Nonlinear response and scaling law in the vortex state of d-wave superconductors

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Abstract. – We study the field dependence of the quasi-particle density of states, the thermodynamics and the transport properties in the vortex state of d-wave superconductors when a magnetic field is applied perpendicular to the conducting plane, specially for the low field and the low temperature compared to the upper critical field and transition temperature, respectively, $H/H_{c2} \ll 1$ and $T/T_c \ll 1$. Both the superfluid density and the spin susceptibility exhibit the characteristic $\sqrt{H}$-field dependence, while the nuclear spin lattice relaxation rate $T_1^{-1}$ and the thermal conductivity are linear in field $H$. With increasing temperature, these quantities exhibit the scaling behavior in $T/\sqrt{H}$. The present theory applies to 2D f-wave superconductors as well; a possible candidate of the superconductivity is Sr$_2$RuO$_4$.

Introduction. – Single crystals of high-$T_c$ cuprate superconductors like YBCO, Bi2212, etc. appear to provide the most useful testing ground for properties of unconventional superconductors [1]. In particular, d-wave superconductivity has been established both in the hole-doped and the electron-doped high-$T_c$ cuprates [2–4]. Due to the nodal structure in the d-wave order parameter, the specific heat [5], the spin susceptibility and the superfluid density [6] have been predicted to behave like $\sqrt{H}$ in the vortex state, where $H$ is the magnetic field. Indeed the $\sqrt{H}$-dependence of the specific heat in the vortex state of YBCO has been established [7–9]. On the other hand, the $\sqrt{H}$-dependence of the superfluid density appears to have not been seen in spite of an elaborate muon spin rotation experiment [10].

In the meantime, the calculational technique greatly improved [11–14]. Therefore, we can study the effect of a magnetic field within the semi-classical approximation almost analytically within the weak-coupling theory of d-wave superconductivity. Here we assume that the Fermi surface in the a-b plane is circle. Then the present model should be also applicable to the recently discovered d-wave superconductors in $\kappa$-(ET)$_2$ salts [15–17]. The present model applies as well to the superconductivity in Sr$_2$RuO$_4$, if the superconductivity is one of 2D f-wave states (i.e. $\Delta(k) \sim e^{\pm i\phi} \cos(2\phi)$, $e^{\pm i\phi} \sin(2\phi)$ or $e^{\pm i\phi} \cos(ck_3)$, where $\phi$ is the angle between the quasi-particle momentum and a-axis in the plane, $k_3$ the quasi-particle momentum in the c-direction and $c$ is the distance between layers of conducting plane in the c-direction) which are considered by Hasegawa et al. [18]. The specific heat [19], NMR [20] and the magnetic penetration depth measurement [21] show clearly the presence of the nodal structure [22]. The
thermodynamics and the planar transport of these \( f \)-wave states are exactly same as the ones in \( d \)-wave superconductors when the field is applied perpendicular to the conducting plane.

The object of the present paper is to study the quasi-particle density of states in the vortex state of \( d \)-wave superconductors when a magnetic field is applied perpendicular to the conducting plane, specially for the low field and the low temperature compared to the upper critical field and transition temperature, respectively, \( H/H_{c2} \ll 1 \) and \( T/T_c \ll 1 \). Then making use of the density of states, we calculate the specific heat, the spin susceptibility, the superfluid density and the nuclear spin lattice relaxation rate \( T_1^{-1} \) in NMR. Also for clarity, we limit ourselves to the superclean limit where the effect of impurity scattering is negligible. For \( E/\Delta \ll 1 \) the quasi-particle density of states is a simple function of \( E/\epsilon \sim E/\sqrt{H} \), where \( E \) is the energy of a quasi-particle, \( \Delta \) is the \( d \)-wave superconducting order parameter, \( \epsilon = v\sqrt{eH}/2 \), \( e \) electron charge and \( v \) the Fermi velocity in the conducting plane. Then in the limit of \( T \to 0 \), the specific heat, the spin susceptibility and the change in the superfluid density behave like \( \sqrt{H} \), while \( T_1^{-1} \) behaves like \( H \). Indeed, the \( \sqrt{H} \)-dependence of the spin susceptibility and the \( H \)-linear-dependence of \( T_1^{-1} \) have been recently observed by NMR in the vortex state of underdoped YBCO [23]. In addition, the thermal conductivity both in the superclean limit and the clean limit will be briefly discussed.

