Is the effective field theory of dark energy effective?

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Abstract. The effective field theory of cosmic acceleration systematizes possible contributions to the action, accounting for both dark energy and modifications of gravity. Rather than making model dependent assumptions, it includes all terms, subject to the required symmetries, with four (seven) functions of time for the coefficients. These correspond respectively to the Horndeski and general beyond Horndeski class of theories. We address the question of whether this general systematization is actually effective, i.e. useful in revealing the nature of cosmic acceleration when compared with cosmological data. The answer is no and yes: there is no simple time dependence of the free functions — assumed forms in the literature are poor fits, but one can derive some general characteristics in early and late time limits. For example, we prove that the gravitational slip must restore to general relativity in the de Sitter limit of Horndeski theories, and why it doesn’t more generally. We also clarify the relation between the tensor and scalar sectors, and its important relation to observations; in a real sense the expansion history $H(z)$ or dark energy equation of state $w(z)$ is $1/5$ or less of the functional information! In addition we discuss the de Sitter, Horndeski, and decoupling limits of the theory utilizing Goldstone techniques.

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Cosmic acceleration is fundamental physics beyond the Standard Model, with potentially revolutionary implications for the nature of gravitation and spacetime, the quantum vacuum, and high energy physics. Its origin may lie somewhere along the spectrum of a single number (the cosmological constant), a single canonical minimally coupled scalar field (quintessence), a scalar-tensor theory, or a multiply coupled, “braided” modification altering the nature of tensor gravity and allowing superluminal propagation. Moreover, there is no clear first principles derivation of any of these, pointing to a particular model. Thus the task of revealing the nature of cosmic acceleration by comparing models to observations seems gargantuan.

An alternative is to take a bottom-up approach and construct a Lagrangian by writing the most general theory possible which can account for the cosmic acceleration, where the allowed operators are restricted by a combination of symmetries and theoretical self-consistency. This is the goal of the Effective Field Theory (EFT) approach [1–5]. EFT comprises two parts: 1) integrating out the physics of the high energy degrees of freedom to derive an effective low energy theory suitable for comparison to observations in our low-
energy world (where low energy is relative, e.g. for inflation this is low relative to the Planck energy), and 2) including all low energy degrees of freedom allowed by the symmetries. When spontaneous symmetry breaking is present, as in a time dependent background, focusing on the Goldstone bosons of the broken symmetry leads to a particularly powerful approach, since the requirement to non-linearly realize the broken symmetry forces relations between the free parameters of the low energy effective theory — the corresponding operators exhibit universal behavior.

The EFT approach has been successfully used in many circumstances, including the early cosmic acceleration of inflation [6, 7] and the theory of dark matter [8, 9]. EFT has also been written down for late cosmic acceleration, i.e. dark energy (EFTDE) [10–13], but it is not clear that its application to dark energy has been successful. Three aspects of the EFTDE present challenges that are somewhat distinct from the cases above, in both theoretical and practical issues: 1) the coefficients of the operators in the action are time dependent functions rather than constants; 2) in the most general EFTDE there are more coefficients than observables; 3) the primary observational constraints on these coefficients are especially challenging: the universe is (nearly) matter dominated, whereas the theory is constructed by using the spontaneous breaking of time translation invariance during (complete) dark energy domination. Indeed, for constraints on modified gravity in the current epoch it is known that for long distance (IR) modifications of gravity near the Hubble scale the time it takes for such modifications to result in local, observational consequences is longer than the current age of the universe [14]. Moreover, in many modified gravity theories screening and/or decoupling from gravity lead to a lack of observational consequences on short distance scales.

We address here aspects of these questions. Basically, can the effective field theory of dark energy be effective? We take care to bridge the theoretical, phenomenological, and observational descriptions of cosmic acceleration and in particular investigate both the scalar and tensor aspects of the theory. One of the recent exciting developments in cosmic acceleration has been the identification of relations between properties of the matter density field (scalar sector) and gravitational wave propagation (tensor sector) [15–17], offering new observational connections.

In section 2 we briefly review the different methods of describing the impact of beyond Einstein physics from the theory, phenomenology, and observational points of view, giving the translations between them. The functions defined in these ways are examined more closely in section 3, with numerical solutions given for the specific case of Galileon gravity, establishing that simple parameterizations as often used in the literature to place observational constraints are insufficient and biased. We also derive some general relations in the early, matter dominated, and late time, asymptotic de Sitter (dS) limits. In section 4 we focus on the Goldstone mode in the EFTDE. We use this to examine the dS, Horndeski, and decoupling limits of the theory. Using these limits and the Goldstone method we are able to reproduce some of the key findings from the previous sections and we discuss how viable modifications of gravity fall into two primary classes based on their decoupling behavior. We discuss the implications for the effectiveness of EFTDE to explore cosmic acceleration in section 5.

2 Describing cosmic acceleration in theory and observations

EFT provides an action with terms of particular forms determined by the allowed symmetries, and coefficients that in the case of dark energy (used generically as including modified gravity)
are time dependent functions. These coefficients have dimensions of mass and so can be illustratively written as $M_i(t)$. Constraining the $M_i(t)$ gives specific information about the class of theory, since different theories predict different terms in the action, i.e. some $M_i$ are zero. Observational quantities on the other hand deal with measurements of density contrasts and particle motions (e.g. velocities or gravitational lensing). These can be phrased in terms of modified Poisson equations and the relation between metric potentials, i.e. the effective gravitational coupling strength (generalized Newton’s constant) and the gravitational slip. Phenomenology attempts to sit between the theory and observational quantities, e.g. dealing with “property” functions characterizing and following from the equations of motion. We briefly review each, and connect the quantities.

2.1 EFT action and functions

The EFTDE quadratic action in the Jordan frame, following the parameter notation of [13] and working with metric signature ($-++,++$), is

$$S_2 = \int d^4x \sqrt{-g} \left[ \frac{m_0^2}{2} \Omega(t) R - \Lambda(t) - c(t) g^{00} + \frac{M_2^2(t)}{2} (\delta g^{00})^2 - \frac{\dot{M}_1^3(t)}{2} \delta K \delta g^{00} 
- \frac{\dot{M}_2^2(t)}{2} \delta K^2 - \frac{\dot{M}_3^2(t)}{2} \delta K_{\mu\nu} \delta K_{\mu\nu} + \frac{\dot{M}_4^2(t)}{2} \delta R^{(3)} \delta g^{00} + m_2^2(t) \partial_{\mu} g^{00} \partial^\mu g^{00} + L_m + \ldots \right],$$

where $\delta g^{00} = g^{00} - 1$ and $\delta R^{(3)}$ are perturbations of the time-component of the metric and the spatial curvature, respectively, and dots represent terms that are suppressed by either derivatives or higher powers in the perturbations.\(^1\)

Given the normal to constant time hypersurfaces

$$n_\mu = \frac{\partial_\mu t}{\sqrt{-\partial_\sigma t \partial^{\sigma} t}} = \delta_\mu^0,$$

we have the perturbation of the extrinsic curvature

$$\delta K_{\mu\nu} = K_{\mu\nu} - K^{(0)}_{\mu\nu} = K_{\mu\nu} + 3H (g_{\mu\nu} + n_\mu n_\nu),$$

where its trace $\delta K = \delta K_{\mu\mu}$, and $H(t)$ is the Hubble expansion parameter.

The action involves background quantities $\Lambda(t)$ and $c(t)$, and linearly perturbed quantities involving the mass coefficients. In the simple case of quintessence the background terms correspond to the potential $\Lambda(t) = V(\phi(t))$ and the kinetic term $c(t) = \frac{\dot{\phi}^2}{2}$ of the scalar field. In more involved models these background expressions can be more complicated.

Note that we wrote the gravitational coupling to the Ricci tensor as $m_0^2 \Omega(t)$, to show the explicit mass dimension in the constant $m_0$, leaving the time dependence in $\Omega(t)$. When $\Omega$ is constant (e.g. unity) we can identify $m_0^2 \rightarrow m_p^2$, where we use reduced Planck mass units $m_p = 1/\sqrt{8\pi G_N} = 2.4 \times 10^{18}$ GeV. In general, $m_0^2 \Omega(t)$ is a single quantity, which we will sometimes denote $p$. While $m_0^2 \Omega(t)$ is a homogeneous background quantity it is important to note that given our bottom-up approach (and without knowledge of the UV completion of the theory) there are not enough equations to fix $\Omega(t)$ uniquely from the background evolution [11]. This parameter must be determined by enforcing symmetries and constraints coming from considering the perturbations which start at quadratic order in the action.

\(^1\)We refer the reader to [6, 7] for more details.
The background equations of motion (which are the tadpole cancellation conditions, discussed below) in the Jordan frame are given by

\begin{align}
3H^2 + 3\dot{\Omega}(t)\dot{\Omega}(t) &= \frac{1}{m_0^2\Omega(t)}(c + \Lambda + \rho_m) \\
2\ddot{H} + \ddot{\Omega}(t) + \frac{2\dot{\Omega}(t)}{\Omega(t)}\dot{H} &= -\frac{1}{m_0^2\Omega(t)}(2c + \rho_m + p_m).
\end{align}

(2.4) \hspace{1cm} (2.5)

Several comments are in order. For arbitrary \(\Omega(t)\) we have a Brans-Dicke theory (which in some sense is \textit{not} a true modification of gravity as it does not imply a change in the propagator of the spin-2 graviton) and the Jordan and Einstein frames are manifestly different although they will lead to the same observables. We use these background equations to eliminate tadpole terms from the action. In other words, when we expand the first (gravitational) term in eq. (2.1) we want the perturbations from that term to be canceled at the linear level by the terms coming from \(c(t)\delta g^{00}\) and \(\Lambda(t)\). This is equivalent to solving the background equations for \(c\) and \(\Lambda\) and plugging them back into the action. After this procedure the action that results from eq. (2.1) is the most general theory possible for the scalar perturbations (at the quadratic level) about a spatially homogeneous background that spontaneously breaks time diffeomorphism invariance [12, 13].

