On the Chiral Quark Soliton Model with Pauli-Villars Regularization

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PACS numbers : 12.39.Fe, 12.39.Ki, 12.38.Lg, 13.40.Em

Abstract

The Pauli-Villars regularization scheme is often used for evaluating parton distributions within the framework of the chiral quark soliton model with inclusion of the vacuum polarization effects. Its simplest version with a single subtraction term should however be taken with some caution, since it does not fully get rid of divergences contained in scalar and pseudoscalar quark densities appearing in the soliton equation of motion. To remedy this shortcoming, we propose here its natural extension, i.e. the Pauli-Villars regularization scheme with multi-subtraction terms. We also carry out a comparative analysis of the Pauli-Villars regularization scheme and more popular proper-time one. It turns out that some isovector observables like the isovector magnetic moment of the nucleon is rather sensitive to the choice of the regularization scheme. In the process of tracing the origin of this sensitivity, a noticeable difference of the two regularization scheme is revealed.

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1 Introduction

The recent calculations of nucleon parton distributions within the chiral quark soliton model (CQSM) exclusively utilizes the so-called Pauli-Villars regularization scheme [1-6]. This is to be contrasted with the fact that most of the past calculations of the nucleon static observables were carried out by using the proper-time regularization scheme [7-8]. There are some reasons for it. The first reason is mainly technical. For obtaining parton distributions, one need to evaluate nucleon matrix elements of quark bilinear operators which is nonlocal in two space-time coordinates. The problem is that we have no unanimous idea about how to generalize the proper-time scheme for the regularization of such unusual quantities. The second but more positive reason for using the Pauli-Villars regularization scheme has been advocated by Diakonov et al. [1,2]. They emphasize that this regularization scheme preserves certain general properties of parton distributions such as positivity, factorization properties, sum rules etc., which are easily violated by other regularization schemes like the proper-time one.

Recently, there was a controversial debate on the stability of soliton solutions in the CQSM regularized with the Pauli-Villars subtraction scheme [10,11]. It seems that the problem has been settled by now, since stable soliton solutions seem to exist at any rate if the Pauli-Villars regularization is applied to the quark seas only, not to the discrete bound state sometimes called the valence quark orbital. Unfortunately, this is not the end of the story. In fact, soliton solutions of the CQSM with use of the Pauli-Villars regularization scheme were obtained many years ago by Döring et al. [12]. (To be more precise, the model used by them is not the CQSM but the Nambu-Jona-Lasinio model. In fact, they were forced to impose an \textit{ad hoc} nonlinear constraint for the scalar and pseudoscalar meson fields at the later stage of manipulation. Otherwise, they would not have obtained any convergent solutions [13].) The fact that the single-subtraction Pauli-Villars scheme cannot regularize the vacuum quark condensate was already noticed in an earlier paper [14] as well as in this paper [12]. To remove this divergence, which is necessary for obtaining a finite gap equation, Döring et al. propose to add some counter terms, which depend on the meson fields, to the original effective action. It is very important to recognize that this procedure is not workable within the CQSM, since their counter terms reduce to mere constants under the chiral circle condition which we impose from the very beginning. Thus, one must conclude that the simplest Pauli-Villars scheme with the single-subtraction term is unable to fully get rid of the divergence of the vacuum quark condensate at least in the nonlinear model. One should take this fact seriously, because it brings about a trouble also in the physics of soliton sector. To understand it, one has only to remember the fact that the scalar quark density appearing in the soliton equation of motion is
expected to approach a finite and nonzero value characterizing the vacuum quark condensate as the distance from the soliton center becomes large [15]. This necessarily means that the scalar quark density appearing in the soliton equation of motion cannot also be free from divergences.

The purpose of the present study is then twofold. On the one hand, we want to show that the single-subtraction Pauli-Villars scheme is not a fully satisfactory regularization scheme, and that at least one more subtraction term is necessary for a consistent regularization of the effective theory. This will be made convinced through the formal discussion given in II and also the explicit numerical results shown in III.A. On the other hand, we also want to know the regularization-scheme dependence of the CQSM through the comparative analysis of typical static observables of the nucleon predicted by the two regularization schemes, i.e. the Pauli-Villars one and the proper-time one. The discussion on this second issue will be given in III.B. We then summarize our conclusion in IV.

2 Pauli-Villars regularization scheme

We begin with the effective lagrangian of the chiral quark model with an explicit chiral symmetry breaking term as

\[ \mathcal{L}_{CQM} = \mathcal{L}_0 + \mathcal{L}', \]

where \( \mathcal{L}_0 \) denotes the chiral symmetric part [16] given by

\[ \mathcal{L}_0 = \bar{\psi} \left( i \not\! \partial - MU_{55}(x) \right) \psi, \]

with

\[ U_{55}(x) = e^{i \gamma_5 \tau \cdot \pi(x)/f_\pi} = \frac{1 + \gamma_5}{2} U(x) + \frac{1 - \gamma_5}{2} U^\dagger(x), \]

while

\[ \mathcal{L}' = \frac{1}{4} f_\pi^2 m_\pi^2 \text{tr}(U(x) + U^\dagger(x) - 2), \]

is thought to simulate a small deviation from the chiral symmetric limit. Here the trace in (4) is to be taken with respect to flavor indices. (One could have taken an alternative choice that introduces explicit chiral-symmetry-breaking effects in the form of quark mass term. We did not do so, because it turns out that this form of action cannot be regularized consistently with the Pauli-Villars subtraction method.)
The idea of the Pauli-Villars regularization can most easily be understood by examining the form of the effective meson action derived from (1) with the help of the standard derivative expansion:

\[ S_{\text{eff}}[U] = S_f[U] + S_m[U], \quad (5) \]

where

\[ S_f[U] = -i N_c \text{Sp} \log(i \partial - MU^n) \]
\[ = \int d^4x \{ 4N_c M^2 I_2(M) \text{tr}(\partial_\mu U \partial^\mu U^\dagger) + \text{higher derivative terms} \}, \quad (6) \]

\[ S_m[U] = \int d^4x \frac{1}{4} f^2 m^2 \pi \text{tr}(U(x) + U^\dagger(x) - 2). \quad (7) \]

In eq.(6), the coefficient

\[ I_2(M) \equiv -i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M^2)^2}, \quad (8) \]

of the pion kinetic term diverges logarithmically. In fact, by introducing a ultraviolet cutoff momentum \( \alpha \) that should eventually be made infinity, one finds that

\[ I_2(M) \sim \frac{1}{16\pi^2} \{ \ln \alpha^2 - \ln M^2 - 1 \}. \quad (9) \]