For \( T \neq 0 \) K, all these quantities exhibit scaling behavior as first discussed by Simon and Lee [24]. Actually, the present model gives the explicit scaling functions, which should be readily accessible experimentally.

**Quasi-particle density of states.** Following the semi-classical approximation by Volovik [5] the quasi-particle density of states in the the vortex state of \( d \)-wave superconductor is given by

\[
N(E, H)/N_0 \approx \frac{1}{\Delta} \langle |E| \vee |v \cdot q| \rangle ,
\]

where \(|E| \vee |v \cdot q| \) means the bigger one among \(|E|\) and \(|v \cdot q|\). Also we assumed \(|E|, |v \cdot q| \ll \Delta\). Here \(|v \cdot q|\) is the Doppler shift [25] associated with the pair momentum \(2q\) and the Fermi velocity \(v\). In a magnetic field \( H || c\), the Doppler shift is given as \(v \cdot q = \frac{v}{2} \cos \phi\), where \(r\) is the distance from the center of the vortex and \(\phi\) is the angle between \(v\) and \(q\). Although \(v\) is parallel to one of the nodal lines at low temperature, \(\phi\) runs from 0 to \(2\pi\). Finally, \(\langle \ldots \rangle\) of eq. (1) means the spatial average over \(r\) and \(\phi\). This average is carried out over a unit cell of a square vortex lattice characteristic to \(d\)-wave superconductors [6] à la Wigner-Seitz:

\[
\langle \ldots \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi \frac{2}{d^2} \int_0^d r dr \ldots ,
\]

where \(2d\) is the distance between vortices, \(d = 1/\sqrt{eH}\) [14]. Then eq. (1) reduces to

\[
N(E, H)/N_0 = \frac{4}{\pi} \frac{\epsilon}{\Delta^2} g \left( \frac{E}{\epsilon} \right) \approx \frac{2v}{\pi} \sqrt{\frac{\pi}{\Phi_0}} \sqrt{H} g \left( \frac{2}{v} \sqrt{\frac{\Phi_0}{\pi}} \frac{E}{\sqrt{H}} \right)
\]

with the scaling function \(g(s)\),

\[
g(s) = \begin{cases} 
\frac{\pi}{4} s(1 + \frac{1}{2s^2}) & \text{for } s = \frac{E}{\epsilon} \geq 1, \\
\frac{3}{4} \sqrt{1 - s^2} + \frac{1}{4s} (1 + 2s^2) \sin^{-1} s & \text{for } s = \frac{E}{\epsilon} \leq 1
\end{cases}
\]

and \(\epsilon = \frac{v}{2} \sqrt{eH} = \frac{v}{2} \sqrt{\frac{\pi}{\Phi_0}} \sqrt{H}\) and \(\Phi_0\) is a quantum of flux \((\simeq 2.07 \times 10^{-11} \text{ T cm}^2)\). For \(E = 0\), we obtain \(N(0, H)/N_0 = \frac{4}{\pi} \frac{\epsilon}{\Delta^2}\), since \(g(s) = 1 + \frac{1}{4} s^2\) for \(s \ll 1\). Also \(g(s) \sim \frac{\pi}{4} s\) for
**Fig. 1** – The scaling function \( g(s) \) in eq. (4) is shown as a function of \( s = E/\epsilon \).

