Quasiparticle poisoning of Majorana qubits

Torsten Karzig,1 William S. Cole,1 and Dmitry I. Pikulin1,2

1Microsoft Quantum, Station Q, Santa Barbara, California 93106, USA
2Microsoft Quantum, Redmond, Washington 98052, USA
(Dated: April 6, 2020)

Qubits based on Majorana zero modes are a promising path towards topological quantum computing. Such qubits, though, are susceptible to quasiparticle poisoning which does not have to be small by topological argument. We study the main sources of the quasiparticle poisoning relevant for realistic devices – non-equilibrium above-gap quasiparticles and equilibrium localized subgap states. Depending on the parameters of the system and the architecture of the qubit either of these sources can dominate the qubit decoherence. However, we find in contrast to naive estimates that in moderately disordered, floating Majorana islands the quasiparticle poisoning can have timescales exceeding seconds.

Majorana zero modes (MZMs) provide a basis for topologically protected qubits [1,3]. The topological protection means that dephasing of the MZM-based qubit (Majorana qubit) is exponentially small in the separation of the MZMs in space and in the energy gap of the separating region. Both of these quantities can be controlled in experiment which leads to the potential of long dephasing times.

Majorana qubits encode information in the joint parity of MZM pairs. Dephasing of a Majorana qubit can thus only happen via incoherent exchange of parity between the MZMs [4] or via processes of uncontrolled exchange of quasiparticles (QPs) between the Majorana subspace and other fermionic modes [5–7]. The latter is called quasiparticle poisoning (QPP) of the Majorana qubit. QPs may have different origins – they may be inside the Majorana qubit at above-gap energies excited by temperature or external perturbations, they may come from the environment, or they may be located in non-topological subgap states. Exponential suppression of QPP in parameters of the system can be shown for a system decoupled from gapless leads under the assumptions of thermal equilibrium and a spectral gap in the system [8]. In the present work we analyze how breaking these assumptions in real systems affects QPP.

Non-equilibrium QPs are present in any realistic physical system and have been explored extensively in the context of superconducting qubits [9–15] and dedicated devices [16–20]. In a conventional superconductor a single QP cannot relax to the condensate consisting of Cooper pairs. Once QPs are created by an external perturbation the only way to get rid of them is to pair them up. This process becomes slow for low densities of the QPs [19,21]. Therefore, even for small rates of QP creation, superconductors typically have a non-equilibrium density of QPs that significantly exceeds the expected thermal occupation.

The effects of non-equilibrium QPs on the Majorana qubits can be described by relaxation events of the QPs into the MZMs. The QPs are described by the density \( n_{qp} \). In order to determine the lifetime (or dephasing time) of the qubit we have to determine the relaxation rate into MZMs given a certain \( n_{qp} \) and then find an expression for typical QP densities. The latter can be obtained from steady state solutions of a model with a certain rate of exciting higher-energy QPs, subsequent relaxation and recombination.

The potential danger of QPP for Majorana qubits is widely acknowledged in the literature [9,27] and the timescales for the poisoning influence the design of the Majorana-based quantum computer [22]. However, there exist few quantitative estimates for the corresponding decoherence times. Moreover, the existing estimates [4–7] rely on values for \( n_{qp} \) that are typical for conventional superconductors and do not take into account how the presence of the MZM itself will change \( n_{qp} \).

Finally, the effect of disorder and subgap states is mainly discussed in the literature in terms of the effect on the topological transport gap and transition from the topological to the trivial phase due to disorder [23–26]. The disorder, however, suppresses the spectral gap faster than the transport gap and thus causes the presence of subgap states well before the topological transition. Such subgap states present a reservoir where parity may leak from the MZMs, thus causing decoherence from equilibrium QPP.

The aim of the present paper is to provide a self-consistent estimate of QPP taking into account the peculiarities of mesoscopic topological superconductors including the finite volume of the superconductor and equilibrium QPP due to the presence of subgap states.

