Deliberative Democracy
with Costly Voting Power Portfolios

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Abstract

We present a collective decision-making model where one or more individuals propose a status quo change that is iteratively updated by a dynamic committee of experts until a point when a referendum is held to decide its finality. Suppose that everyone in the society has some initial voting power. In each iteration there are three stages. In the first stage, each individual decides what percentage of their voting power they want to keep for themselves and how to distribute the rest of it to other individuals in the society. With every change in distribution there is a voting power penalty. In the second stage, the deliberative committee consists of the most powerful individuals, whose role is to bring forward corrections to the proposal. In the third stage, if an individual outside of the committee has kept some voting power for herself and disagrees with a correction, she can vote against it. If more than half of the voting power outside the committee is against a correction then it is discarded. If the committee or the proposal remain unchanged for two consecutive time periods, the deliberation stops and a referendum is held. The sum of the voting power penalties from past redistributions is added to the final negative vote, while the voting power of those who abstain is counted as positive. We show that this process will stop in finite time.

Keywords: deliberation, liquid democracy, social choice, consensus

1 Introduction

In representative democracies, opinion divides between those with legislative power and the electorate may occur if there is no interaction between the two. In the case where the members of the legislative bodies remain constant over
extended periods of time and across diverse subjects, these divides can become detrimental for democratic decision-making. Moreover, these constant members may not be experts in all the possible subjects that they are called to decide upon in their term. We believe that in order for representation to work at its best, there needs to exist a mechanism that lets society pick its representatives according to their perceived expertise on each subject.

Supposing that a group of individuals has drafted a status quo change proposal that affects a society, we suggest that the society forms a dynamic deliberative committee of its experts, whose function is to negotiate the proposal prior to it becoming permanent. We can think of this committee as a dynamic parliament. In our system, the committee’s members are iteratively updated through a modified version of liquid democracy, which we use as a mechanism that helps society identify its experts for the subject in question.

Liquid democracy [1], or more accurately liquid voting, is an extension of proxy voting [2] where a voter \( A \) can either vote for themselves or delegate their voting power to some other voter \( B \). Voter \( B \) can decide to vote with their increased voting power or delegate it all to another voter \( C \) and so on, making these delegations transitive. Liquid voting was designed to help in situations where a voter realises that they do not possess sufficient knowledge to make an informed decision and they feel more comfortable assigning their democratic right to another voter who they deem more capable of making an informed decision.

In our approach, we use a generalization of liquid voting which was briefly discussed in [4]. In this context, voter \( A \) can delegate shares of their voting power to one or more voters and they can transitively delegate shares of their voting power to others and so on. The voters may delegate shares that add up to less than 100% of their voting power and keep the rest for themselves. It can be seen as a hedging strategy for a voter in the sense that it reduces the risk induced by incomplete information about the knowledge possessed by the receiving voters on the subject in question. We use the term voting power portfolios to refer to these types of votes.

We represent status quo change proposals as vectors in \([0, 1]^s\), with \( s \) being the number of subjects that will be affected and \([0, 1]\) the range of values that each subject can attain. Every individual in the society has an opinion regarding the status quo change that depends on her information set and is represented by a strict preference over every possible status quo change that can affect these \( s \) subjects. We assume that for every individual there is a status quo change in \([0, 1]^s\) that she strictly prefers to any other proposal.

Each individual has a few options regarding the distribution of their voting power. If they take no action in the first stage of the initial period \( t = 0\),
they have chosen to keep all of their voting power to themselves. If they
want to distribute their voting power to one or more individuals then they
announce pairs of shares and recipients. These shares, along with those kept
for themselves, need to add up to 100%. In the first stage of every subse-
quent period, inaction means that they stick with their previous distribution.
In the second stage of every period there is no choice regarding the use of
voting power to be made. In the third stage, where a vote needs to be cast,
abstention is counted as positive voting power. If at any $t > 0$ they announce
a change in their distribution, they lose a percentage of their available voting
power. This lost voting power counts as negative in the final referendum.

The members of the dynamic deliberative committee of experts are the
individuals with the most voting power kept for themselves in the second
stage of each period. Each one of them can update the law proposal’s text
to be closer to their opinion by replacing or adding text whose percentage of
the corpus is less than or equal to the percentage of the total available voting
power of the committee that they hold at $t$.

In the third stage of every period, the individuals who are not in the
committee but hold non-zero voting power can still participate in the process
by voting against corrections proposed by the committee’s members. If more
than half of the voting power of the society outside the committee at $t$ is
against a specific correction, it is discarded and the member that proposed
it cannot propose another until the end of that period.