**Fig. 2** – The specific heat data (△) by Nishizaki et al. [19] is fitted with the \( \sqrt{H} \) law.

\( s \gg 1 \), which means \( N(E,H)/N_0 \simeq E/\Delta \) for the high-energy excitations \( E \gg \epsilon \). The scaling function \( g(s) \) vs. \( s = E/\epsilon \) is shown in fig. 1. The same function as eq. (4) has been obtained in [12].

**Thermodynamics.** – Making use of the density of states, i.e. eq. (3), we can determine the thermodynamic quantities at low temperatures, \( T \ll T_c \). First, the specific heat \( C_s(T,H) \) is given by

\[
C_s(T,H)/\gamma_n T = \frac{3}{2\pi^2 T^3} \int_0^\infty dE E^2 N(E,H)/N_0 \text{sech}^2 \left( \frac{E}{2T} \right)
= \frac{4}{\pi} \frac{\epsilon}{\Delta} f(T/\epsilon),
\]

where

\[
f(T/\epsilon) = \frac{3}{2\pi^2} \left( \frac{\epsilon}{T} \right)^3 \int_0^\infty ds s^2 g(s) \text{sech}^2 \left( \frac{\epsilon}{2T} \right)
\]

and \( \gamma_n T \) is the specific heat in the normal state. We have \( f(T/\epsilon) \to 1 \) for \( T/\epsilon \ll 1 \) and \( f(T/\epsilon) = \frac{27\zeta(3)}{4\pi} \frac{T}{\epsilon} \) for \( T/\epsilon \gg 1 \). With this limiting behavior of \( f(T/\epsilon) \), the specific heat takes \( C_s/\gamma_n T = \frac{4}{3\Delta} \) at low temperature \( T \ll \epsilon \) and \( C_s/\gamma_n T = \frac{27\zeta(3)}{\pi^2} \frac{T}{\Delta} \) for \( H = 0 \), respectively.

In the limit of \( T \to 0 \), the \( \sqrt{H} \)-dependence of the specific heat in YBCO has been discussed in [7–9]. When we parameterize \( C_s/T = A_\epsilon \sqrt{H} \) and the specific heat in the absence of the field \( C_s(T,0)/T = \alpha T \), we can deduce \( \nu = \frac{27\zeta(3)}{2\pi} \sqrt{\frac{\pi}{\Phi_0}} \frac{A_\epsilon}{\alpha} \simeq 2.28 \times 10^6 \text{ cm/s} \) and \( 1.76 \times 10^6 \text{ cm/s} \) from [7] and [9], respectively. We may introduce an adjustable parameter \( a \) in front of \( \nu \) as in [12]. Then if we assume the Fermi velocity \( \nu \simeq 10^7 \text{ cm/s} \), \( a \) is about 0.2 which is the same as deduced by Chiao et al. [26].

In the case of Sr$_2$RuO$_4$, the recent specific heat data by Nishizaki et al. [19] exhibits clearly the \( \sqrt{H} \)-dependence as shown in fig. 2. The deviation from the \( \sqrt{H} \) law for \( H < 0.01 \) T might
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Fig. 3 – The scaling function \( F(T/\epsilon) \) of specific heat in eq. (7), the scaling function \( I(T/\epsilon) \) of the spin susceptibility (or the superfluid density) in eq. (10) and the scaling function \( G(T/\epsilon) \) of the nuclear spin lattice relaxation rate in eq. (13) are shown altogether as a function of \( T/\epsilon \).

Fig. 4 – The thermal conductivity data in YBCO by Chiao et al. [26] are fitted with the thermal conductivity equation (15) with \( \Gamma/\Gamma_c = 0.06(\square), 0.188(\triangle), 0.33(\cdots) \), where \( \square \) for \( x = 0 \), \( \triangle \) for \( x = 0.006 \), \( \cdots \) for \( x = 0.03 \) of \( \text{YBa}_2(\text{Cu}_{1-x}\text{Zn}_x)_3\text{O}_{6.9} \). The \( H \ln(\text{const}/H) \) dependence describes very well the experimental data.

be due to the entrance into the Meissner state \( (H_{c1} \approx 5 \text{ mT}) \).

