PredicTor: Predictive Congestion Control for the Tor Network

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Abstract—In the Tor network, anonymity is achieved through a multi-layered architecture, which comes at the cost of a complex network. Scheduling data in this network is a challenging task and the current approach shows to be incapable of avoiding network congestion and allocating fair data rates. We propose PredicTor, a distributed model predictive control approach, to tackle these challenges. PredicTor is designed to schedule incoming and outgoing data rates on individual nodes of the Tor architecture, leading to a scalable approach. We successfully avoid congestion through exchanging information of predicted behavior with adjacent nodes. Furthermore, we formulate PredicTor with a focus on fair allocation of resources, for which we present and proof a novel optimization-based fairness approach. Our proposed controller is evaluated with the popular network simulator ns-3, where we compare it with the current Tor scheduler as well as with another recently proposed enhancement. PredicTor shows significant improvements over the previous approaches, especially with respect to latency.

1. INTRODUCTION

The Tor network allows its users to anonymously access the Internet, and thus serves an important societal role by supporting freedom of press and speech. It consists of an overlay network connecting so-called relay nodes, which can be used to establish anonymous connections. To this end, the Tor client software builds a cryptographically-secured circuit, a path over three relays, where each relay knows its immediate neighbors only.

While an extra delay is inevitable to gain anonymity (due to re-routing the traffic), the performance—in terms of latency, data rates, and fairness—is however neither optimal nor stable [1], [2]. One of the major shortcomings is the lack of fair rate allocation [3] and an effective congestion control [2], [4]. Here, congestion control describes the nontrivial task of scheduling data transmissions in a way that minimizes network load while obtaining the maximum possible throughput. Relaying data over a series of nodes, like in Tor, amplifies the problem; especially when rising delays occur in the network. In particular, Tor relays are unable to react to congestion, for example by signaling upstream to throttle sending rates.

Different methods have been proposed to improve the performance of the Tor network, including the adaptation of standard congestion control algorithms to Tor [1], [5], as well as the development of tailored approaches [4], [6]. Most notably, PCTCP [5], which uses a dedicated TCP connection between each relay for every circuit, has the potential to be actually deployed in Tor. While PCTCP provides some improvements, e.g., in fairness, it still does not provide sufficient congestion control. Other approaches often require changes to the network infrastructure and are therefore not directly applicable.

The problem of congestion in networks has also been studied extensively from a control theoretical perspective in the past. Previous works include classic linear control [7] including PID [8] and state-feedback LQR control [9]. It is well understood that delay is among the main challenges of controlling the network. More recently, especially optimization-based methods have been applied to the problem with promising results [10], [11]. Model predictive control (MPC), as applied in [11], is an advanced control technique that can deal with non-linear systems and explicitly take constraints into consideration. Its predictive control action is particularly suited for systems with significant delay. Furthermore, MPC has received significant attention as a method for distributed control [12], [13], where local controllers interact to jointly control an interconnected system. Distributed MPC is often applied to systems with a complex network character, such as transportation systems [14], energy management [15] or process industry applications [13], where a centralized solution is prohibitive due to the size of the system or privacy concerns. In order to obtain global properties, the local action is often coordinated by exchanging information about predicted future behavior [12].

In this paper, we develop PredicTor (see Figure 1), a distributed MPC congestion control algorithm for the Tor network. The distributed design is imperative, to allow scaling the network and most importantly, maintain anonymity of the users. In contrast to the current behavior of Tor, PredicTor avoids congestion by generating backpressure. This denotes

![Fig. 1: Overview of the proposed method.](image-url)
the strategy of propagating congestion back to the original
sender instead of accumulating it within the network, and
it is achieved via the information exchange of the proposed
distributed MPC. Furthermore, PredicTor is designed with
a focus on fair allocation of resources. While optimization-
based rate allocation is a well researched topic, with equi-
alent formulations for TCP and other methods [10], we
introduce in this work a novel optimization-based maxi-
min fairness formulation. To the best of our knowledge,
PredicTor is the only distributed MPC approach to tackle the
previously mentioned congestion and fairness challenges of
the Tor network. While distributed MPC has been applied
to regular computer networks before [11], our approach
explicitly considers fairness and the applicability to a real
network.

