Computation of the quarkonium and meson-meson composition of the $\Upsilon(nS)$ states and of the new $\Upsilon(10753)$ Belle resonance from lattice QCD static potentials

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We compute the composition of the bottomonium $\Upsilon(nS)$ states (including $\Upsilon(10860)$) and the new $\Upsilon(10753)$ resonance reported by Belle in terms of quarkonium and meson-meson components. We use the Born Oppenheimer approximation, static potentials from a lattice QCD study of string breaking and the unitary emergent wave method to compute the poles of the $S$ matrix. We focus on $I = 0$ bottomonium $S$ wave bound states and resonances, where the Schrödinger equation is a set of two coupled differential equations. One of the two channels corresponds to a confined heavy quark-antiquark pair $b\bar{b}$, the other to a pair of heavy-light mesons $B^{(*)}B^{(*)}$.

We confirm the new Belle resonance $\Upsilon(10753)$ as a dynamical meson-meson resonance with 94% meson-meson content. Moreover, we identify $\Upsilon(4S)$ and $\Upsilon(10860)$ as predominantly quarkonium states, however with sizable meson-meson contents of 30% and 41%, respectively. With these results we contribute to the clarification of ongoing controversies in the vector bottomonium spectrum.

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I. INTRODUCTION

Starting from lattice QCD static potentials, our long term goal is a complete computation of the masses and decay widths of bottomonium bound states and resonances as poles of the $S$ matrix. We expect our technique to be eventually updated to study the full set of exotic $X, Y$ and $Z$ mesons. In this work, however, we focus on the somewhat simpler, but nevertheless controversial $I = 0$ bottomonium $S$ wave resonances.

In Table I we show the available experimental results according to the Review of Particle Physics [1]. Since we work in the heavy quark limit, the heavy quark spins $S^q_{PC}$ do not appear in the Hamiltonian and the relevant quantum numbers $J^{PC}$ are the remaining part of the total angular momentum and the corresponding parity and charge conjugation (also listed in Table I). Notice that we also list several states observed at Belle with large significance [2,3]. These states are not yet confirmed by other experiments, because presently Belle and Belle II are the only experiments designed to study bottomonia.

In particular a new resonance, $\Upsilon(10753)$, possibly another $\Upsilon(nS)$ state or a $Y$ state, since it is a vector but suggested to be of exotic nature, has been recently observed at Belle with a mass around 10.75 GeV [4]. The previously observed resonances $\Upsilon(4S)$ and $\Upsilon(10860)$ approximately match quark model predictions of bottomonia and, thus, this new resonance comes in excess and needs to be understood.

Notice also that the discovery of this resonance by Belle with the process $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$ resulted from the experimental effort to clarify the controversy on the nature of the other excited $\Upsilon$ resonances [3]. The $\Upsilon(4S)$, $\Upsilon(10860)$, and $\Upsilon(11020)$, although having masses approximately compatible with the quark model, have transitions to lower bottomonia with the emission of light hadrons with much higher rates compared to expectations for ordinary bottomonia. A possible interpretation is that these excited $\Upsilon$ states have large

