We consider an 8–dimensional gravitational theory, which possess a principle fiber bundle structure, with Lorentz–scalar fields coupled to the metric. One of them plays the role of a Higgs field and the other one that of a dilaton field. The effective cosmological constant is interpreted as a Higgs potential. The Yukawa couplings are introduce by hand. The extra dimensions constitute a $SU(2)_L \times U(1)_Y \times SU(2)_R$ group manifold. Dirac fields are coupled to the potentials derived from the metric. As result, we obtain an effective four–dimensional theory which contains all couplings of a Weinberg–Salam–Glashow theory in a curved space-time. The masses of the gauge bosons and of the first two fermion families are given by the theory.

1. Introduction

Interest in higher–dimensional theories has never disappeared, it has waned and waxed but never has gone to zero. Recently, we analyzed the problem of the explanation for the bare mass of some elementary particles in the context of higher–dimensional models. In a further work, the fermionic sector of higher–dimensional theories was studied, this fermionic sector consisting of the first fundamental families. The space–time was endowed with the internal symmetry that corresponds to that of the $U(1) \times SU(2)$ group with no scalar fields at all. Unfortunately, the neutrino turns out to be massive whereas the gauge fields associated to the weak interaction are massless and the ratio between the leptonic and hadronic masses is one third, a result clearly denied by the experiment. It must be mentioned that the radius of the $S^1$ circle, is the only parameter of the extra dimensions involved in the masses of the fermionic sector, cf. Ref. , however.

To pursue of this line of thought, we introduce scalar fields that could play the role of a Higgs field and thereby could yield the terms that are necessary to solve some of the aforementioned problems.

To consider non–Abelian symmetries , we could just take the structure of principal fiber bundle for the whole space–time. If we want to introduce the electroweak
interactions the structure of the principal fiber bundle is \((G, P) \xrightarrow{\pi} M^4\), where \(M^4\) is the four–dimensional Riemannian spacetime manifold and the fiber is assumed to be the group manifold of a compact non–Abelian group \(G\). For the particular case that comprises the electroweak interactions, it must be at least four–dimensional. When this is combined with the four–dimensional spacetime, we are led to a gravity theory which is at least eight-dimensional. The line element takes the form:

\[
\begin{align*}
\text{ds}^2 &= g_{\mu\nu}(x)dx^\mu dx^\nu - \varphi(x)^2 \left[ dx^5 + \kappa L^{-1}_i B_i(x)dx^\mu \right]^2 \\
&\quad - \Phi(x)^\dagger \Phi(x) \gamma_{ij}(y) \left[ dy^i + \kappa L^{-1}_i \kappa_{\alpha}^i(y) A_{\mu}^\alpha(x)dx^\mu \right]^2,
\end{align*}
\]

where \(\gamma_{ij}\) is the metric tensor on the group manifold of \(G\) and the functions \(\kappa_{\alpha}^i(y)\) are the components of the Killing vectors on \(G\). The fields \(A_{\mu}^\alpha(x)\) are gauge potentials of an arbitrary non–Abelian gauge group as well as components of the gravitational field in \(4 + n\) dimensions.

In the present paper we construct an eight–dimensional space–time, where the new coordinates \(y^i\) have to be interpreted as a parametrization of the manifold of the non–Abelian group \(SU(2)_L \times U(1)_Y\). The group manifold of \(SU(2)_L\) is the sphere \(S^3\) and that of \(U(1)_Y\) is the circle \(S^1\), therefore the space \(S^1 \times S^3\) has the desired \(U(1)_Y \times SU(2)_L \times SU_R(2)\) symmetry and it is the natural manifold for this group. In this framework we investigate an eight–dimensional gravity theory with two fields coupled to the metric \((1)\), which possesses a principle fiber bundle structure, where \(\varphi(x)\) is a singlet with respect to \(U(1)_Y \times SU(2)_L\), namely, it is a dilaton field with its usual linear vacuum behavior, in which it tends to a constant value, but \(\Phi(x)\) is now endowed with an isospin structure. These two type of fields depend only on the space–time coordinates.