Poisoning due to non-equilibrium QPs.—We start by considering non-equilibrium QPP resulting from relaxation of above-gap non-equilibrium QPs into one of the computational Majoranas with a rate \( \Gamma_{\gamma} \). See Fig. 1 for a schematic representation of the relevant processes. The non-equilibrium QP density in superconductors can result from a steady rate of QP creation \( \Gamma_{cre} = 2\gamma CP N_{CP} \) via breaking of some of the \( N_{CP} \) Cooper pairs. The exact nature of the Cooper pair breaking process may be due to stray radiation, cosmic rays, etc. but is unimportant for our model. In accordance with the literature [9,27], we
Cooper pairs by annihilating with another QP with a rate \( \Gamma_{\text{ann}} \) further assume that the resulting high energy QPs relax sub

subgap states that may be present in the topological superconductor poisoning can be due to the presence of low energy dominated by the more effective \( \Gamma_{\text{sub}} \). Another source of quasiparticle poisoning can be due to the presence of low energy subgap states that may be present in the topological superconductor. Leakage of the purity from the MZMs into such state is described by the rate \( \Gamma_{\text{sub}} \).

We further assume that the resulting high energy QPs relax quickly to the gap edge but only slowly recombine into Cooper pairs by annihilating with another QP with a rate \( \Gamma_{\text{rec}} \). The latter can relax either by pairwise recombination into Cooper pairs at a rate \( \Gamma_{\text{rec}} \) or by relaxing into MZMs at rate \( \Gamma_{\gamma} \). At low densities \( \Gamma_{\text{rec}} \) becomes slow and the QP density close to the MZM is suppressed and dominated by the more effective \( \Gamma_{\gamma} \). Another source of quasiparticle poisoning can be due to the presence of low energy subgap states that may be present in the topological superconductor poisoning can be due to the presence of low energy subgap states that may be present in the topological superconductor. Leakage of the purity from the MZMs into such state is described by the rate \( \Gamma_{\text{sub}} \).

As we detail below, for practical sizes of the superconducting islands, the rate of relaxing into MZMs \( \Gamma_{\gamma} \) would be by far the fastest relaxation channel for QPs. Similar to QP traps based on superconducting vortices \[1\][2][28][32] MZMs act as efficient QP traps. The resulting non-equilibrium density of the QPs are lower by several orders of magnitude than what one would expect from the conventional estimates \[9\][12][14][15][17][38] we find \( \Gamma_{\text{rec}} \sim 10^{-4} \text{ s}^{-1} \) which corresponds to \( 10^{-4} \text{ s}^{-1} \mu \text{m}^{-3} \) being low densities which makes relaxation processes into MZMs that become available in topological superconductors highly relevant.

Let us consider the diffusive dynamics of the above-gap QPs in a system with MZMs located at positions \( x = x_i \) taking into account the above creation and relaxation processes. The diffusion equation reads

\[
\dot{n}_{\text{qp}} = 2 \gamma_{\text{br}} n_{\text{CP}} + D \nabla^2 n_{\text{qp}} - \sum_i \gamma_0 n_{\text{qp}} f(x - x_i) - \gamma_0 n_{\text{qp}}^2 / n_{\text{CP}}.
\]  

(1)

Here, \( \gamma_{\text{br}} \) is the rate of breaking Cooper pairs, \( D \) is the diffusion constant of the above-gap QPs, \( \gamma_0^{-1} \) a characteristic time scale for QP relaxation into a MZM and \( f(x) \) describes the local extent of a MZM located around \( x = 0 \). While in general \( f(x) \) depends non-trivially on the overlap of MZM and QP wavefunctions \[4\], in the limit of a weakly changing QP density \( \gamma_0^{-1} \) relative to the topological coherence length we can write \( f(x) \approx V_{\text{MZM}} \delta(x) \), where \( V_{\text{MZM}} = (f dV/|\psi_{\text{MZM}}|^2 \text{d}x \int dV) \) is a measure of the volume of the MZM with wavefunction \( \psi_{\text{MZM}} \). In an effectively one-dimensional system \( V_{\text{MZM}} = \xi \).

We first review the argument giving the non-equilibrium QP density in an isolated trivial superconductor. To this end we set \( \gamma_0 = 0 \). The steady state solution of Eq. \( (1) \) then yields a constant density \( n_{\text{qp}}(x) = n_{\text{SC}} \), with

\[
n_{\text{SC}} = \sqrt{\frac{2 \gamma_{\text{br}}}{\gamma_0}} n_{\text{CP}}.
\]  

(2)

such that \( \Gamma_{\text{rec}}(n_{\text{SC}}) = \Gamma_{\text{rec}}(n_{\text{SC}}) \).

