I. INTRODUCTION

It was argued that a nuclear system may have bubble configuration [1 2], which is in conflict with the general knowledge that an atomic nucleus is compact. The question of the possibility of the existences of exotic bubble or toroidal configuration of atomic nuclei has long been discussed for more than 60 years [4–12]. The pioneer study of the spherical bubble nuclei was made by Wilson in 1946 [4]. Twenty years later, based on a liquid drop model, Siemens and Bethe studied spherical bubble nuclei [5]. Wong studied known stable nuclei and found spherical bubbles on the basis of a liquid drop model plus a shell correction energy [11]. And Moretto et al. argued that nuclear bubble configuration could be stabilized by the inner vapor pressure [12]. Based on the hydrodynamic equations, Borunda and López argued that the hollow configuration is caused by the liquid-like behavior of compressed nuclear matter [14]. The transport model simulations of nuclear collisions at low incident beam energy also point out the possible bubble configuration in nuclear matter [13–20]. And J. Dechargé et al. recently gave stable bubble solutions of some specific super-heavy nuclei using the approach of the self-consistent microscopic Hartree–Fock–Bogoliubov calculations with the effective Gogny interaction [1 2]. These results are qualitatively consistent with the studies based on the liquid drop model using Strutinsky shell correction method [6 21] plus the phenomenological shell model potentials [22 23]. At the time being, the possible proton bubble configuration of some light nuclei has also been studied [24].

The probe of the bubble configuration formed in heavy-ion collisions has recently been argued in Refs. [13–20], while the hollow configuration of atomic nucleus is in fact hard to probe [25]. This is the reason why the existence of bubble nuclei is still in debate. Besides the possible electronic hyperfine configuration measurement of the nuclear density profile [2 21 27], it is necessary to study whether hollow atomic nuclei exist or not by other completely different methods. Very recently, Najman et al. analyzed the Data from an experiment on the $^{197}$Au+$^{197}$Au reaction at 23 MeV/nucleon and showed that the exotic nuclear configurations such as toroid-shaped objects may exist [23]. This experimental investigation stimulates further studies on the exotic nuclear configurations theoretically. Based on the isospin-dependent Boltzmann nuclear transport model, here we show that the observable $\pi^-/\pi^+$ ratio in heavy-ion collisions at intermediate energies is very sensitive to the initial nuclear bubble configuration. In the following, for the convenience of studying, we use the bubble nucleus $^{274}$X as projectile and target to collide at a beam energy of 0.4 GeV/nucleon to show how to probe the nuclear bubble configuration in nucleus. This example is fictitious, but it could demonstrate the point.

II. THE ISOSPIN-DEPENDENT TRANSPORT MODEL

In the used isospin-dependent Boltzmann-Uehling-Uhlenbeck (BUU) transport model, nucleon coordinates in initial colliding nuclei are given by [28]

$$R = R(x_1)^{1/3}; \cos \theta = 1 - 2x_2; \phi = 2\pi x_3,$$

$$x = r \sin \theta \cos \phi; y = r \sin \theta \sin \phi; z = r \cos \theta. \quad (1)$$

Where $R$ is the radius of compact nucleus, $x_1, x_2, x_3$ are three independent random numbers (limited to be between 0 and 1). If there is a spherical bubble with radius $R_b$ in a nucleus, the radius $R_b$ of the nucleus with bubble configuration is given by conservation of volume

$$R_b^3 = R^3 + R_{\text{bubble}}^3. \quad (2)$$

In the present study we let inner bubble radii $R_{\text{bubble}} = R/2, R/3$ [11]. Due to the short-range correlations of nucleons in nucleus, above Fermi momentum, momentum distribution of nucleon in nucleus exhibits a $1/k^4$ high-momentum tail shape [29], the isospin-dependent nucleon momentum distribution in this study is thus given by