This logarithmic divergence can be removed if one introduces a regularized action as follows:

\[ S_{\text{eff}}^{\text{reg}}[U] = S_f^{\text{reg}}[U] + S_m[U], \quad (10) \]

where

\[ S_f^{\text{reg}}[U] \equiv S_f[U] - \left( \frac{M}{M_{PV}} \right)^2 S_f^{M_{PV}}[U]. \quad (11) \]

Here \( S_f^{M_{PV}} \) is obtained from \( S_f[U] \) with \( M \) replaced by the Pauli-Villars regulator mass \( M_{PV} \). Further requiring that the above regularized action reproduces correct normalization for the pion kinetic term, one obtains the condition:

\[ \frac{N_c M^2}{4\pi^2} \ln \left( \frac{M_{PV}}{M} \right)^2 = f^2 \pi, \quad (12) \]

which can be used to fix the regulator mass \( M_{PV} \). Once the effective action is regularized, the static soliton energy should be a finite functional of the soliton profile \( F(r) \) under the standard hedgehog ansatz \( U(\mathbf{x}) = \exp[i\mathbf{\tau} \cdot \hat{r} F(r)] \). Since the soliton equation of motion is obtained from the stationary condition of the static energy against the variation of \( F(r) \), everything seems to be going well with the above single-subtraction Pauli-Villars regularization procedure. Unfortunately, this is not the case. To understand what the problem is, we first recall the fact that the scalar quark density appearing in the soliton equation of motion is expected to
approach a finite and nonzero constant characterizing the vacuum quark condensate as the
distance from the soliton center becomes large \cite{15}. (This is a natural consequence of our
demand that both of the soliton \((B = 1)\) and vacuum\((B = 0)\) sectors must be described
by the same (or single) equation of motion.) On the other hand, it has been known that the
vacuum quark condensate contains quadratic divergences that cannot be removed by the single-
subtraction Pauli-Villars scheme \cite{12,14}. This then indicates that the scalar quark density
appearing in the soliton equation of motion cannot also be free from divergences.

To get rid of all the troublesome divergences, we propose here to in
crease the number of
subtraction terms, thereby starting with the following action :

\[
S_{\text{eff}}^\text{reg}[U] = S_f^\text{reg}[U] + S_m[U],
\]

where

\[
S_f^\text{reg}[U] \equiv S_f[U] - \sum_{i=1}^{N} c_i S_f^\Lambda_i[U],
\]

with \(N\) being the number of subtraction terms. The logarithmic divergence of the original
action is removed if the condition

\[
1 - \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right)^2 = 0
\]

is fulfilled. Similarly, the normalization condition (12) is replaced by

\[
\frac{N c M^2}{4 \pi^2} \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right)^2 \ln \left( \frac{\Lambda_i}{M} \right) = \frac{f_\pi^2}{\pi}.
\]

The single-subtraction Pauli-Villars scheme corresponds to taking \(N = 1, \Lambda_1 = M_{PV}\), and
\(c_1 = (M/M_{PV})^2\). This is naturally the simplest case that satisfies both conditions (15) and
(16).

To derive soliton equation of motion, we must first write down a regularized expression for
the static soliton energy. Under the hedgehog ansatz \(\pi(x) = f_\pi \hat{r} F(r)\) for the background pion
fields, it is obtained in the form :

\[
E_{\text{static}}^\text{reg}[F(r)] = E_f^\text{reg}[F(r)] + E_m[F(r)],
\]

where the meson part is given by

\[
E_m[F(r)] = -f_\pi^2 m_\pi^2 \int d^3 x (\cos F(r) - 1),
\]

while the fermion (quark) part is given as

\[
E_f^\text{reg}[F(r)] = E_{\text{val}} + E_{\text{vp}}^\text{reg},
\]
with

\[ E_{\text{val}} = N_c E_0 \]

\[ E_{\text{reg}}^{\text{vp}} = N_c \sum_{n < 0} \left( E_n - E_n^{(0)} \right) - \sum_{i=1}^{N} c_i N_c \sum_{n < 0} \left( E_n^{\Lambda_i} - E_n^{(0)\Lambda_i} \right). \]

Here \( E_n \) are the quark single-particle energies, given as the eigenvalues of the static Dirac Hamiltonian in the background pion fields:

\[ H \begin{bmatrix} n \end{bmatrix} = E_n \begin{bmatrix} n \end{bmatrix}, \]

while the energy \( E_n^{(0)} \) denote the energy eigenvalues of the vacuum Hamiltonian given by eq. (23) with \( F(r) = 0 \) or \( U = 1 \). Eq. (19) means that the quark part of the static energy is given as a sum of the contribution of the discrete bound-state level and that of the negative energy Dirac continuum. The latter part is regularized by subtracting from the Dirac sea contribution a linear combination of the corresponding sum evaluated with the regulator mass \( \Lambda_i \) instead of the dynamical quark mass. \( E_n^{\Lambda_i} \) in these subtraction terms are the eigenenergies of the Dirac Hamiltonian (23) with \( M \) replaced by \( \Lambda_i \) and with the same background pion field."

Now the soliton equation of motion is obtained from the stationary condition of \( E_{\text{static}}^{\text{reg}}[F(r)] \) with respect to the variation of the profile function \( F(r) \):

\[ 0 = \frac{\delta E_{\text{static}}[F(r)]}{\delta F(r)} = 4\pi r^2 \left\{ -M [S(r) \sin F(r) - P(r) \cos F(r)] + f_\pi^2 m_\pi^2 \sin F(r) \right\}, \]

which gives

\[ F(r) = \arctan \left( \frac{P(r)}{S(r) - \frac{f_\pi^2 m_\pi^2}{M}} \right). \]

