Mathematical model for the study and prediction of a porous body thermal state

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Abstract. The paper deals with a mathematical model of thermal state of a porous body which is in form of a rectangular parallelepiped. The internal heat generation and the heat exchange between the surfaces of the porous body and surrounding medium were taken into account in the model developing process. In order to solve the problem, there was developed a modification of the finite-difference method based on the method of lines. Also, using the least squares method, there was proposed a sinusoidal formula for describing the annual variation of ambient temperature.

1. Introduction

The basic equations describing the processes of heat and mass transfer under thermal effect on porous media are called the A.V. Lykov equations. [1]. They describe the mode of soft drying at low heat flows, when the drying temperature does not exceed 50–70°C, which is lower than the boiling point of water. The drying process is accompanied by an increase in internal overpressure of water vapor and the displacement of the phase transition region inside the body.

An analytical method for solving Lykov equations of heat and mass transfer with time-dependent boundary conditions was presented by Jen Y. Liu in his work [2].

Ilyasov U.R. and Igoshin D.E. [3] presented a model describing the process of heat and mass transfer at the drying of a moist porous material. Solutions are obtained which describe the distribution of temperature and concentration fields. It is found that, depending on the parameters of external influence, both drying and moistening of the environment can occur.

Fedyaev A., Fedyaev I. and Vidinb Yu. [4] developed a mathematical model of surface evaporation, which allows one to continuously calculate the entire drying process in both hard and soft conditions. Grinchik N., Gishkelyuk I. and Kundaye S. [5] considered a mathematical model of unsteady processes of interconnected heat and mass transfer in capillary-porous media, taking into account the influence of capillary and surface forces, the intensity of mass transfer between the phases and thermo-capillary flows.

It should be noted that when solving a problem of storing a porous body in a three-dimensional statement with the region of complex configuration, the calculation and simultaneous consideration of heat and moisture transfer with small coefficients at the level of molecular exchange, the consideration
of external factors such as solar radiation and ambient temperature, the evaporation process and other factors impose certain difficulties.

2. Problem statement
To analyze, forecast and make a management decision on the process of storing and drying the porous body, for example, raw cotton bale, a three-dimensional mathematical process of heat exchange with the environment is obtained by the equation

\[
\frac{\partial T}{\partial t} = a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + f(x, y, z, t),
\]

where \( T, f = f(x, y, z, t) \) are the temperature of porous bodies and the intensity of internal heat generation (source) \((K \cdot \text{c}^{-1})\); \( a \) is the coefficient of thermal diffusivity of mass \((\text{m}^2 \text{s})\). The intensity of internal heat generation, as considered in [6], is the form of exponential law

\[
f(x, y, z, t) = b(x, y, z)e^{-at}.
\]

The parallelepiped is located in the first octant of the Cartesian coordinate system, and its dimensions along the coordinates are \( l_x, l_y, l_z \) and the initial temperature value in the raw cotton bale is set as

\[
T(x, y, z, 0) = T_0(x, y, z)
\]

Boundary conditions on the faces of rectangular parallelepiped are:

\[
T(0, y, z, t) = \mu_{0y}(y, z, t), \quad T(l_x, y, z, t) = \mu_{l_y}(y, z, t),
\]

\[
T(x, 0, z, t) = \mu_{0x}(x, z, t), \quad T(x, l_y, z, t) = \mu_{l_x}(x, z, t),
\]

\[
\frac{\partial T(x, y, 0, t)}{\partial z} = 0, \quad \frac{\partial T(x, y, l_z, t)}{\partial z} = \eta[T(\omega(t)) - T(x, y, l_z, t)].
\]

