Atomic Size Pearls being Dark Matter and giving Electron Signal

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Abstract

We seek to explain both the seeming observation of dark matter by the seasonal variation of the DAMA-LIBRA data and the observation of “electron recoil” events at Xenon1T in which the liquid-Xe-scintillator was excited by electrons - in excess to the expected background - by the same dark matter model. In our model the dark matter consists of bubbles of a new type of vacuum containing ordinary atomic matter, say diamond, under high pressure ensured by the surface tension of the separation surface (domain wall). This atomic matter is surrounded by a cloud of electrons extending out to about atomic size. We also seek to explain the self interactions of dark matter suggested by astronomical studies of dwarf galaxies and the central structure of galaxy clusters. At the same time we consider the interaction with matter in the shielding responsible for slowing the dark matter down to a low terminal velocity, so that collisions with nuclei in the underground detectors have insufficient energy to be detected. Further we explain the “mysterious” X-ray line of 3.5 keV from our dark matter particles colliding with each other so that the surfaces/skins unite. Even the 3.5 keV X-ray radiation from the Tycho supernova remnant is explained as our pearls hitting cosmic rays in the remnant.

What the DAMA-LIBRA and Xenon1T experiments see is supposed to be our dark matter pearls excited during their stopping in the shielding or the air. The most remarkable support for our type of model is that both these underground experiments see events with about 3.5 keV energy, just the energy of the X-ray line.

We fit numerically the cross section over mass ratio for the self interaction of the dark matter at low velocity $v \to 0$, as observed in the study of dwarf galaxies and find the size of the pearls to be rather close to the smallest possible under the restrictions from our earlier 3.5 keV line fit. However the mass obtained from this dwarf galaxy fit turns out to be too small for allowing the pearls to penetrate down to the DAMA experiment in less than a year, as needed to avoid a washing out of the seasonal variation. This led us to investigate more carefully the velocity relevant to our calculation and to consider pearls of larger size.
Also the total energy of the dark matter pearls stopped in the shield is reasonably matching order of magnitudewise with the absolute observation rates of DAMA-LIBRA and Xenon1T, although the proposed explanation of their ratio requires further development.

It should be stressed that accepting that the different phases of the vacuum could be realized inside the Standard Model, our whole scheme could be realized inside the Standard Model. So then no new physics is needed for dark matter!

1 Introduction

For a long time we have worked on a dark matter model \cite{1 2 3 4 5 6 7 8}, in which the dark matter consisted of cm-size pearls which were in fact bubbles of a new vacuum type surrounded by a skin caused by the surface tension of this new vacuum. This skin kept a piece of usual atomic matter highly compressed inside the bubble. The present article is an extension and update of our Bled-proceedings article 2021 \cite{9} by improving the calculation of the radius of the pearl and of the surrounding electron cloud. Further we have here a discussion of the troubles in our model of getting the dark matter pearls, with the rather large interaction suggested by studies of dwarf galaxies, through the 1400 m shielding of the DAMA experiment in less than a year.

The idea of our model is actually rather similar to the Dark Matter as Color Superconductors proposed by Ariel Zhitnitsky \cite{10} in 2003. This was called to our attention by Konstantin Zioutas with special interest in the solar corona heating \cite{11}.

In fitting data in the old model - in which the pearls were supposed so large and heavy that they could have caused the Tunguska event - the most and almost only successful fit consisted in that we fitted, with a common parameter, both the overall rate and the very 3.5 keV energy of the X-ray line originally observed in several galaxy clusters, Andromeda and the Milky Way Center \cite{12 13 14 15 16 17} and supposedly coming from dark matter. But now it turned out that this successful fitting relation between the 3.5 keV energy and the overall rate of the X-ray radiation only depends on the density of the pearls or equivalently the Fermi momentum or energy of the electrons kept inside the pearls, but not on the absolute size of the pearls. Thus we could change the model to make the pearl sizes much smaller, and that is what we did in our contribution to the Bled workshop proceedings in 2021. So the pearls making up the dark matter are now rather of atomic size. Really we shall consider pearls with radii ranging from $R \sim 10^{-11} m$ to $R \sim 10^{-9} m$. But even such small pearls get stopped to some extent by the shielding into which they must penetrate to reach the underground experiments like the DAMA-LIBRA and Xenon experiments looking for dark matter. Using an astronomical observation based model by Correa \cite{18} especially, we shall construct a rather definite picture of our pearls from which we estimate that the pearls hitting the earth actually get stopped presumably in the atmosphere, but if not there then at least in the earth shielding. The pearls thereby lose so much speed that it becomes quite understandable that the Xenon-experiments, looking for nuclei being hit by them and causing scintillation in fluid xenon, will not see any such events. However the DAMA-LIBRA experiment \cite{19 20} would not distinguish if it is a nucleus that is hit or some energy is released which causes the scintillator to
luminesce. So only the DAMA-LIBRA experiment would be able to get a signal if the dark matter, e.g. our pearls, could be somehow excited and emit their excitation energy when they pass through the detector. In our model we shall indeed suggest that the pearls get excited and emit their energy by electron emission. That would not be easy to distinguish for DAMA-LIBRA but would still of course come with seasonal variation\(^1\) so that it would be observed as dark matter by DAMA-LIBRA. Whether the emission is via electrons or nuclei would not matter. But for the Xenon-experiments such electron emission was effectively not counted for a long time, but now rather recently the Xenon1T experiment has actually observed an excess of “electron recoil events”. So they have now in fact seen an electron emission somehow.

We shall see in section 11 that both the excess of electron recoil events in Xenon1T \(^2\) and the events seen by DAMA-LIBRA \(^19\), \(^20\) have the energy of each event remarkably enough centering about the energy value 3.5 keV of the mysterious X-ray line found astronomically!

This coincidence of course strongly suggests that these events from DAMA-LIBRA and Xenon1T are related to dark matter particles that can be excited precisely by this energy 3.5 keV.

In our earlier papers \(^5\), \(^6\), \(^7\) we have already connected the excitability of our pearls by just this energy 3.5 keV and especially the emission of photons (or here in the present work also electrons) with just this energy with a gap in the single particle electron spectrum of the pearls caused by what we call the homolumo-gap effect.

A very serious warning, which needs an explanation in order to rescue our model, is delivered by the fact that if as we now suggest the Xenon1T electron recoil event excess is coming from just the same decay of dark matter excitations as the DAMA-LIBRA observation, then these two experiments ought a priori to see equally many events, say per kg. However, DAMA-LIBRA sees 250 times as many events as Xenon1T sees excess events.

We shall discuss in section 9 the difficulty due to this ratio not being unity, but the hope for now is that the Xenon1T experiment has the observed decaying pearls falling through a fluid, namely the fluid xenon, while the scintillator in DAMA-LIBRA is a solid made from NaI(Tl). The pearls are likely to form a little Xe-fluid bubble around them and flow or fall through the xenon-fluid, while they will much more easily get caught so as to almost sit still or only move much slower through the NaI scintillator. If so the pearls with their supposed excitations would spend more time in the DAMA-LIBRA NaI than in a corresponding volume of xenon-liquid. Then, hoping for such a difference between the fluid xenon and the solid NaI, the terminal velocity in the gravitational field in the xenon has to be 250 times faster than that in the solid in which it anyway has to be fast enough to reach through in less than a year. Crudely estimating the viscosity of fluid xenon then gives limits for the mass of the pearl.

In the following section 2 we describe how, in the Bled Proceedings paper \(^9\), we imagined the particles making up the dark matter in our model to be bubbles of the size \(R = r_{\text{cloud}} \times 3.3M_{\text{eV}} \approx 8 \times 10^{-12} m\) with heavy atomic matter inside. These bubbles are surrounded by a cloud of electrons, which we show in

\(^1\)We note however that the ANAIS experiment has failed to see an annual modulation with NaI(Tl) scintillators and their results \(^21\) are incompatible with the DAMA-LIBRA results at 3.3\(\sigma\).
section to have a thickness of at least $7.8 \cdot 10^{-13} \text{m}$. But presumably the cloud of electrons outside is rather similar to an ordinary atom, having a thickness of the order of an atomic radius $\sim 10^{-10} \text{m}$. In [9] we supposed that the homolumo gap meant that the electrons only extended out to where the electric potential would equal the value 3.5 keV (the corresponding radius is called $r_{\text{cloud} \text{3.5keV}}$).

Here the quantities 3.3 MeV and 3.5 keV in the subscripts are the numerical electric potentials felt by an electron at the distances mentioned. A special point to note in this section already present in the earlier articles about the big pearls is the above mentioned homolumo-gap effect, causing a band or gap in the energy levels without any single particle electron eigenstates. The width of this gap is fitted to the 3.5 keV line in the observed X-ray spectrum from galaxy clusters, the Milky Way Center etc. [12, 13, 14].

Next in section 3 we briefly review astronomical observations and modelling of the dark matter, which suggests the idea that dark matter interacts with itself (self interacting dark matter SIDM). It is only when the corresponding cross section $\sigma$ is divided by the particle mass $M$ that we have a combination that has any chance of being observed by its effects on the atomic matter.

New material in this paper compared to the Bled proceedings [9] demonstrates that there is a minimal size for a pearl containing electrons with a Fermi momentum $p_f = 3.3 \text{ MeV}$ as determined from the intensity and energy of the 3.5 keV X-ray line emitted by galactic clusters [6, 7, 8, 15]. This is presented in section 4, in which we see that indeed the actual pearl size found in [9] is only a little bigger than the minimal size.

In section 5 we discuss the range out to which the electron cloud may go, a question very important for the cross section $\sigma$ for the self interaction and also for the stopping of the pearls once hitting the earth. We calculate the radii $r_{\text{cloud} \text{3.5keV}}$ and $r_{\text{cloud} \text{3.3MeV}}$ using the same Thomas-Fermi approximation as in [9]. We improve this approximation in section 6.

In section 7 we list a series of numerical successes of our model for the dark matter, hopefully making the reader see that there is really some reason for it being at least in some respects correct.

In section 8 we stress again that our dark matter pearls get stopped and at the same time excited, mainly to emit quanta of energy 3.5 keV, in the air and/or in the shielding above the experiments. It is the braking energy from this slowing down that is supposed to feed the excitations.

