Improved selection of extremely red quasars with boxy CIV lines in BOSS

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Accepted 2022 January 27. Received 2021 November 22; in original form 2021 August 2

ABSTRACT
Extremely red quasars (ERQs) are an interesting sample of quasars in the Baryon Oscillation Spectroscopic Sample (BOSS) in the redshift range of \( z = 2.0 - 3.4 \) and have extreme red colours of \( i - W3 \geq 4.6 \). Core ERQs have strong CIV emission lines with rest equivalent width of \( \geq 100 \)\(^\text{˚A} \). Many core ERQs also have CIV line profiles with peculiar boxy shapes which distinguish them from normal blue quasars. We show, using a combination of kernel density estimation and local outlier factor analyses on a space of the \( i - W3 \) colour, CIV rest equivalent width and line kurtosis, that core ERQs likely represent a separate population rather than a smooth transition between normal blue quasars and the quasars in the tail of the colour-REW distribution. We apply our analyses to find new criteria for selecting ERQs in this 3D parameter space. Our final selection produces 133 quasars, which are three times more likely to have a visually verified CIV broad absorption line feature than the previous core ERQ sample. We further show that our newly selected sample are extreme objects in the intersection of the WISE AGN catalogue with the MILLIQUAS quasar catalogue in the colour-colour space of \( (W1 - W2, W2 - W3) \). This paper validates an improved selection method for red quasars which can be applied to future datasets such as the quasar catalogue from the Dark Energy Spectroscopic Instrument (DESI).

Key words: quasars: general – quasars: emission lines – galaxies: statistics

1 INTRODUCTION
Quasars are high luminosity active galactic nuclei (AGN), fuelled by gas and dust accreting onto a supermassive black hole (SMBH). Observations show that the growth of a SMBH is correlated with the physical properties of the host galaxy, such as velocity dispersion, stellar mass, and star formation rate, although the mechanisms which correlate these properties are not completely clear (Gebhardt, et al. 2000; Tremaine, et al. 2002; Haring and Rix 2004; Gültekin, et al. 2009; Shankar, Bernardi & Haiman 2009; Kormendy & Ho 2013; Azadi, et al. 2015; Graham 2016). The co-evolution of a SMBH and its host galaxy mostly occurs during dusty starbursts resulting in the observation of sub-mm or ultraluminous infrared galaxies (Sanders, et al. 1988; Veilleux, et al. 2009; Simpson, et al. 2014). Kroupa et al. (2020) proposed a model whereby SMBHs form when the dynamical collapse of star clusters is accelerated by the accretion of gas from a host galaxy, which can naturally explain the correlation between a host galaxy and SMBH mass, as well as explain quasars found at high redshift.

Unobscured quasars which exhibit blue thermal continuum are the majority of optically selected quasars. Red quasars, on the other hand, are a small population of quasars that show a variety of redder near infra-red and optical colours. Several studies have investigated the origin of the red colour in red quasars (e.g. Kim & Im (2018) or Klindt, et al. (2019)), however the question still remains unsettled (Calistro Rivera et al. 2021).

One possibility is that red quasars have been obscured and reddened by dust during a brief transition phase between dusty starburst galaxies and blue quasars (e.g. Richards, et al. 2003; Hopkins, et al. 2005; Urrutia, Lacy & Becker 2008; Hopkins et al. 2008; Glikman, et al. 2012, 2015; Banerji M. et al., 2015; Assef, et al. 2015; Ishibashi & Fabian 2016; Hickox & Alexander 2018). In this model, a quasar is buried in the starburst dust when the host galaxy is young, making the colour of that quasar red. Many ERQs also exhibit extreme line properties which may indicate unusually powerful outflows occurring in a young evolution phase. Quasar-driven outflows may clear out the observer's line of sight, and so at the end of this evolutionary phase, we observe an optical and/or UV luminous quasar.

There are other models; for example, the unified AGN model (Antonucci 1993) suggests that red quasars are viewed with intermediate orientations between Type 1 and Type 2 quasars (for a recent review see Hickox & Alexander (2018)). According to this model, unobscured (type 1) AGN are viewed face-on, while obscured (type 2) AGN are observed edge-on. A red colour is produced by a dusty torus blocking part of the nuclear emission. However, this model has diffi-
culty explaining the extreme line properties seen in some red quasars (Urrutia et al. 2009; Klindt, et al. 2019).

Ross et al., (2015) studied a population of red quasars at 0.28 ≤ z ≤ 4.36 in the Baryon Oscillation Spectroscopic Survey (BOSS Dawson et al. 2013) of the Sloan Digital Sky Survey-III (SDSS-III Eisenstein et al. 2011). These red quasars were identified using a simple colour selection originally intended for red galaxies: a magnitude difference of $r - W_4$ ≥ 14 between the infra-red band (W4 in WISE with effective wavelength of 12μm) and the optical band ($r$ in SDSS with effective wavelength of 6231Å).

Hamann et al. (2017) (hereafter H17) used the sample of Ross et al., (2015) but narrowed down the redshift range to 2.0 ≤ z ≤ 3.4 and changed the colour selection to $i - W3$ ≥ 4.6 (~3 magnitudes redder than the typical colour of BOSS quasars), calling the sample thus identified Extremely Red Quasars, or ERQs. Interestingly, ERQs showed exotic spectral properties, which motivated H17 to define a smaller core ERQ (CERQ) subsample defined by $REW(C IV)$ ≥ 100 Å. This criterion was chosen to better correlate red quasars with other extreme line properties: peculiar ‘boxy’ profiles, $N v$ > $Ly\alpha$, a high incidence of blue-shifted broad absorption lines (BALs), and [OIII] 5007Å outflow speeds reaching > 6000 km/s. ERQs also have an unusually flat UV SED considering their extreme red colour (steep Mid-IR to UV SED), although this may be an artifact of the BOSS selection algorithm, which would not target quasars which are red in all SDSS bands for spectroscopic follow-up.

Perrotta et al., (2019) (hereafter P19) studied a sample of 28 ERQs and found an outflow speed for the [OIII] line of 1992 - 6702 km/s. This is on average three times faster than those of luminosity matched blue quasars. This outflow speed is highly correlated with $i - W3$ colour but not with radio loudness nor Eddington ratios. P19 suggests that this correlation may indicate a connection between reddening and the efficiency of energy and momentum injection from ERQs to the interstellar medium. ERQs may produce more effective feedback in their host galaxies, regulating the star formation rate and SMBH growth more effectively. This is again indicative that some ERQs with extreme line values are connected with an early dusty stage of quasar-galaxy evolution where strong quasar-driven outflows provide important feedback to the host galaxies (P19).

We therefore have a working hypothesis identifying ERQs with an intermediate stage of quasar evolution between dusty galaxies and red quasars. This study is an effort to produce quantitative evidence for or against this hypothesis, which is based on the unusual line properties exhibited by the spectra. If such quantitative evidence is forthcoming, we also desire to refine the selection criteria for ERQs in order to better study the outflows connected with this stage of quasar evolution. We will provide selection criteria for objects that exhibit the extreme properties of ERQs, among a sample of quasars with spectroscopic data. We use the existing manually selected sample of ERQs to define a training set, and then provide a modified sample of extremely red quasars in BOSS with more uniform (and more uniformly exotic) properties. In summary, we address the following questions:

| Sample | Selection criteria | Size |
|--------|--------------------|------|
| T1LIM | $10^{46.54}$ erg.s$^{-1}$ ≤ $L_{bol}$ ≤ $10^{48.60}$ erg.s$^{-1}$ | 29,072 |
| TIERQ | $i-W3$ ≥ 4.6 | 154 |
| T1CERQ | $REW(C IV)$ ≥ 100 Å | 72 |

1) To what extent are ERQs separated from the main locus of BOSS quasars (§4.1, §4.2 and §4.3)?
2) If they are, which selection criteria best produce quasars connected with this intermediate stage of quasar evolution (§4.4)?