There are seven free parameters that describe the most general theory [12, 13]:

\[ [m_0^2\Omega(t), \bar{M}_1^2(t), M_2(t), \bar{M}_2(t), M_3(t), \bar{M}(t), m_2(t)] \]  

(2.6)

Given these parameters it is possible to characterize all existing models — see table 1 in appendix A. For example, linearized Horndeski theory, or equivalently generalized Galileon theory, is reproduced by the parameter choices

\[ m_2 = 0; \quad 2\bar{M}^2 = \bar{M}_2^2 = -\bar{M}_3^2 \quad \text{[Horndeski]} \]  

(2.7)

This class of theories has received notable attention because the resulting equations of motion are second order in time and space derivatives, which ensures they do not suffer from an Ostrogradsky instability and there is a well defined Cauchy problem.

The additional parameters in eq. (2.6) however allow for the construction of more general models, raising the question — are these theories sound and stable? We explain a simple rationale for their validity in appendix B. The upshot is that by construction the EFTDE approach renders these theories stable within their regime of applicability regardless of the number of time and/or space derivatives. We emphasize that this does \textit{not} imply the more restricted Horndeski-like theories are uninteresting. Instead, we are pointing out that the EFTDE approach provides a complete framework for establishing which proposals for dark energy are observationally interesting when confronted with existing data. If a class of models is established as favored by the data it will then be an important endeavor to establish the high energy completion of such a model. Moreover, Horndeski-like theories provide an important class of models where a non-perturbative formulation seems possible. It is an important question whether such models can be embedded in a complete theory of quantum gravity, but we see that in the observationally relevant regime these theories are captured within the EFTDE through the parameter selection given in eq. (2.7). Thus, if observations can rule out such models in the EFT regime this would suggest model building should go in a different direction.
We would also like to emphasize that the seven parameters of eq. (2.6) correspond to the most general action at quadratic level in the perturbations. Going beyond the quadratic level leads to additional parameters that need to be determined through a combination of theoretical and observational constraints. For investigations during the linear regime of structure formation (where perturbations are small) these seven parameters should provide the leading contributions, but even then we will see below that there are only four observational quantities to restrict the seven free parameters. That is, there is an inverse problem. This is a familiar situation from particle physics, where collider data places constraints on those EFT parameters, but many different proposals for physics beyond the standard model can lead to the same observational predictions. We have a similar challenge to address here with degeneracies in the map between the EFT parameters and experimental observations. The more general (beyond Horndeski) EFTDE approach most likely will require novel observations to determine fully the nature of dark energy.

2.2 Observational functions

The EFTDE approach allows us to specify the expansion history of the background universe, which is described by the Hubble parameter $H(t)$. The remaining parameters are also connected to direct observables, e.g. the growth and motion of structure (perturbations). The impact of the theory on the perturbed quantities is conveniently described in terms of the modified Poisson equations for non-relativistic and relativistic particles, e.g. galaxies and light.

Working in the conformal Newtonian gauge, the perturbed metric is

$$ds^2 = -(1 + 2\Psi) dt^2 + a^2(t) (1 - 2\Phi) d\vec{x}^2,$$

where we assume vanishing spatial curvature. The modified Poisson equations can then be written as

$$\nabla^2 \Psi = 4\pi a^2 G_{\text{eff}}^\Psi \rho_m \delta_m,$$

$$\nabla^2 (\Psi + \Phi) = 8\pi a^2 G_{\text{eff}}^{\Psi+\Phi} \rho_m \delta_m,$$

where the (time and scale dependent) $G_{\text{eff}}^\Psi$ are modified Newton’s constants. In the quasi-static regime they can be viewed as giving the gravitational coupling strength. The quantity $\rho_m$ is the matter density, and $\delta_m = \delta\rho_m/\rho_m$ is the density perturbation. The first equation is central to the motion of non-relativistic matter and hence the growth of massive structures, while the second governs the motion of relativistic particles, and hence the propagation of light [18].

While $G_{\text{eff}}^\Psi$ and $G_{\text{eff}}^{\Psi+\Phi}$ are tied directly to observables through these equations (leaving aside the step of relating observed galaxies to the underlying matter distribution), it is also common in the literature to use $G_{\text{eff}}^\Psi$ and a gravitational slip defined as

$$\eta = \frac{\Psi}{\Phi} = \frac{G_{\text{eff}}^\Psi}{2G_{\text{eff}}^{\Psi+\Phi} - G_{\text{eff}}^\Psi}.$$

In [17] they use

$$\bar{\eta} = \frac{2\Psi}{\Psi + \Phi} = \frac{2\eta}{1 + \eta} = \frac{G_{\text{eff}}^\Psi}{G_{\text{eff}}^{\Psi+\Phi}},$$

which is somewhat more closely tied to observations, and we will use this version of the slip.
In addition, observation of gravitational waves (e.g. through cosmic microwave background B-mode polarization, pulsar timing arrays, or directly with gravitational wave antennas) would allow the tensor sector to be probed. The wave propagation equation involves three quantities: the propagation speed \( c_T(t) \), the friction term, and modifications to the source term. The last one arises from anisotropic stress, which also sources the gravitational slip, so we basically have two more “observable” functions from the tensor sector. This point has been somewhat under-appreciated in the literature — the scalar sector gives two functions worth of information on the theory and the tensor sector gives two functions worth as well. Together they add up to four functions, equivalent to that of the theory description in the Horndeski framework. In beyond Horndeski theories, the gravitational wave source term gets additional contributions and there can be a mass term as well. As mentioned in the previous section the Horndeski correspondence between the observables and the free parameters is the result of working with a special class of models, at the quadratic level.

### 2.3 Property functions

In [17], they develop “property” functions that describe classes of theories phenomenologically, characterizing properties such as the kineticity — kinetic structure of the scalar field, affecting the sound speed, braiding — mixing of the scalar kinetic terms with the metric, affecting dark energy clustering, running Planck mass — leading to anisotropic stress, and tensor wave speed. These give four time dependent functions \( \alpha_K(t) \), \( \alpha_B(t) \), \( \alpha_M(t) \), and \( \alpha_T(t) \) that enter in the equations of motion.

The property functions play a role midway between the directly theoretical exhibition of the action functions and the ties to actual data of the observational functions. Just as one might approach quintessence through the more closely theoretical potential \( V(\phi) \) or the more phenomenological equation of state \( w(t) \), which type of function is used is mostly a matter of taste. In the next section we give translations between these functions.

### 2.4 Translating between functions

We assemble here the key equations relating the action, property, and observational functions to each other.

The \( \alpha_i \)’s from table 2 of [17] for the action eq. (2.1) above are

\[
\alpha_M = \frac{\dot{\Omega} + \frac{\dot{m}_2^2}{m_0^2}}{H\Omega + H\frac{m_2^2}{m_0^2}} = \frac{\dot{p}/p(1 + N) + \dot{N}}{H(1 + N)} \tag{2.13}
\]

\[
\alpha_K = \frac{2c + 4M_2^4}{m_0^2\left(H^2\Omega + H^2\frac{m_2^2}{m_0^2}\right)} = 2\frac{c + 2M_2^4}{H^2p(1 + N)} \tag{2.14}
\]

\[
\alpha_B = -\frac{\dot{M}_2^3 + m_0^2\dot{\Omega}}{m_0^2\left(H\Omega + H\frac{m_2^2}{m_0^2}\right)} = \frac{\dot{p} + \dot{M}_2^3}{Hp(1 + N)} \tag{2.15}
\]

\[
\alpha_T = -\frac{\dot{M}_2^2}{m_0^2\left(\Omega + \frac{M_2^2}{m_0^2}\right)} = -\frac{N}{1 + N} \tag{2.16}
\]

where we define \( p(t) = m_0^2\Omega(t) \) and \( N(t) = \dot{M}_2^2/p \).
The converse transformation is
\[ N = -\frac{\alpha T}{1 + \alpha T}; \quad 1 + N = (1 + \alpha T)^{-1} = c_T^{-2} \tag{2.17} \]
\[ \frac{\dot{p}}{\dot{p}} = H\alpha M - \frac{\dot{N}}{1 + \alpha T} \quad M_1^3 = -Hp \frac{\alpha B}{1 + \alpha T} - \dot{p} = -Hp \left( \frac{\alpha B}{1 + \alpha T} + \alpha M - \frac{N'}{1 + N} \right) \quad (2.18) \]
\[ c + 2M_2^4 = \frac{1}{2} \alpha K H^2 p(1 + N), \tag{2.19} \]
where prime denotes \(d/d\ln a\).

We see that \(N\) depends only on \(\alpha T\), \(p\) depends only on \(\alpha M\) and \(\alpha T\), and \(\alpha K\) only enters for the combination \(c + 2M_2^4\). Since the observables or their proxies, e.g. the gravitational coupling \(G_{\text{eff}}\) and gravitational slip \(\bar{\eta}\) entering the growth of structure only depends on the \(\alpha_i\), this seems to imply that \(c\) and \(M_2^4\) cannot be separately determined, only the particular combination \(c + 2M_2^4\). Some quantities that are not directly observable, such as the scalar sound speed \(c_s\) and the quantity \(C_3\) below, do depend on \(c\) separately. Beyond the quasistatic approximation (e.g. near horizon scales), \(M_2\) may appear separately [19].