For the remainder of the manuscript we will use parameters for an Al-proximitized nanowire of total volume \( V_{\text{SC}} = 10 \mu \text{m} \times 200 \text{nm} \times 10 \text{nm} = 2 \cdot 10^{-2} \mu \text{m}^3 \) as such system is the closest to practical applications \[3\]. All the estimates are straightforward to perform for other superconductors and host systems \[35–37\]. In the Al-based system \( n_{\text{CP}} = D(E_F) \Delta = 3 \cdot 10^6 / \mu \text{m}^3 \), using \( \gamma_0^{-1}, \gamma_0^{-1} \sim 50 \text{ns} \) \[4\] and \( n_{\text{SC}} = 0.01 \ldots 10 \mu \text{m}^{-3} \) \[9\][12][14][15][17][38] we find \( \gamma_{\text{br}}^{-1} \sim 10^5 \ldots 10^6 \text{s} \). Note that this gives quite small total rates of QP creation even when multiplying by the number of Cooper pairs in a typical sample of size \( V_{\text{SC}} \). Using these numbers we find \( \Gamma_{\text{rec}}^{-1} \sim 0.1 \ldots 1 \text{day} \).

Let us consider the diffusive dynamics of the above-gap QPs in a system with MZMs located at positions \( x = x_i \) taking into account the above creation and relaxation processes. The diffusion equation reads

\[
\dot{n}_{\text{qp}} = 2 \gamma_{\text{br}} n_{\text{CP}} + D \nabla^2 n_{\text{qp}} - \sum_i \gamma_0 n_{\text{qp}} f(x - x_i) - \gamma_0 n_{\text{qp}}^2 / n_{\text{CP}}.
\]  

(1)

Note that the above estimate relies on uniformly distributed completely delocalized QPs and thus provides an upper bound for the creation rate. As noted e.g. in Ref. \[21\] the annihilation itself can reduce the probability for QPs to be close enough to recombine especially if they are localized due to disorder. This effect makes recombination less efficient and leads to estimates with even smaller rates \( \Gamma_{\text{rec}} \) that would be consistent with the observed QP densities. Ref. \[21\] also provided an encouraging estimate for the QP creation rate (per volume) due to cosmic radiation \( \sim 10^{-4} \text{s}^{-1} \mu \text{m}^{-3} \) which corresponds to \( \Gamma_{\text{rec}}^{-1} \sim 10 \text{days} \).

Let us now consider the effect of relaxation via the localized MZMs. To estimate the importance of the latter in comparison to the bulk QP recombination we examine for a given \( n_{\text{qp}} \) the ratio of the rate of relaxation into MZMs \( \Gamma_{\gamma} \) to the rate of pairwise QP annihilation

\[
\frac{\Gamma_{\gamma}}{\Gamma_{\text{rec}}} = \frac{n_{\text{QP}} V_{\text{M}}}{n_{\text{QP}} V_{\text{M}}} n_{\text{CP}}.
\]  

(3)

Using for the estimate for the volume occupied by the Majorana wavefunction \( V_{\text{M}} \sim 200 \text{nm} \times 10 \text{nm} \times 100 \text{nm} = 2 \cdot 10^{-4} \mu \text{m}^3 \), we obtain that at densities similar to the zero field densities of non-equilibrium QPs...
Due to the symmetry of the system it is sufficient to focus on the region of \( x = 0 \) and \( x = L \) with \( f(x) = \xi \delta(x) \), see Fig. 2. Due to the symmetry of the system it is sufficient to focus on the region of \( x \in [0, L/2] \). Using boundary conditions of vanishing current \( \partial_x n_{\text{qp}}(x) = 0 \) at \( x = 0 \) and \( x = L/2 \) we obtain the steady state solution of the diffusion equation

\[
n_{\text{qp}}(x) = \left( 1 + \frac{\gamma_0 \xi}{D} x \right) n_\gamma - \frac{\gamma_{\text{br}} n_{\text{CP}}}{D} x^2
\]

with \( n_\gamma = n_{\text{qp}}(0) = \gamma_{\text{br}} n_{\text{CP}} L / \gamma_0 \xi \). The expression of the QP density at the MZMs reflects the balance of the total relaxation and creation rates.