| name            | $G(J^{PC})$ | $m$ [MeV] | $\Gamma$ [MeV] | $J^{PC}$ |
|-----------------|-------------|-----------|----------------|---------|
| $\eta_b(1S)$    | $0^+(0^{++})$| 9399.0 ± 2.3 | 10 ± 5 | 0^{++} |
| $\Upsilon(1S)$  | $0^-(1^{--})$| 9460.30 ± 0.26 (54.02 ± 1.25)×10^{-3} | 0^{++} |
| $\chi_{bc}(1P)$ | $0^+(0^{++})$| 9859.44 ± 0.73 | -      | 1^{--} |
| $\chi_{b}(1P)$  | $0^+(1^{++})$| 9892.78 ± 0.57 | -      | 1^{--} |
| $h_b(1P)$       | $1^{++}$    | 9893.3 ± 0.8 | -      | 1^{--} |
| $\chi_{bc}(1P)$ | $0^+(2^{++})$| 9912.21 ± 0.57 | -      | 1^{--} |
| $\eta_{b}(2S)_{belle}$ | $0^+(0^{++})$| 9999.0 ± 6.3 | -      | 0^{++} |
| $\Upsilon(2S)$  | $0^-(1^{--})$| 10023.26 ± 0.31 (31.98 ± 2.63)×10^{-3} | 0^{++} |
| $\Upsilon(1D)$  | $0^-(2^{--})$| 10163.7 ± 1.4 | -      | 2^{++} |
| $\chi_{bc}(2P)$ | $0^+(0^{++})$| 10232.5 ± 0.9 | -      | 1^{--} |
| $\chi_{b}(2P)$  | $0^+(1^{++})$| 10255.46 ± 0.77 | -      | 1^{--} |
| $h_{b}(2P)_{belle}$ | $1^{++}$    | 10259.8 ± 1.6 | -      | 1^{--} |
| $\chi_{bc}(2P)$ | $0^+(2^{++})$| 10268.65 ± 0.72 | -      | 1^{--} |
| $\Upsilon(3S)$  | $0^-(1^{--})$| 10355.2 ± 0.5 (20.32 ± 1.85)×10^{-3} | 0^{++} |
| $\chi_{b}(3P)$  | $0^+(1^{++})$| 10512.1 ± 2.3 | -      | 1^{--} |
| $\Upsilon(4S)_{belle}$ | $0^-(1^{--})$| 10579.4 ± 1.2 | 20.5 ± 2.5 | 0^{++} |
| $\Upsilon(10753)_{belle}$ | $0^-(1^{--})$| 10752.7 ± 7.0 | 35.5 ± 21.6 | 0^{++} |
| $\Upsilon(10860)$ | $0^- (1^{--})$| 10889.9 ± 3.2 | 51 ± 7 | 0^{++} |
| $\Upsilon(11020)$ | $0^-(1^{--})$| 10992.9 ± 1.0 | 49 ± 15 | 0^{++} |

Table I. Masses $m$ and decay widths $\Gamma$ of $I = 0$ bottomonium according to the Review of Particle Physics [1]. We also include several states observed at Belle [2,3], but not yet confirmed by other experiments. We add an extra column with the quantum number $J^{PC}$ conserved in the infinite quark mass limit (in the last three lines $J^{PC} = 2^{++}$ is also a possibility). We mark with horizontal lines the opening of the $BB$ and $B^* B^*$ thresholds.

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admixtures of $B^{(*)}\bar{B}^{(*)}$ meson pairs [4,7]. Another scenario is that they do not correspond to the $S$ wave states $\Upsilon(5S)$ and $\Upsilon(6S)$, but instead to the $D$ wave states $\Upsilon(3D)$ and $\Upsilon(4D)$ [7–9]. The Belle experiment was, thus, designed to produce and study $\Upsilon$ states with a large $B^{(*)}\bar{B}^{(*)}$ admixture.

After the observation of the new resonance at Belle, more exotic interpretations have been proposed for the excited $\Upsilon$ states. Most interpretations consider the new $\Upsilon(10753)$ resonance as a non-conventional state, e.g. a tetraquark [10–11] or a hybrid meson [12–14].

In this work, we aim to contribute to the clarification of these controversies on the bottomonium resonances $\Upsilon(4S)$, $\Upsilon(10753)$ and $\Upsilon(10860)$. The low-lying bottomonium spectrum up to the $B^{(*)}\bar{B}^{(*)}$ threshold was studied within full lattice QCD extensively [15,22]. However, it is extremely difficult to study higher resonances with several decay channels in a similar setup. Thus, we follow a different strategy to study systems with both heavy and light quarks. In a first step, lattice QCD is used to compute the potential energy for the heavy quarks by simulating the dynamics of the light quarks and gluons. In this work we do not carry out such simulations, but utilize lattice QCD static potentials from Ref. [23], which were computed in the context of string breaking. Then, in a second step, the dynamics of the heavy quarks is determined by solving the Schrödinger equation. Within this so-called Born-Oppenheimer approach we determine the percentage of a confined pair of heavy quarks $b\bar{b}$ as well as the percentage of a pair of heavy-light mesons $B^{(*)}\bar{B}^{(*)}$.

This Born-Oppenheimer approach was applied before to study exotic mesons containing two bottom quarks. For example, the spectrum of $b\bar{b}$ hybrid mesons was studied extensively (see e.g. Refs. [24,27]), however, mostly using static potentials computed within pure SU(3) lattice gauge theory, which are confining and do not allow decays to pairs of lighter mesons. The first application of this approach to study tetraquarks can be found in [28,29]. For instance the existence of a stable $bbud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ was confirmed [30,31], whereas other flavor combinations do not form four-quark bound states [32]. In this context the approach was also updated by including techniques from scattering theory and a new $bbud$ tetraquark resonance with quantum numbers $I(J^P) = 0(1^+)$ was found [33].