In section 9 we investigate the problem for our model that it must at least get the pearls to reach down through the earth to DAMA in preferably less than a year, since otherwise one would not observe much seasonal variation. But for the pearls to get down fast enough we require pearls of a mass too high to match well with the amount of self interaction observed at low velocity $v \rightarrow 0$ in the study of dwarf galaxies [18]. We are thereby led to consider pearls of size $R \sim 1 \text{nm}$ and to speculate that during their passage through space they may have collected some dirt around them.

A special estimation, based on energy considerations, of whether the number of events seen by DAMA-LIBRA and by the Xenon1T electron recoil excess are of a reasonable order of magnitude is put forward in section 10. The success of such an estimation has to be rather limited in as far as the rates of the two observations - that should have been the same if we do not include the possibility of faster or slower motion through the detectors - deviate by a factor of 250.

In section 11 we call attention to the perhaps most remarkable fact support-
Figure 1: The figure illustrates the bit smaller than atom-size complicated/macroscopic dark matter particle in our model, a pearl.

ing a major aspect of our model: That the energy per event for both DAMA-LIBRA and the Xenon1T-electron recoil excess centers around 3.5 keV, just the energy of the photons in the mysterious X-ray line seen in galactic clusters mentioned above! So all three effects should correspond to the emission of an electron or photon due to the same energy transition inside dark matter.

The values of the parameters characterising our pearls are discussed in section 12. Finally in section 13 we conclude and provide a short outlook.

2 Pearl

Dark Matter Atomic Size Pearls, Electronic 3.5 keV Signal

We here consider minimal size pearls similar to those discussed in Bled [9]; the structure of such small dark matter pearls is sketched in Figure 1. We will later in the article consider larger pearls with a radius $R \sim 1 \text{ nm}$.

- In the middle is a spherical bubble of radius

$$R \approx r_{\text{cloud 3.3MeV}} \approx 8 \times 10^{-12} \text{m}. \quad (1)$$

Here $r_{\text{cloud 3.3MeV}}$ denotes the radius where the electron potential is 3.3 MeV, which is identified with the Fermi energy $E_f$ of the electrons in the bulk of the pearl - i.e. inside the radius $R$. We estimated the value $E_f = 3.3 \text{ MeV}$ in previous papers [6, 7, 8] by fitting the overall rate of the intensity of the 3.5 keV line emitted by galactic clusters and the very frequency 3.5 keV of the radiation in our model.

- The outer radius

$$r_{\text{cloud 3.5keV}} \approx 7 \times 10^{-11} \text{m} \quad (2)$$
is where the electron potential is 3.5 keV. By our story of the “homolumo gap”: the electron density crudely goes to zero at this radius. (It gradually falls in the range between $r_{\text{cloud}3.3\text{MeV}}$ and $r_{\text{cloud}3.5\text{keV}}$).

The electron density and potential in the pearls

- Due to an effect, we call the homolumo-gap effect [5, 22], the nuclei in the bubble region and the electrons themselves become arranged in such a way as to prevent there from being any levels in an interval of width 3.5 keV. So, as illustrated in Figure 2, outside the distance $r_{3.5\text{keV}} = r_{\text{cloud}3.5\text{keV}}$ from the center of the pearl at which the Coulomb potential is $\sim 3.5$ keV deep there are essentially ($\sim$ in the Thomas-Fermi approximation) no more electrons in the pearl-object.

- The radius $r_{3.3\text{MeV}} = r_{\text{cloud}3.3\text{MeV}}$ at which the potential felt by an electron is 3.3 MeV deep, is supposed to be just the radius to which the many nuclei inside the pearl (which replace the single nucleus in ordinary atoms) reach out. So inside the bubble the potential is much more flat.

- The energy difference between the zero energy line and the effective Fermi surface, above which there are no more electrons, is of order 3.5 keV, the energy so crucial in our work.

- Since in the Thomas-Fermi approximation with a homolumo gap there are no electrons outside roughly the radius $r_{3.5\text{keV}} = r_{\text{cloud}3.5\text{keV}}$, this radius will give the maximal cross section, even for very low velocity $\sigma_{v\rightarrow0}$.

However we now believe that there will be a tail of the electron cloud further out responsible for the low velocity cross section or even that the pearl collects some dirt around it. See section 9.

The homolumo gap effect.

Let us consider the spectrum of energy levels for the electrons in a piece of material, e.g. one of our pearls, and at first assume that the positions or distributions of the charged particles in the material are fixed.
Then the ground state is just a state built e.g. as a Slater determinant for the electrons being in the lowest single electron states, so many as are needed to have the right number of electrons.

But now, if the charged particles can be moved due to their interactions, the ground state energy could be lowered by moving them so that the filled electron state levels get lowered.

So we expect introducing such a "back reaction" will lower the filled states.

When the filled levels get moved downwards, then the homo = “highest occupied molecular orbit” level will be lowered and its distance to the next level, the lumo (= lowest unoccupied molecular orbit), will appear extended on the energy axis, as illustrated in Figure 3.

We believe that we can estimate the homolumo-gap $E_H$.

Using the Thomas-Fermi approximation - or crudely just some dimensional argument where the fine structure constant has the dimension of velocity - we calculated the homolumo gap in highly compressed ordinary matter for relativistic electrons [7]:

$$E_H \sim \left(\frac{\alpha}{c}\right)^{3/2}\sqrt{2}p_f$$

where $p_f =$ Fermi momentum

$$\frac{\alpha}{c} = \frac{1}{137.03...}$$

(the $\sqrt{2}$ comes from our Thomas-Fermi calculation).

It is by requiring this homolumo-gap to be the 3.5 keV energy of the X-ray line mysteriously observed by satellites from clusters of galaxies, Andromeda and the Milky Way Center that we estimate the Fermi-energy to be $E_f \approx p_f = 3.3$ MeV in the interior bulk of the pearl.
Brief summary of theoretical ideas underlying our dark matter pearls

- **Principle** Nothing but Standard Model! (Seriously it would mean not in a BSM-workshop.)

- **New Assumption** Several Phases of Vacuum with Same Energy Density; this is the so-called Multiple Point Principle [5, 6, 24, 25, 26, 27, 28, 29].

- **Central Part** Bubble of New Phase of Vacuum with e.g. carbon under very high pressure, surrounded by a surface with tension \( S = \text{domain wall} \) providing the pressure.

- **Outer part** Cloud of Electrons much like an ordinary atom having a nucleus with a charge \( Z \approx \times 10^4 \) to \( \times 10^5 \).

We provide our picture of minimal size dark matter pearls in Figure 4.

### 3 Non-gravitational Interactions

The collisionless cold dark matter model provides a good description of the large scale structure of the Universe. However there are various problems at small scales [30, 31] for the hypothesis that dark matter only has gravitational interactions. Originally Spergel et al [32] suggested that the lack of a peak or cusp in the center of galaxy clusters, as expected for cold dark matter with purely gravitational interactions, required self interacting dark matter with a relatively large cross section. The relevant parameter is in fact the cross section to mass ratio \( \frac{\sigma}{M} \) and for the cores in galaxy clusters, where the collision velocity is \( v \sim 1000 \text{ km/s} \), a value \( \frac{\sigma}{M} \sim 0.1 \text{ cm}^2/\text{g} \) is needed. The self interaction can of course be velocity dependent and the cores in spiral galaxies where \( v \sim 100 \text{ km/s} \)
km/s require $\sigma M \sim 1 \, \text{cm}^2/\text{g}$. In dwarf galaxies around our Milky Way, where dark matter moves more slowly $v \sim 30 \, \text{km/s}$, larger cross section to mass ratios $\sigma M \sim 50 \, \text{cm}^2/\text{g}$ are needed.

Recently Correa [18] made a study of the velocity dependence of self interacting dark matter. In particular she analysed the Milky Way dwarf galaxies and her results are displayed in Figure 5. The extrapolation of Correa’s fit to the data towards zero velocity points to the ratio $\sigma M \rightarrow 150 \, \text{cm}^2/\text{g}$. This ratio can be taken as an experimental estimate of the impact area over the mass as seen for very soft collisions. In our model the cross section in this low velocity limit is given by the extent out to which the electrons surrounding our pearls reach.

4 Minimal Size for the Pearls

It is our hope here to estimate the size of the pearls in our model under the extra assumption that the pearl size is the smallest allowed using the Fermi momentum $p_f = 3.3 \, \text{MeV}$ of the electrons inside the skin of the pearl. The value of $p_f$ was determined in earlier papers [6, 7, 8] from the frequency and intensity of the 3.5 keV line emitted by various clusters of galaxies etc. Basically we want to show that there is a certain thickness crudely of the electronic cloud providing the electrical pressure needed to keep the surface tension skin spanned out. If this needed thickness of the cloud, which really has a lack of electrons or anti-cloud on the inner side of the pearl skin, should have more electrons than the whole pearl, of course something would be wrong. Thus the radius of the pearl must be at least so large as to allow enough protons in the pearl as to make it possible to let a smaller number of electrons be pressed outside and form an electric attraction sufficient to carry the skin with its tension spanned out.

4.1 The Condenser of the Cloud and its Pressure on the Skin

Have in mind that some electrons are pushed out of the skin in the philosophy that the skin acts most strongly on the nuclei and not quite as forcefully on the electrons; the electrons are highly degenerate, while the nuclei need much less energy to achieve the possibility for the required quantum fluctuation in the momentum. So we should imagine that near the skin there is on the outside an excess of electrons while on the inside the corresponding deficit of electrons. The full number of electrons must of course be equal to that of the protons in the nuclei, since the total pearl must be essentially neutral. Looking locally along the skin the electron cloud configuration acts as a condenser with the negative electrons on the outside and the missing electron positive charge on the inside. In such a condenser there is of course an attractive force between the two plates (formed from the electron and hole clouds). The expression for the force $F_{\text{att}}$ attracting the plates in a condenser is

$$F_{\text{att}} = \frac{\epsilon_0 AV^2}{2d^2} \quad (6)$$

$$= \frac{Q^2}{2A\epsilon_0} \quad (7)$$
Table 2. From left to right: name of the dSph galaxy, present-time virial mass, concentration parameter and core size of the subhalo hosting the dSph and range of preferred cross section values that reproduce the observed DM central densities.

| Name  | $M_{200}$ [$10^9 M_\odot$] | $c_{200}$ | $r_{core}$ [pc] | $\sigma/m_D$ [cm$^2$g$^{-1}$] |
|-------|-----------------|---------|----------------|------------------|
| UM    | 6.13            | 54.2    | 180.8          | 40 – 50          |
| Draco  | 1.17            | 26.8    | 472.9          | 20 – 30          |
| Carina | 1.69            | 19.1    | 648.4          | 40 – 50          |
| Sextans | 6.32           | 20.6    | 395.5          | 70 – 120         |
| C/Val  | 0.46            | 25.7    | 356.8          | 50 – 80          |
| Sculptor | 1.65           | 25.8    | 553.2          | 30 – 40          |
| Fornax | 2.29            | 15.3    | 1036.7         | 30 – 50          |
| LeoII  | 0.65            | 30.6    | 148.8          | 90 – 180         |
| LeoI   | 1.17            | 31.1    | 410.8          | 50 – 70          |

Figure 5: Cross section to mass ratio $\sigma/m_D$ of self-interacting dark matter particles as a function of the collision velocity $v$ in dwarf galaxies from reference [18].