With a real application in mind, we design PredicTor as
a modification of the Tor protocol. For evaluation purposes,
we build a prototype based on the ns-3 network simulator
and its extension nstor [6]. Our results indicate that PredicTor
can clearly reduce the latency of data transmission as well as
the load on the network. As an additional contribution, our
implementation of PredicTor and the required adaptations to
nstor are available as an open source software project[1].

The remainder of this paper is structured as follows. In
Section II we present the structure of the Tor network,
including related terminology and mathematical notation.
Our main contribution is presented in Section III First, we
present and proof Theorem [1] which is an optimization-based
method to obtain max-min fairness. We then discuss the dy-
namic system model and our distributed MPC concept before
we present the full PredicTor formulation. In Section IV we
showcase the performance of PredicTor in a ns-3 network
simulation study of an exemplary Tor topology.

II. STRUCTURE OF THE TOR NETWORK

In order to achieve anonymity, the Tor network [16]
provides a set of relay nodes. These relays are used by clients
to tunnel their communication through the network. The
established paths are commonly referred to as circuits and
carry equally-sized packets. Anonymity in Tor is achieved by
the fact that a server cannot tell where client data originates
from, since the server only sees the last relay in the circuit.
The Tor relays form a so-called overlay network, a computer
network that operates on top of the public Internet. The
necessary resources (servers and bandwidth) are provided by
volunteers and are not subject to any central authority.
An exemplary Tor topography is depicted in Figure 2. It
contains three circuits that share a set of six relays. One of
the relays was (randomly) chosen by all three circuits and
thus constitutes a possible bottleneck. This scenario is proto-
typical for commonly observed behavior in the Tor network.
Note that circuits generally carry data bidirectionally. For
simplicity, we only consider one direction in this paper; the
other direction can be realized completely analogously.

Formally, we introduce Tor as an overlay network graph
\( G(N, E) \) where \( N \) denotes the set of nodes and \( E \) the set
of overlay links. The network has a total of \( |N| = n \) nodes
and \( |E| = e \) connections. We denote the set of Tor circuits
\( P \) with \( i \in P \) being the i-th circuit of the set of cardinality
\( |P| = p \). \( P_\alpha \in P \) denotes the subset of circuits traversing
node \( \alpha \in N \). Generally, we refer to circuits with Roman
letters and to nodes with Greek letters. When considering
the network at the circuit level, we denote with \( r_i \) the data
rate (in packets per second) at which a circuit \( i \) is transferring
information. Furthermore, each node \( \alpha \in N \) of the overlay
network has a limited capacity \( C_\alpha \), since overlay connections
share the same physical connection.

Definition 1: A rate vector \( r = [r_1, r_2, \ldots, r_p] \) is feasible if:

\[
\forall i \in P : \quad 0 \leq r_i \quad \text{and} \quad (1)
\]

\[
\forall \alpha \in N : \quad \sum_{i \in P_\alpha} r_i \leq C_\alpha. \quad (2)
\]

We denote \( R_f \) the set of feasible rate vectors.

Each node \( \alpha \in N \) can receive, store and send data from each
circuit \( i \in P_\alpha \). We denote \( s_{\alpha,i} \) the circuit queue (storage in
number of packets) in node \( \alpha \) for circuit \( i \) and the vector
with all queues for each circuit in node \( \alpha \) as \( s_\alpha \in \mathbb{N}^{\left|P_\alpha\right|} \).
Congestion of the network results from high values of these
circuit queues and can be quantified with the data backlog.

Definition 2: The data backlog \( b \) of a network \( G(N, E) \)
is computed for all nodes \( \alpha \in N \) and all circuits \( i \in P \) as:

\[
b = \sum_{\alpha \in N} \sum_{i \in P} s_{\alpha,i}. \quad (3)
\]

III. PREDICTOR

The proposed predictive controller for the Tor network
(PredicTor) is developed with three objectives in mind:
Primarily, we are aiming to avoid congestion of the network
by limiting the data backlog of circuits. Secondly, we seek
to fully utilize the available resources of the network, and
lastly, we require a fair allocation of these resources.