If the committee members and their voting power remain unchanged for
two consecutive time periods or if the proposal remains unchanged for two
consecutive time periods, the deliberation stops. We show that one of the
two conditions will be satisfied in finite time, so for the purposes of the model
we do not need to restrict the number of iterations that need to pass before
the final period of the process.

After the deliberation stops, all eligible voters participate in a referen-
dum in order to decide whether the updated status quo change proposal is
finalized or not. The referendum happens in two stages. In the first stage, all
individuals either do one last redistribution of their voting power or they do
nothing and they keep their last voting power portfolio. In the second stage,
they cast a vote with the voting power that they have kept for themselves.
The proposal is finalized if the positive voting power is more than the nega-
tive. Abstention counts as positive voting power and the loss from portfolio
rebalancings counts as negative.
2 Motivation

The model is inspired by the parliamentary democracy of modern Greece, where, every four years, citizens vote to fill the 300-seat parliament with members of the established political parties. The government is formed by the party, or the coalition of parties, that holds the parliamentary majority. The ruling party has the power to propose laws to the parliament, the parliament deliberates on them and then it votes whether to make them part of the legislation or not.

The most obvious vulnerability of this system is that if the government is formed by a party that holds the majority of the parliament without the need for coalition then said party can essentially legislate without the need for negotiations with its political adversaries.

Another vulnerability, which is common to all democracies with constant members of legislative bodies over extended periods of time, is that the same politicians, each one with their own expertise, deliberate and vote on a diverse set of laws during their term. Can these people be experts in all possible subjects, in order to be able to meaningfully negotiate the proposed laws?

In our system, a government would still have the power to propose laws for a fixed term but the electorate would be able to iteratively choose those who they believe to have sufficient knowledge on the subject of each proposal in order to negotiate it. The liquid part ensures that if a citizen does not wish to actively participate then they can distribute their voting power to the citizens who they deem capable of making a decision for them.

3 Model

Suppose society $N$ of $|N| = n < \infty$ individuals, deliberative committee of experts at time $t$ denoted $k_t$, consisting of $k < n$ individuals and group $G$ of $|G| = g$ individuals proposing the initial status quo change that may or may not be part of $N$, but if $G \subset N$ then $g < \frac{k}{3}$.

The status quo is denoted by the vector $\ell_{-1} \in [0, 1]^s$, where $s$ indicates the subjects that are of interest and $[0, 1]$ is the range of values that each subject can attain. The status quo change proposed by $G$ at time $t = 0$ is represented by the vector $\ell_0 \in [0, 1]^s$ where $\ell_0 \neq \ell_{-1}$.

**Definition 3.1.** Each individual $i \in N$ has an opinion regarding status quo changes, which is described by a total order $\preceq_i$ over the set $[0, 1]^s$ and an optimal proposal $\ell^i \in [0, 1]^s$ such that $\ell^i \preceq_i \ell$ for any $\ell \in [0, 1]^s$.

1 A total order is a binary relation $\succeq$ on a set $X$ which satisfies, for any $a, b, c \in X$, the following:
In every period $t$, there are three stages to the deliberation. In the first stage, each $i \in N$ decides whether to keep their voting power to themselves or to delegate a share between 0% and 100% of their voting power to one or more individuals, who can in turn transitively delegate shares of their voting power to other individuals etc. In the second stage, the top $k$ individuals with respect to their voting power are the experts chosen by society to form the deliberative committee at $t$. We describe the third stage in detail in section 4.

**Definition 3.2.** We denote the share of voting power that $i \in N$ assigns to $j \in N$ in the first stage of period $t$ as

$$w^i_t(j) \in [0, 1] \text{ such that } \sum_{j \in N} w^i_t(j) = 1. \tag{1}$$

If, at any $t$, some $i \in N$ has changed their delegation, or $w^i_t(j) \neq w^{i-1}_t(j)$ for any $j \in N$, she gets an exogenously given penalty $c \in (0, 1)$ that is multiplied by the number of times she has already changed her voting power distribution $\lambda^{i-1}_t$ plus one, with $\lambda^i_0 = 0$ for all $i$.

**Definition 3.3.** The cost multiplier of a voting power distribution change for $i \in N$ at time $t$ is

$$\lambda^i_t = \lambda^{i-1}_t + \mathbf{1}_{\{w^i_{t-1}(j) \neq w^i_t(j) \text{ for some } j \in N\}}. \tag{2}$$

The remaining percentage of voting power for $i \in N$ at time $t$ is

$$\alpha^i_t = [1 - c\lambda^i_t] \mathbf{1}_{\{1 > c\lambda^i_t\}}, \tag{3}$$

where the function $\mathbf{1}_{\{\text{condition}\}}$ is an indicator function that takes the value 1 if the condition is true and 0 otherwise.

