Self–dual Lorentzian wormholes in n–dimensional Einstein gravity

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Abstract: A family of spherically symmetric, static and self–dual Lorentzian wormholes is obtained in n–dimensional Einstein gravity. This class of solutions includes the n–dimensional versions of the Schwarzschild black hole and the spatial–Schwarzschild traversable wormhole. Using isotropic coordinates we study the geometrical structure of the solution, and delineate the domains of the free parameters for which wormhole, naked singular geometries and the Schwarzschild black hole are obtained. It is shown that, in the lower dimensional Einstein gravity without cosmological constant, we can not have self–dual Lorentzian wormholes.

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I. INTRODUCTION

During the past two decades, considerable interest has grown in the field of wormhole physics, which has rapidly grown into an active area of research. Two separate directions emerged: one relating to Euclidean signature metrics and the other concerned with Lorentzian ones.

From the point of view of the Euclidean path integral formulation of quantum gravity Coleman[1], and Giddings and Strominger [2], among others, have shown that the effect of wormholes is to modify the low energy coupling constant, and to provide probability distributions for them.

On the purely gravitational side, the interest has been focused on Lorentzian wormholes. The interest in this area was especially stimulated by the pioneering works of Morris, Thorne and Yurtsever [3, 4], where static, spherically symmetric Lorentzian wormholes were defined and considered to be an exciting possibility for constructing time machine models with these exotic objects, for backward time travel (see also [5, 6]).

Most of the efforts are directed to study static configurations that must have a number of specific properties in order to be traversable. The most striking of these properties is the violation of energy conditions. This implies that the matter that generates the wormholes is exotic [3, 4, 6, 9], which means that its energy density is negative, as seen by static observers. These wormholes have no horizons and thus allow two–way passage through them.

On the other hand, many theories of unification (e.g. superstring theories, M–theory) require extra spatial dimensions to be consistent. Until today a number of important solutions of Einstein equations in higher dimensions have been obtained and studied, and they have led to important generalizations and wider understanding of gravitational fields. For example, Myers and Perry [7] have found the n–dimensional versions of the Schwarzschild, Reissner–Nordstrom and Kerr solutions, and have discussed the associated singularities, horizons and topological properties. Dianyan [8] has considered the n–dimensional Schwarzschild–de Sitter and Reissner–Nordstrom–de Sitter spacetimes.

An exact general solution of Einstein equations for spherically symmetric perfect fluids in higher dimensions was found by Krori, Borgohain and Das [9]. Ponce de Leon and Cruz [10] discuss the question of how the number of dimensions of space–time influences the equilibrium configurations of stars.

Higher dimensional wormholes have also been consid-
erated by several authors. Euclidean wormholes have been studied by Gonzales–Diaz and by Jianjun and Sicong [1] for example. The Lorentzian ones have been studied in the context of the n–dimensional Einstein–Gauss–Bonnet theory of gravitation [12]. Evolving higher dimensional wormholes have been studied by Kar and Sahdev and by A. DeBenedictis and Das [13, 14].

It is the purpose of the present paper to obtain, following the prescription provided by Dadhich, Kar, Mukherjee and Visser [15], a family of spherically symmetric, static and self–dual Lorentzian wormholes in n–dimensional Einstein Gravity.

The outline of the present paper is as follows: In Sec. II we briefly review some important aspects of 4–dimensional Lorentzian wormholes and give the definition of self–dual wormholes developed in [15]. In Sec. III a new class of metrics is presented which represent self–dual Lorentzian wormholes in n–dimensional Einstein gravity. Their properties are studied. We use the metric signature (−+++ and set c = 1.

II. 4–DIMENSIONAL LORENTZIAN WORMHOLES

A. Characterization of Lorentzian wormhole

The metric ansatz of Morris and Thorne [3] for the spacetime which describes a static Lorentzian wormhole is given by

$$ds^2 = -e^{2\phi(r)}dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2(d\theta^2 + \sin^2\theta d\varphi^2),$$  
(1)

where the functions $\phi(r)$ and $b(r)$ are referred to as red–shift function and shape function respectively.

Morris and Thorne have discussed in detail the general constraints on the functions $b(r)$ and $\phi(r)$ which make a wormhole [3]:

Constraint 1: A no–horizon condition, i.e. $e^{\phi(r)}$ is finite throughout the space–time in order to ensure the absence of horizons and singularities.

Constraint 2: Minimum value of the r–coordinate, i.e. at the throat of the wormhole $r = b(r) = b_0$, $b_0$ being the minimum value of $r$.

