Adaptively tuned vibration absorber using cone based continuously variable transmission

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Abstract: In this paper, a new adaptive tuned vibration absorber (ATVA) design is presented for suppressing the resonance of primary system. A cone-based continuously variable transmission (cone CVT) is proposed to integrate a dynamic vibration absorber (DVA) for retuning itself in response to a time-varying excitation frequency. The natural frequency of ATVA can be dynamically tuned over a wide range by varying the gear ratio of the cone CVT, resulting in a change in the system's net mass moment of inertia. To predict the system response for various design parameters, a mathematical model is developed. The simulation of this model is employed through MATLAB software. The analytical model is numerically evaluated by tuning the frequency from 3.27 to 7.43 Hz. The results indicate that the proposed scheme may greatly increase the frequency bandwidth of the absorber while also suppressing the primary system's resonance. This indicates that the work is applicable to ATVA's practical applications.

Subjects: Mechanical Engineering; Structural Mechanical Engineering; Vibration; Acoustical Engineering

Keywords: Adaptive tuned vibration absorber; cone CVT; vibration attenuation effect; normalized bandwidth; neutralizer

1. Introduction
Generally, machines and dynamic system are exposed to a constant alternating force with a constant frequency. If the frequency of excitation is equal to or close to the structure's resonance frequency, this forcing may cause excessive vibrations. These vibrations may result in the destruction or sudden failure of machines or structures. As a result, the use of dynamic vibration absorbers (DVA) is proposed to mitigate the systems' extreme vibrations.

The dynamic vibration absorber application is invented by Frahm (Frahm, 1909). Because of its simple construction and apparent effect in vibration control applications, it has been widely employed in suppressing unwanted vibrations (Chung, 2004; Mori et al., 2010; Viet &
Park, 2011; Webster & Vaicaitis, 1992; Wu, 2008). The DVA, on the other hand, is a single-frequency vibration control device that operates exclusively within a narrow range of exciting frequencies. When the exciting frequency is variable, the DVA's vibration attenuation effect commonly decreases as a result of mistuning (Inman, 1994). As a result, in order to handle various excitation frequencies, the DVA must have an adjustable natural frequency. To achieve this propose, an adaptive tuned vibration absorber (ATVA) has been invented (Brennan, 1999; Sun et al., 1995; Von Flotow, 1994). Because it does not require high-level power signals for an active system, the ATVA consumes less energy than an active vibration absorber. Additionally, the ATVA is a fail-safe system since it can operate as a passive vibration absorber in the event of a power outage.

Many studies have developed new configurations for ATVA in order to increase the operating frequency range and improve the vibration reduction performance. Nguyen et al. (2020) conceived and designed an adaptive neural network controller for a MRE-based absorber to be installed on the suspension. Dong et al. (2020) developed an MR torsional vibration absorber with variable stiffness to control powertrain vibration. Kumbhar et al. (2018) developed an adaptive tuned vibration absorber using magnetorheological elastomer-shape memory alloy composite. Yuan et al. (2019) suppressed mode coupling chatter for robotic machining by using a semi-active magnetorheological elastomer absorber. Hu and Chen (2015) showed that adding an inerter to a damper without a spring has little effect on performance, whereas attaching the inerter to a vibration absorber with both a damper and a spring significantly improves performance. Jin et al. (2019) proposed two inerter-based configurations for beam-type structural vibration suppression. Under harmonic loads, they demonstrated that the inerter base vibration absorbers are more efficient than conventional DVAs, particularly when the mass ratio is modest. Almashhor and Asiri (2021) proposes a control approach based on variable mass tuned vibration absorber to suppress the primary system’s resonance. Homaeinezhad et al. (2020) developed novel intelligent actuators capable of absorbing the MIMO vibrational systems. Brzeski et al. (2015) proposed using a device with an inerter, which allows for precise changes in inertance. They demonstrated that a continuously variable transmission (CVT) with a gear-ratio control system can accomplish such a characteristic. In their later publication (Brzeski et al., 2017), they reported the experimental model of this innovative vibration absorber. To characterize the system, they conducted various experiments. They also used numerical simulation to show that the inerter base vibration absorber prototype with the CVT has significant dampening properties across a wide range of vibration frequencies. Sadeghian et al. (2022) suggest an inerter-based device for structural vibration control in which the inertance may be changed based on the excitation frequency. The suggested device is able to generate a variable inductance and frequency via the CVT’s core in a smooth and seamless manner.

