How does Motility-Induced Phase-Separation interplay with Glassiness in Dense Active Matter?

Matteo Paoluzzi,1 Demian Levis,1,2 and Ignacio Pagonabarraga3,1,2

1Departament de Física de la Matèria Condensada, Universitat de Barcelona, C. Martí Franquès 1, 08028 Barcelona, Spain.
2UBICS University of Barcelona Institute of Complex Systems, Martí i Franquès 1, E08028 Barcelona, Spain.
3CECAM Centre Européen de Calcul Atomique et Moléculaire, École Polytechnique Fédérale de Lausanne (EPFL), Batochime, Avenue Forel 2, 1015 Lausanne, Switzerland.

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INTRODUCTION

Active self-propelled particles have been often employed as a starting and minimal model for capturing collective behaviors in biological and living materials, examples run from biological tissues to dense drops of ants [1–3]. Moreover, active agents might show a high degree of heterogeneities in their microscopic characteristics inside a given population. However, in modeling biological tissues, usually, it is assumed that each cell has the same mechanical and geometrical properties [4]. Cellular differences play a crucial role in many biological processes as in the case of cancer development, where cell heterogeneity is a common feature [5]. Phenotypic heterogeneity has functional consequences that impact the ability of microbes to adapt to different environments [6] and mechanical heterogeneity changes the collective properties in simple models of biological tissues [7].

In the case of self-propelled disks, the total force acting on given particle results from the combination of the self-propulsive force $f_s$ and the mechanical force $f_M$ due, for instance, to steric interactions. Imagine two active particles facing head to head: their equilibrium distance and thus their effective linear size is determined by the balance between these two forces. It follows that heterogeneities in the self-propulsive force change the equilibrium distances and thus the typical particle size. If in the system of interest only average values are available, it is reasonable to introduce suitable probability distributions for those quantities.

In this work, we will focus our attention on the effect of quenched fluctuations in the typical active agent size. To take into account this aspect, we perform numerical simulations of Active Brownian disks of heterogeneous size. To make a contact with what is known on equilibrium systems, we study the phase diagram of repulsive Active Brownian Particles (ABP) composed of a continuous, i. e., polydisperse, mixture [9]. We explore the phase diagram using as a control parameter the persistence length $\ell$ and the packing fraction $\phi$. The model system introduced here is particularly suitable for studying the interplay between Motility-Induced Phase Separation (MIPS) and glassy dynamics [10–15]. We shall see that glassiness occurs only at low activity, although the system in the MIPS phase arranges in dense patches that do not provide any evidence of positional order (and thus glassy behavior might develop).

We study the phase diagram of polydisperse ABP and investigate the properties of the system at high packing fractions, where we document a glass transition driven by the persistence length. The latter is signaled by a dynamical slowing down of the time correlation function of the hexatic order parameter, with a decoupling between density and hexatic fluctuations. We show that, in the glassy region, there is a coexistence of hexatic and...
liquid phases. Systems of polydisperse self-propelled particles have been considered before [16–22], although the interplay between MIPS and glassiness has not been investigated yet, neither its vibrational spectrum at large densities.

We then look at the statistical properties of the inherent structures obtained from stationary configurations at different parent activities. In this way, we can provide an estimate of the distance, computed in a probabilistic sense, between a typical steady-state configuration, that results from the non-equilibrium dynamics in the presence of self-propulsion, and the corresponding configuration that minimizes the mechanical energy. We show that glassy configurations are almost inherent configurations. Comparing dense configurations obtained for higher persistence length, we document a discontinuous crossover approaching MIPS.

Finally, we explore the concept of effective temperature in connection with the vibrational density of states of instantaneous configurations. We show that the magnitude of the largest negative eigenvalues of the dynamical matrix, i. e., the “faster” unstable mode of the system, encodes information about the break down of the concept of effective temperature in active glasses. Those unstable directions can be surfed by the active system for removing geometrical frustration and bringing the system into a fluid phase. This result suggests that, even though glassy configurations at low activity are basically indistinguishable by their equilibrium counterpart, the mechanism of fluidization in active systems might be distinct from that in equilibrium glasses, where mean-field models point the attention out on marginal directions in the potential energy landscape rather than the unstable ones [23].