Constraint 3: Finiteness of the proper radial distance, i.e.

$$\frac{b(r)}{r} \leq 1,$$  
(2)

(for $r \geq b_0$) throughout the space–time. This is required in order to ensure the finiteness of the proper radial distance $l(r)$ defined by

$$l(r) = \pm \int_{b_0}^{r} \frac{dr}{\sqrt{1 - \frac{b(r)}{r}}}.$$  
(3)

The ± signs refer to the two asymptotically flat regions which are connected by the wormhole. The equality sign in (2) holds only at the throat.

Constraint 4: Asymptotic flatness condition, i.e. as $l \to \pm \infty$ (or equivalently, $r \to \infty$) then $b(r)/r \to 0$.

Notice that these constraints provide a minimum set of conditions which lead, through an analysis of the embedding of the spacelike slice of (1) in a Euclidean space, to a geometry featuring two asymptotically flat regions connected by a bridge [13].

In this paper we are not including considerations about the traversability constraints discussed by Morris and Thorne [3].

B. Self–dual Lorentzian wormholes

In order to construct a wormhole, one has to specify or determine the red–shift function $\phi(r)$ and the shape function $b(r)$. In general one of them is chosen by fiat and the other is determined by implementing some physical condition. In this paper we shall use the definition of self–dual wormholes developed in [13] and use it to obtain the n–dimensional self–dual version. The authors of Ref. [14] have proposed the following equation as the equation for wormhole:

$$\rho = \rho_t = 0,$$  
(4)

where $\rho = T_{\alpha\beta}u^\alpha u^\beta$ and $\rho_t = (T_{\alpha\beta} - \frac{1}{2}T g_{\alpha\beta})u^\alpha u^\beta$ are the energy density measured by a static observer and the convergence density felt by a timelike congruence respectively $(u^\alpha u_\alpha = -1)$.

From Eq. (4) we have that $T = 0$. This specific restriction on the form of the energy–momentum tensor automatically leads to a class of wormhole solutions. Since we are interested in Einstein wormholes, we conclude that the scalar curvature $R = 0$. This constraint will be a condition on the shape function $b(r)$ and on the red–shift function $\phi(r)$ and on their derivatives. For obtaining the n–dimensional self–dual wormhole solution, we first demand $\rho = 0$ from which we obtain $b(r)$, and then solve $T = R = 0$ which would determine the function $\phi(r)$.

It is interesting to note that the most general solution of Eq. (4) automatically incorporates the requirement of the existence of a throat without horizons, i.e. Constraint 1 enumerated above is satisfied.

III. N–DIMENSIONAL SELF–DUAL LORENTZIAN WORMHOLES

A. General $R = 0$ solution

For the spherically symmetric wormhole in higher dimensions we shall consider the spacetime given by

$$ds^2 = -e^{2\phi(r)}dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 d\Omega_{n-2}^2.$$  
(5)
where now
\[ d\Omega_{n-2}^2 = d\theta_2^2 + \sin^2 \theta_2 d\theta_3^2 + \ldots + \prod_{i=3}^{n-2} \sin^2 \theta_i d\theta_{n-1}^2. \] (6)

In other words we have replaced the two–sphere of metric \( \Omega^2 \) by a \((n-2)\)–sphere. In our notation the angular coordinates are denoted by \( \theta_i \), where \( i \) runs from 2 to \((n-1)\) and \( n \) is the dimensionality of the space-time. Note that in this notation there is no angular coordinate \( \theta_1 \).

We now introduce the proper orthonormal basis as
\[ ds^2 = -\theta(0)\theta(0) + \theta(1)\theta(1) + \theta(2)\theta(2) + \ldots + \theta(n)\theta(n), \]
where the basis one–forms \( \theta(\alpha) \) (here the index \( \alpha \) runs from 0 to \((n-1)\)) are given by
\[ \theta(0) = e^{\phi(r)} dt, \]
\[ \theta(1) = \frac{dr}{\sqrt{1 - \frac{b(r)}{r}}}, \]
\[ \theta(2) = r d\theta_2, \]
\[ \theta(3) = r \sin \theta_2 d\theta_3, \ldots, \]
\[ \theta(n-1) = r \prod_{i=2}^{n-2} \sin \theta_i d\theta_{n-1}. \]

From these expressions we obtain for the non–null components of the Einstein tensor:
\[ G_{(0)(0)} = -\frac{(n-2)}{2r^3} \left[ (n-4)b + rb' \right], \] (8)
\[ G_{(1)(1)} = -\frac{(n-2)}{2r^3} \left[ (n-3)b + 2r\phi'(b - r) \right], \] (9)
\[ G_{(2)(2)} = \frac{1}{2r^3} \left[ 2r^2(b - r)(\phi'' + \phi'^2) + r(rb' - 7b - 2nr + 2nb + 6r)\phi' + b(15 - 8n) + rb'(n - 3) + n^2b \right], \] (10)
and
\[ G_{(3)(3)} = G_{(4)(4)} = \ldots = G_{(n-1)(n-1)} = G_{(2)(2)}, \] (11)
where the prime denotes a derivative with respect to \( r \).

