Three Spin Spiky Strings in $\beta$-deformed Background

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ABSTRACT: We study rigidly rotating strings in $\beta$-deformed AdS$_5 \times S^5$ background with one spin along AdS$_5$ and two angular momenta along $S^5$. We find the spiky string solutions and present the dispersion relation among various charges in this background. We further generalize the result to the case of four angular momenta along AdS$_5 \times S^5$.

KEYWORDS: AdS-CFT Correspondence, Bosonic Strings.
1. Introduction

AdS/CFT correspondence [1] relates the spectrum of free strings on AdS$_5 \times S^5$ with the spectrum of operator dimensions in the $N = 4$ Supersymmetric Yang-Mills (SYM) theory in four dimensions. This mapping is highly nontrivial and challenging because of our lack of understanding of the full string theory spectrum. Hence, it is instructive to look at both the gauge and gravity theories at certain limits such as large angular momentum limit and then compare the spectrum on both sides. Further, $N = 4$ SYM theory can be described by integrable spin chain model where the anomalous dimension of the gauge invariant operators were found [2]. It was further noticed that string theory has as integrable structure in the semiclassical limit and the anomalous dimension in the $N = 4$ SYM can be derived from the relation between conserved charges of the worldsheet solitonic string solution of the dual string theory on AdS$_5 \times S^5$ background.

In the past few years the integrability of both string theory and gauge theory side has played a key role in proving the duality conjecture better. The most frequently studied cases were rotating and pulsating strings solutions in certain limits of spin waves in long-wave approximation and interesting observations were made in [3], [4], [5], [6], [7]. Another interesting case is the low lying spin states which are equivalent to the so called magnon states. In this connection, Hofman and Maldacena in [8] have derived a mapping between particular state (magnon) of spin chain with the semiclassical string states on $R_t \times S^2$. Further it was realized that magnons are special cases of more general solutions known as spikes and the dual gauge theory operators have been analyzed in detail in [9]. It was also observed in [10] that both giant magnon and single spike solutions can be viewed as two different limits of the same rigidly rotating strings on $S^2$ and $S^3$. In this connection, a large class of multispin spiky string and giant magnon solutions have been studied, in
various backgrounds including the orbifolded and non-AdS backgrounds, for example in \[12\]

We are interested in studying a class of spiky string solution in the so-called Lunin-Maldacena background \[13\]. The integrability of Lunin-Maldacena background has been studied in \[14\], \[15\], \[16\]. The giant magnon and single spike solutions are studied in detail including the integrable models, for example in \[17\], \[18\].

More recently in \[19\], \[20\], \[21\], a more general class of solutions with three divergent angular momenta have been studied and interesting dispersion relations among various conserved charges have been obtained. In the present paper, we wish to generalize the result of \[22\] in a $\beta$-deformed background with one(two) spin in AdS and two spins on the deformed sphere. Knowing such solutions on the gravity side will definitely help us in finding out the nature of corresponding operators on the gauge theory side.

The rest of the paper is organized as follows. In section-2 we write the relevant part of the Lunin-Maldacena background which will be useful for studying the rigidly rotating string on this background. In section 3, we study the rotating open string in $AdS_3 \times S^3_\gamma$ backgrounds with one spin along the AdS$_3$ and two angular momenta along the deformed sphere. We compute all the conserved charges and find two limiting cases corresponding to giant magnon and single spike solutions. We write down the dispersion relation among various divergent momenta in both cases. We further generalize the above solutions to the case of rotating string with two spins along the AdS$_3$ and two angular momenta along the deformed sphere. We write down the corresponding dispersion relation for the giant magnon solution. Finally in section 4, we conclude with some remarks.

