Synthesis of the adaptive continuous system for the multi-axle wheeled vehicle body oscillation damping

M M Zhileykin¹, G O Kotiev² and M V Nagatsev³

Department of Wheeled Vehicles, Bauman Moscow State Technical University, Moscow, Russian Federation

E-mail: ¹jileykin_m@mail.ru, ²kotievgo@yandex.ru, ³ngmax@yandex.ru

Abstract. In order to meet the growing mobility requirements for the wheeled vehicles on all types of terrain the engineers have to develop a large number of specialized control algorithms for the multi-axle wheeled vehicle (MWV) suspension improving such qualities as ride comfort, handling and stability. The authors have developed an adaptive algorithm of the dynamic damping of the MVW body oscillations. The algorithm provides high ride comfort and high mobility of the vehicle. The article discloses a method for synthesis of an adaptive dynamic continuous algorithm of the MVW body oscillation damping and provides simulation results proving high efficiency of the developed control algorithm.

1. Introduction
In order to improve efficiency of the multi-axle wheeled vehicles (MWV) automotive engineers are increasing their cruising speed [1].

As it is known, the maximum speed of a MWV is limited by the road conditions and the parameters of the vehicle. The limitations imposed by the road conditions can be divided into the following groups: traction limitations, and the so-called direct limitations [2]. Increasing the power-weight ratio of the modern vehicles allows eliminating some limitations from the first group. The second group includes:
- speed limitations determined by the kinematic properties of the powertrain;
- steering limitations connected with the ability of the vehicle to negotiate the actual turns of the track;
- limitations connected with the danger of losing control over the vehicle movement;
- limitations imposed by the vehicle body threshold accelerations which may lead to the crew fatigue and injuries and to the vehicle equipment failure when driving over terrain irregularities.

In view of the above, the problem of the improvement of the MVW suspension to provide higher vehicle speed over irregular terrain remains important today.

One of the promising ways to improve the ride comfort of the MWV is the development of the dynamic active suspension systems and control laws for such systems. By the dynamic control systems here we mean the systems operating in real time mode and using current (instantaneous) values of the state variables. This feature distinguishes them from the static systems using the time-averaged signal. Here, by active control systems we mean the systems using energy supplied from external sources.

Theoretical aspects of the active vibration isolation systems for the general case of the rigid body vibration isolation as well as for special cases like driver’s seat suspension engineering have been
studied by many Russian researchers: Balagula V. Ya., Gaytsgori M.M., Genkin M.D., Yablonskiy V.V., Eliseev S.A, Inosov S. V., Sinev A. V., Furman F.A. and others [3 – 8].

Automobile active suspensions have been studied in the works of Kol'tsov V.I., Furunzhiev R.I., Ostanin A.N., Sharapov V.D., V.D., V.D. [9, 10, 11], Sutton H.B., Thompson A.G. and others [12, 13].

The researchers note that active suspensions is the most effective choice in terms of ride comfort.

Introduction of the active control systems into the MWV suspension is hindered by the lack of the continuous control laws for the spring and damping elements of the suspension [1].

The aim of the work is to develop the MWV suspension optimal control laws that would decrease vibrations on the driver’s seat at kinematic excitation.

2. Synthesis of the dynamic continuous control system for the multi-axle vehicle body oscillation damping

Let us assume that the vehicle body is rigid and the suspension of the wheels is a sliding pillar suspension [1]. The linearized model of the MWV body motion has just three degrees of freedom (see figure 1):

- Translation along the vertical axis Z of the coordinate system fixed to the MWV body;
- Rotation about the principal axis X of inertia in the coordinate system fixed to the vehicle body;
- Rotation about the principal axis Y of inertia in the coordinate system fixed to the vehicle body;
- Rotation about the principal axis Y of inertia in the coordinate system fixed to the vehicle body.

![Figure 1. Multi-axle wheeled vehicle modeled as a multiply supported oscillating system.](image-url)
Simplified equations of the multi-axle vehicle body motion:

\[
\begin{align*}
\dot{\omega}_x &= \frac{1}{J_x} U_1 \\
\dot{\omega}_y &= \frac{1}{J_y} U_2 \\
\dot{V}_z &= \frac{1}{M_{spm}} U_3 - g \\
\dot{\phi} &= \omega_x \\
\dot{\psi} &= \omega_y \\
\dot{z} &= V_z
\end{align*}
\]  

(1)