**ABSENCE OF STRUCTURAL ARREST AT HIGH ACTIVITY**

A qualitative view of the phase diagram is provided in Fig. 1, where we report the snapshots of typical stationary configurations. As one can see, the system undergoes MIPS and separates into a dense and dilute phase for large enough values of the persistence length $\ell$. We observe that the MIPS regime extends until the largest packing fraction simulated, i. e., $\phi = 0.78$. As we shall study in the next sections, the high density regime is characterized by a glassy phase at small persistence lengths.

We start our quantitative analysis by looking at the statistical properties of the relevant coarse-grained fields, i. e., the local packing fraction field $\phi(x, y, t)$, and hexatic order parameter $\psi_6(x, y, t)$. For monitoring the presence of MIPS, we look at the behavior of the probability distribution function of $\phi(x, y, t)$, $P(\phi) = \langle \delta(\phi - \phi(x, y, t)) \rangle_t$. To investigate the emergence of a disordered glassy phase, we study the relaxation time of density fluctuations through the intermediate scattering function $F_{\phi}(q, t)$ and the time-correlation of the hexatic order parameter $C_6(t)$ (their definition is provided in SI). The resulting phase diagram is shown in Fig. 2-(A), the behavior of $P(\phi)$ crossing the MIPS region is shown in panel (B). The distribution develops the typical double-peaked structure due to the presence of two coexisting phases. The same is true even at very high densities, as documented in panel (C). The qualitative features of the phase diagram are consistent with those observed in athermal self-propelled disks [23]. We checked that the system remains always in disordered, liquid-like configurations, in the whole range of parameters, by looking at the radial distribution function $g(r)$, as shown in panel (D). $g(r)$ does not reveal any hint of crystallization, even at the largest packing fraction $\phi = 0.78$.

The color map in Fig. 2-(A) indicates the relaxation
The smaller negative eigenvalue of the energy landscape. (B) The color map indicates the relaxation time (in log scale). Green symbols indicate the typical persistent length corresponding to critical point. The blue dashed curve corresponds to the transition between homogeneous fluid and liquid/hexatic coexistence. FIG. 2. Phase diagram. (A) The black dashed curve individuates the MIPS coexistence region. The red star indicates the critical point. The blue dashed curve corresponds to the transition between homogeneous fluid and liquid/hexatic coexistence. The color map indicates the relaxation time (in log scale). Green symbols indicate the typical persistent length corresponding to critical point. The blue dashed curve corresponds to the transition between homogeneous fluid and liquid/hexatic coexistence.

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FIG. 3. Representative stationary configurations in MIPS ($\ell = 100$, $\phi = 0.44$, $L = 120(\sigma)$) for varying fraction of polydisperse disks $f$ (increasing values from left ($f = 0$) to right ($f = 1.0$), see legend).

time of $C_\psi(t)$ that has been computed using the criterium $C_\psi(\tau_\psi) = e^{-1}$. For defining the glassy state we adopt the threshold value $\tau_\psi = 10^4$ [25]. We anticipate that we highlight the dynamical slowing down of $C_\psi$ rather than the one of the self-intermediate scattering function $F_{\text{self}}$ because its typical relaxation time $\tau_\psi$ is larger (see below Fig. 3(C)). From the study of the relaxation time, it turns out that there is no hint of glassy dynamics in the MIPS region, even though the dense component is an assembly of polydisperse disks.