This section starts by deriving an optimization-based
method to obtain global fairness in Subsection III-A. We also
show that this formulation will satisfy our second objective
and utilize the available resources. Congestion control, our
primary objective, can only be achieved by exchanging the
predicted action between connected nodes. The concept for
exchanging information is presented in Subsection III-B.

\[1\]https://github.com/cdoepmann/predictor
We then present the full optimal control problem (OCP) in Subsection III-C. Finally, we discuss the interaction of PredicTor and a Tor relay in Subsection III-D.

A. Optimization-based fairness

In the following we present an optimization-based method (Theorem 1) to achieve max-min fairness. We consider for the derivation the global rate $r_i$ for circuit $i \in P$.

Definition 3: A feasible rate vector $r^f \in R_f$ is called max-min fair, if for all circuits $i \in P$ and for all other feasible rates $\bar{r} \in R_f$ it holds that:

$$\bar{r}_i \geq r^f_i \Rightarrow \exists j \in P : r^f_j \leq \bar{r}_j \wedge r^f_j \leq r^f_i.$$  (4)

This definition means that if a rate $r^f$ is max-min fair, any other feasible rate that increases the rate for the favored circuit $i$ comes at the cost of reducing the rate for the disadvantaged circuit $j$, which is already smaller than the rate of circuit $i$.

Definition 4: For a circuit $i \in P_n$ and a rate vector $r$, we denote node $\alpha \in N$ a bottleneck, if:

$$\sum_{i \in P_{\alpha}} r_i = C_{\alpha}, \forall j \in P_n : r_j \geq r_j.$$  (5)

Lemma 1: Let $r^f$ be a max-min fair rate vector. Each circuit $i \in P$ has exactly one bottleneck. This bottleneck is the global rate-limiting factor of the circuit under stationary conditions.

Proof: The proof is shown in [17].

We can now state one of the main contributions of this work: how to obtain a max-min fair rate $r$ by solving a convex optimization problem.

Theorem 1: An overlay network achieves max-min fairness with rate $r = r^{\max} - \Delta r$ as the optimal solution of:

$$c = \min_{r \in \mathbb{P}} \sum_{i \in P} (\Delta r_i^f)^2$$
subject to:

$$r^{\max} - \Delta r \in R_f,$$

$$0 \leq \Delta r \leq r^{\max}$$

where $r^{\max}$ is an arbitrary upper limit with $\Delta r^{\max} \geq \max(C_1, C_2, \ldots, C_n)$.

Proof: Proof by contradiction. Assume that the optimal solution $r^*$ with optimal cost $c^*$ is not fair. If $\forall i \in P$ it holds that $r^*_i \leq r^f_i$:

$$c^* = \sum_{i \in P_n} (\Delta r_i^*)^2 \geq \sum_{i \in P_n} (\Delta r_i^f)^2 = c^f$$

Since the fair rate $r^f$ is feasible, the assumed solution is not optimal. On the other hand, if we favor circuit $i$ with the rate $r^*_i \geq r^f_i$, then, by Definition 3 we disadvantage circuit $j$ with rate $r^*_j$:

$$\forall p_j : r^*_j \leq r^f_j \wedge r^*_j \leq r^f_j.$$  (6)

This means that $\Delta r^*_j \geq \Delta r^f_j$ and $\Delta r^*_i \geq \Delta r^f_i$. We denote the magnitude of the disadvantage given to circuit $j$ by $\Delta r^*_j - \Delta r^f_j = m$. Considering Definition 4 and Lemma 1, we note that a disadvantage of magnitude $m$ for circuit $j$ is an upper bound for the possible advantage that can be given to circuit $i$:

$$\Delta r^*_j - \Delta r^f_j \geq \Delta r^f_i - \Delta r^*_i.$$  (7)

We can now substitute $m$:

$$r^*_j - r^f_j \geq r^f_i - \Delta r^*_i - m.$$  (8)

Rearranging the terms above leads to:

$$\Delta r^*_i \geq \Delta r^f_i - m.$$  (9)

