Bound states in the continuum in abruptly terminated dielectric slab waveguides

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Abstract. We demonstrate that dielectric slab waveguides terminated with a dielectric ridge (i.e. with a slab waveguide segment having a greater thickness) support high-Q resonances and bound states in the continuum (BICs). The resonances and BICs take place at oblique incidence of TE-polarized guided modes in the total internal reflection geometry, when all the incident radiation is reflected at the end facet of the ridge and is not scattered out of the waveguide. We derive a simple coupled-wave model rigorously proving the BIC existence and predicting its locations. The predictions of the model are in an excellent agreement with the results of the rigorous numerical simulations. We also discuss the influence of the symmetry of the structure on the BIC existence. The obtained results may be useful for the design of novel integrated nanophotonic devices.

1. Introduction

In the last decade, investigation of the bound states in the continuum (BICs) in photonics has attracted considerable attention (see the recent review paper [1] and references therein). The BICs are non-leaky eigenmodes (the modes with an infinite lifetime and an infinite quality factor) supported by a resonant photonic structure having open scattering channels. The leakage to these channels is canceled either due to symmetry reasons or by means of parameter tuning. The phenomenon of BIC is not only of a great theoretical interest, but is also promising for practical applications in lasers, sensors and filters, since a small deviation from the BIC condition makes it possible to obtain resonators with extremely high quality factors.

In a recent paper, we studied BICs arising in a very simple integrated nanophotonic structure consisting of a single dielectric ridge on the surface of a single-mode dielectric slab waveguide [2]. BIC formation in this structure is associated with the coupling between the quasi-TE and quasi-TM modes of the ridge. In the present work, we investigate theoretically and numerically the formation of high-Q resonances and BICs in a similar structure, namely, in a dielectric ridge terminating a slab waveguide. We show that the resonances and BICs occur at oblique incidence of the fundamental TE-polarized mode of the waveguide in the total internal reflection geometry, when the incident wave is fully reflected at the end facet of the ridge terminating the waveguide. We also discuss the influence of
the symmetry of the structure on the BIC existence. The obtained results may be useful for pulse- and beam-shaping applications in integrated nanophotonics.

2. Results and discussion
Let us study the diffraction of guided modes of a dielectric slab waveguide on a ridge (a finite-length segment of a slab waveguide with a greater thickness) terminating the waveguide. We will consider two cases: the “symmetric” case when the ridge is centered with respect to the waveguide and the “asymmetric” case when the ridge is aligned to the lower interface of the waveguide. The cross sections of the corresponding structures are shown in the insets of Fig. 1; we assume that the structure is invariant to translation in the direction perpendicular to the plane of the figure and that the slab waveguide is semi-infinite to the left. In the considered example, the waveguide and the ridge have the same refractive index \( n_{wg} = 3.3212 \) (corresponds to GaP at the free-space wavelength of the incident radiation \( \lambda = 630 \text{ nm} \)), and the thicknesses \( h_{wg} = 70 \text{ nm} \) and \( h_r = 110 \text{ nm} \), respectively. The structure is surrounded by a uniform dielectric with the refractive index \( n = 1.45 \). At these parameters, the slab waveguide supports only the fundamental TE- and TM-polarized modes, which have the effective refractive indices \( n_{TE} = 2.5137 \) and \( n_{TM} = 1.6808 \), respectively. The waveguide with the thickness \( h_r \) is also single-mode and supports the modes with the effective refractive indices \( n_{TE,r} = 2.7722 \) and \( n_{TM,r} = 2.1322 \).

![Figure 1](image.png)

**Figure 1.** Phase of the reflection coefficient of the TE-polarized guided mode from a dielectric ridge terminating the waveguide vs. the ridge length \( l \) and the angle of incidence \( \theta \): the “symmetric” case of a ridge centered with respect to the waveguide (a) and the “asymmetric” case of a ridge aligned to the lower interface of the waveguide (b). The horizontal dashed lines in (b) show the “cut-off” angles of the TM-polarized guided modes \( \theta_{TM} \) and \( \theta_{TM,r} \), corresponding to the slab waveguides with the thicknesses \( h_{wg} \) and \( h_r \), respectively, whereas the black circles depict the BIC positions predicted by the derived coupled-wave model. The insets show the cross-sections of the considered structures.