The curvature scalar is given by
\[ R = \frac{n-2}{2r^3} \left[ 2r^2(b - r)(\phi'' + \phi'^2) + r^2b' + 2r^2(2 - n) + rb(2n - 5) \right] \phi' + (n - 4)(n - 2)b + rb'(n - 2). \] (12)

We define the diagonal energy–momentum tensor components as
\[ T_{(0)(0)} = \rho(r), T_{(1)(1)} = \tau(r), \]
\[ T_{(2)(2)} = T_{(3)(3)} = \ldots = T_{(n-1)(n-1)} = p(r). \] (13)

Here \( T_{(2)(2)} = T_{(3)(3)} = \ldots = T_{(n-1)(n-1)} \), in view that we have a spherically symmetric space–time.

Using the Einstein equations
\[ G_{(\alpha)(\beta)} = R_{(\alpha)(\beta)} - \frac{R}{2} g_{(\alpha)(\beta)} = -\kappa T_{(\alpha)(\beta)}, \] (14)
where \( \kappa = 8\pi G \), we find from \([3]\) that
\[ \kappa\rho(r) = \frac{(n-2)}{2r^3} \left[ (n-4)b + rb' \right], \] (15)
\[ \kappa\tau(r) = \frac{(n-2)}{2r^3} \left[ (n-3)b + 2r\phi'(b - r) \right], \] (16)
\[ \kappa p(r) = -\frac{1}{2r^3} \left[ 2r^2(b - r)(\phi'' + \phi'^2) + r(rb' - 7b - 2nr + 2nb + 6r)\phi' + b(15 - 8n) + rb'(n - 3) + n^2b \right]. \] (17)

Here \( \tau(r) \) is the radial pressure \( p_r \) (and differs by a minus sign from the conventions of Morris and Thorne \([4]\)) and \( p(r) \) is the transverse pressure \( p_t \).

Considering the condition of self–duality \( R = T = 0 \) we find the following equation:
\[ \phi'' + \phi'^2 + \left( \frac{rb' + b(2n - 5) + 2r(2 - n)}{2r(b - r)} \right) \phi' + \frac{(n-4)(n-2)b + rb'(n-2)}{2r^2(b - r)} = 0. \] (18)

This equation can be thought of as the master equation for all \( n \)–dimensional static spherically symmetric \( R = 0 \) geometries.

Given the function \( b(r) \) one can solve equation \([13]\) to obtain \( \phi(r) \). We shall find the function \( b(r) \) from the self–dual condition. This implies that \( \rho(r) = 0 \) and then, from Eq. \([15]\), we have
\[ b(r) = 2mr^{-3-n}. \] (19)

Here we have written the constant of integration as \( 2m \) in order to obtain the mass \( m \) in the four dimensional Schwarzschild case. Then Eq. \([18]\) simplifies to
\[ \phi'' + \phi'^2 + \left( \frac{nnm^3 - mr^3 + 2r^2 - nr^4}{2mr^4 - m^3r^{-3-n}} \right) \phi' = 0. \] (20)
The solution of this nonlinear differential equation may be written as

\[ \phi(r) = \ln \left( k + \lambda \sqrt{1 - \frac{2m}{r^{n-3}}} \right), \tag{21} \]

where \( k \) and \( \lambda \) are constants of integration. This implies that the line element takes the form

\[
\text{d}s^2 = -\left( k + \lambda \sqrt{1 - \frac{2m}{r^{n-3}}} \right)^2 \text{d}t^2 + \frac{\text{d}r^2}{1 - \frac{2m}{r^{n-3}}} + r^2 \text{d}\Omega^2_{n-2}. \tag{22}
\]