Together with $\Delta r^*_j - \Delta r^f_j = m$ we can write the difference between the optimal cost and a a max-min fair cost as:

$$c^* - c^f = (\Delta r^*_i)^2 + (\Delta r^*_j)^2 - (\Delta r^f_i)^2 - (\Delta r^f_j)^2$$

$$\geq (\Delta r^f_i + m)^2 + (\Delta r^f_j - m)^2 - (\Delta r^f_i)^2 - (\Delta r^f_j)^2$$

$$\geq 2\Delta r^f_i m - 2\Delta r^f_j m + 2m^2$$

$$\geq 2m(\Delta r^f_i - \Delta r^f_j) + 2m^2 \geq 0,$$

where the first equality is given by the fact that only the circuits $i$ and $j$ are different for the optimal and fair solutions. The last inequality holds because circuit $j$ is already disadvantaged ($\Delta r^*_j \geq \Delta r^f_j$). The last inequality implies that the fair cost is smaller or equal than the optimal cost, which is a contradiction and proofs that the optimal solution of problem 6 yields the max-min fair rate vector $r$.

B. Distributed MPC

PredicTor is a distributed MPC approach, where an optimal control problem is repeatedly solved at each node $\alpha \in N$ of the network, to obtain local decisions regarding incoming and outgoing rates. To achieve global fairness while maintaining constrained backlogs, adjacent nodes need to exchange information about their predicted future actions.

Predictions are obtained on the basis of a dynamic model, for which we denote $s^{k}_{\alpha,i}$ the queue of a circuit $i$ in node $\alpha$ and at time step $k$. The dynamic model equation can be written as:

$$s^{k+1}_{\alpha,i} = s^{k}_{\alpha,i} + \Delta t(r^k_{\alpha,i} - r^k_{out,\alpha,i}),$$  (10)

where $\Delta t$ denotes the sampling time. We differentiate between incoming ($r_{in,\alpha}$) and outgoing ($r_{out,\alpha}$) rate, which can vary from the overall rate for circuit $i$, due to local storage terms.

For the interaction of multiple nodes, we denote $\alpha \in N$ the currently considered node, with connections to predecessor ($\beta$) and successor ($\gamma$) nodes. For the current node $\alpha$ it is irrelevant whether the incoming data comes from several nodes or only from a single node. For this reason, we assume that all incoming data for all different circuits comes from a single predecessor node $\beta$.

To further facilitate the statement of the optimization problem as well as the investigation of the proposed method, we assume in the following that all connections in $E$ of the network $G(N,E)$ experience a constant delay which is equivalent to the timestep ($\Delta t$) of the control problem.

We want to emphasize that the proposed algorithm is not
restricted to that case and can be easily adapted for the case of varying delays.

Information is exchanged at each MPC time step, such that node $\alpha$ receives messages with predicted trajectories of all connected nodes. The interaction of node $\alpha$ with its incoming node $\beta$ is shown in Figure 3. We differentiate between upstream and downstream information exchange. This is important because downstream messages travel with the data and latency between nodes can be omitted. Upstream messages, on the other hand, are traveling opposed to the data and latency between nodes can be omitted. Upstream information is delayed. The formulation in (8)

$$
\text{subject to :} \\
\Delta s_{\alpha|\beta}^k = \Delta s_{\alpha|\beta}^{k-1} + \Delta t (r_{\text{in},\alpha}^k - r_{\text{out},\beta}^k),
$$

(10d)

$$
0 \leq r_{\text{out},\beta}^k \leq r_{\text{max},\beta},
$$

(10h)

$$
\Delta r_{\text{in},\alpha}^k \leq C_{\text{in}}^\alpha
$$

(10f)

$$
\sum_{i \in P_{\alpha}} (r_{\text{max}} - r_{\text{out},i}^k) \leq C_{\text{out}}^\alpha
$$

(10g)

$$
\sum_{i \in P_{\alpha}} (r_{\text{max}} - r_{\text{out},i}^k) \leq C_{\text{out}}^\alpha
$$

(10i)

$$
s^0_{\text{in}} = s^0_{\text{init}}, \quad |\Delta s_{\alpha|\beta}| = 0,
$$

(10j)

no new packets entering the node.}

The objective in (10a) is motivated by the presented Theorem 1 but with some important adaptations. Most notably, we introduced $\Delta r$ variables for both the incoming and outgoing rates. Introducing the control variable $\Delta r_{\text{in},\alpha}$ allows to control the incoming rate. This is of significant importance for the desired congestion control as it induces backpressure and data will be stopped from entering the network if it cannot be forwarded. The quadratic term in $\Delta r_{\text{out},\alpha}$ ensures that the circuit queue is emptied even if there are no new packets entering the node.

