Harmonic, Intermodulation and Cross-Modulation Distortion in Directly Modulated Quantum Cascade Lasers

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Abstract. Using a simplified rate equation model, expressions for harmonic, intermodulation and cross-modulation distortion for a directly modulated quantum cascade laser can be derived. This paper shows how such derivations can be done and discusses some implications for quantum cascade lasers. It is important to understand such distortion, especially for applications in communication systems.

1. Introduction

With quantum cascade lasers (QCLs) showing great potential in communication systems and the ubiquity of RF communication systems, QCLs can be utilized in these systems through transmission of RF over light carriers. Example of such system is community antenna television (CATV) system. These systems contribute to the emergent of sub-discipline of microwave photonics. Direct intensity modulation of the laser light is the simplest use of light carriers in the RF communication system. Such modulation is obtained by modulating the drive current of a laser with RF/microwave carriers. However, this modulation scheme produces harmonic, intermodulation and cross-modulation distortion. Analysis of these distortion is important for data transmission using amplitude as they will limit the available bandwidth in large signal modulation. Second and third harmonic distortions can be eliminated when the subcarriers are restricted to an octave bandwidth. The intermodulation and cross-modulation distortion, however, falls within the octave bandwidth. Cross-modulation distortion is known as intelligible crosstalk, in contrast with unintelligible crosstalk resulted by intermodulation distortion. It occurs when the modulation of one carrier is unintentionally transferred to another carrier. Hence, as will be done here, it is important to study nonlinear distortion in a direct intensity modulated QCL.

To the best knowledge of our knowledge, no previous work on harmonic, intermodulation and cross-modulation distortions for quantum cascade lasers has been reported. But harmonic and intermodulation distortions for interband laser have been widely investigated. Such investigation is done either using perturbation techniques [1–9] to linearise the rate equations or by a harmonic-balance method [10, 11]
2. Analytical Expressions for Harmonic, Intermodulation and Cross-Modulation Distortion

By the assumptions given by [12], the single-mode, two-level simplified rate equations [13] are given as follows:

\[ \frac{dN_3}{dt} = \frac{I}{e} - \frac{N_3}{\tau_3} - G(N_3 - N_2)P \]  
\[ \frac{dN_2}{dt} = \frac{N_3}{\tau_3} - \frac{N_2}{\tau_{21}} + G(N_3 - N_2)P \]  
\[ \frac{dP}{dt} = N\beta N_3 P \]  

where \( N_2 \) and \( N_3 \) are the electron numbers in level 2 and level 3 respectively; \( G \) is the gain coefficient per stage; \( e \) is the magnitude of electric charge; \( \tau_{21} \) is the lifetime representing the transitions from level 2 to 1 respectively; \( \tau_{sp} \) is the spontaneous lifetime between level 3 and 2; \( \tau_p \) is the photon lifetime; \( P \) is the photon number in the cavity; \( N \) is the number of stages; \( \beta \) is the coupling of spontaneous emission into the lasing mode; \( I \) is the injected current and \( t \) is time. \( \tau_3 \) is given by \( 1/\tau_3 = 1/\tau_{31} + 1/\tau_{32} + 1/\tau_{sp} \) where \( \tau_{31} \) and \( \tau_{32} \) are the lifetimes representing the transitions from level 3 to 1 and 3 to 2 respectively, and \( \tau_{3}' \) is given by \( 1/\tau_{3}' = 1/\tau_{32} + 1/\tau_{sp} \).

2.1. Perturbation analysis

The gain term at the rate equations makes the system nonlinear. The rate equations are linearized for small perturbations by writing various quantities as a perturbation series in powers of a perturbation parameter, \( \xi \). The perturbation series for \( I \) and \( N_3 \) are given by:

\[ I = I^{(0)} + \xi\omega_1 I^{(\omega_1)} + \xi\omega_2 I^{(\omega_2)} + \xi\omega_3 I^{(\omega_3)} \]  
\[ N_3 = N_3^{(0)} + \xi\omega_1 N_3^{(\omega_1)} + \xi\omega_2 N_3^{(\omega_2)} + \xi\omega_3 N_3^{(\omega_3)} + \xi^{-1}\omega_1 N_3^{(\omega_1)^*} + \xi^{-1}\omega_2 N_3^{(\omega_2)^*} \]  
\[ + \xi\omega_3 N_3^{(\omega_3)} + \xi\omega_1 N_3^{(\omega_1)^*} + \xi\omega_2 N_3^{(\omega_2)^*} \]  
\[ + \xi\omega_3 N_3^{(\omega_3)} + \xi\omega_1 N_3^{(\omega_1)^*} + \xi\omega_2 N_3^{(\omega_2)^*} \]  
\[ + \xi\omega_3 N_3^{(\omega_3)} + \xi\omega_1 N_3^{(\omega_1)^*} + \xi\omega_2 N_3^{(\omega_2)^*} \]  
\[ + \xi\omega_3 N_3^{(\omega_3)} + \xi\omega_1 N_3^{(\omega_1)^*} + \xi\omega_2 N_3^{(\omega_2)^*} \]  
\[ + \xi\omega_3 N_3^{(\omega_3)} + \xi\omega_1 N_3^{(\omega_1)^*} + \xi\omega_2 N_3^{(\omega_2)^*} \]  

with expressions similar to \( N_3 \) for \( N_2 \) and \( P \). Here, \( I^{(0)} \) is the DC bias current which is higher than the threshold current, \( I_{th} \). Superscripts of \( \xi \) denotes the powers while the subscripts denotes the angular frequency of the subcarriers. Those in braces of the quantities \( I \), \( N_3 \), \( N_2 \) and \( P \) also denote the angular frequency of the subcarriers. Three sinusoidal drive currents of angular frequencies of \( \omega_1 \), \( \omega_2 \) and \( \omega_3 \) are considered for obtaining three-carrier, third order intermodulation distortion of angular frequency \( \omega_1 + \omega_3 - \omega_2 \).

After equating coefficients with the same power and angular frequencies and some algebra, the equations are then written in matrix form as follows:

\[ [D(\omega)] [U(\omega)] = [F(\omega)] \]  

where \( \omega \) represents \( \omega_1, \omega_2, \omega_3, 2\omega_1, 2\omega_2, 3\omega_1, 3\omega_2, \omega_1 + \omega_2, \omega_1 + \omega_3, \omega_1 - \omega_2, \omega_3 - \omega_2, 2\omega_1 - \omega_2, \omega_1 + \omega_3 - \omega_2, \omega_1 + \omega_2 - \omega_1 \),

\[ [D(\omega)] = \begin{bmatrix} j\omega + \tau_3^{-1} + GP^{(0)} & -GP^{(0)} & (\tau_p N)^{-1} \\ -GP^{(0)} & j\omega + \tau_{21}^{-1} + GP^{(0)} & -(\tau_p N)^{-1} \\ -NGP^{(0)} - \beta N \tau_{sp}^{-1} & NGP^{(0)} & j\omega \end{bmatrix} \]
and $[U(\omega)] = \left[ N_3^{(\omega)} N_2^{(\omega)} P^{(\omega)} \right]^\mathsf{T}$ is the column matrix of the unknown quantities and T indicates transpose operation. $[F(\omega)]$ is the column matrix of the forcing terms and is given in Table 1 and 2. The forcing term matrix is given as the transpose of row matrix to save space. Here, the superscripts in braces of the forcing terms denote the frequency of the phasors and not the order.

Table 1. Forcing matrix for fundamental, harmonic, intermodulation and cross-modulation terms