The components of the energy–momentum tensor are given by

\[ \rho(r) = 0, \tag{23} \]

\[ \kappa\tau(r) = -k(n-3)(n-2) \frac{m r^{1-n}}{k + \lambda \sqrt{1 - \frac{2m}{r^{n-3}}}}, \tag{24} \]

and

\[ \kappa p(r) = \frac{k(n-3) m r^{1-n}}{k + \lambda \sqrt{1 - \frac{2m}{r^{n-3}}}}. \tag{25} \]

These expressions satisfy the constraint for the trace of the energy–momentum tensor:

\[ T = T(\alpha(\beta)g^{(\alpha)(\beta)}) = -\rho(r) + \tau(r) + (n-2) p(r) = 0. \]

Note that from equations (22)–(25), for \( n = 4 \) we obtain the 4–dimensional expressions (9), (10), (11) and (12) of Ref. [15].

For \( n = 3 \) the Eq. (21) does not represent the general solution for the differential equation (21), since the solution depends on only one integration constant. The general solution now will be given by

\[
\text{d}s^2 = -(k + \lambda \ln r)^2 \text{d}t^2 + \frac{\text{d}r^2}{1 - \frac{2m}{r^{n-3}}} + r^2 \text{d}\Omega^2_{n-2}, \tag{26}
\]

where \( k \) and \( \lambda \) are the constants of integration and

\[ \rho = 0, \]

\[ \kappa\tau = -\kappa p = -\frac{\lambda(1-2m)}{(k + \lambda \ln r)r^2}, \]

are the components of the energy–momentum tensor. The resulting spacetime is singular at \( r_1 = 0 \) and \( r_2 = e^{-k/\lambda} \). If \( k \gg 0 \) or \( \lambda \approx 0 \), then the singular radius \( r_2 \to 0 \).

If we set \( \lambda = 0 \), we have from (22) a vacuum solution which is locally flat, but conical in form. This metric is analogous to the Schwarzschild metric around a point mass in four dimensions. However, this metric does not lead to a black hole [17], and then also there is no three-dimensional Schwarzschild wormhole.

For \( n = 2 \) the metric (22) is not the general self–dual solution, since in (1+1)–gravity the Einstein tensor vanishes identically, so that the self–dual condition (21) is identically satisfied.

Thus, in the following subsection we shall consider that \( n \geq 4 \).

### B. Black holes, naked singularities and wormholes

Within the framework of the obtained family of solutions there are three classes of spherically symmetric spacetimes: black hole, nakedly singular and wormhole geometries. For example, the \( n \)–dimensional vacuum Schwarzschild geometry is obtained when \( k = 0 \). If \( m < 0 \), we have a naked singularity (for any value of \( k \) and of \( \lambda \)). The \( n \)–dimensional spatial–Schwarzschild traversable wormhole is obtained when \( \lambda = 0 \).

We are interested in the interpretation of the obtained solution as a Lorentzian wormhole. The weak energy condition \( (\rho \geq 0, \rho + \tau \geq 0, \rho + p \geq 0) \) and null energy condition \( (\rho + \tau \geq 0, \rho + p \geq 0) \) are both violated by Eqs. (23)–(25). Effectively, we have here that \( \rho = 0 \); then we obtain \( \tau \geq 0 \) and \( p \geq 0 \). But \( T = 0 \) implies that \( \tau = -2p \), thus, if \( p \geq 0 \), we have \( \tau \leq 0 \) (and vice-versa).

The extent of the energy condition violation, caused by the behavior of \( r^{1-n} \), for \( n \geq 4 \), is large in the vicinity of the throat. One does have a control parameter \( k \) which can be chosen to be very small in order to restrict the amount of violation.

In the following, we shall follow the procedure of Dadhich et al. [15]. To really make the wormhole explicit we need two coordinate patches:

\[
\text{d}s^2 = -(k + \lambda \ln r)^2 \text{d}t^2 + \frac{\text{d}r^2}{1 - \frac{2m}{r^{n-3}}} + r^2 \text{d}\Omega^2_{n-2}, \tag{27}
\]

where \( (2m)^{1/(n-3)} \leq r_1 \leq \infty \) and

\[
\text{d}s^2 = -\left( k - \lambda \sqrt{1 - \frac{2m}{r^{n-3}}} \right)^2 \text{d}t^2 + \frac{\text{d}r^2}{1 - \frac{2m}{r^{n-3}}} + r^2 \text{d}\Omega^2_{n-2}, \tag{28}
\]

where \( (2m)^{1/(n-3)} \leq r_2 \leq \infty \).