Here \( S(r) \) and \( P(r) \) are regularized scalar and pseudoscalar densities given as

\[ S(r) = S_{\text{val}}(r) + \sum_{n < 0} S_{n}(r) - \sum_{i=1}^{N} c_i \Lambda_i \sum_{n < 0} S_{n}^{\Lambda_i}(r), \]

\[ P(r) = P_{\text{val}}(r) + \sum_{n < 0} P_{n}(r) - \sum_{i=1}^{N} c_i \Lambda_i \sum_{n < 0} P_{n}^{\Lambda_i}(r), \]

with

\[ S_{n}(r) = \frac{N_c}{4\pi} \int d^3 x < n | x > \gamma^0 \frac{\delta(|x| - r)}{r^2} < x | n >, \]

\[ P_{n}(r) = \frac{N_c}{4\pi} \int d^3 x < n | x > i\gamma^5 \gamma^5 \frac{\delta(|x| - r)}{r^2} < x | n >, \]
and $S_{\text{val}}(r) = S_{n=0}(r)$ and $P_{\text{val}}(r) = P_{n=0}(r)$, while $S^\Lambda_n(r)$ and $P^\Lambda_n(r)$ are the corresponding densities evaluated with the regulator mass $\Lambda_i$ instead of the dynamical quark mass $M$. As usual, a self-consistent soliton solution is obtained in an iterative way. First by assuming an appropriate (though arbitrary) soliton profile $F(r)$, the eigenvalue problem of the Dirac hamiltonian is solved. Using the resultant eigenfunctions and their associated eigenenergies, one can calculate the regularized scalar and pseudoscalar quark densities evaluated with the regulator mass $\Lambda_i$ and $\Lambda_i$. (Later, this relation will be checked numerically.) What we must do now is to find necessary conditions for the subtraction constants $c_i$ and $\Lambda_i$ in the multi-subtraction Pauli-Villars scheme to make the vacuum quark condensate finite. This can be achieved by examining the expression of the vacuum quark condensate obtained consistently with the soliton equation of motion:

$$M\langle \bar{\psi}\psi \rangle_{\text{vac}}^{\text{reg}} = M\langle \bar{\psi}\psi \rangle_{\text{vac}} - \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right) \Lambda_i \langle \bar{\psi}\psi \rangle_{\text{vac}}^{\Lambda_i}, \quad (31)$$

or equivalently

$$\langle \bar{\psi}\psi \rangle_{\text{vac}}^{\text{reg}} = \langle \bar{\psi}\psi \rangle_{\text{vac}} - \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right)^2 \langle \bar{\psi}\psi \rangle_{\text{vac}}^{\Lambda_i}, \quad (32)$$

where

$$\langle \bar{\psi}\psi \rangle_{\text{vac}} = -4N_c M \int \frac{d^3k}{(2\pi)^3} \frac{1}{E_k^{(0)}}, \quad (33)$$

with $E_k^{(0)} = (k^2 + M^2)^{1/2}$, while $\langle \bar{\psi}\psi \rangle_{\text{vac}}^{\Lambda_i}$ are obtained from $\langle \bar{\psi}\psi \rangle_{\text{vac}}$ with the replacement of $M$ by $\Lambda_i$. Using the integration formula

$$\int_{-\alpha}^{\alpha} \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{k^2 + M^2}} = \frac{1}{8\pi^2} \left\{ 2\alpha^2 - M^2 \ln \alpha^2 + (1 - 2\ln 2)M^2 + M^2 \ln M^2 \right\}, \quad (34)$$

with $\alpha$ being a ultraviolet cutoff momentum, we obtain

$$\langle \bar{\psi}\psi \rangle_{\text{vac}}^{\text{reg}} = -\frac{N_c M}{2\pi^2} \left\{ \left[ 1 - \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right) \right] \cdot 2\alpha^2 - \left[ M^2 - \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right)^2 \Lambda_i \right] \cdot \ln \alpha^2 \right. \left. + \left[ M^2 - \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right)^2 \Lambda_i \right] \cdot (1 - 2\ln 2) + M^2 \ln M^2 - \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right)^2 \Lambda_i^2 \ln \Lambda_i^2 \right\}, \quad (35)$$

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which clearly shows that \( \langle \bar{\psi}\psi \rangle_{\text{vac}} \) contains quadratic and logarithmic divergences as \( \alpha \) going to infinity. These divergences can respectively be removed if the subtraction constants are chosen to satisfy the following conditions:

\[
M^2 - \sum_{i=1}^{N} c_i \Lambda_i^2 = 0, \tag{36}
\]

\[
M^4 - \sum_{i=1}^{N} c_i \Lambda_i^4 = 0. \tag{37}
\]

Using the first of these conditions, the finite part of \( \langle \bar{\psi}\psi \rangle_{\text{vac}} \) can also be expressed as

\[
\langle \bar{\psi}\psi \rangle_{\text{vac}} = \frac{N_c \Lambda^3}{2\pi^2} \sum_{i=1}^{N} c_i \left( \frac{\Lambda_i}{M} \right)^4 \log \left( \frac{\Lambda_i}{M} \right)^2. \tag{38}
\]

It is now obvious that the single-subtraction Pauli-Villars scheme cannot satisfy both conditions (36) and (37) simultaneously. Although the quadratic divergence may be removed, the logarithmic divergence remains in \( \langle \bar{\psi}\psi \rangle_{\text{vac}} \) and consequently also in \( S(r = \infty) \) in view of the relation (30). To get rid of both these divergences, we need at least two subtraction terms, which contains four parameters \( c_1, c_2 \) and \( \Lambda_1, \Lambda_2 \). The strategy for fixing these parameters is as follows. First by solving the two equations (36) and (37) with \( N = 2 \) for \( c_1 \) and \( c_2 \), we obtain

\[
c_1 = \frac{\left( \frac{M}{\Lambda_1} \right)^2 \Lambda_2^2 - M^2}{\Lambda_2^2 - \Lambda_f^2}, \tag{39}
\]

\[
c_2 = - \frac{\left( \frac{M}{\Lambda_2} \right)^2 \Lambda_1^2 - M^2}{\Lambda_1^2 - \Lambda_f^2}, \tag{40}
\]

which constrains the values of \( c_1 \) and \( c_2 \), once \( \Lambda_1 \) and \( \Lambda_2 \) are given. For determining \( \Lambda_1 \) and \( \Lambda_2 \), we can then use two conditions (16) and (38), which amounts to adjusting the normalization of the pion kinetic term and the value of vacuum quark condensate.