Every $i$'s gross voting power at any period $t$ is equal to her initial voting power $p^i_0 \in [1, +\infty)$ for all $i \in N$, plus the sum of all the power shares that other individuals have transferred to her $\sum_{j \in N \setminus \{i\}} p^j_t(i)$. This is always discounted by the voting power penalty of any change in delegation up until that period.

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1. $a \succsim a$ (reflexivity)
2. if $a \succsim b$ and $b \succsim a$ then $a = b$ (antisymmetry)
3. if $a \succsim b$ and $b \succsim c$ then $a \succsim c$ (transitivity)
4. $a \succsim b$ or $b \succsim a$ (totality).
Definition 3.4. The total voting power that is available to every \( i \in N \) at time \( t \) is

\[
\tilde{p}_i^t = \alpha_t^i \left( p_0^i + \sum_{j \in N \setminus \{i\}} p_j^t(i) \right).
\] (4)

Definition 3.5. The voting power that \( i \in N \) has delegated to \( j \in N \) at time \( t \) is

\[
p_j^t(i) = w_j^t(i) \tilde{p}_i^t.
\] (5)

Definition 3.6. The voting power portfolio of \( i \in N \) at time \( t \) is denoted by

\[
\overline{p}_i^t = (p_j^t(i))_{j \in N}.
\] (6)

Definition 3.7. The voting function of \( i \in N \) regarding any proposal \( \ell \in [0, 1]^s \) is of the form

\[
v_i^t(\ell) = p_i^t(i) \mathbf{1}_{\{\ell \succsim i, \ell_{-i}\}}.
\] (7)

The experts that comprise the committee in the second stage of period \( t \) are the top \( \overline{k} \) individuals in the society with respect to their \( p_i^t(i) \). To break any possible ties, we compare the appeal of the competing experts, meaning that the winner of the tie is the one to whom the most voting power was transferred to directly.

Definition 3.8. The appeal of \( i \in N \) at time \( t \) is the sum of percentages of voting power that were assigned to her directly by every \( j \in N \)

\[
\overline{w}_i^t = \sum_{j \in N} w_j^t(i).
\] (8)

If the appeal of the competing experts is not enough to break a tie, we can use their popularity as a tie-breaking factor.

Definition 3.9. The popularity of \( i \in N \) at time \( t \) is the number of individuals in \( N \) that assigned a non-zero percentage of their voting power directly to her

\[
\bar{w}_i^t = \sum_{j \in N} 1_{\{w_j^t(i) \neq 0\}}.
\] (9)
4 Proposal Minting

At the end of the second stage of each time period there are at most \( \bar{k} \) candidate corrections to the proposal \( \ell_t \), since the members of the committee are the only ones that can propose corrections but they are not obliged to exercise their right. The size of each correction relative to the corpus of \( \ell_t \) is proportional to the percentage of the total voting power of the committee that each expert’s \( p_i^t(i) \) holds.

The updating of the corpus happens in the third stage of each time period and it happens in two steps. First, the members of the committee propose corrections and the citizens that are not in the committee vote for or against them. Second, amendments to some corrections are made and the citizens outside of the committee vote on them.

**Definition 4.1.** The total voting power of the committee at \( t \) is represented by \( p^k_t \), which is equal to

\[
p^k_t = \sum_{i \in k_t} p_i^t(i).
\] (10)

Assume that there is a function

\[ f : [0, 1]^s \times [0, 1]^s \to [0, 1] \]

denoting the percentage of the corpus of \( \ell_t \) that needs to change due to a correction brought forward by a member of the committee.

**Definition 4.2.** A correction to the corpus of \( \ell_t \) brought forward by a member of the committee \( i \in k_t \) is denoted by \( d\ell_{t+1}^i \) and it needs to satisfy the constraint

\[
f \left( d\ell_{t+1}^i, \ell_t \right) \leq \frac{p_i^t(i)}{p^k_t}.
\] (11)

A member of the committee \( i \in k_t \) can also build on the correction of another member of the committee \( j \in k_t \setminus \{i\} \) and their correction also needs to satisfy (11).