We acknowledge the possibility that our sample may be affected by the selection criteria of BOSS, which uses a colour selection to find quasar candidates and thus may discard some red quasars. However, in the absence of another equally large spectroscopic quasar survey this is unavoidable. We will thus analyse BOSS quasars and check for evidence that we are affected by colour selection in Section 4.5.

We use a standard cosmology throughout ($H_0 = 67.3$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.315$, $\Omega_\Lambda = 0.685$) (Planck Collaboration et al. 2014).

2 QUASAR SAMPLES

In this section, we introduce our quasar samples their selection criteria, summarised in Table 1. The primary parent sample is similar to the emission-line catalogue of H17 which results from custom fits of $C IV$ and $N v$ emission lines performed on spectra in the SDSS-III BOSS quasar catalogue. The subsamples follow the selections in H17, to which we refer the reader for a detailed explanation of the criteria adopted.

2.0.1 Type 1 sample

Following the H17 sample selection procedure, we first limit the quasar redshift to 2 ≤ z ≤ 3.4. This redshift range encompasses most of the BOSS survey, while ensuring that $Ly\alpha$ and $N v$ $\lambda 1240$ are within the BOSS spectral range. We require that successful fits to the $N v$ ($nv_{flag} = 0$) and $C IV$ ($q_{flag} = 0$) emission lines are made at reasonable signal to noise (SNR(REW(C IV)) ≥ 3 and SNR(FWHM(C IV)) ≥ 4)). We limit ourselves to Type 1 quasars, defined as FWHM(C IV) ≥ 2000 km s$^{-1}$ (Alexanderoff R. et al., 2013; Ross et al., 2015). We also require that the quasars have a good detection in the W3 band (SNR(ABW3) ≥ 3), do not exhibit artifacts
in the WISE data (cc_flag='0000'), and are not excessively blue ($i - W3 > 0.8$).

2.0.2 T1ERQ and T1CERQ samples

Following H17 we use a colour cut of $i - W3 \geq 4.6$ to extract ERQs from the full sample of type 1 quasars. H17 considered several colour cuts, choosing their boundary to produce the most dramatic differences in the median spectral properties of ERQs as compared to blue quasars. H17 also defined a subsample, core type 1 ERQs (T1CERQs), with the additional condition of $\text{REW}(\text{C}iv) \geq 100\AA$, chosen to be more correlated with the unusual line properties found in some ERQs. These conditions define a natural two parameter space parameter in $i - W3$ and $\text{REW}(\text{C}iv)$, which we will use extensively in what follows.

2.0.3 T1LM sample

ERQs are very luminous, with an average bolometric luminosity for T1CERQs of $10^{42.21\pm0.31}$ ergs$s^{-1}$. For comparison, the full quasar sample has an average luminosity of $10^{46.82\pm0.21}$ ergs$s^{-1}$. This high luminosity is a selection effect. SDSS cannot detect faint ERQs, because it does not detect objects with an $i$ band magnitude $\lesssim 22$.

This large luminosity has implications for our later analysis, as there is an anti-correlation between the REW of the C iv emission line and the continuum luminosity of Type 1 quasars (Baldwin J. A., 1977). We made a sample of Type 1 luminosity matched quasars (T1LM) drawn from the T1 sample, but with luminosities between the minimum and maximum luminosities of T1CERQs (ie. $10^{46.54} \leq L_{bol}(\text{T1CERQ}) \leq 10^{48.00}$).

We derive mean luminosities using the procedure described in P19, using the AB magnitude in the W3 band and the luminosity distance of our standard cosmology.

Bolometric luminosities are difficult to determine for ERQs due to the large and uncertain extinction corrections which must be applied in the rest-frame UV and optical. However, the W3 band is less susceptible to obscuration than the UV and optical bands. We therefore use it as a surrogate for estimating the bolometric luminosity. We add a correction for flux suppression due to obscuration of a factor of 8, following H17. We thus set $L_{bol} = 8 \lambda L_\lambda$ at $\lambda = 3.45\mu$m in the rest frame. WISE W3 photometry measures $\sim 3.45\mu$m in the rest-frame at the typical redshift of ERQs in our study. We first converted the AB magnitude of W3 to units of flux per frequency ($F_W(\nu) = 10^{-0.4(22.6+m_{\text{AB}}})$) and then to observed flux per wavelength ($F_\lambda = \nu F_W(\nu)$). Intrinsic luminosity is computed using the luminosity distance, $D_L$, of our standard cosmology by:

$$L_\lambda \lambda_{rest} = 4\pi D_L^2 F_\lambda \lambda_{obs} \tag{1}$$

Even after luminosity matching, the sample may contain quasars with a range of different black hole masses or Eddington ratios. However, reliable measures of these quantities are not generally available in our dataset, and so to avoid the risk of over-fitting we rely on simple luminosity matching to homogenise our sample.

3 ANALYSIS METHODS

3.1 Kurtosis of the C IV line: a third parameter

H17 investigated a large number of unusual emission line properties in the T1ERQ and T1CERQ samples. In particular, they found boxy C IV line shapes, a large N V to C IV line ratio ($N/\text{C}iv$) and moderately reduced FWHM($\text{C}iv$) compared to a population of normal blue quasars with similar W3 magnitudes to T1CERQs. However, the $N/\text{C}iv$ fits done in H17 did not attempt to deblend the nearby Lyman-$\alpha$ line and so the $N/\text{C}iv$ strength may be overestimated. The $C$ IV line is uniquely powerful for our analysis as it is the strongest metal line in the quasar spectrum. Other promising lines (e.g. Si IV or He II) lines are much weaker blended. For example, C III is blended with S III and Al III.

We focus here on the boxy shape of the C iv line, quantified by the kurtosis. Kurtosis ($k_{80}$) is defined in H17 as the ratio of the velocity width of the C iv line at 80% of the peak height to the velocity width at 20% of the peak height. A high kurtosis C IV line profile occurs in most ERQs and indicates a boxy line. The median $k_{80}$ for T1ERQs is 0.35 and for T1CERQs 0.36, while the larger T1LM sample has a median of 0.25.

Figure 1 show histograms of $k_{80}(\text{C} iv)$. Each panel is labelled by joint thresholds on $\text{REW}(\text{C} iv)$ and $i - W3$ colour and the number of quasars satisfying these conditions. A redder colour skews the $k_{80}(\text{C} iv)$ distribution towards more boxy C iv line quasars (higher $k_{80}(\text{C} iv)$). Increasing the $\text{REW}(\text{C} iv)$ threshold does not change the overall shape of the $k_{80}(\text{C} iv)$ distribution dramatically, nor its most probable value when compared to the unconditioned sample in the top left panel. However, we see a slightly enhanced population of high kurtosis objects when conditioning on $\text{REW}(\text{C} iv)$. High $k_{80}(\text{C} iv)$ is thus highly correlated with red colour, but not with $\text{REW}(\text{C} iv)$, suggesting that it is a good choice for a third parameter, along with $i - W3$ and $\text{REW}(\text{C} iv)$.