We consider the case when the TE-polarized mode of the waveguide is obliquely incident at the ridge. The plane, in which the incidence angle is measured, is horizontal (and is perpendicular to the plane of the cross-sections shown in the insets of Fig. 1). In the general case, reflected TE- and TM-polarized guided modes are generated upon diffraction of the incident mode on the ridge, as well as a
continuum of non-guided waves scattered from the waveguide. However, at angles of incidence greater than the “critical” value $\theta_\text{cr} = \arcsin(n/n_{\text{TE}}) = 35.23^\circ$, the wavevector component of the incident mode perpendicular to the ridge interfaces is greater than the magnitude of the wavevector of the plane waves propagating in the medium surrounding the structure. Therefore, at these angles, the incident wave is totally reflected at the ridge facet terminating the waveguide, and no energy is scattered out of the waveguide [2–4]. Moreover, at the angles greater than the “cut-off” angle $\theta_{\text{TM}} = \arcsin(n_{\text{TM}}/n_{\text{TE}}) = 41.96^\circ$, no propagating reflected TM-polarized modes are generated. Finally, at $\theta > \theta_{\text{TM},r} = \arcsin(n_{\text{TM},r}/n_{\text{TE}}) = 58.02^\circ$, no propagating TM-polarized modes exist in the ridge (in the segment of a waveguide with the thickness $h$). For the details on this “scattering suppression” mechanism, the reader is referred to [2–4].

From now on, let us focus on the angular range $\theta_{\text{TM}} < \theta < \theta_{\text{TM},r}$. As discussed above, in this angular range, the reflected radiation contains only the TE-polarized guided mode, whereas in the ridge region, both the TE- and TM-polarized modes exist. Due to total internal reflection, the reflected wave has a constant (unit) amplitude and it is the phase of the reflection coefficient that has to be investigated. In Fig. 1, the dependences of the phase of the complex amplitude of the reflected TE-polarized guided mode on the ridge length $l$ ($0 \leq l \leq 600$ nm) and the angle of incidence $\theta$ ($\theta_\text{cr} = 35.23^\circ < \theta < 80^\circ$) are shown for the two considered cases: the symmetric case [Fig. 1(a)] and the asymmetric case [Fig. 1(b)]. The plots were calculated using an efficient in-house implementation of the rigorous coupled-wave analysis (RCWA) technique [5] adapted for the solution of integrated optics problems [6]. RCWA, also called the Fourier modal method, is an established technique for the numerical solution of Maxwell’s equations.

Using the symmetry properties of the structure, one can show that in the symmetric case [Fig. 1(a)], no polarization conversion occurs, i.e. the TM-polarized modes in the ridge region are not excited. Therefore, in this case, the phase slowly changes as the ridge length and/or the angle of incidence vary, and there are no high-Q resonances.

In the asymmetric case [Fig. 1(b)], the situation is radically different. In the angular range of interest $\theta_{\text{TM}} < \theta < \theta_{\text{TM},r}$, sharp resonances are present, with the quality factor strongly varying along the dispersion curves. At several points, the resonances vanish (i.e. its width turns to zero), which indicates the presence of BICs. The shape of these resonances is quite similar to the resonances discussed in our previous work [2], with the important difference that they are manifested only in the phase of the reflected radiation, whereas the amplitude of the complex reflection coefficient is strictly unity. We derived a simple coupled-wave model that takes into account the coupling between the TE- and TM-polarized modes in the ridge region, rigorously proves the existence of BICs in the investigated asymmetric structure and allows one to analytically predict their positions in the $(l, \theta)$ space using the numerically calculated “local” reflection and transmission coefficients of the guided modes at the ridge interfaces. For the sake of brevity, we do not present this model here (its detailed description will be published elsewhere), but show the BIC locations predicted for the considered example in Fig. 1(b) (black circles). It is evident from Fig. 1(b) that the predictions of the model are in excellent agreement with the results of rigorous numerical simulations.

3. Conclusion
In this work, we demonstrated that a dielectric ridge terminating an abruptly ended single-mode dielectric slab waveguide operating in the total internal reflection geometry supports high-Q resonances, which are manifested in the phase of the reflection coefficient of the incident TE-polarized guided mode. We have shown that the quality factor of these resonances strongly varies along the dispersion curves and even reaches infinity, which means that in the investigated structure, bound states in the continuum (BICs) exist. The existence of high-Q resonances and BICs in the studied integrated nanophotonic structure makes it promising for the pulse- and beam-shaping applications.
Acknowledgments
This work was funded by Russian Foundation for Basic Research (RFBR) under the projects 18-37-20038 and 16-29-11683 (investigation of the asymmetric structure), by Russian Science Foundation (RSF) under the project 19-19-00514 (investigation of the symmetric structure), and by the Russian Federation Ministry of Science and Higher Education within a state contract with the “Crystallography and Photonics” Research Center of the RAS under agreement 007-GZ/Ch3363/26 (implementation of the simulation software).

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