Chaos under interface oscillations during boiling and sea surface agitation

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Abstract. Results of investigation of the phase interface fluctuations at film and bubble boiling and sea surface agitation are presented. The vapor film thickness and vapor bubble growing on the wall were determined with the aid of laser diagnostics. Oscillations of a growing vapor bubble and simultaneous pressure oscillations in liquid were detected using optical (laser) and acoustic diagnostics. For the very first time volume and shape oscillations of a vapor bubble growing on a wall were determined. An approached model of volume oscillations of a vapor bubble, growing on a heat transfer surface is presented basing on experimental data. The system of model equations and bubble dynamics were presented considered, accounting simultaneous evaporation and condensation at the interface. It has been found that the growth of a vapor bubble is accompanied by its volume oscillations, which become chaotic under certain circumstances.

1. Introduction

According to the model of Bromley [1], at film boiling on a horizontal cylinder the vapor film thickness is assumed constant in time. In [2-4], fluctuations of the phase interface at pool film boiling were investigated, and it was shown that the vapor film thickness varied in time. In [5], the vapor film thickness fluctuations were taken into account when modeling film boiling. According to the model, it was assumed that the vapor film thickness varied in time according to the sinusoidal law, and the local heat flux density, mean vapor film thickness, and the amplitude of the phase interface fluctuations were calculated. The phase portrait obtained for a fragment of vapor film thickness fluctuations in the three-dimensional projection of phase space. It is seen that the phase trajectory of the system belongs to a certain closed region; in this case, in the process of evolution the phase point in a certain interval of time passes near the most probable state marked by a large point in the figure. Thus, the most probable state of the system is the point of its unstable equilibrium. Perturbations resulting from the departure of vapor bubbles in the vicinity of the upper generatrix of the cylinder leads the system to the loss of its stability, but it relaxes into an equilibrium state in a certain time, i.e., the perturbation energy dissipates in the system, as a result perturbations attenuate. Such a process is reproduced in the form of complex vibrations and corresponds to the regime of determined chaos. The appearance of the determined chaos in vapor film thickness fluctuations can be stated strictly, if the leading Lyapunov’s indices calculated according to [6] turn out to be positive.
2. Bubble boiling. Vapor bubble oscillations during its growth on a wall

For the first time oscillations of vapor bubbles during their growth on a wall were fixed in [7] (Fig. 1).

Figure 1. Time dependence of the diameter of a vapor bubble growing on a wall.

With the aid of the spectral analysis of the signals of optical and acoustic probes, the interrelationship of interface fluctuations and pressure in liquid was investigated. It was established that changes of both the shape and volume of a vapor bubble occur. In analyzing the signal of the optical probe, the function of the correlation coefficient was constructed and the Taylor time macro- and microscales were calculated (Fig. 2).

Figure 2. Coefficient of correlation and the Taylor temporal macro- and microscales.

The approximations of dimensions of the vapor bubble during its growth (correlational dimensionality) in the form of $N(r) \sim r_{ph}^v$ were found. Here $N$ is the number of points and $r_{ph}$ is the distance between them in the phase space; $v$ is the correlational dimensionality. Using the value of the time delay $\tau$ equal to the arithmetic mean between the Taylor time scales, the phase portrait was
constructed on the basis of experimental data (Fig. 3) (time delay $\tau = 57$ ms, correlational dimensionality $\nu = 0.49$).

![Figure 3](image_url)

**Figure 3.** Experimental data-based phase portrait of vapor bubble oscillations.

To analyze the vibrations of a vapor bubble during its growth on a wall, the technique based on the calculation of Lyapunov's coefficients was used [8,9]. The maximum Lyapunov's index is found as follows

$$\lambda_1 = \frac{1}{\tau} \sum_{i=1}^{N} \log_2 \frac{L(t_i)}{L(t_{i-1})},$$  \hspace{1cm} (1)

where $L(t_i)$ is the distance between two points on the phase portrait at the $i$th moment; $N$ is the number of measurements; $\tau$ is the value of time delay.

For the given time delay Lyapunov's index equal to $\lambda_1 = 0.0515$ was found. The positive value of Lyapunov's index indicates chaotic oscillations of the phase interface.

In [10-12], a model describing the vibrations of a vapor bubble during its growth on the wall was suggested. The system of model equations described in these works includes the equation of mass balance in a vapor bubble, the equation of motion, and the equation of evolution in time of the thickness of the thermal liquid layer surrounding the bubble.

The numerical solution is obtained for the system of equations (describing the vibrations of a vapor bubble during its growth on the wall) that under certain conditions becomes chaotic. This is indicated by Lyapunov indices two of which are positive (Fig. 4).
Figure 4. Evolution of Lyapunov’s coefficients on the phase portrait.

The agreement between the predicted and experimental characteristics of the phase interface was obtained (Figs. 5).

Figure 5. Projection of the four-dimensional phase portrait of the investigated system of equations.

Conclusion

1. Results of investigation of the phase interface fluctuations at film and bubble boiling are presented. Experiments were carried out at pool boiling of water and Freon-113 under the atmospheric pressure. The vapor film thickness and vapor bubble growing on the wall were registered with the aid of laser diagnostics. The statistical characteristics of the vapor bubble vibrations are analyzed with the aid of phase portraits.

2. Oscillations of a growing vapor bubble and simultaneous pressure oscillations in liquid have been detected using optical (laser) and acoustic diagnostics. For the very first time, the volume and shape of oscillations of a vapor bubble, growing on a wall, have been detected.

3. Maximal Lyapunov’s exponent has been calculated for a limited set of experimental data to determine the nature of interface oscillations. Its positive value $\lambda_1 = 0.0515$ proves occurrence of chaotic interface oscillations under certain circumstances.
4. An approached model of volume oscillations of vapor bubble, growing on a heat transfer surface, has been presented based on experimental data. The system of model equations and bubble dynamics have been considered, accounting simultaneous evaporation and condensation at the interface. It has been found that the growth of a vapor bubble is accompanied by its volume oscillations, which become chaotic under certain circumstances.

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