The $B_c \rightarrow J/\Psi KD$ weak decay and its relation with the $D_{s0}^*(2317)$ resonance

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Abstract. We study the influence of the $D_{s0}^*(2317)^+$ resonance in the decay $B_c \rightarrow J/\Psi DK$. In this process, we consider that the $B_c$ meson decays first into $J/\psi$ and the quark pair $c\bar{s}$, and then the quark pair hadronizes into $DK$ or $D_s\eta$ components. The final state interaction, generating the $D_{s0}^*(2317)^+$ resonance, is described by the chiral heavy meson unitary approach. With the parameters chosen in order to match the pole position of the $D_{s0}^*(2317)^+$, we obtain the $DK$ invariant mass distribution of the decay $B_c \rightarrow J/\psi DK$, and also the rate for $B_c \rightarrow J/\psi D_{s0}^*(2317)$. We predict the ratio of these two magnitudes. This decay mechanism has not been measured yet, and it would provide insight about the nature of this resonance.

1 Introduction

In this talk we show the results given in Ref. [1] where the role of the unmeasured decay $B_c \rightarrow J/\Psi KD$ in the molecular nature of the $D_{s0}^*(2317)^+$ resonance was discussed. Further references and details can be found in this work. The scalar resonance $D_{s0}^*(2317)^+$ was first observed by the BABAR Collaboration [2] and later it was confirmed by several collaborations. Before the BABAR experiment there were some predictions about a similar state. These predictions involved different quark models and potentials, and also some QCD calculations, predicting a meson mass larger than the experimental value. After the BABAR experiment, many theoretical groups performed research on the $D_{s0}^*(2317)^+$ state, see for example Refs. [3, 4]. Since the mass of the $D_{s0}^*(2317)^+$ is close to the threshold of the $DK$ system, being the difference of about 50 MeV, the molecular state interpretation seems natural. The $D_{s0}^*(2317)^+$ state was studied in the frame of molecular, $KD$ mixing with $c\bar{s}$ state, four-quark state, and the mixture of two-meson and four-quark state. It is worth mentioning that the lattice QCD simulations also have provided some hints about this issue after the BABAR discovery. Results for the $DK$ scattering length were extrapolated to physical pion masses with the help of unitarized chiral perturbation theory, and by means of the Weinberg compositeness condition [5] the amount of $KD$ content in the $D_{s0}^*(2317)^+$ was determined, resulting in a sizable fraction of the order of 70% within errors (a more accurate determination of that fraction, also of the order of 70% was done in [6]).

Here we shall give the $DK$ invariant mass distribution in the decay $B_c \rightarrow J/\psi DK$, from which information on the internal structure of the $D_{s0}^*(2317)^+$ state will be obtained.

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2 Formalism

As a first step, we write the leading mechanisms describing the weak process by means of which the heavy quark $b$ in the $B_c$ meson decays into three lighter quarks. This is depicted in Fig. 1, and our first assumption will be that the matrix element for these transitions will be factorized and assumed to be constant in the small range of the $K^*D$ invariant mass that we will study.

A $c\bar{s}$ pair will combine into a $J/\psi$, so we need to split the $c\bar{s}$ quark pair into meson states. The interaction of the $J/\psi$ with the pseudoscalars will not be considered. The hadronization of the $c\bar{s}$ pair is achieved introducing a $q\bar{q}$ combination with the quantum numbers of the vacuum: $c\bar{s}(u\bar{u}+d\bar{d}+s\bar{s}+c\bar{c})$.

The splitting of this product of quark fields into meson states is done introducing the $J$ coefficient. Our product of quark fields in terms of meson fields is

$$
\phi = \begin{pmatrix}
\frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{2}} + \frac{\eta''}{\sqrt{6}} & \pi^+ & K^+ & D^0 \\
\pi^- & \frac{\eta}{\sqrt{3}} - \frac{\eta'}{\sqrt{2}} + \frac{\eta''}{\sqrt{6}} & K^0 & D^+
\\
K^- & \bar{K}^0 & \frac{\eta'}{\sqrt{3}} & D^0
\\
D^0 & \bar{D}^+ & D^0 & \eta_c
\end{pmatrix},
$$

(1)

then our product of quark fields in terms of meson fields is

$$(\phi^2)_{43} = D^0K^+ + D^+K^0 + \eta_D^+D^*_s - \frac{1}{\sqrt{3}}\eta_D^+ + \sqrt{\frac{2}{3}}D^*_s\eta' = \sum j h_j P_j P'_j,$$

(2)

where in Eq. (2) $j$ labels the different channels. We consider only the meson channels whose mass is close to the $K^*D$ threshold, we keep the $D^+K^0$, $D^0K^+$ and $\eta D^*_s$ channel, namely $j = 1, 2, 3$. As shown in Fig. 2, the amplitude for the final $J/\psi K^0 D^*$ state is written in terms of the common $V_P$ constant factor which accounts for the weak production and hadronization mechanisms. There are contributions to the final $K^*D$ state that involve intermediate meson loops of the different channels. The interaction of the meson channels is denoted by the $t_{i,j}$ vertex, which is obtained after the unitarization of the $S$-wave leading order amplitudes, using the heavy meson chiral Lagrangian from Ref. [4]. The expression of these tree level amplitudes can be found in Ref. [1], we will denote them here as $V_{i,j}$. The unitarization is achieved solving the on-shell version of the factorized Bethe-Salpeter equation in coupled channels:

$$r^{-1} = V^{-1} - G.$$

(3)

The $G$ that appears in Eq. (3) is a diagonal matrix in the coupled channels basis, with the loop function of the different meson channels. The loops are integrated using dimensional regularization,
and regularized including a subtraction constant, called \(\alpha\), at some scale \(\mu\) that we take as 1.5 GeV. The analytic expression can be found in Ref. [1]. The meson dynamics will generate the \(D^*_s(2317)\) resonance, manifesting as a pole of the amplitudes in the complex energy plane. In our approach, we fine tune the free parameter \(\alpha\) in order to generate a pole in \(t\) in the physical Riemann sheet of the complex energy, at the position of the mass of the \(D^*_s(2317)\) resonance, below the \(KD\) threshold.