The observational functions are related to the theory and phenomenology functions by the expressions below. Following [13] and working in the Newtonian limit we have
\[ 4\pi G_{\text{eff}} = \frac{C_3 - C_1 B_3}{A_1 (B_3 C_2 - B_1 C_3) + A_2 (B_1 C_1 - C_2) + A_3 (C_3 - C_1 B_3)}, \tag{2.21} \]
where the derivation and the expressions for the \(A_i, B_i,\) and \(C_i\) are given in appendix C.

In the particular case of Horndeski theory and the dS limit we find
\[ 4\pi G_{\text{eff}} = \frac{1}{2m_0^2 \Omega} \left( 1 + \frac{\dot{M}_2}{m_0^2 \Omega} + \frac{\dot{M}_1^3}{2m_0^2 \Omega H} \right)^{-1} \tag{Horndeski, de Sitter limit}\]
\[ \quad [\text{Horndeski, de Sitter limit}] \tag{2.22} \]
If instead of Horndeski and the dS limit one focuses only on the background quantities (all \(\dot{M}_i = 0\), and \(m_2 = 0 = \dot{M}\) we have
\[ 4\pi G_{\text{eff}} = \frac{1}{2m_0^2 \Omega} \left( 1 + \frac{(\Omega/\Omega)^2}{4c/(m_0^2 \Omega) + 3(\Omega/\Omega)^2} \right) \quad [\text{background}] \tag{2.23} \]
This corresponds to Brans-Dicke type modifications of gravity. In either case we recover \(4\pi G_{\text{eff}} = 1/(2m_0^2)\) for the Einstein-Hilbert action.

For the gravitational slip, in the Newtonian limit the general expression is
\[ \bar{\eta} = \frac{2(C_3 - C_1 B_3)}{B_3 C_2 - B_1 C_3 + C_3 - C_1 B_3}. \tag{2.24} \]
Of great interest is that in the dS limit of Horndeski theory one immediately finds that
\[ \bar{\eta} \rightarrow 1 \quad [\text{Horndeski, de Sitter limit}] \tag{2.25} \]
This derivation of a general property for an entire family of theories is a major success for EFTDE. This property had been derived for the restricted case of covariant Galileon theory.
in [15], where it was shown that despite this vanishing of deviations of the gravitation slip, the tensor sector still showed clear departures from general relativity.

Apart from Horndeski theory and the dS limit, if one just turns on the background operators then working in the Newtonian limit gives

\[ \tilde{\eta} = \frac{c/(m_0^2 \Omega) + (\Omega/\Omega)^2}{c/(m_0^2 \Omega) + (3/4)(\Omega/\Omega)^2} \quad [\text{background}] . \]  

(2.26)

As far as the tensor wave speed, that has the simple general expression

\[ c_T^2 = \left( 1 - \frac{M_3^2}{m_0^2 \Omega} \right)^{-1} . \]  

(2.27)

\[ c_T^2 = (1 + N)^{-1} = \left( 1 + \frac{M_3^2}{m_0^2 \Omega} \right)^{-1} \quad [\text{Horndeski}] . \]  

(2.28)

Here \( \Omega(t) \) only enters because it defines the effective Planck mass — no time derivatives of \( \Omega(t) \) appear (though they do, in terms of \( \alpha_M \), i.e. the running of the Planck mass, in the gravitational wave propagation friction term). In the absence of the extrinsic curvature terms, \( \bar{M}_3 \to 0 \) and we recover \( c_T = 1 \). (See appendix D for a discussion of the meaning of modification of gravity.)

Finally, the observable functions are related to the property functions by [17]

\[ \frac{G_{\text{eff}}}{G_N} = \frac{2m_p^2}{M_*^2} \left[ \alpha_B(1 + \alpha_T) + 2(\alpha_M - \alpha_T) \right] + \alpha_B' \]  

(2.29)

where \( M_*^2 = m_0^2 \Omega + \bar{M}_3^2 = m_0^2 \Omega/(1 + \alpha_T) \); note \( \alpha_M = (\ln M_3^2)' \).

The gravitational slip \( \tilde{\eta} \) is given by

\[ \tilde{\eta} - 1 = \frac{(2 + 2\alpha_M)[\alpha_B(1 + \alpha_T) + 2(\alpha_M - \alpha_T)] + (2 + 2\alpha_T)\alpha_B'}{(2 + \alpha_M)[\alpha_B(1 + \alpha_T) + 2(\alpha_M - \alpha_T)] + (2 + \alpha_T)\alpha_B'} , \]  

(2.30)

and the tensor wave speed is

\[ c_T^2 = 1 + \alpha_T . \]  

(2.31)

Whether using the EFT action functions, property functions, or observational functions, one always has another function \( H(t) \) describing the background. In addition, the tensor wave speed \( c_T(t) \) is evident in all three, appearing as itself in the observational, as \( \alpha_T = c_T^2 - 1 \) in the phenomenological, and essentially as \( M_3 \) in the theory approaches.

3 Functions to parameters?

Since EFT provides four or more functions of time in the \( M_i \) or \( \alpha_i \), apart from the background expansion \( H(t) \), constraints from observations will be difficult in full generality. Usually one needs to compress the number of degrees of freedom by a lower dimensional parametrization. Attempts to do so by choosing power law dependence on scale factor \( a \), or proportionality to the contribution of the effective dark energy density \( \Omega_{\text{de}} \) (usually assuming a constant energy density, i.e. a cosmological constant) to the total dynamics as a function of time have appeared in the literature. We find below that even for restricted classes of theories these are vastly oversimplified and hence inaccurate.
3.1 General expressions and numerical results

The property functions $\alpha_i$ are built out of ratios of several Lagrangian terms and there is no obvious reason why they should have a simple time dependence, especially when terms are of comparable magnitude and can interact and cancel as they evolve. The one exception might be at high redshift when the deviations from General Relativity (GR) are small; there one Lagrangian term may initially dominate, and hence also drive the effective dark energy density. This should not generically hold at late times however when cosmic acceleration becomes important — and the vast majority of observations enter.

We illustrate this for the case of covariant Galileons. The Galileon subclass of the Horndeski Lagrangian involves four terms, with constant coefficients $c_2$, $c_3$, $c_4$, $c_5$ all presumably of order unity. These combine into functions of time $\kappa_i$ which can be translated into the property functions $\alpha_i$ (see the appendix of [15]). For example,

$$\alpha_B = \frac{2\kappa_5 x}{\kappa_4} = \frac{4c_3\bar{H}^2x^3 - 24c_4\bar{H}^4x^4 + 30c_5\bar{H}^6x^5 + 8c_G\bar{H}^2x^2}{-2 + 3c_4\bar{H}^4x^4 - 6c_5\bar{H}^6x^5 - 2c_G\bar{H}^2x^2},$$

where $x = (1/m_p)d\phi/d\ln a$ and $\bar{H} = H(a)/H_0$.

At early times, $\bar{H}$ is large and the $c_5$ term may be expected to dominate in both the effective dark energy density and the property functions, i.e. $\alpha_i \propto c_5 \propto \Omega_{de}$. But this only holds when the dark energy density is small. When it begins to contribute significantly to the dynamics, eventually leading to cosmic acceleration, then $\bar{H} \sim \mathcal{O}(1)$ and all terms become of the same order — but with the possibility of opposite signs which can lead to cancellations in the denominator and large swings in the values of the $\kappa_i$ or $\alpha_i$.

As one numerical example of the full evolution, consider the kinetic function $X = \dot{\phi}^2/2 = \bar{H}^2x^2/2$. This not only enters the Galileon Lagrangian terms and the $\alpha_i$ as above, but more generally the Horndeski Lagrangian involves functions of $X$. Figure 1 shows that the functional dependence of $X(a)$ itself is unlikely to be easily parametrized by a power law scale factor dependence or other simple form.

In fact, the situation is even more complicated in that the functions that connect directly to observables, such as the effective gravitational coupling in the modified Poisson equations or the gravitational slip between the metric potentials, are themselves ratios of products of the $\alpha_i$. This is evident from eqs. (2.29)–(2.30).

Essentially, the observables constrain functions (e.g. $G_{\text{eff}}$) that are a ratio of sums of products (of $\alpha_i$) that are a ratio of sums (of $c_i$). Illustratively,

$$\text{Observable} \sim \frac{\sum \Pi \left( \frac{f_1(H,X)}{f_2(H,X)} \right)}{\sum \Pi \left( \frac{f_3(H,X)}{f_4(H,X)} \right)}. \tag{3.3}$$

There is no reason to expect a simple time dependence should provide an accurate parametrization.

Figure 2 shows the numerical solutions for the $\alpha_i(a)$ for several Galileon cases.

Again, the only simple time dependence comes in the early time limit where indeed all quantities are proportional to $\Omega_{de}$, as derived for Galileon gravity in eqs. (57)–(61) of [20] and for EFT in the next subsection (but deriving the proportionality constant requires solving the equations of motion, and so is difficult to do in a model independent manner), and in the
Figure 1. The canonical kinetic term \( X = (1/2)\dot{\phi}^2/m_p^2 \) is plotted vs the log of the scale factor \( a \) for the Galileon gravity cases corresponding to figure 1 (solid black), 2 (dashed blue), and 3 (dotted red) respectively of [15]. Note that \( X \) cannot be easily fit by a low order polynomial or simple function.

late time dS limit where all quantities becomes time independent. During redshifts \( z \approx 0–10 \), where almost all observations lie with constraining power on modified gravity causing cosmic acceleration, simple time dependences fail.