Also Wang et al. [27] studied the scaling behavior of the specific heat of YBCO

\[
\frac{C(T, H) - C(T, 0)}{\gamma_n T} \left[ \frac{4}{\pi} \frac{\epsilon}{\Delta} \right] = F\left( \frac{T}{\epsilon} \right) = f\left( \frac{T}{\epsilon} \right) - \frac{27\zeta(3)}{4\pi} \frac{T}{\epsilon} \\
= \begin{cases} 
1 - \frac{27\zeta(3)}{4\pi} \left( \frac{T}{\epsilon} \right) + \frac{7\pi^2}{30} \left( \frac{T}{\epsilon} \right)^2 + \cdots, & \text{for } \frac{T}{\epsilon} \ll 1, \\
\frac{3}{\pi} (\ln 2)(\frac{T}{\epsilon}) + \cdots, & \text{for } \frac{T}{\epsilon} \gg 1.
\end{cases}
\]

We show in fig. 3 \( F(T/\epsilon) \) vs. \( T/\epsilon \). The present model appears to describe the scaling behavior in YBCO [27] very well. Clearly a similar study of the scaling relation in the vortex state of \( \text{Sr}_2\text{RuO}_4 \) will be of great interest.

Similarly, both the spin susceptibility \( \chi \) and the superfluid density \( \rho_s \) should exhibit a scaling behavior,

\[
\chi(T, H)/\chi_n = 1 - \rho_s(H, T) = \frac{1}{2T} \int_0^\infty dE \frac{N(E, H)}{N_0} \text{sech}^2 \left( \frac{E}{2T} \right) \\
= \frac{4}{\pi} \frac{\epsilon}{\Delta} h(T/\epsilon)
\]

and

\[
h(T/\epsilon) = \frac{1}{2T} \int_0^\infty ds g(s) \text{sech}^2 \left( \frac{s}{2T} \right).
\]

Here \( \chi_n \) is the spin susceptibility in the normal state. We have \( h(T/\epsilon) \to 1 \) for \( T/\epsilon \ll 1 \) and \( h(T/\epsilon) = \frac{\pi}{2} \ln \frac{T}{\epsilon} \) for \( T/\epsilon \gg 1 \). With this limiting behavior of \( h(T/\epsilon) \) both the susceptibility
and superfluid density take \( \sim \frac{2 \nu}{\pi} \sqrt{\frac{\pi}{4 \nu}} \) in the limit of \( T \to 0 \), and \( \sim 2(\ln 2) \frac{\nu T}{\Delta} \) in the limit of \( T \gg \epsilon \). As already mentioned in the introduction, the \( \sqrt{\Pi} \)-dependence of the susceptibility has been observed in slightly underdoped YBCO [23]. However, the reported \( \rho_n(T, H) \) in YBCO by muon spin rotation [10] appears not to exhibit the \( \sqrt{H} \) behavior. A further study of \( \rho_n(T, H) \) is clearly desirable. Again it may be more useful to introduce the scaling function by

\[
I(T/\epsilon) = \frac{\chi(T, H) - \chi(T, 0)}{\chi_n \left[ \frac{4 \epsilon}{\pi \Delta} \right]} = h(T/\epsilon) - \frac{\pi}{2} \ln 2(T/\epsilon)
\]

\[
\approx \begin{cases} 
1 + \frac{\pi^2}{18} \left( \frac{T}{\epsilon} \right)^2 - \frac{\pi}{2} \ln 2 \frac{T}{\epsilon} + \cdots, & \text{for } \frac{T}{\epsilon} \ll 1, \\
\frac{\pi}{10} \ln(3.3518 \frac{T}{\epsilon}), & \text{for } \frac{T}{\epsilon} \gg 1.
\end{cases}
\]

(10)

The scaling function \( I(T/\epsilon) \) vs. \( T/\epsilon \) is shown in fig. 3.