We explore the presence of hexatic patches, typical of two-dimensional systems [26,29], by looking at the distribution $P(|\psi_0|) = \langle \delta(|\psi_0| - |\psi_0(x, y, t)|) \rangle_t$, shown in Fig. 2(E) ($\ell = 0.1$). At high densities, the distribution becomes double peaked indicating the presence of hexatic patterns (see also panels (D) and (E) in Fig. 3). The Fluid/Hexatic transition develops a reentrance in the phase diagram (see SI). This is highlighted in Fig.
**FIG. 4.** The impact of geometrical heterogeneities on MIPS. (A) $P(|\psi_0|)$ by varying the fraction of heterogeneous particles from $f = 0$, to $f = 1$, from black to orange (see legend). The inset shows the height of the second peak $P_M \equiv (\text{at high } |\psi_0| \text{ values})$ as a function of $f$. (B) Spatial correlation function $g_6(r)$ as $f$ increases, from black to orange. The dashed red curve is a power law $r^{-1/4}$, the dashed blue curve is an exponential decay $e^{-r/\xi}$ with $\xi \sim O(1)$. (C) $P(\phi)$ by varying $f$. The inset shows the height of the peak at high $\phi$ values as $f$ increases. (D) $P(\phi)$ for small persistence length ($\ell = 0.001$) and high density ($\phi = 0.79$), blue circles refer to instantaneous configurations, red diamonds to inherent configurations. The dashed black line is the fit to a Gaussian function. (E) $P(\phi)$ of instantaneous and inherent configurations for large persistence length ($\ell = 200$ and $\phi = 0.79$). $P_{IS}(\phi)$ is well captured by a Gaussian fit (dashed black line). (F) Kullback–Leibler divergence $D_{KL}$ between steady-state ($P_S$) and inherent ($P_{IS}$) distributions for the two observables, i. e., $P(\phi)$ (red circles) and $P(|\psi_0|)$ (blue diamonds).

where we report the height of the second peak, i. e., $P_M \equiv \max_{|\psi_0| > 0.5} P(|\psi_0|)$, as a function of $\ell$ at $\phi = 0.79$. $P_M$ is a non-monotonous function of $\ell$, i. e., $P_M$ decreases as $\ell$ increases towards the fluid phase and, eventually, develops a bump in the MIPS region. This behavior is consistent with hexatic patches that are more pronounced in the glassy and MIPS region. Deep in the hexatic/liquid phase, at small activity and high packing fraction, the system behaves as a disordered solid.

**STABILITY OF MIPS AGAINST QUENCHED FLUCTUATIONS**

In order to quantify the effect of particle heterogeneities on MIPS, we have performed numerical simulations deep in the phase-separated region for a larger system size, i. e., $\ell = 100$, $\phi = 0.44$, $L = 120(\sigma)$, and by varying the fraction $f \in [0, 1]$ of polydisperse disks. This means that at $f = 0$, the system is monodisperse, i. e., $\sigma_1 = (\sigma)$, $\forall i$.