Note that there is a possible confounder in the fitting procedure of H17: weak lines will be fit with a single Gaussian rather than two if the second Gaussian does not improve the fit. A single Gaussian has $k_{80} = 0.37$. We have checked that this does not significantly affect our results by making a version of Figure 1 where spectra with $k_{80}(\text{C} iv) > 0.37$ have been removed. This reduces the total size of the sample by 34780 spectra and the number of core ERQs by 57. In practice, since most of the removed spectra are not ERQs this cut moderately strengthens the trends we report. Fig. 2 shows these high $k_{80}(\text{C} iv)$ objects in the low $\text{REW}(\text{C} iv)$ and blue part of the parameter space. The median $\text{REW}(\text{C} iv)$ for $k_{80}(\text{C} iv) > 0.37$ and $k_{80}(\text{C} iv) > 0.36$ are 17A and 19A respectively. This indicates that most of the quasars with high $k_{80}$ are weak C IV line objects, very far from the ERQs in colour space.

3.2 Defining T1CERQs with a wedge or a cone

One of our main goals in this study is to examine variations in quasar spectral properties as one moves in parameter space between the main quasar locus and T1CERQs. We define $\nu_{\text{T1CERQ}}$, the vector in the 2D parameter space of colour-REW(\text{C} IV) between the median of the T1LM sample and the median of the T1CERQ sample, where the black line in

MNRAS 000, 1–14 (2020)
We used the \( \text{MinMaxScaler} \) function from \textit{sklearn}.

\(^2\) It is possible to consider more complex geometries. However this makes the analysis over-complicated and is not necessarily better than a wedge (or cone) which covers an area (or volume), as long as a variety of directions are included.

\(^3\) We used the \text{MinMaxScaler} function from \textit{sklearn}.
It has an opening angle of the cone opening angle will be large, the black line is the normalisation is removed.

The 3D cone with this definition thus includes all quasars with \( i - W3 \geq 4.6, \) \( \text{REW}(Civ) \geq 100A, \) and \( \text{k}80(Civ) \geq 0.33. \) It has an opening angle of \( \theta = 19.6^\circ \) in the normalised 3D space.

Figure 2 visualises the boundaries between the T1LM, T1ERQ, and T1CERQ quasar samples in the parameter space of \( i - W3 \) and \( \text{REW}(Civ). \) Colours denote \( \text{k}80, \) showing again the large \( \text{k}80 \) associated with ERQs. The line in Figure 2 is along \( v_{\text{TICERQ}} \), directed from the median of the T1LM sample to the median of the T1CERQ sample.

### 3.3 Local Outlier Factor Analysis

Wishing to investigate whether T1CERQs are a separate subpopulation of quasars, we applied several clustering methods on our dataset. We tried density-based clustering techniques (e.g. DBSCAN (Ester et al., 1996) and hierarchical clustering algorithms (e.g. agglomerative clustering (Day et al., 1984)). However, clustering algorithms could not handle the very wide disparity in size between the T1LM sample (29,237 quasars) and the T1CERQ sample (72 quasars). Since T1CERQs are a very small portion (0.25%) of the total quasar sample, clustering methods were either not able to find T1CERQs as a separate cluster or, if they could, the uncertainties in the obtained labels were high.

Instead, we used a Local Outlier Factor (LOF)\(^4\) analysis (Breunig et al., 2000) which quantifies the level of distinctness in T1CERQs. The LOF has had other uses in astronomy:

\(^4\) We used the LOF implementation in \texttt{sklearn} (Pedregosa et al., 2011).

for example, detecting unusual spectra in SDSS (Wei et al., 2013) and distinguishing supernovae candidates from massive galaxies (Tu et al., 2010). LOF measures the extent to which a data point is isolated with respect to its neighbours by comparing the local reachability density of an object to the local reachability density of its k-nearest neighbours using the following score:

\[
\text{LOF}(A) = \frac{1}{\rho_k(A)} \sum_{B \in N_k(A)} \frac{\rho_k(B)}{||N_k(A)||},
\]

This is otherwise called the LOF score for the k-nearest neighbours of point A. \( \rho_k(A) \) (or \( \rho_k(B) \)) is the local reachability density of the k-nearest neighbours of A (or B), defined by:

\[
\rho_k(A) = \frac{||N_k(A)||}{\sum_{B \in N_k(A)} RD_k(A,B)},
\]

where \( N_k(A) \) (or \( N_k(B) \)) is the set of all k-nearest neighbours of the point A (or B). \( RD_k(A,B) \) is the reachability distance between point A and B defined by:

\[
RD_k(A,B) = \max\{d_k(B), d(A,B)\}.
\]

\( d(A,B) \) in Eq. 5 is the Euclidean distance between point A and B in the normalised 2 or 3D space. \( d_k(B) \) is the set of all distances between point B and \( N_k(B) \).

For example, the density around a data point, deep in a dense cluster of points, is very similar to the density of its neighbourhood; this results in LOF ≈ 1. If the data point is located somewhere denser than its nearest neighbours, then it has LOF < 1. A point where the average density of the neighbours is higher than that of the point has LOF > 1, corresponding to the expected behaviour for a small cluster separate from the main group.

The LOF is defined as a function of the number of nearest neighbours, \( k \), which sets the scale or resolution of the cluster searched for. Thus translated into our analysis, \( k \) provides information about the size of the putative T1CERQ cluster.

#### 3.3.1 Mock Data Analysis in 2D

To better illustrate the behaviour of the LOF\((k)\) score on known distributions of data points and for different k-nearest neighbours, we created 100 mock 2D data sets by making 100 draws from two overlapping Gaussian distributions, \( G_1 \) and \( G_2 \). To make each mock data set we draw 30000 data points from \( G_1: \mathcal{N}(\mu = [0,0], \sigma = [1,0; 0,1]) \) and 200 data points from \( G_2: \mathcal{N}(\mu = [3,3], \sigma = [1,0; 0,1]) \), which in total gives us 100 mock data sets consisting of 30200 data points each. We chose the same covariance matrix for \( G_1 \) and \( G_2 \) for simplicity. However, the distance between the centers of \( G_1 \) and \( G_2 \) imitates the distance between the median of \( i - W3 \) in T1LM and the median of \( i - W3 \) in T1CERQs. Similar to the \( i - W3 \geq 4.6 \) and \( \text{REW}(Civ) \geq 100A \) cuts which define T1CERQs, we define a core \( G_2 \) sample (c\( G_2 \)) with \( x,y > 2.5\sigma \). The average population of c\( G_2 \) among our 100 mock data sets is 96 (see Figure 3a). Moreover, on average only 1 data point from \( G_1 \) belongs to c\( G_2 \), while on average 104 data points from \( G_2 \) lay outside of the \( x,y > 2.5\sigma \) cuts, showing the level of blending between the \( G_2 \) and \( G_1 \) populations.