This paper proposes new adaptive tuned vibration absorber for neutralization of vertical vibration of primary system. The key design goals are to create a system that can tune itself to a wide range of excitation frequencies and accomplish this tuning operation with low energy consumption. To achieve these goals, the design described in this study installs a cone CVT between absorber mass and primary mass for enabling effective vibration control over a range of frequencies. The natural frequency of the proposed ATVA can be dynamically tuned over a wide range by adjusting the position of the transmission wheel. The suggested concept has the advantages of a passive vibration absorber with low power consumption and the flexibility of an active device. This paper is organized as follows: In Section 2, working principles of the proposed cone CVT are presented. In Section 3, mathematical model and bandwidth of the proposed ATVA are derived. In Section 4, lookup table is proposed to tune the position of the wheel. In Section 5, the results of numerical study are presented, while the last section summarizes the obtained results.
2. Working principle of the proposed cone CVT

Figure 1 shows the isometric view and schematic model of the proposed cone CVT. The cone CVT consists of two cones which are covered with friction material and a wheel which is connected to a slider. The wheel is situated between the input and output cones. The moment is transmitted from input cone to output cone via a wheel. The gear ratio are altered by changing the ratio of the radii between the input and output cones by adjusting the position of the wheel. In this model, the slider is driven by a stepping motor through a ball screw to tune the position of the wheel.

It is assumed that no slip occurs between cones and the wheel; therefore, the gear ratio between the input cone and the output cone can be expressed as:

\[
G = \frac{\theta_o}{\theta_i} = \frac{DL - (D - d)z\cos\alpha}{dl + (D - d)(z\cos\alpha + d_w\sin\alpha)}
\]  

Where

- \( G \) = The gear ratio
- \( D \) = The bigger diameter of the cone
- \( d \) = The smaller diameter of the cone
- \( l \) = The length of the cone
- \( \alpha \) = Taper angle
- \( d_w \) = The diameter of the wheel
- \( z \) = The wheel position
- \( \theta_i \) = The angular displacement of input cone
- \( \theta_o \) = The angular displacement of output cone
It is noticed that the gear ratio of cone CVT depend on the wheel position.

The torque exerted on the shaft of input cone can be written as

\[ T_i = [I_{ci} + (I_{co} + I_{fw})G^2] \dot{\theta} \]  \hspace{1cm} (2)

Where

- \( T_i \) = The torque exerted on input cone
- \( I_{ci} \) = The moment of inertia of input cone
- \( I_{co} \) = The moment of inertia of output cone
- \( I_{fw} \) = The moment of inertia of flywheel

3. Mathematical modelling of a presented ATVA

Figure 2 presents the schematic of primary structure which is coupled with a proposed ATVA. The primary structure is modeled by two springs that are connected in parallel and a mass and excited by harmonic force. The ATVA consists of a vibrating mass and a spring. A CVT gearbox is also mounted between absorber mass and primary mass. The absorber mass is connected to the CVT gearbox by using rack and pinion mechanisms.

The application of Newton's second law of motion to each of the masses gives the equations of motion:

\[ k_s(x_2 - x_1) + \left[ \frac{I_{ci}}{r^2} + \frac{(I_{co} + I_{fw})G^2}{r^2} \right] (x_2 - x_1) - k_p x_1 + F(t) = m_p \ddot{x} \]  \hspace{1cm} (3)

\[ - \left[ \frac{I_{ci}}{r^2} + \frac{(I_{co} + I_{fw})G^2}{r^2} \right] (x_2 - x_1) - k_s(x_2 - x_1) = m_s \ddot{x} \]  \hspace{1cm} (4)

Figure 2. Proposed ATVA attached to 1 dof primary system.
where $x_1$ is the displacement of the primary structure, $x_2$ is the displacement of the absorber mass, $m_s$ is the absorber mass, $m_p$ is the primary structure mass, $k_s$ is the stiffness of ATVA, $k_p$ is the stiffness of the primary structure, $r$ is the pitch diameter of the pinion that cooperates with the moving rack, and $F(t) = F_0 \sin \omega t$.

Holding the primary system fixed, the equation of motion of the absorber mass alone will be as follows:

$$
\left[ m_s + \frac{I_{ci}}{r^2} + \frac{(I_{co} + I_{fw})G^2}{r^2} \right] \ddot{x} + k_s x_2 = 0
$$

(5)

Therefore, the natural frequency $\omega$ of the presented DVA is

$$
\omega_s = \sqrt{\frac{k_s}{m_s + M}}
$$

(6)

Where

$$
M = \frac{I_{ci}}{r^2} + \frac{(I_{co} + I_{fw})G^2}{r^2}
$$

Equation (6) indicates that the natural frequency can be tuned by adjusting the position of the wheel.