Typical stationary configurations are shown in Fig. 3. A quantitative analysis shows that, although $P(|\psi_0|)$ remains double-peaked at any $f$ value, a small fraction of heterogeneous disks causes a huge decrease of the height of the second peak (see Fig. 4-(A)). This fact can be quantified further by looking at $P_M$ defined before. As one can see in the inset of the same panel, $P_M$ dramatically decreases as $f$ increases. The reduction of orientational order can be appreciated by the study of $g_6(r)$. The typical behavior of $g_6(r)$ as $f$ increases from 0 to 1 is presented in Fig. 4-(B). Although, because of finite size effects, it is hard to identify a clear power-law behavior at $f = 0$, it is clear that, polydispersity hinders hexatic order, in agreement with equilibrium studies [30]. As $f$ increases, $g_6(r)$ tends to decay exponentially, i. e., $g_6(r) \sim e^{-r/\xi}$, with a correlation length of the order of a few particle sizes that tends to screen any power-law decay. However, such structural change in the hexatic patches is not accompanied by any dramatic change on the density de-mixing due to MIPS. This is shown in Fig. 4-(C), where we report $P(\phi)$ for different values of $f$. We observe that height of the high-density peak decreases as the fraction of heterogeneous disks increases. However, the position of the two peaks remains much more stable, indicating that both, the dense and the dilute phase, remain at almost the same average densities, i.e., the MIPS coexistence region remains largely unaffected by polydis-
FIG. 5. Relaxation dynamics at high packing fraction. (A) Intermediate scattering function $F(q, t)$ computed at the peak of the static structure factor $q = q_{\text{max}}$. The red arrow indicates increasing values of $\ell = 0.004, 0.006, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035, 0.04, 0.045, 0.05, 0.08, 0.1, 0.5, 1.0, 5.0$ (B) Relaxation dynamics of the correlation $C_\phi(t)$. (C) Structural relaxation time as a function of $\ell$ for different observables and density (see legend). Density fluctuations decay faster than the orientational ones. The map of displacements on a time scale $\tau_\phi$ are shown in (D), and (E), for $\ell = 10^{-3}$ and $10^{-2}$, respectively. The color map indicates the magnitude of the hexatic order field $|\psi_6(x, y)|$. The map reveals the presence of heterogeneous regions in both fields, long-time displacement and local hexatic order. (F) Dynamical susceptibility $\chi(t)$ for different persistent lengths $\ell$ (see legend). The red circles indicate the peak position $\chi_3(\tau_\phi)$ ($\tau_\phi$ is reported in (C) as a function of $\ell$).

The presence of MIPS through a broad tail in $P_{\text{inst}}(\phi)$. On the other hand, the inherent configurations produce a Gaussian distribution centered around $\phi = 0.79$.

For making quantitative progresses we measure the distance between $P_{\text{IS}}$ and $P_{\text{inst}}$ using the Kullback–Leibler divergence $D_{KL}[P_{\text{IS}}|P_{\text{inst}}]$ (see SI). The result is shown in Fig. (E) for both $\phi$ and $\psi_6$. We obtain that $D_{KL}[P_{\text{IS}}(\phi)|P_{\text{inst}}(\phi)]$ is small in the glassy state, at small $\ell$, indicating that instantaneous and inherent configurations are almost identical, and even decreases as $\ell$ increases. However, as the system approaches MIPS, $D_{KL}$ jumps to higher values. This is because inherent configurations are homogeneous while instantaneous ones tend to decompose into two phases due to MIPS. Less informative is $D_{KL}[P_{\text{IS}}(\psi_6)|P_{\text{inst}}(\psi_6)]$ that maintains small values showing a smooth crossover on intermediate persistence length, i. e., $\ell \sim 0.1$.

**ACTIVITY-DRIVEN DYNAMICAL ARREST**

We now explore the high packing fraction region $\phi = 0.79$. We study the behavior of the self-part of the intermediate scattering function $F_{\text{self}}(q, t)$, and the time-correlation function of the hexatic order parameter $C_\psi(t)$. We monitor $F_{\text{self}}(q_{\text{max}}, t)$, where $q_{\text{max}}$ is the wave vector of the first peak of the static structure fac-
FIG. 6. Instantaneous Normal Modes. (A) $\mathcal{D}(\omega)$ of inherent configurations (blue curve) and instantaneous configurations at different persistence length ($\ell = 10^{-4}, 1, 10$, red and green, respectively.) (B) The average number of negative frequencies approaches a plateau value at large $\ell$ and undergoes a crossover for $\ell \sim 1$ at the same point where the structural relaxation time (green squares) starts to grow for decreasing values of $\ell$ (dashed blue area). (C) The largest negative frequency in unit of persistence time $\tau$ as a function of persistence length $\ell$ is order 1 at the crossover. Magenta symbols refer to $\phi = 0.66$.