| $\omega$          | $[F(\omega)]^\mathsf{T}$          |
|-------------------|-----------------------------------|
| $\omega_{1,2,3}$  | $[I^{(\omega_{1,2,3})}/e \ 0 \ 0]$|
| $2\omega_{1,2,3}$ | $[-S^{(2\omega_{1,2,3})} \ S^{(2\omega_{1,2,3})} \ NS^{(2\omega_{1,2,3})}]$|
| $3\omega_{1,2,3}$ | $[-S^{(3\omega_{1,2,3})} \ S^{(3\omega_{1,2,3})} \ NS^{(3\omega_{1,2,3})}]$|
| $\omega_1 + \omega_2$ | $[-S^{(\omega_1+\omega_2)} \ S^{(\omega_1+\omega_2)} \ NS^{(\omega_1+\omega_2)}]$|
| $\omega_1 + \omega_3$ | Exchange $\omega_2$ with $\omega_3$ above. |
| $\omega_1 - \omega_2$ | $[-S^{(\omega_1-\omega_2)} \ S^{(\omega_1-\omega_2)} \ NS^{(\omega_1-\omega_2)}]$|
| $\omega_3 - \omega_2$ | Exchange $\omega_1$ with $\omega_3$ above. |
| $2\omega_1 - \omega_2$ | $[-S^{(2\omega_1-\omega_2)} \ S^{(2\omega_1-\omega_2)} \ NS^{(2\omega_1-\omega_2)}]$|
| $\omega_1 + \omega_3 - \omega_2$ | $[-S^{(\omega_1+\omega_3-\omega_2)} \ S^{(\omega_1+\omega_3-\omega_2)} \ NS^{(\omega_1+\omega_3-\omega_2)}]$|
| $\omega_1 + \omega_2 - \omega_1$ | $[-S^{(\omega_1+\omega_2-\omega_1)} \ S^{(\omega_1+\omega_2-\omega_1)} \ NS^{(\omega_1+\omega_2-\omega_1)}]$|

The forcing terms in the matrix are defined in Table 2.

Note that the forcing terms for the nonlinear distortions of QCLs have a strong resemblance with those for interband lasers, but with difference in the electron number terms. In QCLs, the forcing terms consist of electron number differences instead of absolute electron number as in interband lasers. The forcing terms for the higher order distortions are given in terms of lower order distortions and are found once the lower order distortions are known. The second order intermodulation distortions of frequency $\omega_1 + \omega_2$, $\omega_1 + \omega_3$, $\omega_1 - \omega_2$, and $\omega_3 - \omega_2$ are given here as they are needed for finding the two-carrier, third order intermodulation distortion, three-carrier, third order intermodulation distortion and cross-modulation distortion. These second order intermodulation distortion will not be a hindrance in a communication system as they will be filtered out with a RF bandpass filter following the optical receiver.

2.2. Fundamental response

The optical modulation depth (OMD), $m_d$ is defined as [14]

$$m_d = \frac{P^{(\omega_{1,2,3})}}{P^{(0)}} = \frac{|I^{(\omega_{1,2,3})}|}{I^{(0)} - I_{th}} \tag{8}$$
Table 2. Definition of forcing terms in the matrix