The observationally related functions themselves are even further from a power law or simple form. Figure 3 shows the key functions of the gravitational coupling strength \( G_{\text{eff}}^\Phi \), gravitational slip \( \eta \), and tensor wave speed squared \( c_T^2 \) that directly affect observables such as growth and gravitational wave propagation (including cosmic microwave background B-mode polarization). (This figure puts in a single view some results shown in [15] and [20].) Appendix E discusses the implications of \( c_T < 1 \).

3.2 Early time limit

Without knowing the full equations of motion of the specific theory we cannot make definite statements about the early time limit, i.e. in the matter dominated era before cosmic acceleration takes hold. (In the previous section we did do this by specializing to the covariant Galileon case.) However, we give the following plausibility argument.

Let us assume that in the early time limit all beyond Einstein-Hilbert factors in the action that are nonzero are of the same order of magnitude (as one might expect in a natural theory without hierarchy issues, but see our further justification below based on the Galileon case). Specifically, this means their characteristic scales are the same (if not zero):

\[
\Lambda \sim c \sim M_4^4 \sim H \bar{M}_1^2 \sim H^2 \bar{M}_2^2 \sim H^2 \bar{M}_3^2 \sim H^2 m_2^2 \sim H^2 (m_0^2 \Omega - m_p^2),
\]

(3.4)

this makes the not-unreasonable assumption that the early time evolution of the modifications is driven by the Hubble expansion time scale.

The background, Friedmann eq. (2.4) implies that \( c/(H^2 m_p^2) \sim O(\Omega_{\text{de}}) \), where \( \Omega_{\text{de}} \) is the fractional contribution of the effective dark energy density to the total energy density. This allows us to translate any of the mass terms into an order of magnitude in terms of \( \Omega_{\text{de}} \).
Figure 2. The property functions $\alpha_i$ are plotted vs the logarithm of the scale factor for the Galileon gravity cases corresponding to figures 1, 2, and 3 respectively of [15], showing nonmonotonic and complicated dependence of these functions with scale factor.

Furthermore, as the dominant time scale in the matter dominated epoch is the Hubble time, we assume that time derivatives give rise to a factor $H$, in particular that for the kinetic term $\ddot{X} \sim HX$, i.e. $\ddot{\phi} \sim H\dot{\phi}$, where $H \sim 1/t$ during matter domination. The importance of this scaling is well known from quintessence [21] and is in contrast to the situation in the EFT of Inflation where the inflaton is assumed to dominate the expansion at all times.

We then see that the other fractional background contribution to the Friedmann expansion eq. (2.4) is also

$$\frac{\dot{\Omega}}{H\Omega} \sim \frac{H(m_0^2\Omega - m_p^2)}{Hm_p^2} \sim \mathcal{O}(\Omega_{de}) \ .$$

(3.5)
Figure 3. The observation impacting functions $G_{\Phi}^{\Phi}$, $\eta$, and $c_T^2$ are plotted vs the logarithm of the scale factor for the Galileon gravity cases corresponding to figure 1, 2, and 3 respectively of [15]. These functions can have nonmonotonic dependence and cannot be fit accurately by power law or few parameter forms. The values of $G_{\Phi}^{\Phi}$ asymptote to a constant dS value outside the box in the first two panels.

Let us turn to the observable functions. From eq. (2.27) we readily see that in the early time limit the tensor wave speed

$$c_T^2 \approx 1 - \frac{M_5^2}{m_p^2} \sim 1 + O(\Omega_{\text{de}}) .$$

(3.6)

For the gravitational slip, evaluating the orders of magnitude of the $A_i$, $B_i$, and $C_i$ from appendix C, including our time derivative prescription, gives

$$\eta \sim \frac{m_p^2 H^2 \Omega_{\text{de}} + m_p^2 H^2 \Omega_{\text{de}}^2}{m_p^2 H^2 \Omega_{\text{de}} + m_p^2 H^2 \Omega_{\text{de}}^2} \sim 1 + O(\Omega_{\text{de}}) .$$

(3.7)
In particular this can be verified quickly for the background only case of eq. (2.26). Since 
\( (\dot{\Omega}/\Omega) \sim H\Omega_{\text{de}} \) then \( (\dot{\Omega}/\Omega)^2 \sim H^2\Omega^2_{\text{de}} \ll c/(m_0^2\Omega) \sim H^2\Omega_{\text{de}} \) and thus

\[
\bar{\eta}_{bg} \approx 1 + \frac{(\dot{\Omega}/\Omega)^2}{c/(m_0^2\Omega)} \approx 1 + O(\Omega_{\text{de}}) .
\]

(3.8)

The background only case of eq. (2.23) for the gravitational coupling is similar:

\[
4\pi G_{\text{eff,bg}} \approx \frac{1}{2m_p^2} [1 + O(\Omega_{\text{de}})] .
\]

(3.9)

The general case is

\[
4\pi G_{\text{eff}} \sim \frac{m_p^2H^2\Omega_{\text{de}} + m_p^2H^2\Omega_{\text{de}}^2}{m_p^4H^2\Omega_{\text{de}} + m_p^4H^2\Omega_{\text{de}}^2} \sim \frac{1}{2m_p^2} [1 + O(\Omega_{\text{de}})] .
\]

(3.10)

Let us now reexamine our input assumptions. First consider the “equal magnitude” assumption of the action contributions. Approaching this from the converse direction, starting from eqs. (2.17)–(2.20), one sees that if the property functions \( \alpha_i \) are \( O(\Omega_{\text{de}}) \) then the various action functions (mass terms) do indeed have the relations of eq. (3.4). To explore further, consider the Galileon case. The deviation of the observable functions from the GR values going as \( O(\Omega_{\text{de}}) \) in the early time limit was derived in [20]. This was then extended to the property functions deviating as \( O(\Omega_{\text{de}}) \) in [15]. The key point, emphasized by [20], was that at early times \( \text{one of the Galileon } c_i \text{ terms was dominant over all the others, due to their scaling with } H \) (and \( X \)). This then led to the \( \alpha_i \) being dominated by one \( c_i \) contribution and at the same time the fractional effective dark energy density \( \Omega_{\text{de}} \) was proportional to the same term.

Generalizing this from Galileon to Horndeski, we ask whether this means that one Lagrangian term \( L_i \), involving the \( G_i(\phi, X) \) and their derivatives, is dominant. In the Galileon case this works since each \( G_i \) is proportional to a constant \( c_i \) times a power law function of \( X \). For example, \( G_5 = 3c_5X^2 \) and so \( X G_5, X \sim G_5 \) and \( \dot{G}_5 \sim H G_5 \) since \( \dot{X} \sim HX \). However, the property functions \( \alpha_i \) and the action functions are mixtures of the \( L_i \). (See [17] and [19] for explicit expressions.) One dominant \( c_i \) term, and hence \( L_i \), feeds into multiple \( \alpha_i \) and \( M_i \). In particular, the \( c_4 \) and \( c_5 \) contributions feed into all of them (and we expect the \( c_5 \) term to dominate at early times due to its scaling with the highest power of the expansion rate \( H \)). Thus we have the remarkable result that

The dominance of one \( c_i \) term implies equal orders of magnitude for the action functions.

From this follows, as we have shown, the deviation of the observable functions as \( \Omega_{\text{de}} \) at early times.

In general Horndeski, we have functions \( G_i(\phi, X) \) not restricted to a constant times a power law in \( X \). Still, since the property functions and actions functions are all functions of \( G_{i,\phi} \) and \( G_{i,X} \) then whichever \( G_{i,\phi} \) term dominates should enter and dominate for all of them and we expect equal orders of magnitude (or zero). (The one exception is \( m_0^2\Omega \) which alone depends on \( G_4 \) without derivatives; but a shift in the constant part of \( G_4 \) is simply a redefinition of the Planck mass.) So everything depends on the evolution of the scalar field \( \phi \), and the only timescale in its equation of motion is the background expansion \( H \).
This indicates that indeed in the early time limit of matter domination, the action, property, and observational functions should deviate from their GR values proportionally to the fractional dark energy density contribution $\Omega_{de}$. We can restate the above aphorism more generally as

\[
\text{The natural dominance of one } G_i \text{ implies that in the early time limit, } \{\text{deviations from GR}\} \propto \Omega_{de}.
\]

This raises an interesting question as to whether the $G_i$ should be regarded as more elemental constituents of the action than the $M_i$, which are combinations of derivatives of them. We leave this for future work.

We expect exceptions to this early time behavior when:

1. Fine tuning of the theory parameters makes multiple elements (e.g. $G_i$) comparable to each other,

2. $H$ evolves such that some subdominant term is no longer negligible compared to the dominant term (typically when $H/H_0 = 1$ starts to fail, if the mass scales of the $G_i$ are normalized so the constant coefficients of the $G_i$ magnitudes are all of order unity), or

3. Another time scale enters the physics, such as from nonlinear collapse or couplings to the matter sector.

Note that case 2 occurs even for simple, “natural” theories, and the condition $H/H_0 \gg 1$ breaks well before the present ($z \gtrsim 10$) while still $\Omega_{de} \ll 1$. For example, in $\Lambda$CDM $H(z = 10)/H_0 = 20$ while $\Omega_{de}(z = 10) = 0.002$. Thus using $\alpha_i(t) \propto \Omega_{de}(t)$ or $(\bar{\eta} - 1) \propto \Omega_{de}(t)$ in the late epoch where most observational data exists is not a valid approximation.