Here \( T(\omega(t)) \) is the ambient temperature. The daily average value of the ambient temperature, depending on the date \( \tau_g \), is set in the form:

\[
T(\tau_g) = T_{\min} + (T_{\max} - T_{\min}) \sin 2\pi \frac{\tau_g - 121}{\Pi_\rho},
\]

where \( \Pi_\rho \) is the duration of the astronomical year of Earth; \( T_{\min} \) is the average daily ambient temperature; \( T_{\max} - T_{\min} \) are the daily amplitudes of its changes. The daily change in ambient temperature with an interval of 25°C (continental climate) is given by the following formula:

\[
T(t) = T(\tau_g) + 12.5T_0(t),
\]

where \( T_0(t) = \frac{A_0}{2} \sum_{i=1}^{6} (A_i \cos \frac{2\pi \nu t}{\Pi} + B_i \sin \frac{2\pi \nu t}{\Pi}) \) is the daily change in ambient temperature in the range \([-1; 1]\) with the best order of approximation, according to the results of field observations; the coefficients of a trigonometric polynomial are given in [7]. As the initial temperature of mass, the average temperature per \( \tau \)-th day is taken: \( T_0 = T(\tau_g) \). To set \( f(x, y, z, t) \) in the center of the opening with coordinates \((x, y, z)\) the following integral is used.
\[
\left[ -\left[ \frac{\partial T}{\partial t} \right] \right]_\Gamma = \eta \left( T_{\infty} - T_r \right) l_s, \quad (8)
\]

where \( \Gamma, l_s \) are the boundary of the opening in the plane \( y0z \) and its length; \( T_r \) is the mass temperature at the opening boundary, the value of which is taken as the previous time for a point \( (x, y, z) \). Finally, a mathematical model (1) - (7) is obtained to predict the state of cotton bale, when heat generation inside the bale and heat exchange with the environment and internal condition (8) are taken into account.

3. Solution method

To solve problem (1) - (8) numerically, a grid in \( x, y \) and \( z \) variables and in time \( t_n = n \tau \) is gradually introduced [7]; a gradual transition is made to discrete coordinates and functions according to the statements of the difference-differential method, also known as the straight line method [8]. In equation and conditions we proceed to discrete coordinates in time \( \tau \) and coordinate \( x \), introducing new sought for values of \( u_i^{n+1} (y, z) \) and functions \( f_i^{n+1} (y, z) \), \( \mu_{\nu_0}^{n+1} (y, z) \), \( \mu_{\nu_i}^{n+1} (z) \), \( \mu_{\nu_{i+1}}^{n+1} (z) \), \( \nu_{\nu_0}^{n+1} (y) \) and \( T_{\infty}^{n+1} \). For internal nodes \( i = 1..N_x \) of a segment of \( l_x \) length, the equation (1) is approximated in the form

\[
\frac{1}{\tau} u_i^{n+1} + \frac{a}{h^2_i} (u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}) + a \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)^{n+1} + f_i^{n+1} - \frac{1}{\tau} u_i^n = 0. \quad (9)
\]

Realizing boundary conditions (3) in (8) a matrix equation is derived:

\[
\frac{1}{\tau} U^{n+1} + \frac{a}{h^2_i} M U^{n+1} + a \left( \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right)^{n+1} + F^{n+1} = 0, \quad (10)
\]

where

\[
U^{n+1} = \left( u_1^{n+1}, u_2^{n+1}, \ldots, u_{N_x}^{n+1} \right)^T,
\]

\[
F^{n+1} = \left( f_1^{n+1} - \frac{1}{\tau} u_0^{n+1} + \frac{a}{h^2_i} \mu_0^{n+1}, f_2^{n+1} - \frac{1}{\tau} u_1^{n+1} - \frac{1}{\tau} u_0^{n+1}, \ldots, f_{N_x}^{n+1} - \frac{1}{\tau} u_{N_x-1}^{n+1} + \frac{a}{h^2_i} \mu_{N_x}^{n+1} \right)^T.
\]

\[
M_x = \begin{bmatrix}
-2 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
1 & -2 & 1 & 0 & \ldots & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & \ldots & 0 & 0 & 0 \\
0 & 0 & 1 & -2 & 1 & \ldots & 0 & 0 \\
0 & 0 & 0 & 1 & -2 & \ldots & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \ldots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \ldots & 1 & -2 \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 & -2 \\
\end{bmatrix}_{N_x}.
\]