We have to sew these patches together at \( r = r_1 = r_2 = (2m)^{1/(n-3)} \). To make this clearer, it is convenient to go to isotropic coordinates. The advantage of these coordinates is that in almost all cases a single coordinate patch covers the entire space–time.
In isotropic coordinates, the metric in a slice $t = \text{constant}$ is conformal to the metric of Euclidean (n-1)–space. We consider a transformation in which the angular coordinates $\theta_2 \ldots \theta_{n-1}$ and $t$ remain unchanged while the radial coordinate

$$ r \rightarrow \tau = \tau(r), \quad (29) $$

so that $\tau$ is some other radial coordinate, where the metric (22) takes the form

$$ ds^2 = - \left( k + \lambda \frac{1 - \frac{2m}{(2\tau)^{n-3}}}{1 - \frac{2m}{r^{n-3}}} \right)^2 dt^2 + \lambda^2(\tau)[d\tau^2 + r^2 d\Omega^2_{n-2}]. \quad (30) $$

Comparing this metric with (22) we obtain $r^2 = \lambda^2(\tau) \tau^2$ and the following ordinary differential equation:

$$ \frac{dr}{\sqrt{r^2 - 2mr^{n-3}}} = \pm \frac{d\tau}{\tau}. \quad (31) $$

Since we require $\tau \rightarrow \infty$ as $r \rightarrow \infty$, we take the plus sign. Integrating the latter expression we find

$$ r = \tau \left[ 2^{n-4} \left( 1 + \frac{m}{(2\tau)^{n-3}} \right)^2 \right]^{1/(n-3)}. \quad (32) $$

Note that, when $\tau \rightarrow 0$ or $\tau \rightarrow \infty$, the coordinate $r \rightarrow \infty$ for $n \geq 4$.

When $n = 4$, we obtain exactly the same transformation for going from curvature coordinates to isotropic coordinates as was used for Schwarzschild itself. In this case of course the transformation (22) carries the n–dimensional Schwarzschild metric into isotropic form, since the space part of the metric (22) is identical to the space part of the higher dimensional Schwarzschild version.

Thus by (22) and (32) the line element, in terms of isotropic coordinates, is given by

$$ ds^2 = - \left( k + \lambda \frac{m}{(2\tau)^{n-3}} + 1 \right)^2 dt^2 + \left[ 2^{n-4} \left( 1 + \frac{m}{(2\tau)^{n-3}} \right)^2 \right]^{2/(n-3) \left[ d\tau^2 + r^2 d\Omega^2_{n-2} \right]}. \quad (33) $$

Note that from this equation for $n = 4$, we obtain the 4–dimensional expression (15) of Ref. [15].

It can be shown that both metrics (21) and (22) take the isotropic form (33) with the help of transformation (23). Then we have a single global coordinate chart for the alleged traversable Lorentzian wormhole. In this case $r \approx 0$ is one of the asymptotically flat regions and $r \approx \infty$ is the other one.

In order to have a traversable wormhole, we need to know the ranges of the parameters $k$ and $\lambda$ for which we have no horizon, i.e. the $g_{tt}$ component does not go to zero. We suppose $m > 0$, in order to have no naked singularities. There is a possible horizon when $g_{tt} = 0$, thus

$$ k \left( \frac{m}{(2\tau)^{n-3}} + 1 \right) + \lambda \left( \frac{m}{(2\tau)^{n-3}} - 1 \right) = 0. \quad (34) $$

Solving this equation we obtain

$$ \tau_H = \frac{1}{2} \left( \frac{m(\lambda - k)}{\lambda + k} \right)^{1/(n-3)}. \quad (35) $$

In order to have a horizon, the radius $\tau_H$ must be positive; then we have

$$ \frac{\lambda - k}{\lambda + k} \geq 0. \quad (36) $$

One can show that this horizon is actually a naked curvature singularity.

To see this more clearly, we calculate the radial and transverse pressure:

$$ \kappa \tau = -\kappa(n - 2)p = -\frac{k(n - 3)(n - 2)m \left[ (2\tau)^{n-3} + m \right]^{2(1-n)/(n-3)}}{2^{(1-n)/(n-3)}(2\tau)^{1-n} \sqrt{g_{tt}}}, \quad (37) $$

and we conclude that both pressures diverge as $g_{tt} \rightarrow 0$. Thus, if the parameters $\lambda$ and $k$ satisfy the inequalities

$$ \lambda + k > 0 \quad \text{and} \quad \lambda - k > 0, \quad (38) $$

or

$$ \lambda + k < 0 \quad \text{and} \quad \lambda - k < 0; \quad (39) $$

we have naked singularities. If the parameters $\lambda$ and $k$ do not satisfy Eqs. (38) or (39), then Constraint 1 enumerated above is satisfied, and curvature singularities do not form, the $g_{tt}$ components of the metric never go to zero, and we have a traversable wormhole.

