An Engineering Estimation Method of the Sound Insulation of Massive Partitions on the Base of Design Model with Lumped Parameters

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Abstract. The issue shows theoretical and practical approach to the massive single-layer partition's design on the base of lumped model's method, provided by Zakharov A.V. The expediency of using the "Mass Law's" formula in case of the normal noise wave's incidence on the sound isolating partition, due to the adherence of continuity conditions of sound waves transmission at the interface of two media, is approved. The common simplified equations for the massive partition's sound insulation for the engineering method, which is based on lumped (discrete) models, are derived. The execution sequence of one-layer partition's engineering insulation design is given and the frequency response graph for the concrete one-layer partition based on this design is also represented. The comparison results of the medial three octave deviations of the sound isolation figures from the experimental data between Russian normative document method and the proposed engineering method for the different construction materials are represented.

1. Introduction

The design of single-layer sound insulation partitions with specified acoustical parameters is one of the main tasks of a building acoustics. Nowadays, the semigraphical calculation, which is represented by Russian normative document [1], is a basic computational method for obtaining the frequency response at rated building insulation range.

Along with the normative method, today, there are some others effective alternative techniques for sound insulation of partitions defining, for example [2, 3, 4, 5, 6, 7]; and in particular, the technique with applying a physical model with lumped parameters [8, 9, 10, 11, 12, 13, 14], that was developed at the turn of XX and XXI centuries by Zakharov A. V. Its final calculation formulas for sound insulation of single-layer partitions with the surface density from 100 to 800 kg/m² are simple in use. And that makes this technique sufficient for the practical engineering designs.

2. Materials and methods

It's common knowledge, that the airborne sound isolation of one-layer homogeneous plate, \( R_{M.A.L.1} \), generically is defined by "Mass Action Law" [15, 16, 17, 18], (formula 1):

\[
R_{M.A.L.1} = 10\log_{10} \left( 1 + \left( \frac{\pi f m \cdot \cos \theta}{\rho_0 c_0} \right)^2 \right), \text{ dB};
\]  

(1)
where \( \alpha \) - is the transmission coefficient of the oscillation velocity into the plate; \( f \) - is the current frequency, Hz; \( m \) - is the surface density of the plate, \([\text{kg} \cdot \text{m}^{-2}]\); \( \rho_0 \) - is the air specific weight, \([\text{kg} \cdot \text{m}^{-3}]\); \( c_0 \) - is the sound speed in the air, \([\text{m} \cdot \text{s}^{-1}]\); \( \theta \) - is the angle of the sound wave incidence, [deg].

This formula enables to define the plate sound insulation in cases of various incident angles of sound waves on the plate surface within the range till the wave coincidence frequency. However, there are the limits for formula (1) application [15]. Accordingly the standard sound isolation theory concepts, the equation (1) is not appropriate for incidence angles values from 75 to 90 degrees [15]. The results of isolation by formula (1) are not equal to field measurements in certain practical cases, when the sound incidence on a plate surface is close to the excluded angles. For instance, there are not any difference between the wall sound insulation values at long length corridors with parallel sound waves propagation, from the wall isolation values in situations of another configuration premises with the sound incidence angles near to normal. The explanations of regulations non-compliance in formula (1) can be obtained in course of deep analyzing by its physical parameters.

The formula (1) has the mass of the plate fragment, \( m \), in the numerator, this mass executes a periodic motion with a frequency \( f \); the same time, we see the density of air medium fragment, \( \rho_0 \), with the velocity \( c_0 \), being in denominator. That's means, generally, that the squared quotient can be considered as a two media fragment interaction: the plate and the air, which both have their masses. It has a greater clarity, when equation (1) is written in form (2), if \( \lambda_0 = c_0/f \):

\[
R_{\text{M.A.L.1}} = 10 \log \left( 1 + \left( \frac{\pi m \cos \theta}{\rho_0 \lambda_0} \right)^2 \right), \text{dB};
\]  

(2)

where \( \lambda_0 \) - is the longitudinal sound wave length in the air, [m].

In a such form, the squared summand of the logarithm is a relationship of the surface density of a plate to the media fragment, which is limited by the wave length and the unit area of the sound beam section. Thereat, \( \pi \) is a reduction factor. Thus, it is possible to conclude, that we observe the interaction between two objects with their masses and geometrical dimensions in the event of sound wave incidence on the plate. Sound wave incidence on the plate surface is complied with the rules of energy flow continuity equation at the interface of different media. Accordingly continuity conditions, the sound beam cross section area should match the plate area, which is covered by this beam (should coincide with the trace of the beam). Also by continuity, the media should not be apart or interpenetrated at the interface of interaction [19].