$$n_{\text{proton/neutron}}(p) = \begin{cases} 
C_1_{\text{proton/neutron}} \cdot \frac{2}{A_N^{1/3}} \cdot \frac{p}{p_f}, & p < p_f, \lambda p_f \\
0, & p > \lambda p_f 
\end{cases}. \quad (3)$$
Where \( n(p) \) is nucleon momentum distribution in initial colliding nucleus, \( p_f \) is the nuclear Fermi momentum, 
\[ \lambda = p_{\text{max}}/p_f, \]
the factor of maximum momentum of nucleon relative to the nuclear Fermi momentum. We in this study let \( \lambda = 2 \) [30]. \( A, Z, N \) are the nuclear mass number, proton number and neutron number, respectively. The \( C_1 \) and \( C_2 \) parameters are determined by the normalization condition \( 4\pi \int^\infty_0 n(p)p^2dp = 1 \) and roughly 20 percent of nucleons with momenta above the nuclear Fermi momentum [31], i.e., \( 4\pi \int^{p_{\text{f}}}_0 n(p)p^2dp \times 100\% = 20\% \). The 20 percent of nucleons with high momenta are randomly distributed in the nucleus.

We use the Skyrme-type parametrization for the mean field, which reads [25]

\[ U(\rho) = A(\rho/\rho_0) + B(\rho/\rho_0)^\sigma. \]  (4)

Where \( \sigma = 1.3 \), \( A = -232 \) MeV accounts for attractive potential and \( B = 179 \) MeV accounts for repulsive. With these choices, the ground-state compressibility coefficient of nuclear matter is \( K = 230 \) MeV [32]. We let the kinetic symmetry-energy (the symmetry energy can be divided into the kinetic symmetry energy and the potential symmetry energy) be -6.71 MeV [33], and the symmetry-potential becomes [34]

\[ U_{\text{sym}}^{n/p}(\rho, \delta) = 38.31(\rho/\rho_0)^\gamma \times (\pm 2\delta + (\gamma - 1)\delta^2), \]  (5)

where \( \delta = (\rho_n - \rho_p)/\rho \) is the isospin asymmetry of nuclear medium. In the above, we let the value of the symmetry-energy at saturation density be 31.6 MeV [35, 36]. In the present study, we let \( \gamma = 0.3 \) [37] for the soft symmetry energy choice while \( \gamma = 1.5 \) for stiffer symmetry energy choice as comparison.

The in-medium baryon-baryon (BB) elastic cross sections are factorized as the product of a medium correction factor and the free baryon-baryon scattering cross sections [38], i.e.,

\[ \sigma_{\text{medium}}^{BB, \text{elastic}} = \frac{1}{3} e^{-u/0.54568} \times (1 \pm 0.85\delta) \sigma_{\text{free}}^{BB, \text{elastic}}. \]  (6)

\( u = \rho/\rho_0 \) is the relative density. \( 1 \pm 0.85\delta \) is the isospin dependent factor. Because pion’s production and absorption are in fact mainly determined by the inelastic baryon-baryon scattering cross section, the in-medium correction of the inelastic baryon-baryon scattering cross section has also to be taken into account. In this model, for the inelastic baryon-baryon scattering cross section, we use the form [39]

\[ \sigma_{\text{medium}}^{BB, \text{inelastic}} = (e^{-1.3u}) \times (1 \pm 0.85\delta) \sigma_{\text{free}}^{BB, \text{inelastic}}. \]  (7)

In Eqs. (6) and (7), whether in initial or final states, “+” is for neutron-neutron scattering while “-” for proton-proton scattering. We neglect the isospin dependent factor in other baryon-baryon scattering cases.

III. RESULTS AND DISCUSSIONS

Before studying how to probe the bubble configuration in nucleus in heavy-ion collisions, it is instructive to show the density-evolution in collisions of normal and bubble nuclei. Shown in Fig. 1 is the evolution of the central density \( \rho_{\text{central}} = \rho(0,0,0) \) in center of mass system with the volume of 1 fm\(^3\) in \( {}^{274}_{\text{X}} + {}^{274}_{\text{X}} \) head-on collisions at a beam energy of 0.4 GeV/nucleon with bubble radii \( R_{\text{bubble}} = 0, R/2, R/3 \).