In the effective 4–dimensional theory, the gauge transformations are a remnant of the original coordinate invariance group in \(4 + n\) dimensions, which has been spontaneously broken down through dimensional reduction to the symmetries of the four–dimensional coordinate transformation group and a local gauge group, \(\varphi(x)\) and \(\Phi(x)\) are Lorentzian scalar fields, more precisely \(\varphi\) can be identified with a dilatonic field, while \(\Phi\) has isospin structure. These two type of fields depend only on the space–time coordinates.

Dirac fields are coupled to the metric one, this field contains the first two fermionic families, see below \((15)\). A potential term related to the field \(\Phi(x)\) that contains a mass and quartic self–interaction terms is introduced, it behaves as the effective four–dimensional cosmological constant \((10,11)\). By hand, we introduce Yukawa terms, which consist of two contributions, namely, the bare one, which compensates the bare contribution of the fifth dimension that leads to non–physical results and the usual Yukawa coupling which generates through GIM mechanism the fermionic masses. This means that the group structure of the right–hand part is \(U(1)\). Through the spontaneous symmetry breaking of the \(U(1) \times SU(2)_L\) symmetry of our Lagrangian and employing Weinberg decomposition we achieve mass terms for the gauge fields related to the weak interaction as well as those for the
electron, muon, s, c, u, and d quarks. The gauge field related to the electromagnetic interaction remains massless, the $Z$, $W^+$ and $W^-$ bosons acquire mass and their masses are in accordance with the usual relations in the four-dimensional Weinberg–Salam–Glashow theory, namely, the ratio between the mass of the $Z$ boson and that of the $W^+$ or $W^-$ is $\cos^{-2}\theta_W$, where $\theta_W$ stands for the Weinberg angle. Through the process of symmetry breaking the fermionic sector acquires mass, but there is a mass term related to both of our neutrinos that does not come from symmetry breaking. Again it is the influence of the fifth dimension, which produce the bare masses of the electron, muon, u, s, c and d quarks. We may conclude that for the electron, muon, u, s, c, and d quarks, the mass term contains two contributions, one coming from symmetry breaking and the other one emerging from the presence of the radius of the fifth dimension, as a consequence of the dimensional reduction and must be cancelled by the Yukawa couplings, as well as for the neutrinos.

As a consequence of dimensional reduction, in the four-dimensional effective theory Pauli terms arise in the usual manner, they may be understood as an anomalous weak momentum and an anomalous electromagnetic momentum, in this theory the neutrino has no electric charge, but it generates an electromagnetic field and this fact may be seen in the polarization currents that emerge in the Yang–Mills equations.

This paper is easily extended to include the third fundamental family.

This work is organized as follows: In section II we construct the scalar curvature. In section III we build the eight–dimensional Dirac–Lagrangian density. In section IV we calculate the Yukawa couplings and afterwards carry out the breaking of symmetry. Section V contains Weinberg decomposition of the field equations by means of the mixing angle and the ensuing results are then discussed.

2. Scalar curvature

The local principal fiber bundle line element for the product space-time $M^4 \times G$, is given by $ds^2$ where $\mu, \nu = 0, 1, 2, 3$; $i, j = 5, 6, 7, 8$; $\alpha, \beta, \ldots = \text{group indices}$. We identify $g_{\mu\nu}$ with the metric of the four–dimensional space–time, $\gamma_{ij}(y)$ is the Killing metric on $S^1 \times S^3$, $\kappa_i^\alpha(y)$ are the Killing vectors and $A^\alpha_i(x)$ the corresponding gauge fields, $\varphi(x)$ is a dilaton field, while $\Phi(x)$ is an isospin quadruplet. We adopt the eight–dimensional analogue of the Einstein–Dirac–Higgs action

$$I_8 = \int d^4x \int d^4y \sqrt{\hat{g}} \left[ \frac{1}{16\pi GV} \left( \hat{R} + V(\Phi) + Y_u \right) + \mathcal{L}_D \right],$$