The concept of effective temperature helps to rationalize some features of active systems $^{32}$-$^{40}$. In colloidal glasses driven by thermal noise, because of the caging effect, particles spend most of the time vibrating around their equilibrium position until cooperative rearrangements allow the particle to escape from the cage $^{41}$. This equilibrium-like picture can be reasonably employed also in the case of an active glass whenever the active motion causes only vibrations and thus rattling inside the cage, i.e., in the small persistence length regime $\ell \lesssim (\sigma)$. In order to make progress, we consider harmonic vibrations around an equilibrium inherent configuration. The harmonic vibrations define a set of eigenfrequencies $\omega_\nu = \sqrt{\lambda_\nu}$, with $\lambda_\nu$ the $\nu$–th eigenvalue of the dynamical matrix $^{33}$ $(\text{SI})$. We can thus introduce the following (mode-dependent) effective temperature $^{37}$ (see SI) $T_{\text{eff}}(\lambda_\nu, \ell) = \frac{\mu_\nu}{\mu_\nu + \mu \lambda_\nu}$, with $\mu$ being the particles’ mobility. At variance with early studies, here we are going to provide an alternative interpretation of $T_{\text{eff}}(\lambda_\nu, \ell)$ that allows to gain new insight into the active glass transition.

Let $r^* = (r^*_1, r^*_2, ..., r^*_3)$ be the inherent configuration of the system so that $\partial_r \Phi_{r^*} = 0$. The stability of such a configuration is written in the eigenvalues $\lambda_\nu$ of the Hessian matrix $\mathbb{H}_{ij} = \partial_i \partial_j \Phi$. If all the eigenvalues $\lambda_\nu$ are strictly positive, the configuration lays in a minimum of the potential energy landscape. Let us introduce the
persistent time $\tau = \ell/\nu_0$. As soon as the system develops instabilities, i.e., directions with negative curvature in the energy landscape, there will be a critical value $\tau_c$ that causes a divergence in $T_{\text{eff}}$ so that, for $\tau \geq \tau_c$, the concept of $T_{\text{eff}}$ becomes ill-defined. This critical value corresponds to the largest negative eigenvalue $\lambda_{\text{max}}$, i.e., $\tau_c = |\lambda_{\text{max}}|^{-1/2}$. For typical configurations in the liquid regime, the energy spectrum always develops negative eigenvalues, making the concept of effective temperature meaningful only in the limit $\tau \ll \tau_c$.

Replacing $\ell$ in favor of $\tau$, we thus stress that not only $T_{\text{eff}}(\lambda, \tau)$ makes sense for $\tau \rightarrow 0$, but also that the breaking of the concept of effective temperature is bounded to the structure of the energy landscape. For checking this fact, we have computed the spectrum of instantaneous and inherent normal modes. The corresponding density of states $D_{\text{inst,LS}}(\omega)$ are shown in Fig. (3)-(A). As a standard procedure, imaginary eigenmodes $\lambda^{-} \sim -\sqrt{|N^{-}|}$. At variance with instantaneous configurations, the frequency spectrum $D_{\text{LS}}(\omega)$ of inherent configurations does not contain any negative frequency, meaning that it is linearly stable. As $\ell$ increases, the average number of negative frequencies $\langle n^{-}\rangle$ increases (panel (B)). Interestingly, both $\langle n^{-}\rangle$ and the (slowest) structural relaxation time $\tau_0$ approach a plateau value at the same crossover value $\ell_c \sim 1$.

For connecting the crossover with instabilities in the energy landscape, we look at the behavior of the largest negative frequency $\omega^{(-)}_{\text{max}}$ as a function of $\ell$, as shown in Fig. (4)-(C). The estimate from the effective temperature breaking is $\tau_c |\omega^{(-)}_{\text{max}}| \approx 1$, as one can see, we correctly obtain $\ell_c \sim 1$. We repeated the analysis at $\phi = 0.66$ and the result fairly match the prediction (see Fig. (4)-(C) and Fig. (2)-(A)). As in the case of equilibrium glasses, we can thus relate the topological properties of the energy landscape with dynamical slowing down $\lambda^{-}\sim 1$. However, at variance with equilibrium, our picture suggests that the fluidization of glassy configuration in active system is connected to negative directions in the energy landscape rather than the marginal ones.