Very recently we started to study bottomonium resonances, again using the Born-Oppenheimer approach. This case is more involved, because there are two coupled channels, a confined quarkonium channel with flavor $bb$ and a meson-meson decay channel with flavor $b\bar{b}(u\bar{u} + d\bar{d})$. In Ref. [34] we developed algebraic methods to derive the potentials for the corresponding Schrödinger equation, including a $bb$ potential, a $B^{(*)}\bar{B}^{(*)}$ potential and a mixing potential, from lattice QCD static potentials computed e.g. in studies of string breaking [23,35]. Applying the emergent wave method we determined $I = 0$ bottomonium $S$ wave resonances. Independently of the experimental observation of the resonance $\Upsilon(10753)$ at Belle [3], which we were not aware of at that time, we predicted a similar resonance with mass $10774^{+4}_{-3}$ MeV [34].

This paper is structured as follows. In section II we review the theoretical basics of our approach from Ref. [34]. We discuss, how to utilize lattice QCD static potentials, and how to solve the coupled Schrödinger equation to obtain a quarkonium and a meson-meson wave function. We also review our results for the poles of the $S$ matrix, i.e. for $I = 0$ bottomonium $S$ wave resonances. In section III we propose a technique to determine the percentage of the quark-antiquark and the meson-meson component of a bottomonium state, either a bound state (if we neglect the weak interactions) or a resonance. Then we apply this technique to $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, $\Upsilon(10753)$ and $\Upsilon(10860)$. Finally, in section IV we conclude.

II. SUMMARY OF OUR APPROACH

In this section we briefly summarize our approach from Ref. [34] to study quarkonium resonances with isospin $I = 0$ in the Born-Oppenheimer approximation using lattice QCD static potentials. We also recapitulate the main results from Ref. [34].

A. Theoretical basics

We consider systems composed of a heavy quark-antiquark pair $Q\bar{Q}$ and either no light quarks (quarkonium) or another light quark-antiquark pair $q\bar{q}$ with isospin $I = 0$ (for large $Q\bar{Q}$ separation two heavy-light mesons $M = \bar{Q}q$ and $M = \bar{q}Q$). We treat the heavy quark spins as conserved quantities such that the energy levels of $Q\bar{Q}(q\bar{q})$ systems as well as their decays and resonance parameters do not depend on these spins. Moreover, we assume that two of the four components of the Dirac spinors of the heavy quarks $Q$ and $\bar{Q}$ vanish. These approximations become exact for static quarks and are expected to yield reasonably accurate results for $b$ quarks, possibly even for $c$ quarks.

In Ref. [34] we have derived in detail a coupled channel Schrödinger equation for a 4-component wave function $\psi(r) = (\psi_{QQ}(r), \psi_{MM}(r))$ (Eq. (10) in Ref. [34]). The upper component of this wave function represents the $Q\bar{Q}$ channel, the lower three components represent the $MM$ channel. For the $MM$ channel we consider only the lightest heavy-light mesons with $J^P = 0^-$ and $J^P = 1^-$, i.e. $B$ and $B^*$ mesons for $Q = b$ (as usual, $J$, $P$ and $C$ denote total angular momentum, parity and charge conjugation). Within the approximations stated above these two mesons have the same mass. One can show that the spin of the two light quarks is 1, which is represented by the three components of $\psi_{MM}(r)$. Note that we ignore decays of $Q\bar{Q}$ to lighter quarkonium and a light $I = 0$ meson, e.g. a $\sigma$ or an $\eta$ meson, because they are suppressed by the OZI rule.

$J^{PC}$ denotes total angular momentum excluding the heavy quark spins and the corresponding parity and charge conjugation. It is a conserved quantity. As in Ref. [34] we focus throughout this work on $J^{PC} = 0^{++}$. Thus $J^{PC} = S^{PC}$, where $S_Q$ denotes the heavy quark spin, with only two possibilities, $S^{PC} = 0^+, 1^-$.
The coupled channel Schrödinger equation for the partial wave with $\tilde{J} = 0$ is a 2-channel equation,

$$
\left( -\frac{1}{2} \begin{pmatrix} 1/\mu_Q & 0 \\ 0 & 1/\mu_M \end{pmatrix} \right) \partial_r^2 + \frac{1}{2r^2} \begin{pmatrix} 0 & 0 \\ 2/\mu_M & 0 \end{pmatrix} + V_0(r) + 2m_M - E \begin{pmatrix} u_{0,0}(r) \\ \chi_{1-0,0}(r) \end{pmatrix} = - \begin{pmatrix} V_{\text{mix}}(r) \\ V_{\tilde{M},\|}(r) \end{pmatrix} kr j_1(kr),
$$

where $\mu_Q = m_Q/2$ and $\mu_M = m_M/2$ are the corresponding reduced masses.