### 3.3 de Sitter limit

In the opposite, late-time limit the cosmic acceleration should lead to a nearly dS state. In this limit, $\alpha_M = 0 = \alpha'_B$. From eq. (2.30) this implies that one necessarily has $\bar{\eta} = 1$. However, we see from eq. (2.29) that this does not imply restoration to GR. Not only may $c_T^2 - 1 = \alpha_T$ be nonzero but $G_{eff}^\Phi \neq G_N$ is possible.

In particular, from eq. (2.29)

\[
\frac{G_{eff}}{G_N} - 1 \rightarrow \frac{\alpha_B - 2[1 - (m_p^2/M^2_{*dS})]}{2 - \alpha_B}.
\]  

(3.11)

(We remove the superscript $\Phi$ since $\bar{\eta} = 1$ implies that $G_{eff}^\Phi = G_{eff}$.) We see that several sources of modification of the Poisson equation and growth can occur, from braiding and the asymptotic Planck mass.

Thus, although gravitational slip is guaranteed to vanish in the dS limit, deviations from GR can still occur in the strength of gravitational coupling and in the tensor sector.

In the dS limit, for the Galileon case we can derive

\[
\alpha_{M,dS} = 0,
\]  

\[
\alpha_{B,dS} = \frac{2}{G(2 + A)} \frac{G(2 + A) - 1}{G(2 + A) + 1},
\]  

(3.12)  

(3.13)
\[ \alpha_{K,dS} = 6 \frac{G(2 + 5A) - 1}{G(2 + A) + 1}, \]  
\[ \alpha_{T,dS} = \frac{G[6 + C - (8/3)A] - 1}{G(6 - 4A) + 3}, \]  
(3.14, 3.15)

where \( G = G_{\text{eff,dS}}/G_N, \) \( C = (c_G x^2)_{\text{dS}}, \) and \( A = (c_G \dot{H} x^2)_{\text{dS}}, \) where \( c_G \) allows for derivative coupling. We can also write

\[ \frac{M_*^2}{m_p^2} = \frac{2G + 1}{2G}, \]  
\[ \frac{m_0^2 \Omega}{m_p^2} = \frac{G(2 + A) + 1}{G(2 - (4/3)A) + 1} \frac{G(12 + C - (20/3)A) + 2}{6G}. \]  
(3.16, 3.17)

In the uncoupled case, when \( c_G = 0, \) note that \( \alpha_{B,dS} \) and \( \alpha_{K,dS} \) (and \( M_* \)) are functions of \( G \) alone. That is, the dS value of the gravitational coupling strength determines these parameters. The tensor parameter \( \alpha_T \) has further freedom however.

We are free, for example, to choose \( G = 1 \) (as in the third of our triplets of plots) and this implies \( \alpha_{B,dS} = 2/3 \) and \( \alpha_{K,dS} = 2 \) for the uncoupled case. Earlier we had noted that \( \bar{\eta} = 1 \) in the dS limit does not imply a restoration to general relativity; here we see that even imposing \( G = 1 \) does not do so. We still have \( c_T \neq 1 \) and indeed the Poisson equation for growth is still modified due to the presence of the \( \alpha_i \) (which can be thought of as braiding the scalar and tensor sectors, and giving effective dark energy clustering).

In summary, the EFTDE makes clear the following general results:

1. In the early time limit, the EFTDE action, property, or observational functions deviate from their GR values proportionally to \( \Omega_{\text{de}} \), but only under certain conditions.
2. In the dS limit, the gravitational slip restores to its GR value \( \bar{\eta} = \eta = 1 \) for Horndeski theories.
3. In the regime of most observations, in between these limits, there is no simple, accurate parametrization of the time dependence of the functions. Observations must be confronted theory by theory.

4 Goldstones, decoupling, and the de Sitter limit

We now examine further aspects of EFT. In particular, we demonstrate that the Goldstone approach to the EFTDE can explain the important result of eq. (2.25) that the gravitational slip restores to general relativity in the Horndeski and dS limit, and how this result can be altered in more general cases. We also discuss how the decoupling limit (or lack of it) is useful for grouping models into universality classes and allows one to rule out entire subclasses of models within the EFTDE framework.

4.1 The Goldstone approach and decoupling

One advantage of the EFTDE is that introducing the Goldstone boson associated with the spontaneous breaking of time diffeomorphism invariance by the background evolution establishes relationships between operators of the low-energy theory and helps classify models in a universal way. In addition, often it is possible to take a decoupling limit where the Goldstone...
scalar becomes the most important degree of freedom and the other gravitational (gauge) degrees of freedom can be neglected. These decoupling limits also help to distinguish different classes of models.

We presented the action in unitary gauge in section 2.1, where we wrote the most general theory for the fluctuations about a cosmological background realizing ΛCDM. The differences in classes of dark energy theories are then encoded in the coefficients of the fluctuations\(^2\) given in eq. (2.6). The construction of the Goldstone action from the unitary gauge action eq. (2.1) was performed in [12, 13]. Physically, introducing the Goldstone allows us to capture the symmetry breaking resulting from the expansion of the universe. That is, for observations at small distances/high energy (UV) the cosmic expansion of the background is negligible (and so invariant under time-translations\(^3\)), whereas at large distances/low energy (IR) this symmetry is no longer realized — this is spontaneous symmetry breaking. Introducing the Goldstone gives us a way to keep track of the symmetry breaking, the fluid or field responsible for it, and if a decoupling limit exists it is often easier to study this scalar than the full gravitational theory.

To introduce the Goldstone boson we perform the broken time diffeomorphism \(t \rightarrow t + \xi^0(t, \vec{x})\) on the action eq. (2.1). Because the cosmological background depends on time the parameter \(\xi^0\) will appear explicitly in the action for the perturbations. We then replace \(\xi^0 \rightarrow \pi(t, \vec{x})\) everywhere it appears in the action and require that the Goldstone transforms as \(\pi \rightarrow \pi - \xi^0\) under diffeomorphisms. Under the transformation we have

\[
\begin{align*}
g^{00} &\rightarrow g^{00} + 2g^{0i} \partial_i \pi + g^{\mu\nu} \partial_\mu \pi \partial_\nu \pi \\
g^{0i} &\rightarrow g^{0i} + g^{\mu i} \partial_\mu \pi \\
\delta K_{ij} &\rightarrow \delta K_{ij} - H \pi h_{ij} - \partial_i \partial_j \pi \\
\delta K &\rightarrow \delta K - 3 H \pi - a^{-2} \nabla^2 \pi \\
\nabla_\mu &\rightarrow \nabla_\mu, \ g_{\mu\nu} \rightarrow g_{\mu\nu}, \ R_{\mu\nu\lambda\sigma} \rightarrow R_{\mu\nu\lambda\sigma},
\end{align*}
\]

(4.1)

where we have expanded to linear order in \(\pi\), and we see that covariant quantities remain invariant.

The resulting action appears in [12, 13], but for simplicity let us consider the simple case

\[M_2(t) = \tilde{M}_1(t) = \tilde{M}_2(t) = \tilde{M}_3(t) = \tilde{M}(t) = m_2(t) = 0\]

with the resulting Goldstone action

\[S_\pi = \int d^4x \left[ \frac{1}{2} m_0^2 \Omega(t+\pi) R + \Lambda(t+\pi) - c(t+\pi) \left( \delta g^{00} - 2 \hat{\pi}^2 + 2 \hat{\pi} \delta g^{00} + 2 \nabla_i \pi g^{0i} - \hat{\pi}^2 + a^{-2} \nabla_\mu \nabla^\mu \pi \right) \right] + S_m,\]

(4.2)

where dots indicate terms with higher powers of \(\pi\) and \(S_m\) is the matter action. We see that this action is invariant under time diffeomorphisms if we require the Goldstone to transform as \(\pi \rightarrow \pi - \xi^0(t, \vec{x})\), i.e. the symmetry is non-linearly realized [7]. Requiring the symmetry

\(^2\)The one exception is again the parameter \(m_2^2\), however in moving to the Einstein frame this would indeed correspond to coefficients of perturbations in the matter sector (i.e. the parameters determining the interaction strength and masses of particles).

\(^3\)The reader may be confused by the use of the term Goldstone boson, since gravity is a gauge theory. However, in the decoupling limit the gravitational (gauge) fields decouple from the would-be Goldstone resulting in a non-linear sigma model and making this language appropriate. Moreover, in this limit time diffeomorphism invariance reduces to time translation invariance. This is analogous to the situation for electroweak symmetry breaking where similar methods can be used to prove the Goldstone equivalence theorem [22].
be realized in the UV has forced relationships between the various operators (all the terms in parentheses must have coefficient $c$). In fact, for the case $\Omega = 1$ all of the operators are fixed by the background evolution $c$ and $\Lambda$ and there are no free parameters — recall we must use the equations of motion eqs. (2.4) and (2.5) to eliminate $\Lambda$ and $c$ from the action and fix them in terms of the $\Lambda$CDM history ($H$ and $\dot{H}$). In the general case we find

$$c(t) = -m_0^2\Omega\left(\dot{H} + \frac{\Omega}{2\Omega} + \frac{\dot{\Omega}}{2\Omega} - \frac{1}{2}\rho_m, \right) \quad (4.3)$$

$$\Lambda(t) = m_0^2\Omega\left(\dot{H} + 3H^2 + \frac{\Omega}{2\Omega} + \frac{7\dot{\Omega}}{2\Omega} - \frac{1}{2}\rho_m, \right) \quad (4.4)$$

which we see evolve as we pass from matter domination to dark energy domination, and as $\Omega$ evolves.