| ω               | $S^{(ω)}$                              |
|------------------|----------------------------------------|
| $2ω_{1,2,3}$     | $0.5G \left( N_3^{(ω_{1,2,3})} - N_2^{(ω_{1,2,3})} \right) P^{(ω_{1,2,3})}$ |
| $3ω_{1,2,3}$     | $0.5G \left( N_3^{(ω_{1,2,3})} - N_2^{(ω_{1,2,3})} \right) P^{(ω_{1,2,3})} + \left( N_3^{(2ω_{1,2,3})} - N_2^{(2ω_{1,2,3})} \right) P^{(ω_{1,2,3})}$ |
| $ω_1 + ω_2$      | $0.5G \left[ \left( N_3^{(ω_1)} - N_2^{(ω_1)} \right) P^{(ω_2)} + \left( N_3^{(ω_2)} - N_2^{(ω_2)} \right) P^{(ω_1)} \right]$ |
| $ω_1 + ω_3$      | Exchange $ω_2$ with $ω_3$ above.        |
| $ω_1 - ω_2$      | $0.5G \left( N_3^{(ω_1)} - N_2^{(ω_1)} \right) P^{(ω_2)*} + \left( N_3^{(ω_2)} - N_2^{(ω_2)} \right) ^* P^{(ω_1)}$ |
| $ω_3 - ω_2$      | Exchange $ω_1$ with $ω_3$ above.        |
| $2ω_1 - ω_2$     | $0.5G \left[ \left( N_3^{(ω_1)} - N_2^{(ω_1)} \right) P^{(ω_1 - ω_2)} + \left( N_3^{(ω_{1-ω_2})} - N_2^{(ω_{1-ω_2})} \right) P^{(ω_1)} \right.$ $+$ $\left( N_3^{(ω_2)} - N_2^{(ω_2)} \right) ^* P^{(2ω_1)} + \left( N_3^{(2ω_1)} - N_2^{(2ω_1)} \right) P^{(ω_2)*} \right]$ |
| $ω_1 + ω_3 - ω_2$| $0.5G \left[ \left( N_3^{(ω_1)} - N_2^{(ω_1)} \right) P^{(ω_1 - ω_2)} + \left( N_3^{(ω_{1-ω_2})} - N_2^{(ω_{1-ω_2})} \right) P^{(ω_1)} \right.$ $+$ $\left( N_3^{(ω_2)} - N_2^{(ω_2)} \right) ^* P^{(ω_1 - ω_3)} + \left( N_3^{(ω_{1-ω_3})} - N_2^{(ω_{1-ω_3})} \right) P^{(ω_2)*} \right.$ $+$ $\left( N_3^{(ω_3)} - N_2^{(ω_3)} \right) P^{(ω_1 - ω_2)} + \left( N_3^{(ω_{1-ω_2})} - N_2^{(ω_{1-ω_2})} \right) P^{(ω_3)} \right]$ |
| $ω_1 + ω_2 - ω_1$| $0.5G \left[ \left( N_3^{(ω_1)} - N_2^{(ω_1)} \right) ^* P^{(ω_1 + ω_2)} + \left( N_3^{(ω_{1+ω_2})} - N_2^{(ω_{1+ω_2})} \right) P^{(ω_1)*} \right.$ $+$ $\left( N_3^{(ω_2-ω_1)} - N_2^{(ω_{2-ω_1})} \right) P^{(ω_1)} + \left( N_3^{(ω_1)} - N_2^{(ω_1)} \right) P^{(ω_2-ω_1)} \right]$ |

as the light current characteristic of QCLs is linear at above threshold. Here, $P^{(ω_{1,2,3})}$ is the peak value of the sinusoidal light intensity corresponding to sinusoidal peak current, $I^{(ω_{1,2,3})}$; $P^{(0)}$ is the DC value at the bias current, $I^{(0)}$, and $I_{th}$ is the threshold current. Small OMD ranging from 1% to 10% is considered here.

Using Cramer’s rule, (6) is solved for the expressions of the unknown quantities. The denominator for all the expressions is the determinant of $|D(ω)|$ and is given by

$$|D(ω)| = -jω^3 - \left( \frac{1}{τ_{21}} + \frac{1}{τ_{31}} + \frac{2GP^{(0)}}{τ_{p}} \right) ω^2$$

$$+ \left( \frac{2}{τ_{p}} + \frac{1}{τ_{31}} + \frac{1}{τ_{21}} \right) GP^{(0)} + \frac{1}{τ_{31}τ_{21}} + \frac{β}{τ_{p}τ_{3p}} \right) jω$$

$$+ \left( \frac{1}{τ_{31}} + \frac{1}{τ_{21}} \right) \frac{GP^{(0)}}{τ_{p}} + \frac{β}{τ_{21}τ_{p}τ_{3p}} \right)$$

(9)

The expression of the fundamental component of the photon number solved from (6) for $ω = ω_{1,2,3}$ is given as

$$P^{(ω_{1,2,3})} = m_4 \frac{MP^{(ω_{1,2,3})}}{|D(ω_{1,2,3})|}$$

(10)
where

\[ M_P(\omega_{1,2,3}) = \frac{I^{(0)} - I_{th}}{e} \left[ \left( NGP^{(0)} + \frac{\beta N}{\tau_{sp}} \right) j\omega_{1,2,3} - \left( \frac{1}{\tau_3^2} - \frac{1}{\tau_{21}} \right) NGP^{(0)} \right. \]
\[ \left. + \left( \frac{1}{\tau_{21}} + GP^{(0)} \right) \frac{\beta N}{\tau_{sp}} \right] \] (11)