Here

- \(U_1\) – control moment generated by the damping elements of the suspension about X-axis (applied at the vehicle body center of mass);
- \(U_2\) – control moment generated by the damping elements of the suspension about Y-axis (applied at the vehicle body center of mass);
- \(U_3\) – resultant control force vector generated by the spring and damping elements of the suspension in the vertical direction (applied at the vehicle body center of mass);
- \(\omega_x\) – angular velocity of the vehicle body about x-axis;
- \(\omega_y\) – angular velocity of the vehicle body about y-axis;
- \(V_z\) – linear velocity of the vehicle body along z-axis;
- \(\phi\) – angle of the body roll about x-axis;
- \(\psi\) – angle of the body roll about y-axis;
- \(z\) – translation of the vehicle body center of mass along z-axis;
- \(J_x, J_y\) – MVW sprung mass inertia moments about axes x and y respectively;
- \(M_{spm}\) – sprung mass.

Here control inputs \(U_1\) and \(U_2\) are control torques about the longitudinal axis X and central lateral axis Y of the vehicle respectively. Control input \(U_3\) is a control vector whose components are forces parallel to the vertical axis Z. All the control inputs are generated by the controlled spring and damping elements of the multi-axle vehicle suspension.

System (1) can be divided into three independent control channels: two channels for damping the vehicle body angular oscillations and angular stabilization of the vehicle body, and one channel for damping its vertical oscillations.

During synthesis of the optimal systems the developers widely use quadratic integral performance criteria containing the coordinates of the object and of the control input [14, 15].

Initial system (1) of the state equations is a linear system which can be presented as a vector differential equation

\[
\frac{dX(t)}{dt} = A(t)X(t) + B(t)u(t)
\]

(2)

here \(X(t)\) – n-dimensional state vector; \(A(t), B(t)\) – matrix time functions with dimensions \(n \times n, n \times m\) respectively;
- \(u(t)\) - m-dimensional control vector.

Due to evident physical reasons, control vector \(U(t) = [u_1(t), ..., u_m(t)]\) has limited dimensions, i.e. it belongs to a closed set

\[
|u_j(t)| \leq U_{j_{max}}, j = 1, 2, ..., m.
\]

In order to select the control performance criterion we should define its requirements.
The most universal performance criterion is the criterion based on the optimized integral functional. Use of the integral criteria, the quadratic functionals in particular, allows formulation of the requirements for the transient behavior of the control system by setting large number of their weight coefficients. Practically unrestricted selection of the values of these coefficients provides for the fundamental property of the synthesized system – its asymptotical stability [15]. Iterative directed search of the weight coefficients usually meets the reasonable requirements of the primary transient performance criteria.

The main advantage of this method is that it provides the ability to use results of the optimal control theory and the theory of the analytical design of the optimal controllers for the synthesis of a control system for a complex object. Ever-growing use of the theory of the analytical design of the optimal controllers in the complex industrial object control applications is due to such their advantages as generality, maximum formalization, logical completeness, essential mathematical simplicity.

Thus, for controlled objects (2) we need to solve a problem of the theory of the analytical design of the optimal controllers for the respective integral criterion. Selection of the control performance functional (during the theory of the analytical design of the optimal controllers problem formulation) is performed under the assumption that the control objective for a multi-axle wheeled vehicle suspension is stabilization of the output variables (velocity, acceleration, and displacement of the sprung masses) at a certain level providing lower vibrations and better stability in case of high-amplitude (high power) inputs from the terrain.

At present, in the theoretical and practical optimal control applications, the developers widely use control system performance integral quadratic functionals of the form [16]

\[
I = \int_{t_0}^{t_1} [U^T(t)r(t)u(t) + X^T(t)q(t)X(t)]dt.
\]  

Here, \(U(t)\) – \(m\)-dimensional control vector; \(X(t)\) – \(n\)-dimensional state vector; \(r(t)>0, q(t)>0\) – weight coefficients.

Value of the functional \(I\) depends on the initial state \(X(t_0)\), time value \(t_0\), and control input \(u(t)\) over the interval \((t_0,t_1)\). The objective of the optimal control is to minimize functional (3).

3. Developing optimal control law for damping angular and vertical oscillations of the multi-axle wheeled vehicle

3.1. Damping angular oscillations of the MWV body

Let us assume that \(x_1 = \phi\) – body roll angle; \(x_2 = \omega\) – angular velocity. From equations (1) we can get to the system of equations describing the angular oscillations of the MWV of the form

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= U^* \\
U^* &= \frac{U}{J}
\end{align*}
\]

Here, \(J=J_x\) or \(J=J_y\).

Here, the control input \(U\) is the control torque about the respective central axis of the vehicle. All the control inputs are generated by controlled spring and damping elements of the vehicle suspension.