The objective in (10a) is further modified by introducing a discount factor ($d$). This is necessary because naively implementing our presented fairness formulation also results in unfairness along the prediction horizon, where it is always preferable to increase the rate of the smallest element in a sequence for a given circuit. In practice, however, we want to send and receive as soon as possible as long as instantaneous fairness is achieved. In Appendix A we present a guideline on how to choose an upper bound for $d$ to obtain the desired behavior.

We implement PredicTor based on CasADi [18] in combination with IPOPT [19] and MA27 linear solver for fast state-of-the-art optimization.

### D. Interaction of controller and network

The proposed controller is implemented on the application layer of each node in the Tor network. At each timestep, problem (10) is solved with the most recent measurement of:

$$
\text{maximize} \quad \sum_{i \in P_{\alpha}} (r_{\text{max}} - r_{\text{out},i}^k)
$$

(10b)

$$
\text{subject to :} \\
\Delta s_{\alpha|\beta}^k = \Delta s_{\alpha|\beta}^{k-1} + \Delta t (r_{\text{in},\alpha}^k - r_{\text{out},\beta}^k),
$$

(10c)

$$
0 \leq r_{\text{out},\beta}^k \leq r_{\text{max},\beta},
$$

(10e)

$$
\Delta s_{\alpha|\beta}^k = s^k_{\alpha|\beta} - \Delta s_{\alpha|\beta}^{k},
$$

(9a)

$$
\Delta s_{\alpha|\beta}^k = \Delta s_{\alpha|\beta}^{k} + \Delta t (r_{\text{in},\alpha}^k - r_{\text{out},\beta}^k),
$$

(9b)

Equation (9) states that any value $r_{\text{in},\alpha}^k \neq r_{\text{out},\beta}^k$ will adjust the predicted circuit queue at the predecessor node. This plays an important role for the distributed MPC formulation, as it ensures the circuit queue is emptied even if there are no new packets entering the node.

To solve (10a), the predicted trajectories of adjacent nodes $r_{\text{in},\gamma}^{k-1}$, $r_{\text{out},\beta}$ and $s^k_{\beta}$, as well as the current size of the circuit queue in the current node $s^0_{\text{init}}$ have to be supplied. Note that according to (3), we set $r_{\text{max},\alpha} = r_{\text{min},\gamma}$.

The objective in (10a) is further modified by introducing a discount factor ($d$). This is necessary because naively implementing our presented fairness formulation also results in unfairness along the prediction horizon, where it is always preferable to increase the rate of the smallest element in a sequence for a given circuit. In practice, however, we want to send and receive as soon as possible as long as instantaneous fairness is achieved. In Appendix A we present a guideline on how to choose an upper bound for $d$ to obtain the desired behavior.

We implement PredicTor based on CasADi [18] in combination with IPOPT [19] and MA27 linear solver for fast state-of-the-art optimization.

### C. Optimization problem

We propose the following OCP for congestion control with fairness formulation for node $\alpha$, predecessor node $\beta$ and successor node $\gamma$.

$$
\min \quad \sum_{k=0}^{N_{\text{horz}}} d^k ((\Delta r_{\text{in},\alpha}^k)^2 + (\Delta r_{\text{out},\alpha}^k)^2)
$$

(10a)

subject to:

$$
\Delta s_{\alpha|\beta}^k = s^k_{\alpha|\beta} - \Delta s_{\alpha|\beta}^{k},
$$

(9a)

$$
\Delta s_{\alpha|\beta}^k = \Delta s_{\alpha|\beta}^{k} + \Delta t (r_{\text{in},\alpha}^k - r_{\text{out},\beta}^k),
$$

(9b)

Figure 3: Information exchange between nodes $n_{\alpha}$ and $n_{\beta}$.
the circuit queue $s^\text{init}_\alpha$ of the current node $\alpha \in N$ and with the received information from adjacent nodes. The optimal solution of (10) is converted to trajectories of incoming ($r_{\text{in},\alpha}$) and outgoing ($r_{\text{out},\alpha}$) rates, where the first element of $r_{\text{out},\alpha}$ is used to control at which rate data is sent. In particular, we employ a token-bucket method [17] to shape the outgoing traffic. Note that we are controlling the data rates on a per-circuit basis, which is similar to the aforementioned PCTCP [5].