2. $\beta$-deformed $AdS_5 \times S^5$ background

Here we present the general background for $\beta$-deformed $AdS_5 \times S^5$ found by Lunin and Maldacena \[13\]. This background is obtained from pure $AdS_5 \times S^5$ by a series of STsTS transformations \[23\], which is dual to the Leigh-Strassler marginal deformations of $N = 4$ SYM \[24\]. The deformed parameter $\beta = \gamma + i\sigma_d$ in general is a complex number, but here we restrict $\beta$ to its real part only. Thus, the relevant metric component of the supergravity background dual to real $\beta$-deformations of $N = 4$ SYM is:

$$ds^2 = R^2 \left( ds^2_{AdS_5} + \sum_{i=1}^{3} (d\mu_i^2 + G\mu_i^2 d\phi_i^2) + \tilde{\gamma}^2 G\mu_1^2\mu_2^2\mu_3^2 (\sum_{i=1}^{3} d\phi_i^2) \right), \quad (2.1)$$

which also have the dilaton, Ramond-Ramond (RR) and Neveu-Schwarz-Neveu-Schwarz (NS-NS) fields.

The antisymmetric form of the $B$-field relevant for our classical string analysis is:

$$B = R^2\tilde{\gamma} G(\mu_1^2\mu_2^2 d\phi_1 d\phi_2 + \mu_2^2\mu_3^2 d\phi_2 d\phi_3 + \mu_1^2\mu_3^2 d\phi_1 d\phi_3), \quad (2.2)$$

where

$$\tilde{\gamma} = R^2\gamma, \quad R^2 = \sqrt{4\pi g_s N},$$
\[ G = \frac{1}{1 + \tilde{\gamma}^2 (\mu_1^2 \mu_2^2 + \mu_2^2 \mu_3^2 + \mu_1^2 \mu_3^2)} \]

\[ \mu_1 = \sin \theta \cos \psi, \quad \mu_2 = \cos \theta, \quad \mu_3 = \sin \theta \sin \psi. \]  

(2.3)

3. Semiclassical Strings on \(AdS_3 \times S_3^\gamma\)

We restrict the motion of the string on \(AdS_3 \times S_3^\gamma \subset AdS_5 \times S_5^\gamma\). This space can be achieved by putting \(\mu_3 = 0, \phi_3 = 0\) i.e, \(\psi = 0, \phi_3 = 0\) in (2.1) - (2.3). Thus the metric components of the deformed \(AdS_3 \times S_3^\gamma\) background is:

\[ ds^2 = - \cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2 + G \sin^2 \theta d\phi_1^2 + G \cos^2 \theta d\phi_2^2, \]  

(3.1)

where \(G = \frac{1}{1 + \tilde{\gamma}^2 \sin^2 \theta \cos^2 \theta}\) and the relevant non-zero component of B-field due to the series of T-duality transformations is given by

\[ B_{\phi_1 \phi_2} = \tilde{\gamma} G \sin^2 \theta \cos^2 \theta. \]  

(3.2)

The Polyakov action for the fundamental string in this background is given by

\[ S = \frac{T}{2} \int d\tau d\sigma \left[ - \cosh^2 \rho \dot{t}^2 + \dot{\rho}^2 + \sinh^2 \rho \dot{\phi}^2 + \dot{G} \sin^2 \theta \dot{\phi_1}^2 + \dot{G} \cos^2 \theta \dot{\phi_2}^2 \right] , \]

(3.3)

where the ‘dot’ and ‘prime’ denote the derivatives with respect to \(\tau\) and \(\sigma\) respectively and \(T = \frac{\lambda}{2\pi}\), where \(\lambda\) is the ‘t Hooft coupling constant. We take the following ansatz for the rotating open string

\[ t = \tau + F_1(y), \quad \rho = \rho(y), \quad \phi = \omega_1(\tau + F_2(y)), \]

\[ \phi_1 = \tau + F_3(y), \quad \theta = \theta(y), \quad \phi_2 = \omega_2(\tau + F_4(y)), \]  

(3.4)

where \(y = a\sigma - b\tau\). Solving the equations of motion for \(t, \phi, \phi_1\) and \(\phi_2\), we have the following expression for \(F_1, F_2, F_3\) and \(F_4\)

\[ F_1_y = \frac{1}{a^2 - b^2} \left( \frac{A_1}{\cosh^2 \rho} - b \right), \]

\[ F_2_y = \frac{1}{a^2 - b^2} \left( \frac{A_2}{\sinh^2 \rho} - b \right), \]

\[ F_3_y = \frac{1}{a^2 - b^2} \left( \frac{A_3}{G \sin^2 \theta} - b - a\tilde{\gamma} \omega_2 \cos^2 \theta \right), \]  

(3.4)
\[ F_{4y} = \frac{1}{a^2 - b^2} \left( \frac{A_4}{G \cos^2 \theta} - b + \frac{a \gamma}{\omega_2} \sin^2 \theta \right), \]  

(3.5)

where \( F_y = \frac{\partial F}{\partial y} \).