**Nuclear spin lattice relaxation rate.** – In the superclean limit \( T_1^{-1} \) is given by

\[
T_1^{-1}(T, H)/T_{1n}^{-1} = \frac{1}{2T} \int_0^{\infty} dE \left( \frac{N(E, H)}{N_0} \right)^2 \operatorname{sech}^2 \left( \frac{E}{2T} \right)
\]

\[
= \left( \frac{4 \epsilon}{\pi \Delta} \right)^2 J(T/\epsilon),
\]

(11)

where

\[
J(T/\epsilon) = \frac{\epsilon}{2T} \int_0^{\infty} ds g^2(s) \operatorname{sech}^2 \left( \frac{\epsilon}{2Ts} \right).
\]

(12)

Here \( T_{1n}^{-1} \) is the nuclear spin lattice relaxation rate in the normal state. We have \( J(T/\epsilon) \to 1 \) for \( T/\epsilon \ll 1 \) and \( J(T/\epsilon) \to \frac{1}{3} \left( \frac{\pi^2 T}{4 \epsilon} \right)^2 \) for \( T/\epsilon \gg 1 \). Therefore the nuclear spin lattice relaxation rate \( T_1^{-1}(T, H)/T_{1n}^{-1} \) takes \( \sim H \) behavior in the limit of \( T \to 0 \) and \( \sim (T/\Delta)^2 \) in the limit of \( T \gg \epsilon \). Again it is more convenient to introduce the scaling function by

\[
G(T/\epsilon) = \frac{T_1^{-1}(T, H) - T_1^{-1}(T, 0)}{T_{1n}^{-1} \left[ \frac{4 \epsilon}{\pi \Delta} \right]^2} = J(T/\epsilon) - \frac{\pi^4}{48} (T/\epsilon)^2
\]

\[
\approx \begin{cases} 
1 + \left( \frac{\pi^2}{9} - \frac{\pi^4}{48} \right) \left( \frac{T}{\epsilon} \right)^2 + \cdots, & \text{for } \frac{T}{\epsilon} \ll 1, \\
\frac{\pi^2}{18} + O((\epsilon^2/\epsilon^2)), & \text{for } \frac{T}{\epsilon} \gg 1.
\end{cases}
\]

(13)

We show the scaling function \( G(T/\epsilon) \) in fig. 3. The detection of this scaling function will be very useful.

**Planar thermal conductivity.** – Following [14] the thermal conductivity in the superclean limit (i.e. \( \Gamma/\Delta \ll H/H_{c2} \ll 1 \)) within the conducting plane in the vortex state with the field configuration of \( H \parallel c \) is obtained as

\[
\kappa(T, H)/\kappa_n = \frac{56}{5\pi} \left( \frac{T}{\Delta} \right)^2 \left\{ \frac{20}{7\pi^2} \left( \frac{\epsilon}{T} \right)^2 + \left( \ln \left[ \frac{4\Delta}{3.5T\sqrt{1 + \left( \frac{\epsilon}{1.75T} \right)^2} \right] \right)^2 \right\},
\]

(14)

where \( \kappa_n = \frac{\pi^2}{6} \frac{nT}{\Gamma m} \) is the thermal conductivity in the normal state, \( n \) the density of electrons, \( m \) the electron mass, and \( \Gamma \) the scattering rate by impurity in the unitary scattering limit.
In the limit of $T \to 0$, $\kappa(T, H)/\kappa_n$ reduces to $\frac{8v^2}{\pi^2\Phi_0\Delta^2} H$, linear in $H$ contrary to [12]. The local thermal conductivity $\kappa(r)$ assumed in [12], we think, is unrealistic, since the quasi-particle mean free path in these systems is $\sim \mu m$. This linear field dependence as well as the quasi-scaling behavior of $\kappa/T^3$ vs. $H/T^2$ in eq. (14) have not been seen in high-$T_c$ cuprates.