In order to exchange the trajectories between relays, we extend the Tor protocol with respective control messages. Entry (exit) nodes do not have a predecessor (successor) to exchange data. In this case, we provide reasonable, synthetic trajectories to bootstrap the data transfer, and behave accordingly. For example, the first node in a circuit reads data from its source according to its computed incoming rate.

While PredicTor is generally agnostic to the underlying transport protocol, we implement it using TCP as a reliability mechanism, to avoid packet loss and packet reordering.

IV. RESULTS

Evaluating the proposed controller on the live Tor network is neither feasible nor responsible, due to its sensitive nature. Instead, the performance of PredicTor is investigated in simulation studies. To evaluate the performance of PredicTor, we use ns-3, a discrete-event network simulator that offers a safe simulation environment. It achieves a high degree of realism by emulating the network down to the physical layer, including queueing effects, potential packet loss, and other network effects.

For the evaluation, we focus on several core metrics that are relevant in this context: The amount of data transferred within a given time span gives an indication of how well the available resources are utilized. Comparing these values between circuits constitutes a measure of fairness. On the other hand, the byte-wise latency between data entering and leaving the network is important to characterize the applicability of the system for end users. Latency is strongly influenced by the size of queues within the network. We therefore also consider the backlog, which constitutes a metric for the overall load of the network.

The results presented in the following are obtained with a discount factor as shown in (11), where we choose $d_0 = \frac{1}{3}$, as discussed in Appendix A.

A. Open-loop prediction

We begin this section by investigating the decision-making process of the proposed controller in (10) by studying an open-loop prediction. This allows to highlight several interesting aspects of the behavior of PredicTor, before we present the closed-loop distributed application. In Figure 4, we showcase a result of (10) for the central node of the presented Tor topology in Figure 2. Again, we denote $\alpha$ the current node, $\beta$ its predecessor and $\gamma$ its successor.

We created a synthetic scenario, consisting of trajectories $r^\text{max}_{\text{out},\alpha}$, $r_{\text{out},\beta}$, $s_\beta$, as well as the initial circuit queues ($s^\text{init}_\alpha$). For the incoming connections, the proposed controller can now determine the optimal trajectory $r_{\text{in},\alpha}$. Since we need to consider the time delay (upstream information exchange), changes to the outgoing rate $r_{\text{out},\beta}$ are not immediately possible. The effect of the change $r_{\text{in},\alpha} - r_{\text{out},\beta}$ shows itself in the corrected prediction of the source circuit queue. We see, for example, how $r_{\text{in},1}$ for circuit 1 is chosen such that the queue for that circuit is emptied at the source node 2.

For the outgoing connections, we obtain the optimal trajectory ($r_{\text{out},\alpha}$). We notice that at the beginning of the horizon, all circuits are assigned the same fair rate 3. Over time, the controller first reduces the rate for circuit 1 ($r_{\text{out},1}$), as its circuit queue is emptying 4 and the incoming rate is also vanishing 5. Afterwards, the controller reduces the rate for circuit 2 ($r_{\text{out},2}$) to meet exactly the incoming rate ($r_{\text{in},2}$) 6. We can also see how individual constraints
for \( r_{\text{out}}^{\text{max}} \) are obeyed as well as capacity constraints for incoming and outgoing rates.

**B. Comparison**

In this section, we showcase the closed-loop behavior of PredicTor for two different scenarios and the presented Tor topology from Figure 2. We compare the performance of PredicTor with Tor (current standard) as well as PCTCP. Closed-loop means that at each MPC step we are updating the current state of the system from measurements obtained with ns-3. Furthermore, we receive updated information from all adjacent nodes. Two scenarios are investigated. In scenario 1, circuits 1-3 start sending at the beginning of the simulation window. Circuits 1 and 3 have an infinite source of packets to forward, whereas circuit 2 stops and restarts twice during the simulated window. In scenario 2, all circuits have an infinite source of packets. This scenario is considered for the fairness evaluation because it better approximates stationary behavior.