(2)

to be the basic action. Here $\hat{R}$ is the eight–dimensional scalar curvature, $V(\Phi) = -\frac{\mu^2}{2} \Phi^4 + \frac{1}{4!} (\Phi^4)^2$ is a Higgs potential term, and $\mathcal{L}_D$ is a straightforward generalization in eight dimensions of the well known four–dimensional Dirac–Lagrangian density, which will be seen in the next section, whereas $Y_u = h(\bar{L}\Phi R + \bar{R}\Phi^4 L)$ denotes the Yukawa term and $V$ is the volume of the internal space.
We are going to employ the horizontal lift basis (HLB)
\[ \hat{\theta}^\nu = dx^\nu; \quad \hat{i} = dy^i + \frac{\kappa}{L} K^i_\alpha(y) A^\alpha_\nu(x) dx^\nu, \]
as basis one–forms. The basis dual to (3) is.
\[ \hat{e}_\mu(x, y) = \partial_\mu - \frac{\kappa}{L} A^\alpha_\mu K^i_\alpha \partial_i; \quad \hat{e}_i(y) = \partial_i. \]

On dimensional grounds we introduce the length scales \( L^{-1} \) and \( L_1^{-1} \) of \( S^3 \) and \( S^1 \) respectively. The metric in this basis is simply
\[ g^\hat{\mu}\hat{\nu} = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & -g_{ij}(y) \end{pmatrix}. \]
The eight-dimensional curvature scalar is given by
\[ \hat{R} = R + \frac{R_{s^3}}{\Phi^4} - 2(\partial_\nu \ln \varphi)(\partial^\nu \ln \varphi) + \frac{3}{\Phi^4} (D_\nu \Phi^4)(D^\nu \Phi) - \frac{\kappa^2}{4} (D^\nu \Phi^4 F_{\mu\nu} + \Phi^4 F_{\mu\nu}^a F^a_{\mu\nu}), \]
where \( R \) is the scalar curvature of \( M_4 \) and \( R_{s^3} \) is the scalar curvature of \( S^1 \times S^3 \), which is defined by \( R_{s^3} = -\gamma_{ij} R^k_{ikj} \) so that \( R_{s^3} > 0 \) for the sphere. Moreover, \( g_{55} = \varphi^2 L_1^2 \) and \( g_{ij} = \Phi^2 \gamma_{ij} \)

3. Eight-dimensional Dirac-Lagrangian density

As basis for the tangent bundle of \( S^1 \), we use the vector \( \partial_5 \) and for the tangent bundle of \( S^3 \) the three Killing vectors which can be written in terms of the Euler angles \( (\theta, \rho, \psi) \) as follows:
\[ K_5 = L_1 \partial_5; \quad K_6 = L[\cos \psi \partial_\theta - \sin \psi (\cot \theta \partial_\psi - \csc \theta \partial_\rho)], \]
\[ K_7 = L[\sin \psi \partial_\theta + \cos \psi (\cot \theta \partial_\psi - \csc \theta \partial_\rho)]; \quad K_8 = L \partial_\rho. \]
(Note that \( y^6 = \theta, \ y^7 = \rho, \ y^8 = \psi \). Thus the Killing metric \( \gamma_{ij} = K_\alpha K^\alpha_\beta \) for \( S^1 \times S^3 \) may be easily evaluated. We can now calculate the achtbein necessary to introduce spinors, in the HLB. The achtbein like the metric is simply block diagonal
\[ e^\hat{\hat{A}}_\mu = \begin{pmatrix} e^A_\mu \\ 0 \\ e^{(k)}_j \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} e^A_\mu \\ 0 \\ 0 \end{bmatrix} & 0 & 0 & 0 & 0 \\ 0 & \varphi L_1 & 0 & 0 & 0 \\ 0 & 0 & \bar{\Phi} L & 0 & 0 \\ 0 & 0 & 0 & \bar{\Phi} L \cos \theta & 0 \\ 0 & 0 & 0 & \bar{\Phi} L \sin \theta & 0 \end{bmatrix}, \]
\( \hat{A}, \ldots, \hat{\mu}, \ldots = 0, 1, 2, \ldots, 8 \) and \( e^\hat{\hat{A}}_\mu \) satisfy the usual relation \( \hat{g}^\hat{\mu}\hat{\nu} = e^\hat{\hat{A}}_\mu e^{\hat{\hat{B}}} e_{\hat{\nu}\hat{\alpha}} \eta_{\hat{A}\hat{B}} \) with \( \eta_{\hat{A}\hat{B}} = \begin{pmatrix} +1 & -1 & -1 \\ -1 & +1 & -1 \\ -1 & -1 & +1 \end{pmatrix} \) and \( e^{(k)}_j \) are the vierbeins which satisfy the usual
Dirac matrices on \( M \) 