Let us introduce fundamental \( B_x \) and diagonal \( \Lambda_x \) matrices with the elements

\[
b_{s,t,p} = (-1)^{s+p} \frac{2}{N_x + 1} \sin \frac{s \pi}{N_x + 1}, \Lambda_{s,s} = -2 \left( 1 + \cos \frac{s \pi}{N_x + 1} \right) \] at \( s, p = 1, 2, \ldots, N_x \). It is known that matrices \( M_x \) and \( \Lambda_x \) are mutually similar: \( M_x = B_x \Lambda_x B_x^{-1} \), where \( B_x^{-1} \) is the reverse matrix \( B_x \) equal in this case to matrix \( B_x \). Equation (10) is multiplied from the left-hand side by matrix \( B_x^{-1} \) and, with \( B_x^{-1} M_x U^{n+1} = \Lambda_x U^{(1)} \), compose a new matrix equation
\[
\frac{1}{\tau} U^{(1)} + \frac{a}{h^2} \Lambda_{s} U^{(1)} + a \left( \frac{\partial^2 U^{(1)}}{\partial y^2} + \frac{\partial^2 U^{(1)}}{\partial z^2} \right) + F^{(1)} = 0, \tag{11}
\]

where
\[
U^{(1)} = \left( u_1^{(1)}, u_2^{(1)}, \ldots, u_N^{(1)} \right)^T = B_x U^{n+1} = B_x U^{n+1} = \left( \sum_{p=1}^{N} b_{x,1,p} u_p^{n+1}, \sum_{p=1}^{N} b_{x,2,p} u_p^{n+1}, \ldots, \sum_{p=1}^{N} b_{x,N,N,p} u_p^{n+1} \right)^T,
\]
\[
F^{(1)} = \left( f_1^{(1)}, f_2^{(1)}, \ldots, f_N^{(1)} \right)^T = B_x F^{n+1} = B_x F^{n+1} = \left( \sum_{p=1}^{N} b_{x,1,p} F_p^{n+1}, \sum_{p=1}^{N} b_{x,2,p} F_p^{n+1}, \ldots, \sum_{p=1}^{N} b_{x,N,N,p} F_p^{n+1} \right)^T.
\]

Here \( F^{n+1} \) is the \( p \)-th element of the column bit vector \( F^{n+1} \).

From equation (11) let us proceed to autonomous equations with \( N \) dimensionality
\[
\frac{1}{\tau} U^{(1)} + \frac{a}{h^2} \Lambda_{s} U^{(1)} + a \left( \frac{\partial^2 U^{(1)}}{\partial y^2} + \frac{\partial^2 U^{(1)}}{\partial z^2} \right) + F^{(1)} = 0.
\]

On the second stage of discretization in equation (12) from coordinate \( y \) proceed to finite differences, introducing the functions \( u_{i,j}^{(2)}(z) \) and \( f_{i,j}^{(2)}(z) \) instead of the functions \( u_{i,j}^{(1)}(y,z) \) and \( f_{i,j}^{(1)}(y,z) \).

Now we need to determine the values of the function \( u_{i,j}^{(2)}(z) \) at the boundaries \( j = 0 \) and \( j = N_y + 1 \). Find them according to the introduced change of variables:
\[
\mu_{i,j}^{(2)}(z) = \sum_{p=1}^{N} b_{i,j,p} \mu_{i,p}^{n+1} \left( ph_i, z \right), \quad \mu_{i,j}^{(2)}(z) = \sum_{p=1}^{N} b_{i,j,p} \mu_{i,j}^{n+1} \left( ph_i, z \right). \tag{13}
\]

Further, we need to determine the boundary conditions (5) on the third coordinate, and we write them in the form:
\[
\frac{\partial u_{i,j}^{(2)}}{\partial z} (0) = \sum_{p=1}^{N} b_{i,j,p} v_{i,j}^{n+1} \left( jh_i \right), \quad \frac{\partial u_{i,j}^{(2)}}{\partial z} (l_i) = \eta \left[ T_{i,j}^{(2)} - u_{i,j}^{(2)}(l_i) \right],
\]

where \( T_{i,j}^{(2)} = \sum_{p=1}^{N} b_{i,j,p} T_{i,j}^{n+1} \). The implementation of conditions (13) in (12) and the equation (12), at discretization along the \( y \) coordinate, leads to the matrix equation
\[
\frac{1}{\tau} U_i^{(2)} + \frac{a}{h^2} \Lambda_{s} U_i^{(2)} + a \left( \frac{\partial^2 U_i^{(2)}}{\partial z^2} \right) + F_i^{(2)} = 0, \tag{14}
\]