### 3 Numerical Results and Discussion

#### 3.1 Single- versus double-subtraction Pauli-Villars regularization

The most important parameter of the CQSM is the dynamical quark mass \( M \), which plays the role of the quark-pion coupling constant thereby controlling basic soliton properties. Throughout the present investigation, we use the value \( M = 400 \text{MeV} \) favored from the previous analyses of static baryon observables. In the case of single-subtraction Pauli-Villars scheme,
the regulator mass \( M_{PV} \) is uniquely fixed to be \( M_{PV} = 570.86 \text{ MeV} \) by using the normalization condition (12) for the pion kinetic term, and there is no other adjustable parameter in the model. In the case of double-subtraction Pauli-Villars scheme, we have four regularization parameters \( c_1, c_2, \Lambda_1, \) and \( \Lambda_2 \). From the divergence free conditions (36) and (37), \( c_1 \) and \( c_2 \) are constrained as (39) and (40), while \( \Lambda_1 \) and \( \Lambda_2 \) are determined from (16) and (38) with \( f_\pi = 93 \text{ MeV} \) and \( < \bar{\psi}\psi >_{\text{vac}} = - (286.6 \text{ MeV})^3 \). In spite of their nonlinearity, the two conditions (16) and (38) are found to uniquely fix the two parameters \( \Lambda_1 \) and \( \Lambda_2 \) within the physically acceptable range of parameters. The solution that we found is

\[
c_1 = 0.445, \quad c_2 = -0.00612, \quad \Lambda_1 = 630.01 \text{ MeV}, \quad \Lambda_2 = 1642.13 \text{ MeV}.
\]

(41)

As usual, all the numerical calculations are carried out by using the so-called Kahana and Ripka basis [17]. Following them, the plane-wave basis, introduced as a set of eigenstates of the free hamiltonian \( H_0 = \alpha \cdot \nabla / i + \beta M \), is discretized by imposing an appropriate boundary condition for the radial wave functions at the radius \( D \) chosen to be sufficiently larger than the soliton size. The basis is made finite by including only those states with the momentum \( k \) as \( k < k_{\text{max}} \). The eigenvalue problem (22) is then solved by diagonalizing the Dirac hamiltonian \( H \) in the above basis. We are thus able to solve the self-consistent Hartree problem and also to calculate any nucleon observables with full inclusion of the sea-quark degrees of freedom. If the theory is consistently regularized, final answers must be stable against increase of \( k_{\text{max}} \) and \( D \) (especially against the increase of \( k_{\text{max}} \)).

Now we show in Fig.1 the \( k_{\text{max}} \) dependence of the theoretical pseudoscalar and scalar quark densities in the single-subtraction Pauli-Villars scheme. These curves are obtained for a fixed value of \( D \) as \( MD = 12 \). The corresponding \( k_{\text{max}} \) dependence of the quark densities in the double-subtraction Pauli-Villars scheme are shown in Fig.2. Comparing the two figures, one immediately notices that the quark densities obtained in the single-subtraction Pauli-Villars scheme do not cease to increase in magnitudes as \( k_{\text{max}} \) increases. Undoubtedly, this must be a signal of logarithmic divergences contained in \( S(r = \infty) \) (and generally also in \( P(r) \) and \( S(r) \)). On the other hand, in the case of double-subtraction Pauli-Villars scheme, the magnitudes of \( P(r) \) and \( S(r) \) are seen to grow much more slowly. To convince more clearly the above qualitative difference of the two regularization schemes, we plot in Fig.3 the value of \( S(r = \infty) \), i.e. the scalar quark density at the spatial infinity, as functions of \( k_{\text{max}} \), and also as functions of \( \log(k_{\text{max}}/M) \). Contrary to the case of single-subtraction scheme in which a clear signal of logarithmic divergence is observed, the value of \( S(r = \infty) \) obtained in the double-subtraction scheme is seen to converge to some limiting value. Although the rate of this convergence is rather slow, it appears that this limiting value certainly coincides with the
prescribed value of vacuum quark condensate $\langle \bar{\psi} \psi \rangle_{\text{vac}} = -(286.6 \text{ MeV})^3 = -3.062 \text{ fm}^{-3}$.

Now that one has convinced the fact that the naive Pauli-Villars scheme with the single-subtraction term contains logarithmic divergence in the quark densities appearing in the soliton equation of motion, one may come to the following question. Why could the authors of ref. [12] obtain self-consistent soliton solutions despite the presence of the above-mentioned divergences? The answer lies in the way of obtaining a self-consistent soliton profile in the nonlinear model (not in the original Nambu-Jona-Lasinio model). After evaluating the pseudoscalar and scalar quark densities with some (large but) finite model space (especially with finite $k_{\text{max}}$), a new profile function $F(r)$ to be used in the next iterative step is obtained from (25). Since $P(r)$ and $S(r)$ appears respectively in the numerator and denominator of the argument of arctangent, it can happen that the logarithmic divergence contained in both of $P(r)$ and $S(r)$ are offset each other. (We point out that the effect of the term $f_2^2 m_\pi^2 / M$ accompanying the scalar quark density is rather small, anyway.) In fact, Fig.4 shows the $k_{\text{max}}$ dependence of the self-consistent profile function $F(r)$ in both of the single-subtraction scheme and the double-subtraction scheme. One sees that the resultant $F(r)$ is quite stable against the increase of $k_{\text{max}}$ even in the single-subtraction scheme, in spite of the fact that it shows logarithmically divergent behavior for both of $P(r)$ and $S(r)$. Undoubtedly, this is the reason why the authors of [12] succeeded in obtaining self-consistent soliton profile $F(r)$ despite the divergences remaining in each of $P(r)$ and $S(r)$. Because of this fortunate accident, self-consistent soliton profiles $F(r)$ in the nonlinear model can be obtained with a good accuracy by using a modest value of $k_{\text{max}}$ not only for the double-subtraction scheme but also for the single-subtraction one, and besides the resultant $F(r)$ and not much different in these two schemes. This also applies to most nucleon observables which depend only on $F(r)$ and have no direct dependence on $S(r)$ and/or $P(r)$. The previous calculation of parton distributions with use of the single-subtraction Pauli-Villars scheme may be justified in this sense [1-6]. To verify the validity of this expectation, we investigate the $k_{\text{max}}$ dependence of a typical nucleon observable which contains only a logarithmic divergence, i.e. the isovector axial-vector coupling constant $g^{(3)}_A$. Fig.5 show the $k_{\text{max}}$ dependence of $g^{(3)}_A$ in the single- and double-subtraction Pauli-Villars regularization schemes. One sees that this quantity certainly shows a tendency of convergence in both regularization schemes, though the rate of convergence in the double-subtraction scheme is much faster than for the scalar and pseudoscalar densities in the same regularization scheme. Nonetheless, one must be very careful if one is interested in nucleon observables, which have direct dependence on $S(r)$ or $P(r)$. The most important nucleon observable, which falls into
this category, is the nucleon scalar charge (or the quark condensate in the nucleon) given by

\[ \langle N | \bar{\psi} \psi | N \rangle \equiv \int d^3r \left[ S(r) - S(r = \infty) \right]. \tag{42} \]

The superiority of the double-subtraction scheme to the single-subtraction one must be self-explanatory in this case, since this quantity is convergent only in the former scheme.