Let us consider the particular case, when sound incidence angle is being normal; so \( \cos \theta = 1 \) and the formula (2) is rearranged in form (3):

\[
R_{\text{M.A.L.1}} = 10 \log \left( 1 + \left( \frac{\pi m}{\rho_0 \lambda_0} \right)^2 \right), \text{dB};
\]  

(3)

The geometric interpretation is shown at figure 1. The continuing conditions are fully satisfied, as the beam width \( b \) before striking the plate is coincided both with the unite width of the plate area with the mass \( m \), covered by the beam, and the width of the sound beam after transmission. But there is no possible sufficient geometric model for oblique incidence, that fulfill the requirements of energy flow continuing at the interface media, in case of the unit beam width \( b \), which must be equal to the unit width of the plate fragment and the unit width of transmitted beam. In such approach, the traces width and square for incident and transmitted beams don't coincide with the width and square of the area with a surface density \( m \), figure 1b.

The oblique incidence process under the continuing condition can be assumed to look like as shown at the figure 1c: the width \( b \) should change proportionally with the \( \theta \) angle cosine, that to make the striking and transmitted traces width and unite square equal with such factors of the plate fragment with the mass \( m \). Then there is a necessity in physical analogue for the sound propagation process description, which allows to take into consideration the changing of air-medium fragment with the density \( \rho_0 \), velocity \( c_0 \), limited by the wave length \( \lambda_0 \) at the current frequency in compliance with the sound incident angle. Such model may be described by the momentum conservation and the kinetic energy conservation equations [20]. Their appliance is conventional in classical mechanics.
Figure 1. The scheme of sound propagation through the insulation plate in case of: a - normal beam incidence; b - oblique beam incidence with the discontinuity in massive layer and nonobservance of continuing conditions; c - oblique beam incidence with fulfillment of continuity conditions, with the width coincidence of the beam trace and the plate area.

The important obstacle for notation of momentum equation is an incapability of momentum conservation law in acoustics. This is due to the three reasons:

- the summarized velocity vector value in formula (2) is equal to zero in case of total dilatational wave length;
- the conservation momentum law is appropriate for a closed system only: none of the included bodies shouldn’t have an impacts from other bodies, except of involved in this system [20]; and acoustic medium is not a closed system;
- the interaction of two dissimilar objects is considered in such physical model: one object is an incompressible body, which is represented by the lumped mass with its oscillations, and another object is elastic and inertial medium, confined by the undulatel motion.

As it shown in [14], the first reason of momentum equation inapplicability can be eliminated as follows. Any area of plate layer is virtually limited by the medium fragment with its volume, defined by the wave length at current frequency and by the area of sound beam lateral section. The square of lateral beam section is equal to the unite square at normal incidence. The mentioned volume mass and the mass of unite area of the separation layer are approximated by the material points, which consist the virtual closed system.

The solution of second problem for momentum equation application is possible to obtain, when velocities values of considered virtual masses are being taken as equal to the effective values of oscillating velocities under the condition of all masses oscillations along the one axis.

The third momentum law restriction can be obviated through the reduction of action of the mass fragment at one wave length to the equivalent action of lumped (concentrated) mass. It is feasible to achieve due to the introduction of suitable reduction coefficient. If we rewrite the formula (3) for normal incidence by the way of both numerator and denominator in second brackets multiplication by 2 (4), we shall face the necessity of numerator and denominator multiplication by the reduction coefficient $1/2\pi$, that to attain only the value of concentrated mass in the denominator.

$$R_{M.A.L.1} = 10\log \left(1 + \left(\frac{\gamma m}{2\rho_0} \right)^2\right), \text{dB};$$

Therefore, as in [14], it is appropriate to introduce the concept of "reduced mass": the mass of medium fragment, which is limited by $1/2\pi$ wave length at the current frequency, so $\mu = \rho\lambda/2\pi$. Accordingly this, the equation for the plate sound insulation will have the form (5):

$$R_{M.A.L.1} = 10\log \left(1 + \left(\frac{\gamma m}{2\rho} \right)^2\right), \text{dB};$$

The multiplication of denominator by 2 is explained because of the simultaneous interaction of lumped mass both with the reduced mass of the air medium before the layer and the same mass after the separation layer of the plate. Thus, in compliance with the argumentation of application of momen-
In the case of oblique sound incidence on the plate surface at an angle \( \theta \), in agreement with continuity condition and geometric interpretation at figure 1c, the equation of momentum conservation will be represented as expression (10):

\[
\frac{\mu \cos \theta \nu}{\cos \theta} = \frac{\mu \cos \theta \nu \beta}{\cos \theta} + \frac{\mu \cos \theta \nu \alpha}{\cos \theta} + m \cdot \nu \alpha
\]

This equation shows the reduced masses values, which are corrected by the continuity conditions through the cosines of incident and reflection angles, and the effective values of oscillation velocities projected at axis \( z \). The equation of momentum conservation for normal incidence in form (6), which was represented above, is obtained after the proper transformations and cancellations. Therefore, the sound insulation of a plate can be defined by the Mass Action Law formula for normal sound incidence, independently from the incident wave angles.