Figure 6: Cross section per unit mass, $\sigma/m_D$, as a function of the average collision velocity, $v_{\text{av}}$, of DM particles within each subhalo's core. Symbols show the range of $\sigma/m_D$ needed for the SIDM model to reproduce the central DM densities reported by Kaplinghat et al. (2019). The solid line corresponds to the best-fit relation given by eq. (15) to the MW dSph data.
where $A = 4\pi R^2$ is the area of one of the condenser plates, which in our case for thin clouds is the surface area of the pearl. The electric tension of the condenser is here denoted $V$ and the distance between the plates $d$ is imagined as small. The vacuum permeability is denoted $\epsilon_0$. Further $Q$ is the total charge on the condenser and $q = \frac{Q}{A}$ is the charge density per unit area. Writing the relation per unit area we have

$$F_{att} \frac{A}{A} = \frac{q^2}{2\epsilon_0},$$

(8)

$$= \frac{(Z_{\text{outside}})^2 \epsilon}{2\epsilon_0}.$$  

(9)

where $Z_{\text{outside}}$ is the number of electrons outside the skin.

In our early work [3] we required that the pressure provided by the skin with tension $S$ and radius $R$ should be balanced by the relativistic electron degeneracy pressure

$$P = \frac{2S}{R},$$

(10)

$$\approx \frac{1}{12\pi^2} * p_f^4.$$  

(11)

So when electrons are pushed out of a layer inside the skin, the electron degeneracy pressure is replaced by the electric force per unit area

$$F_{att} \frac{A}{A} \approx P = \frac{1}{12\pi^2} * p_f^4.$$  

(12)

Now we have fitted to the intensity and the very line frequency of the 3.5 keV X-ray line emitted by galactic clusters [8] using only one parameter:

$$\frac{\xi_1}{\Delta V} = 0.6 \text{MeV}^{-1}.$$  

(13)

The Fermi momentum of the electrons is given by this parameter to be

$$p_f = \xi_1^{-1/4} 2\Delta V = 3.3 \text{MeV}.$$  

(14)

Thus the pressure is

$$P = \frac{1}{12\pi^2} * p_f^4,$$

(15)

$$= (3.3 \text{MeV})^4,$$

(16)

$$= 1.00 \text{MeV}^4$$

(17)

Using that the fine structure constant is

$$\alpha = \frac{e^2/(hc)}{4\pi\epsilon_0},$$

(18)

it follows from equations (19), (12) and (17) that:

$$2\pi\alpha \left(\frac{Z_{\text{outside}}}{A}\right)^2 = 1.00 \text{MeV}^4$$

(19)
giving

$$\frac{Z_{\text{outside}}}{A} = \frac{1.00 \, MeV^2}{\sqrt{2\pi \alpha}}$$ \quad (20)

$$= 1.17 \times 10^{26} \, m^{-2}.$$ \quad (21)

Now the electron density in the undisturbed interior of our pearls is

$$n_e = \frac{1}{3\pi^2} \ast p_f^3$$ \quad (22)

$$= \frac{1}{3\pi^2} \ast (3.3 \, MeV)^3$$ \quad (23)

$$= 1.5 \times 10^{38} \, m^{-3}.$$ \quad (24)

So the layer thickness of material of electron number density $n_e = 1.5 \times 10^{38} \, m^{-3}$ having an area density of $\frac{Z_{\text{outside}}}{A} = 1.17 \times 10^{26} \, m^{-2}$ is

$$\text{“layer thickness”} = \frac{1.17 \times 10^{26} \, m^{-2}}{1.5 \times 10^{38} \, m^{-3}}$$ \quad (25)

$$= 7.8 \times 10^{-13} \, m.$$ \quad (26)

This then means that on the inside of the skin at least there must be a range of the size of “layer thickness” = $7.8 \times 10^{-13} \, m$ in which the electrons are removed relative to the usual density in the deep inside of the pearl. But if the skin radius $R$ is not at least of this size, it is nonsense. So we conclude that

$$R \geq \text{“layer thickness”} = 7.8 \times 10^{-13} \, m$$ \quad (27)

is the minimum size for a pearl with an electron Fermi momentum of 3.3 MeV.

This minimal size which we here found is clearly an underestimate of the needed minimal size of the radius for the skin, since the electrons will not be removed exactly in a layer with sharp edges. Therefore the realistic minimal radius is some factor of order unity larger than our number here $7.8 \times 10^{-13} \, m$. In fact the realistic limit cannot be far away from the radius $5 \times 10^{-12} \, m$ found in our Bled-proceedings contribution [9].

4.2 Little more thinking of the lower limit

It is clear that even the possibility that the total number of electrons inside the skin should be pushed out is absurd. There must of course be a density (per unit volume) of electrons inside the skin of at least the same size as in the outside region. So the lowering of the density due to the missing electrons on the inside cannot be more than 50% of the total density before the emptying out. This consideration would make the “layer thickness” two times as large. Taking into account that in the case of minimal size we have to deal with a strongly curved skin compared to this “layer thickness”, we also see that the inner thickness layer may have to be about doubled to compensate for the smaller sphere area as one goes deeper in. So overall we are close to having to scale up this “layer thickness” by a factor 4. That would bring us really very close to the radius $5 \times 10^{-12} \, m$ calculated in the Bled-proceedings paper [9] and in the next section, see eq. (47).
So we might take the point of view that we are thereby effectively calculating the minimal size of the pearl. Indeed in [9] and section 5 below, we assume that the number of electrons \(Z\) outside the central part of the pearl is of order \(\frac{M}{4\pi\hbar^2}\), meaning there are equally many electrons inside and outside as in the limiting case discussed above.

5 Range of Electron Cloud

We can consider the calculation in this section as an estimate of the smallest size of the pearl which is compatible with our parameter \(\frac{1}{4}\) of \(\frac{\xi}{\Delta V} = 0.6 \, \text{MeV}\). We also find that the value of the cross section to mass ratio \(\sigma\) from dwarf galaxies [18] at the lowest velocity is compatible with this minimal size.

The range of the extension of electrons around the pearl is to first approximation (using a Thomas Fermi approximation philosophy) supposed to be given by the requirement that the electron binding energy is of the order of the homolumo gap value 3.5 keV. So we denote this radius by \(r_{\text{cloud}}\). But probably the Thomas Fermi approximation, which we shall use, is not applicable to the very thinnest outskirts of the electron cloud. At the very outermost part of the electron cloud the more usual wave function thinking might be more applicable, and consequently the electron density will not fall off sharply at \(r_{\text{cloud}}\) as the Thomas Fermi approximation together with the homolumo-gap effect would suggest. Thus we should rather believe in a somewhat bigger extension of the cloud, which compared to the use of a sharp cut off at \(r_{\text{cloud}}\) would give a larger low velocity cross section \(\sigma\) and thus (for the same mass) a bigger ratio \(\sigma\).

Similarly the radius of the bubble containing the nucleons inside our dark matter pearl corresponds to a radius \(r_{\text{cloud}}\) at which the potential for the electron is -3.3 MeV (= Fermi energy of the electrons (numerically)). The high velocity hard collisions of our pearls, supposed to result in the unification of two pearls into a single pearl, correspond to interactions between the bubble skins with a cross section of order \(\pi r_{\text{cloud}}^2\).

5.1 Thomas Fermi Approximation, Electron Cloud

We will now consider the electric potential for our pearl using the Thomas-Fermi approximation for a heavy atom [33, 34, 35]. In this approximation the Coulomb potential of the “nuclear” charge \(Z\) - which in our model should be the number of charge quanta outside the skin of radius \(R = r_{\text{cloud}}\) - is multiplied by the Thomas-Fermi screening function \(\chi\) where

\[
b = 0.88 \frac{a_0}{Z^{1/3}}
\]

and \(a_0\) is the Bohr radius. We assume, that the skin of the bubble or “nucleus” of the pearl mainly acts on the nucleons or rather nuclei. So the electrons spread out and some of the electrons - in fact \(Z\) - are outside the central part of the pearl inside the skin. Therefore \(Z\) is also the effective charge of the central part of the pearl or bubble of the new phase.
In the Thomas-Fermi approach we are then led to the following equations for $r_{\text{cloud }3.5\text{keV}}$ and $r_{\text{cloud }3.3\text{MeV}}$:

$$\frac{\alpha \ast Z}{r_{\text{cloud }3.5\text{keV}}} \ast \chi\left(\frac{r_{\text{cloud }3.5\text{keV}}}{b}\right) = 3.5 \text{ keV} \quad (29)$$

$$\frac{\alpha \ast Z}{r_{\text{cloud }3.3\text{MeV}}} \ast \chi\left(\frac{r_{\text{cloud }3.3\text{MeV}}}{b}\right) = 3.3 \text{ MeV} \quad (30)$$

$$b = 0.88 \ast \frac{a_0}{Z^{1/3}} \quad (31)$$

We identify $r_{\text{cloud }3.5\text{keV}}$ with the radius of the electron cloud and $r_{\text{cloud }3.3\text{MeV}}$ with the skin radius $R$ of the pearl.

