Measurement of the fractional orbital angular momentum of asymmetric laser beams by using two cylindrical lenses

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Abstract. Here we propose and study both theoretically and experimentally a simple and high-efficient technique for measuring the orbital angular momentum (OAM) of paraxial laser beams. The technique uses two intensity distributions measured in the foci of two perpendicular cylindrical lenses. For these distributions, first-order intensity moments are calculated, which allow the OAM to be easily obtained. The experimental error increases from ~1% for small fractional OAM (up to 4) to ~8% for large fractional OAM (up to 30). We also show numerically that the proposed technique allows to determine the OAM if the beam is distorted by a phase diffuser in the initial plane.

1. Introduction

Laser beams with the orbital angular momentum (OAM) are actively studied and have numerous practical uses, including optical communications and micromanipulation [1, 2]. Modern reviews [3, 4] and monographs [5] in this area have recently been published. Thus, developing simple and high-efficiency techniques for measuring the OAM of such laser beams has been a topical issue.

There are many tools to measure the OAM of paraxial beams, including diffractive optical elements [6, 7], transformation optics [8], metasurfaces [9], interferograms [10, 11], triangular apertures [12], annular diffraction gratings [13], and cylindrical or astigmatic lenses [11]. Not all these techniques allow measuring fractional OAM values, whereas an arbitrary OAM can be obtained by measuring the intensity with a cylindrical lens [14–16] and computing intensity moments [15–17].

In [15, 16], a technique is described that uses a single cylindrical lens for approximate measuring the OAM of rotationally symmetric beams. Meanwhile, for arbitrary laser beams two cylindrical lenses are needed. Here we generalize the proposed in [15, 16] method of fractional OAM measurement for asymmetric paraxial beams. In addition, we show numerically that the generalized technique with two lenses allows to determine the OAM if the beam passes through a phase diffuser with a Gaussian correlation function.

2. Measuring the OAM from intensity distributions in foci of two cylindrical lenses

In Refs. [15, 16] it was proposed that OAM should be determined from a single measurement of the intensity in the focus of a cylindrical lens. But this method is only suited for optical vortices with a circularly symmetric radial component of the beam, whose amplitude is given by...
where \((r, \varphi, z)\) are the cylindrical coordinates, \(\mu\) is the integer or fractional topological charge. Below, we describe a generalization of this technique to measure the OAM of an arbitrary paraxial beam from two intensity distributions in the focal planes of two cylindrical lenses positioned perpendicularly to each other and placed in identical beam branches outgoing from a splitting cube (Fig. 1).

\[
E_e (r, \varphi, z) = \exp(-i \mu \varphi) \Phi (r, z),
\]

(1)

where \((r, \varphi, z)\) are the cylindrical coordinates, \(\mu\) is the integer or fractional topological charge. Below, we describe a generalization of this technique to measure the OAM of an arbitrary paraxial beam from two intensity distributions in the focal planes of two cylindrical lenses positioned perpendicularly to each other and placed in identical beam branches outgoing from a splitting cube (Fig. 1).
where \( q_i(z) = 1 - iz_0/z \). \( x = (z_0/z)^i (\rho/w_0)^i [j/(2q_i(z))] \), \( I_i(x) \) is a modified Bessel function.

From Eq. (8), an intensity null is seen to occur on the optical axis (\( \rho = 0 \)), while the entire intensity pattern is not circularly symmetric. With the modified Bessel functions in Eq. (8) described by a complex argument, the argument takes real values only in the far-field diffraction zone \((z >> z_0)\). The OAM of the field of Eq. (8) is the same as that of the initial field of Eq. (6) and is equal to Eq. (7).