In Figure 5, we display the outgoing rates \( (r_{\text{out}}) \) of the middle node for scenario 1 over the course of the simulation time. Note that all rates are obtained from the ns-3 simulation. PredicTor shows a desirable behavior with constant, sustainable rates and smooth transitions when circuit 2 stops and restarts. Fair behavior can be observed in these transitions: all circuits share the same rate during activity and circuit 1 and 3 have an infinite source of packets to forward, whereas circuit 2 stops and restarts twice during the simulated window. In scenario 2, all circuits have an infinite source of packets. This scenario is considered for the fairness evaluation because it better approximates stationary behavior.

In Figure 6, we compare the performance of PredicTor with Tor (current standard) as well as PCTCP. Closed-loop means that at each MPC step we are updating the current state of the system from measurements obtained with ns-3. Furthermore, we receive updated information from all adjacent nodes. Two scenarios are investigated. In scenario 1, circuits 1-3 start sending at the beginning of the simulation window. Circuits 1 and 3 have an infinite source of packets to forward, whereas circuit 2 stops and restarts twice during the simulated window. In scenario 2, all circuits have an infinite source of packets.

![Fig. 5: Comparison of outgoing rates \((r_{\text{out}})\) for the central node as shown in the Tor topology from Figure 2 for scenario 1. Rates for all methods are calculated from ns-3 simulation results as the number of packets that are forwarded within one sampling time step \((\Delta t = 0.04\text{s})\).](image)

![Fig. 6: Comparison of total backlog (packets in the network) in the Tor topology from Figure 2 for scenario 1. Backlogs are calculated from ns-3 simulation results.](image)

![Fig. 7: Histogram of latency for received packets in the Tor topology from Figure 2 for scenario 1. Latencies are calculated from ns-3 simulation results and cumulated for all three circuits.](image)

![Fig. 8: Comparison of latency and transferred data.](image)

| circuit | mean latency \(^1\) [ms] | data transferred \([\times 10^5\] packets] |
|--------|----------------|----------------|
|        | PredicTor | Tor | PCTCP | PredicTor | Tor | PCTCP |
| 1      | 103       | 536 | 596   | 8.93      | 6.82 | 8.72  |
| 2      | 117       | 523 | 669   | 8.92      | 6.82 | 9.58  |
| 3      | 105       | 589 | 643   | 8.93      | 13.13| 8.69  |
| Total  | 106       | 558 | 624   | 26.78     | 26.78| 26.98 |

\(^1\)scenario 1, \(^2\)scenario 2

We further compare PredicTor, Tor, and PCTCP in Figure 6 and 7 where we display the backlog and latency. PredicTor succeeds at its primary goal of sustaining a manageable backlog, especially compared to Tor’s and PCTCP’s approach. The importance of this effective congestion control becomes apparent in Figure 7 where we compare histograms for the latencies of received packets. PredicTor significantly improves on Tor and PCTCP with an average latency of 106 ms, in contrast to 558 ms and 624 ms, where a theoretical minimum of 80 ms is possible.

In Table II, we summarize and compare the mean latency as well as the total number of received packets (throughput) for each circuit. Regarding throughput, the three methods perform similarly, with the difference that only PredicTor achieves near perfect fairness. Tor clearly discriminates circuit 1 and 2 which share a connection, while PCTCP, as
expected, manages to revise this effect to some extent.

While being clearly advantageous with respect to backlog and latency, our proposed congestion controller comes at the cost of using network capacity for the exchange of messages for distributed MPC. These messages are not included in the presented figures above. We quantify their effect for the presented scenario and found that this overhead would reduce the throughput by 5.39%.

Furthermore, it is clear that PredicTor introduces significant complexity compared to the previous methods. However, the optimization problem \([10]\) is convex, which guarantees a global solution in polynomial time. For the given scenario, we obtain a solution in around 0.1 ms (laptop-grade CPU), which is sufficient for a timestep of 40 ms. The problem complexity (number of optimization variables and constraints) grows linearly with the number of circuits per node and it is therefore expected that also realistically large topologies can be tackled with the approach in real-time.