The solution of Eq. (4) is depicted in Fig. 2 and is characterized by a minimum in the QP density \( n_{\text{qp}} = n_\gamma \) close to the MZMs and a maximal density \( n_{\text{qp}} = n_{\text{max}} \) in-between the MZMs. We now discuss the important length scales of the problem. From Eq. (4) one can extract the length scale \( L_\gamma = D / \gamma_0 \xi \) over which the density changes only weakly. In a system of size \( L < L_\gamma \) the diffusion time to explore the system is shorter than the typical relaxation time. This lead to a homogeneous density, i.e. \( n_{\text{max}} / n_\gamma \approx 1 \). Using a diffusion constant typical for of Al QPs \( D = 2 \mu m^2 \text{ns}^{-1} \) \cite{11}, \( \xi \approx \text{200nm} \) \cite{10 11}, and \( \gamma_0^{-1} \approx \text{50ns} \) we obtain \( L_\gamma \approx \text{1nm} \). We thus expect that Majorana qubits based on moderately sized islands of topological superconductors \cite{12 43} are in the regime of a small constant density of QPs dominated by relaxation into MZMs. The corresponding QP-limited decoherence times are given by the inverse creation rate of \( \Gamma_{\text{cre}}^{-1} \) which as estimated above can exceed seconds.

To obtain estimates for the case of Majorana qubits based on bulk superconductors \cite{33} we consider system sizes \( L \gg L_\gamma \). Equation (4) suggests that \( n_{\text{qp}}(x) \) saturates at \( n_{\text{max}} = \gamma_{\text{br}} n_{\text{CP}} L^2 / 4D \) which can be understood as the density of broken Cooper pairs (in the absence of relaxation) created over the time it takes a QP to explore half of the system \( L^2 / 4D \). For large systems with \( L > L_{\text{SC}} \) this density will be cut off once it reaches \( n_{\text{max}} = n_{\text{SC}} \) and the formerly neglected pair recombination processes become relevant. Increasing the size of the system beyond \( L_{\text{SC}} \) will not change the density profile around the MZM but only add a larger region of density \( n_{\text{SC}} \) far away from the MZM. We can therefore estimate the density \( n_\gamma \) in the limit of a bulk superconductor by \( n_\gamma(L = L_{\text{SC}}) \) which leads to poisoning rates of \( \Gamma_\gamma = 2 \gamma_{\text{br}} n_{\text{CP}} L_{\text{SC}} \) with \( L_{\text{SC}} = \sqrt{n_{\text{SC}} D / n_{\text{CP}} \gamma_{\text{br}}} \). Using the estimates for \( \gamma_{\text{br}} \) based on Eq. (2) one can rewrite

\[
\Gamma_\gamma = \sqrt{2 \gamma_0 D n_{\text{SC}} / n_{\text{CP}}} \quad \text{and with} \quad n_{\text{SC}} = \gamma_0 \text{ as quoted above we obtain poisoning times} \quad \Gamma_\gamma^{-1} \approx \text{0.1 \mu s...1ms} \quad \text{with corresponding} \quad L_{\text{SC}} \approx \text{1...100nm}.
\]

We therefore see that the poisoning times of Majorana qubits based on bulk superconductors are significantly worse than those of isolated islands. While the above estimates do not make it impossible to build Majorana qubits with bulk superconductors, the resulting coherence times will likely not be able to exceed those of conventional superconducting qubits.

