the number of SKUs included in the picking loop, the objective value, the maximum relative deviation in picks between stations, the runtime until a gap of less than 0.5% is reached and the resulting gap for different values of allowed deviation $\delta$ and a fixed weighting factor $\alpha = 250$. Note that there is a gap slightly above 0.5% for $\delta = 2\%$ as we were not able to find a further solution without running out of memory for this setting.

**Integrated vs sequential storage assignment**

Finally, we compare our integrated model to a sequential two-stage approach, where we first solve the problem akin to the forward reserve allocation problem, i.e. the selection of SKUs to be allocated to the efficient picking loop area (part I of our objective function), with neither taking into account the picking efficiency (part II of our objective function) nor respecting balancing constraints. After a runtime of 24 seconds only the model can be solved with a gap of 0.11%. It suggests to include 1533 SKUs into the picking loop, leading to a total score of 26.57. Comparing these results to Table 1, we find that the score improves only slightly while we allocated 36 SKUs more to the picking loop than when also accounting for picking efficiency and limiting the deviation in picks between stations to $\delta = 1\%$. At the same time, the runtime reduces comprehensively. We then try to balance the workload between stations assuming the set of these 1533 SKUs allocated to the picking loop as given. Even when permitting for a very large deviation of $\delta = 40\%$ we are not able to solve the feasibility problem after a runtime of one hour. Allowing to remove SKUs from the predefined set, however, allows us to solve the model with a score of 26.52 (based on 1510 of the 1533 SKUs) and a total objective value of 73.093, when limiting the deviation to $\delta = 1\%$. As this objective value is smaller than for the integrated approach, the results underline the importance of an integrated solution compared to a sequential approach for the assignment problem of the retailer considered here.
5  Coping with short-term demand variation

The general model developed in the previous section allows the retailer to solve the three-level storage assignment problem, i.e. to decide which SKUs of the assortment should be allocated to the picking loop while also determining the assignment of SKUs to stations and shelves within the warehouse. However, the demand for SKUs (and, therefore, the number of picks) is not at all constant for each day, week or even month of the year. In this section, we discuss the importance of accounting for variation in demand when assigning SKUs to shelves. We start by presenting historical picking data from a European e-grocery retailer with a focus on recurring patterns depending on the day of week. Next, we analyse the quality of the storage assignment determined in the previous section with respect to possible imbalance between stations on the level of days of week. Finally, we extend our model formulation by limiting the deviation of picks between stations on the level of days of week and compare both approaches. This allows us to derive the benefit of explicitly including variation in demand into the storage assignment decision of the retailer. As, on the other hand, the retailer needs to spend effort on data collection, data processing and computational power for the detailed analysis, this analysis forms the basis for the decision whether the benefit of the detailed solution outweighs this effort.

5.1 Historical picking data

In addition to the characteristics of SKUs introduced in Section 2, the data set of the e-grocery retailer covers historical picking data and provides us with information on the average number of picks per month for a specific day of week for the SKUs within the assortment of the retailer, which are appropriate for the picking loop, for the year 2020. The data set covers the ID of the SKU, the day of week (1 corresponds to Monday, 6 to Saturday), the month, and the corresponding average number of picks for this month and day of week. Positive values on Sundays correspond to picks at early Sunday morning if the purchases on the preceding Saturday could not be fully accomplished until midnight. Given 4,693 SKUs with data for 12 months each, we find 2,348 out of these 56,316 combinations with 0 picks for all days of

\footnote{Note that the data is anonymised by multiplying the number of picks by the same constant for each entry, such that the relative relationship remains unchanged.}
the week, while in more than 30% of the combinations we have 0 picks for at least one day of the week. This suggests the presence of variation in demand across days of week and months. Considering the average number of picks per day of week given in Figure 7, we find peaks on Tuesday, driven by demand by business companies, as well as on Friday, where leisure goods are mainly demanded. These findings are supported by Table 2 displaying the relative distribution of picks per day of week and additionally covering the number of SKUs where the highest demand is observed on this specific day of week. Again, we find the highest values for Tuesday, followed by Friday.

![Figure 7: Total number of picks in thousands in the assortment of the retailer appropriate for the picking loop depending on the day of week (1 corresponds to Monday, 6 to Saturday).](image)

| day of week | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|-------------|--------|---------|-----------|----------|--------|----------|
| average relative number of picks on this day of week | 15.6% | 17.3% | 16.8% | 16.9% | 17.1% | 16.3% |
| number of SKUs with highest demand on this day of week | 556 | 1127 | 702 | 746 | 919 | 643 |
| number of SKUs with exceptional high demand on this day of week | 85 | 45 | 28 | 25 | 49 | 35 |

Table 2: Average relative number of picks per day of week, number of SKUs where the highest demand is given on this day of week and number of SKUs with exceptional high demand on a specific day of week.