We choose \( \Psi_fL \) 

The covariant derivative for spinors is defined by 

\[
\hat{\nabla}_\mu \psi_{jR,L} = (\hat{e}_\mu + \hat{\Gamma}_\mu)\psi_{jR,L} .
\]

Here \( \hat{\Gamma}_\mu = \frac{1}{2} e_\bar{\mu} e_{\bar{B}\rho\mu} \sigma^{\bar{A}\bar{B}} \) is the the Clifford–algebra–valued connection, with \( \sigma^{\bar{A}\bar{B}} = \frac{1}{4}[\Gamma^{\bar{A}}, \Gamma^{\bar{B}}]. \)

The size of the eight-dimensional spinors is sixteen. Let \( \gamma^A \) and \( \gamma^k \) denote the Dirac matrices on \( M^4 \) and \( S^1 \times S^3 \) respectively. Then we may take the Dirac matrices on \( M^4 \times S^1 \times S^3 \) to be given by the following tensor products:

\[
\Gamma^A = I \otimes \gamma^A \quad A = 0, 1, 2, 3 \quad \Gamma^k = \gamma^k \otimes \hat{\gamma}^5 \quad k = 5, 6, 7, 8 ,
\]

where \( \hat{\gamma}^5 \) is the usual \( \gamma^5 \)-matrix on \( M_4 \). The matrices in (11) satisfy \( \{\Gamma^\bar{A}, \Gamma^\bar{B}\} = 2\eta^{\bar{A}\bar{B}}. \)

Here \( \gamma^4 \) are the usual 4-dimensional Dirac matrices and \( \gamma^k \) are given by

\[
\gamma^5 = \begin{pmatrix} i \mathbf{1} & 0 \\ 0 & -i \mathbf{1} \end{pmatrix} , \quad \gamma^l = \begin{pmatrix} 0 & \sigma^l \\ -\sigma^l & 0 \end{pmatrix} \quad l = 6, 7, 8 .
\]

The eight-dimensional Dirac-Lagrangian density is defined by

\[
\mathcal{L}_D = \sum_{f=1}^{2} \frac{i}{2} (\bar{\Psi}^{fL} \Gamma^A e_\bar{\mu} \hat{\nabla}_\mu \Psi_{fL} + e_\bar{A} \hat{\nabla}_\mu \bar{\Psi}^{fL} \Gamma^A \Psi_{fL})
\]

\[
+ \sum_{f=1}^{2} \frac{i}{2} (\bar{\Psi}^{fR} \Gamma^A e_\bar{\mu} \hat{\nabla}_\mu \Psi_{fR} + e_\bar{A} \hat{\nabla}_\mu \bar{\Psi}^{fR} \Gamma^A \Psi_{fR}) .
\]

Where \( \Psi_{fL} \) is the left-hand part and \( \Psi_{fR} \) is the right-hand part of our two fundamental families and the covariant derivatives for \( \Psi_{fR} \) and \( \Psi_{fL} \) are given by

\[
\nabla_\nu \Psi_{fR} = [\partial_\nu - \frac{\kappa}{L_1} B_\nu \partial_5 + \Gamma_\nu] \Psi_{fR}
\]

\[
\nabla_\nu \Psi_{fL} = [\partial_\nu - \frac{\kappa}{L_1} B_\nu \partial_5 - \frac{\kappa}{L} A_\nu^\alpha K^i_\alpha \partial_i + \Gamma_\nu] \Psi_{fL} .
\]

We choose \( \Psi_{fL}(x^\mu, y^i) = V^{-1/2} e^{i\beta Y_\nu/2} e^{i\theta^3/2} e^{i\theta^1/2} e^{i\phi^3/2} \psi_{fL}(x^\mu), \ (f = 1, 2) \) as the \( x^i \) dependence for the left-hand part of our Dirac spinor in terms of the Euler angles in \( S^3 \). The left-hand part of our Dirac fields is

\[
\psi_{1a}(x^\mu) = \begin{pmatrix} \nu_\mu \\ e^- \\ u \\ d \end{pmatrix} , \quad \psi_{2a}(x^\mu) = \begin{pmatrix} \nu_\mu \\ \mu^- \\ c \\ s \end{pmatrix} ,
\]