Poisoning due to subgap states.—We now turn to another type of decoherence possible in Majorana devices – leakage of the parity from the MZM subspace into other subgap states. This mechanism does not require non-equilibrium QPs. For simplicity we thus focus on equilibrium QPP due to subgap states. The concern here is that realistic Majorana devices may be disordered, and while the transport gap can be observed in the widely used transport experiments the (potentially zero) spectral gap is less accessible. Thus equilibrium QPP is possible in topological phase if there are enough subgap states inside the wire that are close enough in position to the MZMs. The equilibrium rate of a jump to a subgap state is:

\[
\Gamma = \omega_n \exp \left( -\frac{2x}{\xi} - \frac{\delta E}{k_B T} \right).
\]

Here \( x \) is the distance from the Majorana to the localization center of a subgap state, \( \xi \) is the disordered coherence length, \( \delta E \) is the energy difference to the target state, and \( \omega_n \) is the “attempt frequency” corresponding to the physical process that enables the tunneling event (for example, electron-phonon scattering, charge noise on gates, etc.).

As a specific scenario, we can take the average number of subgap states in a p-wave wire with finite mean free path \( \ell \) arising from Gaussian disorder, as worked out in Ref. \cite{23}:

\[
\langle N(E) \rangle \propto \frac{L}{\xi_0} \left( \frac{E}{\Delta_0} \right)^{\eta}, \quad \eta = 4 \ell / \xi_0 - 2
\]
where the proportionality constant is of order one, $\xi_0$ is the clean coherence length, $\Delta_0$ the gap in the zero-disorder limit, and the parameter $\eta$ describes the approach to a disorder-driven topological phase transition when $\ell \leq \xi_0/2$. The apparent coherence length, which governs the spatial overlap of Majorana wavefunctions and diverges at this transition, is given by $\xi^{-1} = \xi_0^{-1} - (2\ell)^{-1}$. From Eq. (6), we can extract the energy window $[0, E']$, with $E' = \Delta_0 (\xi_0/(2x))^{1/\eta}$, that contains on average 1 state within a distance $x$ from the edge of the system. Thus, to tunnel into a subgap state at distance $x$ an electron typically needs to gain an energy

$$\delta E = \int_0^{E'} E \nu(E) dE = \frac{\Delta_0 \eta}{1 + \eta} \left( \frac{\xi_0}{2x} \right)^{1/\eta}. \tag{7}$$

The optimal tunneling distance is then obtained by maximizing the rate Eq. (5) with respect to $x$,

$$x_{\text{opt}} = \frac{\xi}{2 \left( \frac{\eta + 2}{\eta} \right)} ^ {\frac{\eta + 1}{\eta + 1 - \frac{\eta + 1}{\eta} \left( \frac{\xi}{2 \left( \frac{\eta + 2}{\eta} \right)} \right)}} \left( \frac{1}{\eta + 1} \right)^{\frac{1}{\eta + 1 - \frac{\eta + 1}{\eta} \left( \frac{\xi}{2 \left( \frac{\eta + 2}{\eta} \right)} \right)}} \tag{8}$$

Note that this expression has been written in terms of the disordered coherence length and transport gap, using $\xi/\xi_0 = (\eta + 2)/\eta$. Then, finally, the typical leakage rate is obtained from Eq. (5) at $x_{\text{opt}}$

$$\Gamma_{\text{sub}} = \omega_a \exp \left[ -g(\eta) \left( \frac{\Delta}{k_B T} \right)^\frac{1}{\eta + 1} \right]. \tag{9}$$

with $g(\eta) = (\eta + 1)^{\frac{\eta + 1}{\eta + 1 - \frac{\eta + 1}{\eta} \left( \frac{\xi}{2 \left( \frac{\eta + 2}{\eta} \right)} \right)}}$. Crucially, this rate is suppressed by a stretched exponential in $\Delta/(k_B T)$.

To gain intuition for the time scale in realistic devices we first take a (transport) topological gap of $\Delta \approx 100\mu eV$ and a temperature $k_B T = 5\mu eV$ ($\approx 50mK$). We also assume the attempt frequency is the electron-phonon scattering rate $\omega_a = \gamma_0 \approx (50ns)^{-1}$. For $\ell/\xi = 1/4$ (or $\ell/\xi_0 = 3/4$) we find a typical leakage rate of $\Gamma_{\text{sub}} \approx 30\mu$s, and already for $\ell/\xi = 1/2$ ($\ell/\xi_0 = 1$) we find $\Gamma_{\text{sub}} \approx 30$ms. In Fig. 3 we plot the leakage rate as a function of $\ell/\xi$ and of $\Delta/k_B T$. It is apparent that at high temperatures or at the disorder-driven phase transition the poisoning time is given by the electron-phonon scattering, however the time can exceed seconds (with $\Delta/k_B T \approx 20$) for $\ell/\xi \gtrsim 1$ and this mechanism of QPP becomes essentially inoperative when $\ell/\xi \gtrsim 2$. While these estimates are promising, they rely on a simple model which may be substantially modified in a real topological superconducting heterostructure, for example due to rare but strong impurities like dislocations, or the heterogeneous nature of the system.