We create a wedge, following the same procedure as §3.2, for our mock data. We are interested in the behaviour of data in a wedge directed from the median of the bigger population
(G1 in the mock 2D data, T1LM in the real dataset) towards the smaller population (cG2 in the mock 2D data, T1CERQs in the real dataset). Note that the centre of G2 is 3σ away from the centre of G1. The opening angle for the mock wedge (see §3.2) is 20° to be close to the opening angle in 2D of the real data (18.5°). The corresponding unit vector that is directed from the center of G1 to the center of G2 is \( \hat{v}_{G1} = (1/\sqrt{2}, 1/\sqrt{2}) \). To see how LOF changes along \( \hat{v}_{G1} \), we binned the wedge as a function of distance from the center of G1, with bins shown in Figure 3a.

We then calculated median LOF scores in each bin of the mock 2D data set using nearest neighbours (i.e. cluster size) \( k = 40, 50, 100 \), and 150 for each of our 100 mock data sets. We plotted the corresponding 68% confidence intervals for median LOF scores within each bin in Figure 3a. The LOF scores for the mocks have a local minimum in bin 4, well beyond the 68% confidence intervals of bin 3 and bin 5 and also consistent with the confidence interval of the difference between LOF scores of bin 3 and bin 4 (CI(LOF(3)) - CI(LOF(4))) when \( k = 40 \) or \( k = 50 \), but not when \( k = 100 \) or 150.

This local minimum in LOF score is caused by the local over-density in bin 4 from cG2. A point in bin 6 close to the centre of G2 is located in a denser region compared to the average point located in bin 3, where the transition between G1 and G2 happens; thus the average LOF score in bin 4 is smaller than in bin 3. In the terminology of the literature, bin 4 includes locally less outlier data points. Data points in bin 5 are far from the centre of G2; as a result, data points in bin 4 are on average also locally less outlier than data points in bin 5. These two observations explain the dip in the LOF score of bin 4 for \( k = 40 \) and 50.

However, the local over-density in bin 4 (i.e. local minimum in median LOF score) is less significant when we consider more neighbours (i.e. \( k = 100 \)). This is because the population size of cG2 in the sample is 96 points. A larger cluster includes much of G1 in addition to G2 (see Eq. 3). Thus LOF\(_{100}\) and LOF\(_{150}\), by incorporating more nearest neighbours for a data point in bin 4, do not show a local minimum in the median LOF score. A significant local minimum in the LOF scores in a specific region of parameter space can therefore be used to find the boundary between two populations, even though they have dramatically different sizes.

We confirmed that this local minimum did not occur in other mock datasets without two clearly separated populations. We tested the LOF score variation in a single Gaussian population (G1) with a normal distribution of \( \mathcal{N}(\mu = [0.0], \sigma = [3.0, 0.3]) \). We generated LOF\(_k\) scores for \( k = 40, 50, 100 \) and 150 in 100 draws from G1 each with 30,000 data points, and used the same binning as for the mock dataset containing two Gaussian distributions. As before we looked at the median LOF\(_k\) score and 68% confidence intervals around it. The corresponding plot to Figure 3b never showed a local minimum in the LOF\(_k\) score.

### 3.3.2 Mock Data Analysis in 3D

To confirm that this dip also occurs in a mock 3D dataset, we performed a similar analysis for 3D Gaussian distributions G1 : \( \mathcal{N}(\mu = [0.0, 0.0], \sigma = [1.0, 0.0; 0.1, 0.0, 0.1]) \) with 30,000 points and G2 : \( \mathcal{N}(\mu = [3.3, 3.3], \sigma = [1.0, 0.0; 0.1, 0.0, 0.1]) \) with 150 data points. We generated 100 mock data sets and used the same cuts for building cG2 \((x, y, z \geq 2.5\sigma)\). On average the population of cG2 in our 100 mock data sets is 50. For comparison, the population size of T1CERQs with the additional \( k_{80}(\mathcal{C}IV) \geq 0.33 \) cut is 52. We never have a point from G1 in the \( x, y, z \geq 2.5\sigma \) region, but on average 100 data points from G2.

We used a simple binning procedure similar to the one used for our mock 2D data. We defined a cone along \( \hat{v} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}) \), directed from the centre of G1 to the centre of G2 with an opening angle of 20°. We defined a central bin C where \( r \leq 1\sigma \). Bin 1-5 are within the cone and between two spheres as follows: bin 1, 1σ \( \leq r \leq 1.5\sigma \); bin 2, 1.5σ \( \leq r \leq 2.5\sigma \); bin 3, 2.5σ \( \leq r \leq 4.8\sigma \); bin 4, 4.8σ \( \leq r \leq 7\sigma \); bin 5, \( r \geq 7\sigma \).

Figure 4 shows the median LOF score in each bin for different nearest neighbours of \( k = 70, 100, 150 \), and 200. The 68%
confidence intervals for each median LOF score is shown as the error bar. The decrease in the LOF score from bin 3 to bin 4 is more than the 68% confidence intervals of each bin and also more than the 68% confidence interval of the difference between the average LOF score in bin 3 and bin 4. As a result the local dip in the LOF score of bin 6 is significant to at least the 68% level.

Having demonstrated that a signature of two mixed populations (in 2D and 3D) is a dip in the LOF to at least the 68% level. a result the local dip in the LOF score of bin 6 is significant between the average LOF score in bin 3 and bin 4. As a result the local dip in the LOF score of bin 6 is significant to at least the 68% level.

Having demonstrated that a signature of two mixed populations (in 2D and 3D) is a dip in the LOF to at least the 68% level.

4 RESULTS

4.1 Density in the 2D parameter space

Our first objective is to gather evidence to determine whether T1CERQs are part of a separate population or extreme examples of T1 quasars, probing the tail of the main distribution. As a first, simple attempt to answer this question, Figure 5 visualises the quasars in the 2D parameter space of $i - W3$ colour, REW(CIV), normalised as explained in §3.2. It is visually apparent that the T1CERQs are over-dense compared to other regions of parameter space at a similar distance from the main quasar locus. To quantify how much, we computed the density of quasars in parameter space using kernel density estimation (KDE) with a Gaussian kernel. We want to compare the density of the parameter space to the high density region near the median of T1LM sample, and so we plotted density contours relative to the maximum density. For the KDE smoothing bandwidth we applied Silverman’s rule of thumb to obtain the bandwidth for each dimension separately:

$$h_i = \sigma_i \left( \frac{4}{N(d + 2)} \right)^{\frac{1}{d+4}}. \quad (6)$$

Here, $\sigma_i$ is the standard deviation for the $i$-th dimension of our normalised parameter space, $N$ is the number of objects (29237 for the T1LM sample), and $d$ is the number of dimensions; 2 in 2D parameter space and 3 in 3D parameter space. Given $\sigma_{i-W3} = 0.077$ and $\sigma_{REW(CIV)} = 0.084$, we obtained $h^{2D}_{i-W3} = 0.014$ and $h^{2D}_{REW(CIV)} = 0.015$.

The density contours with $\rho > 0.05\rho_{\text{max}}$ in Figure 5 are similar in shape, and show the shape of the main quasar locus. However, the contours at lower densities are elongated in the direction of the T1CERQs (blue circles in Figure 5), including some mild local density maxima caused by T1CERQs. The low number of samples in this region means that the density contours are somewhat noisy, but it is apparent that the T1CERQs are an over-density in this parameter space. Figure 6 shows a similar density trend in the $i - W3$, $k_{s80}$ plane. Here the over-density near the T1CERQs is even more apparent: the lowest density contour is extended at $k_{s80} \sim 0.35$ towards high $i - W3$.