(15)
whereas the right-hand part is a singlet with respect to SU(2), such that we may choose the following dependence for the right hand part: $\Psi_R(x^\mu, y^i) = V^{-1/2}e^{iy^R/2}\tilde{\psi}_R(x^\mu)$, $f = 1, 2$. Here $\tilde{\psi}_R = \epsilon_R, \mu_R, u_R, c_R, d_R, s_R$ are the right handed singlets, The left and right handed fermionic hypercharge matrices are $Y_L$ and $Y_R$ respectively. The SU(2) generators $\tau^i$ are given by

$$Y_\psi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & -1/3 \end{pmatrix}; \quad \tau^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\tau^2 = \begin{pmatrix} 1 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}; \quad \tau^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

where $\tau^i$ satisfy the usual SU(2) algebra $\{\tau^i, \tau^j\} = 2\delta^{ij}\mathbf{1}$; $[\tau^i, \tau^j] = 2i\epsilon^{ijk}\tau_k$ with $Y_R = -2$ for right handed leptons $\epsilon_R, \mu_R$, and $Y_R = \frac{4}{3}$ for right handed quarks $u_R, c_R$ and $Y_R = -\frac{2}{3}$ for $d_R, s_R$.

The form for the hypercharge matrix, $Y_\psi$ has been selected by convenience to obtain the correct values of the electric charge, using Gell–Mann–Nishima expression $Q = T_3 + \frac{Y}{2}$ being $Q$ the electric charge.

4. Yukawa couplings, GIM mechanism and symmetry breaking.

The effective four-dimensional potential is given by $V(\Phi) = \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + \frac{\lambda}{4}(\Phi^4)^2$ where the isospin quadruplet is given by $\Phi_a = (\phi_1, \phi_2, \phi_3, \phi_4)^T$.

For the hadronic part, we may introduce the well known Cabbibo mixture $d_c = d \cos \theta_c + s \sin \theta_c$ $s_c = s \cos \theta_c - d \sin \theta_c$, where $\theta_c$ is the Cabbibo angle. The ground state for the Higgs field is chosen as $\Phi^0 = (0, am, 0, an)^T, \tilde{\Phi}^0 = (am, 0, an, 0)^T$. Without loss of generality we may assume $m > 0$ and $n > 0$, such that $n^2 + m^2 = 1$, where $a^2 = -6\mu^2/\lambda$. The covariant derivative of the $\Phi$ field is $D_\nu \Phi = [\partial_\nu - \frac{2}{5}g_1 B_\nu Y_l - \frac{2}{5}g_2 A_{\nu}^\alpha \tau_\alpha] \Phi$.

After symmetry breaking, the Yukawa terms for the leptons and hadrons are given as follows

$$\mathcal{L}_{Y_I} = G_1 am(\bar{e}_L e_R + \bar{\epsilon}_R e_L) + G_2 am(\bar{\mu}_R \mu_L + \bar{\mu}_L \mu_R)$$

$$\mathcal{L}_{Y_h} = G_3 an(\bar{d}_L d_R + \bar{\epsilon}_R d_L) + G_6 an(\bar{s}_L s_R + \bar{s}_R s_L) + G_7 an(\bar{u}_L u_R + \bar{u}_R u_L) + G_8 an(\bar{c}_L c_R + \bar{c}_R c_L).$$

In order to achieve the Weinberg–Salam–Glashow model in the low energy limit we will add to the Yukawa terms a bare contribution to compensate the direct influence.
of the fifth dimension on the fermionic masses, which leads to results excluded by the experimental data.