The normalized modulation response has been reported by [13]. As the magnitude of the distortions is of interest, the un-normalized modulation response is considered in this chapter. Fundamental components of \( N_3 \) and \( N_2 \) are needed for obtaining solution for higher order distortions. These components are solved from (6) for \( \omega = \omega_{1,2,3} \) and are given as

\[ N_{3}^{(\omega_{1,2,3})} = m_d \frac{M_{N_3}(\omega_{1,2,3})}{|D(\omega_{1,2,3})|}; \quad N_{2}^{(\omega_{1,2,3})} = m_d \frac{M_{N_2}(\omega_{1,2,3})}{|D(\omega_{1,2,3})|} \] (12)

where

\[ M_{N_3}(\omega_{1,2,3}) = \frac{I^{(0)} - I_{th}}{e} \left[ -\omega_{1,2,3}^2 + \left( \frac{1}{\tau_{21}} + GP^{(0)} \right) j\omega_{1,2,3} + \frac{GP^{(0)}}{\tau_{p}} \right] \] (13)

\[ M_{N_2}(\omega_{1,2,3}) = \frac{I^{(0)} - I_{th}}{e} \left[ \left( \frac{1}{\tau_{31}^2} + GP^{(0)} \right) j\omega_{1,2,3} + \frac{GP^{(0)}}{\tau_{p}} + \frac{\beta}{\tau_{p}\tau_{sp}} \right] \] (14)

2.3. Second harmonic (2\( \omega_{1,2,3} \)) and third harmonic (3\( \omega_{1,2,3} \)) response

Using Cramer’s rule, expressions for second and third harmonic are found by solving (6) for \( \omega = 2\omega_{1,2,3} \) and \( \omega = 3\omega_{1,2,3} \) respectively. The harmonic expressions are given in terms of \( \omega \) as

\[ N_{3}^{(\omega)} = \frac{M_{N_3}(\omega)}{|D(\omega)|}; \quad N_{2}^{(\omega)} = \frac{M_{N_2}(\omega)}{|D(\omega)|} \] (15)

\[ P^{(\omega)} = \frac{M_{P}(\omega)}{|D(\omega)|} \] (16)

where

\[ M_{N_3}(\omega) = \left[ \omega^2 - \left( \frac{1}{\tau_{21}} + \frac{1}{\tau_{p}} \right) j\omega + \frac{1}{\tau_{21}\tau_{p}} \right] S^{(\omega)} \] (17)

\[ M_{N_2}(\omega) = \left[ -\omega^2 + \left( \frac{1}{\tau_{31}} + \frac{1}{\tau_{p}} \right) j\omega + \frac{1}{\tau_{31}\tau_{p}} \right] S^{(\omega)} \] (18)

\[ M_{P}(\omega) = \left[ -\omega^2 + \left( \frac{1}{\tau_{21}} + \frac{1}{\tau_{3}} - \frac{\beta}{\tau_{sp}} \right) j\omega + \frac{1}{\tau_{3}\tau_{21}} - \frac{\beta}{\tau_{21}\tau_{sp}} \right] NS^{(\omega)} \] (19)

Second and third harmonics are found by substituting the appropriate angular frequency and forcing terms into (16). Note that second harmonic can be obtained once the fundamental components of \( N_3 \), \( N_2 \) and \( P \) are found and third harmonic can be obtained once both fundamental component and second harmonic are found. Fundamental component is found using (12) and (10) and second harmonic is found using (15) and (16). Only third harmonic of \( P \) is of interest and the third harmonic of \( N_3 \) and \( N_2 \) need not to be found.
2.4. Two-carrier, third order intermodulation distortion \((2\omega_1 - \omega_2)\)

The expressions for two-carrier, third order intermodulation distortion (IMD2) are obtained by solving (6) for \(\omega = 2\omega_1 - \omega_2\). Using Cramer’s rule, the expression for \(P\) are obtained as given in (16) with \(\omega = 2\omega_1 - \omega_2\). The solution for this distortion can be obtained once the fundamental component, second harmonic and second order intermodulation of \(N_3\), \(N_2\) and \(P\) are found. The fundamental component is found using (12) and (10) and second harmonic and second order intermodulation are found using (15) and (16) with their respective angular frequencies.

Finally, note that expressions for three-carrier, third order intermodulation distortion can be derived in a similar manner.

3. Conclusion

Expression for harmonic and intermodulation distortion in QCLs have been derived and discussed. In future work these expressions will be analysed further including graphs for some typical parameter values.

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