On the other hand, a recent thermal conductivity data of $\text{Sr}_2\text{RuO}_4$ at $T = 0.35 \text{ K}$ exhibits a clear $H$-linear dependence for $H > 0.02 \text{ T}$. At a low magnetic field ($H < 0.015 \text{ T}$) the thermal conductivity appears to be independent of $H$. The thermal conductivity in the superclean limit in the present model for $T > \epsilon$ becomes almost independent of $H$ and increases like $\kappa \sim T^3 \ln(\Delta/T)$.

Further, this kind of behavior should be quite common to the unconventional superconductors with the nodal superconductors (e.g., $E_{2u}$-state in $\text{UPt}_3$ [29]). In particular, when $\epsilon/T \ll 1$, $\kappa/T^3$ decreases with increasing $H$ almost linearly in $H/T^2$. Indeed our result appears to describe the quasi-scaling behavior observed in the B-phase of $\text{UPt}_3$ recently [29].

The low-temperature thermal conductivity in high-$T_c$ cuprates with the field $H \parallel c$ appears to be described by the one in the clean limit (i.e. $H/H_{c2} \ll \Gamma/\Delta \ll 1$) rather than the one in the superclean limit.

The thermal conductivity in the clean limit, on the other hand, is given by [14]

$$\frac{\kappa}{\kappa_{00}} = \frac{\Delta_{00}}{\Delta} \left(1 + \frac{v^2 eH}{6\pi \Gamma \Delta} \ln \left(\frac{2\Delta}{\pi \Gamma} \right) \ln \left(\frac{4\Delta}{v^2 eH} \right)\right),$$

(15)

Here $\Delta_{00}$ is the order parameter in the absence of the impurity scattering at $T = 0$, $\kappa_{00} = \frac{\pi}{3} \frac{T_{c0}}{\Delta_{00} m}$ the universal thermal conductivity in the limit of $T \to 0$ and for $H = 0$. Indeed this $\sim H \ln(\text{const}/H)$ behavior is very consistent with the recent data from YBCO and Bi2212 single crystals [26]. We show such a comparison in fig. 4, where we took $\Gamma/T_c = 0.06, 0.188$, and 0.33. Here $\Gamma_c = 0.8819 T_{c0}$ is the critical scattering rate when the superconductivity disappears.

Concluding remarks. – Limiting ourselves in the configuration $H \parallel c$ (i.e. the magnetic field normal to the conducting plane) and in the superclean limit, we obtain the expression of the thermodynamic quantities and $T_{c-1}$ in NMR in vortex states in $d$-wave superconductors. Some of limiting behaviors for $T \to 0 \text{ K}$ have been well established in the vortex state of YBCO, though little work on the scaling behavior has been done. We have shown also the present model describes some features of the vortex state in $\text{Sr}_2\text{RuO}_4$, specially the presence of the nodal structure in the order parameter. This suggests that single crystals of $\text{Sr}_2\text{RuO}_4$ will provide another testing ground of the present model. At present, among three $2D$ $f$-wave states for $\text{Sr}_2\text{RuO}_4$, only one of them, $\Delta(k) \sim \cos(ck_3)e^{\pm i\phi}$, appears to be viable, since both the extremely small-angle dependence of the upper critical field [30] and the thermal conductivity [28,31] with the field applied parallel to the conducting plane are incompatible to other two candidates. Also, we expect similar behaviors of vortex state in other unconventional superconductors [14,22]. Therefore the exploration of the vortex state will bring further insight into unconventional superconductors.