Now, the two virasoro constraints \( T_{\tau \sigma} = 0 \) and \( T_{\tau \tau} + T_{\sigma \sigma} = 0 \) give the following two equations

\[ \rho_y^2 + \theta_y^2 = \cosh^2 \rho \left( -\frac{1}{b} F_{1y} + F_{1y}^2 \right) - \omega_1^2 \sinh^2 \rho \left( -\frac{1}{b} F_{2y} + F_{2y}^2 \right) - G \sin^2 \theta \left( -\frac{1}{b} F_{3y} + F_{3y}^2 \right) - \omega_2^2 G \cos^2 \theta \left( -\frac{1}{b} F_{4y} + F_{4y}^2 \right) \]

\[ \rho_y^2 + \theta_y^2 = \cosh^2 \rho \left( -\frac{1}{a^2 + b^2} F_{1y} + F_{1y}^2 \right) - \omega_1^2 \sinh^2 \rho \left( -\frac{1}{a^2 + b^2} F_{2y} + F_{2y}^2 \right) - G \sin^2 \theta \left( -\frac{1}{a^2 + b^2} F_{3y} + F_{3y}^2 \right) - \omega_2^2 G \cos^2 \theta \left( -\frac{1}{a^2 + b^2} F_{4y} + F_{4y}^2 \right), \]

(3.6)

Using (3.5) in (3.6) we get the following relation among various constants

\[-A_1 + \omega_1^2 A_2 + A_3 + \omega_2^2 A_4 = 0 \]  

(3.7)

Now, the equation of motion for \( \rho \) becomes

\[(a^2 - b^2)\rho_{yy} + \frac{1}{a^2 - b^2} \sinh \rho \cosh \rho \left( \frac{A_4^2}{\cosh^4 \rho} - a^2 - \frac{\omega_1^2 A_3^2}{\sinh^4 \rho} + \omega_1^2 a^2 \right) = 0 , \]

(3.8)

where \( \rho_{yy} = \frac{\partial^2 \rho}{\partial y^2} \). We can compute equation of motion for \( \theta \) from the first Virasoro constraint (3.6) which is given by the relation

\[ \theta_y^2 = -\rho_y^2 + \cosh^2 \rho \left( -\frac{1}{b} F_{1y} + F_{1y}^2 \right) - \omega_1^2 \sinh^2 \rho \left( -\frac{1}{b} F_{2y} + F_{2y}^2 \right) - G \sin^2 \theta \left( -\frac{1}{b} F_{3y} + F_{3y}^2 \right) - \omega_2^2 G \cos^2 \theta \left( -\frac{1}{b} F_{4y} + F_{4y}^2 \right) . \]

(3.9)

Once we get the form of \( \rho_y \) then we will be able to compute \( \theta_y \). The conserved charges are

\[ E = T \int d\sigma \ \cosh^2 \rho (1 - b F_{1y}), \]

\[ S = \omega_1 T \int d\sigma \ \sinh^2 \rho (1 - b F_{2y}), \]

\[ J_1 = T \int d\sigma \ G \sin^2 \theta (1 - b F_{3y}), \]

\[ J_2 = \omega_2 T \int d\sigma \ G \sin^2 \theta (1 - b F_{4y}). \]

(3.10)
3.1 Giant Magnon Solution

For finding out the giant magnon solution, we choose the integration constants as $A_1 = b, A_2 = 0, A_3 = b$ and $A_4 = 0$. The solution of equation (3.8) becomes

$$\rho_y^2 = \frac{1}{(a^2 - b^2)^2} \left( a^2 (1 - \omega_1^2) - \frac{b^2}{\cosh^2 \rho} \right) \sinh^2 \rho. \quad (3.11)$$