What we have is a bound state, which is not strictly true but it is a good picture if we recall that the \(D^*_s(2317)\) width comes from an isospin violating mode, and is very small. We calculate the coupling \(g_i\) of the resonance to the different channels in terms of the residue of \(t\) at the pole position \(s_p\), since

\[
    t_{i,j} \approx g_i g_j / (s - s_p)
\]

for values of the invariant \(s\) close to the pole. We find a larger coupling to the \(KD\) channels \(g_{K^+D^0} = g_{K^0D^+} = 7.4\) GeV but the \(\eta D_s\) coupling is also sizeable, about \(-6.0\) GeV. With the unitarized amplitudes of Eq. (3) we build the decay amplitude which enters in the invariant \(KD\) mass distribution of the decay,

\[
    \frac{d\Gamma}{dM_{\text{inv}}} = A^2 p_{J/\psi}^3 p_{KD} / (2\pi)^3 m_B^2 \left| h_1 + \sum_{i=1}^3 h_i G_i t_{i1} \right|^2.
\]

In order to derive Eq. (4) we consider that the \(V_p\) vertex must be a \(P\)-wave vertex, \(V_p = \sqrt{3} A \cos \theta\), in order to match total spin conservation, since the pseudoscalars interact in \(S\)-wave in our approach. The amplitude for the decay reads \(V_p \left( h_1 + \sum_{i=1}^3 h_i G_i t_{i1} \right)\), see Fig. 2. In a similar way we are also able to calculate the coalescence, that is, the width for the direct production of the \(D^*_s(2317)\) in the \(B_c\) decay: \(\Gamma(B_c \rightarrow J/\psi D^*_s(2317))\),

\[
    \Gamma(B_c \rightarrow J/\psi D^*_s(2317)) = \frac{A^2 p_{J/\psi}^3}{8\pi m_{B_c}^2} \left| \sum_{i=1}^3 h_i G_i g_i \right|^2_{s=s_p},
\]

and as we see in Eq. (5) it is given in terms of the coupling of the resonance to the different channels. The ratio of the \(KD\) invariant mass distribution (Eq. (4)) and the coalescence (Eq. (5)) provides a free parameter magnitude tied to the molecular nature of the \(D^*_s(2317)\) resonance. We define this ratio removing the phase space and multiplying by the necessary factors in order to make it dimensionless,

\[
    \frac{d\tilde{\Gamma}}{dM_{\text{inv}}} = M_{B_c}^2 \frac{(d\Gamma/dM_{\text{inv}})/p_{J/\psi}^3 p_{KD}}{\Gamma(B_c^+ \rightarrow J/\psi D^*_s(2317))/ p_{J/\psi}^3}.
\]

### 3 Results

In the left panel of Fig. 1 we show the results of the \(KD\) invariant mass distribution, and on the right panel we find the result of the ratio \(d\tilde{\Gamma} / dM_{\text{inv}}\) defined in Eq. (6). It is interesting to compare the
Figure 3. (Left) $K D$ invariant mass distribution for the $B_c \rightarrow J/\psi K D$ decay, the dashed line is the phase space. Both curves are normalized in the energy range showed; (Right) The ratio of the $K D$ invariant mass distribution and the coalescence without the phase space factor.

$K D$ invariant mass distribution with the phase space, the solid and dashed lines respectively. The $A$ constant in Eq. (5) is chosen in a way that the invariant mass distribution is normalized in the energy range of the plot. The phase space is also normalized in the same energy range. We clearly see that both curves have a different behaviour near the threshold. Even if the phase space is relatively small, we observe a huge enhancement in the $K D$ invariant mass distribution. This enhancement has been also observed experimentally in several decays of heavy mesons, see for example in Ref. [7] the $B'$s and $B_s$'s decays into $K D$ final states. We present an unmeasured reaction where this feature is also predicted, but in our approach is due to the dynamically generation of the $D_{s0}^*$($2317$). Furthermore, we give the ratio of the $K D$ invariant mass distribution and the $D_{s0}^*$($2317$) production. In the right panel of Fig. 3 we can observe the features of this magnitude. It has a finite value at threshold, and it looks like the tail of a peak located below. Here this is indeed due to the presence of the pole of the $D_{s0}^*$($2317$) below the threshold, because Eq. (6) is nothing but the ratio of the amplitudes of Fig. 2 and the coalescence. Since we are forcing the presence of the pole with the fine tuning of the subtraction constant $a$, this is what explains the shape of the plot shown in the right panel of Fig. 3. The comparison of our predictions with the measurement results of the process proposed here would provide further information about the molecular nature of the $D_{s0}^*$($2317$) resonance. As a final comment, we mention that possible $q\bar{q}$ small components were considered adding a CDD pole to the meson’s potential and using modified Weinberg compositeness rule to match different $K D$ amounts obtained from the lattice studies, see Refs. [1, 5, 6]. This contribution does not change the conclusions and general features shown here.

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