A possible explanation for these outer contours is the nonlinear effect of dust reddening on the colour distribution of quasars (Richards et al. 2001). However, Fig. 11 of H17 shows that the typical SED of ERQs is very different from the SED of dust-reddened quasars without the strong C IV line characteristic of core ERQs. Core ERQs have SEDs which are much flatter in the rest-frame UV than suggested by their red $i - W3$ colours, while non-core ERQs exhibit a sharp decline in the near UV with only moderately red colours across the near IR, similar to type 1 QSOs reddened by dust extinction (H17).

4.2 Median Spectra

A second intuitive way to examine exotic line properties is to make median spectra for the T1CERQ sample. Since we have, in §3.2, constructed vectors towards the T1CERQ sample, together with geometric cones which contain the T1CERQ quasars. We are now able to bin the cones and make median spectra within these bins and examine how the spectra of T1CERQs change in 2- and 3D parameter space. Me-
Table 2. List of median physical properties in each bin from Figure 7. "C" in the 2nd column denotes the central bin. Column No. shows the number of quasars in each bin. \( f_{\text{BAL}} \) is the fraction of quasars which contain a visually verified BAL feature near the C\( \alpha \) line. Other columns show the median values and their standard deviations in each bin.

| Bin | No. | \( i-W3 \) | \( \text{REW(C}\alpha) \) | \( \text{FWHM(C}\alpha) \) | \( \text{kt}_{80}(C\alpha) \) | \( N_{v}/C\alpha \) | \( f_{\text{BAL}} \) | Luminosity |
|-----|-----|--------|----------------|-----------------|-----------------|--------------|-------------|-----------|
| C   | 21706 | 2.45 ± 0.35 | 35 ± 10 | 5400 ± 1500 | 0.25 ± 0.05 | 1.54 ± 0.55 | 0.14 | 46.83 ± 0.21 |
| 1   | 664  | 3.37 ± 0.15 | 55 ± 8  | 4700 ± 1700 | 0.24 ± 0.06 | 1.04 ± 0.46 | 0.22 | 46.76 ± 0.19 |
| 2   | 273  | 3.76 ± 0.18 | 68 ± 13 | 4300 ± 1700 | 0.23 ± 0.07 | 0.95 ± 0.55 | 0.29 | 46.79 ± 0.22 |
| 3   | 126  | 4.14 ± 0.24 | 89 ± 22 | 3900 ± 1500 | 0.28 ± 0.07 | 0.99 ± 0.60 | 0.32 | 46.86 ± 0.24 |
| 4   | 44   | 4.65 ± 0.17 | 92 ± 30 | 3500 ± 1400 | 0.35 ± 0.07 | 1.66 ± 0.78 | 0.52 | 47.02 ± 0.28 |
| 5   | 41   | 5.02 ± 0.26 | 125 ± 42| 3300 ± 1200 | 0.35 ± 0.06 | 1.28 ± 0.76 | 0.29 | 47.09 ± 0.32 |
| 6   | 41   | 5.90 ± 0.57 | 181 ± 80| 3100 ± 900  | 0.36 ± 0.06 | 1.74 ± 0.58 | 0.12 | 47.30 ± 0.29 |

Figure 6. Density of quasars in a normalised \( i-W3 \), \( \text{kt}_{80} \) space. Density contours are shown relative to the maximum density at \( \rho/\rho_{\text{max}} = 0.5, 0.05, 0.005, 0.0015 \). Blue circles are T1CERQs. Black dots are the other TILMs.

Median spectra are made by stacking, after the following preprocessing steps:

1. Shift the observed flux into the quasar’s rest frame.
2. Normalise the spectrum by the median flux between 1680Å and 1730Å in the rest frame. This region was chosen as the quasar spectrum is mostly free from significant line features.
3. Interpolate all fluxes onto a logarithmic grid defined between 800Å and 3000Å.

Since visualisation is easier in 2D, we perform our first median spectrum analysis by binning in the normalised parameter space of \( i-W3 \) and \( \text{REW(C}\alpha) \) along the \( \vec{v}_{\text{TICERQ}} \) direction described in §3.2. Figure 7 shows the 2D wedge towards \( \vec{v}_{\text{TICERQ}} \) together with the density contours around which we define the bins for our median spectra. The bin boundaries are chosen to bring out specific features of the median spectra. Three inner density contours at 0.3\( \rho_{\text{max}} \), 0.1\( \rho_{\text{max}} \), 0.03\( \rho_{\text{max}} \) show the shape of the core of the TILM sample. The three outermost contours scale the contour at 0.03\( \rho_{\text{max}} \) by 1.35, 1.55, and 1.95, as the number of quasars this far from the main locus is too low to accurately estimate density. The choice of the 1st (1.35) and 2nd (1.55) scale factors gives a bin that covers the boundary of TICERQs suggested by H17 (the lower left corner of \( \text{REW(C}\alpha) > 100\)Å and \( i-W3 \geq 4.6 \) box in Figure 7). The 3rd scale factor (1.95) is chosen so that bins 5 and 6 are equally populated.

Figure 7. A binned wedge along \( \vec{v}_{\text{TICERQ}} \) towards the T1CERQ sample, with bins defined by density contours. The populations of each bin is provided. Bin-C is enclosed by the innermost solid line contour at 0.3\( \rho_{\text{max}} \). 2nd and 3rd contours are at the levels of 0.03 and 0.01 of \( \rho_{\text{max}} \). The three outer dashed line contours are \( \times 1.35, \times 1.55 \), and \( \times 1.95 \) enlarged version of the biggest solid line contour.

Figure 8 and Table 2 show that there is an evolution in the line properties of quasars along \( \vec{v}_{\text{TICERQ}} \). The median C\( \alpha \) emission line in bins 1 through 3 is symmetric, close to the shape of the median spectrum in bin C, the main TILM quasar locus (shown by the thick grey spectrum in Figure 8).

Table 2 shows that there is a jump in the C\( \alpha \) line kurtosis in bin 4: median \( \text{kt}_{80}(C\alpha) \) is 0.28 in bin 3 and 0.35 in bins 4-6. Bin 4 also has an unusually large BAL fraction (0.52). To ensure that these properties are due to the T1CERQ vector and not a function of distance from the quasar locus, we also checked the line properties along \( -\vec{v}_{\text{TICERQ}} \) and confirmed that \( \text{kt}_{80}(C\alpha) \) remained similar to bin C, while the BAL fraction dropped to 0.07. We confirmed these trends by making median spectra along vectors both clockwise and anti-clockwise of \( \vec{v}_{\text{TICERQ}} \), again finding that the line kurtosis remained low and confirming that the \( \vec{v}_{\text{TICERQ}} \) direction is unique.