5. Weinberg decompositions

We proceed to carry out the decomposition of the field equations, in order to do so we employ the Weinberg decomposition

\[
Z_\mu = A_\mu^3 c_W + B_\mu s_W; \quad A_\mu = -A_\mu^3 s_W + B_\mu c_W; \quad W^\pm_\mu = A_\mu^1 \mp iA_\mu^2
\]

with the abbreviation \( c_W = \cos \theta_W \) and \( s_W = \sin \theta_W \) and \( \theta_W \) is the Weinberg angle. \( A_\mu \) is now the electromagnetic field and \( Z_\mu \) the neutral boson. Proceeding as usual in the Weinberg-Salam theory

\[
Z_{\mu \nu} = Z_{\nu \mu} - Z_{\mu \nu}; \quad A_{\mu \nu} = A_{\nu \mu} - A_{\mu \nu}; \quad W^\pm_{\mu \nu} = W^\pm_{\nu \mu} - W^\pm_{\mu \nu},
\]

the electron charge is for \((g_1 = \kappa / L_1 \text{ and } g_2 = \kappa / L)\) given by \( \hat{e} = g_1 c_W = g_2 s_W \). The decomposed field equations are the following:

Dirac equations:

\[
i \gamma^\mu (\partial_\mu + i \hat{e} A_\mu Q + \Gamma_\mu 1_4) \psi_{fa} + \frac{1}{2} g_2 B_f + \frac{1}{2} \gamma^\mu Z_\mu S_f + \frac{i}{2} \varphi e^\mu e_B \sqrt{16 \pi G (s_W Z_{\mu \nu} + c_W A_{\mu \nu}) \sigma^{AB} \psi_{fa} + a M_f \psi_{fR} = 0}
\]

where

\[
Q = \begin{pmatrix} 0 & -1 \\ 2/3 & -1/3 \end{pmatrix}, \quad B_1 = \begin{pmatrix} \gamma^\mu W^\pm_{\mu \nu} e_L \\ \gamma^\mu W^\pm_{\mu \nu} c_L \\ \gamma^\mu W^\mp_{\mu \nu} d_L \\ \gamma^\mu W^\mp_{\mu \nu} u_L \end{pmatrix}, \quad B_2 = \begin{pmatrix} \gamma^\mu W^\pm_{\mu \nu} c_L \\ \gamma^\mu W^\mp_{\mu \nu} u_L \\ \gamma^\mu W^\mp_{\mu \nu} c_L \\ \gamma^\mu W^\pm_{\mu \nu} u_L \end{pmatrix}.
\]

\[
S_1 = \begin{pmatrix} (g_2 c_W + g_1 s_W) \nu_e \\ -(g_2 c_W - g_1 s_W) e_+ \\ (g_2 c_W - \frac{1}{2} g_1 s_W) u_L \\ -(g_2 c_W + \frac{3}{5} g_1 s_W) d_L \end{pmatrix}, \quad S_2 = \begin{pmatrix} (g_2 c_W + g_1 s_W) \nu_\mu \\ -(g_2 c_W - g_1 s_W) c_\mu \\ (g_2 c_W - \frac{1}{2} g_1 s_W) c_\mu \\ -(g_2 c_W + \frac{3}{5} g_1 s_W) s_\mu \end{pmatrix}.
\]

\[
M_1 = \begin{pmatrix} 0 \\ m G_1 e_R \\ n G_2 u_R \\ n G_3 d_R \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 \\ m G_4 \mu_R \\ n G_5 c_R \\ n G_6 s_R \end{pmatrix}.
\]

and \( \psi_{1R}(x^\mu) = (0, e_R, u_R, d_R)^T, \psi_{2R}(x^\mu) = (0, \mu_R, c_R, s_R)^T \).
For the right hand singlets:

\[ i\gamma^\mu (\partial_\mu \lambda + i\bar{v}_\mu A_\mu Q + \Gamma_\mu \lambda)\psi_{f R} + g_1 s_W \gamma^\mu Z_\mu R_f \]

\[ + \frac{i}{2} \Phi_1 e_B e_B' \sqrt{16\pi G(s_W Z_{\mu \nu} + c_W A_{\mu \nu})} \sigma^{AB} \psi_{f R} + a M_A \psi_{f A} = 0 \quad (25) \]

where

\[ R_1(x^\mu) = (e_R, -(1/6)u_R, -(1/6)d_R)^T, \quad R_2(x^\mu) = (\mu_R, -(1/6)c_R, -(1/6)s_R)^T. \]