We also note that in general the Goldstone appearing in the coefficients $\Lambda$ and $c$ in eq. (4.2) would lead to additional terms. During dark energy domination their time dependence is negligible, but in the matter dominated phase and during the transition to dark energy this time dependence could be important.\(^4\)

One of the advantages of the Goldstone approach comes from the decoupling limit. Deep in the dark energy epoch ($\rho_m \ll H^2 m_p^2$) and taking $\Omega = 1$, the leading mixing with gravity comes from an operator

$$\mathcal{O}_{\text{mix}} \sim m_p^2 H \pi \delta g^{00} \sim H^{1/2} \bar{\pi} \delta g^{00} \quad (4.5)$$

as in the case of the EFT of inflation. In the last term we introduced the canonically normalized fields $\pi_c \sim m_p H^{1/2} \pi$ and $\delta g^{00} \sim m_p \delta g^{00}$. This implies that if we are interested in scales with energy $E \gg E_{\text{mix}} \equiv H^{1/2}$ we can neglect the gravitational degrees of freedom and focus entirely on the action for the Goldstone. For example, this approach was utilized to study the stability of quintessence in [10], where the authors constructed the EFT of Quintessence by introducing the additional operator $M_2$. The additional operator implies a shift in the normalization of the field as we introduce the Goldstone [12]

$$-c(t)\delta g^{00} + \frac{1}{2} M_2^4 (\delta g^{00})^2 \rightarrow (c(t) + 2M_2^4)\bar{\pi}^2 - c(t)(\nabla \pi)^2 - 2(c(t) + 2M_2^4)\bar{\pi} \delta g^{00}, \quad (4.6)$$

leading to a sound speed $c_s^2 = c/(c + 2M_2^4)$ and the mixing energy becomes $E_{\text{mix}} \sim (c(t) + M_2^4)\dot{\pi}^2/((c(t) + 2M_2^4)\bar{\pi}^{1/2} m_p)$. In the general case where $c(t)$ is given by eq. (4.3) the behavior of $E_{\text{mix}}$ could become quite involved, similar to the complexity found in previous sections.

Introducing the operator corresponding to $M_2$ along with a general time dependence for $\Omega(t)$ leads to the most general EFT of a scalar-tensor theory allowed (including theories like $F(R)$ gravity). However, these EFTs are not very interesting for a number of reasons [24]. In particular, requiring consistency with the equivalence principle and fifth force/solar system constraints places strong restrictions on the free parameters. This typically implies that either no new observables (predictions) will result, or the models are already ruled out. However, one lesson is that given the EFTDE approach we see that all models of a particular class can be scrutinized by data, in this case characterized by $\Omega(t)$ and $M_2$. These constraints can be further enhanced by also accounting for a correct fit of the background to $\Lambda$CDM as was done in [25] to rule out an entire class of Gauss-Bonnet type models. Moreover, these same

\(^4\)Depending on what scales and observations we are interested in, oscillations in the dark energy could also lead to important corrections; cf. [23] and references within.
parameters enter theoretical considerations like the allowed values of the equation of state and the connection to stability as studied in [10], and the decoupling limit can make such an analysis more tractable.

4.2 de Sitter limit

During dark energy domination we can take the dS limit corresponding to $\dot{H} = 0$ and $\rho_m \ll H^2m_p^2$ and the tadpole constraints eq. (4.3) and eq. (4.4) simplify to

$$c(t) = -m_0^2\Omega \left( \frac{\dot{\Omega}}{2\Omega} + \frac{\ddot{\Omega}}{2\Omega}H \right),$$  (4.7)

$$\Lambda(t) = m_0^2\Omega \left( 3H^2 + \frac{\ddot{\Omega}}{2\Omega} + \frac{7}{2} \frac{\dot{\Omega}}{2\Omega}H \right),$$  (4.8)

which for $\Omega = 1$ would reduce to $c = 0$ and $\Lambda = 3H^2m_p^2$. From above we had the sound speed of the Goldstone $c_s^2 = c/(c+2M_4^2)$, which we see also goes to zero (even in the presence of the $M_4^2$ correction). This demonstrates that there are no propagating scalar degrees of freedom in the dS limit.

However, this conclusion can change if we consider additional corrections from the EFTDE. Consider the additional operators

$$\int d^4x\sqrt{-g} \left( \frac{M_2^2}{2} (\delta K)^2 + \frac{M_3^2}{2} \delta K_{\mu\nu} K^{\mu\nu} \right),$$  (4.9)

with the second term being responsible for the altered tensor wave speed we saw in section 2.4. Introducing the Goldstone leads to a term in the action [6]

$$\int d^4x\sqrt{-g} \left( \frac{M_2^2}{2} + M_2^2 a^{-4} \left( \partial_k\pi \right)^2 \right).$$  (4.10)

This implies that in the dS limit the scalar mode will still propagate even though $c$ vanishes. This is because the higher derivatives in the equation of motion correct the dispersion relation at $O(k^4)$, yet the EFTDE is perfectly consistent as discussed in appendix B. Such terms were first utilized in theories of Ghost Condensation [14]. Now consider the Horndeski theory where we have $M_2^2 = -M_2^2$ and so the correction term in eq. (4.10) vanishes. That is, there is again no propagating scalar degree of freedom. This explains why in the Horndeski dS limit we saw that the slip reduced to the GR result. From this Goldstone approach, we can state:

*Without higher derivative corrections, scalar degrees of freedom will not propagate in pure dS when $c = 0$.*

Alternatively, if we allow $\Omega$ to evolve in time, then from eq. (4.7) we see that $c$ will not vanish in the dS limit. However, in this case there is not a healthy decoupling limit of the theory. Recall that for $\Omega = 1$ we have the mixing energy $E_{\text{mix}} = \dot{H}^{1/2}$ and so in the dS limit the Goldstone decouples from gravity. However, for $\Omega = \Omega(t)$ (or $\alpha_M \neq 0$) we saw above that $E_{\text{mix}} \sim (c(t) + M_4^2)/(c(t) + 2M_4^2)^{1/2}m_p$ and using eq. (4.7) we see that the mixing will rely on the time dependence of $\Omega$. In practice, this means that we must either tune the variation of $\Omega$ to be small (to the point where no modification results) or it will be ruled out by experiment. In addition to this time dependence we note that even in the absence of such
coupling ($\dot{\Omega} \to 0$ or $\alpha_M \to 0$) the dS state does not mean that all time derivatives can be neglected. For example, in the Galileon case the field kinetic term $X = (1/2)\dot{\phi}^2$ goes to a nonzero constant and so $c_s$ doesn’t vanish (eq. (39) of [20] gives the cubic equation for the uncoupled Galileon case); recall that in Horndeski theory many Lagrangian terms involve $X$ or functions of $X$ and these do not vanish in the dS limit. Figure 1 shows this numerically.

Finally, even though in the pure dS limit the scalar will not propagate, this does not mean that corrections resulting from eq. (4.9) won’t alter the tensor sector. Indeed we saw in eq. (2.27) that the tensor speed depends on $\bar{M}_3$ and so the correction in eq. (4.9) can alter $c_T$ while maintaining $\bar{n} = 1$ in the dS limit. We note that this correction survives the dS limit and the Horndeski limit, $c_T^2 = \left(1 - \frac{\bar{M}_3^2}{m_p^2}\right)^{-1}$. (4.11)

However, depending on whether the theory has a healthy decoupling limit this correction may also vanish. For example, in Ghost Condensation we have the decoupling limit $m_p^2 \to \infty$ with $\bar{M}_3$ fixed and this correction vanishes and $c_T \to 1$.

Turning on additional operators in the EFTDE leads to different behaviors, but in taking the decoupling limit two interesting classes of models emerge: those that have a healthy decoupling limit, and those that become strongly coupled and require screening mechanisms. An example of the latter is given by table 1 where the class of models that are DGP-like are captured by introducing the additional operator $\delta g^{00} \delta K$ corresponding to $\bar{M}_1$. It is well known that these theories do not have a healthy decoupling limit. That is, as we try to take the decoupling limit the Goldstone is found to become strongly coupled at solar system scales and one then has to explore possible screening mechanisms to see if GR can be recovered in the UV (see e.g. [26] for a review). In the EFTDE this corresponds to coefficients like $\bar{M}_1$ growing so large that the validity of the EFT fails and a new EFT would need to be constructed.

At this point the reader may wonder what the usefulness of the EFTDE is then, if model dependent features such as screening mechanisms must be considered. However, there is a familiar analog from particle physics. In the search to discover the Higgs boson, we used EFT to parameterize the possible mechanisms behind electroweak symmetry breaking. In particular, we were able to rule out possible models (such as Technicolor) without knowing the UV completion of such theories. Whereas now that we know a scalar Higgs in consistent with the data, we can try to find the UV extension of the model (SUSY, etc.). Here we have a similar situation. In particular, it is well known that DGP type models are not compatible with structure formation [27] when accounting for the cosmic acceleration. This implies that any model that falls into the universal class in the EFTDE given by $[\Lambda, c, \Omega, M_2, \bar{M}_1]$ (and with the relations given in the table) is inconsistent with the data. Thus, as an explanation of cosmic acceleration it is no longer useful to study these models and so their UV completion (screening) is irrelevant. This is the power of the EFTDE. Classes of models can be ruled out in the linear regime and help focus model building.