V. Conclusion

In this work, we have proposed a novel model predictive control formulation to tackle the challenge of congestion in Tor, whilst fairly allocating resources. PredicTor is a distributed approach that relies on exchanging information in the form of predicted actions to all adjacent nodes. We evaluate the proposed method in the state-of-the-art network simulator ns-3 and compare results to the present method of congestion control in Tor, as well as an adaption thereof (PCTCP). PredicTor significantly outperforms both methods in terms of latency. In our test scenario, we reduced the mean latency from 558 ms (Tor) and 624 ms (PCTCP) to just 106 ms. PredicTor is at the same time superior in fairness and has a similar throughput. The exchange of information slightly reduces this last figure by 5.39%, which is found to be an acceptable trade-off.

APPENDIX

A. Discount Factor

Claim 1: Let the discount factor take the form:

\[
d^{k+1} = d_0 \cdot d^k,
\]

with \(d_0 \leq \frac{1}{3}\). Let \(\Delta \tilde{r}_{\text{out}, \alpha,i}, \Delta \tilde{r}_{\text{in}, \alpha,i}\) be a feasible solution of \([10]\). Under the assumption that for circuit \(i \in P_{\alpha}\):

\[
\Delta \tilde{r}^k_{\text{in}, \alpha,i} \leq \Delta \tilde{r}^{k+1}_{\text{in}, \alpha,i},
\]

and if it is feasible, the optimal solution \(\Delta r^*_{\text{in}, \alpha}\) of \([10]\) will have the following property:

\[
\Delta r^*_{\text{in}, \alpha,i} = \Delta \tilde{r}^k_{\text{in}, \alpha,i} - m^k,
\]

\[
\Delta r^{k+1*}_{\text{in}, \alpha,i} = \Delta \tilde{r}^{k+1}_{\text{in}, \alpha,i} + m^k,
\]

where \(0 < m^k \leq \Delta \tilde{r}^k_{\text{in}, \alpha,i}\). This means that it is optimal to reduce the rate at time \(k\) by magnitude \(m^k\) whilst increasing the rate at \(k\) by the same magnitude. The same holds for the outgoing rate \((\Delta r_{\text{out}, \alpha,i})\).

Proof: We denote with \(c_{i,k}(m)\) the cost of \([10]\), where for circuit \(i\) at time \(k\) the rate \(\Delta r^k_{\text{in}, \alpha,i}\) has been decreased by magnitude \(m^k\) at the cost of increasing \(\Delta r^{k+1}_{\text{in}, \alpha,i}\) at \(k + 1\) by the same magnitude. We want to find a value of \(d_0\) for \(0 < m^k \leq \Delta r^k_{\text{in}, \alpha,i}\), such that:

\[
c_{i,k}(m) - c_{i,k}(0) \leq 0.
\]

After subtracting all unchanged terms, we obtain:

\[
d^k (\Delta \tilde{r}^k_{\text{in}, \alpha,i} - m^k)^2 + d^{k+1} (\Delta \tilde{r}^{k+1}_{\text{in}, \alpha,i} - m^k)^2 - d^k (\Delta \tilde{r}^{k+1}_{\text{in}, \alpha,i})^2 - d^{k+1} (\Delta \tilde{r}^{k+1}_{\text{in}, \alpha,i})^2 \leq 0.
\]

We consider \([11]\) and expand the quadratic terms, such that:

\[-2\Delta \tilde{r}^k_{\text{in}, \alpha,i}m^k + (m^k)^2 + d_0 (2\Delta \tilde{r}^{k+1}_{\text{in}, \alpha,i}m^k + (m^k)^2) \leq 0.
\]

Due to \([12]\), the inequality still holds when substituting:

\[
\Delta r^{k+1}_{\text{in}, \alpha,i} = \Delta r^k_{\text{in}, \alpha,i}.
\]

Considering that \(0 \leq m^k\), we can further simplify the inequality:

\[-2\Delta \tilde{r}^k_{\text{in}, \alpha,i} + m^k + d_0 (2\Delta \tilde{r}^{k+1}_{\text{in}, \alpha,i} + m^k) \leq 0.
\]

The inequality still holds when substituting \(\Delta r^{k+1}_{\text{in}, \alpha,i} = m^k\), since \(m^k \leq \Delta r^k_{\text{in}, \alpha,i}\):

\[
d_0 (3m^k) \leq m^k,
\]

\[
d_0 \leq \frac{1}{3}.
\]

The proof is identical for the outgoing rate \((\Delta r_{\text{out}, \alpha,i})\). ■

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