4. Numerical simulation

In this section, we numerically verify the proposed technique, using as illustration the field (8), which is generated with the aid of the phase \( \mu \phi \) formed on the liquid-crystal display of a SLM. The modeling is based on the numerical calculation of a Fresnel transform. The simulation parameters are: wavelength of light, \( \lambda = 532 \) nm, Gaussian beam waist, \( w_0 = 1 \) mm, focal length of the cylindrical lens, \( f = 1 \) m, calculation domain: \(-R \leq x, y \leq R, (R = 5 \) mm), \( z = f \), and discretization step on both coordinates, \( \Delta x = \Delta y = 20 \) \( \mu \)m. Shown in Fig. 2 are phase patterns in the initial plane \( z = 0 \) [Fig. 2(a, d, g, k)] and intensity patterns in the focus of the lens whose axis is parallel to the \( x \)-axis [Fig. 2(b, e, h, l)] and \( y \)-axis [Fig. 2(c, f, i, m)] for a Gaussian beam having passed through a SPP with the topological charge 3.00 [Fig. 2(c-e)], 3.25 [Fig. 2(d-f)], 3.50 [Fig. 2(g-i)], and 3.75 [Fig. 2(j-l)]. The intensity patterns in the focal plane of the cylindrical lens [Fig. 2(b, c, e, f, h, i, k, l)] are shown in the domain \(-R/2 \leq x, y \leq R/2\).

According to Eq. (7), for the above-indicated values of the topological charge, the normalized OAM should be equal to 3.00; 3.09, 3.50, and 3.91, respectively. Actually, by substituting in Eq. (5) intensity values corresponding to the intensity distribution in Fig. 2, we obtain the following OAM values: 2.98, 3.06, 3.45, and 3.87. The respective error is 0.7%, 1%, 1.4%, and 1%.

In a similar way, measurement of the large-valued fractional OAM can be numerically simulated. Since in this case the optical vortex has a stronger divergence, the computation domain \( R \) needs to be increased. The simulation parameters are: wavelength, \( \lambda = 532 \) nm, waist radius, \( w_0 = 1 \) mm, focal length of the cylindrical lens, \( f = 1 \) m, computation domain, \(-R \leq x, y \leq R, (R = 10 \) mm), \( z = f \), the increment on both coordinates, \( \Delta x = \Delta y = 10 \) \( \mu \)m.

Figure 3 shows phase patterns [Fig. 3(a)] in the initial plane \( z = 0 \) and intensity patterns in the focus of the cylindrical lens whose axis is parallel to the \( x \)-axis [Fig. 3(b)] and \( y \)-axis [Fig. 3(c)] for a Gaussian beam having passed through a SPP with the topological charge 30.3.

According to Eq. (7), for the topological charge 30.3, the normalized OAM should be equal to 30.15. Substituting the intensities from Fig. 3 into Eq. (5), we find that OAM is 28.87, with the error amounting to 4%. Properly increasing the computational domain, the error of OAM determination can be further decreased. However, in practice the accuracy cannot be improved this way as when measuring low intensities, the accuracy is not improved due to decreasing signal-to-noise ratio.
Figure 3. (a) Phase pattern in the initial plane $z=0$ and intensity pattern in the focus of the cylindrical lens whose axis is parallel to the (b) $x$-axis and (c) $y$-axis for a Gaussian beam having passed through a SPP with topological charge $30.3$.

5. Experimental determination of the OAM by measuring two intensity distributions

The experiments were conducted with an optical setup in Fig. 1. A Gaussian beam of wavelength 532 nm was expanded and collimated with a 40-$\mu$m circular pinhole PH. Then it was normally incident onto a transmission SLM (a 1024 × 768-pixel HOLOEYE LC 2012, with 36-$\mu$m pixels and a 36.9 × 27.6-mm operating region). Spherical lenses of focal lengths 350, 250, and 150 mm were utilized as lenses L1, L2, and L3. Lenses L2, L3 and a spatial filter F were used to filter the light field from the SLM, thus cutting off an inoperative zero diffraction order. The mutually perpendicular cylindrical lenses CL1 and CL2 had a 500-mm focal length. These lenses were imaged onto the SLM-display, with the Gaussian beam waist being about 6.5 mm. The intensity pattern in the focus of the cylindrical lenses CL1 and CL2 was recorded with 3264 × 2448-pixel CCD-cameras CCD1 and CCD2 (with a 1.67 µm × 1.67 µm pixel).