### 3.2 Pauli-Villars versus proper-time regularization

How to introduce ultraviolet cutoff into our effective chiral theory is a highly nontrivial problem. Diakonov et al. advocated the Pauli-Villars subtraction scheme as a "good" regularization scheme for evaluating leading-twist parton distribution functions of the nucleon within the chiral quark soliton model [1,2]. The reason is that it preserves several general properties of the parton distributions (such as positivity, factorization properties, sum rules etc.), which can easily be violated by a naive ultraviolet regularization. On the other hand, Schwinger’s proper-time regularization has most frequently been used for investigating low energy nucleon properties within the chiral quark soliton model [7-9]. One might then wonder how these predictions obtained by using the proper-time regularization scheme would be altered if one uses the Pauli-Villars one.

Before entering into this discussion, we think it useful to recall some basic properties of the proper-time regularization scheme. In this scheme, the regularized effective meson action takes the same form as (10) except that \( S_f^{\text{reg}}[U] \) is now given in the form :

\[ S_f^{\text{reg}}[U] = \frac{1}{2} i N_c \int_0^\infty \frac{d\tau}{\tau} \varphi(\tau) \text{Sp} \left( e^{\tau D^\dagger D} - e^{-\tau D_0^\dagger D_0} \right), \tag{43} \]

with

\[ D = i \not \! \partial - M \not \! \gamma_5, \quad D_0 = i \not \! \partial - M. \tag{44} \]

The regularization function \( \varphi(\tau) \) is introduced so as to cut off ultraviolet divergences which now appear as a singularity at \( \tau = 0 \). For determining it, we can use a similar criterion as what was used in the Pauli-Villars scheme. That is, we require that the regularized theory reproduces the correct normalization of the pion kinetic term as well as the empirical value of the vacuum quark condensate. This gives two conditions :

\[ \frac{N_c M^2}{4\pi^2} \int_0^\infty \frac{d\tau}{\tau^2} \varphi(\tau) e^{-\tau M^2} = f_{\pi}^2, \]

\[ \frac{N_c M}{2\pi^2} \int_0^\infty \frac{d\tau}{\tau^2} \varphi(\tau) e^{-\tau M^2} = \langle \bar{\psi} \psi \rangle_{\text{vac}}. \tag{46} \]
Schwinger’s original choice corresponds to taking
\[ \varphi(\tau) = \theta \left( \tau - \frac{1}{\Lambda^2} \right), \]  
(47)
with \( \Lambda \) being a physical cutoff energy. However, this simplest choice cannot fulfill the two conditions (45) and (46) simultaneously. Then, we use here slightly more complicated form as
\[ \varphi(\tau) = c \theta \left( \tau - \frac{1}{\Lambda_1^2} \right) + (1 - c) \theta \left( \tau - \frac{1}{\Lambda_2^2} \right), \]  
(48)
which contains three parameters \( c, \Lambda_1 \) and \( \Lambda_2 \) [18]. Although the above two conditions are not enough to uniquely fix the above three parameters, we find that solution sets \( (c, \Lambda_1, \Lambda_2) \) lie only in a small range of parameter space and that this slight difference of regularization parameters hardly affects the soliton properties. We use the following set of parameters in the numerical investigation below:
\[ c = 0.720, \quad \Lambda_1 = 412.79 \text{ MeV}, \quad \Lambda_2 = 1330.60 \text{ MeV}. \]  
(49)
Within the framework of the chiral quark soliton model, which assumes slow collective rotation of a hedgehog soliton as
\[ U_\gamma(x,t) = A(t)U_0^\gamma(x)A^\dagger(t), \quad A(t) \subset SU(2), \]  
(50)
the nucleon matrix element of any quark bilinear operator \( \bar{\psi}O\psi \) is given as a perturbative series in the collective angular velocity operator \( \Omega \) defined by
\[ \Omega = i A^\dagger(t) \frac{d}{dt} A(t). \]  
(51)
It is shown below that a noteworthy difference between the proper-time regularization and the Pauli-Villars one appears at the zeroth order term in \( \Omega \). We recall that, in both schemes, the \( O(\Omega^0) \) contribution to this matrix element is given as
\[ \langle O \rangle_{\Omega^0} = \int \mathcal{D}A \, \Psi^{(J)^r}_{M_J M_T}[A] \langle O \rangle_{\Omega^0}^{\Psi^{(J)}}_A \Psi^{(J)}_{M_J M_T}[A], \]  
(52)
with
\[ \langle O \rangle_{\Omega^0}^{\Psi^{(J)}}_A = \langle O \rangle_{\val}^{\Omega^0} + \langle O \rangle_{\vp}^{\Omega^0}, \]  
(53)
where \( \Psi^{(J)}_{M_J M_T}[A] \) is a wave function describing the collective rotational motion. In eq.(53),
\[ \langle O \rangle_{\val} = N_c \langle 0 | \tilde{O} | 0 \rangle , \quad \text{with} \quad \tilde{O} = A^\dagger O A, \]  
(54)
represents the contribution of the discrete bound state level called the valence quark one. Within the Pauli-Villars scheme, the contribution of the Dirac continuum can be given in either of the following two forms:

\[
\langle O \rangle_{\text{vp}}^{\Omega^0} = N_c \sum_{n<0} \langle n | \tilde{O} | n \rangle - \text{Pauli-Villars subtraction},
\]

\[
= -N_c \sum_{n\geq0} \langle n | \tilde{O} | n \rangle - \text{Pauli-Villars subtraction}. \tag{55}
\]

Note that the first form is given as a sum over the occupied single-quark levels, while the second form given as a sum over the nonoccupied levels. The equivalence of the two expressions follows from the identity

\[
0 = \text{Sp} \tilde{O} = \sum_{n<0} \langle n | \tilde{O} | n \rangle + \sum_{n\geq0} \langle n | \tilde{O} | n \rangle, \tag{56}
\]

which holds for most operators including the isovector magnetic moment operator investigated below, if it is combined with the fact that a similar identity holds also for the corresponding Pauli-Villars subtraction terms. The situation is a little different for the proper-time regularization scheme. The regularized Dirac sea contribution in this scheme is given in the following form [8]:

\[
\langle O \rangle_{\text{vp}}^{\Omega^0} = -\frac{N_c}{2} \sum_{n=\text{all}} \text{sign}(E_n) g(E_n) \langle n | \tilde{O} | n \rangle, \tag{57}
\]

with

\[
g(E_n) = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{d\tau}{\sqrt{\tau}} |E_n| e^{-\tau E_n^2}. \tag{58}
\]