If we decide to look for the minimal size pearl, an assumption that seems to be approximately fulfilled\(^2\), we can add to the above set of equations the assumption that \textit{the number of electrons outside the skin and inside the skin are of similar order of magnitudes}. To be a bit more precise we can say: Thinking of it as at outset that the electrons were all kept inside, like the nuclei, we must have removed a number of electrons from the inside equal to the amount present in the outside cloud, because otherwise the full pearl would not be neutral. But it would be quite unreasonable that there should be a lower electron density in the inside than in the outside. So approximating for a moment the skin by being flat we can at the very most have half of the electrons in the outside. Since we assume that the nuclei inside the skin are so light as to have roughly equally many protons and neutrons we can, remembering that the nuclei dominate the mass $M$, write the above result as an inequality:

$$M/m_N \geq 4Z, \quad (33)$$

where $Z$ is the number of electrons outside the skin in the cloud. The assumption of the minimal size becomes then that there is an equality instead of the inequality \(^{33}\):

$$M/m_N \approx 4Z \text{ for minimal size.} \quad (34)$$

It is going to be a success of our model that using this minimal size assumption we get a similar value to the Thomas-Fermi value \(^{30}\) for $R \approx r_{\text{cloud }3.3\text{MeV}}$ using another method to calculate it. We shall use

$$\frac{\sigma}{M\big|_{v\to0}} = 150 \text{ cm}^2/g \quad (35)$$

and

$$\sigma = \pi \ast r_{\text{cloud }3.5\text{keV}}^2 \quad (36)$$

to determine the mass $M$. Then using the formula for the mass of a pearl in terms of the radius $R$ and the Fermi momentum \(^7\)\(^8\)

$$\frac{M}{m_N} = \frac{8}{9\pi} \ast (R \ast p_f)^3, \quad (37)$$

\(^2\)However we shall consider much bigger pearls in section \(^5\) in order to be consistent with an annual modulation in the DAMA-LIBRA data.
we can calculate another value for \( R \).

In our updated contribution to the Bled Proceedings from 2020 \cite{8} we estimated a pearl mass of \( M \sim 10^5 \) GeV. So we take here \( Z = 5.3 \times 10^4 \) (mainly from the “historical value” under the assumption of minimal size) as a typical charge in the central part of the pearl, for which then \( b = 1.24 \times 10^{-12}m \). Using numerical values for the Thomas-Fermi screening function in the paper \cite{36}, we obtain from (29) the radius of the electron cloud to be

\[
3.3 \text{ MeV} = 4.96 \times 10^{-11}m. \tag{38}
\]

Then assuming - from the dwarf galaxies observations - the low velocity ratio \( \frac{\sigma}{M} = 150 \text{ cm}^2/g \) we obtain

\[
M = \frac{\pi \times (4.96 \times 10^{-11}m)^2}{150 \text{ cm}^2/g} \tag{39}
= 5.2 \times 10^{-19}g \tag{40}
= 3.1 \times 10^5 m_N \tag{41}
\]

As a side remark notice that, using our proposed minimal-size-rule of taking \( Z \) to be a quarter of the number \( M/m_N \), we would get \( Z = 8 \times 10^4 \) to be compared with our input here \( 5.3 \times 10^4 \), which is very well consistent within a factor 2.

Next using \( p_f = 3.3 \text{ MeV} = 1.6 \times 10^{13}m^{-1} \)

\[
(R \times p_f)^3 = 3.1 \times 10^5 \times \frac{9\pi}{8} \tag{42}
= 10.9 \times 10^5 \tag{43}
\]

giving

\[
R = \frac{\sqrt[3]{10.9 \times 10^5}}{1.6 \times 10^{13}m^{-1}} \tag{44}
= 6.4 \times 10^{-12}m. \tag{45}
\]

This is to be compared with the Thomas-Fermi value obtained from \( \chi(r/b) \) in \( \chi \)

\[
R = r_{\text{cloud,3.3MeV}} = 3.66 \times 10^{-12}m. \tag{46}
\]

These two different estimates of the radius \( r_{\text{cloud,3.3MeV}} \) at which the potential is 3.3 MeV essentially coincide to the accuracy of our calculation; they deviate by a factor of order unity \( 6.4/3.7 = 1.7 \). So we could claim that formally our model is able to predict the low velocity limit \( \frac{\sigma}{M} \mid_{v \rightarrow 0} \) in agreement with the value 150 cm\(^2\)/g estimated from the study of dwarf galaxies around the Milky Way.

We shall take the average of the two values \( (45) \) and \( (46) \) as our best estimate of the bubble skin radius, under the assumption of “minimal size”:

\[
r_{\text{cloud,3.3MeV}} = 5.0 \times 10^{-12}m \text{ (assuming “minimal size”)}. \tag{47}
\]

\(^3\)However we now prefer a value of \( M \sim 10^{12} m_N \); see section \( 12 \).
and from \((38)\) we have the radius of the electron cloud
\[
r_{\text{cloud}} \approx 3.5 \text{ keV} = 5.0 \times 10^{-11} \text{m. (assuming “minimal size”) (48)}
\]

We note that these two radii differ by an order of magnitude, which means that the quantity \(\tau\) for our minimal size pearls should differ by two orders of magnitude between low velocities and high velocities, as astronomical observations indicate is the case for self interacting dark matter \([18]\).

6 Correction to have \(\chi\) (“\(\chi\)”) \((x_{\text{cloud}} 3.3 \text{MeV}) (1 - \frac{x dx}{\chi dx}) = 1\) at \(r = R\)

In the previous section we used the Thomas-Fermi screening function \(\chi(r/b)\) satisfying the boundary condition that \(\chi(0) = 1\) corresponding to a nucleus of zero radius compared to the atomic size. This approximation is good for real atoms, and it is the solution of the Thomas-Fermi differential equation with this boundary condition that has been studied and calculated. However, our genuine pearl inside the skin is not so terribly small compared to the atom or in our case the cloud of electrons. We therefore should in principle do a better job by imposing the boundary condition \(\chi = 1\) at that value of the variable \(x = r/b\), which corresponds to the radius of the skin \(R = r_{\text{cloud}} \approx 3.3 \text{MeV}\).

Now, however, what boundary condition to take is a little bit more complicated: At the boundary value of \(r = R\) at which we start the cloud calculation we have the charge \(eZ\) inside the sphere of this radius \(R\). That means that the electric field radiating from the sphere is given by \(eZ\), but the potential (as the reader should have in mind is gauge dependent and only defined by say the potential at infinity being fixed to 0) does not only depend on the charge inside the sphere, but also on what is outside, if the gauge is fixed in the outside, namely at infinity.

The \(\chi\)-function with boundary condition at \(r = R\) rather than at \(r = 0\) is a different function from the “universal function” \(\chi\) as evaluated in say the Parand article \([36]\), but let us here denote it by the letter \(\chi(x)\) anyway, and just remember when we use which boundary condition.

Considering for a moment \(\chi\) as a function of the distance to the center \(r\) we obtain for the potential
\[
V_{\text{eff}}(r) = \frac{e^2Z\chi}{r} \quad (49)
\]
the radial field
\[
eE = \frac{dV_{\text{eff}}(r)}{dr} \quad (50)
\]
\[
eE = e^2Z \frac{-\chi + r \frac{dx}{dr}}{r^2}, \quad (51)
\]
which should equal the Coulomb field
\[
eE_{\text{Coulomb}} = -e^2Z \frac{1}{r^2} \quad (52)
\]
Thus at $r = R$ we have the boundary condition:

$$-1 = -\chi + r \frac{d\chi}{dr}$$

meaning

$$1 = \chi \frac{\left(1 - \frac{rd\chi}{\chi dr}\right)}{1 - \frac{xd\chi}{\chi dx}}.$$  

(53)

(54)

(55)

We found that the $x$-variable value corresponding to the radius $r_{cloud \, 3.3 \, MeV}$, where the potential equals the Fermi energy in the bulk of the interior skin material in our model, namely $3.3 \, MeV$, is $x_{cloud \, 3.3 \, MeV} = 2.95$ (actually using the boundary at $x = 0$, so it is only crude). Near this value $x = 2.95$ the $\chi$-function (with boundary condition at $x = 0$ as usual) is approximated by being proportional to the power-law

$$\chi(x) \propto x^{-1.2} \quad \text{for } x \text{ near } 2.95$$

(56)

and the value is

$$\chi(2.95) = 0.16.$$  

(57)

Thus

$$\chi(2.95)(1 - \frac{x d\chi(x)}{\chi(x) dx}) = 0.16 \ast (1 + 1.2)$$

(58)

$$= 0.35$$

(59)

To get a crude estimate we take it that we can find a solution of the Thomas-Fermi differential equation, which we avoid calculating explicitly, satisfying the boundary condition (55) by scaling the $\chi$ up by the inverse of the value 0.35 at the point $r = R$. Since it is a non-linear differential equation we should solve it again, it is in truth more complicated, but scaling by 0.35$^{-1} = 3$ would then roughly mean that a given value is taken on for a larger $x$ by a factor $f$ such that $f^{(p+1)} = 3$. The power $p$ is a power chosen so that in the region of interest here between the two radii $r_{cloud \, 3.3 \, MeV}$ and $r_{cloud \, 3.5 \, keV}$, the function $\chi$ can be considered proportional to $x^{-p}$. Let us crudely estimate this power $p$, for simplicity taking the corresponding range of $x$-values as 3 and 30. The power $p+1$ used in the formula $f^{(p+1)} = 3$ was due to the potential going proportional to $\chi/r ^{\propto \chi/x}$. Now we have

$$\chi(3) = 0.1566 \quad \text{and} \quad \chi(30) = 2.255 \ast 10^{-3}.$$  

(60)

This gives a fall over one decade of

$$\text{"fall"} = \frac{0.1566}{0.002255} = 69.4.$$  

(61)

So we obtain

$$p = \log("\text{fall}") = 1.84.$$  

(62)

That is to say the factor is $f = x \sqrt[3]{3} = 1.47$. So very crudely the radius say at which the potential is $3.5 \, keV$ and where the electron cloud should end, would be pushed out to a bigger radius by a factor 1.47.

$$r_{cloud \, 3.5 \, keV} = 5 \ast 10^{-11} m \rightarrow r_{cloud \, 3.5 \, keV} = 1.47 \ast 5 \ast 10^{-11} m$$

(63)

$$= 7.4 \ast 10^{-11} m.$$  

(64)
6.1 Also the Skin Radius gets Corrected

For the analogous correction of the radius of the skin \( R = r_{\text{cloud} \, 3.3 \text{MeV}} \) we should use the power 1.2 meaning approximating the \( \chi \) to be proportional to \( x^{-1.2} \), because we work very close to where this power is o.k.. Then the increasing of the value of the \( \chi \) function by a factor 3 means that the value at which the potential has a required value \( 3 \times 3.3 \text{MeV} \), will go up by a factor \( f \), where now this \( f \) must obey \( f^{1+1.2} = 3 \). This means a correction factor for the radius \( r_{\text{cloud} \, 3.3 \text{MeV}} \) of \( f = \sqrt[3]{3} \approx 1.65 \) giving

\[
r_{\text{cloud} \, 3.3 \text{MeV}} = 5 \times 10^{-12} m \quad \rightarrow \quad r_{\text{cloud} \, 3.3 \text{MeV}} = 1.65 \times 5 \times 10^{-12} m \quad (65)
\]

\[
= 8.2 \times 10^{-12} m \quad (66)
\]