We enumerate now some other properties of the obtained geometry:

(1) The geometry is invariant under simultaneous sign flips $\lambda \rightarrow -\lambda$ and $k \rightarrow -k$, independently of the dimensions of the space–time.

(2) $k = 0$, $\lambda \neq 0$ is the n–dimensional Schwarzschild geometry; it is a non–traversable wormhole (in this case the isotropic coordinate does not cover the entire manifold).

(3) $\lambda = 0$, $k \neq 0$ is the zero–tidal–force spatial–Schwarzschild traversable wormhole (the manifold is covered completely by the isotropic coordinate system).

(4) $\lambda = 0$, $k = 0$ is singular, independently of the dimensions of the space–time.

(5) At the throat $g_{tt}(r = [2m]^{1/(n-3)}) = -k^2$, so $k \neq 0$ is required to ensure traversability.

(6) $g_{tt}(\tau = \infty) = -(k - \lambda)^2$ and $g_{tt}(\tau = 0) = -(k + \lambda)^2$; then we see that time runs at different rates in the two asymptotic regions.
IV. CONCLUDING REMARKS

Our aim in this paper has been to obtain self–dual Lorentzian wormholes in the framework of n–dimensional Einstein gravity. For this we have followed the prescription provided by Dadhich et al. [15] for obtaining wormholes. They have proposed one such prescription which is characterized by the equation \( \rho = \rho_t = 0 \), which implies that \( R = 0 \) and, equivalently, a traceless energy–momentum tensor. This constraint gives a condition on the shape function \( b(r) \) and on the red–shift function \( \phi(r) \) and on their derivatives.

For obtaining the n–dimensional self–dual wormhole solution, we first demand \( \rho = 0 \) from which we obtain \( b(r) \), and then solve \( T = R = 0 \) which would determine the function \( \phi(r) \).

The resulting geometry represents a family of space–times which contains Lorentzian wormholes, naked singularities, and the n–dimensional Schwarzschild black hole. However, in lower dimensional Einstein gravity, without cosmological constant, we can not have self–dual Lorentzian wormholes.

The isotropic coordinates are used to display the full structure of the obtained higher dimensional geometry.

In this case, of course the wormhole geometry has exotic matter as the source, i.e. the energy–momentum tensor of the matter explicitly violates the energy conditions. This fact can not be used to rule out wormhole solutions. Today many physical situations are known in which the energy conditions are violated [15, 16].

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[1] S. Coleman, Nucl Phys. 307, 867 (1988).
[2] S.B. Giddings and A. Strominger, Nucl. Phys. B 321, 481 (1988).
[3] M.S. Morris and K.S. Thorne, Am. J. Phys. 56, 395 (1988).
[4] M.S. Morris, K.S. Thorne and U. Yurtsever, Phys. Rev. Lett. 61, 1446 (1988).
[5] J.D. Novikov, Sov. Phys. JETP 68, 439 (1989).
[6] M. Visser, Lorentzian Wormholes: From Einstein to Hawking, (AIP, New York, 1995).
[7] R.C. Myers and M.J. Perry, Ann. Phys. (N.Y.) 172, 304 (1986).
[8] X. Dianyan, Class. and Quantum Grav. 5, 871 (1988).
[9] K.D. Krori, P. Borgha and Kanika Das, J. Math. Phys. 30, 2315 (1989).
[10] J. Ponce de Leon and N. Cruz, Gen. Rel. Grav. 32, 1207 (2000).
[11] P. Gonzales–Diaz, Phys. Lett. B 247, 251 (1990); X. Jianjun and J. Sicong, Mod. Phys. Lett. 6, 251 (1990).
[12] B. Bhawal and S. Kar, Phys. Rev. D 46, 2464 (1992).
[13] S. Kar and D. Sahdev, Phys. Rev. D 53, 722 (1996).
[14] A. DeBenedictis and D. Das, [ar–qc/0207074] (2002).
[15] N. Dadhich, S. Kar, S. Mukherjee and M. Visser, Phys. Rev. D 65, 064004 (2002).
[16] M. Visser and C. Barcelo, Energy conditions and their cosmological implications, [ar–qc/0001099].
[17] A. Staruszkiewics, Acta. Phys. Polin. 24, 735 (1963).