Using (3.11) and the above integration constants in (3.9), we get the following expression for $\theta_y$

$$\theta_y^2 = \frac{1}{(a^2 - b^2)^2} \left[ a^2 (1 - \omega_2^2) \cos^2 \theta + b^2 - \frac{b^2}{G \sin^2 \theta} + 2ab\gamma \omega_2 \cos \theta \right]. \quad (3.12)$$

Note that the above equation can be rewritten as

$$\theta_y = \frac{\Omega_0}{a^2 - b^2} \cot \theta \sqrt{\sin^2 \theta - \sin^2 \theta_0}, \quad (3.13)$$

where, $\sin \theta_0 = \frac{b}{\sqrt{a^2 - \omega_1^2}}$, and $\Omega_0 = \sqrt{a^2 - (a\omega_2 - b\gamma)^2}$. Now the conserved charges (3.10) become

$$E = T \frac{a^2 - b^2}{a^2 - b^2} \int_0^\infty d\rho \ (a^2 \cosh^2 \rho - b^2),$$

$$S_{\omega_1} = T \frac{a^2 - b^2}{a^2 - b^2} \int_0^\infty d\rho \ a^2 \sinh^2 \rho,$$

$$J_1 = T \frac{a^2 - b^2}{a^2 - b^2} \int_0^\infty d\rho \ (a^2 \sin^2 \theta - b^2),$$

$$J_2 = T \frac{a^2 - b^2}{a^2 - b^2} \int_0^\infty d\rho \ a^2 (1 - \frac{b\gamma}{a\omega_2}) \cos^2 \theta. \quad (3.14)$$

It is clear from the above expressions that we have the following relation among various conserved charges

$$E - J_1 = \frac{S}{\omega_1} + \frac{J_2'}{\omega_2}, \quad (3.15)$$

where $J_2' = \frac{a\omega_2}{\sqrt{a^2 - \Omega_0^2}} J_2$. As the conserved charges are divergent, we use the same regularization technique as in [22] to remove the divergent part of the conserved charges. Let us write

$$S_{\omega_1} = 2Ta \frac{a^2 - b^2}{a^2 - b^2} \int_0^\infty d\rho \frac{\sinh^2 \rho}{\rho_y} = 2T \frac{a^2 - b^2}{\sqrt{1 - \omega_1^2}} \int_1^\infty dz \frac{z}{\sqrt{z^2 - z_0^2}}, \quad (3.16)$$

where $z = \cosh \rho$ and $z_0 = \cosh \rho_0 = \frac{b}{a\sqrt{1 - \omega_1^2}}$. Subtracting the divergent part of the integral (3.16), we have the regulated value given as

$$\frac{S_{\text{reg}}}{\omega_1} = -\frac{\sqrt{\lambda}}{2} \sqrt{\frac{1 - z_0^2}{1 - \omega_1^2}}. \quad (3.17)$$
From the above expression, we find the following relation
\[
\frac{S_{\text{reg}}}{\omega_1} = -\sqrt{S_{\text{reg}}^2 + \frac{\lambda}{\pi^2} (1 - \frac{z_0^2}{z_0^2})}.
\] (3.18)

Further, the time difference between two end points of the open string is given by
\[
\Delta t = -\frac{2b}{a\sqrt{1 - \omega_1^2}} \int_{-\infty}^{\infty} d\rho \frac{\tanh \rho}{\cosh^2 \rho - \frac{b^2}{a^2(1 - \omega_1^2)}}
\]
\[
= -2 \tan^{-1} \frac{z_0}{\sqrt{1 - z_0^2}}.
\] (3.19)

Now the equation (3.18) can be written in the following form
\[
\frac{S_{\text{reg}}}{\omega_1} = -\sqrt{S_{\text{reg}}^2 + \frac{\lambda}{\pi^2} \cos^2 \frac{\Delta t}{2}}.
\] (3.20)

Further, the angle difference between two end points of the open string is given by
\[
\frac{\Delta \phi_1}{2} = \int_{\theta_0}^{\pi} d\theta \frac{\gamma \sqrt{a^2 - \Omega_1^2}}{\Omega_0} \cot \theta \frac{\sin^2 \theta + \frac{b}{\sqrt{a^2 - \Omega_1^2}}}{\sin^2 \theta - \sin^2 \theta_0} = \frac{\pi}{2} - \theta_0 + \frac{\gamma \sqrt{a^2 - \Omega_1^2}}{\Omega_0} \cos \theta_0.
\] (3.21)