It should be noted that a constant proportional change in demand over all SKUs included in the picking loop would not affect the balancing between stations significantly. However, if there is high demand for multiple SKUs in the same station compared to other stations on a specific day of week, this would incur congestion at this station and, therefore, affect the operational efficiency of the retailer. As an example, SKUs with high demand at the beginning of a week should be matched with those predominantly demanded right before the
weekend to balance the workload across stations. Table 2 confirms the results of Figure 7 and suggests that SKUs can be categorized into two main groups, one with highest demand at the beginning of the week and one with highest demand at the end of the week, while the relative number of picks per day of week over all SKUs fluctuates between 15.6% and 17.3%. Therefore, we define an exceptional high demand at a certain day of week if more than 25% of the picks per week for the specific SKU are accomplished on this day of week. Again, we find high values for Tuesday and Friday, but also for Monday. Figure 8(a) gives boxplots on the relative number of picks for each SKU on a specific day of week. While there is a higher variance for Monday and Saturday, for the other days we find 50% of the SKUs to have a relative amount of weekly picks between 15% and 19%, whereas on Wednesday there is one SKU with a relative number of picks exceeding 40%. In more detail, Figure 8(b) covers the six SKUs corresponding to those having the maximum relative amount of weekly picks at a specific day of week. We display their relative distribution of picks for each day of week, indicating that there is strong variation in the number of picks for these SKUs across different days of the week. This finding supports that the consideration of balancing demand for each day of the week individually allows the retailer to improve the assignment of SKUs to stations and shelves.

![Boxplots](a) Boxplots

![Detailed description](b) Detailed description

Figure 8: Boxplots of the relative number of picks and detailed description of SKUs corresponding to those with the highest relative number of picks at a specific day of the week in the assortment of the retailer appropriate for the picking loop.

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3We exclude SKUs for which data is not available for single days of week.
5.2 Evaluating variation-agnostic storage assignments

As the descriptive data analysis suggests varying demand patterns for days of week, we consider the degree of imbalance in the number of picks between different stations on the level of days of week. Due to the variation in demand, two different problems may occur: (1) imbalance due to the assignment of SKUs to stations and (2) congestion driven by the assignment of SKUs to shelves within each station. For the purpose of this analysis, we apply the basic model proposed in Section 4, using a weighting factor $\alpha = 250$ and a maximum deviation $\delta = 1\%$ of average picks over all days of week. This allows us to analyse the quality of the basic solution on the level of days of week.

| day of week | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|-------------|--------|---------|-----------|----------|--------|----------|
| deviation at station 1 | -1.38% | 0.05% | -0.56% | -0.18% | -1.21% | 1.22% |
| deviation at station 2 | -2.09% | -1.06% | -0.90% | 0.03% | 1.60% | 0.81% |
| deviation at station 3 | -0.16% | -0.21% | -0.65% | -0.38% | -0.18% | -1.53% |
| deviation at station 4 | 1.17% | 0.43% | 0.72% | 0.61% | 1.19% | 1.08% |
| deviation at station 5 | -2.00% | -0.11% | 0.33% | 0.28% | -2.05% | 0.24% |
| deviation at station 6 | 1.65% | 0.40% | -0.01% | -0.15% | 0.04% | -0.50% |
| deviation at station 7 | 2.31% | -0.32% | 0.56% | 0.22% | -0.41% | -0.54% |
| deviation at station 8 | 0.51% | 1.14% | 0.54% | -0.43% | 1.05% | -0.76% |

Table 3: Relative deviation in the number of picks between single stations and the average number of picks over all stations as well as corresponding minimum and maximum values. Absolute values exceeding intended maximum deviation level of $\delta = 1\%$ are indicated in bold letters.

In the basic model formulation as proposed in Section 4.1, Constraints (5) and (6) limit the maximum deviation in picks between stations. In particular, at each station we allow for a workload deviating up to $\delta = 1\%$ from the average value over all stations. However, as this model only includes the average number of picks over all days of week, the deviation might be distinctly higher for single days of the week due to variation in demand. Table 3 gives an overview on the deviation in picks between each station and the average value over all stations for single days of week. Additionally, the two bottom lines indicate the maximum positive and negative deviation. Our findings show that the intended maximum deviation level of $\delta = 1\%$ is violated for several days even if this constraint is satisfied when averaging over all days of week. In the solution, the permitted deviation is exceeded on Monday, Tuesday, Friday, and Saturday. Falling below a deviation of -1% will not affect the completion time of the whole picking process within one day, but might lead to dissatisfaction of the workforce due to imbalance in the workload. In addition, there is potential to decrease the completion time...
compared to the solution proposed by the basic model for specific days of week. However, exceeding a deviation of 1% directly leads to congestion at some stations and, therefore, inefficiencies in the operational processes of the retailer, which should be avoided. As the deviation from the intended level also holds for days with a high number of picks in total, such as Tuesday and Friday (see Figure 7), our results strongly advise to take into account variation in demand when determining an assignment of SKUs to stations.