It follows from equation (67) that the mass density of the pearl is given by

\[
\rho_B = \frac{2m_N}{3\pi^2} f^3
\]

\[
= \frac{2}{3\pi^2} (940 \text{ MeV}) (3.3 \text{ MeV})^3
\]

\[
= 5.2 \times 10^{11} \text{ kg/m}^3. \quad (69)
\]

Using the density (69) of the part of the pearl inside the skin we obtain the mass of the pearl to be

\[
M = \frac{4\pi}{3} \times 5.2 \times 10^{11} \text{ kg/m}^3 \times (8.2 \times 10^{-12} m)^3 \quad (70)
\]

\[
= 1.19 \times 10^{-21} \text{ kg} \quad (71)
\]

\[
= 7.0 \times 10^5 \text{ GeV} \quad (72)
\]

Here we just improved the calculation for the radius \( R \) of the minimal size pearl and thus got the minimal mass to be \( M = 7.0 \times 10^5 \text{ GeV} \). So we obtain

\[
\frac{\sigma}{M} = \frac{\pi r_{\text{cloud} \, 3.5 \text{keV}}^2}{M} = \frac{\pi (7.4 \times 10^{-11})^2}{7.0 \text{GeV} \times 1.78 \times 10^{-27} \text{ kg/GeV}} \quad (73)
\]

\[
= 140 \text{ cm}^2/\text{g}, \quad (74)
\]

still in agreement with the low velocity estimate \( \frac{\sigma}{M} \approx 150 \text{ cm}^2/\text{g} \) from the study [18] of the Milky Way dwarf galaxies.

7 Achievements

- **Low velocity \( \frac{\sigma}{M}|_{v \rightarrow 0} \) cross section to mass ratio.** The a priori story, that dark matter has only gravitational interactions seems not to work perfectly: Especially in dwarf galaxies (around our Milky Way) where dark matter moves relatively slowly an appreciable self interaction cross section to mass ratio \( \frac{\sigma}{M} \) is needed. According to the fits in [18] this ratio has the low velocity limit \( \frac{\sigma}{M}|_{v \rightarrow 0} = 150 \text{ cm}^2/\text{g} \). We may say our pearl-model “predicts” this ratio in order of magnitude provided that the pearls are of “minimal size”, meaning that the pearls are as small as possible while it is still true that there can be a sensible diminishing of electron number in the inside compared to that we had with completely neutral material inside.
• **Can make the Dark Matter Underground Searches get Electron Recoil Events** Most underground experiments are designed to look for dark matter particles hitting the nuclei in the experimental apparatus, which is then scintillating so that such hits presumed to be on nuclei can be seen. But our pearls are excited in such a way that they send out energetic electrons (rather than nuclei) and this does not match with what is looked for, except in the DAMA-LIBRA experiment. In this experiment the only signal for events coming from dark matter is a seasonal variation due to the Earth running towards or away from the dark matter flow.

• **The Intensity of 3.5 keV X-rays from Clusters etc.** We fit the very photon-energy 3.5 keV and the overall intensity from a series of clusters, a galaxy, and the Milky Way Center [8] with one parameter $\xi_{1/4}^{1/4} = 0.6 \text{MeV}^{-1}$.

• **3.5 keV Radiation from the Tycho Supernova Remnant.** Jeltema and Profumo [38] discovered the 3.5 KeV X-ray radiation coming from the remnant of Tycho Brahe’s supernova, which was unexpected for such a small source. We have a scenario giving the correct order of magnitude for the observed intensity in our pearl model: supposedly our pearls are getting excited by the high intensity of cosmic rays in the supernova remnant [8].

Even though we only need the one parameter $\xi_{1/4}^{1/4} = 2 pf$, it is nice to know the notation:

\[
\Delta V = \frac{\text{difference in potential for a nucleon between the inside and the outside of the central part of the pearl}}{\text{potential difference for a nucleon between the inside and the outside of the central part of the pearl}} \\
\approx 2.5 \text{MeV} 
\]

\[
\xi_{fS} = \frac{R}{R_{crit}} \text{ estimated to be } \approx 5 
\]

where $R = \text{actual radius of the new vacuum part}$

\[
\approx r_{cloud} 3.3 \text{MeV} 
\]

and $R_{crit} = \text{Radius when pressure is so high that nucleons are just about being spit out}$

The subscript $fS$ on the parameter $\xi_{fS}$ indicates that the surface tension $S$ is fixed independent of the radius $R$.

• **DAMA-rate** Estimating observation rate of DAMA-LIBRA from kinetic energy of the incoming dark matter as known from astronomy.

• **Xenon1T Electron recoil rate** Same for the electron recoil excess observed by the Xenon1T experiment.

In order to explain these last calculational estimates it is necessary to know how we imagine the dark matter to interact and get slowed down in the air and the earth shielding; also how the dark matter particles get excited and emit 3.5 keV radiation or electrons.

**About the Xenon1T and DAMA-rates:**
Absolute rates very crudely Our estimate of the absolute rates for the two experiments are very very crude, because we assume that the dark matter particles - in our model small macroscopic systems with tens of thousands (for the larger pearls considered in section 9.4 rather \( \sim 10^{12} \)) of nuclei inside them - can have an exceedingly smooth distribution of lifetimes on a logarithmic scale. These calculations are discussed in section 10.

The ratio of rates The ratio of the rates in the two experiments - Xenon1T electron recoil excess and DAMA - should in principle be very accurately predicted in our model, because they are supposed to see exactly the same effect just in two different detectors in the same underground laboratory below the Gran Sasso mountain! One would therefore expect the rates to be the same, but the Xenon1T rate is 250 times smaller than the DAMA rate. We briefly refer to a possible resolution of this problem, which needs further study, in section 11.

8 Impact

Illustration of Interacting and Excitable Dark Matter Pearls

The dark matter pearls come in with high speed (galactic velocity), but get slowed down to a much lower speed by interaction with the air and the shielding mountains, whereby they also get excited to emit 3.5 keV X-rays or electrons. We shall consider two types of pearl here: the minimal size pearl with radius \( R \sim 8 \times 10^{-12} \) m discussed so far and a pearl introduced in section 9.4 of radius \( R \sim 10^{-9} \) m, which is surrounded by dirt in outer space that is quickly burnt off in the atmosphere.

Pearls Stopping and getting Excited in Earth Shield

What happens when the dark matter pearls in our model hit the earth shielding above the experimental halls of e.g. DAMA?

Stopping Taking it that the pearls stop in the earth: The pearls are slowed down in about \( 5 \times 10^{-6}s \) from their galactic speed of about 300 km/s down to a speed \( \sim 49 \) km/s below which collisions with nuclei can no longer excite the 3.5 keV excitations. Using the value of \( \sigma_M \sim 2 \) cm\(^2/\)g obtained in the fit to the dwarf galaxies in [13] for the minimal size pearl we get the stopping length to be \( d \sim 2/3 \) cm. On the other hand for the pearl of radius 1 nm we get a stopping length \( d \sim 22 \) cm.

But taking it that they stop in the air, which is more likely: They are slowed down over a range of about 7 km - as the density of the atmosphere
goes up by a factor $e = 2.71$. over such a range - in about $2 \times 10^{-2}s$. The above stopping lengths in earth correspond in the penetration into the atmosphere to stopping at the heights

$$h = 44 \text{ km} \quad \text{for minimal size pearls} \quad (79)$$

$$h = 19 \text{ km} \quad \text{for pearls with } R \sim 1 \text{ nm.} \quad (80)$$

- **Excitation** As long as the velocity is still over ca. 49 km/s, collisions with nuclei in the shielding can excite the electrons inside the pearl by 3.5 keV or more and make pairs of quasi electrons and holes say. We expect that often the creation of (as well as the decay of) such excitations require electrons to pass through a (quantum) tunnel and that consequently there will be decay half lives of very different sizes. We hope even up to many hours or days or years ...

- **Slowly sinking:** After being stopped in of the order of 2/3 cm of the shielding for a minimal size pearl or 22 cm for a 1 nm size pearl, the pearls continue with a much lower velocity driven by the gravitational attraction of the Earth. If a high Reynolds number formula would have been valid the pearls might manage to come down to the experimental halls in days. However if - as we now believe - they are so slow as to rather make Stokes law be used it becomes a problem to get them down inside a year, as we require since otherwise the DAMA-seasonal-effect tends to be smeared away.

When the pearls after hopefully less than a year reach down the 1400 m to the laboratories, most of the pearls have returned to their ground states, but some exceptionally long living excitations survive.

*Note that the slowly sinking velocity is so low that collisions with nuclei cannot give such nuclei enough speed to excite the scintillation counters neither in DAMA nor in Xenon-experiments.*

- **Electron or $\gamma$ emission** Typically the decay of an excitation could be that a hole in the Fermi sea of the electron cloud of the pearl gets filled by an outside electron under emission of another electron by an Auger-effect. The electron must tunnel into the pearl center. This can make the decay lifetime become very long and very different from case to case.

**Emission as electrons or photons makes Xenon-experiments not see events, except...**

That the decay energy is released most often as electron energy meaning that such events are discarded by most of the Xenon-experiments, which only expect the nucleus recoils to be dark matter events. This would explain the long standing controversy consisting in DAMA seeing dark matter with a much bigger rate than the upper limits from the other experiments.

Rather recently though Xenon1T looked for potential excess events among the electron recoil events and found 16 events/year/tonne/keV in the lowest keV-bands over a background of the order of $(76 \pm 2) \text{ events/year/tonne/keV}$. In our model this rate should be compatible with the DAMA event rate. However they deviate by a factor of 250. It therefore appears that we need the pearls to run much faster through the xenon-apparatus than through the DAMA one.
9 Consistency of the Picture of Slowed Down Impact Pearls

We postulate - as strongly indicated from the range of energies seen in the two experiments being both concentrating around 3.5 keV - that the Xenon1T electron recoil excess and the DAMA effects should be due to the same effect, namely that dark matter particles although stopped or slowed down de-excite in the detectors and emit energy in quanta of 3.5 keV. Then it seems, as we just said, that the rate per kg of scintillator say should be rather closely the same in the two experiments. But now the DAMA-experiment sees 250 times as many events - even counting only the modulated part of the signal - as Xenon1T sees as excess electron recoil events.

