Face-to-face interactions in a Vicsek Model

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In this study, we investigate statistical properties of face-to-face interactions in a two-dimensional space by focusing on adjacency relationships in a Vicsek model. We compute neighbors for all particles at every time step, and investigate the cumulative lifetime distributions \( P(\tau) \) of adjacent edges. It is found that \( P(\tau) \) changes from an exponential to a power-law distribution when interaction radius \( R_0 \) increases. Remarkably, the power-law exponent \( \alpha \simeq 3/2 \) emerges in large \( R_0 \).

Introduction.— Waiting time distributions are commonly utilized in research aiming to statistically characterize a wide range of stochastic processes in physical and social systems. There are various examples in which waiting time distributions \( p(\tau) \) exhibit exponential decay: arrival of telephone calls or e-mails, decay of radioactive elements, occurrence of car accidents, and scoring of competitive sports events [1, 2]. Such systems can be theoretically described by a homogeneous Poisson process in which each event occurs independently at a constant rate within a certain time interval [3].

Meanwhile, a power-law distribution, \( p(\tau) \sim \tau^{-\alpha} \), is also ubiquitous in nature, as it is associated with the dynamics of earthquakes, solar flares, animal movement, and human activities [4]. Such power-law behavior of waiting time is often referred to as “burstiness”, and attracts significant attention in statistical physics, particularly as it relates to human activities. These human activities can be divided into individual and interaction-driven (or communication-driven), and the latter is further divided into direct (face-to-face) and indirect interactions [4]. Previous studies reported that such human activities exhibit bursty behavior characterized by different power-law exponents. Further, a number of stochastic models yielding such power-law behavior have been developed based on an extended Poisson process; examples include the priority queue model [5], Hawkes process [6], and cascading Poisson process [7].

Recently, experiments involving face-to-face interactions using wearable sensors have been conducted in scientific conferences [8, 9], schools [10, 11], and other settings [12, 13]. These studies measured how long two people were in close proximity within a certain distance (i.e., the duration of the face-to-face interaction). In these cases, it was found that the duration time distribution also obeys a power-law. However, because special environments and equipment are needed in order to conduct face-to-face experiments, obtaining a sufficient number of examples of face-to-face interaction can be difficult.

Because face-to-face interactions, in which individuals are close to each other within a certain space, are common in the real world, the essential features of the burstiness can be extracted not only from human activities but from a more general situation. Hence, in this paper, we investigate statistical properties of face-to-face interactions occurring in a relatively simple and general situation by considering a model of a many-particle system on a two-dimensional space. To this end, we employ a 2D Vicsek model for describing collective motions of self-propelled particles [16, 17]. The interaction in this model can be regarded as the adjacency relationships between particles. We numerically investigated a lifetime distribution of adjacent edges, and found that the distribution is subject to a power law with exponent \( \alpha \simeq 3/2 \).

Model.— Let us consider an \( N \)-particle system in a 2D circular space with a diameter \( L \), which corresponds to the system size. We denote the position and the velocity of the \( j \)-th particle at time \( t \) as \( \vec{r}_j(t) \) and \( \vec{v}_j(t) = v_0 \vec{s}_j(t) \), respectively. Here, \( v_0 \) denotes the speed and \( \vec{s}_j(t) \) denotes the unit vector. Note that \( \vec{s}_j(t) \) is determined by the angle \( \theta_j(t) \) in a polar coordinate. The equation of each particle’s motion is given as follows [18]:

\[
\theta_j(t + \Delta t) = \Arg \sum_{k \sim j}^{N} \left[ (1 - c) \vec{v}_k(t) + c \vec{f}_{jk}(t) \right] + \xi_j(t), \quad (1)
\]

\[
\vec{r}_j(t + \Delta t) = \vec{r}_j(t) + \Delta t v_0 \vec{s}_j(t + \Delta t). \quad (2)
\]