Figure 9: Allocation of SKUs and corresponding average picks per day of week to shelves with given distance to the picker for SKUs not exceeding a height of 250 mm when applying the basic model formulation with $\alpha = 250$ and $\delta = 1\%$.

As the SKUs are assigned to shelves within these stations based on the average number of picks over all days of week, any variation and, therefore, a possible high relative demand at a specific day is ignored in the assignment. This may result in further congestion at this station if a SKU is assigned to an outer shelf even if there is a relatively high demand on a certain day of week. We extend the analysis illustrated in Figure 6 to the level of days by reporting the number of picks in relation to the distance between the picker and the shelves in Figure 9. While there is variation in the figures for the different days of week, only a few outliers can be observed. As mentioned in Section 4.2, these outliers can be explained by their large width and also occur in the solution proposed by the basic model on the level

\footnote{Again we exclude SKUs exceeding a height of 250mm.}
of averages. Therefore, it can be concluded that the assignment of SKUs to shelves within stations does not lead to operational inefficiencies for specific days of week. Still, we need to address the large deviation in the number of picks between different stations on the level of days of week.

5.3 Ensuring robustness against short-term demand variation

Our results in the previous section suggest to limit the deviation in the number of picks between different stations in the picking loop. For this purpose, we introduce a set of days of week $t \in T$ and variable $p^t_v$ representing the number of picks for SKU $v \in V$ and day of week $t$. To distinguish from the deviation on the level of averages, we use $\delta_t$ to denote the allowed relative deviation for each day of week. In general, the retailer could decide on different values for each day individually. In this analysis, we assume $\delta$ to be constant over all days of week. Therefore, we replace Constraints (4) – (6) of the basic model formulation from Section 4.1 with constraints limiting the deviation on the level of days of week:

\begin{align}
  z^t_k &= \sum_{v \in V} \sum_{r \in R_k} p^t_v x_{v,r} \quad \forall k \in K \quad \forall t \in T \\
  z^t_k &\leq (1 + \delta_t) \cdot \frac{1}{|K|} \sum_{k \in K} z^t_k \quad \forall k \in K \quad \forall t \in T \\
  z^t_k &\geq (1 - \delta_t) \cdot \frac{1}{|K|} \sum_{k \in K} z^t_k \quad \forall k \in K \quad \forall t \in T
\end{align}

Computational experiments with the robust model

Due to the increased complexity of the model, in some settings, the runtime increases considerably. Therefore, we run the optimisations in this section until reaching a gap of 1%. Similar to the analysis presented in Table 4, we first compare the results of this extended model depending on the chosen value for $\delta_t$. Again, we present several summary statistics regarding the objective values and the runtime. While all statistics vary slightly, our results do not provide clear patterns depending on the value for $\delta_t$. In particular, we find the highest value of the objective function as well as the highest values for both parts of the objective function individually for $\delta_t = 1.5\%$. As the model gives more flexibility if we allow for a higher
deviation, it could be expected that the objective value increases (as also found in Table 1 in Section 4.2). However, the objective value also depends on the resulting gap in the solution. This is distinctly lower for $\delta_t = 1.5\%$ causing the higher objective value in this setting. In total, we again find a quite small difference between the best and worst objective value of only 0.3\% for a gap of up to 1\% in the optimisation.

| $\delta_t$ | number of SKUs | objective part I | objective part II | objective value | maximum relative deviation | runtime | gap |
|-------------|----------------|------------------|-------------------|-----------------|---------------------------|---------|-----|
| 1.0%        | 1481           | 26.49            | 0.1817            | 72.916          | 1.00\%                    | 1564s   | 0.83\% |
| 1.5%        | 1496           | 26.54            | 0.1865            | 73.157          | 1.49\%                    | 988s    | 0.49\% |
| 2.0%        | 1492           | 26.47            | 0.1859            | 72.955          | 1.93\%                    | 1061s   | 0.77\% |
| 2.5%        | 1495           | 26.49            | 0.1859            | 72.971          | 2.49\%                    | 947s    | 0.75\% |
| 3.0%        | 1504           | 26.39            | 0.1862            | 72.933          | 2.90\%                    | 1054s   | 0.80\% |

Table 4: Summary statistics on the number of SKUs included in the picking loop, the objective value, the maximum relative deviation in picks between stations, the runtime until a gap of less than 1\% is reached and the resulting gap for different values of allowed deviation on the level of days $\delta_t$ and a fixed weighting factor $\alpha = 250$.