FIG. 1: (Color online) Evolution of the central density in \( {}^{274}_{\text{X}} + {}^{274}_{\text{X}} \) head-on collisions at a beam energy of 0.4 GeV/nucleon with bubble radii \( R_{\text{bubble}} = 0, R/2, R/3 \).

FIG. 2: (Color online) Evolution of the contour plots of density distribution in \( {}^{900}_{\text{X}} + {}^{900}_{\text{X}} \) head-on collisions at the beam energy of 0.4 GeV/nucleon in X - Z plane with bubble radii \( R_{\text{bubble}} = 0, R/2, R/3 \).
on collisions at a beam energy of 0.4 GeV/nucleon with bubble radii $R_{\text{bubble}} = 0, R/2, R/3$. It is clearly seen that, compared with the collision of normal nuclei, there is a serious depletion of central density with bubble configurations in projectile and target nuclei. Shown in Fig. 2 is the evolution of contour plots of density distribution in $^{90\text{0}}X + ^{90\text{0}}X$ head-on collisions at the beam energy of 0.4 GeV/nucleon in X - Z plane (the beam axis is in the Z direction). We can see that the maximum compression-density region ($\rho/\rho_0 > 1.8$) is larger for the normal nuclei while it is smaller if there are bubble configurations in nuclei. And the larger the bubble is, the smaller the maximum compression region is seen. This is understandable since with bubbles in colliding nuclei, the collision of the two nuclei becomes two bubble’s collision, the compression is thus not strong. It is interesting to see that, if there are larger bubble configurations in the colliding nuclei, there is also a bubble configuration in the formed compression matter. Fig. 3 shows the evolution of the contour plots of density distribution in X - Y plane. Again, it is seen that there is larger compression if there are no bubble configurations in projectile and target while the compression is smaller if there are bubble configurations in initial colliding nuclei.

Since the bubble configurations in colliding nuclei affect maximum compression in heavy-ion collisions and the ratio of $\pi^-/\pi^+$ is in fact affected by the neutron to proton ratio of compression matter [40], it is thus naturally to think whether the observable $\pi^-/\pi^+$ ratio is affected by the bubble configurations in colliding nuclei or not. Shown in Fig. 4 is the time evolution of the observable $\pi^-/\pi^+$ ratio in the $^{90\text{0}}X + ^{90\text{0}}X$ head-on collisions at the beam energy of 0.4 GeV/nucleon with bubble radii $R_{\text{bubble}} = 0, R/2, R/3$. One can clearly see that the value of $\pi^-/\pi^+$ ratio from the bubble nuclei’s collision is evidently larger than that from the collisions of normal nuclei. The effects of bubble configurations in colliding nuclei on the value of $\pi^-/\pi^+$ ratio reach about 23\% and 68\%, respectively, for $R_{\text{bubble}} = R/3, R/2$. Because the size of the compression region from the bubble nuclei’s collision is smaller than that from the normal nuclei, the symmetry potential at supra-saturation densities would have less time to act on nucleons. The effect of the symmetry energy at supra-saturation densities is thus smaller in the collision of the bubble nuclei’s collision. Therefore...
fore neutrons are less repelled by the symmetry potential at high densities in the collision of bubble nuclei. And the value of the $\pi^-/\pi^+$ ratio reflects the ratio of neutron number and proton number in dense matter, i.e., $\pi^-/\pi^+ \approx (N_{\text{dense}}/Z_{\text{dense}})^2$ \cite{40}, it is not surprising to see larger value of the $\pi^-/\pi^+$ ratio in the collision of bubble nuclei.