There is also a relatively large jump in \( N_{v}/C\alpha \) from \( \sim 1 \) in bins 1–3 to 1.66 in bin 4. We found that \( N_{v}/C\alpha \) also increased along \( -\vec{v}_{\text{TICERQ}} \), perhaps indicating that this is not intrinsic to TICERQs, but in this direction the \( N_{v} \) line is weak and the fit is likely to suffer severely from blending with the Lyman-\( \alpha \) line.
4.2.1 3D parameter space

Motivated by the success of our 2D analysis, we made median spectra in the 3D normalised parameter space of $i-W3$, $\log_{10}(\text{REW}(\text{CIV}))$, and $k_{t}80(\text{CIV})$. The median spectra bins were made within the 3D cone defined in §3.2. The central bins were again defined by density iso-surfaces relative to the maximum density and computed using a KDE as in §4.1. The central quasar locus, bin C, was $\rho > 0.5\rho_{\text{max}}$. Bin 1 has $\rho = 0.5\rho_{\text{max}} - 0.05\rho_{\text{max}}$ and bin 2 is $\rho = 0.05\rho_{\text{max}} - 0.01\rho_{\text{max}}$. As for our 2D binning, we did not use the density iso-surfaces at lower density levels, because the low numbers of spectra in these bins make local density estimates too noisy. Instead, we uniformly enlarged the iso-surface of $0.01\rho_{\text{max}}$ by factors of 1.5, 2.1, and 2.5 to make three extra surfaces and used these enlarged surfaces for bins 4 though 6. The expansion factors of 1.5 and 2.1 were chosen so that bin 4 covers H17’s boundary for ERQs ($i-W3 \geq 4.6$ plane in Figure 9a). The scale factor of 2.5 was chosen so that the last two bins had an equal sized population (23 quasars in each).

All 3D bins are colour coded in Figure 9a. The transparent box in Figure 9a shows $i-W3 \geq 4.6$, $\log_{10}(\text{REW}(\text{CIV})) \geq 100$, and $k_{t}80(\text{CIV}) \geq 0.33$. Median spectra of these 3D bins are plotted in Figure 9b and Table 3 summarises the median physical properties in each bin of Figure 9a. As for the 2D analysis, the kurtosis increments in each bin from 1 to 3 and saturates at 0.33 in bin 4, suggesting bin 4 is a good candidate for a boundary separating a population of red quasars from the main TILM sample. $\text{Nv/CIV}$ is larger in bin 4 – 6 compared to bin 1 though bin 3. It is again large in bin C, but this may again be due to blending with Lyman-$\alpha$. As in 2D, bins 3 and 4 have a high fraction of BALs, although this is not true of bin 6. In general the trends in 3D are similar to those in 2D: this extra parameter, however, will be useful in the next sections.

4.3 Local Outlier Factor Analysis

We showed in §4.1 that T1CERQs are an over-density when compared to other quasars at a comparable distance from the centre of the main population, and in §4.2 we found that there was an increase in kurtosis around the fourth bin from the central TILM sample. In this section, we quantify the distinctness of T1CERQs from the main TILM sample using LOF, and examine the 2D and 3D candidate boundaries we found in §4.2. As a reminder, in §3.3 we showed that a signature of two distinct populations is a dip in the LOF score.

4.3.1 LOF Analysis in 2D

We now proceed to analyse the full TILM sample with LOF along a vector directed towards the T1CERQ, $\vec{v}_{\text{T1CERQ}}$. We use the bins depicted in Figure 7, and compute a median LOF score in the normalised 2D space of $i-W3$ and $\log_{10}(\text{REW}(\text{CIV}))$. There is a (small) dip in the median LOF score for bin 4. This is interestingly consistent with the results from median spectra in §4.2, where we saw that bin 4 was also associated with unusual spectral properties. The magnitude of the dip for $k = 40$ is 0.019, which is somewhat less than the 68% uncertainty of the LOF score in our 2D mock data analysis ($\sigma(\text{LOF(bin 3)}-\text{LOF(bin 4)}) = 0.022$). The 2D LOF analysis thus provides indications that the T1CERQs may be a separate population from the main TILM, but is by no means definitive.

Figure 10 also shows the dependence of LOF score on neighbour number, $k$. For $k = 40, 50, 100$, and 150. The LOF score falls from bin 3 to bin 4 when the number of nearest neighbours is 40 or 50, and monotonically increases for $k = 100$ or 150. The LOF score thus suggests that a putative separate population of T1CERQs would have a population between

![Median spectra for bins in the direction of $\vec{v}_{\text{T1CERQ}}$. The bin number and the number of quasars in each bin are shown. As a reminder, T1CERQs are found in bins 5 and 6 and part of bin 4.](Image)
Figure 9. Top (a): 3D bins along a cone around $\mathbf{v}^{3D}_{\text{ICERQ}}$. The central bin is shown by grey points at the centre. Each bin, separated by density iso-surfaces as described in §4.2.1, is painted a different colour. Dashed lines shows the region of $i-W3 \geq 4.6$, $\log_{10}\text{REW(Civ)} \geq 2$, and $k_{80}\text{(Civ)} \geq 0.33$ in the min-max normalised space. Bottom (b): Median spectra for the corresponding coloured objects in each bin of the top panel. Spectra colours for each bin match those in Figure 9a.

Table 3. List of median physical properties in the bins of Figure 9a. Columns are named as in Table 2.

| Bin | No. | $i$-W3 | REW(Civ) | FWHM(Civ) | $k_{80}$ (Civ) | N\textsc{v}/Civ | $f_{\text{bal}}$ | Luminosity |
|-----|-----|--------|----------|------------|----------------|--------------|-------------|------------|
| C   | 8284| 2.40 ± 0.21 | 35 ± 6 | 5600 ± 1300 | 0.25 ± 0.03 | 1.60 ± 0.50 | 0.11 | 46.85 ± 0.21 |
| 1   | 386 | 3.10 ± 0.20 | 47 ± 7 | 5400 ± 1400 | 0.28 ± 0.01 | 1.22 ± 0.44 | 0.16 | 46.79 ± 0.18 |
| 2   | 93  | 3.69 ± 0.21 | 58 ± 10 | 4500 ± 1800 | 0.30 ± 0.02 | 1.09 ± 0.55 | 0.30 | 46.80 ± 0.22 |
| 3   | 94  | 4.08 ± 0.33 | 78 ± 18 | 3900 ± 1600 | 0.32 ± 0.03 | 1.29 ± 0.58 | 0.52 | 46.91 ± 0.22 |
| 4   | 64  | 4.74 ± 0.33 | 93 ± 24 | 3500 ± 1300 | 0.36 ± 0.02 | 1.18 ± 0.72 | 0.55 | 47.19 ± 0.27 |
| 5   | 23  | 5.43 ± 0.35 | 142 ± 44 | 3000 ± 1400 | 0.37 ± 0.01 | 1.78 ± 0.71 | 0.30 | 47.20 ± 0.33 |
| 6   | 23  | 6.17 ± 0.61 | 209 ± 92 | 3500 ± 1100 | 0.36 ± 0.01 | 1.74 ± 0.65 | 0.04 | 47.49 ± 0.27 |
for bin 4. This fall in the LOF score from bin 3 to bin 4 is shown in Figure 9a once again shows a dip in the median LOF score of the similar bin in the mock 3D data analysis (T1CERQs). Our median spectra and LOF analysis has previously found a sub-population of quasars in 2D space, the separate population may be somewhat different when viewed in the 3D parameter space of \( i - W_3, \log_{10}(REW(C IV)) \) and \( k_{80}(C IV) \). There are also indications in the LOF score that the sub-population is moderately larger than the T1CERQ set found by H17. In this section we will design 3D criteria which optimises the selection of these objects. We call our new subset of quasars Type 1 boxy C IV emission line extremely red quasars (T1BERQs).