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REFERENCES

[1] Maki K., Won H., *Ann. Phys. (Leipzig)*, 5 (1996) 320; *J. Phys. I*, 6 (1998) 2317.
[2] Tsuei C. C. and Kirtley J. R., *Phys. Rev. Lett.*, 85 (2000) 182.
[3] Kokales J. P. et al., *Phys. Rev. Lett.*, 85 (2000) 3696.
[4] Prozorov R. et al., *Phys. Rev. Lett.*, 85 (2000) 3700.
[5] Volovik G. E., *JETP Lett.*, 58 (1993) 469.
[6] Won H. and Maki K., *Europhys. Lett.*, 30 (1995) 421; *Phys. Rev. B*, 53 (1996) 5927.
[7] Moler K. A. et al., *Phys. Rev. Lett.*, 73 (1994) 2744; Moler K. A. et al., *Phys. Rev. B*, 55 (1997) 3954.
[8] Wright D. A. et al., *Phys. Rev. Lett.*, 82 (1999) 1550.
[9] Revas B., Genoud J. Y., Junod A., Neumeier K., Erb A. and Walker E., *Phys. Rev. Lett.*, 80 (1998) 3364.
[10] Sonier J. E. et al., *Phys. Rev. Lett.*, 83 (1999) 4156.
[11] Barash Yu. S., Mineev V. P. and Svidzinskii A. A., *Sov. Phys. JETP*, 65 (1997) 638; Barash Yu. S. and Svidzinskii A. A., *Phys. Rev. B*, 58 (1998) 476.
[12] Kübert C. and Hirschfeld P. J., *Solid State Commun.*, 105 (1998) 439; *Phys. Rev. Lett.*, 80 (1998) 4963.
[13] Vekhter I., Carbotte J. P. and Nicol E. J., *Phys. Rev. B*, 59 (1999) 7123; Vekhter I., Hirschfeld P. J., Carbotte J. P. and Nicol E. J., *Phys. Rev. B*, 55 (1999) R9023.
[14] Won H. and Maki K., *Proceedings of the NATO Workshop on Borocarbides, June 2000*, cond-mat/0004105.
[15] Carrington A. et al., *Phys. Rev. Lett.*, 85 (1999) 4172.
[16] Pinteric M. et al., *Phys. Rev. B*, 61 (2000) 7033.
[17] Ichimura K., Arai T., Nomura K., Takasaki S., Yamada T., Nakatsuji S. and Anzai H., *Synth. Met.*, 103 (1999) 1812.
[18] Hasegawa Y., et al., *J. Phys. Soc. Jpn.*, 69 (2000) 336.
[19] Nishizaki S., Maeno Y. and Mao Z., *J. Phys. Soc. Jpn.*, 69 (2000) 572.
[20] Ishida K. et al., *Nature*, 396 (1998) 658.
[21] Bonalde I. et al., *Phys. Rev. Lett.*, 85 (2000) 4775.
[22] Won H. and Maki K., *Europhys. Lett.*, 52 (2000) 1427.
[23] Zheng G.Q., Clark W. G., Kitaoka Y., Asayama K., Kodama Y., Kuhns P. and Moulton W. G., *Phys. Rev. B*, 60 (1999) R9947.
[24] Simon S. H. and Lee P. A., *Phys. Rev. Lett.*, 78 (1997) 1548, 5029; Volovik G. E. and Kopnin N. B., *Phys. Rev. Lett.*, 78 (1997) 5028.
[25] Maki K. and Tsuneto T., *Prog. Theor. Phys.*, 27 (1960) 228.
[26] Chiao M. et al., *Phys. Rev. Lett.*, 82 (1999) 2943; Chiao M. et al., *Phys. Rev. B*, 62 (2000) 3554.
[27] Wang Y., Revas B., Erb A. and Junod A., cond-mat/0009194.
[28] Izawa K. et al., cond-mat/0012137.
[29] Suderow H., Brison J.P., Huxley A. and Flouquet J., *Phys. Rev. Lett.*, 80 (1998) 165.
[30] Mao Z. Q., Maeno Y., Nishizaki S., Akima T. and Ishiguro T., *Phys. Rev. Lett.*, 84 (2000) 991.
[31] Tanatar M. A., Nagai S., Mao Z. Q., Maeno Y. and Ishiguro T., *Physica C*, 341-348 (2000) 991.