The unknown is the control law providing minimum value of the functional

\[
I(x,U) = \int_{0}^{\infty} \left[ rU^2 + \sum_{i=1}^{2} q_i x_i^2 \right] dt,
\]

here, \(r > 0; q_i > 0\); \(x\) – object state vector; \(U\) – object control vector.

We’ll try to meet the following boundary conditions
\[ x_1(0) = x_{10}; \quad x_2(0) = x_{20}; \]
\[ x_1(\infty) = 0; \quad x_2(\infty) = 0. \]  \hspace{1cm} (6)

So, we’ll be solving the problem without the right-hand boundary since at the dynamic control of the suspension there is no need to impose any limitations on the phase coordinates. This type of control only provides reduction of the phase coordinate values and is not capable of bringing them to zeroes. At this stage of the optimal controller synthesis, we’ll not impose any limitations on the phase coordinates and control inputs.

The above-formulated problem is Lagrange problem of the constraint extremum [16]. According to Lagrange problem solution procedure, intermediary functional

\[ I(x, U) = \int_0^\infty \left[ rU^2 + \sum_{i=1}^2 q_i x_i^2 + \sum_{i=1}^2 \lambda_i(t) \left( x_i - \sum_{i=1}^2 a_{ik} x_k - b_i U_i \right) \right] dt, \]
\[ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 1/f \end{bmatrix} \]  \hspace{1cm} (7)

Euler-Lagrange equations for functional (7)

\[ \frac{\partial \Phi}{dx_i} - \frac{d}{dt} \frac{\partial \Phi}{dx_i} = 0, \quad i = 1, 2, \]
\[ \frac{\partial \Phi}{dU_i} - \frac{d}{dt} \frac{\partial \Phi}{dU_i} = 0, \quad i = 1, 2, \]  \hspace{1cm} (8)

here \( \Phi(x, U) \) – integrand of functional (7). We should add constraint equations (4) to equations (8). The system of the associated equations will look as follows

\[ \begin{cases} \dot{x}_1 = x_2 \\ x_2 = \frac{1}{2f^2 r} \lambda_2 \\ \dot{\lambda}_1 = 2q_1 x_1 \\ \dot{\lambda}_2 = 2q_2 x_1 - \lambda_1 \end{cases} \]  \hspace{1cm} (9)

The characteristic equation for a linear homogeneous system of differential equations (9)

\[ D(\mu) = \begin{bmatrix} -\mu & 1 & 0 & 0 \\ 0 & -\mu & 0 & 1 \\ 2 \cdot q_1 & 0 & -\mu & 0 \\ 0 & 2 \cdot q_2 & -1 & -\mu \end{bmatrix} = 0, \]  \hspace{1cm} (10)

or

\[ \frac{\mu^4 f^2 r - q_2 \mu^2 + q_1}{f^2 r} = 0 \]

Since \( r > 0 \), the roots of the characteristic equation are

\[ \mu_{1,2,3,4} = \pm \left[ \frac{2}{f^2 r} \left( q_2 \pm \sqrt{q_2^2 - 4f^2 r q_1} \right) \right]^{1/2}. \]  \hspace{1cm} (11)

According to boundary conditions (6), eigen values from the right-hand semiplane can be neglected, which brings us to the synthesis of a stable automatic control system. Then, according to the known procedures [16] we arrive to the general solution of system (9).
\[ x_1(t) = C_1 \exp(\mu_3 t) + C_2 \exp(\mu_4 t), \] (12)

\[ x_2(t) = C_1 \mu_3 \exp(\mu_3 t) + C_2 \mu_4 \exp(\mu_4 t), \] (13)

Here \( \mu_3, \mu_4 \) – roots of characteristic equation (10), the roots being located in the left-hand semiplane; \( C_1, C_2 \) - constants.

Let us solve equations (12) and (13) for \( C_1 \exp(\mu_3 t) \) and \( C_2 \exp(\mu_4 t) \). We get the following expressions:

\[ C_1 \exp(\mu_3 t) = \frac{x_1 \mu_4 - x_2}{\mu_4 - \mu_3}, \]

\[ C_2 \exp(\mu_4 t) = \frac{x_2 - \mu_3 x_1}{\mu_4 - \mu_3}. \]

The solution of the second equation of system (4) gives us the expression for the optimal control law of the angular oscillation damping

\[ U^* = \mu_4 \mu_3 \left[ -x_1 + \frac{\mu_4 + \mu_3}{\mu_4 \mu_3} x_2 \right] \] (14)

System (4) with control law (14) is stable since the roots \( \mu_3, \mu_4 \) of its characteristic equation are located in the left-hand semiplane. The derived optimal control law can be used for controlling each roll angle of the vehicle body separately.