One could of course speculate that the DAMA experiment sees some sort of emission not visible in the Xenon1T experiment, but we prefer the explanation that the pearls run faster through the liquid xenon than the solid NaI and thus spend less time in the xenon-detector than in the NaI one per kg.

We shall now look at what is required to make such a picture work and be consistent:

9.1 Need for reaching down in less than one year

Since the DAMA experiment sees the seasonal variation of the signal, it is of course needed that the pearls reach down to the experimental halls faster than or at least not much slower than in one year. If the pearls reach down much slower than in a year the signal would be smeared out in time and the modulated signal would almost be washed away.

But now most of the way down through the shielding there is solid material, not exactly NaI, but something which is similar to that solid.

If we shall explain the ratio of 250 times more particles or events in DAMA compared to the Xenon1T electron recoil excess by the slower passage in the NaI than in the liquid xenon, then we need that the terminal velocity - when the drag force just balances gravity - is 250 times smaller in NaI than in the liquid xenon. But to avoid the modulation being smeared out the strongly slowed down pearls should at least go through the shielding to the detectors in less than one year. The shielding in Gran Sasso contains 1400 m of mountain, and in addition the pearls may be slowed down already in the atmosphere, exactly how high in the atmosphere is of course dependent on the parameters of the pearls, the cross section and mass. However the time for the pearls to pass through the atmosphere will be much smaller than the time to pass through 1400 m of stone and can be safely ignored.

For the pearl to reach through 1400 m in one year requires its velocity to be at least

\[ v_{\text{needed}} = \frac{1400m}{\text{1 year}} \]

(81)

\[ = 4.43 \times 10^{-5} m/s. \]

(82)

In the liquid xenon the pearl should fall with a 250 times larger speed, i.e.

\[ v_{\text{terminal liquid Xe}} \geq 250 \times 4.43 \times 10^{-5} m/s = 1.11 \times 10^{-2} m/s. \]

(83)
9.2 Size requirement from Speed Requirement

Using Stoke’s law for a ball with mass

$$M = \rho_B \ast \frac{4}{3} \pi R^3$$  \hfill (84)

one finds its terminal velocity in a gravitational field of strength \( g \) to be

$$v_{\text{terminal}} = \frac{2g\rho_B R^2}{9\eta},$$ \hfill (85)

where \( \eta \) is the dynamical viscosity of the fluid through which the ball falls. Here \( \rho_B \) is the density of the ball and \( R \) its radius. In this formula the ball is taken to be homogeneous, so it means for our pearls that one has ignored the surrounding electron cloud, which has in the model an essentially zero density, and the radius \( R \) is the radius of the skin preferably.

Hence in our model we rather have an inequality

$$v_{\text{terminal}} \leq \frac{2g\rho_B R^2}{9\eta},$$ \hfill (86)

where the equality will be more and more true the bigger the pearl.

Inserting the minimal required value of the terminal velocity in liquid xenon \[83\] of \( v_{\text{terminal}} = 1.11 \ast 10^{-2} m/s \) in our inequality we obtain

$$1.11 \ast 10^{-2} m/s \leq \frac{2g\rho_B R^2}{9\eta},$$ \hfill (87)

where of course now the dynamic viscosity \( \eta \) should be the one for liquid xenon. For our crude calculations we use instead the data on the viscosity of gaseous and liquid nitrogen \[37\] to estimate a typical value for the dynamical viscosity of the fluid form of a normal gas near its boiling point to be

$$\eta_{\text{liquid gas}} \approx 100 \mu Pa \ s = 0.1 mPa \ s \ \text{near the boiling point.}$$ \hfill (88)

We take the density of the material inside the skin from eq. \[69\]

$$\rho_B = 5.2 \ast 10^{11} kg/m^3.$$ \hfill (89)

The inequality \[87\] then means

$$R^2 \geq \frac{1.11 \ast 10^{-2} m/s \ast 9\eta}{2g\rho_B}$$ \hfill (90)

$$= \frac{1.11 \ast 10^{-2} m/s \ast 9 \ast 10^{-4} Pa s}{2 \ast 9.8 m/s^2 \ast 5.2 \ast 10^{11} kg/m^5}$$ \hfill (91)

$$= 0.98 \ast 10^{-18} m^2$$ \hfill (92)

giving

$$R \geq 0.99 \ast 10^{-9} m.$$ \hfill (93)
The mass corresponding to this borderline radius \( R = 0.99 \times 10^{-9} m \) is

\[
M_{\text{minimal}} = \frac{4\pi}{3} \rho_B R_{\text{minimal}}^3
\]

\[
= \frac{4\pi}{3} \times 5.2 \times 10^{11} \text{kg/m}^3 \times (0.99 \times 10^{-9} m)^3
\]

\[
= 2.1 \times 10^{-15} \text{kg}
\]

\[
= 1.2 \times 10^{12} \text{GeV}.
\]  

This is to be compared with our fit involving the “minimal size” pearls with \( \sigma/M \sim 150 \text{ cm}^2/\text{g} \), which gave \( M = 7.0 \times 10^5 m_N \).

We have formally here an inconsistency of the order \( 1.2 \times 10^{12} m_N = 1.7 \times 10^6 \).

So, in order to avoid wash out of the seasonal effect in DAMA and to have enough difference in the speed of the pearls in the fluid xenon and the shielding, we need \( 1.7 \times 10^6 \) times as big mass \( M \) of the pearl as we get from fitting the dwarf galaxy number \( \sigma/M \sim 150 \text{ cm}^2/\text{g} \).

9.3 Hope for better fit by using \( v \sim 300 \text{ km/s} \) instead of 0

The discrepancy between the above two mass estimates might be resolved if instead of the zero velocity limit \( \frac{E}{M} = 150 \text{ cm}^2/\text{g} \) we could use a smaller value.

We namely in our attempt to fit the ratio assumed that the electron cloud of the pearl only extended out to the distance \( r_{\text{cloud}} 3.5 \text{ keV} \) where the electron potential is 3.5 keV, as expected from the homolumo gap. But it is known that one cannot trust the Thomas Fermi approximation completely, even if the homolumo gap is working, so as to cut off the density of electrons at this \( r_{\text{cloud}} 3.5 \text{ keV} \) distance sharply. Rather there will be a tail of the electron cloud even further out, and at a true zero velocity even such a very thin cloud of electrons could cause some cross section. However, it is rather sure, that if we consider pearls meeting each other with a relative velocity \( v \) such that the kinetic energy of the pearl \( \frac{1}{2} M v^2 \) taken per electron in the cloud, i.e. \( \frac{Mv^2}{2Z} \approx \frac{Mv^2}{2\sqrt{M/M_N}} = 2m_N v^2 \) (if we take \( Z \approx M/(4m_N) \) as in the minimal size case), becomes equal to the binding or gap energy 3.5 keV, then at that velocity the one pearl will penetrate into the other to the potential depth of this order. So at the velocity given by

\[
\frac{1}{2} \times 4m_N v^2 \sim 3.5 \text{ keV}
\]

\[
giving \quad v \sim \sqrt{\frac{0.0035 \text{ MeV}}{2 \times 940 \text{ MeV}}} \]

\[
= 0.00136c
\]

\[
= 4.1 \times 10^5 \text{ m/s} = 410 \text{ km/s}
\]

the kinetic energy per cloud electron is 3.5 keV. Extrapolating the curve in Figure 5 for \( \frac{\sigma}{M} \) as a function of the velocity \( v \) to 410 km/s gives about 1 cm²/g for the ratio. The speculation would then be that for lower velocities the “tail” distribution, which should not have been there in the Thomas Fermi approximation with the homolumo gap cut off, is important and causes a bigger cross section. At very low velocities you could also expect van der Waals forces between the pearls causing them to scatter. We hope in a later article to be able to estimate the long distance and therefore low velocity behaviour better.
Figure 6: Cross section per unit mass $\frac{\sigma}{M}$ of self-interacting dark matter particles as a function of the collision velocity $v$ in dwarf galaxies from reference [18].

Figure 7. Same as Fig. 6, but extended to cover the range of MW- ($\sim 200$ km/s) and cluster-size ($\sim 1000 - 5000$ km/s) haloes' velocities. The figure shows upper and lower limits for $\frac{\sigma}{m}$ taken for substructure abundance studies (e.g. Volgelbarger et al. 2012 and Zavala et al. 2013), as well as based on halo shape/ellipticity studies and cluster lensing surveys (see text).
In fact we find on Figure 6 from [18] that at the velocity $v = 410 \text{ km/s}$ the cross section to mass ratio is very close to

$$\frac{\sigma}{M} \bigg|_{v=410 \text{ km/s}} = 1 \text{ cm}^2/\text{g}$$  \hspace{1cm} (102)

where the kinetic energy per cloud electron is 3.5 keV. Thus taking it that the cross section to mass ratio we have calculated is indeed rather that at 410 km/s, we would fit the mass to be 150 times bigger than the value $7.0 \times 10^9 m_N$ from (72). That would mean we have the mass prediction

$$M = 1.0 \times 10^8 m_N \quad \text{(for } \frac{\sigma}{M} = 1 \text{ cm}^2/\text{g} \text{)}$$  \hspace{1cm} (103)

corresponding to a pearl radius\(^4\)

$$R = 4.3 \times 10^{-11} \text{ m}.$$  \hspace{1cm} (104)

However the mass (103) is still much smaller than the minimal mass $M_{\text{minimal}}$ needed to avoid wash out of the DAMA seasonal effect.

9.4 Improvement by Dirt Surrounding the Pearl?

Taking it seriously that we at least have some observations suggesting a dark matter self interaction, having a ratio $\frac{\sigma}{M}$ which approaches e.g. the value $150 \text{ cm}^2/\text{g}$, we unavoidably need that the dark matter particle has some extension up to a cross section that can be consistent with such a ratio. But now the ratio falls as a function of the mass $M$ or the radius, whether $R$ or $r_{\text{cloud}} 3.5 \text{ keV}$ is used. So if we make the mass and the radius as big as required by (97) and (93), i.e. $M \geq 1.2 \times 10^{12} \text{ GeV}$ and $R \geq 0.99 \times 10^{-9} \text{ m}$, we cannot match the ratio $\frac{\sigma}{M} = 150 \text{ cm}^2/\text{g}$ without changing our model. The idea we propose is to assume that during their passage through space - probably at a rather early stage when there was a lot of ordinary matter around - the pearls have collected some dirt around them. They could have become small grains of dust having used the original pearl as a seed. Such dust would again be washed off / shaken off in the Earth’s atmosphere or the shielding mountains of the laboratories. If so the dust would not influence the passage through the earth material.