Now, we find the giant magnon dispersion relation as
\[
(E - J_1)_{\text{reg}} = -\sqrt{S_{\text{reg}}^2 + \frac{\lambda}{\pi^2} \cos^2 \frac{\Delta t}{2}} + \sqrt{J_2^2 + \frac{\lambda}{\pi^2} \sin^2 \frac{\Delta \phi_1}{2}}.
\] (3.22)

This expression matches with that of [22] even if we are dealing with $\beta$-deformed background and has implicit dependence on the deformation parameter $\gamma$ in the definition of $\Delta \phi_1$.

3.2 Single spike Solution

To obtain the single spike solution, we chose the integration constants as: $A_1 = \frac{a^2}{b} = A_3$ and $A_2 = 0 = A_4$. The solution of equation (3.8) now becomes
\[
\rho_y^2 = \frac{1}{(a^2 - b^2)^2} \left( a^2(1 - \omega_1^2) - \frac{a^4}{b^2 \cosh^2 \rho} \right) \sinh^2 \rho.
\] (3.23)

Using (3.23) and the above integration constants in (3.9), we have the following expression for $\theta_y$:
\[
\theta_y = \frac{a \Omega_1}{a^2 - b^2} \cot \theta \sqrt{\sin^2 \theta - \sin^2 \theta_1},
\] (3.24)

where, $\sin \theta_1 = \frac{a}{\Omega_1}$, and $\Omega_1 = \sqrt{1 - (\omega_2 - \frac{a^2}{b})^2}$. Thus the conserved charges (3.10) becomes:
\[
E = \frac{T}{a^2 - b^2} \int d\sigma \ a^2 \sinh^2 \rho.
\]
\[
\frac{S}{\omega_1} = \frac{T}{a^2 - b^2} \int d\sigma \ a^2 \sinh^2 \rho,
\]
\[
J_1 = -\frac{T}{a^2 - b^2} \int d\sigma \ a^2 \cos^2 \theta,
\]
\[
J_2 = \frac{T}{a^2 - b^2} \int d\sigma \ a^2 (1 - \frac{a\tilde{\gamma}}{b\omega_2}) \cos^2 \theta.
\]

(3.25)

From (3.25), we get the following relation between the conserved charges
\[
E - J_1 = \frac{S}{\omega_1} + \frac{J''_2}{\omega_2},
\]
where \(J''_2 = \frac{b\omega_2}{\sqrt{1 - \Omega_1}} J_2\). For completeness we wish to compute \(J_1\) and \(J_2\) as
\[
J_1 = -\frac{2Ta}{a^2 - b^2} \int_{\frac{\theta_1}{2}}^{\theta_1} \frac{d\theta}{\theta_y} \cos^2 \theta = \frac{2T}{\Omega_1} \cos \theta_1 ,
\]
\[
J_2 = \frac{2Ta}{a^2 - b^2} \sqrt{1 - \Omega_1^2} \int_{\frac{\theta_1}{2}}^{\theta_1} \frac{d\theta}{\theta_y} \cos^2 \theta = -\frac{2T}{\Omega_1} \sqrt{1 - \Omega_1^2} \cos \theta_1 .
\]

(3.27)

From (3.27), we have the following relation between \(J_1\) and \(J_2\)
\[
J_1 = \sqrt{J_2^2 + \frac{\lambda}{\pi^2} \cos^2 \theta_1} .
\]

(3.28)

This expression has already been found in [18]. One can also see that from (3.25) we have the following relation between \(E\) and \(S\),
\[
E - \frac{S}{\omega_1} = 0.
\]

(3.29)

### 3.3 Magnon solution with Four spins

In this section, we would like to generalize the results of the previous section to include two spins along AdS and two angular momenta along the deformed \(S^3\). The relevant metric and B-field is given by
\[
ds^2 = -\cosh^2 \rho dt^2 + d\rho^2 + \sinh^2 \rho (d\psi^2 + \sin^2 \psi d\xi_1^2 + \cos^2 \psi d\xi_2^2)
\]
\[
+ d\theta^2 + G \sin^2 \theta d\phi_1^2 + G \cos^2 \theta d\phi_2^2 ,
\]
\[
B_{\phi_1\phi_2} = \tilde{\gamma} G \sin^2 \theta \cos^2 \theta ,
\]
\[
G^{-1} = 1 + \tilde{\gamma}^2 \sin^2 \theta \cos^2 \theta .
\]