3.2. Damping vertical oscillations of the MVW body

Let’s turn to the development of the control law for damping the vertical oscillations of the vehicle body. Initial differential equation

\[ \dot{V}_z = \frac{1}{M_{spm}} U_3 - g \] (15)

Let us introduce new notation. Let us denote \( U = \frac{1}{M_{spm}} U_3 - g \). Then equations (15) can be rewritten as

\[ \begin{cases} \dot{V}_z = U \\ \dot{z} = V_z \end{cases} \] (16)

Let us denote \( x_1 = z \) – vertical displacement of the vehicle body center of mass; \( x_2 = Vz \) – vertical velocity of the vehicle body center of mass. Then, system (16) can be rewritten as

\[ \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = U \end{cases} \] (17)

Control law \( U_3 \) is the resultant vector force at the center of mass of the vehicle body. The unknown is the control law providing minimum value of the functional

\[ I(x, U) = \int_0^\infty \left[ r U^2 + \sum_{i=1}^{2} q_i x_i^2 \right] dt, \] (18)

Here, \( r > 0; \ q_i > 0; \ x \) – object state vector; \( U \) – object control vector.

We’ll try to meet the following boundary conditions

\[ x_1(0) = x_{10}; \ x_2(0) = x_{20}; \]

\[ x_1(\infty) = 0; \ x_2(\infty) = 0. \] (19)
The above-formulated problem is Lagrange problem of the constraint extremum [16]. According to Lagrange problem solution procedure, intermediary functional
\[ I(x, u) = \int_0^T \left[ r u^2 + \sum_{i=1}^2 q_i x_i^2 + \sum_{i=1}^2 \lambda_i(t) \left( x_i - \sum_{i=1}^2 a_{ik} x_k - b_i u_i \right) \right] dt, \]
\[ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/r \end{bmatrix}. \]
Euler-Lagrange equations for functional (20) have the form of (8). We should add constraint equations (17) to equations (8). The system of the associated equations will look as follows
\[ \begin{cases} \dot{x_1} = x_2 \\ x_2 = \frac{1}{2r} \lambda_2 \\ \lambda_1 = 2q_1 x_1 \\ \lambda_2 = 2q_2 x_1 - \lambda_1 \end{cases} \]
The characteristic equation for a linear homogeneous system of differential equations (21)
\[ D(\mu) = \begin{bmatrix} -\mu & 1 & 0 & 0 \\ 0 & -\mu & 0 & 1 \\ 2 \cdot q_1 & 0 & -\mu & 0 \\ 0 & 2 \cdot q_2 & -1 & -\mu \end{bmatrix} = 0, \]
or
\[ \frac{\mu^4 M^2 r - q_2 \mu^2 + q_1}{r} = 0 \]
Since \( r > 0 \), the roots of the characteristic equation are
\[ \mu_{1,2,3,4} = \pm \left[ \frac{2}{M^2 r} \left( q_2 \pm \sqrt{q_2^2 - 4r q_1} \right) \right]^{1/2}. \]
According to boundary conditions (19), as shown above, eigen values from the right-hand semiplane can be neglected, which brings us to the synthesis of a stable automatic control system. Then, according to the known procedures [16] we arrive to the general solution of system (21)
\[ x_1(t) = C_1 \exp(\mu_3 t) + C_2 \exp(\mu_4 t), \]
\[ x_2(t) = C_3 \mu_3 \exp(\mu_3 t) + C_4 \mu_4 \exp(\mu_4 t), \]
here \( \mu_3, \mu_4 \) – roots of characteristic equation (22), the roots being located in the left-hand semiplane. Let us solve equations (24) and (25) for \( C_1 \exp(\mu_3 t) \) and \( C_2 \exp(\mu_4 t) \). This gives the following expressions
\[ C_1 \exp(\mu_3 t) = \frac{x_1 \mu_4 - x_2}{\mu_4 - \mu_3}, \]
\[ C_2 \exp(\mu_4 t) = \frac{x_2 - \mu_3 x_1}{\mu_4 - \mu_3}. \]
Solution of the second equation of system (17) gives us the expression for the optimal control law of the vertical oscillation damping.
System (17) with control law (26) is stable since the roots $\mu_3, \mu_4$ of its characteristic equation are located in the left-hand semiplane. The final expression of the control law has the following form

$$U_3 = M_{spm}(U + g) = M_{spm}\left[\mu_4\mu_3\left(-x_1 + \frac{\mu_4 + \mu_3}{\mu_4\mu_3} x_2\right) + g\right].$$