If an individual \( i \notin k_t \) with \( p_i^t(i) \neq 0 \) does not agree with a correction to the proposal at \( t \), they can vote against it. If more than half of the total voting power of the rest of the society is against a correction then it is blocked. In order to require minimum engagement by those outside of \( k_t \), we assume that their vote is positive by default unless they explicitly disagree with the correction.
The condition that needs to be satisfied in order for a correction $d\ell_{t+1}^i$ to become part of $\ell_{t+1}$ is

$$\sum_{j \in N \setminus \{k_t\}} v_j^i(d\ell_{t+1}^i) > \frac{1}{2} \sum_{j \in N \setminus \{k_t\}} p_j^i(j). \quad (12)$$

If two conflicting corrections $d\ell_{t+1}^i$ and $d\ell_{t+1}^j$ of some $i, j \in k_t$ both get accepted, $i$ and $j$ need to generate a new correction at the second step of the process, which needs to pass (12) or it will not be included in $\ell_{t+1}$.

If $d\ell_{t+1}^j$ builds upon $d\ell_{t+1}^i$ for any $i, j \in k_t$ and either $j$’s correction or both corrections get rejected we do not have any ambiguous outcomes. If both corrections are accepted then the richest one, in the sense that it builds upon the most corrections, is added to $\ell_{t+1}$. If one or more corrections upon which a correction $d\ell_{t+1}^j$ is built are voted down then $j$ can propose a different correction, whose inclusion in $\ell_{t+1}$ is subject to (12).

5 Referendum

The process stops naturally if no corrections have been made to the proposal between $T - 1$ and $T$. This can mean that either no member of $k_t$ brought forward a correction to the proposal at $T$ or that every correction that was brought forward at $T$ was voted down by the rest of the society. The process also stops if the resulting committee’s members and their voting power remain unchanged between $T - 1$ and $T$.

**Definition 5.1.** The proposal $\ell_T$ is finalized if

$$\sum_{i \in N} v_i^{\ell_T}(\ell_T) > \frac{1}{2} \sum_{i \in N} p_i^0, \quad (13)$$

or if the voting power of positive votes is more than the sum of voting power of negative votes and the voting power that was lost during the deliberation.

6 Results

**Theorem 6.1.** Suppose that if at any time $t$ it is true that $p_t^i(i) = p_t^j(j)$ for all $i, j \in N$ then the deliberation stops. There exists some $T < \infty$ such that the deliberation stops and society holds a referendum for $\ell_T$.

**Proof.** The process stops if the proposal remains unchanged for two consecutive iterations or if the members of the committee and their voting power
remain unchanged for two consecutive iterations. If the experts and their
evoting power change then some $i \in N$ has rebalanced their voting power
portfolio, which induced a voting power penalty. As the process continues,
more and more citizens will have diminished voting power. Since the num-
ber of individuals in the society is finite, and $c \in (0, 1)$, at some point all
citizens will have zero voting power and this will happen in finite time. By
the assumption, this will stop the deliberation.

\section{Future research}

In the future, we plan to work on more economic questions, such as the exis-
tence of Nash equilibrium committees, their stability and their representation
of citizens’ preferences. If we do that, we will be able to answer questions
such as the optimal size $K$ of the deliberative committee with respect to the
size of the society or the optimal penalty $c$ etc.

In future research, we could introduce the possibility to purchase voting
power at some point in the process, maybe with some quadratic cost as in \cite{3},
in order to let the citizens express a stronger feeling regarding a law proposal.

An assumption that can be relaxed is the fully connected societal graph.
In practice, people would not be interested in delegating shares of their voting
power to anyone in the society that is outside of their network unless they
are a ”celebrity”. The proximity of citizens in the societal graph could be a
restricting factor for the number of other citizens that a citizen can assign
their voting power to.

We could also allow for citizens to have a dynamic $\ell^t$ at $t + 1$ that may
depend on the $\ell^t$ of the previous period, the opinion of their representative
and the proposed law at $t$. We could also add a random noise $\epsilon \sim \mathcal{N}(0, \sigma^2)$
to their opinion in every period.

If the system was to be implemented in a society, a state could offer extra
initial voting power to its citizens as an incentive for them to contribute in
some positive way in public life.

\section{Conclusion}

We have introduced a form of governance that can bridge the gap that may
exist between the opinion of legislators and voters. If the legislators remain
invariant over time periods, we proposed the formation of a sort of variable
”parliament”, a dynamic deliberative committee of experts, that negotiates
status quo change proposals in order to ensure that the opinions of the voters
are taken into account during the deliberation. By making this committee
dynamic, we also give the voters the power to mitigate possible adverse effects
of corruption of its members. By getting constant signals on the sentiment
about the corrections of the proposal, the chosen experts can make more in-
formed decisions on what type of change to the status quo do the voters want.
It is our hope that the proposed system will help improve representation of
peoples’ preferences in collective decision making settings.

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