For equation (14), we introduce the fundamental \( B_x \) and diagonal \( \Lambda \) matrices with the corresponding elements, as in the case of equation (10). Multiplying equation (14) on the left-hand side by the matrix \( B_x^{-1} \), we get
\[
\frac{1}{\tau} \tilde{U}_i^{(2)} + \frac{a}{h^2} \Lambda \tilde{U}_i^{(2)} + a \left( \frac{\partial^2 \tilde{U}_i^{(2)}}{\partial z^2} \right) + \tilde{F}_i^{(2)} = 0, \tag{15}
\]
Here we use the notation \( F_i^{(2)} \) for the \( Q \)-th element of the column bit vector \( F_i^{(2)} \). From (15) proceed to the separate \( N_x, N_y \) autonomous equations:

\[
\frac{1}{\tau} \ddot{u}_{i,j}^{(2)} + \frac{a}{h_x^2} \lambda_x u_{i,j}^{(2)} + \frac{a}{h_y^2} \lambda_y u_{i,j}^{(2)} + \frac{d^2 u_{i,j}^{(2)}}{dz^2} + f_{i,j}^{(2)} = 0.
\] (16)

The boundary conditions for these equations are

\[
\frac{d\ddot{u}_{i,j}^{(2)}}{dz} = \eta [T_{i,j}^{(2)} - \ddot{u}_{i,j}^{(2)} (l_z)]
\]

where \( T_{i,j}^{(2)} = \sum_{p=1}^{N_z} b_{j,i,p} T_{i,j}^{(3)} \).

Now, along the \( z \) coordinate, introduce discrete functions \( u_{i,j,k}^{(3)} \) and \( f_{i,j,k}^{(3)} \), and in (16) proceed to the difference equations for the internal nodes:

\[
\frac{1}{\tau} u_{i,j,k}^{(3)} + \frac{a}{h_x^2} \lambda_x u_{i,j,k}^{(3)} + \frac{a}{h_y^2} \lambda_y u_{i,j,k}^{(3)} + \frac{u_{i,j,k+1}^{(3)} - 2u_{i,j,k}^{(3)} + u_{i,j,k-1}^{(3)}}{h_z^2} + f_{i,j,k}^{(3)} = 0.
\]

Let us compose an equation for applying the sweep method: \( a_k u_{i,j,k+1}^{(3)} - b_k u_{i,j,k}^{(3)} + c_k u_{i,j,k-1}^{(3)} = -f_{i,j,k}^{(3)} \), where

\[
a_k = \frac{a}{h_z^2}, \quad b_k = a_k + c_k - \left( \frac{1}{\tau} + \frac{a}{h_x^2} \lambda_x + \frac{a}{h_y^2} \lambda_y \right),
\]

Let us compose an equation for applying the sweep method: \( a_k u_{i,j,k+1}^{(3)} - b_k u_{i,j,k}^{(3)} + c_k u_{i,j,k-1}^{(3)} = -f_{i,j,k}^{(3)} \), where

\[
a_k = \frac{a}{h_z^2}, \quad b_k = a_k + c_k - \left( \frac{1}{\tau} + \frac{a}{h_x^2} \lambda_x + \frac{a}{h_y^2} \lambda_y \right).
\]

According to these notations, the sweep is carried by the formula \( u_{i,j,k}^{(3)} = \alpha_k u_{i,j,k+1}^{(3)} + \beta_k \), where

\[
\alpha_k = \frac{a_k}{b_k - c_k \alpha_{k+1}}, \quad \beta_k = \frac{c_k \beta_{k+1} + f_{i,j,k}^{(3)}}{b_k - c_k \alpha_{k+1}}.
\]

The boundary condition at \( z = 0 \) is realized with a second order of accuracy:

\[
3u_{i,j,0}^{(3)} - 4u_{i,j,1}^{(3)} + u_{i,j,2}^{(3)} = -2h_z \eta T_{i,j}^{(2)} \)