5 Conclusions

Within the last year or two the theoretical structure of dark energy and its relation to observables has been recognized to be far richer than previously realized. The tensor sector aspects (gravitational waves, e.g. CMB B-modes) are important and complementary to the scalar sector (matter density and lensing). In particular, EFTDE demonstrates that the
“classical” view of dark energy in terms of the expansion history $H(z)$ or the equation of state $w(z)$ can be merely $1/5$ — or less — of the functional information!

EFTDE provides a well defined, complete framework to deal consistently with a whole suite of classes of theories, and without imposing by hand constraints such as restricting the number of derivatives. We have related the EFT action terms, or functions, to the property function approach and the observable, or modified Poisson equation, function approach, providing a translation dictionary. This brings together views of theorists, phenomenologists, and observers. The expansion history, or $H(z)$ Hubble function, can be specified separately.

At quadratic level in the action, EFTDE has seven functions of time besides the Hubble function. Unique observation functions are fewer, leading to an inverse problem for how data can constrain theory. We need either further theoretical principles (even beyond, e.g., the null energy condition or Equivalence Principle) or symmetries, or we must search assiduously for new types of observational probes (e.g. beyond the linear or quasistatic regimes, involving screening on astrophysical scales or near horizon scale effects).

While the restriction to the Horndeski class of theories is not justified based on limiting to two derivatives, this class does have a nonperturbative formulation that is of interest. Therefore Horndeski theories are worth studying. This restricts the EFTDE to four free action functions or property functions and a like number of observable functions. However another type of inverse problem arises: the data does not have the leverage to fit arbitrary functions, and we have shown that there is no simple time dependence or low dimensional parametrization expected or in practice. This unfortunately obviates much of the literature on constraining such theories. We demonstrate that the EFT functions are ratios of sums of products of ratios of sums!

We analyze this impasse in some detail, exploring the reasons in the origins of action terms as generically of the same order of magnitude; for the Galileon subclass we show how this arises from mixes of more elementary components. We evaluate the action, property, and observable functions numerically for several Galileon instances to demonstrate agreement with the theoretical reasoning, with the conclusion that during the redshift range $z = 0–10$ where the vast majority of observational data arises, the functions cannot be reduced to a few parameters.

Using EFTDE we can show very general results in two limits: early time matter domination and late time de Sitter asymptote. In the early time limit we demonstrate that for Horndeski theories the deviations of the functions from the GR values will be linearly proportional to the effective dark energy density contribution. We also lay out how and when this breaks down (and in general it does not hold for beyond Horndeski theories).

For the dS limit we prove that within Horndeski theories (but not more generally) the gravitational slip will restore to GR, $\eta = 1$. But this does not mean the physics is that of GR. We give an example where one can even tune the gravitational coupling strength to GR, $G_{\text{eff}} = G_N$, at the same time, so the entire scalar sector appears to be GR, but the tensor sector will show deviations. This again highlights the complementarity of scalar and tensor observations, e.g. galaxy and CMB polarization surveys.

We clarify many of the parameter relationships in both the dS and Horndeski limits by working within the Goldstone approach to the EFTDE. By focusing on this scalar associated with the spontaneous symmetry breaking by the cosmic expansion we argue that the promising models for an alternative to a cosmological constant fall into two groups based on their decoupling behavior. In this way the EFTDE places models into universality classes that can be scrutinized by data. We also saw that although models that rely on screening
mechanisms require analysis beyond the level of EFT to be complete, it is valuable to use the EFTDE to determine whether such models can be consistent with existing data in the linear regime. A notable example is given by the DGP-like class of models where we know they are inconsistent with structure formation.

In the appendices we provide some additional useful details, including a table identifying the presence of action functions for specific theories, a simple rationale for why EFT removes the need to count derivatives (based on work by Weinberg and others), an explicit summary of the EFT equations of motion, what constitutes a “full” modification of gravity, and the use of the gravi-Cherenkov catastrophe to rule out classes of theories.

In summary, the effective field theory of dark energy is highly effective at helping us think deeply about the nature of cosmic acceleration, and derive general results in certain limits, but thus far remarkably ineffective in the quest to use data to constrain the origin of cosmic acceleration! The field is much richer than appreciated a few scant years ago, and even determining $H(z)$ or $w(z)$ exactly is just a waypoint in our knowledge. The endeavor for a fundamental understanding will require new insights in both theoretical and observational techniques.

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A Specific theories within EFTDE

EFTDE covers many different theories of dark energy and modified gravity. Table 1 reviews the connections in terms of which operators from the action enter.

B EFT and validity of beyond Horndeski theories

How much should we worry about higher derivative operators within EFT, for example those arising in beyond Horndeski theories? The arguments below follow those emphasized by [28]. The EFT approach is formulated by utilizing symmetries to perform a local expansion of the degrees of freedom in the theory. The leading term (most relevant operator) is corrected by higher derivative terms, which are suppressed relative to the leading term by powers of the cutoff of the theory $M$. As an example, consider the EFT of a scalar field with

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{c}{M^2} (\Box \phi)^2 + \ldots, \quad (B.1)$$

where the missing terms are suppressed by further powers of $M$. Naively it would seem that the correction term would lead to both higher time and space derivatives. If the equations of motion contain more derivatives this would seem to require more initial conditions — suggesting the presence of new solutions, which are often argued to lead to runaway behavior.
Table 1. A list of operators required to describe various different models (adapted from [13]).

| Model parameter | Ω | Λ | c | $M_2^4$ | $M_2^2$ | $M_2^0$ | $M^0$ | $m_2^2$ |
|-----------------|---|---|---|-------|-------|-------|-------|-------|
| Corresponding Operator | R | $\delta g_{00}$ | $(\delta g_{00})^2$ | $\delta g_{00} K_{\mu}^\nu$ | $(\delta K_{\mu}^\nu)^2$ | $\delta K_{\mu}^\nu K_{\nu}^\sigma$ | $\delta g_{00} \delta R^{(3)}$ | $\overline{\delta g_{00} \delta g_{00}}$ |
| ΛCDM | 1 | ✓ | ✓ | 0 | ✓ | ✓ | ✓ | ✓ |
| Quintessence | 1/✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $f(R)$ | ✓ | ✓ | ✓ | 0 | ✓ | ✓ | ✓ | ✓ |
| $\delta$-essence | 1/✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Galileon (Kinetic Braiding) | 1/✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| DGP | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Ghost Condensate | 1/✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Horndeski (Generalized Galileon) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ |
| Hořava-Lifshitz | 1 | ✓ | ✓ | 0 | ✓ | ✓ | ✓ | ✓ |

✓ Operator is necessary.
- Operator is not included.
1, 0 Coefficient is unity or exactly vanishing.
1/✓ Minimally and non-minimally coupled versions of this model exist.
† Coefficients marked with a dagger are linearly related to other coefficients in that model by numerical coefficients.

It would also seem that the presence of these terms would lead to the emergence of new degrees of freedom due to the correction term. Indeed, the propagator is

$$\Pi(k) = \frac{1}{k^2 + m^2 + c M^2 k^4},$$

(B.2)
suggesting that in addition to the original scalar of mass $m$ we have a new degree of freedom with mass $\sim M$. However, such an interpretation is incorrect. If we restrict our attention to energies $E \ll M$, then the action is a local expansion in powers of $M$, where the dimensionless number $c$ is expected to be of order one. It would then be incorrect to write eq. (B.2) without doing the same expansion, i.e. we should instead write

$$\Pi(k) = \frac{1}{k^2 + m^2} \left[ 1 - \frac{k^2}{M^2} \left( \frac{k^2}{k^2 + m^2} \right) + O \left( \frac{k}{M} \right)^4 \right],$$

(B.3)

so there is only a single particle (pole) with mass $m$ which is corrected in powers of $k^2/M^2$. As discussed in [28] an equivalent way to obtain a consistent set of equations is if we substitute the lower order (derivative) equations of motion into the action to eliminate the higher derivative terms. These are not special constraints, instead by invoking this method we are self-consistently accounting for the fact that the EFT is a local expansion and thus the equations of motion should be as well. Thus, when working within the EFTDE there is no need to enforce special constraints or relations like those assumed in the Horndeski class of models to avoid instabilities. For a recent pedagogical introduction and a discussion of the subtleties with time dependence we refer the reader to [29].

Although the EFTDE always avoids Ostrogradsky instabilities, one may worry that the restriction $E \ll M$ is a severe limitation. However, the utility of the approach is it allows possible explanations for dark energy to be matched to observations. The observations most relevant to constraining dark energy lie between the present Hubble scale ($10^{-33}$ eV) and solar system distances or larger ($r > 1$ AU $\approx 1/(10^{-18}$ eV)) or larger. Thus, we are interested in experiments probing the EFTDE in the energy range $10^{-33}$ eV $\lesssim E_{\text{probe}} \lesssim 10^{-18}$ eV.
For example, in Ghost Condensation (a special case of the EFTDE) if we modify gravity on scales of the solar system or larger $r_c^{-1} \sim 10^{-18}$ eV this corresponds to a scale $M \sim (m_p/r_c)^{1/2} \sim 0.1$ MeV [14]. Whereas for modifications at the Hubble scale (with $\Lambda = 0$) one finds $M \sim (m_p/r_c)^{1/2} \sim 10^{-3}$ eV. We see these both safely satisfy the requirement $E_{\text{probe}} \ll M$. Thus, for using cosmological observations to probe the EFTDE the requirement $E \ll M$ presents no limitations.