In the first term on the right hand side of eq. (1), the notation \( k \sim j \) in the summation indicates that the \( j \)-th particle interacts with others within the circle of radius \( R_0 \), whose center is \( \vec{r}_j \). Here, the alignment (\( \vec{v}_k \)-part) and repulsive (\( \vec{f}_{jk} \)-part) interactions are considered, and \( \vec{f}_{jk}(t) \) between a pair of particles \( j \) and \( k \) is given by

\[
\vec{f}_{jk} = -\vec{e}_{jk} \times \left[ 1 + \exp \left( \frac{|\vec{r}_j - \vec{r}_k|}{R_f} - 2 \right) \right]^{-1},
\]

where \( \vec{e}_{jk} \) denotes the unit vector from \( j \) to \( k \), and \( R_f \) is the typical repulsion distance [18]. The balance of the alignment and repulsive force is controlled by the
FIG. 1. Snapshots of our simulation for \( N = 200 \) and \( \eta = 0.2 \) in the circular reflection boundary with diameter \( L = 1 \). Particles’ directions are represented by arrows. The adjacency networks are defined by (a) the Delaunay triangulation and (b) Euclidean distance \( d < 0.05 \). \( R_0 = 0 \): Particles move randomly because there are no interactions between them. \( R_0 = 0.04 \): Multiple clusters appear. \( R_0 = 0.1 \): All particles form a single cluster and move together along the boundary.

The parameter \( c \). The operator \( \text{Arg} \) converts the vector to the angle. The second term in eq. \( \xi \) represents noise, where \( \xi \) is a uniform random number between \([-\eta\pi, \eta\pi] \) (0 < \( \eta < 1 \)).

At every time step, we define neighbors for all particles by means of: (i) Delaunay triangulation and (ii) the Euclidean distance \( d \). Hereafter, these neighbors are referred to as Delaunay and Euclidean neighbors. The Delaunay triangulation is defined as the adjacency relationships in the Voronoi regions of each particle (A Voronoi region of a particle is a set of locations whose distance to the particle is less than to any other \( [19] \)). On the other hand, for the Euclidean neighbors, we define two particles as adjacent if the Euclidean distance between them is less than \( d = 0.05 \). Thus, at each time step, particles form an adjacency network as shown in Fig. 1. We focus on the duration time \( \tau \) in which the adjacency relationship between two particles continues (i.e., the lifetime of adjacent edges).

Considering the applicability of our simulation to real experiments such as those reported in Ref. \( [20] \), the following calculation was performed in the circular reflection boundary with fixed diameter \( L = 1 \). We set the parameters as follows: \( \Delta t = 1 \), \( v_0 = 0.005 \), \( R_f = 0.003 \) and \( c = 0.5 \). \( R_0 \), \( \eta \), and \( N \) are the controlling parameters. The total time step is set as \( T = 11000 \), and the cumulative distribution of \( \tau \), denoted by \( P(\tau) \), is computed using the data of duration time \( \tau \) that are obtained from all adjacent edges for \( t > 1000 \).

Result.— We first present results for \( R_0 = 0 \). In this condition, each particle moves randomly because no interactions occur among them (see the particles’ directions in Fig. 1). Figure 2 shows the cumulative distribution \( P(\tau) \) for the (a) Delaunay and (b) Euclidean neighbors in a single logarithmic scale. It is found that \( P(\tau) \) for small \( \eta \) exhibits exponential decay, indicating that each edge is rewired randomly (adjacency relationships change randomly).

Next, in Fig. 3(a), we present \( P(\tau) \) for \( R_0 \neq 0 \) obtained from a Delaunay neighbor in a double logarithmic scale. As shown in Fig. 4, each particle has interactions with others and forms clusters; multiple and single clusters appear for \( R_0 = 0.04 \) and \( R_0 = 0.1 \), respectively. Unlike in the case of \( R_0 = 0 \), \( P(\tau) \) becomes the power-law distribution, \( P(\tau) \sim \tau^{-\alpha+1} \). Remarkably, the power-law exponent in large \( R_0 \) becomes \( \alpha \simeq 3/2 \). We have confirmed that power-law behavior with the exponent \( \alpha \simeq 3/2 \) is almost independent of the number of particles \( N \) and \( \eta \). Regarding the Euclidean neighbor, \( P(\tau) \) exhibits almost the same behavior as in the case of the Delaunay neighbor, as shown in Fig. 3(b).