The upper equation represents the $\bar{Q}Q$ channel with orbital angular momentum $L_{\bar{Q}Q} = \bar{J} = 0$. $u_{0,0}(r)$ is the radial part of the $\bar{J} = 0$ partial wave of the wave function

$$
\psi_{\bar{Q}Q}(r) = \sqrt{4\pi i} \frac{u_{0,0}(r)}{kr} Y_0(\Omega) + \ldots
$$

with the dots . . . denoting partial waves with $\bar{J} > 0$. Similarly, the lower equation represents the $\tilde{M}M$ channel with orbital angular momentum $L_{\tilde{M}M} = 1$. $j_1(kr)$ and $\chi_{1-0,0}(r)$ are the radial parts of the $\bar{J} = 0$ partial waves of the incident plane wave and the emergent spherical wave of the 3-component wave function

$$
\tilde{\psi}_{\tilde{M}M}(r) = \sqrt{4\pi i} \left( j_1(kr) + \frac{\chi_{1-0,0}(r)}{kr} \right) Z_{1-0,0}(\Omega) + \ldots
$$

with $Z_{1-0,0}(\Omega) = e_{\tau}/\sqrt{4\pi}$ and the dots . . . denoting partial waves with $\bar{J} > 0$. Moreover, $m_Q$ and $m_M$ are the heavy quark and heavy-light meson masses, respectively, and $\mu_Q = m_Q/2$ and $\mu_M = m_M/2$ are the corresponding reduced masses.

The energy $E$ and the momentum $k$ are related according to $k = \sqrt{2\mu_M E}$. The potentials $V_{\bar{Q}Q}(r)$, $V_{\tilde{M}M,\|}(r)$ and $V_{\text{mix}}(r)$ represent the energy of a pair of heavy quarks, the energy of a pair of heavy-light mesons and the mixing between the two channels, respectively. In Ref. [24] we related these potentials algebraically to lattice QCD correlators computed and provided in detail in Ref. [23] in the context of string breaking for lattice spacing $a \approx 0.083$ fm and pion mass $m_\pi \approx 650$ MeV. The data points for $V_{\bar{Q}Q}(r)$, $V_{\tilde{M}M,\|}(r)$ and $V_{\text{mix}}(r)$ are shown in Fig. 1 together with appropriate parameterizations,

$$
V_{\bar{Q}Q}(r) = E_0 - \frac{\alpha}{r} + \sigma r + \sum_{j=1}^2 c_{\bar{Q}Q,j} r^2 \exp \left( - \frac{r^2}{2l_{\bar{Q}Q,j}^2} \right),
$$

$$
V_{\tilde{M}M,\|}(r) = 0,
$$

$$
V_{\text{mix}}(r) = \sum_{j=1}^2 c_{\text{mix},j} r^2 \exp \left( - \frac{r^2}{2l_{\text{mix},j}^2} \right).
$$

The parameters appearing in Eq. (4) to Eq. (6) are collected in Table I.

The appropriate boundary conditions for the radial wave functions $u_{0,0}(r)$ and $\chi_{1-0,0}(r)$ are

$$
u_{0,0}(r) \propto r \quad \text{for } r \to 0 \quad (7)
$$

$$
u_{0,0}(r) = 0 \quad \text{for } r \to \infty \quad (8)
$$

$$
\chi_{1-0,0}(r) \propto r^2 \quad \text{for } r \to 0 \quad (9)
$$

$$
\chi_{1-0,0}(r) = i t_{1-0,0} kr h_1^{(1)}(kr) \quad \text{for } r \to \infty, \quad (10)
$$

where $h_1^{(1)}$ is a spherical Hankel function of the first kind and $t_{1-0,0}$ is the scattering amplitude and an eigenvalue of the $S$ matrix. We computed $t_{1-0,0}$ as a function of the complex energy $E$. Poles of $t_{1-0,0}$ on the real axis below the $\tilde{M}M$ threshold indicate bound states. Poles of $t_{1-0,0}$ at energies with non-vanishing negative imaginary parts represent resonances with masses $m = \text{Re}(E)$ and decay widths $\Gamma = 2\text{Im}(E)$. $t_{1-0,0}$ is also related to the corresponding scattering phase via $e^{2i\theta_{1-0,0}} = 1 + 2it_{1-0,0}$.