The Yang-Mills equations in the lower energy limit (where \( \varphi = \varphi_0, \) see below) are the following:

for photon:

\[ A_{\mu \lambda} + 2\hat{e}\{i(W^{+(\mu)}_{\lambda\mu}W^-) + \frac{i}{2}[W^{+\mu\lambda}W^- - W^{-\mu\lambda}W^+] \]

\[ - \frac{\hat{e}}{2} [(\frac{C_W}{s_W}Z^\mu - A^\mu)[(W^{+\lambda} + W^{-\lambda})(W^+ - W^-) + i(W^{+\mu} - W^{-\mu})^2]] = \frac{\hat{e}}{\varphi^2}[\bar{e}_L\gamma^\mu e_L - \frac{2}{3}\bar{u}_L\gamma^\mu u_L + \frac{2}{3}\bar{d}_L\gamma^\mu d_L + \bar{\mu}_L\gamma^\lambda \mu_L - 2\bar{c}_L\gamma^\lambda c_L + \frac{1}{3}\bar{s}_L\gamma^\lambda s_L + \bar{e}_R\gamma^\mu e_R - \frac{2}{3}\bar{u}_R\gamma^\mu u_R - \frac{1}{3}\bar{d}_R\gamma^\mu d_R]
\]

\[ - \bar{\mu}_R\gamma^\lambda \mu_R - \frac{2}{3}\bar{c}_R\gamma^\lambda c_R + \frac{1}{3}\bar{s}_R\gamma^\lambda s_R] + \sqrt{16\pi G} \frac{C_W}{\varphi}[\{\bar{e}_\nu \sigma^\mu\lambda \nu e + \bar{e}_L\sigma^\mu\lambda e_L - \bar{u}_L\sigma^\mu\lambda u_L - \bar{d}_L\sigma^\mu\lambda d_L\}]_{\mu}, \quad (26) \]

for the \( Z_\mu \)-boson:

\[ Z_{\mu \lambda} - 2g_2 c_W \{i(W^{+(\mu)}_{\lambda\mu}W^-) + \frac{i}{2}[W^{+\mu\lambda}W^- - W^{-\mu\lambda}W^+] \]

\[ - \frac{g_2 c_W}{2} [(Z^\mu - \frac{s_W}{c_W} A^\mu)[(W^{+\lambda} + W^{-\lambda})(W^+ - W^-) + i(W^{+\mu} - W^{-\mu})^2]] = \frac{3a_2}{16\pi G} \frac{[g_1^2 + g_2^2]}{2} Z^\mu
\]

\[ = \frac{1}{2T_1^2}[(g_1 s_W + g_2 c_W)(\bar{\nu}_e \gamma^\lambda \nu e + \bar{\mu}_L\gamma^\lambda \mu_L) + (g_1 s_W - g_2 c_W)(\bar{e}_L\gamma^\lambda e_L + \bar{\mu}_L\gamma^\lambda \mu_L)]
\]

\[ + (g_2 c_W - \frac{1}{3}g_1 s_W)(\bar{u}_L\gamma^\lambda u_L + \bar{c}_L\gamma^\lambda c_L) + (g_2 c_W + \frac{1}{3}g_1 s_W)(\bar{d}_L\gamma^\lambda d_L + \bar{s}_L\gamma^\lambda s_L)] + g_1 s_W(\bar{e}_R\gamma^\lambda e_R + \bar{\mu}_R\gamma^\lambda \mu_R) - \frac{1}{3}g_1 s_W(\bar{u}_R\gamma^\lambda u_R + \bar{c}_R\gamma^\lambda c_R) - \frac{1}{3}g_1 s_W(\bar{d}_R\gamma^\lambda d_R
\]

\[ + \frac{1}{3}g_1 s_W(\bar{s}_R\gamma^\lambda s_R)] + \sqrt{16\pi G} \frac{C_W}{\varphi} s_W[\{\bar{e}_\nu \sigma^\mu\lambda \nu e + \bar{e}_L\sigma^\mu\lambda e_L - \bar{u}_L\sigma^\mu\lambda u_L - \bar{d}_L\sigma^\mu\lambda d_L
\]

\[ + \bar{\nu}_\mu \sigma^\mu\lambda \nu + \bar{\mu}_L \sigma^\mu\lambda \mu_L - \bar{e}_L \sigma^\mu\lambda e_L - \bar{s}_L \sigma^\mu\lambda s_L]_{\mu}, \quad (27) \]
respectively. These results entitle us to evaluate