(27)

4. Simulation results

The developed optimal control laws were tested by simulation of the straight-line steady motion of a multi-axle 8x8 wheeled vehicle with gross weight 60 ton over an unpaved road with different speeds. The vehicle motion over the terrain was simulated by the mathematical model developed at the Department of Wheeled Vehicles of the BMSTU [17]. In the MWV mathematical model the speed of the vehicle is not a preset value but is determined by the forces of the interaction between the rolling wheels and the terrain. This provides high accuracy of the MWV motion simulation.

Figures 2 – 4 show vibration levels on the driver’s seat in octave bands of frequencies for active and passive suspensions over an unpaved road for speeds 10 – 70 km/h.

![Figure 2. Vibration levels on the driver’s seat (octave I).](image-url)
Analysis of the charts in figures 2 – 4 brings us to the following conclusions.

- The developed optimal control laws for damping the MVW vehicle body are effective for damping oscillations of the sprung mass at all frequencies from the operational range (0 – 5 Hz).
- Over the unpaved road for the active suspension vibration levels at different octaves on the driver’s seat go down by 2 – 6 dB according to the vehicle speed.
- The simulation results show that for a vehicle moving over the unpaved road with the active suspension controlled according to the proposed control laws, maximum speed can be increased from 20 km/h to 70 km/h by ride comfort criterion (according to the standards of GOST 12.1.012-2004).
Conclusion
The authors have developed the optimal control laws for damping the oscillations of the MWV body, which allows reduction of the vibration level on the driver’s seat and increase in the maximum speed of the vehicle, the laws are characterized in that they provide generating the control inputs in real time mode. The authors have demonstrated the efficiency of the proposed control laws by means of mathematical simulation of the MWV driving over an unpaved road with kinematic excitation.

The proposed optimal control laws can be used in the MWV suspension control systems with magnetorheological shock absorbers or controlled hydropneumatic springs. Further evolution of the research line can be the development of the energy-efficient MWV suspension control systems with continuous control input on the vehicle body.

References
[1] Belousov B N and Popov S D 2014 Heavy-duty wheeled vehicles: Design, Theory, Calculations SAE International ISBN 978-0-7680-7723-0
[2] Sukhorukov A V 2003 Controlling the damping elements of the suspension system of a fast-moving tracked vehicle (Moscow, Ph. D. thesis)
[3] Balagula V Ya and Gaytsgori M M 1977 Structuring control system of the active suspension for the earthmoving equipment Russian National Research and Information Centre for Construction Machinery proceedings 75 pp 39 – 48
[4] Vygovskiy V V 1979 On control synthesis for the active vibration isolation system of a rigid body Controlled mechanical systems (Irkutsk) pp 41 – 51
[5] Eliseev S V 1979 Theory of the active vibration isolation systems Author's abstract of the doctoral thesis (Irkutsk: Irkutsk State Technical University)
[6] Eliseev S V 1982 Dynamics of the controlled mechanical systems (Irkutsk: Irkutsk State Technical University)
[7] Kolovskiy M Z 1977 Optimization of the active vibration isolation systems Mashinovedenie 6 pp 42 – 46
[8] Frolov K V and Furman F A 1980 Applied theory of the vibration isolation systems Mashinostroenie (Moscow) 276 p.
[9] Sharapov V D 1980 Active suspensions of the road vehicles RVVKU (Riga)
[10] Furunzhiev P I Ostanin A N 1984 Control of the oscillations of the multi-axle vehicles Mashinostroenie (Moscow)
[11] Kol’tsov V I 1967 Limits of the suspensions of the terrain vehicles Ph. D. thesis (Moscow)
[12] Sutton 1978 Synthesis of active suspension systems Ph. D. thesis University of Salford
[13] Tompson A 1970-71 Design of active suspensions Proc. Inst. Mech. Eng. 185 pp 553 – 563
[14] Andryushchenko V A 1990 Theory of the automatic control systems (Leningrad, Leningrad State University)
[15] Aliev F A, Larin V B and Naumenko K I et al 1978 Optimization of the linear time-invariant control systems Naukova Dumka (Kiev)
[16] Ivanov V A and Faldin F V 1981 Theory of the optimal automatic control systems Nauka (Moscow)
[17] Afanas'ev B A, Belousov B N and Zhegov L F et al. 2008 All-wheel drive vehicle engineering 3 (Moscow: BMSTU)