| potential | parameter | value |
|-----------|-----------|-------|
| $V_{\bar{Q}Q}(r)$ | $E_0$ | $-1.599(269)$ GeV |
| | $\alpha$ | $+0.320(94)$ |
| | $\sigma$ | $+0.253(035)$ GeV$^2$ |
| | $c_{\bar{Q}Q,1}$ | $+0.826(882)$ GeV$^2$ |
| | $\lambda_{\bar{Q}Q,1}$ | $+0.964(47)$ GeV$^{-1}$ |
| | $c_{\bar{Q}Q,2}$ | $+0.174(1.004)$ GeV$^2$ |
| | $\lambda_{\bar{Q}Q,2}$ | $+2.663(425)$ GeV$^{-1}$ |
| $V_{\tilde{M}M,\|}(r)$ | - | - |
| $V_{\text{mix}}(r)$ | $c_{\text{mix},1}$ | $-0.988(32)$ GeV$^2$ |
| | $\lambda_{\text{mix},1}$ | $+0.982(18)$ GeV$^{-1}$ |
| | $c_{\text{mix},2}$ | $-0.142(7)$ GeV$^2$ |
| | $\lambda_{\text{mix},2}$ | $+2.666(46)$ GeV$^{-1}$ |

Table I. The parameters of the potential parametrizations [4] to [6].
B. Main results from Ref. [34]

In Ref. [34] we applied our approach to study bottomonium bound states and resonances with \( I = 0 \). For \( m_M \), which is the energy reference of our system, we use the spin-averaged mass of the \( B \) meson and the \( B^* \) meson, i.e. \( m_M = (m_B + 3m_{B^*})/4 = 5.313 \text{ GeV} \) [1]. \( \mu_Q = m_Q/2 \) in the kinetic term of the coupled channel Schrödinger equation (1) is the reduced mass of the \( b \) quark. Since results are only weakly dependent on \( m_Q \) (see e.g. previous work following a similar approach [30] [56]), we use for simplicity \( m_Q = 4.977 \text{ GeV} \) from quark models [37].

In Ref. [34] we presented both the scattering amplitude \( t_{1→0,0} \) and the phase shift \( \delta_{1→0,0} \) for real energies \( E \) above the \( \bar{B}^{(*)}B^{(*)} \) threshold at 10.627 GeV. We also checked probability conservation by showing the Argand diagram for \( \bar{Q}(10750) \) and \( \bar{Q}(10860) \) in the complex energy plane, which are shown in Fig. 2 and collected in Table III.

There are four poles on the real axis below the \( \bar{B}^{(*)}B^{(*)} \) threshold representing bound states \( (n = 1, \ldots, 4 \text{ in Table III}) \). By comparing them with \( \eta_b(1S) = \Gamma(1S), \Gamma(2S), \Gamma(3S) \) and \( \Gamma(4S) \). We also obtained a resonance around 10.870 GeV, which matches \( \gamma(10860) \) rather well (\( n = 6 \text{ in Table III} \)).

Moreover, in Ref. [34] we predicted a new, dynamically generated resonance close to the \( \bar{B}^{(*)}B^{(*)} \) threshold with mass \( \approx 10.774 \text{ GeV} \) (\( n = 5 \text{ in Table III} \)). Very recently Belle has observed a bottomonium state at \( (10.753 \pm 0.007) \text{ GeV} \) denoted as \( \gamma(10753) \) not yet confirmed by other experiments, which could correspond to our prediction. Higher resonances \( (n \geq 7 \text{ in Table III}) \) have very small widths, significantly smaller than suggested by experimental data. The reason for this is most likely that we consider only the coupling of quarkonium to the lightest meson-meson channel \( \bar{B}^{(*)}B^{(*)} \). To obtain realistic widths for resonances above \( \approx 11.025 \text{ GeV} \), which is the threshold of one heavy-light meson with negative parity and another with positive parity, one has to include all excited meson-meson channels up to the respective resonance masses.