**DISCUSSION**

Living systems as bacterial colonies or biological tissues are typically composed of self-propelled units of different sizes and different self-propulsive forces. In this work, we studied how this quenched disorder impacts the collective behavior of active particles. As a model system, we have considered a mixture of Active Brownian disks of different radii. We have explored the phase diagram using as control parameters the persistence length $\ell$ of the active motion and the packing fraction $\phi$. Similar to the case of monodisperse active systems, we have documented the presence of a MIPS coexistence region for large persistence lengths. The phase separation extends from relatively small packing fractions ($\phi \sim 0.2$) to very high densities $\phi \sim 0.8$.

Geometrical frustration due to geometrical heterogeneities makes the system considered here a good model of glass $[9]$. We showed that this peculiarity did not get lost because of activity. In particular, we documented a glass transition at high density characterized by decoupling between the relaxation time of density fluctuations and that of the hexatic order parameter.

The study of the inherent structures has unveiled a deep connection between stationary configurations at small persistence lengths and the corresponding inherent ones. This connection turns out to be lost as $\ell$ increases. This is because MIPS, although it might be interpreted in terms of effective potential and equilibrium-like spinodal decomposition, it is an intrinsic non-equilibrium phenomenon caused by the self-propelled motion which cannot be completely reduced to an effective equilibrium description.

The model introduced here is particularly suitable for studying the consequence of the competition between glassy dynamics, peculiar of equilibrium dynamics and known to play a pivotal role in dense active systems $[49, 50]$, and MIPS, a typical feature of active systems whose importance in bacterial colonies has been recently documented $[51]$. In this work, we have developed a deeper understanding of the concept of effective temperature in active matter, based on the analysis of the spectrum of vibrations around its inherent structures. Using a simple picture based on the definition of effective temperature through an extension of the energy equipartition theorem $[33]$ that can be applied to dense active systems $[33, 37, 38]$, we found that the breaking of the concept of effective temperature in active matter can be linked to the properties of the energy landscape of the instantaneous configurations. Since an instantaneous configuration is not optimal, the corresponding vibrational spectrum contains always negative eigenvalues. We observed that the magnitude of the largest of them can be used as a proxy, i.e., $\tau_c \sim 1/\omega^{(-)}_{\text{max}}$, for estimating the crossover between active liquid, from the point of view of the relaxation time, and a active glassy regime, i.e., where the structural relaxation time starts increasing dramatically. The estimated value $\tau_c \sim 1$ is also in agreement with a heuristic argument that estimates the validity of the concept of $T_{\text{eff}}$ for vibrations smaller than the typical particles size $\tau \ll \langle \sigma \rangle/\nu_0 \sim 1$. Our approach, which is based on the properties of the vibrational energy landscape, provides an interpretation of the mechanisms leading to the fluidization of frustrated glassy configurations. In particular, our picture suggests that, in spite the similarities between glassy dynamics in active and equilibrium-driven systems, the fluidization of the active glassy state is due to microscopic mechanisms different from those in equilibrium, since the important parameter might be the magnitude of the largest negative eigenvalue rather than the number of quasi-stationary points $[52]$. As a future direction, it might be interesting to study other aspects
of the topological crossover in active glasses [53] and how it is connected with non-trivial spatial correlations of the velocity field [54] [55].

Finally, the lack of any dynamical slowing down in the MIPS phase is consistent with the breaking of the concept of effective temperature. In particular, the MIPS regime lives always at high persistence lengths, and thus high activity, i.e., in a regime where non-equilibrium effects are important. This fact has strong implications, for instance, it follows that density and geometrical frustrations are not the only minimal ingredients for driving a dynamical arrest in Active Matter. More in general, MIPS seems to be incompatible with dynamically arrested phases.