Shown in Fig. 5 is the kinetic energy distribution of the observable $\pi^-/\pi^+$ ratio in the same collisions as Fig. 4. It can be seen that, for the energetic pion mesons (mainly from more denser matter), the effects of the bubble configurations on the value of $\pi^-/\pi^+$ ratio reach about 70% and 300%, respectively, for $R_{\text{bubble}} = R/3$ and $R/2$. Such large bubble effects on the $\pi^-/\pi^+$ ratio could be used to probe the bubble configuration in nucleus.

While in fact the effects of the nuclear symmetry energy on the $\pi^-/\pi^+$ ratio in the collisions for heavy nuclei, especially for super-heavy or hyper-heavy nuclei, are larger than that for lighter nuclei. This is because the heavier nuclei generally have large $N/Z$ ratio. Shown in Fig. 6 is the effects of the symmetry energy on the evolution of the observable $\pi^-/\pi^+$ ratio in the head-on collisions at the beam energy of 0.4 GeV/nucleon with bubble radii $R_{\text{bubble}} = 0$, $R/2$, $R/3$. Because the stiffer symmetry energy ($\gamma = 1.5$) causes more neutrons to emit from the formed dense matter in collision and the neutron-poor dense matter causes a small value of the $\pi^-/\pi^+$ ratio \cite{40}, as expected, one sees in Fig. 6 the value of $\pi^-/\pi^+$ ratio with the stiffer symmetry energy ($\gamma = 1.5$) is smaller than that with the soft symmetry energy ($\gamma = 0.3$). Therefore, the effect of the symmetry energy is one of the largest uncertain factors in probing the bubble configuration in heavier nuclei by the observable $\pi^-/\pi^+$ ratio. Fortunately, the nuclear symmetry energy at high densities has recently been roughly pinned down, i.e., a soft symmetry energy is supported by the FOPI and FOPI-LAND experiments \cite{37}.

FIG. 6: (Color online) Effects of the symmetry energy on the evolution of the observable $\pi^-/\pi^+$ ratio in the same $^{900}\text{X} + ^{900}\text{X}$ head-on collisions at the beam energy of 0.4 GeV/nucleon with the bubble radii $R_{\text{bubble}} = 0$, $R/2$, $R/3$. It can be seen that, for the energetic pion mesons (mainly from more denser matter), the effects of the bubble configurations on the value of $\pi^-/\pi^+$ ratio in the same collisions as Fig. 4. It can be seen that, for the energetic pion mesons (mainly from more denser matter), the effects of the bubble configurations on the value of $\pi^-/\pi^+$ ratio in the same collisions as Fig. 4.

IV. CONCLUSIONS

Based on the isospin-dependent nuclear transport model at intermediate energies, it is found that the bubble configurations in colliding nuclei affect maximum compression in heavy-ion collisions. Compared with the collision of normal nuclei, there is a serious depletion of central density with bubble configurations in projectile and target nuclei. Since the symmetry energy plays important role, the observable $\pi^-/\pi^+$ ratio in heavy-ion collisions is sensitive to the bubble configurations in colliding nuclei. The ratio of $\pi^-/\pi^+$, especially its kinetic-energy distribution, could be used to probe the bubble configuration in nucleus.

However, the detection of the variation of the value of the $\pi^-/\pi^+$ ratio with the size of the bubble depends on the experimental ability of discerning bubble nucleus. Furthermore, since the variation of the value of the $\pi^-/\pi^+$ ratio can be due to other factors (fragmentation of participant nuclei, inelastic cross section of nucleon-nucleon scattering, etc.), from an enhancement of the value of the $\pi^-/\pi^+$ ratio, one can not deduce the existence of bubble configuration in nucleus before reducing theoretical uncertainties.