III. QUARK-QUARK AND MESON-MESON CONTENT OF \( I = 0 \) BOTTOMONIUM

We continue or investigation of bottomonium bound states and resonances with isospin \( I = 0 \) by studying their structure and quark content. In particular we explore, whether the bound states and resonances close to the \( \bar{B}^{(*)}B^{(*)} \), i.e. \( n = 4, 5, 6 \), which could correspond to the experimentally observed \( \gamma(4S), \gamma(10750) \) and \( \gamma(10860) \), are conventional \( \bar{Q}Q \) or \( \bar{Q}q \) four-quark component.

We inspect in detail the percentages of quarkonium and of a meson-meson pair present in each of the bound states and resonances. To this end we compute

\[
\%_{\bar{Q}Q} = \frac{Q}{Q + M}, \quad \%_{\bar{M}M} = \frac{M}{Q + M}
\]  

(11)

with

\[
Q = \int_0^{R_{\text{max}}} dr \left| u_{0,0}(r) \right|^2, \quad M = \int_0^{R_{\text{max}}} dr \left| \chi_{1→0,0}(r) \right|^2.
\]

(12)

\( u_{0,0}(r) \) and \( \chi_{1→0,0}(r) \) are the radial wave functions of the \( \bar{Q}Q \) and the \( \bar{M}M \) channel, respectively, obtained by solving the coupled channel Schrödinger equation (1) with energies \( E \) identical to the real parts of the corresponding poles.

1. Bound states

For bound states \( E < 0 \) and the corresponding momentum is complex, \( k = i\sqrt{2\mu_ME} \). The boundary condition (10) for \( \chi_{1→0,0}(r) \) simplifies to

\[
\chi_{1→0,0}(r) = 0 \text{ for } r \to \infty.
\]

(13)

Thus, both \( Q \) and \( M \) are independent of \( R_{\text{max}} \), if chosen sufficiently large, i.e. \( R_{\text{max}} \geq 2 \text{ fm} \), because also \( u_{0,0}(r) = 0 \) for \( r \to \infty \) (cf. Eq. (8)). The same is true for \( \%_{\bar{Q}Q} \) and \( \%_{\bar{M}M} \), which represent the probabilities to either find the system in a quarkonium configuration or in a meson-meson configuration.

2. Resonances

For resonances things are more complicated. First, resonances are defined by poles in the complex energy plane with non-vanishing negative imaginary parts of \( E \). Evaluating \( \%_{\bar{Q}Q} \) and \( \%_{\bar{M}M} \) at such a complex energy does not seem to be meaningful, because \( \left| u_{0,0}(r) \right|^2/r^2 \) and \( \left| \chi_{1→0,0}(r) \right|^2/r^2 \) are only proportional to probability densities, if \( E \) is real. Thus we compute \( \%_{\bar{Q}Q} \) and \( \%_{\bar{M}M} \) at the real part of the corresponding pole position, \( \text{Re}(E) \), which is the resonance mass.

There is, however, another complication, namely that \( M \) is not constant but linearly rising for large \( R_{\text{max}} \). The reason is that \( \chi_{1→0,0}(r) \) represents an emergent wave (see Eq. (10)). The need to remove part of the meson-meson emergent wavefunction has already been addressed in a momentum space formalism [38] [39]. For the systems we study we found the dependence of \( \%_{\bar{Q}Q} \) and \( \%_{\bar{M}M} \) on \( R_{\text{max}} \) to be rather mild, with an uncertainty of only a few percent in the range 1.8 fm \( \leq R_{\text{max}} \leq 3.0 \text{ fm} \), i.e. where the quarkonium component is already negligible, \( u_{0,0}(r = R_{\text{max}}) \approx 0 \). Thus, we interpret \( \%_{\bar{Q}Q} \) and \( \%_{\bar{M}M} \) as estimates of probabilities to either find the system in a quarkonium configuration or in a meson-meson configuration, as for the bound states discussed before.