(3.30)
We take the following anstaz

\[ t = \tau + G_1(y), \quad \rho = \rho(y), \quad \psi = \text{constant}, \quad \xi_1 = \omega_1(\tau + G_2(y)), \]
\[ \xi_2 = \omega_2(\tau + G_3(y)), \quad \phi_1 = \tau + G_4(y), \quad \theta = \theta(y), \quad \phi_2 = \omega_3(\tau + G_5(y)), \]

\[(3.31)\]

where \( y = a\sigma - b\tau \). Solving the equation of motion for the coordinates \( t, \xi_1, \xi_2, \phi_1 \) and \( \phi_2 \), we have the following expression for \( G_1(y), G_2(y), G_3(y), G_4(y) \) and \( G_5(y) \) respectively.

\[ G_1(y) = \frac{1}{a^2 - b^2} \left( \frac{A_1}{\cosh^2 \rho} - b \right), \]
\[ G_2(y) = \frac{1}{a^2 - b^2} \left( \frac{A_2}{\sinh^2 \rho \sin^2 \psi} - b \right), \]
\[ G_3(y) = \frac{1}{a^2 - b^2} \left( \frac{A_3}{\sinh^2 \rho \cos^2 \psi} - b \right), \]
\[ G_4(y) = \frac{1}{a^2 - b^2} \left( \frac{A_4}{G \sin^2 \theta} - b - a\tilde{\gamma}\omega_3 \cos^2 \theta \right), \]
\[ G_5(y) = \frac{1}{a^2 - b^2} \left( \frac{A_5}{G \cos^2 \theta} - b - \frac{a\tilde{\gamma}}{\omega_3} \cos^2 \theta \right), \]

\[(3.32)\]

where \( A_1, A_2, A_3, A_4 \) and \( A_5 \) are integration constants, which satisfy the following relation, as derived from Virasoro constraints,

\[-A_1 + \omega_1^2 A_2 + \omega_2^2 A_3 + A_4 + \omega_3^2 A_5 = 0 \cdot \]

\[(3.33)\]

The conserved charges derived from Virasoro constraints as explained earlier corresponding to \( t, \xi_1, \xi_2, \phi_1 \) and \( \phi_2 \) are \( E, S_1, S_2, J_1 \) and \( J_2 \) respectively. The charges are shown to satisfy the following dispersion relation among them

\[ E - J_1 = \frac{S_1}{\omega_1} + \frac{S_2}{\omega_2} + \frac{\tilde{J}_2}{\omega_3}, \]

\[(3.34)\]

where \( \tilde{J}_2 = \frac{J_2}{1 - \frac{\omega_3}{\omega_1}} \). While deriving the above relation we have used \( A_1 = b = A_4 \), and \( A_2 = 0 = A_3 = A_5 \). One can further use the same kind of regularization technique as discussed in previous section to get a formal expression for the giant magnon dispersion relation in the presence of two spins along \( \text{AdS}_5 \) and two angular momenta along \( S^5 \). We skip the details here.