As \( a_k u_{i,j,2}^{(3)} - b_k u_{i,j,1}^{(3)} + c_k u_{i,j,0}^{(3)} = -f_{i,j,2}^{(3)} \), eliminating \( u_{i,j,2}^{(3)} \) from two last equations, we get

\[
u_{i,j,0}^{(3)} = \alpha_k u_{i,j,1}^{(3)} + \beta_0, \quad \alpha_k = \frac{4a_k - b_k}{3a_k - c_k}, \quad \beta_0 = \frac{-2h_z \eta T_{i,j}^{(2)} + f_{i,j,1}^{(3)}}{3a_k - c_k}.
\]

The calculation of the sweep coefficients values begins with these formulas and extends to the values of \( \alpha_{N_z} \) and \( \beta_{N_z} \). Subsequently, a reverse sweep is conducted, for the first step of which \( u_{i,j,N_z+1}^{(3)} \) is required. Find it from the approximation of the boundary condition at \( z = l_z \):

\[
3u_{i,j,N_z+1}^{(3)} - 4u_{i,j,N_z}^{(3)} + u_{i,j,N_z-1}^{(3)} = 2h_z \eta T_{i,j}^{(2)} - u_{i,j,N_z}^{(3)}.
\]

With \( u_{i,j,N_z-1}^{(3)} = \alpha_{N_z-1} u_{i,j,N_z}^{(3)} + \beta_{N_z-1} \) the last equation has the form

\[
3u_{i,j,N_z+1}^{(3)} + (4 + \alpha_{N_z+1}) u_{i,j,N_z}^{(3)} = 2h_z \eta T_{i,j}^{(2)} - \beta_{N_z+1}.
\]

Here consider \( u_{i,j,N_z}^{(3)} = \alpha_{N_z} u_{i,j,N_z+1}^{(3)} + \beta_{N_z} \). So

\[
[3 + (4 + \alpha_{N_z+1} + 2h_z \eta) \alpha_{N_z}] u_{i,j,N_z+1}^{(3)} = 2h_z \eta T_{i,j}^{(2)} - \beta_{N_z} - (4 + \alpha_{N_z+1} + 2h_z \eta) \beta_{N_z}.
\]
Hence, we get \( u_{i,j,N_z+1}^{(3)} = \frac{2h_\eta \eta_{\infty}^{(3)} - \beta_{N_z+1} - (-4 + \alpha_{N_z+1} + 2h_\eta) \beta_{N_z}}{3 + (-4 + \alpha_{N_z+1} + 2h_\eta) \alpha_{N_z}} \).

Calculating the values of \( u_{i,j,N_z+1}^{(3)} \) the reverse sweep is started according to the formula
\[
u_{i,j,k}^{(3)} = \alpha_k \nu_{i,j,k+1}^{(3)} + \beta_k \quad \text{for values of } k = N_z, N_z - 1, ..., 2, 1, 0.
\]

The algorithm described above applies to each new time step \( n+1 = 1, 2, ..., \). But for its first step, the values \( u_{i,j,k}^{(3)} \) for the start time \( t = 0 \) are required.

At discretization in time \( t \) and in coordinate \( x \) we introduce \( u_0^0(y,z) \). After the diagonalization of equation (14), we proceed to condition \( u_{i,j}^{(0)}(y,z) = \sum_{p=1}^{N_x} b_{i,j,p} u_p^0(y,z) \). At discretization in \( y \), introduce the substitution \( u_{i,j}^{(0)}(z) = u_{i,j}^{(0)}(y,z) \). At discretization of equation (19) we get
\[
u_{i,j}^{(2)}(z) = \sum_{q=1}^{N_z} b_{i,j,q} u_{i,j}^{(2)}(z).
\]

Substitute \( u_{i,j,k}^{(3)} = \nu_{i,j}^{(2)}(z) \). As a result for the grid function \( u_{i,j,k}^{(3)} \) initial condition is
\[
u_{i,j,k}^{(3)} = \sum_{p=1}^{N_x} \sum_{q=1}^{N_z} b_{i,j,p} b_{i,j,q} u_{p,q,k}^0.
\]

The last stage of calculation is the transfer from \( u_{i,j,k}^{(3)} \) to initial sought for value of \( u_{i,j,k}^{n+1} \).