3. Numerical results and error analysis

We show plots of \( \%_{\bar{Q}Q} \) and \( \%_{\bar{M}M} \) as functions of \( R_{\text{max}} \) for the first seven bottomonium bound states and resonances in Fig. 5.
As expected, for the four bound states, \( n = 1, \ldots, 4 \), both \( \% \bar{Q}Q \) and \( \% \bar{M}M \) are constant for large \( R_{\text{max}} \). For \( \eta_b(1S) \equiv \Upsilon(1S) \) \( (n = 1) \) this is the case already for \( R_{\text{max}} \geq 0.4 \) fm, while e.g. for \( \Upsilon(4S) \) \( (n = 4) \) \( R_{\text{max}} \geq 2.0 \) fm is needed. This is not surprising and just indicates that wave functions for increasing \( n \) are less localized, as usual in quantum mechanics. \( \eta_b(1S) \equiv \Upsilon(1S), \Upsilon(2S) \) and \( \Upsilon(3S) \) have \( \% \bar{Q}Q \approx 89\% \), i.e. are clearly quarkonium states. \( \Upsilon(4S) \), which is close to the \( \bar{B}^0(B^0) \) threshold is still quarkonium dominated \( \% \bar{Q}Q \approx 70\% \), but already has a sizable four-quark component \( \% \bar{M}M \approx 30\% \).

For the resonances there is a dependence of \( \% \bar{Q}Q \) and \( \% \bar{M}M \) on \( R_{\text{max}} \) but it is rather mild, with an uncertainty of 2\% or less in the range \( 1.8 \) fm \( \leq R_{\text{max}} \leq 3.0 \) fm (see also the discussion in section III). The wide resonance with \( n = 5 \) has \( \% \bar{M}M \approx 94\% \), and, thus, is essentially a meson-meson pair. The resonance with \( n = 6 \) is a mix of quarkonium and a meson-meson pair with slightly larger \( \bar{Q}Q \) component \( \% \bar{Q}Q \approx 59\%, \% \bar{M}M \approx 41\% \). Resonances with \( n \geq 7 \) are above the threshold of one heavy-light meson with negative parity and another with positive parity. Since this decay channel is currently neglected, their decay widths are tiny and they are almost stable. Correspondingly, they are strongly quarkonium dominated, i.e. \( \% \bar{Q}Q \gg \% \bar{M}M \). We stress that results for \( n \geq 7 \) should not be trusted until all relevant decay channels are included.

\( \% \bar{Q}Q \) and \( \% \bar{M}M \) for \( R_{\text{max}} = 2.4 \) fm are listed in Table III, together with their statistical errors and, for the resonances, also systematic uncertainties. To estimate statistical errors, we utilize the same 1000 sets of parameters as in Ref. [14], which were generated by resampling the lattice QCD correlators from Ref. [23]. Asymmetric statistical errors are defined via the 16th and 84th percentile of the 1000 samples. We visualize these errors as error bands on \( \% \bar{Q}Q \) and \( \% \bar{M}M \) in Fig. 3. We define the asymmetric systematic uncertainties as \[ \% \bar{Q}Q(R_{\text{max}} = 1.8 \) fm) \( \% \bar{Q}Q(R_{\text{max}} = 2.4 \) fm) \] and \[ \% \bar{Q}Q(R_{\text{max}} = 3.0 \) fm) \( \% \bar{Q}Q(R_{\text{max}} = 2.4 \) fm) \] in the same way for \( \% \bar{M}M \). They are around 2\% for the resonances with \( n = 5 \) and \( n = 6 \), respectively, and negligible for all other \( n \). The total uncertainties on \( \% \bar{Q}Q \) and \( \% \bar{M}M \) are rather small. Thus, our predictions concerning the structure of the bound states and resonances are quite stable within our framework. The columns "\( \% \bar{Q}Q \)" and "\( \% \bar{M}M \)" in Table III represent the main results of this work, since these numbers reflect the quark composition of the bound states and resonances and clarify, which states are close to ordinary quark model quarkonium, and which states are dynamically generated by a meson-meson decay channel.

IV. CONCLUSIONS

In Ref. [14] we recently developed a formalism based on lattice QCD static potentials, to study resonances with a heavy quark-antiquark pair and possibly also a light quark-antiquark...
Table III. Masses and decay widths for $I = 0$ bottomonium with $J^{PC} = 0^{++}$ from the coupled channel Schrödinger equation [1] and the corresponding $\bar{Q}Q$ and $\bar{M}M$ percentages for $R_{\text{max}} = 2.4$ fm. For comparison we also list available experimental results. Relevant $B_s^*(B_s^*)$ thresholds are marked by horizontal lines. Errors on our results for $m$ and $\Gamma$ are purely statistical, while for $\%\bar{Q}Q$ and $\%\bar{M}M$ we additionally show systematic uncertainties for the resonances, as discussed in section III 2.