To compare this with the corresponding expression in the Pauli-Villars scheme, it is convenient to rewrite it as

\[
\langle O \rangle_{\text{vp}}^{\Omega^0} = \frac{1}{2} \left\{ N_c \sum_{n<0} g(E_n) \langle n | \tilde{O} | n \rangle - N_c \sum_{n\geq0} g(E_n) \langle n | \tilde{O} | n \rangle \right\}. \tag{59}
\]

One sees that here the answer is given as an average of the two expressions, i.e. the one given as a sum over the occupied levels and the others given as a sum over the nonoccupied levels. (This feature is a consequence of the starting covariant expression for an operator expectation value in the proper-time scheme.) However, contrary to the previous case in which ultraviolet regularization is introduced in the form of the Pauli-Villars subtraction, now there is no reason to believe that the above two terms give the same answer. In fact, the introduction of the energy dependent cutoff factor \( g(E_n) \) generally breaks the equivalence of the two expressions because of the spectral asymmetry of the positive- and negative-energy levels induced by the background pion field of hedgehog form.
Now we start a comparative analysis of the two regularization schemes on the basis of the numerical results. For reference, we also solve the soliton equation of motion in the chiral limit. By assuming no (or at least weak) $m_\pi$ dependence of $<\bar{\psi}\psi>_{\text{vac}}$ appearing in (16) and (38), this calculation can be done by setting $m_\pi = 0$ in (18) and (25) without changing the sets of regularization parameters given in (41) and (49). Since the way of cutting off the ultraviolet component is totally different for the two regularization schemes, it naturally affects solutions of the soliton equation of motion. Although the detailed contents of the soliton energy are highly model dependent concepts and are not direct observables, they are anyhow very sensitive to this difference of the self-consistent solutions. Table 1 shows this comparison. Comparing the answers of the two regularization schemes, one finds that the Pauli-Villars scheme leads to more strongly deformed soliton, which means a deeper binding of the discrete valence level and larger vacuum polarization energy. One sees that the total soliton energy is lower for the Pauli-Villars scheme than for the proper-time scheme. One also observes that the soliton energy is very sensitive to the pion mass. When one goes from the finite pion mass case to the chiral limit, one obtains much lower soliton energy.

Table 1: The static soliton energy in the proper-time regularization scheme and the (double-subtraction) Pauli-Villars one. $E_{\text{val}}$, $E_{\text{v.p.}}^{\text{reg}}$ respectively stand for the valence quark contribution and the Dirac sea one to the fermionic energy, while $E_m$ represents the mesonic part of the energy. The sum of these three parts gives the total static energy $E_{\text{static}}^{\text{reg}}$.

|                  | $E_{\text{val}}$ [MeV] | $E_{\text{v.p.}}^{\text{reg}}$ [MeV] | $E_m$ [MeV] | $E_{\text{static}}^{\text{reg}}$ [MeV] |
|------------------|------------------------|-------------------------------------|-------------|--------------------------------------|
| proper-time ($m_\pi = 138$ MeV) | 633.0                  | 617.6                              | 37.2        | 1287.9                               |
| Pauli-Villars ($m_\pi = 138$ MeV) | 447.6                  | 569.2                              | 51.3        | 1068.1                               |
| proper-time ($m_\pi = 0$ MeV)    | 555.6                  | 688.6                              | 0           | 1244.2                               |
| Pauli-Villars ($m_\pi = 0$ MeV)  | 351.5                  | 655.4                              | 0           | 1006.9                               |

Probably, the most important observable which has strong sensitivity to the above difference of the self-consistent solutions is the flavor-singlet axial charge or the quark spin content of the nucleon $\langle \Sigma_3 \rangle$. The theoretical predictions for this quantities in the two regularization schemes are shown in Table 2. In evaluating this quantity, we did not introduce any regularization, because it is related to the imaginary part of the (Euclidean) effective action and is convergent itself. This means that the difference between the two schemes purely comes from that of the self-consistent solutions. One sees that the Pauli-Villars scheme leads to smaller quark...
Table 2: The quark spin content of the nucleon $< \Sigma_3 >$ in the proper-time regularization scheme and the Pauli-Villars one.

| Scheme          | $< \Sigma_3 >_{\text{val}}$ | $< \Sigma_3 >_{\text{v.p.}}$ | $< \Sigma_3 >$ |
|-----------------|-----------------------------|-------------------------------|----------------|
| proper-time ($m_\pi = 138$ MeV) | 0.484 | 0.005 | 0.489 |
| Pauli-Villars ($m_\pi = 138$ MeV) | 0.391 | 0.008 | 0.399 |
| proper-time ($m_\pi = 0$ MeV) | 0.374 | 0.007 | 0.380 |
| Pauli-Villars ($m_\pi = 0$ MeV) | 0.286 | 0.011 | 0.298 |

spin content. The reason can easily be understood. Within the framework of the chiral quark soliton model, the rest of the nucleon spin is carried by the orbital angular momentum of quark fields and this latter portion increases as the deformation of the soliton becomes larger [8]. A similar tendency is also observed when one goes from the finite pion mass case to the chiral limit.

Table 3: The $O(\Omega^0)$ contributions to the isovector magnetic moment of the nucleon in the proper-time regularization scheme and the Pauli-Villars one. The second column represents for the valence quark contribution. The third and fourth columns stand for the answers for the vacuum polarization contributions respectively obtained with the occupied and nonoccupied formulas, while the fifth column gives the average of the two answers. The total $O(\Omega^0)$ contributions are shown in the sixth column.

| Scheme          | $\mu_{\text{val}}^{(3)}(\Omega^0)$ | $\mu_{\text{v.p.}}^{(3)}(\Omega^0)$ | $\mu_{\text{av}}^{(3)}(\Omega^0)$ |
|-----------------|-------------------------------|---------------------------------|----------------------------------|
| proper-time ($m_\pi = 138$ MeV) | 1.611 | 1.312 | 0.210 | 0.761 | 2.372 |
| Pauli-Villars ($m_\pi = 138$ MeV) | 1.762 | 0.996 | 0.996 | 0.996 | 2.759 |
| proper-time ($m_\pi = 0$ MeV) | 1.623 | 1.908 | 0.588 | 1.248 | 2.875 |
| Pauli-Villars ($m_\pi = 0$ MeV) | 1.810 | 1.738 | 1.738 | 1.738 | 3.547 |