4. Conclusions

In this paper, we have found a class of giant magnon and single spike string solutions in less supersymmetric real \( \beta \)-deformed Lunin-Maldacena background with three spins along various directions of \( \text{AdS}_5 \) and \( S^5 \). The relation among conserved quantities in (3.15) is
similar to the undeformed $AdS_3 \times S^3$ giant magnon relation obtained in [22]. As expected, for zero deformed parameter i.e, $\tilde{\gamma} = 0$, we get the same value of $J_2$ and the same relation among the charges as derived in [22]. Thus, we get the result as expected in [17], where the authors claimed that the deformed parameter should not appear explicitly in the dispersion relation, however can be absorbed in the definition of the conserved charges. As argued in [17] if the magnon dispersion relation depends explicitly on the deformed parameter then in general we cannot find integrable spin chain systems. We also discarded the divergent terms in (3.22) and found the regularized dispersion relation which is superposition of two magnon bound states where the worldsheet momentum is shifted by a factor $2\pi\gamma$, as in [17],[18]. At this point, it is worth mentioning about [25], where a class of magnon solutions were derived and it was shown that in the limit of $J_2 \to \infty$, the dispersion relation was independent of the deformation parameter. In the present case however we would like to stress that our solutions are very similar to the ones presented in [18], because if we switch off the spin along the $AdS$ space we get back dispersion relation presented there. However it will be interesting to generalize the solutions of [25] to include an extra spin along the $AdS$ direction and check the finite size correction to the magnon and spike dispersion relation. It would also be interesting to look for the dual operators on the boundary, as one would expect from the AdS/CFT duality. The exact nature of the operators are unknown. But the expectations from [27] leads us to believe that such dual operators would exist. It would really be challenging to construct such operators dual to the spiky strings presented in this paper.

References

[1] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [hep-th/9711200].

[2] J. A. Minahan and K. Zarembo, “The Bethe ansatz for N=4 super Yang-Mills,” JHEP 0303, 013 (2003) [hep-th/0212208].

[3] A. A. Tseytlin, “Spinning strings and AdS / CFT duality,” In *Shifman, M. (ed.) et al.: From fields to strings, vol. 2* 1648-1707 [hep-th/0311139].

[4] H. Dimov and R. C. Rashkov, “Generalized pulsating strings,” JHEP 0405, 068 (2004) [hep-th/0404012].

[5] M. Smendback, “Pulsating strings on AdS(5) x S**5,” JHEP 0407, 004 (2004) [hep-th/0405102].

[6] N. Beisert, “The Dilatation operator of N=4 super Yang-Mills theory and integrability,” Phys. Rept. 405, 1 (2005) [hep-th/0407277].

[7] A. A. Tseytlin, “Semiclassical strings and AdS/CFT,” hep-th/0409296.

[8] J. Plefka, “Spinning strings and integrable spin chains in the AdS/CFT correspondence,” Living Rev. Rel. 8, 9 (2005) [hep-th/0507136].

[9] D. M. Hofman and J. M. Maldacena, “Giant Magnons,” J. Phys. A A 39, 13095 (2006) [hep-th/0604135].
[10] M. Kruczenski, “Spiky strings and single trace operators in gauge theories,” JHEP 0508, 014 (2005) [arXiv:hep-th/0410226].

[11] R. Ishizeki and M. Kruczenski, “Single spike solutions for strings on S2 and S3,” Phys. Rev. D 76, 126006 (2007) [arXiv:0705.2429 [hep-th]].

[12] N. P. Bobev, H. Dimov, R. C. Rashkov, “Semiclassical strings in Lunin-Maldacena background,” [hep-th/0506063]. S. Ryang, “Rotating strings with two unequal spins in Lunin-Maldacena background,” JHEP 0511, 006 (2005) [hep-th/0509195]. N. P. Bobev and R. C. Rashkov, “Multispin Giant Magnons,” Phys. Rev. D 74, 046011 (2006) [hep-th/0607018]. M. Kruczenski, J. Russo, A. A. Tseytlin, “Spiky strings and giant magnons on S**5,” JHEP 0610, 002 (2006) [hep-th/0607044]. J. Kluson, R. R. Nayak, K. L. Panigrahi, “Giant Magnon in NS5-brane Background,” JHEP 0704, 099 (2007). [hep-th/0703244]. H. Dimov, R. C. Rashkov, “On the anatomy of multi-spin magnon and single spike string solutions,” Nucl. Phys. B799, 255-290 (2008) [arXiv:0709.4231 [hep-th]].

B. -H. Lee, R. R. Nayak, K. L. Panigrahi, C. Park, “On the giant magnon and spike solutions for strings on AdS(3) x S**3,” JHEP 0806, 065 (2008) [arXiv:0804.2923 [hep-th]]. J. R. David, B. Sahoo, “Giant magnons in the D1-D5 system,” JHEP 0807, 033 (2008) [arXiv:0804.3267 [hep-th]]. G. Grignani, T. Harmark and M. Orselli, “The SU(2) x SU(2) sector in the string dual of N=6 superconformal Chern-Simons theory,” Nucl. Phys. B 810, 115 (2009) [arXiv:0806.4959 [hep-th]]. B. -H. Lee, K. L. Panigrahi, C. Park, “Spiky Strings on AdS(4) x CP**3,” JHEP 0811, 066 (2008) [arXiv:0807.2559 [hep-th]].