Resonances with $n \geq 7$ are above the threshold of one heavy-light meson with negative parity and another with positive parity at around 11.025 GeV and, thus, should not be trusted (indicated by a gray shaded background).

| $n$ | $m$ [GeV] | $\text{Im}(m)$ [MeV] | $\Gamma$ [MeV] | $\%\bar{Q}Q$ | $\%\bar{M}M$ | name | $m$ [GeV] | $\Gamma$ [MeV] |
|-----|-----------|-------------------|---------------|---------------|---------------|------|-----------|---------------|
| 1   | $9.562^{+11}_{-17}$ | 0 | $0.89_{-0.04}^{+0.05}$ | $0.11_{-0.005}^{+0.004}$ | $\eta_b(1S)$ | 9.399(2) | 10(5) |
| 2   | $10.018^{+8}_{-10}$ | 0 | $0.90_{-0.02}^{+0.01}$ | $0.10_{-0.001}^{+0.002}$ | $\Upsilon(1S)$ | 9.460(0) | $\approx 0$ |
| 3   | $10.340^{+7}_{-9}$ | 0 | $0.88_{-0.02}^{+0.01}$ | $0.13_{-0.002}^{+0.002}$ | $\Upsilon(2S)$ | 10.023(0) | $\approx 0$ |
| 4   | $10.603^{+5}_{-6}$ | 0 | $0.70_{-0.02}^{+0.05}$ | $0.30_{-0.025}^{+0.024}$ | $\Upsilon(3S)$ | 10.355(1) | $\approx 0$ |
| 5   | $10.774^{+4}_{-4}$ | $-49.3^{+3.0}_{-4.6}$ | $98.5^{+9.2}_{-5.9}$ | $0.06_{-0.011}^{+0.018}$ | $\Upsilon(4S)$ | 10.579(1) | 21(3) |
| 6   | $10.895^{+7}_{-10}$ | $-11.1^{+3.6}_{-2.4}$ | $22.2^{+7.1}_{-4.9}$ | $0.59_{-0.016}^{+0.018}$ | $\Upsilon(10750)$ | 10.753(7) | 36(22) |
| 7   | $11.120^{+13}_{-18}$ | $-0.0^{+0.0}_{-0.2}$ | $0.0^{+0.0}_{-0.0}$ | $0.87_{-0.002}^{+0.007}$ | $\Upsilon(10860)$ | 10.890(3) | 51(7) |

The most recent controversy concerns the nature of the newly discovered resonance $\Upsilon(10753)$. Model calculations suggest for instance this resonance to be either a tetraquark, a hybrid meson or the more canonical and so far missing $\Upsilon(3D)$ [7,9]. With our lattice QCD based approach we find a pole corresponding to a mass 10.774 GeV, quite close to the Belle measurement of the mass of the $\Upsilon(10753)$ resonance, $(10.753 \pm 0.007)$ GeV. In Ref. [34] we had already anticipated this pole to be dynamically generated by the meson-meson channel. Now we confirm that this resonance is mostly composed of a pair of mesons, $\%\bar{M}M \approx 94\%$. While there is essentially no potential in the meson-meson channel, the mixing potential with the quarkonium channel generates an effective potential sufficiently strong to bind the mesons into a resonance. Thus, since it is not a quarkonium state and the heavy quark spin can be $1^-$, it can be classified as a $Y$ type crypto-exotic state. Notice that it should also be part of the $\eta_b$ family, since the heavy quark spin can also be $0^+$ and there is degeneracy with respect to the heavy quark spin.

As an outlook, we are on the way to extend our study beyond $S$ wave bottomonium, to $P$ wave, $D$ wave and $F$ wave, which is more cumbersome, since in these cases there is an additional meson-meson channel. We expect then to be able to address the controversy on the existence of $D$ wave resonances in more detail. Moreover, in the long term we plan to compute lattice
Figure 3. (Color online.) Percentages of quarkonium $\% \bar{Q} Q$ and of a meson-meson pair $\% \bar{M} M$ present in each of the first seven bound states and resonances as a function of $R_{\text{max}}$. The error bands represent statistical uncertainties.

QCD static potentials ourselves, in order to update our results with more precision and, hopefully, with excited meson-meson channels. The latter would enable us to make predictions also for energies above the $B^{(*)}_s, B^{(*)}$ threshold at $= 11.025$ GeV.

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