Comment on “Phase Diagram of the Random Energy Model with Higher-Order Ferromagnetic Term and Error Correcting Codes due to Sourlas”

In a recent Letter, Dorlas and Wedagedera (DW) have studied the random energy model (REM) with an additional p-spin ferromagnetic interaction, as a guide to the properties of a p-spin Ising model with both random spin glass and uniform ferromagnetic exchange, itself relevant to an error-correcting code. They showed that the non-glassy ferromagnetic phase, found for p = 2, to lie between the paramagnetic and glassy ferromagnetic phases, is squeezed out to larger ferromagnetic exchange as p is increased and is eliminated in the limit of p → ∞. Here we note that (i) we have solved the corresponding problem of a spherical spin system with p-spin glass interactions and r-spin ferromagnetic interactions and have shown that for all r ≥ p > 2 the opposite situation applies, namely glassy ferromagnetism is suppressed and only non-glassy ferromagnetism remains, and (ii) a simple mapping yields the results of DW and generalizations.

The Hamiltonians for both the Ising and spherical models consist of a disordered and a ferromagnetic term:

\[ H = \sum_{i_1 < i_2 \ldots < i_p} J_{i_1 \ldots i_p} \phi_{i_1} \ldots \phi_{i_p} - \frac{J_0(r - 1)!}{N^{r-1}} \sum_{i_1 < i_2 \ldots < i_r} \phi_{i_1} \ldots \phi_{i_r}, \]

(1)

where the \( J_{i_1 \ldots i_p} \) are independent Gaussian random couplings of zero mean and variance \( p! J^2 / 2 N^{p-1} \), and \( \phi_i^2 = 1 \) for Ising or \( 1 / r \sum_i \phi_i^2 = 1 \) for spherical spins. The properties of the system can be found from the free energy \( f_{SG}(M) \) of the system with \( J_0 = 0 \) and a constrained magnetization \( M \). They are obtained by minimizing the free energy

\[ f(M) = f_{SG}(M) - \frac{1}{r} J_0 M^r, \]

(2)

with respect to \( M \), which means solving

\[ f'_M(M) = \frac{df_{SG}(M)}{dM} = J_0 M^{r-1}. \]

(3)

Generally \( f'_M(M) \) is first order in small \( M \), diverges as \( |M| \to 1 \), and is monotonically increasing in between. For \( r = 1 \), corresponding to an applied field \( h = J_0 \), \( f'_M(M) = h \), so the equilibrium magnetization increases monotonically with \( h \) and tends to unity as \( h \to \infty \). For \( r = 2 \), \( f'_M(M) = J_0 M \), so there is always a solution at \( M = 0 \), and a ferromagnetic solution appears continuously when \( J_0 \geq f'_M(0) \). For \( r > 2 \), the transition is to a magnetization \( M_{min} > 0 \), and \( M_{min} \) increases with \( r \).

The true strength of this method is in predicting the onset of glassiness: this depends on which parts of \( f_{SG}(M) \) correspond to glassy solutions and so varies with model and with temperature.

In the upper curve of the Figure we show \( f'_M(M) \) for the REM above the glass transition temperature \( T_s \); below \( T_s \) the solution is glassy everywhere. At the temperature shown, the ferromagnetic transition is to a non-glassy phase for small enough \( r \), while for larger \( r \) \( M_{min} \) is already in the glassy region. As \( r \to \infty \), \( J_0 M^{r-1} \) approaches a function which jumps from zero to \( J_0 \) at \( M = 1 \), so \( M_{min} \to 1 \) and the transition is directly to the glassy ferromagnet.

In the lower curve we show \( f'_M(M) \) for the spherical p-spin model slightly above its \( T_s \); at some higher temperature the glassy region disappears; below \( T_s \) the glassy region extends down to \( M = 0 \). At the temperature shown, for small enough \( r \), \( M_{min} \) lies in the lower non-glassy branch, so increasing \( J_0 \) leads to a non-glassy ferromagnet, then a glassy, then back to a non-glassy. For some larger \( r \) the first non-glassy ferromagnet disappears, and for still larger \( r \) so does the glassy ferromagnet. A full calculation shows that the second critical value is \( r = p \).

The discussion here has been of the static spinodal transition, but it can be easily extended to the thermodynamic transition by comparing the free energies of competing phases: DW concentrate on this latter case.

Peter Gillin and David Sherrington
Theoretical Physics, 1 Keble Rd, Oxford, OX1 3NP, UK.

[1] T. C. Dorlas and J. R. Wedagedera, Phys. Rev. Lett. 83, 4441 (1999).
[2] N. Sourlas, Nature 339, 693 (1989).
[3] B. Derrida, Phys. Rev. B 24, 2613 (1981).
[4] P. Gillin and D. Sherrington, J. Phys. A (to be published).