MATERIALS AND METHODS

Due to their robustness against crystallization, polydisperse mixtures result to be good candidates as glass formers in a wide range of temperatures, even below the dynamical arrest temperature [9]. Here we consider a polydisperse mixture in two spatial dimensions where Active Brownian disks of different diameters σ interact through a pair potential \( v(r_{ij}) = c_0 + c_2 \left( \frac{r_{ij}}{\sigma_j} \right)^2 + c_4 \left( \frac{r_{ij}}{\sigma_j} \right)^4 \), with \( r_{ij} \equiv |r_i - r_j| \). We set the softness exponent to \( n = 12 \).

The coefficients \( c_0 \), \( c_2 \), and \( c_4 \) are chosen in a way that \( v(r_c) = v'(r_c) = v''(r_c) = 0 \), where we have introduced the standard notation \( v'(r) = \frac{dv}{dr} \) and \( v''(r) = \frac{d^2v}{dr^2} \). For suppressing the tendency to demix, we consider non-additive diameters \( \sigma_{ij} = \frac{1}{2} \left( \sigma_i + \sigma_j \right) \left( 1 - \epsilon (\sigma_j - \sigma_i) \right) \), where \( \epsilon \) tunes the degree of non-additivity [9] [56]. The cutoff is \( r_c = 1.25 \sigma_j \), and \( \epsilon = 0.2 \).

The particle diameters \( \sigma_i \) are drawn from a power law distribution \( P(\sigma) \) with \( \langle \sigma \rangle = \int_{\sigma_{\min}}^{\sigma_{\max}} d\sigma P(\sigma) \sigma = 1 \), with \( \sigma_{\min} = 0.73 \), \( \sigma_{\max} = 1.62 \), and \( P(\sigma) = A \sigma^{-3} \), with \( A \) a normalization constant [9].

The dynamical state of the \( i \)-th particle is given by its position \( r_i \) and by the orientation \( e_i \) of the self-propelling force that, in two spatial dimension, is parametrized by the angle \( \theta_i \), i.e., \( e_i = \hat{x} \cos \theta_i + \hat{y} \sin \theta_i \), with \( \hat{x} \) and \( \hat{y} \) the unit vectors of the x and y axis, respectively. The overdamped equations of motion for the \( i \)-th disk read

\[
\dot{r}_i = v_0 e_i + \mu F_i + \zeta_i \tag{1}
\]

and

\[
\dot{\theta}_i = \eta_i \tag{2}
\]

with \( \eta_i = 0 \) and \( \langle \eta_i(t) \eta_j(s) \rangle = \frac{2}{\tau} \delta_{ij} \delta(t-s) \). The term \( \zeta_i \) is a thermal noise, i.e., \( \langle \zeta_i^2 \rangle = 0 \), and \( \langle \zeta_i^2(t) \zeta_j^2(s) \rangle = 2 \mu k_B T \delta_{ij} \delta^2(\tau(t-s)). \) The force \( F_i = \sum_{j \neq i} f_{ij} \), with \( f_{ij} = -\nabla r_i v(r_{ij}) \).

For gaining insight into the stability of inherent and instantaneous configurations, we have computed the normal modes by evaluating the 2N eigenvalues \( \lambda_N \), with \( \kappa = 1, \ldots, 2N \) of the Hessian matrix. The computations have been done using Python NumPy linear algebra functions [58].

The system is enclosed in a square box of side \( \ell = 60 \langle \sigma \rangle \) with periodic boundary conditions. For exploring the phase diagram, we have performed numerical simulations for \( N \in [15^2, 60^2] \) and persistence length \( \ell \in [0.001, 200] \).

The inherent structures have been obtained minimizing the mechanical energy \( \Phi = \frac{1}{2} \sum_{i \neq j} v(r_{ij}) \). Energy minimization is performed using the FIRE algorithm [57].

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