Hydromagnetic instabilities and magnetic field amplification in core collapse supernovae

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Abstract. Some of the most violent events in the universe, the gamma ray burst, could be related to the gravitational collapse of massive stellar cores. The recent association of long GRBs to some class of type Ic supernova seems to support this view. In such scenario fast rotation, strong magnetic fields and general relativistic effects are key ingredients. It is thus important to understand the mechanism that amplifies the magnetic field under that conditions. I present global simulations of the magneto-rotational collapse of stellar cores in general relativity and semi-global simulations of hydromagnetic instabilities under core collapse conditions. I discuss effect of the magneto-rotational instability and the magnetic field amplification during the collapse, the uncertainties in this process and the dynamical effects in the supernova explosion.

1. Introduction

There is strong evidence of the presence of strong magnetic fields in at least some of the compact objects resulting from the collapse of massive stars: the existence of magnetars, strongly magnetized neutron stars, as inferred from SGR and AXP observations [1], and the association of long-GRBs with some type Ic supernovae [2], which supports the collapsar model [3]. In this picture, after the collapse of rapidly rotating cores of massive stars, the magnetic field is amplified extracting energy from the rotation necessary to power the GRB.

Among the different mechanisms to amplify the magnetic field in this scenario the most promising is the magneto-rotational instability (MRI) [4]. In a simplified scenario, accretion discs, the condition for this instability to operate is that the angular velocity decreases outwards, \( \partial_r \Omega^2 < 0 \), where \( \Omega \) is the angular velocity of the fluid. In this case, any magnetic field becomes unstable and grows exponentially in the time-scale of the rotation period. The typical length-scale of the fastest growing mode is \( \lambda \sim 2\pi c_A/\Omega \), being \( c_A \) the Alfvén speed. The more general criterion can be found in [5, 6].

For the typical conditions in core collapse supernovae, the outer layers of the proto-neutron star (PNS) fulfill the instability condition with a growth time of a few milliseconds [7] for the fastest rotating progenitors. This time-scale is comparable with the dynamical evolution time of the PNS, and therefore we expect the MRI to have a strong influence in the evolution of the PNS. However the length-scale of the fastest growing mode in this case is \( \lambda \sim 100(B/10^{14}G) \) m.
Resolving this small length-scale in a multidimensional numerical simulation including the collapse of the iron core (∼2000 km) to the PNS (∼30 km) is computationally challenging and so far only models with unrealistically high initial magnetic field have been able to study the development of the MRI in core collapse [8, 9]. In this paper we present results of some of these global core collapse simulations in general relativity. As an alternative approach to the problem of the smallness of the length scales we present semi-global simulations of the MRI.

2. General relativistic global simulations

We have performed simulations of the collapse of rotating stellar cores using the general relativistic magneto-hydrodynamics (GRMHD) code CoCoNuT [9]. The code solves the evolution of the GRMHD equations using state-of-the-art Godunov-type schemes combined with a constrained transport scheme for the magnetic field. This technique allows for energy-momentum conservation, the preservation of the magnetic flux, and the correct behaviour of shocks and discontinuities. The metric is treated in the CFC approximation [10, 11] and is numerically evolved using spectral methods [12]. We use realistic stellar evolution progenitor models [13], a microphysical equation of state [14] and the deleptonization prescription of [15]. The initial magnetic field configuration is a purely-poloidal dipole-like field with three different initial strengths: 0, 10^{10} and 10^{12} G. We use a differentially rotating angular velocity profile for the progenitor with sufficient angular momentum to form a neutron star with a few milliseconds period. The progenitor core is unstable and it collapses helped by the lose of pressure support due to the deleptonization.

As the central density reaches the nuclear matter density, nuclear forces dominate the pressure and the equation of state stiffens. This leads to the formation of a shock wave which travels outwards and stalls around a few 100 km radius. The central region forms a compact PNS of about 30 km. In the model with highest magnetic field, thanks to the high resolution used in this region (∼100 m), we were able to resolve the fastest growing mode of the MRI and the formation of channel flows. That leads to an exponential spin down of the PNS in a few tens of milliseconds (see figure 1). In the case of weaker magnetic field, 10^{10} G, the spin down is slower. We suspect that in this case the spin-down could be comparable to the strongly magnetized case if we were able to resolve numerically the fastest growing mode of the MRI. Unfortunately the numerical resolution requirements increase linearly with the magnetic field and for this model it would mean increasing the numerical resolution by a factor 100, which is computationally not affordable nowadays. More details about these simulations can be found in [9].

3. Semi-global simulations

Instead of trying to increase the resolution in global simulations we have decided to simulate only a small box with the conditions present in the PNS. The box is about 1 km size which is
sufficiently small so that we can afford the resolution needed to study the MRI, and at the same
time is sufficiently large to study the effect of stratification. We use the Newtonian ideal MHD
code AENUS [6] which uses high order Godunov-type schemes (up to 9th order) and constrained
transport for the magnetic field evolution.

Figure 2. Time evolution of the average Maxwell stress components (different colors) in a typical semi-
global 3D simulation. The exponential growth terminates after about 15 ms and a highly turbulent
saturated state is formed.

The evolution of the magnetic field in these simulations is characterized by three phases as seen in figure 2: exponential growth, termination and saturation. The exponential growth phase can be described using the linear theory [5]. In this phase channel flows grow and amplify the magnetic stresses in the box. We have checked that the growth rates of the instability agree with theory as long as the fastest growing mode is spatially resolved by at least a few points. This holds in all the regimes of the MRI from the pure magneto-shear instability case to the magneto-convective limit. The termination phase occurs as the channel flows are destroyed by parasitic instabilities, mainly tearing modes. Despite of using an ideal MHD code, tearing modes appear due to the presence of numerical resistivity. We have obtained scaling relations between the magnetic stress at termination and the number of cells per channel flow, which is related to the numerical resistivity of the code. The computation shows that lower numerical resistivity produces higher termination values, due to the slower growth rate of the parasitic instabilities. The last regime is the saturation. In this regime 2 and 3 dimensional results differ considerably. In 2D simulations the magnetic stresses increase after the termination forming broader channel flows until they fill completely the box. In the 3D case (see figure 2) a quasi-stationary level, slightly higher than the termination point, is reached. The final state is highly turbulent with reappearance of coherent flow patterns in some cases. More details about these simulations can be found in [6].

4. Conclusions
We conclude that the MRI probably plays a extremely important role in the collapse of rapidly rotating stellar cores, even for weakly magnetized progenitors. However the numerical requirements to solve the GRMHD equations resolving all necessary length scales at the same time is unaffordable with present supercomputers. The only possible approach to this problem is to perform semi-global simulations. Those simulations show that it is possible to amplify
the magnetic field under the conditions present in the PNS. However the saturation level, which ultimately sets the real influence in the global dynamics, depends strongly on numerical resistivity. Therefore we need numerical studies with physical resistivity in order to solve the question of the influence of the MRI in the core collapse scenario. Furthermore, large qualitative differences were found between 2D and 3D simulations. This implies that we cannot rely in the less computationally expensive 2D simulations to study the MRI and future simulations will have to be full three dimensional.

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