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[1] J. Dechargé, J.F. Berger, K. Dietrich, M.S. Weiss, Phys. Lett. B 451, 275 (1999).
[2] J. Dechargé, J.F. Berger, M. Girod, K. Dietrich, Nucl. Phys. A 716, 55 (2003).
[3] W. Nazarewicz et al., Nucl. Phys. A 701, 165c (2002).
[4] H. A. Wilson, Phys. Rev. 69, 538 (1946).
[5] P. J. Siemens, H. A. Bethe, Phys. Rev. Lett. 18, 704 (1967).
[6] M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. 44, 320 (1972).
[7] C. Y. Wong, Phys. Lett. B 41, 446 (1972).
[8] C. Y. Wong, Phys. Lett. B 41, 451 (1972).
[9] K. T. R. Davies, C. Y. Wong, S. J. Krieger, Phys. Lett.
[10] X. Campi, D.W.L. Sprung, Phys. Lett. B 46, 291 (1973).
[11] Y. Yu, A. Bulgac, P. Magierski, Phys. Rev. Lett. 84, 412 (2000).
[12] C.Y. Wong, Ann. Phys. 77, 279 (1973).
[13] L. G. Moretto, K. Tso, and G. J. Wozniak, Phys. Rev. Lett. 78, 824 (1997).
[14] M. Borunda and J. A. López, Il Nuovo Cimento 107A, 2773 (1994).
[15] W. Bauer, George F. Bertsch, and Hartmut Schulz, Phys. Rev. Lett. 69, 1888 (1992).
[16] D. H. E. Gross, Bao-An Li, A. R. DeAngelis, Annalen der Physik, 504, 467 (1992).
[17] Bao-An Li, D. H. E. Gross, Nuclear Physics A 554, 257 (1993).
[18] H. M. Xu, J. B. Natowitz, C. A. Gagliardi, R. E. Tribble, C. Y. Wong, W. G. Lynch, Phys. Rev. C 48, 933 (1993).
[19] K. Cherevko, L. Bulavin, J. Su, V. Sysoev and F. S. Zhang, Phys. Rev. C 89, 014618 (2014).
[20] G. C. Yong, Phys. Rev. C 93, 014602 (2016).
[21] W. D. Myers, W. J. Swiatecki, Nucl. Phys. 81, 1 (1966).
[22] K. Dietrich, K. Pomorski, Nucl. Phys. A 627, 175 (1997).
[23] K. Dietrich, K. Pomorski, Phys. Rev. Lett. 80, 37 (1998).
[24] J. M. Yao, H. Mei, Z. P. Li, Phys. Lett. B 723, 459 (2013).
[25] R. Najman et al., Phys. Rev. C 92, 064614 (2015).
[26] T. Suda et al., Phys. Rev. Lett. 102, 102501 (2009).
[27] A. N. Antonov et al., Nucl. Instrum. Methods Phys. Res., Sect. A 637, 60 (2011).
[28] G. F. Bertsch and S. Das Gupta, Phys. Rep., 160, 189 (1988).
[29] O. Hen et al. (The CLAS Collaboration), Science 346, 614 (2014).
[30] G. C. Yong, arXiv:1503.08523 (2015).
[31] R. Subedi et al. (Hall A. Collaboration), Science 320, 1476 (2008).
[32] B. G. Todd-Rutel, J. Piekarewicz, Phys. Rev. Lett. 95, 122501 (2005).
[33] B. J. Cai, B. A. Li, Phys. Rev. C 92, 011601 (2015).
[34] O. Hen, B. A. Li, W. J. Gou, L. B. Weinstein, E. Piasetzky, Phys. Rev. C 91, 025803 (2015).
[35] J. Xu, L. W. Chen, B. A. Li, H. R. Ma, Astrophys. J. 697, 1549 (2009).
[36] C. Xu, B. A. Li, L. W. Chen, Phys. Rev. C 82, 054607 (2010).
[37] G. C. Yong, arXiv:1504.02528 (2015).
[38] Q. F. Li, Z. X. Li, S. Soff, M. Bleicher and H. Stöcker, J. Phys. G 32, 407 (2006).
[39] T. Song, and C. M. Ko, Phys. Rev. C 91, 014901 (2015).
[40] B. A. Li, G. C. Yong, W. Zuo, Phys. Rev. C 71, 014608 (2005).