Now for a pearl of mass $M = 1.2 \times 10^{12} \text{ GeV}$ and radius $R = 0.99 \times 10^{-9} \text{ m}$ the ratio $\frac{\sigma}{M} = 1.5 \times 10^{-2} \text{ cm}^2/\text{g}$. So in order to increase this ratio to $150 \text{ cm}^2/\text{g}$ we need the speculated dirt to increase the cross section of the dark matter particle by a factor of $10^4$. But now the cross section of course goes as the square of the radius and thus the dirt must increase the radius by a factor 100.

On Figure 7 we have indicated how our pearl now with dimension $R = 0.99 \times 10^{-9} \text{ m}$ or bigger is enhanced by an 100 times bigger in radius dust configuration around this genuine pearl. Since the genuine pearl is about $1 \text{ nm}$ in radius the full pearl radius becomes of the order of $100 \text{ nm}$.

\(^4\) We note that the increase in the value of $R$ compared to (50) leads to a similar increase in $r_{\text{cloud}} 3.5 \text{ keV}$ and consequently to a value of $\frac{\sigma}{M} \sim 2 \text{ cm}^2/\text{g}$. 

---

26
Figure 7: The blueish sphere around the pearl symbolizes the “dirt”, which may be attached to the pearl and is likely to be some form of carbon or carbohydrate or silicate. This dirt would be stripped off when the pearl passes through the atmosphere or the Earth. The inner red sphere is the second vacuum part, and in between one sees a relatively tiny region comprising the electron cloud.

10 Numerical Rates for DAMA and Xenon1T-electron-recoil-excess

10.1 The Kinetic Energy Flux from Dark Matter

The dark matter density $D_{\text{sol}}$ in our part of the Milky Way and its velocity $v$ are of the orders of magnitude

$$D_{\text{sol}} = 0.3 \text{ GeV/cm}^3$$  \hspace{1cm} (105)

$$v = 300 \text{ km/s} \text{ (relative to solar system).}$$  \hspace{1cm} (107)

This gives a kinetic energy density

$$D_{\text{kin energy}} = \frac{1}{2} v^2 D_{\text{sol}}$$  \hspace{1cm} (108)

$$= 0.5 \times (10^{-3})^2 c^2 \times 5.34 \times 10^{-22} \text{ kg/m}^3$$ \hspace{1cm} (109)

$$= 2.40 \times 10^{-11} \text{ J/m}^3,$$ \hspace{1cm} (110)

meaning an influx of kinetic energy

“power per m$^2$” $= vD_{\text{kin energy}}$ \hspace{1cm} (111)

$= 7.2 \times 10^{-6} \text{ W/m}^2.$ \hspace{1cm} (112)
Distributing this energy rate over the amount of matter down to the depth 1400 m with density 3000 kg/m³ we obtain the energy rate per kg

\[
\text{“power to deposit”} = \frac{7.2 \times 10^{-6} \text{W/m}^2}{1400 \text{ m} \times 3000 \text{ kg/m}^3} = 1.7 \times 10^{-12} \text{W/kg.}
\]

However, assuming that all the events from the dark matter - as given by the modulated part of the signal found by DAMA-LIBRA - are just due to decays with the decay energy 3.5 keV, the rate of energy deposition per kg observed by DAMA-LIBRA [20] is

\[
\text{“deposited rate”} = 0.0412 \text{ cpd/kg} \times 3.5 \text{ keV} = 0.0412 \text{ cpd/kg} \times 3.5 \times 1.6 \times 10^{-16} \text{J}
\]

\[
= 2.7 \times 10^{-22} \text{W/kg,}
\]

which is

\[
\frac{2.7 \times 10^{-22} \text{W/m}^2}{1.7 \times 10^{-12} \text{W/m}^2} = 1.6 \times 10^{-10} \text{times as much.}
\]

We can express this by saying that there is a need for a suppression factor \(suppression\) being \(1.6 \times 10^{-10}\) for the DAMA-LIBRA rate. For the excess of the electronic recoil events found by Xenon1T the corresponding suppression factor must be the 250 times smaller number. This is because the event rate of these excess electron recoil events is 250 times smaller than that of the annual modulation part of the DAMA rate and the depth of the experiment under the earth is the same 1400 m. Thus we summarize the experimentally determined suppression factors:

\[
suppression_{\text{DAMA}} = 1.6 \times 10^{-10}
\]

\[
suppression_{\text{Xenon1T}} = \frac{1.6 \times 10^{-10}}{250} = 6.4 \times 10^{-13}.
\]

### 10.2 Estimating “suppression” theoretically

The idea for obtaining theoretical estimates of these suppression factors is to say that the observed events come from excitations of our pearls with a lifetime of the order of the time it takes for the pearl, after its excitation under its stopping in the air or in the stone above the experiments, to reach down to the experimental detectors.

As discussed in section [3], there is a tension between the high mass of the pearl required for it to reach through 1400 m of earth to the detectors in less than a year and the value of \(\sigma M = 150 \text{cm}^2/g\) suggested by the dwarf galaxy observations. Thus we are driven to seek a passage time for the pearl through the earth material close to the upper limit of one year from the condition that the seasonal variation should not wash away.
10.3 Equally hard to excite and to de-excite

In order that there can be any de-excitations of the pearls after such a year, it is of course needed that an appreciable part of the possible 3.5 keV excitations of our pearls have lifetimes of this order of magnitude. A priori these excitations are excitons for which the electron and hole can be close by and decay rapidly or it is possible that one of the partners is outside in the electron cloud and long lived. By arguing that some tunnelling of electrons in or out or around in the pearl may be needed for some (de-)excitations, we can claim that the lifetimes for the various excitation possibilities are smoothly distributed over a wide range in the logarithm of the lifetime; then there will be some pearl-excitations with the appropriate lifetime, although somewhat suppressed by a factor of the order of $1/\text{width}$ where the width here is the width of the logarithmic distribution. We shall take this width to be of order $\ln(\text{suppression}_{\text{DAMA}}) \sim 23$. But more importantly:

If a certain excitation is long-lived, it is also hard to produce. So we shall talk about an effective “stopping” or “filling time” for a pearl passing into the Earth, and imagine that during this “stopping” or “filling time” the excitations of the pearls have to be created. So the probability for excitation or suppression would be expected to be

$$\text{suppression} \approx \frac{\text{“filling time”}}{\text{“lifetime”}}.$$  \hspace{1cm} (121)

If the excitation happens to be of sufficiently long lifetime - say of order one year - then we can expect it to have a sensible chance of de-exciting just in the experimental detectors in Gran Sasso, DAMA or Xenon1T say.

But what shall we take for this “stopping” or “filling time”? A relatively simple idea, which is presumably right, is to say that the stopping takes place high in the atmosphere because a pearl entering the Earth’s atmosphere with galactic speed will be slowed down in the high air with a $\sigma \sim 2 \text{ cm}^2/\text{g}$. Now the density of the atmosphere rises by a factor $e = 2.718...$ per about 7 km. So as the slowing down begins it will, because of this rising density, essentially stop again after 7 km. Thus the time during which the pearl is truly slowing down in speed and forming 3.5 keV excitations is of the order of the time it takes for it to run 7 km. With the pearl velocity of about 300 $\text{km/s}$ (essentially the escape velocity for the galaxy) we then have

$$\text{“stopping time”} \approx \frac{7 \text{ km}}{300 \text{ km/s}} \Rightarrow 0.023 \text{ s}.$$  \hspace{1cm} (122)

The crucial factor, which we believe to be most important, is that in order to excite an excitation with a lifetime of the order $3 \times 10^7$ s = 1 year it would a priori need $3 \times 10^7$ s so that, if we only have 0.023 s, then there will be a suppression:

$$\text{suppression} = \frac{\text{“stopping time”}}{\text{“lifetime”}} \approx \frac{\text{“stopping time”}}{\text{“passage time”}} \approx \frac{0.023 \text{ s}}{3 \times 10^7 \text{s}}.$$  \hspace{1cm} (123)
This crudest estimate has to be compared with the experimental suppressions given above

\[
\frac{\text{suppression}_{\text{DAMA}}}{\text{suppression}_{\text{theory}}} = \frac{1.6 \times 10^{-10}}{6 \times 10^{-10}} = \frac{1.6}{6} = \frac{4}{1.6} = 4
\]

\[
\Rightarrow \text{suppression}_{\text{DAMA}} = \frac{1.6 \times 10^{-10}}{4} = 6 \times 10^{-13}
\]

\[
\frac{\text{suppression}_{\text{Xenon1T}}}{\text{suppression}_{\text{theory}}} = \frac{6 \times 10^{-10}}{250} = 6.4 \times 10^{-13}
\]

\[
\Rightarrow \frac{\text{suppression}_{\text{Xenon1T}}}{\text{suppression}_{\text{theory}}} \approx 1000
\]

But the last number 1000 for the misfitting of the Xenon1T-excess should be corrected for the fact that the time spent in the liquid xenon is shorter than in the solids, in which the pearls move slower.

But here in addition there can be several corrections to \(\text{suppression}_{\text{theory}}\), at least we should correct by the width in logarithm of the supposed distribution of the lifetimes among the different excitations. Above we suggested a factor 23, which would bring the DAMA rate to only deviate by about a factor 6, now to the side that the theoretical suppression suppresses a factor 6 more than the experimental. If it happens as we hope that the pearl’s speed through the liquid xenon is just 250 times faster than through the solids, the deviation for the Xenon1T excess will be just the same as for DAMA. Our estimate is of course extremely uncertain.

We can never get the DAMA rate and the electron recoil excess rate from Xenon1T agree with the same estimate, in as far as they deviate by a factor 250, unless we have some mechanism like the faster passage through the xenon fluid because of its lower viscosity. Our only chance is in a later paper to justify say the story that, because the scintillator in which the Xenon1T events are observed is fluid while the NaI in the DAMA experiment is solid, the pearls pass much faster through the Xenon1T apparatus than they pass through the DAMA instrument. Imagine say that the pearls partly hang and get stuck in the DAMA experiment, but that they cannot avoid flowing down all the time while they are in the fluid xenon in the Xenon1T scintillator.