By assuming harmonic solution,

$$
x_1 = X_1 e^{i \omega t} \quad \text{and} \quad x_2 = X_2 e^{i \omega t}
$$

The steady-state amplitude of primary structure can be expressed as

$$
X_1 = \frac{F_0 (-k_s + (M + m_s) \omega^2)}{[k_s - M \omega^2 + k_p - \omega^2 m_p] [-k_s + (M + m_s) \omega^2] + (k_s - M \omega^2)^2}
$$

(7)

By defining

$$
\alpha^2 = \frac{k_p}{m_p}, \quad \gamma = \frac{\omega}{\omega_p}, \quad g = \frac{\omega_s}{\omega_p}, \quad \delta_w = \frac{F_0}{k_p}, \quad \mu = \frac{m_s}{m_p}, \quad \beta = \frac{M}{m_p}
$$

From the definitions above, the magnitude displacement dynamic amplification factor $\frac{|X_1|}{\delta_w}$ of the primary structure can be derived by

$$
\frac{|X_1|}{\delta_w} = \frac{(g^2 - \gamma^2)}{\sqrt{\gamma^4 + 1}} \sqrt{\gamma^4 - [(1 + g^2) + \mu g^2] \gamma^2 + g^2}
$$

(8)

This system’s bandwidth (BW) can be defined as the range of frequencies at which the magnitude displacement dynamic amplification factor is below a specific threshold, $T_0$. To determine the bandwidth, the Eq. (8) should be equated to the provided value of $T_0$ and solve for the two $\omega^2$ values that will yield the lower and upper bounds of the operating range as follows:
$$\omega_2^2 = \frac{\omega_L^2 \left( \frac{1}{\tau_2} + 1 + g^2 + \mu g^2 \right) - \omega_p^2 \sqrt{\left( \frac{1}{\tau_2} + 1 + g^2 + \mu g^2 \right)^2 - 4 \left( \frac{\omega_L^2}{\omega_p^2} + 1 \right) \left( g^2 + \frac{g^4}{\tau_2^2} \right)}}{2 \left( \frac{\omega_L^2}{\omega_p^2} + 1 \right)}$$

(9)

$$\omega_R^2 = \frac{\omega_p^2 \left( 1 + g^2 + \mu g^2 - \frac{1}{\tau_2} \right) + \omega_L^2 \sqrt{\left( 1 + g^2 + \mu g^2 - \frac{1}{\tau_2} \right)^2 - 4 \left( \frac{\omega_L^2}{\omega_p^2} + 1 \right) \left( g^2 - \frac{g^4}{\tau_2^2} \right)}}{2 \left( \frac{\omega_L^2}{\omega_p^2} + 1 \right)}$$

(10)

The normalized bandwidth of this system can be determined as

$$BW = \frac{\omega_R - \omega_L}{\sqrt{\omega_L \omega_R}}$$

(11)

4. Tuning of the wheel position by lookup table

Vibration of the primary structure is eliminated if the excitation frequency of the primary system is set equal to the absorber natural frequency. The natural frequencies of the ATVT are dependent on the position of the wheel, which should be adjusted according to the excitation frequency. In this research, we propose a lookup table to tune the position of the wheel. The natural frequency versus the position of the wheel \((\omega_s - z)\) table is first construct. By measuring the excitation frequency, the position of the wheel can then be set according to the table. The block diagram of the self-tuning algorithm is sketched in Figure 3.

Another approach for tuning the position of the wheel is to use the conventional PID controller. This method begins by receiving the external excitation frequency measured by the sensor and using Equation 6 to calculate the desired position of the wheel. The calculation result is then forwarded to the PID controller, which produces and delivers the control signal to move the
stepper motor in order to match the natural frequency of the vibration absorber with that of the incoming vibrations.

5. Numerical simulation of the proposed ATVA
Numerical simulations using MATLAB with ODE45 solver was conducted in order to generate data for the dynamical analysis of the system. The simulation parameters are shown in Table 1.

Figure 4 presents the absorber natural frequency (Hz) versus the displacement of the wheel z(mm) for different values of the moment inertia of flywheel. During the experiments, the moment inertia of flywheel can be adjusted by changing flywheel with different moment of inertia. The obtained results show that natural frequency of the proposed ATVA increases with increasing the position of the wheel. Furthermore, increasing the moment inertia of flywheel increases the tunable frequency range. In addition, this parameter is more effective at low frequency range.