The choice of our \( k_{80}(C IV) \) parameter space is also motivated by Figure 1, which suggests that there are a small number of low \( k_{80}(C IV) \) objects within the T1CERQ class and that a minimum \( k_{80} \) condition will produce a purer sample. Here we outline our recipe for selecting T1BERQs, summarizing steps introduced in earlier sections:

1. Normalise the parameter space of \( (i - W_3, \log_{10}(REW(C IV)), k_{80}(C IV)) \) with a Min-Max scaler (discussed in §3.2).

2. Define \( \vec{r}^{3D}_{CERQ} \), a vector from the median of the T1LM sample to the median of those points satisfying \( i - W_3 \geq 4.6, \text{REW}(C IV) \geq 100 \text{Å}, \text{and } k_{80}(C IV) \geq 0.33 \), in the normalised space.

3. Find a cone along \( \vec{r}^{3D}_{CERQ} \), with a tip located at the median point of T1LM, and an opening angle so that the cone includes all quasars satisfying \( i - W_3 \geq 4.6, \text{REW}(C IV) \geq 100 \text{Å}, \text{and } k_{80}(C IV) \geq 0.33 \).

4. Using KDE, find density iso-surfaces and bin the cone in the previous step by successive iso-surfaces. One of the bins passes through an initial guess about the boundary of the desired population (here \( i - W_3 \geq 4.6, \text{REW}(C IV) \geq 100 \text{Å}, \text{and } k_{80}(C IV) \geq 0.33 \)).

5. Calculate the LOF score in each bin.

6. Find the bin showing a local minimum in LOF score.

7. Repeat steps (4) to (6) varying the inner and outer boundaries of the candidate bin found in step (6) and find the bin which shows the largest decrease in LOF scores as compared to the neighbour bin located closer to the centre of the T1LM sample.

8. Find a plane perpendicular to \( \vec{r}^{3D}_{CERQ} \) and tangent to the inner boundary of the optimum bin in step (7).

9. Define the boundaries of the T1BERQs using the common region between the plane of step (8) and the cone of step (3).}

Following this procedure, we found all quasars in a cone with a tip at (in our normalised 3D parameter space) (0.23, 0.54, 0.47) and an opening angle of 19.6°, the same cone as in §3.2. The bin with a minimum LOF score was bin 4, as in §4.4, and the optimised, adjusted boundary between bins 4 and 3 was expanded by a factor of 1.5 from the bin boundaries of §4.2.1. The change in the LOF score across this bin boundary increased moderately to LOF(bin 3) - LOF(bin 4) = 0.130 for \( k = 70 \). Thus, the plane in step (8) of our procedure passes through a point (0.50, 0.72, 0.75) in the normalised space of \( (i - W_3, \log_{10}(REW(C IV)), k_{80}(C IV)) \) with a normal vector of \( \vec{n} = (0.64, 0.41, 0.64) \). We thus define T1BERQs by the following inequalities in the 3D normalised parameter space:

\[
0.64(i - W_3) + 0.41 \log_{10} REW + 0.64 k_{80}(C IV) - 1.10 \geq 0.
\]

\[
(7)
\]

\[
\theta \leq 19.6^\circ.
\]

\[
(8)
\]
where $\theta$ is defined in Eq. 2.

Figure 12 visualises the resulting set of quasars, T1BERQs, in 2D projections of the 3D space. Quasars are colour coded to show those which would be selected by both the T1CERQ and the T1BERQ criteria, by one but not the other, or by neither. Quasars selected by T1CERQ but not T1BERQ (15 quasars) are those with low $k_{80}$; they are red and possess strong but not boxy C iv lines. Quasars selected by T1BERQ but not T1CERQ (76 quasars) are those which our local outlier factor selection algorithm judged to be closer to the ERQ subset than the main quasar locus. They are generally somewhat less red than the other ERQs and have weaker C iv lines, but exhibit the same extreme line properties. Overall, the T1BERQ selection produces 133 quasars.

Figure 13 compares the median spectra of T1BERQs to T1CERQs. As expected given the selection criterion, the T1BERQ sets have higher average $k_{80}$, but lower average $i-W3$ and lower REW(C iv). However, they also exhibit the other unusual line properties associated with T1CERQs, to a stronger extent. In particular, the 76 quasars in T1BERQs but not T1CERQs have a high BAL fraction of $f_{\text{BAL}} = 0.62$, roughly three times larger than T1CERQs (see Fig. 14 for a clearer comparison). The low $k_{80}$ quasars which were removed were also those with the lowest BAL fraction. The FWHM of the C iv line is larger in the newly selected T1BERQs, strengthening the general trend shown in Table 3. Finally the N v line is strong, as shown by the high N v/C iv, and visually in the median spectra, where N v strength is comparable to the Lyman-\(\alpha\) emission line.

4.5 T1BERQs in WISE AGN catalogue

To determine whether our sample of T1BERQs are extremely red only within the SDSS colour selection criteria or are extreme also as a part of other quasar samples, we performed a parallel analysis using the MILLIQUAS catalogue (Flesch 2021). We cross-matched the quasars in MILLIQUAS with the WISE AGN catalogue (Assef et al. 2018), as the infrared flux measurements of WISE are well-suited to studying red quasars like ERQs (H17). MILLIQUAS is a compendium of extant spectra with a high likelihood of being quasars. As SDSS is the largest spectral survey in existence, most, but not all, spectra in MILLIQUAS come from SDSS. If the colour selection function of SDSS was truncating the ERQ distribution, we would expect that the set of objects in MILLIQUAS but not in SDSS would extend substantially further towards the locus of ERQs in WISE colour space.

Figure 15 shows our selected sample of T1BERQs in the $(w1-w2, w2-w3)$ colour-colour space$^5$ of the WISE catalogue. We also show our comparison sample (i.e. cross-matched WISE AGN and MILLIQUAS quasars that have spectroscopic redshift but are not listed in SDSS). MILLIQUAS does extend the quasar locus moderately towards low $w2-w3$, but this is in the opposite direction to the T1BERQs. There is no evidence that colour selection effects are skewing our sample. We also show the histogram of T1BERQs in colour-colour space, as well as the histogram of the MILLIQUAS catalogue. These two histograms clearly have separate centers, indicating that even though T1BERQs are originally selected in the SDSS, they are extreme even in the MILLIQUAS catalogue after excluding SDSS quasars.

5 CONCLUSIONS

We have studied the phenomenon of extremely red quasars (ERQs), found by Hamann et al. (2017) (H17) to have red colour, large $\text{REW(C iv)}$ and unusual emission line properties, including boxy C iv line profiles a high incidence of blue-shifted BALs, and high [OIII] 5007\AA outflow speeds up to 6702 km s$^{-1}$ Perrotta et al., (2019). These properties are consistent with ERQs being consistent with an early dusty stage of quasar-galaxy evolution, where strong quasar-driven outflows provide important feedback to the host galaxies. In this paper, we have used data driven techniques to understand whether the ERQs, when mapped into spectral param-

$^5$ Unfortunately, the $i-$band magnitudes for the quasars in our comparison sample are not available; otherwise it would be very illustrative to compare T1BERQs with our comparison sample in a 3D colour-colour-colour plot of $(i-w3, w1-w2, w2-w3)$.
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Figure 13. The median spectrum of T1CERQs is shown by the blue curve. The median spectrum of T1CERQs which are not among T1BERQs is plotted with the red curve. The median spectrum of those T1BERQs which are not among T1CERQs is shown in orange. The thick grey curve shows the median spectrum of all quasars in the dense region within bin C in Figure 9a.