11 3.5 keV

Order of magnitudewise we see 3.5 keV in 3 different places.
X-ray galaxy cl.

Xenon1T Elec. R.

DAMA-LIBRA

The energy level difference of about 3.5 keV occurring in 3 different places is important evidence motivating our model of dark matter particles being excitable by 3.5 keV:

- **The line** From places in outer space with a lot of dark matter, galaxy clusters, Andromeda and the Milky Way Center, an unexpected X-ray line with photon energies of 3.5 keV (to be corrected for Hubble expansion...) was seen.

- **Xenon1T** The dark matter experiment Xenon1T did not find the standard nucleus-recoil events expected for dark matter, but found an excess of electron-recoil events with energies concentrated crudely around 3.5 keV.

- **DAMA** The seasonally varying component of their events lie in energy between 2 keV and 6 keV, not far from centering around 3.5 keV.

We take it seriously and not as an accident that both DAMA and Xenon1T see events with energies of the order of the controversial astronomical 3.5 keV X-ray line. We are thereby driven towards the hypothesis that the energies for the events in these underground experiments are determined from a decay of an excited particle, rather than from a collision with a particle in the scintillator material. It would namely be a pure accident, if a collision energy should just coincide with the dark matter excitation energy observed astronomically.

*So we ought to have decays rather than collisions! How then can the dark matter particles get excited?*

You can think of the dark matter pearls in our model hitting electrons and/or nuclei on their way into the shielding:

- **Electrons** Electrons moving with the speed of the dark matter of the order of 300 km/s toward the pearls in the pearl frame will have kinetic energy of the order

\[
E_e \approx \frac{1}{2} \times 0.5 \text{ MeV} \times \left(\frac{300 \text{ km/s}}{3 \times 10^5 \text{ km/s}}\right)^2 = 0.25 \text{ eV.} \tag{134}
\]

- **Nuclei** If the nuclei are say Si, the energy in the collision will be 28*1900 times larger \(\sim 5 \times 10^4 \times 0.25 \text{ eV} \approx 10 \text{ keV.}\) That would allow a 3.5 keV excitation.
To deliver such \( \approx 10 \) keV energy the nucleus should hit something harder than just an electron inside the pearls. It should preferably hit a nucleus, e.g. C, inside the pearl.

12 Discussing Parameters

We have considered pearls having a cloud of \( Z = 5.3 \times 10^4 \) electrons as in [9] and improved the calculation of the minimal size pearl, for which we obtained the updated skin radius \( R = r_{\text{cloud}3.3\text{MeV}} \) and mass \( M \):

\[
R = 8.2 \times 10^{-12} \text{m} \quad \text{and} \quad M = 7.0 \times 10^5 \text{GeV} \quad (135)
\]

The cross section to mass ratio for this minimal size pearl is

\[
\frac{\sigma}{M} = \frac{\pi r_{\text{cloud}3.5\text{keV}}^2}{M} \approx 150 \text{cm}^2/\text{g} \quad (136)
\]

in agreement with the low velocity limit estimated from the study of dwarf galaxies around the Milky Way.

We also considered the possibility that the cross section obtained by taking \( r_{\text{cloud}3.5\text{keV}} \) to be the outer radius of the electron cloud corresponds to a collision in which the kinetic energy of the pearl per cloud electron is equal to 3.5 keV. This occurs for a velocity \( v \sim 410 \text{km/s} \) and, according to the dwarf galaxy fit of [18] in Figure 6, corresponds to \( \frac{\sigma}{M} \sim 1 \text{cm}^2/\text{g} \) and hence to a pearl of mass 150 times that of the minimal size pearl with the following parameters:

\[
R = 4.3 \times 10^{-11} \text{m} \quad \text{and} \quad M = 1.0 \times 10^8 \text{GeV} \quad (137)
\]

However neither of the pearls with parameters [135] or [137] is heavy enough to pass through the earth to the DAMA-LIBRA detector in less than one year. In fact one needs a mass of at least \( M_{\text{minimal}} = 1.2 \times 10^{12} \text{GeV} \) in order for the pearl to reach down in less than one year and so we take as our favoured pearl parameters:

\[
R = 1.0 \times 10^{-9} \text{m} \quad \text{and} \quad M = 1.2 \times 10^{12} \text{GeV} \quad (138)
\]

This 1 nm size pearl has a cross section per mass ratio of \( \frac{\sigma}{M} \simeq 1.5 \times 10^{-2} \text{cm}^2/\text{g} \), but we would like this ratio to approach 150 cm\(^2/g\) as suggested by the dwarf galaxy data. So we have speculated that these pearls collect dirt around them, as illustrated in Figure 7 which is stripped off them when they pass through the atmosphere or the Earth. This dirt increases the radius by two orders of magnitude, so that the full radius becomes of order 100 nm.

The pressure from the surface tension \( S \) of the pearl skin is balanced by the degeneracy pressure from the electrons with Fermi momentum \( p_f \):

\[
\frac{2S}{R} = \frac{p_f^3}{12\pi^2}. \quad (139)
\]

Our one parameter fit to the intensity and frequency of the 3.5 keV line from galaxy clusters and the Milky Way Center [6 7 8] determines the Fermi momentum to be \( p_f = 3.3 \text{MeV} \). Hence the surface tension is given by

\[
S = \frac{(3.3 \text{MeV})^4}{24\pi^2R} \quad (140)
\]

\[
= 2.6 \times 10^{12} \text{MeV}^3/\text{m} \times R. \quad (141)
\]
So for the minimal size pearl with $R = 8.2 \times 10^{-12} \text{ m}$ the surface tension is
\[ S^{1/3} = 2.8 \text{ MeV} \] (142)
and for the 1 nm size pearl
\[ S^{1/3} = 14 \text{ MeV}. \] (143)

13 Conclusion

We have described a seemingly viable model for dark matter consisting of atomic size but macroscopic pearls. These pearls consist of a bubble of a new speculated type of vacuum containing some normal material - presumably carbon - under the high pressure of the skin (surface tension). The electrons in a pearl are partly pushed out of the genuine bubble of the new vacuum phase. In order to accommodate the annual modulation of the DAMA-LIBRA data we now favour a heavier pearl than the minimal size pearl considered in the Bled proceedings [9]. These heavier pearls each contain about $10^{12}$ nucleons in the bubble of radius about $R = 1.0 \times 10^{-9} \text{ m}$ and have a surface tension $S^{1/3} = 14 \text{ MeV}$. Unlike the minimal size pearls they have a much smaller cross section to mass ratio than the value $\frac{\sigma}{M} \simeq 150 \text{ cm}^2/\text{g}$ suggested by the dwarf galaxy data. So we were led to speculate that this value could be achieved if “dirt” had collected around them (see Figure 7) increasing the radius from about 1 nm to about 100 nm.

We have compared the model or attempted to fit:

- Astronomical suggestions for the self interaction of dark matter in addition to pure gravity.
- The astronomical 3.5 keV X-ray emission line found by satellites, supposedly from dark matter.
- The underground dark matter searches.

We list below the quantities we have crudely estimated:

1. The low velocity cross section divided by mass.

2. A priori we predict that the event rate per kg seen by DAMA-LIBRA and the electron excess event rate per kg at Xenon1T should be the same; they are both supposed to come from electrons emitted under the de-excitation of our pearls at the same depth. The observed ratio of rates is 250, which we hope to explain as due to our pearls falling appreciably faster through the fluid xenon than through the solid NaI and hence spending a much shorter time in the Xenon1T detector than in the DAMA-LIBRA detector.

3. The absolute rate of the two underground experiments. (But unfortunately unless we explain the ratio of the rates for the two experiments as say due to the different velocities through the scintillator materials, we cannot of course predict the absolute rate to be better than deviating by about a factor of 250 from at least one of them.)
4. The rate of emission of the 3.5 keV X-ray line from the Tycho supernova remnant \cite{38} due to the excitation of our pearls by cosmic rays \cite{8}.

5. Relation between the frequency 3.5 keV and the overall emission rate of this X-ray line observed from galaxy clusters etc.

6. We also previously predicted the ratio of dark matter to atomic matter (=“usual” matter) in the Universe to be of order 5 by consideration of the binding energies per nucleon in helium and heavier nuclei, assuming that the atomic matter at some time about 1 s after the Big Bang was spit out from the pearls under a fusion explosion from He fusing into say C \cite{1}.

13.1 Outlook

At the end we want to mention a few ideas which we hope will be developed as a continuation of the present model:

- **Speculative Phase from QCD.** QCD and even more QCD with Nambu-Jona-Lasinio type spontaneous symmetry breaking is sufficiently complicated, that possibly a new phase appropriate for our pearls could be hiding there. There is already an extremely interesting observation \cite{39}.

- **Relative Rates of DAMA and Xenon1T.** A crucial test for our model is to reproduce the relative event rates in DAMA and in the excess of electron recoils in Xenon1T. This requires a careful study of the viscosity of fluid xenon and the properties of our pearls.

- **Walls in the Cosmos.** If the domain walls have a sufficiently large surface energy density or surface tension say $S^{1/3} \gtrsim 10$ MeV, their effect on cosmology would be phenomenologically unacceptable. The pearls in this paper have a surprisingly small surface tension ranging from $S^{1/3} = 2.8$ MeV to 14 MeV. So astronomically extended domain walls just barely become possible in our model e.g. walls around the large voids between the bands of galaxies; so that these voids could be say formally huge dark matter pearls, though with much smaller density.

- **New Experiments?** According to our estimates the observed rate of decays of our dark matter pearls should be larger the less shielding they pass through. So an obvious test of our model would be to make a DAMA-like experiment closer to the earth surface where we would expect a larger absolute rate than in DAMA, although there might of course be more background. Actually such an experiment is already being performed by the ANAIS group \cite{21}, but they have so far failed to see an annual modulation in their event rate.

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Figure 6. Cross section per unit mass, \( \sigma/m \), as a function of the average collision velocity, \( \langle v \rangle \), of DM particles within each subhalo’s core. Symbols show the range of \( \sigma/m \) needed for the SIDM model to reproduce the central DM densities reported by Kaplinghat et al. (2019). The solid line corresponds to the best-fit relation given by eq. (15) to the MW dSph data.