Another possible concern is whether the EFTDE can account for modified gravity models that rely on a screening mechanism to recover GR at solar system scales. These effects are captured by the time-dependent evolution of the coefficients eq. (2.6). Indeed, for the specific case of screening in DGP gravity it was shown explicitly in [12] that the EFTDE captured the same effects. However, it is true that in models where screening effects are present it is necessary to find the UV completion for the EFT. The advantage of the EFTDE energy scale in that case is that it allows one to see if the proposed model is valid as a low energy explanation of the cosmic acceleration, and if it passes that test, then investigations into its UV completion are made worthwhile as we discuss in section 4. Finally, we comment that the EFTDE can also capture models where higher derivative operators (corresponding to many of the terms in eq. (2.6)) can become more important than a standard kinetic term. In this case the scaling dimension of the EFT changes and this can also lead to new models for dark energy — that nevertheless can be completely stable at both the classical and quantum level.

C EFT equations of motion

The Newtonian limit must take into account the sound horizon rather than simply the Hubble scale, that is, $k \gg aH/c_s$, where the sound speed

$$c_s^2 = \frac{c / (m_0^2 \Omega) + (3/4)(\dot{\Omega}/\Omega)^2}{c / (m_0^2 \Omega) + (3/4)(\dot{\Omega}/\Omega)^2 + 2\dot{M}_2^2 / (m_0^2 \Omega)}.$$  \hfill (C.1)

Note that the presence of $M_2^2$ causes $c_s$ to differ from the speed of light [13]; equivalently, only $\alpha_K$ is affected by $M_2$.

In this limit the equations of motion are

$$-\frac{k^2}{a^2} (A_1 \Phi + A_2 \pi + A_3 \Psi) = \rho_m \delta m,$$  \hfill (C.2)

$$B_1 \Psi + B_2 \Phi + B_3 \pi = 0,$$  \hfill (C.3)

$$\frac{k^2}{a^2} (C_1 \Phi + C_2 \Psi + C_3 \pi) = 0,$$  \hfill (C.4)

where $\delta m \equiv \delta \rho_m / \rho_m$ and the coefficients are given by

$$A_1 = 2m_0^2 \Omega + 4\dot{M}_2^2,$$  \hfill (C.5)

$$A_2 = -m_0^2 \dot{\Omega} - \dot{M}_1^3 + 2H \dot{M}_2^2 + 4H \dot{M}_2^2,$$  \hfill (C.6)

$$A_3 = -8m_2^2,$$  \hfill (C.7)

$$B_1 = -1 - 2\dot{M}_2^2 / m_0^2 \Omega,$$  \hfill (C.8)

$$B_2 = 1,$$  \hfill (C.9)
\[ B_3 = -\frac{\dot{\Omega}}{\Omega} + \frac{M_3^2}{m_0^2\Omega} \left( H + \frac{2\dot{M}_3}{M_3} \right), \quad (C.10) \]

\[ C_1 = m_0^2\dot{\Omega} + 2HM^2 + 4\dot{M}\dot{M}, \quad (C.11) \]

\[ C_2 = -\frac{1}{2}m_0^2\dot{\Omega} - \frac{1}{2}\dot{M}_3^3 - \frac{3}{2}HM_2^2 - \frac{1}{2}HM_3^2 + 2H\dot{M}^2, \quad (C.12) \]

\[ C_3 = c(t) - \frac{1}{2}HM_1^3 - \frac{3}{2}M_1^3\dot{M}_1 + \dot{H} \left( 2\dot{M}^2 - 3\dot{M}_2^2 - \dot{M}_3^2 \right) + 2H \left( H\dot{M}^2 + 2\dot{M}\dot{M} \right), \]

\[ + \frac{k^2}{2\alpha^2} \left( \dot{M}_2^2 + \dot{M}_3^2 \right). \quad (C.13) \]

For Horndeski theories we have

\[ 2\dot{M}^2 = \dot{M}_2^2 = -\dot{M}_3^2; \quad m_2 = 0 \quad [\text{Horndeski}] \quad (C.14) \]

and these coefficients simplify significantly. In particular, the last two coefficients become

\[ C_2 = -\frac{1}{2}m_0^2\dot{\Omega} - \frac{1}{2}\dot{M}_1^3, \quad (C.15) \]

\[ C_3 = c(t) - \frac{1}{2}HM_1^3 - \frac{3}{2}M_1^3\dot{M}_1 + (H^2 - \dot{H})M_2^2 + 2H\dot{M}^2\dot{M}_2. \quad (C.16) \]

In the dS limit of Horndeski theory, where all masses and the Hubble parameter freeze to constant values,

\[ A_1 = 2m_0^2\Omega + 2\dot{M}_2^2, \]

\[ A_2 = -\dot{M}_1^3, \]

\[ A_3 = 0, \]

\[ B_1 = -1 - \frac{\dot{M}_2^2}{m_0^2\Omega}, \]

\[ B_2 = 1, \]

\[ B_3 = -\frac{HM_2^2}{m_0^2\Omega}, \]

\[ C_1 = HM_2^2, \]

\[ C_2 = -\frac{1}{2}\dot{M}_1^3, \]

\[ C_3 = -\frac{1}{2}HM_1^3 + H^2\dot{M}_2^2. \quad (C.17) \]

### D Meaning of modification of gravity

Exactly what should be interpreted as a modification of gravity, rather than a new scalar field, is not generally agreed upon in the literature. For non-trivial \( \Omega(t) \) in front of the Ricci scalar, we have a scalar-tensor theory of the Brans-Dicke type. This does not change the spin-2 nature of the graviton, so some authors would argue that it is not a true modification of gravity. (We merely point this out; we do not take a particular stand on the issue, although
we will sometimes call theories that modify the tensor sector “full modifications”. That said, these theories are still of course interesting.

An example of a full modification comes from turning on the extrinsic curvature $K_{\mu\nu}$ terms, which are parameterized by $\bar{M}_2$ and $\bar{M}_3$. As [17] noted, both types of modification lead to parameters that come together in pairs (and so they argued the $\alpha$ notation is useful). But this pairing helps us to see a key difference between the modifications. For example, when we introduce the Goldstone boson (scalar) as in section 4 we find that there is a decoupling limit where if we take $m_0 \to \infty$ while holding $\bar{M}_2$ fixed that the scalar will decouple from the rest of gravity. This allows proposals like the Ghost Condensate to get around stability problems (although it has other issues). We note this ratio always appears in the $\alpha$’s (and this is why we defined our parameter $N$). Whereas, a similar decoupling limit does not exist for other theories, such as Brans-Dicke theories. In other words, a similar limit for $\Omega(t)$ does not exist. In these cases we will have to resort to screening by turning on other operators.

As an example of why scalar-tensor theories are not full modifications of gravity, consider the gravitational wave sound speed from eq. (2.27). In the absence of the extrinsic curvature terms $\bar{M}_3 \to 0$ we recover $c_T = 1$.

E Gravi-Cherenkov issues of Galileons

Note that the gravi-Cherenkov limit [30, 31] requiring $c_T \gtrsim 1$ (really $> -10^{-15}$ from [32]) puts strong constraints on modified gravity. However, [33] points out that the high energy of the radiated graviton involved, $\sim 10^{10}$ GeV, may put it beyond the scope of the effective field theory. If one does take the limit at face value, it appears to rule out the entire class of standard (uncoupled) covariant Galileons, as seen below.

In the de Sitter limit for Galileons (see eq. (13) of [15]), the tensor speed excess is

$$c_T^2 - 1 = \frac{2E - 2A - 3F}{1 - (3/2)E + A + 3F},$$

where $A$ comes from the derivative coupling term, and $C$, $D$, $E$, $F$ are the standard Galileon terms, e.g. $F = c_5 H^6 x^5$ — see [20]. Using the equations of motion, in the de Sitter limit we have (eqs. (67)–(68) of [20])

$$3F = 2D - C - 4 - 2A,$$  

$$9E = 8D - 3C - 10 - 2A,$$  

so we obtain the condition for $c_T^2 < 1$ to be

$$-2D + 3C - 16 - 4A < 0 \quad \text{if} \quad 2D - (3/2)C - 4 - 2A > 0.$$  

(E.4)

But the second condition, coming from the denominator of $c_T^2$, is related to the no-ghost condition when $A = 0$ (no derivative coupling, as we now assume).

Having no ghosts requires (eq. (71) of [20])

$$2 + (3/4)C < D < 5 + (3/4)C.$$  

(E.5)

Since $C = c_2 x^2 < 0$ (right sign kinetic term, imposed by positive energy density and no ghosts; see the right panel of figure 5 in [20]), then the lower bound of eq. (E.5) implies $2D > 4 + (3/2)C$, satisfying the second condition of eq. (E.4). We are thus left with evaluating
the first condition of eq. (E.4), whether $2D > 3C - 16$. Again we can write the lower bound of the no-ghost condition as

$$2D > (3/2)C + 4 > (3/2)C + 4 - [20 - (9/2)C] = 3C - 16,$$  

(E.6)

where the second inequality follows because the quantity in square brackets is positive ($C$ being negative). But the final expression is exactly the first condition of eq. (E.4) and so we see that for uncoupled covariant Galileons, $c^2_T < 1$. If the gravi-Cherenkov condition is valid, this rules out this entire class of modified gravity.

An alternate, shorter method of reaching this conclusion is to use eq. (3.15) to ask whether $\alpha_T < 0$. For the uncoupled Galileon case this occurs when $G(6+C) < 1$. If we insist that the strength of gravity be positive, then this requires $C < -6 + (1/G)$. From figure 5 of [20] we always have $C < -6$, and so indeed uncoupled Galileons have $\alpha_T < 0$ or $c^2_T < 1$, leading to a gravi-Cherenkov catastrophe.

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