S. Ryang, “Giant Magnon and Spike Solutions with Two Spins in AdS(4) x CP**3,” JHEP 0811, 084 (2008) [arXiv:0809.5106 [hep-th]]. S. Benvenuti and E. Tonni, “Giant magnons and spiky strings on the conifold,” JHEP 0902, 041 (2009) [arXiv:0811.0145 [hep-th]]. M. C. Abbott, I. Aniceto, “Giant Magnons in AdS(4) x CP**3: Embeddings, Charges and a Hamiltonian,” JHEP 0904, 136 (2009) [arXiv:0811.2423 [hep-th]]. R. Suzuki, “Giant Magnons on CP**3 by Dressing Method,” JHEP 0905, 079 (2009) [arXiv:0902.3368 [hep-th]]. D. Arnaudov, H. Dimov and R. C. Rashkov, “On the pulsating strings in AdS_5 x T^1,1,” J. Phys. A A 44, 495401 (2011) [arXiv:1006.1539 [hep-th]]. S. Biswas and K. L. Panigrahi, “Spiky Strings on NS5-branes,” Phys. Lett. B 701, 481 (2011) [arXiv:1103.6153 [hep-th]].

[13] O. Lunin and J. M. Maldacena, “Deforming field theories with U(1) x U(1) global symmetry and their gravity duals,” JHEP 0505, 033 (2005) [hep-th/0502086].

[14] R. Roiban, “On spin chains and field theories,” JHEP 0409, 023 (2004) [hep-th/0312218].

[15] D. Berenstein and S. A. Cherkis, “Deformations of N=4 SYM and integrable spin chain models,” Nucl. Phys. B 702, 49 (2004) [hep-th/0405215].

[16] S. A. Frolov, R. Roiban and A. A. Tseytlin, “Gauge-string duality for superconformal deformations of N=4 super Yang-Mills theory,” JHEP 0507, 045 (2005) [hep-th/0503192].

[17] C. -S. Chu, G. Georgiou and V. V. Khoze, “Magnons, classical strings and beta-deformations,” JHEP 0611, 093 (2006) [hep-th/0606220].

[18] N. P. Bobev and R. C. Rashkov, “Spiky strings, giant magnons and beta-deformations,” Phys. Rev. D 76, 046008 (2007) [arXiv:0706.0442 [hep-th]].

[19] S. Giardino and H. L. Carrion, “Classical strings in AdS(4) x CP(3) with three angular momenta,” JHEP 1108, 057 (2011) [arXiv:1106.5684 [hep-th]].

[20] K. L. Panigrahi, P. M. Pradhan and P. K. Swain, “Rotating Strings in AdS(4) x CP(3) with B(NS) holonomy,” JHEP 1201, 113 (2012) [arXiv:1109.2458 [hep-th]].
[21] S. Giardino, “Divergent energy strings in $AdS_5 \times S^5$ with three angular momenta,” JHEP 1112, 022 (2011) [arXiv:1110.3682 [hep-th]].

[22] S. Ryang, “Three-spin giant magnons in AdS(5) x S**5,” JHEP 0612, 043 (2006) [hep-th/0610037].

[23] S. Frolov, “Lax pair for strings in Lunin-Maldacena background,” JHEP 0505, 069 (2005) [hep-th/0503201].

[24] R. G. Leigh and M. J. Strassler, “Exactly marginal operators and duality in four-dimensional $N=1$ supersymmetric gauge theory,” Nucl. Phys. B 447, 95 (1995) [hep-th/9503121].

[25] D. V. Bykov and S. Frolov, “Giant magnons in TsT-transformed AdS(5) x S**5,” JHEP 0807, 071 (2008) [arXiv:0805.1070 [hep-th]].

[26] T. McLoughlin and I. Swanson, “Integrable twists in AdS/CFT,” JHEP 0608, 084 (2006) [hep-th/0605018].