3. Results and Discussion

The reduced mass of a medium and the lumped (concentrated) mass of a sound insulation plate (partition or slab) fragment are the discrete parameters, which are the base for physical sound insulation model with lumped parameters.

It is known, that the sound insulation increases constantly in the frequency range till the frequency, which lays approximately one octave lower, than so called limiting frequency, \( f_L \). Also, the sharp dip of a curve is observed from one octave lower than \( f_L \) up to \( f_L \), that caused by the process of wave coincidence between the trace of the incident sound wave in air and the length of a flexural wave in the plate. After that, beginning from the frequency \( f_L \) it is seen the sound insulation gradual growth of 6 dB per octave [1], till the ordinate of near 60-70 dB. Such insulation behavior is detailed in publications [15, 16].

Accordingly issues [9, 10, 11, 12, 13], which take into consideration the discrete features of continuum medium and momentum and energy conservation laws, the physical model of sound propagation through the one-layered enclosure plate at the range up to the wave coincidence frequency \( f_L \) can be represented as synchronous elastic collision of reduced medium mass \( \mu \) (air) before the plate against the lumped mass of the plate \( m \) and reduced air medium mass \( \mu \) after this plate. That is expressed by equation (1).

Further, the physical model of sound transition through the enclosure can be revealed as the synchronous elastic strike of reduced medium mass \( \mu \) before the plate against the reduced mass of the plate \( \mu_p \) and reduced air medium mass \( \mu \) after. It means, that the plate in the range of wave coincidence frequencies will be considered not as the concentrated mass, but as the medium of flexural wave propagation. And the insulation formula, with the condition \( \rho_0 \lambda_0 \ll m \), should have an appearance (11):
\[ R_{M.A.L.2} = 10 \log \frac{1}{\alpha^2} = 20 \log \left(1 + \frac{\mu_{pl}}{2\mu}\right) = 20 \log(1 + \frac{f_m}{2\rho_0 c_0}), \text{dB} \] (11)

Conditionally third segment of specified range begins with the frequency 65 dB by the field researches and normative documents. The shear and dilatational waves appear there; their propagation velocity does not depend from the oscillation frequency, and so, the wave coincidence angle will be just one [9]. The insulation growth depending the growth of a frequency stops at this segment.

For the practical engineer calculations purposes, the formulas (4) and (11) are considerably simplified after the substitution of the standard acoustical values: \( \rho_0 = 1.3 \text{ [kg.m}^{-3}] \) and \( c_0 = 340 \text{ [m.s}^{-1}] \). Additionally, sound insulation before the limiting frequency will be also defined due to the resonance phenomena corrections, \( \Delta R_{res.} \), which are approximately equal to 6 dB [8, 9], that's taken into consideration in formula (12):

\[ R_{M.A.L.1} = 20 \log m_f - 48, \text{ dB} \] (12)

The formula (11) will have the ensuing form after the coincidence wave frequency:

\[ R_{M.A.L.2} = 20 \log m_f - 58, \text{ dB} \] (13)

The engineer sound insulation calculations of single-layered partitions is performed in following sequence: the frequency of wave coincidence beginning, \( f_L \), is found, it's magnitude is rounded to the nearest geometric mean octave frequency at the graph; the insulation is defined by the formula (12) at the standard frequency range up to the one octave lower than \( f_L \); the inclined line is plotted till the frequency with the maximum ordinate of 65 dB by the formula (13) from the beginning in \( f_L \), and the horizontal line is constructed right from the highest insulation point to the end of the standard range; the point with the insulation at the frequency, which lays one octave lower than \( f_L \), is joined with the insulation point at the limiting frequency \( f_L \).

The gained in such a way frequency response characterizes the sound insulation properties of the partition at the normalized frequency range reasonably accurate.

**Figure 2.** The sound insulation graphs for reinforced concrete partition of 100 mm thickness with \( m=240 \text{ [kg.m}^{-2}] \): 1 - are the experimental data; 2 - is the line, constructed by the [1]; 3 - is the graph by the formulas (12) and (13).

For example, the graph of reinforced concrete partition sound insulation in comparison with the frequency characteristic by the normative document [1] and by the experimental measurements [18] is shown at the figure 2. The arithmetic average deviation from the experimental data [18] for the different partitions variants by proposed method is 2.84 dB and by the normative method is 3.12 dB. That's can prove the effectiveness of the engineering method, considered in the article.

**4. Conclusions**

The sound insulation of single-layered partitions may be calculated just using the Mass Action Law expression for normal sound incidence on the plate surface.
The formulas for practical engineering sound insulation method, with taking into consideration wave coincidence and resonance phenomena in solid plates, are represented. And their practical application algorithm is provided.

There were found out the proximity of proposed method results with the experiments and the normative technique.

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