Table 1. Parameter values used for MATLAB simulation of the proposed ATVA

| Symbol | Value | Units |
|--------|-------|-------|
| \(m_i\) | 5 | kg |
| \(k_i\) | 16,000 | N/m |
| \(m_p\) | 20 | kg |
| \(k_p\) | 30,126.805 | N/m |
| \(i_{ci}\) | 0.00475 | Kg.m^2 |
| \(i_{co}\) | 0.00475 | Kg.m^2 |
| \(I_{fw}\) | 0.005 | Kg.m^2 |
| \(F_o\) | 30 | N |
| \(D\) | 120 | mm |
| \(d\) | 40 | mm |
| \(l\) | 300 | mm |
| \(\alpha\) | 7.594 | Degree |
| \(d_w\) | 74 | mm |
| \(r_o\) | 0.1 | |
| \(r\) | 50 | mm |

Figure 4. The natural frequencies of the proposed ATVA under different moment inertia of flywheel.
Figure 5 shows the normalized bandwidth as a function of the position of the wheel. It is seen that the higher the wheel position, the greater the normalized bandwidth. On the other hand, decreasing the moment inertia of flywheel improves the normalized bandwidth.

Figure 6 shows the transmissibility of the proposed ATVA. As shown in Figure 6(a), the antiresonance frequency of the system appears at about 3.94 Hz, and the antiresonance frequency slightly moves to higher frequency as the position of the wheel increases. Additionally, as shown in Figure 6, once the wheel's position is fixed, vibration attenuation occurs within a very narrow bandwidth (BW) in the vicinity of the antiresonance frequency. According to Figure 5, BW can be highest when z is highest, given a target transmissibility ($T_{o}$) of 0.1. Even for $z = 290\text{mm}$ and $I_{fw} = 0.005\text{kg.m}^2$, BW turns out to be only 2.4 percent. Thus, in the absence of an adaptive DVA, the system's vibration attenuation performance can decline significantly in the presence of a modest shift in the excitation frequency.

Numerical simulations are conducted on the closed-loop system consisted of the proposed tuned vibration absorber of Eqs. (3), (4), and (6), self-tuning algorithm, and the primary structure which is modeled as a single degree of freedom system. In the simulation, slight damping ratios are assumed. First, the ATVA is turned on with its transmission wheel initially positioned at 180 mm and at excitation frequency of 6.77 Hz, which is close to the natural frequency of primary structure. As shown in Figure 7, the adaptation is activated after 30 s. The wheel position is automatically tuned toward 200 mm, at which the natural frequency of the ATVA matches the excitation frequency. The excitation frequency is then raised to 6.96, and it is seen that the wheel position is automatically re-tuned to 220 mm, at which the natural frequency matches the excitation frequency again.

To evaluate the vibration attenuation performance of the proposed ATVA, a series of experiments are conducted. First, the frequency response of the primary structure without the ATVA attachment is obtained using an external excitation frequency change between 3.27 and 7.43 Hz. Then, the ATVA is appended to the primary structure and the self-tuning algorithm is enabled. Using the same frequency change scenarios, Figure 8 shows RMS values of the primary structure displacement with respect to excitation frequencies. As can be seen in Figure 8, bare primary structure, the system has a large resonance peak at 6.177 Hz. However, in the presence of ATVA, vibration of the primary structure is significantly reduced over the operation range of frequencies.
Figure 6. The amplitude of vibration of primary structure at various wheel position. (a) $z = 30\text{mm}$, (b) $z = 80\text{mm}$, (c) $z = 120\text{mm}$, (d) $z = 160\text{mm}$, (e) $z = 200\text{mm}$, (f) $z = 270\text{mm}$.
Figure 7. Time response of the closed-loop system: (a) displacements of the primary structure and the absorber mass; (b) the wheel position.

(a) Tuned @ 6.77 Hz  Re-Tuned @ 6.96 Hz  Adaptation activated

(b)

Figure 8. Displacement RMS values of the primary structure with/without the ATVA at same excitation states.
6. Conclusions
In this study, a cone CVT was used in a DVA whose natural frequency could be tuned by means of adjusting the position of transmission wheel. The ATVA was used to mitigate the vibration of the structure in conjunction with a table-lookup algorithm. The numerical approach was used to test the proposed ATVA. The effect of ATVA parameters $I_{th}$ and $z$ on the vibration suppression and absorber performance and broadband frequency is studied. The simulation results demonstrated that the structure vibration could be reduced effectively using the proposed ATVA rather than using passive-type absorbers for a wide frequency range. The acquired results can be utilized to guide the development of an appropriate ATVA. Future studies may incorporate a cone CVT into a pendulum-style vibration absorber. In this scheme, the pendulum will be directly coupled to the CVT gearbox to enable natural frequency tuning.

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