In conclusion, we studied the effect of quasiparticle poisoning in Majorana qubits and found that non-equilibrium QPs are less harmful than expected from naive estimates based on the typical bulk quasiparticle concentration in conventional superconductors. We have shown that poisoning due to non-equilibrium QPs is happening at the rate of the QP generation, which we expect to be of the order of seconds (or even days) and thus much slower than the timescales of qubit operations. Another potential source of QPP is the presence of disorder-induced subgap states. Our estimations show that Majorana parity leakage into such states is negligible for weak disorder. Only once the mean free path becomes comparable to the coherence length, QPP may present a problem due to supgap states that proliferate as the system approaches the disorder-induced topological transition.

We acknowledge useful discussions with Chetan Nayak and Gijs de Lange.

[1] A. Y. Kitaev, “Fault-tolerant quantum computation by anyons,” Ann. Phys. 303, 2 (2003).
[2] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, “Non-Abelian anyons and topological quantum computation,” Rev. Mod. Phys. 80, 1083 (2008).
[3] R. M. Lutchyn, E. P. A. M. Bakkers, L. P. Kouwenhoven, P. Krogstrup, C. M. Marcus, and Y. Oreg, “Majorana zero modes in superconductor–semiconductor heterostructures,” Nat. Rev. Mater. 3, 52 (2018).
[4] C. Knapp, T. Karzig, R. M. Lutchyn, and C. Nayak, “Dephasing of Majorana-based qubits,” Phys. Rev. B 97, 125404 (2018).
[5] G. Goldstein and C. Chamon, “Decay rates for topological memories encoded with Majorana fermions,” Phys. Rev. B 84, 205109 (2011).
[6] D. Rainis and D. Loss, “Majorana qubit decoherence by quasiparticle poisoning,” Phys. Rev. B 85, 174533 (2012).
[7] J. C. Budich, S. Walter, and B. Trauzettel, “Failure of protection of Majorana based qubits against decoherence,” Phys. Rev. B 85, 121405 (2012).
[8] G. C. Ménard, F. K. Malinowski, D. Puglia, D. I. Pikulin, T. Karzig, B. Bauer, P. Krogstrup, and C. M. Mar-
curs, “Suppressing quasiparticle poisoning with a voltage-controlled filter,” Phys. Rev. B 100, 165307 (2019).

[9] J. M. Martinis, M. Ansmann, and J. Aumentado, “Energy Decay in Superconducting Josephson-Junction Qubits from Nonequilibrium Quasiparticle Excitations,” Phys. Rev. Lett. 103, 097002 (2009).

[10] D. Ristè, C. Bultink, M. Tiggelman, R. Schouten, K. Lehner, and L. DiCarlo, “Millisecond charge-parity fluctuations and induced decoherence in a superconducting transmon qubit,” Nat. Commun. 4, 1913 (2013).

[11] C. Wang, Y. Y. Gao, I. M. Pop, U. Vool, C. Axline, T. Brecht, R. W. W. Heeres, L. Frunzio, M. H. Devoret, G. Catelani, L. I. Glazman, and R. J. Schoelkopf, “Measurement and control of quasiparticle dynamics in a superconducting qubit,” Nat. Commun. 5, 5836 (2014).

[12] U. Vool, I. M. Pop, K. Sliwa, B. Abdo, C. Wang, T. Brecht, Y. Y. Gao, S. Shankar, M. Hatridge, G. Catelani, M. Mirrahimi, L. Frunzio, R. J. Schoelkopf, L. I. Glazman, and M. H. Devoret, “Non-Poissonian Quantum Jumps of a Fluxonium Qubit due to Quasiparticle Excitations,” Phys. Rev. Lett. 113, 247001 (2014).