In order to quantify the change of distributions, we introduce a coefficient of variation defined as \( CV = \sigma(\tau)/\langle \tau \rangle \), where \( \tau \) and \( \sigma(\tau) \) are the mean and standard deviation of \( \tau \). Here, \( CV \) becomes unity when \( \tau \) follows an exponential distribution, and \( CV \) increases as the deviation increases (\( CV < 1 \) corresponds to a narrower distribution than the exponential). We present the \( R_0 \)-dependence of \( CV \) in the right-hand panel of Fig. 3. It is found that \( CV \) changes from unity to larger values as \( R_0 \) increases, and that \( CV \) increases rapidly at \( R_0 \approx 0.05 \). Therefore, \( P(\tau) \) changes from an exponential to a long tail distribution at \( R_0 \approx 0.05 \).

When \( R_0 \) is large enough, particles form a single cluster and move together as shown in the right-hand panel of Fig. 4. In order to analyze the boundary’s effect on \( P(\tau \rangle \), we performed a simulation with a free boundary condition for \( R_0 = 0.2 \). It is found from Fig. 4(a) that the power-law with \( \alpha \simeq 3/2 \) emerges with an increase in \( \eta \). The deviation from the power law for small \( \eta \) suggests that the rewiring of edges seldom occurs because of the fixed positions of particles. It should be emphasized...
that in the small $\tau$ region, the same power-law exponent $\alpha \simeq 3/2$ is obtained independently of $\eta$. Therefore, interactions with the boundary do not change the power-law exponent, and affect only the tail of $P(\tau)$.

We also check the effect of the repulsive force in large $R_0$ by controlling the repulsion distance $R_f$. As shown in Fig. 4(b), the power-law region with $\alpha \simeq 3/2$ is shortened with decreasing $R_f$. The same result is obtained for the Euclidian neighbor. This indicates that the cohesion of particles in small $R_f$ inhibits the emergence of longevity edges. Thus, the repulsive forces between particles are necessary for the emergence of the power-law behavior of $P(\tau)$.

**Discussion.**

We have shown that the exponential distribution shifts to the power-law distributions with the exponent $\alpha \simeq 3/2$ with an increase in $R_0$. According to the above results, the alignment and repulsive interactions between particles yield the power-law behavior of $P(\tau)$; these interactions prolong the adjacency relationships between pairs of particles. Our results suggest that the burstiness of face-to-face interactions is not a specific feature of human activities.

Regarding the power-law exponent $\alpha \simeq 3/2$ in large $R_0$, the following simplified models for the bursty behavior appear to be related. First, a model for face-to-face human interactions proposed by Starnini et al. assumes a situation similar to that in our system [21]: $N$ agents, who have their own attractiveness, move randomly in 2D space, and each of them stochastically stops moving depending on the attractivenesses around them. It has been reported that the power law with $\alpha \simeq 3/2$ can be obtained in conjunction with the first-passage time problem. Meanwhile, the priority queue model by Barabási is another model generating the same exponent $\alpha \simeq 3/2$ [5, 22]. This model assumes the arrival and execution of a task with constant probabilities. It is known that exponential and power-law distributions with the exponent $\alpha \simeq 3/2$ are analytically obtained depending on the random and priority execution of tasks [23]. While the priority queue model is not an interaction-driven model, we
expect that the queuing process is related to the rewiring of adjacent edges for a single particle; in fact, we have confirmed that $P(\tau)$ for a single particle exhibits the same behaviors as those shown in Fig. 3.

The Vicsek model is known to be the minimal model for collective motions of self-propelled particles. Previous studies have mainly focused on macroscopic properties such as an order-disorder transition of particles’ directions or a giant density fluctuation [17]. On the other hand, it appears that the situation depicted in our numerical simulation has not been sufficiently examined thus far. We expect that our results will be observed in some experimental systems, such as those for bacterial motions in circular pools [20] and self-propelled robots [24].

Furthermore, adjacency relationships of particles are a more detailed characterization of collective motions rather than macroscopic quantities such as the order parameter. Analyses of the adjacency relationships, such as those in Refs. [25] and [26], provide new insights into the research field of collective motion.

**Conclusion.**— A lifetime distribution $P(\tau)$ of adjacent edges in a Vicsek model with repulsive interaction changes from an exponential to a power-law distribution depending on the interaction radius $R_0$, and specific exponent $\alpha \simeq 3/2$ is obtained in large $R_0$. It reflects the transition from a random to an ordered motion of particles. Our simulation simplifies the face-to-face human activities and the burstiness is still observed in such a simplified system. We expect that our results will provide new insights to the research fields of bursty phenomena and collective motion.

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