There are different kinds of nucleon observables, which contain (potential) logarithmic divergence and thus depend directly on how they are regularized. Most typical are the $O(\Omega^0)$ contribution to the isovector axial-vector coupling constant $g_A^{(3)}$ and the isovector magnetic moment $\mu^{(3)}$ of the nucleon. Let us first show the results for the isovector magnetic moment,
since it turns out to have stronger dependence on the choice of the regularization scheme. Table 3 shows the $O(\Omega^0)$ contribution to the isovector magnetic moment. For each regularization scheme, the second column represents the answer obtained with the occupied expression, while the third column does the answer obtained with the nonoccupied one. In the case of Pauli-Villars scheme, the equivalence of the two expressions is nicely confirmed by the explicit numerical calculation. In the case of proper-time scheme, however, we encounter quite a dissimilar situation. First, the answer obtained with the occupied expression is about 30\% larger than the corresponding answer of the Pauli-Villars scheme, while the answer obtained with the nonoccupied expression is about 80\% smaller than the answer obtained with the occupied one. Since the final answer of the proper-time scheme is given as an average of the occupied and nonoccupied expressions, the consequence is that the prediction of the proper-time scheme for the $O(\Omega^0)$ contribution to $\mu^{(3)}$ is about 14\% smaller than the corresponding prediction of the Pauli-Villars scheme. (See the fourth column of the Table 3.) Note that the difference between the two regularization schemes becomes much more drastic when one goes to the chiral limit. This is due to the fact that the $O(\Omega^0)$ vacuum polarization contribution to the isovector magnetic moment is extremely sensitive to the pion mass effect such that it is much larger in the chiral limit.

Table 4: The final predictions for the isovector magnetic moment of the nucleon, given as sums of the $O(\Omega^0)$ and $O(\Omega^1)$ contributions.

|               | $\mu^{(3)}(\Omega^0)$ | $\mu^{(3)}(\Omega^1)$ | $\mu^{(3)}(\Omega^0 + \Omega^1)$ |
|---------------|------------------------|------------------------|-----------------------------------|
| proper-time   | 2.372                  | 1.072                  | 3.445                             |
| $m_\pi = 138$ |                        |                        |                                   |
| MeV           | 2.759                  | 1.211                  | 3.970                             |
| Pauli-Villars |                        |                        |                                   |
| $m_\pi = 0$   | 2.875                  | 1.032                  | 3.907                             |
| MeV           | 3.547                  | 1.182                  | 4.729                             |

Before comparing our theoretical predictions with the observed isovector magnetic moment of the nucleon, we must take account of the $O(\Omega^1)$ contribution, too, since it is known to give sizable correction to the leading-order result \cite{19,20}. Although we do not go into the detail here, it turns out that this $O(\Omega^1)$ piece is not so sensitive to the difference of the regularization scheme as the $O(\Omega^0)$ piece is. The reason is that this $O(\Omega^1)$ term is given as a double sum over the occupied levels and the nonoccupied ones and the formula has some symmetry under the exchange of these two types of single-quark orbitals \cite{21}. The final predictions for the
Table 5: The final predictions for the isovector axial-coupling constant of the nucleon, given as sums of the \( O(\Omega^0) \) and \( O(\Omega^1) \) contributions.

|                     | \( g_A^{(3)}(\Omega^0) \) | \( g_A^{(3)}(\Omega^1) \) | \( g_A^{(3)}(\Omega^0 + \Omega^1) \) |
|---------------------|---------------------------|---------------------------|--------------------------------------|
| proper-time \( m_\pi = 138 \text{ MeV} \) | 0.848                     | 0.412                     | 1.260                                |
| Pauli-Villars \( m_\pi = 138 \text{ MeV} \) | 0.976                     | 0.408                     | 1.384                                |
| proper-time \( m_\pi = 0 \text{ MeV} \) | 0.921                     | 0.348                     | 1.269                                |
| Pauli-Villars \( m_\pi = 0 \text{ MeV} \) | 1.054                     | 0.344                     | 1.398                                |

nucleon isovector magnetic moment obtained as a sum of the \( O(\Omega^0) \) and \( O(\Omega^1) \) contributions are shown in Table 4. After all, the prediction of the Pauli-Villars scheme is about 15% larger than that of the proper-time scheme and a little closer to the observed moment. The effect is much more drastic in the chiral limit. The prediction of the Pauli-Villars scheme is about 20% larger than that of the proper-time scheme and nearly reproduces the observed isovector magnetic moment of the nucleon, i.e. \( \mu_{\text{exp}}^{(3)} \approx 4.71 \).

Finally, we show in Table 5 the predictions for the isovector axial-charge of the nucleon obtained as a sum of the \( O(\Omega^0) \) and \( O(\Omega^1) \) contributions. Also for this quantity, there are some detailed differences between the predictions of the two regularization schemes. Nonetheless, the final answers for \( g_A^{(3)} \) turn out to be not so sensitive to the difference of the regularization schemes as compared with the case of the isovector magnetic moment. Besides, one also notices that the finite pion mass effect hardly influences the final prediction for this particular quantity.

4 Conclusion

In summary, the single-subtraction Pauli-Villars regularization scheme, which is often used in evaluating nucleon structure functions within the framework of the CQSM, cannot be regarded as a fully consistent regularization scheme in that it still contains ultraviolet divergences in the scalar and pseudoscalar quark densities appearing in the soliton equation of motion. However, these divergences can easily be removed by increasing the number of subtraction term from one to two. After this straightforward generalization, the effective theory is totally divergence free. Especially, both the vacuum quark condensate and the isoscalar piece of the nucleon scalar charge becomes finite now. Nonetheless, we find that, owing to the accidental cancellation
explained in the text, one can obtain a finite soliton profile $F(r)$ even in the single-subtraction scheme, and besides the resultant soliton solution is not extreme different from the corresponding one obtained in the double-subtraction scheme. Furthermore, it turns out that, for most nucleon observables, which contain only the logarithmic divergence, the predictions of the two regularization schemes are not much different. The previous calculations of quark distribution functions with use of the single-subtraction Pauli-Villars regularization scheme would be justified in this sense.