[13] L. Grünhaupt, N. Maleeva, S. T. Skacel, M. Calvo, F. Levy-Bertrand, A. V. Ustinov, H. Rotzinger, A. Monfardini, G. Catelani, and I. M. Pop, “Loss mechanisms and quasiparticle dynamics in superconducting microwave resonators made of thin-film granular aluminum,” Phys. Rev. Lett. 121, 117001 (2018).

[14] K. Serniak, M. Hays, G. de Lange, S. Diamond, S. Shankar, L. Burkhart, L. Frunzio, M. Houzet, and M. Devoret, “Hot nonequilibrium quasiparticles in transmon qubits,” Phys. Rev. Lett. 121, 157001 (2018).

[15] K. Serniak, S. Diamond, M. Hays, V. Fatemi, S. Shankar, L. Frunzio, R. Schoelkopf, and M. Devoret, “Direct Dispersive Monitoring of Charge Parity in Offset-Charge-Sensitive Transmons,” Phys. Rev. Appl. 12, 041052 (2019).

[16] J. Aumentado, M. W. Keller, J. M. Martinis, and M. H. Devoret, “Nonequilibrium quasiparticles and e periodicty in single-cooper-pair transistors,” Phys. Rev. Lett. 92, 066802 (2004).

[17] M. Shaw, R. Lutchyn, P. Delsing, and P. Eckernach, “Kinetics of nonequilibrium quasiparticle tunneling in superconducting charge qubits,” Phys. Rev. B 78, 024503 (2008).

[18] D. J. van Woerkom, A. Gersedi, and L. P. Kouwenhoven, “One minute parity lifetime of a NbTiN Cooper-pair transistor,” Nat. Phys. 11, 547 (2015).

[19] M. Hays, G. de Lange, K. Serniak, D. van Woerkom, D. Bouman, P. Krogsstrup, J. Nygård, A. Gersedi, and M. Devoret, “Direct microwave measurement of andreev-bound-state dynamics in a semiconductor-nanowire josephson junction,” Phys. Rev. Lett. 121, 047001 (2018).

[20] J. van Veen, A. Proutski, T. Karzig, D. I. Pikulin, R. M. Lutchyn, J. Nygård, P. Krogsstrup, A. Gersedi, L. P. Kouwenhoven, and J. D. Watson, “Magnetic-field-dependent quasiparticle dynamics of nanowire single-cooper-pair transistors,” Phys. Rev. B 98, 174502 (2018).

[21] A. Bespalov, M. Houzet, J. S. Meyer, and Y. V. Nazarov, “Theoretical Model to Explain Excess of Quasiparticles in Superconductors,” Phys. Rev. Lett. 117, 117002 (2016).

[22] C. Knapp, M. Beverland, D. I. Pikulin, and T. Karzig, “Modeling noise and error correction for majorana-based quantum computing,” Quantum 2, 88 (2018).

[23] P. W. Brouwer, M. Duckheim, A. Romito, and F. von Oppen, “Probability distribution of majorana end-state energies in disordered wires,” Phys. Rev. Lett. 107, 196804 (2011).

[24] P. W. Brouwer, M. Duckheim, A. Romito, and F. Von Oppen, “Topological superconducting phases in disordered quantum wires with strong spin-orbit coupling,” Phys. Rev. B 84, 144526 (2011).

[25] A. M. Lobos, R. M. Lutchyn, and S. D.arma, “Interplay of disorder and interaction in majorana quantum wires,” Phys. Rev. Lett. 109, 146403 (2012).

[26] I. Adagideli, M. Wimmen, and A. Teker, “Effects of electron scattering on the topological properties of nanowires: Majorana fermions from disorder and superlattices,” Phys. Rev. B 89, 144506 (2014).

[27] K. Segall, C. Wilson, L. Li, L. Frunzio, S. Friedrich, M. C. Gaidis, and D. E. Prober, “Dynamics and energy distribution of nonequilibrium quasiparticles in superconducting tunnel junctions,” Phys. Rev. B 70, 214520 (2004).