\[ W^+_{\mu \lambda} - ig_2 \left\{ 2W^{-}[\lambda (c_W Z^\mu] - s_W A^\mu]_{; \mu} - W^+_{\mu \lambda} (c_W Z_{\mu} - s_W A^{\mu \lambda}) \right\} + W^+_{\lambda} (c_W Z^{\lambda \mu} - s_W A^{\mu \lambda}) + ig_2 \left[ W^+_{\mu} (W_{\lambda i} W_3^i + W_{\mu j} W_{3 j}^3) W_{\lambda j}^+ \right] \]

for the \( W^- \)--boson:

\[ W^-_{\mu \lambda} - ig_2 \left\{ 2W^+[-\lambda (c_W Z^\mu] - s_W A^\mu]_{; \mu} + W^-_{\mu \lambda} (c_W Z_{\mu} - s_W A^{\mu \lambda}) \right\} + W^+_{\lambda} (c_W Z^{\lambda \mu} - s_W A^{\mu \lambda}) + ig_2 \left[ W^-_{\mu} (W_{\lambda i} W_3^i + W_{\mu j} W_{3 j}^3) W_{\lambda j}^- \right] \]

Eqs. (27) and (28) give the mass terms for the electron, muon, \( u, d, c, \) and \( s \) quarks. There are two contributions, a dilatonic term proportional to \( 1/\varphi L_1 \), which is a consequence of the dimensional reduction and is cancelled by the Yukawa bare contribution and another one that emerges from symmetry breaking i.e., \( a m G_j \) or \( a n G_j \).

\[ \varphi \rightarrow \varphi_0 \] From Eqs. (26)–(29) we find that the photon stays massless, and that the squared mass for the \( W \) and \( Z \) bosons are given by

\[ M_W^2 = \frac{3a^2 g_2^2}{16\pi G}; \quad M_Z^2 = \frac{3a^2 g_2^2}{16\pi G} \left[ \frac{g_1^2}{\varphi^2} + g_2^2 \right] , \]

respectively. These results entitle us to evaluate \( a \) and the Yukawa constants \( G_j \).

In order to match these theoretical predictions with the experimental results\(^{19,23} \), \( M_W^2 / M_Z^2 = \cos^2 \theta_W \) has to be satisfied. This leads us to conclude that our dilatonic field, \( \varphi \) in its linear vacuum, must have a ground state in which it is a constant \( \varphi_0 \neq 0 \), and \( g_\mu \eta_\mu, F_\mu = 0, F_\mu^\alpha = 0 \) and \( T_{00} = 0 \) is a minimum. This is in fact the case according to our field equations; the ground state is degenerate and exists for any arbitrary constant value of \( \varphi = \varphi_0 \).

\[ \varphi \rightarrow \varphi_0 \] From this point of view, we can choose an appropriate value of \( \varphi_0 \) in order to obtain the true boson masses; one finds: \( \varphi = 1 \).

Beside the usual terms of a Weinberg-Salam-Glashow theory in a curved spacetime, we find that as consequence of the dimensional reduction, there are two anomalous momenta, one related to the electromagnetic gauge field \( A_\mu \) and the other one associated to the weak gauge field \( Z_\mu \). In Gaussian units these momenta have the value \( \langle h/2c \rangle \sqrt{16\pi G} c_W \approx 5.9 \times 10^{-32} \) cm, for the anomalous electromagnetic momentum and \( \langle h/2c \rangle \sqrt{16\pi G} s_W \approx 3.2 \times 10^{-32} \) cm for the weak anomalous momentum. The interaction of these momenta with their corresponding gauge fields appear in the Yang–Mills equations (26)–(28) and they produce additional polarization currents.
This model is readily extended to include the third fundamental family and all our previous conclusions stand.

In conclusion, we have shown that the standard model can be obtained from an eight–dimensional gravity theory taking principal fiber bundle structure with an enlarged Yukawa coupling and interpreting the effective cosmological constant as Higgs potential. The correct gauge bosons masses as well as the fermionic ones are given by the theory. It seems to be that in order to introduce the gravitational interaction in a unified mathematical consistent theory, the extra dimensions are needed.

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