Figure 14. This figure shows a zoom in view of the median spectra around C\textsc{iv} BAL region. Our newly classified objects (i.e. T1BERQs which are not among T1CERQs) are compared to T1CERQs and to the quasars in the central bin of Fig. 9a and Fig. 9b. Our 76 newly classified quasars have a higher visually verified BAL fraction.

eter space, represent a separate population and, if so, where the boundaries of this population lie.

We applied a kernel density estimation in the space of \(i - W3\) colour and C\textsc{iv} rest equivalent width identified by H17 to show that the ERQs produce overdensities. We computed the local outlier factor, previously calibrated on mock data, to assess whether these overdensities could be explained as statistical fluctuations at large distance from the median sampled quasar. The signature of two separate populations, as we showed using mock data, is a dip in the local outlier factor around the boundaries of ERQs, with a cluster size of 100-150. The dip in the local outlier factor provides evidence that ERQs are connected to a distinct phase of quasar formation, rather than being part of a smooth transition from normal blue quasars to the tail of colour-REW distribution towards redder colours and larger REW(C\textsc{iv})s.

We refined the selection criteria for T1CERQs, resulting in a new sample of ‘boxy’ ERQs (T1BERQs). The idea behind these selection criteria is to use line emission properties to better align the boundary of the T1CERQ sample with the onset of the special phase of quasar formation that leads to these exotic quasar properties. To do this, we made use of the ‘boxy’ shape of the C\textsc{iv} line, defining a boundary in 3D which maximised the depth of the dip in the LOF score. Our final sample defined T1BERQs by the inequalities 7 and 8, which refer to the common region between a plane and a cone obtained by finding the largest minimum of an LOF score in a bin.

There are 15 quasars in the sample of T1CERQs which are not in T1BERQs. Despite having very red colour and extremely strong C\textsc{iv} lines, these quasars have much lower \(kt_{80}(C\textsc{iv})\) compared to the average T1CERQ and are thus excluded on the basis of their non-boxy line profile. On the other hand, there are 76 quasars which are within the T1BERQ sample, but are not T1CERQs. Selected on the basis of their \(kt_{80}(C\textsc{iv})\) as well as their red \(i - W3\) and high REW(C\textsc{iv}), these quasars have more extreme spectral properties, exclusive of the selection criteria, than the T1LM sample.
TIBERQ selection criteria produced N\textsubscript{v} lines which were strong compared to the C\textsubscript{iv} and Lyman-\alpha, and C\textsubscript{iv} lines with a greater FWHM than expected for the quasars’ colour. The TIBERQs also had a high BAL fraction of \( f_{\text{BAL}} \approx 0.62 \), roughly three times larger than T1CERQs. If ERQs are associated with an early dusty stage of quasar formation, we would expect strong metal lines and a high fraction of BAL, associated with a dense accretion disc. The final result of our paper is thus improved selection criteria which produce a purer sample of these interesting objects. This will help to identify ERQs more efficiently in up-coming large quasar surveys such as the Dark Energy Spectroscopic Instrument (DESI) DESI Collaboration et al. (2016) or HETDEX Hill et al. (2008), and select the best targets for follow-up observations investigating quasar and galaxy evolution.

DATA AVAILABILITY

Our underlying quasar line catalogue is from Hamann et al. (2017) and is available as the Supplemental BOSS Emission Line Catalog\textsuperscript{6}. Our analysis scripts and the sample of TIBERQs as a fits table are publicly available in GitHub\textsuperscript{7}.

ACKNOWLEDGEMENTS

We are extremely grateful to Fred Hamann for his important contribution to this paper. We are also grateful to Serena Perrotta, Marie Wingyee Lau, Ming-Feng Ho, and Jarred Gillette for their insightful comments and suggestions. The authors appreciate the constructive comments of the anonymous reviewer and we would like to thank Joseph Mazzarella for his guidance about using NED/IPac data bases. SB was supported by NSF grant AST-1817256. RM thanks Fred Hamann for supporting him for part of this work from NSF grant AST-1911066.

REFERENCES

Alexandroff R. et al., 2013, MNRAS, 435, 3306
Antonucci R., 1993, ARA&A, 31, 473
Assef R. J., et al., 2015, ApJ, 804, 27
Assef R. J., Stern D., Noirot G., Jun H. D., Cutri R. M., Eisenhardt P. R. M., 2018, ApJS, 234, 23. doi:10.3847/1538-4365/aaa00a
Azadi M., et al., 2015, ApJ, 806, 187
Baldwin J. A., 1977, ApJ, 214, 679. 1977, ApJ, 214, 679
Banerji M., McMahon R. G., Hewett P. C., Gonzalez-Solares E., Koposov S. E., 2013, MNRAS, 429, L55
Banerji M., Alaghband-Zadeh S., Hewett P. C., McMahon R. G., 2015, MNRAS, 447, 3368
Breunig, M.M., Kriegal, H.P., Ng, R.T. and Sander, J., 2000, Proceedings of the 2000 ACM SIGMOD international conference on Management of data (pp. 93-104).
Dawson K. S., Schlegel D. J., Ahn C. P., Anderson S. F., Aubourg É., Bailey S., Barkhouser R. H., et al., 2013, AJ, 145, 10
DESI Collaboration, Aghamousa A., Aguilar J., Ahlen S., Allen L. E., Allende Prieto C., et al., 2016, arXiv, arXiv:1611.00036
Di Matteo T., Springel V., Hernquist L., 2005, Natur, 433, 604
Day, William HE, and Herbert Edelsbrunner. Journal of classification 1.1 (1984): 7-24.
Eisenstein D. J., et al., 2011, 142, 72
Ester, M., Kriegel, H. P., Sander, J., & Xu, X., Kdd. Vol. 96. No. 34. 1996.
Fawcett V. A., et al., 2020, MNRAS, 494, 4802
Flesch E. W., 2021, arXiv, arXiv:2105.12985
Gebhardt K., et al., 2000, ApJL, 539, L13
Glikman E., et al., 2012, ApJ, 757, 51
Glikman E., Simmons B., Mailly M., Schawinski K., Urry C. M., Lacy M., 2015, ApJ, 806, 218
Graham A. W., 2016, ASSL, 418, 263, ASSL..418
Gültekin K., et al., 2009, ApJ, 698, 198
Hamann, F., Zakamska, N. L., Ross, N., et al. 2017, MNRAS, 464, 3431
Haring, N., and Rix, H. W. (2004). The Astrophysical Journal Letters, 604(2), L89.
Hickox R. C., Alexander D. M., 2018, ARA&A, 56, 625
Hill G. J., Gebhardt K., Komatsu E., Drory N., MacQueen P. J., Adams J., Blanc G. A., et al., 2008, ASPC, 399, 115, arxiv.org/abs/0806.0183
Hopkins P. F., Hernquist L., Cox T. J., Di Matteo T., Martini P., Robertson B., Springel V., 2005, ApJ, 630, 705

\textsuperscript{6} https://datadryad.org/stash/dataset/doi:10.6086/D1H59V
\textsuperscript{7} https://github.com/rezamonadi/ExtremelyRedQuasars