[28] J. N. Ullom, P. A. Fisher, and M. Nahum, “Magnetic field dependence of quasiparticle losses in a superconductor,” Appl. Phys. Lett. 73, 2494 (1998).

[29] J. T. Peltonen, J. T. Muhonen, M. Meschke, N. B. Kopnin, and J. P. Pekola, “Magnetic-field-induced stabilization of nonequilibrium superconductivity in a normal-metal/insulator/superconductor junction,” Phys. Rev. B 84, 220502 (2011).

[30] H. Q. Nguyen, T. Aref, V. J. Kauppila, M. Meschke, C. B. Winkelmalm, H. Courtois, and J. P. Pekola, “Trapping hot quasi-particles in a high-power superconducting electronic cooler,” New J. Phys. 15, 085013 (2013).

[31] I. Nsanzineza and B. L. T. Plourde, “Trapping a Single Vortex and Reducing Quasiparticles in a Superconducting Resonator,” Phys. Rev. Lett. 113, 117002 (2014).

[32] M. Taupin, I. M. Khaymovich, M. Meschke, A. S. Melnikov, and J. P. Pekola, “Tunable quasiparticle trapping in Meissner and vortex states of mesoscopic superconductors,” Nat. Commun. 7, 1 (2016).

[33] T. Hyart, B. Van Heck, I. Fulga, M. Burrello, A. Akhemov, and C. Beenakker, “Flux-controlled quantum computation with majorana fermions,” Phys. Rev. B 88, 035121 (2013).

[34] We confirm that the described system operates in this limit as $L \gg \xi$.

[35] M. Pendharkar, B. Zhang, H. Wu, A. Zarassi, P. Zhang, C. Dempsey, J. Lee, S. Harrington, G. Badawy, S. Gazibegovic, et al., “Parity-preserving and magnetic field resilient superconductivity in indium antimonide nanowires with tin shells,” arXiv:1912.06071.

[36] S. Mi, D. Pikulin, M. Wimmen, and C. Beenakker, “Proposal for the detection and braiding of majorana fermions in a quantum spin hall insulator,” Phys. Rev. B 87, 241405 (2013).

[37] J. Manousakis, A. Altland, D. Bagrets, R. Egger, and Y. Ando, “Majorana qubits in a topological insulator nanoribbon architecture,” Phys. Rev. B 95, 165424 (2017).

[38] A. Vepsäläinen, A. H. Karamlou, J. L. Orrell, A. S. Doria, B. Loer, F. Vasconcelos, D. K. Kim, A. J. Melville, B. M. Niedzielski, J. L. Yoder, S. Gustavsson, J. A. Formaggio, B. A. VanDevender, and W. D. Oliver, “Impact of ionizing radiation on superconducting qubit co-
herence,” arXiv:2001.09190.

[39] Due to the delta function form of $f(x)$, $\partial_x n_{qp}(x)$ jumps at $x = 0$. The boundary condition of vanishing derivative at $x = 0$ should be understood at infinitesimally negative $x$.

[40] S. M. Albrecht, A. P. Higginbotham, M. Madsen, F. Kuemmeth, T. S. Jespersen, J. Nygård, P. Krogstrup, and C. M. Marcus, “Exponential protection of zero modes in Majorana islands,” Nature 531, 206 (2016).

[41] S. Vaitiekėnas, M.-T. Deng, P. Krogstrup, and C. M. Marcus, “Flux-induced Majorana modes in full-shell nanowires,” arXiv:1809.05513.

[42] T. Karzig, C. Knapp, R. M. Lutchyn, P. Bonderson, M. B. Hastings, C. Nayak, J. Alicea, K. Flensberg, S. Plugge, Y. Oreg, C. M. Marcus, and M. H. Freedman, “Scalable designs for quasiparticle-poisoning-protected topological quantum computation with majorana zero modes,” Phys. Rev. B 95, 235305 (2017).

[43] S. Plugge, A. Rasmussen, R. Egger, and K. Flensberg, “Majorana box qubits,” New J. Phys. 19, 012001 (2017).

[44] D. E. Liu, E. Rossi, and R. M. Lutchyn, “Impurity-induced states in superconducting heterostructures,” Phys. Rev. B 97, 161408 (2018).