Figure 4 shows intensity distributions obtained in the foci of the lenses CL1 and CL2 by the SLM-aided optical vortices with the topological charge $3$, $3.25$, $3.5$, $3.75$, and $30.3$. The normalized OAM from Eq. (6) based on the intensities in Fig. 4 was found to be $3.02$, $3.10$, $3.47$, $3.96$, and $27.78$ respectively.

For comparison, Table 1 gives OAM values numerically and experimentally derived using two cylindrical lenses. For the beams outgoing from a SPP with a fractional topological charge of $3.00$; $3.25$; $3.50$; $3.75$; $30.3$, Table 1 gives theoretical OAM values derived from Eq. (7), alongside values obtained from Eq. (5) using numerically and experimentally derived intensity distributions.

According to Table 1, for the topological charges less than $30$, the experimental error in determining the OAM of a fractional-topological-charge laser beam by measuring two intensity distributions in the foci of two cylindrical lenses rotated 90 degrees relative to each other does not exceed $8\%$. 
Table 1. OAM values calculated theoretically using Eq. (7) and numerically using Eq. (5) based on numerically and experimentally derived intensity distributions.

| μ  | 3.00 | 3.25 | 3.50 | 3.75 | 30.3 |
|----|------|------|------|------|------|
| OAM, Eq. (7) | 3.00 | 3.09 | 3.50 | 3.91 | 30.15 |
| OAM, numerically, Eq. (5) | 2.98 | 3.06 | 3.45 | 3.87 | 28.87 |
| Error, numerically, % | 0.7 | 1.0 | 1.4 | 1.0 | 4.25 |
| OAM, experiment, Eq. (5) | 3.02 | 3.10 | 3.47 | 3.96 | 27.78 |
| Error, experimental, % | 0.7 | 0.3 | 0.9 | 1.3 | 7.9 |

6. Determination of the OAM of a light beam passed through a phase diffuser

Figure 5 shows the initial distribution of intensity (Gaussian beam) and phase (fractional optical vortex), as well as intensity distribution after propagation in space. We used the following parameters: free space wavelength $\lambda = 532$ nm, waist radius $w_0 = 0.5$ mm, topological charge $\mu = 10.5$, propagation distance $z = 50$ cm. Calculation area is $-R \leq x, y \leq R$, where $R = 4$ mm.

![Figure 5](image)

Figure 5. Initial distribution of intensity (a) and phase (b) as well as intensity distribution after propagation in space (c) of a Gaussian optical vortex with a fractional topological charge.

Figure 6 shows the same phase in the initial field but after passing a random phase diffuser (with the correlation radius $\sigma = 50$ $\mu$m and with phase ranging from $-\pi/2$ to $\pi/2$) as well as as intensity distribution after propagation in space and intensity distributions in the focal planes of two perpendicular cylindrical lenses with the focal length $f = 25$ cm.

![Figure 6](image)

Figure 6. Initial phase distribution of a Gaussian optical vortex from Fig. 5 passed through a phase diffuser (a), intensity distribution after propagation in space (b), intensity distributions in the focal planes of two perpendicular cylindrical lenses (c, d).

It is seen in Fig. 6(b) that the intensity distribution is rather distorted compared to Fig. 5(c). Theoretical OAM value is 10.5, while the value calculated by the intensity distributions from Figs. 6(c, d) is 10.02 (the error 5%).

7. Conclusion

In this work, we have proposed and studied numerically and experimentally a simple and efficient approach to determine the OAM of paraxial beams. In this approach, two intensity distributions are measured in the focal plane of two cylindrical lenses perpendicular to each other, before calculating the intensity moments. The experimental error increases from about 1% when determining small fractional OAM (up to 4), and to about 8% for large fractional OAM (up to 30). We also show that the technique allows obtaining the OAM if the beam is distorted by a phase diffuser.

8. References

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