With \( B_k^{-1} = B_k \) and \( B_j^{-1} = B_j \) this transition is carried out according to the formula
\[
u_{i,j,k}^{n+1} = \sum_{p=1}^{N_x} \sum_{q=1}^{N_z} b_{i,j,p} b_{i,j,q} u_{p,q,k}^{(3)}.
\]

Thus, the accuracy of approximation of equations and conditions is of the first order in time and of the second order in coordinates.

4. Results and discussion

To carry out a comprehensive study of the thermal state of porous bodies and heat exchange with the environment, an algorithm and software have been developed with the criterion given in [9].

The results of calculations of the temperature of a bale of porous body with a single opening are obtained at an initial temperature of the bale 40°C and at an ambient temperature of 30°C. After 50 days, the internal part of the bale is heated up to 57°C due to internal heat generation. Parts that come in contact with the outside atmosphere - the borderline and parts around the opening are cooled down to 30°C. As the moisture content of the porous body increases, the values of the coefficient of thermal conductivity and the intensity of heat generation increase. This leads to more intense heating of the internal part of the bale (up to 65-70°C and higher). The internal heat generation factor becomes important and it can lead to spontaneous combustion of a bale or to a loss of natural qualities of the porous body after long-term storage.

When the size of the bale decreases, the heat exchange with the environment occurs intensively, and the internal heat generation factor turns out to be in the foreground. After long-term storage (70 days), internal points of the bale, which has dimensions of 15, 9 and 6 m, are heated to 53°C. Calculations have shown that, when the value of the coefficient of thermal conductivity decreases, the role of the internal heat generation factor increases. For example, after 60 days of storage at \( a = 0.01 \, m^2 \, s^{-1} \), the maximum temperature within the bale reaches 58°C.
When storing a bale of porous body without an opening, the inner part is heated more intensively and the temperature inside the bale reaches 58°C. The openings provide the heat exchange of the inner part of the bale of porous body with the environment and contribute to the cooling of the mass. The internal heat generation factor becomes noticeable when storing the bale for a long time. The temperature of the internal points reaches 63°C and higher when the time of storage of a bale of porous body is more than 90 days.

5. Conclusion
The results of calculations show the following features of the thermal state of a bale of a porous body:

- Firstly, with the increase in the size of the bale, the role of heat exchange with the environment becomes insignificant and there is a widespread increase in the temperature of a porous body due to internal heat generation;
- Secondly, at high values of moisture content of the mass, the temperature of a porous body grows more intensively, than at low moisture content;
- Thirdly, the presence of an opening through which the mass of the porous body interacts with the heat of the environment, leads to a decrease in the greatest temperature value in the bale of a porous body.

The results show that, at small sizes of stored material, the role of heat exchange with the environment is appreciable. With large volumes of agricultural products, the tangible role of internal heat generation, which leads to a general increase in the temperature of the mass, becomes significant. The presence of openings contributes to a certain decrease in the temperature of a porous body.

References
[1] Lykov A V 1968 Drying theory (Moskow: Energy)
[2] Jen Y L 1991 Journal of Heat Transfer 113 757-62
[3] Ilyasov U and Igoshin D 2008 J. of Thermophysics and Aeromechanics 15(4) 689-97
[4] Fedyaev A, Fedyaev I and Vidinb Yu 2008 Journal of Siberian Federal University 1 68-75
[5] Grinchik N, Gishkelyuk I and Kundaye S 2001 Collection of Scientific Articles Modern Science 2(7) 145-53
[6] Ravshanov N and Ataullaev A 1988 Dep. in VINITI 1833 53-9
[7] Khuzhayev Zh 2014 Proc. Republican Scientific and Technical Conference 1 157-60
[8] Faddeeva V 1949 Proceedings of the USSR Academy of Sciences 28 73-103
[9] Romansky R and Kirilov K 2018 AIP Conference Proceedings 2048(1) 060006