We have also carried out a comparative analysis of typical nucleon observables based on the Pauli-Villars regularization scheme and the proper-time one. A nice property of the Pauli-Villars regularization scheme, which is not possessed by the proper-time one, is that it preserves a nontrivial symmetry of the original theory, i.e. the equivalence of the occupied and nonoccupied expressions for $O(\Omega^0)$ contributions to nucleon observables. The improvement obtained for the isovector magnetic moment of the nucleon was shown to be related to this favorable property of the Pauli-Villars regularization scheme. How to introduce ultraviolet cutoff into an effective low energy model should in principle be predictable from the underlying QCD dynamics. For lack of precise information about it, however, phenomenology must provides us with an important criterion for selecting regularization schemes. The regularization scheme based on the Pauli-Villars subtraction appears to be a good candidate also in this respect.

Acknowledgement

Numerical calculation was performed by using the workstations at the Laboratory of Nuclear Studies, and those at the Research Center for Nuclear Physics, Osaka University.

References

[1] D.I. Diakonov, V.Yu. Petrov, P.V. Pobylitsa, M.V. Polyakov, and C. Weiss, Nucl. Phys. B480, 341 (1996).

[2] D.I. Diakonov, V.Yu. Petrov, P.V. Pobylitsa, M.V. Polyakov, and C. Weiss, Phys. Rev. D56, 4069 (1997).

[3] M. Wakamatsu and T. Kubota, Phys. Rev. D56, 4069 (1998).
[4] M. Wakamatsu and T. Kubota, Osaka University preprint OU-HET-310/98, hep-ph/9809443.

[5] C. Weiss and K. Goeke, Bochum University preprint RUB-TPII-12/97, hep-ph/9712447.

[6] P.V. Pobylitsa, M.V. Polyakov, K. Goeke, T. Watabe, and C. Weiss, Bochum University preprint RUB-TPII-4/98, hep-ph/9804436.

[7] H. Reinhardt and R. Wünsch, Phys. Lett. B215, 577 (1998); T. Meissner, F. Grümmer, and K. Goeke, Phys. Lett. B227, 296 (1989).

[8] M. Wakamatsu and H. Yoshiki, Nucl. Phys. A524, 561 (1991).

[9] For reviews, see, M. Wakamatsu, Prog. Theor. Phys. Suppl. 109, 115 (1992); Chr.V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, Th. Meissner, E. Ruiz Arriola and K. Goeke, Prog. Part. Nucl. Phys. 37, 91 (1996); R. Alkofer, H. Reinhardt and H. Weigel, Phys. Rep. 265, 139 (1996).

[10] H. Weigel, L. Gamberg, and H. Reinhardt, Phys. Rev. D58, 038501 (1998).

[11] D.I. Diakonov, V.Yu. Petrov, P.V. Pobylitsa, M.V. Polyakov, and C. Weiss, Phys. Rev. D58, 038502 (1998).

[12] F. Döring, A. Blotz, C. Schüren, Th. Meissner, E. Ruiz-Arriola, and K. Goeke, Nucl. Phys. A536, 548 (1992).

[13] T. Watabe and H. Toki, Prog. Theor. Phys. 87, 651 (1992).

[14] M. Jaminon, G. Ripka, and P. Stassart, Phys. Lett. B227, 191 (1989).

[15] M. Wakamatsu, Phys. Rev. D46, 3762 (1992).

[16] D.I. Diakonov, V.Yu. Petrov, and P.V. Pobylitsa, Nucl. Phys. B306, 809 (1988).

[17] S. Kahana and G. Ripka, Nucl. Phys. A429, 462 (1984); S. Kahanna, G. Ripka, and V. Soni, Nucl. Phys. A415, 351 (1984).

[18] A. Blotz, D.I. Diakonov, K. Goeke, N.W. Park, V.Yu. Petrov, and P.V. Pobylitsa, Nucl. Phys. A555, 765 (1993).

[19] M. Wakamatsu and T. Watabe, Phys. Lett. B312, 184 (1993).
[20] Chr. V. Christov, A. Blotz, K. Goeke, P. V. Pobylitsa, V. Yu. Petrov, M. Wakamatsu, and T. Watabe, Phys. Lett. B325, 467 (1994).

[21] M. Wakamatsu, Prog. Theor. Phys. 95, 143 (1996).

Figure caption

Fig. 1. The $k_{\text{max}}$ dependence of the scalar quark density $S(r)$ and the pseudoscalar density $P(r)$ in the single-subtraction Pauli-Villars scheme.

Fig. 2. The $k_{\text{max}}$ dependence of the scalar quark density $S(r)$ and the pseudoscalar density $P(r)$ in the double-subtraction Pauli-Villars scheme.

Fig. 3. The scalar quark densities at the spatial infinity $S(r = \infty)$ as functions of $k_{\text{max}}/M$ and as functions of $\log(k_{\text{max}}/M)$ in the single- and double-subtraction Pauli-Villars schemes.

Fig. 4. The $k_{\text{max}}$ dependence of the self-consistent soliton profiles $F(r)$ in the single- and double-subtraction Pauli-Villars schemes. The curves with different $k_{\text{max}}$ are almost indistinguishable.

Fig. 5. The $k_{\text{max}}$ dependence of the nucleon isovector axial-charges $g_A^{(3)}$ in the single- and double-subtraction Pauli-Villars schemes.
Fig. 1

$-P(r)$ vs $Mr$ for different $k_{\text{max}}$: $k_{\text{max}} M = 7$, $k_{\text{max}} M = 14$, $k_{\text{max}} M = 21$, $k_{\text{max}} M = 28$.

$-S(r)$ vs $Mr$ for different $k_{\text{max}}$: $k_{\text{max}} M = 7$, $k_{\text{max}} M = 14$, $k_{\text{max}} M = 21$, $k_{\text{max}} M = 28$. 
Fig. 2

\[ -P(r) \]

\[ -S(r) \]

- \( k_{\text{max}} M = 7 \)
- \( k_{\text{max}} M = 14 \)
- \( k_{\text{max}} M = 21 \)
- \( k_{\text{max}} M = 28 \)
Fig. 4

\[ M \]

\[ F(r) \]

\[ k_{\text{max}} M = 7 \]
\[ k_{\text{max}} M = 14 \]
\[ k_{\text{max}} M = 21 \]
\[ k_{\text{max}} M = 28 \]
Fig. 5

\[ g_A^{(3)} \]

vs.

\[ \frac{k_{\text{max}}}{M} \]

- Single-subtr.
- Double-subtr.