Finally, we compare our findings to those obtained under the basic model from Section 5.2 that ignores day-of-week demand variation. Note that Table 1 gives results on the basic model for a gap of 0.5\%, while we allow for a gap of up to 1\% in Table 4. For the purpose of comparison, we also provide results on the basic model with a maximum gap of 1\% in Table 5 in the Appendix. We find that the objective values from the robust model accounting for demand variation are barely different to those obtained with the basic model. While the runtime increases from about two minutes to 15-20 minutes, for $\delta = 1\%$ even more, we are able to decrease the deviation between stations from more than 2\% for single days to less than 1\%. As we address a tactical problem of the retailer, which has not to be solved regularly, the benefit of reducing congestion in the picking loop outweighs the increased runtime.

6 Conclusion

In this paper, we develop an integrated approach to address a three-level storage assignment problem arising in a fulfilment centre operated by a leading European e-grocery retailer. The fulfilment centre can be characterised as a hybrid warehouse combining a highly efficient, partially automated picking loop with a less efficient picker-to-parts area. While the demand for e-groceries increased during recent years, at the same time the market became more competitive. This requires e-grocery retailers to improve their operational efficiency. A key
challenge is the assignment of SKUs to shelves within a fulfilment centre. We optimise a bi-objective value function of the retailer by accounting for the importance of SKUs allocated to the high efficient picking loop, while also addressing the picking efficiency depending on the distance between a picker and the shelves. To avoid congestion within the picking loop, we additionally limit the permitted relative deviation in the number of picks between different stations in the constraints of our proposed optimisation model.

Our results show that we can efficiently solve the model with a remaining gap of less than 0.5% within about 90-120 minutes in most settings. As we cover a tactical problem of the retailer, which has not to be solved regularly but only in case of major changes in the assortment of the retailer or customer preferences, such runtimes are reasonable. In addition, we can show that our integrated approach is superior compared to solving the allocation to the picking loop and the assignment to stations and shelves sequentially.

An additional challenge addressed in this paper is the presence of day-of-week-dependent demand variation for certain SKUs. In our business case, the variation of demand is particularly high at the beginning of a week and right before the weekend. In a set of experiments, we show that a storage assignment that is based on day-of-week-agnostic average demand figures tends to exhibit a highly imbalanced workload on certain days of the week. In order to mitigate this problem, we extend the aforementioned storage assignment model to account for day-of-week-dependent demand variation. It turns out that the resulting model yields robust storage assignments that satisfy the permitted workload imbalance without compromising the quality of the solutions in terms of the (efficiency-oriented) objective value.

As this paper deals with an existing fulfilment centre, we are only able to address the tactical storage assignment problem. However, future research could also consider the strategic problem of designing warehouses, i.e. include the decision on the size of the picking loop, the number of stations and the shape of shelves. Regarding the tactical problem considered in this paper, future work may include individual levels of permitted deviation between stations depending on the total number of picks on a certain day of week, that is, varying $\delta_t$ with respect to the day of week $t \in T$. As possible congestion is more critical at a day of week with high workload in total, this extension to the model can further reduce operational inefficiencies for e-grocery retailers. In addition, customer demand varies not only across days of week but also within the year e.g. due to seasonality. In case of data availability for multiple years,
future work could also address the question of rearrangements in the assignment. While those changes require operational effort, they could be beneficial in case of comprehensive changes in customer preferences.

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A Appendix

A.1 Representation of shelves

Figure 10: Representation of the structure of shelves

A.2 Boxplot on the number of units per order line

Figure 11: Boxplot of the average number of units within one order line for the SKUs in the assortment of the retailer appropriate for the picking loop.

A.3 Results on basic model with gap of 1%

| $\delta$ | number of SKUs | objective part I | objective part II | objective value | maximum relative deviation | runtime | gap |
|---------|----------------|------------------|-------------------|-----------------|----------------------------|---------|-----|
| 1.0%    | 1501           | 26.55            | 0.1863            | 73.12           | 0.89%                      | 121s    | 0.55% |
| 1.5%    | 1494           | 26.49            | 0.1864            | 73.09           | 1.50%                      | 121s    | 0.58% |
| 2.0%    | 1503           | 26.56            | 0.1860            | 73.07           | 1.97%                      | 131s    | 0.62% |
| 2.5%    | 1502           | 26.56            | 0.1859            | 73.03           | 2.17%                      | 118s    | 0.67% |
| 3.0%    | 1483           | 26.50            | 0.1862            | 73.04           | 2.95%                      | 116s    | 0.65% |

Table 5: Summary statistics on the number of SKUs included in the picking loop, the objective value, the maximum relative deviation in picks between stations, the runtime until a gap of less than 1% is reached and the resulting gap for different values of allowed deviation $\delta$ and a fixed weighting factor $\alpha = 250$. 